Error Analysis of High-time-resolution Na Lidar Data and Power Spectrum Density of Mesospheric Na Layer

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Abstract. The observed 6-s Na layer data at Beijing (40.2°N, 116°E), China was conducted error analysis and compared with 5-minute Na Lidar data located at Wuhan (30.5° N, 114.4° E), China. It was found that 6-s time resolution fluctuations/change rates of Na atomic density were geophysical phenomenon rather than measurement error. The relative measurement errors between Beijing and Wuhan are comparable around 90 km. Based on the long-duration (longer than 10 hours in an observational night) 6-s Na profiles, power spectral density (PSD) of the Na density fluctuations has been calculated. There exist strong fluctuations at frequencies larger than the typical value (2.5×10⁻³ Hz, equivalent to a period of ~5 min) of the Brunt-Väisälä frequency in the mesopause region.

1. Introduction
Free metal atom layers exist in the mesosphere and lower thermosphere (MLT) region from 80 to 110 km is widely believed to be originated from meteor ablation [2, 3, 6, 7, 10]. Every day, many tons of microgram-sized extraterrestrial particles with velocities between 11 and 72 km/s enter and are ablated in the upper layers of the earth’s atmosphere, which give rise to a globally-covered gas-phase metal atom layer in the mesopause region.

The study of atmospheric turbulence spectra is very important for two aspects. First, the sources and sinks of energy and enstrophy and nonlinear interactions are shown on the shape of the power spectrum in a statistical sense. Second, the limits of predictability of the atmospheric state depend on the form of the power law. Direct measurements have been carried out on turbulence in the upper troposphere and middle atmosphere by balloons, aircrafts, and rockets. However, the observations are sporadic in time, making it difficult to extract temporal power spectra features of turbulence in atmosphere. With continuous and high time resolution Na observation data, temporal Power Spectral Density (PSD) for turbulence in MLT region during only one night has been got by earlier works [4, 9].

2. Geophysical phenomenon or error
Figure 1 shows a time/altitude contour of Na density observed from 18:53 to 19:03 on the night of 2 October 2011 with 6 s time resolution. Na density variation near 92 km in this contour is apparently composed of a series of fluctuations during a few seconds. In order to illustrate the fluctuations/change rates of Na atomic density presented above being geophysical phenomenon rather than measurement error of our lidar. A sequence of Na density profiles is taken out from 18:53 to 19:03 during the night of 2 October 2011, which consists of 60 profiles covering 10 minutes.
Fig. 1. Time/altitude contour of Na density observed from 18:53 to 19:03 during the night of 2 October 2011 with 6 s time resolution.

Figure 2 shows six out of the 60 profiles together with measurement errors (horizontal red bars). Firstly, pay attention to the Na density variations in the circles of a, b, c, d, e and f. They all cover 1.6 km range containing more than 16 sequence vertical observational points and increase/decrease as a whole from a to b, c to d and e to f. Secondly, there gradually appears a density concave and then moves downward from 87 to 88 km. Such big vertical ranges of density enhancement or depletion as a whole and downward concave during the same time wouldn’t be caused by measurement errors.

Fig. 2. Consecutive profiles of Na density from 19:01:09 to 19:01:45 on 2 October 2011 (Short red bars represent measurement errors on the corresponding altitudes).

So as to further confirm fluctuations on a fixed altitude present geophysical phenomenon rather than measurement errors, the 92-km Na atomic density and Na abundance together with measurement errors from 18:56:33 to 19:02:57 local time on 2 October 2011 were revealed. It found that Na density ranged from 3085 cm$^{-3}$ to 3670 cm$^{-3}$ and the Na abundance varied between 3.10×10$^9$ cm$^{-2}$ and 3.36×10$^9$ cm$^{-2}$ during the six minutes. Furthermore, the profile-to-profile variations that larger than the corresponding error bars are often discriminable (e.g., decay stage for b, d and growth stage for a, c and d in Figure 2) though they are comparable to the magnitude of the measurement errors.

In previous work, Chen and Yi [1] analyzed the simultaneous variations of common-volume Na and Fe lidar measurements with 5-min time resolution and found that both of them showed the same variation trend on the bottom layer, which means the magnitude of the variation rates is a measure of metal layer variability (regardless of local production/loss or advection effect of already-existing horizontal structure by large-scale wind) rather than experimental error since the Na and Fe lidars are two completely independent systems. They impossibly have very similar measurement noises. In order
to compare the backscattered photon counts and measurement errors between Beijing (6 s) and Wuhan (5 min) to verify our observation at Beijing is geophysical phenomenon rather than measurement errors by another aspect. With the same calculation method by Chen and Yi [1], the simultaneous and common-volume Na and Fe data during the night of 28 September 2012 over Wuhan is employed to analyze the fluctuations (reflected by change rates) of Na atomic density. The results are shown in Figure 3. Figure 3 (a) and (b) present contours of the observed Na and Fe atomic density variations, which obviously has the similar variation trend. Figure 3 (c) and (d) depict the calculated variation rates of Na and Fe layers together with time variations of their densities at 85 km during the same night. Fe has stronger variation rates compared with Na. the mean profiles of absolute and relative change rates for Na density during this night are shown in figure 3 (e) and (f), respectively. The mean increase/decrease rates during the night for Na have maxima of 1.1/0.9 cm-3 s-1 around 86.5 km, while the corresponding relative increase/decrease rates have minimum of 7.46×10-5/7.42×10-5 s-1 near 90.5 km. Furthermore, we calculate the characteristic time in terms of the data plotted on figure 3 (f) (which is obtained by the reciprocal of relative increase/decrease rate). The largest characteristic time for Na atomic density fluctuation is ~3.7 h, which in general accord with the mean result ~2 h by Chen and Yi [1].

