C Function Representation of the Local Potential Approximation

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Abstract
Within the Local Potential Approximation to Wilson’s, or Polchinski’s, exact renormalization group, and for general spacetime dimension, we construct a function, $c$, of the coupling constants; it has the property that (for unitary theories) it decreases monotonically along flows, and is stationary only at fixed points —where it ‘counts degrees of freedom’, $i.e.$ is extensive, counting one for each Gaussian scalar. Furthermore, by choosing restrictions to some sub-manifold of coupling constant space, we arrive at a very promising variational approximation method.
1. A C Function.

Following Zamalodchikov’s celebrated $c$-theorem\cite{1} for two dimensional quantum field theory, a number of groups have sought to generalise these ideas to higher dimensions\cite{3}. The motivation behind this, is not only to demonstrate irreversibility of renormalization group flows, in some precise sense and under certain general conditions, and thus prove that exotic flows such as limit cycles, chaos, etc. are missing in these cases, but perhaps more importantly to provide an explicit, and useful, geometric framework for the space of quantum field theories. In ref.\cite{1}, Zamalodchikov established three important properties for his $c$ function,

1. There exists a function $c(g) \geq 0$ of such a nature that

$$\frac{d}{dt}c \equiv \beta^i(g) \frac{\partial}{\partial g^i}c(g) \leq 0 \quad ,$$

where the $g^i$ form an (infinite) set of dimensionless parameters, a.k.a. the coupling constants, and the beta functions are defined as $\beta^i = dg^i/dt$. The equality in (1) is reached only at fixed points of the renormalization group $g(t) = g_*$.  

2. $c(g)$ is stationary at fixed points\cite{1} i.e. $\beta^i(g) = 0$ for all $i$, implies $\partial c/\partial g^i = 0$.

3. The value of $c(g)$ at the fixed point $g_*$ is the same as the corresponding (Virasoro algebra) central charge\cite{4}. (This property thus only makes sense in two dimensions.)

Within the Local Potential Approximation\cite{5}–\cite{9} (LPA) to Wilson’s\cite{3} or Polchinksi’s\cite{10} exact renormalization group (which we describe below), we display a $c$-function which has the first two properties in any dimension $D$. This follows by virtue of the property that the $\beta$-functions are the ‘gradient flows’ of $c$ with respect to a positive definite metric. This property in turn follows straightforwardly from an observation by Zumbach\cite{6}. At fixed points, our $c$ function is extensive, viz. additive in mutually non-interacting degrees of freedom, as is also true of the Virasoro central charge (cf. property 3). Furthermore, we may normalise so that our $c$ counts one for each Gaussian scalar and zero for each infinitely massive scalar (corresponding to a High Temperature fixed point), as does the central charge in two dimensions. It is probably not possible within the Local Potential Approximation, to establish a more concrete link to Zamalodchikov’s $c$. Of course, it would be very interesting to understand if these observations generalise to higher orders in the derivative

\footnote{1 We will not here need to restrict to critical fixed points\cite{3}.}
expansion\(^1\)\(^2\) (which likely would allow a direct comparison with Zamalodchikov’s \(c\)), or indeed generalise to an exact expression along the present lines.

In the second part of this letter, we compute exactly within the LPA, some simple illustrative examples. In the third and final part, we point out that this geometrical structure for the LPA leads naturally to a variational approximation scheme, by restricting the flow to some finite dimensional submanifold of coupling constant space, of our choosing. Approximate continuum limits (\(i.e.\) fixed point behaviour), follow from stationarizing with respect to this finite parameter set. We briefly investigate the efficacy of the method, considering, in particular, the simplest polynomial approximation to the non-perturbative fixed point potential for a single scalar field in three dimensions. The resulting form, and critical exponents, lie exceedingly close to the exact\(^3\) answers.

