Nonlinear robust wheel slip rate tracking control for autonomous vehicle with actuator dynamics

Jiaxu Zhang1,2, Shiying Zhou2 and Jian Zhao1

Abstract
This article presents a novel nonlinear robust wheel slip rate tracking control strategy for autonomous vehicle with actuator dynamics. First, a simple yet effective wheel slip rate dynamic model with the lumped uncertainty is established as the basis of the nonlinear robust wheel slip rate tracking control strategy design. Second, a nonlinear robust wheel slip rate tracking control law with lumped uncertainty observer is derived via the Lyapunov-based method. The lumped uncertainty observer is used to estimate and compensate the lumped uncertainty of the system by combining the radial basis function neural network with the adaptive laws for the unknown optimal weight vector of the radial basis function neural network. Then, a novel tracking differentiator is designed to calculate the derivative of the desired wheel slip rate, which is an essential aspect of the proposed nonlinear robust wheel slip rate tracking control law. Finally, the performance of the proposed control strategy is verified based on straight line braking maneuvers with three typical signals.

Keywords
Wheel slip rate tracking control, actuator dynamics, nonlinear robust control, lumped uncertainty observer, tracking differentiator

Date received: 27 January 2020; accepted: 8 April 2020

Handling Editor: James Baldwin

Introduction
Nowadays, autonomous vehicle has become one of the most popular emerging areas, and it will have a far-reaching impact on human society.1 However, fast and stable wheel slip rate tracking control is the important basis for autonomous vehicle to realize the advanced self-driving functions of fully automatic parking control,2 adaptive cruise control,3 and autonomous emergency braking control.4 Therefore, fast and stable wheel slip rate tracking control has been attracted widespread attention by many scholars and automobile manufacturers.

In the early days, the main objective of the wheel slip rate tracking control is to improve the braking stability and minimize the braking distance of the vehicle, and research results adopt mostly logic switching rule to make the wheel slip rate steady in the near desired value corresponding to the maximum value of the friction coefficient between tire and the road surface. Kuo and Yeh5 presented a four-phase anti-lock brake algorithm to accommodate all road conditions, and the proposed algorithm adopted the elapsed time interval and the angular acceleration of the wheel to control the switching of the four modes of the brake actuator, which consisted of the pressure increasing mode, the high-pressure holding mode, the pressure decreasing mode, and the low-pressure holding mode. Fu et al.6 adopted semi-analytical and semi-numerical methods to establish a set of switching rules to make the wheel slip rate tracking control for autonomous vehicle with actuator dynamics.
slip rate steady in the near desired value corresponding to the maximum value of the friction coefficient between tire and the road surface. Kiencke and Nielsen\textsuperscript{7} designed the logic switching rules based on wheel deceleration thresholds to keep the wheel slip rate in the neighborhood of the optimal point. Ait-Hammouda and Pasillas-Lepine\textsuperscript{8} presented an 11-phase anti-lock brake strategy based on wheel deceleration thresholds, and the proposed strategy could accommodate discontinuous road transitions. Pasillas-Lepine\textsuperscript{9} presented a class of five-phase anti-lock brake algorithms based on wheel deceleration logic-based switching and gave the existence and stability conditions of the most efficient limit cycle based on Poincaré maps. Tanelli et al.\textsuperscript{10} proposed a hybrid anti-lock braking system (ABS) control approach to establish a limit cycle around the wheel slip rate corresponding to the maximum value of the friction coefficient between tire and the road surface and proved that the limit cycle had asymptotic stability and structural stability with respect to different road conditions and the actuator rate limit based on Poincaré map. Jing et al.\textsuperscript{11} proposed a switched control approach for ABS based on the Filippov framework. First, the control actions of the brake actuator with on/off valves were divided into increase action, hold action, and decrease action. Then, the switching surfaces were designed to switch the three control actions of the brake actuator with on/off valves to make the wheel slip rate converge to the desired equilibrium set. The above logic switching rule–based wheel slip rate tracking control methods are difficult to achieve high-precision tracking control of any desired wheel slip rate and could not satisfy the requirements of autonomous vehicle for wheel slip rate tracking control.

