Dynamical Constraints on Disk Masses

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Abstract. While the total interior mass of a galaxy is reasonably well determined by a good rotation curve, the relative contributions from disk, bulge and halo are only weakly constrained by one-dimensional data. Barred galaxies are intrinsically more complicated, but provide much tighter constraints on the disk masses and support the idea that most of the mass in the inner parts of bright galaxies is in their stars. There appears to be no systematic difference in dark matter content between barred and unbarred galaxies, consistent with the theoretical result that the global stability of galaxies with dense centers does not depend on their halo fraction. The rotation curve shapes of lower luminosity and low-surface-brightness galaxies, on the other hand, indicate significant mass in the DM halo even near their centers. We find that most DM halos appear to have large cores, inconsistent with the predictions from cosmological simulations. We also show that such large-core halos can result from compression by disk infall of physically reasonable initial halos. Maximum disks, while apparently required by the data, do seem to present some puzzles; most notably they re-open the old disk-halo “conspiracy” issue and incorrectly predict that surface brightness should be a second parameter in the Tully-Fisher relation.

1. Surface brightness and luminosity

The question of what fraction of the central attraction should be attributed to dark matter (DM) within the disk of a spiral galaxy is still unresolved. Most of the controversy surrounds the higher luminosity, high surface brightness (HSB) galaxies which I argue here have very little DM in their inner regions. As the rotation curves of most LSB and low-luminosity galaxies, on the other hand, do not have the shapes predicted from their light distributions, a significant DM contribution is required in their inner parts. While this conclusion has been established from much careful work on individual galaxies, trends suggesting increasing DM content towards both types of galaxy can also be found in a statistical analysis of a large galaxy sample.

If the surface density of a disk of mass $M$ decreases exponentially in its outer parts with scale length $R_0$, the rotation speed arising from the disk alone
may be written

\[ V_{\text{disk}}(R) = \sqrt{\frac{GM_{\text{disk}}}{R_0}} S \left( \frac{R}{R_0} \right) \]  

The (dimensionless) function \( S \), which describes the shape, is plotted in Figure 1 for three different disks having the same total mass and outer exponential scale length. It can be seen that while the shape of the rotation curve arising from the disk is strongly dependent on the surface density profile of the inner disk, the height of the maximum is not; it varies from 0.6 by about 10% only between the three cases. Thus the peak of the disk’s contribution to the rotation speed in most galaxies \( V_{\text{disk,max}} \sim 0.6 \sqrt{GM_{\text{disk}}/R_0} \) even when the inner surface density profile departs significantly from exponential.

We can use this fact to rank galaxies according to the relative contributions of the disk and halo to the observed peak rotation speed, \( V_m \). Following Syer, Mao & Mo (1998), we form the ratio of observable quantities

\[ \epsilon_1 = \frac{V_m}{\sqrt{GL_1/R_0}} \frac{1}{\sqrt{h}} \sim 0.6 \frac{V_m}{V_{\text{disk,max}}} \sqrt{\frac{\Upsilon_1}{h}}, \]  

where \( L_1 \) is the disk luminosity (in Solar units) in the I-band. The second form shows that the values obtained depend both on the disk M/L, \( \Upsilon_1 \), and on \( h (= H_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1}) \), because both luminosity and scale length are distance dependent. More than 2000 galaxies from the sample gathered by Mathewson & Ford (1996) are plotted in Figure 2; galaxies with \( V_{\text{sys}} < 1000 \text{ km s}^{-1} \) are omitted. Combining the information they provide with the assumption that the disk is exponential, allows us to deduce \( R_0 \) and the central surface brightness, \( \mu_0 \), in the manner described by Syer et al.
Figure 2. Evidence for increasing DM fractions in both (a) low-luminosity and (b) LSB galaxies. Galaxies classified as barred are marked with crosses in (b).

The dashed line in Figure 2(a) indicates the slope of simple proportionality between \( V_m \) and \( \sqrt{GL_I/R_0} \); it is clear that the distribution of points towards the low \( V_m \) (low luminosity) end has a shallower slope, indicating increasing DM fractions in these galaxies. The right hand panel shows a trend of increasing DM content from high to low SB galaxies. Both trends are strong enough to show through the scatter which is increased by deviations from exponential, bulge contributions, intrinsic M/L variations, departures from Hubble flow, imperfect inclination and extinction corrections, etc.

