Baryogenesis from axion inflation

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based mainly on
1806.08769, 1905.13318
in collaboration with
Benedict von Harling, Enrico Morgante
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see also 1807.03358, 1812.08021
with Yohei Ema, Ben Mares, Francesco Muia,
Kyohei Mukaida, Mauro Pieroni and Ryosuke Sato
Outline

- Helical gauge field and chiral fermion production during axion inflation
- Baryogenesis from helical gauge fields @ EWPT
- Chiral MHD: from inflation to the EWPT
‘Axion’ inflation

Slow-roll inflation \(\rightarrow\) very flat scalar potential

Reheating after inflation \(\rightarrow\) coupling to the SM

Inflaton as Pseudo Goldstone Boson (PNGB) with shift-symmetric couplings

\[
\phi F_{\mu\nu} \tilde{F}_{\mu\nu} \quad \text{and} \quad (\partial_\mu \phi) \bar{\psi} \gamma^\mu \gamma^5 \psi
\]

related by the chiral anomaly equation:

\[
0 \neq \partial_\mu J_5^\mu = -\frac{1}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}
\]
‘Axion’ inflation

\[ \mathcal{L} = \sqrt{-g} \left[ \frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} Y_{\mu\nu} Y^{\mu\nu} + \sum_\alpha \bar{\psi}_\alpha \left( i \gamma_5 \gamma^5 \right) \left( i \partial_\mu \gamma_5 Y_{\mu\nu} \gamma_5 \right) \psi_\alpha + \frac{\alpha Y_{\mu\nu}}{4\pi f_a} Y^{\mu\nu} \]

after chiral fermion rotation:

\[ (\partial_\mu \psi) \bar{\psi} \gamma^\mu \gamma^5 \psi \]

during inflation we can neglect

- Yukawas not effective
- non-abelian gauge groups will not be relevant for baryogenesis
- SM Higgs assumed to be stabilized at zero vev

PNGB inflaton coupled to SM
helical gauge field production

ignoring fermions for a moment:

\[
\mathcal{L} = -\partial_\mu \phi \partial^\mu \phi - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} - V(\phi) - \frac{\alpha}{4f_a} \phi F^{\mu\nu} \tilde{F}_{\mu\nu}
\]

\[
\frac{d^2 A_\pm}{d\tau^2} + \left[ k^2 \pm 2k \frac{\xi}{\tau} \right] A_\pm(\tau, k) = 0, \quad \xi = \frac{\alpha \dot{\phi}}{2H f_a}
\]

explosive helical gauge boson production

approximately homogeneous gauge field background

Turner, Widrow '88, Garretson, Field, Caroll '92

baryogenesis from axion inflation
fermion production

**helical gauge field production**
- one helicity of gauge field acquires tachyonic mass
- parallel E,B fields; constant & homogeneous on scales $\ll H^{-1}$

**(chiral) fermion production**
- fermion production in constant E,B background
- quantum `Schwinger - type’ production ($\rightarrow$ anomaly equation)

**backreaction on gauge field production**
- fermions are accelerated in gauge field background
- induced current inhibits gauge field production

$$\Box A^\nu - \partial_\mu \left( \frac{\alpha \phi}{\pi f_a} \tilde{F}^{\mu \nu} \right) - g Q_J^\nu = 0$$
fermion production

helical gauge field production

- one helicity of gauge field acquires tachyonic mass
- parallel E,B fields; constant & homogeneous on scales $<< H^{-1}$

(chiral) fermion production

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$$\Box A^\nu - \partial_\mu \left( \frac{\alpha \phi}{\pi f_a} \tilde{F}^{\mu\nu} \right) - g Q J_\psi^\nu = 0$$

baryogenesis from axion inflation
backreaction on gauge field production: 
\[ \Box A^\nu - \partial_\mu \left( \frac{\alpha \phi}{\pi f_a} \tilde{F}^{\mu \nu} \right) - gQ J_\psi^\nu = 0 \]

stationary solution:
\[ 0 = \dot{\rho}_A = -4H \rho_A + 2\xi H \hat{E} \cdot \hat{B} - \hat{E} \cdot gQ \langle J_\psi \rangle \]

upper bound on E- and B-fields
backreaction on gauge field production:

\[ \Box A^\nu - \partial_\mu \left( \frac{\alpha \phi}{\pi f_a} \tilde{F}^{\mu \nu} \right) - gQ J_\psi^\nu = 0 \]

stationary solution:

\[ 0 = \dot{\rho}_A = -4H \rho_A + 2\xi H \mathbf{E} \cdot \mathbf{B} - \mathbf{E} \cdot gQ \langle J_\psi \rangle \]

\[ EY_B H^{-4} \]

\[ \xi = 22 \]

\[ Q = 0 \]

constraints from inflation with

\[ r = 0.1 \]

\[ r = 0.05 \]

upper bounds on E- and B-fields

baryogenesis from axion inflation
conserved currents & charges

\[ \phi \]

**shift symmetry**
\[ \phi \mapsto \phi + \theta \]

**chiral charge:**
left- vs right-handed

**axial U(1)**
\[ \psi_R \mapsto e^{i \theta_A} \psi_R \]
\[ \psi_L \mapsto e^{-i \theta_A} \psi_L \]

**vector U(1)**
\[ \psi_R \mapsto e^{i \theta_V} \psi_R \]
\[ \psi_L \mapsto e^{i \theta_V} \psi_L \]

**Chern Simons charge**
of gauge field configuration

\[ J_\phi^\mu \equiv f_a g^{\mu\nu} \partial_\nu \phi \sim f_a \dot{\phi} \]
\[ J_5^\mu \equiv \bar{\psi} \gamma^\mu \gamma_5 \psi \]
\[ K_{CS}^\mu \equiv \frac{\alpha}{\pi} e^{\mu\nu\rho\sigma} A_\nu \partial_\rho A_\sigma. \]
\[ J_\psi^\mu \equiv \bar{\psi} \gamma^\mu \psi \]

**Dual fermion and gauge field production**
driven by rolling inflaton

\[ 0 = \partial_\mu J_\psi^\mu, \]

\[ \partial_\mu \left( \sqrt{-g} J_\phi^\mu + \frac{1}{2Q^2} J_5^\mu \right) = \partial_\mu \left( \sqrt{-g} J_\phi^\mu - \frac{1}{2} K_{CS}^\mu \right) = -\sqrt{-g} f_a V'(\phi) \approx 0 \]

\[ \phi \neq 0 \]

**baryogenesis from axion inflation**
Helical gauge field and chiral fermion production during axion inflation

Baryogenesis from helical gauge fields @ EWPT

Chiral MHD: from inflation to the EWPT
baryogenesis from helical gauge fields

EW phase transition: \( U(1)_Y \rightarrow U(1)_{em} \) (cross-over, adiabatic)

If\(^(*)\) helicity in gauge fields is preserved until EW phase transition,

anomaly equation dictates generation of fermion asymmetry:

\[
\partial_\mu J_{B+L}^\mu = -\frac{3}{16\pi^2} (g_Y^2 \gamma_{\mu\nu} \tilde{\gamma}^{\mu\nu} - 2g_W^2 W_{\mu\nu}^a \tilde{W}_a^{\mu\nu}) \approx 0
\]

\[
\eta_B = \frac{q_B}{s} \simeq \frac{17}{37} \left[ (g_W^2 + g_Y^2) \frac{f(\theta_W, \hat{T}) S}{\gamma_{W,\text{sph}}} \right] T=135 \text{ GeV}
\]

\(-\)

\(\sim\)

source = helicity

efficiency = change of Weinberg angle

\[ f(\theta_W, \hat{T}) = -\hat{T} d\theta_W/d\hat{T} \sin(2\theta_W) \]

sphaleron washout

Jimenez, Kamada, Schmitz, Xu '17

\(\sim\)

\(\sim\)

(*) large conductivity in thermal plasma \( \delta_{\eta} h \sim \vec{E} \cdot \vec{B} \rightarrow 0 \),

supported by numerical MHD simulations [Banerjee, Jedamzik '04; Tahiashvili, Tevzadze, Brandenburger, Neronov '13; Brandenburger, Schober et al '17, '18]

but requires certain conditions \(\rightarrow\) next part of this talk
baryogenesis from helical gauge fields

