Strong Symmetry Breaking at $e^+e^-$ Linear Colliders

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The study of strong symmetry breaking at an $e^+e^-$ linear collider with $\sqrt{s} = 0.5 - 1.5$ TeV is reviewed. It is shown that processes such as $e^+e^- \rightarrow \nu\bar{\nu}W^+W^-$, $e^+e^- \rightarrow \nu\bar{\nu}H$, and $e^+e^- \rightarrow W^+W^-$ can be used to measure chiral Lagrangian and strong resonance parameters. The linear collider results are compared with those expected from the LHC.

I. INTRODUCTION

Until a Higgs boson with large couplings to gauge boson pairs is discovered, the possibility of strong electroweak symmetry breaking must be entertained. Without such a particle the scattering of gauge bosons will become strong at a scale of order 1 TeV. The most commonly studied class of theories which deals with this scenario is technicolor [1]. A generic prediction of technicolor theories is that there is a vector resonance with mass below about 2 TeV which unitarizes the $WW$ scattering cross section. Scalar and tensor resonances are also possible, along with light pseudo-Goldstone bosons which can be produced in pairs or in association with other particles [2].

Independent of the model, the strong interactions of gauge bosons below the threshold for resonance production can be described by an effective chiral Lagrangian in analogy with $\pi\pi$ scattering below the $\rho$ resonance [3]:

$$\mathcal{L}_{SB} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \cdots$$

$$\mathcal{L}^{(2)} = \frac{\omega^2}{4} \text{Tr} D^\mu \Sigma D_\mu \Sigma + \frac{g}{16\pi} \frac{v^2}{2} b_1 (\text{Tr} \Sigma^\dagger D_\mu \Sigma)^2$$

$$+ \frac{gg'}{16\pi^2} a_1 \text{Tr} \Sigma B^{\mu\nu} \Sigma^\dagger W_{\mu\nu}$$

$$\mathcal{L}^{(4)} = \frac{\alpha_4}{16\pi^2} \text{Tr} \left( D_\mu \Sigma^\dagger D_\mu \Sigma \right) \left( D^\nu \Sigma^\dagger D^\nu \Sigma \right) + \frac{\alpha_5}{16\pi^2} \left[ \text{Tr} \left( D^\mu \Sigma^\dagger D_\mu \Sigma \right) \right]^2$$

$$- ig \frac{L_{BR}}{16\pi^2} \text{Tr} \left( W^{\mu\nu} D_\mu \Sigma D_\nu \Sigma^\dagger \right) - ig' \frac{L_{LR}}{16\pi^2} \text{Tr} \left( B^{\mu\nu} D_\mu \Sigma^\dagger D_\nu \Sigma \right).$$

Here $W^{\mu\nu}$ and $B^{\mu\nu}$ are related to the $SU(2) \times U(1)$ gauge fields as in [3], $D_\mu$ is the covariant derivative, $g$ and $g'$ are the $SU(2) \times U(1)$ coupling constants, and $\Sigma$ is composed of the Goldstone boson fields $w^k$:

$$\Sigma = \exp \left( \frac{iv^{k} \tau^{k}}{v} \right),$$

where the $\tau^k$ are Pauli matrices and $v = 246$ GeV is the Standard Model Higgs vacuum expectation value parameter. The chiral Lagrangian parameters $a_1$ and $b_1$ are tightly constrained by precision electroweak data [3]. The terms with coefficients $\alpha_4$ and $\alpha_5$ induce anomalous quartic gauge boson couplings which can be measured by observing gauge boson scattering in processes such as $e^+e^- \rightarrow \nu\bar{\nu}W^+W^-$ and $\nu\bar{\nu}ZZ$. The terms with coefficients $L_{BR}$ and $L_{LR}$ induce anomalous triple gauge couplings (TGC’s) which can be measured in the reaction $e^+e^- \rightarrow W^+W^-W^-$. In this paper we summarize strong symmetry breaking signals and the measurement of chiral Lagrangian and strong resonance parameters at an $e^+e^-$ linear collider (LC) with a center of mass energy in the range of 0.5 to 1.5 TeV. Many of the results are taken from the strong symmetry breaking sections of Ref. [4], which the reader is invited to consult for further details.

