On the controllability of a quantum object

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Abstract. On the basis of the V-function method, the physical statement of controllability as a wave motion of an object to a new state is carried out. It follows from the V-function method that the wave motion of an object is inextricably linked with its trajectory. It is shown that controllability as an object leaves the state occurs in the direction of the gradient of the wave function.

1. Introduction
The origin of modern quantum theory in the works of M. Planck [1] was the result of a synthesis of the corpuscular and wave approaches to the interpretation of the blackbody spectrum, which led to the discovery of the universal Planck constant \( h \). In subsequent works, A. Einstein [2], N. Bohr [3] and Louis de Broglie [4] continued to study the corpuscular and wave properties in the behavior of the electromagnetic field and electron, which led them to the concept of a photon, quantization of the energy levels of atoms and physical waves of de Broglie. The greatest success in the mathematical generalization of the Louis de Broglie wave theory was achieved by E. Schrödinger [5], having formulated in this way the equation named after him for the wave function of an electron in an arbitrary external field.

The wave approach is generally recognized by the creators of the quantum theory allowed to achieve a deeper physical understanding of the nature of the phenomena of the microcosm, but the wave function of the electron has acquired a non-classical nature and, according to M. Born's proposal, it began to describe the amplitude of the probability of finding an electron in space [6]. In turn, the incompatibility of the trajectory and wave description (wave-particle dualism) was considered as one of the basic principles of the theory, philosophically justified by the Heisenberg uncertainty relation [7].

Recent experimental achievements in studying the behavior of individual microscopic systems, in turn, revive a steady interest in testing the basic principles of quantum theory and stimulate a deeper rethinking of its physical foundations, the role of information in the theoretical description of the behavior of micro-particles [8, 9]. The ongoing attempts to understand the paradoxical manifestation of corpuscular-wave dualism of the electron motion (and other micro-particles) also stimulate the creation of new theories, one way or another develop ideas of wave-pilot Louis de Broglie [10,11]. In this paper, we propose a new approach based on the particle-wave monism to explain the nature of the
electron. The physical controllability of the electron we consider as the exit of the object from the state, overcoming the limits of the uncertainty principle.

The theory we developed [12-15] (the V-function method) uses a description of physical reality, which takes into account the presence of electron trajectories that reflect the fact of the existence of a particle, but it is also accepted that the electron’s motion is determined by the physical wave \( V(x, t) \). It should be noted that in contrast to the positivist approach [16], used in constructing of quantum mechanics and based on the description of reality using only the values observed in the experiment (dipole moments of transitions, frequencies of emitted photons, etc., showing the mode of existence of the electron), below we use the concept of "process-state", which is introduced to describe the nature and mode of existence of the electron. This concept is initially formulated on the basis of the ontology of the dynamism strategy [17], where motion (process) represents the essence of reality, and the trajectory (state) represents the mode of existence of reality.

The trajectory of a quantum object is formed by controllability. As is generally known, there are two different concepts of controllability: controllability in control theory and controllability in flight dynamics. Kalman was the first in the theory of control to pose the problem of controllability of dynamical systems [18]. The concept of controllability in flight dynamics has a completely different meaning [19, 20]. Here, the controllability refers to the compliance of the aircraft to the current disturbances. Such controllability is implied for a quantum object.

2. Local controllability and its relation to the maximum principle in the performance problem

Let us consider a controlled dynamic system.

\[
\dot{x} = f(x, u) \tag{1}
\]

\[
x(t_0) = x_0;
\]

\[
x = (x_1, x_2, \ldots, x_n)^T, \text{ } x - n\text{-dimensional vector of phase coordinates, } x(t) \in X;
\]

\[
u = (u_1, u_2, \ldots, u_r)^T, \text{ } u - r\text{-dimensional vector of control actions, } u(t) \in U.
\]

The right parts of system (1) are functions that are continuous in their arguments and have non-

continuous partial derivatives \( \frac{\partial F}{\partial x} \), \( \frac{\partial F}{\partial u} \), that are bounded in absolute value. The system (1) itself satisfies the conditions of the theorem on the existence and uniqueness of solutions.

Let the control change by a small amount, i.e.

\[
u(t) = \nu_*(t) + \delta \nu(t), \text{ where } \nu_*(t) \text{ is the optimal control minimizing functionality}
\]

\[
I(t) = \int_{t_0}^{t} L(x, u, \tau)d\tau \tag{2}
\]

and taking its value from the internal points of the area \( U \).

Based on the local variational principle, we will study controllability as a component of the wave and determining the output of a dynamical system from a state or the same as compliance with perturbations. We present a formulation of the local variational principle:

Of all the possible transitions to a new state, one is realized in which at each moment of time the rate of change of the wave function \( V(x, t) \) takes a stationary value.
\[ \delta \left( \frac{dV}{dt} \right) = 0 \]  

(3)

where the wave function is a \( V \)-function satisfying the equation:

\[ \frac{\partial^2 V(x, t)}{\partial x^2} - \sum_{i,j=1}^{n} \frac{\partial^2 V(x, t)}{\partial x_i \partial x_j} \tilde{f}_i(x) \tilde{f}_j(x) = 0 \]  

(4)

\( \tilde{f}(x) = f(x, u(x)) \) — vector of the right parts of the equations of motion of the object.

