Alignment Timescale of the Microquasar GRO J1655-40

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1 INTRODUCTION

Microquasars are X-ray binaries which have relativistic radio jets (Mirabel & Rodriguez 1999). They consist of a compact object such as a black hole or a neutron star which is accreting matter from a companion star. GRO J1655–40 is one of only fifteen observed microquasars in our galaxy (Paredes 2005) and has a black hole of mass $M_1 = 6.3 \pm 0.5 M_\odot$ and a companion star of mass $M_2 = 2.4 \pm 0.4 M_\odot$ (Greene, Bailyn & Orosz 2001).

GRO J1655–40 shows transient behaviour in a complex form. It was discovered in July 1994 (Zhang et al. 1994) through observations of hard X-ray outbursts and then observed again several times in the following year. Before 1994 it appeared to be quiescent for some time. The original outbursts were associated with radio flaring and superluminal motion of the radio plasmoids which led Hjellming & Rupen (1995) to estimate the distance to the system to be 3.2 kpc. Israeli et al. (1999) measured the abundances of oxygen, magnesium, silicon and sulphur in the atmosphere of the companion star to be six times greater than in the Sun. The secondary star does not have enough mass to reach the internal temperatures required to create these elements. A stellar wind or mass transfer from the primary during its presupernova evolution could only have provided a small fraction of these observed overabundances because the CNO cycle would have led to a much larger nitrogen to oxygen ratio than is observed. So these abundances are interpreted as evidence that supernova ejecta have been captured by the secondary star, perhaps during fall-back on to the black hole. The relative abundances of the contaminating elements led Israeli et al. (1999) to suggest that the black hole’s progenitor was a star with mass $25 - 40 M_\odot$. It is therefore likely that the system lost more than half its mass in the supernova even if the progenitor of the black hole had had a strong stellar wind. In order for the system to have remained bound the supernova explosion must have been associated with a substantial kick. Such a kick could have altered the spin of the black hole leaving it misaligned with the binary orbit. Even a small velocity kick can lead to a large offset between the spin of the black hole and the orbital axes (Brandt & Podsiadlowski 1995). There is no reason why a kick substantial enough to keep the system bound would leave the spin aligned with the orbit.

GRO J1655–40 has relativistic jets of material leaving the system which we assume are generated in the inner parts of a black hole accretion disc and so are perpendicular to the plane of the disc close to the hole. The combined action of the Lense-Thirring effect and the internal viscosity of the accretion disc causes the angular momenta of the black hole and the inner accretion disc to align. This is known as the Bardeen & Petterson (1975) effect. It affects only the inner regions of the disc because of the short range of the Lense-Thirring effect. The outer parts of the disc tend to remain in their original configuration. Hjellming & Rupen (1995) measured the jet inclination to be $85^\circ \pm 2^\circ$ to the line of sight. This differs significantly from the binary orbital plane inclination of $70.2^\circ \pm 1.9^\circ$ (Greene, Bailyn & Orosz 2001). This implies that there is a misalignment between the inclination of the black hole and the outer parts of the accretion disc and so the accretion

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Figure 1. We consider the position of the jet fixed relative to the line of sight. The cone represents the surface on which the binary orbital angular momentum must lie. The smallest possible inclination between the black hole and the binary angular momentum occurs in position a where \( \eta = 15^\circ \). The largest inclination between the black hole and the binary angular momentum occurs in position c where \( \eta = 165^\circ \).

disc is warped. The inner parts are aligned with the black hole spin and the outer parts are aligned with the binary orbital plane because of tidal torques.

In Figure 1 we show the line of sight and the given position of the jet. Relative to this jet, the angular momentum vector of the binary must lie in one of the cones either pointing towards or away from us. The smallest possible inclination between the black hole and the binary angular momentum, \( \eta \), occurs in position a where \( \eta = 15^\circ \) and the system is close to alignment. The largest possible inclination between the black hole and the binary angular momentum occurs in position c where \( \eta = 165^\circ \) and the system is close to counter-alignment. The inclination of the system is in the range \( 15^\circ < \eta < 165^\circ \).

