OBSERVABILITY AND REDUNDANCY BASED PMU PLACEMENT AT OPTIMAL LOCATION OF POWER SYSTEM

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Abstract

This paper investigates redundancy and observability constrained Sequential Quadratic (SQ) technique for optimal Phasor Measurement Units (PMU) placement. The nonlinear constraints of buses are considered with this approach to optimize the quadratic objective for PMU placement. Zero Injection (ZI) bus constraints are modeled in quadratic formulation to less PMU locations. PMU placement with and without ZI constraints are compared to illustrate the importance of ZI constraint modeling for PMU placement. Redundancy in network is estimated with number of branches connected to bus. Redundancy of bus network is measured by the proposed Bus Redundancy Index (BRI). To estimate observability performance of the complete network, a Complete System Bus Observability Index (CSBOI) is proposed. IEEE-14,30, and 57 bus systems are simulated with the proposed constrained SQ Programming formulation in MATLAB. The comparison of planned way with conventional methods is also considered to show its efficacy.

Keywords: Branches; Observability; Phasor Measurement Units; Redundancy; Sequential Quadratic Programming (SQP); Zero Injection buses

I. Introduction

With application of Synchro phasors in Wide Area Monitoring System (WAMS), it gave a scope to present research scholars to work in the area of power system protection and state estimation. Accurate state of network will be attained by the placement of PMUs in the network buses. The PMUs located at different parts of network or grid is synchronized with a clock signal provided by Global Position System (GPS) [I],[II]. This synchronization of PMUs provides quick results compared to Supervisory Control and Data Acquisition (SCADA) systems which were operated in present power system placement at every bus in network is high economy and is infeasible. PMU placement should be considered in such an approach that every bus in the network is observable. Measurements increases with increase in PMU number thereby increase in redundancy. PMU should be placed in such manner that, with redundant PMU measurements in network, system is completely observable. Observability of network is measured in two ways, one is through...
numerical way that is by finding rank of network and other is by topology of network. In topology of network, incidence matrix is formulated to check the observability with the obtained PMU locations. With consideration of literature survey for Optimal PMU Placement (OPP) methods in last ten years, it is evolved that many authors solved OPP problem in linear form \([V][VIII][XI][XII]\). In \([VII]\) the author suggested Binary Integer Programming (BIP) method for OPP problem. Conventional observability analysis is proposed to estimate system concern and reduce number of PMUs. The author proposed a modified Integer Linear Programming (ILP) method to recognize the channel capacity of PMU and optimal placement to attain complete observability. The author presented Mixed Integer Linear Programming method for OPP which makes easy to measure the PMU channels, effect of zero injection buses and contingencies. In \([X]\) the author suggested Generalized Binary Integer Linear Programming (GBILP) for optimal PMU placement to find the dynamic state estimation with complete observability. In \([IV]\) the author presented Mixed Integer Linear programming and Non Linear programming for OPP problem. The author suggested an improved depth first algorithm for OPP problem considering bus weight \([XIII]\). The author proposed Minimum Connectivity Based Reduction (MCBR) technique is proposed for placement of PMUs to achieve complete observability. In \([VI]\) the author presented a new topology method considering zero injection buses and links between buses and leaf nodes to achieve system observability. The author proposed multi-objective Biogeography Based Optimization (BBO) for OPP problem to make system network completely observable. The author presented improved Binary Artificial Bee Colony Algorithm (BABC) for OPP problem considering conventional flow measurements and zero injection measurements. This paper proposes redundancy and observability constrained OPP with SQ approach. Redundancy is measured considering branches connected to buses in network. Bus redundancy index and Complete System Bus Observability Index are proposed to check performance of proposed optimization method.

II. Identification of Problem

The main objective function is formulated for OPP considering complete observability with optimal redundancy. The non-linear constraints of network are programmed considering SQP method. For example, consider standard 14 bus network is given in Fig.1 for formulation.

![Fig.1. Standard 14-bus network](image-url)
The representation of problem is given as

\[ J(y) = \text{Min} \sum_{i=1}^{14} y_i^T c_i y_i \]  

(1)

Subjected to nonlinear observability constraints

The above formulation is quadratic unconstrained problem for which the PMU cost at every bus is same for all buses. Equality constraints (2) ensure that optimal values will be either \( y_k = 1 \) or \( y_k = 0 \). Cost quadratic equation (1) implies that \( 0 \leq y_k \leq 1 \). \( C \) is the cost function generally considered as \([1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \ 1 \] \( T \).

