Brans-Dicke gravity: from Higgs physics to (dynamical) dark energy

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Abstract

The Higgs mechanism is one of the central pieces of the Standard Model of electroweak interactions and thanks to it we can generate the masses of the elementary particles. Its fundamental origin is nonetheless unknown. Furthermore, in order to preserve renormalizability we have to break the gauge symmetry spontaneously, what leads to a huge induced cosmological constant incompatible with observations. It turns out that in the context of generalized Brans-Dicke theories of gravity the Higgs potential structure can be motivated from solutions of the field equations which carry harmless cosmological vacuum energy. In addition, the late time cosmic evolution effectively appears like an universe filled with mildly evolving dynamical dark energy mimicking quintessence or phantom dark energy.

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I. HIGGS POTENTIAL AND HARMFUL VACUUM ENERGY

In the ΛCDM, or concordance cosmological model, the measured vacuum energy density at present, \( \rho_{\Lambda 0} = \Lambda / (8\pi G) \), is of order \( 10^{-47} \text{ GeV}^4 \sim (10^{-3}\text{eV})^4 \) in natural units. Here \( \Lambda \) is the cosmological constant (CC) and \( G \) is Newton’s constant. The discovery of the Higgs boson \( \phi \) and the measurement of its mass (\( M_{\phi} \simeq 125 \text{ CeV} \)) implies a value of the electroweak (EW) vacuum energy density of order \( |\langle V_{\phi} \rangle| \sim M_{\phi}^2 v^2 \sim 10^8 \text{ GeV}^4 \), where \( v = \mathcal{O}(200) \text{ GeV} \) is the vacuum expectation value of the Higgs doublet of the standard model. The result is some 55 orders of magnitude higher than \( \rho_{\Lambda 0} \). Such phenomenal mismatch is at the root of the famous “CC problem” \([1–4]\) in the context of the standard model of particle physics.

Thus, paradoxically, while the famous Englert-Brout-Higgs mechanism \([5, 6]\) for spontaneous symmetry breaking (SSB) of the gauge symmetry is very much welcome in pure particle physics, it nevertheless carries a profound and unsolved enigma in the context of gravity and cosmology that seems to clash violently with observations. The problem might be suggesting that the vacuum energy in quantum field theory (QFT) is not the same as the cosmological vacuum energy. The two kind of vacua might be conceptually different and in such case there would be no reason to mix them up. While a solution does not seem possible in General Relativity (GR), it may be feasible in Brans-Dicke (BD) type gravity.

II. GENERALIZED BRANS-DICKE ACTION AND FIELD EQUATIONS

Consider the standard BD-action \([7]\) with BD-field, \( \psi \), coupled to the curvature \( R \):

\[
S_{BD}[\psi] = \int d^4x \sqrt{-g} \left[ \frac{1}{2} R \psi - \frac{\omega}{2\psi} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \right],
\]

where \( \omega \) is the standard BD-parameter \([7]\). Recall that in the limit \( \omega \to \infty \) this action is supposed to reproduce Einstein’s GR. In this theory, the effective gravitational coupling at any time of the cosmic expansion is slowly varying with the cosmic time \( t \), and reads

\[
G_{\text{eff}}(t) = \frac{1}{8\pi \psi(t)}.
\]

Equivalently, the effective value of the (reduced) Planck mass squared at any time is just given by the BD-field: \( \psi(t) = M_P^2(t) \). At \( t = t_0 \) (our time) we have \( M_P(t_0) \equiv M_P = 1/\sqrt{8\pi G} \simeq 2.43 \times 10^{18} \text{ GeV} \), hence \( G_{\text{eff}}(t_0) = G \) is the current value of Newton’s coupling.
Let us extend non-trivially the above action by invoking a new scalar field, $\phi$, which is non-minimally coupled to gravity (both derivatively and non-derivatively) and also coupled with the BD-field. The suggested new piece of the action is

\[
S[\phi, \psi] = \int d^4x \sqrt{-g} \left[ \xi R \phi^2 - \frac{1}{2} g^{\mu \nu} \partial_\mu \phi \partial_\nu \phi + \frac{\varsigma}{\phi^2} G_{\mu \nu} \partial^\mu \phi \partial^\nu \phi + \eta \phi^2 \psi - V(\phi) \right].
\] (3)

