Trajectory Prediction in Autonomous Driving with a Lane Heading Auxiliary Loss

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Abstract—Predicting a vehicle’s trajectory is an essential ability for autonomous vehicles navigating through complex urban traffic scenes. Bird’s-eye-view roadmap information provides valuable information for making trajectory predictions, and while state-of-the-art models extract this information via image convolution, auxiliary loss functions can augment patterns inferred from deep learning by further encoding common knowledge of social and legal driving behaviors. Since human driving behavior is inherently multimodal, models which allow for multimodal output tend to outperform single-prediction models on standard metrics; the proposed loss function benefits such models, as all predicted modes must follow the same expected driving rules. Our contribution to trajectory prediction is twofold; we propose a new metric which addresses failure cases of the off-road rate metric by penalizing trajectories that contain driving behavior that opposes the ascribed heading (flow direction) of a driving lane, and we show this metric to be differentiable and therefore suitable as an auxiliary loss function. We then use this auxiliary loss to extend the the standard multiple trajectory prediction (MTP) and MultiPath models, achieving improved results on the nuScenes prediction benchmark by predicting trajectories which better conform to the lane-following rules of the road.

I. INTRODUCTION

To safely navigate complex city traffic, autonomous vehicles need the ability to predict the future trajectories of surrounding vehicles. There is inherent uncertainty in predicting future trajectories, making it a challenging task. In particular, the distribution of future trajectories is multimodal. At a given instant in a traffic scene, a driver could have one of several plausible goals, with multiple paths to each goal.

Recent work has addressed multimodality in trajectory prediction by learning models that output multiple trajectories conditioned on the past motion of agents and the static scene around them. Common approaches include learning mixture models [1]–[8], sampling latent variable models [9]–[22], or sampling stochastic policies trained using inverse reinforcement learning [23]–[26]. However, defining appropriate evaluation metrics for models that output multiple trajectories still remains an open challenge.

The most commonly used evaluation metric for multimodal trajectory prediction is the minimum average displacement error over $K$ trajectories (minADE$_K$). This has the advantage of not penalizing diverse, but plausible trajectories output by models. A limitation of minADE$_K$ is that it fails to penalize models that output a diverse set of trajectories of poor quality (Fig 1). This has been addressed in prior work by additionally reporting sample quality metrics. Of particular interest are the off-road rate and off-road distance metrics partially address this (bottom-left), but fail to penalize trajectories that violate lane direction. Our proposed off-yaw metric and corresponding YawLoss seek to address this (bottom-right).

![Motivating example: A vehicle of interest approaching an intersection (top-left). The commonly used minADE$_K$ metric fails to penalize a diverse set of poor trajectories (top-right). The off-road rate and off-road distance metrics partially address this (bottom-left), but fail to penalize trajectories that violate lane direction. Our proposed off-yaw metric and corresponding YawLoss seek to address this (bottom-right).](image)

In this work, we define a new metric for sample quality of predicted trajectories termed the off-yaw rate. The off-yaw rate measures the adherence of predicted trajectories to conform to the lane-following rules of the road.

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to lane direction, and penalizes predictions that violate lane direction (Fig 1d). Moreover, we show that the off-yaw rate can be used as a differentiable loss function termed YawLoss, which can serve as an auxiliary training loss for multimodal trajectory prediction models. Our formulation of the YawLoss can be applied for training both mixture models as well as latent variable models for trajectory prediction, and leads to predicted trajectories that better conform to the lane direction, while also achieving lower minADE_K values. We report results on the publicly available NuScenes prediction benchmark by incorporating the YawLoss for training two vehicle trajectory prediction models that represent the state of the art, namely MTP proposed by Cui et al. [3] and Multipath proposed by Chai et al. [4].

