Blackbody radiation shift of the Ga$^+$ clock transition

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Abstract
The blackbody radiation shift of the Ga$^+$ $4s^2\,^1S_0 \rightarrow 4s4p\,^3P_0^o$ clock transition is computed to be $-0.0140 \pm 0.0062$ Hz at 300 K. The small shift is consistent with the blackbody radiation shifts of the clock transitions of other group III ions which are of a similar size. The polarizabilities of the Ga$^+$ $4s^2\,^1S_0\, , 4s4p\,^3P_0^o\, , \text{and} 4s4p\,^1P_1^o$ states were computed using the configuration interaction method with an underlying semi-empirical core potential. Quadrupole and non-adiabatic dipole polarizabilities were also computed. A byproduct of the analysis involved calculations of the low-lying spectrum and oscillator strengths, including polarizabilities, of the Ga$^{2+}$ ion.

1. Introduction
The current standard of time is based on the cesium fountain frequency standard [1, 2]. However, recent developments in cold atom physics and improvements in optical frequency measurements make it increasingly likely that an atomic clock based on an optical transition will supplant the current cesium standard and consequently lead to a new definition of the second [3]. At present, the smallest frequency uncertainty has been achieved by an optical clock based on quantum logic technology and using the highly forbidden Al$^+$ $3s^2\,^1S_0 \rightarrow 3S\,^3P_0^o$ transition. The fractional frequency uncertainty of this clock is only $8.6 \times 10^{-18}$ [4, 5]. This clock would only drift by a period of 1 second over a period of 3.7 $\times$ 10$^9$ years.

Optical frequency standards capable of achieving such extreme precisions are however sensitive to very small environmental influences. One of the most important of these influences is blackbody radiation (BBR) emitted by the apparatus containing the atomic or ionic clock. The electromagnetic field associated with this BBR results in an AC Stark shift of the energies of the two states that define the clock transition. The energies of the upper and lower states of the clock transition can shift by different amounts since the polarizabilities of the two levels will not necessarily be the same. This leads to a temperature dependent shift in the frequency of the clock [6–8]. It is expected that the BBR shift will become an increasingly important component of the error budgets for optical frequency standards as other potential sources of uncertainty are eliminated and their overall precision is improved [9–11]. Consequently, clock transitions involving upper and lower states having polarizabilities that are close to each other are attractive since they will have small BBR shifts. Indeed, a very small BBR shift [12] was a primary motivation for the development of the Al$^+$ frequency standard [4, 5, 13, 14]. Just recently, investigations of the $ns^2\,^1S_0 \rightarrow nsnp\,^3P_0^o$ transitions of other group III positive ions of the periodic table, namely, B$^+$, In$^+$ and TI$^+$ have been completed. The BBR shifts of all these ions were found to be small, with In$^+$ having the largest 300 K BBR shift of $-0.017$ Hz [15–17]. All of these BBR shifts were between 1 to 2 orders of magnitude smaller than the BBR shifts of other atoms and ions advanced as atomic frequency standards [7]. Therefore, it is somewhat surprising that there has not yet been a calculation of the BBR shift of the Ga$^+$ clock transition.

Calculations of the polarizabilities of the Ga$^+$ $4s^2\,^1S_0\, , 4s4p\,^3P_0^o\, \text{and} 4s4p\,^1P_1^o$ states have been made using a large configuration interaction (CI) calculation to account for valence correlation. Core-valence correlations were included by adding semi-empirical core-polarization potentials to the core potential based on a Hartree–Fock (HF) wave function for the Ga$^{3+}$ core. The BBR shift is calculated and found to be small and roughly the same size as the shifts for other group III ions.
The CI calculations used to generate the physical and $L^2$ pseudo states were similar in style to those used previously to determine the dispersion parameters and polarizabilities of a number of two electron systems [12, 18–20]. The Hamiltonian for the two active electrons is written

$$H = \sum_{i=1}^{2} \left( -\frac{1}{2} \nabla_i^2 + V_{\text{dir}}(\mathbf{r}_i) + V_{\text{exc}}(\mathbf{r}_i) + V_{\text{p1}}(\mathbf{r}_i) \right) + V_{\text{p2}}(\mathbf{r}_1, \mathbf{r}_2) + \frac{1}{r_{12}}, \quad (1)$$

