"FASTER THAN LIGHT" PHOTONS AND CHARGED BLACK HOLES

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Abstract

Photons propagating in curved spacetime may, depending on their direction and polarisation, have velocities exceeding the “speed of light” c. This phenomenon arises through vacuum polarisation in QED and is a tidal gravitational effect depending on the local curvature. It implies that the Principle of Equivalence does not hold for interacting quantum field theories in curved spacetime and reflects a quantum violation of local Lorentz invariance. These results are illustrated for the propagation of photons in the Reissner-Nordström spacetime characterising a charged black hole. A general analysis of electromagnetic as well as gravitational birefringence is presented.
1. Introduction

Photons propagating in certain curved spacetime backgrounds may, depending on their direction and polarisation, have velocities exceeding the “speed of light” \( c \). This is a quantum phenomenon induced by vacuum polarisation in quantum electrodynamics (QED). It is a tidal gravitational effect depending explicitly on the local curvature. The Principle of Equivalence does not hold for QED in curved spacetime.

These remarkable results were discovered by Drummond and Hathrell in 1980 [1]. They considered the effect of one-loop vacuum polarisation on photon propagation in de Sitter, Schwarzschild, Robertson-Walker and gravitational wave spacetimes. In each case except de Sitter, for which the curvature is totally isotropic, they identified directions and polarisations for which the photon velocity was faster than \( c \).

In this paper, we extend these results to an arbitrary (but slowly varying) background of electromagnetic and gravitational fields, which enables us to discuss the Reissner-Nordström spacetime. Our results therefore describe photon propagation around a charged black hole. We find three classes of contributions to the photon velocity: (a) the basic gravitational effect identical to the Schwarzschild case, (b) the indirect effect of the charge through its modification of the gravitational field, and (c) the contribution of the electromagnetic field itself. The sign of the electromagnetic contribution is always such as to reduce the photon velocity, while the gravitational effect may increase the velocity for certain directions and polarisations. The most striking case is orbital (i.e. \( \phi \)) directed photon propagation, where the velocities of the radial and tangential (i.e. \( \theta \)) polarisation states are

\[
v_r = 1 + \frac{1}{240} \frac{\alpha}{\pi (Mm)^2} \left( \frac{2M}{r} \right)^3 \left[ 1 + \frac{5}{12} \left( \frac{\alpha}{m^2} \right)^{-1} \left( \frac{Q}{Q_0} \right)^2 - \frac{2}{3} \left( \frac{Q}{Q_0} \right)^2 \frac{2M}{r} \right]
\]

\[
v_\theta = 1 + \frac{1}{240} \frac{\alpha}{\pi (Mm)^2} \left( \frac{2M}{r} \right)^3 \left[ -1 + \frac{13}{12} \left( \frac{\alpha}{m^2} \right)^{-1} \left( \frac{Q}{Q_0} \right)^2 - \frac{7}{6} \left( \frac{Q}{Q_0} \right)^2 \frac{2M}{r} \right]
\]

(1.1)

respectively. For radial photon propagation, the velocity is always \( c \). Here, \( Q \) and \( M \) are the charge and mass of the black hole, \( m \) is the electron mass and \( \alpha \) is the fine structure constant. We have parametrised the result in terms of the accretion limit charge* \( Q_0 = Mm/\alpha^2 \). For black holes with charge \( Q \simeq Q_0 \), the indirect type (b) gravitational effect of the charge (the second terms in eq.(1.1) ) is suppressed by an inverse power of the (squared) electron charge to mass ratio, where \( \alpha/m^2 \simeq 10^{42} \). The direct gravitational and electromagnetic effects are then of the same order, the relative size depending on the radial distance \( r \) from the centre of the black hole. Notice that for \( Q \leq Q_0 \), propagation at the horizon \( r \simeq 2M \) in the orbital direction is superluminal.

The physical origin of this phenomenon is relatively clear[1]. In picturesque terms, vacuum polarisation in QED allows the photon to exist as a virtual \( e^+e^- \) pair, thereby acquiring an effective size of \( O(\lambda_c) \), where \( \lambda_c = m^{-1} \) is the Compton wavelength of the

* We use dimensionless units where \( G = \hbar = c = \epsilon_0 = 1 \) throughout.
electron. In an anisotropic gravitational field*, the photon is therefore sensitive to the curvature and its characteristics of propagation may differ from those in flat space. The polarisation dependence (gravitational birefringence[1]) reflects the anisotropy of the background field.

Since superluminal propagation is clearly a delicate issue, we should be perfectly clear what is being described. We find that in a local inertial frame at a given point in spacetime the photon momentum $k^a$ satisfies the light-cone condition

$$ (\eta_{ab} + \alpha \sigma_{ab}) k^a k^b = 0, $$

where $\eta_{ab}$ is the Minkowski metric and $\alpha \sigma_{ab}$ is the one-loop vacuum polarisation correction, which depends on the Riemann curvature tensor at the given point. In the Drummond-Hathrell calculation[1], $\sigma_{ab}$ is of $O((\lambda_c/L)^2)$, where $L$ is the curvature scale. In familiar quantum field theoretic terms, the pole in the photon propagator in the local Lorentz frame is shifted from $k^2 = 0$ to $k^2 + \alpha \sigma_{ab} k^a k^b = 0$.

The condition (1.2) is not Lorentz invariant, since it depends explicitly on the local curvature. As noted in ref.[1], this is the key observation which permits propagation according to eq.(1.2) without leading to the classic paradoxes associated with superluminal propagation in special relativity. Patching together local Lorentz frames to cover the manifold, we find, in contrast to the Principle of Equivalence, that photons do not in general follow spacetime geodesics (even to the extent to which it is possible to describe particle trajectories in quantum theory).

