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Simulation Analysis of Resonant Frequency of Mix Sucker Rod with absorber

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Abstract. Considering the influence of sucker rod absorber (SRA) on longitudinal vibration of sucker rod string (SRS), a new model of mix sucker rod with absorber (MSRWA) in the form of numerical integration is built. In the detail, based on the dynamical theory of continuous systems, the wave equation of instantaneous motion of MSRWA is deduced. The dynamic response is solved out with the mode superposition method. The affecting factors of natural frequency are analyzed, such as the length, diameter, density and elastic modulus of SRS. Then three dimensional curves of natural frequency are obtained. The results are very important for the design and application of SRS.

1. Introduction

The sucker rod pumping system (SRPS) is an important method of the mechanical recovery methods in the domestic and overseas oilfields [1-3]. In order to reduce energy consumption and boost its crude production, the optimal design of SRPS had been done since the nineteen sixties [4, 5]. The resonant frequency of SRPS is foundation of optimal design of SRPS, so the resonant frequency of SRPS has been studied. Two steps in estimating natural frequencies and damping ratios are given by Lu X [6], the first step is from measurement records to the power spectral density function, the second step is from the power spectrum to system parameter estimates. Dong S gave the conclusion that there will be resonance phenomena, when the excited frequency and even multiple of fundamental frequency are equal [7]. Peng Y analyzes the twisting vibration of rotary sucker rod string, a formula of calculating every harmonic vibration inherent frequency is presented [8]. The inherent frequency of longitudinal vibration of single stage rod string is analyzed and calculated from the point of view of vibration theory by Zhu B [9]. The 3-D vibration of sucker rod string in the reciprocating motion is proposed by Chen J, such as horizontal vibration, longitudinal vibration and torsional vibration. The sucker rod string is considered the elastic body whose quality is distributed by using the elastic body vibration theory. The dynamic models for the vibrations of sucker rod string in three directions are established [10]. Although the resonant Frequency of SRPS has been studied for years, the most of aforementioned studies have not been studied sufficiently. In the detail, the formula of natural frequency is not accurate [11], when the free vibration of SRS is researched. When the SRA is installed between polished rod and SRS, the influence of SRA on the SRS’s vibration is usually ignored. In the period of oil field development, the cost of oil production has been the focus of the global attention at present [12-13]. However, with application of SRA, the vibration of SRS will be absorbed and the stress of SRS and the peak of dynamic load will be reduced, at the same time, the motor load and power consumption will decrease. Therefore, the resonant frequency of longitudinal
vibration of MSRWA is necessary to be analyzed, which lays the foundation for the design and application of SRA.

2. Dynamic simulate model of MSRWA

The number stages of SRS assemblage are supposed to be k, and the polished rod displacement is u*. The relative displacement between polished rod and SRS top is u0. Considering the influence of SRA on SRS’s longitudinal vibration, a mechanical model of MSRWA is established, as shown in Fig.1.

![Figure 1. The mechanical model of MSRWA](image)

Based on the mechanical model of longitudinal vibration, a mathematical model is built, as follows

\[
\begin{align*}
\frac{\partial^2 u_i}{\partial t^2} - a_i^2 \frac{\partial^2 u_i}{\partial x_i^2} + c_i \frac{\partial u_i}{\partial t} &= \frac{d^2 u^*}{dt^2} - c_k \frac{du^*}{dt} + g \\
E_i A_i \frac{\partial u_i(0)}{\partial x_i} &= k_0 u_0 + c_0 d_0 \\
E_i A_i \frac{\partial u_i(L_i)}{\partial x_i} &= F_p
\end{align*}
\]

Where,

\[
\begin{align*}
a_i &= \sqrt{\frac{E_i}{\rho_i}} \\
c_i &= \left\{ \left[ 0.20 + 0.39 \frac{D_k}{D_i} \right] + \frac{2.197 \times 10^4}{25} \right\}^{1.257} \left( \frac{D_k^2}{D_i^2} \right)^{-0.381} \frac{12 \pi \mu l_k}{\rho_i A_k} \left( \frac{D_k}{D_i} \right) \left( D_i - D_k \right) \left( \frac{D_k}{D_i} \right)
\end{align*}
\]

Where, \(a_i\) is the velocity of sound in SRS (m/s); \(c_i\) is the friction coefficient (Pa.s); \(F_p\) is the load of pump plunger (N); \(\omega\) is the excitation frequency (rad/s)

3. Free vibration
Ignoring the influence of the damping of SRA on the SRS’s vibration, the equation of free vibration is obtained based on Eq.(1).

\[
\frac{\partial^2 u_k}{\partial t^2} - u_k^2 \frac{\partial^2 u_k}{\partial x_k^2} = 0
\]

(3)

The equations of boundary and continuous conditions are given, as follows.

