1/f noise in hopping conduction: Role of multi-site aggregates

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We propose a mechanism for 1/f-type noise in hopping insulators based on the multi-electron charge redistribution within the specific aggregates of the localized states located in the vicinity of the critical resistors. We predict that the noise with 1/f-type spectrum extends down to practically arbitrarily low frequencies.

A nature of the underlying mechanism of 1/f low-frequency noise spectrum in hopping conductors is a part of the more general problem of the universal origin of 1/f, and had been an unresolved issue for decades.1,2,3,4,5,6,7,8,9,10 A general intrinsic reason for the 1/f spectrum is the exponentially wide distribution of the relevant relaxation times in the system. Thus the issue of the mechanism of 1/f spectrum reduces to the question of the microscopic origin of such a distribution. The low temperature transport in doped semiconductors occurs via electron hopping between the donors (or hole hopping between acceptors when considering p-type semiconductors). The hopping distance is determined by the balance between the tunneling probability over this distance and the probability of the thermal activation necessary to accommodate the difference between the initial and final levels. Optimization of these probabilities cuts the effectively conducting band around the Fermi level. This hopping mechanism is called variable range hopping (VRH) transport. With the decreasing temperature this effective impurity band gets narrower and the electron has to tunnel over a larger distance to find the appropriate destination state, since the levels at the close by donors are separated by a large gap, see Ref. [11] and references therein. In this regime the conducting paths form a dilute percolation cluster, and the system resistance is controlled by the very rare critical hopping resistors $R_c$. The conductivity of the critical resistors are most susceptible to the dynamic fluctuations of charge distribution in a body of the conductor and they become the source of the current noise.

The crucial question associated with the origin of the noise is whether the 1/f law goes indefinitely to low frequencies and if not, what is the nature of its lower bound. This problem was first addressed by Shklovskii and Kogan11 and references therein. In this regime the conducting paths form a dilute percolation cluster, and the system resistance is controlled by the very rare critical hopping resistors $R_c$. The conductivity of the critical resistors are most susceptible to the dynamic fluctuations of charge distribution in a body of the conductor and they become the source of the current noise.

Here we revisit the problem of the low-frequency noise in hopping conductors and develop the approach for calculating the noise spectrum based on the idea of multi-site bistable aggregates. While switching between their pseudo-ground states, the aggregates produce low-frequency electrical noise, which is “read-out” by the sites belonging to the backbone percolation cluster. The read-out mechanism suggested earlier7 attributes the noise to fluctuations of the effective resistors forming bonds of the percolation cluster with correlation length $L$. The fluctuations of each resistor are due to the noise produced by the nearest bistable aggregate. Since the switching times of these aggregates are spread in an exponentially broad interval the noise spectrum turns out to be of 1/f type, Shklovskii10 devoted to the temperature dependence of the noise spectrum. The alternative idea was put forward by Kogan who assumed that a hopping system is in fact the Coulomb glass and can therefore have arbitrarily low intrinsic frequencies due to transitions between the different metastable “pseudo-ground” states separated by unlimitedly large barriers. Accordingly, there is no saturation in the noise spectrum down to extremely low frequencies. Unfortunately, no analytical description for this mechanism was proposed.

In the original approach8 the aggregates consisted of pairs of sites. However, the probability of finding such pairs - fluctuators - is limited by the fact that such fluctuator be located in a “pore” free of other sites able to facilitate electron hopping. Consequently, the maximal switching time are limited that leads to a cut-off in the noise spectrum Here we generalize the approach2 including into the consideration multi-site aggregates that can assume two distinct charge configurations with the nearly same energy. In this respect, the aggregates that can be viewed as two-level systems. It is essential that transitions between these states occur via the multi-electron hops, either coherent or incoherent. The probabilities of such transitions are strongly suppressed comparing to the case of isolated pairs, and this is the source of very large intrinsic switching times. Consequently, the cut-off frequency can be extremely low. The suggested model of multi-site bistable aggregates is a “bridge” between the ideas of small fluctuators and of pseudo-ground state of the whole system10.
I. MODEL