We shall discuss the issues and potential paradoxes raised by these calculations in a little more detail in sect. 4. For the moment, we simply emphasise the generality of the phenomenon. Any interacting quantum field theory (presumably with the exception of conformal field theories) necessarily involves a scale which will enter the propagators through vacuum polarisation. In turn, this means that the theory is sensitive at the quantum level to tidal gravitational effects depending on the local curvature. Local Lorentz invariance is therefore lost at the quantum level. In formal terms, this can presumably be viewed as an anomaly in local Lorentz invariance. The Principle of Equivalence, which depends for its usual formulation on the existence of point particles, is therefore inapplicable to the motion of real elementary particles in quantum field theory**. Its content reduces simply to the statement that the curved spacetime admits everywhere a local Minkowski tangent space.

The paper is arranged as follows. In sect. 2, we describe the effective action for QED in an arbitrary background gravitational and electromagnetic field and, following the techniques of ref.[1], derive the equation of motion governing photon propagation.

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* In this paper, as in ref.[1], we are only able to make explicit calculations in the case of slowly varying background fields. It is therefore the anisotropy rather than the inhomogeneity of the gravitational (or electromagnetic) field which is responsible for the effects found here. For rapidly varying, inhomogeneous fields similar corrections are expected, depending on derivatives of the curvature.

** Similar observations on the limits of the equivalence principle in string theory have been made recently by Mende[2]. As we see here, however, it is not necessary to go beyond quantum field theory to encounter the limits of the point particle concept.
Special emphasis is placed on identifying the expansion parameters and approximations used since, as we discuss in sect. 4, fundamental questions of principle concerning the observability of this phenomenon depend critically on the parameter range of validity of the calculation.

In sect. 3, we consider the special case of the Reissner-Nordström spacetime, which describes (the exterior of) a charged black hole, and derive the light-cone condition (1.2) appropriate to different directions and polarisations of photon propagation. The essential results are described above.

The phenomenon of electromagnetic birefringence (polarisation dependence of the photon velocity in a background electromagnetic field) has been demonstrated by Adler[3] for the case of a constant magnetic field. In that case, the photon velocities corresponding to both transverse polarisations are found to be less than $c$. In appendix A, we show using our formalism that this result remains true for an arbitrary anisotropic electromagnetic background (although we are still forced to assume only weak inhomogeneity). Superluminal propagation is a special feature of gravity.

Finally, in sect. 4, we describe briefly why the causal paradoxes normally associated with faster than light propagation in special relativity are avoided by the tidal nature of the modification (1.2) of the light cone. We discuss the question of whether this superluminal propagation is in principle observable and assess the possibilities of answering some of the conceptual questions surrounding this effect by extending the derivation to short wavelength photons with $\lambda \ll \lambda_c$.

Following the original paper of Drummond and Hathrell[1] there have been surprisingly few papers in the literature developing this topic. The essential results of ref.[1] were rapidly generalised to neutrinos in a Friedmann metric by Ohkuwa[4]. Some questions were raised concerning the validity the superluminal propagation and the compatibility with a dispersion relation (see sect. 4) by Dolgov and Khriplovich[5]. An analogous effect, superluminal propagation in a flat spacetime with boundaries, has been investigated by Scharnhorst[6] and Barton[7,8] who encounter the same conceptual and interpretational questions raised by the curved spacetime effect. Quantum field theory, in particular the Hawking effect, in Reissner-Nordström spacetime has been investigated independently of the question of photon propagation by Gibbons[9].
2. Effective Action and Photon Propagation

The description of photon propagation given here is based on the one-loop effective action for QED in a background gravitational and electromagnetic field. This can be calculated using, e.g., heat kernel techniques as an expansion in powers of the background field curvatures, i.e. the Riemann tensor $R_{\mu\nu\sigma\tau}$ and the electromagnetic field strength $F_{\mu\nu}$. Since the derivation is well-documented elsewhere ([1] and references therein), we simply quote the first few terms in the expansion which are relevant to photon propagation, viz.

$$\Gamma = -\frac{1}{4} \int dx \sqrt{-g} F_{\mu\nu} F^{\mu\nu} + \frac{1}{m^2} \int dx \sqrt{-g} \left[ a RF_{\mu\nu} F^{\mu\nu} + b R_{\mu\nu} F^{\mu\sigma} F^{\nu\delta} + c R_{\mu\nu\sigma\tau} F^{\mu\nu} F^{\sigma\tau} + d D_\mu F^{\mu\nu} D_\nu F^{\sigma\nu} \right] + \frac{1}{m^4} \int dx \sqrt{-g} \left[ z (F_{\mu\nu} F^{\mu\nu})^2 + y F_{\mu\nu} F_{\sigma\tau} F^{\mu\sigma} F^{\nu\tau} \right] + \ldots$$

(2.1)

where

$$a = \frac{5}{720} \frac{\alpha}{\pi}, \quad b = \frac{26}{720} \frac{\alpha}{\pi}, \quad c = -\frac{2}{720} \frac{\alpha}{\pi},$$

$$d = -\frac{1}{30} \frac{\alpha}{\pi}, \quad z = -\frac{5}{180} \alpha^2, \quad y = \frac{14}{180} \alpha^2$$

(2.2)

Of course there are further terms of $O(R^2)$ and $O(R^3)$ independent of the electromagnetic field strength but these do not play any rôle in our analysis.

Eq.(2.1) should be viewed as giving the one-loop, i.e. $O(\alpha)$, quantum corrections to the quadratic $F^2$ action as an expansion both in powers of $R/m^2$ and $\alpha F^2/m^4$. (Here, ‘$R$’ and ‘$F$’ denote generic curvature and field strength terms.) For a Reissner-Nordström black hole of mass $M$ and charge $Q$, these expansion parameters are $\frac{1}{(Mm)^2} \left( \frac{2M}{r} \right)^3$ and $\frac{1}{(Mm)^2} \left( \frac{Q}{Q_0} \right)^2 \left( \frac{2M}{r} \right)^4$ respectively (cf. eq.(1.1)). So, at the horizon and for a charge equal to the accretion limit ($\alpha Q_0^2 = m^2 M^2$), both the expansion parameters reduce simply to $(Mm)^{-2}$.

