Effect of particle shapes on effective strain gradient of SiC particle reinforced aluminum composites

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Abstract. The stress increments depend not only on the plastic strain but also on the gradient of plastic strain, when the characteristic length scale associated with non-uniform plastic deformation is on the order of microns. In the present research, the Taylor-based nonlocal theory of plasticity (TNT plasticity), with considering both geometrically necessary dislocations and statistically stored dislocations, is applied to investigated the effect of particle shapes on the strain gradient and mechanical properties of SiC particle reinforced aluminum composites (SiC/Al composites). Based on this theory, a two-dimensional axial symmetry cell model is built in the ABAQUS finite element code through its USER-ELEMENT (UEL) interface. Some comparisons with the classical plastic theory demonstrate that the effective stress predicted by TNT plasticity is obviously higher than that predicted by classical plastic theory. The results also demonstrate that the irregular particles cause higher effective gradient strain which is attributed to the fact that angular shape particles give more geometrically.

1. Introduction
In recent years, the ceramic particle-reinforced metal-matrix composites (MMCp) have offered designers much advantage on applications under high temperature, for their good strength, good structural rigidity, dimensional stability, lightweight and low thermal expansion. Lots of work has also improved the mechanical properties of MMCp, and the fundamental relationships between microstructure and mechanical properties have become the focus of researches. There are many microstructure characteristics influencing the mechanical properties of MMCp, such as particles’ volume fraction[1], particles’ shape[1-3], particles’ size[4], particles’ topology distribution[5], and relative particle size[6], etc.

There are some reports about particle shape in the open literature. Christman and et al.[1] predicted the stress-strain behavior for whisker aspect ratios of 4 and 5, and cell aspect ratios of 3, 5, and 6, respectively. It’s showed that an increase in the cell aspect ratio results in a decrease in the predicted...
elastic modulus. Weissenbek and et al. [3] have compared different unit cell models (including plane stress, plane strain, axisymmetric, and three-dimensional models), each with different particle shapes (spherical, cubic, and cylindrical). They concluded that reasonable overall mechanical properties (in tension and compression) could be predicted by using the axisymmetric model, and it’s also pointed out that spherical inclusions give rise to the softest response. In view of the irregular shapes of the particulate typically used in real particle-reinforced MMCp, thus spherical inclusions may not be the best choice for unit cell models of real materials. In an excellent comparison of experiment and modeling, Song and et al.[4] performed uniaxial tension tests on particulate-reinforced 6061-Al matrix composites reinforced by spherical particles and by angular particles, and also performed FEM calculations based on an axisymmetric unit cell model. Considering that the particles in the majority of particle-reinforced MMCp are angular in shape, unit cell models that use cylindrical particles may be more appropriate for modeling the overall mechanical properties.

The above discussion was implemented in the framework of classical plastic theory, which only takes into account the strain hardening resulted from the statistically stored dislocations (SSDs). When the characteristic length scale associated with non-uniform plastic deformation is on the order of microns, the stress at a given point depends on not only the current values and possibly the previous history of deformation and temperature, but also the neighborhood of the point. Lloyd[4] investigated the particle size effect for A356 aluminum alloy reinforced by 15 vol.% SiC particles with two different diameter sizes, 7.5 and 16µm. The research showed that the small silicon carbide particles with 7.5µm diameter gave significantly higher plastic work hardening than the large particles with 16µm diameter. The classical plastic theory possesses no internal length scale and cannot explain the size dependence of plasticity observed in experiments, which is reflected by the collective behavior of a vast number of SSDs.

In the study, we have investigated the effect of particle shape on the mechanical property of SiC particle reinforced aluminum composites(SiC/Al composites) by applying the Taylor-based nonlocal theory (TNT) of plasticity [7, 8].The effective strain gradients of matrix in SiC/Al composites with different particle shapes have been discussed by a numerical analysis. In our analysis, an axisymmetric unit cell with the SiC particle size of 13.5µm diameter and particles’ fraction of 20% has been built to investigate the plastic hardening work of MMCp caused by effective strain gradient.

2. TNT plasticity and finite element model

2.1. TNT plasticity

TNT plasticity(developed by Gao and Huang[7]) is based on a multi-scale framework connecting the micro-scale notion of statistically stored and geometrically necessary dislocations to the meso-scale notion of plastic strain and strain gradient. The constitutive equations contain no higher-order stresses nor strain gradients. As a nonlocal integration of strains, strain gradients are introduced in the form of nonlocal variables. The basic principles in the nonlocal constitutive framework can be written as follows: (1) The flow stress obeys the Taylor hardening relations; (2) The density of geometrically necessary dislocations is calculated as nonlocal variables expressed in terms of a weighted average of plastic strain; (3) The essential structure of classical plasticity is preserved. In TNT plasticity, the flow stress \( \sigma_{\text{flow}} \) is obtained from the Taylor dislocation model as in equation (1).

