String Breaking in SU(2) Yang Mills Theory with Adjoint Sources

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Abstract

We compute the static potential in three-dimensional SU(2) Yang Mills Theory with adjoint sources using numerical simulations. By employing a variational approach involving string and gluelump operators, we obtain clear evidence for string breaking in the adjoint potential. The breaking scale $r_b$ is computed and extrapolated to the continuum limit. The result in units of the scalar glueball mass is $r_b m_G = 10.3 \pm 1.5$. We also resolve the structure of higher excitations of the flux-tube and gluelumps. Furthermore we discuss the implications of our findings for the case of the four-dimensional theory.

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1 Introduction

An important concept characterizing the confining force of non-Abelian gauge theories is the potential of a pair of static charges separated by a distance \( r \). For charges in the fundamental representation, confinement in Yang Mills theories in 2+1 and 3+1 dimensions manifests itself in a gauge string formed between the charges, which leads to a potential rising linearly with the charge separation. When fundamental representation matter fields are added, this linear rise extends only to some distance \( r_b \) at which there is enough energy in the gauge string to pair-create matter particles. These screen the confining force and the potential stays at a constant value, corresponding to the energy of two bound states of a static source and a dynamical particle. If the charges are in the adjoint representation, again a linear rise in the potential is observed. In this case, however, screening via string breaking is expected to occur even in pure Yang Mills theory, since the adjoint string can couple to gluons which may be pair-produced.

The static potential itself is only defined up to an arbitrary constant. This is apparent from its divergence (linear in 4 dimensions, logarithmic in 3 dimensions), which cannot be absorbed in a parameter of the theory. However, there are two physical quantities which can be extracted from the potential: (i) the strength of the confining force as a function of distance, \( F(r) \), is specified by the slope of the potential \(^1\) (ii) the string breaking scale \( r_b \), where the screening of the charges sets in, gives the range of the force. While, for a given strength of the force, the string breaking scale in the fundamental representation potential depends on the bare mass of the matter fields (it takes less energy to pair-produce lighter particles), it is a purely dynamical scale in the adjoint representation, which cannot be tuned by any bare parameter of the theory. Hence, like the string tension, it represents an independent, non-perturbative dynamical scale characterizing the physics of confinement.

Despite a lot of effort, the screening phenomenon has not yet been observed in numerical simulations of QCD with dynamical fermions using the standard method of measuring large Wilson loops \([1]\). Likewise, Wilson loop calculations have also failed to exhibit string breaking for the adjoint potential in 2+1 dimensions \([4]\), and the evidence in 3+1 dimensions is somewhat inconclusive \([2, 3]\).

However, recently string breaking was observed in the fundamental potential in the

\(^{1}\)For the potential in the fundamental representation in Yang Mills theory the limit \( F(r \to \infty) = \sigma \) defines the string tension. For screened potentials, this limit is zero and cannot be used to define a string tension in the linear part.
confinement phase of the SU(2) Higgs model in 2+1 \cite{5} and 3+1 \cite{6} dimensions by means of a mixing analysis. In those references it was demonstrated clearly that the Wilson loop has very poor overlap with the screened two-meson final state, and hence is not suitable to extract the static potential for distances larger than the string breaking scale $r_b$. Instead, the simulations have to be supplemented by operators which have a good projection onto the two-meson state, and an analysis of mixing between the string and the two-meson states has to be performed for every distance $r$. This conclusion has been corroborated by two further works. String breaking was also seen in QCD at finite temperatures \cite{7}, where Polyakov loops rather than Wilson loops have been used. At zero temperature, the potential beyond the screening length has been extracted in a quenched QCD simulation employing operators which show good overlap with the screened final state \cite{8}. Finally, a recent calculation in three-dimensional SU(2) gauge theory coupled to staggered fermions has laid claim to the observation of string breaking by measuring Wilson loops on asymmetric lattices \cite{9}.

In this paper, we employ the mixing analysis that has successfully been applied to observe string breaking in the SU(2) Higgs model \cite{5,6} to study the static potential of adjoint representation charges in 2+1 dimensional SU(2) Yang Mills theory. We obtain clear evidence for string breaking and calculate the breaking scale $r_b$ in the continuum limit.

In section 2 we introduce the operators used in the calculation, section 3 summarizes the simulation and the variational calculation used to extract the potential. Our numerical results are presented in section 4, a brief discussion of the results as well as our conclusions are contained in section 5.