II. RELATED RESEARCH

A. Multimodal trajectory prediction

A large body of recent literature has addressed the problem of human and vehicle trajectory prediction. For comprehensive surveys we refer the reader to [29], [30]. Here, we discuss models that output multimodal predictions. A common approach for multimodal trajectory prediction is to learn mixture models. Each mixture component represents a mode of the trajectory distribution. Models typically output mean trajectories for each mode and standard deviations, along with a categorical probability distribution over modes. Early work associated modes of the trajectory distribution with pre-defined maneuvers or intents [1], [2]. The need for pre-defined maneuvers was alleviated by the multiple trajectory prediction (MTP) loss proposed by Cui et al. [3]. The MTP loss has a cross-entropy component for learning the categorical probability distribution over modes, and a regression component that only penalizes the mode that is closest to the ground truth. This formulation has since been used by subsequent works [5]–[8]. More recently Chai et al. [4] extended this idea to learn deviations from anchor trajectories as modes of the trajectory distribution, rather than mean trajectories themselves, and Phan-Minh et al. [31] proposed to discard regression outputs altogether while just assigning probabilities to a discrete trajectory set. Another common approach for multimodal trajectory forecasting is learning latent variable models. Conditioned on input context such as past trajectories and static scene, latent variable models map samples from a simple latent distribution to trajectory samples. Prior works have used generative adversarial networks [9]–[14], conditional variational autoencoders [15]–[18], and more recently normalizing flow based models [19]–[22]. Finally, some approaches output multimodal predictions by sampling stochastic policies learned using inverse reinforcement learning [23]–[26]. While our proposed off-yaw metric and YawLoss can be used in conjunction with any approach that involves regression outputs, here we report results using the MTP and Multipath models as baselines.

B. Sample quality metrics and auxiliary loss functions for trajectory prediction

As described in section I, the commonly used minADE_K metric for trajectory prediction is a good measure for sample diversity, but can be a poor measure of sample quality or precision. There is inherent tension between sample diversity and sample quality or precision [19]. Several works have thus employed metrics in addition to minADE_K for measuring sample quality of trajectories. Rhinehart et al. [19] define a symmetric KL divergence metric with a component that measures sample diversity, and a component that measures sample precision and also use both metrics as loss functions for training. Some works [18], [20], [32] report collision rates for trajectories predicted for multiple actors in the scene, penalizing falsely predicted collisions. Cui et al. [33] report kinematic feasibility of predicted vehicle trajectories. Casas et al. [7] report lane infractions via traffic light or lane divider violations in predicted trajectories, as well as performance metrics for a downstream planner relying on these predictions. They also use prior knowledge of reachable lanes and the route of the autonomous vehicle to define a reward function for training the trajectory prediction model via the REINFORCE algorithm. Finally, closely related to our work, a large number of approaches use the off-road rate and off-road distance metrics [5], [6], [26]–[28], [34] for evaluating predicted trajectories. These metrics compute the proportion of predicted points that lie outside the drivable region of the road and the nearest distance of predicted points to the drivable region respectively. Niedoba et al. [27], Boulton et al. [28] and Messaoud et al. [6] also use the off-road rate as a loss function for training trajectory prediction model. Our off-yaw rate and YawLoss improve upon the off-road rate by explicitly reasoning about the direction of motion of lanes and penalizing predicted trajectories that violate it.

III. OFF-YAW RATE AS A METRIC

A. Off-Yaw Rate

By accepted legal and social convention, when driving in a lane, the vehicle must move in the direction of the lane heading as to not interfere with other traffic. The Off-Yaw Rate is a measure of a trajectory’s ability to orient in the direction of the nearest lane.

Define a vehicle’s initial position on trajectory \( \tau \) as \( (x_0^\tau, y_0^\tau) = (0,0) \), and its initial orientation in the local frame as \( \theta = 0 \) aligned with the standard y-axis. Given a trajectory of points \( \tau = \{ (x_0^\tau, y_0^\tau), (x_1^\tau, y_1^\tau), \ldots, (x_n^\tau, y_n^\tau) \} \), where points 1 through \( n \) correspond to predicted future points, we can estimate the vehicle heading relative to its initial orientation with the following procedure. First, we make use of the assumption that the trajectory sample rate relative to map scale is sufficiently high that we can accept a straight-line approximation between consecutive points. Let \( (\hat{x}_i^\tau, \hat{y}_i^\tau) \) be the midpoint of two consecutive points \( (x_i^\tau, y_i^\tau), (x_{i+1}^\tau, y_{i+1}^\tau) \), defined by the function:
So, using each angular distance, the off-yaw measure of an $n$-point trajectory is:

$$Y(\tau) = \sum_{i=1}^{n} \delta(\hat{x}_i^T, \hat{y}_i^T, \theta_{\tau_i}^G).$$  \hspace{1cm} (4)$$

