Power Constrained Parallel Queuing with Contention*  
Elijah Hradovich, Marek Klonowski and Dariusz R. Kowalski  

Abstract  
We examine deterministic broadcasting on multiple-access channels for a scenario when packets are injected continuously by an adversary to the buffers of the devices at rate \( \rho \) packages per round. The aim is to maintain system stability, that is, bounded queues. In contrast to previous work we assume that there is a strict limit of available power, defined as the total number of stations allowed to transmit or listen to the channel at a given time, that can never be exceeded. We study how this constraint influences the quality of services with particular focus on stability. We show that in the regime of deterministic algorithms, the significance of energy restriction depends strongly on communication capabilities of broadcasting protocols. For the adaptive and full-sensing protocols, wherein stations may substantially adopt their behavior to the injection pattern, one can construct efficient algorithms using very small amounts of power without sacrificing throughput or stability of the system. In particular, we construct constant-energy adaptive and full sensing protocols stable for \( \rho = 1 \) and any \( \rho < 1 \), respectively, even for worst case (adversarial) injection patterns. Surprisingly, for the case of acknowledgment based algorithms that cannot adopt to the situation on the channel (i.e., their transmitting pattern is fixed in advance), limiting power leads to reducing the throughput. That is, for this class of protocols in order to preserve stability we need to reduce injection rate significantly. We support our theoretical analysis by simulation results of algorithms constructed in the paper. We depict how they work for systems of moderate, realistic sizes. We also provide a comprehensive simulation to compare our algorithms with backoff algorithms, which are common in real-world implementations, in terms of queue sizes and energy consumption.  

Keywords: Multiple-access channel, broadcasting, energy constraint, contention, stability, distributed algorithms, queue size, packet latency.  

1 Introduction  
Energy awareness becomes an important factor in design and analysis of communication protocols, especially in the context of distributed systems of mobile devices or Internet of Things. For systems of constrained battery-supplied devices, limiting energy expenditure can be critical and its importance is nowadays comparable to other metrics such as time and computational complexity. In this paper we investigate a classical communication model with \( n \) devices (stations) attached to a single communication medium, called a multiple-access channel, and an adversary injecting (in an arbitrary way) at most \( 0 < \rho \leq 1 \) packets per round, on average, to devices’ buffers. Due to constrains of the channel at most one station can successfully transmit a single packet during one round. The primary goal is to design an algorithm that guarantees stability, that is, a property that the sizes of queues in buffers stay bounded for the highest possible injection rate \( \rho \), which we will be calling throughput. Another important aim is fairness (equal access to the channel for all stations) and the related problem of minimization of packets’ latency.  

*E. Hradovich and M.Klonowski are from Department of Fundamental Problems of Technology, Wroclaw Univeristy of Science and Technology, Poland. E-mail: {Marek.Klonowski, Ilya.Hradovich}@pwr.edu.pl; D.R.Kowalski is from Department of Computer Science, University of Liverpool, Liverpool L69 3BX, UK. E-mail: D.Kowalski@liverpool.ac.uk  
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Our contribution In this paper we consider another factor, namely the energy consumption. In practice, energy in such systems is spent mostly on radio communication. Energy in our paper is measured as the maximal number of devices that transmit or listen to the channel simultaneously in a single round during (possibly infinite) execution of the algorithm. We investigate how limiting the energy of a distributed scheduler, by at most $k$ transmission attempts per round, influences the efficiency of the system. In other words, what injection rate $\rho$ can be held by the system within bounded queues.

For settings with strictly limited energy we construct optimal (w.r.t. throughput/maximal stable injection rate $\rho$) solutions for different classes of protocols discussed in literature: $\rho = 1$ for adaptive protocols, any $\rho < 1$ for full-sensing protocols, and sub-optimal $\rho = \Theta\left(\frac{k}{n \log^2(n)}\right)$ for acknowledgment based ones (the latter result is complemented by the upper bound $\min\left\{\frac{k}{n}, \frac{1}{3 \log n}\right\}$ on stable injection rates for this class). The main conclusion from our results is that for some classes, i.e., adaptive and full sensing protocols, we are able to construct an energy efficient algorithm without decreasing throughput of the system (i.e., comparing to the corresponding protocols without energy restrictions). That is, stability of adaptive protocols essentially does not depend on energy restrictions. In some other classes, e.g., acknowledgment based protocols, limiting the energy restricts the maximal throughput.

Apart from rigid formal analysis, significant part of this paper is devoted to experimental results proving the efficiency of the constructed protocols for realistic sizes of systems. We also show that our algorithms outperform back-off-type protocols both in terms of energy consumption and system stability (queue sizes).

1.1 Previous and related work

In our paper we study distributed broadcasting on multiple access channel in the framework of adversarial queuing and limited energy of stations. The adversary definition, taxonomy of different models of communication channels (w.r.t. stations capabilities) are the same as in [1] by Chlebus et. al, wherein the authors considered maximal possible throughput for different settings without the energy factor. Clearly, the results as well as technical contribution are substantially different. In our approach we first fix an energy limit then we investigate what maximal throughput is achievable.

Problem of limiting energy during execution of distributed algorithms attracted a lot of attention in recent years. Problems of energy-efficient leader election, size approximation and census were studied by Chang et. al in [2] in single-hop networks obtaining energy-optimal algorithms for different models. Earlier research on energy complexity of leader election and related problems include Jurdzinski et. al [3, 4]. Energy-efficient size approximation of a single hop radio networks were discussed by Jurdzinski et. al in [3] and by Kardas et. al in [5]. In [7, 8] the authors considered energy-efficient algorithms for assigning unique identifiers to all stations. Energy efficient broadcast protocols in the related model of multi-hop radio networks was studied e.g., in [6].

To the best of our knowledge, adversarial packet injections on multiple-access channel were considered for the first time by Bender et. al in [10] and Chlebus et. al [11]. The authors of the former paper considered maximal possible throughput of randomized backoff protocols in queue-free model, while in the latter work deterministic distributed broadcast algorithms in the model of stations with queues were studied. Randomized counterparts of this problem can be found in the earlier paper [12]. Further results in this line considering the maximum rate for which stability of queues is achievable include Chlebus et. al [1] and Anantharamu et. al in [13], wherein authors consider a wide spectrum of models with respect to adversary’s limitations and capabilities of stations and the channel (e.g., distinguishing collisions from the silence on the channel).

In [14], Biekowski et. al introduced the model with unlimited adversary that can inject packets into arbitrary stations with no constraints on their number nor rates of injection, and pursued competitive analysis w.r.t. optimal solution. In all aforementioned papers the number of stations was known in advanced. In [15], Anantharamu and Chlebus investigated a system of multiple access channel with unbounded number of stations attached to it, that can be activated by the adversary by injecting packets.

All above-mentioned papers on dynamic packet arrivals did not focus on energy aspects. Recently Bender et. al [16] obtained results closer to our work. They presented an algorithm for contention resolution (where each station has to transmit a message to the shared channel) achieving constant throughput using only constant average energy for transmitting and $O\log(\log^* n)$ for listening per station, where $n$ is the size of the devices’ ID space.
1.2 Organization of this paper

The rest of this paper is organized as follows. In Section 2 we present a formal model of the channel, stations and the adversary. We also define metrics considered in our paper. Section 3 is devoted to the strongest, i.e., adaptive, algorithms, wherein stations can adopt their behavior to the communication channel and add some information to the transmitted packages. We construct an algorithm that needs only a constant number of stations being switched on in each round, which guarantees stability for an adversary even for $\rho = 1$. In Section 4 we discuss a weaker class of protocols, namely the full-sensing protocols. We construct an algorithm with a collision-detection mechanism that is stable for an adversary with any $\rho < 1$. The weakest type of algorithms (acknowledgment based), wherein all actions are set before the execution of the algorithm, are discussed in Section 5. In Section 6 we present various experimental results for the constructed algorithms as well as a comprehensive comparison with commonly used (also in real-life systems) back-off protocols. We conclude in Section 7.

2 Model

General We restrict our attention to the synchronous “slotted” model, in which the stations use local clocks ticking at the same rate and indicate the same round numbers. We motivate it by portability of developed protocol — synchronous slots can be transformed into asynchronous time windows under clock synchronization restriction. We follow the model from [1] and [17], while also enhancing it by consideration of energy consumption. There are $n$ stations attached to a shared transmission medium that make it a synchronous multiple-access channel. The main properties of the channel are:

- Time is divided into slots of equal size, called rounds.
- Each round consist of phases: transmission, listening and data processing. The stations, according to their programs, attempt either to transmit in the first phase or to listen to the channel in the second phase, and finally apply some local computation.
- A packet is successfully received if its transmission does not overlap with any other transmission.
- A packet successfully transmitted by a station reaches all the stations in the listening phase of the same round, and is acknowledged at the transmitter.

Packets arrival We assume that injected packets are kept in individual queues by each station, till they are successfully transmitted. We model the packet arrival by an adversary, who injects packets into the system queues. Such an approach allows flexible close-to-reality algorithms testing, because packet arrival is independent from the protocol. Different protocols can be compared under the same adversarial strategy. In our paper we consider the $(\rho, b)$-leaky-bucket adversary [1]. It constrains the adversary according to two parameters: the injection rate $\rho$, being the maximum average number of packets injected per round, and the burstiness $b$ — the number of packets that can be injected simultaneously in the same round[1].

