\(D\Xi\) and \(D'\Xi\) Molecular States from One Boson Exchange

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We explore the existence of \(D\Xi\) and \(D'\Xi\) molecular states within the one boson exchange model. We regularize the potential derived in this model with a form factor and a cut-off of the order of 1 GeV. To determine the cut-off, we use the condition that the \(X(3872)\) is reproduced as a pole in the \(J^{PC} = 1^{+} D' D\) amplitude. From this we find that the \(J^{P} = \frac{3}{2}^{-}\) \(D' \Xi\) system is on the verge of binding and has an unusually large scattering length. For the \(J^{P} = \frac{5}{2}^{-}\) \(D' \Xi\) systems the attraction is not enough to form a bound state. From heavy quark symmetry and the quark model we can extend the previous model to the \(P\Xi_{Q0}\) and \(P' \Xi_{Q0}\) systems, with \(P = B, D\) and \(\Xi_{Q0} = \Xi_{cc}, \Xi_{bb}\). In this case we predict a series of triply heavy pentaquark-like molecules.

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I. INTRODUCTION

The discovery of the \(X(3872)\) by the Belle Collaboration\(^{11}\) fifteen years ago represented the first hidden charm state that did not fit into the charmonium spectrum. Afterwards experiments have found a series of similar states, informally known as XYZ states. They cannot be easily accommodated in the naive quark model and other component have to be invoked to explain their masses, decay widths and production rates, see \(^{[2]}\) for a recent review. A few are tetraquark-like, such as the \(Z_{c}(3900)\) \(^{[3]}\), \(Z_{c}(4020)\) \(^{[5]}\), \(Z_{b}(10610)\) and \(Z_{b}(10650)\) \(^{[6]}\) \(^{[8]}\)), while recently two pentaquark-like states have been observed, the \(P_{c}(4380)\) and \(P_{c}(4450)\) \(^{[9]}\). A few of them have the interesting feature of being close to an open charm threshold. The most notable example is the \(X(3872)\), located almost on top of the \(D' D^0\) threshold, but the list also includes the \(Z_{c}(3900)\) \((D^0 D)\) and \(Z_{c}(4020)\) \((D' D)\), the \(Z_{b}(10610)\) \((B^0 B)\) and \(Z_{b}(10650)\) \((B' B)\) and the \(P_{c}(4450)\) near the \(D' \Xi\) threshold. This characteristic has led to the conjecture that the previous states might be hadronic molecules.

A hadronic molecule is a loosely bound state or resonance composed of hadrons. They were originally proposed to explain the \(\psi(4040)\) as a \(D^* D^*\) bound state \(^{[10]}\) \(^{[11]}\). Though the \(\psi(4040)\) turned out to be a charmonium state at the end, the idea quickly caught the attention of theoreticians \(^{[12]}\) \(^{[15]}\) and the later discovery of the \(X(3872)\) showed that these speculations were indeed on the right track. Besides the \(X(3872)\), the most prosaic example of a hadronic molecule is the deuteron, which also inspired the Weinberg compositeness condition \(^{[16]}\). Other strong molecular candidates include the \(Z_{b}(10610)\), \(Z_{b}(10650)\) \(^{[17]}\) \(^{[23]}\) and the \(P_{c}(4450)\) \(^{[24]}\) \(^{[28]}\).

Recently the LHCb Collaboration has observed five narrow states \(\Omega_{c}(3000)^{0}\), \(\Omega_{c}(3050)^{0}\), \(\Omega_{c}(3066)^{0}\), \(\Omega_{c}(3090)^{0}\), and \(\Omega_{c}(3119)^{0}\) \(^{[29]}\), where four of them have also been recently confirmed from an analysis of the Belle data \(^{[30]}\). These states can be accomodated as excited \(\Omega_c\) baryons \(^{[31]}\) \(^{[32]}\), as compact baryons in which the ss quark pair forms a diquark \(^{[33]}\) or as molecular states \(^{[34]}\) \(^{[38]}\). The LHCb data also hint at a structure around 3188 MeV, which is near the \(D\Xi\) threshold \((3179 - 3191\) MeV\), and could be a bound state of \(D\Xi\). Wang et al. \(^{[39]}\) used the Bethe-Salpeter formalism to study the \(S\)-wave \(D\Xi\) interaction and they also found two bound states, one isoscalar and one isovector, respectively, which could contribute to the 3188 MeV structure near the five new narrow \(\Omega_c\) states. Debastiani and Liang \(^{[34]}\) \(^{[40]}\) used an extension of the chiral unitary model to calculate the interactions between \(D\Xi, D'\Xi, B\Xi, B'\Xi\) plus other channels, and obtained one zero width state with \(J^{P} = 1/2^{-}, 3/2^{-}\), which couples mostly to \(D'\Xi\) and \(B'\Xi\) and another state with \(J^{P} = 5/2^{-}\), which couples mostly to \(D\Xi\) and \(B\Xi\).

In this work we study possible bound states near the thresholds of \(D\Xi, D'\Xi, B\Xi, B'\Xi\) within the one boson exchange (OBE) model, where we will also consider the replacement of \(\Xi\) by \(\Xi_{cc}\) or \(\Xi_{bb}\) to explore the existence of heavy molecules containing two and three charm/bottom quarks. The OBE model is an intuitive framework in which a few of the most quantitative successful descriptions of the nuclear force have been achieved \(^{[41]}\) \(^{[42]}\). Nowadays it has been superseded by the effective field theory (EFT) approaches \(^{[43]}\) \(^{[44]}\), which have two theoretical advantages over the OBE model: (i) the possibility of making \textit{a priori} error estimates and (ii) the low-energy equivalence with quantum chromodynamics. However the EFT approach requires a large number of data to fix the low energy constants that substitute the exchange of light mesons such as the \(\sigma, \rho\) and \(\omega\). This means that in situations where hadron-hadron scattering data is poor, such as hadron molecules, the OBE potential has the advantage of providing a model of the short-range dynamics of these systems, at the price of sacrificing the systematicity of EFT. As a matter of fact the seminal works that pioneered the idea of hadronic molecules \(^{[10]}\) \(^{[11]}\) were indeed based on the OBE model and a few modern explorations rely heavily on this model \(^{[18]}\) \(^{[19]}\) \(^{[24]}\).

The manuscript is organized as follows: in Section II we review the OBE model as applied to the \(D\Xi, D'\Xi\) plus anal-
ogous systems and derive the potentials in these systems. In Section III we show the predictions we obtain for the molecular states. Finally we present our conclusions in Section IV.

