The interaction of the steel shell and the core concrete element in operation

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Abstract. Pipe concrete as a complex material, which effectively combines the advantages of steel and concrete, is used in the construction of high-rise and long-span structures and has good prospects for expanding the scope. At present, this is prevented by the relatively poor knowledge of the steel shell interaction with the concrete core under operational loads, when, due to the difference in the coefficients of transverse deformation of steel and concrete, the shell can detach from the core and lose its stirrup effect, which significantly increases the concrete strength. In the article in a linear-elastic formulation, the Lame problem is applied to a circular cylinder subjected to internal (for the shell) and external (for the core) uniform radial pressure, as well as an external centrally applied the longitudinal compressive force. The formulas for determining the stresses and radial displacements in the shell wall and core are obtained. The recommendations on providing the conditions for the joint operation of the shell and core, which are relevant, in particular, for the pipe-concrete kinematic racks of foundations protecting the buildings under seismic effects, are given.

Introduction

Concrete laid in a steel pipe shows compressive strength 1.5–3 times higher than when testing the standard concrete samples [1-3]. This is due to the fact that under such conditions, concrete experiences triaxial compression due to the constraint of lateral expansion deformation (the so-called stirrup effect). The effect is stronger, the stiffer the shell, that is, the greater the ratio of the pipe wall thickness to its diameter. Increasing the strength of the concrete core increases the range of the linear elastic work of the structure. Eccentric compression reduces the concrete effectiveness and the stronger, the greater the eccentricity of the load and the flexibility of the structure [1-3, 4-7]. Therefore, it is most advisable to use the pipe concrete with axial or eccentric compression with low eccentricities. Such conditions arise in kinematic-type pipe-reinforced concrete seismic insulating supports [8], which in recent decades have been increasingly used to reduce the seismic effects’ intensity on buildings and structures.

The disadvantage of the pipe-concrete structures is the difficulty of ensuring the joint operation of the steel shell and the concrete core. There are various technological and constructive methods for eliminating this drawback, for example, the device on the inner surface of the stirrup in different versions of special anchors in the zone of force transmission [4-7], the use of concrete in the core on expanding cement during hardening [3]. However, all these methods have mainly experimental justifications of a private nature.
Since the relative shortening strains of concrete in the steel shell can reach a value of 0.01 or more before failure [3], the ultimate load for such elements is of no practical value due to the inadmissibility of the corresponding strains in the supporting structures. The determining factor in this case is the deformation criterion. And at operational loads, both the shell material and the core concrete, due to an increase in its strength under conditions of triaxial compression, have a practically linear relationship between the deformations and stresses. Therefore, the theoretical justification for the interaction of the stirrup and the core of the concrete element can be constructed on the basis of the equations of the elasticity theory, namely, the Lame problem solution for the cylinder [9].

Materials and Methods
A pipe-concrete element in which the shell is a steel circular cylinder and the core is heavy concrete is considered. The steel deformation diagram is linear, the non-linear deformations are possible in concrete, the development measure of which is estimated using an elastic coefficient equal to the ratio of elastic deformation to total. Concrete tightly fills the inner cavity of the shell. The mass of the pipe and the core is evenly distributed throughout the volume, as a result of which the geometric axis is also the physical axis. The structure length ratio to the diameter of the cross section does not exceed 5, as is customary in civilian buildings. This leads to the absence of longitudinal bending and loss of the shape stability under operational loads, up to destructive. The load is compressive vertical, and is transmitted to the structure through the hard dies in the upper and lower ends evenly over the entire area, so that its resultant line coincides with the longitudinal axis of the structure. The method of transferring the load is to the entire section or only to the core. The action of the longitudinal load creates the transverse deformations of the core, shell, and interaction forces arise between them.

The purpose of the study is to determine the stresses and strains in the shell and core of a relatively short concrete element under axial compression based on the Lame problem solution for a circular cylinder [9]. The research method used to perform the research is analytical.

