EDM constraints and CP asymmetries of $B$ processes in supersymmetric models

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Abstract

We demonstrate that electric dipole moments (EDMs) strongly constrain possible SUSY contributions to the CP asymmetries of $B$ processes; $LL$ and/or $RR$ flavour mixings between second and third generations are severely restricted by the experimental limit on the mercury EDM, and so therefore are their possible contributions to the CP asymmetries of $B \rightarrow \phi K$ and $B \rightarrow \eta'K$. We find that SUSY models with dominant $LR$ and $RL$ mixing through non-universal $A$-terms is the only way to accommodate the apparent deviation of CP asymmetries from those expected in the Standard Model without conflicting with the EDM bounds or with any other experimental results.

1 Introduction

The most recent results of BaBar and Belle collaborations [1] on the mixing-induced asymmetries of $B \rightarrow \phi K$ and $B \rightarrow \eta'K$ indicate possible deviation from the Standard Model (SM) expectations. The Belle experimental values of these asymmetries are given by

$$S_{\phi K} = 0.06 \pm 0.33 \pm 0.09,$$
$$S_{\eta'K} = 0.65 \pm 0.18 \pm 0.04.$$ (1)

The BaBar experimental values are

$$S_{\phi K} = 0.50 \pm 0.25^{+0.07}_{-0.04},$$
$$S_{\eta'K} = 0.27 \pm 0.14 \pm 0.03.$$ (3)

Comparison with the world average CP asymmetry $S_{J/\psi K} = 0.726 \pm 0.03$ shows that the average CP asymmetry of all $b \rightarrow s$ penguin modes from the Belle results is $0.43^{+0.12}_{-0.11}$,
which is $2.4\sigma$ away from the SM result, and from the BaBar result is $0.42 \pm 0.10$, a $2.7\sigma$ deviation.

Supersymmetry (SUSY) is one of the most popular candidates for physics beyond the SM, and a natural place to look for explanations of such deviation. Indeed in SUSY models there are many new sources of CP violation besides the CKM phase. However stringent constraints on these phases are usually obtained from the experimental bounds on the electric dipole moment (EDM) of the neutron, electron and mercury atom. Because of this it is a challenge for SUSY models to give a new source of CP violation that can explain the possible discrepancy between CP asymmetry measurements and the expected SM results, whilst at the same time avoiding the overproduction of EDMs.

It is known [2–4] that SUSY models with a large squark mixing and order one phase between the second and third generations can accommodate the CP asymmetry results via gluino exchange. The squark mixings can be classified, according to the chiralities of their quark superpartners, into left-handed or right-handed (L or R) squark mixing. The left-handed mixings for the down-squark are given by the mass insertions $(\delta^d_{LL})_{ij}$ and $(\delta^d_{LR})_{ij}$, and the right-handed mixings by $(\delta^d_{RL})_{ij}$ and $(\delta^d_{RR})_{ij}$. It is remarkable that in order simultaneously to satisfy the measurements of $S_{\phi K}$ and $S_{\eta' K}$ and explain the deviation between them, both left- and right-handed contributions have to be involved. This is because the left- and right-handed contributions have an opposite sign due to the different parity in the final states of $B \rightarrow \phi K$ and $B \rightarrow \eta' K$ [4].

In this paper we argue that a large flavour mixing between the second and third generation via $(\delta^d_{LL})_{23}$ and $(\delta^d_{RR})_{23}$ leads to a large $(\delta^d_{LR})_{22}$, which produces a large strange EDM and consequently overproduces neutron EDM (assuming the Parton model) and mercury EDM. We will show that, taking EDM constraints into account, the possible solution of the $S_{\phi K}$ and $S_{\eta' K}$ discrepancy based on $(\delta^d_{LL})_{23}$ and $(\delta^d_{RR})_{23}$ is disfavoured. This leaves the scenario with large mass insertions $(\delta^d_{LR})_{23}$ and $(\delta^d_{RL})_{23}$ (due to non-universal trilinear A-terms) as the only possible consistent solution.

