Pointlike electrons and muons and the nature of neutrinos

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We show that the form factor of an electron or a muon is unity for the modulus of the momentum transfer away from the Hagedorn temperatures $T_0^q \sim m_e$ and $T_0^\tau \sim m_\tau$, and lower than the hadronic Hagedorn temperature $T_0^{QCD} \sim m_s$. The structure of the electron or the muon is resolved for momentum transfers comparable to $T_0^{QCD}$. This is seen in scattering experiments performed in the 1950s, 1960s, and early 1970s to test the validity of QED. We interpret center-vortex loops with no self-intersection, occurring in the confining phases of each SU(2) gauge-theory factor, as Majorana neutrinos. We offer a possible explanation for a contribution to $\Lambda_{\text{CMB}}$ not generated by the far-off nonconfining, strongly interacting gauge theory SU(2)$_{\text{CMB}}$.

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Introduction. The main purpose of this Letter is to extend and substantialize a thermal theory for charged leptons put forward in [1]. The work in [1] is based on the approach in [2]. The extension is related to the existence and the nature of neutrinos. We will not address the interactions of neutrinos with charged leptons. In the present Universe the gauge dynamics subject to the symmetry SU(2)$_{\text{CMB}} \times SU(2)_e \times SU(2)_\mu \times SU(2)_\tau$ predicts the existence of stable fermionic states (charged leptons) with one unit of electric charge [3]. In addition, it predicts the existence of one (entangled) asymptotic photon. The electric charge is a magnetic charge in each of the SU(2) gauge-theory factors. Its coupling to the photon (the fine-structure constant $\alpha$) can be postdicted to 2.4% accuracy in a tree-level treatment of the thermodynamics. Our theory seems to resolve the long-standing problem of the apparent infinite self-energy of a structureless charged lepton and most probably a number of other important problems [1]. We believe that the gauge theory with symmetry SU(2)$_{\text{CMB}} \times SU(2)_e \times SU(2)_\mu \times SU(2)_\tau$ is underlying Quantum Electrodynamics (QED). Here we substantialize this theory in view of the following apparent contradiction: In scattering, annihilation, and production processes involving purely electrodynamical interactions one would expect to see the structure of the charged lepton unless some so-far unknown mechanism prevents Nature from showing this structure to us most of the time. As we will argue below, this mechanism is rooted in the existence of an exponentially growing density of extremely unstable excitations.

