Analysis of nature of $\phi \to \gamma \pi \eta$ and $\phi \to \gamma \pi^0 \pi^0$ decays *

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Abstract

We study interference patterns in the $\phi \to (\gamma a_0 + \pi^0 \rho) \to \gamma \pi \eta$ and $\phi \to (\gamma f_0 + \pi^0 \rho) \to \gamma \pi^0 \pi^0$ reactions. Taking into account the interference, we fit the experimental data and show that the background reaction does not distort the $\pi^0 \eta$ spectrum in the decay $\phi \to \gamma \pi \eta$ everywhere over the energy region and does not distort the $\pi^0 \pi^0$ spectrum in the decay $\phi \to \gamma \pi^0 \pi^0$ when the invariant mass $m_{\pi^0 \pi^0} > 670$ MeV. We discuss the details of the scalar meson production in the radiative decays and note that there are conclusive arguments in favor of the one-loop mechanism $\phi \to K^+ K^- \to \gamma a_0$ (or $\gamma f_0$). We discuss also distinctions between the four-quark, molecular, and two-quark models and argue that the establishment of the scalar meson production mechanism in the $\phi$ radiative decays gives new strong evidence in favor of the four-quark nature of the scalar $a_0(980)$ and $f_0(980)$ mesons.

I. INTRODUCTION

As was shown in a number of papers, see Refs. [1–6] and references therein, the study of the radiative decays $\phi \to \gamma a_0 \to \gamma \pi \eta$ and $\phi \to \gamma f_0 \to \gamma \pi \pi$ can shed light on the problem of the scalar $a_0(980)$ and $f_0(980)$ mesons. These decays have been studied not only theoretically but also experimentally [4]. Present time data have already been obtained from Novosibirsk with the detectors SND [8–11] and CMD-2 [12], which give the following branching ratios: $BR(\phi \to \gamma \pi \eta) = (0.88 \pm 0.14 \pm 0.09) \cdot 10^{-4}$ [10], $BR(\phi \to \gamma \pi^0 \pi^0) = (1.221 \pm 0.098 \pm 0.061) \cdot 10^{-4}$ [11] and $BR(\phi \to \gamma \pi \eta) = (0.9 \pm 0.24 \pm 0.1) \cdot 10^{-4}$, $BR(\phi \to \gamma \pi^0 \pi^0) = (0.92 \pm 0.08 \pm 0.06) \cdot 10^{-4}$ [12]. DAΦNE also confirms the Novosibirsk results [13].

These data give evidence in favor of the four-quark $(q^2 \bar{q}^2)$ [14, 22] nature of the scalar $a_0(980)$ and $f_0(980)$ mesons. Note that the isovector $a_0(980)$ meson is produced in the radiative $\phi$ meson decay as intensively as the well-studied $\eta'$ meson involving essentially strange quarks $s\bar{s}$ ($\approx 66\%$), responsible for the decay.

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As shown in Refs. [1,3,23], the background situation for studying the radiative decays \( \phi \rightarrow \gamma a_0 \rightarrow \gamma \pi^0 \eta \) and \( \phi \rightarrow \gamma f_0 \rightarrow \gamma \pi^0 \pi^0 \) is very good. For example, in the case of the decay \( \phi \rightarrow \gamma a_0 \rightarrow \gamma \pi^0 \eta \), the process \( \phi \rightarrow \pi^0 \rho \rightarrow \gamma \pi^0 \eta \) is the dominant background. The estimation for the soft, by strong interaction standard, photon energy, \( \omega < 100 \text{ MeV} \), gives \( BR(\phi \rightarrow \pi^0 \rho \rightarrow \gamma \pi^0 \eta, \omega < 100 \text{ MeV}) \approx 1.5 \cdot 10^{-6} \). The influence of the background process is negligible, provided \( BR(\phi \rightarrow \gamma a_0 \rightarrow \gamma \pi^0 \eta, \omega < 100 \text{ MeV}) \geq 10^{-5} \). In Sec. II we show that for the obtained experimental data the influence of the background processes is negligible everywhere over the photon energy region [19].