III. Observability and Redundancy Constrained PMU Placement

III.i. Zero Injection Bus Modelling Sing Network Topology

The bus which is not a either generator or load bus. This bus is considered for optimization of OPP. In this network the exhibiting of ZI variables consider for nonlinear system. At node 4 consider ZI bus as shown in Fig.2.

The phase voltage of 1 to (k-1) buses are identified, then the \( I_{l,1} \) is calculated as

\[ I_{l,1} = Y_{l,1} [e_L - e_i] \]  

(3)

where \( Y_{l,1} = \) line admittance of buses 1, L

For bus \( k \), the voltage is calculated by

\[ e_k = V_k - Z_{1,k} \sum_{l=2}^{k-1} I_{l,1} \]  

(4)
where $Z_{1,k}$ is impedance between buses 1 and $k$.

Each ZI node specifies a supplementary constraint. By decreases the ZI bus in this system is essential to find minimum no. of PMUs based on observability minimization.

As Bus- 2 is ZI bus

$$I_{24} = I_{12} + I_{32}$$

The bus 4 voltage is considered as

$$V_4 = V_2 - (I_{12} + I_{32})Z_{24}$$

From KVL, the voltage of bus 4 is measured, the PMU is not required at bus 4.

**III.ii. Redundancy Constrained PMU Placement**

The channel or branch data of each bus of network is considered for optimal placement to obtain redundant measurements with complete observability of network. For example, consider the branch data of 14-bus system.

The branch data of 14-bus system is as follows

| Bus no | B₁ | B₂ | B₃ | B₄ | B₅ | B₆ | B₇ |
|--------|----|----|----|----|----|----|----|
| Branches | 2  | 4  | 2  | 5  | 4  | 4  | 3  |
| Bus no | B₈ | B₉ | B₁₀| B₁₁| B₁₂| B₁₃| B₁₄|
| Branches | 1  | 2  | 2  | 2  | 2  | 3  | 2  |

Redundancy at bus 4, 2, 5, and 6 is highest compared to other buses. To decrease redundancy of measurements, the constraints related to buses are made compulsory for OPP to achieve complete observability.
To optimize the PMU placements, the nonlinear constraints are reduced with the rule of subsets. Consider A and B are two sets, if $A \subset B$, then $A + B = B$.

By applying the set rule to non-linear constraints of buses

\[
(\text{bus-1 + bus-2}) = \text{bus-3},
\]
\[
(\text{bus-3 + bus-4}) = \text{bus-7},
\]
\[
(\text{bus-8 + bus-7}) = \text{bus-15},
\]
\[
(\text{bus-12 + bus-13}) = \text{bus-13}
\]

The redundancy of measurements can be decreased with set rule useful.

The problem of redundancy and observability constrained PMU location is framed as follows

\[
J(y) = \text{Min} \sum_{k=1}^{14} y_k^T c_k y_k
\]

Subject to nonlinear observability constraints

\[
\begin{align*}
\sum_{k=1}^{14} y_k & = b - 2 = (1 - y_1)(1 - y_2)(1 - y_3)(1 - y_4)(1 - y_5) = 0 \\
& = b - 4 = (1 - y_2)(1 - y_3)(1 - y_5)(1 - y_6)(1 - y_7) = 0 \\
& = b - 5 = (1 - y_2)(1 - y_3)(1 - y_4)(1 - y_6)(1 - y_7) = 0 \\
& = b - 6 = (1 - y_2)(1 - y_3)(1 - y_4)(1 - y_5)(1 - y_7) = 0 \\
& = b - 7 = (1 - y_6)(1 - y_7)(1 - y_8) = 0 \\
& = b - 9 = (1 - y_4)(1 - y_6)(1 - y_7)(1 - y_8) = 0 \\
& = b - 13 = (1 - y_4)(1 - y_6)(1 - y_7)(1 - y_8) = 0
\end{align*}
\]

With application of proposed SQP approach with above nonlinear observability constraints, optimal placements for installation of PMUs attained are 2, 6, 7, and 9.