II. THE ONE BOSON EXCHANGE POTENTIAL

In this section we explain the one boson exchange (OBE) model (see Ref. [12] for a review) and how it applies to the $D\Sigma$ and $D^*\Xi$ systems. The OBE model is a generalization of the idea of Yukawa, namely that nuclear forces arise from the exchange of pions, to shorter distances. For that the exchange of other light mesons (besides the pion) is considered, in particular the $\sigma$ scalar meson and the $\rho$ and $\omega$ vector mesons. The reason for the development of the OBE model was the failure of the original two-pion theories in the fifties (which did not include chiral symmetry), see Ref. [45] for a historical perspective. The fundamental idea is that the bulk of the two-pion exchange potential can be described by the exchange of a heavier meson, which either couples strongly to pions or which can be interpreted as a multi-pion resonance. The idea is physically compelling and has been fairly successful at the phenomenological level, as illustrated by the nuclear potentials based on it [41, 46]. There are limitations however, such as the requirement of form factors to regularize the potentials or the requirement of fine-tuning the coupling constants [47,48] (at least in the two-nucleon system). Yet the OBE potential is still perfectly able to provide a good estimation of the plausibility of hadronic bound states and their location, as attested for instance in the pioneering work of Voloshin and Okun [10].

A. The Lagrangian

We begin with the interaction of the $D$ and $D^*$ fields with the $\pi$, $\sigma$, $\rho$ and $\omega$ mesons. First we group the fields of the $D$ and $D^*$ in the (heavy spin symmetric) superfield

$$H_v = \frac{1 + y}{2\sqrt{2}} (D^\mu \gamma_\mu - D \gamma_5),$$

with $v$ the velocity parameter and where we have used the convenient normalization of Falk and Luke [49]. In this normalization we can write the interaction lagrangian of the $H$ superfield with the light mesons as

$$\mathcal{L}_{HH\pi} = -\frac{g_{H\pi}}{\sqrt{2} f_{\pi}} \text{Tr} \left[ H_v \gamma^\mu \gamma^5 \cdot \partial_{\mu} \bar{H}_\nu \right],$$

$$\mathcal{L}_{HH\sigma} = g_{H\sigma} \text{Tr} \left[ H_v \sigma \gamma_\mu \partial^\mu \bar{H}_\nu \right],$$

$$\mathcal{L}_{HH\rho} = g_{H\rho} \text{Tr} \left[ H_v \gamma_{\mu \nu} \partial^\mu \partial^\nu \bar{H}_\nu \right]$$

$$+ \frac{f_{H\rho}}{4M_1} \text{Tr} \left[ H_v \sigma_{\mu \nu} \gamma^\tau \left( \partial^\tau \partial^\rho - \partial^\rho \partial^\tau \right) \bar{H}_\nu \right],$$

$$\mathcal{L}_{HH\omega} = -g_{H\omega} \text{Tr} \left[ H_v \omega \gamma_\mu \partial^\mu \bar{H}_\nu \right]$$

$$- \frac{f_{H\omega}}{4M_1} \text{Tr} \left[ H_v \sigma_{\mu \nu} \left( \partial^\tau \omega^\rho - \partial^\rho \omega^\tau \right) \bar{H}_\nu \right].$$

where the traces are in the 4x4 space spanned by the Dirac matrices when defining the superfield $H$. In the lagrangian above we take the normalization $f_{\pi} = 132$ MeV, $g_{H\pi}, g_{H\sigma}, g_{H\rho}, f_{H\pi}, f_{H\rho}$ and $f_{H\omega}$ are the different couplings in the OBE model and $M_1$ is a mass scale that we introduce for $f_{H\pi}$ and $f_{H\rho}$ to be dimensionless ($M_1$ is in principle arbitrary, but we will later take it to be the D-meson mass, i.e. $M_1 = m_D$). If we choose the velocity parameter to be $v = (1,0)$, we can substitute the superfield $H_v$ by a non-relativistic superfield $H$

$$H_v \to H = \frac{1}{\sqrt{2}} \left[ P + \vec{p} \cdot \sigma \right],$$

where $H$ is a 2x2 matrix (instead of a 4x4 one). Now the lagrangian can be rewritten as

$$\mathcal{L}_{HH\pi} = -\frac{g_{H\pi}}{\sqrt{2} f_{\pi}} \text{Tr} \left[ H \gamma^\mu \gamma^5 \cdot \partial_{\mu} \bar{H} \right],$$

$$\mathcal{L}_{HH\sigma} = g_{H\sigma} \text{Tr} \left[ H_\sigma \gamma_\mu \partial^\mu \bar{H} \right],$$

$$\mathcal{L}_{HH\rho} = g_{H\rho} \text{Tr} \left[ H_\rho \gamma_{\mu \nu} \partial^\mu \partial^\nu \bar{H} \right]$$

$$- \frac{f_{H\rho}}{4M_1} \text{Tr} \left[ H_\rho \sigma_{\mu \nu} \gamma^\tau \left( \partial^\tau \partial^\rho - \partial^\rho \partial^\tau \right) \bar{H} \right],$$

$$\mathcal{L}_{HH\omega} = -g_{H\omega} \text{Tr} \left[ H_\omega \gamma_\mu \partial^\mu \bar{H} \right]$$

$$+ \frac{f_{H\omega}}{4M_1} \text{Tr} \left[ H_\omega \sigma_{\mu \nu} \left( \partial^\tau \omega^\rho - \partial^\rho \omega^\tau \right) \bar{H} \right].$$

Now for the $\Xi$ field we can write the interaction lagrangian

$$\mathcal{L}_{\Xi \Xi \pi} = -\frac{g_{\Xi \pi}}{\sqrt{2} f_{\pi}} \bar{\psi}_{\Xi} \gamma^\mu \gamma^5 \cdot \partial_{\mu} \bar{\psi}_{\Xi},$$

$$\mathcal{L}_{\Xi \Xi \sigma} = g_{\Xi \sigma} \bar{\psi}_{\Xi} \gamma_\mu \sigma \partial^\mu \bar{\psi}_{\Xi},$$

$$\mathcal{L}_{\Xi \Xi \rho} = g_{\Xi \rho} \bar{\psi}_{\Xi} \gamma_\mu \rho_\mu \bar{\psi}_{\Xi}$$

$$+ \frac{f_{\Xi \rho}}{4M_2} \bar{\psi}_{\Xi} \gamma_\mu \gamma_\nu \partial^\mu \partial^\nu \bar{\psi}_{\Xi},$$

$$\mathcal{L}_{\Xi \Xi \omega} = g_{\Xi \omega} \bar{\psi}_{\Xi} \gamma_\mu \omega_\mu \bar{\psi}_{\Xi}$$

$$+ \frac{f_{\Xi \omega}}{4M_2} \bar{\psi}_{\Xi} \gamma_\mu \gamma_\nu \left( \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \right) \bar{\psi}_{\Xi},$$

which is analogous to the one for the $D$ and $D^*$ (and it is identical in form to the one in the nucleon-nucleon case). In the lagrangian $g_{\Xi \pi}, g_{\Xi \sigma}, g_{\Xi \rho}, f_{\Xi \pi}, f_{\Xi \rho}, f_{\Xi \omega}$ and $M_2$ denote the couplings and the mass scale for the cascade baryon. Here we can use the heavy baryon formulation by making the field redefinition

$$\Xi = e^{i M_\Xi v \cdot x} \psi_{\Xi}.$$

If we choose again $v = (1,0)$, we arrive at the lagrangian

$$\mathcal{L}_{\Xi \Xi \pi} = -\frac{g_{\Xi \pi}}{\sqrt{2} f_{\pi}} \bar{\psi}_{\Xi} \gamma^\mu \gamma^5 \cdot \partial_{\mu} \bar{\psi}_{\Xi},$$

