Generalized transparency in semi-inclusive processes

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Abstract:
It is argued that the transparency of a medium for passage of a nucleon, knocked-out in a semi-inclusive \((e,e'p)\) reaction and subsequently scattered elastically, is not the same as the one measured in purely elastic scattering. Expressions are given for the properly generalized transparency and those are compared with recently proposed, alternative suggestions. Numerical results are presented for selected nuclear targets and kinematic conditions, applying to the Garino et al and the SLAC NE18 experiment.

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1. Introduction

The present note relates to recent analyses of some semi-inclusive $A(e, e'p)X, A(p, 2p)X$ processes and attempts to extend the notion of transparency of a medium as a measure of Final State Interactions (FSI) effects of the knocked-out proton with the medium. There is additional interest because of the predicted anomalously large transparency if, part of the time, the proton would appear as an object of small dimensions.

In the simplest description of semi-inclusive (SI) reactions (the Plane Wave Impulse Approximation (PWIA)), the recoiling proton after knock-out by the projectile exits without undergoing FSI with the remaining core. Of course, scattering with medium particles does occur and contributes to FSI.

An accurate calculation of the latter under general circumstances is close to impossible. As an example of the encumbering complications we mention the fact that, in general, the knocked-out nucleon does not leave the nuclear core in its initial state, i.e. the scattering may in part be inelastic. In addition the nucleon may on its way out be excited and de-excited, or produce particles. Some of these processes have non-negligible amplitudes even for relatively low energies.

It is natural to ascribe the effects of elastic scattering of a knocked-out particle to a medium property as is the transparency. However, it is not at all evident how to link the latter to the standard definition of transparency in elastic scattering. It thus seems useful to start with a brief discussion on the transparency in purely elastic scattering, then to evaluate the same for passage of a knocked-out proton undergoing scattering and finally to compare the two.

* Protracted illness has caused considerable delay in completing this ms. It supersedes a draft which has been circulated well over a year ago.
The transparency of a medium probed in elastic scattering, of say a proton, is defined as the probability to find the latter traveling a distance \(z\), thus
\[
\mathcal{T}^{el}(q, b, z) = |\psi_q^{(+)}(\vec{r})|^2 = |e^{iqz+ix_q^{opt}(\vec{r})}|^2
\]  
(1.1)
Many-body dynamics provides an expression for the above optical (distortion) phase. For instance, if the projectile momentum \(q\) is sufficiently larger than the average momentum \(\langle p \rangle\) of a target particle, the former experiences the influence of essentially fixed scattering centers. In those circumstances the above phase is conveniently calculated in an eikonal approximation, and in the end averaged over \(\rho_A\), the normalized distribution probability of \(A\) scatterers

\[
\rho_A = \left(\frac{A!}{A^A}\right)^{-1}; \rho_1 = A^{-1}\rho_1', \text{ etc.}
\]

\[
\psi_q^{(+)}(\vec{r}) = e^{iqz} \left\langle e^{-i/v_q \sum_i \int_{-\infty}^{z} d\zeta V(\vec{b}_i - \vec{b}; \zeta - z_i)} \right\rangle_{\rho_A},
\]  
(1.2)
where \(v_q = E(q)/q \approx m/q\). The optical phase in (1.1) is determined and subsequently approximated by
\[
e^{ix_q^{opt}(\vec{r})} \equiv \left\langle \prod_i e^{i\tilde{\chi}_q(\vec{r}, \vec{r}_i)} \right\rangle_{\rho_A} \approx \exp \left[A \left(e^{ix_q(\vec{r}_1) - 1}\right)_{\rho_1}\right],
\]  
(1.3)
with
\[
\tilde{\chi}_q(\vec{r}, \vec{r}_1) = \tilde{\chi}_q(\vec{r} - \vec{r}_1; -\infty, z) = -v_q^{-1} \int_{-\infty}^{z} d\zeta V(\vec{b} - \vec{b}_1; \zeta - z_1) = -v_q^{-1} \int_{0}^{\infty} d\zeta' V(\vec{b} - \vec{b}_1; z - z_1 - \zeta'),
\]  
(1.4)
which is the eikonal phase due to the pair interaction \(V(\vec{r} - \vec{r}_1)\).

If the interaction \(V\) has a range, small compared to nuclear dimensions, the general off-shell 2-body eikonal phase \(\tilde{\chi}(\vec{r}; -\infty, z)\) may approximately be related to its on-shell analogue \(\chi(\vec{b}; -\infty, -\infty)\). In a standard parameterization
\[
e^{ix_q(\vec{r}_i - \vec{r}); -\infty, z)} - 1 \rightarrow \theta(z - z_1) \left[e^{ix_q(\vec{b} - \vec{b}_1)} - 1\right]
\]
\[
e^{ix_q(b)} - 1 \approx (2\pi/|q|)f_q(0^\circ) \int d\vec{Q}/(2\pi)^2 e^{i\vec{Q} \cdot \vec{b}}[f_q(\vec{Q})/f_q(0^\circ)]
\]  
(1.5)
\[
\approx (2\pi/|q|)\delta^{(2)}(\vec{b})f_q(0^\circ) \approx \frac{1}{2}\delta^{(2)}(\vec{b})\sigma_{tot}^{(2)}(1 - i\tau_q)
\]
For given momentum $q$, the quantities $f_q(Q_\perp)$, $f_q(0^\circ)$, $\tau_q$ and $\sigma^{tot}_q$ in (1.5) are, respectively the elastic scattering amplitude for momentum transfer $\vec{Q}_\perp$, the same for $\vec{Q}_\perp = 0$, the ratio of the real and imaginary part of the latter and the total $NN$ cross section. Incidentally, Eqs. (1.3), (1.4) (or the parameterization (1.5)) enable a definition of $V^{opt}$, which may be used instead of $\chi^{opt}$.

Returning to Eq. (1.1) one finally has

$$T^{el}(q;b,z) = e^{-A\sigma^{tot}_q \int_{\bar{z}(b)}^{z} dz_1 \rho(b,z_1)} \equiv e^{-(z-\bar{z}(b))/\lambda} \quad (1.6a)$$

$$T^{cl av}(q) = \langle T^{el}(q,z) \rangle_{\rho} = \frac{1}{A\sigma^{tot}_q} \int d\vec{b} \left[ (1 - e^{-At(b)}\sigma^{tot}_q) \right], \quad (1.6b)$$

with $\lambda$, the mean free path and $(\vec{b}, \bar{z}(b))$ the coordinate, where the projectile enters the region of non-vanishing density. Along with $T^{el}$, one defines a transparency $\langle T^{el}(q) \rangle_{\rho}$, averaged over the density $\rho(\vec{r})$, and Eq. (1.6b) is the standard result in terms of the thickness function

$$t(b) = \int_{-\infty}^{\infty} dz \rho(b,z) \quad (1.7)$$

For later reference we note that the densities appearing in Eqs. (1.3)-(1.7) are all diagonal. In addition, Eq. (1.6) does not contain a pair-correlation function: Even if the projectile is a nucleon it is not part of the correlated target.

