A NUMERICAL STUDY ON MULTI-CHAMBER OSCILLATING WATER COLUMNS

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A two-dimensional frequency-domain numerical analysis on the primary conversion efficiency of multi-chamber Oscillating Water Columns (OWCs) is presented. The numerical model is developed by combining the hydrodynamics of the interaction between the wave and the OWC and the thermodynamics of the air chambers. The wave-induced force is calculated using the boundary element method based on the velocity potential theory. The air flow is studied using mass and energy conservation equations and an equation of state. The air pressure in the air chambers, the reflection coefficient, and the primary conversion efficiency of each of the chambers, as well as the combined efficiency, are evaluated using the boundary integral equations. In addition, the behavior of these physical quantities, along with the variations in the nozzle ratio, the relative water depth, the depth of the curtain wall, and the width of the front-chamber are investigated using the calculated results.

\textbf{Key Words} : velocity potential, boundary element, frequency domain, primary conversion efficiency

1. INTRODUCTION

Compared to other wave energy converters (WECs), the simple design, easy installation and operation of Oscillating Water Columns (OWCs) have made them the most popular among wave energy conversion technologies\textsuperscript{(1,2,3)}. In addition, OWCs that utilize air turbines have very few moving parts and there are no moving parts in the water. Several large-scale tests of OWCs have amply demonstrated the reliability of at least the shore-based operations\textsuperscript{(2)}.

In general, OWCs have one water chamber and one air chamber as shown in Fig.1. On the other hand, two-chamber OWCs have also been studied to increase the output power by effective phase control of valves. More recently, Min-Fu Hsieh et al. have proposed a new design with two adjacent chambers, each with a turbine and a generator, aligned in the direction of wave propagation to smooth the output power\textsuperscript{(4)}.

With the objective of enhancing the efficiency and smoothing the output power, the authors have investigated the primary conversion efficiency of OWCs with multiple chambers numerically: two-chamber OWC with two water chambers and two air chambers as shown in Fig.2, and two-chamber OWC with two water chambers and one air chamber as illustrated in Fig.3.

The numerical results are compared with those of a single-chamber OWC (Fig.1). This paper presents...
the results of 2D numerical analysis in frequency domain for the OWCs by Nagata et al.’s method. The major physical quantities examined are the primary conversion efficiency, $\text{EFF}$; the reflection coefficient, $K_R$; and the air pressure in the air chambers, $p/w_0H$. Where the pressure $p$ is nondimensionalized using the specific weight of water, $w_0$, and the incident wave height, $H$. The numerical model is validated by comparing with the experimental data of one-chamber OWC by Ojima et al.\textsuperscript{6,7)}

2. FORMULATION OF THE NUMERICAL MODEL

The numerical model is developed by combining the hydrodynamics between the wave and OWC, and the thermodynamics of the air chamber. The fluid force is calculated using the boundary element method based on the velocity potential theory. Assuming air to be an ideal gas, the air flow is calculated using an equation of state and the equations of conservation of mass and energy. Finally, $\text{EFF}$, $p/w_0H$, and $K_R$ are calculated using the boundary integral equations.

In this section, the numerical model for the OWC with two water chambers and two air chambers (Fig.2) is presented. The numerical model for the OWC shown in Fig.3 can be obtained by a simple modification of this model.

(1) Equations related to wave motion
Assuming the fluid motion to be inviscid, incompressible, and of small amplitude, the potential theory gives the linearized governing equations for the velocity potential $\Phi(x,z;\tau)$ as follows:

$$\nabla^2 \Phi = 0 \quad \text{in the fluid}$$  (1)

$$\frac{\partial \zeta}{\partial t} = \frac{\partial \Phi}{\partial z} \quad \text{on SF}_0, \text{SF}_1, \text{SF}_2$$  (2)

$$\frac{\partial \Phi}{\partial t} + \frac{\mu}{\rho} \frac{\partial \Phi}{\partial z} + g \zeta = -\frac{1}{\rho} (p_0 + p_1) \quad \text{on SF}_1$$  (3)

$$\frac{\partial \Phi}{\partial t} + \frac{\mu}{\rho} \frac{\partial \Phi}{\partial z} + g \zeta = -\frac{1}{\rho} (p_0 + p_2) \quad \text{on SF}_2$$  (4)

$$\frac{\partial \Phi}{\partial t} + \frac{\mu}{\rho} \frac{\partial \Phi}{\partial z} + g \zeta = -\frac{p_0}{\rho} \quad \text{on SF}_0$$  (5)

$$\frac{\partial \Phi}{\partial \nu} = 0 \quad \text{on S}_0, S_B$$  (6)

