NEUTRINOLESS DOUBLE BETA DECAY CONSTRAINTS

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A brief overview is given of theoretical analyses with neutrinoless double beta decay experiments. Theoretical bounds on the “observable”, \( \langle m \rangle_{\beta\beta} \), are presented. By using experimental bounds on \( \langle m \rangle_{\beta\beta} \), allowed regions are obtained on the \( m_l \cdot \cos 2\theta_{12} \) plane, where \( m_l \) stands for the lightest neutrino mass. It is shown that Majorana neutrinos can be excluded by combining possible results of future neutrinoless double beta decay and \( ^3H \) beta decay experiments. A possibility to constrain one of two Majorana phases is discussed also.

1. Introduction

Neutrino oscillation experiments so far supplied us with much knowledge of the lepton flavor mixing. As we know more about that, our desires of knowing better grows also about what can not be determined by oscillation experiments: How heavy are neutrinos? Are they Majorana particles? The former question can be answered directly by \( ^3H \) beta decay experiments. On the other hand, neutrinoless double beta decay (0\( \nu \)\( \beta \beta \)) experiments can give the direct answer to the latter one. 0\( \nu \)\( \beta \beta \) experiments seem to be interesting especially because some of the future 0\( \nu \)\( \beta \beta \) experiments are expected to probe rather small energy scale \( \sim 10^{-2} \) eV, and then will give stringent constraints on neutrino masses and mixing parameters.

The constraints by 0\( \nu \)\( \beta \beta \) experiments have been discussed by many authors. In this talk, I will try to give a brief overview of those in a manner as clear as possible. For generality and to emphasize future perspective, we will not use any specific results of 0\( \nu \)\( \beta \beta \) experiments. Specific constraints can be obtained easily by replacing \( \langle m \rangle_{\beta\beta}^{\text{min}} \) and \( \langle m \rangle_{\beta\beta}^{\text{max}} \), which denote experimental bounds, in equations with actual results of 0\( \nu \)\( \beta \beta \) experiments.

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*For example, see references 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, and the references therein.
2. Theoretical constraints on $\langle m \rangle_{\beta\beta}$

In the standard parametrization the MNS matrix for three neutrinos is

$$U_{\text{MNS}} \equiv \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}. \quad (1)$$

In most parts of this talk we assume that neutrinos are Majorana particles. Then, the mixing matrix includes two extra CP-violating phases (Majorana phases) as

$$U \equiv U_{\text{MNS}} \times \text{diag}(1, e^{i\beta}, e^{i\gamma}).$$

The “observable” of $0\nu\beta\beta$ experiments \textsuperscript{b} is

$$\langle m \rangle_{\beta\beta} \equiv \left| \sum_{i=1}^{3} m_{i} U_{ei}^{2} \right| = \left| m_{1} c_{12}^{2} c_{13}^{2} + m_{2} s_{12}^{2} c_{13}^{2} e^{2i\beta} + m_{3} s_{13}^{2} e^{2i(\gamma-\delta)} \right|. \quad (3)$$

where $U_{ei}$ represent the elements in the first row of $U$ and $m_{i} > 0$ are the neutrino mass eigenvalues. The experimental constraints on mixing angles are $\sin^{2}2\theta_{13} \lesssim 0.1 \equiv \sin^{2}2\theta_{\text{CH}}$ of the CHOOZ bound and $0.2 < \cos 2\theta_{12} < 0.5$ ($0.4 > s_{12}^{2} > 0.25$) with the best fit $\cos 2\theta_{12} = 0.37$ ($s_{12}^{2} = 0.315$).

For simplicity, $\Delta m_{31}^{2} \equiv m_{3}^{2} - m_{1}^{2}$ are fixed as $\Delta m_{31}^{2} = 7.3 \times 10^{-5} \text{eV}$ and $|\Delta m_{23}^{2}| = 2.5 \times 10^{-3} \text{eV}$. Hereafter, we call $m_{1} < m_{2} < m_{3}$ the normal hierarchy and $m_{3} < m_{1} < m_{2}$ case the inverted hierarchy even if masses are almost degenerate. The lightest mass eigenvalue $m_{1}$, which is $m_{1}(m_{3})$ for the normal (inverted) hierarchy, is used as the horizontal axis of all figures in this talk to avoid tedious case studies for whether the mass pattern is hierarchical or not.

