X(4140), X(4270), X(4500), and X(4700) and their $c\bar{s}\bar{c}\bar{s}$ tetraquark partners

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In the simple color-magnetic interaction model, we investigate possible ground $c\bar{s}\bar{c}\bar{s}$ tetraquark states in the diquark-antidiquark basis. We use several methods to estimate the mass spectrum and discuss possible assignment for the X states observed in the $J/\psi\phi$ channel. We find that assigning the Belle X(4350) as a $0^{++}$ tetraquark is consistent with the tetraquark interpretation for the X(4140) and X(4270) while the interpretation of the X(4500) and X(4700) needs orbital or radial excitation. There probably exist several tetraquarks around 4.3 GeV that decay into $J/\psi\phi$ or $\eta_c\phi$.

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I. INTRODUCTION

Recently, several exotic resonances were observed in the invariant mass distributions of $J/\psi\phi$. In the decay process $B^+ \to J/\psi\phi K^+$, the CDF Collaboration found the first evidence of a narrow structure X(4140) with mass $M = 4143.0 \pm 2.9 \pm 1.2$ MeV and width $\Gamma = 11.7^{+8.3}_{-5.0} \pm 3.7$ MeV [1]. Later, the CMS Collaboration [2] and the D0 Collaboration [3] observed structures consistent with the X(4140) in the same process. The D0 Collaboration also observed the structure in $\bar{p}p \to J/\psi\phi +$ anything [4]. However, the Belle [5] and BABAR [6] experiments gave negative results for this state in the B decays. The evidence of a second X(4274) resonance with mass $M = 4274.4^{+8.4}_{-6.7} \pm 1.9$ MeV and width $\Gamma = 32.3^{+21.9}_{-15.3} \pm 7.6$ MeV was also found in $B^+ \to J/\psi\phi K^+$ by the CDF Collaboration [7], which was not confirmed by the BABAR Collaboration [6]. In the $\gamma\gamma \to J/\psi\phi$ process, the Belle Collaboration found the evidence of a narrow state X(4350) [8]. The $B^+ \to J/\psi\phi K^+$ data from D0 also accommodate this structure [3]. In addition, the CMS Collaboration reported the evidence of a state with mass $M = 4313.8 \pm 5.3 \pm 7.3$ MeV and $\Gamma = 38^{+30}_{-15} \pm 15$ MeV in the $B^+ \to J/\psi\phi K^+$ decay [2]. Interested readers may consult the recent review [9].

Very recently, in the $B^+ \to J/\psi\phi K^+$ decay, the LHCb Collaboration confirmed the existence of the X(4140) and X(4274). Their quantum numbers are measured to be $J^{PC} = 1^{++}$ [10]. The mass of the X(4140), $M = 4146.5^{+4.5}_{-2.8}$ MeV, is consistent with the world average $M = 4143.4 \pm 1.9$ MeV, but the width $\Gamma = 83 \pm 21^{+21}_{-14}$ MeV is larger than the existing value $\Gamma = 15.7 \pm 6.3$ MeV. In the same process, the collaboration observed two additional higher resonances, X(4500) and X(4700). Their masses and widths are $M = 4506 \pm 11^{+12}_{-15}$ MeV, $\Gamma = 92 \pm 21^{+21}_{-20}$ MeV and $M = 4704 \pm 10^{+14}_{-21}$ MeV, $\Gamma = 120 \pm 31^{+32}_{-33}$ MeV, respectively. Their quantum numbers are $J^{PC} = 0^{++}$.

It is difficult to understand these X states in the conventional quark-antiquark picture because their decays are expected to be dominated by open charm channels. The proposed theoretical explanations for the X(4140) and the X(4274) include molecules [11–20], compact tetraquark states [21–24], dynamically generated resonances [25, 26], and coupled channel effects [27, 28]. It seems difficult to interpret them with the molecule and cusp scenarios because of the $J^{PC}$ quantum numbers. Compared with the tetraquark configuration, the number of the meson states is reduced.
by half in the molecule configuration since the hidden-color components are ignored. One may turn to the tetraquark picture to understand consistently both the number and the masses of the observed states.

