Pion Exchange Currents in Neutrinoless Double Beta Decay and Limits on Supersymmetry

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Abstract

We examine the pion exchange mode of neutrinoless double beta decay ($0\nu\beta\beta$) induced by the $R$-parity violating quark-lepton operators of the supersymmetric (SUSY) extensions of the standard model of the electroweak interactions. The corresponding nuclear matrix elements are evaluated within the renormalized quasiparticle random phase approximation with proton-neutron pairing, which includes the Pauli effect of fermion pairs and does not collapse for a physical value of the nuclear force strength. It is argued that the pion-exchange mode of $0\nu\beta\beta$-decay dominates over the conventional two-nucleon mode in the case of the SUSY mechanism. As a result sensitivity of $0\nu\beta\beta$-decay to the SUSY contribution turns out to be significantly better that previously expected from the two-nucleon mode calculations. An upper limit on the $R$-parity violating coupling $\lambda_{111}$ is derived from non-observation of neutrinoless double beta decay. This limit is much stronger than that expected from the near future accelerator experiments.

1 Introduction

The observation of neutrinoless nuclear double beta decay ($0\nu\beta\beta$) would undoubtedly indicate the presence of new physics beyond the standard model (SM) of electroweak interactions. However, as yet there is no any experimental evidence for this lepton-number violating exotic process. On the other hand non-observation of $0\nu\beta\beta$-decay at given experimental sensitivity allows one to establish limits on some parameters of new physics. The most famous example is the light effective Majorana neutrino mass which is limited by the $0\nu\beta\beta$-decay experiments \cite{1, 2} at the level $\langle m_{\nu} \rangle \leq O(0.5 \text{–} 1.1 \text{ eV})$ \cite{1, 2} depending on nuclear model.

It is known that the Majorana neutrino exchange is not the only possible mechanism of $0\nu\beta\beta$-decay. The lepton-number violating quark-lepton interactions of the $R$-parity non-conserving supersymmetric extensions of the SM ($R_p$ SUSY) can also induce this process \cite{3, 4, 5}. It is worthwhile to notice that the corresponding 1st generation $R_p$ Yukawa coupling are so stringently...
constrained by non-observation of the 0νββ-process that all possible 1st generation R_p-effects are pushed beyond the reach of the present and the near future accelerator and the other non-accelerator experiments [5, 6].

In searching for tiny effects of the physics beyond the SM the main disadvantage of 0νββ-decay experiments compared to the accelerator ones is the necessity of taking into account nuclear structure.

Although there are many difficulties the study of 0νββ-decay has the advantage of unprecedented accuracy and precision with which the process can be studied experimentally. It is possible to observe large samples of several kilograms of potentially decaying nuclei and to search for the decay of a single nucleus. The lower half-life limit for 0νββ-decay measured in this way is very high. In addition, the 2νββ-decay process predicted by the standard model can be measured at the same time in order to check the nuclear structure calculations.

The nuclear 0νββ-decay is triggered by the 0νββ quark transition d+d → u+u+2e− which is induced by certain fundamental interactions. It was a common practice to put the initial d-quarks separately inside the two initial neutrons of a 0νββ-decaying nucleus. This is the so called two-nucleon mode of the 0νββ-decay. If the above 0νββ quark transition proceeds at short distances, as in the case of R_p SUSY interactions, then the basic nucleon transition amplitude n+n → p+p+2e− is strongly suppressed for relative distances smaller than the mean nucleon radius.

The goal of this paper is to discuss the pion-exchange SUSY mechanism which is based on the one and two pion exchange between the decaying neutrons. The two pion exchange counterpart of this mechanism was first studied in Ref. [6]. At the quark level this mechanism implies the same short-distance R_p MSSM interactions as in Ref.[8]. However, it essentially differs from the previous consideration of the SUSY contribution to the 0νββ-decay at the stage of the hadronization. We will show that in the case of the R_p MSSM induced quark transition the pion-exchange contribution absolutely dominates over the conventional two-nucleon mode. As a result, a significant improvement of the previously known [5] two-nucleon mode 0νββ-decay limit on R_p-Yukawa coupling λ′_{111} becomes possible.

