THE DYNAMICS OF SINKING SATELLITES AROUND DISK GALAXIES: A POOR MAN'S ALTERNATIVE TO HIGH-RESOLUTION NUMERICAL SIMULATIONS

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ABSTRACT

We have developed a simple yet surprisingly accurate analytic scheme for tracking the dynamical evolution of substructure within dark matter halos. The scheme incorporates the effects of dynamical friction, tidal mass loss, and tidal heating via physically motivated approximations. Using our scheme, we can predict the orbital evolution and mass-loss history of individual subhalos in detail. We are also able to determine the impact and importance of the different physical processes on the dynamical evolution of the subhalos. To test and calibrate this model, we compare it with a set of recent high-resolution numerical simulations of mergers between galaxies and small companions. We find that we can reproduce the orbits and mass-loss rates seen in all of these simulations with considerable accuracy, using a single set of values for the three free parameters in our model. Computationally, our scheme is more than 1000 times faster than the simplest of the high-resolution numerical simulations. This means that we can carry out detailed and statistically meaningful investigations into the characteristics of the subhalo population in different cosmologies, the stripping and disruption of the subhalos, and the interactions of the subhalos with other dynamical structures such as a thin disk. This last point is of particular interest given the ubiquity of minor mergers in hierarchical models. In this regard, our method's simplicity and speed makes it particularly attractive for incorporation into semianalytic models of galaxy formation.

Subject headings: dark matter — galaxies: interactions — galaxies: kinematics and dynamics — methods: numerical

On-line material: color figures

1. INTRODUCTION

Over the past two decades, observational and theoretical progress has given rise to an increasingly detailed picture of how structure forms and evolves on galactic scales. The currently favored theoretical models are based on the concept of hierarchical clustering, in a universe dominated by cold dark matter (CDM). Galaxies are embedded within extended halos of dark matter, which form through gravitationally induced mergers of smaller scale structure. The evolution of individual dark matter halos and the formation of galactic structure, if any, inside the halos is strongly dependent on the nonlinear dynamics of gravitational collapse, the stochastic process of merging, and the subsequent evolution of the merged substructure. These processes are extremely challenging to model theoretically. Consequently, the exact role of mergers, especially minor mergers, in the formation and evolution of galaxies is still poorly understood, even though mergers are thought to be a ubiquitous feature of structure formation.

The dynamics of structure formation have been studied extensively using numerical simulations. Simulations have been used to investigate the properties of populations of cluster, group, and galaxy-sized halos in their larger cosmological environment (e.g., Jenkins et al. 1998; Jing & Suto 1998; Governato et al. 1998, 1999; Kauffmann et al. 1999; Pearce et al. 1999; Sigad et al. 2001), the detailed properties of individual halos on scales at which their substructure is resolved (e.g., Ghigna et al. 1998, 2000; Moore et al. 1999; Klypin et al. 1999a, 1999b; Okamoto & Habe 1999; Lewis et al. 2000; Yoshikawa, Jing, & Suto 2000; Jing & Suto 2000; Fukushige & Makino 2001), and the outcome of mergers between halos or subcomponents within halos, both minor (e.g., Quinn & Goodman 1986; Quinn, Hernquist, & Fullagar 1993; Walker, Mihos, & Hernquist 1996; Huang & Carlberg 1997; Velázquez & White 1999) and major (e.g., Barnes 1998; Naab, Burkert, & Hernquist 1999). Existing simulations do not yet have the dynamic range to explore these different scales simultaneously, however, leading to a fragmented treatment of the subject. Moreover, numerical simulations suffer from several other disadvantages: (1) they are very expensive computationally; (2) the detailed statistical properties of bound substructures within a halo (or "subhalos") may depend sensitively on the scheme used to identify them; and (3) the evolution of these objects may be influenced in complex ways by the number of particles used to resolve them, as well as other numerical effects such as finite force resolution (e.g., Ghigna et al. 2000; Knebe et al. 2000).

These limitations are particularly frustrating given the recent suggestions that hierarchical models may be failing to match observations on galactic scales. One apparent discrepancy is between the number of distinct subhalos expected to have survived in a Milky Way-sized halo and the observed number of satellites around the two large galaxies of the Local Group (Klypin et al. 1999b; Moore et al. 1999; Bullock, Klypin, & Weinberg 2000). This excess structure may be implicated in several other problems, including the small disk sizes produced in hydrodynamic simulations (e.g., Navarro & Steinmetz 2000), and the question of disk survival against heating in minor mergers (Toth & Ostriker 1991; Kauffmann & White 1993; Lacey & Cole 1993; Navarro, Frenk, & White 1994; Moore et al. 1999).

To resolve these issues, it is necessary to consider separately the effects of background cosmology, the power spectrum of
density fluctuations, and most importantly, the nature and
dynamics of substructure within galactic halos. Accom-
plishing this goal numerically would entail exploring a large
parameter space with ultrahigh-resolution simulations. This
proposition is, however, prohibitively expensive at present.

An alternate approach to studying structure formation is
to use semianalytic (SA) methods, which combine analytic
theory and numerical results. Semianalytic models of galaxy
formation (e.g., Kauffmann, White, & Guiderdoni 1993;
Cole et al. 1994; Somerville & Kolatt 1999) generate
random realizations of merging (“merger histories” or
“merger trees”) between halos, based on Press-Schechter
statistics (Press & Schechter 1974). The formation and evol-
ution of galactic structure within the halos is then governed
by a set of prescriptions that attempt to describe the effects
of merging, hydrodynamics, shocks, dissipation, star forma-
tion, and feedback.

Semianalytic models of galaxy formation have been used
to study the galaxy luminosity function, the morphology-
density relation, the Tully-Fisher relation, and other global
properties of galaxy populations (see Somerville & Primack
1999 for a recent review, as well as Bullock et al. 2000 for
more recent work on galactic satellites). SA methods have
the advantage of being extremely fast compared to fully
numerical simulations, and although the results depend on the
prescriptions adopted (Benson et al. 2001), these pre-
scriptions are, at least in principle, explicit, so that it is easy
to test the consequences of modifying them. SA methods are
therefore useful for exploring the relative importance of the
various ingredients of the galaxy formation model—the back-
ground cosmology and the shape of the power spectrum,
the density profiles of dark matter halos, the nonlin-
erar dynamics of merging, the gas and radiative physics, star
formation, and feedback algorithms—in determining the
appearance of present-day galaxies.

Until recently, however, SA models of galaxy formation
have focused on star formation and dissipative processes,
and have included only simplified descriptions of merging,
or even ignored the impact of merging altogether. In Dia-
ferio et al. (2001), for example, nearly equal mass mergers
are assumed to destroy any disk present in a halo, but the
effects of uneven-mass mergers are ignored. This makes it
difficult to ascertain whether results related to merging and
morphology are real, or merely artifacts of the over-
simplified dynamics assumed (Kolatt et al. 2000). It also
makes it difficult to relate SA results to studies of the
dynamical evolution of individual galaxies, whether analy-
tic (e.g., Dalcanton, Spergel, & Sommers 1997; Mo, Mao, &
White 1998) or numerical (e.g., Velázquez & White 1999).

To study various issues arising from hierarchical clus-
tering scenarios, including the distribution of satellites
around galactic systems, the impact of these satellites on a
thin disk, and the formation of the stellar halo from tidal
debris, requires a method for modeling the evolution of halo
substructure that synthesizes and generalizes the results of
existing numerical studies, without resorting to expensive
ultrahigh-resolution simulations. The method should take
into account the internal structural properties of a subhalo,
its orbital parameters, and the details of its interaction with
the main galaxy.

We have developed a simple analytic scheme that
addresses this need and complements existing semianalytic
and numerical models of galaxy formation. We consider
subhalos within a galaxy halo individually, following their
orbits and accounting for dynamical friction, mass loss, and
heating, the three main processes that determine the
dynamical evolution of subhalos, using analytic expres-
sions. This approach will allow us to carry out detailed
studies of the properties of subhalos for a variety of cosmol-
ogies and power spectra. Since the physical processes
underlying our scheme are modeled explicitly, we can deter-
mine their relative effects on subhalo evolution directly.
Finally, since our model allows us to generate large
numbers of realizations at little computational cost, we can
do all of the above in a statistically meaningful manner. The
speed of this approach should facilitate the extension of
semianalytic methods to subhalo scales. We also note that
although our method is described in the context of galaxy-
sized halos, it is completely general and can also be used to
study the evolution of substructure in clusters, for instance.

In this paper, we outline our dynamical model for the
evolution of substructure. In § 2, we give a brief synopsis of
the previous investigations of the dynamics of satellites
merging with a larger galactic system, and then describe the
theory underlying our scheme for following orbital decay,
mass loss, and the tidal disruption of subhalos. In § 3, we
test and calibrate our model by comparison with recent
high-resolution numerical simulations of sinking satellites
by Velázquez & White (1999, hereafter VW99). In § 4, we
determine the sensitivity of our results to the model param-
eters, and explore more generally how tidal heating, the
form of the galactic potential, and the subhalo mass profile
affect mass loss and orbital decay. We summarize our
results in § 5. In subsequent papers we will combine our
model of dynamical evolution with semianalytic merger
trees to study the halo substructure and disk heating pro-
duced in a galaxy halo by multiple mergers with cosmo-
logically realistic subhalos on representative orbits.

