Manifestations of a spatial variation of fundamental constants in atomic and nuclear clocks, Oklo, meteorites, and cosmological phenomena

J. C. Berengut and V. V. Flambaum

School of Physics, University of New South Wales - Sydney 2052, Australia

received 7 November 2011; accepted in final form 8 December 2011
published online 18 January 2012

PACS 06.20.Jr - Determination of fundamental constants
PACS 24.30.-v - Resonance reactions
PACS 06.30.Ft - Time and frequency

Abstract – The recent indications of a spatial variation in the fine-structure constant \( \alpha \) from quasar absorption systems (Webb J. K. et al., Phys. Rev. Lett., 107 (2011) 191101) must be independently confirmed by complementary searches. In this letter, we discuss how the spatial variation observed by astronomers can be tested using terrestrial measurements of time variation of the fundamental constants in the laboratory, nuclear decay in meteorites, and analysis of the Oklo natural nuclear reactor. Furthermore, we show that a spatial variation of the fundamental constants may be observable as spatial anisotropy in the cosmic microwave background, the accelerated expansion (dark energy), and large-scale structure of the Universe.

Introduction. – A very large study of quasar absorption systems has recently provided evidence for a spatial variation in the fine-structure constant, \( \alpha = e^2/\hbar c \) [1,2]. The general idea is to compare the wavelengths of atomic spectra measured in the laboratory with those seen in absorption systems at high redshift. Any variation in the value of \( \alpha \) results in well-understood discrepancies between the two spectra. Systematics are controlled to a very high degree by using many atoms and ions [3,4].

Previous studies of quasar absorption spectra had suggested that the fine-structure constant may have been smaller in the past [5–8]. However, these studies all used spectra taken at the Keck telescope in Hawaii, at a latitude of 20°N. Similar studies using the Very Large Telescope in Chile (latitude 25°S) at first showed a stringent null constraint [9]. More careful analysis of the same sample suggested that the errors should be enlarged by a factor of six [10–12], and that a much larger, dedicated VLT survey be performed.

The most recent work, which makes use of both Keck and VLT data, shows a highly significant (~4σ) spatial gradient in the value of \( \alpha \) [1]. That is, \( \alpha \) was larger in the past in one direction and smaller in the past in the opposite direction. This dipole, which we will refer to as the “Australian dipole”, has a declination of around −60°. This explains why the previous studies suggested a time-varying \( \alpha \) that was smaller in the past: they only used data from the Keck telescope, which sees mainly in the Northern Hemisphere. The new results are entirely consistent with the previous ones. The dipole direction and magnitude obtained from the Keck and VLT subsamples are consistent, as are directions obtained from high-redshift and low-redshift quasar subsets (these subsets use different atomic species).

Discovery of a spatial variation in the fundamental constants of nature would have massive implications for the “fine-tuning” problem. This is the question of why the constants of nature seem to be finely tuned for life to exist. While the anthropic principle can be invoked to explain such tuning of the Universe, there remains the question of how it could come about. With the detection of a spatial variation of constants we begin to have a natural explanation for fine tuning: with many possibilities for combinations of constants all occurring within the (possibly infinite in extent) Universe, we simply appear in the part of the Universe that is consistent with our existence.

Extraordinary claims require extraordinary evidence, and the observation of a spatial gradient in \( \alpha \) is no exception. In this letter we discuss how limits from atomic and nuclear clock measurements, meteorite data, and the Oklo nuclear reactor may be interpreted in the
light of the evidence for spatial variation. We show how these independent methods could be used to confirm or contradict the dipole model of $\alpha$-variation that the quasar data suggest. We also briefly discuss the possibility that a spatial variation may be observed via asymmetries in the expansion of the Universe, the cosmic microwave background, and the large-scale structure.

**Model.** – The “Australian dipole” of $\alpha$-variation found by [1] is

$$\frac{\delta\alpha}{\alpha_0} = (1.10 \pm 0.25) \times 10^{-6} r \cos \psi \text{ Gly}^{-1}, \quad (1)$$

where $\delta\alpha/\alpha_0 = (\alpha(r) - \alpha_0)/\alpha_0$ is the relative variation of $\alpha$ at a particular place $r$ in the Universe (relative to Earth at $r = 0$). The function $r \cos \psi$ describes the geometry of the spatial variation: $\psi$ is the angle between the direction of the measurement and the axis of the Australian dipole, $(17.4 \pm 0.9) h, -58(9) ^\circ$ in equatorial coordinates. The distance function is the light-travel distance $r = ct$ measured in giga-lightyears. This is model dependent for large redshifts: we use the standard $\Lambda$CDM cosmology parametrized by WMAP5 [13] to determine the light-travel time $t$. It is assumed here that $\delta\alpha/\alpha_0 = 0$ at zero redshift, which is supported by the data; however this assumption should be tested using the same absorption methods as are used at high redshift (e.g., by using absorbers within our own galaxy).