3.3 Redundancy and Observability Constrained PMU Placement with ZI Modelling

With application of ZI modeling to constraints shown in equation-(5), the problem is formulated as

\[
J(y) = \text{Min} \sum_{k=1}^{14} y_k^T c_k y_k
\]

Subject to nonlinear observability constraints

\[
\begin{align*}
\sum_{k=1}^{14} y_k & = b - 2 = (1 - y_1)(1 - y_2)(1 - y_3)(1 - y_4)(1 - y_5) = 0 \\
& = b - 5 = (1 - y_2)(1 - y_3)(1 - y_4)(1 - y_6)(1 - y_7) = 0 \\
& = b - 6 = (1 - y_2)(1 - y_3)(1 - y_4)(1 - y_5)(1 - y_7) = 0 \\
& = b - 7 = (1 - y_6)(1 - y_7)(1 - y_8) = 0 \\
& = b - 9 = (1 - y_4)(1 - y_6)(1 - y_7)(1 - y_8) = 0 \\
& = b - 13 = (1 - y_4)(1 - y_6)(1 - y_7)(1 - y_8) = 0
\end{align*}
\]
With application of ZI modeling with proposed SQP approach, optimal placements for installation of PMUs attained are 2, 6 and 9.

III.iv. Performance of Observability through CSBOI

For large power system networks, observability of network is determined by two methods, one is by finding rank of the Jacobian matrix determined in SE process other is by topological observability of network. Here we chose topological observability of network. Through this method every bus of network is checked with Bus Redundancy Index (BRI) to find the system is optimal redundant. The limitation of BRI is measured as maximum number of channels connected plus one.

\[ \beta_p \leq \text{SR}_p + 1 \]  

For bus-\( k \), \( BOI (\beta) \) gives the quantity of PMUs placed to measure the bus. Sum of BRI at all buses of network gives Complete System Bus Observability Index (CSBOI) and is given as

\[ \text{CSBOI} = \sum_{p=1}^{N} \beta_p \]  

Maximum redundancy of bus can be formulated as

\[ \text{Max} \sum_{k=1}^{n} b y_k \]  

Subjected to following constraints

\[ \sum_{k=1}^{n} y_k = \mu_0 \]  

where \( \mu_0 \) is minimum quantity of PMUs attained for complete network observability

\( b \) is adjacency matrix

\( y_k \) can be binary decision variable.

\[ y_k = \begin{cases} 1 & \text{if PMU is allocated at bus } k \\ 0 & \text{otherwise} \end{cases} \]

\[ b_{i,j} = \begin{cases} 1 & \text{if } i = j \text{ or connected to each other} \\ 0 & \text{otherwise} \end{cases} \]

PMUs are allocated at redundant buses that can increase bus redundancy of network.

III.v. Sequential Quadratic Programming approach for Complete Network Observability with Optimal Redundancy

Sequential Quadrating Programming (SQP) is an iterative process for constrained and unconstrained non-linear optimization. The quadratic problems subjected to nonlinear constraints are solved using SQP constrained and un-constraints, equal and inequality constraints can formulate using SQP.

The procedure for PMU placement is shown in Fig.1 the basic model of OPP is shown in equation (1-2), ZI constraint modeling is shown in equation (3-4). Redundancy and observability constraint modeling is shown in equation (5-6). The ZI modelling of Redundancy and observability PMU placement constraints are shown in (7-8)
Performance of observability is checked considering CNOI shown in equation (10). The SQP is modeled with these non-linear constraints from equations (1-12) and programmed as shown in Fig.3.

![Diagram of Nonlinear-Constrained Modeling of SQP method](image)

IV. Results and Analysis

The simulation programming of Redundancy and Observability constrained and ZI modeled PMU placement for different test case systems is expressed. Test cases -14- 30 and - 57 bus systems are executed on Intel(R) core (TM), i3 processor at 2.20 GHz, 4 GB of RAM. The branch or channel data of IEEE 14- bus network is shown in Table I. The most redundant and ZI buses are shown in Table II.

| IEEE test cases | Most redundant buses | ZIBs |
|-----------------|-----------------------|------|
| 14 bus          | 4,2,5,6,              | 7    |
| 30 bus          | 6,10,12,27            | 11,12,17,24 |
| 57 bus          | 9,15,38,49            | 6,9,22,25,27,28 |

Redundancy and observability constrained OPP is shown in Table III.
TABLE III. REDUNDANCY AND OBSERVABILITY CONSTRAINED PMU PLACEMENT

| IEEE test cases | PMUs required | Location of PMU          |
|-----------------|---------------|--------------------------|
| 14 bus          | 4             | 2,7,10,13                |
| 30 bus          | 10            | 1,7,8,9,10,12,18,24,26,27|
| 57 bus          | 17            | 1,4,6,13,20,23,27,29,30,33,34,37,41,45,47,51,54,56 |

The ZI modelling of Redundancy and observability constrained OPP is shown in Table IV.