2.1 Protocol families

We consider the following classes of deterministic distributed protocols with respect to stations’ capabilities: 

Adaptive protocols — each station may access the history of transmissions, has the ability to add its ID and a small number of bits to each packet, which other stations can read and make future decisions based on this information.

Full-sensing protocols — each station may access the history of transmissions, has the ability to add its ID to each packet, however it cannot add any extra information to the transmitted packages or transmit a dummy packet (e.g., control message).

Acknowledgment based protocols — The stations can change their transmission sequences only when they receive the acknowledgment of a successfully transmitted message. We assume that a station restarts its sequence in the first round after acknowledgment was received.

[1]There is also a second model of the adversary, the so called window adversary, which is seemingly weaker, c.f., [18] for details.
Protocol classes described above reflect practical limitations of station’s calculation and memory resources and its ability to analyze channel state. BACKOFF protocol can be described as randomized acknowledgment-based algorithm with fixed initial sequence of probabilities.

**Energy factor** In addition to the model from [1], we introduce the energy factor: each station can be at one of two states – online or offline, uses a unit of energy in the first case and no energy in the second one. An online station in a given round can transmit a packet or listen to the channel. An offline (idle) station can only perform some local computations, and in particular synchronization is not lost and an offline station knows when to ”wake-up” and switch to on-line mode. This can be seen as a kind of ”standby” mode. We assume that the adversary can inject packets into the station transmit buffer while it is offline. We introduce the Power Peak Constraint (PPC) energy model defining the maximum available power (energy) \( k \) for the whole system in any single round. It allows to design and classify protocols with overload control in mind and to decrease long-term worst-case risks caused by the lack of one [19]. Notably, there are series of solutions to related problems in different communication contexts [20–22]; for instance, for multi-hop networks designed with cluster overlap [23, 24], saturation naturally degrades performance of neighboring cells and therefore the control of an upper bound on the number of active stations is often implemented.

**Protocol quality measures** We focus on the two following protocol quality measures of an algorithm: Stability — if queues of all stations stay bounded, even in infinite perspective of execution; Energy consumption — maximal number of online stations in one round.

We call an injection rate \( \rho \) critical if for injection rates smaller then \( \rho \) the protocol is always stable while for rates higher then \( \rho \) the number of packages in queues tends to infinity for some injection pattern generated by a \((\rho, b)\)-adversary.

### 3 Adaptive protocol

#### 3.1 Move-Big-To-Front protocol

We first recall a short description of MOVE-BIG-TO-FRONT\((n)\) (MBTF) [1] adaptive algorithm, which is known to be stable against the leaky-bucket adversary even for injection rates as high as \( \rho = 1 \). On the top of it we design its energy efficient adaptive and full-sensing counterparts.

MBTF protocol schedules exactly one station to transmit at each round, thus collisions never occur. This has been implemented by using a conceptual “token” giving the right to transmit, which is assigned in such a way that each round exactly one station holds the token. MBTF protocol enables load balancing by giving stations with larger queues more frequent channel access.

Every station maintains a list of all the stations in its private memory. The list is initialized as sorted in the increasing order by the names of the stations. The operations performed on the lists are determined uniquely by what has been heard on the channel. Hence all these lists at stations are manipulated in exactly the same way. This guarantees that the lists are identical at all stations at each round. Because of this property, we refer to all these lists as (local) copies of the list. Initially the first station in the list holds the token.

The protocol is executed at a given round as follows. A station \( p \) with the token broadcasts a packet, if it has any. If the station with the token does not have a pending packet, then the station does not transmit, which results in a silent round. A station considers itself Big when it has at least \( n \) packets in its queue. A Big station attaches a control bit to indicate this status to each packet it transmits. After a station announces itself as Big, it is moved to the front of the list and it keeps the token for the next round. When Big station finishes its broadcast, the token is moved immediately to the next station in the list. Here being “next” is understood in the cyclic ordering of the list of stations, in that the token from the last station in the list is moved directly to the first one. Note that this list could change any time when Big station is move forward to the beginning of the list.

MOVE-BIG-TO-FRONT algorithm uses \( n \) energy per round, because all of the stations are required to be online all the time. We improve it in the following sections to consume at most 2 energy per round.
3.2 12-O’clock adaptive protocol

The 12-O’CLOCK(n) algorithm (Algorithms 1 and 2) schedules exactly two stations to be operational in a single round, and gives the right to transmit only to one of those stations while the other station is listening. Thus collision never occurs and energy usage never passes the threshold of 2 per round. We call a group of n consecutive rounds a cycle if the last round r of the group satisfies r = 0 mod n. Since it takes at least n – 1 rounds for a station to inform others, due to energy limitation, end-of-cycle rounds play important role in coordination and decision making during the execution; they also motivate the name of the algorithm.

Operational station can be at one of the five states: Idle, Listening, Transmitting, Big or Last-Big. The last three states are given the right to transmit; they could be encoded by two bits when attached to the message by the transmitting station. The Listening state is dedicated to listening, while in the Idle state the station neither transmits nor listens. We describe these states one-by-one later in this section. Recall also that the model assumes, w.l.o.g., that transmission event happens before receiving (listening) event in a round.

```
Procedure transmitToTheChannel()
    switch s.state do
        case Transmitting do
            if s.queue > 3n then
                s.state := Big;
                s.transmit();
            else
                if s.queue > 0 then
                    s.transmit();
                end
                s.state := Idle;
            end
        end
        case Big do
            if round = 0 mod n AND s.queue <= 3n then
                s.state := Last-Big;
                s.moveBigToFront(s.id);
            end
            s.transmit();
        end
        case Last-Big do
            s.transmit();
            if round = n – 1 mod n then
                s.state := Transmitting;
            end
        end
    end
```

Algorithm 1: 12 O’clock adaptive algorithm – transmission phase

High Level Description. The main drawback of the MBTF algorithm is the linear number of listening stations per round; in order to overcome it, we “emulate” each round by a cycle consisting of n rounds. We have to do it in a way that: in each round there are only two stations active, collisions do not occur, and the number of idle rounds (i.e., without transmission) is controlled in a similar way as in the original MBTF algorithm.

More specifically, the algorithm proceeds through n-round cycles. In each round there are two distinguished stations: a station with a “token to transmit” (i.e., in state Transmitting, Big or Last-Big), and another station which is listening (i.e., in state Listening); the rest of stations are (in state) idle. The reason of having three types of states for a potential transmitter is the following: the station could have (re-)discovered that it is Big (i.e., it has more than 3n packets in its queue) during the current cycle, or it wants to inform the others that it lost the status of being Big in the beginning of the current cycle (state Last-Big), or the station has relatively small queue and is just passing the token (state Transmitting) to the listening station.

Stations are changing in the ROUND-ROBIN way: by waking up and adopting state Listening at the round
Procedure `listenToTheChannel()`

```plaintext
switch s.state do
  case Idle do
    if shouldWakeUp() then
      s.state = Listening;
    end
  end
end
```

```plaintext
case Listening do
  switch channel.state do
    case Silence do
      s.state = Transmitting;
    end
    case Normal do
      s.state = Transmitting;
    end
    case Big do
      s.state = Idle;
    end
    case Last-Big do
      s.state = Idle;
      s.moveBigToFront(channel.id);
    end
  end
end
```

Algorithm 2: 12 O’clock adaptive algorithm – listening phase

when its predecessor should transmit, and going back to the idle state in this round (in case it received a message from a Big or Last-Big station) or in the next round right after it transmits (if it received a message from normal Transmitting station or no message at all). If the station has a number of packets in its queue that is bigger than the threshold level 3n, it becomes Big, keeps the token throughout the remainder of the cycle and adds a mark to its packets so that the other (listening) stations may know that they are not allowed to transmit (instead, listening stations immediately go idle). Station can stop being Big only when the number of packets in the queue is lower than the threshold level and when it is the last round of a cycle; the reason for checking this condition only at the end of cycles is to be able to inform all other stations about its Big status throughout the whole cycle (note that such informing process requires at least n − 1 rounds in case of energy 2). After revoking its state Big at the end of some cycle, the station keeps transmitting throughout the next full cycle its packets with a Last-Big mark (as mentioned before, two additional bits are needed to encode whether a transmitting station is in state Transmitting, Big or Last-Big), so that the other stations could adjust their list to the new order in which the transmitting Last-Big station is the first on the list (as the most recent Big station). Note that the threshold 3n for being a Big station consists of term n coming from the criteria for being big in the original MBTF algorithm, and term 2n allows a Big station to make two additional whole cycles in the beginning of becoming Big and when stopping being big (i.e., in the cycle when it is in state Last-Big) without idle rounds.

**Methods.** The algorithm uses the following methods:

- `moveBigToFront(station ID)` — moves station of the input ID to the front of the (local) station list;
- `transmit()` — transmits a packet from the station queue, attaches ID and state information to it;
- `shouldWakeUp()` — checks the idle timeout of the station, that is, the number of rounds left until its predecessor could be in the Transmitting state. It starts from n − 1 when the station drops Listening state, and decreases by 1 each round. Upon becoming 0, the procedure outputs “true” and the station switches to Listening state.

