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Stability analysis of fractional order model on corona transmission dynamics

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A B S T R A C T

In this paper a fractional order mathematical model is constructed to study the dynamics of corona virus in Oman. The model consists of a system of eight non-linear fractional order differential equations in Caputo sense. Existence and uniqueness as well as the stability analysis of the solution of the model are given. The stability analysis is in the frame of Ulam-Hyers and generalized Ulam-Hyers criteria. Numerical simulations are given to support the theoretical results. Many informations on the dynamics of COVID - 19 in Oman were obtained using this model. Also many informations on the qualitative behaviour of the model were obtained.

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1. Introduction

Some epidemic diseases are capable of producing large number of infections starting from a fewer ones, an example of such diseases is COVID-19 [1]. Coronavirus disease is a respiratory and zoonotic disease, caused by a virus of the coronaviridae family which originated in the city of Wuhan China on December 01, 2019 [2]. The Virus strain is severe acute respiratory syndrome coronavirus 2 (SARS-CoV-2), resulting in fever, coughing, breathing difficulties, fatigue, and myalgia. It may transform into pneumonia of high intensity.

Many scientists put their heads together in trying to find answers about the spread and infection of corona virus by examining virus samples [3]. Although the disease strain is known, but, the vaccine is still not available. Hence it is necessary to ensure strict mitigation actions in order to contain the virus. Many countries are taking different measures to cope with the virus.

Taking the China and South Korea cases into consideration, a lot of countries were able to contain the spread of the virus. It is proven that social distancing and testing are some of the key control measures. Another measure responsible for spreading the disease worldwide was regional and global travellers. Although, air travel is almost suspended now, but, this initial shock and countries (and people) not taking it too seriously has taken many countries in Europe and North America to a real bad situation.

Oman has taken various measures to avoid the spread of COVID-19. From mid of March, Schools and Colleges/Universities were closed, consequently other non essential offices and services were closed. The situation is been monitored on daily basis by the government. However, there are many suspected cases which are not been tested so far. By the time of this report, most part of the country is partially locked down. Although, the spread of the disease is not that much as of today (June 17, 2020), but, the new cases are appearing continuously.

We need modelling approach to understand the exact dynamics of the disease [23–25]. This is what motivates this research. It is important to note that, in the classical order model, the state of epidemic does not depend on its history. However, in real life memory plays a vital role in studying the pattern of spread of any epidemic disease. It was found that the waiting times between doctor visits for a patient follow a power law model [4]. It is worth to know that Caputo fractional time derivative is a consequence of power law [5]. When dealing with real world problem Caputo fractional-order derivatives allows traditional initial and boundary conditions [26,27]. Furthermore, due to its non-local behaviour and its ability to change at every instant of time, Caputo fractional-order gives better result than the integer order [28–31].

In recent studies, Khan et al. studied a fractional-order model that describes the interaction among bats and unknown hosts, then among people and seafood market [6]. To predict the trend of the Corona-Virus Yu et al. constructed a fractional time delay dynamic system that studied the local outbreak of COVID-19 [7]. Also, to predict the possible outbreak of infectious diseases like COVID-19 and other diseases in the future Xu et al. proposed a

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generalized fractional-order SEIQRD model [8]. Shaikh et al. used Bats-Hosts-Reservoir - People transmission fractional-order COVID-19 model to estimate the effectiveness of preventive measures and various mitigations, predicting future outbreaks and potential control strategies [9].

The aim of this research is to study fractional - order epidemic model that investigates the dynamics of COVID-19 in Oman. Based on the memorability nature of Caputo fractional-order derivatives, this model can be fitted with data reasonably well. Then, based on the official data given by the Federal Ministry of Health Oman daily, numerical examples will be carried out.

This paper is organized as follows. In Section 2, Preliminary definitions are given. In Section 3, the fractional order model for COVID-19 in Caputo sense is formulated. In Section 4, existence and uniqueness of the solution of the model is established. In chapter 5, stability analysis of the solution of the model in the frame of Ulam-Hyers and generalized Ulam-Hyers is given. Chapter 6, contains the numerical scheme and numerical simulations to illustrate the theoretical results. Finally, conclusion is given in chapter 7.