2 The operators

We consider SU(2) pure gauge theory with the Wilson action

$$S[U] = \beta_G \sum_x \sum_{i<j} \left[ 1 - \frac{1}{2} \text{Tr} P_{ij}(x) \right], \quad (1)$$

where $P_{ij}$ denotes a plaquette of links $U_\mu$ in the fundamental representation; $\beta_G = 4/\alpha g^2$ and $g$ is the bare gauge coupling of mass dimension 1/2. The links in the adjoint representation, $A^{ab}$, are related to the $U_\mu$’s by

$$A^{ab}(x) = \frac{1}{2} \text{Tr} \left( \sigma^a U_\mu(x) \sigma^b U_\mu^\dagger(x) \right), \quad (2)$$
where $\sigma^a$ are the Pauli matrices. With these definitions the operator describing two static adjoint sources with an adjoint representation flux tube between them, i.e. the correlation function of a string of length $r$ over a time interval $t$, is just the adjoint representation Wilson loop,

$$G_{SS}(r, t) = W_{\text{adj}}(r, t) = \left( \left| W(r, t) \right|^2 - 1 \right),$$

(3)

where

$$W(r, t) = \text{Tr} \left[ U(0, r\hat{j}) U(r\hat{j}, r\hat{j} + t\hat{3}) U^\dagger(t\hat{3}, r\hat{j} + t\hat{3}) U^\dagger(0, t\hat{3}) \right]$$

(4)

is the standard Wilson loop and $U(x, y)$ is a shorthand notation for the straight line of fundamental links connecting the sites $x$ and $y$. The static potential in the adjoint representation is then defined in terms of the exponential decay of the Wilson loop,

$$V(r) = -\lim_{t \to \infty} \frac{1}{t} \ln[W_{\text{adj}}(r, t)].$$

(5)

In the region of linear confinement the Wilson loop obeys the area law, whereas for distances beyond the breaking scale $r_b$ a perimeter law is expected. In practical simulations, the limit $t \to \infty$ is not realized, but $t$ is typically less than ten lattice spacings when the signal is lost in noise. As we shall see, due to the poor projection of the Wilson loop onto the screened potential, this is not sufficient to observe string breaking, and one has to use additional operators with good projection onto the final state.

The correlation function for a bound state of a static adjoint colour source and a gluon field, sometimes called gluelump in the literature [10], is given by the non-local gauge-invariant operator

$$G_G(t) = \left\langle \text{Tr} \left[ P(x) \sigma^a \Gamma^{ab}(t) \text{Tr} \left[ P^\dagger(y) \sigma^b \right] \right] \right\rangle$$

$$= \left\langle \text{Tr} \left[ P(x) U(x, y) \left( P^\dagger(y) - P(y) \right) U^\dagger(x, y) \right] \right\rangle$$

(6)

with the adjoint representation Wilson line

$$\Gamma^{ab}(t) = \frac{1}{2} \text{Tr} \left( \sigma^a U(x, y) \sigma^b U^\dagger(x, y) \right), \quad y = x + t\hat{3}. \quad (7)$$

Here, $P(x)$ and $P(y)$ denote the “clover-leaf” of all four plaquettes with the same orientation, which emanate from the endpoints $x, y$ of the adjoint Wilson line into the $(1, 2)$-plane [2, 4].

We now follow the procedure proposed in [2] and construct an operator projecting on two of these bound states at distance $r$ by

$$G_{GG}(r, t) = \left\langle \text{Tr} \left[ P(0) U(0, t\hat{3}) \left( P^\dagger(t\hat{3}) - P(t\hat{3}) \right) U^\dagger(0, t\hat{3}) \right] \right\rangle$$

$$\times \left\langle \text{Tr} \left[ P(r\hat{j}) U(r\hat{j}, r\hat{j} + t\hat{3}) \left( P^\dagger(r\hat{j} + t\hat{3}) - P(r\hat{j} + t\hat{3}) \right) U^\dagger(r\hat{j}, r\hat{j} + t\hat{3}) \right] \right\rangle.$$
Finally, correlations between a string and a gluelump state, and vice versa, may be described by

\[ G_{SG}(r, t) = \langle \text{Tr} \left[ \left( P^\dagger(t\hat{3}) - P(t\hat{3}) \right) U^\dagger(0, t\hat{3}) U(0, r\hat{j}) U(r\hat{j}, r\hat{j} + t\hat{3}) \times P(r\hat{j} + t\hat{3}) U^\dagger(r\hat{j}, r\hat{j} + t\hat{3}) U(0, 0) \right] \rangle, \]

