A CP sensitive asymmetry in the three–body decay $\tilde{t}_1 \to b\tilde{\nu}_\tau \tau^+$

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Abstract

We consider the three–body decay $\tilde{t}_1 \to b\tilde{\nu}_\tau \tau^+$ and propose the asymmetry of the $\tau$ polarization perpendicular to the decay plane as a CP sensitive observable. We calculate this asymmetry in the Minimal Supersymmetric Standard Model with the parameters $\mu$ and $A_t$ complex. In the parameter domain where the decay $\tilde{t}_1 \to b\tilde{\nu}_\tau \tau^+$ is important this asymmetry can go up to $\pm 30\%$. We also estimate the event rates necessary to observe this asymmetry at 90% CL.

1 Introduction

The experimental search for supersymmetric (SUSY) particles will have high priority at the upgraded Tevatron and at LHC. The precision determination of the SUSY parameters will be the main goal of a future $e^+e^-$ linear collider. The analysis of scalar top quarks $\tilde{t}_i$, $i = 1, 2$, will be particularly interesting, because of the large top Yukawa coupling involved in this system. Due to the effects of the top Yukawa coupling the lighter stop may be relatively light if not the lightest charged SUSY particle.

A phenomenological study of production and decays of 3rd generation sfermions at an $e^+e^-$ linear collider with cms energy in the range $\sqrt{s} = 0.5 - 1$ TeV has been given in Ref. There it has been shown that by measuring production cross sections with polarized beams the masses of the top squarks $m_{\tilde{t}_1}, m_{\tilde{t}_2}$ and their mixing angle $\theta_{\tilde{t}}$ can be determined. It has also been shown that with an integrated luminosity of $500 fb^{-1}$ a precision of about 1% can be achieved. The precision to be expected for the underlying SUSY parameters has also been estimated. This analysis has been performed in the Minimal Supersymmetric Standard Model (MSSM) with real parameters.

However, the assumption that all MSSM parameters are real may be too restrictive. In principle, the Higgs–higgsino mass parameter $\mu$ and the trilinear scalar coupling parameters $A_f$ of the sfermions $f$ may be complex. These complex parameters are new sources of CP violation and may provide potentially large SUSY contributions to the electric dipole moments (EDM) of electron and neutron. The very small experimental upper limits of the electron and neutron EDMs, therefore, may lead to restrictions on the complex phases. Recent analyses have shown that in mSUGRA–type models the phase of $\mu$ is restricted to $|\varphi_\mu| \lesssim 0.1 - 0.2$ for a universal scalar mass parameter $M_0 \lesssim 400$ GeV, whereas the phase of the universal trilinear scalar coupling parameter $A_0$ is correlated with $\varphi_\mu$, but otherwise unrestricted. One can conclude that in models with more general parameter specifications the phases of the parameters $A_f$ of the 3rd generation sfermions are not restricted at one–loop level by the electron and neutron EDMs. There may be restrictions at two–loop level. Furthermore, a complex trilinear coupling parameter $A_t$ in the stop system can also lead to interesting CP violating effects in top quark production, as discussed in.
For a complete analysis of the stop sector one has to take into account that the parameters μ and $A_t$ may be complex. The parameter $|μ|$ and its phase $ϕ_μ$ will presumably be determined by measuring suitable observables of the chargino and neutralino sector. For the determination of $|A_t|$ and its phase $ϕ_{A_t}$ appropriate observables in the stop sector have to be measured. However, it may be difficult to define a suitable $CP$ sensitive observable in stop decays if the main decay modes are two–body decays.

