Yangian Symmetries in the $SU(N)_1$ WZW Model and the Calogero-Sutherland Model

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Abstract

We study the $SU(N)$, level 1 Wess-Zumino-Witten model, with affine primary fields as spinon fields of fundamental representation. By evaluating the action of the Yangian generators $Q^a_0, Q^a_1$ and the Hamiltonian $H_2$ on two spinon states we get a new connection between this conformal field theory and the Calogero-Sutherland model with $SU(N)$ spin. This connection clearly confirms the need for the $W_3$ generator in $H_2$ and an additional term in the $Q^a_1$. We also evaluate some energy spectra of $H_2$, by acting it on multi-spinon states.

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In recent years, a number of Conformal Field Theories (CFT’s) have been reconstructed in massless quasi-particle bases of CFT’s, among many equivalent bases of CFT’s. This is better suited for the description of particles in the massive perturbations of CFT’s than in the conventional Verma-module basis. One of very interesting examples which can be studied with the quasi-particle description is the low energy limit of the Calogero-Sutherland (CS) model with $SU(2)$ spin. This new formulation does not resort to affine and Virasoro symmetries, but to Yangian symmetry. The Hilbert space may be regarded as spinon Fock space constructed using the generalized commutation relations. It is now known that Yangian generators can be written in terms of $SU(2)$, level 1 affine currents. Furthermore the action of these generators on the multi-spinon states provides a close relation with the low energy limit of the CS model with $SU(2)$ spin. The generalization to higher level $SU(2)$ Wess-Zumino-Witten (WZW) model has also been constructed in Ref. However, for higher level cases, it is impossible to represent the Yangian generators in terms of affine current.

Regarding the $SU(N)(N > 2)$, level 1 generalization, the additional structures due to the conformal dimension 2 and 3 fields are inevitable. It has been found in Ref. that the Yangian generator $Q_1^a$ and the Hamiltonian $H_2$ need extra terms in terms of $W^a = \frac{1}{2}d^{abc}(J^b J^c)$ and the third order Casimir operator $W_3 = \frac{1}{6}d^{abc}(J^a(J^b J^c))$ (notations for these quantities will be explained later). However it has not been demonstrated that the action of the Yangian generator and the Hamiltonian are indeed those of the CS model with $SU(N)$ spin. This is an interesting problem in light of the work in Ref., where the $SU(N)$ generalization of the Hamiltonian of the dynamical CS model and its Yangian structure was discussed. Another interesting problem is the notion of spinons for $SU(N), N > 2$. In the study of Heisenberg spin chains, Reshetikhin observed that there are in general $N - 1$ types of spinons for $SU(N)$ case. So we expect similar thing in our case. To fully describe the energy spectrum of the model, one has to introduce $N - 1$ species of spinons.
In this letter, we consider the $SU(N)_1$ WZW CFT and construct the Yangian generators and the Hamiltonian $H_2$ in terms of the affine currents of the theory. To determine the coefficients of the extra terms $W^a$ and $W_3$ in $Q^a_1$ and $H_2$ one can check the Yangian algebra $Y(sl_N)$ and the commutativity of $[Q^a_1, H_2] = 0$, which are very complicated [5]. Rather, we apply them to multi-spinon states of $N$-representation, and compare the result with the Hamiltonian and the Yangian generators in the dynamical model [6]. This uniquely fixes the coefficients and gives further physical motivation for the $W$-structures. In general there can be spinons of representation other than the fundamental representation, but these are not considered in this letter.

The Yangian generator $Q^a_1$ and the Hamiltonian $H_2$ consist of two parts, one of which contains the derivative terms, $\sum_i D_i$ and $\sum_i D_i^2$, and the other part contains the interaction term, $\sum_{i\neq j} t_i t_j$, where $D_i = w_i \partial w_i$, $w_i$ being the position of the $i$-th particle in the CS model, and $t_i$ the $SU(N)$ generator acting on the $i$-th particle. We observe that without the contributions from $W^a$ and $W_3$, the derivative part is smaller than that is required from integrability. That is, in CS model the relative coefficient between the derivative term and interaction term is fixed uniquely by integrability [8]. On the other hand, $W^a$ and $W_3$ contribute only to the derivative part up to irrelevant constant terms, and we can therefore determine the coefficients in front of them from the integrability.

