The transverse-momenta distributions in high-energy pp collisions — A statistical-mechanical approach

Leonardo J. L. Cirto, Constantino Tsallis, Cheuk-Yin Wong, and Grzegorz Wilk

1 Centro Brasileiro de Pesquisas Físicas & National Institute of Science and Technology for Complex Systems, Rua Xavier Sigaud 150, 22290-180 Rio de Janeiro-RJ, Brazil
2 Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501, USA
3 Physics Division, Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, USA
4 National Centre for Nuclear Research, Warsaw 00-681, Poland

We analyze LHC available data measuring the distribution probability of transverse momenta $p_T$ in proton-proton collisions at $\sqrt{s} = 0.9$ TeV (CMS, ALICE, ATLAS) and $\sqrt{s} = 7$ TeV (CMS, ATLAS). A remarkably good fitting can be obtained, along fourteen decades in magnitude, by phenomenologically using $q$-statistics for a single particle of a two-dimensional relativistic ideal gas.

The parameters that have been obtained by assuming $dN/dp_Tdy \propto e^{-T_1 p_T/T}$ at mid-rapidity are, in all cases, $q \simeq 1.1$ and $T \simeq 0.13$ GeV (which satisfactorily compares with the pion mass). This fact suggests the approximate validity of a "no-hair" statistical-mechanical description of the hard-scattering hadron-production process in which the detailed mechanisms of parton scattering, parton cascades, parton fragmentation, running coupling and other information can be subsumed under the stochastic dynamics in the lowest-order description. In addition to that basic structure, a finer analysis of the data suggests a small oscillatory structure on top of the leading $q$-exponential. The physical origin of such intriguing oscillatory behavior remains elusive, though it could be related to some sort of fractality or scale-invariance within the system.

PACS numbers: 05.30.-d, 05.90.+m, 25.75.-q, 25.90.+k, 25.75.Ag

The transverse momentum distributions of hadrons produced in collisions involving protons and also heavy ions at RHIC and LHC energies [1] have recently been successfully investigated through nonextensive statistical mechanics [2]. A statistical-mechanical approach to analytically describe these quantities was successfully used phenomenologically by Hagedorn four decades ago [3]. By adopting the idea of some kind of thermal quasi-equilibrium, he introduced the Boltzmann-Gibbs (BG) factor within his theoretical model. The Hagedorn proposal works very well at low beam-energies. However, the experiments at higher energies, which have been achieved with the modern experiments at LHC and other colliders, have produced data that neatly depart more and more from a standard thermal behavior, giving rise to power-law distributions in the asymptotic high-$p_T$ regime.

In order to explain the discrepancy between the BG distribution and experiments at high $p_T$, a number of works have been done through changing the BG thermostatical start-point by its nonextensive generalization. Broadly speaking, this change corresponds to replacing the BG exponential weight by its nonextensive $q$-exponential deformation. Hagedorn himself made heuristically a somewhat similar approach. With the aim of extending his analysis to higher $p_T$ values, power-law functions conforming to pQCD [4] were used instead of exponentials, without resorting to thermostatical justifications [3, 5]. Theories proposed within the nonextensive thermostatics scenario include, for example, the generalized version of the Hagedorn asymptotic bootstrap principle [7-9], reservoir fluctuations [10], micro-canonical jet-fragmentation [11], consequences of thermodynamic consistency [12], and a relativistic hard-scattering model in perturbative QCD [13, 14]. This approach gets also its impact in description of nuclear matter under extreme conditions (as those encountered in present heavy ion collisions), cf., for example, [15] and references therein.

