Distributed leader-follower consensus of nonlinear multi-agent systems with unconsensusable switching topologies and its application to flexible-joint manipulators

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ABSTRACT

This paper investigates the distributed consensus problem of nonlinear multi-agent systems with switching topologies under a leader-follower framework. In particular, the unconsensusable switching topology is considered for modelling communication failures, which is more practical in the applications. By adopting the novel mode-dependent average dwell time, sufficient consensus conditions are established and the corresponding topology-dependent consensus gains are designed in term of linear matrix inequalities. In the end, an illustrative example with application to multiple flexible-joint manipulators is provided to verify the effectiveness of our proposed design method.

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1. Introduction

During the past decade, researches on multi-agent systems have gained great interest from both academic and engineering areas, and have obtained successful achievements. This is due to their distinct advantages compared with a traditional single system, which includes high robustness, high reliability, and high efficiency with relatively low cost (Oh, Park, & Ahn, 2015; Panait & Luke, 2005; Van der Hoek & Wooldridge, 2008; Yuan, Wang, Zhang, & Dong, 2018). With these advanced features, there are many successful applications with multi-agent systems, such as cooperation of intelligent robots (Egerstedt & Hu, 2001), formation of unmanned aerial vehicles (Anderson, Fidan, Yu, & Walle, 2008) and autonomous underwater vehicles (Cui, Ge, How, & Choo, 2010), synchronization of sensor networks (Rault, Bouabdallah, & Challal, 2014), and so on. In particular, the consensus is a fundamental yet significant research front of multi-agent systems, which means that by local information communications, certain agreement can be reached by the group of agents (Guo, Ding, & Han, 2014; Li, Wen, Duan, & Ren, 2015). Generally speaking, there are two types of consensus architecture: the leaderless consensus and the leader-follower consensus (Ding & Guo, 2015; Qiu, Xie, & Hong, 2016). For the leader-follower consensus problem, since it has a very practical background in industrial applications, many effective consensus protocols are proposed to deal with this problem. It should be pointed out that in the real-world applications, the information exchanges of the agents may fail or switch because of communication network problems, which would lead to communication topology jumps. Moreover, the consensus requires certain topology structures such as connected undirected topology, weighted directed topology with directed spanning trees and appropriate coupling strength (Dong & Hu, 2016; Valcher & Zorzan, 2017). Under this context, some research effort has been paid to the switching topologies and some remarkable results can be found in the literature (Ni & Cheng, 2010; Olfati-Saber & Murray, 2004).

On the other hand, substantial progresses have been made towards the analysis and synthesis of switched systems. Burgeoning research results with switched systems have been proposed with several remarkable strategies. Well-known concepts can be found as the dwell time, average dwell time (ADT) and mode-dependent average dwell time (MDADT). Especially, the MDADT method can effectively reduce the conservatism for each subsystems and as thus has been intensively studied. Since the communication topology plays an important role in the consensus problem, it is reasonable to deal with the topology switchings by the novel strategies of switched systems. Encouragingly, some recent preliminary results have been reported for the consensus...
of multi-agent systems based on the MDADT approach (Wang & Yang, 2016; Zheng, Zhang, & Zheng, 2016). However, it should be pointed out that sometimes the topology may be unconsensusable which would result in divergence behaviours of multi-agents. Although this problem is similar to the switched systems with unstable subsystems, it is more difficult for multi-agent systems by graph theory. So far, to the best of the authors’ knowledge, there are still few results for the consensus problem with unconsensusable switching topologies by applying the merit of switched systems, which motivates us for this study.

Following the above issues, in this paper, the leader-follower consensus of nonlinear multi-agent systems with unconsensusable switching topologies is studied with the novel MDADT approach. Compared with the existing literature, the main contributions of our paper can be summarized as follows: (1) this paper makes further attempts to solve the consensus problem of nonlinear multi-agent systems with unconsensusable switching topologies by MDADT. The nonlinear dynamics are more applicable in modelling the practical systems. (2) The switching topologies consisted of both consensusable and unconsensusable topologies are concerned, which is more general than the common assumption on all consensusable topologies, and would bring more design flexibility and robustness for the multi-agents systems.

