Monte-Carlo simulation of the coherent backscattering of electrons in a ballistic system

K. L. Janssens and F. M. Peeters

Department of Physics, University of Antwerp (UIA), B-2610 Antwerpen, Belgium
(August 8, 2018)

We study weak localization effects in the ballistic regime as induced by man-made scatterers. Specular reflection of the electrons off these scatterers results into backscattered trajectories which interfere with their time-reversed path resulting in weak localization corrections to the resistance. Using a semi-classical theory, we calculate the change in resistance due to these backscattered trajectories. We found that the inclusion of the exact shape of the scatterers is very important in order to explain the experimental results of Katine et al. [Superlattices and Microstructures 20, 337 (1996)].

PACS numbers: 73.40.-c, 73.20.Fz, 03.65.Sq

Recently, Katine et al. [1] observed coherent backscattering in a high mobility two dimensional electron gas (2DEG), containing a quantum point contact and two reflector gates (see Fig. 1). The experiment was performed in the ballistic regime. The backscattered trajectories lead to an increase of the resistance which can be removed by the application of a perpendicular magnetic field like in the well-known weak localization effect. The aim of the present work is to explain this experimental result. In contrast to earlier measurements of coherent backscattering, many backscattered trajectories are found in this system, which inclose almost all the same surface area. The constructive interference of these backscattered trajectories leads to an enhancement of the resistance, which is known as weak localization. Previously, weak localization was always measured in the diffuse regime, where it is a consequence of the scattering on impurities. In this work we assume a ballistic regime, where the scattering on impurities is negligible and the reflector gates will now act as big man-made impurities.

For the description of the electron motion we use the semi-classical billiard model [2] in which electrons are point particles. Note that the dimensions of the system under consideration (2µm) are larger than the Fermi wavelength ($\lambda_F = 458Å, n = 3 \times 10^{15}m^{-2}$ in the experiment). Only electrons on the Fermi surface are important for conduction and therefore the relevant electrons all have the same velocity $v_F = \frac{\hbar k_F}{m} = 2 \times 10^5m/s$ but they are injected from the point contact under an angle $\alpha$ which satisfies the distribution $P(\alpha) = \cos(\alpha)/2$. The electron trajectories are determined by Newtons law which, in the absence of a magnetic field, leads to motion in a straight line, while the electrons describe a circular trajectory in the presence of a magnetic field. A condition for the applicability of the ballistic billiard model is that there is no scattering on impurities and that $\lambda_F$ is much smaller than the size of the reflectors. In the experiment the mean free path was $l_e = 5.4\mu m$ which is substantially larger than $2\mu m$, the dimensions of the system. The scattering off the reflector gates is specular. This results in a resistance $R$, determined by the Landauer formula [3]

$$ R = \frac{\hbar}{2e^2}N(1 - r) \quad (1) $$

Hereby $\hbar/e^2 = 25 k\Omega$ represents the fundamental unit of resistance, $N$ is the number of conducting channels and $r$ is the probability for backscattering which is calculated using our numerical simulation.

The Monte Carlo program calculates the trajectory of the electron and verifies whether the electron returns to the point contact or not. The relative number of returning electrons determines $r$. About 10 million electrons were injected from the point contact (which was taken 0.17µm wide) at each value of the magnetic field. The electron flow is shown in Fig. 2 for $B = 0T$. From the Landauer formula the resistance $R$ due to the returning electron orbits is obtained. The total resistance as measured experimentally equals this resistance $R$ plus the point contact resistance $R_{pc}$ and the resistance due to the two-dimensional electron gas (2DEG). The theoretical result for $R$ is shown in Fig. 3. For $|B| < 20mT$ the resistance increases with increasing magnetic field which is opposite to what was found experimentally (see solid curve in Fig. 4). Theoretically, the increase in resistance is a consequence of the fact that the electrons ejected from the point contact have, due to the $P(\alpha) = \cos(\alpha)/2$ distribution, a higher probability to be ejected into the forward direction. With increasing magnetic field, this beam is steered towards one of the reflectors, resulting in an increase of the number of electrons returning to the point contact.

