Quark model study of the $\pi N \rightarrow \pi N$ reactions up to the $N(1440)$ resonance region

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A combined analysis of the reactions $\pi^+ p \rightarrow \pi^+ p, \pi^- p \rightarrow \pi^- p$ and $\pi^- p \rightarrow \pi^+ n$ is carried out with a chiral quark model. The observations are reasonably described from the $\Delta(1232)$ resonance region up to the $N(1440)$ resonance region. Besides the $\Delta(1232)P_{33}$, a confirmed role of $N(1440)P_{11}$ is found in the polarizations of the $\pi^- p \rightarrow \pi^- p$ and $\pi^- p \rightarrow \pi^+ n$. It is found that the $N(1440)N\pi$ and $\Delta(1232)N\pi$ couplings are about 1.7 and 4.8 times larger than the expectations from the simple quark model, respectively, which may suggest the unusual property of $N(1440)P_{11}$ and deficiency of the simple quark model in the description of $N(1440)P_{11}$ and $\Delta(1232)P_{33}$. The $t$- and $u$-channel backgrounds have notable contributions to the $\pi^- p \rightarrow \pi^- p$ reaction, while in the $\pi^- p \rightarrow \pi^- p, \pi^+ n$ reactions, the $s$-channel nucleon and $t$- and $u$-channel backgrounds play crucial roles.

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I. INTRODUCTION

A better understanding of the baryon spectrum and internal structure of excited baryons is a fundamental challenge and goal in hadronic physics [1, 3]. Pion-nucleon ($\pi N$) scattering provides us an important place to study the $\Delta$ and nucleon spectroscopies. Most of our current knowledge about the $\Delta$ and nucleon resonances listed in the Review of Particle Physics by the Partial Data Group (PDG) [4] was extracted from the $\pi N$ scattering. In the past decades, although many efforts have been made by several partial wave analysis groups [5–27], the properties of some $\Delta$ and nucleon resonances are not well understood. Still, strong model dependencies exist in the extracted resonance properties from different groups. For example, the study of $\pi N$ scattering in the literature [11, 12] indicates that the Roper $N(1440)P_{11}$ is dynamically generated from the coupled channel interaction without any excited three-quark core, while in the literature [26–28] the Roper $N(1440)P_{11}$ is suggested to be a three-quark state coupled by a meson cloud. Furthermore, in some literature the $N(1535)S_{11}$ resonance is suggested to be a dynamically generated resonance by analyzing the $\pi N$ reactions [29–33]. Recently, according to our chiral quark model study of the $\pi^- p \rightarrow \eta N, K^+ \Lambda$ [34, 35], and $\gamma N \rightarrow \eta N, \pi^0 N$ reactions [36, 37], the $N(1535)S_{11}$ resonance can be explained as a mixing three-quark state between representations of $[70, 2^+]$ and $[70, 4^+]$. To deepen our understanding of the resonance properties from the $\pi N$ reactions, more partial wave analyses are needed.

In present work, we further extend the chiral quark model to the study of the $\pi^\pm$ elastic reactions $\pi^+ p \rightarrow \pi^+ p, \pi^- p \rightarrow \pi^- p$ and the charge-exchange reaction $\pi^- p \rightarrow \pi^+ n$ up to the $N(1440)$ resonance region. The $\pi^- N \rightarrow \pi^- N$ reactions provide us a good place to study the $\Delta(1232)P_{33}$ and $N(1440)P_{11}$, because the other higher resonances, such as $N(1535)S_{11}$ and $\Delta(1620)S_{31}$ are far from the $\Delta(1232)$ and $N(1440)$ region, their interferences in this low energy region should be strongly suppressed by the phase space. On the other hand, in the energy region what we will consider, there are abundant data, which have been collected by the GWU group [38]. By a combined analysis of these reactions, we hope (i) to further test the validity of the chiral quark model and obtain a better understanding of the reaction mechanism for the $\pi N$ scattering; (ii) to confirm the properties of $\Delta(1232)P_{33}$ extracted from the $\pi^\pm$-meson photoproduction processes in our previous work [36]; (iii) to extract some reliable information of $N(1440)P_{11}$. In our previous quark model analyses of the $\pi^- p \rightarrow \eta N, K^+ \Lambda$ [34, 35] and $\gamma N \rightarrow \eta N, \pi^0 N$ [36, 37] reactions, no obvious evidence of $N(1440)P_{11}$ is found.

In the chiral quark model, an effective chiral Lagrangian is introduced to account for the quark-pseudoscalar-meson coupling. Since the quark-meson coupling is invariant under the chiral transformation, some of the low-energy properties of QCD are retained. There are several outstanding features for this model [35, 39–40]. One is that in this framework only one overall parameter is needed for the nucleon resonances to be coupled to the pseudoscalar mesons in the SU(6)⊗O(3) symmetry limit. This is distinguished from hadronic models where each resonance requires one additional coupling constant as free parameter. Furthermore, the s- and u-channel transition amplitudes at the tree level can be explicitly calculated, and the quark model wavefunctions for the baryon resonances, after convolution integrals, provide a form factor for the interaction vertices. Consequently, all the baryon resonances can be consistently included. The chiral quark model has been well developed and successfully applied to pseudoscalar-meson photoproduction reactions [36, 37, 39–50]. Recently, this model has been extended to $\pi^- p$ [34, 35] and $K^- p$ [51, 53] reactions as well, which provides some novel insights into the observables measured in these reactions.

