Isospin effects on rotational flow in intermediate energy heavy ion collisions

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Within the framework of an isospin-dependent quantum molecular dynamics model, the rotational flow in reactions of 58Fe + 58Fe and 58Ni + 58Ni at 40 MeV/nucleon for different impact parameters is investigated by analyzing the mid-rapidity azimuthal distribution. The rotational observables are also calculated semiquantitatively. For the first time, it is found that the more neutron-rich system (58Fe + 58Fe) exhibits stronger rotational collective flow. This isospin dependence of rotational collective flow is more appreciable in semi-peripheral collisions and it is shown to mainly result from the isospin dependence of nucleon-nucleon cross section rather than the symmetry energy. Meanwhile, it is indicated that the rotational flow depends strongly on the impact parameter.

PACS number(s): 25.70.-z, 25.75.Ld, 02.70.Ns

The azimuthal distribution in heavy ion collisions (HIC’s) has proved very useful in the study of nuclear equation of state (EOS) as well as reaction dynamics because of its sensitivity to collective flow [1]. In particular, the rotational flow may still exist even though the directed flow has disappeared [2]. With the recent advance in radioactive nuclear beam physics, one can investigate isospin degrees of freedom in nuclear reactions at wide energy ranges for different projectile-target combinations [3,4]. Therefore, it is very significant to explore the isospin effects on the azimuthal distribution through studying HIC’s induced by the radioactive nuclei since it may provide people a better probe of isospin dependent reaction dynamics.

We report here results of the first theoretical study on the isospin dependence of the azimuthal asymmetry rotational flow from reactions of 58Fe + 58Fe and 58Ni + 58Ni at 40 MeV/nucleon for different impact parameters within the framework of an isospin dependent quantum molecular dynamics (IQMD) model which includes the symmetry energy, Coulomb interaction, isospin-dependent experimental N-N cross sections, and particularly the isospin dependent Pauli blocking [4,5]. In the present calculations, the so-called soft EOS with an incompressibility of K = 200 MeV is used and the symmetry strength C = 32 MeV without particular consideration [6]. In the initialization process of the IQMD model, the neutron and proton are distinguished from each other [4] and meanwhile the nonphysical rotations in the initialized nuclei have been removed.

In the IQMD model simulation, the reaction plane is known a priori and it is defined as the x-z plane (z-axis corresponds to the beam direction). The azimuthal angle with respect to the reaction plane can be written as

$$\phi = \arctan \left( \frac{P_y}{P_x} \right).$$  \hspace{1cm} (1)

In the present calculations, the calculated results have adopted the average values from 200 to 300 fm/c to accumulate the numerical statistics since the azimuthal distributions have been well stable after 200 fm/c and 200 events are simulated for each impact parameter. Fig. 1 displays the IQMD model predicted normalized azimuthal distribution dN/d\phi for all nucleons from 58Fe + 58Fe (solid circles) and 58Ni + 58Ni (open circles) at 40 MeV/nucleon and impact parameter b = 6 fm. Figures 1 (a), (b) and (c) correspond to the target-like rapidity region, mid-rapidity region and projectile-like rapidity region, namely, the values of reduced center-of-mass (c.m.) rapidity (y/y_{proj})_{c.m.} of nucleons considered belong to [−1.5, −0.5], [−0.5, 0.5] and [0.5, 1.5], respectively. The errors shown are statistical. Also included in Fig. 1 are the Legendre polynomial fits up to the second order for the resulting azimuthal distribution, i.e.,
\[ dN/d\phi = c(1 + a_1 \cos(\phi) + a_2 \cos(2\phi)). \]  

