Anomalous Diffusion at Edge and Core of a Magnetized Cold Plasma

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Abstract. Progress in the theory of anomalous diffusion in weakly turbulent cold magnetized plasmas is explained. Several proposed models advanced in the literature are discussed. Emphasis is put on a new proposed mechanism for anomalous diffusion transport mechanism based on the coupled action of conductive walls (excluding electrodes) bounding the plasma drain current (edge diffusion) together with the magnetic field flux "cutting" the area traced by the charged particles in their orbital motion. The same reasoning is shown to apply to the plasma core anomalous diffusion. The proposed mechanism is expected to be valid in regimes when plasma diffusion scales as Bohm diffusion and at high $B/N$, when collisions are of secondary importance.

1. Introduction
Any phenomena occurring with interfacial systems has a fundamental importance in science and technology [1] (e.g. electrostatic charging of insulators, surface tension, forward conduction in p-n junctions). Specifically, the problem of the plasma-wall interactions is of major importance in plasma physics.

Historically, the anomalously high diffusion of ions across magnetic field lines in Calutron ion sources (electromagnetic separator used by E. Lawrence for uranium isotopes) gave the firsts indications of the onset of a new mechanism [2]. It has been noticed that the plasma moves across the magnetic confining field at a much higher average velocity than it is predicted by classical considerations. The classical diffusion coefficient is given by $D_\perp = \eta p/B^2$, while the anomalous diffusion coefficient by $D_\perp = \alpha kT/B$. Manifestly it is needed a better understanding of the physical laws governing matter in the far nonequilibrium state, and this is a challenging issue for the advancement of this frontier of physics.

General proposed explanations were advanced. The first one was Simon’s "short-circuit" problem, suggesting that the observed losses could be explained by the highly anisotropic medium induced by the magnetic field lines, favoring electron current to the conducting walls [3]. Experiments done by Geissler [4] in the 1960’s have shown that diffusion in a plasma across a magnetic field was nearly classical (standard) diffusion when insulating walls impose plasma ambipolarity, but in the presence of conducting walls charged particles diffused at a much higher rate.

This problem of plasma-wall interaction becomes more complex when a complete description is aimed of a magnetized nonisothermal plasma transport in a conducting vessel. Beilinson
et al. [5] have shown the possibility to control the discharge parameters by applying potential difference to sectioned vessel conducting walls.

In the area of fusion reactors, there is strong indication that for plasmas large but finite Bohm-like diffusion coefficient appears above a certain range of \( B \) [6]. Experiments give evidence of transport of particles and energy to the walls [7]. At the end of the 1960s, experimental results obtained in weakly ionized plasma [4] and in a hot electron plasma [8] (this one proposing a possible mechanism of flute instability) indicated a strong influence that conducting walls have on plasma losses across magnetic field lines. Geissler [4] suggested that the most probable explanation was the existence of diffusion-driven current flow through the plasma to the walls. Concerning fusion reactions, Taylor [9] provided a new interpretation of tokamak fluctuations as due to an inward particle flux resulting from the onset of filamentary currents.

Progress in the understanding of the generation of confinement states in a plasma is fundamental [10] to pursue the dream of a fusion reactor [11, 12]. Anomalous diffusion is a cornerstone in this quest, as recent research with tokamaks suggest that the containment time is \( \tau \approx 10^8 R^2/2D_B \), with \( R \) denoting the minor radius of a tokamak plasma and \( D_B \) is the Bohm diffusion coefficient [13]. Controlled nuclear fusion experiments have shown that transport of energy and particles across magnetic field lines is anomalously large (i.e., not predicted by classical collision theory).

The conjecture made by Bohm is that the diffusion coefficient is \( D_B = \alpha kT/eB \), where \( T \) is the plasma temperature and \( \alpha \) is a numerical coefficient, empirically taken to be 1/16 [2]. Initially, the origin of the anomalous diffusion was assumed to be due to the turbulence of small-scale instabilities (see, for example, Refs. [6, 9, 14]). However, it is now clear that there is a number of different mechanisms that can lead to anomalous diffusion such as, coherent structures, avalanches type processes and streamers, which have a different character than a purely diffusive transport process. Recent experimental results such as scaling of the confinement time in L-mode plasmas and perturbative experiments undermine the previous paradigm built on the standard transport processes [15, 16] showing conclusively that there are many regimes where plasma diffusion does not scale as \( B^{-1} \).

