Primordial density and BAO reconstruction

Hong-Ming Zhu,1, 2 Ue-Li Pen,3, 4, 5, 6 and Xuelei Chen1, 2

1 Key Laboratory for Computational Astrophysics, National Astronomical Observatories, Chinese Academy of Sciences, 20A Datun Road, Beijing 100012, China
2 University of Chinese Academy of Sciences, Beijing 100049, China
3 Canadian Institute for Theoretical Astrophysics, University of Toronto, 60 St. George Street, Toronto, Ontario M5S 3H8, Canada
4 Dunlap Institute for Astronomy and Astrophysics, University of Toronto, 50 St. George Street, Toronto, Ontario M5S 3H4, Canada
5 Canadian Institute for Advanced Research, CIFAR Program in Gravitation and Cosmology, Toronto, Ontario M5G 1Z8, Canada
6 Perimeter Institute for Theoretical Physics, 31 Caroline Street North, Waterloo, Ontario, N2L 2Y5, Canada
7 Center of High Energy Physics, Peking University, Beijing 100871, China

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We present a new method to reconstruct the primordial (linear) density field using the estimated nonlinear displacement field. The divergence of the displacement field gives the reconstructed density field. We solve the nonlinear displacement field in the 1D cosmology and show the reconstruction results. The new reconstruction algorithm recovers a lot of linear modes and reduces the nonlinear damping scale significantly. The successful 1D reconstruction results imply the new algorithm should also be a promising technique in the 3D case.

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I. INTRODUCTION

The observed large-scale structure of the Universe, which contains a wealth of information such as the nature of dark energy, neutrino masses, and primordial power spectrum etc, is a powerful probe of cosmology. The matter power spectrum has been measured to significant accuracy in the current galaxy surveys and the precision will continue to improve with future surveys. However, the nonlinear gravitational evolution is a complicated process and makes it difficult to model the small-scale inhomogeneities. This has led to many theoretical challenges in developing perturbation theories (see e.g. [1] for a brief review). On the other hand, various reconstruction techniques have been proposed to reduce nonlinearities in the density field, in order to obtain better accuracies in the current galaxy surveys and the precision of dark energy, neutrino masses, and primordial power spectrum etc, is a powerful probe of cosmology. The

The standard BAO reconstruction uses the negative Zel’dovich (linear) displacement to reverse the large-scale bulk flows [2]. The nonlinear density field is usually smoothed on the linear scale (∼10 Mpc/h) to make the Zel’dovich approximation valid. Actually, the fully nonlinear displacement which describes the motion beyond the linear order (the Zel’dovich approximation) can be solved from the nonlinear density field. While the algorithm is complicated in the three spatial dimensions, it is quite simple in the 1D case, which is basically the ordering of mass elements (sheets). The 1D cosmological dynamics corresponds to the interaction of infinite sheets of matter where the force is independent of distance [3]. The simplified 1D dynamics provides an excellent means of understanding the structure formation and testing perturbation theories [1]. In this paper we solve the fully nonlinear displacement in 1D and present a new method to reconstruct the primordial density field and hence the linear BAO information.

This paper is organized as follows. In Section II, we present the reconstruction algorithm in the 1D case. In Section III, we briefly describe the 1D N-body simulation. In Section IV, we show the results of reconstruction. In Section V, we discuss the 3D case and future improvements.

II. RECONSTRUCTION ALGORITHM

The Lagrangian displacement Ψ(q) fully describes the motion of mass elements. The Eulerian position x of a mass element is

$$x = q + Ψ(q),$$

(1)

where q is the initial Lagrangian position of this mass element. In simulations, mass elements (sheets) are labeled by their initial Lagrangian coordinates. Once we know their Eulerian positions, the displacement field is obtained. However, in observations we only have the unlabelled Eulerian coordinates. The estimated displacement at the Lagrangian coordinate q = iL/N is

$$s(q) = x_i - iL/N,$$

(2)

where we have ordered the sheet labels i from left to right, L is the box size, and N is the sheet number. Here, q = iL/N is the estimated initial Lagrangian position for the ith sheet at position x_i. If no shell crossing happens, the reconstructed displacement is exact up to a global shift. In the nonlinear regime once shell crossing occurs, the estimated displacement field is quite noisy on the
scale \( \sim L/N \). To reduce stochasticities in the estimated displacement field, we can use the averaged displacement of \( n_p \) particles

\[
s(q) = \frac{1}{n_p} \sum_{j=1}^{i+n_p-1} x_j - i n_p L/N,
\]

where \( q = i n_p L/N \) and \( j \) is the sheet label. Here \( i \) varies from 0 to \( N/n_p \) and \( j \) varies from 0 to \( N \). We take \( n_p = 5 \) to estimate the displacement field in this paper.

