Financial Risk and Returns Prediction with Modular Networked Learning

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Abstract

An artificial agent for financial risk and returns’ prediction is built with a modular cognitive system comprised of interconnected recurrent neural networks, such that the agent learns to predict the financial returns, and learns to predict the squared deviation around these predicted returns. These two expectations are used to build a volatility-sensitive interval prediction for financial returns, which is evaluated on three major financial indices and shown to be able to predict financial returns with higher than 80% success rate in interval prediction in both training and testing, raising into question the Efficient Market Hypothesis. The agent is introduced as an example of a class of artificial intelligent systems that are equipped with a Modular Networked Learning cognitive system, defined as an integrated networked system of machine learning modules, where each module constitutes a functional unit that is trained for a given specific task that solves a subproblem of a complex main problem expressed as a network of linked subproblems. In the case of neural networks, these systems function as a form of an “artificial brain”, where each module is like a specialized brain region comprised of a neural network with a specific architecture.

Keywords: Modular Networked Learning, Long Short-Term Memory, Gated Recurrent Unit, Financial Turbulence, Financial Management
1 Introduction

In the context of artificial intelligence (AI), machine learning allows for the development of intelligent systems that are able to learn to solve problems \cite{1,2,3}. While a problem, where an artificial agent is supplied with an information feature vector, a target variable and a single optimization goal, can be solved by programming the agent with some appropriate single machine learning architecture, when dealing with more complex problems, that can be expressed as a network of linked subproblems, each with its own target and optimization goal, the artificial agent’s cognitive architecture needs to be expanded.

Using the framework of machine learning, this expansion becomes possible by following the networked relations between the different subproblems. In this case, the agent’s cognitive structure can be designed in terms of a Modular Networked Learning (MNL) system, where each module is a cognitive unit that can be comprised of any machine learning architecture. Each module can receive inputs from other modules to which it is linked and can also receive external inputs.

Following the main problem’s networked structure, each learning unit is trained on a module-specific target, thus, instead of a single regression or classification problem, an MNL system works with multiple targets, which are linked to interconnected subproblems, functioning as a modular interconnected system of learning units.

In this sense, an MNL system is an integrated system of machine learning modules, where each learning module constitutes a functional unit that is trained for a given specific task that solves a subproblem of a complex main problem, comprised of different interconnected subproblems. This creates a hierarchical modular cognitive system, where the modular hierarchy comes out of the networked relations between the different modules, that mirrors the networked relations between the different subproblems.

In the case of artificial neural networks, for a general MNL system, each module is a neural network and the MNL system is, itself, a neural network, that is, a modular neural network where each module is a neural network, thus, each learning module functions like a specialized (artificial) “brain region” that is trained for a specific task on a specific target, towards solving a particular subproblem. The links between different neural networks, in the modular system, are, as stated above, related to the networked structure of the main networked problem that the MNL system is aimed at solving.
In finance, the application of MNL systems provides a different way for financial modeling, management and analysis. Namely, traditional financial theory uses two main statistical measures for portfolio management and risk management: the expected returns and the variance around the expected returns \[4, 5, 6\]. However, the main assumption of this theory is that financial markets are efficient, which leads to the assumption that there is no way for which future returns can be predicted based on past returns, with the last period’s price capitalized, using a fixed expected rate of return, being the best predictor for price, this assumption led to the submartingale statistical hypothesis for price movements \[7, 8\].

Contrasting with this theory, financial management schools such as fundamental analysis and technical analysis assumed that there is information which can be used to anticipate price movements, and built different trading rules based on past market data \[5, 9, 10\].

Applications of complex systems science and chaos theory to economics and finance also raised the possibility of emergent nonlinear dynamics coupled with stochastic noise to be present in financial returns’ dynamics, with consequences for financial theory development and financial management \[7, 8, 11, 12, 13, 14, 15, 16, 17\].

Stochastic and deterministic volatility proposals \[7, 8, 14\], including fractal and multifractal risk and returns models \[18, 19, 20, 16, 17\], were introduced and shown to explain, at least to some extent, the presence of emergent nonlinear dependence in financial turbulence, affecting the variance around the expected financial returns.

In the 21st Century, with the rise of the FinTech industry and the integration of machine learning in financial trading, leading to a wave of automation in the financial industry \[22, 23\], the issue of nonlinear market dynamics is a center stage problem for both academia, financial community and regulators \[24, 25, 26\].

In the current work, we apply MNL to financial risk and returns prediction, specifically, we address the two main parameters used as fixed by the efficient market hypothesis proponents: the expected returns and the variance around the expected returns. Without any prior knowledge of the returns distribution, our goal is for an artificial agent to be able to:

1. Based upon a past time window of financial returns, to learn to produce a conditional expectation for the next period’s returns;
2. Based upon the past track record of squared deviations of observed ver-
sus predicted values, learn to produce a conditional expectation for the next period’s squared deviation around the next period’s returns’ prediction, thus, learning to produce an expectation for its own prediction error and, in this way, providing for a machine learning replacement of a probabilistic time-varying variance;

3. Use the two expectations to produce a prediction interval for the next period’s returns that takes into account the predicted volatility risk, measured by the square root of the next period’s predicted squared deviation.

The third task is the main problem, which can be solved by using the outputs of the first and second tasks (these are the two linked subproblems). The first task can be solved by a module trained to predict the next period’s returns based on past returns, for a given time window, this is the main prediction module. The second task, on the other hand, has a hierarchical dependence upon the first task’s module, since it works with the notion of variance as an expected value for the squared deviations around the expected value for a given target random signal.

The evidence of presence of nonlinear volatility dynamics in financial returns’ dynamics \cite{7,8,14,15} means that the conditional variance must change with time, the second task must, therefore, employ a learning module that learns to predict the next period’s squared deviation around the first module’s next period’s prediction, using a past record of squared deviations between observed and predicted values.

Cognitively, this means that the artificial agent is capable of predicting the target variable, in this case, the financial returns (the main prediction module is specialized in this task), but the agent also keeps an internal working memory record of past prediction performance, in terms of the past squared deviations (this is a second module, playing the role of a working memory unit), and uses this record to predict the next period’s squared deviation around the first module’s prediction (this is a third module and corresponds to a dispersion prediction unit, which anticipates market volatility).

We, therefore, have an agent that learns not only to predict the target variable but it also learns to anticipate a squared deviation for its next period’s prediction, quantifying an expected value for the uncertainty associated with the temporal prediction task, which allows the agent to emit a prediction interval centered on the first module’s prediction but that also
takes into account the predicted deviation. In this way, the agent is capable of outputing a volatility-adjusted forward-looking time-varying prediction interval for the financial returns.

While the dispersion prediction unit produces a machine learning version of a conditional variance, for the interval prediction, we need to take the square root of this module’s output, thus, producing an interval, around the expected returns, within a certain number of units of the equivalent of a conditional standard deviation.

In terms of financial theory, we have a modular system that is capable of capturing patterns in the returns’ dynamics to produce an expectation for the next period’s returns’ values, but it is also capable of using the past track record of the squared deviations around its predictions to anticipate the next period’s market volatility, thus, incorporating a second-order nonlinearity associated with market volatility. The main prediction module is trained on the first task, while the dispersion prediction module is trained on the second task.

The work is organized as follows, in section 2, we begin by addressing modular hierarchical cognitive structures in neuroscience and in AI research. Drawing on this research, in subsection 2.1, we introduce the concept of an MNL system, addressing its general structure and computational properties. In subsection 2.2, we address the financial risk and returns’ prediction agent, using the approach defined in subsection 2.1. In subsection 2.3, we introduce two learning architectures for the agent’s modular units, one based on Long Short-Term Memory (LSTM) networks, another based on Gated Recurrent Units (GRUs).

We show that both LSTMs and GRUs are, themselves, modular systems of neural networks, so that, when we connect them in an MNL system we get a more complex modular hierarchy, where each learning module is itself a modular system.

When an MNL system is a hierarchical modular neural network, comprised of learning modules that are, themselves, modular neural networks trained on specific tasks, the cognitive system is not only able to address the subproblem connectivity, associated with a complex problem, but each subproblem is also adaptively addressed by a modular system that is optimized for that specific subproblem. This generates a scaling adaptivity from the local functional modular level up to the higher scale of the whole artificial cognitive system.

In section 3, we compare these two architectures’ performance on three
financial indices: S&P 500, NASDAQ Composite and the Russell 2000.

In section 4, we conclude the work, discussing the implications of the main results for finance.

2 Modular Networked Learning for Financial Risk and Returns Prediction

Modular networked cognition links adaptivity, functionality and information flow towards the solution of complex problems. In a modular networked cognitive system, the adaptive functionality of each module works towards the overall system’s adaptivity.

The information flow between the modules and the way in which the modules are linked play a fundamental role in the whole system’s adaptive responses, since, considering any given module that depends upon other modules for input, its adaptive responses, towards the fulfilling of its functional role in the cognitive system, depend upon the adaptive responses of those other modules, in this way, the evolutionary success of the whole system depends upon the adaptive responses of each module.

In neuroscience, a modular structure in a brain can be identified in terms of a localized topological structure that fulfills some function in the organism’s neural processing and cognition [27, 28, 29]. Modularity can be identified with brain regions or even multi-level structures that are organized in terms of modular hierarchies [27, 28]. In evolutionary terms, modularity, in biological brains, is argued as promoting evolvability, conserving wiring cost, and as a basis for specialized information processing and functionality [29].

In cognitive science, the argument for the importance of modularity is not new, Fodor [30], for instance, proposed a modular thesis of cognition. Changeux and Dehaene also expanded on cognitive science’s modular theses, from the perspective of evolutionary biology and neuroscience [31]. More recently, Kurzweil [32] built a pattern recognition approach to cognition, based upon hierarchical modularity of pattern recognition modules, in order to approach a computational model of the neocortex, aimed at research on AI. While this later work addresses mostly one particular form of machine learning architecture, in the current work, we propose the concept of an MNL system as an integrated cognitive system comprised of modules that can have any machine learning architecture.
Our work draws a computational and cognitive basis from neuroscience \cite{27, 28, 29, 31}, in particular, on the importance of a simultaneous functional specialization and functional interdependence, within neurocognition, an interdependence that, as stated by Changeux and Dehaene \cite{31}, holds for the different levels of neurofunctional organization. Taking into account this framework, we now introduce the concept of an MNL system.