Our goal is to provide a simple interpretation of terrestrial measurements of the variation of constants in terms of a spatial gradient in values of $\alpha$. A minimal hypothesis is to expect all fundamental constants to vary in the same direction. There are some good theoretical justifications for this postulation. For instance, the constants may vary because they are coupled to a (dimensionless) scalar field $\Phi$ which varies over space-time: such a field could be the quintessence field $\Phi/e^\psi$ (see, e.g., [14]) or a dimensionless dilaton field (see, e.g., [15]; more information and references to these ideas may be found in the review [16]). In this case the axis of the dipole is the direction of its gradient $\nabla\Phi$, and a fundamental constant $X$ is coupled to its variation via

$$\frac{\delta X}{X_0} = k_X \delta \Phi, \quad (2)$$

where $k_X$ is a dimensionless coupling coefficient. Our dipole model now requires $\delta \Phi(r) \sim r \cos \psi$ but all constants will vary in the same direction. The evidence for the Australian dipole could be confirmed by independently finding variation of any fundamental constant along its axis.

In this letter we will deal with the constants $\alpha = e^2/hc$, the electron-to-proton mass ratio $\mu = m_e/m_p$, and the dimensionless mass ratio $X_q = m_q e^2/m_m$ where $m_q$ is the light-current quark mass and $\Lambda_{QCD}$ is the position of the Landau pole in the logarithm of the running strong-coupling constant, $\alpha_s \sim 1/\ln(\Lambda_{QCD} r/hc)$. In the Standard Model the electron and quark masses are proportional to the vacuum expectation of the Higgs field, while the proton mass $m_p$ is proportional to $\Lambda_{QCD}$ (if we neglect the $\sim 10\%$ contribution of the quark masses). The relative variation of $X_q$ is then approximately equal to the relative variation of $m_q/m_p$ and $\mu = m_e/m_p$. We can relate the relative variation of different constants by equations like

$$k_{\mu} = R_{\mu}^X k_{\alpha}, \quad (3)$$

where the $R_X^{\mu}$ can be determined from observations and compared with theories of spatial variation. Note that no summation is implied by (3) which simply defines a ratio, $R_{\mu}^X$, of dimensionless coupling constants $k_X$ (2).

**Atomic and nuclear clocks.** – Laboratory-based atomic clocks provide measurements of the time variation of $\alpha$ in the Earth frame. Since the Earth is moving with respect to the rest frame of the cosmic microwave background (CMB), and this motion will have a component along the Australian dipole, we may expect $\alpha$ to vary in our frame. The assumption here is that the dipole is stationary in the CMB rest frame, which is to be expected if the variation is caused by a co-moving scalar field, for example. To start, we neglect the annual motion of the Earth around the Sun, which is averaged out over time. The velocity of the Sun in the CMB rest frame is known to high accuracy from the CMB dipole itself, and is $369 \text{ km s}^{-1}$ in the direction $(168^\circ, -7^\circ)$ [13]. This is almost perpendicular to the direction of the Australian dipole, and is therefore sensitive to the exact angle, which has some uncertainty. We can expect the yearly variation in laboratory measurements to be

$$\dot{\alpha}/\alpha|_{\text{lab}} = 1.35 \times 10^{-18} \cos \psi \text{ y}^{-1}, \quad (4)$$

where $\psi$ is the angle between the motion of the Sun and the dipole. The best fit value for the Australian dipole gives $\cos \psi \sim 0.07$, but this has an uncertainty $\sim 0.1$. This signal will be modulated by the annual motion of the Earth around the Sun; with the angle between the ecliptic plane and the Australian dipole taken to be $35^\circ$, this modulation is

$$\frac{\delta\alpha}{\alpha} = 1.4 \times 10^{-20} \cos \omega t \quad (5)$$

where $\omega$ refers to the angular frequency of the yearly orbit and $\delta\alpha/\alpha$ is maximal (cos $\omega t = 1$) on around 15 June each year. This modulation can be used to show that a drift in $\alpha$ such as (4) is indicative of spatial, rather than temporal, variation.