From the calculation with the QCD sum rule in Ref. [29], one may assign the $X(4500)$ and $X(4700)$ as the two $D$-wave $cs\bar{c}\bar{s}$ tetraquark states of $J^P = 0^+$. In another analysis [30, 31], the former state is assigned as the first radially excited state of the $[cs]_1 + [ar{c}\bar{s}]_{1+}$ tetraquark and the latter one as the ground $[cs]_1 - [ar{c}\bar{s}]_{1-}$ tetraquark, but the assignment of $X(4140)$ as a $1^{++}$ diquark-antidiquark meson is disfavored. In Ref. [32], Maiani et al. proposed that the $X(4140)$ and the $X(4274)$ belong to the ground state 1S-multiplet of diquark-antidiquark tetraquarks while the $X(4500)$ and the $X(4700)$ are radially excited 2S states. Since their tetraquark model allows only one $1^{++}$ state, the quantum numbers of the $X(4274)$ are proposed to be $0^{++}$ or $2^{++}$. In Ref. [33], the hidden charm tetraquarks are investigated systematically in a diquark-antidiquark model, where the $X(4140)$ and $X(4274)$ were explained as the $J^P = 1^+$ hidden charm tetraquarks with quark content $\frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})c\bar{c}$. The scalar $X(4700)$ may be explained as the radial excitation of the hidden charm tetraquarks with the same quark content while the $X(4500)$ is its flavor singlet partner. From a potential quark model calculation with the diquark-antidiquark picture [34], the $X(4140)$ [$X(4700)$] can be assigned as the ground (2S excited) tetraquark state. The $X(4500)$ can be explained as a tetraquark composed of one 2S scalar diquark and one scalar antidiquark, while the $X(4274)$ is a good candidate of the $\chi_{c1}(3P)$ charmonium. A rescattering mechanism is used in Ref. [35] to understand the structure of these four $X$ states. This mechanism may explain the $X(4140)$ and $X(4700)$, but fails to generate the $X(4274)$ and $X(4500)$, which leads to the proposal that they are genuine resonances, e.g., $\chi_{c1}(3P)$. In a coupled-channel quark model calculation, the authors of Ref. [36] find that the $X(4140)$ appears as a cusp while the $X(4274)$, $X(4500)$, and $X(4700)$ appear as $3P_1$, $4S_0$, and $5S_0$ charmonium states, respectively.

To understand the structures of the $X$ states decaying to $J/\psi\phi$, we investigate systematically the mass spectrum of the $S$-wave $cs\bar{c}\bar{s}$ system with the chromomagnetic interaction in this work. We consider several schemes when estimating their masses. The paper is organized as follows. In Sec. II, we present the $C$-parity eigenfunctions of different quantum numbers and give the matrices for the chromomagnetic interaction (CMI). In Sec. III, we extract the needed parameters and list the numerical results of the CMI matrices and the mass spectra with different methods. Section IV is a short summary.

II. FORMALISM

We adopt the diquark-antidiquark basis to analyze the $S$-wave $cs\bar{c}\bar{s}$ system, where the color wave function of the diquark belongs to the $3_c$ or $6_c$ representation and the antidiquark belongs to $3_c$ or $6_c$. In the spin space, both the diquark and the antidiquark can be a singlet or triplet state. When we say diquark in our study, it only means two quarks and the notation is convenient for us to organize the wave functions. The meaning is different from that in the diquark model [37], where the diquark is a strongly correlated quark-quark substructure with color=$3$ and spin=$0$.

For the scalar and tensor states, we have

$$J^{PC} = 2^{++}, \quad \phi_1\chi_1 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 + \frac{1}{3}[\bar{c}\bar{s}]_3), \quad \phi_2\chi_1 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 - \frac{1}{3}[\bar{c}\bar{s}]_3)$$

$$J^{PC} = 0^{++}, \quad \phi_1\chi_3 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 + \frac{1}{3}[\bar{c}\bar{s}]_3), \quad \phi_2\chi_3 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 - \frac{1}{3}[\bar{c}\bar{s}]_3)$$

$$\phi_1\chi_6 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 + \frac{1}{3}[\bar{c}\bar{s}]_3), \quad \phi_2\chi_6 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 - \frac{1}{3}[\bar{c}\bar{s}]_3),$$

where the superscripts on the right side of the equation denote the spin and the subscripts the SU(3)$_c$ representation. The notation $\phi_i$ ($\phi_2$) represents the color wave function with the configuration $[3_c]_i \otimes [\bar{3}_c]_i$ ($[6_c]_i \otimes [\bar{6}_c]_i$). The $\chi_i$ ($i = 1, \cdots, 6$) indicate different configurations for the total spin wave functions coupled with the diquark and the antidiquark. As for the axial vector tetraquarks, one can construct two types of eigenstates with opposite $C$ parities. Two bases naturally have negative $C$ parity,

$$J^{PC} = 1^{++}, \quad \phi_1\chi_2 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 - \frac{1}{3}[\bar{c}\bar{s}]_3), \quad \phi_2\chi_2 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 + \frac{1}{3}[\bar{c}\bar{s}]_3)$$