**Initialization.** In the beginning all but the first two stations are in the Idle state, while the one with the smallest ID is in Transmitting state and its successor is in the Listening state.

**Idle state.** In this state the station does not access the channel, it only keeps updating its idling time until the next wake-up — each round decreases by 1. The starting number of idling rounds is either n
or \(n - 1\) or \(n - 2\), depending on the state from which the station switches to Idle and the message on the channel, see the description of Listening and Transmitting states below. After awaking, i.e., when the idling time decreases to zero, the state switches to Listening.

**Listening state.** Station in the Listening state updates the local station list when the Last-Big transmission occurs on a channel. It changes its state to Transmitting upon receiving a message from a station in the Transmitting state or upon no message received. Otherwise, it becomes idle for the next \(n - 1\) or \(n\) rounds until wake-up — the latter idling time is when the station hears a Last-Big station which is currently located after it on the list of stations (because by shifting the Last-Big station to the beginning, the location of the listening station on the list increases by 1).

**Transmitting state.** The Transmitting state is taken (from Listening state) by a station once per cycle in the round corresponding to its current position on the list of stations, unless there is a Big or Last-Big station in this round. Station in the Transmitting state changes its state to Big and transmits if its queue size is bigger than \(3n\). Otherwise, it transmits being in the Transmitting state, provided it has a packet in its queue, and changes its state to Idle (in order to awake in its listening turn during the next cycle, after \(n - 2\) rounds).

**Big state.** At the end of each cycle, each Big station checks whether its queue size is still bigger than \(3n\); if not, it changes its state to Last-Big. In any prior round, the Big station transmits a packet and remains in the same state. The following property can be easily deducted: once a station changes its state to Big (which happens when being in its regular Transmitting state), it stays there till the end of the cycle; it may then continue throughout whole next cycles, until it changes to Last-Big state at the end of one of them.

**Last-Big state.** Station in the Last-Big state transmits until the end of the cycle. It changes its state to the Transmitting in the end of the cycle after the last transmission happens. Note that, due to the condition of switching from Big to Last-Big state, a station remains in the Last-Big state during the whole one cycle, from the beginning when it switched from the state Big to the end when it switches to the Transmitting.

### 3.3 Analysis and bounds

Consider the total size of the queues in the beginning of the cycle. If it is greater then \(L = n(3n - 1) + 1\) we say that belongs to a dense interval, otherwise it belongs to a sparse interval (here we consider intervals of time). This way the execution of algorithm consists of interleaved dense and sparse intervals, each containing a number of whole cycles.

In relation to a fixed interval, we consider the following terminology: station is pre-big in a given round of the interval if it has not been in the Big state during this interval before that round, and it is post-big if it has been at least once in the Last-Big state during the interval by that round. Station is potentially-big if its queue size is bigger than \(3n\) (i.e., the size allows the station to become Big eventually) or it is in a Big or Last-Big state. Observe that each station is pre-big in some prefix of the interval and post-big in some (disjoint) suffix of the interval; each of these periods could be empty or the whole interval. In-between of being pre-big and post-big, a station is continuously in a Big state.

We define types of cycles depending on availability of Big and Last-Big stations:

**Type-1 cycle:** without any Big or Last-Big station. Token is being passed in the Round-Robin way, by adopting Listening and Transmitting states. This means that at any single round there is one station in the Transmitting state and one in the Listening state.

**Type-2 cycle:** with a Big station \(s\) starting to transmit as Big in some round of the cycle. Here, the token is being passed in the Round-Robin way by applying sequence of Listening and Transmitting states to each station on the list, until \(s\) transmits. Since \(s\) becomes Big, it keeps the token afterwards till the end of the cycle. Note that stations at Big and Transmitting states cannot occur simultaneously in the same round, because once there is a Big station all Listening stations immediately switch to Idle state instead of switching to Transmitting state. Additionally, no Last-Big station could occur in such cycle, due to the type-4 cycle described below.

**Type-3 cycle:** with a Big station \(s\) keeping the “token to transmit” for the whole cycle. All stations after waking-up in the Listening state will learn about the state of \(s\) and become idle until their scheduled wake-up round in the next cycle.

**Type-4 cycle:** with a Last-Big station \(s\) keeping the token for the whole cycle. Station can be in the Last-Big state only for a whole cycle and after being in the Big state (at the end of the previous cycle). All
stations after switching from Idle to the Listening state will learn about the Last-Big state of \( s \) and become idle until their scheduled wake-up round in the next cycle.

The local lists of stations stay synchronized in the beginning of cycles; in fact, only the type-4 cycle changes the order of stations, and the whole cycle is needed to do it consistently in all stations (when they act as listeners) so that they all apply the move of the Last-Big station to the beginning of their local lists by the end of the cycle.

**Lemma 3.1.** Each cycle is of one of the above four types.

**Proof.** Algorithm’s initialization conditions enforce that the first cycle is of type-1 or type-2, as there is no Big or Last-Big station in the beginning. Type-1 can be followed only by the type-1 — if there is no potentially-big station during the cycle, or by the type-2 cycle otherwise. In the type-2 cycle the Big station is chosen during the cycle, and thus the cycle can be followed by the type-3 cycle — if the Big station queue size is above \( 3n \) at the end of the cycle, or by type-4 otherwise. The case of type-3 cycles is the same as the ones of type-2 described above, as in both types there is a Big station at the end (which determines conditions for the next cycle); they can be followed only by a cycle of type-3 or type-4. The type-4 cycle can be followed by type-1 — if there is no potentially-big station, or by type-2 cycle otherwise. Using an iductive argument over cycles, it can be concluded that each cycle is of one of the four defined types. ■

**Lemma 3.2.** In any dense interval, a station can cause a silent round (i.e., is in state Transmitting but has an empty queue) at most \( n - 1 \) times while being pre-big.

**Proof.** Silent rounds occur when some station has a “token to transmit” but has no packets in its queue. Note that it is only possible for stations in Transmitting state, as stations in any of Big states have more than \( n \) packets in their queues. Assume that station \( s \) has no packets in its queue. Within dense interval, in each round there is a potentially-big station. For any cycle, if potentially-big station is before \( s \) in the list, then \( S \) would receive no “token” or receive it and decrease its position in the list. The position of \( S \) can not decrease more then \( n - 1 \) times, as there can be no potentially-big station after \( S \) if it is last in the list. When \( S \) is the last on the list it either never has a possibility to transmit or becomes potentially-big. Pre-big station life-cycle terminates once station is in the Big state by definition. ■

**Lemma 3.3.** In any dense interval, a station causes no silent round while being post-big or in a Big state.

**Proof.** By definition of Big state, a station must have had more than \( 3n \) packets in its queue in the beginning of the current cycle or in the round of the cycle when it turned into the Big state. Therefore, in each round of the cycle it has more than \( 2n \) packets. Hence, it causes no silent round. A post-big station \( s \) could be in a Last-Big state, Big state, Transmitting state or in one of the other two states. In the latter case, it does not attempt to transmit, thus it cannot cause a silent round. The case of Big state was already analyzed. If the station enters Last-Big state, it switches to this state from the Big state in the beginning of the cycle, having more than \( 2n \) packets in its queue; thus in any round of the cycle the number of packets cannot drop below \( n \), and hence no silent round occurs.

It remains to analyze the case when \( s \) is in the Transmitting state. Upon leaving the Last-Big state for the last time, it had at least \( n \) packets in its queues and was placed in the beginning of the list of stations, by the algorithm construction. Then, observe that \( s \) has had an opportunity to transmit only at some type-2 cycle \( C \) when there is no potentially-big station before it on the list or when \( s \) is potentially-big at the time it switches from Listening to Transmitting state. In the latter case, instead of staying in Transmitting state it immediately switches to Big state, which case we already analyzed in the beginning of the proof. Otherwise (i.e., in the former case), either potentially-big station after \( s \) becomes Big, which implies that in some of the next cycles, it switches to Last-Big state and the position of \( s \) on the list decreases without causing any more silent rounds, or \( s \) receives no “token to transmit” and so it cannot cause a silent round.
by default. The position can not decrease more than \( n - 1 \) times, because there can be no potentially-big station after \( s \) if \( s \) is the last on the list (the argument is similar to the one from the proof of Lemma 3.2). Since \( s \) had at least \( n \) packets when switching from its Last-Big state, it can transmit and decrease its position at most \( n - 1 \) times or become Big, whatever comes first; in any case, it has at least one packet when transmitting.

**Theorem 3.4.** The 12 O’clock adaptive protocol is stable against leaky-bucket adversary with injection rate \( \rho = 1 \), and the maximum number of packets stored in a round is at most \( L + (n - 1)^2 + n + b = O(n^2 + b) \).

**Proof.** Within a sparse interval, there can be no more then \( L + n + b \) packets in the stations at the end of any cycle. Indeed, the biggest possible number of packets that the system can start a cycle with is equal to \( L \), and the adversary can inject no more then \( n + b \) packets in \( n \) consequent rounds of the cycle. Once the queue size becomes greater than \( L \) in the beginning of a cycle, the sparse interval terminates and the dense interval begins.