When \(k=1\)

\[
E_1 A_1 \frac{\partial u_1}{\partial x_1} \bigg|_{x=0} = k_0 u_0, \quad E_k A_k \frac{\partial u_k}{\partial x_k} \bigg|_{x=L} = 0
\]

(4)

When \(k=2, 3, 4\ldots\)

\[
\begin{align*}
E_1 A_1 \frac{\partial u_1}{\partial x_1} \bigg|_{x=0} &= k_0 u_0, \quad u_{k-1} \bigg|_{x_{k-1}-x_{k-2}} = u_k \bigg|_{x_{k-2}} \\
E_{(k-1)} A_{(k-1)} \frac{\partial u_{k-1}}{\partial x_{k-1}} \bigg|_{x_{k-1}-x_{k-2}} &= E_k A_k \frac{\partial u_k}{\partial x_k} \bigg|_{x=x_{k-1}} , \quad E_k A_k \frac{\partial u_k}{\partial x_k} \bigg|_{x_{k-1}} = 0
\end{align*}
\]

(5)

The vibration model functions are defined, as follows.

\[
\begin{align*}
\Phi_1(x_i) &= B_1 \sin(b_ix_i) + D_1 \cos(b_ix_i) \quad 0 \leq x_i \leq L_1 \\
\Phi_2(x_i) &= B_2 \sin(b_2x_i) + D_2 \cos(b_2x_i) \quad 0 \leq x_i \leq L_2 \\
\Phi_3(x_i) &= B_3 \sin(b_3x_i) + D_3 \cos(b_3x_i) \quad 0 \leq x_i \leq L_3
\end{align*}
\]

(6)

Combining Eqs. (4) and (5), the transcendental equation of natural frequency is given, as follows

\[
B_2 \cos(b_2L_2) - D_2 \sin(b_2L_2) = 0
\]

(7)

Where,

\[
\begin{align*}
D_1 &= B_1 \frac{E_1 A_1 b_1}{k_0} \\
D_2 &= B_2 \frac{E_1 A_1 b_1}{E_2 A_2 b_2} \left[ \cos(b_1L_1) - \frac{E_1 A_1 b_1}{k_0} \sin(b_1L_1) \right] \\
D_3 &= B_3 \frac{E_2 A_1 b_2}{E_3 A_2 b_2} \left[ \cos(b_2L_2) + \frac{E_1 A_1 b_1}{k_0} \sin(b_1L_1) \right] \\
D_4 &= B_4 \frac{E_3 A_1 b_3}{E_4 A_2 b_3} \left[ \cos(b_3L_3) - \frac{D_3}{B_3} \sin(b_3L_3) \right] \\
D_5 &= B_5 \frac{B_2^2 \sin(b_2L_2) + D_2 \cos(b_2L_2)}{B_3} \\
\end{align*}
\]

(8)

Then the formula of regular function is obtained, as follow.
\[
\phi_i(x_i) = B_i \left[ \sin(h_{i,x_i}) + \frac{E_i A_i b_i}{k_0} \cos(h_{i,x_i}) \right] \quad 0 \leq x_i \leq L_i
\]

\[
\phi_i(x_i) = B_i \left[ \frac{E_i A_i b_i}{E_i A_i b_i} \cos(h_{i,L_i}) \sin(h_{i,x_i}) - \frac{E_i A_i b_i}{E_i A_i b_i} \sin(h_{i,L_i}) \sin(h_{i,x_i}) \right] + \sin(h_{i,L_i}) \cos(h_{i,x_i}) + \frac{E_i A_i b_i}{E_i A_i b_i} \cos(h_{i,x_i}) \cos(h_{i,L_i}) \sin(h_{i,x_i}) \quad 0 \leq x_i \leq L_i
\]

(9)

Where,

\[
B_i = \sqrt{\frac{1}{\rho_i A_i G_i + \rho_i A_i G_i + \rho_i A_i G_i}}
\]

\[
G_i = \left[ \frac{L_i}{2} - \frac{1}{4b_i} \sin(2h_{i,L_i}) \right] + \frac{D_i^2}{B_i^2} \left[ \frac{L_i}{2} + \frac{1}{4b_i} \sin(2h_{i,L_i}) \right] + \frac{D_i^2}{2b_i B_i} \left[ 1 - \cos(2h_{i,L_i}) \right]
\]

(10)

The numerical solutions of the time functions in regular coordinate system are solved out with the fourth order Runge-Kutta method. Then the numerical solutions of the time functions in Natural coordinate system are obtained with the coordinate transformation.