The situation with \( \phi \rightarrow \gamma f_0 \rightarrow \gamma \pi^0 \pi^0 \) decay is not much different. As was shown in [1,3,23] the dominant background is the \( \phi \rightarrow \pi^0 \rho^0 \rightarrow \gamma \pi^0 \pi^0 \) process with \( BR(\phi \rightarrow \pi^0 \rho^0 \rightarrow \gamma \pi^0 \pi^0, \omega < 100 \text{ MeV}) \approx 6.4 \cdot 10^{-7} \). The influence of this background process is negligible, provided \( BR(\phi \rightarrow \gamma f_0 \rightarrow \gamma \pi^0 \pi^0, \omega < 100 \text{ MeV}) \geq 5 \cdot 10^{-6} \).

The exact calculation of the interference patterns between the decays \( \phi \rightarrow \gamma f_0 \rightarrow \gamma \pi^0 \pi^0 \) and \( \phi \rightarrow \rho^0 \pi \rightarrow \gamma \pi^0 \pi^0 \) [19], which we present in Sec. III, shows that the influence of the background in the decay \( \phi \rightarrow \gamma \pi^0 \pi^0 \) for the obtained experimental data is negligible in the wide region of the \( \pi^0 \pi^0 \) invariant mass, \( m_{\pi\pi} > 670 \text{ MeV} \), or in the photon energy region \( \omega < 300 \text{ MeV} \).

In Sec. IV we discuss the mechanism of the scalar meson production in the radiative decays and show that experimental data obtained in Novosibirsk give the conclusive arguments in favor of the one-loop mechanism \( \phi \rightarrow K^+ K^- \rightarrow a_0 \) and \( \phi \rightarrow K^+ K^- \rightarrow f_0 \) of these decays [13]. We explain also why this circumstance gives new strong evidence in favor of the four-quark nature of the scalar \( a_0(980) \) and \( f_0(980) \) mesons.

II. INTERFERENCE BETWEEN THE REACTIONS \( \phi \rightarrow \gamma a_0 \rightarrow \gamma \pi^0 \eta \) AND \( \phi \rightarrow \pi^0 \rho^0 \rightarrow \gamma \pi^0 \eta \)

The fit [19] of the experimental data from the SND detector [10] is shown in Fig. 1. The total branching ratio, taking into account the interference, is \( BR(\phi \rightarrow (\gamma a_0 + \pi^0) \rightarrow \gamma \pi^0 \eta) = (0.79 \pm 0.2) \cdot 10^{-4} \), the branching ratio of the signal is \( BR(\phi \rightarrow \gamma a_0 \rightarrow \gamma \pi^0 \eta) = (0.75 \pm 0.2) \cdot 10^{-4} \) and the branching ratio of the background is \( BR(\phi \rightarrow \rho^0 \pi^0 \rightarrow \gamma \pi^0 \eta) = 3.43 \cdot 10^{-6} \). So, the integral part of the interference is negligible. The influence of the interference on the mass spectrum of the \( \pi \eta \) system is also negligible, see Fig. 1.

III. INTERFERENCE BETWEEN THE \( e^+e^- \rightarrow \gamma f_0 \rightarrow \gamma \pi^0 \pi^0 \) AND \( e^+e^- \rightarrow \phi \rightarrow \pi^0 \rho \rightarrow \gamma \pi^0 \pi^0 \) REACTIONS

The total branching ratio, with interference being taken into account, is \( BR(\phi \rightarrow (\gamma f_0 + \pi^0) \rightarrow \gamma \pi^0 \pi^0) = (1.26 \pm 0.29) \cdot 10^{-4} \), the branching ratio of the signal is \( BR(\phi \rightarrow \gamma f_0 \rightarrow \gamma \pi^0 \pi^0) = (1.01 \pm 0.23) \cdot 10^{-4} \), the branching ratio of the background is \( BR(\phi \rightarrow \rho^0 \pi^0 \rightarrow \gamma \pi^0 \pi^0) = 0.18 \cdot 10^{-4} \).

