Study on Aerodynamic Performance and Lightweight Multiobjective Optimization Design of Wheel With Entropy Weighted Grey Relational Analysis Methods

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ABSTRACT In order to improve the aerodynamic performance and optimization efficiency while wheel lightweight designing, a multi-objective optimization design method of wheel lightweight based on entropy weighted grey relational analysis (EGRA) was proposed in this article. The aerodynamic analysis finite element model of the assembled wheel was established, and the simulation accuracy was verified by experiments. Study the distribution law of performance parameters such as pressure and turbulent kinetic energy in flow field of the car, analyze the variation law of flow field velocity and turbulent intensity in front and rear wheel cavities of the assembled wheel, and analyze the cloud diagram distribution of temperature and surface convective heat transfer coefficient of the brake disc ($h_c$). Research on the influence of wheels with different disc structures on the aerodynamic drag coefficient of the car ($C_d$) and the $h_c$. Combined with grey relational analysis (GRA) and EGRA, the objective evaluation of the comprehensive aerodynamic performance of wheels with different disc structures was given. With the design of experiments (DOEs), 12 important design variables were screened out by contribution analysis method. Using the approximate model method, combined with the RBF surrogate model, a hybrid method combining EGRA and Non-Dominated Sorting Genetic Algorithm-II (NSGA-II) was proposed to lightweight and multi-objective optimize the assembled wheel. Comparing and analyzing the optimization platform recommending scheme, the technique for ordering preferences by similarity to ideal solution (TOPSIS) method preferring scheme and the EGRA method optimum scheme, it was found that the optimal compromise scheme was obtained by the EGRA method, the reduction of the $C_d$ was more obvious, and the improvement rates of performance were also more balanced. After multi-objective optimization, the mass of the assembled wheel was reduced by 10.83%, the $C_d$ was reduced by 5.02%, and the average convective heat transfer coefficient of brake disc ($h_c$) was reduced by 8.02%. The optimized assembled wheel has a weight reduction of 32.74% compared with the same type of cast aluminum alloy wheel, which has a remarkable lightweight effect and significant reduction on the $C_d$. 

INDEX TERMS Aerodynamic drag coefficient, assembled wheel, convective heat transfer coefficient, entropy weighted grey relational analysis, lightweight and multi-objective optimization.

I. INTRODUCTION
The aerodynamic drag of the wheel accounts for 25%~30% of the car and its aerodynamic performance and lightweight...
design have a significant effect on vehicle energy saving and emission reduction. A reasonable wheel structure design can effectively improve the aerodynamic performance and lightweight level of the car [1], [2], [3]. As the space for reduction of the aerodynamic drag coefficient of the car ($C_d$) becomes smaller and the aerodynamics analysis of the car develops toward refinement, the importance of wheels for reducing the $C_d$ is increasing.

In addition, the wheel also affects the flow field near the brake disc, which affects the convective heat transfer coefficient of the brake disc ($h_c$), thereby affecting the heat dissipation performance of the disc. The heat dissipation performance of the brake disc directly affects driving safety, and more than 85% of the heat dissipation of the brake disc is carried out by convection heat transfer [4], [5], [6]. Therefore, in the wheel design, it's necessary to consider the influence of the change of wheel structure on the $h_c$.

Ilea and Iozsa [7] analyzed the effect of body shape in the wheel region on aerodynamic performance by progressively closing the rear wheel opening and used computational fluid dynamics (CFD) methods to obtain the results. The results show that closing the rear wheels has important aerodynamic advantages without affecting other functions of the car. Abandoning the aesthetic problem and designing the wheel area only from the aerodynamic point of view can greatly reduce the aerodynamic drag of the wheel and improve the drag coefficient of the car. Xia et al. [8] used RANS based on SST k-omega turbulence model to study the effect of gap spacing on train aerodynamics and verified by wind tunnel test data. The results show that the gap spacing significantly affects the airflow structure around the compartment gap and the aerodynamic drag of the train model. For the 1/8 scale high-speed train model, the gap spacing results in a significant reduction in the aerodynamic drag of the leading car and an increase in the aerodynamic drag of the trailing vehicle. Huang et al. [9] took the centerline of the two wheels of the symmetrical cross-section of the left wheel as the roll axis, established a simplified roll motion model by using the overlapping grid method, and studied the aerodynamic mechanism of the periodic roll motion of the body. The results show that the roll motion is closely related to the aerodynamic characteristics of the car, and the aerodynamic phase on the car is shifted due to the hysteresis effect, and the most dangerous position of the car can be obtained. This study provides engineering reference for vehicle design and safety evaluation. Yun [10] conducted a detailed analysis of the aerodynamic drag of the KTX-Sancheon and carried out the nose shape optimization using the Broyden-Fletcher-Goldfarb-Shanno optimization method. The results show that the aerodynamic drag of optimized train set is reduced by 15.0% compared with the aerodynamic drag of KTX-Sancheon.

Kothalawala and Gatto [11] studied the influence of applied yaw angle on the aerodynamics of rotating wheels in free air, using unsteady Reynolds-averaged Navier-Stokes simulations to characterize the complexity of the configuration surface and Near Wake Field Physics. The results show that the flow field around the wheel is mainly vortex in nature, and the number and strength of developed vortex structures is strongly dependent on the applied yaw angle level. Su et al. [12] discussed the influence of the spoke characteristics on the $C_d$, studied the improved MIRA model on Fluent, researched the relationship between the $C_d$ under different spoke offset distances and curvatures, and analyzed the reasons for aerodynamic drag changes in different situations. The results show that a smaller spoke offset distance is beneficial to the reduction of drag coefficient. When the wheel spoke offset distance is 10 mm, the minimum value of $C_d$ is 0.2514. Malizia and Blocken [13] accurately modeled two bicycle wheels, investigated the effect of the presence of the ground and type of wheel/ground contact on the wheel’s aerodynamic drag, and provided flow field visualizations to elucidate around spokes wheels in crosswind conditions flow behavior. The results show that the gap between wheel and ground should be a maximum of 10 mm, lower than without crosswind, and the step height should be less than 10 mm. Jia et al. [14] used the stable Reynolds-averaged Navier-Stokes calculation in the simulation, combined with a wind tunnel experiment, to study local flow, surface pressure coefficient, aerodynamic drag coefficient and lift coefficient of wheel under different conditions. The results show that the rotating wheel has a significant effect on the flow around the isolated wheel, and rotation reduces the differential pressure, drag coefficient and lift coefficient, thereby improving aerodynamic performance. Martins et al. [15] studied the aerodynamic interaction of three-element wing and wheels in ground effect by performing a 3D computational fluid dynamics analysis on a simplified quarter model of a Formula One racing car using a detached-eddy simulation approach. The results show that the wheel wake is influenced by flap configuration, and different flap configurations produce different up wash flow fields, resulting in a change in the separation point at the top of the tire. As the separation point moves back, the downwash generated in the central region of the wheel wake gradually increases, resulting in a shorter and longer combined wake.

