Persistent current noise in normal and superconducting nanorings

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Abstract

We investigate fluctuations of persistent current (PC) in nanorings, both with and without dissipation and decoherence. We demonstrate that such PC fluctuations may persist down to zero temperature provided that there exists either interaction with an external environment or an external (periodic) potential produced, e.g., by quantum phase slips in superconducting nanorings. Provided that quantum coherence is maintained in the system, PC noise remains coherent and can be tuned by an external magnetic flux $\Phi_1$ piercing the ring. If quantum coherence gets suppressed by interactions with a dissipative bath, PC noise becomes incoherent and $\Phi_1$-independent.

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(Some figures may appear in color only in the online journal)

1. Introduction

Persistent current (PC) in normal and superconducting meso- and nanorings is a fundamental ground state property of such systems. The basic physical reason behind the existence of PC is quantum coherence of electron wave functions, which may survive at long distances provided that the temperature remains low. Much is known about the average value of PC in such systems. For example, in normal rings this quantity was analyzed in detail both theoretically [1] and experimentally [2]. At the same time, only a few studies of equilibrium fluctuations of PC are available.

While non-vanishing thermal fluctuations of PC can be easily expected, the issue of quantum fluctuations of PC is somewhat less trivial since at $T \to 0$ the system approaches its non-degenerate ground state. Recently, it was demonstrated [3] that in this case, PC does not fluctuate provided that the current operator $\hat{I}$ commutes with the total Hamiltonian $\hat{H}$ of the system. In all other cases, PC fluctuates even though the system remains in its ground state. For instance, quantum fluctuations of PC in mesoscopic rings may persist down to $T = 0$ provided that such systems are coupled to an external dissipative bath [4].

Interestingly enough, quantum fluctuations of PC may also occur in the absence of any dissipation. For instance, it is easy to verify that the operators $\hat{I}$ and $\hat{H}$ do not commute for a quantum particle on a ring with some (periodic) potential [3] and, hence, PC fluctuations do not vanish even in the ground state at $T = 0$. Since quantum coherence remains fully preserved in this case, the magnitude of PC fluctuations should depend on the external magnetic flux $\Phi_1$. It immediately follows that by measuring the equilibrium current noise in meso- and nanorings it is possible to effectively probe quantum coherence and decoherence in such systems.

The goal of this paper is to theoretically analyze PC noise in both dissipative and non-dissipative systems. In the presence of dissipation, the time-reversal symmetry is violated and PC noise in metallic nanorings may be affected by decoherence. If, however, no source of dissipation is available, the time-reversal symmetry is preserved and PC fluctuations remain fully coherent, as we have already explained above. The first situation will be described within a model of a quantum particle on a ring interacting with some quantum dissipative environment [5–7], which could be, e.g., a bath of Caldeira–Leggett oscillators or electrons in a disordered conductor. Within this model, PC fluctuations will be analyzed in section 2 in both the perturbative and in the non-perturbative interaction regime. The second situation will be represented by a model of superconducting nanorings...
with quantum phase slips (QPS) [8], which tend to suppress PC in sufficiently large rings. PC noise in superconducting nanorings will be studied in section 3. In section 4, we will briefly summarize our main observations.

2. A particle on a ring in a dissipative environment

2.1. The model and effective action

Let us consider a quantum particle with mass $M$ and electric charge $e$ moving in a one-dimensional (1D) ring of radius $R$ pierced by magnetic flux $\Phi_s$. This quantum particle interacts with some collective variable $V$ describing voltage fluctuations in our dissipative bath. The total Hamiltonian for this system reads

$$\hat{H} = \frac{(\hat{\phi} - \phi_0)^2}{2MR^2} + \hat{H}_{\text{env}}(V) + \hat{H}_{\text{int}}(\theta, V),$$

(1)

where $\theta$ is the angle variable which controls the position of the particle on the ring, $\phi = -i\frac{\hbar}{\omega}\theta$ defines the angular momentum operator, $\phi_0 = \Phi_s/\Phi_0$, and $\Phi_0 = 2\pi c/e$ is the flux quantum (here and below we set Planck’s constant to be equal to unity, $\hbar = 1$). The first term in equation (1) is just the particle kinetic energy, $\hat{H}_{\text{env}}(V)$ is the Hamiltonian of the bath, and the term

$$\hat{H}_{\text{int}} = e\hat{V}$$

(2)

accounts for the Coulomb interaction between the particle and the bath.

