Nucleon QCD sum rules with the radiative corrections

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Abstract

QCD sum rules for the nucleon are considered in complex $q^2$ plane with inclusion of the radiative corrections of the order $\alpha_s$. It is shown that the radiative corrections affect mainly the residue $\lambda^2$ of the nucleon pole. Their influence on the value of the nucleon mass is much smaller. Following the ideas of Ioffe and Zyablyuk we expand the analysis to complex values of $q^2$. This provides a more stable solution. Varying the weights of the contributions of different dimensions by changing the value of the angle in the complex plane we find the value of the six-quark condensate which insures the best consistency of the right hand sides and left hand sides of the sum rules. The corresponding value of the six-quark condensate appears to be only about 10% smaller than the one, provided by the factorization approximation. The value of the four-quark condensate also appears to be close to the one, corresponding to the factorization assumption. The role of the gluon condensate and its possible values are discussed.

I. INTRODUCTION

The QCD sum rules (SR) method invented by Shifman et al. [1] succeeded in expressing the static properties of the hadrons in vacuum in terms of the expectation values of QCD operators. The SR approach was employed by Ioffe et al. [2, 3] in the case of nucleons. The basic point is the dispersion relation for the polarization operator

$$\Pi(q) = i \int d^4x e^{i(qx)} \langle 0 | T j(x) \bar{j}(0) | 0 \rangle$$

(1)

of the local operator $j(x)$ carrying the quantum numbers of the nucleon. The dispersion relation is considered at large values of $|q^2|$, while $q^2 < 0$, where it
can be presented as a power series of $q^{-2}$ (with the QCD condensates being the coefficients of the expansion). This presentation is known as the operator product expansion (OPE) [4]. At these values of $q^2$ the polarization operator can be presented also as a power series of the QCD coupling constant $\alpha_s$. Several lowest order OPE terms have been obtained in [2, 3]. The leading contribution of the order $q^4 \ln q^2$ comes from the free three-quark loop. The higher terms contain the expectation values

$$\langle 0 | \bar{q}q | 0 \rangle, \tag{2}$$

etc., with $q$ and $G_{\mu\nu}^a$ standing for the quark operators and for the gluon field tensor. The analysis of [3] included also the most important radiative corrections, in which the coupling constant $\alpha_s$ is enhanced by the “large logarithm” $\ln q^2$. The corrections $(\alpha_s \ln q^2)^n$ have been included to all orders for the leading OPE terms [3, 5]. This approach provided good results for the nucleon mass [3] and for the other characteristics of nucleons [6].

There are several traditional points of SR analysis. Following [3] one writes dispersion relations

$$\Pi_i(q^2) = \frac{1}{\pi} \int \frac{\text{Im} \, \Pi_i(k^2)}{k^2 - q^2} \, dk^2 \tag{2}$$

$(i = q, I)$ for the components of the polarization operator

$$\Pi(q) = q_{\mu} \gamma^\mu \Pi^q(q^2) + I \Pi^I(q^2) \tag{3}$$

with $\gamma_\mu$ and $I$ standing for the Dirac and unit matrices. The result of OPE on the left-hand side (LHS) of Eq. (2), is equalled to the hadronic contribution on the right hand side (RHS), which is approximated by the standard “pole+continuum” model. The spectral density $\text{Im} \, \Pi_i(k^2)$ is approximated by the standard “pole+continuum” model

$$\frac{1}{\pi} \text{Im} \, \Pi_i(k^2) = \lambda_N^2 \delta(k^2 - m^2) \tag{4}$$

$$+ \frac{1}{\pi} \theta(k^2 - W^2) \text{Im} \, \Pi_i^{\text{OPE}}(k^2).$$

Thus, the position of the lowest laying pole $m$, the residue $\lambda_N^2$, and the continuum threshold $W^2$ are the unknowns of the SR equations. The standard Borel transform is usually carried out.

However, inclusion of the lowest order radiative corrections beyond the logarithmic approximation made the situation somewhat more complicated. A numerically large coefficient of the lowest radiative correction to the leading OPE of the polarization operator (1) was obtained in [7]. A more consistent
calculation [8] provided this coefficient to be about 6. Thus, the radiative correction increases this term by about 50% at $|q^2| \sim 1 \text{GeV}^2$, which are actual for the SR analysis. This “uncomfortably large” correction is often claimed as the most weak point of the SR approach [9].

The radiative corrections of the order $\alpha_s \ln q^2$ and $\alpha_s$ for the contributions up to $q^{-2}$ have been calculated by Ovchinnikov et al. [10]. These results were used for the calculation of the parameters $m, \lambda^2$ and $W^2$ by finite energy sum rules method (FESR) [11]. Later the influence of the radiation corrections on the values of the nucleon mass was studied by Borel SR technique at fixed values of $W^2$ and $\lambda^2$, taken from FESR calculations [12]. Recently the QCD SR with the radiative corrections included have been used for determination of the value of quark scalar condensate [13]. However, the analysis of the role of the radiative corrections performed totally in framework of the Borel transformed SR have not been done until now. This is done in the present paper. We investigate also the sensitivity of the nucleon characteristics to the variation of the values of the condensates of higher dimensions.

These two points are connected with each other. While the condensate $\langle 0|\bar{q}q|0\rangle$ of dimension $d = 3$ is known with good accuracy, there is no way to obtain the four- and six-quark condensates ($d = 6, 9$) from an experiment. The calculation of these expectation values requires some additional assumptions. As it stands now, the only approach used in the calculations is the factorization hypothesis, which assumed the domination of the vacuum states in the sum over the intermediate states [11]. In this approximation expectation values of the products of four and six quark operators are expressed in terms of the scalar operator $\langle 0|\bar{q}q|0\rangle$. However, the correction of the order $\alpha_s \ln(q^2/\mu^2)$ to this condensate differs from the sum of such corrections to the condensates $\langle 0|\bar{q}q|0\rangle$ [10]. Thus, the factorization hypothesis should be combined with the definition of the normalization point (scale) at which it is assumed to be true.

We study also the dependence of the nucleon parameters on the value of the gluon condensate $\langle 0|\frac{\alpha_s}{\pi}G_{\mu\nu}G_{\mu\nu}|0\rangle$ ($d = 4$). There are indications that this matrix element can be smaller [14] or larger [15] than the standard value obtained in [16].

