Penrose Limits of Branes and Marginal Intersecting Branes

Shijong Ryang

Department of Physics
Kyoto Prefectural University of Medicine
Taishogun, Kyoto 603-8334 Japan
ryang@koto.kpu-m.ac.jp

Abstract

We construct the Penrose limit backgrounds in closed forms along the generic null geodesics for the near-horizon geometries of D1, D3, D5, NS1 and NS5 branes. The Penrose limit metrics of D1, D5 and NS1 have non-trivial dependence of the light-cone time coordinate, while those of D3 and NS5 have no its dependence. We study the Penrose limits on the marginal 1/4 supersymmetric configurations of standard intersecting branes, such as the NS-NS intersection of NS1 and NS5, the R-R intersections of Dp and Dq over some spatial dimensions and the mix intersections of NS5 and Dp over (p -1)-dimensional spaces. They are classified into three types that correspond to the Penrose limits of D1, D3 and D5 backgrounds.
1 Introduction

The Penrose limit \[1, 2\] on the $AdS_5 \times S^5$ solution of type IIB theory has been shown to yield an interesting pp-wave background with maximal supersymmetry \[3, 4\]. The string theory in this background is exactly solvable for the spectrum of oscillators \[5, 6\] so that we can explicitly observe the duality in a particular sector of the four-dimensional $\mathcal{N} = 4$ SYM gauge theory with the type IIB string theory in the pp-wave background beyond the supergravity approximation \[7\]. Moreover, the D-branes on the pp-wave background have been explored \[8\].

According to the three kinds of classifications of null geodesics such as longitudinal, radial and generic ones for a supergravity brane solution, there are three corresponding Penrose limits \[1\]. The Penrose limit along the longitudinal null geodesic yields the trivial flat Minkowski spacetime, while the limit along the radial one is performed to construct the Penrose limit metrics for the backgrounds such as Dp-brane, fundamental string, NS5-brane, M-brane and so on. The near-horizon geometries of D3-brane, M2-brane and M5-brane have themselves AdS structure and turn out to be the pp-wave backgrounds in the Penrose limit along the generic null geodesic. Both M2-brane and M5-brane configurations have the same Penrose limit metric, where there is an isometric symmetry between two configurations. The standard intersecting systems such as three M2-branes transversely intersecting over a 0-brane, two M2-branes and two M5-branes transversely intersecting over a 0-brane have been investigated and the Penrose limit metrics along the generic null geodesics on their near-horizon geometries that include AdS structure have been shown to include the metric of the Cahen-Wallach space. The Penrose limits along the generic null geodesics have been studied for the supersymmetric black holes in four and five dimensions and the supersymmetric string in six dimensions, whose near-horizon geometries also include AdS structure, and shown to consist of the Cahen-Wallach space.

The other pp-wave background has been found by taking the Penrose limit on the near-horizon geometry of a standard intersecting system that two D3-branes transversely intersect over a string, which is also expressed by $AdS_3 \times S^3 \times T^4$ \[9\]. Another pp-wave solution has been constructed by performing the Penrose limit along the generic null geodesic on a non-standard intersection of two NS5-branes over a string, where the harmonic function for each brane component depends on the coordinates of the relative transverse space rather those of the overall transverse space \[10, 11\]. Various types of non-standard intersecting systems whose near-horizon geometries are product spaces including $AdS_p \times S^q$ have been also demonstrated to yield the pp-wave solutions in the Penrose limit \[10\].

The Penrose limit on the near-horizon geometry of NS5-brane has been taken to yield the pp-wave background that is dual to a high energy sector of the little string theory \[12\]. For the geometries of Dp-branes and their near-horizon limits the Penrose limits along the generic null geodesics have been studied \[13, 14\], where the Penrose limits of metric, dilaton and $(p+2)$-form field strength are expressed in terms of the radial coordinate that is related implicitly with a light-cone time coordinate.

Following the framework of Ref. \[4\] we will try to construct the Penrose limit backgrounds explicitly in terms of the light-cone time coordinate itself for the near-horizon geometries of Dp-branes with $p = 1, 3, 5$. Similarly the Penrose limits of the near-horizon geometries...
of NS1-brane and NS5-brane will be derived in closed forms and compared with those of
Dp-branes. Further we will demonstrate the various Penrose limits of the marginal BPS
configurations expressed by the standard intersecting branes such as the intersection of an
NS1-brane and an NS5-brane over a string, that of a Dp-brane and a Dq-brane over a n-
brane with $n = (p + q)/2 - 2$, $n = 0, 1, 2$ and that of an NS5-brane and a Dp-brane over a
$(p - 1)$-brane. Comparing the Penrose limits of these various intersecting branes with those
of Dp-branes with $p = 1, 3, 5$. we will classify the Penrose limits of these marginal BPS bound
states into three types.

