Infrared divergence on the light-front and dynamical Higgs mechanism

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Abstract

Dynamical Higgs mechanism on the light-front (LF) is studied using a \((1 + 1)\)–dimensional model, with emphasis on the infrared divergence problem. The consideration of the zero mode \(k^+ = 0\) is not sufficient for investigating dynamical symmetry breaking on the LF. It also needs to treat properly an infrared divergence caused by internal momentum \(p^+ \rightarrow 0\) \((p^+ \neq 0)\) in the continuum limit. In order to avoid the divergence, we introduce an infrared cutoff function \(F_{IR}(p, \Lambda)\) which is not Lorentz invariant. It is then shown that the gauge boson obtains mass dynamically on the LF.

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1 Introduction

In the light-front (LF) quantized field theory \([1]\), LF momentum \(k^+ \equiv (k^0 + k^1)/\sqrt{2}\) can take only the positive semi-definite value \(k^+ \geq 0\). This property brings attractive features to the LF theory, one of which is the simplicity of a vacuum state. If particles are all massive, the Fock vacuum is the ground state of the system.

The problem of how spontaneous symmetry breaking (SSB) can take place on the LF with such a simple vacuum state has been discussed [2-14], and it has been revealed that the zero mode \((k^+ = 0)\) is responsible for SSB in many cases. There are two types of zero mode, one is the constraint zero mode [15], the other is the dynamical zero mode [16]. In order to treat the zero mode clearly, it is convenient to put the system in a ”box” with finite size, \(-L \leq x^- \leq L\), and finally take the continuum limit \(L \to \infty\) [15]. By solving the zero mode constraint with approximations, it is shown that SSB occurs on the LF in simple models.

However, there are few investigations of the Higgs mechanism on the LF. Therefore, in this paper, we study dynamical Higgs mechanism on the LF using a \((1 + 1)\)–dimensional model once proposed by Gross and Neveu [17]. In this model, SSB occurs by quantum effect (fermion’s one loop) and the gauge boson obtains mass dynamically in the large \(N\) limit. The model has the merit that the zero mode constraint, to leading order in \(1/N\), contains no operator part, so that we can solve it to leading order without worrying about operator ordering.

While the zero mode is important to describe such dynamical SSB on the LF, it is not enough. In addition, we must analyze carefully an infrared divergence caused by internal momentum \(p^+ \to 0\) \((p^+ \neq 0)\) in the continuum limit \(L \to \infty\) [2, 3, 5, 11, 12, 14, 15], which divergence is peculiar to the LF. In our model, such infrared divergences \(p^+ \to 0\) appear in some one-loop fermion’s diagrams, even though massive fermion. In order to avoid the divergence, we introduce an infrared cutoff function \(F_{IR}(p, \Lambda)\) which is not Lorentz invariant. By use of \(F_{IR}\), we show that the gauge boson obtains mass dynamically on the LF.

Here we neglect the dynamical zero mode of the gauge field and the winding number of a complex scalar field. Instead, without these, we investigate dynamical Higgs mech-
anism by considering the constraint zero modes with careful treatment of the infrared divergence on the LF. If instantons or $\theta$ vacuum existed in a model, the dynamical zero mode would play an important role [19, 20]. This problem will be discussed elsewhere in the $(1 + 1)$–dimensional Abelian-Higgs model on the LF [21].

2 The model and zero mode constraint

Once Gross and Neveu proposed a $(1 + 1)$–dimensional model

$$\mathcal{L} = \sum_{a=1}^{N} \bar{\psi}^a (i\partial + e B \gamma_5) \psi^a - \frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu) (\partial^\mu B^\nu - \partial^\nu B^\mu)$$

$$- \frac{N}{2\lambda} (\sigma^2 + \pi^2) - \sum_{a} \bar{\psi}^a (\sigma + i\gamma_5 \pi) \psi^a, \quad (\mu = 0, 1) \quad (1)$$

which exhibits dynamical Higgs mechanism in the large $N$ limit [17]. $\psi^a$ is an $N$–component massless fermion, $B_\mu$ is a $U(1)$ gauge field, $\sigma$ is a scalar and $\pi$ is a pseudoscalar field. A bare coupling constant $\lambda$ is of order $O(N^0)$, and $e$ is $O(1/\sqrt{N})$. This lagrangian (1) is invariant under the local chiral gauge transformations,

$$\psi^a \rightarrow \exp \{ i\gamma_5 \theta(x) \} \psi^a,$$

$$B_\mu \rightarrow B_\mu + \frac{1}{e} \partial_\mu \theta(x),$$

$$\begin{pmatrix} \sigma \\ \pi \end{pmatrix} \rightarrow \begin{pmatrix} \cos 2\theta(x) & \sin 2\theta(x) \\ -\sin 2\theta(x) & \cos 2\theta(x) \end{pmatrix} \begin{pmatrix} \sigma \\ \pi \end{pmatrix}. \quad (2)$$