Wang et al. [16] conducted an experimental study on a 2/5 scale vehicle equipped with 2/5 scale rotating wheels. The results show that the near wake of the wheel has a more local effect on the aerodynamic lift and drag of the car and the low-pressure region of the underbody has an effect on aerodynamic pressure. The rear wheel wake interacts with the car wake, exerting pressure conditions on the bottom of the body and affecting the drag of car. Zhang et al. [17] analyzed the influence of the tire profile on the aerodynamic characteristics of the vehicle, established a parametric model based on the tire size parameters, optimized the design parameters of the tire profile, and reduced the aerodynamic resistance. Zhou et al. [18] analyzed the influence of tire profile and tread pattern structure on tire aerodynamic performance, studied the load characteristics of different tire pattern structures, and revealed the difference between the flow characteristics and the flow field around the tire.
II. METHODOLOGY

A. CONTRIBUTION ANALYSIS METHOD

The contribution analysis method mainly uses the regression of DOEs to calculate the contribution, which is used for ranking the contribution of design variables to the performance target and screening the design variables in high dispersion or high nonlinearity analysis to reduce the computational cost and improve the efficiency of optimal design, and its analytical calculation steps are as follows.

Step 1: Normalize processing

A DOEs approach was used to obtain a sample of experiments between design variables and response characteristics. The design variables have different design spaces, contribution values also change in the design space, and the sample data input needs to be normalized using formula (1):

\[ x_i^* = \frac{x_i - \bar{x}}{\sigma} \]

where \( \bar{x} \) is the average value of the sample data; \( \sigma \) is the standard deviation; \( N \) is the total number of sample data; \( x_i \) is the original input and \( x_i^* \) is the normalized input.

Step 2: Contribution analysis

If there are \( k \) design variables \((x_1, x_2, \ldots, x_k)\), then any response characteristic can be expressed by a multiple regression model as:

\[ P(x_1, x_2, \ldots, x_k) = \mu + \sum_{i=1}^{k} Q_i(x_i) + \ldots + \sum_{i=2}^{k} \sum_{j=1}^{k-1} R_{ij}(x_i, x_j) + \varepsilon \]  

where \( P(x_1, x_2, \ldots, x_k) \) is any response characteristic; \( \sum_{i=1}^{k} Q_i(x_i) \) is the main effect of design variable; \( \sum_{i=2}^{k} \sum_{j=1}^{k-1} R_{ij}(x_i, x_j) \) is the crossover effect of any two design variables; \( \mu \) is a constant term and \( \varepsilon \) is the deviation.

The main effect of the design variable can be expressed by formula (3):

\[ \sum_{i=1}^{k} Q_i(x_i) = \sum_{i=1}^{k} \hat{\beta}_i x_i \]  

Therefore, the contribution of the design variables can be defined by formula (4):

\[ N_{ij} = \frac{100\hat{\beta}_i}{\sum_{i=1}^{k} |\hat{\beta}_i|} \quad i = 1, 2, \ldots, k \]
where $\hat{\beta}$ is calculated coefficient of design variable $x_i$ using the least squares estimation method and $N_x$ is the corresponding contribution of design variable $x_i$.

### B. GREY RELATIONAL ANALYSIS

GRA is a method that uses grey relational grade (GRG) to measure the degree of approximation between experimental sequence and ideal sequence. It is widely used in multi-objective and multi-decision optimization problems, and has comprehensive advantages in solving complex decision-making problems [25]. The specific steps of GRA are as follows:

#### Step 1: Data pre-processing

Due to the different orders of magnitude of experimental data, GRA may not be able to obtain reliable optimal solutions, so it is necessary to convert the experimental data into a set of dimensionless data between 0.00 and 1.00 for further quantitative analysis. According to the characteristics of response characteristics, different data pre-processing techniques can be used.

If the response characteristic has the characteristics of “bigger is better”, the normalization method can be expressed as:

$$
    x_i^*(k) = \frac{x_i(k) - \min_k x_i(k)}{\max_k x_i(k) - \min_k x_i(k)} \\
    \times (k = 1, 2, \cdots; i = 1, 2, \cdots m)  \tag{5}
$$

If the response characteristic has the characteristic of “lower is better”, the normalization method can be expressed as:

$$
    x_i^*(k) = \frac{\max_k x_i(k) - x_i(k)}{\max_k x_i(k) - \min_k x_i(k)} \\
    \times (k = 1, 2, \cdots; i = 1, 2, \cdots m)  \tag{6}
$$

If the response characteristic is ideal with respect to a specific value, the normalization method can be expressed as:

$$
    x_i^*(k) = 1 - \frac{|x_i(k) - T|}{\max |\max_k x_i(k) - T, T - \min_k x_i(k)|} \\
    \times (k = 1, 2, \cdots; i = 1, 2, \cdots m)  \tag{7}
$$

where $x_i^*(k)$ is the k-th response characteristic value of the i-th experiment after normalization; $x_i(k)$ is the initial design value of response characteristic; $\min_k x_i(k)$ and $\max_k x_i(k)$ is the minimum and maximum value of all response characteristics $x_i(k), k = 1, 2, \ldots, n; i = 1, 2, \ldots, m; m$ is the number of experiments; $n$ is the number of response characteristics and $T$ is the specific value.

#### Step 2: Calculate the grey relational coefficient (GRC)

The GRC of the k-th response characteristic of the i-th experiment is expressed as:

$$
    \gamma (x_i^0(k), x_i^*(k)) = \frac{\Delta_{\text{min}} + \xi \Delta_{\text{max}}}{\Delta_{0i}(k) + \xi \Delta_{\text{max}}}  \tag{8}
$$

where $x_i^0(k)$ is the reference experimental sequence; $x_i^*(k)$ is the initial experimental sequence; $\Delta_{0i}(k) = |x_i^0(k) - x_i^*(k)|$ is the absolute difference between $x_i^0(k)$ and $x_i^*(k)$; $\Delta_{\text{max}} = \max_i \max_k \Delta_{0i}(k)$ and $\Delta_{\text{min}} = \min_i \min_k \Delta_{0i}(k)$ are the maximum and minimum values of $\Delta_{0i}(k)$, respectively; $\xi$ is the distinguishing coefficient, $\xi \in [0, 1]$, which is generally defined as 0.5.

#### Step 3: Calculate the GRG

The GRG is calculated by averaging the GRC and expressed as:

$$
    \Gamma (x_i^0, x_i^*) = \frac{1}{n} \sum_{k=1}^{n} \gamma (x_i^0(k), x_i^*(k))  \tag{9}
$$

where $\Gamma$ is the GRG, and $n$ is the number of response characteristic.

According to formula (9), the GRG of each design scheme can be obtained. According to the size of GRG, each design scheme can be sorted to obtain the optimal scheme with comprehensive performance. Through calculation, the design scheme with the highest GRG value represents the scheme with the best comprehensive performance.

### C. ENTROPY WEIGHTED GREY RELATIONAL ANALYSIS (egra)

The entropy weight method is used to determine the weight of objective function. The higher entropy weight, the greater the weight of objective function in optimization process.