where $\zeta$ is the water surface elevation; $p_0$ is the atmospheric pressure; $p_1$ and $p_2$ are the dynamic air pressures in air chambers 1 and 2, respectively; $\rho$ is the density of fluid; $g$ the acceleration due to gravity; and $\nu$ the unit normal vector on the body surface. The external free water surface, the seabed, the wetted surface of the body, and the internal water surfaces in chamber 1 and chamber 2 are denoted by $\text{SF}_0$, $S_0$, $S_B$, $\text{SF}_1$, and $\text{SF}_2$, respectively. The artificial damping force due to Rayleigh is denoted by $\mu$ in Eqs.(3) and (4). Combining Eqs.(2), (3), and (5) and and Eqs.(2), (4), and (5), the following equations are obtained:
The fluid region is divided into four: region 0, region 1, region 2 and region 3 as shown in Fig. 2. The water depth is \( h \) in region 0. Region 0 is the outer region of the OWC. Regions 2 and 3 correspond to the fluid region in chambers 1 and 2 of the OWC, respectively. The velocity potential \( \Phi(x,z;t) \) in region 0 is obtained as the solution of Laplace’s equation, which satisfies the free surface and the bottom boundary conditions at constant water depth \( h \) as follows:

\[
\Phi_0(x,z;t) = \frac{g \zeta_0}{\sigma} \left[ e^{ikx} + K_k e^{-ikx} \right] \frac{\cosh (z+h)}{\cosh kh} e^{-i\tau} \tag{10}
\]

where the first term in the right-hand side corresponds to the incident wave, and the second term has the complex constant \( K_k \) representing the reflected wave. \( \zeta_0 \) in Eq.(10) denotes the amplitude of the incident wave. The wave number \( k \) in the equation is obtained by solving the following dispersion relation:

\[
\sigma^2 = gk \tanh kh \tag{11}
\]

The velocity potential \( \Phi \), air pressure in the air chambers \( p_1 \) and \( p_2 \), and the coordinate system are non-dimensionalized using water depth \( h \) as

\[
\Phi(x,z;t) = \frac{g \zeta_0}{\sigma} \text{Re} \left[ \phi(x',z') e^{i\tau} \right] \tag{12}
\]

\[
p_1(t) = \rho g \zeta_0 \text{Re} \left( p_1 e^{i\tau} \right) \tag{13}
\]

\[
p_2(t) = \rho g \zeta_0 \text{Re} \left( p_2 e^{i\tau} \right) \tag{14}
\]

\[
\zeta(x;t) = \zeta_0 \text{Re} \left( \zeta e^{i\tau} \right) \tag{15}
\]

\[
x' = \frac{x}{h}, \ z' = \frac{z}{h} \tag{16}
\]

These dimensionless terms are used hereafter without the primes.

(2) The boundary value problem

The boundary value problems for the potential functions in region 1 \( \phi^{(1)} \), in region 2 \( \phi^{(2)} \), and in region 3 \( \phi^{(3)} \) can be written as

a) Region 1

\[
\nabla^2 \phi^{(1)} = 0 \quad \text{in the fluid} \tag{17}
\]

\[
\frac{\partial \phi^{(1)}}{\partial \nu} = \Gamma \phi^{(1)} \quad \text{on SF}_0 \tag{18}
\]

\[
\frac{\partial \phi^{(1)}}{\partial \nu} = 0 \quad \text{on} \ S_0, S_B \tag{19}
\]

\[
\phi^{(1)}(l,z) = \left( e^{-i\tau} + K_k e^{i\tau} \right) A(\tau z) \quad \text{on RS} \tag{20}
\]

\[
\frac{\partial \phi^{(1)}}{\partial \nu}(l,z) = -i\tau \left[ e^{-i\tau} - K_k e^{i\tau} \right] A(\tau z) \quad \text{on RS} \tag{21}
\]

where

\[
A(\tau z) = \frac{\cosh \tau (z+1)}{\cosh \tau} \\
\Gamma = \frac{\sigma h}{g} \\
\tau = kh \tag{22}
\]

b) Region 2

\[
\nabla^2 \phi^{(2)} = 0 \quad \text{in the fluid} \tag{23}
\]

\[
\frac{\partial \phi^{(2)}}{\partial \nu} = \Gamma \left[ ip_1 \left( 1 - i - \frac{\mu}{\sigma} \right) \phi^{(2)} \right] \quad \text{on SF}_1 \tag{24}
\]

\[
\frac{\partial \phi^{(2)}}{\partial \nu} = 0 \quad \text{on} \ S_B \tag{25}
\]

d) Kinematical conditions on boundary DC

\[
\phi^{(1)} = \phi^{(2)} \tag{29}
\]

\[
\frac{\partial \phi^{(1)}}{\partial \nu} = \frac{\partial \phi^{(2)}}{\partial \nu} \tag{30}
\]

e) Kinematical conditions on boundary ED

\[
\phi^{(1)} = \phi^{(3)} \tag{31}
\]

\[
\frac{\partial \phi^{(1)}}{\partial \nu} = \frac{\partial \phi^{(3)}}{\partial \nu} \tag{32}
\]