In what follows, we will model a dissipative bath by a three-dimensional diffusive electron gas [6, 7] with the inverse dielectric function

$$\frac{1}{\varepsilon(\omega, k)} = -\frac{i\omega + Dk^2}{4\pi\sigma}.$$  

(3)

Fluctuations of the electric potential $V$ in this dissipative environment are described by the correlator

$$\langle V V \rangle_{\omega, k} = -\coth \frac{\omega}{2T} \text{Im} \frac{4\pi}{k^2 \varepsilon(\omega, k)}.$$  

(4)

Here $\sigma$ is the Drude conductivity of this gas, $D = ve_0/\lambda$ is the electron diffusion coefficient, $v_F$ is the Fermi velocity and $\lambda$ is the electron elastic mean free path, which is assumed to obey the condition $k_F \lambda > 1$ but remain much smaller than the ring radius $l \ll R$. We also point out that equation (3) applies at not too high frequencies $\omega \ll \omega_e = ve_0/l$.

Employing the definition for the current operator

$$\hat{I} = \frac{e}{2\pi} \hat{\theta} = \frac{i e}{2\pi} [\hat{H}, \hat{\theta}] = \frac{e(\hat{\phi} - \phi_0)}{2\pi MR^2}$$

(5)

and making use of the Heisenberg representation $\hat{I}(t) = e^{i\hat{H}t} \hat{I} e^{-i\hat{H}t}$, we introduce the current–current correlation function $\langle \hat{I}(t) \hat{I}(0) \rangle$ and define the PC noise power [3]

$$S(t) = \frac{1}{2} \langle \hat{I}(t) \hat{I}(0) + \hat{I}(0) \hat{I}(t) \rangle - \langle \hat{I} \rangle^2 = \int \frac{d\omega}{2\pi} S_0 e^{-i\omega t}.$$  

(6)

In order to evaluate this correlation function we further introduce the evolution operator $\hat{U}(t, t_0)$ and define the density matrix operator $\hat{\rho}(t) = \hat{U}(t, 0)\hat{\rho}_0\hat{U}^\dagger(t, 0)$, where $\rho_0$ is the initial density matrix. Since our goal here is to analyze the quantum dynamics of the particle rather than that of the bath, it will be convenient to employ the standard influence functional technique [9] and trace out the fluctuating potential $V$. Making use of a simplifying assumption that at the initial time moment the total density matrix is factorized into the product of the equilibrium bath density matrix and that of a particle $\hat{\rho}_0$, one can rewrite the evolution equation for the density matrix in the form of a double path integral over the angle variables $\theta^0$ and $\theta^B$ defined, respectively, on the forward and backward parts of the Keldysh contour,

$$\rho(\theta_1, \theta_2; t) = \sum_{m_1, m_2 = -\infty}^{\infty} e^{i(\theta_1 + 2\pi m_1)\phi_0 - i(\theta_2 + 2\pi m_2)\phi_0} \times \int_0^{2\pi} d\theta'_1 d\theta'_2 e^{-i(\theta'_2 - \theta'_1)\phi_0} \rho_0(\theta'_1, \theta'_2) \int_{\theta'_0(\theta'_1)}^{\theta'_0(\theta'_2)} D\theta^F$$

$$\times \int_{\theta''_0(\theta'_1)}^{\theta''_0(\theta'_2)} D\theta^B e^{i\bar{F}(\theta^F - \theta^B)^2/4E_c} \sin e^{-iS_R - S_i},$$  

(7)

where $\rho(\theta_1, \theta_2; t) = \theta_1(\theta_1(t) | \theta_2)$, $E_c = 1/(2MR^2)$ and $\exp(-iS_R - S_i)$ is the influence functional. Calculation of this functional amounts to averaging over the quantum variable $V$, which is also defined on the Keldysh contour. Such an averaging was performed, e.g., in [10–12] for a degenerate electron gas where the Pauli exclusion principle should be explicitly accounted for. The same procedure can be employed in our present situation of a particle on a ring where no Pauli principle needs to be included. Introducing the new variables $\theta_+ = (\theta^F + \theta^B)/2$ and $\theta_- = \theta^F - \theta^B$, after the standard algebra we obtain

$$S_R[\theta_+, \theta_-] = \pi \alpha \sum_{n = 1}^{\infty} a_n \int_0^t dt' \dot{\theta}_+(t') \sin(n(\theta_+(t') - \theta_-(t'))$$

