The orbital architecture and stability of the $\mu$ Arae planetary system

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ABSTRACT

We re-analyse the global orbital architecture and dynamical stability of the HD 160691 planetary system. We have updated the best-fit elements and minimal masses of the planets based on literature precision radial velocity (RV) measurements, now spanning 15 years. This is twice the RVs interval used for the first characterization of the system in 2006. It consists of a Saturn- and two Jupiter-mass planets in low-eccentric orbits resembling the Earth-Mars-Jupiter configuration in the Solar system, as well as the close-in warm Neptune with a mass of $\approx 14$ Earth masses. Here, we constrain this early solution with the outermost period to be accurate to one month. The best-fit Newtonian model is characterized by moderate eccentricities of the most massive planets below 0.1 with small uncertainties $\simeq 0.02$. It is close but meaningfully separated from the 2e:1b mean motion resonance of the Saturn-Jupiter-like pair, but may be close to weak three-body MMRs. The system appears rigorously stable over a safely wide range of parameter space covering uncertainties of several $\sigma$. The system stability is robust to a five-fold increase in the minimal masses, consistent with a wide range of inclinations, from $\approx 20^\circ$ to $90^\circ$. This means that all planetary masses are safely below the brown dwarf mass limit. We found a weak statistical indication of the likely system inclination $I \simeq 20^\circ$–$30^\circ$. With the well constrained orbital solution, we also investigate the structure of hypothetical debris disks, which are analogs of the Main Belt and Kuiper Belt, and may naturally occur in this system.

Key words: celestial mechanics - planets and satellites: dynamical evolution and stability - stars: individual: HD 160691 - methods: data analysis - methods: observational - techniques: radial velocities

1 INTRODUCTION

HD 160691 ($\mu$ Ara, GJ 691) is a bright ($V = 5.15$ mag) Sun-like, main-sequence G3IV-V dwarf monitored in a few long-term, precision radial velocity (RV) surveys. The Anglo-Australian Telescope team (AAT, UCLES spectrometer) discovered its Jupiter-mass companion HD 160691b in about of 630 days orbit [Butler et al. 2001], and Jones et al. [2002] found a linear trend in the RV data indicating a second, more distant planet. The star was also observed in the Geneva Planet Search program with CORALIE spectrometer. McCarthy et al. [2004] determined the orbital period of the outermost planet HD 160691c $\approx 3000$ days and large eccentricity $e_c \approx 0.57$, however rendering the system unstable. The same year, Santos et al. [2004] detected $\approx 14$ Earth-mass planet HD 160691d in $\approx 9.6$ d orbit with HARPS spectrometer, achieving precision $\approx 1$ m/s, actually below the RV variability (aka stellar jitter) induced by the Sun-like stars themselves. Furthermore, Butler et al. [2006] published 108 new observations of HD 160691, spanning about of 7.5 yr, made after AAT UCLES update, also approaching the measurement uncertainty below 1 m s$^{-1}$ at the end of the observational window. Shortly, Pepe et al. [2007] published RVs from their HARPS followup, and announced the discovery of the fourth, Saturn-mass planet in the system. In parallel, Goździewski et al. [2007] independently used genetic algorithms to re-analyse data in the Butler et al. [2006] catalogue, and they found a very similar solution with small eccentricity orbits, also including the fourth planet with the orbital period $\approx 307$ days. That planet “hided” in the RV signal, because this period is approximately two times shorter as that of the firstly detected planet HD 160691b. Such a planet was unexpected in the paradigm of characterizing planets in order correlated with their RV variability. Goździewski et al. [2007] concluded that the four-planet system may be long term stable in a wide range of the outermost period. However, it could not be constrained very well at that time, in $\approx 3000$–$5000$ days range.

Since then, the star has continued to be RV-monitored. The HARPS measurements are now publicly available in the RV catalogue from archival spectra carefully reduced by Trifonov et al. [2020]. Also, very recently Benedict et al. [2022] published additional 180 measurements from the UCLES spectrometer. The data altogether span 17.3 years ($\approx 6318$ days), between epochs JD 2450915.29 and JD 2457273.2878. Benedict et al. [2022] aimed to derive the new solution for the system based on combined RVs with Hubble Space Telescope (HST) astrometry. They investigated possible astrometric signals of the planets. They conclude that the residuals $\approx 1$-2 mas to the canonical 5-parameter astrometric
model contain marginal or no evidence for any of the planets in the HD 160691 system, making it possible only to constrain lower masses of the planets to $4-7 m_{\text{Jup}}$ (i.e., 2-3 times larger than the minimal masses estimated with the RVs).

Furthermore, Benedict et al. (2022) report their updated Keplerian RV solution including the Saturn-mass planet as catastrophically unstable. They conclude that a notorious instability problem of the system remains unsolved, invoking Pepe et al. (2007), Laskar & Petit (2017); Agnew et al. (2018) and Timpe et al. (2013). This renewed our interest in the dynamics of HD 160691 system, given simultaneously our earlier, extensive investigations (Goździewski et al. 2003; Goździewski et al. 2005), and the results in (Goździewski et al. 2007). We found quite an opposite conclusion that the four-planet architecture, and moderate eccentricity of all planets is crucial to maintain the long-term stability of the system. Actually, we found in (Goździewski et al. 2007) that the 3-planet model involving only two outer Jovian planets is localised at the very border of dynamical stability, with planets in high-eccentricity orbits, and such a feature indicated that the adopted model was incomplete or incorrect.

Extending the RV time series puts the long-term monitored planetary systems deeper in the stability zone. A recent discussion of this heuristic effect can be found in (Stalport et al. 2022). What is more, not only the RV data covers twice the time range in earlier work. The most accurate HARPS data recently been independently reprocessed using a new RV pipeline by (Trifonov et al. 2020). They discovered and removed various systematic errors in a large sample of spectra. In some cases, they claim, the new RVs with improved accuracy can lead to orbital solutions different or more accurate from those found so far, including the hope of detecting additional planets. All of this gives us ample opportunity to test earlier predictions. Our goal is also to update the system’s position in stability diagrams and statistics of multiple systems, studied for example by Timpe et al. (2013) and Laskar & Petit (2017).

In addition to explaining this qualitative discrepancy between the results in (Benedict et al. 2022) and in (Goździewski et al. 2007) the motivation for this work is to answer several open questions which have not been previously addressed in the literature.

Since that the current RV data covers almost twice the observational window since 2006, we want to constrain the orbit of Jupiter’s outermost planet. It was determined with a large uncertainty of 700 days reported in (Pepe et al. 2007) and an even larger uncertainty of ±1300 days in (Goździewski et al. 2007).

Also, it is known that a sufficiently long interval of RVs data makes it possible to detect gravitational interactions between the planets (e.g. Laughlin & Chambers 2001). Until now, the RVs of μ Arae have been modeled in terms of a Keplerian parameterization of the orbital elements, since the interactions of its planets were not measurable at the time. In this kinematic approach, the inclination of the system remains completely unbounded. However, the most accurate Newtonian model can break the mass-inclination degeneracy, or at least constrain the masses of the planets indirectly through the stability requirement.

Our goal is also to resolve the open question of whether the inner Saturn-Jupiter planet pair is involved in the 2:1 MMR, or whether it is only close to this resonance. As far as this is concerned, the conclusions in both Pepe et al. (2007) and Goździewski et al. (2007) were uncertain, as both types (resonance or near-resonance) of solutions were possible. However, this is crucial for explaining the apparent excess of planet pairs near low-order resonances (e.g. Petrovich et al. 2013; Marzari 2018; and references therein). The detailed characterization of multiple planetary systems, including their orbital resonances, is one of the fundamental problems from the point of view of the theory of planet formation and for explaining their observed orbital architectures.

If our early predictions in (Goździewski et al. 2007) hold, and we find a dynamically stable orbital architecture for the planets, it may be possible to study the structure of debris disks in the system, particularly in the broad zone between 1.5 au and 5.2 au, and beyond the outermost planet. According to the packed planetary systems (PPS) hypothesis (Barnes & Greenberg 2007 and references therein), smaller planets may exist in the system, but below the current RV detection level, approximately 1 m s$^{-1}$, which correspond to the Earth’s mass range.

Finally, the highly hierarchical configuration of the HD 160691 planets imposes numerical problems in studying the long-term stability of the system, either through direct numerical integrations or by using the fast indicator approach, which is preferred in this work. Recall that the system contains a warm Neptune in an orbit of 9.6 days, as well as a very distant companion in an orbit of 4100 days, forcing a huge reduction in the discretization step size. To solve this problem, we propose a new numerical algorithm called REM (Panchi et al. 2017), which we proved to be a close analogue of the Maximum Lyapunov Exponent (MLE).

In this work, we compare the results of this fast indicator with the well-tested and widespread MEGNO (Cincotta et al. 2003; Goździewski et al. 2001). We show that despite simplicity of the algorithm, the REM indicator yields 1:1 dynamic maps compared to MEGNO and still outperforms the later variational algorithm in terms of CPU overhead.

We attempt to answer the questions posed above from the perspective of both updated RV time series and constraints provided with astrometric observations, as well as new statistical formulations of the RV model, dynamic and computational tools that have emerged over the time since the studies of Goździewski et al. (2007) and Pepe et al. (2007). We note that Benedict et al. (2022) also modeled the RV using the former, now somewhat “outdated” approach.

