Time and space dependent backgrounds from nonsupersymmetric strings

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Abstract

We investigate maximally symmetric backgrounds in nonsupersymmetric string vacua with D-branes and O-planes localized in the compact space. We find a class of solutions with a perturbative string coupling constant in all regions of spacetime. Depending on the particular model, we find either a time evolution with a big-bang type singularity or a space dependent background with generically orbifold singularities. We show that the result can be interpreted as a supersymmetric bulk with some symmetries broken by the boundaries. We also discuss an interesting connection to Lorentzian and Euclidian orbifolds.
1. Introduction and summary of results

String theory provides a natural setting to address cosmological issues like the cosmological constant problem or the fate of the big-bang singularity. Whereas the first problem still awaits for a qualitatively different perspective, the second one led over the last ten years to more explicit proposals like the pre-big bang model [1], the ekpyrotic scenario [2] or brane-world models [3]. In the second model, objects with negative tensions were important for its realisation. In addition to the positive tension branes, orientifolds allow also for negative tension objects: the O-planes. This renders them candidates for rich possibilities of cosmological backgrounds. The goal of this paper is to study these possibilities.

The string models we consider are vacua with D-branes and orientifold planes, [4, 5] and with broken supersymmetry. Orientifold models with D-branes/O-planes with broken supersymmetry in various sectors of the theory were proposed in the last few years [6, 7, 8, 9]. The classical background of such vacua has typically lower symmetry and was explicitly worked out in some particular examples [10]. An obvious and important worry about such constructions is the issue of classical and quantum stability and their fate. The fact that some of these constructions [8, 9] are tachyon-free in all moduli space of the theory is a promising feature, but gives only a partial answer to the stability question. On the other hand, it is clear that typically, as soon as supersymmetry is broken, D-branes/O-planes start to interact, curve the internal or the noncompact space and generically produce a time-dependent configuration.

The explicit models we consider are nine dimensional string models. At the effective field theory level, supersymmetry breaking is described by non-BPS configurations of D-branes and O-planes, as well as a one-loop bulk cosmological constant.

The maximally symmetric classical background of these models generically depends on two coordinates, which, according to the details of the models, can be $(t, y)$ or $(z, y)$, where $t$ is the time, $y$ is the compact coordinate orthogonal to the branes and $z$ is a noncompact coordinate parallel to the branes. We concentrate on solutions which are perturbative in the string coupling constant so that the classical solution receives small quantum corrections. In the time dependent case, we show by an adequate choice of coordinates that one of the solutions has a static bulk and hence has as much symmetries as the supersymmetric bulk, but the boundaries are moving and thus break the invariance under time translation. The solutions are characterized by a big bang (or big crunch) singularity which is due to the collision of the two boundaries. Interestingly, this solution can be interpreted also as an orbifold by a boost of a static and supersymmetric background. The boost parameter on the Lorentzian side is related to the branes and O-planes and the one-loop cosmological constant. We also show that in models with NS-NS tadpoles, the usual NS-NS tadpole condition is replaced by a sum rule, which is basically the boosted version of the static tadpole condition. In the space dependent
case, the boundaries join at a conical singularity and also break some of the bulk symmetries.
We show that this background can be considered as an orbifold by a two dimensional rotation of the supersymmetric one.

One application of our work is to the big bang type cosmology. In this respect, our results have similar features to the pre-big bang [1] and ekpyrotic [2] scenarios. Another possible application is to the Fischler-Susskind mechanism. The relation we find between the classical solutions of nonsupersymmetric orientifolds and lorentzian orbifolds suggests a deepest relation at the quantum level. In this respect, a severe instability of lorentzian orbifolds was recently pointed in [11] along with some possible ways out [12, 11]. Irrespective of the final fate of lorentzian orbifolds, we believe that connections between seemingly unrelated vacua can be useful for a better understanding of perturbative and nonperturbative aspects of string theory. We hope to come back to this issue in the future.

The paper is organized as follows. Section 2 describes the various classical and perturbative solutions for string models with D-branes and O-planes in nine dimensions and their relation to Lorentzian and Euclidian orbifolds. Section 3 discusses some of their applications to the big bang and big crunch cosmology, whereas Section 4 presents some explicit string examples. Appendix A contains the technical details involved in the resolution of the equations of motion and a more complete set of various classical solutions, including the ones with strong string coupling in some regions of spacetime. Appendix B gives some details on the explicit string models under consideration.

2. Time and space-dependent backgrounds of nonsupersymmetric strings

We consider a generic form of the effective action of the Type II string containing D8 branes (and eventually O8 planes in the case of orientifold models). We include also the bulk one-loop cosmological constant $\Lambda_1$ which typically arise in most nonsupersymmetric strings. The resulting effective action in the string frame reads

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-G} \left[ e^{-2\Phi} \left( R - \frac{1}{2} (\partial \Phi)^2 \right) \right] - \frac{1}{2 \times 10!} F_{10}^2 - 2\kappa^2 \Lambda_1$$

$$- \int_{y=0} d^9x (T_0 \sqrt{-\gamma} e^{-\Phi} + q_0 A_9) - \int_{y=\pi R} d^9x (T_1 \sqrt{-\gamma} e^{-\Phi} + q_1 A_9), \quad (1)$$

where $\Phi$ is the dilaton, $A_9$ the RR nine-form coupling to D8 branes and O8 planes and $\gamma$ is the induced metric. For simplicity of the discussion we placed all branes either at the origin $y = 0$ or at $y = \pi R$ of a compact coordinate $y$ of radius $R$. Since supersymmetry is broken, we do not assume any relation between the RR charges $q_i$ and the NS tensions $T_i$.

We consider in the following string vacua of the type [3, 4], where the RR charge is globally cancelled, as required by consistency arguments, but the NS-NS tadpole conditions are violated.
and there is an induced one-loop cosmological constant

\[ q_0 + q_1 = 0 \quad \text{(RR tadpole conditions)}, \]
\[ T_0 + T_1 \neq 0 \quad \text{(uncancelled NS – NS tadpoles)}, \]
\[ \Lambda_1 \neq 0. \quad (2) \]

The classical field equations have no solution with $SO(9)$ symmetry, in agreement with various arguments presented in the literature \([10]\). We search here for solutions depending on the compact coordinate $y$ and on another coordinate, which can be the time $t$ or another space coordinate $z$. We restrict ourselves in that section to solutions which are smoothly connected to the supersymmetric ones \([19]\) and have a perturbative string coupling throughout the spacetime. We leave to the Appendix A the explicit derivation of the complete set of classical solutions.

### 2.1. Cosmological solutions

The general form of such a solution is of the form

\[
\begin{align*}
    ds^2 &= e^{2A(t,y)} \left( 1 + \frac{kx^2}{4} \right) - 2 \delta_{\mu\nu} dx^\mu dx^\nu + e^{2B(t,y)} (-dt^2 + dy^2), \\
    F_{10} &= f(t,y) \epsilon_{10}, \quad \Phi = \Phi(t,y),
\end{align*}
\]

where $\epsilon_{10}$ is the ten-dimensional volume form. The eight-dimensional metric at fixed $y$ and $t$ is a maximally symmetric one: $k = 0$ for a flat eight dimensional space, $k = 1$ for a closed 8-sphere and $k = -1$ for an open 8-hyperboloid.

