Acceleration of ultra-thin electron layer. Analytical treatment compared with 1D-PIC simulation.

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Abstract. In this paper, we apply an analytical model [V.V. Kulagin et al., Phys. Plasmas 14,113101 (2007)] to describe the acceleration of an ultra-thin electron layer by a schematic single-cycle laser pulse and compare with one-dimensional particle-in-cell (1D-PIC) simulations. This is in the context of creating a relativistic mirror for coherent backscattering and supplements two related papers in this EPJD volume. The model is shown to reproduce the 1D-PIC results almost quantitatively for the short time of a few laser periods sufficient for the backscattering of ultra-short probe pulses.

PACS. 41.75.Jv Laser-driven acceleration, – 52.59.-f Intense particle beams and radiation sources in physics of plasmas, – 29.25.-t Particle sources and targets

1 Introduction

Irradiation of ultra-thin solid foils by high-contrast laser pulses at relativistic intensities may provide a new way to create a relativistic mirror for coherent reflection, generation of high harmonics and compression of drive and probe light \cite{3450}. We consider the regime in which foil electrons are completely separated from ions \cite{6}. The partial reflectivity of such electron layers has been derived and compared with one-dimensional particle-in-cell simulation in \cite{7}. Coherent reflection depends on the square of the layer density and is therefore sensitive to the density profile. The analytical model developed by Kulagin et al. \cite{1} allows to describe the layer evolution almost quantitatively, at least for a few laser periods after expulsion from the foil. This time interval is sufficient to reflect few-cycle probe pulses, to compress them to atto- and zepto-second duration, and to shift their spectra to the VUV- and X-ray regime. Here we apply the Kulagin model to a reference case used in the previous publications in order to develop and explore this method.

2 Analytical Model

Different from \cite{6}, we consider here the electromagnetic field \(\mathbf{E} = \mathbf{E}_L + \mathbf{E}_s\) and \(\mathbf{B} = \mathbf{B}_L + \mathbf{B}_s\) including the self-fields \(\mathbf{E}_s\) and \(\mathbf{B}_s\) of the electron layer in addition to the external plane laser field, propagating in \(x\)-direction and having linear polarization with the components \(E_{Ly}\) and \(B_{Lz}\). Throughout this paper, we use natural units, i.e. time and space coordinates are normalized according to

\(t' = \omega_L t, \ x' = k_L x, \ \text{where} \ \omega_L, k_L \ \text{are laser frequency and wave number, fields} \ E' = eE/(mc\omega_L), B' = eB/(mc\omega_L), \ \text{velocities} \ \beta = v/c, \ \text{and momenta} \ p' = p/mc, \ \text{where} \ e \ \text{is charge unit,} \ m \ \text{electron mass, and} \ c \ \text{velocity of light. In the following, these normalized quantities are used dropping the prime.} \)

The self-fields consist of the longitudinal electrostatic field \(E_{sx}\) due to charge separation between electrons and ions and the transverse electromagnetic fields \(E_{sy}\) and \(B_{sz}\) due to induced electron currents. The laser pulse initially hits a foil which has uniform electron and ion density \(n_i = n_0\) and thickness \(d_0\). While the ions are taken as immobile heavy particles, the electrons move in the fields. Their initial position \(x_0\) in the layer (\(0 \leq x_0 \leq d_0\)) is taken as Lagrangian coordinate, and the goal is to determine the trajectories \(x(t, x_0)\) and \(y(t, x_0)\).

The longitudinal field, felt by electrons of initial position \(x_0\) when at position \(x\), is then given by

\[ E_{sx}(x, x_0) = \begin{cases} N(x - x_0) & (x_0 \leq x \leq d_0) \\ N(d_0 - x_0) & (x \geq d_0) \end{cases}, \] (1)

where

\[ N = n_0/n_{\text{crit}} = \frac{\omega_p^2}{\omega_L^2} \] (2)

is the layer density normalized to the critical density \(n_{\text{crit}}\); it can be conveniently expressed by the squared plasma frequency \(\omega_p^2 = 4\pi e^2 n_0/m\) divided by \(\omega_L^2\). Equation (1) describes the increase of \(E_{sx}\) linear in \(x\) for electrons still inside the ion volume and the constant \(E_{sx}\) after leaving the ion layer. It is valid as long as electron sub-layers with different \(x_0\) keep their relative order.