Compared with the logic switching rule–based wheel slip rate tracking control methods, the dynamic model–based wheel slip rate tracking control methods are easier to achieve high-precision tracking control of any desired wheel slip rate.\textsuperscript{12} Buckholtz\textsuperscript{13} proposed a sliding mode controller with continuous saturation function for wheel slip tracking, and the continuous saturation function was used to reduce the chattering phenomenon of the conventional sliding mode control method at the expense of the robustness and tracking accuracy of the conventional sliding mode control method. Johansen et al.\textsuperscript{14} developed a wheel slip rate tracking control method for the vehicle equipped with electromechanical brake actuator and a brake-by-wire system. First, a family of linearization models was derived from the quarter-vehicle model to use as the nominal model. Then, the gain-scheduled linear quadratic regulator (LQR) approach was adopted to design the wheel slip rate tracking controller based on the nominal model. Park and Lim\textsuperscript{15} proposed an adaptive full state feedback wheel slip rate tracking control method based on the quarter-vehicle model with time delay input, and the unmeasurable vehicle speed is estimated by the nonlinear sliding observer. Harifi et al.\textsuperscript{16} designed a sliding mode controller with integral switching surface for wheel slip rate control based on a two-axle vehicle model, and the integral switching surface could effectively reduce chattering phenomenon in sliding mode control. Corno et al.\textsuperscript{17} established a family of linear models as nominal model by linearizing the wheel slip dynamic model at different vehicle velocities and wheel slip rates and adopted the linear parameter-varying control design technique to develop a fully scheduled wheel slip rate tracking controller scheduled on the vehicle velocity. Amodeo et al.\textsuperscript{18} proposed a nonlinear robust wheel slip rate tracking controller based on the second-order sliding mode control method, which could eliminate the chattering phenomenon of the conventional sliding mode control method. Meanwhile, the sliding mode observer was designed to estimate the tire–road friction coefficient to make the proposed controller accommodate all road conditions. Pasillas-Lépine et al.\textsuperscript{19} presented a continuous wheel slip control approach based on wheel slip rate and wheel acceleration measurements and adopted the Lyapunov stability theory to prove that the state trajectory of the closed-loop system could asymptotically converge to the wheel slip rate around any prescribed setpoint, both in the stable and unstable regions of the tire. Mirzaei and Mirzaeinejad\textsuperscript{20} presented a nonlinear robust wheel slip rate optimal predictive tracking control approach based on the quarter-vehicle model with the Dugoff’s tire model and adopted the Lyapunov stability theory to prove that the proposed approach had strong robustness against modeling uncertainties. Hsu and Kuo\textsuperscript{21} proposed an adaptive exponential-reaching sliding mode wheel slip rate tracking control system by combining an equivalent controller with an exponential compensator. The functional-linked wavelet neural network was used as the equivalent controller to online approximate the system uncertainties and the exponential-reaching sliding mode control method was to design the exponential compensator, which could eliminate the effect of the approximation error caused by the equivalent controller. Mirzaeinejad\textsuperscript{22} presented a robust prediction–based wheel slip rate tracking controller based on nonlinear predictive method and radial basis function neural network (RBFNN). The nonlinear predictive method was used to design an optimal control law, and the RBFNN was used to improve the robustness of the system by estimating the unknown uncertainties of the system. Zhang and Li\textsuperscript{23} proposed an adaptive backstepping sliding mode control approach with RBFNN to
design the wheel slip rate tracking controller, and the RBFNN was used as the uncertainty observer to estimate and compensate the lumped uncertainty of the system. He et al.\textsuperscript{24} presented a robust adaptive wheel slip rate tracking controller based on a quarter-vehicle model and barrier Lyapunov function. Mirzaeinejad\textsuperscript{22} proposed a novel nonlinear robust wheel slip rate tracking controller with RBFNN-based observer, which was used to online estimate and compensate the system uncertainty. Zhang and Li\textsuperscript{25} proposed a new robust backstepping sliding mode wheel slip rate tracking controller based on the $L_2$-gain stability theory, and the simulation results showed that the proposed controller had strong robustness against the system uncertainty and external disturbance. The above dynamic model–based wheel slip rate tracking control methods usually depend on simplified dynamic models to design the controllers, and overly complicated dynamic models will result in the designed controllers with high computational complexity. However, the simplified dynamic models cannot accurately characterize the complex nonlinearity and uncertainty of wheel dynamics. Therefore, the research on nonlinear robust tracking control method for any desired wheel slip rate has profound theoretical and practical value.

In this article, a novel nonlinear robust wheel slip rate tracking control strategy for autonomous vehicle with actuator dynamics is proposed. First, a simple yet effective wheel slip rate dynamic model with the lumped uncertainty is deduced from the quarter-vehicle model with actuator dynamics to use as the basis of the nonlinear robust wheel slip rate tracking control strategy design. Second, a nonlinear robust wheel slip rate tracking control law with lumped uncertainty observer is derived via the Lyapunov-based method; section “Simulation results” validates the performance of the proposed control strategy via the straight line braking maneuvers and finally, section “Conclusion” gives the main conclusion of our work.

The dynamic model

In this section, the quarter-vehicle model with the Burckhardt tire model is established to describe the braking dynamics by ignoring the suspension dynamics, the tire relaxation dynamics, the tire cornering, and camber characteristics. As shown in Figure 1, we choose the wheel angular speed $\omega$ and the vehicle speed $v$ as the system state variables, the dynamic equations of the quarter-vehicle model are given by\textsuperscript{26,27}

$$
\begin{align*}
 J\ddot{\omega} &= rF_x - T_b \\
 mv' &= -F_x
\end{align*}
$$

(1)

where $J$, $r$, $m$, and $T_b$ are the wheel inertia, the effective rolling wheel radius, the mass of the quarter vehicle, and the brake torque, respectively; and $F_x$ is the longitudinal friction force, which is given by

$$
 F_x = F_z \mu(\lambda)
$$

(2)

where $F_z$ is the vertical force at the tire–road contact point; $\mu(\lambda)$ is the longitudinal friction coefficient, and the expression of which in the Burckhardt tire model has the form\textsuperscript{28}

$$
 \mu(\lambda) = \vartheta_1 (1 - e^{-\lambda \vartheta_2}) - \lambda \vartheta_3
$$

(3)

where $\vartheta_1$, $\vartheta_2$, and $\vartheta_3$ are the parameters of Burckhardt tire model, and many different tire–road friction conditions can be modeled by adjusting the three
parameters; \( \dot{\lambda} \) is the wheel slip rate, which is defined by in braking condition

\[
\dot{\lambda} = \frac{v - \omega r}{v}
\]  

(4)

The first derivative of equation (4) with respect to time is given by

\[
\ddot{\lambda} = \frac{1}{v} (1 - \dot{\lambda}) \dot{v} - \dot{\omega} r
\]  

(5)

Substituting equation (5) into equation (1), the state variable \( \omega \) is replaced by the wheel slip rate, and the dynamic equations of the quarter-vehicle model described by equation (1) are modified as

\[
\begin{cases}
\dot{\lambda} = -\frac{1}{v} \left( \frac{1 - \dot{\lambda}}{m} + \frac{r^2}{J} \right) F_{z\mu}(\lambda) + \frac{r}{Jv} T_b \\
m\dot{v} = -F_x
\end{cases}
\]  

(6)

Since the vehicle inertia is far larger than the wheel inertia, the vehicle speed can be regarded as a slowly varying parameter relative to the wheel slip rate. Therefore, the dynamic equations of the quarter-vehicle model described by equation (6) can be simplified as

\[
\dot{\lambda} = -\frac{1}{v} \left( \frac{1 - \dot{\lambda}}{m} + \frac{r^2}{J} \right) F_{z\mu}(\lambda) + \frac{r}{Jv} T_b
\]  

(7)

Aiming at electromechanical brake actuator for autonomous vehicle, the brake actuator dynamics can be described as a first-order inertial link

\[
T_b = \frac{1}{\tau_b s + 1} u
\]  

(8)

where \( \tau_b \) is the dimensionless time constant; \( s \) is the Laplace transform variable; and \( u \) is the actual control input.

