Absence of even-integer \( \zeta \)-function values in Euclidean physical quantities in QCD

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At order \( \alpha_s^4 \) in perturbative quantum chromodynamics, even-integer \( \zeta \)-function values are present in Euclidean physical correlation functions like the scalar quark correlation function or the scalar gluonium correlator. We demonstrate that these contributions cancel when the perturbative expansion is expressed in terms of the so-called \( C \)-scheme coupling \( \hat{\alpha}_s \), which has recently been introduced in Ref. [1]. It is furthermore conjectured that a \( \zeta_4 \) term should arise in the Adler function at order \( \alpha_s^4 \) in the \( \overline{\text{MS}} \)-scheme, and that this term is expected to disappear in the \( C \)-scheme as well.

INTRODUCTION

In the past, it has been noted several times that even-integer values of the Riemann \( \zeta \)-function are absent in the perturbative expansion of some Euclidean physical quantities in quantum chromodynamics (QCD). One prominent example is the Adler function up to order \( \alpha_s^4 \) [2], and explanations for this behaviour were provided in the literature [3]. However, the regularity for example fails in the scalar quark correlation function [4] and the scalar gluonium correlator [5], both also being analytically available up to order \( \alpha_s^3 \) in the \( \overline{\text{MS}} \)-scheme.

In Ref. [6], together with D. Boito, we introduced a novel definition of the QCD coupling, \( \hat{\alpha}_s \), which reflects the simple scheme-transformation properties of the \( \Lambda \)-parameter, such that the scheme dependence of the coupling \( \hat{\alpha}_s \) could be parametrised by a single parameter \( C \). Hence, \( \hat{\alpha}_s \) was named the “\( C \)-scheme” coupling. It was furthermore found that the corresponding \( \beta \)-function is then scheme independent, only depending on the first two \( \beta \)-coefficients \( \beta_1 \) and \( \beta_2 \) in a way that has already been studied previously in a different context [4].

In this work, we shall demonstrate that both, the Euclidean physical scalar correlation function, as well as the scalar gluonium correlator, up to order \( \alpha_s^4 \) are free of even-integer \( \zeta \)-function values when they are appropriately expressed in terms of the \( C \)-scheme coupling. We will also give additional arguments, why even-integer \( \zeta \)-function values have not yet appeared in the Adler function, but we conjecture that a \( \zeta_4 \equiv \zeta(4) \) term should appear at order \( \alpha_s^5 \) in the \( \overline{\text{MS}} \)-scheme. We furthermore conjecture that it should again cancel when the Adler function is expressed in \( \hat{\alpha}_s \) and that the same might hold true for all Euclidean physical quantities in QCD, possibly even for those in quantum field theory in general.

Our article is organised as follows: to begin we collect the required relations for the \( C \)-scheme coupling. Then we briefly discuss the Adler function and argue why a \( \zeta_4 \) term is only expected at order \( \alpha_s^4 \). The scalar quark and gluonium correlators are presented in more detail and it is demonstrated how even-integer \( \zeta \)-function values cancel after rewriting them in the \( C \)-scheme coupling \( \hat{\alpha}_s \).

THE \( C \)-SCHEME COUPLING

The \( C \)-scheme coupling \( \hat{a}_Q \equiv \hat{\alpha}_s(Q)/\pi \) has been introduced in Ref. [1], and is defined by the relation

\[
\frac{1}{\hat{a}_Q} + \frac{\beta_2}{\beta_1} \ln \hat{a}_Q - \frac{\beta_1}{2} C \equiv \frac{\beta_1}{\hat{a}_Q} \frac{Q}{\Lambda} = \frac{1}{\hat{a}_Q} + \frac{\beta_2}{\beta_1} \ln a_Q - \beta_1 \int_0^a \frac{da}{\beta(a)},
\]

where \( C \) is a free parameter,

\[
\frac{1}{\beta(a)} \equiv \frac{1}{\beta(a)} - \frac{1}{\beta_1 a^2} + \frac{\beta_2}{\beta_1^2 a}.
\]

and both, the QCD \( \Lambda \)-parameter, and the coupling \( a_Q \) on the right-hand side are in a conventional renormalisation scheme, like for example the \( \overline{\text{MS}} \) scheme. The coupling \( \hat{a}_Q \) was selected such as to mimic the simple scheme-transformation properties of the \( \Lambda \) parameter.

