Increasing the accuracy of definition torsional geometric properties for rolled and welded beams

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Abstract. The features of determining the torsional geometric properties of the cross-section of a steel beam, rolled or composed of three sheets of arbitrary thickness, are considered. When calculating warping properties, it is suggested to take into account the degree of torsional stiffness of the attached structures, that more accurately reflects the actual operation of the fixed beam in the composition of the decking. The differentiated account of rounding at the flange adjacent to the web leads in the rolled profiles to a significant structured increase in the value of the torsional constant. These factors characterize the special conditions of operation of the steel element with complex resistance, that allows determining more precisely the value of internal forces, namely the design bending moments in two planes and the bimoment, which are proportional to the distribution function of the angle of rotation of the rod along its length. It is noted that in fact the bimoment is not only determined by on the load, the elastic flexural-torsional constant of the cross-section and the span of the beam, an eccentricity, as well as on the stiffness of the structure that attached to the beam.

1. Introduction

Geometric properties that need refinement related to the torsion of the beams with the open profile. These include: torsional constant, warping constant, elastic flexural-torsional constant of the cross-section, warping coordinate (area), shear centre coordinate, mono-symmetricity parameter (Wagner coefficient), stability parameter (auxiliary distance of cross-section). The calculations using the geometric properties take account of: 1 – conventional and alternative (second-order theory) beam design for general stability; 2 – design for the compatible effect of torsion from the eccentricity of the application load and the transverse bending (it can be in one or two planes); 3 – design of beams with distortions in the plane of least stiffness.

Suppose that when the profile is engaged, a variable point that passes through the axis of rotation of the cross-section is a shear centre and can move up or down depending on the degree of restraining, i.e., on the stiffness and location of the attached structures. With sufficient shear stiffness of the attached structures, it is considered that the compressed belt is completely secured from transverse displacements and the rotation axis, which in this case is called restrained, extends over the top of the beam.
The shear stiffness of flooring or bracing is often sufficient to accommodate the restrained axis of rotation of the beam by the criterion of having a reduced required shear stiffness. Due to the high convergence of calculation and simulation results, it is revealed that for a channel with sufficient stiffness of the flooring, the axis of rotation will be located above the shear centre at the level of the top flange, not at the place of flooring attachment as previously thought.

2. The main body

For cross-sections consisting of several narrow rectangles of a certain length and thickness, the torsional constant is determined by the Saint-Venan formula and depends on the size of the rectangles. The work of the authors [1] determines the degree of influence of the rounding between the web and the flange on the empirical coefficient $\eta$, which describes the complexity and type of a cross-section. The coefficient makes it simple to take into account the presence of roundings between broken rectangles of cross-section. Thus, for example, the torsional constant of an I-section $I_t$ by DBN V.2.6-198:2014 is determined taking into account a constant coefficient $\eta=1.29$, for the channel cross-section it is accepted $\eta=1.11$. These data are based on the manual [3]. The results of determining the torsional constant for selected I-beams with parallel edges of flanges (according to GOST 26020-83) and channels with flange slope (in accordance with GOST 8240-89) are presented in the tables. It has been found the degree of approximation by the torsional constant determination can be significant for rolled profiles, particularly for a relatively large radius of rounding. A more accurate estimate of the torsional constant correction for deductions at open ends and its strengthening at the joint is acceptable, so its can be performed in the LTBeam program, which showed the best convergence with FEA, or by the method of the inscribed circle given, for example, in [4], or in ESPRI (engineer’s electronic reference book). The value of the transition coefficient $\eta$ for rolled normal I-beams, based on the most similar results in the table, varies significantly – from 1.23 to 1.60 (for channels – from 1.05 to 1.13) and needs analytical differential description. The greatest effect is achieved for I-beams with medium height (35B1–45B1) and tall channels (22–40). The formulas given in [4] are recommended to be used to obtain $I_t$ by rounding. The problem of geometric optimization in spatial rod structures is included in the article [4]. Other studies on this topic [5–9] are also known.

Consider the geometric properties determined by the Marc Villette formulas embedded in a special free LTBeam program, for the general case – a monosymmetric I-beam with roundings between the web and the flanges (figure 1).