Consider next SI and TI reactions. A convenient definition for a generalized transparency is

$$T^{reac} = \frac{Yield^{exp}}{Yield^{PWIA}} \approx \frac{Yield^{th}}{Yield^{PWIA}} \quad (1.8)$$

In semi-inclusive reactions the detected proton is knocked-out from the nucleus and not produced in an accelerator. Consequently the above generalized transparency cannot without proof be identified with $T^{cl av}$, Eq. (1.6b)\(^1\). In fact, some of the above-mentioned analyses were motivated by an observation on the FSI of the outgoing proton in the $e,e'p$ experiment of Ref. 4: when $T^{reac} \to T^{cl av}$, the extracted mean-free path $\lambda$ exceeded the one estimated from the free $NN$ cross section in Eqs. (1.6a) by a factor of order of 2.
We thus focus in Section 2 on FSI in SI scattering. Assuming that high momentum transfer SI cross sections factorize in the elementary one and a response, we suggest the application of the non-perturbative, first cumulant of the response\textsuperscript{8,9}, which describes FSI generated by binary collisions\textsuperscript{10}. A basic assumption then enables the derivation of a manageable expression for the SI response. In particular for short-range $NN$ forces, a considerable simplification results after parameterization in terms of total and reactive scattering parameters. A different derivation is given for an independent particle model for target ground and excited core states and will be shown to come close to the above result. With the thus derived FSI factors we give expressions for generalized transparencies in SI processes. In Section 3 we make a critical comparison of our approach with the ones of Refs. 1-3. Numerical results are presented in Section 4. In Section 5 we summarize our results and mention their relevance for the delineation of possible colour transparency effects in knock-out reactions.

2. Transparencies in knock-out processes.

Consider semi-inclusive $(e, e'p)$ scattering on a target with $A \gg 1$ when only the outgoing electron and the knocked-out proton are detected. Let $\vec{p}$ be the momentum of the knocked-on particle and $\vec{q}$ the momentum transfer in the reaction. We consider the case, when $\vec{p}'$, the momentum of the knocked-out particle satisfies $p' = |\vec{p} + \vec{q}| \approx q \gg p$. Assuming the $ep$ collision to take place on shell, the SI $(e, e'p)$ yield factorizes in the elastic $ep$ cross section and a generalized SI response $S$ per nucleon, which is the quantity of our interest

$$S^{SI}(q, \omega, \vec{p}') = 1/A \sum_n |\mathcal{F}_{0n}(q)|^2 \delta(\omega + \Delta_n - e(\vec{p} + \vec{q}))$$

$$\mathcal{F}_{0n}(q) = \langle \Phi_A^{(+)} | \rho_{q} | \Psi_{n, \vec{p} + \vec{q}}^{(+)} \rangle$$

(2.1)

In Eq. (2.1) above we assumed for simplicity one kind of nucleons, affected only by the longitudinal part of the $e - N$ interaction $\rho_q$. The squared inelastic form factor $\mathcal{F}$ gives the
transition probability between the ground state and scattering states, induced by the above
density fluctuation. The excited states are asymptotically a proton scattering state with
total on-shell energy $\epsilon(p + q)$ and a $A-1$ core in any conceivable excited state $n$, separated in
energy from the ground state by $\Delta_n$.

For sufficiently high energies only the incoherent part of the density fluctuation matters,
i.e. that part, where one and the same particle (say particle $i$ in $\rho^\dagger_q, \rho_q$) absorbs and emits the
momentum transfer in the elementary collision with the projectile. Consider first the PWIA
approximation to (2.1) where one neglects the interaction between the outgoing proton and
the core

$$V_i(\vec{r}_i) = \sum_{j \neq i} V(|\vec{r}_i - \vec{r}_j|)$$

Its subsequent inclusion generates the FSI between the recoil proton and the medium. Since
an exact description is generally impossible, approximations cannot be avoided. Most use
the high energy of the recoiling proton. We first assume factorization of scattering states in
(2.1) in core states and a proton wave function distorted by the core, i.e.

$$\Psi^{(+)}_{n,p}(\vec{r}_i; \vec{r}_j) \approx \Phi^{A-1}_n(\vec{r}_j) \psi^{(+)}(\vec{p}_i; \vec{r}_j)$$

Eq. (2.3) implies that the distortion of a fast proton in the field $V_i$, Eq. (2.2), is the one
by scatterers, fixed at positions $\vec{r}_j = \vec{b}_j + \vec{z}_j$. As for elastic scattering described by (1.3), the
distorted wave of the knocked-out proton in the eikonal approximation is described by

$$\psi^{(+)}(\vec{b}_i, z_i; \vec{r}_j) = \exp[i(\vec{p} + \vec{q})\vec{r}_j + i\tilde{\chi}^A_q(\vec{b}_i; -\infty, z_i; \vec{r}_j)],$$

That distorted wave is then in a conventional fashion described by an eikonal phase for given
impact parameter $\vec{b}_i$, accumulated on the path $(-\infty, z_i)^7 (v_{p'} = v_{p+q} \approx v_q)$

$$\tilde{\chi}^A_q(\vec{b}_i - \infty, z_i; \vec{r}_j) = -i/v_q \sum_{j \neq i} \int_{-\infty}^{z_i} d\sigma V_i(\vec{b}_i - \vec{b}_j, \sigma - z_i)$$
At this point we envisage the possibility of particle production and absorption in \(NN\) collisions. Although by choice not detected in \((e,e'p)\) reactions, those particles could be present, in which case the states (2.3) are insufficient and others should be introduced explicitly. If absent in final states, intermittent mesons may still be produced and absorbed in intermediate stages. Those may be accounted for, using a complex optical phase in Eqs. (1.1) and (1.3) or, alternatively, a complex \(NN\) potential \(V^3\), effectively describing elastic scattering, without the need to specify inelastic channels containing mesons.

In the actual construction of the SI response we shall follow two routes. One method employs closure, while in the second one exploits the advantages of a single particle model for all states in (2.1).

2a. Closure in the construction of the semi-inclusive response.

