Chiral torsional effect

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We propose the new nondissipative transport effect - the appearance of axial current of thermal quasiparticles in the presence of background gravity with torsion. For the non-interacting model of massless Dirac fermions the response of the axial current to torsion is derived. The chiral vortical effect appears to be the particular case of the chiral torsional effect. The proposed effect may be observed in the condensed matter systems with emergent relativistic invariance (Weyl/Dirac semimetals and the \textsuperscript{3}He-A superfluid).

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\section*{INTRODUCTION}

The non-dissipative transport effects have been widely discussed recent years \cite{1-7}. These effects are to be observed in the non-central heavy ion collisions \cite{8}. They have also been considered for the Dirac and Weyl semimetals \cite{10-17} and in \textsuperscript{3}He-A \cite{18}. It was expected that among the other effects their family includes the chiral separation effect (CSE) \cite{19}, the chiral magnetic effect (CME) \cite{20,21}, the chiral vortical effect (CVE) \cite{22}, the anomalous quantum Hall effect (AQHE) \cite{23,24,25}, the no-go Bloch theorem \cite{26}. All those phenomena have the same origin - the chiral anomaly. The naive ”derivation” of the CME from the standard expression for the chiral anomaly has been presented, for example, in \cite{27,28,29}.

However, the more accurate consideration demonstrates that the original equilibrium version of the CME does not exist. In \cite{30,31} this subject has been elaborated via the numerical lattice methods. In the condensed matter theory the absence of CME was reported in \cite{32}. The same conclusion has been obtained on the basis of the no-go Bloch theorem \cite{33}. The analytical proof of the absence of the equilibrium CME was presented in \cite{34,35}, using the technique of Wigner transformation \cite{36,37,38} applied to the lattice models\cite{39}. The same technique allows to reproduce the known results on the AQHE \cite{40}. It also confirms the existence of the equilibrium CSE \cite{41}. Notice, that in the framework of the naive nonregularized quantum field theory the CSE was discussed, for example, in \cite{42} in the technique similar to the one that was used for the consideration of the CME \cite{13,20}.

The investigation of momentum space topology was developed previously mainly within the condensed matter physics. It allows to relate the gapless boundary fermions to the bulk topology for the topological insulators and to describe the stability of the Fermi surfaces of various kinds. Besides, this technique allows to calculate the number of the fermion zero modes on vortices (for the review see \cite{18,39}). Momentum space topology of QCD has been discussed recently in \cite{43}. The whole Standard Model of fundamental interaction was discussed in this framework in \cite{44}.

Recently the new non-dissipative transport effects were proposed: the rotational Hall effect \cite{45} and the scale magnetic effect \cite{46}. In the present paper we will propose one more non-dissipative transport effect - the chiral torsional effect. Namely, we will discuss the emergence of axial current in the presence of torsion. It will be shown that this effect is intimately related to the chiral vortical effect \cite{21}. In conventional general relativity \cite{47} torsion vanishes identically, it appears only in its various extensions. The background (non-dynamical) gravity with torsion emerges, in particular, in certain condensed matter systems. For example, elastic deformations in graphene and in Weyl semimetals induce the effective torsion experienced by the quasiparticles \cite{31,48}. In \textsuperscript{3}He-A torsion appears dynamically when motion of the superfluid component is non-homogeneous. Below we will consider mostly the massless free Dirac fermions in the presence of the background gravity with torsion. We assume that this model is the relevant ingredient of the low energy effective field theory for the appropriate condensed matter system (either the Weyl/Dirac semimetal or \textsuperscript{3}He-A) or of the theory of elementary particles in the presence of real background gravity with torsion. The other ingredients of the mentioned theories are interactions between the fermions due to the exchange by various interaction carriers including the gauge bosons. On the present level of consideration we omit the interactions completely.

Besides, we disregard the natural anisotropy existing without emergent gravity in \textsuperscript{3}He-A and Weyl semimetals.
Therefore, while applying expression for the chiral torsional effect obtained below to these materials one should take care of this anisotropy. It actually may be restored in the final expression in the straightforward way.

