Two–loop Dirac neutrino mass and WIMP dark matter

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Abstract

We propose a “scotogenic” mechanism relating small neutrino mass and cosmological dark matter. Neutrinos are Dirac fermions with masses arising only in two–loop order through the sector responsible for dark matter. Two triality symmetries ensure both dark matter stability and strict lepton number conservation at higher orders. A global spontaneously broken U(1) symmetry leads to a physical Diracon that induces invisible Higgs decays which add up to the Higgs to dark matter mode. This enhances sensitivities to spin–independent WIMP dark matter search below $m_h/2$.

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I. INTRODUCTION

Two of the main observational shortcomings of the Standard Model is that it lacks neutrino masses [1] as well as a viable candidate for cosmological dark matter [2]. Even though light neutrinos themselves can account only for a very small fraction of the dark matter, they may hold the key to the basic understanding of what causes the dark matter to exist in the first place. Indeed, the existence of neutrino masses and of cosmological dark matter may be closely interconnected in several ways [3]. For example, the mechanism of neutrino mass generation itself can involve the exchange of particles which make up the bulk of the observed dark matter. This is the main idea of scotogenic models [4, 5]. The prototype model is based on the assumption that the dark sector, odd under a parity symmetry, is connected with the neutrino sector through the generation of the light neutrino masses. The dark matter particle plays the role of messenger of radiative neutrino mass generation [6, 7]. In the simplest conventional scenario [4], the dark matter is made up of a weakly interacting massive particle (WIMP), for example, the lightest scalar component of an inert Higgs doublet.

In this letter we explore the possibility of generating scotogenic Dirac neutrino masses radiatively, by forbidding lepton number violation through the cyclic $Z_3$ symmetry. This ensures strict lepton number conservation and the Diracness of neutrinos at higher orders. Another discrete $Z_3$ symmetry is responsible for dark matter stability. Neutrino masses are generated only at the two-loop level, through the same sector responsible for cosmological dark matter. This new realization combines the idea of two-loop scotogenic neutrino masses suggested in [8] with the idea of having a conserved lepton number leading to the Dirac nature of neutrinos. In our model there is a global U(1) symmetry which forbids the usual Dirac mass term of the neutrinos with the standard Higgs [9, 10] 1. This symmetry breaks spontaneously leading to a physical Goldstone boson – a gauge singlet Diracon [12] – which induces invisible Higgs decays. These are analogous to the invisible Higgs decays by Majoron emission in models with Majorana neutrinos [13]. The extra invisible channel adds up to the Higgs boson decays to pairs of dark matter particles at collider experiments, providing tighter limits on WIMP dark matter below $m_h/2$.

1 In contrast to Refs. [9, 10], neutrinos here are Dirac fermions, as opposed to Quasi-Dirac [11].
II. THE MODEL

We will consider a simple extension of the standard $SU(3)_{c} \otimes SU(2)_{L} \otimes U(1)_{Y}$ model with the symmetries and field content indicated in Table I, where $\omega$ and $\alpha$ are cube roots of unity, i.e. $\omega^3 = 1 = \alpha^3$. There are two complex $SU(2)_{L}$ doublets, $H$ and $\eta$ and three complex singlets, $\sigma$, $\chi$ and $\xi$.

\begin{table}[h]
\begin{tabular}{|c|c|c|c|c|c|c|c|}
\hline
 & $L$ & $\nu^c$ & $H$ & $\eta$ & $N$ & $S$ & $\sigma$ & $\xi$ & $\chi$ \\
\hline
$SU(2)_L$ & 2 & 1 & 2 & 1 & 1 & 1 & 1 & \\
$U(1)_D$ & -1 & 3 & 0 & 0 & -1 & 2 & -2 & 0 \\
$Z_3^{DM}$ & 1 & 1 & 1 & $\alpha$ & $\alpha$ & $\alpha$ & $1$ & $\alpha^2$ & $\alpha$ \\
$Z_3$ & $\omega$ & $\omega^2$ & 1 & 1 & $\omega$ & $\omega^2$ & 1 & 1 & 1 \\
\hline
\end{tabular}
\caption{Relevant particle content and quantum numbers of the model.}
\end{table}

The invariant Yukawa Lagrangian is given by

$$\mathcal{L} = y^c \bar{\nu}^c \tilde{\eta} S + y^R \nu^c N \xi + \lambda N S \chi.$$ (1)