Analysis of the SR at complex values of $q^2 = |q^2|e^{i\varphi}$ was employed first by Ioffe and Zyablyuk [17] for investigation of hadronic $\tau$-decay. Varying the value of angle $\varphi$ in complex plane of $q^2$ one can change the weights of different OPE contributions. Some of the terms can be eliminated in such a way. By using this technique one can expect to obtain more reliable and stable results. The hadron parameters should not depend on $\varphi$. We apply this approach for the nucleon channel.

We recall the original form of QCD SR in Sect. II. Corrections of the
order $\alpha_s$ are included in Sect. III. The corresponding equations in complex $q^2$ plane and the analysis are presented in Sect. IV.

II. QCD SUM RULES AT REAL VALUES OF $q^2$

For the specific calculations we use the “current” $j$, which enters Eq. (1) in the form \[ j(x) = u^a(x)C\gamma_\mu u^b(x)\gamma_5\gamma^\mu d^c(x)\varepsilon_{abc}, \] (5)
with $u$ and $d$ standing for the quark fields. $C$ is the charge conjugation operator, while $a, b, c$ are the color indices. The lowest OPE terms of the operators $\Pi^q(q^2)$ and $\Pi^I(q^2)$ introduced by Eq. (3) can be presented as

$$\Pi^q = A_0 + A_4 + A_6 + A_8, \quad \Pi^I = B_3 + B_7 + B_9.$$ (6)

Here the lower indices show the dimensions of the condensates, contained in the corresponding terms, $A_0$ is the contribution of the free quark loop.

The leading OPE terms $A_0, A_4$ and $B_3$ contain divergent integrals. It is sufficient \[2\] to carry out regularization in the simplest way, i.e., just by introducing an ultraviolet cutoff $C_u$. Direct calculation with all the radiative corrections being neglected provides \[2\]

\[
\begin{align*}
A_0 &= -\frac{Q^4}{64\pi^4} \ln \left( \frac{Q^2}{C_u^2} \right), \\
A_4 &= -\frac{1}{32\pi^2} \ln \left( \frac{Q^2}{C_u^2} \right) \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle, \\
B_3 &= -\frac{Q^2}{4\pi^2} \ln \left( \frac{Q^2}{C_u^2} \right) \langle 0 | \bar{q}q | 0 \rangle
\end{align*}
\] (7)
with the standard notation $G^2 = G^{a\mu}_\nu C^{a\mu\nu}$, while $Q^2 = -q^2 > 0$.

The condensates of the lowest dimensions $d = 3, 4$, i.e., $\langle 0 | \bar{q}q | 0 \rangle$ and $\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle$ can be obtained from the Gell-Mann–Oakes–Renner relation \[16\] and from the meson sum rules \[15\]. The higher OPE terms $A_6$ and $B_9$ contain the expectation values of the products of four and six quark operators. There are neither experimental no rigorous theoretical data on the four-quark condensates of the general form $\langle 0 | \bar{q}\Gamma_1 q\bar{q}\Gamma_2 q | 0 \rangle$, with $\Gamma_{1,2}$ acting on Lorentz and color indices. The same refers to the six-quark condensates. The standard approach is the factorization approximation \[11\] expressed by equation

\[
\langle 0 | \bar{q}\Gamma_1 q\bar{q}\Gamma_2 q | 0 \rangle = N^{-2}[(\text{Tr}\Gamma_1 \text{Tr}\Gamma_2) - \text{Tr}(\Gamma_1 \Gamma_2)]\langle 0 | \bar{q}q | 0 \rangle^2
\]
with $q$ standing for $u$ and $d$ quarks and $N = 12$. This equation presents all the four-quark condensates in terms of the scalar expectation values $\langle 0 | \bar{q}q | 0 \rangle$. However all Lorentz structure with $\Gamma_1 = \Gamma_2$ contribute due to the second term in brackets on the RHS of the latter equation (several examples are presented in [11]). Similar relation can be written for the six-quark condensate. Also, the factorization approximation for the quark-gluon condensate, which enters the expression for $B_7$ provides

$$\langle 0 | \bar{q} \frac{\alpha_s}{\pi} G^2 q | 0 \rangle = \langle 0 | \bar{q}q | 0 \rangle \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle.$$ 

Thus, the higher order terms are

$$A_6 = \frac{2}{3Q^2} (\langle 0 | \bar{q}q | 0 \rangle)^2,$$

$$A_8 = -\frac{1}{6Q^4} \mu_0^2 (\langle 0 | \bar{q}q | 0 \rangle)^2,$$

$$B_7 = \frac{1}{12Q^2} \langle 0 | \bar{q}q | 0 \rangle \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle,$$

$$B_9 = 32\pi^2 \frac{17}{81} \frac{\alpha_s}{\pi} Q^4 (\langle 0 | \bar{q}q | 0 \rangle)^3.$$ 

(8)

Here the terms $A_6$, $B_7$ and $B_9$ are presented in the factorization approximation. In $A_8$ we have taken into account that the sum rules for the baryon resonances provide [19]

$$\langle 0 | \bar{q}q \frac{\alpha_s}{\pi} G^a_\mu \lambda^a \frac{\lambda^a}{2} \sigma_{\mu \nu} q | 0 \rangle = \mu_0^2 (\langle 0 | \bar{q}q | 0 \rangle)^2$$ 

(9)

with $\mu_0^2 \approx 0.8\text{ GeV}^2$.

In the analysis, carried out in [3], the most important radiative corrections of the order $\alpha_s \ln q^2$ have been included. These contributions were summed to all orders of $(\alpha_s \ln q^2)^n$, being expressed in terms of the factor

$$L = \left( \frac{\ln Q^2/\Lambda^2}{\ln \mu^2/\Lambda^2} \right)^{4/9},$$

(10)

with $\Lambda \approx 150\text{ MeV}$ is the QCD scale, while $\mu$ is the normalization point. It was assumed in [3] that the physical characteristics are normalized at $\mu = 500\text{ MeV}$. The power $4/9$ reflects the so-called anomalous dimension.

The next traditional point is the Borel transform

$$\hat{B} = \frac{(Q^2)^{n+1}}{n!} \left( -\frac{d}{dQ^2} \right)^n,$$

$$Q^2 = -q^2 \to \infty, \quad n \to \infty, \quad \frac{Q^2}{n} = M^2,$$

(11)
converting a function of \( q^2 \) into the function of \( M^2 \). It removes the terms depending on \( C_u \) in the expressions provided by Eq. (7), and also makes the “pole+continuum” model more reliable, suppressing the contributions of the heavier states by the factor \( e^{-k^2/M^2} \). Actually, in order to deal with the values of the order of unity (in GeV units), it is convenient to use the operator \( B^* = 32\pi^4 \hat{B} \).