Domcke, von Harling, Morgante, Mukaida ‘19

fermion backreaction
Outline

- Helical gauge field and chiral fermion production during axion inflation
- Baryogenesis from helical gauge fields @ EWPT
- Chiral MHD: from inflation to the EWPT

\[ \phi F_{\mu\nu} \tilde{F}_{\mu\nu} \]

inflation

helical gauge fields

chiral fermions

chiral thermal plasma

baryogenesis @ EWPT
chiral MHD

chiral magnetohydrodynamics:

\[ \dot{T}_{\text{diff}} \sim \alpha_Y H_{\text{rh}} \]

- diffusion
- inverse cascade
- chiral plasma instability

\[
\frac{\partial B_Y}{\partial \eta} = \frac{\nabla^2 B_Y}{\sigma_Y} + \nabla \times (\mathbf{v} \times B_Y) + \frac{2\alpha_Y}{\pi} \frac{711 \mu_{e1}}{481 \sigma_Y} \nabla \times B_Y
\]

\[
\frac{\partial \mu_{e1}}{\partial \eta} = -\frac{1}{T^2/3} \frac{\partial q_{CS,Y}}{\partial \eta} - \Gamma_{Y_e} \frac{711}{481} \mu_{e1}
\]

\[
\frac{\partial}{\partial \eta} \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = \nu \nabla^2 \mathbf{v} + \frac{1}{\rho + P} \left( -\frac{1}{2} \nabla B_Y^2 + (B_Y \cdot \nabla) B_Y \right)
\]

[see e.g. Durrer, Neronov '13]

non-linear coupled equations

baryogenesis from axion inflation
chiral MHD

chiral magnetohydrodynamics:

\[ \hat{T}_{\text{diff}} \sim \alpha_Y H_{\text{rh}} \]

diffusion \hspace{1cm} \text{inverse cascade} \hspace{1cm} \text{chiral plasma instability}

\[ \frac{\partial B_Y}{\partial \eta} = \frac{\nabla^2}{\sigma_Y} B_Y + \nabla \times (\mathbf{v} \times B_Y) + \frac{2\alpha_Y}{\pi} \frac{711}{481} \mu_{e1} \nabla \times B_Y \]

\[ \frac{\partial \mu_{e1}}{\partial \eta} = -\frac{1}{T^2/3} \frac{\partial q_{\text{CS},Y}}{\partial \eta} - \Gamma_Y \frac{711}{481} \mu_{e1} \]

conserved quantity
until \( Y_e \) becomes efficient \( @ T \sim 10^5 \text{ GeV} \)

\[ \frac{\partial}{\partial \eta} \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = \nu \nabla^2 \mathbf{v} + \frac{1}{\rho + P} \left( -\frac{1}{2} \nabla B_Y^2 + (B_Y \cdot \nabla)B_Y \right) \]

viscous regime
viscous regime
\[ v^2 \sim R_e B_Y^2 / \rho \]
\[ R_e \approx L_{\text{rh}} v_{\text{rh}} / \nu \]
\[ v^2 \sim B_Y^2 / \rho \]

\[ \sim \text{upper bound for } v \]
\[ \text{sourced by } B \text{- field} \]

baryogenesis from axion inflation
magnetic Reynolds number: \( R_m \approx \sigma_Y L v \)

neglect chemical potential for a moment:

\[
\frac{\partial B_Y}{\partial \eta} = \frac{\nabla^2}{\sigma_Y} B_Y + \nabla \times (\nu \times B_Y)
\]

for \( R_m > 1 \) (large velocity field), non-linear dynamics can prevent diffusion

can occur in turbulent and viscous regime

large \( R_m \) is a necessary requirement to preserve helicity

[Banerjee, Jedamzik ’04]
magnetic Reynolds number

maximal $E_Y B_Y$

equilibrium $E_Y B_Y$

lower bound on scale of inflation and on baryon asymmetry

baryogenesis from axion inflation

Valerie Domcke (DESY, Hamburg)
chiral plasma instability

chiral magnetohydrodynamics:

\[ \dot{T}_{\text{diff}} \sim \alpha_Y H_{\text{rh}} \]

