A Study of Factors Influencing Wood Decay and Fungal Activities Based on Mathematical Modelling Methods

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Abstract. This paper establishes a series of mathematical models to describe the wood decay by fungi and important factors affecting fungi activities. First, it is assumed that the external environment will not change; by doing so, a simplified model was established. In continuous time intervals, the model took two situations into account—only single species involved and only multiple species involved. We then used the logistic model to describe the growth rate of fungi. In the situation of multiple species, we also considered competitions between different species. Given the initial area, combined with the relationship between fungal growth rate and decomposition rate, a simplified lignin decomposition model was established. On the basis of this model, a graph of the expansion rate and decomposition rate of different fungi over time was drawn, describing the decay effect of a single fungus and multiple fungi on wood in two different time ranges, short-term and long-term. When multiple species involved, competition between each fungal strain will also be considered. Secondly, considering the complexity of the environment in reality, we include two factors—temperature and moisture—into our model. These two would significantly affect fungi’s activities. Based on the research of Dianel S. Maynard and others, we establish a linear model -The Temperature and Moisture Change Model- to describe the impact of temperature and moisture. We also apply the model in five regions of different climatic characteristics and compare the growth rate and decomposition rate of different fungi to these five regions.

Keywords: the logistic model, wood decomposition, the role of fungi, ecosystem.

1. Introduction
As an important part of the ecosystem, fungi can be seen everywhere in our daily life. We can see them annoying on the moldy wall or the rotten food, and these fungi can also be used to make delicious dishes. In fact, fungi not only are closely related to our life but also make a great contribution to the stability of the ecosystem. Though fungi can only rely on their organic carbon formed by carbon fixation of other organisms as their carbon source, the evolution of fungi has resulted in diverse metabolic capability, and these powerful fungi can decompose organic matters. The role of decomposers in the ecological system is crucial, without which animal and plant residues will pile up like mountains; the carbon substances
will be locked in organic matter, no longer participating in the biological cycle. The result of this is that the material circulation of the ecosystem will be suspended. The ecosystem will finally collapse. Therefore, it is necessary for us to study fungal decomposition, which is of great significance for understanding the carbon cycle.

2. Notations

| Abbreviation | Description |
|--------------|-------------|
| e            | Natural logarithm |
| t            | The time |
| t₀           | Initial time |
| t_{max(i)}   | Time for the growth of the i-th fungus colony to reach a steady state |
| r_i          | The inherent relative radius growth rate of the i-th fungus |
| r'_i         | The inherent relative radius growth rate of the i-th fungus after introducing environmental changes |
| R₀           | Initial radius of mycelium |
| R_{max}(t)   | The radius of hyphae of the i-th fungus changing by time |
| R_{max(i)}   | The maximum radius that a fungal colony can grow to |
| S_{i}(t)     | The colony area of the i-th fungus changing by time |
| S_{max}(i)   | The largest area that the i-th fungus can grow to |
| C_{i}        | The percentage of wood can be decomposed by the i-th fungus every unit area |
| C_{i}(t)     | At time t, the percentage of wood can be decomposed by the i-th fungus in the total wood in the environment |
| σ_i          | The growth inhibitory effect of the i-th fungus on other fungi |
| n_i          | The water potential |
| T            | The temperature |
| r_{i}(m)     | The extension rate of the i-th fungus varies with moisture |
| r_{i}(T)     | The extension rate of the i-th fungus varies with temperature |
| r_{i}(m,T)   | The extension rate of the i-th fungus varies with moisture and temperature |
| r_{max}(m)   | The average extension rate of the i-th fungus at the optimal moisture |
| r_{max}(T)   | The average extension rate of the i-th fungus at the optimal temperature |
| r_{max}(i)   | The extension rate of the i-th fungus under optimal conditions |

Figure 1. The Notations of this paper

3. The Model of Wood Decomposition

3.1. Single-Colony Model

In ideal conditions, there are only single species living in a given area. In the experiments of Liang zhi and Han baoping, the researchers conducted single-species simulation cultivation of a typical wood-decay fungus—white-rot fungus. The growth of fungus fits the logistic equation well [1]. Therefore, we establish a logistic model based on their experiments. The growth of fungi mainly depends on the extension of their hyphae. For our hypothesis, we use the change of fungal radius to describe the growth of fungi:

\[
\frac{dR}{dt} = r(1 - \frac{R}{R_{max}})R
\]

