Abstract

We consider the bounds for the values of higgs mass $m_H$ and the mass of the extra quarks and leptons $m_{\text{extra}}$ derived from the stability of electroweak vacuum and from the absence of Landau pole in Higgs potential. We find that in the case of the absence of new physics up to the GUT scale the bounds on masses of 4th generation are so strong that one can hope to discover it at LEP2.
Last year CDF and D0 collaborations announced about observation of $t$-quark – the last missing member of the third generation. The first one, $\tau$-lepton, was discovered 20 years ago, i.e. at the time when practically nobody begged for a new quark-lepton generations. From LEP1 experimental results we have learnt already that the sequence of the generations with light neutrino is completed with $N_f$(light neutrino) = 3. As for the theory, nobody suggested yet a good explanation of the fact that $N_f = 3$, or of the fact that neutral leptons are massless. So it is natural to look for new sequential generations $(T_B)$ and $(N_L)$ $(i = 4, 5, ...)$ with heavy neutral lepton $N$: $m_N > \frac{1}{2}m_Z$.

There exist a few direct and indirect bounds on the masses of these new sequential generations.

**Experimental bounds.**

From LEP1.5 searches it follows that

$$m_E > 62\text{GeV}$$

(1)

$$m_N > 58.9\text{GeV}$$

Bounds on the masses of new quarks depend crucially on the assumptions about their decay. For example, for quarks, stable enough to leave the detector, the limit is

$$m_{T,B} > 139\text{GeV}$$

(2)

For unstable quarks this number is lower. It is rather safe to say that at the moment absolute bound for quarks is somewhere near 100 GeV. These are, so to say, direct bounds. They demonstrate that new generations should be rather heavy.

**Theoretical bounds.**

The study of radiative corrections for LEP1 data produces additional indirect bounds. The main facts are that the radiative corrections for LEP1 observables are unexpectedly small (of the order of $10^{-3}$) and that the contributions of heavy quarks and leptons into low-energy observables (e.g. into LEP1 observables) are finite even for heavy mass goes to infinity (the absence of decoupling of chiral matter). So to protect the successful description of the precision data in the framework of the Standard Model from undesirable new contributions we have to impose some bounds on new physics.

First of all following to the Veltman’s arguments we conclude that the masses of isopartner ($m_T$ and $m_B$, $m_N$ and $m_L$) should be almost degenerate. Indeed for the case of very different masses $m_T$ and $m_B$ we effectively get a violation of SU(2)$_L$ symmetry at low energy and, as a result, a large low-energy loop corrections of the order of

$$\left(\frac{\alpha_{W,Z}}{\pi}\right) \frac{m_T^2 - m_B^2}{m_Z^2} \sim 10^{-2} \frac{m_T^2 - m_B^2}{m_Z^2}$$

(3)

where $\alpha_{W,Z}$ are the weak coupling constants. Since there is no room for large corrections to the Standard Model values for electroweak observables, the masses of the new quarks and leptons in SU(2) doublets should be approximately degenerate, i.e.

$$\frac{|m_T^2 - m_B^2|}{m_Z^2} \lesssim 1, \quad \frac{|m_E^2 - m_N^2|}{m_Z^2} \lesssim 1.$$
So hereafter we assume, that

\[ m_T \simeq m_B \simeq m_Q \]

\[ m_E \simeq m_N \simeq m_L \]

(To reduce the number of parameters we assume also that \( m_Q \simeq m_L = m_{\text{extra}} \)).

The study of the radiative corrections with degenerate and heavy masses is a little bit more subtle matter than the previous case. The result is that starting from \( m_{\text{extra}} \simeq 60 \text{ GeV} \) (LEP 1.5 bounds) the additional contribution to LEP observables due to extra generation rather weakly depend on \( m_{\text{extra}} \) and the effect of one new generation can be compensated by increasing of the fitted value of \( m_{\text{top}} \) by 10 GeV [3]. The experimental accuracy for \( m_{\text{top}} \) is of the order of 20 GeV, the accuracy of the fitted value of \( m_{\text{top}} \) from LEP1 data is of the order of 10 GeV. So direct measurements and precision LEP data allow to have one or two new sequential heavy generations.

But there is another way to attack the problem and to get the upper bounds both on the masses and on the number of new generations [4]. The point is that heavy fermions change the vacuum energy (i.e. the higgs potential) through radiative correction [5, 6]. Roughly speaking the heavy fermions give a negative contribution into higgs potential due to Fermi statistic. In one loop approximation this contribution looks like

\[
\Delta V(\phi) = -\frac{1}{16\pi^2} \sum_{f=Q,L} \left( \frac{N_{fc}^4 m_f^4}{\eta^4} \right) \phi^4 \ln \frac{\phi^2}{\eta^2}, \text{ where } N_Q^c = 3, \ N_L^c = 1
\]

So if heavy fermions are not accompanied by scalars (or by vectors) to compensate this negative contribution, the electroweak vacuum \( \phi = \eta \) becomes unstable for large value of \( \phi \). There are rather subtle bounds on the values of the parameters for selfconsistent theory. For the Standard Model the experimental value of top mass \( m_t = 180 \text{ GeV} \) is very close to the edge of the region for allowed values of heavy mass. So for SM the vacuum is already oversaturated by heavy top alone and there is practically no room for new heavy fermions (without new scalars).