In the remainder, we focus on dense intervals. Note that in the beginning of a dense interval, the number of packets in the system is at most \( L + n \) plus the burstiness above the injection rate (upper bounded by \( b \)); indeed, as in the beginning of the preceding cycle the interval was sparse, the number of packets was not bigger than \( L \), and during that cycle the adversary could inject at most \( n \) packets accounted to the injection rate plus the burstiness.

Within any dense interval, a station in the Big or Last-Big state is guaranteed to be in each cycle, by the pigeon-hole principle. It makes type-1 cycle impossible to occur. Consider type-3 and type-4 cycles: during those cycles packet is transmitted in every round, and thus a silent round cannot occur; hence the number of packets does not grow (except of burstiness above the injection rate, but this is upper bounded by \( b \) at any round of the interval, by the specification of the adversary).

In type-2 cycles, post-big stations can not cause silent round, by Lemma 3.3, and stations in Big state cannot cause silent rounds as they always have more than \( 2n \) packets pending. Hence, type-2 cycles may have silent rounds caused only by pre-big stations. However, there can be no more than \( n - 1 \) pre-big stations in the system in the beginning of the dense interval (because there is at least one potentially-big station). Each pre-big station can cause no more than \( n - 1 \) silent rounds, by Lemma 3.2. Observe that in each cycle with a silent round some potentially-big station will change its state to Big — silent round would not occur if there was potentially-big station with higher position in the list than any empty station. Hence, there can be no more than \( n - 1 \) cycles with silent rounds caused by same (pre-big) station. To summarize, there are at most \( n - 1 \) cycles with silent rounds for each of at most \( n - 1 \) pre-big stations, resulting in the upper bound of \( L + (n - 1)^2 + n + b \) on.

### 4 12 O’clock full-sensing protocol with collision detection

12-O’clock full-sensing protocol works similarly to its adaptive counterpart, however a decision to change state is based on information who transmitted packet, since adding bits is not allowed for full-sensing protocols. To overcome the lack of additional information bits, we implement precise control mechanism to the “out-of-order transmissions” and the collisions enforced by them so that Big stations could be identified. Combined with recognition of ID attached to successful transmissions, any station can learn about Big station and adjust to it, with some small waste of transmission and increased delay.

In our algorithm, typically stations in the Listening state discover Big stations by reading the ID of station transmitting on the channel and comparing it to the predecessor ID from the list — if they do not match then the listening station(s) deduct that the transmitter is in Big state. By distinguishing silence from collision, the algorithm is able to manage borderline cases, see the description below. However, due to collisions, the protocol is not universally stable, albeit we will prove its stability against injection rates \( \rho \leq \frac{n + 1}{n} \).

We consider three channel states: **Silence** when there is no transmission, **Transmission** when there is single transmission on the channel, **Conflict** when there is more than one transmission. Stations can be at one of four states: Idle, Listening, Transmitting or Big. The last two states are given the right to transmit; they are distinguished by the order in the list – only Big station can transmit out of the order of the list; in the only one possible case when Big station transmits within the order, collision occurs and later transmissions...
clarify the system state. The Listening state is dedicated to listening, while in the Idle state the station neither transmits or listens. We describe these states later in this section. As previously we assume that transmission happens before the listening phase (Algorithms 3 and 4).

```
Procedure transmitToTheChannel()
    switch s.state do
    case Transmitting do
        if s.queue > 0 then
            s.transmit();
            s.transmitted = true;
        end
    end
    case Big do
        s.transmit();
    end
end
```

Algorithm 3: 12 O’clock full-sensing algorithm — transmission phase.

**Methods.** The algorithm uses the following methods:

- `moveBigToFront(station ID)` — moves station of the input ID to the front of the (local) station list;
- `transmit()` — transmits a packet from the station queue, with attached ID and state of the station;
- `shouldWakeUp()` — checks the idle timeout of the station, that is, the number of rounds left until its predecessor can be in the Transmitting state. It starts from either \( n \) or \( n - 1 \) or \( n - 2 \) when the station drops Listening or Transmitting state, and decreases by 1 each round. Upon becoming 0, the procedure outputs “true” and the station switches to Listening state.

### 4.1 Description of states and transitions

This section analyses in more detail the properties of the four possible states of a station and transitions between them. This is only a more comprehensive description of Algorithms 3 and 4 and possible cases during their executions.

**Idle state.** In this state the station does not access the channel, it only keeps updating its idling time until the next wake-up — it decreases by 1 each round. The starting number of idling rounds is either \( n \) or \( n - 1 \) or \( n - 2 \), depending on the state from which the station switches to Idle and the message on the channel, see the description of Listening and Transmitting states below. After awaking, i.e., when the idling time decreases to zero, the state switches to Listening.

**Listening state.** A station in the Listening state considers all three channel state cases, in the following way. Conflict on the channel occurs only when a Big station \( S \) interrupted its successor. No information is available on the channel, hence the Listening station keeps its state unchanged for one more round in order to hear an ID of the Big station. Note that there will be two stations in the Listening state and one in the Big state next round. Both Listening stations would recognize \( S \) as Big and update their local station lists accordingly.

Upon hearing a silence, the Listening station knows that it will not interrupt a Big station transmission next round and thus it changes its state to Transmitting.

Finally in case of the transmission on the channel, the Listening station checks transmission ID on the channel and either it takes the token from its successor, or becomes idle and updates the local station list if it was not its predecessor’s transmission. It becomes idle for the next \( n - 2 \), \( n - 1 \) or \( n \) rounds until subsequent wake-up; more specifically, the first idling time \( n - 2 \) occurs when station waited additional round after collision on the channel, the second idling time \( n - 1 \) occurs when the station hears a Big station which is currently located after it on the list of stations, and the last idling time \( n \) occurs when the Big station was located before it on the list.

**Transmitting state.** The Transmitting state is taken by a station once per cycle in the round corresponding to its current position on the list of stations, unless there is a Big station in the beginning of that round.
Procedure listenToTheChannel()
switch s.state do
  case Idle do
    if shouldWakeUp() then
      s.state = Listening;
    end
  end
  case Listening do
    switch channel.state do
      case Conflict do
        s.state = Listening;
      end
      case Transmission do
        if channel.id = predecessor.id then
          s.state = Transmitting;
        else
          s.state = Idle;
          s.moveBigToFront(channel.id);
        end
      end
      case Silence do
        s.state = Transmitting;
      end
    end
  end
  case Transmitting do
    switch channel.state do
      case Conflict do
        if s.transmitted then
          if s.queue > 3n then
            s.state := Big;
          else
            s.state = Idle;
          end
        else
          s.state = Idle;
          s.moveBigToFront(predecessor.id);
        end
      case Silence do
        s.state = Idle;
      end
    end
    case Big do
      if mod(round(n) = n-1 AND s.queue ≤ 2n then
        s.state := Transmitting;
        s.moveBigToFront(s.id);
      end
    end
  end
end

Algorithm 4: 12 O’clock full-sensing algorithm — listening phase.

A station in the Transmitting state changes its state to Idle when there is a silence on the channel — it is
possible only when it had no packets and there was no Big station in the beginning of this round. In case of a conflict, it updates its local station list by moving its predecessor from the list to the front, as its is the only station which transmission on the channel would allow the Transmitting station to change its state from Listening to Transmitting.

If a Transmitting station has successfully transmitted, then there is no Big station transmission in this round. Additionally, if the Transmitting station has queue size exceeding $3n$, it changes its state to Big and keeps transmitting accordingly starting from the next round. Otherwise, it changes its state to Idle, in order to awake in its listening turn during the next cycle, after $n - 2$ rounds. If the station has not transmitted but a single transmission occurs on the channel, then this is a transmission from predecessor of Big station (any other Big station would cause the station not to switch to the Transmitting state in the first place, as it would switch directly from the Listening to Idle) which has not caused a conflict only because the Transmitting station has had no packets to transmit. In this case the station behaves accordingly — updates the local list of stations and changes its state to Idle.

**Big state.** At the end of each cycle, a Big station checks whether its queue size is still bigger than $2n$; if not, it changes its state to Transmitting. In any other round, the Big station transmits a packet and remains in the same state. The following property can be easily deducted: once a station changes its state to Big (which happens when being in its Transmitting state), it stays there at least till the end of the next cycle; it may then continue throughout the whole next cycles, until it changes to the Transmitting state at the end of some of them.

### 4.2 Analyses and bounds

Similarly to the Adaptive protocol analysis, we consider the sum of the queues’ sizes in the beginning of a cycle. If it is greater then $L = n(3n - 1) + 1$ we say that it belongs to the dense interval, otherwise it belongs to the sparse interval. This way any execution of the Full-sensing algorithm consists of dense and sparse interleaved intervals.

In relation to a fixed interval we consider the following terminology: station is pre-big if it had never been in the Big state and it is post-big if it was at least once in the Last Big state, during the considered cycle. Station is potentially-big if its queue size allows it to become Big (provided other necessary conditions would hold) or it is in the Big state.

