Abstract—Assembly planning is a difficult problem for companies. Many disciplines such as design, planning, scheduling, and manufacturing execution need to be carefully engineered and coordinated to create successful product assembly plans. Recent research in the field of design for assembly has proposed new methodologies to design product structures in such a way that their assembly is easier. However, present assembly planning approaches lack the engineering tool support to capture all the constraints associated to assembly planning in a unified manner. This paper proposes COMPOSITIONAL PLANNING, a string diagram based framework for assembly planning. In the proposed framework, string diagrams and their compositional properties serve as the foundation for an engineering tool where CAD designs interact with planning and scheduling algorithms to automatically create high-quality assembly plans. These assembly plans are then executed in simulation to measure their performance and to visualize their key build characteristics. We demonstrate the versatility of this approach in the LEGO assembly domain. We developed two reference LEGO CAD models that are processed by COMPOSITIONAL PLANNING’s algorithmic pipeline. We compare sequential and parallel assembly plans in a Minecraft simulation and show that the time-to-build performance can be optimized by our algorithms.

I. INTRODUCTION

Today, mass customization of products such as automobiles and consumer electronics is forcing companies to provide a very large product variety to address the diverse customer requirements. Digital manufacturing technologies make it possible to accommodate mass customization during product design and manufacturing. For example, parametric designs in computer aided design (CAD) software allow for the specification of configurable products, and computer aided manufacturing (CAM) algorithms allow for the fabrication of products on different machines. Unfortunately, there is very limited engineering tool support for product assembly planning. Although some companies use design for assembly (DFA) [1] and design for manufacturing and assembly (DFMA) [2] methodologies that attempt to develop product structures that facilitate their assembly, their implementation is ad-hoc. Therefore, assembly planning is a task that is loosely coupled to the rest of the digital manufacturing pipeline. The objective of this paper is to open new avenues for interoperable assembly planning that is tightly coupled to the upstream design activities, and the downstream assembly tasks.

String diagrams are a powerful graphical calculus for reasoning in category theory [3]. String diagrams have also proven useful in many other domains. They have been shown to provide a mathematically sound graphical language in domains including linguistics [4], systems engineering [5], and computer science [6]. Generally, string diagrams represent processes which require and produce resources. Assembly planning is the discipline of understanding how to optimally chain assembly processes together to craft a whole product from separate parts [7]. Thus, string diagrams are a natural tool for formulating assembly planning problems and constructing their solutions.

To demonstrate this thesis we show string diagrams can be used to build construction schedules for various LEGO models. From each LEGO 3D CAD file we generate a connectivity graph where the nodes represent LEGO pieces and the edges indicate that they are connected in the final model. Given a hierarchical clustering of this connectivity graph, we generate a construction plan represented by string diagrams which is hierarchical, compositional, and interpretable. Using the formalism of string diagrams, complex sub-assemblies can be black boxed into larger string diagrams. Having this hierarchical structure allows us to manipulate or adapt our plan at a desired level of abstraction. Furthermore, there is categorical formalism in place which allows to efficiently generate schedules from these string diagrams. We use topological sorting, Girvan-Newman, and Leiden algorithms to generate assembly plans and schedules with different properties. Finally, we use Minecraft [8] as a simulator to validate the resulting schedules and measure their time-to-build performance.

In this paper, we demonstrate the versatility of this approach with a framework, COMPOSITIONAL PLANNING, that provides a new way of talking about assembly planning. Our category theoretic interpretation provides a flexible mathematical foundation that allows for an end-to-end demonstration from CAD design to assembly simulation. The contributions of this paper are the following:

• we show how string diagrams are an efficient and mathematically sound language to represent an assembly planning domain.
• as large string diagrams can be tedious and cumbersome for humans to work with, our framework automates the creation of large string diagrams and thus eliminates the overhead traditionally associated with them.
• a novel algorithm that converts string diagrams to expressions that result in highly parallel assembly plans
• a Minecraft based simulation environment “mod” to execute LEGO assembly plans.
• we publish the CompositionalPlanning framework as a Julia package for others to reproduce and build upon our work.