To quantify the concept of an aggregate let us start from the simplest one—a fluctuator consisting of the two trapping sites occupied by a single electron. Such pairs have two levels with small energy separation $E \lesssim T$ and their relaxation time, $\tau_p$, is related to the tunneling between the states; $\tau_p^{-1} = \nu_0 e^{-2r_p/a}$ where $\nu$ is some (generally temperature-dependent) pre-exponential factor, $r_p$ is the spatial separation between the sites while $a$ is the localization length. The values of $\tau$ should be much larger than the typical hopping times, $\tau_h = \nu_0^{-1} e^{2r_h/a}$, along the current-carrying percolation cluster, therefore $r_p$ should be larger than the typical hopping distance in the cluster, $r_h = a \xi$. Here $\xi$ is the connectivity parameter of the percolation theory entering the exponent of critical hopping resister, $R_c = R_0 e^\xi$, see Ref. 14 and references therein. We consider an isolated pair, therefore there is no any other site in a close neighborhood (i.e. within the distance $r \ll r_p$) from either of the centers, that could facilitate the processes with the rates $\gg \tau_p^{-1}$. One can express this requirement as the inequality

$$\frac{2r_p}{a} \leq \frac{\Delta}{T} + \frac{2r}{a} = \xi$$

(1)

where $\Delta$ is the relevant energy band width. To relate this bandwidth to the distance $\tilde{r}$ we estimate $\tilde{r}$ as $\tilde{r} = (3/4\pi n)^{1/3}$ where $n$ is the concentration of the relevant centers. Since for a Coulomb-gap controlled system in the VRH regime $n = (2/3)(\kappa \Delta/e^2)^3$ we obtain $\Delta \approx (9/8\pi)^{1/3}(e^2/\kappa r)^3$. Optimizing the r.h.s. of Eq. (1) with respect to $\tilde{r}$ we find $r_p \lesssim \tau_h e^{(\sqrt{2} - 1)}$. Here $\xi = (T_0/T)^{1/2}$, $T_0 = e^2/\kappa a^{13}$.

Now we consider the aggregates consisting of 4 hopping sites occupied by 2 electrons. These close to square arrays are the simplest next hierarchy level aggregates realizing the two level system (the three-site aggregates are reduced to pairs, since even a slight deviation from the perfect arrangement lifts the triple degeneracy and gives rise to an exponentially wide gap separating the split levels of the closest pair from the third level). A degenerate configurations requiring the least energy arise when two electrons are trapped at the diagonally opposite sites. Labelling the sites as 1, 2, 3 and 4, we say that the pairs are formed by the occupied sites 1 $\leftrightarrow$ 2, and 3 $\leftrightarrow$ 4, respectively. For the configuration where sites 2 and 4 are occupied one has

$$\varepsilon_2 + \frac{e^2}{\kappa r_{24}} < \mu, \quad \varepsilon_1 + \frac{e^2}{\kappa r_{12}} + \frac{e^2}{\kappa r_{14}} > \mu,$$

$$\varepsilon_4 + \frac{e^2}{\kappa r_{42}} < \mu, \quad \varepsilon_3 + \frac{e^2}{\kappa r_{43}} + \frac{e^2}{\kappa r_{23}} > \mu;$$

(2)

the configuration where sites 1 and 3 are occupied obeys:

$$\varepsilon_1 + \frac{e^2}{\kappa r_{13}} < \mu, \quad \varepsilon_2 + \frac{e^2}{\kappa r_{12}} + \frac{e^2}{\kappa r_{14}} > \mu,$$

$$\varepsilon_3 + \frac{e^2}{\kappa r_{34}} < \mu, \quad \varepsilon_4 + \frac{e^2}{\kappa r_{43}} + \frac{e^2}{\kappa r_{23}} > \mu.$$

(3)

Here $\varepsilon_i$ are the single-particle energies of the sites, $r_{ij}$ are the sites spacings, and $\mu$ is the chemical potential. The energy splitting between the two configurations is

$$E \equiv E_1 - E_2 = \frac{2e^2}{\kappa} \left( \frac{1}{r_{24}} - \frac{1}{r_{13}} \right) + \varepsilon_2 + \varepsilon_4 - \varepsilon_1 - \varepsilon_3. \quad (4)$$

The relaxation processes associated with the transitions between the two configurations with the almost equal energies are the process that involve the simultaneous exchange of electrons between the sites 1 $\rightarrow$ 2 and 3 $\rightarrow$ 4. All other processes increase the Coulomb energy and therefore are unlikely to occur. The switch can be either due to a multi-electron (a two-electron in our case) tunneling first suggested by Pollak\textsuperscript{15} or due to incoherent processes.