Planets discovered in the μ Arae system are named in different ways. Here we adopt three designations: the first one is based on the star name, as the central object and subsequent Roman letters (“b”, “c”, “d”, and so on) attributed to the planetary companions in the chronological order of their discovery (Goździewski et al. 2007). The second method is to enlist the planets according to their distance from the star, with digits “1”, “2”, “3”, and so on. Finally, we use the names attributed to the planets by the International Astronomical Union (2015) in the NameExoWorld campaign among firstly discovered 19 extrasolar planetary systems. They were inspired by characters from the famous Don Quijote book by Miguel de Cervantes. So the μ Arae system is composed of the host star Cervantes (HD 160691), and planetary companions HD 160691d (Dulcinea, planet “1”), HD 160691e (Rocinante, planet “2”), HD 160691b (Quijote, planet “3”), and HD 160691c (Sancho, planet “4”), respectively.

The paper is structured as follows. After this Introduction, we describe data sources used for this study in Sect. 2. We discuss planet detection limits, based on the astrometric HST data and their analysis reported in (Benedict et al. 2022), as well as our independent simulations of the astrometric signal. In Sect. 3, we brieﬂy recall essential details on the RV modeling in terms of Keplerian and Newtonian parameterization of the initial conditions (ICs) for.
multi-planet configurations, and we point out factors omitted in the prior literature. We report on a comparison of the results based on these two RVs parametrizations. Sect. 3 is devoted to the long-term stability of the system. We aim to bound the inclination of the system with the RVs alone, based on the Newtonian model and statistical and dynamical constraints. Section 4 is devoted to numerical simulations that reveal the dynamical structure of hypothetical debris disks in the system as well as indicate possible localization of additional smaller planets. The work is summarised in Conclusions.

2 THE REFLEX MOTION DATA FOR HD 160691

2.1 Astrometric observations

Benedict et al. (2022) observed HD 160691 with the HST Fine Guidance Sensor (FGS) between dates 2007.5 to 2010.4 (for about 2 orbital periods of HD 160691b). They made a detailed reduction of the observations and reported the results. Overall, the accuracy of the astrometric measurements is 0.6–0.7 mas, and the residuals to 5-elements canonical astrometric solution (no companions present) are estimated on the level of ≃20%. This set has 411 measurements and also spans 6317.5 days. To simplify presentation of the RV offsets, we sub-

\[ \Delta \chi^2 = 0 \]

applies to the free motion of the star. However, the inner Saturn-like planet remains deep below the detection limit (blue-shaded region).

The situation is dramatically worse, if a hypothetical data accuracy ≃0.7 mas is close to the HST FGS astrometry. Even if the system inclination is statistically most likely for \( I = 60^\circ \) or smaller, consistent with the inclination bias reported in Benedict et al. (2022), \( I = 30^\circ \), scaling the minimal masses by a factor of ≃20% and ≃100%, respectively, only the outermost planet could be barely detected with the astrometric time-series.

Unfortunately, these arguments and simulations leave little hope that a re-analysis of the available astrometric data may change the results and conclusions in Benedict et al. (2022) and Brandt et al. (2022). Therefore we abandoned the HST astrometry from further analysis, and we focused on the RV observations only.

2.2 Radial Velocity data

We considered two slightly different sets of the RV measurements for \( \mu \) Arae available in public archives and sources. The RV data set \( D_2 \) consists of 380 measurements spanning 6317.5 days. They are collected with three instruments: CORALIE (\( D_{\text{CORALIE}} \)), UCLES (\( D_{\text{UCLES}} \)) and HARPS (\( D_{\text{HARPS}} \)). This set is literally the same as in Benedict et al. (2022), and we obtained it from the author (private communication). In densely sampled parts of the observational window, the data were binned if there was more than one measurement made during a night. The mean uncertainty is different for individual spectrometers, and varies between \( \sigma \sim 1 \) m s\(^{-1} \) up to a several m s\(^{-1} \) for CORALIE. Moreover, Benedict et al. (2022) considered HARPS observations in two disjoint sets: from Pepe et al. (2007) and the second part of the time-series after that date from Trifonov et al. (2020). They attributed different RV offsets to these sets.

We also compiled a second data set \( D_2 \). Trifonov et al. (2020) derived the RV velocities from spectra obtained prior– and post–the HARPS upgrade in May 2015, and corrected them for various systematics and instrumental effects. Since the available data for HD 160691 contains effectively only two post-upgrade measurements made in nights of June and July 2015, we skipped these points from the orbital analysis. It would be difficult to account for two free parameters, \( \sigma_1 \) and \( \sigma_0 \equiv \sigma_{\text{UCLES}} \), to be statistically determined with the RV subset comprising of only two datum. Moreover, because the post-upgrade HARPS epochs overlap with UCLES measurements, skipping them unlikely may change the model results. We also get rid of two free parameters. Similarly to Benedict et al. (2022), we also binned densely sampled measurements, but with a smaller interval of 0.1 days. Before doing that, we removed several points from the HARPS RV time series in Trifonov et al. (2020), with heavily outlying uncertainties of 10–24 m s\(^{-1} \), given the mean uncertainty \( \sigma_{\text{HARPS}} \sim 1 \) m s\(^{-1} \). The problematic measurements appear around JD 2453169 (mid-June, 2004), when literally hundreds of spectra were taken overnight. Removing these points should not cause any problem, due to the dense sampling and binning. For the binned data in set \( D_2 \), we adopted the uncertainties as the mean uncertainty in a particular bin.

In this way, the data set \( D_2 \) consists of the whole pre-upgrade HARPS measurements \( \Delta \chi_{\text{HARPS}} \), as a homogeneous data set from Trifonov et al. (2020), and \( D_{\text{CORALIE}} \) and \( D_{\text{UCLES}} \) from Benedict et al. (2022). This set has 411 measurements and also spans 6317.5 days. To simplify presentation of the RV offsets, we subtracted the mean value of all RVs in a given subset from individual RVs in this subset.

\[ \Delta \chi^2 = 0 \]
Finally, in some experiments we considered data set $D_3$ composed of 349 measurements from the pre-upgrade HARPS and $D_{coralie}$ from [Benedict et al. 2022]. These RV time-series span the same time interval as $D_5$ does. This data set $D_3$ lacks the less accurate $D_{coralie}$ RVs.

### 2.3 Keplerian vs Newtonian Radial Velocities

The mathematical models for the RV velocities are well known. However, to keep the presentation self-consistent, and to cover some nuances, we will briefly recall the required material.

Since, following the prior literature, we expect that the $\mu$ Arae orbits may be quasi-circular, to get rid of weakly constrained longitudes of pericenter $\varpi_i$ when eccentricities $e_i \sim 0$, we introduce Poincaré elements \[ \{x_i = e_i \cos \varpi_i, y_i = e_i \sin \varpi_i\}, i = 1, 2, 3, 4. \] Also, the mean anomaly $M$ at the initial epoch $t_0$ denoted as $M_i \equiv M_i(t_0)$ is defined through the III law of Kepler, but written for the Jacobian reference frame

\[ P_i = 2\pi \sqrt{\frac{a_i^3}{k^2(m_0 + m_1 + \ldots + m_i)}} - M_i(t) = M_i + \frac{2\pi}{P_i}(t - t_0), \tag{1} \]

where $k$ is the Gauss constant, and $P_i$, $a_i$ stand for the orbital period and semi-major axis for each planet, respectively.

Regarding the Keplerian parameterization of the RV, we apply the well known canonical formulae [Smart 1949] due to the presence of planets

\[ C(t) \equiv V_r^K(t) = \sum_{i} N_{pl} K_i \left[ e_i \cos \omega_i + \cos(v_i + \omega_i) \right], \tag{2} \]

\[ = \sum_{i} N_{pl} K_i \left[ x_i + (x_i^2 + y_i^3)^{-1/2} (x_i \cos v_i - y_i \sin v_i) \right], \tag{3} \]

where $\omega \equiv \varpi$ for a coplanar system, $v \equiv v(t)$ denotes the true anomaly of a planet, $N_{pl}$ is the number of planets in the system, and $v = v(P_i, e_i, M_i(t))$. To characterize the orbit of the $i$-th planet, we need to know five free orbital elements: $\Theta_i = \{ K_i, P_i, e_i \cos \omega_i, y_i \equiv e_i \sin \varpi_i, M_i \}$, where the RV semi-amplitude $K_i$ depends on the minimal mass of the planet $m_i \sin I$, when the inclination $I = 90^\circ$.

Let us note that we interpret the RV signal in terms of the geometric elements inferred in the Jacobian frame of reference. We follow here conclusions and discussion in [Lee & Peale 2003], to properly express parameters of the Keplerian model through the N-body initial condition. We need that to investigate the long-term stability of the system with the numerical integrations. For relatively massive planets, the Jacobian (canonical) elements account for indirect interactions between the planets on Keplerian orbits to the first order in the masses (the ratio of planet masses to the star mass), see also [Goździowski et al. 2012] for more details.