4. Characteristic analysis

The basic parameters are as follows: when the length of SRS \( L_d \) is 1000m, the length of first stage \( L_1 \) is 600m and the length of second stage \( L_2 \) is 400m. When the length of SRS \( L_d \) is 5000m, the length of first stage \( L_1 \) is 3000m and the length of second stage \( L_2 \) is 2000m. The Young’s modulus of SRS \( E_1 \) as well as \( E_2 \) is \( 2.1 \times 10^{11} \) Pa; the density of rod \( \rho_1 \) as well as \( \rho_2 \) is \( 7850 \) kg/m\(^3\); the diameter of first stage \( D_1 \) is \( 22 \) mm and the diameter of second stage \( D_2 \) is \( 19 \) mm.

**Table 1.** The comparison of natural frequency

| older   | Theoretical | Current model | New model |
|---------|-------------|---------------|-----------|
|         | rad/s       | rad/s         | Error %   | rad/s       | Error %   |
| 1       | 8.1         | 8.1           | 0         | 8.1         | 0         |
| 2       | 24.4        | 24.4          | 0         | 24.4        | 0         |
| 3       | 40.6        | 56.9          | 16.3      | 40.6        | 0         |
| 4       | 56.9        | 67.7          | 10.8      | 56.9        | 0         |
| 5       | 73.1        | 89.2          | 16.1      | 73.1        | 0         |
| 6       | 89.4        | 138.1         | 48.7      | 89.4        | 0         |

When \( L_d = 1000m, L_1 = L_2 \), two stages SRS is transformed into one stages SRS. Based on the current model, new model and the analytic expression of natural frequency, the six natural frequencies are obtained, in Tab.1. The table shows that there is a large error in results of current model, but the
simulation results of the new model are found to be in good agreement with the simulation results of theoretical value. Therefore, the new model of natural frequency in the paper is advised to use in actual engineering.

When the elasticity modulus of SRS $E_1=E_2=2.1\times10^{11}$ Pa, the density of SRS $\rho_1=\rho_2=7850$kg/m$^3$, the length of SRS $L_1=5000$m, the diameter of SRS $D_1=0.022$m. The combination ratios of length and diameter are changed and three dimensional curves of natural frequency are obtained, in Fig.2.

As you can see in the above figures, the first natural frequency shows large changes when the diameter ratio of SRS is larger than 1.5 or the length ration of SRS is an interval value 0.3-0.7. The first natural frequency is increasing with the diameter ratio increasing. With the length ratio increasing, the first natural frequency is first increasing and then decreasing. With second natural frequency, the length ratio is major factor of natural frequency.

![Figure 2](image1.png)

(a) First natural frequency          (b) Second natural frequency

**Figure 2.** The natural frequency of SRS with length and diameter

When the elasticity modulus ratio decreasing and density ratio increasing, the natural frequency of SRS is increasing. When the elasticity modulus ratio is smaller areas, the density ratio has a significant impact on natural frequency. When the density ratio is bigger areas, the elasticity modulus ratio has a significant impact on natural frequency. Therefore, the above three-dimensional curves have important implications for the design and application of SRS.

![Figure 3](image2.png)

(a) First natural frequency          (b) Second natural frequency

**Figure 3.** The natural frequency of SRS with elasticity modulus and density
5. Conclusions
Considering the influence of SRA on longitudinal vibration of SRS, a simulation model of MSRWA is built. A computational model of natural frequency is corrected against the deficiency of current model. With the free vibration of SRS, a computational model of natural frequency is corrected against the deficiency of current model. With the forced vibration of MSRWA, a mode superposition method is used to find the numerical solution. The affecting factors of natural frequency are analyzed, such as the length, diameter, density and elastic modulus of SRS. Then three dimensional curves of natural frequency are obtained. The above conclusions are very important to the design and application of SRA in practical applications.

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