Minimalistic Attacks: How Little it Takes to Fool Deep Reinforcement Learning Policies

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Abstract—Recent studies have revealed that neural network based policies can be easily fooled by adversarial examples. However, while most prior works analyze the effects of perturbing every pixel of every frame using full black-box policy access, in this paper we take a more restrictive view towards adversary generation - with the goal of unveiling the limits of a model's vulnerability. In particular, we explore minimalistic attacks by defining three key settings: (1) black-box policy access: where the attacker only has access to the input (state) and output (action probability) of an RL policy; (2) fractional-state adversary: where only a few pixels are perturbed, with the extreme case being a single-pixel adversary; and (3) tactically-chanced attack: where the attacker only significant frames are tactically chosen to be attacked. We formulate the adversarial attack by accommodating the three key settings, and explore their potency on six Atari games by examining four fully trained state-of-the-art policies. In Breakout, for example, we surprisingly find that: (i) all policies showcase significant performance degradation by merely modifying 0.01% of the input state, and (ii) the policy trained by DQN is totally deceived by perturbation to only 1% frames.

Index Terms—Reinforcement Learning, Adversarial Attack.

I. INTRODUCTION

Deep learning [1] has been widely regarded as a promising technique in reinforcement learning (RL), where the goal of an RL agent is to maximize its expected accumulated reward by interacting with a given environment. Although deep neural network (DNN) policies have achieved super human performance on various challenging tasks (e.g., video games, robotics and classical control [2]), recent studies have shown that these policies are easily deceived under adversarial attacks [3]–[5]. These works are however found to make some common assumptions, viz., (1) white-box policy access: where the adversarial examples are analytically computed by back-propagating through known neural network weights, (2) full-state adversary: where the adversary changes almost all pixels in the state, and (3) fully-chanced attack: where the attacker strikes the policy at every frame.

Given that most prior works analyze the effects of perturbing every pixel of every frame assuming white-box policy access, we propose to take a more restricted view towards adversary generation - with the goal of exploring the limits of a DNN model's vulnerability in RL. In this paper, we thus focus on minimalistic attacks by only considering adversarial examples that perturb limited number of pixels in selected frames, and under the restricted black-box policy access. In other words, we intend to unveil how little it really takes to successfully fool state-of-the-art RL policies. Our study is based on three restrictive settings, namely, black-box policy access, fractional-state adversary, and tactically-chanced attack. These concepts are detailed next.

Black-box Policy Access (BPA). Most previous studies focus on a white-box setting [5], that allows full access to a policy network for back-propagation. However, most systems do not release their internal configurations (i.e., network structure and weights), only allowing the model to be queried; this makes the white-box assumption too optimistic from an attacker’s perspective [6]. In contrast, we use a BPA setting, where the attacker only has access to the input and output of a policy.

Fractional-State Adversary (FSA). In the FSA setting, the adversary only perturbs a small fraction of the input state. This, in the extreme situation, corresponds to the single-pixel attack shown in Fig. 1, where perturbing a single pixel of the input state is found to change the action prescription from ‘RIGHT’ to ‘LEFT’. In contrast, most previous efforts [5] are mainly based on a full-state adversary (i.e., the number of modified pixels is fairly large, usually spanning the entire frame).

Tactically-Chanced Attack (TCA). In previously studied RL adversarial attacks [3], [7], [8], the adversary strikes the policy on every frame of an episode; this is a setting termed as the fully-chanced attack. Contrarily, we investigate a relatively restrictive case where the attacker only strikes at a few selected frames - a setting we term as tactically-chanced attack, where...
a minimal number of frames is explored to strategically deceive the policy network.

The proposed restrictive settings are deemed to be significant for safety-critical RL applications, such as in the medical treatment of sepsis patients in intensive care units [9], [10] and treatment strategies for HIV [11]. In such domains, there may exist a temporal gap between the acquisition of the input and the action execution; thereby, providing a time window in which a tactical attack could take place. In the future, as we move towards integrating end-to-end vision-based systems for healthcare, the sensitivity of prescribed medical actions (such as drug prescription or other medical interventions) to perturbations in medical images poses a severe threat to the utility of RL in such domains. Moreover, the restrictive nature of the attacks makes them practically imperceptible, greatly reducing the chance of identifying and rectifying them. With this in mind, the present paper explores the vulnerability of deep RL models to restrictive adversarial attacks, with particular emphasis on image-based tasks (i.e., Atari games).

