Adaptive control for the nonlinear suspension systems with stochastic disturbances and unknown time delay

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ABSTRACT
In this work, an adaptive control problem is investigated for the nonlinear active suspension systems (ASSs) with stochastic disturbances and time delay. Also, in the suspension system, the nonlinearity of the springs and the dampers are considered. In order to be closer to the actual system, the external random disturbances are not neglected in this paper. It is worth mentioning that unknown time-varying delay is considered in this study to facilitate the treatment of transmission time delay. Based on Lyapunov theory and backstepping technique, the presented adaptive controller is able to ensure that all the signals in close-loop system are uniformly ultimately bounded in probability. Finally, the simulation results further illustrate the effectiveness of the proposed approach.

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1. Introduction
The automotive suspension of Montazeri-Gh & Soleymani (2010) is the device that connects the vehicle body to vehicle wheels. It mainly consists of spring systems and damping systems. By constantly adjusting the linkage in the suspension system, the resistance generated by road disturbances is effectively attenuated, thus guaranteeing the stability of the car during normal driving. In active suspension systems, actuators are installed between the body and wheels and adjust the vehicle to a certain state to improve safety and comfort.

Compared with passive suspension systems and semi-active ones, the active ones can be adjusted independently according to different road conditions and have certain advantages in vehicle vibration damping. Therefore, the active suspension system (Li et al., 2014) can bring better control effects and ride experience. In the last decade, active suspension systems have gradually become a hot topic of research in the control field. In Fei et al. (2021), Li et al. (2019) and Huang et al. (2018), the suspension systems were considered in linear form. To ensure the practical performance of the system, the nonlinear suspension systems were considered in Na et al. (2020) and Wen et al. (2017). Note that the main features of the mentioned intelligent adaptive control approaches were that they only can deal with the suspension systems control problem, while random disturbance terms were not taken into account.

Stochastic disturbances are the common phenomena in real systems. Therefore, the researches on stochastic control problems have great promise and attracted widespread attention. To study the adaptive control problem of stochastic nonlinear systems with nonlinear parameters, the authors in Tian and Xie (2007), Xie and Tian (2009) and Li et al. (2012), solved the adaptive calming problem for a class of higher-order nonlinear parameters by transforming them into linear parameterized systems using the parameter separation principle. In addition, the authors in Ye (2005) investigated the adaptive control problem for the stochastic nonlinear systems by constructing the switch-type controller to adjust the controller parameters in a switching manner. In Gao and Yuan (2013), the authors studied the adaptive suppression problem for chained stochastic incomplete systems and obtained an adaptive controller under backstepping control techniques. Also, in Chen et al. (2010), the authors studied the problem of adaptive neural network for stochastic systems with inmeasurable states.

Subsequently, the neuro-based minimal learning back stepping control scheme was developed in Behrouz et al. (2020) for ASSs with additional stochastic terms. In Mao et al. (2017), the authors developed a fault detection scheme for passive suspension systems of rail vehicles with external disturbances and random process signals. However, in the above mentioned literatures, the time-delay phenomena were not considered.

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Time delays are often found in the state, the control input and measurements of many real industrial systems. There exists the Krasovskii and Razumikhin method for Lyapunov function. The former applies to a wider range of problems, and it usually leads to more reliable results than the latter. The authors in Wang et al. (2015) developed adaptive fuzzy tracking controller for the stochastic nonlinear time-delay systems. The problem of adaptive neural intelligent control was investigated in Wang et al. (2017) for the nonlinear time-delay systems with inmeasurable state. Further, the authors in Jia et al. (2018) proposed the control approach for highly systems with unknown state time delays. Recently, the authors in Sun et al. (2021) proposed fuzzy control approach can reduce possible chattering phenomena and achieved better control performance.

However, it is worth mentioning that none of the systems studied above were the ASSs. Inspired by the existing research results, this paper studies an adaptive control strategy based on the backstepping technique for the ASSs to ensure the safety and comfort of the vehicle with stochastic disturbances and unknown time delay. Although the authors in Wang et al. (2017) and Jia et al. (2018) also studied the adaptive control issues with time delays, the ASSs and the stochastic problems are not be fully considered.

2. System descriptions and preliminary knowledge

An electromagnetic active suspension model is shown in Figure 1, which can be used to describe the vertical problem of an active suspension system. \( m_1 \) and \( m_2 \) represent the spring mass and the unspring mass, respectively. \( F_s \) and \( F_d \) are the spring force and damper force, respectively, and \( F_t \) is the force produced by the tire. \( s_1 \in R \) and \( s_2 \in R \) are the displacements of spring and non-spring, and \( s_r \) represents the road displacement input.

The electromagnetic actuator force is defined as \( F_e = 2\pi T_m/p_t = Ai \), \( T_m = K_i \) is the force moment, \( u_d = u_s - u_e = Ld_i/dt + Ri \) is the motor control voltage, and \( A \) = \( 2\pi K_i/p_t \) represents the motor constant.