In order to derive the N-body initial condition from the fitted Keplerian elements $\Theta_i$, $i = 1, \ldots, N_{pl}$, we first determine the minimal $m_1 \sin I \equiv m_1$ and semi-major axes $a_i$ of the planets. The semi-amplitude $K_i$ of the RV signal

\[ K_i \sqrt{1 - e_i^2} = a_i \left( \frac{2\pi}{P_i} \right) \frac{m_i}{(m_0 + m_1 + \ldots + m_i)}, \]

where the $a_i$ constrained by the observationally derived orbital period $P_i$ obeys Eq. [1] and $m_0$ stands for the star mass. Eliminating $a_i$, we obtain a cubic equation for the unknown masses, which may be subsequently solved for $m_i$, $i = 1, 2, \ldots$ based on analytical formulae or with a simple Newton-Raphson scheme (a few iterations suffice to reach the machine accuracy). Then we transform the geometric elements to Cartesian coordinates and velocities with the standard two-body formulae, where the gravitational parameter for the $i$th planet is $\mu_i$ = $k^2(m_0 + m_1 + \ldots + m_i)$.

To determine parameters of the orbital model explaining the RV time-series, we optimized a canonical form of the maximum likelihood function $L$ [Baliev 2009]

\[ \ln L = -\frac{1}{2} \sum_{i} \left( \frac{O \text{C}_{ij}^2}{\sigma^2_{ij}} \right) - \frac{1}{2} \sum_{i} \ln \sigma^2_{ij} - \frac{1}{2} \ln 2\pi \sum_{i} \ln 2 \sigma_{ij}^2, \tag{4} \]

where $(O-C)_{ij}$ is the $(O-C)$ deviation of the observed $i$-th RV observation, with the uncertainty $\sigma^2_{ij}$, $\sigma^2_{ij} = \sigma^2_{ij} + \sigma^2_{ij}$, with $\sigma_i$ parameter scaling the raw error $\sigma_i$, $\sigma_j$ is the total number of the RV observations. We assume that the uncertainties are Gaussian.

The error floor factors $\sigma^2_{ij}$ are different for each telescope, as they may involve not only the intrinsic, chromospheric RV stellar
variability (stellar jitter), but also an instrumental uncertainties inherent to each telescope and the RV pipeline. The RV model also involves individual offsets of the zero-level RV for each instrument. Distinguishing between these two parameters is important even for the same spectrometer and different setups of its work. For instance, the upgrade of HARPS optical fibres around the middle of 2015 changed the instrumental profile and thus the RV offset between the pre- and post-upgrade RVs. To complicate things even more, the RV offset may be not the same for all stars and may even depend on the stellar spectral type (Trifonov et al. 2020).

Therefore fitting the jitter uncertainties as free parameters of the model is crucial to obtain adequate statistical representation of the RV data. We may note here, that in the past, these parameters have been fixed based on the averaged values for chromospherically quiet stars of a given spectral type. That recently outdated (and somewhat incorrect) approach was used by Goździewski et al. (2007) and Pepe et al. (2007). Benedict et al. (2022) tuned the RV uncertainties to obtain $\chi^2 \approx 1$.

Usually, the Keplerian model determines sufficiently accurately the N-body, exact RVs. However, for systems with large mass planets, this equivalence may be questionable, especially if the interval of the RV time series becomes long. Then we have to introduce the self-consistent model that requires solving the Newtonian equations of motion. The RV due to the planets is the velocity component of the star along the z-axis w.r.t. the barycenter of the Solar system

$$C(t) \equiv V_\nu(t) = -\frac{1}{m_0} \sum_{j=1}^{N_p} m_j z_j(t),$$

which is parameterised through planet masses and the osculating orbital elements $\mathbf{\theta}_i = [m_i, a_i, x_i, y_i, M_i]$ for each planet in the system. Here, as the osculating epoch we select the epoch of the first observation in the given time series. In some experiments, we also selected the osculating epoch in the middle of the data window.

Expressions for the RVs, Eq. 2 and Eq. 5 have to be accompanied with the instrumental zero-level offset $V_{0,j}$, $j = 1, \ldots, M$ that makes it possible to compute $(O - C)(t)$ in Eq. 2. For $N_p$ planets forming a coplanar system observed with $M$ instruments, we have therefore $p = 5N + 2M$ free parameters to be fitted to one-dimensional time series of the RV observations.

The definition in Eq. 5 is constructed so the best-fitting models should yield $\chi^2 = \chi^2 / (N_{\text{RV}} - p) \sim 1$. and $\chi^2$ cannot be used to compare the models quality. Instead, [Balbinot] (2009) proposed to use:

$$\ln L = -\ln L / N_{\text{RV}} - \ln(2\pi\sigma)^2 / 2,$$

where $L$ is expressed in ms$^{-1}$. This statistics is suitable to assess the relative quality of fits, since $L \sim (\sigma)$ measures a scatter of measurements around the best-fitting models, similar to the common RMS — smaller $L$ means better fit.

In order to localize the best-fitting solutions in the multidimensional parameter space, we explore it with evolutionary algorithms (GEA from hereafter, Charbonneau [1995], Rucinski et al. [2010]). We then perform the MCMC analysis in the neighborhood of selected solutions using an affine invariant ensemble sampler (Goodman & Weare [2010]) encompassed in a great emcee package (Foreman-Mackey et al. [2013]). The computations were performed in multi-CPU environment, making it possible to evaluate 128,000–256,000 (or more) of 144–384 emcee “walkers” from a small-radius ball around a solution found with the GEA.

We select all priors as flat (or uniform, improper) by sufficiently broad ranges on the model parameters, e.g., $P_i \in [1, 10,000]$ days, $x_i, y_i \in [-0.25, 0.25]$, $m_i \in [0.1, 14]$ m$_{\text{Jup}}$, $(i = 1, 2, 3, 4)$, the error floors (jitters) $\sigma_{j,i} > 0$ m s$^{-1}$, $j = 1, \ldots, M$. In a few experiments with the N-body model, we also tested Gaussian priors for the $(x_i, y_i)$ elements of the innermost planet, with the mean equal to zero and variances $\sigma_{x,y} = 0.05, 0.075, 0.1$, respectively.

In this case, however, the results of sampling did not substantially change, compared to the flat priors.

### 2.4 The best-fitting orbital configurations

We first performed an extensive search for the best-fit solutions using GEA, and we collected $\approx 10^3$ solutions for both data sets and model variants. We found that the best-fit Keplerian and Newtonian models with $L \approx 3.2$ m s$^{-1}$ (RMS $\approx 3.4$ m s$^{-1}$) have well determined extrema of $\ln L$ for orbital periods $P_i$ of roughly 9.64, 308, 645, 4030 days, respectively. Also, all osculating eccentricities are well limited to moderate values, roughly in the range of 0.02–0.1.

The resulting best-fitting parameters for data sets $D_1$ and $D_2$ are given in Tables 1 and 2. The best-fitting Keplerian model Fit IIK in Fig. 2 is illustrated in Fig. 2 (left panel). Using this solution as an example, we checked the consistency of the Keplerian and Newtonian parameterization. We transformed Fit IIK as osculating elements for the epoch of the first observation $t_0 = \text{JD } 2450915.29$ in the UCLES data, as described in Sect. 2.3. We then computed the Newtonian RV signal through of numerical integration of the N-body equations of motion for the entire four-planet system with the IAS15 integrator (Rein & Spiegel [2013]). It turns out that the difference $\Delta \nu RV(t) = V_\nu(t) - V_\nu^C(t)$ increases in an oscillatory manner, reaching about $\pm 10$ m s$^{-1}$, which exceeds more than twice the RV signal from the innermost planet (red curve in the residuals diagram in Fig. 2).

To verify this effect globally in the parameter space, we performed the MCMC sampling with both the Keplerian and Newtonian RV models. The final results for data set $D_2$ are illustrated in Fig. 3. We skip presentation of the results for $D_1$, since they are very similar. This figure shows one- and two-dimensional projections of the posterior probability distribution for selected Keplerian (top row) and Newtonian (bottom row) orbital elements obtained for the innermost (left column) and outermost (right column) planet, respectively. The posterior has well defined extrema along all dimensions. We did not notice significant correlations between the displayed parameters, except for $x,y$ and $M$. The quality of the best-fit configurations, in terms of RMS $\approx 3.4$ m s$^{-1}$, is almost also the same. Surprisingly, the posterior distributions are not only very similar to each other, especially if we compare the two-dimensional shape distributions for $x,y$ and $M$, but also the eccentricities and orbital angles closely overlap, e.g., the best-fit $M_4$ anomaly differs by only 2° in these models.

How to interpret this apparent paradox, given the relatively large masses of Jupiter-like companions and their significant, mutual interactions over the observing interval, illustrated in Fig. 2? A direct comparison of the RV signals may be biased because the accuracy of the formal two-body Keplerian element transformation to Cartesian coordinates is limited to the first order in masses (e.g. Goździewski et al. [2012]). However, the representation of the Keplerian initial condition for the N-body problem may better fit the data if it is tuned within the parameter uncertainties. Therefore, given well bounded orbital elements, the MCMC sampling reveals globally similar posteriors for both models.