### 2.1 Modular Networked Learning Systems

In its general form, an MNL system can be defined as an artificial learning system comprised of a network of modules, where each learning module is a cognitive unit that can have any machine learning architecture. There are two general types of modules that we consider:

- Transformation units: whose task is to perform data transformation, that can be used to extract learning features;
- Learning units: that are aimed at learning to solve a specific task and employ some machine learning architecture.

Each module can receive inputs from other modules to which it is linked and can also receive external inputs. This leads to a second classification for the modules:

- Sensory modules: whose input features are comprised only of external data;
- Internal modules: whose input features are comprised only of other modules’ outputs;
- Mixed modules: whose input features are comprised of external data and other modules’ outputs.

Each learning unit, in an MNL system, is trained on a module-specific target, producing a predicted value for that target, thus, instead of a single regression or classification problem, an MNL system works with multiple targets and (hierarchically) linked problems, functioning as a hierarchical modular interconnected system.

The connections between the modules introduce a form of local hierarchical information processing, where a given module may use other modules’
outputs as inputs for its respective functional effectiveness, which is, in this case, the solution of its specific task. The whole system thus addresses complex problems, involving multiple interconnected subproblems.

The hierarchy comes out of the interconnections between the different modules which, in turn, comes from the interconnection between the different subproblems. Thus, rather than engineering the system by taking the hierarchy as primitive, the modules (which, in the case of a learning unit, is, itself, an artificial learning system) and the connections between them are the primitive basis for the system’s engineering, the hierarchy comes out as a consequence of the connections between the modules, which, in turn, reflects the nature of the complex problems that the MNL system must solve.

The training protocols for the MNL system must follow the inter-module linkages, therefore, the sensory modules must be trained first, since they do not receive any internal input, only working on external data, then, the training must follow the sequence of linkages, so that, at each training stage, a specific learning unit is only trained after all its input learning units’ training, since it must receive the outputs of these units, in order to be able to learn, and it must learn to respond to already optimized input learning units.

Indeed, in the present work, we are dealing with an example of cooperative networks that are trained to solve different interconnected objectives, building on each other’s work. In this context, in order to be properly optimized, a given learning unit needs the learning units, upon which it depends for its functionality, to already be trained.

Since any machine learning architecture can be chosen for the modules, depending on the architectures used, we can have a different modular system. An MNL system comprised only of neural networks has, however, a specific property: the system is, itself, a neural network, that is, we have a modular system of neural networks that is, itself, a neural network, which adds another level to deep learning that can be characterized as form of deep modular hierarchical learning, since, each neural network is specialized in a subproblem, but the networks’ processing is linked, which means that, at the higher levels (higher cognitive functions), the modules need to work with the outputs of the processing of other modules, beginning with the sensory modules. This allows the development of deep modular networked learning, where each learning unit can use a (possibly different) deep learning architecture.

In the case of recurrent neural networks, whose learning units are recurrent neural networks, the connection between two learning units requires an intermediate working memory unit, which can be defined as a transformation
unit. We will show an example of this in the next section.

Any type of neural network can be used for different modules, the network architecture for each module depends upon the specific problem that it is aimed at solving. The same holds for other machine learning architectures.

To exemplify the application of MNL systems to interconnected problems, we will now apply MNL to financial risk and returns prediction.

2.2 A Financial Risk and Returns’ Predictor Agent Using Modular Networked Learning

The efficient market hypothesis (EMH), introduced in 1970 by Fama [34], states that financial assets’ prices always fully reflect all available information, this hypothesis was integrated in modern financial theory, which developed from the 1950s to the 1980s and 1990s [4, 5, 6, 19, 35]. There are three versions of the EMH, as addressed by Fama [34]: the weak version assumes that all historical price information is already reflected in prices, the semi-strong version extends the hypothesis to include all the publicly available information and the strong version extends the hypothesis to include all relevant information (public and private), which means, in this last case, that one cannot consistently profit from insider trading.

The EMH versions are nested, in the sense that the if the semi-strong version fails the strong version fails, even if the weak version may not fail, on the other hand, if the weak version fails, the strong and semi-strong versions also fail, and the EMH itself fails.

The weak version’s main statistical implication is that we cannot use past price information to predict the next period’s price. This implication actually comes out of Fama’s argument regarding the empirical testability of the EMH, which led the author to recover the concept of fair game, and propose the martingale and submartingale models as a working basis for statistically testing the EMH [31].

In the statistical assumption that we cannot use past price information to predict the next period’s price, the EMH is in opposition to other schools of financial management, in particular, to technical analysis, which assumes that there is relevant information in past price trends that can help predict the next period’s price [9, 19, 35].

If the EMH is correct, then, that means that we cannot predict the next period’s price based on past trends, namely, statistically, for a financial price
series $S_t$, given any past price time window $\{S_t, S_{t-1}, S_{t-2}, ..., S_{t-\tau}\}$, the expected value of the price at time $t+1$ obeys the submartingale condition:

$$E[S_{t+1}|S_t, S_{t-1}, ..., S_{t-\tau}] = S_t e^{\mu}$$  \hspace{1cm} (1)

that is, the best prediction for the next period’s price is the current price capitalized by one period, using a fixed exponential rate of return $\mu \geq 0$.

If we consider the logarithmic returns, connecting the price at time $t+1$ to the price at time $t$, we get:

$$S_{t+1} = S_t e^{r_{t+1}}$$ \hspace{1cm} (2)

where $r_{t+1}$ is a random variable, then, the submartingale condition implies that:

$$E[r_{t+1}|r_t, r_{t-1}, ..., r_{t-\tau}] = \mu$$ \hspace{1cm} (3)

Indeed, if the conditional expected value of the returns over a past time window was not equal to a fixed value $\mu$, thus, independent from past returns, then, we could use the price value at $t-\tau$, $S_{t-\tau}$, and the sequence $r_t, r_{t-1}, ..., r_{t-\tau}$ to anticipate the next period’s financial price.

A second early assumption used in modern financial theory, in particular, in modern portfolio theory and option pricing theory, is that the logarithmic returns series have a fixed variance, so that we also get the condition:

$$E[(r_{t+1} - \mu)^2 | r_t, r_{t-1}, ..., r_{t-\tau}] = \sigma^2$$ \hspace{1cm} (4)

With the development of chaos theory and the complexity approach to economics and finance, mainly, in the 1980s and 1990s, the possibility of emergent complex nonlinear dynamics, linking trading strategies and multiagent adaptation, led to other hypotheses and market theories, namely, the coherent market hypothesis (CMH), introduced by Vaga, which assumes that markets undergo phase transitions and addresses these phase transitions applying statistical mechanics, and the adaptive market hypothesis (AMH), introduced by Lo, who applied evolutionary biology, complex systems science and neuroscience, raising the argument for the connection between the traders’ adaptive dynamics and emergent market patterns. Both hypotheses are convergent on one point: the possibility of emergent complex nonlinear dynamics.

In particular, emergent complex nonlinear dynamics can take place, leading to both the expected value and variance changing with time as a basic
consequence of investors’ adaptive responses to risk and returns, so that we get an emergent functional dependence:

$$E[r_{t+1} | r_t, r_{t-1}, ..., r_{t-\tau}] = \mu_{t+1} = f_0(r_t, r_{t-1}, ..., r_{t-\tau})$$ (5)

$$E[(r_{t+1} - \mu_{t+1})^2 | r_t, r_{t-1}, ..., r_{t-\tau}] = \sigma^2_{t+1} = f_1(r_t, r_{t-1}, ..., r_{t-\tau})$$ (6)

These functions are, however, unknown, with different competing models being able to produce similar turbulence patterns [15, 16, 17, 18, 19, 20, 38]. When there is turbulence and strong nonlinearities, the conditional probabilities can change from time period to time period in unstable ways, a major argument raised by Vaga [36]. In this case, without the knowledge of the underlying probability structure and dynamical rules, the human analyst is faced with different possible alternatives that might have generated the same data structure, thus, an underlying problem for financial management arises due to the complex nonlinear dynamics present in the data, since this nonlinear dynamics can have a complex functional form and/or be influenced by stochastic factors and, also, it affects two key parameters: how much one can expect to get as returns on an investment, and the level of volatility risk associated with that investment.

Key methodologies used for financial management, such as equivalent martingale measures applied to financial derivatives’ risk-neutral valuation, can be invalidated by changes in the market volatility, linked to a nonlinear evolution of the variance, which can lead to multiple equivalent measures [6]. Whenever there are strong dynamical instabilities, it becomes difficult to estimate a probability distribution by applying a frequencist approach, so that we are, in fact, dealing with a decision problem under uncertainty, where the financial manager needs to form an expectation for the financial returns and an evaluation of the returns’ variance, producing a calculatory basis for the expected value and variance for time $t+1$, given the information available at time $t$, but has no access to the underlying probability nor to the dynamical rules.

An artificial agent built with an MNL system can be applied to these contexts, since it is effective in capturing complex nonlinear dynamical relations between predictors and target variables.

In this way, the probability-based expected value for the next period’s returns (equation (5)) is replaced, in the context of uncertainty, by the machine learning-based expected value for the next period’s returns, produced by a module that is specialized in predicting the the financial returns’ next
period’s value. We call this module the main prediction unit (MPU), it is a sensory module, since it works with the external data signal, and a learning unit aimed at learning to predict the financial returns’ next period’s value.

Likewise, the variance, which is the expected value for the squared deviation around the expected value (equation (6)), is replaced, in the context of uncertainty, by the machine learning-based expected value for the next period’s squared deviation around the machine learning-based expected returns, which implies the need for another learning unit that is trained on that specific task, we call this module the dispersion prediction unit (DPU).