Mini-review of experimental situation. The elastic differential cross section for $e^-e^-$ (ee) scattering was measured in [3] for electron (positron) incident kinetic energies between 0.6 MeV and 1.2 MeV (0.6 MeV and 1.0 MeV) and an energy transfer of 50% the incident kinetic energy. For Moeller (Bhabha) scattering good agreement of experiment with theory was observed at the high-energy ends. At 0.6 MeV incident kinetic energy the differential cross sections turned out to be significantly lower than the theoretical Moeller (Bhabha) prediction (it is clear that binding effects are entirely negligible). Measurements of $e^-e^-$ elastic scattering at 6.1 MeV and 15.7 MeV incident kinetic electron energy were reported in [3, 5]. Excellent agreement with the Moeller prediction was obtained. In the 1960s and early 1970s violations of (tree-level) QED were parameterized by a cutoff scale in the form factor $F_\Lambda(q^2) = (1 - q^2/\Lambda^2)^{-1}$ of the charged lepton. Elastic $e^-e^-$ scattering experiments gave different values for $\Lambda$: $\Lambda > 0.76$ GeV (4.4 GeV) for $e^-e^-$ scattering at $\sqrt{s} = 600$ MeV ($\sqrt{s} = 1112$ MeV) and 95% confidence [3, 5, 7]. A test of QED by $e^-e^+$ scattering at $\sqrt{s} = 2 \times 510$ MeV yielded $\Lambda > 3.8$ GeV (positive metric) and $\Lambda > 2.0$ GeV (negative metric) [12]. Experiments using a bremsstrahlungsbeam incident on a carbon target to produce $e^-e^+$ pairs at wide angles [4, 10] obtained the ratio $R$ of experimental, differential cross section to the Bethe-Heitler prediction (the contribution of Compton processes is very small). In [8] $R$ was found to deviate strongly from unity for $e^-e^+$ invariant masses $M$ in the vicinity of the muon threshold. In particular, for $M < 170$ MeV $R$ was measured to be significantly below unity whereas for $M > 200$ MeV $R$ values were measured consistent with or larger than unity. In [10] the experiment was repeated, and a much better consistency of $R$ with unity was claimed. Still, $R$ was measured to be significantly below unity at $M \sim 170,310,450$ MeV and above unity at $M \sim 380$ MeV. For $500 < M < 900$ MeV experiment turned out to be consistent with $R = 1$. Similar results were obtained in [11]. In [13] a rather weak lower bound of $\Lambda > 1.3$ GeV was measured in $e^-e^+$ annihilation into $2\gamma$ at $\sqrt{s} = 1$ GeV, and in [14] the QED tree-level prediction for total muon production cross section by $e^-e^- \rightarrow \mu^+\mu^-$ was found to be consistent with experiment at $1020$ MeV < $\sqrt{s}$ < 1340 MeV. In [15] data on wide-angle $e^-e^-$ Bhabha scattering at $1.4$ GeV < $\sqrt{q^2}$ < 2.4 GeV are reported. In the vicinity of the $\tau$ threshold significant deviations of the measured cross section from the QED prediction were seen (three data points at $\sqrt{q^2} = 1.8,1.85,2.4$ were not consistent with the QED prediction). For a compilation of QED tests at higher energy see [16]. A conspicuous feature is that differential cross sections for the reaction $e^+e^- \rightarrow e^+e^-$ deviate significantly from the QED prediction at small scattering angle where the momentum transfer is close to the kinematic threshold for $\tau$ produc-
Hagedorn then argues that the only admissible solutions to this condition are exponentially increasing densities of state. The occurrence of a highest temperature \( T_0 \) is an immediate consequence of this solution. Subsequently, Hagedorn applies his theory to a description of scattering and annihilation processes in strong interactions. For center-of-mass energies considerably (but not too much) larger than the highest temperature \( T_0 \) there is no real difference between bosonic and fermionic statistics. The probability density \( w(p_\perp) \) for generating a particle of mass \( m \) and transverse momentum \( p_\perp \) in the collision is derived as

\[
w(p_\perp) = \text{const} \times p_\perp \sqrt{\frac{T_0}{p_\perp^2 + m^2}} \exp \left[-1/T_0 \sqrt{\frac{p_\perp^2 + m^2}{1/T^2}}\right].
\]  

(2)

For example, a differential elastic cross section for \( 2 \to 2 \) scattering contains three types of factors: (a) \( w(p_\perp) \) in \( w(\mathbf{E}) \), (b) the probability that the number of particles is two, and (c) kinematical and geometrical factors being algebraic in \( p_\perp \) and \( E \). Wu and Yang observe \( 19 \) that the electromagnetic form factor \( F(q^2) \) of the proton (we do not distinguish between electric and magnetic form factors in this Letter) is an exponentially falling function of the momentum transfer \( \sqrt{q^2} \) as

\[
F(q^2) \to \exp \left[-\sqrt{q^2}/(4T_0)\right] \quad (\sqrt{q^2} \gg T_0).
\]  

(3)

Hagedorn interprets Eq. (3) as a supporting fact for his thermodynamical theory for particle scattering in strong interactions. He obtains an estimate for \( T_0 \sim 165 \text{ MeV} \) by taking the inverse pion mass as a natural radius for the fire-ball mediating the scattering of hadrons.

**QED, an exponentially rising density of states, and neutrinos.** Hagedorn developed his thermodynamical approach to particle scattering within the framework of strong interactions. He did not anticipate the possibility that Quantum Electrodynamics (QED) could be generated by a strongly interacting theory in the sense proposed in \( 1 \). This theory possesses an exponentially growing density of states. The spectrum of states in the confining (or center) phase for one of the SU(2) gauge theories in \( SU(2)_{\text{EM}} \times SU(2)_e \times SU(2)_\mu \times SU(2)_\tau \) is depicted in Fig. 1. In \( 1 \) we have interpreted the single center-vortex loop (the first state in Fig. 1) as a dark matter particle since it does not carry electric charge. We did not specify further the nature of this particle. At first sight we have rejected the possibility that this particle is the neutrino associated with the charged lepton. This seemed to be justified by the (erroneous) conclusion that the mass of the single center-vortex loop should be comparable to the mass of the charged lepton. In the meantime we have changed our conclusion. A center-vortex loop with no self-intersection has vanishing energy in the confining phase for a sufficiently large loop size \( 20 \). This is the reason why center-vortices condense in the first place (in \( 21 \) a beautiful discussion is given of the distribution of topological charge in dependence of the gradient of the tangential vector to the loop). Thus, the associated particle has vanishing mass. There is an
The spectrum of excitations of an SU(2) gauge theory in its confining phase. The thick lines denote (large) center-vortex loops, and the dots represent crossings of center vortices. Each crossing carries ± one unit of electric charge. The first two states correspond to stable excitations. States with more than one center-vortex crossing are extremely unstable in the presence of photons which belong to SU(2) gauge-theory factors not being in their center (or confining) phases. The electric charges in such 'hadrons' either (very rarely) annihilate into neutrinos or (much more often) into photons. A 'hadron' is torn apart by Coulomb repulsion if only equal-sign electric charges reside in it.

