QUANTIZING $N = 2$ MATTER–SUPERGRAVITY SYSTEMS

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Abstract

We consider $N = 2$ supergravity coupled to $N = 2$ Yang–Mills matter and discuss the nature of one–loop divergences. Using $N = 1$ superfields and superspace methods, we describe the quantization of the system in the abelian case.
1 Introduction

Recently there has been considerable activity in studying matter–supergravity lagrangians as effective low–energy representations of superstring theories. This has been done both in component and in $N = 1$ superspace formulations, but primarily at the classical level. Of special interest have been theories with $N = 2$ supersymmetry, both from a phenomenological point of view and because of the interesting geometrical structures which emerge there, with particular emphasis on special and quaternionic geometry. It is expected that below a certain energy scale these theories can be used without invoking the full string technology. However, they necessarily contain nonrenormalizable interactions. Thus in order to have realistic and phenomenologically interesting models one has to show that naturalness is not spoiled and the hierarchy problem is not affected by radiative corrections. In particular it is important to study the ultraviolet cutoff dependence for these effective theories derived from strings.

At the quantum level, it is well-known that, whereas pure supergravity is on–shell one– and two–loop finite, in (super)gravity–matter systems divergences appear already at the one–loop level. Thus, if these effective theories are to represent superstring models to moderately high scales, it is of interest to know the nature of these divergences. A priori, one expects to encounter in the one–loop effective actions for matter fields quadratically divergent quantum supergravity corrections, and indeed, recent component calculations in $N = 1$ models have verified their existence. Other studies have indicated that the presence of quadratic divergences, even in a “hidden” sector, can have a destabilizing effect on the hierarchy problem.

Our interest in these issues stems from the expectation that $N = 2$ matter systems coupled to $N = 2$ supergravity may display better high–energy behavior. Since some of the favored string–inspired models have $N = 2$ supersymmetry before breakdown, at a lower scale, to $N = 1$ and ultimately to $N = 0$, this problem is of more than academic interest.

We consider $N = 2$ supergravity coupled to $N = 2$ Yang–Mills matter. First we present a general superspace power counting argument, on the basis of $N = 2$ supersymmetry, to determine the nature of the one-loop divergences. Then we address the problem of quantizing the system. We have to deal with several gauge fields, and their quantization consists primarily in fixing gauges and determining the corresponding ghost actions. We work in $N = 1$ superspace because at the present time no suitable $N = 2$ description of the supergravity sector is available. In this formalism, the gauge–fixing violates $N = 2$ invariance. Consequently, to extract properties which follow from the full
supersymmetry we must look at on–shell quantities which do not depend on the gauge parameters or the form of the gauge–fixing functions. We fix gauges in the simplest and most convenient way, avoiding as much as possible the introduction of couplings of the ghost fields to the physical fields.

Our paper is organized as follows: in section 2, we describe the general power counting argument regarding the nature of one–loop divergences. In section 3, we present our $N = 2$ model consisting of an abelian vector multiplet coupled to supergravity. For this system a complete $N = 1$ superspace description has been given by Labastida et al \cite{7}. It contains an abelian $N = 1$ vector multiplet and a scalar multiplet, coupled to nonminimal $n = −1$ supergravity, as well as to a gravitino multiplet. In order to construct local matter–supergravity interactions, the scalar multiplet has to be described by a complex scalar prepotential rather than the customary chiral scalar superfield.

The quantization of the matter multiplets, i.e. the vector multiplet and the above mentioned complex scalar is straightforward and briefly discussed in section 4. In section 5 we study the quantization of the $N = 2$ supergravity system. We describe the gravitino multiplet by means of a complex spinor superfield, and chiral scalar and real scalar compensating superfields. We fix the various gauge invariances by appropriate choices of gauge–fixing functions, which allow putting the quadratic action in standard form and obtaining simple propagators. Finally we discuss the quantization of nonminimal $n = −1$ supergravity, described by a real vector superfield and a complex linear superfield compensator. In general the gauge–fixing procedure requires introducing corresponding Faddeev–Popov ghosts. We have made all our gauge choices flat with respect to both the Yang–Mills and the supergravity fields, so that we need not worry about coupling of the Faddeev–Popov ghosts to physical multiplets except in the case of nonlinear variations of the fields. For the system under consideration this happens only for the $n = −1$ supergravity superfields. We discuss in detail the quantization of the corresponding Faddeev–Popov lagrangian at the end of section 5.

One feature of the quantization process is the introduction of “catalyst” fields \cite{9}, a procedure for recasting kinetic lagrangians, gauged–fixed but not in convenient form, to a form which leads to standard propagators. Unlike most of the ghost fields mentioned above, many of the catalysts end up being coupled to the physical fields. Also, it is well–known that the quantization of the linear superfield introduces an infinite tower of ghosts. However, since the latter do not couple to the physical fields, we can avoid discussing the difficulties introduced by them. In the Appendix we have listed some useful formulas.

We use superspace notations and conventions as in ref. \cite{9}. 

2 On quadratic divergences for $N = 2$ Yang–Mills in presence of $N = 2$ supergravity

By ordinary power counting – the gravitational coupling constant $\kappa$ has dimensions of $(\text{mass})^{-1}$ – conventional lagrangian models of (super)gravity–matter systems are not renormalizable. The effective action for such systems is on–shell finite at the one–loop level when restricted to the case of external gravitational lines only [4], and at the two–loop level as well for the corresponding supergravity case [3]. However, divergences appear as soon as one considers external matter lines, even on shell. At best, one may hope that the degree of divergence is only logarithmic, so that such models can be used as low–energy effective actions from strings even at moderate energies.

As mentioned in the introduction, it has been shown that supergravity–matter systems with $N = 1$ supersymmetry develop quadratic divergences already at the one–loop level [5], and these can have deleterious effects on the validity of string–inspired lagrangian models beyond energies where classical considerations are sufficient. Specifically, it has been shown that the effective action for scalar multiplets or vector multiplets, with radiative corrections due to the exchange of supergravity fields, depends quadratically on the ultraviolet cutoff and this can destroy properties of supersymmetric systems such as naturalness and the solution to the hierarchy problem, even when they occur in a “hidden” sector [6]. We present here a power counting argument, based on $N = 2$ supersymmetry and gauge invariance, that indicates that the situation may be better in the case of the one–loop effective action for $N = 2$ Yang-Mills with radiative corrections due to $N = 2$ supergravity fields. Whereas terms involving $N = 2$ chiral superspace integrands of the form $\mathcal{F}(W)$ – here $W$ is the $N = 2$ Yang-Mills field strength –are quadratically divergent, full superspace terms are at most logarithmically divergent. In particular, for the chiral multiplet component of the $N = 2$ Yang-Mills superfield, one might encounter quadratic divergences only for terms of the form $\bar{\omega}\mathcal{F}(\omega) + h.c.$. For the model we are considering in this paper, only the two–point function $\bar{\omega}\mathcal{F}(\omega)$ would be quadratically divergent.