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TABLE I: Expected errors for the real and imaginary parts of CP-conserving TGCs assuming $\sqrt{s} = 500$ GeV, $\mathcal{L} = 500$ fb$^{-1}$ and $\sqrt{s} = 1000$ GeV, $\mathcal{L} = 1000$ fb$^{-1}$. The results are for one-parameter fits in which all other TGCs are kept fixed at their SM values.

| TGC         | $\sqrt{s} = 500$ GeV | $\sqrt{s} = 1000$ GeV |
|-------------|-----------------------|------------------------|
|             | Re $\times 10^{-4}$   | Re $\times 10^{-4}$    |
| $g_1^\gamma$| 15.5                  | 12.8                   |
| $\kappa_\gamma$ | 3.5                   | 1.2                    |
| $\lambda_\gamma$ | 5.4                   | 2.0                    |
| $g_2^Z$     | 14.1                  | 11.0                   |
| $\kappa_Z$  | 3.8                   | 1.4                    |
| $\lambda_Z$ | 4.5                   | 1.7                    |

The reaction $e^+e^- \rightarrow \bar{\nu}\nu W^+W^-$ provides unique access to $W^+W^- \rightarrow t\bar{t}$ since this process is overwhelmed by the background $gg \rightarrow t\bar{t}$ at the LHC. Techniques similar to those employed to isolate $W_LW_L \rightarrow W^+W^-$, $ZZ$ can be used to measure the enhancement in $W_LW_L \rightarrow t\bar{t}$ production. Even in the absence of a resonance it will be possible to establish a clear signal. The ratio $S/\sqrt{B}$ is expected to be 12 for a linear collider with $\sqrt{s} = 1$ TeV, 1000 fb$^{-1}$ and 80%/0% electron/positron beam polarization, increasing to 22 for the same luminosity and beam polarization at $\sqrt{s} = 1.5$ TeV.

III. $e^+e^- \rightarrow W^+W^-$

Strong gauge boson interactions induce anomalous TGC’s at tree-level:

\[
\begin{align*}
\kappa_\gamma &= 1 + \frac{e^2}{32\pi^2s^2_w}(L_{9L} + L_{9R}) \\
\kappa_Z &= 1 + \frac{e^2}{32\pi^2s^2_w}(L_{9L} - \frac{s^2_w}{c^2_w}L_{9R}) \\
g_2^Z &= 1 + \frac{e^2}{32\pi^2s^2_w\frac{s^2_w}{c^2_w}}L_{9L}
\end{align*}
\]

where $\kappa_\gamma$, $\kappa_Z$, and $g_2^Z$ are TGC’s, $s^2_w = \sin^2\theta_w$, and $c^2_w = \cos^2\theta_w$. Assuming QCD values for the chiral Lagrangian parameters $L_{9L}$ and $L_{9R}$, $\kappa_\gamma$ is shifted by $\Delta\kappa_\gamma \approx -3 \times 10^{-3}$.

The TGCs can be measured by analyzing the $W^+W^-$ production and decay angles in the process $e^+e^- \rightarrow W^+W^-$. Table contains the estimates of the TGC precision that can be obtained at $\sqrt{s} = 500$ and 1000 GeV for the CP-conserving couplings $g_1^\gamma$, $\kappa_V$, and $\lambda_V$. These estimates are derived from one-parameter fits in which all other TGC parameters are kept fixed at their tree-level SM values. For comparison the LHC with $\mathcal{L} = 300$ fb$^{-1}$ is expected to measure $\kappa_\gamma$ and $\kappa_Z$ with an accuracy of 0.006 and 0.01, respectively. The $4 \times 10^{-4}$ precision for the TGCs $\kappa_\gamma$ and $\kappa_Z$ at $\sqrt{s} = 500$ GeV can be interpreted as a precision of 0.26 for the chiral Lagrangian parameters $L_{9L}$ and $L_{9R}$. Assuming naive dimensional analysis such a measurement would provide a $8\sigma$ ($5\sigma$) signal for $L_{9L}$ and $L_{9R}$ if the strong symmetry breaking energy scale were 3 TeV (4 TeV).