The trends seen in Figure 2 suggest increasing mass discrepancies within the optical disks towards both low-luminosity and LSB galaxies. But because M/L is so hard to pin down, such diagrams do not tell us whether the disks in galaxies having small values of \( \epsilon_I \) are maximal. [A “maximum disk” model is one in which \( V_{\text{disk, max}}/V_m \gtrsim 0.85 \) (e.g. Sackett 1997). This ratio cannot be much larger than 0.85 when a model includes an extended DM halo with a density profile that decreases monotonically with radius.]

Disagreements arise over how much DM to add to the disk contribution in order to fit the observed rotation curve. As pointed out by van Albada et al. (1985; see also Navarro 1998), the appropriate M/L for the disk is not constrained at all by conventional goodness-of-fit estimators, such as \( \chi^2 \), especially since the mass profile of the DM is unknown. Here I review some arguments that bear on the masses of disks and refer the reader to others given by Bosma elsewhere in these proceedings. The DM content of LSB galaxies is discussed by de Blok (also this volume).
Figure 3. Halo rotation curves required for sub-maximum disks. The error bars show the measured circular speed and the curve shows the pure disk fit (the bulge contribution is omitted in e267g29). The symbols show the halo contribution required if the disk has 100% (filled circles), 80% (open circles), 60% (filled squares), 40% (open squares) and 20% (stars) of the maximum value. The horizontal line indicates the vertical zero point for the halo velocities required for 100% disk. Subsequent curves are shifted upwards by 30 km s$^{-1}$ for every 20% decrease in disk mass. Note the similarity in shapes of the curves marked by symbols and the error bars.

2. No halo fits

The constraint on the disk M/L is especially weak when low spatial resolution 21cm data are used to determine the inner rotation curve; tighter constraints are furnished by optical data. Many authors (Kalnajs 1983; Kent 1986; Buchhorn 1992; Broeils & Courteau 1997; Palunas & Williams 1999) have successfully fitted pure disk models to optical rotation curve data. In this approach, one assumes a constant M/L for the disk, and sometimes a separate value for the bulge, and determines the central attraction from an exact solution of Poisson’s equation for the mass distribution thus inferred from the light. The shape of the rotation curve predicted in this manner generally bears a remarkably close resemblance to that observed, at least in the inner disk.

The largest possible M/L value allowed by the observed circular speed in the inner disk leads to an impressive fit, with discrepancies noticeable near the outer edge only in some cases. The “bumps and wiggles” seen in rotation curves from single-slit observations have long been thought to arise from spiral arm
streaming (see also Bosma, this volume). This suspicion has been confirmed in the 2-D velocity maps obtained by Palunas & Williams (1999) which allow most non-axisymmetric flows to be removed.

The principal conclusion from previous work survives, namely that the overall shape of the rotation curve is well predicted by the light distribution. As previously remarked by van Albada & Sancisi (1986), Freeman (1992) and others, reducing the M/L for the disk would require DM mass distributions tailored individually to match the rotation curve shape of each galaxy. Two illustrative examples taken from the Palunas & Williams sample are shown in Figure 3.

Figure 4 shows the distribution of disk M/L values obtained by Palunas & Williams for their no-halo fits. One can see a smaller spread of values for high luminosity galaxies \(V_m > 200\) km/s, with perhaps a hint of a higher M/L in earlier Hubble types. The broader spread for the lower luminosity galaxies could be due to two factors; first, it is harder to determine the inclination for these often strongly non-axisymmetric galaxies and second, there are some galaxies for which large M/L values give acceptable fits, even though one might expect substantial DM fractions in these low-luminosity systems. It is also worth noting that the M/L values obtained for the luminous galaxies (for \(h \sim 0.6\)) are in line with values predicted by Jablonka & Arimoto (1992) and by Worthey (1994) for quite reasonable population models.

