Distinctive signatures of the lowest bottomonium hybrid

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We show that the lowest bottomonium hybrid $H(1P)$ and the conventional bottomonium state $\Upsilon(5S)$, whose masses are close to each other, have very different decay widths to open bottom two-meson channels. We use this fact and the plausible $\Upsilon(5S)-H(1P)$ mixing scenario to infer from current data experimental evidence of the existence of the lowest bottomonium hybrid.

There is nowadays compelling theoretical evidence, from quenched lattice QCD calculations, of the existence of quarkonium hybrids [1]. In contrast, there is not convincing experimental evidence of their existence mostly due to the difficulty of identifying unambiguous distinctive signatures for them. In this regard the lowest bottomonium hybrid state can be an ideal system for trying to disentangle these signatures for several reasons.

First, the mass of the $b$ quark, $M_b$, is much larger than the QCD scale, $\Lambda_{QCD}$, and this supports the use of the Born-Oppenheimer (BO) approximation for its description [1, 2]. In this approach bottomonium, i.e. bound states of $b\bar{b}$, and bottomonium hybrids, i.e. bound states of $b\bar{b}g$ where $g$ stands for a gluon, correspond to solutions of the Schrödinger equation in different potentials: the ground state BO potential $V_{\Pi_{b\bar{b}}^+}(r)$ for bottomonium and the deepest hybrid potential $V_{\Sigma_g^+}(r)$ for the lowest bottomonium hybrid with $J^{PC}=1^{--}$. The fact that the calculated mass of this hybrid is about 10900 MeV, more than 100 MeV below the first $S$-wave $1^{--}$ open flavor meson-meson threshold $B\bar{B}1$, provides an a posteriori justification of the use of the BO potential obtained from quenched lattice calculations. (As for the $P$-wave $B^{(*)}\bar{B}^{(*)}$ and $B^{(*)}\bar{B}^{(*)}$, they do not contribute effectively to the static $b\bar{b}$ configuration assumed in the construction of the BO potentials.)

Second, being the lowest hybrid state it can not decay to other hybrids. Moreover, as the deepest hybrid BO potential $V_{\Sigma_g^+}(r)$ is smaller than the sum of the ground-state BO potential $V_{\Sigma_g^+}(r)$ and the mass of a glueball with the appropriate quantum numbers, decay to a $b\bar{b}$ meson plus a glueball is not expected. Thus, the strong hybrid decays are constrained to final states not involving hybrids or glueballs.

Third, due to the small value of $\Lambda_{QCD}/M_b$ heavy quark spin symmetry (HQSS) can be assumed to be approximately valid. Notice that this follows from the BO approximation and from the fact that the relevant $B\bar{B}1$ threshold is far above in energy.

In this letter we use this reasoning to try to identify unambiguous distinctive signatures of the lowest bottomonium hybrid. This requires a quantitative comparative analysis with alternative physical systems that could eventually produce the same experimental signatures, e.g. bottomonium states with the same quantum numbers $J^{PC}=1^{--}$. A major problem for this comparison can be the use of different models for the calculations. To try to mitigate, at least in part, this inconvenience we shall use the BO framework, from which the spectra of bottomonium and bottomonium hybrids are derived, for a unified treatment of their dominant strong decays. Then, we shall apply it to the the lowest bottomonium hybrid, that we shall call henceforth $H(1P)$, and to the $\Upsilon(5S)$ bottomonium state whose masses are close to that of the experimental $1^{--}$ resonance $\Upsilon(10860)$ [3]. The comparison of the results obtained with the observed decay properties of $\Upsilon(10860)$ will allow us to infer some experimental evidence of $H(1P)$.