The direct, $V_{\text{dir}}$, and exchange, $V_{\text{exc}}$, interactions of the valence electrons with the HF core were calculated exactly. The $1s^22s^22p^63s^23p^63d^{10}$ core wave function was taken from a HF calculation of the Ga$^{2+}$ ground state using a Slater type orbital (STO) basis. The $\ell$-dependent polarization potential, $V_{\text{p1}}$, was semi-empirical in nature with the functional form

$$V_{\text{p1}}(\mathbf{r}) = -\sum_{\ell m} \frac{\alpha_{\text{core}} g_{\ell}^2(r)}{2r^4} |\ell m\rangle \langle \ell m|. \quad (2)$$

The coefficient, $\alpha_{\text{core}}$, is the static dipole polarizability of the core and $g_{\ell}^2(r) = 1 - \exp(-r^2/\rho_{\ell}^2)$ is a cut-off function designed to make the polarization potential finite at the origin. The cut-off parameters, $\rho_{\ell}$, were tuned to reproduce the binding energies of the Ga$^{2+}$ ns ground state and the np, nd and nf excited states. The Ga$^{2+}$ core polarizability was chosen to be $\alpha_{\text{core}} = 1.24 \, \alpha_0$ [21]. The cut-off parameters for $\ell = 0 \rightarrow 3$ were 1.3074, 1.5235, 2.2035 and 1.2977 $\alpha_0$ respectively.

It is essential to include a two body polarization term, $V_{\text{p2}}$, in the Hamiltonian to get accurate energy levels and polarizabilities for Ga$^{2+}$. The polarization of the core by one electron is influenced by the presence of the second valence electron. Omission of the two-body term would typically result in a 4s$^2$ 1S$_0$ state that would be too tightly bound. The importance of the two body polarization potential is discussed in [22]. The two body polarization potential adopted for the present calculation has the form

$$V_{\text{p2}}(\mathbf{r}_1, \mathbf{r}_2) = -\sum_{\ell m} \frac{\alpha_{\text{core}} g_{\ell}^2(r)}{r_{12}} |\ell m\rangle \langle \ell m| g_{\ell 2}(r), \quad (3)$$

where $g_{\ell 2}$ has the same functional form as $g_{\ell}(r)$. The cut-off parameter for $g_{\ell 2}(r)$ was chosen as 1.583 $\alpha_0$, the average of the cut-off parameters for $\ell = 0 \rightarrow 3$. Use of 1.583 $\alpha_0$ for the two-body cut-off parameter resulted in energies that were close to the experimental binding energies for most of the lowest lying states of Ga$^{2+}$. The current approach to solve the Schrödinger equation is termed as configuration interaction plus core polarization (CICP).

There were a total of 195 valence orbitals with a maximum orbital angular momentum of $\ell = 5$. The radial dependence of the orbitals were described by a mixture of STOs and Laguerrre type orbitals (LTOs) [18]. The number of active orbitals for $\ell = 0 \rightarrow 5$ were 50, 30, 30, 30 and 25 respectively. Some $\ell = 0$ and $\ell = 1$ valence orbitals were generated from the STOs used for the core. All the other orbitals were written as LTOs due to their superior linear dependence properties when compared with STO basis sets. LTO basis sets with radial and angular extrapolations have been able to predict the energy of the ground state helium atom to an accuracy of 10$^{-8}$ au [23, 24]. The use of the large orbital basis resulted in wave functions and energies for the low-lying states that were close to convergence.

The length of the CI expansions for the different states of Ga$^+$ ranged from 2000–7000. Some small changes were made to the $\rho_{\ell}$ values that were originally tuned to the Ga$^{2+}$ spectrum to improve the agreement of the Ga$^+$ energies with experiment. The oscillator strengths were computed with operators that included polarization corrections [18, 25, 26]. The cut-off parameter used in the polarization correction to the dipole operator was $1.583 \, \alpha_0$.