The discrepancy in the functional behaviour of $R$ versus $B$ between theory and experiment demonstrates clearly that the wave nature of the electrons has to be included. We will include this by taking into account the phase of the electron. The quantum mechanical contribution to the resistance is determined by the interference of the electron trajectories with their time-reversed paths. The probability for a particle to return to the point contact after collision

1
with the reflector gates is given by \( P = \left| \sum_j A_j \right|^2 \), where \( A_j = |A_j| e^{i\theta_j} \) is the amplitude of the returning trajectory \( j \). The change of phase \( \theta_j \) is determined by the magnetic flux through the surface area \( S_j \) inclosed by the trajectory of the electron, which is given by \( \theta_j = eBS_j/h \). For a time-reversed path, this change of phase has an opposite sign. When two trajectories \( A_j \) and \( A_k \) as in Fig. 1 are considered, we have \( \theta_j \approx -\theta_k \) and \( |A_j| \approx |A_k| \). This implies

\[
P = \left| \sum_j 2 |A_j| \cos \theta_j \right|^2.
\]

(2)

First we checked that for every returning trajectory with \( \alpha_i > 0 \) there exists an equivalent time-reversed path which has \( \alpha_j < 0 \). This is illustrated in Fig. 5 where we show the percentage of returning trajectories as function of the angle under which the electron is ejected from the point contact for two different magnetic fields: (a) \( B = 0T \) and (b) \( B = 10mT \). Notice that the magnetic field results in: 1) a shift of the starting angle of the returning trajectories, and 2) in a small imbalance of the returning trajectories with \( \alpha_i > 0 \) and \( \alpha_i < 0 \). But we found that the latter has almost no effect on the resistance, and therefore will be ignored.

Next we used Eq. (2) in order to calculate \( r = P/(\text{number of simulated electrons}) \), which we inserted in the Landauer formula (Eq. 1). The obtained result is shown in Fig. 4 as the dotted curve. Notice that the positions of the peaks are well reproduced but the height of the central peak is substantially underestimated. The oscillations are determined by the phase difference between the returning trajectories and their time-reversed paths. Constructive interference occurs for \( eBS/h = 2\pi n \) (for \( n = 1, 2, ... \)) and there is a higher probability for returning to the starting point. This results in a peak in the resistance which occurs with a period \( \Delta B = h/2eS_0 = 3.33mT \), where we took \( S_0 = 6.21 \times 10^{-13}m^2 \). For destructive interference, i.e. \( eBS/h = \pi(2n+1) \) (for \( n = 1, 2, ... \)), the probability for returning to the starting point is smaller which determines the position of the dip.

In Ref. 1 a simplified model was proposed which could explain the experimental results. This model assumed a gaussian distribution of surface areas of the backscattered trajectories with average \( S_0 = 6.21 \times 10^{-13}m^2 \) and standard deviation \( \sigma \) which was taken as a fitting parameter where \( \sigma = 0.5S_0 \) was found. Using our billiard model we calculated the distribution of the surfaces of the returning trajectories. The result is shown in Fig. 6 which can be fitted to a Gaussian (solid curve) with average \( S_0 \) and width \( \sigma \). For \( B = 0T \) we found \( S_0 = 6.74 \times 10^{-13}m^2 \) and \( \sigma = 0.68 \times 10^{-13}m^2 \) and for \( B = 10mT \) we found \( S_0 = 6.43 \times 10^{-13}m^2 \) and \( \sigma = 1.13 \times 10^{-13}m^2 \). This corresponds to respectively \( \sigma \approx 0.1S_0 \) and \( \sigma \approx 0.17S_0 \) which is a factor 4 smaller than used in Ref. 1. Therefore, we are forced to conclude that the simple model of Ref. 1 is inadequate to explain their experimental results because a smaller \( \sigma \) will lead to a much smaller damping of the oscillations in \( R \) as a function of the magnetic field.

