UNIVERSAL NEWTON TIME IN CLASSICAL ELECTRODYNAMICS.
ELEMENTS OF PHYSICAL INTERPRETATION

G.A. Kotel’nikov
Russian Research Center "Kurchatov Institute", Moscow 123182, Russia
E-mail: kga@kga.kiae.su

Abstract

It is shown that the universal Newton time may be introduced in the classical electrodynamics. The statement results from an existence of the generalized symmetry of Maxwell equations with respect to Galilei transformations. In the case of the extended Galilei transformations the postulate of invariance of the speed of light may be made compatible with the concept of the universal Newton time. Some physical consequences of the extended Galilei symmetry are considered.

1 Introduction

The postulate on the existence of the universal time was entered into physics by Newton in 1687. Up to the end of the XIXth century this concept perfectly corresponded to the known set of experimental facts, described by the equations of movement in the classical mechanics both on the Earth and within the Sun system. It will suffice to mention one of the remarkable achievements of this concept, namely, the prediction of the Neptune’s orbit parameters on the basis of the Uran’s perturbation study by Le Verrier, 1846. The difficulties have begun at the end of the XIXth century in attempting to make the universal time concept compatible with Maxwell electrodynamics. It is known from a formally mathematical side that the idea of the universal time is realized in Galilei transformations:

\[ x' = x - Vt; \quad y' = y; \quad z' = z; \quad t' = t. \]  

(1)

Here \( x, y, z \) are the space variables, \( t \) is the time, \( V = (V, 0, 0) \) is the velocity of the inertial reference \( K' \) relative to \( K \). While the Newton mechanics equation \( ma = F \) is in accordance with these transformations (invariant with respect to Galilei transformations), Maxwell equations did not seem to have the similar property. In other words, Maxwell equations, by their mathematical nature were presented as Galilei non-invariant equations. This point of view is popularized everywhere. One can find this in any university course of general physics. Moreover, it is from resolving the difficulty of Galilei approach in electrodynamics that the special relativity theory (SR) and the modern relativization of space-time representations have arisen. And now about 90 years after publishing the fundamental works of Voigt, Larmor, Lorentz, Einstein and Poincaré we return to seemingly already resolved problems again.

It was found that a situation is more complicated here, than it is commonly supposed, and the concept of the Newton time may be also accorded with Maxwell equations. It is important to determine what we mean by this. For definiteness, by the words ”the concept of the Newton time may be also accorded with Maxwell equations” we shall mean that vacuum (microscopic) Maxwell equations possess not only the property of the Lorentz invariance, but also the property of the invariance with respect to Galilei transformations in some generalized sense. The Galilei transformations contain the time transformation \( t' = t \), and the universal Newton time will be thereby entered automatically into Maxwell equations.

It should be noted that although the relativistic concept was dominant, the research of Galilei symmetry of Maxwell electrodynamics were carried out after arising SR and are continued at the present time. As example it is possible to note the publications of Le Bellac and Levi-Leblond
and Fushchich and Nikitin [2]. The authors have established that besides the well known
relativistic and conform symmetry, the first and second pair of Maxwell equations, considered
separately, also have the property of invariance with respect to Galilei transformations. The various
linear representations of Galilei group corresponding to the transformations of the fields \( E' = E, \)
\( H' = H \) are realized on the solutions of these equations [1], [2]. (Here the speed of light \( c \) is believed
equal to unit; \( E, H \) are the electrical and magnetic fields). So, the situation has considerably
been cleared, but the attempt at establishing the property of Galilei symmetry of Maxwell
equations in the total set has not been quite successful. It was made later in the works [3] - [5],
which we shall follow below.

2 Galilei symmetry of Maxwell equations

Let us introduce the free Maxwell equations:

\[
\nabla \cdot E = 0; \quad \nabla \cdot H = \frac{1}{c} \partial_t E; \\
\nabla \cdot H = 0; \quad \nabla \cdot E = -\frac{1}{c} \partial_t H.
\]

As it is known, they may be brought to a single equation of the second order namely to the
D’Alembert equation with \( \phi \in (E, H) \):

\[
\square \phi(x) = \left( \frac{1}{c^2} \partial^2_{tt} - \triangle \right) \phi(x) = 0.
\]

The algebraic equation of the light cone may be brought in correspondence with the differential
D’Alembert equation:

\[
c^2 t^2 - x^2 = 0.
\]

Any university course of general physics contains the statement on its Galilei non-invariance. Let
us show by direct calculation that it is not so. For this purpose we use the transformations (1) and
Galilei theorem of velocities addition, recorded for the speed of light:

\[
c'_x = c_x - V = n_x c - V; \quad c'_y = c_y = n_y c; \quad c'_z = c_z = n_z c.
\]

Let us put (1) and (5) into the equation of light cone in frame ‘. We have:

\[
c'^2 t'^2 - x'^2 = \left( c'_x \right)^2 t'^2 - \left( n_x c - V \right)^2 t'^2 + n_y^2 c^2 t'^2 + n_z^2 c^2 t'^2 - (x - V t)^2 - y^2 - z^2 = 0
\]

Here it is taken into account that \( x = c_x t = c n_x t \). It follows from here that the Galilei transformations map the sphere \( x'^2 = c'^2 t'^2 \) with center in the point \( x' = y' = z' = 0 \) onto the sphere \( x^2 = c^2 t^2 \) with the center in the point \( x = y = z = 0 \) just as the Lorentz transformations, which
accords with the relativity principle. The failure in the former considerations consists in the fact
that space and time were transformed according to Galilei but the speed of light was transformed
according to Lorentz. Such inconsistency has resulted to incorrect conclusion that the relativity
principle is violated at joint consideration of the ratios (1) and (3). In the fact the light cone equa-
tion is invariant with respect to Galilei transformations. This circumstance prompts to consider
the symmetric properties of D’Alembert equation more carefully.
In accordance with theoretical-algebraic definition of the symmetry \[7\], let us consider the commutational ratios of the Lie algebra generators of Galilei group with the D’Alambert operator. We have \[8\]:

$$\square, p_0 = [\square, p_k] = [\square, J_k] = 0; \quad [\square | \square, H_k] = 0. \tag{7}$$

