Spontaneous supersymmetry breaking 
in $N = 4$ supergravity with matter

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Abstract

In this paper we consider the problem of spontaneous supersymmetry breaking in $N = 4$ supergravity interacting with vector multiplets. We start with the ordinary version of such model with the scalar field geometry $SU(1,1)/U(1) \otimes SO(6,m)/SO(6) \otimes SO(m)$. Then we construct a dual version of this theory with the same scalar field geometry, which corresponds to the interaction of arbitrary number of vector multiplets with the hidden sector, admitting spontaneous supersymmetry breaking without a cosmological term. We show that supersymmetry breaking is still possible in the presence of matter fields.

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1 Introduction

In this paper we continue our investigation of the problem of spontaneous supersymmetry breaking in extended supergravities started in [1], where $N = 3$ case have been considered. For the $N = 4$ supergravity the only natural candidate for the model of supergravity — vector multiplets interaction is the one with the scalar field geometry $SU(1, 1)/U(1) \otimes SO(6, m)/SO(6) \otimes SO(m)$ [2]. Such a model has indeed been constructed some time ago [3, 4, 5, 6], in this the whole group $SO(6, m)$ turned out to be the symmetry of the Lagrangian and not that of the equations of motion only. So in this aspect the $N = 4$ supergravity seems to be drastically different from $N = 3$ theory, but it is important to note that the $N = 4$ supergravity multiplet itself contains couple of scalar fields, describing the non-linear $\sigma$-model $SU(1, 1)/U(1)$. This leads to the existence of the dual versions for this theory, the most known cases being $SO(4)$ [7] and $SU(4)$ [8, 9] theories. But in turn, this should lead to the existence of dual versions for $N = 4$ supergravity — vector multiplet interaction as well. In [10] the hidden sector for $N = 4$ supergravity was constructed, which was the dual version for the system $N = 4$ supergravity with six vector multiplets and admitted the spontaneous supersymmetry breaking without a cosmological term. In this paper we start with the ordinary version for vector multiplets interaction and then, using the fact that all dual versions have the same scalar field geometry, we manage to construct the model, which turns out to be the generalization of our hidden sector to the case of arbitrary number of vector multiplets. Moreover, we show that the spontaneous supersymmetry breaking is still possible in the presence of matter fields.

2 Ordinary version

Let us start with a well known model of vector multiplets — $N = 4$ supergravity interaction when the scalar fields describe the non-linear $\sigma$-model $[SU(1, 1)/U(1)] \otimes [SO(6, m)/SO(6) \otimes SO(m)]$. We need the following set of fields: graviton $e_{\mu r}$, gravitini $\Psi_{\mu i}$, $i = 1, 2, 3, 4$, vector fields $A^{\mu A}$, $A = 1, 2, ...m + 6$, $g_{AB} = \text{diag}(- - - - - -)$, scalar fields $\Phi_a^A$, $a = 1, 2, ...6$ and complex scalars $z^\alpha$. In this, the spinor and scalar fields must satisfy the following constraints:

\begin{align}
SU(1, 1)/U(1) : & \quad z^\alpha \bar{z}_\alpha = -2, \quad z^\alpha \lambda_\alpha = 0 \\
SO(6, m)/SO(6) \otimes SO(m) : & \quad \Phi_a^A \Phi_{A b} = -\delta_{ab}, \quad \Phi_a^A \Omega_A = 0
\end{align}

In such formulation the theory have local $O(6) \otimes U(1)$ invariance, where the corresponding covariant derivatives look like, e.g.:

\begin{align}
D^\mu \Phi_a^A &= \partial^\mu \Phi_a^A - (\Phi_a^A \partial^\mu \Phi_b^A), \quad \Phi_a^A D^\mu \Phi_b^A = 0 \\
D^\mu z^\alpha &= \partial^\mu z^\alpha + \frac{1}{2}(\bar{z} \partial^\mu z) z^\alpha, \quad \bar{z} D^\mu z^\alpha = 0 \\
D^\mu \Omega^A &= D^\mu_\alpha \Sigma^A - \frac{1}{4}(\Phi_a^A \partial^\mu \Phi_b^A) \Sigma^{ab} \Omega^A + \frac{1}{4}(\bar{z} \partial^\mu z) \Omega^A \\
D^\mu \lambda_\alpha &= D^\mu_\alpha \lambda_\alpha - \frac{1}{4}(\Phi_a^A \partial^\mu \Phi_b^A) \Sigma^{ab} \lambda_\alpha - \frac{1}{4}(\bar{z} \partial^\mu z) \lambda_\alpha
\end{align}
For the case of abelian vector multiplets the full Lagrangian (omitting four fermionic terms) has the form:

$$L_1 = -\frac{1}{2} R + i \epsilon^{\mu \nu \rho \sigma} \bar{\Psi}_\mu \gamma_5 \gamma_\nu D_\rho \Psi_\sigma + i \frac{1}{2} \bar{\Omega} \dot{D} \Omega + i \frac{1}{2} \bar{\lambda} \dot{D} \lambda + \frac{1}{2} D_\mu \Phi_a D_\mu \Phi_a + \frac{1}{2} D_\mu \alpha D_\mu \bar{\gamma}_a - \frac{1}{2} \Omega \gamma_\mu \gamma_\nu \Phi_a \bar{\tau}^a \Psi_\mu - \frac{1}{2} \bar{\lambda}_a \gamma_\mu \gamma_\nu \Phi_a \bar{\gamma} \bar{\gamma}_a \Psi_\mu - \frac{1}{4} \sqrt{|Kz|} \left\{ (A_\mu \nu)^2 + 2 (\Phi_a A_\mu \nu)^2 \right\} - \frac{\gamma_5}{4} \left\{ \frac{Mz}{Kz} - \frac{M \bar{z}}{K \bar{z}} \right\} (A_\mu \nu \bar{A}_{\mu \nu}) + \frac{1}{4} \bar{\Psi}_\mu \frac{1}{Kz} (A_\mu \nu - \gamma_5 \bar{A}_{\mu \nu}) \Phi_a \bar{\tau}^a \Psi_\mu + \frac{i}{4} \bar{\Omega} \gamma_\mu (\sigma A) \frac{1}{Kz} \Psi_\mu - \frac{i}{8} \bar{\lambda}_a \gamma_\mu \bar{\epsilon}_{\alpha \beta} \bar{z}_{\beta} \frac{z_a}{Kz} (\sigma A) \Phi_a \bar{\tau}^a \Psi_\mu + \frac{1}{8} \bar{\Omega} (\sigma A) \Phi_a \bar{\tau}^a \Psi_\mu \frac{1}{Kz} \bar{O} A + \frac{1}{4} \bar{\Omega} (\sigma A) \frac{z_a}{Kz} \bar{\epsilon}_{\alpha \beta} \bar{\lambda}_\beta$$

(4)

where

$$Kz = (K_\alpha z^\alpha), \quad Mz = (\bar{M}_\alpha z^\alpha)$$

(5)

and $K_\alpha, M_\alpha$ - constant vectors, satisfying the relations:

$$K_\alpha = \bar{\epsilon}_{\alpha \beta} \bar{K}_{\beta}, \quad (K_\alpha M_\alpha) = \frac{1}{2}$$

(6)

Here we introduced six antisymmetric matrices $(\bar{\tau}^a)_{ij}$ such that:

$$(\bar{\tau}^a)_{ij} = [(\bar{\tau}^a)_{ij}]^*, \quad \frac{1}{2} \epsilon^{ijkl} (\bar{\tau}^a)_{kl} = \frac{1}{2} \epsilon^{ijkl} (\bar{\tau}^b)_{ik} (\bar{\tau}^a)_{kj} = -2 \delta^i_k \delta^{ab}$$

(7)

Besides, we will need fifteen matrices

$$\left\{ \Sigma^{[ab]} \right\}_{ij} = \frac{1}{2} \left[ (\bar{\tau}^a)_{ik} (\bar{\tau}^b)_{kj} - (\bar{\tau}^b)_{ik} (\bar{\tau}^a)_{kj} \right]$$

(8)

which are generators of the group $O(6) \approx SU(4)$. Let us recall, that we use the $\gamma$-matrix representation where majorana spinors are real, so that in all expressions with spinors (including matrices $\tau^a$ and $\Sigma^{ab}$) matrix $\gamma_5$ plays the role of imaginary unit $i$. This leads, e.g.:

$$\gamma_\mu (\bar{\tau}^a)_{ij} = (\bar{\tau}^a)_{ij} \gamma_\mu, \quad (\Sigma^{ab})_{ij} \gamma_\mu = -\gamma_\mu (\Sigma^{ab})_{ij}$$

(9)

The Lagrangian (4) is invariant under the following local supertransformations:

$$\delta e_{\mu \nu} = i (\bar{\Psi}_\mu \gamma_5 \eta)$$

$$\delta \Psi_\mu = 2 D_\mu \eta + \frac{i}{4Kz} (\sigma A) \Phi_a \bar{\tau}^a \gamma_\mu \eta$$

$$\delta A_\mu^A = (\bar{\Psi}_\mu \bar{K} \Phi_a \bar{\tau}^a \eta) - i (\bar{\lambda}_a \gamma_\mu \phi_a \bar{\tau}^a K_\alpha \eta) + i (\bar{\Omega} A \gamma_\mu \bar{K} \eta)$$

