Forward Modeling of Magnetic Field Measurements at the Bases of Stellar Coronae through Extreme-ultraviolet Spectroscopy

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Received 2022 June 26; revised 2022 August 23; accepted 2022 September 12; published 2022 October 10

Abstract

Measurements of the stellar coronal magnetic field are of great importance for understanding stellar magnetic activity, yet such measurements have been extremely difficult to obtain. Recent studies proposed a new method of obtaining magnetic field measurements based on the magnetic-field-induced transition (MIT) of the Fe X ion. Here we construct a series of stellar coronal magnetohydrodynamics models and synthesize several Fe X emission lines at extreme-ultraviolet wavelengths, and then diagnose the magnetic field strength at the bases of the coronae using the MIT technique. Our results show that the technique can be applied to some stars with magnetic fields more than three times higher than that of the Sun at solar maximum. Furthermore, we investigate the uncertainty of the derived magnetic field strength caused by photon counting errors and find that a signal-noise ratio of ~50 for the Fe X 175 Å line is required to achieve effective measurements of the stellar coronal magnetic field.

Unified Astronomy Thesaurus concepts: Magnetohydrodynamics (1964); Magnetohydrodynamical simulations (1966); Stellar coronae (305); Stellar magnetic fields (1610); Space weather (2037)

1. Introduction

The magnetic activity of host stars plays a pivotal role in determining the habitability of exoplanets (e.g., Aigrain et al. 2017; Linsky 2019). Stellar magnetic fields are the energy source for a variety of explosive magnetic activities including superflares (e.g., Maehara et al. 2012) and coronal mass ejections (e.g., Argiroffi et al. 2019; Namekata et al. 2021; Veronig et al. 2021). These events could significantly affect the physical properties and chemical composition of exoplanetary atmospheres through various processes (e.g., Cohen et al. 2014; Cherenkov et al. 2017; Tilley et al. 2019; Hazra et al. 2020; Chen et al. 2021a; Alvarado-Gómez et al. 2022), which has put our understanding of stellar magnetic fields at the core of investigating the habitability of exoplanets.

Despite such importance, there have been only limited ways to measure stellar magnetic fields. Most previous attempts only measured the stellar photospheric magnetic field based on the Zeeman effect of spectral lines (e.g., Johns-Krull & Valenti 2000; Donati et al. 2008; Morin et al. 2008; Reiners 2012; Kochukhov & Lavai 2017; Kochukhov & Shulyak 2019). Compared with the photospheric magnetic field, the coronal magnetic field is even more difficult to measure, which is the case for both the Sun and other stars (e.g., Yang et al. 2020a, 2020b; Jiang et al. 2022; Zhu et al. 2022). There have been several attempts to make stellar coronal magnetic field measurements based on radio observations (e.g., Gary & Linsky 1981; Mutel et al. 1985; Güdel 2002), which could be subject to uncertainties because the radio emission mechanisms are often not easy to determine. Indirect reconstructions of the stellar coronal magnetic field may be achieved through extrapolation from a photospheric magnetogram obtained with Zeeman–Doppler imaging (ZDI; see Semel 1989; Hebrard et al. 2016) technique. However, this method can only reveal large-scale field structures, and uncertainties exist in both the extrapolation and ZDI techniques due to assumptions.

Recently, Li et al. (2015) and Li et al. (2016) proposed that the intensity ratios of several Fe X lines at extreme-ultraviolet wavelengths can be used to measure the solar corona (SC) magnetic field strength based on the magnetic-field-induced transition (MIT) theory. Chen et al. (2021b) validated this technique for magnetic field measurements of solar active regions through forward modeling. Besides, this technique has been applied to actual spectral observations of solar active regions (e.g., Landi et al. 2020, 2021; Si et al. 2020; Brooks & Yardley 2021). Inspired by these studies, our previous work (Chen et al. 2021c) suggested that the MIT technique can be extended to measurements of the magnetic field strengths at the coronal bases of active stars in which the average photospheric magnetic flux density is at least one order of magnitude larger than that of the Sun. However, in our previous study the input photospheric magnetograms of our models were scaled from a solar photospheric magnetogram taken during a relatively inactive phase of the solar cycle, which may not be appropriate for investigations of magnetic fields on active stars or solar-type stars at the peak of their long-term activity cycles (e.g., Wilson 1978; Bondar 2019).

