Vector field localization and negative tension branes

Massimo Giovannini *

Institute of Theoretical Physics, University of Lausanne

BSP-1015 Dorigny, Lausanne, Switzerland

Abstract

It is shown that negative tension branes in higher dimensions may lead to an effective lower dimensional theory where the gauge-invariant vector fields associated with the fluctuations of the metric are always massless and localized on the brane. Explicit five-dimensional examples of this phenomenon are provided. Furthermore, it is shown that higher dimensional gauge fields can also be localized on these configurations with the zero mode separated from the massive tower by a gap.

To appear in Physical Review D

*Electronic address: Massimo.Giovannini@ipt.unil.ch
I. INTRODUCTION

Suppose that the space-time contains non-compact extra-dimensions and consider, for sake of concreteness the following $D$-dimensional geometry

$$ds^2 = a^2(w)[dt^2 - dx_1^2 - ... - dx_d^2 - dw^2],$$

with one bulk coordinate $w$ and with $D = d + 2$ space-time dimensions. Even in the absence of gravitational interactions, five-dimensional domain-wall solutions allow the localization of fermionic zero modes [1]. This type of multi-dimensional domain-wall solutions can be obtained by setting the warp factors to a constant. From a strictly gravitational point of view the four-dimensional Planck mass would not be finite. In this sense the increase of the dimensionality of space-time represents a tool in order to obtain an effective lower dimensional theory where chiral fermions may be successfully localized. Indeed, this approach led to a very interesting development in the analysis of chiral gauge theories on the lattice. In particular, it was shown that the massless fermionic zero mode is still present if the five-dimensional continuous space is replaced by a lattice [2]. Hence, it was realized that chiral symmetry can be realized in different ways avoiding the known problem of doubling of fermionic degrees of freedom [3]. A crucial ingredient, in this context, was the use of the so-called Wilson-Ginzparg relation [4].

In spite of the fact that an increase in the dimensionality of space-time may lead to localized chiral fermions a problem may still remain as far as vector fields are concerned [5]. In particular, a higher-dimensional domain-wall solution of the type of [1] does not lead to the localization of vector fields. More specifically, neither the vector fields generated as fluctuations of the higher-dimensional metric, nor the gauge fields can be localized on the same wall to which the fermionic zero-modes are attached. The localization of vectors fields is the subject of the present analysis. It is well known that there are various mechanisms aiming at the localization of gauge fields in higher-dimensional contexts [6–10]. In most of these approaches the idea is to analyze the situation where the four-dimensional Planck mass
\[ M_P^2 \simeq M^3 \int dwa^d(w), \quad (1.2) \]

is finite. In such a background it can be shown, in general terms, that the graviton zero mode is also localized leading to ordinary gravity on the wall [11,12]. A particularly simple example of this phenomenon is provided by five-dimensional AdS space where, however, neither the gauge fluctuations coming from the geometry [13–15] nor the minimally coupled higher dimensional gauge fields can be simply localized \(^{1}\).

Suppose now to drop this hypothesis. Suppose, in other words, that the four-dimensional Planck mass is not finite. Is it possible to localize spin one fields on such a configuration? As in [1,2] gravitational interactions are used here as an effective tool in order to localize higher spins but not as an interaction which should be, by itself, localized. In this sense, the present approach is more modest than the one proposed in [6–10] where all the higher spins should be, in principle, localized.

Our logic is, in short, the following. Consider, for sake of concreteness, a five-dimensional warped geometry of the type of Eq. (1.1) with \(d = 3\). In this type of warped geometry vector modes may come both from the gauge field living in the bulk and from the vector fields arising as fluctuations of the five-dimensional metric. In qualitative terms, the possible behavior of spin one fluctuations of the geometry and of bulk gauge fields will be now separately examined.

The spin one fluctuations of a five-dimensional geometry in the presence of a brane configuration can be expressed in terms of two equations which are written in fully gauge-invariant [i.e. coordinate independent] terms\(^{2}\):

\[ \partial_\alpha \partial^\alpha V_\mu = 0, \]

\(^1\) The reason for this statement is that the squared modulus of the zero mode leads, in this case, to a logarithmically divergent normalization integral.