In the present paper we define a $CP$ sensitive asymmetry in the decay $\tilde{t}_1 \rightarrow b\tilde{ν}_ττ^+$. As this is a three–body decay the polarization of the $τ^+$ normal to the decay plane is sensitive to $CP$ violation. The appropriate $CP$ sensitive observable is defined by the asymmetry of the $τ$ polarization perpendicular to the decay plane. As we will show this asymmetry can go up to 30%. Moreover, we show the existence of parameter regions where the decay $\tilde{t}_1 \rightarrow b\tilde{ν}_ττ^+$ has a sufficient branching ratio allowing for the measurement of this asymmetry. We perform our analysis in the MSSM with $μ$ and $A_t$ complex. We focus on scenarios where only the decays $\tilde{t}_1 \rightarrow b\tilde{ν}_τℓ$, $\tilde{t}_1 \rightarrow b\tilde{ν}_ττL$, $\tilde{t}_1 \rightarrow c\tilde{χ}_1^0$ are kinematically allowed. We assume that the lightest neutralino $\tilde{χ}_1^0$ is the lightest SUSY particle (LSP). The $CP$ asymmetry defined above is analogous to that defined in Ref. |11| in case of top quark decays.

In Section 2 we shortly review stop mixing in the presence of complex parameters. In Section 3 we give the formulae of the $CP$ violating observable. In Section 4 we present numerical results for the phase dependences of the $CP$ asymmetry. We give an theoretical estimate of the event rates necessary to observe the $CP$ sensitive asymmetry at an $e^+e^−$ linear collider with $\sqrt{s} = 0.5 – 1$ TeV. Section 5 contains a short summary.

## 2 $\tilde{t}_L – \tilde{t}_R$ Mixing

We first give a short account of $\tilde{t}_L – \tilde{t}_R$ mixing in the case the parameters $μ$ and $A_t$ are complex. The masses and couplings of the $\tilde{t}$–squarks follow from the hermitian $2 \times 2$ mass matrix which in the basis $(\tilde{t}_L, \tilde{t}_R)$ reads

$$L^2_M = - (\tilde{t}_L^T, \tilde{t}_R^T) \begin{pmatrix} M^2_{t_{LL}} & e^{-iϕ_2} |M^2_{t_{LR}}| \\ e^{iϕ_2} |M^2_{t_{LR}}| & M^2_{t_{RR}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix},$$  \hspace{1cm} (1)$$

where

$$M^2_{t_{LL}} = M^2_{Q} + \left( \frac{1}{2} - \frac{2}{3} \sin^2 Θ_W \right) \cos 2ϕ \ m_Z^2 + m_t^2, \hspace{1cm} \tag{2}$$

$$M^2_{t_{LR}} = M^2_{Q} + \frac{2}{3} \sin^2 Θ_W \cos 2ϕ \ m_Z^2 + m_t^2, \hspace{1cm} \tag{3}$$

$$M^2_{t_{RR}} = (M^2_{t_{LR}})^* = m_t(A_t - μ^* \cot β), \hspace{1cm} \tag{4}$$

$$ϕ_2 = \arg[A_t - μ^* \cot β], \hspace{1cm} \tag{5}$$

where $\tan β = v_2/v_1$ with $v_1(v_2)$ being the vacuum expectation value of the Higgs field $H_1^0(H_2^0)$, $m_t$ is the mass of the top quark and $Θ_W$ is the weak mixing angle, $μ$ is the Higgs–higgsino mass parameter and $M_Q, M_Γ, A_t$ are the soft SUSY–breaking parameters of the stop system. The mass eigenstates $\tilde{t}_i$ are $(\tilde{t}_1, \tilde{t}_2) = (\tilde{t}_L, \tilde{t}_R) R^T$ with

$$R^i = \begin{pmatrix} e^{iϕ_i} \cos θ_i & \sin θ_i \\ -sin θ_i & e^{-iϕ_i} \cos θ_i \end{pmatrix}, \hspace{1cm} \tag{6}$$

|2|
with
\[
\cos \theta_t = \frac{-|M_{t_{LR}}^2|}{\sqrt{|M_{t_{LR}}^2|^2 + (m_{t_1}^2 - M_{t_{LL}}^2)^2}}, \quad \sin \theta_t = \frac{M_{t_{RR}}^2 - m_{t_1}^2}{\sqrt{|M_{t_{LR}}^2|^2 + (m_{t_1}^2 - M_{t_{LL}}^2)^2}}.
\]