\[ G_{GS}(r, t) = \langle \text{Tr} \left[ \left( P^\dagger(0) - P(0) \right) U(0, t\hat{3}) U(t\hat{3}, r\hat{j} + t\hat{3}) U^\dagger(r\hat{j}, r\hat{j} + t\hat{3}) \times P(r\hat{j}) U(r\hat{j}, r\hat{j} + t\hat{3}) U^\dagger(t\hat{3}, r\hat{j} + t\hat{3}) U(0, t\hat{3}) \right] \rangle. \]  

The static potential, its excitations and the mixing between gauge string and two-meson state can then be extracted from measurements of the matrix correlator

\[ G(r, t) = \begin{pmatrix} G_{SS}(r, t) & G_{SG}(r, t) \\ G_{GS}(r, t) & G_{GG}(r, t) \end{pmatrix}. \]  

We also keep the single gluelump operator \( G_G \) from (6) in our simulations, in order to check whether \( G_{GG} \) defined in (8) indeed has a good projection onto a two-gluelump state, for which one expects \( E_{GG} \approx 2E_G \). Hence, our procedure is entirely analogous to the one applied in our earlier work [5]. There, the operator with explicit projection onto a two-meson state, \( G_{MM} \), plays the same role as the two-gluelump operator \( G_{GG} \) considered in this work.

Note that the energy levels extracted from the operator \( G_G \) in eq.(6) are logarithmically divergent with decreasing lattice spacing and hence do not have a continuum limit. This divergence is due to the self-energy of the static source which, although perturbatively computable, cannot be absorbed by renormalization into a parameter of the theory. For a detailed discussion of this point, as well as a perturbative computation and subtraction of the divergence, see [14]. For the computation of the string breaking scale, however, this divergence does not pose a problem. First, we observe that all operators \( G_{ij} \) used in the matrix correlator \( G(r, t) \) contain two temporal Wilson lines and hence the same divergence (it is the temporal lines whose length is taken to infinity that represent the propagation of the static sources in the definition of the potential, eq.(5)). This divergence appears in the energy values of the static potential, which therefore do not have a continuum limit. This is well known, and exactly the same holds if only Wilson loops are used. It reflects the fact that the static potential is defined only up to an arbitrary (infinite) constant, and does not itself constitute a finite physical quantity. On the other hand, the confining force and the string breaking scale are finite physical quantities. The force is defined by the slope of the static potential, and the string breaking scale by the equality of the energy stored in the string and the energy needed to pair-produce the constituents needed to form gluelump states. Hence, both quantities are defined by
energy differences such that the divergence cancels out and these quantities do have a continuum limit. All of these features can be seen explicitly in our calculations.

3 Simulation and analysis

We now describe the details of our numerical work. It is well known that the projection properties of operators can be significantly improved by using smeared link variables in the spatial directions. For this purpose, and to create a larger basis of operators, we have employed the standard fuzzing algorithm \[1\] to obtain smeared spatial link variables of unit length, which were then used instead of the original ones in constructing the correlation functions defined above. All links in the time directions were left unsmeared such that the transfer matrix remains unaffected by our smearing procedure. As a basis of operators used in the matrix correlator eq. (10), we chose three different link fuzzing levels for the spatial Wilson lines and three or five fuzzing levels for the clover-leaves \(P\). With increasing number of iterations in the fuzzing algorithm we have also increased the size of the clover-leaves by powers of two, starting from the simplest choice of \(1 \times 1\) plaquettes at the lowest fuzzing level, up to \(16 \times 16\) at the highest. Depending on the number of fuzzing levels used to compute the clover-leaves, our correlator \(G(r, t)\) represents a \(6 \times 6\) or \(8 \times 8\) matrix. This size of the basis has been sufficient to yield good projection onto the first two lowest states in the calculation of the fundamental representation potential \([5, 6]\). For the single gluelump operator our basis consists of five different fuzzing levels.