Based on Newton’s second law, the dynamic equation of the suspension system is

\[
\begin{align*}
\dot{s}_1 &= F_d + F_s + F_u \\
\dot{s}_2 &= -F_d - F_s - F_u + F_t
\end{align*}
\]

where \( F_u = Ai \) and \( F_s = k_{s1}(s_1 - s_2) + k_{s2}(s_1 - s_2)^3 \), \( F_d = k_d(s_1 + s_2) \), \( F_t = k_t(s_2 - s_r) \) are the forces produced by spring, damper and tire, respectively. Defining the state variables \( x_1 = s_1, x_2 = \dot{s}_1, x_3 = s_2, x_4 = \dot{s}_2, x_5 = i \). Rewritten the dynamic equation (1), it becomes

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{1}{m_1} [k_d(x_2 - x_4) + k_{s1}(x_1 - x_3) \\
&+ k_{s2}(x_1 - x_3)^3 + Ai] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{1}{m_2} [-k_d(x_2 - x_4) - k_{s1}(x_1 - x_3) \\
&- k_{s2}(x_1 - x_3)^3 - Al + k_t(x_3 - s_r)] \\
\dot{x}_5 &= \frac{u - Ri}{L} \\
y &= x_1
\end{align*}
\]

Then, an active suspension system with random disturbances and unknown time-varying delay as follows

\[
\begin{align*}
\dot{x}_1 &= x_2 \ dt + g_1(X)^T \ dw \\
\dot{x}_2 &= \left( \frac{1}{m_1} [k_d(x_2 - x_4) + k_{s1}(x_1 - x_3) \\
&+ k_{s2}(x_1 - x_3)^3 + A x_3] \right) \ dt + g_2(X)^T \ dw \\
\dot{x}_3 &= x_4 \ dt + g_3(X)^T \ dw \\
\dot{x}_4 &= \left( \frac{1}{m_2} [-k_d(x_2 - x_4) - k_{s1}(x_1 - x_3) \\
&- k_{s2}(x_1 - x_3)^3 - A x_5 + k_t(x_3 - s_r)] \right) \ dt + g_4(X)^T \ dw \\
\dot{x}_5 &= \left( \frac{u - R i}{L} + h(y_d(t - v_1(t))) \right) \ dt + g_5(X)^T \ dw \\
y &= x_1
\end{align*}
\]

where \( X = (x_1, x_2, x_3, x_4, x_5)^T \) are system states, \( x_1 \) and \( x_3 \) represent the displacement of the body and the wheels, respectively. \( u \) is the system control input. Assume that \( g_i(X) \ (i = 1, \ldots, 5) \) as an unknown smooth function and the incremental covariance \( E(dw \cdot dw^T) = b(t)b(t)^T \ dt \). \( h(\cdot) \) is the unknown nonlinear function, \( v_1 \) is unknown.
time-varying delay function satisfies $|\nu_1| \leq \nu$, the derivative of $\nu_1$ satisfies $\dot{\nu}_1 \leq \nu^r \leq 1$, where $\nu$ and $\nu^r$ are known constants.

**Control Objective:** For the electromagnetic active nonlinear suspension system with unknown time delay, an adaptive controller is designed to keep the vertical motion of the vehicle stable.

**Assumption 2.1 (Li et al., 2016):** The unknown nonlinear smooth function $h_j(y)$ satisfies

$$|h_j(y)|^2 \leq \sigma r(t)H_j(\sigma r(t)) + \bar{H}_j(\nu_d(t)) + \zeta_j(t) \quad (4)$$

where $\sigma r$ is the tracking error, $H_j(\cdot)$ and $\bar{H}_j(\cdot)$ ($j = 1, \ldots, 5$) are bounded functions that satisfy $H_j(0) = 0$, $\zeta_j(\cdot) \geq 0$.

**Assumption 2.2 (Ji & Xi, 2006):** The disturbance covariance $g_i^T b_i(t)b_i^T(t)g_i \leq \psi (i = 1, \ldots, 5)$ is bounded and $\psi$ is a positive constant.

**Lemma 2.1 (Young’s inequality):** For any $x, y \in \mathbb{R}^n$, there exists an inequality such that

$$x^T y \leq \frac{a^p}{p} \|x\|^p + \frac{1}{qa^q} \|y\|^q \quad (5)$$

where $a > 0$, the constants $p > 1$ and $q > 1$ satisfy $(p - 1)(q - 1) = 1$.

**Lemma 2.2 (Sun et al., 2016):** Let $\varphi(X_m) = [\varphi_2(X_m), \varphi_3(X_m), \varphi_4(X_m), \varphi_5(X_m)]^T$ be the basis function vector of a RBFNN and $X_m = [x_1, x_2, x_3, x_4, x_5]^T$. For any positive integer $n \leq m$, then

$$\|\varphi(X_m)\|^2 \leq \|\varphi(X_n)\|^2 \quad (6)$$

### 3. Adaptive control design

Based on the backstepping technique, an adaptive controller for the ASSs with time delay and stochastic disturbances is designed. Then, the stability of the control scheme is proved by Lyapunov theory and backstepping technique.

**Step 1:** Define $\nu_j (j = 1, \ldots, 5)$ as the tracking errors, $y_d$ as the tracking signal, one has

$$\nu_1 = x_1 - y_d \quad (7)$$

where $y_d$ represents the tracking error signal, and $y_d, \dot{y}_d$ are bounded.

