Heavy baryons in hot stellar matter with light nuclei and hypernuclei

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The production of light nuclei and hypernuclei together with heavy baryons, both hyperons and \( \Delta \)-baryons, in low density matter as found in stellar environments such as supernova or binary mergers is studied within relativistic mean-field models. Five light nuclei were considered together with three light hypernuclei. The presence of both hyperons and \( \Delta \)-baryons shift the dissolution of clusters to larger densities and increase the abundance of clusters. This effect is larger the smaller the charge fraction and the higher the temperature. The couplings of the \( \Delta \)-baryons were chosen imposing that the nucleon effective mass remains finite inside neutron stars.

I. INTRODUCTION

Neutron stars (NS) are born in highly energetic events called core-collapse supernovae (CCS). Right after the core-collapse, the proto-neutron star reaches high temperatures of the order of tens of MeV. However, in a matter of a few seconds, neutrons and photons diffuse out of the star, and the star cools down to less than 1 MeV, reaching its ground-state configuration in chemical equilibrium (also known as \( \beta \)-equilibrium) \cite{1}. The star will remain in equilibrium unless it is perturbed by some external phenomena, such as a collision with another NS. In these type of events, both CCS and neutron star mergers (NSM), \( \beta \)-equilibrium is not necessarily achieved, and temperatures as high as 50 to 100 MeV may be attained \cite{2}. At such high temperatures, exotic degrees of freedom such as hyperons and \( \Delta \)-isobars may appear at much lower densities, as compared to the NS case. In fact, a finite temperature allows for the presence of excited states of the nucleons, which can then be converted into heavier baryons at lower densities. Therefore, to describe such events, it is necessary to consider a wide range of charge fractions, temperatures and densities.

In the NS inner crust, heavy neutron-rich clusters (pasta phases) \cite{3,4} should form, immersed in a gas of neutrons and electrons \cite{5}. Light clusters, such as \(^2\)H, \(^3\)H, \(^3\)He, \(^4\)He, \(^6\)He, are also expected to be present for temperatures above 1 MeV \cite{6}. As the density increases even further, these heavy clusters will eventually melt at densities of \( \sim 0.5n_0 \). This sets the transition to the core of the star. In this region, the composition of the star corresponds to uniform nuclear matter made of neutrons, protons, electrons and muons \cite{6}. In the inner core of the star (densities of the order of \( \sim 2n_0 \)), exotic degrees of freedom such as hyperons and delta isobars, or even deconfined quark matter, may appear \cite{7}.

Hyperons, together with the nucleons, form the spin−1/2 baryonic octet. \( \Delta \)-isobars are spin−3/2 baryons formed by \( u \) and \( d \) quarks, that usually decay via the strong force into a nucleon and a pion. These exotic degrees of freedom will appear at high densities, reducing the pressure of the system, when the increasing chemical potentials of the nucleons approach the effective mass of hyperons and \( \Delta \)s, so that the nucleons start to be converted into these new degrees of freedom \cite{1,8,11}.

Besides reducing the Fermi pressure, the introduction of hyperons decreases the free energy of matter \cite{12,13}. These authors also showed that, at low densities, hyperons can compete with light clusters, implying that the minimization of the free energy should also allow for the appearance of hyperons at these densities. In Ref. \cite{14}, the possible appearance of hyperons in the density region of the non-homogeneous matter that forms the inner crust of a NS was analyzed. Temperatures below the melting temperature of the heavy clusters that form this region were considered, i.e \( T \lesssim 15 \) MeV. It was found that only very small amounts of hyperons, like \( \Lambda \) fractions below \( 10^{-5} \), were present in the background gas. The low-density EoS of stellar matter including light clusters and heavy baryons was also studied in Ref. \cite{15}. In addition to hyperons, the author also considered \( \Delta \)-baryons, pions, and the presence of a representative heavy cluster. It was shown that, depending on temperature and density, the composition of matter may shift from a greater abundance of light clusters to a heavy-baryon predominance.

In a recent work \cite{16}, the calculation of the abundance of purely nucleonic light clusters (\(^2\)H, \(^3\)H, \(^4\)He, \(^6\)He) as well as hyperons was performed in the framework of relativistic mean-field models for finite temperature and fixed proton fraction. In the present work, we intend to include \( \Delta \)-isobars and use two relativistic mean-field models, FSU2H \cite{17} and DD2 \cite{18}. The introduction of clusters is going to follow the approach first presented in Ref. \cite{19}, where the effect of the medium on the binding energy of the clusters is considered through the introduction of a binding energy shift, together with a universal coupling of the scalar \( \sigma \)-meson to the different clusters, that was chosen so that the equilibrium constants of the NIMROD experiment \cite{20} were reproduced. In Refs. \cite{21,22}, this theoretical approach \cite{19} was applied to the description of the INDRA data \cite{23}, where an experimental analysis of data was also done including in-medium effects. It was verified that, due to the inclusion of the in-medium effect in the experimental analysis, the equilibrium constants were reproduced with a larger \( \sigma \)-meson coupling. The calibration of the scalar meson to the clusters coupling was later performed for other models in Ref. \cite{24}.

