Leptoproduction of charm revisited

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ABSTRACT

We calculate the energy–momentum distribution of the charmed quarks produced in neutrino reactions on protons, quantifying the importance of mass and current non–conservation effects. We study the strange and charm distributions probed in neutrino interactions in the presently accessible kinematical region. Some ambiguities inherent to the extraction of the parton densities from dimuon data are pointed out.
Charm excitation in neutrino scattering is a primary source of information on the strangeness in nucleons. At moderate values of $Q^2$ (the momentum transfer squared in deep inelastic scattering), subtle threshold and current non-conservation effects are at work \[1, 2, 3, 4\], which lead to the non-universality of the charm contribution to the structure functions probed by muons and by neutrinos. The driving term of the neutrino-excitation of charm is the $W$-gluon fusion process $W^+g \to c\bar{s}$ (Fig. 1), which consists of two partonic subprocesses: excitation of charm on the strange sea and excitation of anti-strangeness on the anti-charmed sea (we shall consider only the Cabibbo unsuppressed reactions). Both subprocesses are of course an integral part of the gauge-invariant QCD cross section and gauge invariance requires both amplitudes (Figs. 1a,1b) to be taken into account. In principle, the two subprocesses can be separated according to which parton, $c$ or $\bar{s}$, is produced in the $W$-boson hemisphere in the Breit reference frame (other conventions are possible, as well). Such a separation becomes well defined at $Q^2 \gg (\mu + m)^2$, which is not yet the case with the present experiments (hereafter, $m$ and $\mu$ will denote the mass of the strange and of the charmed quark, respectively).

From an experimental point of view, the extraction of what is called the strange structure function of the nucleon is a multi-stage process (see \[5\] and the references therein). The signature of the excitation of the charmed quark is the semileptonic decay $c \to s\mu^+\nu$, which leads to the opposite-sign muon pairs $\mu^+\mu^-$ in the final state (here $\mu^-$ comes from the primary weak interaction vertex). The main source of background are the semileptonic decays of pions and kaons. This forces one to put stringent lower cuts on the energy of $\mu^+$. Because of these cuts the measured cross section is sensitive to the energy of the produced charmed quarks. In the limit where the masses of the quarks can be neglected, the primary produced quark carries $\approx 100\%$ of the $W$’s energy, and then fragments with some fragmentation function (usually supposed to be independent of $Q^2$) into the charmed particles, the semileptonic decays of which produce the desired signature of the charm production. At moderate values of $Q^2$, the mass effects deeply modify this scenario. In this paper we shall show that, at moderate $Q^2$, the primary charmed quarks have quite a broad energy distribution. Furthermore, this energy distribution depends on $Q^2$, and understanding this dependence is important for unfolding the acceptance effects and for the determination of the primary charm production rate.

We shall focus on the region of small values of the Bjorken variable $x = Q^2/2m_\nu\nu$ and on the lowest-order QCD subprocesses, which simplifies the analysis. However, it will be evident that our main conclusions have a broader applicability.

We have to calculate the $(T)$ and $(L)$ absorption cross section $d\sigma_{T,L}/dzd^3k$ for the transverse ($T$) and longitudinal ($L$) $W$ and $\gamma^*$. We denote by $k$ the transverse momentum of the charmed quark, and by

$$
z = \frac{\omega + \sqrt{\omega^2 - \mu^2 - k^2}}{\nu + \sqrt{\nu^2 + Q^2}}
$$

(1)
the fraction of the light–cone momentum of the W ($\gamma^*$) carried by the charmed quark. Here we concentrate on the $(z, k)$ distribution of the produced quarks, i.e. of the jets generated by these quarks, leaving aside the issue of quark fragmentation into charmed hadrons.

The analysis of the energy and momentum distribution in the case of massless quarks and/or asymptotically large $Q^2$ was first performed by Altarelli and Martinelli [6]. More recently Ellis and Nason [7] gave a very detailed treatment of the real photoproduction of charm. One important finding was that the higher–order QCD corrections only slightly change the energy and transverse momentum distribution of the produced quarks.