Then, we can get

$$d\nu_1 = (x_2 - \dot{y}_d) dt + g_1(X)^T dw \quad (8)$$

Consider the following Lyapunov function candidate

$$V_1 = \frac{1}{4} \nu_1^4 \quad (9)$$

Since $\nu_2 = x_2 - x_d$, calculating the infinitesimal generator $\ell$ by (9), we can obtain

$$\ell V_1 = \nu_i^2 [\nu_2 + x_d - \dot{y}_d] + \frac{3}{2} \nu_1^2 g_1 b_1^T g_1 \nu_2 \quad (10)$$

According to the Young’s inequality, we can get

$$\frac{3}{2} \nu_1^2 g_1 b_1^T g_1 \nu_2 \leq \frac{3\psi^2}{4n_1^3} + \frac{3n_1^2}{4} \nu_1^4 \quad (11)$$

where $n_1$ is a positive design constant. Then, (10) becomes

$$\ell V_1 \leq \nu_i^2 \left[\nu_2 + x_d - \dot{y}_d + \frac{3n_1^2}{4} \nu_1 - \frac{3\psi^2}{4n_1^3}\right] + \frac{3\psi^2}{4n_1^3} \quad (12)$$

The virtual controller $x_d$ is designed as

$$x_d = -c_1 \nu_1 + \dot{y}_d - \frac{3n_1^2}{4} \nu_1 \quad (13)$$

Substituting (13) into (12), we can get

$$\ell V_1 \leq -c_1 \nu_1^4 + \nu_1^3 \nu_2 + D_1 \quad (14)$$

where $D_1 = \frac{3\psi^2}{4n_1^3}$.

**Step 2:** Choose the Lyapunov function $V_2$ as

$$V_2 = V_1 + \frac{1}{4} \nu_2^4 + \frac{1}{2} \Theta_2^2 \quad (15)$$

where $\Theta_2$ is the estimation of $\Theta_1^*$ which will be defined later and $\Theta_2 = \Theta_1^* - \Theta_1$.

Calculating the infinitesimal generator $\ell$ by (15), we can obtain that

$$\ell V_2 = -c_1 \nu_1^4 + \nu_1^3 \nu_2 + D_1 + \nu_2^3 d\nu_2 + \frac{3}{2} \nu_1^2 g_1 b_1^T g_2 - \Theta_2^T \Theta_2 \quad (16)$$

Then

$$d\nu_2 = \left\{\frac{1}{m_1} \left[k_d(x_2 - x_4) + k_s(x_1 - x_3) + k_s(x_2(x_1 - x_3))^3 + Ax_5 - m_1 \dot{x}_d\right] dt + g_2(X)^T dw \quad (17)$$

Substituting (17) into (16), one has

$$\ell V_2 = -c_1 \nu_1^4 + \nu_1^3 \nu_2 + D_1 - \Theta_1^T \Theta_2 + \nu_2^3 [k_d(x_2 - x_4) + k_s(x_1 - x_3) + k_s(x_2(x_1 - x_3))^3 + Ax_5 - m_1 \dot{x}_d] dt + g_2(X)^T dw \quad (18)$$
Since \( f_2 \) is an RBF NN \( \theta_i^T \phi(X) \) (\( i = 2, \ldots, 5 \)) such that \( f_i = \theta_i^T \phi(X) + \varepsilon_i, i = 2, \ldots, 5 \), where \( X = (x_1, x_2, x_3, x_4, x_5) \), and \( \varepsilon_i \) denotes the approximation error which satisfies \( |\varepsilon_i| < \delta_i \) and \( \delta_i \) is a positive constant. Then, (18) becomes

\[
\ell V_2 = -c_1 \sigma_1^4 + \sigma_2^3 \sigma_2 + D_1 - \dot{\Theta}_2^T \dot{\Theta}_2 + \sigma_2^3 [\theta_2^T \varphi_2(X)] \\
+ \varepsilon_2 + x_3 + \frac{3}{2} \sigma_2^2 g_1 b_2 b_2^T g_2
\]  

(19)

According to Lemmas 2.1 and 2.2, one gets

\[
\sigma_2^3 \varphi_2^T \varphi_2(X) \leq \| \sigma_2^2 \| \| \varphi_2(X) \| \\
\leq \sigma_2^3 \| \varphi_2(X) \| \\
\leq \frac{1}{2a_2^2} \sigma_2^5 \theta_2^T \varphi_2(X) + \sigma_2^2 \frac{a_2^2}{2}
\]  

(20)

\[
\sigma_2^3 \sigma_2 \leq \frac{2}{3} \sigma_2^3 + \frac{1}{3} \sigma_2^9 \sigma_2^3
\]  

(21)

\[
\frac{3}{2} \sigma_2^2 g_1 b_2 b_2^T g_2 \leq \frac{3}{4} \sigma_2^3 + \frac{3}{4} \sigma_2^4
\]  

(22)

where \( \Theta_2^* = \| \theta_2^* \|^2 \), and \( \xi_2, n_2, a_2 \) are positive design constants.

