Kaluza-Klein Burst: a New Mechanism for Generating Ultrahigh-Energy Cosmic Rays

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Abstract

By invoking small extra dimensions as a good energy bearer, a new scenario for understanding the origin of ultrahigh-energy cosmic rays (UHECRs), from both the bottom-up and the top-down viewpoints, is proposed. We explore the possibility of generating UHECRs via Kaluza-Klein (KK) bursts, a violent energy transfer from extra dimensions to ordinary dimensions through collisions between KK modes, in particular, within clumps of KK modes. The possible scales of these clumps range from the astronomical, e.g. KK stellar compact objects, to the microscopic, e.g. “KKONIUM”. Advantages of this KK burst model of UHECRs and possible signatures of clumped KK modes are discussed.

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I. INTRODUCTION

More than one thousand ultrahigh-energy cosmic ray (UHECR) events of energies above $10^{19}$ eV have been detected in the past ten years. In particular the events above $10^{20}$ eV have fired the controversy about the existence and the origin of the Greisen-Zatsepin-Kuzmin (GZK) cutoff. No matter whether the cutoff exists or not, the detection of UHECRs has brought various challenges and inspired a lot of ideas and imaginations for an ultrahigh-energy part of the world. (For a review, see [4].)

The difficulties in UHECR models mainly stem from two observational results, ultrahigh energies and roughly isotropic arrival directions. Ultrahigh energies make it difficult, in the bottom-up models (for a review, see [5, 6]), to find powerful enough sources, in particular, under continuous energy losses via the GZK mechanism. This difficulty leads to the GZK puzzle. In addition, isotropic arrival directions make it unlikely to explain the UHECR events via only a few sources, unless their trajectories can have been significantly deflected by cosmic magnetic fields.

The situation is different in the top-down models (for a review, see [5]), where UHECRs are produced via decays or annihilation of very massive particles (or topological defects) so that energy is not an issue. These very massive particles might behave like dark matter and reside in the local dark halo. In this case the roughly isotropic distribution of arrival directions is not an issue, either. Nevertheless, we know little yet about such exotic, very massive particles. Their existence is unconfirmed.

In this paper we propose a new mechanism for generating UHECRs by invoking small compact extra (spatial) dimensions as a good energy bearer. We explore the feasibility of generating UHECRs via Kaluza-Klein (KK) bursts, a violent energy transfer from extra dimensions to ordinary dimensions through collisions of KK modes. We will show how KK bursts can serve and benefit the construction of UHECR models, including both the bottom-up and the top-down models, without introducing new particle species.

The paper is organized as follows. Sec. II sketches the basic idea of the KK burst model of UHECRs. Sec. III contains the study of the spectrum of UHECRs originated from KK bursts involving uniformly distributed KK modes, clumped KK modes, and KK compact objects (in particular, “kkonium”), respectively. A summary and discussions follow in Sec. IV.
II. KALUZA-KLEIN BURST

As a quantum nature, an excitation in an extra dimension of a finite size $l_{\text{ED}}$, i.e. a Kaluza-Klein (KK) mode, carries quantized KK momentum: $P_{\text{kk}} = 2\pi n_{\text{kk}}/l_{\text{ED}} \equiv n_{\text{kk}} \hat{P}_{\text{kk}}$, where the KK (quantum) number $n_{\text{kk}} = 0, \pm 1, \pm 2, \ldots$, and $\hat{P}_{\text{kk}}$ denotes one unit of KK momentum. We will use the symbol $\text{KK}^n$ to denote a KK mode with a KK number $n$. KK$^n$ and KK$^{-n}$ will be called KK mode and anti-KK mode. (Note that KK modes and anti-KK modes need not to be particles and anti-particles.) A bound state consisting of a KK mode and an anti-KK mode will be called “kkonium”.