Before discussing the LF formalism, we will see briefly how the gauge boson $B_\mu$ obtains mass dynamically in the equal-time quantization formalism. Our interest is now the condensation of $\sigma$ or $\pi$, so we integrate (1) over the fermion field $\psi^a$

$$\mathcal{L} = -\frac{1}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu) (\partial^\mu B^\nu - \partial^\nu B^\mu) - \frac{N}{2\lambda} (\sigma^2 + \pi^2)$$

$$- iN \text{ Tr log } (i\partial - \sigma - i\gamma_5 \pi + e B \gamma_5) \quad (3)$$

To leading order in $1/N$, the effective potential of $\sigma$ and $\pi$ becomes double well type due to fermion one loop correction. Hence $\sigma$ field has a vacuum expectation value $\nu$, and $\pi$ field is now a would-be-Goldstone boson. The massless gauge field $B_\mu$ combines with this $\pi$ field to become a massive vector field with mass $e\sqrt{N}/\sqrt{\pi}$ [17].
Now, within the framework of the LF formalism, we study the lagrangian \((3)\) in which the fermion field has been integrated. This is because the condensation of \(\sigma\) or \(\pi\) is of interest when investigating the Higgs mechanism in our model. If the system is put in a “box” (i.e. \(-L \leq x^- \leq L\)), momentum \(k^+\) is discretized and one can isolate the zero mode clearly \([15]\). We use such a discretization method for the simple reason that the zero mode should be separated from other modes in order to examine SSB on the LF. Therefore, it should be taken into account that the continuum limit \(L \to \infty\) is taken after calculations are over. Boundary conditions of \(\sigma(x), \pi(x)\) and \(B_\mu(x)\) are chosen to be periodic.

Let us parametrize the fields \((\sigma, \pi)\) in polar variables \((\xi, \eta)\) such that \(\sigma + i\pi = \xi e^{i\eta}\), and furthermore rescale the gauge field \(B_\mu\) and the coupling constant \(e\) as \(B_\mu \to \sqrt{N}B_\mu\) and \(e \to e/\sqrt{N}\). Consequently, the lagrangian is rewritten as

\[
L = -\frac{N}{4} (\partial_\mu B_\nu - \partial_\nu B_\mu)(\partial^\mu B^\nu - \partial^\nu B^\mu) - \frac{N}{2\lambda} \xi^2 \\
- iN \text{ Tr log} (i\partial - \xi \exp(i\gamma_5 \eta) + eB\gamma_5)
\]

(4)

where \(e \sim O(N^0)\).

Hereafter, we shall derive the zero mode constraints for \(\xi(x)\) and \(\eta(x)\) to leading order in \(1/\sqrt{N}\). As mentioned in Section 1, we neglect the dynamical zero mode of the gauge field \(B_\mu\) and the winding number \(2\pi m = \eta(L) - \eta(-L)\) of the phase field. Hence, the polar variables \(\xi(x)\) and \(\eta(x)\) are also taken to satisfy periodic boundary conditions. Separate c-number parts \(\xi_0\) and \(\eta_0\) of the zero mode \((k^+ = 0)\) of \(\xi(x)\) and \(\eta(x)\), respectively,

\[
\xi(x) = \xi_0 + \tilde{\xi}(x), \\
\eta(x) = \eta_0 + \tilde{\eta}(x).
\]

(5)