\(^2\) The Greek indices label the \((3 + 1)\) (Poincaré invariant) dimensions. The Latin (uppercase) indices run over the whole five-dimensional space-time.
\[ \mathcal{V}_\mu' + \frac{3}{2} \mathcal{H} \mathcal{V}_\mu = 0, \quad (1.3) \]

where the prime denotes the derivative with respect to \( w \) and \( \mathcal{H} = (\ln a)' \). Eq. (1.3) will be specifically derived in Section II and generalized in Section III. Here some qualitative remarks will be made.

Eqs. (1.3) tell that the vector modes of the geometry are always massless and that the zero mode is localized provided

\[ \int_0^\infty \frac{dw}{a^2(w)}, \quad (1.4) \]

is convergent. By looking, simultaneously, at Eq. (1.4) and at the expressions of the curvature invariants pertaining to the geometry (1.1)

\[
R = \frac{4}{a^2}(2\mathcal{H}' + 3\mathcal{H}^2),
\]

\[
R_{MN}R^{MN} = \frac{4}{a^4}(9\mathcal{H}^4 + 6\mathcal{H}'\mathcal{H}^2 + 5\mathcal{H}'^2),
\]

\[
R_{MNAB}R^{MNAB} = \frac{8}{a^4}(2\mathcal{H}'^2 - 5\mathcal{H}^4), \quad (1.5)
\]

it can be argued that there may exist warped backgrounds where the vector modes of the geometry are localized and the bulk geometry is regular. An example of this class of backgrounds is the warp factor

\[ a(w) = e^{K|w|}, \quad (1.6) \]

with \( K > 0 \). One of the questions which will be investigated in this paper is the actual possibility of realizing such a model. As it will be shown in Section II this type of behavior will never be realized in original Randall-Sundrum set-up. This statement will be made clear by a detailed calculation. Here it is enough to notice that in the Randall-Sundrum

\[ ^3 \text{By regular we mean that the curvature invariant are analytic functions of the bulk coordinate for every } w > 0. \text{ For } w = 0 \text{ we will allow the possibility of a singularity which comes from the thin nature of the brane configuration.} \]
model the $Z_2$ symmetry, together with the requirement that the four-dimensional Planck mass is finite, implies that while the integral appearing in Eq. (1.2) is always convergent, the integral of Eq. (1.4) is always divergent (in particular for $w \to \infty$). Hence, the graviphoton is never normalizable.

The gauge fields leaving in the bulk lead to spin one fluctuations obeying equations which are different from the ones of the spin one fluctuations arising directly from the geometry. In order to have an explicit example, consider, for instance, the case of an Abelian gauge field coupled to the geometry in five dimensions. The action of such a field can be written as

$$S = -\frac{1}{4} \int d^5x \sqrt{-G} F_{AB} F^{AB}, \quad (1.7)$$

where $F_{AB} = \nabla_{[A} A_{B]} \equiv \partial_{[A} A_{B]}$ and $A_B$ is the vector potential. From Eq. (1.7), a Schrödinger-like equation leading can be obtained for the mass eigenstates. Defining $\Delta_1 = \sqrt{a}$, the normal modes of the action (1.7) are $A_\mu = \Delta_1 A_\mu$. In terms of these normal modes the equation of motion is

$$A_\mu'' - \frac{\Delta_1''}{\Delta_1} A_\mu - \partial_\alpha \partial^\alpha A_\mu = 0, \quad (1.8)$$

where $\partial_\mu A^\mu = 0$ and where $A_w = 0$. In the specific case of the action (1.7) $\Delta_1 = \sqrt{a}$ but, in more general terms, some couplings to the higher-dimensional dilaton field could be expected. This case will be studied in Section IV. Here only notice that the normalization condition of Eq. (1.8) for the gauge zero mode leads to the following integral