The mapping function $f_i: [0,1] \rightarrow [0,1]$ applied in the entropy must satisfy the following three conditions $f_i(0) = 0, f_i(x) = f_i(1-x)$, and $f_i(x)$ must be monotonically increasing in interval $x \in (0,0.5)$; Therefore, the function $w_e(x)$ is defined as the mapping function of the entropy weight method:

$$
    w_e(x) = xe^{1-x} + (1-x)e^x - 1  \tag{10}
$$

where $w_e(x)$ takes its maximum value at $x = 0.5$, that is, $w_e(0.5) = 0.6487$. At the same time, in order to ensure that the mapping function can take values within the range $[0,1]$, the following new entropy is defined:

$$
    W = \frac{1}{(e^{0.5} - 1)n} \sum_{i=1}^{n} w_e(x_i)  \tag{11}
$$

According to the above definition and the GRC, the steps for determining the weight of each objective function are as follows:

1. In all design schemes, the sum of the GRC corresponding to the k-th response characteristic:

$$
    T_k = \sum_{i=1}^{m} \gamma (x_i^0(k), x_i^*(k))  \tag{12}
$$

2. Normalize the coefficients:

$$
    N_e = \frac{1}{(e^{0.5} - 1)m} = \frac{1}{0.6487m}  \tag{13}
$$

3. The entropy of each response characteristic can be written as:

$$
    e_k = N_e \sum_{i=1}^{m} \left( \frac{\gamma (x_i^0(k), x_i^*(k))}{T_k} \right)  \tag{14}
$$
(4) The sum entropy can be expressed as:

\[ E = \sum_{k=1}^{n} e_k \]  

(5) The relative weight can be calculated as:

\[ \beta_k = \frac{1}{m - E} (1 - e_k) \]  

(6) The weight of the \( k \)-th response characteristics can be written as:

\[ \omega_k = \frac{\beta_k}{\sum_{i=1}^{n} \beta_i} \]  

Since the relative significance of each response characteristic may be different, the simple averaging method of formula (9) may lead to inaccurate evaluation of the GRG. Therefore, according to formula (17), the weight of each response characteristic can be obtained, and different weights are assigned to the response characteristic to carry out EGRA:

\[ \Gamma (x_0^*, x^*_k) = \sum_{k=1}^{n} \omega_k \gamma (x_0^*(k), x^*_k(k)) \]  

where \( \omega_k \) is the weight of the \( k \)-th response characteristic.

According to formula (18), EGRA sorting can be performed to obtain optimal solution for comprehensive performance. The design with the highest EGRA value represents the best overall performance.

III. CALCULATION MODELS AND MESHING

A. WHEEL MODEL

The separate design and processing of rim and disc can break through the limitations of traditional wheel manufacturing in terms of structural optimization design, mixed material application and advanced manufacturing technology, and improve the mechanical properties, manufacturing process and production efficiency of the wheel. Moreover, in the author’s previous research, due to the separate design and processing of the disc, the design space of the disc shape becomes larger, and the aerodynamic drag of the car can be reduced through the optimization of the disc shape design. This article takes the 16 × 6/2J assembled wheel as the research object, which is connected by bolts, as shown in Figure 1.

B. DRIVER STANDARD CAR MODEL

The car models based on aerodynamics research are mainly divided into two categories: one is simplified car models, such as SAE standard car models and Ahmed blunt-body car models, the other is mass-produced car models. The application scope of simplified model research results is limited and there is a certain gap with real car. However, due to the limited production cycle, high research cost, and difficulty in obtaining research and test data by the public, mass-produced cars also limit the use of researchers.

The Institute of Aerodynamics and Fluid Mechanics, which is affiliated to the Technical University of Munich, has launched a new car model, the DrivAer model, for the study of the outer flow field of car. Common modular standard components are provided for researchers to choose, which can be freely combined to form different configuration models according to needs: the chassis are divided into Detailed (D) and Smooth (S); the rearview mirror are divided into with Mirrors (wM) and without Mirrors (woM); wheels are divided into with Wheels (wW) and without Wheels (woW); ground conditions are divided into Ground Simulation (with GS) and without Ground Simulation (woGS). And all of them provide corresponding wind tunnel test data for researchers to use in CFD numerical simulation for validation.

In order to achieve the accuracy of CFD simulation analysis, it is necessary to adjust the calculation scheme of numerical simulation based on wind tunnel test data to determine the appropriate meshing strategy, boundary condition parameters and solution method [26], [27]. Thus, this article uses the DrivAer model and its wind tunnel test data to verify the correctness of the simulation model. Considering the research focus of wheel aerodynamic performance, the combination configuration of a smooth chassis, removing the rear-view mirror, keeping the wheels and mobile ground were selected [28]. As shown in Figure 2.

C. WIND TUNNEL MODEL

Theoretically, the scope of the virtual wind tunnel of the flow field outside the car should be infinite, but it is
unexpected in practical engineering, as long as the boundary of the virtual wind tunnel does not interfere with the flow field, and the incoming flow is approximated as no interference.

Create a rectangular computational domain, make 3 times the car length from the entrance of the computational domain, 7 times the car length from the computational domain exit, 5 times the car width from the left and right borders of the computational domain, and 5 times the height of the car from the top boundary of the computational domain. Then the blockage ratio of the virtual wind tunnel is 1.6%, and the computational domain basically eliminates the influence of the blockage effect of wind tunnel.

**D. MESHING STRATEGY**

Computational grids are the basis for spatial discretization of control equations, and the type and size of grids affect computational accuracy and computation time. According to the relationship among grid points, it can be divided into unstructured grids, structured grids and composite grids. The composite grid scheme is an ideal compromise scheme at present, which can give full play to the advantages of unstructured grids and structured grids, while avoiding the shortcomings of each. In this article, the composite mesh scheme of triangular prism mesh, tetrahedral mesh and hexahedral mesh is used.

1) **BOUNDARY LAYER MESHING**

The wall function is used to represent the distribution of physical quantities such as velocity, temperature, and turbulent energy in the boundary layer to solve the influence of the wall on the flow, and its wall equation can be a good correction of the turbulence model for the region affected by viscous forces. The wall function method needs to determine the height of the first layer grid node of the boundary layer [29], as shown in formula (19):

\[
y = \frac{y^+ \mu}{\sqrt{0.037 \rho^2 U_\infty^2 \left( \frac{\mu}{\rho U_\infty} \right)^{0.2}}}
\]

(19)

where \(y^+\) is the wall distance, \(\mu\) is the dynamic viscosity of air, \(\rho\) is the air density, \(U_\infty\) is the inflow velocity, \(u\) is the characteristic velocity of flow, \(L\) is the characteristic length of car.

When the test condition is set to 20 °C, \(\rho = 1.205 \text{ kg/m}^3\), \(\mu = 1.81 \times 10^{-5} \text{ kg/(m-s)}\), \(u = 30 \text{ m/s}\), \(L = 4.612 \text{ m}\), \(U_\infty = 30 \text{ m/s}\), \(y^+ = 35\). According to formula (19), \(y = 0.45 \text{ mm}\).

The triangular prism meshes parallel to the body and wheels were stretched to meet the wall function and accurately simulate the boundary layer flow on the car surface. Part of the boundary layer meshes of the body and wheel parts are shown in Figure 3, which has 3 layers of the grids and the growth rate is 1.2 [30].

2) **COMPUTATIONAL DOMAIN MESHING**

In the sensitive area of the car, the gradient of the flow field parameters is large, and mesh refinement is required. Therefore, the grid near the boundary of the computational domain can be divided sparsely, and the grid near the car needs to be refined, and three gradually encrypted density boxes were set up. The research focus of this article is the aerodynamic performance of wheel, so it is necessary to refine the meshes around the front, rear and wheels, and the mesh refinement strategy is shown in Figure 4. The flow field near the boundary of computational domain is divided by a hexahedral structured mesh, and the flow field near car outside the boundary layer is divided by a tetrahedral unstructured mesh by the center interpolation method [31]. The half-car model flow field has a total of 5.324 million calculation grids, as shown in Figure 5.