We calculate the nuclear matrix elements governing the two-nucleon, one pion-exchange and two-pion exchange SUSY contributions to 0νββ-decay within the renormalized Quasiparticle Random Phase Approximation with proton-neutron pairing (full-RQRPA). This nuclear structure method has been developed from the proton-neutron QRPA method, which has been frequently used in the 0νββ-decay calculations. The full-RQRPA is an extension of the pn-QRPA by considering the effect of the proton-neutron pairing and the Pauli effect of the fermion pairs in an approximate way. In this way the sensitivity of the nuclear matrix elements to the details of the nuclear Hamiltonian is reduced considerably and more reliable values on the lepton number non-conserving parameters are obtained.
2 The Nuclear Structure Method

Most of the double beta decaying nuclei are not yet accessible for the detailed shell model treatment because of the complexity of this approach. Therefore the pn-QRPA is mostly used for the nuclear structure calculations. In general the half-life for a $0\nu\beta\beta$-process can be factorized in the form

$$\frac{1}{T_{1/2}} = G |ME|^2 \epsilon$$

with the leptonic phase space factor $G$, the nuclear matrix element $ME$ and the factor $\epsilon$ that is given by the special extension of the standard model under consideration.

The nuclear matrix element $ME$ is ruled by the nuclear structure of the involved nuclei, but also by the considered $0\nu\beta\beta$-decay mechanism, which determines the transition operators. We calculate the matrix element in the intermediate nucleus approach, which requires the construction of a complete set of the intermediate nuclear states.

In the framework of the QRPA or renormalized QRPA the $m^{th}$ excited states with angular momentum $J$ and projection $M$ is created by a phonon-operator $Q$ with the properties

$$Q_{JM}^{|0^+_\text{RPA}}\rangle = |m, JM\rangle$$

and $$Q|0^+_\text{RPA}\rangle = 0.$$ (2)

Here, $|0^+_\text{RPA}\rangle$ is the ground state of the initial or the final nucleus. The phonon-operator $Q$ takes the following form

$$Q_{JM}^m = \sum_{k\mu \leq \nu} X_{(k\mu\nu),J}^m A_{(k\mu\nu),J}^\dagger + Y_{(k\mu\nu),J}^m \tilde{A}_{(k\mu\nu),J}^\dagger$$ (3)

where the tilde indicates time-reversal. $A_{(k\mu\nu),J}^\dagger$ is the two quasi-particle-creation and annihilation operator coupled to good angular momentum $J$ with projection $M$ namely

$$A_{(k\mu\nu),J}^\dagger = n(k\mu, l\nu) \sum_{m_k, m_l} C_{j_ka_kj_lmi_l}^{JM} a_{\mu k m_k}^\dagger a_{\nu l m_l}^\dagger.$$ (4)

The indices $\mu$ and $\nu$ denote the isospin structure of the bifermion operator. In the case of a BCS definition of the quasi-particles without pn-pairing it would distinguish between proton and neutron type quasi-particles. For the case of a HFB definition of the quasi-particles with pn-pairing the quasi-particles have no definite isospin anymore and the index becomes 1 or 2. The factor $n(k\mu, l\nu)$ guarantees the normalization for the case of two identical particles $k\mu$ and $l\nu$.

From Eq. (2) and (3) one can derive the RQRPA equation

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}_{j^\pi} \begin{pmatrix} X \\ Y \end{pmatrix} = \Omega_{j^\pi} \begin{pmatrix} U & 0 \\ 0 & -U \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix},$$

(5)
where \( \Omega_m^J \) denotes the excitation energy of the \( m \)th excited state with angular momentum \( J \) and parity \( \pi \) with respect to the ground state \(| 0^+_\text{RPA} \rangle \). The three matrices \( \mathcal{A}, \mathcal{B}, \mathcal{U} \) are given by the expectation values of the following commutators in the correlated ground state

\[
\mathcal{A}_{k\mu l\nu, J}^{a\alpha b\beta} = \langle 0^+_\text{RPA} | [A(a\alpha b\beta, JM), [H, A^\dagger(k\mu l\nu, JM)]] | 0^+_\text{RPA} \rangle ,
\]

\[
\mathcal{B}_{k\mu l\nu, J}^{a\alpha b\beta} = \langle 0^+_\text{RPA} | [A(a\alpha b\beta, JM), [\tilde{A}(k\mu l\nu, JM), H]] | 0^+_\text{RPA} \rangle ,
\]

\[
\mathcal{U}_{k\mu l\nu, J}^{a\alpha b\beta} = \langle 0^+_\text{RPA} | [A(a\alpha b\beta, JM), A^\dagger(k\mu l\nu, JM)] | 0^+_\text{RPA} \rangle .
\]