In the BO approximation the state of a color singlet system made of a $b$ quark, a $\bar{b}$ antiquark, and a flavor-singlet configuration of light fields can be written as [2]

$$|E_{Lm_Ls_{\bar{b}b}m_{s_{\bar{b}b}}};\Lambda,\eta,\epsilon\rangle$$

1

$$=\int d\vec{r}'R_{nL}(r')Y_{Lm_L}(\hat{r}')|r',\Lambda,\eta,\epsilon(r')\rangle|s_{\bar{b}b}m_{s_{\bar{b}b}}\rangle$$

where $E$ stands for the energy, $r$ for the $b-\bar{b}$ vector distance, and $R_{nL}(r)$ and $Y_{Lm_L}(\hat{r})$ for the hybrid radial and angular wave functions respectively. The spherical harmonic $Y_{Lm_L}(\hat{r})$ is an eigenstate of $L^2$ and $L_z$, being $L$ an angular momentum of the system defined as $L=L_{\bar{b}b}+J_\beta$ where $L_{\bar{b}b}$ is the orbital angular momentum of $b\bar{b}$ and $J_\beta$ is the total angular momentum of the light fields. These fields are characterized by quantum numbers $\beta=(\Lambda,\eta,\epsilon)$ which are conserved in the presence of static $b$ and $\bar{b}$ sources (for the physical meaning of these quantum numbers, see for example [2]). We shall center on the vacuum configuration, called $\Sigma_g^+$, specified by

$$\Sigma_g^+ \equiv \left(\Lambda_{\Sigma_g^+} = 0, \eta_{\Sigma_g^+} = +1, \epsilon_{\Sigma_g^+} = +1\right)$$

and its BO potential $V_{\Sigma_g^+}(r)$, and on the lowest gluon field configuration, named $\Pi_u^+$, specified by

$$\Pi_u^+ \equiv \left(\Lambda_{\Pi_u^+} = 1, \eta_{\Pi_u^+} = -1, \epsilon_{\Pi_u^+} = +1\right)$$

and its BO potential $V_{\Pi_u^+}(r)$.

From (1), bottomonium and bottomonium hybrid states characterized by $J^{PC}$ where $J=L+s_{\bar{b}b}$ is the total angular momentum ($s_{\bar{b}b}$ is the spin of $b\bar{b}$),

$$P = \epsilon(-1)^{\Lambda+L+1}$$

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is the parity, and
\[ C = \eta \epsilon (-1)^{\Lambda + L + s_{bb}} \]
is the charge conjugation of the system, can be easily built as
\[ |ELs_{bb}Jm_J; \Lambda, \eta, \epsilon \rangle = \sum_{m_L, m_s} \langle Lm_Ls_{bb}m_{s_{bb}} | Ls_{bb}Jm_J \rangle |ELm_{bb}m_{s_{bb}}; \Lambda, \eta, \epsilon \rangle. \]

For 1−− bottomonium, with the vacuum configuration \( \Sigma^+_g \), one has \( L = l_{bb} = 0, 2, s_{bb} = 1 \) and \( J = j_{bb} = 1 \) where \( j_{bb} \) is the total angular momentum of \( bb \). For the lowest 1−− bottomonium hybrid, with gluon configuration \( \Pi^+_u \), one has \( L = 1, s_{bb} = 0 \) and \( J = 1 \).

If kinematically allowed, the dominant strong decays for bottomonium are known to be to open bottom two-meson states. It is usually assumed that the decay takes place in two steps. The first step is the emission of the vacuum configuration of a flavor singlet light quark-antiquark pair, \( q \bar{q} \). In the BO framework this emission corresponds to a transition
\[ |E, l_{bb}, s_{bb}, J = j_{bb}, m_J = m_{j_{bb}}; \Sigma^+_g \rangle \rightarrow |E, l_{bb}, s_{bb}, l_{\bar{q}q}, j_{\bar{q}q}, s_{\bar{q}q}, j_{\bar{q}q}; m_J; \Sigma^+_g \rangle \]
where \( |q \bar{q}\rangle \equiv |l_{\bar{q}q}, s_{\bar{q}q}, j_{\bar{q}q}, m_{j_{\bar{q}q}}\rangle \). Conservation of parity and charge conjugation implies
\[ (-1)^{j_{\bar{q}q} + 1} = 1 \quad \text{and} \quad (-1)^{j_{\bar{q}q} + s_{\bar{q}q}} = 1 \]
respectively. Hence, \( j_{\bar{q}q} = \text{odd} \) and \( s_{\bar{q}q} = \text{odd} \) \( \Rightarrow s_{\bar{q}q} = 1 \). If we reasonably assume that the most favored emission is for \( j_{\bar{q}q} \) having its minimal value then \( j_{\bar{q}q} = 1 \) and \( s_{\bar{q}q} = 0 \) so that the emitted \( q \bar{q} \) pair is in a \( ^3P_0 \) or \( 0^+ \) state. The second step is the combination of the color singlet \( q \bar{q} \) with the color singlet \( bb \) giving rise to \( (b \bar{q}) \) and \( (b \bar{q}) \) mesons. This second step process defines the so called \(^3P_0 \) decay model which has been very successful in dealing with quarkonium decays to open bottom two-meson states. This model was proposed in [4] and detailed for bottomonium decays in [5].

