A composite dual-porosity fractal model for channel-fractured horizontal wells

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ABSTRACT

The channel fracturing technique has been proven to provide much higher conductive fracture networks by placing discontinuously proppant inside fracture packs. Despite the great success of this new fracturing technique, there is still a lack of models and methods to characterize non-uniform placement of fracture proppant and heterogeneous permeability distribution within stimulated-reservoir-volume (SRV). In this article, a dual-porosity model is coupled with the tri-linear flow model to quantify the production performance of channel fractured horizontal wells. As a consequence of channel fracturing, the non-uniform distribution of fracture networks, and the heterogeneities of both matrix/fracture porosity and permeability are characterized using fractal theory. By implementing the Bessel Function and Laplace transform techniques, analytical solutions are derived by integrating fluid flow across primary hydraulic fractures, SRV and unstimulated reservoir matrices. Through quantitative comparisons of well bottom-hole pressure history, a synthetic fine-grid numerical simulation example is implemented to verify the accuracy of analytical solutions. Sensitivity analysis of fractal dimension, fractal connectivity-index and primary fracture connectivity is carried out to quantify the temporal and spatial consequences of channel fracturing on both reservoir/fracture heterogeneity and well productivity throughout well life.

1. Introduction

In recent years, shale oil and gas have greatly contributed to the US energy portfolio and are expected to be the main source of energy within the next 20 years. The majority of commercial production in shale play requires intensive fracturing along with long horizontal well drilling (Abdulal, Samandarli, & Wattenbarger, 2011; Britt & Smith, 2009; Wang & Wu, 2013). As a widespread and effective approach, multi-stage fractured horizontal wells have made oil and gas production more efficient and economical in shale plays. In addition, in view of incidences of natural fractures, hydraulic fracturing cannot only create high-conductive and long primary fractures but also reactivate sealed natural fractures to generate intensively complex fracture networks around wells (Clarkson, 2013; Mayerhofer et al., 2010; Wang et al., 2015; Wang, Su, Zhang, Sheng, & Ren, 2015; Yuan et al., 2015).

The primary goal of hydraulic fracturing is to create highly conductive flow paths for the reservoir fluids coming out. Therefore, it is necessary to create enough high conductivity for fractures to guarantee excellent performance from well production (Britt & Bennett, 1985; Economides & Nolte, 2000). Inside hydraulic fracturing, the proppant pack serves as a mechanical support to open hydraulic fractures, which form a highly conductive porous medium for fluid flow. Proppant has evolved from nutshells and wood chips, to natural sands, and to strong ceramic particles. Fracturing fluid includes gelled oils, water-based fluids, and viscoelastic surfactant-based solutions (Denney, 2010; Medvedev, Kraemer, Pena, & Panga, 2013; Stephens et al., 2007; Vincent, 2009). The majority of existing published works have focused on enhancing the total flow rates inside hydraulic fractures; however, the optimization of fracture proppant placement is still in progress.

The channel fracturing technique is a novel approach that replaces the homogenous placement of proppants inside hydraulic fractures using the heterogeneous distribution of proppants (Gillard et al., 2010). The objective of channel fracturing is to enhance fracture conductivity by creating a more stable network with open channels. The heterogeneous conglomerations of injected proppant can support the openness of hydraulic fractures for the transport of reservoir fluids with much smaller flow resistance. The conglomerates of degradable fibrous materials...
This work introduces a quantitative characterization of fracture networks with open channels after channel fracturing. Fractal theory is combined with the tri-linear flow model in a dual-porosity matrix-fracture system to present the production of multistage fractured horizontal wells. The non-uniform distribution of fracture permeability and porosity are also characterized using the fractal permeability–porosity relationship. Our analytical solutions are verified using numerical simulations.

2. Fracture conductivity with open channels

The objective of the channel fracturing technique is to introduce a heterogeneous fracture structure with open channels inside fractures instead of the conventional homogenous distribution of proppant pack to enhance the flow capacity of reservoir fluids. As shown in Figure 1, after the dispersion, mixing and settling process of fracturing proppant pulse, the open channels are formed inside the fractures, with the help of the fracturing fluid viscosity and fiber concentration properly tailored inside the fractures. In-house laboratory experiments were conducted to estimate the enhancement of fracture conductivity with channel fracturing (Medvedev et al., 2013). The results indicated that the enhancement of fracture conductivity by a non-uniform proppant cluster is several orders of magnitude larger than that of a 20/40-mesh sand proppant pack with even distribution (Figure 2).