We also see the posteriors for the near 2e:1b MMR pair of a
Table 1. Best-fit parameters of the µ Arae (Cervantes) system for the Keplerian (Fit IK) and Newtonian (Fits IN) parameterization, data set $D_1$. The osculating epoch is the date of the first observation in the UCLES data set. The system is coplanar with the inclination $i = 90^\circ$ and nodal longitudes $\Omega = 0^\circ$. The stellar mass is $1.13 M_\odot$ (Bonfanti et al. 2015) as used in (Benedict et al. 2022), close to 1.10±0.02 $M_\odot$ in Soriano & Vaucouleurs 2010. The RV offsets are computed w.r.t. the mean RV in each individual data set. Uncertainties are estimated around the median values $\mu$, i.e., $|\mu - \sigma, \mu + \sigma|$ as the 16th and 86th percentile of the samples. Numerical values for Fit IK selected from the MCMC samples with low RMS are quoted to the 7th digit after the dot, to make it possible to reproduce the dynamical maps and direct numerical integrations. The mean longitude $\lambda = \varpi + M$ at the epoch was computed from the MCMC samples.

| Planet          | HD 160691d (Dulcinea, 1) | HD 160691e (Rocinante, 2) | HD 160691b (Quijote, 3) | HD 160691c (Sancho, 4) |
|-----------------|--------------------------|---------------------------|-------------------------|------------------------|
| $K$ [m s$^{-1}$] | $2.95\pm0.19$            | $13.22\pm0.34$            | $36.47\pm0.22$          | $23.17\pm0.33$        |
| $P$ [d]         | $9.638\pm0.001$          | $308.75\pm0.29$           | $645.00\pm0.36$         | $4060\pm27$           |
| $e \cos \varpi$ | $-0.104\pm0.063$         | $-0.093\pm0.014$          | $0.058\pm0.011$         | $0.022\pm0.012$       |
| $e \sin \varpi$ | $-0.059\pm0.063$         | $-0.014\pm0.017$          | $0.023\pm0.008$         | $0.032\pm0.013$       |
| $e$             | $0.137\pm0.056$          | $0.096\pm0.014$           | $0.063\pm0.010$         | $0.040\pm0.013$       |
| $\varpi$ [deg]  | $210\pm32$               | $189\pm10$                | $21.6\pm(9.4,8.2)$      | $55.9 \pm 17.5$       |
| $M$ [deg]       | $223.3\pm32$             | $66.7\pm10.5$             | $272.5\pm(8.3,9.4)$     | $185.9 \pm 17.2$      |
| $\lambda$ [deg] | $73.3\pm10.5$            | $255.4\pm3.9$             | $294.0\pm1.0$           | $241.9 \pm 2.2$       |
| $V_0$ [m s$^{-1}$] | CORALIE: $13.04\pm0.42$, UCLES: $-7.80\pm1.20$, HARPS: $1.0\pm0.3$, HARPS: $-4.20\pm0.32$ | CORALIE: $1.30\pm0.21$, UCLES: $6.1\pm1.1$, HARPS: $0.62\pm0.46$, HARPS: $1.67\pm0.40$ |
| $\sigma_f$ [m s$^{-1}$] | CORALIE: $1.10\pm0.43$, UCLES: $-7.74\pm1.14$, HARPS: $1.10\pm0.30$, HARPS: $-3.94\pm0.32$ | CORALIE: $1.23\pm0.20$, UCLES: $5.88\pm(1.07,0.93)$, HARPS: $0.45\pm0.40$, HARPS: $1.51\pm0.36$ |

Saturn-Jupiter-like planets exhibiting some significant differences (see on-line Supplementary Material, Fig. A1). This can be explained by their relatively shorter periods, covering $\approx 20$ and $\approx 10$ times the observational window, respectively, and the 2:1 MMR proximity, which strengthens the mutual gravitational interactions.

The MCMC experiment implies that, keeping in mind the limitation for representing individual ICs, we can still use Keplerian MCMC sampling to efficiently explore the parameter space, in terms of the posterior distribution, especially for highly hierarchical configurations with large period ratio. Note that $P_2/P_1 \approx 400$ for HD 160691. However, parameterization in terms of the N-body dynamics is obviously more accurate approach to explain the RV variability when considering individual (local) best-fit models.

To justify the above explanation, we compared the outcomes of the Keplerian and Newtonian fits for data set $D_2$ in Table 1 and the results are illustrated in the O-C diagram in the right panel in Fig. 5. This time, the difference between the signals plotted as a red curve in the residuals diagram has much less variability, with the largest differences $\approx 5$ m s$^{-1}$ appearing for epochs without data.

As noted above, an important feature of the posterior distribution is well bounded parameters for all planets. In particular, the semi-major axes of the middle pair, near 2e:1b-MMR (Rocinante–Quijote) are constrained to $\approx 0.0015$–0.002 au, and for the outermost Sancho planet to just $\approx 0.02$ au, i.e., its orbital period may be determined with the uncertainty of one month (25–50 times better than with the data in 2006). That seems to be quite surprising, since the observational window covers only about 1.5 times the period of this companion. Similarly, the Poincaré elements (likely $\approx 0.9376\pm0.0015$) of the Saturn- and Jovian planets may be determined to within $\approx 0.01$, with uncertainties of the arguments of pericenter and the mean anomalies at the osculating epoch $t_0$ on the level of $\approx15^\circ$. This translates to the mean longitude at the epoch $\lambda_t$ that may be determined to $\approx 4^\circ$. The eccentricities in the Keplerian and Newtonian parameterizations (Tables 1 and 2) are at the 0.05 level with small uncertainties, as we will show below, may be crucial for maintaining the long-term stability of the system.

We should also comment on similarities and difference between solutions derived for data sets $D_1$ and $D_2$ in this work, and with the Keplerian model in (Benedict et al. 2022).

We obtained very similar eccentricities of the planets, particu-
We conducted direct numerical integrations of the system with all mechanisms opposing gravitational tides, such as thermal atmospheric tides, evaporation of the atmosphere, and the eccentricity excitation from a distant companion. The later seems to be not the cause of the moderate eccentricity of HD 160691d, but the presence of atmospheric tides may be sufficient to explain its moderate value.

The most significant difference between the solutions in [Benedict et al. 2022] and in this work is relatively shorter orbital period of HD 160691c, by ≃ 100 days (yet only ≃ 2%) in [Benedict et al. 2022]. They report this solution as strongly unstable in 100 Kyr time scale, in contrast to our models, which appear safely stable in extended regions of the parameter space, for at least 6.7 Gyr, as discussed below.

We attempted to address outlying UCLES measurements, visible on the right end of the observation window (Fig. 2). There are systematic deviations from the synthetic model, reaching ≃ 10 m s\(^{-1}\), and unlikely they can be eliminated with the standard RV drift model for the UCLES data RV effect. In order to account for it, we added a periodic drift to the RV model. This can be explained by a long-term instrumental UCLES deviate as systematically as the UCLES data from the common RV ephemeris. The HARPS and UCLES epochs overlap almost throughout the time window, but the HARPS measurements do not deviate as systematically as the UCLES data from the common model. This can be explained by a long-term instrumental UCLES effect. In order to account for it, we added a periodic drift to the RV model for the UCLES data RV drift(t) = A cos(\(nt + \phi_0\)) where A, n and \(\phi_0\) are the semi-amplitude, frequency and relative phase of the signal, respectively.

As the result of the MCMC sampling of the Keplerian model with this modification, we show (O–C) for the best-fit model in Fig. 4 and a section of the corner plot for the posterior with offsets,
error floors, and drift parameters (on-line Supplementary Material, Fig. A2). Note that in this case we analyzed only the concurrent HARPS and UCLES RV series (data set $D_2$). It turns out that the drift component can significantly reduce the UCLES outliers. The drift correction reduces the RMS value to $2.5$ m s$^{-1}$, which is almost $1$ m s$^{-1}$ less than the value for the unmodified model. However, the posterior distributions reveal that the drift’s long period $P = 2\pi/n \approx 36$ yrs cannot be meaningfully constrained. Moreover, its half-amplitude $A \approx 12–15$ m s$^{-1}$ is weakly limited on the right end, and strongly correlated with the RV offset $V_{0,2} \equiv V_{0,UCLES}$, as it is labeled in the corner plot for the UCLES data. At the same time, the orbital parameters have not changed except for the period of $P_2 \approx (3944 \pm 27)$ days, significantly shorter than $P_2 \approx 4020–4060$ days in our models without drift, but similar to $P_2 \approx 3947$ days in the solution of Benedict et al. (2022).

Given some variability in the residuals to the Keplerian and Newtonian models in Fig. 2, we analyzed them with the Lomb-Scargle periodogram, in the period window from 2 days to 64,000 days. The results are shown in Fig. 3. Indeed, the (O-C) in the left panel for the 4-planet model to the data set $D_2$ shows some signature of the long-term drift. However, we did not detect any significant peak at the 1% false alarm probability estimated by the bootstrap method at a level of $\approx 0.07$. We performed the same test on the residuals to the 4-planet model with the sinusoidal drift. It is clear that the long-term drift period has disappeared, and there are still no significant peaks in the high frequency range. The (O-C) analysis suggests that we could not detect any significant RV signal that can be attributed to a new planet in the system.

These results are consistent with the conclusions in the work of Benedict et al. (2022). They did not detect any correlation of the RV variability attributed to the planets with the periodicity of the spectral line profile distortion indicators. They found peaks of the bisector with low significance, around 357–368 days and 497 days, which can be explained by stellar activity.