FIG. 1: The spectrum of excitations of an SU(2) gauge theory in its confining phase. The thick lines denote (large) center-vortex loops, and the dots represent crossings of center vortices. Each crossing carries ± one unit of electric charge. The first two states correspond to stable excitations. States with more than one center-vortex crossing are extremely unstable in the presence of photons which belong to SU(2) gauge-theory factors not being in their center (or confining) phases. The electric charges in such 'hadrons' either (very rarely) annihilate into neutrinos or (much more often) into photons. A 'hadron' is torn apart by Coulomb repulsion if only equal-sign electric charges reside in it.

The gauge group describing electrodynamics was proposed in [2] to be SU(2)_{CMB} × SU(2)_{e} × SU(2)_{μ} × SU(2)_{τ}. For momentum transfers \sqrt{q^2} much smaller than the Hagedorn temperature \( T_0^CMB \) but larger than \( T_0^CMB \sim 10^{-4} \text{eV} \) there is only one massless photon in the theory [1]. This photon is quantum entangled with two very heavy photons which belong to the two gauge-theory factors in SU(2)_{e} × SU(2)_{μ} being in their confining phases [1]. In an on-shell state (no restriction on its energy) the massless photon can be considered as a part of the thermal ensemble, characterized by the temperature \( T_{CMB} \). This is true because the photon, possibly a radio wave or visible light or a \( \gamma \) ray, only generates a temperature fluctuation \( \Delta T/T_{CMB} \ll 1 \) which is much smaller than the primordial fluctuations seen in the CMB. We thus may view the entire visible Universe as a fire-ball of temperature \( T_{CMB} \). The mechanism which decreases the temperature of this fire-ball is gravitational expansion. For elastic scattering of charged leptons the photon couples to the respective lepton via a form factor which is written as a sum (recall postulate (iii) above, the scattering is a statistical process involving all purely leptonic resonances) of the type

\[ F(q^2) \sim \frac{1}{2} \left( \exp \left[ -\sqrt{q^2}/(4T_0^c) \right] + \exp \left[ -\sqrt{q^2}/(4T_0^\mu) \right] \right), \]