- diffusion
- inverse cascade
- chiral plasma instability

\[ \frac{\partial B_Y}{\partial \eta} = \frac{\nabla^2}{\sigma_Y} B_Y + \nabla \times (\mathbf{v} \times B_Y) + \frac{2\alpha_Y}{\pi} \frac{711}{481} \frac{\mu e_1}{\sigma_Y} \nabla \times B_Y \]

\[ \frac{\partial \mu_{e1}}{\partial \eta} = -\frac{1}{T^2/3} \frac{\partial q_{CS,Y}}{\partial \eta} - \Gamma_{Ye} \frac{711}{481} \mu e_1 \]

\[ \frac{\partial}{\partial \eta} \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = \nu \nabla^2 \mathbf{v} + \frac{1}{\rho + P} \left( -\frac{1}{2} \nabla B_Y^2 + (B_Y \cdot \nabla)B_Y \right) \]

non-linear coupled equations

baryogenesis from axion inflation

[ see e.g. Durrer, Neronov '13 ]
chiral plasma instability

neglect velocity field for a moment, re-write eom for the two helicity modes of $B$:

\[ \partial_\eta h_k = -\frac{2k^2}{\sigma_Y} h_k + \frac{711}{481} \frac{8\alpha_Y \mu_{e_1}}{\pi} \rho_{B,k} \]

\[ \partial_\eta \rho_{B,k} = -\frac{2k^2}{\sigma_Y} \rho_{B,k} + \frac{711}{481} \frac{2\alpha_Y \mu_{e_1} k^2 h_k}{\pi} \]

with

\[ h_k = \frac{k^3}{2\pi^2} \left( |A_Y^+(\eta, k)|^2 - |A_Y^-(\eta, k)|^2 \right) \]

\[ \rho_{B,k} = \frac{k^4}{4\pi^2} \left( |A_Y^+(\eta, k)|^2 + |A_Y^-(\eta, k)|^2 \right) \]

\[ \eta_{\text{CPI}} \sim \frac{481}{711} \frac{\pi \sigma_Y}{\alpha_Y \mu_{e_1} k_{\text{CPI}}} \]

baryogenesis from axion inflation
neglect velocity field for a moment, re-write eom for the two helicity modes of $B$:

$$\partial_\eta h_k = - \frac{2k^2}{ \sigma_Y} h_k + \frac{711}{481} \frac{8 \alpha_Y}{\pi} \frac{\mu_{e_1}}{\sigma_Y} \rho_{B,k}$$

$$\partial_\eta \rho_{B,k} = - \frac{2k^2}{ \sigma_Y} \rho_{B,k} + \frac{711}{481} \frac{2 \alpha_Y}{\pi} \frac{\mu_{e_1}}{\sigma_Y} k^2 h_k$$

with

$$h_k = \frac{k^3}{2\pi^2} \left( |A^+_Y(\eta, k)|^2 - |A^-_Y(\eta, k)|^2 \right)$$

$$\rho_{B,k} = \frac{k^4}{4\pi^2} \left( |A^+_Y(\eta, k)|^2 + |A^-_Y(\eta, k)|^2 \right)$$

$$\eta_{CPI} \sim \frac{481}{711} \frac{\pi \sigma_Y}{\alpha_Y \mu_{e_1} k_{CPI}}$$
chiral plasma instability

maximal $E_Y B_Y$

equilibrium $E_Y B_Y$

window for scale of inflation and baryon asymmetry
Conclusion

- shift-symmetric coupling of inflaton to SM 
  - dual production of 
    - helical gauge fields & chiral fermions
- avoid erasure of helicity by 
  - diffusion 
  - chiral plasma instability 
  - lower bound on H
- upper bound on H
- helicity is converted to baryon asymmetry @ EWPT 
  - predicts baryon asymmetry ~ observed value
- uncertainties mainly governed by SM physics
Outlook

uncertainties related to SM physics
- change of Weinberg angle during EWPT
- MHD with `symmetric’ initial conditions for chem. potential and helicity
- MHD with large magnetic Reynolds number but small kinetic Reynolds number

uncertainties in the Early Universe physics
- spectrum of hyper charge fields with fermion backreaction
- reheating dynamics
  - [Cuissa, Figueroa ’18]

model building questions
- Higgs vev during inflation
- inflation model
- UV completion
- dark matter

