Robust Control of Fluidized Bed Layering Granulation

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Abstract: This contribution is concerned with control of fluidized bed layering granulation with external sieve-mill-cycle. It is well-known that this configuration is sensitive with respect to variations in the operating conditions and supplied material properties. Therefore, control is needed to achieve desired particle properties over a wide range of process conditions. In this contribution $H_\infty$-control is applied in order to control the process. In contrast to preceding works, the coupling between the particle and the fluid phase is taken into account. Furthermore, in addition to the Sauter diameter of the particle size distribution, the apparent particle porosity is controlled, which is of great importance for practical applications.

Keywords: Particulate processes, robust control

1. INTRODUCTION

Fluidized bed layering granulation is an important particulate process, whose purpose is to transform a liquid raw material into a solid product. It is operated in a fluidized state, i.e. the particle bed is fluidized by supplying an air stream. Injecting a solution or suspension to the fluidized particle bed results in a wetting of the particle surface. This liquid layer is then dried and forms therefore a new solid layer resulting in a particle growth. In order to achieve a constant particle production the granulation process can be combined with a sieving box, separating the desired product size from to small or to big particles, and a mill, generating new nuclei from oversized particles.

As has been investigated for example by Dreysschultze et al. (2015), Schmidt et al. (2017), Neugebauer et al. (2017) and Neugebauer et al. (2019) the described configuration may lead to instabilities, resulting in the occurrence on nonlinear oscillations of the particle size distribution. These are in general undesired. To overcome this problem different feedback control approaches have been proposed. In Cotabarren et al. (2015) decentralized PI-controllers are designed for a multi-chamber granulation process. Robust $H_\infty$-loop shaping controllers have been investigated in Palis and Kienle (2012, 2013) for two continuous granulation processes with internal and external product classification. For the later a nonlinear discrepancy-based controller has been proposed in Palis and Kienle (2014). In order to cope with uncertainties and reduce controller complexity a simple adaptive model-free control scheme has been proposed in Palis (2018) for a continuous granulation process with internal classification.

It should be mentioned, that the focus in previous contributions has been on the particle phase not taking into account the heat and mass transfer with the gas and liquid phase. Although, this is a reasonable first step, newer experimental investigations (e.g. Rieck et al. (2015) and Diez et al. (2018)) show that the coupling between these phases may have a great influence on product properties. For instance, shell porosity $\epsilon_{\text{shell}}$ and apparent particle porosity $\epsilon_p$, two important particle properties, depend on the thermal process conditions. A first model accounting for the bidirectional coupling between the particles and the fluid phase has been proposed in Neugebauer et al. (2018).

Therefore, in this contribution, a robust control approach is applied to control a fluidized bed layering granulation taking into account the heat and mass transfer, i.e. the interchange between the gas phase of the fluidization air, the liquid phase of the solution or suspension and the solid phase of the particle bed.

2. DYNAMIC PROCESS MODEL

Depending on the focus, particulate processes can be described by different model paradigms. In this contribution, the focus is on the time-evolution of the whole population of particles, in contrast to the behavior of a single particle. This can be described by a population balance model, where the internal coordinate is the characteristic particle size $L$. In fluidized bed layering granulation, the change in the particle size distribution $n(t, L)$ is mainly due to particle growth and the feed $\dot{n}_{\text{in}}$ and withdrawal $\dot{n}_{\text{out}}$ of particles. Additional effects like nucleation due to spray drying, agglomeration or particle breakage can in general been neglected.

$$\frac{\partial n(t, L)}{\partial t} = -G \frac{\partial n}{\partial L} + \dot{n}_{\text{in}} - \dot{n}_{\text{out}} \quad (1)$$
In the given configuration, particles are continuously withdrawn from the granulation chamber and fed to a sieving box to remove product and oversized particles. The oversized particles are then ground and fed back into the granulation chamber. The overall process scheme is depicted in Fig. 1.

Assuming spherical particles and an equal distribution of the injected liquid on the particle surface the growth rate $G$ (Mörl et al. (2007)) is given as follows

$$G(n, \dot{m}_{\text{inj}}, \epsilon_{\text{shell}}) = \frac{2x_{\text{inj}}\dot{m}_{\text{inj}}}{(1 - \epsilon_{\text{shell}})\rho_s \mu_2(n)}.$$  

(2)

It depends on the effective mass flow, i.e. the injection rate $\dot{m}_{\text{inj}}$ and the corresponding mass fraction $x_{\text{inj}}$, the mass density of the solid $\rho_s$, the porosity of the particles’ shell $\epsilon_{\text{shell}}$ and the overall surface of the particle bed $A_{\text{bed}}(t) = \pi \mu_2(n)$

$$\mu_2(n) = \int_{0}^{\infty} L^2 n(t,L) \ dL.$$  

(4)