**IV. BOUNDARY CONDITIONS AND EXPERIMENTAL VERIFICATION**

**A. TURBULENCE MODEL**

The Reynolds-Averaged Navier-Stokes (RANS) method based on the time-averaged velocity field is the basic method used in turbulence calculation. The Standard \(k-\varepsilon\) model, RNG \(k-\varepsilon\) model and Realizable \(k-\varepsilon\) model are most widely used, especially for the aerodynamics analysis of
car which the turbulent Reynolds number region is large enough. In this article, the Realizable $k-\varepsilon$ model is selected, which effectively avoid distortion when simulating strong swirl or flow with curved walls, the convergence is greatly improved [32], [33].

$$\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho ku_i)}{\partial x_i} = \left[ \left( \mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k - \rho \varepsilon$$ \hspace{1cm} (20)

$$\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho u_i \varepsilon)}{\partial x_i} = \frac{\partial}{\partial x_j} \left[ \left( \mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + \rho C_1 \varepsilon - \rho C_2 \frac{\varepsilon^2}{k + \sqrt{\varepsilon}}$$ \hspace{1cm} (21)

where $k$ is turbulent kinetic energy, $\rho$ is air density, $\varepsilon$ is dissipation rate, $\mu_t$ is turbulent viscosity coefficient, $G_k$ is generation term of turbulent kinetic energy caused by the mean velocity gradient, $\sigma_k$ and $\sigma_\varepsilon$ are the Prandtl function, $C_1$ and $C_2$ are constants.

**B. NUMERICAL SOLUTION METHOD**

Assuming that the air flow is incompressible, steady-state numerical calculations are performed using a pressure-based solution using an uncoupled implicit method. The Realizable $k-\varepsilon$ model is used for the turbulent region, and the standard wall function method is used for the wall region. The numerical solution method settings are shown in Table 1.

**C. BOUNDARY CONDITIONS**

Numerical calculation needs to give boundary conditions on the boundary of a finite area, which not only satisfies the solution of mathematical equations, but also has clear physical meaning. When setting the ambient temperature $T = 293$ K, the boundary condition parameters of the computational domain are shown in Table 2.

In Table 2, $I$ is the turbulence intensity; $D$ is the hydraulic diameter; $u$ is the inlet wind speed and the speed of the car; $\omega$ is the wheel rotation angular velocity; and $p$ is the outlet pressure of the computational domain.

The roughness height and the roughness constant settings of each boundary are shown in Table 3.

**D. SIMULATION AND EXPERIMENTAL VERIFICATION**

1) **VERIFICATION OF $C_D$**

The test data of DrivAer in different combinations were provided by the Technical University of Munich. Figure 6 shows the $C_d$ in the wind tunnel test of DrivAer under different configurations.

Import the finite element model into ANSYS/FLUENT 14.0 for solution (Hardware: PowerEdge R7525; AMD 7763 2.45 GHz 64 C; 1.92TB NVME; NVIDIA Ampere A40 48GB GPU), and the simulated value of the $C_d$ is compared with the experimental value, as shown in Table 4.

The error between the simulation value and the test value is 0.40%, which shows that the set calculation scheme is correct and feasible.

2) **VERIFICATION OF SURFACE PRESSURE COEFFICIENT OF DrivAer MODEL**

In order to further verify the correctness of the CFD calculation scheme, the pressure coefficient $c_p$ distribution on...
the surface of DrivAer model is selected for simulation and experimental comparison verification. The $c_p$ distribution on the surface of the $z = 150$ mm plane, as shown in Figure 7(a) and 7(b). The $c_p$ distribution on the surface at top and bottom of the symmetry plane, as shown in Figure 7(c) and (d).

![FIGURE 7. Distribution of the $c_p$ on DrivAer.](image)

In this article, the pressure coefficients of two DrivAer configuration models (FSwMwW with GS and FSwMwW with GS) are calculated. The results show that the presence or absence of rear-view mirrors has little effect on the pressure coefficients of the $z = 150$ mm plane, symmetry plane and wheel of the car model, in figure 7(b)-(d). The comparison with the experimental value shows that the simulated value of the surface pressure coefficient distribution of the DrivAer model is in good agreement with the experimental value, the numerical simulation result is accurate.

**V. AERODYNAMIC ANALYSIS OF WHEEL**

**A. BRAKE DISC MODEL AND BOUNDARY CONDITIONS**

In order to study the influence of wheel structure on the $h_t$, it is necessary to add a brake disc to the DrivAer model. The gray cast iron (HT250) ventilated brake disc was selected, and other accessories such as brake calipers were omitted. The 3D model and main structural parameters are shown in Figure 8 and Table 5, respectively.

![FIGURE 8. 3D model of ventilated brake disc.](image)

The heat flux density of the brake disc is under the condition of constant speed braking on a long downhill [34].

**TABLE 5. Parameters of disc model.**

| Parameter name               | Parameter value |
|------------------------------|-----------------|
| Total surface area /mm$^2$   | 239800          |
| Outer diameter /mm           | 275             |
| Inner diameter /mm           | 170             |
| Thickness /mm                | 26              |
| Number of Fins /             | 45              |

The boundary conditions of the brake disc are set as shown in Table 6.

**TABLE 6. Boundary settings of brake disc.**

| Property                               | Value | Property                               | Value |
|----------------------------------------|-------|----------------------------------------|-------|
| Density $\rho$(kg/m$^3$)               | 7280  | Spin speed $\omega$(rad/s)            | 92    |
| Specific heat $c_p$/(J/(kg·K))         | 480   | area heat flux $q$/(W/m$^2$)          | $2.5\times10^4$ |
| Thermal conductivity $\lambda$(W/(m·K))| 45    | Roughness height $K$/mm              | 0.5   |
| Temperature $T$(K)                    | 293   | Roughness constant $C_t$             | 0.5   |

**B. FEM FOR ASSEMBLED WHEEL AERODYNAMIC ANALYSIS**

In the DrivAer model, the original wheel was replaced with the assembled wheel, and the above ventilated brake disc model was added, the 3D model of DrivAer after reconfiguration is shown in Figure 9(a). The FEM is shown in Figure 9(b).

![FIGURE 9. Reconfigured DrivAer model.](image)

The same meshing strategy as in Section III is used to divide the reconfigured DrivAer model and flow field, half-car flow field calculation grids is 5.669 million. The refined part and the finite element model of the flow field are shown in Figure 10 and Figure 11, respectively. The finite element model is set up using the same boundary conditions and solution method as in Section IV. Since heat transfer needs to be calculated, the energy equation is opened and the calculation is performed in ANSYS/FLUENT.

**C. $y^+$ VALIDATION OF THE FIRST LAYER GRID**

The $y^+$ value of the first layer grid in the near-wall region of body satisfies $30 < y^+ < 300$, in order to indicate that the numerical calculation can obtain good simulation results. Figure 12 shows the $y^+$ value distribution cloud map of reconfigured DrivAer boundary layer grid when the car speed is 30 m/s.