The major challenges, therefore, lie in how to delicately accommodate the three restricted settings for adversarial attacks in DRL. To this end, we design a mathematical program with a novel objective function to generate the FSA under the black-box setting, and propose an entropy-based uncertainty measurement to achieve the TCA. Note that, the optimization variables are defined as the discrete 2-D coordinate location(s) and perturbation value(s) of the selectively attacked pixel(s), and the designed mathematical program guarantees a successful deception of the policy as long as a positive objective value can be found. The optimization procedure, for a given input frame, is then carried out by a simple genetic algorithm (GA) [12], [13]. In this regard, the Shannon entropy of the action distribution is specified as the attack uncertainty, whereby only a few salient frames are tactically attacked. We demonstrate the sufficiency of the three restrictive settings on four pre-trained state-of-the-art deep RL policies on six Atari games.

To sum up, the contributions of this paper are listed as:

- We unveil how little it takes to deceive an RL policy by considering three restrictive settings, namely, black-box policy access, fractional-state adversary, and tactically-chanced attack;
- We formulate the RL adversarial attack as a black-box optimization problem comprising a novel objective function and discrete FSA optimization variables. We also propose a Shannon entropy-based uncertainty measurement to sparingly select the most vulnerable frames to be attacked;
- We explore the three restrictive settings on the policies trained by four state-of-the-art RL algorithms (i.e., DQN [1], PPO [14], A2C [15], ACKTR [16]) on six representative Atari games (i.e., Pong, Breakout, SpaceInvaders, Seaquest, Qbert, BeamRider);
- Surprisingly, we find that on Breakout: (i) with only a single pixel (≈ 0.01% of the state) attacked, the trained policies are completely deceived; and (ii) by merely attacking around 1% frames, the policy trained by DQN is totally fooled.

The remainder of this paper is organized as follows. In Section II, we provide an overview of related works in RL adversarial attacks, highlighting the novelty of our paper. Section III presents preliminaries on reinforcement learning and adversarial attack. In section IV, we describe our proposed optimization problem formulation and the associated procedure to generate adversarial attacks for deep RL policies. This is followed by Section V, where we report the numerical results to prove the effectiveness of our attacks. Finally, Section VI concludes this paper.

II. RELATED WORK

Since Szegedy et al. [17], a number of adversarial attack methods have been investigated to fool deep neural networks (DNNs). Most existing approaches generate adversarial examples through pursuing human invisible perturbation on images [18]–[20], with the goal of deceiving trained classifiers. In other words, they mainly focus on the adversarial attacks for supervised learning tasks. Adversarial attacks in RL have been relatively less explored to date.

In RL, Huang et al. [3] was the first to demonstrate that neural network policies are vulnerable to adversarial attack by adding small modifications to the input state of Atari games. A full-state adversary (i.e. adversarial examples that change almost every pixel in the input state) has previously been generated by a white-box policy access based approach [21], where the adversarial examples are computed via backpropagation. Lin et al. [4] proposed the strategically-timed attack and the so-called enchanting attack, but the adversary generation is still based on a white-box policy access assumption and full-state perturbation. Besides, Kos et al. [22] compared the influence of full-state perturbations with random noise, and utilized the value function to guide the adversary injection.

In summary, existing works are largely based on white-box policy access, together with assumptions of a full-state adversary and fully-chanced attack. There is little research studying the potency of input perturbations that may be far less extensive. Therefore, our goal in this paper is to investigate an extremely restricted view towards the vulnerability of deep RL models, viz., based on black-box policy access, fractional-state adversary, and tactically-chanced attack. It is contended that studying such restrictive scenarios might give new insights on the geometrical characteristics and overall behavior of deep neural networks in high dimensional spaces [23].

III. PRELIMINARIES

This section first provides the background of RL and several representative approaches to learn RL policies. It then illustrates the basic concepts and definition of adversarial attack.

A. Reinforcement Learning

Reinforcement learning (RL) [24]–[33] can be formulated as a Markov Decision Process (MDP) [24], where the agent interacts with the environment based on the reward and state transition. This decision making process is shown in Fig. 2 where s_t and r_t are the state and reward received from
environment at step $t$, and $a_t$ is the action selected by the agent. Based on $a_t$, the agent interacts with the environment, transitioning to state $s_{t+1}$ and receiving a new reward $r_{t+1}$. This procedure continues until the end of the MDP.

The aim of RL [35] is to find an optimal policy $\pi(\theta^*)$ that maximizes the expected reward:

$$\theta^* = \max_\theta \mathbb{E}\left[ \sum_{t=0}^{T} \gamma^t r(t) | \pi_\theta \right],$$  

where $\theta$ represents the training parameters (e.g., the weights of a neural network); $\sum_{t=0}^{T} \gamma^t r(t) | \pi_\theta$ is the episode rollout; and $\gamma \in (0, 1)$ denotes the discount factor that balances the long- and short-term rewards.