The procedure we follow to diagonalize \(G(r, t)\) by means of a variational calculation has been described in detail in the literature \([12, 13]\), and its application to the calculation of the adjoint potential has already been discussed \([11, 2]\). Here we outline the procedure once more for the current problem. For the sake of clarity we assume that all links have been transferred to the temporal gauge, where the links in the time direction are set to unity. This choice greatly simplifies the notation, and since all \(G_{ij}\) are manifestly gauge-invariant, the physics remains unchanged. We then note that each of the \(G_{ij}\) represents a correlation function which can be written as

\[
G_{ij}(r, t) = \langle \phi_i(t)\phi_j(0) \rangle .
\]

Here the \(\phi_i\)'s represent a spatial gauge string of length \(r\) at different fuzzing levels for \(i = 1, \ldots, 3\), and a two-gluelump operator at different fuzzing levels for \(i = 4, \ldots, N\), with \(N = 6, 8\). The variational diagonalization of \(G(r, t)\) consists in solving the generalized

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eigenvalue problem \cite{13, 14}

\[ G(r, t) v_i(t, t_0) = \lambda_i(t, t_0) G(r, t_0) v_i(t, t_0), \quad t > t_0. \] (12)

From the eigenvectors \( v^i \) one may construct the corresponding eigenstates

\[ \Phi_i = c_i \sum_k v^i_k \phi_k = \sum_k a_{ik} \phi_k, \] (13)

which are superpositions of the string and gluelump operators used in the simulation. The constants \( c_i \) are chosen such that \( \Phi_i \) is normalized to unity. The diagonalized correlation matrix may then be written as

\[ \Gamma_i(r, t) = \langle \Phi_i(t) \Phi_i(0) \rangle = \sum_{j, k=1}^{N} a_{ij} a_{ik} G_{jk}(t), \] (14)

and represents the (approximate) correlation functions of the eigenstates of the Hamiltonian. We extract the energies corresponding to the states \( \Phi_i \) by fitting a single exponential to these correlation functions \cite{14},

\[ \Gamma_i(r, t) \sim e^{-E_i t}. \] (15)

To check the stability of the procedure we have performed the same calculation for \( t_0 = 0, \ t = a, 2a, 3a \) and obtained consistent results in all cases. Due to the normalization of the \( \Phi_i \), the coefficients \( a_{ik} \) take values between zero and one. They quantify the overlap of each individual correlator \( G_{ik} \) with the correlators of the mass eigenstates, \( \Gamma_i \), and play a crucial role in the mixing analysis of the numerical results. Since all operators are gauge-invariant, the same considerations apply without fixing a gauge, only the notation becomes more involved. In this case adjoint Wilson lines have to be inserted between the \( \phi \)'s in eq. (14), but the \( \Gamma_i \) are still expressed by the last of eq. (14).

We have worked at three values of the bare gauge coupling, \( \beta_G = 9, 12, 15 \). Our chosen lattice sizes correspond to \( L/\beta_G = 4 \). At \( \beta_G = 9, 12 \) we also considered \( L/\beta_G \geq 5 \) in order to check for finite size effects.

For all values of \( \beta_G \) we have employed a maximum of 18 iterations of the fuzzing algorithm with a link/staple mixing ratio of 2.0, which was sufficient to observe saturation in the projection onto the ground state. At \( \beta_G = 9 \) measurements were taken after every compound update consisting of 5 over-relaxed and one heat-bath sweeps. For \( \beta_G = 12, 15 \) we increased the number of over-relaxed sweeps between measurements to 10. Typically, it was sufficient to collect between 500 and 1000 measurements in bins of 20-50 to obtain a satisfactory signal. Statistical errors were estimated using a jackknife procedure.
4 Numerical results

We begin the presentation of our numerical results with the spectrum of a single gluelump state, as given in Table 1. This operator has been previously computed in [3, 14], and we observe good agreement with the values reported in [14]. Upon conversion to physical units, i.e. $M/g^2 = aM\beta_G G/4$, one observes a slight increase in $M/g^2$ as the continuum limit is approached, which may signal the onset of the weak logarithmic divergence discussed in Section 2.