From Eq. (1) it is an easy matter to derive the corresponding \( \beta \)-function, which was found to only depend on the scheme-invariant coefficients \( \beta_1 = 11/2 - N_f/3 \) and \( \beta_2 = 51/4 - 19/12 N_f \), \( N_f \) the number of quark flavours:

\[
-Q \frac{d\hat{a}_Q}{dQ} \equiv \hat{\beta}(\hat{a}_Q) = \frac{\beta_1 \hat{a}_Q^2}{1 - \frac{\beta_2}{\beta_1} \hat{a}_Q} = -2 \frac{d\hat{a}_Q}{dC}.
\]

Additionally, also \( C \)-evolution is governed by the same \( \beta \)-function. This implies that in the \( \hat{a}_Q \) coupling, scale and scheme variation can be considered on an equal footing.

Hence, transforming from a general scheme \( a_Q \) to \( \hat{a}_Q \) can be performed in two steps. From Eq. (1) at \( C = 0 \),
defining $\tilde{a}_Q \equiv \tilde{a}_C^{C=0}$, one finds the relation

$$a_Q = \tilde{a}_Q + \left( \frac{\beta_1}{\beta_1} - \frac{\beta_2^2}{\beta_1^2} \right) \tilde{a}_Q^2 + \left( \frac{\beta_1}{2\beta_1} - \frac{\beta_3^2}{2\beta_1} \right) \tilde{a}_Q^3 + \left( \frac{\beta_5}{3\beta_1} - \frac{\beta_2 \beta_4}{6\beta_1^2} + \frac{7\beta_1^2}{6\beta_1} \right) \tilde{a}_Q^4 + \ldots$$

(4)

Then, in a second step, the C evolution can be employed to transform from $\tilde{a}_Q$ to the general C-scheme coupling:

$$\bar{a}_Q = \tilde{a}_Q + \frac{1}{2} C \tilde{a}_Q^2 + \left( \frac{\beta_2}{2} C + \frac{\beta_1^2}{4} C^2 \right) \tilde{a}_Q^3 + \left( \frac{\beta_3}{2} C + \frac{9\beta_1 \beta_2}{8} C^2 + \frac{13\beta_2^2 \beta_3}{24} C^3 + \frac{\beta_1^3}{16} C^4 \right) \tilde{a}_Q^4 + \ldots$$

(5)

This summarises all required relations for the C-scheme coupling $\bar{a}_Q$.

THE ADLER FUNCTION

The resummed perturbative Adler function $D(Q^2)$, $Q^2 > 0$, which results from the $Q^2$-derivative of the vector correlator, and is a physical, scale- and scheme-independent quantity, assumes the simple expression

$$D(Q^2) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} c_{n,1} a_Q^n.$$  

(6)

The independent coefficients $c_{n,1}$ are known analytically up to order $\alpha_s^4$. Further details regarding our notation and additional references can be found in Ref. 10. For definiteness, but to keep the expressions simple, we will only quote results for $N_f = 3$. At $N_c = 3$ and in the $\overline{\text{MS}}$-scheme, the $c_{n,1}$ were found to be:

$c_{0,1} = c_{1,1} = 1$, $c_{2,1} = \frac{299}{24} - 9\zeta_4$,

$c_{3,1} = \frac{58957}{228} - \frac{779}{4} \zeta_3 + \frac{79}{2} \zeta_5$,

$c_{4,1} = \frac{7836154}{20736} - \frac{1704247}{432} \zeta_3 + \frac{4185 \zeta_2^2}{8} \zeta_3 + \frac{34165 \zeta_5}{96} - \frac{1995}{16} \zeta_7$,  

(7)

where $\zeta_n \equiv \zeta(n)$. As is seen explicitly, up to order $\alpha_s^4$, even in the $\overline{\text{MS}}$-scheme, the $c_{n,1}$ only contain the odd $\zeta$-function values $\zeta_3$, $\zeta_5$, and $\zeta_7$.

Next, we transform the Adler function into the C-scheme coupling $\bar{a}_Q$ at $C = 0$. The corresponding expansion assumes the form

$$D(Q^2) = \frac{N_c}{12\pi^2} \sum_{n=0}^{\infty} \bar{c}_{n,1} \bar{a}_Q^n.$$  

(8)

Employing Eq. (4), only the coefficients $\bar{c}_{3,1}$ and $\bar{c}_{4,1}$ turn out different from the $\overline{\text{MS}}$ coefficients, and read:

$$\bar{c}_{3,1} = \frac{262955}{1296} - \frac{779}{4} \zeta_3 + \frac{79}{2} \zeta_5,$$

$$\bar{c}_{4,1} = \frac{357259199}{93312} - \frac{1713103}{432} \zeta_3 + \frac{4185 \zeta_2^2}{8} \zeta_3 + \frac{34165}{96} \zeta_5 - \frac{1995}{16} \zeta_7.  

