PPN parameters for multiscalar-tensor gravity without a potential

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Abstract. A generic multiscalar-tensor gravity action functional is proposed in the Jordan conformal frame. After redefining the scalar fields, the parameterised post-Newtonian parameters are calculated, neglecting the potential term. The field equations are shown to be nearly identical with those of scalar-tensor gravity up to the first post-Newtonian approximation, differing only in the definitions of constant terms. This work is a part of a pursuit to chart the landscape of observationally viable scalar-tensor theories.

1. Introduction
The Brans-Dicke theory was developed half a century ago in an attempt to introduce the elusive Mach’s principle in a mathematically rigorous relativistic description of gravity [1]. Since then scalar-tensor gravity (STG) theories have been put to use in various other contexts, such as theories of inflation and more recently, to give an account of the accelerating expansion of the universe [2, 3, 4, 5]. Scalar-tensor gravity introduces a new degree of freedom not present in general relativity. When formulating a STG, one has to specify the conformal frame. Common classification distinguishes between the Jordan frame and the Einstein frame; their equivalence in describing nature has remained a subject of academic discussion [2, 3, 6].

A natural generalisation of STG theories is obtained by considering multiple scalar fields. Multiscalar-tensor gravity (MSTG) theories have been first formulated and thoroughly studied in the Einstein conformal frame, including a simplified parameterised post-Newtonian (PPN) analysis revealing the PPN parameters $\gamma$ and $\beta$ [7, 8]. From a theoretical point of view, the MSTG action can also be viewed as a special case of the more general multiscalar Horndeski theory action [9, 10]. This paper presents a full PPN analysis of MSTG in the Jordan conformal frame. As a simplification, no potential term is considered. It is commonly argued that a potential would assume the role of the cosmological constant and therefore its contribution to solar system physics would be negligible in comparison with the effects of local matter [11, 12].

The paper is organised as follows. In Sec. 2 the multiscalar-tensor action functional is introduced. The scalar fields are suitably redefined and the equations of motion are presented. In Sec. 3 the parameterised post-Newtonian formalism is introduced and all relevant quantities are assigned orders of magnitude. The field equations are presented up to the first post-Newtonian order in Sec. 4 and solved in Sec. 5, revealing the full set of PPN parameters. The results are summarised in Sec. 6.
2. Action functional and equations of motion

We start with the following general form of the multiscalar-tensor gravity Jordan frame action with $N$ scalar fields $\Phi^\alpha$ [13],

$$S = \frac{1}{2\kappa^2} \int_{V_4} d^4x \sqrt{-g} \left( FR - Z_{\alpha\beta\rho} \partial^\alpha \Phi^\rho \partial^\beta \Phi^\beta - 2\kappa^2 U \right) + S_m[g_{\mu\nu}, \chi_m] \quad (2.1)$$

We have adopted a system of units where the speed of light and Planck’s constant are set to equal one, $c = h = 1$. The action (2.1) is a natural extension to multiple scalar fields of the general Jordan frame STG action [6, 14, 15, 16, 17]. Here indices $\alpha, \beta, \gamma, \ldots = 1, 2, \ldots, N$ label the scalar fields $\Phi$, indices $\mu, \nu \ldots = 0123$ belong to spacetime coordinates, while $ij \ldots = 123$ are reserved for spatial coordinates. The arbitrary functions

$$F = F(\Phi^1, \Phi^2, \ldots, \Phi^N), \quad Z_{\alpha\beta} = Z_{\alpha\beta}(\Phi^1, \Phi^2, \ldots, \Phi^N), \quad U = U(\Phi^1, \Phi^2, \ldots, \Phi^N) \quad (2.2)$$

determine a distinct MSTG if specified. Matter is represented by the matter action $S_m[g_{\mu\nu}, \chi_m]$, where $\chi_m$ denotes matter fields.