2. DYNAMICS OF MERGING SUBSTRUCTURE

2.1. Background

The first detailed study of the dynamics of satellites orbit-
ing in a halo containing a disk galaxy was carried out by
Quinn & Goodman (1986). This work was subsequently
improved upon by Quinn et al. (1993). Using numerical
simulations, these authors confirmed that dynamical fric-
tion from the disk particles leads to rapid orbital decay for
massive satellites, as expected from analytic arguments.
They also found that orbital decay times vary with inclina-
tion, prograde orbits close to the plane of the disk decaying
the fastest, although in the end they could not account for
the anisotropy in the orbits of satellites around spiral gal-
axies observed by Holmberg (1969). The sinking of their
satellites, initially on circular orbits, took place in two steps:
(1) a relatively slow decay of the orbital radius largely domi-
nated by loss of altitude with respect to the disk, and (2) a
rapid decay in the radius once the satellite was in the disk
plane. Their results also suggested that massive satellites
would heat the disk appreciably, although the effect was
complex and depended on the orbit and the internal struc-
ture of the satellite. Finally, the authors noted that numeri-
cal noise was a significant problem in their simulations, and
that even in simulations with ~ 500,000 particles, noise
would make it difficult to study the dynamics of a minor
merger over the Hubble time.

In the period between Quinn & Goodman (1986) and
Quinn et al. (1993), Tóth & Ostriker (1992, hereafter TO92)
studied the effects of minor mergers on the structure of the Milky Way’s disk, using an analytic model for the evolution of the satellites. The orbital energy a satellite lost through dynamical friction was assumed to go into heating the disk locally. Comparing their predictions to the observed scale height of the disk and to the local value of Toomre’s stability parameter $Q$, they concluded that the Milky Way could not have accreted more than 4% of its mass interior to the solar circle in the past 5 Gyr.

Several more recent studies have sought to evaluate the results of TO92. Walker et al. (1996, hereafter WMH96) and Huang & Carlberg (1997, hereafter HC97) used numerical simulations, improving on previous work by including a responsive halo. WMH96 opted to use a very large number of particles to reduce numerical noise, and in turn had to start their satellites fairly close in, at a distance of 21 kpc from the center of the galaxy. They found that the evolution of the satellite was similar to that described by Quinn & Goodman (1986) and Quinn et al. (1993). With regard to heating, they found that accretion of a satellite with 10% of the disk mass would result in a significant thickening of the disk at the solar radius. HC97, on the other hand, considered satellites starting out at larger distances with masses of 10%–30% of the disk mass. The internal structure of their satellites consisted of a small, tightly bound core, embedded in an extended low-density envelope containing most of the mass. Perhaps as a result, the satellites tended to be disrupted by tidal forces before they could heat the disk significantly. For completeness, we note that Weinberg (1995, 1998) and Sellwood, Nelson, & Tremaine (1999) have also studied the detailed response of the disk in satellite-disk interactions, while Johnston, Hernquist, & Bolte (1996) and Johnston, Sigurdsson, & Hernquist (1999) have studied the properties of tidally stripped debris in minor mergers.

The most recent numerical study of satellite-disk interactions is that of VW99, who studied mergers involving several different satellites, intermediate in density between those of TO92 and those of HC97, on various different orbits. They confirmed that Chandrasekhar’s formula (Chandrasekhar 1943) gives a useful approximation to the drag force exerted by the halo on the satellite, provided that the Coulomb logarithm is adjusted separately for each orbit. VW99 also found that the response of the disk depends partly on the orientation of the satellite orbit, prograde encounters tending to heat the disk preferentially, while retrograde encounters tend to tilt it. They concluded that TO92 overestimated the magnitude of disk heating by a factor of 2–3 overall.

Although much progress has been made in studying the effect of minor mergers on galactic structure, it is difficult to determine the cosmological implications of merger simulations, given the lack of clear, consistent, and robust results across the different studies. This is, first and foremost, due to the fact that each study has considered satellites with different internal properties. Structural characteristics such as the satellite’s density profile affect the rate of mass loss, and therefore the evolution of the satellite’s orbit, not to mention the response of the disk to the satellite. In addition, the satellites in the different studies were introduced at different radii, and early studies did not include the dynamical friction produced by a live halo. On a more practical level, the simulations were subject to numerical effects such as finite force resolution, shot noise, relaxation, and artificial heating from interactions between particles of different masses, which differed from one study to another. Finally, all studies prior to that of VW99 focused on satellites on circular or nearly circular orbits, making it difficult to generalize their results to satellites accreted by galaxies in self-consistent cosmological settings. To analyze this prior body of numerical work and overcome the limitations mentioned previously requires an alternative method that follows the relevant physical processes explicitly, and can generate many realizations of merging at little computational expense.

### 2.2. The Analytic Model

In the simplest approximation, a subhalo or satellite moving within a larger halo can be represented as a point mass with no internal structure, orbiting in a smooth, static potential. This approximation, however, ignores important dynamical effects due to the subhalo’s internal structure, and the interactions between the subhalo and the matter that constitutes the halo. The latter, for example, give rise to a drag or dynamical friction force that can make the orbital evolution of the satellite very different from that of a point mass in a static potential. Over the course of their orbits, extended subhalos are also likely to experience mass loss through tidal stripping. Accounting for this mass loss is particularly important, since the magnitude of dynamical friction, and hence the rate at which the satellite’s orbit decays, depends on its mass. In addition, satellites may occasionally be subjected to gravitational shocks as they pass through regions of higher density. Such shocks will result in internal heating that not only alters a subhalo’s internal structure, but also accelerates mass loss. To describe the evolution of a subhalo accurately, all of the above processes need to be taken into account. Our goal, then, is to provide analytic descriptions of these phenomena that can be implemented in our model at little computational expense.

We treat each subhalo as a spherically symmetric satellite, with structural properties that change over time. The state of the satellite is fully specified at any moment by its bound mass, the original form of its density distribution, and the amount of heating it has experienced. The density distribution is initially set to a standard form such as a King model, or any one of several common analytic density profiles. If the satellite experiences tidal heating, however, its density distribution may change. We do not track these changes explicitly, since we are only concerned with changes in the mean density of the satellite within its limiting radius in our description of heating and mass loss.

To determine the satellite’s orbit to first order, we ignore its spatial extent and calculate the trajectory of a point particle with the same total mass, moving in the smooth, axisymmetric potential generated by the dark halo, the stellar spheroid or bulge, and the disk. This approximation is entirely sufficient as long as the scale of the satellite’s orbit is larger than the satellite itself; if the satellite falls into the central region of the potential, we consider it to be disrupted in any case. The background potential is taken to be static in the present study, although in general it can be allowed to vary in a self-similar way in our code.

To account for dynamical friction, the response of the halo and disk to the satellite, we impose a drag force on the satellite, which we calculate using Chandrasekhar’s formula. As mentioned above, we also adjust the satellite’s total mass and modify its internal structure in response to
tidal stripping and tidal heating, respectively, during the course of its orbit. The satellite is considered disrupted when it has been stripped down to its core radius, or when it falls into the central region of the potential. Each of these processes is described separately below.

2.2.1. Dynamical Friction

Chandrasekhar (1943) showed that a massive particle moving through a distribution of background particles will generate a wake. The collective gravitational force from the wake will act back on the massive particle, causing a drag force known as dynamical friction.

Dividing the background potential into its two kinematically distinct components, we use Chandrasekhar’s formula to estimate the dynamical friction exerted by the halo/bulge system and by the disk on an orbiting satellite:

\[ F_{df} = F_{df,\text{halo}} + F_{df,\text{disk}} \]

\[ = -4\pi G^2 M_{\text{sat}} \sum_{i=h,d} \rho_i(<V_{\text{rel},i}) \ln \Lambda_i \frac{V_{\text{rel},i}}{|V_{\text{rel},i}|^3}, \quad (1) \]

where \( V_{\text{rel},h} = V_{\text{sat}} - V_{\text{rot},h} \)

\( V_{\text{rel},d} = V_{\text{sat}} - V_{\text{rot},d} \),

\( \rho_i(<V_{\text{rel},i}) = \rho_i(r) \left[ \text{erf}(X_i) - X_i \text{erf}(X_i) \right], \)

and \( X_i = \frac{V_{\text{rel},i}}{\sqrt{2} \sigma_i} \).