Our power counting arguments assume that the amplitudes under consideration do have manifest global $N = 2$ supersymmetry. In practice, however, since a suitable $N = 2$ superspace formulation of supergravity is not available, explicit calculations have to be performed in an $N = 1$ (or component) formalism. In general, any calculation which is not manifestly $N = 2$ supersymmetric (i.e. not done in terms of $N = 2$ superfields) will involve breaking of the (extended) supersymmetry, either because auxiliary fields have been eliminated, or because the fixing of the local gauge invariances has to be done in a manner which does not respect it. Specifically for the case under consideration, as we
will discuss below, the fixing of the various gauge invariances has to be done separately for the $N = 1$ members of the $N = 2$ multiplets and this leads to some explicit breaking of the $N = 2$ global invariance. Therefore, in practice we have to restrict ourselves to the computation of on–shell, gauge invariant and gauge independent quantities, i.e. S–matrix elements.

For amplitudes involving external Yang–Mills fields, $N = 2$ supersymmetry and gauge invariance imply that the divergent part has to be a local expression depending on the Yang–Mills field strength, the $N = 2$ chiral superfield $W$, in an integral which is either over chiral superspace with a $d^4\theta$ measure, or over full superspace, with a $d^8\theta \equiv d^4\theta d^4\bar{\theta}$ measure. We are using spinor coordinates $\theta^a_1, \theta^a_2$ and their complex conjugates, with $\bar{D}_i \bar{W} = 0$. Besides the fields and integration measures (including the space-time measure $d^4x$), other dimensionful quantities are the gravitational coupling $\kappa$, and an ultraviolet cutoff $\Lambda$. The (mass) dimensions of the various quantities are listed below:

\[
\begin{align*}
[d^4x] &= -4 & [d^4\theta] &= 2 \\
[W] &= 1 & [\kappa] &= -1 \\
[\Lambda] &= 1
\end{align*}
\] (2.1)

Each (super)gravitational internal propagator brings with it a power of $\kappa^2$ and it is easy to see that, in the absence of any matter self–interactions, a one–loop diagram with $2n$ external $W$–lines involves $n$ supergravity propagators. Thus, allowed dimensionless local expressions are of the form

\[
\int d^4xd^4\theta \ W^{2n} \ \kappa^{2n} \Lambda^2
\] (2.2)

involving chiral (or antichiral) integrals, and

\[
\int d^4xd^8\theta \ W^n \bar{W}^n \ \kappa^{2n} \ln \Lambda
\] (2.3)

with full superspace integrals, or expressions with (spinor or space-time) derivatives, which are more convergent.

Upon reduction to $N = 1$ superspace, these expressions can be rewritten in term of the superfields of the $N = 1$ vector and scalar multiplets, $V, \omega$, which make up the $N = 2$ Yang–Mills multiplet. We are particularly interested in the contributions to the scalar multiplet effective action. The results of this reduction are well known: one defines $N = 1$ components by

\[
\omega = W|, \quad W^\alpha = -D_2^\alpha W|
\] (2.4)

where the bar indicates evaluation at $\theta^a_2 = \bar{\theta}^a_2 = 0$ and rewrites the integration measure in terms of $N = 1$ measures, with the replacement $d^2\theta_2 \rightarrow \frac{1}{2} D_2^a D_2^a$, etc. In particular,
as far as the pure scalar multiplet is concerned, the chiral superspace integral in (2.2) leads to expressions of the form $2n\bar{\omega}\omega^{2n-1}$. The full superspace expression in (2.3) on the other hand, leads to chiral superfield contributions of the form (in the abelian case, for the nonabelian case see also ref. [10])

$$n^2\bar{\omega}^{n-1}\omega^n D^2\bar{\omega}D^2\omega + n\bar{\omega}^{n-1}\omega^n \Box \bar{\omega} + \frac{n}{2}(n-1)\omega^n\bar{\omega}^{n-2}\partial^a\bar{\omega}\partial_a\omega$$  

(2.5)

For theories such as we will consider below in which the Green’s functions have equal number of external $\omega$ and $\bar{\omega}$ fields, quadratic divergences are possibly present only in the chiral superfield two–point function. The higher–point terms are at most logarithmically divergent and contain derivatives of the chiral superfields.

These conclusions are applicable whenever one can argue that the results of a calculation are gauge invariant and $N = 2$ supersymmetric. In our context, this means on–shell, gauge invariant and gauge independent quantities. Thus, in an $N = 1$ calculation they apply to scalar multiplet (and by $N = 2$ supersymmetry vector multiplet) scattering amplitudes which should have at most logarithmic divergences.

3 $N = 1$ superfield description of the $N = 2$ vector multiplet coupled to $N = 2$ supergravity

$N = 2$ extended supergravity in $N = 1$ superspace is described by the nonminimal, $n = -1$, version of $N = 1$ supergravity, and a gravitino multiplet. The former consists of the vector superfield $H_{a\dot{a}}$ and a complex linear compensator $\Upsilon$. For the latter, a convenient description is by means of a spinor superfield $\phi_\alpha$ and scalar compensators $V$ (real), and $\Phi$ (chiral). The full $N = 2$ action has been constructed in ref. [7].

The $N = 2$ abelian vector multiplet consists of an $N = 1$ vector multiplet, described by a scalar (prepotential) superfield $\Omega$, and a scalar multiplet described by a chiral superfield $\omega$. However, in order to couple this matter system to $N = 2$ supergravity it is necessary to solve the chirality constraint in terms of a (gauge) prepotential $\Psi$,

$$\omega = \nabla^2 \Psi$$

The complete $N = 1$ action for the coupled system, as given in refs. [7], [8] is

$$S = -\frac{1}{2\kappa^2} \int d^4x d^4\theta \ E^{-1} \left\{ 2C + [\phi^\alpha(\lambda_\alpha + \mathcal{W}_\alpha)C + \bar{\phi}^\alpha T_\alpha + \Phi] + \frac{i}{2} W^\alpha \nabla_\alpha \mathcal{V} + h.c. \right\}$$
\begin{equation}
+ \int d^4 x d^4 \theta E^{-1} \left\{ \omega \bar{\omega} [C + \frac{1}{4}(x \bar{\nu} + \bar{x} \nu)]
+ \frac{1}{4} \left[ \Gamma^\alpha (N^\beta_\alpha \omega_\beta + C \lambda_\alpha \omega + \bar{\nu}(\nabla_\alpha + T_\alpha) \omega) + \frac{1}{2} \omega \bar{\nu} \nabla^\alpha \Gamma_\alpha + h.c. \right] \right\} \tag{3.1}
\end{equation}