Here $G_{\mu \nu} = R_{\mu \nu} - (1/2) g_{\mu \nu} R$ is the Einstein tensor and $\xi, \varsigma, \eta$ are dimensionless coefficients. Derivative interactions ($\varsigma \neq 0$) with gravity have been considered in the literature [8–10]. As we shall see, they are crucial for our purposes. While $\psi(t)$ varies very slowly with the cosmic time, $\phi(t)$ can evolve much faster. To comply with homogeneity and isotropy, none of them should vary with space. The field $\phi$ will effectively behave as the Higgs scalar. The total action that we propose is the sum $S_{\text{tot}} = S_{\text{BD}}[\psi] + S[\phi, \psi] + S_m$, where $S_m$ is the matter action. For $\xi = \varsigma = \eta = 0$ the scalar field part of the action boils down to the sum of two decoupled actions $S_{\text{BD}}[\psi] + S[\phi]$, with $S[\phi]$ minimally coupled to gravity. This case is of course uninteresting.

Let us first explore the early epoch when the electroweak (EW) phase transition occurs. This epoch is characterized by the scalar field dominance and we can neglect $S_m$. Within the (flat) FLRW metric ($ds^2 = -dt^2 + a^2(t) d\mathbf{x}^2$), the variation of the total action, $S_{\text{tot}}$, with respect to the BD-field $\psi$ yields

\[
3\dot{H} + 6H^2 - \omega \frac{\dot{\psi}}{\psi} + 3H \frac{\omega}{2} \frac{\dot{\psi}^2}{\psi^2} = 0,
\] (4)

where $H = \dot{a}/a$ is the Hubble rate. Similarly, the variation with respect to the metric gives the following 00-component

\[
3H^2 \psi + 3H \dot{\psi} - \frac{\omega}{2} \frac{\dot{\psi}^2}{\psi} + \eta \phi^2 \psi - \frac{1}{2} \psi^2 + 6\xi H^2 \phi^2 + 12\xi H \dot{\phi} \phi - 9\varsigma H^2 \dot{\phi}^2 = V(\phi),
\] (5)

and the variation with respect to $\phi$ renders

\[
\ddot{\phi} + 3H \dot{\phi} - 12\xi \dot{H} \phi - 24\xi H^2 \phi + \frac{dV}{d\phi} + 18\xi H^3 \frac{\dot{\phi}}{\phi} + 12\xi H \ddot{\phi} + 2\eta \psi \phi = 0.
\] (6)

Let us search for power-law solutions of the above equations, namely

\[
H = \frac{n}{t} \quad (n > 0), \quad \phi(t) = \phi_1 \left( \frac{t}{t_1} \right)^\alpha, \quad \psi(t) = \psi_1 \left( \frac{t}{t_1} \right)^\gamma.
\] (7)

The first expression is equivalent to $a \propto t^n$. Here $n, \alpha, \gamma$ are dimensionless parameters and $\phi_1$ and $\psi_1$ are the values of the scalar fields at some early time $t = t_1$ in the past when the
power-law solutions hold good, e.g. during the EW phase transition. From its interpretation we must have $\psi_1 > 0$, and $|\gamma| \ll 1$. The above scaling solutions are associated to asymptotic states of the different phases of the cosmic evolution, e.g. the radiation-dominated ($n = 1/2$) and the matter-dominated ($n = 2/3$) epochs, inflation ($n \gg 1$) \[10, 11\].