Technically this problem reduces to the solution of renormgroup equations for running coupling constants. With the account of radiative corrections the renormalization group improved higgs potential can be presented in the following form [6]:

\[
V[\phi] = -\frac{1}{2} \mu^2(t)[G(t)\phi]^2 + \frac{1}{4} \lambda(t)[G(t)\phi]^4
\]

where \( \phi \) is the higgs field, \( t = \ln(\phi/\eta) \), \( \eta = 246 \text{ GeV} \) is v.e.v. of \( \phi \) in electroweak vacuum, \( \mu(t) \) and \( \lambda(t) \) are running constants and \( G(t) \) is determined by anomalous dimension of the field \( \phi \).

For \( \phi \sim \eta \) the initial values of \( \mu \) and \( \lambda \) govern \( V(\phi) \) behaviour, while for \( \phi \gg \eta \) the radiative corrections to \( \lambda(t) \) becomes essential. For \( \phi \gg \eta \) the term \( \sim \phi^2 \) is negligible and only the behaviour of \( \lambda(t) \) is crucial for the selfconsistency of the theory.

If \( \lambda(t) \) becomes negative at some value \( \phi_0 \), then the \( V(\phi) \) minimum \( \phi = \eta \), that corresponds to our electroweak vacuum, becomes unstable. So if we wish to live in the stable Universe, we have to avoid such values of the parameters that lead to vacuum instability.
On the other hand for large higgs mass $m_H$ the coupling constant $\lambda(t)$ becomes infinite at some value $t_0 = \ln \phi_0/\eta$. This is so called Landau pole in $\lambda(t)$. (Such behaviour for selfinteraction of scalar and for Yukawa coupling was first discovered in perturbation theory in 50th. Later it was confirmed by computer calculations and by more rigorous arguments). So to avoid strong interaction in Higgs sector we demand the absence of Landau poles in $\lambda(t)$.

So our restriction is: $0 < \lambda(t) < \infty$ up to the scale $\Lambda$ at which new physics begins. In order to get $\lambda(t)$ behavior one needs the renormalization group equations which determine the behavior of $\lambda$, Yukawa coupling constants of $t$-quark and new quarks and leptons with Higgs doublet and gauge coupling constants. These equations can be found in literature [4]. Let us present here the renormalization group equation for the running value $\lambda(t)$ in a theory with $N$ generations of heavy fermions with the degenerate masses:

$$
\frac{d\lambda}{dt} = \frac{3}{2\pi^2} \lambda^2 + \frac{\lambda}{4\pi^2} \left[ 3g_t^2 + 3N(g_T^2 + g_B^2) + N(g_E^2 + g_N^2) \right] - \frac{1}{8\pi^2} \left[ 3g_t^4 + 3N(g_T^4 + g_B^4) + N(g_E^4 + g_N^4) \right] - \frac{3}{16\pi^2} \lambda(3g_t^2 + g^2) + \frac{9g_t^4}{384\pi^2} g^2 + \frac{9g_t^2}{192\pi^2} g^3 + \frac{27}{384\pi^2} g^4,
$$

where $g$ and $g'$ are SU(2) and U(1) coupling constants and constants $g_i$ determine the masses of the corresponding fermions by the formula: $m_i = \frac{g_i(0)}{\sqrt{2}}$, $g_t(0) = 1.035$ corresponds to $m_t = 180$ GeV. We should add to equation (4) the renormgroup equations for $g, g'$, SU(3) coupling constant $g_3$ and $g_i$ and numerically integrate the system of the coupled differential equations. Results of the integration for the case of one heavy generation are presented in Fig. 1. If the value of $m_4$ is small, then we approximately get the allowed interval of the values of $m_H$ for $m_t = 180$ GeV in the Standard Model, well-known from literature [5].
For all values of ultraviolet cutoff $\Lambda$ the low lines which represent $\lambda = 0$ stability bound go up with growing of $m_4$. The physical reason for such behavior is clear – the third term in equation (6) becomes larger and a heavier Higgs is required to get a positive potential for heavier fermions. The upper lines which represent Landau pole bound are governed by the first term in (6) and are almost constant for $\Lambda > 10^{10}$ GeV. However, for $\Lambda = 10^5$ GeV new phenomena occur – the second term in (6) becomes essential and an upper curve goes down for increasing $m_4$. So the allowed interval of $m_4$ values shrinks.

From Fig. 1 we see that for traditional Grand Unified Theories, for which new physics does appear only at $\Lambda \sim 10^{15}$ GeV, the bounds for the mass of 4th generation is very low, $m_4 \leq 100$ GeV. This is a region available for LEP2 research. As for higgs mass, in this case it is fixed between 160 and 180 GeV.

If we decrease the bounds for new physics down to $\Lambda = 10^{10}$ GeV, the bounds for $m_4$ increases up to $m_4 < 140$ GeV, i.e. that is exactly CDF bound for stable quarks (see eq. (2)).

The introduction of the additional heavy generations will restrict the allowed values of $m_{extra}$ and $m_{Higgs}$. For example, for $N = 3$ and $\Lambda = 10^{10}$ GeV only $m_{extra} < 90$ GeV is allowed (see Fig. 2). In our analysis we use the one loop renormalization group potential. It is not difficult to take into account the second loop as well. In this case we expect the change of the bounds by approximately 10 GeV, i.e. not very drastically.

Conclusions.

We see that in the SM we are allowed to add only one (or two) new sequential generation with rather low masses. They could be discovered at LEP2.

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