THE WEAK ISOSPIN AND THE GRAVITY

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Abstract: The Clifford pentad of 4X4 complex matrices defines the currents of the particles. The weak isospin transformation scatters the particle on two components into the 2-dimensional space of the antidiagonal Clifford matrices. The physics particles move in the 3-dimensional space of the diagonal Clifford matrices. Such sectioning of the 5-dimensional space on two subspaces (2-dimensional and 3-dimensional) defines the Newtonian gravity principle.

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1. Introduction

In the Quantum Theory the fermion behavior is depicted by the spinor $\Psi$. The probability current vector $\vec{j}$ components of this fermion are the following:

$$j_x = \Psi^\dagger \cdot \beta^1 \cdot \Psi, j_y = \Psi^\dagger \cdot \beta^2 \cdot \Psi, j_z = \Psi^\dagger \cdot \beta^3 \cdot \Psi.$$  \hspace{1cm} (1.1)

Here

$$\beta^1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}, \beta^2 = \begin{bmatrix} 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \\ 0 & 0 & 0 & i \\ 0 & 0 & -i & 0 \end{bmatrix}, \beta^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

are the members of the Clifford pentad, for which other members are the following:

$$\gamma^0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \beta^4 = \begin{bmatrix} 0 & 0 & i & 0 \\ 0 & 0 & 0 & i \\ -i & 0 & 0 & 0 \\ 0 & -i & 0 & 0 \end{bmatrix}$$

Let this spinor be expressed in the following form:

$$\Psi = |\Psi| \cdot \begin{bmatrix} \exp(i \cdot g) \cdot \cos(b) \cdot \cos(a) \\ \exp(i \cdot d) \cdot \sin(b) \cdot \cos(a) \\ \exp(i \cdot f) \cdot \cos(v) \cdot \sin(a) \\ \exp(i \cdot q) \cdot \sin(v) \cdot \sin(a) \end{bmatrix}.$$  \hspace{1cm} (1.2)

In this case the probability current vector $\vec{j}$ has got the following components:
\[ j_x = |\Psi|^2 \cdot \left[ \cos^2(a) \cdot \sin(2 \cdot b) \cdot \cos(d - g) - \sin^2(a) \cdot \sin(2 \cdot v) \cdot \cos(q - f) \right], \]
\[ j_y = |\Psi|^2 \cdot \left[ \cos^2(a) \cdot \sin(2 \cdot b) \cdot \sin(d - g) - \sin^2(a) \cdot \sin(2 \cdot v) \cdot \sin(q - f) \right], \]
\[ j_z = |\Psi|^2 \cdot \left[ \cos^2(a) \cdot \cos(2 \cdot b) - \sin^2(a) \cdot \cos(2 \cdot v) \right]. \]  

(1.2) 

LAB: w1

If
\[ \rho = \Psi^\dagger \cdot \Psi, \]

then \( \rho \) is the probability density, i.e. \( \int \int \int_V \rho(t) \cdot dV \) is the probability to find the particle with the state function \( \Psi \) in the domain \( V \) of the 3-dimensional space at the time moment \( t \). In this case, \( \{\rho, \vec{j}\} \) is the probability density 3 + 1-vector.

If
\[ \vec{u} = \rho \cdot \vec{u}, \]  

(1.3) 

LAB: w2

then \( \vec{u} \) is the average velocity for this particle.

Let us denote:
\[ J_0 = \Psi^\dagger \cdot \gamma^0 \cdot \Psi, J_4 = \Psi^\dagger \cdot \beta^4 \cdot \Psi, J_0 = \rho \cdot V_0, J_4 = \rho \cdot V_4. \]  

(1.4) 

LAB: w3

In this case:

\[ V_0 = \sin(2 \cdot a) \cdot [\cos(b) \cdot \cos(v) \cdot \cos(g - f) + \sin(b) \cdot \sin(v) \cdot \cos(d - q)], \]
\[ V_4 = \sin(2 \cdot a) \cdot [\cos(b) \cdot \cos(v) \cdot \sin(g - f) + \sin(b) \cdot \sin(v) \cdot \sin(d - q)]; \]  

(1.5) 

LAB: w4

and for every particle:
\[ u_x + u_y + u_z + V_0^2 + V_4^2 = 1. \]  

(1.6) 

LAB: VV

For the left particle (for example, the left neutrino): \( a = \frac{\pi}{2}, \)

\[ \Psi_L = |\Psi_L|^2 \cdot \begin{bmatrix} 0 \\ 0 \\ \exp(i \cdot f) \cdot \cos(v) \\ \exp(i \cdot q) \cdot \sin(v) \end{bmatrix} \]

and from (1.2), and (1.3): \( u_x + u_y + u_z = 1 \). Hence, the left particle velocity equals 1; hence, the mass of the left particle equals to zero.