Here \( M_{\text{sat}} \) is the mass of the satellite, \( r \) is its position, \( V_{\text{sat}} \) is its velocity, \( V_{\text{rot}} \) is the local circular velocity of the disk, \( \rho_h \) is the local density of the spherical (halo/bulge) component, \( \rho_d \) is the local density of the disk, \( \ln \Lambda_h \) and \( \ln \Lambda_d \) are the Coulomb logarithms for the halo/bulge and the disk, and \( \sigma_h \) and \( \sigma_d \) are the one-dimensional velocity dispersions of the halo/bulge particles and the disk particles, respectively (Binney & Tremaine 1987).

The derivation of this formula assumes a massive point particle, moving through an infinite, homogeneous background of much lighter particles with an isotropic Maxwellian velocity distribution of zero mean. Numerous detailed studies of satellite dynamics (Weinberg 1986; Cora, Muzzio, & Vergne 1997; Bontekoe & van Albada 1987; van den Bosch et al. 1998; VW99; Colpi, Mayer, & Governato 1999) have shown the formula to be more widely applicable, however, in the sense that it gives a useful approximation to the drag force on an extended satellite in a finite halogalaxy system, provided that the Coulomb logarithms are adjusted appropriately. In general, Binney & Tremaine (1987) suggest that Chandrasekhar’s formula will be fairly accurate provided that the mass of the satellite does not exceed 20% of the mass of the larger system, and that the orbit of the satellite lies neither outside the larger system nor completely within its core.

The argument of the Coulomb logarithm can be expressed as \( \Lambda = \frac{r_{\text{max}}}{b_{\text{min}}} \), where \( b_{\text{max}} \) and \( b_{\text{min}} \) are measures of the maximum and minimum impact parameters, respectively, of the background particles contributing to the wake. The various approximations made in deriving equation (1) introduce some uncertainty into the precise definition of these parameters, however. For a finite background system, \( b_{\text{max}} \) is conventionally taken to be the characteristic scale of the system. Possible choices for a spherically symmetric system include the half-mass radius of the system (e.g., Quinn & Goodman 1986), the distance over which the background density changes by a factor of 2 (Binney & Tremaine 1987), and the tidal radius of the halo or the distance between the satellite’s position and the center of background system (Colpi et al. 1999).

The value of \( b_{\text{min}} \) is equally ambiguous. For a point-mass satellite, \( b_{\text{min}} \equiv \frac{G(M_{\text{sat}} + m)}{V^2} \), where \( m \) is the mass of the background particles and \( V \) is a velocity “typical” of the encounter, such as the rms velocity of the background particles (Chandrasekhar 1943), or their mean velocity relative to the satellite (Binney & Tremaine 1987). For extended satellites, White (1976) derived an expression for \( b_{\text{min}} \) that is approximately equal to 0.2\( r \) (or very roughly the half-mass radius) for a wide range of King profiles, while Quinn & Goodman (1986) take \( b_{\text{min}} \) to be the larger of the half-mass radius of the satellite and the point-mass value \( \frac{G(M_{\text{sat}} + m)}{V^2} \), with \( V \) taken as the mean velocity of the satellite with respect to the background particles.

The choice of an appropriate Coulomb logarithm to describe friction from the disk, and more generally the applicability of Chandrasekhar’s formula to an inhomogeneous distribution of background particles, is even less clear. Maoz (1993) and Domínguez-Tenreiro & Gómez-Flechoso (1998) derived formulae for the magnitude of the energy loss produced by an arbitrary distribution of uniform velocity dispersion, but could not specify the direction of the corresponding frictional force. It is also possible to calculate dynamical friction for a uniform background of particles with an ellipsoidal velocity distribution (Binney & Tremaine 1987), in which case the frictional force is strongest in the direction of the smallest principal axis of the distribution. Since disk friction is of secondary importance compared with halo friction, however, we limit ourselves to using Chandrasekhar’s formula to calculate its approximate direction and magnitude.

There is one important correction to the expression for disk friction given above. In deriving equation (1), we have taken the density of the satellite’s wake to be constant, and equal to the background density at the center of the satellite. Since the wake has a finite extent, this approximation may result in errors in the drag force if the background density changes over small scales. This is likely to occur, for example, when the satellite is in the plane of the disk, because of the latter’s small vertical scale height. To correct for this, the disk density \( \rho_d \) used in equation (1) should be smoothed in the vertical direction. In principle, the smoothing length ought to be related to the characteristic scale of the wake or the satellite; however, this would mean using a different smoothing scale for each individual satellite. At present, this level of complication does not seem warranted, nor would it fully account for finite-size effects (see Domínguez-Tenreiro & Gómez-Flechoso 1998 for a more detailed discussion of the drag force on extended objects).

We have therefore chosen to smooth the disk density in the vertical direction by a fixed length corresponding to 2 times the disk scale height, noting that this smoothing length is on the order of the half-mass radius for satellites with masses in the range where dynamical friction has a substantial effect.

Given the uncertainties associated with calculating the Coulomb logarithm, on the one hand, and the fact that, on the other hand, Chandrasekhar’s formula with an appropriately adjusted Coulomb logarithm gives an excellent approximation to the drag force measured in numerical studies, we treat \( \ln \Lambda_h \) and \( \ln \Lambda_d \) as free parameters, to be determined by comparison with simulations. In principle, each simulation should be characterized by slightly different values of \( \ln \Lambda_h \) and \( \ln \Lambda_d \), since these quantities depend weakly on a satellite’s mass and internal structure, and on
the nature of the orbit. Since our main objective is to establish the simplest accurate description of satellite dynamics that can be used in general studies of halo substructure, we have chosen to determine average values for the Coulomb logarithms, using the 15 satellite simulations carried out by VW99, the most extensive and detailed set of such simulations currently available. We will also start by assuming that these parameters remain constant over a satellite’s lifetime; the effect of varying the logarithms as the satellite loses mass is discussed further in §4.1.

We recognize that in currently favored theories of structure formation, subhalo properties sample a much larger volume of parameter space than explored by VW99. The ratio \( M_{\text{sat}}/M_{\text{halo}} \), for example, spans at least 5 orders of magnitude in groups and clusters, whereas the VW99 simulations only sample the upper end of the spectrum. This does raise a question about the applicability of the average values of \( \ln \Lambda_s \) and \( \ln \Lambda_d \) determined from the VW99 simulations to the more general problem of subhalo evolution in hierarchical scenarios. The importance of this point depends on the specific issue under consideration. It is the evolution of the most massive satellites, whose orbits decay over a few dynamical times, that will determine whether the frequent mergers predicted by hierarchical models of structure formation result in excessive heating of the disk, for instance. In this case, the relevant values of the Coulomb logarithms will be very similar to the average values determined from the VW99 simulations.

More generally, from the previous discussion of the definitions of \( b_{\text{min}} \) and \( b_{\text{max}} \), we expect that the argument of the Coulomb logarithm may scale as \( \Lambda \propto (M_{\text{sat}}/M_{\text{halo}})^{-1} \). The logarithms of two satellites of masses \( M_1 < M_2 \), orbiting within the same large halo, should therefore differ by \( \Delta \ln \Lambda \sim \ln (M_2/M_1) \). Thus, for instance, the values predicted for a \( \sim 10^5 \) and a \( \sim 10^{10} \) solar mass satellite would differ by \( \Delta \ln \Lambda \approx 9-10 \), consistent with the differences in the values for massive globular clusters and for the Magellanic clouds estimated by Tremaine, Ostriker, & Spitzer (1975) and by Tremaine (1976), respectively. In addition to implying that the Coulomb logarithm is systematically larger for less massive satellites, this scaling also suggests that its value will increase over the course of a given satellite’s evolution, as the satellite loses mass as a result of tidal stripping. We are unable to test the former implication, since VW99 satellites span a very narrow range in initial mass; in §4.1, however, we compare the orbits of satellites under the assumptions that (1) the Coulomb logarithm remains constant, and (2) it increases as the satellite loses mass.

### 2.2.2. Mass Loss

As noted previously, a satellite with a finite extent will lose mass because of tidal stripping as it orbits within the halo-galaxy system. Since the drag force the satellite experiences due to dynamical friction varies as \( M_{\text{sat}}^2 \), to first order, the magnitude of the deceleration acting on the satellite will decrease correspondingly. As a result, mass loss can significantly alter the dynamics of the satellite. To account for this, we need to estimate the amount of mass that remains bound to the satellite throughout its orbit.

Material becomes unbound from the satellite through the action of tidal forces. Slowly and rapidly varying tidal forces will affect the satellite differently. In a slowly varying system, material outside some limiting, “tidal,” radius will be stripped from the satellite, while in a rapidly varying system, material throughout the satellite will be tidally heated. These two regimes have been studied previously by making the approximation that the system is static in the first case (that is, that the satellite is on a circular orbit), or that the satellite undergoes a very short perturbation, but is otherwise isolated in the second case (the impulse approximation). In this section, we consider tidal stripping on general orbits. Tidal heating will be treated separately in the section that follows.