As required by momentum conservation in extra dimensions, the KK number is conserved module two, i.e., KK modes must be created or annihilated in pairs. This feature helps extra dimensions be a good energy bearer against the energy pillage by the GZK mechanism and other similar energy degrading processes (as to be called GZK-like mechanisms). In a usual way of thinking about the GZK-like mechanisms, ultrahigh-energy particles lose energy through the scattering with background particles or fields. Nevertheless, due to KK momentum conservation, the energy stored in extra dimensions is hardly dissipated through the interactions with background particles that are mostly KK zero modes.

This feature can help the bottom-up scenario get rid of two essential difficulties. In the bottom-up models, particles are accelerated to ultrahigh energies within extreme astrophysical environments, which are usually very dense. How ultrahigh-energy particles escape from these dense regions without losing much energy through the scattering with particles therein is a serious intrinsic problem. Another difficulty stems from the GZK mechanism that makes it unlikely for UHECRs to maintain energies beyond the GZK threshold (around $10^{20}$ eV) after travelling a distance longer than 50 Mpc. Unfortunately, there are very few powerful enough sources within the GZK zone, a region with a radius of 50 Mpc around the earth. These few sources can hardly explain the UHECR spectrum and the distribution of arrival directions. Nevertheless, if these ultrahigh energies are stored in KK momentum of KK modes, they will have much better chance to be carried out of the sources and to travel a long distance even much larger than 50 Mpc without significant loss.

In addition, a KK (nonzero) mode might collide with an anti-KK mode, through which the energy stored in extra dimensions may be transferred to ordinary dimensions. If the size of extra dimensions is extremely small such that $\hat{P}_{\text{kk}}$ is extremely large correspondingly, this
energy transfer will be very violent. For a three-dimensional observer, this process looks like a burst caused by a collision between two very heavy particles (with large effective mass from KK momentum), through which a large amount of energy is released from the mass of incoming particles to the kinetic energy of outgoing particles. This burst is dubbed “Kaluza-Klein (KK) burst” hereafter. This violent energy transfer from extra dimensions to ordinary dimensions provides a mechanism for generating UHECRs.

KK modes with ultra-large KK momentum may be created, in the form of free KK modes or kkonium, from the very early universe or within extreme astrophysical environments (which may locate outside the GZK zone). The UHECRs detected on the earth are those generated via KK bursts involving KK modes which have come into or are always within the GZK zone.

KK bursts serve both the top-down and the bottom-up models. In the top-down point of view, the process of generating UHECRs from the KK burst of two freely moving KK modes is similar to that involving annihilation of superheavy particles [7], while the case of KK bursts in kkonium, i.e. decays of kkonium, is similar to decays of superheavy particles [8, 9].

For the bottom-up scenario, the detected UHECRs are generated via KK bursts within the GZK zone, involving KK modes or kkonium which are created in extreme astrophysical environments, whose locations may be outside the GZK zone. Note that in this model the extreme astrophysical environments are sources of KK modes and kkonium that generate UHECRs within the GZK zone, but not the “first-hand” sources of UHECRs. Also note that in the KK burst model UHECRs are KK zero modes, but not KK (nonzero) modes. Thus, in this model UHECRs do still suffer the energy pillage by the GZK-like mechanisms. It is the KK (nonzero) modes, the source of UHECRs, but not UHECRs themselves, that can defend the energy they carry against the GZK-like mechanisms during the journey to the earth.

The way of circumventing the GZK-like energy pillage via small extra dimensions in the KK burst model is similar to that in the Z-burst model of UHECRs [10, 11, 12]. The Z-burst model explains the detection of the super-GZK events by invoking (extremely-high-energy) neutrinos as a good energy messenger, which, in the KK burst model, is played by KK modes (including kkonium).
III. SPECTRUM

In calculating the differential flux of UHECRs originated from KK bursts, we consider the simplest channel: a KK$^{+1}$ mode collides with a KK$^{-1}$ mode, creating two outgoing KK zero modes with large kinetic energy, which then produce two leading jets through fragmentation. We assume that the distributions of KK$^{±1}$ modes are both described by the number density $n_{kk}(r)$, where $r$ is the position vector measured by observers on the earth. In addition, we assume that KK momentum dominates the energy of each KK$^{(±1)}$ mode, such that the gain of large kinetic energy for each outgoing KK zero mode mainly comes from KK momentum.