2
\[
\delta \Omega^A = - \frac{1}{2Kz} \left[ (\sigma A)^A + \Phi_a^A \Phi_b^B (\sigma A)^B \right] \eta - i \hat{D} \Phi_a^A \bar{\tau}^a \eta \tag{10}
\]
\[
\delta \lambda_\alpha = \frac{1}{4Kz} \varepsilon_{\alpha \beta \gamma} \bar{z}^\beta (\sigma A) \Phi_a^A \bar{\tau}^a \eta - i \hat{D} z^\alpha \eta
\]
\[
\delta \Phi_a = (\Omega \bar{\tau}^a \eta) \quad \delta \bar{z}_\alpha = 2(\bar{\lambda}_\alpha \eta)
\]

Now we can switch on a non-abelian gauge interactions. As usual, we will assume that fields \(\Phi_a^A\), \(\Omega^A\), \(A^A_\mu\) are transformed under the adjoint representation of some gauge group \(G\) with structure constants \(f^{ABC}\). In this, we have to replace all the derivatives by covariant ones, e.g.:

\[
\begin{align*}
\partial_\mu \Phi_a^A & \rightarrow D_\mu \Phi_a^A = \partial_\mu \Phi_a^A - f^{ABC} \Phi_b^B A^C_\mu \\
\partial_\mu \Omega^A & \rightarrow D_\mu \Omega^A = \partial_\mu \Omega^A - f^{ABC} \Omega^B A^C_\mu \\
A^A_\mu & \rightarrow D_\mu A^A_\mu - \partial_\mu A^A_\mu - f^{ABC} A^B_\mu A^C_\nu
\end{align*}
\tag{11}
\]

Then, in order to restore the invariance of the Lagrangian spoiled by such replacement, one have to introduce the following additional terms to the Lagrangian:

\[
\Delta L = f^{ABC} \left\{ \frac{1}{24} \bar{\Psi}_\mu \sigma^{\mu \nu} \Gamma^{ABC}(\bar{z}K) \Psi_\nu + \frac{i}{12} (\bar{\Psi} \gamma) \Gamma^{ABC} K^\alpha \lambda_\alpha + \frac{1}{2} \bar{\lambda}_\alpha K^\alpha \Sigma^{AB} \Omega^C + \right.
\]
\[
+ \frac{i}{4} (\bar{\Psi} \gamma) \Sigma^{AB}(\bar{z}K) \Omega^C - \frac{1}{2} \bar{\Omega} (\bar{z}K) \bar{\tau}^a \Phi_a^B \Omega^C + \frac{1}{24} \bar{\Omega} D^{ABC}(\bar{z}K) \Omega^D \right\} - \left. (\bar{z}K)^2 \left\{ \frac{1}{4} (f^{ABC} \Phi_a^A \Phi_b^B)^2 + \frac{1}{6} (f \Phi_a^A \Phi_b^B)^2 \right\} \right.
\tag{12}
\]

as well as complete the supertransformation laws with:

\[
\begin{align*}
\delta' \Psi_\mu &= - \frac{i}{12} \gamma_\mu f^{ABC} \Gamma^{ABC}(\bar{z}K) \eta \\
\delta' \Omega^A &= - \frac{1}{2} (\bar{z}K) \{ f^{ABC} + \Phi_a^A \Phi_b^D f^{DBC} \} \Sigma^{BC} \eta \\
\delta' \lambda_\alpha &= - \frac{1}{12} \varepsilon_{\alpha \beta \gamma} \bar{z}^\beta (\bar{z}K) f^{ABC} \Gamma^{ABC} \eta
\end{align*}
\tag{13}
\]

Here we introduce the notations:

\[
\Sigma^{AB} = \Phi_a^A \Phi_b^B \Sigma^{ab} \quad \Gamma^{ABC} = \Phi_a^A \Phi_b^B \Phi_c^C \Gamma^{abc}
\tag{14}
\]

where matrices \((\Gamma^{abc})_{ij}\), which are antisymmetric on \(a, b, c\) and symmetric on \(i, j\), are determined by the relation:

\[
(\Gamma^{abc})_{ij} = (\tau^{ia})_{ik}(\bar{\tau}^b)^{kl}(\tau^{lc})_{lj}
\tag{15}
\]

As in the \(N = 3\) supergravity case, global symmetry group \(O(6, m)\) is non-compact, so one can consider a lot of different gaugings in this theory. The first class consists of usual non-abelian gaugings with the gauge groups like \(O(3) \otimes O(3) \otimes H\), \(O(3) \otimes O(2, 1) \otimes H\), \(O(3) \otimes O(3, 1) \otimes H\) and so on, where \(H\) is some compact group. In general such a gauging leads to the non-zero value of a cosmological term, but in some cases cosmological term can be fine.
tuned to zero by adjusting the values of different gauge coupling constant. One of the most interesting models of this type is the one constructed in [11], which gives \( N = 4 \rightarrow N = 1 \) breaking.