2 Penrose limit along the generic null geodesic

We start to review the relevant aspects of the Penrose limit in Ref. [4]. We consider the
Penrose limit of a ten-dimensional background with a metric
\[
\begin{align*}
    ds^2 &= A^2(-dt^2 + ds^2(E^p)) + B^2dr^2 + B^2r^2(d\psi^2 + \sin^2\psi d\Omega^2_{7-p}),
\end{align*}
\]
where the metric on $S^{8-p}$ has been written out as $d\Omega^2_{8-p} = d\psi^2 + \sin^2\psi d\Omega^2_{7-p}$. We choose
a null geodesic to lie in the $(t, r, \psi)$ plane and perform the coordinate transformation from
$(t, r, \psi)$ to $(u, v, \tilde{z})$
\[
\begin{align*}
    u &= u(r), \\
v &= t + l\psi + a(r), \\
\tilde{z} &= \psi + b(r),
\end{align*}
\]
where $u$ is the affine parameter along the generic null geodesic and $l$ is constant. If $a(r), u(r)$
and $b(r)$ are specified by $da/dr = (B^2/A^2 - l^2/r^2)^{1/2}$,
\[
\begin{align*}
    u &= \int^r \frac{B^2dr}{\sqrt{\frac{B^2}{A^2} - \frac{l^2}{r^2}}} \\
\end{align*}
\]
and
\[
\begin{align*}
    b &= -\int^r \frac{l/r^2 dr}{\sqrt{\frac{l^2}{A^2} - \frac{l^2}{r^2}}} \\
\end{align*}
\]
and the Penrose limit is taken along this null geodesic after the rescaling of coordinates, then
the metric is expressed in terms of Rosen coordinates as
\[
\begin{align*}
    ds^2 &= 2du dv + (B^2r^2 - l^2/A^2)d\tilde{z}^2 + A^2\sum_{a=1}^p(d\tilde{x}^a)^2 + B^2r^2\sin^2\psi \sum_{i=1}^{7-p}(d\tilde{y}^i)^2.
\end{align*}
\]
This is the metric of spacetime in the neighbourhood of the generic null geodesic in the
specific Penrose scaling limit. For the Dp-brane solution characterized by a harmonic function
$H = C^{-1} = 1 + Q_p/r^{7-p}$ with the Dp-brane charge $Q_p$, the R-R $(p + 2)$-form field strength
$F_{p+2} = dvol(E^{1,p}) \wedge dC(r)$ becomes in the Penrose limit to be
\[
\begin{align*}
    \tilde{F}_{p+2} &= C' \frac{l}{B} \frac{1}{A^2} - \frac{l^2}{B^2r^2} du \wedge d\tilde{x}^1 \wedge \cdots \wedge d\tilde{x}^p \wedge d\tilde{z},
\end{align*}
\]
where \( A^{-2} = B^2 = H^{1/2} \). Using the following change of coordinates

\[
\begin{align*}
    u &= x^- , \\
    v &= x^+ + \frac{\partial_- A}{2A} x^2 + \frac{\partial_-(rB \sin b)}{2rB \sin b} y^2 + \frac{\partial_- \sqrt{B^2 r^2 - l^2 A^2}}{2\sqrt{B^2 r^2 - l^2 A^2}} z^2 , \\
    \tilde{x}^a &= \frac{x^a}{A} , \\
    \tilde{y}^i &= \frac{y^i}{rB \sin b} , \\
    \tilde{z} &= \frac{z}{\sqrt{B^2 r^2 - l^2 A^2}} ,
\end{align*}
\]

(7)

where \( \partial_- = d/dx^- , A = A(r(x^-)), B = B(r(x^-)) , r = r(x^-) \) that is implicitly given by inversing the Eq. (3), we can transform the metric (5) in the Rosen form into in the Brinkman form

\[
ds^2 = 2dx^- dx^- + A(dx^-)^2 + \sum_{a=1}^p (dx^a)^2 + \sum_{i=1}^{7-p} (dy^i)^2 + dz^2 ,
\]

(8)

where

\[
A = \frac{\partial^2 A}{A} x^2 + \frac{\partial^2 (rB \sin b)}{rB \sin b} y^2 + \frac{\partial^2 \sqrt{B^2 r^2 - l^2 A^2}}{\sqrt{B^2 r^2 - l^2 A^2}} z^2 .
\]

(9)

\section{Penrose limits of elementary branes}

We are ready to consider the near-horizon geometry of D3-brane configuration characterized by \( A^{-2} = B^2 = \sqrt{Q_3}/r^2 \) for the metric (1). The original radial coordinate is explicitly determined from (3) as

\[
r = \frac{\sqrt{Q_3}}{l} \sin \left( \frac{l}{\sqrt{Q_3}} u \right) ,
\]

(10)

where we have simply chosen an integration constant in such a way as \( u \) becomes zero at \( r = 0 \). This solution yields a restriction \( Q_3/l^2 \geq r^2 \). Since \( B^2 r^2 - l^2 A^2 = \sqrt{Q_3} \cos^2(lu/\sqrt{Q_3}) \) and \( b = -lu/\sqrt{Q_3} \) where \( b \) in (3) is also chosen to be zero at \( r = 0 \), the non-trivial factor \( \mathcal{A} \) (4) in the metric is calculated by

\[
\mathcal{A} = -\frac{l^2}{Q_3} \left( \sum_{a=1}^3 (x^a)^2 + \sum_{i=1}^4 (dy^i)^2 + z^2 \right) ,
\]