Here \(p_0 = i\partial_t, p_k = -i\partial_k, J_k = (\text{xxp})_k, H_k = -t\partial_k, k = 1, 2, 3, x_{1,2,3} = x, y, z\). It follows from here that the Lie algebra of Galilei group is the invariance algebra of the free Maxwell equations. Hence, the set of field transformations should exist which will transform into themselves the equations \(8\) in combination with the Galilei transformations \(10\). The task is to find them. For this we record the sought transformations as:

$$E'_1 = \Phi(x, t)E_1; \quad H'_1 = \Phi(x, t)H_1;$$
$$E'_2 = \Phi(x, t)a(E_2 + h_{23}H_3); \quad H'_2 = \Phi(x, t)a(H_2 + e_{23}E_3);$$
$$E'_3 = \Phi(x, t)a(E_3 + h_{32}H_2); \quad H'_3 = \Phi(x, t)a(H_3 + e_{32}E_2), \tag{8}$$

where \(\Phi(x, t)\) is some weight function. Let us express the Galilei transformations in variables \(x_0 = ct, x = (x_1, x_2, x_3)\):

$$x'_0 = \gamma x_0; \quad x'_1 = x_1 - \beta x_0; \quad x'_2 = x_2; \quad x'_3 = x_3; \quad c' = \gamma c, \tag{9}$$

where \(\gamma = (1 - 2\beta n_1 + \beta^2)^{1/2}, n = c/c = (n_1, n_2, n_3), \beta = V/c, \) and produce the replacement of the variables in the equations \(8\), \(10\) taking into account the relations \(9\) and the relationships between the derivatives:

$$\partial'_0 = (\partial_0 + \beta \partial_1)/\gamma; \quad \partial'_k = \partial_k; \quad k = 1, 2, 3;$$
$$\partial_{00}' = (\partial_0 + \beta \partial_1)^2/\gamma^2 =$$
$$= (\partial_{00}^2 + 2\beta \partial_0 \partial_1 + \beta^2 \partial_{11}^2)/\gamma^2; \quad \partial_{kk}' = \partial_{kk}^2. \tag{10}$$

Let us use the concept of the generalized symmetry \[8\] and require that the initial equations should be transformed into themselves due to the compatibility of the set of engaging equations (the conditions of transformation into themselves and the final equations), where the conditions of transformation into themselves are obtained by means of replacement of the variables \(8\) and \(10\) in Maxwell equations \(8\).

**Initial Equations**

| Conditions of Transformation into Themselves. Final Equations |
|---------------------------------------------------------------|
| \(\nabla \cdot \mathbf{E}' = 0;\)                             |
| \(\nabla \cdot \mathbf{H}' = 0;\)                             |
| \(\nabla \times \mathbf{H}' - \partial'_0 \mathbf{E}' = 0;\)  |
| \(\nabla \times \mathbf{E}' + \partial'_0 \mathbf{H}' = 0;\)  |
| \(\nabla \cdot \mathbf{E} = 0;\)                             |
| \(\nabla \cdot \mathbf{H} = 0;\)                             |
| \(\nabla \times \mathbf{H} - \partial_0 \mathbf{E} = 0;\)     |
| \(\nabla \times \mathbf{E} + \partial_0 \mathbf{H} = 0.\)    |

(11)
The weight function $\Phi$ entering into this set may be found from the condition of transformation of D’Alembert equation into himself \[3\]:

**Initial Equation**  
**Condition of Transformation into Himself. Final Equation**

\[
\Box \phi' = 0 \quad \text{and} \quad \Box \phi = 0. 
\]

For the plane waves

\[
E = l e^{-ik \cdot x}, \quad H = m e^{-ik \cdot x},
\]

where $l$ and $m$ are the polarization vectors; $k \cdot x = k_0 x_0 - k x$, $k_0 = \omega / c$, $k = k_0 n$, $\omega$ is the electro-magnetic frequency; $n$ is the guiding vector of wave front, the weight function $\Phi(x,t)$ is \[3\]

\[
\Phi = e^{-(1-\gamma)k \cdot x - \beta k_0 (n_1 x_0 - x_1)}/\gamma.
\]

Let us put the function $\Phi$ into the set of equations (13), taking into account that $\Phi = e^{i(k_0 x_0 - k x - \beta k_0 (n_1 x_0 - x_1))}/\gamma$, $\partial_t \Phi = -i(-k_1 + \beta k_0)\Phi/\gamma$, $\partial_t \Phi = -i(-k_2)\Phi/\gamma$, $\partial_3 \Phi = -i(-k_3)\Phi/\gamma$, $(\partial_0 + \beta \partial_1)\Phi = -ik_0 \gamma \Phi$, $k = nk_0$. As a result we have the eight algebraic equations of the second order for finding the five unknown group parameters $a$, $e_{23}$, $e_{32}$, $h_{23}$, $h_{32}$:

\[
\begin{align*}
-n_2 m_3(a h_{23}) & -n_3 m_2(a h_{32}) & & -(n_2 l_2 + n_3 l_3)(a) & & l_1(-n_1 + \beta) = 0; \\
-n_2 l_3(a e_{23}) & -n_3 l_2(a e_{32}) & & -(n_2 m_2 + n_3 m_3)(a) & & m_1(-n_1 + \beta) = 0; \\
-n_2 l_2(a e_{32}) & +n_3 l_3(a e_{23}) & & -(n_2 m_3 - n_3 m_2)(a) & & \gamma l_1 = 0; \\
(-n_1 + \beta) l_2(a e_{32}) & -\gamma m_3(a h_{23}) & & -[(n_1 + \beta) m_3 + \gamma l_2](a) & & m_1 n_3 = 0; \\
(-n_1 + \beta) l_3(a e_{23}) & -\gamma m_2(a h_{32}) & & -[(n_1 + \beta) m_2 + \gamma l_3](a) & & m_1 n_2 = 0; \\
-n_2 m_2(a h_{32}) & +n_3 m_3(a h_{23}) & & -(n_2 l_3 - n_3 l_2)(a) & & \gamma m_1 = 0; \\
(-n_1 + \beta) m_2(a h_{32}) & +\gamma l_3(a e_{23}) & & -[(n_1 + \beta) l_3 - \gamma m_2](a) & & l_1 n_3 = 0; \\
(-n_1 + \beta) m_3(a h_{23}) & +\gamma l_2(a e_{32}) & & -[(n_1 + \beta) l_2 + \gamma m_3](a) & & l_1 n_2 = 0. \\
\end{align*}
\]

By analogy with relativistic theory we will find the solution of the set (15) from the requirements

\[
E'H' = 0 \rightarrow EH = 0, \quad E'^2 - H'^2 = 0 \rightarrow E^2 - H^2 = 0 \text{ in the relations:}
\]

\[
E'H' = \Phi^2 EH + \\
+ \Phi^2(a^2 - 1)(E_2 H_2 + E_3 H_3) + \Phi^2 a^2[(e_{23} + e_{32})E_2 E_3 + \\
(h_{23} + h_{32})H_2 H_3 + +e_{32} h_{32} E_2 H_2 + e_{23} h_{23} E_3 H_3) = \\
\Phi^2(1 - a^2 - a^2 h_{23} h_{32})E_1 H_1 = 0; \\
E'^2 - H'^2 = \Phi^2(E^2 - H^2) + \\
+ \Phi^2(a^2 - 1)(E_2^2 + E_3^2 - H_2^2 - H_3^2) + \Phi^2 a^2[2(h_{23} - e_{23})E_2 H_3 + \\
2(h_{32} - e_{32})E_3 H_2 + +h_{32}^2 H_2^2 + h_{23}^2 H_3^2 - e_{32}^2 E_2^2 - e_{23}^2 E_3^2] = \\
\Phi^2(1 - a^2 + a^2 h_{23}^2)(E_1^2 - H_1^2) = 0.
\]

These relations will be true if the group parameters possess the following properties:

\[
e_{23} = -e_{32}; \quad h_{23} = -h_{32}; \quad e_{23} = h_{32}; \quad e_{32} = h_{23};
\]

\[
e_{23} h_{23} = (1 - a^2)/a^2; \quad h_{23}^2 = (a^2 - 1)/a^2;
\]

\[
h_{23} = -\sqrt{a^2 - 1}/a; \quad e_{32} = +\sqrt{a^2 - 1}/a.
\]

Putting (17) into the set (13), we obtain four equations for finding the unknown parameter $a$:

\[
\begin{align*}
\sqrt{a^2 - 1} & & -n_1 a & & +n_1 - \beta = 0; \\
n_1 \sqrt{a^2 - 1} & & -a & & +\gamma = 0; \\
\gamma \sqrt{a^2 - 1} & & -(-n_1 + \beta)a & & -n_1 = 0; \\
(-n_1 + \beta)\sqrt{a^2 - 1} & & -\gamma a & & +1 = 0.
\end{align*}
\]
From this and the set (17) we have $\sqrt{a^2 - 1} = [n_1(\gamma - 1) + \beta]/(1 - n_1^2)$ and

$$a = [n_1(\beta - n_1) + \gamma]/[1 - n_1^2];$$
$$e_{23} = +[n_1(\gamma - 1) + \beta]/[n_1(\beta - n_1) + \gamma];$$
$$h_{23} = -[n_1(\gamma - 1) + \beta]/[n_1(\beta - n_1) + \gamma]. \quad (19)$$

As a result the formulas (8) take the form:

$$E_1' = \Phi E_1;$$
$$E_2' = \Phi [n_1(\beta - n_1) + \gamma]E_2 - [n_1(\gamma - 1) + \beta]H_3;$$
$$E_3' = \Phi [n_1(\beta - n_1) + \gamma]E_3 + [n_1(\gamma - 1) + \beta]H_2;$$

$$H_1' = \Phi H_1;$$
$$H_2' = \Phi [n_1(\beta - n_1) + \gamma]H_2 + [n_1(\gamma - 1) + \beta]E_3;$$
$$H_3' = \Phi [n_1(\beta - n_1) + \gamma]H_3 - [n_1(\gamma - 1) + \beta]E_2. \quad (20)$$

(For comparison, in relativistic theory we have $\Phi = 1$, $a = 1/\sqrt{1 - \beta^2}$, $e_{23} = -e_{32} = h_{32} = \beta$, $h_{23} = -h_{32} = e_{32} = -\beta$). It can be shown that the transformations (20) form a group by virtue of the Galilei theorem of velocities addition $\beta'' = \beta + \gamma \beta'$ and transformation properties of the guiding cosines of a wave front

$$n_1' = (n_1 - \beta)/\gamma; \ n_2' = n_2/\gamma; \ n_3' = n_3/\gamma. \quad (21)$$

