Heuristic Optimization of Electrical Energy Systems: A Perpetual Motion Scheme and Refined Metrics to Compare the Solutions

Gianfranco Chicco, Andrea Mazza*

Politecnico di Torino, Dipartimento Energia “Galileo Ferraris”, corso Duca degli Abruzzi 24, 10129 Torino, Italy
(e-mail: gianfranco.chicco@polito.it, andrea.mazza@polito.it)

* Corresponding author: A. Mazza (andrea.mazza@polito.it)

Abstract

Many optimization problems admit a number of local optima, among which there is the global optimum. For these problems, various heuristic optimization methods have been proposed. Comparing the results of these solvers requires the definition of suitable metrics. In the electrical energy systems literature, simple metrics such as best value obtained, the mean value, the median or the standard deviation of the solutions are still used. However, the comparisons carried out with these metrics are rather weak, and on these bases a somehow uncontrolled proliferation of heuristic solvers is taking place. This paper addresses the overall issue of understanding the reasons of this proliferation, showing that the assessment of the best solver can be cast into a perpetual motion scheme. Moreover, this paper shows how the use of more refined metrics defined to compare the optimization result, associated with the definition of appropriate benchmarks, may make the comparisons among the solvers more robust. The proposed metrics are based on the concept of first-order stochastic dominance and are defined for the cases in which: (i) the globally optimal solution can be found (for testing purposes); and (ii) the number of possible solutions is so large that practically it cannot be guaranteed that the global optimum has been found. Illustrative examples are provided for a typical problem in the electrical energy systems area – distribution network reconfiguration. The conceptual results obtained are generally valid to compare the results of other optimization problems.
Keywords
Electrical energy systems, optimization, meta-heuristics, reconfiguration, solution ranking, first-order stochastic dominance.

1. Introduction

For many optimization problems, the solution space is composed of a number of local optima, among which there is the global optimum. Depending on the size of the system under analysis, the global optimum can be calculated (e.g., through exhaustive search over all the feasible solutions, or with a search over a limited number of solutions with the certainty that the non-searched solutions cannot provide better results). Otherwise, it is practically impossible to guarantee that it has been reached. In the latter case, the optimization method in many cases is derived from a metaheuristic framework based on the generation of solutions depending on random number extractions, and just provides pseudo-optimal or best-so-far solutions.

In general, during the execution of the optimization procedure, a heuristic method is run several times. A number of pseudo-optimal solutions can be obtained in different ways:

- by running a given method with different parameters, also with the aim of performing sensitivity analysis to determine the best values of the parameters;
- by running different methods (or variants of the same method), each one with a given set of parameters;
- for methods whose outcomes depend on random number extractions, different solutions are also found by using the same parameters and changing just the seed for random numbers extraction.

In the sequel, the term solver is generally used to represent the execution of an optimization method or variant with a specific structure and set of parameters.
For a given optimization problem, the determination of the best solver\(^1\) is one of the topics widely addressed in the literature. For this purpose, suitable metrics are needed in order to carry out effective comparisons among the methods or variants.

The Evolutionary Computation community is addressing the comparison among different solvers in a wide way, by considering a predefined set of problems to which the solvers are applied, and defining dedicated metrics. For example, the *performance ratio* is defined by dividing the computation time of the solver by the minimum computation time obtained from all the solvers. Then, the cumulative distribution function (CDF) of the performance ratio is called *performance profile* of the solver \([1]\). When the optimization problem has a particularly high computational burden, the *data profile* \([2]\) is constructed by using as ingredients the computation time, the number of function evaluations, and a user-defined target on the value of the objective function. In these approaches, one of the main limitations is the selection of the set of problems to analyze, for which there is no general criterion defined yet.

Other types of comparisons are carried out on the outcomes of two algorithms run on the same problem. These include non-parametric statistical hypothesis tests (e.g., the unpaired Wilcoxon rank-sum test), calculating the confidence interval around the mean (or the median) of the solutions \([3]\). However, focusing on the intervals around the mean or median may be limitative, as the details on the specific location of the solutions are not included.

In the electrical energy systems domain, simple indicators such as the best and worst solutions obtained, as well as the mean value, standard deviation and median of the solutions, are still used in a number of contributions published in scientific journals. However, none of these simple metrics is able to really provide significant outcomes. In fact:

- The best (globally optimal) solution could be obtained by chance at any time from any solver, regardless of the overall set of solutions reached; the best solution could be even found (without being aware of it) in one of the solutions indicated by the operator to form the initial population used by the heuristic method, namely, before running the procedure. In this case, running the

\(^1\) The inappropriateness of the concept “determining the best solver” is discussed in Section 2.
procedure would provide the best result. However, would anybody conclude that the heuristic is perfect, or it was just a lucky initialization?

- The mean value of the solutions could be the same for solvers providing very different solutions (among which some solutions could be close to the global optimum), or for solvers leading to concentrating most solutions around the mean value (in this case the best solution could be far from the global optimum). In this case, how to decide which solver is better?

- Using the standard deviation in addition to the mean value seems to improve the situation, but still with insufficient indications on the statistical distribution of the solutions.

In order to obtain more robust statistical information on the quality of the solutions, it would be possible to get information from calculating further probabilistic moments (e.g., skewness, kurtosis, etc.). However, the interpretation of the contribution of these probabilistic moments to establishing whether the results obtained from one solver are better with respect to the results obtained from another solver is not straightforward.

In practice, the mean and median values are still used to rank the algorithms also in International competitions based on heuristic optimizations. For example, in the series of conferences on Evolutionary Computation (CEC), the criteria to rank the algorithms for constrained real parameter optimizations executed on one problem with multiple runs (with a predefined number of runs) are based on the mean and median values [4]. In particular, the procedure for ranking the algorithms based on the mean values includes a first ranking based on the feasibility rate (to exclude algorithms that provide unfeasible solutions), a second ranking based on the mean violation amounts (to ensure that the solutions obtained do not violate the constraints), and a third ranking based on the mean value of the objective function. Likewise, the procedure for ranking the algorithms based on the median solutions ranks the feasible solutions based on the value of the objective function, and the unfeasible solutions based on the amount of constraint violation. The two rankings are then summed up to provide the overall ranking for the problem considered. Since the competition is based on multiple problems, the rankings obtained from each problem are then summed up to obtain the final ranking.
Specific competitions on heuristic optimization have been launched also in the power and energy systems area. For example, in the 2017 IEEE competition on modern heuristic optimizers for smart grid operation [5], two testbeds were constructed (specific comments are reported in the next section). Each testbed had to be run for a given number of scenarios. Each scenario required the execution of a given number $N_{MC}$ of Monte Carlo assessments of the objective function, resulting in $N_{MC}$ best solutions. Then, each scenario was characterized by a score, calculated as the mean value of the corresponding $N_{MC}$ best solutions. The total score was determined by the sum of the scores obtained for each scenario. Again, the best solution and the mean values were used to establish the scores.

This paper introduces a different rationale, with the aim of constructing performance indicators simple to be calculated and interpreted, and statistically more effective than the best value or the first probabilistic moments to represent the nature of the solutions.

The main contributions of this paper are:

a) An overview on some problems in the power and energy systems area that are addressed with heuristics methods, to show that several papers have used different heuristic methods, variants and hybrid versions to solve these problems, in most cases with the focus on testing a new method, without providing advances or insights on the problem itself.

b) A wide discussion on the reasons why the comparison of the performance of different optimization algorithms carried out with simple metrics may lead to an undue and somehow uncontrolled proliferation of heuristic solvers. A perpetual motion scheme is introduced to represent the unsolvable nature of the problem of limiting the introduction of “new best solvers”.

c) The formulation of more refined metrics defined to compare the optimization results in a statistically significant way for problems in which finding the global optimum cannot be guaranteed (extending the previous findings of the authors presented in [6]) and, as a new addition to the same framework, for problems with computable global optimum.

