Almost Maximal Lepton Mixing with Large T Violation in Neutrino Oscillations and Neutrinoless Double Beta Decay

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Abstract

We point out two simple but instructive possibilities to construct the charged lepton and neutrino mass matrices, from which the nearly bi-maximal neutrino mixing with large T violation can naturally emerge. The two lepton mixing scenarios are compatible very well with current experimental data on solar and atmospheric neutrino oscillations, and one of them may lead to an observable T-violating asymmetry between $\nu_\mu \to \nu_e$ and $\nu_e \to \nu_\mu$ transitions in the long-baseline neutrino oscillation experiments. Their implications on the neutrinoless double beta decay are also discussed.

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I. INTRODUCTION

Recent observation of atmospheric and solar neutrino anomalies, particularly that in the Super-Kamiokande experiment [1], has provided robust evidence that neutrinos are massive and lepton flavors are mixed. Analyses of the atmospheric neutrino deficit favor $\nu_\mu \to \nu_\tau$ as the dominant oscillation mode with the mass-squared difference $\Delta m_{21}^2 \sim 10^{-3}$ eV$^2$ and the mixing factor $\sin^2 2\theta_{atm} > 0.88$ at the 90% confidence level. As for the solar neutrino anomaly, there are four possible solutions belonging to two categories: (a) solar $\nu_e$ neutrinos changing to active $\nu_\mu$ or sterile $\nu_s$ neutrinos due to the long-wavelength vacuum oscillation with the parameters $\Delta m_{31}^2 \sim 10^{-10}$ eV$^2$ and $\sin^2 2\theta_{atm} \approx 1$ [2]; (b) the matter-enhanced $\nu_e \to \nu_\mu$ or $\nu_e \to \nu_s$ oscillations via the Mikheyev-Smirnov-Wolfenstein (MSW) mechanism with $\Delta m_{32}^2 \sim 10^{-5}$ eV$^2$ and $\sin^2 2\theta_{sun} \sim 1$ (large-angle solution), with $\Delta m_{32}^2 \sim 10^{-6}$ eV$^2$ and $\sin^2 2\theta_{sun} \sim 10^{-2}$ (small-angle solution), or with $\Delta m_{32}^2 \sim 10^{-7}$ eV$^2$ and $\sin^2 2\theta_{sun} \sim 1$ (low solution) [3]. Although the large-angle MSW solution seems to be somehow favored by the present Super-Kamiokande and SNO data [1,4], the other three solutions have not been convincingly ruled out. To pin down the true solution to the solar neutrino problem remains a challenging task of the next-round solar neutrino experiments.

The strong hierarchy between $\Delta m_{21}^2$ and $\Delta m_{32}^2$, together with the small $\nu_3$-component in $\nu_e$ configuration [5], implies that atmospheric and solar neutrino oscillations decouple approximately from each other. Each of them is dominated by a single mass scale [6], which can be set as

$$\Delta m_{21}^2 \equiv m_2^2 - m_1^2 \approx \pm \Delta m_{31}^2 ,$$
$$\Delta m_{32}^2 \equiv m_3^2 - m_2^2 \approx \pm \Delta m_{21}^2 .$$

(1)

Of course $\Delta m_{31}^2 \approx \Delta m_{32}^2$ holds in this approximation. As a consequence, the mixing factors of solar and atmospheric neutrino oscillations in the disappearance-type experiments (i.e., $\nu_e \to \nu_e$ and $\nu_\mu \to \nu_\mu$) are simply given by

$$\sin^2 2\theta_{sun} = 4|V_{e1}|^2|V_{e2}|^2 ,$$
$$\sin^2 2\theta_{atm} = 4|V_{\mu3}|^2 \left(1 - |V_{\mu3}|^2\right) ,$$

(2)

where $V$ is the $3 \times 3$ lepton flavor mixing matrix linking the neutrino mass eigenstates ($\nu_1, \nu_2, \nu_3$) to the neutrino flavor eigenstates ($\nu_e, \nu_\mu, \nu_\tau$). The present experimental data seem to favor the large-angle MSW solution to the solar neutrino problem. In this case, $\sin^2 2\theta_{atm} \sim \sin^2 2\theta_{sun} \sim \mathcal{O}(1)$. Then two large mixing angles can be drawn from Eq. (2): one between the 2nd and 3rd lepton families and the other between the 1st and 2nd lepton families [7].

1 Throughout this paper we do not take the LSND evidence for neutrino oscillations [6], which has not been independently confirmed by other experiments [7], into account.