For a satellite on a circular orbit of radius \( r \) within a spherically symmetric mass distribution, the combined potential of the entire system is static in the rotating frame. In this case, we can identify the tidal radius with the distance to the saddle point in the point interior to the satellite’s orbit, since this is the point at which the radial forces on a test particle cancel out (von Hoerner 1957; King 1962; Binney & Tremaine 1987). The distance \( R_t \) from the satellite center to this point is

\[
R_t \approx \left( \frac{GM_{\text{sat}}}{\omega^2 - d^2 \Phi/dr^2} \right)^{1/3}
\]

(King 1962), where \( M_{\text{sat}} \) is the mass of the satellite, \( \omega \) is its angular velocity, and \( \Phi \) is the potential of the main system.

This estimate of the tidal radius is only formally valid when \( M_{\text{sat}} \) is much smaller than the mass of the main system, \( R_t \) is much smaller than the orbital radius, and the satellite is corotating at its orbital frequency. Even under these restricted assumptions, the mass inside \( R_t \) is only approximately equal to the bound mass, because there exist orbits that extend beyond \( R_t \), but remain bound to the satellite (Binney & Tremaine 1987, and references therein). Furthermore, even in this simple case, the tidal boundary is not spherical, and thus the use of equation (2) is approximate.\(^1\)

General satellite orbits are not circular, nor is the external potential in which they move necessarily spherical. We can still use equation (2) to define an instantaneous tidal limit for the system, where \( \omega \) is now the instantaneous angular velocity of the satellite. For noncircular orbits, or orbits out of the plane of the disk, \( R_t \) changes with time, however, and mass outside \( R_t \) will become unbound as a result of successive accelerations over the course of the orbit, rather than being stripped immediately. While equation (2) represents a steady-state solution to mass loss, the characteristic timescale for transient changes in mass on general orbits should be the orbital period of the satellite.

To model this type of mass loss, we therefore assume that the satellite mass beyond \( R_t \) is lost over the course of one orbital period, and scale the mass loss in each time step accordingly.

In calculating \( d^2 \Phi/dr^2 \), we average over the asphericity of the potential due to the disk component. We set

\[
\frac{d^2 \Phi}{dr^2} = \frac{d^2 \Phi_{\text{ph}}}{dr^2} = \frac{d}{dr} \left[ \frac{-GM(<r)}{r^2} \right],
\]

where \( \Phi_{\text{ph}} \) is the potential produced by a spherically symmetric distribution, with the same total mass \( M(<r) \) interior to \( r \) as the axisymmetric distribution. This will be very close to the radial gradient of the actual force on the satellite when it is far from the disk, or when it is in the plane of

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\(^1\) Innanen, Harris, & Webbink (1983), for instance, calculate a slightly different value for the limiting radius based on the length of the short axis of a tidally distorted satellite.
the disk. Only when the satellite is close to the disk, but on an inclined orbit, will the true gradient differ substantially from this value, and in practice we expect tidal shocking to dominate the physics of the mass loss in these cases.

We can write the stripping condition in terms of densities; the tidal limit occurs at the radius \( R_t \), within which the mean density of the satellite, \( \bar{\rho}_{\text{sat}} \), exceeds the density of the galaxy interior to its orbital radius \( r \), \( \bar{\rho}_{\text{gal}} \) by a factor \( \eta \):

\[
\bar{\rho}_{\text{sat}}( < R_t ) = \eta \bar{\rho}_{\text{gal}}( < r ) ,
\]

where

\[
\eta = \frac{\bar{\rho}_{\text{sat}}( < R_t )}{\bar{\rho}_{\text{gal}}( < r )} = \frac{r^3}{R_t^3} \frac{M_{\text{sat}}}{M( < r )} = \frac{r^3}{GM( < r )} \left( \frac{\omega^2}{\omega^2} - \frac{2}{\omega^2} \frac{d^2 \Phi}{dr^2} \right) = \frac{r^3}{\omega^2} \frac{d^2 \Phi}{dr^2} ,
\]

where \( \omega \) is the instantaneous angular velocity of the satellite, and \( \omega_c \) is the angular velocity of a circular orbit of radius \( r \).

This leads to a particularly simple algorithm for stripping satellites. First, we divide the satellite’s orbital path into discrete sections corresponding to fixed time steps. In each time step, we determine the tidal radius of the satellite using equation (4). Of the material outside this radius, we remove a fraction \( \Delta t / t_{\text{orb}} \), where \( \Delta t \) is the length of the time step and \( t_{\text{orb}} = 2\pi/\omega \) is the instantaneous orbital period, which we assume to be the characteristic timescale for mass loss. Finally, we treat the satellite as disrupted and set its bound mass to zero when the tidal radius becomes smaller than the core radius of its initial density profile, although by this point it has normally lost so much mass that the exact disruption criterion is unimportant in practice. As mentioned above, we also treat satellites which have fallen into the core of the bulge as disrupted, to avoid instabilities in the orbital calculation.

2.2.3. Tidal Heating

Whereas a steady or slowly varying tidal field will result in the stripping of loosely bound mass, a rapidly changing gravitational field, caused, for example, by fast encounters with the galactic disk or bulge, will induce gravitational shocks that can add energy to the satellite, changing its structure and accelerating mass loss (Ostriker, Spitler, & Chevalier 1972; Spitler 1987; Kundic & Ostriker 1995; Gnedin & Ostriker 1997, 1999; Gnedin, Hernquist, & Ostriker 1999). Tidal heating from shocks changes both the mean and the dispersion of particle energies within the satellite; to model heating fully requires a Fokker-Planck code to track the changing distribution function. In keeping with the simple method for estimating tidal mass loss developed above, we derive a first-order correction for tidal heating, and scale it to match the mass-loss rates seen in the simulations. To do this we first identify rapid shocks by comparing the shock timescale to the satellite’s internal orbital period. Specifically, we heat the satellite only when \( t_{\text{shock}} < t_{\text{orb, sat}} \), where \( t_{\text{shock}} = (t_{\text{sh}} + t_{\text{sh,b}})^{-1} \) is an average of the disk and bulge shock times \( t_{\text{sh}} = z/V_{\text{sat}} \) and \( t_{\text{sh,b}} = r/V_{\text{sat}} \), weighted so that the shorter time dominates, and \( t_{\text{orb, sat}} = 2\pi r_g / V( r_g ) \) is the orbital period of the satellite at its half-mass radius, \( r_g \). This corresponds to the range of shock timescales considered by Gnedin & Ostriker (1999). Over the course of each rapid shock, we then calculate the first-order change in energy within the satellite, and estimate how the satellite’s mass-loss rate will vary as a result of this energy input.

To derive the energy change, we use the formalism of Gnedin et al. (1999). Consider an element of unit mass, with coordinates \( x \) with respect to the satellite center. In the impulse approximation, tidal acceleration acting over the course of a rapid encounter, of duration \( t \), will induce a velocity change,

\[
\Delta V = \int_0^t A_{\text{tid}}(t') dt' ,
\]

relative to the satellite’s center of mass, where \( A_{\text{tid}} \) is the tidal acceleration. The resulting first-order change in its energy is simply equal to the work done by the tidal acceleration,

\[
\Delta E(t) = W_{\text{tid}}(t) = \frac{1}{2} \Delta V^2 = \frac{1}{2} \int_0^t A_{\text{tid}}(t') dt' \cdot \int_0^t A_{\text{tid}}(t'') dt''.
\]

If we divide the shock into a series of \( n \) discrete time steps of length \( \Delta t \), then the work done is

\[
W_{\text{tid}}(t_n) = \frac{1}{2} \Delta t^2 \left[ \sum_{i=0}^{n-1} A_{\text{tid}}(t_i) \cdot \sum_{j=0}^{n-1} A_{\text{tid}}(t_j) \right] .
\]

In going from \( t_n \) to \( t_{n+1} \), the energy change in a single time step is

\[
\Delta W_{\text{tid}}(t_n \to t_{n+1}) = \frac{1}{2} \Delta t^2 A_{\text{tid}}(t_n) \cdot \left[ 2 \sum_{i=0}^{n-1} A_{\text{tid}}(t_i) + A_{\text{tid}}(t_{n+1}) \right] .
\]

If the satellite is sufficiently small, we can express the tidal acceleration in terms of the gradient of the gravitational acceleration due to the external potential, evaluated at the center of the satellite:

\[
A_{\text{tid}}(t) = x(t) \cdot [ \nabla g_{\text{sat}}(x=0) ] = g_{a,b} \partial_x^n = a \cdot ( \partial_x^n ) b ,
\]

where \( g \) is the external gravitational field, \( g_{a,b} = \partial g / \partial x^a \) evaluated at \( x = 0 \), \( e_a \) is the unit vector in the \( x_a \)-direction, and repeated indices \( a, b \) indicate summation over the three Cartesian coordinates. Thus, taking the dot product in equation (9) and averaging over a sphere of radius \( r \) gives

\[
\Delta W_{\text{tid}}(t_n \to t_{n+1}) = \frac{1}{6} r^2 \Delta t^2 \left[ 2g_{a,b}(t_n) \sum_{i=0}^{n-1} g_{a,b}(t_i) + g_{a,b}(t_n) g_{a,b}(t_{n+1}) \right] ,
\]

with 18 terms from the two summations over \( a \) and \( b \), where we have used the fact that

\[
\langle x_i x_j \rangle = \frac{1}{2} r^2 \delta_{ij}
\]

averaged over a sphere. [As explained below, this is only strictly true if the shock is rapid, so that \( x_i(t_0) \approx x_i(t_f) \approx x_i(t_n) \) for all \( t_f < t_n \).