According to the Laplace inverse transform, equation (8) can be transformed to the following form

\[
\tilde{T}_b = -\frac{1}{\tau_b} \tilde{T}_b + \frac{1}{\tau_b} u
\]  

(9)

Let \( x_1 = \dot{\lambda} \) and \( x_2 = T_b \) denote the system state variables, and let \( d = [d_1 \quad d_2]^T \) denotes the lumped uncertainty of the system caused by ignoring the suspension dynamics, the tire relaxation dynamics, the tire cornering, and camber characteristics. According to equations (7) and (9), the simple yet effective wheel slip rate dynamic model with the lumped uncertainty is given by

\[
\begin{cases}
\dot{x}_1 = f(x_1) + Gx_2 + d_1 \\
\dot{x}_2 = -\frac{1}{\tau_b} (x_2 - u) + d_2
\end{cases}
\]  

(10)

where

\[
f(x_1) = -\frac{1}{v} \left( \frac{1 - x_1}{m} + \frac{r^2}{J} \right) F_{z\mu}(x_1) \quad \text{and} \quad G = \frac{r}{Jv}
\]

The proposed control strategy

Control law design

In this section, the RBFNN is used as the lumped uncertainty observer to suppress the effect of the lumped uncertainty described by equation (10) on system performance since it can approximate and compensate any nonlinear function with arbitrary accuracy.\(^{30}\) Let \( \phi(x) = [\phi_1(x), \phi_2(x), \ldots, \phi_l(x)]^T \) denotes the basis function vector of RBFNN, and the basis function is given by

\[
\phi_i(x) = \exp \left( -\frac{\|x - a_i\|^2}{2b_i^2} \right) \quad i = 1, \ldots, l
\]  

(11)

where \( a_i \) and \( b_i \) are the central vector and the width of the basis function, respectively; and \( l \) is the number of the basis functions.

According to the basis function vector of RBFNN, the lumped uncertainty of the system is given by

\[
d_j = \hat{d}_j(x, W^*_j) + e_j = (W^*_j)^T \phi(x) + e_j \quad j = 1, 2
\]  

(12)

where \( e_j \) is the approximation error satisfying \( |e_j| \leq \tilde{e} \); and \( W^*_j \) is the unknown optimal weight vector of RBFNN, which is given by

\[
W^*_j = \arg \min_{W_j \in \{W_j \leq M\}} \left\{ \sup_{x \in M_x} \{ |d - \hat{d}(x, \tilde{W}_j)| \} \right\} \quad j = 1, 2
\]  

(13)

where \( \tilde{W}_j \) is the estimate of the unknown optimal weight vector of RBFNN; \( M \) is the positive design parameter; and \( M_x \) is the compact set of the input vector of RBFNN.

Supposing that \( \lambda_d \) denotes the desired wheel slip rate, the equilibrium point of the system is shifted to the origin via the change of the system states described
by equation (10). Taking into account that the equilibrium point is at the origin, we can get Theorem 1 based on the control objective that the closed-loop system can quickly and accurately track the desired wheel slip rate under the condition of the unknown optimal weight vectors of RBFNN

$$\begin{align*}
\dot{z}_1 &= x_1 - \dot{\lambda}_d \\
\dot{z}_2 &= f(x_1) + Gx_2 + \kappa_1 z_1
\end{align*}$$  \tag{14}

where $\kappa_1$ is the positive design parameter.

**Theorem 1.** Consider the system with actuator dynamics described by equation (10), the following nonlinear robust wheel slip rate tracking control law is designed to construct the closed-loop system

$$u = x_2$$

where $\kappa_2$ is the positive design parameter; $\text{sgn}(\cdot)$ is the sign function. The estimates of the unknown optimal weight vectors are updated by the following modified adaptive laws

$$\begin{align*}
\dot{\tilde{W}}_1 &= \Gamma_1(|z_1| + |z_2|)\phi(x)\left(\text{sgn}(z_1) + \kappa_1 \text{sgn}(z_2)\right) - \sigma_1(\tilde{W}_1 - \tilde{W}_1^0) \\
\dot{\tilde{W}}_2 &= \Gamma_2(|z_1| + |z_2|)G\phi(x)\text{sgn}(z_2) - \sigma_2(\tilde{W}_2 - \tilde{W}_2^0)
\end{align*}$$  \tag{16}

where $\Gamma_1$ and $\Gamma_2$ are the positive definite matrices; $\sigma_1$ and $\sigma_2$ are the positive design parameters; $\tilde{W}_1^0$ and $\tilde{W}_2^0$ are the initial estimates of the unknown optimal weight vectors. If the design parameter $\kappa_2$ satisfies $\kappa_2 > 1/2$, then all signals of the closed-loop system are uniformly ultimately bounded, and the wheel slip rate tracking error satisfies

$$\lim_{t \to \infty} \sup(|z_1|) \leq \sqrt{\frac{1}{\eta_1} (\kappa_1 + G + 1) \dot{z}^2 + \sigma_1(\tilde{W}_1^0)^T \Gamma_1^{-1} \tilde{W}_1^0 + \sigma_2(\tilde{W}_2^0)^T \Gamma_2^{-1} \tilde{W}_2^0)}$$  \tag{18}

where $\tilde{W}_1^0 = W_1^0 - \tilde{W}_1^0$ and $\tilde{W}_2^0 = W_2^0 - \tilde{W}_2^0$ are the initial estimation errors of the unknown optimal weight vectors.

**Proof.** Defining the Lyapunov function as

$$V_1 = \frac{1}{2} (|z_1| + |z_2|)^2 + \frac{1}{2} \tilde{W}_1^T \Gamma_1^{-1} \tilde{W}_1 + \frac{1}{2} \tilde{W}_2^T \Gamma_2^{-1} \tilde{W}_2$$  \tag{19}

where $\tilde{W}_1 = W_1^0 - \tilde{W}_1$ and $\tilde{W}_2 = W_2^0 - \tilde{W}_2$ are the estimation errors of the unknown optimal weight vectors.