In addition, as the hydraulic fracture propagates into the deep reservoir, the ‘stimulation intensity’ gradually attenuates, which leads to a decrease of permeability within SRV. Hence, in this article, the concept of threshold permeability (TP) is proposed to represent permeability at the outer boundary of stimulated reservoir volume (which is still much higher than matrix permeability). In addition, three scenarios with different threshold fracture conductivity – $K_{FT1}$, $K_{FT2}$, $K_{FT3}$ –

**Figure 1.** Schematic diagram of conventional fracturing and channel fracturing technology, modified from Gillard et al. (2010).
are compared. Variations in the fracture conductivity of the proppant pack and permeability along the SRV are shown in Figure 3. As the fracture networks propagate within the region of SRV, the average permeability decreases gradually and reaches $K_{T3}$ at the boundary of SRV, very close to the initial matrix permeability. This means that less fracture proppant is pumped into the induced fracture network remote from the primary fracture. In Figure 3, $K_{T1}$ represents the case with constant fracture permeability with uniform fracture proppant placement throughout SRV, which is described by

\begin{align}
k_m(y) &= k_m \left( \frac{y}{b_f} \right)^{d_m - \theta_m - 2} \\
\phi_m(y) &= \phi_m \left( \frac{y}{b_f} \right)^{d_m - 2} \\
k_f(y) &= k_f \left( \frac{y}{b_f} \right)^{d_f - \theta_f - 2} \\
\phi_f(y) &= \phi_f \left( \frac{y}{b_f} \right)^{d_f - 2} \\
k_F(x) &= k_F \left( \frac{x}{x_f} \right)^{d_F - \theta_F - 2} \\
\phi_F(x) &= \phi_F \left( \frac{x}{x_f} \right)^{d_F - 2}
\end{align}

where $d_i$ denotes the fractal dimension, representing the dimension of the fractal fracture network embedded in the Euclidean matrix ($i = f$), fractal matrix porous media ($i = m$) and proppant pack ($i = F$); $b_f$ is half the width of the hydraulic fracture; $\theta_m, \theta_f$ is the connectivity index characterizing the diffusion process or diffusivity capacity – this parameter is related to the spectral exponent; $k_f, k_m, \phi_f, \phi_m$ are the properties at the boundary shared by the SRV; $k_F$ and $\phi_F$ are the properties at the boundary shared by the primary fracture; $x_f$ is half the length of the hydraulic fracture.
3. Mathematical model

3.1. Physical assumptions

The finite-conductive primary fracture is located centrally in a closed rectangular system, with a half-length of $x_f$, with $y_f$, and penetrates the entire formation vertically. The spacing among neighboring fractures is $2y_c$, and no flow boundary exists in the middle of them. The lateral boundaries perpendicular to the primary fracture plane are assumed at the distance of $x_e$ from the center of fractures. The number of fractures is $N = L_h/2y_e$, where $L_h$ is length of SRV. Therefore, the drainage area of facing left fracture is equal to $1/N$ of the total drainage area of MFHW with $N$ identical transverse hydraulic fractures, as shown in Figure 4. Therefore, the pressure-transient flow model of a horizontal well can be described as facing left fracture with constant production rate equal to $q_f = q/N$. Some basic assumptions commonly used for MFHW are applied, as follows:

1. Local thermal equilibrium, and single phase oil or gas flow are applied.
2. No flow interaction is considered between matrix systems and wellbore.
3. The interference among a transverse fracture is modeled using no-flow boundaries.

Figure 5 presents the scheme of the tri-linear flow model with fractal permeability and porosity for a HiWAY-Channel Fractured horizontal well. It includes linear flow in the primary fracture (region 1), the SRV (region 2) and the outer-reservoir region (region 3). The fluid flow in each region is represented by the diffusivity equation with two boundary conditions. In addition, Euclidean diffusivity equations in each region are replaced by the fractal diffusivity model incorporated with fractal permeability and porosity relationships. In Figures 4 and 5 the subscripts $m$ and $f$ denote the fractal matrix and fractal fracture, respectively, within the dual-porosity SRV region.

Figure 4. Arrangement of SRVs for fractured horizontal well with $N$ identical transverse hydraulic fractures.