Note that \(\tilde{\xi}(x)\) and \(\tilde{\eta}(x)\) contain both operator valued zero mode and oscillation modes. With the equation \((3)\), the Euler-Lagrange equation for \(\xi\) is expressed as \([3, 12]\)

\[
0 = \frac{N}{\lambda} \left( \xi - iN \text{ Tr exp}(i\gamma_5 \eta) \left\{ i\partial - \xi \exp(i\gamma_5 \eta) + eB\gamma_5 \right\}^{-1} \\
- \frac{N}{\lambda} \left( \xi_0 + \tilde{\xi}(x) \right) - iN \text{ Tr exp}(i\gamma_5 (\eta_0 + \tilde{\eta}))
\]
\[
\times \{ S_0 + S_0 \left[ \exp(i\gamma_5\eta_0)\{\xi_0\exp(i\gamma_5\tilde{\eta}) - 1\} + \tilde{\xi}\exp(i\gamma_5\tilde{\eta}) - eB_5 \right] S_0 \\
+ S_0 [\cdots] S_0 [\cdots] S_0 + \cdots \} ,
\]
where \( S_0(x, y) \) is defined as
\[
S_0(x, y) \equiv (i\partial - \xi_0 \exp(i\gamma_5\eta_0) + i\epsilon)^{-1} .
\]

After substituting (3) into the lagrangian (4), one can easily find that the \( \tilde{\xi}(x) \) propagator is proportional to \( 1/N \). Accordingly, the order of \( \tilde{\xi}(x) \) is \( O(1/\sqrt{N}) \) at most, and we expand the field in terms of \( 1/\sqrt{N} \) as [6, 12]
\[
\tilde{\xi}(x) = \tilde{\xi}^{(1)}(x) + \tilde{\xi}^{(2)}(x) + \cdots ,
\]
where the order of \( \tilde{\xi}^{(1)}(x) \) is \( 1/\sqrt{N} \), the order of \( \tilde{\xi}^{(2)}(x) \) is \( 1/N \), and so on. For the same reason, \( \tilde{\eta}(x) \) and \( B_\mu(x) \) are expanded in terms of \( 1/\sqrt{N} \) as
\[
\tilde{\eta}(x) = \tilde{\eta}^{(1)}(x) + \tilde{\eta}^{(2)}(x) + \cdots , \\
B_\mu(x) = \tilde{B}_\mu^{(1)}(x) + \tilde{B}_\mu^{(2)}(x) + \cdots .
\]
By use of (8) and (9), the Euler-Lagrange equation for \( \xi \) expanded in terms of \( 1/\sqrt{N} \) is obtained [6, 12].

Zero mode constraint for \( \xi \) is derived from the Euler-Lagrange equation (3) by integrating over \( x^- \) [4]. To leading order in \( 1/\sqrt{N} \), this becomes [6, 12]
\[
\int_{-L}^{L} dx^- \left[ N \frac{\xi_0}{\lambda} - iN \text{tr} \exp(i\gamma_5\eta_0) S_0(x, x) \right] = (2L)N \left[ \frac{\xi_0}{\lambda} - i \text{tr} \exp(i\gamma_5\eta_0) S_0(x, x) \right] = 0 .
\]
In the same manner, to leading order, zero mode constraint for \( \eta \) is
\[
\text{tr} i \gamma_5 \xi_0 \exp(i\gamma_5\eta_0) S_0(x, x) = 0 .
\]

3 Infrared divergence on the light-front

Since \( S_0 \), (7), can be regarded as a fermion’s propagator, momentum in \( S_0 \) is discretized such as \( p_n^+ = \pi n/L \ (n = \pm 1/2, \pm 3/2, \cdots) \) where there is no zero mode. This
corresponds to the choice of the antiperiodic boundary condition for the fermion field. The zero mode constraint for $\xi$ (10) is then given by

$$\frac{\xi_0}{\lambda} = 2i \xi_0 \left( \frac{1}{2L} \sum_{n = \frac{1}{2}, \frac{3}{2}, \ldots} \right) \int_{-\infty}^{\infty} \frac{dp^-}{2\pi} \frac{1}{2p_n^+p^- - \xi_0^2 + i\epsilon} , \quad (12)$$

where internal momentum $p_n^+$ does not have zero mode. The r.h.s corresponds to the one loop diagram of the fermion with mass $\xi_0$ in the equal-time quantization, but we should compute it with the LF metric.