The parameter $Mm$ controls the nature of the black hole. For $Mm \gg 1$, it is essentially a classical, general relativistic object and quantum effects such as we are discussing here are strongly suppressed by powers of $(Mm)^{-2} = O((\lambda_c/L)^2)$. On the other hand, for $Mm \simeq 1$, quantum effects are important and phenomena such as black hole evaporation are significant. In particular, for small $Mm$, the Hawking temperature $T_H = 1/(8\pi M) > m$ and electrons may be produced as Hawking radiation.

Since the Reissner-Nordström solution has an electric field, $e^+e^-$ pair creation will occur. The rate of production, first calculated by Schwinger[10], is suppressed (at $r \simeq 2M$) by an exponential factor $\exp - (Mm(Q/Q_0))$. Again, we see that for $Q \simeq Q_0$ the relevant
parameter is $Mm$ and that for $Mm \gg 1$, pair creation is strongly suppressed. For $Q \gg Q_0$, the regime in which the effect of the charge on the gravitational field becomes significant, the black hole would normally lose charge rapidly through pair creation.

This double expansion is typical of quantum field theory calculations in a background field or at finite temperature, density etc. The general form of the perturbation expansion in the coupling constant is $f_0(s) + \alpha f_1(s) + \alpha^2 f_2(s) + \ldots$, where $f_i(s)$ are functions of a dimensionless parameter $s$ defined as the ratio of the background scale to a physical mass in the quantum theory. Here, $s = \mathcal{R}/m^2$ or $\alpha F^2/m^4$, whereas, for example, in calculations of the finite temperature effective action in electroweak theories (see, e.g., ref.[11]) $s = T/m_W$. Critical behaviour, such as phase transitions, occurs generally when this parameter $s$ is of $O(1)$. However, in many cases, the present calculation being in this category, the best we can do is give an expansion of the coefficient functions $f_i(s)$ for small and/or large $s$ (for example, the low and high temperature expansions of the effective potential). We emphasise that the restriction here to weak curvature $\lambda_c \ll L$ is purely technical, arising from our ability to calculate the effective action. For strong curvatures of the order of the elementary particle masses, the phenomena described here will be enhanced, becoming ordinary $O(\alpha)$ quantum effects.

There is a further expansion inherent in eq.(2.1) in the number of derivatives acting on the electromagnetic field strength. This is related to the dependence of the photon propagation on its wavelength $\lambda$ and is discussed carefully below.

At this point, we could in principle compute the photon propagator from the effective action (2.1) and derive the modified light-cone condition (1.2) directly. However, it is in practice much simpler to follow the method of ref.[1] and compute the equation of motion for the electromagnetic field corresponding to the photon in the geometric optics approximation.

The equation of motion of the electromagnetic field,

$$\frac{\delta \Gamma}{\delta A_\nu} = 0, \quad (2.3)$$

becomes

$$D_\mu F^{\mu\nu} + \frac{1}{m^2} \left[ 4a \left( F^{\mu\nu} D_\mu R + RD_\mu F^{\mu\nu} \right) + 2b \left( F^{\sigma\nu} D_\mu R^{\mu\sigma} + R^{\mu\sigma} D_\mu F^{\sigma\nu} - F^{\sigma\mu} D_\mu R^{\nu\sigma} - R^{\nu\sigma} D_\mu F^{\sigma\mu} \right) + 4c \left( F^{\sigma\tau} D_\mu R^{\mu\sigma\tau} + R^{\mu\sigma\tau} D_\mu F^{\sigma\tau} \right) + 2d \left( D^2 D_\sigma F^{\sigma\nu} - D_\mu D^\nu D_\sigma F^{\sigma\mu} \right) \right] + \frac{1}{m^4} \left[ -8z \left( F^{\sigma\tau} F_{\sigma\tau} D_\mu F^{\mu\nu} + 2F^{\mu\nu} F_{\sigma\tau} D_\mu F^{\sigma\tau} \right) - 8y \left( F^{\nu\tau} F_{\sigma\tau} D_\mu F^{\mu\sigma} + F^{\mu\sigma} F_{\sigma\tau} D_\mu F^{\nu\tau} + F^{\mu\sigma} F^{\nu\tau} D_\mu F_{\sigma\tau} \right) \right] = 0 \quad (2.4)$$
To study the motion of a photon, we now write
\[ F_{\mu\nu} = \bar{F}_{\mu\nu} + \hat{f}_{\mu\nu}, \] (2.5)

where \( \bar{F}_{\mu\nu} \) is the background electromagnetic field satisfying \( \delta \Gamma / \delta A|_{\bar{F}} = 0 \), and linearise in \( \hat{f}_{\mu\nu} \). To leading order in \( \alpha \), we can simply take \( \bar{F} \) and the curvature to be the solutions of the classical field equations. The “back reaction” on the metric is a higher order correction to the results we find here. The characteristics of the photon propagation are then given by the geometric optics approximation where we set
\[ \hat{f}_{\mu\nu} = f_{\mu\nu} e^{i\theta}, \] (2.6)

where \( f_{\mu\nu} \) is a slowly varying amplitude and \( \theta \) the rapidly varying phase, with \( k_\mu = D_\mu \theta \) corresponding to the photon momentum.

The electromagnetic Bianchi identity
\[ D_\lambda F_{\mu\nu} + D_\mu F_{\nu\lambda} + D_\nu F_{\lambda\mu} = 0 \] (2.7)

applied to eq.(2.5) implies
\[ k_\lambda f_{\mu\nu} + k_\mu f_{\nu\lambda} + k_\nu f_{\lambda\mu} = 0 \] (2.8)

and so
\[ f_{\mu\nu} = k_\mu a_\nu - k_\nu a_\mu. \] (2.9)

The field strength \( f_{\mu\nu} \) therefore has just three independent components. The vector \( a_\mu \) gives the direction of polarisation of the photon.