$$\int_0^\infty dw \Delta_1^2. \quad (1.9)$$

In the case of five-dimensional AdS space-time (i.e. $a(w) \simeq 1/|w|$, $\Delta_1^2 \sim a \sim 1/|w|$) the integral (1.9) is logarithmically divergent. Is it possible to envisage the situation where $\Delta \sim e^{-K_1|w|}$? In this case the gauge zero mode would be localized and it would be separated from the massive tower by a mass gap of the order $K_1$. The problem of the gauge zero modes and the possible occurrence of a mass gap in the spectrum will be one of the issues related to the present analysis.
A perspective similar to the one discussed in this paper has been recently adopted in a partially related problem, namely the possibility of implementing the Higgs mechanism without invoking a fundamental scalar [16]. Suppose [16] that the higher dimensional theory gives rise to a weight function multiplying the gauge kinetic term by a factor which depends upon the bulk coordinate \( w \). In the simplest realization this weight function can be related to the volume of the extra-dimensional space but there may be different situations like the ones where some other field (like the dilaton) gives important contributions. If the weight function satisfies a number of properties, then, the pure higher dimensional theory may lead to a lower-dimensional theory of massive vector bosons with a gap in the spectrum. Another important analogy is that the backgrounds leading to the effects analyzed in [16] do not necessarily lead to a finite four-dimensional Planck mass.

The plan of the paper is the following. In Section II arguments will be provided in order to show that the gauge modes of the geometry may be localized provided the four-dimensional Planck mass is not finite. In Section III an explicit model of negative tension branes will be provided. In Section IV the localization of the spin one fluctuations will be discussed. Section V contains some concluding remarks.

II. BASIC CONSIDERATIONS

Consider, as a warm-up, the case of gauge-invariant vector fluctuations in the case where the bulk action contains the usual Einstein-Hilbert term supplemented by a bulk cosmological constant in the presence of a thin brane configuration. This is exactly the case of a Randall-Sundrum type of background and it will be shown that, in this case the vector fluctuations of the geometry cannot be localized, as anticipated in the introduction.

The action for the system is given as

\[
S = \int d^5 x \sqrt{|G|} \left[ -R - \Lambda \right] - \frac{\lambda}{2} \int d^4 \rho \sqrt{|\gamma|} \left[ \gamma^{\alpha \beta} \partial_\alpha X^M \partial_\beta X^N G_{MN} - 2 \right],
\]

where \( \gamma^{\alpha \beta} \) is the induced metric on the brane and where \( \rho^\mu \) are the coordinates on the brane. It is important to write down the action in rigorous terms not so much for the analysis of
the background but for the analysis of the vector fluctuations. By functional derivation of the action with respect to the space-time and induced metric we get a set of two (coupled) equations that can be simply written as

\[ Q_{AB} = T_{AB}, \quad (2.2) \]
\[ \gamma_{\alpha\beta} = \partial_\alpha X^M \partial_\beta X^N G_{MN} \quad (2.3) \]

where

\[ Q_{AB} = R_{AB} + \frac{\Lambda}{3} G_{AB} \quad (2.4) \]
\[ T_{AB} = \frac{\lambda}{2} \int d^4 \rho \sqrt{\frac{|\gamma|}{|G|}} \gamma^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N (G_{AM} G_{BN} - \frac{1}{3} G_{MN} G_{AB}) \delta(w). \quad (2.5) \]

In the background (1.1) with \( d = 3 \) the induced metric is

\[ \gamma_{\alpha\beta} = \delta_{\alpha}^A \delta_{\beta}^B \eta_{AB} a(w)^2, \quad (2.6) \]
and the equations for the background are

\[ 6(\mathcal{H}^2 - \mathcal{H}') = \lambda a \delta(w), \quad (2.7) \]
\[ 6[3\mathcal{H}^2 + \mathcal{H}'] + 2\Lambda a^2 + \lambda a \delta(w) = 0. \quad (2.8) \]

In the case of five-dimensional AdS space-time, the only solution to Eqs. (2.7)–(2.8) is given by

\[ a(w) = \frac{1}{(b|w| + 1)} \quad (2.9) \]

where \( b = \sqrt{-\Lambda/12} \).