In this paper, we start from a solar photospheric magnetogram taken during solar maximum, and construct a series of stellar coronal models to explore the capability of the MIT technique in stellar coronal magnetic field measurements. We also investigate what signal-to-noise ratio (S/N) is required to achieve effective measurements of the field strength. Section 2 describes the models used in this study, line synthesis, and methodology. Our magnetic field diagnostic results are
presented in Section 3. In Section 4 we investigate the uncertainties involved in this technique. Conclusions are given in Section 5.

2. Models and Emission Line Synthesis

2.1. Models

A series of stellar coronal models were constructed based on the Alfvén Wave Solar Model (AWSoM; van der Holst et al. 2014) within the Space Weather Modeling Framework (SWMF; Tóth et al. 2012). In this study, we only used the SC component since the emission of Fe X lines is mainly formed in the lower corona or coronal bases of stellar atmospheres. This model assumes that an Alfvén wave is injected at the inner boundary and the corona is heated through Alfvén wave turbulence dissipation. The model also considers the radiative cooling effect and electron heat conduction.

The inner boundary of the model was specified as the top of the chromosphere, where the electron temperature and number density were set to $5 \times 10^6$ K and $2 \times 10^{17}$ m$^{-3}$, respectively. The Poynting flux $S_A$ of the outgoing wave at the inner boundary is proportional to the radial magnetic field strength. This setting was made to satisfy the power-law relationship between the total coronal heating power and unsigned magnetic flux (Pevtsov et al. 2003; Sokolov et al. 2013). For this reason, the radial magnetic field distribution at the inner boundary is required as input. As ZDI maps can reveal the large-scale magnetic field in the stellar photosphere, they have been chosen as input in some stellar simulations (e.g., Cohen et al. 2014; Alvarado-Gómez et al. 2018, 2019). However, small-scale field structures are missed in ZDI maps, which would inevitably lead to an underestimation of the heating rate in the lower corona. To avoid this problem, in this work we used a synoptic solar magnetic field map obtained by the Global Oscillation Network Group to resolve small-scale field structures. The magnetograms for the stellar models are obtained through increasing the magnetic flux density by different factors.

In Chen et al. (2021c) we used a synchronous field map centered at 2011 February 15, which corresponds to Carrington Rotation (CR) 2106 (during the rising phase of solar cycle 24). In this study, we used the synoptic map of CR 2155 taken from 2014 September 17 to October 14 during solar maximum. Both maps have a resolution of 1° at the disk center. The two magnetograms are shown in Figure 1 for comparison. We first constructed a solar CR 2155 model with the original CR 2155 magnetogram, then rescaled the CR 2155 magnetogram by different factors ranging from 2 to 9 and constructed a series of stellar coronal models. Each of the models is named “xN”, in which “N” is the rescaling factor (e.g., x1 represents the solar model). The transverse correlation length ($L_x$) of the Alfvén waves is proportional to $1/\sqrt{B}$ (Hollweg 1986), which means $L_x \sqrt{B}$ is a constant in each model. In the x5, x6, x7, x8, and x9 models, $L_x \sqrt{B}$ was set to the default value of $1.5 \times 10^5$ m$\sqrt{T}$. For models with weaker magnetic fields (the x1, x2, x3, and x4 models), to keep the models stable $L_x \sqrt{B}$ was adjusted to $3.0 \times 10^5$ m$\sqrt{T}$. This ensures that the energy of an Alfvén wave is dissipated in a wider range of space, which can prevent the devastating accumulation of energy near the inner boundary and chromospheric evaporation. The Alfvén wave Poynting flux at the inner boundary is assumed to be proportional to the radial magnetic field, and the proportionality $(S_A/B)_0$ is another important parameter in the models. In the solar case, Sokolov et al. (2013) estimated $(S_A/B)_0$ to be $1.1 \times 10^6$ W m$^{-2}$. Here we assume that $(S_A/B)_0$ in the stellar models is the same as in the solar model. This leads to more energy being injected into the corona in the more active models. For more detailed descriptions of the numerical setups, we refer the reader to Jin et al. (2012) and Oran et al. (2013).

