Hidden Neutrino Gauge Symmetry

HoSeong La*

Department of Physics and Astronomy,
Vanderbilt University,
Nashville, TN 37235, USA

A new gauge force that acts exclusively on neutrinos is proposed. This new force violates neutrino flavors while masses are diagonal, potentially opening a door for a new field theoretical treatment of the neutrino oscillation. The basic idea and a framework are presented.

*hsla.avt@gmail.com
Nonzero neutrino masses clearly indicate the existence of physics beyond the Standard Model (SM). The oscillation data imply neutrino masses are of $\mathcal{O}(1)$ eV or less and non-degenerate\[1]. There are various attempts of extending the SM to accommodate the current experimental data (see some recent reviews [2, 3, 4] among many others), but it is safe to say that none has been definitively successful due to lack of an explicit neutrino flavor violating structure\[1]. So, there is a room for another proposal, which is drastically different from others. In this Letter, we will briefly lay out the basic idea and a framework to extend the SM with a new gauge force exclusively acting on neutrinos. More details about the neutrino physics in this context will be reported elsewhere in the future.

The basic ingredients we need for potentially successful neutrino physics, which can explain the neutrino oscillation phenomenon field theoretically\[6, 7, 8, 9\], are that the new gauge force violates neutrino flavors and neutrino masses are non-degenerate\[2\]. We assume the new gauge force is abelian, $U(1)_\nu$, which is spontaneously broken at some energy scale above the electroweak (EW) scale. As a typical consequence, this spontaneous symmetry breaking generates a mass for $U(1)_\nu$ gauge boson, but the pure massless neutrino sector still reveals the gauge symmetry. So, we can still take advantage of the gauge invariance in the pure neutrino sector. Then a flavor-violating Lagrangian for the pure neutrino sector can be constructed as

$$\mathcal{L}_\nu = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i \sum_{i=1}^{N} \bar{\psi}_i \gamma^\mu (\partial_\mu + ig_i A_\mu) \psi_i - \sum_{i,j=1}^{N} \alpha_{ij} A_\mu \bar{\psi}_i \gamma^\mu \psi_j - \sum_{i=1}^{N} m_i \bar{\psi}_i \psi_i,$$  \hspace{1cm} (1)

where $\alpha_{ij} = \alpha_{ji}$, $\alpha_{ii} = 0$, and $\psi_i$ are neutrino mass eigenstates such that $m_i$ is the tree-level physical masses. $g_i$’s are preferably the same, but we reserve the possibility of different $g_i$’s. We assume that tree-level masses are generated by a mechanism outside the pure neutrino sector such that they are free parameters here. Looking at this, one may hastily conclude that, even in the massless case, this Lagrangian is not $U(1)_\nu$ gauge invariant because of the mixing terms. However, being mass eigenstates does not guarantee they are also $U(1)_\nu$ charge eigenstates so that one cannot just gauge transform field variables in eq.(1).

In terms of a proper orthogonal transformation $\tilde{\psi}_i = O_{ij} \psi_j$, where $O^T O = 1$ for $O = (O_{ij})$, we can diagonalize the gauge coupling terms as

$$\mathcal{L}_\nu = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \sum_{i=1}^{N} \bar{\tilde{\psi}}_i \gamma^\mu (\partial_\mu + i\tilde{g}_i A_\mu) \tilde{\psi}_i - \sum_{i,j=1}^{N} \tilde{m}_{ij} \bar{\tilde{\psi}}_i \tilde{\psi}_j,$$  \hspace{1cm} (2)

\[1\] See [5] for an earlier example of the neutrino flavor violation.
\[2\] Even if one starts with tree-level massless neutrinos, the degeneracy has to be broken to generate the neutrino oscillation[10].

1
where $\tilde{\psi}_i$ are now $U(1)_\nu$ charge eigenstates and

$$
(g_i \delta_{ij} + \alpha_{ij}) = O^T \text{diag}(\tilde{g}_i) O,
$$

(3a)

$$
\text{diag}(m_i) = O^T (\tilde{m}_{ij}) O.
$$

(3b)

The specific values of $O_{ij}$ can be easily derived from the above equations.