As already noted, these “no halo” fits frequently reveal a mass discrepancy near the edge of the optical rotation curve. Conventional “maximum disk” models require a halo with a large, low-density core (e.g. Broeils 1992). Figure 3 shows that fits with a non-hollow halo would require at most a slight reduction in the disk M/L.

None of these arguments is either new or truly compelling. Others (e.g. van der Kruit 1995; Navarro 1998; Courteau & Rix 1999) stress that less than maximum disk models also yield acceptable fits. It has been argued (e.g. Blumenthal
et al. 1986) that halo compression as the disk forms leads to featureless rotation curves (see also §6), and this argument is extended somehow to account for the similarity of the shape of halo contribution to that of the disk. The population synthesis argument is weak because minor changes to the low-mass end of the IMF can lead to significant changes in the predicted M/L.

3. Barred galaxy velocity fields

The above argument is not decisive because we have considerable freedom to decompose a 1-D rotation curve into contributions from the sum of two, or more, other 1-D functions. Our group at Rutgers has therefore embarked on a program to use the 2-D velocity fields of barred spiral galaxies to provide extra constraints.

A massive bar in a disk galaxy distorts the usual circular flow pattern, leading to characteristic ‘S’-shaped iso-velocity contours when such a galaxy is observed in a suitable projection. The strength of the non-axisymmetric flow pattern can be modeled to determine the mass required in the barred component of the potential, leading to an estimate of the disk M/L that is independent of rotation curve fitting.

The first such study has been completed for the southern barred spiral galaxy NGC 4123. Weiner (1998) has collected broad-band photometry and velocity maps both for the inner galaxy, using Fabry-Perot measurements of the Hα emission, and for the outer galaxy, using 21 cm data from the VLA. He has constructed models based on full 2-D hydrodynamical simulations of a massless gas layer in a rotating potential derived from the observed light distribution.

In order to construct the model potential from his I-band image, Weiner subtracts an unresolved source at the center, rectifies the image to face on, assumes a constant thickness and computes the gravitational field, modulo a single unknown M/L. For a number of assumed M/L values, he constructs axisymmetric DM halos which, when combined with the disk contribution, fit the rotation curve well outside the barred region.

He has run a grid of models covering the parameter space of two unknowns: the M/L and the pattern speed of the bar and compares the projected velocity distribution with the high spatial resolution Fabry-Perot velocity field in the inner parts of NGC 4123. He finds (with a very high degree of confidence) $2.6 \leq M/L_{\odot} h^{-1} \leq 3.3$ is required to produce a flow pattern with strong enough non-circular motions to match the data.

This range of M/L values requires that 72% to 90% of the mass within 10 kpc is in the stars – the contribution from DM can be little more than the minimal amount required to prevent a hollow halo. Work on other galaxies is in hand to check that this is not just a freak case. It should be noted that Englmaier & Gerhard (this volume) reach a similar conclusion from similar work on the Milky Way.
4. Dynamical friction on bars

A second argument from barred galaxies relates to the prediction (Weinberg 1985) that bars should be slowed dramatically by dynamical friction. This prediction has been confirmed (Debattista & Sellwood 1998) for moderate halo masses in N-body simulations of a barred disk in a responsive halo. The strong retarding torque acting on the bar causes the pattern speed to decrease rapidly to about one fifth of its initial value. Since this change occurs while the bar length increases only marginally, the ratio, $\mathcal{R}$, of the corotation radius to the bar semi-major axis, increases from $\gtrsim 1$ at a time soon after the bar formed, to some $\mathcal{R} \sim 2.5$ late in the simulation.

On the other hand, $\mathcal{R}$ remains close to 1.3 in a “maximum disk” model having a realistic rotation curve for as long as the calculation was run.

Further work (Debattista & Sellwood, in preparation) has shown that friction is only moderately reduced in halos that rotate in the same sense as the disk, even when halo rotation is cranked up to a perhaps unrealistic extent. Friction is also largely independent of whether the velocity distribution of a non-rotating halo is isotropic, radially or azimuthally biased, but friction was reduced when an anisotropic halo was given a high degree of rotation.