(8)

and

$$S_I[\theta_+, \theta_-] = -2\pi \alpha \sum_{n = 1}^{\infty} a_n \int_0^t dt' \int_0^{t'} dt'' \pi T^2 \sinh(\pi T(t' - t''))$$

$$\times \cos(n(\theta_+(t') - \theta_-(t''))) \sin \frac{n(\theta_+(t') - \theta_-(t''))}{2} \times \sin \frac{n(\theta_+(t'') - \theta_-(t''))}{2},$$  

(9)

where $\alpha = 3/(8k_F^2l^2)$ is the effective coupling constant in our problem, and $a_n$ are the Fourier coefficients and are equal to $a_n = (2/(\pi r))\ln(n/r)$ if $n < r = R/l \gg 1$ and to $a_n = 0$ otherwise. The weak disorder condition $k_F l \gg 1$ implies a small effective coupling constant $\alpha \ll 1$. It is also worth pointing out that the above influence functional reduces to one derived within the Caldeira–Leggett model provided that one chooses $\alpha = \eta R^2/\pi$ and $a_n = \delta_{1n}$, where $\eta$ defines the effective friction produced by the bath and $\delta_{1n}$ is the Kronecker symbol.

In the case of the Caldeira–Leggett environment, decoherence of a quantum particle on a ring was investigated with the aid of both real-time [5] and Matsubara [6]
techniques that yield similar results, i.e. exponential suppression of quantum coherence down to $T = 0$ at sufficiently large ring radii. This result is by no means surprising since the model is exactly equivalent to that of Coulomb blockade in a single electron box where an exponential reduction of the effective charging energy at large conductances is also well established [13, 14]. The model of a particle in a diffusive electron gas was employed by different authors [6, 7, 15–18] investigating the effect of interaction-induced decoherence on the average value of PC. Below we will make use of this model to analyze PC fluctuations in the presence of quantum decoherence.

2.2. Perturbation theory

Provided that the ring radius $r$ is sufficiently small one can proceed perturbatively in $\alpha$. Consider the kernel of the evolution operator $U$ which establishes a relation between the density matrix elements at different moments of time

$$\tilde{\rho}(m_1, m_2; t) = \sum_{m_1', m_2'} U_{m_1', m_1}^{m_2', m_2}(t) \tilde{\rho}(m_1', m_2'; 0),$$

where

$$\tilde{\rho}(m, n; t) = \int_0^{2\pi} \frac{d\theta_1}{2\pi} \int_0^{2\pi} \frac{d\theta_2}{2\pi} \rho(\theta_1, \theta_2; t) e^{i m \theta_1 + i n \theta_2}$$

is the density matrix in the momentum representation which remains diagonal for the problem in question. Therefore, one can rewrite the above evolution equation as a matrix one for the diagonal elements of the density matrix.

In order to account for interaction effects, we will employ a perturbation theory similar to that developed for the Coulomb blockade problem [19, 20]. Expanding the influence functional $\exp(-i S_R - S_0)$ in powers of the coupling constant $\alpha$ and taking into account first-order diagrams depicted in figure 1, we arrive at the evolution kernel

$$\tilde{U}_{m,m'}^{m',m}(\omega) \equiv \int_0^\infty dt e^{i\omega t} \tilde{U}_{m,m'}^{m',m}(t) = \left[ \frac{i}{\omega + i \Gamma_{m,m'}} \right],$$

where

$$[\tilde{\Gamma}_{\omega}]_{m+n, m} = \frac{\pi \alpha a m}{2} \left( \frac{E_{m+n} - E_m + \omega}{e^{\Delta E_{m+n} - \omega} - 1} + \frac{E_{m+n} - E_m - \omega}{e^{\Delta E_{m+n} + \omega} - 1} \right),$$

Here $I_n = 2E_C(m - \phi_x)$ and $P_n = e^{-E_C(n-\phi_x)^2/\Gamma}$ define, respectively, the current and the distribution function in the absence of interactions. The latter quantity also involves the partition function for our system $Z = \sum_n e^{-E_C(n-\phi_x)^2/\Gamma}$. We have numerically evaluated both the kernel of the evolution operator and the PC noise power. Our results are displayed in figure 2 at different values of temperature and magnetic flux.