RESPONSE OF THE AXIAL CURRENT TO TORSION

We are considering the model of massless Dirac fermions, which serves as the low energy approximation for the electronic excitations in Weyl semimetals and also for the fermionic excitations in $^3$He-A. For simplicity we will restrict ourselves by one flavor of Dirac fermions, and do not take into account the anisotropy of the Fermi velocity. In order to take it into account we should simply rescale the coordinates in an appropriate way.

Following \cite{49,50} we will calculate here the response of the axial current $j^5(x) = \bar{\psi}(x)\gamma^5\gamma^5\psi(x)$ to torsion in terms of the Wigner transformation of the Green function. We consider the case of vanishing spin connection and the nontrivial torsion encoded in the vielbein $e^a_{\mu}(x) = \delta^a_{\mu} + \delta e^a_{\mu}(x)$. The inverse vielbein is denoted here by $E^a_{\mu} = \delta^a_{\mu} + \delta E^a_{\mu}$, it obeys equation $e^a_{\mu}(x)E^b_{\mu}(x) = \delta^a_b$, which gives $\delta e^a_{\mu}(x) \approx -\delta E^a_{\mu}$. The momentum operator is to be substituted by $p_a \rightarrow (\delta^a_b + \delta E^a_{\nu}(x))p_b \approx (p_a - \delta e^a(x)p_b)$. Product of the coordinate - depending vielbein and momentum operator should be symmetrized in order to lead to the Hermitian Hamiltonian of the quasiparticles \cite{51,52}. Suppose, that $G(p)$ is the two point fermion Green function in momentum space for the system of Dirac spinors. In order to obtain expressions for the axial current we deform it as follows:

$$G(p) \rightarrow G(p - B(x)\gamma^5)$$

Next, the axial current is to be expressed as the functional derivative of the partition function with respect to $B$. The partition function is built using the Dirac operator defined in momentum space as $G^{-1}(p - B(x)\gamma^5)$. In this consideration $x$ is to be treated as parameter. Following the same steps as those of \cite{50} in the leading order in the derivative of the vielbein we obtain the following expression for the axial current:

$$j^{5k} = \frac{i}{2}T^{a}_{ij} \int \frac{d^3p}{(2\pi)^3} p_a \text{Tr} G\partial_p G^{-1} \partial_p G \gamma^5 \partial_p G^{-1}$$

$$T^{a}_{ij} = \partial_i e^a_j - \partial_j e^a_i$$

Let us consider the situation when the only nonzero components of torsion are given by:

$$T^{0}_{ij} = e^0_{fij} S^f$$

where $S$ is a certain three - vector. In case of the non-interacting theory of massless Dirac fermions in continuous space in the presence of finite temperature $T$ we obtain the following expression:

$$j^{5k} = 4S^k T \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3} \frac{\omega_n^2}{(\omega_n^2 + p^2)^2}$$

Here the sum is over the discrete Matsubara frequencies $\omega_n = 2\pi(n + 1/2)/T$, $n \in Z$. In the expression of Eq. (2) we do not see the signature of the topological protection from various corrections. Therefore, we do not exclude that the small deformations of the Green’s function may lead to the variation of the axial current. Thus, there will, possibly, appear corrections to this expression due to interactions.

SUM OVER THE MATSUBARA FREQUENCIES

The obtained above expression (4) is divergent, and requires regularization. Below we present the results of our calculations made using the two complementary methods: via the zeta-regularization and via the sum over the Matsubara frequencies with the contribution of vacuum subtracted. In both cases the results coincide. Let us consider first the direct summation over the Matsubara frequencies:

$$j^{5z} = 4S^k T \sum_{\omega_n} \int \frac{d^3p}{(2\pi)^3} \frac{\omega_n^2}{(\omega_n^2 + p^2)^2} = j^{5_{\text{vac}}} + j^{5_{\text{medium}}}$$

where $\omega_n = 2\pi T(n + 1/2)$. Let us perform summation over $n$:

$$4T \sum_{\omega_n} \frac{\omega_n^2}{(\omega_n^2 + p^2)^2} = -2 \int \frac{dz}{2\pi i} \frac{z^2}{(z^2 - p^2)^2} \text{th} \left(\frac{z}{2T}\right)$$

$$= 2\frac{z^2}{(z + p)^2} \frac{d}{dz} \text{th} \left(\frac{z}{2T}\right) \bigg|_{z = -p} + 2\frac{z^2}{(z - p)^2} \frac{d}{dz} \text{th} \left(\frac{z}{2T}\right) \bigg|_{z = -p} + \frac{p}{2T} \text{th} \left(\frac{p}{2T}\right)$$

$$= -\frac{1}{2T \text{ch}^2 \left(\frac{p}{2T}\right)} + \frac{1}{p} \text{th} \left(\frac{p}{2T}\right)$$

In the first row the integral is over the contour surrounding all poles of the hyperbolic tangent. This contour is then deformed to surround points $\pm p$. In this expression we separate the contribution of thermal quasiparticles subtracting the contribution of vacuum (the expression of Eq. (3) for $T = 0$). We denote $x = \frac{p}{2T}$ and obtain:

$$j^{5_{\text{medium}}} = 2T^2 S^k \int_0^{\infty} \frac{x^2 dx}{\pi^2} \left(\frac{1}{x^2} + \frac{1}{x} \text{th} \left(\frac{1}{x}\right) - \frac{1}{x}\right)$$

Vacuum part is formally divergent, and we do not consider it here. After integration over $x$ we obtain the

\footnote{In this form gravity emerges in graphene and Weyl semimetals.}
following expression for the separate contribution of medium (i.e. of the thermal quasiparticles):
\[
\tilde{J}^{5k}_{medium} = \frac{T^2}{12} S^{5k} = -\frac{T^2}{24} \epsilon^{0kij} T_{ij}^0
\]  
(8)

It is well-known that ζ - regularization subtracts systematically all divergencies. It allows to include all logarithmic divergencies to the renormalization of coupling constants automatically. At the same time the power-like divergencies are subtracted from all expressions completely. Formally integration over momenta in Eq. (5) gives \( \tilde{j}^5 = \frac{1}{\pi} \sum_{n=\infty}^{0} |\omega_n| \). In zeta-regularization instead of this expression we obtain
\[
\tilde{j}^5 = \frac{1}{\pi} \sum_{n=\infty}^{0} \frac{1}{\omega_n}\]

Here the sum over \( n \) determines the function of \( s > 1 \). The final answer is given by its analytical continuation to \( s = -1 \). This results in the following resummation
\[
\tilde{j}^5 = 2\tilde{S}\tilde{T}^2 \left( \sum_{n=0}^{\infty} \frac{1}{(n+1/2)^s} \right)_{s=-1} = 2\tilde{S}\tilde{T}^2 \zeta_H(-1,1/2) = \frac{T^2}{12} \tilde{S}
\]

(9)

In the second row \( \zeta_H(s,\nu) \) is the Hurwitz zeta-function.

One can see that the expressions for the current produced by the thermal quasiparticles of (5) and (4) coincide.

CVE EFFECT AS THE PARTICULAR CASE OF THE CHIRAL TORSIONAL EFFECT

Let us consider the particular non-relativistic system (having the emergent relativistic symmetry at low energies) in the presence of macroscopic motion with velocity \( v \). When \( v \) is constant or slowly varying, the one-particle Hamiltonian is given by \( \hat{H} = \hat{H}_0 + \hat{p}v \). Here \( \hat{H}_0 \) is the Hamiltonian in the reference frame accompanying the motion. For the case of emergent massless Dirac fermions \( \hat{H}_0 = \gamma^a \sum_{k=1,2,3} \gamma^k p_k \). Then the combination \( p_0 - \hat{H} \) may be written as \( \gamma^0 E^0_a \gamma^a p_k \) with the inverse vierbein \( E^0_a = 1 \), \( E^a_k = \delta^a_k \), \( E^a_0 = -v_k \), \( E^0_a = 0 \) for \( a, k = 1, 2, 3 \). Therefore, we are able to treat the velocity of the macroscopic motion as the components of the vierbein: \( \epsilon^0_k = v_k \), \( k = 1, 2, 3 \). Correspondingly, torsion appears with the nonzero components \( T^0_{ij} = \partial_i v_j - \partial_j v_i = 2 \epsilon_{0ijk} \hat{\omega}^k \), where \( \hat{\omega} \) is vector of angular velocity. Eq. (5) gives
\[
\tilde{j}^{5k}_{medium} = -\frac{T^2}{24} \epsilon^{0kij} T^0_{ij} = \frac{T^2}{6} \omega^k
\]