Discussions and Results
The action of the axial longitudinal load on the concrete element creates a transverse expansion along with longitudinal deformations in the core and shell. Provided that the transverse displacement of the outer contour of the core exceeds the transverse displacement of the inner shell contour, interaction forces arise between them. The nucleus, in an effort to expand, stretches the shell, which, resisting this, compresses the nucleus in the radial direction. Let $\rho$ be arising forces of radial interaction, tensile for the stirrup and compressive for the core. In the absence of a shell, the transverse absolute deformation of the core would be $\Delta r_{b,0}$, the presence of a shell limits its value by $w<\Delta r_{b,0}$, and the difference between these values is shown by $\Delta r_{b,0}$ due to compression of the core by the required efforts $p$. The modules quantities sum $w$ and $\Delta r_{b,0}$ should be equal to the absolute value $\Delta r_{b,0}$. Thus, in displacements, the equilibrium equation has the form:

$$w + \left[\Delta r_{b,0}(p)\right] = \Delta r_{b,0},$$

where all the terms are described above; we can only add to these descriptions that $w$ is nothing more than an increment of the inner radius of the shell under the influence of the radial pressure $p$ from the core side during its transverse expansion from the longitudinal compressive load.

To write the terms of equation (1) as the effort $p$ functions we use the Lame solution for a circular cylinder under the radial pressure action [9]:

$$\sigma_r = \frac{1}{b^2-a^2} \left( a^2 p - a^2 \frac{b^2}{r^2} p \right) = -\frac{(b/r)^2 - 1}{(b/a)^2 - 1} p,$$

$$\sigma_\theta = \frac{1}{b^2-a^2} \left( a^2 p + a^2 \frac{b^2}{r^2} p \right) = \frac{(b/r)^2 + 1}{(b/a)^2 - 1} p,$$
\[ w(r) = \frac{(1-2\nu)a^2rp + a^2b^2 p/r}{2(b^2 - a^2)G} = 1 - 2\nu + \frac{b^2}{r^2} \frac{rp}{2(b^2 / a^2 - 1)G}, \]  
where \( \sigma_r \) is the normal stress across the sites, over the sites perpendicular to the shell radius; 
\( \sigma_\theta \) is normal stress across areas parallel to the shell radius and horizontal plane; 
\( r \) is the radius modulus - vector of the point in question; 
a \( \) is the radius of the shell’s inner surface and the outer surface of the core; 
b \( \) is the radius of the outer surface of the shell; 
\( G = \frac{E}{2(1 + \nu)} \) shows the shear modulus (Lame coefficient); 
\( E \) defines the material elasticity modulus (\( E_b \) – concrete, \( E_s \) – steel); 
\( \nu \) denotes the lateral deformation coefficient (\( \nu_b \) – for concrete, \( \nu_s \) – for steel). 

Substituting \( a \) instead \( r \), we obtain from the formulas (2) - (4) the following expressions of stresses and displacements for the inner surface of the shell:

\[ \sigma_r(a) = -p, \quad \sigma_\theta(a) = \frac{(b/a)^2 + 1}{(b/a)^2 - 1} p = \frac{(a + t)^2 / a^2 + 1}{(a + t)^2 / a^2 - 1} p = \frac{1 + 2t/a + t^2 / a^2 + 1}{1 + 2t/a + t^2 / a^2 - 1} p = \frac{\eta + 1}{\eta} p, \] 
\[ w(a) = \frac{1 - 2\nu_s + b^2 / a^2}{2(b^2 / a^2 - 1)G} \frac{ap}{\frac{2}{(1 + 2t/a + t^2 / a^2 - 1)E_s / [2(1 + \nu_s)]}} \approx \frac{1 - \nu_s^2 + \eta(1 + \nu_s)}{\eta E_s} ap, \] 

where \( t = b - a \), \( \eta = t/a \) – the shell wall thickness and its relative value. In the formulas (5) and (6), the ratio \( (b/a)^2 = (1 + \eta)^2 = 1 + 2\eta + \eta^2 \) was replaced by an approximate \( 1 + 2\eta \) due to smallness \( \eta \) compared to unit. 

In addition to displacement (6), the shell wall will receive a transverse expansion deformation \( w_t \) also under vertical compressive load \( N_t \), if the external load \( N \) transmitted to the entire section:

\[ w_1 = \frac{N_s}{2\pi R t E_s} = \frac{N_s}{2\pi t E_s}, \] 

where \( R=(a+b)/2 \) – is a radius of the shell’s median surface; 
\( N_t \) – is a part of the total external compressive load \( N \) perceived by the shell wall. 