Consider now the localization properties of the vector fluctuations pertaining to this geometry. In order to address this question Eqs. (2.2) and (2.3) have to be perturbed with respect to the spin one degrees of freedom appearing in the fluctuations of \( G_{AB} \), namely,

\[ \delta G^{(v)}_{AB} = a^2(w) \left( \begin{array}{cc} 2\partial_{(\mu} f_{\nu)} & D_\mu \\ D_\mu & 0 \end{array} \right). \quad (2.10) \]
where $\partial_{(\mu} f_{\nu)} = \frac{1}{2}(\partial_{\mu} f_{\nu} + \partial_{\nu} f_{\mu})$ and where $\partial_{\alpha} f_{\alpha} = 0$ and $\partial_{\alpha} D_{\alpha} = 0$. Eq. (2.10) represents the most general decomposition of the spin one fluctuations of the geometry. The remaining nine degrees of freedom correspond either to scalar or to tensor fluctuations.

From Eqs. (2.2)–(2.3) the perturbed equations are:

$$\delta Q_{AB} = \delta T_{AB}, \quad (2.11)$$
$$\delta \gamma_{\mu\nu} = \delta^M_{\mu} \delta^N_{\nu} \delta G_{MN}. \quad (2.12)$$

As far as the vector fluctuations are concerned, the relevant components are the $(\mu, w)$ and the $(\mu, \nu)$. Direct calculations show that

$$\delta Q_{\mu\nu} = \left[ \frac{2}{3} \Lambda a^2 + 2(\mathcal{H}' + 3\mathcal{H}^2) \right] \partial_{(\mu} f_{\nu)} + [\partial_{(\mu} f_{\nu)}]' + 3\mathcal{H} \partial_{(\mu} f_{\nu)}']$$
$$- [\partial_{(\mu} D_{\nu)}]'- 3\mathcal{H} [\partial_{(\mu} D_{\nu)}], \quad (2.13)$$

$$\delta Q_{\mu w} = \left[ \frac{\Lambda}{3} a^2 + \mathcal{H}' + 3\mathcal{H}^2 \right] D_{\mu} - \frac{1}{2} \partial_{\alpha} \partial^{\alpha} [D_{\mu} - f_{\mu}'], \quad (2.14)$$

and that

$$\delta T_{\mu\nu} = -\frac{\lambda}{3} a(w) \delta(w) \partial_{(\mu} f_{\nu)}, \quad (2.15)$$
$$\delta T_{\mu w} = -\frac{\lambda}{6} a(w) \delta(w) D_{\mu}. \quad (2.16)$$

Hence, the relevant equations for the spin one fluctuations of the metric can be written as

$$-\frac{1}{2} \partial_{\alpha} \partial^{\alpha} [D_{\mu} - f_{\mu}'] + [\mathcal{H}' + 3\mathcal{H}^2 + \frac{\Lambda}{3} a^2 + \frac{\lambda}{6} a \delta(w)] D_{\mu} = 0, \quad (2.17)$$
$$- [\partial_{(\mu} D_{\nu)}]'- 3\mathcal{H} [\partial_{(\mu} D_{\nu)}] + [\partial_{(\mu} f_{\nu)}]'+ 3\mathcal{H} [\partial_{(\mu} f_{\nu)}]$$
$$+ \left[ \frac{2}{3} \Lambda a^2 + 2(\mathcal{H}' + 3\mathcal{H}^2) + \frac{\lambda}{6} a \delta(w) \right] \partial_{(\mu} f_{\nu)} = 0. \quad (2.18)$$

