Chiral vibrations in the A=135 region

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Chiral vibrations are studied with the RPA plus self-consistent tilted axis cranking formalism in the A=135 region. In this method chiral vibrations appear as a precursor to the static chiral regime. The properties of the RPA phonons are discussed and compared to experimental data. We discuss the limits the chiral region and the transition to the non harmonic regime.

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I. INTRODUCTION

Chirality is an important symmetry in many physical systems. In molecules it is a static property of the geometry that appears in molecules with more than four different atoms, which can have left-handed and right-handed enantiomers. In these cases the left- and right-handed geometry is connected by reflection in a plane. In particle physics chirality is a dynamical feature of mass-less particles, which indicates orientation of the intrinsic spin relative to the linear momentum. In a rotating nucleus it arises as a combination of a dynamical property, the angular momentum, with a static property, the reflection symmetric triaxial shape of the nucleus. If the angular momentum vector lays outside any of the three principal planes, there are a left-handed and a right-handed arrangement, which are connected by the time reversal operation. This kind of chirality is manifested as a pair of (almost) degenerate rotational bands with the same parity [1]. Pairs of such bands have been seen experimentally in the mass regions A=105, A=135 and A=190 and theoretically described using mean field tilted axis cranking (TAC) models [2,3], two-particle-rotor models [4,5] or extensions of the IBA model [6,7].

Chirality appears both in molecules and nuclei as a spontaneously broken symmetry: The many body Hamiltonian describing these systems is invariant with respect to the chiral operation (space inversion or time reversal, respectively), which has the consequence that the exact eigenfunctions are achiral. However, there exist very good approximate solutions, which are chiral, where the left-handed and right-handed configurations have the same energy. The exact eigenfunctions are odd and even superpositions of these chiral solutions. The energy splitting between these states is given by the matrix element between the two chiral configurations, which is the inverse time for tunneling from one to the other. In most molecules the tunneling time is so long that they stay in one of the enantiomers and the level splitting is unmeasurable. However, for CH$_3$NHF the tunneling is rapid. The splitting is in the order of 100 meV, i.e. the tunneling frequency is in the order of 3000 GHz, which is somewhat larger than the average rotational frequency of the molecule at room temperature. For the known nuclear cases, the left-right conversion is always rapid. The observed energy distance between the chiral partners is in the order of 100 keV [8], except in very narrow spin regions, where the bands cross. This is comparable with the rotational frequency in the observed spin range.

TAC is a microscopic mean field method that has been shown to very well describe the energy and the intra-band transition rates of the lower of the two chiral partner bands, see e.g. [9]. However, it gives either one achiral self-consistent solution or two degenerate chiral ones. It cannot describe the left-right mode, which is a well known deficiency of the mean field solution when it breaks spontaneously a symmetry (cf. e.g. [1,10]). In a finite system, as the nucleus, the symmetry breaking develops in a gradual way. First a precursor appears as a slow vibration around the symmetric configuration. Becoming increasingly anharmonic, the vibration changes into tunneling between two asymmetric configurations, which is progressively inhibited.

In order to describe the splitting between the two bands one has to go beyond the mean field approximation. So far this has only be done in the framework of two-particle-core coupling models [4,6,7]. These studies show a development from chiral vibrations toward tunneling between static left- and right-handed configurations with increasing angular momentum. Although the quantal nature the core-particle models accounts well for this aspect of symmetry breaking, they are based on several assumptions the validity of which is not assured. Examples are: Rigid shape and irrotational-flow moments of inertia for a triaxial rotor core [4], which are not consistent with microscopic cranking calculations [5], the application of the IBA core, and the IBAFF coupling scheme [6,7]. A microscopic treatment starting from the TAC mean field seems important for

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a better understanding of nuclear chirality. Obviously the transition from chiral vibrations to chiral rotation involves large amplitude collective motion, a treatment of which goes beyond the scope of this paper. However the regime of chiral vibration can be treated by the random phase approximation (RPA) approach as long as the energy splitting between the bands is large enough for the assumption of a harmonic vibration remaining reasonable. As the energy splitting decreases the anharmonic effects will become more important until the transition point where the RPA energy becomes zero and the TAC mean field becomes chiral. The present paper is devoted to a study of the spin range before the transition point where the RPA energy becomes zero and the TAC effects will become more important until the transition point where the RPA approximation will allow us investigating the chiral vibrations and provide insight into the nature of nuclear chirality.

