Performance Analysis of CFAR Detector Based on Censored Mean and Cell Average

Jian Ma, Jianjun Xiang, Fang Peng
Aeronautics Engineering College, Air Force Engineering University, Xi’an 710038, China
majiankgy@163.com

Abstract. It is proposed in this paper that a new constant false alarm rate detector based on censored mean and cell average. The detector takes the mean value of censored mean and cell average as noise or clutter power estimation. Under homogeneous background and Swerling II target models, the detection probability analytic expressions and the false alarm probability analytic expressions of the detector are derived. In homogeneous background and multiple interfering targets situation, the detection performance of the detector is analyzed. Compared with the classical detectors, such as the cell average detector, the greatest of detector and the smallest of detector, the new detector has better detection performance both in homogeneous background and in multiple interfering targets situation.

1. Introduction
Constant false alarm rate(CFAR) processing is the automatic selection detection threshold based on noise or clutter power. The choice of detection threshold affects the detection probability of radar. The CFAR processing usually estimates the power of the noise or clutter and then multiplies it by the threshold factor to obtain the detection threshold. The threshold factor is decided by false alarm probability ($P_{fa}$). It is the key technical to estimate the power of noise and clutter. A new CFAR detector called the CMCA-CFAR detector is proposed in this paper. It uses the logic operation of censored mean (CM) [1-2] in leading reference windows and cell average (CA) [3-4] in lagging reference windows to estimate the power of noise and clutter. Under homogeneous background and Swerling II target models, the detection probability ($P_d$) analytic expressions and $P_{fa}$ analytic expressions of the detector are derived. Under the homogeneous background and multiple interfering targets situation, the detector performance is analyzed. Compared with classical detectors [5-6], such as CA, greatest of (GO) [7], smallest of (SO) [8], and ordered statistics (OS) [9] detectors, the new detector has better detection performance both in homogeneous background and in multiple interfering targets situation.

2. Structure of Detector
The structure of CMCA-CFAR detector is show in Fig.1. $v$ is the sample of the test cell. The two shadow cells are protection cell which can prevent energy leakage into adjacent cells. Both sides of the test cell are called reference windows. The number of reference cells is $m+n$. The $m$ is length of leading reference window, and the $n$ is length of lagging reference window. $Z$ is estimation value of noise power. $T$ is a threshold factor. $x_i$ ($i=1,2,\cdots,m$) is sample value of leading reference windows. $y_j$ ($j=1,2,\cdots,n$) is sample value of lagging reference windows. In leading reference window, it use CM
method to estimate noise power $X$. In lagging reference window, it use CA method to estimate noise power $Y$. So the total noise power is $Z=X+Y$.

![Fig.1 Structure of the CMCA-CFAR detector](image)

### 3. Formulation of Detector

Here it can be considered that the noise in the receiver is white Gaussian noise, and the detection envelope is Rayleigh distributed. In single-pulse and square-law detection, the test cell and reference cell variables are always assumed to be independent and identically distributed. So in homogeneous background, the sample value of leading reference windows $x_i$ ($i=1,2,\cdots,m$) and sample value of lagging reference windows $y_j$ ($j=1,2,\cdots,n$) are identically distributed, their probability density function (PDF) as follows

$$f_{x_i}(x) = f_{y_j}(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}} \quad x > 0$$

The noise power estimation of leading reference windows is $X$, It takes the mean value of CM estimations as a noise power estimation. The noise power estimation of lagging reference windows is $Y$, It takes the mean value of CM estimations as a noise power estimation. The value of $X$ and $Y$ as follows

$$X = \frac{1}{m-r} \sum_{i=1}^{m-r} x_i \quad Y = \frac{1}{n} \sum_{j=1}^{n} y_j$$

The PDF of $X$ in leading reference window is \[5,10\]

$$f_x(x) = \sum_{j=1}^{m-r} a_j e^{-\frac{c_j}{\mu}}$$

with

$$a_j = \frac{m-r}{\prod_{i\neq j} (c_i-c_j)} = \left(\frac{m-r}{r}\right)_{j-1}^{-1} \left(\frac{m-j+1-r}{r}\right)^{m-r-1}, \quad c_j = \frac{m-j+1}{m-r-j+1}$$

The PDF of $Y$ in lagging reference window is \[5,10\]

$$f_y(y) = \frac{y^{n-1} e^{-\frac{y}{\mu}}}{\Gamma(n)}$$

The detection probability of the detector is
\[ P_d = \int_0^{\infty} e^{-\frac{z}{\mu}} f_z(z)dz = M_z(u) \bigg|_{\text{SNR}} \]  

(6)

\( f_z(z) \) is PDF of the noise power estimation \( Z \), \( M_z(u) \) is the moment generation function (MGF) of the noise power estimation \( Z \). So the false alarm probability of the detector is

\[ P_{fa} = M_z(u) \bigg|_{u=T} \]  

(7)

Because the total noise power estimation is \( Z = X + Y \)

(8)

So the PDF and MGF of the \( Z \) can be written as

\[ f_z(z) = \int_0^{\infty} f_x(x)f_y(z-x)dx \]  

(9)

\[ M_z(u) = M_x(u) \cdot M_y(u) \]  

(10)

With

\[ M_x(u) = \int_0^{\infty} e^{-u} f_x(z)dz = \sum_{j=1}^{\infty} \frac{a_j}{\mu u + c_j} \]  