King et al. (2005) find that for a black hole that is massive compared to a misaligned disc, then the disc aligns with the black hole. If the disc is massive compared with the black hole then the black hole aligns with the disc. In GRO J1655–40 the disc is much less massive than the black hole. However because the disc is in, and is tidally linked to the orbital motion of, a binary system, what matters is the amount of angular momentum in the binary orbit, not just in the disc. The orbital angular momentum is several hundred times that of the black hole so we expect the hole to tend to eventually align with the orbit.

Maccarone (2002) uses the model of Natarajan & Pringle (1998) to calculate the timescale on which the black hole aligns with the outer disc and hence the binary orbit. This assumes that the surface density and viscosities are constant. He finds that the ratio of alignment timescale to binary lifetime is about 0.3. He assumes the binary lifetime is the time to accrete the whole star, not the stellar evolution time. Using the outburst history of GRO J1655–40, Maccarone (2002) finds the alignment timescale to be \( 8 \times 10^8 \) yr with a companion of main-sequence lifetime of \( 7 \times 10^8 \) yr. He concludes that the system should be approximately one alignment timescale in age and so the black hole should not be fully aligned with the disc because it takes more than one alignment timescale for full alignment.

We examine in more detail the stellar evolution of the companion star to estimate how long the system can remain in this steady state and whether the system should be aligned or not. We make use of the analysis of Martin, Pringle & Tout (2007) and use power laws in distance from the central black hole for the surface density and viscosities to find the alignment timescale of the system. We consider the possibility that the disc was initially counter-rotating.

2 MISALIGNMENT

Because the binary is not visually separated we only know the inclination of the orbit relative to the line of sight. We do not know its projection on to the sky. Given the observed inclination of the binary orbit and the jets to the line of sight we can find the probability distribution of the angle between the two momenta.

If the position angle, \( \phi \), of the binary relative to the line of sight (see Figure 1) is randomly distributed in \( 0 < \phi < 2\pi \) then

\[
P(\phi) d\phi = \frac{1}{2\pi} d\phi.
\]

Since

\[
P(\eta) d\eta = P(\phi) d\phi
\]

we deduce

\[
P(\eta) = P(\phi) \frac{d\phi}{d\eta} = \frac{1}{2\pi} \frac{d\phi}{d\eta}
\]

Note that each value of \( \eta \) maps to two values of \( \phi \). If the binary angular momentum is pointing towards us we have the standard spherical trigonometric cosine formula

\[
\cos \eta = \cos i \cos j + \sin i \sin j \cos \phi.
\]

where \( i = 85^\circ \) is the inclination of the jet to the line of sight and \( j = 70^\circ \) is the inclination of the binary orbit to the line of sight. Differentiating we find

\[
\frac{d\phi}{d\eta} = \frac{\sin \eta}{\sin i \sin j \sin \phi}.
\]

If the binary orbital angular momentum is pointing away from us then we have the relation

\[
\cos \eta = \cos i \cos(\pi - j) + \sin i \sin(\pi - j) \cos \phi
\]

and we find

\[
\frac{d\phi}{d\eta} = \frac{\sin \eta}{\sin i \sin(\pi - j) \sin \phi}.
\]

In Figures 2 and 3 we plot the probability distributions for the two cases where the binary angular momentum is pointing towards and away from us. We assume either is equally likely. We see that if the binary points towards us, the most likely misalignment angles are the extremes of \( \eta = 15^\circ \) (case a in Figure 1) or \( 155^\circ \) (case b) and if the binary points away then the most likely angles are \( \eta = 25^\circ \) (case d) or \( 165^\circ \) (case c).
3 WARPED ACCRETION DISCS

There are two viscosities in the disc, $\nu_1$ corresponds to the azimuthal shear (the viscosity normally associated with accretion discs) and $\nu_2$ corresponds to the vertical shear in the disc which smooths out the twist. The second viscosity acts when the disc is non-planar. We assume that we have a steady state disc in which $\nu_1 \Sigma = \text{const}$ and that the surface density is a power law

$$\Sigma = \Sigma_0 \left( \frac{R}{R_0} \right)^{-\beta}$$  \hspace{1cm} (8)

(Shakura & Sunyaev 1973), where $R$ is the spherical radial coordinate, $R_0$ is some fixed radius and $\Sigma_0$ is a constant. To be in steady state, the first viscosity must obey

$$\nu_1 = \nu_{10} \left( \frac{R}{R_0} \right)^{\beta},$$  \hspace{1cm} (9)

where $\beta$ and $\nu_{10}$ are constants. We assume that the second viscosity also obeys the power law

$$\nu_2 = \nu_{20} \left( \frac{R}{R_0} \right)^{\beta}$$  \hspace{1cm} (10)

and $\nu_{20}$ is a constant.