For solving the above equation it is necessary to introduce some approximation scheme for the calculation of the \( \mathcal{A}, \mathcal{B}, \mathcal{U} \) matrices. The simplest and the most frequently used one is the quasiboson approximation (QBA) scheme, which implies the two-quasiparticle operator \( A^\dagger(k\mu l\nu, JM) \) to be a boson operator. However, it is well-known that the QBA violates the Pauli exclusion principle because we have neglected terms coming from the commutator of the two bifermion operators by replacing the exact expression for this commutator with its expectation value in the uncorrelated BCS/HFB ground state, which is the vacuum for the quasi-particle operators. It turns out that the QBA is a poor approximation and leads to too strong ground state correlations, an unphysically large occupation of quasi-particle states in the correlated ground state, close to a collapse of the QRPA solution. But the ground state correlations influence the nuclear matrix elements severely and in general the use of the QBA leads to a very sensitive dependence of the nuclear matrix elements on the strength of the residual interaction in the particle-particle channel.

To overcome this problem the Pauli-principle has to be incorporated in the approach \cite{[1]} \cite{[10]}, by which the occupation of the quasi-particle states in the correlated ground state would be limited. The Pauli-principle can be incorporated to large extend by calculating explicitly the commutator of Eq. (6) with the single quasi-particles obeying Fermi-anti-commutation rules. Neglecting the non-diagonal part the commutator is not anymore boson like, but obtains corrections to its bosonic behavior due to the fermionic constituents.

\[
\mathcal{U}_{k\mu l\nu, J}^{a\alpha b\beta} = \langle 0^+_\text{RPA} | [A(a\alpha b\beta, JM), A^\dagger(k\mu l\nu, JM)] | 0^+_\text{RPA} \rangle 
\]

\[
\simeq n(k, l, \nu)n(a, \alpha, b, \beta) \left( \delta_{k\alpha} \delta_{l\mu} \delta_{\nu\beta} - \delta_{k\alpha} \delta_{\nu\alpha} \delta_{l\mu} \delta_{\mu\beta} (-1)^{j_k + j_l - J} \right) 
\]

\[
\times \left\{ 1 - \frac{1}{n} \left( 0^+_\text{RPA} | [a_{\alpha}^\dagger a_{\beta}^\dagger, 00] | 0^+_\text{RPA} \rangle \right) 
\right\} =: D_{k\mu, l\nu; J^\pi},
\]

with the known abbreviation \( \hat{j} = \sqrt{(2j + 1)} \). This expression is still diagonal in the quasi-particle configuration indices, and therefore the QRPA Eq. \cite{[6]} can...
easily be brought to eigenvalue form

\[
\begin{pmatrix}
A & B \\
-B & -A
\end{pmatrix} \begin{pmatrix}
\mathcal{D}^{-1/2} & \\
\mathcal{D}^{-1/2}
\end{pmatrix} \begin{pmatrix}
X^m \\
Y^m
\end{pmatrix} = \Omega^m_{j^*} \begin{pmatrix}
\mathcal{D}^{1/2} & \\
\mathcal{D}^{1/2}
\end{pmatrix} \begin{pmatrix}
X^m \\
Y^m
\end{pmatrix}.
\]

(10)

As in this form the matrices \(A\) and \(B\) and the eigenvectors are renormalized by \(\mathcal{D}\) this approach is called renormalized QRPA (RQRPA). To solve this equation the quasi-particle occupation in the correlated ground state needs to be known. But the occupation can only be derived from the back-going amplitudes \(Y\) of the RQRPA diagonalization. An implicit formulation for the \(\mathcal{D}\)-matrix can be given, if one express the one-body densities of Eq. (9) in terms of mapping on two boson operators. Then one arrives at the formula

\[
D_{(k\mu l\nu),J} = 1 - \frac{1}{\hat{J} k^2} \sum_{k'\mu' l'\nu'} D_{(k\mu k'\mu'),J', J^*} |Y^m_{(k\mu k'\mu'), J^*}|^2
\]