The process of a KK burst is similar to pair annihilation of two heavy particles with (effective) mass about $\hat{P}_{kk}$. (Note that, unlike pair annihilation, KK bursts may involve different species of particles, e.g., an electron and a photon.) The possibility that UHECRs originate from the annihilation of superheavy dark matter has been considered by Blasi, Dick, and Kolb [7]. It is straightforward to employ the formulation therein to study the spectrum of UHECRs generated by KK bursts. Following [7], the formula for the spectrum of UHECRs originated from KK bursts can be written as

$$\mathcal{F} = \frac{dN(E, E_{\text{jet}} = \hat{P}_{kk})}{dE} \cdot F = \frac{dN(E, E_{\text{jet}} = \hat{P}_{kk})}{dE} \cdot 2 \langle \sigma_b v \rangle \int d^3r \frac{n_{kk}^2(r)}{4\pi |r|^2},$$

where $dN(E, E_{\text{jet}})/dE$ is the fragmentation spectrum from a jet of energy $E_{\text{jet}}$ [13] and $F$ denotes the UHECR flux. As indicated in the above equation, for calculating the spectrum $\mathcal{F}$ we need to know the KK burst cross-section $\sigma_b$ and the number density of KK$^{±1}$ modes $n_{kk}$. So far the knowledge about the cross-section at such high energies is poor. In the following calculations, we will introduce the unitary bound, $\hat{P}_{kk}^{-2}$, to the KK burst cross-section and utilize the ratio $\alpha_{kk} \equiv \langle \sigma_b v \rangle / \hat{P}_{kk}^{-2}$, which can be regarded as an effective coupling constant associated with KK bursts. As to $n_{kk}$, we will introduce the energy density of dark matter to give an upper limit to the energy density of KK modes $\rho_{kk}$ (i.e. $n_{kk} \hat{P}_{kk}$).

As indicated in [7], for the consistency with the observed UHECR spectrum in the model of superheavy dark matter annihilation, the preferred value of the mass of superheavy dark matter is in the range $10^{21}$–$10^{23}$ eV, and the corresponding value of the flux $F$ ranges from 0.4 km$^{-2}$yr$^{-1}$ (for mass $10^{21}$ eV) to 7 km$^{-2}$yr$^{-1}$ (for mass $10^{23}$ eV). These results can be applied
to the KK burst model. Accordingly, the preferred value of one-unit KK momentum \( \hat{P}_{kk} \) is in the range \( 10^{21} - 10^{23} \) eV, i.e. the corresponding size of extra dimensions ranges from \( 10^{-27} \) cm to \( 10^{-25} \) cm, and the flux \( F \) provided by KK bursts needs to reach the value mentioned above, i.e.,

\[
F = 0.4 \text{ km}^{-2}\text{yr}^{-1} \quad \text{for} \quad \hat{P}_{kk} = 10^{21}\text{eV} \sim 10^{25}\text{cm}^{-1},
\]

\[
F = 7 \text{ km}^{-2}\text{yr}^{-1} \quad \text{for} \quad \hat{P}_{kk} = 10^{23}\text{eV} \sim 10^{27}\text{cm}^{-1}.
\] (2)

### A. Uniformly Distributed KK Modes

In the following we study two parts of contributions of KK bursts to UHECRs, one from KK modes in the local dark halo and the other from those in the GZK zone (excluding the effect of the overdensity in the local halo). We first consider a toy model where KK±1 modes are uniformly distributed with a constant number density \( n_{kk}(\text{halo}) \) in the local halo. In this model, the flux \( F \) is simply

\[
F_{\text{halo}} = 2 \langle \sigma_b v \rangle n_{kk}(\text{halo}) \int_{\text{halo}} \frac{d^3r}{4\pi |r|^2}.
\] (3)