(11)

which is negative definite. Even if we take account of the integration constants as \( r = (\sqrt{Q_3}/l) \sin(lu/\sqrt{Q_3} + C_1) , b = -lu/\sqrt{Q_3} + C_2 \) , we get the same result. Applying the coordinate transformation (7) to the R-R field strength (6) for the near-horizon geometry of Dp-brane with \( H = Q_p/r^{7-p} \) we have

\[
\tilde{F}_{p+2} = C' \frac{l}{rB^2 A^{p+1}} dx^- \wedge dx^1 \wedge \cdots dx^p \wedge dz - \sum_{a=1}^3 (a^a)^2 + \sum_{i=1}^4 (dy^i)^2 + dz.
\]

(12)

The dilaton is simply given by

\[
e^{2\phi} = \left( \frac{Q_p}{r^{7-p}} \right)^{\frac{3-p}{2}} .
\]

(13)
The near-horizon geometry of D3-brane is special since the coefficient in (12) becomes a constant $4l/\sqrt{Q_5}$, where we have to add the Hodge dual for the $p=3$ case. This background gives the solution of the maximally supersymmetric pp-wave in the type IIB theory. When $l$ is taken to be zero, both the 5-form field strength and the factor $\mathcal{A}$ vanish. Consequently the pp-wave background reduces to the ten-dimensional Minkowski spacetime that is the Penrose limit on the near-horizon geometry of D3-brane along the radial null geodesic.

Now we consider the near-horizon limit of the D5-brane metric, that is (1) with $A^2 = B^{-2} = r/\sqrt{Q_5}$. From (3) $r$ is expressed in terms of $u$ as

$$r = \sqrt{\frac{Q_5 - l^2}{Q_5}}u, \quad (14)$$

where an integration constant is chosen such that $u = 0$ at $r = 0$, and $l$ is bounded as $l^2 < Q_5$. The integration in (4) leads to

$$b = -\frac{l}{\sqrt{Q_5 - l^2}} \ln \frac{r}{r_1} \quad (15)$$

with an integration constant $r_1$. Substituting (14) and (15) into (4) we can derive

$$\mathcal{A} = -\frac{1}{4(x^-)^2} \left[ \sum_{a=1}^{5} (x^a)^2 + z^2 + \left( 1 + \frac{4l^2}{Q_5 - l^2} \right) \sum_{i=1}^{2} (y^i)^2 \right], \quad (16)$$

which is negative definite due to $Q_5 > l^2$. It is noted that the result is independent of the integration constant $r_1$ for (4). If we choose the other integration constant for $u = u(r)$ (3), the resulting expression of $\mathcal{A}$ simply shows the shift of $x^-$. Therefore any integration constant for (3) can be absorbed into the definition of the light-cone time coordinate. From (12) the R-R 7-form field strength is specified by the non-constant value

$$\frac{2lQ_5}{Q_5 - l^2} \frac{1}{(x^-)^2}, \quad (17)$$

while the dilaton in (13) is also a function of $x^-$

$$e^{2\phi} = \frac{Q_5 - l^2}{Q_5^2} (x^-)^2. \quad (18)$$

For the near-horizon geometry of D1-brane specified by $A^2 = B^{-2} = r^3/\sqrt{Q_1}$, we have to manipulate the integration

$$u = \int_0^r dr \frac{1}{\sqrt{1 - \frac{r^2}{Q_1} r^4}}, \quad (19)$$

where an integration constant is fixed as $u = 0$ at $r = 0$. We restrict the region of $r$ to $Q_1/l^2 \geq r^4$ where the null geodesic line is well defined, and then the Eq. (19) can be described as

$$-\frac{\sqrt{2l}}{Q_1^{1/4}} u + K \left( \frac{1}{\sqrt{2}} \right) = F \left( \cos^{-1} \left( \frac{\sqrt{7}}{Q_1^{1/4}} r \right), \frac{1}{\sqrt{2}} \right), \quad (20)$$
where \( K(1/\sqrt{2}) \) is complete elliptic integral of the first kind and \( F(\phi, 1/\sqrt{2}) \) is incomplete one. It is possible to invert this equation as

\[
\sqrt{1 - \frac{l^2}{Q_1} r^2} = \text{sn} \left( w, \frac{1}{\sqrt{2}} \right),
\]

(21)

where \( w = -\sqrt{2l} u/Q_1^{1/4} + K \) and \( \text{sn}z \) is one of Jacobi’s elliptic functions with a property \( \text{sn}K = 1 \), that yields \( u = 0 \) at \( r = 0 \). The original radial coordinate \( r \) is explicitly described in terms of \( u \) as

\[
r = \frac{Q_1^{1/4}}{\sqrt{l}} \text{cn} w,
\]