The rest of this paper is arranged as follows. In Section 2, some preliminaries are introduced on graph theory and the leader-follower consensus problem is formulated. Section 3 gives the main theoretical findings with MDADT in detail and Section 4 provides the consensus simulation results with flexible-joint manipulators to show the effectiveness of the developed theoretical results. In the end, Section 5 draws the conclusions and our future work.

Notation: $\mathbb{R}^n$ and $\mathbb{R}^{m \times n}$ stand for the $n$-dimensional Euclidean space and the space of $m \times n$ real matrices, respectively. $A - B > 0$ $(A - B < 0)$ means that $A - B$ is positive definite (negative definite). $A \otimes B$ denotes the Kronecker product. $\text{sym}(A)$ denotes $A + A^T$ and $\text{diag} \{ \cdots \}$ represents the block-diagonal matrix. Finally, all matrices are compatible with algebraic operations.

2. Problem formulation and preliminaries

2.1. Graph theory

The directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$, $\mathcal{I} = \{1, \ldots, N\}$ is adopted to describe the information topology of the agents. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ represents the weighted adjacency matrix with

$$a_{ij} > 0, \quad (v_i, v_j) \in \mathcal{E},$$

$$a_{ij} = 0, \quad \text{Otherwise.} \quad (1)$$

$\mathcal{V}(\mathcal{G}) = \{v_1, \ldots, v_N\}$ and $\mathcal{E}$ denote the sets of nodes and edges, respectively. The $L = (l_{ij})_{N \times N}$ denotes the Laplacian matrix with

$$l_{ij} = -a_{ij}, \quad i \neq j$$

$$l_{ii} = \sum_{j=1, i \neq j}^{N} a_{ij}. \quad (2)$$

If $\mathcal{G}$ has a directed spanning tree, then 0 is the eigenvalue of $L$. More details of the algebraic graph theory can be found in Yu, Chen, and Cao (2010).

In this paper, it is assumed that there are $N$ followers whose communication topology is $\mathcal{G}$ and one leader. As a consequence, the adjacency matrix between the leader and the followers is defined by $B = \text{diag}(b_1, b_2, \ldots, b_N)$ with

$$b_i > 0, \quad \text{the ith follower is connected with the leader,}$$

$$b_i = 0, \quad \text{Otherwise.} \quad (3)$$

Without loss of generality, there is at least one follower connected with the leader.

Furthermore, since the communication among the agents may fail in practical applications, the switching topologies are studied. Let $\{\mathcal{G}_p, B_p : p \in S = 1, 2, \ldots, n\}$ denote all possible switching graphs, where $\sigma(t)$ represents the index set for all graphs. The switching signal $\sigma(t) : [0, \infty) \rightarrow p$ and its value is the index of graph at time $t$. The switching sequence is $t_1, t_2, \ldots$, where $[t_h, t_{h+1}), h = 1, 2, \ldots$ is uniformly bounded non-overlapping.

2.2. Nonlinear multi-agent system

Consider a nonlinear multi-agent system under a leader-follower framework. The followers are described by the following dynamics:

$$\dot{x}_i(t) = Ax_i(t) + f(x_i(t)) + Bu_i(t), \quad i \in \mathcal{I}, \quad (4)$$

where $x_i(t) \in \mathbb{R}^n$ denotes the state of the $i$th follower; $f(x_i(t))$ is a nonlinear Lipschitz function satisfying $\|f(x_i(t)) - f(x_j(t))\| \leq \|x_i(t) - x_j(t)\|, i \neq j, i, j \in \mathcal{I}$; $u_i(t) \in \mathbb{R}^m$ is the control input; $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are known constant matrices.

The dynamics of leader are given as follows:

$$\dot{x}_0(t) = Ax_0(t) + f(x_0(t)),$$

where $x_0(t) \in \mathbb{R}^n$ denotes the state of the leader.
Consequently, define $e_i(t) := x_i(t) - x_0(t)$ and it can be obtained that
\[ \dot{e}_i(t) = A e_i(t) + f_\sigma(e_i(t)) + B u_i(t), \]
where $f_\sigma(e_i(t)) = f(x_i(t)) - f(x_0(t))$.

Moreover, the following consensus protocol is designed for the followers:
\[ u_i(t) = -K_\sigma(t) \left\{ \sum_{j \in N_i} a_{ij}(t)[x_j(t) - x_i(t)] + b_{ij}(t)[x_i(t) - x_0(t)] \right\}, \tag{5} \]
where $K_\sigma(t)$ denotes the topology-dependent consensus gain to be determined.