The $SU(N)_1$ WZW model has the affine Kac-Moody current $J^a(z)$ which satisfies the operator product expansion (OPE),

$$J^a(z)J^b(w) = \frac{\delta^{ab}}{(z-w)^2} + \frac{1}{(z-w)}f^{abc}J^c(w) + \cdots,$$

where $a = 1, 2, \cdots, N^2 - 1$, and $f^{abc}$ are the structure constants of $SU(N)$ with the normalization $f^{abc} f^{bcd} = -2N \delta^{ad}$. We do not write down explicitly the regular parts of OPE. The chiral vertex operators of $N$-representation of this model are identical as the spinon fields $\phi^a(z)$, $\alpha = 1, 2, \cdots, N$, which transform according to

$$J^a(z)\phi^\alpha(w) = \frac{(t^a)_{\beta}^{\alpha} \phi^\beta(w)}{(z-w)} + \cdots,$$
where \((t^a)^\alpha_{\beta}\) are the \(\alpha \beta\) elements of the \(SU(N)\) matrices in the fundamental representation, \([t^a, t^b] = f^{abc} t^c\).

The first two generators of \(Y(\mathfrak{sl}_2)\) Yangian can be written in terms of the affine Kac-Moody current. This can be motivated from a limiting procedure which takes the number of lattice sites of a spin system to infinity\[9\]. In case of \(Y(\mathfrak{sl}_N)(N > 2)\) one has to be more careful since new local operators can arise from this limiting procedure\[5\]. There is of course no correction for \(Q^a_0\). Thus we write down the Yangian generators in terms of affine currents and also a contribution from \(W^a\) as follows:

\[
Q^a_0 = J^a_0, \\
Q^a_1 = Q^a_1,\text{unc} + \mu Q^a_1,\text{corr} \\
= \frac{1}{2} f^{abc} \sum_{m > 0} (J^b_{-m} J^c_m) + \mu d^{abc}(J^b J^c)_0,
\]

(3)

where \(J^a_m, m \in \mathbb{Z}\) is the Laurent mode of \(J^a(z) = \sum_m J^a_m z^{-m-1}\) and \(d^{abc}\) is the completely symmetric tensor, \(\{t^a, t^b\} = d^{abc} t^c + \frac{2}{N} \delta^{ab}\) with the normalization \(d^{abc} d^{bcd} = \frac{2(N^2-4)}{N} \delta^{ad}\). The brackets mean normal ordering between fields\[10\]. In principle, we may think of \(f^{abc}(J^b J^c)_0\) as the other possible candidate for correction term. But this is proportional to the zero mode of \(\partial J^a\) which does not give rise to the derivative part of \(Q^a_1\). Similarly, the Hamiltonian \(H_2\) can be written as follows:

\[
H_2 = H_{2,\text{unc}} + \nu H_{2,\text{corr}} \\
= \sum_{m > 0} (m J^a_{-m} J^a_m) + \nu d^{abc}(J^a (J^b J^c))_0.
\]

(4)

In the above ‘unc’ and ‘corr’ stand for uncorrected and corrected parts, where we mean by the uncorrected term which is the same form as in \(Y(\mathfrak{sl}_2)\). The parameters, \(\mu, \nu\) will be determined later.

First of all, we consider the action of \(H_2\) on the two spinon states given by the following contour integrals:

\[
H_2 \phi^\alpha(w_2) \phi^\beta(w_1) |0\rangle = \left[ \int_{|z_1| > |z_2|} \frac{dz_1}{2\pi i} \int_{C_i} \frac{dz_2}{2\pi i} \frac{z_1 z_2}{(z_1 - z_2)^2} J^a(z_1) J^a(z_2) \right] \phi^\alpha(w_2) \phi^\beta(w_1) |0\rangle.
\]
\[ + \nu \, d^{abc} \oint_{C_i} \frac{dz}{2\pi i} z^2 (J^a(J^bJ^c))(z) \phi^\alpha(w_2)\phi^\beta(w_1)|0\rangle = \left[ \frac{N^2 - 1}{2\Delta(2\Delta + 1)N} \left( 1 + \nu \frac{6(N^2 - 4)}{N} \right) \right] \sum_i ((D_i)^2 + 2\Delta D_i) \]
\[ - \sum_{i \neq j} \theta_{ij} \theta_{ji} \theta^a_i \theta^a_j + 2\nu C_3(N) \right] \phi^\alpha(w_2)\phi^\beta(w_1)|0\rangle, \tag{5} \]