There are two important corrections to equation (11). First, our calculation is based on the impulse approximation, that is, the mass element is assumed to remain stationary over the course of the shock. This approximation is expected to break down in the central regions of the satellite, where the dynamical timescales can be comparable to, or shorter than, the shock duration. In these regions the effects of the shock will be greatly reduced. To account for this, we adjust the heating during rapid shocks using the
first-order adiabatic correction discussed by Gnedin & Ostriker (Gnedin & Ostriker 1999, and references therein):

\[ \Delta E_1 = A_1(x)\Delta E_{1,\text{imp}}, \]  

(12)

where \( A_1(x) = (1 + x^2)^{-\gamma} \), and \( x \), the adiabatic parameter, is the ratio of the shock duration, \( t_{\text{shock}} \), and the orbital period of the satellite at its half-mass radius, \( t_{\text{orb, sat}} \). Since most of the heating in our model comes from fairly rapid disk shocks, we use a value of 5/2 for \( \gamma \), in keeping with the results of Gnedin & Ostriker (1999).

Second, heating also leads to a change in the internal velocity dispersion of the satellite, as discussed by Kudić & Ostriker (1995). Both the average energy gain and the increase in the dispersion will cause most of the mass to become unbound. In keeping with the simplicity of our analytic approach, we only compute the first-order change in the energy distribution, and account for the higher order effects through the introduction of a heating coefficient, \( \epsilon_h \), that we adjust to yield reasonable overall matches to the VW99 simulations:

\[ \Delta E = \epsilon_h\Delta E_1, \]  

(13)

where

\[ \Delta E_1 = A_1(x)\Delta E_{1,\text{imp}} = A_1(x)\Delta W_{\text{tid}}. \]

Kudić & Ostriker (1995) estimate that the second-order heating term has an effect comparable to or greater than that of the first-order term, so we expect \( \epsilon_h \) to be greater than 2. From the disruption timescale arguments in Gnedin & Ostriker (1997), for instance, we might expect that \( \epsilon_h \approx 7/3 \). The value of \( \epsilon_h \) used in practice, however, will also depend on the dependence criterion and the adiabatic parameters discussed previously.

To determine how heating affects the satellite, we assume that the change in its mass distribution does not involve shell crossings, and that the potential energy of a mass element remains proportional to its total energy (as it would in virial equilibrium). The total energy \( E(r) \) of a mass element at a radius \( r \) will thus be proportional to \(-1/r\), so that an injection of energy \( \Delta E(r) \) will result in an eventual change in radius \( \Delta r \propto \Delta E(r)/r^2 \). In the absence of shell crossings, the mean density inside radius \( r \) will therefore change as

\[ \Delta \rho_r \propto \Delta \left( \frac{M(<r)}{r^3} \right) \propto -\frac{\Delta r}{r^2} \propto -\frac{\Delta E(r)}{r^2}. \]  

(14)

As equation (14) suggests, heating will cause the effective density of the bound part of the satellite to decrease, accelerating mass loss. The decrease in density of the bound component may correspond to an overall expansion of the satellite, or it may simply reflect an increase in the satellite’s velocity dispersion; either way, the predicted change in the bound mass is the same. The actual “size” of the satellite (say, the radius of the outermost bound material) will depend on both the expansion timescale and the stripping timescale. Since \( \Delta W_{\text{tid}}(r)/r^2 \) is independent of \( r \), however, we do not need to know this outer radius to account for heating. If we keep a running total of the 18 terms in equation (11), we can calculate the effective density change produced by tidal heating at some arbitrary radius \( r \) as a function of time, and then apply tidal stripping (eq. [4]) to the new, heated density profile to determine how much mass is lost. The calculation of the outer radius of the satellite is more complex, and since it should not affect satellite dynamics, and cannot be tested with the data available from the VW99 simulations, we will postpone it to a subsequent paper.

Our method for describing heating suffers from two limitations. First, we have used an average adiabatic correction for the system in equation (12). The actual correction for an orbit of radius \( r \) will depend on the orbital period at that radius, so the density change produced by heating will also depend weakly on \( r \). If we use a single scalar quantity to track \( \Delta E(r^2) \) for a given satellite, we will overestimate the heating experienced in its inner regions as a result. Second, we have assumed that the internal structure of the satellite does not change in the derivation of equation (14). On slow orbits, satellite structure may be partially revirialized as the system relaxes between shocks, producing a tightly bound core that is resistant to subsequent tidal effects. In practice, we expect these effects to be secondary, and to be partly masked by uncertainties in our choice of values for \( \ln \Lambda \) and \( \epsilon_h \).

3. COMPARISON WITH NUMERICAL RESULTS

To determine appropriate values for the three free parameters in our analytic model, \( \ln \Lambda_h \), \( \ln \Lambda_b \), and \( \epsilon_h \), and to test how accurately it predicts the evolution of a subhalo moving inside a larger halo, we compare our analytic results to 15 recent high-resolution simulations by VW99 of the evolution of a single satellite within a larger halo containing a disk galaxy. Reproducing the results of these simulations offers a good test of our simplified description of merging, since each one follows the orbital evolution and mass-loss history of the satellite in detail, and together they cover a range of different orbits and satellite densities. The satellites also have large masses and small orbital pericenters; thus, if we can match these simulations reasonably well, we expect the agreement to be even better for the more common case of small satellites orbiting at large distances in the halo.

3.1. Simulation Parameters

For the purpose of comparison, we evolved orbits in a static potential identical to the one adopted by VW99, which consists of three components: a truncated isothermal halo with a core, a stellar bulge, and an exponential disk. The density profiles of the three components are

\[ \rho_h(r) = \frac{M_h a}{2\pi^{1/2} r_{\text{cut}}^3} \exp \left(-\frac{r^2}{r_{\text{cut}}^2} \right), \]

\[ \rho_b(r) = \frac{M_b}{2\pi} a (a + r)^3, \]

\[ \rho_d(r) = \frac{M_d}{4\pi R_d^2 z_0} \exp \left( -\frac{R}{R_d} \right) \text{sech}^2 \left( \frac{z}{z_0} \right), \]

where the masses and scale lengths of the components are

\[ M_h = 7.84 \times 10^{11} \, M_\odot, \quad \gamma = 3.5 \, \text{kpc}, \quad r_{\text{cut}} = 84 \, \text{kpc}, \]

\[ M_b = 1.87 \times 10^{10} \, M_\odot, \quad a = 525 \, \text{pc}, \]

\[ M_d = 5.60 \times 10^{10} \, M_\odot, \quad R_d = 3.5 \, \text{kpc}, \quad z_0 = 700 \, \text{pc}. \]

The disk density used in the calculation of dynamical friction was smoothed in the vertical direction by two disk scale heights, as explained in § 2.2.1, to reflect the finite size
of the satellite, and we similarly smoothed the vertical component of the tidal field used in calculating heating by the disk. For the disk velocity dispersion, we used

$$\sigma_d = V_{c,d}/\sqrt{2} = \sigma_0 \exp \left( -R/2R_d \right),$$

where $V_{c,d}$ is the circular velocity of the disk, the disk scale length is $R_d = 3.5$ kpc as above, and $\sigma_0 = 143$ km s$^{-1}$. This expression is the expected functional form for the exponential disk model considered here, normalized to the value measured by VW99.

The sum of the halo and bulge densities was used to calculate the other friction term, since these components are kinematically similar. For the halo-bulge velocity dispersion, we used

$$\sigma_h = \left( \frac{V_{c,h}^2 + V_{c,b}^2}{2} \right)^{1/2},$$

where $V_{c,h}$ and $V_{c,b}$ are the circular velocities of the halo and the bulge, respectively. This expression is approximately valid over the range of radii of interest here.

Fifteen different orbits were calculated, with initial conditions corresponding to those of VW99 (see Table 1). Our satellites S1, S2, and S3 were King models identical to those used by VW99, with the initial masses, core radii, and tidal radii listed Table 2.

### 3.2. Results

Figure 1 shows the evolution of satellite S1 on five orbits of different inclination with respect to the disk. The open circles show the results of the simulation, and the solid curves show the results from our analytic model. Figure 2 shows similar results for the more concentrated satellite S2. In each figure, the left-hand plots show the position of the satellite versus time, while the right-hand plots show the mass. The angle $i$ indicated on the plots is the angle between the initial angular momentum vectors of the satellite and of the disk, so that orbits G1S2 and G1S9 are coplanar with the disk and prograde with respect to disk rotation, while orbits G1S6 and G1S13 are coplanar and retrograde.