2. Preliminaries

Definition 2.1 ([10]). The fractional integral of order \( \nu \) with the lower limit \( 0^+ \) for a function \( g \) is defined as

\[
\mathcal{J}_0^\nu g(s) = \frac{1}{\Gamma(\nu)} \int_0^s (s - \tau)^{\nu - 1} g(\tau)d\tau, \quad \nu > 0,
\]

provided the right side is point-wise defined on \([0, \infty)\), where \( \Gamma(\cdot) \) denotes the gamma function.

Definition 2.2 ([10]). The Caputo fractional derivative of order \( \nu \) with the lower limit \( 0^+ \) for a function \( g \) is defined as

\[
\mathcal{C}D_0^\nu g(s) = \frac{1}{\Gamma(n - \nu)} \int_0^s (s - \tau)^{n-\nu-1} g^{(n)}(\tau)d\tau, \quad n-1 < \nu < n, \quad n \in \mathbb{N},
\]

provided the function \( g \) differentiable on \([0, +\infty)\), where \( \Gamma(\cdot) \) denotes the gamma function.

Theorem 2.1 ([10]). Let \( \Re(\nu) > 0 \), \( n = [\Re(\nu)] + 1 \). Then

\[
(\mathcal{J}_0^\nu \mathcal{C}D_0^\nu g)(s) = g(s) - \sum_{k=1}^{m} \left( \mathcal{D}_0^\nu g(0^+) \right) \frac{s^k}{k!}.
\]

3. Formulation of the model

Let the total population be \( N(t) \). The population is divided into eight compartments, namely; Susceptible population \( S(t) \), Exposed population \( E(t) \), Asymptomatic Infective population \( I_a(t) \), Symptomatic Infective Population \( I_s(t) \), Isolated Infective Population \( I_i(t) \), Hospitalised Infective Population \( I_h(t) \), Recovered population \( R(t) \) and then Dead Individuals \( D(t) \). The dynamics of this population is represented by the following system of fractional order differential equations (FODE), and the meaning of parameters is given in Table 1.

\[
\begin{align*}
\mathcal{C}D_0^\nu S(t) &= -\beta S(\alpha I_a + \xi I_s + I_i), \\
\mathcal{C}D_0^\nu E(t) &= \beta S(\alpha I_a + \xi I_s + I_i) - kE, \\
\mathcal{C}D_0^\nu I_a(t) &= (1 - p)KE - \gamma A I_a, \\
\mathcal{C}D_0^\nu I_s(t) &= pkE - qE, \\
\mathcal{C}D_0^\nu I_i(t) &= q(1 - \Psi)I_s - (\gamma_A + \mu_I)I_i, \\
\mathcal{C}D_0^\nu I_h(t) &= q\Psi I_s - (\gamma_H + \mu_H)I_h, \\
\mathcal{C}D_0^\nu R(t) &= \gamma_A + \gamma_H, \\
\mathcal{C}D_0^\nu D(t) &= \mu I_s I_s + \mu H I_h.
\end{align*}
\]

4. Existence and uniqueness results

The theory of existence and uniqueness of solutions is one of the most dominant fields in the theory of fractional-order differential equations. The theory has recently attracted the attention of many researchers, we are referring to [11,12] and the references therein for some of the recent growth. In this section, we discuss the existence and uniqueness of solution of the proposed model using fixed point theorems. Let us reformulate the proposed model (2) in the subsequent form

\[
\begin{align*}
\mathcal{C}D_0^\nu S(t) &= -\beta S(\alpha I_a + \xi I_s + I_i), \\
\mathcal{C}D_0^\nu E(t) &= \beta S(\alpha I_a + \xi I_s + I_i) - kE, \\
\mathcal{C}D_0^\nu I_a(t) &= (1 - p)KE - \gamma A I_a, \\
\mathcal{C}D_0^\nu I_s(t) &= pkE - qE, \\
\mathcal{C}D_0^\nu I_i(t) &= q(1 - \Psi)I_s - (\gamma_A + \mu_I)I_i, \\
\mathcal{C}D_0^\nu I_h(t) &= q\Psi I_s - (\gamma_H + \mu_H)I_h, \\
\mathcal{C}D_0^\nu R(t) &= \gamma_A + \gamma_H, \\
\mathcal{C}D_0^\nu D(t) &= \mu I_s I_s + \mu H I_h.
\end{align*}
\]