String Diagrams for Assembly Planning

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II. RELATED WORK

Assembly planning problems and approaches have been widely investigated. A comprehensive survey of them can be found in [7]. In [9], survey of assembly design and planning systems is presented. In particular, the Assembly Sequence Planning (ASP) problem that we target in this paper is an NP-hard problem. ASP’s goal is to find a collision-free sequence of assembly operations that put together individual parts given the geometry of the final product and the relative positions of parts in the final product. ASP is considered a combinatorial problem and therefore representations of the space of possible sequences has been an active area of research [10]. While various representations ranging from AND-OR graphs [11] to Petri nets [12] have been proposed, we are the first to study ASP under a category theoretic framework. In this paper, we show that category theory provides both an explicit and an implicit representation on assembly sequences. On the one hand, string diagrams provide an explicit ASP representation of partial and full assemblies. On the other hand, expressions provide us mathematical soundness and rigor as they implicitly encode precedence assembly relationships.

III. CATEGORICAL ASSEMBLY PLANNING FRAMEWORK

Our Compositional Planning framework, shown in Figure 1 consists of five reusable software components. The first step is to infer a connectivity graph from the CAD model that describes how the different parts come together in terms of their geometry (e.g., orientation) and assembly operations (e.g., snap, glue, weld, insert). This step is necessary because most of CAD models list quantities of all parts and detailed structural functions in relation to a finished product, and this information does not directly map to their assembly.

The second step consists of plan generation [13] – described by different string diagrams – that have different properties of order and parallelism. These plans, although expressed by different expressions of string diagrams, generate the same LEGO model as they: (i) bring an initial world to a goal world using a set of assembly operators, and (ii) minimally impose ordering constraints.

The third step generates a schedule using a plan and a detailed knowledge of the execution environment (e.g., number of workers, machines). The job of the schedule generation [13] is to impose further ordering constraints on the assembly operator application to achieve a robust (e.g., against failures) and time-efficient execution of the assembly task (e.g., time-to-assembly).

The fourth and final step consists of executing the schedule in a simulator to visualize the assembly task, and to generate performance metrics. Although this paper focuses on LEGO CAD models as an input, and Minecraft simulations as an output, our components are domain agnostic, and they can be easily adapted for use in other domains.

A. Connectivity Diagram Generation from CAD

Compositional Planning parses text-based LDraw files [14] that describe all the bricks in the model by type (e.g., 2×2, 2×4), color, center coordinates (x, y, z), and a 3×3 rotation matrix. This LDraw file represents the BOM of the LEGO model and does not contain any information about the connectivity of the bricks.

Vertical stacking is the most common operation with LEGO bricks as shown in the example in Figure 1. Therefore, the first step in our framework is to infer the vertical connectivity in the LEGO CAD model. In LDraw’s coordinate system −y is “up”. Therefore, two bricks are connected if the top face of one brick has the same y coordinate as the other brick’s down face, and their boxes (defined by (x, z) center coordinates and the brick’s width and length) intersect. Other less common operations such as horizontal stacking, and operations involving other LEGO pieces such as pegs are left for future work. However, the connectivity inference would follow a similar principle as the one described above. Similarly, extending our framework beyond LEGO would require new parsers to read CAD file formats, and new inference algorithms to derive the connectivity between parts.

As an illustrative example consider the LEGO model and its connectivity diagram shown in Figure 1. It consists of seven bricks sequentially numbered from 1, ..., 7. In addition, our algorithm also includes the “ground” nodes to facilitate the model construction using the base build plate. In this example, the ground nodes 9 and 8 connected to 6 and 1, respectively.

B. String Diagrams

String diagrams are diagrams where resources are represented by strings (wires) and processes are represented by boxes. For example a process which snaps a peg into a hole is represented by

\[ \text{L1} \quad \text{snap} \quad \text{base} \]

String diagrams can be composed in sequence

\[ \text{L1} \quad \text{snap} \quad \text{base} \quad \text{snap} \quad \text{C} \]

indicating that the processes must be performed in sequence.