As follows from the above mentioned correlation, the aggregate formed by the sites with large single-particle energies would have short intersite distances and, accordingly, high rates of the multi-electron hops. Since the slow relaxing aggregates should have relatively large larger intersite separations, their single-particle energies should be relatively small.

To proceed further it is convenient to map the aggregates onto a spin system. A single pair is mapped on the 1/2-spin (the spin direction is taken from the occupied to the unoccupied site), the four-site aggregates are represented as a pair of the interacting antiparallel spins. The interaction between the spins forming an aggregate is antiferromagnetic since the Coulomb repulsion pushes electrons belonging to the adjacent pairs apart. Adding more pairs we map larger aggregates to clusters of 3, 4, ...$N$ spins. This model is a minimal model for the general Coulomb glass since we allow only for the transitions that are represented by flips of spins. In the absence of randomness the system is antiferromagnet, disorder may, in principal, drive it to the spin glass.

The 2$N$-site aggregates (or antiferromagnet $N$-spin clusters) must obey restrictions similar to those given by Eqs. (4) and (5). To estimate the density of states for the aggregates we shall take into account that in course of the reversal of the “spins” the energy of each site crosses the Fermi level. Thus we should compare the change of the site energy resulting from the aggregate rearrangement and the “static” scatter of the site energies.

To do so, let us specify some energy $\varepsilon_i$ within the Coulomb gap and consider the sites with energy band $\varepsilon_i < \varepsilon_f$. Let us consider the aggregates formed from these sites in the same way as it was considered above. For the site energy we have, cf. with Ref. [14]

$$\varepsilon_i = \varphi_i + V_i, \quad V_i = \frac{e^2}{\kappa} \sum_{j \neq i} \frac{(1 - n_j)}{r_{ij}}. \quad (5)$$

Here $\varphi_i$ is the potential induced on the site $i$ by the background charges not included in the aggregate; the sum in $V_i$ is calculated over the sites forming the aggregate. Thus $\varphi_i$ is due to the disorder while $V_i$ depends on the state of the aggregate.
To estimate $V_i$ let us split it into the parts having different symmetry with respect to rearrangement of the aggregate between its two metastable states: $V_i = (1/2)(V_i^+ + V_i^- s_i)$. Here $s_i = \pm 1$. The symmetric part can be treated as a contribution to the potential $\varphi_i$. Assuming the potentials $\varphi_i$ and $V_i^{\pm}$ to be random and uncorrelated, we write

$$\tilde{\varepsilon}_i = \varphi_i^2 + \frac{1}{4} (V_i^+)^2 + \frac{1}{4} (V_i^-)^2. \quad (6)$$

Since we are interested in the states within the Coulomb gap, the density of states for the single-particle energies $\varepsilon_i$ is given by the Efros-Shklovskii law, $g \propto \varepsilon_i^2$. Thus the typical distance between the sites as

$$\tilde{r} = \left[ 2 \int_0^{\varepsilon_f} g(\varepsilon) d\varepsilon \right]^{-1/3} = (3/2)^{1/3} e^2 / \kappa f. \quad (7)$$

One expects that the potential formed on the site $i$ by the other sites forming the aggregate can be estimated as

$$V_\pm \equiv \left( \frac{V_i^{\pm}}{\varepsilon_f} \right)^{1/2} = \zeta_\pm \varepsilon_f \quad \text{where} \quad \zeta_\pm \sim 1 \quad \text{are numerical parameters depending on the aggregate geometry}. \quad \text{Making use of Eq. (6) one has}$$

$$\tilde{\varphi}^2 = \varepsilon_f^2 (1 - \zeta_-^2 / 4 - \zeta_-^2 / 4). \quad (8)$$