Similar to Ref. [38], we may combine the four bases

$$\phi_1\chi_4 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 + \frac{1}{3}[\bar{c}\bar{s}]_3), \quad \phi_1\chi_5 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 - \frac{1}{3}[\bar{c}\bar{s}]_3),$$

$$\phi_2\chi_4 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 - \frac{1}{3}[\bar{c}\bar{s}]_3), \quad \phi_2\chi_5 = \frac{1}{\sqrt{2}}(\frac{1}{3}[cs]_3 + \frac{1}{3}[\bar{c}\bar{s}]_3),$$

to get the $C$-parity eigenstates

$$J^{PC} = 1^{++}, \quad \phi_1\chi_\rho = \frac{1}{\sqrt{2}}(\phi_1\chi_4 + \phi_1\chi_5), \quad \phi_2\chi_\rho = \frac{1}{\sqrt{2}}(\phi_2\chi_4 + \phi_2\chi_5)$$

$$J^{PC} = 1^{--}, \quad \phi_1\chi_\eta = \frac{1}{\sqrt{2}}(\phi_1\chi_4 - \phi_1\chi_5), \quad \phi_2\chi_\eta = \frac{1}{\sqrt{2}}(\phi_2\chi_4 - \phi_2\chi_5).$$
Where the base is \((\phi)\) which reads \(H\) in mind.

Here the subscript \(p\) \((n)\) means “positive” \((negative)\) \(C\) parity.

In this paper, we adopt the simple chromomagnetic Hamiltonian to estimate the mass spectrum of the \(cs\bar{c}\bar{s}\) system, which reads
\[
H = \sum_i m_i + H_{CM} = \sum_i m_i - \sum_{i<j} C_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j \sigma_i \cdot \sigma_j.
\]

Here, the effective mass \(m_i\) for the \(i\)th constituent quark incorporates not only the usual constituent quark mass but also the effects of the kinetic energy, color confinement, and so on. The effective coupling constants \(C_{ij}\) reflect the strength for the contact interaction. The \(\sigma_i\) \((i = 1, 2, 3)\) are the Pauli matrices while \(\vec{\lambda}_i = \lambda_i (-\lambda_i^*)\) for quark (antiquark) with \(\lambda_i\) \((i = 1, \cdots, 8)\) being the Gell-Mann matrices. With the constructed wave functions, it is not difficult to get the matrices \(\langle H_{CM}\rangle\) for different states.

The above chromomagnetic Hamiltonian should work well in the calculation of the mass splittings between the member states within the same spin-flavor multiplet. If the mass of one state is known, the masses of all the other member states within the same multiplet can be predicted quite reliably. The above chromomagnetic Hamiltonian can also be used to estimate the mass splittings of the two systems when they have similar color-flavor configurations. For example, one can use this Hamiltonian to calculate the mass splitting of two tetraquark states quite reliably.

In the very beginning, we emphasize that the above Hamiltonian is oversimplified. The kinetic energy and confinement interaction are replaced by the constituent quark mass. With such a crude approximation, there certainly exist large systematical errors in the estimate of the overall hadron mass. However, the mass splittings of the two hadron states remain reliable since most of the inherent uncertainties cancel each other. The readers should keep this point in mind.

For the tetraquark states with \(J^{PC} = 2^{++}\), we have
\[
\langle H_{CM}\rangle = \left( \begin{array}{ccc}
\frac{4}{3}(4C_{cs} + C_{c\bar{c}} + 2C_{c\bar{s}} + 2C_{s\bar{s}}) & \frac{2}{3}(4C_{cs} - 2C_{c\bar{c}} + 2C_{s\bar{s}}) & \frac{2}{3}(4C_{cs} - 5C_{c\bar{c}} - 10C_{c\bar{s}} - 5C_{s\bar{s}}) \\
\frac{2}{3}(4C_{cs} + 2C_{c\bar{c}} + 2C_{s\bar{s}}) & \frac{8}{3}(C_{c\bar{c}} - C_{s\bar{s}}) & \frac{20}{3}(C_{c\bar{c}} - C_{s\bar{s}}) \\
\frac{2}{3}(4C_{cs} - 5C_{c\bar{c}} + 2C_{s\bar{s}}) & \frac{20}{3}(C_{c\bar{c}} - C_{s\bar{s}}) & \frac{20}{3}(C_{c\bar{c}} - C_{s\bar{s}})
\end{array} \right),
\]

where the base is \((\phi_1\chi_1, \phi_2\chi_1)^T\).