2 The conjecture, that two of the three lepton flavor mixing angles could be extraordinarily large (i.e., equal or close to 45°) had been made by several authors [8] before the first-round Super-Kamiokande data appeared in 1998.
A particularly interesting limit is \( \sin^2 2\theta_{\text{atm}} = \sin^2 2\theta_{\text{sun}} = 1 \), corresponding uniquely (up to a trivial sign or phase rearrangement) to

\[
V_0 = \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}
\end{pmatrix},
\]

the so-called “bi-maximal” flavor mixing pattern \([4]\). There have been a lot of discussions about the bi-maximal and nearly bi-maximal neutrino mixing scenarios \([10]\). While the latter could straightforwardly be obtained from slight modifications of the former, the arbitrariness in doing so has to be resolved by imposing simple flavor symmetries or dynamical constraints on the charged lepton and neutrino mass matrices. In Ref. \([11]\), for example, it has been shown that a nearly bi-maximal neutrino mixing pattern can naturally arise from the explicit breaking of the lepton flavor democracy.

The present paper aims to discuss two simple but instructive possibilities to construct the lepton mass matrices, from which two almost bi-maximal neutrino mixing patterns can directly be derived. We find that these two scenarios have practically indistinguishable consequences on solar and atmospheric neutrino oscillations, but their predictions for leptonic CP or T violation are quite different. To be specific, we calculate the deviation of solar neutrino mixing from maximal mixing in each scenario. We illustrate that one of the two lepton mixing patterns may lead to an observable T-violating asymmetry between \( \nu_\mu \rightarrow \nu_e \) and \( \nu_e \rightarrow \nu_\mu \) transitions in the long-baseline neutrino oscillation experiments. The implications of our phenomenological models on the neutrinoless double beta decay are also discussed in some detail.

II. NEARLY BI-MAXIMAL MIXING

The fact that the masses of three active neutrinos are extremely small is presumably attributed to the Majorana feature of the neutrino fields \([12]\). In this picture, the light (left-handed) neutrino mass matrix \( M_\nu \) must be symmetric and can be diagonalized by a single unitary transformation:

\[
U_\nu^\dagger M_\nu U_\nu^* = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}.
\]

The charged lepton mass matrix \( M_l \) is in general non-Hermitian, hence the diagonalization of \( M_l \) needs a bi-unitary transformation:

\[
U_l^\dagger M_l U_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}.
\]
\[ V = U^\dagger \nu . \] (6)

Note that \((m_1, m_2, m_3)\) in Eq. (4) and \((m_e, m_\mu, m_\tau)\) in Eq. (5) are physical (positive) masses of light neutrinos and charged leptons, respectively.

In the flavor basis where \(M_l\) is diagonal (i.e., \(U_l = 1\) being a unity matrix), the flavor mixing matrix is simplified to \(V = U_\nu\). The bi-maximal neutrino mixing pattern \(U_\nu = V_0\) can then be constructed from the product of the Euler rotation matrices

\[
R_{12}(\theta_x) = \begin{pmatrix}
\cos \theta_x & \sin \theta_x & 0 \\
-\sin \theta_x & \cos \theta_x & 0 \\
0 & 0 & 1
\end{pmatrix}
\] (7)

and

\[
R_{23}(\theta_y) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \theta_y & \sin \theta_y \\
0 & -\sin \theta_y & \cos \theta_y
\end{pmatrix}
\] (8)

with special rotation angles \(\theta_x = \theta_y = 45^\circ\):

\[
V_0 = R_{23}(45^\circ) \otimes R_{12}(45^\circ)
\]

\[
= \begin{pmatrix}
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\
-\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\
\frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\] (9)

Obviously the vanishing of the (1,3) element in \(V_0\) assures an exact decoupling between solar \((\nu_e \rightarrow \nu_\mu)\) and atmospheric \((\nu_\mu \rightarrow \nu_\tau)\) neutrino oscillations. The corresponding neutrino mass matrix \(M_\nu\) turns out to be

\[
M_\nu = V_0 \begin{pmatrix}
m_1 & 0 & 0 \\
0 & m_2 & 0 \\
0 & 0 & m_3
\end{pmatrix} V_0^T
\]

\[
= \begin{pmatrix}
A_\nu & B_\nu & C_\nu \\
C_\nu & A_\nu & -B_\nu \\
-C_\nu & B_\nu & A_\nu
\end{pmatrix},
\] (10)

where

\[
A_\nu = \frac{m_3}{2} + \frac{m_1 + m_2}{4}, \\
B_\nu = \frac{m_3}{2} - \frac{m_1 + m_2}{4}, \\
C_\nu = \frac{m_2 - m_1}{2\sqrt{2}}.
\] (11)

A trivial sign or phase rearrangement for \(U_\nu = V_0\) may lead to a slightly different form of \(M_\nu\), but the relevant physical consequences on neutrino oscillations are essentially
unchanged. If the masses of \( \nu_1 \) and \( \nu_2 \) neutrinos are nearly degenerate (i.e., \( m_1 \approx m_2 \)), one can arrive at a simpler texture of \( M_\nu \), in which \( A_\nu \approx (m_3 + m_1)/2 \), \( B_\nu \approx (m_3 - m_1)/2 \), and \( C_\nu \approx 0 \) hold.