This work is organized as follows. The model is reviewed in Sec.II. Then, in Sec.III, our numerical results and analysis are presented and discussed. Finally, a summary is given in Sec.IV.

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II. FRAMEWORK

In this section, we give a brief overview of the chiral quark model. In this model, the meson-quark interactions are adopted by the effective chiral Lagrangian \[ H_m = \frac{1}{f_m} \bar{\psi}_j \gamma^\mu \gamma_5 \psi_j \partial_\mu \phi_m, \] (1)

where \( \psi_j \) represents the \( j \)-th quark field in a hadron, \( f_m \) is the meson’s decay constant, and \( \phi_m \) is the field of the pseudoscalar-meson octet. Then the \( s \) and \( u \)-channel transition amplitudes \( M_s \) and \( M_u \) can be worked out with the relations [58]:

\[
M_s = \sum_j \langle N_f | H_m^j | N_i \rangle \frac{1}{E_i + \omega_i - E_j} \langle N_i | H_m^j | N_f \rangle,
\]
(2)

\[
M_u = \sum_j \langle N_f | H_m^j | N_i \rangle \frac{1}{E_i - \omega_i - E_j} \langle N_i | H_m^j | N_f \rangle.
\]
(3)

In the above equations, \( \omega \) and \( \omega_j \) are the energies of the incoming and outgoing mesons, respectively. \( |N_i\rangle, |N_f\rangle \) and \( |N_j\rangle \) stand for the initial, intermediate, and final states, respectively, and their corresponding energies \( E_i, E_j \), and \( E_f \) are the eigenvalues of the nonrelativistic Hamiltonian of the constituent quark model \( \hat{H} \). In our previous work \[58,59\], the amplitudes \( M_s \) and \( M_u \) have been worked out in the harmonic oscillator basis.

The \( t \)-channel backgrounds might play an important role in the reactions, thus, the \( t \)-channel contributions of vector exchange and the scalar exchange are considered in this work. The vector meson-quark and scalar meson-quark interactions are adopted by \[54,55\]

\[
H_V = \frac{g_{Vq} b m}{2m_H} (a \gamma^\mu \bar{\psi}_j \gamma_5 \psi_j) V_i \phi_m,
\]
(4)

\[
H_S = \frac{g_{Sqq} \bar{\psi}_j \gamma_5 \psi_j} {2m_H} S.
\]
(5)

Meanwhile, the \( VPP \) and \( SPP \) couplings are adopted as

\[
H_{VPP} = -ig_{V} T_r \langle \phi_m, \phi_m | V | \phi_m \rangle,
\]
(6)

\[
H_{SPP} = \frac{g_{SPP}}{2m_H} \partial_\mu \phi_m \partial^\mu \phi_m
\]
(7)

where \( V, P \) and \( S \) stand for the vector-, pseudoscalar-, scalar-meson fields, respectively. The coupling constants \( a, b, g_{Vq}, \) \( G_V, \) and \( g_{SPP} \) are to be determined by experimental data. In this work, both the scalar \( \sigma \) and \( \sigma \)-vector \( \rho \)-meson exchanges are considered for the \( \pi^+ p \rightarrow \pi^+ p \) and \( \pi^- p \rightarrow \pi^- p \) processes, while the vector \( \rho \)-meson exchange is only considered for the \( \pi^- p \rightarrow \pi^0 p \) process. The details of the \( t \)-channel transition amplitude can be found in our previous work \[58\].

Furthermore, the backgrounds from the Coulomb interactions and the contract term may play some roles in the reactions at low energies. To include the contributions from the contract term (meson-meson-quark-quark interaction), we adopt an effective chiral Lagrangian \[57\]:

\[
H_{contact} = \frac{i}{4f_m} \bar{\psi}_j \gamma^\mu \{ \phi_m, (\partial_\mu \phi_m) \}, \]
(8)

To include the contributions of Coulomb interactions, we follow the method developed in Refs. \[58-61\]. The details of the amplitudes for the Coulomb term can be found in Ref. \[61\].

In this work, we focus on the contributions of the \( s \)-channel resonances, which are degenerate within the same principle number \( n \). To obtain the contributions of individual resonances, we need to separate out the single-resonance-excitation amplitudes within each principle number \( n \) in the \( s \) channel. Taking into account the width effects of the resonances, the resonance transition amplitudes of \( s \) channel can be generally expressed as \[55,56\]

\[
M_R = \frac{2M_R}{s - M_R^2 + iM_R \Gamma_R} O_R e^{-k \sigma^2/(2s^2)},
\]
(9)

where \( \sigma \) is the spin operator of the nucleon, \( n \equiv q \times k/(|k| \times q) \). \( f(\theta) \) and \( g(\theta) \) stand for the non-spin-flip and spin-flip amplitudes, respectively, which can be expanded in terms of the familiar partial wave amplitudes \( T_{ls} \) for the states with \( J = l \pm 1/2 \):

\[
f(\theta) = \sum_{l=0}^{\infty} [(l + 1)T_{ls} + iT_{ls}] \rho_l (\cos \theta),
\]
(11)

\[
g(\theta) = \sum_{l=0}^{\infty} [T_{ls} - T_{ls}] \sin \theta \rho_l (\cos \theta).
\]
(12)