![Figure 1](image_url)

**Figure 1.** Monosymmetric I-section to determine the geometric properties of the over flange, web, under flange, over and under roundings; and possible diagram of normal stresses in it under complex load.
The total cross-sectional area will be:

\[ A = A_{f_1} + A_{f_2} + A_w + 2A_{r_1} + 2A_{r_2}. \]  

(1)

The ordinate of the centre of gravity (centroid) \( G \) relative to the cross-section bottom will be:

\[ z_G = A^{-1} \left[ A_{f_1} \left( h - \frac{t_{f_1}}{2} \right) + 2A_{f_1} \left( h - t_{f_1} - v_{r_1} \right) + A_w \left( t_{f_2} + \frac{h_r}{2} \right) + 2A_{r_2} \left( t_{f_2} + v_{r_2} \right) + A_{f_2} \frac{t_{f_2}}{2} \right]. \]  

(2)

Auxiliary distances for over and under flanges, web, over and under roundings according to figure 1 will be equal to:

\[ z_{f_1} = h - 0.5t_{f_1} - z_G; \quad z_{f_2} = h - 0.5t_{f_2} - z_G; \]  

(3)

\[ z_w = t_{f_2} - 0.5h_w - z_G; \]  

(4)

\[ z_{r_1} = h - t_{f_1} - v_{r_1} - z_G; \quad z_{r_2} = t_{f_2} + v_{r_2} - z_G. \]  

(5)

The cross-section has moments of inertia that will be equal to:

\[ I_y = I_{y_{f_1}} + A_{f_1} z_{f_1}^2 + 2I_{f_1} + 2A_{f_1} z_{r_1}^2 + I_{y_{w}} + A_w z_w^2 + 2I_{r_2} + 2A_{r_2} z_{r_2}^2 + I_{y_{f_2}} + A_{f_2} z_{f_2}^2. \]  

(6)

\[ I_z = I_{z_{f_1}} + 2I_{f_1} + 2A_{r_1} \left( 0.5t_{w} + v_{r_1} \right)^2 + I_{z_{w}} + 2I_{r_2} + A_{r_2} \left( 0.5t_{w} + v_{r_2} \right)^2 + I_{z_{f_2}}. \]  

(7)

To determine the torsional constant of a rectangular cross-section \( a \times b \), with \( a < b \), the approximate formula is used:

\[ R_a(a,b) = \frac{1}{3} ba^3 \left[ 1 - \frac{a}{b} \left( 0.633 - 0.055 \frac{a^3}{b^3} \right) \right]. \]  

(8)

The components of the torsional constant for individual rectangular elements will be:

\[ I_{i_1} = R_i \left( t_{f_1}, b_{f_1} \right); \quad I_{i_2} = R_i \left( t_{f_2}, b_{f_2} \right); \quad I_{i_3} = R_i \left( t_{w}, h \right); \]  

(9)

\[ I_{i_4} = R_i \left( \min \left(t_w, t_{f_1} \right), \max \left(t_w, t_{f_1} \right) \right); \quad I_{i_5} = R_i \left( \min \left(t_w, t_{f_2} \right), \max \left(t_w, t_{f_2} \right) \right); \]  

(10)

\[ I_{i_6} = R_i \left( \min \left(t_w + 0.4r_1, t_{f_1} + 0.4r_1 \right), \max \left(t_w + 0.4r_1, t_{f_1} + 0.4r_1 \right) \right); \]  

(11)

\[ I_{i_7} = R_i \left( \min \left(t_w + 0.4r_2, t_{f_2} + 0.4r_2 \right), \max \left(t_w + 0.4r_2, t_{f_2} + 0.4r_2 \right) \right). \]  

(12)

The total torsional constant of the cross-section of the steel element will be equal to:

\[ I_t = I_{i_1} + I_{i_2} + I_{i_3} - I_{i_4} \left( \frac{t_w}{b_{f_1}} \right)^2 - I_{i_5} \left( \frac{t_w}{b_{f_2}} \right)^2 - \alpha_1 \left( I_{i_4} - I_{i_6} \right) - \alpha_2 \left( I_{i_5} - I_{i_7} \right). \]  

(13)

Auxiliary coefficients \( \alpha \) by \( i = 1; 2 \) will be for \( r_i > 0 \) (for \( r_i = 0 \) \( \alpha_i = 0 \)):

\[ 6t_w + r_i \leq b_{f_i} \rightarrow \alpha_i = 4; \quad 6t_w + t_{f_i} > b_{f_i} \rightarrow \alpha_i = \frac{8}{1 + \frac{6t_w + t_{f_i}}{b_{f_i}}}. \]  