The first derivative of the Lyapunov function along the trajectory of the system is given by

$$\dot{V}_1 = (|z_1| + |z_2|) \left( \dot{z}_1 \text{sgn}(z_1) + \dot{z}_2 \text{sgn}(z_2) \right) - \tilde{W}_1^T \Gamma_1^{-1} \dot{\tilde{W}}_1 - \tilde{W}_2^T \Gamma_2^{-1} \dot{\tilde{W}}_2$$  \tag{20}

Substituting equation (10) into equation (20) yields

$$\dot{V}_1 = (|z_1| + |z_2|) \left( f(x_1) + G(z_2 - \kappa_1 z_1) + d_1 - \dot{\lambda}_d \right) \text{sgn}(z_1) + \left( g(x_1) + G \left( -\frac{1}{\tau_0} (x_2 - u) + d_2 \right) + \kappa_1 \left( f(x_1) + G(z_2 - \kappa_1 z_1) + d_1 - \dot{\lambda}_d \right) \right) \text{sgn}(z_2) - \tilde{W}_1^T \Gamma_1^{-1} \dot{\tilde{W}}_1 - \tilde{W}_2^T \Gamma_2^{-1} \dot{\tilde{W}}_2$$  \tag{21}

where

$$g(x_1) = f'(x_1) = \frac{1}{v} \left( \frac{2 F_{\text{C}} \mu(x_1)}{m} \dot{x}_1 - \left( \frac{1 - x_1}{m} + \frac{\phi^2}{J} \right) F_{\text{C}} \mu(x_1) \right)$$

Substituting equation (12) into equation (21) yields

$$\dot{V}_1 = (|z_1| + |z_2|) \left( f(x_1) + G(z_2 - \kappa_1 z_1) + (W_1^0)^T \phi(x) + e_1 - \dot{\lambda}_d \right) \text{sgn}(z_1) + \left( g(x_1) + G \left( -\frac{1}{\tau_0} (x_2 - u) + (W_2^0)^T \phi(x) + e_2 \right) + \kappa_1 \left( f(x_1) + G(z_2 - \kappa_1 z_1) + (W_1^0)^T \phi(x) + e_1 - \dot{\lambda}_d \right) \right) \text{sgn}(z_2) - \tilde{W}_1^T \Gamma_1^{-1} \dot{\tilde{W}}_1 - \tilde{W}_2^T \Gamma_2^{-1} \dot{\tilde{W}}_2$$  \tag{22}

Substituting the nonlinear robust wheel slip rate tracking control law described by equation (15) into equation (22) yields
\[
\dot{V}_1 = (|z_1| + |z_2|)
\]
\[
\left(-\kappa_2(|z_1| + |z_2|) + \left(\hat{W}_1^T \phi(x) + \varepsilon_1\right)\text{sgn}(z_1)
\right)
\]
\[
+ \left(G(\hat{W}_2^T \phi(x) + \varepsilon_2) + \kappa_1(\hat{W}_1^T \phi(x) + \varepsilon_1)\right)\text{sgn}(z_2)
\]
\[
- \hat{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1 - \hat{W}_2^T \Gamma_2^{-1} \dot{\hat{W}}_2
\]
(23)

Substituting the modified adaptive laws for the unknown optimal weight vectors into equation (23) yields
\[
\dot{V}_1 = -\kappa_2(|z_1| + |z_2|)^2 + (|z_1| + |z_2|)
\]
\[
\left(\text{sgn}(z_1) + \kappa_1 \text{sgn}(z_2)\right)\varepsilon_1 + G\text{sgn}(z_2)\varepsilon_2
\]
\[
+ \sigma_1 \hat{W}_1^T \Gamma_1^{-1} (\dot{\hat{W}}_1 - \dot{W}_1^0)
\]
\[
+ \sigma_2 \hat{W}_2^T \Gamma_2^{-1} (\dot{\hat{W}}_2 - \dot{W}_2^0)
\]
(24)

Substituting the constraint of the approximation error that \(|\varepsilon_0| \leq \bar{\varepsilon}\) into equation (24) yields
\[
\dot{V}_1 \leq -\kappa_2(|z_1| + |z_2|)^2 + (|z_1| + |z_2|)(\kappa_1 + G + 1)\bar{\varepsilon}
\]
\[
+ \sigma_1 \hat{W}_1^T \Gamma_1^{-1} (\dot{\hat{W}}_1 - \dot{W}_1^0)
\]
\[
+ \sigma_2 \hat{W}_2^T \Gamma_2^{-1} (\dot{\hat{W}}_2 - \dot{W}_2^0)
\]
\[
\leq -\kappa_2(|z_1| + |z_2|)^2
\]
\[
+ (|z_1| + |z_2|)(\kappa_1 + G + 1)\bar{\varepsilon}
\]
\[
- \sigma_1 \hat{W}_1^T \Gamma_1^{-1} \dot{\hat{W}}_1 + \sigma_1 \hat{W}_1^T \Gamma_1^{-1} \dot{W}_1^0
\]
\[
- \sigma_2 \hat{W}_2^T \Gamma_2^{-1} \dot{\hat{W}}_2 + \sigma_2 \hat{W}_2^T \Gamma_2^{-1} \dot{W}_2^0
\]
(25)

According to Young’s inequality, we can get
\[
(|z_1| + |z_2|)(\kappa_1 + G + 1)\bar{\varepsilon} \leq \frac{1}{2}(|z_1| + |z_2|)^2
\]
\[
+ \frac{1}{2} (\kappa_1 + G + 1)^2 \bar{\varepsilon}^2
\]
(26)

\[
\sigma_1 \hat{W}_1^T \Gamma_1^{-1} \dot{W}_1^0 \leq \frac{\sigma_1}{2} \hat{W}_1^T \Gamma_1^{-1} \hat{W}_1 + \frac{\sigma_1}{2} (\hat{W}_1^0)^T \Gamma_1^{-1} \hat{W}_1^0
\]
(27)

\[
\sigma_2 \hat{W}_2^T \Gamma_2^{-1} \dot{W}_2^0 \leq \frac{\sigma_2}{2} \hat{W}_2^T \Gamma_2^{-1} \hat{W}_2 + \frac{\sigma_2}{2} (\hat{W}_2^0)^T \Gamma_2^{-1} \hat{W}_2^0
\]
(28)