For the lowest bottomonium hybrid state decays to open bottom two-meson states, if kinematically allowed, can be expected to be dominant as well. In parallel with the bottomonium case we shall assume that the decay takes place in two steps. The first step is the emission out of the gluon configuration of a flavor singlet and color octet light quark-antiquark pair. In the BO framework, the emission corresponds to a transition from the hybrid system to a color octet \( bb \) plus a color octet \( q \bar{q} \) with the vacuum configuration
\[ |E, L = 1, s_{bb} = 0, J = 1, m_J; \Pi^+_u \rangle \rightarrow |E, l_{bb}, s_{bb} = 0, j_{\bar{q}q}; l_{\bar{q}q}, s_{\bar{q}q}, j_{\bar{q}q}; m_J; s_{\bar{q}q}, j_{\bar{q}q}; 1, m_J; \Sigma^+_g \rangle. \]

Conservation of parity implies
\[ \epsilon_{\Pi^+_u} (-1)^{\Lambda + s_{bb} + L + 1} = \epsilon_{\Sigma^+_g} (-1)^{\Lambda + s_{bb} + l_{bb} + 1} (-1)^{J_{\bar{q}q} + 1}, \]
so that \( l_{bb} + l_{\bar{q}q} = \text{odd} \), and conservation of charge conjugation
\[ \eta_{\Pi^+_u} \epsilon_{\Pi^+_u} (-1)^{\Lambda + l_{bb} + s_{bb}} = \eta_{\Sigma^+_g} \epsilon_{\Sigma^+_g} (-1)^{\Lambda + l_{\bar{q}q} + s_{\bar{q}q}} (-1)^{l_{\bar{q}q} + s_{\bar{q}q}} \]
so that \( l_{bb} + l_{\bar{q}q} + s_{\bar{q}q} = \text{odd} \). Hence \( s_{\bar{q}q} = \text{even} \) \( \Rightarrow s_{\bar{q}q} = 0 \). Besides, the conservation of the component of the total angular momentum of the light fields along the \( bb \) axis [2] can be expressed in this case as \( j_{\bar{q}q} \geq \Lambda_{\Pi^+_u} = 1 \). If we reasonably assume that the most favored emission is for \( j_{\bar{q}q} \) having its minimal value then \( j_{\bar{q}q} = 1, s_{\bar{q}q} = 0 \) and \( l_{\bar{q}q} = 1 \) so that the emitted color octet \( q \bar{q} \) pair is in a \( ^1P_1 \) or \( 1^+ \) state. Then, \( l_{\bar{q}q} = \text{even} \). For the lowest hybrid it is quite natural to assign \( j_{bb} = l_{bb} = 0 \) so that the color octet \( bb \) pair is in a \( 0^+ \) state. Notice that the quantum numbers of the emitted pair \( 1^- \) are the same quantum numbers characterizing the ground state gluehump, which is the limit of the gluon configuration \( \Pi^+_u \) when \( r \rightarrow 0 \) (in this limit \( J_{P_0} \) and \( C \) are conserved). In other words, when \( r \rightarrow 0 \) the hybrid can be seen as composed of a \( 0^+ \) color octet \( bb \) and a \( 1^+ \) color octet glue. Hence, the physical picture of the emission process when \( r \rightarrow 0 \) is that of a spectator \( 0^+ \) color octet \( bb \) and a \( 1^+ \) glue that converts into the color octet \( q \bar{q} \).