This paper put emphasis on a mechanism of wall current drain set up together with the magnetic field ”cutting” lines across the area traced by the charged particles trajectories. The proposed mechanism of anomalous diffusion is expected to be valid in purely diffusive regimes when plasma diffusion scales as Bohm diffusion, both in the edge and core of a cold magnetized plasma. At his stage it was considered of secondary importance the role of collisions in randomizing the particle’s distribution function. From collisional low temperature plasmas to a burning fusion plasma subject the plasma confinement vessel to strong wall load, both in stellarator or tokamak operating modes, this explanation could be of considerable interest, particularly when diffusive transport process are dominant.

We would like to stress that this work is not free from omission of important contributions.

2. Simon’s ”short-circuit” theory

The first attempt to explain why the plasma diffuse at a much higher average velocity than it is predicted by classical theory has been advanced by A. Simon [3]. The magnetic field lines structure a highly anisotropic medium. Any fluctuation of the space charge builds up an electric field, which has a strong effect on the currents parallel to the magnetic field lines. In fact, the classical equation for conductivity across a magnetic field is given by

\[
\sigma_\perp = \frac{\sigma_0}{1 + \frac{n e^2}{\nu_c \Omega_c}}
\]

where \( \Omega_c = eB/m \) is the electron cyclotron frequency, \( \nu_c \) is the electron collision frequency, and \( \sigma_0 = e^2 n e/m \nu_c \) is the conductivity in the absence of a magnetic field. By the contrary,
due to $\Omega_c/\nu_e \gg 1$, this electric field is too small to have any importance on the crossed-field conductivity. From this results that there is a strong current to the wall without a concomitant current to different regions of the plasma, making of this situation a kind of circuitual "short-circuit" problem. Although Simon attempted to explain the anomalously high rate of diffusion in Calutron ion sources in the frame of the classical diffusion theory calculating the coefficient $D_\perp$ as being approximately equal to the transverse diffusion coefficient of the ions. His proposal is not suitable, however, because the experimental determination of $D_\perp$ by means of a decaying plasma have shown that according to the magnetic field strengthen the transverse diffusion coefficient can be much higher than the classical one or smaller than the transverse diffusion coefficient of the ions [4].

3. Plasma turbulence and transport

Purely diffusive transport models cannot give convincing explanations for a variety of experiments in magnetically confined plasmas in fusion engineering devices, particularly the scaling of the confinement time in L-mode plasmas. The assumed underlying instabilities are driven by either the pressure gradient or the ion temperature gradient. It is well established fact that transport in high temperature confined plasmas is driven by turbulence and plasma profiles, and are subject to transition from L-mode to H-mode (characterized by a a very steep gradient near the plasma surface) [17]. The non-linearity in the gradient-flux relation is the source of turbulence and turbulence-driven transport. The fluxes contain all the dynamic information on the transport process. Accordingly, changes in the gradient trigger local instabilities in the plasma. This local instability induces an increase in the nearby gradients, thus causing a propagation of the instability all across the plasma. In particular, an excessive pressure on the core propagates to the edge in a kind of avalanche.

In weakly turbulent cold magnetized plasmas, besides the Calutron ion sources and the magnetron, the study of particle transport in crossed electric and magnetic fields results from applications to electromagnetic space propulsion (Hall thrusters). Those plasma accelerators work with a radial magnetic field that prevents electron flow toward the anode and forcing the electrons closed-loop drift around the axis of the annular geometry. Neutrals coming from the anode are ionized in this rotating electrons cloud, while ions are accelerated by an axial electric field that freely accelerate them out from the device. This effect develops in the so called extended acceleration zone (or electric-magnetic region plasma). In this acceleration zone the electron gyro-radius and the Debye shielding length are small relative to the apparatus dimension, while the ion gyro radius is larger than the apparatus typical length. From these spatial scales results that the electron motions are $[E \times B]$ drifts, but ions are accelerated by the electric field that develops in the plasma. The first observations of a large amount of electron transport toward the anode have been noticed in the 60’s (e.g., Ref. [18]) and they have been related to electric field fluctuations since they were correlated with the density variations in order to produce anomalous transport. Other possible mechanism that could explain the high electrons transverse conductivity were advanced: collisions with the wall [19, 20]. Electrons moving freely along the lines of forces of the magnetic field collide with the wall more frequently than with ions and neutrals, being reflected at the wall and enhancing emission of low-energy secondary electrons from the wall. As referred, the other strong candidate, which could possibly be the source of a higher axial electron current than predicted by the standard classical kinetic theory, is the turbulent plasma fluctuations. But it seems that there is no clear consensus on this issue [21, 22, 23].