other observables
- intergalactic magnetic field
- gravitational waves
- non-gaussianities & mu-distortions in CMB
backup slides
overview
coupling to gauge fields

\[ \mathcal{L} = -\partial_{\mu}\phi\partial^{\mu}\phi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - V(\phi) - \frac{\alpha}{4f_a}\phi F_{\mu\nu}\tilde{F}^{\mu\nu} \]

\[ \frac{d^2 A_{\pm}(\tau, k)}{d\tau^2} + \left[ k^2 \pm 2k \frac{\xi}{\tau} \right] A_{\pm}(\tau, k) = 0, \quad \xi = \frac{\alpha \dot{\phi}}{2Hf_a} \]

production of PBHs and UCMHs
Linde, Mooji, Pajer '13
Muia, VD, Pieroni '17

explosive helical gauge boson production
additional friction modifies dynamics of inflation

polarized SGWB at LISA and LIGO
Cook, Sorbo '11/12
Barnaby, Pajer, Peloso '12,
Binetruy, VD, Pieroni '16

strongly blue-tilted non-gaussian scalar and tensor power spectrum

baryogenesis from decaying helical gauge fields
Jiminez, Kamada, Schmitz, Xu '17

inflation on steep potentials
Anber, Sorbo '09

relaxation mechanism
Hook, Marques-Tavares '16

binetruy, vd, pieroni '16
linde, mooji, pajer '13
muia, vd, pieroni '17
The most natural scalar potential for an axion is a periodic potential, breaking the shift symmetry of the scalar sector. Both the scalar and tensor sector preserve the usual scaling behaviour of de Sitter space. In the far super-horizon modes to both the energy and variance of the gauge fields is negligible, due to a significant time scale separation.\[ \tau_{\text{H}} \gg \tau_{\text{ph}} \]

\[ \frac{\alpha}{f_a} \]

This parameter choice ensures the correct normalization of the scalar power spectrum\[ \Delta_s^2 \sim \frac{1}{N(2\pi \xi)^2} \]

Note that the gauge fields $a_j$ contribute to the tensor power spectrum of the standard vacuum fluctuations during inflation. At small scales, the gauge fields $a_j$ and the scalar fields $\phi$ contribute to the tensor power spectrum in the form of a factor of $1/2$ for the far super-horizon modes.\[ V = V_0 \left[ 1 - \cos \left( \frac{\phi}{f_\phi} \right) \right] \]

\[ f_\phi \approx 9.2 \, M_P \]

The coupling to gauge fields can be calculated by comparing the two terms in the Hamiltonian and using the relation between the energy density and the variance of the gauge fields. For the parameter point of Fig. 4.3 this yields\[ \lambda [\text{Mpc}] \]

\[ \Delta_s^2(k) \sim \frac{1}{N(2\pi \xi)^2} \]

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\[ \lambda [\text{Mpc}] \]
coupling to fermions

\[ \mathcal{L} = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + i \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{\phi}{2 f_a} \partial_\mu \left( \bar{\psi} \gamma^\mu \gamma_5 \psi \right) \]

\[ \phi \partial_\mu J_5^\mu \rightarrow \dot{\phi} J_5^0 \]

add. contribution to scalar and tensor power spectrum

Anber, Sabancilar '16
Adshead, Pearce, Peloso, Roberts, Sobrbo '18

chiral fermion production

spontaneous CPT violation

spontaneous baryogenesis

Kusenko, Schmitz, Yanagida '14
Adshead, Sfakianakis '15/'16
two frames

U(1) gauge symmetry + massless Dirac fermion
+ pseudo Goldstone boson + chiral anomaly:

\[ S = \int d^4x \left\{ \sqrt{-g} \left[ \frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left( i \gamma^\mu - g Q A^\mu \right) \psi + \frac{\alpha \phi}{4\pi f_a} F_{\mu\nu} \tilde{F}^{\mu\nu} \right\}. \]

