A study on the extrusion processes of hollow products in the die with a movable matrix

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Abstract. Calculation formulas of general type and the developed mathematical model of the direct extrusion process are presented, which makes it possible to determine the force mode of deformation. The examples of calculations and graphs showing the possibility of determining the optimal tool geometry and determining the amount of decrease in deformation force when using a die with a movable matrix are given.

1. Introduction
The hollow parts having conical surfaces find wide application in various designs and products. The manufacture of such parts by cold extrusion forging provides an improvement in performance due to the possibility of obtaining parts with the oriented fiber structure, and also makes it possible to reduce or completely eliminate the products finishing by cutting, which ensures their competitiveness.

The use of dies with a moving matrix (figure 1) for the manufacture of parts with conical cavities by direct extrusion from annular or cylindrical workpieces allows the deforming force to be reduced, since in this case the frictional forces acting on the surface between the workpiece and the matrix play an active role.

2. Methods of research
The magnitude of the deforming extrusion force can be determined by the theoretical upper bound method [1, 2] using the process scheme and the velocity hodograph, adopting a planar deformation scheme.

The velocity hodograph (figure 2b), where the velocity vectors are constructed from the pole point O, with the numbering corresponding to the blocks, will be the same for both forming schemes shown in figure 2a and figure 2c.

At the same time, when extruded in a die with a movable matrix (figure 2c), the costs for overcoming the frictional forces acting on the contact surface between the deformed workpiece and the matrix are substantially lower due to the decrease in velocity by the value \( V_i \).
Figure 1. Scheme of workpiece deformation between the fixed counterpunch and the movable matrix: (a) extruding the annular preform; (b) extruding a continuous blank.

Figure 2. Calculation schemes: a – extrusion in the die with a fixed matrix; b – velocity hodograph corresponding to this scheme; c – extrusion in the die with a movable matrix.

Note that the design of the hodograph is the most time-consuming calculation stage of the known theoretical method [1]. However, it is possible to determine the magnitude of the deforming force of extrusion even without the velocity hodograph construction, using only the design scheme of the process [2]. In this case, the relative specific force $q$ acting on the punch is represented by the sum calculated from equations (1) and (2), the fractions of this force expended on shear and friction along the boundaries of the blocks in accordance with the scheme of the plastic deformation process.

\[ q_i = \mu \frac{l_i}{h_i}, \]  
\[ q_j = \frac{l^2_i \cdot \sin \gamma}{h_i \cdot h_j}, \]  
\[ (1) \]  
\[ (2) \]
where $\mu = \tau_y/\tau_s$; $\tau_y$ – the metal yield strength on shear; $\tau_s$ – the contact shear stress, $l_{ij}$ – the lengths of boundaries between blocks $i$ and $j$; $l_i$ and $l_j$ – the lengths of the sides of the $i$-th and $j$-th blocks parallel to the current lines in these triangular blocks, $h_i$ and $h_j$ – the block heights; $\gamma$ – is the angle of current lines rotation on the adjacent boundary of the blocks $i$ and $j$.

For the considered schemes, the lengths of the boundaries of triangular blocks (with regard to the accepted notations) can be expressed through coordinates of six nodes (table 1) and trigonometric functions of two angles $\alpha$ and $\beta$.

| Table 1. Coordinates of nodes. |
|-------------------------------|
| Node numbers and node coordinates | 1 | 2 | 3 | 4 | 5 | 6 |
|----------------------------------|---|---|---|---|---|---|
| $x_1 = 0$ | $x_2 = R$ | $x_3 = r$ | $x_4 = R$ | $x_5 = r_5$ | $x_6 = R$ |
| $y_1 = 0$ | $y_2 = 0$ | $y_3 = h_0$ | $y_4 = l_3$ | $y_5 = L$ | $y_6 = L$ |

Equation for calculating the power mode of the process:

$$ q = q_1 + q_2 + q_3 + q_4 + q_5 + q_{12} + q_{23} + q_{34} + q_{45}. \quad (3) $$