2.2. Emission Line Synthesis

Similar to Chen et al. (2021b), we first synthesized the emissions of the Fe x 174, 175, 177, 184, and 257 Å lines from our models. The contribution functions were calculated using the CHIANTI code together with the atomic data from the CHIANTI database (version 10.0; Dere et al. 1997; Del Zanna et al. 2021), in which the collision data originate from Del Zanna et al. (2012). For the radiative transition data, we used the results obtained from the multiconfiguration Dirac–Hartree–Fock and relativistic configuration interaction calculations by Wang et al. (2020) and Li et al. (2021). The MIT transition rates as a function of magnetic field strengths were obtained using the values given by Li et al. (2021). We first created the contribution functions for the related Fe X lines at a large number of electron temperatures ($\log T/K$, from 5.6 to 6.6 with an interval of 0.004), electron densities ($\log N/cm^{-3}$, from 6 to 15 with an interval of 0.04), and magnetic field strengths ($B/$Gauss, from 0 to 6500 with an interval of 10 Gauss). We then interpolated the contribution functions to the $\log T/K$, $\log N/cm^{-3}$, and $B/G$ values of the grid points in the models, and synthesized the spectral line emissions from different locations on the stars. Figure 2 presents the Fe X 174 Å images synthesized from different models by assuming that the line of sight (LOS) is along the $+$x-axis. We also calculated the average temperature and intensity as a function of height in each model.
The most significant difference among the models is the different coronal temperatures resulting from different heating rates. In the x1 (solar) model, the temperature reaches $10^{6.2-6.3}$ K at a height of 30 Mm. In those active stellar models, the more intense heating at the inner boundary leads to higher coronal temperatures (e.g., $10^{6.5}$ MK in the x9 model). Because of this, the emissions at higher heights in the models with stronger magnetic fields are considerably weakened, due to the fact that

Figure 2. Fe X 257 Å intensity images assuming that the LOS is along the +x-axis in the different models. The intensity images are shown in logarithmic scale and arbitrary units. The +z-axes are parallel to the stellar spin axis. The average temperature (black curve) and intensity (red curve) as a function of height above the stellar surface in each model are also shown. The vertical dashed line represents the emissivity-weighted height, as defined in Equation (1).
the contribution functions of the Fe X lines peak at a temperature of \( \sim 1 \) MK. We defined the average formation height of the Fe X 174 Å emission line as the emissivity-weighted height:

\[
H_0 = \frac{\int_{V_0} H \cdot e_{174} dV}{\int_{V_0} e_{174} dV},
\]

where \( e_{174} \) is the emissivity of the Fe X 174Å line and \( V_0 \) represents the whole simulation box. The average formation height of the 174Å line in each model is shown as a vertical dashed line in Figure 2. In all models, the average formation height is below 11 Mm, which indicates that the integrated intensities of the FeX emissions mainly come from the low coronae or coronal bases.

To mimic different viewing angles, we selected a series of LOS directions every 30° in both the inclinational and azimuthal directions. Since the stars are spatially unresolved, the emissivity of a spectral line was integrated over the whole domain except the far side of the stellar disk. In this way, we obtained the intensities of different Fe X lines and line ratios for each LOS. As predicted by MIT theory, the intensity ratio between the FeX 257 Å line and another FeX reference line (e.g., the 174, 177, or 184 Å line) is a function of electron temperature \( T \), density \( N \), and magnetic field strength \( B \). In this study, we chose a fixed temperature of 10^6.02 K, at which the contribution functions of the Fe X lines peak (Li et al. 2021). The electron density was derived from the intensity ratio of the density-sensitive 175/174 Å line pair (Brosius et al. 1998; Del Zanna & Mason 2018). After these, the magnetic field strength \( B_1 \) was derived from the intensity ratios of the Fe X lines. Another MIT diagnosing method called “weak field strength technique” was introduced and used by Landi et al. (2020) to measure the relatively weak magnetic field strength (i.e., below \( \sim 150\)–200 Gauss) in the SC. However, since most of the measured magnetic field strengths of our models exceed the upper limit of the measurable range (as one will see in Figure 3), this method is not considered in this study.