$$\mathcal{L}_{\Xi \Xi \sigma} = g_{\Xi \sigma} \bar{\psi}_{\Xi} \gamma_\mu \sigma \partial^\mu \bar{\psi}_{\Xi},$$

$$\mathcal{L}_{\Xi \Xi \rho} = g_{\Xi \rho} \bar{\psi}_{\Xi} \gamma_\mu \rho_\mu \bar{\psi}_{\Xi}$$

$$- \frac{f_{\Xi \rho}}{4M_2} \bar{\psi}_{\Xi} \gamma_\mu \gamma_\nu \partial^\mu \partial^\nu \bar{\psi}_{\Xi},$$

$$\mathcal{L}_{\Xi \Xi \omega} = g_{\Xi \omega} \bar{\psi}_{\Xi} \gamma_\mu \omega_\mu \bar{\psi}_{\Xi}$$

$$- \frac{f_{\Xi \omega}}{4M_2} \bar{\psi}_{\Xi} \gamma_\mu \gamma_\nu \left( \partial^\mu \omega^\nu - \partial^\nu \omega^\mu \right) \bar{\psi}_{\Xi}.$$
### B. The OBE Potential

The general form of the $P\Xi$ and $P^*\Xi$ OBE potential is

$$V = \zeta V_\pi + V_\sigma + V_\rho + \zeta V_\omega,$$

where the subscript indicates from which meson comes the contribution ($\pi$, $\sigma$, $\rho$ or $\omega$) and with $\zeta = \pm 1$ a sign. We take the convention that

$$\zeta = +1 \quad \text{for } P\Xi \text{ and } P^*\Xi,$$

$$\zeta = -1 \quad \text{for } \bar{P}\Xi \text{ and } \bar{P}^*\Xi.$$

If the vector meson and the hadrons are point-like, their vertices can be directly computed from the Lagrangian and we end up with the following potentials in momentum space

$$
\begin{align*}
V_\rho(q) &= \bar{q}_1 \cdot \bar{q}_2 \frac{g_{\rho 1} g_{\rho 2}}{2 m_\rho^2} (\sigma_1 \cdot \sigma_2) (\vec{q}_1 \cdot \vec{q}_2) q^2 + m_\rho^2, \\
V_\omega(q) &= \bar{q}_1 \cdot \bar{q}_2 \left( \frac{g_{\omega 1} g_{\omega 2}}{2 m_\omega} - \frac{f_{\omega 1} f_{\omega 2}}{2 m_\omega} \right) (\vec{q}_1 \cdot \vec{q}_2) (\vec{q}_1 \times \vec{q}_2), \\
V_\omega(q) &= -\frac{g_{\omega 1} g_{\omega 2}}{q^2 + m_\omega^2} + \frac{f_{\omega 1} f_{\omega 2}}{m_\omega} \left( \frac{\vec{q}_1 \cdot \vec{q}_2}{q^2 + m_\omega^2} \right),
\end{align*}
$$

In the expressions the subscript 1 and 2 is used for the $P/P^*$ heavy meson and $\Xi$ baryon respectively. For $\bar{d}_1$ we take

$$\bar{d}_1 = 0 \quad \text{for } P,$$

$$\bar{d}_1 = S_1' \quad \text{for } P^*,$$

with $S'$ the spin-1 angular momentum matrices. For the pion decay constant we take the $f_\pi = 132 \text{ MeV}$ normalization. The choice of sign for the momentum space potential is such that the Lippmann-Schwinger equation reads $T = V + V g_0 T$, where the $T$-matrix is in turn normalized so that for zero-energy scattering $T \rightarrow 2 \pi a_0/\mu$, with $a_0$ the scattering length and $\mu$ the reduced mass of the system. That is, we are using the standard non-relativistic normalization which is also used in the two-nucleon system, see for instance Ref. [42].

We can take into account the finite size of hadrons by including form factors in the calculation, that is

$$V_M(q, \Lambda) = V_M(q) F_1(q, m, \Lambda_1) F_2(q, m, \Lambda_1),$$

where $F_1$ and $F_2$ are the form factors corresponding to vertex 1 and 2, i.e. the $P/P^*$ heavy meson and the $\Xi$ baryon. The form factor can depend on the momentum transfer $q$, the mass of the exchanged meson $m$ and a cut-off $\Lambda$. Here we will use a monopolar form factor of the type

$$F(q, m, \Lambda) = \frac{\Lambda^2 - m^2}{\Lambda^2 - q^2},$$

for both vertices, where $q^2 = q_0^2 - \vec{q}^2$ is the 4-momentum of the exchanged meson.

In configuration space and for point-like interactions the components of the OBE potential take the form

$$V_\rho(r) = -\bar{r}_1 \cdot \bar{r}_2 \frac{g_{\rho 1} g_{\rho 2}}{6 f_\rho^2} \left[ -\bar{d}_1 \cdot \sigma_2 \delta(r) + \bar{d}_1 \cdot \sigma_2 m_\rho^2 W_T(m_\rho r) + S_{12}(r) m_\rho^2 W_T(m_\rho r) \right],$$

$$V_\omega(r) = -g_{\omega 1} g_{\omega 2} m_\omega W_T(m_\omega r),$$

$$V_\omega(r) = \frac{f_{\omega 1} f_{\omega 2}}{2 m_\omega} \left( \frac{2}{3} \bar{d}_1 \cdot \sigma_2 \delta(r) + \frac{1}{3} S_{12}(r) m_\rho^2 W_T(m_\rho r) \right),$$

where the functions $W_T(x)$ and $W_T(x)$ are defined as

$$W_T(x) = \frac{e^{-x}}{4 \pi x^3},$$

$$W_T(x) = \left( 1 + \frac{3}{x} + \frac{3}{x^2} \right) \frac{e^{-x}}{4 \pi x}. $$

The effects of the finite size of the hadrons is easy to take into account by making the changes

$$\delta(r) \to m^3 d(x, \lambda),$$

$$W_T(x) \to W_T(x, \lambda),$$

$$W_T(x) \to W_T(x, \lambda), $$

where $\lambda = \Lambda/m$. For the monopolar form factor of Eq. 30, we end up with

$$d(x, \lambda) = \frac{(\lambda^2 - 1)^2}{2 \lambda^2} e^{-\lambda x} \frac{4 \pi}{4 \pi},$$

$$W_T(x, \lambda) = W_T(x) - \lambda W_T(\lambda x) \frac{(\lambda^2 - 1)^2}{2 \lambda} - \lambda^2 \frac{4 \pi}{4 \pi},$$

$$W_T(x, \lambda) = W_T(x) - \lambda^2 W_T(\lambda x) - \frac{(\lambda^2 - 1)^2}{2 \lambda} \frac{4 \pi}{4 \pi}. $$