The $y^+$ value of the boundary layer grid on the surface of the car is distributed around 34.50, and this article estimates...
y^+ = 35.0 in formula (19). The numerical calculation of the flow field truly reflects the flow characteristics of wall boundary layer.

**D. FLOW FIELD CHARACTERISTICS ANALYSIS FOR CAR**

Replacing wheels and adding brake discs increases the $C_d$ of the DrivAer model, which is 0.2332. The distribution of each performance parameter of the car flow field is shown in Figure 13.

As shown in Figure 13(a), the airflow forms a positive pressure region at the front of the body and the front windshield, which constitutes the main $C_d$, and the positive pressure at the front of the body reaches 102.25 kPa.

The vortex system in the wheel cavity is most distributed, especially in the front wheel, which has a great influence on the car, which in turn affects the $C_d$ and the $h_c$. Meanwhile, the formation of negative pressure areas at the bottom and rear of the body consumes lots of energy, resulting in increased $C_d$.

As shown in Figure 13(b), the turbulent kinetic energy is large when the airflow passes through the front of the body, the windshield, the front of the bottom of the front wheels and the rear wheels, resulting in a lot of energy consumption. When the airflow passes through the front of the body, due to the pressure difference, the airflow enters the wheel cavity, resulting in turbulent kinetic energy increase and energy consumption.

**E. FLOW FIELD CHARACTERISTICS IN WHEEL CAVITY**

Due to the rotation of the wheel and the negative pressure, the airflow flowing into the wheel cavity will form strong turbulence or vortex core inside the wheel, which will affect the flow field near the car, dissipate energy and increase aerodynamic drag. The velocity vector of the flow field and the distribution of turbulence intensity at the wheel cavity are shown in Figure 14.

As shown in Figures 14(a) and (b), the airflow hits the front of the lower part of the wheel to form positive pressure and wheel cavity vortex, which increases the $C_d$. The airflow velocity at the front wheel is the largest, and it also has the greatest impact on the $C_d$ and the $h_c$.

As shown in Figures 14(c) and (d), the airflow forms a vortex core at front and rear wheels off the ground, and the vortex core expands into a vortex, which is separated in the direction of the car leaving. The maximum turbulence intensity at the front wheel is 23.62%, and the rear is 17.83%.

Because the front wheel is directly impacted by the incoming flow, the turbulent flow intensity is greater; while the turbulent flow intensity of the rear wheel is smaller, but it has a great impact on the flow field at the rear of the body; the tail of the body forms two wake vortices that revolve around its own vortex core, and a back flow phenomenon occurs. Therefore, the flow field near the wheel cavity can be changed by optimizing the design of the disc structure to reduce the $C_d$.

**F. TEMPERATURE AND $H_C$ ANALYSIS FOR BRAKE DISC**

The high-speed rotating wheel causes the change of the flow field near the wheel cavity, and the change of the flow field determines the $h_c$. The cloud map of temperature and $h_c$ of front and rear brake discs are shown in Figure 15; the average temperature and average surface convective heat transfer coefficient of brake disc ($h_a$) are shown in Table 7.
As shown in Figure 15, the $h_c$ at the bottom and the surface near the wheel is better than other parts, which is related to the turbulence intensity of the flow field near the disc.

As shown in Table 7, the average temperature of the front disc is 20.82% lower than that of the rear disc, and the $h_a$ of the front disc is 23.18% higher than that of the rear disc.

The above analysis shows that the greater the airflow velocity and turbulence intensity of the flow field in the wheel cavity, the better the $h_c$, but it will cause the increase of the $C_d$.

To sum up, the reduction of the $C_d$ and the improvement of the $h_c$ are in conflict. By optimizing the wheel structure, especially the disc structure, the impact of the airflow on the side of the body on the rotating disc can be reduced, thereby reducing the $C_d$, but it will cause the reduction of the $h_c$. Therefore, it is necessary to study the aerodynamic performance of wheels with different disc structures to explore the influence rules and compromise points of these two performances.

### TABLE 7. Average temperature and $h_c$ comparison of brake discs.

|                | T/K   | $h_c$(W/(m$^2$K)) |
|----------------|-------|--------------------|
| Front brake disc | 658.22 | 76.80              |
| Rear brake disc  | 831.26 | 62.35              |

### G. AERODYNAMIC PERFORMANCE ANALYSIS FOR WHEELS WITH DIFFERENT DISC

To further study the influence law of wheel structure on the $C_d$ and the $h_c$, different disc structures were designed and analyzed to study the aerodynamic performance of the wheel.

By changing the wheel disc structure, 18 different wheel models were designed and divided into 3 groups for research. The first group of wheel models has the same disc opening area, but different number of spokes: keep the disc opening area of 29927 mm$^2$ unchanged, and design 5-spokes, 6-spokes, 7-spokes, 8-spokes, 9-spokes and 10-spokes wheels. The second group of wheel models has the same disc opening area, but different spoke styles: keep the disc opening area of 29927 mm$^2$ unchanged, and design common 6 different spoke style wheels. The third group of wheel models has the same number of spokes, but different disc opening area: the number of spokes is designed to be 5, and the spoke widths are 100 mm, 85 mm, 70 mm, 55 mm, 40 mm and 25 mm wheels. Three groups of wheel models with different disc structures are shown in Figure 16.

Keep the brake disc unchanged in the reconfigured Dri/vAer model, replace wheels with different spoke structures, adopt the same CFD analysis preprocessing method and calculation scheme as in Section III and Section IV. Using ANSYS/FLUENT for numerical calculation, the $C_d$ and the $h_a$ are obtained (the average value of front and rear brake discs is taken). Combined with GRA and EGRA in Section II, the aerodynamic performance of wheels with different disc structures is ranked, as shown in Table 8.

As shown in the first group of wheel models in Table 8, the 6-spokes wheel has the smallest $C_d$, and the 5-spokes wheel has the largest $h_a$. The opening area of disc of each wheel is the same, the air flow entering the wheel cavity, vortex formation and turbulence intensity are basically close, so the $C_d$ does not change significantly due to the change of the number of spokes. However, wheels with fewer spokes have larger holes between individual spokes, which is conducive to the concentration of airflow into the wheel cavity and enhances the heat convection effect of brake disc. As the number of spokes increases, the $h_a$ decreases.

Based on the GRA ranking, the 5-spokes wheel has the highest comprehensive performance index of 0.8571; based on the EGRA ranking, the 5-spokes wheel has the highest comprehensive performance index of 0.8795. Both methods
can give an optimal solution with a high comprehensive performance index based on the conflicting the $C_d$ and the $h_a$.

After sorting and comparing the two methods, it is pointed out that the 5-spokes wheel has high comprehensive performance, which avoids the blindness of subjective selection. The results show that when the opening area of disc is the same, reducing the number of spoke is beneficial to reduction of the $C_d$ and the improvement of the comprehensive aerodynamic performance of wheel.

As shown in the second group of wheel models in Table 8, the wheel with the f-style disc has smallest $C_d$, and the wheel with the a-style disc has the largest $h_a$. The opening area of disc of each wheel is the same, and the wheel with a large closed area at the top of spoke connected to the rim is not conducive to the airflow into the wheel cavity, the formation of the vortex system and the increase of the turbulent intensity, so the $C_d$ is small, but it also leads to the increase of the $h_a$.