After these manipulations the Borel transformed SR are \[ L^q(M^2) = R^q(M^2), \quad L^I(M^2) = R^I(M^2) \] \[ (12) \]
with
\[ L^q = \tilde{A}_0 + \tilde{A}_4 + \tilde{A}_6 + \tilde{A}_8, \quad L^I = \tilde{B}_3 + \tilde{B}_7 + \tilde{B}_9. \] \[ (13) \]
Here, as well as in Eq. (6), the lower indices show the dimensions of the condensates. The terms on the RHS of Eqs. (13) are
\[ \tilde{A}_0 = \frac{M^6 E_2}{L}, \quad \tilde{A}_4 = \frac{bM^2 E_0}{4L}, \quad \tilde{A}_6 = \frac{4}{3} a^2 L, \]
\[ \tilde{A}_8 = -\frac{1}{3} \frac{\mu_0^2}{M^2} a^2, \quad \tilde{B}_3 = 2aM^4 E_1, \]
\[ \tilde{B}_7 = -\frac{ab}{12}, \quad \tilde{B}_9 = \frac{272}{81} \frac{\alpha_s}{\pi} \frac{a^3}{M^2} \] \[ (14) \]
with the notations \( E_i = E_i(W^2/M^2), i = 0, 1, 2 \) and
\[ a = -(2\pi)^2 \langle 0 | \bar{q}q | 0 \rangle \] and \( b = (2\pi)^2 \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle \). \[ (15) \]
The conventional values of these condensates at \( \mu = 0.5 \) GeV are \( a = 0.55 \) GeV³ and \( b = 0.50 \) GeV⁴. The functions
\[ E_0(x) = 1 - e^{-x}, \quad E_1(x) = 1 - (1 + x)e^{-x}, \]
\[ E_2(x) = 1 - \left( 1 + x + \frac{x^2}{2} \right) e^{-x} \] \[ (16) \]
account for the contribution of the continuum, \( E_i(x) \to 1 \) for \( W^2 \to \infty \). Thus, the RHS of Eqs. (12)
\[ R^q(M^2) = \lambda^2 e^{-m^2/M^2}, \quad R^I(M^2) = m\lambda^2 e^{-m^2/M^2} \] \[ (17) \]
describe only the contributions of the nucleon pole with \( \lambda^2 = 32\pi^4 \lambda_N^2 \).

The LHS of Eq. (12), being calculated as OPE series, becomes the better approximation, while the value of \( M^2 \) increases. On the other hand, “pole+continuum” model for the RHS becomes increasingly accurate for smaller values of \( M^2 \). The interval
\[ 0.8 \text{ GeV}^2 < M^2 < 1.4 \text{ GeV}^2, \] \[ (18) \]
where both approximations are expected to be true was found in [3]. The values of the parameters $m$, $\lambda^2$ and $W^2$ have been obtained by minimization of the functional

$$f_1(M^2) = \sum_{i=q,t} \left( \frac{L^i(M^2) - R^i(M^2)}{L^i(M^2)} \right)^2$$  \hspace{1cm} (19)$$

in the interval (18), providing

$$m = 0.931 \text{ GeV} , \quad \lambda^2 = 1.86 \text{ GeV}^6,$$

$$W^2 = 2.09 \text{ GeV}^2.$$  \hspace{1cm} (20)

Account of the anomalous dimensions is not very important. Putting $L = 1$ one can find

$$m = 0.930 \text{ GeV} , \quad \lambda^2 = 1.79 \text{ GeV}^6,$$

$$W^2 = 2.00 \text{ GeV}^2.$$  \hspace{1cm} (21)

Note that straightforward using of this approach can provide misleading results. The solution of (19) corresponding to minimization appears to be unstable with respect to the values of the QCD condensates. Moreover, sometimes the functional (19) may have few different minima. This is illustrated by Fig. 1, where the solution corresponding to the second minimum is shown by the dashed line. For the values of $b$ which do not exceed strongly the conventional value $b = 0.50 \text{ GeV}^4$ the best $\chi^2$ fitting is provided by the first solution shown by the solid line. However, for larger values of $b$ the minimization procedure makes us to jump to the second solution with smaller value of the nucleon mass $m = 0.6 \text{ GeV}$. Consider, for example, the SR with somewhat larger value of the gluon condensate. At $b = 0.65$ (which is 30% larger than the conventional value) minimization of $f_1$ (19) provides much smaller values $m \approx 0.6 \text{ GeV}$ and $W^2 = 1 \text{ GeV}^2$—see Fig. 1. Such solution cannot be treated as a physical one because of the small value of $W^2$. Indeed, the contribution of the continuum (treated approximately) exceeds the contribution of the pole, which is treated exactly. Thus the “pole+continuum” model for the RHS of the sum rules has no sense. Although the accuracy of the physical solution does not change much with $b$, it will not be noticed by $\chi^2$ minimization procedure since unphysical solution is even more accurate — see Fig. 1. The unphysical solution with small values of $m$ and $W^2$ can be traced by successive inclusion of the condensates of higher dimension. Including only the condensates of dimensions $d = 3, 4$ we find a trivial solution $m = 0, \lambda^2 = 0, W^2 = 0$. Inclusion of the condensate with $d = 6$ still keeps $m = 0, W^2 = 0$, but $\lambda^2 = \frac{4}{3}a^2 = 0.4 \text{ GeV}^6$. Inclusion of the higher condensates provides small nonzero values of $m$ and $W^2$. 

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To avoid this situation we add analysis of the functions

\[
\begin{align*}
  m_1^2(M^2) &= \frac{M^4}{\mathcal{L}^4(M^2)} \frac{d\mathcal{L}^4(M^2)}{dM^2}, \\
  m_2^2(M^2) &= \frac{M^4}{\mathcal{L}^4(M^2)} \frac{d\mathcal{L}^4(M^2)}{dM^2}, \\
  m_3(M^2) &= \frac{\mathcal{L}^4(M^2)}{\mathcal{L}^4(M^2)}.
\end{align*}
\]

(22)

Note that these functions depend also on the threshold value \( W^2 \). For the solution of the SR equations (12) it should be

\[
  m_1^2(M^2) = m_2^2(M^2) = m_3(M^2) = \text{const} = m^2.
\]

(23)

Note different meaning of the masses \( m_{1,2} \) and \( m_3 \). While \( m_{1,2} \) determine the position of the lowest pole, which approximates the \( q^2 \) dependence of (3), the difference \( m_3 - m \) reflects rather the admixture of the negative parity state \( 1/2^- \) to the nucleon.