Choosing the characteristic size of the local halo to be 30 kpc and the distance of the solar system from the halo center to be 8 kpc, we obtain

\[
F_{\text{halo}} \simeq \left( 6 \times 10^{-26} \text{ km}^2\text{yr}^{-1} \right) \cdot \alpha_{kk} \xi_{kk(\text{halo})}^2 \cdot \left( \frac{\hat{P}_{kk}}{10^{21}\text{eV}} \right)^{-4},
\] (4)

where \( \alpha_{kk} \equiv \langle \sigma_b v \rangle / \hat{P}_{kk}^2 \) and \( \xi_{kk(\text{halo})} \equiv \rho_{kk(\text{halo})}/\rho_{\text{halo}} \) (i.e. \( n_{kk(\text{halo})}/\rho_{\text{halo}} \)), which is the fraction of the energy density contributed from KK modes in the local halo. Note that in the above equation we have introduced the unitary bound \( \hat{P}_{kk}^2 \) to the KK burst cross-section \( \langle \sigma_b v \rangle \) and the energy density of the local halo, \( \rho_{\text{halo}} = 2-13 \times 10^{-25} \text{ g cm}^{-3} \), as an upper limit to the energy density of KK modes in the halo \( \rho_{kk(\text{halo})} \). As indicated in Eq. (4), the flux \( F \) is proportional to \( \hat{P}_{kk}^{-4} \) for fixed \( \alpha_{kk} \) and \( \xi_{kk(\text{halo})} \). The value \( 6 \times 10^{-26} \text{ km}^2\text{yr}^{-1} \), which can be regarded as an upper bound of the flux \( F \) for \( \hat{P}_{kk} \geq 10^{21} \text{ eV} \), is far below the required values in Eq. (2).

In the same way, we can calculate the flux contributed from the uniformly distributed KK modes within the GZK zone (ignoring the overdensity in the local halo). We obtain

\[
F_{\text{GZK}} \simeq \left( 7 \times 10^{-33} \text{ km}^2\text{yr}^{-1} \right) \cdot \alpha_{kk} \xi_{kk(\text{GZK})}^2 \cdot \left( \frac{\hat{P}_{kk}}{10^{21}\text{eV}} \right)^{-4},
\] (5)
where \( \xi_{\text{GZK}} \equiv \rho_{\text{GZK}}/\rho_{\text{DM}} \), which is the fraction of the energy density of dark matter contributed from KK modes within the GZK zone. Here we have introduced the energy density of the dark matter in the universe, \( \rho_{\text{DM}} = 1.4 \times 10^{-30} \) g cm\(^{-3} \), as an upper limit to the energy density of KK modes within the GZK zone, \( \rho_{\text{KK(GZK)}} \). This flux is in general much smaller than that originated from the local halo, and is also far below the required values in Eq. (2).

**B. Clumped KK modes and KK Compact Objects**

KK modes may clump together in small regions and form compact objects through gravity or other interactions. The large overdensity in these clumps can enhance the flux \( F \) because of the non-linearity with respect to \( n_{\text{KK}} \) in Eq. (1).