For a more straightforward physical interpretation of the theory it is convenient to define a new set of scalar fields $\{\phi^1, \phi^2, \ldots, \phi^{N-1}, \phi^N \equiv \Psi\}$ by setting $\Psi = F(\Phi^1, \Phi^2, \ldots, \Phi^N)$. Taking into account that

$$Z_{\alpha\beta} \partial_{\rho} \Phi^\alpha \partial^\rho \Phi^\beta = Z_{\alpha\beta} \left( \frac{\partial \Phi^\alpha}{\partial \phi^a} \frac{\partial \Phi^\beta}{\partial \phi^b} \partial_{\rho} \phi^a \partial^\rho \phi^b + 2 \frac{\partial \Phi^\alpha}{\partial \phi^a} \frac{\partial \Phi^\beta}{\partial \phi^b} \partial_\rho \phi^a \partial^\rho \phi^b + \frac{\partial \Phi^\alpha}{\partial \phi^a} \partial_\rho \phi^a \partial^\rho \Phi^\beta \right) \quad (2.3)$$

we denote

$$Z_{ab} = Z_{\alpha\beta} \left( \frac{\partial \Phi^\alpha}{\partial \phi^a} \frac{\partial \Phi^\beta}{\partial \phi^b} \right), \quad Z_{aN} = Z_{\alpha\beta} \frac{\partial \Phi^\alpha}{\partial \phi^a} \frac{\partial \Phi^\beta}{\partial \phi^N}, \quad Z_{NN} = Z_{\alpha\beta} \frac{\partial \Phi^\alpha}{\partial \phi^a} \frac{\partial \Phi^\beta}{\partial \phi^N} \quad (2.4)$$

$$U(\phi^1, \phi^2, \ldots, \phi^{N-1}, \Psi) = U(\Phi^1, \Phi^2, \ldots, \Phi^N) \quad (2.5)$$

where $a, b, \ldots = 1, 2, \ldots, N - 1$ index the scalar fields $\phi$. The target space of scalar fields $\Phi^\alpha$ can be considered as a $N$-dimensional space with a metric tensor $Z_{\alpha\beta}$ and we can use suitable transformations of the coordinates $\Phi^\alpha$ for imposing the following $N - 1$ conditions:

$$Z_{aN}(\phi^1, \phi^2, \ldots, \phi^{N-1}, \Psi) = 0 \quad (2.6)$$

Denote

$$Z_{NN} = \omega(\phi^1, \phi^2, \ldots, \phi^{N-1})/\Psi \quad (2.7)$$

The action (2.1) now reads

$$S = \frac{1}{2\kappa^2} \int_{V_4} d^4x \sqrt{-g} \left( \Psi R - Z_{ab} \partial_\rho \phi^a \partial^\rho \phi^b - \omega \frac{\partial \phi^a}{\partial \phi^b} \partial_\rho \Psi \partial^\rho \Psi - 2\kappa^2 U \right) + S_m[g_{\mu\nu}, \chi_m] \quad (2.8)$$

Note that the functions

$$\omega = \omega(\phi^1, \phi^2, \ldots, \Psi), \quad Z_{ab} = Z_{ab}(\phi^1, \phi^2, \ldots, \Psi), \quad U = U(\phi^1, \phi^2, \ldots, \Psi) \quad (2.9)$$

depend on all scalar fields. The scalar field $\Psi$ acts as a variable part of the gravitational constant, and thus it will be assumed that $\Psi > 0$, while the conditions $2\omega + 3 > 0$ and $Z_{ab} > 0$ are imposed to make the redefinition of the scalar fields in the Einstein conformal frame possible [13]. In this
work, the potential term $-2\kappa^2U$ is neglected in the following. The field equations corresponding to the action (2.8) are

$$\Psi R_{\mu\nu} = \kappa^2 \left( T_{\mu\nu} - \frac{\omega + 1}{2\omega + 3} g_{\mu\nu} T \right) + \nabla_{\mu} \partial_{\nu} \Psi + Z_{ab} \partial_{\mu} \phi^{a} \partial_{\nu} \phi^{b} + \frac{\omega}{\Psi} \partial_{\mu} \Psi \partial_{\nu} \Psi + \frac{g_{\mu\nu}}{4\omega + 6} \left[ \frac{\Psi}{\Psi} \frac{\partial Z_{ab}}{\partial \Psi} - Z_{ab} \right] \partial_{\rho} \phi^{a} \partial_{\rho} \phi^{b} - \frac{\partial \omega}{\partial \phi^{a}} \partial_{\rho} \Psi \partial_{\rho} \Psi - 2 \frac{\partial \omega}{\partial \phi^{a}} \partial_{\rho} \phi^{a} \partial_{\rho} \Psi$$