The scalar potential can be separated as follows

$$V = V + V(H, \eta) + V(\xi, \sigma, \chi, H, \eta),$$ (2)

where the first term $V$ in the scalar potential contains the relevant terms for the generation of the neutrino masses, namely,

$$V = \lambda_1^\chi H^\dagger H \eta^* \sigma \chi + \lambda_3^\chi \chi^3 + h.c.$$ (3)

while the second term $V(H, \eta)$ is the Higgs potential associated to the $\eta$ doublet

$$V(H, \eta) = \mu_1^2 H^\dagger H + \mu_2^2 \eta^\dagger \eta + \lambda_1 |H|^4 + \lambda_2 |\eta|^4 + \lambda_3 |H|^2 |\eta|^2$$

$$+ \lambda_4 |H^\dagger \eta|^2 + \lambda_5 [(H^\dagger \eta)^2 + h.c].$$ (4)
The last term \( V(\xi, \sigma, \chi, H, \eta) \), is given by
\[
V(\xi, \sigma, \chi, H, \eta) = \mu_\xi^2 \xi \xi^* + \mu_\sigma^2 \sigma \sigma^* + \mu_\chi^2 \chi \chi^* + \lambda_\chi (\xi \xi^*)^2 + \lambda_\sigma (\sigma \sigma^*)^2 + \lambda_\chi \chi^* \xi \xi^* \\
+ \lambda_\chi \xi \chi^* \xi \xi^* + \lambda_\sigma \chi \chi^* \sigma \sigma^* + \lambda_\chi H \chi^* H^\dagger H + \lambda_\chi \eta \chi^* \eta^\dagger \eta + \lambda_\xi \xi \chi^* H^\dagger H \\
+ \lambda_\xi \xi \chi^* \xi \xi^* + \lambda_\sigma \sigma^* \sigma \sigma^* + \lambda_\sigma \sigma^* \eta \eta^\dagger \eta.
\] (5)

After spontaneous symmetry breaking the fields are shifted as follows,
\[
H = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}} (v_h + h^0 + iA^0) \end{pmatrix}, \quad \eta = \begin{pmatrix} \eta^+ \\ \frac{1}{\sqrt{2}} (\eta_R + i\eta_I) \end{pmatrix}, \\
\sigma = \frac{1}{\sqrt{2}} (v_\sigma + \sigma_R + i\sigma_I), \quad \chi = \frac{1}{\sqrt{2}} (\chi_R + i\chi_I), \quad \xi = \frac{1}{\sqrt{2}} (\xi_R + i\xi_I).
\] (6)

Notice that there are no vacuum expectation values (vevs) for scalars \( \eta, \chi, \xi \) which are charged under \( Z_3^{DM} \).

![FIG. 1: Two–loop generation of Dirac neutrino mass.](image)

The fermions \( N \) and \( S \) will form heavy Dirac neutrinos by pairing with their corresponding partners \( \bar{N} \) and \( \bar{S} \). The light neutrinos acquire their masses via the loop in Fig. 1. From the minimization of the scalar potential, the scalar fields charged under the \( Z_3^{DM} \) do not acquire vacuum expectation values (vevs), while for the Higgs and the \( \sigma \) fields, there are two relevant tadpole equations:
\[
\mu_1^2 = \lambda_1 v_\eta^2 + \frac{1}{2} \lambda_{\sigma H} v_\sigma^2, \\
\mu_2^2 = \lambda_2 v_\sigma^2 + \frac{1}{2} \lambda_{\sigma H} v_\eta^2.
\] (7)
The corresponding mass matrix for the CP-even “active” scalars is

\[ M^2_R = \begin{pmatrix} 2\lambda_1 v^2_h & \lambda_H v_h v_{\sigma} \\ \lambda_H v_h v_{\sigma} & 2\lambda_\sigma v^2_{\sigma} \end{pmatrix}. \]  

(8)

The pseudoscalars include the unphysical Goldstone boson \( G^0 \) and a physical one, \( D \), namely the Diracon. In contrast to that of Ref. [12] the Diracon here is a pure singlet under weak SU(2) and hence is not subject to the strong astrophysical bound coming from stellar cooling considerations [14].

The mass matrices for the scalar and pseudoscalar “dark” sector (charged under \( Z^{DM}_3 \)) in the basis \( \eta, \chi, \xi \) are given as

\[ M^2_R = \begin{pmatrix} \frac{1}{2} \left( \lambda_{345}^+ v^2_h + \lambda_\sigma v^2_{\sigma} - 2\mu_{\eta}^2 \right) & \frac{\lambda_1 v_h}{2\sqrt{2}} & 0 \\ \frac{\lambda_1 v_h}{2\sqrt{2}} & \frac{1}{2} \left( \lambda_H v^2_h + \lambda_\xi v^2_{\sigma} \right) & \frac{\lambda_3 v_{\eta}}{\sqrt{2}} \\ 0 & \frac{\lambda_3 v_{\eta}}{\sqrt{2}} & \frac{1}{2} \left( \lambda_H v^2_h + \lambda_\sigma v^2_{\sigma} \right) \end{pmatrix}, \]  