Substituting inequalities (26)–(28) into inequality (25) yields
\[
\dot{V}_1 \leq -\frac{\kappa_2}{2} (|z_1| + |z_2|)^2
\]
\[
- \frac{\sigma_1}{2} \dot{W}_1^0 \Gamma_1^{-1} \dot{W}_1 - \frac{\sigma_2}{2} \dot{W}_2^0 \Gamma_2^{-1} \dot{W}_2
\]
\[
+ \frac{1}{2} (\kappa_1 + G + 1)^2 \bar{\varepsilon}^2 + \frac{\sigma_1}{2} (\hat{W}_1^0)^T \Gamma_1^{-1} \hat{W}_1^0
\]
\[
+ \frac{\sigma_2}{2} (\hat{W}_2^0)^T \Gamma_2^{-1} \hat{W}_2^0
\]
(29)

According to the constraints that \(\sigma_1 > 0, \sigma_2 > 0,\) and \(\kappa_2 > 1/2,\) we can define the positive constant as
\[
\eta_1 = \min\{2\kappa_2 - 1, \sigma_1, \sigma_2\}
\]
(30)

Substituting the positive constant defined by equation (30) into inequality (29) yields
\[
\dot{V}_1 \leq -\eta_1 \dot{V}_1 + \frac{1}{2} (\kappa_1 + G + 1)^2 \bar{\varepsilon}^2 + \frac{\sigma_1}{2} (\hat{W}_1^0)^T \Gamma_1^{-1} \hat{W}_1^0
\]
\[
+ \frac{\sigma_2}{2} (\hat{W}_2^0)^T \Gamma_2^{-1} \hat{W}_2^0
\]
(31)

According to comparison principle, we can get based on inequality (31)
\[
\dot{V}(t) = e^{-\eta_1 t} \dot{V}_1(0) + \frac{1}{2\eta_1} \left[ (\kappa_1 + G + 1)^2 \bar{\varepsilon}^2
\right.
\]
\[
+ \sigma_1 (\hat{W}_1^0)^T \Gamma_1^{-1} \hat{W}_1^0 + \sigma_2 (\hat{W}_2^0)^T \Gamma_2^{-1} \hat{W}_2^0
\]
(32)

where \(V(0)\) denotes the initial value of the Lyapunov function.

Therefore, all signals of the closed-loop system are uniformly ultimately bounded. Moreover, according to the definition of the Lyapunov function, the wheel slip rate tracking error satisfies
\[
\lim \sup_{t \to \infty} |z_1| \leq \frac{1}{\eta_1} \left[ (\kappa_1 + G + 1)^2 \bar{\varepsilon}^2 + \sigma_1 (\hat{W}_1^0)^T \Gamma_1^{-1} \hat{W}_1^0 + \sigma_2 (\hat{W}_2^0)^T \Gamma_2^{-1} \hat{W}_2^0 \right]
\]
(33)

According to inequality (33), reducing the initial estimation errors of the optimal weight vectors can reduce the upper bound of the wheel slip rate tracking error.

**Tracking differentiator design**

In the process of the above nonlinear robust wheel slip rate tracking control law design described by Theorem
1, the first derivative of the desired wheel slip rate is an essential aspect of the nonlinear robust wheel slip rate tracking control law. Usually, the first-order inertial link with small time constant is used to calculate the first derivative of the input signal, but it will lead to excessive noise in the first derivative of the input signal because of its inherent noise amplification effect. In order to avoid the excessive noise caused by the first-order inertial link with small time constant, a novel tracking differentiator described by Theorem 2 is designed in this section to calculate the first derivative of the desired wheel slip rate.

**Theorem 2.** Supposing that the second derivative of the desired wheel slip rate satisfies \(|\dot{\lambda}_d| \leq C\), the tracking differentiator is given by

\[
\begin{align*}
\dot{v}_1 &= -\gamma(v_1 - \lambda_d) + v_2 \\
\dot{v}_2 &= -\gamma(v_1 - \lambda_d) - (v_1 - \lambda_d)^p
\end{align*}
\]

(34)

where \(p > 2\) is the odd number; and \(\gamma > 2\) is the design parameter. If the positive design parameter \(\gamma \rightarrow +\infty\), then \(v_1 \rightarrow \lambda_d\), \(\dot{v}_1 \rightarrow \ddot{\lambda}_d\).

**Proof.** Shifting the equilibrium point of the system to the origin based on the following change of the system states

\[
\begin{align*}
y_1 &= v_1 - \lambda_d \\
y_2 &= \dot{y}_1 + y_1
\end{align*}
\]

(35)

According to equation (35), the system described by equation (34) is transformed into the following form

\[
\begin{align*}
\ddot{y}_1 &= y_2 - y_1 \\
\ddot{y}_2 &= -(\gamma - 1)y_2 - y_1 - y_1^p - \dddot{\lambda}_d
\end{align*}
\]

(36)

Defining the Lyapunov function as

\[
V_2 = \frac{1}{p+1}y_1^{p+1} + \frac{1}{2}y_2^2 + \frac{1}{2}\lambda_d^2
\]

(37)

The first derivative of the Lyapunov function along the trajectory of the system is given by

\[
\dot{V}_2 = y_1^p\dot{y}_1 + y_1\dot{y}_2 + y_2\ddot{y}_2
\]

(38)

Substituting equation (36) into equation (38), we can obtain

\[
\dot{V}_2 = y_1^p\dot{y}_1 + y_1(y_2 - y_1) + y_2\left(\dot{y}_2 - (\gamma - 1)y_2 - y_1 - y_1^p - \dddot{\lambda}_d\right)
\]

\[
= -y_1^{p+1} - y_1^2 - (\gamma - 1)y_2^2 - y_2\lambda_d
\]

(39)

According to Young’s inequality,\(^2\) we can get

\[
y_1^{p+1} \leq \gamma_\frac{1}{2}y_1^2 + \frac{1}{2}\gamma^2\lambda_d^2
\]