$$+ (2\omega + 3) \Box \Psi = \kappa^2 T + \left( \frac{\Psi}{\Psi} \frac{\partial Z_{ab}}{\partial \Psi} - Z_{ab} \right) \partial_{\rho} \phi^{a} \partial_{\rho} \phi^{b} - \frac{\partial \omega}{\partial \phi^{a}} \partial_{\rho} \Psi \partial_{\rho} \Psi - 2 \frac{\partial \omega}{\partial \phi^{a}} \partial_{\rho} \phi^{a} \partial_{\rho} \Psi$$

$$Z_{ac} \Box \phi^{a} = \left( 1 \frac{\partial Z_{ab}}{\partial \phi^{c}} - \frac{\partial Z_{ac}}{\partial \phi^{b}} \right) \partial_{\rho} \phi^{a} \partial_{\rho} \phi^{b} + \frac{1}{2} \frac{\partial \omega}{\partial \phi^{a}} \partial_{\rho} \Psi \partial_{\rho} \Psi - \frac{\partial Z_{ac}}{\partial \Psi} \partial_{\rho} \phi^{a} \partial_{\rho} \Psi \right)$$

In the rest of the paper Eqs. (2.10a) - (2.10c) will be our principal object of investigation.

3. PPN formalism

The parameterised post-Newtonian formalism is a useful calculation tool to confront metric gravity theories with solar system observations [11, 12]. The gravitational field is assumed to be slowly varying and weak. In a metric description of gravity these assumptions translate to

$$\rho = \text{the pressure and } \Pi = \text{the specific internal energy, } p = \text{the pressure and } u^\mu = \text{the four-velocity of matter. Orders of magnitude are assigned to these quantities relative to the velocity } u^i = u^i/u^0 \text{ of the source matter, which is taken to be a first order small quantity in units where the speed of light is equal to one. Denoting } O(n) \propto |\vec{v}|^n, \text{ we assign orders } O(2) \text{ to } \rho \text{ and } \Pi \text{ and } O(4) \text{ to } p. \text{ Time derivatives } \partial_\mu \text{ of the metric components and other fields are weighed with a velocity order } O(1). \text{ The space-time metric is taken to be a perturbed Minkowski metric}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

The Newtonian potential

$$U = \int \rho'/|\mathbf{x} - \mathbf{x}'| d^3x'$$

has the same order of magnitude $O(2)$ as $\rho$. The Newtonian description of gravity can be cast in metric form by setting $h_{00}^{(2)} = 2GNU$ and setting all other perturbations to zero. Here and
in the following a superscript in parentheses denotes the order of magnitude of the quantity it is attached to. Thus, in order to describe gravity at the first post-Newtonian level, higher order perturbations have to be considered. The detailed analysis shows that the non-vanishing post-Newtonian order metric components are \( h_{ij}^{(2)} \), \( h_{0j}^{(3)} \) and \( h_{00}^{(4)} \). In standard PPN formalism the so-called geometrised units are chosen such that the coefficient of \( 2U \) in \( h_{00}^{(2)} \), acting as an effective Newtonian constant, is set to equal one. The generic PPN metric is postulated with components

\[
\begin{align*}
    g_{00} &= -1 + 2U - 2\beta U^2 - 2\xi\Phi_W + (2\gamma + 2 + 3\epsilon + \zeta_1 - 2\xi)\Phi_1 \\
    g_{0j} &= -\frac{1}{2} [(4\gamma + 3 + \alpha_1 - \alpha_2 + \zeta_1 - 2\xi)\Psi_j + (1 + \alpha_2 - \zeta_1 + 2\xi)W_j] \\
    g_{ij} &= (1 + 2\gamma U)\delta_{ij}
\end{align*}
\]

(3.4a)

(3.4b)

(3.4c)