We write (3) in this form:

\[ \delta \left( \frac{dV}{dt} \right) = \frac{d}{dt} \left( \frac{\partial V^T}{\partial x} \frac{\delta x}{\delta t} \right) = 0, \]  

(5)

as well as system (1) in variations:

\[ \delta \dot{x} = \delta f = \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial u} \delta u. \]  

(6)

Then, introducing the vector \( \lambda = \frac{\partial V}{\partial x} \), where \( \frac{\partial V}{\partial x} \) is the gradient of the wave function, we have from (5) taking into account (6):

\[ \frac{d}{dt} \left( \lambda^T \delta x \right) = \dot{\lambda}^T \delta x + \lambda^T \dot{\delta x} = \dot{\lambda}^T \delta x + \lambda^T \delta f = \dot{\lambda}^T \delta x + \]  

\[ + \lambda^T \left( \frac{\partial f}{\partial x} \delta x + \frac{\partial f}{\partial u} \delta u \right) = 0 \]  

(7)

By introducing a function into consideration \( H = \lambda^T f \) and accepting:

\[ \dot{\lambda} = - \frac{\partial f}{\partial x} \lambda = - \frac{\partial H}{\partial x}; \]  

(8)

from (7) taking into account (8) we get:

\[ \frac{\partial f}{\partial u} \lambda = \frac{\partial H}{\partial u} = 0. \]  

(9)

Let the motion occur in the direction of the gradient of the wave function \( V(x, t) \). Now let's look at the function \( H = \lambda^T f = \lambda^T \dot{x} \). This function is the scalar product of two vectors in an \( n \)-dimensional phase space. If we consider the controllability as the output of the dynamic system from the state, as the moment of formation of the trajectory, then this representation makes it possible to determine the direction of the phase velocity vector. This direction is formed not as an action of the control law \( u(x) \), but as a possibility of control, as a result of the action of all unaccounted disturb-
ances. And it is not arbitrary, but such that the direction of the velocity vector tends to the direction of the wave gradient, i.e. exit from the state occurs in the direction of the gradient of the wave function and the function $H$ reaches its maximum in $u$. In other words, in support of the concept of the wave-pilot of Louis de Broglie, postulated by him to explain the corpuscular wave dualism [21], the wave controls the movement of the object.

Otherwise, the scalar product of vectors $\lambda$, $\dot{x}$ tends to the value of the product of the moduli of the vectors, i.e. $\left< \frac{\partial V}{\partial x}, \dot{x} \right> = \left| \frac{\partial V}{\partial x} \right| |\dot{x}|$.

As an example, consider the linear motion of an electron $\dot{x} = u(x)$

Make a function $H = \lambda u$, where $\lambda = \frac{\partial V}{\partial x}$. The maximum $\max H$ control is achieved if $u = \lambda = \frac{1}{m} \frac{\partial V}{\partial x}$, where $m = \text{const}$. According to the $V$-function method, the trajectory motion of the particle (10) corresponds to the wave motion satisfying equation (4), which in our case takes the form

$$\frac{\partial^2 V(x,t)}{\partial x^2} - \left( \frac{1}{m} \frac{\partial V}{\partial x} \right)^2 \frac{\partial^2 V(x,t)}{\partial x^2} = 0$$

and this equation is solved with the following boundary conditions

$$V(x,t) \big|_{t=0} = V(x,0) = 0,$$
$$\frac{\partial V(x,t)}{\partial t} \big|_{t=0} = \frac{\partial V(x,0)}{\partial t} = \tilde{C}_1,$$
$$V(x,t) \big|_{x=0} = V(0,t) = 0,$$
$$\frac{\partial V(x,t)}{\partial x} \big|_{x=0} = \frac{\partial V(0,t)}{\partial x} = \tilde{C}_2.$$

3. Discussion and conclusion

The controllability problem in this formulation is similar to the Pontryagin maximum principle in the speed [22, 23]. But there are introduced already at the formulation of the optimal control problem the criterion $I = \int_{t_0}^T l dt$, which requires its minimization and the vector of auxiliary variables $\lambda_i(t)$, which satisfies the conjugate system (8). To prove the maximum principle $H(x(t), u(t), \lambda(t))$, a convex analysis apparatus is used, and namely the method of cones. In this case, restrictions are imposed on the control area and the control itself belongs to the class of piecewise continuous functions undergoing discontinuities of I kind, i.e. it can take its values on the border of the control area. In our case, the conjugate system (8) and the condition $\max H$ are obtained from the necessary and sufficient theorem [12-15] and the notion of local controllability described above. In this case, the problem of speed in optimal control as if pre-laid in the local variational principle. And the essence of controllability as a physical task consists in the co-directionality of vectors $\frac{\partial V}{\partial x}$, $\dot{x}$ at each moment of time, and the quality criterion (2) determines the measure of controllability, the structure of which is not selected, but is constructed during the movement of the object. In addition, the controllability of the
object in this formulation allows us to resolve the uncertainty principle, the uncertainty now reduces to solving the boundary value problem of equation (4).

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