III. COSMOLOGICALLY HARMLESS HIGGS

One of the above field equations contains the potential $V$ and the other the derivative $dV/d\phi$. The final expression for $V$ can be derived self-consistently by requiring that the field equations determine the same form when we substitute in them the power-law solutions (7). Let us consider arbitrary $n$ values\(^1\). From Eq. (4), which does not depend on $V$, we find

$$
\eta \frac{t^2}{t_1^2} \left( 3n(\gamma + n) - \frac{\omega}{2} \gamma^2 + \eta \frac{\phi^2}{t_1^2} \right) t^{-2} + \left( \frac{\phi^2}{t_1^2} \left[ 6\xi n (n - 2) - \frac{1}{2} \right] - 9n^2 \varsigma \right) t^{-4}
$$

and the field equation for $\phi$ leads to an alternative expression for $V$:

$$
V = -\frac{\psi_1}{t_1^2} \left( 2n \frac{\phi^2}{t_1^2} \frac{\phi^2}{\gamma - 2} \left( 12\xi n (1 - 2n) - 3n + 2 \right) + 9n^2 (1 - n) \varsigma \right) t^{-4}.
$$

In both of them the two powers $t^{-2}$ and $t^{-4}$ are consistently present. When we revert back to $\phi$ through $t/t_1 = \phi_1/\phi$ (using $\alpha = -1$), these powers become associated to $\phi^{2-\gamma}$ and $\phi^4$, respectively. Therefore it is possible to match the coefficients of each power. We obtain the following unique form for the effective potential (up to an additive constant):

$$
V = \frac{2\eta \psi_1}{\gamma - 2} \left( \frac{t}{t_1} \right)^\gamma \phi^2 + \lambda \phi^4 \approx \eta \psi_1 \phi^2 + \lambda \phi^4.
$$

In the second equality we used the fact that $|\gamma| \ll 1$ and thus the coefficient of $\phi^2$ remains essentially constant. The obtained potential is essentially a Higgs-like potential for $\phi$. The coefficient $\lambda$ of the quartic coupling is given by

$$
\lambda = -\frac{3\xi n}{\phi^4 t_1^4 (12\xi + 1)} \left[ 12n\xi (n^2 - n + 1) + 2n - 1 \right].
$$

For instance, for $\xi > 0$ and $\varsigma < 0$ the necessary condition $\lambda > 0$ for vacuum stability is fulfilled both for $n = 1/2$ and $n = 2/3$. The form (10) of the potential tells us that there will be SSB of

\(^1\) For the sake of simplicity, in the original essay I presented the result only for $n = 1/2$ (radiation epoch.)
the EW symmetry provided $\eta < 0$, so this crucial sign is uniquely determined. Note that $|\eta| \ll 1$ because $\psi_1 \sim M^2_P$ and the physical Higgs mass squared is of order $M^2_H \sim |\eta|\psi_1 \sim 10^4 \text{GeV}^4$. The fact that $|\eta|$ must be a very small parameter has nothing to do with fine-tuning. As pointed out long ago by Bjorken, it is instead a well-known feature expected of any theory claiming a possible connection between the gravity scale and the EW scale.

It is also interesting to mention that the consistency conditions leading to the unique expression of the Higgs potential $V$ indicated above imply a noticeable relation between the power $\gamma$ that governs the cosmic time evolution of $\psi$ and the BD-parameter $\omega$. For $\omega \gg n$ it takes the simple form

$$\gamma^2 \omega \simeq 2n.$$ (12)

It follows that $\gamma$ is naturally small for large $\omega$. A counterpart to this relation for the present universe will be elucidated in the next section, which will provide also a connection between $\omega$ and the (different) power-law evolution of the BD-field in the current epoch.

**IV. LATE TIME UNIVERSE AND EFFECTIVE DYNAMICAL DARK ENERGY**

As we have seen, in the present BD-gravity context the standard solutions $H^2 \sim a^{-2/n} (n = 1/2, 2/3)$ for matter and radiation can perfectly support SSB with large EW vacuum energy but essentially zero cosmological vacuum energy. At late time the scalar field $\phi$ becomes suppressed: $\phi = \phi_1 (t_1/t) \to 0$, and with it $V \to 0$. The field equation for $\phi$, Eq. (6), completely decouples. The matter component now becomes relevant and is described as usual by a perfect fluid with proper density $\rho_m$ and pressure $p_m$. Around the present epoch we can just replace the r.h.s. of Eq. (5) by $\rho_m + \rho_\Lambda$ and set $p_m = 0$. Since $\phi \to 0$ we recover the standard BD equation carrying a remnant of constant vacuum energy density $\rho_\Lambda$, which is necessary to match the current observation of a CC term:

$$3H^2 + 3H\frac{\dot{\psi}}{\psi} - \frac{\omega}{2} \frac{\dot{\psi}^2}{\psi^2} = \frac{\rho_m + \rho_\Lambda}{\psi}.$$ (13)