One can see from Fig. 2 that the influence of the background process on the spectrum of the \( \phi \rightarrow \gamma \pi^0 \pi^0 \) decay is negligible in the wide region of the \( \pi^0 \pi^0 \) invariant mass, \( m_{\pi\pi} > 670 \text{ MeV} \), or when photon energy less than 300 MeV.
FIG. 1. Fitting of $dBR(\phi \rightarrow \gamma\pi\eta)/dm \times 10^4$GeV$^{-1}$ with the background is shown with the solid line, the signal contribution is shown with the dashed line.

FIG. 2. Fitting of $dBR(\phi \rightarrow \gamma\pi^0\pi^0)/dm \times 10^4$GeV$^{-1}$ with the background is shown with the solid line, the signal contribution is shown with the dashed line. The dotted line is the interference term. The data are from the SND detector.
The difference from the experimental data, observed in the region $m_{\pi\pi} < 670$ MeV, is due to the fact that in the experimental processing of the $e^+e^- \rightarrow \gamma\pi^0\pi^0$ events the background events $e^+e^- \rightarrow \omega\pi^0 \rightarrow \gamma\pi^0\pi^0$ are excluded with the help of the invariant mass cutting and simulation, in so doing the part of the $e^+e^- \rightarrow \phi \rightarrow \rho\pi^0 \rightarrow \gamma\pi^0\pi^0$ events is excluded as well.

IV. CONCLUSION

The experimental data give evidence [19] not only in favor of the four-quark model but in favor of the dynamical model suggested in Ref. [1], in which the discussed decays proceed through the kaon loop, $\phi \rightarrow K^+K^- \rightarrow \gamma f_0(a_0)$, see Refs. [1,3].

Indeed, according to the gauge invariance condition, the transition amplitude $\phi \rightarrow \gamma f_0(a_0)$ is proportional to the electromagnetic field strength tensor $F_{\mu\nu}$ (in our case to the electric field). Since there are no pole terms in our case, the function $g(m)$ in

\[
\frac{d\Gamma(\phi \rightarrow \gamma a_0 \rightarrow \gamma\pi\eta m)}{dm} = \frac{2m^2\Gamma(\phi \rightarrow \gamma a_0(m))\Gamma(a_0(m) \rightarrow \pi\eta)}{\pi |D_{a_0}(m)|^2}
\]

and

\[
\frac{d\Gamma(\phi \rightarrow \gamma f_0 \rightarrow \gamma\pi^0\pi^0 m)}{dm} = \frac{|g(m)|^2\sqrt{m^2 - 4m^2_f(m^2_{\phi} - m^2)}}{3(4\pi)^3m^3_{\phi}} \left| \sum_{R,R'} g_{RK^+K^-}G^{-1}_{RR'} g_{R'R\pi^0\pi^0} \right|^2
\]

is proportional to the energy of photon $\omega = (m^2_{\phi} - m^2)/2m_{\phi}$ in the soft photon region. To describe the experimental spectra in Figs. [1] and [2], the function $|g(m)|^2$ should be smooth (almost constant) in the range $m \leq 0.99$ GeV, see Eqs. (1) and (2). Stopping the function $\omega^2$ at $\omega_0 = 30$ MeV, using the form-factor of the form $1/(1+R^2\omega^2)$, requires $R \approx 100$ GeV$^{-1}$. It seems to be incredible to explain the formation of such a huge radius in hadron physics. Based on the large, by hadron physics standard, $R \approx 10$ GeV$^{-1}$, one can obtain an effective maximum of the mass spectra under discussion only near 900 MeV. In the meantime, the $K^+K^-$ loop gives the natural description to this threshold effect, see Fig. 3.

To demonstrate the threshold character of this effect we present Fig. 4 and Fig. 5 in which the function $|g(m)|^2$ is shown in the case of $K^+$ meson mass is 25 MeV and 50 MeV less than in reality.

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1Eq. (2) takes into account the mixing of $f_0(980)$ meson with other scalar resonances, see Refs. [3-5].
FIG. 3. The function $|g(m)|^2$ is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

One can see from Figs. 4 and 5 that the function $|g(m)|^2$ is suppressed by the $\omega^2$ low in the region 950-1020 MeV and 900-1020 Mev respectively.