The magnetron is a sputtering tool, used for reactive deposition and etching. The magnetron effect is applicable to different geometries and only need a closed-loop $[E \times B]$ drift to work. Rossnagel et al. [24] have shown that the Hall-to-discharge current ratio measured in those configurations could be explained if the high collision frequencies for electrons were associated
to Bohm diffusion. In particular, Kaufman [25] argues that anomalous diffusion in closed-loop \([E \times B]\) thrusters could shift from core diffusion to edge diffusion (or wall effects) with increasing magnetic fields.

4. Circuital model of anomalous diffusion

In a seminal paper [26] a conjecture was proposed based on the principle of minimum entropy-production rate, stating that a plasma will be more stable whenever the internal product of the current density \(j\) by an elementary conducting area \(dA\) at every point of the boundary - excluding the surface collecting the driving current - is null, \((j \cdot A) = 0\) at any point of the boundary (and excluding the surfaces collecting the discharge current), independently of the resistance \(R_i\). The general idea proposed by Robertson [26] assumes that the plasma boundary is composed of small elements of area \(A_i\), each one isolated from the others, but each one connected to the exterior common circuit through its own resistor \(R_i\) and voltage \(V_i\). The entropy production rate in the external circuits is given by:

\[
\frac{dS}{dt} = \sum_i \frac{1}{T} (j_i \cdot A_i)^2 R_i, \tag{2}
\]

where \(T\) is the temperature of the resistors, supposed to be in thermal equilibrium with all the others. It is important to remark that the summation is over the different conducting areas eventually confining the plasma, excluding the electrodes areas. Fig.1 illustrates this concept.

![Schematic diagram of the plasma boundary connected to the common circuit through conducting walls.](image)

**Figure 1.** Schematic diagram of the plasma boundary connected to the common circuit through conducting walls.

We consider a simple axysymmetric magnetic configuration with magnetic field lines parallel
to z-axis with a plasma confined between two electrodes (see Fig. 1). In general terms, a particle’s motion in the plasma results in a massive flux. As long as the flux is installed, the flux will depend naturally on a force \( F \) - in this case the pressure gradient-driven process of diffusion to the wall - responsible of the wall driven current \( j \). According to the fundamental thermodynamic relation, the plasma internal energy variation \( dU \) is related to the amount of entropy supplied or rejected and the work done by the driven force, through the equation:

\[
\frac{dU}{dt} = (j \cdot A)^2 R + \left( \mathbf{F} \cdot \frac{dr}{dt} \right).
\]  

(3)

The last term we identify with the macroscopic diffusion velocity \( \mathbf{v}_d \) depicting the process of plasma expansion to the wall. To simplify somehow the calculations we assume a single plasma fluid under the action of a pressure gradient \( \mathbf{F} = A L dp/dy \), where \( \mathbf{j} \) is a unit vector directed along the Oy axis.