We observe that the bi-maximal neutrino mixing pattern will be modified, if \( U_l \) deviates somehow from the unity matrix. This can certainly happen, provided that the charged lepton mass matrix \( M_l \) is not diagonal in the flavor basis where the neutrino mass matrix \( M_\nu \) takes the form given in Eq. (10). As \( U_\nu = V_0 \) describes a product of two special Euler rotations in the real \((2,3)\) and \((1,2)\) planes, the simplest form of \( U_l \) which allows \( V = U_l^T U_\nu \) to cover the whole \( 3 \times 3 \) space should be \( U_l = R_{12}(\theta_x) \) or \( U_l = R_{31}(\theta_x) \) (see Ref. [14] for a detailed discussion). To incorporate T violation in neutrino oscillations, however, the complex rotation matrices

\[
R_{12}(\theta_x, \phi_x) = \left( \begin{array}{ccc} \cos \theta_x & \sin \theta_x e^{i \phi_x} & 0 \\ -\sin \theta_x e^{-i \phi_x} & \cos \theta_x & 0 \\ 0 & 0 & 1 \end{array} \right) \quad (12)
\]

and

\[
R_{31}(\theta_z, \phi_z) = \left( \begin{array}{ccc} \cos \theta_z & 0 & \sin \theta_z e^{i \phi_z} \\ 0 & 1 & 0 \\ -\sin \theta_z e^{-i \phi_z} & 0 & \cos \theta_z \end{array} \right) \quad (13)
\]

should be used [15]. In this case, we arrive at lepton flavor mixing of the pattern

\[
V_{(x)} = \left( \begin{array}{ccc} c_x & -s_x e^{i \phi_x} & 0 \\ s_x e^{-i \phi_x} & c_x & 0 \\ 0 & 0 & 1 \end{array} \right) \left( \begin{array}{ccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 1 & 0 & 0 \end{array} \right) = \left( \begin{array}{ccc} \frac{c_x}{\sqrt{2}} + \frac{s_x}{\sqrt{2}} e^{i \phi_x} & \frac{c_x}{\sqrt{2}} - \frac{s_x}{\sqrt{2}} e^{-i \phi_x} & 0 \\ -\frac{c_x}{\sqrt{2}} + \frac{s_x}{\sqrt{2}} e^{-i \phi_x} & -\frac{c_x}{\sqrt{2}} + \frac{s_x}{\sqrt{2}} e^{i \phi_x} & c_x \sqrt{2} \end{array} \right) \quad (14)
\]

or of the pattern

\[
V_{(z)} = \left( \begin{array}{ccc} c_z & 0 & -s_z e^{i \phi_z} \\ 0 & 1 & 0 \\ s_z e^{-i \phi_z} & 0 & c_z \end{array} \right) \left( \begin{array}{ccc} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ 1 & 0 & 0 \end{array} \right) = \left( \begin{array}{ccc} \frac{c_z}{\sqrt{2}} - \frac{s_z}{\sqrt{2}} e^{i \phi_z} & \frac{c_z}{\sqrt{2}} + \frac{s_z}{\sqrt{2}} e^{-i \phi_z} & 0 \\ -\frac{c_z}{\sqrt{2}} + \frac{s_z}{\sqrt{2}} e^{-i \phi_z} & -\frac{c_z}{\sqrt{2}} + \frac{s_z}{\sqrt{2}} e^{i \phi_z} & c_z \sqrt{2} \end{array} \right) \quad (15)
\]

where \( s_x \equiv \sin \theta_x, c_x \equiv \cos \theta_x, \) and so on. It is obvious that \( V_{(x)} \) and \( V_{(z)} \) represent two nearly bi-maximal flavor mixing scenarios, if the rotation angles \( \theta_x \) and \( \theta_z \) are small in magnitude.