As a matter of fact the structure of the OBE potential presented here is exceedingly simple. We can write it as a sum of a central, spin-spin and tensor component

$$V(r) = V_C(r) + \bar{d}_1 \cdot \sigma_2 V_S(r) + S_{12}(r) V_T(r),$$

for \( V_C(r) \) the standard non-relativistic normalization which is also used
where for point-like interactions we have

\[
V_C(r) = -g_{\sigma 1} g_{\sigma 2} m_\pi W_T(m_\sigma r) \\
+ \frac{2}{3} \tau_1 \cdot \tau_2 g_{\rho 1} g_{\rho 2} m_\rho W_T(m_\rho r) \\
- \frac{2}{3} \zeta g_{\omega 1} g_{\omega 2} m_\omega W_T(m_\omega r),
\]

(44)

\[
V_S(r) = -\zeta \tau_1 \cdot \tau_2 \frac{g_{\rho 1} g_{\rho 2}}{M_\rho^2} \left[ -\delta(r) + m_\rho^3 W_T(m_\rho r) \right] \\
+ \frac{2}{3} \tau_1 \cdot \tau_2 \frac{f_\rho 1 f_\rho 2}{2 M_1} \left[ -\delta(r) + m_\rho^3 W_T(m_\rho r) \right] \\
- \frac{2}{3} \frac{f_\omega 1 f_\omega 2}{2 M_1 M_2} \left[ -\delta(r) + m_\omega^3 W_T(m_\omega r) \right],
\]

(45)

\[
V_T(r) = -\zeta \tau_1 \cdot \tau_2 \frac{g_{\rho 1} g_{\rho 2}}{M_\rho^2} m_\rho^3 W_T(m_\rho r) \\
- \frac{1}{3} \tau_1 \cdot \tau_2 \frac{f_\rho 1 f_\rho 2}{2 M_1} m_\rho^3 W_T(m_\rho r) \\
+ \frac{1}{3} \zeta \frac{f_\omega 1 f_\omega 2}{2 M_1 M_2} m_\omega^3 W_T(m_\omega r),
\]

(46)

while for finite-size hadrons we substitute \( \delta(r) \), \( W_T(x) \) and \( W_T(x) \) by their finite-size versions.

C. Couplings

For the \( D \) and \( D^* \) heavy mesons the axial coupling with the pion we take

\[ g_1 = 0.60, \]

(47)

which is compatible with \( g_1 = 0.59 \pm 0.01 \pm 0.07 \) as extracted from the \( D^* \to D\pi \) decay \[50\] [51].

The coupling to the \( \sigma \) meson, in the case of the nucleon-nucleon interaction, can be determined from the non-linear sigma model \[52\] yielding

\[ g_{\sigma NN} = \sqrt{\frac{M_N}{f_\pi}} \approx 10.2. \]

(48)

For the case of the \( D, D^* \) mesons and \( \Xi \) baryons we can estimate the coupling to the \( \sigma \) from the quark model. If we assume that \( \sigma \) only couples to the \( u \) and \( d \) quarks, we expect

\[ g_{\sigma 1} = g_{\sigma 2} = \frac{g_{\sigma NN}}{3} \approx 3.4. \]

(49)

We can also deduce from \( SU(3) \) flavour symmetry and the OZI rule that

\[ g_{\rho 1} = g_{\omega 1}, \]

(50)

\[ g_{\rho 2} = g_{\omega 2}. \]

(51)

From the universality of the \( \rho \) couplings (Sakurai’s universality \[53\]) and the KSFR (Kawarabayashi-Suzuki-Fayyazuddin-Riazuddin) relation \[54,55\] we expect

\[ g_{\rho 1} = g_{\rho 2} = \frac{m_\rho}{2 f_\rho^2} \approx 2.9. \]

(52)

Yet there might be deviations from this value. Regarding the \( \rho \) coupling to the heavy mesons, Casalbuoni et al. \[56\] indicate that

\[ g_{\rho 1} = \beta \frac{m_\rho}{2 f_\rho^2} \approx 2.6, \]

(53)

where \( \beta = 0.9 \). The \( \rho \) coupling thus obtained actually coincides with lattice QCD calculations in the heavy quark limit \[57\], which yield \( g_{\rho 1} = 2.6 \pm 0.1 \pm 0.4 \). For the \( \rho \) and \( \omega \) coupling to the cascade, there is also the possibility of obtaining it from the nucleon-nucleon case. In that case the relevant relations are (see the Appendix)

\[ g_{\rho 2} = g_{\rho NN}, \]

(54)

\[ g_{\omega 2} = \frac{1}{3} g_{\omega NN}. \]

(55)

In the nucleon-nucleon case the \( SU(3) \) + OZI relation is

\[ g_{\omega NN} = 3 g_{\rho NN}, \]

(56)

which is compatible with the analogous relation for the cascade baryon once we take into account the quark model. Yet nuclear potentials usually violate the previous relation, requiring a \( g_{\omega NN} \sim 20 \) or larger, a discrepancy which has been long known in OBE models and usually attributed to the fact that the \( g_{\omega NN} \) used in nuclear potentials might indeed also account for some of the short-range quark-gluon exchanges \[42\]. Yet, this discrepancy can be understood in more modern terms within the renormalization ideas that have become commonplace after the advent of chiral EFT. The explanation lies in the fine-tuning nature of the nucleon-nucleon interaction, which translates into a fine-tuning of the \( \omega \) coupling. In fact the \( \omega \) coupling provides a very important central contribution to the nuclear force, which is responsible in a large part for the concrete values of the scattering lengths of the singlet and triplet channels. By combining the OBE model with modern renormalization techniques this discrepancy disappears and the \( SU(3) \) relation is recovered \[48\]. These findings indicate that the use of the \( SU(3) \) relations is the most judicious choice to determine the \( g_{\omega} \) couplings, at least for exploratory studies of prospective hadron molecules where we are not trying to fit fine-tuned systems.

For \( f_\rho \) and \( f_\omega \) the estimations in the case of the charmed mesons are as follows. \( SU(3) \) and the OZI rule imply that

\[ f_{\rho 1} = f_{\omega 1}. \]

(57)

This relation appears automatically if the lagrangians are written in terms of the vector meson nonet. Meanwhile vector meson dominance applied to the weak decays of the charmed mesons \[56\] bring us to

\[ \frac{f_{\rho 1}}{2 M_1} = 2 \frac{m_\rho}{2 f_\rho^2} = \frac{\kappa_{\rho 1}}{2 m_H}, \]

(58)

with \( |\kappa| = 0.60 \pm 0.11 \text{ GeV}^{-1} \) and where in the second line we have written the coupling of the \( \rho \) in the normalization for
which we take $M_1 = m_H$ with $m_H$ the mass of the charmed meson. If we take $m_H = m_D$, $g_{ρ1} = 2.6$ and assume that $λ$ is positive we obtain
\[ \kappa_{ρ1} = 4.5 \pm 0.8 . \]  

(60)

For the cascade the estimations are more involved. The reason is that the coupling of the $ρ$ meson to the octet baryons depends in general on two coupling constants, the symmetric and antisymmetric octet couplings $1$. In the case of the nucleon-nucleon interaction it is possible to use single vector meson dominance to obtain the relation
\[ f_{ρ^NN} = \kappa_{ρNN} g_{ρNN} , \]  
\[ f_{ω^NN} = \kappa_{ωNN} g_{ωNN} , \]  