In the presence of steady and uniform magnetic field lines (this simplifies the equations, but do not limit the applicability of the model), the particles stream freely along them. From magnetohydrodynamics we have a kind of generalized Ohm’s law (see, for example, Ref. [27]):

\[
\nabla p = -enE - en[\mathbf{v} \times \mathbf{B}] + [\mathbf{j} \times \mathbf{B}] - \frac{en\mathbf{j}}{\sigma},
\]  

(4)

where \( \sigma = e^2 n \tau_e m_e \) is the electric conductivity, with \( \tau_e \) denoting the average collision time between electrons and ions. The force balance equation is given by:

\[
\nabla p = [\mathbf{j} \times \mathbf{B}],
\]  

(5)

valid whenever the Larmor radius is smaller than the Debye radius. This assumption simplifies further the extension of our model to high enough magnetic fields. Therefore, after inserting Eq. 5 into Eq. 4 the y component of velocity is obtained:

\[
v_y = -\frac{E_x}{B} - \frac{1}{\sigma B^2} \frac{dp}{dy}.
\]  

(6)

From Eq. 6 we have the classical diffusion coefficient scaling with \( 1/B^2 \) and thus implying a random walk of step length \( r_L \) (Larmor radius). To get the anomalous diffusion coefficient and as well understand better its related physics, we must consider the process of diffusion to the wall - in the presence of an entropy source - with the combined action of the wall current drain, as already introduced in Eq. 3.

Therefore, using the guiding center plasma model the particle motion is made with velocity given by:

\[
\mathbf{j} = en \mathbf{v}_d = -\frac{[\nabla p \times \mathbf{B}]}{B^2}.
\]  

(7)

This equation forms the base of a simplified theory of magnetic confinement. In fact, the validity of Eq. 7 is restrained to the high magnetic field limit, when the Larmor radius is shorter than the Debye radius.

Considering the motion along only one direction perpendicular to the wall (y-axis), it is clear that

\[
(j \cdot A)^2 = \frac{A^2}{B^2} \left( \frac{dp}{dy} \right)^2.
\]  

(8)

If we consider a quasi-steady state plasma operation, the plasma total energy should be sustained. Hence, \( dU/dt = 0 \), and the power associated with the driven pressure-gradient is just
maintaining the dissipative process of plasma losses on the wall. Eq. 3 governs the evolution of the diffusion velocity. Hence, we have

\[ n v_d = - \frac{n R A k T}{L} \frac{D n}{B^2 dy} = -D_T \frac{dn}{dy}, \]  

(9)

with \(D_T\) denoting the transverse (across the magnetic field) diffusion coefficient given by:

\[ D_T = \frac{n R A k T}{L} \frac{1}{B^2}. \]  

(10)

This new result coincides with the classical diffusion coefficient [28] whenever \(n R A / L \equiv n v_{el} / e^2\), containing a dependence on collision frequency and particle number density. Other theoretical approaches to this problem were advanced by Bohm [2], who proposed an empirically-driven diffusion coefficient associating plasma oscillations as the source of the enhanced diffusion, while Tonks [29] have shown that the current density that is present in a magnetically immobilized plasma is only generated by the particle density gradient, not being associated with any drift of matter. Simon electron “short-circuit” [3] scheme attempt to explain the different rates of diffusion, electrons and ions do experiment across the magnetic field. While the ion flux dominates the radial diffusion, the electron flux dominates axial losses, due to an unbalance of currents flowing to the wall.

In the absence of collisions, the guiding centers of charged particles behave as permanently attached to the same lines of force. On the contrary, as a result of collisions with others charged particles the guiding centers shift from one line of force to another resulting in a diffusion of plasma across the field lines. In our model, each orbit constitutes an elementary current \(I\) eventually crossing the wall.

However, the particle diffusion coefficient as shown in Eq. 10 gives evidence of an interplay with the resistance that the elementary circuit offer when in contact with the walls in the presence of the frozen-in effect. In fact, for sufficiently strong magnetic fields apparently a hydrodynamic behavior of the plasma is installed [6, 30], with the appearance of “convective cells” and the \(1/B\) behavior dominates, giving birth to the anomalous diffusion mechanism. The onset of freezing magnetic lines is valid whenever the Lundquist number \(S \gg 1\) (convection of the magnetic field dominated medium). In this case the magnetic field lines are frozen-in in the medium (consequence of a vortex type of character of the magnetic field \(B\)) and the flux of them across a given surface is constant:

\[ \Phi = B A' = B L^2 \alpha. \]  

(11)