To find out the optimal policy parameters $\theta^*$, many different RL algorithms have been proposed. We select several representative ones for demonstration, including DQN [1], PPO [14], A2C [15], ACKTR [16].

- **Deep Q-Networks (DQN)** [1]: Instead of predicting the probability of each action, DQN computes the Q value for each available action. Such Q value $Q^\pi(s, a)$ represents the expected accumulated discounted reward that is approximated from the current frame. Based on such approximation, DQN is trained via minimizing the temporal difference loss (i.e., squared Bellman error). In this paper, the corresponding policy for DQN is achieved by selecting the action with maximum Q value. Besides, to keep DQN-trained policy consistent with other three algorithms, a soft-max layer is added at the end of DQN.

- **Proximal Policy Optimization (PPO)** [14]: PPO is an off-policy method using policy gradient, and it strikes a balance among ease of implementation, sample complexity, and ease of tuning. PPO is proposed based on trust region policy optimization (TRPO) that solves a constrained optimization problem so as to alleviate performance instability. However, PPO handles this issue in a different manner as compared to TRPO: it involves a penalty term that indicates the Kullback-Leibler divergence between the old policy action prediction and the new one. This operation computes an update at each step, minimizing the cost function while restricting the updating step to be relatively small.

- **Advantage Actor-Critic (A2C)** [36]: A2C is an actor-critic method that learns both a policy and a state value function (i.e., also called critic). In A2C, the advantage $A$ represents the extra reward will get if the action $a_t$ is taken, which is formulated by $A = Q(s_t, a_t) - V(s_t)$, where $Q(s_t, a_t)$ is the state-action value and $V(s_t)$ is the state value. Such design is to reduce variance of the policy gradient and increase stability by involving a baseline term $V(s_t)$ in $A$ for the gradient estimation.

- **Actor Critic using Kronecker-Factored Trust Region (ACKTR)** [16]: ACKTR is an extension of the natural policy gradient, which optimizes both the actor and the critic using Kronecker-factored approximate curvature (K-FAC) with trust region. ACKTR is the first scalable trust region natural gradient method for actor-critic methods. The investigation on ACKTR suggests that Kronecker-factored natural gradient approximations in RL is a promising framework.

Although remarkable performance has been achieved by these algorithms on many challenging tasks (e.g., video games and board games [37], [38]), recent studies have revealed that the policies trained by these algorithms are easily fooled by adversarial perturbations [3][4][20], as introduced next.

B. Adversarial Attack

Recent studies find that deep learning is vulnerable against well-designed input perturbation (i.e., adversary) [6]. These adversaries can easily fool even seemingly high performing deep learning models with human imperceptible perturbations. Such vulnerabilities of deep learning models have been well studied in supervised learning, and also to some extent in RL [3][4][20].

In RL, the aim of an adversarial attack is to find the optimal adversary $\delta_t$ that minimizes the accumulated reward. Let $T$ represent the length of an episode, and $r(\pi, s_t)$ indicate a function that returns the reward given the state $s_t$ and DNN based policy $\pi$. Accordingly, the general formulation of adversarial attack in RL can be formulated as:

$$\min_{\delta_t} \sum_{t=1}^{T} r(s_t, a_t + \delta_t) : \forall t \|\delta_t\| \leq L$$  

where the adversary generated at time-step $t$ is represented by $\delta_t$, and its norm (i.e., $\|\delta_t\|$) is bounded by $L$. The basic assumption of Eq. (2) is that the misguided action selection of policy $\pi$ will result in a reward degradation. In other words, an action prediction $a_t^\pi$ obtained from the perturbed state $s_t + \delta_t$ may differ from the originally unperturbed action $a_t^\pi$, thus threatening the reward value $r(\cdot)$. This corresponds to the definition of untargeted attacks, which is stated as $a_t^\pi \neq a_t^\pi^\prime$.

To solve the optimization problem formulated in Eq. (2), many white-box based approaches (e.g., FSGM [18]) have been applied in previous efforts [3][4]. These white-box methods are essentially gradient-based methods, as they generate adversarial examples by back-propagating through the policy network to calculate the gradient of a cost function with respect to the input state, i.e., $\nabla_{s_t} J(\pi, \theta, \Delta a_t, s_t)$. Here, $\theta$ represents the weights of the neural network; $\Delta a_t$ is the change in action space; $J$ indicates the loss function (e.g., cross-entropy loss).

However, an essential precondition of white-box based approaches is the complete knowledge of the policy, including the structure and the model parameters $\theta$. Contrarily, a more
restrictive setting is black-box policy access \([6]\), that only allows an attacker to present the input state to the policy and observe the output. This serves as an oracle that only returns the output action prediction.