As has been shown in experiments by Diez et al. (2018); Hoffmann et al. (2015); Rieck et al. (2015), the shell porosity $\epsilon_{\text{shell}}$ correlates linearly to the drying potential $\eta$ for the layering granulation of sodium benzoate.

$$\epsilon_{\text{shell}} = -\Delta \epsilon_{\text{shell}} \eta + \epsilon_{\text{shell},0}.$$  

(5)

Assuming an ideally mixed fluidization medium, the drying potential $\eta$ is given by the moisture content of the fluidization medium at the inlet $Y_{\text{in}}$, inside the process chamber $Y$, and its saturation moisture $Y_{\text{sat}}$.

During the continuous process operation particles are withdrawn from the granulation chamber.

$$\dot{n}_{\text{out}}(t,L) = Kn$$  

(7)

The withdrawn particles are then classified into fines $\dot{n}_{\text{fines}}$, product $\dot{n}_{\text{product}}$, and oversized fraction $\dot{n}_{\text{over}}$ using a sieving box.

$$\dot{n}_{\text{over}} = T_1 \dot{n}_{\text{out}}$$  

(8)

$$\dot{n}_{\text{product}} = (1 - T_1)T_2 \dot{n}_{\text{out}}$$  

(9)

$$\dot{n}_{\text{fines}} = (1 - T_1)(1 - T_2) \dot{n}_{\text{out}}$$  

(10)

Here, $T_1$ and $T_2$ are the separation functions for the two sieves.

$$T_i = \frac{\int_{0}^{L} \exp\left(\frac{-(L - \mu_i)^2}{2\sigma_i^2}\right)}{\int_{0}^{\infty} \exp\left(\frac{-(L - \mu_i)^2}{2\sigma_i^2}\right)}.$$  

(11)

While the product fraction is removed from the process, oversized particles are milled and then fed back to the process together with the particles from the fines fraction. It is assumed that the mill generates a Gaussian size distribution with a mean diameter $\bar{L}_{\text{mill}}$ and variance $\sigma_{\text{mill}}^2$. The mass of the oversize fraction is hereby conserved.

$$\dot{n}_{\text{mill}} = 6 \epsilon \frac{(\bar{L} - L_{\text{mill}})^2}{\sqrt{2\pi} \sigma_m} \int_{0}^{\infty} L^3 \dot{n}_{\text{over}} \ dL.$$  

(12)

The particle flows $\dot{n}_{\text{in}}$ and $\dot{n}_{\text{out}}$ in eq. 1 are then given as follows:

$$\dot{n}_{\text{in}} = \dot{n}_{\text{mill}},$$  

(13)

$$\dot{n}_{\text{out}} = \dot{n}_{\text{product}} + \dot{n}_{\text{over}}.$$  

(14)

In contrast to previous works, the mean diameter $L_{\text{mill}}$ is assumed to consist of two terms: $L_{\text{mill},0}$ accounting for the mill operating conditions, e.g. the mill power, and $\Delta L_{\text{mill}}$ representing the influence of the particle properties on the breakage.

$$L_{\text{mill}} = L_{\text{mill},0} + \Delta L_{\text{mill}}.$$  

(15)

As has been shown in Diez et al. (2018) the later, i.e. $\Delta L_{\text{mill}}$, is strongly influenced by the apparent porosity $\epsilon_p$.

$$\epsilon_p = 1 - \frac{m_{p,\text{dry}}/\rho_s}{(\pi/6) \mu_3(n(t,L))}.$$  

(16)

In the following, it will be assumed that this influence can be reflected by a linear correlation.

$$\Delta L_{\text{mill}}(\epsilon_p) = a_{\text{break}} + b_{\text{break}} \epsilon_p.$$  

(17)

In order to keep the bed mass constant over time a mass controller is applied using the drain $K$ as a control handle.
This behavior can be reflected by an ideal mass controller, i.e. choosing the drain \( K \) such that the time-derivative of the bed mass vanishes.

\[
K = -\frac{(1 - \epsilon_{\text{shell}}) \int_0^\infty L^3 \frac{\partial \dot{n}_i}{\partial t} \, dL}{(1 - \epsilon_p) \int_0^\infty L^3 \dot{n}_{\text{prod}} \, dL}
\]  

(18)