The mass eigenvalues are
\[
m_{t_{1,2}}^2 = \frac{1}{2} \left( (M_{t_{LL}}^2 + M_{t_{RR}}^2) \pm \sqrt{(M_{t_{LL}}^2 - M_{t_{RR}}^2)^2 + 4|M_{t_{LR}}^2|^2} \right).
\]

\(\hat{t}_L - \hat{t}_R\) mixing is naturally large because of the large top quark mass entering in the off–diagonal elements of the mass matrix (see Eqs. (10) and (12)). This is important for the CP sensitive observable discussed below, because it is proportional to \(\sin \theta_t \cos \theta_t\). Note further that for \(|A_l| \gg |\mu| \cot \beta\) we have \(\varphi_t \approx \varphi_A\).

### 3 Tau Polarization Asymmetry

The parts of the Lagrangian relevant for the three–body decay \(\hat{t}_1 \to b\tilde{\nu}_r\tau^+\) are
\[
\mathcal{L}_{\hat{t}_1\hat{t}_1} = g\hat{b}(k_{1j}^r P_L + l_{1j}^r P_R)\tilde{\chi}_j^+ \hat{t}_1 + h.c.,
\]
\[
\mathcal{L}_{\tilde{\chi}_1\tilde{\nu}_r} = g\tilde{\chi}_j^+(k_{j}^r P_R + l_j^r P_L)\tilde{\nu}_r + h.c.,
\]
where
\[
l^r_{1j} = -e^{-i\varphi_i} \cos \theta_1 V_{1j} + Y_1 \sin \theta_1 V_{1j}, \quad k_{1j}^r = Y_b e^{-i\varphi_i} \cos \theta_1 U_{1j}^*,
\]
\[
l^r_j = -V_{1j}, \quad k_j^r = Y_r U_{1j}^*.
\]
g is the weak coupling constant, \(P_{R,L} = 1/2(1 \pm \gamma_5)\) and the Yukawa couplings are \(Y_1 = m_t/\sqrt{2} m_W \sin \beta, \quad Y_b = m_b/\sqrt{2} m_W \cos \beta, \quad Y_r = m_r/\sqrt{2} m_W \cos \beta\). The unitary \(2 \times 2\) matrices \(U\) and \(V\) diagonalize the chargino mass matrix \(\tilde{M}_1\). A convenient parametrization of \(U\) and \(V\) for complex parameters can be found in [13].