III. CORRECTIONS OF THE ORDER \( \alpha_s \) AT REAL VALUES OF \( q^2 \)

Inclusion of the corrections of the order \( \alpha_s \) to the polarization operator modifies the expressions on the RHS of Eqs. (7),(8) to

\[
\begin{align*}
  A_0 &= -\frac{1}{64\pi^4} Q^4 \ln \frac{Q^2}{\mu^2} \\
  &\quad \times \left( 1 + \frac{71 \alpha_s}{12 \pi} - \frac{1}{2 \pi} \ln \frac{Q^2}{\mu^2} \right), \\
  A_6 &= \frac{2}{3} \frac{\langle 0 | \bar{q}q\bar{q}q | 0 \rangle}{Q^2} \left( 1 - \frac{5 \alpha_s}{6 \pi} - \frac{1}{3 \pi} \ln \frac{Q^2}{\mu^2} \right), \\
  B_3 &= -\frac{\langle 0 | \bar{q}q | 0 \rangle}{4\pi^2} Q^2 \ln \frac{Q^2}{\mu^2} \left( 1 + \frac{3}{2} \frac{\alpha_s}{\pi} \right),
\end{align*}
\]

(24)

obtained in [8, 10] (see Appendix A). The radiative corrections to the other terms are not included since the values of the corresponding condensates are known with poor accuracy. Strictly speaking this is true for the four-quark condensate as well, which is obtained in the factorization approximation only. However, we shall assume the latter hypothesis for the four-quark condensate, advocated recently in [20]. Note that the correction of the type \( \alpha_s \ln(q^2/\mu^2) \) to the term \( B_3 \) vanishes due to the cancellation of the correction \( \frac{1}{2 \pi} \ln \frac{Q^2}{\mu^2} \) to the condensate \( \langle 0 | \bar{q}q | 0 \rangle \) and that coming from the free quark system. If the factorization approximation would have been true for all values of \( \mu^2 \), we
would find $\frac{\alpha_s}{\pi} \ln \frac{Q^2}{\mu^2}$ for the last term in brackets of the formula for $A_6$. One can see that the value of the coefficient is another one. Thus, the factorization assumption cannot be true for all values of $\mu^2$.\footnote{Since the value of $\ln(M^2/\mu^2)$ is not large and we are studying the role of the whole $O(\alpha_s)$ corrections, and not only the part (given by the renormgroup) which is responsible for the scale dependence of the polarization operator, here we keep the logarithmic $\alpha_s \ln Q^2$ contribution but, to avoid the double counting, neglect the anomalous dimension factor $L$.}

In Ref. [10] these formulas have been used for investigation of the nucleon parameters with the finite energy sum rules technique [11]. It was shown that inclusion of radiative correction leads to a noticeable reduction of the nucleon mass. Similar behavior was obtained in the analysis carried out in [12] in framework of the Borel transformed sum rules at fixed values of $W^2$.

Using (24), we find for the corresponding contributions to the Borel transformed SR (see Appendix B)

\begin{align}
\tilde{A}_0(M^2, W^2) &= M^6 E_2 \left[ 1 + \frac{\alpha_s}{\pi} \left( \frac{53}{12} - \ln \frac{W^2}{\mu^2} \right) \right] \\
&\quad - \frac{\alpha_s}{\pi} \left[ M^4 W^2 \left( 1 + \frac{3W^2}{4M^2} \right) e^{-W^2/M^2} \right. \\
&\quad \left. + M^6 \mathcal{E}(-W^2/M^2) \right], \\
\tilde{A}_6(M^2, W^2) &= \frac{4}{3} a^2 \\
&\quad \times \left[ 1 - \frac{\alpha_s}{\pi} \left( \frac{5}{6} + \frac{1}{3} \left( \ln \frac{W^2}{\mu^2} + \mathcal{E}(-W^2/M^2) \right) \right) \right], \\
\tilde{B}_3(M^2, W^2) &= 2aM^4 E_1 \left( 1 + \frac{3}{2} \frac{\alpha_s}{\pi} \right)
\end{align}

with $E_i = E_i(W^2/M^2)$, $i = 0, 1, 2$ and

$$\mathcal{E}(x) = \sum_{n=1} E_i \frac{x^n}{n \cdot n!}.$$
bution to the structure $L^q$. Hence, neglecting the influence of radiative corrections on the value of $W^2$, one can see that the denominator of the function $m_3(M^2)$ (22) increases due to the radiative corrections. Thus, in agreement with [10, 12], the latter diminish the value of nucleon mass. However, the present analysis which includes possible modification of all the parameters $(m, \lambda^2, W^2)$ by the radiative corrections, provides somewhat different result.

To carry out quantitative analysis, we need to clarify the argument of the running coupling constant

$$\alpha_s(k^2) = \frac{4\pi}{9\ln(k^2/\Lambda^2)}. \hspace{1cm} (26)$$

In [10] the radiative corrections have been obtained at $\alpha_s = \text{const}$. Since the momenta in the loops corresponding to the radiative corrections are of the order of $q$, it is reasonable to assume $\alpha_s = \alpha_s(Q^2)$ in Eq. (24). After the Borel transform we obtain $\alpha_s = \alpha_s(M^2)$ in Eq. (25). Since we are considering $M^2$ of the order 1 GeV$^2$, we can put $\alpha_s = \alpha_s(1\text{ GeV}^2) = 0.35$. Minimization of the functional (19) provides

$$m = 0.94 \text{ GeV}, \quad \lambda^2 = 2.00 \text{ GeV}^6,$$

$$W = 1.90 \text{ GeV}, \hspace{1cm} (27)$$

while for $\alpha_s = \alpha_s(M^2)$

$$m = 0.94 \text{ GeV}, \quad \lambda^2 = 2.11 \text{ GeV}^6,$$

$$W^2 = 2.00 \text{ GeV}^2. \hspace{1cm} (28)$$

In Fig. 2 we present the values of the parameters as the functions of $\alpha_s$. One can see that the radiative corrections affect mostly the value of the residue $\lambda^2$. The consistency of the LHS and RHS of the SR is shown in Fig. 3(a).

