Grand unification of $\mu - \tau$ Symmetry

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(Dated: February, 2006)

Abstract

Near maximal neutrino mixing needed to understand atmospheric neutrino data can be interpreted to be a consequence of an interchange symmetry between the muon and tau neutrinos in the neutrino mass matrix in the flavor basis. This idea can be tested by a measurement of the neutrino mixing parameter $\theta_{13}$ and looking for its correlation with $\theta_{23} - \pi/4$. We present a supersymmetric SU(5) grand unified model for quarks and leptons which obeys this exchange symmetry and is a realistic model that can fit all observations. GUT embedding shifts $\theta_{13}$ from its zero $\mu - \tau$ symmetric value to a nonzero value keeping it under an upper limit.
I. INTRODUCTION

Observation of nonzero neutrino masses and determination of two of their three mixing parameters by experiments have raised the hope that neutrinos may provide a clue to flavor structure among quarks[1]. In order to make progress in this direction however, one needs knowledge of the detailed nature of the quark-lepton connection e.g. whether there is an energy scale where quarks and leptons are unified into one matter (or grand unification of matter). While there are similarities between quarks and leptons that make such an unification plausible, there are also many differences between them which may apriori point the other way: for instance, the mixing pattern among quarks is very different from that among leptons and the neutrino mass matrices in the flavor basis exhibit symmetries for which there apparently is no trace among quarks. Two examples of such apparent lepton-exclusive symmetries are: (a) discrete $\mu - \tau$ symmetry[2, 3] of the neutrino mass matrix in the flavor basis indicated by maximal atmospheric mixing angle and small $\theta_{13}$ and (b) continuous $L_e - L_\mu - L_\tau$[4] symmetry, which will be indicated if the mass hierarchy among neutrinos is inverted.

If neutrinos are Majorana fermions, they are likely to acquire masses from very different mechanisms e.g. one of the various seesaw mechanisms which involve completely independent flavor structure (say for example from right handed neutrinos) than quarks. The apparent disparate pattern for quark and leptons mixings then need not argue against eventual quark-lepton unification. In fact there are now many grand unification models (where quarks and leptons are unified at short distances) where small quark mixings and large lepton mixings along with all their masses can be understood with very few assumptions in a seesaw framework[1].

In this paper we address the question as to whether there could be an apparently pure leptonic symmetry such as $\mu - \tau$ symmetry in the neutrino mass matrix in the flavor basis (i.e. the basis where charged leptons are mass eigenstates), which is part of a general family symmetry within a quark-lepton unified framework such as a grand unified model. We particularly focus on this symmetry since there appears to be some hint in favor of this from the present mixing data. In the exact symmetry limit, the mixing parameter $\theta_{13} = 0$[2] and breaking of the symmetry not only implies a small nonzero value for $\theta_{13}$ but also leads to a correlation between $\theta_{13}$ with $\theta_{23} - \pi/4$, which can be used to test for this idea[3]. This
question has been discussed at a phenomenological level in several recent papers but to
the best of our knowledge no full-fledged gauge model has been constructed. Indeed most
gauge models for $\nu_\mu - \nu_\tau$ symmetry discussed in the literature treat leptons separately from
quarks.

One simple way to have quark flavor structure completely separated from that of leptons
and yet have quark-lepton unification is to use the double seesaw framework where neu-
trino flavor texture from “hidden sector” singlet fermions (e.g. SO(10) singlets ) which are
completely unrelated to quarks (for examples of such models, see [7, 8]). One can then have
any pure “leptonic” symmetry on the hidden singlets without at the same time interfering
with quark flavor texture. A necessary feature of such models is that one must introduce
new fermions into the model. A question therefore remains as to whether one could do this
without expanding the matter sector. In this paper, we propose such an approach without
introducing new fermions within a realistic SU(5) GUT framework that unifies quarks and
leptons. We demand the full theory prior to symmetry breaking to obey a symmetry be-
tween the second and third generation (or a generalized version of “$\mu - \tau$” symmetry). The
neutrino masses are assumed to arise from a triplet seesaw (type II) mechanism, which
disentangles the neutrino flavor structure from the quark flavor structure. The quark mass
matrices are however constrained by the $\mu - \tau$ symmetry. The quark mixing angles then
introduce departures from exact $\mu - \tau$ symmetry results and lead to nonzero $\theta_{13}$ as well as
departures from maximal atmospheric mixing.