where

\[
\begin{align*}
\Theta_1(t, S, E, I_a, I_s, I_i, I_h, R, D) &= -\beta S(\alpha I_a + \xi I_s + I_i), \\
\Theta_2(t, S, E, I_a, I_s, I_i, I_h, R, D) &= \beta S(\alpha I_a + \xi I_s + I_i) - kE, \\
\Theta_3(t, S, E, I_a, I_s, I_i, I_h, R, D) &= (1 - p)KE - \gamma A I_a, \\
\Theta_4(t, S, E, I_a, I_s, I_i, I_h, R, D) &= pkE - qE, \\
\Theta_5(t, S, E, I_a, I_s, I_i, I_h, R, D) &= q(1 - \Psi)I_s - (\gamma_A + \mu_I)I_i, \\
\Theta_6(t, S, E, I_a, I_s, I_i, I_h, R, D) &= q\Psi I_s - (\gamma_H + \mu_H)I_h, \\
\Theta_7(t, S, E, I_a, I_s, I_i, I_h, R, D) &= \gamma_A + \gamma_H, \\
\Theta_8(t, S, E, I_a, I_s, I_i, I_h, R, D) &= \mu I_s I_s + \mu H I_h.
\end{align*}
\]

Thus, the proposed model (2) takes the form

\[
\begin{align*}
\mathcal{C}D_0^\nu \Phi(t) &= \mathcal{K}(t, \Phi(t)), \quad t \in J = [0, b], \quad 0 < \alpha \leq 1, \\
\Phi(0) &= \Phi_0 \geq 0.
\end{align*}
\]
on condition that
\[
\begin{align*}
\Phi(t) &= (S, E, I_a, I_h, I_{h_2}, R, D)^	op, \\
\Phi(0) &= (S_0, S_0, I_{h_0}, I_{h_0}, I_{h_0}, R_0, D_0)^	op, \\
K(t, \Phi(t)) &= (\Theta(t, S, E, I_a, I_h, I_{h_2}, R, D))^	op, \quad i = 1, \ldots, 8,
\end{align*}
\]
which implies that \(\|P\Phi_1) - (P\Phi_2)\| \leq \Omega L_k \|\Phi_1 - \Phi_2\|.\) Therefore, as a consequence of Banach contraction principle, proposed model (2) possess a unique solution. □

Next, we prove the existence of solutions of the proposed model (2) by employing the concept of well-known Krasnoselskii's fixed point theorem.

**Theorem 4.2** ([13]), Let \(M \neq \emptyset\) be a closed, bounded and convex subset of a Banach Space \(E\). Let \(P_1, P_2\) be two operators that obey the given relations

- \(P_1, P_2 \in \mathcal{M}\), whenever \(\Phi_1, \Phi_2 \in M\);
- \(P_1\) is compact and continuous;
- \(P_2\) is a contraction mapping.

Then there exists \(u \in M\) such that \(u = P_1u + P_2u\).

**Theorem 4.3.** Suppose that the function \(K : J \times \mathbb{R}^8 \rightarrow \mathbb{R}\) is continuous and satisfies condition (A1). In addition, assume that

\(A_2) = |K(t, \Phi)| \leq \psi(t), \quad \forall t \in J, \quad \forall \Phi \in \mathcal{M}\).

Then the proposed model (2) has at least one solution provided \(\Omega L_k < 1\),

\[
\Omega = \frac{b^\nu}{\Gamma(\nu + 1)}.
\]

**Proof.** Consider the operator \(P : \mathcal{M} \rightarrow \mathcal{M}\) defined by

\[
(P\Phi)(t) = \Phi_0 + \frac{1}{\Gamma(\nu)} \int_0^t (t - \tau)^{\nu - 1}K(\tau, \Phi(\tau)) d\tau.
\]

