Learning trajectory of three dimensions’ topic through analytical geometry approach

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Abstract. The purpose of this research is to create a learning trajectory of the distance, angle, and angular measure concept in three dimensions. The underlying problem of this research was the discovery of learning obstacles in using the learning trajectory which was built in the Euclidean geometry approach that already existed, such as the lack of students’ spatial sense, teachers’ capability, and learning activities’ habit. The method used in this research is a qualitative research, particularly prospective analysis part of the didactical design research through observational learning, literary studies of topics’ related material, and interview. Based on the results of research and discussion, the vector topic learning activity was too emphasized on the algebraic form, so the learning trajectory that was recently compiled through analytic geometry approach, required an emphasis on exploration of the vector geometric representation.

The new learning trajectory, i.e. through analytical geometry approach, that has been formed can be useful to address the problem of learning trajectory before. For future research, we consider to develop a didactical design with this learning trajectory as the main learning framework.

1. Introduction

Mathematics is one of the subjects in school which is considered quite difficult for students. The topic of three dimensions is part of mathematics which proved difficult for high school students [1]. Based on some research results, the low result of learning all topics that related to three-dimensional objects, caused by the lack of student’s spatial sense [1–4]. Low learning outcomes from this topic is indeed caused by the lack of student’s spatial sense, but on the other hand, this lack is caused by their prerequisites knowledge of geometry postulate, theorem, and concept, including congruence and similarity, and trigonometry [1, 5]. Problems related to low results in learning this topic that caused by the lack of student’s spatial sense and their lack in prerequisites knowledge has always been the main problems that become a learning obstacle of this topic in Indonesia.

The didactical design of an already composed for concept of distance, angle and angular measure in three dimensions’ topic, use a learning trajectory that composed through Euclidean geometry approaches ranging from postulates, theorems, concepts, and procedures as well as the capability of the spatial sense and prerequisites knowledge [6–8]. Learning trajectory with this approach is the learning trajectory that always used in learning this topic in Indonesia. Starting from the findings that the spatial sense and
prerequisite knowledge issue in learning three dimensions topic, meaning that it needs to create and develop another learning trajectory that were built in different approaches.

Speaking on the topic of three dimensions, it also discusses about the field of plane and space. Cartesian coordinate system includes both fields in this topic. Based on this similarity it can be concluded that the Cartesian coordinate system can be used as an approach to study the topic of three dimensions. Vector is a topic that broached this topic, it contains the vector length and the angular measure between the two vectors. The length of a vector can be used to represent the distance between objects of geometry, and the angular measure between two vectors can be used to determine the angular measure that is formed between objects of geometry. Departing from the commonality between the Cartesian coordinate system, the vector with three dimensions, it means that it can be used to compose learning trajectory of three dimensions’ topic through the analytical approach that is by moving the objects of study of three-dimension into Cartesian coordinate system in space. The learning trajectory of this topic through analytical geometry approach which will be drawn up, requiring information on how students learn vector and possible barriers to learning that emerged from the process of learning. The process of obtaining information activities will provide valuable experience in the process of repersonalization and recontextualization to develop the learning trajectory, and at times, the difficulties in this process can occur in the process of learning [9].

2. Experimental method
The method used in this research is a qualitative research, particularly prospective analysis part of the didactical design research [10] through observing student’s learning activities, literary studies of topics’ related material and interview to some students and teachers. Prospective analysis is analysis of prior learning didactical situation including re-contextualisation, re-personalization, and prediction of responses in the form of didactical design hypothesis [10], but for this research, it will be resulting only the formed learning trajectory that will used to develop the didactical design. Observation focused on how the learning activity and process happens related to how teachers teach and how students learn about vectors. The study of the literature focused to find how the topic of vector and three dimensions was presented in text books, and the content contained therein. The study of the literature was conducted to find the link between the topics as well as the deficiencies that must be addressed. The interview was conducted to examine the activity more in related problems experienced by teachers and students in learning vector’s topic. The research conducted at Pilot Laboratory Senior High School of Universitas Pendidikan Indonesia.

