Performance of exponential similarity measures in supply of commodities in containment zones during COVID-19 pandemic under Pythagorean fuzzy sets

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Abstract
Following the breakout of the novel coronavirus disease 2019 (COVID-19), the government of India was forced to prohibit all forms of human movement. It became important to establish and maintain a supply of commodities in hotspots and containment zones in different parts of the country. This study critically proposes new exponential similarity measures to understand the requirement and distribution of commodities to these zones during the rapid spread of novel coronavirus (COVID-19) across the globe. The primary goal is to utilize the important aspect of similarity measures based on exponential function under Pythagorean fuzzy sets, proposed by Yager. The article aims at finding the most required commodity in the affected areas and ensures its distribution in hotspots and containment zones. The projected path of grocery delivery to different residences in containment zones is determined by estimating the similarity measure between each residence and the various necessary goods. Numerical computations have been carried out to validate our proposed measures. Moreover, a comparison of the result for the proposed measures has been carried out to prove the efficacy.
Owing to the existence of varied types of ambiguities and vagueness in the information data, decision-making difficulties are far too complicated and difficult. Zadeh introduced the theory of fuzzy sets to properly model uncertain/vague information. Many generalizations and expansions of fuzzy sets (FSs) have been offered by scholars after Zadeh’s remarkable drive and used in a wide range of applications. Atanassov’s intuitionistic fuzzy sets (IFSs) are one of the extremely important expansions of the FSs that have been broadly explored and used in several areas. IFSs theory has been successfully applied to a variety of real-world challenges, such as decision-making. Remarkable outcomes on IFSs have been carried out by many researchers.

Pythagorean fuzzy set (PFS) is a generalization of IFS that has the requirement that the sum of squares of association and non-association degrees ≤1. The idea is developed specifically to describe uncertainty mathematically and to provide a structured instrument for dealing with imprecision in real-world circumstances. Garg introduced an improved ranking order interval-valued PFSs using the Technique for Order of Preference by Similarity to Ideal Solution technique. Indeed, the hypothesis of PFSs has been widely considered, as demonstrated by various researchers. In association with the uses of PFSs, Rahman et al. proposed a few ways to deal with multiattribute group decision-making through aggregation operator (AO). Overall, the possibility of PFSs has pulled in incredible considerations from numerous researchers, and the idea has been pragmatic to a few application regions, namely, AOs, multicriteria decision-making (MCDM) problems, information measures, and many more.

The research of IFS distance and similarity measures yields a plethora of measures, each of which represents unique traits and behavior in real-world decision-making and pattern identification. For determining the degree of similarity between two collections of data, PFS similarity measures have been investigated from several angles in recent times. Some formulae of Pythagorean fuzzy information measures on similarity measures and corresponding transformation relationships were also developed. Similarity measures for trigonometric function for FSs, IFSs, and PFSs were also proposed. The similarity measures of the IFSs and PFSs are extensively utilized in various disciplines, comparable to the pattern identification, the clinical finding, decision-making. However, Lu and Ye offered a similarity measure of IVFSs on log function. Jana et al. proposed the MCDM method in a single-valued trapezoidal neutrosophic environment based on few operators. Jana and Pal suggested dynamic new AOs for dealing with IFNs or IVIFNs information. However, Dombi operations have been introduced on two single-valued trapezoidal neutrosophic numbers. Jana and Karaaslan proposed trapezoidal neutrosophic FSs under an MCDM environment.

As the virus is spreading beyond the borders of the countries, we try looking for any possible escape route. Medical science is doing its best in the search for a possible situation. Doctors and medical staff over the world have truly proved themselves to be the unsung hero in this time of chaos by putting themselves on the front line, and by making sacrifices for the sake of humanity. So, while being in the nonmedical field, we still have tried to do our part by trying
to contribute a little to the quest. Many researchers have proposed mathematical models and analyzed to understand the transmission dynamics of the coronavirus disease 2019 (COVID-19) pandemic but (i) no one has proposed exponential similarity/distance measures for PFSs and (ii) application of such measures in COVID-19 is also not studied.

In this article, we are exploring the resourcefulness of exponential similarity measures of PFSs in the application of choosing the ideal supply of goods to different households with respect to many crucial attributes based on family requirements. This article is structured as follows: Section 2 begins with preliminaries of FSs, IFSs, and PFSs. Section 3 comprises the concept of proposed exponential similarity measures of PFSs. We introduce exponential similarity measures of the PFSs and their numerical computations to validate our measures. Application of the proposed measures in finding the best technique for distributing a range of commodities to various residences using PFSs in a containment zone is provided in Section 4. The comparison has been done with the existing similarity measure by an illustration in Section 5. The numerical findings suggest that the designed similarity measures are well suited to use with any fuzzy variables. Finally, Section 6 summarizes the document and delivers directions for future experiments.