When $WW$ scattering becomes strong the amplitude for $e^+e^- \rightarrow W_LW_L$ develops a complex form factor $F_T$ in analogy with the pion form factor in $e^+e^- \rightarrow \pi^+\pi^-$. To evaluate the size of this effect the following
expression for $F_T$ can be used:

$$F_T = \exp \left[ \frac{1}{\pi} \int_0^\infty ds' \delta(s', M_\rho, \Gamma_\rho) \left( \frac{1}{s' - s - i\epsilon} - \frac{1}{s} \right) \right]$$

where

$$\delta(s, M_\rho, \Gamma_\rho) = \frac{1}{96\pi v^2} \frac{s}{s^2 - M_\rho^2 + i\Gamma_\rho s} + \frac{3\pi}{8} \left[ \tanh \left( \frac{s - M_\rho^2}{4\Gamma_\rho} \right) + 1 \right].$$

Here $M_\rho, \Gamma_\rho$ are the mass and width respectively of a vector resonance in $W_L W_L$ scattering. The term

$$\delta(s) = \frac{1}{96\pi v^2}$$

is the Low Energy Theorem (LET) amplitude for $W_L W_L$ scattering at energies below a resonance. Below the resonance, the real part of $F_T$ is proportional to $L_{9L} + L_{9R}$ and can therefore be interpreted as a TGC. The imaginary part, however, is a distinct new effect.

The real and imaginary parts of the form factor $F_T$ are measured in $e^+e^- \rightarrow W^+W^-$ in the same manner as the TGCs. The expected 95% confidence level limits for $F_T$ for $\sqrt{s} = 500$ GeV and a luminosity of 500 fb$^{-1}$ are shown in Figure 1, along with the predicted values of $F_T$ for various masses $M_\rho$ of a vector resonance in $W_L W_L$ scattering. The signal significances obtained by combining the results for $e^+ e^- \rightarrow \nu\bar{\nu} W^+ W^-$, $\nu\bar{\nu} ZZ$ with the $F_T$ analysis of $e^+ e^- \rightarrow W^+ W^-$ are displayed in Fig. 1 along with the results expected from the LHC [15]. At all values of the center-of-mass energy a linear collider provides a larger direct strong symmetry breaking signal than the LHC for vector resonance masses of 1200, 1600 and 2500 GeV. Only when the vector resonance disappears altogether (the LET case in the lower right-hand plot in Fig. 1) does the direct strong symmetry breaking signal from the $\sqrt{s} = 500$ GeV linear collider drop below the LHC signal. At higher $e^+ e^-$ center-of-mass energies the linear collider signal exceeds the LHC signal.

![Graph](image)

**FIG. 1:** 95% C.L. contour for $F_T$ for $\sqrt{s} = 500$ GeV and 500 fb$^{-1}$. Values of $F_T$ for various masses $M_\rho$ of a vector resonance in $W_L W_L$ scattering are also shown. The $F_T$ point ‘LET’ refers to the case where no vector resonance exists at any mass in strong $W_L W_L$ scattering.

### IV. STRONG WW SCATTERING BENCHMARK PROCESSES

The Snowmass 2001 working group on experimental approaches at linear colliders used a series of benchmarks to help evaluate the physics program of a future $e^+ e^-$ linear collider [14]. Strong $WW$ scattering in the presence of scalar and vector resonances was simulated using the model of Han et al. [20], with resonance masses of
FIG. 2: Direct strong symmetry breaking signal significance in $\sigma'$s for various masses $M_{\rho}$ of a vector resonance in $W_L W_L$ scattering. The numbers below the “LC” labels refer to the center-of-mass energy of the linear collider in GeV. The luminosity of the LHC is assumed to be $300$ fb$^{-1}$, while the luminosities of the linear colliders are assumed to be $500$, $1000$, and $1000$ fb$^{-1}$ for $\sqrt{s}=500$, $1000$, and $1500$ GeV respectively. The lower right hand plot “LET” refers to the case where no vector resonance exists at any mass in strong $W_L W_L$ scattering.