Substituting (20)–(22) into (19), one has

\[
\ell V_2 \leq -c_1 \sigma_1^4 - \frac{2}{3} \sigma_2^3 + \frac{3}{4} \sigma_2^3 + D_1 \\
+ \sigma_2^3 \left[ \frac{1}{2a_2^2} \sigma_2^5 \theta_2^T \varphi_2(X) + \sigma_2^2 \frac{a_2^2}{2} \right] \\
+ \varepsilon_2 + \frac{1}{3} \sigma_2^9 \sigma_2^3 + \sigma_2^3 \sigma_2 \\
+ \frac{3}{2} \sigma_2^2 g_1 b_2 b_2^T g_2
\]  

(23)

Since \( \sigma_3 = x_3 - x_{2d} \), the virtual controller \( x_{2d} \) is designed as

\[
x_{2d} = -c_2 \sigma_2 - \frac{1}{3} \sigma_2^9 \sigma_2^3 - \frac{1}{2a_2^2} \sigma_2^5 \theta_2 \varphi_2(X) \varphi_2(X) \\
- \frac{3n_2^2}{4} \sigma_2 - \frac{1}{2} \sigma_2^3
\]  

(24)

The parameter adaptive law \( \dot{\Theta}_2 \) is designed as

\[
\dot{\Theta}_2 = \frac{1}{2a_2^2} \sigma_2^5 \theta_2 \varphi_2(X) \varphi_2(X) - \sigma_2 \dot{\Theta}_2
\]  

(25)

where \( \sigma_2 > 0 \) is design parameter.

Substituting (24)–(25) into (23), one has

\[
\ell V_2 \leq -c_1 \sigma_1^4 - c_2 \sigma_2^4 + \frac{2}{3} \sigma_2^3 + D_1 + \sigma_2^2 \sigma_3 \\
+ \sigma_2 \dot{\Theta}_2 \dot{\Theta}_2 + \frac{3}{4} \sigma_2^2 \sigma_3 + \frac{1}{2} \sigma_2^2 + \frac{a_2^2}{2}
\]  

(26)

Based on Lemma 2.1, we can get

\[
\sigma_2 \dot{\Theta}_2 \dot{\Theta}_2 \leq - \sigma_2^2 \theta_2^2 + \sigma_2^2 \theta_2^2
\]  

(27)

From (27), one has

\[
\ell V_2 \leq -c_1 \sigma_1^4 - c_2 \sigma_2^4 + \sigma_2^3 \sigma_3 - \sigma_2^2 \theta_2^2 + D_2
\]  

(28)

where \( D_2 \) can be written as \( D_2 = \frac{3}{4} \sigma_2^2 + \frac{3}{4} \sigma_2^2 + \frac{a_2^2}{2} \).

Step 3: Choosing the Lyapunov function \( V_3 \) as

\[
V_3 = V_2 + \frac{1}{4} \sigma_2^4 + \frac{1}{2} \Theta_2^2
\]  

(29)

Calculating the infinitesimal generator \( \ell \) by (29), we can obtain

\[
\ell V_3 \leq -c_1 \sigma_1^4 - c_2 \sigma_2^4 + \sigma_2^3 \sigma_3 - \sigma_2^2 \theta_2^2 + D_2 + \frac{3}{2} \sigma_2^2 \sigma_3 g_1 b_2 b_2^T g_3 - \dot{\Theta}_2^T \dot{\Theta}_3
\]  

(30)

Similarly, we can obtain that

\[
d \sigma_3 = (x_4 - x_{2d}) dt + g_3(X)^T dw
\]  

(31)

Substituting (31) into (30), one gets

\[
\ell V_3 \leq -c_1 \sigma_1^4 - c_2 \sigma_2^4 + \sigma_2^3 \sigma_3 - \sigma_2^2 \theta_2^2 + D_2 + \sigma_2^3 \dot{\sigma}_4 + \sigma_2^2 \dot{\Theta}_3
\]  

(32)

where \( f_3 = -x_{2d} \theta_3^T \psi(X) + \varepsilon_3 \), and \( \sigma_4 = x_4 - x_{2d} \).

According to the Young’s inequality and Lemma 2, we can obtain

\[
\sigma_3^2 \theta_2^T \psi(X) \leq \frac{1}{2a_3^2} \sigma_3^6 \theta_2^T \psi(X) \psi_2(X) + \sigma_2^2 \frac{a_2^2}{2}
\]  

(33)

\[
\sigma_2^3 \sigma_3 \leq \frac{2}{3} \sigma_2^3 + \frac{1}{3} \sigma_3^2 \sigma_2^3
\]  

(34)

\[
\frac{3}{2} \sigma_2^3 g_3 b_3 b_3^T g_3 \leq \frac{3}{4} \sigma_2^2 \sigma_2^3 + \frac{3n_2^2}{4} \sigma_2^3
\]  

(35)

where \( a_3, c_3, n_3 \) are positive definite constants.