where the contours \( C_i \)'s encircle the points \( w_1, w_2 \) and \( D_i = w_i \partial_{w_i}, \theta_{ij} = \frac{w_i}{w_i - w_j} \) and \( C_3(N) = \frac{(N^2 - 1)(N^2 - 4)}{N^2} \). We used the following fact;
\[ (t^a J^b)^\alpha_\beta = \left( \frac{\delta^{ab}}{N} + \frac{1}{2} f^{abc} t^c + \frac{1}{2} d^{abc} t^c \right)^\alpha_\beta, \tag{6} \]

and many identities among \( f^{abc} \) and \( d^{abc} \). (See Ref.[10] for conventions.) We can raise and lower spinor indices with antisymmetric symbols \( \epsilon_{\alpha\beta} \) and \( \epsilon^{\alpha\beta} = -\epsilon_{\alpha\beta} \).

The correction term vanishes automatically for \( N = 2 \) and the rest is in agreement with the one in Refs.[3, 4]. In this calculation, one should know the normal ordering of \( (J^a\phi^\alpha)(z) \), which is the next term of the right hand side of Eq.(2). It can be written as
\[ (J^a\phi^\alpha)(z) = \frac{1}{2\Delta} (t^a)^\alpha_\beta \partial^\beta \phi^\alpha(z), \tag{7} \]

where \( \Delta = \frac{N^2 - 1}{2N(N + 1)} \) is the conformal dimension of the primary field \( \phi^\alpha(z) \). One can evaluate explicitly the OPE between stress-energy tensor in Sugawara form \( T = \frac{1}{2(N + 1)} (J^a J^a)(z) \) (with the central charge \( c = N - 1 \)), and spinon field \( \phi^\alpha(w) \), according to Eq.(3) and recognize that \( \Delta \) is the coefficient of \( \frac{1}{(z-w)^2}\phi^\alpha(w) \). The mode of \( T(z) \), i.e. \( L_n \), is given by
\[ L_n = \frac{1}{2(1 + N)} \sum_{m=-\infty}^{\infty} (J^a_m J^a_{n-m}). \tag{8} \]

By acting the spinon field \( \phi^\alpha \) on both sides, we have the following relation:
\[ L_{-2}\phi^\alpha = \frac{1}{4\Delta} \partial^2 \phi^\alpha + \frac{N(4\Delta - 1)}{2(N^2 - 1)} t^a J^a_{-2} \phi^\alpha. \tag{9} \]

Then the normal ordering of \( (\partial J^a\phi)(z) = J^a_{-2}\phi(z) \) leads to
\[ (\partial J^a\phi^\alpha)(z) = \frac{1}{2\Delta(2\Delta + 1)} t^a \partial^2 \phi^\alpha(z) + \frac{2(N^2 - 1)}{N(2N - 1)} \Phi^\alpha(z), \tag{10} \]
where

\[ \Psi^{a,\alpha}(z) \equiv (\partial J^a \phi^\alpha)(z) - \frac{N^2}{2(N^2 - 1)} t^a \partial^2 \phi^\alpha(z) + \frac{N}{N^2 - 1} t^a (T \phi^\alpha)(z), \]

(11)

which is a primary field of conformal dimension \(2 + \Delta = \frac{5N-1}{2N}\). The second term of Eq.(10) vanishes when we calculate \(t^a(\partial J^a \phi^\alpha)(z)\). Finally we get the closed form of Eq.(5) by exploiting the above arguments. We would like to stress that the contribution from correction term does not give rise to the interaction term, \(t^a t^b\), but the derivative terms, \(D_i\) and \(D_i^2\). This is due to the fact that this corrected term is purely the zero mode of a conformal field in the theory. In other words, it acts on only one spinon field, not on two.

The Hamiltonian of the dynamical model[8] is given by

\[ H_2^{\text{dyn}} = \sum_i (D_i)^2 + \sum_{i\neq j} \lambda (P_{ij} + \lambda) \theta_{ij} \theta_{ji} \]

\[ = \sum_i (D_i)^2 + \sum_{i\neq j} \lambda (t^a_i t^a_j + \frac{1}{N} + \lambda) \theta_{ij} \theta_{ji}, \]

(12)

where we used that the permutation operator \(P_{ij}\) for \(SU(N)\) can be written as

\[ P_{ij} = t^a_i t^a_j + \frac{1}{N}. \]

(13)