For the \(J^{PC} = 1^{++}\) case,
\[
\langle H_{CM}\rangle = \left( \begin{array}{ccc}
\frac{4}{3}(4C_{cs} - C_{c\bar{c}} + 2C_{c\bar{s}} - C_{s\bar{s}}) & -2\sqrt{2}(C_{c\bar{c}} - 2C_{c\bar{s}} + C_{s\bar{s}}) & -2\sqrt{2}(C_{c\bar{c}} - 2C_{c\bar{s}} - C_{s\bar{s}}) \\
-2\sqrt{2}(C_{c\bar{c}} - 2C_{c\bar{s}} + C_{s\bar{s}}) & \frac{8}{3}(C_{c\bar{c}} - C_{s\bar{s}}) & \frac{20}{3}(C_{c\bar{c}} - C_{s\bar{s}}) \\
-2\sqrt{2}(C_{c\bar{c}} - 2C_{c\bar{s}} - C_{s\bar{s}}) & \frac{20}{3}(C_{c\bar{c}} - C_{s\bar{s}}) & \frac{20}{3}(C_{c\bar{c}} - C_{s\bar{s}})
\end{array} \right),
\]

where the base is \((\phi_1\chi_p, \phi_2\chi_p)^T\). In the case \(J^{PC} = 1^{+-}\), one gets
\[
\langle H_{CM}\rangle = \left( \begin{array}{ccc}
\frac{4}{3}(4C_{cs} - C_{c\bar{c}} + 2C_{c\bar{s}} - C_{s\bar{s}}) & 2\sqrt{2}(C_{c\bar{c}} - 2C_{c\bar{s}} + C_{s\bar{s}}) & 2\sqrt{2}(C_{c\bar{c}} - 2C_{c\bar{s}} - C_{s\bar{s}}) \\
-2\sqrt{2}(C_{c\bar{c}} - 2C_{c\bar{s}} + C_{s\bar{s}}) & \frac{8}{3}(C_{c\bar{c}} - C_{s\bar{s}}) & \frac{20}{3}(C_{c\bar{c}} - C_{s\bar{s}}) \\
-2\sqrt{2}(C_{c\bar{c}} - 2C_{c\bar{s}} - C_{s\bar{s}}) & \frac{20}{3}(C_{c\bar{c}} - C_{s\bar{s}}) & \frac{20}{3}(C_{c\bar{c}} - C_{s\bar{s}})
\end{array} \right),
\]

where the base is \((\phi_1\chi_2, \phi_2\chi_2, \phi_1\chi_n, \phi_2\chi_n)^T\).

As for the \(J^{PC} = 0^{++}\) case, we have
\[
\langle H_{CM}\rangle = \left( \begin{array}{ccc}
\frac{4}{3}(2C_{cs} - C_{c\bar{c}} + 2C_{c\bar{s}} - C_{s\bar{s}}) & \frac{4}{\sqrt{3}}(C_{c\bar{c}} - 2C_{c\bar{s}} + C_{s\bar{s}}) & \frac{10}{\sqrt{3}}(C_{c\bar{c}} - 2C_{c\bar{s}} - C_{s\bar{s}}) \\
-\frac{4}{3}(2C_{cs} + 5C_{c\bar{c}} + 2C_{c\bar{s}} + 5C_{s\bar{s}}) & 2\sqrt{6}(C_{c\bar{c}} + 2C_{c\bar{s}} + C_{s\bar{s}}) & -\frac{10}{\sqrt{3}}(C_{c\bar{c}} - 2C_{c\bar{s}} - C_{s\bar{s}}) \\
-16C_{cs} & 8C_{cs} & 0
\end{array} \right),
\]

where the base is \((\phi_1\chi_3, \phi_2\chi_3, \phi_1\chi_6, \phi_2\chi_6)^T\).

When deriving these matrices, we have adopted the approach used in Refs. [39, 40]. The spin and color matrix elements are calculated separately with \(H_S = \sum_{i<j} C_{ij} \sigma_i \cdot \sigma_j\) and \(H_C = -\sum_{i<j} C_{ij} \vec{\lambda}_i \cdot \vec{\lambda}_j\), respectively. Then one performs a type of “tensor product” of \(\langle H_S\rangle\) and \(\langle H_C\rangle\) to get the final \(\langle H_{CM}\rangle\). For example, if one obtains...
The matrix elements in spin space are easy to calculate [37]. We here give those in color space
\[
\left( \frac{\langle \phi_1 | H_C | \phi_1 \rangle}{\langle \phi_2 | H_C | \phi_2 \rangle} \right) = \left( \frac{4}{3}(4C_{cs} + 2C_{c\bar{s}} + C_{c\bar{c}} + C_{s\bar{s}}) - \frac{2}{3}(2C_{c\bar{s}} - C_{c\bar{c}} - C_{s\bar{s}}) + \frac{2}{3}(2C_{c\bar{s}} - C_{c\bar{c}} - C_{s\bar{s}}) \right).
\] (10)

With this matrix, it is easy to check the consistency between our formulas and those in the diquark model [37].

