Maser mechanism of optical pulsations from anomalous X-ray pulsar 4U0142+61

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ABSTRACT

Based on the work of Luo & Melrose (1992), a maser curvature emission mechanism in the presence of curvature drift is used to explain the optical pulsations from anomalous X-ray pulsars (AXPs). The model comprises a rotating neutron star with a strong surface magnetic field, i.e., a magnetar. Assuming the space charge-limited flow acceleration mechanism, in which the strongly magnetized neutron star induces strong electric fields that pull the charges from its surface and flow along the open field lines, the neutron star generates a dense flow of electrons-positions (relativistic pair plasma) by either two photon pair production or one photon pair creation resulting from inverse Compton scattering of the thermal photons above the pulsar polar cap (PC). The motion of the pair plasma is essentially one-dimensional along the field lines. We propose that optical pulsations from AXPs are generated by curvature-drift-induced maser developing in the PC of magnetars. Pair plasma is considered as an active medium that can amplify its normal modes. The curvature drift which is energy dependent is another essential ingredient in allowing negative absorption (maser action) to occur. For the source of AXP0142+61, we find that the optical pulsation triggered by curvature drift maser radiation occurs at the radial distance $R(\nu_M) \sim 4.75 \times 10^9$ cm to the neutron star. The corresponding curvature maser frequency is about $\nu_M \approx 1.39 \times 10^{14}$ Hz, and the pulse component from the maser amplification is about 27%. The result is consistent with the observation of the optical pulsations from the anomalous X-ray pulsar 4U0142+61.

Key words: masers-radiation mechanism-pulsars:individual(4U0142+61)-stars:neutron-X-rays:stars
INTRODUCTION

Anomalous X-ray pulsars (AXPs) are a class of rare X-ray emitting pulsars whose energy source have been perplexing for some 20 years (Arons 1983; Fahlman & Gregory 1981; Van Paradijs et al. 1995; Mereghetti & Stella 1995). Unlike other x-ray emitting pulsars, the luminosity of AXPs is orders of magnitude greater than their rotational spin-down power, and so they require an additional energy source. One possibility is that AXPs are thought to be solitary magnetic rotating neutron stars with a magnetic field stronger than $10^{14}$ G (Thompson & Duncan 1996). This would make them similar to the soft γ ray repeaters (SGRs) (Kouveliotou et al. 1998; Kaspi 2004), but alternative models of binary system scenario with a very low mass companion that do not require extreme magnetic fields also exist (for a review see, Israel et al. (2002b); Mereghetti et al. (2002)).

It has been reported that the spectral properties of AXPs are characterized by the sum of power-law and black-body components and a small pulse fraction in X-ray band; some AXPs have been observed with optical emission, which in some cases, i.e., in AXP 4U0142+61, has a larger optical pulse fraction than that in the X-ray band by five to ten times (Hulleman et al. 2000; Kern & Martin 2002; Israel et al. 2003). The optical emission has been related to the accretion disk model (Chatterjee et al. 2000) owing to reprocessing of the X-ray irradiation (Perna et al. 2000). Assuming that beamed X-rays are emitted at the neutron star surface, then the rotation of the neutron star leads to the observed X-ray pulsations. In the accretion disk model optical pulsations at the X-ray pulsation period can arise from thermal reprocessing of X-rays illuminating a disk. Another possible model for 4U0142+61 is a white dwarf with the magnetic field $B = 5 \times 10^8$ G and the temperature $T = 4 \times 10^5$ K, possibly the result of a double-degenerated merger (Hulleman et al. 2000). Not only the accretion model and the white dwarf model are inconsistent with the measurement of optical pulsations, but more importantly, they are also inconsistent with the observed optical/IR spectrum which indicates that AXP 4U0142+61 is a magnetar (Kern & Martin 2002). The magnetar model, originally proposed by Duncan & Thompson (1992) to explain SGRs, appears to be successful at interpreting most of the properties of AXPs. A surface dipolar magnetic field of order $B_{ns} \sim 10^{14} - 10^{16}$ G naturally accounts for the long periods $P$ and high spin-down rates $\dot{P}$ in the magnetic braking model (Kouveliotou et al. 1998). The luminous X-ray emission may be explained as the decay of super-strong magnetic fields (Thompson & Duncan 1996).
Further evidence has recently come from the burst activity of two AXPs, as predicted by the magnetar model (Gavriil et al. 2002; Kaspi et al. 2003). The magnetar model for AXPs has been spectacularly successful in explaining their most important phenomenology, including the optical pulsations of AXPs could arise if the magnetosphere emission of the magnetar is self-absorbed at optical frequency (Hulleman et al. 2000), and the magnetars could radiate coherent emission from plasma instabilities in the infrared and optical bands, as discussed recently by Eichler, Gedalin, & Lyubarsky (2002) (hereafter EGL02). However, there are so far no detailed models to account for the anomalous behavior noted for the optical pulsations of AXP 4U0142+61 (Israel et al. 2003).