String diagrams can also be composed in parallel

\[ \text{A1} \quad \text{buildcolumn} \quad \text{B1} \]

\[ \text{A2} \quad \text{buildroof} \quad \text{B2} \]

indicating that the order in which the tasks are performed does not matter. String diagrams also have algebraic expressions called morphisms. For example, the first example has a corresponding algebraic expression given by

\[ \text{snap} : \text{L1} \otimes \text{L2} \to \text{base} \]
Note that this expression is both functional and typed. This makes \textsc{Compositional Planning} an efficient and type-safe framework. In a similar way, the expressions which string diagrams represent can be composed in sequence and parallel using the two operations

\[
(f: x \rightarrow y, g: y \rightarrow z) \mapsto f \circ g: x \rightarrow z \quad (1)
\]

\[
(f: x \rightarrow y, f': x' \rightarrow y') \mapsto f \otimes f': x \otimes x' \rightarrow y \otimes y' \quad (2)
\]

Rather complicated expressions can be built using these operations, e.g., see the string diagrams in Figure 1. A natural question to ask is when two expressions correspond to the same string diagram. The answer to this question is essential to understanding how planning domain dependencies represented in string diagrams can be algebraically manipulated. It turns that if the algebraic expressions satisfy the right set of axioms, then the way that a string diagram is drawn is independent of the expression it generates. A structure of algebraic expressions satisfying these axioms is a well-known structure in category theory called a symmetric monoidal category. In [15] it is shown that string diagrams unambiguously represent morphisms in a given symmetric monoidal category. The following theorem, whose proof will be omitted due to space limitations, ensures the soundness of string diagrams in the LEGO assembly domain.

**Theorem** Let \( L \) be finite set of LEGO pieces and let \( X \) be the set of all possible snaps between sub-assemblies in \( L \). Then, there is a symmetric monoidal category generated by \( X \) and \( L \) whose morphisms represent all possible assembly plans using the LEGOs in \( L \).

The following sections describe our implementation of symmetric monoidal categories for planning and scheduling.

**C. Planning**

A LEGO CAD model and its connectivity graph describe an object in its final, assembled state, but not how to assemble it. A plan consists of step-by-step instructions on how to assemble the atomic parts (LEGO bricks) into the desired object. In general, there are many possible plans for assembling the same object, corresponding to different ways of forming intermediate sub-assemblies.

For us, plans are string diagrams. In such a diagram, the strings represent sub-assemblies and the boxes represent operations of joining together sub-assemblies to make a larger sub-assembly. Formally, a sub-assembly is a subset of the atomic parts, interpreted as being assembled. Each join operation takes two disjoint sub-assemblies \( A \) and \( B \) as inputs and produces a single output, the union \( A \cup B \). A plan is a string diagram that takes all the singleton sets (sub-assemblies consisting of a single part) as inputs and produces as output the set of all parts (full assembly). Although every plan is valid at this level of description, not every plan will be physically feasible.

As proof of concept, we implemented two simple algorithms for assembly planning. In the sequential algorithm, we topologically order the edges of the connectivity graph, i.e., we topologically order the nodes and then lexicographically order the edges, viewed as ordered pairs of nodes. For each edge, taken in this order, we join the two sub-assemblies containing the source and target, if they are distinct; otherwise, we do nothing. We continue in this way until all the edges have been exhausted, at which point the object is fully assembled. When the connectivity graph is a path graph, the sequential plan is the obvious plan that joins the parts together one-at-a-time.

In the parallel algorithm, we create more opportunities for parallelism by partitioning the connectivity graph into components, making plans on each component, and then treating these plans as black boxes in a higher-level plan. This meta-algorithm has several knobs to tune, and it can be applied recursively. To partition the graph, we can apply any community detection algorithm that finds non-overlapping communities. In our experiments, we use the Girvan-Newman algorithm [16] and a variant of the Louvain method [17], the Leiden algorithm [18]. We perform only one level of partitioning and we do sequential planning within each partition and also to assemble the resulting sub-plans.