The equations of motion of the nine-form can be readily solved and the solution, in the Einstein frame, reads

\[ f = -q_0 \kappa^2 e^{5\Phi/2} \epsilon(y), \quad (4) \]

where $\epsilon(y)$ is an odd $2\pi R$-periodic function and $\epsilon(y) = 1$ when $y$ is between 0 and $\pi R$. Since most of our results ask for the existence of negative tension objects, we mostly refer in the following to orientifold models \([4]\) and consequently we choose the ten-form to be odd under $y \rightarrow -y$. Since the nine-form potential has no physical degrees of freedom, we replace it in the Lagrangian by its classical expression \([12]\). As a result, we get an effective one loop cosmological constant given by

\[ \Lambda_e = \Lambda_1 + \frac{q_0^2}{4} \kappa^2. \quad (5) \]

We define for later convenience $\beta_0 = -T_0 \kappa^2$ and $\beta_1 = T_1 \kappa^2$. The Einstein and dilaton equations for the remaining functions are given by\(^2\)

\[
-\Phi_{tt} + \Phi_{yy} - 8A_t \Phi_t + 8A_y \Phi_y - 5\kappa^2 \Lambda_e e^{2B + 5\Phi/2} = -\frac{5}{2} S,
\]

\(^2\)The notations we use are $\Phi_{tt} = \partial^2 \Phi / \partial t^2$, etc.
\[ -7A_{tt} + 7A_{yy} - 28A_t^2 + 28A_y^2 - B_{tt} + B_{yy} = \frac{\Phi_t^2}{4} + \frac{\Phi_y^2}{4} - 21ke^{2B-2A} + \kappa^2\Lambda_ke^{2B+5\Phi/2} = S, (7) \]

\[ -8A_{tt} - 36A_t^2 + 28A_y^2 + 8A_tB_t + 8A_yB_y - \frac{\Phi_t^2}{4} - \frac{\Phi_y^2}{4} - 28ke^{2B-2A} + \kappa^2\Lambda_ke^{2B+5\Phi/2} = 0, \quad (8) \]

\[ -8A_{yy} + 28A_t^2 - 36A_y^2 + 8A_tB_t + 8A_yB_y - \frac{\Phi_t^2}{4} - \frac{\Phi_y^2}{4} + 28ke^{2B-2A} - \kappa^2\Lambda_ke^{2B+5\Phi/2} = -S, \quad (9) \]

\[ A_{ty} + A_tA_y - A_tB_y - A_yB_t + \frac{\Phi_t\Phi_y}{16} = 0, \quad (10) \]

where we have defined

\[ S = e^{5\Phi/4+B} [\beta_0\delta(y) - \beta_1\delta(y - \pi R)]. \quad (11) \]

All the functions \( A, B \) and \( \Phi \) are even in \( y \) and \( 2\pi R \)-periodic. The sources in the right-hand side of the equations determine the \( y \) derivatives of these functions at 0 and \( \pi R \) as

\[ A_y(0^+,t) = \frac{\beta_0}{16} e^{B(0,t)+5\Phi(0,t)/4}, \quad A_y(\pi R^-,t) = \frac{\beta_1}{16} e^{B(\pi R,t)+5\Phi(\pi R,t)/4}, \quad (12) \]

and

\[ B_y(0^+,t) = A_y(0^+,t), \quad \Phi_y(0^+,t) = -20A_y(0^+,t), \quad B_y(\pi R^-,t) = A_y(\pi R^-,t), \quad \Phi_y(\pi R^-,t) = -20A_y(\pi R^-,t). \quad (13) \]

The boundary conditions (13) imply the relations:

\[ \Phi(y,t) = -20A(y,t) + \phi(t), \quad B(y,t) = A(y,t) + b(t), \quad (14) \]

where \( \phi \) and \( b \) are for the time being arbitrary functions of time. The relations (14) have the virtue of reducing the boundary conditions to (12). Using the relations (14) in equation (10), one gets

\[ [e^{24A-b-5\phi/4}]_{yt} = 0, \quad (15) \]

which is readily solved by

\[ e^{24A-b-5\phi/4} = F(y) + G(t). \quad (16) \]

The relations (14) and (16) allow to transform the partial differential equations (8-9) into ordinary differential equations for \( b, \phi, F \) and \( G \). The boundary conditions (12) translate into

\[ F'(0^+) = \frac{3\beta_0}{2}, \quad F'((\pi R^-)) = \frac{3\beta_1}{2}. \quad (17) \]

The equations resulting from the substitution in (8-9) of the relations (14) and (16) are discussed in some details in Appendix A. We show there that for \( k = 0 \) it is possible to solve exactly and to find all the solutions of the equations of motion. There are two classes of solutions. The first one, denoted \( a \) in the Appendix A, is characterized by

\[ \dot{\phi} = 0, \quad \dot{b}^2 = \lambda^2, \quad \dot{G}b + \lambda^2G = 0, \quad \dot{F}^2 - \lambda^2F^2 = 9\kappa^2\Lambda_e, \quad (18, 19) \]
where \( \lambda \) is a positive constant. In (17), (18) and (19) the prime (the dot) denote differentiation with respect to \( y \) \((t)\). There is a second class of solutions, called \( b \) in the Appendix, which are characterized by \( G = 0 \). It is shown in the Appendix that the solutions in the first class are the only ones with a perturbative value of the string coupling in the whole spacetime. In addition, they are the only ones which are smoothly connected to the supersymmetric solutions [19]. Notice the remarkable fact that eqs. (18) and (19) are of first order, even though they are not derived from BPS-type conditions.

The boundary conditions are not always compatible with equations (19). Evaluating (19) at the origin and at \( \pi R \) we get the two conditions

\[
T_0^2 \geq q_0^2 + 4 \frac{\Lambda_1}{\kappa^2} , \quad T_1^2 \geq q_1^2 + 4 \frac{\Lambda_1}{\kappa^2} .
\]

(20)

These conditions are necessary but, as we will see, not sufficient to insure the existence of a solution in this class. Before analysing in detail the solutions for the different models, notice that if the effective cosmological constant \( \Lambda_e \) is negative eqs. (20) are automatically satisfied. This is to be compared to the sum rules [15] where staticity and compactness impose severe restrictions (or fine tuning) on the tensions.

The SUSY configuration is a special case of the above system: it corresponds to a vanishing one loop cosmological constant, \( \Lambda_1 = 0 \), and to the equality of the RR charges and NS charges \( q_0 = T_0 = -T_1 \). This implies that \( \lambda = 0, F' \) is constant and the space is flat, \( k = 0 \).

If neither of the two conditions (20) are satisfied then the only solutions for the background of the form (3) have singularities in the string coupling and are displayed for completeness in the Appendix. However, a non-singular solution with the same number of isometries exists, it amounts to interchange the time coordinate with one of the eight coordinates \( x \), called \( z \) later on. If one the conditions (20) is true but not the other, then one has to look for a solution with lower symmetries. There are three qualitatively distinct cases to consider, depending on the value of the effective one-loop cosmological constant (5):

- **i)** If \( \Lambda_e > 0 \), then the solution has the form

\[
F(y) = \frac{3 \kappa \sqrt{\Lambda_e}}{\lambda} \sinh(\lambda |y| + \omega) ,
\]

(21)

where the boundary conditions (17) determine the parameters \( \lambda \) and \( \omega \)

\[
\text{ch}(\omega) = -T_0 \kappa/(2 \sqrt{\Lambda_e}) , \quad \text{ch}(\pi \lambda R + \omega) = T_1 \kappa/(2 \sqrt{\Lambda_e}) .
\]

(22)

Notice that (22) can have a solution only if the tensions in \( y = 0 \) and \( y = \pi R \) have opposite signs. Therefore we need objects of negative tension in the theory, which in our explicit string examples later on are orientifold planes.