This paper is organized as follows. In section 2 we introduce the supersymmetric contributions to the strange quark EDM which could be enhanced by large mixing between the second and third generation and leads to a large Hg EDM. Section 3 is devoted to imposing the EDM constraints on the SUSY phases in a model independent analysis, and the impact of these constraints on the SUSY contribution to the CP asymmetries of $B \rightarrow \phi K$ and $B \rightarrow \eta' K$. In section 4 we give numerical results and show correlations among the Hg EDM and the CP asymmetries of B-decays. Our conclusions are given in section 5.

## 2 Supersymmetric contributions to strange quark EDM

As mentioned in the introduction, SUSY models have several possible sources of CP violation in addition to the CKM phase. These CP phases can have important implications
for CP violating phenomenology. In particular they can induce large EDMs of quarks and leptons at the one-loop level that far exceed the experimental limits, and stringent constraints on SUSY CP phases are found [5]. The most recent measurements for the neutron ($d_n$) and mercury ($d_{Hg}$) EDMs lead to the following limits:

\[ d_n = 6.3 \times 10^{-26} \text{e cm}, \]

\[ d_{Hg} = 2.1 \times 10^{-28} \text{e cm}. \]

The neutron EDM receives contributions of different sources and the predicted value in any particular model depends quite strongly on the particular model of the neutron used for the calculation. Because of this it is worth briefly summarizing the calculation.

The major contributions to the EDMs come from electric and chromoelectric dipole operators and the Weinberg three-gluon operator:

\[
L = -\frac{i}{2} d^E q \bar{q} \gamma_5 q F^{\mu\nu} - \frac{i}{2} d^C q \bar{q} \gamma_5 T^a q G^{a\mu\nu} - \frac{1}{6} d^G f_{abc} G_{a\mu\rho} G_{b\rho\nu} G_{c\lambda\sigma} \epsilon^{\mu\nu\lambda\sigma}. \quad (7)
\]

In order to evaluate the neutron EDM, one needs to make some assumptions about the internal structure of the neutron. The models can be classified as follows;

1- The chiral quark model

In this model the neutron EDM is related to the EDMs of the valence quarks;

\[ d_n = \frac{4}{3} d_d - \frac{1}{3} d_u. \quad (8) \]

The quark EDMs are given by

\[ d_q = \eta^E d_q^E + \eta^C \frac{e}{4\pi} d_q^C + \eta^G \frac{e \Lambda}{4\pi} d_q^G, \]

where the QCD correction factors are given by $\eta^E = 1.53$, $\eta^C \simeq \eta^G \simeq 3.4$ and where $\Lambda \simeq 1.19$ GeV is the chiral symmetry breaking scale.

2- The parton quark model

Here one assumes that the quark contributions to neutron EDM are weighted by the factor $\Delta_q$ defined as $\langle n | \frac{1}{2} \bar{q} \gamma_\mu \gamma_5 q | n \rangle = \Delta_q S_\mu$, where $S_\mu$ is the neutron spin;

\[ d_n = \eta^E \left( \Delta_d d_d^E + \Delta_u d_u^E + \Delta_s d_s^E \right), \quad (10) \]

where the individual quark contributions are given in terms of the gluino, chargino and neutralino contributions

\[ d_q = d_q^G + d_q^{\tilde{\chi}^+} + d_q^{\tilde{\chi}^0}. \quad (11) \]
The following values for $\Delta_q$ are usually used: $\Delta_d = 0.746$, $\Delta_d = -0.508$, and $\Delta_s = -0.226$. The main difference between the parton quark model and the chiral quark model is the large strange quark contribution in the parton model. Also in this model, the relevant contribution is only due to the electric EDM of the quarks in contrast with the chiral quark model where the chromoelectric and three-gluon operators contribute as well.