Then, the closed-loop dynamics of the multi-agent system can be obtained by
\[ \dot{e}(t) = A e(t) + f_\sigma(e(t)) - BK_\sigma(t) \]
\[ \times \left\{ \sum_{j \in N_i} a_{ij}(t)(e_i(t) - e_j(t)) + b_{ij}(t)e_i(t) \right\}, \]
which can be further rewritten as follows:
\[ \dot{e}(t) = [(I \otimes A) - ((L_\sigma(t) + B_\sigma(t)) \otimes BK_\sigma(t))] e(t) + F(e(t)), \]
where
\[ e(t) = [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T \]
and
\[ F(e(t)) = [f_\sigma^T(e_1(t)), f_\sigma^T(e_2(t)), \ldots, f_\sigma^T(e_N(t))]^T. \]

As a result, the leader-follower consensus can be achieved if it holds that
\[ \lim_{t \to \infty} \|e(t)\| = 0, \]
which implies that
\[ \lim_{t \to \infty} \|x_i(t) - x_0(t)\| = 0, \quad i \in \mathcal{I}. \]

Before proceeding, the following definitions are given.

**Definition 2.1:** For a fixed $\sigma(t)$, the communication topology $\mathcal{G}_\sigma(t)$ is said to be consensusable if the consensus can be achieved with $\mathcal{G}_\sigma(t)$, otherwise the topology is called unconsensusable.

**Remark 2.1:** It should be pointed out that for both leader-follower consensus or leaderless consensus problems, there are requirements for the structure of directed or undirected communication topologies. Typical conditions can be found as topologies with directed spanning trees. By the above definition, it can be obtained that the topology can be unconsensusable if there is no follower connected to the leader or there is no directed spanning tree in $\mathcal{G}$. Under this context, the topology switchings are more complicated, which is similar to the cases of switched systems with unstable subsystems.

Without loss of generality, suppose that there are $r$ consensusable topologies with $\sigma(t) = 1, 2, \ldots, r$ and $n-r$ consensusable topologies with $r + 1, r + 2, \ldots, n$. All the possible communication topology can be divided into the consensusable set denoted as $\mathcal{C}T$ and the unconsensusable set denoted as $\mathcal{UC}T$, respectively.

To this end, some well-known concepts of switched systems are introduced for later analysis and synthesis.

**Definition 2.2 (ADT, Niu, Karimi, Wang, & Liu, 2017):**
For a switched system
\[ \dot{z}(t) = f_\sigma(t)(z(t)), \quad z(0) = z_0, \tag{6} \]
if
\[ N_\sigma(t, T) \leq N_0 + \frac{T - t}{\tau_\sigma} \]
holds for $\tau_\sigma > 0$ with an integer $N_0 \geq 0$, then $\tau_\sigma$ is called ADT and $N_0$ is called the chattering bound.

**Definition 2.3 (MDADT, Zhao, Zhang, Shi, & Liu, 2012):**
For any $T > t \geq 0$, denote $N_{\sigma p}(T, t)$ as the number of switching of the $p$th topology over $[t, T]$ and the total running time of the $p$th topology is $T_p(T, t)$. $\sigma(t)$ is said to have an MDADT $\tau_{ap}$, if there exist positive numbers $N_{ap}(T, t)$ such that
\[ N_{\sigma p}(T, t) \leq N_{ap}(T, t) + \frac{T_p(T, t)}{\tau_{ap}}. \]

**Remark 2.2:** Compared with ADT, it can be found that the MDADT switching can reduce the conservativeness by considering the mode-dependent features for each subsystems.

### 3. Main results

In this section, sufficient consensus conditions are derived for the multi-agent systems with unconsensusable switching topologies.