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3There is also the solution \( F \to -F \) in (21). This is equivalent, however, to a reflection \( y \to -y + \pi R \) which exchanges the two fixed points, accompanied by the replacement \( \omega \to -\omega - \pi \lambda R \).
The final solutions of the classical field equations, in the Einstein frame, read

\[
e^{24A} = e^{b_0 + 5\phi_0/4} \left[ G_0 + \frac{3\kappa \sqrt{\Lambda_e}}{\lambda} e^{\lambda t} sh(\lambda |y| + \omega) \right],
\]

\[
e^{24B} = e^{24\lambda t + 25b_0 + 5\phi_0/4} \left[ G_0 + \frac{3\kappa \sqrt{\Lambda_e}}{\lambda} e^{\lambda t} sh(\lambda |y| + \omega) \right],
\]

\[
e^{\Phi} = e^{-5b_0/6 - \phi_0/24} \left[ G_0 + \frac{3\kappa \sqrt{\Lambda_e}}{\lambda} e^{\lambda t} sh(\lambda |y| + \omega) \right]^{-\frac{3}{2}}.
\]  

(23)

We did carefully keep track in (23) of the integration constants \( \phi_0, b_0 \) and \( G_0 \). Notice that for \( G_0 < 0 \), there are singularities in the \((t, y)\) plane. We restrict in this section to safer values \( G_0 \geq 0 \). Notice that for \( T \_1 < |T_0| \), we get \( \omega < 0 \) and therefore there are singularities if \( G_0 + (3\kappa \sqrt{\Lambda_e}/\lambda) \exp(\lambda t) \, sh(\omega) \leq 0 \), whereas for \( T \_1 > |T_0| \), \( \omega \) is positive and therefore there are no singularities in the compact space. In the limit of vanishing \( \lambda \) and \( \omega \), we recover the supersymmetric solution [19].

Let us consider in more details the resulting space-time metric and choose for simplicity \( b_0 = 0, \phi_0 = 0 \). We find

\[
ds^2 = \left[ G_0 + \frac{3\kappa \sqrt{\Lambda_e}}{\lambda} e^{\lambda t} sh(\lambda |y| + \omega) \right]^{\frac{1}{12}} \left[ \delta_{\mu \nu} dx^\mu dx^\nu + e^{2\lambda t} (-dt^2 + dy^2) \right].
\]  

(24)

Finally, by making the change of variables

\[
T = \frac{1}{\lambda} e^{\lambda t} \, ch(\lambda y + \omega), \quad X = \frac{1}{\lambda} e^{\lambda t} \, sh(\lambda y + \omega),
\]  

(25)

we get the spacetime metric

\[
ds^2 = \left[ G_0 + 3\kappa \sqrt{\Lambda_e} X \right]^{\frac{1}{12}} \left[ \delta_{\mu \nu} dx^\mu dx^\nu - dT^2 + dX^2 \right],
\]  

(26)

when \( y > 0 \). The \( Z_2 \) identification \( y \rightarrow -y \) is mapped in terms of the coordinates \((X, T)\) to a parity \( \Pi_X \) times a boost \( K \) with a parameter \( 2\omega \). This means that the orientifold operation acts in the \((T, X)\) plane as

\[
\Omega' = \Omega \, \Pi_X \, K_{2\omega}.
\]  

(27)

In addition, the identification of points on the circle \( y = y + 2\pi R \) results in \((T, X)\) coordinates in the orbifold identification

\[
\begin{pmatrix} T \\ X \end{pmatrix} \rightarrow \begin{pmatrix} ch(2\pi \lambda R) & sh(2\pi \lambda R) \\ sh(2\pi \lambda R) & ch(2\pi \lambda R) \end{pmatrix} \begin{pmatrix} T \\ X \end{pmatrix},
\]  

(28)

which is nothing but a two-dimensional boost \( K_{2\pi \lambda R} \) with a velocity \( v = \text{th}(2\pi \lambda R) \) in the \((T, X)\) space.
The final result (26) is quite surprising. Indeed, (26) coincides with the spacetime metric obtained in [19] in the supersymmetric Type I' string with N D8 branes at the origin $X = 0$ of a compact coordinate of radius $R$ and $32 - N$ D8 branes at $X = \pi R$! The supersymmetric Polchinski-Witten solution and our non-supersymmetric solution appear to be two different orbifolds of the same ten-dimensional background. More precisely, the SUSY solution uses the translation group, while the non-SUSY one uses the two-dimensional Lorentz group.

The metric (26) and the identifications (27), (28) allow a simple physical interpretation of our configuration in the $(X, T)$ coordinates. Indeed, the fixed points of the two orientifold operations are

$$
\Omega' : X = \th \omega T
$$

$$
\Omega' \mathcal{K}_{2\pi\lambda R} : X = \th (\pi \lambda R + \omega) T .
$$

(29)

Consequently, the negative tension O-planes and the branes located at the origin move with a constant velocity

$$
v_0 = \th \omega ,
$$

(30)

in the static background (24), whereas the positive tension branes and O-planes at $y = \pi R$ move at a constant velocity

$$
v_1 = \th (\pi \lambda R + \omega) .
$$

(31)

Moreover, the boundary conditions (22) encode the dynamics of the two boundaries in the condition

$$
T_0 \sqrt{1 - v_0^2} + T_1 \sqrt{1 - v_1^2} = 0 .
$$

(32)

The interpretation of (32) is quite simple. In the supersymmetric case, the branes and O-planes are at rest and (32) reduces to the standard NS-NS tadpole condition $T_0 + T_1 = 0$. In the case with broken supersymmetry, the NS-NS tadpoles are “boosted” according to the velocity of the branes and O-planes in the background (26). The boost is the one appropriate for a lagrangian density, since the dilaton field in (1) couples to the lagrangian, instead of the energy.

An interesting particular example of the above results is the one in which all global tadpoles are cancelled and we have a negative bulk cosmological constant

$$
T_0 + T_1 = 0 , \quad \Lambda_1 < 0 .
$$

(33)

In this case it turns out that (22) is satisfied for

$$
\omega = - \frac{\pi \lambda R}{2} , \quad \ch \left( \frac{\pi \lambda R}{2} \right) = \frac{T_1 \kappa}{2 \sqrt{\Lambda_e}} ,
$$

$$
v_0 = - \th \left( \frac{\pi \lambda R}{2} \right) , \quad v_1 = \th \left( \frac{\pi \lambda R}{2} \right) .
$$

(34)
Therefore even in the absence of disk NS-NS tadpoles, one-loop cosmological constant is sufficient to generate a constant velocity dynamics, which can be interpreted in terms of orientifolds of a Lorentzian orbifold. Notice, however, that in this case the metric and the dilaton can be singular between the O-planes. From this perspective, models with NS-NS tadpoles seem to be the only ones free of singularities in the compact space.