(3) Boundary integral equations

The boundary enclosing the fluid region is divided into \( N \) elements by \( N \) points (Fig.4). If we denote the
center and the length of each element by \( j = (\xi_j, \eta_j) \) and \( \Delta S_j (j=1-N) \), respectively, relationships between the potentials on the boundary \( \phi(j) = \phi(\xi_j, \eta_j) \) and their normal derivatives \( \partial \phi(j) = \partial \phi(\xi_j, \eta_j) \) are given by Green’s theorem as

\[
\sum_{j=1}^{N} \left[ F_{mj} \phi(j) - E_{mj} \bar{\phi}(j) \right] = 0 \tag{33}
\]

where

\[
\begin{align*}
F_{mj} &= -\delta_{mj} + \overline{E}_{mj} \quad \text{for region 1} \\
F_{mj} &= \delta_{mj} + \overline{E}_{mj} \quad \text{for regions 2 and 3} \\
E_{mj} &= \frac{1}{\pi} \int_{M_j} \log R_{mj} ds \\
\overline{E}_{mj} &= \frac{1}{\pi} \int_{M_j} \frac{\partial}{\partial \nu} \log R_{mj} ds \\
R_{mj} &= \left[ (\xi_j - \xi_m)^2 + (\eta_j - \eta_m)^2 \right]^{-1/2}
\end{align*}
\]

a) Region 1

\[
\begin{align*}
\sum_{j=1}^{N_{1}} \left[ F_{mj}^{(1)} - \Gamma E_{mj}^{(1)} \right] \phi_{1}^{(j)}(j) + \sum_{j=1}^{N_{1}} \left[ F_{mj}^{(1)} \phi_{1}^{(j)}(j) \right] \\
+ \sum_{j=1}^{N_{2}} \left[ F_{mj}^{(2)} \phi_{2}^{(j)}(j) - E_{mj}^{(2)} \phi_{2}^{(j)}(j) \right] \\
+ \sum_{j=1}^{N_{3}} \left[ F_{mj}^{(3)} \phi_{3}^{(j)}(j) - E_{mj}^{(3)} \phi_{3}^{(j)}(j) \right] \\
+ \sum_{j=1}^{N_{4}} \left[ F_{mj}^{(4)} \phi_{4}^{(j)}(j) - E_{mj}^{(4)} \phi_{4}^{(j)}(j) \right] + \sum_{j=1}^{N_{5}} F_{mj}^{(5)} \phi_{5}^{(j)}(j) \tag{34}
\end{align*}
\]

\[
+ K_R \sum_{j=1}^{N_{1}} \left[ F_{mj}^{(1)} + i \tau E_{mj}^{(1)} \right] e^{i \tau l} A(kz_j)
\]

\[
= - \sum_{j=1}^{N_{1}} \left[ F_{mj}^{(1)} - i \tau E_{mj}^{(1)} \right] e^{-i \tau l} A(kz_j)
\]

The number of source point \( m \) on the boundary SR is one. Hence the total number of source points \( m \) in Eq.(34) is \((N_{1}^{(1)} + N_{2}^{(1)} + N_{3}^{(1)} + N_{4}^{(1)} + N_{5}^{(1)} + 1)\).

b) Region 2

\[
\begin{align*}
\sum_{j=1}^{N_{2}} \left[ F_{mj}^{(2)} + \Gamma E_{mj}^{(2)} \left( 1 - i \frac{\mu}{\sigma} \right) \right] \phi_{2}^{(j)}(j) \\
+ \sum_{j=1}^{N_{2}} \left[ F_{mj}^{(2)} \phi_{2}^{(j)}(j) \right] \\
+ \sum_{j=1}^{N_{3}} \left[ F_{mj}^{(2)} \phi_{3}^{(j)}(j) - E_{mj}^{(2)} \phi_{3}^{(j)}(j) \right] \\
+ \sum_{j=1}^{N_{3}} \left[ F_{mj}^{(2)} \phi_{4}^{(j)}(j) - E_{mj}^{(2)} \phi_{4}^{(j)}(j) \right] + \sum_{j=1}^{N_{5}} F_{mj}^{(2)} \phi_{5}^{(j)}(j)
\end{align*}
\]

\[
\left( \sum_{j=1}^{N_{2}} \left[ F_{mj}^{(2)} \phi_{3}^{(j)}(j) - E_{mj}^{(2)} \phi_{3}^{(j)}(j) \right] + \sum_{j=1}^{N_{3}} \left[ F_{mj}^{(2)} \phi_{4}^{(j)}(j) - E_{mj}^{(2)} \phi_{4}^{(j)}(j) \right] \right) \rho_{a}^{2} = 0
\]