Note that in [13] the SR have been considered at fixed value $W^2 = 2.5 \text{ GeV}^2$, providing somewhat larger residue value $\lambda^2 \approx 3 \text{ GeV}^2$. Assuming $W^2 = 2.5 \text{ GeV}^2$ in Eqs. (12), we would move to larger values $\lambda^2 = 2.6 \text{ GeV}^2$ in our solution. This is close to the result of [13], although our procedures of inclusion of radiative corrections differ in several points.

We can look for the solution in which the discrepancy between the values of $m_i(M^2)$ defined by Eq. (22) is minimized. It can be obtained by minimization of the functional

$$f_2(M^2) = f_1(M^2) + f_m(M^2) \hspace{1cm} (29)$$

with

$$f_m(M^2) = \sum_i \frac{(m_i(M^2) - m)^2}{3m^2}, \hspace{1cm} (30)$$
while $f_1(M^2)$ is determined by (19). For $\alpha_s = 0$ the procedure provides $m = 0.93$ GeV, $\lambda^2 = 1.82$ GeV$^6$, $W^2 = 2.02$ GeV$^2$, while for $\alpha_s = 0.35$ we find

$$m = 0.94 \text{ GeV}, \quad \lambda^2 = 1.98 \text{ GeV}^6,$$
$$W^2 = 1.88 \text{ GeV}^2. \quad (31)$$

Thus, minimization of the functionals $f_1(M^2)$ (19) and $f_2(M^2)$ (29) leads to close results. The functions $m_i(M^2)$ are presented in Fig. 3(b).

IV. SUM RULES IN THE COMPLEX PLANE OF $Q^2$

Now we consider the SR at complex values of $Q^2 = S^2 e^{i\varphi}$ (32) with the real values of $S^2 > 0$ and $-\pi < \varphi < \pi$. The Borel transform (11) will be carried out with respect to $S^2$. It is reasonable to consider the SR for the real part of the operators $\Pi^i(q^2)$ (3).

A. Possible values of $\varphi$

Note first that our “pole+continuum” model makes sense only for $\cos \varphi > 0$, i.e., $-\frac{\pi}{2} < \varphi < \frac{\pi}{2}$. This becomes clear while considering the Borel transform of contribution of the nucleon pole to the RHS of Eq. (2)

$$B^*_{S^2} \int \frac{\lambda^2_2 \delta(k^2 - m^2)}{m^2 + S^2 e^{i\varphi}} \, dk^2 = \lambda^2 e^{\frac{-m^2}{M^2} \cos \varphi} e^{i\theta} \quad (33)$$

with $\theta = \frac{m^2}{M^2} \sin \varphi$. Thus the contributions of heavier states are suppressed with respect to the lowest one only if $\cos \varphi > 0$.

Also, the values of $\varphi$ should not be too close to $\pm \pi/2$. This is because the leading OPE term $B_3$ obtains the factor $\cos \varphi$ in the complex plane, while the higher term $B_5$ is multiplied by $\cos 2\varphi$. Hence, the convergence of OPE series becomes worse at $\varphi$ close to $\pm \pi/2$. That’s why we focus on the values

$$0 \leq \varphi \leq \frac{\pi}{4}. \quad (34)$$

Analysis at negative values of $\varphi$ will not provide new data, since all the functions involved are the even functions of $\varphi$.

B. Basic equations in the complex plane
Following the previous discussion, we must present the Borel transformed dispersion relations for the real parts of the operators $\Pi^q(q^2)$ and $\Pi^I(q^2)$. For the Borel transforms of the contributions (7), (8) we find

$$\hat{B}A_0 = \frac{M^6 e^{2i\varphi}}{32\pi^4}, \quad \hat{B}A_4 = \frac{M^2}{32\pi^2} \langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle,$$

$$\hat{B}A_6 = \frac{2}{3} e^{-i\varphi} \langle 0 | \bar{q}qq | 0 \rangle,$$

$$\hat{B}A_8 = \frac{-1}{6M^2} e^{-2i\varphi} \langle 0 | \bar{q}qq | 0 \rangle \frac{\alpha_s}{\pi} G^2 \sigma_{\mu\nu} \frac{\lambda^a}{2} q | 0 \rangle,$$

$$\hat{B}B_3 = -\frac{M^4}{4\pi^2} e^{i\varphi} \langle 0 | \bar{q}q | 0 \rangle,$$

$$\hat{B}B_7 = \frac{e^{-i\varphi}}{24} \langle 0 | \bar{q} \frac{\alpha_s}{\pi} G^2 q | 0 \rangle,$$

$$\hat{B}B_9 = \frac{272 e^{-2i\varphi}}{81M^2} \frac{\alpha_s}{\pi^3} \langle 0 | \bar{q}qqq | 0 \rangle. \quad (35)$$

The contributions $A_0, A_4$ and $B_3$ contain the quark loops, thus requiring inclusion of the contribution of the continuum. The latter can be evaluated as continuum, which takes the form

$$R_n = \int_{W^2}^{\infty} \frac{dk^2(k^2)^n}{k^2 + S^2 e^{i\varphi}} = \int_{W^2_c} \frac{dk_c^2(k_c^2)^n e^{i\varphi}}{k_c^2 + S^2 e^{i\varphi}} \quad (36)$$

with $n = 0, 1, 2$, and

$$W^2_c = W^2 e^{-i\varphi}. \quad (37)$$

The corresponding Borel transforms are thus

$$\hat{B}R_n = e^{i\varphi} M^2 \left( 1 - E_n \left( \frac{W^2_c}{M^2} \right) \right). \quad (38)$$

Using these results, and including the radiative corrections (25) we can present the SR in the form

$$\text{Re} (\mathcal{L}_c e^{i\varphi}) = R^q_c, \quad \text{Re} (\mathcal{L}^I_c e^{i\varphi}) = R^I_c \quad (39)$$

with

$$R^q_c = \lambda^2 e^{-\frac{m^2}{M^2} \cos \varphi} \cos \left( \frac{m^2}{M^2} \sin \varphi \right),$$

$$R^I_c = m\lambda^2 e^{-\frac{m^2}{M^2} \cos \varphi} \cos \left( \frac{m^2}{M^2} \sin \varphi \right). \quad (40)$$
while

\[ \mathcal{L}_c^q(M^2, W_c^2) = e^{2i\varphi} \tilde{A}_0(M^2, W_c^2) + \tilde{A}_4(M^2, W_c^2) + e^{-i\varphi} \tilde{A}_6(M^2) + e^{-2i\varphi} \tilde{A}_8(M^2), \]