• ii) If $\Lambda_e < 0$, then the conditions (21) are verified and the solution for $F$ is given by

$$F(y) = \pm \frac{3\sqrt{-\Lambda_e}}{\lambda} \text{ch}(\lambda|y| + \omega) ,$$

where $\omega$ and $\lambda$ are determined by

$$sh(\omega) = \mp T_0 \kappa / (2\sqrt{-\Lambda_e}) , \quad sh(\pi \lambda R + \omega) = \pm T_1 \kappa / (2\sqrt{-\Lambda_e}) .$$

The solution of the classical field equations reads

$$e^{24A} = e^{b_0 + 5\phi_0 / 4} \left[ G_0 \pm \frac{3\sqrt{-\Lambda_e}}{\lambda} e^{\lambda t} \text{ch}(\lambda|y| + \omega) \right] ,$$

$$e^{24B} = e^{24\lambda t + 25b_0 + 5\phi_0 / 4} \left[ G_0 \pm \frac{3\sqrt{-\Lambda_e}}{\lambda} e^{\lambda t} \text{ch}(\lambda|y| + \omega) \right] ,$$

$$e^\Phi = e^{-5b_0 / 6 - \phi_0 / 24} \left[ G_0 \pm \frac{3\sqrt{-\Lambda_e}}{\lambda} e^{\lambda t} \text{ch}(\lambda|y| + \omega) \right]^{-\frac{5}{6}} .$$

By the same change of variables (25) and by setting for simplicity $b_0 = \phi_0 = 0$, we get for $y > 0$ the spacetime metric

$$ds^2 = \left[ G_0 \pm \frac{3\sqrt{-\Lambda_e}}{\lambda} \right]^{\frac{1}{12}} \left[ \delta_{\mu\nu} dx^\mu dx^\nu - dT^2 + dX^2 \right] .$$

As in the case i), the $Z_2$ identification $y \rightarrow -y$ is mapped in terms of the coordinates $(X, T)$ to a parity times a boost with a parameter $2\omega$, while the identification of points on the circle $y = y + 2\pi R$ gives for the $(T, X)$ coordinates the identification (28). The boundary conditions (36) in this case imply the following condition on the spacetime boundary velocities

$$\frac{T_0}{v_0} \sqrt{1 - v_0^2} + \frac{T_1}{v_1} \sqrt{1 - v_1^2} = 0 .$$

Notice that in this case it is possible to obtain solutions to (39) with tensions of the same sign in the two fixed points, provided that the two velocities (30), (31) have opposite signs.

• iii) If $\Lambda_e = 0$, then there are time-dependent solutions provided that the tensions at $y = 0$ and $y = \pi R$ have opposite signs. The final form of the solution is

$$ds^2 = \left[ G_0 + F_0 e^{\lambda (t \pm |y|)} \right]^{\frac{1}{12}} \left[ \delta_{\mu\nu} dx^\mu dx^\nu + e^{2(\lambda t + b_0)} (-dt^2 + dy^2) \right] ,$$

$$e^\Phi = e^{\phi_0} \left[ G_0 + F_0 e^{\lambda (t \pm |y|)} \right]^{-\frac{5}{6}} .$$
where $F_0$ is a constant and the parameter $\lambda$ in (25) is determined by the condition

$$e^{\pi \lambda R} = -\frac{T_1}{T_0}.$$  \hspace{1cm} (41)

In this case, by introducing the coordinates (25) with $\omega = 0$, we find the spacetime metric

$$ds^2 = \left[ G_0 + \lambda F_0 X^\pm \right]^{1/2} \left[ \delta_{\mu\nu} dx^\mu dx^\nu - dT^2 + dX^2 \right],$$  \hspace{1cm} (42)

where we introduced the light-cone coordinates $X^\pm = T \pm X$. Notice from (41) that by taking $T_0$ very small and negative $T_0 \to 0^-$, we can generate an infinite boost parameter $\lambda$.

2.2. Static solutions

One may wonder whether there are similar solutions which do not depend on time but on an additional space coordinate $z$, that is with the ansatz

$$ds^2 = e^{2A} g_{\mu\nu} dx^\mu dx^\nu + e^{2B} (dz^2 + dy^2),$$  \hspace{1cm} (43)

where $g_{\mu\nu}$ is the eight dimensional flat, dS or AdS metric and with $A$, $B$ and $\Phi$ functions of $y$ and $z$. Here also we have two classes of solutions. The first one is also characterized by a finite string coupling and is smoothly connected to the supersymmetric solution [19]. In this case, a similar analysis as before shows that the eight-dimensional metric $g_{\mu\nu}$ is flat and, with the replacement $t \to z$ equations (16), (17) and (18) hold true. The nontrivial modification is a crucial sign in equation (19) which becomes

$$F'^2 + \lambda^2 F^2 = 9\kappa^2 \Lambda_e.$$  \hspace{1cm} (44)

If we allow for a negative effective one loop cosmological constant, we see at once that (44) cannot be satisfied and the time dependent solutions are the only ones. Evaluating (44) at the origin and at $\pi R$ we get the two conditions

$$T_0^2 \leq q_0^2 + 4\frac{\Lambda_1}{\kappa^2}, \quad T_1^2 \leq q_1^2 + 4\frac{\Lambda_1}{\kappa^2}.$$  \hspace{1cm} (45)

Unlike in (20), these conditions are necessary and sufficient to insure the existence of a solution. The explicit form of the solution in this case is

$$F(y) = \frac{3\kappa\sqrt{\Lambda_e}}{\lambda} \sin(\lambda |y| + \omega),$$  \hspace{1cm} (46)

where the boundary conditions (17) determine the parameters $\lambda$ and $\omega$

$$\cos(\omega) = -T_0 \kappa/(2\sqrt{\Lambda_e}), \quad \cos(\pi \lambda R + \omega) = T_1 \kappa/(2\sqrt{\Lambda_e}).$$  \hspace{1cm} (47)
The final solutions of the classical field equations, in the Einstein frame, read

\[ e^{24A} = e^{b_0 + 5\phi_0/4} \left[ G_0 + \frac{3\kappa \sqrt{\Lambda_e}}{\lambda} e^{\lambda z} \sin(\lambda |y| + \omega) \right], \]

\[ e^{24B} = e^{24\lambda z + 25b_0 + 5\phi_0/4} \left[ G_0 + \frac{3\kappa \sqrt{\Lambda_e}}{\lambda} e^{\lambda z} \sin(\lambda |y| + \omega) \right], \]

\[ e^{\Phi} = e^{-5b_0/6 - \phi_0/24} \left[ G_0 + \frac{3\kappa \sqrt{\Lambda_e}}{\lambda} e^{\lambda z} \sin(\lambda |y| + \omega) \right]^{-\frac{5}{6}}. \] (48)

This solution is continuously connected to the supersymmetric solution [19] in the limit of vanishing \( \lambda \) and \( \omega \).

The analog of the equation (32) in this case is

\[ \frac{T_0}{\cos \omega} + \frac{T_1}{\cos(\lambda \pi R + \omega)} = 0. \] (49)

Notice that, contrary to the previous time-dependent case (32), eq. (49) does not give any restriction on the tensions.

The \( z \) coordinate is noncompact and the Planck mass in this background is infinite. There are singularities for \( z = \infty \) (or \( z = -\infty \) for the negative branch solution \( z \to -z \)). Depending on the sign of \( G_0 \) and the numerical values of \( \lambda \) and \( \omega \), this solutions can also have singularities at a finite distance from the origin in the \((z, y)\) plane.

Interestingly, this solution can also be related to the supersymmetric solution [19]. Indeed, by the change of coordinates

\[ Y = \frac{1}{\lambda} e^{\lambda z} \sin(\lambda y + \omega), \quad Z = \frac{1}{\lambda} e^{\lambda z} \cos(\lambda y + \omega), \] (50)

we get for \( y > 0 \) the spacetime metric

\[ ds^2 = \left[ G_0 + 3\kappa \sqrt{\Lambda_e} Y \right]^{\frac{1}{12}} \left[ \eta_{\mu\nu} dx^\mu dx^\nu + dY^2 + dZ^2 \right], \] (51)

which is the one derived by Polchinski and Witten [19], except that here the \( Y \) coordinate is noncompact. The periodicity \( y = y + 2\pi R \) reflects in the new coordinate system \((Z, Y)\) in the orbifold identification

\[ \begin{pmatrix} Z \\ Y \end{pmatrix} \to \begin{pmatrix} \cos(2\pi \lambda R) & -\sin(2\pi \lambda R) \\ \sin(2\pi \lambda R) & \cos(2\pi \lambda R) \end{pmatrix} \begin{pmatrix} Z \\ Y \end{pmatrix}, \] (52)

which is nothing but a two-dimensional rotation \( \mathcal{R}_\theta \) in the \((Z, Y)\) space, with an angle \( \theta = 2\pi \lambda R \).