The vectors $f_{\mu}$ and $D_{\mu}$ are not invariant with respect to infinitesimal coordinate transformations preserving the vector nature of the fluctuations. The coordinate transformation preserving the vector nature of the fluctuations is a nothing but a vector shift in the coordinates which can be parametrized in terms of the (divergence-less) Poincaré vector $\zeta_{\mu}$:

$$x_{\mu} \to \bar{x}_{\mu} = x_{\mu} + \zeta_{\mu}. \quad (2.19)$$
On the fixed background defined by Eqs. (2.7)–(2.8), Eq. (2.19) induces a computable transformation in $f_\mu$ and $D_\mu$ while we move from the original coordinate system to the tilded one. More specifically, the shift is given by

\begin{align}
\tilde{f}_\mu &= f_\mu - \zeta_\mu, \\
\tilde{D}_\mu &= D_\mu - \zeta'_\mu.
\end{align}

(2.20)

(2.21)

Now looking together at Eqs. (2.20)–(2.21) we can define a gauge-invariant vector fluctuation which is exactly

$$V_\mu = a^{3/2}[D_\mu - f'_\mu].$$

(2.22)

This quantity is clearly invariant under infinitesimal coordinate transformations. The factor $a^{3/2}$ in Eq. (2.22) comes about since we want to work with fluctuations which have canonical kinetic terms in the action. In terms of the gauge-invariant fluctuations Eqs. (2.11) and (2.12) can be written in a fully coordinate independent form since the gauge-dependent part vanishes once the background equations (2.7)–(2.8) are used inside the perturbed equations of motion (2.17)–(2.18). The result is that

$$\partial_\alpha \partial^\alpha V_\mu = 0,$$

$$V'_\mu + \frac{3}{2} \mathcal{H} V_\mu = 0.$$  

(2.23)

Hence, as anticipated in the introduction, the gauge-invariant vector fluctuations are always massless and the normalization condition for the zero mode implies that

$$\int_0^\infty \frac{dw}{a^3(w)},$$

(2.24)

should be finite. In the case of the set-up discussed in the present Section, inserting Eq. (2.9) into (2.24), the resulting integral is never convergent. Moreover, the form of Eq. (2.7)–(2.8) prevents a solution of the type $a(w) \simeq e^{K|w|}$ for the warp factor.

---

4 Since the kinetic term of the combination $D_\mu - f'_\mu$ always appears, in the action, multiplied by $a^3(w)$, $V_\mu$ is correctly normalized.
As discussed in the introduction, the evolution equation for the canonical gauge modes corresponding to the action of an Abelian field strength can be simply obtained
\[ A''_{\mu} - \left( \frac{\sqrt{a}}{a} \right)'' A_{\mu} - \partial_{\alpha} \partial^\alpha A_{\mu} = 0, \] (2.25)
and the normalization condition for the zero mode implies that the integral
\[ \int_0^\infty a(w)dw \] (2.26)
should be convergent. Again, using Eq. (2.9) it can be checked that the above integral is always logarithmically divergent. The conclusion is that, in the case of the typical Randall-Sundrum set-up neither the vector fields of the geometry nor the bulk gauge fields can be normalized.

III. NEGATIVE TENSION BRANES

Consider now a slightly different situation, namely the case where the brane action also contain a dilatonic contribution. The action of the system, in analogy with the notations of the previous Section can be written as:
\[ S = \int d^5x \sqrt{|G|} \left[ -R + \frac{1}{2} G^{AB} \partial_A \varphi \partial_B \varphi - V(\varphi) \right] - \frac{\lambda}{2} f(\varphi) \int d^4 \rho \sqrt{|\gamma|} \left[ \gamma^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N G_{MN} - 2 \right], \] (3.1)
whose related equations of motion can be written as
\[ Q_{AB} = T_{AB}, \] (3.2)
\[ G^{AB} (\partial_A \partial_B \varphi - \Gamma^C_{AB} \partial_C \varphi) + \frac{\partial V}{\partial \varphi} + \frac{\lambda}{2} \frac{\partial f}{\partial \varphi} \int d^4 \rho \frac{\sqrt{|\gamma|}}{\sqrt{|G|}} \gamma^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N G_{MN} \delta(w) = 0, \] (3.3)
\[ \gamma_{\alpha\beta} = f(\varphi) \partial_\alpha X^A \partial_\beta X^B G_{AB}, \] (3.4)
where, now,
\[ Q_{AB} = R_{AB} - \frac{1}{2} \partial_A \varphi \partial_B \varphi + \frac{V}{3} G_{AB}, \] (3.5)
\[ T_{AB} = \frac{\lambda}{2} f(\varphi) \int d^4 \rho \frac{\sqrt{|\gamma|}}{\sqrt{|G|}} \gamma^{\alpha\beta} \partial_\alpha X^M \partial_\beta X^N [G_{AN} G_{BM} - \frac{1}{3} G_{MN} G_{AB}] \delta(w). \] (3.6)
Consider now the case where the dilaton field only depends upon the bulk coordinate. Then Eqs. (3.2)–(3.3) give