Equation (4) gives a useful benchmark for comparing laboratory measurements to the recent spatial $\alpha$-variation data. The current best limit on the rate of $\alpha$-variation in laboratory measurements is $\dot{\alpha}/\alpha = (-1.6 \pm 2.3) \times 10^{-17} \text{ y}^{-1}$, obtained by comparison of Hg$^+$ and A$^+$ clocks over the course of a year [17] using the proposal and calculations of [3,4]. Therefore, this limit will have to be improved by two orders of magnitude to compete with the astronomical spatial-variation data. Fortunately,
atomic clocks are improving rapidly, and there are several new schemes that could allow measurement at this level of precision, including the proposed $^{229}$Th nuclear clock [18,19], highly charged ion clocks [20,21], and using the accidental degeneracy in atomic dysprosium [4,22].

To avoid misunderstanding, we note that the astronomical observations do not exclude temporal variation of $\alpha$ below the rate $10^{-16}$ per year. That is, when considering temporal variations, the laboratory observations are already competitive at their present level of accuracy.

**Oklo.** – The Oklo natural nuclear reactor is well known to give limits on variation of the fine-structure constant over the period since the reactor was running ($\sim 1.8$ billion years) [23,24]. We can get an estimate for the kind of

\[
\frac{\delta \lambda / \lambda}{10^{-9}} \times 0.01 \text{Gy} = 10 \frac{\delta X_q}{X_q} - \frac{\delta \alpha}{\alpha} \times 10 \text{meV}, \quad (6)
\]

we expect that $E_r$ would have been different at the time the Oklo reactor was running by

\[
\delta E_r \mid_{\text{Oklo}} \approx 1 \text{meV} - R_q^{\alpha} 10 \text{meV}, \quad (7)
\]

where $R_q^{\alpha}$ is defined by $k_q = R_q^{\alpha} k_{\alpha}$ as in eq. (3). The Sm resonance has an order of magnitude more sensitivity to the variation in the dimensionless light-quark mass $X_q$ than it does to $\alpha$. The current best $2\sigma$ limits from Oklo data are $-12 < \delta E_r < 26 \text{meV}$ [27] and $-73 < \delta E_r < 62 \text{meV}$ [28].

**Meteorites.** – Meteorites can be used to determine average rates of decay of long-lived isotopes over the last 4.6 gigayears, which can be compared with laboratory rates to test for changes in fundamental constants [29,30]. The energy difference in the $\beta$-decay of $^{187}$Re to $^{187}$Os is very small, $\omega = 2.66 \text{keV}$. It is the result of cancellation between the relatively large Coulomb and asymmetry energy differences (which are $\sim 20 \text{MeV}$ [29,31]) of the two nuclei, and so it is very sensitive to the possible variation of constants.

We define our dimensionless observable by $y = \lambda_{R_e}/\lambda_U$: the ratio of the decay rate of the $^{187}$Re isotope to that of a relatively insensitive isotope, such as uranium. The relative variation of $y$ is $\delta y / y = \delta \lambda_{R_e} / \lambda_{R_e} - \delta \lambda_{U} / \lambda_U$. Since $\lambda_{R_e} \ll \lambda_{U}$, the uranium isotope serves to calibrate the time during which the meteorite was formed. Therefore, taking $\lambda_U$ as fixed, we can express the sensitivity of $\lambda_{Re}$ to the variation of $\alpha$ as $\delta \lambda / \lambda = K_\alpha \delta \alpha / \alpha$, with $K_\alpha \approx 2 \times 10^4$ [31,32].

The decay rate determined from the Re-Os isochrons [33] is an average value over the time since the meteorites were formed,

\[
\bar{\lambda} = \frac{1}{t_0 - t_1} \int_{t_1}^{t_0} \lambda(t) \, dt, \quad (8)
\]

where $t_1$ and $t_0$ are the meteorite time and present time, respectively. As the solar system moves through the CMB rest frame along the Australian dipole, we expect the fine-structure constant to vary as (1) with $r \cos \psi = vt = 4.3 \times 10^{-4} t$, where $t$ and $r$ are measured in Gy. As in the Oklo case, this movement is that of the galaxy as a whole. Then

\[
\frac{\delta \lambda(t)}{\lambda} = 4.8 \times 10^{-10} K_\alpha t \quad (9)
\]

and integrating (8) with $t_0 - t_1 = 4.6 \text{Gy}$ gives

\[
\bar{\lambda} = \lambda_0 \left(1 - 1.1 \times 10^{-9} K_\alpha \right), \quad (10)
\]

where $\lambda_0 = \lambda(t_0)$. Therefore, the relative difference between the decay rates measured from meteorites and that measured in the lab is expected to be at the level

\[
\frac{\bar{\lambda} - \lambda_0}{\lambda_0} = -2.2 \times 10^{-5}. \quad (11)
\]

Unfortunately, this is far below the current accuracy, $\sim 10^{-2}$, which comes from $\bar{\lambda} = 1.666 \times 10^{-11} \text{y}^{-1}$ [33] and $\lambda_0 = 1.685 \times 10^{-11} \text{y}^{-1}$ [34] (see also [35]). Both of these measurements will need to improve significantly before the expected range of variation is reached.