The model consists of a minimal set of Higgs bosons which are anyway required to recon-
cile the charged fermion masses in the minimal SU(5) model. We find that the requirement
of $\mu - \tau$ symmetry for neutrinos can be imposed on the model without contradicting observed
charged fermion masses and mixings. As noted, the model predicts a nonzero value for $\theta_{13}$
correlated with the departure of $\theta_{23}$ from its maximal value.

This paper is organized as follows: in sec. 2, we present the SU(5) model with $\mu - \tau$
symmetry; in sec. 3, we discuss coupling unification in the model since we have a new
scale around $10^{14}$ GeV to implement the type II seesaw for neutrino masses. We close with
concluding remarks in sec. 4.
II. SUSY SU(5) MODEL WITH $\mu - \tau$ SYMMETRY

As in the usual SU(5) model, matter fields are assigned to $5 \equiv F_\alpha$ and $10 \equiv T_\alpha$ (with $\alpha = 1, 2, 3$ denotes the generation index). We choose the Higgs fields to belong to the multiplets $24$ (denoted by $\Phi$ and used to break the SU(5) symmetry down to the standard model); $5 \oplus \bar{5}$ (denoted by $h + \bar{h}$) and $45 \oplus \bar{45}$ (denoted by $H + \bar{H}$) used to give masses to fermions) and $15 \oplus \bar{15}$ (denoted by $S + \bar{S}$) to give masses to neutrinos via the type II seesaw mechanism[13].

The matter and Higgs fields transform under the $\mu - \tau$ discrete flavor symmetry as follows:

\[
\begin{align*}
F_\mu & \leftrightarrow F_\tau \\
(h, \bar{h}) & \leftrightarrow (h, \bar{h}) \\
(H, \bar{H}) & \leftrightarrow (-H, -\bar{H})
\end{align*}
\] (1)

and all other fields are singlets under this transformation. In this model, the matter part of the superpotential can be written as

\[W = Y_{15}FFS + Y_5TT\bar{h} + Y_5TF\bar{h} + Y_{45}TFH.\] (2)

After the electro-weak symmetry breaking, the mass matrices for the standard model fermions are given by

\[
M_\nu = Y_{15} < S > = \begin{bmatrix} X & Y & Y \\ Y & Z & W \\ Y & W & Z \end{bmatrix}
\] (3)

\[
M_u = Y_5 < h > = \begin{bmatrix} A & B & C \\ B & D & E \\ C & E & F \end{bmatrix}
\] (4)

\[
M_d = Y_5 < \bar{h} > + Y_{45} < H > = \begin{bmatrix} A_1 & B_1 & C_1 \\ E_1 & D_1 & F_1 \\ E_1 & D_1 & F_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ E_2 & D_2 & F_2 \\ -E_2 & -D_2 & -F_2 \end{bmatrix}
\] (5)

\[
M_e = Y_5^T < \bar{h} > - 3Y_{45}^T < H > = \begin{bmatrix} A_1 & E_1 & E_1 \\ B_1 & D_1 & D_1 \\ C_1 & F_1 & F_1 \end{bmatrix} - 3 \begin{bmatrix} 0 & E_2 & -E_2 \\ 0 & D_2 & -D_2 \\ 0 & F_2 & -F_2 \end{bmatrix}
\] (6)
where the various parameters characterising the mass matrices are given in terms of the
Yukawa couplings and vacuum expectation values of fields as follows: $<S>, <h>, \langle \bar{h} \rangle$, $<H>$ are vevs of $S, h, \bar{h}, H$ respectively.