Obviously, the operator \(P\) is well defined and the unique solution of model (2) is just the fixed point of \(P\). Indeed, let us take \(\sup_{t \in J} \|K(t, 0)\| = M_1\) and \(\kappa \geq \|\Phi_0\| + \Omega M_1\). Then, it is enough to show that \(P\mathbb{B}_\kappa \subset \mathbb{B}_\kappa\), where the set \(\mathbb{B}_\kappa = \{\Phi \in \mathcal{M} : \|\Phi\| \leq \kappa\}\) is closed and convex. Now, for any \(\Phi \in \mathbb{B}_\kappa\), yields

\[
\|P\Phi(t)\| \leq \|\Phi_0\| + \frac{1}{\Gamma(\nu)} \int_0^t (t - \tau)^{\nu - 1} \|K(\tau, \Phi(\tau))\| d\tau
\]

\[
\leq \Phi_0 + \frac{1}{\Gamma(\nu)} \int_0^t (t - \tau)^{\nu - 1} \|K(\tau, \Phi(\tau)) - K(\tau, 0)\| + \|K(\tau, 0)\| d\tau
\]

\[
\leq \Phi_0 + \frac{(L_k + M_1)}{\Gamma(\nu)} \int_0^t (t - \tau)^{\nu - 1} d\tau
\]

\[
\leq \Phi_0 + \frac{(L_k + M_1)}{\Gamma(\nu + 1)} b^\nu
\]

\[
\leq \Phi_0 + \Omega (L_k + M_1)
\]

\[
\leq \kappa.
\]

Hence, the results follows. Also, given any \(\Phi_1, \Phi_2 \in \mathbb{B}_\kappa\), we get

\[
\|(P\Phi_1)(t) - (P\Phi_2)(t)\| \leq \frac{1}{\Gamma(\nu)} \int_0^t (t - \tau)^{\nu - 1} |K(\tau, \Phi_1(\tau)) - K(\tau, \Phi_2(\tau))| d\tau
\]

\[
\leq \frac{L_k}{\Gamma(\nu)} \int_0^t (t - \tau)^{\nu - 1} |\Phi_1(\tau) - \Phi_2(\tau)| d\tau
\]

\[
\leq \Omega L_k \|\Phi_1(t) - \Phi_2(t)\|.
\]
5. Stability results

In this section, we drive the stability of the proposed model (2) in the frame of Ulam-Hyers and generalized Ulam-Hyers stability. The concept of Ulam stability was introduced by Ulam [14,15]. Then, in several research papers on classical fractional derivatives, the aforementioned stability was investigated, see for example, [16–19]. Moreover, since stability is fundamental for approximate solution, we strive to use nonlinear functional analysis on Ulam-Hyers and generalized stability of the proposed model (2). Thus the following definitions are needed. Let $\epsilon > 0$ and consider the inequality given below

$$
|\mathcal{D}^\nu_0 \Phi(t) - K(t, \Phi(t))| \leq \epsilon, \quad t \in J.
$$

(14)

where $\epsilon = \max(e_j^T)$, $j = 1, \ldots, 8$.

**Definition 5.1.** The proposed problem (2) is Ulam-Hyers stable if there exist $C_\nu > 0$, such that for every $\epsilon > 0$ and a solution $\Phi \in \mathcal{E}$ satisfying (5.1), there exists a unique solution $\Phi \in \mathcal{E}$ of equation (2), with

$$
|\Phi(t) - \Phi(t)| \leq C_\nu \epsilon, \quad t \in J.
$$

where $C_\nu = \max(C_{\nu_j})^T$.

**Definition 5.2.** Problem(2) is referred to generalized Ulam-Hyers stable if there exist a continuous function $\psi_\nu : \mathbb{R}_+ \to \mathbb{R}_+$, with $\psi_\nu(0) = 0$, such that for every solution $\Phi \in \mathcal{E}$ of the equation (14), there a solution $\Phi \in \mathcal{E}$ of equation (2), such that

$$
|\Phi(t) - \Phi(t)| \leq \psi_\nu(\epsilon), \quad t \in J.
$$

where $\psi_\nu = \max(\psi_{\nu_j})^T$.

**Remark 5.1.** A function $\Phi \in \mathcal{E}$ satisfy the inequality (14), if and only if there exist a function $h \in \mathcal{E}$ with the property below:

(i) $|h(t)| \leq \epsilon, \quad h = \max(h_j)^T, \quad t \in J.$

(ii) $\mathcal{D}^\nu_0 \Phi(t) = K(t, \Phi(t)) + h(t), \quad t \in J.$

**Theorem 5.1.** Assume that $\Phi \in \mathcal{E}$ satisfies inequality (14), then $\Phi$ satisfies the integral inequality describe by

$$
|\Phi(t) - \Phi_0 - \frac{1}{\Gamma(\nu)}\int_0^t (t - \tau)^{\nu-1}K(\tau, \Phi(\tau))d\tau| \leq \Omega \epsilon.
$$

