Low Frequency Vibration Isolation Performance of the Plate with Periodic Cylindrical Oscillators

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Abstract: The paper is devoted to the vibration reduction and isolation performance of a periodic structure with cylindrical vibrators. A plate with periodic oscillators was investigated using finite element technic. In the finite element analysis, test specimens were modelled with finite element software. With appropriate geometric and material parameters, the structure can obtain elastic wave band gaps, where vibrations are prohibited. So, the structure can be regarded as a kind of vibration isolation material. The dispersion curve and transmission spectrum of the structure were calculated to find the range of the band gaps. Both the longitudinal vibration and flexural vibration of the plate are studied. The results show that the designed plate with proper parameters has a low frequency range of band gaps. It has a wide application prospect in the field of low frequency vibration isolation.

1. Introduction
During the past decade, propagation of elastic wave in periodic structures, so-called Phononic crystals (PCs), has received considerable attention for their potential applications in vibration isolation [1][2][3]. The structure has elastic wave band gap, in which the elastic wave propagation is prohibited. If we change the geometric or material parameters of the periodic structure, the position of the band gap will also change. Over the past decades, a significant amount of researches have been studied on the band gap characteristics of periodic structure. Periodic structure can be divided into the types of Bragg scattering and local resonance according to the band gap formation mechanism. The periodic structure based on Bragg scattering mechanism is studied firstly. Vasseur et al.[4] tested the transmission spectrum of two-dimensional solid/solid triangle lattice phononic crystal in the direction of ΓX and ΓJ. The plane wave expansion method and the theory of finite difference time domain method is used for validation. The wavelength of band gap frequency of Bragg scattering type corresponds to the same order of magnitude as lattice constant. The first locally resonant phononic crystal proposed by professor Liu et al.[5] It is pointed out that the wave length corresponding to the low frequency band gap is larger than its lattice constant through a three-component periodic structure (epoxy resin-soft rubber-lead), that is, the low-frequency band gap can be obtained with a smaller structure size.

In recent years, more and more researches focus on vibration suppression of periodic structural plates. Pennec et al [6]. found that the opening of its gap requires appropriate geometrical parameters by the investigation on a two-component pillared PC plate. Hsu [7] proposed the pillars with a “neck” in the PC slab with stepped resonators and proved that there are both local resonance band gaps and Bragg band gaps in the pillars. Zhao et al.[8][9] studied the vibration band gaps in pillared phononic crystal plate and a series of parameter analysis was carried out. However, at present, phononic crystal plates are
generally small in size and high in vibration isolation frequency, and are mainly used in aviation and mechanical fields. In this paper, a large scale periodic structural plate with a low frequency band gap is proposed. Therefore, it can be more applicable to other low frequency vibration isolation fields. The longitudinal and flexural vibration band gaps of the periodic structure plate is concluded throughout the calculation of dispersion curve of cell structure and analysis of its displacement mode. The longitudinal and flexural transmission spectrum is also calculated for verification.

2. Model and method

2.1 Model

Figure 1 depicts the plate with periodic vibrator pillars. The oscillator is arranged in 6 rows and 6 columns periodically on the plate. The unit cell of the periodic structure is shown in Figure 2. The vibrator pillar in the unit cell is composed of two layers of cylindrical slices of rubber and steel. The diameter of the rubber layer and steel layer are \( d_1 \) and \( d_2 \) respectively. Since the rubber layer is generally seen like a spring, \( d_1 \) is set smaller than \( d_2 \) to save materials. Treating one rubber stub and one steel cap as a vibrator, such vibrators are periodically arranged on the surface of the plate. The geometric parameters of the structure are defined as follows: The distance between adjacent cells (lattice constant) is \( a \), so the plate is \( 6a \) in length and \( 6a \) in width. The thickness of the plate is denoted by \( e \); and the thickness of the rubber layer and steel layer are \( h_1 \) and \( h_2 \) respectively. The \( z \)-axis is perpendicular to the plate. All materials used in this study are assumed to be elastically isotropic for simplicity. The materials parameters used in the calculation are shown in Table 1.

![Figure 1. The plate with periodic cylindrical oscillators](image)

![Figure 2. (a) Unit cell of the plate, (b) front view of (a).](image)

| Material  | Mass density (kg/m\(^3\)) | Young’s modulus (10\(^6\)N/m\(^2\)) | Poisson’s ratio |
|-----------|--------------------------|----------------------------------|----------------|
| Aluminum  | 2800                     | 72000                           | 0.35           |
| Steel     | 7780                     | 210000                          | 0.3            |
| Rubber    | 1300                     | 0.118                           | 0.47           |

2.2 Method

In order to obtain the dispersion curve and frequency band gap of the plate, the finite element method is used to calculate the characteristic frequency and resonance mode, which has been proven to be an efficient method in previous research [8]. Compared with other analytical methods, FEM is easier to solve PCs with complex shape of scatters. Considering the periodicity of the structure, only one cell is needed
to calculate the dispersion curve. In the finite element algorithm, the discrete form of the eigenvalue equations in the unit cell can be written as

\[(K - \omega^2 M) \mathbf{u} = 0\]  

(1)

where \(K\) and \(M\) are the stiffness and mass matrices of the unit cell respectively, \(\mathbf{u}\) is the displacement of the nodes and \(\omega\) is the circular frequency. Computations are performed with software COMSOL Multiphysics. The unit cell is meshed with the “fine” grade of the predefined mesh sizes to get good convergence. Free boundary conditions are applied to the top and bottom surfaces of the unit cell. With the Bloch-Floquet theorem, periodic boundary conditions are used for the interfaces between the nearest unit cells:

\[u_j(x + a, y + a) = u_j(x, y) e^{i(k_xa + k_ya)}, (j = x, y, z)\]  

(2)

where \(u_j\) is the elastic displacement vector; \((x, y,\) and \(z)\) are the position vectors; \(k_x\) and \(k_y\) are Bloch wave vectors limited in the irreducible first Brillouin zone (BZ). The dispersion relations can be obtained by changing the value of \(k_x\) and \(k_y\) along the boundary of the first Brillouin zone and calculating the eigenvalue problem generated by the FEM algorithm.