### 3. Energies and oscillator strengths

#### 3.1. Energy levels

The energy levels of the present calculations are compared with experiment in table 1. The cut-off parameters of the polarization potential were tuned to reproduce the experimental binding energies of the lowest states of each table 1. Theoretical and experimental energy levels (in Hartree) for some of the low-lying states of the Ga$^{2+}$ and Ga$^+$ ions. The energies are given relative to the energy of the Ga$^{2+}$ core. The experimental energies for the spin–orbit doublets of Ga$^+$ are averages with the usual $(2\ell + 1)$ weighting factors. The CICP energies for Ga$^+$ are those computed after additional tuning of the $\rho_{\ell}$ parameters. The experimental data were taken from the National Institute of Standards and Technology [27].

| State      | Experiment | CICP     |
|------------|------------|----------|
| Ga$^{2+}$  |            |          |
| 4s$^2$ 1S$_0$ | $-1.883$0675 | $-1.883$0676 |
| 4s4p 1P$_0$   | $-1.667$2451 | $-1.667$2449 |
| 4s4p 1P$_1$   | $-1.560$9281 | $-1.560$9281 |
| 4s5s 1S$_1$   | $-1.424$0174 | $-1.424$0171 |
| 4s5s 1S$_0$   | $-1.397$0779 | $-1.399$0705 |
| 4p$^2$ 3D$_1$ | $-1.392$2588 | $-1.392$2558 |
| 4p$^2$ 3D$_0$ | $-1.364$4842 | $-1.364$4842 |
| 4p$^2$ 1F$_1$ | $-1.358$0662 | $-1.358$0660 |
| 4s5p 1P$_0$   | $-1.343$4609 | $-1.343$3985 |
| 4s5p 1P$_1$   | $-1.333$7993 | $-1.333$0456 |
| 4s5d 1D$_1$   | $-1.308$1145 | $-1.311$3844 |
| 4s6s 1S$_0$   | $-1.277$0280 | $-1.287$1275 |
| 4s6s 1S$_0$   | $-1.273$6978 | $-1.278$1214 |
| 4p$^2$ 3D$_0$ | $-1.265$0309 | $-1.266$6402 |
| 4s5d 1D$_1$   | $-1.258$1318 | $-1.258$1396 |
| 4s4f 1F$_2$   | $-1.257$3356 | $-1.257$3336 |
| 4s4f 1F$_3$   | $-1.257$2887 | $-1.257$1514 |
symmetry. The energies of the lowest Ga\textsuperscript{2+} states are all in agreement with experiment since this was the criteria used to tune the cut-off parameters. The excited states tend to under-bind the experimental energies by about 0.001–0.002 Hartree.

Small adjustments to the cut-off parameters were made for the calculations of the Ga\textsuperscript{+} states. For example, the value of $\rho_0$ was reset to 1.2187 $\alpha_0$ for the calculation of the states with $^1S_0^+$ symmetry. The value of $\rho_0$ was fixed by requiring that the theoretical and experimental energies for the 4s\textsuperscript{2} $^1S_0^+$ state be the same. Other fine tunings of the cut-off parameters were made for all symmetries. The parameters were tuned to the energies of the spin–orbit states that are relevant to the calculation of the BBR shift of the clock transition. The most important levels for the calculation of the polarizabilities are the most tightly bound levels. The agreement between the theoretical and experimental energies for these levels minimizes the impact that differences in the long range behaviour of the wave functions (which are influenced by the energy) will have on radial matrix elements that are part of the polarizability calculation.

3.2. Oscillator strengths

The oscillator strengths for the transitions between the low-lying states are listed in Table 2 for Ga\textsuperscript{2+} and Table 3 for Ga\textsuperscript{+}. The absorption oscillator strength from state $\psi_i$ to state $\psi_j$ is calculated according to the identity [18, 34],

$$f_{ij}^{(a)} = \frac{2i\langle \psi_i; L_i \parallel \epsilon^j C^j (\hat{F}) \parallel \psi_j; L_j \rangle}{(2k + 1)(2L_i + 1)}.$$  (4)

In this expression, $\epsilon_{ji} = (E_j - E_i)$ is the energy difference between the initial state and final state, while $k$ is the polarity of the transition, and $C^j (\hat{F})$ is a spherical tensor. Experimental energy differences were used for the calculation of oscillator strengths. The angular momentum weighted average energy difference was used for the Ga\textsuperscript{2+} transitions. The energy differences of individual levels with the specified total angular momentum, $J$, were used for the triplet states of Ga\textsuperscript{+}.

There have been a number of calculations of the energy levels and oscillator strengths for Ga\textsuperscript{2+} [28, 30, 31, 33, 45–47]. Not all of the calculations of oscillator strengths have been tabulated. Table 2 gives oscillator strengths that are deemed to be the most accurate or of particular relevance to the present calculations. The most comprehensive calculations for Ga\textsuperscript{2+} appear to be a model potential (MP) calculation [28] and most recently a relativistic coupled cluster (RCC) calculation [29]. The reliability of the MP calculation from [28] is questionable since the oscillator strength for the resonance 4s $\rightarrow$ 4p transition is at variance with the CICP calculation and experiment. The CICP and RCC give oscillator strengths that mostly lie within 5% of each other with the exceptions occurring for oscillator strengths that are small.