The two modules are linked in a modular network. This network must actually be comprised of three modules: two learning units (the MPU and the DPU) plus one additional sliding window memory unit which is a transformation unit, keeping track of the past record of squared deviations. The first unit is aimed at producing an expectation for the financial returns $r_{t+1}$, given the last $\tau + 1$ observations, so that we replace the expected value in equation (5) by the output of the learning unit:

$$\mu_{t+1} = \Phi_{MPU}(r_t)$$ (7)

where $r_t \in \mathbb{R}^{\tau+1}$ is a sliding window vector of the form:

$$r_t = \begin{pmatrix} r_{t-\tau} \\ \vdots \\ r_{t-1} \\ r_t \end{pmatrix}$$ (8)

and $\Phi_{MPU}$ is the nonlinear mapping produced by a neural network linking the past returns information, expressed in the vector $r_t$, to the expected value for the next period’s returns $\mu_{t+1}$, approximating the unknown function $f_0$ in equation (5).

Now, since the input $r_t$ has an internal sequential structure, as a sliding window vector signal, this means that, by using a recurrent neural network for the MPU, the processing of the input will take into account the sequential structure present in the input to the MPU. This is relevant because of the evidence that financial returns go through regime changes, a point raised in complex systems science approaches to finance [13, 14, 15, 16, 17, 18, 36], which means that the sequence of values in $r_t$ matters, recurrent neural networks (RNNs) capture this type of sequential dependence in data. The MPU may, thus, predict positive, negative or null returns, which means that...
the market price can be expected to rise, remain unchanged or fall, depending on the sequential patterns exhibited by the past returns. This is in stark contrast to the sub-martingale property assumed by the EMH.

Let, then, for a decreasing sequential index \( i = \tau, ..., 1, 0 \), \( \hat{\Pi}_i \) be the projection operator on the index \( i \) component of a \( \tau + 1 \) size vector, so that for the input \( \mathbf{r}_t \), we have:

\[
\hat{\Pi}_i \mathbf{r}_t = r_{t-i}
\]  

(9)

and let \( \mathbf{h}_{1,t} \in \mathbb{R}^{d_1} \) be the \( d_1 \)-dimensional hidden state of the RNN for the first module (the MPU) at time \( t \), where we use the lower index 1 as the module’s index. For a time window of \( \tau + 1 \) size, the RNN generates a sequence of hidden states, so that we have the mapping of the sequence \( \{\hat{\Pi}_i \mathbf{r}_t\}_{i=\tau}^0 \) to a corresponding sequence of hidden states \( \{\mathbf{h}_{1,t-i}\}_{i=\tau}^0 \), where \( \mathbf{h}_{1,t-i} \) is updated in accordance to some pointwise nonlinear activation function \( \Phi_1 \) that results from the computation of the RNN, namely, we have the general update rule:

\[
\mathbf{h}_{1,t-i} = \Phi_1 \left( \mathbf{h}_{1,t-i-1}, \hat{\Pi}_i \mathbf{r}_t \right)
\]  

(10)

that is, the network sequentially projects each component of the input data vector and updates the hidden state, depending upon its previous hidden state and the projected value of the input vector.

Now, given the sequence of hidden states, the last element of the sequence is fed forward to a single output neuron, so that the final prediction is obtained by the feedforward rule:

\[
\mu_{t+1} = \Theta_1 \left( \mathbf{W}_1^T \mathbf{h}_{1,t} + b_1 \right)
\]  

(11)

where \( \mathbf{W}_1 \in \mathbb{R}^{d_1} \) is a synaptic weights vector, \( b_1 \in \mathbb{R} \) is a bias and \( \Theta_1 \) is a nonlinear activation function for the final feedforward connection. In this way, the MPU is trained on a regression problem, such that the unknown function in equation (5) is replaced by the module’s predictions. Formally, using the general scheme in equations (10) and (11), we get the nested structure:

\[
\mu_{t+1} = \Phi_{MPU} (\mathbf{r}_t) = \\
\Theta_1 \left[ \mathbf{W}_1^T \Phi_1 \left( \left( ... \Phi_1 \left( \left( \Phi_1 \left( \mathbf{h}_{1,t-\tau-1}, \hat{\Pi}_\tau \mathbf{r}_t \right), \hat{\Pi}_{\tau-1} \mathbf{r}_t \right), ..., \hat{\Pi}_0 \mathbf{r}_t \right) \right) + b_1 \right]
\]  

(12)

\(^1\)By considering a decreasing index sequence we are moving forward in time from the earliest element in the sequence to the latest.
The RNN scheme incorporates a basic financial adaptation dynamics, where the patterns in the past returns’ sequential values affect the trading patterns and, therefore, the next returns’ value. This is a basic feedback dynamics in the emergent market cognition and activity. The artificial agent incorporates this in its processing of its expected value for the next period’s returns.

Now, since financial time series exhibit evidence of time-varying volatility in the returns’ series, with long memory and clustering of turbulence [13, 14, 15, 16, 17, 18, 19], the deviations around expected returns must be time-varying and patterns may be present in the squared deviations around an expected value, so that we do not get a stationary variance [13, 14, 15, 18, 19, 20].

Financially, this means that while market agents trade based on expectations regarding returns, they also incorporate market volatility risk in trading decisions, which introduces a feedback with regards to returns and volatility, linked to the risk of fluctuations around the expected value. This also means that the market will be sensitive to changes in volatility and these changes end up influencing trading decisions and, thus, the volatility dynamics itself.

Therefore, a second level of expectations needs to be computed by the artificial agent, in order to reflect a basic market computation of patterns in volatility, that is, in order to capture emergent dynamical patterns in volatility, the artificial agent needs to have a cognitive system that is capable of producing an expectation for volatility, which we will measure in terms of an expected squared deviation around the next period’s agent’s prediction.

In this way, the agent must be capable of incorporating past volatility data into a prediction of the next period’s squared deviation around its own expectation, such that the artificial agent is capable of producing a machine learning equivalent of a forward looking variance, replacing equation (6) with a machine learning-based variance. In order to do this, we need the other two modules, the working memory module and the DPU.

The working memory module is a transformation unit, in the form of a sliding window working memory neural network, that keeps track of the past prediction performance in terms of squared deviations. This network has $\tau + 1$ neurons linked in chain. The state of the network at time $t$ is described
by a vector $s_t^2 \in \mathbb{R}^{\tau+1}$ defined as:

$$s_t^2 = \begin{pmatrix} (r_{t-\tau} - \mu_{t-\tau})^2 \\ \vdots \\ (r_{t-1} - \mu_{t-1})^2 \\ (r_t - \mu_t)^2 \end{pmatrix}$$

(13)

Now using the projection operators, so that $\hat{\Pi}_i s_t^2 = (r_{t-i} - \mu_{t-i})^2$, the state transition from $s_t^2$ to $s_{t+1}^2$ is, locally, described as:

$$\hat{\Pi}_i s_{t+1}^2 = \begin{cases} \hat{\Pi}_{i-1} s_t^2, & i \neq 0 \\ (\hat{\Pi}_0 r_{t+1} - \mu_{t+1})^2, & i = 0 \end{cases}$$

(14)

for $i = 0, 1, 2, ..., \tau$, which leads to the sliding window transition:

$$\begin{pmatrix} (r_{t-\tau} - \mu_{t-\tau})^2 \\ \vdots \\ (r_{t-1} - \mu_{t-1})^2 \\ (r_t - \mu_t)^2 \end{pmatrix} \rightarrow \begin{pmatrix} (r_{t+1-\tau} - \mu_{t+1-\tau})^2 \\ \vdots \\ (r_{t+1-1} - \mu_{t+1-1})^2 \\ (r_{t+1} - \mu_{t+1})^2 \end{pmatrix}$$

(15)

Therefore, the last neuron in the chain records the last squared deviation, and the remaining neurons implement a sliding window operation, leading to a mixed module, which always keeps track of the past prediction performance, in terms of squared deviations, over the time window. The agent is therefore, keeping track of its own past squared prediction errors.

The third module, is an internal module, that uses the input from the sliding window working memory neural network in order to predict the next period’s squared deviation. Thus, given a training sequence of pairs of the form $(s_t^2, s_{t+1}^2)$, with $s_{t+1}^2 = (r_{t+1} - \mu_{t+1})^2$, the third module learns to predict the next period’s dispersion around the expected value produced by the first module, which justifies the name dispersion prediction unit (DPU).

Since the sequential patterns of volatility matter for the task of predicting volatility, in particular, changes in volatility and the clustering of turbulence need to be processed by the agent, then, as in the case of the MPU, we also need to work with a recurrent neural network, therefore, as in the case of the MPU, we also get the sequentially nested structure:

$$\sigma_{t+1}^2 = \Phi_{DPU} (s_t^2) = \Theta_3 \left[ W_3^T \Phi_3 \left( \left( \ldots \Phi_3 \left( \Phi_3 \left( h_{3,t-\tau-1}, \hat{\Pi}_\tau s_t^2 \right), \hat{\Pi}_{\tau-1} s_t^2 \right), \hat{\Pi}_0 s_t^2 \right) + b_3 \right) \right]$$

(16)
where $\mathbf{W}_3 \in \mathbb{R}^{d_3}$ is a synaptic weights vector, $b_3 \in \mathbb{R}$ is a bias and $\Theta_3$ is a pointwise nonlinear activation function for the final feedforward connection. The training of the third unit replaces the unknown function in equation (6) by the learning unit’s predictions.

Regarding the last nonlinear activation, for the MPU, we will set $\Theta_1$, in equation (12), to the tanh activation, since its prediction target (the financial returns) can be both positive and negative, financially, also, the tanh structure may be useful to capture market correction movements. In the case of the DPU, the target cannot be negative, so we will set $\Theta_3$ to a ReLU activation\(^2\).

A relevant point, characteristic of a modular system, is that there is a scaling of nonlinearities in equation (16), indeed, since the squared errors, received from the sliding window (working) memory block, incorporate the past outputs of the first module, we have a hierarchical scaling of nonlinear transformations of the original returns signal. This is a useful point, because it allows the MNL system to capture patterns associated with strong nonlinearities in the original signal, in order to synthesize an expected returns value and a volatility risk expectation.