Next we consider the electromagnetic fields due to the currents induced in the electron layer by the driving laser.
An electron, that originated from \(x_0\) and has position \(x(t, x_0)\) and velocities \(\beta_x(t, x_0), \beta_y(t, x_0)\) at time \(t\), experiences approximately

\[
E_{sy}(t, x_0) = \frac{N\beta_y}{2}\left[\frac{x_0}{1 - \beta_x} + \frac{d_0 - x_0}{1 + \beta_x}\right],
\]

\[
B_{sy}(t, x_0) = \frac{N\beta_y}{2}\left[\frac{x_0}{1 - \beta_x} - \frac{d_0 - x_0}{1 + \beta_x}\right].
\]

Here the first terms in the square brackets stem from electron currents to the left (\(x'_0 < x_0\)) and the second terms from those to the right (\(x'_0 > x_0\)) of the considered electron at \(x_0\). The approximation made in the present model is that the velocities \(\beta_x\) and \(\beta_y\) are assumed spatially uniform.

The equations of motion then are

\[
\frac{dp_x}{dt} = -E_x - \beta_y B_z,
\]

\[
\frac{dp_y}{dt} = -E_y + \beta_x B_z,
\]

\[
\gamma^2 = 1 + p_x^2 + p_y^2,
\]

\[
\frac{d\gamma}{dt} = -\beta_x E_x - \beta_y E_y,
\]

\[
\frac{dx}{dt} = \beta_x = p_x/\gamma,
\]

\[
\frac{dy}{dt} = \beta_y = p_y/\gamma.
\]

From this we find

\[
\frac{d}{dt}(\gamma - p_x) = (1 - \beta_x)E_x - \beta_y(E_y - B_z),
\]

\[
\frac{dp_y}{dt} = -(1 - \beta_x)E_y - \beta_x(E_y - B_z).
\]

For a plane wave moving in vacuum in \(x\)-direction, the laser fields satisfy the dispersion relation \(\omega_L = \beta c k_L\) and \(E_{Lx}(\tau) = B_{Lz}(\tau)\) with propagation coordinate \(\tau = t - x\).

For an electron moving along \(x(t)\), this implies \(\dot{x}/\dot{t} = 1 - \beta_x\). Introducing \(\kappa = \gamma - p_x\), making use of \(\kappa = \gamma(1 - \beta_x) = \gamma\dot{\tau}/\dot{t}\), and recalling \(E_x = E_{sx}, E_y = E_{sy}, B_z = B_{Lx} + B_{sz}\), we derive, after some algebra, the coupled equations for \(\kappa(\tau)\) and \(p_y(\tau)\):

\[
\frac{d\kappa}{d\tau} = E_{sx} - N(d_0 - x_0)\frac{p_y^2}{1 + p_y^2},
\]

\[
\frac{dp_y}{d\tau} = -E_{Lx} - \frac{Nd_0 p_y}{2\kappa}.
\]

They are solved by numerical integration. From \(\kappa = \gamma - p_x, \gamma^2 = 1 + p_x^2 + p_y^2,\) and the solutions \(p_y(\tau)\) and \(\kappa(\tau),\) one can obtain

\[
\gamma(\tau) = \frac{1 + p_y^2}{2\kappa} + \frac{\kappa}{2},
\]

\[
p_x(\tau) = \frac{1 + p_y^2}{2\kappa} - \frac{\kappa}{2},
\]

and the particle trajectories then follow from

\[
\frac{dx}{d\tau} = p_x/\kappa, \quad \frac{dy}{d\tau} = p_y/\kappa,
\]

in parametric form with time given by \(t(\tau) = \tau + x(\tau)\).