(61)

(62)

with $κ_{ρNN} = μ_ρ - μ_ω - 1$, $κ_{ωNN} = μ_ρ + μ_ω - 1$, yielding $κ_{ρNN} = 3.7$ and $κ_{ωNN} = -0.1$. This idea can be adapted to the $D$ and $D^*$ charged mesons and the $Ξ$ baryon, in which case we have $κ_ρ = μ_{ω} - μ_{ρ} - 1$ and $κ_ω = μ_{ω} + μ_{ρ} + 1$. The application of the previous idea implicitly assumes the convention $M_2 = m_ω$, which we will follow onwards. From the experimental values $μ_ω = -0.6507(25)$ and $μ_{ρ} = -1.250(14)$ listed in the PDG $53$ we obtain $κ_ρ = -0.401$ and $κ_ω = -0.901$. Other possibility is to constrain them from the quark model, in which case we obtain
\[ 1 + κ_ρ = \frac{μ_ω}{m_ω} \left( 1 + κ_{ρNN} \right) , \]  
\[ 1 + κ_ω = \frac{μ_ω}{m_ω} \left( 1 + κ_{ρNN} \right) , \]  

(63)

(64)

which yield $κ_ρ = -3.2$ and $κ_ω = -1.3$, in stark contrast with the previous estimations. We can also consider the family of phenomenological soft-core potentials by the Nijmegen group $53,54$ which also contain estimations for the electric and magnetic couplings of the vector mesons with the cascade. In this case we have $κ_ρ = -2.0, -0.7, -0.3$ $κ_ω = -1.1, -1.9, -2.3$ for the ESC04a, ESC04d and ESC08c potentials respectively $2$, though it is worth noticing that $κ_ω$ is obtained from a value of the omega coupling $g_{ω2} \sim (2 - 3) g_{ρ2}$ that is considerably larger than the SU(3) + OZI rule expectation. From the previous discussion it is apparent that there is a considerable level of uncertainty in $κ_ρ$ and $κ_ω$. For simplicity we will use the values
\[ κ_ρ = κ_ω = -1.5 . \]  

(65)

This choice is similar to the geometric mean of the previous determinations ($κ_ρ = -1.3$ and $κ_ω = -1.5$) and to the values we obtain when we compute the cascade magnetic moments at tree level in chiral perturbation theory ($μ_{ω} = -1.60$ and $μ_{ρ} = -0.97$ yielding $κ_ρ = -1.63$ and $κ_ω = -1.57$). We review the set of parameters we use in the OBE potential in Tables I and II.

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1 The electric-type coupling of the $ρ$ meson — the $g_ρ$ coupling — is an exception because of its universal character.

2 We have simply divided the magnetic and electric couplings $κ = f/g$ for these potentials.

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**TABLE I:** Masses and quantum numbers of the hadrons from which we form molecules in the present work (plus the nucleon). For the $N$ and heavy mesons we use the isospin average of the masses listed in the PDG $53$. For the $Ξ_{cc}$, we use the experimental value of the $Ξ_{cc}$ mass from the LHCb collaboration $66$, and for the $Ξ_{bb}$ we use the lattice QCD determination of Ref. $67$. We round the numbers at the MeV level.

| Hadron | $I^G (J^P)$ | $M$ (MeV) |
|--------|-------------|------------|
| $N$    | $\frac{1}{2}^-(\frac{3}{2}^+)$ | 938        |
| $Ξ$    | $\frac{1}{2}^-(\frac{3}{2}^+)$ | 1318       |
| $D$    | $\frac{1}{2}^-(0^-)$ | 1867       |
| $D^*$  | $\frac{1}{2}^-(1^-)$ | 2009       |
| $B$    | $\frac{1}{2}^-(0^-)$ | 5279       |
| $B^*$  | $\frac{1}{2}^-(1^-)$ | 5325       |
| $Ξ_{cc}$ | $\frac{1}{2}^+(\frac{1}{2}^+)$ | 3621       |
| $Ξ_{bb}$ | $\frac{1}{2}^+(\frac{1}{2}^+)$ | 10127      |

**TABLE II:** Masses, quantum numbers and couplings of the light mesons of the OBE model ($π, σ, ρ, ω$). For the magnetic-type coupling of the $ρ$ and $ω$ vector mesons we have used the decomposition $f_{(ρ,ω)} = g_{(ρ,ω)} g_{(ρ,ω)}$. $M$ refers to the mass scale involved in the magnetic-type couplings.

| Coupling | $D/D^*$ | $Σ$ |
|----------|---------|-----|
| $g$      | 0.60    | -0.25 |
| $g_σ$    | 3.4     | 3.4  |
| $g_ρ$    | 2.6     | 2.9  |
| $g_ω$    | 2.6     | 2.9  |
| $κ_ρ$    | 4.5     | -1.5 |
| $κ_ω$    | 4.5     | -1.5 |
| $M$      | 1867    | 1318 |

**D. $DΣ$ and $D^*Ξ$ Wave Function**

For the molecules we are considering here, the total wave function is the product of the isospin and spin-spatial wave functions
\[ |Ψ⟩ = |IM_1⟩ |ψ_{JM}(|r⟩)⟩ . \]  

(66)

where $J$ refers to the total angular momentum of the molecule under consideration. The isospin wave function comes from the coupling of the isospin of the two hadrons in the molecule
\[ |IM_1⟩ = \sum_{M_{11}M_{12}} ⟨I_1 M_{11} I_2 M_{12} |IM⟩ |I_1 M_{11}⟩ |I_2 M_{12}⟩ . \]  

(67)

The only subtly in the isospin wave function is when the hadron contains a light anti-quark $q$, for which there will be a minus sign for one of the components of the isospin multiplet.
For instance, if we consider the $\bar{D} (\bar{c}q)$ and the $D (c\bar{q})$ we have

$$D = \begin{pmatrix} \bar{D} \\ D \end{pmatrix} \text{ and } \bar{D} = \begin{pmatrix} D^* \\ -\bar{D} \end{pmatrix},$$

(68)

where the upper and lower components represent the $|\frac{1}{2} + \frac{1}{2}\rangle$ and $|\frac{1}{2} - \frac{1}{2}\rangle$ isospinors respectively. In contrast, for the cascade we simply have

$$\Xi = \begin{pmatrix} \Xi^0 \\ \Xi^- \end{pmatrix},$$

(69)

where we are using the same prescription for the isospinors.