\[ \varphi'^2 = 6(\mathcal{H}^2 - \mathcal{H}') - \lambda a(w) f^2(\varphi) \delta(w), \]  
\[ \mathcal{H}' + 3\mathcal{H}^2 + \frac{V}{3} a^2 + \frac{\lambda}{6} a f^2(\varphi) \delta(w) = 0, \]  
\[ \varphi'' + 3\mathcal{H} \varphi' - \frac{\partial V}{\partial \varphi} a^2 - 2\lambda \frac{\partial f}{\partial \varphi} a \delta(w) = 0, \]  

with

\[ \gamma_{\alpha\beta} = a^2(w) f(\varphi) \delta^M_{\alpha} \delta^N_{\beta} \eta_{AB}. \]  

With the same algebra discussed in the previous Section we can obtain the equation describing the gauge-invariant vector fluctuations. Indeed, recalling that

\[ \delta Q_{\mu\nu} = \left[ \frac{2}{3} V a^2 + 2(\mathcal{H}' + 3\mathcal{H}^2) \right] \partial_{(\mu} f_{\nu)} + \left[ \partial_{(\mu} f'_{\nu)} \right]' + 3\mathcal{H} \left[ \partial_{(\mu} f'_{\nu)} \right], \]  
\[ \delta Q_{\mu w} = \left[ \frac{V}{3} a^2 + \mathcal{H}' + 3\mathcal{H}^2 \right] - \frac{1}{2} \partial_{(\mu} \partial^a [D_\mu - f'_\mu], \]  

and that

\[ \delta T_{\mu\nu} = -\frac{\lambda}{3} a(w) f^2(\varphi) \delta(w) \partial_{(\mu} f_{\nu)}, \]  
\[ \delta T_{\mu w} = -\frac{\lambda}{6} a(w) f^2(\varphi) \delta(w) D_\mu, \]  

exactly the same equations for the gauge-invariant vector fluctuations can be obtained.

The analysis of the background equations leads in the case examined in the present Section to the possibility of the solutions postulated in the introduction on the basis of more general arguments. More specifically, consider the following parameterization of the solution to Eq. (3.7)–(3.9):

\[ a(w) = e^{B[w]}, \quad \varphi(w) = A[w], \]  
\[ V(\varphi) = \Lambda e^{b\varphi}, \quad f(\varphi) = e^{c\varphi} \]
Inserting Eqs. (3.15)–(3.16) into Eqs. (3.7)-(3.9) we obtain the following matching conditions for the discontinuities:

\[ B = -\frac{\lambda}{12}, \quad (3.17) \]
\[ A = c\lambda, \quad (3.18) \]

and the following algebraic equations from the remaining terms:

\[ A^2 = 6B^2, \quad B^2 = -\frac{\Lambda}{9}, \quad (3.19) \]
\[ 2B + bA = 0, \quad 3AB - b\Lambda = 0. \quad (3.20) \]

The algebraic relations (3.19) and (3.20) together with the junction condition (3.17)–(3.18) give rise to two sets of solutions depending upon the sign of \( A \). If \( A > 0 \) we will have increasing dilaton solutions (denoted by \( A^> \)). If \( A < 0 \) we will have decreasing dilaton solutions (denoted \( A^< \)).