In this case the group parameters and the weight function $\Phi$ have the following transformation properties:

$$e_{23}'' = (e_{23}' + e_{23})/(1 + e_{23}' e_{23});$$
$$e_{32}'' = (e_{32}' + e_{32})/(1 + e_{32}' e_{32});$$
$$h_{23}'' = (h_{23}' + h_{23})/(1 + h_{23}' h_{23});$$
$$h_{32}'' = (h_{32}' + h_{32})/(1 + h_{32}' h_{32});$$
$$a'' = a' a(1 + e_{23}' e_{23}) = a' a(1 + h_{23}' h_{23}) = \ldots ;$$
$$\Phi'' = \Phi \Phi', \quad (23)$$

which correspond to the matrix law of multiplying the matrices of field transformations in going to primed variables. The formulas (20) are completely analogous to the formula of the relativistic theorem of velocities addition and also arise in multiplying the matrices from the Lorentz group. From this and due to the presence of the local weight function $\Phi(x, t)$ (14) it can be concluded that the matrices of the field transformations (20) form the projective representation of the Lorentz group [3] on the solutions as plane waves of D'Alembert equation. It is similar to the projective Galilei group representations realizing on the solutions as plane waves of Schrödinger equation in Quantum theory [9], [10], and it is also the unexpected circumstance in the symmetry theory of Maxwell equations. But the similar cases are known.

For example, as early as 1909, Cunningham [11] showed that the inversion group $I$: $x'' = x''/x^2 \ (x^2 = x^{02} - x^2, \ x^0 = ct)$ in Minkowski space induces the electromagnetic field transformations, which can be described through a matrix of Lorentz group representation $D(L)$ with the local parameters $\beta = 2x^{0}/(x^{02} + r^2)$, $\sqrt{1 - \beta^2} = (x^{02} - r^2)/(x^{02} + r^2)$:

$$\phi'_p(x') = x^4 D_{pq}(L) \phi_q(x), \quad (25)$$
where \( r^2 = x^2 + y^2 + z^2 \), \( x^4 = (x^0 - x^2)^2 \); \( \phi_p \in (E, H) \), \( p, q = 1, 2, \ldots, 6 \).

In 1970 Isham, Salam and Strathdee generalized this result for the special conformal group \( C_4 \):
\[ x'^\mu = (x^\mu - a^\mu x^2)/\sigma, \quad \sigma = (1 - 2a \cdot x + a^2 x^2) \]
and fields of different conformal dimensions \([12]\). We write their result, as applied to 4-potential \((A^0, A)\) of electromagnetic field
\[
\phi_p(x') = \left| \frac{\text{det} \partial x'/\partial x} {\left( \frac{1}{4} D_{pq}(L) \phi_0(x) \right)^l} \right| \sigma D_{pq}(L) \phi_q(x), \quad (26)
\]
where \( \phi \in (A^0, A) \), \( p, q = 0, 1, 2, 3, \ l = -1 \) is the conformal dimension of 4-potential. (The conformal dimension is equal −2 for the electromagnetic fields \( E, H \) in the case of Cunningham).

The same result \((l = -1)\) may be also received by the method \([7]\).

Now let us turn to the our case. One can see that the formulas \((20)\) for electromagnetic field transformations may be written as:
\[
\phi_p'(x') = \Psi(x) D_{pq}(L) \phi_q(x) \quad (27)
\]
with the weight function \( \Psi \) and the group parameters \( a, \ e_{23}, h_{23} \) from the expressions \((14)\) and \((19)\).

We have an analogy with the result \([11]\) exception that the weight function \( x^4 \) in the expression \((23)\) has a universal character, but in the formula \((27)\) the weight function is determined by a concrete solution of Maxwell equations; besides the Lorentz, Inversion groups are the subgroups of the conformal group \( O(2, 4) \) and the Lorentz, Galilei groups are the subgroups of the \( O(2, 5) \) group \([13]\).

In sum, the formulas \((20)\) and \((27)\) reflect the fact that the electromagnetic fields may be classified through their transformation properties under the Lorentz group. The given property of electromagnetic field was also investigated in the paper \([3]\) as evidence of the symmetry of Maxwell equations, conditioned by existence of the second order commutational ratios \([\Box[\Box, H_k]] = 0 \), where \( H_k \) are the generators of Galilei group, \( k = 1, 2, 3 \). \( \Box \) is D’Alembert operator.

So, according to the result received, we may say that the well known Lorentz-symmetry of Maxwell equation is realized on the true representations of Lorentz group \([8]\); on the projective representations of Lorentz group the Galilei-symmetry of these equations is realized.

We also note that if the guiding vector is \( n = (1, 0, 0) \), the field transformations is reduced to formulas
\[
E_1' = E_1 = 0; \quad E_2' = \frac{[(1 - \beta + 0.5\beta^2) E_2 - \beta(1 - 0.5\beta) H_3]} {1 - \beta}; \quad E_3' = \frac{[(1 - \beta + 0.5\beta^2) E_3 + \beta(1 - 0.5\beta) H_2]} {1 - \beta};
\]
\[
H_1' = H_1 = 0; \quad H_2' = \frac{[(1 - \beta + 0.5\beta^2) H_2 + \beta(1 - 0.5\beta) E_3]} {1 - \beta}; \quad H_3' = \frac{[(1 - \beta + 0.5\beta^2) H_3 - \beta(1 - 0.5\beta) E_2]} {1 - \beta}; \quad (28)
\]
were \( \Phi = 1, \ \gamma = (1 - \beta), \ a = (1 - \beta + 0.5\beta^2)/(1 - \beta), \ e_{23} = \beta(1 - 0.5\beta)/(1 - \beta + 0.5\beta^2) \).