If we naively calculate the $p^-$ integration in (12) using a Lorentz invariant cutoff function such as $\theta(\Lambda^2 - |2p_n^+p^-|)$, information of mass $\xi_0$ is lost. Furthermore, after the continuum limit $L \to \infty$, there arises an infrared divergence $p^+ \to 0 (p^+ \neq 0)$ (13),

$$\frac{1}{2L} \sum_{n = \pm \frac{1}{2}, \pm \frac{3}{2}, \ldots} \int_{-\infty}^{\infty} \frac{dp^-}{2\pi} \frac{1}{2p_n^+p^- - \xi_0^2 + i\epsilon} \theta(\Lambda^2 - |2p_n^+p^-|)$$

where a regularization of an ultraviolet divergence $p_n^+ \leq \Lambda$ has been done. These unfavorable points (i) a loss of mass information, and (ii) the infrared divergence $p^+ \to 0 (p^+ \neq 0)$, are already known as fundamental problems in the continuum theory of the LF formalism [18, 8]. We will comment here that these are common problems on the LF and have nothing to do with SSB itself.

The renormalization of the infrared divergence was studied by Thies and Ohta [7] in the chirally invariant Gross-Neveu model on the LF with no gauge field ($e = 0$). They derive the self-consistency condition (the Hartree equation)

$$1 = Ng^2 \frac{1}{2\pi} \int_{\epsilon}^{\Lambda} \frac{dp^+}{p^+} , \quad (14)$$

which is identical to our constraint equation for $\xi$ obtained using the cutoff function $\theta(\Lambda^2 - |2p_n^+p^-|)$ as (13). (Our coupling constant $\lambda$ is related to the coupling constant $g$ in Ref [7] by $\lambda = Ng^2/2$.) The integral in the Hartree equation (14) is regularized by ultraviolet and infrared cutoffs. The ultraviolet divergence is renormalized to an effective coupling constant $g_{\text{eff}}$. The Hartree equation (14) is regarded as the infrared
renormalization condition for the effective coupling constant $g_{\text{eff}}$. It should be noted that the ultraviolet cutoff $\Lambda$ and infrared cutoff $\epsilon$ are introduced independently. Thies and Ohta show that physical quantities do not depend on these cutoffs by use of the infrared renormalization condition (the Hartree equation).

If one, however, takes notice of the first problem (i) the loss of mass information, another management of the infrared divergence will be possible. Namely, the constraint (12) can be regarded as the gap equation having a nontrivial solution, while it is regarded as the infrared renormalization condition in the work by Thies and Ohta. We now consider the infrared cutoff function by which one can interpret (12) as the gap equation. As discussed previously, the Lorentz invariant cutoff function $\theta(\Lambda^2 - |2p^+_n p^-|)$ can not prevent the infrared divergence. At the pole $p^- = \xi_0^2 / 2p^+$, any Lorentz invariant cutoff function $G(2p^+ p^-, \Lambda)$ can not depend on $p^+$ because $G(2p^+ \times (\xi_0^2 / 2p^+), \Lambda) = G(\xi_0^2, \Lambda)$. Thereby, $G(2p^+ p^-, \Lambda)$ does not restrict the region of internal momentum $p^+$ in the Feynman integral, and there arises the infrared divergence as seen from (13). As long as a Lorentz invariant regularization is used, we can not overcome these difficulties. In order to avoid these, it is almost inevitable to violate the Lorentz invariance [18].

Therefore, we introduce a Lorentz noninvariant infrared cutoff function $F_{\text{IR}}(p, \Lambda)$ such as

$$F_{\text{IR}}(p, \Lambda) = \left\{ \frac{-\Lambda^2}{(p^-)^2 - \Lambda^2 + i\epsilon} \right\}. \quad (15)$$

Since this $F_{\text{IR}}$ cuts off $p^-$ at $\Lambda$, it effectively brings an infrared cutoff $p^+ \sim \xi_0^2 / 2\Lambda$ on mass shell $2p^+ p^- = \xi_0^2$. It is similar to the parity invariant regularization [14, 22] where the ultraviolet and infrared cutoffs are related to each other. Note that the effective infrared cutoff of $p^+$ depends on the mass $\xi_0$, but $F_{\text{IR}}$ (15) does not involve $\xi_0$.