(10)

which is nothing but the conventional expression for the chiral vortical effect (see also [52]). Recall that the latter effect for vanishing chemical potential is the appearance of the axial current in the system of rotating thermal quasiparticles. Typically in the consideration of this effect the vacuum as a whole is assumed to be at rest, only the excitations over vacuum are rotated. Therefore, indeed, the subtraction performed above (of the vacuum contribution to the chiral torsional effect) effectively results in the description of the chiral vortical effect for vanishing \( \mu \).

CONCLUSIONS AND DISCUSSIONS

To conclude, we have calculated the axial current produced by thermal quasiparticles in the presence of torsion for the system of the noninteracting massless Dirac fermions. Eq. (8) is obtained in two ways: via zeta-regularization and via the summation over Matsubara frequencies with the vacuum contribution subtracted. Although we did not consider the effects of interactions directly, there are certain indications, that the coefficient in Eq. (8) is subject to the renormalization due to interactions. Notice, that in Eq. (8) there is no general coordinate invariance. First of all, this invariance is absent because we consider the system with vanishing spin connection, which corresponds to the case of emergent gravity in real Weyl materials [56]. This corresponds to the partial gauge fixing, which eliminates the general coordinate invariance. Moreover, we considered the system with finite temperature. Therefore, even the Lorentz invariance is broken and Eq. (8) actually possesses the \( O(3) \) symmetry of spatial rotations.

It is also worth mentioning that we considered only the components of torsion \( T^0_{ij} \) with \( i, j = 1, 2, 3 \). The possible appearance of the axial current in the presence of the other components of torsion remains open and requires an additional consideration, which is out of the scope of the present paper. The resulting current of Eq. (8) represents the new anomalous transport effect. The corresponding current is non-dissipative, at least in the approximation of the slowly varying vierbein/torsion. The explanation for this is the mentioned above analogy between the composition \( \delta \epsilon^0_\omega \) and the electromagnetic vector-potential. According to this analogy \( \hat{H}^k = T^0_{ij} \epsilon^{0ijk} \hat{\omega} \) plays the role of the effective magnetic field. The latter does not perform the work and thus cannot lead to dissipation.

The divergence of the vacuum axial current in the presence of background torsion prompts that the quantum field theory in the presence of background gravity with torsion is ill-defined. Presumably, there are the aspects of the theory, in which it depends on regularization. In the other words, suppose, that the microscopically different real physical systems have naively the same continuous low energy effective field theory. In the presence of emergent torsion these effective theories become dif-
ferent. In particular, we expect the appearance of such differences between the low energy description of electrons in Weyl semimetals (in the presence of elastic deformations) and the description of the quasiparticles in the $^3$He-A superfluid (in the presence of the in-homogeneous motion of the superfluid component). This difference may, possibly, be related to the appearance of the Nieh-Yan-like term \cite{5} in the chiral anomaly for the solids and its absence for $^3$He-A. Notice, that experiments with vortices in $^3$He-A prompt that the chiral anomaly in this superfluid does not contain the Nieh-Yan term (see \cite{18} and references therein). At the same time the recent theoretical consideration of Weyl semimetals indicates the appearance of this term in the chiral anomaly for these materials. \cite{57}

It would be interesting to apply the lattice regularization in the spirit of \cite{41} for the calculation of axial current of massless Dirac fermions in the presence of nontrivial vielbein. Such a calculation may shed led on various questions related to the vacuum contribution to the chiral torsional effect and to the possible appearance of the Nieh-Yan term in the chiral anomaly.

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