Considerable for the theory is the question on the invariants of the space-time and field transformations. Let us write them in comparison with the relativistic case.
These transformations being neither Galilean, nor relativistic are the limit ratios both for the Galilei components \((k_0, k)\) in the Galilean and relativistic variants of the theory are different:

**The Galilei group** \(G_1\)

\[
c^2 t'^2 - x'^2 = c^2 t^2 - x^2 = 0;\]
\[
t' = t; \ c < \infty;\]
\[
k_0' = k_0; \ k^2' = k^2;\]
\[
n'^2 = n^2 = 1;\]
\[
n_1' x_1' - x_1 = n_1 x_0 - x;\]
\[
E' H' = \Phi^2 E H = 0;\]
\[
E'^2 - H'^2 = \Phi^2 (E^2 - H^2) = 0;\]
\[
E_1' = \Phi E_1; \ H_1' = \Phi H_1;\]
\[
\Box' = [(\partial_0 + \beta \partial_1)^2/\gamma^2 - \Delta];\]

**The Lorentz group** \(L_1\)

\[
c^2 t'^2 - x'^2 = c^2 t^2 - x^2 = s^2;\]
\[
c' = c; \ t < \infty;\]
\[
k_0'^2 - k'^2 = k_0^2 - k^2 = 0;\]
\[
k_0' = k_0; \ k^2' = k^2;\]
\[
n'^2 = n^2 = 1;\]
\[
n_1' x_1' - x_1 = n_1 x_0 - x;\]
\[
E' H' = \Phi^2 E H = 0;\]
\[
E'^2 - H'^2 = \Phi^2 (E^2 - H^2) = 0;\]
\[
E_1' = E_1; \ H_1' = H_1;\]
\[
\Box' = \Box.\]

It is not difficult to see that the part of Galilei-invariants coincides with Lorentz-invariants. However, the distinctions are also observed. They are caused by the distinction in transformational properties of the appropriate values. For example, formulas of transformation of the zero vector \(k\) in the Galilean and relativistic cases respectively.

\[
t_1 x_1' = t_1 x_1;\]
\[
E_1' = \Phi E_1; \ H_1' = \Phi H_1;\]
\[
\Box' = [(\partial_0 + \beta \partial_1)^2/\gamma^2 - \Delta];\]

They differently describe the Doppler effect in electrodynamics.

Let us consider the approximation of the small speeds. At the small speeds, when \(\beta << 1\), for both the relativistic and Galilean cases the common formulas of field transformations are realized:

\[
E' = E + V x H/c; \ H' = H - V x E/c\]

They are known as the Galilean limit. In the present work this term has a conditional character, as the formulas are the limit both for the relativistic and nonrelativistic field variables transformations. The situation here is analogous to the case with independent variables:

\[
x' = x - V t; \ y' = y; \ z' = z; \ t' = t; \ c' = c.\]

These transformations being neither Galilean, nor relativistic are the limit ratios both for the Galilei and relativistic cases.

In the approximation of ultra high velocities, when \(\beta \rightarrow 1\), the electromagnetic fields \(E = (0, 1, 0)\phi, \ H = (0, 0, 1)\phi\) with \(n = (1, 0, 0), \ \phi = exp(-ik \cdot x)\) are transformed as

\[
\phi' = (1 - \beta)\phi; \ \phi' = \sqrt{(1 - \beta)/(1 + \beta)}.
\]

in the Galilean and relativistic cases respectively. Their principle distinction consists in the absence of real fields in the relativistic case if \(\beta > 1\).

We also note, that in the relativistic theory the transformation parameters of the field variables do not depend on the kind of field. Owing to this fact these transformations have global nature in
are the symmetry operators of Maxwell equations because of the existence of the commutational ratios
The extended Galilei group $E_3$ is defined in 4-dimensional real space $E^3 \otimes T^1$ with the metric
$$\begin{align*}
ds^2 &= dx'^2 + dy'^2 + dz'^2 = \rho^2(dx^2 + dy^2 + dz^2) = \rho^2 ds^2
\end{align*}$$
in the subspace $E^3 \subset E^3 \otimes T^1$. The extended Galilei group Lie algebra is generated by the set of operators: $p_0 = i\partial_t, p_k = -i\partial_k$ of the translations group $T^1_k$; $J_k = (xxp)_k$ of the 3-spatial rotations group $SO_3$; $H_k = -tp_k$ of the the pure Galilei transformations group $G_3$; $D = -x_k p_k$ of the scale transformation group $\Delta_1$, where $k = 1, 2, 3$; $x_{1,2,3} = x, y, z$. The commutational ratios of these generators are:
$$\begin{align*}
\{J_k, p_0\} &= 0; & \{H_k, p_0\} &= ip_k; & \{p_k, p_0\} &= 0; \\
\{J_k, p_l\} &= i\epsilon_{klm}p_m; & \{H_k, p_l\} &= 0; & \{p_k, p_l\} &= 0; \\
\{J_k, H_l\} &= i\epsilon_{klm}H_m; & \{H_k, H_l\} &= 0; & \{p_0, D\} &= 0; \\
\{J_k, J_l\} &= i\epsilon_{klm}J_m; & \{H_k, D\} &= iH_k; & \{p_k, D\} &= ip_k; \\
\{J_k, J_k\} &= 0;
\end{align*}$$
where $\epsilon_{klm}$ is the completely antisymmetrical tensor with $\epsilon_{123} = 1$. The Lie algebra generators are the symmetry operators of Maxwell equations because of existence of the commutational ratios
$$\begin{align*}
\{\square, p_0\} &= \{\square, p_k\} = \{\square, J_k\} = 0, & \{\square[\square, H_k]\} &= 0, & \{\square[\square, D]\} &= 0.
\end{align*}$$
Bearing this in mind, let us choose the parameter \( \varphi \) as

\[
| = 1/(1 - 2\beta R_{kl} n_k n_l + \beta^2)^{1/2}
\]  