In addition to the above, most prior studies on attacking RL policies only analyze the effects of perturbing every pixel of every frame, which is deemed too intensive to be of much practical relevance. Therefore, this paper aims to provide a far more restricted view towards the impact of adversarial attacks, with the goal of unveiling the limits of a models vulnerability.

IV. THE PROPOSED METHODOLOGY

This section provides details of the three key ingredients of the proposed restrictive attack setting, namely black-box policy access (BPA), fractional-state adversary (FSA), and tactically-chanced attack (TCA).

A. Black-box Policy Access

We adopt the commonly used black-box definition \([6, 40]\) from supervised learning, where the attacker can only query the policy. In other words, the attacker is unable to analytically compute the gradient \(\nabla_{s_t} \pi(\cdot|s_t, \delta_t)\), but only has the privilege to query a targeted policy so as to obtain useful information for crafting adversarial examples. However, such a setting has been rarely investigated in RL adversarial attacks.

We realize such a BPA setting by formulating the adversarial attack as a black-box optimization problem. To this end, we propose and utilize a measure \(D(\cdot)\) to quantify the discrepancy between the original action distribution \(\pi(\cdot|s_t)\) (produced by an RL policy without input perturbation) and the corresponding perturbed distribution \(\pi(\cdot|s_t + \delta_t)\). Assuming a finite set of \(m\) available actions \(a_{1}^t, a_{2}^t, \ldots, a_{m}^t\), the probability distribution over actions is represented as \(\pi(\cdot|s_t) = [p(a_{1}^t), p(a_{2}^t), \ldots, p(a_{m}^t)]\), where \(\sum_{j=1}^{m} p(a_{j}^t) = 1\). Typically, a deterministic policy selects the action \(\pi = \arg \max_j p(a_{j}^t)\).

With this, the black-box attack considers the discrepancy measure as the optimization objective function, where the state \(s_t\) is perturbed by adversary \(\delta_t\) such that the measure \(D(\cdot)\) is maximized.

The overall problem formulation can thus be stated as:

\[
\max_{\delta_t} D(\pi(\cdot|s_t), \pi(\cdot|s_t + \delta_t)) : \forall t \|\delta_t\| \leq L. \tag{3}
\]

The above mathematical program changes the action distribution to \(\pi(\cdot|s_t + \delta_t)\), from the original (optimal) action distribution \(\pi(\cdot|s_t)\). If the change in action distribution leads to a change in action selection, then, based on Bellman’s principle of optimality \([30]\), it can then be concluded that the perturbed action will lead to a sub-optimal reward as a consequence of the attack. Under this hypothesis, we propose to replace the reward minimization problem of Eq. (2) with a discrepancy maximization formulation proposed in Eq. (3).

The exact choice of discrepancy measure \(D(\cdot)\) is expected to have a significant impact on the attack performance, as different measures shall capture different patterns of similarities. Previous works \([3, 4, 20]\) apply the Euclidean norm (e.g., \(L_1, L_2, L_{\infty}\)) between \(\pi(\cdot|s_t + \delta_t)\) and \(\pi(\cdot|s_t)\). However, such \(L_p\) norm cannot guarantee a successful untargeted attack, since maximizing the \(L_p\) norm does not ensure that the action selection for the perturbed state has been altered. Thus, a successful untargeted attack under deterministic action selection must ensure that \(\arg \max_j \pi(\cdot|s_t + \delta_t) = \arg \max_j \pi(\cdot|s_t)\), where \(\pi(\cdot|s_t) = p(a_{j}^t)\).

To enable a more consistent discrepancy measure for untargeted attack, we design the following function \(\tilde{D}\) based on the query feedback of the policy:

\[
\tilde{D} = \max_{\delta_t \neq 0} \pi(\cdot|s_t + \delta_t) - \pi(\cdot|s_t + \delta_t)_{o} \tag{4}
\]

where: \(o = \arg \max_j \pi(\cdot|s_t)\).