(15)

**Proof.** Thanks to (i) of Remark 5.1,

$$
\mathcal{D}^\nu_0 \Phi(t) = K(t, \Phi(t)) + h(t)
$$

and Theorem [10], gives

$$
\Phi(t) = \Phi_0 + \frac{1}{\Gamma(\nu)}\int_0^t (t - \tau)^{\nu-1}K(\tau, \Phi(\tau))d\tau + \frac{1}{\Gamma(\nu)}\int_0^t (t - \tau)^{\nu-1}h(\tau)d\tau.
$$

(16)

Using (i) of Remark 5.1 and (A2), we get

$$
|\Phi(t) - \Phi_0 - \frac{1}{\Gamma(\nu)}\int_0^t (t - \tau)^{\nu-1}K(\tau, \Phi(\tau))d\tau| \leq \frac{1}{\Gamma(\nu)}\int_0^t (t - \tau)^{\nu-1}|h(\tau)|d\tau
$$

$$
\leq \Omega \epsilon.
$$

(17)

Hence, the desired results. \(\square\)

**Theorem 5.2.** Suppose that $K : J \times \mathbb{R}^8 \to \mathbb{R}$ is continuous for every $\Phi \in \mathcal{E}$ and hypotheses (A1) hold with $1 - \Omega \mathcal{L}_\nu > 0$. Thus, problem (2) is Ulam-Hyers and consequently, generalized Ulam-Hyers stable.

**Proof.** Suppose that $\Phi \in \mathcal{E}$ satisfies the inequality (14) and $\Phi \in \mathcal{E}$ be a unique solution of (2). Thus, for any $\epsilon > 0$, $t \in J$ and Lemma 5.1, gives

$$
|\Phi(t) - \Phi(t)| = \max_{t \in J} |\Phi(t) - \Phi_0 - \frac{1}{\Gamma(\nu)}\int_0^t (t - \tau)^{\nu-1}K(\tau, \Phi(\tau))d\tau|
$$

$$
\leq \max_{t \in J} |\Phi(t) - \Phi_0 - \frac{1}{\Gamma(\nu)}\int_0^t (t - \tau)^{\nu-1}K(\tau, \Phi(\tau))d\tau|
$$

$$
+ \max_{t \in J} |\int_0^t (t - \tau)^{\nu-1}|K(\tau, \Phi(\tau)) - K(\tau, \Phi(\tau))|d\tau|
$$

$$
\leq \frac{1}{\Gamma(\nu)}\int_0^t (t - \tau)^{\nu-1}|\Phi(\tau) - \Phi(\tau)|d\tau
$$

$$
\leq \Omega \epsilon.
$$

So,

$$
|\Phi - \Phi| \leq C_\nu \epsilon,
$$

where

$$
C_\nu = \frac{1}{1 - \Omega \mathcal{L}_\nu}.
$$

So, setting $\psi_\nu(\epsilon) = C_\nu \epsilon$ such that $\psi_\nu(0) = 0$. We conclude that the proposed problem (2) is both Ulam-Hyers and generalized Ulam-Hyers stable. \(\square\)

6. Numerical Simulations and Discussion

Here we use COVID-19 data obtained from Federal Ministry of Health Oman on 18th June 2020 for the Numerical simulations. The parameter values are given in Table 2. We can observe that the number of infection get to zero with time. This is true as per as any epidemic disease is concern. Herein, the fractional variant of the model under consideration via Caputo fractional operator is numerically simulated via first order convergent numerical techniques as proposed in [20–22]. These numerical techniques are accurate, conditionally stable, and convergent for solving fractional-order both linear and nonlinear system of ordinary differential equations.

Now we discuss the obtained numerical outcomes of the governing model in respect of the approximate solutions. To this aim, we employed the effective Euler method under the Caputo fractional operator to do the job. The initial conditions are taken as $S(0) = 4, 602, 296, E(0) = 26818, I_s(0) = 300, I_l(0) = 169, I_f(0) = 113, r(0) = 56, R(0) = 12364, D(0) = 119$ and the parameters values are as in Table 2 below:

Considering the values in the above table, we carry out the following numerical simulations with the fractional order value $\nu = 0.67$.