FIG. 4. The function $|g(m)|^2$ for $m_{K^+} = 469$ MeV is drawn with the solid line. The contribution of the imaginary is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.
FIG. 5. The function $|g(m)|^2$ for $m_{K^+} = 444$ MeV is drawn with the solid line. The contribution of the imaginary part is drawn with the dashed line. The contribution of the real part is drawn with the dotted line.

In the mass spectrum this suppression is increased by one more power of $\omega$, see Eqs. (1) and (2), so that we cannot see the resonance in the region 980-995 MeV. The maximum in the spectrum is effectively shifted to the region 935-950 MeV and 880-900 MeV respectively.

In truth this means that $a_0(980)$ and $f_0(980)$ resonances are seen in the radiative decays of $\phi$ meson owing to the $K^+K^-$ intermediate state, otherwise the maxima in the spectra would be shifted to 900 MeV.

Thus the mechanism of production of the scalar mesons in the $\phi$ radiative decays is established at a physical level of proof. It is the rarest case in hadron physics.

Both real and imaginary parts of the $\phi \rightarrow \gamma a_0(f_0)$ amplitudes are caused by the $K^+K^-$ intermediate state. The imaginary parts are caused by the real $K^+K^-$ intermediate state while the real parts are caused by the virtual compact $K^+K^-$ intermediate state, that is, we are dealing here with the four-quark transition. Needless to say radiative four-quark transitions can happen between two two-quark states as well as between two-quark and four-quark states but their intensities depend strongly on a type of the transitions. A radiative four-quark transition between two two-quark states requires creation of an additional $q\bar{q}$ pair, that is, such a transition is forbidden according to the Okuba-Zweig-Izuka (OZI) rule, while a radiative four-quark transition between two-quark and four-quark states happens

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2It is worth noting that the $K^+K^-$ loop model is practically accepted by theorists, compare, for example, Ref. [24] with Ref. [25]; true there is exception [26].

3It will be recalled that the imaginary part of every hadronic amplitude describes a multi-quark transition.
without creation an additional \( q\bar{q} \) pair, that is, such a transition is allowed according to the OZI rule.

Let us discuss this problem from two points of view: i) from point of view of intermediate states and ii) from point of view of the \( 1/\Nc \) expansion.

i) It was noted already in paper \[1\] that the imaginary part of the \( K^+K^- \) loop is calculated practically in a model independent way making use of the coupling constants \( g_{\phi K^+K^-} \) and \( g_{a_0(f_0)K^+K^-} \) due to the Low's theorem \[27\] for the photons with energy \( \omega < 100 \text{ MeV} \) which is soft by the standard of strong interaction.

In the same paper it was noted that the real part of the loop (with accuracy up to 20% in the width of the \( \phi \rightarrow \gamma f_0(a_0) \) decay) is practically not different for the point-like particle and the compact hadron with form-factor which has the cutting radius in the momentum space about the mass of \( \rho \) meson (\( m_\rho = 0.77 \text{ GeV} \)).

In contrast to the four-quark state which is the compact hadron \[14\], the bound \( K\bar{K} \) state is the extended state with the spatial radius \( R \sim 1/\sqrt{m_\rho \epsilon} \), where \( \epsilon \) is the binding energy. Corresponding form-factor in the momentum space has the radius of the order of \( \sqrt{m_\rho \epsilon} \approx 100 \text{ MeV} \) for \( \epsilon = 20 \text{ MeV} \). The more detailed calculation \[2\] gives for the radius in the momentum space the value \( p_0 = 140 \text{ MeV} \). As a result, the contribution of the virtual intermediate \( K^+K^- \) states in the \( K^+K^- \) loop is suppressed by the momentum distribution in the molecule, and the real part of the loop amplitude is negligible \[4\]. It leads to the branching ratio much less than the experimental one. In addition, the spectrum is much narrower in the \( K\bar{K} \) molecule case that contradicts to the experiment, see the behavior of the imaginary part contribution in Fig. 3 and in corresponding figures in \[4\].