In particular, more refined metrics are defined to compare the optimization result in two specific cases:

- **Case G (Global)** – It is possible to calculate the globally optimal solution, for example by running
exhaustive search over all the solutions of the problem. In this case, conceptually there would be no need for running other optimization methods or meta-heuristics. However, if the globally optimal solution can be found only with very long computation time, much faster alternative optimization solvers could be available. In this case, it may be useful to assess the performance of the alternative solvers in specific situations, in order to avoid running exhaustive search each time the solution has to be calculated for a similar problem (e.g., when the parameters of the system under analysis change). In addition, testing some solvers on test systems for which the globally optimal solution has been already found may be useful for a preliminary assessment of these solvers before applying them to large-scale problems. This is for example the case of using predefined benchmark functions to test the solvers [4][7]. However, it has to be noted that the classical benchmark functions are rather different with respect to the problems addressed in practical applications on electrical systems. In fact, the electrical networks introduce challenging aspects, such as discrete problem formulations, and especially the presence of constraints difficult to handle (e.g., the equality constraints given by active and reactive power flow balances, and the constraint of maintaining a radial network structure [8][9]).

• Case R (Relative) – The number of feasible solutions is so large that the global optimum cannot be found in a reasonable computation time. In this case, the pseudo-optimal solution can be identified only in a relative way as the best solution found so far. This situation is frequently found in many problems, typically because of the combinatorial explosion of the number of possible solutions for large-scale systems.

The metrics introduced in this paper are based on the calculation of a number of solutions from different solvers, on the construction of the cumulative distribution function (CDF) of these solutions, and on the definition of specific indicators, having an intuitive geometric meaning. These indicators are based on the CDF of the solution obtained from each solver and on a reference CDF constructed by considering the global optimum for Case G or a selected set of solutions for Case R.

The next sections of this paper are organized as follows. Section 2 recalls some typical problems referring to power and energy systems that are addressed by using heuristic methods, and provides
specific comments on the variety of methods adopted. Section 3 discusses the issue of finding out the best solver, casting the problem into a perpetual motion scheme. Section 4 illustrates the use of more refined performance metrics based on first-order stochastic dominance concepts. Section 5 introduces, without loss of generality, the specific problem (distribution system optimal reconfiguration) addressed in the illustrative examples. Section 6 shows the results of specific distribution network optimal reconfigurations run as case study applications. The last section contains the concluding remarks.

2. Typical power and energy problems solved with heuristic methods

Various problems in the power and energy systems area have been typically solved in the last decades by using traditional optimization methods (e.g., gradient-based approaches, linear and non-linear programming, quadratic programming, dynamic programming, branch and bound, Newton-based methods, Lagrangian relaxation, and interior point methods). The traditional methods have been used very efficiently to obtain a rapid convergence to the optimum in case of optimization problems whose objective functions are defined in a polynomial way and the variables are defined in a convex domain. An example of this kind of problems is the basic economic dispatch, in which the objective function is the total cost composed of quadratic individual cost functions for the generation units. However, if specific aspects (such as valve-point effects and prohibited zones) are considered and the number of decision variables increases, the traditional methods suffer from the dimensionality problem and can fail the search of the optimum.

With respect to the basic formulation, different aspects can make the formulation of optimization problems addressing real-world problems more complex. For example:

- objective functions defined in highly non-linear forms, leading to the presence of several local optima;
- presence of linear or non-linear constraints that make the domain of definition of the variables mathematically non-convex;
- problems in which it is not possible to compute derivatives that could be sent to algorithms that use this type of information;
discrete combinatorial problems, affected by the curse of dimensionality when applied to large-scale systems.

For problems in which the traditional methods exhibit difficult convergence or require very long computation times, the application of heuristic methods has provided a viable alternative. Sometimes, heuristic methods could lead to results that were not previously found because the exploration of the domain of definition of the variables was inherently limited by the characteristics of the solver. The heuristics have been also used to solve problems that had no known solution, providing satisfactory (even though non-optimal) solutions.

There are some favorable aspects of the use of heuristic methods, for example:

- the heuristics are generally simple to implement in their solution schemes, even though in some cases a careful implementation of the solution strategy is needed to guarantee the effective incorporation of the equality and inequality constraints (as a matter of example, the radiality of the distribution system should be correctly implemented for avoiding the creation of a huge number of unfeasible solutions) [8];
- the heuristics contain in their underlying principles some specialized operators (some examples are parallelism, elitism, selection with probability-based acceptance, topology, memory, immunity, and self-adaptation [10]) that could be particularly useful to enable wider exploration of the space of the solutions;
- the heuristics can be hybridized to benefit from the different characteristics of two or more methods, for example adding an effective local search strategy to a method that is based on exploring a wide range of solutions, or using an internal optimization to find the most suitable parameters to be used by the solver. A successful example is the Evolutionary Particle Swarm Optimization (EPSO) [11], which uses an evolutionary model together with a particle movement operator to formulate a self-adaptive algorithm.

Various applications of heuristic methods to electrical power and energy systems have been reviewed in [12]. A selected set of illustrative applications is considered in this paper, in order to show the diffusion of the application of the heuristic methods to these problems, also in recent years:
• **Distribution system reconfiguration (DSR):** selection of the open/closed condition of the distribution network branches (or of the switches at the terminals of the branches) in order to optimize a predefined objective (or multi-objective function). DSR is a combinatorial optimization problem that uses non-linear mathematical models. Specific constraints are given by the network connectivity, the need of considering only radial configurations, and the technical limits on the node voltages, branch currents, and others [10]. More details are indicated in Section 5.2, as the DSR problem has been chosen to show the application examples in this paper.

• **Economic dispatch (ED):** calculation of the electrical power outputs of the generation units in the power system in order to obtain the minimum total operating cost, subject to a set of constraints. The equality constraints are given by the power balance equations. The minimum and maximum power, prohibited operating zones, multiple fuel options, and transmission limits are typically found as inequality constraints. In addition, depending on the type of generation unit (hydro or thermal) there are other constraints such as the valve-point effect, ramp rates, discharge limits, reservoir volume limits, and water dynamic balance equation with transport delay time. The main points for the use of heuristic methods rather than traditional methods are the large number of decision variables involved, difficulties in handling non-linear characteristics, non-smooth cost functions, presence of several local minima, non-convexity or non-connection of the domain of definition of the decision variables, and long computation times requested. Further accuracy issues are indicated in [13]. Moreover, in some cases multi-objective functions are considered, especially when there are objectives with different nature (e.g., emission, cost and so on) and these objectives are not incorporated into a single (weighted) objective function.

• **Load forecasting (LF):** forecast of the demand at different time scales (short-term, medium-term, and long-term) and at different levels of aggregation (individual or aggregate loads). The demand is related to a number of exogenous factors (e.g., temperature, humidity, economic and social aspects) in a complex and non-linear way, and other random components do appear as “noise”. For this reason, conventional techniques such as linear correlation models for time series analysis and
regression models could be poorly accurate. The heuristic methods are seen as alternatives to artificial neural networks and other machine learning approaches.

- **Maintenance scheduling (MS):** definition of the sequence in time to perform maintenance of the generation units in order to optimize a given objective function under the system operational constraints. This problem is non-linear and stochastic, and may have a number of conflicting objectives. MS is closely linked to reliability aspects and costs, and these objectives can be handled within a risk analysis framework. The constraints are manifold, and include technical aspects (e.g., load demand, system reserve requirements, availability of equipment, and duration of the maintenance period), personnel-related constraints (e.g., availability of maintenance teams) and economic aspects (budget constraints).