As the mixing angle \( \theta_x \) or \( \theta_z \) arises from the diagonalization of \( M_l \), it is expected to be a simple function of the ratios of charged lepton masses. Then the strong mass hierarchy of charged leptons naturally assures the smallness of \( \theta_x \) or \( \theta_z \), as one can see later on.
III. CONSTRAINTS ON $\sin^2 2\theta_{\text{sun}}$ AND $\sin^2 2\theta_{\text{atm}}$

Indeed the proper texture of $M_l$ which leads to the flavor mixing pattern $V_{(x)}$ is

$$M^{(x)}_l = \begin{pmatrix} 0 & C_l & 0 \\ C_l^* & B_l & 0 \\ 0 & 0 & A_l \end{pmatrix},$$

(16)

where $A_l = m_\tau$, $B_l = m_\mu - m_e$, and $C_l = \sqrt{m_e m_\mu} e^{i\phi_x}$. The mixing angle $\theta_x$ in $V_{(x)}$ is then given by

$$\tan(2\theta_x) = 2 \frac{\sqrt{m_e m_\mu}}{m_\mu - m_e}.$$  

(17)

On the other hand, the proper texture of $M_l$ which gives rise to the mixing pattern $V_{(z)}$ reads as follows:

$$M^{(z)}_l = \begin{pmatrix} 0 & 0 & C_l \\ 0 & B_l & 0 \\ C_l^* & 0 & A_l \end{pmatrix},$$

(18)

where $A_l = m_\tau - m_e$, $B_l = m_\mu$, and $C_l = \sqrt{m_e m_\tau} e^{i\phi_z}$. The mixing angle $\theta_z$ in $V_{(z)}$ turns out to be

$$\tan(2\theta_z) = 2 \frac{\sqrt{m_e m_\tau}}{m_\tau - m_e}.$$ 

(19)

Taking the hierarchy of charged lepton masses (i.e., $m_e \ll m_\mu \ll m_\tau$) into account, one obtains

$$s_x \approx \sqrt{\frac{m_e}{m_\mu}},$$

$$s_z \approx \sqrt{\frac{m_e}{m_\tau}},$$

(20)

to a good degree of accuracy. Numerically, we find $\theta_x \approx 3.978^\circ$ and $\theta_z \approx 0.972^\circ$ with the inputs $m_e = 0.511$ MeV, $m_\mu = 105.658$ MeV, and $m_\tau = 1.777$ GeV [16].

Now let us calculate the mixing factors of solar and atmospheric neutrino oscillations in the disappearance-type experiments. Using Eq. (2), we arrive straightforwardly at

$$\sin^2 2\theta_{\text{sun}} = 1 - s_x^2 \left(1 + 2 \cos^2 \phi_x\right),$$

$$\sin^2 2\theta_{\text{atm}} = 1 - s_z^4$$

(21)

for $V_{(x)}$; and

$$\sin^2 2\theta_{\text{sun}} = 1 - s_z^2 \left(1 + 2 \cos^2 \phi_z\right),$$

$$\sin^2 2\theta_{\text{atm}} = 1$$

(22)
for $V(z)$. Allowing $\phi_x$ and $\phi_z$ to take arbitrary values, we find that the magnitude of $\sin^2 2\theta_{\text{sun}}$ lies in the following range:

$$1 - 3s_i^2 \leq \sin^2 2\theta_{\text{sun}} \leq 1 - s_i^2,$$

where $i = x$ or $z$. Numerically, we obtain $0.986 \leq \sin^2 2\theta_{\text{sun}} \leq 0.995$ for $V(x)$ and $0.999 \leq \sin^2 2\theta_{\text{sun}} \leq 1.000$ for $V(z)$. Note that $\sin^2 2\theta_{\text{atm}} = 1.000$ holds in both cases. Therefore the two nearly bi-maximal neutrino mixing patterns are practically indistinguishable in the experiments of solar and atmospheric neutrino oscillations. They may be distinguished from each other with the measurements of $|V_{e3}|$ and CP or T violation in the long-baseline neutrino oscillation experiments.

It is worth mentioning that Gonzalez-Garcia, Peña-Garay, Nir, and Smirnov have recently defined a small real parameter $\epsilon$ to describe the deviation of solar neutrino mixing from maximal mixing \[17\]:

$$\sin^2 \theta_{\text{sun}} \equiv 1 - \frac{\epsilon^2}{2}$$

with $|\epsilon| \ll 1$. This parameter proves very useful for phenomenological studies of the solar neutrino problem \[17\]: the probabilities of solar neutrino oscillations depend quadratically on $\epsilon$ in vacuum, and linearly on $\epsilon$ if matter effects dominate. It is then our interest to calculate $\epsilon$ in the nearly bi-maximal neutrino mixing scenarios under discussion. We notice that