Recently, the experimental data reveal surprisingly that the transverse momentum spectrum from the very low energy regime of many tenths of a GeV to the very high energy regime of hundreds of GeV, in high-energy pp collisions at central rapidities, is characterized by a small number of degrees of freedom [13]. However, one expects on physical grounds at least three different physical mechanisms contributing to the hadron production process. In the low $p_T$ region at central rapidities, the mechanism of non-perturbative string-fragmentation is expected to play an important role [16, 17]. In this mechanism, the hadrons are produced from the string stretched between the colliding nucleons with a plateau structure in rapidity. In the low $p_T$ region near the beam and target rapidities, the mechanism of direct-fragmentation is expected to play a dominant role [18]. In this mechanism, the produced hadron fragments directly from the nucleon without making a collision with the partons of the other colliding nucleon. The transverse spectrum then depends on the probability of the hadron fragmenting out of the nucleon. In the high $p_T$ region, the relativistic hard-scattering process becomes im-

1 Actually, such approach was firstly proposed in the analysis of the uncorrelated jet model, see [6].
portant. In this mechanism, a parton from the projectile nucleon scatters elastically with a parton from the target nucleon, and the parton subsequently cascades and fragments into the produced hadrons. The transverse spectrum therefore depends on many factors, including the parton distribution in the nucleons, the parton-parton hard-scattering amplitude, the cascade and fragmentation of the scattered parton to the hadrons, as well as the running of the coupling constant, as described in [14].

The three mechanisms mentioned above (string- and direct-fragmentations, and hard-scattering) are expected to give rise to different shapes of the transverse distributions as they depend on the transverse momentum in different ways. Indeed, the string fragmentation is governed by the parton intrinsic contributions as they depend on the transverse momentum to give rise to different shapes of the transverse distributions as functions of $\Delta y$, and the running of the coupling constant. Therefore, one would normally expect that the three different production mechanisms will lead to different behaviors as functions of $p_T$, and there would consistently be a breakdown of a single description at some point of the complete spectrum.

What interestingly emerges from the analysis of the data in high-energy pp collisions is that the good agreement of the present phenomenological fit extends to the whole $p_T$ region (or at least for $p_T$ greater than 0.5 GeV/c) where reliable experimental data are available: in fact, the ALICE 0.9 TeV data reliably extend even to lower values, namely down to $p_T \simeq 0.1$ GeV/c [13]. This being achieved with very few free parameters implies the simplicity of the underlying structure and the dominance of one of the three mechanisms over virtually the whole $p_T$ range in the central rapidity region. It is reasonable to consider the dominant mechanism to be the hard-scattering process because the other two mechanisms are unlikely to produce hadrons with high $p_T$. Moreover, the dominance of hard-scattering also for the production of low-$p_T$ hadron in the central rapidity region is supported by two-particle correlation data where the two-body correlations in minimum $p_T$-biased data reveal that a produced hadron is correlated with a “ridge” of particles along a wide range of $\Delta \eta$ on the azimuthally away side centering around $\Delta \phi \sim \pi$ [19–21]. The $\Delta \phi \sim \pi$ (back-to-back) correlation indicates that the correlated pair is related by a collision, and the $\Delta \eta$ correlation in the shape of a ridge indicates that the two particles are partons from the two nucleons and they carry fractions of the longitudinal momenta of their parents, leading to the ridge of $\Delta \eta$ at $\Delta \phi \sim \pi$.

While the basic hard-scattering process is relatively simple in the jet production level, there are many layers of stochastic processes in the production of hadrons from jets which mask this simplicity. The hard-scattering process exhibits the power-law behavior of the transverse momentum in its basic framework for jet production [13]. However, many other stochastic elements are involved in this process which definitively increase its complexity.

In spite of this fact, it turns out, remarkably enough, to be reasonable to assume, in the lowest-order description, a “no-hair” statistical-mechanical hypothesis for the transverse-momentum distribution in the $p_T$ region above 0.5 GeV/c. The hadron production process appears to be well characterized by a single-particle statistical-mechanical description of QCD quanta in an relativistic $d = 2$ system. All other information about the produced hadron matter, such as the parton distribution, the hard-scattering mechanism of jet production, jet-parton cascade process followed by parton fragmentation, and the intermediary possible quark-gluon plasma, “disappear” behind the stochastic process.