- **Optimal power flow (OPF):** determination of the operating point at steady state to minimize a predefined objective (or multi-objective) function (e.g., considering costs, emissions, system losses, and reactive power support) under a wide set of network constraints. The OPF is a non-linear optimization problem. Classical optimization methods are highly sensitive to the initial conditions and frequently converge to local solutions or even diverge.

- **Power system planning (PSP):** the related problems include operational planning (i.e., changes occurring in the system at constant load, such as increasing the level of system automation, replacement of network components, or reactive power planning) and expansion planning of network and generation (considering scenarios of demand growth, also taking into account the evolution of distributed generation and storage). The objective function for planning is typically economic and is composed of discounted investment costs, operation and maintenance costs, reliability costs, and salvage value. The constraints are numerous and may involve technical, economic, environmental and social aspects. Planning problems are by nature large-scale, non-linear combinatorial problems, with integer and real variables. The number of solutions to be evaluated could grow exponentially with the system size, and there is typically a large number of local optima.

Among the contents presented in [12] in 2008, the heuristic methods surveyed in [14] include evolutionary algorithms (e.g., genetic algorithms, evolution strategies, evolutionary programming, and
genetic programming), simulated annealing, tabu search (which is not a probability-based method), and particle swarm optimization. However, a number of other heuristic techniques have been introduced in the last decade.

A text search has been carried out in the Science Direct database, considering the set of problems indicated above and different heuristic methods. Table 1 shows the number of articles in which each specific heuristic has been mentioned or applied (in its original version or in a variant), or has been used as a benchmark to compare the results obtained by the proposed solver. The cases in which the heuristic appears for at least ten times for a given problem are listed in the table. Other heuristics appearing less than ten times are not considered. A basic reference is included for each heuristic method.

From Table 1 it appears clearly that some classical heuristics are widely considered, but also many heuristics appeared recently have been applied to the various problems. Overall, hundreds of articles are available in the literature to deal with or solve the set of problems indicated. For this reason, a comprehensive review of the contents of these contributions is outside the scope of this paper. However, in most cases these articles do not provide any advance or insight on the specific problem, but merely aim at testing a “new” heuristic, or a variant of an existing heuristic, or a hybridization of different methods (e.g., different heuristics, or heuristics and conventional algorithms, in the latter case typically resorting to the capabilities of the conventional algorithms to perform a deterministic local search). In the most significant cases, some parts of the methods are customized in order to deal with specific aspects of the problem under analysis (e.g., the treatment of the radial network constraint in the DSR problem). This customization can be seen as a methodological insight. Nevertheless, there is no certainty that the changes introduced lead systematically to better results with respect to other variants proposed. This lack of certainty gives room to the proliferation of new methods, variants and hybridizations.

Another reason why the number of contributions on testing heuristics in the power and energy systems area is so high is the lack of widely accepted benchmarks defined for the specific problems. This situation is different with respect to the evolutionary computation community area, where specific benchmark functions are used to test the algorithms. However, the properties of these benchmark
functions are typically different with respect to the properties of the objective functions and constraints encountered in the problems referring to power and energy systems. In particular, the major challenges appearing in the power and energy systems area are due to the need for handling the non-linear equality constraints that describe the power balances in large-scale applications, the topological constraints of the electrical networks, and the non-connected domain of definition of the decision variables.

On the basis of the above considerations, the next section presents a consistent scheme to explain the main reasons why there are so many heuristics proposed to solve the optimization problems, claiming each time that the solver tested is better than the existing ones.

3. Best Solver or Best Solution? A Perpetual Motion Scheme and the Emergence of a Paradox

Let us consider an optimization problem admitting several local optima, among which the global optimum cannot be practically identified in a reasonable computation time. This kind of problem is conceptually suitable for testing new solvers. Let us start from the best solution found so far, and construct a framework to represent the overall activity carried out by testing different solvers.

Fig. 1 shows how the identification of the best solver can be cast into a perpetual motion scheme, with no exit point. The *testing* block is the one in which the solutions (i.e., objective function values) are determined. On the basis of this scheme, the number of scientific publications appearing on the same subject by changing the solver is continuously increasing. Once a new best solver has been identified, the process continues by testing new solvers. Due to the intractable number of possible solutions and of the uncertainty given by the random search embedded in the solvers, it is possible to find out another solver providing better results according with the metric established for comparison. Of course, if the metric is *weak*, better solutions may be found more easily, as it will be remarked below.

An interesting question may be: is this growth in the number of publications continuously claiming to have found the best solver conceptually sound? The notes provided in this section aim at showing that in various cases the reasons indicated by the authors to declare their solver as the best one are somehow questionable.
Let us consider the following situation, which could emerge from applying the *perpetual motion* scheme. From successive testing on the same problem and different system or method parameters, the following propositions could hold (where better means strictly better, that is, the equality case is not considered):

- Solver D is better than Solvers \{A, B, C\}
- Solver E is better than Solvers \{B, D, F\}
- Solver G is better than Solvers \{A, E, D\}
- Solver F is better than Solvers \{A, D, G\}

By rewriting some parts of the last proposition, it is possible to recognize the following chain:

- Solver F is better than Solver G
- Solver G is better than Solver E
- Solver E is better than Solver F

A consequence of this chain of propositions is the absurd proposition

\[ \text{Solver F is better than Solver F}, \]

in which Solver F is qualified as strictly better than itself. The latter proposition is clearly a true paradox. However, situations like this one are possible in practical applications in which a new solver is tested on a given problem against a number of other solvers that provide worse performance in the executions carried out with the set of parameters adopted. The main problem of this approach is that it is only driven by the objective of obtaining better results, without trying to understand why the solver is performing better than others. This approach is making the meta-heuristics field vulnerable, as the undue and somehow uncontrolled proliferation of heuristic solvers makes it difficult to recognize the true innovation occurring in the field [50].

In the framework of analysis indicated in Fig. 1 for *Case R* optimization problems with no known global optimum applied to large-scale systems, the crucial point is: *if the solutions found on a specific problem by using one solver are better than with another solver, this does not mean that the solver is better.* In other terms, “the solver is better” is a *global* property that cannot be demonstrated in a complete way. Thereby, the conclusions in many contributions that claim to have found the “best method” by comparing sets of solutions obtained from different methods should be at least restated to
indicate that the method tested has provided better solutions for the specific problem and case study application, with the specific initialization and in terms of the indicator used to assess the solutions. Of course, in this case the strength of the scientific contribution would sound poorer, almost vanishing.

Nevertheless, in the absence of a global optimum there is no way to break the perpetual motion scheme in a scientifically rigorous way. What can be done in this respect is to improve the mechanism of comparison among different solvers, in order to require more significant information to declare that the solutions obtained from a solver are better than the solutions given by other solvers. In this case, the core of the problem is to reinforce the meaning of the test block “better solution?” appearing in Fig. 1.

This can be done by resorting to more refined metrics for ranking the solvers on the basis of a statistically significant set of solutions.

A dedicated metric has been introduced by [51], using two criteria to evaluate an algorithm, namely, the number of objective function calculations occurred before satisfying the stop criterion, and the value of the best objective function found by the optimization algorithm. These criteria are used in a Pareto-dominance analysis, concluding that the best algorithms are the ones producing non-dominated solutions in the plane defined by the number of objective function calculations and by the objective function value itself. An approximation to the first-order stochastic dominance concept [52] has been used in [51] to rank the solvers in each evaluation criterion. Further studies in [53] considered also one-way ANOVA for ranking each evaluation criterion.