Following the work of Martin, Pringle & Tout (2007), we consider a black hole of mass $M_1$ at $R = 0$ with spin angular momentum $J$. We consider the disc to be made up of annuli of width $dR$ and mass $2\pi \Sigma R dR$ at radius $R$ from the central object of mass $M_1$ with surface density $\Sigma(R, t)$ at time $t$ and with specific angular momentum density $L = (GM_1 R)^{1/2} \Omega = RL$. The unit vector describing the direction of the angular momentum of a disc annulus is $l = (l_x, l_y, l_z)$ with $|l| = 1$. We use equation (2.8) of Pringle (1992) setting $\partial L / \partial t = 0$ and add a term to describe the Lense-Thirring precession (the last one) to give

$$0 = \frac{1}{R} \frac{\partial}{\partial R} \left( \frac{3R \partial}{L \partial R} (\nu_1 L) - \frac{3}{2} \nu_1 \right) L + \frac{1}{2} \nu_2 RL \frac{\partial l}{\partial R} - \omega_p \times L R^3.$$  \hspace{1cm} (11)

The Lense-Thirring precession is given by

$$\omega_p = \frac{2GJ}{c^2}$$  \hspace{1cm} (12)

(Kumar & Pringle 1985), where the angular momentum of the black hole can be expressed in terms of the dimensionless spin parameter $a$ such that

$$J = acM_1 \left( \frac{GM_1}{c^2} \right).$$  \hspace{1cm} (13)

The black hole spin is also evolving because of the torques exerted by the disc so we have

$$\frac{dJ}{dt} = -2\pi \int_{R_{\text{out}}}^{R_{\text{in}}} \frac{\omega_p \times L}{R^3} R dR,$$  \hspace{1cm} (14)

where the integration is done over the surface of the disc. Martin, Pringle & Tout (2007) solve equation (11) to find the disc profile and then use equation (13) to find the timescale for alignment of the black hole with the binary orbit. For $\beta = 0$ this is

$$t_{\text{align}}(0) = \frac{1}{\sqrt{2\pi \Sigma}} \left( \frac{acM_1}{\nu_2 G} \right)^\beta,$$  \hspace{1cm} (15)

(Scheuer & Feiler 1996), where $\Sigma$ and $\nu_2$ must be evaluated at the warp radius. The warp radius is found by balancing the Lense-Thirring precession term with the viscous term associated with the second viscosity in equation (11) to find

$$R_{\text{warp}} = \frac{2}{\nu_2} \frac{\omega_p}{(R_{\text{warp}}).}$$  \hspace{1cm} (16)
We know that the inclination of the black hole to the orbital plane is in the range $15^\circ < \eta < 165^\circ$. We let $\eta$ be the current inclination of the black hole to the outer disc. The outer disc is in the binary orbital plane. We let $\eta_0$ be the inclination when the system first started transferring mass. If $\eta_0 < \pi/2$ the system started closer to alignment whereas if $\eta_0 > \pi/2$ the system started closer to counter-alignment than alignment. The system is always aligning so $\eta < \eta_0$ even if it is counter-aligned. We use the work of Martin, Pringle & Tout (2007) to find that

$$\sin \eta = \sin \eta_0 e^{-t_{\text{accrete}}/t_{\text{align}}},$$

(21)

where $t_{\text{accrete}}$ is the time for which the system has been steadily transferring mass, if $\eta_0 < \pi/2$ and the system started nearer to alignment than counter-alignment. However, if the system started closer to counter-alignment then

$$\sin \eta = \sin \eta_0 e^{t_{\text{accrete}}/t_{\text{align}}},$$

(22)

and $\eta_0 > \pi/2$. We estimate $t_{\text{accrete}}$ from binary star models in Section 6 below.