(22)

which indeed satisfies \( Q_1/l^2 \geq r^4 \) and correctly reduces to \( r = u \) in the \( l \to 0 \) limit. The radial coordinate \( r \) is the periodic function of \( u \), which is similar to the D3-brane case. Choosing an integration constant in (4) such that \( b \) vanishes at \( r = 0 \), we have

\[
\sin b = -\frac{1}{\sqrt{2}} \left( 1 - \sqrt{1 - \frac{l^2}{Q_1} r^4} \right)^{1/2},
\]

(23)

Using this expression and \( dr/du = \sqrt{2} \text{sn}w \text{dn}w \) that reduces to \( (1 - l^2 r^4/Q_1)^{1/2} \) through (21), which is also given from (19), we compute the factor \( \mathcal{A} \) as a function of \( r \)

\[
\mathcal{A} = -\frac{3}{4r^2 Q_1}[(5l^2 r^4 - Q_1)(x_1^2 + z^2) + (l^2 r^4 - Q_1) \sum_{i=1}^{6} (y_i)^2],
\]

(24)

which is further expressed in terms of \( x^- \) as

\[
\mathcal{A} = -\frac{3l}{4\sqrt{Q_1} \text{cn}^2 w}[(5c^4 w - 1)(x_1^2 + z^2) + (c^4 w - 1) \sum_{i=1}^{6} (y_i)^2]
\]

(25)

with \( w = -\sqrt{2l} x^-/Q_1^{1/4} + K \). The factor \( \mathcal{A} \) is singular at \( x^- = 0 \) due to \( \text{cn}K = 0 \). The R-R 3-form field strength and the dilaton in the Penrose limit are given by

\[
\tilde{F}_{x^-x^1x^2} = \frac{6}{l} \text{cn}^4 w, \quad e^{2\phi} = \frac{l^3}{\sqrt{Q_1} \text{cn}^6 w},
\]

(26)

which show the non-trivial \( x^- \) dependences.

Here we describe the general expression of the Penrose limit metric for the near-horizon geometry of Dp-brane in terms of the original radial coordinate. The non-trivial component of the metric (9) is evaluated as

\[
\mathcal{A} = -\frac{7-p}{16r^2} \left[ \left( (p - 3) + \frac{13 - 3p}{Q_p} l^2 r^{5-p} \right) \left( \sum_{a=1}^{p} (x^a)^2 + z^2 \right) + \left( (p - 3) + \frac{p + 1}{Q_p} l^2 r^{5-p} \right) \sum_{i=1}^{7-p} (y_i)^2 \right],
\]

(27)
where we have used \( du/dr = 1/\sqrt{1 - l^2 r^5 / Q_p} \), \( db/dr = -l/\sqrt{Q_p r^{p-3} - l^2 r^2} \) and we have observed that the \( \cos b \) terms in \( \partial^2 u(rB \sin b) \) cancel out. In Ref. [13] the same implicit expression of the factor \( A \) for the Dp-brane was presented and expressed in terms of a variable that is defined to be proportional to \( r^{(p-5)/2} \) for \( p \neq 5 \). Recently the same expression described in terms of \( r \) itself has been presented in Ref. [14]. For \( p = 3 \) and \( p = 5 \) the factor \( A \) in (27) turns out to be fairly simplified. For \( p = 1 \) it reduces to (24). In the \( l \rightarrow 0 \) limit \( u \) is equal to \( r \), that is seen in (10), (14), (22), and the factor \( A \) becomes a symmetric expression

\[
A = -\frac{(7 - p)(p - 3)}{16(x^-)^2} \left( \sum_{a=1}^{p} (x^a)^2 + z^2 + \sum_{i=1}^{7-p} (y^i)^2 \right),
\]

(28)

which gives the Penrose limit metric of the near-horizon geometry of Dp-brane along the radial null geodesic. Thus we have performed the explicit integration in \( u = u(r) \) and derived its inversion \( r = r(u) \) so that the factor \( A \) together with the dilaton and the R-R field strength has been expressed in terms of the Brinkman coordinate \( x^- \) for the D1, D3 and D5 configurations. It is noted that there is a common structure between (16) the factor \( A \) of D5-brane and (25) the factor \( A \) of D1-brane that each \( A \) is singular at \( x^- = 0 \), which is obliged to \( 1/r^2 \) factor in (27). On the other hand there is a reciprocal structure that \( \bar{F}_3 \) and \( e^{2\phi} \) in (26) for the D1-brane vanishes and diverges at \( x^- = 0 \) respectively, while \( \bar{F}_7 \) in (17) and \( e^{2\phi} \) in (18) for the D5-brane diverges and vanishes respectively. This reciprocal behavior of the \( x^- \) dependence can be read from (12) and (13), where \( p = 3 \) is the critical number.

Now we turn our attention to the near-horizon geometry of NS5-brane. This geometry is so specified by \( A = 1 \), \( B = Q_{5NS}^5/r^2 \) with the charge of NS5-brane \( Q_{5NS}^5 \) in (1) that we have

\[
r = C_1 e^{\sqrt{Q_{NS}^5 - l^2 u}}, \quad b = -\frac{l}{Q_{NS}^5}(u + C_2)
\]

(29)

with the integration constants \( C_1, C_2 \). The \( u \) dependence of \( b \) is the same as the D3-brane case. Combining these expressions leads to the metric in the Penrose limit characterized by

\[
A = -\frac{l^2}{(Q_{5NS}^5)^2} \sum_{i=1}^{2} (y^i)^2,
\]

(30)

where there are no \( x^2 \) and \( z^2 \) terms. This result agrees with the expression presented in Ref. [12]. The \( x^- \) dependence as well as the \( C_1 \) and \( C_2 \) dependences do not appear in the same way as the previous D3-brane case, where \( rB = constant \) in both cases.