\[ \mathcal{L}_c^f(M^2, W_c^2) = e^{i\varphi} \tilde{B}_3(M^2, W_c^2) + e^{-i\varphi} \tilde{B}_7 + e^{-2i\varphi} \tilde{B}_9(M^2) \]

with \( W_c^2 \) defined by Eq. (37), while the contributions \( \tilde{A}_i \) and \( \tilde{B}_i \) are given by Eq. (25). The functions

\[ m_1^2(M^2) = \text{Re} \frac{M^4 e^{i\varphi}}{L^q(M^2)} \frac{dL^q(M^2)}{dM^2}, \]

\[ m_2^2(M^2) = \text{Re} \frac{M^4 e^{i\varphi}}{L^f(M^2)} \frac{dL^f(M^2)}{dM^2}, \]

\[ m_3^2(M^2) = \frac{\text{Re} L^f(M^2)}{\text{Re} L^q(M^2)} \]

generalize Eq. (22) for the complex plane, and Eq. (23) should have been true for the exact solution.

C. Dependence of hadron parameters on \( \varphi \)

Now we try to find the nucleon parameters by minimization of the functional

\[ f_c^c(M^2) = \sum_i \left( \frac{\text{Re} L_i^c(M^2) - R_i^c(M^2)}{\text{Re} L_i^c(M^2)} \right)^2, \]

which is generalization of Eq. (19) for the complex plane of \( Q^2 \). Since the \( M^2 \) interval of stability may vary with \( \varphi \), we present the results for the two intervals, e.g., the one given by Eq. (18), and also

\[ 1.0 \text{ GeV}^2 < M^2 < 1.8 \text{ GeV}^2. \]

The results are shown in Fig. 4. One can see that the largest stability is obtained at \( \varphi \leq \frac{\pi}{4} \). Also, the nucleon residue appears to be more sensitive to the value of \( \varphi \), then the values of the nucleon mass and of the continuum threshold \( W^2 \).

D. Dependence on the value of six-quark condensate

As we said earlier, the less reliable value of the condensates involved is that of the six-quark condensate, which determines the contribution \( \tilde{B}_9 \) on
the RHS of Eq. (42). While all the results obtained above are obtained in factorization approximation, now we put

$$\langle 0 | (\bar{q}q)^3 | 0 \rangle = (\langle 0 | \bar{q}q | 0 \rangle)^3 \eta_{6q}, \quad (46)$$

with $\eta_{6q} = 1$ under the factorization assumption and check dependence of the parameters on $\varphi$ at various values of $\eta_{6q}$. The results are presented in Fig. 5. One can see that the value of the nucleon mass $m$ exhibits larger stability with respect to changes of $\varphi$ and $\eta_{6q}$ than the values of $\lambda^2$ and $W^2$.

At $\varphi = \frac{\pi}{4}$ the contribution of the six-quark condensate vanishes, and the result is free from uncertainties in the value of $\eta_{6q}$. For $\varphi = \frac{\pi}{4}$

$$m = 0.95 \text{ GeV}, \quad \lambda^2 = 1.49 \text{ GeV}^6, \quad W^2 = 1.57 \text{ GeV}^2. \quad (47)$$

Note that at $\varphi = \frac{\pi}{4}$ the contribution $\tilde{A}_0$ on the RHS of Eq. (41), corresponding to the free-quark loop also turns to zero. This is not necessary a weak point, since the system of three free quarks contributes rather to continuum then to the lowest laying pole.

At $\varphi = \frac{\pi}{4}$ the consistency of LHS and RHS of Eqs. (39) is not very good. It becomes much better at smaller values of $\varphi$ in the domain of stability in $(M^2, \varphi)$ plane.

E. Stability of solution in $(M^2, \varphi)$ plane

Now we shall look for the solution which provides the best consistency of the LHS and RHS of Eqs. (39) in certain domains of the values of $M^2$ and $\varphi$. In Table I we present result obtained by minimization of the functional (44). One can see that the consistency (“$\chi^2$ per point”) becomes much better if we limit the interval of the values of the angles to

$$0 \leq \varphi \leq \frac{\pi}{8}. \quad (48)$$

The results obtained in the interval (45) are close to those found for the interval (18). In this case ”$\chi^2$ per point” is somewhat smaller than in the latter one. However we consider the results obtained in traditional interval to be more reliable, since on the upper end of (45) the contribution of continuum is not suppressed stronger than that of the pole. Thus, the limits of stability of SR in $(M^2, \varphi)$ plane are given by Eqs. (18) and (48) with the values of the parameters being

$$m = 0.94 \text{ GeV}, \quad \lambda^2 = 2.0 \text{ GeV}^6, \quad W^2 = 1.9 \text{ GeV}^2. \quad (49)$$
Consistency of LHS and RHS of Eqs. (39) is illustrated by Fig. 6 for the hadron parameters presented by Eq. (49) for several values of $\varphi$. In Fig. 7 we show the functions $m_i(M^2)$ defined by Eq. (43) for several values of $\varphi$ and dependence of the values $m_i(M^2)$ on $\varphi$ for several values of $M^2$. This illustrates the weak dependence of the values of the hadron parameters on the values of $M^2$ and $\varphi$ in this region.

Note that similar results can be obtained by minimization of the functional

$$f_2^c(M^2) = f_1^c(M^2) + f_m(M^2)$$

(50)

with $f_m$ and $f_1^c$ defined by Eqs. (30) and (44). These data are presented in Table II. The functions $m_i(M^2)$ are shown in Fig. 8.

F. Values of the condensates of high dimension

Now we can try to use the SR for diminishing of the uncertainties of the values of the QCD condensates. We put

$$\langle 0 | (\bar{q}q)^2 | 0 \rangle = (\langle 0 | \bar{q}q | 0 \rangle)^2 \eta_4^q,$$

$$\langle 0 | \frac{\alpha_s}{\pi} G^2 | 0 \rangle = (2\pi)^{-2} b_0 \eta_G$$

(51)

with $b_0 = 0.50 \text{GeV}^4$, and try to determine the values of $\eta_4^q$, $\eta_G$, and $\eta_6^q$ (46) from the SR. Deviations of the parameters $\eta_4^q$ and $\eta_6^q$ from unity characterize the violation of factorization hypothesis. Possible deviation of parameter $\eta_G$ from unity is due to uncertainties in the value of the gluon condensate. The terms $\tilde{A}_4$, $\tilde{A}_6$ and $\tilde{B}_6$ on the RHS of Eqs. (41) and (42) obtain the factors $\eta_G$, $\eta_4^q$ and $\eta_6^q$ correspondingly.

