Gravitons and the Drell-Hearn-Gerasimov Sum Rule

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Abstract

One-loop diagrams containing a graviton provide a finite contribution to the anomalous magnetic moment $a_\ell$ of a lepton, whether or not the graviton propagates in $n$ large extra compact dimensions. In the present work, the tree graph photoproduction of a graviton, integrated up to an arbitrary cutoff, is shown to violate the Drell-Hearn-Gerasimov sum rule for $a_\ell^2$. The possibility of resurrecting the sum rule from high energy contributions originating in string excitations is discussed in a qualitative manner, and various problems associated with such a program are pointed out.
1 Introduction

Years ago, Berends and Gastmans [1] calculated the one-loop contribution of virtual massless gravitons to $a_\ell = \frac{1}{2}(g-2)\ell$, the anomalous magnetic moment of a lepton $\ell$, and obtained a finite result. Recently, Graesser [2] has redone this calculation in the context of a revised picture of gravity [3] which is of much current interest: in this version, the gravitational sector lives in an expanded $D = (4 + n)$-dimensional spacetime, with the extra $n$ dimensions compactified on surfaces whose characteristic sizes may be as large as a millimeter. Graesser has considered the contribution to $a_\ell$ of the resulting Kaluza-Klein tower of spin-2 gravitons (and spin-0 partners). The contribution of each KK mode is again finite, as is the total if the sum on modes is appropriately cut off at or near the $D$-dimensional fundamental scale $M_D$ [4]. The finiteness of these results raises an interesting question: is a well-known sum rule for $a_\ell^2$, the Drell-Hearn-Gerasimov (DHG) sum rule [5] satisfied in quantum gravity, at least at the one-loop level? In accord with a general argument by Brodsky and Schmidt [6], it will be seen that, at the one-loop level, this sum rule requires the vanishing of a certain integral involving tree-level contributions to a difference of polarized cross sections for the photoproduction of gravitons. As a result of a tedious but straightforward computation, it will be shown that this sum rule is not satisfied, neither in 4-dimensional nor in $(4 + n)$-dimensional gravity. At one level, this may be a plausible result: gravity, being a non-renormalizable theory, does not satisfy the finiteness criteria (to be detailed below) necessary for validity of the sum rule. From a different perspective, the finiteness of the gravitational contribution to $a_\ell$ at the one-loop level, combined with a string-based belief in Reggeization of amplitudes (including gravitational) at very high energies, suggest that perhaps the sum rule would regain validity upon inclusion of all the high energy string excitations. In conformance with the original $s$- and $t$-channel string duality, this suggested an examination of the constraints on the Regge behavior in order that high energy contributions at at least be consistent with restoring the validity of the sum rule. This could only be done in a rough and speculative fashion, and a discussion of the problems associated with imposing the resulting constraints will be discussed in the context of Type $I'$ string theories. Speculations aside, the principal concrete result remains that, to lowest order, the DHG sum rule is not obeyed by the tree-level contributions involving photoproduction of gravitons.
2 The DHG Sum Rule and Quantum Gravity

Under certain conditions on the high energy behavior of both the real and imaginary part of the forward spin-difference Compton amplitudes, there exists a sum rule for $a_\ell^2$. For a spin-1/2 target, it reads

$$a_\ell^2 = \frac{m_\ell^2}{2\pi^2\alpha} \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu} \Delta\sigma(\nu) ,$$

(1)

where $\Delta\sigma(\nu) \equiv \sigma_P(\nu) - \sigma_A(\nu)$ is the difference between photoabsorption cross sections for the scattering of a photon of lab energy $\nu$ from a target lepton in the cases where the initial photon and lepton spin components along the incident photon direction are parallel and anti-parallel, respectively. The validity of the sum rule is predicated on both the vanishing of $\Delta\sigma$ at high energies and on the absence of polynomial terms in the real part of the forward Compton spin-difference amplitude $f_2$. Both conditions would obtain if the full Compton amplitude were to Reggeize; the details of this Reggeization at string energies will be critical in resolving the problem raised by the calculation which follows.