Assume that the energy transfer \(\omega\) is larger than all excited energies which contribute significantly to the inelastic form factors in (2.1). One may then apply closure over core states, replacing the separation energies there by an appropriate average \(\langle \Delta \rangle^*\). Putting without impunity \(i \rightarrow 1\), the response (2.1) can be recast into the form (recall \(\vec{p}' = \vec{p} + \vec{q}\))

\[
S_{SI}^{SU}(q,y_0,\vec{p}') = (m/q)\delta(y_0 - p_z) \int d\vec{r}_1 \int d\vec{r}_1' \rho_1(1,1')\exp[i\vec{p'}(1-1')][\Pi_{j \geq 2} \int dj][\rho_A(1,1';j)/\rho_1(1,1')]
\]

\[
\exp \left[ i\tilde{\chi}_q^A(\vec{b}_1; -\infty, z_1; j) - i\tilde{\chi}_q^{*A}(\vec{b}_1'; -\infty, z_1'; j) \right],
\]

(2.6)

where the energy transfer \(\omega\) has been replaced by the PWIA scaling variable

\[
y_0 = -q + \{2m(\omega - \langle \Delta \rangle)\}^{1/2}
\]

(2.7)

Again we meet in (2.6) \(\rho_A\), a \(A\)-particle density matrix, but now diagonal in all coordinates \(j\), except the one of the struck nucleon. This reflects the single-particle nature of the excitation operator: Inclusion of multi-particle exchange densities or currents, would lead to generalized densities, non-diagonal in a corresponding number of coordinates.

* Whenever not leading to confusion we shall replace \(\vec{r}_k \rightarrow k; j(= \vec{r}_j)\) denotes all coordinates except the one which is singled out: \(i\) in the case above.
Since the definition (1.8) of transparency in a knock-out reaction relates to the response without FSI we start with the PWIA when $V,\chi \to 0$ in (2.5), thus

$$S^{SI,\text{PWIA}}(q,y_0,p') \approx \frac{m}{q} \delta(p_z - y_0)n(p)$$ \hspace{1cm} (2.8)

Returning to Eq. (2.6) for a system where the recoiling proton interacts with the core, the exact SI response may be written as

$$S^{SI}(q,y_0,p') = \left(\frac{m}{q}\right)\delta(y_0 - p_z) \int d\vec{r}_1 d\vec{r}'_1 e^{i\vec{p}(\vec{r}_1 - \vec{r}'_1)} \rho_1(\vec{r}_1,\vec{r}'_1)\tilde{R}^{SI}(q,\vec{r}_1,\vec{r}'_1)$$ \hspace{1cm} (2.9)

For homogeneous matter the non-diagonal single-particle density in Eq. (2.9) is a function of the difference of the arguments, and its Fourier transform is simply related to the single particle momentum distribution

$$\frac{\rho_1(s,0)}{\rho} = \int \frac{d^3p}{(2\pi)^3} e^{-isp} n(p)$$ \hspace{1cm} (2.10)

Although well-defined, (see for instance Section 2b below) no simple expression exists for $\rho_1(\vec{r},\vec{r}')$ for finite systems. A suitable interpolation is \textsuperscript{3,12} $(\vec{S} = (\vec{r}_1 + \vec{r}'_1)/2); \vec{s} = \vec{r}_1 - \vec{r}'_1; \rho_1(r,r) \to \rho(r)$

$$\rho_1(\vec{r}_1,\vec{r}'_1) \approx \rho(\vec{S}) \int dS^3 \rho_1(\vec{s},\vec{S}') = \rho(S)\Sigma(s)$$ \hspace{1cm} (2.11)

Assuming (for large nuclei) in (2.11) $\rho(S) \to \rho(r_1)$, substitution into Eq. (2.9) leads to

$$S^{SI}(q,y_0,p') = \left(\frac{m}{q}\right)\delta(y_0 - p_z) \int d\vec{r}_1 \rho(r_1) \int d^3p''/(2\pi)^3 n(p'' - \vec{p}') R(q,\vec{r}_1,\vec{p}'')$$

$$= \int d\vec{r}_1 \rho(r_1) \int d^3p''/(2\pi)^3 S^{SI,\text{PWIA}}(q,y_0,p'' - \vec{p}') R(q,\vec{r}_1,\vec{p}''),$$ \hspace{1cm} (2.9')

where $R$ is the Fourier transform of $\tilde{R}$ in the variable $s$.

We notice that in both (2.8) and (2.9) (or (2.9')) the assumption of an average separation energy (or closure) has caused the replacement of the standard single nucleon spectral function\textsuperscript{13} by the momentum distribution, as if the spectral strength is concentrated in one
energy. In contradistinction, the second line in (2.9′) holds independent of that assumption. In the ultimate expression for the transparency, Eq. (1.8) one needs the ratio of both expressions. It is plausible that that ratio is less sensitive to the closure assumption than the same for numerator and denominator separately. Next we observe that the SI response contains a convolution and not a product of the momentum distribution and the FSI factor. Below we shall discuss examples and exceptions to general behaviour.

From a comparison with (2.8) one sees that all FSI effects appear concentrated in the factor $\tilde{R}^{SI}(q, \vec{r}_1, \vec{r}_1')$. The latter is far too complicated for an exact evaluation. As above for elastic scattering (cf. Eqs. (1.2), (1.3)), we exploit also here the high energy of the knocked-out nucleon and assume that binary collisions of that nucleon with partners in the core dominate the FSI in the SI response. In the appropriate first cumulant approximation to $R$ one can perform the integrals over $A − 3$ coordinates in (2.6). This permits a reduction of the $A$-body density matrix in (2.6) to a (off-diagonal) 2-particle density matrix, which upon introduction of an off-diagonal 2-body correlation function can be written as $^8,^9$

\begin{align}
\rho_2(1, 2; 1', 2) &= \rho(2)\rho_1(1, 1')\zeta_2(1, 1'; 2) \\
\zeta_2(1, 1'; 2) &\approx \sqrt{g(1, 2)g(1', 2)},
\end{align}

(2.12)

where in the second line we give an interpolation, suggested by Gersch et al$^9$ and which shall be used throughout. Notice the differences with the elastic case discussed in Section 1 and the relevant remark there after Eq. (1.7): In a SI process the proton to be knocked-out is part of the correlated target, hence the appearance of a correlation function. In contradistinction to the elastic case, an essentially different mechanism described above causes here densities and generalized correlation functions to be non-diagonal.