Now we try to solve SR equations treating the hadron parameters $m$, $\lambda^2$, $W^2$ and also the values $\eta_4^q$, $\eta_6^q$ and $\eta_G$ as the unknowns.

**Four-quark condensate.** Here we put $\eta_6^q = \eta_G = 1$ and try to determine the value of $\eta_4^q$. Minimization of (44) provides

$$m = 0.964 \text{ GeV}, \quad \lambda^2 = 2.11 \text{ GeV}^6,$$

$$W^2 = 1.99 \text{ GeV}^2, \quad \eta_4^q = 0.93.$$  

(52)

Thus, the QCD sum rules prefer only small deviation from the results of the factorization approximation.

**Six-quark condensate.** Now we try to determine the value of $\eta_6^q$, putting $\eta_4^q = \eta_G = 1$. Minimization of (44) provides

$$m = 0.937 \text{ GeV}, \quad \lambda^2 = 1.97 \text{ GeV}^6,$$

$$W^2 = 1.90 \text{ GeV}^2, \quad \eta_6^q = 0.90.$$  

(53)
demonstrating small deviations of the value $\eta_{6q}$ from the factorization hypothesis value $\eta_{6q} = 1$.

**Gluon condensate.** Now we fix $\eta_{4q} = \eta_{6q} = 1$ and look for the dependence of the solution on the value of the gluon condensate. The value of the nucleon mass appears to be rather sharp function of $\eta_G$. For example, at $\eta_G = 2$ we find $m = 0.74$ GeV.

Note that the functions $m_3(M^2)$ (22) is much more sensitive to the value of the gluon condensate than the “pole masses” $m_{1,2}(M^2)$. Variation of the parameter $\eta_G$ in the interval $0 \leq \eta_G \leq 2$ changes the values of $m_{1,2}$ by about 60 MeV, while $m_3$ changes by a factor 1/2.

Treating $\eta_G$ as an unknown parameter of the SR equations we find that minimization of (44) takes place at $\eta_G = 2.2$ and at the underestimated value of the nucleon mass $m \approx 0.67$ GeV. On the other hand, one needs $\eta_G \approx 1$ to obtain the value of the nucleon mass close to the observable one.

**V. SUMMARY**

We carried out the analysis of QCD sum rules for nucleons including the lowest order radiative corrections. This is the first analysis performed totally in framework of SR approach. We show that the radiative corrections modify mainly the values of the nucleon residue, while that of the nucleon mass suffers minor changes.

Our analysis was carried out for the real values of $q^2$ and in the complex $q^2$ plane. We found the region of stability in $(M^2, \varphi)$ plane in the latter case. Our main result is expressed by Eq. (49), being illustrated by Fig. 6.

We used the nucleon sum rules to clarify the values of the condensates of high dimension. We showed that the sum rules require the deviations of the four-quark condensates from the factorization value to be very small. Similar deviations of the six-quark condensates do not exceed 10%. As to the uncertainties of the gluon condensate, the values, which exceed the standard one by 30% provide the value of the nucleon mass $m < 0.9$ GeV. The greater values of the gluon condensate may cause problems in description of the nucleon. A more detailed analysis of the limitations on the gluon condensate value, coming from the nucleon sum rules will be published elsewhere.

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Here we describe the renormalization procedure, which leads to Eqs. (24). The point have not been considered in detail in previous publications. Until we do not include the radiative corrections to the quark loops (terms $A_0$, $A_4$ and $B_3$ in Eqs. (7), (8)), renormalization is not important, since the contribution $Q^4 \ln(C_2^2)$ is eliminated by the Borel transform. In other words, the Borel transform carries out the renormalization automatically. However, the situation becomes more complicated if the corrections of the order $\alpha_s \ln Q^2$ are included.

Consider first the three-quark loop. While the radiative corrections are neglected, we can carry out the renormalization of the function $A_0(Q^2)$ by subtracting the two lowest terms of the Taylor expansion at a normalization point $Q^2 = \mu^2$. This provides for the renormalized contribution

$$A_{0r} = -\frac{1}{64\pi^4} Q^4 \ln \frac{Q^2}{\mu^2}. \quad (A1)$$

(we omitted the terms, which will be killed by the Borel transform). The contributions which include the radiative corrections to Eq. (A1) are renormalized in the same way. This leads to the equality, presented by the upper line of Eq. (24).

Similar procedure can be applied to the calculation of the renormalized contributions $A_6$ and $B_3$. The result is presented by Eq. (24). There are no loops in the case of $A_6$. Thus, only the radiative corrections are renormalized. There are no corrections of the order $\alpha_s \ln Q^2$ in the case of $B_3$. Hence, only the two quark loop was renormalized. Unlike the case of $A_0$, the renormalization was not important here due to the Borel transform.

Here we calculate the contribution of the term $A_0$ determined by Eq. (24) to the LHS of Eq. (12). This means that we must calculate the Borel transform, subtracting the contribution of the continuum.

The renormalized contribution is

$$A_{0r} = -\frac{1}{64\pi^4} Q^4 \ln \frac{Q^2}{\mu^2}$$

$$\times \left( 1 + \frac{71}{12} \frac{\alpha_s}{\pi} - 2 \frac{\alpha_s}{\pi} \ln \frac{Q^2}{\mu^2} \right). \quad (B1)$$

Employing [2, 12] we find

$$\hat{B}(Q^4 \ln Q^2) = -2M^6,$$
\[
\hat{B}(Q^4 \ln^2 Q^2) = -4M^2 \left( \ln M^2 - \gamma_E + \frac{3}{2} \right)
\]  

(B2)

with \(\gamma_E \approx 0.577\) being the Euler constant. Thus

\[
B^* A_{0r} = M^6 \left( 1 + \frac{71}{12} \frac{\alpha_s}{\pi} \right) - M^6 \frac{\alpha_s}{\pi} \left( \ln \frac{M^2}{\mu^2} - \gamma_E + \frac{3}{2} \right)
\]

(B3)

with the last term of the RHS originating from the last term in brackets on the RHS of Eq. (B1). Recall that \(B^* = 32\pi^4 \hat{B}\).