Towards such a goal of description, we can postulate that for the whole range of $p_T$ from about 0.5 GeV/c to very high values, the transverse distribution of the produced hadrons obeys the “no-hair” statistical-mechanical description with very few degrees of freedom of the hard-scattering process. Motivated by the good agreement of previous works, we use here the nonextensive thermostatistical background to examine the experimental high-energy pp data and we describe the distribution of hadronic transverse momenta at central rapidity $y \simeq 0$ with the following nonextensive ansatz\(^2\)

\[
\frac{dN}{dy dp_T} = \frac{1}{2\pi p_T} \frac{dN}{dy dp_T} = A e^{-E_T/T}, \tag{1}
\]

where the $q$-exponential function $e_q^x$ is defined by

\[
e_q^x \equiv [1 + (1 - q) x]^{1/(1-q)} \quad (e_1^x = e^x). \tag{2}
\]

Here, $E_T$ is the relativistic energy associated with the transverse momentum of a single particle of the beam, namely, $E_T = \sqrt{p_T^2 + m^2}$. As most of the produced particles are pions, $m$ was taken equal to the meson mass $m_\pi$. Notice that, if the distributions are properly normalized, the constant prefactor $A$ is not an independent parameter but a straightforward function of $(q, T)$.

The experimental results for two energies (0.9 and 7 TeV) and various detectors (CMS, ALICE, ATLAS) [13, 14] together with the proposed distribution (1) are indicated in Fig. 1. We verify that $q$ increases slightly with the beam energy, but, for the present energies, remains always $q \simeq 1.1$. Some indications exist that, in the limit of extremely high energies, $q$ approaches a limiting value (possibly close to 1.2 [9, 22]). The effective

\(^2\) We are adopting unity for the Boltzmann constant $k_B$.\n
temperature is, in all cases, \( T \approx 0.13 \text{ GeV} \), neatly compatible with the meson mass \( m_q = 0.135 \text{ GeV}/c^2 \). The dashed line (an ordinary exponential of \( E_T \)) illustrates how important can be the discrepancy with the analogous Boltzmann-Gibbs approach as \( p_T \) increases.

What we may extract from the behavior of the experimental data is that the Hagedorn scenario appears to be essentially correct excepting for the fact that we are not facing thermal equilibrium but a different type of stationary state, typical of violation of ergodicity (for a discussion of the kinetic and effective temperatures see [23, 24]). These results reinforce the pioneering connection between quantum chromodynamics and nonextensive statistics first suggested by Walton and Rafaelski [25]. The successful fitting of Eq. (1) along impressive 14 decades strongly points at the present “no-hair” assumption (namely a two-dimensional relativistic single-particle system) as the simplest one on which further sophisticated models can be built. We emphasize also that, in all cases, the temperature turns out to be one and the same, namely \( T = 0.13 \text{ GeV} \).

As a concluding remark, we note that the data/fit plot in the bottom part of Fig. 1 exhibits intriguing (rough) log-periodic oscillations, which suggest some hierarchical fine-structure in the quark-gluon system where hadrons are generated. This behavior is possibly an indication of some kind of (multi)fractality in the system. Indeed, the concept of self-similarity, one of the landmarks of fractal structures, has been used by Hagedorn in his very definition of fireball, as was previously pointed out by Beck [9] and has been found in the analysis of jets produced in \( pp \) collisions at LHC [26]. These small oscillations have already been preliminary discussed in [27, 28], where the authors were able to mathematically accommodate them by essentially allowing the index \( q \) in Eq. (1) to be a complex number\(^3\) (see also Refs. [29, 30]).

Two of us (L.J.L.C. and C.T.) have benefited from partial financial support from CNPq, Faperj and Capes (Brazilian agencies). The research of CYW was supported in part by the Division of Nuclear Physics, U.S. Department of Energy, and that of GW was supported by the National Science Center (NCN) under contract DEC-2013/09/B/ST2/02897.