where \( \nabla_A \), with \( A = \alpha, \dot{\alpha}, \alpha \equiv (\alpha \dot{\alpha}) \), denote suitably defined \( N = 1 \) supergravity covariant derivatives with connections including some gravitino multiplet contributions. \( W_\alpha \equiv i \bar{\nabla}^2 \nabla_\alpha \nu \) is the \( \nu \) field strength, \( E \) is the \( N = 1 \) supergravity vielbein determinant, \( \Gamma_\alpha \) and \( \omega_\alpha \) are supercovariantized vector multiplet \( U(1) \) spinor connection and field strength, respectively, and the remaining quantities, \( B \), \( C \), \( v \) and \( x \), and \( N^\beta_\alpha \), \( \lambda_\alpha \) and \( T_\alpha \) are composed of fields in the gravitino multiplet and their derivatives. We give a summary of some relevant quantities:

\begin{align*}
\Gamma_\alpha &= \nabla_\alpha \Omega - 2 \psi_\alpha \beta \nabla^\beta \bar{\Psi} \\
\psi_\alpha \beta &= \nabla_\alpha \phi^\beta + \dot{\psi} \beta \Phi \\
x &= \psi_\alpha \alpha \\
B &= \frac{1}{2} \psi_\alpha \beta \psi_\alpha \beta \\
v &= x + \bar{x} B \\
C &= 1 - B \bar{B} \\
N^\beta_\alpha &= C \delta^\beta_\alpha + \bar{\nu} \psi_\alpha \beta \\
\lambda_\alpha &= - \nabla_\alpha \nabla_\beta \bar{\phi} \dot{\alpha} + \bar{\nabla}^2 \phi_\alpha + \nabla_\alpha \Phi + W_\alpha + \text{higher order terms} \\
\omega_\alpha &= \frac{i}{4} \bar{\nabla}^\alpha \Gamma_\alpha \dot{\alpha} - \frac{i}{4} \nabla_\alpha \bar{\Gamma} \dot{\alpha} - \lambda_\alpha \omega - \frac{1}{2} [\psi_\alpha \beta \nabla_\alpha - \nabla_\alpha \psi_\alpha \beta] \omega \\
&\quad + \text{higher order terms} \omega \bar{\omega} \tag{3.2}
\end{align*}

The complete definitions can be found in ref. [8].

In addition to the \( N = 1 \) supersymmetry invariance which is implicit in our use of \( N = 1 \) superfields, the action in (3.1) is invariant under a second supersymmetry transformation. With spinor parameter \( \epsilon_\alpha \), it acts on the matter fields as

\begin{align*}
\delta \omega &= - \epsilon^\alpha \omega_\alpha \\
\delta \omega_\alpha &= \epsilon_\alpha \Sigma - i \epsilon^\alpha [\nabla_\alpha \omega + \psi_\alpha \beta \omega_\beta] \\
\delta \Gamma_\alpha &= 2 \bar{\omega} \epsilon_\alpha + \epsilon^\dot{\alpha} \left[ \frac{1}{2} \lambda_\alpha \Gamma_\alpha + i \psi_\alpha \gamma_\alpha \right] \tag{3.3}
\end{align*}

with \( \Gamma_\alpha \dot{\alpha} \) the \( U(1) \) Yang-Mills vector connection, and \( \psi_\alpha \beta \dot{\alpha} \) expressible in terms of derivatives of the gravitino fields. \( \Sigma \) is a function of the fields in the matter and gravitino multiplets. Their expressions to lowest order in the supergravity fields are given by

\begin{equation}
i \Gamma_\alpha \dot{\alpha} = \nabla_\alpha \Gamma_\dot{\alpha} + \bar{\nabla}_\dot{\alpha} \Gamma_\alpha + \ldots
\end{equation}
\[
\dot{\psi}^{\dot{\alpha}}_\alpha = -\frac{1}{2} \delta^{\dot{\beta}}_\dot{\alpha} \bar{\lambda}_{\dot{\alpha}} + \bar{\nabla}_{\dot{\alpha}} \dot{\psi}^{\dot{\beta}}_\alpha + \ldots
\]

\[
\Sigma = 2\nabla^2 \bar{\omega} - \lambda^\alpha \omega_\alpha + \nabla^\dot{\alpha} \dot{\psi}^{\dot{\beta}}_{\dot{\alpha}} \bar{\omega}_{\dot{\beta}} - \psi^{(\dot{\alpha}\dot{\beta})} \nabla_{\dot{\alpha}} \bar{\omega}_{\dot{\beta}} + \ldots
\] (3.4)

The second supersymmetry transformation laws for the supergravity fields can be found in ref. [7].

To quantize the theory, since we have gauge fields, \(\Omega\) and \(\Psi\) as matter prepotentials, \(H_a, \phi_\alpha\) as fundamental supergravity fields, we need fix the various gauge invariances. In principle one would like to perform the quantization in a way that maintains explicit \(N = 2\) supersymmetry. This could be achieved by introducing a quantum–background splitting for the various superfields so that gauges could be fixed in a \(N = 2\) background covariant way. In practice the analysis of the quantum and background separate invariances becomes so cumbersome that this approach is not easy to implement. Thus we apply a low–brow procedure by separately gauge–fixing the various \(N = 1\) prepotentials. This is sufficient for calculating \(S\)–matrix elements. These on–shell quantities are the relevant ones at the quantum level: they are uniquely defined, being independent of the definite gauge choices made in the course of the quantization. Our starting point is the quadratic part of the action in (3.1)

\[
S^{(2)} \equiv S^{(2)}_m + S^{(2)}_G + S^{(2)}_g
\] (3.5)

where the subscripts \(m, G\) and \(g\) indicate matter, gravity and gravitino fields respectively. We present the details of the gauge–fixing procedure in the next two sections.

4 Quantization of \(N = 2\) Yang–Mills

We concentrate here on the matter part of the action. As already mentioned, the basic superfields are two unconstrained complex prepotentials \(\Omega\) and \(\Psi\) which appear in the action (3.1) only through the quantities

\[
\omega = \bar{\nabla}^2 \Psi \quad \quad \quad \Gamma_\alpha = \nabla_\alpha \Omega - 2\psi_\alpha^\beta \nabla^\beta \bar{\Psi}
\] (4.1)

In addition to the usual gauge invariance of the vector multiplet

\[
\delta \Omega = \bar{\Lambda} \quad \quad \nabla_\alpha \bar{\Lambda} = 0
\] (4.2)

and the \(U(1)\) invariance

\[
\delta \Omega = \kappa \quad \quad \delta \Psi = 0
\] (4.3)
with $\kappa$ real, the definitions in (4.1) exhibit the extra invariance under the following transformations with spinor parameter $\chi$:

$$
\delta \Psi = \bar{\nabla}^\dot{\alpha} \bar{\chi}_\alpha \\
\delta \Omega = 2\phi_\beta \nabla^\beta \nabla^\gamma \chi_\gamma - 2\Phi \nabla^\gamma \chi_\gamma 
$$

(4.4)

The $U(1)$ $\kappa$–transformation can be used to reach a gauge in which $\Omega$ is purely imaginary; once this has been achieved every transformation that would take one out of this gauge has to be followed by a compensating $\kappa$–transformation, with $\kappa$ chosen so as to cancel the real part of $\Omega$. In the following we can then set $\Omega = -\bar{\Omega} \equiv iV$.