(14)
Consider an alternative approach to determining the torsional constant, taking into account the rounding. For a cross-section with two flanges and one web without rounding between them, we get the expression:

$$I_t = \frac{2bt_f^3 + (h - 2t_f)t_w^3}{3}. \quad (15)$$

In DBN V.2.6-198: 2014 the rounding is taken into account by introducing an empirical coefficient $\varepsilon$ without explaining its purpose. For a rolled I-beam, the cross-section of which is divided into rectangles quite clearly (two full flanges and a web), the formula is written as follows

$$I_t = \varepsilon \left[ 2bt_f^3 + (h - 2t_f)t_w^3 \right], \quad (16)$$

where $\varepsilon$ – coefficient to be taken from table K.1 of DBN V.2.6-198: 2014; for rolled I-beam it is equal $\varepsilon = 0.43$ (1.29/3), for rolled channel $\varepsilon = 0.37$ (1.11/3).

It would seem that the formula for the channel should have the same appearance as recorded in the guide for the selection of elements cross-sections of steel structures [2]. However, it is recorded as follows (Formula (K.5) of DBN V.2.6-198: 2014)

$$I_t = 0.37 \left( h t_w^3 + b t_f^3 \right). \quad (17)$$

This form of recording is devoid of content because, given both the full height $h$ and the full width of the cross-section $b$, the angle among the web and the flange is taken twice. But apart from splitting into a web and two full flanges, the channel cross-section can also be expanded into a full web whose height is equal to the height of the cross-section and incomplete flanges adjacent to the web. The first method gives a slightly higher result of calculating the torsional constant if the thickness of the flange is greater than the thickness of the web. But there is another method of dividing the cross-section into rectangles, which leads to an average result – at the midline. It looks the most reasonable in terms of the torsion theory. For the channel without roundings in this case

$$I_t = \frac{2bt_f^3 + h_s t_w^3}{3}. \quad (18)$$

Consider an alternative approach to determining the torsional constant, taking into account roundings. The methodology below was based on the reference book [3]. Increasing the torsional constant due to roundings is performed in the following scheme. $D$ is the diameter of the largest circle that can be inscribed on the joint, and $\alpha$ is the dimensionless coefficient obtained by the graph or empirical formula. There are different formulas and graphs for various cross-sections. The deduction is defined as $0.105t_f^3$ on each free end. There is the necessary to make four deductions if there are four ends, for example as in an I-beam, and two deductions are made (the link above mistakenly shows four deductions for the channel) if there are only two ends, for example as in a channel. The diameter of an inscribed circle for an I-beam (for a T-shaped connection) is given by the following formula:

$$D_1 = \frac{(t_f + r)^2 + (r + 0.25t_w)t_w}{2r + t_f}. \quad (19)$$

Specifying dimensionless empirical coefficient $\alpha_1$:

$$\alpha_1 = -0.042 + 0.2204\frac{t_w}{t_f} + 0.1355\frac{r}{t_f} - 0.0865\frac{r t_w}{t_f^2} - 0.0725\frac{t_w^2}{t_f^2}. \quad (20)$$
Given the refinement coefficients $D_1$ and $\alpha_1$, as well as the understatement at the ends of the flanges, the formula for determining the torsional constant will take the following form:

$$I_t = \frac{2}{3}bt_f^3 + \frac{1}{3}(h-2t_f)t_w^3 + 2\alpha_1D_1^4 - 0.420t_f^4. \quad (21)$$

The diameter of the inscribed circle for the channel (for the L-shaped connection) is given by the following formula:

$$D_3 = 2\left\{\left(3r + t_w + t_f\right) - \sqrt{2\left(2r + t_w\right)\left(2r + t_f\right)}\right\}. \quad (22)$$

Specifying dimensionless empirical coefficient $\alpha_3$:

$$\alpha_3 = -0.0908 + 0.2621\frac{t_w}{t_f} + 0.1231\frac{r}{t_f} - 0.0752\frac{r^2t_w}{t_f^2} - 0.0945\frac{t_w^2}{t_f^2}. \quad (23)$$

Given the refinement coefficients $D_3$ and $\alpha_3$, as well as the understatement at the ends of the flanges, the formula for determining the torsional constant for the channel will take the following form:

$$I_t = \frac{2}{3}bt_f^3 + \frac{1}{3}(h-2t_f)t_w^3 + 2\alpha_3D_3^4 - 0.210t_f^4. \quad (24)$$

