Spatial High Precision Scanning Control Method Based on Feedforward Closed-Loop Model Reference Adaptive

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Abstract. The time-modulated Fourier transform spectrometer realizes spectrum detection by scanning the optical path of the corner mirror. During the scanning process, the servo system is required to have high-precision and low-speed characteristics. Aiming at the fluctuation of scanning speed caused by spatial micro-vibration during scanning, a closed-loop model reference adaptive control algorithm based on feedforward is studied. The permanent magnet synchronous linear motor is used to drive the angle mirror to move back and forth along the guide rail to achieve large optical path and high-precision scanning with the maximum optical path difference of ± 34cm, the speed stability ≥ 99%.

1. Introduction
Fourier transform infrared spectrometer is widely used to explore the composition of planetary atmosphere and the characteristics of solid surface due to its wide spectral range, high spectral resolution, accurate wavenumber and radiance calibration and high detection reliability[1-2]. The high-resolution spectrometers "Fy- 3", "Fy- 4" and "GF-5" launched in China in recent years are all time-modulated Fourier transform spectrometers. Because of their different scanning methods and maximum scanning optical path difference, different spectral resolutions are obtained [3-4]. Compared with the above loads, the direct driving method of permanent magnet synchronous linear motor and linear guide rail is adopted in this paper, which overcomes the disadvantages of limited swing angle of swing arm scanning and insufficient to provide large optical path difference, so that the spectral resolution and moving mirror travel are greatly improved [5-6]. In view of the interferometer working in a space environment, the micro-vibration in space causes velocity fluctuations, and it is difficult to achieve 99% speed stability. This paper studies the adaptive control method to achieve low-speed smooth motion of the corner mirror, and proposes a closed-loop model based on feedforward.

2. Controlled Object
The voltage balance equation of permanent magnet synchronous linear motor is

\[ L_s \frac{d^2 l_s(t)}{dt^2} + R_s l_s(t) = u(t) - K_e \frac{dl_s(t)}{dt} \]  

(1)

and the motor equation of motion is

\[ \frac{d^2 l_s(t)}{dt^2} = \frac{K_i l_s(t)}{m} - \frac{1}{m} \frac{dl_s(t)}{dt} - \frac{F_s(t)}{m} \]  

(2)
where $i_a(t)$ is current, $R_a$ is resistance, $L_a$ is inductance, and $u(t)$ is motor voltage; $K_i$ and $K_e$ are thrust constant and back EMF constant respectively; $m$, $I_a(t)$, $B_m$ are load mass, load moving displacement and viscous friction coefficient respectively; $T_m(t) = K_i i_a(t)$ is the motor thrust, $T_L(t)$ is the load resistance. Let $v = \frac{dl(t)}{dt}$ be the speed of load movement, $a = \frac{d^2l(t)}{dt^2}$ be the acceleration of load movement. Therefore, equation (2) can be rewritten as,

$$ma = K_i l_a(t) - T_L(t) - B_m v$$

(3)

The state equation of the motor is established according to equations (1) ~ (3).

Define the position variable and speed variable are system state variables, let $x_1(t) = l_a(t)$ and $x_2(t) = v(t)$ the expression of the state is

$$\dot{x}(t) = Ax(t) + B(g(t) + u(t))$$

and the expression of the output is

$$y(t) = Cx(t)$$

(5)

where

$$A = \begin{bmatrix} 0 & 1 \\ 0 & \frac{-K_i K_e}{R_m} \end{bmatrix} \in \mathbb{R}^{2 \times 2}, \quad x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \in \mathbb{R}^2, \quad B = \begin{bmatrix} 0 \\ \frac{K_i}{R_m} \end{bmatrix} \in \mathbb{R}^2, \quad C = [1 \ 0] \in \mathbb{R}^2;$$

$$g(t) = -\frac{R_m}{K_i} - \frac{B_m R_m}{K_i} x_2(t).$$

Next, the control strategy will be designed based on the above controlled object.