To compare measured values with the values in the models, the average magnetic field strength in each model is needed. Since the emission lines only carry information of the magnetic field in the source regions of the FeX emissions, the average magnetic field strength in the simulation domain should be weighted by the FeX emissivity. Therefore, we use the FeX 174 Å line emissivity-weighted average field strength, i.e.:

\[
B_0 = \frac{\int_{V(i)} B \cdot e_{174} dV}{\int_{V(i)} e_{174} dV},
\]

Figure 3 shows a comparison of the magnetic field strengths calculated using the intensity ratios of the Fe X 257/174 Å, 257/177 Å, and 257/184 Å line pairs (\( B_1 \)) and values in the source regions. Here \( i \) is the \( i_{th} \) LOS direction, and \( V(i) \) is the whole simulation domain excluding the far-side region for each LOS.

3. Magnetic Field Diagnostic Result

Figure 3 shows a comparison of the magnetic field strengths calculated using the intensity ratios of the Fe X 257/174 Å, 257/177 Å, and 257/184 Å line pairs (\( B_1 \)) and values in the source regions. Here \( i \) is the \( i_{th} \) LOS direction, and \( V(i) \) is the whole simulation domain excluding the far-side region for each LOS.
model ($B_0$). In each panel of the upper row, we present the results derived from the same line pair for all models. The black dashed line in each panel indicates $B_1 = B_0$, i.e., the magnetic field strength derived from the line ratio is equal to the value in the model. The ranges of the magnetic field strength in models x1–x9 ($B_0$) are 16–30, 34–71, 60–128, 86–169, 127–234, 163–300, 196–348, 228–394, and 259–441, Gauss, respectively. As a representation of the average magnetic strength in the source regions of the Fe X emissions, $B_0$ not only depends on the rescaling factor, but also depends on the distance from the stellar disk to the source regions, which is revealed by the formation height of Fe X. Thus, $B_0$ is not proportional to the rescaling factor since the formation height of the Fe X lines differs in the different models. The density values ($\log N$/cm$^{-3}$) along different LOS directions of the models, as calculated using the Fe X 175/174 Å line ratio, are 8.6–8.73, 8.74–8.91, 8.88–9.06, 8.94–9.11, 9.13–9.27, 9.21–9.34, 9.25–9.37, 9.28–9.4, and 9.3–9.41, respectively. For models with a surface magnetic flux density more than three times larger than the solar case (x3–x9 models), the MIT method was found to be capable of yielding reasonable estimations of the coronal magnetic field strengths for all LOS directions. However, this technique fails to provide reasonable estimations for almost all LOS directions in the x1 and x2 models, since the obtained intensity ratios of Fe X 257 Å and the reference lines do not fall in the range predicted by MIT theory, a phenomenon that is most likely due to the low densities ($N < 10^{8.8}$ cm$^{-3}$) in these two models (see a more detailed discussion in Chen et al. 2021c).

As one will see in Section 4, the Fe X 175 Å line used for the density measurements is relatively weaker compared to the other Fe X lines, which means that in actual observations the S/N ratio of the Fe X 175 Å line will be smaller. Landi et al. (2020) used the Fe XI 182.17/(188.22+188.30) Å intensity ratio as a substitute for the Fe X 175/174 Å line ratio to measure the density. To explore the suitability of this method for our stellar investigations, we calculated the contribution functions of the Fe XI 182.17, 188.22, and 188.30 Å lines, and synthesized their emissions in the model. We then used the Fe XI intensity ratio to measure the electron density. Figure 4 shows a comparison between the densities measured by the Fe X and Fe XI intensity ratios. The density measured by the Fe XI intensity ratio is only 69% of the density of the Fe X line pair on average. This is because the contribution functions of the Fe X lines peak at a higher temperature ($10^6.12$ K) compared to those of the Fe X lines, which means that the Fe X lines are formed at higher heights where the density is smaller. We also found that the derived magnetic field strength using the density from the Fe XI intensity ratio is underestimated by $\sim$200 Gauss compared with $B_0$, and the correlation between $B_1$ and $B_0$ is weaker compared to that using the density from the Fe X 175/174 Å line pair. Therefore, the magnetic field measurement result using the density from the Fe XI intensity ratio is not shown here. Below we still use the Fe X 175/174 Å line pair to measure the density.