**Cosmological implications.** – Variations of the fundamental constants will lead to changes in the masses and binding energies of elementary particles, including leptons and baryons as well as dark-matter particles. If one assumes that cosmological conservation laws apply in a flat Universe, then

\[
\Omega_{\text{baryons}} + \Omega_{\text{dark matter}} + \Omega_{\text{radiation}} + \Omega_{\text{dark energy}} = 1,
\]

and there must be an exchange between the light- and dark-matter energy densities and the “dark energy”.  

20006-p3
Therefore, a cosmological anisotropy in the fundamental constants will result in an anisotropic energy exchange between these contributions to the total energy density of the Universe.

The contribution of baryonic matter to gravitational processes will be approximately proportional to the dimensionless constant $Gm_b^2/hc^2 \sim g_{QCD}^2/hc$, where $G$ is Newton's gravitational constant. In the absence of a confirmed theory for dark matter, we may write a similar expression for the dark-matter contribution, $Gm_{DM}^2/hc$, and a cross-term $Gm_pm_{DM}/hc$, which depend on an unknown mass $m_{DM}$. The effect of the variation of these contributions to the energy density of the Universe may be related to varying $G$ theories.

We see that the observed $\alpha$-variation is related to mass density and the cosmological “constant”, and hence the accelerated expansion of the Universe. Therefore, it may be possible to see a spatial anisotropy in the redshift-luminosity relationships of SNIa supernovae data. Also, the properties of the supernovae themselves depend on fundamental constants; this effect may contribute to any observable signal. Additionally the temperature anisotropy of the cosmic microwave background and large-scale structure formation may have observable anisotropies.

**Conclusion.** – We have shown how laboratory experiments of time variation of fundamental constants and limits from the Oklo nuclear reactor and meteorites can be compared to the spatial variations seen by astronomers. This interpretation is essential if one wishes to independently corroborate the spatial variation seen by astronomers. None of the current terrestrial limits contradict the reported observation of a spatial gradient in $\alpha$.

We have also tested other astrophysical systems where one could find a spatial variation of fundamental constants [2]. Briefly, the existing $H_2$ data from quasar absorption spectra shows hints that there may be a dipole in $\mu$-variation with an axis corresponding to the $\alpha$-variation of ref. [1]. On the other hand the variation of $x = \alpha^2 \mu_g_\rho$, inferred from 21 cm data, has a best-fit dipole whose axis does not correspond to that of the Australian dipole, although in this case systematics heavily dominate. It is also possible to infer a spatial variation of fundamental constants during big-bang nucleosynthesis from high-redshift measurements of primordial abundances. The existing deuterium data does not support the dipole interpretation statistically, but the preferred axis is aligned with the Australian dipole. There is a strong impetus now to perform measurements of relative primordial abundance at high redshifts of as many elements as possible in as many different spatial directions as possible.

Finally we note that it may be possible to observe a spatial dipole in other cosmological systems (CMB, supernovae, and large-scale structure). Although the results of [1] (interpreted as strictly spatial variation) suggest that accuracy at the level $10^{-5}$ will be required, if the hints from big-bang nucleosynthesis turn out to be real, then there is an additional redshift (time)-dependence that could substantially increase the variation at the time of the CMB.

***

We thank G. t’Hooft, J. A. King, J. K. Webb, M. T. Murphy, and D. Budker for useful discussions. This work is supported by the Australian Research Council, Marsden grant, and ECT*.