(11)

\[
- \frac{1}{\hat{J}^2} \sum_{l'\nu'} D_{(l\nu l'\nu'),J', J^*} |Y^m_{(l\nu l'\nu'), J^*}|^2.
\]

Eq. (11) and (10) have to be solved self-consistently in an iterative procedure. Note that an eigenvalue equation has to be written down separately for every multipolarity \(J^*\) which are then coupled by Eq. (11). The method presented above include both proton-neutron pairing and the Pauli effect of fermion pairs and is denoted the full-RQRPA. In the limit the proton-neutron pairing is switched off, one obtains the pn-RQRPA method.

In order to calculate double beta transitions two fully independent RQRPA calculations are needed, one to describe the beta transition from the initial to the intermediate nucleus and another one for the beta transition from the intermediate to the final nucleus. For that purpose the one-body transition densities of the charge changing operator has to be evaluated. They take the following form:

\[
< J^* m_i | [c^+_{pk} \tilde{c}_{nl}] | J^+ 0^+ > = \sqrt{2 J + 1} \sum_{\mu, \nu = 1, 2} m(\mu k, \nu l) \times
\]

\[
\left[ u_{k\mu p}^{(i)} v_{l\nu n}^{(i)} \tilde{X}^m_{\mu \nu} (k, l, J^* ) + v_{k\mu p}^{(i)} u_{l\nu n}^{(i)} \tilde{Y}^m_{\mu \nu} (k, l, J^* ) \right] \sqrt{D^{(i)}_{k\mu l\nu J^*}},
\]

(12)

\[
< 0^+_f | [c^+_{pk} \tilde{c}_{nl'}] | J^* m_f > = \sqrt{2 J + 1} \sum_{\mu, \nu = 1, 2} m(\mu k', \nu l') \times
\]

\[
\left[ v_{k'\mu p}^{(f)} u_{l'\nu n'}^{(f)} \tilde{X}^m_{\mu \nu} (k', l', J^* ) + u_{k'\mu p}^{(f)} v_{l'\nu n'}^{(f)} \tilde{Y}^m_{\mu \nu} (k', l', J^* ) \right] \sqrt{D^{(f)}_{k'\mu l' \nu J^*}},
\]

(13)
with \( m(\mu a, \nu b) = \frac{(-1)^{1+\delta_{\nu b}} \delta_{\nu b}}{(1+\delta_{\nu b})^{1/2}} \). We note that the \( X^{m}_{\mu \nu}(k, l, J) \) and \( Y^{m}_{\mu \nu}(k, l, J) \) amplitudes are calculated by the renormalized QRPA equation only for the configurations \( \mu a \leq \nu b \) (i.e., \( \mu = \nu \) and the orbitals are ordered \( a \leq b \) and \( \mu = 1, \nu = 2 \) and the orbitals are not ordered). For different configurations \( X^{m}_{\mu \nu}(k, l, J) \) and \( Y^{m}_{\mu \nu}(k, l, J) \) in Eqs. (13) and (14) are given by following the prescription in Eqs. (65) and (66) of Ref. [11]. The index \( i \) (f) indicates that the quasiparticles and the excited states of the nucleus are defined with respect to the initial (final) nuclear ground state \( |0_i> \) (\( |0_f> \)). \( c_p \) and \( c_n \) denote proton particle creation and neutron hole annihilation operators, respectively.