The second class of models corresponds to the possibility to have a gauge group which contains translations as well as rotations. An example of such gauging was constructed some time ago [12] and appeared to be the first (as far as we know) model giving spontaneous supersymmetry breaking with two arbitrary scales. But the gravitini masses turn out to be

\[
\mu_1 = \mu_2 = \frac{m_1 + m_2}{2} \quad \text{and} \quad \mu_3 = \mu_4 = \frac{m_1 - m_2}{2},
\]

so that only \( N = 4 \rightarrow N = 2 \) partial Higgs effect is possible.

The last possibility is the "abelian" gaugings corresponding to pure translations similar to the model we have constructed for the \( N = 3 \) supergravity [13]. But in the \( N = 4 \) case such models give non-zero value of a cosmological term.

### 3 Dual version

In this section we are going to construct the dual version which will be the generalization of the hidden sector [14] to the case of arbitrary number of vector multiplets. The crucial point is that as the scalar field geometry remains unchanged we may use all the terms without vector fields from the previous section. Let us start by rewriting the corresponding parts of the Lagrangian (4) and supertransformations (10) in terms of independent spinor and scalar fields. For that purpose we introduce a kind of light cone variables:

\[
\Phi_a^A = (x_a^m + E_{am}, x_a^m - E_{am}, \phi_a^A), \quad a, m = 1, 2, ..., 6 \tag{16}
\]

Now, by introducing a new field

\[
\pi^{nm} = (E^{-1})^{am}x_a^n - (E^{-1})^{an}x_a^m \tag{17}
\]

we can solve the constraint (2) for \( x_a^m \):

\[
x_a^m = \frac{1}{4}(2\delta_a^m + \Phi_{a\hat{A}}\Phi_{b\hat{A}})(E^{-1})^{bm} + \frac{1}{2}E_{en}\pi^{nm}; \tag{18}
\]

Besides, in order all the scalar fields kinetic terms to be diagonal, one have to make a change \( \Phi_a^A \rightarrow E_{am}\Phi^{mA} \).

Analogously, we solve the constraints (2) for spinor fields introducing new fields \( \Omega^A = (\xi^m + \chi_m; \xi^m - \chi_m, \Omega^A) \). This gives:

\[
\xi^m = -(E^{-1})^{am}x_a^n\chi_n + \frac{1}{2}\Phi^{mA}\Omega_A. \tag{19}
\]

Here two changes of variables \( \chi_m \rightarrow \frac{1}{\sqrt{2}}E_{am}\chi^a \) and \( \Omega \rightarrow (\Omega + \Phi^{mA}\chi_m) \) are necessary to have canonical kinetic terms for spinors.

Now we are ready to rewrite all the terms without vector fields in our new variables. Let us start with the hidden sector, the part of the Lagrangian without matter fields \( \Phi^{mA} \) and
The corresponding part of the supertransformations (10) has the form:

\[ \Omega^\Lambda: \]

\[
L_1 = -\frac{1}{2} R + \frac{1}{2} D_\mu z D_\mu \tilde{z} + \frac{1}{2} (S_\mu^+)^2 + \frac{1}{2} (P_\mu)^2 +
\]

\[
+ i \bar{\Psi}_\mu \gamma_5 \gamma_\mu \left\{ D_\alpha + \frac{1}{4} (S_\alpha^- + P_\alpha)_{ab} \Sigma^{ab} \right\} \Psi_\beta +
\]

\[
+ \frac{i}{\sqrt{2}} \chi^a \gamma_\mu \left\{ \delta a D_\mu - (S_\mu^- - P_\mu)_{ab} + \frac{1}{4} \delta a (S_\mu^- + P_\mu)_{cd} \Sigma^{cd} \right\} \chi^b +
\]

\[
+ \frac{i}{\sqrt{2}} \lambda_\alpha \gamma_\mu \left\{ D_\mu - \frac{1}{4} (S_\mu^- + P_\mu)_{ab} \Sigma^{ab} \right\} \lambda_\alpha -
\]

\[
- \frac{1}{2} \lambda_\alpha \gamma_\mu \gamma_\nu D_\mu z^a \Psi_\mu - \frac{1}{2} \lambda_\alpha \gamma_\mu \gamma_\nu (S_\mu^+ + P_\nu)_{ab} \tilde{\tau}^b \Psi_\mu
\]

(20)

where we have introduced the following notations:

\[
(S_\mu^\pm)_{ab} = \frac{1}{2} (\partial_\mu E_{am} (E^{-1})_m^b \pm (a \leftrightarrow b))
\]

\[
(P_\mu)_{ab} = E_{am} \partial_\mu \pi^{mn} E_{bn}
\]

(21)