$$\sin^2 2\theta_{\text{sun}} = 1 - \epsilon^2$$

results from Eq. (24) exactly. Comparing Eq. (25) with Eqs. (21) and (22), we obtain

$$|\epsilon| = s_x \sqrt{1 + 2 \cos^2 \phi_x}$$

for $V(x)$; and

$$|\epsilon| = s_z \sqrt{1 + 2 \cos^2 \phi_z}$$

for $V(z)$. Given $\phi_x$ and $\phi_z$ of arbitrary values, the allowed region of $|\epsilon|$ turns out to be $0.069 \leq |\epsilon| \leq 0.120$ in the scenario of $V(x)$, and $0.017 \leq |\epsilon| \leq 0.029$ in the scenario of $V(z)$. Both ranges of $|\epsilon|$ are phenomenologically interesting for solar neutrino oscillations, as comprehensively discussed in Ref. \[17\].

### IV. LEPTONIC T VIOLATION

The strength of CP or T violation in neutrino oscillations, no matter whether neutrinos are Dirac or Majorana particles, is measured by a universal and rephasing-invariant parameter $J$ \[14\], defined through the following equation:

$$\text{Im} \left( V_{\alpha i} V_{\beta j}^* V_{\alpha j}^* V_{\beta i}^* \right) = J \sum_{\gamma,k} (\varepsilon_{\alpha \beta \gamma} \varepsilon_{ijk})$$
in which the Greek subscripts run over \((e, \mu, \tau)\), and the Latin subscripts run over \((1, 2, 3)\). Considering the two lepton mixing scenarios proposed in section 2, we obtain

\[
\mathcal{J} = \begin{cases} 
\frac{c_x s_x}{4\sqrt{2}} \sin \phi_x & \text{[for } V(x)\text{]}, \\
\frac{c_z s_z}{4\sqrt{2}} \sin \phi_z & \text{[for } V(z)\text{]}. 
\end{cases}
\]  

(29)

For illustration, we typically take \(\phi_x = \phi_z = 90^\circ\). Then we arrive at \(\mathcal{J} \approx 0.012\) and \(\mathcal{J} \approx 0.003\), respectively, for \(V(x)\) and \(V(z)\). The former could be determined from the probability asymmetry between \(\nu_\mu \rightarrow \nu_e\) and \(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\) transitions (CP-violating asymmetry), or that between \(\nu_\mu \rightarrow \nu_e\) and \(\nu_e \rightarrow \nu_\mu\) transitions (T-violating asymmetry) in a long-baseline neutrino oscillation experiment \[18\], if the earth-induced matter effects were assumed to be absent or negligible:

\[
\Delta P = P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \\
= P(\nu_\mu \rightarrow \nu_e) - P(\nu_e \rightarrow \nu_\mu) \\
= 16 \mathcal{J} \sin F_{12} \sin F_{23} \sin F_{31} \\
\approx 16 \mathcal{J} \sin F_{21} \sin^2 F_{32},
\]  

(30)

where \(F_{ij} = 1.27\Delta m^2_{ij} L/E\) with \(L\) being the distance between the neutrino source and the detector (in unit of km) and \(E\) being the neutrino beam energy (in unit of GeV). In realistic long-baseline neutrino oscillation experiments, however, the terrestrial matter effects are by no means small and must be taken into account.

It is generally expected that the T-violating asymmetry between \(\nu_\mu \rightarrow \nu_e\) and \(\nu_e \rightarrow \nu_\mu\) transitions is less sensitive to matter effects than the CP-violating asymmetry between \(\nu_\mu \rightarrow \nu_e\) and \(\bar{\nu}_\mu \rightarrow \bar{\nu}_e\) transitions \[19\]. For simplicity, we concentrate only on T violation in the following. In analogy to Eq. (30), the matter-corrected T-violating asymmetry can be expressed as

\[
\Delta \tilde{P} = \tilde{P}(\nu_\mu \rightarrow \nu_e) - \tilde{P}(\nu_e \rightarrow \nu_\mu) \\
= 16 \mathcal{J} \sin \tilde{F}_{12} \sin \tilde{F}_{23} \sin \tilde{F}_{31} \\
\approx 16 \mathcal{J} \sin \tilde{F}_{21} \sin^2 \tilde{F}_{32},
\]  