(9)

and

\[ M^2_I = \begin{pmatrix} \frac{1}{2} \left( \lambda_{345}^- v^2_h + \lambda_\sigma v^2_{\sigma} - 2\mu_{\eta}^2 \right) & -\frac{\lambda_1 v_h}{2\sqrt{2}} & 0 \\ -\frac{\lambda_1 v_h}{2\sqrt{2}} & \frac{1}{2} \left( \lambda_H v^2_h + \lambda_\xi v^2_{\sigma} \right) & -\frac{\lambda_3 v_{\eta}}{\sqrt{2}} \\ 0 & -\frac{\lambda_3 v_{\eta}}{\sqrt{2}} & \frac{1}{2} \left( \lambda_H v^2_h + \lambda_\sigma v^2_{\sigma} \right) \end{pmatrix}, \]  

(10)

where the parameter \( \lambda_{345}^\pm \) is given by

\[ \lambda_{345}^\pm \equiv \lambda_3 \pm \lambda_4 \pm \lambda_5. \]  

(11)

Finally, the mass for the “inert” electrically charged scalar is given by

\[ M^2_{\eta^+} = \frac{1}{2} \left( -2\mu_{\eta}^2 + \lambda_3 v^2_h + \lambda_\sigma v^2_{\sigma} \right). \]  

(12)

### III. DARK MATTER ANNIHILATION

As usual in scotogenic models [4] [6, 7] dark matter in our model can be either scalar or fermionic. Here we focus on the first case, where the dark matter candidate is the lightest scalar eigenstate of \( M^2_{R,I} \) in Eqs. (9) and (10), which can be a general mixture of \( Z^{DM}_3 \)-charged doublet and singlet scalars \( \eta, \chi \) and \( \xi \). An important requirement for the dark matter
interpretation of such candidate is that its relic abundance matches the value observed by the Planck collaboration. There are in principle three possibilities:

- mainly doublet dark matter
- generic doublet–singlet dark matter combination
- mainly singlet dark matter

The first case can be arranged if the coupling $\lambda^3_\chi$ is suppressed and/or the vev of $\sigma$ is large. In this case one loses the signature corresponding to invisible Higgs decay to the $\text{Dirac}_1$, Eq. (13). The dark matter candidate is well studied in other similar scenarios such as the scotogenic model [4] and the Inert Doublet Model [15–17]. In this context, the sign of the dimensionless coupling $\lambda_5$, determines whether the dark matter has a either CP-odd or CP-even nature, and the correct relic abundance constrains the parameter $\lambda^{345}_5$ in Eq. (11) [17].

In the second and most general case the situation is analogous to that of sneutrino dark matter in the inverse seesaw model described in Ref. [18, 19]. The dark matter candidate is made up of a singlet-doublet combination with potentially “comparable” components, and can lead both to an adequate relic density as well as to a detectable signal in nuclear recoil.

Finally, the last and simplest of the three cases, corresponds to that in which the dark matter candidate is mainly singlet and is detected primarily by the Higgs portal interaction. In the present model, the dark matter singlet would be given mainly by a combination of the fields $\chi$ and $\xi$. Without loss of generality we will denote as $X$ the lightest combination of these singlets.

However, thanks to the $Z_3^{DM}$ nature of our dark matter candidate and to the presence of the $\text{Dirac}_1$, there are other distinctive features in our case. Indeed, due to the cubic terms in the scalar potential, one finds that, besides annihilations, semi-annihilation processes play an important role in determining the dark matter relic density, as explained carefully in Ref. [20]. In contrast to the case of $Z_2$ dark matter, the dark matter spin–independent direct detection cross section is no longer directly related to the annihilation cross section.

In the case of interest, the limit in which the dark matter candidate $X$ is stabilized by the $Z_3^{DM}$ symmetry has been studied in detail in Ref. [20, 21]. In this case the dimensionful term

\[ 2 \text{In order to generate nonzero neutrino mass through Fig. 1 none of the } \lambda^3_\chi \text{ couplings can vanish exactly. Hence the dark matter candidate is necessarily a combination of the triality–carrying scalars.} \]

\[ 3 \text{ We assume that the doublet-singlet mixing is negligible. Then, we define } X \equiv c_\alpha \chi - s_\alpha \xi \text{ and } \bar{X} \equiv s_\alpha \chi + c_\alpha \xi. \]
\( \lambda_X^3 \) contributes to the semi-annihilation processes like, for instance, \( XX \to X^* h \) that can dominate in the determination of the relic density. As a result the \( \lambda_{XH} \) coupling no longer links the annihilation rate to the spin independent nuclear recoil detection cross section, in contrast to the more familiar case in which dark matter is stabilized by the \( Z_2 \) symmetry \cite{22}.