Substituting (33)–(35) into (32) results in

\[
\ell V_3 \leq -c_1 \sigma_1^4 - c_2 \sigma_2^4 + \frac{2}{3} \sigma_2^3 \sigma_3 - \sigma_2^2 \theta_2^2
\]  

(36)
Substituting (40) into (39) results in

\[ \sigma_{\ell x}d_{j} \leq -c_1 \sigma_{4}^4 - c_2 \sigma_{2}^4 + \frac{2}{3} \sigma_{3}^{3/2} - \frac{\sigma_2}{2} \dot{\Theta}_3^2 \]

\[ + x_{3d} + \frac{\sigma_2}{2} \Theta_3^2 + D_2 + \sigma_3^3 \left( \sigma_4 \right) \]

\[ + \sum_{j=1}^{3} c_j \sigma_j^2 - \sum_{j=2}^{3} \frac{g_j}{2} \dot{\Theta}_j^2 + \sigma_3^3 \sigma_4 + D_3 + \sigma_3^3 d_{\sigma_4} \]

\[ + \sum_{j=1}^{3} \frac{\sigma_2}{2} \Theta_3^2 + \Theta_3^3 \dot{\Theta}_3 + \sigma_3^2 \Theta_3^3 \]

(36)

The virtual controller \( x_{3d} \) is designed as

\[ x_{3d} = -c_3 \sigma_3 - \frac{\sigma_2}{2} \Theta_3 + \frac{1}{2} \sigma_3 \Theta_3^2 \]

\[ - \frac{1}{2} \sigma_3 \Theta_3^2 \]

(37)

The parameter adaptive law \( \dot{\Theta}_3 \) is designed as

\[ \dot{\Theta}_3 = \frac{1}{2} \sigma_3^2 \Theta_3^2 (x_3) \sigma_2 (x_3) - \sigma_3 \dot{\Theta}_3 \]

(38)

where \( \sigma_3 > 0 \) is design parameter.

Substituting (37)–(38) into (36), one gets

\[ \ell V_3 \leq -c_1 \sigma_4^4 - c_2 \sigma_4^4 - c_3 \sigma_4^4 + \frac{2}{3} \sigma_3^{3/2} - \frac{\sigma_2}{2} \dot{\Theta}_3^2 \]

\[ + D_2 + \frac{1}{2} \sigma_3^2 + \sigma_3^4 \sigma_4 + \sigma_3^3 \dot{\Theta}_3 + \frac{a_3^2}{2} \]

\[ + \sum_{j=1}^{3} c_j \sigma_j^2 - \sum_{j=2}^{3} \frac{g_j}{2} \dot{\Theta}_j^2 + \sigma_3^3 \sigma_4 + D_3 \]

\[ + \sigma_3^3 d_{\sigma_4} \]

(39)

Similar to (27), it follows that

\[ \sigma_3 \dot{\Theta}_3 \dot{\Theta}_3 \leq -\sigma_3 \dot{\Theta}_3^2 + \sigma_3^2 \Theta_3^3 \]

(40)

Substituting (40) into (39) results in

\[ \ell V_3 \leq -c_1 \sigma_4^4 - c_2 \sigma_4^4 - c_3 \sigma_4^4 + \frac{\sigma_2}{2} \dot{\Theta}_3^2 \]

\[ - \frac{\sigma_3}{2} \Theta_3^2 + \sigma_3^3 \sigma_4 + D_2 + \frac{1}{2} \sigma_3^3 + \frac{2}{3} \sigma_3^{3/2} \]

\[ + \sum_{j=1}^{3} c_j \sigma_j^2 - \sum_{j=2}^{3} \frac{g_j}{2} \dot{\Theta}_j^2 + \sigma_3^3 \sigma_4 + D_3 \]

\[ + \sum_{j=1}^{3} \frac{\sigma_2}{2} \Theta_3^2 + \Theta_3^3 \dot{\Theta}_3 + \sigma_3^2 \Theta_3^3 \]

\[ + \sum_{j=1}^{3} \frac{\sigma_2}{2} \Theta_3^2 + \Theta_3^3 \dot{\Theta}_3 + \sigma_3^2 \Theta_3^3 \]

(41)

where

\[ D_3 = \sum_{j=1}^{2} \frac{\sigma_2}{2} \Theta_3^2 + \sum_{j=1}^{3} \frac{\sigma_2}{2} \Theta_3^2 + \sum_{j=1}^{3} \frac{a_3^2}{2} \]

\[ + \sum_{j=1}^{3} \frac{\sigma_2}{2} \Theta_3^2 + \sigma_3^2 \Theta_3^3 \]

Step 4: Choosing the Lyapunov function \( V_4 \) as

\[ V_4 = V_3 + \frac{1}{4} \sigma_4^4 + \frac{1}{2} \dot{\Theta}_4^2 \]

(42)

The time derivative of \( V_4 \) along the solution of \( \sigma_4 \) is

\[ \ell V_4 \leq -\sum_{j=1}^{3} c_j \sigma_j^2 - \sum_{j=2}^{3} \frac{g_j}{2} \dot{\Theta}_j^2 + \sigma_3^3 \sigma_4 + D_3 + \sigma_3^3 d_{\sigma_4} \]

\[ + \frac{3}{2} \sigma_4^3 g_4^T b_4 b_4^T g_4 - \dot{\Theta}_4^T \dot{\Theta}_4 \]

(43)

The time derivative of \( \sigma_4 \) is

\[ d_{\sigma_4} = \left\{ \frac{1}{m_2} \left[ -k_d (x_2 - x_4) - k_1 (x_1 - x_3) - k_2 (x_1 - x_3) \right] - x_3 \right\} \]

\[ + g_4 (X)^T d \dot{w} \]

(44)