Based on the GRA ranking, the comprehensive performance index of wheel with the f-style disc is the highest of 0.6796; based on the EGRA ranking, the comprehensive performance index of wheel with the f-style disc is the highest of 0.7483. Both methods give an optimal solution with a high comprehensive performance index, after sorting and comparison, both methods point out that the wheel with f-style disc has higher comprehensive performance, which avoids the blindness of subjective selection. The results show that when the opening area of the spokes is the same, increasing the closed area of the top of the spokes connected to the rim is beneficial to the reduction of the $C_d$, but is not conducive to the improvement of the $h_a$. 

### Table 8. Aerodynamic performance parameters of wheels with different disc structures.

| First group      | 5-spokes | 6-spokes | 7-spokes | 8-spokes | 9-spokes | 10-spokes |
|------------------|----------|----------|----------|----------|----------|-----------|
| $C_d$            | 0.24251  | 0.24250  | 0.24253  | 0.24252  | 0.24254  | 0.24255   |
| $h_a/(W/(m^2\cdot K))$ | 73.67     | 71.29    | 72.08    | 69.52    | 69.03    | 68.51     |
| (GRG)            | 0.8571   | 0.7601   | 0.5366   | 0.4695   | 0.3710   | 0.3333    |
| (EGRA)           | 0.8795   | 0.7225   | 0.5495   | 0.4560   | 0.3688   | 0.3333    |
| Second group     |          |          |          |          |          |           |
| $C_d$            | 0.2425   | 0.2391   | 0.2433   | 0.2421   | 0.2385   | 0.2331    |
| $h_a/(W/(m^2\cdot K))$ | 76.86     | 73.12    | 74.65    | 67.35    | 73.82    | 68.38     |
| (GRG)            | 0.6759   | 0.5096   | 0.5080   | 0.3475   | 0.5479   | 0.6796    |
| (EGRA)           | 0.6064   | 0.4989   | 0.4706   | 0.3506   | 0.5345   | 0.7483    |
| Third group      |          |          |          |          |          |           |
| $C_d$            | 0.2298   | 0.2305   | 0.2335   | 0.24253  | 0.2405   | 0.2319    |
| $h_a/(W/(m^2\cdot K))$ | 62.68     | 69.21    | 71.26    | 75.58    | 77.82    | 78.24     |
| (GRG)            | 0.6667   | 0.682    | 0.5797   | 0.5393   | 0.6609   | 0.8760    |
| (EGRA)           | 0.7062   | 0.7078   | 0.5860   | 0.5150   | 0.6268   | 0.8613    |
As shown in the third group of wheel models in Table 8, the 5-spokes wheel with a 100 mm spoke width has the smallest \( C_d \), and the 5-spokes wheel with a 25 mm spoke width has the largest \( h_a \). Under the premise that the 5-spokes remain unchanged, as the width of the spoke is smaller, the opening area of spoke is larger, and the airflow entering the wheel cavity, the vortex system formed and the turbulence intensity increase, which increases the \( C_d \) and the \( h_a \).

Based on the GRA ranking, the comprehensive performance index of the 5-spoke wheel with a 25 mm spoke width is the highest of 0.8760; based on the EGRA ranking, the comprehensive performance index of the 5-spoke wheel with a 25 mm spoke width is the highest at 0.8613. Both methods give an optimal solution with a high comprehensive performance index, after sorting and comparison, both methods point out that a 5-spokes wheel with a 25 mm spoke width has a high comprehensive performance, which avoiding the blindness of subjective selection. The results show that with the same number of spoke, increasing the spoke width is beneficial to the reduction of the \( C_d \), reducing the spoke width is beneficial to the increase of the \( h_a \), and the improvement of the comprehensive aerodynamic performance of wheel depends on the compromise between the two performances.

To sum up, reducing the \( C_d \) and improving the \( h_a \) are not in conflict in all cases. Under certain conditions, the optimization of the two performances can be sought. To a certain extent, the GRA and EGRA methods used in this article can select the optimal solution combining the two performances, which provides a method basis for the wheel optimization design and scheme selection in the following.

VI. MULTI-OBJECTIVE OPTIMIZATION MODEL FOR ASSEMBLED WHEEL

A. OPTIMIZE THE DESIGN PROCESS

In order to comprehensively consider and meet the requirements of various performance index of the wheel, and optimize the most important objectives as much as possible, based on the previous aerodynamic performance research of the wheel, the parametric modeling technology and the approximation model method are used, and combined with the multi-objective optimization method, the lightweight multi-objective optimization of the wheel is carried out [35]. Under the condition of meeting the requirements of each performance index of wheel, the optimal lightweight and aerodynamic performance of the wheel is achieved. The flowchart of wheel lightweight multi-objective optimization is shown in Figure 17.

B. PARAMETRIC MODELING

In order to improve the deformation ability and design space of the wheel and increase the sampling space of the experimental design, the wheel model needs to be parametrically designed. Based on the advanced mesh deformation technology, the parameterization of wheel structure is realized by the translation and scaling of the finite element model control nodes, so as to construct the parameterized variables of the wheel model. The assembled wheel parametric model in this article defines 21 design variables, denoted as DV1, DV2,…, DV21, as shown in Figure 18. Table 9 gives the description, initial value and range of each design variable.

C. EXPERIMENT DESIGN AND CONTRIBUTION ANALYSIS

Due to the complex structure of the wheel and the large number of initial design variables, in order to improve the efficiency of multi-objective optimization of the assembled wheel, the initial design variables should be screened in combination with the contribution analysis. Based on the initial value and range of design variables in Table 9, the optimal Latin hypercube design (OLHD) was adopted, and 100 samples were selected to analyze the contribution of 21 initial design variables of the assembled wheel. Figure 19 shows the contribution values of 21 design variables to the performance of the wheel. In the contribution analysis graph, a positive bar value indicates a positive correlation, and a negative bar value indicates a negative correlation.

According to the ranking of contribution results, 12 initial design variables with greater influence are retained, namely DV1, DV2, DV3, DV4, DV5, DV6, DV7, DV8, DV16, DV17, DV18, DV21, and the 12 variables are represented as \( x_1 \sim x_{12} \) in sequence. The description, initial value and range of each design variable are shown in Table 9.

D. APPROXIMATE MODEL AND ACCURACY VERIFICATION

The relationship between design variables and performance index can be obtained using approximate model. The optimization based on the approximate model method, combined with the multi-objective optimization algorithm, can realize multi-objective optimization. Kriging and RBF surrogate
FIGURE 18. Schematic diagram of the design variables of the assembled wheel.