The orientifold identification \( y \to -y \) is mapped in terms of the coordinates \((Z, Y)\) to a parity \( \Pi_Y \) times a rotation \( \mathcal{R}_{2\omega} \) with an angle \( 2\omega \)

\[ \Omega' = \Omega \Pi_Y \mathcal{R}_{2\omega}. \] (53)
The metric (48) and the orbifold and orientifold operations (52), (53) allow a physical interpretation of our configuration in the \((Z,Y)\) coordinates. Indeed, the fixed points of the two orientifold operations are

\[
\begin{align*}
\Omega' : Y &= \tan \omega Z, \\
\Omega' \mathcal{R}_{2\pi\lambda R} : Y &= \tan(\pi\lambda R + \omega) Z.
\end{align*}
\]

Consequently, the orientifolds and branes located at the origin have an angle \(\theta_0 = \omega\) with respect to the \(Z\) axis, whereas the branes and orientifolds at \(y = \pi R\) are at an angle \(\theta_1 = \pi\lambda R + \omega\). In the new coordinate system, the interpretation of (49) is quite simple. It corresponds to the dilaton NS-NS tadpole condition obtained from the action (1), when we correctly take into account in the Born-Infeld action the different rotation of the two sets of D-branes and O-planes. Notice that the orbifold identification (52) imply that the two-dimensional \((Z,Y)\) plane has singularities. For discrete values of the rotation angle \(\lambda = 1/(NR)\) or \(M/(NR)\) with \(M\) and \(N\) coprime, these singularities are of a conical type and the resulting model before the orientifold operation (54) is the noncompact \(C/Z_N\) orbifold model.

2.3. Freely-acting Lorentzian orbifold models

It was shown in [11] that Lorentzian orbifolds are unstable, since adding sources changes completely the spacetime which collapses into a large black hole. It was also pointed out in [11, 12] that making the orbifold boost to be freely-acting by combining it with a shift in an additional coordinate can cure this problem in some cases. We would like here to point out that such freely-acting boost operations can also be obtained from some specific nonsupersymmetric orientifold models. For definiteness we restrict ourselves again to the case of the two-dimensional boost. Let us start with Type IIB, orbifolded by \((-1)^F\delta\), where \((-1)^F\) is the spacetime fermion number and \(\delta\) a shift acting simultaneously in two compact coordinates \(y_1, y_2\)

\[
\delta y_1 = y_1 + \pi R_1 , \quad \delta y_2 = y_2 + \pi R_2 .
\]

This orbifold breaks completely supersymmetry and generates a negative one-loop bulk cosmological constant \(\Lambda_1\), but has no fixed points. We now orientifold by the operation \(\Omega' = \Omega\Pi_{y_1}\), where \(\Pi_{y_1}\) is a parity in \(y_1\). The fixed points under \(\Omega'\) will generate O8 planes. By consistency, we must introduce D8 branes in the theory. The low-energy effective action is of the form (1).

By the chain of arguments discussed in the previous paragraphs, the classical background of this model is equivalent to a two-dimensional orbifold operation, supplemented by a shift

\[
\begin{align*}
T + X &\rightarrow e^{2\pi\lambda R}(T + X) , \quad T - X \rightarrow e^{-2\pi\lambda R}(T - X) , \\
y_2 &\rightarrow y_2 + \pi R_2 ,
\end{align*}
\]

Perturbative orientifold models of this type can be explicitly constructed [21].
Figure 1: The spacetime diagram of the cosmological solution. In the region $T < 0$ the time reversed evolution is drawn. The spacetime is contained in the cone defined by the two orientifold planes, denoted by dash-dotted lines.

where $(T, X)$ are defined in terms of $(t, y_1)$ as in (25). In addition, the orientifold operation acting in the $(T, X, y_2)$ coordinates includes a boost, $\Omega' = \Omega \Pi_X K_{2\omega}$, analogously to (27).

This type of smoothing of the singularity for the usual Milne space turns out to be not sufficient to cure the instability problem discussed in [11] and presumably also in the context discussed here. However, in other examples, like the “null-brane” orbifold [12], it does help. It would therefore be interesting to find nonsupersymmetric vacua related in the sense described in the previous section to the null brane orbifold [12].

A similar construction can be applied to models with a background depending on two space coordinates (48). This can help in order to give a simple meaning to the case where the parameter $\lambda$ is generic, by combining an irrational angle in the $(Z, Y)$ plane with a translation along an additional circle. This actually defines a Melvin-type model [22]. There seems therefore to be a surprising connection between nonsupersymmetric orientifolds with NS-NS tadpoles and Melvin type string models.

3. Cosmological applications

The spacetime diagram representing the solution (26) is depicted in figure 1. The bulk is the region between the two dashed lines which represents the two moving boundaries. At $T = 0$ there is a big bang type singularity and the two boundaries coincide. Near this region of spacetime the effective field theory approach we are pursuing is not valid due to the higher order $\alpha'$ corrections. Note however that the string coupling is small in this region so that higher genus corrections are expected to be negligible. With the help of (26), the proper distance separating the two fixed points under the orientifold involution is easily calculated and is given in the
Einstein frame by

\[ \frac{8}{25\kappa\sqrt{\Lambda_e}} \left[ (G_0 + 3\kappa\sqrt{\Lambda_e} v_1 T)^{\frac{24}{25}} - (G_0 + 3\kappa\sqrt{\Lambda_e} v_0 T)^{\frac{24}{25}} \right], \quad (57) \]

where we recall that \( v_0, v_1 \) are the velocities of the two boundaries \([30], [31]\).

The distance becomes arbitrarily small for small \( T \) signalling a breakdown of the classical effective description. In the lower part of the diagram in figure 1 we draw the time reversed solution which represent two collapsing boundaries. Assuming, as in \([1]\) and \([2]\) that string corrections allow a smooth transition between the two branches, one suppresses the big bang type singularity and gets an eternal spacetime. This could offer a new perspective on the proposals of \([1], [2]\), since our string model realizes a scenario similar to the one proposed there and hopefully the string corrections are easier to handle.

Since matter and Yang-Mills interactions are confined to the boundaries, the relevant metric to consider, from the point of view of an observer on the branes, is the induced one on the boundaries. Let us for simplicity suppose that \( \omega = 0 \). The metric induced at the origin \( y = 0 = X \) is just a flat metric, and the one induced at \( y = \pi R \) is given by

\[ ds^2 = \left[ G_0 + 3\kappa sh(\pi \lambda R)\sqrt{\Lambda_e} T \right]^{\frac{1}{12}} \delta_{\mu\nu} dx^\mu dx^\nu - d\tilde{T}^2 + dX^2, \quad (58) \]

where we performed the change of coordinates \( \tilde{T} = T/\text{ch}(\pi \lambda R) \). The geometry of spacetime experienced by an observer on the boundaries does not necessarily reveal the spacetime singularities of the full ten dimensional geometry. This is particularly obvious for the induced metric on \( y = 0 \) which is completely flat and does not “see” the singularity at \( X = T = 0 \). On the other boundary, where the metric is (58) the observer experiences an expanding universe with a singularity in the past at \( \tilde{T} = -G_0/(3\kappa sh(\pi \lambda R)\sqrt{\Lambda_e}) \). However, this singularity is just an illusion since the metric looses its validity before that time at \( \tilde{T} = 0 \), where the true singularity takes place.