The three body decay \(\hat{t}_1 \to b\tilde{\nu}_r\tau^+\) proceeds via exchange of charginos \(\tilde{\chi}_i^\pm, i = 1, 2\). We consider the polarization of the outgoing \(\tau^+\) perpendicular to the decay plane, which is sensitive to CP violation. We define the unit vector
\[
\hat{e}_N = \frac{\vec{p}_{\tau} \times \vec{p}_{b}}{|\vec{p}_{\tau} \times \vec{p}_{b}|}.
\]
The average polarization of the outgoing \(\tau^+\) in the direction \(\hat{e}_N\) in the decay \(\hat{t}_1 \to b\tilde{\nu}_r\tau^+\) is given by
\[
P_N^+ = \frac{B(\hat{t}_1 \to b\tilde{\nu}_r\tau^+(\hat{e}_N)) - B(\hat{t}_1 \to b\tilde{\nu}_r\tau^-(\hat{e}_N))}{B(\hat{t}_1 \to b\tilde{\nu}_r\tau^+(\hat{e}_N)) + B(\hat{t}_1 \to b\tilde{\nu}_r\tau^-(\hat{e}_N))},
\]
For the CP conjugated process we get
\[
P_N^- = \frac{B(\hat{t}_1 \to b\tilde{\nu}_r\tau^-(\hat{e}_N)) - B(\hat{t}_1 \to b\tilde{\nu}_r\tau^+(\hat{e}_N))}{B(\hat{t}_1 \to b\tilde{\nu}_r\tau^-(\hat{e}_N)) + B(\hat{t}_1 \to b\tilde{\nu}_r\tau^+(\hat{e}_N))}
\]
Note that \(P_N^- = -P_N^+\) since the couplings are complex conjugate to each other. The observables \(P_N^+\) and \(P_N^-\) are odd under naive time reversal \(T_N\), where only the polarization and momentum vectors are reversed but initial and final states are not interchanged. We can define a CP sensitive asymmetry of the form
\[
A_{CP} = \frac{1}{2}(P_N^+ - P_N^-).
\]
In order to obtain a $CP$ asymmetry which for practical reasons is more useful, we have to take into account the subsequent decay $\tilde{\nu}_i \rightarrow \tilde{\chi}_i^0 \nu$. The complete decay chain is then $\tilde{t}_1 \rightarrow b \tilde{\nu}_i \tau^+ \rightarrow b \tilde{\tau}_i \chi_1^0 \nu$. Another decay chain leading to the same final state is $\tilde{t}_1 \rightarrow b \tilde{\nu}_i \nu \rightarrow b \tau_1 \chi_1^0 \nu$, where in the second step the $\tilde{\tau}_i$ decays into $\tau \chi_1^0$. In the $\tilde{\tau}_i$ rest system $\tilde{\tau}_i \rightarrow \tau \chi_1^0$ has an isotropic decay distribution. The decay mode $\tilde{t}_1 \rightarrow b \tilde{\tau}_i \nu \rightarrow b \tau_1 \chi_1^0 \nu$ can be easily incorporated in our consideration.

However, the decay $\tilde{t}_1 \rightarrow b W \chi_1^0 \rightarrow b \tau \chi_1^0 \nu$ also leads to the same final state $\chi_1^0$. The decay $\tilde{t}_1 \rightarrow b W \chi_1^0 \rightarrow b \tau \chi_1^0 \nu$ is more involved, because the $W$ polarization leads to a non-vanishing correlation between the $\vec{p}_i - \vec{p}_0$ plane and the $\tau$ polarization. While this may also lead to $CP$ sensitive effects, in the present paper we confine ourselves to the discussion of the $CP$ asymmetry in the decay chains $\tilde{t}_1 \rightarrow b \tilde{\nu}_i \tau \rightarrow b \tau \chi_1^0 \nu$ and $\tilde{t}_1 \rightarrow b \tilde{\tau}_i \nu \rightarrow b \tau \chi_1^0 \nu$, assuming that $m_{\tilde{t}_1} < m_W + m_{\chi_1^0} + m_b$.

We define a $CP$ asymmetry similar to Eq. (14), but for the final state $b \tau \chi_1^0 \nu$:

$$\mathcal{A}_{CP} = \frac{1}{2}(\mathcal{P}_{N}^{\tau^+} - \mathcal{P}_{N}^{\tau^-}),$$

with

$$\mathcal{P}_{N}^{\tau^+} = \frac{B(\tilde{t}_1 \rightarrow f(\vec{e}_N)) - B(\tilde{t}_1 \rightarrow f(-\vec{e}_N))}{B(\tilde{t}_1 \rightarrow f(\vec{e}_N)) + B(\tilde{t}_1 \rightarrow f(-\vec{e}_N))}, \quad \mathcal{P}_{N}^{\tau^-} = \frac{B(\tilde{t}_1 \rightarrow \bar{f}(\vec{e}_N)) - B(\tilde{t}_1 \rightarrow \bar{f}(-\vec{e}_N))}{B(\tilde{t}_1 \rightarrow \bar{f}(\vec{e}_N)) + B(\tilde{t}_1 \rightarrow \bar{f}(-\vec{e}_N))},$$

where we have introduced a shorthand notation $f(\vec{e}_N) \equiv \tilde{\chi}_i^0 \tilde{\nu}_i b \tau^+(\vec{e}_N)$ and $\bar{f}(\vec{e}_N) \equiv \tilde{\chi}_i^0 \nu \tilde{b} \tau^-(\vec{e}_N)$. The branching ratio for the decay $\tilde{t}_1 \rightarrow \tilde{\chi}_i^0 \nu \tau$ is to a good approximation given by