Since the inclusion of RV drift appears problematic due to the strong $V_{0,2}-A$ correlation, and the drift-modified model does not actually qualitatively change the orbital architecture and stability of the system (as justified below), rather than shortening the outermost orbital period by $\approx 2\%$, we have abandoned this model. However, the likely instrumental nature and origin of the UCLES RV-outliers remains unexplained.

3 LONG-TERM STABILITY OF THE SYSTEM

The well bounded best-fit parameter ranges make is possible to simplify the analysis of the dynamical character of the system. We conducted it with two fast dynamical indicators, the Mean Exponential Growth factor of Nearby Orbits (MEGNO, $Y$) Cincotta et al. 2003 and the Reversibility Error Method (Panchin et al. 2017) REM. These numerical tools are CPU-efficient variants of the Maximal Lyapunov Exponent (MLE) that make it possible to detect unstable solutions and visualize the structure of the phase space.

The usefulness of the MEGNO method in analyzing the dynamics of planetary systems with strongly interacting companions has been proven for a long time (e.g., Goździewski et al. 2012 and references therein). We have also shown in Panchin et al. (2017) that the REM indicator is not only equivalent to MEGNO, but may be also much more CPU-efficient. Briefly recalling the idea of this algorithm, computing REM relies in comparing the difference between the Cartesian initial condition $x_0$ after integrating it numerically forward and back, for the same number $n$ of time steps $\Delta t$, using a time-reversible numerical scheme, to obtain the final state $x(\pm n\Delta t)$. Then the REM indicator is

$$REM = ||x_0 - x(\pm n\Delta t)||.$$
Figure 3. One- and two-dimensional projections of the posterior probability distribution for orbital parameters of the innermost (left column) and the outermost (the right column) planet, respectively. The top row is for the Keplerian model, and the bottom row is for the Newtonian model to data set $D_2$. The parameters are expressed in units consistent with Table 2. The semi-amplitude $K_i$ is equivalent to the mass $m_i$, and the orbital period $P_i$ is equivalent to the semi-major-axis $a_i$. The MCMC chain length is 180,000 iterations for each of 384 different instances (walkers) selected in a small ball around a best-fitting solution found with the evolutionary algorithms for the Keplerian model, and 294,000 iterations in each of 176 walkers for the Newtonian model. Parameter uncertainties are estimated as 16th and 84th percentile samples around the median values at 50th percentile.

This difference grows exponentially with integration time for chaotic systems, and at a polynomial rate for regular (stable) configurations. Such a simple algorithm can be implemented with a symplectic discretization scheme. In practice, for systems with small and moderate eccentricities, which $\mu$ Arae systems appear to be, we use the classic leap-frog algorithm (e.g. Laskar & Robutel 2001) with symplectic correctors of the order 5 (Wisdom 2006), offering numerical accuracy and efficiency comparable to higher order methods (Wisdom 2018), see also (Panichi et al. 2017) for details. As we have shown, in the later paper, this REM algorithm is particularly useful in regions of phase space with predominantly stable solutions and outperforms then any MEGNO variant in terms of CPU-efficiency.

In this work, to speed up computations, we conducted the nu-
3.1 Stability of the model based on data set \( D_1 \)

We first computed the two-dimensional dynamical maps in the neighborhood of the Newtonian Fit IN in Table 1 based on the original data set \( D_1 \) from Benedict et al. (2022). Figure 6 illustrates the \((a_\ell, e)\)-plane. In these scans, all other orbital elements are kept at their best-fit values listed in Table 1. To make possible reproduce the results, we quote exact numerical values of the elements and masses. For each initial condition in the grid, the equations of motion were integrated up to 200 Kyr, corresponding to \( \approx 1.8 \times 10^4 P_4 \). This time interval allows for the detection of short-term chaotic motions for the time scale of the MMRs instability (e.g. Goździewski & Migaszewski 2018).

Some of the dynamical maps were computed for 3-planet systems with the most massive planets, omitting the innermost warm Neptune. Its very short orbital period of 9 days compared to that one of the outermost planet \( \approx 4000 \) days) causes a huge CPU overhead. Before that, we investigated whether the presence of Dulcinea could affect the orbital evolution of the other massive companions and such 3-planet maps. To this end, we numerically integrated the systems described by Fit IN, with and without the warm Neptune, for several Myr, when secular effects may already play a role. Fig. 5 illustrates the resulting oscillating semi-major and eccentricity over a narrow time interval around 2.8 Myr for Quijote (HD 160691b). Clearly, the elements span the same ranges and evolve along curves with very similar shapes. Their de-phasing is due to a small change of the mean motion and other elements. The most significant shift can be seen for Sancho (HD 160691c, not shown here), yet its semi-major axes is shifted by \( \approx 0.002 \) au, roughly 10 times less than \( 1 \sigma \) uncertainty for this orbital element.

To study whether the innermost planet can be omitted from the system for long-term integrations, Farago et al. (2009) averaged the model for the fast orbiting innermost planet. Obviously, such an analytical model is numerically as CPU efficient, as the 3-planet model. Moreover, they found for the particular \( \mu \) Arae case the results from three formulations of the orbital evolution: the exact one, the 3-planet model with omitted warm Neptune, and the 3-planet model with its mass added to the mass of the star lead to barely distinct results.

![Figure 4](image-url)  
**Figure 4.** Lomb-Scargle periodograms for residuals to the Keplerian Fit IIK in Fig. 2 left panel, data symbols are the same as in Fig. 2 and to the residuals to the best-fitting Keplerian model with a hypothetical, instrumentally induced periodic term \( A \cos(n t + \phi_0) \) in the UCLES measurements, over-plotted on the RV measurements from UCLES and HARPS spectrometers (right panel). Brown circles are for HARPS and blue diamonds are for UCLES instrument, respectively. Red filled circles mark the orbital periods of the detected planets.

![Figure 5](image-url)  
**Figure 5.** Temporal evolution of the osculating semi-major axis (top panel) and eccentricity (bottom panel) for planet HD 160691b in a narrow time window around 2.8 Myr. In each panel, curves with different colour illustrate solutions for two ICs, with and without the innermost planet. In the later case, we added its mass to the mass of the star. Elements of the planets included in the integrated system in both experiments are the same (Fit IN, Table 1). The systems were integrated with the SABA3 symplectic scheme with the step size of 0.5 days.
Figure 6. Dynamical maps for the best-fitting N-body Fit IN (Tab. 1) to data set D1. Top-right and bottom panels are for a close-up of the scan shown in top-left panel. The fast indicators $\log |\text{REM}| \lesssim -4$ and $\langle Y \rangle \simeq 2$ characterise regular (long-term stable) solutions, which are marked with black/dark blue colour; chaotic solutions are marked with brighter colors, up to yellow. The integration time of each initial condition is 200 Kyr ($\sim 1.8 \times 10^4 \times P_d$). Panels in the right column are for the 3-planet model omitting the warm Neptune, and for the full 4-planet configuration, respectively. The REM indicator was computed with the leap-frog with the step size of 8 days (3-planet map) and 0.33 days, respectively (bottom scan). The MEGNO scan (bottom-left panel) was computed for 3-planet model with the Gragg-Bulirsch-Stoer (GBS) algorithm ([Hairer et al. 1993][Hairer & Wanner 1995]). The asterisk symbol means the position of the nominal model. Diamond and triangle symbols are mark the ICs tested with the direct numerical integrations for 6.7 Gyr, see the text. Resolution for the top and bottom-left plots is 640 × 360 points, and 360 × 200 points for the bottom-right scan.

Figure 7. Evolution of a selected critical angle $\theta_{2e:3b}$ of the three-body MMR of the outer planets for the initial condition marked in dynamical maps in Fig. 6 with a white diamond.

To test this independently, and without any simplifications of the equations of motion, we used the REM indicator directly and compared dynamical maps for the 3- and 4-planet configurations, respectively, for the same ranges of orbital parameters.

We start with the upper-left panel in Fig. 6 for a relatively broad region of the ICs marked with a star symbol. That map was computed without the innermost Neptune, using the leap-frog scheme and a time step of 8 days. A wide structure around $a_3 \simeq 1.47$ au on the left of this IC corresponds to the 2b:1c MMR of the inner pair of Saturn-Jupiter–mass planets. Given the small 1σ uncertainty 0.001 au of the nominal semi-major axis, the separation of the best-fitting configuration from this MMR is meaningful (the error bars are smaller than the symbol radius). Simultaneously, the ICs is located between three narrow strips of unstable solutions that may be identified with higher-order resonances. Close-up maps in the remaining panels of Fig. 6 reveal a very close proximity of the ICs to one of these strips.

Panels in the bottom row are for the same $(a_3, e_3)$-plane, but scanned with $\langle Y \rangle$ for the 3-planet model (bottom-left panel) and with REM calculated for the full 4-planet configuration (bottom-right panel), but with a much smaller step size of 0.33 days and lower resolution compared to the 3-planet REM-map computed with the leap-frog step-size 8 days (upper-right panel). Of course, this is forced by the short orbital period of HD 160691d. The maps
clearly illustrate the one to one results, in a region with weakly unstable configurations and different, very fine dynamical structures. We may note that the ICs is negligibly shifted by \(\sim 10^{-5}\) au with respect to the unstable structure, between the 3-planet and 4-planet scans.