Two different frames describing the same physics

\[ S = \int d^4x \left\{ \sqrt{-g} \left[ \frac{g^{\mu\nu}}{2} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left( i \gamma^\mu - g Q A^\mu \right) \psi - \frac{\phi}{2Q^2 f_a} \partial_\mu J^\mu_5 \right\}. \]
fermion production*

\[ 0 = (i \partial_\eta \pm i \nabla \cdot \sigma \pm gQA \cdot \sigma) \psi_{R/L} \equiv D_{R/L} \psi_{R/L} \]

differentiate with \[ \tilde{D}_{R/L} \equiv i \partial_\eta \pm i \nabla \cdot \sigma \mp gQA \cdot \sigma \]

assume constant E,B in z-direction: \[ (A_\mu) = (0, 0, -Bx, Et) \]

auxiliary eom:

\[ 0 = \tilde{D}_{R/L} D_{R/L} \psi_{R/L} = \left[ -\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial x^2} - (gQBx + p_y)^2 - (gQEt - p_z)^2 - gQ (B \pm iE') \sigma_z \right] \Psi_{R/L} \]

separable differential equation with \textbf{discrete energy levels} (Landau levels):

\[ \vec{E} = 0 \quad \omega_L = \begin{cases} \pm \sqrt{p_z^2 + 2ngQB} & \text{for } n = 1, 2, \ldots, \\ -p_z & \text{for } n = 0, \end{cases} \quad \omega_R = \begin{cases} \pm \sqrt{p_z^2 + 2ngQB} & \text{for } n = 1, 2, \ldots, \\ p_z & \text{for } n = 0, \end{cases} \]

determine particle production induced by E-field

baryogenesis from axion inflation
fermion production (LLL)*

Figure 3: The Landau levels for left-/right-handed fermions for $s = +$, i.e., $\dot{Q} > 0$, are shown in the left/right figure. The lowest Landau level is depicted by the blue line while the higher ones are drawn as black lines. One can readily see that the higher ones are symmetric but the lowest one is asymmetric with respect to the interchange of left- and right-handed fermions.

Landau levels. First, we study the spectrum of Eq. (3.26) when we turn off the electric field. This consideration is useful for understanding the relation of two fermion production channels via the anomalous equation and via the Schwinger-like effect. Eq. (3.26) then becomes:

$$0 = \sqrt{\frac{p_z^2 + 2|Q||B|}{2g}}$$

Let us focus on the right-handed fermion. Its dispersion relation is obtained from Eq. (3.28):

$$\omega_R = \left( < \Delta p_z^2 + 2ng|Q||B| \right)^{1/2}$$

for $n = 1, 2, \ldots$, and

$$s p_z$$

for $n = 0$,

where $s = \pm$ for $Q \neq 0$. One can see that the energy spectrum is discretized, which is known as Landau levels. Intuitively, this is because the uniform magnetic field restricts the transverse motion of a charged particle by the Lorentz force. Note here that, for given $Q$ and $B$, the lowest Landau level (LLL) with $n = 0$ has only one frequency while the higher Landau levels (HLLs) with $n \geq 1$ have positive/negative frequencies. To understand the reason, let us move back to the definition of the auxiliary field, Eq. (3.21). Evidently, Eq. (3.28) allows two independent solutions $g_R = Ne^{ip_z t}$ for $n = 0$ with $N$ being a normalization factor. However, one of them yields $R = 0$ if we insert the solution into Eq. (3.21):
Figure 3: The Landau levels for left- and right-handed fermions for $s_0 = +1$, i.e., $\dot{Q} > 0$, are shown in the left-right figure. The lowest Landau level is depicted by the blue line while the higher ones are drawn as black lines. One can readily see that the higher ones are symmetric but the lowest one is asymmetric with respect to the interchange of left- and right-handed fermions.

Landau levels. First, we study the spectrum of Eq. (3.26) when we turn off the electric field. This consideration is useful for understanding the relation of two fermion production channels via the anomalous equation and via the Schwinger-like effect. Eq. (3.26) then becomes

$$0 = \frac{\partial^2}{\partial t^2} + \frac{p_z^2}{g^2 |Q|} + \frac{2ng|Q|}{|B|}.$$  

(3.28)