(3.4d)

where the post-Newtonian potentials have their standard definitions:

\[
\begin{align*}
    \Phi_W &= \int \frac{\rho^2(x-x')}{|x-x'|^3} \left( \frac{x-x'}{|x-x'|^2} - \frac{x-x''}{|x'-x''|^2} \right) d^3x'd^3x'' \\
    \Phi_1 &= \int \frac{\rho U}{|x-x'|} d^3x' \\
    \Phi_2 &= \int \frac{\rho U^2}{|x-x'|^3} d^3x' \\
    \Phi_3 &= \int \frac{\rho}{|x-x'|} d^3x' \\
    \Phi_4 &= \int \frac{\rho}{|x-x'|} d^3x' \\
    V_i &= \int \frac{\rho v_i}{|x-x'|} d^3x' \\
    W_i &= \int \frac{\rho (v' \cdot (x-x'))}{|x-x'|^3} d^3x'
\end{align*}
\]

(3.5a)

(3.5b)

(3.5c)

(3.5d)

while the PPN parameters \( \alpha_1, \alpha_2, \alpha_3, \gamma, \beta, \zeta_1, \zeta_2, \zeta_3, \zeta_4 \) and \( \xi \) remain to be determined by the theory [11, 12].

The perturbed energy-momentum tensor has components

\[
\begin{align*}
    T_{00} &= \rho \left( 1 + \Pi + v^2 - h_{00}^{(2)} \right) + \mathcal{O}(6), \quad T_{0j} = -\rho v_j + \mathcal{O}(5), \quad T_{ij} = \rho v_i v_j + p\delta_{ij} + \mathcal{O}(6)
\end{align*}
\]

(3.6)

The scalar fields also have to be expanded around their constant background values. The order of magnitude of source terms in equations (2.10b) and (2.10c) limits the possible perturbations to

\[
\Psi = \Psi_0 + \psi^{(2)} + \mathcal{O}(4), \quad \phi^a = \phi_0^a + \varphi^{(4)} + \mathcal{O}(6)
\]

(3.7)

Now we are set to expand the functions \( \omega \) and \( Z_{ij} \) in their Taylor series. We assume the leading terms and constant coefficients to be of order \( \mathcal{O}(0) \). For \( \omega \) the first terms in the Taylor series are

\[
\omega(\Psi, \phi^a) = \omega(\Psi_0, \phi_0^a) + \frac{\partial \omega}{\partial \Psi} \psi^{(2)} + \mathcal{O}(4) \equiv \omega_0 + \frac{\partial \omega}{\partial \Psi} \psi^{(2)} + \mathcal{O}(4)
\]

(3.8)

We could expand \( Z_{ab} \) in the same manner, but from Eqs. (2.10a)–(2.10c) it is evident that \( Z_{ab} \) or its derivatives appear only as coefficients of quantities that are beyond the order of magnitude necessary for calculations at the first post-Newtonian level. That simplifies the calculations significantly and it will be shown in the next section that the resulting approximate field equations are nearly identical to those of the case with just one scalar field.
4. Approximate field equations

Now all relevant quantities have been expanded in terms of perturbations with known orders of magnitude and Eqs. (2.10a)–(2.10c) can be written up to the first post-Newtonian order. We begin by writing Eq. (2.10b) up to the second order,

\[ \nabla^2 \psi^{(2)} = -\kappa^2 \rho / (2\omega_0 + 3) \]  

(4.1)

which is readily solved by

\[ \psi^{(2)} = \kappa^2 U / 4\pi (2\omega_0 + 3) \]  

(4.2)

It is useful to choose the following gauge to fix the coordinates [18]:

\[ h_{ij,j} - \frac{1}{2} \eta_{ij} \psi,0 = \frac{1}{\Psi_0} \psi,0 h_{0j,j} = \frac{1}{\Psi_0} \psi,0 \]  

(4.3)

Making use of Eqs. (4.3) the components of the Ricci tensor are calculated for the metric (3.2) from their definition,

\[ R_{00} = -\frac{1}{2} \nabla^2 h_{00}^{(2)} - \frac{1}{2} \nabla^2 h_{00}^{(4)} + \frac{1}{\Psi_0} \psi,0^{(2)} + \frac{1}{2\Psi_0} h_{00,j}^{(2)} \psi,j^{(2)} - \frac{1}{2} h_{00,j}^{(2)} h_{00,j}^{(2)} + \frac{1}{2} h_{jk}^{(2)} h_{00,jk}^{(2)} + \mathcal{O}(6) \]  