2 | PRELIMINARIES

Some fundamental theories related to FSs, IFSs, and PFSs are discussed in this segment.

**Definition 2.1** (Zadeh\(^1\)). Let \( X \) be a nonempty set. A fuzzy set \( P \) in \( X = \{x_1, x_2, ..., x_n\} \) is characterized by a membership function:

\[
P = \{ (x, \delta_p(x)) | x \in X \},
\]

where \( \delta_p(x) : X \to [0,1] \) is a measure of belongingness of degree of membership of an element \( x \in X \) in \( P \).

**Definition 2.2** (Atanassov\(^2\)). An IFS \( P \) in \( X \) is given by

\[
P = \{ (x, \delta_p(x), \zeta_p(x)) | x \in X \},
\]

where \( \delta_p(x), \zeta_p(x) : X \to [0,1] \), where \( 0 \leq \delta_p(x) + \zeta_p(x) \leq 1 \), \( \forall x \in X \). The number \( \delta_p(x) \) and \( \zeta_p(x) \) represents, respectively, the participation degree and nonparticipation degree of the element \( x \) to the set \( P \).

For each IFS \( P \) in \( X \), if

\[
\eta_p(x) = 1 - \delta_p(x) - \zeta_p(x), \quad \forall x \in X.
\]

Then \( \eta_p(x) \) is called the degree of indeterminacy of \( x \) to \( P \).

**Definition 2.3** (Yager\(^37\) and Yager and Abbasov\(^38\)). An IFS \( \tilde{A} \) in \( X \) is given by

\[
P = \{ (x, \delta_p(x), \zeta_p(x)) | x \in X \},
\]

where \( \delta_p(x), \zeta_p(x) : X \to [0,1] \),
with the condition that \( 0 \leq \delta_p^2(x) + \zeta_p^2(x) \leq 1, \ \forall x \in X \) \hspace{1cm} (4)

and the degree of indeterminacy for any PFS \( \tilde{A} \) and \( x \in X \) is given by

\[
\eta_p(x) = \sqrt{1 - \delta_p^2(x) - \zeta_p^2(x)}.
\] \hspace{1cm} (5)

3 \quad \text{NOVEL EXPONENTIAL SIMILARITY MEASURES}

To begin, the axiomatic preposition of similarity for PFSs is discussed.

**Preposition 1** (Ejegwa\(^{20}\)). Let \( X \) be a nonempty set and \( P, Q, R \in \text{PFS}(X) \). The similarity measure \( \text{Sim} \) between \( P \) and \( Q \) is a function \( \text{Sim}: \text{PFS} \times \text{PFS} \rightarrow [0,1] \) satisfies

\[(P1) \text{ Boundedness: } 0 \leq \text{Sim}(P, Q) \leq 1.\]

\[(P2) \text{ Separability: } \text{Sim}(P, Q) = 1 \iff P = Q.\]

\[(P3) \text{ Symmetric: } \text{Sim}(P, Q) = \text{Sim}(Q, P).\]

\[(P4) \text{ Inequality: If } R \text{ is a PFS in } X \text{ and } P \subseteq Q \subseteq R, \text{ then } \text{Sim}(P, R) \leq \text{Sim}(P, Q) \text{ and } \text{Sim}(P, R) \leq \text{Sim}(Q, R).\]

In several circumstances, the weight of the components \( x_i \in X \) must be considered. For instance, in decision-making, the traits usually have distinct significance, and hence ought to be designated unique weights. As a result, we submit two weighted logarithmic similarity measures also between \( P \) and \( Q \), as follows:

Let \( P, Q \in \text{PFS}(X) \) such that \( X = \{x_1, x_2, ..., x_n\} \) then

\[
S_{PFSE1}(P, Q) = 1 - \frac{e}{n} \sum_{i=1}^{n} \left\{ \frac{\left| \delta_p^2(x_i) - \delta_Q^2(x_i) \right| e^{ \left| \delta_p(x_i) - \delta_Q(x_i) \right|} + \left| \zeta_p^2(x_i) - \zeta_Q^2(x_i) \right| e^{ \left| \zeta_p(x_i) - \zeta_Q(x_i) \right|} }{2} \right\},
\] \hspace{1cm} (6)

\[
S_{PFSE2}(P, Q) = 1 - \frac{e}{n} \sum_{i=1}^{n} \left\{ \frac{\left| \delta_p^2(x_i) - \delta_Q^2(x_i) \right| e^{ \left| \delta_p(x_i) - \delta_Q(x_i) \right|} + \left| \zeta_p^2(x_i) - \zeta_Q^2(x_i) \right| e^{ \left| \zeta_p(x_i) - \zeta_Q(x_i) \right|} }{3} \right\},
\] \hspace{1cm} (7)