By choosing phase factors and $\theta_{13}$ so as to maximize the right-hand side of (3), we obtain theoretical upper bounds on $\langle m \rangle_{\beta\beta}$ as

$$\langle m \rangle_{\beta\beta} \leq (m_{1} c_{12}^{2} + m_{2} s_{12}^{2}) c_{\text{CH}}^{2} + m_{3} s_{\text{CH}}^{2} \quad (4)$$

for the normal hierarchy, and

$$\langle m \rangle_{\beta\beta} \leq m_{1} c_{12}^{2} + m_{2} s_{12}^{2} \quad (5)$$

for the inverted hierarchy, where $s_{\text{CH}} (c_{\text{CH}})$ is the largest (smallest) value of $s_{13}$ ($c_{13}$) determined by the CHOOZ experiment. In the similar way a

\textsuperscript{b}The actual observable is the half life $T_{1/2}^{0\nu} = (G^{0\nu} |M^{0\nu}|^{2} (m_{\beta\beta}^{2})^{-1}$ with the nuclear matrix elements $M^{0\nu}$ and the calculable phase space integral $G^{0\nu}$.
Figure 1. Shown are theoretical bounds on $\langle m \rangle_{\beta\beta}$ with $\cos 2\theta_{12} = 0.2$ which is the bound of LMA: $0.2 < \cos 2\theta_{12} < 0.5$. (See the text in Sec. 2.) Figs. (a) and (b) are for the normal and inverted hierarchy, respectively. The region surrounded by those lines is allowed. Note that $\langle m \rangle_{\beta\beta}$ has the absolute lower bound, $\approx 0.05 \times \cos 2\theta_{12}$ eV, for the inverted hierarchy although does not for the normal one.

Theoretical lower bound is obtained as

$$\langle m \rangle_{\beta\beta} \geq s_{CH}^2 |m_1 \cos^2 \theta_{12} - m_2 \sin^2 \theta_{12}| - m_3 \sin^2 \theta_{CH}. \quad (6)$$

Figs. 1 present allowed regions on the $m_l$-$\langle m \rangle_{\beta\beta}$ plane determined by (4), (5), and (6) for fixed $\theta_{12}$. Fig. 1(a) shows that $\langle m \rangle_{\beta\beta}$ can be zero for the normal hierarchy. It occurs around $m_1 = s_{12}^2 \sqrt{\Delta m_{12}^2 / \cos 2\theta_{12}}$ which is obtained by setting the right-hand side of (6) zero with $\theta_{CH} = 0$. In the inverted hierarchy, almost maximal $\theta_{12}$ is necessary for $\langle m \rangle_{\beta\beta}$ to vanish because $m_1 \simeq m_2$, but it is outside of the LMA region.

On the other hand, Fig. 1(b) shows that $\langle m \rangle_{\beta\beta}$ has the absolute lower bound for the inverted hierarchy with the value of $\theta_{12}$. The approximate form of the absolute lower bound, $\langle m \rangle_{\beta\beta} \geq 0.05 \times \cos 2\theta_{12}$ eV, is extracted by setting $m_1 = \sqrt{\Delta m_{12}^2} = 0$ in the right-hand side of (6) for the inverted hierarchy. Considering $0.2 \lesssim \cos 2\theta_{12}$ of LMA, $\langle m \rangle_{\beta\beta}$ must be larger than about 0.01 eV. Therefore, the inverted hierarchy is rejected for Majorana neutrinos if experiments show $\langle m \rangle_{\beta\beta} \lesssim 0.01$ eV.

3. Constraints on $m_l$ by a $0\nu\beta\beta$ result

A neutrinoless double beta decay experiment puts experimental bounds on $\langle m \rangle_{\beta\beta}$ as $\langle m \rangle_{\beta\beta}^{\text{min}} \leq \langle m \rangle_{\beta\beta} \leq \langle m \rangle_{\beta\beta}^{\text{max}}$, although $\langle m \rangle_{\beta\beta}^{\text{min}}$ vanishes for a negative result. From now on, we try to utilize those experimental bounds.