For the part of the wavefunction containing the spatial and spin pieces, we can express it as a sum of different components with the same parity and total angular momentum, i.e. a partial wave expansion

$$\langle \psi_{JM}(\hat{r}) \rangle = \sum_{LS} \psi_{LSJ}(r) |^{LS+1}L_J\rangle,$$

(70)

where the sum over angular momentum only comprises even or odd $L$ depending on the parity of the molecule $P = (-1)^L$. For the partial wave projection we have adopted the spectroscopic notation $^{LS+1}L_J$, where $S$ is the total spin, $L$ the orbital angular momentum and $J$ the total angular momentum. We define the $|^{LS+1}L_J\rangle$ as follows

$$|^{LS+1}L_J\rangle = \sum_{M_L,M_S} \langle LML_S M_S |JM\rangle |S M_S\rangle Y_{LM}(\hat{r}),$$

(71)

where $\langle LML_S M_S |JM\rangle$ is a Clebsch-Gordan coefficient, $Y_{LM}(\hat{r})$ a spherical harmonic and $|S M_S\rangle$ the spin function, which in turn can be defined as

$$|S M_S\rangle = \sum_{M_{S1},M_{S2}} \langle S_1 M_{S1} S_2 M_{S2} |S M_S\rangle |S_1 M_{S1}\rangle |S_2 M_{S2}\rangle,$$

(72)

with $|S_1 M_{S1}\rangle, |S_2 M_{S2}\rangle$ are the spin function of particle 1 and 2.

The mixing of partial waves with the same $J$ but different $S/L$ requires the tensor force. The coupling only happens in the $D\Xi$ and $D^*\Xi$ cases, because for $D\Xi$ and $D^*\Xi$ the tensor force disappears. As molecular states are expected to be more probable for S-waves, we will consider only the following partial waves:

$$|D\Xi(J = \frac{1}{2})\rangle = \frac{1}{2} |S_{\frac{1}{2}}\rangle,$$

(73)

$$|D^*\Xi(J = \frac{1}{2})\rangle = \{ |S_{\frac{1}{2}}\rangle, |D_{\frac{1}{2}}\rangle \},$$

(74)

$$|D^*\Xi(J = \frac{3}{2})\rangle = \{ |S_{\frac{3}{2}}\rangle, |D_{\frac{3}{2}}\rangle, |D_{\frac{1}{2}}\rangle \}.$$

(75)

The evaluation of the spin-spin and tensor operators for these channels can be found in Table [III].

### Table III: Matrix elements of the spin-spin and tensor operator for the partial waves we are considering in this work, see Eqs. (73), (74) and (75) for the definitions.

| $D\Xi(J = 1)$ | $D^*\Xi(J = \frac{1}{2})$ | $D^*\Xi(J = \frac{3}{2})$ |
|----------------|---------------------|---------------------|
| $d_{11}$, $d_{22}$, $d_{12}$ | $\begin{pmatrix} -2 & 0 \\ 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ |

### E. The Extension to $\Xi_{cc}$ and $\Xi_{bb}$ Baryons

We can extend the OBE model to the doubly heavy baryons in two ways. The first is the quark model, in which case we derive the interactions of the $\Xi_{cc}$ and $\Xi_{bb}$ with the light mesons from the ones for the $\Xi_q$. The second is heavy antiquark-diquark symmetry [68, 70], in which case the derivation is from the heavy meson interactions.

In the quark model we expect the strange quark to act as an expector in what refers to the couplings to the $\pi, \sigma, \rho$ and $\omega$ light mesons. From this the OBE lagrangian for the $\Xi_{cc}$ and $\Xi_{bb}$ baryons is

$$\mathcal{L}_{\Xi_{cc}\Xi_{bb}q} = \frac{g_{\Xi\Xi\pi}}{\sqrt{2} f_\pi} \Xi_{cc} \pi^\dag \cdot \nabla (\hat{\pi} \cdot \hat{r}) \Xi_{bb},$$

(76)

$$\mathcal{L}_{\Xi_{cc}\Xi_{bb}f} = g_{\Xi\Xi\sigma} \Xi_{cc} \sigma \Xi_{bb},$$

(77)

$$\mathcal{L}_{\Xi_{cc}\Xi_{bb}g} = g_{\Xi\Xi\rho} \Xi_{cc} \rho \Xi_{bb},$$

(78)

$$\mathcal{L}_{\Xi_{cc}\Xi_{bb}g} = g_{\Xi\Xi\omega} \Xi_{cc} \omega \Xi_{bb},$$

(79)

where $M_2$ is the same as in the original lagrangian for the cascade. In short, the couplings are the same as for the $\Xi$.

Heavy heavy antiquark-diquark symmetry (HADS) is a manifestation of heavy quark symmetry which states that a heavy quark pair behaves as a heavy antiquark [68]. According to this symmetry the couplings of the $\Xi_{cc}$ and $\Xi_{bb}$ baryons can be deduced from those of the $\bar{D}, D^*$ and $\bar{B}, B^*$ heavy antimesons [69, 70]. The application of HADS can actually be encapsulated in the following two relations between the lagrangian for the $P$ and $P^*$ heavy antimesons and the $\Xi_{QQ}$ doubly heavy baryons

$$\text{Tr}[\hat{H}^\dag \hat{H}] \rightarrow \Xi_{QQ} \hat{\sigma} \Xi_{QQ},$$

(80)

$$\text{Tr}[\hat{H}^\dag \hat{\sigma} \hat{H}] \rightarrow -\frac{1}{3} \Xi_{QQ} \hat{\sigma} \Xi_{QQ},$$

(81)

where the bar over the $H$ field indicates that we are dealing the heavy antimeson superfield. From this, the OBE lagrangian
for the heavy mesons and changing the sign of the $\pi$ and $\omega$ contributions to take into account their G-parity, we arrive at

$$L_{\Xi_0\Xi_0\pi} = -\frac{1}{3} \frac{g_1}{\sqrt{2} f_\pi} \Xi_0^i \cdot \nabla (\varphi_i \cdot \dot{\varphi}) \Xi_0^i,$$  \hspace{1cm} (82)

$$L_{\Xi_0\Xi_0\sigma} = g_{\pi\sigma} \Xi_0^i \sigma \Xi_0^i,$$  \hspace{1cm} (83)

$$L_{\Xi_0\Xi_0\rho} = \frac{1}{3} \frac{f_{\rho}}{4 M_1} \epsilon_{ijkl} \Xi_0^i \sigma_k (\partial_j \rho_l - \partial_l \rho_j) \Xi_0^j,$$  \hspace{1cm} (84)

$$L_{\Xi_0\Xi_0\omega} = g_{\omega\sigma} \Xi_0^i \sigma_k (\partial_j \omega_l - \partial_l \omega_j) \Xi_0^j,$$  \hspace{1cm} (85)