This formulation is different from the Euclidean norm, guaranteeing a successful untargeted attack if \(\tilde{D}\) is positive. To support this claim, we refer to the theorem and proof below:

**Theorem 1.** Suppose discrepancy measure \(\tilde{D}\) in Eq. (4) is positive; policy \(\pi\) is a deterministic, i.e., choosing action \(a_{j}^t\) is chosen such that \(o = \arg \max_j \pi(\cdot|s_t)\). Then the adversarial example \(\delta_t\) for policy \(\pi\) at state \(s_t\), is guaranteed to make a successful untargeted attack, i.e.,

\[
\arg \max_j [\pi(\cdot|s_t + \delta_t)_{j}] \neq \arg \max_j [\pi(\cdot|s_t)]_{j}.
\]

**Proof.** We use the symbol \(o\) and \(p\) to represent the action index from state \(s_t\) and perturbed state \(s_t + \delta_t\), respectively. As the policy \(\pi\) is deterministic, \(o\) and \(p\) are computed by:

\[
o = \arg \max_j [\pi(\cdot|s_t)]_{j},
\]

\[
p = \arg \max_j [\pi(\cdot|s_t + \delta_t)]_{j}.
\]

The discrepancy measure \(\tilde{D}\) is positive, then we have

\[
\tilde{D} > 0 \Rightarrow \max_{\delta_t \neq o} \pi(\cdot|s_t + \delta_t) - \pi(\cdot|s_t + \delta_t)_{o} > 0
\]

\[
\Rightarrow \max_{\delta_t \neq o} \pi(\cdot|s_t + \delta_t) > \pi(\cdot|s_t + \delta_t)_{o}
\]

\[
\Rightarrow \max_{\delta_t \neq o} \pi(\cdot|s_t + \delta_t)_{j} = \max_{\delta_t \neq o} \pi(\cdot|s_t + \delta_t)_{j}.
\]

Therefore, the perturbed action index \(p\) is given by:

\[
p = \arg \max_j [\pi(\cdot|s_t + \delta_t)]_{j} = \arg \max_j [\pi(\cdot|s_t + \delta_t)]_{j} \neq o
\]

\[
\Rightarrow \max_{\delta_t \neq o} \pi(\cdot|s_t + \delta_t)_{j} \neq \arg \max_j [\pi(\cdot|s_t)]_{j}.
\]

With the discrepancy measure \(\tilde{D}\), the mathematical program for generating adversarial attacks is given by,

\[
\max_{\delta_t \neq o} \pi(\cdot|s_t + \delta_t)_{j} - \pi(\cdot|s_t + \delta_t)_{o} \tag{5}
\]

where: \(\forall t \|\delta_t\| \leq L, o = \arg \max_j \pi(\cdot|s_t)_{j} \)

When resolving the optimization problem in Eq. (5), \(\delta_t\) has to be determined by its parameterisation. In this paper, \(\delta_t\) is limited to perturbing only a small fraction of the input state (i.e., fractional-state adversary), as described next.
B. Fractional-State Adversary

To explore adversarial attacks limited to a few pixels in RL, we investigate the fractional-state adversary (FSA) setting. In comparison with the full-state adversary depicted in Fig. 3(a), FSA only perturbs a fraction of the state; shown in Fig. 3(b). The extreme scenario for FSA is merely a single-pixel attack, which is deemed to be more physically realizable for the attacker than a full-state one. For instance, pasting a sticker or a simple fading of color in the “STOP” sign could easily lead to a FSA in Fig. 3(b).

To achieve the FSA setting, we propose a different parameterization for the adversary \( \delta_f \) with the discrete representations of perturbed pixels. In doing so, we parameterize the adversary \( \delta_f \) by its corresponding pixel coordinates (i.e., \( x_t, y_t \)) and perturbation value \( p_t \):

\[
\delta_f = P(x_t, y_t, p_t) = P(x_t^1, y_t^1, p_t^1, \ldots, x_t^n, y_t^n, p_t^n)
\]

where \( n \) is the number of pixels that are attacked, \( x_t^i, y_t^i \) are coordinate values of the perturbed pixel, and \( p_t^i \) is the adversarial perturbation value of the \( i \)-th pixel. With Eq. (6), the final black-box optimization problem is reformulated as:

\[
\max_{x_t, y_t, p_t} \max_{j \neq 0} [\pi(s_t + P(x_t, y_t, p_t))] - [\pi(s_t + P(x_t, y_t, p_t))]
\]

where: \( \forall t \quad o = \arg \max_j [\pi(s_t)]_j \);

\[
\forall t \quad 0 \leq x_t^i \leq I_x, \quad I_x = [1, 2, \ldots, n];
\]

\[
\forall t \quad 0 \leq y_t^i \leq I_y, \quad I_y = [1, 2, \ldots, n];
\]

\[
\forall t \quad 0 \leq p_t^i \leq I_p, \quad I_p = [1, 2, \ldots, n];
\]

The process repeats until the maximum objective evaluation number \( f_m \) is reached.