3. Numerical results and analyses

During the calculation, lattice constant \(a = 1m\), plate thickness \(e = 0.02m\), \(h_1 = h_2 = 0.04m\) and \(d_1 = 0.6m\), \(d_2 = 0.8m\) were kept unchanged. The dispersion curve of the unit cell is reported in Figure 3. The propagation of multiple vibrations is coupled in this dispersion curve. We need to find out the starting frequency \(f_s\) (from which frequency the gap starts) and the cutoff frequency \(f_c\) (at which frequency the gap ends) of the flexural vibration band gaps and the longitudinal vibration band gaps. We can pick them out by distinguish the modes from the others. The displacement fields of the eigenmodes labelled in Figure 3 are shown in Figure 4.

Figure 3. Dispersion curve of the unit cell
Figure 4. Displacement field of eigenmode at point B1, B2, B3 and B4.

Figure 5. Layout of measuring points of calculated transmission power spectrum

Mode B1 (18 Hz) concentrates the vibration energy in the steel cap in the vibrator pillar which vibrates along the z-axis. This indicates that flexural vibration band gaps begin to form. So, this mode corresponds with the starting frequency of the flexural vibration band gap (FVBG). In mode B2 (44 Hz), the vibration of the base plate and the steel cap is inverse along the z-axis to achieve the dynamic balance. This mode corresponds with the cut-off frequency of the FVBG. The dominant vibration of B1 and B2 are along the z-axis and the vibration couples with the antisymmetric Lamb mode of the base plate, as a result the modes are involved in the formation of the FVBGs.

For longitudinal vibration band gap (LVBG), we follow the same analytical thinking. In mode B3 (7 Hz), the vibration energy concentrates on the whole body movement of the vibrator pillar along the xy plane and keep their base plates stationary, which indicates the start of a LVBG. While Mode B4 (13 Hz) achieves dynamic balance of the inverse vibration of the lateral steel cap and the base plate in the xy plane. So, Modes B3 and B4 correspond with the starting and cut-off frequencies of the LVBG. The dominant vibration of B3 and B4 are in the xy plane and the vibration couples with the symmetric Lamb mode of the base plate, as a result the modes are involved in the formation of the LVBGs.

Through the analysis of the above band structure mode, we found that the FVBGs start from mode B1 at 18 Hz and cut-off at mode B2 at 44 Hz. The LVBGs start from mode B3 at 7 Hz and cut-off at mode B4 at 13 Hz similarly.

To further demonstrate the existence of the FVBG and LVBG of the studied structure, we also calculated the transmission power spectrums. Firstly, a displacement excitation $d_{in}$ was imposed on the middle of one end of the plate, and set up the domain point probe on the middle of the opposite end of the plate (Figure 5), the displacement response $d_{out}$ was picked up. The transmission spectra $L_d$ is defined as

$$L_d = 20 \log \left| \frac{d_{out}}{d_{in}} \right|.$$  (3)

Where $d_{in}$ and $d_{out}$ are the vibration displacement of the input and output sides of the plate, respectively. The response spectra of the plate are shown in Figure 6.
The transmission spectrum depicts that the band gap range of flexural vibration is between 19 Hz to 43 Hz. Meanwhile, the longitudinal vibration enjoys the band gap between 7 Hz to 13 Hz. The frequency range of the attenuated transmission spectrums of the vibrations is nearly consistent with the results obtained from the band diagram (Figure 3). The correctness of the calculation results is further proved. In order to show the propagation of flexural vibration with different frequencies in the plate, the displacement modes of the plate within the band gap and out of the band gap are plotted (Figure 7). It can be clearly seen that the vibration can pass through the plate without any loss if its frequency is outside of the band gap. However, the vibration within the band gap will be significantly suppressed when it propagates in the plate, and the vibration displacement is mainly limited near the excitation point.

**Figure 6.** Transmission power spectrum of flexural vibration and longitudinal vibration propagating in the plate. The gray shadow regions denote the band gaps.

**Figure 7.** The modes corresponding to the frequencies of periodic structural plates outside the elastic wave flexural band gap (12 Hz) (a), and within the elastic wave flexural band gap (30 Hz) (b).

### 4. Conclusion

A plate with periodic cylindrical oscillators is proposed in this paper to realize the vibration isolation with a low frequency. Each oscillator consists of a small rubber cap and a large steel cap. By introducing the concept of band structure in solid mechanics, the band structure of a cell is analysed, and the range of flexural band gap and longitudinal band gap is concluded with the modal displacement diagram. The results are further verified with the calculation of transmission curves of the whole plate. Finally, the displacement modes of the plate are plotted. It shows that vibrations within the band gap are prohibited.
structures or large mechanical equipment, the research results are of great significance for vibration reduction and isolation in low frequency domain. Further research should concentrate on how to achieve a lower and wider band gap with smaller oscillators through proper design.

5. References
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