We can fix the parameter \(\nu\) by identifying the Hamiltonian in Eq.(3) with that in Eq.(12) up to an overall factor. Then, we must first have,

\[ \lambda = -\frac{1}{N}, \]

(14)

and also

\[ \frac{N^2 - 1}{2\Delta(2\Delta + 1)N} \left( 1 + \nu \frac{6(N^2 - 4)}{N} \right) = N. \]

(15)

So we get

\[ \nu = \frac{N}{6(N + 1)(N + 2)}. \]

(16)

This coefficient is exactly the same as the one in Ref.[5]. It is important to notice that the reason for the extra \(\nu\) term is evident in Eq.(15). Putting these together, the
Hamiltonian $H_2$ acts on two spinon states by a linear combination of $H_1^{dyn} = \sum_i D_i$ and $H_2^{dyn}$ as follows:

$$H_2 = \frac{(N - 1)(N - 2)}{3N} + N \left( H_2^{dyn} + 2\Delta H_1^{dyn} \right),$$  \hspace{1cm} (17)$$

with

$$\lambda = -\frac{1}{N}.$$ \hspace{1cm} (18)$$

The action of $Q_a^1$ on two spinon states is given by the following contour integrals:

$$Q_a^1 \phi^\alpha(w_2)\phi^\beta(w_1)|0\rangle = \left[ \frac{1}{2} \oint_{C_1} \frac{dz_1}{2\pi i} \oint_{C_2} \frac{dz_2}{2\pi i} \frac{z_2}{z_1 - z_2} f^{abc} J^b(z_1) J^c(z_2) \right] \phi^\alpha(w_2)\phi^\beta(w_1)|0\rangle,$$ \hspace{1cm} (19)$$

where the contours $C_i$’s encircle the points $w_1, w_2$. We arrive at the following expressions after doing the integrals:

$$Q_a^1 \phi^\alpha(w_2)\phi^\beta(w_1)|0\rangle = \left[ \left( -\frac{N}{4\Delta} + \mu \frac{N^2 - 4}{\Delta N} \right) \sum_i D_i t_i^a + \mu \frac{N^2 - 4}{N} Q_0^a + \frac{1}{2} f^{abc} \sum_{i \neq j} \theta_{ij} t_i^b t_j^c \right] \phi^\alpha(w_2)\phi^\beta(w_1)|0\rangle.$$ \hspace{1cm} (20)$$

Again the correction terms vanish for $N = 2$. The contribution due to the presence of second term in Eq.(19) does not give rise to the last term of Eq.(20). It is easy to check that the Yangians given by the differential operators satisfy Yangian algebra. Comparing with the Yangian generator of dynamical model given in Ref.[12], we can determine the parameter $\mu$ as follows using the relation $\lambda = -\frac{1}{N}$ obtained in Eq.(15)

$$-\frac{N}{4\Delta} + \mu \frac{N^2 - 4}{\Delta N} = \frac{1}{\lambda}(= -N).$$ \hspace{1cm} (21)$$

That is, we must have

$$\mu = -\frac{N}{4(N + 2)}.$$ \hspace{1cm} (22)$$

This gives an independent confirmation of the Ansatz given in Ref.[1], where the coefficient was fixed by the Yangian algebra $(Y1) - (Y4)$ and the commutativity of $[Q_1^1, H_2] = 0$. 


0. Here checking the Yangian algebra in terms of differential operators is easier than in terms of affine currents.

The final form of $Q_1^a$ on two spinon states is given by

$$Q_1^a \phi^\alpha(w_2)\phi^\beta(w_1)|0\rangle = \left[-N \sum_i D_i t_i^a + \frac{1}{2} f^{abc} \sum_{i \neq j} \theta_{ij} t_i^b t_j^c - \frac{N - 2}{4} Q_0^a\right] \phi^\alpha(w_2)\phi^\beta(w_1)|0\rangle.$$  \hspace{1cm} (23)

Just as in the $H_2$ case, the term of $-\frac{N - 2}{4} Q_0^a$ can be absorbed in $Q_1^a$ by redefining it because it induces only a spectral parameter shift.