In the following section, the stochastic dominance concepts are exploited to formulate intuitively meaningful indicators to create stronger comparisons among the optimization solutions. The testing of these indicators is then carried out on a problem referring to electrical distribution networks, whose characteristics are more elaborated than the classical benchmark functions.

4. Refined Metrics and Indicators to Assess the Results of Optimization Methods

4.1. Optimization formulation and deterministic dominance
The general framework for optimization is set up by considering the minimization case (without loss of generality) for a system with a given vector of parameters $p$, a given vector of decision variables $d$, and a scalar objective function $y$. In mathematical terms:

$$\bar{y} = \min_{(p,d)} g(p,d)$$

s.t. $r(p,d) = \ell$

$$w(p,d) \leq \ell$$

where $g(\cdot)$, $r(\cdot)$, and $w(\cdot)$ indicate the functions representing the objective, the vector of equality constraints and the vector of inequality constraints, respectively.

In order to compare the solutions obtained by executing different solvers taken from a given set of solvers $S$, each solver is executed for a given number of times, denoted as $H_s$. The dependence on $s$ is indicated because the number of solutions can be either the same for each solver (in order to set up the comparisons under the same conditions for all the solvers), or it may be different from solver to solver (for example, when the operator intends to highlight the difference among the execution times of each solver; in this case, $H_s$ can be determined as the number of solutions provided by the solver $s$ within a predefined total execution time).

The metrics considered here are based on the concept of dominance between two sets of solutions. Let us consider the $H_A$ and $H_B$ solutions provided by two solvers $\{A, B\} \in S$ run on the same optimization problem. The simplest case is to check whether the solutions obtained from the two solvers are completely separate; in this case, deterministic dominance does exist. By denoting with $\bar{y}_h^A$ the $h^{th}$ solution obtained with the solver A, and with $\bar{y}_h^B$ the $h^{th}$ solution obtained with the solver B, the dominance condition for which the solutions obtained from the solver A exhibit deterministic dominance over the solutions obtained from the solver B in case of objective function minimization (Fig. 2) is expressed as:

$$\max_{h \leq n} \bar{y}_h^A < \min_{h \leq n} \bar{y}_h^B$$

However, in practical applications, situations leading to deterministic dominance rarely occur. A more complete characterization of the dominance concept is provided in the literature by resorting to the stochastic dominance concepts illustrated in the next section.
4.2. Stochastic dominance

The stochastic dominance concepts introduced in [52] are revisited in the light of their application to the solutions of an optimization problem obtained from different solvers, extending the approach recently introduced by the authors in [6]. This section summarizes the basic stochastic dominance concepts.

To compare two or more solvers, let us consider the same number \( H \) of solutions taken for each solver. Checking the stochastic dominance requires constructing the Cumulative Distribution Function (CDF) of the \( H \) solutions of the optimization (1) obtained from each solver under analysis. These solutions are sorted in the ascending order of the variable \( y \). Let us denote with \( F_s^{(y)} \) the CDF corresponding to the solver \( s \in S \). Fig. 3 shows an example of construction of the CDF for an example with \( H = 50 \) solutions and objective function expressed in per units (p.u.).

Different stochastic dominance formulations can be introduced by considering two solvers \( \{A, B\} \in S \), run \( H \) times each one on the same minimization problem:

1. **First-order** stochastic dominance: the solutions obtained from the solver \( A \) exhibit first-order stochastic dominance over the solutions obtained from the solver \( B \) if and only if the following condition is satisfied on the CDFs for any value of the output variable \( y \), with strict inequality existing for at least one value of \( y \):

\[
F_A^{(y)}(y) > F_B^{(y)}(y) \quad \text{(3)}
\]

In graphical terms (Fig. 4a), the condition (3) means that no entry of the CDF referring to the solutions of solver \( B \) has to lie on the left-hand side of the CDF referring to the solutions of solver \( A \). The strict inequality is required to exclude that all the entries in the two CDFs are identical.

2. **Second-order** stochastic dominance: the solutions obtained from the solver \( A \) exhibit second-order stochastic dominance over the solutions obtained from the solver \( B \) if and only if the following condition is verified on the CDFs for any value of the output variable \( y \), with strict inequality existing for at least one value of \( y \):

\[
F_A^{(y)}(y) > F_B^{(y)}(y) \quad \text{(4)}
\]
In graphical terms (Fig. 4b and Fig. 4c), the second-order stochastic dominance condition is based on an integral quantity, and as such it cannot be excluded that one or more entries of the CDF referring to the solutions of solver B are located on the left-hand side of the CDF referring to the solutions of solver A (as in Fig. 4b). The case of Fig. 4b satisfies the second-order stochastic dominance conditions (as the area in Fig. 4c is always positive). However, it does not satisfy the first order stochastic dominance (as there are intersections between the CDFs).

On the basis of their definitions, the first-order stochastic dominance condition is sufficient to ensure second-order stochastic dominance. However, the reverse case does not hold, because in the second-order stochastic dominance, based on an integral condition, the CDFs of the solutions under analysis could cross with each other, thus not respecting the first-order stochastic dominance condition. For the same reason, higher-order stochastic dominance is not considered. Thereby, the first-order stochastic dominance is considered here as the relevant metric to be exploited for constructing suitable indicators to compare the solutions obtained from different solvers. The definition of these indicators is addressed in the next sections.

4.3. Determination of the reference CDF

The first aspect to develop performance indicators that can be used for appropriately comparing the outcomes of two (or more) optimization solvers is the definition of a reference CDF. This reference CDF should satisfy the following property: none of the CDFs originated from solutions of the solvers under comparison has to pass on the left-hand side of the reference CDF. Partial or total superposition of the CDFs under test with the reference CDF is allowed. This property can be seen as the declaration that the reference CDF exhibits first-order stochastic dominance with respect to all the CDFs under test, with the addition of the possibility that the reference CDF is exactly equal with one of the CDFs (not included in the first-order stochastic dominance conditions).

This reference CDF is constructed in a different way in the following two cases:

- **Case G**: when it is possible to construct the whole set of solutions and calculate the global optimum,
the reference CDF is equal to zero until the global optimum is reached, then it jumps to unity (Fig. 5). This behavior takes into account the fact that the best case for the solutions obtained from a solver would be to reproduce the global optimum in all its executions. The reference CDF for this case is denoted as $\overline{F}_{\text{ref},G}$ to identify the use of the global optimum.

- **Case R**: when the set of solutions is too large to be evaluated, the procedure introduced in [6] is used. The reference CDF is constructed by calculating the objective function during the execution of the optimization with each solver (by using the penalized objective function in order to take into account possible constraint violations, as indicated in Section 4.2). Then, a predefined number $H$ of solutions is taken from each CDF under test. If the number $H_s$ of solutions for each solver $s \in S$ has not been set to the same value $H$ during the executions (e.g., because successive optimizations have been for each solver run until the redefined computation time limit has been reached), the number of solutions is taken as the minimum $\mu = \min_s \{H_s\}$, and the best $H$ solutions are considered for each solver, for the benefit of the solvers providing faster executions. If the solutions obtained from $M$ solvers have to be compared, the number of solution points available is $M \cdot H$. These points are ordered in the ascending order all together, then the best $H$ points are taken to construct the reference CDF. Fig. 6 summarizes the procedure by using three solvers $\{A, B, C\} \in S$ and $H = 100$ points for each solver. The reference CDF for this case is denoted as $\overline{F}_{\text{ref},R}$ to identify the fact that it comes from a relative assessment.

There is a conceptual difference between the rationale used in the definition of the reference CDF for Case G and Case R. In Case G a fixed reference entry (the global optimum) is available, thereby the reference CDF is set up in a fixed way and the comparison between the actual CDF and the reference CDF is conducted in **absolute** terms. In Case R the entries of the reference CDF may vary depending on $H$ and on the context in which the calculations are carried out (e.g., the computation time limit indicated above), so that the comparison between the actual CDF and the reference CDF is conducted in **relative** terms.