3- QCD sum rules

The QCD sum rules analysis of ref.[6] leads to the following relation between the neutron EDM and the electric EDMs and chromoelectric EDMs of $u$ and $d$ quarks:

$$d_n = 0.7(d^E_d - 0.25d^E_u) + 0.55e(d^C_d + 0.5d^C_u),$$

where the value of quark vacuum condensate $\langle \bar{q}q \rangle = (225 \text{GeV})^3$ has been used. It can be seen from the above equation that the QCD sum rules cannot incorporate the effect of the strange quark in the neutron EDM.

4- The Chiral Lagrangian approach

In ref.[7], the chiral lagrangian approach was adopted to try to incorporate the strange quark chromoelectric EDM contribution to the neutron EDM. This analysis leads to the following result for the neutron EDM in terms of the quark chromoelectric EDM:

$$d_n = (1.6d^C_u + 1.3d^C_u + 0.26d^C_s)e \text{ cm}.$$

Passing to the mercury atom EDM, the major contribution here comes from $T$-odd nuclear forces in $\pi^0$ and $\eta$ couplings to the nucleus, which is generated by the chromoelectric EDMs of the constituent quarks. The resulting EDM of the mercury atom is given by ref.[8] as

$$d_{Hg} = -e(d^C_d - d^C_u - 0.012d^C_s) \times 3.2 \times 10^{-2}.$$

Although the coefficient for the $d^C_s$ is much smaller than the coefficients of the chromoelectric EDM of the down and up quarks, this contribution is still important since $d^C_s$ itself is enhanced by the heavy strange quark mass and by the relatively large mixing in the second generation. Recently the mercury EDM has been reconsidered in the light of the QCD sum rule calculations, with the result that the coefficients multiplying the first generation quarks could be reduced by a factor 2.5-3 [6] (see ref.[9]) for a recent discussion). Our study will depend mainly on the strange quark EDM so this uncertainty will not effect our conclusions. We will therefore use the older bound for this study, and comment at the end.

The dominant 1-loop gluino contribution to the EDMs is given by

$$d_{d,u}^E = -\frac{2}{3} \frac{\alpha_s}{\pi} Q_{d,u} \frac{m_d}{m^2} \text{Im}(\delta_{LR})_{11} M_1(x),$$
\[ d_s^E = -\frac{2}{3} \frac{\alpha_s}{\pi} Q_s \frac{m_\tilde{g}}{m_\tilde{q}} \text{Im}(\delta_{LR}^d)_{22} M_1(x), \tag{16} \]
\[ d_s^C = \frac{g_s \alpha_s}{4\pi} \frac{m_\tilde{g}}{m_\tilde{q}} \text{Im}(\delta_{LR}^d)_{22} M_2(x), \tag{17} \]

where \( x = m_\tilde{g}^2/m_\tilde{q}^2 \). The current experimental bounds using the parton model for the neutron EDM imply the following constraints on the relevant mass insertions [5]:

\[ \text{Im}(\delta_{LR}^d)_{11} < 1.9 \times 10^{-6}, \quad \text{Im}(\delta_{LR}^d)_{22} < 6.6 \times 10^{-6}, \tag{18} \]

where to illustrate we have taken \( m_\tilde{q} \simeq 500 \text{ GeV} \) and \( x = 1 \). The experimental limit on the mercury EDM leads to a stronger bound on the imaginary part of \( (\delta_{LR}^d)_{11} \) and about the same bound on the imaginary part of \( (\delta_{LR}^d)_{22} \):

\[ \text{Im}(\delta_{LR}^d)_{11} < 6.7 \times 10^{-8}, \quad \text{Im}(\delta_{LR}^d)_{22} < 5.6 \times 10^{-6}. \tag{19} \]

As alluded to above, the mass insertion \( (\delta_{LR}^d)_{22} \) is more sensitive to the mixing between the second and the third generation, so the bound on its imaginary part is the relevant one for our analysis. It is remarkable that the bounds obtained on this quantity from the mercury EDM and neutron EDM are almost the same, however, as we emphasized, the computation of the neutron EDM is more model dependent. Therefore in our analysis we will concentrate on the constraint obtained from the mercury EDM.