In the Standard Model, the validity of the sum rule requires the vanishing of the integral on the RHS of Eq. (1) when the cross section is calculated from the lowest order tree graphs, namely $\gamma + \ell \rightarrow \gamma + \ell$, $W^- + \nu_\ell$. This follows from the observation that the LHS of (1) when calculated in the Standard Model is of $O(\alpha^2)$ (for fixed $\sin\theta_W$), whereas in Born approximation the RHS is of $O(\alpha)$, and hence must vanish. This is indeed found to be the case, as shown in the explicit calculation of Altarelli, Cabibbo, and Maiani. Using loop expansion techniques, the result has been generalized by Brodsky and Schmidt: the validity of the DHG sum rule requires the vanishing of the integral on the RHS for the sum of $2 \rightarrow 2$ processes $\gamma\ell \rightarrow bc$ in the Born approximation. This then raises the interesting question: since the one-loop quantum gravity contributions to $a_\ell$ are finite and calculable, is the DHG sum rule satisfied to the appropriate order in this case? If not, can we learn anything from the failure to satisfy?

In order to simplify the discussion, I will limit the physics to QED and gravity (the rest of the Standard Model can be included with little complication). In this case, at the one loop level, the corrections to $a_\ell$ from these sources are additive:

$$a_\ell = a_\ell^{QED} + a_\ell^{QG} \ (1 \ loop) .$$

(2)

Thus, to one-loop level, the sum rule may be written as
\[
\left( a_{\ell}^{\text{QED}} \right)^2 + 2 \, a_{\ell}^{\text{QED}} \, a_{\ell}^{\text{QG}} + \left( a_{\ell}^{\text{QG}} \right)^2 = \frac{m_{\ell}^2}{2\pi^2\alpha} \int_{\nu_{\text{th}}}^{\infty} \frac{d\nu}{\nu} \left( \Delta\sigma(\nu)|_{\text{QED}} + \Delta\sigma'(\nu) \right) .
\] (3)

The first term under the integral is the pure QED contribution, up to the appropriate order, and the second is the mixed gravity-QED cross section. Since QED by itself obeys the DHG sum rule, the first terms on the LHS and RHS will cancel. With this cancellation, and to lowest order in \(m_{\ell}^2\), the sum rule reads

\[
2 \, a_{\ell}^{\text{QED}} \, a_{\ell}^{\text{QG}} + \left( a_{\ell}^{\text{QG}} \right)^2 = \frac{m_{\ell}^2}{2\pi^2\alpha} \int_{s_{\text{th}}}^{\infty} \frac{ds}{s} \Delta\sigma'(s) .
\] (4)

I will now consider separately the cases with gravity propagating in \(n > 0\) \((D > 4)\) and \(n = 0\) \((D = 4)\) dimensions, respectively.

\(n > 0\)

I first briefly review the properties of \(4 + n\)-dimensional gravity I will need for the discussion which follows. For compactification on an \(n\)-torus, the \(D\)-dimensional Planck scale \(M_D\) is related to the large radius \(R_n\) and the reduced Planck mass via \cite{3, 9, 10}

\[
\left( \frac{M_P}{M_D} \right)^2 = (M_D R_n)^n ,
\] (5)

so that \(M_D\) can range from \(\sim\) TeV to \(M_P\) for \(R \leq 1\) mm as long as \(n \geq 2\). The resulting Kaluza-Klein tower of spin-2 gravitons (and spin-0 partners, the radions) can have very small mass splitting \(\Delta m^2 = (1/R_n)^2\). This scale also characterizes the surface tension of the soliton (say a D-brane) to which is tied the open string containing ordinary matter. Feynman rules have been developed \cite{10, 11} for these couplings, and a large number of authors have explored the phenomenological implications of this view of gravity.