(31)

where \(\tilde{F}_{ij} = 1.27\Delta \tilde{m}^2_{ij} L/E\) and \(\Delta \tilde{m}^2_{ij} \equiv \tilde{m}_i^2 - \tilde{m}_j^2\) with \(\tilde{m}_i\) being the effective neutrino masses in matter. The relation between \(\mathcal{J}\) and \(\tilde{\mathcal{J}}\) reads \[20\]

\[
\tilde{\mathcal{J}} = \mathcal{J} \frac{\Delta m^2_{21}}{\Delta \tilde{m}^2_{21}} \cdot \frac{\Delta m^2_{31}}{\Delta \tilde{m}^2_{31}} \cdot \frac{\Delta m^2_{32}}{\Delta \tilde{m}^2_{32}},
\]  

(32)

where

\[
\Delta \tilde{m}^2_{21} = \frac{2}{3} \sqrt{x^2 - 3y} \sqrt{3(1 - z^2)},
\]

\[
\Delta \tilde{m}^2_{31} = \frac{1}{3} \sqrt{x^2 - 3y} \left[3z + \sqrt{3(1 - z^2)}\right],
\]

\[
\Delta \tilde{m}^2_{32} = \frac{1}{3} \sqrt{x^2 - 3y} \left[3z - \sqrt{3(1 - z^2)}\right],
\]  

(33)
and

\[ \begin{align*}
  x &= \Delta m_{21}^2 + \Delta m_{31}^2 + A , \\
  y &= \Delta m_{21}^2 \Delta m_{31}^2 + A \left[ \Delta m_{21}^2 \left( 1 - |V_{e2}|^2 \right) + \Delta m_{31}^2 \left( 1 - |V_{e3}|^2 \right) \right] , \\
  z &= \cos \left[ \frac{1}{3} \arccos \frac{2x^3 - 9xy - 27A \Delta m_{21}^2 \Delta m_{31}^2 |V_{e1}|^2}{2 (x^2 - 3y)^{3/2}} \right].
\end{align*} \]

(34)

The terrestrial matter effects are described by the parameter \( A = 2\sqrt{2} G_F N_e E \) [21], with \( N_e \) being the background density of electrons and \( E \) being the neutrino beam energy. Assuming the matter density of the earth’s crust to be constant, one may get \( A \approx 2 \cdot 10^{-4} \text{ eV}^2 \text{E}/[\text{GeV}] \) as a good approximation [22].

To illustrate, let us calculate \( \tilde{J} \) and \( \Delta \tilde{P} \) for two scenarios of the long-baseline neutrino oscillation experiments: \( L = 730 \text{ km} \) and \( L = 2100 \text{ km} \). The former baseline corresponds to a neutrino source at Fermilab pointing toward the Soudan mine or that at CERN toward the Gran Sasso underground laboratory, and the latter corresponds to a possible high-intensity neutrino beam from the High Energy Proton Accelerator in Tokaimura to a detector located in Beijing [23]. We typically take \( \Delta m_{21}^2 \approx 5 \cdot 10^{-5} \text{ eV}^2 \) (the large-angle MSW solution to the solar neutrino problem) and \( \Delta m_{32}^2 \approx 3 \cdot 10^{-3} \text{ eV}^2 \), as well as \( \phi_x = 90^\circ \) based on the almost bi-maximal lepton mixing pattern \( V(x) \). The numerical results of \( \tilde{J} \) and \( \Delta \tilde{P} \) as functions of the neutrino beam energy \( E \) are shown in Figs. 1 and 2, respectively. We observe that the magnitude of \( \tilde{J} \) can significantly be suppressed due to matter effects. This feature of \( \tilde{J} \) makes the measurement of leptonic CP- and T-violating asymmetries more difficult in practice. Indeed the T-violating asymmetry \( \Delta \tilde{P} \) is quite small in the chosen range of the neutrino beam energy (1 GeV \( \leq E \leq 20 \) GeV), at most at the percent level. The terrestrial matter effects on \( \Delta \tilde{P} \) are in general insignificant and negligible, except the case of the resonance enhancement at \( E \approx 1.5 \text{ GeV} \) for \( L = 730 \text{ km} \) or at \( E \approx 4 \text{ GeV} \) for \( L = 2100 \text{ km} \). It should be noted that \( \Delta \tilde{P} \approx \Delta P \) has no way to lead to \( \tilde{J} \approx J \).

Therefore a relatively clean signal of T violation, even measured in the future long-baseline neutrino experiments, does not mean that the fundamental T-violating parameter \( J \) or \( \phi_x \) can directly be determined. To pin down those genuine parameters of flavor mixing and T violation, we must first of all understand the terrestrial matter effects to a high degree of accuracy. More reliable knowledge of the earth’s matter density profile is unavoidably required for our long-baseline neutrino oscillation experiments.