C. Tactically-Chanced Attack

To explore the adversarial attacks on a restricted number of frames, we design the tactically-chanced attack (TCA) where the attacker strategically strikes only salient frames that are likely to be most contributing to the accumulated reward. In contrast, most existing approaches [3] apply adversarial examples on every frame, which is referred to as fully-chanced attack. Our proposed TCA is clearly more restrictive, as can further be highlighted from three different perspectives: (1) due to the communication budget restriction, the attacker may not be able to strike the policy in a fully-chanced fashion; (2) a tactically-chanced attack is less likely to be detected by the defender; and (3) only striking the salient frames improves the attack efficiency, as many frames contribute trivially to the accumulated reward.

To this end, we define the normalized Shannon entropy of the action distribution \( \pi(s_t) = [p(a_1^n), p(a_2^n), \ldots, p(a_m^n)] \) as a measure of the attack uncertainty. Such attack uncertainty \( \zeta_t \) of each frame is given by,

\[
\zeta_t = - \sum_{i=0}^{m} \frac{p(a_i^n) \cdot \log p(a_i^n)}{\log m}
\]

where \( p(a_i) \) is the probability of action \( a_i \); \( m \) is the dimension of the action space. As probability \( p(a_i) \in [0, 1] \), \( \zeta_t \) is constrained in \((0, 1]\). In such a formulation, the frame with
Fig. 4: The six representative Atari games (i.e., Pong, Breakout, SpaceInvaders, Seaquest, Qbert, BeamRider) applied in the experiments

Fig. 5: The attack uncertainty $\zeta_t$ on Pong.

relatively low $\zeta_t$ indicates that the policy has a high confidence in its prescribed action. Hence, we assume that attacking such frames is expected to effectively disrupt the policy.

We demonstrate the attack uncertainty on Pong in Fig. 5; similar trends can be observed on the other games as well. In the figure, the frames with smaller $\zeta_t$, marked by brown rectangles, depict that the ball is close to the paddle. On the other hand, the blue rectangle marked ones with larger $\zeta_t$ (around 1) indicate that the ball is distant from the paddle. Attacking the brown marked frames are intuitively more efficient to fool the policy, as they are likely to lead to a more considerable reward loss. In contrast, an attack may be relatively inconsequential for the blue marked frames. As the ball is distant from the paddle, attacking such frames will have little impact on the reward.

Therefore, we formulate the TCA by defining a TCA threshold ($\zeta^*$) as shown by the red line in Fig. 5; this controls the proportion of attacked frames, where only those frames with an uncertainty value below $\zeta^*$ are perturbed. In the experimental section, we analyze the effect of threshold $\zeta^*$ by varying its values; this also helps us to explore how little it actually takes to deceive a policy (from the perspective of the number of frames attacked). The adversary $\delta_t$ under TCA setting is thus given by:

$$
\delta_t = \begin{cases} 
\mathcal{P}(s_t, y_t, p^*_{t}), & \text{if } \zeta_t < \zeta^*, \\
\text{none}, & \text{if } \zeta_t \geq \zeta^*. 
\end{cases}
$$

Eq. 9 suggests that if the attack uncertainty ($\zeta_t$) is smaller than $\zeta^*$, an adversary shall be generated by solving the optimization problem in Eq. 7. Otherwise, the attacker will tactically hide without wasting sources on trivial frames.

V. EXPERIMENTS AND ANALYSIS

We evaluate the three restrictive settings on six Atari games in OpenAI Gym with various difficulty levels, including Pong, Breakout, SpaceInvaders, Seaquest, Qbert, and BeamRider. These games are shown in Fig. 4. For each game, the policies are trained by four start-of-the-art RL algorithms, including DQN, PPO2, A2C, ACKTR. The network structure is adopted from [51], and it keeps same for all the four RL algorithms.

A. Experiment Setup

We utilize the fully-trained policies from the RL baseline zoo. Each policy follows the same pre-processing steps and neural network architecture (i.e., shown as Fig. 6 in [1]). The input state $s_t$ of the neural network is the concatenation of the last four screen images, where each image is resized to $84 \times 84$. The pixel value of the grey scale image is in the range of $[0, 255]$ stepped by 1. The output of the policy is a distribution over candidate actions for PPO2, A2C, ACKTR, and an estimation of Q values for DQN.

To calculate the attack uncertainty for DQN, the soft-max operation is applied to normalize the output. Moreover, given the image size and pixel value, the constraints in Eq. 7 are set as $I_x = 84$, $I_y = 84$, $I_p = 255$. In the GA, the population size $\lambda$ is 10, and the maximum number of objective evaluations $f_m$ is set as 400. However, as our objective function has a guarantee for successful untargeted adversarial attack, the optimization process will terminate when a positive function value is found. To investigate the impacts of FSA size $n$, we...
We investigate the effectiveness of the fractional-state adversary (FSA) via varying the value of FSA size $n$ in the range of $[1, 10]$ scaled by 1, where $n = 1$ corresponds to the extreme case that only one pixel is perturbed. For a fair comparison with respect to different algorithms and games, we set the TCA threshold $\zeta^*$ as the mean of all frames’ $\zeta$ that are obtained from the unattacked policy testing. Fig. 7 illustrates the performance (i.e. accumulated reward) drop on the policies trained by the four RL algorithms on the six Atari games. Several interesting observations can be noted.