Assuming an ideally mixed system, the model equations for the fluid phase can be derived from the mass and enthalpy balances given in Neugebauer et al. (2018). The states are here the temperature, moisture content and mass of the particle and fluid phase, respectively. The moisture content of the particle and fluid phase, \( X \) and \( Y \), are defined as the ratios of solvent to dry mass.

\[
X = \frac{m_{p,\text{solvent}}}{m_{p,\text{dry}}},
\]

(19)

\[
Y = \frac{m_{f,\text{solvent}}}{m_{f,\text{dry}}},
\]  

(20)

The time behavior of the moisture contents is thus given by:

\[
\dot{X} = \frac{\dot{m}_{p,\text{solvent}} - X \dot{m}_{p,\text{dry}}}{m_{p,\text{dry}}},
\]

(21)

\[
\dot{Y} = \frac{\dot{m}_{f,\text{solvent}} - Y \dot{m}_{f,\text{dry}}}{m_{f,\text{dry}}},
\]

(22)

where \( \dot{m}_{p,\text{solvent}}, \dot{m}_{p,\text{dry}}, \dot{m}_{f,\text{solvent}}, \dot{m}_{f,\text{dry}} \) are the mass changes resulting from the mass balances for the solvent and the dry mass of the particle and fluid phase. The mass balances for the dry mass and solvent of the particle phase consist of the mass addition from the injected spray \( m_{\text{inj}} \) in and \((1 - x_{inj}) m_{\text{inj}} \), the external supply of particles \( m_{p,\text{dry},\text{in}} \) and \( X_{inj} m_{p,\text{dry},\text{in}} \), e.g. external seeding particles with a moisture content \( X_{inj} \), and the product removal \( \dot{m}_{p,\text{dry},\text{out}} \) and \( X \dot{m}_{p,\text{dry},\text{out}} \). Furthermore, does the solvent mass balance of the particle phase contain a sink for the liquid evaporation \( \dot{m}_{\text{evap}} \). The mass balances for the solvent and dry mass of the liquid phase contain the terms for the incoming and outgoing gas stream, \( \dot{m}_{f,\text{dry},\text{in}}, \dot{m}_{f,\text{dry},\text{out}}, X_{inj} \dot{m}_{f,\text{dry},\text{in}} \) and \( Y \dot{m}_{f,\text{dry},\text{out}} \), and the solvent supply from evaporation in the particle phase.

\[
\dot{m}_{p,\text{dry}} (t) = x_{inj} m_{\text{inj}} + \dot{m}_{p,\text{dry},\text{in}} - \dot{m}_{p,\text{dry},\text{out}}
\]

(23)

\[
\dot{m}_{p,\text{solvent}} (t) = (1 - x_{inj}) m_{\text{inj}} + X_{inj} \dot{m}_{p,\text{dry},\text{in}} - X \dot{m}_{p,\text{dry},\text{out}} - \dot{m}_{\text{evap}}
\]

(24)

\[
\dot{m}_{f,\text{dry}} (t) = \dot{m}_{f,\text{dry},\text{in}} - \dot{m}_{f,\text{dry},\text{out}}
\]

(25)

\[
\dot{m}_{f,\text{solvent}} (t) = Y_{inj} \dot{m}_{f,\text{dry},\text{in}} - Y \dot{m}_{f,\text{dry},\text{out}} + \dot{m}_{\text{evap}}
\]

(26)

Here, the mass flow rates \( \dot{m}_{p,\text{dry},\text{in}}, \dot{m}_{p,\text{dry},\text{out}} \) represent the coupling with the population balance model.

\[
\dot{m}_{p,\text{dry},\text{i}} = (1 - \epsilon_{p,i}) \rho_s \frac{\pi}{6} \int_0^\infty L^3 \dot{n}_i \, dL
\]

(27)

where \( i = \{ \text{in}, \text{out} \} \). The evaporation rate \( \dot{m}_{\text{evap}} \) depends mainly on the overall surface of the particles, the drying velocity \( \dot{\nu} \) and the difference between the liquid phase moisture content \( Y \) and its saturation level \( Y_{\text{sat}} \). Here, the saturation moisture \( Y_{\text{sat}} \) reflects the maximum vapor amount which can be carried by the fluid. For the given granulation process it is a function of the inlet moisture content \( Y_{in} \) and temperature \( \theta_{\text{f,in}} \), i.e. \( Y_{\text{sat}} = Y_{\text{sat}} (Y_{in}, \theta_{\text{f,in}}) \).

\[
\dot{m}_{\text{evap}} = \dot{\nu} (\delta) \beta_p (A_{\text{bed}} Y_{\text{sat}} - Y) \theta_{\text{f,dry}}
\]

(28)

The coefficient \( \beta_p \) describes the mass transport between fluid and particles as proposed in Gnielinski (2013). On the macroscopic scale the drying process can be divided into three stages (e.g. van Meel (1958)) shown in Fig. 2. Here, each stage depends on the particle moisture content \( X \) and its relation to the thermodynamic adsorption equilibrium \( X_{eq} \) and the critical particle moisture content \( X_{crit} \):

(1) for a low moisture content \( X \) below the thermodynamic adsorption equilibrium \( X_{eq} \), i.e. \( X < X_{eq} \), no evaporation takes place, thus the normalized drying velocity \( \dot{\nu} \) is zero.