With the initial radius of the bacteria \(R(0) = R_0\), we can get the equation below by separating the variables:

\[
R(t) = \frac{R_{max}}{1 + (\frac{R_{max}}{R_0} - 1)e^{-rt}}
\]
Remember that we assume fungal hyphae have the same growth ability; the fungus grows horizontally in a circular outward expansion trend; the height of the fungus is maintained as a unit height. From there, we can calculate the area of the fungal colony at time $t$ and the largest area it can grow to:

$$S(t) = \pi R^2(t)$$

$$S_{\text{max}} = \pi R_{\text{max}}^2$$

Also, we assume the colony's ability per unit area to decompose lignin is constantly the same and that the temperature and moisture are constant. Therefore, we can calculate the amount of wood that the colony can decompose at time $t$:

$$C(t) = cS(t)$$

Then the total amount of wood decomposed by the colony at time $t$ is:

$$dC = \int_{t_0}^{t} C(t) \, dt$$

### 3.2. Multi-Colony Model

When only single-species colonies live in the given area, the environment can provide sufficient nutrients and space for them to grow. However, in reality, a situation like this is rare. Therefore, our model will increase the number of fungal colonies. We take three kinds of colonies as an example and establish a multi-colony model based on the model. In our further analysis, we would use three fungi with different extension rates and moisture tolerances to discuss how their competitive interaction affects their growth and decomposition.

Similarly, we use the hyphal extension to represent the growth of fungi. When there are different kinds of fungi living in the same environment, Parvathy Venugopal believes, “The competitive interactions predominated synergistic interactions between wood-decay species at all stages of wood decomposition, thereby significantly affecting wood decay.”[2] Therefore, we only consider the competitive interaction of the three fungi and introduce the retardation of the three fungi on each other in the logistic equation:

$$\begin{align*}
\frac{dR_1}{dt} &= r_1 \left(1 - \frac{R_1}{R_{\text{max}(1)}} - \sigma_2 \frac{R_2}{R_{\text{max}(2)}} - \sigma_3 \frac{R_3}{R_{\text{max}(3)}}\right)R_1 \\
\frac{dR_2}{dt} &= r_2 \left(1 - \frac{R_2}{R_{\text{max}(2)}} - \sigma_1 \frac{R_1}{R_{\text{max}(1)}} - \sigma_3 \frac{R_3}{R_{\text{max}(3)}}\right)R_2 \\
\frac{dR_3}{dt} &= r_3 \left(1 - \frac{R_3}{R_{\text{max}(3)}} - \sigma_1 \frac{R_1}{R_{\text{max}(1)}} - \sigma_2 \frac{R_2}{R_{\text{max}(2)}}\right)R_3
\end{align*}$$

Therefore, the area of the fungal colony at time $t$ and the maximum area that it can grow to can be calculated:

$$\begin{align*}
S_1(t) &= \pi R_1^2(t), S_{\text{max}(1)} = \pi R_{\text{max}(1)}^2 \\
S_2(t) &= \pi R_2^2(t), S_{\text{max}(2)} = \pi R_{\text{max}(2)}^2 \\
S_3(t) &= \pi R_3^2(t), S_{\text{max}(3)} = \pi R_{\text{max}(3)}^2
\end{align*}$$

At time $t$, the amount of wood they can decompose is:

$$\begin{align*}
C_1(t) &= c_1 S_1(t) \\
C_2(t) &= c_2 S_2(t) \\
C_3(t) &= c_3 S_3(t)
\end{align*}$$

Then the total amount of wood decomposed by each colony at time $t$ is:
\[
\begin{align*}
\frac{dC_1}{dt} &= \int_{t_0}^{t} C_1(t)dt \\
\frac{dC_2}{dt} &= \int_{t_0}^{t} C_2(t)dt \\
\frac{dC_3}{dt} &= \int_{t_0}^{t} C_3(t)dt
\end{align*}
\]

Finally, we can calculate the total amount of wood decomposed by three fungi at time \( t \):
\[
dC_1 + dC_2 + dC_3
\]

### 3.3. The Relationship Between Hyphal Extension and Decomposition Rate

Created by Nicky Lustenhouwer et al., the graph of the relationship between different fungal species’ decomposition rate and hyphal extension rate shows a linear correlation [3]; the hyphal extension rate of the fungus is calculated by mean, and the decomposition rate is the percentage of the total wood decomposition at \( t_1 \). It is not difficult for us to get the following formula:
\[
\begin{align*}
&\frac{R(t_1) - R(0)}{t_1 - t_0} - \frac{R(t_1) - R(0)}{t_1 - t_0} = k(dC_1(t_1) - dC_2(t_1)) \\
&\frac{R(t_1) - R(0)}{t_1 - t_0} - \frac{R(t_1) - R(0)}{t_1 - t_0} = k(dC_1(t_1) - dC_3(t_1))
\end{align*}
\]

Where \( k \) is the linear correlation coefficient between decomposition rate and hyphal extension rate.