We shall regularize the ultraviolet divergence by the replacement $(p^2 - \xi_0^2 + i\epsilon)^{-1} \rightarrow (p^2 - \xi_0^2 + i\epsilon)^{-1} - (p^2 - \Lambda^2 + i\epsilon)^{-1}$, then a properly regularized expression of the zero mode constraint (12) is

$$\frac{\xi_0}{\lambda} = 2i \xi_0 \left( 1 + \sum_{n=\pm\frac{1}{4}, \pm\frac{3}{4}, \cdots} \right) \int_{-\infty}^{\infty} \frac{dp^-}{2\pi} \left( \frac{1}{2p^+_n p^- - \xi_0^2 + i\epsilon} - \frac{1}{2p^+_n p^- - \Lambda^2 + i\epsilon} \right) \times \left\{ \frac{-\Lambda^2}{(p^-)^2 - \Lambda^2 + i\epsilon} \right\}.$$
where we have taken the continuum limit $L \to \infty$ because of the infrared cutoff function $F_{IR}$. Physical quantities such as a vacuum expectation value $\xi_0$ should not depend on the artificial parameter $L$; indeed, $\xi_0$ has no $L$ dependence in the continuum limit.

This zero mode constraint (16) in the continuum limit is nothing but the gap equation [6, 12]. It has a nontrivial solution

$$\xi_0 = M \exp(1 - \pi/\lambda_R) > 0,$$

where $\lambda_R$ is a renormalized coupling constant and $M$ is a renormalization point [17, 13]. (Our notation $\xi_0$ corresponds to $M_F$ in Ref [17].) Although a trivial solution $\xi_0 = 0$ also exists, it is not suitable because of a tachyon pole of the $\tilde{\xi}(x)$ propagator.

On the other hand, the $\eta$’s zero mode constraint (11) whose l.h.s is always zero allows $\eta_0$ to have any real value. This reflects the fact that all the points on a circle with radius $\xi_0$ in the $(\sigma, \pi)$ plane are physically equivalent due to the chiral invariance of the model. We choose a solution

$$\eta_0 = 0,$$

of the zero mode constraint for simplicity.

In the equal-time quantization formalism, the chiral equivalent vacua are degenerate and one chooses a vacuum state among them. On the other hand, in the LF formalism, a vacuum state is the Fock vacuum and one chooses a solution of the zero mode constraint. Multi-vacuum states in the equal-time formalism correspond to multi-solutions of the zero mode constraint in the LF formalism [4, 5].
4 Dynamical Higgs mechanism

To leading order in $1/\sqrt{N}$, the effective lagrangian involving the gauge field $\tilde{B}_\mu(x)$ and the phase $\tilde{\eta}(x)$ is obtained from the lagrangian (4) with the help of (5), (17) and (18)

$$
\mathcal{L}_{\text{eff}}(\tilde{B}_\mu, \tilde{\eta}) = -\frac{N}{4}(\partial_\mu \tilde{B}_\nu^{(1)} - \partial_\nu \tilde{B}_\mu^{(1)})^2 \\
+ iN \left[ \frac{1}{2} \xi_0 \text{Tr} S \tilde{\eta}^{(1)} \tilde{\eta}^{(1)} + \frac{1}{2} \xi_0^2 \text{Tr}( S i \gamma_5 \tilde{\eta}^{(1)} S i \gamma_5 \tilde{\eta}^{(1)}) \right. \\
- \frac{1}{2} e \xi_0 \text{Tr}( S i \gamma_5 \tilde{\eta}^{(1)} S \tilde{B}^{(1)} \gamma_5 ) - \frac{1}{2} e \xi_0 \text{Tr}( S \tilde{B}^{(1)} \gamma_5 S i \gamma_5 \tilde{\eta}^{(1)} ) \\
+ \frac{1}{2} e^2 \text{Tr}( S \tilde{B}^{(1)} \gamma_5 S \tilde{B}^{(1)} \gamma_5 ) \right] + \text{(constant term)}, \quad (\mu, \nu = +, -) \quad (19)
$$

where $S$ is defined as

$$
S(x, y) \equiv S_0(x, y)|_{\eta_0 = 0} = (i\slashed{\partial} - \xi_0 + i\epsilon)^{-1}. \quad (20)
$$

To derive $\mathcal{L}_{\text{eff}}$, the $1/\sqrt{N}$ expansion (8) and (9) also have been used. One loop Feynman integrals on the LF in (19) are calculated as follows.