We first consider the sub-galactic clumps of KK modes in the local halo. For simplicity we make the following assumptions: (i) KK modes are uniformly distributed with a constant number density \( n'_{\text{KK}} \) in each clump. (ii) Clumps are uniformly distributed in the halo. (iii) All clumps possess the same (effective) mass. In this case, the formula for the flux \( F \) is:

\[
F_{\text{halo}} = \left( \frac{n'_{\text{KK}}}{n_{\text{KK}}} \right)_{\text{halo}} \cdot 2 \langle \sigma v \rangle n_{\text{KK(halo)}}^2 \int_{\text{halo}} \frac{d^3 r}{4\pi |r|^2} \nonumber \]  
\[
\simeq \left( \frac{n'_{\text{KK}}}{n_{\text{KK}}} \right)_{\text{halo}} \cdot \left( 6 \times 10^{-26} \text{ km}^2 \text{ yr}^{-1} \right) \cdot \alpha_{\text{KK}} \xi_{\text{KK(halo)}}^2 \left( \frac{\hat{P}_{\text{KK}}}{10^{21} \text{ eV}} \right)^{-4} \nonumber \]  

\[ \tag{6} \]

Compared with Eqs. (3) and (4), the above formula contains an extra factor \( \left( \frac{n'_{\text{KK}}}{n_{\text{KK}}} \right)_{\text{halo}} \) that is the ratio of the over-density \( n'_{\text{KK}} \) in each clump to the mean density \( n_{\text{KK(halo)}} \) in the halo. This factor can enhance the flux substantially if the sub-clumps of KK modes are over-dense significantly.

The required value of the flux \( F \) in Eq. (2) gives a lower bound to the ratio \( \left( \frac{n'_{\text{KK}}}{n_{\text{KK}}} \right)_{\text{halo}} \) and the corresponding energy density of KK modes in each clump in the halo, \( (\rho'_{\text{KK(halo)}}) \) [i.e. \( (n'_{\text{KK}}\hat{P}_{\text{KK}})_{\text{halo}} \)], as follows:

\[
\left( \frac{n'_{\text{KK}}}{n_{\text{KK}}} \right)_{\text{halo}} \quad (10^{21} \text{ eV}) \simeq 7 \times 10^{24} \cdot \alpha_{\text{KK}}^{-1} \xi_{\text{KK(halo)}}^{-2} \geq 7 \times 10^{24}, \quad (7a) \\
\left( \frac{n'_{\text{KK}}}{n_{\text{KK}}} \right)_{\text{halo}} \quad (10^{23} \text{ eV}) \simeq 10^{34} \cdot \alpha_{\text{KK}}^{-1} \xi_{\text{KK(halo)}}^{-2} \geq 10^{34}, \quad (7b) 
\]
or, equivalently, in unit of \( g \text{ cm}^{-3} \),

\[
\begin{align*}
\rho'_{kk}^{(10^{21} \text{eV})} & \simeq 4 \cdot \alpha_{kk}^{-1} \xi_{kk(halo)}^{-1} \geq 4, \\
\rho'_{kk}^{(10^{23} \text{eV})} & \simeq 6 \times 10^9 \cdot \alpha_{kk}^{-1} \xi_{kk(halo)}^{-1} \geq 6 \times 10^9,
\end{align*}
\]

(8a)

(8b)

where the superscripts denote the conditions for \( \hat{P}_{kk} \).

In the same way, the contribution from the clumps of KK modes in the GZK zone (excluding the effect of the overdensity in the local halo) is:

\[
F_{GZK} \simeq \left( \frac{n_{kk}'}{n_{kk}} \right)_{GZK} \cdot \left( \frac{7 \times 10^{-33}}{\text{km}^2 \text{ yr}} \right) \cdot \alpha_{kk} \xi_{kk(GZK)}^2 \left( \frac{\hat{P}_{kk}}{10^{21} \text{eV}} \right)^{-4},
\]