3. Result and discussion
Based on the results of the interview, observation, and literary studies related to three dimensions’ topic, found three problems that can become an obstacle and noteworthy learning to devise the learning trajectory through analytical geometry approach. First problem comes from the math curriculum in Indonesia and the syllabus is used, the absence of topics related to the equations of line and plane in space. Second, the learning process of vector’s topic is more emphasis on algebraic form of vector thus resulting in a lack of learning materials from the vector’s geometric representation, it is apparent from the learning materials, learning activities, and an interview. Third, as a result of the second problem, the students never draw some solid geometry form (cube, cuboid, prism, etc.) in the Cartesian coordinate system in space. Fourth, the lack of exploration activities in learning vector.

The problems already mentioned earlier, provide three vital information in drawing up the learning trajectory of three dimensions’ topic through analytical geometry approach. First, the exploration of geometric representation activities and drawing vector and solid form including another geometry object in Cartesian coordinate system became the main focus. It was done in this way because it is easier to derive the algebraic formula, calculations and procedures of the topic from the geometric one than the other way around [11]. The results of these exploration activities are used to further exploration that corporate the sub-topic associated to three dimensions’ topic, which are the concept of distance, angle, and angular measure in space. Second, the activities to draw the object of study in three dimensions
through analytical geometry approach, carried out with and without the help of 3D software. Third, utilize vector as a representation of the objects of study, for example, namely the normal vector to represent the plane and polygons. These activities are part of the instructional tasks that are indeed necessary in the learning trajectory that composed through analytic geometry approach [12].

**Figure 1.** Learning trajectory of three dimensions through analytical geometric approach.

Figure 1 shows the formed learning trajectory. The exploration holds the main key in this learning trajectory, since it will be used through the learning process with this learning trajectory. This learning trajectory doesn’t lose the distance, angle, and angular measure concept of learning three dimensions in traditional way, i.e. through Euclidean geometry approach. The learning trajectory explains the number of meetings needed in class to learn this topic, which is eight meetings, start from exploration 1 until angle between two planes. But, if the students learn the vector’s topic in a correct way, then, it only needs 6 meetings in class, which is the green part in this learning trajectory.

The formed learning trajectory contain one main framework on each meeting in learning three dimensions’ concept. Figure 2 shows the flow of the main framework.
Figure 2. The main framework of each meeting in learning trajectory through analytical geometry approach.

Figure 2 shows the important role of drawing the three dimensions related problem in Cartesian coordinate and identifying the related vector through the process. This framework can also be used not only to solve a problem, but also to find the procedure to determine the distance, angle, and angular measure in three dimensions’ topic. Another advantage of this learning trajectory, the spatial sense that student need is only to explore the vector directions and the geometry objects in space, they don’t need to have a certain spatial sense like in Euclidean geometry approach, where they had to dissect the three-dimensional object to two-dimensional object, in order to solve the problem.

Figure 3. Example of routine problem related to distance between two points in solid and the comparison of the solving process between the analytical and Euclidean geometry approach.
Figure 3 shows the comparison and difference between the analytical and Euclidean geometry approach in solving the problem. The analytical geometry side shows how the spatial sense work in solving the problem. It only works in determining the vector, cuboid, and the formed segment as the distance to calculate it. While the Euclidean geometry side, the spatial sense is needed not only to find the segment as the distance, but also needed to dissect the cuboid into two-dimensional object to help the calculation.

The learning trajectory of three dimensions’ topic through analytical geometry approach, as a whole did not emphasize on the spatial sense. The question is, is this allowed? A variety of sources that discuss about learning geometry and geometric thinking itself, only discuss about the geometric thinking levels, yet no sources that discuss about the level of spatial ability or spatial sense in learning geometry [13–16]. This is due to problem of whether that ability is something gained through the process of the exercise, or a talent, so it can be argued that this ability is the field of psychology, related to mental rotation in human mind [17], because if it was a talent, it will be difficult to teach three-dimensions’ topic that emphasizes on the spatial sense or spatial ability as the main tool for students to learn it. In addition, there is also the theory of multiple intelligence that contains the visual-spatial intelligence describing that kind of intelligence as a talent [18].