The solid (dashed) line is the result of fit for $^{58}\text{Fe} + ^{58}\text{Fe}$ ($^{58}\text{Ni} + ^{58}\text{Ni}$). In Eq. (2) the coefficient $a_1$ represents the strength of the directed flow with preferred emission at $\phi = \pm 180^\circ$ (i.e., $a_1 < 0$) for low rapidity values (in the backward hemisphere) and $\phi = 0^\circ$ (i.e., $a_1 > 0$) for high rapidity values (in the forward hemisphere) if the incident energy is above the balance energy. The coefficient $a_2$ reflects a flattening of the ellipsoid. A negative value of $a_2$ (i.e., the azimuthal distribution peaks at $\phi = \pm 90^\circ$, simultaneously) reflects the squeeze-out effects and a positive value (i.e., the azimuthal distribution peaks at $\phi = 0^\circ$ and $\pm 180^\circ$, simultaneously) the rotational collective motion. One can see from Fig. 1 that for low (target-like) rapidity region the azimuthal distribution only peaks at $\phi = 0^\circ$ while it only peaks at $\pm 180^\circ$ for high (projectile-like) rapidity region, which imply that these reaction systems display negative deflections (corresponding to negative scattering angles) since the incident energy is below the balance energy.  

It is indicated in Fig. 1 (b) that the azimuthal distribution peaks at $\phi = 0^\circ$ and $\pm 180^\circ$ simultaneously for mid-rapidity nucleons, which are indicative of a rotation-like behavior. The strength of the rotational flow, $a_2$, equals to 0.20151 and 0.18047 for $^{58}\text{Fe} + ^{58}\text{Fe}$ and $^{58}\text{Ni} + ^{58}\text{Ni}$, respectively. In order to observe the impact parameter dependence of rotational flow and explore its isospin dependence at different impact parameters, we show values of $a_2$ for mid-rapidity nucleons in Fig. 2 as a function of impact parameter $b$ for $^{58}\text{Fe} + ^{58}\text{Fe}$ (solid circles) and $^{58}\text{Ni} + ^{58}\text{Ni}$ (open circles) at 40 MeV/nucleon. The errors shown are statistical. A strong isospin dependence is observed in Fig. 2, namely, the more neutron-rich system $^{58}\text{Fe} + ^{58}\text{Fe}$ displays stronger collective flow than the system $^{58}\text{Ni} + ^{58}\text{Ni}$, especially in the semi-peripheral collisions ($b = 6$ fm). In addition, one can see that the rotational flow increases with increment of impact parameter $b$ from near central to peripheral collisions and then decreases, and finally disappears in the most peripheral collisions since at such large impact parameter the interaction between the projectile and target is very weak.  

In order to see more clearly the rotational flow and its isospin dependence, one can calculate some characteristic physical quantities of the rotation for mid-rapidity nucleons, such as the angular momentum, the moment of inertia, the angular velocity and its rotational energy, with the help of classical mechanics. In the present IQMD model calculations, we simply consider the nucleon system as a rigid body at every time point and then the angular momentum about the normal of the reaction plane (y-axis) can be calculated as follows,

\[ L_y = \sum L_y(i) = \sum (z_i p_{x_i} - x_i p_{z_i}), \]  

where $x_i$, $z_i$, $p_{x_i}$, $p_{z_i}$ are components on the reaction plane of $\mathbf{r}_i$ and $\mathbf{p}_i$ in the c.m. system. The moment of inertia about the y-axis can be written as

\[ I_y = \sum I_y(i) = \sum m_i (r_i^2 - y_i^2). \]  

Therefore, the angular velocity around the y-axis can be expressed as

\[ \omega_y = L_y / I_y, \]  

and its collective rotational energy as

\[ E_{\text{rot}} = \frac{1}{2} I_y \omega_y^2. \]  