Consider now Eq. (2.6) in the first cumulant approximation $^{10}$ and concentrate on the
the approximation $\vec{b}_1 = \vec{b}_2$. Using (2.5) and (2.12) one finds $(\vec{r} = \vec{r}_1 - \vec{r}_2)$

$$[\Pi_{j\geq 2}\int dq\{\rho_A(1,1';j)/\rho_1(1,1')\} \exp[i\hat{\chi}_q^{A}(\vec{b}_1; z_1, -\infty; \vec{r}_j) - i\hat{\chi}_q^{A*}(\vec{b}_1; z_1', -\infty; \vec{r}_j)]$$

$$= [\Pi_{j\geq 2}\int dq\{\rho_A(1,1';j)/\rho_1(1,1')\} \exp[i\hat{\chi}_q^{A}(\vec{b}_1 - \vec{b}_j; z_1, -\infty; \vec{r}_j) - i\hat{\chi}_q^{A*}(\vec{b}_1 - \vec{b}_j; z_1', -\infty; \vec{r}_j)]$$

$$\approx 1 + (A-1)\int d^2\rho_2(1,1';2)/\rho_1(1,1') \exp\left\{-i/v_q \int_{z_1}^{z_1'} d\sigma V(\vec{b},\sigma - z_2) - 2/v_q \int_{-\infty}^{z_1} d\sigma \text{Im} V(\vec{b},\sigma - z_2) - 1\right\}$$

$$\approx \exp[(A-1)\int d^2\rho_2(1,1';2)/\rho_1(1,1') \exp\left\{-i/v_q \int_{0}^{s_z} d\sigma' V(\vec{b},z - \sigma') - 2/v_q \int_{s_z}^{\infty} d\sigma' \text{Im} V(\vec{b},z - \sigma') - 1\right\}]$$

(2.13)

We shall use the parameterization (1.5) for short-range interactions in the limit $Q_0 \to \infty$ and shall assume that, as for nuclear matter, $\zeta_2(\vec{r}_1, \vec{r}_2, \vec{s}) \approx \zeta_2(\vec{r}, \vec{s}_1)$. One may then rewrite the SI response Eq. (2.9), containing binary collision FSI effects in the manageable form

$$S^{SI}(q_0, \vec{p}) = (m/q)\delta(q_0 - p_z) \int d\vec{r}_1 d\vec{s}_e d\vec{s}_r \hat{\rho}_1(\vec{r}_1, \vec{s}) \hat{R}^{SI}(q_0, \vec{r}_1, \vec{s})$$

$$\hat{R}^{SI} = \hat{R}^{tot} \hat{R}^{reac}$$

$$\hat{R}^{tot}(q, \vec{r}_1, \vec{s}) \approx \exp\left[-(A-1)(\sigma_q^{tot}/2)(1 - i\tau_q) \int_{0}^{s_z} dz \rho(\vec{b}_1, z_1 - z)\zeta_2(0, z, \vec{s})\right]$$

(2.14a)

$$\hat{R}^{reac}(q, \vec{r}_1, \vec{s}) \approx \exp\left[-(A-1)(\sigma_q^{reac}) \int_{s_z}^{\infty} dz \rho(\vec{b}_1, z_1 - z)\zeta_2(0, z, \vec{s})\right]$$

(2.14b)

Eqs. (2.14) feature the total $NN$ total cross section, as well as the 'partial' and total reaction cross sections

$$\sigma_q^{p,\text{reac}}(b) \equiv 1 - e^{-2i\chi_q(b)} \approx \delta^{(2)}(b)\sigma_q^{\text{reac}}$$

$$\sigma_q^{\text{reac}} = \sigma_q^{tot} - \sigma_q^{el}$$

We note that the first part in the exponent of $\hat{R}^{tot}$ in Eq. (2.14a) exists for real as well as for complex $V$ (or $\chi$) and decreases with increasing $NN$ total cross section. The absorptive part of $\chi$, produces a component (2.14b) in $\hat{R}$ and $T$, which decreases with the $NN$ (total) reaction (inelastic) cross section.

We shall return to the results of Section 2a, but elaborate first on the above mentioned alternative approach.

2b. An extreme single particle model
Using (2.3) we consider the part $F_i$ of the inelastic form factor in (2.1) contributing to the response (2.1), which is excited by the density fluctuation of the $i^{th}$ particle

$$F_{0n}(q) = \langle \Phi_0^A(i;j)|e^{-i\vec{q}\cdot\vec{r}_i}|\Phi_n^{A-1}(j)\psi_{n,p+q}^{(+)}(i;j) \rangle$$  \hspace{1cm} (2.16)

For its evaluation we formulate the following single particle model. If the knock-out is fast, i.e. if rearrangement can be neglected, the orbital of the knocked-out particle $\phi_{\nu_i}(i)$ also determines the excited state of the remaining $A-1$ nucleons. One then finds for the product of the two bound state wave functions in (2.16)

$$\Phi_0^A(i;j)\Phi_{\nu_i}^{A-1}(j) \approx \phi_{\nu_i}(i)\rho_{A-1}^\nu(j) \approx \phi_{\nu_i}(i)\rho_{A-1}(j)$$  \hspace{1cm} (2.17)

It has been assumed above that various $A-1$ particle diagonal densities for states which differ from the ground state by one particle in orbital $\nu_i$, may all be replaced by the density $\rho_{A-1} \equiv \rho_{A-1}^\nu$ in the ground state. Of course, in order for Eq. (2.17) to hold for every $i$, one must have an extreme single-particle model for both target and core states.

Next we treat again the many-body eikonal phase in (1.2) in the first cumulant (1.3) and the short-range approximation (1.5), and obtain for the contribution of the $i^{th}$ particle to the inelastic form factor (2.16)

$$\langle \Phi_0^A|e^{-i\vec{q}\cdot\vec{r}_i}|\Phi_{\nu_i}^{(+)}\rangle \approx \int d\vec{r}_i \phi_{\nu_i}(\vec{r}_i)e^{i\vec{p}\cdot\vec{r}_i}\exp\left[-(A-1)(1-i\tau_q)\sigma_q^{tot}/2\int_{-\infty}^{Z_i} dz_2 \rho(b_1,z_2)\right]$$  \hspace{1cm} (2.18)

Substitution in Eq. (2.1) gives

$$S_{SI}(q,\omega,\vec{p}) \approx A^{-1} \sum_{\nu} \delta(\omega + \Delta \nu - e_0(\vec{p} + \vec{q})) \int d\vec{r}_i d\vec{r}'_i \phi_{\nu_i}(\vec{r}_i)\phi_{\nu_i}(\vec{r}'_i)e^{i\vec{p}\cdot(\vec{r}_i-\vec{r}'_i)} \exp\left[-(A-1)\sigma_q^{tot}/2\left\{(1-i\tau_q)\int_{-\infty}^{Z_i} dz_2 \rho(b_1,z_2) + (1+i\tau_q)\int_{-\infty}^{Z_i} dz_2 \rho(b_1',z_2)\right\}\right]$$  \hspace{1cm} (2.19)