The analytic orbits were calculated using fixed Coulomb logarithms of $\ln \lambda_h = 2.4$ for the halo and $\ln \lambda_d = 0.5$ for the disk, which are in the range predicted by the theoretical estimates mentioned in § 2.2.1 ($\ln \lambda_h \approx 1.9$–2.6 and $\ln \lambda_d \approx 0.6$–1.3, depending on the orbit and the satellite). The heating coefficient used, $\epsilon_2 = 3.0$, is also in the expected range. We see that for this choice of parameter values, we obtain a very good match to the orbital decay and mass loss in all ten cases.

Examining the orbital evolution in detail, we note that the analytic model matches the numerical results remarkably well, given that we used a single set of parameter values to fit results for three different satellites and eight different sets of initial conditions. Our prescription for dynamical friction reproduces the decay in the amplitude and period of the orbit, and the analytic orbit remains in phase with the numerical results for as long as the mass loss is well matched, typically five or six orbital periods. We show below that varying the Coulomb logarithms by 10%–20% produces a slightly better match to some of the orbits. This is not surprising, since the theoretical estimates for the Coulomb logarithms discussed in § 2.2.1 predict a small variation with orbital parameters and satellite properties. Varying the parameters by less than 10% does not affect the results substantially.

Comparing the mass-loss rates, we see that the analytic model gives an excellent estimate of the timescale for mass loss, and predicts the bound mass in the simulations to within 20%, up to the point where the satellite has lost most of its mass. Our model also reproduces the dependence of mass loss on the orientation of the orbit for prograde and retrograde orbits in the disk, predicting faster mass loss on prograde orbits. This appears to be mainly the result of the stronger dynamical friction experienced by satellites in this case. Orbits out of the plane of the disk show a weak dependence on inclination in the simulations. We reproduce this marginally, although the amplitude of the effect is much smaller in our model than in the simulations. This may indicate that dynamical friction from the disk is more important than we predict for these orbits.

As well as comparing results for a particular satellite model on orbits of different inclinations, we can also compare the results for different satellites on similar orbits, or for the same satellite on orbits of different circularity. Figures 3 and 4 show the orbital evolution of three different
Fig. 1.—Orbits and mass-loss histories for satellite S1 (solid curves), compared with numerical results from Velázquez & White (1999) (open circles). The parameter values used were $\ln \Lambda_h = 2.4$, $\ln \Lambda_d = 0.5$, $v_h = 3.0$. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 2.—Same as Fig. 1, but for a more concentrated satellite (Velázquez & White 1999, model S2). [See the electronic edition of the Journal for a color version of this figure.]
Fig. 3.—Orbits and mass-loss histories for three different satellite models with the same initial orbital parameters. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 4.—Same as Fig. 3, but for a retrograde orbit. [See the electronic edition of the Journal for a color version of this figure.]
satellites: the fiducial satellite (S1), a satellite that is more concentrated (S2), and one that is more massive and more concentrated (S3), on prograde (Fig. 3) and retrograde orbits (Fig. 4), inclined by 45° to the plane of the disk. For both prograde and retrograde orbits, the more massive satellite experiences more dynamical friction, falls in faster, and is quickly disrupted. The more concentrated satellite retains its mass longer than S1, despite having fallen further into the potential. The analytic model accurately reproduces these trends, although for the more concentrated satellite the mass-loss rates are a bit slow, as discussed further below.

Dynamical friction, tidal stripping, and tidal heating will also depend on the circularity of a satellite’s orbit. Figure 5 shows results for the same satellite model (S1), on inclined orbits of three different circularities. Here again we achieve a good match to the simulation results, reproducing the trend of faster mass loss for more radial orbits, and getting an accurate estimate of the disruption times for the three different orbits. This particular comparison also demonstrates the limitations of the approximation that satellites lose an equal fraction of their mass at each pericentric passage (e.g., Johnston 1998). This approximation correctly describes mass loss for the satellite on the orbit of intermediate circularity. For the satellite on a more radial orbit, however, the fraction decreases slightly with time, while for the more circular orbit it increases with time, as dynamical friction causes the orbit to decay. As we discuss in §4.4, the same is true of low-density versus high-density satellites.

Some systematic differences are apparent in the comparison between the numerical and analytic results. The mass loss in the simulations is smoother than in the analytic model, showing less variation in rate at pericentric passage. We also underestimate the mass-loss rates for the more concentrated satellite, on orbits that are inclined with respect to the disk (G1S10–G1S12). Varying the parameter values suggests that this is due to a slight underestimate of the dynamical friction in these cases. This may indicate that the Coulomb logarithm increases as the satellite loses mass, as discussed in §4.1. Some of the theoretical estimates for dynamical friction mentioned in §2.2.1 also predict a larger Coulomb logarithm for more concentrated satellites, so we may be seeing evidence of this in our results. In the absence of more numerical results to confirm this dependence on concentration, however, we limit ourselves to using a single set of average values for the Coulomb logarithms. Finally, our most circular orbit (G1S8) experiences slightly less mass loss than predicted. In this case, the characteristic timescale for the tidal shocks is very close to the internal orbital period of the satellite. Using a more restrictive definition of rapid shocks in this case produces results that match the numerical behavior exactly. Here again, however, we have insufficient numerical data to justify a general modification to our scheme.

We also note that the details of numerical mass-loss histories may depend on the precise definition of the bound mass adopted by the authors, as well as on the mass resolution, the softening lengths, and the time-stepping

![Fig. 5.—Orbits and mass-loss histories for satellite S1, on orbits of different circularity. [See the electronic edition of the Journal for a color version of this figure.]](image)
algorithms used in the simulations. Johnston et al. (1996, 1999), for example, find that mass loss is more sharply concentrated around pericentric passage in their simulations, compared to the results of VW99, and VW99 themselves find that changing the resolution of a simulation by a factor of 4 can have an effect comparable to the discrepancy we see between the analytic and numerical results (VW99, Fig. 10). Investigating mass loss in detail would require a careful reanalysis of the simulations. The fact that we find good agreement with numerical results over a range of orbits and for several satellite models, however, gives us some confidence in our description of orbital evolution.

In summary, using a simple model of dynamical friction, tidal heating, and tidal mass loss, we can reproduce analytically the results of high-resolution numerical simulations of mergers, predicting accurate timescales for satellite infall and disruption, with the correct dependence on satellite mass, satellite concentration, and on the inclination and circularity of the orbit. Our model has three free parameters: \(\ln \Lambda_h\), \(\ln \Lambda_d\), and \(\epsilon_s\). Of the three, the overall evolution of a satellite depends most strongly on \(\ln \Lambda_h\), to a lesser degree on \(\epsilon_s\), and only weakly on \(\ln \Lambda_d\), as discussed further below. Adjusting these parameters to match the results of VW99, we obtain values that fall within the range predicted by analytic calculations.

4. DISCUSSION

Having established that our method is capable of reproducing the simulation results of VW99, we have used it to carry out a series of controlled experiments, in order to gain a better understanding of the processes that determine the dynamical evolution of a satellite orbiting within a larger halo. This evolution is essentially driven by two competing processes: dynamical friction causes the satellite’s orbit to decay into the center of the larger halo, where it experiences more tidal stripping and heating, while tidal stripping and heating reduce the bound mass of the satellite, and thus the magnitude of the frictional force, slowing down the orbital decay. Which of the two processes will dominate depends on the mass and density of the satellite. Satellites that are both massive and dense, for instance, will fall into the center of the potential quickly, before they experience much mass loss, while low-density satellites will lose mass quickly, and fall into the potential more slowly as a result. We return to this point in § 4.4, since it may help to explain the widely varying conclusions regarding mass accretion and disk heating that have appeared in the literature.

Since the tidal heating and dynamical friction produced by the disk, halo, and bulge are specified explicitly in our model, we first test the sensitivity of its predictions to the values of the parameters \(\ln \Lambda_h\), \(\ln \Lambda_d\), and \(\epsilon_s\), which modulate these effects. This will allow us to determine how strongly the VW99 simulations constrain the average values of the parameters used here. Furthermore, we can also use these results to estimate how systematic changes in the Coulomb logarithms or in the heating coefficient might affect satellite evolution, when considering more general problems. This is particularly important, since current theories of structure formation predict a much broader range of satellite properties than is represented in the VW99 simulations. We also comment on the overall importance of tidal heating, since this process is often neglected when modeling satellite dynamics. Finally, by removing the disk and bulge components from our potential, we can estimate separately the effect of these components, which have not yet been included in most cosmological simulations, on the dynamical evolution of galactic satellites.