The second step is the combination of the color octet \( q \bar{q} \) with the color octet \( bb \) giving rise to \( b \bar{q} \) and \( b \bar{q} \) mesons. This two step process defines the \( ^3P_0 \) model for the decay of the lowest bottomonium hybrid into open bottom two-meson states.

It is worth to emphasize that the \( ^3P_0 \) decay model is essentially different from the decay models built from constituent glue or flux tube hybrid models, see [6] and references therein. In essence, in these hybrid models the created pair creation is assumed to be spin triplet whilst in the \( ^3P_1 \) decay model is spin singlet. This difference is crucial to establish the forbidden and allowed decays from the lowest bottomonium hybrid to open bottom two-meson states, as we show next.

Let us consider the decay \( H (1^P) \rightarrow C + F \) where \( C \) is a \( b \bar{q} \) meson state \( (B, \bar{B}, B_u, \bar{B}_u) \), and \( F \) is a \( bq \) meson state \( (B, \bar{B}, B_u, \bar{B}_u) \). In parallel with the \( ^3P_0 \) decay model for bottomonium we shall characterize the \( q \bar{q} \) emission by a real constant probability amplitude: \( \sqrt{2} \gamma_1 \) for \( u \bar{q} \) or \( d \bar{q} \) and \( \sqrt{2} \gamma'_1 \) for \( s \bar{q} \) where the \( \sqrt{2} \) is a color normalization factor \( \gamma^2 > \gamma'^2 \) because the emission of a \( u \bar{q} \) or \( d \bar{q} \) pair is more probable than that of a \( s \bar{q} \) pair). Let us note that this is a simplification, since we expect \( \gamma_1 \) and \( \gamma'_1 \) to have some dependence on the momentum of
the produced mesons, which is different for the several 
$C + F$ final states. Notice also that the color matrix ele-
ment in the combination of the emitted $1^{-+}$ color octet $q\bar{q}$ with the $0^{-+}$ ($s_{bb} = 0 = s_{\bar{b}b}$) color octet $b\bar{b}$ is $1/\sqrt{2}$ so that the total (emission + combination) color factor is $\sqrt{3}/2 = 1$ as it corresponds to the decay of an ini-
tial color singlet into final color singlet states. As for the 
radial wave function of the color octet $b\bar{b}$ we shall approx-
imate it by that of the hybrid $R_H(1P) (r) = R_{a=1,L=1} (r)$. 
This approximation is justified in the limit $r \to 0$, where 
the hybrid wave function factorizes in the product of the $b\bar{b}$ and glue wave functions. This last one, from which 
$q\bar{q}$ pair is produced, does depend on $r$ through the 
interaction potential that becomes negligible against the 
centrifugal barrier when $r \to 0$. Then the approximation 
holds as long as the $\Pi^+_b$ configuration remains close to 
the glue lump. As a matter of fact, this is expected to 
occur up to a distance around 0.5 fm [2].

The calculation of the width follows exactly the same 
procedure used in the $3P_0$ model detailed in [4, 5]. In 
the rest frame of $H (1P)$ and for the emission of a $u\pi$ 
or $d\bar{s}$ pair it can be expressed as (we follow the PDG 
conventions [3])

$$\Gamma(H (1P) \to C + F) = \frac{\gamma^2 2\pi E_C E_F}{M_H} k |\mathcal{M}|^2$$

(12)

where $M_H$ is the mass of the hybrid, $E_C$ is the energy 
of the $C$ meson given by $E_C = \sqrt{M_F^2 + \vec{k}^2}$ being $k$ the 
modulo of the three-momentum of $C$ (or $F$), and

$$|\mathcal{M}|^2 = \frac{1}{16\pi^2} \left| \langle I_C m_{Ic} I_F m_{IF} | I_{bb} m_{\bar{b}b} \rangle \right|^2 \left[ \begin{array}{ccc} I_1 & I_2 & I_{bb} \\ I_3 & I_4 & 0 \end{array} \right] \left[ \begin{array}{c} I_{C} \\ I_{F} \end{array} \right] \left[ \begin{array}{c} I_{bb} \end{array} \right]$$