Thus, on the RHS of (4), the lowest order contributions are the tree processes \(\gamma \ell \rightarrow G\ell, \gamma \ell \rightarrow \Phi\ell\) where \(G\) is one of the spin-2 graviton, and \(\Phi\) is its scalar partner (the radion). The cross section for the latter process is suppressed by a factor of \(m_{\ell}^2\), so it does not enter the present consideration\footnote{In accordance with the scaling argument in \cite{3}, the radion contribution should cancel when combined with \(O(m_{\ell}^2)\) corrections to the tree-level \(\Delta\sigma(\gamma \ell \rightarrow G\ell)\).}

The calculation of \(\Delta\sigma(\gamma \ell \rightarrow G\ell)\) is straightforward \cite{12}. I fix the kinematics so that the photon is incident along the \(+z\)-axis with momentum \(q\), helicity +1, the lepton
with momentum $p_1$ along the $-z$-axis. For massless leptons, the amplitude \( \mathcal{M}_{P(A)} \) is helicity conserving, and the polarization amplitudes for $\gamma \ell \rightarrow G \ell$ are then written as

\[
\mathcal{M}_{P(A)} = \bar{u}_{L(R)}(p_2) \mathcal{O}^{\mu
u\rho} u_{L(R)}(p_1) \epsilon^{(+1)}_\rho(q) \left( \epsilon^{A}_\mu(k) \right)^*, \tag{6}
\]

Squaring and summing over the final state graviton helicity, one finds (for all particles massless except for the graviton, with mass $m$) the differential cross sections in the c.m.

\[
d\sigma_{P(A)}/d\cos\theta = \sum_{\Lambda} \left| \mathcal{M}_{P(A)} \right|^2 / (8\pi s)^2 \cdot (1 - x). \tag{7}
\]

where $x \equiv m^2/s$, and

\[
\sum_{\Lambda} \left| \mathcal{M}_{P(A)} \right|^2 = (\epsilon^{(+1)}_\rho(q))^* \text{Tr} \left( \bar{\mathcal{O}}^{\mu'\nu'\rho'}(p_2) \mathcal{O}^{\mu\nu\rho}(p_1) (1 + (-)\gamma_5)/2 \right) \epsilon^{(+1)}_\rho(q) \mathcal{P}_{\mu'\nu';\mu\nu}(k), \tag{8}
\]

The graviton spin-2 projection operator $\mathcal{P}$ is given in Refs.\[10, 11\], and $\bar{\mathcal{O}} = \gamma^0 \mathcal{O}^\dagger \gamma^0$.

Finally, from (7) and (8),

\[
\frac{d\sigma_P}{d\cos\theta} - \frac{d\sigma_A}{d\cos\theta} = (\epsilon^{(+1)}_\rho(q))^* \text{Tr} \left( \bar{\mathcal{O}}^{\mu'\nu'\rho'}(p_2) \mathcal{O}^{\mu\nu\rho}(p_1) \gamma_5 \right) \epsilon^{(+1)}_\rho(q) \cdot \mathcal{P}_{\mu'\nu';\mu\nu}(k) \cdot (1 - x) / (8\pi s)^2. \tag{9}
\]

The contributing diagrams are given in Refs.\[10, 11\] and are shown in Fig.1.

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**Figure 1:** Graphs contributing to $\gamma \ell \rightarrow G \ell$. 

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The amplitude can be obtained from these references, and I write it here for completeness (kinematics in Fig. 1):

\[ i\mathcal{O}_{\mu\nu} = \left( \frac{ieQ}{4M_P} \right) \left[ \gamma_\mu (P + p_2)_\nu \left( \frac{P}{s} \right) \gamma_\rho + \gamma_\rho \left( \frac{K}{u} \right) \gamma_\mu (p_1 + K)_\nu \right] \\
- \left( \frac{ieQ}{M_P} \right) \left[ q_\mu q_\nu \gamma_\rho + (q \cdot Q) \eta_{\mu\rho} \gamma_\nu - \eta_{\mu\rho} q_\nu - \gamma_\mu q_\nu Q_\rho \right] / t \\
- \left( \frac{ieQ}{2M_P} \right) \gamma_\mu \eta_{\nu\rho} + \mu \leftrightarrow \nu \cdot (10) \]

In Eq. (10), \( Q_\ell \) is the charge on the lepton, \( \bar{M}_P = (8\pi G_N)^{-1/2} \), \( P = p_1 + q \), \( Q = k - q \), and \( K = p_1 - k \) (see Fig. 1).