From Figure 3, one can clearly see that $B_1$ monotonically increases with $B_0$ in general. However, one may also notice that $B_1$ deviates from $B_0$ for almost all the cases. The lower row of Figure 3 shows the frequency histograms of the relative error ($B_1 - B_0$)/$B_0$. In each histogram, ($B_1 - B_0$)/$B_0$ is around 0.4 on average. It is worth mentioning that $B_0$ is empirically defined, thus $B_0$ and $B_1$ are not theoretically identical. For this reason, the plots of $B_1$ versus $B_0$ only serve to reveal the general relevance of the measured field strengths with the real values. In order to investigate the actual physical meaning of the measured field strength $B_1$, we attempted to find out the region in which the emissivity-weighted average field strength is equal to $B_1$. To do this, we chose different emissivity thresholds $e_{th}$ and delimited the corresponding spaces $V'$ in which the emissivity is higher than $e_{th}$. For the $i_{th}$ LOS and an emissivity threshold of $e_{th}$, the emissivity-weighted average field strength within $V'$ is then defined as:

$$B_0(i, e_{th}) = \frac{\int_{V'(i,e_{th})} B \cdot e_{174} dV}{\int_{V'(i,e_{th})} e_{174} dV}. \quad (3)$$

The difference between Equation (2) and Equation (3) is that Equation (2) calculates the average magnetic field in the whole space, while Equation (3) calculates the average magnetic field only in regions with strong Fe X emissivity. One can derive $e_{th}$ by solving $B_0(i, e_{th}) = B_1(i)$. In Figure 5, we present $V'$ as gray isosurfaces above the photosphere for six different LOS directions in the x3, x6, and x9 models. Note that the radial extension of each isosurface is very small and the height of the isosurface varies with location, which cannot be shown in these two-dimensional images. Nevertheless, we can still see that in most of the cases the isosurface of $V'$ significantly overlaps with the active regions. Thus, it appears that the measured field strength largely reflects the average field strength in stellar active regions.

From the analyses above, we may conclude that the MIT method can be applied to stars with a field strength of at least 3 times larger than that of the Sun during solar maximum (i.e., stars with a $B_0$ of at least $\sim$60 Gauss). In Chen et al. (2021c), we suggested that the MIT method could be applied to stars with a field strength of at least 20 times larger than that of the Sun (i.e., stars with a $B_0$ of at least $\sim$160 Gauss). This discrepancy is mainly due to the different densities in the models used by the two studies. Chen et al. (2021c) found that...
the MIT measurability of the magnetic field strength significantly increases with the stellar coronal density. As a comparison, the x3 model in this study and the x10 model in the previous study both have an average $B_0$ of $\sim$80–90 Gauss, while the average measured coronal density of the former model ($10^{8.04}$ cm$^{-3}$) is more than 1.7 times larger than that of the latter model ($10^{8.80}$ cm$^{-3}$). This indicates that the stellar models in this study are generally denser in the corona as compared to the models in the previous study under the same magnetic field strength. The reason for such a difference is that compared to the CR 2106 magnetogram used by the previous study, the CR 2155 magnetogram in this study has more active regions, in which the density is higher than that in the quiet region. Therefore, the higher stellar densities can facilitate measurements of the magnetic field strength when the stellar activity level is high.

4. Effect of the S/R on Magnetic Field Measurements

So far, we have not yet considered the unavoidable uncertainties in real observations, which may affect the application of the MIT technique for magnetic field measurements. In real observations, uncertainties could come from photon counting, calibration, and the accuracy of the atomic database. Here, we focus on the effect of noise caused by photon counting.