In all situations in which $\Omega$ (or $V$) and $\omega = \nabla^2 \Psi$ propagate in the loops, but no bare quantum $\Psi$ is present, there is no need to solve the chirality constraint for the quantum $\omega$. In this case one only has to gauge fix the standard invariance in (4.2). This can be done, while maintaining the extra invariance in (4.4), by choosing a gauge–fixing term of the form

$$
S_{GF}^m = -\frac{1}{4} \int d^4x d^4\theta \, \bar{\nabla}^\dot{\alpha} \bar{\Gamma}_\dot{\alpha} D^\alpha \Gamma_\alpha 
$$

(4.5)

and combining this term with the corresponding classical part of the action

$$
S_m = \frac{1}{16} \int d^4x d^4\theta \, \Gamma^\alpha [\bar{\nabla}^\dot{\alpha} (D^\alpha \bar{\nabla}_\dot{\alpha} + \bar{\nabla}_\dot{\alpha} D^\alpha) - i\partial_{\alpha\dot{\alpha}} \bar{\Gamma}^\alpha] 
$$

(4.6)

We note that substituting in (4.6) the explicit forms of $\Gamma^\alpha$, $\bar{\Gamma}^{\dot{\alpha}}$, the gauge field $V$ appears with the conventional lagrangian $\frac{1}{2} V D^\alpha \bar{\nabla}^2 D_\alpha V$. In the same manner (4.5) contains the standard gauge-fixing function $-D^2 V \bar{\nabla}^2 V$ which converts it into the Feynman gauge form $-\frac{1}{2} V \Box V$.

If we are in a situation in which the field $\Psi$ itself has to be treated as a quantum field, then necessarily we have to break the symmetry in (4.4). This is achieved by choosing as gauge–fixing function

$$
F_\alpha = D_\alpha \Psi 
$$

(4.7)

and introducing a gauge–fixing term of the form

$$
- \int d^4x d^4\theta \, F^\alpha \left( \frac{1}{4} \bar{D}_\dot{\alpha} D_\alpha - i\partial_{\alpha\dot{\alpha}} \right) F^\alpha = \int d^4x d^4\theta \, \bar{\Psi} (\bar{D}^2 D^2 - \bar{D}^\dot{\alpha} D^2 \bar{D}_\dot{\alpha}) \Psi 
$$

(4.8)

Again, when combined with the classical term $\bar{\omega} \omega = D^2 \bar{\Psi} \bar{D}^2 \Psi$, it gives rise to a standard $\bar{\Psi} \Box \Psi$ kinetic term.

Faddeev–Popov ghosts should be introduced corresponding to the gauge–fixing in (4.4), but since $V$ is abelian they do not couple to the physical fields and therefore they are irrelevant. The same conclusion can be reached for the ghosts of the $\Psi$ superfield. In this case the gauge variation of $\Psi$ has zero–modes, $\delta \bar{\chi}_\alpha = \bar{D}^\beta \bar{\chi}_{(\dot{\alpha}\dot{\beta})}$ (see (4.4)) and the
gauge-fixing of the ghost lagrangian will introduce an infinite tower of ghosts. The same situation occurs in the quantization of the supergravity multiplet, to be discussed below. However, with the flat gauge fixing in (4.7), they are completely decoupled and do not contribute to physical amplitudes.

5 Quantization of $N = 2$ supergravity

5.1 Quantization of the gravitino multiplet

We begin by studying the gauge-fixing procedure for the $N = 1$ gravitino multiplet, described by the general spinor superfield $\phi^\alpha$, and a pair of compensators that allow us to write a local action (see ref. [9], sec. 4.5.e; for our purpose, the most convenient choice of compensators uses a real vector $V$ and a chiral scalar $\Phi$). The corresponding quadratic part of the action contained in (3.1), (also cf. [9], eq. (4.5.36)), can be written in the form

$$S_g^{(2)} = -\frac{1}{2} \int d^4x d^4\theta \left[ \phi^\alpha (\bar{D}^\dot{\alpha} D_\alpha \bar{\phi}_{\dot{\alpha}} + \bar{D}^2 \phi^\alpha + 2W_\alpha + D_\alpha \Phi) + \bar{\phi}^{\dot{\alpha}} (D^\alpha \bar{D}_{\dot{\alpha}} \phi^\alpha + D^2 \bar{\phi}_{\dot{\alpha}} + 2\bar{W}_{\dot{\alpha}} + \bar{D}_{\dot{\alpha}} \bar{\Phi}) \right.$$  \left.$$ + (D_\alpha \phi^\alpha + 2\Phi) \Phi + (\bar{D}_{\dot{\alpha}} \bar{\phi}^{\dot{\alpha}} + 2\bar{\Phi}) \bar{\Phi} + \frac{i}{2} (D^\alpha \mathcal{V} W_\alpha - D^\alpha \mathcal{V} W_{\dot{\alpha}}) \right] \tag{5.1}$$

with $\mathcal{W}_\alpha \equiv i\bar{D}^2 D_\alpha \mathcal{V}$. It is invariant under the gauge transformations

$$\delta \phi^\alpha = \Lambda^\alpha + D_\alpha Z \quad \bar{D}_{\dot{\alpha}} \Lambda_{\dot{\alpha}} = 0$$
$$\delta V = i(Z - Z) + \Lambda + \bar{\Lambda} \quad D_\alpha \Lambda = 0$$
$$\delta \Phi = \bar{D}^2 \bar{Z} \tag{5.2}$$

where $Z$ is a general superfield.

We use the following gauge-fixing functions:

$$F^\alpha = \bar{D}^2 \phi^\alpha + a W_\alpha$$
$$F = D^\alpha \phi^\alpha + b \Phi$$
$$G = D^2 V \tag{5.3}$$

and the gauge-fixing lagrangian

$$L_{GF} = c F^\alpha \frac{D^2}{\square} F_\alpha + c \bar{F}^{\dot{\alpha}} \frac{\bar{D}^2}{\bar{\square}} \bar{F}_{\dot{\alpha}} + d \bar{F} F + e \bar{G} G \tag{5.4}$$
(Nonlocal gauge-fixing terms are a common feature of the quantization procedure in superspace.) After some algebra, we find that with the choice

$$a = d = 1 \quad b = e = -1 \quad c = \frac{1}{2}$$ (5.5)

the lagrangian takes the form

$$L_g^{(2)} + L_{GF} = i\phi^\alpha \partial_\alpha \bar{\phi}^\dot\alpha - \bar{\Phi}\Phi - \frac{1}{2} \nabla \Box \nabla$$ (5.6)

Thus we obtain standard propagators

$$\langle \phi_\alpha(x, \theta) \bar{\phi}_{\dot\alpha}(x', \theta') \rangle = i \partial_\alpha \delta^4(x - x') \delta^4(\theta - \theta')$$

$$\langle \Phi(x, \theta) \bar{\Phi}(x', \theta') \rangle = \frac{1}{2} \delta^4(x - x') \delta^4(\theta - \theta')$$

$$\langle V(x, \theta) V(x', \theta') \rangle = \frac{1}{2} \delta^4(x - x') \delta^4(\theta - \theta')$$ (5.7)

Since we have considered flat (with respect to Yang–Mills and supergravity) gauge-fixing functions and the gauge transformations in (5.2) are all linear, the ghost fields are not interacting and can be dropped.