(40)

Substituting inequality (40) into equation (39) yields

\[
\dot{V}_2 \leq -\eta_2 y_2^2 - \left(\frac{\gamma}{2} - 1\right)y_2^2 - \frac{1}{2}\gamma^2\lambda_d^2
\]

(41)

Defining the positive constant as

\[
\eta_2 = \min\{p + 1, 2, \gamma - 2\}
\]

(42)

Substituting the positive constant defined by equation (42) into inequality (41) yields

\[
\dot{V}_2 \leq -\eta_2 y_2^2 + \frac{1}{2}\gamma^2\lambda_d^2
\]

(43)

According to comparison principle,\(^2\) we can get based on inequality (43)

\[
V_2(t) \leq e^{-\eta_2 t}V_2(0) + \frac{C}{2\eta_2}\lambda_d^2
\]

(44)

where \(V_2(0)\) denotes the initial value of the Lyapunov function.

Substituting assumption that \(|\dot{\lambda}_d| \leq C\) into inequality (44) yields

\[
V_2(t) \leq e^{-\eta_2 t}V_2(0) + \frac{C}{2\eta_2}
\]

(45)

According to inequality (45), we can get

\[
\lim_{t \to \infty} \sup |y_1| \leq \min\left\{p + 1, \sqrt{\frac{(p + 1)C}{2\eta_2}}, \sqrt{\frac{C}{\eta_2}}\right\}
\]

(46)

\[
\lim_{t \to \infty} \sup |y_2| \leq \sqrt{\frac{C}{\eta_2}}
\]

(47)

Therefore, when \(\gamma \rightarrow +\infty\), we can obtain \(y_1 \rightarrow 0\) and \(y_2 \rightarrow 0\). Moreover, we can deduce that \(v_1 \rightarrow \lambda_d\) and \(\dot{v}_1 \rightarrow \ddot{\lambda}_d\) based on equation (35).

According to Theorem 2, the nonlinear robust wheel slip rate tracking control law described by equation (15) can be rewritten as
Table 1. The parameters of the vehicle and the proposed control strategy.

| Meaning                                                                 | Symbol | Value          |
|-------------------------------------------------------------------------|--------|----------------|
| The mass of the quarter vehicle                                         | \(m\)  | 354.00 (kg)    |
| The wheel inertia                                                       | \(J\)  | 0.90 (kg m\(^2\)) |
| The effective rolling wheel radius                                      | \(r\)  | 0.31 (m)       |
| Time constant                                                           | \(\tau_b\) | 0.01 (s)    |
| The number of the basis functions                                       | \(l\)  | 20             |
| The central vector of the basis function                                | \(a_i\) | [0, 1, 0, 3000] |
| The width of the basis function                                         | \(b_i\) | 1025           |
| The positive design parameters                                          | \(\kappa_1, \kappa_2\) | 350, 20       |
| The positive definite matrixes                                          | \(\Gamma_1, \Gamma_2\) | 10 \times E\(_{20}\) |
| The positive design parameters                                          | \(\sigma_1, \sigma_2\) | 0.01          |
| The positive design parameter                                           | \(\gamma\) | 80            |
| The positive design parameter                                           | \(p\)  | 3              |

Figure 2. The simulation results of straight line braking maneuver with desired step signal: (a) the desired wheel slip rate and its estimated value, (b) the derivative of the desired wheel slip rate, (c) the vehicle and wheel speeds, (d) the desired wheel slip rate and its actual value, (e) the wheel slip rate tracking error, (f) the brake torque, (g) the relationship between wheel slip rate and longitudinal friction coefficient, (h) the uncertainty \(d_1\) of front left wheel, (i) the uncertainty \(d_2\) of front left wheel, (j) the uncertainty \(d_1\) of rear right wheel, and (k) the uncertainty \(d_2\) of rear right wheel.
In this section, the straight line braking maneuvers on a flat road with desired step signal, desired sinusoidal signal, and desired ramp signal are carried out based on vehicle dynamics simulation software (MSC CarSim®) to validate the performance of the proposed control strategy. In order to investigate the effect of load transfer during braking on the performance of the proposed control strategy, we show the simulation results of front left wheel and rear right wheel. Let $E_{20\times 20}$ denotes $20 \times 20$ identity matrix, and the parameters of the vehicle and the proposed control strategy are listed in Table 1.

**Simulation results**

In this section, the straight line braking maneuvers on a flat road with desired step signal, desired sinusoidal signal, and desired ramp signal are carried out based on vehicle dynamics simulation software (MSC CarSim®) to validate the performance of the proposed control strategy. In order to investigate the effect of load transfer during braking on the performance of the proposed control strategy, we show the simulation results of front left wheel and rear right wheel. Let $E_{20\times 20}$ denotes $20 \times 20$ identity matrix, and the parameters of the vehicle and the proposed control strategy are listed in Table 1.

**Straight line braking maneuver with desired step signal**

The straight line braking maneuver with desired step signal is carried out on a flat ice-snow road to test the steady and dynamic characteristics of the wheel slip rate tracking closed-loop system. In the straight line
braking maneuver with desired step signal, the initial vehicle speed and the gear position of the transmission are set to 33.33 m/s and neutral. Meanwhile, the final value of the desired step signal is set to 0.09. Figure 2 (a)–(k) shows the desired wheel slip rate and its estimated value, the derivative of the desired wheel slip rate, the vehicle and wheel speeds, the desired wheel slip rate and its actual value, the wheel slip rate tracking error, the brake torque, the relationship between wheel slip rate and longitudinal friction coefficient, the uncertainty $d_1$ of front left wheel, the uncertainty $d_2$ of front left wheel, the uncertainty $d_1$ of rear right wheel, and the uncertainty $d_2$ of rear right wheel. As shown in Figure 2(a)–(b), the proposed tracking differentiator can effectively smooth the desired wheel slip rate and calculate the first derivative of the desired wheel slip rate without excessive noise. As shown in Figure 2(c)–(f), both the front left and rear right wheel slip rates can quickly and accurately track the desired value under the condition that the rear wheel vertical load is transferred to front wheel. Therefore, the proposed control strategy has strong robustness against the suspension dynamics and the tire relaxation dynamics. As shown in Figure 2(h)–(k), the lumped uncertainty observer can accurately estimate the lumped uncertainty of the system.
As shown in Figure 2(g), the longitudinal friction coefficients of front left wheel and rear right wheel can change with their wheel slip rates, and finally stabilize in the near desired value corresponding to the final value of the desired step signal. Therefore, the proposed control strategy can effectively improve the braking smoothness.