The explicit dependencies of \( (\delta_{LR}^d)_{22} \) and \( (\delta_{RL}^d)_{22} \) on the LL and RR mixing between the second and the third generations are given by

\[ (\delta_{LR}^d)_{22} = (\delta_{LL}^d)_{23} (\delta_{LR}^d)_{33} (\delta_{RR}^d)_{32} + [(\delta_{RR}^d)_{23} (\delta_{RL}^d)_{33} (\delta_{LL}^d)_{32}]^*, \tag{20} \]

where \( (\delta_{LR}^d)_{33} = (\delta_{RL}^d)_{33} \sim \frac{m_b(A_b - \mu \tan \beta)}{m_\tilde{q}^2} \). Recall that the EDM is proportional to the imaginary part of the coefficients of the \( d_R^*d_R \) term in the Lagrangian. In the MSSM the relevant part of the Lagrangian is given by \( \mathcal{L} \sim (YA - \mu \tan \beta) d_L^*d_R + \text{h.c.} \), where h.c. refers to \( (YA - \mu \tan \beta)^* d_L d_R^* \).

The EDM imposes stringent constraints on the flavour conserving CP phases of the \( A_b \) and \( \mu \) terms. It is reasonable therefore to assume that these phases are suppressed, in which case \( (\delta_{LR}^d)_{33} \sim m_b/m_\tilde{q} \sim 10^{-2} \). Also, due to the hermiticity of the LL and RR sectors of the squark mass matrix, \((\delta_{LL(RR)}^d)_{32} = (\delta_{LL(RR)}^d)_{23}^* \). Thus one finds

\[ (\delta_{LR}^d)_{22} \simeq 10^{-2} \left[ (\delta_{LL}^d)_{23} (\delta_{RR}^d)_{23} + [(\delta_{RR}^d)_{23} (\delta_{LL}^d)_{23}]^* \right] \tag{21} \]

Furthermore, \( (\delta_{LR}^d)_{22} \) can also be expressed as

\[ (\delta_{LR}^d)_{22} = (\delta_{LL}^d)_{23} (\delta_{LR}^d)_{32} + [(\delta_{RR}^d)_{23} (\delta_{RL}^d)_{32}]^*, \tag{22} \]

where \( (\delta_{LR}^d)_{32} = (\delta_{RL}^d)_{32}^* \). In the next section, we will determine the values of \( \text{Im}(\delta_{LR}^d)_{22} \) and \( \text{Im}(\delta_{RL}^d)_{22} \) within the regions of the parameter space that satisfy the experimental results of \( S_{\phi K} \) and \( S_{\eta K} \). We will show that the EDM of the strange quark allows the possibility of SUSY models with large LR (RL) mixing only.
3 SUSY contributions to the CP asymmetries $S_{\phi K}$ and $S_{\eta'K}$

Including the SUSY contribution, the effective Hamiltonian $H_{\text{eff}}^{\Delta B=1}$ for these processes can be expressed via the Operator Product Expansion (OPE) as

$$H_{\text{eff}}^{\Delta B=1} = \left\{ \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left( C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_7 \gamma_7 \gamma_7 + C_{8g} Q_{8g} \right) + \text{H.c.} \right\} + \left\{ Q_i \to \tilde{Q}_i, C_i \to \tilde{C}_i \right\},$$

(23)

where $\lambda_p = V_{pb} V_{ps}^*$, with $V_{pb}$ the unitary CKM matrix elements satisfying $\lambda_t + \lambda_u + \lambda_c = 0$, and $C_i \equiv C_i(\mu_b)$ are the Wilson coefficients at low energy scale $\mu_b \simeq m_b$.