**Theorem 3.1:** Consider multi-agent system (4) with switching topologies, for giving constants $\alpha_i, \mu_i > 1, i \in \mathcal{S}$ and the topology-dependent consensus gains $K_i$, if there exist matrices $P_i > 0, \forall (i, j) \in \mathcal{S} \times \mathcal{S}, i \neq j$, such that
\[ \Phi_i := \begin{bmatrix} \Phi_{ii} & (I \otimes P_i) \\ -I \end{bmatrix} < 0, \]
\[ \Phi_i := \text{sym}(l \otimes P_i A) - (L_i + B_i) \otimes P_BK_i \]
\[ - \alpha_i (l \otimes P_i) + \bar{P}_i I, \] (7)

and
\[ P_i \leq \mu_i P_i, \] (8)

Then, the leader-follower consensus can be achieved by any switching signal with MDADT
\[ \frac{T^-}{T^+} \geq \frac{\gamma^+ + \gamma^*}{\gamma^- - \gamma^*}, \quad 0 < \gamma^+ < \gamma^- \]
\[ \tau_{a_i} > \tau_{a_i}^*, \quad \alpha_i < 0, \quad i \in \mathcal{C}T, \]
\[ \tau_{a_i} > \tau_{a_i}^*, \quad \alpha_i > 0, \quad \forall \tau_{a_i}^* > 0, \quad i \in \mathcal{U}C \mathcal{T}. \]

Proof: Choose the following mode-dependent multiple Lyapunov functions:
\[ V_{\sigma(t)}(e(t)) = e^T(t)(l \otimes P_{\sigma(t)})e(t). \]

Then, it can be derived that
\[ \dot{V}_{\sigma(t)}(e(t)) = e^T(t)(l \otimes P_{\sigma(t)})e(t) + e^T(t)(l \otimes P_{\sigma(t)})\dot{e}(t) \]
\[ - \alpha_{\sigma(t)} V_{\sigma(t)}(e(t)) \]
\[ = 2e^T(t)(l \otimes P_{\sigma(t)})e(t) - 2e^T(t)((L_{\sigma(t)} + B_{\sigma(t)}) \otimes P_{\sigma(t)}BK_{\sigma(t)})e(t) \]
\[ + 2e^T(t)(l \otimes P_{\sigma(t)})F(e(t)) - F^T(e(t))F(e(t)) \]
\[ = \xi^T(t) \Pi_{\sigma(t)} \xi(t), \]

where \( \xi(t) := [e^T(t), F(e(t))]^T \) and
\[ \Pi_{\sigma(t)} := \frac{\Pi_{\sigma(t)}}{\Pi_{\sigma(t)}}, \]
\[ \Pi_{\sigma(t)} := \text{sym}((l \otimes P_{\sigma(t)} A) + (L_{\sigma(t)} - B_{\sigma(t)} \otimes P_{\sigma(t)} BK) \]
\[ - \alpha_{\sigma(t)} (l \otimes P_{\sigma(t)}) + \bar{P}_i I. \]

Consequently, for \( \forall t > 0 \) and \( \forall t \in [t_k, t_{k+1}] \), it can be obtained by (7) and (8) that

\[ V_{\sigma(t)}(e(t)) \leq \exp(\alpha_{\sigma(t)}(t - t_k))V_{\sigma(t)}(e(t_k)) \]
\[ \leq \mu_{\sigma(t_k)} \exp[\alpha_{\sigma(t_k)}(t - t_k)]V_{\sigma(t_{k-1})}(e(t_k)) \]
\[ \leq \mu_{\sigma(t_k)} \mu_{\sigma(t_{k-1})} \exp[\alpha_{\sigma(t_k)}(t - t_k)] \alpha_{\sigma(t_{k-1})} \]
\[ \leq \mu_{\sigma(t_k)} \mu_{\sigma(t_{k-1})} \exp[\alpha_{\sigma(t_k)}(t - t_k) + \alpha_{\sigma(t_{k-1})}] \]
\[ \leq \prod_{i=1}^{k} \mu_{\sigma(t_i)} \exp[\alpha_{\sigma(t_i)}(t - t_k) + \alpha_{\sigma(t_{k-1})}] \]
\[ \leq \prod_{i=1}^{r} N_{\sigma(t_i)}(t_0) \exp \left( \sum_{i=1}^{n} \alpha_i T_i(t, 0) \right) \]
\[ \leq \exp \left( \sum_{i=1}^{n} N_{\sigma(t_i)} \ln \mu_i \right) \exp \left( \sum_{i=1}^{r} T_i(t, 0) \right) \]

Moreover, one has
\[ \bar{P}_i e^T(t)e(t) - F^T(e(t))F(e(t)) \geq 0. \]