The possibility of maser mechanism through the inverse population of particles over energy levels (Chiu & Canuto 1971; Kaplan & Tsytovich 1973; Zheleznyakov 1973) has been widely discussed to address the origin and the main properties of observed radio emission (Sturrock 1971; McCray 1966; Zheleznyakov 1966; Beskin et al. 1988). It is maser in a sense that the radiation intensity exceeds the total spontaneous radiation intensity of all individual particles in the source, i.e., coherent emission mechanism acting in astrophysical conditions (Ginzburg, & Zheleznyakov 1975). In most models for a pulsar magnetosphere (Goldreich & Julian 1969; Sturrock 1971; Michel 1982), the curvature emission as power mechanism is associated with such a system. Three theories for coherent curvature emission have been suggested: coherent curvature emission by bunches (Sturrock 1971; Ruderman & Sutherland 1973; Cheng & Ruderman 1980), curvature maser via inhomogeneity due to curvature of magnetic field lines (Beskin et al. 1988), and curvature maser by curvature drift (Zheleznyakov & Shaposhnikov 1979).

The maser mechanism attributed to curvature drift was discussed in detail by Luo & Melrose (1992, hereafter LM92). With an appropriate choice of the electron energy spectrum, the curvature absorption coefficient including the curvature drift can become negative, and the drift motion allows amplification to occur (LM92). Furthermore, Luo & Melrose (1995) considers the curvature maser emission due to field line torsion in pulsar magnetospheres. Such maser emission was also applied to pulsar radio emission. For both curvature-drift-induced maser emission (hereafter, CDIME) and torsion-induced-maser emission (hereafter, TIME), (i) the maser action relies on the spectral asymmetry due to the dependence of the drift angular shift on the Lorentz factor \( \gamma \), (ii) maser emission occurs only in a limited angular range; and (iii) the Lorentz factor must exceed a threshold (Luo & Melrose 1995).

There is as yet no generally accepted mechanism for explaining why the optical emis-
sion is more strongly pulsed than the soft X-ray emission. We investigate how the CDIME model works in the magnetar environment and associate it with the properties of the optical pulsations from AXP sources. The organization of the paper is as follows: in Sect.2, we first make a brief review of the necessary conditions for CDIME proposed by LM92 and discuss the properties of relativistic pair plasma along the open field lines in magnetars. In section 3, we discuss the parameters of the region where optical emission are supposed to be generated and apply the theoretical framework developed in the previous section for a detailed calculation of optical pulsations in magnetar sources. It is shown that the CDIME mechanism makes it possible to explain the characteristics of optical pulsations in AXPs 4U0142+61. We should emphasize that the model is based only on the general considerations concerning the properties of the flux of relativistic electron-positron plasma flowing in the magnetosphere of a magnetar. Finally, summary and discussion of the results are given.

2 CURVATURE-DRIFT-INDUCED MASER EMISSION AND PAIR PLASMA

2.1 Necessary conditions for CDIME

The maser curvature emission triggered by curvature drift has been investigated by LM92. This mechanism is outlined as following: the maser emission is possible only when there is beam-type distribution of outflowing pairs with the Lorentz factor $\gamma$ satisfying a certain value, which is determined by the geometry of the pulsar magnetosphere. Due to the strong magnetic field, the electrons and positrons lose all their perpendicular energy rapidly to synchrotron radiation and fall to the lowest Landau levels. Therefore the particle motion in a pulsar magnetosphere is essentially one-dimensional. In the one-dimensional approximation, for maser action to occur, both the following conditions must be satisfied:

$$
\frac{df(\gamma)}{d\gamma} > 0 ,
$$

$$
\frac{d\eta}{d\gamma} < 0 .
$$

where $f(\gamma)$ is the particle distribution function normalized to unity, and $\eta$ is the spectral power of a single particle, which is formulated by

$$
\eta(\omega, \theta, \gamma) = \frac{q^2 \omega^2 R_B}{6\pi^3 c^2} \left\{ (\theta - \theta_d)^2 [\xi^{-1} K_{1/3}(y)]^2 + [\xi^{-2} K_{2/3}(y)^2] \right\} ,
$$

where $\theta$ is a polar angle with $|\theta| << 1$, $\omega$ is the curvature emission circular frequency,

$$
\xi = \left\{ 2(1 - n) + n[\gamma^{-2} + (\theta - \theta_d)^2] \right\}^{-1/2} ,
$$
$n$ is the refractive index, and $y$ is defined by

$$y = \omega/(3n^{1/2}\omega_{R}\xi^3),$$

with $\omega_{R} \approx c/R_{B}$. $R_{B}$ is the the radius of curvature on a field line. Considering a beam of electrons propagation in pure dipole curved magnetic field, the curvature radius on the field line can be written as (Lyutikov et al. 1999),

$$R_{B} = \frac{4}{3}\frac{\sqrt{R_{ns}R}}{\alpha_{*}}, \quad (4)$$

here $\alpha_{*}$ is the angle at which a given field line intersects the neutron star surface. $R$ and $R_{ns}$ is the radial distance to the star and the surface radius of the star, respectively. For the case of the last open field, one has (Lyutikov et al. 1999)

$$\alpha_{*} = \sqrt{2\pi R_{ns}/c P} \approx 5.91 \times 10^{-3} P_{6}^{-1/2}, \quad (5)$$

where $P$ is the spin period of the star, and $P_{6} = P/6$ s.