The derivative (actually the divergence) of \( s(q) \) gives the reconstructed density field

\[
\delta_r(q) = -\frac{\partial s(q)}{\partial q},
\]

i.e., the differential motion of mass elements. Reconstruction from the gridded density field can be implemented following the same principle, which we adopt in the following calculations.

### III. SIMULATIONS

The 1D \( N \)-body dynamics can be simulated using the particle-mesh (PM) method. The 1D simulations we use are run with the 1D PM code in Ref. [1]. The 1D simulation involves \( 3 \times 10^8 \) sheets with \( 3 \times 10^8 \) PM elements in a \( 10^8 \) Mpc box. The 1D simulation assumes a matter-dominated background cosmology (\( \Omega_m = 1 \)) and have the same dimensionless power spectrum as the concordance cosmology, i.e.,

\[
k P_{1D}(k)/\pi = k^3 P_{3D}(k)/(2\pi^2),
\]

where \( P_{3D} \) is the 3D linear power spectrum from the linear Boltzman code.

The initial condition is generated using the Zel’ dovich approximation. Since the Zel’dovich approximation is exact up to shell crossing, the PM calculation is started at \( z = 10 \). In the analysis, we use the output at \( z = 0 \).

We also scale the initial density field by the linear growth factor to get the linear density field at \( z = 0 \).

Note that the nonlinear evolution in 1D is more significant than the 3D case. The nonlinear evolution in the concordance (3D) cosmology at \( z = 0 \) is only comparable to the 1D cosmology at \( z = 1 \) [1].

### IV. RESULTS

Figure [1] shows the linear, nonlinear and reconstructed correlation functions. Since the BAO feature in 1D is sharper than that in 3D, the smearing of the BAO peak in 1D is more substantial [1]. Nevertheless, the new reconstruction method sharpens the peak significantly. The nonlinear density field \( \delta(x) \) is given on the Eulerian position \( x \), while the reconstructed density field \( \delta_r(q) \) is calculated on the Lagrangian position \( q \).

To conveniently quantify the linear information \( \delta_L \) in the nonlinear density field \( \delta \), we decompose the nonlinear density field \( \delta \) as

\[
\delta(k) = b(k)\delta_L(k) + n(k).
\]

Here, \( b \delta_L \) is completely correlated with the linear density field \( \delta_L \). Correlating the nonlinear density field with the linear density field,

\[
\langle \delta(k)\delta_L(k) \rangle = b(k)\langle \delta_L(k)\delta_L(k) \rangle,
\]

we obtain

\[
b(k) = \frac{P_{\delta\delta_L}(k)}{P_{\delta_L}(k)}.
\]
Nonlinear evolution drives $b(k)$ to drop from unity, reducing the linear signal. Separating the part correlated with the linear density field, we have $n(k) = \delta(k) - b(k)\delta_L(k)$. $n(k)$ is generated in the nonlinear evolution and thus uncorrelated with the linear density field $\delta_L$, further reducing $b\delta_L$ with respect to $\delta$. This part induces noise in the measurement of BAO. Such decomposition helps to write the nonlinear power spectrum as

$$P_\delta(k) = D(k)P_{\delta_L}(k) + P_n(k),$$

where $D(k) = b^2(k)$ is the nonlinear damping factor. Here, $b(k)$ is often referred to as the “propagator” and $P_n$ is usually called the “mode-coupling” term \cite{4, 5}. For the reconstructed field $\delta_r(q)$, we also have

$$\delta_r(q) = b_r(q)\delta_L(q) + n_r(q),$$

where $b_r(q) = P_{\delta_r\delta_L}(q)/P_{\delta_L}(k)$. Similarly, the reconstructed power spectrum is given by

$$P_{\delta_r}(k) = D_r(k)P_{\delta_r\delta_L}(k) + P_{n_r}(k),$$

where $D_r(k) = b_r^2(k)$. Here, the subscript “$r$” denotes that the reconstructed field is given on the Lagrangian coordinate. In Fig. 2, we plot the linear components and the noise terms of the nonlinear and reconstructed fields.