The “photon equation of motion” is now found by substituting (2.5) and (2.6) into eq.(2.4). Imposing several simplifying approximations, we find
\[ k_\mu f^{\mu\nu} + \frac{1}{m^2} \left[ 2bR^{\mu\sigma}k_\mu f^{\sigma\nu} + 4cR^{\mu\nu}\sigma\tau k_\mu f^{\sigma\tau} \right] \]
\[ + \frac{1}{m^4} \left[ -16z\bar{F}^{\mu\nu}\bar{F}_{\sigma\tau}k_\mu f^{\sigma\tau} - 8y(\bar{F}^{\mu\sigma}\bar{F}_{\sigma\tau}k_\mu f^{\nu\tau} + \bar{F}^{\mu\sigma}\bar{F}^{\nu\tau}k_\mu f^{\sigma\tau}) \right] \]
\[ = 0. \] (2.10)

The dominant quantum corrections to the photon propagation are of \( O(\alpha(\lambda_c/L)^2) \) from the curvature terms and \( O(\alpha(\alpha\bar{F}^2/m^4)) \) from the electromagnetic terms. The approximations are:

(i) Assuming the background gravitational and electromagnetic fields vary with the typical curvature scale \( L \), terms involving \( D_\lambda R_{\mu\nu} \), \( D_\lambda F_{\mu\nu} \) etc. are suppressed relative to the leading correction by \( O(\lambda/L) \), where \( \lambda \) is the photon wavelength.

(ii) Terms with derivatives acting on \( f_{\mu\nu} \) are suppressed by \( O(\lambda/L_A) \), where \( L_A \) is the scale of variation of the “photon” amplitude. This is the standard leading-order geometric optics approximation.
(iii) Terms involving \( D_\mu F^{\mu\nu} \) within the square brackets in eq.(2.4) simply give \( O(\alpha) \) corrections to the \( k_\mu f^{\mu\nu} \) term in eq.(2.10) and can be omitted provided QED perturbation theory is reliable.

The final approximation we have made is to neglect the series of terms in the effective action involving successively higher powers of derivatives compensated by inverse powers of mass, i.e., symbolically, \( \sum_p \left( \frac{R}{m^2} \right)^p \left( \frac{D^p}{m^p} \right) F^2 \) terms. This is valid provided the photon wavelength is large compared to the electron Compton wavelength, i.e. \( \lambda_c \ll \lambda \). As we discuss later, this is a restriction we would like to circumvent and indeed recent developments[12] in the heat kernel technique for calculating the effective action may make this possible.

For the moment, however, combining these approximations we see that our results will be valid provided \( \lambda_c \ll \lambda \ll L \). Since the dominant quantum effect is \( O(\alpha(\lambda_c/L)^2) \), it is necessarily small.

The geometric optics approximation (ii) also deserves further scrutiny since, as noted by Dolgov and Khriplovich[5], the classical motion of a localised wave packet is subject to tidal forces producing \( O(\lambda/L)^2 \) corrections to the phase and group velocities which could dominate the quantum effect on the photon velocity calculated here. However, to decide whether this is relevant to the full quantum theory would require recasting our discussion in terms of the curved space photon propagator and we leave this for further work.

To solve eq.(2.10), we use the following general method[1]. Multiplying by \( k^\lambda \), antisymmetrising on \( \lambda \) and \( \nu \), and using the Bianchi identity gives
\[
k^2 f^{\lambda \nu} - 2k^{[\lambda} \{ \ldots \}^{\nu]} = 0,
\]
(2.11)
where \( \{ \ldots \} \) denotes the square bracket terms in eq.(2.10). These are a set of linear equations for the components of \( f^{\lambda \nu} \), with coefficients of \( O(k^2) \).

Now introduce a local Lorentz frame via the vierbeins \( e^a_\mu \), \( a = 0, 1, 2, 3 \), so that
\[
g_{\mu \nu} = \eta_{ab} e^a_\mu e^b_\nu,
\]
(2.12)
where \( \eta_{ab} = \text{diag}(1, 1, 1, 1) \). In this frame, eq.(2.11) becomes a set of linear equations for the field strength components \( f^{ab} = k^a a^b - k^b a^a \), where the directions of \( a^a \) determine the polarisation.

It is useful to introduce the following notation for the antisymmetric combination of vierbeins,
\[
U^{ab}_\mu = e^a_\mu e^b_\nu - e^b_\mu e^a_\nu.
\]
(2.13)
Contracting the field tensor \( f^{\lambda \nu} \) with \( U^{ab}_\mu \) projects the Lorentz components, i.e. \( f^{ab} = (1/2) f^{\mu \nu} U^{ab}_\mu \). The appropriate choice of a linearly independent set of three components will depend on the specific background considered. For example, contracting eq.(2.11) with \( U^{01}_\mu \), \( U^{02}_\mu \) and \( U^{23}_\mu \) provides a set of three linearly independent equations for \( f^{01} \), \( f^{02} \) and \( f^{23} \). Writing these equations in matrix form, the eigenvalues give the light-cone conditions (1.2) on the photon momentum while the eigenvectors determine the polarisations.

In the following section we carry through this programme for the Reissner-Nordström metric characterising a charged black hole. Appendix A contains the analogous discussion for a general anisotropic electromagnetic background.
3. Photon Propagation in Reissner-Nordström Spacetime

The Reissner-Nordström metric is an exact classical solution of the coupled Einstein-Maxwell equations. The exterior region beyond the horizon describes the gravitational and electromagnetic fields of a charged black hole.