Putting $\alpha_{*}$ into Eq.(4), we have

$$R_{B} = 2.25 \times 10^{8} P_{6}^{1/2}, \quad (6)$$

$$\omega_{R}(R_{B}) \approx c/R_{B} = \frac{4}{3} \times 10^{2} P_{6}^{-1/2}. \quad (7)$$

The condition in Eq.(1) requires an effective particle population inversion which provides free energy to drive the maser emission. The second condition in Eq.(2) is obtained by partial integration of the absorption coefficient $\Gamma$, which is written as (Melrose 1978)

$$\Gamma(\omega, \theta) = -\frac{(2\pi c)^3}{2\omega^2mc^2} \int d\gamma \frac{df(\gamma)}{d\gamma} \eta(\omega, \theta, \gamma), \quad (8)$$

where $N_{0}$ is the number density of primary relativistic electron beam ejected from the PC, and is roughly equal to the Goldreich-Julian density $n_{GJ}$ at the stellar surface. $n_{GJ}$ is defined by (Goldreich & Julian 1969)

$$N_{0} = n_{GJ} = 1.15 \times 10^{12} B_{ns,14} P_{6}^{-1},$$

where $B_{ns}$ is the magnetic field strength at the stellar surface, and $B_{ns,14} = B_{ns}/10^{14}$ G.

Once the condition in Eqs.(1) and (2) are satisfied, maser action can occur. It should be noted that maser emission corresponds to the absorption coefficient Eq.(8) being negative for a certain range of parameters (e.g. within a certain angular range). In order to determine in which frequency range the amplification is important, one needs to estimate the optical depth. According to LM92, the maximum modulus of optical depth for negative absorption
is estimated to be
\[ \tau(a) \approx \frac{1}{8\pi} \left( \frac{\omega_B}{\gamma \omega} \right) \left( \frac{W_p}{W_m} \right) \Delta \theta_0 , \]
where \( \omega_B = |e|B/m_e c \) is the non-relativistic gyro-frequency, \( e \) is the charge of the pair particle, \( m_e \) is its mass and \( c \) is the speed of light. \( W_p = m_e c^2 \gamma N_0 \) is the particle energy density, and \( W_m = B^2/8\pi \) is the magnetic energy density. The one-dimensional motion requires that \( W_p/W_m \leq 1 \). \( \Delta \theta_0 \) is defined by
\[ \Delta \theta_0 = \begin{cases} (3\omega_R/\omega)^{1/3} & ; \text{for } \omega \ll \omega_c , \\ 1/\gamma & ; \text{for } \omega \approx \omega_c , \\ (\omega_c/\omega)^{1/2}\gamma^{-1} & ; \text{for } \omega \gg \omega_c , \end{cases} \]
where \( \omega_c \) is the characteristic frequency of the curvature radiation,
\[ \omega_c = \gamma^3 \omega_R . \]
Considering Eq.(10), \( \omega_R, W_p \) and \( W_B \), we rewrite Eq.(9)
\[ \tau(a) \approx \begin{cases} 4.48 \times 10^3 P^{-1} R_B^{-1/3} \omega^{-4/3} & ; \text{for } \omega \ll \omega_c , \\ \gamma^{-1}\omega^{-1}P^{-1} & ; \text{for } \omega \approx \omega_c , \\ 1.73 \times 10^5 \gamma^{1/2} R_B^{-1/2} \omega^{-3/2} P^{-1} & ; \text{for } \omega \gg \omega_c , \end{cases} \]
Given the the optical depth \( |\tau(\nu_M)| \), one can estimate the corresponding amplification ratio \( \eta \) at maser action occurring
\[ \eta \equiv \frac{\Delta I_r}{I_r} = \exp(-|\tau(\nu_M)|) , \]
where \( I_r \) is the intensity of curvature emissions, \( \Delta I_r \) is the amplified intensity at maser frequency \( \nu_M \). Consequently appreciable amplification requires that the modulus of the effective optical depth \( |\tau(\nu_M)| \) be greater than unity. According to the conclusions of LM92, the amplification is important only when the frequencies satisfy \( \omega \leq \gamma^{-1}\omega_B \). This only takes place in the case \( \omega \ll \omega_c \), the corresponding maser frequency \( \nu_M \) and the maser optical depth \( |\tau(\nu_M)| \) are, respectively
\[ \nu_M = (2\pi \gamma)^{-1} \omega_B , \]
\[ |\tau(\nu_M)| = 4.48 \times 10^3 \gamma^{4/3} P^{-1} R_B^{-1/3} \omega_B^{-4/3} . \]
To apply the analytic CDIME model described in the above subsection in the magnetar environment, we need to know the pair plasma density \( n_{\pm} \) and the Lorentz factor \( \gamma \).
2.2 Pair plasma generation in magnetar environment