(9)

Like $c_{3,1}$ and $c_{4,1}$, the coefficients $\bar{c}_{3,1}$ and $\bar{c}_{4,1}$ still only include odd-integer $\zeta$ values up to $\zeta_7$, because the transformation (11) only includes $\beta$-coefficients up to $\beta_5$, which have $\zeta_2$ as the sole irrational component. This changes at order $\alpha_s^5$, since $\beta_5$ also contains a $\zeta_4$ term (11 [12]). However, below we shall demonstrate that the $\zeta_4$ term in $\beta_5$ precisely cancels a corresponding term in the $\overline{\text{MS}}$ coefficients of scalar quark and gluonium correlators, such as to make the C-scheme coefficients independent of $\zeta_4$.

Therefore, we conjecture that the same should also happen for the Adler function: we suspect that the coefficient $c_{3,1}$ will contain a $\zeta_4$ term which cancels against the corresponding term in $\beta_5$ once the Adler function is reexpressed in the C-scheme coupling $\bar{a}_Q$. Under this assumption, we can predict the component of $c_{3,1}$ which is proportional to $\zeta_4$. At $N_c = 3$, but for arbitrary number of quark flavours $\Lambda_f$, it is found to be

$$c_{3,1}^c = \left( \frac{2673}{512} - \frac{1995}{16} \Lambda_f + \frac{809}{2361} \Lambda_f^2 \right) \zeta_4.  

(10)

THE SCALAR QUARK CORRELATOR

In the case of the scalar quark correlator, the Euclidean physical quantity is given by the second derivative of the correlation function $\Psi(Q^2)$. Its definition and further details can be found in Ref. 13. Furthermore, the scalar current has an anomalous dimension which is inverse to that of the quark mass. Hence, a scale- and scheme-invariant correlator can be obtained by multiplying with two powers of a generic quark mass $m_q \equiv m(Q)$. The physical scalar correlator then takes the form

$$\Psi''(Q^2) = \frac{N_c \, m_q^2}{8\pi^2} \frac{2}{Q^2} \left( 1 + \sum_{n=1}^{\infty} d''_{n,1} a_Q^n \right),  

(11)

where both, the running quark mass, as well as the running QCD coupling are to be evaluated at the renormalisation scale $Q$. At $\Lambda_f = \Lambda_c = 3$, the perturbative coefficients $d''_{n,1}$ take the explicit values

$$d''_{1,1} = \frac{11}{3}, \quad d''_{2,1} = \frac{2971}{144} - \frac{35}{2} \zeta_3,$$

$$d''_{3,1} = \frac{1995097}{5184} - \frac{65869}{216} \zeta_3 - \frac{5}{2} \zeta_4 + \frac{715}{72} \zeta_5,$$

$$d''_{4,1} = \frac{236129559}{497664} - \frac{25214381}{5184} \zeta_3 + \frac{192155}{216} \zeta_2^2 \zeta_3 - \frac{14575}{768} \zeta_4 + \frac{59875}{108} \zeta_5 - \frac{625}{48} \zeta_6 - \frac{52255}{256} \zeta_7.$$
It is observed that in this case $d_{4,1}''$ contains a $\zeta_4$ term and $d_{4,1}'$ both $\zeta_4$ and $\zeta_6$.

For the ensuing discussion it will be essential to remove the running effects of the quark mass from the remaining perturbative series. This can be achieved by rewriting the running mass $m_\ell$ in terms of an invariant quark mass $\hat{m}$ which is defined through the relation

$$m_\ell \equiv \hat{m} [\alpha_s(Q)\gamma_{m}^{(1)}/\beta_1 \exp \left\{ \int_0^{a_0} \frac{d\alpha}{\beta_1} [\gamma_m(a) - \gamma_{m}^{(1)}/\beta_1 a] \right\} ,$$  

(13)

where $\gamma_m(a)$ is the quark-mass anomalous dimension and $\gamma_{m}^{(1)}$ its first coefficient. Accordingly, we define a modified perturbative expansion with new coefficients $r_n$,

$$\Psi''(Q^2) = \frac{N_c}{8\pi^2} \frac{\hat{m}_Q^2}{Q^2} [\alpha_s(Q')\gamma_{m}^{(1)}/\beta_1 \left\{ 1 + \sum_{n=1}^{\infty} r_n a_0^n \right\} ,$$

(14)

which now contain contributions from the exponential factor in eq. (13). The order $\alpha_s^4$ coefficient $r_4$ depends on $\beta$-function coefficients as well as quark-mass anomalous dimensions up to five-loops [14]. Let us remark that the $\zeta_4$ term that is present in $d_{4,1}'$, as well as the $\zeta_6$ term in $d_{4,1}''$, are cancelled by the additional contribution, while $\zeta_4$ still remains in $r_4$. The respective cancellations have also been observed in ref. [3] for a related quantity.