Let us now consider, separately, these two classes of solutions. In the first case the solution of Eqs. (3.17)–(3.18) together with Eqs. (3.19)–(3.20) can be written as

\[ A^> = \sqrt[6]{\frac{2}{3}} \sqrt{-\Lambda}, \quad (3.21) \]
\[ c^> = \mp \frac{1}{4} \sqrt[6]{\frac{2}{3}} \equiv \frac{1}{4} b^>, \quad (3.22) \]
\[ B^>_{\pm} = \pm \sqrt[6]{-\frac{\Lambda}{9}}, \quad (3.23) \]
\[ \lambda^>_{\pm} = \mp 4 \sqrt{-\Lambda}, \quad (3.24) \]

If \( A < 0 \) the corresponding solution reads

\[ A^< = -\sqrt[6]{\frac{2}{3}} \sqrt{-\Lambda}, \quad (3.25) \]
\[ c^< = \pm \frac{1}{4} \sqrt[6]{\frac{2}{3}} \equiv \frac{1}{4} b^<, \quad (3.26) \]
\[ B^<_{\pm} = \pm \sqrt[6]{-\frac{\Lambda}{9}}, \quad (3.27) \]
\[ \lambda^<_{\pm} = \mp 4 \sqrt{-\Lambda}. \quad (3.28) \]
Notice that the relation among the couplings $c$ and $b$ [i.e. $c = b/4$] holds in both sets of solutions. In Eqs. (3.21)–(3.24) and also in Eqs. (3.25)–(3.28) the $\pm$ refers to the sign of $B$. Hence, for both sets of solutions $B$ is positive for negative tension branes [i.e. $\lambda < 0$] and it is negative for positive tension branes [i.e. $\lambda > 0$]. Notice that our conventions differ slightly from the ones usually employed in the case of positive tension dilatonic walls [17].

IV. LOCALIZATION PROPERTIES OF SPIN ONE FLUCTUATIONS

Let us now analyze the case of negative tension branes in light of the localization properties of the vector fields of the geometry. If the brane has negative tension the gauge-invariant vector zero mode is always localized. In fact, since the zero mode is

$$\mathcal{V}_\mu^0 = \frac{\mathcal{N}_1}{a(w)^{3/2}},$$

the corresponding normalization condition

$$\mathcal{N}_1^2 \int_{-\infty}^{\infty} \frac{dw}{a^3(w)} = \frac{2}{3B_+} \equiv \frac{2}{\sqrt{-\Lambda}},$$

implies that $\mathcal{N}_1 \sqrt{2} (-\Lambda)^{-1/4}$. Clearly, if the brane has positive tension the gauge-invariant vector zero mode is never localized. Consider now the evolution equation for the gauge fluctuations. In the presence of the dilaton field the kinetic term of the gauge fields picks up a coupling to the higher-dimensional dilaton field. In particular the action will be written as

$$S = -\frac{1}{4} \int d^5x \sqrt{|G|} e^{-\phi} F_{AB} F^{AB}.$$  

The evolution equation for the gauge modes has the form anticipated in the introduction namely [6]

5 Notice that the energy-momentum tensor derived from the gauge kinetic term of Eq. (1.7) has been neglected in the solution of the background equations. This is correct, in the present case,
\[ A'' - \frac{\Delta_2''}{\Delta_2} A_\mu - \partial_\alpha \partial^\alpha A_\mu = 0, \quad (4.4) \]

where, now,

\[ \Delta_2(w) = e^{-\frac{\phi}{2}} \sqrt{a}. \quad (4.5) \]

Consider, first, the case where the dilaton is a decreasing function of the modulus of the bulk coordinate. Then, in this case, the gauge zero-mode is

\[ A^0_\mu = N_2 \Delta(w) = e^{\gamma_\prec |w|}, \quad (4.6) \]

where \( \gamma_\prec = \frac{1}{2} (B_+ - A^-) \). From the obtained solutions we can immediately say that \( \gamma_\prec > 0 \). Hence, in this case, the zero mode is not localized.