\(^3\) It should be noted here that an alternative to complex \( q \) would be a log-periodic fluctuating scale parameter \( T \); such possibility was discussed in [28].

\[ \frac{1}{2\pi p_T dy dp_T} = A e^{-E_T/T} \]

**FIG. 1.** Comparison of Eq. (1) with the experimental transverse momentum distribution of hadrons in \( pp \) collisions at central rapidity \( y \). Herein the temperature is being measured in [\( T \)] = GeV and the normalization constant in [\( A \)] = GeV\(^{-2}c^4\). For a better visualization both the data and the analytical curves have been divided by a constant factor as indicated. The ratios data/fit are shown at the bottom, where a nearly log-periodic behavior is observed on top of the \( q \)-exponential one. Data from [13, 14].

[1] V. Khachatryan et al. (CMS Collaboration), Phys. Rev. Lett. 105, 022002 (2010); J. High Energy Phys. 02, 041 (2010); S. Chatrchyan et al. (CMS Collaboration) ibid. 08, 086 (2011); G. Aad et al. (ATLAS Collaboration), New J. Phys. 13, 053033 (2011); K. Aamodt et al. (ALICE Collaboration), Phys. Lett. B 693, 53 (2010); Eur. Phys. J. C 71, 1594 (2011); 71 1655 (2011); A. Adare et al. (PHENIX Collaboration), Phys. Rev. D 83, 052004 (2011); Phys. Rev. C 83, 064903 (2011); B. Abelev et al. (ALICE Collaboration), Phys. Lett. B 722, 262 (2013).

[2] C. Tsallis, J. Stat. Phys. 52, 479 (1988); Introduction to Nonextensive Statistical Mechanics – Approaching A Complex World (Springer, New York, 2009); Entropy, in Encyclopedia of Complexity and Systems Science, edited by R. A. Meyers (Springer, Berlin, 2009) p. 2859; M. Gell-Mann and C. Tsallis eds., Nonextensive Entropy – Interdisciplinary Applications (Oxford University Press, New York, 2004). A regularly updated bibliography on nonadditive entropies and nonextensive statistical mechanics is available at http://tsallis.cat.cbpf.br/biblio.htm.

[3] R. Hagedorn, Revuo Nuovo Cimento 6, 1 (1983).
[4] R. Blankenbecler and S. J. Brodsky, Phys. Rev. D 10, 2973 (1974); A. L. S. Angelis et al., Phys. Lett. B 79, 505 (1978); for the history of the power law, see J. Rak and M. J. Tannenbaum, High-\( p_T \) Physics in the Heavy Ion Era (Cambridge University Press, Cambridge, 2013).

[5] G. Arison et al. (UA1 Collaboration), Phys. Lett. B 118, 167 (1982).
[6] C. Michael and L. Vanryckeghem, J. Phys. G 3, L151 (1977); C. Michael, Prog. Part. Nucl. Phys. 2, 1 (1979).
[7] I. Bediaga, E. M. F. Curado, and J. M. de Miranda,
Physica A 286, 156 (2000).

[8] A. Deppman, Physica A 391, 6380 (2012); J. Phys. G. 41, 055108 (2014); L. Marques, E. Andrade-II, and A. Deppman, Phys. Rev. D 87, 114022 (2013).

[9] C. Beck, Physica A 286, 164 (2000).

[10] T. S. Biró, G. G. Barnaföldi, P. Ván, and K. Úrömösy, arXiv:1404.1256 [hep-ph].

[11] K. Úrömösy, G. G. Barnaföldi, and T. S. Biró, Phys. Lett. B 718, 125 (2012); T. S. Biró, K. Úrömösy, and Z. Schram, J. Phys. G 37, 094027 (2010); T. S. Biró and P. Ván, Phys. Rev. E 83, 061147 (2011); T. S. Biró and Z. Schram, EPJ web conf. 13, 05004 (2011); T. S. Biró, Is there a temperature? (Springer, New York, 2011).