Rotating magnetized neutron stars are unipolar inductors that generate huge potential drops across the open field line region. Under certain conditions, a part, or even the total amount, of this potential will drop across a charge-depleted region (or a gap) formed in the PC area of the pulsar. Generally, depending on the boundary conditions at the surface, there are two kinds of inner accelerator models that are involved to produce the acceleration of particles and the production of electron-positron pairs. One is the vacuum (V) type gap if $\Omega \cdot B < 0$ (Ruderman & Sutherland 1975): positive ions are strongly bounded to the surface so a vacuum gap develops along open field lines above PC, and pairs are required to provide current flow through the gap, which can then operate as a stable accelerator. Another is the space-charge-limited flow (SCLF) type accelerator in polar cap models (Arons & Scharlemann 1979; Daugherty & Harding 1996; Harding & Muslimov 1998): acceleration occurs in the region of open field near the magnetic poles. On the open field lines, the neutron star generates a dense flow of electron-positron pairs penetrated by a highly relativistic primary electrons. This provides copious secondary pairs flowing out along the open field lines above the PC of pulsars. As an underlining PC accelerator model, we employ in this paper the general relativistic version of a SCLF model developed earlier by Muslimov & Tsygan (1992) and advanced in a number of important aspects by Harding & Muslimov (1998, 2001, 2002), because this kind of SCLF accelerator can also work in a magnetar environment (Zhang & Harding 2000a). Although numerical simulations (Baring & Harding 2001) show that the pair yields drop steeply with increasing magnetic field, so that in magnetar environments where photon splitting may effectively suppress pair production at the stellar surface and consequently the V-type accelerators cannot develop. However, SCLF gaps could be extremely long and narrow so that pair formation fronts (PFFs) could be formed at much higher altitudes above the neutron star surface (Harding & Muslimov 1998, Zhang et al. 2000). This means that at the top of SCLF gap when pair production starts to overcome photon splitting, the pair-production rate also rises steeply to provide copious pairs (Zhang & Harding 2000a). Here we assume that in magnetar environments copious pairs is possible if an SCLF accelerator is formed. Alternatively, even if photon splitting could completely suppress one-photon pair production in superstrong magnetic fields, two-photon pair production more likely occurs near the threshold, therefore, the magnetar environment may not be pairless (Zhang 2001).

We first investigate the Lorentz factor $\gamma_0$ of a primary electron accelerated from the
For a given distribution of voltage, the primary particles are accelerated through the potential drop up to the energies corresponding to the Lorentz factor $\gamma_0$, whose value is uncertain (Daugherty & Harding 1982) but is assumed to be a free parameter here. We follow the acceleration model of Harding et al. (2002) and consider the deceleration owing to the curvature-radiation reaction as Lyutikov et al. (1999) to determine the ranges of $\gamma_0$

$$3 \times 10^6 \leq \gamma_0 \leq 3 \times 10^7 .$$

Pair creation in SCLF model allows an entirely different relation between charge and current densities $n_{GJ}$ and $J$, the total number of charge carriers is greatly amplified (Thompson et al. 2002). And the pair plasma density $n_\pm$ can be scaled roughly by $n_{GJ}$ at the stellar surface (Beskin et al. 1988; Lyutikov et al. 1999),

\begin{equation}
 n_\pm(R_{ns}) = \lambda_0 n_{GJ},
 = 1.15 \times 10^{12} \lambda_0 B_{ns,14} P_6^{-1},
\end{equation}

where $\lambda_0$ is the multiplicity factor, which varies theoretically in the range $\lambda_0 \sim 10^3 - 10^6$ (Arons et al. 2004; Lyutikov 2004; Lyutikov et al. 1999; Zhang & Loeb 2004; Hibschman & Arons 2001). According to the relation between the parameters of the plasma and the beam comes from the energy argument that the primary particles stop producing pairs when the energy in the pair plasma becomes equal to the energy in the primary beam (Lyutikov et al. 1999), we have

$$1.5 \times 10^5 < \lambda_0 \approx 0.05 \gamma_0 < 1.5 \times 10^6 ,$$

In the above equation it has been assumed that the initial density, temperatures and velocities of the plasma components are equal.

### 3 CDIME IN MAGNETAR

We generalize the analytic expressions for the CDIME model, of the type described in section 2.1, to a magnetar environment and then apply it to optical emission of AXPs. The treatment is to assume that the parameters of CDIME model involved to describe the maser action in the magnetar environment change with the radial distance to the neutron star surface.
3.1 Radius–to–parameters mapping

The radial dependence of the parameters is assumed to follow the dipole geometry of the magnetic field as treated by Lyutikov et al. (1999), consequently,

\[ B(r_*) = B_{ns} r_*^{-3}, \]
\[ \lambda(r_*) = \lambda_{r_*}^{-3}, \]
\[ \omega_B(r_*) = \omega_B(R_{ns}) r_*^{-3}, \]
\[ \gamma(r_*) = \gamma_0 r_*^{-3}, \]
\[ \omega_R(r_*) = \omega_R r_*^{-3}, \]

where \( r_* = R/R_{ns}, R \) is the radial distance to the star, \( R_{ns} = 10^6 \) cm is the stellar radius.