Each cycle can be only of one of the three types:

**Type-1.** without any Big station. Token is being passed in the Round-Robin way, by adopting Listening and Transmitting states. This means that at any single round there is one station in the Transmitting state and one in the Listening state.

**Type-2.** with a Big station $S$ starting to transmit as Big in some round of the cycle. Here, the token is being passed in the Round-Robin way by applying the sequence of Listening and Transmitting to consecutive stations on the list, until $S$ transmits for the second time. The successor of station $S$ can not recognize $S$ as Big since $S$ is supposed to transmit by the default Round-Robin way of passing the token within the list order. Conflict occurs if the successor of $S$ has a packet to transmit. Otherwise, in the case of successful transmission, stations in Listening and Transmitting states active at this round would read the Big station ID from the transmission, both changing their states to Idle afterwards. Otherwise the station in the Transmitting state learns from the conflict about the state of $S$, and then it changes its state to Idle and updates the local station list. The station which was in the Listening state at that time learns about the state of $S$ a round after the conflict, since it could not be a successor of any Big station.

**Type-3.** with a Big station $S$ keeping the “token to transmit” for the whole cycle. All but one stations after waking-up in the Listening state will learn about the state of $S$ and become Idle until the next cycle. One station would not recognize $S$ as Big, but it will be interrupted by its transmission. Through the conflict on the channel it would however learn about the state of $S$, an then it changes its state to Idle and updates the local station list.

The following two lemmas justify the usage of cycles defined above and provide the limit on the number of collisions. They will be used implicitly in the analysis.

**Lemma 4.1.** Each cycle is of one of the three above types.

**Proof.** The starting conditions of the algorithm enforce the system to start in the type-1 cycle.
Type-1 cycle can be followed by another type-1 cycle, if there is no potentially-big station, or by a type-2 cycle otherwise.

In a type-2 cycle the Big station is chosen, and therefore it can only be followed by a type-3 cycle — this is because the Big station needs to transmit more than \( n \) packets in order to start to consider changing its state, which may happen only at the end of some cycle.

A type-3 cycle, with a Big station keeping the token (to transmit) for the whole cycle, can be followed either by the same type of a cycle if an adversary keeps injecting packets into the Big station, or by a type-1 cycle if there is no potentially-big station, or by a type-2 cycle otherwise.

\[ \text{Lemma 4.2. No more than one collision per cycle can occur.} \]

\[ \text{Proof. Note that in a type-1 cycle collision may not occur, as at any single round there is station in the Transmitting state and another one in the Listening state.} \]

\[ \text{In a type-2 cycle no collision occurs until the second transmission of a station in the Big state, by the same reasoning as for type-1 cycles. If the Big station successor has packets in its queues there is a collision on the channel. The station in the Transmitting state becomes Idle at the end of this round until the next cycle. Stations further down on the list can not have the Big station as predecessor and would wake up in the Listening state, learn about the Big station from its transmission and change their state directly to Idle, hence there can be no more collisions.} \]

\[ \text{A type-3 cycle with a Big station } S \text{ keeping the “token to transmit”. Consider the case, when type-2 cycle precedes. We divide stations of the system into two groups: group-A consists of stations after the Big station } S \text{ on the list, which have already learned about the state of } S \text{ and updated their local lists of stations. Group-B are stations before } S \text{ on the list, which had no occasion to do so. If group-A is empty, then there is a single succeeding to } S \text{ station in the group-B. It causes one collision due to assumption of default ROUND-ROBIN predecessor, which is } S \text{. The rest of the stations in this cycle will switch directly from the Listening to Idle state, thus no more collision occur. If both group-A and group-B are not empty, then no station in the group-B can have } S \text{ as predecessor, because } S \text{ is down in the list for any station in group-B by definition, and its not last on their outdated list version since group-A is not empty. Due to group-A stations having their lists updated, } S \text{ is the first station in their lists, what together with nonempty group-B assumption makes it impossible to any station from the group-A to have } S \text{ as predecessor. It follows that all of the group-A and group-B stations would change state directly from Listening to Idle, thus no collision occur. If group-B is empty or type-3 cycle precedes the current cycle, than cyclic order of the list does not change (i.e. each station has the same successor and predecessor in the beginning and the end of the cycle), so there is a single succeeding to } S \text{ station which causes one collision due to assumption of default ROUND-ROBIN predecessor, which is } S \text{. No more stations can have } S \text{ as predecessor, thus the rest of the stations would change state directly from Listening to Idle and no more collision occurs.} \]

We call a round with collision caused by station in the Big state an assertion round. In relation to cycles we assume that there is an assertion round in every cycle, since this is the worst possible case — no more than one collision in a cycle can occur by Lemma above. By a silent round we understand any non-assertion round with no successful transmission. We say that a station causes a silent round if during this round it is in state Transmitting; note that it may occur only if the station has empty queue in this round. Observe also that there cannot be a Big station in a silent round, as stations in Big state have more than \( n \) packets in their queues.

\[ \text{Lemma 4.3. In any dense interval, a station can cause a silent round at most } n - 1 \text{ times while being pre-big.} \]

\[ \text{Proof. Silent rounds occur when some station has a “token to transmit” but has no packets in its queue. Assume that a station } S \text{ has no packets in its queue. Within dense interval, in each round there is a potentially-big station. For any cycle, if potentially-big station is before } S \text{ on the list, then } S \text{ would receive no “token” or receive it and decrease its position on the list. The position of } S \text{ can not decrease more then } n - 1 \text{ times, because there can be no potentially-big station after } S \text{ if it is the last on the list. Since in the dense interval there is always a Big station, } S \text{ as the last station in the list either has no possibility to cause silent round (when some other station } S' \text{ before it in the list changes state to Big), or becomes Big} \]
itself. Pre-big station life-cycle terminates once station is in the Big state by our definition, hence through the whole pre-big life-cycle station $S$ may cause no more than $n - 1$ silent round.

**Lemma 4.4.** In any dense interval, a station causes no silent round while being post-big or in a Big state.

**Proof.** A post-big station $S$ could be in a Big state, Transmitting state or in one of the other two states. In the latter case, it does not attempt to transmit, hence it cannot cause a silent round. If the station enters Big state, it switches from the Transmitting state at some round of the cycle, having more than $3n$ packets in its queue; it’ll switch back to the Transmitting state when having less than $2n$ packets in the end of the cycle. Thus, in any round of the cycle the number of packets cannot drop below $n$, and hence no silent round occurs.

It remains to analyze the case when $S$ is in the Transmitting state. Upon leaving the Big state for the last time, it had at least $n$ packets in its queues and was placed in the beginning of the list of stations, by the algorithm construction. Then, observe that $S$ has had an opportunity to transmit only at some type-2 cycle when there is no potentially-big station before it on the list or when $S$ is potentially-big at the time it switches from Listening to Transmitting state. In the latter case, instead of staying in Transmitting state it immediately switches to Big state, which case we already analyzed in the beginning of the proof. Otherwise (i.e., in the former case), either some potentially-big station after $S$ becomes Big, which implies that in some of the next cycles, it will switch back to Transmitting state and the position of $S$ on the list decreases without causing any more silent rounds, or $S$ receives no “token to transmit” and so it cannot cause a silent round by default. The position can not decrease more than $n - 1$ times, because there can be no potentially-big station after $S$ if $S$ is the last on the list (the argument is similar to the one from the proof of Lemma 4.3). Since $S$ had at least $n$ packets when switching from its Big state, it can transmit and decrease its position at most $n - 1$ times or become Big, whatever comes first; in any case, it has at least one packet when being in Transmitting state.

**Theorem 4.5.** The 12 O’clock full-sensing protocol is stable against leaky-bucket adversary with injection rate $\rho = 1 - \frac{1}{n}$, and the maximum number of packets stored in a round is at most $L + (n - 1)^2 + n + b = O(n^2 + b)$.

**Proof.** Within a sparse interval, there can be no more then $L + n + b$ packets in the stations at the end of any cycle. Indeed, the biggest possible number of packets that the system can start a cycle with is equal to $L$, and the adversary can inject no more then $n + b$ packets in $n$ consequent rounds of the cycle. Once the queue size becomes greater than $L$ in the beginning of a cycle, the sparse interval terminates and the dense interval begins.

In the remainder, we focus on dense intervals. Note that in the beginning of a dense interval, the number of packets in the system is at most $L + n$ plus the burstiness above the injection rate (upper bounded by $b$); indeed, as in the beginning of the preceding cycle the interval was sparse, the number of packets was not bigger than $L$, and during that cycle the adversary could inject at most $n$ packets accounted to the injection rate plus the burstiness.

Within any dense interval, a station in the Big state is guaranteed to be in each cycle, by the pigeon-hole principle. It makes type-1 cycle impossible to occur. Consider type-3 cycles: during those cycles a packet is transmitted in every round, and thus a silent round cannot occur; hence the number of packets does not grow (except of burstiness above the injection rate, but this is upper bounded by $b$ at any round of the interval, by the definition of the adversary).