The structure of this paper is as follows: in the next
Section, we briefly describe the formalism used, in Section [11] the results are discussed, and finally, in Section [15] some conclusions are drawn.

II. RELATIVISTIC MEAN FIELD DESCRIPTION OF HADRONIC MATTER

We briefly present the formalism used to describe warm matter which, besides nucleons, also includes light clusters, light hyperclusters, both considered as point-like particles, and heavy baryons, both hyperons and the Δ-baryons. We, therefore, generalize the study performed in [16], in order to include the Δ baryons.

The description of the hadronic matter will be carried out within a relativistic mean-field (RMF) approach, see for instance [1]. The interaction is described by the exchange of virtual mesons, in particular, the following mesons will be introduced: the usual isoscalar-scalar σ meson, isoscalar-vector ωμ and isovector-vector ρμ, together with the isoscalar-vector φμ meson field with hidden strangeness responsible for an extra repulsion between hyperons.

We consider both a model with density-dependent couplings (DD2) and a model with nonlinear meson terms (FSU2H), two different approaches of introducing the density dependence on the symmetric nuclear matter EoS and on the symmetry energy. Both models satisfy constraints from observations (they are able to reach 2 M⊙ stars), experimental data, and theoretical calculations.

The light clusters considered in the present study are the usual purely nucleonic light nuclei (2H, 3H, 3He, 4He) together with the neutron rich nucleus 6He. Concerning light hypernuclei nuclei, we introduce the three loosely bound hypernuclei: the 3H hypertriton [25], the 4H hyperhydrogen 4 [26] and the hyperhelium 4 3He [27]. The spin and isospin of these clusters have been summarized in [16]. Only clusters with a charge \( Z \leq 2 \) are introduced, because we will essentially consider temperatures above 15 MeV in order to have noticeable fractions of heavy baryons at low densities. Under these conditions, it was shown in [28] that heavy clusters have already dissolved and the light clusters that remain have a charge \( Z \leq 2 \).

The approximation introduced concerning the description of light clusters as point-like particles sets some limitations, in particular, the density of clusters should be low. Above a given density, defined by the interaction and the temperature, clusters melt and matter is only constituted by nucleons and heavy baryons. In our approach, the ω-meson is the main responsible for this process, although other descriptions are possible such as the introduction of an excluded volume [29,33].

The Lagrangian density for this system reads [1,16,18,19]

\[
\mathcal{L} = \sum_{b=baryonic \ octet, \Delta} \mathcal{L}_b + \sum_{i=light \ nuclei} \mathcal{L}_i + \sum_{m=\sigma, \omega, \phi, \rho} \mathcal{L}_m + \mathcal{L}_{nl},
\]

The sum over \( b \) extends over the spin-1/2 baryonic octet and the spin-3/2 Δ quadruplet. The second term is the sum over light nuclei and light hypernuclei, and the last two terms refer to the mesonic terms, where \( \mathcal{L}_{nl} \) includes all the nonlinear mesonic terms and is only present in FSU2H.

The baryonic term in Eq. (1) reads

\[
\mathcal{L}_b = \bar{\Psi}_b [i \gamma_\mu \partial^\mu - m_b + g_{\sigma b} \gamma_5 \sigma - g_{\omega b} \gamma_\mu \omega^\mu - g_{\rho b} \gamma_\mu \rho^\mu] \Psi_b + \mathcal{L}_{nl},
\]

where \( \Psi_b \) is the baryon field, \( \tilde{\Psi}_b \) is the isospin operator and the parameters \( g_{\sigma b}, g_{\omega b}, g_{\rho b} \) are the couplings parameters of the baryons to the mesons. The other parameters of the model are the nucleonic vacuum mass \( m = m_n = m_p = 939 \) MeV, the hyperon masses, \( m_\Lambda = 1115.683 \) MeV, \( m_{\Sigma^-} = 1197 \) MeV, \( m_{\Sigma^0} = 1193 \) MeV, \( m_{\Sigma^+} = 1189 \) MeV, \( m_{\Xi^-} = 1321 \) MeV, and \( m_{\Xi^0} = 1315 \) MeV, and the Δ masses taken to be equal to 1232 MeV.

In Table I we summarize the symmetric nuclear matter properties of the two models considered.

| Model   | \( \rho_0 \) (fm\(^{-3}\)) | \( B/A \) (MeV) | \( K \) (MeV) | \( E_{sym} \) (MeV) | \( L \) | \( M^*/M \) |
|---------|------------------------|----------------|---------|----------------|-------|-------------|
| FSU2H   | 0.15                   | 16.28          | 238     | 30.5           | 45    | 0.59        |
| DD2     | 0.149                  | 16.02          | 243     | 31.7           | 58    | 0.56        |

For the DD2 model [18] with density-dependent coupling parameters, the couplings \( g_{m,N} \) of the nucleons \((N = n, p)\) to the \( \sigma, \omega \) and \( \rho \) mesons are given by

\[
g_{m,N}(n_B) = g_{m,N}(n_0)h_M(x), \quad x = n_B/n_0,
\]

with \( n_B \) the baryonic density, and \( n_0 \) the saturation density. The isoscalar couplings depend on the function \( h_M \) given by [18],

\[
h_M(x) = \frac{1 + b_M(x + d_M)^2}{1 + c_M(x + d_M)^2},
\]

while for the isovector coupling, \( h_M \) has the form

\[
h_M(x) = \exp[-a_M(x - 1)],
\]

with the parameters \( a_M, b_M, c_M, \) and \( d_M \) defined in Ref. [18]. Other parameters of the model such as the meson masses are also given in [18]. For the other model, FSU2H, the couplings are defined in [17].
The mesonic Lagrangian densities are given by
\[
\mathcal{L}_m = \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_s^2 \sigma^2) + \frac{1}{4} W_{\mu \nu} W^{\mu \nu} - \frac{1}{4} F_{\mu \nu} F^{\mu \nu} - \frac{1}{4} \tilde{R}_{\mu \nu} \tilde{R}^{\mu \nu}
\]
\[
+ \frac{1}{2} m_\omega^2 \omega_{\mu \nu} \omega^{\mu \nu} + \frac{1}{2} m_\phi^2 \phi_{\mu \nu} \phi^{\mu \nu} + \frac{1}{2} m_{\rho}^2 \rho_{\mu \nu} \rho^{\mu \nu}
\]
(6)
and
\[
\mathcal{L}_{nl} = -\frac{\kappa}{3!} g_{\sigma N}^3 \sigma^3 - \frac{\lambda}{4!} g_{\omega N}^4 \sigma^4 + \frac{\zeta}{4!} g_{\omega N}^4 \omega_0^4
\]
\[
+ \Lambda_s g_{\sigma N}^2 g_{\omega N}^2 \rho_{0}^2 \omega_0^2
\]
(7)
The meson masses and couplings \(\kappa, \lambda, \zeta, \Lambda_s\) are defined for each parametrization, see Ref. [18] for DD2, [17] for FSU2H.

### Table II. Coupling constants of the vector mesons \(m = \omega, \phi, \rho\) to the different hyperons, normalized to the respective \(\omega, \phi, \rho\) meson nucleon coupling, i.e. \(x_{mb} = g_{mb} / g_{N N}\), except for the \(\phi\)–meson where the \(g_{\phi N}\) is used for normalisation.

| \(b\) | \(x_{\omega b}\) | \(x_{\phi b}\) | \(x_{\rho b}\) |
|---|---|---|---|
| \(\Lambda\) | \(2/3\) | \(-\sqrt{3}/3\) | 1 |
| \(\Sigma\) | \(2/3\) | \(-\sqrt{3}/3\) | 1 |
| \(\Xi\) | \(1/3\) | \(-2\sqrt{3}/3\) | 1 |

Similarly to what we have done for the hyperons, we can write the couplings of the \(\Delta\) particles to the mesons, \(g_{m \Delta}\), in terms of the nucleon couplings as:
\[
g_{m \Delta} = x_{m \Delta} g_{m N}, \quad m = \sigma, \omega, \rho,
\]
(9)
with \(x_{m \Delta}\) being the ratio between the \(\Delta\) and nucleon couplings to the mesons.

Due to limited experimental observations, the couplings of the \(\Delta\) particles to the mesons are still poorly constrained. Some phenomenological analyses from pion–nucleon scattering [36], electron scattering on nuclei [37] and electromagnetic excitations of the \(\Delta\) particles [38] have set the following constraints on the values of the coupling constants, as summarized in Ref. [11]: (i) the \(\Delta\) potential in nuclear matter could be slightly more attractive than the nucleon potential implying that the ratio \(x_{\sigma \Delta}\) should be above 1; (ii) \(x_{\sigma \Delta}\) is larger than \(x_{\omega \Delta}\):
\[
0 < x_{\sigma \Delta} - x_{\omega \Delta} \lesssim 0.2 \quad (10)
\]

(iii) there are no experimental constraints on \(x_{\rho \Delta}\). We will take into account the uncertainties associated to these couplings, allowing the couplings to vary within a large interval of values, as done by other authors [9–11, 39]. In the present work, we will adopt the following intervals:
\[
0.9 \leq x_{\sigma \Delta} \leq 1.2 \quad (11)
\]
\[
0.9 \leq x_{\omega \Delta} \leq 1.2 \quad (12)
\]
The lower limit is set to 0.9 because the nucleon effective mass goes to zero at quite low densities for smaller values. Stronger constraints will be defined by imposing that the effective mass cannot go to zero at densities below the central density of the maximum mass star. For the coupling to the \(\rho\)-meson we will consider \(x_{\rho \Delta} = 0.8, 1,\) and \(2\).

### Table III. Coupling constants of the \(\sigma\) meson to the different hyperons, normalized to the \(\sigma\) meson nucleon coupling, i.e. \(x_{\sigma b} = g_{\sigma b} / g_{\sigma N}\), for the DD2 and FSU2H models.

| \(x_{\sigma b}\) | DD2 | FSU2H |
|---|---|---|
| \(x_{\sigma \Lambda}\) | 0.621 | 0.620 |
| \(x_{\sigma \Xi}\) | 0.474 | 0.452 |
| \(x_{\sigma \Xi}\) | 0.320 | 0.310 |

### B. Chemical equilibrium

In order to impose chemical equilibrium, the chemical potentials of baryons and light clusters and hyperclusters are needed. The chemical potential of baryon \(b\) is given by
\[
\mu_b = g_{\omega \omega 0} g_{\phi \phi 0} + \phi_{\omega 0} + g_{\phi \phi 0} + \gamma^{R} + \sqrt{k_F^2 + m_b^2},
\]
(13)
where \(k_F\) is the Fermi momentum of baryon \(b\), and \(\gamma^{R}\) is the rearrangement term, only present in the DD2 model, defined as
\[
\gamma^{R} = \sum_c \left( \frac{\partial \mu_c}{\partial \rho_c} \omega_c + \frac{\partial \mu_c}{\partial \rho_c} \phi_c + \frac{\partial \mu_c}{\partial \rho_c} \sigma_c \right),
\]
(14)
where the sum is over the baryons, \(c = b\), for \(T = 0\) MeV, and over the baryons and clusters, \(c = b, i\), for
finite temperatures. The effective chemical potentials \( \mu_i^e \) that enter the Fermi distributions are given by

\[
\mu_i^e = \mu_b - g_{\omega b} \phi_0 - g_{\rho b} \phi_0^R - g_{\phi b} \phi_0 - \Sigma_i^R .
\]  

(15)

In the following, we consider matter in equilibrium with a fixed charge fraction \( Y_b \). All particle chemical potentials can be written in terms of the chemical potentials corresponding to two conserved charges, baryonic charge and electrical charge. We consider that the strangeness chemical potential is zero, because the weak force does not conserve strangeness. Therefore the chemical potential of each particle \( c \) is a linear combination of the baryon and electric charge chemical potentials:

\[
\mu_c = b_c \mu_n - q_c \mu_e
\]  

(16)

where \( b_c \) is the baryon number of particle \( c \); \( q_c \) is the electrical charge (in units of +e); and \( \mu_n, \mu_e \) the baryon and electrical charge chemical potentials, respectively. Since \( \mu_e = \mu_n - \mu_p \), the hyperon chemical potentials can be written in terms of the nucleon chemical potentials:

\[
\mu_{\Sigma} = \mu_n
\]  

(17)

\[
\mu_{\Sigma^+} = 2\mu_n - \mu_p, \quad \mu_{\Sigma^-} = \mu_n, \quad \mu_{\Sigma^0} = \mu_p
\]  

(18)

\[
\mu_{\Xi} = 2\mu_n - \mu_p, \quad \mu_{\Xi^0} = \mu_n .
\]  

(19)

Similarly, for the \( \Delta \) isobars we have:

\[
\mu_{\Delta^-} = 2\mu_n - \mu_p, \quad \mu_{\Delta^0} = \mu_n, \quad \mu_{\Delta^+} = \mu_p, \quad \mu_{\Delta^{++}} = 2\mu_p - \mu_n .
\]  

(20)

In equilibrium, the cluster or hypercluster \( i \) chemical potential must satisfy

\[
\mu_i = N_i \mu_n + Z_i \mu_p + A_i \mu_A .
\]  

(21)

The corresponding effective chemical potential \( \mu_i^e \) is given by

\[
\mu_i^e = \mu_i - g_{\omega i} \phi_0 - g_{\rho i} \phi_0^R - g_{\phi i} \phi_0 - \Sigma_i^R .
\]  

(22)

The terms involving \( A_i \) in Eq. (21) and the meson field \( \phi_0 \) in Eq. (22) only contribute to the chemical potential of hypernuclei.

The total charge fraction \( Y_Q \) of the system is defined as

\[
Y_Q = \sum_b q_b Y_b + \sum_i \frac{q_i}{A_i} Y_i
\]  

(23)

where \( q_b \) and \( q_i \) are the electric charge (in units of +e) of baryon \( b \) and light cluster or hypercluster \( i \). The mass fraction \( Y_i \) of the cluster or hypercluster \( i \) is given by

\[
Y_i = A_i \frac{\rho_i}{\rho_B} .
\]  

(24)

The introduction of light clusters follows the formalism first presented in [19], and the inclusion of hyperons in these clusters, termed hyperclusters, was introduced in a recent work [13], where the details on the calculations can be found.

### III. Results

In the present section we present our main results. In the first subsection we constrain the \( \Delta \)-meson couplings imposing that the effective mass must be non-zero. In a second subsection, we analyse the effect of the heavy baryons on the cluster abundances.

#### A. Constraining the \( \Delta \) couplings

In order to introduce the \( \Delta \)-isobars, \( \Delta^{-0,+++,+} \), it is necessary to constrain the \( \Delta \) couplings to the mesons. The uncertainty on the \( \Delta \) couplings can be accounted for by allowing them to vary within a large interval of values. As explained in Sec.II A, we take them to be \( 0.9 \leq \sigma \Delta \leq 1.2 \), \( 0.9 \leq \omega \Delta \leq 1.2 \), and \( \rho \Delta = 0.8, 1, 2 \).

Further constraints are obtained from observations: the EoS must be able to describe 2 \( M_\odot \) stars, and the effective mass of nucleons must remain finite inside the star. In order to build a complete EoS, it is necessary to match the core EoS to the crust EoS. We have considered for the outer crust the BPS EoS [40], and for the inner crust, we take the inner crust EoS obtained within a Thomas-Fermi calculation, including non-spherical heavy clusters [11, 12]. The inner crust EoS for DD2 [43] and for FSU2H [44] can be found in the CompOSE database [45], an online, free and public repository for EoS. For the core EoS, the full spin-1/2 baryonic octet, the four \( \Delta \) isobars, electrons and muons were included in \( \beta \)-equilibrium and at \( T = 0 \) MeV.

In the left panels of Fig. 1 we plot the Mass-Radius relations obtained with the EoSs corresponding to a few representative sets of the \( \Delta \) couplings: \( x_{\pi \Delta} = x_{\omega \Delta} = 0.9; x_{\pi \Delta} = x_{\omega \Delta} = 1; x_{\pi \Delta} = 1.1, x_{\omega \Delta} = 1; x_{\pi \Delta} = 1.2, x_{\omega \Delta} = 1.05; x_{\pi \Delta} = 1.2, x_{\omega \Delta} = 1.1; x_{\pi \Delta} = x_{\omega \Delta} = 1.2; x_{\rho \Delta} = 0.8 \). For comparison, we also show (in black) the EoS without \( \Delta \)-isosbars. We also include two horizontal bands corresponding to two of the most massive pulsars ever observed, PSR J0740+6620 [14, 17], and PSR J0348+0432 [15], with masses \( M = 2.08 \pm 0.07 M_\odot \) and \( M = 2.01 \pm 0.04 M_\odot \), respectively.

For both models, the EoS which include the \( \Delta \) isobars show a significant decrease in the radius of the intermediate-mass stars (\( \sim 1.4 M_\odot \)), compared to the stars without \( \Delta \). The appearance of the \( \Delta \) softens the EoS, and, therefore, the star is further compressed due to gravity and, consequently, has a smaller radius, as discussed in Ref. [11]. On the other hand, the presence of the \( \Delta \)s does not seem to significantly affect the maximum masses.

There is also a visible difference in the intermediate-mass radius between the EoSs with different values of the \( \Delta \) couplings to mesons, denominated \( \Delta \nu = x_{\pi \Delta} - x_{\omega \Delta}. \) The EoSs with the same difference in the \( \Delta \nu \) have similar intermediate-mass radii. This is easily understood if we
consider the $\Delta$ potential in nuclear matter:

$$U_N^\Delta(\rho_0) = -g_{\sigma\Delta} \sigma + g_{\omega\Delta} \omega_0 + g_{\rho\Delta} I_{3\Delta} \rho_0$$  \hspace{1cm} (25)$$

If $g_{\rho\Delta}$ increases, the $\Delta$ potential decreases and becomes more attractive, which leads to an increase in the abundances of the $\Delta$s. On the other hand, if $g_{\omega\Delta}$ increases, the potential becomes less attractive and the $\Delta$s are less favored. Therefore, the larger the $\Delta x$, the higher the abundances of the $\Delta$s and the earlier their onset (see Table IV). An earlier onset of $\Delta$s and a higher abundance leads to a more significant softening of the EoS and a larger reduction of its intermediate-mass radius. Looking at Eq. (25), it is also reasonable to conclude that different EoSs with the same difference $\Delta x$ (e.g., $x_{\sigma\Delta} = x_{\omega\Delta} = 0.9$ and $x_{\sigma\Delta} = x_{\omega\Delta} = 1$) will have similar potentials and, therefore, similar intermediate-mass radii since the increase in $x_{\sigma\Delta}$ is approximately compensated by a similar increase in $x_{\omega\Delta}$.

Some of the Mass-Radius relations in the left panels of Fig. 1 do not reach the maximum mass star. The nucleon effective mass corresponding to the EoSs with $\Delta$s decreases much faster than the ones without $\Delta$s, and eventually becomes zero, see right panels of Fig. 1. For some parametrizations the effective mass drops so fast that it becomes zero before the maximum mass star is reached. These EoSs are not appropriate to describe NSs if no phase transition to quark matter is considered, and, therefore, they will be discarded in the present study.

Considering the couplings tested for the DD2 model satisfying Eqs. (12), the following sets of $\Delta$ couplings are not valid: $x_{\sigma\Delta} = x_{\omega\Delta} = 0.9$; $x_{\sigma\Delta} = 1.1$, $x_{\omega\Delta} = 1$; $x_{\sigma\Delta} = 1.2$, $x_{\omega\Delta} = 1.05$. For the FSU2H model only the pair $x_{\sigma\Delta} = x_{\omega\Delta} = 1.2$ corresponds to a valid EoS. The valid $\Delta$ couplings, i.e., the ones that are able to reach the maximum mass before the nucleon effective mass becomes zero, are shown in Table IV.
TABLE IV. Maximum mass \( M_{\text{max}} \), correspondent radius \( R(M_{\text{max}}) \), central density \( \rho_c \), 1.4\( M_\odot \) radius \( R(1.4 M_\odot) \), onset density of \( \Delta^- \) \( \rho_{\Delta^-} \), onset density of the \( \Lambda \) hyperon \( \rho_\Lambda \), for the DD2 and FSU2H models and considering only the valid \( \Delta \) couplings.

| Baryonic Octet | \( M_{\text{max}}(M_\odot) \) | \( R(M_{\text{max}}) \text{(km)} \) | \( \rho_c(\text{fm}^{-3}) \) | \( R(1.4 M_\odot) \text{(km)} \) | \( \rho_{\Delta^-}(\text{fm}^{-3}) \) | \( \rho_\Lambda(\text{fm}^{-3}) \) |
|----------------|------------------|------------------|------------------|------------------|------------------|------------------|
| DD2            |                  |                  |                  |                  |                  |                  |
| \( x_{\sigma\Delta} = x_{\omega\Delta} = 1, x_{\rho\Delta} = 1 \) | 2.04             | 11.45            | 0.99             | 13.91            | -                | 0.33             |
| \( x_{\sigma\Delta} = 1.2, x_{\omega\Delta} = 1.1, x_{\rho\Delta} = 1 \) | 2.02             | 11.11            | 1.05             | 12.93            | 0.28             | 0.36             |
| \( x_{\sigma\Delta} = x_{\omega\Delta} = 1.2, x_{\rho\Delta} = 1 \) | 2.06             | 10.95            | 1.05             | 12.26            | 0.23             | 0.39             |
| \( x_{\sigma\Delta} = x_{\omega\Delta} = 1.2, x_{\rho\Delta} = 2 \) | 2.05             | 11.31            | 1.01             | 12.97            | 0.27             | 0.35             |
| \( x_{\sigma\Delta} = x_{\omega\Delta} = 1.2, x_{\rho\Delta} = 0.8 \) | 2.04             | 11.32            | 1.01             | 13.13            | 0.32             | 0.34             |
| FSU2H          |                  |                  |                  |                  |                  |                  |
| \( x_{\sigma\Delta} = x_{\omega\Delta} = 1.2, x_{\rho\Delta} = 1 \) | 1.99             | 12.39            | 0.79             | 13.29            | -                | 0.33             |
| \( x_{\sigma\Delta} = x_{\omega\Delta} = 1.2, x_{\rho\Delta} = 2 \) | 1.98             | 11.73            | 0.91             | 12.97            | 0.26             | 0.35             |
| \( x_{\sigma\Delta} = x_{\omega\Delta} = 1.2, x_{\rho\Delta} = 0.8 \) | 1.97             | 11.97            | 0.87             | 13.26            | 0.30             | 0.34             |

B. Clustered matter with \( \Delta \)-baryons

In the following we discuss the effect of the presence of \( \Delta \)-baryons on the properties of clustered matter for temperatures above 10 MeV, when heavy clusters are not expected anymore. We are mainly going to work with the DD2 model with the \( \Delta \) couplings equal to the nucleons, i.e. \( x_{\sigma\Delta} = x_{\omega\Delta} = x_{\rho\Delta} = 1 \). However, whenever we compare the DD2 and FSU2H models we have to set \( x_{\sigma\Delta} = x_{\omega\Delta} = 1.2 \), since these are the only FSU2H valid couplings that remained from the initial six sets of \( \Delta \) couplings.

In the left panel of Fig. 2 we show the nucleon and purely nucleonic light cluster abundances with (thick lines) and without (thin lines) hyperons as a function of the temperature for a charge fraction of \( Y_Q = 0.1 \) and density of \( n_B = 0.1 \text{ fm}^{-3} \). The presence of both hyperons and \( \Delta \)s increases even further the abundances of light clusters. This is justified by the reduction of the nucleon density in the presence of hyperons and \( \Delta \)s which leads to smaller binding energy shifts.

In Fig. 3, we show again the abundances of clusters and unbound nucleons in a system with \( \Delta \)-isobars, but
The ∆ abundances are also displayed with thick pink lines. The main effect of introducing ∆s is a reduction of the neutrons as well as of the neutral and negatively charged hyperons, whereas the abundances of protons and positively charged Σ hyperon increase. The most abundant ∆-isobar is clearly the Σ− which is negatively charged, so its appearance is compensated by a reduction of the neutral and negatively charged particles and an increase of the positively charged ones. Except for the neutrons, all particles increase their abundances with the temperature. At finite temperature new channels open and the interaction, the mass and the charge define the abundances. It is energetically favorable to convert highly energetic neutrons into other particles. The more attractive couplings of the ∆s compared with the hyperons explains why they are more abundant than their equally charged hyperon counterparts.

In Fig. 4 we show the impact on the total mass fractions and dissolution densities of the clusters caused by the inclusion of hyperons only (solid lines) and by the inclusion of both hyperons and ∆s (dash-dotted lines) for two charge fractions $Y_Q = 0.3$ (orange) and 0.1 (blue), and for the DD2 model. The dashed lines were obtained for nucleonic matter, and have been included for comparison. Like in previous Figs, the scalar cluster-meson coupling fraction is set to $x_s = 0.93$, and the ∆ couplings are fixed to $x_{\sigma\Delta} = x_{\omega\Delta} = x_{\rho\Delta} = 1$. On the left panel, the behavior of the total cluster mass fraction is plotted for $T = 50$ MeV and two charge fractions as a function of density. For $Y_Q = 0.3$, the largest cluster fractions are reached for densities below saturation density. Besides, in these range of densities the three different scenarios consider do not differ much. The larger differences occur for $\rho \gtrsim 0.2$ fm$^{-3}$, for densities above the maximum in the cluster distribution and close to the dissolution. The presence of the heavy baryons shifts the dissolution density to larger densities. This effect is present considering only hyperons but it is intensified when ∆-baryons are also included. The presence of ∆s reduces the nucleon fraction, and this is reflected on the medium effects felt by the clusters through a smaller binding energy shift. For the smaller charge fraction, $Y_Q = 0.1$, the difference between the distributions occurs also for densities at the abundance peak, with the largest mass fractions occurring for matter with ∆s and hyperons, followed by mat-
FIG. 5. Total mass fraction $Y_{\text{tot}}$ of the light clusters as a function of the density at $T = 50$ MeV (left) and the dissolution density of the clusters, $n_d$, as a function of the temperature (right) for a calculation without hyperons and $\Delta$s (dashed), with hyperons and without $\Delta$s (solid), and with hyperons and $\Delta$s (dash-dotted). The charge fraction is fixed to $Y_Q = 0.3$ (orange) and 0.1 (blue), and the DD2 model is considered.

FIG. 6. Mass fractions of the unbound nucleons (red), $\Lambda$, $\Sigma$ and $\Xi$ hyperons (green), $\Delta$-isobars (orange), light clusters (blue), and light hypernuclei (pink) with (thick) and without (thin) $\Delta$-particles as a function of the density for $T = 50$ MeV and $x_s = 0.93$, with $Y_Q = 0.3$ (left) and 0.1 (right). The calculation is performed for the DD2 RMF model.

ter with hyperons, and the smallest fraction for nucleonic matter. It is clear that the smaller the charge fraction, the larger the difference between the three distributions. The right panel of Fig. 5 summarizes the effect of the heavy baryons on the dissolution density of clusters: the differences occur for temperatures above 25 MeV, with the largest dissolution densities occurring for matter with the smallest charge fraction and containing both hyperons and $\Delta$-baryons. These effects are all understood by realizing that the presence of heavy baryons reduces the nucleonic background gas, and, therefore, the binding energy shift, preventing clusters to melt.

In Fig. 6 we plot together the fractions of unbound nucleons, light clusters, light hypernuclei, the $\Lambda$ fraction, the total $\Sigma$ fraction corresponding to the sum of the $\Sigma^{+,0,-}$ fractions, the total $\Xi$ fraction corresponding to the sum of the $\Xi^{0,-}$ fractions, the total $\Delta$ fraction corresponding to the sum of $\Delta^{-,0,+}$, for a charge fraction of $Y_Q = 0.3$ (left) and 0.1 (right) in a calculation with (thick lines) and without (thin lines) $\Delta$s. The $\Delta$
The & couplings were chosen equal to the ones of the nucleons, $x_{\sigma\Delta} = x_{\omega\Delta} = x_{\rho\Delta} = 1$. We see that the inclusion of $\Delta$s increases the abundances of the purely nucleonic light clusters above their maxima through the reduction of the binding energy shift of the clusters, so we would expect a similar increase for the hyperclusters. In fact, that is exactly what Fig. 6 shows. Once again, the effect is higher for the charge fraction $Y_Q = 0.1$, since a smaller charge fraction favours negatively charged particles, which is the case of the $\Delta^-$ (the most abundant of the $\Delta$s). Therefore, if the $\Delta$s are more abundant for $Y_Q = 0.1$, the reduction of the binding energy shifts of the hyperclusters after their maxima will be larger, resulting in higher dissolution densities and fractions. On the other hand, for densities below the hyperclusters maxima, the introduction of $\Delta$s actually slightly reduces the abundances of hyperclusters, which may be due to a drop in the $\Delta$s, which are essential to build hyperclusters.