The oscillator strengths reported in Table 3 are taken from a variety of sources [28, 35, 39–44]. There is a good deal of variety in values of the oscillator strengths for the resonance 4s\textsuperscript{2} $^1S_0^+$ $\rightarrow$ 4s4p $^3P_0^1$ transition, with values ranging from 1.69 to 1.983. However, two of the most recent calculations, a multi-configuration Dirac–Fock (MCDF) and a CI calculation gave values of 1.71 [35] and 1.704 [43] respectively. These are at a 1% level of agreement with the CICP oscillator strength. The MCDF calculation explicitly allowed for core–valence correlation, but only allowed for excitations from the 3d\textsuperscript{10} orbital. The more tightly bound orbitals account for about 20% of the core polarizability. The oscillator strength for the 4s\textsuperscript{2} $^1S_0^+$ $\rightarrow$ 4s4p $^3P_0^1$ inter-combination transition is small and will not make a significant contribution to the polarizability.

Transitions originating on the 4s4p $^3P_0^1$ multiplet will determine the polarizability of this state. The overall level of agreement between the present CICP oscillator strengths and those of the MCDF calculation [35] is good, with only a 2% difference in oscillator strengths for the two strongest transitions. There is a 6% disagreement for the 4s4p $^3P_0^1$ $\rightarrow$ 4s5s $^3S_0^1$ transition, but this oscillator strength is small and it only makes a 10% contribution to the polarizability of the 4s4p $^3P_0^1$ state. There is no explicit statement regarding the size of the orbital space used in the MCDF calculation, but it is likely to be significantly smaller than that used for the present calculations.

4. Polarizabilities and BBR shifts

4.1. Scalar and tensor polarizabilities

This analysis is performed under the assumption that spin–orbit effects are small and the radial parts of the wave functions are the same for the states with different $J$. All of the polarization parameters reported here are calculated using oscillator strength sum rules. The multipole oscillator strengths $f_{ij}^{(a)}$ are defined in equation (4). Then the adiabatic multipole polarizabilities $\alpha_k$ from the state $i$ are written as [7, 48]

$$\alpha_k = \sum_j f_{ij}^{(a)} \frac{\epsilon_{ji}}{\epsilon_{ji}}.$$  (5)
where Ga potentials were tuned to give the energies of the identified spin–orbit state. The notation relativistic random phase approximation. The CICP oscillator strengths were obtained from calculations where the core-polarization

A related sum rule is the non-adiabatic multipole polarizability \( \beta_k \) [18, 49], which is defined as

\[
\beta_k = \frac{1}{2} \sum_i f_{ij}^{(k)} \left( \frac{\epsilon_i}{\epsilon_j} \right).
\]

This is useful for the analysis of resonant excitation Stark ionization spectroscopy (RESIS) [50] experiments. A RESIS experiment would be able to determine the polarizabilities of Ga\(^+\) and Ga\(^{2+}\) to better than 1% accuracy.

The dynamic polarizability to lowest order variations in the frequency can be written [7] as

\[
\alpha_k(\omega) \approx \alpha_k(0) + \omega^2 S_k(-4) + \cdots,
\]

where \( S_k(-4) \) is

\[
S_k(-4) = \sum_i f_{ij}^{(k)} \left( \frac{\epsilon_i}{\epsilon_j} \right).
\]

States with a non-zero angular momentum will also have a tensor polarizability [7, 51, 52]. For a state with angular momentum \( L_0(M_0) \), this is defined as the polarizability of the magnetic sub-level with \( M = L_0(M = J_0) \). The total polarizability is written in terms of both a scalar and tensor polarizability. The scalar polarizability represents the average shift of the different \( M \) levels while the tensor polarizability gives the differential shift.