The agent’s MNL system is, thus, not only capable of producing a prediction for a target, but it is also able to address its own past prediction performance and to anticipate by how much it may fail in its next step prediction. This is the result of the linkage between the three modules, where the first module provides the prediction for the target, the second module produces a working memory record of the past squared deviations and the third module uses that record to anticipate the next step squared deviation, effectively incorporating a concept of expected squared dispersion around an expected value (the concept of variance). Using this expected squared dispersion, a machine learning-based standard deviation can be extracted from equation (16) by taking the square root:

$$\sigma_{t+1} = \sqrt{\sigma_{t+1}^2} = \sqrt{\Phi_{DPU} (s_t^2)}$$

A risk and returns’ prediction agent, defined as an artificial agent with

\(^2\)Another alternative would be a sigmoid activation, however, the sigmoid dampens for higher values of the input, while the ReLU obeys a linear rule for a positive input, which is better for capturing jumps in volatility. The squared deviation along with the last ReLU layer allow one to incorporate, in a single dynamical recurrent model, the scaling of nonlinearities \cite{13,19} and generalized auto-regressive conditional heteroskedasticity (GARCH)-type squared dynamical dependence patterns \cite{13,14,15}.\]
the above built-in MNL system can, thus, reduce the uncertainty associated with the functional dynamical relations in equations (5) and (6), and provide a calculatory basis for expected returns and risk assessment.

In particular, in what regards the prediction of the next period’s financial returns, the agent, at time $t$, can use the outputs of the MPU and DPU to produce a prediction interval for time $t+1$, that takes into account both the returns’ expected value as well as the time-varying machine learning-based standard deviation defined as the square root of the DPU’s output, so that, instead of a single point prediction, the agent provides an interval prediction that also incorporates volatility risk prediction, formally:

$$I_{l,t+1} = [\mu_{t+1} - l\sigma_{t+1}; \mu_{t+1} + l\sigma_{t+1}]$$

(18)

The agent is, thus, reporting a prediction within $l$ units of $\sigma_{t+1}$. The output of the MPU sets the center for the interval, while the output of the DPU sets the scale for the reported uncertainty in the agent’s prediction and, thus, for the predicted market volatility. While the uncertainty level $l$ is fixed, $\sigma_{t+1}$ is an anticipated uncertainty given a time window of data, which means that the interval’s amplitude adapts to the market data.

In terms of interval prediction performance, given a sample sequence of length $T$, the proportion of cases that fall within the interval bounds can be calculated as follows:

$$p_l = \frac{\# \{ t : r_t \in I_{l,t}, t = t_1, t_2, ..., T \}}{T}$$

(19)

The success in prediction depends upon the agent’s MNL system’s success in expectation formation, which, in turn, depends upon the ability of the agent’s MNL system to capture main nonlinear patterns in the returns and squared deviations’ dynamics, producing expectations based upon these patterns. We will compare, in the next section, the performance of two main recurrent architectures LSTM and GRU, which we now address.

### 2.3 Learning Units’ Architectures

The architectures LSTM [39, 40] and GRU [41, 42] are two recurrent neural network architectures that are, themselves, modular networks comprised of interacting neural networks. When the RNNs assigned to the MPU and DPU are LSTM or GRU neural networks, we get a modular system whose learning
units are, thus, also, modular systems, so that a linked scaling modularity is obtained from the individual learning units up to the entire MNL system.

While, at the level of the individual learning units, the modularity, in this case, is aimed at the adaptive memory management that captures main temporal patterns in the input data and is trained at predicting a single target, at the level of the entire modular networked cognitive system the agent’s learning units are each trained on a different target. This distinction between target specific modularity and multitarget modularity is important when dealing with an MNL system comprised of different target-specific modular neural networks.

The artificial agent’s cognitive system, thus, becomes an integrated neural network with a scaling modular organization where each specialized learning unit is itself a modular neural network. To place this point more formally, let us consider the main equations for the LSTM.

2.3.1 Long Short-Term Memory Architecture

Considering an LSTM architecture, the hidden state update rule depends upon a memory block, comprised of a number of $d_k$ memory cells, where, as previously, we set the module index $k = 1$ for the MPU and $k = 3$ for the DPU, formally, for a decreasing index $i = \tau, ..., 1, 0$ the update rule depends upon output gate neurons and the memory cells as follows:

$$h_{k,t-i} = o_{k,t-i} \odot \tanh(c_{k,t-i})$$

where $\odot$ stands for the pointwise multiplication and $\tanh$ is the pointwise hyperbolic tangent activation function, $c_{k,t-i} \in \mathbb{R}^{d_k}$ is the vector for the LSTM memory cells’ state at time $t - i$ and $o_{k,t-i} \in \mathbb{R}^{d_k}$ is an output gate vector that modulates the hidden layer’s response to the neural activation resulting from the memory cells’ signal. Both the output gate and the memory cells will depend upon the input data and the previous hidden layer’s state.

The output gate is an adaptive neural response modulator, comprised of $d_k$ neurons, that modulates (pointwise) multiplicatively the response of the hidden layer neurons to the LSTM memory cells, thus, the neurons that comprise this output gate play the role of what can be considered memory response modulator neurons. These neurons also form the output layer of a neural network that receives inputs from: external input data, the previous hidden state and the memory cells’ state (peephole connection) plus a bias,
so that we have the following update rules for the MPU and DPU’s memory response modulator neurons, respectively:

\[
o_{1,t-i} = \text{sigm} \left( \hat{\Pi}_i r_t W_{1,o} + R_{1,o} h_{1,t-i-1} + p_{1,o} \odot c_{1,t-i} + b_{1,o} \right) \tag{21}
\]

\[
o_{3,t-i} = \text{sigm} \left( \hat{\Pi}_i s_t^2 W_{3,o} + R_{3,o} h_{3,t-i-1} + p_{3,o} \odot c_{3,t-i} + b_{3,o} \right) \tag{22}
\]

where \( \text{sigm} \) is the sigmoid activation function\(^3\).

At each update, the local network, supporting the memory response modulator neurons’ computation for the MPU, processes the past financial returns at step \( t-i \), with associated synaptic weights vector \( W_{1,o} \in \mathbb{R}^{d_1} \), which explains the first term in equation (21). Formally, we have:

\[
\left( \hat{\Pi}_i r_t \right) W_{1,o} = r_{t-i} W_{1,o} = \begin{pmatrix}
  r_{t-i} w_{1,o}^1 \\
  r_{t-i} w_{1,o}^2 \\
  \vdots \\
  r_{t-i} w_{1,o}^{d_1}
\end{pmatrix}
\tag{23}
\]

The first term for the DPU, on the other hand, processes the past squared deviations at step \( t-i \), stored in the sliding window memory block, with associated synaptic weights vector \( W_{3,o} \in \mathbb{R}^{d_3} \), so that we get, for the first term in equation (22), the following result:

\[
\left( \hat{\Pi}_i s_t^2 \right) W_{3,o} = (r_{t-i} - \mu_{t-i})^2 W_{3,o} = \begin{pmatrix}
  (r_{t-i} - \mu_{t-i})^2 w_{3,o}^1 \\
  (r_{t-i} - \mu_{t-i})^2 w_{3,o}^2 \\
  \vdots \\
  (r_{t-i} - \mu_{t-i})^2 w_{3,o}^{d_3}
\end{pmatrix}
\tag{24}
\]

The remaining structure of the two modules is the same: each module’s memory response modulator neurons receive a recurrent input from the previous hidden state \( h_{k,t-i-1} \), with a corresponding synaptic weights matrix \( R_{k,o} \in \mathbb{R}^{d_k \times d_k} \). Assuming a peephole connection, the memory response modulator neurons also receive an input from the memory cells with peephole weights \( p_{k,o} \in \mathbb{R}^{d_k} \). The pointwise multiplication means that the relation

\(^3\)We use this notation since the symbol \( \sigma \) is already being used for the standard deviation.
for peephole connections is one-to-one. Finally, both modules add a bias $b_{k,o} \in \mathbb{R}^{d_k}$, before applying the pointwise sigmoid activation function.

This is the main structure for the output gate. Now, for each learning unit, the corresponding memory cells update their state in accordance with the following rule:

$$c_{k,t-i} = i_{k,t-i} \odot z_{k,t-i} + f_{k,t-i} \odot c_{k,t-i-1}$$  \hspace{1cm} (25)$$

In this case, $z_{k,t-i} \in \mathbb{R}^{d_k}$ is an input layer to the cells that makes the memory cells update their state, processing a new memory content, the impact of this input layer is modulated by the input gate, $i_{k,t-i} \in \mathbb{R}^{d_k}$, which determines the degree to which the cells update their state, incorporating the new memory content in their processing.

Likewise, there is a tendency of the memory cells not to process new memories, like a short-term memory forgetfulness which is incorporated in the second pointwise product that comprises the memory cells’ update rule, which is comprised of the previous step memory cells’ state $c_{k,t-i-1}$, pointwise multiplied by the forget gate $f_{k,t-i} \in \mathbb{R}^{d_k}$, which modulates the degree of response of the memory cells to their previous state. In the limit case, if $i_{k,t-i} = \mathbf{0}_{d_k}$ and $f_{k,t-i} = \mathbf{1}_{d_k}$, where $\mathbf{0}_{d_k}$ is a $d_k$-sized column vector of zeroes and $\mathbf{1}_{d_k}$ is a $d_k$-sized column vector of ones, the memory cells’ state would remain unchanged $c_{k,t-i} = c_{k,t-i-1}$, which means that the network would be unable to incorporate new memory contents in its dynamical memory processing.