The corresponding part of the supertransformations (11) has the form:

\[
\delta \Psi_\mu = 2 D_\mu \eta + \frac{1}{2} (S_\mu^- + P_\mu)_{ab} \Sigma^{ab} \eta
\]

\[
\delta \chi^a = - i \gamma_\mu (S_\mu^+ + P_\mu)_{ab} \tilde{\tau}^b \eta
\]

\[
\delta \bar{\chi}^a = 2 (\bar{\lambda}_a \eta)
\]

\[
\delta \lambda_\alpha = - i \gamma_\mu D_\mu z^a \eta
\]

\[
\delta \bar{\lambda}_a = \frac{1}{2} (\bar{\chi}^a E_{bn} \tilde{\tau}^b \eta)
\]

\[
\delta \pi^{mn} = \frac{1}{2} (\chi^a [(E^{-1})_a^m (E^{-1})_m^n - (E^{-1})_b^m (E^{-1})_a^n] \tilde{\tau}^b \eta)
\]

(22)

Note, that all the derivatives are the \(U(1)\)-covariant ones and contain \((\bar{z} \partial_\mu z)\) terms in according to (3).

Part of the Lagrangian (12) with matter fields looks like:

\[
L_2 = \frac{1}{2} E_{am} E_{an} \partial_\mu \Phi^m \partial_\mu \Phi^n + \frac{i}{\sqrt{2}} \bar{\Omega} \gamma_\mu \left\{ D_\mu + \frac{1}{4} (S_\mu^- + P_\mu)_{ab} \Sigma^{ab} \right\} \Omega -
\]

\[
- i \bar{\chi}_a \gamma_\mu E_{am} \partial_\mu \Phi^m \Omega - \frac{1}{2} \Omega \gamma_\mu \gamma_\nu E_{am} \partial_\mu \Phi^m \tilde{\tau}^a \Psi_\mu
\]

(23)

and the corresponding supertransformations:

\[
\delta \Omega = - i \gamma_\mu E_{am} \partial_\mu \Phi^m \bar{\tau}^a \eta
\]

\[
\delta \Phi^m = \bar{\Omega} (E^{-1})_a^m \bar{\tau}^a \eta
\]

(24)

In this, the expression for the \((P_\mu)_{ab}\) takes the form:

\[
(P_\mu)_{ab} = E_{am} [\partial_\mu \pi^{mn} + \frac{1}{2} (\Phi^m \tilde{\tau}^a \Phi^n)] E_{bn}
\]

(25)

The essential part of the dual version is the hidden sector (14). Note that our current notations differ from whose used in (15). The main difference is that for the non-linear \(\sigma\)-model \(SU(1, 1)/U(1)\) we use doublet of scalar fields \(z_\alpha\), satisfying the constraint (15). Besides,
let us denote twelve vector fields as \((A_\mu)_m^\hat{\alpha}\), \(\hat{\alpha} = 1, 2\). In this, part of the Lagrangian containing vector fields will have the form:

\[
L_3 = -\frac{1}{4} U^\alpha_{\mu\nu} \tau^a_{\mu\nu} - \frac{1}{2} \pi^{mn}(\bar{A}_\mu)_m^\hat{\alpha} \varepsilon_{\hat{\alpha}\hat{\beta}} (A_\mu)_n^\hat{\beta} - \frac{i}{4\sqrt{2}} \chi^a \gamma^\mu (\sigma U)^a \Psi_\mu - \frac{1}{4\sqrt{2}} \bar{\Psi}_\mu (U^\mu_{\mu\nu} - \gamma_5 \bar{U}^\mu_{\mu\nu}) \tau^a \Psi_\nu - \frac{i}{8\sqrt{2}} \bar{\lambda}_\alpha \gamma^\mu \varepsilon_{\alpha\beta} \chi^\beta (\sigma U^a)^{\hat{\alpha} \tau^a} \Psi_\mu - \bar{\lambda}_\alpha \varepsilon^{\alpha\beta} \bar{\xi}_\beta (\sigma U)^a \chi^a + \frac{1}{8\sqrt{2}} \bar{\chi}^a (\sigma U^a)^{\beta \tau^b} \chi^a
\]

and the appropriate part of the supertransformations:

\[
\delta \Psi_\mu = \frac{i}{4\sqrt{2}} (\sigma U^a)^{\hat{\alpha} \tau^a} \gamma_\mu \eta
\]

\[
\delta (A_\mu)_m^\hat{\alpha} = \frac{E_{ma}}{\sqrt{2}} \left\{ (\bar{\Psi}_\mu \bar{\tau}^a z^\alpha G_\alpha^\hat{\beta} \eta) + i(\bar{\chi}^a \gamma_\mu z^\alpha G_\alpha^\hat{\beta} \eta) - i(\bar{\lambda}_\alpha \gamma_\mu \bar{\tau}^a G_\alpha^\hat{\beta} \eta) \right\}
\]

\[
\delta \chi^a = \frac{1}{2\sqrt{2}} (\sigma U)^a \eta \quad \delta \lambda_\alpha = \frac{1}{4\sqrt{2}} \varepsilon_{\alpha\beta} \chi^\beta (\sigma U^a)^{\hat{\alpha} \tau^a} \eta
\]