In type-2 cycles, post-big stations cannot cause silent rounds, by Lemma 4.4 and stations in Big state cannot cause silent rounds as they always have more than $2n$ packets pending. Hence, type-2 cycles may have silent rounds caused only by pre-big stations. However, there can be no more than $n - 1$ pre-big stations in the system in the beginning of the dense interval (because there is at least one potentially-big station). Each pre-big station can cause no more than $n - 1$ silent rounds, by Lemma 4.3. Observe that in each cycle with a silent round some potentially-big station will change its state to Big — a silent round would not occur if there was a potentially-big station with higher position on the list. Hence, there can be no more than $n - 1$ cycles with silent rounds caused by same (pre-big) station. To summarize, there are at most $n - 1$ cycles with silent rounds for each of at most $n - 1$ pre-big stations, resulting in the upper bound of $L + (n - 1)^2 + n + b$ on the sum of the queue sizes in a round.
4.2.1 Stability bound improvement

It was proved in [1] that it is not possible to construct a full-sensing stable protocol against leaky-bucket adversary $\rho = 1$ for a system with a number of stations bigger than 3. However, we could still improve the 12 O’clock full-sensing protocol:

**Lemma 4.6.** For any given $\rho < 1$, the 12 O’clock full-sensing protocol can be modified to be stable against the leaky-bucket adversary with injection rate $\rho$.

**Proof.** Algorithm may handle any injection rate $\rho$ smaller than 1 by following the strategy:

- Transmitting station considers itself Big when it has more then $2n + kn$ packets, where $k \geq \frac{1}{n(1-\rho)}$ is a positive integer;
- Transmitting station remembers of being interrupted by its predecessor, and instead of waking up after the subsequent nearly $n$ rounds, as in the original 12 O’clock full-sensing protocol, it wakes up after $kn$ rounds.

This way interruption may happen only once in $kn$ rounds and the adversary with injection rate of $\rho = 1 - \frac{1}{kn}$ can be handled. We adjust the sparse/dense border value to $L’ = n((2+k)(n-1) + 1$, since the Big station definition has changed. Following the logic of the Theorem 4.5 proof, in the dense interval there are at most $k(n-1)$ cycles for each of at most $(n-1)$ pre-big stations, resulting in the upper bound of total queue sizes of $L’ + k(n-1)^2 + n + b = O(kn^2 + b)$.

5 Acknowledgment-based protocols

In this section we consider acknowledgment based protocols in k-PPC model. First we state two simple bounds of this model. Then we present our algorithm that is optimal up to a multiplicative polylogarithmic factor with respect to the injection rate that can be handled.

5.1 Bounds

**Lemma 5.1.** There is no correct, acknowledgment-based algorithm in the k-PPC model with power limit $k < n$ without global-clock mechanism for any $\rho > 0$.

**Proof.** We say that protocol with power limit $k$ and injection rates $\rho$ is correct, when queues of all stations stay bounded at all times independently from adversary strategy and for any round the number of online stations is at most $k$.

Assume that $P$ is correct deterministic acknowledgment-based protocol, within k-PPC model without global clock in the system of $n$ stations. Then for each station $S_i$, there is a default starting sequence $p_i$, where $i$ is the index of the station. Because $P$ is correct, each $p_i$ contains a first occurrence of transmission bit 1. Let $t_i$ be the position of the first transmitting bit in the sequence $p_i$. Because system is not equipped in the global clock mechanism, stations’ starting rounds are set by adversary. Let us say that $s_i$ is a global timeline start moment of station $S_i$. It follows that first transmission of station $i$ occurs at round $s_i + t_i$. In order to overload the system adversary follows the strategy: choose round $e$ as $e = \max\{t_1, \ldots, t_n\}$; start station $S_i$ at round $s_i = e - t_i$. Then all $n > k$ stations transmit at round $e$ and thus $P$ has to overflow power limit $k$.

**Theorem 5.2.** Any acknowledgment-based algorithm in the k-PPC model with power limit $k < n$ and with $(b, \rho)$-leaky bucket adversary cannot be stable for $b > 0$ and $\rho > \min\{\frac{k}{n}, \frac{1}{3\log n}\}$.

To prove the theorem, assume first that $\frac{k}{n} \leq \frac{1}{3\log n}$. Suppose, to a contradiction, that during $T$ rounds the adversary could inject more than $T \cdot k/n$ packets. During those $T$ rounds there is a station that is allowed to transmit at most $T \cdot k/n$, independently on the state of queues. That is, in each round at most $k$ stations can be active, thus during $T$ rounds at most $T \cdot k$ can be active in total. Since there is $n$ stations in total, there must be a station that is allowed to transmit at most $T \cdot k/n$ messages during $T$ rounds. Thus it is enough if the adversary adds more than $T \cdot k/n$ messages to this station and after $T$ rounds
there is at least one package retained in the queue of this station. Iterating the procedure the adversary can generate queues of arbitrary length, thus the system cannot be stable, which results in contradiction. This proves that $\rho$ cannot exceed $\frac{k}{n}$.

The second case when the minimum formula equals to $\frac{1}{3\log n}$ follows directly from Thm. 5.1 in [1].

5.2 Algorithm

In this section we present an algorithm working in $k$-PPC model that is stable for leaky bucket adversary with any $b$ and some $\rho = \Theta(\frac{k}{n \log^*(n)})$. We start with definitions.

5.2.1 $k$-light Selectors

Let us consider a set $N = \{1, \ldots, n\}$ and its subsets $S, X, Y \subset N$. We say that $S$ hits $X$ if $|S \cap X| = 1$. We say that $S$ avoids $Y$ if $|S \cap Y| = 0$.

**Definition 5.1.** We say that a family $S \subset 2^N$ is a $(n, \omega)$-selector if for any subset $X \subset N$ such that $\omega/2 \leq |X| \leq \omega$ there are $\omega/4$ elements hit by subsets from $S$.

Note that this definition is a special case of a selective family introduced in [25]. The intuition behind $S$ is as follows: we can “separate” at least a fraction of elements of any subset $X$ (of appropriate size) using sets that belong to $S$.

**Definition 5.2.** We say that $S = (n, \omega)$-selector is $k$-light if any $S_i \in S$ has at most $k$ elements.

**Theorem 5.3.** There exists a $k$-light $(n, \omega)$-selector of size $m = O(\omega \log n)$ for $k \geq \frac{n}{\omega}$, and $m = O\left(\frac{n}{k} \log n\right)$ for $k < \frac{n}{\omega}$.

**Proof.** The first part of the proof is a modified reasoning from [26]. Let us assume that $\omega > 1$ and $\omega|n$. Let $m$ be the size of a selector to be fixed later. Let us choose independently $m$ random subsets of $\{1, \ldots, n\}$ of size $l = \frac{\omega}{2}$. That is, $S = (S_1, \ldots, S_m)$ is a random family. Let us consider any fixed sets $X, Y \subset \{1, \ldots, n\}$, such that $\omega/4 \leq |X| \leq \omega; |Y| \leq \omega/4$ and a random $S_i$.

$$
\Pr[S_i \text{ avoids } Y \text{ and hits } X] = \frac{\binom{|X|}{1} \binom{n-|X|-|Y|}{l-1}}{\binom{n}{l}}
$$

$$
= |X| \cdot l \cdot \frac{(n-|X|-|Y|)!}{n!}
$$

$$
= \frac{|X| \cdot l \cdot \prod_{i=0}^{l-2} \frac{n-|X|-Y-i}{n-i}}{n-l+1}
$$

$$
> \frac{\omega}{n} \cdot l \cdot \prod_{i=0}^{n-\frac{5}{4}\omega} \frac{n-i}{n-i}^{l-1}
$$

$$
\geq \frac{\omega}{n} \cdot \left(1 - \frac{5/4 \omega}{n-l+2}\right)^{l-1}
$$

$$
\geq \frac{\omega}{n} \cdot \frac{n}{\omega} \left(1 - \frac{5}{n/4}\right)^{n-1}
$$

$$
\geq \frac{1}{4} \left(1 - \frac{5}{n/4}\right)^{n-1}
$$

$$
= \frac{1}{4} \exp(-5)
$$

Let us bound the probability that for any sets $X, Y$ such that $\omega/4 \leq |X| \leq \omega$ and $|Y| \leq \omega$ there exists an $i$ such that $S_i$ hits $X$ and avoids $Y$. The probability of complementary event can be roughly bounded as
follows:

\[
\sum_{|X| = \lfloor \omega \log n \rfloor}^\omega \left( \frac{n!}{|X|!} \right) \sum_{y=0}^\omega \left( \frac{n!}{|Y|!} \right) (1 - c)^m \leq \omega^2 n^{2\omega} (1 - c)^m \leq n^{4\omega} (1 - c)^m \leq e^{4\omega \ln n - m \ln(1-c)} < 1.
\]

Note that the last inequality holds for some \( m = O(\omega \log n) \). That is, for such \( m \) the random structure \( S = (S_1, S_2, \ldots, S_n) \) with probability greater then zero hits any \( X \) and avoids any \( Y \) of an appropriate sizes. Thus such structure must exist and in consequence we can take \( S \) and use it for the reminder of the proof.