### 4.1 Co-rotating Disc and Black Hole

If the system started closer to alignment than counter-alignment so that $\eta_0 < \pi/2$ then we find the time that the accretion must last for, by inverting equation (22), to be

$$t_{\text{accrete}} = t_{\text{align}} \log \left( \frac{\sin \eta_0}{\sin \eta} \right).$$

(23)

However if the disc was initially counter-rotating so that $\eta_0 > \pi/2$ and now $\eta < \pi/2$ then we must find the time that it takes to move from $\eta_0$ to $90^\circ$ using equation (22) and then add on the time it takes to move from $90^\circ$ to $15^\circ$ using equation (21). Then we find

$$t_{\text{accrete}} = -t_{\text{align}} \log \left( \frac{\sin \eta_0 \sin \eta}{\sin \eta} \right).$$

(24)

As an example, in Fig. 4 we see how the initial inclination of the disc varies with the length of time it has been steadily accreting for a disc which currently is at an inclination of $15^\circ$.

### 4.2 Counter-rotating Disc and Black Hole

If the disc is now closer to counter-alignment than alignment so that $\eta > \pi/2$ then, using equation (22), we find the initial inclination of the system as a function of the time for which it has been accreting to be

$$t_{\text{accrete}} = t_{\text{align}} \log \left( \frac{\sin \eta}{\sin \eta_0} \right).$$

(25)

In Fig. 5 we see how the initial inclination of the disc varies with the length of time it has been steadily accreting for for a disc which is now at an inclination of $165^\circ$.

As illustrations we examine further these two extreme cases. If the inclination of the black hole relative to the binary orbit is random then the probability that it formed at $15^\circ$ or less is only 0.017. Similarly the probability that it formed at $165^\circ$ or more is only 0.017. It is therefore likely that it has undergone some but not excessive alignment and we would expect $t_{\text{accrete}} \approx t_{\text{align}}$. We note that a completely counter-aligned black hole is in an unstable equilibrium so it is possible that we now observe the hole counter-aligned at an inclination of $165^\circ$. However this would have required both the unlikely event of forming very close to counteralignment and the accretion timescale to be small compared with the alignment timescale. It is therefore more likely that the
hole is now only 15° from alignment. We cannot tell how misaligned it started but it may well have been counter-aligned in the past.

5 Warped Disc Model of GRO J1655–40

In this section we find the viscosities, the warp radius and the alignment timescale of GRO J1655–40. The period of the system is 2.624 d (Greene, Bailyn & Orosz 2001). By Kepler’s law the separation is \( R_B = 1.14 \times 10^{12} \) cm for a total mass of \( 8.7 \, M_\odot \). Abramowicz & Kluzniak (2001) estimate that the dimensionless spin of the black hole \( a = 0.2 - 0.67 \), while Wagoner, Silbergleit & Ortega-Rodriguez (2001) suggest that it could be as high as 0.92.

We take the outer truncation radius of the disc, \( R_{\text{out}} \), to be the largest possible stable orbit of a test particle in this binary system. We use parameters derived by Paczynski (1971) for the three body system. In Fig. 6 we plot the truncation radius against the mass ratio of the two stars. GRO J 1655–40 has mass ratio \( M_2/M_1 = 2.4/6.3 = 0.38 \) which corresponds to a truncation radius of \( R_{\text{out}} = 0.37 R_B = 4.22 \times 10^{11} \) cm which is the outer edge of our disc. This outer edge is used later to find timescales in the disc.

5.1 Viscosities

We use the \( \alpha \)-prescription to find the viscosity of the disc (Shakura & Sunyaev 1973). In bright X-ray binaries the dominant radiative opacity is bound free. Using the steady state mass accretion rate, \( \dot{M} = 3 \pi \nu \Sigma \), Wijers & Pringle (1999) find the viscosity to be

\[
\nu_j = 6.40 \times 10^{15} \alpha_j^4 \left( \frac{M_1}{M_\odot} \right)^{\frac{1}{2}} \left( \frac{\dot{M}}{10^{-8} \, M_\odot \, \text{yr}^{-1}} \right)^{\frac{3}{4}} \times \left( \frac{R}{10^{11} \text{cm}} \right)^{\frac{5}{4}} \text{cm}^2 \text{g}^{-1},
\]