The Einstein-Maxwell equations are

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi T_{\mu\nu}, \]  

where the electromagnetic energy-momentum tensor is

\[ T_{\mu\nu} = F_{\mu\sigma} F^{\sigma\nu} - \frac{1}{4} g_{\mu\nu} F^{\sigma\tau} F_{\sigma\tau}, \]

and

\[ D_{\mu} F^{\mu\nu} = 0. \]

The Reissner-Nordström metric in standard Schwarzschild coordinates is \[ ds^2 = -V(r)^2 dt^2 + V(r)^{-2} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

where

\[ V(r) = 1 - \frac{2M}{r} + \frac{Q^2}{4\pi r^2}. \]

There is an event horizon at \( r_+ = M \left(1 + \sqrt{1 - \frac{Q^2}{4\pi M^2}}\right). \) For a charge \( Q \simeq Q_0, \) \( r_+ \) is extremely close to the Schwarzschild radius \( 2M. \)

A local Lorentz frame can be defined with vierbeins

\[ e^a_\mu = \text{diag} (V(r), V(r)^{-1}, r, r \sin \theta) \]

and a basis of 1-forms \( e^a = e^a_\mu dx^\mu \) orthonormal w.r.t. \( \eta_{ab}. \) The connection is found from the torsion-free Cartan equation

\[ de^a + \omega^a_{\; b} \wedge e^b = 0 \]

and the curvature 2-form \( R^a_{\; b} \equiv \frac{1}{2} R^a_{\; bcd} e^c \wedge e^d \) from

\[ R^a_{\; b} = d\omega^a_{\; b} + \omega^a_{\; c} \wedge \omega^c_{\; d}. \]

The non-zero components of the curvature tensor are

\[ R^0_{\; 101} = -(V V'' + V/2) \]
\[ R^0_{\; 202} = R^0_{\; 303} = R^1_{\; 212} = R^1_{\; 313} = -\frac{1}{r} VV' \]
\[ R^2_{\; 323} = \frac{1}{r^3} (1 - V^2) \]
plus other components related to these by standard symmetries. The Riemann curvature can therefore be expressed as

\[ R_{\mu \nu \sigma \tau} = -(A - B)(g_{\mu \sigma} g_{\nu \tau} - g_{\mu \tau} g_{\nu \sigma}) - (3A - 4B)U_{\mu \nu}^0 U_{\sigma \tau}^0 + (3A - 2B)U_{\mu \nu}^{23} U_{\sigma \tau}^{23}, \]  

(3.10)

where

\[ A = \frac{M}{r^3}, \quad B = \frac{1}{4\pi} \frac{Q^2}{r^4}. \]  

(3.11)

The electromagnetic field strength is

\[ F_{\mu \nu} = -\frac{1}{4\pi} \frac{Q}{r^2} U_{\mu \nu}^0, \]  

(3.12)

describing a radial electric field.

Now consider the equation of motion for photon propagation. To simplify notation, define

\[ \ell_\nu = k^\mu U_{\mu \nu}^{01}, \quad n_\nu = k^\mu U_{\mu \nu}^{02}, \quad m_\nu = k^\mu U_{\mu \nu}^{23}. \]  

(3.13)

Now contract eq.(2.11) successively with \( U_{\nu \lambda}^{01}, U_{\lambda \nu}^{02} \) and \( U_{\lambda \nu}^{23} \) (or, equivalently, contract eq.(2.10) with \( \ell_\nu, n_\nu \) and \( m_\nu \)). This gives the following system of linear equations*,

\[
\begin{bmatrix}
  k^2 + (\alpha + \beta + \delta + 2\epsilon) \ell^2 & 0 & \gamma \ell m \\
  (\beta + \delta + \epsilon) \ell n & k^2 + (\alpha + \epsilon) \ell^2 & \gamma m n \\
  (\beta + \delta + \epsilon) \ell m & 0 & k^2 + (\alpha + \epsilon) \ell^2 + \gamma m^2 \\
\end{bmatrix}
\begin{bmatrix}
  f_{01}^0 \\
  f_{02}^0 \\
  f_{23}^0 \\
\end{bmatrix}
= 0,
\]

(3.14)

where

\[
\begin{align*}
\alpha &= \frac{4b}{m^2} B \\
\beta &= -\frac{8c}{m^2} (3A - 4B) \\
\gamma &= \frac{8c}{m^2} (3A - 2B) \\
\delta &= -\frac{32z}{m^4} \frac{1}{4\pi} B \\
\epsilon &= -\frac{8y}{m^4} \frac{1}{4\pi} B.
\end{align*}
\]

(3.15)

In terms of Lorentz frame components, \( \ell^2 = k^0 k^0 - k^1 k^1 \), \( n^2 = k^0 k^0 - k^2 k^2 \), \( m^2 = k^2 k^2 + k^3 k^3 \) and \( \ell m = 0 \), \( \ell n = -k^1 k^2 \), \( m n = -k^0 k^3 \). Notice that \( \ell m = 0 \) in eq.(3.14).

* In fact, the \( k^2 \) terms in the diagonal entries are all multiplied by a factor \( 1 + \frac{2bB}{m^2} - \frac{8c(A-B)}{m^2 B} \), but this merely gives a higher order correction of \( O(\alpha^2) \) in the light-cone condition and must be dropped for consistency.

The following formulae, derived using the Bianchi identity, are useful in simplifying these equations:

\[
\begin{align*}
k_{\mu} \ell_{\nu} f_{\mu \nu} &= k^2 f_{01}^0 \\
\ell_{\nu} \ell_{\sigma} U_{\nu \tau}^{01} f_{\sigma \tau} &= -\ell^2 f_{01}^0 \\
\ell_{\nu} \ell_{\sigma} U_{\nu \tau}^{01} f_{\sigma \tau} &= -\ell^2 f_{01}^0 \\
\ell_{\nu} \ell_{\sigma} U_{\nu \tau}^{01} f_{\sigma \tau} &= \ell n f_{01}^0 \\
m_{\nu} \ell_{\sigma} U_{\nu \tau}^{01} f_{\sigma \tau} &= k^2 f_{23}^0 \\
m_{\nu} \ell_{\sigma} U_{\nu \tau}^{01} f_{\sigma \tau} &= -\ell^2 f_{23}^0 \\
m_{\nu} \ell_{\sigma} U_{\nu \tau}^{01} f_{\sigma \tau} &= \ell m f_{01}^0.
\end{align*}
\]
Setting the determinant to zero in eq.(3.14) gives the condition

$$(k^2 + (\alpha + \beta + \delta + 2\epsilon)\ell^2) \left( k^2 + (\alpha + \epsilon)\ell^2 + \gamma m^2 \right) \left( k^2 + (\alpha + \epsilon)\ell^2 \right) = 0.$$  (3.16)

So we have in general three roots, viz.