The layer of finite thickness is divided into 100 sub-layers, and each sub-layer is assumed to move according to Eqs. (10). Finally, the density is calculated, using \(x(t, x_0)\) and

\[
n = n_0/(\partial x/\partial x_0).
\]

From this we obtain spatial density distributions, as shown in Fig. 1 and 3.

### 3 Comparison of model with PIC simulation

The model is now applied to the reference case, already discussed in paper I [6]. It is compared with 1D-PIC sim-
ulation in Figs. 1 and 2. A planar foil with density $N = n_0/n_{crit} = 159$ and $\varepsilon_0 = N d_0 = 1$ is irradiated by a single-cycle laser pulse $E_{Ly} = a_0 \sin \tau (0 \leq \tau \leq 2\pi)$ with $a_0 = 5$. At this high intensity, we consider foil ionization to happen very rapidly; actually we use the approximation of an initially completely ionized plasma layer. The 1D-PIC simulations have been performed using the code LPIC++ [8].

The results in Fig. 1 show profiles of laser field and electron density at times $t/\tau_0 = 1$ and $t/\tau_0 = 3$ after interaction with the foil. The laser pulse has expelled all electrons from the foil. They are seen as a dense layer carried along by the first half-wave of the pulse, while foil ions are considered immobile and are located at $x = 0$ (not shown). Apparently, the electron layer is transparent to the light, the front has penetrated the layer, but is depleted due to interaction with the layer. Different from the results in paper I, the analytical model [1] is now well reproducing the 1D-PIC results. There are two reasons for this agreement: (1) correct account for the self-radiation of the electron layer which may be viewed as the effect of forward Thomson scattering reducing the wave amplitude in front of the layer; (2) correct account of initial electron acceleration inside the ion layer.

A conspicuous deviation between model and PIC results concerns the side of the incident light pulse. It appears in simulations with different PIC codes and may be of numerical origin. This needs further study. Smaller deviations in the layer thickness develop at later times (see Fig. 1b at $t/\tau_L = 3$) and may be attributed to approximations made in Eqs. 3 and 4 for the self-radiation fields.

In Figs. 2a and 2b, the temporal evolution of the electron $\gamma$-factor is depicted. Again we find good agreement between model and PIC results, both for values of $\langle \gamma \rangle$ averaged over the layer (Fig. 2a) and spatial $\gamma$ gradients (Fig. 2b). The average rises up to a maximum of $\langle \gamma \rangle \approx 15$, and then falls again when the layer drifts into the decelerating phase of the second half-wave. Notice that the maximum of $\langle \gamma \rangle$ is much lower than the pure single-particle estimate of $\gamma_{max} = 2a_0^2 = 50$. This is because of the electrostatic and electromagnetic self-fields of the layer, well described by the model.

![Fig. 2.](image-url) (a) Temporal evolution of the $\langle \gamma \rangle$-factor averaged over electron layer. (b) Spatial distribution of $\gamma$-factor over electron layer at different times. Model results (solid lines) are compared with 1D-PIC results (dashed lines).

**Fig. 3.** (a) Density distribution at $t/\tau_L = 1$ with (solid line) and without (dashed line) accounting for the linear increase of $E_{ax}$ inside the ion volume (compare Eq. 11). (b) Thickness of electron layer $d$ at time $t/\tau_L = 1$ plotted versus initial thickness $d_0$; model results (solid line) compared to 1D-PIC results (dashed line).