Eq.(2) is manifestly gauge invariant if the mass term is diagonal. However, since the Weak interaction violates $U(1)_\nu$ charge conservation so that the mass generating mechanism can break $U(1)_\nu$ as well, the requirement of gauge invariance of mass term can be relaxed. Our intention is that, eq.(2) has the gauge coupling terms diagonal in the charge eigenstates, while eq.(1) has mass terms diagonal in the mass eigenstates, but both cannot be diagonal at the same time. Note that $U(1)_\nu$ charges are not quantized and different neutrinos can carry different amount of charges. Since neutrino flavor eigenstates and mass eigenstates are related by a unitary mixing matrix, $\tilde{\psi}_i$ are not necessarily the same as the flavor eigenstates. So, eq.(1) implies neutrino flavor violation.

To demonstrate the idea, let us first examine the $N = 2$ case. Nevertheless, it should reveal some of characteristics for the three neutrino case. Solving eqs.(3a-3b), we can obtain

$$
\alpha_{12} = \frac{\tilde{m}_{12}}{\tilde{m}} (\tilde{g}_1 - \tilde{g}_2),
$$

(4)

where $\tilde{m} \equiv \sqrt{(\tilde{m}_{11} - \tilde{m}_{22})^2 + 4\tilde{m}_{12}^2}$. This tells us that the flavor-violating coupling constants depend on the neutrino masses and that, to have flavor-violating interactions, the off-diagonal mass term for charge eigenstates must not vanish and different charge eigenstates must have different $U(1)_\nu$ charges. The diagonalized physical neutrino masses are given by

$$
m_1 = \frac{1}{2}(\tilde{m}_{11} + \tilde{m}_{22} + \tilde{m}),
$$

(5a)

$$
m_2 = \frac{1}{2}(\tilde{m}_{11} + \tilde{m}_{22} - \tilde{m}).
$$

(5b)

This does not really tell us about any pattern of neutrino masses even if we assume off-diagonal mass is much smaller than the diagonal ones, and there is no other theoretical constraint we can impose (at least in the $N = 2$ case).

However, if we extend the gauge symmetry, we can demand the gauge invariance of the off-diagonal mass terms under $U(1)_\nu$ to obtain an extra constraint. Consider $U(1)_\nu \times U(1)' = U(1)_1 \times U(1)_2$ for the pure neutrino sector, where $U(1)'$, which is broken only by non-degenerate neutrino masses, is a source of mixing.
In terms of charge eigenstates under general $U(1)_1 \times U(1)_2$, $\mathcal{L}_\nu$ can be written in a manifestly gauge invariant form as, for $N=2$,

$$\mathcal{L}_\nu = -\frac{1}{4} \sum_{i=1}^{N} F_{\mu\nu}^{(i)} F^{(i)\mu\nu} + i \sum_{i=1}^{N} \bar{\psi}_i \gamma^\mu \left( \partial_\mu + i \tilde{g}_i A_\mu^{(i)} \right) \psi_i - \sum_{i=1}^{N} \bar{\psi}_i m_{ij} \psi_j. \quad (6)$$

Note that the mass terms are not diagonal and the off-diagonal parts are not invariant under $U(1)_1 \times U(1)_2$. But they are invariant under the symmetric combination of $U(1)_1 \times U(1)_2$, which we identify as $U(1)_\nu$ with a gauge field $A_\mu$. Then

$$\left( \begin{array}{cc} g A_\mu & g' A'_\mu \\ g' A'_\mu & g A_\mu \end{array} \right) \equiv O^T \left( \begin{array}{cc} \tilde{g}_1 A_\mu^{(1)} & 0 \\ 0 & \tilde{g}_2 A_\mu^{(2)} \end{array} \right) O \quad (7)$$

with

$$O = \frac{1}{\sqrt{2}} \left( \begin{array}{cc} 1 & 1 \\ 1 & -1 \end{array} \right). \quad (8)$$

This implies that $\tilde{m}_{11} = \tilde{m}_{22}$ for $\tilde{m}_{12} \neq 0$ to have diagonalized masses for mass eigenstates. Now eq.(6) becomes

$$\mathcal{L}_\nu = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F_{\mu\nu}^' F'^{\mu\nu} + i \sum_{i=1}^{2} \bar{\psi}_i \gamma^\mu \left( \partial_\mu + i g A_\mu \right) \psi_i - g A'_\mu (\bar{\psi}_1 \gamma^\mu \psi_2 + \bar{\psi}_2 \gamma^\mu \psi_1) - \sum_{i=1}^{2} m_i \bar{\psi}_i \psi_i, \quad (9)$$

where, from the gauge kinetic energy terms, we obtain $g = g'$ and $\tilde{g}_1 = \tilde{g}_2 = \sqrt{2} g$. Eq.(9) is invariant under $U(1)_\nu$, while $U(1)'$ gauge field violates the flavor. Once the gauge symmetry is extended, we can obtain flavor violating interaction even if $\tilde{g}_1 = \tilde{g}_2$. In this case, physical masses are $m_1 = \tilde{m}_{11} + \tilde{m}_{12}$, $m_2 = \tilde{m}_{11} - \tilde{m}_{12}$. So, one can see that the compatibility of eq.(1) and eq.(2) (or eq.(6) and eq.(9)) can provide nontrivial constraints on the properties of neutrinos.