According to the calculation scheme: the total height of the forging ($l_1 + L$), as well as the coordinates of the $5$th and $6$th nodes, correspond to the required forging dimensions at the final extrusion time; $l_{12} = R$ and parallel to $X$ axis; the origin of coordinates coincides with the first node; $h_0$ – height of the 0th hard block of the workpiece; $l_1$ – length of the contact boundary of the 1st rigid block of the workpiece and the matrix. Given this, we find the length of the blocks boundaries:

$$ l_2 = \frac{r}{\sin \beta}, \quad l_{23} = \left[ h_0^2 + (R - r)^2 \right]^{1/2}, \quad l_{34} = \left[ (l_3 - h_0)^2 + (R - r)^2 \right]^{1/2}, \quad l_4 = \frac{r_5 - r}{\sin \alpha}, $$

$$ l_{45} = \left[ l_5^2 + (R - r_5)^2 \right]^{1/2}, \quad l_5 = L - l_3. $$

Let us calculate the block heights:

$$ h_2 = \frac{R \cdot h_0}{l_2}, \quad h_3 = R - r, \quad h_4 = \left[ R - r - (l_3 - h_0) \cdot \tan \alpha \right] \cos \alpha, \quad h_5 = R - r_5. $$

Further, using equations (1) and (2), we define the components of the right-hand side of equation (3) that make up:

$$ q_1 = \frac{\mu \cdot l_1}{R}, \quad q_2 = \frac{\mu \cdot l_2}{h_2}, \quad q_3 = \frac{\mu \cdot l_3}{h_3}, \quad q_4 = \frac{\mu \cdot l_4}{h_4}, \quad q_5 = \frac{\mu \cdot l_5}{h_5}, \quad q_{12} = \frac{R \cdot \sin \beta}{h_2}, \quad q_{23} = \frac{l_{23}^2 \sin \beta}{h_2 h_3}, \quad q_{34} = \frac{l_{34}^2 \sin \alpha}{h_3 h_4}, \quad q_{45} = \frac{l_{45}^2 \sin \alpha}{h_4 h_5}. $$

Equation (3), after substituting all found components, is a mathematical model designed to calculate the process force mode of the direct extrusion in the die with a fixed matrix (figure 2a). It allows us to find the theoretical value of the deformation force and optimize the process, determining the geometric relationships of the forming tool, in which extrusion is carried out with the lowest energy costs. Note that varying the coordinates of the 1st, 2nd and 3rd nodes increases the accuracy of calculations in determining the optimal ratios.

When extruded in the die with a movable matrix (figure 2c), due to the decrease (by value $V_1$) in the relative speed of movement of the formed metal on the surface of its contact with the matrix, the
relative specific force acting on the punch decreases, which improves the resistance of the deforming tool. When using the developed mathematical model, this decrease in force is taken into account as follows: \( q_1 = 0 \); \( q_3 \) and \( q_5 \) are multiplied by \((R - h_s) \cdot h_i^{-1}\) and by \((R - h_s) \cdot h_i^{-1}\), respectively.

The constructed mathematical model of the process, taking into account several variable parameters, allows their optimal values to be calculated and the optimum tool ratios to be found for which the minimum deformation power is ensured, which is shown in figures 3-6, where the coefficient \( \mu \), taking into account friction conditions on the tool, changed within 0 to 0.5.

3. Results and discussion

The dependencies in figure 3 show that when the ordinate node 4 is varied in the deformation area, expressed through \( l_3 \), it is always possible to find its position (for \( r = 5 \), \( \alpha = 30^\circ \)), in which the minimum deformation power is reached. However, for different instrument ratios this position can vary significantly. For example, figures 4 and 5 show that for \( R = 30 \), \( r_s = 20 \), \( l_1 = 15 \), \( \beta = 45^\circ \), the optimal values of \( l_3 \) and the cone angle \( \alpha \), depending on the radius \( r \) determining the initial gap, and the coefficient of friction \( \mu \), vary much.

Figure 6 shows the theoretical values of the relative specific deformation force at the optimum values of the variable parameters.

**Figure 3.** Graphs of the change in the relative specific force \( q \) as a function of \( l_3 \) and \( \mu \).

**Figure 4.** Graphs of the change in the optimal value \( l_3 \) as a function of \( r \) and \( \mu \).

**Figure 5.** Graphs of the change in the optimal angle \( \alpha \) (deg.) as a function of \( r \) and \( \mu \).

**Figure 6.** Graphs of the change in the specific force \( q \) at the optimal \( l_3 \) and \( \alpha \).
4. Conclusion
The obtained results show that the direct extrusion of the workpiece in the die with the movable matrix helps to reduce the technological load improving the resistance of the deforming tool, in comparison not only with direct extrusion in the fixed matrix, but also with the reverse extrusion, due to the reduction of the prevailing influence of frictional forces acting on the workpiece contact surface with the matrix.

The method of calculation, which does not require the construction of a velocity hodograph, shown in the example of the extrusion process in the die with a movable matrix, reduces the complexity of developing mathematical models of various forming processes and can be used to calculate the technological load and optimize the geometry of the forming tool.

References
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