For $\psi = 1/(8\pi G) = \text{const.}$ it reduces to the standard Friedmann’s equation of GR. However, at this stage of the cosmic evolution we need also the $ii$-component of the BD field equations emerging from variating the action with respect to the metric, as this equation is linked to the pressure components. While $p_m$ can be neglected, $p_\Lambda = -\rho_\Lambda$ cannot be neglected anymore. The relevant
pressure equation reads
\[ 2\dot{H} + 3H^2 + \frac{\ddot{\psi}}{\psi} + 2H\frac{\dot{\psi}}{\psi} + \frac{\omega}{2}\frac{\dot{\psi}^2}{\psi^2} = -\frac{1}{\psi}p_{\Lambda}. \] (14)

Correspondingly, for \( \psi = 1/(8\pi G) = \text{const.} \) it boils down to the standard pressure equation in GR. However, \( \psi \) is not exactly constant in our time. It still evolves mildly with the expansion, albeit at a slightly different rate that we can estimate. Once more we seek power-law solutions of the form
\[ \psi = \psi_0 a^{-\epsilon} = M_P^2 a^{-\epsilon}, \] (15)
where we have normalized the scale factor at present to \( a_0 = 1 \). The power \( \epsilon \) is of course small in absolute value and is the analogue in the modern epoch of the power \( \gamma \) used earlier. The relevant BD field equations for the current epoch are the following: Eq. (4) -- dropping \( \phi \) in it -- (13) and (14). Substituting the ansatz (15) in (13) and using the fact that \(|\epsilon| \ll 1\) we can put the result in an approximate \( \Lambda \)CDM-like form, namely \( 3H^2 = 8\pi G \left( \rho_m a^{-3+\epsilon} + \rho_{\Lambda}(H) \right) \), where
\[ \rho_{\Lambda}(H) = \rho_\Lambda + \frac{3\nu}{8\pi G}H^2 \] (16)
and we have defined \( \nu \equiv \epsilon(1 + \omega\epsilon/6) \). Recall that \( 1/G \equiv 1/G(t_0) = 8\pi\psi(t_0) \). The expression (16) emulates an effective dynamical dark energy (DDE), which is induced by the slow dynamics of the BD field. The present physical value is \( \rho_{\Lambda 0} \equiv \rho_{\Lambda}(H_0) = \rho_\Lambda + 3\nu/(8\pi G)H_0^2 \), very close to \( \rho_\Lambda \) \((|\epsilon| \ll 1)\). The anomalous matter conservation law \( \rho_m = \rho_m a^{-3+\epsilon} \) mimics an interaction of matter with the DDE. For \( \epsilon = 0 \) (hence \( \nu = 0 \)) we recover the exact situation of the \( \Lambda \)CDM. Remarkably, for \( \epsilon \neq 0 \) the expression (16) adopts the form of the running vacuum model (RVM), see [4, 14] and references therein. In Ref. [15–19] it was shown that the RVM fits the overall observations better than the \( \Lambda \)CDM.