One observes a strong dependence of PC noise power on the external magnetic flux $\phi_x$. This feature indicates the coherent nature of PC noise [3], which clearly persists also in the presence of dissipation provided that the effect of the latter is sufficiently weak. It turns out that PC noise grows with increasing $\phi_x$ and formally diverges in the vicinity of the point $\phi_x = 0.5$. This divergence has to do with the fact that the distance between the two lowest energy levels $\delta E(\phi_x) = E_C(1 - 2\sin \phi_x)$ becomes small in this limit. Hence, the system undergoes rapid transitions between these states corresponding to different PC values. As a result, PC fluctuations in our system get greatly enhanced as soon as the flux approaches the value $\phi_x = 0.5$.

We also emphasize that PC noise does not vanish even in the limit $T \to 0$. In this case, $S_\omega$ is equal to zero only at frequencies below the inter-level distance $\omega < \delta E(\phi_x)$ and remains non-zero at higher values of $\omega$. We also point out that PC noise vanishes at $\phi_x = 0$ if evaluated perturbatively in the lowest order in $\alpha$. However, non-zero PC noise at $\phi_x = 0$ is recovered if one goes beyond perturbation theory in the interaction, as will be demonstrated below. The latter property also follows from the general expressions formulated in terms of the exact eigenstates of the total Hamiltonian [3].

Finally, we observe that at non-zero $T$ there appears an additional zero frequency peak in $S_\omega$. This peak grows with increasing temperature and eventually merges with all the
other peaks, forming a wide hump at a sufficiently high \( T \). In this case, quantum coherence gets essentially suppressed and PC noise becomes flux-independent.

2.3. Non-perturbative regime

Let us now go beyond perturbation theory in the effective coupling constant \( \alpha \). At first sight this step might be considered unnecessary since within the applicability range of our model this coupling constant always remains small \( \alpha \ll 1 \). However, it turns out [7] that the actual parameter that controls the strength of interaction effects is \( ar \) rather than \( \alpha \). Hence, should the ring radius be sufficiently large, i.e.

\[
4\pi ar \gg 1, \tag{16}
\]

perturbation theory in the interaction fails and a non-perturbative analysis of the problem becomes inevitable.

In the limit (16) and not too low temperature, one can employ the semiclassical approximation, which amounts to expanding the action \((8)\) and \((9)\) up to quadratic in \( \theta \) terms. The resulting effective action can be exactly reformulated in terms of the quasi-classical Langevin equation [21–23] for the ‘center-of-mass’ variable \( \theta \). For the model under consideration, this equation reads

\[
-\frac{1}{2E_C} \ddot{\theta}(t) - \frac{\gamma}{2} \dot{\theta}(t) = \sum_{n=1}^{\infty} \left( \xi_n(t) \cos(n\theta(t)) \right) + \lambda_n(t) \sin(n\theta(t)), \tag{17}
\]

where we introduced the parameter

\[
\gamma = 2\pi a \sum_{n=1}^{\infty} a_n n^2 = 4\pi ar^2 \tag{18}
\]

and defined Gaussian stochastic fields \( \xi_n(t) \) with the correlators

\[
\langle \xi_n(t) \xi_m(t') \rangle_{\xi,\lambda} = \langle \lambda_n(t) \lambda_m(t') \rangle_{\xi,\lambda} = \frac{\pi T^2}{\sinh^2(\pi T|t - t'|)} = -\delta_{m,n} \pi a^2 a_n n^2 \frac{\pi T^2}{\sinh^2(\pi T|t - t'|)}, \tag{19}
\]

\[
\langle \xi_n(t) \lambda_m(t') \rangle_{\xi,\lambda} = 0. \tag{20}
\]

At high temperatures the white noise limit is realized,

\[
\langle \xi_n(t) \xi_m(t') \rangle_{\xi,\lambda} = 2\delta_{m,n} \pi a^2 a_n^2 T \delta(t - t'), \tag{21}
\]

and equation (17) can be solved exactly. In this limit, we obtain

\[
S_w = \frac{e^2 \gamma T E_C^2}{\pi^2 (\omega^2 + (\gamma E_C)^2)^2}. \tag{22}
\]