In the static chiral picture on would expect the two bands to have very similar electromagnetic transition rates. Recent experiment have shown different band transition rates in the two chiral partner bands in $^{134}$Pr [7, 11] while the bands in $^{128}$Cs [12] and $^{135}$Nd [13] have similar transition rates. The different transition rates in $^{134}$Pr have been interpreted as being due to coupling of shape degrees of freedom to the orientation degree of freedom [7] or even that the bands are not chiral partner bands at all [11].

Chiral pairs of rotational bands have mainly been suggested for nuclei with odd proton and odd neutron number. In the A=135 region, they are build on the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration. Candidate bands have been seen in the odd-odd nuclei $^{126-132}$Cs, $^{130-134}$La, $^{132-134}$Pr, $^{136}$Pm and $^{138-140}$Eu. In this work we focus on the N=75 isotope chain and the Z=57 isotope chain which represent the central part of this region. We have performed calculations in the framework of tilted axis cranking (TAC) and random phase approximation (RPA) for the twin bands in the N=75 isotones and the Z=57 isotopes built on the $\{\pi h_{11/2}, \nu h_{11/2}\}$ configuration. Examples for chiral partner bands exist also in odd-even nuclei [13, 14]. The case of $^{135}$Nd has been studied in [13] by means of the method presented in this paper. In principle, chiral partner bands should also exist in even-even nuclei build on two-particle two-hole like configurations. For these and even more complex configurations our method can be directly applied. We describe the formalism in section II.1. The results for the energy, amplitudes and transition rates are presented in section II.2. An analysis of the structure of the phonons is given in section II.3, where it will be demonstrated that for most of the cases the chiral character prevails, i.e. the coupling to the shape degrees of freedom is weak.

### II. FORMALISM

Most earlier TAC calculations for chiral bands have used the Strutinsky renormalization for calculating the energies (SCTAC [15]). Since we intend to carry out RPA calculations we need an interaction Hamiltonian. Hence, we choose using the self-consistent TAC with the QQ-force and a constant pair gap (PQTAC [15]) in two major N-shells ($N_{\text{low}}=4$ and $N_{\text{up}}=5$):

$$H' = h_0 + \sum_{m=-2}^{2} \sum_{N=4}^{5} \frac{\kappa_N}{2} Q^{(N)}_{m} Q^{(N)}_{-m} (-)^{m+1} - \sum_{\tau=n,p} \Delta_{\tau} (P^+_{\tau} + P^{-}_{\tau}) - \omega \cdot J$$

where $h_0$ is the spherical Woods-Saxon energy [2] and $Q^{(N)}_{m}$ are the dimensionless quadrupole operators for each N-shell with an N-dependent force strength [16, 24]$

\kappa_N = \kappa_0 \frac{N_{\text{low}} - B}{N - B} \left( \frac{2Z(N)}{A} \right)^{1/3}$

The value of the parameter $B$ is chosen to approximately reproduce the Strutinsky potential energy surface [15]. Setting the parameter $B = 0.5$ gives a similar prolate oblate mass difference and $\gamma$-softness as the SCTAC calculations in this mass region. In other mass regions a different value of $B$ may be needed. The parameters $\kappa_0$, listed in table I, are adjusted to reproduce the moment of inertia of the SCTAC calculations. The angular velocity in the cranking term, $\omega \cdot J$, is defined as

$$\omega_1 = \omega \sin \vartheta \cos \varphi, \quad \omega_2 = \omega \sin \vartheta \sin \varphi, \quad \omega_3 = \omega \cos \vartheta,$$

where $\vartheta$ and $\varphi$ are the tilt angles [2].