In this paper we shall be concerned with the region of moderate $Q^2$ relevant to neutrino experiments, where the strong unequality of masses of the charmed and strange quarks manifests its effects. Two important points are the onset of the parton model interpretation of heavy flavors when passing over the threshold value of $Q^2 \sim 4\mu^2, (m+\mu)^2$, and the transition from the dominance of the longitudinal cross section and the strong breaking of the Callan–Gross relation at small $Q^2$ to the dominance of the transverse cross section and the (approximate) restoration of the Callan–Gross relation at large $Q^2$.

To the lowest order in QCD, the longitudinal and transverse absorption cross sections in the charged current case are given by a generalization of the formulas provided in [8] – see eqs. (11) and (12) – and read ($\alpha_{\text{ew}}$ is the appropriate electroweak coupling constant for the specific process considered)

$$\frac{d\sigma_{T,L}}{dz \, d^2k} = \frac{16}{\pi^2} \alpha_{\text{ew}} \alpha_S(k^2) \int d^2\kappa \frac{V(\kappa)\alpha_S(\kappa^2)}{(\kappa^2 + \mu_E^2)^2} \left\{ \frac{N_{T,L}(k, k)}{[k^2 + \varepsilon^2]^2} + \frac{N_{T,L}(k, \kappa, k + \kappa)}{[(k + \kappa)^2 + \varepsilon^2]^2} - 2 \frac{N_{T,L}(k, k + \kappa)}{[(k + \kappa)^2 + \varepsilon^2][k^2 + \varepsilon^2]} \right\}$$

(2)

where the gluon-gluon-nucleon vertex function $V(\kappa)$ is related to the charge form factor of the proton $G_{em}(q^2)$ through $V(\kappa) = 1 - G_{em}(3\kappa^2)$,

$$\varepsilon^2 = z(1 - z)Q^2 + zm^2 + (1 - z)\mu^2,$$

and the functions $N_{T,L}(k_1, k_2)$ are explicitly given by

$$N_T(k_1, k_2) = \left[ z^2 + (1 - z)^2 \right] (g_V^2 + g_A^2) k_1 \cdot k_2 + g_V^2 [zm + (1 - z)\mu]^2 + g_A^2 [zm - (1 - z)\mu]^2,$$

(4)

$$N_L(k_1, k_2) = \frac{1}{Q^2} \left[ (g_V^2 (m - \mu)^2 + g_A^2 (m + \mu)^2) k_1 \cdot k_2 + g_V^2 \left\{ 2Q^2 z(1 - z) + (m - \mu) [zm - (1 - z)\mu] \right\}^2 ight.$$  

$$+ g_A^2 \left\{ 2Q^2 z(1 - z) + (m + \mu) [zm + (1 - z)\mu] \right\}^2 \right)$$

(5)
In muon scattering $g_V = e_i$ (the charge of the quark involved in units of the electron charge), $g_A = 0$ and $\mu = m$. In the charged current (CC) neutrino interactions $g_A = -g_V = -1$ and $\mu$ and $m$ stand for the strange and the charm quark masses. In the neutral current (NC) neutrino interactions $\mu = m$ and the corresponding vector and axial coupling are given by the Standard Model \[9\]. The strong coupling $\alpha_S(\tilde{k}^2)$ in front of the differential cross section enters at the virtuality of the quark $\tilde{k}^2 = \varepsilon^2 + k^2$.

The $z$–distributions of the charmed quark can be calculated by integrating eq. (2) over the transverse momentum.

Let us start with the $z$-distribution for $\sigma_T$. Here $N_T$ is a mild function of $z$, and the $z$ dependence comes from $\varepsilon^2$. To a crude approximation, neglecting the scaling violations, we find

$$\frac{d\sigma_T}{dz} \sim \frac{z^2 + (1 - z)^2}{\varepsilon^2} = \frac{z^2 + (1 - z)^2}{z(1 - z)Q^2 + zm^2 + (1 - z)/\mu^2}. \tag{6}$$

Consider first the muon scattering, $\mu = m$. The distribution (6) is nearly flat at $Q^2 \lesssim 4\mu^2 \sim 10 \text{GeV}^2/c^2$. Only at asymptotically high $Q^2 \gg 4\mu^2$, it develops the parton model peaks at $z \to 0$ and $z \to 1$ (see \[3\]), so that the charmed (anti)quark carries a fraction $z \sim 1 - \mu^2/Q^2$ of the photon’s light-cone momentum. The result of the exact calculation of $(1/\sigma_T)d\sigma_T/dz$ for the $\bar{c}c$ excitation in electromagnetic scattering is shown in Fig. 2a. The situation is similar for the NC neutrino scattering (Fig. 2b).