From (9) and (10) I find

\[ \frac{d\sigma_P}{d\cos \theta} - \frac{d\sigma_A}{d\cos \theta} = \frac{1}{16} \frac{\alpha}{M_P^2} (1 - x) \left[ \left( \frac{s}{t} \right)^2 (4x^2 - 8x) + \left( \frac{s}{u} \right) (x - 2x^2) \\
+ (2x^2 - 4x - 4) + \left( \frac{u}{s} \right) (4 - x) \right], \quad (11) \]

where \( t = Q^2 \), \( u = K^2 \). The cross section has a smooth massless limit \((x = 0)\) which coincides with that obtained from starting with massless gravitons.

With the substitutions \((t, u) = -\frac{1}{2}s(1 - x)(1 \mp \cos \theta)\), and the imposition of a collinear cutoff \(-1 + \delta \leq \cos \theta \leq 1 - \delta\), the total polarization cross section is obtained by integrating (14) over \( \cos \theta \):

\[ \Delta \sigma_{\gamma \ell \rightarrow G\ell} (s, m^2) = \frac{1}{16} \frac{\alpha}{M_D^2} \left[ \log \left( \frac{x}{x - 1} \right) (14x - 4x^2) - (12 - 9x - 6x^2 + 3x^3) \right]. \quad (12) \]

The contribution to \( \Delta \sigma_{\gamma \ell \rightarrow G\ell} (s) \) from an entire KK tower of gravitons is given (approximately) by integrating over the density of states [10, 11, 12] up to the kinematic limit

\[ \Delta \sigma_{\gamma \ell \rightarrow G\ell} (s) = \frac{2\pi^{n/2}}{\Gamma(n/2)} \int_0^{\sqrt{s}} \Delta \sigma_{\gamma \ell \rightarrow G\ell} (s, m^2) m^{n-1} \, dm \\
= \frac{\pi^{n/2}}{\Gamma(n/2)} \frac{\bar{M}_P^2}{M_D^2} \left( \frac{s}{M_D^2} \right)^{n/2} \int_0^1 \Delta \sigma_{\gamma \ell \rightarrow G\ell} (x) x^{n/2-1} \, dx \\
= \frac{1}{16} \frac{\alpha}{M_D^2} \cdot \left( \frac{s}{M_D^2} \right)^{n/2} A_n, \quad (13) \]
where, e.g.,

\[ A_2 = \pi \left( -25/4 + (17/3) \log(2/\delta - 1) \right) \]
\[ A_4 = \pi^2 \left( -21/10 + (11/3) \log(2/\delta - 1) \right) \]

Finally, the contribution of (13), integrated up to some upper limit \( \bar{s} \leq M_D^2 \) to the RHS of the sum rule (4) is

\[ \frac{m_{\ell}^2}{2\pi^2 \alpha} \int_{s_{th}}^{\bar{s}} \frac{ds}{s} \Delta \sigma_{\gamma \ell \rightarrow G\ell}(s) = \frac{1}{16\pi^2} \left( \frac{m_{\ell}^2}{M_D^2} \right) \left( \frac{A_n}{n} \right) \kappa^{n/2}, \] (14)

where \( \kappa \equiv \bar{s}/M_D^2 \).

On the LHS of (4), \( a_{\ell}^{QED} \sim O(\alpha) \) and (using the same cutoff at \( \bar{s} \)) \( a_{\ell}^{QG} \sim \kappa^{n/2}O(m_{\ell}^2/M_D^2) \) [2]; thus (14) by itself violates the DHG sum rule at the one-loop level. Before discussing this result, I will present the analogous result for the case with no extra dimensions.