where \( j = 1, 2 \), \( \alpha_j \) is the usual \( \alpha \)-prescription for viscosity \( \nu = \alpha c_s H \) and \( \dot{M} \) is the accretion rate on to the black hole. We take \( \alpha_1 = 0.2 \) and \( \alpha_2 = 2 \) (Lodato & Pringle 2007). This is equivalent to equations (26) and (27) with \( R_0 = 10^{11} \) cm and

\[
\nu_{10} = 1.11 \times 10^{16} \left( \frac{M_1}{0.2} \right)^{\frac{1}{2}} \left( \frac{M_1}{0.3 \, M_\odot} \right)^{\frac{1}{4}} \left( \frac{\dot{M}}{10^{-8} \, M_\odot \, \text{yr}^{-1}} \right)^{\frac{3}{4}} \text{cm}^2 \text{g}^{-1},
\]

and

\[
\nu_{20} = 7.03 \times 10^{15} \left( \frac{M_1}{0.2} \right)^{\frac{1}{2}} \left( \frac{M_1}{0.3 \, M_\odot} \right)^{\frac{1}{4}} \left( \frac{\dot{M}}{10^{-8} \, M_\odot \, \text{yr}^{-1}} \right)^{\frac{3}{4}} \text{cm}^2 \text{g}^{-1}.
\]

The viscous timescale associated with \( \nu_1 \) is the timescale on which matter is moved through the disc. For GRO J1655–40 this is

\[
t_{\nu_1} = \frac{R^2}{\dot{M}} = 1.7 \left( \frac{\alpha_1}{0.2} \right)^{\frac{3}{4}} \left( \frac{M_1}{0.3 \, M_\odot} \right)^{\frac{1}{4}} \left( \frac{\dot{M}}{10^{-8} \, M_\odot \, \text{yr}^{-1}} \right)^{\frac{3}{4}} \left( \frac{R}{R_{\text{out}}} \right)^{5/4} \text{yr}.
\]

This is a relatively short timescale and so the disc reaches steady state quickly.
5.2 Warp Radius

We find the warp radius to be

$$R_{\text{warp}} = \frac{2 \omega_p}{\nu_2(R_{\text{warp}}/\nu_2)\frac{\nu_2}{c}}$$

(Scheuer & Feiler 1996; Martin, Pringle & Tout 2007) so that

$$R_{\text{warp}} = 4.33 \times 10^8 \left( \frac{a}{0.5} \right) \frac{\nu}{\nu_2} \left( \frac{a_2}{a} \right)^{-\frac{3}{2}} \left( \frac{M_1}{6.3 M_{\odot}} \right)^{\frac{7}{4}} \left( \frac{M}{10^{-6} M_{\odot} \text{yr}^{-1}} \right)^{-\frac{3}{4}} \text{cm.}$$

We note that this is not strongly dependent on $\dot{M}$ and so the outbursts of the system do not affect it greatly. We find the viscosities and surface density at the warp radius so that we can find the alignment timescale of the black hole with the disc.

5.3 Timescale of Alignment

In Section 3 we derived a formula for the timescale on which the black hole aligns with the disc. This assumes that the angular momentum of the disc is much greater than that of the black hole. In GRO J1655–40 the outer parts of the disc are locked to the binary orbital plane by tidal torques. It is therefore the angular momentum of the binary orbit, which is much greater than that of the black hole, that matters because the disc is constantly fed with matter at its outer edge.