The total number of source points \( m \) in Eq.(35) is \((N_{1}^{(2)} + N_{2}^{(2)} + N_{3}^{(2)} + N_{4}^{(2)} + N_{5}^{(2)} + N_{6}^{(2)})\).

c) Region 3

\[
\begin{align*}
\sum_{j=1}^{N_{3}} \left[ F_{mj}^{(3)} + \Gamma E_{mj}^{(3)} \left( 1 - i \frac{\mu}{\sigma} \right) \right] \phi_{3}^{(j)}(j) \\
+ \sum_{j=1}^{N_{3}} \left[ F_{mj}^{(3)} \phi_{3}^{(j)}(j) \right] \\
+ \sum_{j=1}^{N_{3}} \left[ F_{mj}^{(3)} \phi_{4}^{(j)}(j) - E_{mj}^{(3)} \phi_{4}^{(j)}(j) \right] \\
+ \sum_{j=1}^{N_{3}} \left[ F_{mj}^{(3)} \phi_{5}^{(j)}(j) - E_{mj}^{(3)} \phi_{5}^{(j)}(j) \right] + \sum_{j=1}^{N_{5}} F_{mj}^{(3)} \phi_{5}^{(j)}(j)
\end{align*}
\]

\[
\left( \sum_{j=1}^{N_{3}} \left[ F_{mj}^{(3)} \phi_{3}^{(j)}(j) - E_{mj}^{(3)} \phi_{3}^{(j)}(j) \right] + \sum_{j=1}^{N_{3}} \left[ F_{mj}^{(3)} \phi_{4}^{(j)}(j) - E_{mj}^{(3)} \phi_{4}^{(j)}(j) \right] \right) \rho_{a}^{2} = 0
\]

The total number of source points \( m \) in Eq.(36) is \((N_{1}^{(3)} + N_{2}^{(3)} + N_{3}^{(3)} + N_{4}^{(3)} + N_{5}^{(3)})\).

(4) Thermodynamics in air chambers

The equations presented in this section are applicable to both chambers. Therefore, subscripts 1 and 2 denoting the respective chambers are dropped here and will be reintroduced later. Assuming air as a perfect gas, the equation of state, the equation of continuity, and the equation of conservation of energy are given as

\[
\frac{P_a}{\rho_a} = RT_a \tag{37}
\]

\[
\frac{d}{dt} (\rho_a V_a) + \frac{dm_a}{dt} = 0 \tag{38}
\]
\[ p_a \frac{dV_a}{dt} + C_v \frac{d(p_a V_a)}{dt} + C_p \frac{dV_a}{dt} + \rho_a \frac{dm_a}{dt} = 0 \]  
(39)

where \( p_a \) is the mass density of air in the chamber, \( R \) the gas constant of air, \( T_a \) the temperature in the air chamber, \( T_e \) the absolute temperature of air, \( V_a \) the volume of air in the air chamber, \( C_v \) the specific heat at constant volume, and \( C_p \) the specific heat at constant pressure, \( \frac{dm_a}{dt} \) is the rate of the mass of the air outflow (or inflow) through the nozzle on the ceiling of the air chamber and is expressed by

\[ \frac{dm_a}{dt} = \pm \rho_a \epsilon C_d C_s A W \sqrt{2C_p(T_a - T_0)} \]  
(40)

where \( C_d \) is the coefficient of contraction, \( C_s \) the coefficient of velocity, and \( T_0 \) the absolute temperature of air in open air. The area of the water chamber at still water level is denoted by \( A_W \) where \( l \) is the length of the air chamber (equal to the total length of OWC, \( B \) in Figs. 1~3) and \( W \) the width.

The nozzle ratio \( \epsilon \) is defined as the ratio of the area of the nozzle opening \( A_n \) to \( A_W \). \( \rho_e \) is the mass density of air, \( \rho_e = \rho_a \) for outflow and \( \rho_e = \rho_0 \) for inflow. \( \rho_0 \) is the mass density of atmospheric air.