(9)

where \( (n_{kk}'/n_{kk})_{GZK} \) is the ratio of the overdensity in each clump to the mean density in the GZK zone. The requirement in Eq. (2) gives a lower bound to the ratio \( (n_{kk}'/n_{kk})_{GZK} \) and the corresponding energy density of KK modes in each clump in the GZK zone, \( \rho'_{kk}^{GZK} \) [i.e. \( (n_{kk}' \hat{P}_{kk})_{GZK} \)], as follows:

\[
\begin{align*}
(n_{kk}'/n_{kk})^{(10^{21} \text{eV})}_{GZK} & \simeq 6 \times 10^{31} \cdot \alpha_{kk}^{-1} \xi_{kk(GZK)}^{-2} \geq 6 \times 10^{31}, \\
(n_{kk}'/n_{kk})^{(10^{23} \text{eV})}_{GZK} & \simeq 10^{41} \cdot \alpha_{kk}^{-1} \xi_{kk(GZK)}^{-2} \geq 10^{41},
\end{align*}
\]

(10a)

(10b)

or, equivalently, in unit of \( g \text{ cm}^{-3} \),

\[
\begin{align*}
\rho'_{kk}^{(10^{21} \text{eV})}_{GZK} & \simeq 200 \cdot \alpha_{kk}^{-1} \xi_{kk(GZK)}^{-1} \geq 200, \\
\rho'_{kk}^{(10^{23} \text{eV})}_{GZK} & \simeq 3 \times 10^{11} \cdot \alpha_{kk}^{-1} \xi_{kk(GZK)}^{-1} \geq 3 \times 10^{11}.
\end{align*}
\]

(11a)

(11b)

The number density ratios in Eqs. (7) and (10) are not so large as they look like. For comparison, we list in the following table the minimal values of the above ratios as well as the nucleon number density ratios of several stellar objects (the sun, the earth, white dwarfs, and neutron stars) to the background (the galactic disk and the universe).

| in unit of: | \( n_{kk(halo)} \) | \( n_{kk(GZK)} \) |
|-------------|-----------------|-----------------|
| \( n_{kk}'(\hat{P}_{kk}=10^{21} \text{eV}) \) | \( 7 \times 10^{24} \) | \( 6 \times 10^{31} \) |
| \( n_{kk}'(\hat{P}_{kk}=10^{23} \text{eV}) \) | \( 10^{34} \) | \( 10^{41} \) |

| in unit of: | \( n_{N} \text{(disk)} \) | \( n_{B} \text{(universe)} \) |
|-------------|-----------------|-----------------|
| \( n_{N} \text{(sun)} \) | \( 1-4 \times 10^{23} \) | \( 3-8 \times 10^{30} \) |
| \( n_{N} \text{(earth)} \) | \( 0.4-2 \times 10^{24} \) | \( 1-3 \times 10^{31} \) |
| \( n_{N} \text{(white dwarf)} \) | \( 10^{24.5-10^{32}} \) | \( 10^{32-10^{39}} \) |
| \( n_{N} \text{(neutron star)} \) | \( 10^{33-10^{38}} \) | \( 10^{40-10^{45}} \) |
\[ n_N(\text{disk}) = 2 - 7 \text{ cm}^{-3} \]
\[ n_B(\text{universe}) = 1.1 - 2.6 \times 10^{-7} \text{ cm}^{-3} \]

Here \( n_N(\text{disk}) \) is the nucleon number density of the galactic disk, and \( n_B \) is the baryon number density of the universe. We can see that, in both the case of the local galaxy and the case of the GZK zone, the minimal value of the ratio \( n'_{kk}/n_{kk} \) is comparable to the nucleon number density ratio of the earth or a low-density white dwarf to the background (the galactic disk or the universe) for \( \hat{P}_{kk} = 10^{21} \text{ eV} \), and is comparable to that of a low-density neutron star for \( \hat{P}_{kk} = 10^{23} \text{ eV} \). This is a hint that the detected UHECRs are generated via KK bursts within some sorts of stellar compact objects consisting of KK modes: “KK STARS”, “KK DWARFS”, “KK MACHOS” (massive compact halo objects made of KK modes), etc.