(13)

where $I$ and $m_I$ stand for isospin and its third compo-
nent, $s$ for spin, $J$ for total angular momentum of the 
initial state, $I_{bb} = 0$, $s_{bb} = 0$, $j_{bb} = 0$, $s_{\bar{b}b} = 0$, $I_{\bar{b}b} = 1$, $j_{\bar{b}b} = 1$ and the subscripts $1, 2, 3, 4$ refer to $b, \bar{b}, q, \bar{q}$ re-
spectively. The square brackets are related to the $9j$ symbols:

$$\begin{pmatrix} j_a & j_h & j_c \\ j_c & j_a & j_f \\ j_f & j_h & j_i \end{pmatrix} = \sqrt{\gamma_c \gamma_j \gamma_h \gamma_k} \begin{pmatrix} j_a & j_h & j_c \\ j_c & j_a & j_f \\ j_f & j_h & j_i \end{pmatrix}$$

(14)

with $j \equiv 2j+1$. The spatial integral $\mathcal{J}+$ is given by

$$\mathcal{J}+(k) = i^{3b_\bar{b}} \sqrt{\frac{3(l_{bb} + 1)}{2l_{\bar{b}\bar{b}} + 3}} I_+(k)$$

(15)

with

$$I_+(k) = \int_0^\infty r^2 dr p^2 dp u_C^*(p) u_F(p) u_H(r) [p j_1 (pr) j_{1b,\bar{b}} + h_q k j_0 (pr) j_{1b} (h_b kr)]$$

(16)

where $h_b \equiv \frac{M_B}{M_B + M_H}$ and $h_q \equiv \frac{M_B}{M_B + M_H}$ and $u$ stands for the 
Fourier transform of the radial wave function.

Notice that from these expressions one can easily recover 
the corresponding ones to the $3P_0$ model for the 
decay of an $S$-wave $1^{-+}$ bottomonium state $\Upsilon$ by substi-
tuting $\gamma_0^2 \to \gamma_0^2$, $s_{\bar{b}b} = 0 \to s_{\bar{b}b} = 1$, $j_{\bar{b}b} = 1 \to j_{\bar{b}b} = 0$, 
$s_{\bar{b}b} = 0 \to s_{\bar{b}b} = 1$, $j_{\bar{b}b} = 0 \to j_{bb} = 1$, and $H \to \Upsilon$.

For the sake of simplicity we shall use henceforth the 
notation:

$$\Gamma_{H} = \Gamma_{H(1P) \to B\bar{B}}, \quad \Gamma_{H'} = \Gamma_{H(1P) \to B_s\bar{B}_s'}$$

(17)

From (13) and taking into account that the three el-
ements in the same column in the $9j$ symbol have to 
satisfy the triangular rule for the symbol not to vanish 
we immediately infer that the lowest bottomonium hybrid 
can only decay to $CF$ open bottom two-meson channels with

$$s_{CF} = s_{bb} = 0.$$ 

(18)

From this spin selection rule, the decays to $B^+\bar{B}$ and 
$B^0_s\bar{B}_s'$ (we use this notation instead of $B^+\bar{B} + B^0\bar{B}'$ and $B^0_s\bar{B}_s + B^0_s\bar{B}_s'$), although kinematically allowed, are for-
bidden

$$\Gamma_{H^*} = 0, \quad \Gamma_{H^*'} = 0.$$ 

(19)

As for the calculation of the widths for the other kinematically 
allowed decays to $B\bar{B}$, $B^0\bar{B}'$ and $B^0_s\bar{B}_s'$ we shall use for the lowest 
hybrid state the mass 10888 MeV and the radial wave 
function calculated in reference [7] and for the final 
mesons their experimental masses and for simplicity, as usual, Gaussian radial functions

$$\psi_{C,F} (r) = \frac{2}{\pi^{3/2}} e^{\frac{-r^2}{2v_{C,F}^2}}$$

(20)

with $v_{C,F} \simeq 0.45$ fm (this value corresponds to an aver-
age rms radius for open bottom mesons from a standard 
Cornell like potential fitting their masses). For the heavy 
and light quark masses we have chosen standard values 
$M_b = 4793$ MeV, $M_s = 500$ MeV and $M_{u,d} = 340$ MeV. 
Thus, we get