As emphasized in refs. [2, 4], the dominant gluino contributions are due to the QCD penguin diagrams and chromo-magnetic dipole operators. At the first order in MIA, the gluino contributions to the corresponding Wilson coefficients at the SUSY scale are given by

$$C_{3g} = -\frac{\alpha_s^2}{2\sqrt{2}G_F m^2_q} \left( \delta_{LL}^d \right)_{23} \left[ -\frac{1}{9} B_1(x) - \frac{5}{9} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right],$$

$$C_{4g} = -\frac{\alpha_s^2}{2\sqrt{2}G_F m^2_q} \left( \delta_{LL}^d \right)_{23} \left[ -\frac{7}{3} B_1(x) + \frac{1}{3} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right],$$

$$C_{5g} = -\frac{\alpha_s^2}{2\sqrt{2}G_F m^2_q} \left( \delta_{LL}^d \right)_{23} \left[ \frac{10}{9} B_1(x) + \frac{1}{18} B_2(x) - \frac{1}{18} P_1(x) - \frac{1}{2} P_2(x) \right],$$

$$C_{6g} = -\frac{\alpha_s^2}{2\sqrt{2}G_F m^2_q} \left( \delta_{LL}^d \right)_{23} \left[ -\frac{2}{3} B_1(x) + \frac{7}{6} B_2(x) + \frac{1}{6} P_1(x) + \frac{3}{2} P_2(x) \right],$$

$$C_{8g} = \frac{\alpha_s \pi}{\sqrt{2}G_F m^2_q} \left[ \left( \delta_{LL}^d \right)_{23} \left( \frac{1}{3} M_3(x) + 3M_4(x) \right) + \left( \delta_{LR}^d \right)_{23} \frac{m_{\tilde{g}}}{m_b} \left( \frac{1}{3} M_1(x) + 3M_3(x) \right) \right],$$

(24)

where $\tilde{C}_{i,8g}$ are obtained from $C_{i,8g}$ by exchanging $L \leftrightarrow R$ in $(\delta_{AB}^d)_{23}$. It is clear that the part proportional to LR mass insertions in $C_{8g}$ which is enhanced by a factor $m_{\tilde{g}}/m_b$ would give a dominant contribution. Using the QCD factorization mechanism to evaluate the matrix elements, the decay amplitude of $B \to \phi K$ can be presented in terms of the relevant Wilson coefficients as follows [2]:

$$A(B \to \phi K) = -i \frac{G_F}{\sqrt{2}} m_B^2 F_{B \to K} f_\phi \sum_{i=1,10,7,8g} H_i(\phi) (C_i + \tilde{C}_i),$$

(25)

where $H_i(\phi)$ are given in Ref.[2] and the Wilson coefficients $C_i$ and $\tilde{C}_i$ are defined according to the parametrization of the effective Hamiltonian in Eq.(23)

$$H_{\text{eff}}^{\Delta B=1} = \frac{G_F}{\sqrt{2}} \sum_i \left\{ C_i Q_i + \tilde{C}_i \tilde{Q}_i \right\}$$

(26)
Therefore, the contributions of the $RR$ and $RL$ terms in $R_\phi$ have the same sign as the $LL$ and $LR$ ones. For instance, with $m_{\tilde{q}} = m_{\tilde{g}} = 500$ GeV, one obtains

$$R_\phi \simeq -0.14 e^{-i0.1(\delta_{LL}^{d})_{23}} - 127 e^{-i0.08(\delta_{LR}^{d})_{23}} - 0.14 e^{-i0.1(\delta_{RR}^{d})_{23}} - 127 e^{-i0.08(\delta_{RL}^{d})_{23}}. \quad (27)$$

From this result, it is clear that the largest SUSY effect is provided by the gluino contribution to the chromomagnetic operator which is proportional to $(\delta_{LR}^{d})_{23}$. However, the $b \to s\gamma$ constraints play a crucial role in this case. For the above SUSY configurations, the $b \to s\gamma$ decay constrains the possible gluino contributions since it sets $|(\delta_{LR}^{d})_{23}| < 0.016$. Despite this, on implementing the bound in Eq. (27), we see that the gluino contribution (proportional to $(\delta_{LR}^{d})_{23}$) is still able to generate large values for $R_\phi$, consequently driving $S_{\Phi K}$ towards the region of small values.