Now we show that \( S \) is a \( (n, \omega) \)-selector. Let us take any \( X \) such that \( \omega/2 \leq |X| \leq \omega \) and \( Y = \emptyset \). By the property of \( S \) there exists \( S_i \) such that it hits \( X \). Let \( \{r_1\} = |S_i \cap X| \). Now let us construct \( X = X \setminus \{r_1\} \) and \( Y = Y \cup \{r_1\} \). Since still \( \omega/4 \leq |X| < \omega \) and \( |Y| \leq \omega/4 \) we can find \( S_{i_2} \in S \), such that it hits the truncated \( X \) and avoids \( Y = \{r_1\} \) thus there exists \( r_2 = |S_{i_2} \cap X| \). Then we set \( X = X \setminus \{r_2\} \) and \( Y = Y \cup \{r_2\} \). We iterate such septation \( \omega/4 \) times to get \( \omega/4 \) distinct elements that are chosen from the initial \( X \). Thus we get the first case of the theorem.

To prove the second case, first we need to construct a \( \frac{n}{\omega} \)-selector \( S' \) of size \( m = O(\omega \log n) \). Clearly, this is possible using the above construction. Then we need to partition each \( S_i \in S' \) into \( \lceil \frac{n}{\omega} \rceil \) sets of size at most \( k \) to obtain a “diluted” selector. This results in \( m = O \left( \frac{n}{\omega^2} \omega \log n \right) = O \left( \frac{n}{\omega} \log n \right) \) sets of size at most \( k \).

**5.2.2 k-light Interleaved Selectors protocol**

We assume existence of a global clock.

Let us assume that \( n \) is a power of 2 and thus \( \log n \) is an integer. We consider a sequence of \( S_1, \ldots, S_{\log(n)} \), where \( S_i \) is \( k \)-light \( (n, 2^i) \)-selector of length \( m_i \). Moreover, let \( S^j_i \) be the \( j \)-th set of the \( i \)-th selector. That is, \( S_i = \{ S^1_i, \ldots, S^{m_i}_i \} \).

Let us consider the slot number \( t \) that can be uniquely represented as \( t = j \log n + i \) for \( 1 \leq i \leq \log n \) and \( j \geq 0 \).

Station \( x \) transmits in the \( t \) slot if and only if \( x \) has a packet to be transmitted and \( x \in S^j_i \mod m_i + 1 \).

The order sets of selectors “activating” stations is crucial for performance of the algorithm and motivate its name. This order is depicted on the Figure 1.

![Figure 1: Interleaved Selectors: A = \{S_1, S_2, S_3\}, where S_1 = \{S^1_1, S^2_1\}, S_2 = \{S^1_2, \ldots, S^2_2\} and S_3 = \{S^1_3, \ldots, S^2_3\}.](image)

**5.3 Protocol analysis**

Obviously in a single round at most \( k \) stations can transmit, since the sets \( S^j_i \) consist of at most \( k \) elements. Let us now investigate the performance of the protocol.

**Theorem 5.4.** Let us assume that in round \( t \) there are \( r \) stations with nonempty queues, such that \( 2^i \leq r < 2^{i+1} \). The system will transmit at least \( 2^j/16 \) packages before the round \( t' = t + 8 \sum_{i=1}^j m_i \log n \) for some \( j \geq i \).

**Proof.** Let us first consider a set \( X_0 \subset \{1, \ldots, n\} \) of stations such that \( |X_0| = r \) and \( 2^{i-1} \leq r < 2^i \). Let \( S_i = \{S^1_i, \ldots, S^{m_i}_i\} \) be a \((n, 2^i)\)-selector and \( S_{i+1} = \{S^1_i, \ldots, S^{m_{i+1}}_i\} \) be a \((n, 2^{i+1})\)-selector for some \( i < \log(n) \). We assume that stations from \( X_0 \) have nonempty queues of messages. We observe all stations during \( T = m_i + m_{i+1} \) rounds. We assume that the adversary can add packages to queues (even to initially
empty queues) during the execution of the algorithm. Let \( X_t \) be the set of nonempty stations in the round \( t \). In the \( j \)-th round stations from \( X_j \cap S_j^t \) transmit for \( j < m_i \) and \( X_j \cap S_j^{t-m_i} \) for \( j \geq m_i \). In other words, in consecutive rounds transmit nonempty stations pointed by sets from \( S_t \), then stations from \( S_{t+1} \).

**Lemma 5.5.** If less then \( 2^i/16 \) different stations has transmitted during \( T \) rounds of the process then \( |X_T| \geq \min \{r + 2^i/8, 2^i/2^{i+1}\} \).

**Proof.** Let \( Y = \bigcup_{i=1}^{T} X_i \setminus X_0 \) be the set of all stations filled by the adversary during the process. Let \( O^* \) be the set of stations that transmitted during the process. Moreover, let \( T(X) \) denote the set of the stations that transmitted at least once in the static case with the initial set \( X \) of nonempty stations, i.e. when the adversary does not add any messages. Clearly, \( |T(X_0 \cup Y)| \leq |O^*| + |Y| \). Indeed, adding \( Y \) to the set of nonempty stations can increase the number of transmitting stations only by \( |Y| \). On the other hand if a transmission of a station is blocked in the original process it must be also blocked in the case if all \( X_0 \cup Y \) stations are nonempty at the beginning. Let us consider two cases. In the first we assume \( |X_0 \cup Y| < 2^{i+2} \). In follows that \( |T(X_0 \cup Y)| \geq 2^i/4 \) because of the properties of selectors. Thus \( 2^i/4 \leq |O^*| + |Y| \). We assumed however that \( |O^*| < 2^i/16 \), thus \( |Y| > 3/16 \cdot 2^i \). That is, the adversary added messages to at least \( 3/16 \cdot 2^i \) initially nonempty stations but less then \( 2^i/16 \) has transmitted. Finally in he round \( T \) a least \( r + 2^i/8 \) are nonempty.

In the remaining case, if \( |X_0 \cup Y| > 2^{i+2} \) and only at most stations \( 2^i/16 \) transmitted, the lemma holds trivially. ■

Note that in any contiguous segment of \((m_i + m_{i+1}) \log n\) rounds all sets of stations with nonempty queues from \( S_i, S_{i+1} \) are allowed to transmit (see Fig 1). Following Lemma 1 after \((m_i + m_{i+1}) \log n\) executed rounds at least one of the three events occurred: (1) \( 2^i/16 \) transmitted; (2) the number of stations with nonempty queues increased by \( 2^i/8 \); (3) there is at least \( 2^{i+1} \) nonempty queues.

Note that event (3) may occur at most \( \log n - i \) times, similarly event (2) may occur at most \( 8(\log n - i) \) times till reaching the state of at least \( 2^{n-1} \) nonempty stations. Thus, after at most \( \sum_{i=1}^{n-1} (m_i + m_{i+1}) \log n + m_{\log n} \log n = O(\frac{n}{\rho} \log^2 n) \) rounds at least a fraction of nonempty stations will transmit at least one package. ■

**Corollary 5.5.1.** The protocol is stable for the leaky bucket adversary with any \( b \) and some \( \rho = \Theta(\frac{b}{n \log^2 n}) \).

6 Algorithms simulations

In order to evaluate efficiency of developed protocols, we performed simulations for both new and existing algorithms and compared the results. We analyzed the impact of the execution length, system size and injection rates on the queue sizes and energy consumption.

We collate Adaptive and Full-sensing versions of the 12-O’CLOCK algorithm with mathematical Backoff algorithms, exponential and polynomial, which are used as a base in most of radio-network technologies today. Additionally, we take into account acknowledgment-based algorithms: ROUND-ROBIN and 8-light INTERLEAVED-SELECTORS.

Our main simulation goals are to analyze and compare across the considered protocols:

(i) **General workflow** for stable injection rates;

(ii) **Critical injection rates**, that is - the lowest injection rates where queue size or latency begin to grow in exploding tempo (because in practice such rates indicate no stability);

(iii) **Energy** usage below critical injection rates, so that energy consumption in stable executions could be evaluated.

A digest of the obtained results is presented in Figures 2.9.

6.1 Implementation

We have implemented 12 O’CLOCK adaptive and full-sensing versions, 8-light INTERLEAVED-SELECTORS, ROUND-ROBIN, BACKOFF exponential, linear and square polynomial versions in Java and Julia languages. Their relationship to colors used on plots in Figures 5.9 is described in Figure 7. Each recorded result is an average of 120 experiments of one million rounds each.
Backoff protocols In general we follow the model from [17]. That is, we use synchronous model with a message length limited to the length of the transmission phase in a single round, and we do not terminate undelivered messages. However, we employ an upper bound on Backoff counter, as in real-world applications. A station attempts to transmit and learns about transmission success or failure within a single round. Analogously to the description in [17], the limits on the Backoff contention window size function (without window reset) are $2^i$, $2i$, $2^2i$ accordingly for exponential, linear and square polynomial versions of Backoff protocol, where $i$ denotes the unsuccessful transmission counter. The size of the window is limited by 2048, as the biggest system size studied in this work is equal to 32. That allows to protect protocols from unnecessary increase of the window size and thus improves their worst-case stability.