At lower values of \( T \) the approximation (21) fails and more accurate equations (19) and (20) should be employed. Treating the noise terms in equation (17) perturbatively [23] and taking into account only the zeroth- and the first-order contributions, we arrive at the result

\[
\theta_\ast(t) = \theta_0^{(0)} + \theta_\ast^{(1)}(t), \tag{23}
\]

where \( \theta_0^{(0)} \) is some physically irrelevant constant and \( \theta_\ast^{(1)}(t) \) obeys the equation

\[
-\frac{1}{2E_C} \ddot{\theta}^{(1)}(t) - \frac{\gamma}{2} \dot{\theta}^{(1)}(t) = \sum_{n=1}^{\infty} \xi_n(t), \tag{24}
\]

which allows us to immediately recover the noise power

\[
S_w = \frac{e^2 \gamma E_C^2}{2\pi^2 (\omega^2 + (\gamma E_C)^2)^2} \omega \coth \frac{\omega}{2T}. \tag{25}
\]

Obviously, this result reduces back to equation (22) in the limit \( T \gg \omega \). At \( \omega \ll \gamma E_C \) the parameter \( E_C \) drops out from the expression for the noise power and we obtain

\[
S_w = \frac{e^2 \omega}{2\pi^2 \gamma} \coth \frac{\omega}{2T}. \tag{26}
\]

i.e. in this case \( S_w \propto 1/\alpha \). For \( \omega \to 0 \) this expression further reduces to \( S_w \propto T/\gamma \). The function \( S_w \) (25) is displayed in figure 3 at different temperatures.

Comparing these results with those derived perturbatively in section 2.2, we observe that, while at weak interactions the PC noise remains coherent and, hence, depends on the external magnetic flux \( \Phi_x \), in the limit of strong interactions (16) this dependence is practically absent. This is due to the strong decoherence effect produced by our dissipative bath. As a result, in the limit (16) the average value of PC gets exponentially suppressed, while PC noise does not vanish but becomes essentially incoherent.

Note that, strictly speaking, the flux-dependent contribution to \( S_w \) survives also in this limit, but it remains exponentially small, as was demonstrated by the analysis [24]. Technically, the presence of such a \( \Phi_x \)-dependent correction to the result (25) is related to the fact that the angle variable \( \theta \) is defined on a ring, i.e. is compact. The Langevin equation approach employed here ‘decompactifies’ this variable, thereby capturing only the \( \Phi_x \)-dependent contributions to PC noise power. In order to estimate the leading \( \Phi_x \)-dependent correction to equation (25), one can follow the analysis initially developed for the problem of weak Coulomb blockade in metallic quantum dots [25]. This approach allows us to obtain the relation between the density matrices and expectation values evaluated for the problems described.
by the same Hamiltonian but, respectively, compact and non-compact variables. Without going into corresponding details here we only quote the result [24]

\[ S_\omega = \frac{\epsilon^2 y E_c^2 \omega \coth(\omega/2T)}{2\pi^2(\omega^2 + (\gamma E_c)^2)} \left( 1 - A e^{-4\pi\alpha r} \cos(2\pi \varphi_s) \right), \]

(27)

where \( A \propto \alpha r \). Thus, in the non-perturbative limit (16) the coherent flux-dependent contribution is indeed exponentially small and can be safely neglected as compared to the main incoherent term (25).

3. PC noise in thin superconducting nanorings

Let us now turn to the analysis of PC noise in a dissipativeless system which will be a superconducting nanoring. As long as the ring remains sufficiently thick, superconducting fluctuations can be ignored and, hence, there exists no physical mechanism that could cause PC fluctuations. If, however, the ring becomes thin, superconductivity may be disrupted in various places in the ring due to fluctuations of the order parameter. At low temperatures, most important fluctuations of that kind are QPS [8]. Below we will demonstrate that the properties of superconducting nanorings in the presence of QPS are described by the effective theory equivalent to that for a quantum particle on a ring in a periodic potential.