Although $B \to \phi K$ and $B \to \eta'K$ are very similar processes, the parity of the final states can vary the result. In $B \to \phi K$ the contributions from $C_i$ and $\tilde{C}_i$ to the decay amplitude are identically the same (with the same sign), whereas in $B \to \eta'K$ they have opposite signs. This can be easily understood by noting that

$$\langle \phi K | Q_i | B \rangle = \langle \phi K | \tilde{Q}_i | B \rangle. \quad (28)$$

which is due to the invariance of strong interactions under parity transformations, and to the fact that initial and final states have the same parity. However, in the case of the $B \to \eta'K$ transition, where the initial and final states have opposite parity, we have

$$\langle \eta'K | Q_i | B \rangle_{QCDF} = -\langle \eta'K | \tilde{Q}_i | B \rangle_{QCDF}. \quad (29)$$

As a result, the signs of the $C_i$ and $\tilde{C}_i$ in the decay amplitude are different for $B \to \eta'K$, and so the sign of the $RR$ and $RL$ in $R_{\eta'}$ are different from the sign of $LL$ and $LR$ in contrast with the $R_\phi$ case. Using the same SUSY inputs adopted in Eq. (27), we have

$$R_{\eta'} \simeq -0.07 e^{i0.24(\delta_{LL}^{d})_{23}} - 64(\delta_{LR}^{d})_{23} + 0.07 e^{i0.24(\delta_{RR}^{d})_{23}} + 64(\delta_{RL}^{d})_{23} \quad (30)$$

Following the parametrization of the SM and SUSY amplitudes in Ref.[4], $S_{\phi K}$ can be written as

$$S_{\phi(\eta')K} = \sin 2\beta + 2R_{\phi(\eta')} \cos \delta_{12} \sin (\theta_{\phi(\eta')} + 2\beta) + R_{\phi(\eta')}^2 \sin (2\theta_{\phi(\eta')} + 2\beta) \left/ \left(1 + 2R_{\phi(\eta')} \cos \delta_{12} \cos \theta_{\phi(\eta')} + R_{\phi(\eta')}^2 \right) \right., \quad (31)$$

where $R_{\phi} = |A_{\text{SUSY}}/A_{\text{SM}}|$, $\theta_{\phi} = \arg(A_{\text{SUSY}}/A_{\text{SM}})$, and $\delta_{12}$ is the strong phase. In order to accommodate the experimental results of $S_{\phi K}$ and $S_{\eta'K}$ we should have at least one of the following two scenarios [2, 4]: large mixing between the second and the third generations in $LL$ and $RR$ sectors or large mixing between the second and the third generations in $LR$ and $RL$ sectors.

As can be seen from Eq.(31), the deviation of $S_{\phi(\eta')K}$ from $\sin 2\beta$ strongly depends on the size of $R_{\phi(\eta')}$. The minimum values of $S_{\phi K}$ and $S_{\eta'K}$ can be obtained by large
values of $|\langle \delta_d^d\rangle_{23}| \sim \mathcal{O}(1), |\langle \delta_d^d\rangle_{23}| \sim \mathcal{O}(1)$ and phases of $\langle \delta_d^d\rangle_{23}$ and $\langle \delta_d^d\rangle_{23}$ of order one or $|\langle \delta_d^d\rangle_{23}| \sim \mathcal{O}(10^{-3}), |\langle \delta_d^d\rangle_{23}| \sim \mathcal{O}(10^{-3})$ and phases of $\langle \delta_d^d\rangle_{23}$ and $\langle \delta_d^d\rangle_{23}$ of order one. It is important to note that to have deviation between $S_{\phi K}$ and $S_{\eta' K}$, the contributions from $\langle \delta_d^d\rangle_{23}$ and $\langle \delta_d^d\rangle_{23}$ should be different, so that the gluino contribution to $S_{\phi K}$ becomes larger than its contribution to $S_{\eta' K}$ [4]. It is also worth mentioning that, as can be seen from Eq.(31), the effects of LL and RR mixing on $S_{\phi(\eta')K}$ remain limited compared to the effect of LR and RL.