As the last step, similarly to the Adler function, we reexpress the QCD coupling in terms of $\alpha_s$. The perturbative expansion of $\Psi''$ then assumes the form

$$\Psi''(Q^2) = \frac{N_c}{8\pi^2} \frac{\hat{m}_Q^2}{Q^2} [\alpha_s(Q')\gamma_{m}^{(1)}/\beta_1 \left\{ 1 + \sum_{n=1}^{\infty} \tilde{r}_n a_0^n \right\} ,$$

(15)

defining the coefficients $\tilde{r}_n$, which take the particular values

$$\tilde{r}_1 = \frac{442}{81} , \quad \tilde{r}_2 = \frac{2510167}{2488} - \frac{325}{3} \zeta_3 ,$$

$$\tilde{r}_3 = \frac{1276567259}{25509168} - \frac{67361}{1944} \zeta_3 + \frac{18305}{324} \zeta_5 ,$$

$$\tilde{r}_4 = \frac{222689651973}{8264970432} - \frac{1667932911}{189568} \zeta_3 + \frac{601705}{644} \zeta_5^2 ,$$

$$+ \frac{117847345}{20952} \zeta_5 - \frac{3285415}{20936} \zeta_7 .$$

As has already been pointed out above, now even the $\zeta_4$ term remaining in $r_4$ got cancelled by a corresponding contribution in $\beta_5$, originating from the global $\alpha_s$ prefactor, such that only odd-integer $\zeta$-function contributions persist. Even though we have just derived results for $N_f = 3$, we have convinced ourselves that the cancellation of even $\zeta$ values does in fact take place for an arbitrary number of flavours and a general gauge group. Furthermore, since the transformation [3] only contains the $\beta$-function coefficients $\beta_1$ and $\beta_2$ which are rational, the absence of even $\zeta$ values also remains true for a general C-scheme coupling $\alpha_s$. It is hence a scheme-independent statement.

**THE SCALAR GLUONIUM CORRELATOR**

A basic two-point correlation function that is relevant for the study of scalar gluonium can be defined as

$$\Pi_{G^2}(q^2) \equiv i \int dxe^{iqx} \langle 0| T\{J_G(x)J_G(0)\}|0 \rangle ,$$

(17)

where the gluonic current is represented by $J_G(x) \equiv G_{\mu\nu}^a(x) G^{\mu\nu,a}(x)$ and $G_{\mu\nu}^a(x)$ is the QCD field-strength tensor.

In order to be able to define a physical quantity, one should work with a renormalisation group invariant current. In the chiral limit, where the operator $J_G(x)$ does not mix with $m\bar{q}(x)q(x)$ or $m^4$, such a current can be chosen to be

$$\hat{J}_G(x) \equiv \frac{\beta(a)}{\beta_1 a} J_G(x) = \frac{\beta(a)}{\beta_1 a} G_{\mu\nu}^a(x) G^{\mu\nu,a}(x) .$$

(18)

In analogy to $\Pi_{G^2}(q^2)$, we can then define the two-point correlator for the current $\hat{J}_G(x)$, which expressed in terms of $\Pi_{G^2}(q^2)$ takes the form ($Q^2 = -q^2$):

$$\Pi_{G^2}(Q^2) = \left( \frac{\beta(a)}{\beta_1 a} \right)^2 \Pi_{G^2}(Q^2) .$$

(19)

A Euclidean physical quantity in analogy to the Adler function can be obtained by taking three derivatives of $\Pi_{G^2}(Q^2)$, leading to the definition

$$D_{G^2}(Q^2) \equiv -Q^2 \frac{d^3 \Pi_{G^2}(Q^2)}{d(Q^2)^3} .$$

(20)

The corresponding perturbative expansion then takes the following general form [18]:

$$D_{G^2}(Q^2) = \left( \frac{N_f^2 - 1}{2\pi^2} \right) a_2^2 \sum_{n=0}^{\infty} g_n a_0^n .$$

(21)