Here we introduced the following notation:

\[
U^\alpha_{\mu\nu} = (E^{-1})^{ma} z^\alpha \bar{H}_\alpha^\hat{\alpha} (A_\mu)_m^\hat{\alpha},
\]

while matrices \(G\) and \(H\) have the form:

\[
G_\alpha^\hat{\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & \gamma_5 \\ 1 & -\gamma_5 \end{pmatrix}, \quad H^{\hat{\alpha} \alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -\gamma_5 \\ 1 & \gamma_5 \end{pmatrix}.
\]

They satisfy the following useful relations:

\[
\varepsilon_{\alpha\beta} G^{\beta\hat{\alpha}} = G_\alpha^\hat{\alpha}, \quad \varepsilon^{\alpha\beta} H_\beta^\hat{\alpha} = -H^{\hat{\alpha} \alpha}, \quad H_\alpha^\hat{\alpha} G^\hat{\beta} = -\varepsilon^{\hat{\alpha}\hat{\beta}}
\]

\[
\bar{G}^{\alpha\hat{\alpha}} G_\beta^\hat{\beta} = -\gamma_5 \varepsilon^{\hat{\alpha}\hat{\beta}}, \quad H^{\hat{\alpha} \alpha} H_\beta^\hat{\beta} = -\gamma_5 \varepsilon^{\hat{\alpha}\hat{\beta}}, \quad H^{\hat{\alpha} \alpha} G_\beta^\hat{\beta} = \delta^{\alpha\beta}
\]

Now let us turn to the matter fields. Having in our disposal all the terms without vector fields it is a relatively easy task to complete the Lagrangian using ordinary Noether procedure. It turns out that the only additional terms to the Lagrangian are:

\[
L_4 = -\frac{i}{2} \bar{\Omega}^\hat{A} \frac{1}{|Kz|^2} \left\{ (\sigma V)^{\hat{A}} + (\sigma C)^{\hat{A}} \right\} K^\alpha \lambda_\alpha + \frac{1}{8\sqrt{2}} \bar{\Omega}^\hat{A} (\sigma U)^a \bar{\tau}^a \Omega^\hat{A} +
\]

\[
+ \frac{i}{4} \bar{\Omega}^\hat{A} \gamma^\mu \frac{1}{Kz} \left\{ (\sigma V)^{\hat{A}} + (\sigma C)^{\hat{A}} \right\} \Psi_\mu -
\]

\[
- \frac{1}{4|Kz|^2} (V^\hat{A}_{\mu\nu} + C_{\mu\nu}^{\hat{A}})^2 + \frac{\gamma_5}{4 \left| \frac{Mz}{Kz} - h.c. \right|} (C_{\mu\nu}^{\hat{A}})^2 -
\]

\[
- \frac{1}{4} (V^\hat{A}_{\mu\nu} + 2C_{\mu\nu}^{\hat{A}}) \bar{W}^\hat{A}_{\mu\nu}
\]

and to the supertransformations, correspondingly:

\[
\delta \bar{\Omega}^{\hat{A}} = -\frac{1}{2Kz} \left\{ (\sigma C)^{\hat{A}} + (\sigma V)^{\hat{A}} \right\} \eta
\]

\[
\delta C^{\hat{A}}_\mu = i(\bar{\Omega}^\hat{A} \gamma_\mu (Kz) \eta) + i(\bar{\chi}^a \gamma_\mu E_{am} \Phi^{m\hat{A}} (Kz) \eta) + \bar{\Psi}_\mu \bar{\tau}^a E_{am} \Phi^{m\hat{A}} (Kz) \eta) - i(\bar{\lambda}_\alpha \gamma_\mu \bar{K}_\alpha \bar{\tau}^a E_{am} \Phi^{m\hat{A}} \eta)
\]
where $K_\alpha$, $M_\alpha$ are the same as in the (4) and we have introduced:

$$V_\mu^A = \frac{1}{\sqrt{2}} \{(K \bar{\tau}) U_{\mu}^{a} + (\bar{K} \tau) U_{\mu}^{*a}\} E_{am} \Phi_{m^A}$$

$$W_\mu^A = \frac{\gamma_5}{\sqrt{2}} \{(K \bar{\tau}) U_{\mu}^{*a} - (\bar{K} \tau) U_{\mu}^{a}\} E_{am} \Phi_{m^A}$$

\[ \text{(33)} \]