We now turn to examine the backgrounds of different nonsupersymmetric models.

4. Explicit string examples

4.1. The \( SO(N) \times USp(32 - N) \) model

The model we consider here is an orientifold of the nine dimensional Scherk-Schwarz deformation of Type IIA strings \([17]\). There are several consistent orientifold operations already considered in the literature \([7]\). The example we are studying here is based on the involution \([16]\) \( \Omega' = \Omega(-1)^{F_L}\Pi_y \), where \((-1)^{F_L}\) is the left world-sheet fermion number and \( \Pi_y \) is the parity in a compact coordinate \( y \). The virtue of this projection, similar in spirit with the one used in
Type O orientifolds \cite{18}, is that it eliminates the closed string tachyon present in the Scherk-Schwarz compactification. The fixed points of the orientifold operation are \( y = 0 \), containing an \( O8_+ \) plane with \((-16, -16)\) units of RR and NS-NS charges and \( y = \pi R \) containing an \( O8_- \) plane of charges \((-16, +16)\).

As shown in Appendix B, RR tadpole cancellation in this model asks for a net number of \( N = 32 \) D8 branes placed in fixed positions on the compact coordinate \( y \). A nice feature of this model, which was one original motivation for studying it, is that if all D8 branes are at the origin, the open massless spectrum is supersymmetric with gauge group \( SO(32) \), while if all D8 branes are placed in the other fixed point \( y = \pi R \), the massless open spectrum precisely coincides with the one of the 10d model \cite{8}, with a gauge group \( USp(32) \). Moving continuously the branes from one fixed point to the other interpolates therefore continuously between the supersymmetric \( SO(32) \) and the nonsupersymmetric \( USp(32) \) Type I strings. Of course, this statement is strictly speaking incorrect, since the closed (bulk) sector has softly broken supersymmetry governed by the radius \( 1/R \), but for large enough radius the main supersymmetry breaking appears due to the simultaneous presence of \( D8 \) branes and \( O8_- \) planes, which breaks supersymmetry at string scale, if they are in top of each other, or if not at a scale proportional to the distance between them.

One of the puzzles of this model which motivated our investigation is the interaction pattern between D branes and O planes of this model. In flat space, the \( O8_+ \) plane at \( y = 0 \) and the \( O8_- \) at \( y = \pi R \) repel each other, a rather surprising feature, whereas O-planes and anti O-planes of the same type attract each other. Moreover, the D-branes do not interact with the \( O8_+ \) planes by supersymmetry, whereas they are attracted by the \( O8_- \) planes. Consequently, if we start with an initial configuration of D8 branes on top of \( O8_+ \) planes, with massless supersymmetric spectrum and gauge group \( SO(32) \), the dynamics of the system seems to push the D8 branes to move towards \( y = \pi R \) and the final state of the system is the \( USp(32) \) nonsupersymmetric string.\footnote{In this case, supersymmetry is however nonlinearly realized in the open sector \cite{3}.}

In analogy with Section 2, we place \( N \) D8 branes at \( y = 0 \) and the rest 32-N at \( y = \pi R \). The effective action of the system is (1), with \( \beta_0 = (16 - N)\kappa^2 T_8 \), \( \beta_1 = (48 - N)\kappa^2 T_8 \) and \( q_0 = T_0 \). There is a bulk one-loop cosmological constant, which is small in the large-radius limit \( R >> \sqrt{\alpha'} \), such that the effective cosmological constant is positive \( \Lambda_e > 0 \) and we are in the case i) of section 2. To start with, we neglect the one loop cosmological constant. The equations are readily solved and the solution is (23) with \( \omega = 0 \). In this case, according to (30), the boundary at the origin has zero velocity \( v_0 = 0 \), whereas (32) fixes the boost parameter \( \lambda \) by the equation \( \beta_0 \cdot \text{ch}(\lambda \pi R) = \beta_1 \). This relation shows that solutions with the symmetry displayed in (3) exist provided that the number of branes at the origin is less than 16 \( (\beta_0 > 0) \) and therefore the tension due to the orientifolds and the branes at the origin must be negative.
It turns out that the sign of the one-loop cosmological constant is crucial. Indeed, since $T_0 = q_0$, eq. (19) implies that there are no solutions for a positive one loop cosmological constant, no matter how small it is. Since supersymmetry is broken in the closed sector by a Scherk-Schwarz mechanism, the resulting one-loop cosmological constant is negative and (19) does have solutions.

We leave for future work a complete analysis of the dynamics of a probe brane in this background. Here we just note that, since the bulk is supersymmetric, a particular solution for the position of the test brane is $X = \text{cst.}$. In this case, since the one-loop bulk cosmological constant is small, the velocity of the supersymmetric boundary at $y = 0$ is much smaller than the positive velocity of the nonsupersymmetric boundary at $y = \pi R$. Therefore, for this particular solution the test brane will stay closer to the supersymmetric boundary, despite the naive flat space arguments presented above.

4.2. Other models

The solutions we found in Section 2 can be used for other string models with broken supersymmetry.

An interesting model to analyse is the nine dimensional model with 16 $O_8^+$ planes at the origin and 16 anti-orientifold $\overline{O}_8^+$ planes at the other fixed point $y = \pi R$. In fact, this model is a different orientifold of the same Scherk-Schwarz deformation of the type II string. The model do not need branes for consistency, since the RR charges add up to zero. In addition, the tachyon is not removed by the orientifold projection for any values of the radius. Let us however add an equal number $N$ of brane-antibrane pairs, the branes being placed at the origin and the antibranes at $y = \pi R$, in order to avoid the occurrence of open-string tachyons. For this configuration, we obtain

$$T_0 = (N - 16)T_8, \quad T_1 = (N - 16)T_8,$$

$$q_0 = -(N + 16)T_8, \quad q_1 = (N + 16)T_8.$$

The bulk one-loop cosmological constant in this model is small in the large radius limit and therefore the conditions (45) are satisfied. We obtain therefore solutions of the type (48), with the parameter $\lambda = 1/R$. In the $(Z, Y)$ coordinate system, the orbifold identification (52) becomes trivial and therefore we get a regular two-dimensional noncompact plane.

Since the brane-antibrane pairs are not needed by the consistency of the theory, we can also analyse the particular case $N = 0$. The classical background can be easily deduced from section 2. In fact this case corresponds to $\beta_0 = -\beta_1$ and $\beta_0 > 0$. If we take into account the negative one loop cosmological constant we can get solutions of the form (37), but only provided that the effective cosmological constant $\Lambda_e$ is negative. This corresponds however to a radius $R$ of the order the string scale, where our effective theory analysis is not reliable.
Another model which can be similarly analysed is the T-dual of the \( USp(32) \) nonsupersymmetric string compactified on a circle. The model has 16 \( O8_\perp \) planes at each fixed point and 32 \( \overline{D}8 \) branes which for simplicity can be distributed among the fixed points, \( N \) at \( y = 0 \) and \( 32 - N \) at \( y = \pi R \). Here, the one loop bulk cosmological constant is zero. A quick inspection of (19) supplemented with the boundary conditions (17) shows that there are no solutions for any values of \( N \).

We can also analyse the case of the supersymmetric Type I' string supplemented by a certain number \( N \) of brane-antibrane pairs. In order to avoid open-string tachyons, we place all \( 32 + N \) \( D8 \) branes at the origin and all \( N \overline{D}8 \) antibranes at \( y = \pi R \). In this case, we have

\[
T_0 = (N + 16)T_8, \quad T_1 = (N - 16)T_8 \\
q_0 = (N + 16)T_8, \quad q_1 = -N + 16)T_8
\]  

and the bulk one-loop cosmological constant is zero. The conditions (20) in this case are violated, while (15) are satisfied. Therefore, a solution exists depending on two space coordinates \((y, z)\) given explicitly by (18), with \( \omega = 0 \) and \( \lambda \) determined by \( \sin(\pi \lambda R) = (16 - N)/(N + 16) \).