1) Overall, the FSA size $n$ is positively related to the performance drop. That is to say, the FSA with larger $n$ is able to deceive the policy more easily. 2) On most of the games, the policies are almost completely fooled with $n \leq 4$. This indicates a small FSA size is sufficient to achieve a successful adversarial attack, indicating the efficiency of FSA. 3) There is a relatively huge performance drop for DQN based policy in comparison with policies trained by other RL algorithms, especially on Breakout and Qbert. It suggests that DQN based policy is more vulnerable; whereas, the ACKTR based policy is more robust under the attack with FSA setting. 4) The

Fig. 7: The results of adversarial attack with different FSA size $n$ (i.e. pixel number), where the line and shaded area illustrate the mean and standard deviation of 30 independent runs respectively.

B. Results for Fractional-State Adversary

apply a grid search in $[1, 10]$ scaled by 1. We also apply a grid search for tactically-chanced attack (TCA) threshold $\zeta^*$ in the range of $[0, 1]$. This corresponds to the different proportion of attacked frames as depicted by the x-axis in Fig. 8.

Fig. 8: The results of adversarial attack with different TCA boundary $\zeta^*$, where the line and shaded area illustrate the mean and standard deviation of 30 independent runs respectively.

(1) Overall, the FSA size $n$ is positively related to the performance drop. That is to say, the FSA with larger $n$ is able to deceive the policy more easily. (2) On most of the games, the policies are almost completely fooled with $n \leq 4$. This indicates a small FSA size is sufficient to achieve a successful adversarial attack, indicating the efficiency of FSA. (3) There is a relatively huge performance drop for DQN based policy in comparison with policies trained by other RL algorithms, especially on Breakout and Qbert. It suggests that DQN based policy is more vulnerable; whereas, the ACKTR based policy is more robust under the attack with FSA setting. (4) The
results also imply that the high performance of unattacked policy cannot guarantee a strong robustness to resist attack. For instance, in SpaceInvaders, the original performance of A2C is higher than DQN, but the performance of A2C drops faster than that of DQN as shown in Fig. 7(c). (5) Most importantly, we surprisingly find that: on Breakout, with single pixel attacked, the policy trained by all the four RL algorithms are completely deceived with the accumulated reward close to 0. On Qbert and BeamRider, the policy trained by DQN is also completely deceived with only a single pixel attacked. Such single pixel attack corresponds to an approximate perturbation proportion of 0.01% (i.e., 1/(84 x 84)) of the total number of pixels in a frame.

C. Results for Tactically-Chanced Attack

We study the tactically-chanced attack (TCA) by altering the TCA threshold $\zeta^*$. Different $\zeta^*$ corresponds to different proportion of attacked frames. In our exploration, we set the $\zeta^*$ in the range of $(0, 1]$, where $\zeta^* = 0$ and $\zeta^* = 1$ correspond to 0% and 100% proportion of attacked frames, respectively. Recall that according to the results for FSA size analysis shown in Fig. 7 most of the policies are sufficiently fooled with no more than 4 pixels attacked. Hence, for a fair comparison, we set the FSA size $n = 4$ in subsequent experiments. The results are displayed in Fig. 8 where we observe that in general the performance (i.e., accumulated reward) decreases when higher proportion of frames are attacked. This demonstrates that the
policies are more easily fooled with more frames attacked. In addition, we make several other fascinating findings.

1. In Fig. 8(a) on Pong, only with 10% of frames attacked, the models trained by DQN and PPO2 are totally fooled with the accumulated reward around −20. This also suggests that the model trained by A2C and ACKTR are more robust than those trained by DQN and PPO2. 2. In Fig. 8(b) on Breakout, the model trained by DQN is totally deceived by only perturbing less than 5% of frames on average. 3. In Fig. 8(f) on Beamrider, even when the proportion of attacked frames is about 20%, all policies can only obtain a reward with less than 200. Especially for the policies trained by DQN and PPO2, the accumulated reward drops from 14000 to 2000.