TABLE IV. REDUNDANCY AND OBSERVABILITY CONSTRAINED PMU PLACEMENT WITH ZI MODELING

| IEEE test cases | PMUs required | Location of PMU          |
|-----------------|---------------|--------------------------|
| 14 bus          | 3             | 2,6,9                    |
| 30 bus          | 8             | 2,8,9,12,19,22,25,27     |
| 57 bus          | 15            | 4,6,12,15,20,23,25,28,32,37,41,48,51,55,56 |

From Table III and IV, the location of PMUs using ZI modelling is decreased, then the installation cost is decreased. PMU placement with ZI modeling is observable with optimal number PMU measurements. The performance of redundancy is checked with BRI proposed. BRI shows measurement redundancy that how many times PMU measures states of network.

TABLE V. BRI OF 14-BUS NETWORK

| Bus No | B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 | B9 | B10 | B11 | B12 | B13 | B14 |
|--------|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|
| BRI    | 1  | 1  | 1  | 2  | 1  | 1  | 1  | 2  | 1  | 1   | 1   | 1   | 1   | 1   |

TABLE VI. BRI OF 30-BUS NETWORK

| Bus No | B1 | B2 | B3 | B4 | B5 | B6 | B7 | B8 | B9 | B10 | B11 | B12 | B13 | B14 | B15 | B16 | B17 | B18 | B19 | B20 | B21 | B22 | B23 | B24 | B25 | B26 |
|--------|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| BRI    | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1  | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   | 1   |

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Bus Redundancy Index of 14, 30 and 57 bus networks is shown in Table V, VI and VII. BRI shows how many times, individually bus is measured by PMU. From BRI tables, it is observed that bus with more number of branches is measured number of times than bus with less number of branches connected. Table VIII shows CNOI. It is observed that, redundancy is high for without ZI modeling and is optimal with ZI modeling.

**TABLE VIII.** Complete Network Observability (CNOI) with and Without ZI modeling

| IEEE bus Networks | Complete Network Observability Index |
|-------------------|-------------------------------------|
|                  | ZI modeling | Conventional modeling |
| 14bus             | 15          | 16                      |
| 30bus             | 32          | 40                      |
| 57bus             | 61          | 70                      |

The evaluation of proposed process with conventional methods as shown in Table IX for the PMU placement with complete observability. With application of ZI modelling to redundancy and observability constrained PMU placement, the number of PMUs are minimized in IEEE standard power networks.
TABLE IX. EVALUATION OF PLANNED OBSERVABILITY CONSTRAINED
METHOD WITH EXTRA METHODS FOR COMPLETE OBSERVABILITY

| Methods                                      | 14-bus case | 30-bus case | 57-bus case |
|----------------------------------------------|-------------|-------------|-------------|
| ILP with Multi scheduling [4]                | 4           | 10          | 17          |
| BIP [7]                                      | 4           | 10          | -           |
| BBO [14]                                     | 4           | 10          | -           |
| Improved BABCA [15]                          | 4           | 10          | -           |
| Constrained SQ approach                      | 4           | 10          | 17          |
| Constrained SQ approach considering ZI modeling | 3           | 8           | 15          |

II. Conclusion

This paper presented sequential quadratic programing method, formulated considering redundancy and observability constraints with, without ZIBs. for placement of PMU. ZI constraint modeling decreases the PMU locations in IEEE standard bus networks thereby minimizing the cost of PMUs for installation. Redundancy constraint modeling decreases the measurements to achieve complete observability of system network thereby achieving optimal redundant measurements for state estimation. Redundancy and observability constraint modeling with and without zero injection modeling shows the effectiveness of constrained modeling. Complete Network Observability Index (CNOI) is proposed to evaluate performance of complete network. CNOI with and without ZI modeling for redundancy constrained PMU locations is evaluated. Redundancy and observability constrained sequential quadratic approach attains complete observability with optimal PMU locations.
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