| $^{130}$Cs | $0.0367$ |
| $^{132}$La | $0.0359$ |
| $^{134}$Pr | $0.0348$ |
| $^{136}$Pm | $0.0355$ |
| $^{138}$Eu | $0.0363$ |
| $^{140}$Tb | $0.0357$ |
| $^{138}$La | $0.0372$ |
| $^{134}$La | $0.0337$ |

The monopole pair operators are denoted by $P^\pm_{\tau}$, and we use a proton and neutron pairing gap, $\Delta_{n,p}$, that is 80% of the odd-even mass difference in this mass region.

In the TAC calculations for the A=135 region, chiral solutions appear as a transient phenomenon where the angular momentum vector moves from the principal plane axis spanned by the 1- (short) and 3- (long) axes ($\varphi = 0\,^{\circ}$) to the principal plane spanned by the 2- (intermediate) and 3-axes ($\varphi = 90^{\circ}$) [2]. The RPA method describes the regime when the TAC angular momentum lays in one of the principal planes. The excited chiral partner band can be described as a chiral vibration, which is a periodic motion of the orientation of the deformed potential relative to angular momentum vector, i.e. a mode in the $\vartheta$ and $\varphi$ degrees of freedom centered about its TAC equilibrium values. There are two critical frequencies. The first
is associated with the transition of the TAC mean field solution into the chiral regime at $\varphi = 0$, and the second is associated with the transition out of chiral regime at $\varphi = 90^\circ$. These transitions of the mean field are accompanied by an instabilities of the RPA, where the energy of the lowest RPA phonon becomes zero.

When the TAC solution has $0 < |\varphi| < 90^\circ$ (static chiral regime) our present approach gives zero energy splitting between the two chiral configurations with $\varphi = \pm |\varphi|$. However, higher order terms in our Hamiltonian would give rise to tunneling between the left-handed and the right-handed solution which causes an energy splitting between the two bands in this region too. Figure 1

![FIG. 1: Potential energy in the rotating frame for $^{134}$Pr at $\omega = 0.4$ MeV/$\hbar$. The two solid curves are calculated with $\gamma$ or $\varphi$ fixed while the dotted curve is calculated with a full minimization.](image)

shows the TAC energy in the rotating frame, spanned by the three principal axes of the triaxial density distribution, as a function of the tilt angle $\varphi$ and the triaxial deformation parameter $\gamma$. The calculation is done for the $\pi h_{11/2}, \nu h_{11/2}$ configuration in $^{134}$Pr at $\omega = 0.4$ MeV. The energy surface is very flat especially in the $\varphi$ direction. When we allow for the full minimization in the orientation degrees of freedom the surface becomes symmetric in $\pm \gamma$, because the change of sign is equivalent with a reorientation of the axes. It is therefore enough to consider $\gamma > 0$ only. Figure 2 shows the potential energy surface in the rotating frame (total routhian) as a function of $\theta$ and $\varphi$. One can see the very soft nature of the potential in the $\varphi$ direction.

The RPA calculates the harmonic excitations around the mean field minimum. It will describe the system as long as we are in the chiral vibrational regime well before the transition to static chirality. The transition point corresponds to where the RPA solution goes to zero energy and the tilted mean field acquires a nonzero $\varphi$. After solving the mean field problem the Hamiltonian can be written as

$$H = h_{\text{mf}} + H_{\text{res}}$$  \hspace{1cm} (3)

where $h_{\text{mf}}$ is the diagonal mean field Hamiltonian

$$h_{\text{mf}} = E_{\text{mf}} + \sum_k e_k \alpha_k^\dagger \alpha_k$$  \hspace{1cm} (4)