For a complete presentation of the renormalized QRPA method we have to discuss also limitations of this nuclear structure method. There are no doubts that RQRPA offers advantages over QRPA:

i) There is no collapse of RQRPA solution for a physical value of the particle-particle interaction strength.

ii) The RQRPA Hamiltonians demonstrate better mutual correspondence like the QRPA ones [12]. Nevertheless the RQRPA has also several shortcomings:

i) In the framework of the RQRPA the Ikeda sum rule [13] (here given for Gamow-Teller \( \beta \)-transitions)

\[
S = S_-(1^+) - S_+(1^+) = \frac{1}{2J+1} \sum_{m} |\langle m, 1^+ | \beta_+ | 0 \rangle|^2 - \frac{1}{2J+1} \sum_{m} |\langle m, 1^+ | \beta_- | 0 \rangle|^2
\]

\[= \frac{3(N-Z)}{16} \]

is not fulfilled. It is worthwhile to notice that the completeness of the excited \( 1^+ \) states is the only condition needed to derive the Ikeda sum rule. In the QRPA this rule is identically fulfilled, while in RQRPA the violation is proportional to the renormalizing matrix \( D \), which depends on the strength of the residual interaction. In the framework of the RQRPA the Ikeda sum rule is exhausted only to 70-80%.

It is supposed that the omission of the scattering terms in both the construction of the excited states and the evaluation of one-body densities could be the reason of it. At the moment there is work under progress including these operators in the framework of extended RQRPA [14]. There is a hope that in such a model the Ikeda sum rule could be restored.

ii) The two sets of intermediate nuclear states generated from the initial and final ground states are not identical because of the considered QBA or RQBA schemes. Therefore the overlap factor of these states is introduced in the theory as follows [12]:

\[
< J_{m_i}^+ | J_{m_i}^+ > \approx |Q_{JM_i}^{m_i} Q_{JM_i}^{m_i} | \approx \sum_{\mu \leq \nu \leq l} \sum_{\mu' \leq \nu' \leq l'} \delta_{\mu \mu'} \delta_{\nu \nu'} \tilde{u}_{l l'} \times \]

\[
(16)
\]

\[
(16)
\]
Figure 1: One example diagram for a possible contribution to neutrinoless double beta decay in $R$-parity breaking SUSY-extensions of the standard model.

with

$$\tilde{u}_{k\mu\mu'} = u^{(i)}_{k\mu}u^{(f)}_{k\mu'} + v^{(i)}_{k\mu}v^{(f)}_{k\mu'}.$$  \hspace{1cm} (17)

Here, $Q_{JM}^{m}$ and $Q_{JM}^{+m}$ are respectively phonon annihilation and creation operators for the initial and final nuclear states. We note that in the previous calculations $\tilde{u}$ was approximated by the unity. However, in that case the overlap factor is dependent on the phases of the occupation BCS/HFB amplitudes $u$’s and $v$’s, which are in principal arbitrary. A negligible difference between the results with the above overlap containing $\tilde{u}$ and with the overlap without $\tilde{u}$ one obtains only if the phases of the BCS/HFB amplitudes are chosen so that $\tilde{u}$ is positive for each level.

3 $R_p$ SUSY induced $0\nu\beta\beta$-decay. Matrix elements and experimental constraints.

$R$-parity is a discrete multiplicative symmetry defined as $R_p = (-1)^{3B+L+2S}$, where $S$, $B$ and $L$ are the spin, the baryon and the lepton quantum number. This symmetry is conserved in the minimal supersymmetric models (MSSM). A consequence of this symmetry would be, that SUSY-partners can only be produced in associated production and the lightest SUSY particle is stable.

However, $R$-parity might not be conserved. We consider presently popular models with the explicit $R_p$-violation. The $R_p$-violating part of the superpotential breaking lepton number conservation and relevant to $0\nu\beta\beta$-decay is

$$W_{H_p} = \chi_{ijk} L_i Q_j \bar{D}_k.$$  \hspace{1cm} (18)

Here $L$, $Q$ are lepton and quark doublets while $\bar{E}$, $\bar{U}$, $\bar{D}$ are lepton and up, down quark singlet superfields. Indices $i, j, k$ denote generations. An example
of the diagram contributing to $0\nu\beta\beta$-decay is shown in Fig. 1. It involves the lepton number violating interactions originated from the superpotential (18).

The supersymmetric model only gives the underlying transition of a down-quark to an up-quark. Then this transition has to be transformed to one going from a neutron to a proton. Looks natural and most straightforward to incorporate the quarks in the nucleons. In this way, we come up with the well known two-nucleon mode. The corresponding effective $R_p$ SUSY induced nucleon-nucleon interaction is shown in Fig. 2a). But the intermediate SUSY-partners are very heavy particles, and therefore in the two-nucleon mode the two decaying neutrons must come very close to each other, what is suppressed by the nucleon repulsion.