Of course, the two-quark state is as compact as four-quark one. The question arises, why is the branching ratio in the two-quark model suppressed in comparison with the branching ratio in the four-quark model? There are two reasons. First, the coupling constant of two-quark states with the \( K\bar{K} \) channel is noticeably less \[3,15\] and, second, there is the OZI rule that is more important really.

If the isovector \( a_0(980) \) meson is the two-quark state, it has no strange quarks. Hence \[1,3,17\], the decay \( \phi \rightarrow \gamma a_0 \) should be suppressed according to the OZI rule. On the intermediate state level, the OZI rule is formulated as compensation of the different intermediate states \[29,31\]. In our case these states are \( K\bar{K}, K\bar{K}^* + \bar{K}K^*, K^*\bar{K}^* \) and so on. Since, due to the kinematical reason, the real intermediate state is the only \( K^+K^- \) state, the compensation in the imaginary part is impossible and it destroys the OZI rule. The compensation should be in the real part of the amplitude only. As a result, the \( \phi \rightarrow \gamma a_0 \) decay in the two-quark model is mainly due to the imaginary part of the amplitude and is much less intensive than in the four-quark model \[14,15\]. In addition, in the two-quark model, \( a_0(980) \) meson should appear in the \( \phi \rightarrow \gamma a_0 \) decay as a noticeably more narrow resonance than in other processes, see the behavior of the imaginary part contribution in Fig. 3.

As regards to the isoscalar \( f_0(980) \) state, there are two possibilities in the two-quark model. If \( f_0(980) \) meson does not contain the strange quarks the all above mentioned arguments about suppression of the \( \phi \rightarrow \gamma a_0 \) decay and the spectrum shape are also valid for the \( \phi \rightarrow \gamma f_0 \) decay. Generally speaking, there could be the strong OZI violation for the isoscalar \( q\bar{q} \) states (mixing of the \( uu, dd \) and \( ss \) states) with regard to the strong mixing of the quark and gluon degree of freedom which is due to the nonperturbative effects of QCD \[32\]. But, an almost exact degeneration of the masses of the isoscalar \( f_0(980) \) and isovector
a_0(980) mesons excludes such a possibility. Note also, the experiment points directly to the weak coupling of f_0(980) meson with gluons, \( B(J/\psi \to \gamma f_0 \to \gamma \pi \pi) < 1.4 \cdot 10^{-5} \).\footnote{It will be recalled that the OZI allowed \( \phi \to \gamma \eta' \) amplitude has the order of \( N_c^0 \).}

If f_0(980) meson is close to the s\bar{s} state\footnote{It will be recalled that the OZI allowed \( \phi \to \gamma \eta' \) amplitude has the order of \( N_c^0 \).}, there is no suppression due to the the OZI rule. Nevertheless, if a_0(980) and f_0(980) mesons are the members of the same multiplet, the \( \phi \to \gamma f_0 \) branching ratio, \( BR(\phi \to \gamma \pi^0 \pi^0) = (1/3) BR(\phi \to \gamma \pi \pi) \approx 1.8 \cdot 10^{-5} \), is significantly less than that in the four-quark model, due to the relation between the coupling constants with the KK, \( \eta \) and KK, \( \pi \pi \) channels inherited in the two-quark model, see Refs. [1][3].

In addition, in this case there is no natural explanation of the a_0 and f_0 mass degeneration. Only in the case when the nature of f_0(980) meson in no way related to the nature of a_0(980) meson (which, for example, is the four-quark state) the experimentally observed branching ratio of the \( \phi \to \gamma f_0 \) decay could be explained by s\bar{s} nature of f_0(980) meson. But, from the theoretical point of view, such a possibility seems awful [17].

ii) What is more, the OZI allowed transition is bound to have a small weight in the \( 1/N_c \) expansion in case of s\bar{s} nature of f_0(980) meson. Indeed, the main term of the \( 1/N_c \) expansion of the \( \phi \to \gamma f_0 \) amplitude, i.e, the OZI allowed transition, has the order of \( N_c^0 \) but does not contains the \( K^+K^- \) intermediate state. This state emerges only in the next to leading term of the \( 1/N_c \) expansion, i.e., in the OZI forbidden transition, which has the order of \( 1/N_c^2 \).