### 5.2 Quantization of $N = 1$ nonminimal supergravity

We discuss now gauge-fixing for the $N = 1$ supergravity sector. Nonminimal $n = -1$ supergravity is described by the real vector superfield $H^a$, and the complex linear superfield compensator $\Upsilon$. The relevant material can be found in ref. [9]. The action is simply (we set $\kappa = 1$ in eq. (3.1))

$$S_G = - \int d^4x d^4\theta \ {E^{-1}} = - \int d^4x d^4\theta \ {\hat{E}^{-1}} (\bar{\Upsilon} \Upsilon)^{-1}$$ (5.8)

where $\Upsilon$ satisfies the linearity condition $\bar{D}^2 \Upsilon = 0$ and, in ”chiral” representation, $\bar{\Upsilon} = e^{-H} \Upsilon e^H$, with $H \equiv i H^a \partial_a$. We write $\Upsilon = 1 + \tau$ and solve the linearity condition in terms of a spinor superfield $\bar{\tau}_\alpha$

$$\Upsilon = 1 + \tau = 1 + \bar{D}^\dot{\alpha} \bar{\tau}_\dot{\alpha}$$ (5.9)

Also,

$$\hat{E}^{-1} = [\det (1 + \Delta)]^{-1} = e^{-tr \ ln(1 + \Delta)}$$ (5.10)

where

$$\Delta_a^b = -i \bar{D}_a \Delta_\alpha^b$$

$$\Delta_\alpha^b i \partial_b = e^{-H} D_\alpha e^H - D_\alpha$$ (5.11)
The action in (5.8) is invariant under local supersymmetry transformations. The complete gauge variation of the vector superfield $H_a$ can be written as

$$e^{H'} = e^{i\Lambda} e^H e^{-i\Lambda} \quad (5.12)$$

where $\Lambda \equiv \Lambda^i A_i D_A$ and

$$\begin{align*}
\Lambda^a &= -i \bar{D}^{\dot{\alpha}} L^\alpha \\
\Lambda^a &= \bar{D}^2 L^a \\
\Lambda^a &= e^{-H} D^2 \bar{L}^{\dot{\alpha}} e^H \\
\bar{\Lambda}^\alpha &= e^{H} \bar{D}^2 L^{\alpha} e^{-H}
\end{align*} \quad (5.13)$$

with arbitrary complex $L_\alpha$. From (5.12) the infinitesimal variation is obtained as an infinite expansion of the form

$$\delta H = i(\bar{\Lambda} - \Lambda) + \frac{1}{2} i(\bar{\Lambda} + \Lambda), H] + \frac{1}{12} [i(\bar{\Lambda} - \Lambda), H], H] + \ldots \quad (5.14)$$

In the same way the infinitesimal variation of the linear superfield $\Upsilon$ is given by

$$\delta \Upsilon = (\bar{D}_a \Lambda^{\dot{a}}) \Upsilon + [i \Lambda, \Upsilon] \quad (5.15)$$

Using the explicit expressions in (5.13), from (5.14) and (5.15) we obtain respectively

$$\begin{align*}
\delta H_a &= D_a \bar{L}_{\dot{a}} - \bar{D}_{\dot{a}} L_a + \frac{i}{2} (D^\beta \bar{L}^{\dot{\beta}} + \bar{D}^{\dot{\beta}} L^\beta) \partial_\beta H_a - \frac{i}{2} H^b \partial_b (D_a L_{\dot{a}} + \bar{D}_{\dot{a}} L_a) \\
&\quad - \bar{D}^2 L^\beta D_\beta H_a - D^2 \bar{L}^{\dot{\beta}} D_{\dot{\beta}} H_a + \ldots
\end{align*} \quad (5.16)$$

and

$$\begin{align*}
\delta \tau_{\alpha} &= -\bar{D}^2 L_{\alpha}(1 + D^\beta \tau_{\beta}) - \bar{L}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} D^2 \tau_{\alpha} - \frac{1}{2} D_{\alpha}(\bar{L}^{\dot{\alpha}} \bar{D}_{\dot{\alpha}} D^\beta \tau_{\beta}) \\
&\quad - H^b i \partial_b D^2 L_{\alpha} + \ldots
\end{align*} \quad (5.17)$$

Having solved the linearity constraint on $\Upsilon$ in terms of $\tau_{\alpha}$, we have introduced an additional gauge freedom under

$$\delta \tau_{\alpha} = D^\beta \Lambda_{(\alpha \beta)} \quad (5.18)$$

with arbitrary complex $\Lambda_{\alpha \beta}$. Both invariances in (5.16, 5.17) and (5.18) require gauge fixing.

Substituting in (5.10) the explicit expressions in (5.11), to quadratic order in $H_a$ we have

$$\hat{E}^{-1} = 1 - \bar{D}_{\dot{a}} D_a H^a - \frac{i}{2} \bar{D}_{\dot{a}}(D_a H^{c \dot{c}} \partial_c H^a - H^{c \dot{c}} \partial_c D_a H^a)$$

$$+ \frac{1}{2} \bar{D}_{\dot{a}} D_a H^b \bar{D}_{\dot{b}} D_{\beta} H^a + \frac{1}{2}(\bar{D}_{\dot{a}} D_a H^a)^2 \quad (5.19)$$

11
Thus, to this order in the supergravity fields, the action (5.8) becomes

\[ S^{(2)}_G = \int d^4x d^4\theta \left[ \frac{1}{2} H^{\alpha\dot{\alpha}} \Box H_{\alpha\dot{\alpha}} - \frac{1}{2} H^{\alpha\dot{\alpha}} \{ D^2, \bar{D}^2 \} H_{\alpha\dot{\alpha}} \right. \]
\[ \left. + (D^\alpha \tau_\alpha)(\bar{D}^{\dot{\alpha}} \bar{\tau}_{\dot{\alpha}}) + (D^\alpha \tau_\alpha)^2 + (\bar{D}^{\dot{\alpha}} \bar{\tau}_{\dot{\alpha}})^2 - H^{\alpha\dot{\alpha}}(\bar{D}_\alpha D^2 \tau_\alpha - D_\alpha \bar{D}^2 \bar{\tau}_{\dot{\alpha}}) \right] \] (5.20)

In order to fix the gauge invariances and obtain standard kinetic terms we first introduce instead of \( \sigma_\alpha \) the field

\[ \tau_\alpha = \sigma_\alpha + z \bar{D}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} \] (5.21)

and then introduce two gauge-fixing functions, corresponding to the invariances in (5.16), (5.17) and (5.18)

\[ F_\alpha = \bar{D}^{\dot{\alpha}} H_{\alpha\dot{\alpha}} + a \sigma_\alpha \]
\[ F_{\alpha\beta} = D_{(\alpha} \sigma_{\beta)} + b i \partial_{(\alpha} \bar{D}^{\dot{\beta}} H_{\beta)\dot{\beta}} + c D_{(\alpha} \bar{D}^{\dot{\beta}} H_{\beta)\dot{\beta}} \] (5.22)

with suitable constants \( a, b, c \), and suitable gauge-fixing terms. We want to achieve the double goal of cancelling cross-terms between \( H_{\alpha\dot{\alpha}} \) and \( \sigma_\alpha \), and putting the quadratic kinetic terms in a form which leads to simple propagators.