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A Appendix: Equations of motion and more solutions

In this appendix we look for solutions to the equations of motion (6-9) supplemented with the boundary conditions (17). We will be able to find all the solutions with \( k = 0 \) and special solutions with \( k = -1 \).

The combination (8)+(9)+2/5(3) using the relations (14) gives

\[
\ddot{\phi} + 8\dot{A}\dot{\phi} + 140ke^{2b} = 0.
\]  

From (16) we get \( \dot{A} \) in terms of \( F \) and \( G \) and their derivatives. The previous equation becomes

\[
\ddot{\phi} + \frac{1}{3} \left( \frac{\dot{G}}{G + F} + \dot{b} + \frac{5}{4}\dot{\phi} \right) \dot{\phi} + 140ke^{2b} = 0 .
\]  

In this equation only \( F \) depends on \( y \). If we assume that at least one among \( \beta_0 \) and \( \beta_1 \) is not zero than due to (17) \( F \) cannot be constant. This implies that \( \dot{\phi}\dot{G} = 0 \). We have to consider two cases: a) \( \dot{\phi} = 0 \) and b) \( \dot{G} = 0 \).
• **Case a:** Equation (62) with $\dot{\phi} = 0$ gives $k = 0$, so only flat eight dimensional spaces are possible in this case. The combination (7)+(9) gives

$$4\ddot{b} + (b\dot{G} + \dot{G})(F + G)^{-1} = 0 .$$

This equation gives $\ddot{b} = 0$ and $b\dot{G} + \dot{G} = 0$, so we have $\dot{b} = \lambda$, $\lambda$ being a constant and 

$$\lambda\dot{G} + \ddot{G} = 0 .$$

Next, consider the equation (8). After using (64), it gives

$$(\dot{G} + \lambda G)^2 - (F'^2 - \lambda^2 F^2 - 9k^2\Lambda_\epsilon) + 2\lambda F'(\dot{G} + \lambda G) = 0 .$$

This can hold provided that

$$\dot{G} + \lambda G = 0 , \quad F'^2 - \lambda^2 F^2 = 9k^2\Lambda_\epsilon .$$

We have obtained all the equations (19) and (18). It is now possible to verify that the other equations are identically satisfied. This case leads to solutions where the dilaton never diverges and they are smoothly connected to supersymmetric solutions in the limit of zero boost (rotation) and was discussed in great detail in the text.

• **Case b:** Since $\dot{G} = 0$, we can assume without loss of generality that $G = 0$ because an eventual constant $G$ can be absorbed into $F$. Equation (62) now becomes

$$\ddot{\phi} + \frac{1}{3} \left( \dot{b} + \frac{5}{4} \dot{\phi} \right) \dot{\phi} + 140ke^{2b} = 0 .$$

In this case (16) implies

$$\dot{A} = \frac{1}{24} \left( \dot{b} + \frac{5}{4} \dot{\phi} \right) , \quad A' = \frac{F'}{24F} ,$$

so that $\dot{A}$ ($A'$) depends only on $t$ ($y$). The four equations (14) have thus the form

$$\mathcal{F}_i(y) + \mathcal{G}_i(t) = 0 \quad , \quad i = 1, \ldots, 4$$

for some functions $\mathcal{F}_i$ and $\mathcal{G}_i$. This implies that both must be constants, that is

$$\mathcal{F}_i = a_i , \quad \mathcal{G}_i = -a_i ,$$

where $a_i$ are four constants. The $y$ dependent part of the equations is quite simple. For instance equation (8) gives

$$F'^2 - 9a_3F^2 = 9\kappa^2\Lambda_\epsilon .$$
Inserting this equation in the other ones \( \mathcal{F}_i \) determines the constants \( a_1, a_2 \) and \( a_4 \) in terms of \( a_3 \). The remaining equations read

\[
(\ddot{A} + \ddot{b}) + 8 \dot{A}(\ddot{A} + \dot{b}) = \frac{25a_3}{8},
\]
\[
\dot{A} + 8 \ddot{A}^2 + \dot{A} = \frac{1}{8} \dot{b} + \frac{49}{8}ke^{2b} = \frac{a_3}{2},
\]
\[
\ddot{A} + 8 \ddot{A}^2 + 7ke^{2b} = \frac{a_3}{8},
\]
\[
32 \times 7 \dddot{A}^2 - \dot{b}^2 + 48 \dot{A} \dddot{b} + 7 \times 25ke^{2b} = \frac{25a_3}{2}.
\]

Notice that all the equations are not independent since for instance \((72) - 8(73) + 7(74)\) is identically zero.

Let us first consider the \( k = 0 \) case. If we define \( h = \exp \left( \left( b + \frac{5\phi}{4} \right) / 3 \right) \), then the equation \((74)\) reads

\[
\ddot{h} - a_3 h = 0.
\]

A direct consequence of this equation is

\[
\dot{h}^2 - a_3 h^2 = E,
\]

where \( E \) is a constant. Equation \((77)\) now can be solved as

\[
\dot{\phi} = \frac{c}{h},
\]

where \( c \) is a constant. Equation \((72)\) and \((73)\) are identically satisfied and equation \((75)\) gives \( E = c^2 / 8 \). If \( a_3 = \lambda^2 / 9 > 0 \), then the solution reads

\[
h(t) = \frac{3c}{2\sqrt{2a}} \, \text{sh} \left( \frac{\lambda(t - t_0)}{3} \right).
\]

The spacetime metric and the string coupling are given by

\[
ds^2 = \left[ \text{sh} \left( \frac{\lambda(t - t_0)}{3} \right) \right]^3 F(y) \frac{1}{\text{sh} \left( \frac{\lambda(t - t_0)}{3} \right)},
\]

where \( t_0 \) is an integration constant and the parameters \( \lambda \) and \( \omega \) are determined by \((22)\).

The physical time-region can be taken for example between \( t_0 \) and \( \infty \). The final solution depends, as in the solutions discussed in the text, on the sign of the effective cosmological constant. For the three different values of \( \Lambda_e \), the \( y \)-dependent function \( F(y) \) is

\[
\Lambda_e > 0 \quad , \quad F(y) = \frac{3\sqrt{\Lambda_e}}{\lambda} \text{sh}(\lambda|y| + \omega),
\]
\[
\Lambda_e < 0 \quad , \quad F(y) = \frac{3\sqrt{\Lambda_e}}{\lambda} \text{ch}(\lambda|y| + \omega),
\]
\[
\Lambda_e = 0 \quad , \quad F(y) = F_0e^{\lambda|y|},
\]

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where the parameters $\lambda, \omega$ are determined by (22) if $\Lambda_e > 0$, by (36) if $\Lambda_e < 0$ and by (46) if $\Lambda_e = 0$.

These solutions have large string coupling close to the big bang singularity. Moreover, they are not smoothly connected in the $\lambda = \omega = 0$ limit to the supersymmetric solution [19].