Table 1: Mass estimates for the gluelump ground state and first excitation.

| $\beta_G$ | $L/a$ | $aM$  | $M/g^2$ | $aM^*$ | $M^*/g^2$ |
|-----------|-------|-------|---------|--------|-----------|
| 9.0       | 36    | 0.689(4) | 1.550(9) | 0.983(12) | 2.212(27) |
| 9.0       | 52    | 0.683(4) | 1.537(9) | 0.974(8)  | 2.192(18) |
| 12.0      | 48    | 0.524(3) | 1.572(9) | 0.729(8)  | 2.187(24) |
| 12.0      | 60    | 0.525(3) | 1.575(9) | 0.737(13) | 2.211(39) |
| 15.0      | 60    | 0.426(3) | 1.598(11)| 0.615(12) | 2.306(45) |

In Figure 1 we display the result obtained for the potential and its first few excitations at $\beta_G = 12$ on a $48^3$ lattice using a $6 \times 6$ operator basis. Let us first consider the ground state potential. At small distances the familiar linear rise corresponding to area law behaviour of the Wilson loop is observed. However, at $r/a \approx 20$ saturation of the ground state potential at a value corresponding to twice the gluelump energy is clearly visible, in accord with the expectation that the adjoint string should break at large distances. This picture is confirmed by analyzing the operator content of the ground state, as shown in Figure 2. As in the case of the potential in the fundamental representation [3, 4], we observe that the Wilson loop has nearly full projection onto the ground state potential for distances $r < r_b$, but practically no projection onto the screened final state at $r > r_b$. Rather, it keeps projecting onto the unbroken string state, which for $r \gtrsim r_b$ corresponds to the first excited state of the potential. As in the fundamental representation, this offers an explanation for the failure to observe string breaking in previous calculations employing Wilson loops only [3, 4]. On the other hand, the maximal projection of the operator type $G_{GG}$ onto the ground state is always significantly smaller than that of the Wilson loops for $r < r_b$, but rapidly reaches nearly full projection for $r > r_b$. Again, in analogy to the fundamental representation, the region where both operator types have comparable projection onto the ground state potential in the vicinity of the string breaking scale $r_b$ (i.e. the mixing region) is rather narrow.
Next, consider the first excited state. Here we note that excitations can be identified unambiguously for \( r/a \geq 4 \). The energy of the first excited state rises linearly for \( 4 \leq r/a \leq 10 \), which suggests that it is an excitation of the gauge string. Curiously, this state nevertheless receives its main contributions from the \( G_{GG} \) operator, as Figure 2 shows. This is in marked contrast to the fundamental representation potential studied in [5], where the states with maximum projections of \( G_{SS} \) and the two-meson correlation \( G_{MM} \) can always be interpreted as string or two-meson states, respectively. In other words, apart from the string breaking region itself, there is no significant mixing between purely gluonic operators and operators containing scalar fields, in accordance with the same observations made for the spectrum of the SU(2) Higgs model [13, 16]. In pure Yang Mills theory, the operator \( G_{GG} \) involves only gluonic degrees of freedom, and the mixing between the operator types can be expected to be more complicated. Closer to \( r_b \) however, where the contribution of the operator \( G_{GG} \) to the first excitation becomes maximal, it is apparent from the constant energy that the first excited state of the system corresponds to a two-gluelump state, until its energy is crossed by that of the lowest string state at \( r_b \), which subsequently corresponds to the first excited state.
Figure 2: The coefficients $a_{ik}$ defined in eq. (13) for the ground state ($i = 1$) and the first excited state ($i = 2$). At each value of $r/a$ we plot the maximum overlap in the string ($k = S$) and gluelump channels ($k = G$).

In order to illustrate the necessity of including the gluelump operators and performing the mixing analysis, we have also performed the diagonalization of the $3 \times 3$ sub-matrix $G_{SS}$, i.e. using only the Wilson loops of various fuzzing levels. The results for the ground state and the first excited state are displayed in Figure 3. Due to the poor projection of the string operator onto the two-gluelump system, the ground state energy for $r \gtrsim r_b$ continues to rise linearly, but shows signs of strong downward fluctuations as indicated by the large lower error bars in Figure 3. We conclude that for $r \gtrsim r_b$ any calculation employing Wilson loops only, is either likely to miss the true ground state altogether, or unable to determine its energy with sufficient precision for string breaking to be unambiguous. Indeed, the energies for the first excited state extracted from the $3 \times 3$ sub-matrix $G_{SS}$ show a significant linear rise for all values of $r$, which is yet another manifestation of the bad projection of the string operator onto the two-gluelump state, but this time in the case of the excited string. For small distances, this result of the $3 \times 3$ Wilson loop sub-block confirms our interpretation of the first excited state of the full $6 \times 6$ calculation as a string excitation, despite its large admixture of the two-gluelump operator, cf. Figure 4.
From Figure 3 it is apparent that the level crossing between the gauge string and the two-gluelump system is repeated for the higher excitations of the potential as well. This is not surprising, since any excitation of the gauge string will still rise linearly with $r$, only at a higher level. On the other hand, the excitations of the two-gluelump system at large separations are at constant energy levels. One would expect the first excitation of the asymptotic final state to consist of one ground state gluelump and one first excited gluelump, the second excitation of two first excited gluelumps etc. We have indicated the location of the energies of these consecutive combinations, as taken from Table 1, by the dashed lines in Figure 3. It is easy to see that the expectation concerning the excitation spectrum of the screened asymptotic potential is indeed confirmed by our calculation.

