Review of stripline beam impedance: application to the extraction kicker for the CLIC damping rings

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Abstract. The beam coupling impedance of the stripline kicker for beam extraction from the CLIC Damping Rings (DRs) has been studied analytically, numerically with Computer Simulation Technology (CST) Particle Studio (PS) and measured in the laboratory, although not all the results were previously understood. In order to have a better knowledge about the beam coupling impedance of a stripline kicker, a simple model has now been studied, with flat electrodes and a cylindrical beam pipe. From this preliminary study, a new approach for the dipolar component of the horizontal impedance has been derived, when considering both odd and even operating modes of the striplines. This new approach has been used to understand the differences found between the predicted transverse impedance and the two wires measurements carried out in the laboratory for the prototype CLIC DR striplines. Future tests of beam coupling impedance with beam in the ALBA Synchrotron Light Source will complete this study.

1. Preliminary study: a simple approximation

Several approaches for calculating the beam coupling impedance of stripline pickups and kickers can be found in the literature [1, 2, 3, 4, 5], most of them based on the image current analysis of Lambertson and Goldberg, which is only valid at frequencies below $c/L$ [3], where $L$ is the striplines length and $c$ is the speed of light in vacuum. These approaches are similar, but two main differences can be noted: some of them use an analytical approach for the geometrical factors [1, 2], with an accuracy greater than 95% for electrodes with $h > 2a$, where $h$ is the electrodes height and $a$ is the aperture, i.e. the distance between the electrodes. Other authors use a more precise geometric factor calculated numerically with an electrostatic solver [3, 4, 5], which can be applied to striplines of any shape and dimensions. In addition, in [1, 2, 3] the two different operation modes of striplines (differential and common, also known as odd and even, respectively) are not considered, whereas in [4, 5] the difference between the odd and even characteristic impedances has been taken into account. However, the question of which operation mode is related to the longitudinal and transverse beam coupling impedances, for a stripline kicker, is not well documented and is studied in more detail in this paper.

In [5], analytical equations for the longitudinal and transverse beam coupling impedances were compared with numerical simulations and laboratory measurements: a good agreement was found for the longitudinal beam coupling impedance. However there was a disagreement in the dipolar component of the transverse beam coupling impedance, which was not understood. In [1, 2, 3], the transverse beam coupling impedance is related to the even mode characteristic impedance, whereas in [4, 5] it is related to the odd mode. Hence, the starting point of this
preliminary study was to analyse the horizontal beam coupling impedance in the two operation modes, and compare these results with numerical simulations and measurements, to better understand which operation mode was excited.

The simple model used (Figure 1), consists of flat electrodes of 500 mm length and 25 mm height, with an aperture of 20 mm and a cylindrical beam pipe radius of 25 mm. For these striplines, the predicted odd and even mode characteristic impedances are 44.6 Ω and 60.6 Ω, respectively.

![Figure 1. Simple model used for studying the beam coupling impedance and its relationship to the two operation modes of a stripline kicker.](image)

### 1.1. Longitudinal beam coupling impedance

The longitudinal beam coupling impedance can be analytically calculated from the image currents in the electrodes, created by a beam passing through the aperture of the striplines, i.e. in the common or even mode configuration, with the following equation [4, 5]:

\[
Z_{||} = Z_{\text{comm}} g_{||}^2 \left( \sin^2 \left( \frac{\omega L}{c} \right) + j \sin \left( \frac{\omega L}{c} \right) \cos \left( \frac{\omega L}{c} \right) \right)
\]  

(1)

where \(Z_{\text{comm}} (= Z_{\text{even}}/2)\) is the common mode characteristic impedance, \(g_{||} (= 0.89)\) is the longitudinal geometric factor, and \(\omega\) is the angular frequency. This equation is valid under the assumption that the beam is passing through the centre of the stripline aperture, or with an offset \(x_0/b \ll 1\) [2], where \(x_0\) is the beam offset and \(b\) is the half-aperture of the striplines. The longitudinal geometrical factor has been calculated with the electrostatic solver of Opera 2D as \(g_{||} = V(0,0)\), i.e. it is equal to the voltage at the centre of the stripline aperture, when considering that the electrodes are driven by a voltage of 1 V each. The result has been compared with
the longitudinal beam coupling impedance predicted with CST PS, when modelling a bunch of 20 mm length passing through the centre of the stripline aperture, and found to be in good agreement, as will be shown below.