The starting point of our derivation is the expression for the grand partition function \( Z \). This expression can be represented in terms of a path integral over the phase \( \varphi \) of the superconducting order parameter. Employing the low-energy effective action for a quasi-1D superconducting wire [26–28], one finds that

\[ Z = \sum_{\varphi, \varphi'} \int \mathcal{D}\varphi e^{-S} \int dx \frac{1}{2\pi} \int dr_1 \ldots dr_N \int dx_N \frac{1}{2\pi} \int \frac{dx_1}{2\pi} \ldots \int \frac{dx_N}{2\pi}, \]

(28)

where we defined \( \lambda = \frac{\pi^2 N_s D \Delta s}{2s} \), \( s \) is the ring cross section, \( N_s \) is the density of states at the Fermi level, \( \Delta s \) is the absolute value of the superconducting order parameter, \( D \) is the diffusion coefficient and \( v = \sqrt{\pi \sigma \Delta s / C} \) is the velocity of low-energy plasmon mode propagating along the wire (the so-called Mooji–Schön mode). Here \( \sigma \) is the Drude normal state conductivity of our metallic ring and \( C \) is the wire capacity per unit length.

The path integral in equation (28) should be performed with periodic (in imaginary time) boundary conditions \( \psi(x, 0) = \psi(x, \beta) + 2\pi m \), where \( m \) is an arbitrary integer number (the so-called winding number). The boundary conditions should also be periodic with respect to the spatial coordinate along the ring and, in addition, should be sensitive to the magnetic flux piercing the ring, i.e. \( \psi(L, \tau) = \psi(0, \tau) + 2\pi \varphi_s \). Here \( L = 2\pi R \) is the ring perimeter, \( \varphi_s = \Phi_s / \Phi_0 \) and \( \Phi_0 = \Phi_0 / 2 \) is the superconducting flux quantum.

The partition function (28) can be evaluated semiclassically. In the main approximation it suffices to take into account all relevant saddle point configurations of the phase variable \( \varphi \) that satisfy the equation

\[ (\partial^2_x + v^2 \partial^2_\tau)\psi(x, \tau) = 0. \]

(29)

Apart from trivial solutions of this equation linear in \( \tau \) and \( x \) there exist non-trivial ones which correspond to virtual phase jumps by \( \pm 2\pi \) at various points of a superconducting ring. These quantum topological objects can be viewed as vortices in space–time and represent QPS events [8, 26, 27]. All these configurations can be effectively summed up with the aid of the approach involving the so-called duality transformation.

In order to proceed let us express the general solution of equation (29) in the form

\[ \psi(x, \tau) = a_\mu \tau + b_\mu x + \psi^{\text{QPS}}(x, \tau), \]

(30)

where \( a_\mu \) and \( b_\mu \) are some constants fixed by the boundary conditions. We also introduce the vorticity field \( \sigma(x, \tau) \) with the aid of the relations

\[ v \partial_x \sigma = \partial_t \psi^{\text{QPS}}, \quad \partial_t \sigma = -v \partial_x \psi^{\text{QPS}}. \]

(31)

This field is single-valued and obeys the equation

\[ \partial^2_x \sigma + v^2 \partial^2_\tau \sigma = 2\pi v \sum_j v_j \delta(x - x_j) \delta(\tau - \tau_j), \]

(32)

where \( x_j, \tau_j \) and \( v_j \) denote, respectively, the space and time coordinates of the \( j \)th phase slip and its topological charge (phase winding). The partition function for a given saddle point solution can be rewritten as a path integral over the \( \sigma \)-field containing the functional delta function which follows from the above equation. Performing a summation over all possible QPS configurations, we obtain

\[ Z = \sum_{N=0}^{\infty} \sum_{\nu_0, \ldots, \nu_N} \int_0^{2\pi} \frac{dx}{2\pi} \int dx_1 \ldots dx_N \frac{1}{2\pi} \int \frac{dx_1}{2\pi} \ldots \int \frac{dx_N}{2\pi}, \]

(33)

where \( \beta = 1 / T \).

\[ \gamma_{\text{QPS}} \sim (g_\xi \Delta / \xi) \exp(-ag_\xi) \]

(34)

is the QPS rate [27]. \( g_\xi = 4\pi N_s D s / \xi \) is the dimensionlessness conductance of the wire segment of length equal to the superconducting coherence length \( \xi \) and \( a \) is a numerical prefactor of the order of 1. Equations (33) and (34) are applicable provided that \( g_\xi \) is sufficiently large, i.e. \( g_\xi e^{-\pi a/2} \ll 1 \).