Extending over all $m$ predicted modes, we reach the per-sample average off-yaw expression:

$$Y = \sum_{\tau=1}^{m} Y(\tau)$$ \hspace{1cm} (5)

**B. Lane Change Approximations**

There is a small margin of expected angular error, $\epsilon$, which can account for minor adjustments to the vehicle heading in order to stay within the lane. In addition to lane-correcting error $\epsilon$, a second exception to the assumption of driving in alignment with the nearest lane occurs when a driver changes lanes, during which their vehicle may orient at an angle no more than (and typically much less than) 90$^\circ$ to perform the lane change maneuver, with a 90$^\circ$ lane change occurring only when traffic is at a stop. Typical lane changes occur at angles relative to the flow of traffic and vehicle dynamics such as turning radius and velocity. Since a trajectory should not off-yaw during a legal lane change, nor small-angle lane corrections, we therefore constrain the measure function to only penalize angular differences which exceed a threshold, $\alpha$. The modified angular difference, $\delta$, has the following formula:

$$\delta(x, y, \theta) = \begin{cases} 
0 & \delta(x, y, \theta) \leq \alpha \\
\delta(x, y, \theta) & \delta(x, y, \theta) > \alpha 
\end{cases}$$  \hspace{1cm} (6)

For our experiments, we selected a threshold of 45$^\circ$.

**C. Off-Yaw in Intersections**

When a vehicle passes through an intersection, the vehicle must cross over lanes which flow in discordant directions (look no further than the existence of stoplights as proof). For this reason, the measure should not count any deviation from the heading of the closest lane for midpoints which lie in an intersection. Thus, the measure is modified as the following function:

$$Y^\alpha(\tau) = \frac{1}{n} \sum_{i=1}^{n} I(x_i^T, y_i^T) \delta^\alpha(x_i^T, y_i^T),$$  \hspace{1cm} (7)$$

where

$$I(x_i^T, y_i^T) = \begin{cases} 
0 & (x_i^T, y_i^T) \text{ in intersection} \\
1 & \text{otherwise} 
\end{cases}$$  \hspace{1cm} (8)

Extending over all $m$ predicted modes, we reach the modified per-sample off-yaw measure expression:

$$\hat{Y}^\alpha(T) = \sum_{\tau=1}^{m} Y^\alpha(\tau)$$  \hspace{1cm} (9)
The Off-Yaw Rate for a set of samples and their predicted trajectory sets is the average fraction of trajectories which contain off-yaw events, defined in the following equation:

\[ R_{\text{off-yaw}} = \frac{1}{N} \sum_{i=1}^{N} \frac{1}{m} \sum_{\tau=1}^{m} Y^\alpha(\tau) \]  

(10)

IV. YAWLOS

A. Off-Yaw Metric as a Loss Function

In this section, we show that the Off-Yaw Metric in Eq. (9) is differentiable, and is therefore suitable as an auxiliary loss function which penalizes vehicle trajectories that move against the flow of traffic, which we name YawLoss.

We begin with (9) and differentiate with respect to network output set of trajectories \( T = \{\tau_1, \tau_2, \ldots, \tau_m\} \). For brevity, we abbreviate \( x_1^i, y_1^i, x_{i+1}^i, y_{i+1}^i \) as \( \bar{x}_i^i \).

\[ \nabla Y^\alpha(T) = \frac{1}{m} \sum_{\tau=1}^{m} \nabla Y^\alpha(\tau) \]

(11)

\[ = \frac{1}{mn} \sum_{\tau=1}^{m} \sum_{i=1}^{n} \nabla I(x_1^i, y_1^i)\delta^\alpha(\bar{x}_i^i, \bar{y}_i^i, \theta(\bar{x}_i^i)) \]

(12)

Since the sum of differentiable functions is differentiable, we continue our analysis with the sum term:

\[ g(\bar{x}_i^i) = \nabla I(\bar{x}_i^i, \bar{y}_i^i)\delta^\alpha(\bar{x}_i^i, \bar{y}_i^i, \theta(\bar{x}_i^i)) \]

(13)