Since the typical variation of the site energy, $\delta V \equiv \tilde{V}_- = \zeta_- \varepsilon_f$ we obtain the following estimates

$$\tilde{V}_- = \zeta_- \varepsilon_f, \tilde{V}_+ = \zeta_+ \varepsilon_f, \tilde{\varphi} = \varepsilon_f \sqrt{1 - \zeta_-^2 / 4 - \zeta_-^2 / 4}. \quad (9)$$

Note that the relations between $\tilde{V}_-$, $\tilde{V}_+$ and $\tilde{\varphi}$ do not depend on $\varepsilon_f$ provided we deal with the states within the Coulomb gap. One can expect that these ratios are of the order of unity.

Basing on the discussion given above let us estimate the aggregates density of states. First, let us note for all the sites belonging to the aggregate the inequality $V_i^{\pm} > V_i^{\pm} + 2\tilde{\varphi}_i$ must be met. Indeed, only such states cross the Fermi level. The probability to form an aggregate depends on two parameters - the ratios $\zeta_- / \zeta_+$ and $\zeta_- / 2\tilde{\varphi}$. Assuming that $\zeta_- / \zeta_+ \lesssim 1$ and $2\tilde{\varphi} \gtrsim \zeta_-$ one concludes that to form an aggregate one should deal with the sites with the potentials $\varphi_i$ less than their average scatter. Since the Coulomb gap exists only for the single-electron energies, $\varepsilon_i$ rather than for disorder-induced potential $\varphi$, the distribution of the random potential $\varphi$, $P(\varphi)$, is smooth for small $\varphi$ and one can put $P(\varphi) \approx 1 / 2\tilde{\varphi}$. Consequently, he relative weight of the aggregate sites is in this case given by

$$\frac{1}{2\tilde{\varphi}} \int_0^{\tilde{V}_- + \tilde{V}_+ / 2} P(\varphi) d\varphi = \frac{\zeta_- - \zeta_+}{\sqrt{4 - \zeta_-^2 - \zeta_+^2}} \equiv \lambda. \quad (10)$$

These considerations are valid until $\lambda < 1$. In this case relative weight of the aggregate formed by $2N$ sites is

$$\lambda^{2N} = e^{-2N \ln 1 / \lambda}. \quad \text{Thus at } \lambda < 1 \text{ is the probability to form a metastable aggregate is exponentially small in terms of the number of the sites involved in the aggregate. This will be mainly discussed in what follows.}$$

However, there is no fundamental principle requiring $\lambda \leq 1$. This is why later we will also discuss the scenarios where the multistable aggregates can be realized with a probability close to unity.

**II. THE DISTRIBUTION OF THE RELAXATION RATES.**

To calculate noise in the system one needs density of states, $P(N, r, E)$, of finding the aggregate with $N$ sites and distance $r$ between the sites per unit energy interval, $E$. This density can be expressed in the form

$$P(N, r, E) = \frac{\lambda^N}{\sqrt{\pi N \rho N^3 \cdot e^{2 / \kappa r}}} = \frac{2.7\lambda^N}{T_{ES} \sqrt{\pi N^3 / 2 \lambda^2 r^2}}. \quad (11)$$

Here $N^3$ is the volume of the aggregate while $\sqrt{\pi e^{2 / \kappa r}}$ is the energy bandwidth for small energies.

Let us estimate the distribution function, $P(\nu)$, of the relaxation rates, $\nu$, for the aggregate rearrangement. We define the rate as

$$\nu = \nu_0 \max \left\{ e^{-N^2 / 3 \xi^2 / 2.7 \rho} e^{-2N \rho} \right\}. \quad (12)$$

Here $\rho \equiv r / a$, $\xi \equiv (T_{ES} / T)^{1 / 2}$. The first term in parentheses describes formation of a “domain wall” in the aggregate and the second term corresponds to coherent tunneling transitions leading to re-charging of all aggregate sites.