V. NEUTRINOLESS DOUBLE BETA DECAY

So far we have only introduced a Dirac-type T-violating phase into the lepton flavor mixing matrix \( V \). The latter may in general consist of two additional T-violating phases of the Majorana type; i.e.,

\[ V \rightarrow \hat{V} = V P_{\nu} , \]

(35)

where \( P_{\nu} = \text{Diag}\{1, e^{i\rho}, e^{i\sigma}\} \) is a diagonal Majorana phase matrix. Although \( \rho \) and \( \sigma \) have no effect on CP or T violation in normal neutrino-neutrino and antineutrino-antineutrino
oscillations, they are expected to play an important role in the neutrinoless double beta decay, whose effective mass term is given as

$$\langle m_{\nu_e} \rangle = \left| \sum_{i=1}^{3} (m_i V_{ei}^2) \right|.$$  \hfill (36)

The current experimental bound is $\langle m_{\nu_e} \rangle < 0.34$ eV, obtained by the Heidelberg-Moscow Collaboration at the 90\% confidence level \[24\]. For the two nearly bi-maximal lepton mixing scenarios under discussion, $\langle m_{\nu_e} \rangle$ reads as follows:

$$\langle m_{\nu_e} \rangle_{(x)} = \left| \frac{\alpha}{2} c_x^2 + \frac{\beta}{\sqrt{2}} s_x c_x e^{i\phi_x} + \frac{\gamma}{4} s_x^2 e^{i2\phi_x} \right|,$$

$$\langle m_{\nu_e} \rangle_{(z)} = \left| \frac{\alpha}{2} c_z^2 - \frac{\beta}{\sqrt{2}} s_z c_z e^{i\phi_z} + \frac{\gamma}{4} s_z^2 e^{i2\phi_z} \right|,$$

where

$$\alpha = m_1 + m_2 e^{i2\rho},$$

$$\beta = m_1 - m_2 e^{i2\rho},$$

$$\gamma = m_1 + m_2 e^{i2\rho} + 2m_3 e^{i2\sigma}.$$  \hfill (37)

Note that $s_x \approx \sqrt{m_e/m_\mu} \approx 0.069$ and $s_z \approx \sqrt{m_e/m_\tau} \approx 0.017$, therefore $c_x \approx c_z \approx 1$ is an excellent approximation. If the spectrum of neutrino masses were known, one would be able to simplify the expression of $\langle m_{\nu_e} \rangle_{(x)}$ or $\langle m_{\nu_e} \rangle_{(z)}$ and confront it with the present experimental bound. Subsequently let us take four specific but interesting cases of the neutrino mass spectrum for example.

(a) $m_1 \approx m_2 \approx m_3$. In this case, the third term of $\langle m_{\nu_e} \rangle_{(x)}$ or $\langle m_{\nu_e} \rangle_{(z)}$ is negligible. We then arrive at

$$\langle m_{\nu_e} \rangle_{(x)} \approx m_1 \left| \frac{1}{2} \left(1 + e^{i2\rho}\right) + \frac{s_x}{\sqrt{2}} \left(1 - e^{i2\rho}\right) e^{i\phi_x} \right|,$$

$$\langle m_{\nu_e} \rangle_{(z)} \approx m_1 \left| \frac{1}{2} \left(1 + e^{i2\rho}\right) - \frac{s_z}{\sqrt{2}} \left(1 - e^{i2\rho}\right) e^{i\phi_z} \right|. \hfill (39)$$

If $\rho \approx \pm 90^\circ$ holds, we obtain $\langle m_{\nu_e} \rangle_{(x)} \approx \sqrt{2} s_x m_1$ and $\langle m_{\nu_e} \rangle_{(z)} \approx \sqrt{2} s_z m_1$. The experimental bound $\langle m_{\nu_e} \rangle < 0.34$ eV is then assured for $m_1 \leq 3$ eV of pattern $\hat{V}_{(x)}$ or for $m_1 \leq 15$ eV of pattern $\hat{V}_{(z)}$. If the value of $\rho$ is not close to $\pm 90^\circ$, one may obtain $\langle m_{\nu_e} \rangle \approx m_1 |\cos \rho|$ for both lepton mixing patterns. Such a constraint could provide some information on the Majorana phase $\rho$, provided that the magnitude of $m_1$ were already known.

(b) $m_1 \approx m_2 \gg m_3$. One may easily check that the results of $\langle m_{\nu_e} \rangle_{(x)}$ and $\langle m_{\nu_e} \rangle_{(z)}$ in this case are essentially the same as those in case (a).