We work with \(N\) Bose fields \(\varphi_a\) in \(D\) Euclidean dimensions. We have checked that the constructions can be generalised to Fermi fields, but for simplicity we omit this. In a condensed notation\(^1\)\(^2\)\(^3\), Polchinski’s form of the Wilson renormalization group is given by\(^1\)\(^2\)\(^4\)

\[
\frac{\partial S_\Lambda}{\partial \Lambda} = \frac{1}{2} \frac{\delta S_\Lambda}{\delta \varphi_a} \cdot \frac{\partial \Delta_{UV}}{\partial \Lambda} \cdot \frac{\delta S_\Lambda}{\delta \varphi_a} - \frac{1}{2} \text{tr} \frac{\partial \Delta_{UV}}{\partial \Lambda} \cdot \frac{\delta^2 S_\Lambda}{\delta \varphi_a \delta \varphi_a} .
\]

Here \(S_\Lambda[\varphi]\) is the interaction part of a Wilsonian effective action \(S_\Lambda^{eff} = \frac{1}{2} \varphi_a \Delta_{UV}^{-1} \varphi_a + S_\Lambda\), and \(\Delta_{UV} = C_{UV}(q^2/\Lambda^2)/q^2\) is a massless propagator whose momentum \(q\) is cutoff by an effective ultra-violet cutoff \(\Lambda\). We require that the cutoff function \(C_{UV}\) is analytic at \(q = 0\) and that \(C_{UV}(0) = 1\) (so that physics is unchanged at scales much less than \(\Lambda\)), and \(C_{UV} \to 0\) as \(q \to \infty\) sufficiently rapidly that all momentum integrals in (2) are well-defined (\(i.e.\) well regulated). Wilson’s flow equation\(^3\) is identical to (2), after the transformation\(^1\)\(^3\)\(^9\)

\[
\varphi \mapsto \sqrt{C_{UV}} \varphi , \quad \text{and identification} \quad \mathcal{H} \equiv -S_\Lambda .
\]

In the Local Potential Approximation, we restrict the interactions to that of a general potential: \(S_\Lambda = \int d^Dx V(\varphi, \Lambda)\), and discard from the right hand side of (2) all terms that correspond to higher derivative corrections (on expansion of \(C_{UV}\) where necessary). Thus,

\[
\frac{\partial}{\partial \Lambda} V(\varphi, \Lambda) = \frac{\alpha}{\Lambda^3} \left( \frac{\partial V}{\partial \varphi_a} \right)^2 - \gamma \Lambda^{D-3} \frac{\partial^2 V}{\partial \varphi_a \partial \varphi_a} ,
\]

where \(\alpha = -C_{UV}'(0)\) and \(\gamma = -\int \frac{d^Dq}{(2\pi)^D} C_{UV}'(\tilde{q}^2)\) .

\(^2\) within LPA!
We assume that the dimensionless coefficients $\alpha$ and $\gamma$ are positive; this is the case, for example, if $C_{UV}$ is strictly monotonically decreasing. To express all quantities in a form appropriate for the continuum limit (i.e., approach to fixed points), we change to dimensionless variables $\varphi \mapsto \varphi \Lambda^{D/2-1} \sqrt{\gamma}$, $V \mapsto V \Lambda^D \gamma/\alpha$, and $t = \ln(\mu/\Lambda)$ (where $\mu$ is some arbitrary physical mass scale), and by thus also absorbing the coefficients $\alpha$ and $\gamma$, obtain the LPA of Polchinski’s equation

$$
\frac{\partial}{\partial t} V(\varphi, t) + \frac{1}{2}(D-2)\varphi_a \frac{\partial V}{\partial \varphi_a} - D V = \frac{\partial^2 V}{\partial \varphi_a \partial \varphi_a} - \left( \frac{\partial V}{\partial \varphi_a} \right)^2 ,
$$

(4)
in a manifestly ‘scheme’ independent form\(^3\). Since the LPA corresponds to discarding all momentum dependent terms from (2), it is immediate to realise from (3) that the above is also the LPA for Wilson’s equation.