3. Closed-Loop Model Reference Adaptive Control Algorithm Based on Feedforward

According to the closed-loop model reference adaptive control principle, the control quantity is adjusted according to the deviation between the reference model output and the system output. There is a certain time delay, which reduces the robustness of the system. Based on the known information of interferometer control system - reference velocity and acceleration, a closed-loop model reference adaptive control strategy based on feedforward is proposed quantity with a certain gain. The whole control strategy block diagram is shown in figure 1.

**Figure 1.** Block diagram of feedforward based closed-loop model reference adaptive control.
For equation (4), the state feedback adaptive rate is designed so that system $x(t)$ globally uniformly asymptotically tracks $x_{ref}(t) \in \mathbb{R}^2$ of the reference model, and the reference model is written as

$$\dot{x}_{ref}(t) = A_{ref}x_{ref}(t) + B_{ref}r(t) + \gamma e(t)$$

(6)

Where $A_{ref} \in \mathbb{R}^{2 \times 2}$ is Hurwitz matrix, $r(t) \in \mathbb{R}$ is bounded reference information, $\gamma \in \mathbb{R}^{2 \times 2}$ is positive feedback coefficient to be designed, and $\gamma^T = \gamma$. It is assumed that there are ideal known control gains $K_r \in \mathbb{R}$ and $K_x \in \mathbb{R}^2$, which satisfy $A_{ref} = A + BK_x$ and $B_{ref} = BK_r$.

The system perturbation is

$$g(t) = \theta^T(t)\phi(x(t)), \quad g(t) \in \mathbb{R}$$

(7)

where $\theta(t) \in \mathbb{R}^n$ is unknown constant matrix, $\phi(x(t)) \in \mathbb{R}^m$ is an uncertain regression vector.

Adaptive control uses the adaptive law to make $x(t)$ globally consistent and asymptotically track the reference state $x_{ref}(t)$. It is necessary to ensure that the output of system remains globally consistent and bounded during servo tracking. Therefore, given any bounded instruction $r(t)$, the choice of control input $u(t)$ must satisfy $e(t) = x(t) - x_{ref}(t)$ asymptotically approaching 0 globally, that is, $\lim_{t \to \infty} \|x(t) - x_{ref}(t)\| = 0$ is established.

The conventional control law is

$$u(t) = K_x x(t) + K_r r(t) + K_v r_v + K_a r_a$$

(8)

The adaptive control law is $u_a(t) = -\hat{\theta}^T(t)\phi(x(t))$, and the total control law of the system is $u(t) = u_{ref}(t) + u_a(t)$, where, $r_v$ and $r_a$ represent the reference speed and reference acceleration respectively, $K_v$ and $K_a$ represent the feedforward gains. $\hat{\theta}(t)$ is the estimation of the ideal weight vector $\theta(t)$, defined $\dot{\hat{\theta}}(t)$ as the first derivative of the weight vector $\hat{\theta}(t)$, the update rate of the weight vector is $\hat{\theta}(t) = \Gamma \phi(x(t))e^T PB$. $\Gamma = \Gamma^T > 0$ is the adaptive rate, $P = P^T > 0$ and $Q = Q^T > 0$ satisfy the algebraic Lyapunov equation satisfying equation (8),

$$PA_{ref} + A_{ref}^T P = -Q$$

(9)

Bring equations (7) ~ (8) into equation (4-5), let $\Delta \theta(t) = \hat{\theta}(t) - \theta(t)$.

Define Lyapunov energy function:

$$V(e(t), \Delta \theta(t)) = e^T(t)Pe(t) + \Delta \theta^T(t)\Gamma^{-1}\Delta \theta(t)$$

(10)

Where (10) is satisfied $P = P^T > 0$, and the first derivative of Lyapunov energy function is obtained

$$\dot{V}(e(t), \Delta \theta(t)) = -e^T(t)Qe(t) - 2e^T(t)\gamma Pe(t) \leq 0$$

(11)

so the $x(t)$ is uniformly bounded, $\dot{x}(t)$ and $e(t)$ are bounded

As

$$V(e(0), \Delta \theta(0)) \leq \lambda_{max}(P)e^T(0)e(0) + \frac{1}{\lambda_{min}(\Gamma)} \Delta \theta^T(0)\Delta \theta(0)$$