$$\Gamma_{H^*} = 1.7 \text{ MeV}, \quad \Gamma_{H^*'} = 25.7 \text{ MeV},$$

(21)

$$\Gamma_{H^*} = 30.3 \text{ MeV}, \quad \Gamma_{H^*'} = 114.3 \text{ MeV},$$

so that

$$\Gamma_{H^*} = 15.1, \quad \Gamma_{H^*'} = 3.8.$$
Let us emphasize that the hybrid decay pattern resulting from Eqs. (18) and (20) is very different from the one predicted by constituent glue or flux tube models. In these models, as a consequence of the assumption of a spin triplet light quark pair, the hybrid decays to two $S$-wave mesons, e.g. $B^* \to \bar{B}^*(s)$, are forbidden [8].

It is very illustrative to compare our results with the corresponding decay widths from the $1^{-+}$ bottomonium state $\Upsilon (5S)$ with a calculated mass of 10865 MeV, quite close to the hybrid one. In this case, using the $3P_0$ decay model and the same kind of self-explained simplified notation we get

\[
\frac{\Gamma_{\Upsilon^0}}{\Gamma_0} = 0.8 \text{ MeV}, \quad \frac{\Gamma_{\Upsilon^{++}}}{\Gamma_0} = 0.5 \text{ MeV}, \quad \frac{\Gamma_{\Upsilon^+}}{\Gamma_0} = 1.9 \text{ MeV},
\]

\[
\frac{\Gamma_{\Upsilon^0}}{\Gamma_0} = 0.5 \text{ MeV}, \quad \frac{\Gamma_{\Upsilon^{++}}}{\Gamma_0} = 0.3 \text{ MeV}, \quad \frac{\Gamma_{\Upsilon^+}}{\Gamma_0} = 1.4 \text{ MeV},
\]

from which

\[
\frac{\Gamma_{\Upsilon^s}}{\Gamma_s} = 2.4, \quad \frac{\Gamma_{\Upsilon^{s*}}}{\Gamma_s} = 2.8,
\]

\[
\frac{\Gamma_{\Upsilon^s}}{\Gamma_{s'}} = 0.6, \quad \frac{\Gamma_{\Upsilon^{s*}}}{\Gamma_{s'}} = 0.6.
\]

The comparison of Eqs. (18), (21) with Eq. (23) makes clear the very different decay pattern from $H (1P)$ and $\Upsilon (5S)$. Therefore, Eqs. (18), (20) and (21) constitute a distinctive signature of the lowest bottomonium hybrid.

From the experimental point of view the $1^{-+}$ resonance $\Upsilon (10860)$ produced in $e^+e^-$ annihilation has a measured mass of 10889.9$^{+3.2}_{-2.6}$ MeV pretty close to the calculated masses of the lowest bottomonium hybrid $H (1P)$ and the $\Upsilon (5S)$ bottomonium state, and not far from the $B_s^* \to \bar{B}_s$ threshold. This resonance has dipion decays $\Upsilon (10860) \to \pi^+\pi^- h_0^+(1,2) P$ and $\Upsilon (10860) \to \pi^+\pi^- \Upsilon ((1,2,3) S)$ with a similar production rate. As $s_{h_0} = 0$ and $s_{\pi^+\pi^-} = 1$, approximate HQSS implies that $\Upsilon (10860)$ must have $s_{h_0}$ = 0 and $s_{\pi^+\pi^-}$ = 1 components. This has led to different proposals about its nature. In Ref. [9], following HQSS arguments, a mixture of $\Upsilon (5S)$ and a $P$-wave $B_s^* \to \bar{B}_s$ has been suggested. Taking into account that the decays $B_s^* \to B^* \to B^{(*)} \bar{B}^{(*)}$ are suppressed, $\Upsilon (10860) \to B^{(*)} \bar{B}^{(*)}$ should then proceed through the $\Upsilon (5S)$ component. However, the comparison of the calculated ratio $\frac{\Gamma_{\Upsilon^{s*}}}{\Gamma_{\Upsilon^s}} = 0.6$ with data $\frac{\Gamma_{\Upsilon (10860)} \to B \bar{B}}{\Gamma_{\Upsilon (10860)} \to \bar{B} \pi} = 6.9 \pm 1.4$ seems not to support that mixing (notice that in [9] the ratios are different than here because the spatial integrals have not been taken into account). Indeed, in the BO approximation that mixing is strongly suppressed because $b$ and $\bar{b}$ in the $P$-wave $B_s^* \to \bar{B}_s$ are not static. This suppression has also been inferred in a recent analysis of the meson-meson components in $\Upsilon (10860)$ [10].