### 4.4. First-order stochastic dominance-based indicators
In this section, the first-order stochastic dominance concepts are adapted to the optimization outcomes, considering objective function minimization.

Specific indicators are constructed on the basis of the reference CDF and of the CDFs under test. An indicator for each test CDF is calculated. For this calculation, the reference CDF and the test CDF must be represented with the same number of points $H$ on the vertical axis, corresponding to the width $\Delta c = 1/H$. Thereby, in Case $G$ of Section 3.3, the reference CDF is discretized into $H$ points having the same value of the objective function, located at successive vertical steps of width $\Delta c$.

By definition of the reference CDF, no point of the CDF under test can be located on the left-hand side of the reference CDF. Thereby, it is possible to formulate indicators that may assume values belonging to the interval $(0,1)$. The test CDFs may then be ranked according with these indicators, with higher values representing better performance, occurring when the area between the reference CDF and the test CDF, calculated by using the $H$ solution points, is smaller. The unity value is reached when the CDF of the test method is exactly equal to the reference CDF.

The $H$ objective function values located on the horizontal axis of the CDF, $z = 1,\ldots, H$, are denoted with $y_{s(z)}$ for the test CDF of the solver $s$, and with $y_{ref}(z)$ for the reference CDF in Case $R$. The following areas are calculated by summing up the horizontal areas defined on the basis of the reference and test CDFs (since the vertical steps in the CDF have the same length):

- Area referring to the test CDF obtained with the solutions of the solver $s \in S$ in Case $G$, calculated with respect to the reference CDF when the global optimum $y_{s(z)}$ is known:
  \[
  [\mu_{s(z)}] = \sum_{z=1}^{H} f_{ref}(y_{s(z)}) - y_{ref}(z) \]

- Area referring to the test CDF obtained with the solutions of the solver $s \in S$ in Case $R$, calculated with respect to the reference CDF when the global optimum is not known, considering the common partitioning of the vertical axis (Fig. 7):
  \[
  [\mu_{s(z)}] = \sum_{z=1}^{H} f_{ref}(y_{s(z)}) - y_{ref}^{(0)}(z) \]

(5)
The indicators are then defined under the general name \textit{OPISD} (Optimization Performance Indicator based on Stochastic Dominance). In order to take into account the two cases described in Section 3.3, two versions of the indicator are formulated:

- \textit{OPISDG} (\textit{global OPISD}), to be applied when the global optimum of the objective function under analysis is known (\textit{Case G}) by considering a number \( H \) of CDF values:
  \[
  \text{OPISDG} = \frac{1}{1+H^2}
  \]  
  \text{(7)}

- \textit{OPISDR} (\textit{relative OPISD}), to be applied when the global optimum of the objective function under analysis is not known (\textit{Case R}) [6] by considering a number \( H \) of CDF values:
  \[
  \text{OPISDR} = \frac{1}{1+H^2}
  \]  
  \text{(8)}

The expression used for the definition of the indicators comes from the need to take into account cases in which an area could be null, as well as to work when the objective function may assume both positive and negative values, providing results in the interval \((0,1)\) at all times.

In spite of their similar definition, the two indicators are kept separate because of a fundamental concept: the reference CDF for \textit{global OPISD} never changes, being the global optimum, so that the values assumed by the indicator \textit{OPISDG} for a given value of \( H \) can be assumed as an absolute reference that can be used to rank the results of different contributions prepared at different times. Conversely, the reference CDF for \textit{relative OPISD} generally changes each time a comparison among two or more solvers is carried out, so that the numerical results for \textit{OPISDR} are only valid for the corresponding ranking obtained, and cannot be used in successive comparisons.

The test methods are then \textit{ranked} on the basis of the relevant \textit{OPISD} indicator from the most effective one (i.e., with the highest \textit{OPISD} value) to the less effective one (with the lowest \textit{OPISD} value).

The new \textit{OPISD} indicators are a general-purpose metric that can be used to rank the performance of different optimization methods. It is particularly suitable to rank \textit{heuristic} methods, for which many solutions may be available from multiple executions even with the same number of parameters, by just changing the seed for random number extractions. Applications of the proposed metric to a discrete optimization problem are illustrated in Section 4.
4.5. Further indicators

In addition to simple indicators such as the mean, the median and other probabilistic moments, further indicators can be defined by taking into account how many solutions are located on the reference CDF. In this case, the different nature of Case G and Case R emerges again. In fact:

- The indicator $\frac{\text{PERC}_{G}}{\text{PERC}_{G}}$ consists of the percentage of solutions reaching the global optimum (and by definition the reference CDF) in Case G. This indicator has an absolute meaning, as the global optimum for the problem under analysis is fixed.
- The indicator $\frac{\text{PERC}_{R}}{\text{PERC}_{R}}$ consists of the percentage of solutions located on the reference CDF in Case R. It is an indicator with relative meaning, as the reference CDF in this case changes depending on the test CDFs in the comparison carried out.

5. Optimization Problem and Solution Methods

5.1. Stochastic dominance applications to power and energy systems

Stochastic dominance concepts have been used to represent the constraints in some problems in the power and energy systems area. The most typical application of stochastic dominance concepts refers to risk-aversion modelling in electricity trading and portfolio optimization. The second-order stochastic dominance constraint has been imposed in [54] to reduce the risk of low profit for an electricity retailer in deciding the contracts and selling prices to offer to potential customers, and in [55] for self-scheduling of large consumers. In [56] portfolio optimization is carried out by setting up a minimum tolerable CDF used as the reference; then, the CDF of the portfolio model is accepted if it exhibits second-order stochastic dominance over the reference CDF.

Furthermore, first-order stochastic dominance constraints have been applied in [57] to the expansion planning problem for optimizing the introduction of new generation capacity, and in [58] to the risk-based approach for energy trading in a virtual power plant. Energy scheduling in a residential system...
with renewable energy sources has been addressed in [59] by choosing the renewable energy distributions according with the first-order stochastic dominance principle.

Generally, the indicators $OPISD_G$ and $OPISD_R$ can be used to deal with various optimization problems. An application of the first-order stochastic dominance to compare the CDFs of the solutions obtained from different heuristics has been presented by the authors in [6], defining the $OPISD$ indicator corresponding to what is called $OPISD_R$ in this paper. Without loss of generality, the results presented have been obtained on a classical optimization problem in the electricity distribution area – the distribution network reconfiguration with minimum losses. This problem is conceptually simple and can be easily understood by the readers with different backgrounds. At the same time, it is a discrete optimization problem for which it is possible (for relatively small networks) to calculate the global optimum and construct the reference CDF for Case G, while for large networks the combinatorial explosion of the number of solutions does not enable us determine the global optimum in a reasonable computation time, so that the definitions valid for Case R have to be applied. The solutions of the optimal distribution network reconfiguration for Case G and Case R are considered in this section.

5.2. Description of the DSR optimization problem

The Medium Voltage (MV) electricity distribution systems are structurally composed of a number of nodes interconnected by branches forming a weakly meshed structure. The network is typically operated with a radial configuration in order to simplify the protection schemes.