**D. Scheduling**

A plan, in the form of a string diagram, says what steps to perform and how the steps depend on each other. A schedule
extends the information in a plan by assigning the steps a definite order; formally, a schedule is any linear extension of the topological ordering of the operations (boxes) in the plan. For simplicity, we take a resource-agnostic view of scheduling, in which the number of workers is unknown at the time of planning and scheduling. The aim in scheduling is therefore to maximize the opportunities for parallelism, given the constraints imposed by the plan [19].

Our scheduling algorithms have two major phases. First, we create a syntactic expression representing the plan. In general, a single string diagram can be represented by many different expressions; we construct one of them. We then linearize the expression, using a simple recursive algorithm, to obtain a schedule.

A small example will illustrate the relationship between string diagrams and syntactic expressions. Consider the string diagram shown in Figure 2 the composite of \( f \) and \( g \) in parallel with composite of \( h \) and \( k \). We can represent this diagram by either of the expressions \((f \cdot g) \otimes (h \cdot k)\) (read “\( f \) then \( g \), and \( h \) then \( k \)”) and \((f \otimes h) \cdot (g \otimes k)\) (read “\( f \) and \( h \), then \( g \) and \( k \)”).

\[
\begin{array}{c}
\text{Fig. 2. Relationship between string diagrams and syntactic expressions}
\end{array}
\]

We have developed an algorithm to find an expression for any string diagram representing a morphism in a symmetric monoidal category. As the algorithm is fairly elaborate, we will not digress to present it carefully, except to say that it is inspired by existing algorithms that recognize in a DAG, or reduce a DAG to, a series-parallel digraph [20], [21].

The second phase of scheduling is more straightforward. Having formed an expression for the plan, we schedule the plan by recursively linearizing the expression tree. Given a composition \( f \cdot g \), we simply concatenate the schedules for \( f \) and \( g \). Given a product \( f \otimes g \), we interleave the schedules for \( f \) and \( g \), meaning that we take the first element of the \( f \)-schedule, then the first element of the \( g \)-schedule, then the second element of the \( f \)-schedule, and so on, until the both schedules have been exhausted. For example, both of the above expressions \((f \cdot g) \otimes (h \cdot k)\) and \((f \otimes h) \cdot (g \otimes k)\) yield the same schedule \((f, h, g, k)\). This procedure can be seen as a special case of an existing algorithm for optimally scheduling series-parallel digraphs [22].

E. Simulation

Minecraft is an immensely popular 3D open-world video game where players can build their own structures [23]. The game world and most of its elements are made of different kinds of blocks. These blocks can be used to create structures of any complexity. This versatility makes Minecraft a good fit to represent the CAD model and to simulate their assembly process. It has already proven to be a well-suited simulation tool in other robotics domains [24]. To execute the schedule generated by our framework we extended Minecraft by a new mod. With this mod we can simulate the whole assembly process of the CAD model described by the schedule. The simulation not provides us with a comprehensible visual representation of the process, but also allows us to quantify execution time and worker occupancy of different schedules.

Our open source Minecraft mod executes the assembly operations in the correct order as dictated by the generated schedule. The geometric CAD information is encoded in the connectivity diagram and it is parsed by our mod. Each LEGO brick is represented by a single or multiple Minecraft blocks. The schedule and the operations specified in it determine which and how many bricks can be connected to each other per step. In addition to the precedence constraints encoded in the schedule, we can define a number of workers. Each worker is allowed to perform a single operation per step. Operations are dispatched to workers in the order they appear in the schedule. Only operations for which a worker is available can be executed. Hence, the level of parallelism of the schedule and the number of workers determine the time it takes to complete the assembly. Disjoint sub-assemblies are assembled in their own area respectively until they are connected to each other forming a new (sub-)assembly. The assembly process is finished when all of the schedule’s operations have been completed by the available workers.