In order to enable a comparison with the approach in Section 2a we replace also here the separation energies by an average $\langle \Delta \rangle$. The remaining sum over $\nu$ in (2.19) is then just the
expansion of the non-diagonal density matrix $\rho_{i}^{sp}(\vec{r}_{1},\vec{r}_{1}')$ in the extreme single particle model, i.e. with orbitals which have unit occupation probability

$S^{SI}(q,y_{0},\vec{p}') \approx \frac{m}{q}\delta(p_{z} - y_{0}) \int \int d\vec{r}_{1}'d\vec{r}_{1}\rho_{i}^{sp}(\vec{r}_{1},\vec{r}_{1}')e^{i\vec{p}'(\vec{r}_{1}-\vec{r}_{1}')}$

$\exp \left[ - (A - 1)\sigma_{q}^{tot}/2 \left\{ (1 - i\tau_{q}) \int_{-\infty}^{z_{i}} dz_{2}\rho(\vec{b}_{1},z_{2}) + (1 + i\tau_{q}) \int_{-\infty}^{z_{i}'} dz_{2}\rho(\vec{b}_{1}',z_{2}) \right\} \right] \quad (2.20)$

Finally taking also here $\vec{b}_{1}' = \vec{b}_{1}$ (cf. (2.9))

$S^{SI}(q,y_{0},\vec{p}') \approx \frac{m}{q}\delta(p_{z} - y_{0}) \int \int d\vec{r}_{1}'d\vec{r}_{1}\rho_{i}^{sp}(\vec{r}_{1},\vec{r}_{1}')e^{i\vec{p}'(\vec{r}_{1}-\vec{r}_{1}')}$

$\exp \left[ - (A - 1)\sigma_{q}^{tot}/2 \left\{ (1 - i\tau_{q}) \int_{-\infty}^{z_{i}} dz_{2}\rho(\vec{b}_{1},z_{2}) + (1 + i\tau_{q}) \int_{-\infty}^{z_{i}'} dz_{2}\rho(\vec{b}_{1}',z_{2}) \right\} \right]$

$\tilde{R}(q,\vec{r}_{1},\vec{s}) = \exp \left[ - (A - 1)\sigma_{q}^{tot}/2 \left\{ (1 - i\tau_{q}) \int_{0}^{s_{z}} dz\rho(\vec{b}_{1},z_{1} - z) + 2 \int_{s_{z}}^{\infty} dz\rho(\vec{b}_{1},z_{1} - z) \right\} \right]$

$\rho_{i}^{sp}(\vec{r}_{1},\vec{r}_{1}') = \sum_{\nu} \phi_{\nu}^{*}(\vec{r}_{1})\phi_{\nu}(\vec{r}_{1}') \quad (2.21)$

Eq. (2.21) resembles its counterpart Eqs. (2.14) of section 2a. Based on a strict single particle model, it manifestly lacks reference to pair correlations, non-diagonal or diagonal. Otherwise it has the total reaction cross section in (2.14b) replaced by the total $NN$ cross section. The lack of correlations is understandable since we are using an extreme single particle model.

The other difference with Eq. (2.13b), namely the replacement of the reaction cross-section by $\sigma^{tot}$ is more subtle. The assumption (2.17) prescribes core excited states to be the target ground state from which one particle is removed. No other excited core states are permitted, whereas in the closure approximation of Section 2, particle knock-out from the core or general break-up is included. In experiments with sufficient energy resolution to guarantee that only one nucleon is knocked out of the nucleus, one expects the appearance of the total, rather than of reaction cross-section. This may for instance be the case for the NE18 SLAC experiment\textsuperscript{15}. In 'less' exclusive reaction the partial use of the reaction cross-section may well be more appropriate.
2c. The generalized transparency in SI reactions.

We are now in a position to give expressions for the desired SI transparency \( T^{SI}(q, \vec{p}') \equiv \frac{d\sigma(q, \omega, \vec{p}')^{\text{data}}}{d\sigma(q, \omega, \vec{p}')^{\text{PWTA}}} = \frac{S(q, \omega, \vec{p}')^{\text{SI} \, \text{data}}}{S(q, \omega, \vec{p}')^{\text{SI} \, \text{PWTA}}} \) (2.22)

and in particular when in the FSI factor (cf. (2.12)) only binary collision approximation are retained

\[
T(q, \vec{p}') = \frac{\int d1 \int d1' \rho_1(1, 1')e^{i\vec{p}(1-1')}}{\int d1 \int d1' \rho_1(1, 1')e^{i\vec{p}(1-1')}} \Rightarrow [n(p)^{-1}] \int d1 \int d\vec{p}' / (2\pi)^3 n(\vec{p}' - \vec{p}'') R(q, 1, \vec{p}'') \]  

As had already been observed after Eq. (2.9'), the SI transparency (the same holds for the TI response) is generally expressed as a convolution in momentum space. A common product in (2.23a) would reduce the generalized transparency to the appropriate FSI factor \( R \) and this indeed happens in some simplified models (see Section 3). Note that closure apparently renders \( T \) independent of \( \omega \).

Eqs. (2.23) will be the basis for actual numerical calculations. We remark that, in principle, we could have exploited the Fourier transform of the FSI factor Eq. (2.9) before application of the short-range approximation (1.5). It is the most general form in the first cumulant approximation, and the corresponding FSI factor (or transparency) there include, in principle calculable off-shell \( NN \) scattering effects. Those are lost in the measurable, on-shell expressions in Eq. (2.14), which are much easier to handle, give direct insight in parametric dependence and also happen to be most convenient for making comparisons with other approaches. An example with the result (2.23) has already been discussed in the previous subsection: Others will be discussed in Section 3.

3. Comparison

The reasoning which, through the parameterization (1.5) transforms the FSI factor in Eqs. (2.6), (2.13) to the simple forms (2.14) and (2.23), has in part been suggested before
by Kohama, Yazaki and Seki (KYS). It is thus appropriate to make first a comparison with their work. In a first and inessential assumption, the \( A \)-particle density in (2.6) is taken by KYS in a mean field approximation without correlations (cf. Section 2b) \(^3\)

\[
\rho_A(1,1';j) = \rho_1(1,1')\Pi_{j\geq 2}\rho(j),
\]

with the Negele-Vautherin Ansatz (2.11) for the non-diagonal single particle density \( \rho_1(1,1') \).

In Ref. 3b a non-diagonal pair-distribution function (2.11) is used, as had been suggested by Gersch et al\(^9\).

