Ignition in tokamaks with modulated source of auxiliary heating

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Abstract. It is shown that the ignition may be achieved in tokamaks with the modulated power source. The time-averaged source power may be smaller than the steady-state source power, which is sufficient for the ignition. Nevertheless, the maximal power must be large enough, because the ignition must be achieved within a finite time interval.

1. Introduction

In order to achieve the ignition in tokamaks, usage of a stationary source of auxiliary heating has been suggested. On the other hand, the pulsed sources, more simple and cheap, existed earlier. The maximal power of such sources is higher than it is necessary for the ignition but the averaged power is not sufficient to sustain the steady-state fusion. However, the non-linear dependence of the fusion output power on the temperature can lead to ignition. The harmonic oscillations of the power source produce the non-linear increase of the temperature and, as the consequence, the additional fusion output. Similar effect has been predicted in [1], where it was shown that the usage of the modulated power source can lead to the decrease of the impurity radiation during the tokamak discharge start-up.

The energy confinement time degradation $\tau_e \sim P_{\text{output}}^{0.69}$ decreases the effect. However, recent investigations show the degradation to be essentially small [2]. The time dependence of the temperature is found in the present paper for the oscillating heating power. Three stages of the temperature growth were found. At the first stage the average temperature rises rapidly. At the second stage the rise is slow. The third stage is related to the thermal instability and extremely rapid rise of the temperature. A simple qualitative model of the temperature evolution is presented below.

2. Qualitative model

The zero-dimensional model is used in the present paper:

$$3n \frac{dT}{dr} = -\frac{3nT}{\tau_e} + \frac{P_{\text{fus}}}{\tau_e} + P_{\text{aux}}.$$

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Here \( n \) is the plasma density, \( \tau_E \) is the energy confinement time, \( P_{\text{fus}} \) and \( P_{\text{aux}} \) are the specific fusion heating by \( \alpha \)-particles and the auxiliary heating power, respectively. The fusion specific power has the form

\[
P_{\text{fus}} = \frac{n^2}{4} \langle \sigma v \rangle \varepsilon.
\]

(2)

Deuterium and tritium densities are supposed to be equal, \( n_D = n_T = n / 2 \), \( \varepsilon = 3.52 \) MeV. The factor \( \langle \sigma v \rangle \) is presented in [3] with a good accuracy. However, inside the wide temperature interval one can use approximate expression

\[
P_{\text{fus}} = A \tau^2.
\]

(3)

The accuracy of the expression (3) is worse than that of (2) but it is acceptable for qualitative analysis.

The energy confinement time for \( H \)-regime has the form [3]

\[
\tau_E = 0.0562 I_p^{0.09} B_t^{0.15} W^{-0.69} M_i^{0.19} \bar{n}_{19}^{0.41} \left( \frac{a}{R} \right)^{0.58} k^{0.78} \equiv \Omega \bar{n}^{0.41} W^{-0.69}.
\]

(4)

Here \( I_p \) is the plasma current expressed in MA, \( B_t \) is the toroidal magnetic field (T), \( W = (P_{\text{fus}} + P_{\text{aux}}) V \) is the input power (MW), \( V \) is the plasma volume (\( m^3 \)), \( M_i = 2.5 \) is the effective atomic mass in a.m.u., \( \bar{n}_{19} \) is the volume-average plasma density (10^{19} m^{-3}), \( k \) is the ellipticity.

The auxiliary heating power consists of the basic and oscillating parts,

\[
W_{\text{aux}} = W_{\text{aux}}^0 + \tilde{W}.
\]

(5)

The equation (1) may be rewritten as follows:

\[
\frac{dT}{dt} = -\frac{T}{\Omega \bar{n}^{0.41}} (W_{\text{fus}} + W_{\text{aux}}^0 + \tilde{W})^{1.4} + \frac{n}{12} A \tau^2 + \frac{P_{\text{aux}}}{3n}.
\]

(6)