### 4 Layer compression depending on initial thickness

A particularly interesting observation is a tendency for layer compression during the first stage of acceleration stage. Compression may even overcome Coulomb expansion. It originates from the initial phase of acceleration, when the electrons are still inside the ion layer. There the $E_x$ field, felt by an electron initially at $x_0$ and given by Eq. 1, rises linearly $\propto (x - x_0)$ before reaching a constant $E_x$ value when leaving the ion volume. During this initial phase, electrons on the left side of the layer facing the incident laser pulse gain more energy than those to the right. This leads to layer bunching, as it is demonstrated for time $t/\tau_L = 1$ in Fig. 3a. Here the dashed profile corresponds to using $E_{ax}(x, x_0) = N(d_0 - x_0)$ for all $x$ and $x_0$, while the solid profile corresponds to the correct treatment using full Eq. 1.

The compression depends on the initial thickness of the foil. It is more pronounced when starting with thicker foils. In Fig. 3a, we show the thickness $d$ at time $t/\tau_L = 1$ as a function of initial thickness $d_0$, keeping the areal density $N d_0 = 1$ constant. For $d_0/\lambda_L = 0.02$ and $t/\tau_L = 1$, layer expansion is not only reduced relative to the reference case with $d_0/\lambda_L = 0.001$, but is actually slightly compressed. This effect is seen in both model and PIC results. This is good news for experiments using laser pulses with more realistic shapes. In case of Gaussian shapes rather than the sharp-front flat-top pulses considered here, one expects some extent of foil expansion before the pulse maximum hits. The compression then leads to high-density layers during the first laser cycles of layer acceleration.

### 5 Expansion of a freely propagating layer

Finally we discuss a simple solution which is important for the late stage of a freely propagating layer when it is not in contact any more with the laser field. The solution describes the longitudinal decay of the relativistic mirror
due to Coulomb explosion. Initially the layer has uniform density $n_0$ and plasma frequency $\omega_p$. It consists of electrons only (no ion layer) and propagates in $x$-direction with momentum $\gamma_0\beta_0$. The equation of motion then reduces to

$$dp_x/dt = N x_0$$

(14)

keeping definitions and normalization the same as before. The coordinate $x_0$ now denotes the initial position of a test electron in the plane layer extending from $-d_0$ to $+d_0$. The longitudinal momentum is

$$p_x(t, x_0) = \beta_0 \gamma_0 + N x_0 t,$$

(15)

and the corresponding energy

$$\gamma = \gamma_0 \sqrt{1 + 2\beta_0 \frac{x_0}{d_0} \frac{t}{T_0} + \left(\frac{x_0}{d_0} \frac{t}{T_0}\right)^2},$$

(16)

involves a characteristic time $T_0 = \gamma_0/(Nd_0)$. In dimensional units, it is given by

$$T_0 = \frac{\gamma_0}{\omega_p^2 d_0/c}.$$  

(17)

Apparently, it is the relativistic plasma frequency which sets the longitudinal decay time. The electron trajectories $x(t, x_0)$ can be easily inferred from energy conservation

$$\gamma - \gamma_0 = N d_0 (x - x_0)$$

(18)

From this the evolution of the density profile $n(x, t)$ is obtained with the help of Eq. (15). Profiles of an initially uniform electron layer are plotted in Fig. 4. In this case the characteristic decay time is $T_0 = 4.3$ fs.

### 6 Conclusion

In conclusion, we have considered electron acceleration from ultra-thin foils by high-contrast ultra-short laser pulses in the regime in which all electrons are expelled from the foil. They form a dense electron layer that can be used as a relativistic mirror for Thomson backscattering, at least over a short time interval of a few laser periods after foil interaction. This paper supplements a companion paper discussing the backscattered spectra [6]. Here we have shown that the analytical model of Kulagin et al. [1] well reproduces corresponding particle-in-cell simulations. Analytical theory describing layer dynamics is important for basic understanding as well as developing and analyzing future experiments.

As a particular result, we have identified a regime of layer compression related to initial acceleration inside the ion volume. It is found that foils somewhat expanded initially are superior to foils of same areal density, but thinner and denser. Also an analytical formula is given for longitudinal layer expansion after it disconnects from the driving laser field and is freely propagating.

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