Both the isospin-\(1/2\) and isospin-\(3/2\) resonances contribute to the \( \pi^+ p \rightarrow \pi^+ p, \pi^0 n \) reactions. Thus, we need separate out the isospin-\(1/2\) and \(3/2\) resonance contributions from these reaction amplitudes. As we know, the partial wave amplitudes \( T_{ls} \) for the \( \pi N \rightarrow \pi N \) reactions can be decomposed into the linear combinations of \( s \)-channel isospin amplitudes with the relations

\[
T_{ls}(\pi^+ p \rightarrow \pi^+ p) = \frac{1}{2} T_{ls}^{1/2},
\]
(13)

\[
T_{ls}(\pi^- p \rightarrow \pi^- p) = \frac{1}{2} (2T_{ls}^{1/2} + T_{ls}^{3/2}),
\]
(14)

\[
T_{ls}(\pi^- p \rightarrow \pi^0 n) = \frac{3}{4} (T_{ls}^{1/2} - T_{ls}^{3/2}),
\]
(15)

where \( T_{ls}^{1/2} \) and \( T_{ls}^{3/2} \) correspond to the isospin-\(1/2\) and \(3/2\) resonance contributions, respectively. Using these relations, we can separate out \( s \)-channel isospin contributions from the \( \pi N \rightarrow \pi N \) amplitudes.

In the SU(6)\(\otimes\)O(3) symmetry limit, we have extracted the amplitudes for each \( s \)-channel resonances within \( n \leq 2 \) shell for the \( \pi^+ p \rightarrow \pi^+ p, \pi^- p \rightarrow \pi^- p \) and \( \pi^+ p \rightarrow \pi^0 n \) processes. Our results are listed in Tables \[1\] and \[4\]. Comparing the amplitudes of different resonances with each other, one can easily...
TABLE I: Parameters.

| Constituent quark mass | $M_u$ | 330 MeV |
|------------------------|-------|-------|
| $M_d$                   | 330 MeV |
| $M_s$                   | 450 MeV |
| Harmonic oscillator parameter | $\alpha$ | 400 MeV |
| degenerate masses of the $n = 1, 2$ shell resonances | $M_1$ | 1650 MeV |
| $M_2$ | 1750 MeV |
| Parameters in $t$ channel | $G_{\pi\alpha}(\pi\pi)$ | 12 |
| $g_{\pi\rho\pi}^{\sigma}$ | 65 |
| $m_\rho$ | 770 MeV |
| $m_\sigma$ | 450 MeV |
| $\pi NN$ coupling | $g_{\pi NN}$ | 13.48 |

In the $t$ channel, there are two parameters, the coupling constants $G_{\pi\alpha}$ from the $\rho$-meson exchange and $g_{\pi\rho\pi}^{\sigma}$ from the $\sigma$-meson exchange. We determine these parameters by fitting the data, which are listed in Table I. It should be mentioned that the coupling constants $G_{\pi\alpha}$ and $g_{\pi\rho\pi}^{\sigma}$ bear about a 30% uncertainty.

In our framework, the $s$-channel resonance transition amplitude, $O_R$, is derived in the SU(6)$\otimes$O(3) symmetry limit. In reality, the symmetry of SU(6)$\otimes$O(3) is generally broken owing to some reasons. To accommodate the symmetry-breaking and hadronic dressing effects, following the idea of Ref. [47], we introduce a set coupling strength parameters, $C_R$, for each resonance amplitude,

$$O_R \rightarrow C_R O_R,$$  

where $C_R$ should be determined by fitting the data. The deviations of $C_R$ from unity imply the SU(6)$\otimes$O(3) symmetry breaking. The determined $C_R$ values are listed in Table I. It is found that the $C_R$ parameters for the $\Delta(1232)$ and $N(1440)$ resonances are notably larger than 1. Thus, the SU(6)$\otimes$O(3) symmetry of these states is seriously broken by some effects, which will be discussed later in details.

Furthermore, the masses and widths for the $s$-channel resonances are important input parameters in the calculations. For the main resonances $\Delta(1232)P_{33}$ and $N(1440)P_{11}$, we vary their masses and widths in a proper range to better describe the data. To be consistent with our previous study, we take the masses and widths of $N(1535)S_{11}$ and $N(1520)D_{13}$ from [54], where the resonances parameters of $N(1535)S_{11}$ and $N(1520)D_{13}$ are well constrained. The other resonances have few effects on the reactions, thus, their masses and widths are taken from the PDG [4], or the constituent quark model predictions [54,56] if no experimental data are available. The masses and widths for some low-lying resonances have been listed in Table II. It is found that the mass and width for the $\Delta(1232)P_{33}$ are $M \approx 1212$ MeV and $\Gamma \approx 100$ MeV, respectively, which are consistent with those extracted from the neutral pion photoproduction processes in our previous work [56]. It should be emphasized that our extracted mass and width for the $\Delta(1232)P_{33}$ are quite close to the values of the pole parameterization from the PDG [4]. The reason is that, when we fit the data a constant resonance width $\Gamma_R$ is used, which is similar to the pole parameterization. Furthermore, we find that the $N(1440)P_{11}$ resonance seems to favour a narrow width $\Gamma \approx 200$ MeV, which is also comparable to the values of the pole parameterization from the PDG [4].