(12)

in $[0, t]$, take integral at both ends of $\dot{V}(e(t), \Delta \theta(t)) = 2e^T(t)(A_{ref} - \gamma)e(t)$, we have
\[
\int_0^t \dot{V}(e(\tau), \Delta \theta(\tau))d\tau = \int_0^t (2e^T(\tau)P(A_{\text{ref}} - \gamma)e(\tau))d\tau
\] (13)

Define \( \dot{V}(e(t), \Delta \theta(t)) = -e^T Q e - 2\gamma P \leq 0 \), the left side of equation (13) is less than zero, so

\[
V(e(0), \Delta \theta(0)) - V(e(t), \Delta \theta(t)) \geq 2\lambda_{\text{min}}(P)\lambda_{\text{min}}(A_{\text{ref}} - \gamma) \int_0^t \|e(\tau)\|^2 d\tau
\] (14)

Simultaneous (13) ~ (14), we have

\[
\lambda_{\text{max}}(P)e^T(0)e(0) + \frac{1}{\lambda_{\text{min}}(\Gamma)} \Delta \theta^T(0)\Delta \theta(0) \geq 2\lambda_{\text{min}}(P)\lambda_{\text{min}}(A_{\text{ref}} - \gamma)
\] (15)

\[
\int_0^t \|e(\tau)\|^2 d\tau \leq \frac{\lambda_{\text{max}}(P)e^T(0)e(0) + \frac{1}{\lambda_{\text{min}}(\Gamma)} \Delta \theta^T(0)\Delta \theta(0)}{2\lambda_{\text{min}}(P)\lambda_{\text{min}}(A_{\text{ref}} - \gamma)}
\] (16)

Therefore, the error \( L_2 \) is expressed as

\[
\|e(t)\|_{L_2} \leq \sqrt{\frac{\lambda_{\text{max}}(P)e^T(0)e(0) + \frac{1}{\lambda_{\text{min}}(\Gamma)} \Delta \theta^T(0)\Delta \theta(0)}{2\lambda_{\text{min}}(P)\lambda_{\text{min}}(A_{\text{ref}} - \gamma)}}
\] (17)

From equation (17), it can be seen that \( \|e(t)\|_{L_2} \) depends on parameters \( \Gamma \) and \( \gamma \), the influence of initial value \( \Delta \theta(0) \) of parameter estimation error on tracking error is reduced by increasing \( \Gamma \), and by adjusting \( \gamma \) to reduce system tracking error, which is also the aspect that the closed-loop model reference adaptive control is better than that of the open-loop.

Next, the steady-state characteristics of the feedforward based closed-loop model reference adaptive control algorithm will be obtained by analyzing \( L_\infty \).

The control quantity of equation (17) is brought into the state space model of the controlled object

\[
x(t) = Ax(t) + B \left( K_x x(t) + K_r r(t) + K_\theta \dot{\theta}(t) \varphi(x(t)) \right)
\]

\[
= (A + BK_x)x(t) + BK_r r(t) + B \left( K_r r(t) + K_\theta \dot{\theta}(t) \right) \varphi(x(t))
\] (18)

The solution of the above formula is

\[
x(s) = (sl-A_{\text{ref}})^{-1}B_{\text{ref}} r(s) + (sl-A_{\text{ref}})^{-1}B \left( K_r r(s) + K_\theta \dot{\theta}(s) \varphi(x(s)) \right)
\]

\[
+ (sl-A_{\text{ref}})^{-1}x(0)
\] (19)

The solution of the reference model \( \dot{x}_{\text{ref}}(t) = A_{\text{ref}} x_{\text{ref}}(t) + B_{\text{ref}} r(t) + \gamma e(t) \) is

\[
x_{\text{ref}}(s) = (sl-A_{\text{ref}})^{-1}B_{\text{ref}} r(s) + (sl-A_{\text{ref}})^{-1}\gamma e(s) + (sl-A_{\text{ref}})^{-1}x_{\text{ref}}(0)
\] (20)

Suppose that \( x_{\text{ref}}(0) = x(0) \), subtract equations (19) and (20) to obtain

\[
e(s) = \frac{B \left( K_r r(s) + K_\theta \dot{\theta}(s) \varphi(x(s)) \right)}{(sl-A_{\text{ref}} + \gamma)}
\] (21)

For a uniformly asymptotically stable MIMO system, when the input and output are bounded, it is finite.
The upper limit of tracking error $e(t)$ can be reduced by feedforward control parameters $K_1$ and $K_2$.