In the case $a_3 = -\lambda^2/9 < 0$, the solution is

$$h(t) = \frac{3c}{2\sqrt{2}\lambda} \sin\left(\frac{\lambda(t - t_0)}{3}\right).$$  \hspace{1cm} (82)

The solution exists only when the effective cosmological constant is positive. The final solution in this case is

$$e^\Phi = e^{-5b_0/6 - \phi_0/24}[\cos\left(\frac{\lambda(t - t_0)}{3}\right)]^{-5/2}[\tan\left(\frac{\lambda(t - t_0)}{6} + \pi/4\right)]^{2\sqrt{2}} \frac{3\sqrt{\Lambda_e}}{\lambda} \sin(\lambda|y| + \omega)]^{-5/6},$$

$$ds^2 = \left[\frac{3\sqrt{\Lambda_e}}{\lambda} [\cos\left(\frac{\lambda(t - t_0)}{3}\right)]^3 \sin(\lambda|y| + \omega)\right]^{1/2} \times$$

$$\left[\delta_{\mu\nu} dx^\mu dx^\nu + \cos\left(\frac{\lambda(t - t_0)}{3}\right)^{2/3} \tan\left(\frac{\lambda(t - t_0)}{6} + \pi/4\right)]^{5/4} \left(-dt^2 + dy^2\right),$$  \hspace{1cm} (83)

where the parameters $\lambda$ and $\omega$ are determined here by (47).

We were able to find particular curved solutions, $k \neq 0$ of the system (72-75) for $k = -1$. The solution has a constant $b$ and $\dot{\phi}$:

$$\dot{\phi}^2 = 9 \times 16a_3, \hspace{1cm} e^{2b} = \frac{3}{7}a_3.$$  \hspace{1cm} (84)

The solution, which exists only for $a_3 > 0$, is explicitly given by

$$ds^2 = e^{5\phi_0/4}[e^{5\lambda t} F(y)]^{1/12} \left[1 - \frac{x^2}{4}\right]^{-2} \delta_{\mu\nu} dx^\mu dx^\nu + \frac{\lambda^2}{21} (-dt^2 + dy^2),$$

$$e^\Phi = e^{-\phi_0/24} F(y)^{-5/6} e^{-\lambda t/6},$$  \hspace{1cm} (85)

where for the three different values of $\Lambda_e$, the $y$-dependent function $F(y)$ is given by (81).

Contrary to the flat solutions discussed in the text, all the backgrounds discussed in the Appendix cannot be transformed by a change of coordinates into static ones.

There are also static solutions, depending on coordinates $(z, y)$, with $z$ being parallel to the branes. There are also two cases to consider, and the analog of case a was already discussed in the text. In case b, eqs. (72)-75) still hold provided we flip the sign in the right hand side of the equations. Here $k = 1$ corresponds to the dS eight-dim. space, whereas $k = -1$ corresponds to the AdS eight-dim. space. The solutions have a form similar to the one appearing in (81) and (83) with the time-dependence replaced by a $z$-dependence such that hyperbolic functions are replaced by trigonometric functions and vice-versa.
B Appendix: Nine dimensional nonsupersymmetric orientifold

In this appendix we give the technical details for the string model \[16\] considered in section 4. It is simpler to construct its T-dual version. We start from a Scherk-Schwarz deformation of IIB which is obtained by modding IIB on a circle of radius $r$ by $(-1)^F \times \sigma$, where $F$ is the spacetime fermion number and $\sigma$ acts on a circle $S^1$ as an asymmetric shift $y_R \rightarrow y_R + \pi r$, $y_L \rightarrow y_L - \pi r$.

The resulting torus partition function is:

$$T = \frac{1}{2} \left\{ \left| V_8 - S_8 \right|^2 \Lambda_{m,n} + \left| V_8 + S_8 \right|^2 (-1)^n \Lambda_{m,n} \right\} + \frac{1}{2} \left\{ \left| O_8 - C_8 \right|^2 \Lambda_{n,m+\frac{1}{2}} + \left| O_8 + C_8 \right|^2 (-1)^n \Lambda_{m+\frac{1}{2},n} \right\} ,$$

(87)

where the $SO(8)$ characters are defined as:

$$O_8 = \frac{\theta_3 + \theta_4}{2\eta^4}, \quad V_8 = \frac{\theta_3 - \theta_4}{2\eta^4}, \quad S_8 = \frac{\theta_2 - \theta_1}{2\eta^4}, \quad C_8 = \frac{\theta_2 + \theta_1}{2\eta^4} ,$$

(88)

with $\theta_i$ the Jacobi functions and $\eta$ the Dedekind function and

$$\Lambda_{m,n} = \sum_{m,n} \frac{\alpha'(m + \eta r)}{q^{m + \eta r}} \frac{\alpha'(m - \eta r)}{q^{m - \eta r}} .$$

(89)

The resulting model interpolates between the IIB theory in the limit $r \rightarrow 0$ and the 0B theory in the limit $r \rightarrow \infty$.

After performing a rescaling of the radius $r \rightarrow 2r$ the torus amplitude reads:

$$T = \left\{ \left| V_8 - S_8 \right|^2 \Lambda_{2m,n} + \left| V_8 + S_8 \right|^2 \Lambda_{2m+1,n} \right\} - \left\{ (V_8 \bar{S}_8 + S_8 V_8) \Lambda_{2m,n+\frac{1}{2}} + (O_8 \bar{C}_8 + C_8 \bar{O}_8) \Lambda_{2m+1,n+\frac{1}{2}} \right\} .$$

Next, we consider the orientifold obtained by gauging the discrete symmetry $\Omega' = \Omega (-1)^{F_L}$, where $\Omega$ is the standard worldsheet parity operator and $(-1)^{F_L}$ is the worldsheet fermion number. The resulting Klein bottle is given by:

$$K = \frac{1}{2} \left\{ (V_8 - S_8) P_{2m} - (O_8 - C_8) P_{2m+1} \right\} ,$$

with $P_m = \sum_m \frac{\alpha'_m}{q^{|m|}}$. The tachyon is antisymmetrised in the Klein bottle and removed from the spectrum. Notice that the limit where the radius goes to infinity reduces to the closed sector of the O'B model.

The tadpoles are obtained from the transverse channel amplitude

$$\tilde{K} = \frac{25}{2} (V_8 - S_8) W_n - (V_8 + S_8)(-1)^n W_n = \frac{25}{2} (V_8 W_{2n+1} - S_8 W_{2n}) ,$$

(90)
where \( W_n = \sum_n q^{\frac{n^2 - \mu^2}{\alpha'}} \).

We can see that there is no NS-NS tadpole, but there is a R-R tadpole. The cancelation of the tadpoles will require the introduction of \( D9 \) branes. In a T-dual spacetime interpretation, the model contains 16 \( O8^+ \) and 16 \( O8^- \) planes.

The open sector amplitudes for the \( D9 \) branes are

\[
A = \left\{ \frac{N_1^2 + N_2^2}{2} \sum_m q^{\frac{m^2 + \mu^2}{\alpha'}} + N_1 N_2 \sum_m q^{\frac{m(m+1)/2 + \mu^2}{\alpha'}} \right\} (V_8 - S_8), \\
M = -\frac{N_1 - N_2}{2} \sum_m (-1)^m q^{\frac{m^2 + \mu^2}{\alpha'}} V_8 + \frac{N_1 + N_2}{2} \sum_m q^{\frac{m^2}{\alpha'}} S_8,
\]

(91)

The RR tadpole cancellation asks for \( N_1 + N_2 = 32 \) and the resulting gauge group is \( SO(N_1) \times USp(N_2) \). In the T-dual version we have \( N_1 \) \( D8 \) branes at the origin and \( N_2 = 32 - N_1 \) branes at the other fixed point. When \( N_2 = 0 \) the open spectrum is that of the supersymmetric type I, whereas the \( N_1 = 0 \) open spectrum is the one of the \( USp(32) \) non-susy model.

Notice that in the limit \( R \to \infty \) the RR tadpole cancellation condition is modified and the resulting gauge group is \( U(32) \).

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