As a check of the stability of our variational calculation, we have also considered an $8 \times 8$ correlation matrix, whose operator basis was supplemented by adding one smaller and one larger fuzzing level to those contained in the $6 \times 6$ basis. The operator content of all $\Phi_i$ after diagonalization is shown in Figure 4 for a small and a large value of $r/a$, at $\beta_G = 12$, where $\phi_4$ and $\phi_8$ represent the operators for the additional fuzzing levels. At large distances, it is apparent that there are no low-lying states which would receive their
Figure 4: Histogram of overlaps $a_{ik}$ in the $8 \times 8$ operator basis for (a): $r/a = 9$ and (b): $r/a = 22$ at $\beta_G = 12$ on $48^3$. The additional operators compared to the $6 \times 6$ basis are $\phi_4$ and $\phi_8$, which have dominant contributions to the eigenstates $\Phi_8$, $\Phi_7$ and $\Phi_6$ only.

dominant contribution from these operators. This observation holds for other values of $r/a$ as well, with the exception of small distances, where the operator $\phi_8$ has significant projection onto the ground state. Since it is the most extended operator, it is plausible to think that overlapping smeared links will at small distances mimic a gauge string and hence lead to this increased projection. However, the values for the breaking scale $r_b$ extracted using the $8 \times 8$ basis are in complete agreement with those of the $6 \times 6$ basis for all lattices. We conclude that our $6 \times 6$ basis contains the relevant operators with good projection onto the low-lying states, and furthermore the numerical results for $r_b$ are stable under a variation of the operator basis.

The qualitative features discussed in the preceding paragraphs for $\beta_G = 12$ are repeated in the simulations at other lattice spacings. We now discuss the determination of the string breaking scale $r_b$. As in our previous work we can estimate $r_b$ at the point of maximal mixing, which is determined by minimizing the projections of the string and two-gluelump operators on the ground state, viz.

$$\Delta \equiv a_{1,S} - a_{1,G} \bigg|_{r=r_b} = 0. \quad (16)$$

This definition can be used at any fixed value of the lattice spacing to determine $r_b/a$. It is, however, not obvious that the estimates for $r_b/a$ obtained in this manner show the correct scaling behaviour as the continuum limit is approached. The reason is that
the operators $\phi_i$ of the original basis cannot be chosen such that their projection properties remain entirely unchanged as $\beta$ is varied. As a consequence the coefficients $a_{ik}$ for a given eigenstate $i$ will in general not be the same for different lattice spacings, and, moreover, an exactly computable relation between them cannot be found. However, physical quantities derived from the overlaps will show the correct scaling behaviour if the linear combinations corresponding to a given eigenstate can be related exactly at different lattice spacings.

A more robust prescription to determine $r_b$ is to minimize the splitting between the energies of the ground and first excited states, so that

$$\Delta_E \equiv E_1(r) - E_2(r) \bigg|_{r=r_b} \equiv \min.$$  \hspace{1cm} (17)

From the discussion at the end of section 2 it is obvious that $\Delta_E$ has a continuum limit. In practice the minimization is realized through an interpolation of the energy difference of the string and the two-gluelump state to the point where it vanishes. Systematic uncertainties in this determination $r_b$ can be estimated by varying the number of data points entering the interpolation. Furthermore, eq. (17) is also applicable in cases where the information about the composition of a given state as extracted from the overlaps alone is somewhat ambiguous. An example is the breaking of the excited adjoint string discussed above (see also Figure 2), where there is a sizeable, even dominant, contribution of the two-gluelump operator to the ground state below the breaking scale (which one would estimate to be around $r/a \approx 10$).