For a distribution system with $N$ nodes, $B$ total branches and $S$ supply points, any radial configuration starting from the supply nodes without isolating any of the other nodes has $A = B - N + S$ open branches. This condition is necessary to define the radial network, but it is not sufficient, because by cutting $A$ branches at random it would be very likely to create loops in the network and isolate some nodes. Moreover, the total number of $\Psi$ radial configurations that can be obtained by starting from a given network structure is computable by using the Kirchhoff’s tree matrix theorem [60]. However, this number of configurations could be so high that even the generation of these radial configurations (that is not straightforward and would need to apply a graph-search algorithm) could be practically intractable.
As an example, the real distribution network shown in [61] has $N = 207$ nodes, $B = 213$ branches and $S = 1$ supply point, resulting in $A = 7$ open branches for each radial network, and in $\mathcal{V} = 151,641,612$ possible radial configurations. For larger networks the situation is even more extreme. For example, in the urban distribution network used in [62] with $N = 535$ nodes, $B = 554$ branches and $S = 4$ supply points, there are $A = 23$ open branches for each radial network, and the number of possible radial configurations is as high as $\mathcal{V} = 7.197 \cdot 10^{32}$. With these numbers, it is clearly not practically feasible to determine the global optimum for this problem with certainty in a reasonable computation time. Thus, a number of deterministic and meta-heuristic methods have been proposed in the literature to calculate pseudo-optimal solutions with an acceptable computational burden. Reviewing the state of the art of these methods is outside the scope of this paper (some recent reviews may be found in [63] and in the introductory sections of [61]), as the focus is set here on the metrics to compare the results obtained from different methods.

A further challenge of this problem is that the solutions have to satisfy a set of constraints. These constraints are of two types:

1. All configurations created in the solution process have to be radial. This aspect is addressed by applying the close-open branch-exchange mechanism [64], in which each configuration change is applied by closing an open branch, detecting the closed loop formed in the network, then opening a branch in the loop to restore the radial structure.

2. Operational limits, referring to various aspects, such as maximum and minimum voltage magnitude limits at each node, maximum fault currents at each node, maximum current magnitude at each branch, and so forth [65]. This aspect is addressed by introducing specific penalty terms in the problem formulation, in such a way that a penalized objective function can be used to ensure that any radial configuration corresponding to a numerical solution (discarding only the configurations leading to lack of convergence of the power flow calculations) is represented among the configurations to be analyzed. This fact enables the use of solvers taken from meta-heuristics for which a temporary solution worsening may be acceptable (provided that a probability-based check is passed) in order to open the search space to reach more solutions.
For a given radial configuration $X$, belonging to the set $X$ of the radial configurations for which the power flow calculations is solvable, the total network losses $P_{TOT}(X)$ are expressed in terms of the resistance $R_b$ and current magnitude $I_b$ of any branch $b \in B$, where $B$ is the set of network branches:

$$\overline{P_{tot}}(X) = \sum_{b \in B} R_b I_b^2$$

(9)

Considering the set $\bar{\nu}$ of variables subject to constraint violation, the amount of violation $\bar{\nu}_v$ and the penalty factor $\bar{\alpha}$ that assumes positive values only when the corresponding violation does exist for each $v \in B$, the penalized objective function is formulated as:

$$\bar{f}_P(x) = \overline{P_{tot}}(x) + \sum_{v \in B, \bar{\nu}_v > 0} \bar{\alpha}_v \bar{\nu}_v^2$$

(10)

The optimization problem becomes

$$\min_{x \in \mathcal{X}} \bar{f}_P(x)$$

(11)

Since this paper is focused on the definition of the performance indicators, without loss of generality, the minimum losses problem addressed here is based on a single loading condition. More generally, with the current diffusion of distributed resources in the distribution systems, the minimum losses problem is formulated by taking into account the evolution in time of generation and demand patterns, assessing the losses on a given time interval, also with the possibility of determining an optimal set of network configurations for intra-day time periods [61]. The concepts introduced in this paper can be directly applied to the results of these more detailed problem formulations.

5.3. Solution methods

For the purpose of this paper, three solvers implemented according to specific meta-heuristics, namely, Simulated Annealing (SA), Genetic Algorithms (GA) and Particle Swarm Optimization (PSO), have been selected. These solvers have been already widely used in the literature for distribution system optimal reconfiguration problems [63]. For each method, there is a set of parameters to be defined before its execution. In addition, these methods require random choices; thereby, to ensure repeatability of the results it is necessary to fix the seed for random number extraction to a given value $s_0$. On another
point of view, different solutions can be found even with the same set of parameters, just changing the seed for random number extraction. The seed for random number extraction is the entry point to access the sequence of random numbers implemented in the specific function to extract random numbers from a uniform probability distribution in $(0,1)$, and cannot be considered as a parameter of the solver.

The same *adaptive* stop criterion is used for the three methods, concluding the execution when the objective function does not change for a predefined number $N_s$ of successive iterations \([65]\). The other parameters to be set up for the three methods are recalled below.

The SA method \([41]\) is composed of a main cycle and an internal cycle. The main cycle is driven by a control parameter $c$ having an initial value $c_0$ and decaying in the successive iterations (using $m$ as the iteration counter) according with the progression $c_m = \alpha c_{m-1}$, with a constant cooling rate $\alpha$ chosen in the range $(0,1)$, until the stop criterion (defined on the iterations of the main cycle) is satisfied. The initial value $c_0$ can be estimated by running the SA procedure with given values of the parameters until a predefined number of worse solutions $N_w$ has been reached, then calculating the average worsening $\overline{w}$ and imposing a percentage of acceptance $p_0$ for that worsening. Thereby, $c_0 = \overline{w}/\ln(1/p_0)$. At any iteration of the main cycle, an internal cycle is run, performing successive branch-exchanges driven by random choices of the branches to close and open, and stopping the corresponding iterations when the maximum number $M_A$ of configurations analysed or the maximum number $M_C$ of configurations accepted are reached.

In the GA method \([66]\), the information on the network configuration is coded in a string (called chromosome) containing $B$ binary values (called genes) set to 0 for representing an open branch and to 1 for representing a closed branch. A population of $C_{GA}$ strings is formed and used in the GA solution. The objective function is calculated for each string. The solution process is composed of a cycle, in which successive steps perform selection (with a user-defined criterion based on the fitness of each chromosome calculated by using its objective function), crossover (with crossover probability $p_c$) and mutation (with mutation probability $p_m$) until the stop criterion is satisfied.

In the PSO method \([40]\), the information is coded as in the GA. A population of $C_{PSO}$ particles is used. The solution process is composed of a cycle, in which the three terms forming the velocity of the
particles (inertia, memory, and cooperation) are updated and the new string (corresponding to the particle position) is determined for each particle. The objective function is calculated for each string, updating the local best and the global best each time the corresponding solution is improved. According with [67], the inertia weight \( w \) is progressively changed during the iterations from a maximum (initial) value \( w_{\text{init}} \) to a minimum (final) value \( w_{\text{final}} \). A list of possible strategies for varying the inertia weight \( w \) is shown in [68]. In this paper, the strategy e1-PSO from [69] has been applied.

In summary, the sets of parameters considered are:

- for SA: \( p_{SA} = \{ \alpha, N, p_0, M_A, M_C, N_s \} \);
- for GA: \( p_{GA} = \{ C_{GA}, p_c, p_m, N_s \} \);
- for PSO: \( p_{PSO} = \{ C_{PSO}, w_{\text{init}}, w_{\text{final}}, N_s \} \).

6. Application Examples

6.1. Test network with known global optimum (Case G)

Let us consider a test network used in various literature contributions, with 70 nodes [70]. The characteristics of this network are summarized in Table 2. The total number of radial configurations has been calculated, resulting in \( \Psi = 407,924 \) configurations [61].

By considering the initial network structure and loading provided in [71], in the reference system\(^2\) used the base voltage is set to the rated voltage, the base power is equal to 10 kVA, and the supply voltage is set to 1 per unit (p.u.).

The global optimum for this network and loading conditions has minimum total losses equal to 9.859 p.u.