Remark that \(A'\) is now the surface delimited by the elementary circuit \(\gamma\) (see Fig. 2) and \(\alpha \leq 1\) is just a geometrical factor (e.g. \(\alpha = \pi/4\) at the limit of a circular orbit). This situation is fundamental to the onset of anomalous diffusion. Free electrons orbits are helical, but as Fig. 2 shows, their projections at right angles to the field are circular. Each particle orbit constitute an elementary circuit with \(B\)-field cutting its surface being associated with it an elementary flux \(\Phi\). At the same time we can envisage each orbit as constituting by itself an elementary circuit, some of them intersecting the wall and thus the circuit is closed inside the wall. Therefore a resistance \(R\) drags the charged flow at the conducting wall. It is therefore plausible to associate to this elementary circuit a potential drop \(V\) and all the process being equivalent to a current \(I\) flowing through the elementary circuit.

Assuming the plasma is a typical weakly coupled, hot diffuse plasma with a plasma parameter (number of particles in Debye sphere) \(\Lambda = n \lambda_D^3 \approx 1\), it is more likely to expect nearly equal average kinetic and potential energy. However, the typical plasma parameter encountered in glow discharges or in nuclear fusion is \(\Lambda \gg 1\). This means that the average kinetic energy is
Figure 2. Schematic of the geometry for the plasma-wall current drain model. The uniform magnetic field points downward along Oz. Particles describe orbits in the plane xOy intersecting the wall (plan xOz). Orbits are represented by a semi-circular line for convenience. L is the maximum distance the trajectory attains from the wall.

larger than the average potential energy. To contemplate all range of Λ we can relate them through the relationship

\[ \rho V = (\mathbf{J} \cdot \mathbf{A}) \delta. \]  

(12)

Here, \( \rho \) is the charge density, \( \mathbf{A} \) is the vector potential, \( \mathbf{J} \) is the current density and \( \delta \leq 1 \) is just a parameter representing the ratio of potential to kinetic energy. Of course, when \( \Lambda \geq 1 \), then \( \delta \leq 1 \). This basic assumption is consistent with the hydrodynamic approximation taken in the development of equations. The limitations of the model are related with the unknowns \( \Lambda \) and \( \delta \) that can be uncovered only through a self-consistent model of the plasma. However, our analysis of anomalous diffusion remains general and added new insight to the phenomena.

Now suppose that the diffusion current is along y-axis \( \mathbf{J} = -J_y \mathbf{u}_y \) (see Fig.1). Consequently, \( \mathbf{A} = -A_y \mathbf{u}_y \), and then the potential drop will depend on x-coordinate:

\[ \rho[V(x_1) - V(x_0)] = J_y[A_y(x_1) - A_y(x_0)] \delta. \]  

(13)

Multiplying both members by the area \( \mathcal{A}' = x_1z_1 \) and length \( L = y_1 \), we have

\[ Q \Delta V = I y_1[A_y(x_1) - A_y(x_0)] \delta = I \Phi \delta. \]  

(14)

\( \Phi = \oint \gamma (\mathbf{A} \cdot d\mathbf{x}) \) is the flux of the magnetic field through the closed surface bounded by the line element \( d\mathbf{x} \) (elementary circuit \( \gamma \), see also Fig.2). By other side, naturally, the total charge...
present on the volume $V = x_1 y_1 z_1$ is such as $Q = ie$, with $i$ an integer. This integer must be related to ions charge number. From Eq. 14 we obtain

$$R = \frac{\Delta V}{I} = \delta \frac{\Phi}{Q} = \alpha \delta \frac{BL^2}{ie}.$$  \hfill (15)

But, the particle density is given by $n = N/LA$, with $N$ being now the total number of charged particles present in volume $V = AL$. Since $i = N$, we retrieve finally the so-called Bohm-diffusion coefficient

$$D_B = \alpha \delta \frac{kT}{eB}.$$  \hfill (16)

So far, our arguments were applied to edge anomalous diffusion. But they can be generalized to the core anomalous diffusion processes, provided that diffusive transport processes are dominant. For this purpose consider instead of a conducting surface a virtual surface delimiting a given volume, as shown in Fig. 3.