For the near-horizon geometry of the fundamental string specified by \( B = 1 \), \( A^2 = H^{-1} \), \( H = Q_{1NS}^1/r^6 \) with the charge of NS1-brane \( Q_{1NS}^1 \), we can describe \( r \) explicitly in terms of \( u \) by performing the integration in (3) as

\[
u = \sqrt{Q_{1NS}^1 - \sqrt{Q_{1NS}^1 - l^2 r^4}}
\]

(31)

and inverting it

\[
r = \frac{1}{l} \left[ (Q_{1NS}^1 - (2l^2 u - \sqrt{Q_{1NS}^1})^2)^{1/4},
\]

(32)
which vanishes at \( u = 0 \). The null geodesic line is well defined for \( Q_{1}^{NS}/l^2 \geq r^4 \geq 0 \). Through (31) this region is mapped to \( 0 \leq u \leq \sqrt{Q_{1}^{NS}/2l^2} \). On the other hand by carrying out the integration in (4) we have the same expression as the D1-brane case

\[
\sin 2b = -\frac{l r^2}{\sqrt{Q_{1}^{NS}}},
\]

where an integration constant is also chosen as \( b = 0 \) at \( r = 0 \). Similarly to the D1-brane case we can extract \( \sin b \) in the same expression as (23). The substitution of (32) into (23) with \( Q_{1}^{NS} \) yields

\[
\sin b = -lr\sqrt{\frac{Q_{1}^{NS}}{2l^2}},
\]

(33)

Gathering together we derive the factor \( \mathcal{A} \) in the Penrose limit

\[
\mathcal{A} = -\frac{3l^4}{(Q_{1}^{NS} - (2l^2x^- - \sqrt{Q_{1}^{NS}})^2)^2}\left[ (2Q_{1}^{NS} - (2l^2x^- - \sqrt{Q_{1}^{NS}})^2)(x_1^2 + z^2) + Q_{1}^{NS} \sum_{i=1}^{6}(y_i)^2 \right],
\]

(34)

for \( 0 \leq x^- \leq \sqrt{Q_{1}^{NS}/2l^2} \), which again shows the coincidence of coefficients of the \( x_1^2 \) and \( z^2 \) terms and the singular behavior at \( x^- = 0 \). Here we can rewrite (34) in terms of \( r = (Q_{1}^{NS} - (2l^2x^- - \sqrt{Q_{1}^{NS}})^2)^{1/4}/\sqrt{l} \) as

\[
\mathcal{A} = -\frac{3}{r^8}\left[ (Q_{1}^{NS} + r^4) (x_1^2 + z^2) + Q_{1}^{NS} \bar{y}^2 \right],
\]

(35)

whose \( 1/r^8 \) behavior is compared with \( 1/r^2 \) in (27) for the Dp-branes. This expression in terms of \( r \) agrees with the result in Ref. [14].

The Penrose limit for the NS-NS 3-form field strength \( F_{3}^{NS} = dt \wedge d\tilde{x}_1 \wedge dH^{-1} \) and the dilaton leads to

\[
\bar{F}_{x^-x_1z}^{NS} = \frac{6l^2}{[Q_{1}^{NS} - (2l^2x^- - \sqrt{Q_{1}^{NS}})^2]^{1/2}},
\]

\[
e^{-2\phi} = \frac{Q_{1}^{NS} l^3}{[Q_{1}^{NS} - (2l^2x^- - \sqrt{Q_{1}^{NS}})^2]^{3/2}}.
\]

(36)

When \( l \) is taken to be zero, we have

\[
\mathcal{A} = -\frac{3}{16(x^-)^2}(x_1^2 + z^2 + \bar{y}^2), \quad e^{-2\phi} = \frac{(Q_{1}^{NS})^{1/4}}{8}(x^-)^{3/2}, \quad \bar{F}_{x^-x_1z}^{NS} = 0,
\]

(37)

which reproduce the Penrose limit of the near-horizon geometry of NS1-brane along the radial null geodesic [4].

4 Penrose limits of intersecting branes

Let us consider the Penrose limits on the near-horizon geometries of intersecting brane configurations along the generic null geodesics whose tangent vectors have a component tangent
to the overall transverse sphere. There are the following standard intersections representing
the marginal 1/4 supersymmetric bound states: (a) NS-NS intersections: NS1||NS5 (the
internal dimensions of an NS1-brane and an NS5-brane are parallel); (b) R-R intersections:
Dp ⊥ Dq (n), n = (p + q)/2 − 2 (a Dp-brane overlaps a Dq-brane in a n-dimensional space);
(c) mixed intersections: NS5 ⊥ Dp (n), n = p − 1 [15, 16].