Nevertheless, the semiquantitative description above may provide us some intuitive information about the in-plane collective rotational behavior although it is classical and very rough for HIC’s at intermediate energies. Fig. 3 illustrates the time evolution of the absolute value of angular momentum per nucleon $|L_y/A|$, moment of inertia per nucleon $I_y/A$, absolute value of angular velocity $|\omega_y|$, and rotational energy per nucleon $E_{\text{rot}}/A$ from $^{58}\text{Fe} + ^{58}\text{Fe}$ (solid circles) and $^{58}\text{Ni} + ^{58}\text{Ni}$ (open circles) at 40 MeV/nucleon and $b = 4$ fm for mid-rapidity nucleons. The errors shown are statistical. Meanwhile, also included in Fig. 3 (a) are the time evolution of the absolute value of angular momentum per nucleon around the y-axis for the whole system of reactions $^{58}\text{Fe} + ^{58}\text{Fe}$ (solid squares) and $^{58}\text{Ni} + ^{58}\text{Ni}$ (open squares) and its amplified window figure for clarity, which indicate that the conservation of
angular momentum is destroyed slightly, namely, the angular momentum per nucleon for $^{58}\text{Fe} + ^{58}\text{Fe}$ ($^{58}\text{Ni} + ^{58}\text{Ni}$) changes slightly, i.e., from 1.388 (1.388) h/nucleon to 1.348 (1.343) h/nucleon in the whole reaction process, and meanwhile the isospin dependence and the statistical errors are very small. The values for $^{58}\text{Ni} + ^{58}\text{Ni}$ have been offset with 2 fm/c in the horizontal direction for clarity. In fact, both $L_y/A$ and $\omega_y$ are negative by Eqs. (3) and (5) since the reactions display negative deflections. From Fig. 3 (a), one can see clearly that the increment of angular momentum becomes more slowly after about 125 fm/c and tends to its asymptotic value after a longer time when the reaction system may have not any further interaction. The moment of inertia reaches its minimum at about 40 fm/c when the reaction systems reach their maximum overlap and then increases with the reaction systems expanding. The angular velocity peaks at about 50 fm/c when the system has just undergone the strongest exchanges of all kinds of degrees of freedom and then becomes saturate after about 200 fm/c. Similarly, $E_{rot}/A$ has maximum at about 100 fm/c and then becomes saturate after about 200 fm/c. From above analysis, one can conclude that there actually exists in-plane collective rotational motion for mid-rapidity nucleons at such incident energy and impact parameter. The similar results are observed for other impact parameters. More importantly here, one can find from Fig. 3 that the more neutron-rich system $^{58}\text{Fe} + ^{58}\text{Fe}$ exhibits clearly stronger rotational energy and angular momentum than system $^{58}\text{Ni} + ^{58}\text{Ni}$, which confirms again the conclusion obtained in Fig. 2. Meanwhile, these semi-quantitative calculations also support the view about extracting the rotational flow from the azimuthal distribution analysis.

The isospin dependence of the rotational flow may be a result of the competition among several mechanisms in the reaction dynamics, such as the symmetry energy, isospin dependent N-N cross sections, Coulomb energy, the surface properties of the colliding nuclei, and so on. Here, we investigate the influence of the symmetry energy and isospin dependent N-N cross section on the rotational flow. Using different symmetry energy strength $C$ and parametrizations of N-N cross sections, we show in Fig. 4 the IQMD model predicted azimuthal distribution from $^{58}\text{Fe} + ^{58}\text{Fe}$ (solid circles) and $^{58}\text{Ni} + ^{58}\text{Ni}$ (open circles) at 40 MeV/nucleon and $b = 4$ fm for mid-rapidity nucleons. The errors shown are statistical. Meanwhile, the results of Legendre polynomial fits according to Eq. (2) for $^{58}\text{Fe} + ^{58}\text{Fe}$ (solid line) and $^{58}\text{Ni} + ^{58}\text{Ni}$ (dashed line) as well as the resulting $a_1$ and $a_2$ are also included in Fig. 4. In Fig. 4 (a) we use $C = 0$ (no symmetry energy) and Cugnon’s N-N cross section $\sigma_{\text{Cug}}$, which is isospin independent. It is indicated that $a_2$ of $^{58}\text{Fe} + ^{58}\text{Fe}$ is slightly less than that of $^{58}\text{Ni} + ^{58}\text{Ni}$, which may result from the difference of Coulomb interaction and surface properties between the two systems. The case of using $C = 32$ MeV and $\sigma_{\text{Cug}}$, is plotted in Fig. 4 (b). For the results shown in Fig. 4 (c) we use $C = 32$ MeV and experimental N-N cross section $\sigma_{\text{exp}}$, which is isospin dependent. One can see from Fig. 4 that the values of $a_1$ are very small since the transverse momentum disappears at mid-rapidity and meanwhile can find that the symmetry energy enhances the rotational flow ($a_2$) while $\sigma_{\text{exp}}$ reduces it more strongly. It is also indicated that the influence of $\sigma_{\text{exp}}$ on system $^{58}\text{Ni} + ^{58}\text{Ni}$ is stronger than that on the system $^{58}\text{Fe} + ^{58}\text{Fe}$, which just results in the observed isospin dependence. This is easy to understand since the neutron-proton cross section is about three times larger than the neutron-neutron or proton-proton cross section for $\sigma_{\text{exp}}$, which results in more N-N collisions for $^{58}\text{Ni} + ^{58}\text{Ni}$. From above analysis, one can conclude that the isospin dependence of rotational flow seems to mainly result from the isospin dependence of N-N cross section and the symmetry potential has little influence on it.