In this paper, Nakagawa and Ueki method is used in which the pressure and the average water surface elevation in the air chamber is given based on Eqs. (37)~(40) and by linearizing the nonlinear term in Eq. (40). We consider the case of outflow first. The following equation is obtained by using Eq. (37) in Eq. (38):

\[ \frac{d}{dt} \left( \frac{p_a V_a}{RT_a} \right) + \frac{dp_a}{dt} + \frac{dm_a}{dt} = 0 \]  
(41)

Eliminating \( \frac{dm_a}{dt} \) in Eq. (39) using Eq. (41) leads to

\[ \frac{1}{T_a} \left( \frac{dT_a}{dt} \right) = \frac{R}{C_p} \frac{dp_a}{dt} \]  
(42)

Eq. (43) is obtained from Eqs. (37), (39), and (40).

\[ \frac{C_e}{V_a} \frac{dV_a}{dt} + \frac{C_d}{p_a} \frac{dp_a}{dt} + \frac{C_d C_s A_W}{V_a} \sqrt{2C_p(T_a - T_0)} \]  
(43)

We assume the magnitude of the variations in \( p_a, V_a \) and \( T_a \) are small enough so that they can be written as

\[ p_a(t) = p_0 + p_\epsilon(t) = p_0 + \text{Re} \left( \hat{p}_e e^{i\omega t} \right) \]

\[ V_a(t) = V_0 + \text{Re} \left( \hat{v} e^{i\omega t} \right) \]

\[ T_a(t) = T_0 + \text{Re} \left( \hat{T}_e e^{i\omega t} \right) \]

and comparing the coefficient of \( e^{i\omega t} \) we obtain

\[ \left( 1 - \frac{1}{\gamma} \right) \left( \frac{\hat{p}_e}{p_0} \right) - \frac{\hat{T}_e}{T_0} = 0 \]  
(45)

\[ \text{Re} \left[ i\sigma \left( \frac{\hat{v}}{V_0} \right) e^{i\omega t} \right] + \frac{1}{\gamma} \text{Re} \left[ i\sigma \left( \frac{\hat{p}_e}{p_0} \right) e^{i\omega t} \right] \]  
(46)

\[ + \frac{\epsilon C_d C_s A_W}{V_0} \sqrt{2C_p T_0} \sqrt{\text{Re} \left( \frac{\hat{T}_e}{T_0} e^{i\omega t} \right)} = 0 \]

where \( \gamma = \frac{C_p}{C_v} \) is the specific heat ratio. The nonlinear term in Eq. (46) is linearized as follows:

\[ \sqrt{\text{Re} \left( \frac{\hat{T}_e}{T_0} e^{i\omega t} \right)} = \alpha \text{Re} \left( \frac{\hat{T}_e}{T_0} \right) e^{i\omega t} \]  
(47)

where

\[ \alpha = \frac{2}{\pi} \frac{1}{T_0} \int_{-\pi/2}^{\pi/2} \cos \chi \sqrt{\cos \chi} d\chi \]  
(48)

We obtain the following equation after substituting Eq. (47) in Eq. (46)

\[ i\sigma \left( \frac{\hat{v}}{V_0} \right) + \frac{i\sigma}{\gamma} \left( \frac{\hat{p}_e}{p_0} \right) + \beta \left( \frac{\hat{T}_e}{T_0} \right) = 0 \]  
(49)

where

\[ \beta = \frac{\beta_0}{\sqrt{\left( \frac{\hat{T}_e}{T_0} \right)}} \]  
(50)

\[ \beta_0 = \frac{\epsilon C_d C_s A_W}{V_0} \sqrt{2C_p T_0} \alpha_0 \]  
\[ \alpha_0 = \frac{2}{\pi} \int_{-\pi/2}^{\pi/2} \cos \chi \sqrt{\cos \chi} d\chi \]

From Eq. (45) and Eq. (49) we obtain
\[
\hat{p}_s = \frac{-i\sigma}{p_0 \cdot i\sigma + \beta(y - 1) \cdot V_0} \hat{v} \quad (51)
\]

Eq.(51) is valid for the case of inflow as well. The volume of air in the air chamber is expressed as

\[
V_a = V_0 - \int_{S_w} \zeta dxdy \quad (52)
\]

where \(S_w\) is the water surface in the air chamber at still water level. Substituting Eq.(15) in Eq.(52) and comparing with the second equation of Eq.(44), we obtain

\[
\hat{v} = -\int_{S_w} \zeta_0 \zeta^a dxdy \quad (53)
\]