We start with a quadratic combination of these gauge-fixing functions corresponding to a general 't Hooft gauge averaging,

\[ L_{GF} = p \bar{F}_\alpha D^\alpha \bar{D}^{\dot{\alpha}} F_\alpha + q F_\alpha D^\alpha \bar{D}^{\dot{\alpha}} \bar{F}_{\dot{\alpha}} + \left[ r F^\alpha D^2 F_\alpha 
\right. \]
\[ \left. + \frac{1}{2} s F^{\alpha\beta} F_{\alpha\beta} + \frac{1}{2} t D^{(\beta} F^{\alpha)} F_{\alpha\beta} + h.c. \right] \] (5.23)

and work out all linearly independent quadratic terms in \( H^{\alpha\dot{\alpha}}, \sigma_\alpha \) and \( \bar{\sigma}_{\dot{\alpha}} \), to be added to the classical lagrangian. In the Appendix we have listed some identities that we have used in order to obtain a minimal set of independent quadratic quantities. We find that with the following choice of constants,

\[ s = r = 0 \]
\[ c = -\frac{5}{6} \]
\[ p = b = 1 \]
\[ q = \frac{5}{4} \]
\[ t = z = -\frac{1}{2} \]
\[ a = -\frac{2}{3} \] (5.24)

the quadratic supergravity action becomes

\[ S^{(2)}_G + S_{GF} = - \int d^4x d^4\theta \left[ \frac{1}{2} H^{\alpha\dot{\alpha}} \Box H_{\alpha\dot{\alpha}} + \frac{4}{9} \bar{\sigma}_{\dot{\alpha}} (\bar{D}^{\dot{\alpha}} D^\alpha - D^\alpha \bar{D}^{\dot{\alpha}}) \sigma_\alpha \right] \] (5.25)

Although the gauge has been fixed, and the kinetic operators are invertible, the one for the \( \sigma_\alpha \) field will not lead to a convenient propagator. It can be recast in standard form
by a (non–local) field redefinition, or, equivalently, by the use of catalyst fields \[4, 11\]. We perform the shift

\[ \sigma_\alpha \to \sigma_\alpha + \bar{D}^2 D_\alpha \psi + D_\alpha \chi \quad \bar{D}_\alpha \chi = 0 \]  

(5.26)

where \( \psi \) is a general scalar and \( \chi \) a chiral scalar superfield. This leads to new gauge invariances

\[ \delta\sigma_\alpha = \bar{D}^2 D_\alpha K + D_\alpha \Lambda \quad (\bar{D}_\alpha \Lambda = 0) \]
\[ \delta\psi = -K \quad \delta\chi = -\Lambda \]  

(5.27)

Correspondingly we choose as gauge–fixing functions

\[ F_1 = D^\alpha \sigma_\alpha + a \, D^\alpha \bar{D}^2 D_\alpha \psi + b \, D^2 \chi + c \, \bar{D}^2 D^\alpha \psi \]
\[ F_2 = \bar{D}^2 \psi \]  

(5.28)

and we introduce the gauge–fixing lagrangian

\[ h \, \bar{F}_1 F_1 + k \, \bar{F}_2 \Box F_2 \]  

(5.29)

With constants \( a = 1/2, b = 3/2, h = 8/9, k = -2/9 \) the total quadratic lagrangian becomes

\[ -\frac{1}{2} H^a \Box H_a + \frac{4}{9} \bar{\sigma}_\alpha i \partial^\alpha \sigma_\alpha - \frac{2}{3} \chi \Box \chi + \frac{2}{9} \bar{\psi} \Box D^\alpha \bar{D}^2 D_\alpha \psi + \frac{8}{9} c^2 \bar{\psi} \Box \bar{D}^2 D^\alpha \psi - \frac{2}{9} \bar{\psi} \Box D^2 \bar{D}^2 \psi \]  

(5.30)

A final shift, with a new catalyst field, is needed in order to obtain a good kinetic term for \( \psi \):

\[ \psi \to \psi + \rho \quad \bar{D}_\alpha \rho = 0 \]  

(5.31)

The new invariance under a chiral, opposite shift of \( \psi \) and \( \rho \), is fixed simply by a gauge–fixing function \( F = D^2 (\psi + u \rho) \) and a gauge–fixing term \( \psi \Box F \). Appropriate choices of the constants lead to a cancellation of cross terms and to kinetic terms

\[ -\frac{2}{9} \bar{\psi} \Box^2 \psi + \frac{2}{9} u \, \bar{\rho} \Box^2 \rho \]  

(5.32)

We note that the shifts in (5.21) and (5.26), but not the one in (5.31), introduce additional couplings in the action (3.1) between \( \psi \) and \( \chi \) and the physical fields.

Although the gauge fixing in (5.23, 5.29) solves the problem of providing good kinetic terms for \( H^a \) and \( \sigma^\alpha \), we should caution the reader that the quantization of \( \sigma^\alpha \) is more subtle \[12\]. The gauge transformation in (5.18) has zero modes, \( \Lambda_{(\alpha \beta)} = D^\gamma \Lambda_{(\alpha \beta \gamma)} \) and eventually one generates an infinite tower of ghosts. However since these ghosts do not
interact, one cannot close a loop whenever the $\Lambda_{\alpha\beta}$ to consider the part of the Faddeev–Popov lagrangian in which only the role. Indeed, the kinetic matrix has a triangular structure and since the $\Lambda_{\alpha\beta}$ the latter, using also (5.21) with $z$ variations are nonlinear. The relevant gauge variations are in (5.16) and in (5.17). From $\alpha\beta$ terms for the $\Lambda_{\alpha\beta}$ are the corresponding antighosts. By direct inspection of the quadratic and interaction we construct the Faddeev–Popov lagrangian

\[
\langle H^{\alpha\dot{\alpha}}(x, \theta) H_{\beta\dot{\beta}}(x', \theta') \rangle = \frac{\delta_{\alpha\dot{\alpha}}}{4} \delta^4(x - x') \delta^4(\theta - \theta')
\]

\[
\langle \sigma_\alpha(x, \theta) \bar{\sigma}_{\dot{\alpha}}(x', \theta') \rangle = \frac{9}{4} \delta^4(x - x') \delta^4(\theta - \theta') \]

\[
\langle \chi(x, \theta) \bar{\chi}(x', \theta') \rangle = \frac{3}{2} \delta^4(x - x') \delta^4(\theta - \theta')
\]

\[
\langle \psi(x, \theta) \bar{\psi}(x', \theta') \rangle = \frac{9}{2} \delta^4(x - x') \delta^4(\theta - \theta')
\]

