On the importance of the Bohmian approach for interpreting CP-violation experiments

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Abstract

We argue that the inference of CP violation in experiments involving the $K^0 - \bar{K}^0$ system in weak interactions of particle physics is facilitated by the assumption of particle trajectories for the decaying particles and the decay products. A consistent explanation in terms of such trajectories is naturally incorporated within the Bohmian interpretation of quantum mechanics.

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I. Introduction

The Bohm model is able to provide a causal interpretation of quantum mechanics in a consistent manner [1]. At the same time, the predictions of Bohmian mechanics are in exact agreement with the standard quantum mechanical predictions for observable probabilities in all usual experimental situations. In this paper we shall be concerned with examining the possible importance of the Bohmian approach in interpreting certain experiments whose understanding in terms of the standard interpretation is rather ambiguous.

For the purpose of reinterpreting the standard quantum formalism using the Bohmian scheme, a wave function \( \psi \) is not taken to provide a complete specification of the state of an individual system; an additional ontological “position” coordinate (an objectively real “position” existing irrespective of any external observation) is ascribed to an individual particle. The “position” coordinate of the particle evolves with time obeying an equation which can be derived from the Schrödinger equation (considering the one dimensional case)

\[
i\hbar \frac{\partial \psi}{\partial t} = H\psi \equiv -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x)\psi \tag{1}\]

by writing

\[
\psi = Re^{iS/\hbar} \tag{2}
\]

and using the continuity equation

\[
\frac{\partial}{\partial x} (\rho v) + \frac{\partial \rho}{\partial t} = 0 \tag{3}
\]

for the probability distribution \( \rho(x, t) \) given by

\[
\rho = |\psi|^2. \tag{4}
\]

It is important to note that \( \rho \) is ascribed an ontological significance by regarding it as representing the probability density of “particles” occupying actual positions. In contrast, in the standard formulation \( \rho \) is interpreted as the probability density of finding a particle around a certain position. Setting \( (\rho v) \) equal to the quantum probability current leads naturally to the Bohmian interpretation where the particle velocity \( v(x, t) \) is given by

\[
v \equiv \frac{dx}{dt} = \frac{1}{m} \frac{\partial S}{\partial x} \tag{5}
\]

The particle “trajectory” is completely deterministic and is obtained by integrating (5) with the appropriate initial conditions. The essential significance of Bohm’s model lies in providing an elegant solution to the measurement problem (which has been described by Weinberg [2] as
“the most important puzzle in the interpretation of quantum mechanics”) without requiring wave function collapse, since according to the Bohmian interpretation, in any measurement a definite outcome is singled out by the relevant ontological position coordinate.

In view of the importance of the Bohm model in providing not only an internally consistent alternative interpretation of the standard quantum formalism, but also perhaps the neatest solution to the measurement problem [1], it should be worthwhile to look for specific situations where the conceptual superiority of Bohm’s model over the standard interpretation may become easily transparent. To this end, we now proceed to examine the analysis of a fundamentally important experiment of particle physics, namely, the discovery of CP-violation [3].

II. The CP-violation experiment

C(charge conjugation) and P(parity) are two of the fundamental discrete symmetries of nature, the violations of which have not been empirically detected in phenomena other than weak interactions. If a third discrete symmetry T(time reversal) is taken into account, there exists a fundamental theorem of quantum field theory, viz., the CPT theorem which states that all physical processes are invariant under the combined operation of CPT. Nevertheless, there is no theorem forbidding the violation of CP symmetry, and indeed, there have been several experiments to date [4], starting from the pioneering observation of Christenson, Cronin, Fitch and Turlay [3], that have revealed the occurrence of CP violation through weak interactions of particle physics involving the particles $K^0$ and $\bar{K}^0$. The eigenstates of strangeness $K^0$ ($s = +1$) and its CP conjugate $\bar{K}^0$ ($s = -1$) are produced in strong interactions, for example, the decay of $\Phi$ particles. Weak interactions do not conserve strangeness, whereby $K^0$ and $\bar{K}^0$ can mix through intermediate states like $2\pi$, $3\pi$, $\pi\mu\nu$, $\pi\epsilon\nu$, etc. The observable particles, which are the long lived $K$-meson $K_L$, and the short lived one $K_S$, are linear superpositions of $K^0$ and $\bar{K}^0$, i.e.,