in terms of the self-consistent mean field quasi particle operators $\alpha_k$ and $H_{\text{res}}$ is the residual interaction. We introduce the quasi-boson approximation $b^\dagger_\mu = \alpha^\dagger_\mu \alpha^\dagger_j$, where the $b^\dagger_\mu$ are treated as exact bosons and $\mu \equiv i > j$. The Hamiltonian $H$ is rewritten in RPA order by only keeping terms up to second order in the boson operators \[10\],

$$H_{\text{RPA}} = E_{\text{mf}} + \sum_{\mu,\nu} A_{\mu\nu} b_\mu^\dagger b_\nu + \frac{1}{2} \sum_{\mu,\nu} (B_{\mu\nu} b_\mu^\dagger b_\nu + \text{h.c.})$$  \hspace{1cm} (5)

where matrices $A$ and $B$ are hermitian and determined from the residual interaction \[10\]. The matrix elements for the QQ-interaction are

$$A_{\mu\nu} = -\delta_{\mu\nu} E_\mu + \sum_{m=-2}^{2} \kappa \ q_\mu^m q_\nu^{m*};$$  \hspace{1cm} (6)

$$B_{\mu\nu} = \sum_{m=-2}^{2} \kappa \ q_\mu^m q_\nu^{m*};$$  \hspace{1cm} (7)

where $q_\mu^m$ are the quadrupole matrix elements in quasi-boson, approximation \[10\]

$$Q^m = \sum_\mu q_\mu^m b_\mu^\dagger + q_\mu^{m*} b_\mu,$$  \hspace{1cm} (8)

$$q_\mu^m = \langle b_\mu, Q^m \rangle,$$  \hspace{1cm} (9)

and

$$\kappa = \frac{e_0}{\hbar} \langle \delta_0, Q^m \rangle.$$

![FIG. 2: (Color online) Potential energy surface in the rotating frame for $^{134}$Pr at $\omega = 0.4$ MeV/$\hbar$ as a function of $\theta$ and $\varphi$ with constant deformation. The equipotential lines are separated by 25 keV. Darker color represent lower energy.](image)
and $E_{\mu} = \epsilon_i + \epsilon_j$ are the two quasi-particle energies. (We suppressed the summation over the shell number $N$.) We solve the RPA equations

$$[H_{RPA}, O_{\lambda}^\dagger] = E_{RPA} O_{\lambda}^\dagger,$$

using the strength function method of [10, 17]. The RPA eigenmode operators $O_{\lambda}^\dagger$ are

$$O_{\lambda}^\dagger = \sum_{\mu} X_{\mu}^\lambda b_{\mu}^\dagger - Y_{\mu}^\lambda b_{\mu},$$

where the RPA amplitudes $X_{\mu}^\lambda$ and $Y_{\mu}^\lambda$ are obtained by solving the standard set of linear equations resulting from Eq. (10) together with the normalization condition

$$[O_{\lambda}, O_{\lambda'}^\dagger] = \sum_{\mu} X_{\mu}^\lambda X_{\mu}^{\lambda'} - Y_{\mu}^\lambda Y_{\mu}^{\lambda'} = \delta_{\lambda\lambda'}.$$

Since we use a separable force, this set of linear equations is strongly simplified [17].

There are two rotational spurious solutions in the RPA spectrum. One at zero energy induced by the $J_z$ operator and one at the rotational frequency $\omega$ induced by the $J_+$ operator. Numerically the spurious solutions decouple from the physical RPA solutions in a stable manner if the mean field problem is solved accurately enough. In the discussion below we will only refer to the physical RPA solutions in a stable manner if the mean field problem is solved accurately enough. In the discussion below we will only refer to the physical RPA solutions in a stable manner if the mean field problem is solved accurately enough.