If in place of $V$, we introduce a Gibbsian-like measure $\rho(\varphi, t) = \exp \{-V(\varphi, t)\}$, and $G(\varphi) = \exp \{-\frac{1}{4}(D-2)\varphi_a^2\}$, and define the functional

$$
\mathcal{F}[\rho] = a^N \int d^N \varphi \; G \left\{ \frac{1}{2} \left( \frac{\partial \rho}{\partial \varphi_a} \right)^2 + \frac{D}{4} \rho^2 (1 - 2 \ln \rho) \right\} ,
$$

(5)
then (4) may be rewritten manifestly as a ‘gradient flow’ \(^6\):

$$
a^N G \frac{\partial \rho}{\partial t} = -\frac{\delta \mathcal{F}}{\delta \rho} .
$$

(6)

In (5) and (6), $a > 0$ is a normalisation factor for the $\varphi$ measure, to be determined later. [Alternatively, changing variables $\varphi \mapsto \varphi/a$, the factors $a^N$ can be absorbed and appear instead as factors of $a^2$ in front of the $(\partial/\partial \varphi)^2$ terms in (4) and (5).] Substituting the fixed point equation $\delta \mathcal{F}/\delta \rho = 0$ back into (5), we see that fixed points $\rho(\varphi, t) = \rho_*(\varphi)$ satisfy,

$$
\mathcal{F}[\rho_*] = \frac{D}{4} a^N \int d^N \varphi \; G \rho_*^2 .
$$

(7)

Now let $g^i(t)$ be a complete set of parameters (a.k.a. coupling constants) for $V$. Away from fixed points, these will be infinite in number. Asymptotically approaching a particular fixed point, it is possible to identify a finite number of parameters (namely the relevant and marginal couplings) and express all the rest (the ‘irrelevant’ parameters) in terms of

\(^3\) in general using $\Lambda$ raised to the power of their scaling dimensions, however in LPA there is no anomalous dimension for $\varphi$. This fact also allowed our scaling-dimensionless choice for $C_{UV}$ \(^3\).
them. (This ‘reduction of parameters’ corresponds to the continuum limit defined around this fixed point, see later and e.g.\cite{12}\cite{16}; universality arises because perturbations in the irrelevant parameters about these values, exponentially decay away as $t \to \infty$.) Following ref.\cite{1}, we define the operators

$$
\Phi_i(g) = \partial_i V(\varphi, g) ,
$$

(8)

where we have written $\partial_i \equiv \partial/\partial g^i$. We define the metric

$$
\mathcal{G}_{ij}(g) = a^N \int d^N \varphi \rho^2 \Phi_i \Phi_j .
$$

(9)

It is positive definite providing $V$ is real, which we assume (corresponding to a unitary theory in Minkowski space). Multiplying (8) by $\partial_i \rho = -\rho \Phi_i$, we thus obtain

$$
\mathcal{G}_{ij} \beta^j = -\partial_i \mathcal{F}(g) .
$$

(10)

We define the $c(g)$ function through

$$
\mathcal{F} = DA^c/4 ,
$$

(11)

where $A > 1$ is a normalisation factor to be determined. Redefining the metric by a positive factor, $\mathcal{G}_{ij} = (\mathcal{F} \ln A) \hat{\mathcal{G}}_{ij}$, we obtain from (10):

$$
\partial_i c(g) = -\hat{\mathcal{G}}_{ij} \beta^j (g) .
$$

(12)

From (12) and the positive-definiteness of $\hat{\mathcal{G}}_{ij}$, we see that (1), and the properties 1 and 2 hold. Now suppose that the fields form two mutually non-interacting sets. Let us write $\varphi_a \equiv \varphi_a^{(1)}$ when the field belongs to the first set, and $\varphi_a \equiv \varphi_a^{(2)}$ when it belongs to the second set. Similarly, the couplings $g$ split into two sets $g^{(1)}$ and $g^{(2)}$. The potential $V(\varphi, t)$ can be written $V(\varphi, t) = V^{(1)}(\varphi^{(1)}, t) + V^{(2)}(\varphi^{(2)}, t)$. Thus $\rho$ factorizes: $\rho = \rho^{(1)} \rho^{(2)}$, and from (7), at fixed points we have $\mathcal{F}[\rho_*] = \frac{4}{D} \mathcal{F}[\rho_*^{(1)}] \mathcal{F}[\rho_*^{(2)}]$. Therefore from (11) we have, at fixed points, that our $c$ is extensive:

$$
c(g_*) = c(g_*^{(1)}) + c(g_*^{(2)}) .
$$

(13)

At the Gaussian fixed point $V_* = 0$, the Virasoro central charge counts one degree of freedom per scalar, \textit{i.e.} is here equal to $N$ \cite{4}. At the High Temperature fixed point $V_* = V_*^{HT} = \frac{1}{2} \varphi_a^2 - N/D$, the Virasoro central charge vanishes, because $V_*^{HT}$ corresponds
to a non-critical fixed point, i.e. infinitely massive fields and thus no propagating degrees of freedom. This interpretation of \( V_{*}^{HT} \) follows directly from the Legendre transform relation between the Polchinski and Legendre effective potentials \([13][12]\), or by the explicit flow derived below (see also ref.[17]).