**Figure 3.** Volume control and particle’s trajectory submitted to magnetic field. Magnetic field lines point downward.

Our coefficient is time-dependent and can be written under the form:

$$D_\perp = \delta \frac{kT(t)}{eB(t)^2} \frac{\Phi(t)}{L^2}.$$  \hfill (17)

The nonrelativistic solutions of dynamical equation of a charged particle in time-dependent but homogeneous electric and magnetic field give the following approximative expressions for the
width trajectories along the xOy plane (see, e.g. Ref.[31]) by:

\[ \Delta x = \frac{v_\perp}{c} \cos(\Omega_c t + \chi), \]
\[ \Delta y = \mp \frac{v_\perp}{c} \sin(\Omega_c t + \chi), \] (18)

where \( \chi \) is the initial phase, \( v_\perp \) denotes the component of the velocity perpendicular to the magnetic field and the \( \mp \) sign applies to electrons (\(-\)) or positive ions (\(+\)). From them we can retrieve the flux "cutting" area:

\[ A \approx \Delta x \Delta y = -\frac{1}{2} \left( \frac{v_\perp}{c} \right)^2 \sin(2\Omega_c t + 2\chi). \] (19)

Then the anomalous diffusion coefficient is just given by:

\[ D_\perp \approx \mp \delta \frac{kT(t)}{eB(t_0)} \frac{1}{2} \left| \cos(\omega t) \right|. \] (20)

As we can see in Fig.4 this last expression describes fairly well the diffusion process for high enough \( B/N \) values (the magnetic field to gas number density ratio) and explains the main processes building-up such effects as: i) the negative diffusion, which results from the contraction of the flux "cutting" area; ii) the ciclotronic modulation imprint on the transverse diffusion coefficient; iii) and the anomalous diffusion, due to the fast flux rate of the magnetic field through the area \( A \). All this signs can be seen on Fig. 4 where it is shown a comparison of numerical results (5000 Hx, \( 1 \) Hx=\(10^{-27}\) T.m\(^3\)) obtained with Monte Carlo simulations of electron transport in crossed magnetic and electric fields by Petrović et al. [32, 33] with the theoretical prediction given by our Eq. 20. As long as only a self-consistent model could give us an exact value of the ratio of potential to kinetic energy \( \delta \), we assume here \( \delta = 1/40 \). The full agreement with the numerical calculations is not obtained due to neglecting effects related to the electric field variation in time and of the assumed collisionless approximation. This explains the big discrepancy shown in Fig. 4 when compared with the diffusion coefficient at 1000 Hx when collisions begin to be far more important to randomize individual trajectories and our approach is no more valid (at low enough \( B/N \) values).

5. Discussion and Summary

However, Eq. 16 suffers from the indetermination of the geometrical factor \( \alpha \). This factor is related to the ions charge number, it depends on the magnetic field magnitude and as well as on the external operating conditions (due to increased collisional processes, for ex.). The exact value of the product \( \alpha \delta \) can only be determined through a self-consistent plasma model, but we should expect from the above discussion that \( \alpha \delta < 1 \). For a 100-eV plasma in a 1-T field, we obtain \( D_B = 1.67 \) m\(^2\)/s (using the thermal to magnetic energy ratio with particle’s density \( n = 10^{14} \) cm\(^{-3} \)). Furthermore, Eq. 15 can be used as a boundary condition (simulating an electrically floating surface) imposed when solving Poisson equation.

Also it worth to emphasize that when inserting Eq. 15 into Eq. 10, and considering the usual definition of momentum transfer cross section, then it can be obtained a new expression for the classical diffusion coefficient as a function of the ratio of collisional \( \nu \) and cyclotron frequency \( \Omega \), although (and in contrast with the standard expression), now also dependent on the geometrical factor \( \alpha \) and energy ratio \( \delta \):

\[ D_T = (\alpha \delta) \frac{\nu kT}{\Omega m}. \] (21)