The mass matrices depend on nineteen parameters if we ignore CP phases and there
are seventeen experimental inputs (6 quark masses, 3 charged lepton masses, two neutrino
mass difference squares plus five mixing angles values and an upper limit on $\theta_{13}$). For the
sake of comparison, we note that if we generated neutrino masses in the standard model
using a Higgs triplet field, there would be 18 parameters in the absence of CP violation (9
from the quark sector, 3 from the charged lepton mass matrix and six from the neutrino
sector). When one embeds the standard model into a GUT SU(5), to be realistic, one needs
to introduce 45 Higgs and its associated Yukawa couplings. In this case, the total number
of parameters in the Yukawa sector is 24. In our model the requirement of $\mu - \tau$ symmetry
has first led to a reduction in the total number by three and furthermore grand unification
has strongly correlated the down quark and charged lepton mass matrix, as expected. It is
therefore not obvious that the model will be consistent with known data on fermion masses.

To see if the model is phenomenologically acceptable, we first fit the masses of the charged
leptons and down type quarks using the mass values of leptons and quarks at GUT scale
given in Ref. [10]:

| input observable | $\tan \beta = 10$ |
|------------------|------------------|
| $m_u$ (MeV) | $0.7238^{+0.1365}_{-0.1467}$ |
| $m_c$ (MeV) | $210.3273^{+19.0036}_{-21.2264}$ |
| $m_t$ (GeV) | $82.4333^{+30.2676}_{-14.7686}$ |
| $m_d$ (MeV) | $1.5036^{+0.2335}_{-0.2304}$ |
| $m_s$ (MeV) | $29.9454^{+4.3001}_{-4.5444}$ |
| $m_b$ (GeV) | $1.0636^{+0.1414}_{-0.0865}$ |
| $m_e$ (MeV) | $0.3585^{+0.0003}_{-0.0003}$ |
| $m_\mu$ (MeV) | $75.6715^{+0.0578}_{-0.0501}$ |
| $m_\tau$ (GeV) | $1.2922^{+0.0013}_{-0.0012}$ |

The values of parameters in the model are found by scanning the whole parameter space
under the constraint that we satisfy the current experiment requirements of $\theta_{13}$ and $\theta_{23}$.
Note that since in this model, neutrino mass matrix in the flavor basis is $\mu - \tau$ symmetric,
it is diagonalized by the matrix:

$$U_\nu = \frac{1}{\sqrt{2}} \begin{bmatrix} \sqrt{2} \cos \theta_\nu & \sqrt{2} \sin \theta_\nu & 0 \\ -\sin \theta_\nu & \cos \theta_\nu & 1 \\ -\sin \theta_\nu & \cos \theta_\nu & -1 \end{bmatrix},$$

(7)

where \( \theta_\nu \) is the solar mixing angle. The deviations of \( \theta_{13} \) and \( \theta_{23} \) from 0 and \( \frac{\pi}{4} \) respectively should come from left-handed charged leptons mixing matrix. Since these deviations have upper bounds, this puts a nontrivial constraint on the charged lepton mass matrix of the model; but since the charged lepton mass matrix is already constrained by \( \mu - \tau \) symmetry, it is nontrivial to get all masses and mixings to fit. It turns out that the fitting for the masses of leptons and quarks does not provide any bound on \( \theta_{23} \), however it gives quite stringent bound on \( \theta_{13} \). Using the relation \( U_{MNS} = U_l^\dagger U_\nu \), one can write \( \sin \theta_{13} \) and \( \tan \theta_{23} \) as

$$\sin \theta_{13} = \frac{1}{\sqrt{2}} |U_{l21} - U_{l31}|$$

(8)

$$\tan \theta_{23} = \frac{|U_{l22} - U_{l32}|}{|U_{l23} - U_{l33}|}$$

(9)

The 3\( \sigma \) experimental bounds of \( \theta_{13} \) and \( \theta_{23} \) are [11]

$$0.34 \leq \sin^2 \theta_{23} \leq 0.68$$

(10)

$$\sin^2 \theta_{13} \leq 0.051.$$  

(11)

The scatter plot in Fig.1 gives \( \sin^2 \theta_{13} \) as a function of \( \sin^2 \theta_{23} \) allowing for 3 \( \sigma \) uncertainty in all masses except \( m_e \) (chosen to be 0.3 – 0.4 MeV), \( m_\mu \) (chosen to be 73 – 76 MeV) and \( m_d \) left free and \( \theta_{23} \) within 3 \( \sigma \).