The comparison with the lagrangian derived from the quark model entails the following HADS predictions

$$(g_2)_{\text{HADS}} = -\frac{g_1}{3},$$  \hspace{1cm} (86)

$$(g_{\rho\sigma})_{\text{HADS}} = g_{\sigma\rho},$$  \hspace{1cm} (87)

$$(g_{\rho\omega})_{\text{HADS}} = g_{\rho\omega} = \frac{f_{\rho}}{2 M_2},$$  \hspace{1cm} (88)

$$(g_{\omega\omega})_{\text{HADS}} = g_{\omega\omega} = \frac{f_{\omega}}{2 M_2},$$  \hspace{1cm} (89)

which can actually be checked against the quark model expectations. Owing to the choices of the couplings made before, we only have to compare five couplings: $g_{\rho\sigma}, g_{\rho\omega}, g_{\omega\omega}, f_{\rho}, f_{\omega}$ and $f_{\omega\omega}$. These comparisons are reduced to three as $g_{\rho\omega} = g_{\omega\omega}$ from SU(3) + OZI and $f_{\rho} = f_{\omega}$ because of the choice we have made. For the axial coupling we have

$$(g_{2\omega})_{\text{QM}} = -0.25 \quad \text{vs} \quad (g_2)_{\text{HADS}} = -0.20,$$  \hspace{1cm} (90)

which are actually very similar. For the $\rho$ electric-type couplings we have

$$(g_{\rho\rho})_{\text{QM}} = 2.9 \quad \text{vs} \quad (g_{\rho\rho})_{\text{HADS}} = 2.6,$$  \hspace{1cm} (91)

which differ by a $10\%$ only. For the magnetic-type couplings, if we employ $M_2 = m_{\Xi}$ in the doubly heavy sector, the comparison can be directly made in terms of $\kappa_{\rho\rho}$ and $\kappa_{\omega\omega}$ instead of $f_{\rho}$ and $f_{\omega}$, yielding

$$(\kappa_{\rho\rho})_{\text{QM}} = -1.5 \quad \text{vs} \quad (\kappa_{\rho\rho})_{\text{HADS}} = -1.1 \pm 0.2,$$  \hspace{1cm} (92)

plus identical predictions for $\kappa_{\omega\omega}$. In this case the difference is bigger, but both set of values remain compatible. It is important to notice there that the HADS predictions are expected to be subjected to a sizeable error of $\Delta_{\text{QCD}}/m_\pi \approx 30 - 40\%$ in the charm sector (instead of the standard $\Delta_{\text{QCD}}/m_\pi \approx 10 - 15\%$ for HQSS) $^{[78, 79]}$, owing to its status as a model (as they usually lack reliable error estimations). We warn however that the apparent similarity of both set of predictions is not necessarily due to a compatibility between the two models: the choice that we have made for the couplings of the cascade have also played a role.

### III. Predictions of Molecular States

Now we solve the Schrödinger equation for the $H\Xi$ and $\tilde{H}\Xi$ potentials with the coupling constant choices we have made in the previous section. For the cut-off in the form factor we will fix $\Lambda$ as to reproduce the $X(3872)$ in the isospin symmetric limit. In this limit the $X(3872)$ is a $1^{++} D\bar{D}$ molecule with a binding energy of about 4 MeV, which corresponds to a binding energy of about 0 MeV if we consider isospin symmetry breaking in the masses of the charmed mesons. With the choices of the couplings previously made, we obtain the value $\Lambda = \Lambda_X \approx 1.04$ GeV. For this cut-off the charmed meson - cascade molecules do not bind but are pretty close to binding. The $J = \frac{1}{2} \ D\bar{D}$ is the most attractive case. It binds for $\Lambda \geq 1.05$ GeV, which is just a tiny fraction above $\Lambda_X$. Concrete calculations indicate a scattering length of $a_0 = -18.7$ fm, which is indeed larger than any other scale in the system. For the other two configurations of the charmed meson - cascade system we find that the $J = \frac{1}{2} \ D\bar{D}$ system is the most attractive. The previous numbers are subject to theoretical errors, but probably a $30\%$ could be a good guess. For $g_{\rho\rho}$ and $g_{\omega\omega}$, the error depends on how much do we expect the KSF rule to fail, but probably a $10\%$ is enough. The axial coupling and its error $g_1$ are known experimentally, while for $g_2$ the er-
FIG. 1: Binding energies (red line) and root mean square radii
(black line) for the $D\Xi$ and $D^*\Xi$ molecules depending on the reduced mass. The upper, middle and lower panel correspond to the $1/2\ D\Xi/D\Xi_{Q0}$, $1/2\ D^*\Xi/D^*\Xi_{Q0}$ and $1/2\ D^*\Xi/D^*\Xi_{Q0}$ molecules respectively. The vertical dotted lines indicate the reduced masses of the different molecules considered. The calculations are made with a form factor cut-off $\Lambda = \Lambda_X = 1.04$ GeV, which corresponds to the cut-off for which the $X(3872)$ is reproduced in the OBE model we use.

ror is again determined from the quark model. However the independent variation of each of the couplings followed by the subsequent addition of the errors in quadrature is cumbersome because of the large number of parameters to vary (not to mention that it is not so easy to determine the error of all of them). We find instead much more convenient to simply assume a global uncertainty for the $D/D^*$ and $\Xi$ couplings in the following way

$$g_{M1}(1 \pm \Delta_1) \text{ and } g_{M2}(1 \pm \Delta_2),$$

where $M$ stands for the $\pi, \sigma, \rho$ and $\omega$ mesons and with $\Delta_1$ and $\Delta_2$ the relative error we expect in each of the vertices. If we assume all the couplings to vary in the same direction, i.e. correlated errors, then the outcome is that there is an overall uncertainty in the $X(3872)$ and $D\Xi/D^*\Xi$ potentials. In this picture for a potential with a vertex of type $i$ and $j$ we will assign the error

$$V_{ij}(1 \pm \Delta_j)(1 \pm \Delta_j),$$

where vertex type 1 refers to a $D/D^*$ and vertex type 2 to a cascade. We will assume the uncertainties on vertex type 1 and 2 to be uncorrelated. In the previous notation, the $X(3872)$ potential will be $V_{11}$ and the $D\Xi/D^*\Xi$ potential will be $V_{12}$. Besides it is important to stress that the role of the couplings to the charmed mesons and the cascade play a fundamentally different role in the calculations. We are using the $X(3872)$ and hence the couplings of the charmed mesons as a way to fix the unkown parameter $\Lambda$ in the OBE model, i.e. as a sort of renormalization condition. That is, a change in the charmed meson vertex piece of the potential entails a change in $\Lambda_X$ from which to redo the predictions of the binding energy:

$$V'_{11} = V_{11}(1 \pm \Delta_1)^2 \to \Lambda'_{X} \to B'_{D\Xi}$$

where $B'_{D\Xi}$ is the binding energy for $\Lambda'_{X}$, where the parameters for vertex 1 in $V_{12}$ have to change congruently as how they change in $V_{11}$. After this the error of the cascade baryon vertex $(1 \pm \Delta_2)$ should be added in quadrature. If we follow this procedure and assume a global $\Delta_1 = 0.15$, i.e. a 30% global error in the $X(3872)$ potential, we get $\Lambda_X = 1.04^{+0.10}_{-0.09}$ MeV. If we apply this idea to vertex 2 with $\Delta_2 = 0.15$, the predictions and uncertainties for inverse of the $D\Xi, D^*\Xi, \bar B\Xi$ and $B^*\Xi$ scattering lengths can be found in Table. Notice the choice of the inverse scattering length: the reason is that the scattering length diverges and then changes sign when there is a bound state. Its inverse however changes smoothly, and hence the choice. Actually the uncertainties are still compatible with the existence of bound states in the $D\Xi$ and $D^*\Xi$ systems. For the $P\Xi_{Q0}$ and $P^*\Xi_{Q0}$ bound states, their binding energies and uncertainties can be consulted in Table. We stress that the previous conclusions are derived from the hypothesis that the $X(3872)$ is molecular at the distances in which the OBE model applies. Besides the circumstantial fact that the $X(3872)$ is located close to the $D^0\bar D^0$ threshold, the most convincing evidence that the $X$ is molecular is the ratio of its isospin breaking decays $\Gamma(X \to J/\Psi 2\pi) \text{ and } \Gamma(X \to J/\Psi 3\pi)$ \cite{71}. It is relatively easy to explain this branching ratio within the molecular picture \cite{72,73}, but not so if the $X(3872)$ is a compact charmonium-like state \cite{74}. However the radiative decays $\Gamma(X \to J/\Psi \gamma)$ and $\Gamma(X \to \Psi(2S)\gamma)$ \cite{75}...
also been predicted in Ref. [34]. The $\frac{1}{2}^− D^+ \Xi$ and $\frac{3}{2}^− D^* \Xi$ are unlikely to bind, though their interaction is indeed attractive as can be deduced from their scattering lengths. As a consequence the interpretation of the $\Omega_c (3188)$ enhancement as a $D^* \Xi$ bound state [39] is disfavoured. The conclusion about a possible $\frac{3}{2}^− D^* \Xi$ molecule is also different from Ref. [34], where it is predicted, but the previous work includes a series of coupled channels that increase the binding by a small amount. Besides, there is the possibility that these two molecules bind within the uncertainties of our model.

The previous findings can be easily extended to systems in which the $D$ and $D^*$ are substituted by a $B$ and $B^*$ or where instead of the cascade $\Xi$ we have a doubly heavy baryon $\Xi_{cc}$ or $\Xi_{bb}$. These systems have a large reduced mass and are thus more likely to bind. For the $\Omega_b$-like molecular state, $\bar{B} \Xi$ and $\bar{B}^* \Xi$, we find that the $\frac{1}{2}^− \bar{B} \Xi$ binds with the errors of the present model, while the state above have also been predicted in [40]. The other two configurations might bind as well, but this is contingent on the uncertainties. For the triply heavy molecules we find the $\frac{1}{2}^− P^* \Xi_{QQ}$ system would be the most attractive, binding in all cases. The other two configurations $\frac{1}{2}^− P^\ast_{\Xi QQ}$ and $\frac{3}{2}^− P^\ast_{\Xi QQ}$ are less attractive. For the triply charmed pentaquark case the previous two configurations are probably close to the unitary limit, where their central values for the binding energy are 0.3 and 0.2 MeV respectively. For the triply bottom pentaquarks, all configurations bind within the theoretical uncertainties of the present model. Triply heavy pentaquarks have been considered previously in the literature. In Ref. [70] HADS is applied to the $X(3872)$ as a $D^* \bar{D}$ molecule to deduce the existence of possible $P^\ast_{\Xi QQ}$, $P^{\ast \ast}_{\Xi QQ}$, $P^{\ast \ast \ast}_{\Xi QQ}$ and $P^{\ast \ast \ast \ast}_{\Xi QQ}$ bound states. From the $X(3872)$ the existence of isoscalar $\frac{1}{2}^− P^\ast_{\Xi QQ}$ and $\frac{3}{2}^− P^\ast_{\Xi QQ}$ pentaquark-like molecules can indeed be deduced, while for the other $J^P$ combinations the information that can be obtained from the $X(3872)$ is insufficient to predict more states. In this context the OBE model provides a phenomenological estimation of this missing dynamics, which allows us to fully explore the $P^{\ast}_{\Xi QQ}$ and $P^{\ast \ast}_{\Xi QQ}$ cases.

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**Appendix A: Couplings in the Quark Model**

Here we present how to compute the couplings of the light mesons to different hadrons in the quark model. At the quark level the Lagrangians describing quark interactions with light
mesons can be written as [79]:

\[ \mathcal{L}_{Mqq} = g_{\sigma qq}(\bar{u}\gamma_\mu p^\mu - \bar{d}\gamma_\mu q^\mu) + \frac{4}{3} g_{\omega qq}(\bar{u}\gamma_\mu p^\mu + \bar{d}\gamma_\mu q^\mu) + g_{\rho qq}(\bar{u}\gamma_\mu p^\mu q^\mu - \bar{d}\gamma_\mu p^\mu q^\mu) + g_{\sigma qq}(\bar{u}\sigma + \bar{d}\sigma), \]  

where \( g_{\sigma qq}, g_{\omega qq}, g_{\rho qq} \) and \( g_{\sigma qhh} \) are the couplings of the light quark and hadron levels be the same, i.e.,

\[ \langle h, s| \mathcal{L}_{Mhh} | h, s \rangle \equiv \langle h, s| \mathcal{L}_{Mqq} | h, s \rangle, \]

where \( H \) denotes a hadron, \( s \) its spin and \( \mathcal{L}_{MHH} \) is the OBE lagrangian for the hadron \( H \). For instance, let us consider the case of the coupling of nucleon and the cascade to the pion

\[ \langle p | \mathcal{L}_{NNN} | p \rangle = \frac{g_{\sigma NN}}{m_N} q_3, \]

\[ \langle \Xi^0 | \mathcal{L}_{\Xi\Xi\Xi} | \Xi^0 \rangle = \frac{g_{\omega \Xi\Xi}}{m_\Xi} q_3, \]

where \( q \) refers to the momentum of the pion. We can directly compare the previous matrix elements to the ones we obtain from the SU(6) quark model wave functions [79] yielding

\[ \langle p | \mathcal{L}_{pqq} | p \rangle = \frac{5}{3} \frac{g_{\sigma qq}}{m_q} q_3, \]

\[ \langle \Xi^0 | \mathcal{L}_{\Xi\Xi\Xi} | \Xi^0 \rangle = \frac{1}{3} \frac{g_{\omega \Xi\Xi}}{m_\Xi} q_3. \]

A direct comparison gives us the relation between \( g_{\sigma \Xi\Xi} \) and \( g_{\omega \Xi\Xi} \):

\[ g_{\sigma \Xi\Xi} = -\frac{1}{5} m_\Xi g_{\sigma NN} . \]

Repeating this procedure for the other light mesons, we obtain

\[ g_{\sigma \Xi\Xi} = \frac{1}{3} g_{\rho\Xi\Xi}, \]

\[ (g_{\omega \Xi\Xi} + f_{\omega\Xi\Xi}) = -\frac{1}{3 m_N} (g_{\rho\Xi\Xi} + f_{\rho\Xi\Xi}), \]

\[ g_{\rho\Xi\Xi} = g_{\rho\Xi\Xi}, \]

\[ (g_{\rho \Xi\Xi} + f_{\rho\Xi\Xi}) = -\frac{1}{3 m_N} (g_{\rho\Xi\Xi} + f_{\rho\Xi\Xi}). \]

Thus we can relate the nucleon coupling constants with the ones for the cascade or with the ones for other hadrons.

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