\( n = 0 \)

As mentioned after Eq. (11), the \( D = 4 \) case is obtained by setting \( x = 0 \) in that equation, or in Eq. (12):

\[ \Delta \sigma_{\gamma \ell \rightarrow G\ell}(s) = \Delta \sigma_{\gamma \ell \rightarrow G\ell}(s, 0) = -\frac{3}{4} \frac{\alpha}{M_P^2} = -\frac{6\pi \alpha}{M_P^2}. \] (15)

Then, integration up to \( \bar{s} \leq M_P^2 \) gives a contribution to the RHS of the sum rule

\[ \frac{m_{\ell}^2}{2\pi^2 \alpha} \int_{s_{th}}^{\bar{s}} \frac{ds}{s} \Delta \sigma_{\gamma \ell \rightarrow G\ell}(s) = -\frac{3}{\pi} \frac{m_{\ell}^2}{M_P^2} \cdot \log \left( \frac{\bar{s}}{s_{th}} \right). \] (16)

Again, since \( a_{\ell}^{QED} \sim O(\alpha) \) and \( a_{\ell}^{QG} \sim O(m_{\ell}^2/M_P^2) \) [2], this contribution by itself violates the sum rule (4).

I now turn to discuss these results.

3 Discussion

The failure of the gravitational contributions integrated to a large scale \( \bar{s} \) to satisfy the DHG sum rule at the one-loop level suggests at least two possibilities: (1) the sum rule is not valid for processes involving quantum gravity (2) there are contributions from \( s > \bar{s} \) which cancel the low energy contribution and render the sum rule valid.

As mentioned in the introduction, and discussed at length in Ref. [7], the sum rule can fail because the spin-difference forward Compton amplitude \( f_2(s) \) has fixed poles
(i.e., a polynomial piece to the real part), or an imaginary part whose asymptotic behavior requires a subtraction for the dispersion relation. One or both of these is certainly possible: for example, in the $n = 0$ case, there may be at order $\alpha / M_P^2$ a gravity-induced “seagull” term in $f_2$. It is not immediately apparent, however, how this can cancel against the logarithmic cutoff in (16). As far as the imaginary part is concerned, one may certainly conceive of spin-dependent gravitational contributions (e.g., spinning black holes) whose contributions vitiate the sum rule. In such cases, within the present state of knowledge about non-perturbative quantum gravity, there is not much more to say. It is then interesting to speculate about the second possibility above, the possibility of cancellation.

### Possible Role of String Theory

String theory suggests that $f_2$ Reggeizes for $s$ larger than the string scale $M_s^2$, at once eliminating the possibility of the fixed poles and the non-convergence. Moreover, the required Reggeization takes place at the tree-level (the Veneziano amplitude being equivalent to a sum of poles), so that according to the loop-counting criteria discussed in [8], the additive form of (1) is valid. I will discuss in turn the two cases of large extra dimensions ($n > 0$) and no large extra dimensions ($n = 0$).

#### $n > 0$

In this case, I will adopt the string description presented in [13], with reference to previous work in [14] and [15], and detailed in [9, 16]. This is based on a type I theory of open and closed strings, with a $T$-duality transformation allowing a large radius in the $n$ extra dimensions, as well as a weak coupling description. Matter resides in open strings tied to D3-branes and $\lambda = 2\alpha$ serves as the string coupling expansion parameter [13]. The 4-point open string Compton amplitude is then written as in [17], with the appropriate kinematic factors assuring the behavior (for $s \gg M_s^2$)

$$\text{Im} f_2(s) \sim \alpha \cdot \text{const}, \quad \Delta \sigma' \sim \alpha / s$$

which allows convergence. The contribution to the DHG sum rule from the Regge region is

$$\frac{m_t^2}{2\pi^2 \alpha} \int_{s_\ell}^{s_\ell} \frac{ds}{s} \Delta \sigma'(s) \sim \left( \frac{m_t^2}{M_D^2} \right) \left( \frac{1}{\kappa} \right).$$