(11)

\[ M_y(u) = \int_0^{\infty} e^{-u} f_y(z)dz = \left( \frac{n}{\mu u + n} \right)^n \]  

(12)

the MGF of the detector is

\[ M_z(u) = \left( \frac{n}{\mu u + n} \right)^n \sum_{j=1}^{\infty} \frac{a_j}{\mu u + c_j} \]  

(13)

Substituting (13) in (6), the analytic expressions of \( P_d \) for the detectors are obtained. And Substituting (13) in (7), the analytic expressions of \( P_{fa} \) for the detectors are obtained. The analytic expressions of \( P_d \) and \( P_{fa} \) for the detector as

\[ P_d = \left( \frac{n}{T + n} \right) \sum_{j=1}^{m-r} \frac{a_j}{T + c_j} \]  

(14)

\[ P_{fa} = \left( \frac{n}{T + n} \right) \sum_{j=1}^{m-r} \frac{a_j}{T + c_j} \]  

(15)

4. Detector Performance

4.1. Homogeneous background

In homogeneous background, detector performance is determined by detection probability. Figure 2 shows the detection performance of the five detectors in homogeneous background. These detectors are CA, GO, SO, OS and CMCA CFAR detectors. The simulation parameters are selected as \( P_{fa}=10^{-6} \), \( m=n=8 \), the \( k \) of the OS detector is taken as 12[9], and the \( r \) of CMCA detector is taken as 1. Figure 2 explanation of the performance of CA detector is optimal, which is due to participation the number of reference windows estimated by noise is the largest[11]. The performance of CMCA detector and the GO detector are similar. The performance of OS detector is poor, and the performance of the SO detector is the worst.
For comparison, figure 3 shows the difference in performance of the other four detectors relative to the CA detector in homogeneous background with the same parameters. As can be seen from figure 3, the performance of the CMCA and GO detector is slightly lower than the performance of CA detector. The performance of CMCA detector is superior to that of the GO detector. The performance of OS and SO detectors is far from CA.

4.2. Multiple target situations

In multiple target situation, the echoes of the interfering targets influence threshold estimation. Detector performance will drop. Figure 4 shows the detection performance of five detectors in multiple target situations. These detectors are CA, GO, SO, OS and CMCA CFAR detectors. Here it is assumed that the interfering targets are in the leading reference window, and the number of interfering targets is from 1 to 4. The simulation parameter is selected as $P_{fa}=10^{-6}$, $m=n=8$, the $k$ of the OS detector is taken as 12, and the $r$ of CMCA detector is taken as 1. As can be seen from figure 4, CMCA detector performance is optimal in multiple target situations, followed by OS and SO detectors. The CA and GO detectors perform is poor. If the number of interference targets in the leading reference window increases from 1 to 4, the performance of CMCA detector remains basically unchanged, the performance of SO and OS detector decreases slightly, and the performance of CA and GO detectors decreases significantly.
5. Conclusion
A new CFAR detector is proposed in this paper. It uses the logic operation of censored mean leading reference windows and cell average in lagging reference windows to estimate the power of noise and clutter. Under homogeneous background and Swerling II target models, the analytic expressions of detection probability and false alarm probability for the detector are derived. Under the homogeneous background and multiple interfering targets situation, the detector performance is analyzed. The analytic results show that detection performance of CMCA is good, and the structure of detector is simple. The detector is easy to accomplish, it has good application value.

References
[1] Ritcey J A. Performance analysis of the censored mean level detector[J]. IEEE Trans on AES, 1986,22(4):443-454.
[2] Yuan Hui, Tao Jianfeng, An Lei. Two CFAR detectors based on cell selection[J]. Computer Measurement and Control. 2013, 21(4):1057-1059. (in Chinese).
[3] Finn H M, Johnson R S. Adaptive detection mode with threshold control as a function of spatially sampled clutter level estimates[ J]. RCA Review,1968,29:414-464.
[4] Qin Yuhua, Hao Chengpeng. A CFAR detector based on automatic censoring cell averaging and cell averaging[J]. Journal of Detection and Control, 2011, 33(1):14-17. (in Chinese).
[5] He You, Guan Jian, Meng Xiangwei. Radar target detection and CFAR processing[M]. Tsinghua University Press, 2011:36-67.(in Chinese).
[6] Guo Yulan, Ou Jianping, Zhang Jun. A switching CFAR detector based on greatest selection[J]. Journal of National University of defense technology, 2010, 32(5):92-97. (in Chinese).
[7] Ritcey J A, Hines J H. Performance of max mean level detector with and without censoring[J]. IEEE Trans on AES,1989,25(2):213-223.
[8] Weiss M. Analysis of some modified cell averaging CFAR processors in multiple target situations[J]. IEEE Trans on AES, 1982,18(1):102-114.
[9] Rohling H. Radar CFAR thresholding in clutter and multiple target situation[J]. IEEE Trans on AES, 1983,19(4):608-621.
[10] Ma Jian, Wu Lan, Xu Songtao, et al. A new type CFAR detectors based on censored mwen and cell average[J]. Lecture Notes in Computer Science, 2012, 7202:706-712.
[11] Gandhi P P, Kassam S A. Analysis of CFAR processors in homogeneous background[J]. IEEE Trans on AES,1988,24(4):427-445.