### III. MODEL PARAMETERS AND NUMERICAL RESULTS

To estimate the masses of these tetraquark states, we need to extract six parameters: the effective masses \(m_c\) and \(m_s\), and effective coupling constants \(C_{cs}, C_{c\bar{s}}, C_{c\bar{c}},\) and \(C_{s\bar{s}}\). They may be extracted from the known baryons and mesons, where we have assumed that they do not change much from system to system.

The coupling constants rely only on the mass splittings of hadrons. We summarize the adopted hadrons and the obtained coupling constants in Table I. When extracting \(C_{s\bar{s}}\), one has to use the mass of the ground pseudoscalar meson. Since it is affected significantly by chiral symmetry, we use \(C_{s\bar{s}} = C_{s\bar{s}} = 6.4\) MeV for the calculation. This value is determined through \(2m_{q_1} \pm m_{q_1} = (2m_{q_2} \mp m_{q_2}) = 8C_{s\bar{s}} + 8C_{qq}\). With the above parameters, one gets the numerical values for the \(\langle H_{CM} \rangle\) matrices and their eigenvalues and eigenvectors. The results are collected in Table II.

**TABLE I:** Color-magnetic interaction (\(\langle H_{CM} \rangle\)) for various hadrons and the obtained effective coupling constants in units of MeV through the mass differences between the hadrons in the two columns.

| Hadron | CMI | Hadron | CMI | Value |
|--------|-----|--------|-----|-------|
| \(N\)  | \(-8C_{qq}\) | \(\Delta\) | \(8C_{qq}\) | \(C_{qq} = 18.4\) |
| \(\Sigma\) | \(\frac{8}{3}C_{qq} - \frac{4}{3}C_{qq}\) | \(\Sigma^*\) | \(\frac{8}{3}C_{qq} + \frac{4}{3}C_{qq}\) | \(C_{qq} = 12.4\) |
| \(D\) | \(-16C_{qq}\) | \(D^*\) | \(\frac{16}{3}C_{qq}\) | \(C_{qq} = 6.8\) |
| \(D_s\) | \(-16C_{qq}\) | \(D_s^*\) | \(\frac{16}{3}C_{qq}\) | \(C_{qq} = 6.8\) |
| \(\eta_{c}\) | \(-16C_{qq}\) | \(J/\psi\) | \(\frac{16}{3}C_{qq}\) | \(C_{qq} = 5.3\) |
| \(\Sigma_{c}\) | \(\frac{8}{3}C_{qq} - \frac{4}{3}C_{qq}\) | \(\Sigma_{c}^*\) | \(\frac{8}{3}C_{qq} + \frac{4}{3}C_{qq}\) | \(C_{qq} = 4.0\) |
| \(\Xi_c\) | \(\frac{4}{3}C_{qq} - \frac{16}{3}C_{qq}\) | \(\Xi_{c}^*\) | \(\frac{4}{3}C_{qq} + \frac{16}{3}C_{qq}\) | \(C_{qq} = 4.6\) |

**TABLE II:** Color-magnetic interactions for the \(cs\bar{s}\) system in units of MeV.

| \(J^P\) | \(\langle H_{CM} \rangle\) | Eigenvalue | Eigenvector |
|--------|-----------------|------------|-------------|
| 2++    | (58.3, 5.4)     | (73.9)     | (0.32, 0.95) |
|        | (72.1)          | (56.4)     | (0.95, 0.32) |
| 1++    | (27.1, 17.6)    | (5.9)      | (0.62, 0.78) |
|        | (64.9)          | (62.8)     | (0.78, 0.62) |
| 1+-    | (9.2, 0.4)      | (6.2)      | (0.03, 0.93) |
|        | (96.6)          | (7.3)      | (0.12, 0.35) |
|        | (7.2)           | (18.6)     | (0.04, 0.01) |
|        | (18.6)          | (8.3)      | (0.99, 0.07) |
| 0++    | (3.9, 0.4)      | (123.9)    | (0.08, 0.83) |
|        | (180.9)         | (11.0)     | (0.08, 0.83) |
|        | (73.6)          | (0)        | (0.59, 0.01) |
|        | (36.8)          | (7.9)      | (0.05, 0.55) |