3.2. Model descriptions

Dimensionless variables are defined as follows:

$$x_D = \frac{x}{x_f} \quad \text{(Dimensionless space variable)} \quad (7)$$

$$y_D = \frac{y}{y_f} \quad \text{(Dimensionless space variable)} \quad (8)$$

$$b_D = \frac{b_f}{x_f} \quad \text{(Dimensionless space variable)} \quad (9)$$

$$p_D = \frac{k_f h}{q B \mu} (p_i - p) \quad \text{(Dimensionless pressure variable)} \quad (10)$$

$$t_D = \frac{k_f t}{\phi c_t \mu x_f^2} \quad \text{(Dimensionless time variable)} \quad (11)$$

The diffusivity equations of incompressible fluid flow within different regions are derived as shown below:

- **Linear flow in outer unstimulated reservoir**

  The outer unstimulated reservoir is regarded as a single-continuum matrix medium. The diffusivity equation with dimensionless variables to describe the fluid linear-flow along $x$ direction is,

  $$\frac{\partial^2 p_{3D}}{\partial x_D^2} = \frac{k_2(\phi c_t)_2}{k_3(\phi c_t)_3} \frac{\partial p_{3D}}{\partial t_D} \quad (12)$$

  Using Laplace transform, the equation could be expressed as,

  $$\frac{\partial^2 p_{3D}}{\partial x_D^2} = sc_3 p_{3D} \quad (13)$$

  where $c_3 = (k_2(\phi c_t)_3)/k_3(\phi c_t)_2)$. Analytical solutions of Equation (12) under Laplace space can be obtained as

  $$\frac{\partial p_{3D}}{\partial x_D} = \sqrt{sc_3} \tanh \left( \sqrt{sc_3} - \sqrt{sc_3 x_D} \right) p_{2D}|_{x_D=1} \quad (14)$$

  Inner-boundary condition: $p_{3D}|_{x_D=1} = p_{2D}|_{x_D=1}

  Outer-boundary condition: $(\partial p_{3D}/\partial x_D)|_{x=x_D}=0$

- **Linear flow within dual-porosity SRV with fractal theory**

  With the assumption of a pseudo-steady state flow between matrix systems and fracture systems, the fluid flow within the matrix reservoir with fractal permeability and fractal porosity is described as shown below.
**Figure 5.** Schematic of HiWAY-Channel Fracturing fractal flow physical model.

Fluid flow in the matrix,

$$\frac{\alpha k_{2m} \rho}{\mu} k_m(y)(p_m - p_f) + \frac{\partial}{\partial t} (\rho \phi) = 0 \quad (15)$$

Introducing the fractal permeability and fractal porosity,

$$\frac{\alpha k_{2m} \rho}{\mu} \left( \frac{y}{b_f} \right)^{d_m-\theta_m-2} (p_f - p_m)$$

$$= \rho c_{l2m} \Phi_2 m \left( \frac{y}{b_f} \right)^{d_m-\theta_m-2} \frac{\partial p_m}{\partial t} \quad (16)$$

Combining with dimensionless variables,

$$\lambda (p_{fD} - p_{mD}) = (1 - \omega) y_D^2 \frac{\partial p_{mD}}{\partial t_D} \quad (17)$$

Where,

$$\lambda = \frac{k_3}{k_f} x_f^2, \omega = \frac{c_{l2f} \Phi_2 f}{c_{l2m} \Phi_2 m + c_{l2f} \Phi_2 f}$$

The governing flow equation along y direction in a dual-porosity system is,

$$- \frac{\partial}{\partial y} (\rho v_y) + \frac{\alpha k_{2m} \rho}{\mu} \left( \frac{y}{b_f} \right)^{d_m-\theta_m-2} (p_m - p_f) + q_{32}$$

$$= \frac{\partial}{\partial t} (\rho \phi) \quad (18)$$

where, $q_{32} = \rho \left( \frac{y}{b_f} \right)^{d_f-2} (1/x_f) (k_3/\mu) (\partial p_3/\partial x) |_{x=x_f}$ is fluid rate from the outer region 3 into region 2:

$$\frac{\partial^2 p_{fD}}{\partial y_D^2} + \frac{D_f - \theta_f}{y_D} \frac{\partial p_{fD}}{\partial y_D} + \lambda \frac{\theta_f}{b_D^2} \frac{\partial^2 p_{mD}}{\partial y_D^2} + \frac{\partial}{\partial x_D} \left|_{x_D=x_f} \right. = b_f^2 \omega y_D \frac{\partial p_{fD}}{\partial t_D} \quad (19)$$
Applying Laplace transform,
\[
\frac{\partial^2 p_{2D}}{\partial y_D^2} + \frac{d_f - \theta_f - 2}{y_D} \frac{\partial p_{2D}}{\partial y_D} + y_D^2 s_2 p_{2D} = 0
\]  
where,
\[
s_2 = \left[ \frac{(1 - \omega)s}{(1 - \omega)s(y_e/2)^{\alpha_m} + \lambda} - sw + \frac{k_3}{k_2} m_3 \right] \beta_D^2,
\]

Based on the formulas of Bessel Functions in Bowman (1985), this article defines the following coefficients thus,
\[
\alpha_2 = \frac{\theta_f + 3 - d_f}{2}, \quad \gamma_2 = \frac{\theta_f + 2}{2},
\]
\[
n_2 = \frac{\theta_f + 3 - d_f}{\theta_f + 2}, \quad \beta_2 = \frac{\sqrt{\gamma_2}}{\gamma_2}
\]

The analytical solution of Equation (20) can be obtained as,
\[
p_{2D} = y_D^{\alpha_2} \left\{ A_{l_{2m}} \left[ \frac{\sqrt{\gamma_2}}{\gamma_2} y_D^{\gamma_2} \right] + B_{K_{2n}} \left[ \frac{\sqrt{\gamma_2}}{\gamma_2} y_D^{\gamma_2} \right] \right\}
\]  
(21)

The first derivative of the above dimensionless pressure is derived as,
\[
\frac{\partial p_{2D}}{\partial y_D} = \sqrt{\gamma_2} y_D^{\gamma_2 + \gamma_2 - 1} \left\{ A_{l_{2m - 1}} \left[ \frac{\sqrt{\gamma_2}}{\gamma_2} y_D^{\gamma_2} \right] - B_{K_{2n - 1}} \left[ \frac{\sqrt{\gamma_2}}{\gamma_2} y_D^{\gamma_2} \right] \right\}
\]  
(22)

Inner-boundary condition: \( p_{2D} |_{y_D=1} = p_{1D} |_{y_D=1} \)

Outer-boundary condition: \( (\partial p_{2D} / \partial x_D) |_{y_D=y_e} = 0 \)
Combining with the given boundary conditions,
\[
\frac{\partial p_{2D}}{\partial y_D} \bigg|_{y_D=1} = m_2 p_{2D} \bigg|_{y_D=1}
\]  
(23)

where,
\[
m_2 = \sqrt{\frac{\gamma_1}{\gamma_1}} \left\{ I_{n_1-1} \left( \frac{\sqrt{\gamma_2}}{\gamma_2} \right) K_{n_1-1} \left( \frac{\sqrt{\gamma_2}}{\gamma_2} \right) y_D^{\gamma_2} \right\} - I_{n_1-1} \left( \frac{\sqrt{\gamma_2}}{\gamma_2} \right) K_{n_1-1} \left( \frac{\sqrt{\gamma_2}}{\gamma_2} \right) y_D^{\gamma_2} + I_{n_2-1} \left( \frac{\sqrt{\gamma_2}}{\gamma_2} \right) K_{n_2-1} \left( \frac{\sqrt{\gamma_2}}{\gamma_2} \right) y_D^{\gamma_2}
\]

- **Linear flow in hydraulic fracture with fractal theory**

The dimensionless governing equation for fluid flow inside a hydraulic fracture with fractal structures is expressed as,
\[
\frac{\partial^2 p_{FD}}{\partial x_D^2} + \frac{d_F - 2 - \theta_E \frac{\partial p_{FD}}{x_D}}{x_D} + \frac{2x_D^2 k_f}{k_1 b_f^2} \frac{\partial p_{2D}}{\partial y_D} \bigg|_{y_D=1} = \frac{x_D^2}{k_1 (\phi c_1)} \frac{\partial p_{FD}}{\partial y_D} \bigg|_{y_D=1}.
\]