Figure 3 shows the damping factors for the nonlinear (dotted line) and filtered reconstructed (solid line) fields. The wiggles in the reconstructed power spectrum are much more apparent than the nonlinear power spectrum.

Above $0.9$ for $k \lesssim 0.1 \text{ Mpc}^{-1}$. However, the 100 percent reconstruction, cancelling any nonlinear effects, is still unachievable, as some information has been irreversibly lost.

Reconstruction reduces the nonlinear damping $D(k)$ as well as the noise term $P_n(k)$. To quantify the overall performance, we can use the cross-correlation coefficient

$$r(k) = \frac{P_{\delta\delta_L}(k)}{\sqrt{P_\delta(k)P_{\delta_L}(k)}} = \frac{1}{\sqrt{1 + \eta(k)}},$$

where $\eta = P_n/(b^2P_{\delta_L})$ quantifies the relative amplitude of $n$ with respect to $b\delta_L$. We plot the cross-correlation coefficients in Fig. 4. The correlation of $\delta_r$ with $\delta_L$ is even better than that of $\delta$ at $z = 3$.

The raw reconstructed field $\delta_r$ is still noisy on small scales ($k_q \gtrsim 0.24 \text{ Mpc}^{-1}$). To optimally filter out the
The raw nonlinear density field is clearly non-Gaussian. As a result, the reconstructed density field is also non-Gaussian. The power spectrum of the optimal reconstructed field \( \delta_r(k_q) \) is given by

\[
P_{\delta_r}(k_q) = \frac{P_{\delta_L}(k_q)}{b_r(k_q)} W_r(k_q).
\]

The raw nonlinear field \( \delta \) is also filtered. In Fig. 5 we plot the power spectra of the optimal filtered reconstructed and nonlinear fields. The wiggles in the reconstructed power spectrum are much more apparent than the nonlinear power spectrum.

The density fluctuation probability distribution function (PDF) quantifies the Gaussianity of the density field. Figure 6 shows the PDFs of the nonlinear and reconstructed density fields. We also plot the PDFs of the linear component \( b_r \delta_L \) and the noise part \( n_r \) of the reconstructed density field \( \delta_r \). Of course the linear component is Gaussian, while the noise part is non-Gaussian. As a result, the reconstructed density field is also non-Gaussian. The raw nonlinear density field is clearly non-Gaussian.

V. DISCUSSIONS

The new reconstruction method successfully recovers the lost linear information on the mildly nonlinear scales through the Ministry of Research & Innovation. The result in 1D provides an intuitive view of the algorithm and motivates us to develop the reconstruction method in 3D. The nonlinear displacement beyond the Zel’dovich approximation in 3D can be solved using the multigrid iteration scheme [10]. The algorithm for solving the 3D nonlinear displacement is originally introduced for the adaptive particle-mesh N-body code [7] and the moving mesh hydrodynamic code [8].

The reconstructed nonlinear displacement field is also important for the current BAO reconstruction [2], where the linear continuity equation is adopted to solve the displacement under the Zel’dovich approximation. However, the nonlinear displacement retains much more information, describing the motion of dark matter fluid elements beyond the linear order. The reconstructed displacement field \( s(q) \) is given on the Lagrangian coordinate instead of the final Eulerian coordinate. This helps to correct the effect due to the use of \( s(x) \) instead of \( s(q) \) in the BAO reconstruction [3, 11]. As more nonlinear effects will be removed using the nonlinear displacement, we expect the modeling of the reconstructed density field will be simplified.

The Wiener filter is optimal for the case both the signal and the noise are Gaussian random fields. In Fig. 6 the PDFs of the reconstructed density field and the noise are apparently non-Gaussian. The reconstruction can be further improved using the nonlinear filter rather than the Wiener filter [12]. We plan to study this in future.

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