1.0 and 1.5 TeV. The scalar resonance in this model was basically the SM Higgs. The widths of the vector resonances were 0.055 and 0.077 TeV for resonance masses of 1.0 and 1.5 TeV, respectively. For non-resonant strong $W W$ scattering the unitarized K-matrix LET model\cite{21} was used.

When estimating the mass scale reach of the K-matrix LET model and the mass resolution of the resonance model in the presence of a scalar (I=0) or tensor (I=2) resonance, we use the leading order modifications to the LET cross sections\cite{22}: 

$$\sigma(M_0) = \left(1 + \frac{8}{3 M_0^2}\right) \sigma_{\text{LET}}$$
Studies of strong electroweak symmetry breaking are enhanced by an $e^+e^-$ linear collider with $\sqrt{s} = 0.5 - 1.5$ TeV. An LC complements a hadron collider nicely in providing better measurements of the chiral Lagrangian parameters $L_{9L}$ and $L_{9R}$ which affect triple gauge boson vertices. Also, the LC provides competitive measurements of the chiral Lagrangian parameters $\alpha_9$ and $\alpha_9$, which affect quartic gauge boson vertices.

A non-resonant strong symmetry breaking signal will be slightly larger at a $\sqrt{s} = 1.0$ TeV LC than at the LHC, and will be significantly larger if the $e^+e^-$ CMS energy is raised to $\sqrt{s} = 1.5$ TeV. Less energy is required for strong vector resonance detection. A $\sqrt{s} = 0.5$ TeV LC provides a larger vector resonance signal than the

| Collider | Final State | $\sqrt{s}$ (TeV) | $\mathcal{L}$ (fb$^{-1}$) | $\Delta M_0$ (GeV) | $\Delta M_1$ (GeV) | $\Delta M_1$ (GeV) | $\Delta \Gamma_1$ (GeV) |
|----------|-------------|------------------|------------------------|-------------------|-------------------|-------------------|-------------------|
| LC       | $W^+W^-$    | 0.5              | 500                    | -                 | -                 | 5.8               | 19.0              |
| LC       | $W^+W^-$    | 1.0              | 1000                   | 89                | 249               | 0.01              | 0.03              |
| LC       | $W^+W^-$    | 1.5              | 1000                   | 14                | 46                | -                 | -                 |

TABLE II: Expected error $\Delta M_0$ for the mass of a scalar resonance, and expected errors $\Delta M_1$ and $\Delta \Gamma_1$ for the mass and width, respectively, of a vector resonance. Results are shown for vector resonances of mass 1.0 and 1.5 TeV.

\[
\sigma(M_2) = \left(1 + 2 \frac{s}{M_1^2}\right) \sigma_{\text{LET}} ,
\]

where $M_0$ and $M_2$ are the resonance masses in the $I = 0, 2$ channels, respectively. (The tensor resonance formula is used to estimate LHC mass scale sensitivity.) For detecting vector resonances we use the technipion form factor, which to leading order in $s/M_1^2$ is given by

\[
F_T = \frac{M_1^2 - i\Gamma_1 M_1}{M_1^2 - s - i\Gamma_1 M_1} ,
\]

where $M_1$ and $\Gamma_1$ are the vector resonance mass and width, respectively. In order to evaluate the vector mass scale reach in the K-Matrix LET model we use the expression

\[
\text{Re}(F_T) = 1 + \Delta_{\text{LET}} + \frac{s}{M_1^2} ,
\]

where $\Delta_{\text{LET}}$ is the contribution to $F_T$ from strong $WW$ scattering in the absence of a vector resonance. The dependence of $\Delta_{\text{LET}}$ on the details of the unitarization scheme grows as $\sqrt{s}$ grows; the systematic uncertainty due to our lack of knowledge of these details is included in our calculations.

The expected errors for the mass of the scalar resonances are shown in Table II, along with the expected errors for the mass and width of the vector resonances. The measurement of the scalar mass $M_0$ is assumed to come solely from the measurement of the cross section $\sigma$, with $\sigma(M_0)$ defined above. For the measurement of the scalar mass there is a clear advantage in going to the higher CMS energy of 1.5 TeV. In contrast, the masses and widths of the vector resonances are measured very well at all CMS energies. Even the most poorly measured vector resonance parameter – the width of the 1.5 TeV resonance at $\sqrt{s} = 0.5$ TeV – is measured with an accuracy of 6%. At $\sqrt{s} = 1.0$ and 1.5 TeV the vector mass and width resolutions are typical of an $e^+e^-$ collider sitting on top of the resonance.