From Eqs.(51), (53), and \(\hat{p}_s = \rho g \zeta_0 p_s^a\), we obtain

\[
p_s^a = \frac{iC_E}{l_w} \int_{h_a} \zeta^a dx / l_c \quad (54)
\]

where \(l_w\) is the water line along \(x\)-axis in the air chamber and

\[
C_E = \frac{p_0}{\rho gh D_0} \cdot \frac{\sigma \gamma}{i\sigma + \beta(y - 1)} \quad (55)
\]

where \(D_0\) is the height of the air chamber. Eqs.(3)–(4) can be used to express the complex amplitude of the free water surface \(\zeta^a\) as

\[
\zeta^a = \left[ i \left(1 - i \frac{\mu}{\sigma}\right) p + p_s^a \right] \quad (56)
\]

Finally, by substituting Eq.(56) into Eq.(54) and discretizing, we obtain the following for chambers 1 and 2, respectively, where the corresponding subscripts have been reintroduced:

\[
p_{s1}^a = \frac{C_{E1}}{(1 + iC_{E1})l_{c1}} \sum_{j=1}^{N_{c1}} \left(1 - i \frac{\mu}{\sigma}\right) \phi^{(1)}(j) \Delta x_j \quad (57)
\]

\[
p_{s2}^a = \frac{C_{E2}}{(1 + iC_{E2})l_{c2}} \sum_{j=1}^{N_{c2}} \left(1 - i \frac{\mu}{\sigma}\right) \phi^{(2)}(j) \Delta x_j \quad (58)
\]

Here, \(l_{c1}\) and \(l_{c2}\) are the lengths of the back and the front air chambers. For the OWCs under consideration (Figs.2–3), \(l_{c2} = h_c\).

(5) Primary conversion efficiency

The equations presented in this section are applicable to both the chambers and therefore subscripts 1 and 2 denoting chamber 1 and chamber 2 are dropped. The total efficiency of the system is obtained by adding the contributions from the individual chambers. The wave power of the incident wave \(E_W\) is expressed as

\[
E_W = \frac{\rho g^2 \zeta^2 W}{4\sigma f(kh)} \quad (59)
\]

where

\[
f(kh) = \frac{2\cosh^2 kh - \sinh 2kh}{2kh + \sinh 2kh} \quad (60)
\]

The absorbed power by OWC, \(E_{air}\), can be written as

\[
E_{air} = \hat{p}_a(t)Q(t) \quad (61)
\]

where \(Q(t)\) is the air outflow (or inflow) rate through the nozzle of the air chambers and the overhead bar denotes time average over a period. \(Q(t)\) can be written as

\[
Q(t) = \pm \varepsilon C_d C_s A_w \sqrt{2C_p \left|T_a - T_0\right|}
\]

\[
= e\alpha C_d C_s A_w \sqrt{2C_p T_0} \text{Re} \left[ \frac{\hat{p}}{T_0} e^{i\alpha} \right] \quad (62)
\]

Finally, \(E_{air}\) can be written as

\[
E_{air} = \frac{1}{2} \rho^2 \zeta_0^2 \beta_1 \text{Re} \left[ p_s^a p_s^{a*} \right] \quad (63)
\]

where

\[
\beta_1 = e\alpha C_d C_s A_w \sqrt{2C_p T_0} \left(1 - \frac{1}{\gamma}\right) \frac{1}{p_0} \quad (64)
\]

The asterisk mark in Eq.(63) denotes the complex conjugate. The primary conversion efficiency can now be written as

\[
EFF = \frac{E_{air}}{E_W} \quad (65)
\]
3. VALIDATION OF THE NUMERICAL MODEL

Owing to the lack of experimental data for two-chamber OWCs, the validation of the proposed numerical model was done using the experimental data of one-chamber OWC by Ojima et al.\(^6\) The numerical model proposed in section 2 was modified and applied to the one-chamber OWC (Fig.1).

The specifications of the OWC used in the numerical calculation is given in Table 1. The values for \(C_d\) and \(C_s\) are obtained from Ojima et al.\(^5\) The primary conversion efficiency, the reflection coefficient, and the air pressure have been plotted for various values of the nozzle ratio, \(\varepsilon\), and the depth-to-wavelength ratio, \(h/\lambda\). Figure 5 shows that for a given wave period, EFF increases with the nozzle ratio, peaks and then decreases again.

### Table 1 Specifications of OWC used for validation.

| Items                  | Values |
|------------------------|--------|
| Length of OWC (\(B=lc\)) | 0.40m  |
| Water depth (\(b\))    | 0.60m  |
| Curtain wall depth (\(d_c\)) | 0.20m  |
| Height of air chamber (\(D_0\)) | 0.40m  |

\(\gamma\) 1.4015

![Fig.5](image_url) Primary conversion efficiency vs nozzle ratio.

![Fig.6](image_url) Reflection coefficient vs \(h/\lambda\).