In addition, the longitudinal beam coupling impedance can be measured in the laboratory, by measuring the transmission through a wire placed in the centre of the stripline aperture. The longitudinal beam coupling impedance can be calculated using the formula [8]:

\[
Z_{||} = -2Z_{\text{line}} \ln(S_{21}) \tag{2}
\]

where \(Z_{\text{line}}\) is the characteristic impedance of the transmission line formed by the stripline and the wire and \(S_{21} = S_{21,\text{DUT}}/S_{21,\text{REF}}\) is the corrected transmission parameter. A more detailed explanation of this measurement can be found in [5]. Although the results for the longitudinal beam coupling impedance agreed well, for this preliminary study, we have now included predictions from a new model: the wire setup has been simulated in CST Microwave Studio (MWS), and the longitudinal beam coupling impedance has been calculated from the transmission parameter through the wire, as would be done in the laboratory. For this simulation, a \(Z_{\text{line}}\) of 220.7 Ω has been numerically calculated with CST MWS. The results of this model have been compared with Equation (1) and the CST PS simulation, as shown in Figure 2.

**Figure 2.** Real component of longitudinal beam coupling impedance results for Equation (1) (blue line), the CST PS simulation without a wire (red line), and the CST MWS simulation with a wire (green line).

Figure 2 shows the real component of the longitudinal beam coupling impedance. Only the real components of the beam coupling impedance are shown to keep the same notation through the paper –with the two wire method only the real component of the transverse beam coupling impedance was derived and studied. Equation (1) and the CST PS simulation match well in the frequency range studied, although some differences are observed already below \(\sim 200\) MHz. The peaks resulting from the wire method simulation (CST MWS) are slightly larger, but can still be considered a good approximation.
1.2. Horizontal beam coupling impedance

The transverse (horizontal or vertical) impedance of striplines can be divided into two different components: the dipolar component and the quadrupolar component [8]. The dipolar component of the horizontal beam coupling impedance can be directly calculated from the longitudinal beam coupling impedance (Equation (1)), with the Panofsky-Wenzel theorem [3]:

$$Z_{dip,x} = \frac{cZ_{||}}{\omega b^2} \left( \frac{g_\perp}{g_{||}} \right)^2$$

(3)

In Equation (3) $b$ ($= 10 \text{mm}$) is half of the stripline aperture, and $g_\perp$ ($= 0.98$) is the transverse geometrical factor. The transverse geometrical factor has been calculated with the electrostatic solver of Opera 2D, as $g_\perp = V(x,0)b/x$, where $x$ is the considered beam displacement, when driving one electrode with 1 V and the other one with -1 V. The analytical results have been compared with the CST PS predictions, when offsetting the beam horizontally by 1 mm from the centre of the stripline aperture. The results of the comparison are shown below.

The two wire measurement set-up has been modelled with CST MWS, where a dipolar field is created by driving the two wires with equal but opposite polarity voltages. The longitudinal beam coupling impedance is calculated using Equation (2), where now $Z_{\text{line}}$ ($= 369.2 \Omega$) is the differential characteristic impedance of the transmission line formed by the stripline and the two wires, calculated with CST MWS. The dipolar component of the horizontal beam coupling impedance is [8]:

$$Z_{dip,x} = \frac{cZ_{||}}{\omega \Delta^2}$$

(4)