Also KYS have to address the intractable eikonal phases in (2.6). Their approximation centers on the oscillating exponent in (2.9) which KYS claim to vary much faster than any other function in the integrand of (2.6), thus permitting one to put \( \vec{s} = \vec{r}_1 - \vec{r}_1' = 0 \) everywhere. An exception is made for the non-diagonal single particle density (2.11), thereby recognizing the basic role of that density for SI reactions. When the above substitution is actually applied in (2.14) one obtains for the SI response (2.6)

\[
S^{SI\ KYS}(q,y_0,p') = \frac{m}{q} \delta(y_0 - p_z) \int d\vec{s} e^{i\vec{s}\vec{R}} \Sigma(s) \int d\vec{r}_1 \rho(1) e^{-\sigma_{r\text{eac}}}(\vec{r}_1) e^{-\int_\infty^1 d\vec{s} \rho(h,\vec{s})}
\]

The above crucial assumption renders \( \tilde{R} \) independent of \( s_z \) and explicitly proportional to \( n(p) \) and produces a transparency

\[
T^{SI\ KYS}(q) = [A - 1] \sigma_{q\text{eac}}^{-1} \int \bar{b}[1 - e^{-(A-1)\sigma_{q\text{eac}}(b)}]
\]

Eq. (3.3) is not a convolution as the general result (2.23b) predicts. Possibly anticipating (3.3), Kohama et al, in fact, define the transparency by (2.23b), but assume the involved ratio to be independent of the knocked-out proton momentum \( p \) which, as demonstrated, is not generally the case.

Eq. (3.3) for the SI response in the KYS approximation has obvious drawbacks, in part related to the fact that the FSI factor Eq. (3.3) contains the reaction cross section. For
energy losses below the production threshold, \( \sigma^{rec}(q) = 0 \) and the manifest absence of an 'elastic' FSI contribution (2.14a) leads to \( R = 1 \), which is clearly incorrect. Since well above that threshold \( NN \) reaction and total cross sections are of the same order, it is likely that the above shortcoming is significant in general.

In addition \( T^{SI,KYS} \), Eq. (3.3) is independent of \( p \), the momentum of the outgoing nucleon. Thus, in spite of the fact that KYS retained the non-diagonal single particle density matrix and obtained a non-trivial SI response (3.2), the corresponding transparency (3.3) is barely different from the elastic transparency (1.6b) and requires only the replacement of the reaction by the total cross section. The simple cause of all points raised is the exponent in (2.18). In fact, since \( p < p_F \), \( p|\vec{r}_1 - \vec{r}'_1| = \mathcal{O}(1) \), its oscillations are not conspicuously fast and consequently there is no compelling reason to put \( \vec{r}'_1 = \vec{r}_1 \). The assumption \( \vec{b}'_1 = \vec{b}_1 \), but \( z'_1 \neq z_1 \) applied to (2.13) is apparently a much weaker one and gives radically different results. In particular it leaves in (2.14a) below the lowest inelastic threshold an elastic part which is missing in (3.3).

Next we cite an expression for the transparency given by Pandharipande and Pieper

\[
T^{SI,PP}(q) = (A-1)^{-1} \int d\vec{b}'\rho(\vec{b},\vec{z}') \exp\left[ -\int_{z'}^{\infty} d\bar{z} \sigma_{q}^{tot}(\rho(\vec{b},\bar{z})) \right] \approx [(A-1)\langle g \rangle (\sigma_{q}^{tot})]^{-1} \int d\vec{b} e^{-\langle g \rangle (\sigma_{q}^{tot})} \approx \left[ (A-1) \right]^{-1} \int d\vec{b} e^{-\langle g \rangle (\sigma_{q}^{tot})} \tag{3.4}
\]

The steps used to produce (3.4) again lead to a transparency independent of the out-going proton momentum and which is similar to (1.6b) for elastic scattering (See Ref. 16 for a derivation emphasizing the above).

From the above it should have become clear that there are basic differences between the transparency of a medium for an elastically scattered proton and one which in a reaction is removed from the target before scattering. In the 'elastic' case the proton is not correlated at all with nucleons in the nucleus. The interaction with the latter is through the diagonal density and this remains the case for the cross section: Indeed 'elastic' transparency is a
classical concept.

In contradistinction, the emerging proton before knock-out in either a SI reaction, is part of the target. It is therefore correlated with a core nucleon it interacts with, and the nature of the process causes the densities in the cross section to be non-diagonal in a non-trivial manner. No heuristic reasoning can bridge these intrinsically different pictures. *

Next we compare the approach of the authors in Refs. 1 and 3. Indeed, in spite of the similarities, Eqs. (3.3) and (3.4) are basically different. KYS realized the essential appearance of non-diagonal densities: The resemblance of (3.3) to (3.4) is the result of the above-discussed specific approximation.

In concluding this section we recall that, disregarding differences, the SI transparencies (3.3) and (3.4) result if in (2.23a), based on Eqs. (2.14) or (2.21), $s_z \to 0$. On may then ask whether the FSI factors $\tilde{R}$ peak at $s_z = 0$. An estimate can be made if the density and $\zeta$ are replaced by averages, which results in a $z$-integral $\propto s_z$. The FSI factor thus decreases exponentially from $s_z \approx 0$, but the maximum is hardly pronounced 17.

4. Numerical results.

In the following we report on results for the transparency $T$, Eq. (2.23), in its dependence on the one-body density matrix $\rho_1(1,1')$ and the semi-inclusive FSI factor $\tilde{R}(q,1,1')$. Two versions have been tested, namely the closure method (CL) based on Eqs. (2.11) and (2.14), and the extreme single particle model version (SP), using (2.21).

Input elements in the former are total and total reactive $pN$ cross sections, the ratio’s $\tau$ of real and imaginary part of the forward elastic $NN$ amplitude in the parametrization (1.5)

* It is argued in Ref. 1 that a pair-distribution function in (3.4) is needed to account for modifications of the particle density due to short-range correlations. A self-consistent theory of the target accounts treats correlations in derived observables and for instance the density depends implicitly on them.
and the non-diagonal pair distribution function $\zeta_2$ in the approximation (2.12). The required density matrix has been computed from (2.10) and (2.11), with single particle densities and momentum distributions as in Ref. 3. In the SP model the density matrix $\rho_{1p}$, required in (2.21) is constructed from single-particle wave functions, chosen to correspond to levels in a Saxon-Woods potential and which are occupied with unit probability till a given $A(Z,N)$ is reached.