Here, we give two typical fitting points for our model:

(i) Case 1:

\[
\begin{align*}
m_d &= 0.355117 \text{ MeV} & m_s &= 34.0438 \text{ MeV} & m_b &= 985.857 \text{ MeV} \\
m_e &= 0.356047 \text{ MeV} & m_\mu &= 75.1597 \text{ MeV} & m_\tau &= 1336.14 \text{ MeV}
\end{align*}
\]

(12)

(13)

\[
U_l = \begin{bmatrix} 0.999327 & 0.036688 & 0.0000316411 \\ 0.0366849 & -0.999231 & -0.0138381 \\ 0.000476075 & -0.01383 & 0.999904 \end{bmatrix}
\]

(14)
FIG. 1: Scatter plot in the $\sin^2 \theta_{23}$ and $\sin^2 \theta_{13}$ plane.

FIG. 2: Value distribution of $\sin^2 \theta_{13}$. 67 percent of fitting points have $\sin^2 \theta_{13} \leq 0.03$ and 80 percent have $\sin^2 \theta_{13} \leq 0.05$.

For this case, we predict the following values for the neutrino mixing parameters $\theta_{13}$ and $\theta_{23}$:

\[
\theta_{13} \simeq 0.026 \quad (15)
\]
\[
\theta_{23} \simeq 44.3^\circ \quad (16)
\]

(ii) Case 2:

\[
m_d = 0.336552 \text{ MeV} \quad m_s = 38.4364 \text{ MeV} \quad m_b = 926.78 \text{ MeV} \quad (17)
\]
\[
m_e = 0.381779 \text{ MeV} \quad m_\mu = 73.112 \text{ MeV} \quad m_\tau = 1288.52 \text{ MeV} \quad (18)
\]
\[
U_l = \begin{bmatrix}
0.959961 & 0.280133 & 0.000326329 \\
0.279872 & -0.959014 & -0.0443148 \\
0.0121011 & -0.0426319 & 0.999018 \\
\end{bmatrix}
\] (19)

giving us

\[
\theta_{13} \simeq 0.19788 \quad (20)
\]

\[
\theta_{23} \simeq 41.2^\circ \quad (21)
\]

We therefore note that the value of the most probable value for $\theta_{13}$ is in the range from $0.02 - 0.19$ with (as indicated in Fig. 2) values below 0.1 being much more probable.

Note that mass $m_d$ in both cases has almost same magnitude as $m_e$ and is smaller than the central value at the GUT scale by about $\sim 1$MeV. The reason for this is that $H$ is $\mu - \tau$ odd, leading to zero entries in the $M_e, M_d$. Note that in this model, we also have additional threshold correction from the exchange of the gauginos, which make larger contribution to quarks relative to the charged leptons of the corresponding generation due to strong coupling of the gluinos. In particular, the gluino contribution to the tree level masses of the quarks can be significant if the assumption of proportionality between the A-terms and the Yukawa couplings is abandoned. Fig. 3 gives a typical Feynman diagram contributing to the quark masses\cite{15}. The generic contribution to the $(i,j)$ element of the down quark mass matrix is given by:

\[
\delta m_{d,ij} \simeq \frac{2\alpha_s}{3\pi} \frac{M_\tilde{g}^2}{m_{\tilde{g}}^2} (m_{0,ij}^d \mu \tan \beta + A_{ij}^{(d)} m_0) \quad (22)
\]

Including this radiative correction only in the 11 element of the down quark mass matrix, one can get the down quark mass to be in agreement with observations. We also note that
the process of fitting the charged lepton and down quark masses gives a definite rotation
matrix that diagonalizes the down quark mass matrix and contributes to the $V_{CKM}$. We
then appropriately choose the parameters in the symmetric up-quark mass matrix so that
we get the correct $V_{CKM}$.