\[
\sigma_{\text{flow}} = M b \mu \sqrt{\rho} = M b \mu \sqrt{\rho_s + \rho_G} = \sigma_0 \left( f^\gamma (\varepsilon) + l \eta \right)
\]

The dislocation density \( \rho \) is composed of the density \( \rho_s \) for statistically stored dislocations (SSDs), which accumulate by trapping each other in a random way, and the density \( \rho_G \) for geometrically necessary dislocations (GNDs), which are required for compatible deformation of various parts of the non-uniformly deformed material. Where \( M \) is the Taylor factor which acts as an isotropic interpretation of the crystal line anisotropy at the continuum level, \( \alpha \) is an empirical material constant in the Taylor dislocation model ranging between 0.1 and 0.5, \( \mu \) is the shear modulus, and \( b \)
is the magnitude of Burgers vector, $\sigma_{\text{ref}}$ is a reference stress in uniaxial tension, the function $f(\varepsilon)$ is a non-dimensional function determined from the uniaxial stress-strain curve, $\eta$ is the effective strain gradient, and $l$ is identified as the intrinsic material length introduced by Fleck and Hutchinson\cite{9} as in equation (2).

$$l = 18\alpha^2(\mu / \sigma_{\text{ref}}^2)^2$$

In equation (1), the effective strain gradient $\eta$ is obtained from the deviatoric strain gradient tensor $\eta'_{ik}$ by

$$\eta = \sqrt{\eta'_{ik} \eta'_{jk} / 4}$$

where

$$\eta'_{ik} = \varepsilon_{ik,j} + \varepsilon_{jk,i} - \varepsilon_{ij,k} - \frac{1}{4}(\delta_{ik}\varepsilon_{pp,j} + \delta_{jk}\varepsilon_{pp,i})$$

The gradient term, $\varepsilon_{ij,k}$ in equation (4) is calculated as nonlocal variables expressed in terms of a weighted average of strains. An eight-node isoparametric element, have been used to evaluate strains, strain gradients and stresses. The isoparametric transformation is shown as figure 1. There are two different levels of Gaussian integration in the finite element analysis for TNT plasticity. The first is at the element level, which is the same as the classical plasticity theory. The second is at the mesoscale cell level, which is special for TNT plasticity. There is a square (for a two-dimensional problem) mesoscale cell surrounding each element-level Gaussian integration point. The stresses and effective strain gradients of each element-level Gaussian integration point can be expressed in terms of the nodal displacements via Gaussian integration in the mesoscale cell.

Figure 1. A schematic diagram of the isoparametric transformation.
where \( (\int_{\text{cell}} \xi_m \xi_2 dV)^{-1} \) is the inverse of \( \int_{\text{Vcell}} \xi_m \xi_k dV \).

2.2. Finite element model

We adopt a unit cell model to investigate the effect of particle shape on the quasi-static mechanical behavior of the SiCp/Al composites. Both particles and matrix are meshed by the conventional eight-node isoparametric elements. The elements of the matrix have been implemented in the ABAQUS finite element code through its USER-ELEMENT (UEL) interface for TNT plasticity. Those elements, which are similar to the conventional eight-node isoparametric elements, also have eight nodes and nine Gaussian integration points. The bonding between the particles and the matrix is assumed to be perfect in the analysis. The computational meso-mechanics models for various particle shapes are shown in figure 2.

The SiC particle is assumed to be linearly elastic and isotropic, with Young’s modulus \( E_{\text{SiC}} = 427 \text{GPa} \) and Poisson’s ratio \( v_{\text{SiC}} = 0.17 \). The matrix is characterized by TNT plasticity in section 2.1 by using a piecewise elastic-power law hardening relation to model the uniaxial stress–strain behavior for unreinforced A359 Al alloy as in equation (8). Where \( E_{\text{Al}} = 76 \text{GPa} \) is the aluminum Young’s modulus, \( \sigma_{y0} = 208 \text{MPa} \) and \( \epsilon_{y0} \) are the initial yield stress and yield strain, \( n = 0.136 \) is the power exponent, and Poisson’s ratio of aluminum is \( v_{\text{Al}} = 0.33 \).

\[
\sigma = \begin{cases} 
E_{\text{Al}} \epsilon & \text{if } \epsilon > \epsilon_{y0} \\
K \cdot \epsilon^n & (K = E_{\text{Al}}^{n-1} \sigma_{y0}^{n-1}) \text{ if } \epsilon > \epsilon_{y0} 
\end{cases}
\]  
\( (8) \)

Figure 2. Computational meso-mechanics models for various particle shapes: (a) circle; (b) square; (c) octagon.