The above relationships in Eq.(19) are regarded as a 'radius to the parameters' mapping. Because the height of the pair-formation front (PFF), the location where the first pairs are produced in the SCLF model is larger compared to the scale of the pair cascade multiplicity growing (Harding et al. 2002; Lyutikov et al. 1999), so we can assume approximately,

\[ \lambda(r_*) = \lambda_{r_*}^{-3} = \begin{cases} \lambda_0 & \text{if } r_* \leq r_{*,LS} \\ \lambda_0 r_*^{-3} & \text{if } r_* > r_{*,LS} \end{cases} \]

where \( r_{*,LS} = R_{LS}/R_{ns}, R_{LS} = P_c/2\pi \simeq 3 \times 10^{10} P_6 \) cm is the light cylinder radius with the rotational period of the star \( P_6 \). And \( \omega_R(r_*) \) is assumed to satisfy

\[ \omega_R(r_*) = \omega_R r_*^{-3} = \begin{cases} \omega_R(R_B) & \text{if } r_* \leq r_{*,LS} \\ \omega_R(R_B) r_*^{-3} & \text{if } r_* > r_{*,LS} \end{cases} \]

Taking into account of the fact that each primary particles accelerated in the gap generates many secondary electron-positron pairs (Beskin et al. 1988; Arons 1983; Beskin 1982; Daugherty & Harding 1983), the density of the pair plasma is substantially higher than the primary beam density. Consequently, we rewrite Eqs.(9) and (10) as a function of \( r_* \) by considering Eqs.(19) to (21) and replacing \( N_0 \) by \( n_\pm \):

\[ \tau[a(r_*))] \approx \lambda(r_*) P^{-1} \omega^{-1} \Delta \theta(r_*), \]

where \( \Delta \theta(r_*) \)

\[ \Delta \theta(r_*) = \begin{cases} 7.36 \omega^{-1/3} r_* P_6^{-1/6} & \text{for } \omega \ll \omega_c \\ \gamma_0^{-1/3} r_*^3 & \text{for } \omega \approx \omega_c \\ 1.15 \times 10^1 \gamma_0^{1/2} \omega^{-1/2} r_*^{-1/4} P_6^{-1/4} & \text{for } \omega \gg \omega_c \end{cases} \]

Correspondingly, in magnetar environment, the maser frequency \( \nu_M, \omega_c \) in Eqs.(11) and (14)
\[ \nu_M = (2\pi)^{-1}\nu^{-1}\omega_B = (2\pi)^{-1}\gamma_0^{-1}\omega_B(R_{ns}) , \]
\[ = 0.286 \times 10^{15}\gamma_{0.6}^{-1}B_{ns,14} , \]  
\[ \nu_c = \gamma^3\omega_R(r_*) = \gamma_0^3\omega_R(R_B) , \]
\[ = 1.34 \times 10^{19}\gamma_{0.6}P_6^{-1/2} , \]  
\[ \text{where } \gamma_{0.6} = \gamma_0/10^6. \]

Also, the corresponding optical depth for the maser action in magnetar is

\[ |\tau(\nu_M)| = 1.23 \times 10^9\lambda_0^{4/3}\omega_B^{-4/3}(R_{ns})P_6^{-7/6}r_*^4 \]
\[ = 5.6 \times 10^9\lambda_{0.5}^{4/3}\gamma_{0.6}^{-4/3}B_{ns,14}^{-1/3}P_6^{-7/6}r_*^4 , \]  
\[ \text{where } \lambda_{0.5} = \lambda_0/10^5, \text{ and } r_{*4} = r_*/10^4. \]

Based on the observed spin period \( P \) and spin-down rate \( \dot{P} \) of a pulsar, the strength of the surface magnetic field can be estimated by

\[ B_{ns,14} = 6.4 \times 10^5(P\dot{P})^{1/2} , \]
\[ \simeq 1.57(P_6\dot{P}_{-12})^{1/2} , \]  
\[ \text{where } \dot{P}_{-12} = \dot{P}/10^{-12} \text{ s s}^{-1}. \]

One can eliminate \( B_{ns,14} \) by substituting Eq.(27) into Eqs.(24) to (26), consequently,

\[ \nu_M = 0.45 \times 10^{15}\gamma_{0.6}^{-1}P_6^{1/2}\dot{P}_{-12}^{1/2} , \]
\[ \nu_c = \omega_c/2\pi = 4.98 \times 10^{16}\gamma_{0.6}P_6^{-1} , \]
\[ |\tau(\nu_M)| = 3.06 \times 10^9\lambda_{0.5}^{4/3}\gamma_{0.6}^{-11/6}P_6^{-2/3}r_*^4 , \]  
\[ (28) \]
\[ (29) \]
\[ (30) \]

Although curvature maser emission requires the curvature drift, it is driven by the free energy in the particle beam. Hence, for maser emission to occur, the Lorentz factor of the particles must be larger than a certain value. This minimum energy requirement may be estimated in terms of LM92,

\[ \gamma_0 > (\omega_B/\omega_R)_{r_{*0}} , \]
\[ \approx 5.12 \times 10^6P_6^{1/6} , \]  
\[ \text{where } r_{*0} = r_*/10^6 = 1. \]