4 EDM constraints on $S_{\phi K}$ and $S_{\eta' K}$

We now come to the main point of this letter, which is that such large values for the magnitudes and the phases of $\langle \delta_d^d\rangle_{23}$ and $\langle \delta_d^d\rangle_{23}$ may significantly enhance the strange quark EDM thereby overproducing mercury and possibly neutron EDMs. It is interesting to ask therefore whether SUSY is still able to accommodate such large magnitudes and phases, and if so, are they restricted.

As mentioned in the previous section, there are two possible sources of enhancement: the first is the combined effect of $\langle \delta_d^d\rangle_{23}$ and $\langle \delta_d^d\rangle_{23}$, the second source is either $\langle \delta_d^d\rangle_{23}$ or $\langle \delta_d^d\rangle_{23}$ combining with $\langle \delta_d^d\rangle_{32}$ or $\langle \delta_d^d\rangle_{32}$. However, within minimal flavour models such as minimal supergravity (where the trilinear couplings are universal), the size of the mass insertions $\langle \delta_d^d\rangle_{23}$ and $\langle \delta_d^d\rangle_{23}$ are of order $10^{-6}$ and $10^{-7}$ respectively. Therefore the imaginary part of the induced mass insertion $\langle \delta_d^d\rangle_{22}$ can easily be below the bound obtained from the experimental limit on Hg-EDM. In Fig. 1, we plot both $S_{\phi K}$, $S_{\eta' K}$ and $d_{Hg}/(d_{Hg})_{\text{Exp}}$ as functions of $|\langle \delta_d^d\rangle_{23}|$ and $|\langle \delta_d^d\rangle_{23}|$. We assume that $\arg[\langle \delta_d^d\rangle_{23}] \approx \arg[\langle \delta_d^d\rangle_{23}] \approx \pi/2$, in order to enhance their effects on the CP asymmetries of $B$-decays. We also fixed $m_0 = m_g = 400$ GeV.

As can be seen from the figure, in these scenarios the values of Hg EDM are well below
the current experimental limit. However, we cannot account for the CP asymmetries $S_{\phi K}$ and $S_{\eta'K}$, particularly $S_{\eta'K}$ which has been the subject of recent measurements by the BaBar collaboration. In this class of models with dominant $(\delta_{LL}^{d})_{23}$ or $(\delta_{RR}^{d})_{23}$ mass insertions, the value of $S_{\eta'K}$ is close to the SM prediction of $\sin 2\beta$. Therefore if the present $S_{\eta'K}$ result is confirmed, these models will be disfavoured.

In Fig. 2, we display scattering plots for $S_{\phi K}$ and $S_{\eta'K}$ versus the ratio of Hg EDM to its experimental limit. We set $m_0 = m_g = 400$ GeV. The other relevant parameters are scanned as follows: $|(\delta_{LL}^{d})_{23}|$ varies from 0 to 1, $\text{arg}[(\delta_{LL}^{d})_{23}]$ and $\text{arg}[(\delta_{RR}^{d})_{23}]$ are in the region $[-\pi, \pi]$. As can be easily seen from Fig. 2, within the region of the parameter space where $S_{\phi K}$ and $S_{\eta'K}$ fit the experimental data, the Hg EDM exceeds with many order of magnitudes its experimental bound. This imposes severe constraints on this scenario of simultaneous contribution from $LL$ and $RR$ mixing to accommodate both the experimental results of $S_{\phi K}$ and $S_{\eta'K}$. This result is in agreement with that of Ref.[10]. Returning to the question of the precise numbers in the bound, it is clear from the figures that even if the strange quark contribution to the mercury EDM were reduced by a whole order of magnitude (rather than the factor 2.5-3 reduction implied for the first generation contributions to the Hg-EDM from the sum-rule calculations) this conclusion is unchanged.

