Mathematical modelling of the influenced of diffusion rate on macro nutrient availability in paddy field

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Abstract. Nutrition is the chemical compounds that needed by the organism for the growth process. In plants, nutrients are organic or inorganic compounds that are absorbed from the roots of the soil. It consist of macro and micro nutrient. Macro nutrients are nutrition that needed by plants in large quantities, such as, nitrogen, calcium, potassiacium, magnesium, and sulfur. The total soil nutrient is the difference between the input nutrient and the output nutrients. Input nutrients are nutrient that derived from the decomposition of organic substances. Meanwhile, the output nutrient consists of the nutrients that absorbed by plant roots (uptake), the evaporated nutrients (volatilized) and leached nutrients. The nutrient transport can be done through diffusion process. The diffusion process is essential in removing the nutrient from one place to the root surface. It will cause the rate of absorption of nutrient by the roots will be greater. Nutrient concept in paddy filed can be represented into a mathematical modelling, by making compartment models. The rate of concentration change in the compartment model forms a system of homogeneous linear differential equations. In this research, we will use Laplaces transformation to solve the compartment model and determined the dynamics of macro nutrition due to diffusion process.

1. Introduction

Nutrition is the chemical compounds needed by the organism for growing process and normal function in the body system. In plants, nutrients are organic or inorganic compounds that are absorbed from the roots of the soil. Research has shown that plants need 17 nutrients, called "essential elements" [4]. Based on the level of needs, nutrients in plants can be classified into two parts namely macro nutrients and micro nutrients. Nutritional needs are absolute for every plant, and can not be replaced by other elements, of course with different levels according to the type of plant. Lack of nutrients will inhibit the plant life cycle itself. In rice fields, the dynamics of the quantity of macro nutrients in the soil is strongly influenced by several factors. In [3] it is mentioned that soil texture or soil composition over sand, mud and clay contents greatly affects the soil’s ability to retain water and nutrients. In addition, the amount of decomposed organic matter also determines the quantity of nutrients in the soil. The more organic matter contained the more macro nutrients are produced. The process of decomposition of organic matter can not be separated from the role of microorganisms that act as decomposers. Nutrient concept in paddy filed can be represented into a mathematical modelling, by making compartment models. From concept of balance of dynamics of nutrition at paddy field, Ardiansyah, et al [1], formulate dynamic model of macro nutrition using system of differential equation. In the model, it is explained that the available macro nutrients in the soil are the difference between input...
nutrients (organic matter decomposition) with nutrient output (nutrients absorbed by plants, yawning nutrients, and nutrients that precipitate). The nutrient transport can be done through diffusion process. The diffusion process has an important role in spreading the macro nutrient in soil. Diffusion is very important in transferring macro nutrient from one place to another on the surface of the root, so that the nutrient can be absorbed by the plants [2].

2. Mathematics modelling
In building the model, we assume that this model consists of two compartments, which are compartment of available concentration of available nutrient \((c_{ava})\) and compartment concentration nutrient in the form of organic materials \((c_{org})\). The model compartment diagram can be seen below:

Based on Figure (1), the rate of change of nutrient concentration in the form of organic materials can be increased because of the organic input at the constant rate of \(A\). Meanwhile, the rate of change of nutrient concentration in the form of organic materials can be decreased because of decomposition process by the microorganism, that will be spread in ready-to-absorb form, at a rate of \(pN\). We claimed that \(N\) is the diffusion rate of macro nutrient.

Then, the rate of change of nutrient concentration in ready-to-absorb form can be increased because of the spreading of decomposition process of the microorganism at a rate of \(pN\). It also can be decreased because there is nutrient absorption in the form of ready to be absorbed (uptake) by the roots at a rate of \(uc_{ava}\), leaching process at a rate of \(lc_{ava}\) and evaporation process at a rate of \(vc_{ava}\). Thus, we have the following differential equation

\[
\frac{dc_{org}(t)}{dt} = A - pNc_{org}(t)
\]

\[
\frac{dc_{ava}(t)}{dt} = \frac{p}{N}c_{org}(t) - uc_{ava}(t) - vc_{ava}(t) - lc_{ava}(t)
\]

The parameters can be seen in the following table

| Parameter | Definition |
|-----------|------------|
| \(A\)     | Input rate of organic materials |
| \(p\)     | Decomposition rate of organic materials |
| \(N\)     | diffusion rate of macro nutrient |
| \(u\)     | The rate of nutrient uptake by the roots |
| \(v\)     | The rate of nutrient evaporation into the air (volatilized) |
| \(l\)     | The rate of nutrient leaching by the water |
3. Main results
Suppose \( u + v + l = m \), then equation system (1) and (2), can be rewritten as

\[
c_{\text{org}}' = A - \frac{p}{N}c_{\text{org}}(t) \tag{3}
\]

\[
c_{\text{ava}}' = \frac{p}{N}c_{\text{org}}(t) - mc_{\text{ava}}(t) \tag{4}
\]

Suppose that we have initial condition \( c_{\text{org}}(t_0) \) and \( c_{\text{ava}}(t_0) \). Using Laplace transformation in the both side of equation (3) and (4), we obtain equation

\[
\mathcal{L}\{c_{\text{org}}'\} = \mathcal{L}\left\{A - \frac{p}{N}c_{\text{org}}\right\}
\]

\[
\mathcal{L}\{c_{\text{ava}}'\} = \mathcal{L}\left\{\frac{p}{N}c_{\text{org}} - mc_{\text{ava}}\right\} \tag{5}
\]