\[
S_{WPSE1}(P, Q) = 1 - \frac{e}{n} \sum_{i=1}^{n} \omega_i \left\{ \frac{\left| \delta_p^2(x_i) - \delta_Q^2(x_i) \right| e^{ \left| \delta_p(x_i) - \delta_Q(x_i) \right|} + \left| \zeta_p^2(x_i) - \zeta_Q^2(x_i) \right| e^{ \left| \zeta_p(x_i) - \zeta_Q(x_i) \right|} }{2} \right\},
\] \hspace{1cm} (8)
\[ S_{WPFSE2}(P, Q) = 1 - \frac{e}{n} \sum_{i=1}^{n} \omega_i \left\{ \left| \delta_P(x_i) - \delta_Q(x_i) \right| e^{-|\delta_P(x_i) - \delta_Q(x_i)|} \right. \]
\[ \left. + \left| \zeta_P(x_i) - \zeta_Q(x_i) \right| e^{-|\zeta_P(x_i) - \zeta_Q(x_i)|} \right\} \]
\[ \left. + \left| \eta_P(x_i) - \eta_Q(x_i) \right| e^{-|\eta_P(x_i) - \eta_Q(x_i)|} \right\} \right) \],
\]
where \( \eta_P(x_i) = \sqrt{1 - \delta^2_P(x_i) - \zeta^2_P(x_i)} \) and \( \eta_Q(x_i) = \sqrt{1 - \delta^2_Q(x_i) - \zeta^2_Q(x_i)} \); \( \omega = (\omega_1, \omega_2, ..., \omega_n)^T \) is the weight vector of \( x_i \) (\( i = 1, 2, ..., n \)), with \( \omega_k \in [0,1], k = 1, 2, ..., n, \sum_{k=1}^{n} \omega_k = 1 \). If \( \omega = \left( \frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n} \right)^T \), then the weighted exponential similarity measure reduces to proposed exponential similarity measures, that is, if we take \( \omega_k = 1, k = 1, 2, ..., n \), then \( S_{WPFSE1}(P, Q) = S_{PFSE1}(P, Q) \). Similarly, it can be verified that \( S_{WPFSE2}(P, Q) = S_{PFSE2}(P, Q) \).

**Theorem 3.1.** The Pythagorean fuzzy similarity measures \( S_{PFSE1}(P, Q) \) and \( S_{PFSE2}(P, Q) \) defined in Equations (6)–(9) are valid measures of Pythagorean fuzzy similarity.

**Proof.** All the essential conditions are satisfied by the novel similarity measures as follows:

(P1) **Boundedness:** \( 0 \leq S_{PFSE1}(P, Q), S_{PFSE2}(P, Q) \leq 1 \)

Proof. For \( S_{PFSE1}(P, Q) \): As \( 0 \leq |\delta^2_P(x_i) - \delta^2_Q(x_i)| \leq 1 \) and \( 0 \leq |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| \leq 1 \), therefore,

\[ 0 \leq |\delta^2_P(x_i) - \delta^2_Q(x_i)| e^{-|\delta^2_P(x_i) - \delta^2_Q(x_i)|} \leq 1 \] (10)

and

\[ 0 \leq |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| e^{-|\zeta^2_P(x_i) - \zeta^2_Q(x_i)|} \leq 1. \] (11)

From (10) and (11), we have

\[ 0 \leq \frac{|\delta^2_P(x_i) - \delta^2_Q(x_i)| e^{-|\delta^2_P(x_i) - \delta^2_Q(x_i)|} + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| e^{-|\zeta^2_P(x_i) - \zeta^2_Q(x_i)|}}{2} \leq 1 \]

\[ \Rightarrow 0 \leq 1 - \frac{e}{n} \sum_{i=1}^{n} \left( \frac{|\delta^2_P(x_i) - \delta^2_Q(x_i)| e^{-|\delta^2_P(x_i) - \delta^2_Q(x_i)|} + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| e^{-|\zeta^2_P(x_i) - \zeta^2_Q(x_i)|}}{2} \right) \leq 1. \]

Thus, \( 0 \leq S_{PFSE1}(P, Q) \leq 1 \). \( \square \)

Measure \( S_{PFSE2}(P, Q) \) can be proved similarly.
(P2) Separability: \( S_{PFSE1}(P, Q), S_{PFSE2}(P, Q) = 1 \Leftrightarrow P = Q \)

**Proof.** For \( S_{PFSE1}(P, Q) \): For two PFSs \( P \) and \( Q \) in \( X = \{ x_1, x_2, \ldots, x_n \} \), if \( P = Q \), then \( \delta^2_P(x_i) = \delta^2_Q(x_i) \) and \( \zeta^2_P(x_i) = \zeta^2_Q(x_i) \). Thus, \( |\delta^2_P(x_i) - \delta^2_Q(x_i)| = 0 \) and \( |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| = 0 \).