In the $N=3$ case we further assume that the mixing between the first and third neutrinos are secondary so that it can be smaller than others, hence $\alpha_{13}$ should be smaller compared to others. But for the purpose of explicit examples, we will assume $\alpha_{13} = 0$. Otherwise, it becomes rather cumbersome. We also assume $g_i$'s are the same. Then from eq.(3a) we obtain

$$\tilde{O} = \frac{1}{\sqrt{2}} \left( \begin{array}{ccc} \cos \xi & -\sqrt{2} \sin \xi & \cos \xi \\ -1 & 0 & 1 \\ \sin \xi & \sqrt{2} \cos \xi & \sin \xi \end{array} \right) = O^T \quad (10)$$

where the latter equality is up to an over-all sign, $\tan \xi \equiv \alpha_{23}/\alpha_{12}$, and

$$\tilde{g}_1 = g - \sqrt{\alpha_{12}^2 + \alpha_{23}^2}, \quad \tilde{g}_2 = g, \quad \tilde{g}_3 = g + \sqrt{\alpha_{12}^2 + \alpha_{23}^2}. \quad (11)$$
such that $\alpha_{12}^2 + \alpha_{23}^2 = (\tilde{g}_1 - \tilde{g}_3)^2$. Applying this to eqs.(3a-3b), we obtain $\tan \xi = \tilde{m}_{23}/\tilde{m}_{12} = 1$ and that $\tilde{m}_{ii}$'s must be identical to be consistent, provided $\tilde{m}_{13} = 0$. The diagonal masses are given by

$$m_1 = \tilde{m}_{11} - \sqrt{\tilde{m}_{12}^2 + \tilde{m}_{23}^2}, \quad m_2 = \tilde{m}_{11}, \quad m_3 = \tilde{m}_{11} + \sqrt{\tilde{m}_{12}^2 + \tilde{m}_{23}^2},$$

(12)
such that

$$m_2 = \frac{1}{2}(m_1 + m_3).$$

(13)

In this case, the flavor-violating prefactors are

$$\alpha_{12} = \alpha_{23} = \frac{1}{2\sqrt{2}}(\tilde{g}_3 - \tilde{g}_1)$$

(14)

and coupling constants

$$g_1 = g_3 = \tilde{g}_2, \quad g_2 = \frac{1}{2}(\tilde{g}_1 + \tilde{g}_3) = \tilde{g}_2,$$

(15)

where we have imposed $\alpha_{13} = \tilde{g}_1 \cos^2 \xi + \tilde{g}_3 \sin^2 \xi - \tilde{g}_2 \simeq 0$.

If we extend the gauge symmetry to $U(1)^3$, twisting gauge fields becomes

$$O^T \begin{pmatrix} \tilde{g}_1 A^{(1)}_\mu & 0 & 0 \\ 0 & \tilde{g}_2 A^{(2)}_\mu & 0 \\ 0 & 0 & \tilde{g}_3 A^{(3)}_\mu \end{pmatrix} O = \begin{pmatrix} gA_\mu & g'A'_\mu & B_\mu \\ g'A'_\mu & g''A''_\mu & g'A'_\mu \\ B_\mu & g'A'_\mu & gA_\mu \end{pmatrix},$$

(16)

where

$$gA_\mu \equiv \frac{1}{4} \left( \tilde{g}_1 A^{(1)}_\mu + \tilde{g}_3 A^{(3)}_\mu + 2\tilde{g}_2 A^{(2)}_\mu \right),$$

(17a)

$$g'A'_\mu \equiv \frac{1}{2} \left( \tilde{g}_1 A^{(1)}_\mu - \tilde{g}_3 A^{(3)}_\mu \right),$$

(17b)

$$g''A''_\mu \equiv \frac{1}{2} \left( \tilde{g}_1 A^{(1)}_\mu + \tilde{g}_3 A^{(3)}_\mu \right),$$

(17c)

$$B_\mu \equiv g''A''_\mu - gA_\mu.$$  

(17d)