REFERENCES

[1] Webb J. K., King J. A., Murphy M. T., Flambaum V. V., Carswell R. F. and Bainbridge M. B., Phys. Rev. Lett., 107 (2011) 191101.
[2] Berengut J. C., Flambaum V. V., King J. A., Curran S. J. and Webb J. K., Phys. Rev. D, 83 (2011) 123506.
[3] Dzuba V. A., Flambaum V. V. and Webb J. K., Phys. Rev. Lett., 82 (1999) 888.
[4] Dzuba V. A., Flambaum V. V. and Webb J. K., Phys. Rev. A, 59 (1999) 230.
[5] Webb J. K., Flambaum V. V., Churchill C. W., Drinkwater M. J. and Barrow J. D., Phys. Rev. Lett., 82 (1999) 884.
[6] Murphy M. T., Webb J. K., Flambaum V. V., Dzuba V. A., Churchill C. W., Prochaska J. X., Barrow J. D. and Wolfe A. M., Mon. Not. R. Astron. Soc., 327 (2001) 1208.
[7] Murphy M. T., Webb J. K. and Flambaum V. V., Mon. Not. R. Astron. Soc., 345 (2003) 609.
[8] Murphy M. T., Flambaum V. V., Webb J. K., Dzuba V. A., Prochaska J. X. and Wolfe A. M., Lect. Notes Phys., 648 (2004) 131.
[9] Srianand R., Chand H., Petitjean P. and Aracil B., Phys. Rev. Lett., 92 (2004) 121302.
[10] Murphy M. T., Webb J. K. and Flambaum V. V., Phys. Rev. Lett., 99 (2007) 239001.
[11] Murphy M. T., Webb J. K. and Flambaum V. V., Mon. Not. R. Astron. Soc., 384 (2008) 1053.
[12] Srianand R., Chand H., Petitjean P. and Aracil B., Phys. Rev. Lett., 99 (2007) 239002.
[13] Hinshaw G., Weiland J. L., Hill R. S., Odegard N., Larson D., Bennett C. L., Dunkley J., Gold B., Greason M. R., Jarosik N., Komatsu E., Nolta M. R., Page L., Spergel D. N., Wollack E., Halpern M., Kogut A., Limon M., Meyer S. S., Tucker G. S., Weiland J. L., Wright E. L., Astrophys. J. Suppl. Ser., 180 (2009) 225.
[14] Wetterich C., JCAP, 10 (2003) 002.
[15] Damour T. and Polyakov A. M., Nucl. Phys. B, 423 (1994) 532.
[16] Úzan J.-P., Living Rev. Relativ., 14 (2011) 2.
[17] Rosenband T., Hume D. B., Schmidt P. O., Chou C. W., Brusch A., Lorini L., Oskay W. H., Druyllinger R. E., Fortier T. M., Stalnaker J. E., Diddams S. A., Swann W. C., Newbury N. R., Itano W. M., Wineland D. J. and Bergquist J. C., Science, 319 (2008) 1808.
Terrestrial manifestations of a spatial $\alpha$-variation

[18] Peik E. and Tammi Chr., Europhys. Lett., 61 (2003) 181.
[19] Flambaum V. V., Phys. Rev. Lett., 97 (2006) 092502.
[20] Berengut J. C., Dzuba V. A. and Flambaum V. V., Phys. Rev. Lett., 105 (2010) 120801.
[21] Berengut J. C., Dzuba V. A., Flambaum V. V. and Ong A., Phys. Rev. Lett., 106 (2011) 210802.
[22] Dzuba V. A. and Flambaum V. V., Phys. Rev. A, 77 (2008) 012514.
[23] Shlyakhter A. I., Nature, 264 (1976) 340.
[24] Damour T. and Dyson F., Nucl. Phys. B, 480 (1996) 37.
[25] Kogut A., Lineweaver C., Smoot G. F., Bennett C. L., Banday A., Boggess N. W., Cheng E. S., Amici G. D., Fixsen D. J., Hinshaw G., Jackson P. D., Janssen M., Keegstra P., Loewenstein K., Lubin P., Mather J. C., Tenorio L., Weiss R., Wilkinson D. T. and Wright E. L., Astrophys. J., 419 (1993) 1.
[26] Flambaum V. V. and Wiringa R. B., Phys. Rev. C, 79 (2009) 034302.
[27] Gould C. R., Sharapov E. I. and Lamoreaux S. K., Phys. Rev. C, 74 (2006) 024607.
[28] Petrov Y. V., Nazarov A. I., Onegin M. S., Petrov V. Y. and Sakhinovsky E. G., Phys. Rev. C, 74 (2006) 064610.
[29] Dyson F. J., Phys. Rev. Lett., 19 (1967) 1291.
[30] Fuji Y. and Iwamoto A., Phys. Rev. Lett., 91 (2003) 261101.
[31] Olive K. A., Pospelov M., Qian Y.-Z., Coc A., Cassé M. and Vangioni-Flam E., Phys. Rev. D, 66 (2002) 045022.
[32] Dent T., Stern S. and Wetterich C., Phys. Rev. D, 78 (2008) 103518.
[33] Smoliar M. I., Walker R. J. and Morgan J. W., Science, 271 (1996) 1099.
[34] Galeazzi M., Fontanelli F., Gatti F. and Vitale S., Phys. Rev. C, 63 (2000) 014302.
[35] Lindner M., Leich D. A., Russ G. P., Bazan J. M. and Borg R. J., Geochim. Cosmochim. Acta, 53 (1989) 1597.