6.1.1. Comparison among the solutions obtained from different solvers

For the execution of the solvers, \( H_s = 100 \) solutions are extracted from each solver, and \( H = 100 \) solutions are used to form the reference CDF. Furthermore, \( N_s = 20 \) is used in the stop criterion, and the

\(^2\) This reference is adopted here in the absence of uniform definitions of base power and base voltages in the literature papers using this test network.
seed for random number extraction is fixed \((s_0 = 1)\) for all executions to guarantee repeatability of the results. The parameters of the three solvers are set up in such a way to leave only one or a few parameters variable in a given range of values (Table 3). For the other parameters, some general choices are made, as follows:

- SA: by using \(N_v = 10\) and \(p_0 = 0.5\), \(c_0\) is defined on the basis of these values and of the average worsening \(\bar{\Delta}\) obtained empirically. Constant values for \(M_A = 200\) and \(M_C = 50\) are also assumed. The variable parameter is the cooling rate \(\alpha\); the values are chosen with regular steps.

- GA: the mutation probability (applied to each gene) is set to \(p_m = 0.001\). The variable parameters are the number of chromosomes \(C_{GA}\) and the crossover probability \(p_c\), each of which with 10 values chosen at regular steps. The solver is run with all the combinations of the two parameters.

- PSO: The variable parameters are the number of particles \(C_{PSO}\) and the initial value of the inertia weight \(w_{init}\) (with the final value of the inertia weight fixed to \(w_{final} = 0.4\) \([67]\)), each of which with 10 values chosen at regular steps. The solver is run with all the combinations of the two parameters.

Note that the parameters used for this example have been chosen with the aim to show various solutions originated from the solvers and calculate the indicator \(\text{primo}\) resulting from these solutions. There is no attempt to find the optimal set of parameters for each method.

Fig. 8 shows the reference CDF and the test CDFs of the results obtained from the methods to be compared. Table 4 reports the performance indicators. The solutions provided by the SA and PSO methods cover 100% of the reference CDF, reaching the global optimum. This is a reasonable way to consider these methods as viable to be tested on large-scale cases. The example shown for the GA solutions suggests that the setting used for GA is inappropriate to be used, so this GA is not suggested to be used any further.

6.2. Large real network with unknown global optimum (Case R)

The 207-nodes real distribution network mentioned in Section 4.2 is used for the calculations. The characteristics of this network are summarized in Table 5. The base power is 1 MVA, and the base
voltage is equal to the rated voltage of the network (10.5 kV). The global optimum is unknown. The methods SA, GA and PSO have been tested with the same parameters indicated in Table 3. The number of solutions extracted for each solver is $H_s = H = 100$.

6.2.1. Comparison among the solutions obtained from different solvers

Fig. 9 shows the reference CDF and the test CDFs of the results obtained from the methods to be compared. Table 6 reports the performance indicators. In the loading conditions considered, the solutions provided by the SA method are the best ones in the OPISD ranking, followed by the PSO and GA. Looking at Fig. 9, it can be seen that the SA results are not uniformly located at the left-hand side of the CDFs referring to PSO and GA, but the SA solution with the higher objective function is worse than the PSO solution with the higher objective function. The use of the first-order stochastic dominance concepts provides the conceptually sound framework to execute the comparison among these solutions.

In the case studied, the PERC ranking provides the same results, with the performance of SA and PSO indicated as significantly better than the performance of the GA. However, the results shown in Table 6 indicate the PSO results to be closer to the ones of the GA rather than to the SA. This happens because the comparison is just based on the number (and percentage) of occurrences of the best solution, without taking into account the distribution of all the other solutions.

6.2.2. Variation of the solutions in SA for different seeds for random number extraction

As a further illustrative example, one of the solvers (SA) is taken, all its parameters are set up to given values, and only the initial seed for random number extraction is changed. In these conditions, the solutions generally change in the different executions, especially when the solver is run on a large-scale system. In order to show an example of parametric analysis conducted according with the principles indicated in this paper, four cooling rate values are considered, namely, $\alpha = \{0.2, 0.5, 0.7, 0.95\}$, leaving all the other parameters constant at the values used in Section 5.1.1. These values are intentionally very different, also going down to the values 0.5 and 0.2, well-known from the general
theory of SA as poorly appropriate, since they may provide excessively fast decay of the control parameter in the external cycle. The rationale of this choice is indeed to test whether the proposed indicators correctly represent the appropriateness of the parameter setting. For each cooling rate, $H = 120$ solutions have been found, building the corresponding test CDFs. Fig. 10 shows the reference CDF and the test CDFs for the four cooling rate values, whereas Table 7 shows the corresponding performance indicators defined in the previous sections. The results confirm the better suitability of the cooling rate 0.95, while with the other cooling rate values the SA reaches the pseudo-optimal solution for a lower number of times and falls into worse solutions in most cases. As expected, \textit{OPISD} ranks the CDFs from the one having the higher cooling rate to the one with the lower cooling rate.

7. Concluding Remarks

This paper has discussed a number of aspects concerning the comparison among different solvers for heuristic optimization. On the conceptual point of view, a perpetual motion scheme has been drawn to represent the process used for claiming that a new “best solver” has been found. From this scheme, it may be seen that the mechanism of creation and testing of new heuristics cannot be formally stopped. However, the definition of comparison metrics more robust than simple but weak indicators (such as mean value, median, standard deviation, best value, worst value, as well as the \textit{PERC} indicator shown in this paper) may help reduce the proposal of further solvers that do not exhibit a clearly superior performance with respect to other solvers used for the problem under analysis. Establishing a sound comparison requires the definition of a benchmark solver for each problem, whose outcomes may be expressed as the CDF containing a given number $H$ of solutions. On these bases, the comparisons with the results obtained by other proposed solvers may be carried out by calculating the \textit{OPISD} indicators corresponding to the benchmark solver and the proposed solvers, setting up the CDF of the solutions given by the top ranked solver as the (unchanged or new) benchmark.

On the practical point of view, improving the mechanism of comparison among the results of different solvers, combined with the identification of suitable benchmarks for each problem analyzed (to be carried out by a widely recognized task force in the power and energy system community), is
then suggested as one of the possible ways to limit the proliferation of declarations that a new best solver has been found based on weak metrics to compare the solvers. An example is the use of the $OPISD_G$ indicator to carry out a pre-testing of a proposed heuristic on cases with known global optimum, to verify that 100% of the solutions are found, that is, $OPISD_G = 1$, before using that heuristic for further studies. The examples shown in this paper have confirmed that some solvers that exhibit good properties on large networks have no problem in finding the global optimum in 100% of the executions on networks for which the global optimum is known.

In summary, the concepts presented in this paper, supported by the application cases, suggest the following remarks:

- In order to filter out the contributions based on poorly meaningful definitions of “better solution”, the papers that show comparisons only based on the classical simple indicators do not provide effective advancements of the state of the art.

- Robust testing of the results obtained by the solver has to be carried out by using indicators with statistical meaning, such as the ones based on the first order stochastic dominance presented in this paper.

- For a problem with known global optimum, if the solver used (i.e., the solution method with its parameter settings) generates the globally optimal solution in less than 100% of the executions, this solver could be deemed as inadequate to be used on the same problem for systems with larger size. Likewise, contributions presenting only the testing of a heuristic algorithm on a problem with known global optimum have an excessively limited scope and do not provide sufficient insights on the viability of application of the heuristic.

