Measurement of the Mass Difference between $t$ and $\bar{t}$ Quarks

T. Aaltonen, B. Álvarez González, S. Amerio, D. Amidei, A. Anastassov, A. Anno, J. Antos, G. Apollinari, J. A. Appel, A. Apresyan, T. Arisawa, A. Artikov, J. Asaadi, W. Ashman, B. Auerbach, A. Aurisano, F. Azfar, W. Badgett, A. Barbaro-Galtieri, V. E. Barnes, B. A. Barnett, P. Barria, P. Bartos, M. Bause, G. Bauer, F. Bedeschi, D. Beecher, S. Behari, G. Bellettini, J. Bellinger, D. Benjamin, A. Beretvas, B. M. Blinkley, D. Biscello, B. Bizjak, K. R. Bland, B. Blumenfeld, A. Bocci, A. Bodek, D. Bortoletto, J. Boudreau, A. Boveia, B. Brau, R. Brignoli, A. Brusa, C. Bromberg, E. Brucken, M. Buecciantonio, J. Budagov, H. S. Budd, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmith, R. Carosi, S. Carrillo, B. Casal, M. Casarsa, A. Castro, P. Catastini, D. Cauz, V. Cavaliere, B. Brau, L. Brigliadori, A. Brisuda, C. Bromberg, E. Brucken, M. Bucciantonio, J. Budagov, H. S. Budd, K. Burkett, G. Busetto, P. Bussey, A. Buzatu, C. Calancha, S. Camarda, M. Campanelli, M. Campbell, F. Canelli, A. Canepa, B. Carls, D. Carlsmit
We present a direct measurement of the mass difference between $t$ and $\bar{t}$ quarks using $t\bar{t}$ candidate events in the lepton $+\text{jets}$ channel, collected with the CDF II detector at Fermilab's 1.96 TeV Tevatron $p\bar{p}$ Collider. We make an event by event estimate of the mass difference to construct templates for top quark pair signal events and background events. The resulting mass difference distribution of data is compared to templates of signals and background using a maximum likelihood fit. From a sample corresponding to an integrated luminosity of $5 \pm 0.6$ fb$^{-1}$, we measure a mass difference, $M_{\text{top}} - M_{\text{top}} = 3.3 \pm 1.4 \text{(stat)} \pm 1.0 \text{(syst)}$ GeV/$c^2$, approximately 2 standard deviations away from the $CPT$ hypothesis of zero mass difference.

DOI: 10.1103/PhysRevLett.106.152001 PACS numbers: 14.65.Ha, 12.15.Ff, 13.85.Ni, 13.85.Qk

Discrete symmetries reflecting the invariance under discrete transformations, such as charge conjugation ($C$), space reflection or parity ($P$), and time reversal ($T$), are not always exact. Examples include the $C$ and $P$ symmetries and their $CP$ combination, which are violated by the weak interactions [1]. $CPT$ symmetry, which reflects the invariance under the combined operation of $C$, $P$, and $T$ transformations, has not been found to be violated in any experiment so far [2,3]. However, it is important to examine the possibility of $CPT$ violation in all sectors of the
standard model, as there are well-motivated extensions of the standard model allowing for CPT symmetry breaking [4]. In the CPT theorem, particle and antiparticle masses must be identical; thus, a mass difference between a particle and its antiparticle would indicate a violation of CPT. The mass equality has been verified to high precision for leptons and hadrons, but not for quarks. With the exception of the top quark, it is impossible to measure quark masses directly, because a newly created quark dresses itself with other quarks and gluons to form a hadron, and hadron masses yield, at best, only rough estimates of the quark mass. The top quark is by far the most massive quark and, masses yield, at best, only rough estimates of the quark mass.