IV. Results

To validate the CompositionalPlanning’s pipeline we designed the two LEGO CAD models shown in Figure 3(a-b). The design objective was to have two LEGO models of around 100 bricks each with a rich set of features and a few human-intuitive sub-assemblies to validate our approach. The Columns model (Figure 3(a)) is inspired by roman temples and consists of 77 bricks. This model consists of four column sub-assemblies, each composed of 12 vertically stacked \( 2 \times 2 \) bricks each. The roof sub-assembly consists of two pairs of support beams, each arranged in a stair configuration supporting a flat roof. The House model (Figure 3(b)) consists of 86 bricks. The house foundation sub-assembly consists of eight \( 4 \times 10 \) green bricks and supports the house sub-assembly and an electric pole. The house sub-assembly consists of four non-identical walls. We designed each wall using different compositions of bricks (i.e., \( 1 \times 1, 1 \times 8, 1 \times 10, 1 \times 2 \times 2 \), etc.) that result in different connectivity features as highlighted by the different shades of brown. The front wall has a square window and a door. The two side walls have two windows each while the back wall is solid. The four walls support a two layer roof. The pole sub-assembly consists of eleven \( 1 \times 1 \) bricks, and one \( 1 \times 6 \) brick on top.

A. Connectivity Diagrams

The generated connectivity diagrams for the two CAD models are shown in Figure 3(c-d). The Columns model was designed with regularity and symmetry in mind. These features are explicit in its connectivity diagram in Figure 3(c).
with the four long strands representing the columns, and the dense layer on top representing the roof. On the other hand, the House model was designed with asymmetry and irregularity in mind. These features are clearly visible in its connectivity diagram in Figure 3(d). In the House connectivity diagram, the walls are irregular and asymmetric with gaps representing the windows and the door. The pole is represented by the long strand.

Typically, LEGO models come with assembly instructions, or build instructions [25]. These instructions are, most likely, made for human enjoyment and therefore must be intuitive. One natural way to organize these instructions is by sub-assemblies such that humans can relate to the structure they are constructing (e.g., a house). In some cases, these sub-assemblies are obvious. For example, the column and roof sub-assemblies in the Columns connectivity diagram (Fig. 3(c)) are easily distinguishable and therefore can be decomposed into a reasonable build plan. However, there are other cases when these sub-assemblies are not obvious. For example, decomposing the interconnected wall sub-assembly in the House model (Fig. 3(d)) into a reasonable plan is not trivial. For both examples, our plan generation pipeline on string diagrams can be used to generate sequential and highly parallel assembly plans.

### B. Plan Generation

In this paper, the quality of an assembly plan is determined in terms of how many operations can be executed in parallel, rather than on maximizing human enjoyment. For each LEGO CAD model, we generate two schedules. A sequential schedule is generated by topologically sorting the connectivity diagram, and a parallel schedule is generated with the algorithms introduced in Section III-C. Figure 4 shows the sequential schedules generated for the Columns and the House models.

The column symmetry in the Columns model allows the sequential schedule to expose some parallelism as shown in Figure 4(a). If several workers are available, this natural parallelism can be exploited to reduce the time-to-build. The sequential schedule for the House model, as shown in Figure 4(b), exposes very little parallelism due to the asymmetry and irregularity in the model. These two examples help us illustrate the inherent limitations of sequential schedules.

Using the parallel plan generation algorithms described in Section III-C our framework exposes higher levels of parallelism as shown in Figure 4. Here, the black boxes corresponding to a partitioning of the connectivity diagram are shown by the labeled black boxes. The number in each black box represents the number of bricks within the black box sub-plan. Even in the case of a purely sequential plan of the black boxes (represented by the width of the black boxes), it can be observed that the amount of parallelism is higher compared to the sequential schedules in Figure 4. In particular, the House model’s parallel plan in Figure 5(b) exposes much higher parallelism when compared to the its sequential plan in Figure 4(b) consisting of a stairs configuration with minimum parallelism.

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**Fig. 3.** LEGO CAD models and their inferred connectivity diagrams.