Substituting (44) into (43), one has

\[ \ell V_4 \leq -\sum_{j=1}^{3} c_j \sigma_j^2 - \sum_{j=2}^{3} \frac{g_j}{2} \dot{\Theta}_j^2 + \sigma_3^3 \sigma_4 + D_3 + \sigma_3^3 \frac{1}{m_2} \]

\[ \times \left[ -k_d (x_2 - x_4) - k_1 (x_1 - x_3) - k_2 (x_1 - x_3)^3 \right] \]

\[ - A x_5 + k_1 (x_3 - s_r) - m_2 x_{3d} \]

\[ + \frac{3}{2} \sigma_4^3 g_4^T b_4 b_4^T g_4 - \dot{\Theta}_4^T \dot{\Theta}_4 \]

(45)

Due to \( f_4 = -k_4 (x_2 - x_4) - k_1 (x_1 - x_3) - k_2 (x_1 - x_3)^3 \)

\[ + k_1 (x_3 - s_r) - m_2 x_{3d} \) is the unknown nonlinear smooth function, we can assume that \( f_4 = \theta_4^T \psi_4 (X) + \varepsilon_4 \).

Then, the derivative of \( V_4 \) can be written as

\[ \ell V_4 \leq -\sum_{j=1}^{3} c_j \sigma_j^2 - \sum_{j=2}^{3} \frac{g_j}{2} \dot{\Theta}_j^2 + \sigma_3^3 \sigma_4 + D_3 \]

\[ + \sigma_4^3 \left[ -A x_5 \right] \]

\[ + \theta_4^T \psi_4 (X) + \varepsilon_4 \]

\[ + \frac{3}{2} \sigma_4^3 g_4^T b_4 b_4^T g_4 - \dot{\Theta}_4^T \dot{\Theta}_4 \]

(46)

Since \( \sigma_5 = x_5 - x_{Ad} \), the derivative of \( V_4 \) can be written as

\[ \ell V_4 \leq -\sum_{j=1}^{3} c_j \sigma_j^2 - \sum_{j=2}^{3} \frac{g_j}{2} \dot{\Theta}_j^2 + \sigma_3^3 \sigma_4 + D_3 \]

\[ + \sigma_4^3 \left[ \theta_4^T \psi_4 (X) + \varepsilon_4 - \frac{A}{m_2} x_5 \right] \]

\[ - \frac{A}{m_2} x_{4d} \]

\[ + \frac{3}{2} \sigma_4^3 g_4^T b_4 b_4^T g_4 - \dot{\Theta}_4^T \dot{\Theta}_4 \]

(47)
Based on the Young's inequality and Lemma 2.2, one has
\[
\sigma_4^3 \dot{\varphi}_4^T \varphi_4(X) \leq \frac{1}{2\sigma_4^2} \sigma_4^5 \dot{\varphi}_4 \varphi_4^T (\dot{\varphi}_4) \varphi_4 + \frac{\sigma_4^2}{2} \tag{48}
\]
\[
\sigma_4^3 \varphi_4 \leq \frac{2}{3} \frac{3}{2} s_4 + \frac{1}{3} \varepsilon_4 \tag{49}
\]
\[
\frac{3}{2} \sigma_4^2 g \dot{b} \dot{b} \dot{b} \leq \frac{3}{4} \psi^2 \frac{2}{4n_4} + \frac{3n_4^2}{2} \tag{50}
\]
where \(a_4, \varepsilon_4, n_4\) are positive definite constants.

Substituting (48)–(50) into (47), it follows that
\[
\ell V \leq -\sum_{j=1}^{3} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 + \frac{2}{3} s_4^{3/2} + D_3 + \frac{3}{2} \psi^2 \frac{2}{4n_4} \tag{51}
\]
\[
+ \frac{\sigma_4^3}{3s_4} + \frac{3n_4^2}{4 \sigma_4} + \frac{\sigma_4^2}{2} \leq -\dot{\varphi}_4 \dot{\varphi}_4 \tag{52}
\]

The virtual controller \(x_{4d}\) is designed as
\[
x_{4d} = \frac{m_2}{A} \left[ c_4 \sigma_4 + \frac{1}{2} \sigma_4 \dot{\varphi}_4 (\dot{\varphi}_4) \varphi_4 + \frac{1}{3} \frac{3}{4} s_4^3 \right] \tag{53}
\]

The parameter adaptive law \(\dot{\sigma}_4\) is designed as
\[
\dot{\sigma}_4 = \frac{1}{2\sigma_4^3} \sigma_4^2 \dot{\varphi}_4 (\dot{\varphi}_4) \varphi_4 - \sigma_4 \dot{\varphi}_4 \tag{54}
\]
where \(\sigma_4 > 0\) is design parameter.

Substituting (53) into (52), one has
\[
\ell V \leq -\sum_{j=1}^{4} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 - \frac{A}{m_2} \sigma_4 \varphi_4 + D_3 + \frac{\sigma_4^3}{2} \frac{3}{2} s_4^{3/2} \tag{55}
\]
\[
+ D_4 + \frac{3}{4} \psi^2 \frac{2}{4n_4} \tag{56}
\]

Similar to (27), it follows that
\[
\sigma_4 \dot{\varphi}_4 \leq -\sigma_4 \dot{\varphi}_4 + \frac{\sigma_4^2}{2} \frac{3}{2} s_4 \tag{57}
\]

Substituting (53) into (54), one gets
\[
\ell V \leq -\sum_{j=1}^{4} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 - \frac{A}{m_2} \sigma_4 \varphi_4 + D_4 \tag{58}
\]
where
\[
D_4 = \sum_{j=2}^{4} \frac{\sigma_j}{2} \frac{3}{2} s_4^{3/2} + \sum_{j=1}^{4} \frac{\sigma_j^2}{2} \tag{59}
\]

Step 5: Choosing the Lyapunov function \(V\) as
\[
V = V_4 + \frac{1}{4} \sigma_4^2 + \frac{1}{2} \dot{\varphi}_4^2 + M \tag{60}
\]
where \(M = \frac{e^{\left(\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\right)}}{\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\})\) is a positive definite function and \(\delta\) is a constant.