Let us focus on the right-handed fermion. Its dispersion relation is obtained from Eq. (3.28):

$$\omega_R = \pm \sqrt{\frac{p_z^2}{g^2 |Q|} + \frac{2ng|Q|}{|B|}}$$

for $n = 1, 2, \ldots$ and

$$\omega_L = \pm \sqrt{\frac{p_z^2}{g^2 |Q|} + \frac{2ng|Q|}{|B|}}$$

for $n = 0$. One can see that the energy spectrum is discretized, which is known as Landau levels. Intuitively, this is because the uniform magnetic field restricts the transverse motion of a charged particle by the Lorentz force. Note here that, for given $Q$ and $B$, the lowest Landau level (LLL) with $n = 0$ has only one frequency while the higher Landau levels (HLL) with $n \geq 1$ have positive/negative frequencies. To understand the reason, let us move back to the definition of the auxiliary field, Eq. (3.21). Evidently, Eq. (3.28) allows two independent solutions $g_R = N e^{ip_z t}$ for $n = 0$ with $N$ being a normalization factor. However, one of them yields $R = 0$ if we insert the solution into Eq. (3.21):

$$R = \pm \sqrt{\frac{p_y^2}{g^2 |Q|} + \frac{p_z^2}{g^2 |Q|} + \frac{p_z^2}{g^2 |Q|} + \frac{2ng|Q|}{|B|} - \frac{2ng|Q|}{|B|}}.$$  

(3.30)

where $s = \pm$ for $Q \neq 0$. Now it is clear that we need $\omega_R = \pm \sqrt{\frac{p_z^2}{g^2 |Q|} + \frac{2ng|Q|}{|B|}}$ to have non-vanishing $R$. More intuitively, since the LLL can be regarded as a fermion moving along with the magnetic field ($z$-direction), the right-handed fermion must have a spin, $s$, parallel to its motion.
fermion production (LLL)*

left-handed fermions

right-handed fermions

anomaly equation!

Nielsen, Ninomiya ’83

\[
\dot{q}_5 = \dot{q}_R|_{n=0} - \dot{q}_L|_{n=0} = -\frac{\alpha Q^2}{2\pi} F_{\mu\nu} \tilde{F}^{\mu\nu}
\]

\[
\dot{n}^{\text{LLL}}_{\psi} = 2 \times \frac{g^2 Q^2}{4\pi^2} E B
\]

asymmetric fermion production

baryogenesis from axion inflation

Valerie Domcke (DESY, Hamburg)
Figure 3: The Landau levels for left-/right-handed fermions for $s^\pm$, i.e., $\dot{Q} > 0$, are shown in the left/right figure. The lowest Landau level is depicted by the blue line while the higher ones are drawn as black lines. One can readily see that the higher ones are symmetric but the lowest one is asymmetric with respect to the interchange of left- and right-handed fermions.

Landau levels. First, we study the spectrum of Eq. (3.26) when we turn off the electric field. This consideration is useful for understanding the relation of two fermion production channels via the anomalous equation and via the Schwinger-like effect. Eq. (3.26) then becomes

$$0 = \sqrt{2g|Q|B} \frac{\partial^2}{\partial t^2} + p_z^2 + 2ng|Q|B.$$  

Let us focus on the right-handed fermion. Its dispersion relation is obtained from Eq. (3.28):

$$\omega^R = \pm \frac{\Delta p^2}{2} + 2ng|Q|B$$  

for $n = 1, 2, \ldots$, and $s = \pm$ for $Q \neq 0$. One can see that the energy spectrum is discretized, which is known as Landau levels. Intuitively, this is because the uniform magnetic field restricts the transverse motion of a charged particle by the Lorentz force. Note here that, for given $Q$ and $|Q|B$, the lowest Landau level (LLL) with $n = 0$ has only one frequency while the higher Landau levels (HLLs) with $n \geq 1$ have positive/negative frequencies. To understand the reason, let us move back to the definition of the auxiliary field, Eq. (3.21). Evidently, Eq. (3.28) allows two independent solutions $g_R = Ne^{-ipzt}$ for $n = 0$ with $N$ being a normalization factor. However, one of them yields $R = 0$ if we insert the solution into Eq. (3.21):

$$R = \pm p_z + sp_z,$$

where $s = \pm$ for $Q \neq 0$. Now it is clear that we need $\omega^R = \pm p_z$ for $s = \pm$ to have non-vanishing $R$. More intuitively, since the LLL can be regarded as a fermion moving along with the magnetic field ($z$-direction), the right-handed fermion must have a spin, $s$, parallel to its motion.
Figure 3: The Landau levels for left-/right-handed fermions for $s = \pm$, i.e., $\dot{Q} > 0$, are shown in the left/right figure. The lowest Landau level is depicted by the blue line while the higher ones are drawn as black lines. One can readily see that the higher ones are symmetric but the lowest one is asymmetric with respect to the interchange of left- and right-handed fermions.