![Fig.7](image_url) Air pressure vs \(h/\lambda\).
Table 2 Specifications for the analysis of two-chamber OWC.

| Items                              | Values       |
|------------------------------------|--------------|
| Total length of OWC ($B=l_c$)      | 0.40m        |
| Length of back air chamber ($l_1$) | 0.20m        |
| Length of front air chamber ($l_2$)| 0.20m        |
| Opening height of the back chamber ($d_1$) | 0.20m |
| Opening height of the front chamber ($d_2$) | 0.20m |
| Water depth ($h$)                  | 0.60m        |
| Curtain wall depth ($d_c$)         | 0.20m        |
| Height of air chambers ($D_0$)     | 0.40m        |
| $\gamma$                          | 1.4015       |

Figure 6 shows that for a given nozzle ratio, $K_R$ decreases with the relative depth, reaches a minimum, and then increases again with a further increase of the relative depth. As shown in Fig. 7, the air pressure increases with the relative depth first, reaches a maximum value, and decreases again with a further increase of the relative depth. It is found from Figs. 5–7 that the numerical results are in good agreement with the experiment results for EFF, $K_R$, and $p/w_0H$.

4. NUMERICAL ANALYSIS

(1) Primary conversion efficiency, air pressure and reflection coefficient of two-chamber OWC with two air chambers

The variations in the primary conversion efficiency, the reflection coefficient, and the peak air pressure for the two-chamber OWC shown in Fig. 2 are analyzed using the numerical model presented in section 2. The specifications of the device under consideration is given in Table 2. The values for $C_d$ and $C_s$ are obtained from Ojima et al.6)

Each of the chambers of the two-chamber OWC contributes to the overall efficiency of the OWC. The total efficiency is estimated as the sum of these contributions. The combined efficiency is between 60 to 90 percent and is larger for short wave periods as shown in Figs. 8(a)–(d). In these figures, Chr 1 and Chr 2 denote the back chamber and the front chamber, respectively. The contribution of chamber 2 is much greater than that of chamber 1 for short wave periods and decreases with the increase of the period. For $T=1.15$s and 1.5$s, EFF is highest for nozzle ratio slightly greater than 1/100. For $T=2.0$s it is greatest at $\varepsilon=1/100$ and for longer wave periods it is highest at smaller values of the nozzle ratio. The peak air pressures in the air chambers plotted against $h/\lambda$ ratio are shown in Figs. 9(a)–(d). The pressures in chamber 1 and 2 peak at different values of $h/\lambda$. In general, the pressure increases with increasing $h/\lambda$, peaks and then decreases again. The reflection coefficient is
plotted against \( h/\lambda \) for various values of the nozzle ratio as shown in Figs.10(a)–(d). For nozzle ratio 1/100, the minimum value of \( K_R \) is less than 0.4. For nozzle ratios 1/200 and 1/300, \( K_R \) has the smallest magnitude for the relative depth ratio of about 0.15 to 0.2.

(2) Effect of curtain wall depth and front chamber width in two-chamber OWC

In this section, the effects of the variations in the curtain wall depth, \( d_c \), and the front chamber width, \( b_c \), on EFF, \( p/w_0H \), and \( K_R \) are investigated. The wave height and the nozzle ratio in the calculation are 10.0 cm and 1/100, respectively.

Figure 11 shows that the primary conversion efficiency of chamber 1 is independent of \( d_c/h \) for all wave periods. In the case of chamber 2, for \( T=1.15s \) and 1.50s, the primary conversion efficiency decreases with \( d_c/h \). This effect is more pronounced for \( T=1.15s \). However, for longer wave periods there is slight increase with \( d_c/h \). Figure 12 shows that the reflection coefficient increases with \( d_c/h \) for \( T=1.15s \) and 1.5s and decreases for \( T=2.0s, 2.5s \) and 3.0s. Figure 13 shows that no significant effect of \( d_c/h \) on the air pressure in chamber 1 is noted. In chamber 2, for \( T=1.15s \) and 1.5s, the air pressure decreases with \( d_c/h \), and for \( T=2.0s, 2.5s, \) and 3.0s, it increases by a small amount.

Figures 14–16 show the effect of the variations in \( b_c/h \) on various physical quantities. Figure 14 shows that for \( T=1.15s \) and 1.5s, EFF increases with the increase of \( b_c/h \). For these shorter wave periods, chamber 2 is more significant than chamber 1. For longer wave periods, EFF decreases slightly with \( b_c/h \). As for \( K_R \), it decreases with \( b_c/h \) for \( T=1.15s \) and 1.5s, and increases slightly for other wave periods (Fig.15). From Fig.16, it is found that for chamber 1, the pressure increases with \( b_c/h \) for \( T=1.15s \) and 1.5s, and decreases slightly for other
wave periods. For chamber 2, the air pressure decreases with \( \frac{b_c}{h} \) for \( T=1.15s \) and 1.5s, and increases slightly for other wave periods.