III. RESULTS

The results for the TAC and RPA calculations for the $N=75$ isotones and the $Z=57$ isotopes is presented below. We solve the self-consistent TAC mean field problem plus RPA of the Hamiltonian in Eq. (1) for the $\pi h_{11/2} \otimes \nu h_{11/2}$ configuration.

A. Energies

We present the results of the TAC+RPA calculations in Fig. 3 and 4 which show the energy of the lowest RPA phonon and the experimental energy splitting between the chiral bands, which are identified with each other. For reference we also plot the energy of the lowest two-quasiparticle excitation as well as the energy of the second collective RPA phonon. The nuclei studied here represent the chiral region for odd-odd nuclei in the $N=75$ isotope chain and the $Z=57$ isotope chain. For lower $Z$ or larger $N$ we approach the shell closing where the deformation disappears. Smaller $N$ leads to disappearance of the triaxiality. Larger $Z$ approaches the proton drip line, where very little high spin data are available. In the lower-$Z$ part of the $A=135$ region the TAC energy minimum sits at $\varphi = 0$ at low spin. As we move towards larger $Z$ the minimum moves over to $\varphi = 90^\circ$. With increasing $\omega$, $\varphi = 90^\circ$ is preferred, because the intermediate axis has the largest moment of inertia [7].

In the lighter nuclides the energy of the lowest RPA phonon is substantially smaller than the mean field two-quasiparticle energy and the energy of the second RPA phonon. In the heavier nuclides the difference is smaller. For the lowest phonon being a well developed collective excitation it must have a substantially smaller energy than the next excitations. When the lowest phonon sits in a region where other excitations are located the collective strength will become fragmented, and the observed bands will be much like two-quasiparticle excitations.

Figure 3 shows the results for the $N=75$ isotones. The RPA results show a clear trend which reflects properties of the TAC mean field solutions. In the lighter systems, where the TAC solutions has $\varphi = 0$, the phonon energy decreases with spin, reaching zero at the critical frequency for the onset of static chirality ($\varphi > 0$). In the heavier systems, where the TAC solution has $\varphi = 90^\circ$...
already at the band head, the phonon energy increases with increasing angular momentum. $^{136}$Pm is the limiting case where we have a chiral vibration around $\varphi = 0$ and decreasing phonon energy at low spin, then a static chiral regime $0 < \varphi < 90^\circ$, and finally a second chiral vibration around $\varphi = 90^\circ$ and increasing phonon energy. Comparing with experiment, $^{130}$Cs, $^{134}$Pr, $^{138}$Eu have the right trend and magnitude of the energy splitting. $^{132}$La, $^{136}$Pm have a much more constant energy splitting in the experiment than given by the RPA calculations.

Figure 4 show the results for the $Z=57$ isotopes. The magnitude of the energy splitting is reproduced but there is a problem with the angular frequency trend in $^{132}$La. In experiment $^{132}$La looks similar to $^{130}$La while in the calculations it looks more like $^{134}$La.

Our RPA calculations give the correct energy splitting away from the region where the RPA energy approaches zero, which is where we can expect the RPA to work well. However, spin dependence is not always well reproduced. The RPA phonon energy tends to approach zero faster and more often than in experiment, which indicates that anharmonic effects are needed to understand the data in this regime. The RPA phonon energy is generally increasing or decreasing with rotational frequency. The experimental energy splitting, on the other hand, does not change much with $\omega$ in some cases, which seem to appear predominantly in the frequency regions where the TAC gives static chirality, $0 < \varphi < 90^\circ$, i.e. TAC+RPA predicts zero splitting. The constant energy splitting in these regions should be attributed to the tunneling between the left- and right-handed solutions. Its relatively large value of about 300keV indicates strong mixing between the two chiral configurations, which is consistent with the flat potential in $\varphi$ direction in Fig. 2 and our analysis of the composition of the RPA phonon wave function discussed in section $\text{LV}$ below. In $^{134}$Pr the two band cross over in experiment. In $^{130}$Cs and $^{135}$Nd (not shown here, see [13]) one sees in experiment an avoided crossing with some interaction. In all three cases, the crossing frequency correlates with the frequency where the phonon energy goes to zero. At the moment, it remains unclear why these nuclei are different from $^{132}$La and $^{136}$Pm with a nearly frequency-independent energy difference between the bands. To understand this phenomenon further it is clearly necessary to take into account large amplitude anharmonic effects, which will be done in a forthcoming publication $^{22}$.