Putting Eq.(31) into Eq.(18), we have

\[ \lambda_{0.5} = 2.56P_6^{1/6} . \]  
\[ (32) \]
Substituting the values of $\gamma_0$ and $\lambda_0$ in the above equation into Eqs.(28) to (30), we have

$$\nu_M = 8.79 \times 10^{13} P_6^{1/3} \dot{P}_{-12}^{1/2},$$

$$\nu_c = \omega_c / 2\pi = 2.54 \times 10^{17} P_6^{-5/6},$$

$$|\tau(\nu_M)| = 69.12 \times P_6^{-13/9} \dot{P}_{-12}^{-2/3} r_{*A}^4.$$  

With the observed optical amplification determined by $|\tau(\nu_M)|$, we obtain the corresponding maser position $R(\nu_M)$,

$$r_{*A}(\nu_M) = 3.47 \times 10^{-1} |\tau(\nu_M)|^{1/4} P_6^{13/36} \dot{P}_{-12}^{1/6},$$

where $r_{*A}(\nu_M) = r_*(\nu_M)/10^4$, and $r_*(\nu_M) = R(\nu_M)/R_{ns}$.

The CDIME model developed for the magnetar environment is applied to different AXPs/SGRs sources, as shown in Table 1.

### 3.2 Optical pulsation from AXP 4U0142+61 sources

Evidence for the optical pulsation has been reported in AXP sources (Kern & Martin 2002). AXP 4U0142+61 is the brightest AXP in X-rays, has no associated supernova remnant and with its spin-down timescale of $\sim 10^5$ yr (Wilson et al. 1999). This source is observed at optical band ranging from $10^{14}$ Hz to $10^{15}$ Hz. The remarkable aspect of the optical light curve is that its modulation amplitude is very large compared to that of the X-ray light curves. The pulsed fraction of the optical light is about 27% of its total optical flux, which is five to ten times greater than that of soft X-rays. The basic data of all AXPs with known spin periods and spin-down rate detected with ASCA are summarized in Table 1.

For the source of AXP 4U142+61, $P_6 = 1.45$, $\dot{P}_{-12} = 1.98$, and $|\tau(\nu_M)| \approx 1.31$ which is estimated from the pulsed fraction of $\Delta I_r / I_r \approx 27\%$. We obtain the maser frequency and the corresponding maser position $r_*(\nu_M)$ from Eqs.(33) and (36), respectively

$$r_*(\nu_M) = 4.75 \times 10^3,$$

$$\nu_M = 1.39 \times 10^{14} \text{ Hz}.$$  

The model prediction shows that the optical pulsations from AXPs 4U0142+61 occurs at the radius of $r_*(\nu_M) = 4.75 \times 10^3$ and its maser frequency is $\nu_M = 1.39 \times 10^{14}$ Hz.

Hulleman et al. (2001) presented near-infrared and optical observations of the field of the AXP 2259+586 taken with the Keck telescope. They have identified the near-infrared counterpart to AXP 2259+586 at $\sim 10^{14}$ Hz, but no emission was found at frequencies between $(2 - 5) \times 10^{14}$ Hz. Associating the model investigated in this paper to the source of...
AXP 2259+586, we find that if curvature-drift maser action occurs at $r_\ast(\nu_M) = 4.75 \times 10^3$, from Eqs.(35), we have $|\tau(\nu_M)| \approx 4.61$. The corresponding maser frequency is about $\nu_M = 6.45 \times 10^{13}$ Hz. Our model further predicts that pulse component of the optical pulsation is $\sim 0.99\%$. The results show that our model prediction is consistent with the near-infrared observation of the source AXP 2259+586.

In addition, our model predicts that the maser emission should be detectable at K and J bands, because the maser frequency is $\nu_{M,13} \approx 8.79P_{12}^{1/3}P_{6}^{1/2}$, where $\nu_{M,13} = \nu_M/10^{13}$ Hz. As a matter of fact, infrared and near-infrared emissions have also been detected in other two AXPs, namely, AXP 1708-4009, AXP1048-5937 (Israel et al. 2003; Wang & Chakrabarty 2002), but no pulse components have been detected in these sources so far.

Finally, in comparing the emission properties of HBPs with AXPs in this discussion, we take into account of two young isolated radio pulsars, $PSR\ J119 - 6127$ and $PSR\ J1814 - 1744$ (Camilo et al. 2000), which have spin parameters similar to the AXPs and very high inferred magnetic fields, i.e., high magnetic field pulsars (HBPs). Table 1 shows that no optical maser should occur in the sources of HBPs. The possible reason is that although both AXPs and HBPs are rotating high field neutron stars, they have different orientations of the magnetic axes with respect to the rotation axes (Zhang & Harding 2000a). Consequently, for HBPs, one can safely ignore the two-photon pair-production mechanism (Zhang 2001) that is involved to produce copious pairs for triggering optical maser actions in AXPs.