In the CC neutrino scattering the charmed quark is much heavier than the strange quark, $\mu \gg m$. At large $Q^2$ the $z$ distribution (shown in Fig. 2c) again develops peaks at $z \to 1$ and $z \to 0$. However, the $Q^2$ evolution of the forward, $d\sigma_T/dz \sim (1 - z - m^2/Q^2)$, and the backward, $d\sigma_T/dz \sim (1 - z - \mu^2/Q^2)$, peaks is quite different: the forward peak appears sooner.

Now, the CC excitation of charm and (anti)strangeness are inseparable. In the parton model language, at very large $Q^2$, the peak at $z \to 1$ can be identified with the excitation of the charm on the strange sea, while the peak at $z \to 0$ describes the excitation of $\bar{s}$ on the $\bar{c}$ distribution in the nucleon. Thus, only asymptotically the separation of the two subprocesses depicted in Fig. 1 is well defined. As we have already mentioned, in the extraction of the dimuon data, a cut on $z$ is implicitly made, which produces an acceptance–dependent separation of the strange and charm contributions to the $W$–absorption cross sections. However, at moderate $Q^2$ the sensitivity to the cutoff $z_c$ is rather strong and the parton model reinterpretation of the $c\bar{s}$ excitation in terms of the two partonic subprocesses is by no means unique and becomes matter of convention. In this region of $Q^2$ an assumption of the form $d\sigma/dz \sim \delta(z - 1)$, in conjunction with a $Q^2$–independent fragmentation function, is untenable because it would neglect the crucial $Q^2$–dependence of the $z$–distributions.

The problem is complicated by the presence of the longitudinal contribution, which contribute significantly to the “non-partonic” domain of $z$ around 1/2 (for a discussion of the massless case see \[3\]).
In muon interactions, again neglecting the scaling violations, one finds a broad symmetric \( z \)-distribution, shown in Fig. 2a

\[
\frac{d\sigma_L}{dz} \sim \frac{Q^2 z^2 (1-z)^2}{\varepsilon^4}, \tag{7}
\]

which, in proximity of the forward peak, increases with \( Q^2 \). The ratio \( R = \sigma_L/\sigma_T \) is of course rather small, \( R \sim Q^2/4\mu^2 \).

By contrast, in neutrino scattering, because of the nonconservation of the weak axial current \( (g_A \neq 0) \) and of the flavor changing vector current \( (\mu \neq m) \), one has \( N_L \sim N_T \mu^2/Q^2 \) and the longitudinal contribution is rather significant at moderate \( Q^2 \) \( (R \sim 4\mu^2/Q^2) \). In the NC interactions the \( z \)-distribution is symmetric around \( z = \frac{1}{2} \) (Fig. 2b). In the CC interactions, \( d\sigma_L/dz \) develops two peaks at \( z \to 1 \) and \( z \to 0 \), and is asymmetric around \( z = \frac{1}{2} \) (Fig. 2c).

An interesting quantity is \( R(z) = \sigma_L(z)/\sigma_T(z) \) which shows how the weak-current non-conservation effects depend on \( z \). In Fig. 3 we compare \( R(z) \) at \( Q^2 = 4 \text{GeV}^2/\varepsilon^2 \) for \( cc \) excitation in muon scattering and in NC neutrino scattering, and for \( \bar{c}s \) excitation in CC neutrino scattering. Shown for comparison are the values of \( R \) for the \( z \)-integrated cross sections. In virtual photoabsorption both \( R \) and \( R(z) \) are small. In the NC and CC neutrino scattering \( R(z) \) decreases as \( z, (1-z) \to 0 \). Notice that in the CC case \( R(z) \) is slightly asymmetric.