The normalisation factors \( A \) and \( a \) may be uniquely determined by requiring that our \( c \) agree with this counting in any dimension \( D \). Substituting \( c = 0 \) in (11) and \( \rho = \exp -V_{*}^{HT} \) in (7), we obtain

\[
a = e^{-2/D} \sqrt{\frac{D+2}{4\pi}} .
\]

(14)

For the Gaussian fixed point we substitute \( c = N \) in (11) and \( \rho = 1 \) in (7), and thus

\[
A = e^{-2/D} \sqrt{\frac{D+2}{D-2}} .
\]

(15)

Note that \( A > 1 \), as required, for all \( D \geq 2 \). The precise values for \( a \) and \( A \) may be expected to change beyond the LPA.

Perturbing the couplings to first order about a fixed point: \( g^i(t) = g^i_* + \epsilon v^i e^{\lambda t} \), we obtain from (12) the eigenvalue equation

\[
\partial_i \partial_j c(g_*) v^j = -\lambda \hat{G}_{ij}(g_*) v^j ,
\]

determining the eigenoperators \( \Phi = v^i \Phi_i \) and their scaling dimensions \( D - \lambda \). Equivalently, from (10),

\[
\partial_i \partial_j F(g_*) v^j = -\lambda G_{ij}(g_*) v^j .
\]

(16)

2. Examples.

Consider the simple example of the Gaussian fixed point perturbed by the mass operator, for a single scalar field. Thus we set \( V(\varphi, t) = \frac{1}{2} \sigma(t) \varphi^2 + \mathcal{E}(t) \). The \( \beta \) functions for \( \sigma \) and \( \mathcal{E} \) follow easily from (4):

\[
\frac{\partial}{\partial t} \sigma = 2\sigma(1 - \sigma)
\]

\[
\frac{\partial}{\partial t} \mathcal{E} = D\mathcal{E} + \sigma.
\]

(17)

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4 in units of \( \Lambda \)
The general solutions are

\[ \sigma(t) = \frac{1}{1 + r e^{-2t}} \]

\[ \mathcal{E}(t) = \mathcal{E}_0 e^{Dt} - \frac{1}{2} \int_0^1 du \frac{u^{D/2 - 1}}{1 + ur e^{-2t}} , \]

where \( r \) and \( \mathcal{E}_0 \) are integration constants. Considering \( t \to -\infty \), one sees that this solution indeed emanates from the Gaussian fixed point, while a change of variables back to dimensionful (i.e. physical) variables shows that \( r \approx \mu^2/m^2 \), where \( m \) is the mass of the still-Gaussian scalar field, while \( \mathcal{E}_0 \mu^D \) is an added vacuum energy term. We normalised the special solution in (18b) so that with \( \mathcal{E}_0 = 0 \), (18) tends to \( V_{HT}^* \) as \( t \to \infty \).

Since \( \rho \) is Gaussian, (5) is readily determined;

\[ \mathcal{F} = \frac{a}{2} e^{-2\mathcal{E}} \left\{ 2D\mathcal{E} + \frac{6D\sigma + 4\sigma^2 + D^2 - 2D}{D - 2 + 4\sigma} \right\} \sqrt{\frac{\pi}{D - 2 + 4\sigma}} . \]

Combining this, (14), (15) and (11), yields \( c(t) \) and one verifies that when \( \mathcal{E}_0 = 0 \), \( c(t) \) flows from 1 to 0 as \( t \) runs from \(-\infty\) to \(+\infty\). Note that if \( \mathcal{E}_0 \neq 0 \), then \( c(t) \to \mp\infty \) as \( t \to \infty \), depending on the sign of \( \mathcal{E}_0 \). This seems in contradiction with the idea that \( c \) counts degrees of freedom, since evidently the vacuum energy should not figure per se in this counting. However, one must recall from (13), that \( c \) is extensive (i.e. ‘counts’) only at fixed points, and with \( \mathcal{E}_0 \neq 0 \) the system never reaches another fixed point as \( t \to \infty \).