Consider then the situation where the dilaton is an increasing function of the bulk coordinate. In this case the gauge zero mode is given by

\[ A^0_\mu = N_3 \Delta(w) = e^{-\gamma_\succ |w|}, \quad (4.7) \]

where, now,

\[ \gamma_\succ = \frac{1}{2} (A_\succ - B_+) = \left( \frac{\sqrt{6} - 1}{6} \right) \sqrt{-\Lambda}. \quad (4.8) \]

From Eqs. (4.7)–(4.8) the gauge zero mode is clearly normalizable [i.e. \( \gamma_\succ > 0 \)] since \( A_\succ > B_+ \). The normalization constant turns out to be

\[ N_3^2 = \frac{\sqrt{6} - 1}{6} \sqrt{-\Lambda}. \quad (4.9) \]

since the background gauge field vanishes. However, there are brane models where the background gauge field does not vanish. In this case the full Einstein-Maxwell system should be consistently studied and perturbed. Furthermore, it would be interesting to study the possible effect of back-reaction of the zero mode on the obtained solution. Also in this case the full Einstein-Maxwell system should be analyzed. This interesting aspect is left for future work.
The zero mode of Eq. (4.7) is separated from the other states of the continuum by a mass gap. This aspect can be understood by writing down the equation for the mass eigenstates, namely

\[ \left( -\frac{d^2}{dw^2} + V(w) \right) A_\mu = m^2 A_\mu, \]  

where \( V(w) = \Delta''/\Delta \equiv \gamma_\gamma^2 - 2\gamma_\gamma \delta(w). \) The massive wavefunctions start for \( m^2 > \gamma_\gamma^2. \)

V. CONCLUDING REMARKS

In this paper the localization properties of the gauge modes coming from the geometry has been analyzed under the hypothesis that the four-dimensional Planck mass is not finite. It has been shown that the spin one fluctuations can be localized on negative tension branes. The gauge-invariant vector modes of the geometry are always massless. Furthermore the localization of spin one fluctuations coming from the gauge sector has been analyzed on the same type of configurations. It has been shown that solutions describing negative tension branes can simultaneously localize the gauge fields and the vector modes of the geometry.

ACKNOWLEDGMENTS

It is a pleasure to thank M. E. Shaposhnikov for important hints which motivated the present analysis.
REFERENCES

[1] V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. B 125, 136 (1983).

[2] D. B. Kaplan, Phys. Lett. B 288, 342 (1992).

[3] H. B. Nielsen and M. Ninomya, Phys. Lett. B 105, 219 (1985).

[4] P. H. Ginsparg, and K. G. Wilson, Phys. Rev. D 25, 2649 (1982).

[5] V. A. Rubakov, Usp.Fiz.Nauk 171, 913 (2001) [Phys.Usp. 44, 871 (2001)].

[6] G. Dvali and M. Shifman, Phys. Lett. B 496, 113 (2000).

[7] I. Oda, Phys. Lett. B 496, 113 (2000).

[8] S. Dubovsky and V. Rubakov, Int. J. Mod. Phys. A 16, 4331 (2001).

[9] G. Dvali, G. Gabadadze, and M. Shifman, Phys. Lett. B 497, 113 (2000).

[10] A. Kehagias and K. Tamvakis, Phys. Lett. B 504, 38 (2001).

[11] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 3370 (1999).

[12] L. Randall and R. Sundrum, Phys. Rev. Lett. 83 4690 (1999).

[13] M. Giovannini, Phys. Rev. D 64, 064023 (2001); Phys.Rev.D 64, 124004 (2001).

[14] M. Giovannini, Phys. Rev. D 65, 064008 (2002).

[15] M. Giovannini, Graviphoton and Graviscalar delocalization in brane world scenarios, Proc. of Cosmo-01, hep-th/0111218.

[16] M. E. Shaposhnikov and P. Tinyakov, Phys.Lett. B 515, 442 (2001).

[17] M. Cvetic, H. Lü, and C.N. Pope, Phys. Rev. D 63, 086004 (2001).

[18] S. Randjbar-Daemi and M. E. Shaposhnikov, Phys.Lett.B 491, 329 (2000).