Revised formulas for determining the warping constant of a cross-section with flange slope can be found in the manual [2]. Due to the sufficiently small effect of the rounding between the flanges and the web on the value of the warping constant of the cross-section, established in [10], in the case of a monosymmetric I-beam, the formula for finding it relative to the centre of the shear has the simplified form:

$$I_w = \left[I_{zf1} - I_{zf2}\left(\frac{h-t_{f1}+t_{f2}}{2}\right)\right]^2, \quad (I_{zf1} \neq 0) \quad (25)$$

The warping constant of the cross-section for a partially restrained I-beam at a random shear stiffness $S$ of the attached structure relative to the rotation point is suggested to be determined by the chosen formula, which almost completely corresponds to the results of numerical calculations:

$$I_{w,a} = I_w\left[1 + \frac{I_{zf2}}{I_{zf1}\left(\frac{S}{S_a}\right)^2}\right], \quad (S \leq S_a) \quad (26)$$

where $S_a$ – the shear stiffness of attached structures required to accept the restrained axis of rotation (for the rolled beam ratio of the moments of inertia for under and over flanges $I_{zf2}/I_{zf1} = 1$).

It is advisable to use the formula to find the warping constant of the cross-section of the channel but provided that the height and width of the cross-section are taken in the middle line

$$I_w = \frac{t_fb_t^3h_s^2}{12} \frac{3 + 2\eta\psi}{6 + \eta\psi}, \quad (27)$$

where $\eta = \frac{h_s}{b_s}$; $\psi = t_w/t_f$ – the ratio of the sizes of the cross-section (figure 2).
The maximum warping coordinate of the cross-section can be easily determined by the formula:

\[ \omega_{\text{max}} = 0.25b_f h_y, \]  

(28)

where \( b_f \) – flange width; \( h_y \) – the distance between the centres of flanges.

For the monosymmetric I-beam, the warping coordinates of the cross-section \( \omega_1 \) and \( \omega_2 \) without restraining, as well as the coordinates \((\omega_{1,D}, \omega_{2,D})\) relative to the point (figure 2), which is located on the axis of rotation at the restraining, are determined by the formulas:

\[ \omega_1 = 0.5b_f e_1; \quad \omega_2 = 0.5b_f e_2; \]  

(29)

\[ \omega_{1,D} = 0.5b_f e_{1,D}; \quad \omega_{2,D} = 0.5b_f e_{2,D}, \]  

(30)

where \( b_f(e_1, e_2) \) – over (under) flange width; \( e_1(e_2) \) – distance from shear centre to centre of over (under) flange; \( e_{1,D}(e_{2,D}) \) – distance from the point of rotation to centre of over (under) flange.

**Figure 2.** To determine the torsional geometric properties; diagrams of warping coordinates for rolled and welded beams without restraining and with restraining (theoretical and modeled using FEM).

The position of the axis of rotation relative to the shear centre at a random shear stiffness \( S \) of the structures that attached is established, supposing that the distance between them is zero at zero stiffness, and when the desired stiffness \( S_a \), the axis of rotation of the beam passes through the centre of the over flange, where the lateral support is connected. Then the distances \( e_1, e_2, e_{1,D}, e_{2,D} \) (for the rolled I-beam the distance \( e_1 \) equal \( e_2 \) and is defined as \( 0.5h_y \)) will be:
\[ e_1 = h_s \frac{I_{x_2}}{I_z}; \quad e_2 = h_s \frac{I_{x_1}}{I_z}; \]

\[ e_{1,D} = e_1 \left(1 - \frac{S}{S_a}\right); \quad e_{2,D} = e_2 \left(1 + \frac{e_s}{e_s} \frac{S}{S_a}\right). \]  

(31)

(32)

For the channel, the warping coordinates of the cross-section \( \omega_1 \) and \( \omega_2 \) (figure 2) without taking into account the restraining and the coordinate \( (\omega_{1,D}, C; \omega_{2,D}, C; \omega_{3,D}, C) \) (they include their own coordinates \( \omega_{1,D}, \omega_{2,D}, 1 \omega_{3,D} \) relative to the point of rotation \( D \) and the coordinate of the rotation point \( D \) relative to the centre of gravity \( C; \omega_{D,C} \)) in the case of a restrained axis of rotation extending above the shear centre, it is easily determined by the formulas:

\[ \omega_1 = 0.5h_y e; \quad \omega_2 = 0.5h_y (b_s - e); \]

\[ \omega_{1,C,D} = \omega_{1,D} + \omega_{D,C} = 0.5h_y (e - e_y + 0.5t_w); \]

\[ \omega_{2,C,D} = \omega_{2,D} + \omega_{D,C} = -h_y e + 0.5h_y (e - e_y + 0.5t_w); \]

\[ \omega_{3,C,D} = \omega_{3,D} + \omega_{D,C} = -h_y e + h_y b_s + 0.5h_y (e - e_y + 0.5t_w), \]