Computing the gradient, first for \( x_1^i \), we find:

\[ \frac{\partial g}{\partial x_1^i} = \frac{\partial I(\bar{x}_i^i, \bar{y}_i^i)}{\partial x_1^i} \delta^\alpha(\bar{x}_i^i, \bar{y}_i^i, \theta(\bar{x}_i^i)) + I(\bar{x}_i^i, \bar{y}_i^i) \frac{\partial \delta^\alpha(\bar{x}_i^i, \bar{y}_i^i, \theta(\bar{x}_i^i))}{\partial x_1^i} \]

(14)

Because the value of the function \( I \) in the expression

\[ \frac{\partial I(\bar{x}_i^i, \bar{y}_i^i)}{\partial x_1^i} \]

(15)

can only take on values of 0 or 1, the gradient function is simply 0 when the vehicle remains on-road or off-road, and the positive or negative reciprocal of the displacement of \( x_1^i \) otherwise; in any case, defined for all input.

The function \( \delta^\alpha \) of

\[ \frac{\partial \delta^\alpha(\bar{x}_i^i, \bar{y}_i^i, \theta(\bar{x}_i^i))}{\partial x_1^i} \]

(16)

will always give a value in the range \( [0, 360) \), so the rate change relative to any distance that the \( x_1^i \) coordinate is displaced will be defined for all input. The same cases can be extended to the remaining three variables of differentiation \( (y_i, x_{i+1}, y_{i+1}) \), thus making the function \( \tilde{Y}^\alpha(T) \) differentiable and therefore a suitable loss function.

Ultimately, this auxiliary loss function encourages trajectories to stay near lanes whose headings they align with, and to adjust their own headings to more closely match that of the nearest lane.

V. EXPERIMENTAL ANALYSIS AND EVALUATIONS

A. Dataset

We use the public nuScenes dataset [35] to train and evaluate our model. We use from each sample a rasterized image of the surrounding map, vehicle state information (velocity, acceleration, heading), and target trajectory. The data was collected over inner-city drives conducted in Boston and Singapore. We train and evaluate our model using the official benchmark split for the nuScenes prediction challenge; in total, we used 29889 instances in the train set, 7905 instances in the validation set, and 8397 instances in the test set.

B. Network Architecture

We perform experiments using both the MTP network defined in [36] and the MultiPath network defined in [4], consisting of map input to a base CNN, whose output is flattened and concatenated with agent vehicle state information (velocity, acceleration, heading change rate), followed by two fully-connected layers. For our experiments, we use a network output of 15 modes with 12 predicted points per mode (representing 6 seconds of travel) for the case of MTP, and 12 predicted offsets per anchor for the case of MultiPath. We use a base CNN of ResNet-50 [37]. In accordance with the expected input to ResNet with ImageNet dataset pretraining, we normalize our rasterized map images in RGB space prior to training. We use the classification and regression loss functions as defined in [36], with an additive term for lane heading auxiliary loss defined in this work (YawLoss). We use a hyperparameter of 1 as a scalar multiplier for the YawLoss term.

C. Implementation Details

With earlier described rasterized map physical dimensions of 50 meters x 50 meters, using a scale of 0.1 meters per pixel, we assume the lane and trajectory to be approximately straight (i.e. of single uniform heading) on the pixel scale. Each scene map contains information on lane placement and heading, drivable area, and surrounding vehicles and pedestrians. Vehicle state is provided as a three-dimensional input. We use a batch size of 16 and Adam optimizer [38], implemented using PyTorch [39].

D. Reducing Network Training Time & Memory Requirements with Secondary Maps

Calculating this loss per-sample can be computationally expensive. For every predicted mode of each sample instance, it is required to find the \( L2 \)-nearest lane point to each midpoint on the predicted trajectory, with predictions changing on every iteration.

This computational hurdle can be lowered through preprocessing; for each instance map, which in our case extends 10 meters behind the vehicle, 40 meters ahead, 25 meters left, and 25 meters right, we generate a secondary orientation map, covering a larger area to account for trajectories which leave the original map. This secondary map extends 20 meters behind the vehicle, 80 meters ahead, 50 meters left, and 50 meters right. On this map, each pixel location is
assigned a value which equals the orientation of the nearest lane point.