Then, it can be derived that
\[ \dot{V}_{\sigma(t)}(e(t)) - \alpha_{\sigma(t)} V_{\sigma(t)}(e(t)) \]
\[ \leq e^T(t)\text{sym}((l \otimes P_{\sigma(t)} A))e(t) - e^T(t) \]
\[ \text{sym}((L_{\sigma(t)} - B_{\sigma(t)}) \otimes P_{\sigma(t)} BK_{\sigma(t)})e(t) \]
\[ - \alpha_{\sigma(t)} (l \otimes P_{\sigma(t)}) + \bar{P}_i e^T(t)e(t) \leq 0. \]

By assuming \( \gamma^+ T^+ - \gamma^- T^- < -\gamma^* t \), it can be verified that when the MDADT conditions can be satisfied, then
\[ V_\sigma (t) (e(t)) \text{ converges to zero as } t \to \infty. \] Therefore, one has \[ \lim_{t \to \infty} \| e(t) \| \to 0 \text{ as } t \to \infty, \] which completes the proof.

 Remark 3.1: It is worth mentioning that the common Lyapunov functions may be difficult to find in most cases, such that the mode-dependent multiple Lyapunov functions are chosen for the consensus problem. This can considerably reduce the conservatism.

 Remark 3.2: It can be observed that the established conditions are applicable for the slow switching communication topologies, which is more practical than the fast switching in multi-agent systems.

 Based on the results of Theorem 1, the following theorem is given to calculate the desired consensus gains in terms of matrix transformation.

 **Theorem 3.2:** Consider multi-agent system (4) with switching topologies, for giving constants \( \alpha_i, \mu_i > 1, i \in S \), if there exist matrices \( \tilde{P}_i > 0 \) and \( W_i, \forall (i,j) \in S \times S, i \neq j \), such that
\[
\hat{\Phi}_i := \begin{bmatrix} \hat{\Phi}_{i1} & (I \otimes \tilde{P}_i) \\ \ast & -I \\ \ast & \ast \end{bmatrix} < 0,
\]
\[
\hat{\Phi}_{i1} := \text{sym}(I \otimes \hat{P}_i) - (L_i + B_i) \otimes BW_i - \alpha_i (I \otimes \tilde{P}_i),
\]
and
\[
\tilde{P}_i \leq \mu_i \tilde{P}_i,
\]
Then, the leader-follower consensus can be achieved by any switching signal with MDAD
\[
\frac{T^-}{T^+} \geq \gamma^+ + \gamma^-, \quad 0 < \gamma^+ < \gamma^-,
\]
\[
\tau_{ai} > \tau_{ai}^* = -\frac{\ln \mu_i}{\alpha_i}, \quad \alpha_i < 0, \quad i \in \mathcal{CT},
\]
\[
\tau_{ai} > \tau_{ai}^*, \quad \alpha_i > 0, \quad \forall \tau_{ai}^* > 0, \quad i \in \mathcal{UT},
\]
where
\[
\gamma^+ := \max_{i \in \mathcal{UT}} \left\{ \sum_{i=r+1}^{n} \left( \alpha_i + \frac{\ln \mu_i}{\tau_{ai}} \right) \right\},
\]
\[
\gamma^- := -\max_{i \in \mathcal{CT}} \left\{ \sum_{i=1}^{r} \left( \alpha_i + \frac{\ln \mu_i}{\tau_{ai}} \right) \right\},
\]
\[
T^+ := \sum_{i=r+1}^{n} T_i(t,0),
\]
\[
T^- := \sum_{i=1}^{r} T_i(t,0).
\]

\[ \text{In addition, the desired consensus gains can be obtained by } K_i = W_i \tilde{P}_i^{-1}. \]

 **Proof:** Let \( \tilde{\alpha}_i := P_i^{-1}, K_i \tilde{\alpha}_i = W_i \) and perform congruent transformation with \( I \otimes \tilde{\alpha}_i \) and \( \tilde{\alpha}_i \) to (7) and (8), respectively. Then, the rest of the proof can follow directly by Theorem 1.

 Based on the above results, the corresponding distributed consensus algorithm can be given as follows.