We first consider the near-horizon limit of a fundamental string smeared over a solitonic
NS5-brane with metric

$$ds^2 = H_1^{-1}(-dt^2 + dx_1^2) + \sum_{a=2}^{5}(dx^a)^2 + H_5^{NS}[dr^2 + r^2(d\psi^2 + \sin^2\psi d\Omega_2^2)]$$  (38)

with $$H_1^{NS} = Q_1^{NS}/r^2, H_5^{NS} = Q_5^{NS}/r^2$$. Since the $$u$$ in (3) and the $$b$$ in (4) show the same
behaviors as the D3-brane case, we derive the Penrose limit metric through the appropriate
coordinate transformations

$$ds^2 = 2dx^+dx^- + A(dx^-)^2 + \sum_{a=1}^{5}(dx^a)^2 + \sum_{i=1}^{2}(dy^i)^2 + dz^2,$$  (39)

where

$$A = -\frac{l^2}{(Q_5^{NS})^2}(x_1^2 + z^2 + \sum_{i=1}^{2}(y^i)^2)$$  (40)

and the $$Q_1^{NS}$$ dependence does not emerge. It is observed that this $$x^-$$-independent metric
describes a lorentzian symmetric or Cahen-Wallach space. The constant $$l^2/(Q_5^{NS})^2$$ can be
absorbed into a boost of $$(x^+, x^-)$$.

For the R-R intersections with $$n = 1$$, D5||D1, D4⊥D2(1), D3⊥D3(1), we see that $$u$$ and $$b$$ are also identical to those of the D3-brane case and evaluate the factor $$A$$ of the Penrose
limit metric to be the same form as (40)

$$A = -\frac{l^2}{k}(x_1^2 + z^2 + \sum_{i=1}^{2}(y^i)^2),$$  (41)

where $$k = Q_5Q_1, Q_4Q_2$$ and $$Q_3Q'_3$$ respectively, and $$x_1$$ is the coordinate of the common spatial
direction of two intersecting D-branes. In the starting metric the coefficients of $$\sum_{a=2}^{5}(dx^a)^2$$
for the internal coordinates of the Dp-brane and the Dq-brane are constant and characterized
by the ratio $$Q_p/Q_q$$ so that there are no $$x_a$$ terms for $$a = 2, \cdots 5$$ in $$A$$. The result (41) for the
D3 ⊥ D3 (1) configuration is in agreement with the Penrose limit metric studied in Ref. [9].

The near-horizon geometries of these R-R intersections are represented by $$AdS_3 \times S^3 \times T^4$$
in the same way as the near-horizon geometry of the NS1||NS5 intersection, whose $$AdS_3 \times S^3$$
part becomes the six-dimensional Cahen-Wallach space in the Penrose limit along the generic
null geodesic. For the other $$n = 0$$ case, D4||D0, D3||D1, D2||D2 the integrations in $$u(r)$$
and $$b(r)$$ are the same as those for D1-brane case and then the factor $$A$$ can be expressed as

$$A = -\frac{3l}{4kcn^2w}[(5cn^4w - 1)z^2 + (cn^4w - 1)\sum_{i=1}^{3}(y^i)^2]$$  (42)

9
with \( w = -\sqrt{2l}x^-/\sqrt{k} + K \), where \( k = \sqrt{Q_4Q_6}, \sqrt{Q_5Q_1} \) and \( \sqrt{Q_2Q_3} \) respectively. There is another \( n = 2 \) case where D6∥D2, D5⊥D3(2) and D4∥D4(2) configurations are the threshold BPS bound states. In this case the overall transverse space is three-dimensional so that \( u \) is linear to \( r \) and \( b \) is a logarithmic function of \( r \) in the same form as the D5-brane case [14], [13]. The factor \( A \) in the Penrose limit metric is similarly provided by

\[
A = -\frac{1}{4(x^-)^2} \left[ \sum_{i=1}^{2} (x^i)^2 + z^2 + \left( 1 + \frac{4l^2}{k - l^2} \right) y^2 \right],
\]

where \( k = Q_6Q_2, Q_5Q_3 \) and \( Q_4Q_4 \) respectively and the Dp-brane and the Dq-brane orthogonally overlap in the \( x^1 \) and \( x^2 \) directions. In the \( n = 3 \) case the overall transverse space of D5⊥D5(3), D4⊥D6(3) is two-dimensional. Since the relevant harmonic function is expressed by a logarithmic function, we cannot explicitly perform the integration in \( u(r), b(r) \) and then it is impossible to obtain the Penrose limit metric in a closed form.