The position of corotation is not easily determined in real barred galaxies. Weiner (§3) finds $1 \lesssim \mathcal{R} \lesssim 1.4$ for NGC 4123. A value $\mathcal{R} \simeq 1.2$ was deduced by Lindblad et al. (1996) from a very similar study of NGC 1365. More direct estimates can be made for SB0 galaxies using the technique proposed by Tremaine & Weinberg (1984) which requires that the observed material, stars in this case, obey an equation of continuity. Application of this method to NGC 936 by Merrifield & Kuijken (1995) and to NGC 4596 by Gersson et al. (this volume) also places corotation at a radius $\lesssim 1.5$ times the bar semi-major axis. The shapes and locations of dust lanes in many other barred galaxies also suggest a ratio of 1.2 (Athanassoula 1992) and finally some still more model-dependent studies of ringed galaxies (Buta & Combes 1995) suggest a similar value.

While this heterogeneous collection of estimates is not as solid as one would like, all evidence is consistent with $\mathcal{R} \lesssim 1.5$, implying that real bars have not experienced strong braking. Once again, therefore, the DM halo must have a large, low-density core if the radius of corotation is to stay as close to the bar end as observations seem to imply.

5. Stability

Since local stability arguments bearing on the question of the appropriate disk mass are used by Fuchs (this volume) and his arguments are critically reviewed by Bosma (also this volume), I avoid repeating them here and confine this part of my discussion to the question of global bar stability only.

It has been known ever since the pioneering N-body simulations by Miller et al. (1971) and Hohl (1971) that self-gravitating galaxy disk models are prone to a bar-forming instability. In a widely cited paper, Ostriker & Peebles (1973) argued that the global stability of unbarred galaxies required a significant DM content which, as stressed by Kalnajs (1987), must reside in the inner galaxy. This idea is too simple, however. Toomre (1981) argued that galaxies can be
Figure 5. Simulation of a massive, cool and bar-stable disk. The top panels show the second half of the evolution in which strong spiral patterns are present but no bar. The rotation curves (below left) are measured at the times illustrated and show the gradual increase in disk mass; the fixed contributions from the rigid halo (dashed) and central mass (dotted) are shown. The values of $Q$ are also plotted (below right).
stabilized by other means; his argument is restated by Binney & Tremaine (1987, §6.3).

The fact that \( m = 2 \) modes in an almost fully self-gravitating disk can be stabilized by a dense center has been shown in linear stability analyses by Zang (1976), Evans & Read (1998) and Toomre (unpublished). I was able (Sellwood 1989) to confirm Toomre’s linear theory predictions, but I found that the stability of the extreme model he chose was rather delicate, since bars were still triggered by quite modest finite-amplitude effects. These extreme cases also suffer from \( m = 1 \) instabilities.

Not all models stabilized in this way are delicate, however. Figure 5 shows a model that is robustly stable to bar formation, even though it has a moderately cool massive disk in a diffuse halo. This model closely resembles those described by Sellwood & Moore (1999), but the radial distribution of the added particles is different. By concentrating all the infalling particles into an rather narrow annulus, Sellwood & Moore were able to show that spiral patterns could be strong enough to re-arrange the surface density and alter the rotation curve shape. Since the strong spirals that achieved this result led to a rather hot disk (\( Q \sim 4 \)), I added particles in the model presented here over a wide radial range, which allowed the disk to remain cool as shown.

Almost all the central attraction in this model comes from the disk; it supports strong two-arm spiral patterns yet does not form a bar. The key difference between models of this kind and previous bar-unstable disks is the steep inner rise of the rotation curve, caused mostly by mobile particles, but seeded by the introduction of a fixed mass, having \( \sim 1\% \) of the final disk mass.

The steep central rise in the rotation curve of this model resembles that of many galaxies of both late (Rubin, Ford & Thonnard 1980) and early (Rubin, Kenney & Young 1997) Hubble types.

The conclusion of this section is that considerations of global bar-stability do not require a high central density of DM in every galaxy. There is no universal stability criterion; as Ostriker & Peebles argued, galaxies with gently rising curves are unstable unless the disk is significantly sub-maximum, but we now know that fully self-gravitating disks with steeply rising curves are stable.