For the LSTM architecture, the new memory contents plus the input and forget gates are, each, the output layer of a respective neural network. The input and forget gates follow the same rules as the output gate $o_{k,t-i}$, with the exception that the peephole connection depends upon the previous rather than the updated state of the memory cells, thus, for the MPU, we have:

$$i_{1,t-i} = \text{sigm} \left( \left[ \tilde{\Pi}_i r_{t} \right] \ W_{1,in} + R_{1,in} h_{1,t-i-1} + p_{1,in} \odot c_{1,t-i-1} + b_{1,in} \right)$$  \hspace{1cm} (26)$$

$$f_{1,t-i} = \text{sigm} \left( \left[ \tilde{\Pi}_i r_{t} \right] \ W_{1,f} + R_{1,f} h_{1,t-i-1} + p_{1,f} \odot c_{1,t-i-1} + b_{1,f} \right)$$  \hspace{1cm} (27)$$

while, for the DPU, we have:

$$i_{3,t-i} = \text{sigm} \left( \left[ \tilde{\Pi}_i s^2_i \right] \ W_{3,in} + R_{3,in} h_{3,t-i-1} + p_{3,in} \odot c_{3,t-i-1} + b_{3,in} \right)$$  \hspace{1cm} (28)$$

$$f_{3,t-i} = \text{sigm} \left( \left[ \tilde{\Pi}_i s^2_i \right] \ W_{3,f} + R_{3,f} h_{3,t-i-1} + p_{3,f} \odot c_{3,t-i-1} + b_{3,f} \right)$$  \hspace{1cm} (29)$$
The new memory contents for the two modules’ memory cells are, respectively, updated in accordance with the rules:

\[ z_{1,t-i} = \tanh \left( \left( \hat{\Pi}_i r_t \right) W_{1,z} + R_{1,z} h_{1,t-i-1} + b_{1,z} \right) \quad (30) \]

\[ z_{3,t-i} = \tanh \left( \left( \hat{\Pi}_i s_{t}^2 \right) W_{3,z} + R_{3,z} h_{3,t-i-1} + b_{3,z} \right) \quad (31) \]

thus, \( z_{1,t-i} \) is the output layer of a neural network that receives, as inputs, the external data \( r_{t-i} \), multiplied by the synaptic weights \( W_{1,z} \in \mathbb{R}^{d_1} \), a recurrent internal connection \( h_{1,t-i-1} \), with associated synaptic weights \( R_{1,z} \), plus a bias \( b_{1,z} \). For the DPU module \( z_{3,t-i} \), we have the same structure except that the input data is the squared deviation \( \hat{\Pi}_i s_{t}^2 = (r_{t-i} - \mu_{t-i})^2 \).

Considering the whole set of equations for the memory cells update, for each of the two modules, the corresponding LSTM has a sequential processing that begins by outputing new candidate memory contents for the LSTM cells \( z_{k,t-i} \), as well as for the input and forget gates, in this way, the new memory contents are adaptively processed, by the memory cells’ layer so that the cells incorporate the new memory contents to a greater or smaller degree, depending upon the input and forget gates, that are, themselves, output layers of respective neural networks.

For the MPU, this means that the memory cells are able to adapt to the order and values of the elements in the sequence contained in the time window vector \( r_t \), rather than computing the external data as a single block, the values contained in \( r_t \) and the sequence in which they occur are incorporated in the neural processing, which means that the network is able to adapt to changes in patterns, including seasonalities or complex nonlinearities, that may introduce (nonlinear) recurrence structures in data as well as nonlinear long memory.

Likewise, the DPU memory cells are able to adapt to the order and values of the past squared deviations in the time window vector \( s_t^2 \), which is particularly important for dealing with processes with nonstationary variance that exhibit a past dependence such as nonlinear dynamical heteroskedasticity and long memory in volatility, which are characteristic of financial markets \[18, 19\].

In the update of the memory cells’ states, the LSTM recurrent network is using three modules: a neural network, for processing the new memory contents \( z_{k,t-i} \), and the neural networks for the input \( i_{k,t-i} \) and forget \( f_{k,t-i} \)
gates, which lead to an adaptive management of the degree to which each memory cell incorporates the new memory contents.

Once the memory cells’ new state is processed, the LSTM network proceeds to update the memory response modulator neurons $o_{k,t-i}$, which are output neurons of another neural network (another module). Finally, the hidden layer is updated by the pointwise product between the memory response modulator neurons’ state and the memory cells’ state.

After processing each element in the sequence, the last state of the LSTM network’s hidden layer is fed forward to a final neuron which updates its state in accordance with the scheme introduced previously, namely, in this case, applying equation (20), we get, for the expected value and variance at time $t + 1$, the following relations:

$$\mu_{t+1} = \tanh \left\{ W_1^T [o_{1,t} \odot \tanh (c_{1,t})] + b_1 \right\}$$

$$\sigma^2_{t+1} = \max \left\{ (W_3^T [o_{3,t} \odot \tanh (c_{3,t})] + b_3), 0 \right\}$$

where, as defined before, we used the hyperbolic tangent for the next period’s returns expected value, and the ReLU activation for the next period’s expected squared deviation (the next period’s variance).

Addressing the modularity of the LSTM architecture, within the context of an MNL system, is key in both illustrating the development of scaling artificial neural modular structures, as well as distinguishing between two types of modularity, characterized in terms of their respective functionality, a single target modularity and a multiple linked targets’ modularity.

In an MNL system, with learning modules comprised of LSTM learning units, the two types of modularities are linked. Indeed, in the above example, both the MPU and the DPU are modular recurrent networks that are aimed each at predicting a specific target, they are each specialized learning units, so that the modules that comprise their respective recurrent neural networks are functionally connected towards each of these learning units’ specific objective, that is, predicting these learning units’ respective targets.

However, while the MPU is a sensory unit, in the MNL system, with a target that does not depend on any other modular unit, the DPU depends upon the MPU to solve its specific task. In this case, the MPU feeds forward its outputs (its predictions) to a sliding window memory block in the MNL system, which keeps the record of past squared deviations updated and always matching the time window of values, in this way, the past prediction performance of the MPU, measured in terms of the squared deviations, can
be used by the DPU to predict its target, but this means that the DPU is working with the outputs of another module in the system to solve its specific task.

There is a scaling functionality in a general MNL system, which is illustrated here, where a module (in this case, the DPU) works with a function of the outputs of another module (the MPU) to solve a different task from the MPU. On the other hand, the DPU, like the MPU, has an internal modular structure which is linked to its LSTM architecture.

The LSTM architecture of the DPU, as shown in the above equations, is actually working with the past squared deviations between $r_{t-i}$ and $\mu_{t-i}$ as inputs. This shows that the entire neural processing of the MPU is incorporated at each step of the LSTM updates of the DPU, introducing a hierarchical modularity where the DPU depends upon the MPU, which is characteristic of an MNL system.

On the other hand, since the LSTM architecture is itself a modular network trained on a specific target, we have a scaling modularity down to the individual modules, where each learning unit is, itself, a modular network, with the difference that, while at the level of the MNL system, each module is specialized in a specific task, and trained to solve that task, at the level of each learning unit, under the LSTM architecture, the (sub)modules are connected in a network that is trying to solve a single task and the (sub)modules are being trained conjointly towards solving that single task, which makes the LSTM learning unit a specialized unit in this MNL system, comprised of (sub)modules that work towards a single specialized goal.

The effectiveness of a general MNL system depends upon the training of each module, in the sense that a module that is at a level higher in the modular hierarchy must learn to adapt to other modules that are already adapted/fit to solve their respective (specialized) tasks. As stated before, this means that the learning units must be trained following the hierarchy of connections between the subproblems, so that, in this case, the DPU is trained after the MPU is trained.

In the present case, there is an additional specific relation between the two modules, which is linked to the target of the DPU. Since the DPU is aimed at predicting market volatility, in terms of a forward looking expected squared deviation, not only does it work with outputs produced by the MPU, but its target is also dependent on those outputs, since it is learning to predict the next period’s dispersion around the MPU’s next period prediction, this is specific of the current example and not a general feature of MNL systems.
2.3.2 Gated Recurrent Unit Architecture

The GRU is an alternative to the LSTM, working without memory cells \[41, 42\]. The hidden state update rule is such that the GRU has an incorporated (adaptive) recurrent dynamics that modulates the flow of information inside the unit, such that the state update for the hidden layer is a linear interpolation between the previous hidden state activation plus a new candidate activation update. Formally, for the MPU and DPU, we have the following general rule:

\[
h_{k,t-i} = (1_d - u_{k,t-i}) \odot h_{k,t-i-1} + u_{k,t-i} \odot \tilde{h}_{k,t-i}
\]

(34)

where the update gates, for the MPU and DPU, are, respectively, defined as follows:

\[
u_{1,t-i} = \text{sigm} \left( \left( \tilde{\Pi}_1 r_t \right) W_{1,u} + R_{1,u} h_{1,t-i-1} \right)
\]

(35)

\[
u_{3,t-i} = \text{sigm} \left( \left( \tilde{\Pi}_3 s_t^2 \right) W_{3,u} + R_{3,u} h_{3,t-i-1} \right)
\]

(36)

On the other hand, the new candidate activations are also the outputs of neural networks which, following the GRU architecture \[42\], for the MPU and DPU, are defined, respectively, as:

\[
\tilde{h}_{1,t-i} = \text{tanh} \left( \left( \tilde{\Pi}_1 r_t \right) W_{1,h} + R_{1,h} \left( e_{1,t-i} \odot h_{1,t-i-1} \right) \right)
\]

(37)

\[
\tilde{h}_{3,t-i} = \text{tanh} \left( \left( \tilde{\Pi}_3 s_t^2 \right) W_{3,h} + R_{3,h} \left( e_{3,t-i} \odot h_{3,t-i-1} \right) \right)
\]

(38)

where \( e_{1,t-i} \) and \( e_{3,t-i} \) are reset gates defined as:

\[
e_{1,t-i} = \text{sigm} \left( \left( \tilde{\Pi}_1 r_t \right) W_{1,r} + R_{1,r} h_{1,t-i-1} \right)
\]

(39)

\[
e_{3,t-i} = \text{sigm} \left( \left( \tilde{\Pi}_3 s_t^2 \right) W_{3,r} + R_{3,r} h_{3,t-i-1} \right)
\]

(40)

Considering the above equations, both for the MPU and DPU, the corresponding GRU is comprised of three modules, the first two regard the computation of the candidate activation \( \tilde{h}_{k,t-i} \) and the update gate \( u_{k,t-i} \), the third is the reset gate \( e_{k,t-i} \).