Predictions have been made for two sets of kinematic conditions. We considered first the experiment of Garino et al on the targets $^{12}$C, $^{27}$Al, $^{58}$Ni and $^{181}$Ta. The momentum transfer was approximately constant, $q \approx 610$ MeV, while the outgoing protons had energies $E_{p'} = 180 \pm 50$ MeV. We refer to Ref. 4 for the extraction method for $T_{SI}$, Eq. (2.22), from rather coarsely binned data.

Calculations have been made for varying $p_z$, with average $p_z \approx 0$; small $p_\perp$ have been neglected. Fig. 1 contains data and predictions for the transparency $T$, Eq. (2.23), for CL and SP at $p_z = 0$. Drawn, dashed and dot-dashed curves correspond to CL and refer respectively, to results based on (2.14) with and without (off-diagonal) pair correlations and for $s_z = 0$, correlations included. The dotted curve is the SP prediction (2.21). Compared to our ‘best’ prediction, the data appear underestimated by 15-20%. Fig. 2 shows for $^{58}$Ni the dependence of $T$ on the direction of the proton momentum, varying with respect to the momentum transfer. That dependence is qualitatively the same for all targets.

The second set of conditions corresponds to the SLAC NE18 $(e,e'p)$ experiment on $^{12}$C, $^{27}$Al, $^{63}$Cu and $^{208}$Pb. Table I gives the much higher 3-momentum and energy transfers, the square of the 4-momentum transfer and the momentum of the outgoing proton, which is again close to $q$. Figs. 3a-d show predictions for $Q^2$=1.04, 3.00, 5.00, and 6.77 GeV: data have as yet not been released. One observes:

i) Inclusion of correlations reduces by roughly 15-20% the opacity of each data point
calculated with CL.

ii) When compared with CL predictions under comparable circumstances, (i.e. with correlations included) the KYS assumption \( s_z = 0 \) increases \( \mathcal{T} \) by 4-7% for the kinematics of the Garino experiment and \( p_z = 0 \), when \( A \) increases from 12 to 181. Slightly smaller increases have been observed for NE18.

iii) A drastic reduction of \( \mathcal{T} \) by a factor of order 1.5-2.0, and in addition a larger \( A \)-dependence results when comparing SP and CL, both lacking correlations. Part of this reduction is due to the sole appearance in (2.21) of total cross sections, which always exceed total reaction cross sections in (2.14).

iv) Only moderate 10% reductions occur in the predicted \( \mathcal{T} \) when \( q \) in NE18 changes as much a factor 4. Similar changes occur when going from the Garino to the NE18 conditions and all can be predicted. In a NR theory the relevant eikonal phases in (2.5) are proportional to \( v_q \propto m/q \), which for relativistic kinematics are replaced by \( E(q)/q \), which has a much weaker \( q \) dependence. This is also clear from the high-\( q \) parametrization (1.5) of the on-shell eikonal phase or amplitude: The \( E(q)/q \) dependence of the phase \( \chi \) cancels out in the chain \( v \rightarrow t \rightarrow \sigma \) and leaves only a moderate implicit \( q \)-dependence in cross sections. Otherwise there are for the NE18 kinematics no conspicuous changes in the trends already observed for lower \( q \) regarding \( A \)-dependence, influence of correlations and for the results of the KYS assumption.

Finally we report on computations related to the basic assumption \( \vec{b}_1' \rightarrow \vec{b}_1 \) in (2.6), and which through (2.13) enable evaluation of the SI response in the form (2.13). It appears difficult to make a reliable test. We have done so approximately for the SP model using harmonic oscillator wave functions. For those, we calculated the ratio of transparencies under the Garino conditions, once for \( \vec{b}_1' \neq \vec{b}_1 \), then with the two equal. From positive to negative \( p_z \), the above ratio decreases, passing 1 in the neighbourhood of the quasi-elastic peak, \( p_z \approx 0 \). At least there the tested approximation seems to be fully justified.
Out of the alternative approaches mentioned, we already discussed a comparison with KYS. One would like to do the same with the predictions of Ref. 1. However, in view of the discussion in Section 3 on the seizable differences in approach, such a step by step comparison with the work of Pandharipande and Pieper\(^1\) does hardly make sense. An exception should be made for the effect of correlations. Although disagreeing on the way \(g(r)\) has been introduced in \(\mathcal{T}\), a similar substantial increase in \(\mathcal{T}\) has been reported. It is of interest to note that (formally!) a different \(\zeta_2\) has been used in \(^1\). The same holds if the pair distribution function for NM, available to us, is replaced by one for variable density (cf. Fig. 6, Ref. 1). This does not make it likely that uncertainties in the off-diagonal \(\rho_2\) can be blamed for the observed discrepancy.

Next we come to the high \(q, Q^2\) predictions by Benhar \textit{et al}\(^2\). Since based on Ref. 1, it is again difficult to make comparisons with our results. We only note that for the Garino experiment their predictions exceed ours by 10-15\%, but those fall short by about the same amount for high \(q\).

5. Summary and conclusion.

We addressed above the transparency of a nuclear medium for a nucleon knocked-out in semi-inclusive reactions, defined as the ratio of the experimental yield and the same, calculated in the PWIA and clearly related to Final State Interaction of the knocked-out nucleon with the core. In view of the relatively high momentum transfers involved, we used the first cumulant approximation to the SI response, which focuses on binary collisions of the knocked-out proton with a core nucleon. Production and absorption of undetected pions etc., are approximately included, implying a complex interaction \(V_{NN}\). In a simple parameterization for \(V\), ultimately both the total and the total reaction (inelastic) \(NN\) cross section appear in expressions for the transparency.

We then emphasized that from the single-particle nature of the excitation mechanism of
an electron, colliding with a single nucleon, it follows that the SI response or cross section is ultimately proportional to a two-particle density matrix, non-diagonal in the coordinate of the struck nucleon. That density matrix can alternatively be expressed in terms of a non−diagonal single particle density and a pair-distribution function.

The above observation is basically different for elastic scattering from a medium, whether the projectile is a nucleon or otherwise. For it, the cross section appears related to diagonal densities. As a consequence, and notwithstanding intuitive reasoning, the transparency of a medium for passing of a proton removed in a reaction is basically dissimilar to the same in elastic scattering. In particular the correct expression depends on the momentum of the outgoing proton.

Throughout we have treated the first cumulant approximation in two versions. In the first, based on closure over excited states, one explicitly includes (non-diagonal) pair-correlations between the knocked-out proton and nucleons in the core. The second approach is based on a strict, single-particle model for the nuclear ground state as well as for excited states, and the treatment thus foregoes pair-correlations effects.