**Fig. 4.** Sequential plans generated for the two LEGO CAD models.

**Fig. 5.** Parallel plans generated for the two LEGO CAD models. The width of the black boxes represent a purely sequential sub-plan (i.e., a stairs configuration).
C. Simulated Schedule Execution

The schedule and the number of workers determines how many operations can be executed in parallel. This parallelism can be visualized in the simulation when multiple sub-assemblies are constructed at the same time. Figure 6 shows a time-lapse of the parallel assembly process for the Columns and House models with an unlimited number of workers. The top row shows the assembly process at an early stage. Note the parallel construction areas of the assemblies highlighted by the red squares. These construction areas directly correspond to the black boxes of the plans. Furthermore, the main construction area where pieces and sub-assemblies will eventually be connected to each other can be recognized. The middle row shows the half completed assemblies. The structures are more advanced, and some sub-assemblies have already been connected to other assemblies. The bottom row shows the fully assembled Columns and House LEGO models.

![Columns model][1]

![House model][2]

Fig. 6. Time-lapse of the execution of parallel plans. Black box sub-assemblies are built simultaneously on different construction areas highlighted by the red squares.

To assess the quality of the generated plans we simulate the assembly process for the sequential and the parallel schedules for both models. The simulation is run with a varying number of workers – 1, 2, 4, 8 and 16. For each configuration we measured the total number of steps it takes to build the assembly and the average occupancy ratio of the workers. Table I summarizes the results. Our baselines are simulations run with a single worker. While pieces can be snapped to the ground or to an already existing assembly, two single pieces can also be combined to a new sub-assembly. In the latter case we must first connect a LEGO brick to the ground and this occupies a worker for a time step. For this reason, the time-to-built for the parallel schedule with a single worker is higher than the one for the sequential schedule. Unsurprisingly, the parallel schedule with more than one worker always requires fewer steps to complete the assembly while maintaining a higher occupancy rate than its sequential counterpart. The performance difference between the sequential and parallel schedules for the Columns model is rather small because of its architecture. Its sequential plan (Figure 4(a)) already exposes some parallelism. However, our parallel schedule is able to exploit additional opportunities and provides slightly better performance over the sequential schedule.

| Workers | Columns model | House model |
|---------|---------------|-------------|
|         | Steps | Occupancy | Steps | Occupancy |
| Sequential schedule |
| 1       | 92    | 1.00       | 93    | 1.00       |
| 2       | 50    | 0.92       | 75    | 0.62       |
| 4       | 33    | 0.70       | 69    | 0.34       |
| 8       | 25    | 0.46       | 65    | 0.18       |
| 16      | 23    | 0.25       | 65    | 0.09       |
| Parallel schedule |
| 1       | 95    | 1.00       | 98    | 1.00       |
| 2       | 50    | 0.95       | 55    | 0.89       |
| 4       | 30    | 0.79       | 36    | 0.68       |
| 8       | 19    | 0.63       | 26    | 0.47       |
| 16      | 17    | 0.35       | 21    | 0.29       |

TABLE I

| Execution time and worker occupancy for different schedules with varying number of workers. |

For the House model on the other hand, the parallel schedule yields significantly faster assembly times with an increasing number of workers; up to 3 times with 16 workers. Additionally, the workers are used to a higher capacity and the occupancy ratio deteriorates at a slower rate as the number of workers increases. This model shows that our algorithms are able to identify and exploit non-obvious parallelism in less regular and symmetric models.

V. Conclusion and Future Work

In this paper, we studied the use of string diagrams for assembly planning and developed a framework to demonstrate this approach in the LEGO domain. This new perspective gives us multiple advantages. First, it provides us with a powerful graphical calculus for reasoning about the assembly planning domain in category theory. Second, this formalism allows string diagrams to be easily manipulated with a computer algebra system to generate plans and schedules. This allows us to seamlessly interconnect the different disciplines involved in assembly planning. Third, with a novel hierarchical planning approach using black boxes, we demonstrated that the resulting plans expose high-degrees of parallelism that result in efficient assembly. Our COMPOSITIONAL PLANNING framework has several limitations that prescribe future research. (i) As a proof of concept, we focused on the most popular LEGO assembly operations. Since our framework is domain agnostic, extending to other domains is an important direction for future work. (ii) We implemented three planning and one scheduling algorithm. Implementing other algorithms will help us validate the full potential of string diagrams for assembly planning. (iii) The planning algorithms yield different string diagrams. Developing new algorithms to morph a string diagram to another with different properties (e.g., more parallelism) is a challenging but very interesting research direction.
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