### 3.4. The Result of Single-Colony Model

We set the value of relative growth rate of the radius of a single colony \( r \) to be 0.06, the maximum colony radius that the environment can accommodate \( R_{max} \) to be 500mm, and the decomposition rate at \( t = 150 \) days to be 25. Then, we use Matlab to make Figure 2 and Figure 3.

![Figure 2. Radius-time curve of a single colony](image-url)
Figure 3. Decomposition rate-time curve of a single colony

In the single-colony model, from the colony radius-time curve shown in Figure 2, we can get the curve of the decomposition rate of the single colony over time, as shown in Figure 3. It shows that in a relatively short period of time, the colony area would be relatively small, so the decomposition efficiency of the colony is relatively low, leading to the slower growth of the colony's decomposition rate. After a long time, since the radius of the colony is very close to the maximum radius it can reach, the area and decomposition efficiency of the colony tend to stabilize, and the growth rate of the decomposition rate also tends to stabilize at its maximum.

3.5. Result and Analysis of The Decomposition of Multiple Colonies

Refer to the equation in 3.3, we set $dC_a(150) = 23$ and $dC_b(150) = 28$, and we generate Figure 4.

Figure 4. The decomposition rate-time curve of each and the entire fungal community when the colonies coexist

As shown in Figure 5, when the three types of colonies coexist, the time-varying rule of the decomposition rate of the entire fungal community is roughly the same as the rate when only one colony
exists. For each type of colony, in a short period of time, the larger the \( r_i \) of the colony, the faster its area increases, so the cumulative decomposition rate \( dC_t \) overtime is greater. After a long period of time, the areas of the three types of colonies gradually stabilize at their maximum values. Therefore, the decomposition rate of the three types of colonies is greatly affected by the decomposition rate every unit area \( c_{ij} \), that is, the decomposition rate and the rate of increase of decomposition rate of the colony with larger \( c_{ij} \) is larger. Therefore, after a long time, the order of the decomposition rate of the three types of fungi 1 and fungi 2 is opposite to the rate in a short time. The decomposition rate of fungi 3 is much higher than the other two. Our team believes that fungi 3 has lower density and shows stronger competitiveness, which conforms the results in Nicky Lustenhouwer’s research.

4. Muti-Colony Model with Environmental Changes

4.1. The Temperature Moisture Change Model

Regarding the weather change, we mainly consider the influence of temperature and moisture on fungal activities, we consult the research results from researchers Dianel S. Maynard [4]. We develop the models through the linear approximation method on their results and analyze the impacts of temperature and moisture based on our model. Below are the specific processes:

Firstly, in the study of Dianel S. Maynard and others, they defined a Moisture Niche Width. Within this moisture range, half of the fungus can maintain their maximum extension rate. Therefore, we assume that the maximum moisture is \( m_{max} \) and the minimum of Moisture Niche Width is \( m_{min} \). In the range of the maximum moisture and the minimum moisture, all fungi can maintain their maximum extension rate \( r_{max(mi)} \). Similarly, we define the Temperature Niche Width and assume that the maximum temperature and minimum temperature of Temperature Niche Width are \( T_{max} \) and \( T_{min} \). In the range of maximum temperature and minimum temperature, all fungi can maintain their maximum extension rate \( r_{max(Ti)} \).

Secondly, we consider the effects of moisture changes. Different fungi have different hyphal extension rates under different water potentials. Dianel S. Maynard et al. measured the hyphal extension rates (obtained by calculating the average values of different temperatures) of different fungus under different moisture conditions in their experiments. From their graphical result (figure 1c), we can easily find that when the water potential is -5 MPa, the hyphal extension rate of all the fungus≈0, and the hyphal extension rate of each fungus raises with the increase of the water potential until the water potential reaches \( m_{min} \). When the water potential exceeds \( m_{max} \), the hyphal extension rate of all fungi quickly decreases to 0. Hence, the relationship between the hyphal extension rate of each fungus and the water potential can be approximately expressed as:

\[
\begin{align*}
    r_i(m) &= k_{mi}(m + 5), m < m_{min} \\
    r_i(m) &= r_{max(mi)}, m_{min} \leq m \leq m_{max} \\
    r_i(m) &= 0, m > m_{max}
\end{align*}
\]

By determining the slope \( k_{mi} \), we can get the following equation when \( m < m_{min} \):

\[
r_i(m) = r_{max(mi)} - k_{mi} \Delta m
\]

Where \( \Delta m \) represents the difference between the environment moisture and \( m_{min} \).