$$B(\tilde{t}_1 \rightarrow \tilde{\chi}_i^0 \nu \tau) \approx B(\tilde{t}_1 \rightarrow b \tilde{\nu}_i \tau)B(\tilde{\nu}_i \rightarrow \chi_1^0 \nu) + \sum_{i=1}^2 B(\tilde{t}_1 \rightarrow b \tilde{\tau}_i \nu)B(\tilde{\tau}_i \rightarrow \chi_1^0 \tau).$$

This can also be seen by using the formulas for the 4-body stop decays given in [10]. Relation (19) holds if $|m_{\tilde{\tau}_i} - m_{\tilde{\nu}_i}| \gg \Gamma_{\tilde{\tau}_i} + \Gamma_{\tilde{\nu}_i}$, which is naturally fulfilled. If $B(\tilde{\nu}_i \rightarrow \tilde{\chi}_1^0 \nu) \approx 1$ and $B(\tilde{\tau}_i \rightarrow \chi_1^0 \tau) \approx 1$, then Eq. (14) can be rewritten as

$$\mathcal{A}_{CP} \approx \frac{\Gamma(\tilde{t}_1 \rightarrow b \tilde{\nu}_i \tau^+(\vec{e}_N)) - \Gamma(\tilde{t}_1 \rightarrow b \tilde{\nu}_i \tau^-(\vec{e}_N)) - \Gamma(\tilde{t}_1 \rightarrow b \tilde{\tau}_i \tau^-(\vec{e}_N)) + \Gamma(\tilde{t}_1 \rightarrow b \tilde{\tau}_i \tau^+(\vec{e}_N))}{2 \Gamma_{\text{unpol}}},$$

where $\Gamma_{\text{unpol}} = \Gamma(\tilde{t}_1 \rightarrow b \tilde{\nu}_i \tau) + \sum_{i=1}^2 \Gamma(\tilde{t}_1 \rightarrow b \tilde{\tau}_i \nu)$. A very useful approximation for $\mathcal{A}_{CP}$ can be obtained in the limit $\varphi_\mu \rightarrow 0$

$$\mathcal{A}_{CP} \approx \frac{g^4}{\Gamma_{\text{unpol}}} \frac{2 \Gamma(\tilde{t}_1 \rightarrow b \tilde{\nu}_i \tau) |\sin 2\theta_{\tilde{\tau}}| |\mu| \sin \varphi_\mu}{2 \Gamma_{\text{unpol}}},$$

where

$$I = \frac{1}{2m_{\tilde{t}_1}} \int \frac{d^3 p_{\tilde{b}}}{(2\pi)^3 2E_\tilde{b}} \frac{d^3 p_\tau}{(2\pi)^3 2E_\tau} \frac{d^3 p_{\tilde{\nu}_i}}{(2\pi)^3 2E_{\tilde{\nu}_i}} \frac{d^3 p_{\tilde{\tau}_i}}{(2\pi)^3 2E_{\tilde{\tau}_i}} \frac{|p_\tau^2| |p_{\tilde{\nu}_i}| |\sin \theta_{\tilde{\tau}}| (p_\tau^2 - m_{\tilde{\tau}_i}^2)(p_\tau^2 - m_{\tilde{\tau}_i}^2)}{(p_{\tilde{b}}^2 - m_{\tilde{b}}^2)(p_{\tilde{b}}^2 - m_{\tilde{b}}^2)}.$$
4 Numerical Results