$$
\text{Tr} S(x, y) \text{ having both ultraviolet and infrared divergence is similar to the r.h.s of the gap equation (16). It then becomes } (-i/2\pi) \xi_0 \log\Lambda^2/\xi_0^2 \text{ in the continuum limit. Next, Tr}( S i \gamma_5 \tilde{\eta}^{(1)} S i \gamma_5 \tilde{\eta}^{(1)}) \text{ has no infrared divergence but it diverges in the ultraviolet region. We regularize it by the replacement } (\slashed{p} - \xi_0 + i\epsilon)^{-1} \rightarrow (\slashed{p} - \xi_0 + i\epsilon)^{-1} - (\slashed{p} - \Lambda + i\epsilon)^{-1}, \text{ so }
$$

$$
\text{tr} \left( \frac{1}{2L} \sum_{n=\pm \frac{1}{2}, \ldots} \right) \int_{-\infty}^{\infty} \frac{dp^-}{2\pi} \left( \frac{1}{\slashed{p} - \xi_0 + i\epsilon} - \frac{1}{\slashed{p} - \Lambda + i\epsilon} \right) i\gamma_5 \times \left( \frac{1}{\slashed{p} - \slashed{k} - \xi_0 + i\epsilon} - \frac{1}{\slashed{p} - \slashed{k} - \Lambda + i\epsilon} \right) i\gamma_5, \quad (21)
$$

with discretized internal LF momentum $p_n^+ = \pi n/L$ and external LF momentum $k^+ > 0$. Positions of $p^-$ poles of the integrand in (21) depend on the value of momentum $p_n^+$ [23]. (i) $p_n^+ < 0$ ; all poles are in the upper half-plane, (ii) $0 < p_n^+ < k^+$ ; two poles are in the upper half-plane and the other two poles are in the lower half-plane, (iii) $p_n^+ > k^+$ ; all poles are in the lower half-plane. Then, in the continuum limit, (21)
Among four components, only \( S (i \gamma_5 \tilde{\eta}^{(1)} S \tilde{\eta}^{(1)} \gamma_5)\) suffers from no divergence and can be calculated with no problem.

Since the vacuum polarization term \( \text{Tr}(S_i \gamma_5 \tilde{\eta}^{(1)} S \tilde{\eta}^{(1)} \gamma_5) \) appears to diverge ultravioletly, we take the regularization \( \rho \rightarrow \Lambda \), the effective lagrangian \( \mathcal{L}_{\text{eff}} \) as in equation (22) with \( F_{IR} \) as in equation (15). In order to avoid it, we make use of the infrared cutoff function \( F_{IR} \) as in equation (15). In order to avoid it, we make use of the infrared cutoff function

\[
\lim_{L \to \infty} \left( \frac{1}{2L} \sum_{n=\pm \frac{1}{2}, \ldots} \int^\infty_{-\infty} \frac{dp^-}{2\pi} \theta(p^+_n) \theta(k^+ - p^+_n) \right) \left( \frac{1}{p^2 - \xi_0^2 + i\epsilon} \right)
\]

\[
\times \left( \frac{1}{(p - k)^2 - \xi_0^2 + i\epsilon} \right) - \frac{1}{(p - k)^2 - \Lambda^2 + i\epsilon} + O \left( \frac{\log\Lambda}{\Lambda} \right).
\]

The first term which diverges ultravioletly cancels with \( \text{Tr} \ S \) in \( \mathcal{L}_{\text{eff}} \). The term \( \text{Tr}(S_i \gamma_5 \tilde{\eta}^{(1)} S \tilde{\eta}^{(1)} \gamma_5) \) contains infrared divergent terms in the continuum limit,

\[
\frac{1}{2L} \sum_{n=\pm \frac{1}{2}, \ldots} \int^\infty_{-\infty} \frac{dp^-}{2\pi} \left( 2p^- \right)^{\frac{1}{2}} \left( p^2 - \xi_0^2 \right) \left( p^2 - \Lambda^2 \right) \left( p - k \right)^2 - \xi_0^2 + i\epsilon \left( p - k \right)^2 - \Lambda^2 + i\epsilon \right) \]

\[
\times \left( \frac{1}{(p - k)^2 - \xi_0^2 + i\epsilon} - \frac{1}{(p - k)^2 - \Lambda^2 + i\epsilon} \right).
\]

After \( p^- \) integration in (23), one can find there are infrared divergent terms as \( L \to \infty \).