Another possibility is to incorporate quarks undergoing the $R_p$ SUSY transition $d + d \rightarrow u + u + 2e^{-}$ not in nucleons but in virtual pions [6]. There are two possibilities to set up the pion contributions. Namely, only one quark-antiquark pair $\bar{ud}$ is placed in an intermediate pion leading to a diagram shown in Fig. 2b) or both $\bar{ud}$ pairs are placed separately in two intermediate pions, as in Fig. 2c). The scale on which the pion-contribution is enhanced compared to the two-nucleon mode is the ratio of the nuclear form factor cut-off to the pion mass. As this factor favors for each nuclear decay process the pion emission, the two-pion mode is expected to be dominant over the two-nucleon and also the one-pion mode.

The half-life for neutrinoless double beta decay regarding all the three possibilities of hadronization of the quarks can be written in the form

$$\left[T^{\nu\beta}_{1/2}(0^+ \rightarrow 0^+)\right]^{-1} = G_{01} \left(\frac{m_A}{m_p}\right)^4 \left|\eta_q \cdot M^2_q N + \eta_f \cdot M^2_f N + (\eta_q + \eta_f) \cdot M^{N N} + (\eta_q + \eta_f) \cdot M^{N P} + \eta_q \cdot \eta_f \cdot M^{NP}\right|^2.$$

Here $G_{01}$ is the standard phase space factor tabulated for various nuclei in Ref. [1] and $m_A = 850$ MeV. The factors $\eta_q$ and $\eta_f$ are ruled by the SUSY-
parameters and by the strength of $R$-parity violating coupling. This factors will be constrained by the non-observation of neutrinoless double beta decay, and under reasonable assumptions for the SUSY-parameters (especially the SUSY masses), a limit on the $R$-parity violating coupling $\lambda_{111}$ can be derived. The nuclear matrix elements $M_{\pi}^{\pi N}$ governing the two-nucleon mode were presented in Ref. [3]. The one- and two-pion modes contribute to the nuclear matrix element $M_{\pi}^{\pi N}$ which consists of the four partial matrix elements

$$M_{\pi}^{\pi N} = \frac{m_\pi}{m_e} \left[ \alpha^{1\pi} (M_{GT,1\pi} + M_{T,1\pi}) + \alpha^{2\pi} (M_{GT,2\pi} + M_{T,2\pi}) \right],$$

(20)

two (Gamow-Teller and Tensor type) elements for each pionic mode. The two types of matrix elements are given by the expression

$$M_{GT,\pi} = <0^+_j| \sum_{i \neq j} \tau_i^+ \tau_j^+ \tilde{\sigma}_i \cdot \tilde{\sigma}_j \frac{F_2^{(k)}(x_\pi) R}{|\vec{r}_i - \vec{r}_j|} |0^+_i>, \quad \text{with } k = 1, 2$$

(21)

$$M_{T,\pi} = <0^+_j| \sum_{i \neq j} \tau_i^+ \tau_j^+ \left[ 3(\tilde{\sigma}_i \cdot \tilde{r}_ij)(\tilde{\sigma}_j \cdot \tilde{r}_ij) - \tilde{\sigma}_i \cdot \tilde{\sigma}_j \right] \frac{F_2^{(k)}(x_\pi) R}{|\vec{r}_i - \vec{r}_j|} |0^+_i>,$$

with $x_\pi = m_\pi r_{ij}$, $r_{ij} = |\vec{r}_i - \vec{r}_j|$ and $\tilde{r}_{ij} = (\vec{r}_i - \vec{r}_j)/r_{ij}$. The structure functions $F_1^{(k)}(x_\pi)$ and $F_2^{(k)}(x_\pi)$ have their origin in the integration of the propagator of the intermediate particles and take the following form:

$$F_1^{(1)}(x) = e^{-x}, \quad F_2^{(1)}(x) = (3 + 3x + x^2) \frac{e^{-x}}{x^2},$$

$$F_1^{(2)}(x) = (x - 2)e^{-x}, \quad F_2^{(2)}(x) = (x + 1)e^{-x}.$$  

(22)

(23)