4 Supersymmetry breaking

Thus we have managed to construct the dual version of the $N = 4$ supergravity — vector multiplets system which indeed generalize our hidden sector (4) to the case of arbitrary number of vector multiplets. As we have already mentioned, the scalar field geometry are the same as in the ordinary version, but the global symmetry of the Lagrangian and the transformation properties of the vector fields differ in these two models. Now, instead of $O(6, m)$ group, we have $O(m - 6) \otimes \text{GL}(6)$ global symmetry, as well as fifteen translations $\pi^{mn} \to \pi^{mn} + \Lambda^{mn}$, under which all the vector fields are inert. Therefore, by analogy with the $N = 3$ case, one can try to introduce the masses of the vector fields $(A_\mu)_m^\dot{\alpha}$ by using $\pi^{mn}$ as Goldstone fields and replacing

$$\partial_\mu \pi^{mn} \to \partial_\mu \pi^{mn} - M_{\alpha}^{mn}(A_\mu)_p^\dot{\alpha}$$

\[ \text{(34)} \]

The only difficulty that arises here is related to the ”axion” term $\frac{1}{2} \pi^{mn} \epsilon_{\dot{\alpha}\dot{\beta}}(\bar{A}_\mu)_m^\dot{\alpha}(A_\mu)_n^\dot{\beta}$ in the Lagrangian. It can be completed to a gauge invariant expression only provided the tensors $M_{\dot{\alpha}\dot{\beta}np}$ are fully skew-symmetric in their indices. Besides, as will become clear below, one has to impose the condition

$$M_{\dot{\alpha}\dot{\beta}np} = -\frac{1}{6} \epsilon_{\dot{\alpha}\dot{\beta}} \epsilon_{npqr} M_{\dot{\beta}rs}$$

\[ \text{(35)} \]

In this the gauge invariant combination has the form

$$\frac{1}{2} \pi^{mn} \epsilon_{\dot{\alpha}\dot{\beta}}(\bar{A}_\mu)_m^\dot{\alpha}(A_\mu)_n^\dot{\beta} + \frac{2}{3} M_{\dot{\alpha}\dot{\beta}np} \epsilon_{\dot{\beta}\dot{\gamma}}(A_\mu)_p^\dot{\alpha}(A_\nu)_m^\dot{\beta}(\bar{A}_\mu)_n^\dot{\gamma}$$

\[ \text{(36)} \]

which corresponds to the self-interaction of abelian massive vector fields!

As usual, substitution (34) breaks the invariance of the Lagrangian under the supertransformations. To compensate for this noninvariance, it is necessary to complete the Lagrangian of the hidden sector with

$$L' = -\frac{1}{6} \langle \bar{K}^\delta \rangle Q_{abc} Q_{abc} \langle z K \rangle^\delta + \frac{1}{72} \epsilon_{abcd} \epsilon_{\dot{\alpha}\dot{\beta}Q_{abc} Q_{def}} +$$

$$+ \frac{1}{24 \sqrt{2}} \bar{\psi}_{\mu} \sigma_{\mu} (z K)^{\dot{\delta}} Q_{abc} \Gamma_{abc} \psi_{\nu} + \frac{i}{4 \sqrt{2}} \bar{\psi}_{\mu} \gamma_{\mu} (\bar{z})^{\dot{\alpha}} \tilde{Q}_{abc} \Sigma^{ab} \chi_{c} +$$

$$+ \frac{i}{12 \sqrt{2}} \bar{\psi}_{\mu} \gamma_{\mu} Q_{abc} \Gamma_{abc} \bar{K} \alpha \alpha \chi_{c} - \frac{1}{2 \sqrt{2}} \tilde{\chi}_{a}^{\dot{\delta}} Q_{abc} (\bar{z} K)^{\dot{\delta}} \chi_{b}^{c} +$$

$$+ \frac{1}{24 \sqrt{2}} \bar{\chi}_{a}^{\dot{\delta}} Q_{abc} (\bar{z} K)^{\dot{\delta}} \chi_{c} + \frac{1}{2 \sqrt{2}} \tilde{\chi}_{\alpha}^{\dot{\alpha}} Q_{abc} \Sigma^{ab} \chi_{c}$$

\[ \text{(37)} \]
where the following notation is introduced:

\[ Q^{\hat{a}}_{\hat{a}bc} = E_{am}E_{bn}E_{cp}M^{mnp}_{\hat{a}} \]  \hspace{1cm} (38)

In this the total Lagrangian is invariant under supertransformations with additional terms of the form

\[
\begin{align*}
\delta' \Psi_\mu &= -\frac{i}{12\sqrt{2}} \gamma_\mu (zK)^{\hat{a}} Q_{abc} \bar{\Gamma}^{abc} \eta \\
\delta' \chi^a &= -\frac{1}{2\sqrt{2}} (zK)^{\hat{a}} Q_{abc} \Sigma^{bc} \eta \\
\delta' \lambda_\alpha &= -\frac{1}{12\sqrt{2}} \varepsilon^{\alpha\beta} z_\beta (zK)^{\hat{a}} Q_{abc} \bar{\Gamma}^{abc} \Omega^{\hat{a}}
\end{align*}
\]  \hspace{1cm} (39)