In summary, by using the IQMD model, we studied for the first time the in-plane rotational flow in reactions of $^{58}\text{Fe} + ^{58}\text{Fe}$ and $^{58}\text{Ni} + ^{58}\text{Ni}$ at 40 MeV/nucleon for different impact parameters by analyzing the mid-rapidity azimuthal distribution and calculating semi-quantitatively the rotational observables. A strong isospin dependence of the rotational flow has been found, namely, the more neutron-rich system exhibits stronger rotational flow. This isospin dependence is more appreciable in semi-peripheral collisions and it is shown to mainly result from the isospin dependence of N-N cross section rather than the symmetry energy. Meanwhile, it is indicated that the rotational flow depends strongly on the impact parameter, namely, it increases with increment of impact parameter from near central to peripheral collisions and then decreases, and finally disappears in the most peripheral collisions.

The authors would like to thank W. Q. Shen for interesting discussions. This work was supported by the National Natural Science Foundation of China under Grant Nos. 19609033, 19875068, and 19847002, and the Foundation of the Chinese Academy of Sciences.
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FIGURE CAPTIONS

Fig. 1 The IQMD model predicted normalized azimuthal distribution \( dN/d\phi \) for all nucleons from \( {}^{58}\text{Fe} + {}^{58}\text{Fe} \) (solid circles) and \( {}^{58}\text{Ni} + {}^{58}\text{Ni} \) (open circles) at 40 MeV/nucleon and impact parameter \( b = 6 \) fm for different rapidity regions, i.e., the target-like rapidity region (a), mid-rapidity region (b) and projectile-like rapidity region (c). The solid (dashed) line represents the Legendre polynomial fit up to second order for the calculated result of \( {}^{58}\text{Fe} + {}^{58}\text{Fe} \) (\( {}^{58}\text{Ni} + {}^{58}\text{Ni} \)).

Fig. 2 The values of \( a_2 \) for mid-rapidity nucleons as a function of impact parameter \( b \) for \( {}^{58}\text{Fe} + {}^{58}\text{Fe} \) (solid circles) and \( {}^{58}\text{Ni} + {}^{58}\text{Ni} \) (open circles) at 40 MeV/nucleon. The lines are plotted to guide the eye.