In order to explain the existing discrepancy between our theoretical result and the experiment we investigated the effect of the shape of the reflectors on the number of returning electrons. In the above analysis we assumed that the reflector gates are straight lines. However in reality they are part of parabolas given by

\[
x = f(\theta) \cos \theta \\
y = f(\theta) \sin \theta \\
f(\theta) = \frac{2r_0}{1 + \sin \theta} \quad \text{where} \quad 12^\circ < \theta < 30^\circ.
\]

At first sight our previous assumption seemed very reasonable because these parabolas are very well approximated by straight lines. Nevertheless, further study learns that the backscattering is much better with parabolic reflector gates, because the opening of the point contact is right in the focus of the parabolas. An electron, starting in this point and directed towards one of the reflectors, is scattered back to exactly the same point by the parabolic reflector gates in the absence of a magnetic field. This is clearly illustrated in the current flow diagram of Fig. 6 when compared with Fig. 2. An applied magnetic field will much more strongly defocus the system resulting in a smaller number of backscattered electrons as compared to the \( B = 0T \) situation. The resistance obtained with the parabolic reflectors is shown in Fig. 4 by the dashed curve. The central peak and the second neighbour peaks are nicely reproduced. The height of the first side peak is still overestimated which may be due to the small influence of impurities on the backscattering which was neglected in the present work.

In summary we studied the weak localization effect in a ballistic 2DEG with man-made scatterers, i.e. parabolic reflectors, by means of computer simulations. In the studied system the electrons could, after passing through the point contact, collide with the reflector gates and be focussed back to the point contact. The influence of this backscattering on the magnetoresistance was studied as a function of the applied magnetic field. We found that a pure classical treatment could not explain the experimental results of Katine et al. and in fact has even the wrong magnetic field dependence for the resistance. This was a clear indication that quantum mechanical effects are in play.
The quantum mechanical contribution was included through the interference of the time-reversed paths. We found an enhancement of the magnetoresistance as a consequence of the constructive interference of backscattered electrons with their time-reversed paths. This interference leads to oscillations in the resistance as function of the magnetic field which are nicely reproduced by our theoretical results. In order to find the correct qualitative behaviour of these oscillations, it turned out to be necessary to include the correct shape (i.e. parabolic reflector gates) of the reflectors in our simulation.

Acknowledgements. This work was supported by IUAP-IV, the Inter-University Micro-Electronics Center (IMEC, Leuven), and the Flemish Science Foundation (FWO-Vl).

* Electronic address: peeters@uia.ua.ac.be

1 J. A. Katine, M. A. Eriksson, R. M. Westervelt, K. L. Campman, and A. C. Gossard, Superlattices and Microstructures 20, 337 (1996).

2 B. L. Altshuler and P. A. Lee, Physics Today 41, December 1988, p. 36.

3 C. W. J. Beenakker and H. van Houten, Phys. Rev. Lett. 63, 1857 (1989).

4 F. M. Peeters and X. Q. Li, Appl. Phys. Lett. 72, 572 (1998).

5 R. Landauer, IBM J. Res. Dev. 1, 223 (1957).

FIG. 1. Schematic view of the experimental system and an example of the trajectory of a returning electron and its time-reversed path.

FIG. 2. The electron flow out of the point contact (situated at \( x = 0, \ |y| < 0.17\mu m \)) through the studied system at \( B = 0T \).

FIG. 3. The magnetoresistance as function of the applied magnetic field, as obtained within a pure classical treatment.

FIG. 4. The experimental result (solid curve) for the resistance as function of the magnetic field, together with the theoretical traces for straight (dotted curve) and parabolic (dashed curve) reflector gates.

FIG. 5. The percentage of returning electrons as function of the angle at which the electrons are ejected from the point contact for (a) \( B = 0T \) and (b) \( B = 10mT \).

FIG. 6. The distribution of the inclosed surface areas of the backscattered trajectories for (a) \( B = 0T \) and (b) \( B = 10mT \). The solid curve is a Gaussian fit to the results obtained from our simulation.

FIG. 7. The same as Fig. 2 but now for parabolic reflector gates.
The graph shows the ratio $R / R(0)$ as a function of magnetic field $B$ (in mT). The graph includes three distinct curves:

- **exp.** (solid line)
- **parabolas** (dashed line)
- **straight lines** (dotted line)

The peaks and troughs in the $R / R(0)$ ratio indicate the magnetic field's effect on the system, with the solid line showing experimental data, the dashed line representing parabolic fits, and the dotted line depicting straight line approximations.
The figure shows the distribution of starting angles $\alpha_i$ for different magnetic field strengths.

- **Panel (a)**: $B = 0\text{ T}$
  - The distribution is centered around $\alpha_i = 0\%$.

- **Panel (b)**: $B = 10\text{ mT}$
  - The distribution is shifted from $\alpha_i = 0\%$ to $\alpha_i = 10\%$.