(5.33)

Whereas the ghost fields introduced so far have been effectively ignored having no interaction with the quantum fields, this is no longer the case for the Faddeev–Popov ghosts of the supergravity multiplet. They interact with $H_\alpha$ and $\sigma_\alpha$ because the gauge variations are nonlinear. The relevant gauge variations are in (5.16) and in (5.17). From the latter, using also (5.21) with $z = -\frac{1}{2}$, one has

\[
\delta \sigma_\alpha = \delta \tau_\alpha + \frac{1}{2} \bar{D}^\delta \delta H_{\alpha\dot{\alpha}}
\]

(5.34)

Given the field transformations, from the variation of the gauge–fixing functions in (5.22), we construct the Faddeev–Popov lagrangian

\[
L_{FP} = L^\alpha \delta F_\alpha + \Lambda^{\alpha\beta} \delta F_{\alpha\beta} + \text{h.c.}
\]

(5.35)

where the variations are with respect to the Faddeev–Popov ghosts $L_\alpha$, $\Lambda_{\alpha\beta}$ and $L'_\alpha$, $\Lambda'_{\alpha\beta}$ are the corresponding antighosts. By direct inspection of the quadratic and interaction terms for the $\Lambda_{\alpha\beta}$, $\Lambda'_{\alpha\beta}$ fields, it is possible to establish that they do not play any quantum role. Indeed, the kinetic matrix has a triangular structure and since the $\Lambda_{\alpha\beta}$ do not interact, one cannot close a loop whenever the $\Lambda_{\alpha\beta}$’s are present. Therefore it is sufficient to consider the part of the Faddeev–Popov lagrangian in which only the $L_\alpha$ and $L'_\alpha$ fields appear. With a trivial rescaling of the antighosts one obtains

\[
L_{FP} = -\bar{L}^\alpha D^2 \bar{L}_\alpha - \bar{L}^\alpha D_\alpha \bar{D}_\dot{\alpha} L^\dot{\alpha} - L^\alpha \bar{D}^2 L_\alpha - L'^\alpha \bar{D}_\dot{\alpha} \bar{D}_\dot{\alpha} \bar{L}^\dot{\alpha} \\
+ (D^\alpha \bar{L}^\alpha - \bar{D}^\alpha L'^\alpha) \left[ \frac{1}{2} H_\beta i \partial_\beta (D_\alpha \bar{L}_\dot{\alpha} + \bar{D}_\dot{\alpha} L_\alpha) - \frac{i}{2} \partial_\beta H_\alpha (D^\beta \bar{L}^\beta + \bar{D}^\beta L^\beta) \right] \\
+ \bar{D}^2 L^\beta D_\beta H_\alpha + D^2 \bar{L}^\beta \bar{D}_\beta H_\alpha \right] + \left[ L^\alpha \left( \bar{D}^2 L_\alpha D^\beta \sigma_\beta + \bar{L}^\dot{\alpha} \bar{D}_\dot{\alpha} D^2 \sigma_\alpha \right) \\
+ \frac{1}{2} D_\alpha (\bar{L}^\dot{\alpha} \bar{D}_\dot{\alpha} D^\beta \sigma_\beta) + H_\beta i \partial_\beta \bar{D}^2 L_\alpha \right] - \frac{1}{2} L'^\alpha \left( \bar{D}^2 L_\alpha D^\beta \bar{D}^\beta H_\beta \\
+ \bar{L}^\dot{\alpha} \bar{D}_\dot{\alpha} D^2 \bar{D}^\beta H_{\alpha\beta} + \frac{1}{2} D_\alpha (\bar{L}^\dot{\alpha} \bar{D}_\dot{\alpha} D^\beta \bar{D}^\beta H_\beta) \right) + \text{h.c.} \right]
+ \text{higher order terms}
\]

(5.36)
This action has the linearized gauge invariance $\delta L_\alpha = \Lambda_\alpha$ with $\bar{D}_\alpha \Lambda_\alpha = 0$ and will introduce ghosts–for–ghosts. In order to obtain a standard quadratic gauge–fixed kinetic term, it is convenient to already introduce catalysts \[9\] as we did earlier, with the shifts

$$L_\alpha \rightarrow L_\alpha + D_\alpha U \quad \quad \quad L'_\alpha \rightarrow L'_\alpha + D_\alpha U'$$ \hfill (5.37)

The enlarged gauge invariance of the resulting action is then

$$\delta L_\alpha = \Lambda_\alpha + D_\alpha L \quad \quad \quad \bar{D}_\alpha \Lambda_\alpha = 0$$ \hfill (5.38)

We choose as gauge–fixing functions

$$F = D^\alpha L_\alpha + D^2 U + \bar{D}^2 (dU + e\bar{U})$$

$$F_\alpha = \bar{D}^2 [L_\alpha + D_\alpha (U + \bar{U})]$$ \hfill (5.39)

and similar terms $F', F'_\alpha$, and add to (5.36) the gauge–fixing term

$$-\bar{F}' F - F' \bar{F} + \bar{F}' \alpha \frac{\bar{D}^2}{\Box} \bar{F}_\alpha + F' \alpha \frac{D^2}{\Box} F_\alpha$$ \hfill (5.40)

In this way the quadratic part of the lagrangian becomes

$$L_{FP}^{(2)} = \bar{L}' \alpha \iota \partial_\alpha \bar{L}' + L'^{\alpha} \iota \partial_\alpha \bar{L}' + \left[ -\bar{U}' \bar{D}^2 D_\alpha \bar{U}' - \bar{U}' \bar{D}^2 D_\alpha \bar{U}' + (1 - ee') \bar{U}' \bar{D}^2 D_\alpha \bar{U}' \right]$$ \hfill (5.41)

Again the shifts above introduce couplings between the $U$ fields and $H^a$. The U lagrangian is not yet in standard form, but this can be easily achieved introducing some chiral catalyst fields. We refer the interested reader to Appendix B of ref. \[11\], where the procedure is spelled out in detail. We emphasize that the shifts in (5.37) and the subsequent shifts in the catalysts $U$, do lead to unavoidable couplings of these new fields to the supergravity field.

The quantization of the system is now complete.