We can now evaluate the timescale of alignment because we know the viscosities and the warp radius at which we evaluate them. For a steady state disc we have $\beta = \frac{7}{4}$ (from equations (9) and (20)) and so we find

$$t_{\text{align}}(3/4) = t_{\text{align}}(0) \left( \frac{4}{7} \right)^{\frac{3}{4}} \frac{\Gamma(2/7)}{\sqrt{2} \pi (5/7) \cos \left( \frac{\pi}{7} \right)}$$

$$= 1.5233 t_{\text{align}}(0)$$

and

$$t_{\text{align}}(0) = 9.75 \times 10^9 \left( \frac{a}{0.5} \right)^{\frac{7}{4}} \left( \frac{a_1}{a_2} \right)^{\frac{3}{4}} \left( \frac{a_2}{2} \right)^{-\frac{4}{7}}$$

$$\times \left( \frac{M_1}{6.3 M_{\odot}} \right)^{\frac{7}{4}} \left( \frac{M}{10^{-6} M_{\odot} \text{yr}^{-1}} \right)^{\frac{3}{4}} \text{yr} \quad (33)$$

so that

$$t_{\text{align}}(3/4) = 1.49 \times 10^7 \left( \frac{a}{0.5} \right)^{\frac{7}{4}} \left( \frac{a_1}{a_2} \right)^{\frac{3}{4}} \left( \frac{a_2}{2} \right)^{-\frac{4}{7}}$$

$$\times \left( \frac{M_1}{6.3 M_{\odot}} \right)^{\frac{7}{4}} \left( \frac{M}{10^{-6} M_{\odot} \text{yr}^{-1}} \right)^{\frac{3}{4}} \text{yr}. \quad (34)$$

In Fig. 7 we plot the timescale for alignment against the spin of the black hole, $a$, for a range of accretion rates. In Fig. 8 we plot the timescale of alignment for the black hole against the accretion rate for a range of spins. In the next section we consider the time for which the mass transfer can take place to see whether the black hole should be aligned or not.

6 BINARY EVOLUTION MODELS

For our equilibrium warped disc model to be valid, the viscous timescale in the disc (about 1.7 yr as found in section 5.1) must be much less than the time for which material has been accreting through it. We note that there has been time since the 1994 outburst for the disc to regain equilibrium. Outbursts have occurred more frequently in re-
cent years but the system generally returns to quiescence for sufficiently long periods for the disc to regain equilibrium. Because we still see the jet misaligned, the timescale for alignment must be long enough relative to the accretion timescale. We therefore investigate the evolutionary state of the system to determine both the expected mass-transfer rate and the time since mass transfer began.

Early attempts to make detailed evolution models of GRO J1655–40 met with the problem that stars at the secondary’s position in the Hertzsprung-Russell diagram ought to be in the Hertzsprung gap and so evolving from the end of their main sequence to the base of their giant branch on a thermal timescale (Kolb et al. 1997). As a consequence the mass-transfer rate should be too high for the standard soft X-ray transient explanation of the outbursts for which van Paradijs (1996) found that the average accretion rate on to the black hole in GRO J1655–40 should be less than 1.26 × 10^{-10} M_⊙ yr^{-1}. Though we show that we can easily make models in which the secondary star is still on the main sequence, we still find that such a low mass-transfer rate is inconsistent with any Roche-lobe filling system.

Greene, Bailyn & Orosz (2001) have carefully observed the secondary star in GRO J1655–40 during quiescence. Their observations confirm that the secondary star is filling its Roche lobe and are consistent with somewhat lower masses for both the secondary star and the black hole. Indeed we find that we must put the secondary star right at the lower end of their estimated mass range (M_2 = 2.4±0.4 M_⊙) to fit its position in the Hertzsprung-Russell diagram at all well. The observed mass function (Shahbaz et al. 1999) is

\[ f(M_2) = \frac{(M_1 \sin i)^3}{(M_1 + M_2)^2} = 2.73 ± 0.09 M_⊙, \]  

(35)

where M_1 is the mass of the black hole and i = 70.2° ± 1.9° (Greene, Bailyn & Orosz 2001) is the inclination of the binary orbit. In our models we choose the current secondary mass M_2 and determine the current mass of the black hole M_1 from the mass function and hence the total mass M_1 + M_2 of the system. We choose the initial secondary mass M_2, the mass just before mass transfer begins. Assuming conservation of mass and angular momentum during mass transfer we have

\[ P(M_1 M_2)^3 = \text{const} \]  

(36)

so we can calculate the initial period P_i needed to give the current period of 2.62168 ± 0.00014 d (van der Hooft et al. 1998). Greene, Bailyn & Orosz (2001) measure a radius for the secondary star of 5.0 ± 0.3 R_⊙ which we combine with their chosen effective temperature of 336 K to give a luminosity L in the range 1.427 < log_{10}(L/L_⊙) < 1.667.