While the REM map for three planets was calculated several times faster than the \(\langle Y \rangle\) map, the full REM calculation for four planets was more than 15 times slower per pixel. Such overhead is acceptable, however, given that the calculations were performed without any simplification of the Newtonian equations of motion.

The detection of fine unstable structures and tiny islands of stable resonances confirms once again a good sensitivity of the REM algorithm for stable and unstable solutions. To show this better, we interpreted the unstable strip structure through the numerical analysis of the fundamental frequencies (NAFF, Laskar & Robutel 2001) of a particular system marked with a white diamond symbol in a small stable island around \((a_1, e_1) \simeq (1.5211\text{au}, 0.01)\). This island is a part of the three-body MMR 2e:4b:1c structure (one of the strips spanning \(e_1 \in [0, 0.1]\)). We plotted evolution of a selected critical angle of this resonance \(\theta_2, -4, -1 = 2\lambda_2 - 4\lambda_3 - \lambda_4 + \delta_2 + \delta_3 + \delta_4\) in Fig. 7. This critical angle librates with large amplitude around 180°, and the orbital configuration is perfectly stable for at least 1 Gyr, consistently with its location in the stable island.

In contrast, we selected formally unstable ICs by shifting the nominal semi-major axis to the right (to the unstable strip) and marked with a black triangle symbol in Fig. 6. We integrated this ICs for 6.7 Gyr with the SABA4 scheme and for 1 Gyr with the variable step-size IAS15 integrator. Also in this case the system does not reveal any signature of geometric instability, in spite of its formally chaotic character in the sense of MLE (it is not illustrated here, but we invoke a similar example in Sect. 3.2.2). The width of this third-order MMR is very small, \(\Delta a_1 \simeq 0.003\) au, and the diffusion is likely so slow that it does not lead to a change or disruption of the system.

We remark here that Benedict et al. (2022) found quite an opposite, catastrophic instability of the system. In their Keplerian solution, \(P_3 \simeq (3947 \pm 23)\) days is apparently the only significant difference with our fits (Table 1). The origin of this discrepancy may be a subtly different parameterization of the RV signal. For instance, Benedict et al. (2022) did not fit the jitter uncertainties as free parameters, but tuned it posteriori for each data set to obtain \(\chi^2 \simeq 1\). Moreover, our models yield smaller RMS \(\simeq 3.4\text{m s}^{-1}\) rather than \(\simeq 3.8\text{m s}^{-1}\) in the prior work. A shorter period of \(P_3 \simeq 3947\) days may be pointing to an unstable structure close to \(a_4 \simeq 5.12\) au (similar to that one visible in the top-left panel in Fig. 10). We integrated the system with the outermost planet Sancho placed in this unstable zone, but the system survived for at least 1 Gyr. We could not reproduce the strong instability reported in [Benedict et al. 2022], and we cannot find any convincing explanation of this discrepancy.

### 3.2 Stability of the Newtonian model based on data set \(D_2\)

As mentioned above, we also conducted the GEA and MCMC analysis for data set \(D_2\). The results are very similar to the \(D_1\) case. However, there are some subtle qualitative changes with respect to the models for \(D_1\). The eccentricities of the Jovian planets tend to be systematically even smaller than for the \(D_1\)-systems. Also the semi-major axes and orbital periods locate the systems in even more “safe”, stable zone displaced from the 6b:1c MMR by more than 0.1 au, which corresponds to \(\simeq 5\sigma\) in terms of the semi-major axis uncertainty.
3.2.1 The 2e:1b MMR proximity

[Goździewski et al. (2007), Pepe et al. (2007) and Farago et al. (2009) investigated the proximity of the inner pair HD 160691e–b to the 2e:1b MMR. In the two later papers, they found the best-fitting model close to the separatrix, unstable zone of this resonance. Contour levels of $\chi^2$ encompass both the near-resonance and the resonant configuration (Pepe et al. 2007) their Fig. 7). In (Goździewski et al. 2007), we also found that the relative position of the ICs and the shape of the 2e:1b resonance in the ($a_\nu$, $e_\nu$)-plane strongly depend on the semi-major axis of HD 160691e that could be only weakly constrained to ±1300 days (4 au–7 au) and eccentricity $e_\nu$ as large as 0.2 at the time.

We can now revisit this issue with a significantly updated Fit IIN. To do so, we calculated the dynamical maps illustrated in Fig. 5 for the 3-planet (upper panel) and 4-planet (middle panel) configurations, respectively. For the 3-planet model, we added the mass of the outermost Neptune to that of the star. It can be clearly seen that the two maps coincide in each detail, and any shift in the position of the ICs relative to the fine structures is barely noticeable.

The coordinates of the dynamical maps were chosen to match the NAFF maps in (Pepe et al. 2007) their Fig. 7) and in (Farago et al. 2009) their Fig. 3). Since a direct comparison of the maps is not possible, due to changes in elements in the ICs, we have marked with a diamond a qualitative position of the former initial state relative to the approximate shape of MMR 2e:1b and its separatrix zone. Clearly, the Fit IIN is separated from the separatrix region by $\geq 5\sigma$. This statistically proves that the nominal system is not resonant and is in a safely stable zone. The narrow stripes of unstable motions can be identified with weak, higher-order 3-body MMRs with very long diffusion time scales, similar to the 2e:4b:1c MMR analyzed above.

These conclusions can be reinforced with a REM map for the three outer planets in the semi-major axes space, represented in the orbital period ratios ($P_3/P_2, P_4/P_1$)-plane, as the astrocentric Keplerian representation of the semi-major axes, see the bottom panel of Fig. 5. Here, we marked 1$\sigma$ and 3$\sigma$ uncertainties the same as in the previous panels. We computed them based on the MCMC samples. In this map, the 2-body MMRs are marked with vertical (some of them labelled) and horizontal curves. Skewed curves and lines are for 3-body MMRs and could be identified with a method described in (Guzzi 2005). Also this REM map reveals the Fit IIN safely separated from the 2e:1b MMR by several $\sigma$.

3.2.2 Stability limits depending on inclination

Finally, we performed direct MCMC sampling with the inclination added as a free parameter to the Newton co-planar model. As expected, since the RV time series are relatively short covering $\approx 1.5$ periods of the outermost planet, the inclination may be only weakly constrained in the assumed interval $[3^\circ, 90^\circ]$. There should be also strong, almost linear correlations between the masses and mass-inclination correlation due to the $m\sin i$ degeneracy.

However, this intuition seems insufficient in light of the MCMC sampling results for data set $D_2$, illustrated in Fig. 9 (upper plot). It shows posterior histograms for all masses $m_{1,2,3,4}$ and for the inclination $i$ as a free parameter. In addition to the predicted strong mass-inclination correlation, we found a clear, well-defined posterior maximum for $i \approx 30^\circ$. We tested this effect in multiple MCMC sampling experiments, varying the initial solution and sampling conditions.

Since, due to parameter correlations, the estimated auto-

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Figure 10. Dynamical maps for the best-fitting coplanar N-body Fit IIN (Tab. 2) to data set D2, extended to the inclination I space. Subsequent panels are for solutions selected from MCMC samples for the Newtonian model with varied inclination I, illustrated in Fig. 9, upper plot. The color scale is the same, as in Fig. 6. The integration time of each initial condition is 300 Kyr (≈ 2.7 × 10^4 × P_2); we used the leap-frog scheme with the step size of 8 days. The asterisk symbol means the elements of the nominal fits. The inclination of the orbital plane is described in the top-right corner of each panel. An approximate position of the 6b:1c MMR is labeled. Resolution of the plots is 720 × 360 points.

correlation time is as many as 25,000 iterations, we sampled up to 400,000 steps for each of 144 walkers, corresponding to 15–20 auto-correlation times. As a starting point for the sampling, we took Fit IIN in Table 2 supplemented with I = 20°, 45°, 60°, and 75°, respectively. Interestingly, in all cases, regardless of the initial I, the extremum is robust and occurs around I ≃ (30° ± 10°). At the same time, we monitored the RMS > 3.4 m s^{-1} for best-fitting solutions, which rises significantly to RMS ≃ 3.6–3.8 m s^{-1} below I > 30°. This means that the RV data predicts all planetary masses safely below the brown dwarf limit, i.e., the physical masses can be at most 2–3 times the minimum masses.

To assess the statistical significance of this result, we computed the Bayesian information criterion (BIC) defined as (e.g. Claeskens & Hjort 2008)

\[ BIC = p \ln N_{RV} - 2 \ln \mathcal{L}_{\text{max}} \]

for the Newtonian model, for the edge-on system with I = 90° and for a model with variable I, with p = 26 and p = 27 of free parameters, respectively; N_{RV} = 411, and ln \mathcal{L}_{\text{max}} is the value of ln \mathcal{L} evaluated at the posterior extremum. For the two models, we found ln \mathcal{L}_{\text{max}}(\theta, I = 90°) = −987.07 and ln \mathcal{L}_{\text{max}}(\theta, I) = −987.7, respectively, hence BIC(\theta, I = 90°) = 2130.62, and BIC(\theta, I) = 2137.96, respectively. Therefore

\[ \Delta BIC = BIC(\theta, I = 90°) - BIC(\theta, I) \simeq -7 < 2, \]

indicating that there is no evidence of the model with free inclination against the edge-on model with a smaller value of BIC, see Claeskens & Hjort 2008. However, if we apply the second-order Akaike information criterion (AIC) for small sample sizes (N_{RV}/p ≃ 15 < 40),

\[ AIC = 2p + 2(p + 1)/(N_{RV} - p - 1) - 2\ln \mathcal{L}_{\text{max}}, \]

then \Delta AIC < 2 for the two concurrent fit models, and that the candidate model is indicated almost as good as the best edge-on model Claeskens & Hjort 2008. We consider this as a marginal indication of the significance of the inclined model, which needs to be addressed with longer RV time series.