Adding (6) and (7), we obtain the total displacement \( w \) of the shell wall from the vertical load and the radial pressure:

\[ w = w(a) + w_1 = \frac{1 - \nu_s^2 + \eta(1 + \nu_s)}{\eta E_s} ap + \frac{\nu_s N_s}{2\pi t E_s} = k_{s1} ap + k_{s2} \frac{N_s}{t E_s}, \] 

where \( k_{s1} = 1 - \nu_s^2 + \eta(1 + \nu_s), \quad k_{s2} = \nu_s / (2\pi) \).

For a concrete core, which is a solid cylinder, in which \( r = a \) there is an external border, and \( r = b \), like a pipe turned inside out, it has an internal boundary, in the equations (2) - (4) \( b = 0 \) should be taken, since the internal boundary of a solid is absent:

\[ \sigma_{rb}(a) = \sigma_{o,b}(a) = -p \]

\[ w_{rb}(r) = \frac{1 - 2\nu_b}{2G_b} rp = - \frac{1 - 2\nu_b}{2\lambda_b E_b / [(1 + \nu_b)]} rp = \frac{1 - \nu_b - 2\nu_b^2}{\lambda_b E_b} rp = -k_{b1} \frac{r}{E_b} p, \] 

where \( k_{b1} = (1 - \nu_b - 2\nu_b^2) / \lambda_b; \quad \lambda_b = \varepsilon_{o,e} / \varepsilon_b \) - is the coefficient of concrete elasticity equal to the ratio of the deformation elastic part \( E_b \) to complete \( E_b \), including inelastic deformation. 

Using (10) we determine the difference \( \Delta r_{rb}(p) \) between the free \( \Delta r_{b,0} \) and the constrained \( w \) extension of the kernel from the equation (1):
\[
\Delta r_{p, b}(p) = w_p(a) = -k_{a b}(a/E_b)(-p) = k_{a b} \frac{ap}{E_b}
\] (11)

We will find the free (not constrained by the shell) transverse deformation of the concrete core in the radial direction, using the generalized Hooke’s law, so the core undergoes the volume compression;

\[
\varepsilon_{r, b} = [\sigma_r - \nu_b (\sigma_{\theta} + \sigma_z)]/(\lambda_b E_b) \approx \nu_b N_b / (\lambda_b E_b \pi a^2)
\] (12)

where according to (9) \(\sigma_r=\sigma_\theta=-p\), \(\sigma_z=N_0/(\pi a^2)\) – is the vertical normal stresses in the core from the external load part \(N_0\) falling on it.

As experiments show (for example, [3]), \(\sigma_c\) an order of magnitude or more exceeds \(\sigma_r\) and \(\sigma_\theta\). For this reason, in (12) only \(\sigma_r\) is left.

Taking into account the last expression, the free transverse deformation of the core will be obtained as the product of relative deformation \(\varepsilon_{r, b}\) to the length \(a\) base of its action:

\[
\Delta r_{p, 0} = \varepsilon_{r, b} a = \nu_b N_b / (\lambda_b E_b \pi a^2) = k_{a b} N_b / (aE_b)
\] (13)

where \(k_{a b} = \nu_b / (\pi a b)\).

Substitution of (8), (11) and (13) into the equilibrium equation (1) leads to its form:

\[
k_{s 1} \frac{ap}{\eta E_s} + k_{s 2} \frac{N_s}{E_s} + k_{b 1} \frac{ap}{E_b} = k_{b 2} \frac{N_b}{aE_b}
\]

After multiplying all the terms of this equation by \(\eta E_s\), the input designations \(a = E_b/E_s\) and the simple transformations, we find the interaction stresses of the nucleus and the wall:

\[
p = \frac{k_{b 2} \alpha \eta N_b - k_{s 2} N_s}{(k_{s 1} + k_{b 2} \alpha \eta) a^2}
\] (14)

Further calculations depend on how the load is applied to the concrete element.