 **Algorithm 1 Distributed consensus algorithm**

 **Input:** \( A, B, l, L_\sigma(t) \) and \( B_\sigma(t) \)

 **Output:** \( K_\sigma(t) \)

 1. Given constants \( \alpha_\sigma(t), \mu_\sigma(t) > 1, \sigma(t) \in S \)
 2. Solving the LMIs in Theorem 3.2
 3. Obtaining feasible solutions of \( \tilde{\alpha}_\sigma(t) > 0 \) and \( W_\sigma(t) \)
 4. Obtaining the MDADT \( \tau_\sigma(t) \)
 5. return \( K_\sigma(t) = W_\sigma(t) \tilde{\alpha}_\sigma(t)^{-1} \).

4. Illustrative example

In this section, an application example of flexible-joint manipulators is provided to demonstrate the effectiveness of our proposed method.

In the simulation, the nonlinear dynamics of flexible-joint manipulators (see Figure 1) can be given by the form of (4) as Ma and Qiao (2017):
\[
A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -48.6 & -1.25 & 48.6 & 0 \\ 0 & 0 & 0 & 1 \\ 19.5 & 21.6 & 0 & -19.5 \end{bmatrix},
\]
\[
B = \begin{bmatrix} 0 & 21.6 & 0 & 0 \end{bmatrix}^T,
\]
\[
f(x_i(t)) = \begin{bmatrix} 0 & 0 & 0 & -3.33 \sin(x_{3j}(t)) \end{bmatrix}^T, \quad l = 3.33,
\]
where \( x_1(t) \) is the angular rotation of the motor, \( x_2(t) \) is the angular velocity of the motor, \( x_3(t) \) is the angular position of the link and \( x_{3j}(t) \) is the angular velocity of the link.

It is assumed that there are four manipulators and their switching communication topologies are illustrated in Figure 2.

It can be verified that \( G_1, G_2 \) are consensusable and \( G_3 \) is unconsensusable.

For giving parameters \( \alpha_1 = -2, \alpha_2 = -1, \alpha_3 = 2 \) and \( \mu_1 = 1.5, \mu_2 = 2, \mu_3 = 3 \) and \( \tau_{ai} = 2 \), the feasible solutions of the conditions in Theorem 2 can be obtained by \( K_1 = [116.801925.4326 - 45.625630.4458], \)
\[
K_2 = [110.383822.2018 - 48.708628.8315] \quad \text{and} \quad K_3 = [19.562421.404119.730922.3006] \] with \( \tau_{ai} \geq 0.2027, \tau_{a2} \geq \)
Figure 1. Robotic systems with flexible joints.

Figure 2. The switching communication topologies. (a) $G_1$, (b) $G_2$ and (c) $G_3$.

Figure 3. State trajectories of the open-loop manipulators with $x_{i1}$.

Figure 4. State trajectories of the open-loop manipulators with $x_{i2}$.

Figure 5. State trajectories of the open-loop manipulators with $x_{i3}$.

Figure 6. State trajectories of the open-loop manipulators with $x_{i4}$.

0.6931 and $T^-/T^+ \geq 4.4253$, respectively. Thus, with random initial conditions, the open-loop and closed-loop state trajectories of the leader and the followers are shown in Figures 3–10, respectively. It can be seen that all the followers can well track the state trajectory of the leader, which supports our theoretical results. Moreover, it should be pointed out that since the developed consensus conditions are with the $l$, $f(x_i(t))$ will affect the solvability of the conditions. In detail, with same other...
parameters, the feasible solution of Theorem 3.2 may not be obtained while $l$ increases. This implies that in the practical applications, the value of $l$ should be carefully addressed in the consensus problem.

5. Conclusions

In this paper, the distributed consensus problem for nonlinear multi-agent systems with switching topologies is concerned in a leader-follower architecture. Based on the results of model transformation, the MDADT method combined with the multiple Lyapunov method is applied to deal with both slow switching consensusable and unconsensusable topologies. Sufficient consensus criteria are further established in the form of LMIs such that the followers can track the leader. Finally, an application example of consensus for multiple flexible-joint manipulators is presented to illustrate our obtained results. Our future work will be extending the obtained results to the cases with finite-time or fixed-time requirements, which can further reduce the conservatism of the consensus conditions.

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References

Anderson, B. D., Fidan, B., Yu, C., & Walle, D. (2008). UAV formation control: Theory and application. In Vincent D. Blondel, Stephen P. Boyd, & Hidenori Kimura (Eds.), Recent advances in learning and control (pp. 15–33). London: Springer.