Furthermore, we examined this effect for the D3 data set, consisting of only the most accurate HARPS and UCLES RVs, and also changed the osculating epoch of the Newtonian model to the middle of the RV time series. In this experiment, we also increased the number of iterations to 500,000 steps for each of the 144 walkers. As a starting ICs, we chose Fit IIN from Tab. 2 with an initial value of I = 45°, but without any prior tuning of this solution. The results are shown in Fig. 9, lower plot. In this case, the posterior
distribution is shifted toward $I = 20^\circ$. This may further indicate a systematic but weak dependence of the Newtonian model on the inclination, which is also sensitive to the RVs changes.

The stability zone and fine unstable structures for inclined co-planar systems are illustrated in dynamical maps in the $(a_3, e_3)$-plane (Fig. 11) constructed for different inclinations of the co-planar system. We selected the best-fitting solutions from the MCMC samples with lowest RMS $\simeq 3.35$ m/s$^{-1}$ detected, and close to particular, a\'priori fixed inclinations. Subsequent panels are for such best-fitting models with the inclination equal to $I = 90^\circ$ (the nominal Fit IIN in Tab. 2), $I = 60^\circ$, $I = 45^\circ$, and $I = 33^\circ$, respectively. In the later case, the planet masses are twice as large as in the nominal, edge-on system. Moreover, the orbital elements selected from the MCMC samples are slightly different, thus introducing variability consistent with parameter uncertainties to the elements behind the map coordinates.

To effectively illustrate the region of stability with respect to $I$ in a more global way, we scaled the minimal masses in Fit IIN according to the minimum mass rule $m_s \sin I = \text{const}$, recalling the mass-inclination correlation. We then calculated the dynamical maps in the $(I, e_3)$ plane (Fig. 11). For reference, the second upper $x$ axis in these maps is for the mass of HD 160691e scaled with $I$.

Although, as we have shown, the influence of the warm Neptune is negligible for the dynamical evolution of the outer planets when their masses are minimal, this may not be the case for small inclinations. We therefore calculated two versions of the REM maps, for three- (top panel) and four-planets (middle panel), respectively (the later with lower resolution to save CPU time). It can be clearly seen that in the range of $I \in [5^\circ, 90^\circ]$, which covers the variation of masses spanning one order of magnitude, all, even very fine features of the phase space remain the same.

Finally, we constructed a REM map in the orbital period ratios plane shown in Fig. 11 (bottom panel) around $I = 20^\circ$, similar to the scan in Fig. 8. In this case, the masses of the planets are $(1.34, 4.91, 5.85) m_{\text{Jup}}$, i.e., the minimum masses scaled by a factor 3. We integrated each point for 300 kyr forward and back with the leap-frog scheme and the step size of 8 days. The ICs is located in a denser network of 2-body and 3-body MMRs, but still well separated from the 2e:1b MMR. We can also observe the high sensitivity of REM to interacting MMRs, indicated by in their regions of overlap (crossings).

Since the ICs is very close to an unstable 3-body MMR, we
performed a comparative integration of the nominal system and a configuration slightly shifted so that it is located in this nearby unstable MMR region (yellow strip in the lower panel of Fig. 11). We used the SABA scheme and the step size of 16 days, keeping the energy integral to $10^{-10}$ on the relative scale. In both cases, the system survived integrations for the lifetime of the star (6.7 Gyr). Such narrow chaotic 3-body MMRs, similar to that one analysed in Fig. 4 do not appear “dangerous” for the long-term stability. The chaotic configuration reveals only weak diffusion of $a_3$ and $e_3$. This is illustrated in Fig. 12.

The general conclusion of this experiment is a relatively wide stable zone preserved despite the enlarged minimal masses of the planets 2-3 times. The limit of stable solutions for $I = 15^\circ-20^\circ$ roughly coincides with the shape of statistically detected posterior extremum for $I = 30^\circ$ (data set $D_2$) and $I = 20^\circ$ for data set $D_3$, as we found with the MCMC sampling. Systems with the most probable inclinations $I = 60^\circ$ in purely random sample would be in the middle of a broad, stable zone. Such the likely inclination increases the planet masses by only 15%.

Moreover, the clear posterior maxima for $I = 30^\circ$ and $I = 20^\circ$ found here (still, in the stable zone) may confirm the marginally detected bias toward small inclinations of multiple systems, investigated with the HST astrometry in [Benedict et al. 2022]. We should also note that for μ Arae very small inclinations $I \lesssim 10^\circ$ can apparently be ruled out on both statistical as well as on dynamical grounds.

4 POSSIBLE DEBRIS DISKS AND SMALLER PLANETS

Based on the updated, rigorously stable and well constrained orbital solutions collected in Table 3, we simulated the dynamical structure of hypothetical debris disks in the system. The large “gap” between the two outer planets, at $\approx 1.52$ au and 5.2 au, respectively, we can predict orbitally stable objects with masses that are below the present detection levels. This region may be an analogue of the Main Belt in the Solar System, given the striking similarity of the most planet influence for the Main Belt and Kuiper Belt disks, and the mutual gravitational influence between the planets and the test objects. In all experiments, the probe object interacts gravitationally with the three most massive planets.

To calculate the $(Y)$ values for the synthetic systems, we integrated the $N$-body equations of motion and their variational equations with the GBS integrator [Hairer et al. 1995] for $\approx 10^9$ yrs. Such an interval covers $\approx 10^9$ orbital periods of outermost planet, which makes it possible to detect unstable motions associated with strongest two-body and three-body MMRs. This integration time is also consistent with the typical characteristic time-scale required to achieve $(Y)$ convergence for a stable configuration. The GBS integrator is the best choice in the case of collisional dynamics that is frequently expected in this setup.

4.1 Hypothetical asteroidal belts

The results for small-mass asteroids are illustrated in Fig. 13. Cartesian coordinates in the orbital plane of the system shown in the top-right panel are accompanied by plots for canonical elements of the test particles. We gathered $\approx 10^9$ stable solutions with $(|Y| - 2) < 0.007$ for this case. The probe particles are marked with different colors, depending on their dynamical status: brown dots are for objects involved in 1:1c MMR with the outermost planet HD 160691c, orange dots are for stable orbits between HD 160691b and HD 160691c, and blue dots are for the Kuiper belt-like zone beyond the outermost planet.

The edges of the debris disk formed in these regions are highly asymmetric. Also, their non-random distribution in the plane of the osculating elements $(\dot{a}_3, \dot{e}_3)$ is shown in the bottom-right panel in Fig. 13. It was constructed based on the canonical elements determined in the Jacobi reference frame. The use of canonical ele-
Figure 13. **Left column:** dynamical maps for a test particle (Vesta-like asteroid with the mass of $3 \times 10^{-7} m_{\text{Jup}}$) in three distinct regions between the planets. The MEGNO $\langle Y \rangle \sim 2$ indicates a regular (long-term stable) solution marked with black/dark blue colour, $\langle Y \rangle$ much larger than 2, up to $\gtrsim 5$ indicates a chaotic solution (yellow). The integration time of each initial condition is $10^5$ yr ($\simeq 10^4 \times P_e$). **Top-right panel:** Debris disks in the HD 160691 system revealed by $\simeq 10^6$ stable orbits with $|\langle Y \rangle - 2| < 0.007$ gathered in the $\langle Y \rangle$-disk simulation. They are illustrated as a snapshot of astrocentric coordinates $(x, y)$ at the initial epoch $t_0$. Colors of test particles injected with random elements, $a_0 \in [0.9, 10]$ au and $e_0 \in [0, 0.6]$ into the system of three outer planets, correspond to their dynamical status marked also in the panel with orbital elements, below. The initial positions of the planets are marked with filled circles. Gray rings illustrate their orbits integrated in a separate run for 0.2 Myr. **Bottom-right panel:** the orbital structure of hypothetical debris disks in the system, in terms of canonical Jacobi elements in the $(a_0, e_0)$-plane. Some two-body, lowest order MMRs with the planets are labeled, and stable orbits in their regions are marked with different colors, consistent with a snapshot of these stable solutions in the above panel.

4.2 Earth-like planets and the habitable zone

Although we considered low-mass asteroids in this test, stable regions can potentially host larger planets as well, in the Earth mass range. As the mass of the probing objects increases, the regions may decrease in size, both in the coordinate- and orbital element–planes. This is illustrated in Fig. 14 for Earth-mass objects (the left column) and super-Earths (the right column), respectively. That case we should interpret in terms of a potential location of the small planets rather than a representation of a physical debris disk.
The distribution of Earth objects is very similar to the experiment for Vesta-type asteroids, as could be predicted from the similarity of this system to the restricted problem (with zero-mass asteroids). For more massive super-Earth “asteroids” and the inclinations of the system $I=60^\circ$ the stable zones shrink considerably, but the overall disks structure is still preserved. We can conclude that the $\langle Y \rangle$-model scales for several orders of magnitude of the probe masses.