This Letter reports a measurement of the mass difference ($\Delta M_{\text{top}} = M_t - M_J$) between $t$ and $\bar{t}$ quarks using a sample of $t\bar{t}$ candidates in the lepton + jets final state. The data correspond to an integrated luminosity of 5.6 fb$^{-1}$ in proton-antiproton collisions at the Tevatron with $\sqrt{s} = 1.96$ TeV, collected with the CDF II detector [6]. Assuming unitarity of the three-generation Cabibbo-Kobayashi-Maskawa matrix, $t$ and $\bar{t}$ quarks decay almost exclusively into a $W$ boson and a bottom quark ($t \rightarrow bW^+$ and $\bar{t} \rightarrow \bar{b}W^-$) [1]. The case where one $W$ decays into a charged lepton and a neutrino ($W^+ \rightarrow \ell^+\nu$ or $W^- \rightarrow \ell^-\bar{\nu}$) and the other into a pair of jets defines the lepton + jets decay channel. The electric charge of the lepton ($-1$ for $\ell$ and $+1$ for $\ell$) determines the flavor of top quarks with event reconstruction. To select $t\bar{t}$ candidate events in this channel, we require one electron (muon) with $E_T > 20$ GeV ($p_T > 20$ GeV/$c$) and pseudorapidity $|\eta| < 1.1$ [7]. We also require high missing transverse energy [8], $E_T > 20$ GeV, and at least four jets. Jets are reconstructed with a cone algorithm [9] with radius $R = \sqrt{\Delta \eta^2 + \Delta \phi^2} \approx 0.4$. Jets originating from $b$ quarks are identified using a secondary vertex tagging algorithm [10]. In order to optimize the background reduction process and improve the statistical power of the events, we divide the sample of $t\bar{t}$ candidate events into subsamples with zero, one, and two or more $b$-tagged jets.

When an event has zero or one $b$-tagged jet, we require exactly four jets with transverse energy $E_T > 20$ GeV and $|\eta| < 2.0$. If an event has two or more $b$ jets, three jets are required to have $E_T > 20$ GeV and $|\eta| < 2.0$, and a fourth jet is required to have $E_T > 12$ GeV and $|\eta| < 2.4$, with no restriction on the total number of jets. To reject backgrounds, we require the scalar sum of transverse energies in the event, $H_T = E_{T,\text{lepton}} + E_T + \sum_{\text{four jets}} E_{T,jet}^j$, to be greater than 250 GeV.

The primary sources of background events are $W +$ jets and QCD multijet production. Contributions from $Z +$ jets, diboson, and single top production are expected to be small. To estimate the contribution of each process, we use a combination of data and Monte Carlo (MC) based techniques described in Ref. [11]. For the $Z +$ jets, diboson, and single top quark events, we normalized MC simulation events using their respective theoretical cross sections. The QCD multijet background is estimated with a data-driven approach. We model $W +$ jets background events using MC simulation, but the overall rate is determined using data after subtracting the rate of all the other backgrounds and $t\bar{t}$. Table I shows the expected background composition and the expected number of $t\bar{t}$ events.

We assume selected events to be $t\bar{t}$ events in the lepton + jets channel and reconstruct them to form estimators of $\Delta M_{\text{top}}$, using a special purpose kinematic fitter, in which we modify the standard fitter [12] to allow a mass difference between $t$ and $\bar{t}$. Measured four-vectors of jets and lepton are corrected for known effects [13], and resolutions are assigned. The unclustered transverse energy ($U_T$), which is the sum of all transverse energy in the calorimeter that is not associated with the primary lepton or one of the leading four jets, is used to calculate the neutrino transverse momentum. The longitudinal momentum of the neutrino is a free (unconstrained) parameter which is effectively determined by the constraint on the invariant mass of the leptonic $W$. We then define a kinematic fit $\chi^2$ having a free parameter $d m_{\text{reco}}$.