(3) Comparison between one-chamber OWC, two-chamber OWC with two air chambers and two-chamber OWC with one air chamber

The primary conversion efficiency, the reflection coefficient, and the peak air pressure are computed for one-chamber OWC, two-chamber OWC with two
Fig. 13 Air pressure vs $d_c/h$.

Fig. 14 Primary conversion efficiency vs $b_c/h$.

Fig. 15 Reflection coefficient vs $b_c/h$.

Fig. 16 Air pressure vs $b_c/h$.

Fig. 17 Primary conversion efficiency vs $\lambda/B$.

Fig. 18 Reflection coefficient vs $\lambda/B$.

Fig. 19 Air pressure in the air chamber vs $\lambda/B$.

Air chambers and two-chamber OWC with one air chamber and plotted against $\lambda/B$ ratio where $B$ is the total length of the OWC. The dimensions of the device in the calculation are the same as in Table 1 for one-chamber OWC and Table 2 for two-chamber OWC with two air chambers. The dimensions of the two-chamber OWC with one air chamber are also the same as those given in Table 2. However, in this case, there is only one air chamber with one nozzle as shown in Fig. 3. The wave height was 5cm and the nozzle ratio was 1/100 in the calculation.

The primary conversion efficiency increases with $\lambda/B$, reaches a maximum, and decreases again for the one-chamber OWC and two-chamber OWC with two
air chambers as shown in Fig.17. However, for the OWC with two chambers and one air chamber, EFF shows two peaks attaining a lowest value at \( \lambda/B = 8 \). The reflection coefficient shows a similar trend where the two-chamber OWC with one air chamber has double troughs as shown in Fig.18. This result is consistent with the result for EFF. The variations in air pressure in the air chamber as plotted in Fig.19 also show two peaks and a trough for the OWC with two chambers and a single air chamber. The peak pressure for this OWC is greater than that for the one-chamber OWC for larger values of \( \lambda/B \).

The result of our analysis suggests that the two-chamber OWC with two air chambers and one-chamber OWC with one air chamber provide a more smooth power output compared to the two-chamber OWC with one air chamber. In addition, the peak value of EFF of two-chamber OWC with two air chambers is slightly greater for longer wave periods. It was also observed that front chamber was more effective at shorter wave periods. As for the nozzle area ratio, the value of \( \varepsilon \) for which the primary conversion efficiency is highest lies in the vicinity of 1/100, and the exact values differ depending on the incident period. In the case of the back chamber, EFF decreased with the increase of \( \varepsilon \) for \( T = 1.15 \)s and 1.5s, and increased for other wave periods. The total EFF also showed a similar behavior. For the case of the width of the front chamber, the total EFF increased with the increasing value of \( \alpha/h \) for \( T = 1.15 \)s and 1.5s, and decreased for other periods. As expected, EFF for the front chamber decreased rapidly with \( \alpha/h \) and vice versa for the back chamber.

5. CONCLUSIONS

This study investigated two-chamber OWCs using a two-dimensional numerical method based on the velocity potential theory in frequency domain. Calculations were made for three physical quantities: primary conversion efficiency, air pressure, and reflection coefficient. Comparison with one-chamber OWC with the same physical dimensions showed that primary conversion efficiency, EFF, for the two-chamber OWC with two air chambers was slightly greater for longer wave periods. It was also observed that front chamber was more effective at shorter wave periods. As for the nozzle area ratio, the value of \( \varepsilon \) for which the primary conversion efficiency is highest lies in the vicinity of 1/100, and the exact values differ depending on the incident period. In the case of the two-chamber OWC with one air chamber, EFF showed two peaks when plotted against the wavelength-to-length of OWC ratio, \( \lambda/B \). Though the peak values were almost equal to the double chamber OWC with two air chambers, EFF was very small around \( \lambda/B = 8 \).

The effect of nondimensionalized curtain wall depth and front chamber width, \( d_c/h \) and \( b_c/h \), respectively, on the three physical quantities was studied. No significant change was found in the value of EFF of the front chamber with the increase of \( d_c/h \). In the case of the back chamber, EFF decreased with the increase of \( d_c/h \) for \( T = 1.15 \)s and 1.5s, and increased for other wave periods. The total EFF also showed a similar behavior. For the case of the width of the front chamber, the total EFF increased with the increasing value of \( b_c/h \) for \( T = 1.15 \)s and 1.5s, and decreased for other periods. As expected, EFF for the front chamber decreased rapidly with \( b_c/h \) and vice versa for the back chamber.

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