The structure coefficients $\alpha^{1\pi}$ and $\alpha^{2\pi}$ depend on the hadronization of the quarks. For the two-pion coefficient $\alpha^{2\pi}$ a hadronic matrix element of the type $<\pi^+| J_i J_i |\pi^- \rangle$ ($i = P, S, T$) for the hadronic currents $J_i$ needs to be evaluated. This can be done in either a non-relativistic quark model (QM) or by inserting the vacuum state in between the two currents (vacuum insertion approximation, VIA). As the VIA neglects contributions from other intermediate states than the vacuum, it will give a more conservative limit. The values for the structure coefficients $\alpha^{1\pi} = -4.4 \times 10^{-2}$ and $\alpha^{2\pi} = 0.2\,(\text{VIA}), 0.64(\text{QM})$ reveal the above made statement on the dominance of the two-pion over the one-pion mode. Further it will be seen that not only the structure coefficient but also the corresponding nuclear matrix elements $M_{GT,\pi}$ and $M_{T,\pi}$ of the two-pion mode is larger than those of the one-pion mode.

For further calculation the nuclear matrix element governing the SUSY-mechanism of $0\nu\beta\beta$-decay are transformed to ones containing two-body matrix element in relative coordinate. One arrives at the expression:

$$< O_{12} > = \sum_{ijkl} (-)^{j_i + j_{k'}} + J + \mathcal{J} (2\mathcal{J} + 1) \left\{ \begin{array}{ccc} j_k & j_i & J \\ j_l & j_{k'} & \mathcal{J} \end{array} \right\} \times$$
Table 1: Nuclear matrix elements for SUSY two-nucleon, one-pion and two-pion mechanisms of neutrinoless double beta decay for $^{76}\text{Ge}(0^+) \rightarrow ^{76}\text{Se}(0^+)$ nuclear transition within the pn-RQRPA and full-RQRPA. The 12-level (the full $2\hbar\omega$ major oscillator shells) model space has been considered. The presented values has been calculated for $g_{\text{pp}} = 1.0$. The BM, QM and VIA denote bag model, non-relativistic quark model and vacuum insertion approximation, respectively.

| M.E.       | $M_{GT,N}$ [10^{-2}] | $M_{F,N}$ [10^{-2}] | $M_{GT'}$ [10^{-2}] | $M_{F'}$ [10^{-3}] | $M_3$ (BM) (NQM) | $M_4$ (BM) (NQM) |
|------------|----------------------|---------------------|---------------------|---------------------|-----------------|-----------------|
| pn-RQRPA   | 7.05                | -2.48               | -1.04               | 3.76                | -2.38           | -116            |
|            |                      |                     |                     |                     | 5.7             |                 |
| full-RQRPA | 4.32                | -1.26               | -0.65               | 1.94                | -0.76           | -75             |
|            |                      |                     |                     |                     | 5.5             |                 |

| M.E.       | $M_{GT,1\pi}$       | $M_{F,1\pi}$        | $M_{1\pi}$         | $M_{GT,2\pi}$       | $M_{F,2\pi}$    | $M_{2\pi}$ (VIA) (QM) | $M_{2\pi,N}$ (VIA) (QM) |
|------------|---------------------|---------------------|---------------------|---------------------|---------------------|------------------------|------------------------|
| pn-RQRPA   | 1.296               | -1.023              | -22                 | -1.341              | -0.653             | -1.99                  | -754                   |
|            |                     |                     |                     |                     |                     |                        |                        |
| full-RQRPA | 0.840               | -0.450              | -31                 | -0.810              | -0.385             | -439                   | -470                   |
|            |                     |                     |                     |                     |                     |                        |                        |

\[
<0^+_f\| \left[ c_{\text{pl}}\, \tilde{c}_{nl'} \right]_J \| J^\pi m_f \rangle < J^\pi m_i \rangle < J^\pi m_i \| \left[ c_{\text{pl}}\, \tilde{c}_{nl} \right]_J \| 0^+_i > \\
\times < pk, pk'; J\| f(r_{12})\, \tau_1^+, \tau_2^+ \| O_{12} f(r_{12}) \| nl, nl' > , \]  \tag{24}

with a short-range correlation function

\[
f(r) = 1 - e^{-\alpha r^2} (1 - br^2) \quad \text{with} \quad \alpha = 1.1 \text{fm}^2 \quad \text{and} \quad b = 0.68 \text{fm}^2, \tag{25}\]

which takes into account the short range repulsion of the nucleons.