Similarly, we consider the effects of temperature changes. Different fungi have different hyphal extension rates at different temperatures. Dianel S. Maynard et al. measured the hyphal extension rates (obtained by calculating the average values of varying moisture) of different fungus at different temperatures in their experiments. From their graphical result (fig. 1d), it is not difficult to find: When the temperature is 0, the hyphal extension rate of all the fungi≈0, and the hyphal extension rate of each fungus raises with the increase of temperature until the temperature reaches \( T_{max} \). When the temperature exceeds \( T_{max} \), the hyphal extension rate of all the fungi quickly decreases to 0. Therefore, the relationship between the hyphal extension rate of each fungus and the environment temperature can be approximately expressed as:
\[
\begin{aligned}
    r_i(T) = k_{r_i} T, & \quad T < T_{\text{min}} \\
    r_i(T) = r_{\text{max}(T_i)}, & \quad T_{\text{min}} \leq T \leq T_{\text{max}} \\
    0, & \quad T > T_{\text{max}}
\end{aligned}
\]

When \( T < T_{\text{min}} \), the formula could be expressed as:

\[
r_i(T) = r_{\text{max}(T_i)} - k_{r_i} \Delta T
\]

\( \Delta T \) represents the difference between the environment temperature and \( T_{\text{min}} \).

Finally, we combine the effects of temperature and moisture on the hyphal extension rate and create a formula to determine the extension rates of different fungi at different temperatures and moisture conditions:

\[
r_i(m, T) = r_{\text{max}(i)} - k_{m(i)} \Delta m - k_{r_i} \Delta T
\]

4.2. The Result of The Temperature&Moisture Change Model

According to our temperature & moisture change model, when the moisture and temperature are higher than the upper limit of the Niche Width, the hyphae of the colony cannot grow. Therefore, we need to know the growth of the three colonies when the moisture and temperature are lower than the lower limit of the Niche Width. Figure 5, 6, 7 show the results, respectively.

**Figure 5.** The relationship between radius of colony 1 and time as well as moisture

**Figure 6.** The relationship between radius of colony 2 and time as well as moisture
Figure 7. The relationship between radius of colony 3 and time as well as moisture

Obviously, due to the decrease in moisture, the hyphal extension rates of these three types of colonies all slow down in a short period of time. After a long time, their radii gradually stabilize, and there is a trend that the radius of the slowest growing colony 2 increases with the decrease in moisture. The radius of colony 2 dramatically reduces in size, especially when the moisture drops to a certain level. On the contrary, the radii of colonies 1 and 3 increase slightly. Besides, the fastest-growing colony 3 has more evident advantages in competition, while the radius of colony 1 gradually exceeds colony 2 as the moisture decreases.

Since the relative growth rate $r_i$ of the hyphae has a positive linear correlation with temperature as well as moisture, it can be inferred that the influence of temperature reduction on the growth of the three colonies is consistent with that of moisture reduction on the growth of the three colonies.

We take arid, semi-arid, temperate, arboreal and tropical rain forests, five different environments as examples, and predict the possible combinations of fungi that are able to survive. In order to reflect the difference in the environment, we set different temperature or moisture for these 5 environments based on reality. Then we obtain the number of the three colonies’ radii as a percentage of the total number at $t = 3000$ days. The results are shown in Table 1.

Table 1. The combination of species in five different environments

|            | Arid       | Semi-arid  | Temperate  | Arboreal   | Rain forests  |
|------------|------------|------------|------------|------------|---------------|
| Fungi 1    | 20.35%     | 18.43%     | 18.43%     | 18.25%     | 18.19%        |
| Fungi 2    | 18.30%     | 26.53%     | 26.53%     | 27.09%     | 27.26%        |
| Fungi 3    | 61.34%     | 55.04%     | 55.04%     | 54.66%     | 54.55%        |

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