To calculate the distribution of switching rates let us introduce the function $L(N, \rho)$ as

$$L(N, \rho) = -\ln \max \left\{ e^{-N^2 / 3 \xi^2 / 2.7 \rho}, e^{-2N \rho} \right\}. \quad (13)$$

Thus $L(N, \rho) = \ln(\nu_0 / \nu) \approx M$. Denoting $\rho_N$ by the equality $N^2 / 3 \xi^2 / 2.7 \rho_N = 2N \rho_N$,

$$\rho_N = 0.43\xi / N^{1 / 6}, \quad (14)$$

one can express $L(N, \rho)$ as

$$L(N, \rho) = -\ln \max \left\{ e^{-0.86N^{5 / 6} (\rho / \rho_N)}, e^{-0.86N^{5 / 6} (\rho / \rho_N)} \right\}. \quad (15)$$

Let us now invert the above dependence. Since

$$L(N, \rho) \approx \begin{cases} 0.86N^{5 / 6} (\rho / \rho_N), & \rho \gg \rho_N \\ 0.86N^{5 / 6} (\rho_N / \rho), & \rho \gg \rho_N. \end{cases} \quad (16)$$

the inverted function, $\rho(L)$, has two branches:

$$\rho(L) = \begin{cases} (L / L_N) \rho_N, & \rho \gg \rho_N \\ (L_N / L) \rho_N, & \rho \gg \rho_N. \end{cases} \quad (17)$$

$$\rho \equiv \rho_N \left\{ 1, \frac{\partial \rho}{\partial L} \right\} \rho_N \left\{ \frac{L}{L_N} \rho_N \right\}. \quad (18)$$
Here $L_N \equiv 0.86N^{5/6}\xi$, thus $\rho_N/L_N = 1/2N$. Note that $L(N, \rho) \leq L_N$. Now we are ready to calculate the distribution function

$$\mathcal{P}(\nu, E) = 4\pi \int dN \, r^3 dr \, P(N, r, E) \delta \left( \nu - \nu_0 e^{-L(r,N)} \right)$$

$$= \frac{4\pi a^2}{\nu} \int dN \rho^2 d\rho \rho P(N, \rho, E) \left[ \frac{\partial}{\partial \rho} \right]_{\rho = \rho(L)}.$$  

Substituting $P(N, r, E)$ from Eq. (10) we obtain

$$\mathcal{P}(\nu) = \frac{1}{\nu} \frac{2\pi \cdot 2.7}{T_E S \sqrt{\pi}} \int_{N_c}^\infty dN \frac{e^{-\gamma N}}{N^{5/2}} \left[ 1 + \left( \frac{L_N}{M} \right)^2 \right].$$  

(16)

where $\gamma \equiv \ln 1/\lambda$, while $N_c(M) = (M/0.86\xi)^6/5$. Substituting these values to Eq. (16) we get,

$$\mathcal{P}(\nu) \equiv \frac{1}{\nu} \frac{1}{T_E} \frac{4\sqrt{\pi} \cdot 2.7}{\gamma N_c^{5/2}} e^{-\gamma N_c}.$$  

(17)

In this approximation the density of states $\mathcal{P}$ does not depend on the energy splitting $E$. The expression (17) is valid at $\gamma N_c = [(\gamma/5/6)/0.86\xi] \ln(\nu_0/\nu) \geq 1$, or

$$\ln(\nu_0/\nu) \gtrsim \frac{\xi}{\gamma^{5/6}}.$$  

(18)

The product $\gamma N_c$ can be expressed as $\ln(\nu_0/\nu)^\alpha$ where

$$\alpha(\nu) = \frac{\gamma^{5/6}}{0.86\xi} \left[ \frac{\gamma^{5/6}}{0.86\xi} \ln(\nu_0/\nu) \right]^{1/5}.$$  

From realistic estimates one can expect that the second factor does not significantly change within the experimentally accessible frequency condition. For example, assuming $\nu_0 = 10^{10}$ Hz we get $M^{1/5} = 1.87$ at $\nu = 1$ Hz and $M^{1/5} = 2.06$ at $\nu = 10^{-6}$ Hz. Consequently, for reasonable experimental conditions the quantity $\alpha(\nu)$ can be replaced by $\tilde{\alpha} = \alpha(\nu)$ where $\nu$ is some characteristic frequency within the measurement interval. As follows from the above estimate, for any feasible frequency $\tilde{\alpha} \ll 1$ and the noise spectrum is of the $1/f$ type.