In this equation $\Delta$ corresponds to the half-aperture of the striplines, instead of being the distance between the two wires, as it was supposed in [5]: for striplines with small aperture, the dipolar field extends along the whole striplines aperture. The authors suggest that results from Equation (4) cannot be directly compared with Equation (3) and CST PS simulation results, because the two wire measurement excite the odd (or differential) mode, whereas both Equation (3) and the CST PS simulation results refer to the horizontal beam coupling impedance in the even (or common) mode. Nevertheless, a modified version of Equation (3) is proposed by the authors to consider the dipolar component of the horizontal beam coupling impedance in the odd mode. For this mode, the Panofsky-Wenzel theorem cannot be directly applied to Equation (1), since it is related to the even mode. To consider the odd mode, the additional factor $Z_{\text{diff}}/Z_{\text{comm}}$ should be added:

$$Z_{dip,x} = \frac{cZ_{||}}{\omega \Delta^2} \left( \frac{g_\perp}{g_{||}} \right)^2 \left( \frac{Z_{\text{diff}}}{Z_{\text{comm}}} \right)$$

(5)

where $Z_{\text{diff}}$ ($= 2Z_{\text{odd}}$) is the differential characteristic impedance. All the results are shown and compared in Figure 3.

CST PS calculates the transverse beam coupling impedance from the transverse wakefields created by the image currents of the beam, i.e. in the even mode configuration of the striplines, whereas the two wire simulation has been made by considering currents with opposite direction in each wire, i.e. the odd mode configuration of the striplines. Figure 3 shows an excellent agreement between Equation (3) for the even mode and the CST PS simulation, and between Equation (5) for the odd mode and the 2 wire simulation.

2. Application to the compact linear collider damping ring striplines

2.1. Longitudinal beam coupling impedance

The longitudinal beam coupling impedance of the extraction kicker for the CLIC DRs was calculated and also measured in the laboratory, with a good agreement. The simulation of the
Figure 3. Dipolar component of the horizontal beam coupling impedance from Equation (3) (blue line) and Equation (5) (red line), the CST PS simulation (green line) and the CST MWS simulation of the two wire measurement set-up (magenta line).

For good visualization of the beam impedance behaviour, the frequency range is plotted up to 500 MHz.

Figure 4. Longitudinal beam coupling impedance from Equation (1) (blue line), the CST PS simulation (red line), the single wire measurement (green line) and the CST MWS single wire simulation (magenta line).

The longitudinal beam coupling impedance measured (green line) agrees well with the CST
PS (red line) up to around 450 MHz, whereas the single wire simulation (magenta line) is slightly different but still quite good. In addition, Equation (1) (blue line) is a good approximation up to 300 MHz.

2.2. Horizontal beam coupling impedance

The dipolar component of the horizontal beam coupling impedance of the extraction kicker for the CLIC DRs was calculated and also measured in the laboratory, although a disagreement was previously found [5]. We have made new calculations considering the approaches defined in the previous section, and also included a simulation of the single wire set-up with CST MWS. The results are shown in Figure 5.

Figure 5. Dipolar component of the horizontal beam coupling impedance from Equation (3) (blue line) and Equation (5) (red line), the CST PS simulation (green line), the simulation of the two wire measurement set-up (magenta line) and the two wire measurement result (cyan line).

Figure 5 shows that Equation (3) (blue line) agrees very well with the CST PS simulation (green line): both are for the even mode. Furthermore, Equation (5) agrees with the 2 wire simulation (cyan line) and the 2 wire measurement (magenta line): all three are for the odd mode. There is a frequency shift in the measurements, probably due to the reduced velocity of the current through the wires, which were considered to be lossless in the simulations.

3. Conclusions

Analytical equations for estimating the beam coupling impedance of stripline pickups and kickers have been widely used in the past. However, some confusion arises from the different notation and approximations which depend upon the author. In this paper we have tried to solve one of the main issues, i.e. to identify which stripline mode is excited when the beam passes through the centre of the aperture, when the beam is displaced from the centre, and when one or two wires are inserted into the stripline aperture. The conclusion of the preliminary study is that the beam excites the even mode of the stripline, whereas the two wire measurement excites the odd mode. Therefore, these two results cannot be compared directly, and a modified equation...
(Equation (5)) has been presented to consider this difference. Further studies of the quadrupolar contribution to the transverse impedance will be done in the future, as well as measurements with beam in ALBA.

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