Having found the differential forms of the Hamiltonian and the Yangian generators, we now look for the values of low lying energy spectrum of $H_2$. It was found in Ref.[1] that the eigenfunctions of the models are grouped into multiplets characterized by sets of distinct positive integers \{m_i\}, called motifs, which can be transformed into sequences of 0’s and 1’s. The rules on the sequences are that at most $N - 1$ consecutive 1’s can occur, and that the asymptotic behavior of the sequence falls into $N$ distinct cases, each of which corresponds to the $N$ primary sectors including the identity, of the $SU(N)$ level 1 WZW model. Some of low lying energy levels of $SU(3)$ were found in Ref.[5]. We will recover these energy levels by considering the actions of $H_2$ and $H_1$ on states with various modes of spinon fields of fundamental representation acted on the vacuum $|0\rangle$.

We can define a spinon field of representation $\lambda$ corresponding to Young diagram of $[1]^r$ ($r = 1, 2, \ldots N - 1$) as chiral vertex operators transforming in the irreducible $sl_N$ representation of $\lambda$, i.e. $\Phi : L_\sigma \rightarrow L_\rho$. The mode expansion of the spinon is given by following:

$$\Phi\left(\begin{array}{c} \lambda \\ \rho \sigma \end{array}\right)(z) = \sum_{n \in \mathbb{Z}} \Phi\left(\begin{array}{c} \lambda \\ \rho \sigma \end{array}\right)^{-n-(\Delta(\rho)-\Delta(\sigma))} z^{n+\Delta(\rho)-\Delta(\sigma)-\Delta(\lambda)}. \hspace{1cm} (24)$$

In the above $\Delta(\lambda)$ etc. are conformal dimensions of representation $\lambda$, i.e. $\Delta(\lambda) = \frac{C_\lambda}{2(N+1)}$, $C_\lambda$ being the second Casimir in the $\lambda$-representation. The single spinon of $\overline{\lambda}$
representation $\phi^\alpha(z)$ on $|0\rangle$ gives the \( \overline{N} \) state and the action of $H_2$ is as follows:

$$H_2 \phi^-_{-\Delta-n}|0\rangle = \left[ \frac{NC_3(N)}{6(N+1)(N+2)} + Nn(n-1) + (2N-1)n \right] \phi^-_{-\Delta-n}|0\rangle. \quad (25)$$

The lowest one is when $n = 0$ and for $N = 3$ we have the energy eigenvalue of $\frac{1}{9}$. Note that there is a contribution from the third Casimir of $SU(N)$ in the energy. For the $N$-representation, the energy of a single spinon state is obtained by changing the sign of the contribution of the third Casimir, i.e. $C_3(N) = -C_3(\overline{N})$. We can move up to higher energy states by considering multi-spinon states. First let us consider the two spinon states. The decomposition of product of two spinon states of representation $\overline{N}$ can be written as $\overline{N} \times \overline{N} = \frac{N(N-1)}{2} + \frac{N(N+1)}{2}$, i.e. we have a decomposition into an antisymmetric one and a symmetric one. Now we apply the action of $H_2$ on the antisymmetric part of two spinon states to find its eigenvalues:

$$\Phi_{\alpha_3...\alpha_N}(n_2,n_1) \equiv \epsilon_{\alpha_1\alpha_2...\alpha_N} \phi^-_{-\Delta_2-n_2} \phi^-_{-\Delta_1-n_1}|0\rangle, \quad n_2 \geq n_1 \geq 0. \quad (26)$$

It is now straightforward to compute the action of $H_2$ on these states. We just write down the results.

$$H_2 \Phi_{\alpha_3...\alpha_N}(n_2,n_1) = \left[ 2 \times \frac{NC_3(\overline{N})}{6(N+1)(N+2)} 
+ Nn_1(n_1-1) + N(n_2 + \Delta_2 - \Delta_1)(n_2 + \Delta_2 - \Delta_1 - 1) 
+ (2N-1)(n_1 + n_2 + \Delta_2 - \Delta_1) \right] \Phi_{\alpha_3...\alpha_N}(n_2,n_1) 
+ 2(t_1^\alpha t_2^\alpha) \sum_{l>0} l \Phi_{\alpha_3...\alpha_N}(n_2+l,n_1-l). \quad (27)$$

In the above $t_i^\alpha$ acts on $\phi^\alpha_i$. Since the action of the Hamiltonian is lower triangular, we can immediately read off the energy eigenvalues of $H_2$ given as follows:

$$E(n_1,n_2;N) = \frac{(N-1)(N-2)}{3N} + (2N-1)(n_1 + n_2 + \Delta_2 - \Delta_1) 
+ N \left[ n_1(n_1-1) + (n_2 + \Delta_2 - \Delta_1)(n_2 + \Delta_2 - \Delta_1 - 1) \right]. \quad (28)$$