In the above we have used the constraints due to the diagonalization of kinetic energy term such that $\tan \xi = 1$, i.e. $\tilde{m}_{12} = \tilde{m}_{23}$, and that $g = \tilde{g}_2$, $\tilde{g}_1 = \tilde{g}_3$, $g' = \tilde{g}_1/2$, and $1/g'' = 1/g_2 + 2/g_1$. Since we would like to have all diagonal components of the rhs of eq.(16) the same, which sets $g''A''_\mu = gA_\mu$, i.e. $B_\mu = 0$, equivalently $\alpha_{13} \simeq 0$. So, we can identify $U(1)_\nu$ with a gauge field given by eq.(17a).

Note that to make $\tilde{m}_{ij}\bar{\psi}_i\psi_j$, $(i \neq j)$, gauge invariant under $U(1)^3$, $U(1)_\nu$ must be identified as $A_\mu$ given by eq.(17a) and for that $\tilde{m}_{ii}$'s must be the same. So, with one assumption $\tilde{m}_{13} = 0$, identical $\tilde{m}_{ii}$'s is necessary and sufficient condition for the flavor-violating lagrangian to be fully
gauge invariant even with the mass terms. In this sense, $m_2 = (m_1 + m_3)/2$ is a non-trivial outcome.

In both case, this is possible if and only if

$$O = \frac{1}{2} \begin{pmatrix} 1 & -\sqrt{2} & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ 1 & \sqrt{2} & 1 \end{pmatrix} = \tilde{O}^T.$$  \hspace{1cm} (18)

Once extra $U(1)_\nu$ is introduced, we need to worry about new anomalies. We assume both chiralities of neutrinos carry $U(1)_\nu$ charges so that $U(1)_\nu$ is vector-like. The only new anomalies we need to worry about are those involving $U(1)_\nu$, as shown in Figs.1-2. (Fig.3 vanishes because neutrinos are nonchiral under $U(1)_\nu$.) Fig.2 cannot vanish in the SM, so $U(1)_\nu$ has to be a broken symmetry in the SM energy scale and one needs to extend beyond the SM at the energy scale where $U(1)_\nu$ is unbroken. The cancellation of this anomaly requires left-handed neutrinos with opposite (hyper)charge, or the right-handed neutrinos need to carry (hyper)charges. The latter can be easily achieved by extending the SM to the left-right symmetric model\cite{11, 12, 13}, then all anomalies cancel regardless of $\alpha_{ij}$. If the gauge symmetry is extended, since each $U(1)$ acts on one generation of leptons only, the same argument works.

Since the Weak interaction violates conservation of $U(1)_\nu$ charge, this also indicates that $U(1)_\nu$ must be broken at some scale above the EW scale. Furthermore, for $U(1)_\nu$ to couple to neutrinos but not to charged leptons, we need a larger unbroken gauge group structure. For example, the unbroken symmetry could be vector-like $SU(2)'_V \times U(1)_X$ acting only on lepton doublets with $X$-charge $1/2$, while quarks are neutral under them, and $U(1)_\nu$ is the diagonal combination such that $Q_\nu = I_{V3} + X$. The fact that $SU(2)'_V$ does not act on quarks distinguishes it from the vector part of $SU(2)_L \times SU(2)_R$. $U(1)_\nu$ coupling to lepton doublets can be achieved in terms of an isospin projection operator $I^+ \equiv \frac{1}{2}(1 + I_3)$ and $U(1)_\nu$ gauge boson (or $SU(2)_{L,R}$ gauge bosons) transforms covariantly under $SU(2)_{L,R}$ (or $U(1)_\nu$, respectively) as
vector matter[14]. This property is inherited from the equivalent relationship between SU(2)′_V and SU(2)_L,R for SU(2)′_V × SU(2)_L × SU(2)_R gauge symmetry. U(1)_ν symmetry must be broken before the breaking of the left-right symmetry. The neutrino masses generated by the seesaw mechanism in [13] are non-degenerate and can be easily accommodated into the framework presented here.

Although we have not performed detailed analysis of the system provided for neutrino mixing and oscillations yet, there are rather desirable results.