In Fig. 1, dynamics of all the populations involved in the model are presented. It can be seen that with time, all the populations will tend to zero except the recovered and Dead populations. This

| Parameter values. |
|-------------------|
| \(\beta\) calibration |
| \(\alpha\) 0.5 |
| \(\xi\) calibration |
| \(k\) 0.2174 |
| \(\rho\) 0.5 |
| \(q\) 0.3448 |
| \(\psi\) data |
| \(\gamma_s, \gamma_f, \gamma_h\) 0.1961 |
| \(\alpha_s, \alpha_h\) data |
requires more effort to be put in place in order to reduce the number of death and increase the recovery of the infected individuals. In Fig. 2, Asymptomatic and Symptomatic cases are shown. It can be seen that there are more asymptomatic cases as compared to the symptomatic cases. This means, there are many positive individuals without any sign of the disease. Infection through this lane could only be stopped through contact trace. In Fig. 3, Recovered case were depicted against Death cases. It can clearly be seen that there are more recovery cases than death cases. This is a good news and various means should be put in place to maintain the trend. Fig. 4 presents the total Infected cases against total Death cases. It can be seen that indeed many infected individuals recovered. The death cases is very low, which shows that the disease is not fatal in Oman. Fig. 5 shows the relationship between isolated infected cases and hospitalised infected cases. It is clear that as time goes on both populations will tend to zero. Finally Fig. 6 shows the dynamics of various infection cases. It can be seen that isolated and hospitalised populations are very small compared to the asymptomatic and symptomatic cases. Hence there is need for more effort in tracing out the infected individuals.

Many informations on the dynamics of COVID -19 in Oman were obtained using this model. Also many informations on the qualitative behaviour of the model were obtained.

7. Conclusions

In conclusion, this paper consists of a system of eight nonlinear fractional order differential equations in Caputo sense. The existence and uniqueness of solution of the proposed model using fixed point theorems is discussed. Stability analysis in the frame of Ulam-Hyers and generalized Ulam-Hyers criteria is established. Numerical simulations were carried out using real data from Federal ministry of health Oman. It was numerically shown that although the disease is not fatal in Oman, but there will be many.
death cases. Hence there is need for relevant authorities to take every available measure to curtail the spread of the disease.

Declaration of Competing Interest

None.

CRediT authorship contribution statement

Evren Hincal: Visualization, Investigation, Supervision, Software, Validation, Writing - review & editing. Sultan Hamed Alsaadi: Conceptualization, Methodology, Software, Data curation, Writing - original draft.