Let \( U \) be the weak global isospin (SU(2)) transformation with the eigenvalues \( \exp(i \cdot \lambda) \).
In this case for this transformation eigenvector \( \psi \):

\[
U\psi = |\psi| \cdot \begin{bmatrix}
\exp(i \cdot g) \cdot \cos(b) \cdot \cos(a) \\
\exp(i \cdot d) \cdot \sin(b) \cdot \cos(a) \\
\exp(i \cdot \lambda) \cdot \exp(i \cdot f) \cdot \cos(v) \cdot \sin(a) \\
\exp(i \cdot \lambda) \cdot \exp(i \cdot q) \cdot \sin(v) \cdot \sin(a)
\end{bmatrix}
\]

and for \( 1 \leq \mu \leq 3 \) from (1.2):

\[
(U\psi)^\dagger \cdot \beta^\mu \cdot (U\psi) = \psi^\dagger \cdot \beta^\mu \cdot \psi,
\]

(1.7) LAB: w5

but for \( \mu = 0 \) and \( \mu = 4 \) from (1.3):

\[
\psi^\dagger \cdot \gamma^0 \cdot \psi = |\psi|^2 \cdot \sin(2 \cdot a) \cdot \\
[\cos(b) \cdot \cos(v) \cdot \cos(g - f - \lambda) + \sin(b) \cdot \sin(v) \cdot \cos(d - q - \lambda)],
\]

\[
\psi^\dagger \cdot \beta^4 \cdot \psi = |\psi|^2 \cdot \sin(2 \cdot a) \cdot \\
[\cos(b) \cdot \cos(v) \cdot \sin(g - f - \lambda) + \sin(b) \cdot \sin(v) \cdot \sin(d - q - \lambda)];
\]

(1.8) LAB: w6

2. THE WEAK ISOSPIN SPACE

In the weak isospin theory we have got the following entities (Global Symmetries, Standard Model):
- the right electron state vector \( e_R \),
- the left electron state vector \( e_L \),
- the electron state vector \( e \) (\( e = \begin{bmatrix} e_R \\ e_L \end{bmatrix} \)),
- the left neutrino state vector \( \nu_L \),
- the zero vector right neutrino \( \nu_R \),
- the unitary \( 2 \times 2 \) matrix \( U \) of the isospin transformation (\( \det(U) = 1 \)) (Gauge Symmetry).

This matrix acts on the vectors of the kind: \( \begin{bmatrix} \nu_L \\ e_L \end{bmatrix} \).

Therefore, in this theory: if

\[
U = \begin{bmatrix}
 u_{1,1} & u_{1,2} \\
 u_{2,1} & u_{2,2}
\end{bmatrix}
\]

then the matrix

\[
\begin{bmatrix}
 1 & 0 & 0 & 0 \\
 0 & u_{1,1} & 0 & u_{1,2} \\
 0 & 0 & 1 & 0 \\
 0 & u_{2,1} & 0 & u_{2,2}
\end{bmatrix}
\]
operates on the vector
\[
\begin{bmatrix}
e_R \\
e_L \\
\nu_R \\
\nu_L
\end{bmatrix}.
\]