Fig. 3 The time evolution of the absolute value of angular momentum per nucleon \( |L_y/A| \) (a), moment of inertia per nucleon \( I_y/A \) (b), absolute value of angular velocity \( |\omega_y| \) (c), and rotational energy per nucleon \( E_{rot}/A \) (d) from \( {}^{58}\text{Fe} + {}^{58}\text{Fe} \) (solid circles) and \( {}^{58}\text{Ni} + {}^{58}\text{Ni} \) (open circles) at 40 MeV/nucleon and \( b = 4 \) fm for mid-rapidity nucleons. Also included in (a) are the time evolution of the absolute value of angular momentum per nucleon for the whole system of reactions \( {}^{58}\text{Fe} + {}^{58}\text{Fe} \) (solid squares) and \( {}^{58}\text{Ni} + {}^{58}\text{Ni} \) (open squares) and its amplified window figure.

Fig. 4 The IQMD model predicted azimuthal distribution for \( {}^{58}\text{Fe} + {}^{58}\text{Fe} \) (solid circles) and \( {}^{58}\text{Ni} + {}^{58}\text{Ni} \) (open circles) at 40 MeV/nucleon and \( b = 4 \) fm for mid-rapidity nucleons by using different symmetry energy strength \( C \) and parametrizations of \( N-N \) cross sections: \( C = 0 \) (no symmetry energy) with Cugnon’s \( N-N \) cross section \( \sigma_{\text{Cug}} \) (a), \( C = 32 \) (MeV) with \( \sigma_{\text{Cug}} \) (b), and \( C = 32 \) (MeV) with experimental \( N-N \) cross section \( \sigma_{\exp} \) (c). Meanwhile, the results of Legendre polynomial fits according to Eq. (2) for \( {}^{58}\text{Fe} + {}^{58}\text{Fe} \) (solid line) and \( {}^{58}\text{Ni} + {}^{58}\text{Ni} \) (dashed line) as well as the resulting \( a_1 \) and \( a_2 \) are also included.
FIG. 1

(a) (y/y_{proj})_{c.m.}: [-1.5, -0.5]

(b) (y/y_{proj})_{c.m.}: [-0.5, 0.5]

(c) (y/y_{proj})_{c.m.}: [0.5, 1.5]

E=40 MeV/nucleon, b=6 fm

Fe$^{58}$ + Fe$^{58}$

Ni$^{58}$ + Ni$^{58}$
FIG. 2

- $^{58}\text{Fe}^+^{58}\text{Fe}$
- $^{58}\text{Ni}^+^{58}\text{Ni}$

$E = 40\text{ MeV/nucl.}$
$(y/y_{proj\text{.c.m.}} : [-0.5,0.5])$
FIG. 3

58Fe+58Fe
58Ni+58Ni

E=40 MeV/nucleon, b=4 fm
(y/yproj)c.m. : [-0.5,0.5]

|L_y/A| (10^{-3}/nucleon)

|ω|^2 (10^{-3}/fm)

E_{\text{rel}}/A (MeV/nucleon)

time (fm/c)
\[ 58^\text{Fe}^+ \text{Fe}^+: a_1 = 0.01143, a_2 = 0.14593 \]
\[ 58^\text{Ni}^+ \text{Ni}^+: a_1 = -0.00171, a_2 = 0.13634 \]

(a) \( C = 0 \) with \( \sigma_{\text{cross}} \)

\[ 58^\text{Fe}^+ \text{Fe}^+: a_1 = 0.01081, a_2 = 0.16003 \]
\[ 58^\text{Ni}^+ \text{Ni}^+: a_1 = -0.01764, a_2 = 0.16601 \]

(b) \( C = 32 \text{ MeV} \) with \( \sigma_{\text{cross}} \)

\[ 58^\text{Fe}^+ \text{Fe}^+: a_1 = -0.01506, a_2 = 0.17152 \]
\[ 58^\text{Ni}^+ \text{Ni}^+: a_1 = -0.01089, a_2 = 0.17249 \]

(c) \( C = 32 \text{ MeV} \) with \( \sigma_{\text{exp}} \)

\[ 58^\text{Fe}^+ \text{Fe}^+: a_1 = 0.01143, a_2 = 0.14593 \]
\[ 58^\text{Ni}^+ \text{Ni}^+: a_1 = -0.00171, a_2 = 0.13634 \]

FIG. 4