\[
\Rightarrow |\delta^2_P(x_i) - \delta^2_Q(x_i)| e^{-|\delta^2_P(x_i) - \delta^2_Q(x_i)|} + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| e^{-|\zeta^2_P(x_i) - \zeta^2_Q(x_i)|} = 0
\]

\[
\Rightarrow 1 - \frac{e}{n} \sum_{i=1}^{n} |\delta^2_P(x_i) - \delta^2_Q(x_i)| e^{-|\delta^2_P(x_i) - \delta^2_Q(x_i)|} + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| e^{-|\zeta^2_P(x_i) - \zeta^2_Q(x_i)|} = 1.
\]

Therefore, \( S_{PFSE1}(P, Q) = 1 \).

If \( S_{PFST}(P, Q) = 1 \), this implies,

\[
\frac{e}{n} \sum_{i=1}^{n} |\delta^2_P(x_i) - \delta^2_Q(x_i)| e^{-|\delta^2_P(x_i) - \delta^2_Q(x_i)|} + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| e^{-|\zeta^2_P(x_i) - \zeta^2_Q(x_i)|} = 0
\]

\[
\Rightarrow |\delta^2_P(x_i) - \delta^2_Q(x_i)| e^{-|\delta^2_P(x_i) - \delta^2_Q(x_i)|} + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| e^{-|\zeta^2_P(x_i) - \zeta^2_Q(x_i)|} = 0.
\]

Either \( |\delta^2_P(x_i) - \delta^2_Q(x_i)| = 0 \) or \( |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| = 0 \). Therefore \( \delta^2_P(x_i) = \delta^2_Q(x_i) \) and \( \zeta^2_P(x_i) = \zeta^2_Q(x_i) \). Hence \( P = Q \).

Measure \( S_{PFSE2}(P, Q) \) can be proved similarly.

(P3) Symmetric: \( S_{PFSE1}(P, Q) = S_{PFSE1}(Q, P) \) and \( S_{PFSE2}(P, Q) = S_{PFSE2}(Q, P) \)

Proofs are self-explanatory and straightforward.

(P4) Inequality: If \( R \) is a PFS in \( X \) and \( P \subseteq Q \subseteq R \), then \( S_{PFSE1}(P, R) \leq S_{PFSE1}(P, Q) \); \( S_{PFSE2}(P, R) \leq S_{PFSE2}(P, Q) \) and \( S_{PFSE2}(P, R) \leq S_{PFSE2}(P, Q) \); \( S_{PFSE2}(P, R) \leq S_{PFSE2}(Q, R) \).

**Proof.** For \( S_{PFSE1}(P, Q) \): If \( P \subseteq Q \subseteq R \), then for \( x_i \in X \), we have \( 0 \leq \delta_P(x_i) \leq \delta_Q(x_i) \leq \delta_R(x_i) \leq 1 \) and \( 1 \geq \zeta_P(x_i) \geq \zeta_Q(x_i) \geq \zeta_R(x_i) \geq 0 \).

This implies that \( 0 \leq \delta^2_P(x_i) \leq \delta^2_Q(x_i) \leq \delta^2_R(x_i) \leq 1 \) and \( 1 \geq \zeta^2_P(x_i) \geq \zeta^2_Q(x_i) \geq \zeta^2_R(x_i) \geq 0 \).

This we have \( |\delta^2_P(x_i) - \delta^2_Q(x_i)| \leq |\delta^2_P(x_i) - \delta^2_R(x_i)| \), \( |\delta^2_Q(x_i) - \delta^2_R(x_i)| \leq |\delta^2_P(x_i) - \delta^2_R(x_i)| \), and \( |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| \leq |\zeta^2_P(x_i) - \zeta^2_R(x_i)| \). \( |\zeta^2_Q(x_i) - \zeta^2_R(x_i)| \leq |\zeta^2_Q(x_i) - \zeta^2_R(x_i)| \).

From the above we can write

\[
|\delta^2_P(x_i) - \delta^2_Q(x_i)| e^{-|\delta^2_P(x_i) - \delta^2_Q(x_i)|} + |\zeta^2_P(x_i) - \zeta^2_Q(x_i)| e^{-|\zeta^2_P(x_i) - \zeta^2_Q(x_i)|}
\]
\[
\leq \left| \delta^2_R(x_i) - \delta^2_R(x_i) \right| e^{-|\delta^2_R(x_i) - \delta^2_R(x_i)|} + \left| \xi^2_P(x_i) - \xi^2_Q(x_i) \right| e^{-|\xi^2_P(x_i) - \xi^2_Q(x_i)|}
\]