4.1. The Disk and Halo Coulomb Logarithms

To explore the sensitivity of our results to the values assumed for the halo and disk Coulomb logarithms, we have recalculated the VW99 orbits using values that are 20% larger and 20% smaller. In Figure 6, we show as an example the orbits for satellite S2 calculated with \(\ln \Lambda_h = 1.92, 2.4, \) and 2.88, while \(\ln \Lambda_d\) and \(\epsilon_s\) are set to their fiducial values. These values of \(\ln \Lambda_h\) cover the range expected from theoretical considerations, and in some cases produce comparable or improved fits to the simulations, so the variation in the orbits shown in Figure 6 gives a measure of the uncertainty in our results. In particular, since the timescale for orbital decay varies more or less linearly with the magnitude of dynamical friction, we expect that any such timescales derived using our model will be correct to within about 20%.

In Figure 7, we show the orbits of satellite S2 calculated with \(\ln \Lambda_d = 0.4, 0.5, \) and 0.6, while the other two parameters are set to their fiducial values. We see that in general, the satellite orbits are relatively insensitive to the value of the disk Coulomb logarithm. The only exceptions are orbits in the plane of the disk, where a 20% variation in \(\ln \Lambda_d\) results in a 10% variation in the satellite disruption time.

Finally, thus far we have held the values of the Coulomb logarithms fixed over the lifetime of a satellite. As discussed in § 2.2.1, the argument of the Coulomb logarithm, \(\Lambda\), may in fact scale as \((M_{\text{sat}}/M_{\text{halo}})^{-1}\). Figure 8 shows the effect of increasing the Coulomb logarithms according to this scaling, as the satellite loses mass. The initial values of \(\ln \Lambda_h\) and \(\ln \Lambda_d\) were set to 2.4 and 0.4, respectively. The primary effect of the scaling is to produce slightly faster orbital decay and more mass loss in the latter stages of each orbit. On the whole, scaling the logarithms with mass results in a comparable or slightly improved match between the analytical model and the numerical results, but the overall effect is small. In particular, the orbital decay and disruption times remain similar, since dynamical friction decelerates the satellite most effectively before it has lost much mass, and thus before its Coulomb logarithm can change substantially.

4.2. The Importance of Tidal Heating

Mass loss and tidal heating should depend primarily on the density profile of a satellite, rather than its total mass or its tidal radius. This hypothesis is confirmed by the simulations of Johnston et al. (1996), where satellites with the same mean density, but masses ranging over 2 orders of magnitude, show identical mass-loss histories. While the magnitude of dynamical friction may depend on the structure of the satellite in ways we have not accounted for, our model explicitly includes the dependence of heating and mass loss on the density profile. We do not expect the third free parameter in our model, the heating coefficient, to depend on satellite properties, since it simply accounts for the relative effects of first-order and higher order heating terms on the total mass-loss rate. In any case, the dependence of our results on \(\epsilon_s\) is weak: varying \(\epsilon_s\) by 20% has only a slight effect on the satellite orbits, as shown in Figure 9. This raises the question of whether heating can be neglected altogether. To address this question, we recalculated the satel-
Fig. 6.—Set of orbits calculated with constant values of the Coulomb logarithm \( \ln \Lambda = 1.92 \) and 2.88 (solid and dashed curves), compared with the fiducial set for which \( \ln \Lambda = 2.4 \) (dotted curves). [See the electronic edition of the Journal for a color version of this figure.]

Fig. 7.—Set of orbits calculated with constant values of the Coulomb logarithm, \( \ln \Lambda = 0.4 \) and 0.6. (solid and dashed curves, respectively), compared with the fiducial set for which \( \ln \Lambda = 0.5 \) (dotted curves). [See the electronic edition of the Journal for a color version of this figure.]
Fig. 8.—Set of orbits calculated with Coulomb logarithms which scale as \( \ln(M_{\text{halo}}/M_{\text{sat}}) \) (solid curves), compared with the fiducial orbits from Fig. 2 (dotted curves). [See the electronic edition of the Journal for a color version of this figure.]

Fig. 9.—Set of orbits calculated with heating coefficients \( \epsilon_3 = 3.6 \) (solid curve) and \( \epsilon_4 = 2.4 \) (dashed curve), and the Coulomb logarithms set to their fiducial values \( \ln \Lambda_3 = 2.4 \) and \( \ln \Lambda_4 = 0.5 \). [See the electronic edition of the Journal for a color version of this figure.]
lite orbits with heating turned off and the Coulomb logarithms set to their (constant) fiducial values.

The solid curves in Figure 10 show the evolution of satellite S1 on several orbits calculated without tidal heating, compared to the heated orbits (dotted curves) and the simulations (open circles). We see that the overall effect of heating is to increase mass loss, which in turn reduces dynamical friction. In general, the simulation results are better matched by including heating, although the importance of heating varies with circularity and inclination. On inclined orbits, satellites are strongly affected by heating, while its effect on orbits in the plane of the disk is minor. This indicates that for the orbits we have considered, disk shocks dominate over bulge shocks as a source of heating.

If we consider a satellite to be disrupted when it has lost \( \sim 90\% \) of its mass, the disruption times we measure for our satellites are up to 40% shorter because of heating. One might expect an even stronger effect. Comparing the mass-loss and orbital decay curves, however, we note that dynamical friction conspires to reduce the difference between the heated and unheated disruption times for the satellites and orbits considered here. In the no-heating runs, the satellites retain more of their mass initially, and therefore experience more dynamical friction. Their orbits decay faster as a result, and the satellites fall into the center of the potential and are disrupted. This accounts for the sharper cutoff to some of the mass-loss curves. In the runs with heating, early mass loss slows down the subsequent orbital decay, so that the disruption timescale is longer than it would be otherwise. We expect that the difference in disruption times would be greater for satellites with masses or densities smaller than those considered here, however. Heating will also produce a quite different distribution of stripped material from satellites. This is of interest when considering the tidal features of the dark and stellar halos that may be formed by the accretion and disruption of satellites. Overall, tidal heating cannot be neglected when studying minor mergers with semianalytic models.

4.3. The Effect of the Disk and the Bulge

The results discussed in the previous section suggest that the galactic disk strongly affects satellite evolution, by heating satellites on inclined orbits as they pass through it. It also generates dynamical friction, which is important for orbits in the plane of the disk. The effect of the bulge is less clear. Understanding the role of these structures is important in relating small-scale simulations such as those of VW99 to larger cosmological simulations, which do not yet include disks or bulges. To test the effect of these components on satellite orbits, mass-loss rates, and disruption times, and to determine any systematic trends affecting simulation results, we ran our model with one or both of these components removed.

Removing the bulge from the potential has little effect on the satellite orbits, acting only to decrease friction slightly. One would expect the disk to have a much greater effect, because of its greater mass and steep vertical density gra-

![Fig. 10.—Set of orbits calculated with and without heating (dotted and solid curves, respectively), compared with simulations (open circles). The parameter values used were \((\ln \rho_0, \ln \rho_s, \epsilon) = (2.4, 0.5, 3.0)\) and \((2.4, 0.5, 0.0)\) for the heated and unheated cases, respectively. [See the electronic edition of the Journal for a color version of this figure.]
The latter is about 10 times larger than that of the bulge or halo, and hence should contribute roughly 100 times more energy than the other components to heating satellites as they cross the disk plane.

In Figure 11, we show satellite orbits G1S2–G1S9, recalculated in the same potential without a disk (solid curves), as well as the previous results for orbits in the presence of a disk, but with heating turned off (dashed curves), and with both a disk and heating (dotted curves). We use the fiducial values for the Coulomb logarithms and the heating coefficient, and assume that the Coulomb logarithms are independent of the satellite’s mass, to facilitate comparison with other figures in the paper.

When the disk is absent, we see that the dependence of orbital evolution on inclination vanishes, as expected. Furthermore, the initial mass-loss rate is reduced, so that satellites fall farther into the potential without losing as much mass. Turning off the disk or turning off heating produces similar results for orbits G1S3–G1S5, indicating that the effect of the disk is mainly to heat satellites on inclined orbits. In the prograde, coplanar orbit G1S2, on the other hand, the disk contributes mainly to dynamical friction. For this orbit, turning off heating does not change the results substantially, whereas turning off the disk does, increasing the disruption time by almost a factor of 2.

Overall, we conclude that the presence of the disk will have an important effect on the evolution of satellites on orbits similar to those considered here, with pericenters of 20 kpc or less (about 6 scale radii). The satellite orbits seen in cosmological simulations are typically very eccentric. The average apo-to-pericenter ratio found by Ghigna et al. (1998) is 6:1, for instance. Consequently, even orbits with apocenters as large as 120 kpc are very likely be affected by the disk. The effect of the disk on the disruption times measured above is to reduce them by 20%–30%, but as with heating, this difference could be much larger for satellites of different masses or concentrations. To produce realistic distributions of galactic satellites or study the formation of the halo by semianalytic means (e.g., Bullock et al. 2000), it is therefore important to account for the effects of a disk in the model.