If f_0(980) meson is the two-quark state without the strange quarks, the \( 1/N_c \) expansion of the \( \phi \to \gamma f_0(980) \) amplitude starts with the OZI forbidden transition of the order of \( 1/N_c \). But a weight of this term is bound to be small, because it does not contain the \( K^+K^- \) intermediate state, which emerges only in the next to leading term of the order of \( 1/N_c^2 \).

In the two-quark model of a_0(980) meson the \( 1/N_c \) expansion of the \( \phi \to \gamma a_0(980) \) amplitude starts also with the OZI forbidden transition of the order of \( 1/N_c \), whose weight is bound to be small, because this term does not contain the \( K^+K^- \) intermediate state, which emerges only in the next to leading term of the order of \( 1/N_c^2 \).

In the meantime, if a_0(980) and f_0(980) mesons are compact KK states, i.e., four-quark states, the \( 1/N_c \) expansions of the \( \phi \to \gamma a_0(980)(f_0(980)) \) amplitudes start with the OZI allowed transitions of the order of \( N_c^{-1/2} \), which contain the \( K^+K^- \) intermediate state.

As we see, the knowledge of the mechanism of the scalar meson production in the \( \phi \) radiative decays gives the new very strong (if not crucial) evidence in favor of the four-quark nature of the scalar a_0(980) and f_0(980) mesons.

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The authors of this paper use the amplitude of the $\phi \to \gamma(f_0 + \text{background}) \to \gamma\pi^0\pi^0$ decay which does not vanish when $\omega \to 0$, i.e. which does not satisfy the gauge invariance condition. This amplitude is not adequate to the physical problem since the mass spectrum under discussion should have the behavior $\omega^3$ at $\omega \to 0$ and not $\omega$ as in [hep-ph/0011191]. With the same result one can study the electromagnetic form-factor of $\pi$ meson in the $e^+e^- \to \pi^+\pi^-$ reaction near the threshold considering that the cross-section of the process is proportional to the momentum of $\pi$ meson while it is proportional to the momentum in the third power.

Hereinafter the comment could not be included in Physical Review D 63, 094007 (2001) (published 30 March 2001) for the temporal reasons.

To provide the spectrum behavior $\omega^3$ at $\omega \to 0$ the authors, correcting some
typos and undoing some references in [hep-ph/0011191 v4 27 Mar 2001],
inserted a crazy common factor \( F_{\text{thresh}}(\omega) = \sqrt{1 - \exp\{- (\omega/36 \text{ MeV})^2\} \) in the \( \phi \to \gamma(f_0 + \text{background}) \to \gamma\pi^0\pi^0 \) amplitude without any explanations, see Eq. (39) in [hep-ph/0011191 v4]. But the real trouble is that the calculation in [hep-ph/0011191] is not gauge invariant. The calculation of the \( \phi \to q\bar{q} \to \gamma f_0 \) amplitude requires a gauge invariant regularization (for example, the substraction at \( \omega = 0 \)) in spite of the integral convergence. A text-book example of such a kind is the \( \gamma\gamma \to e^+e^- \) (or \( q\bar{q} \to \gamma\gamma \)) scattering. The authors of the paper under discussion obtained that \( A_{\phi\to\gamma f_0} = (\text{in our symbols}) \ g(m)(g_{f_0K^+K^-}/e) \neq 0 \) at \( \omega = (m_\phi^2 - m^2)/2m_\phi = 0 \) \( (A_{\phi\to\gamma f_0} \) does not depend on \( m \) at all), see Eq. (30) in [hep-ph/0011191]. This means that the authors created the false pole in the invariant amplitude free from kinematical singularities: \( (eA_{\phi\to\gamma f_0}/(m_\phi^2 - m^2)) \times (\phi_\mu p_\nu - \phi_\nu p_\mu) (\epsilon_\mu q_\nu - \epsilon_\nu q_\mu), \) compare with Eq. (9) in [hep-ph/0011191]. So, once again, the calculation of [hep-ph/0011191] is not adequate to the physical problem!

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