In the following we present numerical results for the $CP$ sensitive asymmetry $A'_{CP}$ defined in Eqs. (17) and (18). Our input parameters are $m_{\tilde{t}_1}, m_{\tilde{g}}, m_{\tilde{\nu}_e}, |A_t|, \varphi_{A_t}, \tan \beta, M_2, |\mu|$, assuming $M_1 = 5/3 \tan^2 \Theta_W M_2$, with $M_1$ and $M_2$ real. For simplicity we set $\varphi_{A_t} = 0$. We impose the approximate necessary condition for the tree--level vacuum stability $|A_t|^2 < 3(M_Q^2 + M_U^2 + (m_{\tilde{\nu}}^2 + m_{\tilde{\mu}}^2) \cos^2 \beta - m_{1/2}^2 / 2)$ [10]. For the pseudoscalar Higgs mass which appears in this condition we choose for definiteness $m_A = 150$ GeV. We have checked that in the numerical examples studied below the restrictions from the electron and neutron EDMs at two--loop level [3] are fulfilled.

First we consider the influence of the parameter $\varphi_{A_t}$. In Figs. 1a and 1b we show $A'_{CP}$ as a function of $\varphi_{A_t}$ for $M_Q > M_U$ ($|\cos \theta_1| < |\sin \theta_1|$) and $M_Q < M_U$ ($|\cos \theta_1| > |\sin \theta_1|$), respectively. We display the asymmetry for the four scenarios ($|\mu| = 400$ GeV, $\tan \beta = 3$), ($|\mu| = 400$ GeV, $\tan \beta = 10$), ($|\mu| = 700$ GeV, $\tan \beta = 3$) and ($|\mu| = 700$ GeV, $\tan \beta = 10$), taking $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{\nu}_e} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $M_2 = 350$ GeV. We fix the mass of $\tilde{\tau}_1$ by taking $M_{\tilde{\tau}_1} = 0.9 M_\tau$. As can be seen, the $CP$ asymmetry $A'_{CP}$ is much larger for $M_Q > M_U$ than for $M_Q < M_U$. The reason is, that for $M_Q < M_U$ the $t_1$ has a larger $t_L$ component than for $M_Q > M_U$, implying a stronger coupling to the gaugino component of the charginos. This in turn implies a larger $\Gamma_{\text{unpol}}$ for $M_Q < M_U$. Clearly $A'_{CP}$ has to vanish for the $CP$ conserving case $\varphi_{A_t} = 0, \pm \pi$ (see the corresponding factor in Eq. (21)). For better understanding of the value of $A'_{CP}$ in the region $|\varphi_{A_t}| \lesssim \pi/4$, we show in Fig. 1c the branching ratio $B(\tilde{t}_1 \to \tilde{\chi}_1^0 \nu_{\tau} \bar{\nu}_{\tau})$ as a function of $\varphi_{A_t}$, for the two scenarios ($|\mu| = 400$ GeV, $\tan \beta = 3$) and ($|\mu| = 700$ GeV, $\tan \beta = 3$), keeping the other parameters as in Fig. 1a. The decay width $\Gamma(\tilde{t}_1 \to \tilde{\chi}_1^0 \nu_{\tau} \bar{\nu}_{\tau})$ has a similar behaviour.

The minimum at $\varphi_{A_t} = 0$ can be traced back to the fact that $|\theta_1^*|$ (see Eq. (11)) has a minimum there, due to a negative interference between the gaugino and higgsino contribution.