- For a problem with unknown global optimum, the ranking of the solutions by using $OPISD$ may be useful to compare different solvers, or the same solver with different parameters, trying to create a solid benchmark. However, it has to be noted that, in heuristic methods depending on random extractions, even the change of the seed for random number extractions generates different outcomes. As such, the results obtained on any benchmark that may be constructed cannot be considered as an absolute reference.
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Table 1.
Number of articles from Science Direct (at 25 August 2018) that mention or apply a heuristic solver on a given problem.

| Heuristic name | Problem | DSR | ED | LF | MS | OPF | PSP |
|----------------|---------|-----|----|----|----|-----|-----|
| Ant colony optimization (ACO) [15] | 225 | 458 | 226 | 129 | 293 | 91  |
| Artificial bee colony (ABC) [16] | 86 | 336 | 111 | 24 | 247 | 55  |
| Artificial immune system (AIS) [17] | 38 | 64 | 44 | 33 | 84 | 17  |
| Bacterial foraging (BF) [18] | 75 | 210 | 62 | 9 | 136 | 24  |
| Bat algorithm (BA) [19] | 32 | 71 | 31 | 8 | 58 | 10  |
| Big-bang big-crunch (BBBC) [20] | 32 | 26 | 9 | 2 | 25 | 8   |
| Biogeography based optimization (BBO) [21] | 21 | 171 | 22 | 13 | 128 | 31  |
| Charged system search (CSS) [22] | 3 | 28 | 6 | 2 | 17 | 4   |
| Cuckoo search algorithm (CSA) [23] | 45 | 140 | 73 | 16 | 97 | 21  |
| Differential evolution (DE) [24] | 122 | 702 | 217 | 70 | 456 | 115 |
| Evolutionary algorithms (EA) [25] | 221 | 838 | 445 | 70 | 534 | 157 |
| Evolution strategies (ES) [26] | 32 | 136 | 48 | 31 | 61 | 19  |
| Firefly algorithm (FA) [27] | 48 | 163 | 96 | 18 | 109 | 31  |
| Flower pollination algorithm (FPA) [28] | 8 | 40 | 13 | 2 | 31 | 6   |
| Fruit fly optimization (FFO) [29] | 2 | 22 | 49 | 4 | 4 | 3   |
| Genetic algorithms (GA) [30] | 611 | 1629 | 1092 | 696 | 1164 | 407 |
| Grey wolf optimization (GWO) [31] | 10 | 66 | 14 | 5 | 49 | 8   |
| Gravitational search algorithm (GSA) [32] | 42 | 212 | 78 | 14 | 191 | 37  |
| Group search optimization (GSO) [33] | 15 | 96 | 16 | 4 | 59 | 7   |
| Harmony search algorithm (HSA) [34] | 106 | 340 | 82 | 31 | 237 | 50  |
| Imperialist competitive algorithm (ICA) [35] | 7 | 29 | 4 | 2 | 18 | 5   |
| Invasive weed optimization (IWO) [36] | 7 | 38 | 4 | 3 | 33 | 6   |
| Krill herd algorithm (KHA) [37] | 10 | 42 | 14 | 3 | 49 | 3   |
| Memetic algorithms (MA) [38] | 22 | 78 | 32 | 25 | 27 | 11  |
| Moth-flame optimization (MFO) [39] | 0 | 13 | 4 | 0 | 9 | 2   |
| Particle swarm optimization (PSO) [40] | 346 | 1281 | 706 | 208 | 895 | 259 |
| Simulated annealing (SA) [41] | 255 | 650 | 333 | 263 | 410 | 149 |
| Scatter search (SS) [42] | 5 | 24 | 4 | 12 | 10 | 5   |
| Seeker optimization algorithm (SOA) [43] | 7 | 48 | 11 | 2 | 33 | 11  |
| Shuffled frog leaping algorithm (SFLA) [44] | 50 | 91 | 26 | 11 | 102 | 31  |
| Social spider algorithm (SSA) [45] | 2 | 12 | 4 | 0 | 6 | 0   |
| Symbiotic organisms search algorithm (SOS) [46] | 5 | 13 | 5 | 1 | 13 | 2   |
| Teaching-learning-based optimization (TLBO) [47] | 44 | 167 | 31 | 9 | 158 | 30  |
| Tabu search (TS) [48] | 198 | 398 | 139 | 184 | 284 | 91  |
| Whale optimization algorithm (WOA) [49] | 2 | 9 | 5 | 2 | 16 | 2   |

3 The TS has been inserted as a matter of comparison, even if it is not a probability-based method

Table 2
Test Network Data

| Parameter                  | Value |
|----------------------------|-------|
| rated voltage [kV]        | 12.66 |
| number of nodes N         | 70    |
| number of branches B      | 74    |
| number of supply points S | 1     |
| number of open branches A | 5     |
| number of radial configurations \( \varphi \) | 407,924 |
### Table 3
Parameters Used for Computing the Set of Solutions

| solver | parameter | range of values | step |
|--------|-----------|-----------------|------|
| SA     | $\alpha$  | 0.900 ÷ 0.999   | 0.001|
| GA     | $C_{GA}$  | 100 ÷ 190       | 10   |
|        | $p_c$     | 0.35 ÷ 0.44     | 0.1  |
| PSO    | $C_{PSO}$ | 100 ÷ 190       | 10   |
|        | $w_{init}$| 0.81 ÷ 0.90     | 0.01 |

### Table 4
Performance Indicators Calculated for the Test System

| network | solver | $\lambda_{max}$ |
|---------|--------|------------------|
| 70 nodes| SA     | 100%             |
|         | PSO    | 100%             |
|         | GA     | 94%              |

### Table 5
Real Network Data

- rated voltage [kV]: 10.5
- number of nodes $N$: 207
- number of branches $B$: 213
- number of supply points $S$: 1
- number of open branches $A$: 7
- number of radial configurations $\mathcal{P}$: $1.516 \times 10^5$

### Table 6
Performance Indicators Calculated for the Real Network

| network | solver | $\lambda_{max}$ |
|---------|--------|------------------|
| 207 nodes| SA     | 59%              |
|         | PSO    | 20%              |
|         | GA     | 0%               |

### Table 7
Performance Indicators Calculated with the SA Method (Real Network)

| network | cooling rate $\alpha$ | $\lambda_{max}$ | $\lambda_0$ | $\lambda_{max}$ |
|---------|-----------------------|------------------|--------------|-----------------|
| 207 nodes| 0.2                   | 57%              | 0.0024       | 0.9976          |
|         | 0.5                   | 53%              | 0.0019       | 0.9981          |
|         | 0.9                   | 69%              | 0.0011       | 0.9989          |
Figures

Fig. 1. Comparing optimization solvers on the basis of their solutions – A perpetual motion scheme.

Fig. 2. Deterministic dominance. For the objective function minimization, the solutions from solver A dominate the solutions from solver B on a deterministic way.

Fig. 3. Construction of the CDF.
a) First-order (point-to-point)

b) Second-order (example of CDFs)

c) Positive areas $P_{i=1}^{n} (x^w - x^w)$ satisfying the second-order stochastic dominance conditions

Fig. 4. Stochastic dominance.

Fig. 5. Reference CDF for Case G.

Fig. 6. Reference CDF for Case R.

Fig. 7. Representation of the areas contributing to the OPISD definition.
Fig. 8. Reference CDF and test CDFs for the methods run on the 70-node test system.

Fig. 9. Reference CDF and test CDFs for the three methods (207-node network). The internal zoom refers to SA and PSO close to the lowest objective function values.

Fig. 10. Reference CDF and test CDFs for the three methods (207-node network, SA with different cooling rate $\alpha$ and 120 executions for each cooling rate with different initial seed for random number extraction).
Highlights

- The search of the best solver for heuristic methods is not a well-posed problem
- The perpetual motion scheme presented explains the “best solver” paradox
- Two first-order stochastic dominance-based metrics are proposed
- These metrics allow a more significant comparison among optimization solutions
- Using these metrics can limit the uncontrolled proliferation of new “best solvers”