B. Transition rates

In addition to the energy splitting, the RPA also gives the transition rates between the two chiral partner bands. In Fig. 5-10 we show the calculated $B(E2)$ and $B(M1)$ inter-band rates. The intra-band values calculated by means of TAC $^{13}$ are also included. Within the present approach they are the same for the two bands. The inter-band reduced transition probabilities are given by

$$B(E2, I \rightarrow I - m) = \left( |<\varphi | r^2 Y_{2m} | 0 \rangle|^2 \right)_{m = -2, \ldots, 2}$$

and

$$B(M1, I \rightarrow I - m) = \left( |<\varphi | \mu_{1n} | 0 \rangle|^2 \right)_{m = -1, 0, 1}$$

where $| 0 \rangle$ is the RPA ground state and $| 1 \rangle$ is the first excited RPA state. The charges $e_r$ are for 1 protons and 0 neutrons. The spin parts of the magnetic operator are attenuated by the factor of 0.7. We include terms up to linear RPA order in the quadrupole, $r^2 Y_{2m}$ and magnetic $\mu_{1n}$ transition operators. The transition operators are transformed from the intrinsic frame into the lab frame by means of the relation

$$T_{lm}^{\text{lab}} = \sum_\nu D_{\nu m}^l (\varphi, \vartheta - \pi/2, 0) T_{l\nu}$$

where the $-\pi/2$ rotation brings the intrinsic system from x-axis to the z-axis representation.

The inter-band M1 transitions are much stronger in the lighter $N=75$ isotopes where the $B(M1)$ values are typically around 1/3 of the intra-band values. In the heavier $N=75$ isotopes as well as in $^{134}$La the inter-band $B(M1)$ are weak. For the E2 transitions in the lighter $N=75$ isotopes the dominant inter-band transition is $I \rightarrow I - 2$, except approaching the critical frequency the transition $I \rightarrow I \pm 1$ increases in strength. In the heavier $N=75$ isotopes $B(E2, I \rightarrow I + 1)$ becomes the largest inter-band value. However, the $I \rightarrow I + 1$ or $I + 2$ will often not be seen in experiment due to their low or negative energy. The ratios of the different transition probabilities vary substantially over the studied region. The
$B(E2, I \to I \pm 1)$ and $B(M1, I \to I \pm 1)$ are typically the strongest.

There are experimental data for the inter-band transition rates in $^{134}$Pr [23]. Our calculated rates are somewhat larger than what is seen in experiment. There are also data available for the ratio of $B(M1,\text{out})/B(M1,\text{in})$ in $^{136}$Pm, but unfortunately not in the frequency range where we have RPA results.

**FIG. 5:** The RPA inter-band transition rates in $^{130}$Cs, $^{132}$La and $^{134}$Pr.

**FIG. 6:** The TAC intra-band transition rates in $^{130}$Cs, $^{132}$La and $^{134}$Pr.

**FIG. 7:** The RPA inter-band transition rates in $^{136}$Pm, $^{138}$Eu and $^{140}$Tb.

**FIG. 8:** The TAC intra-band transition rates in $^{136}$Pm, $^{138}$Eu and $^{140}$Tb.

**IV. SHAPE AND ORIENTATION AMPLITUDES**

The energy of the RPA phonon and the transition rates do not give direct information about the degree the RPA phonon is dominated by orientation or shape fluctuations. The potential energy surfaces as Fig. 4 and 5 give some hint of which are the important degrees of freedom but they do not provide conclusive information, because the missing collective mass parameter is as important as
the curvature of the potential. In order to analyze the nature of the RPA phonons we calculated the fluctuations of the five quadrupole operators, which are then decomposed into fluctuations of the Euler angles and shape fluctuations.