Rewriting the delta function in equation (33) as a path integral of the exponent and performing summation over all
QPS configurations as well as integration over $\sigma$, we obtain

$$Z = \sum_{m,n=\pm \infty}^{\infty} e^{2\pi i m \phi} \int D\theta e^{-S_{\text{eff}}[\theta]},$$  

(35)

where

$$S_{\text{eff}} = \int dx \, dt \left( \frac{(\partial_\theta \phi)^2 + v^2(\partial_\theta \phi)^2}{8\pi v \lambda} - \gamma_{\text{QPS}} \cos \theta \right).$$  

(36)

In contrast to the original problem, here the path integration is performed over the single-valued field $\theta$ with periodic boundary conditions

$$\theta(x, \beta) - \theta(x, 0) = 2\pi n,$$

$$\theta(L, \tau) - \theta(0, \tau) = 2\pi m.$$  

(37)

PC noise power can be evaluated directly making use of the above equations and the expression for the current $I$, which is just proportional to the phase difference around the ring,

$$I(\tau) = \frac{2\pi e v \lambda}{L} (\psi(L, \tau) - \psi(0, \tau)).$$  

(38)

Having evaluated the Matsubara current–current correlation function, one performs an analytic continuation to real times and, taking into account the fluctuation–dissipation theorem, arrives at the PC noise power spectrum $S_{\omega}$.

In the limit of low temperature $T \ll v/L$ and provided that the ring perimeter is not too large, one can ignore the spatial dependence of the field $\theta$ and hence neglect the term $v^2(\partial_\theta \phi)^2$ in the effective action (36). After that, our problem becomes effectively zero dimensional and exactly equivalent to that of a particle on a ring in the presence of the cosine external potential. In other words, we have mapped our problem onto that described by the Hamiltonian

$$\hat{H} = \frac{(\hat{\phi} - \phi_s)^2}{2MR^2} + U_0(1 - \cos(\kappa \theta)), \quad \kappa = 1,$$

(39)

where one should now identify $\kappa = 1$,

$$\frac{1}{MR^2} \to E_R = \frac{\hbar^2 N_0 D \Delta_s}{R} \sim g_\xi \Delta \xi \frac{\hbar}{R},$$  

(40)

and

$$U_0 \to 2\pi R \gamma_{\text{QPS}} \sim g_\xi \Delta R \frac{\hbar}{\xi} e^{-\eta g_\xi}.$$  

(41)

An analysis of PC fluctuations for the model (39) in the limit of a strong external potential $U_0 \gg E_R$ was carried out in [3]. In the case of superconducting nanorings, it yields

$$S_{\omega} = \frac{e^2}{2\pi U_0} \Omega^3 \frac{\delta(\omega - \Omega - \Lambda \cos(2\pi \phi_s))}{\delta(\omega - \Omega - \Lambda \cos(2\pi \phi_s))},$$  

(42)

where $\Omega = \sqrt{\pi N_0 D \Delta \gamma_{\text{QPS}} g_\xi \Delta e^{-\eta g_\xi}/2}$ is the frequency of oscillations of the ‘particle’ near the bottom of cosine potential and

$$\Lambda = 256 \sqrt{\frac{U_0}{\pi \Omega}} e^{-\frac{\eta g_\xi}{\pi}}.$$  

(43)

The result (42) demonstrates that in the low-temperature limit the PC noise spectrum $S_{\omega}$ differs from zero due to the effect of QPS and has the form of two sharp peaks at frequencies well below the superconducting gap $\Delta$. The exact positions of these peaks can be tuned by the flux $\phi_s$ piercing the ring, although only weakly, since in this case $\Omega \gg \Lambda$. Note that the condition $U_0 \gg E_R$ is equivalent to $R \gg R_c \sim \frac{\xi}{\exp(\alpha g_\xi)}$ in which case the average value of PC is exponentially suppressed [8]. $(I) \propto \exp(-R/R_c)$. The PC noise power spectrum peaks also remain small in this case, although they decrease only as $S_{\omega} \propto R_c/R$ with increasing ring radius.