Over and above this observation, our model has further distinctively novel features associated to the presence of the \textit{Diracon}. This leads to genuinely new interactions absent in previous dark matter models, including the simplest benchmark model studied in \cite{22} as well as the possibilities analyzed in Refs. \cite{20,21}. Indeed, concerning dark matter annihilation, there are new semi-annihilation channels involving the \textit{Diracons}, as illustrated in Fig. 2 \cite{4}. These which should allow one to suppress the \( X \) relic density with respect to the cases considered in these references. In addition, the \textit{Diracon} plays a role in detection, see next.

IV. \textbf{DARK MATTER DETECTION}

Encouraged by the above arguments concerning dark matter annihilation and semi-annihilation processes and in view of the positive results of Ref. \cite{21}, here we take for granted that an adequate relic abundance of the dark matter candidate particle can be ensured. We focus, instead, on another most salient feature of our model, namely, the presence of the invisible Higgs boson decays into \textit{Diracons}, i.e.

\[
h \to \mathcal{D}\mathcal{D}, \quad (13)
\]

\footnote{A detailed determination of the relic density lies outside the scope of this paper.}
and its impact upon the dark matter detection prospects. Such decays through Diracon emission are the exact analogue of the invisible Higgs decays by Majoron emission in models with Majorana neutrinos [13]. As long as the $h \rightarrow \mathcal{DD}$ coupling is non-zero, this Higgs decay mode also contributes in the range $m_X < m_h/2$, that is, when the Higgs decay into dark matter is kinematically allowed. The current bound on the invisible Higgs decays is given by $\mathcal{BR}_{\text{inv}} \equiv \frac{\Gamma_{\text{inv}}}{\Gamma_{\text{inv}} + \Gamma_{\text{vis}}} < 17\%$ [23]. In this scenario, the invisible Higgs decay width, $\Gamma_{\text{inv}}$, "always" has a contribution coming from its decay into Diracons, $\Gamma_{\text{inv}}^D \equiv \Gamma(h \rightarrow \mathcal{DD})$. As a result, for $m_X < m_h/2$, where $m_X$ is the dark matter mass, the invisible decays have two sources, the $h \rightarrow \mathcal{DD}$ and $h \rightarrow XX$, i.e. $\Gamma_{\text{inv}} = \Gamma_{\text{inv}}^X + \Gamma_{\text{inv}}^D$. The Standard Model Higgs is in general a combination of the doublet $H$ and the singlet $\sigma$, if we assume that the mixing between them is small, then $\Gamma_{\text{vis}} = \Gamma_{\text{Total}}^{SM} = 4.434$ MeV, so that the bound on the invisible width is $\Gamma_{\text{inv}} < 0.908169$ MeV. In this region there is a stronger constraint for the quartic coupling of the Higgs with the dark matter, as seen in Fig. 3. In this figure we display the constraints on $\lambda_{HX}$ from the invisible decays of the Higgs (red region) as well as from the LUX [24] and PandaX [25] direct detection spin–independent cross section (purple and blue region, respectively).

![FIG. 3: Exclusion regions in the $(m_X, \lambda_{HX})$-plane. The constraint from invisible Higgs decays is indicated in red. The continuous purple line correspond to the recent limit reported by LUX [24] from dark matter searches while the dot-dashed blue one indicates the current bound given by PandaX [25]. The different shades in red for the invisible decays define different contributions of $\Gamma_{\text{inv}}^D$ to $\mathcal{BR}_{\text{inv}}$ (see text).](image-url)

The different shades in red in Fig. 3 correspond to different contributions of decays of
Higgs into Diracons, $\Gamma_{\text{inv}}^D$, the smaller the contribution of $\Gamma_{\text{inv}}^D$, the darker the region. For instance, the darkest red corresponds to the “standard” case with $\Gamma_{\text{inv}}^D = 0$, while the lightest one is for $\Gamma_{\text{inv}}^D = 0.9$ MeV. As a result the region excluded by the invisible Higgs decays in the $(m_X, \lambda_{HX})$–plane can be broader than the exclusion region set by the LUX data for the mass range $m_X < m_h/2$. In other words, the presence of the extra invisible decay channel into Diracons effectively increases the sensitivities to spin–independent WIMP dark matter searches below $m_h/2$.

V. SUMMARY

In this letter we have proposed a low–scale mechanism for naturally small Dirac neutrino masses generated only at the two–loop level. The sector responsible for cosmological dark matter acts as messenger of neutrino mass generation. Both dark matter stability and strict lepton number conservation are “symmetry protected”. The presence of a global spontaneously broken U(1) symmetry leads to a physical Goldstone boson, dubbed Diracon, that induces new invisible Higgs decays detectable at LHC and other collider experiments. The coexistence of such decays with the Higgs to dark matter channel, if kinematically allowed, leads to stronger sensitivities which we have quantified using current constraints from the LHC. Detailed analysis of the primordial WIMP dark matter density lies outside the scope of the present letter and will be presented elsewhere.

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