Calculations have been performed for both versions using simple parameterizations for the effective $V_{NN}$, thereby avoiding the usual encumbering off-shell effects. The kinematic conditions considered are those for the experiments by Geesaman and Garino et al, and also for the high energy SLAC NE18 experiment. For the former, results have been compared with the above defined experimental quantities. The predictions are 10-15% in excess of observed transparencies.

As to generalities around the predictions, we obviously found transparencies decreasing with mass number and could show that the use of parametrized, on-shell eikonal scattering phases produce only moderate energy dependence in $\tau$. The inclusion of non-diagonal pair-correlations leads to a sizable reduction of opacity.
As regards the two versions considered, the exclusive appearance of total reaction cross sections in the strict single particle model in contrast to total and (smaller) total reaction cross sections in the closure approach, causes an appreciable reduction in $\mathcal{T}$ when compared to the closure approach.

We then compared our approach with previously suggested ones. Ours is in spirit close to the one by Kohama, Yazaki and Seki who also emphasize the use a non-diagonal single-particle density matrix. Unfortunately a strong assumption in the actual evaluation, removes the dependence of the SI transparency on the momentum of the outgoing proton and in addition, renders it solely dependent on the reaction cross section. When the latter is replaced by the total cross section, the resulting SI transparency is indistinguishable from the elastic one.

We then discussed the approach of Pandharipande and Pieper, which differs basically from ours. Their expression is essentially the elastic transparency with modifications we discussed and criticized on the basis that the projectile proton in elastic scattering is essentially not correlated to the target particles, whereas the knocked-out proton in a SI reaction is.

At this point we only briefly mention the issue of colour transparency, which envisages objects of smaller than hadronic size to be produced. Those subsequently exit with anomalously large transparency before reverting to their ‘normal’ hadronic identity. For instance, Komara et al emphasize that in their theory (cf. Eqs. (3.3) and (1.6b), reaction cross sections replace larger total $NN$ cross sections, thus reducing SI transparencies without invoking exotic colour transparency of sub-hadronic constituents. For $(e,e'p)$ experiments under current conditions the issue appears to be academic: The most recent analysis of the dedicated SLAC NE18 $(e,e'p)$ experiment for $Q^2$ up to 7GeV$^2$, shows no trace of colour transparency. The latter had already been called in, in order to explain totally inclusive electron scattering experiments in the $Q^2 \approx 2 - 3$ GeV region. In on-going and planned $(e,e'p)$ and $(p,2p)$
reactions, the detection of colour transparency requires a maximally accurate theory for the regular, nucleonic transparency, which in part motivated the study above.

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References

1V.R. Pandharipande and S. C. Pieper, Phys. Rev. C45 (1992) 780
2O. Benhar, A. Fabrocini, S. Fantoni, G.A. Miller, V.R. Pandharipande and I. Sick, Phys. Rev. C44 (1991) 2328; Phys. Rev. Lett. 69 (1992) 881
3A. Kohama, K. Yazaki and R. Seki, Nucl. Phys. A536 (1992) 716; ibid A551 (1993) 687
4D.F. Geesaman et al, Phys Rev. Lett. 63 (1989) 734; G. Garino et al, Phys Rev. C45 (1992) 780;
5A.S. Carroll et al, Phys Rev. Lett. 61 (1988) 1698
6E.g. G. R. Farrar, H. Liu, L. L. Frankfort and M. I. Strikman, Phys. Rev. Lett. 61 (1988) 686
7R.J. Glauber, Lectures in Theoretical Physics. Ed. W.E. Brittin et al (Interscience, NY., 1959) p. 315; Proceedings Int’l Conf. on Particle and Nuclear Physics, Rehovot, Israel (1957) (Ed. G. Alexander, NHPC)
8See for instance A.S. Rinat and M.F. Taragin, Phys. Rev. B41 (1990) 4247
9H.A. Gersch, L.J. Rodriguez and Phil N. Smith, Phys. Rev. A5 (1972) 1547
10H.A. Gersch and L.J. Rodriguez, Phys. Rev. A8 (1973) 905
11I. Sick, D. Day and J.S. McCarthy, Phys Rev. Lett. 45 (1980) 871
12J. Negele and D. Vautherin, Phys. Rev. C5 (1971) 1472; ibid C11 (1974) 1031
13E.g. A.E.L. Dieperink, T. deForest, I. Sick and R.A. Brandenburg, Phys. Letters B63
(1976) 261; E. Pace and G. Salme, Phys. Lett. B110 (1982) 411

14B.K. Jennings and G. Miller, Phys. Rev. D44 (1992) 692

15J. van den Brand et al, SLAC-NPAS proposal NE18 (1990); R.D. McKeown, CalTech Preprint OAP-719(1992)

16I. Mardor, Y. Mardor, E. Piasetzki, J. Alster and M. Sargsyan, Phys. Rev. C46 (1992) 761

17E.g. C. Carraro and A.S. Rinat, Phys. Rev. B45 (1992) 2945

18P.W. Fillipone, TRIUMF Workshop on Colour Transparency, 6-9 Jan. 1993

19D.B. Day et al, Phys. Rev. C40 (1989) 1011

**Figure captions**

Fig. 1. Transparencies for passage of knocked-out protons in semi-inclusive $(e,e'p)$ reactions on different targets. Data are from Ref 4. Drawn, dashed and dot-dashed curves correspond to CL model based on Eq. (2.14) and correspond respectively to calculations with and without (non-diagonal) pair correlations, and $s_z = 0$ (no correlations). The dotted curve is for the SP model (2.14).

Fig. 2 Dependence of the transparency of $^{58}$Ni in its dependence on the direction of the outgoing momentum with respect to the direction of the momentum transfer.

Fig. 3a The same as Fig. 1 for the NE18 experiment with $Q^2=1.04$ GeV (see Table I).

Fig. 3b The same as Fig. 3a: $Q^2=3.00$ GeV (see Table I).

Fig. 3c The same as Fig. 3a: $Q^2=5.00$ GeV (see Table I).

Fig. 3d The same as Fig. 3a: $Q^2=6.77$ GeV (see Table I).

**Table I**

Kinematic conditions for SLAC NE18 experiment. In units GeV$^{-1}$ the four columns give
momentum, energy transfer, square of 4-momentum and momentum of outgoing proton.

| $q$  | $\omega$ | $Q^2$ | $p'$  |
|------|----------|-------|-------|
| 1.198| 0.630    | 1.038 | 1.201 |
| 2.439| 1.719    | 2.995 | 2.454 |
| 3.538| 2.742    | 4.997 | 3.543 |
| 4.484| 3.652    | 6.772 | 4.490 |