The calculated nuclear matrix elements for the $0\nu\beta\beta$-decay of $\Lambda$=76 isotope within the pn-RQRPA and the full-RQRPA are presented in Table 1. The considered single-particle model space has been the 12-level model space (the full $2 - 4\hbar\omega$ major oscillator shells) introduced in Ref.\cite{2}. One should note that the calculation of the matrix elements in QRPA collapses with increasing interaction strength, while a calculation with RQRPA stays stable in the whole
range of physical interaction strength. The nuclear matrix elements listed in Table 1 have been obtained for the $g_{pp} = 1$ ($g_{pp}$ - parameter used to renormalized particle-particle interaction of the nuclear Hamiltonian). By glancing the Table 1 we note less important role of the one-pion exchange SUSY mechanism and that the two-pion exchange nuclear matrix elements clearly dominate over nuclear matrix elements of the two-nucleon mechanism. There are two sources of this enhancement. The first source has pure nuclear origin as the potential determined by the pion-exchange with mass about 140 MeV is favored in comparison with the potential determined by the cut-off with value about 850 MeV. The second source of enhancement has its origin in the hadronization of the $R_p$ SUSY effective vertex operator $\bar{u}\gamma_5 d \cdot \bar{d}\gamma_5 d \cdot \bar{e}P_R e^c$ replaced by its hadronic image $\pi^2 \cdot \bar{e}P_R e^c$. The enhancement occurs due to the coincidence of the pseudoscalar quark bilinears $\bar{u}\gamma_5 d$ with $\pi$-meson field.

For deducing constraints on the $R$-parity violating SUSY parameters from the non-observation of $0\nu\beta\beta$-decay we shall use values of nuclear matrix elements obtained within the full-RQRPA. The current experimental lower bound on the $^{76}$Ge $0\nu\beta\beta$-decay half-life \cite{15} is

$$T_{1/2}^{0\nu\beta\beta-\exp}(0^+ \rightarrow 0^+) \geq 1.1 \times 10^{25} \text{ years } 90\% \text{ c.l.} \quad (26)$$

With the above calculated nuclear matrix elements lower limit can be transformed to the following upper limits for the 1st generation $R_p$ Yukawa coupling constant

$$\lambda'_{111} \leq 1.3(0.8)10^{-4} \left( \frac{\tilde{m}_d}{100\text{GeV}} \right)^2 \left( \frac{\tilde{m}_e}{100\text{GeV}} \right)^{1/2} \quad (27)$$

$$\lambda'_{111} \leq 9.1(5.2)10^{-4} \left( \frac{\tilde{m}_d}{100\text{GeV}} \right)^2 \left( \frac{\tilde{m}_e}{100\text{GeV}} \right)^{1/2} \quad (28)$$

We point out at that the uncertainties of the nuclear structure calculations are smaller than those from the hadronic matrix elements. The limits in Eqs. (27) - (28) correspond to the VIA (QM) calculations of the hadronic matrix element.

These limits are much stronger than those previously known and lie beyond the reach of the near future accelerator experiments (though, accelerator experiments are potentially sensitive to other couplings than $\lambda'_{111}$). To constrain the size of $\lambda'_{111}$ one needs to make assumptions on the masses of the SUSY-partners. If the masses of the SUSY-partners would be at their present limit \cite{16}, one could constrain the coupling to $\lambda'_{111} \leq 6.0(3.3) \cdot 10^{-5}$. A conservative bound can be set by assuming all the SUSY-masses being at the ”SUSY-naturalness” bound of 1 TeV, leading to $\lambda'_{111} \leq 8.2(4.4) \cdot 10^{-2}$.

These results show that the non-observation of $0\nu\beta\beta$-decay strongly limits extensions of the standard model of electroweak interaction. Although a many-body problem needs to be solved the improvement of the limits is so large that it overcomes the uncertainties in the nuclear and hadronic matrix elements and leads to limits that are much stronger than those from accelerator and non-accelerator experiments.
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