\[
\Rightarrow 1 - \frac{e}{n} \sum_{i=1}^{n} \left| \delta^2_R(x_i) - \delta^2_R(x_i) \right| e^{-|\delta^2_R(x_i) - \delta^2_R(x_i)|} + \left| \xi^2_P(x_i) - \xi^2_Q(x_i) \right| e^{-|\xi^2_P(x_i) - \xi^2_Q(x_i)|}
\]

\[
\geq 1 - \frac{e}{n} \sum_{i=1}^{n} \left| \delta^2_R(x_i) - \delta^2_R(x_i) \right| e^{-|\delta^2_R(x_i) - \delta^2_R(x_i)|} + \left| \xi^2_P(x_i) - \xi^2_Q(x_i) \right| e^{-|\xi^2_P(x_i) - \xi^2_Q(x_i)|}
\]

\[
\Rightarrow S_{PFSE1}(P, R) \leq S_{PFSE1}(P, Q). \text{ Similarly, } S_{PFSE1}(P, R) \leq S_{PFSE1}(Q, R).
\]

Similar proofs can be made for \( S_{PFSE2}(P, R) \leq S_{PFSE2}(P, Q) \) and \( S_{PFSE2}(P, R) \leq S_{PFSE2}(Q, R) \).

Analogous to the proofs done above, we can also validate properties depicted in Preposition 1 for weighted similarity measures \( S_{WPFSE1}(P, Q) \) and \( S_{WPFSE2}(P, Q) \) accordingly.

### 3.1 Numerical verification of the similarity measures

Based on the parameters suggested by Wei and Wei,\(^{25}\) we verify whether proposed similarity measures satisfy the above four properties:

**Example 1.** Let \( P, Q, R \in PFS(X) \) for \( X = \{x_1, x_2, x_3\} \). Suppose

\[
P = \{(x_1, 0.6, 0.2), (x_2, 0.4, 0.6), (x_3, 0.5, 0.3)\},
\]

\[
Q = \{(x_1, 0.8, 0.2), (x_2, 0.7, 0.3), (x_3, 0.6, 0.3)\}, \text{ and }
\]

\[
R = \{(x_1, 0.9, 0.1), (x_2, 0.8, 0.2), (x_3, 0.7, 0.1)\}.
\]

Computing similarity as

\[
S_{PFSE1}(P, Q) = 1 - \frac{e}{6} \left\{ 0.6^2 - 0.8^2 |e^{-0.6^2 - 0.8^2}| + 0.2^2 - 0.2^2 |e^{-0.2^2 - 0.2^2}| + 0.4^2 - 0.7^2 |e^{-0.4^2 - 0.7^2}| + 0.6^2 - 0.3^2 |e^{-0.6^2 - 0.3^2}| + 0.5^2 - 0.6^2 |e^{-0.5^2 - 0.6^2}| + 0.3^2 - 0.3^2 |e^{-0.3^2 - 0.3^2}| \right\},
\]

\[
S_{PFSE1}(P, Q) = 1 - \frac{e}{6} \left[ 0.211619 + 0 + 0.2372448 + 0.2061124 + 0.0985417 + 0 \right] = 1 - 0.341374002 = 0.6586209.
\]

\[
S_{PFSE1}(P, R) = 1 - \frac{e}{6} \left\{ 0.6^2 - 0.9^2 |e^{-0.6^2 - 0.9^2}| + 0.2^2 - 0.1^2 |e^{-0.2^2 - 0.1^2}| + 0.4^2 - 0.8^2 |e^{-0.4^2 - 0.8^2}| + 0.6^2 - 0.2^2 |e^{-0.6^2 - 0.2^2}| + 0.5^2 - 0.7^2 |e^{-0.5^2 - 0.7^2}| + 0.3^2 - 0.1^2 |e^{-0.3^2 - 0.1^2}| \right\},
\]

\[
S_{PFSE1}(P, R) = 1 - \frac{e}{6} \left[ 0.286932 + 0.0291133 + 0.297016 + 0.232367 + 0.188790 + 0.073849 \right] = 1 - 0.50200759 = 0.49799241,
\]

\[
S_{PFSE1}(Q, R) = 1 - \frac{e}{6} \left[ 0.211619 + 0 + 0.2372448 + 0.2061124 + 0.0985417 + 0 \right] = 1 - 0.341374002 = 0.6586209.
\]
\[ S_{PFSE_1}(Q, R) = 1 - \frac{\varepsilon}{6} \left[ |0.8^2 - 0.9^2|e^{-|0.8^2 - 0.9^2|} + |0.2^2 - 0.1^2|e^{-|0.2^2 - 0.1^2|} \\
+ |0.7^2 - 0.8^2|e^{-|0.7^2 - 0.8^2|} + |0.3^2 - 0.2^2|e^{-|0.3^2 - 0.2^2|} \\
+ |0.6^2 - 0.7^2|e^{-|0.6^2 - 0.7^2|} + |0.3^2 - 0.1^2|e^{-|0.3^2 - 0.1^2|} \right], \]

\[ S_{PFSE_1}(Q, R) = 1 - \frac{\varepsilon}{6} \left[ 0.1434230 + 0.0291133 + 0.1291061 + 0.0475612 \\
+ 0.114152 + 0.073849 \right] \\
= 1 - 0.243379436 = 0.756620564. \]