Option 1. Let the axial load create uniform compression of the entire cross section and the same longitudinal deformation in the core \(\varepsilon_b\) and wall \(\varepsilon_s:\n\]

\[
\varepsilon_b = \frac{N_b}{A_b \lambda_b E_b} = \varepsilon_s = \frac{N_s}{A_s E_s} = \frac{N - N_b}{A_s E_s}.
\]

where \(N_b, N_s\) – are the part of the total external load \(N=N_b+N_s\), perceived by the core and the shell wall; \(A_b=\pi a^2\) – is the core cross-sectional area; \(A_s=\pi(b^2-a^2)\) – is the cross-sectional area of the shell wall.

From the equality

\[
\frac{N_b}{A_b \lambda_b E_b} = \frac{N - N_b}{A_s E_s}
\]

we find:

\[
N_b = \left(\frac{A_s E_s}{A_b \lambda_b E_b} + 1\right) N = \frac{\lambda_b}{\mu \alpha + \lambda_b} N, \quad N_s = N - N_b = \frac{\mu \alpha}{\mu \alpha + \lambda_b} N,
\] (15)

where \(\mu = A_s / A_b = \pi(b^2-a^2)/(\pi a^2) = b^2 / a^2 - 1 = (a + t)^2 / a^2 = 2 \eta + \eta^2 \approx 2 \eta\) – is the reinforcement coefficient if there is no reinforcement in the core concrete (at \(\eta < 0.05\) its square can be neglected, since in this case the error in comparison with \(2 \eta\) less 2.5%).

Substituting (15) into (14), we obtain

\[
\beta_1 = \frac{p_1}{\sigma_0} = \frac{\nu_b \eta - 0.5 \nu_s \mu}{(\mu \alpha + \lambda_b)(k_{s 1} / \alpha + k_{b 2} \eta)}
\] (16)

where \(\beta_1\) – is the dimensionless radial interaction parameter \(p_1\) cores and shells related to the conditional parameter of the external load \(\sigma_0 = N/(\pi a^2)\); index \(i\) at \(\beta\) and \(p\) indicates that these parameters belong to the first loading variant, i.e., both the core and the shell.

Let us pay attention to the numerator of the formula (16). Substituting its expression instead of \(\mu\) (15) shows the following:
\[ v_b \eta - 0.5v_s \mu = v_s \eta - 0.5v_s (2 \eta + \eta^2) = \eta (v_b - v_s - 0.5 \eta v_s). \]

Obviously, under the ordinary conditions, when \( v_b \sim 0.2, \ v_s \sim 0.3, \ v_b - v_s - 0.5 \eta v_s < 0 \), there is no interaction of the core with the stirrup, they perceive the external load separately, there is no stirrup effect that increases the concrete strength. It will appear only after increasing the concrete \( v_b \) transverse deformation coefficient due to the formation of the multiple longitudinal microcracks [10]. The contact of the core and the holder will occur provided that

\[ v_b > v_s (1 + 0.5 \eta), \]

and it occurs at a sufficiently high level of the concrete core stress state (order \( \sigma_c / R_p \sim 0.5 \)), therefore, to ensure the joint operation of the core and the shell from the first stages of loading applied to the entire cross section, it is necessary to apply the initial stress state using the expanding cements during hardening, supply of concrete mixture under pressure, or other technological methods.

Option 2. Another option for transferring the external load to the concrete structure is through a hard stamp directly to the core. Vertical forces from the core to the shell wall are transmitted by the friction forces of concrete on steel in the form of the shear stresses, the distribution of which is assumed to be uniform over the entire contact surface. In this case, the longitudinal forces in the shell wall and in the core will be described by the formulas

\[ N_s = 2 \pi R_s \tau_z = 2 \pi R_s \zeta_0 \rho = 2 \pi \rho_0 \zeta a^2 p, \quad N_b = N - N_s, \]

\[
\tau = f_0 \rho - \text{is the shear stress at the contact of the shell and core, due to Coulomb friction of concrete over the surface of the shell when it is displaced under the influence of an external load } N; \text{ the shear stress is proportional to radial pressure } p \text{ with a coefficient of proportionality equal to the coefficient of friction } f_0;
\]

\[ z, \zeta = z/a - \text{is the coordinate along the longitudinal axis of the concrete element with the beginning at the upper end, directed down; } \zeta - \text{shows its relative value.} \]