In Fig. 3 we show $A'_{CP}$ for three values of $|A_t| = 600$ GeV, 1000 GeV and 1300 GeV, assuming $M_Q > M_U$. The other parameters are $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{\nu}_e} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $A_t = 0$, $M_2 = 350$ GeV, $|\mu| = 600$ GeV and $\tan \beta = 3$. For $|A_t| \lesssim 1000$ GeV $A'_{CP}$ increases with increasing $|A_t|$, because $|\sin 2 \theta_1|$ increases (see Eq. (21)). The decrease of $A'_{CP}$ for $|A_t| \gtrsim 1000$ GeV is explained by the fact that $\Gamma_{\text{unpol}}$ increases stronger than $|\sin 2 \theta_1|$. The polarization of the $\tau$ is analysed through its decay distributions. Usually the decay modes $\tau \to \nu_{\tau}, \bar{\nu}_{\tau}, a_{\nu}, \bar{a}_{\nu}, \mu \nu_{\tau}, \mu \bar{\nu}_{\tau}, e \nu_{\tau}, e \bar{\nu}_{\tau}$ are used as analyzers. As we are interested in the transverse polarisation of the $\tau$ lepton, we take only the $\nu_{\tau}$ and $a_{\nu}$ final states for our analysis. The sum of the branching ratios of these two decay modes is $\approx 34\%$. We take for the sensitivities for measuring the polarization of the $\tau$ lepton the values quoted in Ref. [17] for an ideal detector. Moreover, the numbers quoted are for longitudinal tau polarization and it is expected that the sensitivities for transversely polarized tau leptons are somewhat smaller. To account for both effects we assume a reduction of the sensitivity $S$ of 10% [18]. Furthermore, we assume that the direction of flight of the $\tau$ lepton can be reconstructed. Following Ref. [18], the error in measuring the polarization asymmetry is given by

$$\delta P^{\tau} = \frac{1}{S_{\text{red}}} \sqrt{N_{\tau}}$$

where $S_{\text{red}} = 0.3, N_{\tau} = B(t \to \tilde{\chi}_1^0 \nu_{\tau} \bar{\nu}_{\tau}) N_{\tilde{t}_1}$, with $N_{\tilde{t}_1}$ being the number of produced $\tilde{t}_1 \tilde{\tau}_1$ pairs in the process $e^+ e^- \to \tilde{t}_1 \tilde{\tau}_1$.

The minimum number $N_{\tilde{t}_1}$ of produced $\tilde{t}_1$ pairs necessary to observe $A'_{CP}$ at 90% confidence level (CL) is then given by

$$N_{\tilde{t}_1} = \frac{1}{B(t \to \tilde{\chi}_1^0 \nu_{\tau} \bar{\nu}_{\tau})} \left( \frac{1.64}{S_{\text{red}} A'_{CP}} \right)^2.$$  

In Table 1 we display the values of the asymmetry $A'_{CP}$ and the numbers $N_{\tilde{t}_1}$ needed to observe this
asymmetry at 90% CL.

Table 1: The CP asymmetry $A'_{CP}$ defined in Eq. (17) and the number $N_{\tilde{t}_1}$ of $\tilde{t}_1\bar{\tilde{t}}_1$ pairs required to measure this CP asymmetry at 90% CL, choosing $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $M_2 = 350$ GeV and $|A_t| = 1000$ GeV. For $M_{\tilde{Q}} > M_{\tilde{U}}$ we take $|\mu| = 700$ GeV, $\tan \beta = 3$ and for $M_{\tilde{Q}} < M_{\tilde{U}}$ we take $|\mu| = 400$ GeV, $\tan \beta = 10$.

| $A'_{CP}$ | $N_{\tilde{t}_1} \times 10^{-3}$ | $\varphi_{A_t}$ | $M_{\tilde{Q}} > M_{\tilde{U}}$ | $M_{\tilde{Q}} < M_{\tilde{U}}$ |
|-----------|----------------|-----------|----------------|----------------|
| 0.05      | 31             | $\pi/2$  | $M_{\tilde{Q}} > M_{\tilde{U}}$ | $M_{\tilde{Q}} < M_{\tilde{U}}$ |
| 0.11      | 7              | $\pi/4$  | $M_{\tilde{Q}} > M_{\tilde{U}}$ | $M_{\tilde{Q}} < M_{\tilde{U}}$ |
| 0.21      | 4              | $\pi/8$  | $M_{\tilde{Q}} > M_{\tilde{U}}$ | $M_{\tilde{Q}} < M_{\tilde{U}}$ |
| 0.033     | 48             | $\pi/2$  | $M_{\tilde{Q}} < M_{\tilde{U}}$ | $M_{\tilde{Q}} < M_{\tilde{U}}$ |
| 0.030     | 58             | $\pi/4$  | $M_{\tilde{Q}} < M_{\tilde{U}}$ | $M_{\tilde{Q}} < M_{\tilde{U}}$ |
| 0.018     | 165            | $\pi/8$  | $M_{\tilde{Q}} < M_{\tilde{U}}$ | $M_{\tilde{Q}} < M_{\tilde{U}}$ |