The detailed computation for the proposed measures can be summarized in Table 1:

From the above computations, it supports that \( S_{PFSE_j}(P, R) \leq S_{PFSE_j}(P, Q) \) and \( S_{PFSE_j}(P, R) \leq S_{PFSE_j}(Q, R) \). Also, \( SWPFE_j(P, R) \leq SWPFE_j(P, Q) \) and \( SWPFE_j(P, R) \leq SWPFE_j(Q, R) \) \( \forall j = 1, 2 \).

4 | EXPONENTIAL SIMILARITY MEASURES IN SUPPLY OF COMMODITIES

To demonstrate the legitimacy of the exponential similarity measures for PFSs proposed in Section 3, a numerical example is presented to illustrate the usage of the proposed measures.

Since the first instances of coronavirus illness (COVID-19) were discovered in Wuhan, China in December 2019, a growing number of cases have been reported in every country including India. It surpassed the number of COVID-19 patients, causing the World Health Organization to proclaim COVID-19 an epidemic. To combat the spread of COVID-19, the Indian government has devised a plan in every state for the timely delivery of critical supplies in hotspots and containment zones. When any region is designated as a containment zone, private vehicles are banned from entering, and those who leave their homes must wear masks. Vegetable vendors/agriculture-related transportation, basics such as groceries and medical, institutional, and government vehicles are prohibited for the planned duration of the containment. Different commodities are required by different members of the families who are in containment zones, and their distribution is a major goal for the administrative department. As a result, it is necessary to provide the administrative department with appropriate information about the requirements of various containment zones to improve suitable planning, preparation, and competency for optimal commodity distribution. We use PFS sets as a measure because they include the proportion of members of a containment zone who choose a particular commodity (\( \delta \)), the proportion of members of a zone who do not choose a particular commodity (\( \zeta \)), and the proportion of members who choose a commodity that is not available (\( \eta \)). Let there be six affected residents/house owners that represent the set of homes in a quarantine zone and are denoted by Moksha (\( R_1 \)), Daksh (\( R_2 \)), Radhika (\( R_3 \)), Monika (\( R_4 \)), Rahul (\( R_5 \)), and Harsh (\( R_6 \)). Daily demand of four essentials represent the needs provided by the homeowner be denoted by \( D_1 \), \( D_2 \), \( D_3 \), and \( D_4 \) and commodities \( \hat{G}_1 \), \( \hat{G}_2 \), \( \hat{G}_3 \), and \( \hat{G}_4 \) represent the goods. We presume that the above-mentioned house owners communicate their needs for the above goods via telephone, messaging through phone or e-mail, or through WhatsApp.

Weight vector of the attributes is \( \omega = (0.4, 0.3, 0.2, 0.1) \). PFSs are used to express the relationship between the above commodities requirements \( \hat{G}_j \) of various residents \( R_i \). In the processes that follow, the decision-maker will assess the households using the four criteria listed above.
| Proposed measure 1 | Numerical values | Proposed measure 2 | Numerical values | Proposed measure 3 | Numerical values | Proposed measure 4 | Numerical values |
|--------------------|------------------|--------------------|------------------|--------------------|------------------|--------------------|------------------|
| $S_{PFSE_1}(P, Q)$ | 0.658621         | $S_{PFSE_2}(P, Q)$ | 0.661669         | $S_{WFSE_1}(P, Q)$ | 0.882876         | $S_{WFSE_2}(P, Q)$ | 0.878887         |
| $S_{PFSE_1}(P, R)$ | 0.497992         | $S_{PFSE_2}(P, R)$ | 0.49962          | $S_{WFSE_1}(P, R)$ | 0.83266          | $S_{WFSE_2}(P, R)$ | 0.826176         |
| $S_{PFSE_1}(Q, R)$ | 0.756621         | $S_{PFSE_2}(Q, R)$ | 0.759293         | $S_{WFSE_1}(Q, R)$ | 0.91987          | $S_{WFSE_2}(Q, R)$ | 0.917128         |
Step 1: A relation between house owners and their requirements in the form of PFSs is presented in Table 2.

Step 2: A relation between commodities and requirements in the form of PFSs is shown in Table 3.

Step 3: Determine the degree of similarity using exponential similarity measures (Equations 6–9). The obtained measure values are presented in Tables 4–7.

Taking into account of numerical computations of the above tables, it is determined that the most required commodity is $\hat{G}_1$ and $\hat{G}_3$.