Then, we can obtain
\[
\ell V \leq -\sum_{j=1}^{4} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 - \frac{A}{m_2} \sigma_4 \varphi_4 + D_4 \tag{61}
\]

Substituting (59)–(60) into (58), one has
\[
\ell V \leq -\sum_{j=1}^{4} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 - \frac{A}{m_2} \sigma_4 \varphi_4 + D_4 \tag{62}
\]

where \(M = \frac{e^{\left(\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\right)}}{\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\})\) is a positive definite function and \(\delta\) is a constant.

Then, we can obtain
\[
\ell V \leq -\sum_{j=1}^{4} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 - \frac{A}{m_2} \sigma_4 \varphi_4 + D_4 \tag{63}
\]

where \(M = \frac{e^{\left(\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\right)}}{\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\})\) is a positive definite function and \(\delta\) is a constant.

Then, we can obtain
\[
\ell V \leq -\sum_{j=1}^{4} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 - \frac{A}{m_2} \sigma_4 \varphi_4 + D_4 \tag{64}
\]

where \(M = \frac{e^{\left(\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\right)}}{\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\})\) is a positive definite function and \(\delta\) is a constant.

Then, we can obtain
\[
\ell V \leq -\sum_{j=1}^{4} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 - \frac{A}{m_2} \sigma_4 \varphi_4 + D_4 \tag{65}
\]

where \(M = \frac{e^{\left(\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\right)}}{\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\})\) is a positive definite function and \(\delta\) is a constant.

Then, we can obtain
\[
\ell V \leq -\sum_{j=1}^{4} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 - \frac{A}{m_2} \sigma_4 \varphi_4 + D_4 \tag{66}
\]

where \(M = \frac{e^{\left(\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\right)}}{\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\})\) is a positive definite function and \(\delta\) is a constant.

Then, we can obtain
\[
\ell V \leq -\sum_{j=1}^{4} \frac{c_j \sigma_j^2}{2} - \sum_{j=2}^{4} \frac{\sigma_j}{2} \dot{\varphi}_j^2 - \frac{A}{m_2} \sigma_4 \varphi_4 + D_4 \tag{67}
\]

where \(M = \frac{e^{\left(\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\right)}}{\psi^2 \varphi_4^T \varphi_4 + \psi_5^T \psi_5 + \varepsilon_5\})\) is a positive definite function and \(\delta\) is a constant.
According to the Young's inequality and Lemma 2.2, we can get

\[
\sigma_s^2 \theta_s^T \varphi_s(X) \leq \frac{1}{2a_s^2} \sigma_s^6 \theta_s^T \varphi_s(X) \psi_s(X) + \frac{a_s^2}{2}
\]

(63)

\[
-\frac{A}{m_2} \sigma_s^2 \sigma_s \leq \frac{2}{3} \frac{A}{m_2} \sigma_s^{3/2} + \frac{1}{3} \frac{A}{m_2} \sigma_s^3
\]

(64)

\[
\frac{3}{2} \frac{\sigma_s^2}{m_2} \sigma_s g_s b_s^T g_s \leq \frac{3\psi^2}{4n_5^2} + \frac{3n_5^2}{4} \sigma_s^4
\]

(65)

\[
\sigma_s^2 h(y_d(t - v_1)) \leq \frac{1}{2} \sigma_s^6 + \frac{1}{2} h_2(y_d(t - v))
\]

\[
\leq \frac{1}{2} \sigma_s^6 + \frac{1}{2} \sigma_s^3(t) H(\sigma_s(t))
\]

\[
+ \frac{1}{2} \sigma_s^3 h(y_d(t - v)) + \zeta(t - v)
\]

(66)

The parameter adaptive law \( \dot{\vartheta}_s \) is designed as

\[
\dot{\vartheta}_s = \frac{1}{2a_s^2} \sigma_s^6 \theta_s^T \varphi_s(X) \psi_s(X) - \sigma_s \dot{\vartheta}_s
\]

(70)

where \( \sigma_s > 0 \) is the design parameter.