Prior to training, these secondary maps are generated and saved for each data sample. Each grid location on the map represents a heading from the continuous range $[0, 360)$ degrees in the global frame. To represent each grid location as a 64-bit floating point value can quickly become storage intensive for a large set of 500x500 maps. However, only a coarse precision of the angle is required for this problem; we would never consider a driver to be going the ‘wrong way’ if their heading was off by just a few degrees. For this reason, a representation with precision only to the scale of degrees is appropriate for this problem. With this in mind, we can create a data-efficient representation which encodes each heading as an 8-bit grayscale integer pixel value in the range $[1, 255]$, with the value of 0 reserved for map locations corresponding to intersections. Headings are mapped from range $[0, 360)$ degree values to $[1, 255]$ grayscale values as follows:

$$\theta_{\text{map}} = 1 + \left\lfloor \frac{254}{360} \theta \right\rfloor.$$  

(17)

Using the above function, we assign to each point on the secondary map the mapped value of the heading of the $L_2$-nearest lane, illustrated in Fig. 3. During training, when inversely mapping from grayscale integer to degrees, there is a loss of precision that occurs, since the 360 degrees are mapped to 254 values. In this sense, each ‘bin’ of the data representation actually represents a span of approximately $1.417^\circ$, a reasonable precision for this task.

E. Baselines

Results are shown in comparison to the following baselines:

- Constant Velocity, Yaw: The predicted trajectory is a continuation of the vehicle’s current velocity and heading.
- Physics Oracle: As introduced in [31], the proposed trajectory is selected as the best trajectory from four dynamics models:
  - constant velocity and yaw,
  - constant velocity and yaw rate,
  - constant acceleration and yaw, and
  - constant acceleration and yaw rate.
Note that this method is not used to make predictions, but rather provides a reference benchmark to four simple physical models, to illustrate improvement from models which account for more complex maneuvers.
- MultiPath: The predicted trajectories are the output of the MultiPath model, without auxiliary loss, as described in [4].
- MTP: The predicted trajectories are the output of the original MTP model, without auxiliary loss, as described in [36].

F. Metrics

Reported metrics include the following:

- Minimum average displacement error over $k$ most probable trajectories ($\text{MinADE}_k$), for $k = 1, 5, 10$
- Minimum final displacement error over $k$ most probable trajectories ($\text{MinFDE}_k$), for $k = 1, 5, 10$
- Miss rate at 2 meters over $k$ most probable trajectories ($\text{Miss Rate}_{k,2}$), for $k = 5, 10$
- Off-road rate
- Off-yaw rate, the new metric as defined in this paper, measuring the amount of positive angular difference (beyond a threshold) of predicted trajectories from the nearest lane yaw, averaged over all agents.

G. Quantitative Results

We compare our extension of the MTP and MultiPath models to the various baselines in Table I. Our MTP model outperforms or matches the non-extended MTP model on 8 of the 9 reported metrics, the exception being a .01 increase of Miss Rate at 2 m for $k = 10$. In contrast, the MTP model with off-road loss outperforms the baseline on just 4 of the 9 reported metrics. Our MultiPath model outperforms or
In this paper we presented an auxiliary loss function which may be used to augment the performance of existing models.
for vehicle trajectory prediction in urban environments. This lane heading loss function leverages the expectation that vehicles follow the direction ascribed to roadway lanes at all times, with exception for corrective maneuvers, lane changes, and intersection crossings. This loss function applies to all predicted modes, since no mode should predict driving opposite the lane direction. Experiments showed that extending the benchmark MTP model with the lane heading auxiliary loss outperforms the model's original classification and regression losses.

A possible extension of this work would be the application of the lane heading auxiliary loss to other existing deep learning models, in tandem with other auxiliary losses such as off-road loss. Another possibility for future investigation is the tuning of the angular difference threshold and weighting using agent dynamics and scene context. Finally, in our future work, we intend to design a methodology for quantifying nearest lane heading within an intersection or outside of the drivable area to reduce the effect of this non-linearity on training and metric reporting.

As stated by Daily et al. [40], “Self-driving and highly automated vehicles must navigate smoothly and avoid obstacles, while accurately understanding the highly complex semantic interpretation of scene and dynamic activities.” While convolutional neural networks and other data-driven approaches may be effective at repeating known patterns, there is a lost element of explainability which is crucial towards public safety and adoption. By encoding familiar driving expectations through the introduced Off-Yaw Rate metric and YawLoss, we initiate a step towards autonomous vehicle computational models which can both learn and explain.

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