Table 2  Relation between homeowners and their demand

| Relation 1 | $\mathcal{D}_1$ | $\mathcal{D}_2$ | $\mathcal{D}_3$ | $\mathcal{D}_4$ |
|------------|----------------|----------------|----------------|----------------|
| Moksha ($R_1$) | 0.74, 0.39  | 0.62, 0.44  | 0.55, 0.75  | 0.29, 0.62  |
| Daksh ($R_2$) | 0.48, 0.52  | 0.56, 0.65  | 0.85, 0.37  | 0.90, 0.20  |
| Radhika ($R_3$) | 0.15, 0.25  | 0.66, 0.28  | 0.32, 0.42  | 0.71, 0.27  |
| Monika ($R_4$) | 0.66, 0.45  | 0.60, 0.31  | 0.26, 0.50  | 0.35, 0.72  |
| Rahul ($R_5$) | 0.32, 0.59  | 0.88, 0.25  | 0.92, 0.18  | 0.12, 0.49  |
| Harsh ($R_6$) | 0.36, 0.55  | 0.33, 0.15  | 0.25, 0.40  | 0.65, 0.37  |
| Weights       | 0.4         | 0.3         | 0.2         | 0.1         |

Table 3  Relation between commodities and their demand

| Relation 2 | $\mathcal{D}_1$ | $\mathcal{D}_2$ | $\mathcal{D}_3$ | $\mathcal{D}_4$ |
|------------|----------------|----------------|----------------|----------------|
| $\hat{G}_1$ | (0.44, 0.40) | (0.60, 0.20) | (0.50, 0.59) | (0.50, 0.50) |
| $\hat{G}_2$ | (0.64, 0.36) | (0.28, 0.78) | (0.81, 0.15) | (0.49, 0.29) |
| $\hat{G}_3$ | (0.41, 0.62) | (0.40, 0.66) | (0.90, 0.20) | (0.31, 0.22) |
| $\hat{G}_4$ | (0.44, 0.66) | (0.54, 0.38) | (0.26, 0.89) | (0.60, 0.42) |

Table 4  Similarity measure between residents and their requirements for $S_{PFSE1}(P, Q)$

| Similarity measure | $\hat{G}_1$ | $\hat{G}_2$ | $\hat{G}_3$ | $\hat{G}_4$ |
|-------------------|-------------|-------------|-------------|-------------|
| Moksha ($R_1$)    | 0.696666549 | 0.612282393 | 0.429016259 | 0.544711076 |
| Daksh ($R_2$)     | 0.525925853 | 0.612282393 | 0.71764525  | 0.49234501  |
| Radhika ($R_3$)   | 0.674798332 | 0.472819984 | 0.462715779 | 0.601800592 |
| Monika ($R_4$)    | 0.713855773 | 0.488181683 | 0.454671332 | 0.582066715 |
| Rahul ($R_5$)     | 0.56181972  | 0.474857993 | 0.673125507 | 0.471972474 |
| Harsh ($R_6$)     | 0.675977854 | 0.537029059 | 0.588424612 | 0.70290864  |
This analysis is done on the grounds that the higher value of the house owners against every similarity measure demonstrates the greater likelihood of having the option to choose the commodity.

5 | COMPARATIVE STUDY

To demonstrate the dominance of the anticipated logarithmic similarity measures, a comparison is conducted with the existing measures. We first demonstrate some existing similarity measures for the sake of comparison as defined in Table 8.

| TABLE 5 | Similarity measure between residents and their requirements for $S_{PFSE2}(P, Q)$ |
|------------------------------------------|
| Similarity measure | $\hat{G}_1$ | $\hat{G}_2$ | $\hat{G}_3$ | $\hat{G}_4$ |
| Moksha ($R_1$) | 0.655357144 | 0.617937402 | 0.530217563 | 0.638117583 |
| Daksh ($R_2$) | 0.499318721 | 0.617937402 | 0.691952433 | 0.533643797 |
| Radhika ($R_3$) | 0.644937183 | 0.445094033 | 0.421763511 | 0.564213734 |
| Monika ($R_4$) | 0.673013151 | 0.487740238 | 0.451898106 | 0.622183799 |
| Rahul ($R_5$) | 0.532491271 | 0.550408293 | 0.689616815 | 0.500891754 |
| Harsh ($R_6$) | 0.649912825 | 0.489770959 | 0.497372097 | 0.632982483 |