Then, one has

\[
\ell V \leq -\sum_{j=1}^{5} c_j \sigma_j^2 - \sum_{j=2}^{4} \frac{a_j}{2} \hat{\vartheta}_j^2 + D_4 + \frac{1}{2} \delta_2^2 + \frac{2}{3} \frac{A}{m_2} \sigma_s^{3/2}
\]

\[
+ \frac{3\psi^2}{4n_5^2} + \frac{\vartheta^3/2}{3(1 - v^*)} - \delta M + \frac{1}{2} \tilde{H}(y_d(t - v))
\]

\[
+ \zeta(t - v) + \sigma_s \dot{\vartheta}_s \tilde{\vartheta}_s + \frac{1}{2} \sigma_s^2
\]

(71)

According to Young's inequality, the terms in (66) get

\[
\sigma_s \dot{\vartheta}_s \tilde{\vartheta}_s \leq \frac{\sigma_s^2}{2} \hat{\vartheta}_s^2 + \frac{\sigma_s^2}{2} \hat{\vartheta}_s^2
\]

(72)

Substituting (72) into (71) results in

\[
\ell V \leq -\sum_{j=1}^{5} c_j \sigma_j^2 - \sum_{j=2}^{4} \frac{a_j}{2} \hat{\vartheta}_j^2 - \delta M + D_5
\]

(73)

where \( D_5 = \sup_{t \geq 0} |D_5(t)| \) and \( D_5(t) = \sum_{j=2}^{5} \frac{a_j}{2} \hat{\vartheta}_j^2
\]

\[
+ \frac{3\psi^2}{4n_5^2} + \frac{\vartheta^3/2}{3(1 - v^*)} + \frac{1}{2} \tilde{H}(y_d(t - v)) + \zeta(t - v).
\]

Let \( C = \min\{2c_1, 2c_2, 2c_3, 2c_4, 2c_5, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \delta_2\} \) and \( D = \max(D_i) \), we can obtain that

\[
\ell V \leq -CV + D
\]

(74)

Based on (74), one has

\[
E[\ell V(t)] \leq -e^{-Ct} V(0) + \frac{D}{C}
\]

(75)

From (75), it can be shown that the signals \( x_{ij}, (j = 1, 2, 3, 4) \) and \( \vartheta, \dot{\vartheta}, (j = 2, 3, 4, 5) \) are bound in probability. Meanwhile, the tracking error \( \sigma_1 \) satisfies that

\[
|\sigma_1| \leq \sqrt{2(V(0)e^{-Ct} + D/C)}
\]

Therefore, the suspension system (3) is proved to be stable and the movement limitation can be fulfilled.
4. Simulation study

In this part, the validity and feasibility of the designed solution is verified.

Considered changes in the road surface

\[ s_1 = \begin{cases} 
0.1 \sin t & 30 \leq t \leq 40 \\
0 & \text{other} 
\end{cases} \]  \hspace{1cm} (76)

In the simulation, system parameters in (3) are chosen as:

\[ m_1 = 300 \text{ kg}, m_2 = 60 \text{ kg}, k_{s1} = 16000 \text{ N/m}, k_{s2} = 8000 \text{ N/m}, k_d = 1000 \text{ N/m}, k_i = 200000 \text{ N/m}, K_i = 0.314 \text{ Nm/A}, R = 1.2 \Omega, L = 6 \text{ H}. \]

Then, we can obtain that

\[ F_d = 1000(v_1 + v_2), F_s = 16000(s_1 - s_2) + 800(s_1 - s_2)^3, F_u = 65.7i, F_t = 200000(s_2 - s_r). \]

The initial value \( x_1(0) = 0.01 \) and the rest of the initial values are 0. In addition, the design parameters in this paper are chosen as follows:

\[ c_1 = 0.2, c_2 = 0.4, c_3 = 1, c_4 = 2, c_5 = 4, n_1 = 0.15, n_2 = 0.25, n_3 = 0.5, n_4 = 0.15, n_5 = 0.1, \sigma_2 = 10, \sigma_3 = 14, \sigma_4 = 2, \sigma_5 = 3, \]

\[ \varsigma_2 = 2, \varsigma_3 = 1, \varsigma_4 = 0.5, \varsigma_5 = 1, \gamma_2 = 2, \gamma_3 = 3, \gamma_4 = 1, \gamma_5 = 1, \]

\[ a_2 = 0.5, a_3 = 2, a_4 = 1, a_5 = 0.4, \nu = 0.2, \nu^* = 0.6, \text{ and } \tau = 0.5. \]

In addition, the function \( g(y) = \sin^2(x_1) \cos(x_1) \), and the reference signal \( y_d = 0 \) is selected.

The simulation results are shown in Figures 2–9. Among them, Figures 2–6 express the vertical displacements trajectories of the car’s body and wheels, their vertical vibration speeds, and current intensity, respectively. The bounded trajectory of the force generated by the actuator is given in Figure 7. Figure 8 depicts the curve of the tracking error and the adaptive laws curves are expressed in Figure 9. From the above simulation results, it can be seen from the simulation results in Figures 2–9 that the control scheme proposed in this paper ensures the stability of the vertical vibration of the car, thus improving the comfort of the passenger ride.
5. Conclusion

This paper develops an adaptive control scheme for ASSs with unknown time delay and stochastic disturbances. The main objective is to enhance the comfort of the passenger ride. Based on the Lyapunov theory and the backstepping technique, the adaptive controller has been constructed. In addition, the stochastic disturbance and unknown time delay have been considered. Furthermore, all signals in the close-loop have been shown to be UUB in probability. The simulation results verify the effectiveness of the proposed control method. In future work, perhaps failure and finite time control will be considered on the basis of a full vehicle active suspension.

Disclosure statement

No potential conflict of interest was reported by the author.

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