As for the matter fields, it turns out that no new terms in the supertransformations appear, while the Lagrangian have to be completed with the only new term

\[
\frac{1}{24\sqrt{2}} \bar{\Omega}^{\hat{a}} (zK)^{\hat{a}} Q_{abc} \bar{\Gamma}^{abc} \Omega^{\hat{a}}
\]  \hspace{1cm} (40)

So far, we have not given the tensors \( M^{mnp}_{\hat{a}} \) explicitly. Clearly, in the general case one gets a family of tensors related via the O(6) - transformations. It would be convenient to choose a parameterization in which the gravitino mass matrix is diagonal. Then in the concrete representation of \( \tau \)-matrices (see the Appendix) one has

\[ M^{mnp}_{\hat{a}} = m_1 \delta_1^{[m} \delta_2^{n} \delta_3^{p]} + m_2 \delta_1^{[m} \delta_2^{n} \delta_4^{p]} + m_3 \delta_2^{[m} \delta_4^{n} \delta_6^{p]} - m_4 \delta_3^{[m} \delta_4^{n} \delta_5^{p]} \]  \hspace{1cm} (41)

Here the gravitino masses turn out proportional to the combinations

\[
\begin{pmatrix}
    m_1 + m_2 + m_3 + m_4 \\
    m_1 - m_2 - m_3 + m_4 \\
    m_1 - m_2 + m_3 - m_4 \\
    m_1 + m_2 - m_3 - m_4
\end{pmatrix}
\]  \hspace{1cm} (42)

Analysing potential (the first two terms in the (37)), one can show that its minimum corresponds to \(< z_0 > = 1, < z_1 > = 0, (E_{am} = \delta_{am})\). Besides, by virtue of the properties (35) and (44), the value of the potential at the minimum equals zero which corresponds to the absence of a cosmological term.

Hence, the theory constructed really admits a spontaneous supersymmetry breaking without a cosmological term and with \textit{four} arbitrary scales, including the partial super-Higgs effect \( N = 4 \rightarrow N = 3, N = 4 \rightarrow N = 2 \) and \( N = 4 \rightarrow N = 1 \). But, due to the constraint (35), which is essential for the absence of the cosmological term, vacuum expectation value of \((zK)^{\hat{a}} Q_{abc}^{\hat{a}}\) turns out to be zero and the spontaneous supersymmetry breaking does not lead to the appearance of the matter field mass terms.
5 Conclusion

Thus we have seen that in the "minimal" version for $N = 4$ supergravity with matter till now no one have managed to construct a fully satisfactory model i.e. the model which gives spontaneous supersymmetry breaking with different scales, a zero value for cosmological term without fine tuning and the possibility to have partial super-Higgs effect $N = 4 \rightarrow N = 1$. The dual version, constructed in this paper, indeed has all these desired properties but failed to generate soft breaking terms for matter fields. Let us stress, however, that in both cases there exist a lot of possible gaugings, which have not been fully explored so far.

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A

We have used the following \( \tau \)-matrix representation

\[
\tau^1 = \begin{pmatrix}
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 \\
0 & -1 & 0 & 0 \\
-1 & 0 & 0 & 0
\end{pmatrix}, \quad \tau^4 = \begin{pmatrix}
0 & 0 & 0 & \gamma_5 \\
0 & 0 & -\gamma_5 & 0 \\
0 & \gamma_5 & 0 & 0 \\
-\gamma_5 & 0 & 0 & 0
\end{pmatrix}
\]

\[
\tau^2 = \begin{pmatrix}
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0
\end{pmatrix}, \quad \tau^5 = \begin{pmatrix}
0 & 0 & \gamma_5 & 0 \\
0 & 0 & 0 & \gamma_5 \\
-\gamma_5 & 0 & 0 & 0 \\
0 & -\gamma_5 & 0 & 0
\end{pmatrix}
\]

\[
\tau^3 = \begin{pmatrix}
0 & 1 & 0 & 0 \\
-1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0
\end{pmatrix}, \quad \tau^6 = \begin{pmatrix}
0 & -\gamma_5 & 0 & 0 \\
\gamma_5 & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_5 \\
0 & 0 & -\gamma_5 & 0
\end{pmatrix}
\]

Here the matrices \( \Gamma^{abc} \) satisfy the relation

\[
\Gamma^{abc} = \frac{1}{6} \gamma_5 \varepsilon^{abcdef} \Gamma^{def}
\]

(43)

In this representation, the following matrices are diagonal:

\[
\Gamma^{123} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}, \quad \Gamma^{156} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

\[
\Gamma^{246} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}, \quad \Gamma^{345} = \begin{pmatrix}
-1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(45)

as well as the matrices \( \Gamma^{456} \), \( \Gamma^{234} \), \( \Gamma^{135} \) and \( \Gamma^{126} \) related with them through relation (44).
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