We use the latest version of the Cambridge STARS code to construct detailed evolutionary models of the secondary star. The code was originally written by Eggleton (1972, 1973). The equation of state, which includes molecular hydrogen, pressure ionization and coulomb interactions, is discussed by Pols et al. (1995). The initial composition is taken to be uniform with a hydrogen abundance X = 0.7, helium Y = 0.28 and metals Z = 0.02 with the meteoritic mixture determined by Anders & Grevesse (1989). Hydrogen burning is allowed by the pp chain and the CNO cycles. Reaction rates are taken from Caughlan & Fowler (1988).

Figure 9. The evolutionary track in the Hertzsprung-Russell diagram of a 2.5 M_⊙ star in a binary and which is forced to transfer mass to its companion when it fills its Roche lobe which occurs while crossing the Hertzsprung gap. The initial period and primary mass are chosen so that the period and masses fit those of GRO J1655–40 when the secondary reaches 2 M_⊙. The dotted line and right-hand scale give the mass-transfer rate as a function of temperature. The point with error bars is the observed secondary in GRO J1655–40 and the open circle is the model with M_2 = 2 M_⊙ at the observed period.

Opacity tables are those calculated by Iglesias & Rogers (1996) and Alexander & Ferguson (1994).

Foellmi et al. (2006) argue the source must be much nearer than typical estimates of 3.2 kpc. However, Bailyn (private communication) suggests that this is not possible. Foellmi et al. (2006) assumed that the secondary is a normal F6 IV star and so has the radius of an isolated F6 IV star. As we see below it is very easy to fit the secondary with a somewhat more massive and luminous but Roche-lobe filling star.

After experimenting with a variety of initial conditions we find that, to fit its position in the Hertzsprung-Russell diagram, the current mass of the secondary star must be at the low end of the predicted range, between about 2.0 M_⊙ and 2.1 M_⊙ and that the mass of the black hole also lies at the low end of its predicted range, between 5.88 M_⊙ and 5.98 M_⊙. We illustrate two typical models in which the secondary is in different evolutionary states.

The first (Figs 9 and 10) has a secondary star, of initial mass 2.5 M_⊙, and initial period 1.648 d. It fills its Roche lobe while evolving across the Hertzsprung gap. Evolution during this phase is on a thermal timescale so that the mass transfer rate is relatively high, 2.35 × 10^{-7} M_⊙ yr^{-1}, and lasts about 3 × 10^6 yr with M > 10^{-7} M_⊙ yr^{-1}. It fits the observations when the secondary’s mass has fallen to 2 M_⊙ and the black hole mass is 5.88 M_⊙. A total of ∆M = 0.5 M_⊙ of material has passed through the disc. Similar results are found for lower masses but both the accretion rate and the time for which the black hole has been accreting fall.
Figure 10. The mass-transfer rate as a function of time for the system illustrated in Fig. 9.

Figure 11. As Fig. 9 but with a secondary star initially of 2.8 M⊙. This time mass transfer begins on the main sequence when it is more gentle and lasts longer.

Figure 12. The mass-transfer rate as a function of time for the system illustrated in Fig. 11.

The second (Figs [11] and [12]) has a secondary star, of initial mass 2.8 M⊙, and initial period 1.481 d. This fills its Roche lobe while still evolving on the main sequence. Evolution at this phase is on a nuclear timescale so that the mass transfer rate is somewhat lower, 5.30 × 10⁻⁹ M⊙ yr⁻¹, and lasts for about 5 × 10⁷ yr. Once again this fits the observations when M₂ = 2 M⊙ and M₁ = 5.88 M⊙. In this time ∆M = 0.8 M⊙ of material has passed through the disc. Secondary stars of initially higher mass transfer material at a higher rate but not for much longer. We also experimented with including convective overshooting (c.f. Regős, Tout & Wickramasingha 1998) to prolong the main-sequence lifetime but found very little difference in the models that fit at these masses.

7 DISCUSSION

We have successfully modelled GRO J1655–40 with mass transfer by Roche-lobe overflow. We need the masses of the black hole and its companion to be at the low end of their observed ranges. We find the average mass-transfer rate and the length of time that it lasts for different binary systems.