After substituting (18) in (14) and the simple transformations, we have:

\[ \beta_2 = \frac{p_2}{\sigma_0} = \frac{v_b \alpha \eta}{k_{p1}}, \]

where \( k_{p1} = k_{\lambda} + k_{\mu} \lambda_0 \alpha \eta + 2v_b \alpha \eta f_0 \zeta + v_\beta \lambda_0 f_0 \zeta; \)

\[ \beta_2 \text{ – is the dimensionless parameter of radial interaction } p_2 \text{ of the core and shell; index } 2 \text{ at } \beta \text{ and } p \text{ indicates that these parameters in this case relate to the second variant of loading, i.e., only the core, with a uniform distribution of friction over the entire contact surface of the core and shell.} \]

The assumption of a uniform distribution of the friction forces over the entire contact surface of the core and shell is too approximate. According to the principle of Saint-Venant concrete movements relative to the shell wall at some distance \( z \approx 2a \) or \( \zeta \approx 2 \) from the application location \( \sigma_0 \) (that is, from the upper end) will not, and the distribution of friction forces will be close to triangular with a maximum \( \tau = f_0 \rho \) at the upper end.

In this case, the vertical forces \( N_s \) and \( N_b \) will have the form:

\[ N_s = \begin{cases} 2 \pi R_s \left(1 - 0.5 \zeta \zeta / \zeta_0 \right) \zeta a^2 p, & \zeta \leq \zeta_0, \\ 2 \pi \rho_0 \zeta a^2 p, & \zeta > \zeta_0, \end{cases} \quad N_b = N - N_s, \]

where \( \zeta = l/a - \text{is the relative length of the plot } l \text{ pipe-concrete element, counted from the upper end, within which the friction forces act.} \]

Substitution of (20) into (14) gives the following result:

\[ \beta_3 = \frac{p_3}{\sigma_0} = \frac{v_b \alpha \eta}{k_{p2}}, \]

where \( \beta_3 \text{ – is the dimensionless radial interaction parameter } p_3 \text{ cores and shells; index } 3 \text{ of } \beta \text{ and } p \text{ indicates that in this case these parameters relate to the second load case with a maximum of the shear stresses at the upper end and a linear decrease in them to zero at a distance } l \text{ from the loaded end;} \]
$k_{p^2} = \begin{cases} k_1 \alpha_0 + k_2 \alpha_0 \alpha \eta + (v_b \alpha \eta + 0.5v_b \alpha \eta)(2 - \zeta / \zeta_t) f_0 \zeta, & \zeta \leq \zeta_t \\ k_3 \alpha_0 + k_4 \alpha_0 \alpha \eta + (v_b \alpha \eta + 0.5v_b \alpha \eta) f_0 \zeta_t, & \zeta > \zeta_t \end{cases}$

Figures 1 and 2 below show some results of numerical calculations using the formulas (16), (19), (21). They allow to determine the main parameters of the interaction of the core and the shell, including taking into account the inelastic concrete work.

**Figure 1.** Dependence of the core and shell interaction intensity on the thickness $\eta$ shell walls and the concrete class (blue line - less durable concrete; $\beta_2(V_b, \lambda_b, \alpha, \eta, \zeta, f_0)$)

**Figure 2.** The dependence of the core and shell interaction intensity along the structure axis; ordinate axis designations $\beta_2(V_b, \lambda_b, \alpha, \eta, \zeta, f_0)$, $\beta_3$ – with a uniform distribution of friction forces over the core and shell contact; $\beta_4$ – with a variable distribution of friction forces within the area of their action; $\beta_4$ – outside the friction range

**Summary**

The study of the pipe-concrete elements’ operational stage is important in theoretical and practical terms, since the conditions of interaction of the shell wall and the concrete core are of great importance for determining the magnitude of the destructive forces. Here, the methods of transferring the load to the structure, the ratio of wall thickness, strength and deformation properties of concrete, the variability of the coefficient transverse from the level of the stress state are important.
The formulas obtained in the article make it possible to take into account many of these factors for assessing the concrete element shell and the core interaction.

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