Applying Laplace transform, and substituting the boundary condition of Equation (23) leads to,
\[
\frac{\partial^2 p_{FD}}{\partial x_D^2} + \frac{d_F - \theta_F - 2}{x_D} + \frac{2x_D^2 k_f}{k_1 b_f^2} d_1 x_D \frac{\partial p_{FD}}{\partial y_D} = 0,
\]

(25)

where
\[
d_1 = (c_1 s - a_1 m_2), \quad a_1 = \frac{2x_D^2 k_f}{k_1 b_f^2}, \quad c_1 = \frac{k_2 (\phi c_1)}{k_1 (\phi c_1)}.
\]

Similar to the above derivation processes for fluid flow within SRV, the following coefficients are defined:
\[
\alpha_1 = \frac{\theta_F + 3 - d_F}{2}, \quad \gamma_1 = \frac{\theta_F + 2}{2},
\]

\[
n_1 = \frac{\theta_F + 3 - d_F}{\theta_F + 2}, \quad \beta_1 = \frac{\sqrt{\delta_1}}{\gamma_1}
\]

The analytical solution of Equation (25) is obtained as:
\[
p_{FD} = x_D^{\alpha_1} \left[ A_1 I_{n_1} \left( \frac{\sqrt{\delta_1}}{\gamma_1} x_D \right) + B_1 K_{n_1} \left( \frac{\sqrt{\delta_1}}{\gamma_1} x_D \right) \right]
\]

The first derivative of the above dimensionless pressure is,
\[
\frac{\partial p_{FD}}{\partial x} = \sqrt{\frac{\gamma_1}{\gamma_1}} x_D^{\alpha_1+\gamma_1-1} \left\{ A_1 I_{n_1-1} \left[ \frac{\sqrt{\delta_1}}{\gamma_1} x_D \right] - B_1 K_{n_1-1} \left[ \frac{\sqrt{\delta_1}}{\gamma_1} x_D \right] \right\}
\]

(27)

By applying the inner and outer conditions, the transient-pressure solution is obtained as,
\[
p_{FD}|_{x_D=0} = \frac{b_1}{s} m_0
\]

(28)

where
\[
x_{rD} = \frac{b_f}{x_f}, \quad b_1 = - \frac{\pi k_f x_f}{k_1 b_f},
\]

\[
m_0 = x_{rD}^{1-\gamma_1} \left\{ I_{n_1} \left[ \left( \frac{\sqrt{\delta_1}}{\gamma_1} \right) x_{rD} \right] K_{n_1-1} \left[ \left( \frac{\sqrt{\delta_1}}{\gamma_1} \right) x_{rD} \right] + I_{n_1-1} \left[ \left( \frac{\sqrt{\delta_1}}{\gamma_1} \right) x_{rD} \right] K_{n_1-1} \left[ \left( \frac{\sqrt{\delta_1}}{\gamma_1} \right) x_{rD} \right] \right\} - I_{n_1-1} \left[ \left( \frac{\sqrt{\delta_1}}{\gamma_1} \right) x_{rD} \right] K_{n_1-1} \left[ \left( \frac{\sqrt{\delta_1}}{\gamma_1} \right) x_{rD} \right]
\]

Due to the assumption of pressure continuity at the interface between hydraulic fractures and the wellbore, the dimensionless wellbore pressure can be written as follows,
\[
p_{wd} = 1
\]

(29)

Where \( C_D \) denotes dimensionless wellbore storage coefficient and \( s \) denotes skin factor. Considering the constant-pressure condition, the bottom-hole flow rate \( Q_D \) is obtained as,
\[
Q_D = \frac{1}{s^2 p_{wd}}
\]

(30)

This solution can be inverted numerically from Laplace space using the Stehfest algorithm.

### 3.3. Model verifications

To verify the accuracy of the above mathematical model and analytical solutions, a synthetic numerical example is applied with a multi-fractured horizontal well in tight oil using ECLIPSE. To mimic the features of the above