**Straight line braking maneuver with desired sinusoidal signal**

The straight line braking maneuver with desired sinusoidal signal is carried out on a flat wet asphalt road to test the delay characteristic of the wheel slip rate tracking closed-loop system. In the straight line braking maneuver with desired sinusoidal signal, the initial vehicle speed and the gear position of the transmission are set to 33.33 m/s and neutral. Meanwhile, the amplitude, bias, and frequency of the desired sinusoidal signal are set to 0.04, 0.05, and 9.42 rad/s, respectively. Figure 3(a)–(k) shows the desired wheel slip rate and its estimated value, the derivative of the desired wheel slip rate, the vehicle and wheel speeds, the wheel slip rate tracking error, the brake torque, the relationship between wheel slip rate and longitudinal friction coefficient, and the uncertainties of front left wheel and rear right wheel, respectively. As shown in Figure 3(a)–(b), the proposed tracking differentiator can effectively smooth the desired wheel slip rate and calculate the first derivative of the desired wheel slip rate without excessive noise. As shown in Figure 3(c)–(e), both the front left and rear right wheel slip...
rates can quickly and accurately track the desired value, and the front left wheel slip rate has higher tracking accuracy than the rear right wheel slip rate. As shown in Figure 3(h)–(k), the lumped uncertainty observer can accurately estimate the lumped uncertainty of the system. As shown in Figure 3(f)–(g), the brake torques and the longitudinal friction coefficients of front left wheel and rear right wheel can change periodically with their wheel slip rates, and the brake torque of front left wheel is much more larger than that of rear right wheel because of the effect of load transfer. Therefore, the proposed control strategy has strong robustness against the uncertainty caused by the periodic fluctuation of the wheel slip rate.

Figure 4. The simulation results of straight line braking maneuver with desired ramp signal: (a) the desired wheel slip rate and its estimated value, (b) the derivative of the desired wheel slip rate, (c) the vehicle and wheel speeds, (d) the desired wheel slip rate and its actual value, (e) the wheel slip rate tracking error, (f) the brake torque, (g) the relationship between wheel slip rate and longitudinal friction coefficient, (h) the uncertainty $d_1$ of front left wheel, (i) the uncertainty $d_2$ of front left wheel, (j) the uncertainty $d_1$ of rear right wheel, and (k) the uncertainty $d_2$ of rear right wheel.

Straight line braking maneuver with desired ramp signal
The straight line braking maneuver with desired ramp signal is carried out on a flat dry asphalt road to test the tracking characteristic of the wheel slip rate tracking closed-loop system. In the straight line braking maneuver with desired ramp signal, the initial vehicle speed and the gear position of the transmission are set to 33.33 m/s and neutral, respectively. Meanwhile, the
slopes and final values of the four desired ramp signals are set to [0.05, 0.03], [0.1, 0.06], [−0.1, 0.03], and [−0.05, 0], respectively. Figure 4(a)–(k) shows the desired wheel slip rate and its estimated value, the derivative of the desired wheel slip rate, the vehicle and wheel speeds, the desired wheel slip rate and its actual value, the wheel slip rate tracking error, the brake torque, the relationship between wheel slip rate and longitudinal friction coefficient, and the uncertainties of front left wheel and rear right wheel, respectively. As shown in Figure 4(a)–(b), the proposed tracking differentiator can effectively smooth the desired wheel slip rate and calculate the first derivative of the desired wheel slip rate without excessive noise. As shown in Figure 4(c)–(f), both the front left and rear right wheel slip rates can quickly and accurately track the desired value, and the brake torques of front left wheel and rear right wheel generate minor fluctuations at time 1.8–2.2 s to reduce the steady-state tracking error of the wheel slip rate. Therefore, the proposed control strategy has higher tracking accuracy for both fast and slow changing signals. As shown in Figure 4(h)–(k), the lumped uncertainty observer can accurately estimate the lumped uncertainty of the system. As shown in Figure 4(g), the longitudinal friction coefficients of front left wheel and rear right wheel can smoothly change with their wheel slip rates. Therefore, the proposed control strategy can effectively improve the braking smoothness.
Conclusion

This article has proposed a novel nonlinear robust wheel slip rate tracking control strategy for autonomous vehicle with actuator dynamics. First, a simple yet effective wheel slip rate dynamic model is deduced from the quarter-vehicle model with actuator dynamics by regarding the suspension dynamics, the tire relaxation dynamics, the tire cornering, and camber characteristics as the lumped uncertainty of the system, and the wheel slip rate dynamic model is used as the basis of the nonlinear robust wheel slip rate tracking control strategy design. Second, a nonlinear robust wheel slip rate tracking control law with lumped uncertainty observer is derived via the Lyapunov-based method. The lumped uncertainty observer is designed to suppress the effect of the lumped uncertainty on the system performance by adopting the RBFNN with the unknown optimal weight vector adaptive adjust to approximate and compensate the lumped uncertainty. Then, a novel tracking differentiator is designed to calculate the derivative of the desired wheel slip rate, which is an essential aspect of the proposed nonlinear robust wheel slip rate tracking control law. Finally, the performance of the proposed control strategy is verified through simulations of the straight line braking maneuvers on a flat road with desired step signal, desired sinusoidal signal, and desired ramp signal on vehicle dynamics simulation software, and the simulation results show that the proposed control strategy can quickly and accurately track the desired wheel slip rate, and satisfy the requirements of autonomous vehicle for wheel slip rate tracking control.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the Jilin Province Science and Technology Development Plan Projects (20180201056GX) and the Jilin Provincial Development and Reform Commission Science and Technology Projects (2019C036-6).