The Penrose limits of the R-R field strength and the dilaton will be demonstrated for two examples, D2⊥D2 and D4⊥D4(2). For the D2⊥D2 configuration the R-R 4-form

\[
F_4 = dt \wedge (dx^1 \wedge dx^2 \wedge dH_2^{-1} + dx^3 \wedge dx^4 \wedge dH_4^{-1})
\]

is scaled in the Penrose limit to be

\[
\bar{F}_4 = \frac{3cn^{5/2}w}{l^{1/4}(Q_2Q_4)^{1/2}} dx^- \wedge (dx^1 \wedge dx^2 + dx^3 \wedge dx^4) \wedge dz
\]

by taking account of the appropriate constant transformations of \( x^i (i = 1, \cdots 4) \) coordinates. The dilaton specified by \( e^{-2\phi} = (H_2H_2)^{-1/2} = r^3/\sqrt{Q_2Q_2} \) turns out to be a \( x^- \)-dependent function \( e^{-2\phi} = (Q_2Q_2)^{1/4}cn^3w/l^{3/2} \), which shows some similar behavior to the D-1brane case, that is, the square root of [20] accompanied with \( \phi \to 2\phi, Q_1 \to Q_2Q_2 \). Similarly we consider the near-horizon geometry of D4⊥D4(2). The R-R 6-form field strength is represented by

\[
F_6 = dt \wedge dx^1 \wedge dx^2 \wedge (dx^3 \wedge dx^4 \wedge dH_4^{-1} + dx^5 \wedge dx^6 \wedge dH_4^{-1}),
\]

where one intersecting D4-brane extends to \((x^1, x^2, x^3, x^4)\) directions and the other intersecting D4-brane to \((x^1, x^3, x^5, x^6)\) directions. In the Penrose limit it is given by the \( x^- \)-dependent expression

\[
\bar{F}_6 = \frac{l^{1/4}Q_4}{(Q_4Q_4 - l^2)^{3/4}(x^-)^{3/2}} dx^- \wedge dx^1 \wedge dx^2 \wedge (dx^3 \wedge dx^4 + dx^5 \wedge dx^6) \wedge dz.
\]

The dilaton described by \( e^{-2\phi} = (H_4H_4)^{1/2} \) becomes in the Penrose limit to be specified by \( e^{-2\phi} = Q_4Q_4/((x^-)^3 \sqrt{Q_4Q_4 - l^2}) \) which is similar to the D5-brane case, that is, the square root of [18] accompanied with \( \phi \to 2\phi, Q_5 \to Q_4Q_4 \).

Now let us consider the remaining mix intersection, NS5⊥Dp(p − 1) for \( p = 1, \cdots 6 \) with a metric

\[
ds^2 = H_p^{-1/2}[-dt^2 + \sum_{i=1}^{p-1} (dx^i)^2 + H_5^NS (dx^p)^2] + H_p^{1/2} \left[ \sum_{i=p+1}^{6} (dx^i)^2 + H_5^NS (dr^2 + r^2d\Omega_2^2) \right]
\]
with $H_p = Q_p/r, H_5^{NS} = Q_5^{NS}/r$, where the overall transverse space is three-dimensional. From the observation that $u$ is proportional to $\sqrt{r}$ and $b$ is characterized by $\ln r$, whose logarithmic behavior is the same as the D5-brane case, we obtain the Penrose limit metric specified by

$$A = -\frac{1}{4(x^-)^2}(\sum_{i=1}^{p-1}(x^i)^2 + z^2 + \left(1 + \frac{16l^2}{Q_pQ_5^{NS} - l^2}\right)y_1^2 - 3\sum_{i=p}^6(x^i)^2). \quad (49)$$

This factor with the characteristic $x^-$-dependence is similar to that for the D5-brane case, however with a slight difference between the coefficients of $y_1^2$ here and $\sum_{i=1}^2(y_i)^2$ in (16), which is caused by the different behaviors of $u$ as $u \propto r$ and $u \propto \sqrt{r}$.

5 Conclusion

We have constructed the Penrose limit metrics in closed forms for the near-horizon geometries of the D1, D3, D5, NS1 and NS5 branes by carrying out explicitly the integration in the relation that defines the affine parameter $u$ along the generic null geodesic in terms of the radial coordinate $r$, and extracting its inverse relation with an analytic expression. Specially it is observed that the radial coordinate is a periodic function of $u$ for the D1 and D3 branes. The Penrose limit metrics for the D3 and NS5 backgrounds have no dependence of the light-cone time coordinate $x^-$, while those for the D1, D5 and NS1 backgrounds have its dependence and show a common structure that they have a singular behavior at $x^- = 0$.