| Table 1: Data used in the comparative analysis. |
|-----------------------------------------------|
| Matrix bulk permeability, \( k_m \), md       | 0.01 |
| Matrix bulk porosity, \( \phi_m \)           | 0.07 |
| Matrix bulk compressibility, \( c_m \), psi^{-1} | 3.6E-07 |
| Fracture bulk permeability, \( k_f \), md     | 0.05 |
| Fracture bulk porosity, \( \phi_f \)          | 1.5E-5 |
| Fracture bulk compressibility, \( c_f \), psi^{-1} | 5E-10 |
| Fracture length, \( x_f \), m                | 100  |
| Matrix shape factor, \( \alpha \)             | 3E-02 |
| Hydraulic fracture conductivity, \( k_f \), md | 2E03 |
| Dimensionless hydraulic fracture width, \( w_D \) | 1.43E-04 |
| Dimensionless reservoir in \( x \), \( x_D \)   | 1.55 |
| Dimensionless reservoir in \( y \), \( y_D \)   | 107.14 |
| Storativity ratio, \( \omega \)               | 2.98E-07 |
| Fracture space, \( y_D \), m                  | 200  |
description of the heterogeneity of both primary fracture conductivity and permeability within SRV, a logarithmic grid refinement (LGR) method is employed by defining each grid with different values of permeability and porosity. Figure 6 shows the comparison of bottom-hole pressure between the numerical simulation and semi-analytical solutions for a case with \( d_f = 1.75, \phi_f = 0 \). The excellent agreement indicates reasonable accuracy of the above analytical solutions.

In addition, the above analytical solutions are verified by comparing them to the results of Ozkan et al. (2009), with the assumption of uniform fracture properties along MFHW, where \( d_f = d_F = d_m = 2 \) and \( \theta_m = \theta_f = \theta_F = 0 \). As shown in Figure 7, comparative analysis reveals reasonable agreement. The values

**Figure 8.** Permeability distribution with respect to fractal dimension \( d_i \).

**Figure 9.** Effect of primary fracture fractal dimension \( d_f \) on well performance (hydraulic fracture).

**Figure 10.** Effect of SRV fractal dimension \( d_f \) on well performance (stimulated reservoir volume).
of parameter inputs for the model verification has been added in Table 1.

4. Results and discussion

Figure 8 shows the cases with different fractal dimensions. The characteristic length represents the distance from the starting points (i.e. wellbore or center of primary fracture) to the location of study points (i.e. inside primary fracture or SRV). The fractal dimension ‘2’ indicates the case with homogeneous distribution of fracture permeability, which is the example of conventional hydraulic fracturing. However, after implementing channel fracturing, due to the heterogeneous placement of proppant pack inside the primary fracture, the fractal dimension of the hydraulic primary fracture becomes larger than ‘2’. As a result, an increasing trend of fracture permeability with respect to characteristic length is induced for the channel-fracturing case. In addition, with the increase of fractal dimensions, the average values of fracture permeability also increase, which means the enhancement of the conductivity of average hydraulic fractures. However, within SRV, because of both the attenuation of fracturing energy and amounts of packed proppant as SRV propagates, the permeability shows a decreasing trend along the direction of SRV width. Hence, in this region, the fractal dimension becomes less than ‘2’.

Figure 11. Effect of connectivity index on fracture network system $\theta_f$ (stimulated reservoir volume).

Figure 12. Effect of connectivity index on fractal matrix system $\theta_m$ (stimulated reservoir volume).
The effects of the fractal dimension \((d_F \text{ and } d_f)\) of both primary fracture and SRV on production rates are evaluated in Figures 9–10. Figure 9 presents the impacts of the fractal dimension of the hydraulic fracture on well productivity. The larger the fractal dimension of the primary hydraulic fracture, the stronger the packed extent of proppant leading to higher well production. In addition, the fractal dimension of the primary fracture also has significant effects on long-term well production throughout the entire production period. Figure 10 indicates the impacts of SRV fractal dimension on well performance. The larger the fractal dimension of SRV, the more complex the fracture networks within SRV. With the increase of the fractal dimension, the increased average permeability inside SRV results in more oil production.

Figures 11 and 12 show the effects of the fractal connectivity-index of both the matrix system, \(\theta_m\), and SRV system, \(\theta_f\). The results indicate that the connectivity-index has negative effects on fluid flow capacity. The higher connectivity index of both the matrix and the fracture network, the lower the oil production performance. Figure 11 indicates that the effects of the fractal connectivity-index of the primary fracture, \(\theta_f\), can have a significant effect on well performance from the early to the intermediate production period. As for long-term production, the effects of the connectivity-index would disappear. Figure 12 represents the impact of the fractal matrix system connectivity index, \(\theta_m\), on well performance. It has significant effects on production rate from the intermediate to the late production period.