For the $N \times N$ case, we have to change the sign of the first term. We can apply $H_2$ on various multi-spinon states. The explicit form of the energy eigenvalues requires explicit forms of conformal dimensions and fusion rules of the spinon fields.
Let us illustrate this result for the simplest case, i.e $SU(3)$. In this case, there exist three primary motifs, 1 for vacuum, $3,3$ for single spinon states. The eigenvalues for these states are $0, \frac{1}{9}, -\frac{1}{9}$ respectively by putting $n = 0$ in Eq.(25). The excited states for $3,3$ can be obtained by putting $n = 1$. Then the eigenvalues lead to $\frac{46}{9}, \frac{44}{9}$ respectively. For the two spinon state of $3 \times 3$, we have the eigenvalues of $H_2, -\frac{1}{9}$ with $n_1 = n_2 = 0$ and $\frac{26}{9}$ with $n_1 = 0, n_2 = 1$ in Eq.(28). Similarly, the eigenvalues of $H_2$ in the case of $3 \times 3$ are $-\frac{5}{9}$ with $n_1 = n_2 = 0$ and $\frac{22}{9}$ with $n_1 = 0, n_2 = 1$. Other interesting two spinon cases are $N \times N :$

$$\phi^{\alpha}_{-n_2-\Delta_2} \phi^{\beta}_{-n_1-\Delta_1} |0\rangle,$$

where we have $\Delta_2 = \frac{N-1}{2N}$ and $\Delta_1 = \frac{N-1}{2N}$. For $n_1 = 0$ and $n_2 = 2$ we get energy eigenvalue $2N + 2$ and for $n_1 = 0$ and $n_2 = 3$ we get the energy eigenvalue $6N + 3$, consistent with the result from the motif picture in Ref.[5].

We can also consider three spinon state $\frac{N(N-1)}{2} \times N \times N$ with

$$\phi^A \left( \frac{N(N-1)}{2} \right)^{-n_3-\Delta_3} \phi^\beta \left( \frac{N}{N(N-1)} \right)^{-n_2-\Delta_2} \phi^\gamma \left( \frac{N}{N-1} \right)^{-n_1-\Delta_1} |0\rangle,$$

where $A = 1, \ldots, \frac{N(N-1)}{2}$, and

$$\Delta_1 = \Delta(N) = \frac{N-1}{2N}, \quad \Delta_2 = \Delta \left( \frac{N(N-1)}{2} \right) - \Delta(N) = \frac{N-3}{2N},$$

$$\Delta_3 = -\Delta \left( \frac{N(N-1)}{2} \right) = -\frac{N-2}{N}.$$

We can calculate the energy eigenvalue of the state explicitly for the $N = 3$ case as follows:

$$E(n_1, n_2, n_3; 3) = 3 \times \frac{NC_3(N)}{6(N+1)(N+2)}$$

$$+ N \left[ n_1(n_1 - 1) + (n_2 + \Delta_2 - \Delta_1)(n_2 + \Delta_2 - \Delta_1 - 1) + (n_3 + \Delta_3 - \Delta_0)(n_3 + \Delta_3 - \Delta_0 - 1) \right]$$

$$+ (2N - 1) \left[ n_1 + n_2 + n_3 + \Delta_2 + \Delta_3 - \Delta_1 - \Delta_0 \right],$$

(32)
where $\Delta_0 = \Delta_1$, and $C_3(R)$ are the third Casimirs of $R$ representation. So for the low lying case of $n_1 = 0$, $n_2 = 1$, and $n_3 = 2$, we get $E(0,1,2;3) = 11$, consistent with the result from motif picture \[J].

The main observation in this letter is that it is possible to reinterpret the Yangian generators $Q^a_0, Q^a_1$ and the Hamiltonian $H_2$ for $SU(N)_1$ WZW CFT by exploiting the integrability of CS model. We can apply the forms of the Hamiltonian $H_2$ on spinon states, to obtain some of the energy spectra, which is consistent with the results obtained from the motif picture, justifying it. It is also straightforward to apply the technique we made here to a higher Hamiltonian, for example, $H_3$ and find what is it for WZW model from the viewpoint of CS model. Also, the spinon basis of $SU(N)_1$ WZW model should be useful for writing down the corresponding character formulas.

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