$$|K_L⟩ = (p|K^0⟩ - q|\bar{K}^0⟩)/\sqrt{|p|^2 + |q|^2} \quad (6)$$

$$|K_S⟩ = (p|K^0⟩ + q|\bar{K}^0⟩)/\sqrt{|p|^2 + |q|^2} \quad (7)$$

which obey the exponential decay law $|K_L⟩ → |K_L⟩ exp(-\Gamma_L t/2) exp(-i m_L t)$ and analogously for $|K_S⟩$, where $\Gamma_L$ and $m_L$ are the decay width and mass respectively of the $K_L$ particle. It follows from (6) and (7) that

$$⟨K_L|K_S⟩ = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} \quad (8)$$

CP violation takes place if the states $|K_L⟩$ and $|K_S⟩$ are not orthogonal. Through weak interactions the $K_S$ particle decays rapidly into channels such as $K_S → \pi^+\pi^-$ and $K_S → 2\pi^0$ with a mean lifetime of $10^{-10}$ s, whereas, the predominant decay modes of $K_L$ are $K_L → \pi^\pm e^\pm$ (with branching ratio $\sim 39\%$), $K_L → \pi^\pm\mu^\pm\nu$ ($\sim 27\%$), and $K_L → 3\pi$ ($\sim 33\%$) [4]. The CP violating decay mode $K_L → 2\pi$ is extremely rare (with branching ratio $\sim 10^{-3}$) in
the background of the other large decay modes. Considering the Schrödinger evolution, if the analysis of the term corresponding to $K_s$ in the relevant initial wave function shows that it cannot contribute significantly to the emission of two pions with suitable momenta and locations, then one can infer the occurrence of CP violation in this particular situation. In other words such $2\pi$ can only arise through the $K_L$ decay mode. The momenta and locations of the emitted pions are important since the key experimental issue is to detect the $2\pi$ particles coming from the decay of $K_L$ and identify them as coming from $K_L$ and not $K_S$.

In a typical experiment to detect CP violation, an initial state of the type

$$|\psi_i\rangle = (a|K_L\rangle + b|K_S\rangle)$$

is used which is a coherent superposition of the $K_L$ and $K_S$ states. Such a state has been produced by the technique of ‘regeneration’ [5] which has been used in a large number of experiments [6]. The common feature of all these experiments is the measurement of the vector momenta $\vec{p}_i$ of the charged decay products $\pi^+\pi^-$ or $2\pi^0$ from the decaying pions. It is only the type of instrument used for actually measuring the momenta that varies from experiment to experiment.

III. Bohmian trajectories

To see how the Bohmian interpretation helps in drawing the relevant inference from this experiment, we concentrate on the analysis of a single event in which the two emitted pions from a decaying kaon are detected by two detectors respectively along two different directions. From the measured momenta $\vec{p}_1$ and $\vec{p}_2$, the “trajectories” followed by the individual pions are retrodictively inferred assuming that they have followed linear “trajectories”. The point of intersection of these retrodicted “trajectories” is inferred to be the point from which the decay products have emanated from the decaying system; in other words, what is technically known as the “decay vertex” is determined in this way. The value of the momentum of the decaying kaon is obtained by $\vec{p}_k = \vec{p}_1 + \vec{p}_2$. Once the decay vertex and the kaon momentum is known, one estimates the time taken by the kaon to reach the decay vertex from the source, again using at this stage the idea of a linear “trajectory”. If this time turns out to be much larger than the $K_S$ mean lifetime ($\sim 10^{-10}\text{s}$), one infers that the detected $2\pi$ pair must have come from $K_L$, which, as already mentioned, is the signature of CP violation.

It is thus evident from the above discussion that the assumption of a linear “trajectory” of a freely evolving particle (kaon or pion) provides a consistent explanation in support of CP violation in such an experiment. Within the standard interpretation of quantum mechanics, there is no way one can justify assigning a “trajectory” to a freely evolving particle. Moreover, assuming such a “trajectory” to be linear is an additional ad hoc input. One possible argument could be to assign localized wave packets to emitted pions and kaons, and to use the fact that their peaks follow classical trajectories in the case of a free evolution. However, in the standard quantum mechanical description of decay processes, the decay products are regarded as asymptotically free, and hence should be represented by plane wave states. Moreover, even
if they are approximated in some sense by localized wave packets, there would be inevitable
spreading of the wave packets. Even if this spreading is regarded as negligible within the time
interval concerned, a ‘literal identification’ of the wave packet with the particle is conceptually
impermissible without an additional input at the fundamental level in the form of the notion
of a “particle” with a definite position even when unobserved (“particle” ontology).