Landau levels. First, we study the spectrum of Eq. (3.26) when we turn off the electric field. This consideration is useful for understanding the relation of two fermion production channels via the anomalous equation and via the Schwinger-like effect. Eq. (3.26) then becomes

$$0 = \sqrt{2g|Q|B} \omega^2 + p^2 z - 2ng|Q|B \cdot g_{R/L}.$$  (3.28)

Let us focus on the right-handed fermion. Its dispersion relation is obtained from Eq. (3.28):

$$\omega_R = \pm \sqrt{\Delta p^2 z + 2ng|Q|B}$$ for $n = 1, 2, \ldots$,  

$$sp^2 z$$ for $n = 0$,  

(3.29)

where $s = \pm$ for $Q \neq 0$. One can see that the energy spectrum is discretized, which is known as Landau levels. Intuitively, this is because the uniform magnetic field restricts the transverse motion of a charged particle by the Lorentz force. Note here that, for given $Q$ and $g$, the lowest Landau level (LLL) with $n = 0$ has only one frequency while the higher Landau levels (HLLs) with $n \geq 1$ have positive/negative frequencies. To understand the reason, let us move back to the definition of the auxiliary field, Eq. (3.21). Evidently, Eq. (3.28) allows two independent solutions $g_R = Ne^{-ipz/t}$ for $n = 0$ with $N$ being a normalization factor. However, one of them yields $\dot{R} = 0$ if we insert the solution into Eq. (3.21):

$$\dot{R} = \pm p_z i \frac{\partial}{\partial x} x + p_y y + p_z z s g |Q| B_x y a_n h_0 (x_s) e^{ipz/t} s = \pm p_z + sp_z Nh_0 (x_s) e^{ipz/t},$$ (3.30)

where $s = \pm$ for $Q \neq 0$. Now it is clear that we need $\omega_R = \pm p_z$ for $s = \pm$ to have non-vanishing $\dot{R}$.
More intuitively, since the LLL can be regarded as a fermion moving along with the magnetic field Eq. (3.17), justifying the flat spacetime approximation of this section.

We now have all the ingredients to confirm this a posteriori. Let us focus in the limit of $2\pi B \gg m^2 + |eE|$. (3.26) The Schwinger effect given in Eq. (3.28) then becomes

$$\tilde{n}_\psi^\text{HLL} = 4 \times \frac{g^2 Q^2}{8\pi^3} \left( E^2 - \pi E B + \frac{\pi^2}{3} B^2 + \ldots \right).$$

Figure 3: The Landau levels for left-direction, the right-handed fermion must have a spin, fermion production (HLL)*

**symmetric fermion production**

baryogenesis from axion inflation
induced current

**backreaction** on gauge field production:

\[ \Box A^\nu - \partial_\mu \left( \frac{\alpha \phi}{\pi f_a} \tilde{F}^{\mu \nu} \right) - g Q J_\psi^\nu = 0 \]

- reduction of gauge field production

[VD, Mukaida ’18]
induced current

**backreaction** on gauge field production:

- reduction of gauge field production

\[ A^\nu - \partial_\mu \left( \frac{\alpha \phi}{\pi f_a} \tilde{F}^{\mu \nu} \right) - g Q J^\nu = 0 \]

- changes predictions for power spectra

\[ Q = 1, \quad g = 1/\sqrt{2} \]

Graph showing the scalar power spectrum $\Delta_s^2$ as a function of $N$ [e-folds] and $\lambda$ [Mpc].

- Vacuum
- $U(1)$ gauge fields
- Fermions

Upper bound and estimate for $\Delta_s^2$.

Graph showing the evolution of $\Omega_{GW}/\nu^2$ as a function of frequency $f$ [Hz].

- PTA
- LISA
- LIGO
- CMB

Baryogenesis from axion inflation.