(c) $m_1 \approx m_2 \ll m_3$. In this case, we obtain $m_3 \approx \sqrt{\Delta m_{21}^2} \approx \sqrt{\Delta m_{3\text{atm}}^2} \leq 0.1$ eV. Then $m_1$ and $m_2$ should be of $\mathcal{O}(10^{-2})$ eV or smaller. Note that the contribution of $m_3$ to $\langle m_{\nu_e} \rangle$ is always suppressed by $s_x$ or $s_z$. Therefore the magnitude of $\langle m_{\nu_e} \rangle$ is at most of $\mathcal{O}(10^{-2})$ eV for either $\hat{V}_{(x)}$ or $\hat{V}_{(z)}$, much smaller than the present experimental bound.
(d) $m_1 \ll m_2 \ll m_3$. This “normal” neutrino mass hierarchy \[\] leads to $m_3 \approx \sqrt{\Delta m_{32}^2} \approx \sqrt{\Delta m_{atm}^2} \leq 0.1$ eV as well as $m_2 \approx \sqrt{\Delta m_{21}^2} \approx \sqrt{\Delta m_{sun}^2} \leq 0.01$ eV, where the upper limit of $m_2$ corresponds to the large-angle MSW solution to the solar neutrino problem. In this case, Eq. (37) can be simplified as
\[
\langle m_{\nu_e} \rangle (x) \approx \frac{1}{2} \left| m_2 e^{i2(\rho - \sigma)} + m_3 s_x^2 e^{i2\phi_x} \right|,
\]
\[
\langle m_{\nu_e} \rangle (z) \approx \frac{1}{2} \left| m_2 e^{i2(\rho - \sigma)} + m_3 s_z^2 e^{i2\phi_z} \right|.
\]
We see that $\langle m_{\nu_e} \rangle \leq O(10^{-2})$ eV must hold for both nearly bi-maximal lepton mixing patterns.

The neutrinoless double beta decay itself is certainly not enough to determine the two Majorana T-violating phases $\rho$ and $\sigma$. One may in principle study some other possible lepton-number-nonconserving processes, in which the Majorana phases can show up, to get more constraints on $\rho$ and $\sigma$. However, all such processes are suppressed in magnitude by an extremely small factor compared to normal weak interactions [15,25]. Hence it seems practically impossible to measure or constrain $\rho$ and $\sigma$ in any experiment other than the one associated with the neutrinoless double beta decay.

VI. SUMMARY

We have discussed two simple possibilities to construct the charged lepton and neutrino mass matrices, from which two almost bi-maximal neutrino mixing patterns can naturally emerge. Both scenarios are favored by the atmospheric neutrino oscillation data, and are compatible with either the large-angle (or low) MSW solution or the vacuum oscillation solution to the solar neutrino problem. While the two lepton mixing patterns have practically indistinguishable consequences on solar and atmospheric neutrino oscillations, their predictions for leptonic CP or T violation are different and distinguishable. Only one of them is likely to yield an observable T-violating asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ transitions in the long-baseline neutrino oscillation experiments. To be specific, we have taken two typical baselines ($L = 730$ km and $L = 2100$ km) to illustrate the magnitude of T violation and its dependence on the terrestrial matter effects. The implications of our nearly bi-maximal neutrino mixing scenarios on the neutrinoless double beta decay have also been discussed in some detail. We expect that a variety of neutrino experiments in the near future could provide crucial tests of the existing lepton mixing models and give useful hints towards the symmetry or dynamics of lepton mass generation.

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\[\]\[The “inverse” neutrino mass hierarchy $m_1 \gg m_2 \gg m_3$, which is apparently in conflict with our choices $\Delta m_{21}^2 \approx \pm \Delta m_{sun}^2$ and $\Delta m_{32}^2 \approx \pm \Delta m_{atm}^2$ in Eq. (1), will not be taken into account in this paper.\]
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FIG. 1. Illustrative plot for matter effects on the universal T-violating parameter $\tilde{J}$, where $\Delta m^2_{21} \approx 5 \cdot 10^{-5} \, \text{eV}^2$, $\Delta m^2_{31} \approx 3 \cdot 10^{-3} \, \text{eV}^2$, and $\phi_x = 90^\circ$ have typically been input.
FIG. 2. Illustrative plot for matter effects on the T-violating asymmetry $\Delta \tilde{P}$ between $\nu_\mu \rightarrow \nu_e$ and $\nu_e \rightarrow \nu_\mu$ transitions, where $\Delta m_{21}^2 \approx 5 \cdot 10^{-5}$ eV$^2$, $\Delta m_{31}^2 \approx 3 \cdot 10^{-3}$ eV$^2$, and $\phi_x = 90^\circ$ have typically been input.