First, because the true value of $\langle m \rangle_{\beta\beta}$ must be less than $\langle m \rangle_{\beta\beta}^{\text{max}}$ and larger...
than the theoretical lower bound (6), we can construct an inequality

\[ \langle m \rangle^\text{max}_{\beta\beta} \geq \langle m \rangle_{\beta\beta} \geq c_{\text{CH}}^2 |m_1 c_{12}^2 - m_2 s_{12}^2| - m_3 s_{23}^2. \]  

(7)

Similarly, other inequalities with \( \langle m \rangle^\text{min}_{\beta\beta} \) are obtained as

\[ \langle m \rangle^\text{min}_{\beta\beta} \leq \langle m \rangle_{\beta\beta} \leq (m_1 c_{12}^2 + m_2 s_{12}^2) c_{\text{CH}}^2 + m_3 s_{23}^2. \]  

(8)

for the normal hierarchy, and

\[ \langle m \rangle^\text{min}_{\beta\beta} \leq m_1 c_{12}^2 + m_2 s_{12}^2. \]  

(9)

for the inverted hierarchy. These bounds determine allowed regions on the \( m_1 \cos 2\theta_{12} \) plane which are shown in Fig. 2.

As an example \( \langle m \rangle^\text{min}_{\beta\beta} = 0.1 \text{ eV} \) and \( \langle m \rangle^\text{max}_{\beta\beta} = 0.3 \text{ eV} \) are considered in Fig. 2(a). The bounds for the normal (solid lines) and inverted (dashed lines) hierarchies are almost same as each other because the relevant scale of energy is large enough compared with \( \sqrt{|\Delta m^2_{23}|} \) so that the degenerate mass approximation applies. The bounds by (8) and (9) are almost vertical because of small \( \Delta m^2_{21}/m_l^2 \). Thus, those bounds put a lower bound on \( m_l \) almost independently of \( \cos 2\theta_{12} \), which can be written approximately

\[ \langle m \rangle^\text{min}_{\beta\beta} \lesssim m_l. \]  

The upper bound on \( m_l \) can be extracted from (7), and the curve has an asymptotic line \( |\cos 2\theta_{12}| = t_{\text{CH}}^2 \). Consequently, the upper bound on \( m_l \) does not exist if \( |\cos 2\theta_{12}| < t_{\text{CH}}^2 \). Fortunately, it is not the case for LMA (gray band). Eventually, if \( \langle m \rangle^\text{min}_{\beta\beta} \) and \( \langle m \rangle^\text{max}_{\beta\beta} \) are large enough compared with \( \sqrt{|\Delta m^2_{23}|} \), the bounds on \( m_l \) are roughly

\[ \langle m \rangle^\text{min}_{\beta\beta} \leq m_l \leq \frac{1}{0.9 \cos 2\theta_{12}} \times \langle m \rangle^\text{max}_{\beta\beta}. \]  

(10)

The effect of nonzero \( \theta_{\text{CH}} \) is the reason why the coefficient of \( \cos 2\theta_{12} \) is shifted from unity to 0.9. The most conservative upper bound is obtained by \( \cos 2\theta_{12} = 0.2 \) and then the coefficient of \( \langle m \rangle^\text{max}_{\beta\beta} \) is approximately 6.

Next, the case of \( \langle m \rangle^\text{min}_{\beta\beta} = 0.01 \text{ eV} \) and \( \langle m \rangle^\text{max}_{\beta\beta} = 0.03 \text{ eV} \) is considered in Fig. 2(b) as another example. Future experiments are expected to be able to probe such a small energy scale. The bounds in Fig. 2(b) for two hierarchies are no longer similar to each other because \( \Delta m^2_{23}/m_l^2 \) is not negligible in the energy scale. The smaller \( \langle m \rangle^\text{min}_{\beta\beta} \) experiments achieve, the larger the excluded region of small \( \theta_{12} \) for the inverted hierarchy. It will be inconsistent with LMA if \( \langle m \rangle^\text{min}_{\beta\beta} \lesssim 0.01 \text{ eV} \), and then the inverted hierarchy is excluded as discussed in the previous section. Another interesting feature of the bounds in Fig 2(b) is that \( m_l = 0 \) is allowed for the inverted hierarchy although that is excluded for the normal hierarchy. It is because that \( \langle m \rangle^\text{min}_{\beta\beta} = 0.01 \text{ eV} \) is less than the theoretical upper bound on \( \langle m \rangle_{\beta\beta} \) at \( m_l = \)}
Figure 2. Presented are allowed regions determined by two example cases of 0νββ results. (See the text in Sec. 3.) The solid (dashed) lines are the bound for the normal (inverted) hierarchy. LMA region is superimposed by the band of shadow. Note that large mixing is preferred by the inverted hierarchy. Note also that $m_l = 0$ can be allowed even if $\langle m \rangle_{\beta\beta}^{\min} \neq 0$.

