The effect of controlled vibrations on Rayleigh-Benard convection

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Abstract. The paper presents the results of a numerical study of convective heat transfer in a long horizontal layer heated from below with and without the vibration effect of the lower wall. The simulation was carried out on the basis of solving the Navier-Stokes 2D equations for an incompressible fluid in the Boussinesq approximation. It is shown that the influence of vibrations of the lower heated wall on the wave number of the convective flow roll structure, on the time and on the critical Rayleigh number of convection. The influence of controlled harmonic vibrations of wall on the structure of convective flow in the Rayleigh-Benard problem has been investigated. It is shown that the wave number of the periodic convective structure, the critical Rayleigh number, and the time of occurrence of Rayleigh-Benard convection under the vertical vibration effect on the horizontal layer from the lower wall are reduced.

1. Introduction
The beginning of the study of convective fluid flows in horizontal layers heated from below are the experiments of Benard [1], who observed the occurrence of spatially periodic convection in a horizontal layer of liquid heated from below (later they were called Benard cells). Convection when the liquid layer is heated from below has a threshold character and occurs after a certain time when the critical value of the heat flux is exceeded. Rayleigh theoretically investigated the stability of the equilibrium in the horizontal layer and determined the critical values of the parameters of convection occurrence for a layer with both free boundaries [2].

The problem of convective flow in a horizontal layer heated from below is called the Rayleigh-Benard problem, and many papers have been devoted to its study because of the simplicity and variety of its formulations, the abundance of flow modes and various applications. The paper [3] shows the strong sensitivity of characteristics of the Rayleigh-Benard convective flow (depending on the governing parameters, initial and boundary conditions of the problem), analyzes the formation of various spatial-time modes of convective structures, changes in their scales and shapes. The stability of convection in horizontal layers is studied in detail in [4-5], in particular, it is shown that the presence of a free boundary leads to an increase in the instability threshold of the system. The paper [6] presents the results of numerical simulation of gravitational and capillary convection in horizontal layers under normal and microgravity conditions for crystal growth processes.

In addition to technical and technological applications, the Rayleigh-Benard problem has implications for medicine and healthcare. In particular, pathogenic viruses live in a liquid environment and spread by spraying droplets during coughing and sneezing, as well as a result of human contact with films and drops of liquid located on various solid objects. The study of possible ways to control
the hydrodynamics in these thin liquid volumes is also necessary to combat the spread of pathogenic viruses. One of the possible ways to control hydrodynamics and mass transfer in thin liquid layers can be a high-frequency vibration effect with a small amplitude. The investigation of the convective stability of an unevenly heated liquid under the influence of high-frequency vibrations was initiated by the authors of [7] showing that high-frequency vibrations of the horizontal liquid layer heated from below have a stabilizing effect. This conclusion was obtained by another variational method by the same authors in their subsequent works, for example, in [8], as well as by authors other papers [9-12].

In thesis [9], the controlling influence of vibrational accelerations on the stability of mechanical equilibrium and convective flows in systems of various configurations is considered. Maps of convection modes in vertical and horizontal fluid layers under high-frequency longitudinal and transverse vibrations are created. The stabilizing effect of high-frequency vertical oscillations on the convective stability of the equilibrium in a horizontal layer heated from below is experimentally confirmed. In the paper [10], a critical value of the vibration parameter, above which it is impossible to excite convection in the liquid layer by any temperature differences, is found.

In [11], it is shown that under the vibration effect in the conditions of weightlessness in a liquid, an averaged vibration flow (AVF) other than zero may occur. The paper [12] presents the results of further study of AVF for different convective problems. In [13-15], based on the solution of the non-averaged Navier-Stokes equations, the occurrence of AVF and the effect of vibration on the width of the boundary layer for single crystal growth processes are shown.

Marangoni convection in liquid volumes with a free surface can lead to unsteady perturbations of the free surface. As a result, the position of the free surface can have a metastable character and lead to significant changes in the shape of the liquid volume and the position of the free surface of the liquid volume. The free surface can change its position depending on the heat flux vector or mass, taking an energetically favorable position (non-stationary, but on average perpendicular to the concentration (or temperature) gradient) [16]. The paper [16] shows the dynamics of metastable changes in the free surface, which has an oscillatory character, and its average position in zero gravity and terrestrial conditions. It is also shown that gravitational convection has a stabilizing factor for the position of the free surface (even in the presence of thermo-capillary convection).

This paper investigates the influence of controlled vibrations on the wave number of a periodical structure of the convective flow in the Rayleigh-Benard problem under vibration action from one of the boundaries of the horizontal layer. The influence of the vertical vibration from the lower wall on the structure of the convective flow in the horizontal layer, the reduction of the critical Rayleigh number and the time of occurrence of Rayleigh-Benard convection are shown.