We define the fluctuation of the quadrupole operator \( Q_m \) generated by the RPA phonon as the difference of the dispersions in the one-phonon state \( |1\rangle = O_1^\dagger |0\rangle \) and zero-phonon state \( |0\rangle \), where \( O_1^\dagger \) creates the lowest phonon, i.e.

\[
\Delta Q_m^2 = \langle Q_m - \langle Q_m | 1 \rangle | 1 \rangle^2 \rangle - \langle Q_m - \langle Q_m | 0 \rangle | 0 \rangle^2 \rangle | 0 \rangle.
\]

Using that in RPA approximation \( \langle Q_m | 1 \rangle = \langle Q_m | 0 \rangle = q_m \) we find

\[
\Delta Q_m^2 = 2 \left[ Q_m, O_1^\dagger \right]^2.
\]

We define \( \Delta Q_m \) for \( m = -2, \ldots, 2 \) as

\[
\Delta Q_m = \sqrt{2} \left[ Q_m, O_1^\dagger \right]
\]

where the over all sign is ambiguous due to the freedom of the phase of the wave-function, but the relative signs between the different components are relevant. The body-fixed frame is attached to the TAC solution, the shape of which is given by the two self-consistent mean field quadrupole moments \( q_0 = (Q_0) \) and \( q_2 = q_{-2} = (Q_2) \). Its orientation in the laboratory frame is given by the three Euler angles \( \psi, \theta, \varphi \). We decompose the \( \Delta Q_m \) into a part that is generated by a reorientation of the body-fixed frame with constant \( q_m \), which is expressed in terms of changes of the deformation parameters \( \varepsilon \) and \( \gamma \). By means of the Wigner functions \( D_{m_m}^2(\psi, \varphi, \varepsilon, \gamma) \), we express the five components of the quadrupole operators \( Q_m \) in the laboratory system in terms of the two intrinsic quadrupole moments \( Q_0 = Q \varepsilon \cos \gamma - \gamma \) and \( Q_2 = Q \varepsilon \sin \gamma / \sqrt{2} \), i.e.

\[
Q_m(\psi, \theta, \varphi, \varepsilon, \gamma) = D_{m_2}^2(\psi, \theta, \varphi, \varepsilon, \gamma) \bar{Q}_0 + (D_{m_2}^2 + D_{m_2}^{1-2}) \bar{Q}_2,
\]

(19)

where \( Q \) is a constant, \( \varepsilon, \gamma \) are the standard deformation parameters, and the three Euler angles, \( \varphi, \psi, \theta \), specify the orientation of the body-fixed coordinates in the laboratory frame. We derive linear expressions for \( \Delta Q_m \) in terms of the fluctuation in Euler angles and shape parameters \( \gamma \) and \( \varepsilon \)

\[
\Delta Q_m = \sum_i \frac{\partial Q_m}{\partial \alpha_i} \Delta \alpha_i, \quad \alpha_i \in \{ \psi, \theta, \varphi, \gamma, \varepsilon \}.
\]

(20)

Inverting this system of linear equation, we express the fluctuation of Euler angles and of the shape parameters in terms of \( \Delta Q_m \) and mean field expectation values,

\[
\Delta \psi = \frac{\Delta Q_1}{2q_2}, \quad \Delta \theta = \frac{\Delta Q_{-1}}{q_2 + \sqrt{2} q_0}, \quad \Delta \varphi = \frac{\Delta Q_{-2}}{q_2 - \sqrt{2} q_0},
\]

\[
\Delta \varepsilon = \frac{(\sqrt{3} q_2 - q_0) \Delta Q_0 + (\sqrt{3} q_2 + q_0) \Delta Q_2}{2(q_0^2 + q_2^2)}
\]

\[
\Delta \gamma = \frac{(\sqrt{3} q_2 + q_0) \Delta Q_0 - (\sqrt{3} q_2 - q_0) \Delta Q_2}{2(q_0^2 + q_2^2)}.
\]