In the $u$ channel, it is found that contributions from the $n \geq 1$ shell resonances are negligibly small and insensitive to their masses. Thus, the degenerate masses for the $n = 1, 2$ shell resonances are taken in our calculations. The values have been listed in Table II.

Finally, it should be pointed out that all the adjustable parameters are determined by globally fitting the measured differential cross sections, which are obtained from [38]. For the $\pi^+ p \rightarrow \pi^+ p$ reaction, we fit the differential cross sections in the incoming pion-meson momentum range $P_\pi = 260 \sim 420$ MeV/c, while for the $\pi^+ p \rightarrow \pi^+ p, \pi^+ n$ reactions, we fit the measured differential cross sections in the range $P_\pi = 260 \sim$...
FIG. 1: Differential cross sections of the $\pi^+ p \rightarrow \pi^+ p$ reaction compared with the experimental data (open circles) from [64–68] and the solutions (dotted curves) from the GWU group [38]. The first and second numbers in each figure correspond to the incoming $\pi$ momentum $P_\pi$ (MeV) and the $\pi N$ center-of-mass (c.m.) energy $W$ (MeV), respectively.

FIG. 2: Differential cross sections of the $\pi^- p \rightarrow \pi^- p$ reaction compared with the experimental data (open circles) from [64–68] and the solutions (dotted curves) from the GWU group [38]. The first and second numbers in each figure correspond to the incoming $\pi$ momentum $P_\pi$ (MeV) and the $\pi N$ c.m. energy $W$ (MeV), respectively.
FIG. 3: Differential cross sections of the reaction $\pi^- p \rightarrow \pi^0 n$ compared with the experimental data (open circles) from [69–74] and the solutions (dotted curves) from the GWU group [38]. The first and second numbers in each figure correspond to the incoming $\pi$ momentum $P_\pi$ (MeV) and the $\pi N$ c.m. energy $W$ (MeV), respectively.

FIG. 4: Polarizations of the $\pi^+ p \rightarrow \pi^+ p$ reaction compared with experimental data (open circles) from [75–82] and the solutions (dotted curves) from the GWU group [38]. The first and second numbers in each figure correspond to the incoming $\pi$ momentum $P_\pi$ (MeV) and the $\pi N$ c.m. energy $W$ (MeV), respectively.
TABLE II: Masses $M_R$ (MeV) and widths $\Gamma_R$ (MeV) of $s$-channel intermediate states, and their $C_R$ strength parameters.

| Resonance          | $\pi^+ p \rightarrow \pi^+ p$ | $\pi^+ p \rightarrow \pi^+ p$ | $\pi^+ p \rightarrow \pi^+ p$ |
|--------------------|--------------------------------|--------------------------------|--------------------------------|
|                    | $\Gamma_R$ | $M_R$ | $C_R$ | $\Gamma_R$ | $M_R$ | $C_R$ | $\Gamma_R$ | $M_R$ | $C_R$ |
| $N(938)P_{11}$     | ...        | ...   | ...   | ...        | ...   | ...   | ...        | ...   | ...   |
| $\Delta(1232)P_{33}$ | $100^{+7}_{-3}$ | $1212^{+13}_{-5}$ | $2.95^{+0.02}_{-0.20}$ | $100^{+7}_{-3}$ | $1212^{+13}_{-5}$ | $3.00^{+0.02}_{-0.15}$ | $100^{+7}_{-3}$ | $1212^{+13}_{-5}$ | $3.10^{+0.02}_{-0.15}$ |
| $N(1440)P_{11}$   | ...        | ...   | ...   | ...        | ...   | ...   | ...        | ...   | ...   |
| $N(1535)S_{11}$   | ...        | ...   | ...   | ...        | ...   | ...   | ...        | ...   | ...   |
| $\Delta(1620)S_{11}$ | $140$     | $1630$ | $1.0$  | $140$     | $1630$ | $1.0$  | $140$     | $1630$ | $1.0$  |
| $N(1650)S_{11}$   | ...        | ...   | ...   | ...        | ...   | ...   | ...        | ...   | ...   |
| $\Delta(1700)D_{33}$ | $300$     | $1745$ | $1.0$  | $300$     | $1745$ | $1.0$  | $300$     | $1745$ | $1.0$  |
| $N(1520)D_{13}$   | ...        | ...   | ...   | ...        | ...   | ...   | ...        | ...   | ...   |
| $N(1700)D_{13}$   | ...        | ...   | ...   | ...        | ...   | ...   | ...        | ...   | ...   |
| $N(1675)D_{15}$   | ...        | ...   | ...   | ...        | ...   | ...   | ...        | ...   | ...   |