4.4. Results for Different Satellite Profiles

One complication, when comparing numerical studies of disk heating through minor mergers, is the fact that different authors have used satellites with different density profiles in their simulations. We have argued previously, however, that mass loss depends strongly on the density profile of the satellite. Since the mass-loss history of the satellite will affect its orbital evolution via dynamical friction, and also, presumably, its heating of the disk, the use of different density profiles may explain some of the variation between the results reported for different simulations. In this section, we recalculate a few orbits using various satellite models that have appeared in the literature, to test the effect of a satellite’s density profile on its orbital evolution. We then comment on the possible consequences of these results for disk heating.

![Figure 11](https://example.com/f11.png)

**Fig. 11.**—Set of orbits calculated without a disk component in the potential (solid curves), compared with the no-heating case (dashed curves), with the full model (dotted curves), and with the simulations (open circles). The parameter values used were $\ln \Lambda_1 = 2.4$, $\ln \Lambda_2 = 0.5$, and $\epsilon_s = 3.0$ or 0.0. [See the electronic edition of the Journal for a color version of this figure.]
Figures 12 and 13 show several orbits from the set used previously, recalculated for five different satellite models similar to those used in recent merger simulations. In Figure 12, the solid curves are for the VW99 satellite S1, the dotted curves are for a more concentrated King model, similar to the one used by HC97, and the short-dashed curves are for the more concentrated of the two satellites considered by TO92. In Figure 13, the solid curves are for VW99 S1, as before; the long-dashed curves are for the satellite model used by WMH96; and the dot-dashed curves are for a satellite with a NFW profile (Navarro et al. 1996, 1997) of comparable concentration. All the satellite models have been given the same mass for purposes of comparison. Their density profiles and structural parameters are listed in Table 2.

We see, from the mass-loss curves, the strong effect that the satellite’s density profile has on its dynamical evolution. The HC97 satellite is much more concentrated than S1, but has a similar core radius, and by implication an extended, diffuse envelope containing most of the satellite’s mass. This diffuse material is stripped off early on in its orbit, leading to much slower orbital decay thereafter. The TO92 satellite behaves in the opposite way—its mass is so tightly bound that it experiences almost no tidal stripping, falling directly into the center of the potential with little mass loss. Finally, the WMH96 satellite loses about as much mass as S1 in the first pericentric passage, but its overall mass-loss history leads to a much slower orbital decay. The same is true for the satellite with an NFW profile.

These results are consistent with those of the original studies. HC97 found that their satellites were stripped of most of their mass before they hit the disk, while TO92’s more concentrated satellite retained 90% of its mass as it fell in on a circular orbit to a final radius of 4 kpc. WMH96 found that a fair amount of mass was stripped off their satellite in the outer regions of the halo-disk system, but that it managed to carry as much as half of its mass into the central few kiloparsecs. Here we have compared satellites that differ only in density profile, on the same orbit in the same potential. Since the authors mentioned above consider different satellite masses, different orbits, and different forms of the galactic potential in their simulations, the analytic mass-loss rates shown in Figures 12 and 13 will differ in detail from their numerical results, but we certainly reproduce all the trends mentioned.

One way of understanding these different mass-loss rates is in terms of the fraction of a satellite’s mass that lies within a given mean density contour. This structural property is related to the concentration of a satellite and to the form of its density profile. Figure 14 shows the mass fraction as a function of mean density, plotted for the different profiles considered here. We see that all the mass in the HC97 model is at lower mean densities than in the VW99 model S1. Almost all the mass in the TO92 profile, on the other hand, is at mean densities much higher than S1, and higher than the central density of the main galaxy (which is roughly $1 M_\odot \text{pc}^{-3}$). For the WMH96 profile, half the mass is at densities lower than VW99 model S1, but the core of
the satellite is at higher densities. These different profiles lead to different mass-loss rates throughout the orbit of the satellite, and as a result, very different dynamical histories.

In particular, the amount of mass a satellite loses in the outer part of its orbit, before it hits the disk, can vary tremendously from one model to another. Figure 15 shows all the disk crossings recorded in three orbits (G1S1, G1S3, and G1S8), plotted in terms of the fraction of the satellite’s original mass that is still bound to it at that point, versus the radius at which it crosses the disk. The different symbols indicate the satellite models of TO92 (squares), HC97 (triangles), and WMH96 (circles). We see that while the TO92 satellite crosses the disk many times with almost all of its mass intact, less dense satellites such as that of HC97 have been stripped of most of their mass after a few orbits. The filled symbols in Figure 15 indicate the average mass fraction for all disk crossings between 8 and 12 kpc. While TO92’s satellite encounters the disk at this radius with 96% of its mass intact on average, the satellites of HC97 and WMH96 have only 20% of their mass intact at this point. Given that TO92 saw more disk heating in their study than HC97 or WMH96, these results suggest that heating may be simply related to the mass of material accreted by the disk, once tidal stripping has been taken into account. We shall investigate this possibility in detail in a subsequent paper.

In any case, when studying minor mergers, accretion, and disk heating, it is clearly important to use cosmologically

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**FIG. 13.—** Same as Fig. 12, for Velázquez & White’s satellite S1 (solid curve), the satellite of Walker, Mihos, & Hernquist (long-dashed curve), and a satellite with an NFW profile (dot-dashed curve). See Table 2 for the details of the models. The parameter values used were $\ln A_1 = 2.4$, $\ln A_2 = 0.5$, $\epsilon_1 = 3.0$, as above. [See the electronic edition of the Journal for a color version of this figure.]

**FIG. 14.—** Plot of the fraction of mass within a given radius, as a function of mean density within this radius, for the various satellite models. (Line styles are as in Figs. 12 and 13.) The midplane galactic density in the inner disk is around $1 M_\odot$ pc$^{-3}$. [See the electronic edition of the Journal for a color version of this figure.]
motivated density profiles, and to consider how different satellite models may affect the final results.

5. CONCLUSION

While there has been much progress recently in understanding how dark matter halos form through hierarchical merging on cluster, group, and galaxy scales, the dynamical evolution of substructure within galaxy halos is still not very well understood. Numerical simulations typically lack the resolution and statistics to follow the formation and evolution of substructure across the range of scales involved, and the dynamical effect of the baryonic components within halos remains uncertain. Several observed features of galaxies, such as thin disks, seem difficult to explain in hierarchical models of galaxy formation. This may be the result of the computational limitations restricting current numerical studies, or it may indicate a genuine problem with the underlying cosmological models. To explore the parameter space relevant to these issues requires a method that is faster and less computationally intensive than numerical simulation. To this end, we have developed a simple model of the dynamical evolution of halo substructure on galactic scales.

The evolution of small subhalos, or satellites, within a larger halo is the result of two competing processes: dynamical friction, which causes orbital decay, and tidal stripping and heating, which reduce satellite masses, and hence the drag force from dynamical friction. In our model, we follow the dynamics of individual subhalos numerically, but include dynamical friction, stripping, and heating explicitly, using analytic expressions from Chandrasekhar (1943), King (1962), Gnedin & Ostriker (1999), and Gnedin et al. (1999). We calibrate the model by comparison with

fully numerical simulations. In particular, we find that we can accurately reproduce the results of the most recent set of high-resolution simulations of satellite infall, by Velázquez & White (1999), and that comparison with these simulations fixes the values of our three free parameters to within roughly 10%. The values we obtain are all in the range predicted by first-order estimates of friction and heating.

Varying the shape of background potential, the amount of tidal heating, and the density profile of the satellite in our model, we can start to extract from these simulations the factors contributing to mass loss and orbital decay. In general, tidal heating increases the mass-loss and orbital decay times of our satellites substantially, although these effects are partly masked by dynamical friction for the satellites and orbits we consider. We find in particular that the presence of a thin disk causes substantial heating and increased mass loss for satellites on inclined orbits, thereby affecting their overall evolution considerably, while the presence of a central bulge has little effect. For the orbital eccentricities typically seen in cosmological simulations, satellites on orbits with apocenters as large as 120 kpc are likely to pass through the disk repeatedly within a Hubble time, so it is important to consider its effects when studying galactic satellites.

The overall evolution of a satellite is also very sensitive to its density profile. In the tidal truncation approximation, for instance, the satellite’s mass-loss history is determined by its mass distribution as a function of mean density. Satellites with dense cores and extended envelopes will lose much more mass initially than satellites with flatter density profiles. This may explain some of the discrepancies found between different simulations of disk heating through satellite infall.

Having achieved an excellent overall match to simulations of minor mergers, using a simple, physically motivated model of satellite dynamics, we can proceed to consider the evolution of the large numbers of subhalos that a galactic halo will accrete over its lifetime. In the next paper in this series, we describe how to construct a merger tree that determines the mass-accretion history of a halo, and use it as the input to our model of dynamical evolution. We shall then apply this method to several outstanding problems in galaxy formation, notably the question of disk survival in hierarchical models.

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