We have found that the Penrose limits for the near-horizon geometries of the marginal 1/4 supersymmetric bound states consisting of two standard intersecting branes, are classified into three families that are represented by the D1, D3 and D5 types. The marginal intersecting system of $D_p \perp D(4-p)$ with $p = 0, 1, 2$ shows the same Penrose limit metric as the D1-brane type, whereas those of $D_p \perp D(8-p)(2)$ with $p = 2, 3, 4$ and NS5$\perp Dp(p-1)$ with $p = 1, \ldots, 6$ give the Penrose limit metrics similar to the D5-brane type. The other marginal configurations of $D_p \perp D(6-p)(1)$ with $p = 1, 2, 3$ and NS1$\parallel$NS5 are so special as to have the same Penrose limit metrics of Cahen-Wallach form as the D3-brane type. The overall transverse spaces of these special configurations are four-dimensional so that each harmonic function behaves as $r^{-2}$ with the radial coordinate $r$. Its square $r^{-4}$ for the $D_p \perp D(6-p)(1)$ configuration is associated with the behavior of harmonic function in the D3-brane configuration. Similarly the five-dimensional and three-dimensional overall transverse spaces of the $D_p \perp D(4-p)$ and $D_p \perp D(8-p)(2)$ respectively yield the $r^{-3}$ and $r^{-1}$ behaviors to each harmonic function, whose squares are the behaviors of harmonic functions in the corresponding D1 and D5 configurations. From these viewpoints, since the backgrounds of the D2, D4 and D6 branes have the harmonic functions with odd powers in $r^{-1}$, they are not associated with the intersecting systems of two D-branes in the Penrose limit. But the standard intersecting system of three D2-branes, $D2 \perp D2 \perp D2$ that is a marginal 1/8 supersymmetric bound state, has three-dimensional overall transverse space to give a $r^{-1}$ behavior for each harmonic function, whose cube indicates the harmonic function for the D4-brane configuration. Therefore the Penrose limit metric of $D2 \perp D2 \perp D2$ background is the same as that of the D4 background. It would be interesting to construct the Penrose
limits in closed forms on the various non-marginal intersecting systems of two branes or the various non-standard intersecting systems and investigate how they are classified.

References

[1] Penrose, *Any space-time has a plane wave as a limit*, Differential geometry and relativity, Reidel, Dordrecht (1976) pp. 271-275.

[2] R. Güven, *Plane wave limits and T-duality*, Phys.Lett. B482 (2000) 255, [hep-th/0003061](#).

[3] M. Blau, J. Figueroa-O’Farrill, C. Hull and G. Papadopoulos, *Penrose limits and maximal supersymmetry*, Class. Quant. Grav. 19 (2002) L87, [hep-th/0201081](#). A new maximally supersymmetric background of IIB superstring theory, JHEP 0201 (2001) 047, [hep-th/0110242](#).

[4] M. Blau, J. Figueroa-O’Farrill and G. Papadopoulos, *Penrose limits, supergravity and brane dynamics*, [hep-th/0202111](#).

[5] R.R. Metsaev, *Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, Nucl. Phys. B625 (2002) 70, [hep-th/0112044](#).

[6] R.R. Metsaev and A.A. Tseytlin, *Exactly solvable model of superstring in plane wave Ramond-Ramond background*, Phys.Rev. D65 (2002) 126004, [hep-th/0202109](#).

[7] D. Berenstein, J. Maldacena and H. Nastase, *Strings in flat space and pp waves from $\mathcal{N} = 4$ super Yang Mills*, JHEP 0204 (2002) 013, [hep-th/0202021](#).

[8] A. Dabholkar and S. Parvizi, *Dp branes in pp-wave background*, Nucl.Phys. B641 (2002) 223, [hep-th/0203231](#); D. Bak, *Supersymmetric branes in pp wave background*, [hep-th/0204033](#); K. Skenderis and M. Taylor, *Branes in AdS and pp-wave spacetimes*, JHEP 0206 (2002) 025, [hep-th/0204054](#); P. Bain, P. Meessen and M. Zamaklar, *Supergravity solutions for D-branes in Hpp-wave backgrounds*, [hep-th/0205106](#); M. Alishahiha and A. Kumar, *D-brane solutions from new isometries of pp-waves*, Phys.Lett. B542 (2002) 130, [hep-th/0205134](#); S. Seki, *D5-brane in Anti-de Sitter space and Penrose limit*, [hep-th/0205266](#); D.M. Mateos and S. Ng, *Penrose limits of the baryonic D5-brane*, JHEP 0208 (2002) 005, [hep-th/0205291](#).

[9] M. Cvetič, H. Lü and C.N. Pope, *Penrose limits, pp-waves and deformed M2-branes*, [hep-th/0203082](#).

[10] H. Lü and J.F. Vázquez-Poritz, *Penrose limits of non-standard brane intersections*, [hep-th/0204001](#).

[11] A. Kumar, R.R. Nayak and Sanjay, *D-brane solutions in pp-wave background*, [hep-th/0204025](#).
[12] V.E. Hubeny, M. Rangamani and E. Verlinde, *Penrose limits and non-local theories*, hep-th/0205258.

[13] E. Gimon, L.A. Pando Zayas and J. Sonnenschein, *Penrose limits and RG flows*, hep-th/0206033.

[14] H. Fuji, K. Ito and Y. Sekino, *Penrose limit and string theories on various brane backgrounds*, hep-th/0209004.

[15] A.A. Tseytlin, *Composite BPS configurations of p-branes in 10 and 11 dimensions*, hep-th/9702163.

[16] G. Papadopoulos and P.K. Townsend, *Intersecting M-branes*, Phys.Lett. **B380** (1996) 273, hep-th/9603087; A.A. Tseytlin, *Harmonic superpositions of M-branes*, Nucl.Phys. **B475** (1996) 149, hep-th/9604035; J.P. Gauntlett, D.A. Kaster and J. Traschen, *Overlapping branes in M-theory*, Nucl.Phys. **B478** (1996) 544, hep-th/9604179.