Strong evidence that the existence or absence of a bar has nothing whatsoever to do with DM content can be seen in Figures 2(b) and 4 in which the barred and unbarred galaxies are marked by different symbols. There is no apparent tendency for barred galaxies to have lower \( \epsilon_1 \) or higher M/L than their unbarred counterparts.

6. Halo compression

As the principal source of central attraction switches from disk to halo matter, we might expect the rotation curve of a maximum disk galaxy to have a feature near the disk edge. Bahcall & Casertano (1985), among others, stressed the absence of such a feature, which became known as the “disk-halo conspiracy.” In fact, it is a serious overstatement to suggest that there is no such feature – quite a number of galaxies are known in which a marked decrease in circular speed is observed near the disk edge; Bosma (this volume) gives a list of some of the well-established cases and other examples can be found (e.g. Verheijen...
Figure 6. Compression of a spherical halo by a maximum disk. The solid curve which peaks near $R = 20R_0$ is the rotation curve of the halo before the disk was added. The upper solid curve shows the combined rotation curve of the disk and halo, after adding the disk, and the dot-dashed and dashed curves show the contributions of the halo and disk respectively. The dotted curve shows the prediction of the initial halo rotation curve required by HCF.

1997). Nevertheless, a weaker conspiracy remains because the circular speed well beyond the optical disk is very rarely less than 90% of that in the disk region.

Amongst enthusiasts for dynamically significant DM even near the centers of galaxies, the flatness of galaxy rotation curves is regarded as a natural consequence of halo compression as the disk forms within it (Blumenthal et al. 1986; Flores et al. 1993). If disks are maximum, however, the close correspondence between the circular speeds in the disk and halo still requires a conspiracy.

Navarro (1998) has taken the argument further, to suggest that halos with large cores are unphysical because the standard formula for halo compression implies a hollow, or even negative density, halo before the disk formed. The problem discovered by Navarro, however, is not an argument against large cores, but merely reveals the severe limitations of the standard halo compression formula (hereafter HCF).

The HCF, as derived by Barnes & White (1984), Blumenthal et al. (1986) and Ryden & Gunn (1987), embodies three principal assumptions. (1) The mass distributions of both the halo and the disk(!) are spherical. (2) The disk forms slowly enough that the halo response is adiabatic. Assumptions (1) and (2) imply that halo particles conserve their actions, in particular, the component of their angular momentum normal to the orbit plane, $J_z$, throughout. (3) The mean radius of a halo particle’s orbit can be computed from its “home radius” – the radius of a circular orbit of the same $J_z$. This circular orbit assumption is sometimes stated that shells of halo matter do not cross. These drastic approx-
imations do lead to a single (implicit) relation relating the radial mass profiles of the halo before and after the disk formed. Barnes (1987) conducted a direct test and concluded that the HCF “overestimates the halo response by as much as a factor of two.” Despite his warning, the naïve HCF is still widely used.

Figure 6 demonstrates its failure for a maximum disk case. The plot shows the formation of a maximum disk galaxy model as an exponential disk builds up within an initially spherical, low-concentration halo. The halo is composed of 100 000 particles that move freely in their own gravitational well supplemented by that of the disk. The final disk has a total mass of one tenth that of the halo and, to avoid any angular momentum redistribution, I held the disk particles fixed once they were placed in position. The disk was grown gradually by adding two particles per time-step. The resulting maximum disk model has a flat rotation curve out to $\sim 15R_0$.

The dotted curve shows the rotation curve of the pre-compressed halo that would be required by the HCF to yield the final maximum disk model. The fact that velocities become imaginary at radii $\lesssim 8R_0$ confirms Navarro’s finding that the naïve HCF predicts nonsense. That the HCF fails badly in this case is evidenced by the reasonable initial halo actually required for the final maximum disk model.

I conducted further experiments to determine which approximation above is most responsible for this gross error. The simulation shown in Figure 6 was, in fact, run backwards; I started with an equilibrium disk-halo model and evaporated the disk to find the required “initial” halo. As a check of the adiabatic assumption (2), I re-grew the same disk and recovered the rotation curve from which I had started to impressive precision. In order to test assumption (1), I ran a new calculation with the mass of the flat disk replaced by the equivalent, but now spherical, $M(R)$ profile. This change did alter the rotation curve of the halo after evaporation of the “spherical disk” but still it was far from correctly predicted by the HCF, implying that assumption (3) is the principal source of error in the simple formula.