Each of the three modules that determine the GRU’s hidden layer update, is a neural network, with output neurons, respectively, \( \tilde{h}_{k,t-i} \), \( u_{k,t-i} \), and \( e_{k,t-i} \).
and $e_{k,t-i}$. Again, and as in the LSTM case, the specific connectivity introduced by the MNL system, leading to a modular (neural) hierarchy, becomes explicit, namely, for each of the three modules that comprise the GRUs, integrated, respectively, in the MPU and the DPU neural networks, there is a computation of a signal external to the respective unit.

In the case of the MPU, since it is a sensory module, this signal is given by the sequence of values $\{\Pi_t r_t = r_{t-i}\}_{i=\tau}$, which corresponds to a past sequence observations of the financial returns $r_t$, over a $\tau + 1$ time window.

In the case of the DPU, on the other hand, this signal comes from the sliding window memory module that computes the squared deviations between the past observations $r_t$ and the corresponding MPU predictions, over the same temporal window, so that, for the DPU, the signal is given by the sequence of values $\{\Pi_t s_t^2 = (r_{t-i} - \mu_{t-i})^2\}_{i=\tau}$.

Thus, for each module that comprises the DPU’s GRU, we effectively have a processing of a past time window of squared deviations around the predictions from the MPU, stored in the sliding window working memory module. At each level of the GRU processing, in the case of the DPU, the MNL system’s connectivity introduces an internal processing based upon the outputs of another learning unit of the MNL system, which also has a corresponding GRU architecture.

As in the LSTM architecture, each GRU is a modular system of neural networks optimized to predict a specific target, while the entire MNL system can be addressed as a modular system of modular neural networks.

Considering the GRU modules, we have a content updating rule with less modules than the LSTM. This is because the GRU does not have memory cells, rather, to compute the new hidden layer’s state $h_{k,t-i}$, the network uses an adaptive linear interpolation between the previous hidden layer’s state $h_{k,t-i-1}$ and a candidate hidden layer’s new state $\tilde{h}_{k,t-i}$, exposing the whole state each time [42] and such that the candidate hidden layer’s state $\tilde{h}_{k,t-i}$ is computed using the new signal in the sequence ($\Pi_t r_t = r_{t-i}$, in the case of the MPU, and $\Pi_t s_t^2 = (r_{t-i} - \mu_{t-i})^2$, in the case of the DPU), with synaptic weights $W_{k,h}$, plus a signal that comes from a connection with the hidden layer, therefore, processing the previous hidden layer’s state $h_{k,t-i-1}$ modulated by a reset gate $e_{k,t-i}$. If the reset gate is equal to a vector of zeroes, then, in the case of the MPU, the candidate hidden state neural network just feeds forward the new signal to the candidate hidden layer, with associated synaptic weights $W_{k,h} \in \mathbb{R}^{d_1}$, applying a pointwise tanh activation to update
the candidate hidden state.

In general, the reset gate neurons adaptively modulate the exposure of the candidate hidden state neural network to the previous hidden state, with associated neural weights \( R_{k,h} \in \mathbb{R}^{d_k \times d_k} \). The adaptive modulation results from the fact that the reset gate is updated at each sequence step, in accordance with the output of a respective neural network that, in the case of the MPU, processes, as input, the sequence element \( \hat{\Pi}_{r,t} = r_t - i \), with associated neural weights \( W_{1,r} \in \mathbb{R}^{d_1} \), and the previous hidden state \( h_{1,t-i-1} \), with associated neural weights \( R_{1,r} \in \mathbb{R}^{d_1 \times d_1} \), applying, for the reset state update, the pointwise sigmoid activation. Likewise, for the DPU, it processes the sequence element \( \hat{\Pi}_{s,t} = (r_t - \mu_{t-i})^2 \), with associated neural weights \( W_{3,r} \in \mathbb{R}^{d_3} \), and the previous hidden state \( h_{3,t-i-1} \), with associated neural weights \( R_{3,r} \in \mathbb{R}^{d_3 \times d_3} \), applying, for the reset state update, the pointwise sigmoid activation.

Finally, regarding the update gate, if \( u_{k,t-i} \) is close to a vector of zeroes, the hidden state remains unchanged with respect to its previous value, on the other hand, when \( u_{k,t-i} \) is close to a vector of ones, the hidden state update is dominated by the new candidate activation, as per equation (34).

The update gate is, in turn, the output of a neural network, such that, for the MPU, the pointwise sigmoid activation is applied to an input signal where the sequence element \( \hat{\Pi}_{u,t} = r_t - i \) is processed, with associated neural weights \( W_{1,u} \in \mathbb{R}^{d_1} \), and the previous hidden state \( h_{1,t-i-1} \) is processed, with associated neural weights \( R_{1,u} \in \mathbb{R}^{d_1 \times d_1} \). Likewise, for the DPU, the corresponding neural network is such that the pointwise sigmoid activation is applied to an input signal where the sequence element \( \hat{\Pi}_{s,t} = (r_t - \mu_{t-i})^2 \) is processed, with associated neural weights \( W_{3,u} \in \mathbb{R}^{d_3} \), and the previous hidden state \( h_{3,t-i-1} \) is processed, with associated neural weights \( R_{3,u} \in \mathbb{R}^{d_3 \times d_3} \).

As in the LSTM case, after processing each element in the sequence, the last state of the GRU network’s hidden layer is fedforward to a final neuron which updates its state in accordance with the scheme introduced previously, namely, applying equation (34), we get, for the expected value and variance at time \( t+1 \), the following relations:

\[
\mu_{t+1} = \tanh \left\{ W_{1}^T \left[ (1_{d_1} - u_{1,t}) \odot h_{1,t-1} + u_{1,t} \odot \tilde{h}_{1,t} \right] + b_1 \right\} \tag{41}
\]

\[
\sigma_{t+1}^2 = \max \left\{ W_{3}^T \left[ (1_{d_3} - u_{3,t}) \odot h_{3,t-1} + u_{3,t} \odot \tilde{h}_{3,t} \right] + b_3, 0 \right\} \tag{42}
\]
which can be contrasted with those of the LSTM in equations (32) and (33).

Having introduced the two main architectures, we will now address the empirical results from applying these two architectures to different financial data.

3 Empirical Results

We now apply the risk and returns’ predictor agent to the following financial series:

- Dividend adjusted closing value for the S&P 500 from January, 3, 1950 to January, 4, 2018, leading to 17112 data points in the logarithmic returns series, with the first 8000 used for training, and the remaining data points used for testing;

- Dividend adjusted closing value for the NASDAQ Composite from February, 5, 1971 to January, 4, 2018, leading to 11834 data points in the logarithmic returns series, with the first 5000 data points used for training, and the remaining data points used for testing;

- Dividend adjusted closing value for the Russell 2000 from September, 10, 1987 to January 4, 2018, leading to 7642 data points in the logarithmic returns series, with the first 3500 data points used for training, and the remaining data points used for testing;

The first two series are long time series and a benchmark for EMH proponents. A main argument for the EMH would be that the S&P 500 is based on the market capitalization of 500 large companies and that it is free of possible trends in price that might be used by technical analysts. The NASDAQ Composite is also a reference, in this case, mostly linked to technology-based companies.

The third series is the Russell 2000. While the S&P 500 index is a benchmark for large capitalization stocks, the Russell 2000 is a benchmark for small capitalization stocks (small caps), used mainly by small cap funds. Although it is a smaller time series than the previous two indices, it is still a large dataset.

For training, we need to take into account the performance of both modules. In order to do so, we set up a randomly initialized agent population,
train each agent and select the agent with a highest score with the score defined in terms of prediction accuracy.

Therefore, for each architecture, we build a population of randomly initialized agents, with glorot uniform initialization. We calculate for each agent’s MPU and DPU the Root Mean-Squared Error (RMSE) and the respective prediction accuracy defined, for the MPU, as:

\[ p_{\epsilon, MPU} = \frac{\# \{|\mu_t - r_t| < \epsilon : t = 1, 2, ..., T\}}{T} \]  

while, for the DPU, we have:

\[ p_{\epsilon^2, DPU} = \frac{\# \{|\sigma_t^2 - s_t^2| < \epsilon^2 : t = 1, 2, ..., T\}}{T} \]  

where \( T \) is the size of the training dataset.

Thus, the prediction accuracy is defined as the proportion of times in which the prediction falls within an open neighborhood of the target variable, with size \( \epsilon \), in the case of the MPU, and \( \epsilon^2 \), in the case of the DPU.

We set the DPU radius to \( \epsilon^2 \), in order to have comparable results between the MPU and DPU respective scales. Namely, the DPU’s target is in squared units of the MPU’s target, which means that the comparable neighborhood needs to be rescaled accordingly, so that if we took the square root, then, we would get the same neighborhood size, with respect to the returns’ target. The two accuracies are combined in the score which takes the mean accuracy:

\[ \text{score} = \frac{p_{\epsilon, MPU} + p_{\epsilon^2, DPU}}{2} \]

the agent with the highest prediction score in training is chosen for testing.

For optimization we use RMSProp and sequential minibatch training. The RMSProp is used with a squared error loss function, L2 norm regularization and gradient clipping with truncation. Sequential minibatch is used since we are dealing with a time series, which means that we need to use a non-overlapping sequence of minibatches for training the modules, in order to preserve the sequential nature of the data and capture possible nonlinear patterns in past returns.

The hidden layer’s width used is 2 in both architectures, this size was set because in repeated experiments we found that the turbulent nature of the data, and the fact that the neural network is fed sequential 1D signals, leads to a network performance such that an increase in the hidden layer width
makes the agent fit better the laminar periods, while losing predictability with respect to the turbulent bursts, so that the overall performance measures tended to drop on all the series, for a hidden layer width larger than two. Therefore, we report the results obtained for the hidden layer width equal to 2.

The main goal for the artificial agent, as defined previously, is to produce an adaptive interval prediction for the logarithmic returns, incorporating the volatility risk, therefore, in order to specifically test the EMH, we begin by using a small prediction window, namely we use the last two trading sessions and choose the best performers of each architecture in the training data, using the score defined above, evaluating the success of these agents in interval prediction, measuring, for the training and test data, the proportion of cases that fall within the interval bounds $p_l$ (as per equation (19)).