Because \(e_R, e_L, \nu_R, \nu_L\) are the two-component vectors then
\[
\begin{bmatrix}
e_R \\
e_L \\
\nu_R \\
\nu_L
\end{bmatrix}
\begin{bmatrix}
e_{R1} \\
e_{R2} \\
e_{L1} \\
e_{L2} \\
0 \\
0 \\
\nu_{L1} \\
\nu_{L2}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & u_{1,1} & 0 & 0 & u_{1,2} & 0 & 0 \\
0 & 0 & u_{1,1} & 0 & 0 & u_{1,2} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & u_{2,1} & 0 & 0 & u_{2,2} & 0 & 0 \\
0 & 0 & u_{2,1} & 0 & 0 & u_{2,2} & 0 & 0
\end{bmatrix}
\]

and
\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & u_{1,1} & 0 & u_{1,2} \\
0 & 0 & 1 & 0 \\
0 & u_{2,1} & 0 & u_{2,2}
\end{bmatrix}
\begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}
= \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & u_{1,1} & 0 & 0 & u_{1,2} & 0 & 0 \\
0 & 0 & u_{1,1} & 0 & 0 & u_{1,2} & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & u_{2,1} & 0 & 0 & u_{2,2} & 0 & 0 \\
0 & 0 & u_{2,1} & 0 & 0 & u_{2,2} & 0 & 0
\end{bmatrix}.
\]

This matrix has got eight orthogonal normalized eigenvectors of kind:
\[
s_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},
\quad s_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},
\quad s_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \bar{\nu} \\ \bar{\nu} \\ 0 \\ \bar{\nu} \end{bmatrix},
\quad s_4 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \chi \\ \chi \\ 0 \\ -\bar{\chi} \end{bmatrix},
\quad s_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},
\quad s_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},
\quad s_7 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},
\quad s_8 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}.
\[ s_5 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, s_6 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, s_7 = \begin{bmatrix} 0 \\ 0 \\ \chi^* \\ 0 \\ 0 \\ 0 \\ 0 \\ \chi \end{bmatrix}, s_8 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \overline{\chi}^* \\ 0 \\ 0 \\ \chi \end{bmatrix}. \]

The corresponding eigenvalues are: 1, 1, \(\exp(i \cdot \lambda)\), \(\exp(i \cdot \lambda)\), 1, 1, \(\exp(-i \cdot \lambda)\), \(\exp(-i \cdot \lambda)\).

These vectors constitute the orthogonal basis in this 8-dimensional space.

Let \(\gamma^0 = \begin{bmatrix} \gamma^0 & O \\ O & \gamma^0 \end{bmatrix}\), if \(O\) is zero 4 \times 4 matrix, and \(\beta^4 = \begin{bmatrix} \beta^4 & O \\ O & \beta^4 \end{bmatrix}\).

The vectors \(e^{R1}, e^{R2}, e^{L1}, e^{L2}\), correspond to the state vectors \(e, e^R\) and \(e^L\) resp.

In this case \((1.4)\) \(e^\dagger \cdot \gamma^0 \cdot e = J_{0e}, e^\dagger \cdot \beta^4 \cdot e = J_{4e}, J_{0e} = e^\dagger \cdot e \cdot V_{0e}, J_{4e} = e^\dagger \cdot e \cdot V_{4e}\).

For the vector \(e\) the numbers \(k_3, k_4, k_7, k_8\) exist, for which: \(\epsilon = (e_{R1} \cdot s_1 + e_{R2} \cdot s_2) + (k_3 \cdot s_3 + k_4 \cdot s_4) + (k_7 \cdot s_7 + k_8 \cdot s_8)\).

Here \(e_{Rb} = (e_{R1} \cdot s_1 + e_{R2} \cdot s_2)\) if \(e_{La} = (k_3 \cdot s_3 + k_4 \cdot s_4)\) and \(e_{Lb} = (k_7 \cdot s_7 + k_8 \cdot s_8)\) then \(U \cdot e_{La} = \exp(i \cdot \lambda) \cdot e_{La}\) and \(U \cdot e_{Lb} = \exp(-i \cdot \lambda) \cdot e_{Lb}\).

Let for all \(k\) \((1 \leq k \leq 8)\): \(h_k = \gamma^0 \cdot s_k\). The vectors \(h_k\) constitute the orthogonal basis, too. And the numbers \(q_3, q_4, q_7, q_8\) exist, for which: \(\epsilon = (q_3 \cdot h_3 + q_4 \cdot h_4) + (q_7 \cdot h_7 + q_8 \cdot h_8)\).

Let \(e_{Ra} = (q_3 \cdot h_3 + q_4 \cdot h_4), e_{Ra} = (q_7 \cdot h_7 + q_8 \cdot h_8), e_a = e_{Ra} + e_{La}\) and \(e_b = e_{Ra} + e_{Lb}\).