We further analyze the exact number of attacked frames with different TCA threshold values that represent different proportion of attacked frames. As shown in Fig. 9, the number of attacked frames increases when the proportion value is higher, and the exact number of attacked frames is relatively low considering the number of total frames. In Fig. 2(b), on Breakout, when the proportion value is smaller than 20%, less than 50 frames are attacked. The same finding is also observed on Beamrider, where less than 100 frames are attacked when the proportion value is smaller than 20%. On Pong, to achieve a same reward degradation, the policy trained by A2C requires more frames attacked than that of the other three RL policies.

For a better demonstration, we also analyze the relation between attacked frames and total frames in Fig. 10. We find similar trends on the six games, where the number of total frames decreases when more frames are attacked. In particular, such relation is significantly observed on Breakout, where attacking less than 50 frames considerably reduces the number of total frames to a small value from 2000. This probably results from the fact that the attacked frames may cause (i) the termination of the game, or (ii) a loss of life. Both outcomes greatly shorten the episode, resulting in a smaller number of total frames. For instance, if the paddle in Breakout is deceived to miss the ball, the agent would lose 1/5 life.

To explore a more restrictive setting, we additionally examine the single-pixel \((n = 1)\) attack on limited frames. We illustrate the results on Breakout as shown in Fig. 11, and similar trends can be observed on other games. From this figure, we surprisingly find that by only attacking on six frames on average, the policy trained by DQN is totally fooled with reward decreasing from 224 to 13.7. Here, six frames correspond to an attack ratio of \(6/535 \approx 1\%\), where 535 is the averaged number of total frames. Similar surprising findings are also observed on policies trained by PPO2, A2C and ACKTR. For instance, the policy trained by PPO2 shows a significant reward decline with only 27 frames attacked under the same TCA setting as other policies.

D. Initialization Analysis

The GA population initialization plays an important role in efficiently generating successful adversarial examples, and thus highlights the practicality of the approach. To this end, we study the effects of random initialization versus warm starting based on prior data. In RL, the simplest form of prior originates from the correlation between states in a in the Markov Decision Process (MDP); this also distinguishes RL from other supervised learning tasks (e.g., classification), where the predictions on different data points are independent. Thus, with the existence of correlations between adjacent frames in mind, we set up different types population initialization in the GA: (1) GA-RI - random initialization (RI) of \(pop_0\); (2) GA-WSI - warm start initialization (WSI) of \(pop_0\) by adopting the optimal adversarial example \(x^*_p\) found in the previous attacked frame.

To ensure a fair comparison and for ease of presentation, we randomly attack four frames for all the six games. On each attacked frame, the GA optimization is executed 10 times independently on both types of initialization. The comparison results are shown in Fig. 12, where the dash line and shaded area respectively represent the mean and standard deviation of loss values for the 10 runs. In general, we can clearly see that GA-WSI significantly speeds up the optimization convergence while finding the successful adversarial examples (i.e., corresponding to positive objective function values). In particular, on many frames (e.g., Pong: \(ξ_1, ξ_2, ξ_4\); Breakout: \(ξ_2\); Space Invaders: \(ξ_3\); Qbert: \(ξ_3, ξ_4\)), the GA-RI takes relatively more time (or is even unable) to obtain a positive objective value; whereas the GA-WSI achieves it easily.

We interestingly find that, in some cases, GA-WSI provides a successful adversary without the need of any optimization. This observation underlines the real possibility of launching successful adversarial attacks on deep RL policies in real-time at little/no computational cost, a threat that raises severe concerns about their deployment especially in safety critical applications.

In summary, our experimental results and examinations suggest that by moderately setting the FSA size \(n\) and TCA threshold \(ζ^*\), the attacker could successfully disrupt a seemingly high performing policy. Moreover, our investigations on different initialization manners (i.e., GA-RI and GA-WSI) indicate that even restrictive black-box settings for adversarial attack can pose a big challenge to the utility of state-of-the-art RL methods, thus highlighting the need to focus on robustness in the future.
Fig. 12: The comparison between random initialization and case-injected initialization, where the line and shaded area illustrate the mean and standard deviation of 10 independent runs respectively.
CONCLUSION

This paper explores the minimalistic scenarios for adversarial attack in deep reinforcement learning (RL), comprising (1) black-box policy access where the attacker only has access to the input (state) and output (action probability) of an RL policy; (2) fractional-state adversary where only a small number of pixels are perturbed, with the extreme case of one-pixel attack; and (3) tactically-chanced attack where only some significant frames are chosen to be attacked. We verify these settings on policies that are trained by four state-of-the-art RL policies on six Atari games. We surprisingly find that: (i) with only a single pixel ($\approx 0.01\%$ of the state) attacked, the trained policies can be completely deceived; and (ii) by merely attacking around 1% frames, the policy trained by DQN is totally fooled on certain games.

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