$$\chi^2 = \sum_{i=t,\bar{t},4\text{jets}}(p_T^{i,\text{fit}} - p_T^{i,\text{meas}})^2/\sigma_i^2 + \sum_{k=0,1}(U_T^{k,\text{fit}} - U_T^{k,\text{meas}})^2/\sigma_k^2 + (M_{jj} - M_W)^2/\Gamma_W^2 + (M_{W,\mu} - M_\mu)^2/\Gamma_\mu^2 + (M_{b}\nu - (M_\mu - d m_{\text{reco}}/2)^2)/\Gamma_\nu^2$$

where $d m_{\text{reco}}$ the $d m_{\text{reco}}$ value at the lowest $\chi^2$, represents the reconstructed mass difference between the hadronic and leptonic top decay ($M_{bjj} - M_{b\ell \nu}$). In this $\chi^2$ formulation, the first term constrains the $p_T$ of the lepton and four leading jets to their measured values within their uncertainties ($\sigma_i$); the second term does the same for both transverse components $x$ and $y$ of the unclustered transverse energy. In the remaining four terms, the quantities $M_{jj}$,

| TABLE I. | Expected and observed numbers of signal and background events assuming $t\bar{t}$ production cross section $\sigma_{t\bar{t}} = 7.4$ pb and $M_{\text{top}} = 172.5$ GeV/$c^2$. |
|-----------------|-----------------|-----------------|-----------------|
| | 0 b-tag | 1 b-tag | $\geq 2$ b-tag |
|-----------------|-----------------|-----------------|-----------------|
| $W +$ jets | 596 $\pm$ 98 | 883 $\pm$ 23.0 | 11.1 $\pm$ 3.6 |
| QCD multijet | 95.8 $\pm$ 74.4 | 14.7 $\pm$ 12.1 | 2.4 $\pm$ 3.2 |
| $Z +$ jets | 48.8 $\pm$ 9.4 | 5.7 $\pm$ 1.3 | 0.8 $\pm$ 0.2 |
| Diboson | 50.1 $\pm$ 4.7 | 6.6 $\pm$ 0.8 | 1.0 $\pm$ 0.2 |
| Single top | 4.0 $\pm$ 0.4 | 5.5 $\pm$ 0.5 | 2.2 $\pm$ 0.2 |
| Background | 795 $\pm$ 124 | 121 $\pm$ 24 | 17.3 $\pm$ 4.8 |
| $t\bar{t}$ signal | 426 $\pm$ 57 | 578 $\pm$ 72 | 282 $\pm$ 44 |
| Expected | 1220 $\pm$ 137 | 699 $\pm$ 76 | 299 $\pm$ 44 |
| Observed | 1278 | 720 | 296 |
CDF detector is simulated using a GEANT-based software.

The use of different detector components and the different resolutions of the measured values for jet, lepton, and unclustered energy make the reconstructed mass distribution of hadronic top quarks differ from that of leptonic top quarks. Because the sign of $\Delta m_{\text{reco}}$ depends on the lepton charge, $\Delta m_{\text{reco}}$ distributions for the positive and negative lepton events are different. We divide the sample into six subsamples, two samples with positively and negatively charged leptons for each of 0 b-tag, 1 b-tag, and 2 b-tag samples.

With the assumption that the leading four jets in the event come from the four final quarks at the hard scattering level, there are 12, 6, and 2 possible assignments of jets to quarks for 0 b-tag, 1 b-tag, and 2 b-tag, respectively. The minimization of $\chi^2$ is performed for each jet-to-parton assignment, and $\Delta m_{\text{reco}}$ is taken from the assignment that yields the lowest $\chi^2$ ($\chi^2_{\text{min}}$). Events with $\chi^2_{\text{min}} > 9.0$ ($\chi^2_{\text{min}} > 3.0$) are removed from the sample to reject poorly reconstructed events for b-tagged (zero b-tagged) events. To increase the statistical power of the measurement, we employ an additional observable $\Delta m_{\text{reco}}^{(2)}$ from the assignment that yields the 2nd lowest $\chi^2$. Although it has a poorer sensitivity, $\Delta m_{\text{reco}}^{(2)}$ provides additional information on $\Delta M_{\text{top}}$ and improves the statistical uncertainty by approximately 10%.