As an alternative, in reference [7] it has been proposed that $\Upsilon (10860)$ could be a mixing of the lowest bottomonium hybrid $H (1P)$ and the $\Upsilon (5S)$ bottomonium state (mixing has been also analyzed in nonrelativistic effective field theories [11]). In this regard, it is worth to recall that the probability of $H (1P)$ in $\Upsilon (10860)$ is required to be at most of a few percent in order to get a good description of the leptonic widths (this is also in line with the order of magnitude of the HQSS breaking interaction responsible for the mixing). Let us examine now whether this proposal may give or not quantitative account of the observed dominant decays of $\Upsilon (10860)$ to open bottom meson-meson channels.

Following reference [7] we write

\[
\Upsilon (10860) = \cos \theta |\Upsilon (5S)| + \sin \theta |H (1P)|.
\]

As $\Gamma_{H^+} = 0$ we have from (22) $\Gamma_{\Upsilon (10860) \to B \bar{B}} = 1.9 \gamma_0^2 \cos^2 \theta \frac{\Gamma_{\Upsilon^0}}{\Gamma_0} = 7.0 \pm 1.3$ MeV we obtain $\gamma_0^2 \cos^2 \theta = 3.7 \pm 0.7 \Rightarrow \gamma_0 \cos \theta = \pm (1.92 \pm 0.18)$, where the quoted errors come from data uncertainty only. In an analogous manner we use $\Gamma_{H^+} = 0$ and $\Gamma_{\Upsilon (10860) \to B \bar{B}} = 1.4 \gamma_0^2 \cos^2 \theta$ to get $\gamma_0^2 \cos^2 \theta = 0.50 \pm 0.21 \Rightarrow \gamma_0 \cos \theta = \pm (0.71 \pm 0.14)$.

The obvious way to continue is to use these values altogether with Eqs. (20) and (22) for writing any of the other widths as the modulus square of the sum of the corresponding amplitudes $(\sqrt{\Gamma_{\Upsilon^{s*}}^2} + \sqrt{\Gamma_{H^{s*}}^2})^2$ and then to compare with data to extract the possible values of $\gamma_1 \sin \theta$ and $\gamma_1' \sin \theta$. It can be easily checked (choosing $\gamma_0 \cos \theta > 0$) that for $\gamma_1 \sin \theta \approx 0.40 \pm 0.20$ one gets $(\Gamma_{\Upsilon (10860) \to B \bar{B}})_{\text{Theor}} = 5.0 \pm 1.9$ MeV and $(\Gamma_{\Upsilon (10860) \to B \bar{B}})_{\text{Theor}} = 12 \pm 9$ MeV. Despite being compatible with the experimental values, $(\Gamma_{\Upsilon (10860) \to B \bar{B}})_{\text{Theor}} = 2.0 \pm 0.8$ MeV and $(\Gamma_{\Upsilon (10860) \to B \bar{B}})_{\text{Theor}} = 19 \pm 3$ MeV, these results show some tension with data. This tension points out that for $BB$ the best value for $\gamma_0 \cos \theta$ may be close to its lower limit, whereas for $B^* \bar{B}^*$ it may be close to its upper limit (see below). This could be attributed to the approximations we have followed, among them not having considered the possible momentum dependence of $\gamma_0$.