TABLE III: Reduced $\chi^2$ per data point of the full model and that with one resonance or one background switched off obtained in a fit of the measured differential cross sections of the $\pi^+ p \rightarrow \pi^+ p$ and $\pi^+ p \rightarrow \pi^+ p, \pi^+ n$ reactions. Their corresponding $\chi^2$ s are labeled with $\chi^2_{s,p}$, $\chi^2_{t,p}$ and $\chi^2_{\text{pole}}$, respectively.

|                      | full model | $\pi$ pole | $\Delta(1232)P_{33}$ | $N(1535)S_{11}$ | $N(1520)D_{13}$ | $N(1440)P_{11}$ | $u$ channel | $t$ channel |
|----------------------|------------|------------|----------------------|-----------------|-----------------|-----------------|-------------|-------------|
| $\chi^2_{s,p}$       | 3.21       | 74.84      | ...                  | ...             | ...             | ...             | 5.10        | 7.71        |
| $\chi^2_{t,p}$       | 2.48       | 9.89       | 59.40                | 3.29            | 2.49            | 4.55            | 8.36        | 4.33        |
| $\chi^2_{\text{pole}}$ | 3.60       | 5.70       | 147.21               | 8.15            | 3.87            | 4.80            | 19.14       | 7.77        |

540 MeV/c. The data sets used in our fits are shown in Figs. 1, 2 and 3. The reduced $\chi^2$s per data point obtained in our fits are listed in Table III. To clearly see the role of one component in the results, the $\chi^2$s with one resonance or one background switched off are also given in the Table III.

B. $\pi^+ p \rightarrow \pi^+ p$

The $\pi^+ p \rightarrow \pi^+ p$ process provides us a rather clear channel to study the $\Delta$ resonances, because only the isospin 3/2 resonances contribute here for the isospin selection rule. The low-lying $\Delta$ resonances classified in the quark model are listed in Table IV. From the table we can see that in a rather wide $\pi N$ center-of-mass (c.m.) energy range $W < 1.6$ GeV, only the $\Delta(1232)P_{33}$ resonance lies. The higher resonances are the $S$-wave state $\Delta(1620)S_{31}$ and the $D$-wave state $\Delta(1700)D_{33}$, which may mainly contribute to the reaction in the higher energy range $W > 1.5$ GeV. Thus, the description of the $\pi^+ p \rightarrow \pi^+ p$ reaction in the low energy region becomes relatively simple.

The chiral quark model allows us study the $\pi^+ p \rightarrow \pi^+ p$ reaction from the $\Delta(1232)$ resonance region up to $W \approx 1.4$ GeV. Our fits of the differential cross sections and polarizations compared with the data are shown in Figs. 1 and 2 respectively. From these figures, it is found that our fits are in a global agreement with the experimental data in the c.m. energy range $W \approx 1.2 - 1.4$ GeV, although in our calculations the polarizations are overestimated slightly at the backward angles, and the cross sections are overestimated slightly in the region $W > 1.3$ GeV. New precise measurements with a good angle coverage are hoped to be carried out in the future. For the limitations of the present model, our study cannot cover the higher energy region $W > 1.4$ GeV.

To clearly understand the reaction mechanism of $\pi^+ p \rightarrow \pi^+ p$, we show the main contributors one by one in Fig. 2. It is found that the interferences between the $\Delta(1232)P_{33}$ resonance and the backgrounds of the $u$ and $t$ channels can roughly explain the $\pi^+ p \rightarrow \pi^+ p$ reaction up to $W \approx 1.4$ GeV. The Coulomb interactions may play an obvious role at the extremely forward angles. Slight effects from the contact term can also be seen at the forward and backward angles. The behavior of the contact term is similar to that of the $t$ channel. No obvious effects of the higher resonances, such as $\Delta(1620)S_{31}$ and $\Delta(1700)D_{33}$, are found in the low energy region what we consider.

The cross sections around $W = 1.2$ GeV are sensitive to the mass and width of $\Delta(1232)P_{33}$, which provide us a good place to constrain the resonance parameters of $\Delta(1232)P_{33}$. By fitting the measured total cross section with a momentum independent width (see Fig. 3), we obtain that the mass and width of $\Delta(1232)P_{33}$ are $M \approx 1212$ MeV and $\Gamma \approx 100$ MeV, respectively, with a uncertainty of several MeV. These determined mass and width of $\Delta(1232)P_{33}$ are consistent with our recent analysis of the pions photoproduction reactions [36] and also are quite close to the values of the pole parametrization from the PDG [3].

Finally, it should be pointed out that to obtain a better description of the data, we should enhance the $\Delta(1232)P_{33}$ contribution with a factor of $C_\Delta \approx 3.0$, which indicates that the
$\Delta(1232) N\pi$ coupling may be underestimated by a factor of $\sim \sqrt{3}$ in the SU(6)$\otimes$O(3) symmetry limit. This underestimation is also found in the pions photoproduction reactions [36] and the strong decays of $\Delta(1232) P_{33}$ [86]. The $\Delta(1232) P_{33}$ might not be a pure three-quark state [87–92], some other contributions, such as meson-baryon component, may alter the $\Delta(1232) N\pi$ coupling.