(4.4a)

\[ R_{0j} = -\frac{1}{2} \nabla^2 h_{0j}^{(2)} - \frac{1}{4} h_{00,0j}^{(2)} + \frac{1}{\Psi_0} \psi,0^{(2)} + \mathcal{O}(5) \]  

(4.4b)

\[ R_{ij} = -\frac{1}{2} \nabla^2 h_{ij}^{(2)} + \frac{1}{\Psi_0} \psi,ij^{(2)} + \mathcal{O}(4) \]  

(4.4c)

On the other hand, expanding Eq. (2.10a) results in the following equations for the Ricci tensor components,

\[ R_{00} = \kappa^2 \Psi_0 \left[ \frac{\omega_0 + 2}{2\omega_0 + 3} \left( 1 + \Pi - h_{00}^{(2)} - \psi^{(2)} \right) + v^2 + \frac{3\omega_0 + 3 \rho}{2\omega_0 + 3 \rho} - \frac{\partial \psi}{\partial \Psi_0} \right] + \mathcal{O}(6) \]  

\[ R_{0j} = \frac{1}{\Psi_0} \left( \psi,0^{(2)} \right) + \mathcal{O}(5) \]  

\[ R_{ij} = \kappa^2 \Psi_0 \left[ \frac{\omega_0 + 1}{2\omega_0 + 3} \rho \delta_{ij} + \frac{\psi,ij^{(2)}}{\kappa^2} \right] + \mathcal{O}(4) \]  

(4.5a), (4.5b), and (4.5c) respectively, one arrives at equations for the metric perturbations, which are solved in the next section. Since the equations we are about to solve are identical to the corresponding equations of the single field case up to the definitions of constants, it is reasonable to expect the solutions to be identical with the same exceptions. It is nevertheless instructive to present the calculation, as it forms a basis for doing the calculation for the more general case with a potential.
5. PPN parameters
In this section the approximate field equations are solved for the metric perturbations. The solutions have to be obtained in steps of growing orders of smallness. In the end the metric is compared to the generic PPN metric (3.4d) and the PPN parameters are presented. The interpretation of the parameters is not presented here, since it is thoroughly analysed elsewhere [11, 12]. From Eqs. (4.4a) and (4.5a) it is straightforward to derive the equation for $h^{(2)}_{00}$,

$$\nabla^2 h^{(2)}_{00} = -\frac{2\kappa^2 \omega_0 + 2}{\Psi_0} \frac{2\omega_0 + 3}{2} \rho + O(4)$$

(5.1)

which is solved by

$$h^{(2)}_{00} = \frac{\kappa^2 \omega_0 + 2}{2\pi \Psi_0} \frac{2\omega_0 + 3}{2} U$$

(5.2)

revealing the effective gravitational constant

$$G_{\text{eff}} = \frac{\kappa^2 \omega_0 + 2}{4\pi \Psi_0} \frac{2\omega_0 + 3}{2}$$

(5.3)

In choosing geometrised units, $G_{\text{eff}}$ is set to equal one, resulting in an expression for $\Psi_0$

$$\Psi_0 = \frac{\kappa^2 \omega_0 + 2}{4\pi \omega_0 + 3}$$

(5.4)

In the following the expression (5.4) will be used to eliminate $\Psi_0$.

From Eqs. (4.4c) and (4.5c) one arrives at an equation for $h^{(2)}_{ij}$,

$$\nabla^2 h^{(2)}_{ij} = -8\pi \frac{\omega_0 + 1}{\omega_0 + 2} \rho \delta_{ij}$$

(5.5)

which can be readily solved, yielding

$$h^{(2)}_{ij} = \frac{2\omega_0 + 2}{\omega_0 + 2} \delta_{ij} U$$

(5.6)

Equations (4.4b) and (4.5b) are combined to arrive at

$$\nabla^2 h^{(3)}_{0j} = \frac{2\kappa^2}{\Psi_0} \nabla_j \rho - \frac{1}{2} h^{(2)}_{0j,0j}$$

(5.7)

solved by

$$h^{(3)}_{0j} = -\frac{7\omega_0 + 10 \Psi_0}{2\omega_0 + 4} U_j - \frac{1}{2} W_j$$