## 6 Conclusions

The motivation for undertaking this work was to provide all the ingredients necessary for quantitative, perturbative calculations in $N = 2$ matter–supergravity systems. Having reached this goal, the next step would be to perform an actual calculation. We are
For the gauge fixing in (5.23) we obtain

$$H = H^{\alpha} \partial^\alpha \partial^\beta H_{\beta} + \alpha^2 \sigma^\alpha \partial^\alpha \bar{D}^\beta \sigma_{\alpha} + a \sigma^\alpha \bar{D}^\alpha D^2 H_{\alpha} - a \sigma^\alpha \bar{D}^\alpha D^2 H_{\alpha}$$

(A.1)

We have obtained

$$H_{\alpha} \partial^\alpha \partial^\beta H_{\beta} = H_{\alpha} \partial^\alpha \partial^\beta H_{\beta} - H^{\alpha} \square H_{\alpha}$$

$$H_{\alpha} \partial^\alpha \bar{D}^\beta \bar{D}^\beta H_{\beta} = H_{\alpha} \partial^\alpha \bar{D}^\beta \bar{D}^\beta H_{\beta} - H_{\alpha} \partial^\alpha \bar{D}^\beta \bar{D}^\beta H_{\beta}$$

$$H_{\alpha} \partial^\alpha \bar{D}^\beta \bar{D}^\beta \partial^\alpha H_{\beta} = H_{\alpha} \partial^\alpha \bar{D}^\beta \bar{D}^\beta \partial^\alpha H_{\beta} - H_{\alpha} \partial^\alpha \bar{D}^\beta \bar{D}^\beta \partial^\alpha H_{\beta}$$

$$- i H^{\alpha} \square H_{\alpha} + i H^{\alpha} D^2 \bar{D}^2 H_{\alpha} - i H_{\alpha\beta} \bar{D}^\alpha D^2 \bar{D}^\beta H^{\beta}$$

$$H_{\alpha} \partial^\alpha \bar{D}^\beta \bar{D}^\beta \partial^\alpha H_{\beta} = H_{\alpha} \partial^\alpha \bar{D}^\beta \bar{D}^\beta \partial^\alpha H_{\beta} - i H^{\alpha} \square H_{\alpha} - i H^{\alpha} D^2 \bar{D}^2 H_{\alpha} - i H_{\alpha\beta} D^\alpha \bar{D}^2 \bar{D}^\beta H^{\beta}$$

$$H_{\alpha\beta} \bar{D}^\alpha \bar{D}^\beta H_{\alpha} = H_{\alpha\beta} \bar{D}^\alpha \bar{D}^\beta H_{\alpha} - H^{\alpha \beta} \bar{D}^\alpha \bar{D}^\beta D^2 H^{\alpha \beta}$$

(A.2)

For the gauge fixing in (5.23) we obtain

$$F_{\alpha} D^\alpha \bar{D}^\alpha F_{\alpha} = H^{\alpha} D^2 \bar{D}^2 H_{\alpha} + a^2 \bar{D}^\alpha \bar{D}^\beta \sigma_{\alpha} + a \sigma^\alpha \bar{D}^\alpha D^2 H_{\alpha} - a \sigma^\alpha \bar{D}^\alpha D^2 H_{\alpha}$$

We hope to report on this calculation in a not too distant future [13].

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A Appendix

We collect in this Appendix some identities that allow one to identify a minimal set of independent expressions quadratic in the supergravity field $H_{\alpha}$. Besides the terms $H^{\alpha} \square H_{\alpha}$, $H^{\alpha} D^2 \bar{D}^2 H_{\alpha}$, $H^{\alpha} \bar{D}^\beta \bar{D}^\beta H_{\alpha}$ we have included in the minimal set the following expressions: $H^{\alpha} \partial_\alpha \partial_\beta H^{\beta}$, $H^{\alpha} \partial_\alpha \bar{D}^\beta \bar{D}^\beta H_{\alpha}$ and $H_{\alpha\beta} D^{\alpha} \bar{D}^2 \bar{D}^\beta H^{\alpha \beta}$. In terms of these quantities we have obtained

(A.1)
\[
F_\alpha D_\alpha \bar{D} \bar{\alpha} \bar{F}_\bar{\alpha} = H_\alpha i \partial^\alpha D^\beta \bar{D} \bar{\beta} H_b - H_{\beta \bar{\alpha}} \bar{D}^\alpha D^2 \bar{D} \bar{\beta} H_b + a^2 \sigma_\alpha D^\alpha \bar{D} \bar{\alpha} \bar{\sigma}_\bar{\alpha} - a \sigma_\alpha D^\alpha i \partial^\beta H_b \\
+ a \sigma^\alpha D^2 \bar{D} \bar{\alpha} H_a + a \bar{\sigma}_\bar{\alpha} D^\alpha i \partial^\beta H_b - a \bar{\sigma}^\alpha \bar{D}^2 D^\alpha H_a
\]  
(A.3)

\[
F^\alpha D^2 F_\alpha = - H_\alpha \bar{D}^\alpha D^2 \bar{D} \bar{\beta} H^{\alpha \bar{\beta}} + 2 a \sigma^\alpha D^2 \bar{D} \bar{\alpha} H_a + a^2 \sigma^\alpha D^2 \sigma_\alpha
\]  
(A.4)

\[
\frac{1}{2} F^{\alpha \beta} F_{\alpha \beta} = 3 \sigma^\alpha D^2 \sigma_\alpha + 2 b \sigma^\alpha D_\alpha i \partial_\beta H^b - 4 b \sigma^\beta \bar{D}^\beta D^2 H_b + (4 b + 6 c) \sigma^\beta D^2 \bar{D} \bar{\beta} H_b \\
+ 2 b^2 H^\alpha \Box H_a - (b^2 + 2 b c) H_\alpha i \partial^\alpha \partial^\beta H_b - (3 c^2 + 2 b c) H_b \bar{D}^\beta D^2 \bar{D} \bar{\alpha} H^{\beta \bar{\alpha}} \\
+ 4 b c H^\alpha D^2 D^2 H_a - 2 b c H_a i \partial^\alpha D^\beta \bar{D} \bar{\beta} H_b - 2 b c H_{\alpha \beta} \bar{D}^\alpha D^2 D_\beta H^b
\]  
(A.5)

\[
\frac{1}{2} D^\beta F^{\alpha \beta} F_{\alpha \beta} = 3 a \sigma^\alpha D^2 \sigma_\alpha + a b \sigma^\alpha D_\alpha i \partial_\beta H^b + (3 + 2 a b + 3 a c) \sigma^\alpha D^2 \bar{D} \bar{\alpha} H_a \\
- 2 a b \sigma^\alpha D^2 H_a + 2 b H^\alpha D^2 D^2 H_a - (2 b + 3 c) H_b \bar{D}^\beta D^2 \bar{D} \bar{\alpha} H^{\beta \bar{\alpha}} \\
+ b H_a i \partial^\alpha D^\beta \bar{D} \bar{\beta} H_b
\]  
(A.6)

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