The peculiar \( Q^2 \)-dependence of the transverse and longitudinal contributions to the \( z \)-distributions manifests itself in a detectable way at the level of the sea parton densities. We must say that neither the introduction of the density of partons at \( Q^2 \lesssim 4\mu^2, (m + \mu^2)^2 \), nor the comparison of the \( \sigma_T \)-dominated muoproduction with the \( \sigma_L \)-dominated neutrino–production do make much sense. Nonetheless, in order to clarify the issue of the non–universality, let us proceed with such a comparison.

The well-defined quantities are the cross sections \( \sigma_{T,L} \), which can be converted into the structure functions \( F_{L,T}(x, Q^2) = Q^2 \sigma_{L,T}/4\pi\alpha_{ew} \). In terms of the more familiar structure functions \( F_1, F_2 \), one has \( F_1 = F_T/2x \) and \( F_2 = F_T + F_L \). At very large \( Q^2 \), when all quarks can be regarded as massless, the structure functions can be decomposed in terms of the parton densities as \( [9] \) (we consider an isoscalar nucleon)

\[
F_2^{(\mu)}(x, Q^2) = \frac{4}{9}[u_\mu + \bar{u}_\mu + c_\mu + \bar{c}_\mu] + \frac{1}{9}[d_\mu + \bar{d}_\mu + s_\mu + \bar{s}_\mu] \tag{8}
\]

\[
F_2^{(\nu)}(x, Q^2) = u_\nu + \bar{u}_\nu + 2\bar{c}_\nu + d_\nu + \bar{d}_\nu + 2s_\nu \tag{9}
\]

These formulas can be taken as the operational definition of the parton densities even at \( Q^2 \lesssim 4\mu^2, (m + \mu^2)^2 \). Similarly, one can introduce the parton densities \( q_{\nu,\mu}^{(T)} \) defined in terms of \( F_T(x, Q^2) \), which would have been identical to the ones in Eqs. \( \[8,9] \) were it not for the breaking of the Callan-Gross relation. Notice that we have supplied the parton densities by subscripts \( \nu \) and \( \mu \). The non–universality of the charm and strange densities is quantified by the ratio \( r_{\nu/\mu} = (\bar{c}_\nu + s_\nu)/(\bar{c}_\mu + s_\mu) \) (however one has to keep
in mind that there is no direct measurement of $s_\mu$). We have evaluated $r_{\nu/\mu}$ within the model of Ref. [1, 2, 3, 4, 10] taking $(m + \mu)^2 = 4 GeV^2/c^2$. The result is shown in Fig. 4a, where both the $x$– and the $Q^2$–dependence of $r_{\nu/\mu}$ is exhibited. Notice that, whereas at large $Q^2$ the ratio $r_{\nu/\mu}$ tends to the asymptotical (and naively assumed) value of unity, at small $Q^2$ ($\lesssim 10 GeV^2/c^2$) it increases with $Q^2$ before flattening down. In the kinematical region probed by the CCFR experiment [5] the deviation of $r_{\nu/\mu}$ from unity is as large as $\sim 25\%$ and is a possible explanation of the observed discrepancy between different determinations of the strange density [2]. The numerator in $r_{\nu/\mu}$ is dominated by the longitudinal component: taking only the transverse contribution, the departure of $r_{\nu/\mu}$ from unity would be even stronger. For completeness we present in Fig. 4b the ratio $c_\mu/(c_\mu + s_\mu)$ evaluated at two different values of $Q^2$.

Whereas in the determination of $F_2^{(\nu)}$ one deals with the sum $\bar c_\nu + s_\nu$, which is free from ambiguities, being related to the integral of $d\sigma_{T,L}/dz$ over the whole $z$–range, the experimental analysis of the dimuon data introduces a separation of $s_\nu$ and $\bar c_\nu$. The $c\bar s$ production cross section is thus splitted between the two partonic subprocesses of Fig. 1. This amounts to introducing a cutoff $z_c$ in the $z$–distributions, so that $s_\nu$ and $\bar c_\nu$ are defined in terms of the $c\bar s$ production cross section subject to the (arbitrary) cuts $z > z_c$ and $z < z_c$, respectively. In the treatment of the dimuon data the cutoff $z_c$ is implicitly posited and the strange density consequently obtained. Were the data taken at asymptotically large $Q^2$, when the peak at $z = 1$ is delta–like, the choice of the cutoff would be irrelevant. However, the $Q^2$ region of interest, especially at small $x$, is that of small and moderate values $Q^2 \lesssim 20 GeV^2/c^2$, where the choice of $z_c$ necessarily introduces some arbitrariness.