As a whole, from the $\Delta(1232)$ resonance region up to the $N(1440)$ resonance region, the $\pi^+ p$ elastic scattering can be reasonably understood with the interferences between the $\Delta(1232) P_{33}$ resonance and the backgrounds of the $u$ and $t$ channels. The extracted mass and width of $\Delta(1232) P_{33}$ are quite close to the values of the pole parametrization [4]. The large $\Delta(1232) N\pi$ coupling out of the quark model prediction may indicate that $\Delta(1232) P_{33}$ may not be a pure three-quark state.

C. $\pi^- p \rightarrow \pi^- p, \pi^0 n$

Both the isospin-1/2 and -3/2 resonances contribute to the $\pi^- p \rightarrow \pi^- p, \pi^0 n$ reactions. From the spectrum classified in the quark model (see Table IV), we find that only the $\Delta(1232) P_{33}$ and $N(1440) P_{11}$ resonances lie within the $N(1440) P_{11}$ resonance region. The higher resonances

\[ N_{\pm} = N_{1535, 1650} S_{11}, N_{1700} D_{13} \]

are far from the $N(1440) P_{11}$ resonance region, thus, their affects on these reactions should be small within this energy region. In this sense, the $\pi^- p \rightarrow \pi^- p, \pi^0 n$ reactions might be good places to study the properties of the $\Delta(1232) P_{33}$ and $N(1440) P_{11}$ resonances.

Based on our good understanding of $\pi^+ p \rightarrow \pi^+ p$, we further study the $\pi^+ p \rightarrow \pi^+ p, \pi^0 n$ reactions. Our fits of the differential cross sections, total cross sections, and polarizations compared with the data are shown in Figs. 6. From these figures, it is found that the experimental data from $\Delta(1232)$ resonance region up to the $N(1440)$ resonance region are reasonably described within the chiral quark model. It should be mentioned that there are remaining discrepancies.
in the polarizations of $\pi^- p \rightarrow \pi^- p$ below $W = 1.3$ GeV. To gain more knowledge of these reactions, new precise measurements of the polarizations with a good angle and energy coverage is hoped to be carried out in the future.

To clearly understand the low energy reactions $\pi^- p \rightarrow \pi^- p, \pi^0 n$, we show the main contributions to the differential cross sections and polarizations in Figs. 10 and 11 respectively. From these figures, it is found that besides $\Delta(1232)P_{33}$, the Roper $N(1440)P_{11}$ plays a crucial role in the $\pi^- p \rightarrow \pi^- p, \pi^0 n$ reactions. Switching off their contributions, we find that the differential cross sections and polarizations have a notable change at both forward and backward angles. It should be emphasized that a confirmed role of $N(1440)P_{11}$ can be more obviously seen from the polarizations. Slight contributions from the $N(1535)S_{11}$ and $D(1520)D_{13}$ resonances can extend to the $N(1440)$ resonance region as well, for simplicity, we do not show them in the figures. No obvious effects of the higher resonances, such as $\Delta(1620)S_{11}$, $\Delta(1700)D_{33}$, $D(1700)D_{13}$, and $D(1675)D_{15}$ in the low energy regions. The backgrounds from the $s$-channel nucleon pole, $u$ and $t$ channels play important roles in the reactions. The Coulomb interactions may play an obvious role at the extremely forward angles. Slight effects from the contact term can also be seen at the forward and backward angles.

Furthermore, to better understand the properties of the $\Delta(1232)P_{33}$ and $N(1440)P_{11}$ resonances, we also show our fits of the $P_{11}$ and $P_{33}$ amplitudes to the solution WI08 \cite{10} from the GWU group \cite{38} in fig. 9. Our results show a good agreement with the solution WI08. Beyond the mass threshold of $N(1440)P_{11}$, although the real part of the $P_{11}$ amplitude is overestimated in our quark model, its tendency is similar to the solution WI08. It should be mentioned that in the higher energy region $W \approx 1.4$ GeV, our quark model begins to lose its prediction ability, thus, our extracted properties of $N(1440)P_{11}$ may be less reliable than those of $\Delta(1232)P_{33}$.

In the $\pi^- p \rightarrow \pi^- p, \pi^0 n$ reactions, to obtain a good description of the data we also need enhance the contribution of $\Delta(1232)P_{33}$ from the symmetric quark model with a rather large factor $C_{N(1440)} \approx 23$, i.e., the $N(1440)\pi$ coupling is a factor $\approx 4.8$ larger than the prediction with the simple three-quark model, which was also found by analyzing the strong decays of $N(1440)P_{11}$ in Refs. \cite{102, 103}. The unexpected large $N(1440)\pi$ coupling indicates the exotic nature of the $N(1440)P_{11}$ resonance. About the unusual properties of $N(1440)$, there are many discussions in the literature \cite{11, 26, 104–114}.}

As a whole, besides the $\Delta(1232)P_{33}$ resonance, confirmed evidence of the $N(1440)P_{11}$ resonance is found in the polarizations of the $\pi^- p \rightarrow \pi^- p, \pi^0 n$ reactions. The couplings of the $\Delta(1232)\pi$ and $N(1440)\pi$ predicted from the simple three-quark model are about 1.7 and 4.8 times smaller than the values extracted from the experimental data, respectively, which indicates that the $\Delta(1232)P_{33}$ and $N(1440)P_{11}$ resonances cannot be pure three-quark states. Finally, it should be mentioned that the $s$-channel nucleon pole, $u$- and $t$-channel backgrounds play important roles in the reactions.