The product \( \omega\epsilon \) becomes determined by the remaining two BD-equations (4) and (14) upon using the ansatz (15). After a detailed calculation and evaluation at \( a = 1 \) (redshift \( z = 0 \)) one finds
\[ \omega\epsilon = -\frac{4 - 3\Omega_m}{2 - \Omega_m} \simeq -1.8 \quad \text{(for } \Omega_m \simeq 0.3 \text{)}. \] (17)
Thus, \( \omega\epsilon^2 \sim \epsilon \) and the two terms defining \( \nu = \epsilon + \omega\epsilon^2/6 \) are of the same order. The \( \sim \omega\epsilon^2 \) part of (16) is easily seen to originate from the non-canonical kinetic term of the BD-field, cf. Eq. (11), interpreted roughly in the manner of an effective potential contribution to the vacuum dynamics:
\[ \frac{\omega}{\psi} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi \rightarrow \frac{\omega}{\psi} \dot{\psi}^2 \sim \omega\epsilon^2 \psi H^2 \sim \omega\epsilon^2 M_P^2 H^2, \] (18)
where we have used (15). The precise and rigorous contribution, however, appears at the level of the field equations and stems from the third term on the l.h.s of Eq. (13). Together with the second term in that equation (which is linear in $\epsilon$ within the context of the power-law solution we are considering) it leads to the running vacuum form (13) [4, 14]. On comparing (12) with (17) we can see that in both cases the power-law evolution of the BD-field becomes progressively smaller (hence closer to a constant) for larger and larger values of $\omega$, but at different rates: for the current epoch $\epsilon \sim 1/\omega$, whereas in the early universe $\gamma \sim 1/\sqrt{\omega}$.

The above cosmological solution is obviously different from GR and has distinct properties. The pressure equation (14) can also be put in approximate $\Lambda$CDM fashion using (15). Neglecting the small parameter corrections except for those in the dynamical terms, such as $\sim a^{-3+\epsilon}$ and $\sim \nu M_p^2 H^2$, we find the leading expression for the effective acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho_m a^{-3+\epsilon} + \rho_\Lambda (H) + 3p_\Lambda \right).$$

(19)

It follows that the equation of state (EoS) for the effective DDE reads

$$w = \frac{p_\Lambda}{\rho_\Lambda (H)} \approx -1 - \frac{3\nu}{8\pi G \rho_\Lambda H^2} = -1 - \frac{\nu}{\Omega_\Lambda H_0^2}.$$ (20)

Thus, for $\epsilon < 0$ ($\epsilon > 0$) we have $\nu < 0$ ($\nu > 0$) and the effective DDE behaves quintessence (phantom)-like. For $\epsilon = 0$ (i.e. $\nu = 0$) we have $w = -1$ ($\Lambda$CDM) and only then the BD-parameter is forced $|\omega| \rightarrow \infty$ from (17).

Needless to say, matter is locally and covariantly conserved in BD-theory, as there is actually no interaction of matter with the BD-field $\psi$. However, when we try to encapsulate the observational behavior of BD-gravity in a strict GR-form, we find that the former may effectively appear as a model of interactive quintessence or phantom DE. The eventual detection of this kind of “anomalies” in the behavior of the $\Lambda$CDM could be signaling the possible presence of BD-gravity at a more fundamental level. This does not preclude, of course, that other sources of fundamental DE dynamics associated to new fields might also be concomitant with the same physical phenomenon, see e.g. [20]. However, BD-gravity alone does indeed have the capacity to mimic dynamical DE.

Let us finally note that if the tiny time dependence in the coefficient of $\phi^2$ in the first expression of the potential in Eq. (10) had not been neglected, the expectation value of the Higgs field derived from BD-gravity would carry a mild time-evolution, $\langle \phi \rangle \sim t^{\gamma/2}$ ($|\gamma| \ll 1$), which would be transferred to all the particle masses in the universe, a conclusion that has also been reached from alternative considerations in [21–23].
V. CONCLUSIONS

Generalized BD-gravity theories admit cosmologically viable solutions which lead to the precise structure of the Higgs potential. The inherent QFT vacuum energy of the potential is as large as usual but the cosmological vacuum energy density remains very small. The latter emerges only at late times through the usual constant term \( \rho_A \), but accompanied with an additional (albeit dynamical) contribution induced by the slow time evolution of the BD-field \( \psi \). Effectively it leads to \( \Lambda(t)_{\text{CDM}} \), with a moderate time-evolving \( \Lambda(t) = \Lambda(H(t)) \) and EoS very close to \(-1\). More specifically, it mimics the running vacuum model, which has recently been shown to provide a fit to the overall cosmological data better than the ACDM (with rigid cosmological constant \( \Lambda \)). Therefore, finding mild vacuum dynamics, accompanied with unsuspected time-evolving particle masses, could be the “smoking gun” signaling that the underlying gravity theory is not GR but BD. It could provide the missing link between gravity and particle physics.

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