As for the decays to strange mesons we can only reliably use $\Gamma_{\Upsilon (10860) \to B \bar{B}} = 9 \pm 3$ since data for $\Gamma_{\Upsilon (10860) \to B \bar{B}}$ are very uncertain. Then, for $\gamma_0 \cos \theta > 0$ we get $\gamma_1 \sin \theta = 0.25 \pm 0.05$.

In order to go further we may reasonably assume

\[
\gamma_1 \approx \gamma_0, \quad \gamma_1' \approx \gamma_0',
\]

since the different color matrix elements have been normalized in the definition of pair creation vertices through an explicit color factor. Then $\frac{\gamma_1 \sin \theta}{\gamma_0 \cos \theta} = \tan \theta = \frac{0.21 \pm 0.13}{0.92} \Rightarrow \sin^2 \theta \lesssim 0.08$. (If instead we had used $\frac{\gamma_1 \sin \theta}{\gamma_0 \cos \theta} = \tan \theta$ we would have obtained looser contraints.) From these values we calculate $3.7 \pm 0.7 \lesssim \gamma_0 \lesssim 4.0 \pm 0.8$, or

\[
1.92 \pm 0.18 \lesssim \gamma_0 \lesssim 2.00 \pm 0.19.
\]
in good accord with the values commonly used in the literature, see for instance [5], and

\[
0.71 \pm 0.14 \lesssim \gamma_0 \lesssim 0.74 \pm 0.14. \tag{27}
\]

Then, from (25), using the calculated value of \( \gamma_1 \sin \theta \) we get \( 0.008 \lesssim \sin^2 \theta \lesssim 0.12 \), and using the calculated value of \( \gamma_1' \sin \theta \) we get \( 0.04 \lesssim \sin^2 \theta \lesssim 0.25 \). Thus, putting all the constraints together we conclude

\[
0.04 \lesssim \sin^2 \theta \lesssim 0.08 \tag{28}
\]

in agreement with the requirement of a few percent probability of the hybrid in \( \Upsilon(10860) \).

Therefore a fully consistent quantitative description of data comes out. This provides strong support to the explanation of \( \Upsilon(10860) \) as being mainly a mixing of \( \Upsilon(5S) \) with the lowest hybrid state \( H(1P) \). It can be easily inferred that this mixing is also unavoidable to explain data when an additional \( \Upsilon(5S)-\Upsilon(4D) \) mixing is implemented, since the decays to \( B^*\bar{B}^* \) and \( B_s^*\bar{B}_s^* \) from \( \Upsilon(4D) \) are even more suppressed than from \( \Upsilon(5S) \).

Then the need for the \( \Upsilon(5S)-H(1P) \) mixing can be interpreted as a strong experimental evidence for the lowest bottomonium hybrid.

The remaining question is whether a direct detection of the hybrid, or more precisely of the orthogonal combination to \( \Upsilon(10860) \) that we shall call \( H(10860) \) is feasible or not (the chosen name comes from the fact that the mass of \( H(10860) \) has to be close to that of \( \Upsilon(10860) \)). From our results and using \( \gamma_0 \approx \gamma_1 \approx 1.96 \pm 0.19, \gamma'_0 \approx \gamma'_1 \approx 0.72 \pm 0.14 \) and \( \sin^2 \theta \approx 0.06 \pm 0.02 \), we immediately infer that its decays to \( B^*\bar{B} \) and \( B_s^*\bar{B}_s \) will be suppressed and that it will have very dominant decays to \( B^*\bar{B}^* \) and \( B_s^*\bar{B}_s^* \):

\[
\begin{align*}
\Gamma(H(10860) \to B^*\bar{B}^*) &\approx 99 \pm 25 \text{ MeV}, \\
\Gamma(H(10860) \to B_s^*\bar{B}_s^*) &\approx 57 \pm 26 \text{ MeV}.
\end{align*}
\tag{29}
\]

Therefore we can establish as a very conservative lower bound \( \Gamma(H(10860)) > 105 \text{ MeV} \). Such a large width will presumably make prominent the overlap with the \( 2P \) hybrid state, with a mass around \( 11080 \text{ MeV} \) and a larger width, preventing a clean experimental signature.

This points out to the indirect analysis we have carried out as the only current available method to disentangle from data the presence of the lowest hybrid.

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