TABLE 9. Initial value and range of design variables.

| Design variable | Variable description             | Initial value | Lower bound | Upper bound |
|-----------------|---------------------------------|---------------|-------------|-------------|
| DV1/mm          | Width of bottom spokes          | 68.0          | 61.0        | 75.0        |
| DV2/mm          | Width of top spokes             | 65.0          | 60.0        | 70.0        |
| DV3/mm          | Thickness of bottom spokes      | 18.5          | 13.5        | 23.5        |
| DV4/mm          | Arc radius of front spokes      | 150.0         | 135.0       | 165.0       |
| DV5/mm          | Arc radius of rear spokes       | 156.3         | 140.7       | 171.9       |
| DV6/mm          | Thickness of top spokes         | 11.5          | 8.5         | 14.5        |
| DV7/mm          | Width of spoke slots            | 12.0          | 0.0         | 17.0        |
| DV8/mm          | Length of spoke slots           | 44.0          | 40.0        | 48.0        |
| DV9/mm          | Height of spoke slots boss flat | 3.5           | 3.5         | 3.5         |
| DV10/mm         | Width of root spoke grooves     | 45.0          | 35.0        | 50.0        |
| DV11/mm         | Width of top spoke grooves      | 45.0          | 35.0        | 50.0        |
| DV12/mm         | Length of spoke grooves         | 70.0          | 65.0        | 75.0        |
| DV13/mm         | Depth of spoke grooves          | 3.5           | 0.0         | 7.0         |
| DV14/°          | Angle of hub grooves            | 30.0          | 25.0        | 40.0        |
| DV15/mm         | Depth of hub grooves            | 29.0          | 23.0        | 35.0        |
| DV16/mm         | Thickness of hub                | 48.0          | 36.0        | 52.0        |
| DV17/mm         | Thickness of disc ring          | 8.0           | 6.0         | 10.0        |
| DV18/mm         | Thickness of rim ring           | 8.0           | 6.0         | 10.0        |
| DV19/mm         | Inside thickness of rim         | 6.0           | 5.0         | 7.0         |
| DV20/mm         | Outside thickness of rim        | 5.5           | 4.5         | 5.5         |
| DV21/mm         | Middle thickness of rim         | 4.5           | 3.5         | 5.5         |

FIGURE 19. Contribution analysis of design variables.

FIGURE 20. Approximate model accuracy for each performance index.

models are easy to obtain ideal fitting results when solving nonlinear problems, and have stronger predictive ability than single-parameter models, and are widely used in engineering.

Based on the DOE method, the OLHD is used to sample in the design variable space, and a total of 200 sample points are selected to fit the Kriging and RBF surrogate models of each performance index. Within the range of the design variables, another 20 sample points were selected by central composite design, and the cross-validation method was used to verify the accuracy of the Kriging and RBF surrogate models [36], [37]. The approximate model accuracy verification results of the performance index are shown in Figure 20.

Use the coefficient of determination ($R^2$), root mean square error (RMSE) and Mean Absolute Percentage Error (MAPE) to evaluate the accuracy of the approximate model. If the $R^2$ value is closer to 1 and the RMSE value is closer to 0, it indicates that the overall prediction accuracy of approximate model is higher, and when the MAPE value is smaller, it indicates that the local prediction accuracy of approximate model is higher. Table 10 shows the specific index values of the accuracy test results of each approximate model.
TABLE 10. Accuracy verification results of each approximate model.

| Performance response | Kriging surrogate model | RBF surrogate model |
|----------------------|-------------------------|---------------------|
|                      | $R^2$    | RMSE  | MAPE   | $R^2$  | RMSE  | MAPE   |
| $M$/kg               | 0.98114 | 0.04139 | 9.412% | 0.9802 | 0.04736 | 9.361% |
| $C_d$                | 0.99181 | 0.02144 | 8.131% | 0.9972 | 0.01305 | 2.393% |
| $h_a$/($W/(m^2*K)$) | 0.99066 | 0.0239  | 5.787% | 0.9909 | 0.02232 | 8.223% |

By comparing the Kriging surrogate model and the RBF surrogate model, the prediction accuracy of the Kriging surrogate model is higher than that of the RBF surrogate model in terms of wheel mass, the RBF surrogate model has higher accuracy in terms of nonlinear responses such as the $C_d$ and the $h_a$. And the RBF surrogate model has smaller RMSE and MAPE, which better balances the global prediction and local prediction ability of the approximate model. Therefore, after comprehensive consideration, the RBF surrogate model is selected as the approximate model of multi-objective optimization in this article.

E. OPTIMIZING MATHEMATICAL MODEL

The reduction of the $C_d$ is the most important evaluation index in the analysis of aerodynamic performance of wheel, so the $C_d$ is selected as the objective function; The $h_a$ can quantitatively reflect the relationship between the aerodynamic performance of wheel and the $h_a$. Therefore, the $h_a$ is selected as the objective function; The lightweight of wheel has a great influence on the driving performance of car, so the mass of assembled wheel is selected as the objective function. The design variables and range in Table 9 are selected as constraints, and the range of design variables ensure that the wheel can pass tests of structural strength, stiffness, fatigue and impact performance. The mathematical model of the lightweight multi-objective optimization of the assembled wheel can be expressed as:

$$
\begin{align*}
\min & \quad (M(x), C_d(x), -h_a(x)) \\
\text{s.t.} & \quad x \in (x_L, x_U)
\end{align*}
$$

where $M(x)$ is the wheel mass in multi-objective optimization, kg; $C_d(x)$ is the aerodynamic drag coefficient of car; $h_a(x)$ is the average surface heat transfer coefficient of brake disc; $x$ is the design variable, $x_L$ and $x_U$ are the lower and upper limits of design variable, respectively.

VII. OPTIMIZATION RESULTS AND VALIDATION

A. MULTI-OBJECTIVE OPTIMIZATION RESULTS

In the multi-objective optimization platform Isight, based on the constructed RBF surrogate model, the NSGA-II optimization algorithm is used to carry out the lightweight multi-objective optimization design of the assembled wheel. The optimization platform is shown in Figure 21.

The group size of the NSGA-II optimization algorithm is set to 40, the generation is 100, and the crossover probability is 0.90. After 4000 iterations, the Pareto frontier consisting of 269 non-dominated optimal solutions is obtained, as shown in Figure 22.

B. SORTING OF PARETO FRONTIER

It can be seen from Pareto that the performance of $M$, $C_d$, and $h_a$ is difficult to achieve optimal at the same time, so different compromise solutions need to be selected according to different optimization requirements. The objective function data corresponding to the 269 non-dominated optimal solutions in Pareto front are normalized, and then the GRC of each non-dominated optimal solution are calculated. Using the entropy weight method, the weights of three objective functions of $M$, $C_d$, and $h_a$ can be obtained as 0.2077, 0.5121 and 0.2803, respectively. According to the weight of each objective function and the GRC of each non-dominated optimal solution, EGRA sorting can be performed.

Figure 23 shows the Pareto frontier GRG obtained by using EGRA, and the non-dominated optimal solution with the largest GRG is selected as scheme A. Therefore, the 232nd non-dominated optimal solution with the largest GRG
of 0.7181 is considered as the design solution with the best comprehensive performance.

![FIGURE 23. GRG of pareto frontier based on EGRA.](image)

In order to verify the effectiveness and feasibility of EGRA in the process of selecting the optimal compromise optimization scheme, this article also uses TOPSIS method [38] to select the set of solutions with the largest correlation degree from 269 non-dominated optimal solutions as scheme B, and the non-dominated optimal solution with the smallest lightweight coefficient obtained within the Isight platform as scheme C. Among them, when using the TOPSIS method to rank the Pareto fronts, in order to ensure the consistency of each condition, each objective function still adopts the weight obtained by the entropy weight method.

![FIGURE 24. Pareto frontier correlation obtained by TOPSIS method.](image)

Figure 24 shows the Pareto frontier correlation degree obtained by the TOPSIS method, in which the 183rd non-dominated optimal solution has the largest correlation degree of 0.6851, so scheme B is the 183rd non-dominated optimal solution obtained by the TOPSIS method.