In the opposite case $E_R \gg U_0$ the cosine potential term remains small as compared to the particle kinetic energy. Hence, in this limit the effect of QPS may be considered perturbatively in $\gamma_{\text{QPS}}$. In the absence of QPS the noise power $S_{\omega}$ shows only one peak at zero frequency with the amplitude equal to the current dispersion, i.e.

$$S_{\omega} = 2\pi (\langle \hat{I}^2 \rangle - \langle \hat{I} \rangle^2) \delta(\omega).$$  

(44)

The amplitude of this peak decreases with temperature and tends to zero in the limit $T \to 0$. Employing the standard quantum mechanical perturbation theory, in the lowest non-vanishing order in $U_0$ one recovers additional peaks at frequencies corresponding to the transitions between neighboring energy levels. These peaks survive even at zero temperature $T = 0$ in which case one finds that

$$S_{\omega} = \frac{e^2}{\pi (1 + 2\phi_s^2)} \left[ \delta(\omega - \Omega) (1/2 + \phi_s) \right]$$

$$+ \frac{e^2}{\pi (1 - 2\phi_s^2)} \left[ \delta(\omega - \Omega) (1/2 - \phi_s) \right]$$

$$+ \frac{e^2}{\pi (1/2 - \phi_s)} \left[ \delta(\omega - \Omega) (1/2 + \phi_s) \right].$$  

(45)

This equation is applicable as long as the condition

$$\xi g_\xi \ll R \ll \min \left( \frac{R_c}{\gamma}, \frac{v}{2\pi T} \right)$$  

(46)

remains fulfilled. We observe that in this case, PC noise power $S_{\omega}$ has the form of four sharp peaks at frequencies that strongly depend on the external flux $\phi_s$. For example, by tuning the flux to a value close to one-half of the superconducting flux quantum $\phi_s \approx \pm 1/2$, one should observe a strong enhancement of noise peaks that occur in the vicinity of zero frequency $\omega = 0$. The physical reason for this enhancement is the same as that already discussed in section 2.2: the energies of the two lowest levels become close to each other at such values of $\phi_s$, implying the possibility for rapid transitions between these states. Such intensive transitions, in turn, imply strong fluctuations of PC. Exactly at resonance $\phi_s = \pm 1/2$ the second-order perturbation theory fails and a more accurate treatment becomes necessary.

4. Conclusions

In this paper, we have studied fluctuations of PC in nanorings with and without dissipation. Specifically, we restricted our attention to PC noise and evaluated the symmetric
current–current correlation function. Comparing the results obtained within two different models analyzed in sections 2 and 3, we observed both similarities and important differences in the behavior of these systems.

To begin with, in the absence of interactions and dissipation in the model of a particle on a ring (section 2) as well as in the absence of phase slips in superconducting nanorings (section 3), PC fluctuates only at non-zero $T$ and no such fluctuations could occur provided that the system remains in its ground state at $T = 0$. In the presence of interactions in the first model or QPS in the second model, the current operator does not anymore commute with the total Hamiltonian of the system and fluctuations of PC do not vanish down to zero temperature. Yet, another qualitative similarity between these systems is that in both cases PC noise decreases with increasing ring radius $R$.

The most important physical difference between the models considered in sections 2 and 3 is the presence of dissipation and, hence, decoherence in the first model and their total absence in the second model. Accordingly, at low temperatures PC noise always remains coherent in the second case, which implies that the PC noise power spectrum essentially depends on the magnetic flux $\Phi_1$ piercing the ring. In the absence of dissipation at $T \to 0$, PC noise has the form of sharp peaks at frequencies corresponding to energy differences between the system states for which quantum mechanical transitions are possible. At finite values close to one-half of the flux quantum, some energy levels also become close to each other, which means strong enhancement of PC fluctuations.

Coherent fluctuations of PC are also possible in the presence of dissipation provided its effect remains sufficiently weak and the ring radius remains small. In this limit the decoherence effect of the external dissipative bath is still insignificant. Narrow peaks in PC noise get somewhat broadened even at $T \to 0$ due to the presence of dissipation, but the dependence of $S_\omega$ on the magnetic flux persists also in this case. In rings with larger radii, in contrast, fluctuations in the dissipative bath strongly suppress quantum coherence down to $T = 0$ and induce incoherent $\Phi_1$-independent current noise in the ring, which persists even at $\Phi_1 = 0$ when the average PC is absent. Thus, quantum coherence and its suppression by interactions in meso- and nanorings can be experimentally investigated by measuring PC noise and its dependence on the external magnetic flux. It would be interesting to carry out such experiments in the near future.

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