(37)

with \( s = V/V, \ n = c/c, \ \beta = V/c. \) Then the extended Galilei transformations \( \text{(34)} \) will be compatible both with the concept of the universal Newton time \( t' = t \) and with the postulate of invariance of the speed of light because we may conclude from the theorem of velocities addition that \( c' = c \) in the given case. Owing to this fact, for the particular case when \( R_{kl} = \delta_{kl}, \ V = (V, 0, 0), \ b = 0, \) the transformations of space and time are \( \text{(38)} \):

\[
x' = \frac{x - Vt}{\sqrt{1 - 2\beta n_x + \beta^2}}; \quad y' = \frac{y}{\sqrt{1 - 2\beta n_x + \beta^2}}; \quad z' = \frac{z}{\sqrt{1 - 2\beta n_x + \beta^2}}; \quad t' = t; \quad c' = c,
\]  

(38)

where group parameters have the following properties:

\[
\gamma' = 1/\gamma; \quad \beta' = -\beta/\gamma;
\]

\[
\gamma'' = \gamma\gamma'; \quad \beta'' = \beta + \gamma\beta'.
\]  

(39)

3.1 Invariants of the extended transformations

We note invariants of extended transformations induced once again.

The speed of light. We have from the transformational properties of velocity \( v_x' = (v_x - V)/\gamma, \ v_y' = v_y/\gamma, \ v_z' = v_z/\gamma:\)

\[
c' = \sqrt{c_x'^2 + c_y'^2 + c_z'^2} = \sqrt{(c_x - V)^2 + c_y^2 + c_z^2}/\sqrt{1 - 2\beta n_x + \beta^2} = c.\]  

(40)

The equation of propagation of a spherical wave. Suppose \( c^2 t'^2 - x'^2 = 0. \) Then

\[
c^2 t'^2 - x'^2 = (n_x'^2 + n_y'^2 + n_z'^2)c^2 t'^2 - x'^2 = (c^2 t'^2 - x'^2)/\sqrt{1 - 2\beta n_x + \beta^2},\]  

(41)

where \( x = cn_x t, \ n_x' = (n_x - \beta)/\gamma, \ n_y' = n_y/\gamma, \ n_z' = n_z/\gamma. \) Hence we obtain from \( c^2 t'^2 - x'^2 = 0 \) that \( c^2 t^2 - x^2 = 0. \)

The radius of spherical wave. Suppose \( R^2 = x^2 + y^2 + z^2 = c^2 t^2, \ R = ct. \) Then

\[
R' = \sqrt{x'^2 + y'^2 + z'^2} = \sqrt{x^2 - 2Vxt + V^2 t^2 + y^2 + z^2}/\gamma = \gamma ct/\gamma = R.
\]  

(42)

The sum of square of guiding cosines. Suppose \( n^2 = 1. \) Then

\[
n^2 = (n_x^2 - 2\beta n_x + \beta^2 + n_y^2 + n_z^2)/(1 - 2\beta n_x + \beta^2) = 1.
\]  

(43)

Hence, in accordance with the principle of relativity the extended Galilei transformations \( \text{(38)} \) transform the sphere of radius \( R' = c't' \) with center \( x' = y' = z' = 0 \) to the sphere with the same radius \( R = ct = c't = R' \) and center \( x = y = z = 0. \) The speed of light \( c' \) transforms to the same speed of light \( c = c'. \) The sum of square of guiding cosines turns to unit. We obtain an analogy with SR. We also note that in the framework of extended Galilei transformations \( \text{(38)} \) the consecutive explanation of the basic relativistic experiments is possible. Let us consider these explanations following the papers \( \text{(3), (4).} \)
4 The elements of physical interpretation

4.1 Similar to SR interpretation of observations

The Michelson experiment. Let the test run with the terrestrial light source, System K. Then by virtue of isotropy of 3-space the speed of light \( c \) will be the same in all directions. Therefore, for a terrestrial observer immobile with respect to an interferometer, the interference pattern will not be changed when the interferometer is rotated. Then let the test run with an extraterrestrial source of light, System K'. In this case, due to the transformational property \( c' = c - v \) the speed of light in the system K, i.e. on the Earth, turns out to be equal \( c \) and will coincide with the speed of light from the terrestrial source. Therefore, experiments with terrestrial and extraterrestrial sources of light are indistinguishable physically, and the interference pattern will not be changed in our case as in SR.

Test to check independence of the speed of light from velocity of the light source. As in SR, all these experiments will lead to negative results since \( c' = c \).

The Fizeau experiment. Let us suppose that the speed of light in moving medium (water) is equal

\[
v' = \frac{c}{n} + V \left( \frac{1}{n} - \frac{1}{n^2} \right),
\]

where \( n \) is the refractive index, \( V \) is the velocity of water. Then according to the velocity addition theorem (following from the transformations (38)) in the laboratory frame the speed of light turns out to be

\[
v = v' \sqrt{1 - 2\beta n_1 + \beta^2} + V = (1 - \beta) \left[ \frac{c}{n} + V \left( \frac{1}{n} - \frac{1}{n^2} \right) \right] \approx \frac{c}{n} + V \left( 1 - \frac{1}{n^2} \right)
\]

at \( n_1 = 1 \) in agreement with the experiment. Hence, the explanation of the Fizeau experiment is conventional. For example, as compared with the formula (44) we have \( v' = c/n \) in SR, in classical physics - \( v' = (c/n - V/n^2) \).