As a result, the distribution of the relaxation rates can be expressed as

$$\mathcal{P}(\nu) \approx \frac{C}{\nu^{1-\tilde{\alpha}}} \nu^{\tilde{\alpha}} e^{3} \frac{1}{T_E}.$$  

(19)

where $C \approx 12.2/\gamma$ is a numerical factor. The temperature dependence $\tilde{\alpha} \propto \xi^{-6/5} \propto T^{3/5}$ can be used for verification of the proposed mechanism.

III. ESTIMATE OF THE NOISE INTENSITY

Switching between the aggregate configurations leads to the change in the energies, $\delta\varepsilon_i^{(c)}$, of the sites belonging to the percolation cluster and induces the current noise. If $|\delta\varepsilon_i^{(c)}| \gtrsim T$ then the fluctuations of the hopping resistor, $\rho_i$, are large, $|\delta\rho_i| \sim \rho_i$ and can change the percolation cluster structure (see Ref. 3).

To be specific we consider an $N$-spin aggregate coupled to the hopping resistor $\rho_i$ formed by the two hopping sites which we label by indices 1 and 2. The fluctuations of the resistance are estimated as

$$|\delta R_i/R_i| \sim \min(1, |\delta\varepsilon_i|/T), \quad \delta\varepsilon_i \equiv \delta\varepsilon_i^{(c)} \equiv \delta\varepsilon_i^{(c)}.$$  

(20)

If the distance between the resistor $i$ and the nearest aggregate, $r_i$, is much larger than the typical distance between the sites belonging to the hopping cluster, $r_i \gg r_h$, the fluctuation in energy $\delta\varepsilon_i \approx (\partial\delta\varepsilon^{(c)}/\partial r_i) r_h$ where $\delta\varepsilon$ is the potential induced by the aggregate. The latter can be estimated assuming that the total dipole moment of the relevant aggregate is $\delta R_i = \frac{a\xi N_c}{c}$ where $r_i \equiv \frac{a\rho N_c}{c} = 0.43a\xi N_c^{-1/6}$. Consequently, $r_i = 0.43r_h N_c^{-1/6}$ and

$$\delta\varepsilon^{(c)} \sim \frac{e^2 r_i \sqrt{N_c}}{\kappa r_i^3}, \quad \delta\varepsilon \sim \frac{e^2 r_i^2 N_c^{1/3}}{\kappa r_i^3} \sim \frac{T_E S N_c^{1/3} \xi^2 a^{-3}}{r_i^3}.$$  

(21)

As we have already mentioned the noise is formed at the exponentially rare critical resistors. Let us assume for a while that the fluctuations of their resistances are small, $|\delta R_i|/R_i \ll 1$. In a linear approximation the total resistance fluctuation can be written as

$$\frac{\delta R}{R} \sim \frac{1}{N} \sum_i \frac{\delta R_i}{R_i}.$$  

(22)

Here $N$ is the number if critical resistors in the sample which can be expressed through its volume, $V$, and the correlation length of the percolation cluster, $L = r_h \xi = a \xi^2$, as $N = V/L^3$. Assuming the aggregates acting upon different critical resistor to be statistically independent we get

$$\frac{(\delta R_i \delta R_j)}{R_i^2} \sim \frac{1}{N^2} \sum_i (\delta R_i, \delta R_j).$$  

(23)

Now let us estimate of $\delta R_i$. One has in mind that the aggregates are also rarely distributed. This means that all the critical resistors have at most one neighboring aggregate and the fluctuation spectrum is:

$$S(\omega) \equiv (\delta R_i)^2 \omega^2 \sim \nu^2 \frac{\nu_i}{\nu_t^2 + \omega^2} \frac{1}{4 \cosh^2 E_i/2T}.$$  

Here $\nu_i$ is the switching rate of the nearest aggregate with energy splitting $E_i$, $\nu^2 \equiv \min \left[ (\delta R_i/T)^2, 1 \right]$ is the squared dimensionless coupling. Since $\delta R_i \propto r_i^{-3}$ there exists the specific distance, $r_T$, at which the energy variation given by Eq. (21) is equal to $T$:

$$r_T \approx \frac{a \xi^4 N_c^{1/6}}{r_h} \approx r_i^{3/4} N_c^{1/9} \ll L.$$  

(24)