Therefore, we may safely conclude that SUSY models with dominant $LL$ and/or $RR$ large mixing between second and third generations will be ruled out if the experimental results of $S_{\phi K}$, $S_{\eta'K}$ are confirmed. SUSY models with dominant $LR$ and/or $RL$ mixing via non-universal $A$-terms, seem to be the simplest way to account for CP asymmetry $S_{\phi K}$ and $S_{\eta'K}$ without conflicting with EDMs or any other experimental results.

Before we conclude, we give a quantitative prediction for the Hg-EDM due to the effect of large $(\delta_{LR}^{d})_{23}$. As shown in Ref.[2], in order to accommodate the experimental result of the CP asymmetry $|(\delta_{LR}^{d})_{23}|$ should be of order $10^{-3}$ and $\text{arg}[(\delta_{LR}^{d})_{23}] \sim \pi/3$. These values lead to $S_{\phi K} \simeq 0.2$. Assuming (the minimal assumption) that the soft scalar
masses are universal at the SUSY breaking scale, one finds that at the electroweak scale 
\((\delta_{LL}^d)_{23}\) is of order \(10^{-3}\) and \((\delta_{RR}^d)_{23}\) \(\sim 10^{-6}\). Hence one finds that \((\delta_{LR}^d)_{22}\) \(< 10^{-6}\) which implies that 
\(d_{Hg} \sim 0.2(d_{Hg})_{Exp}\). We find it intriguing that the Hg EDM experiment is so close to testing CP violation in the flavour changing sector.

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References

[1] M. A. Giorgi (BaBar collaboration), plenary talk at XXXII Int. Conference on High Energy Physics, Beijing, China, August 16-22, 2004, http://ichep04.ihep.ac.cn; Y. Sakai (Belle collaboration), plenary talk at XXXII Int. Conference on High Energy Physics, Beijing, China, August 16-22, 2004, http://ichep04.ihep.ac.cn.

[2] E. Gabrielli, K. Huitu and S. Khalil, hep-ph/0407291;

[3] E. Lunghi and D. Wyler, Phys. Lett. B 521 (2001) 320; M. B. Causse, arXiv:hep-ph/0207070; G. Hiller, Phys. Rev. D 66 (2002) 071502; M. Ciuchini and L. Silvestrini, Phys. Rev. Lett. 89 (2002) 231802; S. Khalil and E. Kou, Phys. Rev. D 67 (2003) 055009; K. Agashe and C. D. Carone, Phys. Rev. D 68 (2003) 035017; G. L. Kane, P. Ko, H. b. Wang, C. Kolda, J. h. Park and L. T. Wang; Phys. Rev. Lett. 90 (2003) 141803; C. Dariescu, M.A. Dariescu, N.G. Deshpande, D.K. Ghosh, Phys. Rev. D 69 (2004) 112003; M. Ciuchini, E. Franco, G. Martinelli, A. Masiero, M. Pierini, L. Silvestrini, hep-ph/0407073; Z. Xiao and W. Zou, hep-ph/0407205; D. Chakraverty, E. Gabrielli, K. Huitu and S. Khalil, Phys. Rev. D 68 (2003) 095004; S. Khalil and R. Mohapatra, Nucl. Phys. B 695 (2004) 313.

[4] S. Khalil and E. Kou, Phys. Rev. Lett. 91 (2003) 241602.

[5] S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B 606 (2001) 151.

[6] M. Pospelov and A. Ritz, Phys. Rev. Lett. 83 (1999) 2526; M. Pospelov, Phys. Lett. B530 (2002) 123 O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Phys. Rev. D70 (2004) 016003

[7] J. Hisano and Y. Shimizu, hep-ph/0406091.

[8] T. Falk, K. A. Olive, M. Pospelov and R. Roiban, Nucl. Phys. B 560 (1999) 3.
[9] D. Demir, O. Lebedev, K. A. Olive, M. Pospelov and A. Ritz, Nucl. Phys. B 680 (2004) 339.

[10] J. Hisano and Y. Shimizu, hep-ph/0308255, hep-ph/0406091; M. Endo, M. Kakizaki and M. Yamaguchi, hep-ph/03011072;