We can evaluate the mass spectrum of the \(cs\bar{s}\) system if we know the effective masses \(m_i\). Since they incorporate the quark kinetic energy and confinement effects, in principle, their values are different for various systems and cannot be determined uniformly. However, as a rough estimation, we would extract the effective quark masses from the known baryons. For example, the color-magnetic interaction for the nucleon is \(\langle H_{CM} \rangle = -8C_{qq} (q = u, d)\). From the mass...
formula \( M = \sum_i m_i + \langle H_{CM} \rangle \), one gets \( m_\pi = 361.8 \text{ MeV} \). Similarly, we get \( m_\sigma = 540.4 \text{ MeV} \) from \( M_\Omega = 3m_\pi + 8C_{ss} \). With the values of the coefficients \( C_{qq} \) and \( C_{cq} \) and the formula \( m_c = (3M_{\Sigma^+} - 2M_\Delta - 16C_{qq} + 8C_{cq})/3 \), one obtains \( m_c = 1724.8 \text{ MeV} \).

Before estimating the masses of the \( cs\bar{c}\bar{s} \) system, we take a look at the conventional hadrons with the determined parameters. The calculated masses are listed in Table III. From the values, it is obvious that the obtained hadron masses are larger than the experimental data. The discrepancy can even reach 377 MeV for the mesons. Therefore, the resultant estimations with these effective masses should be taken as a theoretical upper limit.

### TABLE III: Comparison for hadron masses between experimental data and theoretical estimation. All the values are in units of MeV.

| Hadron | Theory | Experiment | Deviation |
|--------|--------|-------------|-----------|
| \( D \) | 1975.9 | 1864.8 | 111.1 |
| \( D_s \) | 2154.5 | 1968.3 | 186.2 |
| \( \eta_c \) | 3361.0 | 2983.6 | 377.4 |
| \( \Sigma_c \) | 2452.9 | 2454.0 | 1.1 |
| \( \Xi_c \) | 2525.9 | 2471.0 | 54.9 |

We here use two methods to discuss the tetraquark masses: (1) substitute all the obtained parameters into the formula \( M = \sum_i m_i + \langle H_{CM} \rangle \); and (2) estimate the results with some reference parameter, i.e. \( M - M_{\text{ref}} = \langle H_{CM} \rangle - \langle H_{CM} \rangle_{\text{ref}} \). It is not necessary to use the effective quark masses with the latter method, where the quark mass effects are partly eliminated. In the present study, one estimates the tetraquark masses with three reference parameters, the threshold of \( D_s^* D_s^{*-} \), the threshold of \( J/\psi \phi \), and the mass of the \( Y(4140) \).

### TABLE IV: Mass spectrum of the \( cs\bar{c}\bar{s} \) system in the effective quark mass method in units of MeV.

| \( J^{PC} \) | Tetraquark mass |
|-------------|-----------------|
| \( 2^{++} \) | \( 4600.5 \) 4583.0 |
| \( 1^{++} \) | \( 4589.5 \) 4442.6 |
| \( 1^{+-} \) | \( 4599.4 \) 4518.3 4453.3 4426.3 |
| \( 0^{+-} \) | \( 4653.8 \) 4534.5 4394.8 4262.6 |

In the effective quark mass method, the mass spectrum for the \( cs\bar{c}\bar{s} \) system is given in Table IV. The highest and the lowest tetraquarks are both scalars. Although the two highest masses 4654 and 4533 MeV in the \( J^{PC} = 1^{++} \) case are not far from the observed \( X(4700) \) and \( X(4500) \), it is difficult to assign the observed scalars as ground tetraquarks since our results are overestimated numbers. The two \( J^{PC} = 1^{++} \) tetraquarks are also 300 MeV higher than the \( X(4140) \) and \( X(4274) \). However, the mass splitting between the two \( 1^{++} \) tetraquarks is consistent with experiments and Stancu’s result [22]. If the overestimation is 300 MeV, it is possible to interpret the \( X(4140) \) and \( X(4274) \) as \( cs\bar{c}\bar{s} \) tetraquark states. As a byproduct, although the matrices \( \langle H_{CM} \rangle \) in the diquark-antidiquark basis are different from those in Ref. [22], the eigenvalues are the same [after correcting typos in Eq. (15) in Ref. [22]] if we use the same effective coupling constants. So one does not need to distinguish the two pictures for the compact \( cs\bar{c}\bar{s} \) system once the diagonalization is performed.