4. Simulation Results

$\begin{bmatrix} 0 & 1 \\ 0 & -6.1615 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.5187 \end{bmatrix} K_s = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta_\omega_n \end{bmatrix} \begin{bmatrix} 0 \\ 0.5187 \end{bmatrix} K_r = \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix}$

Let the feedback gains are $K_s = [-30.8482 -1.2311], K_r = [30.8482]$, and the adaptive law gains are $\Gamma = 10, Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$.

The planning curves of the angle mirror movement are shown in the following figure, which includes the displacement curve, the velocity curve, and the acceleration curve. Next, curves will be used as the input of the control system to verify the control algorithm.

Figure 2. Displacement, velocity and acceleration planning curves.

The thermal flutter disturbance is selected as the micro vibration model, and the micro vibration is composed of micro vibration sources with different frequencies and amplitudes. Set 20Hz as the dominant frequency of micro vibration, superimpose two micro vibration components of 10Hz and 30Hz, the amplitudes are 0.25 times and 0.5 times of the dominant frequency, and the initial phases are shifted by 30° and 45°, respectively. The expression is

$d_i(t) = \sum_{i=1}^{k} A_i \sin(\Omega_i t + \Phi_i)$

$= A_{11} \sin(40\pi t) + \frac{3}{4} A_{11} \sin(20\pi t + \frac{\pi}{6}) + \frac{1}{2} A_{11} \sin(60\pi t + \frac{\pi}{4})$ (24)

where $A_{11}$ is the undetermined constant.

In the simulation of suppressing micro vibration disturbance, different $A_{11}$ values are taken to simulate the two algorithms. This paper analyzes and compares the two algorithms through the
displacement output and velocity output, and estimate the maximum space micro-vibration disturbance torque that can be suppressed by the control algorithm.

**Figure 3.** Velocity error curve of the closed-loop model reference adaptive control under different micro vibration amplitudes.

Figure 3 shows the control effects of the closed-loop model reference adaptive control algorithm on these disturbances when $A_1$ is 0.001N, 0.005N, 0.01N and 0.03N are taken in the micro vibration model (24). Figure a shows the velocity error curve of the corner mirror movement in the u-turn section, and the curve is locally enlarged, as shown in Figure b; Figure c is the velocity error of intercepting the uniform velocity section, and the curve is locally enlarged, as shown in Figure d. Because the disturbance is always in a fluctuating state, when the system is stable, the angle mirror cannot achieve complete uniform scanning, and the velocity error is the fluctuating value. The velocity fluctuations in the uniform section caused by different micro vibration disturbance torque amplitudes are 99.63%, 99.58%, 97.57% and 96.24%.

Figure 4 shows the velocity errors of the control algorithm with feedforward to suppress different micro vibrations under the same simulation environment as the above conditions. The velocity fluctuations in the uniform section caused by different micro vibration disturbance torque amplitudes are 99.73%, 99.62%, 98.14% and 97.02%.

Comparing figure 2 and figure 3, the two control algorithms in the uniform speed section are not much different in suppressing chattering. The advantage of the adaptive algorithm with feedforward control in the u-turn section is more obvious, and the speed error is one order of magnitude lower than that of the non-feedforward control algorithm. The u-turn angle mirror adopts sine function motion, the u-turn section velocity fluctuation with feedforward control strategy is small, the response is fast, the time to adjust to the uniform speed is short, the optical path scanning efficiency is improved, and the system has good robustness.
5. Conclusion

The translational optical path reciprocating scanning system based on adaptive control strategy studied in this paper has long scanning stroke, small motion quality and short optical path, which is more suitable for large field of view space-borne detection instruments. It has laid the foundation for the development of time-modulated Fourier spectrometers with high-precision and high-resolution spectral detection capabilities.

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