While this experiment disposes of the objection that a compressed halo having a large core is unphysical, it also highlights the disk-halo conspiracy. In order to yield a reasonably flat rotation curve after the disk has formed, a $V_{\text{max}}$ for the initial halo similar to that for the disk which will form within it seems to be required. It is far from obvious that such well-tuned initial conditions would arise naturally.

7. Surface Brightness and the Tully-Fisher Relation

The rotation curve of any galaxy in which the DM contribution in the inner disk is negligible, will have the form given in equation (1). If disks are maximal, equation (1) predicts that the observed circular speed should vary as $R_0^{-1/2}$ at fixed disk mass, or luminosity if the M/L is approximately constant. Thus one might expect that surface brightness variations to give rise to scatter about the Tully-Fisher relation (TFR). This prediction is well known to fail for LSB galaxies; one of the principal surprises from studies of LSB galaxies is that they (or at least the larger ones) lie on the same TFR as do the HSBs (Zwaan et al.
1995; Sprayberry et al. 1995). This result is one strand of evidence for large mass discrepancies in their inner parts (de Blok, this volume).

If all bright HSB galaxies are maximal, on the other hand, the above predicted correlation between circular velocity in the inner disk and scale length should hold. Courteau & Rix (1999) tested for this in a sample of bright galaxies with well measured $R_0$ and circular speed at $R = 2.2R_0$, but found that the residuals from a TF-like relation do not correlate with surface brightness. Thus even within a sample of HSB galaxies with tightly controlled properties, surface brightness was not a second parameter in the TFR.

As was similarly inferred for the LSBs (de Blok & McGaugh 1997), their result requires either that the disk M/L varies systematically with surface brightness, or that the halo contribution picks up by just enough to compensate for a decreasing disk contribution, or that Newtonian dynamics breaks down. The same conclusions can be drawn from the trend in Figure 2(b), albeit from data of lower quality. Courteau & Rix favor the second alternative, which evidently requires that at least some higher SB galaxies have significant DM contributions to their inner rotation speeds.

8. Conclusions

It is now well-established that LSB and low luminosity galaxies have larger mass discrepancies in their inner parts than do the bright HSB galaxies (Figure 2). The controversial question is how small is the DM contribution to the inner rotation curves of the larger HSB galaxies?

Studies of barred galaxies, in particular, have yielded strong new evidence suggesting that most mass in their inner parts is in stars. The completely independent arguments presented in §§3 & 4 both suggest low upper limits to the DM content of inner parts of barred galaxies. These limits are not so extreme as to violate the conventional maximum disk constraint that the DM halo density should not decrease towards the center. Combined with the apparent absence of any systematic offsets between barred and unbarred galaxies in Figures 3(b) and 5, it seems reasonable to argue that all HSB galaxies are similarly deficient in DM in their inner parts. This conclusion is supported by the older evidence from rotation curve fitting (§2), by some recent evidence for the Milky Way – especially the result obtained by Englmaier & Gerhard (1999, and this volume). Bosma (this volume) presents further arguments for disk masses ranging up to maximum.

The case for maximum disks in bright HSB galaxies is therefore strong, though still not decisive, partly because it leaves at least two serious puzzles: First, why is the circular speed from the disk in the inner galaxy generally so similar to that from the halo further out? Second, why does DM gradually become more important as the disk surface brightness declines? Courteau & Rix (1999) argue that even HSB galaxies have, on average, substantial DM fractions in their inner parts and that barred galaxies have generally smaller fractions. This last suggestion is, however, inconsistent with the evidence in Figures 3(b) and 5.

The DM halos of LSB and low luminosity galaxies are well known to have large cores with low central densities and the evidence presented here suggests
this is also true for bright HSBs. DM halos of this type are quite different from those predicted in many simulations of hierarchical clustering in an expanding universe (but see Primack, this volume).

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