Following Eq. (4) we find for the contribution to the RHS of Eq. (13) for \(\mathcal{L}^q\)

\[
\bar{A}_0(M^2, W^2) = M^6 E_2 \left( 1 + \frac{71}{12} \frac{\alpha_s}{\pi} \right) - \frac{\alpha_s}{\pi} \left( M^6 \left( \ln \frac{M^2}{\mu^2} - \gamma_E + \frac{3}{2} \right) - T(M^2, W^2) \right),
\]

(B4)

with

\[
T(M^2, W^2) = \frac{1}{2} \int_{W^2}^{\infty} dk^2 k^4 \ln \frac{k^2}{\mu^2} e^{-k^2/M^2}
\]

(B5)

describing the contribution of continuum, corresponding to the last term on the RHS of Eq. (B3).

Introducing

\[
z = \frac{W^2}{M^2},
\]

(B6)

and denoting \(k^2 = W^2 x\), we evaluate

\[
T(M^2, W^2) = M^6 \times \left[ \left( 1 + z + \frac{z^2}{2} \right) e^{-z} \ln \frac{W^2}{\mu^2} + t(z) \right]
\]

(B7)

with

\[
t(z) = \frac{z^3}{2} \int_1^\infty dx x^2 \ln x e^{-zx}.
\]

(B8)

Employing integration by parts three times, we obtain

\[
t(z) = \int_1^\infty dx \frac{e^{-zx}}{x} + \frac{3 + z}{2} e^{-z}
\]

\[
= -Ei(-z) + \frac{3 + z}{2} e^{-z},
\]

(B9)
with

$$Ei(x) = \text{V.p.} \int_{-\infty}^{x} \frac{e^{-y}}{y} \, dy = \sum_{n=1}^{\infty} \frac{x^n}{n \cdot n!} + \gamma_E + \ln x$$  \hspace{1cm} (B10)

being the integral exponential function [21]. Combining Eqs. (B4), (B7) and (B9), we come to Eq. (25) of the text.
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Figure captions

FIG. 1. Dependence of solutions of Eq. (12) on the value of the gluon condensate. The solid lines correspond to \( b = 0.50 \), the dashed lines are for the case \( b = 0.65 \). The nucleon parameters are related to those, determined by Eq. (21), i.e., \( m_0 = 0.93 \) GeV, \( \lambda_0^2 = 1.79 \) GeV\(^6\), \( W_0^2 = 2.00 \) GeV\(^2\).

FIG. 2. Dependence of ratios \( m/m_0, \lambda^2/\lambda_0^2 \), and \( W^2/W_0^2 \) on the values of \( \alpha_s \). Long-dashed line shows "\( \chi^2 \) per point". Parameters \( m_0, \lambda_0^2 \) and \( W_0^2 \) are the same as in Fig. 1.

FIG. 3. (a) Consistency of LHS and RHS of Eq. (12). Hadron parameters are given by Eq. (28). Solid and dashed lines show the ratios \( \frac{L_i(M^2)}{R_i(M^2)} \) and \( \frac{L_i(M^2)}{R_i(M^2)} \), correspondingly. (b) The functions \( m_i(M^2), i=1,2,3 \), defined by Eq. (22).

FIG. 4. Dependence of the hadron parameters on the value of \( \varphi \). Figures (a) and (b) correspond to the values of Borel masses in the intervals (18) and (45). Parameters \( m_0, \lambda_0^2 \) and \( W_0^2 \) are the same as in Fig. 1.

FIG. 5. Dependence of the hadron parameters \( m \) (a), \( \lambda^2 \) (b) and \( W^2 \) (c) on the value of \( \varphi \) for several values of \( \eta_{hq} \). The latter are shown by the numbers on the lines.

FIG. 6. Consistency of LHS and RHS of Eq. (39) with the hadronic parameters presented by Eq. (49) for \( \varphi = 0 \) (solid line), \( \varphi = \frac{\pi}{16} \) (dashed line) and \( \varphi = \frac{\pi}{8} \) (dotted line).

FIG. 7. (a) The functions \( m_i(M^2), i=1,2,3 \), for \( \varphi = 0 \) (solid line) and \( \varphi = \frac{\pi}{8} \) (dashed line). The functional \( f_1(M^2) \) (19) is minimized. (b) Dependence of the functions \( m_i(M^2) \) on the value of \( \varphi \) for \( M^2 = 0.8 \) GeV\(^2\) (solid line) and for \( M^2 = 1.4 \) GeV\(^2\) (dashed line).

FIG. 8. The functions \( m_i(M^2), i=1,2,3 \), for \( \varphi = 0 \) (solid line) and \( \varphi = \frac{\pi}{8} \) (dashed line). The functional \( f_2(M^2) \) (50) is minimized.
TABLE I. Nucleon parameters obtained by minimization of the function $f_1^c$ defined by Eq. (44).

| $M^2$ (GeV$^2$) | $\phi$ | $\chi^2$ | $m$ (GeV) | $\lambda^2$ (GeV$^6$) | $W^2$ (GeV$^2$) |
|----------------|--------|----------|-----------|------------------------|----------------|
| 0.8 - 1.4      | 0-$\pi$ | 6.5      | 0.93      | 1.88                   | 1.84           |
| 1.0 - 1.8      | 0-$\frac{\pi}{4}$ | 2.0      | 0.96      | 1.98                   | 1.86           |
| 0.8 - 1.4      | 0-$\frac{\pi}{2}$ | 0.8      | 0.94      | 2.02                   | 1.93           |
| 1.0 - 1.8      | 0-$\frac{3\pi}{4}$ | 0.4      | 0.96      | 2.00                   | 1.88           |

TABLE II. Nucleon parameters obtained by minimization of the function $f_2^c$ defined by Eq. (50).

| $M^2$ (GeV$^2$) | $\phi$ | $\chi^2$ | $m$ (GeV) | $\lambda^2$ (GeV$^6$) | $W^2$ (GeV$^2$) |
|----------------|--------|----------|-----------|------------------------|----------------|
| 0.8 - 1.4      | 0-$\frac{\pi}{4}$ | 7.9      | 0.94      | 1.93                   | 1.88           |
| 1.0 - 1.8      | 0-$\frac{\pi}{4}$ | 3.7      | 0.97      | 1.99                   | 1.87           |
| 0.8 - 1.4      | 0-$\frac{3\pi}{4}$ | 2.5      | 0.94      | 2.04                   | 1.95           |
| 1.0 - 1.8      | 0-$\frac{\pi}{4}$ | 2.7      | 0.96      | 2.02                   | 1.90           |
Figure 1:
Figure 2:

Figure 3:
Figure 4:
Figure 5:
