Nonradial modes in RR Lyrae stars

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ABSTRACT

We present a survey of nonradial mode properties in evolutionary sequences of RR Lyrae star models. Attention is focused on the modes that may be driven by the opacity mechanism and on those that may be excited as a consequence of the 1:1 resonance with the radial pulsation.

Qualitatively, all the models share the same properties of the nonradial modes. At the quantitative level, the properties are to a large extent determined by the radial mode periods. There is only weak dependence on the star metallicity and no apparent dependence on the evolutionary status, that is on the helium exhaustion in the convective core.

In the whole range of RRab and RRc star parameters we find unstable nonradial modes driven by the opacity mechanism. An instability of radial pulsation to a resonant excitation of nonradial oscillations is also a common phenomenon in both types. We discuss a possible role of nonradial modes in amplitude modulation observed in certain RR Lyrae stars.

1 Introduction

Whether or not nonradial modes play a role in RR Lyrae pulsation has been a matter of speculation for some time. Recently, Olech et al. (1999) presented first circumstantial evidence for nonradial modes presence in three RRc variables in M55. The evidence was based on the power spectra which revealed presence of additional modes whose frequencies could not be attributed to radial modes. Similar power spectra were subsequently found in several other RRc and RRab stars (Olech et al. 1999b, Kovacs et al. 1999, Moskalik, P. 1999).

Earlier (Kovacs, 1993; Van Hoolst and Waelpens, 1995) proposed the 1:1 resonant excitation of nonradial modes in a radially pulsating star as an explanation of the Blazhko type modulation. Manifestation of this effect in periodograms is an occurrence of the equally-spaced side peaks around
the main frequency. Calculations provided by Van Hoolst et al (1998) confirmed the plausibility of this idea. These authors studied stability of radial pulsation with use of the third amplitude equation formalism. They found that there is a high probability of a resonant excitation of a low $\ell$-degree mode. However, their calculations were done only on one stellar model. In this paper we apply the same formalism, with one additional simplification, to investigate stability of radial pulsation in a large set of RR Lyrae star models. An outline of the formalism and the results are given in section 4.2.

Still earlier (Dziembowski, 1977) instability of static models of RR Lyrae stars to certain nonradial modes has been demonstrated. The driving effect is the same as for radial pulsation. A linear instability, however, is not a sufficient condition for excitation. Nonlinear calculations are required to determine the ultimate outcome of the linear instability. Because of enormous numerical complexity, for the nonradial modes such calculations have never been done. The instability could be saturated with excitation of a single mode, which seems the most common situation among RR Lyrae stars. However, for instance among $\delta$ Sct stars typical is excitation of many modes. We do not understand why is it so.

In section 4.1 of this paper we present a survey of the unstable nonradial modes in RR Lyrae stars and we discuss potential identifications of the modes detected by Olech et al. (1999).

2 Evolutionary models

All the stellar models adopted in the present investigation, have been computed adopting the latest version of the FRANEC evolutionary code, which includes several upgrades of the input physics. Major improvements are the opacity tables for the stellar interiors as given by Rogers & Iglesias (1992) and low-temperature molecular opacities for outer stellar layers by Alexander & Ferguson (1994). Both high- and low-temperature opacity tables have been computed by adopting the Grevesse (1991) solar chemical mixture. The equation of state is the OPAL one (Rogers et al. 1996), implemented in the temperature-density region not covered by OPAL, with the equation of state of Straniero (1988), plus a Saha Eos in the outer stellar layers (see Cassisi et al. 1998, 1999, for more details). As for the calibration of the superadiabatic envelope convection, the mixing length calibration provided by Salaris & Cassisi (1996) has been adopted.

For the present work, we have computed Horizontal-Branch models for
Figure 1: Horizontal-branch evolutionary sequences. In S1 and S2 sequences the ZAHB composition is characterized by $Z = 0.001, Y_{HB} = 0.243$ and that in S3 by $Z = 0.0002, Y_{HB} = 0.24$. Stellar masses are 0.65, 0.67 and 0.74$M_\odot$, respectively, in S1, S2, S3. The symbols mark the models selected for the pulsational analysis.
two different assumptions on the heavy element abundance: \( Z = 0.0002 \) and 0.001 namely. In both cases, an initial Helium abundance equal to \( Y = 0.23 \) has been adopted. All the HB models have as Red Giant Branch progenitor a structure with mass equal to \( 0.8M_\odot \). This means that when computing the Zero Age Horizontal Branch (ZAHB) models we have accounted for the evolutionary values for the size of the He core mass and the surface He abundance \( (Y_{HB}) \) at the He ignition corresponding to a \( 0.8M_\odot \) progenitor as provided by our own evolutionary computations for the previous H-burning phases.

In Fig. 1, we show the selected evolutionary tracks in the H-R diagram. The symbols along each track indicate the models adopted for the following pulsational analysis.

3 Linear nonadiabatic calculations

Oscillation properties of the selected models were studied with the method developed by one of us (Dziembowski, 1977). Its recent updated description may be found in Van Hoolst et al. (1998). For nonradial modes the equation of nonadiabatic oscillations are solved numerically in the envelope and matched to the asymptotic solution for g-modes, which is valid in the deep interior of RR Lyrae stars. The reason is that beneath the matching point the Brunt-Väisälä frequency is much larger than the oscillation frequencies. The Cowling approximation is assumed, which is well justified for the modes considered. The weakest point in the adopted method is the one related to the treatment of convective transport, whose Lagrangian perturbation is simply ignored. This is certainly a poor approximation but it is not essential for the main aim of this work because effects of convection on radial and nonradial modes are nearly the same.

As an introductory example we plot in Fig. 2 the growth rates, \( \gamma = \Im(\omega) \), and frequencies, \( f = \Re(\omega)/2\pi \) for modes at the selected degrees \( \ell \) for one of the models we chose for the pulsation analysis. The temporal dependence of oscillations is assumed in the form \( \exp(-i\omega t) \). Effects of rotation has been ignored. Thus, each point represents \( 2\ell + 1 \) normal modes.

Let us note two types of unstable modes. There are isolated rapidly unstable modes with the growth rates \( \gamma > 0.01 \; \text{d}^{-1} \) and sequences of modes with much lower \( \gamma \)’s. In the former group we find radial modes and modes with \( \ell = 6 \) and 10 which belong to the class of strongly trapped unstable (STU) modes defined by Van Hoolst et al. (1998). These modes have
Figure 2: Growth rates and frequencies for modes of indicated spherical harmonic degrees in a selected model from S3 sequence. The model is characterized by the following parameters $\log T_{\text{eff}} = 3.822$, $\log\left(L/L_\odot\right) = 1.717$, $Y_c = 0.17$. 
no counterpart in the adiabatic approximation. In the interior the eigen-
function of such modes are – to good approximation – described as inward 
propagating internal gravity waves with exponentially decreasing amplitude.

The growth rate behavior in the sequences of low degree modes reflect 
the trapping properties of the acoustic cavity. Still at \( \ell = 1 \) even for the best 
trapped modes more than 80 percent of the kinetic energy is contributed by 
the g-mode propagation. The trapping effect is weaker at \( \ell = 2 \) and 3 but 
then it begins to increase. Note the sharp peak of \( \gamma \) in the \( \ell = 6 \) sequence 
near the first overtone frequency. The STU modes occur always between 
the two best trapped ordinary modes.

For the occurrence of STU mode a sufficient trapping in the evanescent 
zone separating p- and g-mode propagation zone is needed. In our selected 
mode the STU fundamental modes appear at \( \ell = 8 \). With increasing \( \ell \) 
they tend to Kelvin (f or surface) modes. The instability continues well 
above \( \ell = 100 \). We hesitate to give the upper limit because of the increasing 
uncertainty due to our crude treatment of convection. Near the first overtone 
the STU modes begin at \( \ell = 5 \) and end at \( \ell = 15 \).

\section{4 Survey of pulsational properties}

The H-R positions of stellar models selected for this survey were shown in 
figure 1. The models cover various stages of the central helium burning. This 
can be seen in figure 3, where central helium content is plotted as function 
of the effective temperature. In the same figure, the periods of the first two 
radial modes are plotted. Solid symbols are used to denote linearly unstable 
modes. Second overtone is also unstable in some of our models. However, 
because three is no observational evidence for second overtone excitation in 
RR Lyrae stars, in present survey we consider only modes in the vicinity of 
the first two radial modes.

\subsection{4.1 Opacity-driven modes}

The general property of all models considered is that the trapping effect is 
weak in the \( \ell = 2 - 4 \) range. In the vicinity of an unstable fundamental 
radial there are always unstable \( \ell = 1 \) modes. In some models, like the 
one used in Fig. 2, there is a frequency range where the \( \ell = 2 \) modes are 
unstable as well but with much lower growth rates. Rapid instability occurs 
only for the STU modes, which began in most of the models at \( \ell = 8 \). The 
trapping pattern near first overtone is similar to that near the fundamental
Figure 3: Central helium content ($Y_c$) in selected models is plotted as function of their effective temperature in the left panel. In the right two panels periods of the radial fundamental and first overtone modes are given for the selected models. The empty symbols are used if the mode is stable.
mode. Again the most unstable are modes of the $\ell = 1$ degree and then STU modes which begin at $\ell = 5$ or 6. The main difference is instability at all low degrees.

In Figs. 4 and 5 we show, respectively for the fundamental mode and first overtone ranges, the frequency distances to corresponding radial modes and the relative growth rates for most unstable $\ell = 1$ modes and for the selected STU modes. The most unstable $\ell = 1$ modes as well as all the STU modes have always higher frequencies than the corresponding radial modes. The two plotted parameters vary in rather narrow ranges and their values are determined by the radial mode periods. The dependence on the abundance ($Z$) is most easily seen in the distances of the STU modes. The dependence on the evolutionary status ($Y_c$), for which one should consult Fig. 3, is not recognizable.

The growth rates of STU modes are similar to those of the radial modes. If the instability is saturated by one of these relatively high degree modes
Table 1: Amplitude-modulated RRc stars in M55 (from Olech et al. 1999)

| Star | \( P_p \) | \( d \) | \( (f_p - f_s)/f_p \) | \( A_s/S_p \) |
|------|---------|-------|----------------|---------|
| V9   | 0.316   | -0.028| 0.19           |         |
| V10  | 0.332   | +0.004| 0.44           |         |
| V9   | 0.316   | -0.098| 0.57           |         |

The star would appear as a nonpulsating object. At \( \ell = 5 - 10 \) the cancellation of the opposite sign contribution would reduce the disc-averaged light amplitude to at most few millimagnitude. There is no firm evidence for occurrence of nonpulsating stars in RR Lyrae stars in the RR Lyrae domain of the H-R. The linear theory does not yield us a hint why radial modes are so much preferred over the STU modes by stars.

The secondary peaks in the three amplitude-modulated RRc stars discovered by Olech et al. (1999) cannot be explained in terms STU mode excitations. In Table 1 we provide data on the distances between the primary and secondary peaks and the relative V-amplitudes.

The secondary peak amplitudes are still too large and in two cases the frequencies are lower than those of the main peaks. Interpretation in terms of the \( \ell = 1 \) modes is more plausible though not free of difficulties. Also in this case the secondary peaks position present certain problem. However, the problem is not so essential because always we find unstable \( \ell = 1 \) modes on both sides of the radial modes. Furthermore, we do not have arguments why radial modes should always have higher amplitudes.

4.2 Resonant modes

The criterion for the instability of radial pulsation to excitation of a resonant nonradial mode may be written in the following form

\[
\left( \frac{\delta R}{R} \right)^2 > \sqrt{\frac{D^2 + \kappa^2}{C}},
\]

where \( \delta R \) is the amplitude of radius variations in radial pulsation; \( D \) denotes the frequency distance between the radial and the nonradial mode; \( \kappa \) is the damping rate of the nonradial mode in the limit cycle of the radial mode; and \( C \) is the coupling coefficient. The criterion is from Van Hoolst et al. (1998) only the notation is different.

Evaluation of the quantities occurring in the r.h.s. of Eq.(1) requires, in principle, nonlinear calculations, which we have not done. However, for
Figure 5: Similar to Fig.4 but for the first overtone vicinity. Here $\ell = 7$ is the chosen STU mode.
Figure 6: Properties of the resonant modes (closest to the fundamental radial mode). From top to bottom, the relative moment of inertia, the relative frequency separation between consecutive modes, the fractional damping rate are plotted against radial mode period for the fundamental mode vicinity. The parameters plotted are important for the resonant excitation of the nonradial modes. Modes with degrees $\ell = 1$, and 5 or 6 are most likely excited.
a crude evaluation of the probability that a radial mode of specified amplitude is unstable to parametric excitation of a nonradial mode, we need only linear mode characteristic provided in Figs. 6 and 7 and certain coupling coefficients. These coefficients were evaluated by Van Hoolst et al. (1998) for the stellar model they selected. Here we rely on a simple scaling of their numbers which we will explain below.

Let $P_\ell(A)$ denotes the excitation probability at the radial mode amplitude $A = \delta R/R$. Then, if effect of rotation are ignored, we have

$$P_\ell(A) = \begin{cases} 0 & \text{if } A^4C^2 \leq \kappa^2 \\
\text{Min} \left( 1, \frac{1}{2} \frac{A^4C^2 - \kappa^2}{\Delta \omega} \right) & \text{if } A^4C^2 > \kappa^2 \end{cases}$$

where $\Delta \omega = \omega_{\ell,n-1} - \omega_{\ell,n}$ denotes the frequency distance between consecutive g-modes of degree $\ell$. Note that $\Delta \omega/2$ is the maximum frequency distance between the radial and the nearest nonradial mode and that $\sqrt{A^4C^2 - \kappa^2}$ is the distance at the onset of the instability.

The coupling coefficients, $C$, for various radial & nonradial mode pairs in the model of RR Lyrae star was explicitly calculated by Van Hoolst et al. (1998). From their numbers we found an approximate relation

$$C_{k,\ell} = b_k I_{0,k}/I_\ell,$$  \hspace{1cm} (3)

with

$$b_0 = 27 \quad \text{and} \quad b_1 = 172d^{-1},$$

where $k$ denote radial mode order (here $k = 0$ and 1 for the fundamental and the first overtone, respectively). $I$ denotes mode inertia evaluated assuming the same amplitude at the surface. Our additional simplification consists in adopting the same $b_k$ values for all our models.

Final simplification, which we adopted after Van Hoolst et al. (1998), is the assumption that $\kappa = -\gamma_g$, where $-\gamma_g$ is the damping rate due to dissipation in the g-mode propagation zone. This seems well justified because we consider the situation when the opacity driven instability is saturated by the radial mode and the resonant nonradial modes have almost the same properties in the outer layers. Consequently, there should be also an exact balance between the driving and damping also for the nonradial mode. Then $-\gamma_g$ is all what remains.

The joint probability of the instability is given by

$$P(A) = 1 - \prod_{\ell} \left[ 1 - P_\ell(A) \right].$$  \hspace{1cm} (4)
Figure 7: Similar to Fig. 6 but for the first overtone. In this case the $\ell = 1$ and 4 are most likely excited.

For rotating stars we have to consider modes of different azimuthal numbers and evaluate probabilities $P_{\ell, m}$. The effect increases probability of the resonant instability (Dziembowski et al. 1988).

Very much like Van Hoolst et al. (1998) we find the maximum of probability of the resonant excitation of an $\ell = 1$ mode, both for the fundamental and the overtone radial modes and then an $\ell = 5$ or 6 mode for the fundamental and an $\ell = 4$ for the overtone. This is why we selected these $\ell$-values in Figs. 6 and 7. In addition, there are data for $\ell = 2$ modes. Excitation of these modes is less likely than the $\ell = 1$ modes because of higher inertia. The data on $I_2/I_0$ are important for evaluation of effects of rotation and magnetic field on radial pulsation. We will not discuss these effects here.