(i) $k^2 + (\alpha + \beta + \delta + 2\epsilon)\ell^2 = 0,$ corresponding to $f_{ab} \propto (k_a\ell_b - k_b\ell_a)$

(ii) $k^2 + (\alpha + \epsilon)\ell^2 + \gamma m^2 = 0,$ corresponding to $f_{ab} \propto (k_a m_b - k_b m_a)$

(iii) $k^2 + (\alpha + \epsilon)\ell^2 = 0,$ corresponding to $f_{ab} \propto (k_a a_b - k_a a_b),$ where $a_a = m^2 \ell.m\ell_a + \ell^2 m.m_a - \ell^2 m^2 n_a.$

These three roots are the generalised light-cone conditions anticipated in sect. 1. To understand their implications, it is simplest to consider two special cases:

(a) Radial photon motion:
Set $k^2 = k^3 = 0.$ The solution (i) just produces an overall factor multiplying $\eta_{ab}k^a k^b,$ i.e.

$$(1 - (\alpha + \beta + \delta + 2\epsilon))(-k^0 k^0 + k^1 k^1) = 0.$$  (3.17)

So the light cone is unchanged and the photon velocity $|k^0 / k^1| = 1.$

The solutions (ii) and (iii) are degenerate and also give $|k^0 / k^1| = 1.$ We therefore find that for radial photon motion there is no change in the light cone or the photon velocity for either physical polarisation.

(b) Orbital photon motion:
Now set $k^1 = k^2 = 0$ and consider photon propagation in the orbital ($\phi$) direction $k^3 \neq 0.$

(i) The first root gives the modified light cone

$$-(1 - (\alpha + \beta + \delta + 2\epsilon))k^0 k^0 + k^3 k^3 = 0,$$  (3.18)

i.e. a photon velocity

$$\left| \frac{k^0}{k^3} \right| = 1 + \frac{1}{2} (\alpha + \beta + \delta + 2\epsilon),$$  (3.19)

corresponding to radial polarisation, i.e. $a^a \propto \delta^{a1}.$

(ii) The second root gives a light cone

$$-(1 - (\alpha + \epsilon))k^0 k^0 + (1 + \gamma)k^3 k^3 = 0,$$  (3.20)

i.e.

$$\left| \frac{k^0}{k^3} \right| = 1 + \frac{1}{2} (\alpha + \gamma + \epsilon),$$  (3.21)
corresponding to the other transverse polarisation, $a^a \propto \delta^{a2}$.

(iii) The third root, giving $|k^0/k^3| = 1 + \frac{1}{2}(\alpha + \epsilon)$, corresponds to the unphysical photon polarisation $a^a \propto (k^1\delta^{a0} - k^0\delta^{a1})$.

Substituting back using eqs. (3.15) and (2.2), we therefore find

$$|k^0/k^3|_{r \text{ pol}} = 1 + \frac{\alpha}{30 \pi m^2} + \frac{\alpha}{36 \pi m^2} - \frac{2 \alpha \alpha B}{45 \pi m^4}$$  \hspace{1cm} (3.22)

and

$$|k^0/k^3|_{\theta \text{ pol}} = 1 - \frac{\alpha}{30 \pi m^2} + \frac{13 \alpha}{180 \pi m^2} - \frac{7 \alpha \alpha B}{90 \pi m^4}$$  \hspace{1cm} (3.23)

These are the results quoted in the introduction, eq.(1.1). The three correction terms correspond respectively to the gravitational contribution identical to the Schwarzschild case, the indirect effect of the charge via its induced gravitational field and the direct electromagnetic contribution. The latter is always negative – the electromagnetic field reduces the velocity of both photon polarisations (see Appx. A). The induced gravitational contribution in this case is positive for both polarisations. However, as described earlier, this is hugely suppressed by an inverse power of $\alpha/m^2$ relative to the direct electromagnetic effect.

The remarkable feature of eqs. (3.22) and (3.23) is the opposite sign of the gravitational contribution for the two different polarisation states. This was the result discovered in ref.[1]. It can be traced back to the anisotropy in the curvature tensor, in particular the relative sign of the $U^{01}_{\mu\nu}U^{01}_{\sigma\tau}$ and $U^{23}_{\mu\nu}U^{23}_{\sigma\tau}$ terms in eq.(3.10).

The question of whether photon propagation is or is not superluminal in the Reissner-Nordström metric therefore depends on the relative magnitude of the corrections $\frac{1}{30 \pi m^2}$ and $-\frac{2}{45 \pi m^4}$ in eq.(3.22). This is best phrased in the parametrisation of eq.(1.1) – superluminal propagation requires

$$\frac{r}{2M} > \frac{2}{3} \left( \frac{Q}{Q_0} \right)^2,$$  \hspace{1cm} (3.27)

which is satisfied everywhere outside the horizon for charges less than the accretion limit, $Q < Q_0$. 

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4. Discussion

In this paper, we have shown how photon propagation is modified in the Reissner-Nordstr"om spacetime by vacuum polarisation effects depending on the anisotropy of the background gravitational and electromagnetic fields. The gravitational field may either increase or decrease the photon velocity from $c$ depending on its direction and polarisation, whereas the electromagnetic field always reduces the velocity. For black hole charges approximately equal to the accretion limit, there is a balance between the two corrections with superluminal propagation possible for orbital directed photons at or outside the horizon.