ORCID iD

Jiaxu Zhang https://orcid.org/0000-0001-6159-1965

References

1. Li WF, Xie ZC, Wong PK, et al. Adaptive-event-trigger-based fuzzy nonlinear lateral dynamic control for autonomous electric vehicles under insecure communication networks. *IEEE Trans Indus Elec*. Epub ahead of print 7 February 2020. DOI: 10.1109/TIE.2020.2970680
2. Vorobieva H, Glaser S, Minou-Ename N, et al. Automatic parallel parking in tiny spots: path planning and control. *IEEE Trans Intell Transport Syst* 2015; 16: 396–410.
3. Kim W, Kang C, Son Y, et al. Vehicle path prediction using yaw acceleration for adaptive cruise control. *IEEE Trans Intell Transport Syst* 2018; 19: 3818–3829.
4. Segata M and Cigno RL. Automatic emergency braking: realistic analysis of car dynamics and network performance. *IEEE Trans Vehicular Tech* 2013; 62: 4150–4161.
5. Kuo CY and Yeh EC. A four-phase control scheme of an anti-skid brake system for all road conditions. *Proc Inst Mech Eng, Part D: J Automobile Engineering* 1992; 206: 275–283.
6. Fu WP, Fang ZD and Zhao ZG. Periodic solutions and harmonic analysis of an anti-lock brake system with piecewise-non-linearity. *J Sound Vib* 2001; 246: 543–550.
7. Kiencke U and Nielsen L. *Automotive control systems*. New York: Springer, 2003.
8. Ait-Hammouda I and Pasillas-Lepine W. On a class of eleven-phase anti-lock brake algorithms, robust with respect to discontinuous transitions of road characteristics. *IFAC Proc Vol* 2004; 37: 551–556.
9. Pasillas-Lépine W. Hybrid modelling and limit cycle analysis for a class of five-phase ABS algorithms. *Vehicle Syst Dyn* 2006; 44: 173–188.
10. Tanelli M, Osorio G, Bernardo MD, et al. Existence, stability and robustness analysis of limit cycles in hybrid anti-lock braking systems. *Int J Control* 2009; 82: 659–678.
11. Jing HH, Liu ZY and Chen H. A switched control strategy for antilock braking system with on/off valves. *IEEE Trans Vehicular Tech* 2011; 60: 1470–1484.
12. Liang ZC, Zhao J, Dong Z, et al. Torque vectoring and rear-wheel-steering control for vehicle’s uncertain slips on soft and slope terrain using sliding mode algorithm. *IEEE Trans Vehicular Tech* 2020; 69: 3805–3815.
13. Buckholz KR. Reference input wheel slip tracking using sliding mode control. SAE technical paper 2002-01-0301, 2002.
14. Johansen TA, Petersen I, Kalkkuhl J, et al. Gain-scheduled wheel slip control in automotive brake systems. *IEEE Trans Control Syst Tech* 2003; 11: 799–811.
15. Park KS and Lim JT. Wheel slip control for ABS with time delay input using feedback linearization and adaptive sliding mode control. In: *International conference on control, automation and systems*, Seoul, South Korea, 14–17 October 2008. New York: IEEE.
16. Harifi A, Aghagolzadeh A, Alizadeh G, et al. Designing a sliding mode controller for wheel slip control of antilock brake systems. *Transp Res Part C* 2008; 16: 731–741.
17. Corno M, Savaresi SM and Balas GJ. On linear-parameter-varying (LPV) slip-controller design for two-wheeled vehicles. *Int J Robust Nonlinear Control* 2009; 19: 1313–1336.
18. Amodeo M, Ferrara A, Terzaghi R, et al. Wheel slip control via second-order sliding-mode generation. *IEEE Trans Intell Transp Syst* 2010; 11: 122–131.
19. Pasillas-Lépine W, Loría A and Gerard M. Design and experimental validation of a nonlinear wheel slip control algorithm. *Automatica* 2012; 48: 1852–1859.

20. Mirzaei M and Mirzaeinejad H. Optimal design of a nonlinear controller for anti-lock braking system. *Transp Res Part C* 2012; 24: 19–35.

21. Hsu CF and Kuo TC. Adaptive exponential-reaching sliding-mode control for antilock braking systems. *Nonlinear Dyn* 2014; 77: 993–1010.

22. Mirzaeinejad H. Robust predictive control of wheel slip in antilock braking systems based on radial basis function neural network. *Appl Soft Comp* 2018; 70: 318–329.

23. Zhang JX and Li J. Adaptive backstepping sliding mode control for wheel slip tracking of vehicle with uncertainty observer. *Meas Control* 2018; 51: 396–405.

24. He YG, Lu CD, Shen J, et al. Design and analysis of output feedback constraint control for antilock braking system based on Burckhardt’s model. *Assembly Autom* 2019; 39: 497–513.

25. Zhang JX and Li J. Robust backstepping sliding mode control with L2-gain performance for reference input wheel slip tracking of vehicle. *Infor Tech Control* 2019; 48: 660–672.

26. Rajamani R. *Vehicle dynamics and control*. New York: Springer, 2006.

27. Liang ZC, Chen J and Wang YF. Equivalent acceleration imitation for single wheel of manned lunar rover by varying torque on earth. *IEEE/ASME Tran Mechatron* 2020; 25: 282–293.

28. Savaresi SM and Tanelli M. *Active braking control systems design for vehicles*. London: Springer, 2010.

29. Jo C, Hwang S and Kim H. Clamping-force control for electromechanical brake. *IEEE Trans Vehicular Tech* 2010; 59: 3205–3212.

30. Dash PK, Mishra S and Panda G. A radial basis function neural network controller for UPFC. *IEEE Trans Power Syst* 2000; 15: 1293–1299.

31. Alzer H, Fonseca CMD and Kovacec A. Young-type inequalities and their matrix analogues. *Linear Multilinear Algebra* 2015; 63: 622–635.

32. Khalil HK. *Nonlinear systems*. 3rd ed. Upper Saddle River, NJ: Prentice Hall, 2001.