At $r_i \gg r_T$ the interaction strength $\nu^2 \approx r_i^{-6}$. Consequently, only the nearest aggregate is important.
Replacing summation over the critical resistors (and their nearest aggregates) by averaging and integration over $E$ we obtain

$$S(\omega) \approx \frac{v^2}{N} \int_0^{v_0} d\nu P(\nu) \frac{\nu}{\nu^2 + \delta^2} = \frac{\pi v^2 T}{2N} \mathcal{P}(\omega).$$

(25)

Here $v^2 \equiv r^2 \int_{|r|<r_c} d^3r$, while the distribution function $P(\omega)$ is given by Eq. (17) or (19). Using Eq. (21) we conclude that $v^2$ is of the order of unity, and

$$S(\omega) \approx \frac{\xi}{N^2} \frac{\nu_0^2}{\omega^{1-\alpha}} \ln^3(\nu_0/\omega).$$

(26)

The factor $\xi/N = \xi \mathcal{L}^3/\mathcal{V} = (a^3/\mathcal{V}) \xi^7 \propto T^{-7/2}$. Thus the the considered mechanism leads to a strongly decreasing temperature dependence of the noise intensity. The suggested procedure to estimate the noise intensity is valid if each cell of the backbone cluster contains only one aggregate. This implies the limitation to the cluster size

$$N_c r_c^2 \leq \mathcal{L} \quad \rightarrow \quad \ln(\nu_0/\omega) \leq \xi^6.$$  

(27)

This requirement hold for any realistic frequency since $\xi \gg 1$. One can imagine another restriction relevant to the percolation mechanism behind the resistivity. According to the $1/\omega$ spectrum, one expects that the magnitude of the fluctuations increases with an increase of the observation time. If the relative resistance fluctuation for any relevant resistor,

$$\overline{\delta R_i^2}/R_i^2 = N \frac{\delta R_i^2}{R_i^2} \equiv (N/R_i^2) \int d\omega(\delta R_i^2)$$

becomes comparable with unity, then one expects that the percolation cluster is completely reconstructed by the fluctuations. As a result, the fluctuations with lower frequencies will not be observed in the sample resistance. As follows from Eq. (26), this requirement provides the frequency limit for the validity of our calculation, $\ln(\nu_0/\omega) \gg \sqrt{\xi}$, which is automatically fulfilled for all the frequencies less than the typical hopping frequency, $\nu_h = v_0 e^{-\xi}$.

IV. COULOMB GLASS SCENARIOS

The above considerations demonstrate many-site multistable configurations in the hopping insulator. The probability of finding such configurations depends on the interplay between the Coulomb interaction and the random potential produced by stray disorder, e. g. by charged acceptors. The question which arises here is whether there is a critical value of disorder discriminating between the cases of exponentially low, $\propto e^{-\gamma N}$, probability of finding such configurations and the probability close to one. Earlier we relied upon rather strong disorder assuming exponential decay of the probability with the size of the metastable aggregate.

Now we will discuss the consequences of the intrinsically correlated distribution of charges where there is no exponential decay of the probability with the exponent proportional to the number of sites in the aggregate. Namely, let us assume that there is a critical value $\gamma_c$ of the parameter $\gamma = \ln 1/\lambda$ such as that the systems with $\gamma < \gamma_c$ allow to form the arbitrarily large metastable configurations. The independent support of this idea (although again for model systems) is given by theoretical considerations given in Ref. 14. Namely, it is demonstrated that the sites within a Coulomb gap demonstrate replica-symmetry breaking which can be related to a presence of extensive number of metastable states within a thermodynamic limit $N \rightarrow \infty$. We also note that multistable character of the ground states was also demonstrated by several numerical simulations.

In principle, one can consider the following 3 scenarios:

(i) Strong disorder, $\gamma > \gamma_c$. Only rare multistable aggregates are available due to the effect of disorder, the probability exponentially decreasing with an increase of the aggregate size.

