Lotteries for Shared Experiences

NICK ARNOSTI, University of Minnesota
CARLOS BONET, Columbia University

We consider a model with $k$ identical tickets. The set of agents $N$ is partitioned into a set of groups, and agents have dichotomous preferences: an agent is successful if and only if members of her group receive enough tickets for everyone in the group. We treat the group structure as private information, unknown to the designer. Because there are only $k$ tickets, there can be at most $k$ successful agents. We define the efficiency of a lottery allocation to be the expected number of successful agents, divided by $k$. If this is at least $\beta$, then the allocation is $\beta$-efficient. A lottery allocation is fair if each agent has the same success probability, and $\beta$-fair if for any pair of agents, the ratio of their success probabilities is at least $\beta$.

Given these definitions, we seek lottery allocations that are both approximately efficient and approximately fair. Although this may be unattainable if groups are large, in many cases group sizes are much smaller than the total number of tickets. We define a family of instances characterized by two parameters, $\kappa$ and $\alpha$. The parameter $\kappa$ bounds the ratio of group size to total number of tickets, while $\alpha$ bounds the supply-demand ratio. For any $\kappa$ and $\alpha$, we provide worst-case performance guarantees in terms of efficiency and fairness.

We first consider a scenario where applicants can identify each member of their group. Here, the mechanism typically used is the Group Lottery, which orders groups uniformly at random and processes them sequentially. We show that this mechanism incentivizes agents to truthfully report their groups. Moreover, we prove that the Group Lottery is $(1-\kappa)$-efficient and $(1+2\kappa)$-fair. It is not perfectly efficient, as tickets might be wasted if the size of the group being processed exceeds the number of remaining tickets. It is not perfectly fair, since once only a few tickets remain, a large group can no longer be successful, but a small group can. Furthermore, we show that these guarantees are tight.

Could there be a mechanism with stronger performance guarantees than the Group Lottery? We answer this question by establishing the limits of what can be achieved. Specifically, there always exists an allocation $\pi$ that is $(1-\kappa)$-efficient and fair, but for any $\epsilon > 0$, there are examples where any allocation that is $(1-\kappa+\epsilon)$-efficient is not even $\epsilon$-fair. To show the existence of the random allocation $\pi$, we use a generalization of the Birkhoff-von Neumann theorem proved by Nguyen et al. [1]. By awarding groups according to the allocation $\pi$, we can obtain a mechanism that attains the best possible performance guarantees. Therefore, the $2\kappa$ loss in fairness in the Group Lottery can be thought of as the “cost” of using a simple procedure that orders groups uniformly, rather than employing a Birkhoff-von Neumann decomposition to generate the allocation $\pi$.

In many applications, developing an interface that allows applicants to list their group members may be too cumbersome. This motivates the study of a second scenario, where applicants are only allowed to specify the number of tickets they need. The natural mechanism in this setting is the Individual Lottery. Unfortunately, we show that the Individual Lottery may lead to arbitrarily inefficient and unfair outcomes. It is perhaps not surprising that the Individual Lottery will be inefficient if agents request more tickets than needed, or if each agent has a large chance of success. However, we show that the waste due to over-allocation may be severe even if all agents request only their group size and demand far exceeds supply. Furthermore, because the probability of success will be roughly proportional to group size, small groups are at a significant disadvantage.

Can we achieve approximate efficiency and fairness without asking applicants to identify each member of their group? We show that this is possible with a minor modification to the Individual Lottery which gives applicants with larger requests a lower chance of being allocated. This eliminates the incentive to inflate their group size, we show that this is possible with a minor modification to the Individual Lottery which gives applicants with larger requests a lower chance of being allocated.
Table 1. Summary of main results: worst-case guarantees for the efficiency and fairness of instances in $I(\kappa, \alpha)$.

| Mechanism                  | Action Set   | Efficiency     | Fairness        |
|----------------------------|--------------|----------------|-----------------|
| Benchmark                  |              | $1 - \kappa$   | 1               |
| Group Lottery              | $2^N$        | $1 - \kappa$   | $1 - 2\kappa$   |
| Individual Lottery         | $\{1, 2, \ldots, k\}$ | 0              | 0               |
| Weighted Individual Lottery| $\{1, 2, \ldots, k\}$ | $1 - \kappa - \alpha/2$ | $1 - 2\kappa - \alpha/2$ |

demand, and reduces the possibility of multiple winners from the same group. To make the allocation fair, we choose a particular method for biasing the lottery against large requests: sequentially select individuals with probability inversely proportional to their request. We call this approach the Weighted Individual Lottery. In the Weighted Individual Lottery, a group of four individuals who each request four tickets has the same chance of being drawn next as a group of two individuals who each request two tickets. As a result, outcomes are similar to the Group Lottery. We prove that the Weighted Individual Lottery is $(1 - \kappa - \alpha/2)$-efficient and $(1 - 2\kappa - \alpha/2)$-fair (in fact, we provide slightly stronger guarantees). Notice that these guarantees coincide with those of the Group Lottery when demand far exceeds supply ($\alpha$ is close to 0).

Our main results are summarized in Table 1. Our conclusion is that the Individual Lottery can be arbitrarily unfair and inefficient. These deficiencies can be mostly eliminated by using a Group Lottery. Perhaps more surprisingly, approximate efficiency and fairness can also be achieved while maintaining the Individual Lottery interface, by suitably biasing the lottery against agents with large requests.

The full paper is available at: https://arxiv.org/abs/2205.10942

CCS Concepts: • Theory of computation → Algorithmic mechanism design; Design and analysis of algorithms.

Additional Key Words and Phrases: market design, mechanism design, lottery mechanisms

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