For the inverted hierarchy. (See Fig. 1(b).) It can occur for the normal hierarchy also if $\langle m \rangle_{\beta\beta}^{\min}$ is even smaller. (See Fig 1(a).) The sufficient conditions of $\langle m \rangle_{\beta\beta}^{\min}$ for the exclusion of $m_l = 0$ are obtained by using the theoretical upper bound on $\langle m \rangle_{\beta\beta}$ with $m_l = 0$. Those are

$$\langle m \rangle_{\beta\beta}^{\min} \gtrsim 0.005 \text{eV} \sim \sqrt{\Delta m_{12}^2}, \quad (11)$$

for the normal hierarchy, and

$$\langle m \rangle_{\beta\beta}^{\min} \gtrsim \sqrt{|\Delta m_{23}^2|} \sim 0.05 \text{eV} \quad (12)$$

for the inverted hierarchy.

4. A possibility of excluding Majorana neutrinos

Without information of the mass hierarchy, the hypothesis of Majorana neutrinos can not be rejected by 0νββ experiments even if $\langle m \rangle_{\beta\beta} = 0$. (See Fig. 1(a).) For the rejection, some help of other experiments is necessary. In this section we consider a possibility of the rejection with a help of $^3H$ beta decay experiments which give direct measurement of the neutrino mass. The sensitivity limit is expected to be 0.3 eV, which is still in the degenerate mass regime, in the future KATRIN experiment. Here, we consider the situation that a positive result with $m_l > 0.3 \text{ eV}$ is discovered. On the other hand, a negative result of 0νββ experiments put an upper bound $\langle m \rangle_{\beta\beta}^{\max}$ on $\langle m \rangle_{\beta\beta}$, and the bound is translated to an upper bound on $m_l$. 
as discussed in Sec. 3. Those two bounds on $m_l$ will be inconsistent with each other if $\langle m_l \rangle^{\text{max}}_{\beta\beta}$ becomes small enough. It means the rejection of the Majorana neutrino hypothesis. The critical value of $\langle m_l \rangle^{\text{max}}_{\beta\beta}$ for the rejection is obtained by setting the right-hand side of (10) equal to $0.3 \text{ eV}$ which is the expected sensitivity of the future $^3\text{H}$ beta decay experiment. Consequently, the necessary condition for the rejection with $\cos 2\theta_{12} = 0.2$ is

$$\langle m_l \rangle^{\text{max}}_{\beta\beta} < 0.05 \text{ eV}. \quad (13)$$

The critical value $0.05 \text{ eV}$ seems to be very important goal of $0\nu\beta\beta$ experiments because it is accidentally the same as $\sqrt{|\Delta m^2_{23}|}$ at the SK best fit value where the difference between two hierarchies starts to arise.

5. Placing a constraint on the Majorana phase

When Majorana neutrinos are considered, it is quite natural to hope that one day we will will be able to access the Majorana phases. In general, it is impossible even if we have precise values of $\langle m_{\beta\beta} \rangle$, $m_{\beta\beta}$, $\delta$, and mixing angles because two unknown Majorana phases are involved in (3). Nature, however, gives us a possibility to obtain information of a Majorana phase $\beta$ at the price of the possibility for another phase $\gamma$. It can be seen in (3) by noting that $\theta_{13}$ is tiny and $\theta_{12}$ is not.
By choosing $\theta_{13}$ and $\gamma - \delta$ appropriately in (3), we obtain an inequality which includes $\beta$ as

$$\langle m \rangle_{\beta\beta}^{\text{max}} \geq \langle m \rangle_{\beta\beta}$$

$$\geq c_{CH} \sqrt{m_{12}^2 c_{12}^4 + m_{2}^2 s_{12}^4 + 2 m_{1} m_{2} c_{12}^2 s_{12}^2 \cos 2\beta - m_{3} s_{CH}^2}. \quad (14)$$

Similarly, we obtain

$$\langle m \rangle_{\beta\beta}^{\text{min}} \leq \langle m \rangle_{\beta\beta}$$

$$\leq c_{CH} \sqrt{m_{12}^2 c_{12}^4 + m_{2}^2 s_{12}^4 + 2 m_{1} m_{2} c_{12}^2 s_{12}^2 \cos 2\beta + m_{3} s_{CH}^2}. \quad (15)$$

for the normal hierarchy, and

$$\langle m \rangle_{\beta\beta}^{\text{min}} \leq \langle m \rangle_{\beta\beta} \leq \sqrt{m_{12}^2 c_{12}^4 + m_{2}^2 s_{12}^4 + 2 m_{1} m_{2} c_{12}^2 s_{12}^2 \cos 2\beta} \quad (16)$$

for the inverted hierarchy. Note that (7), (8), and (9) are reconstructed by (14), (15), and (16) with $\cos 2\beta = 1$ or $-1$. Those inequalities determine allowed regions on the $m_l$-$\cos 2\beta$ plane for given $\theta_{12}$.