In the example we presented for the three neutrino flavor case, eq.(10) with tan ξ = 1 and identical ˜m_{ii}’s are the compatibility conditions between eq.(1) with identical g_i’s and eq.(2) (or eq.(6) and eq.(9) extended for three flavors) so that eq.(12) is in some sense not entirely arbitrary. The only assumption we make is ˜m_{13} = 0, then the equality of ˜m_{ii}’s follows. Perhaps, a full analysis with ˜m_{13} ≠ 0 may reveal some incompatibility between eq.(1) and eq.(2). With extended gauge symmetry to U(1)^3, the equality of ˜m_{ii}’s is better clarified at the expense of having more symmetries. Anyhow, if m_2 = (m_1 + m_3)/2 survives at least approximately after all higher order radiative corrections, neutrino masses can be estimated as |m_1| ≃ 0.016 eV, |m_2| ≃ 0.018 eV, and |m_3| ≃ 0.051 eV, based on the current experimental data[1]. This is an interesting result, but we would rather not call it a prediction at this moment because the uniqueness is not clear and it will obviously change in the case of α_{13} ≠ 0, although it is possible α_{13} could be negligible as indicated below eq.(17d).

Our model as it is has an unfamiliar structure because U(1)_ν only acts on neutrinos and the isospin doublet structure is not respected in the way we assign U(1)_ν charges. Interestingly enough, U(1)_ν charge behaves totally opposite to the electric charges as far as leptons are concerned. To make it compatible we have to assume SU(2)′_V × U(1)_X, whose charged gauge bosons carry both U(1)_ν and U(1)_{EM} charges (with the same sign) at low energy. It will be interesting to ask if there is any direct way of checking the existence of U(1)_ν. For example, charged leptons as well as the SM gauge bosons will have SU(2)′_V × U(1)_X interactions at high energy well above the EW symmetry breaking. For another example, a small directional variation of neutrino signals could indicate any deflection of neutrinos due to the emission of or scattering with U(1)_ν gauge bosons. The mass of U(1)_ν gauge boson is expected to be very light while the symmetry breaking occurs above the SM scale. Nevertheless its coupling to neutrinos can be significant enough such that the deflection of neutrinos from a faraway source could be “measurable” in principle. So, it will be worth to look into the possibility of unique processes due to U(1)_ν.
We have not analyzed the consequence of neutrino flavor violating terms due to nonvanishing $\alpha_{ij}$, but we expect they will play important roles in mixing and oscillation of neutrinos. The examples we considered have $\alpha_{12}$ and $\alpha_{23}$, but no $\alpha_{13}$. So, the coupling between $\nu_1$ and $\nu_3$ will occur at higher order processes to make their effective couplings smaller than the other two. Since $\alpha_{ij}$'s can be different in principle, different amounts of neutrino species will come out from the same neutrino source and this process can be alternating to lead to the neutrino oscillation. In addition to neutrino masses, the whole process can be adjusted with parameters, $g, \alpha_{ij}$, which could account for the three degrees of freedom of mixing angles.

I thank Tom Weiler for many illuminating conversations and for reading the manuscript. I also thank Stanley Deser and Jose Valle for helpful communications.

References

[1] K. Nakamura and S.T. Petcov, in Particle Data Group, Phys. Rev. D 86, 010001 (2012) (http://pdg.lbl.gov/2012/reviews/rpp2012-rev-neutrino-mixing.pdf) and references therein.
[2] G. Altarelli, arXiv:1111.6421 [hep-ph].
[3] S. Morisi and J. W. F. Valle, arXiv:1206.6678 [hep-ph].
[4] P. Langacker, arXiv:1112.5992 [hep-ph].
[5] J. Schechter and J. W. F. Valle, Phys. Rev. D 25, 774 (1982).
[6] C. Giunti, C. W. Kim, J. A. Lee and U. W. Lee, Phys. Rev. D 48, 4310 (1993) [hep-ph/9305276].
[7] M. Beuthe, Phys. Rept. 375, 105 (2003) [hep-ph/0109119].
[8] E. K. Akhmedov and J. Kopp, JHEP 1004, 008 (2010) [arXiv:1001.4815 [hep-ph]].
[9] J. Wu, J. A. Hutasoit, D. Boyanovsky and R. Holman, Int. J. Mod. Phys. A 26, 5261 (2011) [arXiv:1002.2649 [hep-ph]].
[10] J. W. F. Valle, Phys. Lett. B 199, 432 (1987).
[11] J. Pati and A. Salam, Phys. Rev. D 10 (1974) 275.
[12] R.N. Mohapatra and J.C. Pati, Phys. Rev. D 11 (1975) 566; G. Senjanović and R.N. Mohapatra, Phys Rev. D 12 (1975) 1502.
[13] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
[14] H. S. La, hep-ph/0306223.