In the second method, we first use the threshold of \( D_s D_s^* \) as a reference parameter. The color-magnetic interaction for the reference system reads \( \langle H_{CM} \rangle_{D_s D_s^*} = -\frac{1}{2} (3C_{cs} - C_{\bar{c}s}) = -72.5 \text{ MeV} \) and the mass of a tetraquark is given by the formula \( M_{\text{tetra}} = m_{D_s} + m_{D_s^*} - \langle H_{CM} \rangle_{D_s D_s^*} + \langle H_{CM} \rangle_{\text{tetra}} \). In the diquark-antidiquark model, one may assume a \([cs]\) substructure with the fixed color representation \( 3 \), (or \( 6_c \)) or consider a general picture that the color representation can also be \( 6_c \) (or \( 3_c \)). The resulting spectra are different. We show both results in Table V. If one uses \( D_s D_s \) or \( D_s^* D_s^* \) as the reference system, one gets the same results. From the table (also Table II), we know that the off-diagonal matrix elements influence significantly the eigenvalues except for the \( J^{PC} = 2^{++} \) case. It is obvious that the mass splitting (33 MeV) between the two \( 1^{++} \) tetraquarks without color mixing is much smaller than experiments. Therefore, one cannot understand the two \( 1^{++} \) states in the ground diquark-antidiquark picture if the color mixing is not included. If the observed mesons are really tetraquark states, the obtained masses are around 70 MeV lower than experimental measurements. Recall that the \( H\)-dibaryon was found to be stable when one uses only the color-spin interaction [41] while no evidence for its existence is observed. This situation indicates that the present method needs improvement. A possible contribution to fix this discrepancy is the additional kinetic energy in forming a compact
TABLE V: Mass spectrum of the $cs\bar{s}\bar{s}$ system in units of MeV by using the threshold of $D_cD^{*}_c$ as a reference parameter. Part 1 (Part 2) is the case in which the mixing between $3_c$ and $6_c$ for the diquark $[cs]$ is (not) considered.

| $J^{PC}$ | Part 1   | Part 2   |
|---------|-----------|-----------|
| $2^{++}$ | 4226.9  | 4209.4  |
| $1^{++}$ | 4215.8  | 4068.9  |
| $1^{+}$  | 4225.8  | 4144.6  |
| $0^{++}$ | 4280.2  | 4160.8  |

Now we evaluate the masses of the possible $cs\bar{s}\bar{s}$ tetraquark states with the $J/\psi\phi$ threshold. Since $\langle H_{CM} \rangle_{J/\psi\phi} = \frac{16}{3} (C_{cc} + C_{ss}) = 46.2$ MeV, the obtained tetraquark masses are about 99 MeV below the values of part 1 in Table V. This number is from the change of the reference parameter that results in $\delta = (m_{D_c} + m_{D^{*}_c} - \langle H_{CM} \rangle_{D_cD^{*}_c} - (m_{J/\psi} + m_{\phi} - \langle H_{CM} \rangle_{J/\psi\phi}) \approx 99$ MeV. In principle, the tetraquark masses should not change with the choice of the reference parameter. However, the structures of a quarkonium and a heavy-light meson are different, and thus the kinetic energies and the confinement strengths are different. One cannot consider such differences in the present method.

In the diquark-antidiquark picture, the charm quark and anticharm quark can be a color-singlet state or a color-octet state. We list the obtained values in Table VII. For comparison, we present these tetraquarks, the observed mesons with the same quantum numbers, and the quark model predictions with relevant $J^{PC}$ in Fig. 1. Various meson-antimeson...
thresholds are also shown. From the figure, the two $1^{++}$ states, $X(4140)$ and $X(4274)$, are consistent with a tetraquark interpretation. It is very interesting to note that the $X(4350)$ state observed by the Belle collaboration [8] is consistent with the highest scalar tetraquark with the mass 4358 MeV. If the quantum numbers of this state can be confirmed to be $J^{PC} = 0^{++}$, it is very likely that more states in the $J/\psi \phi$ or $\eta_c \phi$ invariant mass distribution could be observed. At least four states exist around 4.3 GeV.

TABLE VII: Mass spectrum of the $cs\bar{c}\bar{s}$ system in units of MeV by assigning the $X(4140)$ as the lowest $1^{++}$ state.

| $J^{PC}$ | Tetraquark mass |
|----------|-----------------|
| $2^{++}$ | 4304.4 4286.9   |
| $1^{++}$ | 4293.4 4146.5   |
| $1^{+-}$ | 4303.3 4222.2 4157.2 4130.2 |
| $0^{++}$ | 4357.7 4238.4 4098.7 3966.5 |

In the simple color-magnetic interaction model, we have analyzed the spectrum of the possible ground $cs\bar{c}\bar{s}$ tetraquark system. We use a diquark-antidiquark basis and find that the obtained mass splittings are the same as those in the $(c\bar{c}) - (s\bar{s})$ basis [22], which indicates that it is not necessary for us to care about the substructure for the compact $cs\bar{c}\bar{s}$ system. However, this conclusion is not applicable to the other compact multiquark systems, which rely on the coupling strengths between (anti)quarks.