IV. SUMMARY

In this work, a combined study of the $\pi^- p \rightarrow \pi^- p, \pi^- p \rightarrow \pi^- p$ and $\pi^- p \rightarrow \pi^0 n$ reactions have been carried out within a chiral quark model. Our results show a good global agreement with the data within the $N(1440)$ resonance region.

In these reactions, the resonance properties of $\Delta(1232)P_{33}$ are constrained. By fitting the data with a momentum independent width, we obtain the mass and width of $\Delta(1232)P_{33}$ are $M \approx 1212$ MeV and $\Gamma \approx 100$ MeV, which are consistent with our recent analysis of the pions photoproduction reactions \cite{36}, and are quite close to the values determined with the pole parametrization \cite{4}. The $\Delta(1232)N\pi$ coupling from the quark model is about a factor of $\approx 1.7$ smaller than that extracted from the data. Some exotic components, such as the meson-baryon component, may alter the $\Delta(1232)N\pi$ coupling, which should be studied further.

Confirmed roles of $N(1440)P_{11}$ resonance are found in both the $\pi^- p \rightarrow \pi^- p$ and $\pi^- p \rightarrow \pi^0 n$ reactions. The $N(1440)P_{11}$ has notable contributions to the polarizations, although no obvious effects can be seen in the total cross sections. The extracted mass and width for $N(1440)P_{11}$ are $M \approx 1400$ MeV and $\Gamma \approx 200$ MeV, respectively, which are close to the values determined with the pole parametrization \cite{4}. The $N(1440)\pi\pi$ coupling extracted from the data is a factor of $\approx 4.8$ larger than the symmetric quark model prediction. The unexpected large $N(1440)\pi\pi$ coupling suggests the uncommon properties of the $N(1440)P_{11}$ resonance.

Starting from the incoming $\pi$-meson momentum $p_\pi \approx 440$ MeV/c, slight contributions of $N(1535)S_{11}$ and $N(1520)D_{13}$ are seen in both the $\pi^- p \rightarrow \pi^- p$ and $\pi^- p \rightarrow \pi^0 n$ reactions. The backgrounds play remarkable roles in these three strong interaction processes. The $t$- and $u$-channel backgrounds have notable contributions to the $\pi^- p \rightarrow \pi^- p$ reactions. While in the $\pi^- p \rightarrow \pi^- p, \pi^0 n$ reactions, the $s$-channel nucleon pole and $t$- and $u$-channel backgrounds play an important role.

Finally, it should be pointed out that with present model
FIG. 7: Polarizations of the $\pi^- p \rightarrow \pi^- p$ reaction compared with experimental data (open circles) from $[75, 76, 93–95]$ and the solutions (dotted curves) from the GWU group $[38]$. The first and second numbers in each figure correspond to the incoming $\pi$ momentum $P_\pi$ (MeV) and the $\pi N$ c.m. energy $W$ (MeV), respectively.

FIG. 8: Polarizations of the $\pi^- p \rightarrow \pi^0 n$ reaction compared with the experimental data (open circles) from $[74, 94, 96–98]$ and the solutions (dotted curves) from the GWU group $[38]$. The first and second numbers in each figure correspond to the incoming $\pi$ momentum $P_\pi$ (MeV) and the $\pi N$ c.m. energy $W$ (MeV), respectively.
we cannot deliver higher accuracy descriptions of the data because there are only a few adjustable parameters based on the SU(6)⊗O(3) symmetry, and all of the interactions are limited at the tree level. Furthermore, our present study is difficult to cover the whole \(N(1440)P_{11}\) resonance region, thus, the extracted properties of \(N(1440)P_{11}\) may bear a large uncertainty although confirmed roles of \(N(1440)P_{11}\) are found in the polarizations. To uncover the uncommon nature of \(N(1440)P_{11}\), new precise measurements of the polarizations with a good angle and energy coverage are expected to be carried out in the future.

![Graph of πN partial amplitudes]

**FIG. 9:** The \(πN\) partial amplitudes of \(P_{33}\) and \(P_{11}\). Solid (dashed) curves give the real (imaginary) parts of amplitudes extracted from our quark model. The filled (open) circles stand for the real (imaginary) parts (solution WI08 \([10]\)) extracted by the GWU group \([38]\).

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FIG. 10: Effects of the main contributors on the differential cross sections (left side) and polarizations (right side) of the reaction \( \pi^- p \rightarrow \pi^- p \). The experimental data are taken from [64–68, 76]. The bold solid curves correspond to the full model result. The results by switching off the one of the main contributors in the resonances and non-resonance backgrounds are indicated explicitly by the legend in the figures. The first and second numbers in each figure correspond to the incoming \( \pi^- \) momentum \( P_\pi \) (MeV) and the \( \pi^- \) c.m. energy \( W \) (MeV), respectively.