The fundamental questions, however, remain those confronted by Drummond and Hathrell in their original paper[1]. The most serious is the question of whether a spacelike photon momentum given by the light-cone condition (1.2) necessarily involves a causal paradox, i.e. the possibility of a spatially closed motion backwards in time. As emphasised in ref.[1], two conditions have to be met to establish a violation of causality in the usual context of special relativity, viz. the existence of signal propagation with spacelike momentum and Lorentz invariance. Briefly, the argument is as follows. Suppose $A$ sends a spacelike signal to $B$. To establish a causal paradox, $B$ must be able to return a signal to the past world line of $A$, which requires a signal backwards in time in this frame. Now, since the first signal $A \rightarrow B$ is spacelike, it is possible to find a new frame in which it is indeed backwards in time. However, in order to conclude from this that a signal $B \rightarrow A$ can be sent which is backwards in time in the original frame, we must assume Lorentz invariance. This final assumption is not valid in our context, since eq.(1.2) determining the nature of the signal propagation is a frame-dependent, Lorentz non-invariant condition.

Another crucial issue is whether the phenomenon of “faster than light” propagation is observable, even in principle. The essential point here[1] is that since the maximum time available in the curved spacetime over which to measure a signal is ordinarily of $O(L)$, any length discrepancy due to the corrections to the photon velocity is roughly of order $\delta s \sim L \delta v \sim \alpha \frac{\lambda}{c^2} \lambda \ll \lambda_c$. But the photon wavelength $\lambda$ is restricted in our calculation to be much greater than $\lambda_c$, so it is not at all clear how this distance $\delta s$ can be resolved. We are therefore forced to conclude that a direct measurement of superluminal propagation may be impossible in the parameter range $\lambda_c \ll \lambda \ll L$.

This gives added motivation to try and extend the analysis of photon propagation to the short wavelength regime $\lambda \ll \lambda_c$ and/or to the strong curvature regime $L \simeq \lambda_c$. In terms of the effective action, relaxing the restriction $L \gg \lambda_c$ would involve summing the series of higher powers in the curvature, i.e. terms of the form $\sum_p \left( \frac{R^p}{m^{2p}} \right) F^2$. Unfortunately, this seems to be beyond all known techniques, except perhaps for very special symmetric spaces. On the other hand, relaxing the condition $\lambda \gg \lambda_c$ requires summing terms of the type $\sum_p \left( \frac{R}{m^2} \right) \left( \frac{D^p}{m^p} \right) F^2$. This resummation has been discussed in a recent paper on the heat kernel derivation of the effective action by Barvinsky et al.[12] (see also refs.[14]). We are currently investigating whether these results enable us to describe photon propagation for arbitrary $\lambda$.

Opinions in the literature[1,4-8] are divided as to whether the photon velocity for the superluminal polarisations would increase or decrease as $\lambda$ becomes small. Based
on a preliminary analysis of ref.[12] or a consideration of the large external momentum behaviour of the appropriate Feynman diagrams for vacuum polarisation, we tend to the view that for small photon wavelengths, the rôle of the electron mass is taken over by the photon momentum so that the velocity correction for the parameter range $\lambda \ll \lambda_c \ll L$ is of $O(\alpha(\lambda/L)^2)$. All polarisations would therefore travel with velocity equal to $c$ in the limit $\lambda \to 0$. The objection to this scenario is that it requires the absorptive part in the usual dispersion relation for the refractive index to be negative for the superluminal polarisations while remaining conventionally positive for the subluminal polarisation states. Such admittedly unconventional behaviour does not, however, appear to us to be ruled out although we cannot at present give a mechanism to understand to it. In any case, the issue should be settled by an explicit calculation.

Finally, we emphasise again what we see as ultimately the most important aspect of this phenomenon, viz. the violation of local Lorentz invariance by the tidal gravitational effects arising through vacuum polarisation in an interacting quantum field theory. Such a fundamental result must surely have far-reaching implications for quantum gravity.

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Appendix A. Electromagnetic Birefringence

We show here how to calculate the velocity of light in an arbitrary anisotropic (but only weakly inhomogeneous) electromagnetic field in flat spacetime. This generalises Adler’s original calculation of electromagnetic birefringence\cite{Adler}. From the perspective of this paper, it is interesting to consider this general case in order to check that the resulting photon velocity is always less than $c$ independent of the polarisation, despite the background field anisotropy. This confirms that the “faster than light” phenomenon is a gravitational effect.

We use the same techniques described in sects. 2 and 3. The relevant effective action is the familiar Euler-Heisenberg expression, viz. eq.\((2.1)\) without the curvature terms. In the geometric optics approximation, the “photon equation of motion” (cf. eq.\((2.10)\)) is simply

$$k_a f^{ab} + \frac{1}{2} \delta^{ab} F_{cd} k_a f^{cd} + \epsilon \left(F^{ac} F_{cd} k_a f^{bd} + F^{ac} F^{bd} k_a f_{cd}\right) = 0, \quad (A.1)$$

where here we define

$$\delta = -\frac{32 z}{m^4}, \quad \epsilon = -\frac{8 y}{m^4}. \quad (A.2)$$

The background electromagnetic field is

$$F_{cd} = \begin{pmatrix} 0 & -E_1 & -E_2 & -E_3 \\ E_1 & 0 & B_3 & -B_2 \\ E_2 & -B_3 & 0 & B_1 \\ E_3 & B_2 & -B_1 & 0 \end{pmatrix}, \quad (A.3)$$

that is,

$$F_{cd} = -E_1 U^0_{cd} - E_2 U^{02} - E_3 U^{03} + B_3 U_{12}^{13} - B_2 U_{13}^{12} + B_1 U_{23}^{12}, \quad (A.4)$$

where in analogy to eq.\((2.13)\) we have defined

$$U^{ab}_{cd} = \delta^a_c \delta^b_d - \delta^b_c \delta^a_d. \quad (A.5)$$

From the Bianchi identity, we have

$$f^{ab} = k^a a^b - k^b a^a. \quad (A.6)$$

This shows that for given $k^a$, only three of the $f^{ab}$ are independent. We take these to be $f^{01}$, $f^{02}$ and $f^{03}$ and express the others as