We present the bounds on $\beta$ in Figs. 3; The allowed regions are insides of the two lines with respective width. Presented in Fig. 3(a) are only for the normal hierarchy, but bounds for the inverted hierarchy are almost the same as those of the normal hierarchy because the relevant energy scale is large enough for the degenerate mass approximation to apply. Note that projecting the allowed region upon $m_l$ axis results in the allowed region of $m_l$ which is nothing but the one obtained in Sec. 2. In Fig. 3(a), the bounds determined by $\langle m \rangle_{\beta\beta}^{\text{max}}$ exclude large values of $\cos 2\beta$ for large $m_l$, and those bounds cross the line of $\cos 2\beta = 1$ at around $m_l = \langle m \rangle_{\beta\beta}^{\text{max}}$ almost independently of $\cos 2\theta_{12}$. Thus, an upper bound on $\cos 2\beta$ is extracted if $^3H$ beta decay experiments show that $m_l$ is larger than $\langle m \rangle_{\beta\beta}^{\text{max}}$.

On the other hand, the bounds determined by $\langle m \rangle_{\beta\beta}^{\text{min}}$ exclude small values of $\cos 2\beta$ for small $m_l$ in Fig. 3(a). In principle, it is possible to put a lower bound on $\cos 2\beta$ if other experiments give a stringent upper bound on $m_l$. In practice, however, it seems to be too difficult because the values of $m_l$ that give a lower bound on $\cos 2\beta$ tend to lie within too narrow region of too small energy scale; For example, the region is 0.1-0.2 eV in Fig. 3(a) for the most conservative case of $\cos 2\theta_{12} = 0.5$.

The difference between bounds for two hierarchies can be seen in Fig. 3(b). For the inverted hierarchy, the region of large $\cos 2\beta$ is excluded for all $m_l$. When $\langle m \rangle_{\beta\beta}^{\text{max}}$ becomes less than 0.01 eV, whole region of $\cos 2\beta$ is excluded for the inverted hierarchy. It means the exclusion of the inverted hierarchy as discussed in Sec. 3.
6. Summary

In this talk, I presented an overview of constraints on neutrino masses and mixing imposed by neutrinoless double beta decay ($0\nu\beta\beta$). First, it was shown that $\langle m \rangle_{\beta\beta}$ for the inverted hierarchy has an absolute lower bound $\simeq 0.05 \times \cos 2\theta_{12}$ eV, and stringent experimental upper bounds on $\langle m \rangle_{\beta\beta}$ can exclude the hierarchy. Second, constraints on the lightest neutrino mass $m_l$ was extracted by using a $0\nu\beta\beta$ result. If the energy scales of a result is large enough compared to $\sqrt{|\Delta m^2_{23}|}$, the constraints are $\langle m \rangle_{\beta\beta}^{\text{min}} \lesssim m_l \lesssim \langle m \rangle_{\beta\beta}^{\text{max}} / (0.9 \cos 2\theta_{12})$. Furthermore, considering the expected sensitivity limit on $m_l$ of the future $^3H$ beta decay experiment and assuming a positive result, we uncovered a possibility of rejecting the Majorana neutrino hypothesis. It is necessary for the rejection that $0\nu\beta\beta$ experiments show $\langle m \rangle_{\beta\beta} < 0.05$ eV. Next, it was shown that a bound on a Majorana phase $\beta$ can be placed if a positive result of $^3H$ beta decay experiments is obtained as $m_l > \langle m \rangle_{\beta\beta}^{\text{max}}$.

Finally, I would like to emphasize the importance of precise determination of the nuclear matrix elements $M^{0\nu}$. They are crucial to obtain stringent constraints on neutrino properties in a manner fully utilizing the accuracy of $0\nu\beta\beta$ experiments.

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