(5.8)

Making further use of the definitions of the post-Newtonian potentials (3.5) and the lower order solutions (4.2), (5.2) and (5.6), Eq. (4.4a) assumes the form

$$R_{00} = -\nabla^2 U - \frac{1}{2} \nabla^2 h^{(4)}_{00} + \frac{1}{\Psi_0} \psi_{,00}^{(2)} + \frac{1}{\omega_0 + 2} U_{,i} U_{,i} - \nabla^2 U^2 + \frac{4\omega_0 + 6}{\omega_0 + 2} \nabla^2 \Phi_2$$

(5.9)

while Eq. (4.5a) is rewritten as

$$R_{00} = -\nabla^2 U + \frac{\kappa^2}{16\pi (\omega_0 + 2)(2\omega_0 + 3)^2} \nabla^2 U^2 + \frac{1}{\Psi_0} \psi_{,00}^{(2)} + \frac{1}{\omega_0 + 2} U_{,i} U_{,i} - \frac{2\omega_0 + 3}{\omega_0 + 2} \nabla^2 \Phi_1$$
\[
\frac{\kappa^2}{8\pi(\omega_0 + 2)(2\omega_0 + 3)^2} + \frac{1}{\omega_0 + 2}\nabla^2 \Phi_2 - \nabla^2 \Phi_3 - \frac{3\omega_0 + 3}{\omega_0 + 2} \nabla^2 \Phi_4
\]  

We have made use of the identities

\[ U_i U_j = \nabla U \cdot \nabla U = \frac{1}{2} \nabla^2 U^2 - U \nabla^2 U = \frac{1}{2} \nabla^2 U^2 - \nabla^2 \Phi_2 \]  

Equating (5.9) and (5.10) results in an equation for \( h_{00}^{(4)} \), from which the solution can be read as

\[ h_{00}^{(4)} = -2(1 + \Lambda) U^2 + 4 \left( \frac{2\omega_0 + 3}{2\omega_0 + 4} \right) \Phi_1 + 4 \left( \frac{2\omega_0 + 1}{2\omega_0 + 4} - \frac{\Lambda}{\omega_0 + 2} \right) \Phi_2 + 2\Phi_3 + 6 \left( \frac{\omega_0 + 1}{\omega_0 + 2} \right) \Phi_4 \]  

where

\[ \Lambda = \frac{\kappa^2}{8\pi(2\omega_0 + 4)(2\omega_0 + 3)^2} \]  

Eqs. (5.2), (5.6), (5.8) and (5.12) make up the complete perturbed space-time metric up to the first post-Newtonian order of magnitude. Lastly, the coefficients of the respective PPN potentials in the standard PPN metric (3.4d) and our solution are equated to reveal the full set of PPN parameters

\[ \beta = 1 + \Lambda, \quad \gamma = \left( \omega_0 + 1 \right) / \left( \omega_0 + 2 \right), \quad \xi = \alpha_1 = \alpha_2 = \alpha_3 = \zeta_1 = \zeta_2 = \zeta_3 = \zeta_4 = 0 \]  

It was pointed out earlier that the PPN analysis of a potential-free multiscalar-tensor gravity theory should coincide with the single scalar field case, save for the definitions of the constants \( \omega_0 \) and \( \frac{\partial \omega}{\partial \Psi} \big|_0 \), embedded here in the constant \( \Lambda \). The result (5.14) confirms that claim.

6. Concluding remarks

The parameterised post-Newtonian parameters were calculated for a general multiscalar-tensor theory of gravity with no potential term. Since, after suitably redefining the scalar fields, the structure of the field equations restricts the possible orders of magnitude of perturbations of all but one scalar field to beyond what is relevant to PPN calculations, the equations that determine the PPN parameters are nearly identical to those of the case with a single scalar field, differing only by the definitions of constants \( \omega_0 \) and \( \frac{\partial \omega}{\partial \Psi} \big|_0 \), which here encode the background values of all scalar fields. Solving the perturbed field equations up to the first post-Newtonian order confirms the close analogy of MSTG with the one field case. The presented calculation forms a foundation for generalising the presented results to the case with an arbitrary potential, extending the PPN analysis of the single field case with a potential [19] to multiple scalar fields.

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