Table 11 shows the performance comparison among optimization schemes A, B, and C and all three schemes can satisfy the performance requirements of the wheel. The optimization scheme A has a great improvement on the $C_d$, and the improvement rate is 5.02%; the wheel weight is improved by 10.83%, second only to the scheme C; and the $h_a$ decreases the most, which is 8.02%. Although the optimization schemes B and C are better than the scheme A in terms of the $h_a$, but they are obviously insufficient in the improvement of the $C_d$. Therefore, the optimization scheme A has better comprehensive performance, and the improvement rate among its performances is relatively balanced. Compared with the TOPSIS method, the EGRA is simple and flexible in operation, and can provide guidance in the process of selecting the optimal compromise solution.

Table 12 shows the comparison between the optimization results of design variables of scheme A and the initial values before optimization.

According to Scheme A in Table 11, the mass of the assembled wheel after multi-objective optimization is 5.524 kg, which is 10.83% lower than the initial value; the $C_d$ is 0.2215, which is 5.02% lower than the initial value; the $h_a$ is 64.00, which is 8.02% lower than the initial value. The mass of a 16 × 61.2 J cast aluminum alloy wheel on the market is 8.213 kg. Compared with this wheel, the weight of the assembled wheel after multi-objective optimization is reduced by 2.689 kg, which is 32.74%.

As shown in Table 12, the optimized value of width of spoke grooves ($x_7$) is 4.54, which basically does not interfere with air flow due to its smaller width, which can reduce the $C_d$, but also weakens the $h_a$. The width and thickness of the bottom spokes and the thickness of hub are reduced, while the optimized assembled wheels use magnesium alloy rims, which contribute significantly to the reduction of wheel mass. Ultimately, the material and structural changes resulted in a multi-objective optimized assembled wheel with improved overall performance.

![FIGURE 25. Turbulence intensity cloud map of the assembled wheel cavity after optimization.](image)

C. PERFORMANCE VERIFICATION BEFORE AND AFTER OPTIMIZATION

The change of the disc structure of assembled wheel after optimization affects the flow field of wheel cavity and car, and then affects the $C_d$ and the $h_a$. Therefore, it is necessary to compare and analyze the aerodynamic performance of the optimized front and rear assembled wheels. In the reconfigured DrivAer model in Section V.B, the pre-optimized assembled wheel is replaced with an optimized assembled wheel, and the same calculation scheme as in Section V is used for simulation analysis.

As shown in Figure 25, the maximum turbulent intensities of the X-direction section of front and rear wheel cavity of optimized assembled wheel are 20.65% and 15.81%; Before optimization, they were 23.62% and 17.83% respectively,
which were both lower than those before optimization. As shown in Figure 26 and Table 13, after optimization, the local and average temperatures of front and rear brake discs both increased, and the $h_c$ decreased significantly.

After optimization, the width and thickness of the bottom spokes of assembled wheel was decreased, which reduces the impact of the rotating disc on the airflow and the interference of the outer flow field of car; at the same time, the spoke slot becomes smaller, which increases the closed area of the disc, especially the closed area close to the rim, reduces the air flow entering the wheel cavity, and weakens the interference of multiple airflows, the generation of vortices and the intensity of turbulence. These structural changes reduce aerodynamic drag, but also result in a reduction in the convective heat transfer performance of brake disc.

### VIII. CONCLUSION

This article proposes a multi-objective optimization design method for wheel lightweight based on EGRA. The aerodynamic analysis finite element model of the assembled wheel was established, and the simulation accuracy was verified by experiments. The distribution law of performance parameters such as pressure and turbulent kinetic energy of car flow field is studied, and the variation law of the flow field velocity and turbulent flow intensity at front and rear wheel cavity of assembled wheel was analyzed. The influence of wheels with different disc structures on the $C_d$ and the $h_c$ was researched, and an objective evaluation of the comprehensive aerodynamic performance of wheels with different disc structures was given. Using the approximate model method, the lightweight multi-objective optimization based on the aerodynamic performance of assembled wheel was carried out, and the lightweight effect and the $C_d$ are significantly reduced.

1. Based on the computational fluid dynamics method, the finite element model of the aerodynamic analysis of assembled wheel was established. Using the DrivAer standard model of the Technical University of Munich and its wind tunnel test data, the meshing strategy, boundary conditions and solution method of CFD simulation analysis were determined, and the simulation accuracy of the model was verified by experiments. The results show that the simulation values are in good agreement with the experimental values, the numerical simulation results are accurate, and the virtual wind tunnel can accurately simulate the flow field of DrivAer model.

2. Analyzed the distribution changes of performance parameters such as pressure and turbulent kinetic energy of
car flow field, discussed the change law of the flow field velocity and turbulent intensity of front and rear wheel cavities of assembled wheel, and demonstrated the temperature and $h_c$ cloud map distribution of front and rear brake discs. The results show that the greater the pressure difference and turbulent kinetic energy of the car flow field, the airflow velocity and turbulence intensity of the flow field in the wheel cavity, the better the $h_g$ and the greater the $C_d$.

3. The wheels with different number of spokes, different spoke styles and different spoke widths are designed for CFD simulation calculation, and the influence mechanism of wheel disc structure on the $C_d$ and the $h_c$ was studied. Combined with GRA and EGRA, the objective evaluation of the comprehensive aerodynamic performance of wheels with different disc structures was given. It is concluded that the 5-spokes wheel, the f-spoke style and the 5-spokes wheel with a spoke width of 25mm have better comprehensive performance. The results show that reducing the $C_d$ and increasing the $h_c$ can find the optimal solution that combines the two performance, which provides a method basis for wheel optimization design and scheme selection.

4. Combined with experimental design, 12 important design variables were screened out by contribution analysis method. The approximate model method is used to fit the Kriging surrogate model and the RBF surrogate model. It is found that the RBF surrogate model has higher accuracy in terms of nonlinear responses such as $C_d$ and $h_c$, and better balance the global and local prediction ability of the approximate model. A hybrid method combining EGRA and NSGA-II is proposed for lightweight multi-objective optimization of assembled wheels. Comparing and analyzing the optimization platform recommending scheme, the TOPSIS method preferring scheme and the EGRA method optimum scheme, it was found that the optimal compromise scheme was obtained by the EGRA method, the reduction of the $C_d$ was more obvious, which is 5.02%, and the improvement rates of performance were also more balanced. The feasibility of the sorting method was verified, the blindness of the optimal solution selection was avoided, and the objective evaluation method of the multi-objective optimization design results was established.

5. The mass of the assembled wheel after multi-objective optimization was reduced by 10.83%; the $C_d$ was reduced by 5.02%, and $h_g$ was reduced by 8.02%. Compared with a $16 \times 6^{1/2}$ J type cast aluminum alloy wheel on the market, the wheel has been optimized by multiple objectives to reduce the weight by 32.74%. The maximum turbulence intensity of front and rear wheel cavity of the assembled wheel after optimization is lower than that before optimization, the local and average temperature of front and rear brake discs are both increased, and the $h_g$ is significantly reduced.

6. Further research should focus on the synergistic multidisciplinary multi-objective optimal design of wheel structural strength design and aerodynamic performance, which will result in more conflicting optimization solutions, which makes the EGRA introduced in this article more important in decision making. We can expect the joint improvement of wheel lightweight design and aerodynamic performance from future papers.

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