This divergence is the peculiar character of the LF formalism, which is already observed in the gap equation (13). In order to avoid it, we make use of the infrared cutoff function \( F_{IR} \) as in equation (15). In order to avoid it, we make use of the infrared cutoff function

\[
\lim_{L \to \infty} \{ \text{Eq.}(23) \times F_{IR}(p, \Lambda) \}
\]

\[
= \frac{i}{\pi} \frac{k^- k^-}{k^2} + \frac{2i}{\pi} \frac{k^- k^-}{k^2} \frac{(k^2 - 2\xi_0^2)}{k^2(4\xi_0^2 - k^2)} \arctan \sqrt{\frac{k^2}{4\xi_0^2 - k^2}} + O \left( \frac{\log\Lambda}{\Lambda} \right).
\]

Consequently, in the continuum limit \( L \to \infty \) and \( \Lambda \to \infty \), the effective lagrangian \( \mathcal{L}_{\text{eff}} \) in momentum space results

\[
\mathcal{L}_{\text{eff}} / N = \frac{1}{2} \tilde{\eta}^{(1)} \left( k^2 - \frac{\xi_0^2}{2e} \right) \left( \frac{k^2 - \xi_0^2}{\pi} + \frac{k^2 k^2}{k^2} \right) \tilde{\eta}^{(1)}
\]

\[
- e^2 \xi_0^2 \frac{U(k^2)}{k^2} \left( k^\mu \right) \left\{ \tilde{\eta}^{(1)} \frac{i}{2e} \left( k^\mu \right) \tilde{\eta}^{(1)} \right\} \left( -k^\nu \right) \left\{ \tilde{\eta}^{(1)} \frac{i}{2e} \left( -k^\nu \right) \tilde{\eta}^{(1)} \right\}.
\]

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with a function $U(k^2) \equiv 2 \left\{ \pi \sqrt{k^2(4\xi_0^2 - k^2)} \right\}^{-1} \arctan \sqrt{k^2/(4\xi_0^2 - k^2)}$.

We choose the unitary gauge

$$\tilde{A}_\mu^{(1)} \equiv \tilde{B}_\mu^{(1)} + \frac{1}{2e} \partial_\mu \tilde{\eta}^{(1)} ,$$

which is useful to find what are physical particles. This gives the lagrangian

$$L_{\text{eff}}/N = \frac{1}{2} \tilde{A}_\mu^{(1)} \left\{ (k^2 - \frac{e^2}{\pi}) \left( -g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \right) + 2e^2 \xi_0^2 U(k^2) \frac{k^\mu k^\nu}{k^2} \right\} \tilde{A}_\nu^{(1)} .$$

The phase field $\tilde{\eta}^{(1)}$ disappears from $L_{\text{eff}}$. The equation of motion for $\tilde{A}_\mu^{(1)}$ is derived from (27),

$$(\partial^\mu \partial_\mu + \frac{e^2}{\pi}) \tilde{A}_\nu^{(1)} = 0 , \quad \partial^\mu \tilde{A}_\mu^{(1)} = 0 ,$$

where $U(k^2) \neq 0$ has been used.

Thus we have shown that the gauge boson obtains mass $e/\sqrt{\pi}$ (with the original $e$, $e\sqrt{N}/\sqrt{\pi}$ ) dynamically on the LF.

## 5 Summary

To summarize, we studied dynamical Higgs mechanism on the light-front. The complexity of the vacuum is carried by the constraint zero mode. Zero mode constraint for the (pseudo) scalar field is solved by use of the $1/N$ expansion. Its nontrivial solution breaks the symmetry spontaneously. The choice of a vacuum state among degenerated vacua in the equal-time formalism corresponds to the choice of a solution of the zero mode constraint in the LF formalism. With the unitary gauge, it is shown that the gauge field obtains mass dynamically on the LF in the large $N$ limit.

In calculating the massive fermion’s one loop integral, we face the infrared divergence $p^+ \to 0$ ($p^+ \neq 0$) in the continuum limit $L \to \infty$. In order to avoid this infrared divergence, which is the peculiar character on the LF, we have introduced the Lorentz noninvariant cutoff function $F_{\text{IR}}(p, \Lambda) = -\Lambda^2/\{(p^-)^2 - \Lambda^2 + i\epsilon\}$. On the mass shell, this $F_{\text{IR}}$ effectively brings the infrared cutoff $p^+ \sim \xi_0^2/2\Lambda$ depending on the mass $\xi_0$ . In the limit $L \to \infty$ and $\Lambda \to \infty$, the regularized theory recovers the Lorentz invariance as seen from (23).
It is essential that, if a cutoff function is Lorentz invariant, it is difficult to resolve the problems of the loss of mass information and the infrared divergence on the LF. Careful treatment of the infrared problem is necessary for investigating dynamical Higgs mechanism in our model.

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