We first attempted to find out the uncertainty of the derived magnetic field strengths as a function of S/N, density, and magnetic field strength. For simplification, we assumed that the source region of the FeX emissions has uniform physical parameters (i.e., magnetic field strength $B$, density $N$, and temperature $T$). $T$ is assumed to be $10^{6.02}$ K. The MIT technique uses the intensity ratio of the FeX 175/174 Å line pair ($r_1$) to calculate the density $N$, and uses the intensity ratio of the FeX 257/174 Å line pair ($r_2$) to calculate the magnetic field strength $B$, i.e.:

$$B = B(r_2, N), N = (r_1).$$

(4)

The temperature is not explicit in the functions above since it is fixed to be $10^{6.02}$ K. The measured density and magnetic field strength can thus be estimated by their corresponding values in the source region. Differentiating Equation (4) gives:

$$dB = \left(\frac{\partial B}{\partial r_2}\right)_N dr_2 + \left(\frac{\partial B}{\partial N}\right)_{r_2} dN,$$

(5)

$$dN = \frac{dN}{dr_1} dr_1$$

(6)

Here $dr_1$ and $dr_2$ represent the differentials of line ratios $r_1$ and $r_2$, respectively. $dN$ and $dB$ are the corresponding differentials of $N$ and $B$. $dr_1$ and $dr_2$ are expressed by the differentials of line intensities as:

$$dr_1 = d\left(\frac{I_{175}}{I_{174}}\right) = r_1\left(\frac{dl_{175}}{I_{175}} - \frac{dl_{174}}{I_{174}}\right),$$

(7)

$$dr_2 = d\left(\frac{I_{257}}{I_{174}}\right) = r_2\left(\frac{dl_{257}}{I_{257}} - \frac{dl_{174}}{I_{174}}\right).$$

(8)

Combining Equation (5)–Equation (8) we have:

$$\frac{dB}{B} = a\left(\frac{dl_{175}}{I_{175}} - \frac{dl_{174}}{I_{174}}\right) + b\left(\frac{dl_{257}}{I_{257}} - \frac{dl_{174}}{I_{174}}\right),$$

(9)

where $a = \frac{r_1}{B}\left(\frac{\partial B}{\partial N}\right)_{r_2}/\left(\frac{dr_1}{dN}\right)$, and $b = \frac{r_2}{B}\left(\frac{\partial B}{\partial N}\right)_{r_2}/\left(\frac{dr_2}{dN}\right)$. Since $B$ is a binary function of $N$ and $r_2$, we can rewrite $(\partial B/\partial N)_{r_2}$ and $(\partial B/\partial N)_{r_2}$ as $-(\partial r_2)/B(\partial r_2/N)$ and $1/(\partial r_2/N)$, respectively. Here $(\partial r_2/N)_B$, $(\partial r_2/N)_B$, and $\frac{dr_2}{dN}$ can be calculated from the intensity ratios of FeX lines as a function of magnetic field strength and density.
thus both \(a\) and \(b\) are derived. We assume that the intensity of each line is subject to an uncertainty (e.g., the relative uncertainty of \(I_{174}\) is \(\delta_{174}\)) caused by photon counting. The corresponding relative error of magnetic field strength is given by:

\[
\delta_B = \sqrt{a^2 (\delta_{174}^2 + \delta_{175}^2) + b^2 (\delta_{x}^2 + \delta_{257}^2)}.
\]

The relative uncertainty of each line equals the reciprocal of the S/N. As the 175 Å line is the weakest line among all the FeX lines we used in this study, the S/N of the 175 Å line (\((S/N)_{175}\)) was used as a representation of the noise level of actual observations. Since the S/N equals the square root of the photon number \(n\), the S/N of another line \(y\) (\(y\) can be the 174, 177, 184, or 257 Å line) is:

\[
(S/N)_y = (S/N)_{175} \cdot \sqrt{\frac{n_x}{n_{175}}}.
\]

After these, \(\delta_B\) is then rewritten as:

\[
\delta_B = \frac{1}{(S/N)_{175}} \cdot \sqrt{a^2 \left(\frac{n_{175}}{n_{174}} - 1\right) + b^2 \left(\frac{n_{175}}{n_x} + \frac{n_{175}}{n_{257}}\right)}.
\]

Here \(a\), \(b\), and all the photon number ratios are functions of \(B\) and \(N\); thus \(\delta_B\) is a function of \((S/N)_{175}\), \(B\), and \(N\).