Let \(e_a^\dagger \cdot \gamma^0 \cdot e_b = J_{0a}, e_a^\dagger \cdot \beta^4 \cdot e_b = J_{4a}, J_{0a} = e_a^\dagger \cdot e_a \cdot V_{0a}, J_{4a} = e_a^\dagger \cdot e_a \cdot V_{4a}\).

Let \(e_b^\dagger \cdot \gamma^0 \cdot e_b = J_{0b}, e_b^\dagger \cdot \beta^4 \cdot e_b = J_{4b}, J_{0b} = e_b^\dagger \cdot e_b \cdot V_{0b}, J_{4b} = e_b^\dagger \cdot e_b \cdot V_{4b}\).

In this case: \(J_0 = J_{0a} + J_{0b}, J_4 = J_{4a} + J_{4b}\).

Let \((U \cdot e_a)^\dagger \cdot \gamma^0 \cdot (U \cdot e_a) = J_{0a}', (U \cdot e_a)^\dagger \cdot \beta^4 \cdot (U \cdot e_a) = J_{4a}', J_{0a}' = (U \cdot e_a)^\dagger \cdot V_{0a}', J_{4a}' = (U \cdot e_a)^\dagger \cdot V_{4a}', (U \cdot e_b)^\dagger \cdot \gamma^0 \cdot (U \cdot e_b) = J_{0b}', (U \cdot e_b)^\dagger \cdot \beta^4 \cdot (U \cdot e_b) = J_{4b}', J_{0b}' = (U \cdot e_b)^\dagger \cdot V_{0b}', J_{4b}' = (U \cdot e_b)^\dagger \cdot V_{4b}'.\)
In this case from (1.8):

\[ V'_{0a} = V_{0a} \cdot \cos (\lambda) + V_{4a} \cdot \sin (\lambda), \]
\[ V'_{4a} = V_{4a} \cdot \cos (\lambda) - V_{0a} \cdot \sin (\lambda); \]
\[ V'_{0b} = V_{0b} \cdot \cos (\lambda) - V_{4b} \cdot \sin (\lambda), \]
\[ V'_{4b} = V_{4b} \cdot \cos (\lambda) + V_{0b} \cdot \sin (\lambda). \]

Hence, every isospin transformation divides an electron on two components, which scatter on the angle \(2 \cdot \lambda\) in the space of \((J_0, J_4)\).

These components are indiscernible in the space of \((j_x, j_y, j_z)\) (1.7).

Let \(o\) be the \(2 \times 2\) zeros matrix. Let the \(4 \times 4\) matrices of kind:

\[
\begin{bmatrix}
P & o \\
o & S
\end{bmatrix}
\]

be denoted as the diagonal matrices, and

\[
\begin{bmatrix}
o & P \\
S & o
\end{bmatrix}
\]

be denoted as the antidiagonal matrices.

Three diagonal members \((\beta_1^1, \beta_2^2, \beta_3^3)\) of the Clifford pentad define the 3-dimensional space \(\mathbb{R}\), in which \(u_x, u_y, u_z\) are located. The physics objects move in this space. Two antidiagonal members \((\gamma^0, \beta_4^4)\) of this pentad define the 2-dimensional space \(\hat{A}\), in which \(V_0\) and \(V_4\) are located. The weak isospin transformation acts in this space.

3. GRAVITY

Let \(x\) be the particle average coordinate in \(\mathbb{R}\), and \(m\) be the average coordinate of this particle in \(\hat{A}\). Let \(x + i \cdot m\) be denoted as the complex coordinate of this particle.

From (1.6) this particle average velocity, which proportional to \(x + i \cdot m\), is:

\[ v = \frac{x + i \cdot m}{\sqrt{x^2 + m^2}}. \]

\(|v| = 1\), but for the acceleration:

\[ a = \frac{dv}{dt} = \frac{dv}{dx} \cdot v = -i \cdot m \cdot \left(\frac{x + i \cdot m}{x^2 + m^2}\right)^2. \]

And if \(m \ll x\), then

\[ |a| \approx \frac{m}{x^2}. \]

This is very much reminds the Newtonian gravity principle (Classical Theories of Gravity).