4. Conclusion
The formed learning trajectory of three dimensions’ topic through analytical geometry approach can be useful to solve the learning obstacles that occurred in learning this topic through Euclidean geometry approach. This learning trajectory also gives a meaning and a field for the application of vector’s topic. The geometry representation of vector’s topic plays an important role rather than the algebraic form. The analytical approach makes the process in solving three-dimension problem easier without losing any concept. This learning trajectory also implies the need to change the way of teaching and learning the vector’s topic to not only the algebraic form but also the geometry representation, and also to develop a new learning trajectory and a didactical design for the vector’s topic. For future research, we consider to develop a didactical design with this learning trajectory as the main learning framework.

5. Acknowledgments
We thank all the mathematics teachers and the headmaster of Pilot Laboratory Senior High School of Universitas Pendidikan Indonesia, for giving us permission and help during our research.

6. References
[1] Setiadi D R, Suryadi D and Mulyana E 2017 Didactical Design Enrichment of Angle in Geometry J. Phy. Conf. Series. 895 012060
[2] Simatupang D E 2014 Penerapan Model Pembelajaran Van Hiele Dengan Bantuan Media Software Geogebra Untuk Meningkatkan Kemampuan Pemahaman Geometri Siswa SMP (Theses Universitas Pendidikan Indonesia)
[3] Subroto T 2011 Penggunaan Software Cabri 3d Sebagai Alat Peraga Maya Dalam Pembelajaran Bangun Ruang Di SMP Untuk Meningkatkan Kemampuan Spasial (Theses Universitas Pendidikan Indonesia)
[4] Saha R A, Ayub A F M and Tarmizi R A 2010 The Effects of GeoGebra on Mathematics Achievement: Enlightening Coordinate Geometry Learning Procidia. Soc. Behavioral. Sci. 8 686–693
[5] Kaufmann H and Schmalstieg D 2003 Mathematics and geometry education with collaborative augmented reality Comp. Graph. 27 339–45
[6] Andriatna R 2016 Desain Didaktis Konsep Jarak Dalam Ruang Dimensi Tiga Pada Pembelajaran Matematika SMA (Theses Universitas Pendidikan Indonesia)
[7] Putri W K H W 2016 Desain Didaktis Konsep Jarak Pada Bangun Ruang Sisi Datar (Theses Universitas Pendidikan Indonesia)
[8] Setiadi D R 2015 Desain Didaktis Untuk Mengembangkan Kompetensi Siswa Terhadap Konsep Sudut Pada Bangun Ruang Berdasarkan Learning Trajectory (Theses Universitas Pendidikan Indonesia)

[9] Suryadi D 2013 Didactical Design Research (DDR) to improve the teaching of mathematics Far East. J. Math. Edu. 10 145–59

[10] Suryadi D 2010 Metapedadidaktik dan Didactical Design Research (DDR): Sintesis Hasil Pemikiran Berdasarkan Lesson Study Teori, Paradigma, Prinsip, dan Pendekatan Pembelajaran MIPA dalam Konteks Indonesia (Bandung: FPMIPA UPI) pp 55–75

[11] Dray T and Manogue C A 2006 The Geometry of the Dot and Cross Products J. Online. Math. And. Its. Application. 6 1–13

[12] Clements D and Sarama J 2009 Learning and Teaching Early Math (The Learning Trajectories Approach) (New York: Routledge)

[13] Van Hiele P M 1999 Developing Geometric Thinking through Activities that Begin with Play, Teaching Children Mathematics 5 310–16

[14] Vojkuvkova 2012 The Van Hiele Model of Geometric Thinking. WDS’12 Proceedings of Contributed Papers 1 72–5

[15] Usiskin Z 1982 Van Hiele Level and Achievement in Secondary School Geometry (University of Chicago)

[16] Clements D 2003 Teaching and Learning Geometry A research companion to principles and standards for school mathematics 151–77

[17] Piaget J and Inhelder B 1971 Mental Imagery in Child: A Study of the Development of Imaginal Representation (London: Routledge & Kegan Paul)

[18] Gardner H E 2008 Multiple Intelligences New Horizons (New York: Basic Books)