$$f^{23} = \frac{1}{k_0^2} \left(k^2 f^{03} - k^3 f^{02}\right),$$

$$f^{13} = \frac{1}{k_0^2} \left(k^1 f^{03} - k^3 f^{01}\right),$$

$$f^{12} = \frac{1}{k_0^2} \left(k^1 f^{02} - k^2 f^{01}\right). \quad (A.7)$$

We also define

$$\ell_d = k^c U^{01}_{cd}, \quad n_d = k^c U^{02}_{cd}, \quad p_d = k^c U^{03}_{cd}. \quad (A.8)$$
Again, the other possibilities are not independent:

\[ m_d = k^c U_{cd}^{23} = \frac{1}{k^0} (k^2 p_d - k^3 n_d) \]
\[ q_d = k^c U_{cd}^{13} = \frac{1}{k^0} (k^1 p_d - k^3 \ell_d) \]
\[ r_d = k^c U_{cd}^{12} = \frac{1}{k^0} (k^1 n_d - k^2 \ell_d). \]  
(A.9)

We therefore find the following useful formulae:

\[-\frac{1}{2} F_{ab} f^{ab} = \tilde{E}_1 f^{01} + \tilde{E}_2 f^{02} + \tilde{E}_3 f^{03} \]  
(A.10)

and

\[-F_{ab} k^a = \tilde{E}_1 \ell_a + \tilde{E}_2 n_b + \tilde{E}_3 p_b, \]  
(A.11)

where

\[ \tilde{E}_1 = E_1 - \frac{k^3}{k^0} B_2 + \frac{k^2}{k^0} B_3, \]  
(A.12)

e tc., together with

\[ f^{ab} \ell_a = -k^b f^{01} \quad f^{ab} n_a = -k^b f^{02} \quad f^{ab} p_a = -k^b f^{03}, \]  
(A.13)

which follow from the Bianchi identity.

With these preliminaries, a straightforward calculation along the lines of sect. 3 shows that the equation of motion (A.1) for \( f^{ab} \) reduces to the following matrix equation for the independent components \( f^{01}, f^{02} \) and \( f^{03} \):

\[ \begin{pmatrix} k^2 + (\delta + \epsilon) \tilde{E}_1 (\ldots) + \epsilon X \\ (\delta + \epsilon) \tilde{E}_1 [\ldots] \\ (\delta + \epsilon) \tilde{E}_1 \{\ldots\} \end{pmatrix} \begin{pmatrix} k^2 + (\delta + \epsilon) \tilde{E}_2 (\ldots) + \epsilon X \\ (\delta + \epsilon) \tilde{E}_2 [\ldots] \\ (\delta + \epsilon) \tilde{E}_2 \{\ldots\} \end{pmatrix} \begin{pmatrix} k^2 + (\delta + \epsilon) \tilde{E}_3 (\ldots) + \epsilon X \\ (\delta + \epsilon) \tilde{E}_3 [\ldots] \\ (\delta + \epsilon) \tilde{E}_3 \{\ldots\} \end{pmatrix} \begin{pmatrix} f^{01} \\ f^{02} \\ f^{03} \end{pmatrix} = 0, \]  
(A.14)

where

\[ (\ldots) = \tilde{E}_1 \ell^2 + \tilde{E}_2 \ell.n + \tilde{E}_3 \ell.p \]
\[ [\ldots] = \tilde{E}_1 \ell.n + \tilde{E}_2 n^2 + \tilde{E}_3 n.p \]
\[ \{\ldots\} = \tilde{E}_1 \ell.p + \tilde{E}_2 n.p + \tilde{E}_3 p^2 \]  
(A.15)

and

\[ X = \tilde{E}_1 (\ldots) + \tilde{E}_2 [\ldots] + \tilde{E}_3 \{\ldots\} \]
\[ = (k^0 E_3 - k^2 B_1 + k^1 B_2)^2 + (k^0 E_2 - k^1 B_3 + k^3 B_1)^2 \]
\[ + (k^0 E_1 - k^3 B_2 + k^2 B_3)^2 - (k^1 E_1 + k^2 E_2 + k^3 E_3)^2. \]  
(A.16)
Setting the determinant to zero, we find the following equation analogous to eq. (3.16):

\[(k^2 + \epsilon X)^2 (k^2 + (\delta + 2\epsilon)X) = 0.\]  \hspace{1cm} (A.17)

There are two coincident roots, corresponding to the modified light cone

\[k^2 + \epsilon X = 0\]  \hspace{1cm} (A.18)

and one other corresponding to

\[k^2 + (\delta + 2\epsilon)X = 0.\]  \hspace{1cm} (A.19)

To analyse the light velocities, we can now with no loss of generality choose a specific direction of photon propagation, say \(k_1^1 = k_2^2 = 0\) with \(k_0^0, k_3^3 \neq 0\). A further short calculation now shows that the photon velocities corresponding to the two distinct roots (A.18) and (A.19) are

\[\left|\frac{k^0}{k^3}\right| = 1 + \frac{1}{2} \epsilon \left[(E_1 - B_2)^2 + (E_2 + B_1)^2\right]\]  \hspace{1cm} (A.20)

and

\[\left|\frac{k^0}{k^3}\right| = 1 + \frac{1}{2} (\delta + 2\epsilon) \left[(E_1 - B_2)^2 + (E_2 + B_1)^2\right],\]  \hspace{1cm} (A.21)

the only difference being the coefficients.

The important observation is that the dependence on the background electric and magnetic fields takes the form of a sum of two perfect squares and is therefore always positive. Since both the coefficients \(\epsilon\) and \(\delta + 2\epsilon\) from the Euler-Heisenberg effective action are negative, we find that the photon velocity for an arbitrary anisotropic electromagnetic background field is always less than \(c\).
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