3. Results and discussion

3.1. Particle shape effect of the reinforcement

The stress–strain curves for various particle shapes predicted by TNT theory and classical elastic-plastic theory are shown in figure 3. The equivalent axial strain in the cell is evaluated by the ratio of the uniform displacement \( u_0 \) on the top surface to the half-cell height \( h \). The equivalent axial stress in the cell is determined by the total axial force and the cell cross section area, where the total axial force is obtained from the nodal force of all nodes on the top surface. Some conclusions can be given as follows: (1) Both TNT plasticity and classical plasticity show that the strengthening effect of square is better than that of octagon and circle, and the effect of octagon is slightly stronger than that of circle, which is consistent with the other numerical research[10]. (2) The flow stresses of all particle shapes predicted by TNT plasticity are significantly higher than those predicted by the classical elastic-plastic theory, which is consistent with the result predicted by mechanism-based strain gradient plasticity[11].

The contours of the equivalent stresses predicted by TNT for various particle shapes are in figure 4. The particle volume fraction is 20%. It’s known by observing the von mises stress contours that there
is obvious stress concentration at sharp corners of square model, whose area of high stress region (the stress range from 0 to 420 MPa for all contours) is also obviously lager than that of circle and octagon.

Figure 4. The contours of von mises stresses predicted by TNT for various particle shapes: (a) circle; (b) square; (c) octagon.

3.2. Effect of particle shape on the effect strain gradient

The overall equivalent strain gradient $\eta_{ew}$ in the compressive direction of the composite are calculated by using a area average approach as follows,

$$
\eta_{ew} = \frac{1}{S} \int_S \eta dS = \frac{1}{S} \sum_{j=1}^n \left( \sum_{i=1}^m \eta^{(i)} \left| J^{(i)} \right|^H \right) (9)
$$

where $\eta_{ew}$ is the average equivalent stain gradient, $S$ is the total area, $n$ is the total number of Gaussian integration points of each element, $m$ is the total number of elements, $\sigma^{(i)}$ and $\eta^{(i)}$ are the equivalent stress and the average equivalent strain gradient at the Gaussian integration point, $H^{(i)}$ and $\left| J^{(i)} \right|^H$ are the Gaussian integrations weight coefficient and Jacobian determinant at the integration point, respectively. The average strain gradient versus strain curve predicted by TNT for various particle shapes is presented in figure 5. From the curves, it can be found that the average equivalent strain gradient for square shape is higher than that for circle and octagon. In accordance with the Taylor hardening law in equation (1), higher equivalent strain gradient contributes to higher flow stress of SiCp/Al composites for square shape, compared with circle and octagon model.

The contours of the equivalent strain gradients predicted by TNT plasticity for various particle shapes are presented in figure 6. It can be seen that the distribution of effective strain gradient in matrix is highly inhomogeneous due to the mismatch of elastic module between particles and matrix.
The largest strain gradient of the matrix is located at the region near particles. There is an obvious concentration phenomenon at sharp corners of square model, and the area of the high strain gradient region is also larger than that of octagon and circle. It is attributed to the fact that square shape particles give more geometrically necessary dislocations on the macroscopic plastic work hardening of alloys or metal–matrix composites.

![Graph: Mean strain gradient versus strain curve predicted by TNT for various particle shapes.](image)

**Figure 5.** The equivalent strain gradient versus strain curve predicted by TNT for various particle shapes.

It’s noteworthy that the present study does not account for cracking and debonding between the particles and matrix. The high level of stress concentration often means that the fracture of particles is more likely to happen. Generally speaking, broken particles are unable to afford tension and shear effectively, and the macroscopic mechanical properties of SiCp/Al composites will decrease with the increase of particles’ fracture. Therefore, although the strengthening effect of square inclusions is better than that of circle and octagon inclusion before particles’ fracture happens, the square particle may not be the best inclusion.

![Graphs: The contours of equivalent strain gradients predicted by TNT for various particle shapes.](image)

**Figure 6.** The contours of equivalent strain gradients predicted by TNT for various particle shapes: (a) circle; (b) square; (c) octagon.

4. Summary

In this investigation, a two-dimensional axial symmetry model based on the TNT plasticity is applied to investigate the effect of particle shape on the plastic hardening work of SiC particle reinforced aluminum matrix composites. We have reached the following conclusions:

1. With the predicted numerical results of various particles shapes, it’s observed that the strengthening effect of square model is more effective than circle and octagon (whether TNT plasticity or classical theory).

2. The effective stresses predicted by TNT plasticity for various particle shapes are significantly higher than those predicted by classical theory, since TNT plasticity includes the influence of strain gradient on plastic hardening work of Al matrix.
(3) The effective strain gradient for square shape (predicted by TNT plasticity) is significantly higher than that for circle and octagon shape. It is attributed to the fact that the sharp corners of square give more geometrically necessary dislocations, which cause higher effective strain gradient.

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