Then, in order to evaluate the effects of the temporal window size, and get reproducible results, we store the highest scoring agent’s initial pre-training parameters and just change the temporal window size, analyzing the effects of that increase on $p_l$ for both the training and test data.

This process allows us to evaluate a fundamental point with respect to financial theory: if the EMH is right, then the MNL system cannot perform well on the short-term window (the two trading days), but even if short-term deviations to efficiency were to occur, as we increase the temporal window, the agent should perform increasingly worse, since long-term returns’ windows cannot have relevant information to the next period’s returns. Therefore, the artificial agent would be adjusting to unrelated noise and including increasingly unrelated noise in its training, such a drop in performance should be visible, in particular, with respect to the main purpose of the artificial agent: the interval prediction of financial returns, which uses both the expected value and the one-period ahead standard deviation, produced by the agent’s modular networked system. A drop in interval prediction performance, with increasing time window, should, in this case, be more evident in the test data.

If a two-day window provides good performance and the interval prediction performance does not drop by much with the window size, then, this is strong evidence against the EMH and in favor of the CMH and AMH proponents, as well as those working on machine learning-based algorithmic investing since there is a higher predictability of financial returns from past returns than that predicted by the EMH, which means that there may be algorithmically exploitable patterns. Furthermore, if rise in interval predic-
tion performance, with window size, is obtained then this is even stronger evidence against the EMH.

Considering, then, first, the training data results, these vary for the two architectures and datasets. In table 1, the amplitude of the RMSE in training, for 100 randomly initialized agents is shown to be larger in the GRU than in the LSTM architecture, for the MPU, while, for the DPU, it is the GRU that has the lower amplitude. Although, in regards to the minimum, we get, for each index, similar results for the two architectures. The performance in training seems also to decrease with the size of the time series, so that the time series with the longest training data exhibits lower error in training than the time series with the shortest training data.

|         | Min       | Max       | Mean       |         | Min       | Max       | Mean       |
|---------|-----------|-----------|------------|---------|-----------|-----------|------------|
| LSTM    | S&P 500   |           |            | GRU     |           |           |            |
| MPU     | 0.007567  | 0.052401  | 0.014251   | MPU     | 0.007608  | 0.141089  | 0.023221   |
| DPU     | 0.000155  | 0.048826  | 0.002587   | DPU     | 0.000160  | 0.042688  | 0.003458   |
|         | NASDAQ    |           |            |         | NASDAQ    |           |            |
| MPU     | 0.007880  | 0.056915  | 0.015267   | MPU     | 0.007789  | 0.152901  | 0.025152   |
| DPU     | 0.000277  | 0.101699  | 0.006076   | DPU     | 0.000282  | 0.056534  | 0.005397   |
|         | Russell 2000 |           |            |         | Russell 2000 |           |            |
| MPU     | 0.010023  | 0.067908  | 0.018810   | MPU     | 0.010051  | 0.175307  | 0.030350   |
| DPU     | 0.000409  | 0.139479  | 0.008414   | DPU     | 0.000414  | 0.075175  | 0.007461   |

Table 1: Training data RMSE, from 100 randomly initialized agents using glorot uniform distribution, a time window of 2, and hidden neurons layer width of 2, for LSTM and GRU architectures. The L2 norm regularization parameter was set, in both cases, to 0.001, the minibatch size was set to 100, the trade-off factor for current and previous gradients was set to 0.95, the increasing factor for learning rate adjustment was set to 1.5 and the decreasing factor was set to 0.1. The minimum and maximum scales allowed for the initial learning rate were, respectively, 0.001 and 0.1.

In regards to the prediction accuracy, for a 1% neighborhood size, in the case of the MPU, and 0.01%, in the case of the DPU, as shown in table 2, the amplitude is very large, for each index and architecture, ranging from low prediction accuracy to high prediction accuracy. The two architectures, however, exhibit similar results with respect to each index.

The accuracy results already show a deviation with respect to the EMH.
In the case of the three indices, the maximum MPU accuracy in the training data is higher than 80%, for a 1% neighborhood size, which means that the MPU is capturing a pattern in the training data, linking the next period’s returns to the last two trading days’ returns, being capable of predicting the next period’s returns, within a 1% range, with a higher than 80% accuracy. A similar pattern holds for the DPU accuracy, which implies that there are patterns in the market volatility that are also being captured by the MNL system in the training data.

| LSTM | Min | Max | Mean | GRU | Min | Max | Mean |
|------|-----|-----|------|-----|-----|-----|------|
| S&P 500 |     |     |      | S&P 500 |     |     |      |
| MPU   | 0.192673 | 0.855464 | 0.669282 | MPU   | 0.066767 | 0.857089 | 0.573361 |
| DPU   | 0.016633 | 0.871436 | 0.567719 | DPU   | 0.066783 | 0.862181 | 0.534920 |
|NASDAQ |     |     |      |NASDAQ |     |     |      |
| MPU   | 0.181273 | 0.872549 | 0.671791 | MPU   | 0.058423 | 0.873749 | 0.573601 |
| DPU   | 0.014211 | 0.886309 | 0.577956 | DPU   | 0.058447 | 0.878102 | 0.534037 |
|Russell 2000 |     |     |      |Russell 2000 |     |     |      |
| MPU   | 0.175243 | 0.817038 | 0.642004 | MPU   | 0.054889 | 0.818754 | 0.545563 |
| DPU   | 0.014874 | 0.833238 | 0.544640 | DPU   | 0.054920 | 0.815503 | 0.508249 |

Table 2: Accuracy measures for table 1’s agents, for an open neighborhood of 1% for the MPU and of 0.01% for the DPU.

Now, storing the pre-training parameters for the agent with highest score (as per equation (45)) for each index and architecture, and considering the interval prediction results, using one unit of standard deviation (as per equation (18)), in the case of the LSTM architecture, for a memory window of 2 trading days, the prediction success rates, measured in terms of the proportion of cases where the returns fall within the interval bounds, are low for each of the three indices, in both training and testing, as shown in table 3. The values of the training and test data are close to each other, which shows some robustness in terms of the stability of the results.
| $\tau + 1$ | S&P 500 | NASDAQ | Russell |
|---------|---------|---------|---------|
|        | Train   | Test    | Train   | Test    | Train   | Test    |
| 2       | 0.347174| 0.361480| 0.401521| 0.412798| 0.346682| 0.346628|
| 5       | 0.426658| 0.442039| 0.512425| 0.520446| 0.422923| 0.426531|
| 10      | 0.444862| 0.457045| 0.542369| 0.546455| 0.438218| 0.441398|
| 15      | 0.445922| 0.458320| 0.544064| 0.550345| 0.440058| 0.443201|
| 20      | 0.445980| 0.458604| 0.544355| 0.550861| 0.439595| 0.443063|
| 25      | 0.445660| 0.458448| 0.544242| 0.550641| 0.439420| 0.442923|
| 30      | 0.445718| 0.458402| 0.544737| 0.551011| 0.438953| 0.442784|

Table 3: Interval prediction success in training and test data for agent with LSTM architecture, with increasing window size, using the same initial pre-training parameters as the two trading sessions window highest scoring agent.

Even though each module exhibits good results, in terms of predicting their respective targets, the interval prediction fails more, which means that the interplay between the expected value and volatility, for a time window of 2 trading days, is not capable of capturing the majority of returns swings.

This might be assumed, in first sight, to be supportive of the EMH, however, this is, in fact, related to the ability of the neural network to capture the turbulent patterns in the market, since there is a basic tension between two dynamics, characteristic of financial returns’ turbulence: a laminar dynamics, where the market shows small returns movements, and turbulence bursts, where the market exhibits sudden and clustering volatility with large price jumps.

A neural network may capture very well the smaller swings, and even present good results on average in the prediction, but fail in capturing the joint expected value and volatility dynamics, so that the interval prediction fails more often than it is right. For a two period window, in the case of the LSTM, the prediction success rates are less than 40% (near 35%) in training and testing for the S&P 500 and Russell 2000 indexes, and it is slightly higher than 40% in the case of the NASDAQ.

Thus, while some predictability may be present, for a two-day window, an interval prediction fails more often than not. If the time window increases, however, we can see a tendency for the increase in the interval prediction success rates.

Therefore, for agents with a cognitive architecture given by the LSTM modular system, with the same initial parameters, increase in the window
size, leads, in this case, to a rise in the interval prediction success rates, in both training and testing, which goes against the hypothesis that long term data should have no bearing in the prediction, and, due to increasing noise, that it should in fact lead to a drop in prediction accuracy. The rise in prediction success, however, still leads to poor performance in the case of the S&P 500 and the Russell 2000, which exhibit success rates lower than 50% and higher than 40%. The only index for which the corresponding agent exhibits an accuracy higher than 50% is the NASDAQ Composite, with accuracy rates of 54.4737% (in training) and 55.1011% (in testing).

These results change, in terms of values, for the GRU architecture as shown in table 4. Indeed, while, for a window size of 2 trading days, the GRU shows a worse performance in interval prediction than the LSTM, that performance rises as the window size is increased, in both training and testing, and it rises higher than the LSTM. Indeed, for a 20 day window, the GRU architecture exhibits interval prediction success rates higher than 60%, in both training and testing data, for all the three indices, and for a 30 day window, the prediction success rate is higher than 80% for all the three indices, in both training and testing.

| $\tau + 1$ | S&P 500 | NASDAQ | Russell |
|------------|---------|---------|---------|
|            | Train   | Test    | Train   | Test    | Train   | Test    |
| 2          | 0.310530| 0.307346| 0.309047| 0.303119| 0.313215| 0.303602|
| 5          | 0.389987| 0.387760| 0.381964| 0.380771| 0.388252| 0.379085|
| 10         | 0.494361| 0.494775| 0.491566| 0.484662| 0.495690| 0.477979|
| 15         | 0.590715| 0.590794| 0.589537| 0.586947| 0.593948| 0.577475|
| 20         | 0.675251| 0.677875| 0.679435| 0.671868| 0.685638| 0.664472|
| 25         | 0.762516| 0.761285| 0.766465| 0.758219| 0.772464| 0.752872|
| 30         | 0.830982| 0.834825| 0.835223| 0.835080| 0.840698| 0.826268|

Table 4: Interval prediction success in training and test data for agents with GRU architecture, with increasing window size, using the same initial pre-training parameters as the two trading sessions window highest scoring agent.