This tensor polarizability can be expressed in terms of \( f \)-value sum rules. For an \( L_0 = 1 \) initial state, one can write the tensor polarizability for a dipole field as [7, 51]

\[
\alpha_{2L_0M_0} = \left( \sum_{n,L_n=0} f_{0n}^{(M_0)} \right)^2 - \frac{1}{2} \sum_{n,L_n=1} f_{0n}^{(M_0)} + \frac{1}{10} \sum_{n,L_n=2} f_{0n}^{(M_0)}.
\]

The core does not make a contribution to the tensor polarizability. Expressions for the general tensor polarizabilities have been given elsewhere [51].

| Transition | CICP | MP [28] | MCDF [35] | CI | RRPA | MCHF | Experiment |
|------------|------|---------|-----------|----|------|------|------------|
| \( 4s^2 \, ^1S_0 \rightarrow 4s4p \, ^1P_1 \) | 1.723 | 1.890 | 1.71 | 1.76 [36] | 1.862 [37] | 1.75 [38] | 1.85(15) [39] |
| \( 4s^2 \, ^3S_0 \rightarrow 4s5p \, ^3P_0 \) | 0.00478 | 0.017 | 0.00546 | 0.0066 [43] | 0.00683 [44] | 8.1[−4] [40] | 4.75[−4] [37] |
| \( 4s^2 \, ^3S_0 \rightarrow 4s4p \, ^3P_0 \) | 5.92[−4] | 6.37[−4] [43] | 3.80[−4] [41] |

Table 3. Absorption oscillator strengths for some low-lying transitions of Ga\(^+\). The RRPA column lists calculations performed in the relativistic random phase approximation. The CICP oscillator strengths were obtained from calculations where the core-polarization potentials were tuned to give the energies of the identified spin–orbit state. The notation \( a[b] \) means \( a \times 10^b \).

| Transition | CICP | MP [28] | MCDF [35] | CI | RRPA | MCHF | Experiment |
|------------|------|---------|-----------|----|------|------|------------|
| \( 4s^2 \, ^1S_0 \rightarrow 4s5s \, ^3P_0 \) | 0.1403 | 0.154 | 0.149 | 0.141 [43] |
| \( 4s^2 \, ^1S_0 \rightarrow 4s5p \, ^3P_0 \) | 0.00275 | 0.003 | 0.0275 [43] |
| \( 4s^2 \, ^1S_0 \rightarrow 4p^2 \, ^3P_0 \) | 0.2051 | 0.222 | 0.2051 [43] |
| \( 4s^2 \, ^1S_0 \rightarrow 4p^2 \, ^3P_0 \) | 0.0120 | 0.005 | 0.0172 | 0.029 [36] | 0.58[−3] [42] |
| \( 4s^2 \, ^1S_0 \rightarrow 4p^4 \, ^1D_2 \) | 1.220 | 1.31 | 1.08 [36] | 1.21 [42] |

Table 4. The pseudo-oscillator strength distribution for the Ga\(^{3+}\) ion core. The dipole polarizability of the core was 1.24 au [21] while the quadrupole polarizability was 2.345 au [21].

4.1. The polarizability of the Ga\(^{3+}\) core. The energy distribution of the oscillator strengths originating from core excitations was estimated using a semi-empirical technique [18]. This approach utilizes \( f \)-value sum rules and identities to construct the pseudo-oscillator strength distributions. The sum rules and identities are

\[
f_i^k = kN_i(r_i^{2k-2}).
\]

\[
\alpha_{k,\text{core}} = \sum_i f_i^k \left( \frac{\epsilon_i}{\epsilon_j} \right)^2.
\]
4.1.2. Polarizabilities. Table 5 gives the multipole polarizabilities of the lowest five states of the Ga$^{2+}$ ion and the lowest three states of the Ga$^+$ ion. The energies of the lowest lying states in the Ga$^{2+}$ polarizability calculations were adjusted to be the same as the spin–orbit averaged experimental energies listed in table 1. The polarizabilities of excited states are more sensitive to small errors in calculated energies since the energy differences can be much smaller.

Energy adjustments were made when performing the polarizability calculations of the Ga$^+$ ion states. First, for the singlet states, the energies of the lowest excited states were adjusted to be the same as the experimental binding energies. The purpose of the triplet state calculations was to determine the polarizability of the 4s4p 3Po state. The cut-off parameters for the core-polarization potential were adjusted so that the CICP 4s4p 3Po state energy was the same as the experimental binding energy. Further, the cut-off parameters for other symmetries were adjusted so that the excited state energies were those of the spin–orbit states that could undergo a direct multipole transition with the 4s4p 3Po state. For example, the parameters were tuned so that the 4p2 3Pe and 4s4d 3Df excited state energies were those of the J = 1 state, and the energies of the 3Fg states were set to those of the J = 2 state. In effect, the CICP matrix elements were calculated using wave functions that have the energies of the appropriate spin–orbit states. Exact agreement between the tuned CICP energies and the experimental energies was only achieved for the lowest energy state of each symmetry. For the second and third excited state of each symmetry, CICP matrix elements without any further adjustment were used with the experimental binding energies of the appropriate spin–orbit component.