Results for the K-matrix LET model are shown in Table II. The signal significance is displayed along with the 95% C.L. mass scale limits in the $I = 0, 1$ isospin channels. For comparison, results are also shown for the LHC in the $I = 2$ channel [18]. The tensor mass scale lower limit from the LHC is comparable to the scalar mass scale limits from the LC. Not suprisingly, the largest mass scale limits are the vector limits obtained in $e^+e^- \rightarrow W^+W^-$. Note that the vector mass scale lower limit $M_1$ does not improve as the CMS energy is raised from 1.0 to 1.5 TeV; this is due to the systematic uncertainty in the calculation of $\Delta_{\text{LET}}$, which becomes important near $\sqrt{s} = 1.5$ TeV. The only way to reduce this particular systematic uncertainty is to actually do strong scattering experiments at the LHC and at an $e^+e^-$ LC.

V. SUMMARY

Studies of strong electroweak symmetry breaking are enhanced by an $e^+e^-$ linear collider with $\sqrt{s} = 0.5 - 1.5$ TeV. An LC complements a hadron collider nicely in providing better measurements of the chiral Lagrangian parameters $L_{9L}$ and $L_{9R}$ which affect triple gauge boson vertices. Also, the LC provides competitive measurements of the chiral Lagrangian parameters $\alpha_9$ and $\alpha_9$, which affect quartic gauge boson vertices.

A non-resonant strong symmetry breaking signal will be slightly larger at a $\sqrt{s} = 1.0$ TeV LC than at the LHC, and will be significantly larger if the $e^+e^-$ CMS energy is raised to $\sqrt{s} = 1.5$ TeV. Less energy is required for strong vector resonance detection. A $\sqrt{s} = 0.5$ TeV LC provides a larger vector resonance signal than the
Collider | Final State | √s (TeV) | L (fb⁻¹) | Signal | M₀ (TeV) 95% C.L. | M₁ (TeV) 95% C.L. | M₂ (TeV) 95% C.L. |
|-----------|------------|---------|---------|--------|--------------|--------------|--------------|
| LC        | W⁺W⁻      | 0.5     | 500     | 3σ     | –            | 4.8          | –            |
| LC        | W⁺W⁻      | 1.0     | 1000    | 7σ     | –            | 6.4          | –            |
| LC        | W⁺W⁻      | 1.5     | 1000    | 8σ     | –            | 6.4          | –            |
| LC        | ννW⁺W⁻, ZZ | 1.0     | 1000    | 7σ     | 1.7          | –            | –            |
| LC        | ννW⁺W⁻, ZZ | 1.5     | 1000    | 20σ    | 4.3          | –            | –            |
| LHC       | qqW⁺W⁺     | 14      | 300     | 9σ     | –            | –            | 3.0          |

TABLE III: Signal significance and 95% C.L. mass scale lower limits for the LET model with the K-matrix unitarization scheme. The mass scales M₀, M₁, M₂ correspond to structure in WW scattering in the I=0,1, and 2 isospin channels, respectively.

LHC for masses up to at least 2.5 TeV. The mass and width of a strong vector resonance can be measured at a LC with at least a few percent accuracy, even when the resonance lies well above the e⁺e⁻ CMS energy.

Another important aspect of strong symmetry breaking is the study of W⁺W⁻ → t̄t. This reaction can probably only be studied at a LC. Good strong symmetry breaking signals can be obtained in this channel at a LC, and these results should prove valuable in understanding electroweak symmetry breaking in the fermion sector.

Finally, we note that the systematic errors in signal and background calculations will be smaller at a LC than at a hadron collider, since the production mechanisms and backgrounds are limited to electroweak processes. However, we cannot at this time quantify this advantage since detailed studies of theoretical systematic errors in strong WW scattering have not been performed for either the LHC or the LC. This issue could be important given the size of some of the strong symmetry breaking signals and the paucity of sharp resonances in many strong symmetry breaking scenarios.

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