| TABLE 6 | Similarity measure between residents and their requirements for $S_{WPFSE1}(P, Q)$ |
|------------------------------------------|
| Similarity measure | $\hat{G}_1$ | $\hat{G}_2$ | $\hat{G}_3$ | $\hat{G}_4$ |
| Moksha ($R_1$) | 0.925398455 | 0.904788949 | 0.851657873 | 0.885936827 |
| Daksh ($R_2$) | 0.902338377 | 0.904788949 | 0.939885731 | 0.892448263 |
| Radhika ($R_3$) | 0.927250658 | 0.860611314 | 0.862565482 | 0.894176188 |
| Monika ($R_4$) | 0.936076935 | 0.886770888 | 0.863793085 | 0.901839893 |
| Rahul ($R_5$) | 0.892722516 | 0.855800689 | 0.915977976 | 0.880944795 |
| Harsh ($R_6$) | 0.923861105 | 0.877920633 | 0.909857504 | 0.9230315 |

| TABLE 7 | Similarity measure between residents and their requirements for $S_{WPFSE2}(P, Q)$ |
|------------------------------------------|
| Similarity measure | $\hat{G}_1$ | $\hat{G}_2$ | $\hat{G}_3$ | $\hat{G}_4$ |
| Moksha ($R_1$) | 0.908005642 | 0.916112073 | 0.882440368 | 0.907956336 |
| Daksh ($R_2$) | 0.892356642 | 0.916112073 | 0.938864501 | 0.897754729 |
| Radhika ($R_3$) | 0.913856467 | 0.854472149 | 0.855750564 | 0.880328249 |
| Monika ($R_4$) | 0.922012898 | 0.887323074 | 0.87282183 | 0.916007523 |
| Rahul ($R_5$) | 0.887376243 | 0.878836638 | 0.919257046 | 0.883175908 |
| Harsh ($R_6$) | 0.91575506 | 0.870929682 | 0.88901674 | 0.902985252 |
TABLE 8  Similarity measure proposed by various authors

| Authors          | Similarity measures                                                                 |
|------------------|-------------------------------------------------------------------------------------|
| Peng and Garg    | $Sim^1(P, Q) = 1 - \frac{1}{n}\sum_{i=1}^{n} \left| \left( \delta^2_P(x_i) - \delta^2_Q(x_i) \right) \right| + \left| \left( \xi^2_P(x_i) - \xi^2_Q(x_i) \right) \right|$ |
| Gao and Wei      | $Sim^2(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \cos \left( \left( \delta^2_P(x_i) - \delta^2_Q(x_i) \right) \right) + \left| \left( \xi^2_P(x_i) - \xi^2_Q(x_i) \right) \right|$ |
| Ejegwa           | $Sim^6(P, Q) = 1 - \frac{1}{n}\sum_{i=1}^{n} \left| \left( \delta_P(x_i) - \delta_Q(x_i) \right) \right| + \left| \left( \xi_P(x_i) - \xi_Q(x_i) \right) \right| + \left| \left( \eta_P(x_i) - \eta_Q(x_i) \right) \right|$ |
| Zhang et al.     | $Sim^9(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \left( \left( \delta^2_Q(x_i) - \delta^2_P(x_i) \right) \right) - 1$, |
|                  | $Sim^{10}(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \left( \left( \delta^2_P(x_i) - \delta^2_Q(x_i) \right) \right) - 1$, |
|                  | $Sim^{11}(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \left( \left( \delta^2_Q(x_i) - \delta^2_P(x_i) \right) \right) - 1$, |
|                  | $Sim^{12}(P, Q) = \frac{1}{n}\sum_{i=1}^{n} \left( \left( \delta^2_P(x_i) - \delta^2_Q(x_i) \right) \right) - 1$, |

From the mathematical computation presented in Table 9, the comparison has been done between the similarity measures proposed by the authors shown in Table 8 and the results attained using our proposed similarity measures for PFSs. It has been noticed that the results obtained by using our proposed similarity measures are analogous to the existing measures.

6  | CONCLUSION

In recent times, numerous similarity measures have been established for measuring the level of similarity between PFSs. Nevertheless, it appears that there have been no examinations of similarity measures based on exponential functions for PFSs. By considering the usual parameters of PFSs, we presented some novel exponential similarity measures for PFSs that satisfied the properties of similarity measures. We confirmed the credibility of the projected similarity measures through numerical computations as well. This application of PFSs is applied in the application of supplying appropriate information to the admin department of the most ideal order of commodity distribution in a containment zone as per the house owners’ requirements during COVID-19. The various tables may be altered to meet the needs of the commodities in various containment zones. Also, a comparative analysis of the investigated similarity measures was performed to determine the effectiveness of the proposed measures. Recommended PFSs for similarity measure are a significant device to address the vulnerabilities in the data in a more productive way when contrasted with the other existing sets. The advantage of the proposed methodology is that it not only enables the study of the
second wave of COVID-19 but also gives an insight into the conditions in the coming third wave. These intended measures can be applied to medical diagnosis, complex decision-making, and risk analysis in the future course of action.

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CONFLICT OF INTEREST

The authors declare no conflict of interest.

DATA AVAILABILITY STATEMENT

No. Research data are not shared.

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