In this section we chose the 174 Å line as the reference line when diagnosing the magnetic field strength. We chose six different values of \(\log(N/\text{cm}^3)\) ranging from 9.0 to 10.0, and plot \(\delta_B\) as a function of the \((S/N)_{175}\) and \(B\) under each specified density value in Figure 6. The most obvious feature is that \(\delta_B\) decreases with magnetic field strength, which would facilitate observations of stars with stronger fields. Meanwhile, the \(\delta_B\) does not change significantly with density. For stars with weaker magnetic fields, the derived magnetic field strengths are expected to be subject to larger uncertainties.

To investigate what S/N is required when applying the MIT technique, we added different levels of noise above the emissions of the FeX lines. The intensity of a spectral line was then modified to:

\[
I' = I_s \cdot \left(1 + \frac{R}{S/N_{175}}\right),
\]

where \(I_s\) is the line intensity primarily calculated in Section 3 and \(I'_s\) is the modified intensity after considering the noise. The
subscript $x$ ($x = 174, 175...$) represents the wavelength of the Fe X line in units of Å. $R$ is a random value given by Gaussian distribution that has a standard deviation of 1 and an average value of 0. We chose a series of $(S/N)_{175}$ as 25, 50, 100, and 150, and in each case the $S/N$ values of other lines were determined by the $(S/N)_{175}$ and the photon number ratios. The $S/N$ values of all the Fe X lines when assuming that $(S/N)_{175} = 50$ and the LOS direction is along $+x$-axis are shown in Table 1 as an example. The diagnosing procedure described in Section 3 was then repeated for all the x3–x9 models. To increase the number of data points, the LOS directions were taken every 5° for both the inclination and azimuth. Figure 7 shows the results of the joint probability density functions of $B^1$ and $B^0$. We also calculated the Spearman correlation coefficient $\rho$ between $B^1$ and $B^0$ when choosing the 174 Å line as the reference line. We found that $\rho$ is 0.40, 0.73, 0.89, and 0.93 when the $(S/N)_{175}$ is 25, 50, 100, and 150, respectively. We can see that a $(S/N)_{175}$ of 25 appears to be insufficient for effective measurements, while a $(S/N)_{175}$ of 50 or higher is enough to maintain a good correlation between the measured and real field strengths, as it is also suggested by Figure 6 wherein $\delta_B$ is around 20%–30% when the $(S/N)_{175}$ is around 50. Based on these analyses, we propose that a $(S/N)_{175}$ of at least 50 should be guaranteed in real observations to achieve effective measurements of the coronal magnetic field.

5. Summary and Conclusions

In this work, we have investigated the potential of the MIT technique to measure the stellar coronal magnetic field strength and discussed the associated uncertainty caused by photon counting errors. A series of stellar coronal models were constructed based on a solar photospheric synoptic magnetogram at solar maximum. By comparing the measured field strengths with values in the models, we concluded that the MIT technique can be used to measure the magnetic field strengths at the coronal bases of some stars with average photospheric field strengths of at least 3 times larger than that of the Sun at solar maximum.

We have also derived an analytical expression for the dependence of the relative error of the measured field strength on the S/N. By considering photon counting errors, we found that a $(S/N)_{175}$ of ~50 is needed to achieve effective measurements using the MIT technique. It is worth mentioning that the effect of other uncertainty sources (e.g., the uncertainties of radiometric calibrations) are not included in this study, which should be investigated in the future.

Figure 7. Joint probability density functions of the measured magnetic field strength vs. magnetic field strength in the model. Each column represents the results under a specific noise level.
The authors are supported by the National Key R&D Program of China No. 2021YFA0718600 and NSFC grants 11825301, 12103066 and 11790304. This work was carried out using the SWMF/BATSRUS codes developed at the University of Michigan Center for Space Environment Modeling (CSEM) and made available through the NASA Community.

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