4 DISCUSSION AND CONCLUSIONS

4.1 Discussion

We first need to calculate the optical luminosity of the objects given the estimated distance. Because the total energy source of the maser coherent optical emission come from the non-thermal leptons which include the electrons pulled off from the neutron star surface and the pairs generated through the secondary cascade processes, it should therefore be limited by the total spin-down power of the star. The total luminosity available to convert into optical emission is the spin down luminosity $L_{sd}$ of the magnetar (Eichler, Gedalin, & Lyubarsky 2002), which is

$$L_{sd} = 4\pi^2 IP^{-3} \dot{P},$$

$$\sim 1.83 \times 10^{32} I_{45} P_{-6}^{-3}\dot{P}_{-12} \text{ erg s}^{-1},$$

(39)
where $I$ is the moment of the star inertia, and $I_{45} = I/10^{45} \text{ cgs}$. Observationally, a luminosity of $10^{33} \text{ erg s}^{-1}$ ($\sim 10^{-2}$ of the persistent, pulsed X-ray flux) in coherent optical emission from AXPs could be detectable at 2.2 $\mu$m with imminent technology (EGL02), if the optical emission is beamed into a solid angle with $\delta\Omega/4\pi \sim 0.1$, the observed coherent optical emission $L_{\text{obs}}$ from AXPs corresponds to

$$L_{\text{obs}} = \left(\frac{\delta\Omega}{4\pi}\right)10^{33},$$

$$\sim 10^{32} \text{ erg s}^{-1},$$

which is consistent with the model prediction in Eq.(39).

Secondly, we address why the maser process is so powerful that it can use up essentially all the spin-down energy. One of the most important differences from magnetars and pulsars is the former current carries a much larger fraction of the total energy budget than that of the latter. EGL02 argued that most of the energy budget passes through the magnetosphere currents, assuming that the emission is non-thermal and probably inverse-Comptonized in the magnetosphere (Thompson et al. 2002). Thus, a considerable fraction of the long-term magnetic energy dissipation in magnetars could be used up as coherent electromagnetic emission. The coherent emission of the pulsar is only a small fraction of the spin-down power, but it can be a much higher fraction of the power in polar currents (EGL02).

In addition, a $10^5$ pair multiplicity is invoked in this paper. The required multiplicity is beyond the predicted values from the current pulsar/magnetar cascade theories, but not impossible: the study of the eclipsing behavior system PSR J0737-3039 A&B requires that the pair multiplicity in the pulsar A is of order of $10^6$ (Arons et al. 2004; Lyutikov 2004; Zhang & Loeb 2004). A value of multiplicity factor involved for the Crab is $\lambda = 10^6$ (Kennel & Coroniti 1984; Arons & Scharlemann 1979; Muslimov & Harding 2003). For magnetars, the pair production is potentially suppressed by the processes such as splitting (Baring & Harding 2001). However, we adopt the SCLF model developed by Harding & Muslimov (1998) and Zhang & Harding (2000b) in which a pair formation front occurs at a higher altitude, where the effects of the photon-photon pair production and the inverse Compton off the thermal photons for high-order pairs become more important than the photon splitting effect (Zhang & Harding 2000b; Zhang 2001).

Finally, we address the low frequency turnovers for curvature maser emission. Regardless of the details of any particular model for coherent emission, escaped coherent radiation probably has a minimum frequency of $\nu_p$ in the frame of the outflowing plasma, which gives
it a frequency in the observe frame of $4\pi \nu_p \gamma^{1/2}$, where \( \nu_p \) is defined by \( \nu_p = 9 \times 10^3 n_{\pm}^{1/2} \) (EGL02). Then, the low frequency turnovers for coherent emission can be inferred as

$$\nu_{\text{min}} \approx 1.13 \times 10^5 n_{\pm}^{1/2} \gamma^{1/2}. \quad (40)$$

Combining Eqs.(17),(27),(31),(32) and the assumption \( \gamma = \gamma_o r_o^{-3} \), we rewrite \( \nu_{\text{min}} \) at \( r_o = 4.75 \times 10^3 \)

$$\nu_{\text{min}} \approx 10^{12} P_{6}^{-1/12} \dot{P}_{-12}^{1/4} \text{ Hz}. \quad (41)$$

The low cutoff frequency for maser emission of different AXPs/SGRs sources are listed in Table 2. Table 2 shows that \( \nu_{\text{min}} \) for AXPs/SGRs is of the order \( \sim 10^{12} \text{ Hz} \), making it plausible that they are radio-quiet. However emission at wavelengths between optical and radio bands may not be prohibited, as Zhang (2001) predicted.

### 4.2 Conclusions

In this paper, we focus on some examination of the AXP 4U0142+61 optical emission properties. We have demonstrated that our treatment allows us to explain the observed optical pulsations in AXPs. The fundamental differences between the curvature-drift-induced maser mechanism presented in this paper and the original one proposed by LM92 are as follows:

1. Our model takes into account the curvature-drift maser action in a magnetar environment and explains the optical pulsations of AXPs by arguing that the parameters describing the influences of pair plasma on the curvature emission are as a function of the radius from pulsar PCs.

2. Our model incorporates the SCLF accelerator above the pulsar’s PC where copious pairs can be provided due to steeply rising of the pair-production rate (Zhang 2001; Zhang et al. 2000; Lyutikov et al. 1999).