III. OTHER COMMENTS ON THE MODEL:

A. Gauge coupling unification

This type-II seesaw requires that we have a medium scale for the mass of the SM triplet
Higgs which is present in $15$-Higgs i.e. $M_T \sim 10^{14}$GeV; this is satisfied if we tune the
coupling $\lambda$ of $\lambda \Phi S \tilde{S}$ to $\sim 10^{-2}$ or so since $M_T \sim \lambda v_U$. Once $\Phi$ get vev and breaks $SU(5)$
to Standard Model, it also can induce the mass splitting of multiplets of $S, \tilde{S}$. This will
affect the unification of coupling. We display the effect of these mass splittings to the
gauge coupling running as a threshold correction, in Fig.4 and show that the unification of
couplings is maintained and we get a slight increment in the value of $M_U \simeq 2.36 \times 10^{16}$ GeV.
B. 45 vrs its higher dimensional equivalent

We also like to comment that a more economical possibility is to consider a model that uses a high dimension operator involving with \( \Phi \) instead of the \( \mathcal{H} \). The matter part of the superpotential in this case is given by:

\[
W = Y_{15}FFS + Y_TTH + Y_5FT\tilde{h} + \frac{1}{M_P}Y_{24}FT\Phi\tilde{h},
\]  

(23)

where \( M_P \) is Planck scale and \( H_{24} \) is the \( SU(5) \) adjoint representation used to break \( SU(5) \) to \( SU(3) \times SU(2) \times U(1) \). \( M_P \sim 10^{19}\text{GeV}, \text{vev of } H \sim 10^{16}\text{GeV and vev of } \tilde{h} \sim 10^{2}\text{GeV} \), thus the overall scale of the contribution of this higher dimensional operator to fermion mass matrices \( \sim 100\text{MeV} \). We have tried a fitting of data for this model and find it to be unacceptable, since it gives very large \( \sin^2 \theta_{23} \sim 0.76 - 0.8 \) which is around 4-5 \( \sigma \).

C. Possible \( S_3 \) embedding

An interesting possibility is to embed the \( \mu - \tau \) symmetry into an \( S_3 \) symmetry. There are two reasons one may consider such an extension. First is that the neutrino mass matrix of the form that originates from the \( 15 \) coupling is then given by:

\[
\mathcal{M}_\nu = \begin{pmatrix} a & b & b \\ b & a & b \\ b & b & a \end{pmatrix}
\]  

(24)

which reproduces the interesting tri-bi-maximal mixing pattern \[14\] which seems to be in very good accord with current data. \( SU(5) \) embedding could then provide corrections to the tri-bi-maximal mixing. However, mixing matrix obtained from Eq.\[24\] is arbitrary up to a rotation in the 1-3 space due to the fact that the first and the third generations are degenerate. If we add to this matrix the following \( \mu - \tau \) symmetric but \( S_3 \) breaking matrix of the form:

\[
\delta\mathcal{M}_\nu = c \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}
\]  

(25)

This however breaks the \( S_3 \) symmetry by a large amount.
The second interesting point about the $S_3$ embedding is that in the symmetry limit, the SU(5) model conserves R-parity automatically, making the dark matter naturally stable since the term $FFT$ is forbidden by the symmetry.

IV. CONCLUSION

In summary, we have discussed the grand unification of apparently pure leptonic symmetries such as $\mu - \tau$ symmetry into the quark-lepton unifying Supersymmetric SU(5) model for quarks and leptons and studied its implications for neutrino mixing angles. We find that it is possible to have a completely viable SU(5) model of this type. In this model the neutrino masses arise from a triplet vev induced type II seesaw mechanism. The presence of quark lepton unification leads to small deviations from maximal atmospheric mixing angle and vanishing $\theta_{13}$ implied in the exact symmetry limit.

This work is supported by the National Science Foundation grant no. Phy-0354401

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