where \( T_0^c \) and \( T_0^\mu \) denote the Hagedorn temperatures belonging to the factors in SU(2)_{e} × SU(2)_{μ}. We have omitted the form factor due to \( T_{CMB}^CMB \) since for any so-far performed QED elastic scattering, annihilation or production experiment the momentum transfer are much larger than this temperature, and thus the exponential suppression is extremely large. The \( \sim \) sign in [1] indicates that the exponential dependence is only valid sufficiently far above or below a kinematical threshold for the production of a charged lepton (in practice a factor of two or three is sufficient). Close to the threshold the exponential should be replaced by a Hankel function [15]. The normalization \( 1/2 \) defines the fine-structure constant to be \( \alpha = 1/137 \) for momentum transfer considerably smaller than \( T_0^c \) [1]. As for the \( \tau \) lepton, its mass \( m_\tau \sim 1777 \text{MeV} \) is much larger than the hadronic threshold \( m_\tau \sim 140 \text{MeV} \), and the theory SU(2)\_τ is 'contaminated' by the hadronic world and weak interactions: Fire-balls generated by momentum transfers higher than \( m_\tau \), which rather often occur in processes involving the \( \tau \) lepton, do contain strongly and weakly decaying hadronic resonances. As a consequence, the factor SU(2)\_τ should be treated separately. So far we do not understand why quarks and charged hadrons couple to the same photon as the electron does. It is possible that the three (al-
most) massless photons of $SU(2)_{\text{CMB}} \times SU(2)_{e} \times SU(2)_{u}$, which contribute to scattering processes at momentum transfers larger than $T_0^\mu$ but lower than $T_0^{\text{QCD}}$, are quantum entangled with very heavy photons originating from a pure SU(N) gauge theory disguising itself as QCD in its confining phase. The three massless photons would then see the same electric charge as the QCD photons saw at temperatures considerably larger than $T_0^{\text{QCD}}$. So far we can only speculate about the pure gauge theory which generates quarks. More definite investigations will be performed in the future. The exponentials parametrized by $T_0^\mu \sim 1/200 T_0^e$ are practically unity for $\sqrt{q^2}$ sufficiently below $T_0^e$. As a consequence, the form factors of $e^\pm$ or $\mu^\pm$ are unity. Both the electron and the muon appear to be pointlike in elastic scattering. This statement should generalize to annihilation, pair production, or Compton scattering. For momentum transfer much larger than $T_0^e$, but smaller than $T_0^\mu$ the first exponential in (4) is strongly suppressed as compared to the second one. At first sight one would conclude that the effective form factor is $1/2$ instead of the measured value unity. However, the loss in form factor is precisely compensated by the occurrence of an additional photon in the fire-ball (for momentum transfer considerably larger than $T_0^e$) the $SU(2)_{e}$ gauge dynamics is in addition to $SU(2)_{\text{CMB}}$ in its electric phase, see [1]). This effectively multiplies the left-over exponential by a factor of two. The resulting form factor for $e^\pm$ or $\mu^\pm$ is again, unity – in perfect agreement with experiment [2, 3]. Approaching $T_0^e$ from below, the fall-off of the exponential is not yet compensated by an additional, massless photon: the cross section is below the QED prediction. This situation is, indeed, an experimental result [4, 5].

For $\sqrt{q^2}$ much larger than $T_0^e$ but lower than $T_0^\mu$ the electron should appear structureless again. This is verified in an experiment [6]. For momentum transfers $\sqrt{q^2}$ considerably larger than $T_0^e$ we run into hadronic contaminations which we are not able to handle at present. For $\sqrt{q^2}$ slightly higher than $T_0^\mu$ the left over exponential in (4) starts to become active. This is, again, compensated by the kinematical liberation of an additional photon in the fire-ball. As a consequence, we do not expect a drastic deviation from unity of the effective form factor for $e^\pm$ as we cross the muon threshold. This expectation is in agreement with the experimental data in [11, 12] but not in [6]. In conclusion, we have made sufficiently clear that the structure of the electron and the muon is only resolved in elastic scattering experiments for momentum transfers close to the masses of these charged leptons.

**Possible explanation of missing $\Lambda_{\cosmo}$.** Although topically not in the main line of this Letter we use the remaining space to speculate on the mechanism which generates the part in $\Lambda_{\cosmo}$, not generated by the strongly interacting gauge theory underlying electromagnetism [1]. Let us assume that all particles and their interactions are generated by products of SU(N) Yang-Mills theories. As a consequence, stable and massive particles are solitons of these theories in their confining phases. Particle collisions with momentum transfers larger than the associated Hagedorn temperature $T_0^{\text{SU(N)}}$ generate local fire-balls where the gauge theory is not in its confining phase. Locally, the theory generates negative pressure [2] shortly before the fire-ball dissolves. For example, shortly before the formation of the $e^+e^-$ pair the theories $SU(2)_{e}$ or $SU(2)_{\mu}$ generate negative pressure inside the local fire-ball generated in elastic $e^+e^-$ collisions at $\sqrt{s}$ higher than $T_0^{e\mu}$. A similar situation holds in relativistic hadron or heavy-ion collisions. Space expands quasi-exponentially in time within fire-balls of equation of state close to $\rho = -P$. This is in contrast to the power-law behavior within a radiation dominated fire-ball occurring in the very early stages of a collision with momentum transfer much larger than the Hagedorn temperature of the associated SU(N) gauge theory. Averaging the local equation of state over space at a given time in a gravitationally gauge invariant way, we thus expect a negative, global equation of state to be generated [22]. This coarse-grained contribution to $\Lambda_{\cosmo}$ could make up for the too small homogeneous part in $\Lambda_{\cosmo}$ originating from $SU(2)_{\text{CMB}}$.

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