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TABLE IV: The s-channel resonance amplitudes within n=2 shell for the \( \pi^+ p \to \pi^+ p \), \( \pi^- p \to \pi^- p \) and \( \pi^- p \to \pi^0 n \) processes. We have defined \( M_5 \equiv \frac{1}{x} + |A_{\text{in}}\bar{\nu}\bar{\sigma}| \frac{1}{x} + |A_{\text{out}}\bar{\nu}\bar{\sigma}| \), \( M_6 \equiv \frac{1}{x} + |A_{\text{in}}\nu\sigma| \frac{1}{x} + |A_{\text{out}}\nu\sigma| \), \( M_{30} \equiv \frac{1}{x} + |A_{\text{in}}\nu\sigma| \frac{1}{x} + |A_{\text{out}}\nu\sigma| \), \( M_{D1} \equiv |A_{\text{out}}\nu\sigma| \), \( M_{D2} \equiv |A_{\text{out}}\nu\sigma| \). The functions \( A_{\text{in}} \) and \( A_{\text{out}} \) are defined by \( A_{\text{in}} \equiv -(\frac{q_{\text{out}}}{q_{\text{in}}}) + 1 \) and \( A_{\text{out}} \equiv -(\frac{q_{\text{in}}}{q_{\text{out}}}) + 1 \), respectively. The \( \mu_6 \) is a reduced mass at the quark model level, which equals to \( \frac{1}{\mu_6} = 2/m_0 \) for \( \pi N \) scattering processes.

| resonance | \([N_\pi, N_\Delta, N_\eta] \) | \( O_8 \) | \( \pi^+ p \to \pi^+ p \) | \( \pi^- p \to \pi^- p \) | \( \pi^- p \to \pi^0 n \) |
|-----------|----------------|--------|-----------------|-----------------|-----------------|
| \( N(938)P_{11} \) | \([56, 2, 0, 0] \) | \( f(0) \) | \( \frac{1}{x} - \frac{\mu_6}{x} |A_{\text{in}}\nu\sigma| \frac{1}{x} + |A_{\text{out}}\nu\sigma| \) | \( \frac{1}{x} - \frac{\mu_6}{x} |A_{\text{in}}\nu\sigma| \frac{1}{x} + |A_{\text{out}}\nu\sigma| \) | \( \frac{1}{x} - \frac{\mu_6}{x} |A_{\text{in}}\nu\sigma| \frac{1}{x} + |A_{\text{out}}\nu\sigma| \) |
| \( \Delta(1232)P_{33} \) | \([56, 4, 10, 0, 0] \) | \( g(0) \) | \( \frac{1}{2} |A_{\text{in}}\nu\sigma| \) | \( \frac{1}{2} |A_{\text{in}}\nu\sigma| \) | \( \frac{1}{2} |A_{\text{in}}\nu\sigma| \) |
| \( \Delta(1620)S_{31} \) | \([70, 2, 10, 1, 1] \) | \( f(0) \) | \( \frac{1}{2} M_5 a^2 \) | \( \frac{1}{2} M_5 a^2 \) | \( \frac{1}{2} M_5 a^2 \) |
| \( \Delta(1650)S_{11} \) | \([70, 4, 8, 1, 1] \) | \( f(0) \) | \( \frac{1}{2} M_5 a^2 \) | \( \frac{1}{2} M_5 a^2 \) | \( \frac{1}{2} M_5 a^2 \) |

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TABLE V: A continuation of Table IV.

| Resonance  | $|N_{6},2S+1\ N_{5},n,f|$ | $Q_{R}$ | $\pi^{+}p \rightarrow \pi^{0}n$ | $\pi^{+}p \rightarrow \pi^{0}p$ | $\pi^{+}p \rightarrow \pi^{0}n$ |
|------------|--------------------------|--------|----------------|----------------|----------------|
| $\Delta(1920)P_{33}$ | $[56,4\ 10,2,2]$ | $f(0)$ | $\frac{1}{\sqrt{2}}\frac{M_{F_{2}}}{M_{P_{2}}}kq|qP_{1}(z)$ | $-\frac{1}{\sqrt{2}}\frac{M_{F_{2}}}{M_{P_{2}}}kq|qP_{1}(z)$ | $-\frac{1}{\sqrt{2}}\frac{M_{F_{2}}}{M_{P_{2}}}kq|qP_{1}(z)$ |
| | | $g(0)$ | $\frac{1}{\sqrt{2}}\frac{M_{F_{2}}}{M_{P_{2}}}kq|qP_{1}(z)$ | $-\frac{1}{\sqrt{2}}\frac{M_{F_{2}}}{M_{P_{2}}}kq|qP_{1}(z)$ | $-\frac{1}{\sqrt{2}}\frac{M_{F_{2}}}{M_{P_{2}}}kq|qP_{1}(z)$ |
| $N(1900)P_{13}$ | $[70,2\ 8,2,2]$ | $f(0)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| | | $g(0)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $N(?P)_{33}$ | $[70,4\ 8,2,2]$ | $f(0)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| | | $g(0)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| $\Delta(1920)F_{15}$ | $[56,4\ 8,2,2]$ | $f(0)$ | $\cdots$ | $\cdots$ | $\cdots$ |
| | | $g(0)$ | $\cdots$ | $\cdots$ | $\cdots$ |

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