(33)

(34)

(35)

(36)

where \( e \) – the distance from the centre of the web to the shear centre; \( b_s \) – the width of the flange from the free end to the centre of the web; \( e_y \) – distance along the y-axis from the centre of gravity \( C \) to the left side of the web (table value).

The distance from the centre of the web to the shear centre is logically determined by the rules of strength of materials for determining geometric properties

\[ e = \frac{b_s}{2 + \frac{h_s t_w}{3b_s t_f}}. \]

(37)

Then the warping constant of the cross-section for the restrained channel beam with sufficient shear stiffness \( S \geq S_a \) of the attached structures relative to the rotation point is suggested to be determined by the deduced formula, which almost completely corresponds to the results of numerical calculations:

\[ I_{\omega,D} = b_s t_f \omega_{1,D,C} + \frac{1}{3} h_y t_w \left( \omega_{1,D,C}^2 + \omega_{2,D,C}^2 + \omega_{3,D,C}^2 \right) + \frac{1}{3} b_s t_f \left( \omega_{2,D,C}^2 + \omega_{3,D,C}^2 + \omega_{4,D,C}^2 \right). \]

(38)

The ordinate of the shear centre \( S \) relative to the centre of gravity \( G \) will be:

\[ z_S = \frac{1}{I_z} \left[ I_{z_1} z_{f_1} + I_{z_2} z_{f_2} + I_w z_w + 2I_{r_1} \bar{z}_{r_1} + 2I_{r_2} \bar{z}_{r_2} \right]. \]

(39)

The monosymmetric parameter is:

\[ z_j = -\beta_z, \quad z_j = z_s - 0.5r_y \quad \text{(without rounding)}. \]

(40)

The Wagner coefficient for vertical asymmetry will be:
\[
\beta_z = \frac{1}{2I_y} \left[ z_f (I_{z_f} + A_{z_f} z_{f1}^2 + 3I_{y_f}) + 2z_{r1} \left( 4I_{z_f} + A_{z_f} z_{r1}^2 + z_w (I_{zw} + A_w z_w^2 + 3I_{yw}) \right) + z_{r2} \left( 4I_{z_f} + A_{z_f} z_{r2}^2 + z_{f2} (I_{z_{f2}} + A_{z_{f2}} z_{f2}^2 + 3I_{y_{f2}}) \right) \right] - z_S. \quad (41)
\]

Stability parameter (auxiliary cross-sectional distance at monosymmetricity) without rounding:

\[
r_y = \frac{1}{I_y} \left[ z_y I_{z_f} + A_{z_f} e^3 - A_{f,2} (h_y - e)^3 + \frac{t_w}{4} \left( e^4 - (h_y - e)^4 \right) \right], \quad (42)
\]

where \( e = h_y + t_{f,2} + 0.5t_{f,1} - z_G; \ h_y = h_y + 0.5t_{f,1} + 0.5t_{f,2}. \)

Together, this affects the overall stress-strain state of the beams and determines the calculated ratio by normal stresses, and therefore positively reflects on the level of use and margin of strength of the material while ensuring spatial stability and reliable operation of the structure without failures.

3. Conclusions

Considering the stiffness of attached structures in determining the torsional geometric properties of the beam cross-section more accurately reflects the actual operation of the beam in the decking composition. Taking into account the roundings at the flange adjacent to the web allows for the rolled profiles to significantly increase the torsional constant value. The effect of lateral restraint is detected and justified. The phenomenon of character change of the torsional geometric properties distribution is confirmed theoretically and using the finite element method at warping rotation with high convergence.

These factors characterize the peculiarities of the work of the steel element in complex resistance, allow to more accurately determine the value of internal forces through detailed analysis. Warping geometric properties by rigid restraining of profile are suggested to be determined relative to the attachment point of the lateral support, which is located on the axis of rotation of the beam.

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