[12] J. Cleymans and D. Worku, Eur. Phys. J. A 48, 160 (2012); J. Phys. G 39, 025006 (2012); M. D. Azmi and J. Cleymans, ibid. 41, 065001 (2014).

[13] C. Y. Wong and G. Wilk, Acta Phys. Polonica B 43, 2043 (2012).

[14] C. Y. Wong and G. Wilk, Phys. Rev. D 87, 114007 (2013).

[15] A. P. Santos, F. I. M. Pereira, R. Silva, and J. S. Alcaniz, J. Phys. G 41, 055105 (2014); J. Rożynek and G. Wilk, ibid. 36, 125108 (2009); EPJ web conf. 13, 05002 (2011).

[16] B. Andersson, G. Gustafson, and T. Sjöstrand, Z. Phys. C 20, 317 (1983); B. Andersson, G. Gustafson, G. Ingelman, and T. Sjöstrand, Phys. Rep. 97, 31 (1983); T. Sjöstrand and M. Bengtsson, Comput. Phys. Commun. 43, 367 (1987); B. Andersson, G. Gustavson, and B. Nilsson-Alvqvist, Nucl. Phys. B 281, 289 (1987).

[17] C. Y. Wong, Introduction to High-Energy Heavy-Ion Collisions (World Scientific Publishing Company, Singapore, 1994).

[18] C. Y. Wong and R. Blankenbecler, Phys. Rev. C 22, 2433 (1980).

[19] J. Adams et al. (STAR Collaboration), Phys. Rev. D 74, 032006 (2006).

[20] B. Abelev et al. (ALICE Collaboration), Phys. Rev. D 86, 112007 (2012).

[21] R. L. Ray, Phys. Rev. D 84, 034020 (2011); T. A. Trainor and D. J. Prindle, arXiv:1310.0408 [hep-ph].

[22] T. Wibig, J. Phys. G: Nucl. Part. Phys. 37, 115009 (2010).

[23] W. Niedenzu, T. Grießer, and H. Ritsch, Europhys. Lett. 96, 43001 (2011); L. A. Gougam and M. Tribeche, Phys. Plasmas 18, 062102 (2011); L. A. Rios, R. M. O. Galvão, and L. Cirto, ibid. 19, 034701 (2012); L. J. L. Cirto, V. R. V. Assis, and C. Tsallis, Physica A 393, 286 (2014).

[24] J. S. Andrade Jr., G. F. T. da Silva, A. A. Moreira, F. D. Nobre, and E. M. F. Curado, Phys. Rev. Lett. 105, 260601 (2010); M. S. Ribeiro, F. D. Nobre and E. M. F. Curado, Eur. Phys. J. B 85, 399 (2012); Phys. Rev. E 85, 021146 (2012); E. M. F. Curado, A. M. C. Souza, F. D. Nobre, and R. F. S. Andrade, ibid. 89, 022117 (2014).

[25] D. B. Walton and J. Rafelski, Phys. Rev. Lett. 84, 31 (2000).

[26] G. Wilk and Z. Włodarczyk, Phys. Lett. B 727, 163 (2013).

[27] G. Wilk and Z. Włodarczyk, Physica A 413, 53 (2014).

[28] G. Wilk and Z. Włodarczyk, ArXiv:1403.3508 [hep-ph].

[29] C. Tsallis, L. R. da Silva, R. S. Mendes, R. O. Vallesjos, and A. M. Mariz, Phys. Rev. E 56, R4922 (1997); L. R. da Silva, R. O. Vallesjos, C. Tsallis, R. S. Mendes, and S. Roux, ibid. 64, 011104 (2001).

[30] D. Sornette, Phys. Rep. 297, 239 (1998).