In Fig. 8 we present results of calculation of the excitation probabilities for modes of selected degrees which yield the dominant contribution to the joint probability. We chose $A = 0.075$ for the fundamental mode and $A = 0.025$ for the first overtone. These values correspond to the mean amplitudes
Figure 8: Probability of instability of radial pulsation in the fundamental and first overtone mode relative to excitation of nonradial modes of indicated degrees.
of radius variations in, respectively, RRab and RRc stars (e.g. Jones et al. 1988, Capricorni et al. 1989, Liu & Janes, 1990, Jones et al. 1992). The lower probability of the first overtone instability is a direct consequence of the lower value of $A$.

The probability of the resonant instability in most cases increases with pulsation period. The exception is the instability of the fundamental mode to higher degree modes. In this case damping in the g-mode propagation zone plays an important role. The value of $\kappa$ increases with $P_0$ and $\ell$. The increase reduces the chances for the instability and ultimately prevents it (see Eq. 2).

At typical amplitude of RRab the probability of excitation of $\ell = 1$ mode is between 0.25 and 0.5. This is not so different from the incidence of Blazkho effect which is estimated to be between 20% and 30%. The joint probability of the instability is always higher than 0.5 and close to 1 in most cases. However, excitation of modes with $\ell > 2$ may not lead to amplitude modulation. The incidence of Blazkho effect amongst RRc stars is lower. Kovacs et al. (1999) who analyzed data on a large sample of RRc stars from LMC found the effect in 1.4% of the objects. Our analysis suggest lower chances for the first overtone instability than the fundamental mode but not in such a disproportion.

5 Conclusions and discussion

Our survey shows that all RR Lyrae star models share all qualitative properties of the nonradial modes. There is always a large number of unstable low degree modes with frequencies close to unstable radial modes. However, owing to higher mode inertia, for most of nonradial modes the driving rates are much lower than those for radial modes. The exceptions are the strongly trapped (STU) modes which begin with $\ell$ degrees 7 to 10 (depending on the model) at frequencies somewhat above the fundamental radial mode and with $\ell = 5$ or 6 with at frequencies somewhat above the radial mode overtones. These modes are characterized by growth rates similar to radial modes. However, we argued that these modes are not likely candidates for identification of oscillation detected in some RR Lyrae stars. More likely candidates are the $\ell = 1$ modes. Their driving rates are by nearly an order of magnitude lower than radial modes but it is well known that the growth rate is not necessarily a good predictor of the finite amplitude pulsation.

We found also that parameters which determine the chances of the exci-
tation of nonradial radial modes through the 1:1 resonance do not vary much over the range of RR Lyrae stars parameters. According to our estimate the excitation has a high probability. In fact some nonradial modes should be excited in majority of the RRab pulsators and in a significant fraction of (30%) of RRc pulsators. The actual number should be greater because we ignored effect of rotation. Our crude estimate, which we did not detail here, shows that the effect is significant already at equatorial velocities of few km/s.

Why then the incidence of the anomalous behavior among RR Lyrae stars is relatively rare? We should stress, that a significant amplitude modulation is not automatically implied by the nonradial mode excitation. If the nonlinear interaction between radial and nonradial modes leads to a steady pulsation with constant amplitude then the presence of the nonradial mode will not be easy detectable. A Blazhko-type amplitude modulation may arise then only if the nonradial mode is not symmetric about the rotation axis and it is of low degree. In this case the Blazhko period is equal to the rotation period. Another possibility is a periodic limit cycle in which the amplitudes of the two modes vary intrinsically with the period determined by the nonlinear interaction.

The ultimate answer regarding the presence of nonradial modes in RR Lyrae stars may be expected only from spectropy. A signature of such modes should be searched in the line-profile variations. Thus, high-resolution spectroscopic observations of amplitude modulated RR Lyrae stars are encouraged.

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