![Fig.3. Simultaneous and common-volume Na and Fe lidar observations during the night of 28 September 2012 at Wuhan, China: (a) Contour plot of Na density versus time and altitude, (b) contour plot of Fe density, (c) temporal variations of Na and Fe densities at 85-km altitude, (d) temporal variations of the apparent change rates for Na and Fe at 85-km altitude, (e) profiles of the mean absolute increase and decrease rates together with the mean Na density profile, (f) profiles of the mean relative increase and decrease rates for Na.](image-url)

Most important of all, the comparison of Lidar echo photon counts and relative measurement errors on different altitudes between Wuhan and Beijing are shown in Figure 4. The mean lidar echo photon count profiles on the night of 2 October 2011 at Beijing with 6-s time resolution has the same magnitude with that at Wuhan on the night of 28 September 2012 with 5 min time resolution shown in
Figure 4 (a). Relative measurement errors in Figure 4 (b) between Beijing and Wuhan are comparable around 90 km. Since the results calculated with 5 min time resolution data over Wuhan are believable, fluctuations of Na observed at Beijing with 6 s time resolution might be geophysical phenomenon rather than measurement errors. The amount of fluctuations in the Na density change was expected to be greater at higher temporal resolution as less data was smoothed out unless saturation is reached. However, previous study by Salter et al. [8] showed that the experiments confirm the empirical observations. The laser fluorescence scattering from dense sodium vapour does not saturate as expected. Their experiments confirm that the fluorescence scattering continues to rise with increasing laser intensity even when the latter is well above the nominal saturation value. Furthermore, the higher power (150 Wm$^2$) and time resolution (20 ms) Na lidar by Pfrommer et al. [4, 5] did not reach the saturation.

![Fig.4.](image)

Fig.4. Comparison between the Na lidar measurements for the 6-s (made at Yanqing on 2 October 2011) and 5-min (at Wuhan on 28 September 2012) samplings: (a) lidar photon count profiles and (b) relative measurement error profiles.

3. Power spectral density of Na layer

Based on the long-duration (longer than 10 hours in an observational night) 6-s Na profiles, power spectral density (PSD) of the Na density fluctuations has been calculated at given altitudes. The raw spectrum is well fitted by the model

$$P(f) = a f^\beta + \gamma$$  (1)

where $f$ is the fluctuation frequency, and $\gamma$ represents the noise floor, which is treated as a free parameter [4]. For the purpose of the statistical significance, the power spectral densities derived from 13 nights of the long-duration lidar measurements are averaged and then fitted. The mean PSD together with their linear fittings are shown in Figure 5. The mean spectral indexes are -1.64±0.20, -1.49±0.19, -1.57±0.21 respectively in the three different altitude ranges of 86-88, 89-91 and 92-94 km between the frequencies of 3.38×10$^{-5}$ and 3.5×10$^{-3}$ Hz. The errors represent the standard deviation from the mean power spectra from the 13-night observations from September to November 2010. As seen in Figure 5, there exist strong fluctuations at frequencies larger than the typical value (2.5×10$^{-3}$ Hz, equivalent to a period of ~5 min) of the Brunt-Väisälä frequency in the mesopause region. Obviously, these strong small-scale Na-density fluctuations with periods less than the Brunt-Väisälä period are not due to atmospheric gravity waves. This is consistent with the magnitude of the characteristic time (~120 s at 90.5 km) estimated above. Note that the strong small-scale Na fluctuations are ubiquitous in the Na main layer according to our high-time-resolution lidar observations.
Fig. 5. Mean power spectrum densities (PSD) of Na density for three different altitude ranges (86-88, 89-91 and 92-94 km), derived from the 13 nights of long-duration Na lidar measurements at Wuhan. The straight lines denote the linear fittings to the spectral data.

4. Summary and conclusions

Rapid fluctuations in mesospheric Na layer density were observed with lidar measurements over Beijing(40.2°N, 116°E), China. Profiles of lidar measurement were taken with 1.5 s time resolution but integrated to 6 s resolution together with 96 m spatial interval. They were then conducted error analysis and compared with 5-minute Na Lidar data located at Wuhan (30.5° N, 114.4° E), China. It was found that 6-s time resolution fluctuations/change rates of Na atomic density were geophysical phenomenon rather than measurement error.

Power Spectrum Density (PSD) for thirteen nights (from September to November with observation period longer than 10 hours in one night) covering three altitude segments totally from 86 to 94 km with frequency from \(3.69 \times 10^{-5}\) to \(8.3 \times 10^{-1}\) Hz together with their fitting straight lines in MLT region was showed. The mean spectral indexes are \(-1.64 \pm 0.20\), \(-1.49 \pm 0.19\), \(-1.57 \pm 0.21\) respectively in the three different altitude ranges of 86-88, 89-91 and 92-94 km between the frequencies of \(3.38 \times 10^{-5}\) and \(3.5 \times 10^{-3}\) Hz.

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