Because the effective quark masses contain contributions from kinetic energy, confinement, etc., it is almost impossible to find a universal set of values for different hadron systems. We first tried to estimate tetraquark masses with...
the effective quark masses derived from the conventional hadrons. One gets the overestimated results which can be treated as the upper limits of the tetraquark masses.

In order to partly cancel the uncertainty from the quark masses, we use the threshold of $D_1D_1^*$ (or $D_sD_s$) as a reference parameter. The results are about 70 MeV lower than the experimental masses. If one uses the threshold of $J/\psi\phi$, much lower masses are obtained that can be treated as the lower limits of the tetraquark masses. Probably the inclusion of corrections from kinetic energy and confinement may fix the discrepancies.

If the $X(4140)$ is identified as the lowest $J^{PC} = 1^{++}$ $cs\bar{c}\bar{s}$ tetraquark, we get a consistent assignment that the $X(4274)$ could be another $1^{++}$ tetraquark and the $X(4350)$ could be the highest $0^{++}$ tetraquark.

Although the overestimation for the masses is around 300 MeV in the first method, underestimation is around 70 MeV in the second scheme, and underestimation is around 160 MeV in the third scheme, the mass splittings between these $cs\bar{c}\bar{s}$ tetraquark states in different approaches are consistent with each other.

In determining the parameters, we have used the hypothesis that the effective coupling constants in the conventional hadrons can be applied to multiquark states. One should note that whether this direct extension is appropriate needs further investigations. The reason is that the couplings are proportional to the overlap function of the two constituents, $|\psi(0)|^2$, and no principle says that the functions are the same for all types of hadrons. To investigate the spectrum of possible tetraquarks without this hypothesis, a “type-II” diquark was proposed in Ref. [44], where the spin-spin interaction inside the diquark is much stronger than other possible pairing and diquarks are more compact ingredients.

Even with this hypothesis, the different methods to estimate the $C_{cs}$ may lead to the slightly different tetraquark masses. With the formulae in Sec. II, one obtains the correspondence between our parameters and those used in Ref. [37] is: $(\kappa_{cs})_3 \leftrightarrow \frac{16}{3} C_{cs}$ (22 MeV $\leftrightarrow$ 24.5 MeV), $\kappa_{s\bar{s}} \leftrightarrow \frac{5}{3} C_{s\bar{s}}$ (30 MeV $\leftrightarrow$ 17.1 MeV), $\kappa_{c\bar{s}} \leftrightarrow \frac{8}{3} C_{c\bar{s}}$ (18 MeV $\leftrightarrow$ 18.1 MeV), and $\kappa_{c\bar{c}} \leftrightarrow \frac{5}{3} C_{c\bar{c}}$ (15 MeV $\leftrightarrow$ 14.1 MeV). If we use the parameters consistent with that work, one gets the overestimated results which can be treated as the uncertainty around tens of MeV. Considering the uncertainties in both theoretical estimation and experimental measurement, the discussions in this paper are not affected largely.

To summarize, the $X(4500)$ and $X(4700)$ are not good candidates of the $S$-wave $cs\bar{c}\bar{s}$ tetraquark states. A $D$-wave excitation, two $P$-wave excitations, or a radial excitation is needed to understand the structure of these two higher scalars. On the other hand, the $X(4140)$, $X(4274)$, and $X(4350)$ are consistent with the tetraquark assignment.

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**FIG. 2:** Same as Fig. 1. The masses of the partner states of the LHCb $X(4140)$ in the tetraquark picture are estimated with $C_{cs} = 4.2$ MeV, $C_{s\bar{s}} = 11.3$ MeV, $C_{c\bar{s}} = 6.8$ MeV, and $C_{c\bar{c}} = 5.6$ MeV.

To summarize, the $X(4500)$ and $X(4700)$ are not good candidates of the $S$-wave $cs\bar{c}\bar{s}$ tetraquark states. A $D$-wave excitation, two $P$-wave excitations, or a radial excitation is needed to understand the structure of these two higher scalars. On the other hand, the $X(4140)$, $X(4274)$, and $X(4350)$ are consistent with the tetraquark assignment.
There exist at least three states around 4.3 GeV, which may be observed in the J/ψφ or ηcφ channel. Below the J/ψφ threshold (4120 MeV), there may exist two scalar cs¯s¯s tetraquark states as shown in Table VII. These states should be narrow and can be searched for in the J/ψπππ or radiative decay channels.

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