Using MADGRAPH [15], we generate $t\bar{t}$ signal samples with $\Delta M_{\text{top}}$ between $-20$ and 20 GeV/$c^2$ using almost 2 GeV/$c^2$ step size, where we take the average mass value of $t$ and $\bar{t}$ to be $M_{\text{top}} = 172.5$ GeV/$c^2$. Parton showering of the signal events is simulated with PYTHIA [16], and the CDF detector is simulated using a GEANT-based software package [17].

We estimate the probability density functions (PDFs) of signal and background templates using the kernel density estimation (KDE) [18,19]. For the $\Delta M_{\text{top}}$ measurement with two observables ($\Delta m_{\text{reco}}$ and $\Delta m_{\text{reco}}^{(2)}$), we use the two-dimensional KDE that accounts for the correlation between them. First, at discrete values of $\Delta M_{\text{top}}$ from $-20$ to 20 GeV/$c^2$, we estimate the PDFs for the observables from the above-mentioned $t\bar{t}$ MC samples. We interpolate the MC distributions to find PDFs for arbitrary values of $\Delta M_{\text{top}}$ using the local polynomial smoothing method [20]. We fit the signal and background PDFs to the measured distributions of the observables in the data using an unbinned maximum likelihood fit [21], where we minimize the negative logarithm of the likelihood with MINUIT [22]. Likelihoods are built for each of six subsamples separately, and an overall likelihood is then obtained by multiplying them together. We evaluate the statistical uncertainty on $\Delta M_{\text{top}}$ by searching for the points where the negative logarithm of the likelihood exceeds the minimum by 0.5. References [18,23] provide detailed information about this technique.

We test the fitting procedure using 3000 MC pseudoexperiments (PEs) for each of 11 equally spaced $\Delta M_{\text{top}}$ values ranging from $-10$ to 10 GeV/$c^2$. The distributions of the average residual of measured $\Delta M_{\text{top}}$ (deviation from the input $\Delta M_{\text{top}}$) for simulated experiments is consistent with zero. However, the width of the pull (the ratio of the residual to the uncertainty reported by MINUIT) is 4% greater than unity. We therefore increase the measured uncertainty by 4%.

We examine a variety of systematic effects that could change the measurement by comparing results from PEs in which we vary relevant systematic parameters within their uncertainties. All systematic uncertainties are summarized in Table II. The dominant source of systematic uncertainty is the signal modeling, which we estimate using PEs with events generated with MADGRAPH and PYTHIA. We also estimate a parton showering uncertainty by applying different showering models (PYTHIA and HERWIG [24]) to a sample generated with ALPGEN [25]. We address a possible difference in the detector response between $b$ and $\bar{b}$ jets by comparing data and MC simulation events [26]. We add a systematic uncertainty due to multiple hadron interactions to account for the fact that the average number of interactions in our MC samples is not exactly equal to the number observed in the data. The jet energy scale, the dominant uncertainty in most of the top quark mass measurements, is partially canceled in the measurement of $\Delta M_{\text{top}}$.

| Source                        | Uncertainty (GeV/$c^2$) |
|-------------------------------|-------------------------|
| Signal modeling               | 0.7                     |
| $b$ and $\bar{b}$ jets asymmetry | 0.4                     |
| Jet energy scale              | 0.2                     |
| Parton distribution functions | 0.1                     |
| $b$-jet energy scale          | 0.1                     |
| Background shape              | 0.2                     |
| Gluon fusion fraction         | 0.1                     |
| Initial and final state radiation | 0.1                  |
| Monte Carlo statistics        | 0.1                     |
| Lepton energy scale           | 0.1                     |
| Multiple hadron interaction   | 0.4                     |
| Color reconnection            | 0.2                     |
| Total systematic uncertainty  | 1.0                     |
Figure 1 shows the measured distributions of the observables used for the $\Delta M_{\text{top}}$ measurement overlaid with density estimates using $t \bar{t}$ signal events with $\Delta M_{\text{top}} = -4$ and 0 GeV/$c^2$ and the full background model. The choice of $\Delta M_{\text{top}} = -4$ GeV/$c^2$ (solid line) gives better agreement with the data than that of 0 GeV/$c^2$ (dashed line).