The results are therefore universal in the sense that we can predict the locations of e.g., Earth-like planets that are below the current detection limits. It turns out that such small planets could be found in the habitable zone, despite Rocinante and Quijote prevent stable orbits of terrestrial planets unless they are placed beyond roughly 2 au (see Fig. [14]), or interior to 0.3–0.4 au.

Given the luminosity of $\mu$ Arae $L = 1.9 L_\odot$ and the spectral temperature $T = 5820$ K [Soriano & Vauclair 2010], the outer limiting distance roughly correspond to the orbit of Mars in the Solar system. Indeed, for an Earth-like planet, the inner radius of the runaway greenhouse effect is $r_{\text{min}} = 1.31$ au, the radius of maximum greenhouse effect $r_{\text{max}} = 2.30$ au, and the radius for early Mars zone $r_{\text{EM}} = 2.42$ au [Kopparapu et al. 2014, their habitable zone calculator]. Therefore, habitable Earth-like planets could be found in a small region of Lagrangian (Trojan) 1:1b orbits around HD 160691b as well on the inner edge of the Main Belt, up to the 3:1c MMR gap (see the elements distribution in Fig. [14]).

5 CONCLUSIONS

The HD 160691 planetary system is one of the first detected multi-planet configurations with a mass-diverse planets, and it comprises of a warm Neptune, a Saturn-mass planet, and two massive Jupiter-mass objects. The precision RV data available in public archives, spanning at least 1.5 outermost periods, makes it already possible to tightly constrain the orbits and minimal masses of the planetary companions to $1\sigma \approx 0.02 m_{\text{Jup}}$. Unfortunately, given a low accuracy of the HST astrometry reported in [Benedict et al. 2022], and insufficient detection limits (estimated here independently), we restricted the analysis to the RV data only.

We improved kinematic (Keplerian) models reported more than 15 years ago [Goździewski et al. 2007, Pepe et al. 2007], as well as in the very recent paper by Benedict et al. [2022]. Our Newtonian RV models of the HD 160691 system imply its long-term stable, Solar system-like orbital architecture. The planets revolve...
in low-eccentricity orbits determined with significantly reduced uncertainties \( \simeq 0.01 \) w.r.t. the prior literature, closely resembling the Earth–Mars–Jupiter sequence. Other orbital elements, and particularly the semi-major axes are bounded 0.02 au for the outermost planet, and to just 0.001–0.002 au for remaining inner massive companions. Limiting uncertainty of the outermost semi-major axis to \( \simeq 27 \) days means a qualitative improvement, compared to uncertainties of 700–1300 days reported in (Gozdziewski et al. 2007) and (Pepe et al. 2007). Using the dynamical maps technique, we found that the nominal ICs cover regions in the phase space within several \( \sigma \) error bars that correspond to long-term stable evolution. The direct numerical integrations indicate stable orbital evolution of the best-fitting models for at least 6.7 Gyr (i.e., the lifetime of the star).

The present RV data do not make it possible to fully constrain the system inclination. However, it does not influence the stability in a wide range between 90\(^\circ\) and \( \simeq 20\)\(^\circ\). In this range, coplanar systems remain in similarly wide and safe zones of stable motions, despite of planet masses enlarged a few times, in accord with the \( m \sin i \) relation. Moreover, we found a close overlap of the dynamical stability with the best-fitting models in the sense that there is clear maximum of the posterior distribution for \( \ln L \) and a steep increase of the RMS at \( I \simeq 20^\circ–30^\circ \). This means that all the masses would remain certainly below the brown dwarf mass range. It also proves that the analysed RV data bring information on the mutual interactions between the system components.

The meaningfully constrained orbits make it possible to globally investigate the global dynamical structure of the system. The inner pair of Saturn-Jupiter–mass planets is close to the 2:1 MMR, but is significantly and systematically separated from this resonance. Similarly, the outer pair is close to the 6:1 MMR but also is meaningfully far from it. This result may be important since it adds a new observational evidence on a near-resonant, well characterised multiple system with Jovian-mass planets.

Multiple planetary systems, especially in the lower mass range detected by the Kepler mission, exhibit excess of planets close within the Jovian planets. Their short-term MMR structure closely resembles the Main Belt and the Kuiper Belt in the Solar system.

Prospects to detect relatively massive, super-Earth–mass objects in the zone around 0.3 au–0.5 au or in other parts of the system, where stable orbits of are possible, are uncertain but unlikely. The semi-amplitude of their RV signals would be comparable with the intrinsic stellar jitter variability. We did not detect significant periods in the residuals of the RV models other than those identified with the known planets.

Because \( \mu \) Arae has a fairly large parallax (\( \simeq 65 \) mas), it may be an interesting and promising target for ALMA and other instruments to detect dust emissions, and set additional limits on the presence of small planets in outer parts of the system. In addition, the detection of debris disks, especially the outer one, can help better constrain the inclination of the system.

Monitoring the RV variability of the star still seems plausible, as it may permit to characterise the system even better, once the Gaia DR4 catalogue is released. Our simulation of the IAD measurements with the help of htof package (Brandt et al. 2021) reveal that the two outer planets will be astrometrically detectable with very high S/N, provided the uncertainty of the IAD time series on the level of 0.1 mas. Moreover, we have shown that the mutual gravitational interactions can be detected in the RV data up to the middle of 2015. Additional precision RV observations might greatly help to break or reduce the \( m \sin i \) degeneracy, and confirm or rule out the inclination of the system \( I \simeq 20^\circ–30^\circ \) indicated by our Bayesian MCMC sampling experiments.

Finally, the highly hierarchical configuration of \( \mu \) Arae is a new test-bed for our new fast indicator REM (Panichi et al. 2017) that helps to analyse the structure of the phase space in terms of the most accurate, Newtonian representation of the data. Despite analytical approximations for the motion of the innermost planet may be constructed (Farago et al. 2009), the simple REM algorithm based on the canonical leap-frog scheme offers a sufficient numerical efficiency to derive the dynamical maps through integrating the exact equations of motion of the whole system. It is also fully compatible with more CPU demanding MEGNO technique, especially for systems in regions of the the phase space which are filled with mostly stable solutions.

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7 DATA AVAILABILITY

The Radial Velocity time series referenced in this paper as data set \( \mathcal{D}_1 \) are available in their source form, as published by [Benedict et al. (2022)](https://doi.org/10.1051/0004-6361/201936686) and as data sets \( \mathcal{D}_2 \) and \( \mathcal{D}_3 \) from [Trifonov et al. 2020](https://github.com/3fon3fonov/HARPS_...)
All other data presented in Tables 1 and 2 and Figures, underlying this article will be shared on reasonable request to the corresponding author.

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ON-LINE SUPPLEMENTARY MATERIAL

The following Section contains supplementary MCMC corner plots illustrating posterior probability distribution for three RVs models investigated in the paper and the numerical initial conditions to reproduce some figures in the paper.

Astrocentric Keplerian Elements to reproduce Fig. 11 (bottom panel) and Fig. 12

Stellar mass 1.13002842 Solar masses

| # | m[mJup] | a[au] | e | Inc[deg] | Om[deg] | pm[deg] | M[deg] |
|---|---------|-------|---|----------|---------|---------|--------|
| # Planet d | 0.0922327 | 0.0923196 | 0.1369002 | 19.0 | 0.0 | -166.3286582 | 144.888 7559 |
| # Planet e | 1.3473597 | 0.9290769 | 0.0816879 | 19.0 | 0.0 | -173.2692105 | 153.9525959 |
| # Planet b | 4.9087852 | 1.5262249 | 0.0472992 | 19.0 | 0.0 | 22.3406827 | 234.2662176 |
| # Planet c | 5.8499813 | 5.1882224 | 0.0363553 | 19.0 | 0.0 | 73.8847971 | 86.8947565 |
Figure A1. One– and two–dimensional projections of the posterior probability distribution for orbital parameters of the innermost (left column) and the outermost (the right column) planet, respectively. The top row is for the Keplerian model, and the bottom row is for the Newtonian model. The parameters are expressed in units consistent with Table 2. The semi-amplitude $K_i$ is equivalent to the mass $m_i$, and the orbital period $P_i$ is equivalent to the semi-major-axis $a_i$. The MCMC chain length is 128,000 iterations in each of 384 different instances selected in a small ball around a best-fitting solution found with the evolutionary algorithms. Parameter uncertainties are estimated as 16th, and 84th percentile samples around the median values (50th percentile).
Figure A2. A fragment of the corner plot for posterior samples for the Keplerian model with an instrumental drift attributed to UCLES measurements. Offsets $V_{0,1}$, $V_{0,2}$ and jitters $\sigma_1$ and $\sigma_2$ for HARPS and UCLES, respectively, as well as the semi-amplitude $A$ of the drift signal are expressed in m s$^{-1}$. The $n$ parameter is the drift frequency converted to and expressed in years, and the phase shift of the drift $\phi_0$ is expressed in degrees.