We have described two typical, but not unique, models in the previous section. If the companion is crossing the Hertzsprung gap a typical mass-transfer rate is 2.35 × 10⁻⁷ M⊙ yr⁻¹. For this mass-transfer rate we predict an alignment timescale, equation (34), of tₐₐₐₐ = 7.8 × 10⁵ yr. The accretion rate lasts for about 3 × 10⁶ yr which is about 3.8 tₐₐₐₐ. For a secondary star still on the main sequence a typical mass-transfer rate is 5.30 × 10⁻⁹ M⊙ yr⁻¹. The alignment timescale for this lower rate is tₐₐₐₐ = 2.5 × 10⁷ yr. This accretion rate lasts for about 5 × 10⁷ yr which is about 2 tₐₐₐₐ. We note that the alignment timescales also depend on the spin of the black hole which is not known. In these estimates we have taken α = 0.5.

In calculating the timescales we have assumed conservative mass transfer and a constant accretion rate. Given that mass is expected to accumulate in the disc until an outburst the actual mass transfer might be expected to take place in bursts that last for less than the viscous timescale of the disc. During the intervening quiescence, when our equilibrium model can be applied, the mass-transfer rate would be lower and the alignment timescale correspondingly longer. However, a few tenths of a solar mass of material
must pass through the disc. The angular momentum of this material is much less than that of the black hole. Because \( t_{\text{align}} \propto M^{-32/35} \), \( \frac{t_{\text{accr}}}{t_{\text{align}}} \propto \Delta M M^{-3/35} \) depends on \( M \) so changes in mass-transfer rate do not affect the expected alignment very much.

We can also entertain the possibility that the average mass-transfer rate might be low enough to be consistent with the van Paradijs (1996) SXT model. In this case the alignment timescale would be much longer. We do not have a consistent evolutionary model in this case but cannot expect the mass transfer to last longer than the main-sequence lifetime of a 2 M\(_{\odot}\) star, about 8 \( \times 10^8 \) yr. We do not however expect that such a low mass-transfer rate is possible from a Roche-lobe filling secondary star other than for a short time at the onset and the end of mass transfer. It is unlikely that we are catching the system in either of these brief periods and it is more likely that the condition for transience needs to be reconsidered.

Alternatively the mass transfer might be highly non-conservative, though there is no observational evidence for this. In this case the alignment timescale would be much longer while the evolutionary timescales would be unchanged. Thus the models we have discussed would lead to the fastest alignment and this is just slow enough for us to expect to find the black hole still misaligned with the binary orbit.

We see that, whether the secondary fills its Roche lobe on the main sequence or while evolving across the Hertzsprung gap, the accretion rate lasts for a few alignment timescales and so the fact that we see the black hole still misaligned with the orbital plane now is consistent. We are unable to distinguish whether the black hole is 15° from alignment or counteralignment. Given that the misalignment is likely to be created by a strong kick at the time of the supernova explosion we cannot expect either case to be more likely than the other at the start of the accretion.

For constant mass transfer our model predicts exponential decrease of the misalignment. We find that if \( \eta = 15^\circ \) now and if mass transfer has been going on for \( 2 t_{\text{align}} \) then the misalignment would have initially been \( \eta_0 = 148^\circ \). If the black hole has been accreting for \( 3.8 t_{\text{align}} \) then the system would have initially been at \( \eta_0 = 175^\circ \). However these are only indicative models and it is relatively easy to find others in which the accretion has been going on for less time, but still similar to the alignment timescale. We conclude that the system may or may not have been counter-aligned after its supernova kick but that it is most likely to be close to alignment rather than counteralignment.

8 CONCLUSIONS

We have found that the secondary star is most likely evolving either at the end of the main sequence or across the Hertzsprung gap. The mass-transfer rate is typically fifty times higher in the latter case but we find that, in both cases, the lifetime of the mass transfer state is at most a few times the alignment timescale. The fact that the black hole has not yet aligned with the orbital plane is therefore consistent with either model. The system may or may not have been counter-aligned after its supernova kick but we conclude that it is now more likely to be close to alignment rather than counteralignment.

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