Aberration of light. Let \( K' \) be the Sun, \( K \) be the Earth. For explanation the effect, as in SR, we assume that \( V = (V, 0, 0) \) is the velocity of the Sun and that the light propagate in the direction \( n' = (0, 1, 0) \) in frame \( K' \). Then we have from the transformational properties of the guiding vector \( n \) of the speed of light

\[
n_1' = (n_1 - \beta)/\gamma = 0; \ n_2' = n_2/\gamma = 1; \ n_3' = n_3/\gamma = 0; \ \gamma = \sqrt{1 - 2\beta n_1 + \beta^2};
\]

\[
n_1 = \cos\theta = \beta; \ n_2 = \sin\theta = \gamma; \ n_3 = 0; \ \gamma = \sqrt{1 - \beta^2}.
\]

Since the half of aberration angle is equal to \( \alpha = (\pi/2 - \theta) \) we obtain \( \sin\alpha = \beta, \ \alpha \approx \beta = V/c = 10^{-4} \) in agreement with the experiment. In more general case with \( n' = (\cos\theta', \sin\theta', 0) \) the angle of view on a star may be found from the expression \( \tan\theta' = \sin\theta/\cos(\theta - \beta) \). This formula coincides with the relativistic one \( \tan\theta' = \sin\theta \cdot \sqrt{1 - \beta^2}/(\cos\theta - \beta) \) with an accuracy \( \beta^2 \).

The Doppler effect. Suppose that an emitter moves with a velocity \( V \) along the \( x \)-axis relative to an observer \( K \). Let us attach the frame \( K' \) to the emitter, and take into account the formulas \( k_0' = \gamma k_0, \ k_1' = (k_1 - \beta k_0), \ k_2' = k_2, \ k_3' = k_3 \) (instead of the the formulas (38)). Then we have:

\[
k_0' = \gamma k_0 \to (\omega_0'/c') = \sqrt{1 - 2\beta n_1 + \beta^2} (\omega/c); \ c' = c.
\]
From this we obtain

\[
\omega = \omega_0 / \sqrt{1 - 2\beta n_1 + \beta^2}; \quad \lambda = \lambda_0 \sqrt{1 - 2\beta n_1 + \beta^2},
\]

where \( \omega_0' = \omega_0 \) is the proper frequency and \( \lambda_0 \) is the proper wavelength of radiation; \( \omega, \lambda \) are respectively the frequency and wavelength, measured experimentally. From the results (48) we have the following frequencies for the longitudinal \((n_1 \approx 1)\) and transversal \((n_1 = 0)\) Doppler effect:

\[
\omega_{||} = \omega_0 [1 + \beta n_1 - \beta^2(1 - 3n_1^2)/2]; \quad \omega_{\perp} = \omega_0 [1 - \beta^2 / 2 - \beta^4 / 8].
\]

For comparison, in the relativistic case the analogous formulas are:

\[
\omega = \omega_0 \sqrt{1 - \beta^2 / (1 - \beta n_1)}; \quad \lambda = \lambda_0 (1 - \beta n_1) / \sqrt{1 - \beta^2};
\]

\[
\omega_{||} = \omega_0 [1 + \beta n_1 - \beta^2 (1 - 2n_1^2)/2]; \quad \omega_{\perp} = \omega_0 [1 - \beta^2 / 2 - 3\beta^4 / 8].
\]

These formulas coincide with the formulas (48) and (49) for the longitudinal and the transverse Doppler effects in Galilean case with an accuracy \( \beta^2 \) and \( \beta^4 \).

4.2 Distinction from SR interpretation of observations

We note some of them.

The twin paradox. In Galilean approach the twin paradox do not exist because of universal character of time.

Tests to measure lifetimes of fast nonstable particles. In the framework of transformations (38) it is necessary to admit, that instead of the effect of the time retardation the movement with superrelativistic velocity should exist. For example, the velocity of atmospheric \( \mu \)-mesons should be equal to \( \sim \frac{6 \cdot 10^6}{2 \cdot 10^{-6}} = 3 \cdot 10^{12} \text{cm/s} \).

Superrelativistic objects with real mass. In the relativistic physics such objects can not exist. In the Galilean case their existence is not forbidden. Besides the fast nonstable particles, such objects may be space objects with a large value of the redshift

\[
z = \frac{\lambda - \lambda_0}{\lambda_0} = \sqrt{1 - 2\beta n_1 + \beta^2} - 1
\]

In the case of a radial movement the parameter of the redshift is \( z \approx \beta (\beta - n_1) / (\beta + 1) \approx \beta \) when \( n_1 \approx -1 \), and for \( z > 1 \) the superrelativistic velocity of object will be equal \( v_{||} \approx zc > c \). For the transversal movement when \( n_1 = 0 \), the redshift is \( z = \sqrt{1 + \beta^2} - 1 \) and the velocity of object will be equal \( v_{\perp} = \sqrt{z(z + 2)} c \). Then the superrelativistic movements may be if \( z > \sqrt{2} - 1 = 0.414 \). The quasar NRAO 140 with redshift \( z=1.258 \) may be an object of such type. In the framework of Galilei approach the radial calculated velocity of this object may be equal \( v_{||} \approx zc = 1.258c \); the transversal calculated velocity may be equal \( v_{\perp} \approx \sqrt{z(z + 2)} = 2.02c \). The surprising thing is the fact that the value 2.02c is close to the lower limit of the superrelativistic expansion velocity of QSO NRAO 140 in the framework of the Friedmann cosmological model: from 3c to 10c, depending upon assumptions [14].

Let us also note, that the considered example is not the only case when a quasar has the value of redshift \( z > 1 \). According to [16], the number of these quasars is great enough, for example, the quasars LB8796 \( z=1.320 \), PKS \( z=2.170 \), PHL957 \( z=2.690 \), OQ172 \( z=3.530 \), etc. It is known
more than 100 objects with redshift $z > 3$ too \[17\], among which the quasar Q1158+4635 with $z=4.73$ is present. From standpoint of the work all these objects may move with superrelativistic velocities $v_\parallel \simeq zc$ and superrelativistic motion is an ordinary phenomenon in astrophysics. Hubble’s distance for these objects is equal $D_H \simeq zc/H$ and may be connected with Friedmann’s distance (Robertson-Walker metric) by the relation: $D_F = D_H\left\{q_0 + (q_0 - 1)\left[(1+2q_0z)^{1/2} - 1\right]/z\right\}/q_0^2(1+z)$, where $H$ is the Hubble constant, $q_0$ is the deceleration parameter \[18\]. For the case of $q_0 = 1$ we have $D_F = D_H/(1+z) = zc/H(1+z)$ and the horizon $c/H$, which is absent in the Galilean approach, occurs in Friedmann’s model. If $q \rightarrow 0$, the Friedmann’s distance $D_F \rightarrow z^2c/H(1+z)$ is close to the Galilean value $D_H \simeq zc/H$ for the great values of the red shift parameters $z \gg 1$.