**Figure 13.** Permeability distribution within SRV caused by the increase of primary fracture conductivity.

**Figure 14.** Effect of hydraulic fracture permeability on dimensionless well production rate.
The different influencing periods of the fractal connectivity-index in the matrix system or fracture system may be attributed to their different contributions to well production at different production times. Due to the rapid fluid outflows from the highly conductive fracture system, the contribution of the fracture system mainly occurs during the early to intermediate production periods. After this, the fluid flow in the matrix reservoir starts to work.

Figure 13 shows the effects of primary hydraulic fracture conductivity on the distribution of permeability within SRV. As the values of primary fracture conductivity increase, the threshold permeability of $k_f$ at the boundary of SRV is also enhanced. As well performance with different primary fracture conductivity (see Figure 14) demonstrates, the choice of a highly-conductive primary fracture has a significant impact on production performance throughout well life. This is because enhancement of primary fracture conductivity not only leads to a decrease in fluid flow resistance inside the primary fracture but also results in significant improvement in fluid flow capacity within SRV in reservoirs.

**Conclusions**

In this article, unlike the conventional fracture model with uniform distribution of fracture proppant, a novel analytical solution incorporated with the fractal theory in the tri-linear flow model was presented for a channel-fractured horizontal well. The new model is verified by both numerical simulations and existing simplified models. The main findings and conclusions can be concluded as follows:

- Our model can account for both the heterogeneities of fracture/matrix systems within dual-porosity SRV and non-uniform distribution of proppant pack inside hydraulic primary fractures.
- The different influencing periods of the fractal connectivity-index in the matrix and fracture systems on well production are characterized.
- An increase of the fractal dimension within SRV or a primary fracture results in greater oil production.
- The enhancement of primary fracture conductivity not only leads to a decrease in fluid flow resistance inside a primary fracture, but also results in significant improvement in fluid flow capacity within SRV in reservoirs.

**Nomenclature**

- $B$: Volume factor under initial condition, dimensionless
- $b_f$: Hydraulic fracture half-width (cm)
- $b_D$: Dimensionless hydraulic fracture half-width (cm)
- $C_D$: Dimensionless wellbore storage coefficient
- $c_{im}$: Matrix bulk compressibility (atm$^{-1}$)
- $c_{if}$: Fracture bulk compressibility (atm$^{-1}$)
- $c_{i1}$: Total compressibility of hydraulic fracture, region 1 (atm$^{-1}$)
- $c_{i2}$: Total compressibility in SRV, region 2 (atm$^{-1}$)
- $d$: Fractal dimension (dimensionless)
- $\theta$: Connectivity index (dimensionless)
- $h$: Reservoir thickness (cm)
- $k_m$: Matrix bulk permeability (D)
- $k_f$: Fracture bulk permeability at the edges of the hydraulic fracture, region 2 (D)
- $k_F$: Hydraulic fracture permeability (D)
- $p_D$: Dimensionless pressure
- $p_m$: Matrix pressure in SRV, region 2 (atm)
- $p_f$: Fracture pressure in SRV, region 2 (atm)
- $q$: Production rate (cm$^3$/s)
- $q_D$: Production rate (cm$^3$/s)
- $S$: Skin factor
- $t_D$: Dimensionless time
- $t$: Time (s)
- $x_f$: Hydraulic fracture half-length (cm)
- $x_e$: Reservoir size in x-direction (cm)
- $x_D$: Dimensionless hydraulic fracture half-length in x-direction
- $y_e$: Reservoir size in y-direction (cm)
- $y_D$: Dimensionless hydraulic fracture half-length in y-direction
- $\lambda$: Flow capacity coefficient
- $\omega$: Storability ratio
- $\phi_f$: Total formation porosity in SRV, region 2
- $\phi_F$: Hydraulic fracture porosity in region 1
- $\phi_m$: Matrix porosity
- $\rho$: Formation oil density (g/cm$^3$)
- $\mu$: Oil viscosity (cp)
- $\alpha$: Matrix shape factor in dual-porosity model

**Acknowledgements**

The authors would like to acknowledge the technical support of ECLIPSE.

**Disclosure statement**

No potential conflict of interest was reported by the authors.

**Funding**

This work was supported by the National Basic Research Program of China [2014CB239103], the National Natural Science Foundation of China [51674279] and the China Postdoctoral Science Foundation funded project [2016M602227].
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