On the other hand, the assumption of linear “trajectories” followed by the decaying particles
and the decay products is amenable to a natural explanation within the Bohmian framework.
The decaying kaons as well as the asymptotically free decay products are represented by plane
waves
\[ \psi \sim e^{ikx}. \]  (10)

Hence it follows that in the Bohmian scheme the velocity equation (5) is in this case given by
\[ \frac{dx}{dt} = \frac{\hbar k}{m} \]  (11)

which when integrated provides the linear “trajectories” of the particles. These trajectories are
ontological and deterministic. Therefore, in this interpretation, the exact position coordinates
of the “decay vertex” can be assigned in a natural way by retrodicting the pion “trajectories”
without any inconsistencies of the type inherent in the standard interpretation. Hence, it seems
necessary that the standard formalism of quantum mechanics needs to be supplemented with
the Bohmian interpretation of ontological particle “trajectory” (in the sense that the particle
has traversed a well defined path even when unobserved) to enable for the consistent inference
of the observation of CP violation in the actual experiments involving kaon decays.

IV. Concluding remarks

The main reasons for choosing, in particular, the CP violation experiment for this purpose
are the following. First, unlike other common high energy experiments this particular experiment
involves not merely the measurement of some physical quantities but inferring from the
measured quantities the violation of a fundamental symmetry property of the pertinent physical
interactions. Secondly, again unlike other common high energy experiments, the effects of
particle creation and annihilation are not relevant for the important part of the experiment
involved with the prediction of CP violation, and no second quantized treatment is required
for the theoretical framework. The crucial phenomena of particle decays which this experiment
is concerned with, is appropriately described in terms of the Schrodinger equation (see [4] and
references therein) for which there exists a consistent Bohmian interpretation. Note that ignor-
ing interpretational nuances, if one tries to follow a very pragmatic approach and approximates
the plane wave states of the decay products by wave packets whose peaks follow classical tra-
jectories with finite speeds, careful estimates need to be done to quantify the resulting errors
or fluctuations due to spreading of wave packets by taking into account the actual distances
involved in the performed experiments. (Of course, the estimates of these distances related to

5
the particle trajectories are fundamental from the Bohmian perspective.) This is important because the CP violation effect is exceedingly small; the branching ratio of the CP violating decay mode $K_L \rightarrow 2\pi$ is $10^{-3}$. In none of the CP violation experiments performed to date has this point been considered in the relevant analysis.

We conclude by noting that this analysis suggests that it should be worthwhile to look for more such appropriate examples where the inadequacy or ambiguity of the standard formalism in comprehending the results of the concerned experiments can be avoided by using the Bohmian interpretation. It should be appreciated that since there is no measurement problem in the Bohmian interpretation, a Bohmian analysis is useful for all experiments in quantum mechanics, and in particular scattering experiments where it is required to know why particles are detected where they are at the end of the experiment. The answer to this is clear from the Bohmian perspective—the particles are detected where they actually are. However, from the viewpoint of the standard interpretation the explanation is rather obscure, as long as the Schrodinger wave function is regarded as the complete description of the physical system. In this context it has been recently argued [7] that the concept of quantum probability current, a full understanding of which is provided by Bohmian mechanics, is fundamental for a genuine understanding of scattering phenomena. Apart from this, it has been claimed [8] that a special significance of Bohmian mechanics lies in experiments related to the measurement of time of flight of particles, and tunnelling time in particular for which it is difficult to find a consistent or unambiguous definition within the standard framework of quantum mechanics. All this is of course different from empirically verifying a new consequence, if any exists, of the Bohmian interpretation which is not obtainable from the standard interpretation. Nevertheless, such investigations like the one reported in this paper could be helpful in understanding more clearly the relative merits of the standard and Bohmian interpretations.

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