The limiting speed of propagation of interactions. According to the addition velocities theorem, we have the following value $v$ for movement along of the $x$- axis at $n = (1, 0, 0)$:

\[
v = \gamma v' + V = v' + V(1 - v'/c).
\]

Here the value $v' = c$ has the invariance property only for the case of such phenomenon as the propagation of the light. If $v' \neq c$, the velocity $v$ may be any amount great by appropriate value $V$. Therefore the limiting speed of propagation of all interactions in the approach \[38\] do not exist. The interactions having the nature other than the electromagnetic one may propagate with velocity distinct from the value $c = 3 \cdot 10^{10} cm/s$.

The point-behaviour of elementary particles. The elementary particles should be points in the relativistic physics because of the finite speed of propagation of all interactions. In Galilean physics this requirement may be removed because of the absence of the limiting speed of interactions propagation.

5 Conclusion

It is shown that in the frame of the generalized symmetry approach \[1\] Maxwell equations are invariant with respect to Galilei transformations and admit the existence of the universal Newton time. In the case of the extended Galilei transformations the postulate of the universal Newton time may be made compatible with the concept of invariance of the speed of light. The Galilei symmetry of Maxwell equation means that the Galilei relativity principle may be realized not only in the classical mechanics but in the classical electrodynamics too.

References

[1] Le Bellac M., Levi - Leblond J.M. Galilean electromagnetism, Nuovo Cim. B, 1973, V. 14, N 2, P. 217-235.

[2] Fushchich W.I., Nikitin A.G. Symmetry of Maxwell Equations. Kiev, Naukova Dumka, 1983, P. 119, 141-156.

[3] Kotel’nikov G.A. Invariance of Maxwell Homogeneous Equations Relative to the Galilei Transformations, in Book: Group Theoretical Methods in Physics. V. 1. Chur, London, Paris, New York, Harwood Acad. Publ., 1985, P. 521-535.

[4] Kotel’nikov G.A. The Galilei Group in Investigation of Symmetry Properties of Maxwell Equations, in Book: Group Theoretical Methods in Physics. V. 1. Moscow, Nauka, 1986, P. 466-495.
[5] Kotel’nikov G.A. Galilei Symmetry in Classical Electrodynamics, in Book: Proceedings of 18 Group Theoretical Colloquium. Part 2. Symmetries and Algebraic Structures in Physics. Ed. V.V. Dodonov, V.I. Man’ko. New York, Nova Science, 1991, P. 19-24.

[6] Leznov A.N., Man’ko V.I., Saveliev M.V. Soliton Solutions of Nonlinear Equations and Theory of Group Representations, in Book: Transactions of FIAN. V. 165. Moscow, Nauka, 1986, P. 75-77.

[7] Kotel’nikov G.A. New Symmetries in Mathematical Physics Equations, in Book: VII International Conference ”Symmetry Methods in Physics”, V. 2. Ed. A. N. Sissakian, G. S. Pogosyan. Dubna, 1996, P. 358-363; LANL/abs/physics/9701006; Symmetries of the free Schrödinger Equation, Preprint IAE-5778/1, Moscow 1994, 21 p., LANL/abs/quant-ph/9612049.

[8] Hamermesh M. Group Theory and Its Application to Physical Problems. Moscow, Mir, 1966, P. 549-559.

[9] Hagen C. R. Scale and Conformal Transformations in Galilei-Covariant Theory, Phys. Rev. D, 1972, V. 5, N 2, P. 377-388.

[10] Niederer U. The Maximal Kinematical Invariance Group of the Free Schrödinger Equation, Helv. Phys. Acta, 1972, V. 45, N 5, P. 802-810.

[11] Cunningham E. The Principle of Relativity in Electrodynamics and an Extension There Of, Proc. Lond. Math. Soc., 1909, V. 8, 11 Feb., P. 77-98.

[12] Isham C.J., Salam A., Strathdee J. Spontaneous Breakdown of Conformal Symmetry, Phys. Lett., 1970, V. 31B, N 5, P. 300-302.

[13] Kuznetsov G.I., Moskalyuk S.S., Smirnov Yu.P., Shelest V.P. Graphical Theory of Orthogonal and Unitary Group Representations. Kiev, Naukova Dumka, 1992, P. 183-184.

[14] Pauli W. Theory of Relativity. Moscow-Leningrad, Gostexizdat, 1947, P. 17.

[15] Six ”Superluminal” Quasars Identified, Science News, 1981, V. 120, N 8, P. 118.

[16] Ku W. H.-M., Helfand D. J., Lucy L. B. X-ray Properties of Quasars, Nature, 1980, V. 288, 27 November, P. 323-328.

[17] Carswell B., Hewett P., The Universe at High Redshift, Nature, 1990, V. 343, 11 January, P. 117-118.

[18] Cohen M.H., Cannon W., Parcell G.H., Shaffer D.B., Broderick J.J., Kellermann K.I., Jauncey D.L., The Small-Scale Structure of Radio Galaxies and Quasi-Stellar Sources at 3.8 Centimeters, Astrophys. J., V. 170, December 1, P. 207-217.