The cross section for $e^+e^- \rightarrow \tilde{t}_1\bar{\tilde{t}}_1$ for $m_{\tilde{t}_1} = 240$ GeV is about $5 \text{ fb}$ at $\sqrt{s} = 500$ GeV and in the range $15 - 120 \text{ fb}$ at $\sqrt{s} = 800$ GeV, depending on stop mixing and beam polarization. If the branching ratio $B(\tilde{t}_1 \rightarrow b\tilde{\nu}_\tau\tau^+) = \text{order of 5%}$ or larger, then there are good prospects to measure the CP asymmetry $A'_{CP}$ at a linear collider with an integrated luminosity of $500 \text{ fb}^{-1}$.

5 Summary

In this paper we have studied the CP violating observable defined by the asymmetry of the $\tau$ polarization perpendicular to the decay plane in the three–body decay $\tilde{t}_1 \rightarrow b\tilde{\nu}_\tau\tau^+$. In the parameter domain $m_{\tilde{t}_1} < m_{\tilde{\chi}_1^+} + m_b$, $m_{\tilde{t}_1} < m_W + m_{\chi_1^0} + m_b$ the branching ratio for this decay mode can be quite large. We have calculated this CP asymmetry and the branching ratios in the MSSM with the parameters $\mu$ and $A_t$ complex. We give numerical predictions for the case of an $e^+e^-$ linear collider with cms energy $\sqrt{s} = 0.5 - 1$ TeV. The asymmetry can reach values up to $\pm 30\%$. We give a theoretical estimate of the number of produced $\tilde{t}_1\bar{\tilde{t}}_1$ pairs necessary for measuring this asymmetry at 90% CL. There are good prospects to measure this CP asymmetry at a linear collider with an integrated luminosity of $500 \text{ fb}^{-1}$, if the branching ratio of the decay $\tilde{t}_1 \rightarrow b\tilde{\nu}_\tau\tau^+$ is about 5% or larger.
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Figure 1: The $CP$ sensitive asymmetry $A'_{CP}$ as a function of $\varphi_{A_t}$. The input parameters are $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{\nu}_t} = 200$ GeV, $M_2 = 350$ GeV, $|A_t| = 1000$ GeV, ($\tan \beta = 3, |\mu| = 400$ GeV; thick solid line), ($\tan \beta = 10, |\mu| = 400$ GeV; thin solid line), ($\tan \beta = 3, |\mu| = 700$ GeV; thick dashed line), ($\tan \beta = 10, |\mu| = 700$ GeV; thin dashed line), for the cases: a) $M_{\tilde{Q}} > M_{\tilde{G}}$ and b) $M_{\tilde{Q}} < M_{\tilde{G}}$. 

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Figure 2: The branching ratio of $\tilde{t}_1 \to \tau b \tilde{\chi}_1^0 \nu_\tau$ as a function of $\varphi_{A_t}$, with $M_{\tilde{Q}} > M_{\tilde{U}}, m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $\tan \beta = 3, M_2 = 350$ GeV, $|A_t| = 1000$ GeV for the two cases $|\mu| = 400$ GeV (solid line), $|\mu| = 700$ GeV (dashed line).

Figure 3: The $CP$ sensitive asymmetry $A'_{CP}$ as a function of $\varphi_{A_t}$. The input parameters are $m_{\tilde{t}_1} = 240$ GeV, $m_{\tilde{t}_2} = 800$ GeV, $m_{\tilde{\nu}_\tau} = 200$ GeV, $M_2 = 350$ GeV, $\tan \beta = 3, |\mu| = 600$ GeV for $|A_t| = 600$ GeV (solid line), $|A_t| = 1000$ GeV (dashed line), $|A_t| = 1300$ GeV (dotted line).