2. Problem statement and mathematical model
The problem of thermal convection of a viscous incompressible fluid is considered for a horizontal layer (with an aspect ratio of 1:10) in a gravity field with specified temperatures on horizontal walls and with thermally insulated vertical walls. It is assumed that the liquid is incompressible, and the Prandtl number Pr=1. The system of two-dimensional nonstationary Navier-Stokes equations for an incompressible fluid in the Boussinesq approximation and the heat transfer equation are used to solve the problem. The top wall is free. On the top wall, the boundary condition of slip is set for velocity. For the velocity on solid walls, the conditions of no-slip are accepted.

The analysis of solutions is carried out in a steady-state mode (or in the presence of vibrations, on a quasi-stationary mode). The scheme of the calculated geometry of the mathematical model of the Rayleigh-Benard problem, the boundary conditions and the results of modeling in the form of isolines of the stream function (in the upper figure) and isotherms (in the lower figure) of the convective flow are shown in Fig. 1.
Figure 1. Scheme of the calculated geometry, boundary conditions and isolines of the stream function and isotherms at $Ra = 1.8 \times 10^4$, $Pr = 1$.

The mathematical model is based on the numerical solution of a system of non-stationary planar 2D Navier-Stokes equations for natural convection of an incompressible liquid in the Boussinesq approximation (1-4) [4, 6, 18]:

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + F_1,
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + F_2,
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,
\]

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + F_3,
\]

where $x, y$ are the horizontal and vertical Cartesian coordinates; $u, v$ are the components of the velocity vector; $t$ is the time; $T$ is the temperature; $P$ is the pressure; $\rho$ is the density; $F_1 = 0$ and $F_2 = -g\beta(T - T_0)$ are the components of the vector of external forces; $g$ is the gravitational acceleration of the earth’s free fall; $\beta, \nu, \alpha$ are the coefficients of temperature expansion of the liquid, kinematic viscosity and thermal conductivity, respectively.

The results presented in this paper were obtained using different numerical methods: the finite-difference scalar method [18], the fully implicit matrix finite-difference method [6, 19], and the conservative control volume method [20]. The good accuracy of numerical results was confirmed by comparison with experimental data and comparison of numerical results obtained by different numerical models [21].

3. Results
The results of numerical simulation presented in Fig. 1 show the influence of the lower horizontal wall oscillations on the structure of the convective flow in the Rayleigh-Benard problem. The number of Rayleigh-Benard rollers decreases from 10 to 9 during vertical harmonic vibrations of the lower wall (on law $y = A \sin(2\pi ft)$ with a frequency $f = 10$ Hz and an amplitude $A = 10^{-4}$ m, $Re_{vib} = A^2 2\pi f / \nu = 0.007$), which indicates a decrease in the wave number of the periodic convective structure.
Figure 2. Pictures of isolines of the stream function and isotherms with and without vibrations of the lower wall with $Re_{vibr} = 0.007$, $Ra = 4 \cdot 10^7$, $Pr = 1$.

It should be noted that the critical Rayleigh number of convection decreases significantly. The time of occurrence and establishment of the quasi-stationary mode of convection is significantly reduced, as shown in Figure 3. Fig. 4 shows the vorticity and heat flux isolines, demonstrating that they also have a periodic structure (the maximum friction on the lower wall and the heat source flows at the bottom and the drain on the upper wall).

Figure 3. The dependence of the maximum values of the stream function on time ($Ra = 4 \cdot 10^7$, $Pr = 1$): a) without vibrations and b) with vibrations of the wall with $Re_{vibr} = 0.007$, $Ra = 4 \cdot 10^7$, $Pr = 1$.

Figure 4. Pictures of isolines of the vorticity and heat flux with vibrations of the lower wall with $Re_{vibr} = 0.007$, $Ra = 4 \cdot 10^7$, $Pr = 1$. 

Figure 5. Pictures of isolines of MEAN and RMSD of: the velocity component (u, v), magnitude vorticity (V) and isotherms (T) with vibrations of the wall with $Re_{vib} = 0.007$, $Ra = 4 \times 10^3$, $Pr = 1$.

For the quasi-stationary flow mode (with vibrations of the wall with $Re_{vib} = 0.007$, $Ra = 4 \times 10^3$, $Pr = 1$) Fig.5 shows the mean values (in column "MEAN") and the root-mean-square deviation (in column "RMSD") from the mean values of the following functions: the component of the velocity vector (u, v), the velocity modulus (V), and the temperature (T). These results demonstrate the ordered structures of not only the mean values of the functions, but also their deviations, each having its own distribution in space.

Conclusions

The influence of controlled vibrations on the structure of convective flow in the Rayleigh-Benard problem has been investigated. It is shown that the wave number of the periodic convective structure, the critical Rayleigh number, and the time of occurrence of Rayleigh-Benard convection under the vertical vibration effect on the horizontal layer from the lower wall are reduced.

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