From the Gaussian form of \( G \), we recognize that at both the Gaussian and High Temperature fixed points, the metric (3) (and thus \( \tilde{G}_{ij} \)) is diagonalized by choosing the operators \( \Phi_i \) to be products of Hermite polynomials \( H_n \) in the \( \varphi_a \). Since these also turn out to diagonalize \( \partial_i \partial_j \mathcal{F}(g_*) \), they are the eigenperturbations, and the corresponding eigenvalues follow straightforwardly. Choosing the Gaussian fixed point \( \rho_* = 1 \) for example, and again specializing to the case of one scalar field for simplicity, we thus take \( \Phi_n = H_n(\sqrt{D - 2}) \), \( n = 0, 1, \cdots \). The metric has non-zero components[13] \( G_{nn} = 2^{n+1} |a| \sqrt{\frac{\pi}{D - 2}} \). From (8) and (3), we obtain

\[ \partial_i \partial_j \mathcal{F} \bigg|_{\rho=1} = -D G_{ij} + a \int d\varphi G \frac{\partial \Phi_i}{\partial \varphi} \frac{\partial \Phi_j}{\partial \varphi} . \]

Thus, using\[ H'_n = 2nH_{n-1} , \]
we have that \( \partial_i \partial_j \mathcal{F} \) is also diagonal, and we recover the expected Gaussian spectrum of eigenvalues \( \lambda = -\partial_n \partial_n \mathcal{F}/G_{nn} = D + \frac{1}{2}(2 - D)n \).

\(^5\) prime being differentiation with respect to the argument
We may also construct perturbatively, the continuum limit about the Gaussian fixed point, and $c$ and $G_{ij}$, to any desired order in the relevant and marginal couplings. Thus for example in $D = 4$ dimensions, the massless continuum limit is constructed by solving the $\beta$ functions for $g^2$, $g^6$, $g^8$, etc., iteratively in terms of a power series in $g^4(t)$ (in direct generalisation of the case for “$\lambda(t)$” and “$\gamma_1(t)$” given in ref. [12]), reducing (12) to the one beta function $\beta^4(g^4) = -\partial_4 c(g^4)/\hat{G}_{44}$.

3. Variational Approximations.

The gradient flow form (6) of the LPA, suggests the possibility of approximating $\rho$ by a variational ansatz, that is setting $\rho = \tilde{\rho}(\varphi; g_1, \ldots, g_M)$, $M < \infty$, where $\tilde{\rho}$ is some finitely parametrized set of functions of our choosing. Interpreting the functional derivative in (3) to include only variations in this restricted set, we arrive again at (10) [or (12)] where however, here and from now on, indices run only from 1 to $M$. Thus geometrically, we restrict the flows to the sub-manifold $M$ parametrized by $g_1, \ldots, g_M$.

Approximations to the fixed points of (3) follow, by (10), from solutions to the variational conditions $\partial_i F = 0$, and correspond geometrically to those points $g = \tilde{g}_*$ on $M$ where $T_g M$ (the tangent space to $M$ at $g$) is perpendicular to the exact flows. From this, it is intuitively clear that good results can be expected generically, $e.g.$ for the shape of $\rho$, if $M$ passes sufficiently ‘close’ to the true fixed points, and good results will be obtained for the scaling dimensions and shape, of any such eigenoperators that are almost parallel with $T_{\tilde{g}_*} M$. We can expect decent approximations for the full flows, from say, one fixed point to another – under similar conditions.

It is clear that it is possible to be unlucky and find in this way ‘spurious fixed points’ that do not well approximate the exact solutions, but these will not be relatively stable under changing or improving the ansatz manifold $M$, $e.g.$ by enlarging its dimension $M$. In fact as $M$ improves, such spurious fixed points (if indeed there are any) will disappear entirely. Equally, it is clear that numerical results can (at least in principle) be obtained to any desired degree of accuracy.