(2) for moisture contents above the thermodynamic adsorption equilibrium, but below a certain critical moisture, i.e. \( X_{eq} < X < X_{crit} \), the normalized drying velocity \( \dot{\nu} \) increases with an increasing moisture content.

(3) if the moisture content \( X \) is greater than the critical particle moisture content \( X_{crit} \), i.e. \( X \geq X_{crit} \), the evaporation rate is limited by the state of the fluidization medium and the normalized drying velocity \( \dot{\nu} \) is at its maximum, i.e. \( \dot{\nu} = 1 \).

![Fig. 2. Normalized drying velocity \( \dot{\nu} \) over particle moisture content \( X \)](image)

The described behavior is reflected using the normalized drying velocity (van Meel (1958)).

\[
\dot{\nu} (\delta) = \begin{cases} 
1 & \text{if } X \geq X_{crit} \\
\frac{p \delta (X)}{1 + (p - 1) \delta (X)} & \text{if } X_{crit} > X \geq X_{eq} \\
0 & \text{if } X_{eq} > X 
\end{cases}
\]

(29)

where \( \delta (X) = \frac{X - X_{eq}}{X_{crit} - X_{eq}} \) represents the normalized moisture content. The differential equations for the fluid and particle temperature, \( \theta_{\text{f}} \) and \( \theta_{\text{p}} \), can be derived from the enthalpy balances for the fluid and particle phase (Neugebauer et al. (2018)). Due to the coupling between both phases and the number of input/output streams both equations are rather long and thus omit here.

The overall model, as depicted in Fig. 1, consists of the population balance equation for the particle size distribution and the equations describing the heat and mass balance for the fluid phase.
transfer between the particle bed and the gas phase. It should be mentioned, that most of the parameters and heuristic characteristics are subject to considerable uncertainty and vary with the operating conditions. This results in a corresponding large plant-model mismatch. Therefore, it is reasonable to incorporate appropriate uncertainties in the control design procedure.

3. CONTROLLER DESIGN

From an application point of view, it is of interest to control the Sauter diameter $d_{32,\text{bed}}$, i.e. the ratio of the third and second moment of the particle size distribution, and the moisture content of the fluid phase $\eta$. The first gives a measure for the average particle diameter, whereas the second describes the efficiency of the drying process. 

As control handles the milling diameter $L_{\text{mill,0}}$ and the fluid inlet temperature $\theta_{\text{in}}$ are used. Both can be easily actuated at the real plant. The resulting control problem is hence a 2x2 MIMO system, where the coupling is due to the bidirectional dependencies between the particle and fluid phase. The nonlinear system model has been discretized using a finite volume scheme and linearized at a operating point. 

The resulting $2 \times 2$ transfer function $G(s)$ depicted in Fig. 3 is of order 208 and has thus been further reduced using balanced model truncation (Fig. 4).

The resulting 8-th order transfer function has been used as the nominal model for control design. In order to cope for the above described uncertainties a set of transfer functions for different operating points and parameters has been derived. The maximum and minimum singular value of the resulting transfer functions and the chosen nominal system are depicted in Fig. 5. Calculating the relative deviations to nominal systems, results in the according multiplicative errors, which have been overestimated by a low order transfer function $\Delta_M$ representing an unstructured multiplicative output model uncertainty (Fig. 5).

For convenience two different types of weighting functions $W_1$ and $W_2$ have been used.

![Fig. 3. MIMO control scheme](image)

![Fig. 4. Bode plots of the full-order transfer function $G(s)$ and the reduced system $G_{\text{red}}(s)$.](image)

![Fig. 5. Singular values of the open-loop frequency response of the uncertain system (upper row) and the corresponding multiplicative model uncertainty (lower row).](image)

![Fig. 6. Structure of the $H_\infty$-mixed-sensitivity design problem.](image)
A robust controller has been designed for a fluidized bed layering granulation with external sieve-mill-cycle. From the chosen nominal plant and the weightings, yields a $H_{\infty}$-controller that stabilizes the plant. The according closed-loop singular values are depicted in Fig. 7. As can be seen, the design requirements are fulfilled.