Choosing the cutoff $z_c = 1/2$ is equivalent, in the muon interaction and in the weak NC interaction, to simply equating the charm and anticharm densities, since the $z$–distributions are symmetric. Also in the CC interaction at asymptotically large $Q^2$, where the two parton model peaks tend to become delta–like, taking $z_c = 1/2$ gives $s_\nu = \bar c_\nu = (\bar c_\nu + s_\nu)/2$. By contrast, in the CC case at small $Q^2$ the $z$–distributions are strongly asymmetric, the backward and the forward peaks are barely visible and the splitting of $s_\nu$ and $\bar c_\nu$ cannot be unambiguously done. Hence some care must be exerted in analyzing the dimuon data in terms of the strange density. The effect of two different choices of the cutoff $z_c$ on $s_\nu$ ($z_c = 1/2$ and $z_c = 0.8$, the latter corresponding approximately to taking $s_\nu = \bar c_\nu = (s_\nu + \bar c_\nu)/2$) is displayed in Fig. 5. Our curves are presented for $Q^2 = 10 GeV^2/c^2$ and compared to the dimuon data around the same value of $Q^2$. An exact comparison with the data would require the knowledge of the experimental value of $z_c$, which is lacking.

In conclusion, the calculation of the energy–momentum distribution of the quarks produced in the charm leptoproduction has shown that, in the presently explored kinematical region, mass and current non–conservation effects play a decisive role, making the extraction of the heavy–quark parton densities from dimuon data subject to some (solvable) ambiguity. We have expressed in a quantitative way the deviations from the naive parton model expectations and stressed the importance of a correct
interpretation of the data coming from neutrino interactions.
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Figure captions

Fig. 1 - The excitation of charm in the $W$–gluon fusion process.

Fig. 2 -

(a) The $z$–distributions $(1/\sigma_{T,L})d\sigma_{T,L}/dz$ of the charmed quark produced in the transverse (T) and the longitudinal (L) virtual photoabsorption. The curves are as follows: T at $1\,GeV^2/c^2$, dotdashed; T at $10\,GeV^2/c^2$, solid; L at $1\,GeV^2/c^2$, dotted; L at $10\,GeV^2/c^2$, dashed.

(b) Same as (a), but for the weak neutral current case ($Z^0$–absorption cross sections).

(c) Same as (a), but for the charged current case ($W$–absorption cross sections).

Fig. 3 - The ratio $R(z) = \sigma_L(z)/\sigma_T(z)$ at $Q^2 = 4\,GeV^2/c^2$ in muon scattering (solid curve), CC neutrino scattering (dashed curve), NC neutrino scattering (dotdashed curve).

Fig. 4 -

(a) The ratio $(\bar{c}_\nu + s_\nu)/(\bar{c}_\mu + s_\mu)$ plotted as a function of $x$ at three different values of $Q^2$ (solid curve, $4\,GeV^2/c^2$; dashed curve, $10\,GeV^2/c^2$; dotdashed curve, $30\,GeV^2/c^2$).

(b) The ratio $c_\mu/(c_\mu + s_\mu)$ at two different values of $Q^2$: $10\,GeV^2/c^2$ (dashed curve), $30\,GeV^2/c^2$ (dotdashed curve).

Fig. 5 - The strange density $s_\nu$ probed in neutrino interactions calculated at $Q^2 = 10\,GeV^2/c^2$ with two different cutoffs: $z_c = 0.5$, solid curve (T+L), dashed curve (only T); $z_c = 0.8$, dotdashed curve (T+L), dotted curve (only T). The data are from Ref. [5].
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