A necessary (but certainly not sufficient!) condition that (18) could cancel the lower energy integral (14) is that $\kappa \sim O(1)$, i.e., the perturbative treatment of $\gamma \ell \to G\ell$ is reliable for $\sqrt{s} \lesssim M_D$. Some discussion of this point is given in [10]. An important ingredient is that the D-brane surface tension, which controls the “soft” scale for
gravitons emitted transverse to the brane, is larger than the string scale \( \bar{s} \sim M_D^2 \) vs. the expected \( \bar{s} \sim M_s^2 \) is perhaps disconcerting, and some discussion will be presented in a following section. All of this is a long way from claiming that cancellation occurs: for example, the the collinear cutoff dependence (which occurs only in this extra-dimension case) in (13) has no counterpart in (18). One is assured by the Lee-Nauenberg theorem [18] that including all channels will regulate such singularities to measurable quantities, but how (and whether) this happens smoothly over the whole energy range in the present case certainly remains to be shown.

**\( n = 0 \):** I use the same model here as above, except with no large dimensions. In addition to \( \lambda = 2\alpha \), one also has (after the T-duality transformation on all six compact coordinates) [16]

\[
M_s = (1/\sqrt{2})\alpha M_P/r^3 ,
\]

where \( r \geq 1 \), the compactification radius in units of \( M_s^{-1} \), can be used to adjust \( M_s = M_{GUT} \). The Compton amplitude in the Regge region is the same as in (17), so that the contribution from \( s \geq \bar{s} \) to the DHG integral is given by

\[
\frac{m_t^2}{2\pi^2\alpha} \int_{\bar{s}}^\infty \frac{ds}{s} \Delta s'(s) \sim \frac{\lambda}{\alpha} \frac{m_t^2}{\bar{s}} \sim \left( \frac{m_t^2}{M_P^2} \right) \left( \frac{M_P^2}{\bar{s}} \right) .
\]  

(20)

In this case, cancellation of the lower energy contribution (16) is possible only for \( \bar{s} \sim M_P^2 \gg M_s^2/\alpha^2 \). Again, this is merely a necessary condition – there is no demonstration of cancellation, and problems attached to it as well: for example, the logarithmic cutoff factor in (16) does not appear in the purported cancelling term (20).

**\( M_D \) (or \( M_P \)) vs. \( M_s \)**

To pursue the possibility of cancellation, one must reconcile the appearance of \( \bar{s} = M_D^2(M_P^2) \) with the expected \( \bar{s} = M_s^2 \) as the cutoff. As discussed above, there may be plausible arguments why graviton emission into the bulk may be adequately described by perturbation theory for energies up to \( M_D \); however, it is difficult to understand why the open string contribution to Compton scattering would be delayed for energies above \( M_s \), until \( M_D \). It might be thought that the difference is not significant, since in the string theory model \( M_s \) and \( M_D \) are numerically close [13, 1]; however, this is not the case. The insertion of \( \bar{s} = M_s^2 \) as the cutoff would cause a mismatch of a factor
of $\alpha^2$ between the contributions below and above $\bar{s}$: in the case $n > 0$, this occurs when the relation $M_D \sim \alpha^{-2/(n+2)}M_s$ \cite{13, 9} (along with $\bar{s} = M_s^2$) is inserted into (14) and $\bar{s} = M_s^2$ in (18); for $n = 0$, one has a similar situation, with $M_P \sim M_s/\alpha$ in (16) and $\bar{s} = M_s^2$ in (20). There is only one parameter available to play with, namely the compactification radius $R_{6-n}$ of the $6-n$ remaining dimensions in the bulk. Can this be of a form such as to remove this parametric disparity between $M_D(M_P)$ and $M_s$?