In Table 2 we collect our results for $r_b$ using both eqs. (17) and (16). By comparing different lattice sizes at the same $\beta_G$-value, we see that finite size effects are fully controlled. Furthermore, one observes that the estimates for $r_b/a$ using either eq. (16) or (17) are compatible within errors. We conclude that for our level of accuracy and for our range of lattice spacings the intrinsic systematic uncertainty in the definition eq. (16) is dominated by the statistical error.

Using the estimates for $r_b/a$ as determined from eq. (17) we have extrapolated $r_bg^2$ linearly in $1/\beta_G$ to the continuum limit, using the data points for which $Lg^2 = 16$. We find

$$r_bg^2 = 6.50 \pm 0.94.$$  \hspace{1cm} (18)

Expressing $r_b$ in units of the scalar glueball mass, for which we take $m_G/g^2 = 1.584(17)$ the result is

$$r_b m_G = 10.3 \pm 1.5.$$  \hspace{1cm} (19)
Table 2: Estimates for the string breaking scale.

| $\beta_G$ | $L/a$ | $Lg^2$ | eq. (17) $r_b/a$ | eq. (17) $r_bg^2$ | eq. (18) $r_b/a$ | eq. (18) $r_bg^2$ |
|----------|--------|--------|-----------------|-----------------|-----------------|-----------------|
| 9.0      | 36     | 16.0   | 15.0 ± 1.0      | 6.66 ± 0.44     | 14.5 ± 1.0      | 6.46 ± 0.44     |
| 9.0      | 52     | 23.1   | 14.8 ± 1.3      | 6.59 ± 0.58     | 14.5 ± 0.7      | 6.44 ± 0.33     |
| 12.0     | 48     | 16.0   | 19.9 ± 0.6      | 6.65 ± 0.21     | 19.4 ± 1.0      | 6.47 ± 0.33     |
| 12.0     | 60     | 20.0   | 19.8 ± 1.6      | 6.59 ± 0.54     | 19.3 ± 1.0      | 6.44 ± 0.33     |
| 15.0     | 60     | 16.0   | 24.8 ± 1.0      | 6.60 ± 0.27     | 23.5 ± 1.0      | 6.25 ± 0.27     |

In other words, the energy scale $r_b^{-1}$ turns out to be smaller by an order of magnitude compared with the lightest physical state of the theory.

We can now interpret our findings in the framework of the string picture. One would expect that at the breaking scale the energy of the flux tube is roughly equal to the energy of the lightest bound state consisting of the constituent fields which are pair-produced. For the adjoint string one would naively expect that $\sigma^{adj} r_b \approx m_G$, where $\sigma^{adj}$ is the slope of the linear part in the potential, and $m_G$ is, as before, the lightest glueball. Therefore, in order to test the string picture we have compared the ratio $m_G/r_b$ to the estimate for $\sigma^{adj}$ which was extracted from the linear part of the potential. Using our result in eq. (19) we find that the relation $m_G/r_b \approx \sigma^{adj}$ is satisfied at the 20% level. In view of the many caveats surrounding this analysis, such as the determination of $\sigma^{adj}$ itself and the effects of the binding energy between the pair-produced gluons, one cannot expect a much better quantitative agreement. However, the main purpose of this discussion is to argue that even only a roughly quantitative confirmation of $\sigma^{adj} r_b \approx m_G$ would suggest that the situation in the four-dimensional theory is not much different. Therefore one might expect to find a similar estimate of the breaking scale in units of $m_G$ in four dimensions.

To summarize, we have shown that the same type of variational calculation which has previously been successful in detecting string breaking in Yang Mills theory with fundamental matter, can also be applied to confirm the phenomenon in the potential with adjoint sources. In addition to calculating the breaking scale of the adjoint string in pure Yang Mills theory, we have computed the energies of a few excited states. We found a repeating pattern of potentials whose energies increase with the separation $r$ up to the point where the sources support two gluelumps, at least one of which may be excited.

In view of these results there is little doubt that a similar picture will be obtained in
the four-dimensional theory. In order to work towards more realistic models, one could also study the influence of fundamental matter fields on the value of the breaking scale obtained with adjoint sources. We leave these issues for future work, noting that recent lattice simulations have also studied similar systems \[18, 19\]. Indeed, there is considerable phenomenological interest in these models, since adjoint fermion fields are contained as gluinos in supersymmetric extensions of the Standard Model \[20\].

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**Note added:** On the day of the completion of this work, a paper about string breaking in the adjoint representation was submitted \[21\], in which very similar findings were reported.

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