This hint is also provided by comparing energy densities in the following table.

| \( \rho'_{kk} \) (\( 10^{21} \text{eV} \)) | \( \rho_{\text{halo}} \) (\( 10^{21} \text{eV} \)) | \( \rho_{\text{GZK}} \) (\( 10^{23} \text{eV} \)) | \( \rho_{\text{halo}} \) (\( 10^{23} \text{eV} \)) | \( \rho_{\text{GZK}} \) (\( 10^{23} \text{eV} \)) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| sun             | earth           | white dwarf     | neutron star    |                 |
| \( \rho_{\text{mean}} \) | 1.4             | 5.5             | 30–10^{9}       | 10^{10}–10^{15} |

This table indicates that for \( \hat{P}_{kk} = 10^{21} \text{ eV} \) the minimal energy density of clumped KK modes is comparable to the mass density of the earth or a low-density white dwarf, and for \( \hat{P}_{kk} = 10^{23} \text{ eV} \) it increases to the value comparable to that of a high-density white dwarf or a low-density neutron star.

In addition to KK stellar compact objects, the production of UHECRs via KK bursts of clumped KK modes may also occur within much smaller compact objects, even as tiny as KKONIUM.

### C. KKONIUM

During a KK burst within KKONIUM, two originally bound KK modes get a large amount of kinetic energy from extra dimensions and become two unbound, extremely energetic KK zero modes. For a three-dimensional observer, this process looks like the decay of a superheavy particle (of mass about \( 2\hat{P}_{kk} \) for the lightest KKONIUM). The possibility that UHECRs are produced at the decays of metastable superheavy particles has been proposed.
by Berezinsky, Kachelrieß and Vilenkin [8] and by Kuzmin and Rubakov [9]. The decay spectrum was calculated by Birkel and Sarkar [14] using HERWIG program. (For further calculations using DGLAP evolution equations, see [15].) If these superheavy particles account for the dark matter in the local halo and their decays are responsible for the detected UHECRs, their mass $M_X$ should be within $10^{21} - 10^{23}$ eV and the corresponding decay width $\Gamma_X$ is:

$$\Gamma_X^{-1} = 10^{20} \text{ yr for } M_X = 10^{21} \text{ eV}, \quad (12a)$$

$$\Gamma_X^{-1} = 8 \times 10^{20} \text{ yr for } M_X = 10^{23} \text{ eV}. \quad (12b)$$

From the above information and the relation between the decay width $\Gamma$, the mass $M$ and the characteristic size $R$ of a kKonium,

$$\Gamma \sim M^{-2} R^{-3}, \quad (13)$$

we can estimate the requisite value of $R$ for the case where the decays of kKonium account for the generation of UHECRs. For kKonium systems consisting of KK$^{\pm 1}$ modes, we obtain

$$R \sim 10^{-5} \alpha_{kk}^{2/3} \text{ cm for } \hat{P}_{kk} = 10^{21}/2 \text{ eV}, \quad (14a)$$

$$R \sim 10^{-6} \alpha_{kk}^{2/3} \text{ cm for } \hat{P}_{kk} = 10^{23}/2 \text{ eV}. \quad (14b)$$

The above sizes are much larger than the Bohr radius of kKonium, which is characterized by the inverse of the KK momentum. Consequently kKonium as a UHECR source should be in a highly excited state.

For the other case in which kKonium systems contribute only a tiny fraction of dark matter, the requisite decay width should be much larger and accordingly the size of kKonium should be much smaller. This issue is of our great interest and is under investigation.