These characteristics are in marked contrast to the general situation for truncations (of the renormalization group) to a finite set of operators [14][19], where spurious fixed points multiply at higher orders, and numerical results cease to converge.

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6 since the (restriction of the) metric $G_{ij}$ is still positive definite

7 $i.e.$ with $M = \infty$, or from [4]
To test the practicality of this variational method, we tried the following very simple even polynomial ansätze. The simplest two cases, \( V(\varphi, t) = \mathcal{E}(t) \) and \( V(\varphi) = \mathcal{E}(t) + \frac{1}{2}\sigma(t)\varphi^2 \), both correspond to exact reductions of the full flow equations \( (\text{a.k.a. } \beta \text{ functions}) \) from (4), and are given by (17). Therefore it is immediate to realise that we obtain only—and exactly—the Gaussian and, when \( \sigma \) is included, High Temperature fixed points. Similarly, we obtain the eigenvalue \( \lambda = D \) for the unit operator \( \Phi_{\mathcal{E}} = \partial_{\mathcal{E}}V = 1 \) at both fixed points, and \( \lambda = 2 \) (-2) for the second eigenvalue at the Gaussian (High Temperature) fixed point when \( \sigma \) is included. This may be checked directly through (19), (16), and (9).

In fact it is straightforward to confirm that \( \partial_{\mathcal{E}}V = 1, \lambda = D \), is always a solution in any approximation that includes \( \mathcal{E} \) as a parameter. Equally, from our previous exact analysis of the Gaussian and High Temperature fixed points, it is clear that a general polynomial ansatz of order \( \varphi^n, n > 0 \), with unconstrained coefficients, will find exactly these fixed points and exactly the first \( n \) eigenvalues.

The next-simplest even polynomial is \( V(\varphi, t) = \mathcal{E}(t) + \frac{1}{2}\sigma(t)\varphi^2 + s(t)\varphi^4 \). Since the case \( s = 0 \) has already been analysed, we fix \( s > 0 \). We set \( D = 3 \). We take advantage of coordinate invariance to write \( s = (g^4)^2 \) and \( \sigma = g^4g^2 - 1/4 \), and change variables \( \varphi = x/\sqrt{g^4} \), so that \( \mathcal{F} \) and its derivatives may be expressed in closed form in terms of the integrals \( I_n(g^2) = \int_{-\infty}^{\infty} dx \, x^n e^{-g^2x^2 - 2x^4}, n = 0, 2 \). In this way, on the basis of analytic (for large \( \pm g^2 \)) and numeric estimates, we establish that there is only one non-trivial solution of the variational equations \( \partial_i \mathcal{F} = 0, i = \mathcal{E}, 2, 4 \). It corresponds to \( s = .00772624, \sigma = -.13488 \) and \( \mathcal{E} = .054794 \). In fig.1, we plot the resulting form for \( \rho_* \) and compare it to the one exact non-trivial fixed point solution\(^8\) from (4) (which itself is an approximation to the Ising model fixed point in three dimensions). Solving (16), we obtain, apart from \( \lambda = 3, \nu = 1/\lambda = .6347 \) from the positive eigenvalue, and \( \omega = -\lambda = .6093 \) from the remaining eigenvalue. These are 2% and 8% off the exact values from (4), namely \( \nu = .6496 \) and \( \omega = .6557 \), respectively \(^7\). Overall, we find these results, for this simplest possible variational approximation to the non-trivial fixed point (and/or \( \omega \)), truly impressive.

Since the fixed point behaviour of (4) for a single field—or more generally—a single invariant, is straightforward to solve directly numerically, the true potential of this variational method lies in the relative ease in which approximations for global flows may be solved, and approximate solutions of LPAs found for more than one invariant, which thus correspond to (possibly high dimensional\(^{20}\)) partial differential equations.

Should the \( c \) function be extended beyond the LPA, we expect that the variational method will prove to be even more powerful.

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\(^7\) Solved by shooting as described in refs.\(^{19}\)\(^{16}\)\(^{18}\).
Fig. 1. Plotted as $\rho_*$, the simplest polynomial variational approximation to the non-trivial fixed point potential (dashed line) is compared to the exact solution to (4) (full line).

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