System (5), can be write in the following equation

\[
(s + \frac{p}{N})C_{\text{org}}(s) = \frac{A}{s} + c_{\text{org}}(t_0) \tag{6}
\]

\[
(s + m)C_{\text{ava}}(s) - \frac{p}{N}C_{\text{org}}(s) = c_{\text{ava}}(t_0) \tag{7}
\]

From equation (6), we obtained the following equation

\[
C_{\text{org}}(s) = \frac{A}{s(s + \frac{p}{N})} + \frac{c_{\text{org}}(t_0)}{(s + \frac{p}{N})} \tag{8}
\]

If we substituted equation (8) into (7), we get the following equation

\[
(s + m)C_{\text{ava}}(s) - \frac{p}{N}\left(\frac{A}{s(s + \frac{p}{N})} + \frac{c_{\text{org}}(t_0)}{(s + \frac{p}{N})}\right) = c_{\text{ava}}(t_0)
\]

\[
\Leftrightarrow (s + m)C_{\text{ava}}(s) = \frac{pA}{Ns(s + \frac{p}{N})(s + m)} + \frac{pc_{\text{org}}(t_0)}{N(s + m)(s + \frac{p}{N})} + \frac{c_{\text{ava}}(t_0)}{(s + m)} \tag{9}
\]

The solution of \( c_{\text{org}}(t) \) can be obtained from equation (8) by using the invers of Laplace transform,

\[
c_{\text{org}}(t) = \mathcal{L}^{-1}\left\{c_{\text{org}}(s)\right\}
\]

\[
= \mathcal{L}^{-1}\left\{\frac{A}{s(s + \frac{p}{N})} + \frac{c_{\text{org}}(t_0)}{(s + \frac{p}{N})}\right\}
\]

\[
= \frac{A}{p/N}\mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{1}{(s + \frac{p}{N})}\right\} + c_{\text{org}}(t_0)\mathcal{L}^{-1}\left\{\frac{1}{(s + \frac{p}{N})}\right\}
\]
Then, the general solution of \( c_{org}(t) \) is given by

\[
c_{org}(t) = \frac{AN}{p} \left(1 - e^{-\frac{p}{N}t}\right) + c_{org}(t_0)e^{-\frac{p}{N}t}
\]  

(10)

The invers of Laplace transform, from equation (9),

\[
c_{ava}(t) = \mathcal{L}^{-1}\{C_{ava}(s)\}
\]

\[
= \mathcal{L}^{-1}\left\{\frac{pA}{Ns (s + \frac{p}{N}) (s + m)} + \frac{pc_{org}(t_0)}{N (s + m) (s + \frac{p}{N})} + c_{ava}(t_0)\right\}
\]

\[
= \frac{pA}{N} \mathcal{L}^{-1}\left\{\frac{1}{s (s + \frac{p}{N}) (s + m)}\right\} + \frac{p}{N} c_{org}(t_0) \mathcal{L}^{-1}\left\{\frac{1}{(s + m) (s + \frac{p}{N})}\right\}
\]

\[
+ c_{ava}(t_0) \mathcal{L}^{-1}\left\{\frac{1}{(s + m)}\right\}
\]

We then have the general solution of \( c_{ava}(t) \), namely

\[
c_{ava}(t) = A \left(\frac{c_{ava}(t_0)}{m} - \frac{A}{m - \frac{p}{N}} - \frac{p}{N (m - \frac{p}{N})} c_{org}(t_0)\right) e^{-mt}
\]

\[
+ \left(\frac{A}{m - \frac{p}{N}} + \frac{p}{N (m - p)} c_{org}(t_0)\right) e^{-\frac{p}{N}t}
\]

(11)

The solution of the systems for a long periods of time is

\[
\lim_{t \to \infty} c_{org}(t) = \lim_{t \to \infty} \left(\frac{AN}{p} \left(1 - e^{-\frac{p}{N}t}\right) + c_{org}(t_0)e^{-\frac{p}{N}t}\right)
\]

\[
= \frac{AN}{p}
\]

and

\[
\lim_{t \to \infty} c_{org}(t) = \lim_{t \to \infty} \left(\frac{A}{m} + \left(\frac{c_{ava}(t_0)}{m} - \frac{A}{m - \frac{p}{N}} - \frac{p}{N (m - \frac{p}{N})} c_{org}(t_0)\right) e^{-mt}\right)
\]

\[
+ \lim_{t \to \infty} \left(\frac{A}{m - \frac{p}{N}} + \frac{p}{N (m - p)} c_{org}(t_0)\right) e^{-\frac{p}{N}t}
\]

\[
= \frac{A}{m}
\]

4. Simulation
In this simulation, we used the following parameter values \( c_{org}(0) = 1000, c_{ava}(0) = 50, p = 0.02, u = 0.3, I = 0.02, v = 0.05, A = 10 \). We used different values of diffusion rate, \( N \), namely \( N = 0.02, 0.1, 0.2, 0.3, 0.8 \). Then the illustration of macro nutrient in organic form can be presented in the Figure (2). From the figure, we could see that the higher diffusion rate, the macro nutrient concentration in organic form is higher. By using the same parameter values, we will see the graphics of the influence of different diffusion rate into the availability of macro nutrient in 120 days, as shown in Figure (3). We can see that, as the diffusion rate increase, then the availability of macro nutrient in soil will decrease. It is reasonable, that the macro nutrient will be uptake by the plant roots or volatilized into the air.
5. Conclusion
The concentration of macro nutrient availability in soil will be decreased as the diffusion rate value increased. In long time periods, the concentration of macro nutrient availability will decrease into $A/m^2$.

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