This explains the strong dependence of the classical diffusion coefficient on \( \nu/\Omega \) showing signs of anomalous diffusion as discussed in Ref. [32] (obtained with a time resolved Monte Carlo simulation in an infinite gas under uniform fields) and, in addition, the strong oscillations shown
Figure 4. Comparison between numerical results for 5000 Hx obtained by Monte Carlo simulations Refs.[32,33] of electron transport in crossed electric and magnetic rf fields in argon and Eq. 20 -theo. Dashed line: external time-dependent magnetic field. Parameters used: $\delta = \frac{1}{20}$; $\chi$; applied frequency, $f = 100$ MHz; cyclotron frequency, $\Omega_c \approx 10^{10}$ Hz; $kT_e = 5.4$ eV; $p=1$ Torr, $T_g = 300$ K, $\chi = 0$.

up in the calculations of the time dependence of the transverse component of the diffusion tensor for electrons in low-temperature rf argon plasma. Those basic features result on one side from its dependence on $R$, which is proportional to the flux. Therefore, a flux variation can give an equivalent effect to the previously proposed mechanism: whenever a decrease (or increase) in the flux is onset through time dependence of electric and magnetic fields, it occurs a strong increase (or decrease) of the diffusion coefficient. By other side, when the resistance increases it occurs a related decrease of charged particles tangential velocity and its mean energy. So far, this model gives a new insight into the results referred in [32] and also explains why the same effect is not obtained from the solution of the non-conservative Boltzmann equation as applied to an oxygen magnetron discharge with constant electric and magnetic fields [34].

A further application of Eq. 2 to a cold plasma can give a new insight into the "ambipolar-like" diffusion processes. Considering just one conducting surface (besides the electrodes driving the main current into the plasma) and the plasma build-up of electrons and one ion component to simplify matters, we obtain:

$$\frac{dS}{dt} = \frac{e^2}{T} (-n_e \mu_e E + D_e \nabla n_e + n_i \mu_i E - D_i \nabla n_i)^2 A^2 R. \quad (22)$$

Under the usual assumptions of quasi-neutrality and quasi-stationary plasma (see, for example,
Ref. [28]), the following conditions must be verified:
\[
\frac{n_i}{n_e} = \epsilon = \text{const.}; \quad n_e \mathbf{v}_e = n_i \mathbf{v}_i,
\] (23)
and hence, Eq. 2 becomes:
\[
\frac{dS}{dt} = \frac{e^2}{T} \left[ E(\epsilon \mu_e - \mu_i) n_e + \nabla n_e (D_e - D_i \epsilon) \right]^2 A^2 R. \tag{24}
\]
For a stable steady-state plasma with no entropy sources the condition \( \dot{S} = 0 \) prevails and then an "ambipolar-like" electric field is recovered [28]:
\[
\mathbf{E} = \frac{D_e - \epsilon D_i}{\mu_e - \mu_i \epsilon} \nabla n_e. \tag{25}
\]
It means that the conducting surface must be at its floating potential. Such conceptual formulation provides new insight into "ambipolar-like" diffusion processes. In a thermal equilibrium state, a plasma confined by insulating walls will have an effective coefficient given by the above Eq. 25, a situation frequently encountered in industrial applications. This example by itself relates ambipolar diffusion with no entropy production in the plasma. However, allowing plasma currents to the walls, entropy production is greatly enhanced generating altogether instabilities and plasma losses [26]. As long as confined plasmas are in a far-nonequilibrium state (with external surroundings) it is necessary to establish a generalized principle that rule matter, and this circuital model for anomalous diffusion represents some progress in the physics of plasmas as nonequilibrium systems.

To summarize, we introduced in this study a simple circuital mechanism providing an interpretation of the anomalous diffusion in a magnetized confined plasma in a purely diffusive transport regime. The coupled action of the magnetic field "cutting" flux through the areas traced by the charge carriers elementary orbits, together with the elementary electric circuit constituted by the charged particle trajectory itself are at the basis of the anomalous diffusion process. Whenever conducting walls bounding the plasma drain the current (edge diffusion) or, at the plasma core, the magnetic field flux through the areas traced by the charged particles varies, a Bohm-like behavior of the transverse diffusion coefficient can be expected. Eq. 20 can be used as an analytical formula when simulating plasma behavior at high \( B/N \). In the near future we hope to generalize this model taking into account random collisions. The suggested mechanism could lead to a better understanding of the mechanism of plasma-wall interaction and help to develop a full-scale numerical modeling of present fusion devices or collisional low-temperature plasmas.

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