The tensor polarizabilities and non-adiabatic polarizabilities as well as the related sum rules $S_k(-4)$ of these states are also listed in table 5. The contributions from different transitions to the dipole polarizabilities of the 4s 2 1S0, 4s4p 3Po, and 4s4p 1P0 states are detailed in table 6. The 4s4p 3Po state does not have a tensor polarizability since it is the J = 0 spin–orbit component.

The 4s 2 1S0 ground state polarizability is dominated by the resonant transition which contributes about 92% of the polarizability (refer to table 6). The next most significant contribution to the polarizability comes from the Ga$^{2+}$ core. The uncertainty in the CICP line strength for the resonant transition is assessed to be ±2%. This was based on the variation between the CICP, MCDF [35] and CI [36, 43] oscillator strengths for this transition. The uncertainty in the RRPA core polarizability of 1.24 is assessed to be ±1%. This uncertainty is based on an estimate of the uncertainty in the core polarizability of Ca$^+$ [53]. The total uncertainty in the ground state dipole polarizability of 17.95 is 0.34 au.

There has been an estimate of the Ga$^+$ dipole polarizability by using oscillator strength sum rules and regularities in the 4s 2 1S0 → 4s4p 1P0 line strengths between members of the iso-electronic series [54]. They report a value of 18.14(44) au. This polarizability appears to be for the valence only part of the polarizability.
The CICP calculation of the ground state polarizability did not take into consideration the contribution from the $4s^2 \, ^1S^0 \rightarrow 4s4p \, ^3P^0$ transition. The oscillator strength for this transition is only $6.0 \times 10^{-4}$ [35], so this transition can be safely omitted from the determination of the polarizability. This justifies the omission of the spin–orbit interaction from the effective Hamiltonian for the valence electrons.

The $4s4p \, ^3P^0_0$ state polarizability was computed to be 19.58 au. Table 6 details the contributions of different transitions to this polarizability. The excitations to the lowest three states make a contribution of 86% to the total polarizability with the remainder being split in a roughly equal manner between the core and higher valence excitations. The electric dipole (E1) induced BBR energy shift of an atomic state can be approximately calculated as \[ \Delta \nu \approx \frac{40 \pi^2 \alpha^2}{21 \alpha_1(0)} T^2 \] (15)

The value of $\alpha$ was found to be quite small. In the present CICP calculation, it was $-1.51 \times 10^{-4}$ for the $4s^2 \, ^1S^0_0$ state and $-1.85 \times 10^{-4}$ for the $4s4p \, ^3P^0_0$ state. The net change in frequency due to these two corrections would be $2.3 \times 10^{-5} -3.1 \times 10^{-5} = -8 \times 10^{-6}$ Hz. This change in frequency is much smaller than the uncertainty in the BBR frequency shift.

There is one transition rate that is relevant to the operation of a Ga$^+$ optical frequency standard, namely the rate of the $4s^2 \, ^1S^0 \rightarrow 4s4p \, ^3P^0_0$ transition. A MCDF calculation has obtained the value of 0.334 s$^{-1}$ [35]. The natural line-width of the clock transition is 0.053 Hz.

5. Conclusions

The dipole and quadrupole polarizabilities of the $4s$, $4p$, $4d$, and $5s$ states of Ga$^+$ have been determined by diagonalizing the effective Hamiltonian in a large basis. The dipole and quadrupole polarizabilities of the $4s^2 \, ^1S^0_0$, $4s4p \, ^3P^0_0$, and $4s4p \, ^3P^0_1$ states of Ga$^+$ have been determined from large dimension configuration interaction calculations.

The blackbody radiation (BBR) shift for the $4s^2 \, ^1S^0_0 \rightarrow 4s4p \, ^3P^0_0$ clock transition has been determined to be...
while the BBR shift remains relatively small. The small BBR shift is consistent with the small values reported for other group III ions [12, 15–17, 57]. Table 7 is a summary table of polarizability differences and BBR shifts for the clock transitions of the group III ions. The main trend is for the natural line-width to steadily increase for the heavier atoms while the BBR shift remains relatively small.

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