References

[1] Nishiura H, Oshitani H, Kobayashi T, Saito T, Sunagawa T, Matsu T, Wakita T, COVID M, Suzuki M. Closed environments facilitate secondary transmission of coronavirus disease 2019 (COVID-19). medRxiv 2020.
[2] Lippi G, Plebani M. Laboratory abnormalities in patients with COVID-2019 infection. Clinical Chemistry and Laboratory Medicine (CCLM) 2020; 0(0):20200198.
[3] Callaway E. China coronavirus: labs worldwide scramble to analyse live samples. Nature 2020.
[4] Smithurst DP, Williams HC. Are hospital waiting lists self-regulating? Nature 2001; 410(6829):652–3.
[5] Meerschaert MM, Scikorski A. Stochastic models for fractional calculus, vol 43. Walter de Gruyter; 2011.
[6] Riley S, Fraser C, Donnelly CA, Ghani AC, Abu-Raddad LJ, Hedley AJ, Leung GM, Ho L-M, Lam T-M, Thach TQ, et al. Transmission dynamics of the etiological agent of SARS in Hong Kong: impact of public health interventions. Science 2003; 300(5627):1061–6.
[7] Chen Y, Cheng J, Jiang X, Xu X. The reconstruction and prediction algorithm of the fractional TDD for the local outbreak of COVID-19. 2020. ArXiv preprint arXiv:2002.10302.
[8] Xu C, Yu Y, Yang Q, Lu Z. Forecast analysis of the epidemics trend of COVID-19 in the United States by a generalized fractional-order SEIR model. 2020. ArXiv preprint arXiv:2004.12541.
[9] Ali Z, Kumam P, Shah K, Zada A. Investigation of Ulam stability results of a coupled system of nonlinear implicit fractional differential equations. Mathematics 2019; 7(4):341.
[10] Kilbas A, Srivastava H, Trujillo J. Theory and applications of fractional derivatival equations. North-Holland Mathematics Studies, 204; 2006.
[11] Abdo MS, Shah K, Wahash HA, Panchal SK. On a comprehensive model of the novel coronavirus (COVID-19) under Mittag-Leffler derivative. Chaos Solitons Fractals 2020; 109867.
[12] Shah K, Abdeljawad T, Mahariq I, Jarad F. Qualitative analysis of a mathematical model in the time of COVID-19. BioMed Res Int 2020;2020.
[13] Yong Z, Jinrong W, Lu Z. Basic theory of fractional differential equations. World Scientific; 2016.
[14] Ulam SM. A collection of mathematical problems, vol 8. Interscience Publishers; 1960.
[15] Ulam SM. Problems in modern mathematics. Courier Corporation; 2004.
[16] Ahmed I, Kumam P, Jarad F, Borisut P, Sithithawornkriji K, Ibrahim A. Stability analysis for boundary value problems with generalized nonlocal condition via Hilfer-Katugampola fractional derivative. Adv Differ Equ 2020;2020(1):1–18.
[17] Ahmed I, Kumam P, Shah K, Borisut P, Sithithawornkriji K, Demba MA. Stability results for implicit fractional pantograph differential equations via φ-Hilfer fractional derivative with a nonlocal Riemann-Liouville fractional integral condition. Mathematics 2020;8(1):94.
[18] Ali Z, Kumam P, Shah K, Zada A. Investigation of Ulam stability results of a coupled system of nonlinear implicit fractional differential equations. Mathematics 2019;7(4):341.
[19] Agarhina A, Ntouyas SK, Tariboon J. Existence and Ulam-Hyers stability for Caputo conformable differential equations with four-point integral conditions. Adv Differ Equ 2019;2019(1):139.
[20] Baleanu D, Jajarmi A, Hajipour M. On the nonlinear dynamical systems within the generalized fractional derivatives with Mittag-Leffler kernel. Nonlinear Dyn 2018;94:397–414.
[21] Jajarmi A, Baleanu D. A new fractional analysis on the interaction of HIV with CD4+ t-cells. Chaos Solitons Fractals 2018;113:221–9.
[22] Li C, Zeng F. Numerical methods for fractional calculus. CRC Press; 2015. 24
[23] Kumar D, Singh J, Al Qurashi M, et al. A new fractional SIRS-SI malaria disease model with application of vaccines, antimalarial drugs, and spraying. Adv Differ Equ 2019;2019:278; doi:10.1186/s13662-019-2199-9.
[24] Kumar S, Ahmadian A, Kumar R, Kumar D, Singh J, Baleanu D, Salimi M. An efficient numerical method for fractional SIR epidemic model of infectious disease by using Bernstein wavelets. Mathematics 2020;8:558.
[25] Kumar S, Kumar R, Singh J, Nisar KS, Kumar D. An efficient numerical scheme for fractional model of HIV-1 infection of CD4+ t-cells with the effect of antiviral drug therapy. Alex Eng J 2020;59(4):2053–64.
[26] Singh J, Kumar D, Baleanu D. A new analysis of fractional fish farm model associated with Mittag-Leffler-type kernel. Int J Biomath 2020;13(02):2050010.
[27] Kumar D, Singh J, Baleanu D. On the analysis of vibration equation involving a fractional derivative with Mittag-Leffler law. Math Methods Appl Sci 2020;43(1).
[28] Ahmed J, Baba IA, Yusuf A, et al. Analysis of Caputo fractional-order model for COVID-19 with lockdown. Adv Differ Equ 2020;2020:394. doi:10.1186/s13662-020-02853-0.
[29] Baba IA, Olanifilekan IL, Yusuf A, Baleanu D. Analysis of meningitis model: a case study of northern nigeria. AtMS Bioeng 2020;7(4):179–93. doi:10.3934/bioeng.2020036.
[30] Qureshi S, Yusuf A, Shaikh AA, Inc M, Baleanu D. Mathematical modeling for adsorption process of dye removal nonlinear equation using power law and exponentially decaying kernels. Chaos 2020;30:043106. doi:10.1063/1.5121845.
[31] Qureshi S, Yusuf A. Modeling chickenpox disease with fractional derivatives: from Caputo to Atangana-Baleanu. Chaos Solitons Fractals 2019;122:111–18.