Consider again the Type I$'$ theory \cite{13, 9, 16}, with the standard model fields residing in open strings tied to a D3-brane, and gravity propagating in the 10-dimensional bulk. Allowing (as before) $n$ of the compact dimensions to be large, of equivalent toroidal radius $R_n$, and $6-n$ to be small (of radius $R_{6-n} \sim M_s^{-1}$), one obtains the relation \cite{13, 9}

$$\left(\frac{M_D}{M_s}\right)^{n+2} = \frac{1}{4\pi \alpha^2} \left(\frac{R_{6-n} M_s}{6-n}\right)^{6-n}. \quad (21)$$

Thus, for $\alpha \sim 0.1$ it is possible to simultaneously have $R_{6-n} M_s > 1$ and $M_D \sim O(\alpha^0)M_s$ if

$$\left(\frac{R_{6-n} M_s}{6-n}\right)^{6-n} \gtrsim 100 \alpha^2 = 25 \lambda_{I'}^2. \quad (22)$$

where $\lambda_{I'}$ is the Type I$'$ string coupling constant. This is a statement that the compactification scale is parametrically tied to the dilaton expectation value \cite{19}. It is not meant to be a perturbative statement, but as a constraint on the non-perturbative minima in the $S$–$T$ modular space. Needless to say, this is very ad hoc – I only present it as a hypothetical way of reconciling the $M_s/M_D$ problem.

This scenario is more circumscribed when there are no large extra dimensions. In that case one has, in analogy to (21), $(M_P/M_s)^2 = (2/\alpha^2)(R_6 M_s)^6$ \cite{16}. However, now $M_s$ is identified with $M_{GUT}$, so that $M_P/M_s \simeq 500$. If this be due to modular geometry rather than a factor of $\alpha$, we must impose $(R_6 M_s)^6 \simeq 30,000 \lambda_{I'}^2$, on the compactification volume, or $R_6 M_s \simeq 6 \lambda_{I'}^{3/2}$, on the compactification radius.

4 Concluding Remarks

The finiteness of the one-loop gravitation contribution to the anomalous magnetic moment $a_\ell$ led naturally to the question of whether the DHG sum rule for $a_\ell^2$ is satisfied. The failure to satisfy the sum rule for perturbative contributions below an arbitrary scale $\bar{s}$ could be ascribed to the failure to understand quantum gravity in the strong coupling (high energy) region. Alternatively, there were presented some speculations
concerning possibly compensating contributions above the string scale. In both cases
\((n > 0 \text{ and } n = 0)\) this possibility was beset with uncertainty concerning various log-
arithmetic cutoff factors, not to speak of the absence of an exact calculation. In the
simplest (Regge) approximation, the possibility of a string ‘fix’ for the validity of the
sum rule imposed a necessary parametric condition: namely, that the ordinary tree
level perturbative contribution be included all the way to the respective Planck scales
\(M_D, M_P\) (rather than the expected string scale \(M_s\)), and that the string Compton
amplitude used thereafter. Identifying the string and Planck scales is possible through
a certain dependence of the compactification volume for the \(6 - n\) ‘small’ dimensions
on the dilaton expectation value. The simple analysis presented here, using only an
open string Compton amplitude, ignores specifically non-perturbative gravitational
contributions (such as black hole formation \([20]\)) which may set in at \(M_D (M_P)\). Per-
haps these do not contribute to the forward spin-difference amplitude \(f_2\) (the leading
graviton trajectory considered in \([20]\) does not contribute to \(f_2(0)\)); however, no such
statement is possible in the presence of spinning black holes. It is certainly an open
question whether these have been eliminated in the strong-weak duality transformation
particular to the model considered.

In sum, the principal finding is that the DHG sum rule is not satisfied at the one-
loop level. Although one may speculate along certain stringy fixes to this situation,
there is at present no compelling reason to adopt these in preference to simply pleading
ignorance about the convergence properties of \(f_2\) in the non-perturbative regime of
quantum gravity.

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