Cui, R., Ge, S. S., How, B. V. E., & Choo, Y. S. (2010). Leader–follower formation control of underactuated autonomous underwater vehicles. Ocean Engineering, 37(17–18), 1491–1502.

Ding, L., & Guo, G. (2015). Sampled-data leader-following consensus for nonlinear multi-agent systems with Markovian switching topologies and communication delay. Journal of the Franklin Institute, 352(1), 369–383.

Dong, X., & Hu, G. (2016). Time-varying formation control for general linear multi-agent systems with switching directed topologies. Automatica, 73, 47–55.

Egerstedt, M., & Hu, X. (2001). Formation constrained multi-agent control. IEEE Transactions on Robotics and Automation, 17(6), 947–951.

Guo, G., Ding, L., & Han, Q. L. (2014). A distributed event-triggered transmission strategy for sampled-data consensus of multi-agent systems. Automatica, 50(5), 1489–1496.

Li, Z., Wen, G., Duan, Z., & Ren, W. (2015). Designing fully distributed consensus protocols for linear multi-agent systems with directed graphs. IEEE Transactions on Automatic Control, 60(4), 1152–1157.

Ma, C., & Qiao, H. (2017). Distributed asynchronous event-triggered consensus of nonlinear multi-agent systems with disturbances: An extended dissipative approach. Neurocomputing, 243, 103–114.

Ni, W., & Cheng, D. (2010). Leader-following consensus of multi-agent systems under fixed and switching topologies. Systems & Control Letters, 59(3–4), 209–217.

Niu, B., Karimi, H. R., Wang, H., & Liu, Y. (2017). Adaptive output-feedback controller design for switched nonlinear stochastic systems with a modified average dwell-time method. IEEE Transactions on Systems, Man, and Cybernetics: Systems, 47(7), 1371–1382.

Oh, K. K., Park, M. C., & Ahn, H. S. (2015). A survey of multi-agent formation control. Automatica, 53, 424–440.

Olfati-Saber, R., & Murray, R. M. (2004). Consensus problems in networks of agents with switching topology and time-delays. IEEE Transactions on Automatic Control, 49(9), 1520–1533.

Panait, L., & Luke, S. (2005). Cooperative multi-agent learning: The state of the art. Autonomous Agents and Multi-Agent Systems, 11(3), 387–434.

Qiu, Z., Xie, L., & Hong, Y. (2016). Quantized leaderless and leader-following consensus of high-order multi-agent systems with limited data rate. IEEE Transactions on Automatic Control, 61(9), 2432–2447.

Rault, T., Bouabdallah, A., & Challal, Y. (2014). Energy efficiency in wireless sensor networks: A top-down survey. Computer Networks, 67, 104–122.

Valcher, M. E., & Zorzan, I. (2017). On the consensus of homogeneous multi-agent systems with arbitrarily switching topology. Automatica, 84, 79–85.

Van der Hoek, W., & Wooldridge, M. (2008). Multi-agent systems. Foundations of Artificial Intelligence, 3, 887–928.

Wang, X., & Yang, G. H. (2016). Distributed reliable H-infinity consensus control for a class of multi-agent systems under switching networks: A topology-based average dwell time approach. International Journal of Robust and Nonlinear Control, 26(13), 2767–2787.

Yu, W., Chen, G., & Cao, M. (2010). Some necessary and sufficient conditions for second-order consensus in multi-agent dynamical systems. Automatica, 46(6), 1089–1095.

Yuan, Y., Wang, Z., Zhang, P., & Dong, H. (2018). Nonfragile near-optimal control of stochastic time-varying multiagent systems with control-and state-dependent noises. IEEE Transactions on Cybernetics. doi:10.1109/TCYB.2018.2829713

Zhao, X., Zhang, L., Shi, P., & Liu, M. (2012). Stability and stabilization of switched linear systems with mode-dependent average dwell time. IEEE Transactions on Automatic Control, 57(7), 1809–1815.

Zheng, D., Zhang, H., & Zheng, Q. (2016). Consensus analysis of multi-agent systems under switching topologies by a topology-dependent average dwell time approach. IET Control Theory & Applications, 11(3), 429–438.