Both results place in question the EMH in the sense that, not only do past returns’ data have an impact in interval prediction, but, for the case of the GRU architecture, we can predict the financial returns, within an interval bound comprised of an expected value and square root of the expected squared deviation, with more than 80% success, that is, we have an agent
that is capable of using a forward looking expected returns and volatility expectation to produce an interval prediction that is right more than 80% of the time.

This result was obtained setting $l$ to 1 in the interval (as per equation (18)). If we increase $l$, for the same 30 trading days window, we get higher success rates. Namely, as shown in table 6, the interval prediction, for $l = 1.5$, has a success rates near 95% in test data for all the three indices, and when $l = 2$, we get success rates near 98%, in test data, for all the three indices. The training and test data success rates also show values close to each other which is evidence of robustness of the results.

| #SD | Data | S&P 500 | NASDAQ | Russell |
|-----|------|---------|--------|---------|
| 1   | Train| 0.830982| 0.835223| 0.840698|
|     | Test | 0.834825| 0.835080| 0.826268|
| 1.5 | Train| 0.945340| 0.941093| 0.940116|
|     | Test | 0.945641| 0.946995| 0.950502|
| 2   | Train| 0.980605| 0.979150| 0.976163|
|     | Test | 0.979892| 0.979625| 0.984073|

Table 5: Interval prediction success rates for training and test data for agents with GRU architecture with different units of standard deviation, using the same initial pre-training parameters as the two trading sessions window highest scoring agent.

From a financial theory perspective, these results have a high relevance. In the case of the S&P 500, the data goes from the 1950s to 2018, this means that we have different economic and financial environments.

However the agent consistently performs with approximately 98% performance over these different periods. In the training data, such performance can be expected, as long as the learning approximates well the main features of the data. However, for the test data, in the case of the S&P 500, the agent was able to exhibit a higher than 90% prediction success rate (for the case of an interval prediction using a unit of $1.5\sigma_{t+1}$ and $2\sigma_{t+1}$ ) in a period that goes from 1981 to 2018. Furthermore, the agent was able to achieve this performance, having been trained in the period that goes from 1950s up to 1981.

This indicates the presence of some structural nature in the stock market that remains the same and is captured by the MNL system, allowing it to
consistently perform over long time periods. Figure 1 shows the series and prediction interval bounds for the S&P 500 in both training (left graph) and test data (right graph).

![Figure 1: S&P 500 prediction interval bounds (in red) and observed returns data (in black), for both training (left) and test data (right) series, with $l = 1.5$ and 30 days past window.](image)

The two graphs show the main reason for the success of the agent, namely, whenever there is a turbulence burst, with volatility clustering, the agent’s predicted interval also becomes wider, so that the volatility bursts and price jumps tend to be within the interval’s bounds. Likewise, whenever the market enters into a low volatility period, the agent’s predictions also anticipate low volatility, and, thus, the interval bounds tend to become smaller.

The agent has thus captured an emergent nonlinear dynamics present in the index that links the last 30 days’ returns and volatility to the next period’s returns and volatility. This dynamics must be structural, since the agent’s predictions, in the case of the S&P 500 hold, with a similar success measure, for different samples (training and test) that cover different historical periods, both economic and financial, which is consistent with the hypothesis that the nonlinear dynamics, linked to financial turbulence, may be intrinsic to the trading patterns and the nature of the market as a very large open network of adaptive agents that process financial information and trade based on their expectations for market dynamics.

Although, microscopically, different agents may trade based on different rules, at the macro-level there is an emergent pattern in the market that
leads to a nonlinear dynamics that makes turbulence patterns in financial returns predictable enough to be captured effectively by an agent equipped with a modular networked system of GRUs.

Although the LSTM architecture is also effective in capturing some pattern, the GRU approximates better the large jumps and buildup associated to the volatility bursts, as well as the gradual reduction in volatility associated with laminar periods. These results are consistent with the AMH, which addresses complex nonlinear returns’ dynamics as a result of collective emergent patterns from evolutionary trading rules and actions of financial agents.

It is important to stress that these agents are trying to anticipate the result of their own collective trading behavior, in the sense that they form expectations of market rise (bullish) or market fall (bearish), so that the market exhibits an emergent adaptive dynamics with complex feedback, this makes emerge complex collective behaviors that are structurally linked to the market as an adaptive systemic network.

Furthermore, the test data includes the period of financial automation, which took off since 2008. So that, the agent’s training, which ended in 1981, works well even in a transition period and a post-automation wave trading ecosystem.

The results obtained for the other two indices reinforce these findings, namely, in the case of the NASDAQ, as shown in figure 2, the buildups in volatility tend to be accompanied by widening of the prediction interval, and the less volatile periods also tend to be periods with smaller prediction interval amplitudes, so that the agent’s interval predictions exhibit an anticipatory adaptive response to market conditions, anticipating the returns’ turbulent and laminar dynamics, therefore, the majority of returns fluctuations fit within the interval bounds.
Figure 2: NASDAQ Composite prediction interval bounds (in red) and observed returns data (in black), for both training (left) and test data (right) series, with $l = 1.5$ and 30 days past window.

The NASDAQ focuses strongly on technology stocks, thus, the agent performs well in both a benchmark for *large caps*, that also constitutes a benchmark for market efficiency, as well as in a more sector focused benchmark. This is strong evidence against the EMH, and in favor of complexity science-based market theories.

The last index, reinforces these results, in that we have a *small caps* index, and a smaller series, but we still get a high interval prediction success rate, and a similar coincidence between the agent’s prediction interval amplitudes and the market volatility patterns.
Figure 3: Russell 2000 prediction interval bounds (in red) and observed returns data (in black), for both training (left) and test data (right) series, with $l = 1.5$ and 30 days past window.

The Russel 2000 index provides an important addition to the above results, since we might expect the agent to perform better in small caps and sectorial indices than in the large caps reference, that is the S&P 500 index, but this is not the case, in all three indices, we get similar results both in terms of interval prediction success rate as well as in terms of turbulent and laminar periods’ anticipation.

4 Conclusion

Machine learning allows for the development of AI systems with interconnected learning modules, these are MNL systems that are able to solve complex problems comprised of multiple linked subproblems, where each cognitive module is specialized and trained on a specific task, but the modules are linked in a network that maps the network of connections between the different subproblems.

In the present work, we illustrated an application of AI with an MNL cognitive system to finance. In financial markets, the market dynamics is driven by expectations on returns on one’s investment and expectations on the risk associated with the returns on one’s investment. These are the two fundamental drivers of traders’ decision making.

An MNL system is capable of integrating these two drivers in an expectation for the next period’s returns, based on a time window of past returns,
and an expectation for the next period’s squared deviation around the next period’s returns expectation, based on a time window of past squared deviations.

We introduced an artificial agent with such a cognitive system, which learns to predict the next period’s returns and the next period’s squared deviation around its next period’s returns prediction and uses the two parameters to provide an interval prediction for the next period’s returns, calculating the square root on the next period’s predicted squared deviation as a time-varying volatility parameter that is used to build a prediction interval centered on the expected returns. In this way, we have an interval that incorporates both the expectation around the returns and the expectation around risk.

According to the EMH, such an artificial intelligent system should not be successful in returns’ prediction, that is, such an interval-based prediction should not work. On the other hand, in accordance with the CMH and AMH, this interval-based prediction should be possible, since there are emergent collective trading patterns coming from financial agents’ adaptive trading dynamics, which lead to a nonlinear dynamics with stochastic and deterministic components that explain the presence of long memory in volatility and financial turbulence.

The application of the MNL system to the S&P 500, NASDAQ Composite and Russel 2000 shows that, not only, does interval prediction work, but the predictability increases with an increase in the temporal window. Thus, for a GRU-based MNL system and a 30 trading days window there is a high predictability of returns, which is higher than what we get for smaller time windows, in both training and test data. Although the LSTM does not perform as well as the GRU, we also get this effect of increasing predictability with window size.

The prediction performance is linked to the ability of the MNL system to capture main emergent nonlinearities and patterns in volatility, so that the prediction interval produced by the artificial agent is a dynamical interval that is capable of adapting to market conditions contracting when it anticipates low volatility periods and expanding when it anticipates high volatility periods.

Regarding financial management, these results help explain the success of algorithmic investing, and it lends support to alternative trading schools, in particular technical analysis, which assumes that patterns are present in past financial time series, and that has been a key component in recent efforts
of application of machine learning, including recurrent neural networks, to finance [43, 44, 45].

However, there is an important point to be made: the results hold for each index, independently of the trading school, indeed, we did not explicitly use any technical analysis elements, which would deviate from our goal of pattern discovery, and EMH testing, since it would introduce, top-down, a specific trading school, deviating the agent from the basic pattern discovery which was under test.

The agent is, therefore, not addressing technical analysis specifically, but is, instead, working with two major financial parameters used by the investment management school that is associated with the EMH: the expected value and the variance. This is a purposeful key feature, since the goal was to explicitly test the EMH, especially the weak efficiency hypothesis’ statistical submartingale assumption, against a machine learning alternative. If the weak form of the EMH fails, the semi-strong and the strong versions of the EMH also fail.

The main results that we obtained, with an artificial agent equipped with a GRU-based MNL cognitive system, show that there are long-term patterns in past financial returns that allow an artificial agent to successfully produce an interval prediction of returns, adjusting the expected value and the volatility predictions to the market conditions, consistent with the main arguments of the complexity approaches to finance [11, 12, 36, 37, 46].

The success of this machine learning alternative means that the instruments and techniques used by the EMH-based investment management school need to be revised and transformed, incorporating machine learning, and working with forward-looking expected values and variances. In this sense, traditional financial theory may not hold with respect to the EMH assumption but some of its techniques, especially in regards to risk management and portfolio management, including the expected value and variance-based portfolio optimization techniques, developed in the 1950s by Markowitz [47, 48], may still survive to the degree to which they can be incorporated in successful machine learning algorithms, capable of capturing the markets’ main emergent nonlinear dynamics.
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