Solving the mixed-sensitivity $H_{\infty}$-problem (eq. 30) for the chosen nominal plant and weightings, yields a $H_{\infty}$-controller. The resulting closed-loop singular values in comparison to the according weighting functions are depicted in Fig. 7. As can be seen, the design requirements are fulfilled.

$W_I(s) = \frac{(T_p s + 1)}{(M T_p s + A)}$ with $T_p = 1/\omega_h M$

$W_{II}(s) = \frac{A (T_d s + 1)^n}{(Ts + 1)^2}$

The chosen parameters are given in Table 3. Here, diagonal weightings have been chosen for both channels.

In the second scenario, the reference value of particle porosity $\epsilon_{p,ref}$ is increased from 0.39 to 0.425 at time $t_{sp} = 2$ h. However, at time $t_{dist} = 15$ h, the injection rate $\dot{m}_{inj}$ is decreased from 40.0 kg/h to 34.0 kg/h, which has a direct influence on the particle growth rate. The resulting particle size distribution is shown in Fig. 10. Again the controlled outputs and the control action stay within reasonable limits (Fig. 11).

**Table 1. Weighting parameters**

| Type | $W_1$ | $W_{II}$ | $W_{II}$ | $W_{III}$ | $W_{III}$ |
|------|-------|----------|----------|-----------|-----------|
| $A$  | $5.0 \times 10^{-3}$ | $1.0 \times 10^{-4}$ | $2.0 \times 10^{-3}$ | $4.0 \times 10^{-1}$ | $3.25 \times 10^{-1}$ |
| $\omega_h$ | $7.5 \times 10^{-5}$ | — | — | — | — |
| $M$  | $1.5 \times 10^{0}$ | — | — | — | — |
| $T_d$ | $1.0 \times 10^{-5}$ | $1.0 \times 10^{-6}$ | $1.0 \times 10^{-6}$ | $3.5 \times 10^{-3}$ | $1.25 \times 10^{-3}$ |

The first scenario at time $t_{sp} = 39$ h the moisture content $Y_{inj}$ changes due to an external disturbance from $6 \, \text{kg}\cdot\text{water/kg}\cdot\text{dry}$ to $15 \, \text{kg}\cdot\text{water/kg}\cdot\text{dry}$. The results for the controlled outputs and the control action are shown in Fig. 9. The according particle size distribution is depicted in Fig. 8. As can be seen, the process remains stable and the output values follow their set-points with smaller oscillations in the Sauter diameter. The control effort is reasonable and the external disturbance is well-suppressed.

**Fig. 8. Normalized particle size distributions $q_0(t, L)$ (scenario 1)**

**Fig. 9. Controlled outputs and control action during (scenario 1)**

In the second scenario, the reference value of particle porosity $\epsilon_{p,ref}$ is increased from 0.39 to 0.425 at time $t_{sp} = 2$ h. However, at time $t_{dist} = 15$ h, the injection rate $\dot{m}_{inj}$ is decreased from 40.0 kg/h to 34.0 kg/h, which has a direct influence on the particle growth rate. The resulting particle size distribution is shown in Fig. 10. Again the controlled outputs and the control action stay within reasonable limits (Fig. 11).

**Fig. 10. Normalized particle size distributions $q_0(t, L)$ (scenario 2)**

5. CONCLUSION

A robust controller has been designed for a fluidized bed layering granulation with external sieve-mill-cycle. From

![Fig. 7. Singular values of the closed-loop transfer functions $S(s)$, $T(s)$, and $K(s)$ $S(s)$ compared to the weighting functions $W_i(s)$](image-url)

![Fig. 8. Normalized particle size distributions $q_0(t, L)$ (scenario 1)](image-url)

![Fig. 9. Controlled outputs and control action during (scenario 1)](image-url)

![Fig. 10. Normalized particle size distributions $q_0(t, L)$ (scenario 2)](image-url)

![Fig. 11. Controlled outputs and control action during (scenario 1)](image-url)
Fig. 11. Controlled outputs and control action during (scenario 2)

a practical point of view, it is important to guarantee certain product properties as for example particle size or porosity. Due to the strong coupling between the particle and fluid phase, external disturbances and high process uncertainties, this is a challenging task. As has been shown in this contribution the $H_\infty$-mixed-sensitivity control approach allows for a reasonable solution of this problem. The set-point and disturbance scenarios are of practical relevance and represent typical challenges in production.

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