Acknowledgment-based protocols Round-Robin protocol allows any station $i$ to transmit alone in rounds $i$ modulo $n$. 8-light Interleaved-Selectors are based on randomly generated binary matrices, tested to satisfy the definition of $k$-light $(n,\omega)$-selector.

Adversary An adversary is defined by three parameters used at each round $r$: injection rate $\rho$ – the probability that an adversary will have one more packet in its stock, burst-probability $p$ – the probability of adversary making a decision to inject all of the stock packets at once, and finally the stock size limit $b$ – a constant forcing the adversary to inject all of its stock packets once the stock size is equal to $b$. For each packet decided to be injected to the system, the adversary selects a station $S_i$ for injection with probability $P_i$, where $i \in \{1, 2, 3, \ldots, n\}$: $P_1 = P_2 = \frac{1}{\pi} + \frac{1}{\pi^2}$; $P_{i>2} = \frac{1}{\pi^2}$.

Injection rate $\rho$ and burst-parameter $p$ have values in $(0, 1)$. The burst-probability models the adversary injection behavior: between rare bursts of large numbers of packets (close to 0) and steady flow (close to 1). The stock-size $b$ is a constant equal to 256, basing on operational buffer size limits. After performing some preliminary experiments for different values of $p$, we have chosen $p = 0.5$ for this presentation – it occurred not to influence the system performance as much as we had expected.

6.2 Boundaries on stable injection rates
6.2.1 Boundaries measurement

We took into consideration several measurements of queues of a protocol (at round $r$): a maximal queue size of a single station occurring up to round $r$ (max-max); an average, taken over $r$ rounds, of a maximal queue size of stations at a round (avg-max); a maximal over $r$ rounds of an average queue size of all stations at a round (max-avg); and finally, an average over $r$ rounds of an average queue size of all stations in a
Comparison of those measurements for 12 O’clock full-sensing and BACKOFF exponential protocols is shown in Figures 2 and 3 for system size $n = 32$ against flat packet distribution. The 12 O’clock full-sensing protocol started with $3n = 96$ packets per station (i.e., the total system queue size equal to $3n^2$) stabilizes against injection rate $\rho = 0.968$, which is slightly smaller than the theoretical stability boundary $\rho = \frac{31}{32} = 0.96875$, for all four measurements and its both avg-max and avg-avg measurements decrease after handling the starting queues burst (Figure 2). The adaptive version of the protocol behaves in a similar way when started with full queues against any injection rate $\rho < 1$ and is also bounded against injection rate $\rho = 1$. For exponential BACKOFF protocol, c.f., Figure 3, the max-max and avg-max measurements seem to diverge, while the other two measurements converge but to much higher values than in case of 12 O’clock full-sensing protocol.

Based on the above results, we have chosen the avg-max measurement for further comparison of protocols. This is because when considering other three ways of measuring: max-max is highly volatile for the randomized protocols (and thus it would not be fair for comparison randomized and deterministic protocols) while avg-avg and max-avg do not envision the worst case scenario we are focused on in this work. Note that the avg-avg measurement, studied in [17] and in many other previous papers considering stochastic injections, may yield stability while having some queues many times larger than the guaranteed average.

### 6.2.2 Boundaries for system sizes $n \in \{4, 5, \ldots, 32\}$

In order to see how system queues behave for different system sizes, we have combined simulation results for system sizes $n \in \{4, 5, \ldots, 32\}$ on a single plot (Figure 5). We have excluded the full-sensing version of 12 O’clock since its results are similar to the adaptive version in most of the considered scenarios. In this section we discuss the combined boundaries in Figure 5 and the stable injection rates depicted in Figure 6 defined as minimal injection rates $\rho$ for system size $n$ required to make the value of avg-max measurement to exceed the constant value $\delta = 1024$.

Our results are similar in shape with results from [17], with difference in values. It could be explained by the following: we implemented more adversarial behavior instead of Poisson distribution, used 1 million instead of 10 millions iterations for experiment length, avg-max measurement instead of avg-avg (to better capture worst-case behavior), and finally we set-up a maximal window size limit to comply with real applications of Backoff. Specifically, the maximal window size limit improves the efficiency of exponential BACKOFF protocol in comparison to other versions of BACKOFF protocols in our context.

**Acknowledgment-based** protocols have the same ROUND-ROBIN implementation of selectors for system
sizes $n \in \{4, 5, \ldots, 15, 17, 18\}$, because we were unable to generate better $(n, \omega)$-selectors for $\omega \leq n/2$ required for INTERLEAVED-SELECTORS in those cases. It follows that their plots overlap. The best achieved stability bound is around $\rho = 0.6$ for system size $n = 4$, and it gradually decreases with the increasing system size (in a pace resembling hyperbola). On the other hand, we can observe an improvement of INTERLEAVED-SELECTORS over ROUND-ROBIN protocol for bigger systems: for some system sizes its stability range is even a few times bigger than the stability range of ROUND-ROBIN. The irregular shape of INTERLEAVED-SELECTORS stable injection rates in Figure 6 is caused by selectors being generated independently for each (larger) system size, which leaves a clear scope for further optimization of the quality of selectors.

Note that despite of worse stability bounds, acknowledgment-based algorithms are literally tapes with binary information whether to-transmit or not-to-transmit, which makes them better hardware-tolerant algorithms.

**Backoff** protocols are system size dependent, and the following two interesting phenomena can be observed. First, the lower rank polynomial/function of Backoff protocol the wider extremes in stable injection rates it achieves for different system sizes, e.g., $\rho \in [0.55, 0.7]$ for exponential version versus $\rho \in [0.45, 0.8]$ for square and $\rho \in [0.4, 0.85]$ for linear version, c.f., the values of $\rho$ at the top boundaries of corresponding regions in Figure 5. The second observation is that for smaller system sizes the protocols with lower rank function achieve higher stable injection rates while for larger systems (starting from some size specific for the considered functions) the tendency is opposite c.f., Figure 6.

**Backoff exponential** and **12 O’clock adaptive** protocols show the most system-size independent behavior, with 12 O’clock adaptive protocol being a champion in terms of queue size stability, c.f., Figure 6. Note also that the stable injection rates of 12 O’clock full-sensing protocol improve with increasing system size.

### 6.3 Energy and stability

In order not to discriminate randomized BACKOFF protocols, which may obtain large energy peaks from time to time (unlike our deterministic protocols that guarantee bounded energy at any round), we measured average energy consumption of studied protocols by counting how many stations were not idle on average (over rounds). In Figure 8 we show the ratios of energy and queue size of the considered protocols to the corresponding performances of 12 O’clock adaptive protocol.

Observe that for BACKOFF protocols the energy consumption levels are relatively high and close to the system size when these protocols work within their stable boundaries. In contrary, the 12 O’clock and ACKNOWLEDGMENT-BASED protocols energy usage is low and upper bounded by a constant (i.e., independently on all system parameters).

In order to better illustrate bi-criteria comparison of protocols, we compare them with the **State-aware** protocol, which has full knowledge about all of the queues in the beginning of each round and transmits a packet from a station with the biggest queue. This protocol models close-to-optimal queues and energy usage for given injection patterns. Figure 9 presents our results in logarithmic scale: more efficient protocols...
in energy-queue dimensions are closer to the STATE-AWARE protocol, what makes 12 O’CLOCK adaptive protocol our champion for all injection rates and 8-light INTERLEAVED-SELECTORS to be the second for injection rates lower than $\rho = 0.3$. (Full-sensing version of the 12 O’CLOCK protocol has been omitted from the graphs as it behaved similarly to its adaptive version in our experiment.)

7 Conclusion

We have proposed the $k$-PPC energy model for multiple access channels and studied queue stability of deterministic broadcasting protocols. We have developed protocols with guaranteed constant upper bound on power consumption. 12 O’CLOCK protocol maintains stability for all injection rates $\rho \leq 1$ in its adaptive version and for all $\rho < 1$ in its full-sensing version. K-LIGHT SELECTORS algorithm, though stable for slightly smaller range of injection rates, opens a door for software-less communication solutions within low-level transmission density.

Simulations have shown stability from close to real-world environment perspective (comparing to pure stochastic models). BACKOFF protocols were studied as the most commonly used contention-resolution approach. Experiments have repeated [17] results in regard of tendencies, with some differences in actual values of measurement, most likely caused by few differences in implementation (as discussed earlier in this work). BACKOFF protocols have shown limited stability and inability to have small queues and effective energy usage at the same time. 12 O’CLOCK protocols have shown stability combined with power efficiency and can be applied to both high transmission density and power-limited applications.

8 Discussion and future work

Dynamic system size. It is possible to modify 12 O’CLOCK protocols to handle an increase of the system size. A new station may learn about the beginning of the list by listening to the communication on the channel, and then force its way into the list by a transmission causing collision. The decreasing system size may be handled by removing stations from the end of the list when those stations hold their positions without transmitting anything longer than some time $\tau$.

Adversarial model. An alternative approach would be to allow the adversary to control dynamicity of the system within some pre-defined “budget” for keeping stations “alive”.

Noise resistance. Another interesting dimension would be to study more complex physical models, e.g., with possible transmission interference: when there is a small probability $p$ that transmission fails even in the lack of collision.

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