In Fig. 7 the particle fractions obtained with the models DD2 (thin lines) and FSU2H (thick lines) are compared. Although, the overall behavior is similar in both models, there are visible differences, in particular, after the peak of the cluster distributions: FSU2H model gives smaller cluster fractions and smaller dissolution densities. FSU2H also shows smaller fractions of neutrons and hyperons but larger $\Delta$ fractions. FSU2H favors the appearance of $\Delta$-baryons with respect to DD2, probably due to the difference on the $\rho$-meson couplings, see Eq. 29. A higher $\Delta$ abundance for the FSU2H results in smaller fractions of hyperons compared to the DD2, especially negatively charged ones since the appearance of $\Delta^-$ disfavors negatively charged particles. The fractions of light clusters and hyper-clusters are also smaller for the FSU2H. This may be due to a smaller value of the fraction $x_s$ for the FSU2H ($x_s = 0.91$) compared to the DD2 ($x_s = 0.93$). In fact, the larger the $x_s$ the larger the $\sigma$-cluster couplings, resulting in a stronger binding, and therefore higher abundances.

Finally, let us now compare the total fraction of $\Delta$ isobars, $Y_\Delta$, corresponding to the sum of $\Delta^{-,0,+,++}$ fractions, as a function of the density for $T = 50$ MeV and a charge fraction of $Y_Q = 0.3$, in a calculation for different values of the $\Delta$ couplings using the DD2 and FSU2H models that includes unbound nucleons, hyperons, light clusters and hyperclusters. In the left panel of Fig. 8, we fix $x_{\rho\Delta} = 1$ and perform the calculation for the three previously validated DD2 EoSs and the only valid FSU2H EoS. As we have mentioned before, the larger $x_{\sigma\Delta} = x_{\omega\Delta}$, the higher the abundances of $\Delta$s. On the other hand, DD2 parameterizations with $x_{\rho\Delta} = 0$ show similar abundances of $\Delta$s: the model with higher $\sigma$ and $\omega$ couplings produces slightly higher abundances at smaller densities (where the $\sigma$ coupling is dominant [11]) and lower abundances at higher densities (where the repulsion associated with the $\omega$ coupling dominates [11]). As for the difference between the DD2 and FSU2H models with $x_{\rho\Delta} = 1$ and 2 for DD2 and FSU2H. As we can see from Eq. 29, the larger the value of $x_{\rho\Delta}$, the less attractive the $\Delta^-$ potential is, making its presence less favorable, which is observed for both models. All these different parameterizations affect the fractions of the various particles, since a parameterization with more $\Delta$s than the one presented in Fig. 6 accentuates the effects mentioned in the discussion whereas a smaller abundance reduces the impact of the $\Delta$s.

The effect of heavy baryons on the presence of light clusters at low densities has also been discussed in Ref. [12]. In that study, the author includes pions, the $\Delta$-quadruplet and $\Lambda$ hyperons, besides nucleons and the classical light clusters ($^3$H, $^3$He, $^4$He). The calculation is performed in the dilute limit within a Green’s function formalism. Medium effects on the distribution of particles are included through the definition of the particle self-energies. For the nucleons, the self-energies are approximated by the nucleon effective masses and for the pions the leading contribution to the self-energy within a chiral perturbation theory was considered. With the simplified description of the heavy baryons, the effect of the clusters on the heavy cluster fractions is not seen. In particular, the heavy baryon fractions are insensitive to the cluster formations. An-
other effect is the fact that in Ref. [15], the Λ fraction is larger than the Δ fraction because they are defined by the baryon mass and the interaction with the medium is not considered. In our system, all particles interact with the medium in a self-consistent way, therefore the introduction of the heavy baryons such as hyperons and Δs does have an effect on the clusters abundance. Since our heavy baryons interact with the medium, their abundances do not depend only on their masses, which allows the Δ isobars to be more abundant than the Λ hyperon for certain conditions.

In our study we did not include pions. Since the Δ isobars decay into a nucleon and pion through the strong force if there are available states, the presence of pions is expected in a finite temperature scenario. This will be analyzed in a future study.

IV. CONCLUSIONS

We have performed a calculation of clusterized matter including five light clusters (\(^2\)H, \(^3\)H, \(^3\)He, \(^4\)He, \(^6\)He) and three light hyperclusters (\(^3\)ΛH, \(^4\)ΛH, \(^4\)ΛHe), all hyperons belonging to the baryonic octet and the isospin multiplet of Δ-baryons. The calculation was undertaken in the framework of relativistic mean-field theory, in particular, the models DD2 [18] and FSU2H [17] have been used. Light clusters and hyperclusters were described as pointlike particles, that interact with the mesons of the model, and besides feel a binding energy shift due to the presence of the medium as introduced in [19]. The binding energy shift is important to take into account in an effective way Pauli blocking effects. The hyperon-meson couplings were obtained from a calibration to hypernuclear properties [49]. In order to choose adequate Δ-meson couplings we have considered experimental constraints as summarized in [20] and imposed as well observational constraints. Many parametrizations for the Δ-baryons had to be disregarded because nucleon effective masses would become zero before a maximum star mass would be reached.

The main conclusions of the present work are: (i) the presence of heavy-baryons, both hyperons and Δs favour the formation of clusters and shift their dissolution to larger densities; (ii) a larger number of clusters decreases the fraction of free nucleons, and, in particular, the difference between the fractions of neutrons and protons decrease, which favors processes like direct Urca reactions. In the future, it is important to implement the presence of heavy baryons in the low density warm EoS used in core-collapse supernova or neutron star merging simulations.

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