**IV. DISCUSSION AND SUMMARY**

In this paper the feasibility of generating UHECRs via KK bursts has been discussed. Basically we make use of extra dimensions to overcome difficulties in UHECR models. The most essential difficulty is the energy, its unusual largeness under the energy pillage by the GZK-like mechanisms. Regarding the unusual largeness of energy, the violent energy transfer
from extremely small extra dimensions to ordinary dimensions through KK bursts can easily reach such high energies. Against the energy pillage, extra dimensions, as benefiting from KK momentum conservation, can protect the energy therein when KK (non-zero) modes travel through the universe. This feature is of great benefit to the bottom-up models because it can help particles of large energies (in the form of KK momentum) escape from the source without losing much energy and make it possible for UHECR sources (more precisely, sources of KK modes and kKonium that generate UHECRs within the GZK zone) to be located outside the GZK zone. In this regard, KK modes are messengers of ultrahigh energies for UHECRs generated within the GZK zone, and accordingly they play a similar role to the neutrinos in the Z-burst model [10, 11, 12]. In addition, KK bursts might also be an origin of extremely-high-energy neutrinos invoked in the Z-burst model. This is an interesting possibility worthy of further studies.

In contrast to smoothly distributed KK modes that produce too few UHECRs, clumped KK modes, of various sizes from KK stellar compact objects to those as micro as kKonium, are possible UHECR origins. In this regard, KK bursts provide a possibility, without introducing new particle species, of realizing both UHECR models invoking annihilation of superheavy particles [7] and those invoking decays [8, 9]. For the consistency with the observed spectrum, the minimal energy density of KK stellar objects should range from the mass density of a low-density white dwarf to that of a low-density neutron star for the corresponding size of extra dimensions ranging from \(10^{-27}\) cm to \(10^{-25}\) cm. In addition, kKonium should be in highly excited states with sizes nearly twenty orders of magnitude larger than its Bohr radius. The detailed properties of the kKonium decay and the resultant UHECR spectrum are important issues for a full understanding of kKonium-originated UHECRs, and therefore merit further investigations.

A homogeneous distribution of clumped KK modes within the GZK zone can explain the isotropic arrival directions. In contrast, UHECRs from clumped KK modes with an isotropic distribution in the galaxy should be anisotropic: more from the side of the galactic center, while less from the other side. Furthermore, UHECRs from KK compact objects should exhibit the small-scale clustering in arrival directions, which has been indicated by AGASA data [16, 17, 18], and maybe also the dispersion in arrival times of these clustering events. These are possible characteristic signatures of KK compact objects.

In this work we adopt QCD fragmentation to play a role in the production of UHECRs...
after KK bursts. UHECRs originating from such fragmentation are mostly photons and neutrinos, which seem to be disfavored by the present data. Nevertheless, in the KK burst scenario QCD fragmentation is not a necessary ingredient if the KK modes involved are neither quarks nor gluons. It is possible that KK modes can maintain their identities in particle species during KK bursts, while only transferring momentum from extra dimensions to ordinary dimensions. In this case, different species of KK modes in KK bursts provide different composition of UHECRs. For clarification, possible composition of KK modes and other fragmentation processes are under investigation.

To sum up, the KK burst model of UHECRs invokes extra dimensions as a good energy bearer against the energy pillage occurring within powerful sources of ultrahigh-energy particles and in the journey to the earth, thereby benefiting the bottom-up models. KK bursts can generate extremely energetic particles that result in observed UHECR spectrum, thereby making it possible to build a top-down model without introducing new particle species. In particular, the KK burst model simultaneously realizes the following three scenarios for the origin of UHECRs:

(a) **Decays of superheavy particles** [8, 9], which, in the KK burst model, are played by **kkonium**.

(b) **Annihilation of superheavy particles** [7], which, in the KK burst model, are played by KK modes and anti-KK modes. (Note that the particles involved in a KK burst need not to be a pair of a particle and an anti-particle.)

(c) **Ultrahigh energies carried by messengers** (e.g. the Z-burst model [10, 11, 12]), which, in the KK burst model, are played by KK modes and kkonium.

Thus, from both the bottom-up and the top-down viewpoints, the KK burst model presents a new feasible way to understanding the origin of UHECRs. On the other hand, UHECRs may encode information about extra dimensions, and therefore might be a good probe of, not only the ultrahigh-energy part, but also the extra-dimensional part of the world.

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