3. We consider the multiplicity of the pair plasma density at the emission region of \( r_o (\nu_M) \sim 4.75 \times 10^3 \) from pair cascades, while LM92 considers only the influence of the pairs from the stellar surface.

In summary, it is possible to account for the optical pulsations of AXP 4U0142+61 in terms of curvature-drift-induced maser from relativistic electron (and positrons) at the radial distance to the neutron star surface about \( R = 4.75 \times 10^9 \) cm. The maser component is about 27% of the optical light, and the corresponding optical pulsation frequency is about \( \nu_M = 1.39 \times 10^{14} \) Hz. We predict that significant optical/IR pulsations should exist in
other five AXP/SGR sources, but not in AXPs 2259+586 and HBPs as shown by previous observations (see. Table 2).

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Table 1. SGR AND AXP TIMING PARAMETERS

| Sources       | P(s)<sup>(1)</sup> | ˙P<sub>−12</sub>(ss<sup>−1</sup>)<sup>(2)</sup> | References<sup>(3)</sup> | ν<sub>c,17</sub>(Hz)<sup>(4)</sup> | ν<sub>M,14</sub>(Hz) | |τ(ν<sub>M</sub>)| η     |
|---------------|---------------------|---------------------------------|-------------------------|-------------------------|---------------------|----------------|----------------|
| SGR 1900+14...| 5.16(7)             | 60.5                            | 1, 2                    | 2.88                    | 6.50                | 0.283          | 75.28%         |
| SGR 1806-20...| 7.47(3)             | 115.5                           | 4, 5                    | 2.12                    | 10.1               | 0.108          | 89.75%         |
| AXP 1048-5937 | 6.45(1)             | 32.6                            | 6, 7                    | 2.39                    | 5.14               | 0.311          | 73.30%         |
| AXP 1841-045  | 11.77(6)            | 41.6                            | 8, 9                    | 1.45                    | 7.09               | 0.112          | 89.58%         |
| AXP 2259+586  | 6.98(2)             | 0.488                           | 10,11                   | 2.25                    | 0.65               | 0.212          | 81.39%         |
| AXP 0142+61   | 8.69(7)             | 1.98                            | 12                      | 1.86                    | 1.39               | 1.307          | 26.98%         |
| AXP 1708-4009 | 11 (6)              | 19                              | 13,14                   | 0.60                    | 4.68               | 0.212          | 81.39%         |
| PSR J1119-6127| 0.41 (15)           | 4.0                             | 15                      | 7.18                    | 0.72               | 67.35          | ~ 0.0          |
| PSR J1814-1744| 3.98 (15)           | 0.74                            | 15                      | 3.57                    | 0.66               | 7.776          | 0.04%          |

<sup>(1)</sup> Measured period (1 σ error in last digit).
<sup>(2)</sup> Assumed period derivative (from references).
<sup>(3)</sup> References.—(1) Hurely et al. 1999; (2) Woods et al. 1999; (3) Murakami et al. 1999; (4) Sonobe et al. 1994; (5) Woods et al. 2000; (6) Corbet & Mihara 1997; (7) Paul et al. 2000; (8) Gotthelf & Vasisht 1997; (9) Gotthelf, Vasisht, & Dotani 1999; (10) Kaspi, Chakrabarty, & Steinberger 1999; (11) Cobet et al. 1995; (12) White et al. 1996; (13) Sugizaki et al. 1997; (14) Israel et al. 1999; (15) Camilo et al. 2000

<sup>(4)</sup> ν<sub>c,17</sub> = ν<sub>c</sub>/10<sup>17</sup>; ν<sub>M,14</sub> = ν<sub>M</sub>/10<sup>14</sup>

Table 2. The conditions of synchrotron maser for the optical pulsation in magnetar

| Sources       | ν<sub>min,12</sub><sup>(1)</sup> | ν<sub>M</sub> << ν<sub>c</sub><sup>(2)</sup> | Observation | Model (pulsed) |
|---------------|----------------------------------|-------------------------------|-------------|----------------|
| Optical/IR    | Optical/IR                       |                               |             |                |
| SGR 1900+14...| 2.83                             | yes                           | no/no       | optical        |
| SGR 1806-20...| 3.22                             | yes                           | no/no       | optical        |
| AXP 1048-5937 | 2.38                             | yes                           | no/yes      | optical        |
| AXP 1841-045  | 2.40                             | yes                           | no/no       | optical        |
| AXP 2259+586  | 0.83                             | yes                           | no/yes      | no             |
| AXP 0142+61   | 1.15                             | yes                           | yes/no      | optical        |
| AXP 1708-4009 | 1.98                             | yes                           | no/yes      | optical        |
| PSR J1119-6127| 1.76                             | yes                           | no/no       | no             |
| PSR J1814-1744| 0.96                             | yes                           | no/no       | no             |

<sup>(1)</sup> ν<sub>min,12</sub> = ν<sub>min</sub>/10<sup>12</sup> Hz

<sup>(2)</sup> Refer to Table 1