(ii) Weak disorder, $\gamma < \gamma_c$. The multistable configurations are inherent for the system, which in this case can be called the Coulomb glass. The results depends on the typical distance of charge transfer:

a. the charge transfer within the metastable aggregates is restricted by neighboring sites. Correspondingly, the interactions are dominated by the short range dipole-dipole forces ("dipolar glass"), and the short range ordering in the sites occupation numbers still exists. In this case one expects that the local glassy dynamics is not significantly affected by coupling with the remote regions. Namely, the dynamics of a critical resistor is dominated by the nearest aggregate with a specific relaxation rate. One can expect that in this regime the system would have the $1/\omega$ spectrum down to any practically achievable frequency, and the exponent $\tilde{\alpha} = 0$.

b. The charge transfer at large distances is important. In this case the local short-order configuration of the sites occupation numbers can depend on the state of the remote regions ("large-scale Coulomb glass"), and as a result a hierarchical dynamics becomes possible. In this case the low-frequency noise cannot be regarded as a superposition of statistically-independent Poissonian telegraph-like fluctuations occurring in different parts of the sample and acting on different backbone resistors.

Because of hierarchical dynamics of the aggregates one can expect that fluctuation of each site energy possesses $1/f$-type spectrum. To the best of our knowledge, this situation has not been analyzed yet in a convincing way.
The difference between the “dipolar” and “large-scale” Coulomb glasses can be in principle revealed experimentally by analyzing local occupation fluctuations by means of scanning tunnelling spectroscopy. For the scenario (iia) the telegraph noise with a given relaxation time is expected, while for (iib) a noise with a complex spectrum should be observed provided the large enough observation times are possible.

For both scenarios, the $1/f$ noise spectrum should hold down to arbitrary small (from practical point of view) frequencies.

Since the characteristic switching times for large aggregates can be too long to be observable, at realistic noise frequencies one deals only with switchings of relatively small metastable aggregates. To consider these aggregates as compact and statistically independent one has to ensure that the aggregate’s switching between the metastable states does not cause repopulation of the states outside the aggregate. Consequently, the signs of their energies (counted from the chemical potential) should not change. In addition, the Coulomb interactions with the sites outside the aggregate should not affect the aggregate dynamics. Both contributions are proportional to the surface of the aggregate, i.e. $N^{2/3}$. Repeating the analysis of Sec. II and ascribing the probability factor $e^{-\gamma N^{2/3}}$ to allow for the surface contribution one concludes that $S(\omega) \propto \omega^{-1}$ and $\alpha = 0$.

The controversial experimental results on the temperature dependence of the flicker noise in the Coulomb glass, see e.g. Refs. 14, can, in principle, be of fundamental nature. Indeed, in the previous discussion we did not consider the time fluctuations of the single-particle energies of the sites forming the aggregates. These fluctuations are due to correlated electronic hops, which can be more important for the noise than for the stationary transport. An attempt to consider the role of the fluctuations of site energies in hopping transport was made in Ref. 17. Somewhat later an important role of strong fluctuations in the Coulomb glass was also emphasized in Ref. 12. We feel that correlated hopping may indeed introduce a frequency cut-off to the noise spectrum; this cut-off can be considered as a hallmark of correlated hopping. This may be the case in the materials where experimental values of $T_0$ are significantly smaller than those conventionally expected 18.

Our conclusion regarding the absence of the frequency lower cut-off agrees favorably with available experimental results. The temperature dependence is more problematic since different temperature dependences were observed, see, e.g., analysis in Ref. 11.

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13 Note that even if $\lambda > 1$, for some sites the inequality $\Delta V_i > \varphi$ can be violated. Thus, to be sure that the chosen sites indeed form the aggregate obeying the criteria mentioned above, one should impose some additional restriction on the values of $\varphi$ for such “improper” sites leading to some statistical factor $\lambda$ for each of these sites. One notes that the relative number $\beta$ of such sites depends on the distribution functions of $\varphi_i$ and $V_i$ and does not depend on $N$. As a result, the probability to form the aggregate with $N$ sites is $\propto \lambda^N$.
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