In conclusion, we examine the mass difference between $t$ and $\bar{t}$ quarks in the lepton + jets channel using data corresponding to an integrated luminosity of $5.6$ fb$^{-1}$ from $p \bar{p}$ collisions at $\sqrt{s} = 1.96$ TeV. We measure the mass difference to be $\Delta M_{\text{top}} = M_t - M_{\bar{t}} = -3.3 \pm 1.4$(stat) $\pm 1.0$(syst) GeV/$c^2$. This result is consistent with CPT-symmetry expectation, $\Delta M_{\text{top}} = 0$ GeV/$c^2$, with approximately $2\sigma$ level deviations. It is consistent with the recent result from the D0 Collaboration [28], but is 2.2 times more precise. This is the most precise measurement of the mass difference between $t$ and $\bar{t}$ quarks.

We thank the Fermilab staff and the technical staffs of the participating institutions for their vital contributions.

This work was supported by the U.S. Department of Energy and National Science Foundation; the Italian Istituto Nazionale di Fisica Nucleare; the Ministry of Education, Culture, Sports, Science and Technology of Japan; the Natural Sciences and Engineering Research Council of Canada; the National Science Council of the Republic of China; the Swiss National Science Foundation; the A.P. Sloan Foundation; the Bundesministerium für Bildung und Forschung, Germany; the Korean World Class University Program, the National Research Foundation of Korea; the Science and Technology Facilities Council and the Royal Society, UK; the Institut National de Physique Nucleaire et Physique des Particules/CNRS; the Russian Foundation for Basic Research; the Ministerio de Ciencia e Innovación, and Programa Consolider-Ingenio 2010, Spain; the Slovak R&D Agency; the Academy of Finland; and the Australian Research Council (ARC).

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aDeceased.
bVisitor from University of Massachusetts Amherst, Amherst, MA 01003, USA.
cVisitor from Istituto Nazionale di Fisica Nucleare, Sezione di Cagliari, 09042 Monserrato (Cagliari), Italy.
dVisitor from University of California Irvine, Irvine, CA 92697, USA.
eVisitor from University of California Santa Barbara, Santa Barbara, CA 93106, USA.
fVisitor from CERN, CH-1211 Geneva, Switzerland.
gVisitor from Cornell University, Ithaca, NY 14853, USA.

hVisitor from University of California Santa Cruz, Santa Cruz, CA 95064, USA.
We use a right-handed spherical coordinate system with the origin at the center of the detector. $\theta$ and $\phi$ are the polar and azimuthal angles, respectively. The pseudorapidity is defined by $\eta = -\ln \tan(\theta/2)$. The transverse momentum and energy are defined by $p_T = p \sin(\theta)$ and $E_T = E \sin(\theta)$, respectively, where $p$ and $E$ are the momentum and energy of the particle.

[8] The missing transverse energy, an imbalance of energy in the transverse plane of the detector, is defined by $E_T = -\sum_{\text{tower}} E_T n_T$, where $n_T$ is the unit vector normal to the beam and pointing to a given calorimeter tower and $E_T$ is the transverse energy measured in that tower.

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[26] We select a $b\bar{b}$ sample by requiring exactly two $b$ jets per event using a sample triggered on jet ($E_T > 20$ GeV). In addition, one $b$ jet is required to contain a soft muon from leptonic decays so that we can determine the charge of the $b$ quark associated with the jet. The energy scale of $b$ and $\bar{b}$ jets in the data is compared with dijet MC events to estimate the uncertainty of $b$ and $\bar{b}$ jet energy scale independently. We perform PEs by varying the $b$ and $\bar{b}$ energy separately within their uncertainties.

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