Here \( \lambda = 0.69 \) for the standard ITER scaling, and may be smaller \( (0.25 < \lambda < 0.69) \) for the experiments described in [2]. For qualitative evaluation, one can replace the value of \( W_{\text{aux}}^{\text{max}} \) by the value related to the maximum of \( \langle \sigma v \rangle \) (see Figure 1). Usually \( W_{\text{aux}} << W_{\text{fus}}^{\text{max}} \). Hence, one can obtain:

\[
\Omega \bar{n}^{-0.41} (W_{\text{fus}}^0 + W_{\text{aux}}^0)^{-2} = \Omega W_{\text{fus}}^{-2} \bar{n}_0^{0.41} \left( \frac{\bar{n}}{\bar{n}_0} \right)^{0.41} \equiv \tau_{fus}^f \left( \frac{\bar{n}}{\bar{n}_0} \right)^{0.41}.
\]

(7)

Here \( \bar{n}_0 \) is the reference plasma density. In particular, one can put \( \bar{n} = 10^{19} m^{-3} \).

The equation (6) may be rewritten as follows

\[
\frac{dT}{dt} = -\frac{T}{\tau_{fus}^f} \left( \frac{n}{\bar{n}_0} \right)^{2.4} + \frac{n < \sigma v > E_0}{12} + \frac{P_{\text{aux}}}{3n}.
\]

(8)

Here

\[
\tau_{fus}^f = \Omega \bar{n}_0^{0.41} \left( W_{\text{fus}}^{\text{max}} (n = n_0) \left( \frac{n_0}{n} \right)^2 \right)^{-1}.
\]

(9)

The equation (8) takes the form
\[ \frac{dT}{d\xi} = -\frac{T}{\varepsilon_E} + n_0 \left( \frac{n}{n_0} \right)^{1.41 - 2.4} <\sigma v> E_0 + \frac{P_{\text{aux}}}{3n_0} \left( \frac{n}{n_0} \right)^{2.4 + 0.59}. \]  

Here \( \xi = t \left( n / n_0 \right)^{2.4 - 0.41} \).

**Figure 1.** \( <\sigma v> \) versus the plasma temperature.

Qualitative properties of the solution may be described with a model equation (11)

\[ \frac{dT}{dt} = -1.5T + T^2 + 0.56 + \eta \sin(10t). \]  

Equation (11) for \( \eta = 0 \) has two solutions, the first, \( T = 0.7 \) is stable, the second, \( T = 0.8 \) is unstable. The solution for \( \eta = 0.4 \) is shown in Figure 2. The average temperature \( <T> \approx 0.71 \) is stable. It is higher than the equilibrium temperature for the steady-state regime \( T = 0.7 \). Hence the fusion power output is higher than that in the steady-state regime.

The increase of \( \eta \) must increase the nuclear energy output and provide a transition to ignition. The case \( \eta = 0.7 \) is shown in Figure 3. One can see that the average temperature rises in time. The time scale of the transition into ignition is small. The transition is shown separately in Figure 4.

One can see that for ignition the input power must exceed energy losses not permanently but in a short time interval in contrast to the steady-state regime. Hence, the average auxiliary heating power may be significantly smaller than that in the steady-state regime. The process of the ignition has three stages. The first stage is related to the transition to the quasi-steady state regime with slow rise of the average temperature. The third stage is characterized by a fast increase of the temperature and the transition to the ignition.
Figure 2. Steady-state discharge.

Figure 3. Ignition. The first two stages.
3. Conclusion
In order to increase the fusion energy output and to decrease the heating power required for the ignition the modulation of the power of auxiliary heating is proposed. The harmonic oscillations of the auxiliary heating power produce the fusion power increase. The effect is a consequence of the nonlinear dependence of the fusion power output on the temperature. Two effects are predicted. The first one is the appearance of the additional power output in steady-state regime. Secondly, the ignition is possible even if the averaged power input is not sufficient for ignition in the steady-state regime. Hence, simpler and cheaper power sources may be used for a fusion reactor. The transition to ignition has three stages. The first stage is relatively fast. It is related to the transition from the initial low temperature to the quasi-stationary state with slow temperature rise. The third stage is the extremely fast one. The thermal instability is observed with the ignition as a result. The effect is predicted with the help of a qualitative model based on zero-dimensional approximation. Surely, the problem needs a more detailed analysis.

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