(21-25)

The results are shown in Fig. 11 and 12. One can clearly see that the lowest phonon in this region is dominated by orientation fluctuations especially in the \( \varphi \) direction, which is consistent with the TAC potential energy surface in Fig. 2. The amplitude of \( \Delta \varphi \) is typically 30-40°. The fluctuations in \( \varphi \) diverge at the point where the RPA energy goes to zero, i.e. at the transition to aplanar chiral rotation. The fluctuations in \( \vartheta \) are 10-20° while the fluctuations in \( \psi \) are half that. In all these cases, the shape fluctuations are small in amplitude compared to the orientation fluctuations. The fluctuations...
in $\varepsilon$ are few percent of the equilibrium values and the fluctuations $\Delta \gamma < 6^\circ$ for all cases but $^{134}$La (cf. next paragraph). This is in contrast to Ref. [7], which, taking into account the shape degrees of freedom in the framework of the the IBAFF two-particle-core approach, found a strong coupling between the shape and orientation degrees of freedom. In our microscopic approach, the lowest phonon is a rather clean chiral vibration.

Figures 13 and 14 show how the structure of the lowest RPA phonon develops when moving away from the N=75 chain. In $^{130}$La it is dominated by orientation fluctuations but not in the $\phi$ direction as for the N=75 cases. This nuclide also has relatively strong $\gamma$ fluctuations especially at large $\omega$. In $^{134}$La with N=77 we are approaching the N=82 shell closure. This leads to a reduction in the deformation and increased amplitudes of the shape fluctuations in the RPA phonon. The case of $^{134}$La can no longer be thought of as a chiral vibration even though it has large components of orientation fluctuations in the wave function. The large jump in several of the $^{134}$La curves is due to the level crossing between the first and second RPA solution seen in Fig. 4.

V. CONCLUSIONS

We have used the RPA method to study the energy splitting between chiral partner bands. The RPA phonon energy gives the correct size of the energy splitting at low spin in most cases but the trend with increasing angular momentum is not always reproduced. The RPA calculations also give the inter-band transition rates. As the energy splitting between the chiral partner bands becomes small the anharmonic effects are expected to get more and more important.
By analyzing the RPA amplitudes we found that near \(Z=57\) and \(N=75\) the lowest phonon is a collective chiral vibration. It is dominated by large amplitude fluctuations in the orientation angle \(\varphi\). The amplitudes of the shape oscillations are found to be much smaller than the ones of the orientation oscillations. The large amplitude nature of the \(\varphi\)-fluctuations indicate that for a full understanding of the chiral bands one has to take into account effects that go beyond the harmonic RPA approximation in the orientation degrees of freedom. Anharmonicities become dominant where the energy of the first RPA solution goes to zero and the fluctuation amplitude in \(\varphi\) reaches \(90^\circ\). Studies of the consequences of anharmonicities for the nature of the band crossing and the source of the different in band transition rates seen in \(^{134}\)Pr are to be published in the near future [22].

The quite pure chiral nature of the RPA solution seems to be localized around the \(N=75\) isotones. Moving to more neutron-rich nuclei structure of the first RPA phonon changes to a complex mixture of shape and orientation degrees of freedom, reflecting the approach of the shell closure at \(N=82\). In the more neutron deficient nuclei the \(\epsilon\) deformation is increasing and the \(\gamma\) deformation is decreasing which reduces the collectivity of the lowest RPA solutions that takes on two-quasiparticle character.

The calculation of the inter-band transition rates show that the ratios of the different \(\Delta I\) components of the transitions vary relatively fast with \(N\) and \(Z\).

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