Up to order $\alpha_s^4$, the coefficients $g_n$ can be extracted from the results provided in Ref. [3]. Again at $N_f = 3$, they are obtained as follows:

$$g_0 = 1 , \quad g_1 = \frac{104}{9} , \quad g_2 = \frac{87605}{648} - \frac{465}{8} \zeta_3 ,$$

$$g_3 = \frac{52031155}{31032} - \frac{216701}{144} \zeta_3 + \frac{10205}{24} \zeta_5 ,$$

$$g_4 = \frac{8122573939}{1482992} - \frac{1838382867}{62208} \zeta_3 + \frac{264375}{64} \zeta_5^2 ,$$

$$+ \frac{1335}{128} \zeta_4 + \frac{1478075}{128} \zeta_5 - \frac{2016175}{576} \zeta_7 .$$

As anticipated above, the coefficient $g_3$ in the $\overline{\text{MS}}$ scheme contains a $\zeta_4$ term. Like for the scalar correlator, this is
due to the anomalous dimension of the scalar gluonium current, which leads to the global factor $\alpha_s^2$, multiplying the correlator. As an aside, we also remark that the leading-$N_f$ contributions are in agreement with the large-$N_f$ results derived in Ref. [13].

As in the two examples above, we conclude by rewriting the perturbative series in terms of the $C$-scheme coupling $\bar{a}_Q$:

$$D_{C^2}(Q^2) = \frac{(N_c^2 - 1)}{2\pi^2} \bar{a}_Q^2 \sum_{n=0}^{\infty} g_n \bar{a}_Q^n.$$  

(23)

For this expansion, the coefficients $g_n$ are found to be:

$$\bar{g}_0 = 1, \quad \bar{g}_1 = \frac{104}{9}, \quad \bar{g}_2 = \frac{178607}{1296} - \frac{465}{8} \zeta_3,$$

$$\bar{g}_3 = \frac{20134253}{11064} - \frac{23979}{16} \zeta_3 + \frac{10205}{24} \zeta_5,$$

$$\bar{g}_4 = \frac{116204856235}{5638848} - \frac{690836641}{23328} \zeta_3 + \frac{264275}{64} \zeta_3^2 + \frac{59594845}{5184} \zeta_5 - \frac{2016175}{192} \zeta_7.$$  

(24)

As expected, once again the $\zeta_4$ term in $\bar{g}_4$ has been cancelled by the corresponding contribution in $\bar{g}_5$. Also in this case we have verified that the respective cancellation is independent of the number of flavours $N_f$ and the gauge group.

**CONCLUSIONS**

In this work, we have demonstrated that the Euclidean physical correlation functions corresponding to the scalar quark and scalar gluonium correlator do not contain even-integer $\zeta$-function values in their perturbative coefficients up to the presently analytically available order $\alpha_s^4$ when the perturbative expansion is performed in terms of the $C$-scheme coupling $\bar{a}_s$. We have shown this explicitly for the coupling $\bar{a}_s$ at $C = 0$, but the statement remains true for an arbitrary $C$ since the relation [10] only contains $\beta_1$ and $\beta_2$ which are rational numbers.

In the case of the Adler function, even the perturbative coefficients in the $\overline{MS}$ scheme up to order $\alpha_s^4$ do not contain even-integer $\zeta$-function values. This is related to the fact that the vector current has no anomalous dimension, and hence no prefactor depending on $\alpha_s$ arises. It is conjectured, that a $\zeta_4$ term will appear in the order $\alpha_s^5$ coefficient $c_{5,1}$. Assuming that this term is again cancelled in the $C$-scheme by a corresponding term in the $\beta$-function coefficient $\beta_5$, we predict the respective component $c_{5,1}^f$ proportional to $\zeta_4$ for $\overline{MS}$ in eq. (10).

To our knowledge, at this moment, the cancellation of even-integer $\zeta$-function values for perturbative expansions of Euclidean physical correlators in the $C$-scheme coupling $\bar{a}_s$ can only be checked for the scalar quark and scalar gluonium correlation functions, as only these functions are available up to the required order $\alpha_s^3$. Nonetheless, we conjecture that the same cancellation should also take place for other quantities. It will be exciting to see if this claim is indeed confirmed in the future.

As a last remark, we note that compared to the Adler function coefficients, the ones for scalar quark and gluonium correlators are substantially larger. Already in Ref. [13], we showed how the $C$-scheme coupling can be employed in order to improve the expansion for the scalar quark correlator. In future work, we plan to also return to this question for the scalar gluonium correlator and furthermore intend to demonstrate how the $C$-scheme coupling $\bar{a}_s$ can be utilised for constructing models for the Borel transforms of the investigated correlators, along the lines of Ref. [10].

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**NOTE ADDED IN PROOF**

Meanwhile, the cancellation of even-integer $\zeta$-function values has been demonstrated for a substantial number of additional quantities by Davies and Vogt in Ref. [10], as well as by Chetyrkin in unpublished work (see footnote 2 in Ref. [17]).

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