Earth’s extensive Bekenstein bound

Andreas Martin Lisewski*
Baylor College of Medicine, One Baylor Plaza, Houston, TX 77030, USA
October 15, 2007

Abstract

It is pointed out that two geophysical quantities have comparable values. First, Earth’s total negative entropy flux integrated over geological time and, second, its extensive Bekenstein entropy bound, which follows if entropy is an extensive function of volume and energy. The similarity between both numbers suggests that the formation of young, i.e. non-primordial, planetary mass black holes may not be impossible.

Planet Earth is an open thermodynamic system far from equilibrium. Its global thermodynamic state is to a large part determined by incoming solar energy and by a negative flux of radiation entropy, so-called exergy. Low entropy electromagnetic radiation from the Sun is absorbed, its free energy is successively dissipated in the Earth’s atmosphere and surface, and higher entropy thermal radiation is released into space, which is a vast low temperature reservoir at $T_B \approx 3$ K. Life and civilization, in their highly evolved and complex form, would be unlikely without this heat process.

The exergy flux $\dot{\sigma}$ can be estimated when Earth and Sun are approximated as black bodies. In this case, the Stephan-Boltzmann law gives

$$\dot{\sigma} = I_0(1 - \alpha)(1/T_E - 1/T_\odot) \approx 900 \text{ mW m}^{-2}\text{K}^{-1},$$

where $\alpha = 0.3$ is the Earth’s planetary albedo, $I_0 \approx 340 \text{ Wm}^{-2}$ is the net energy flux from the Sun to the Earth’s surface, and $T_E = 255$ K and $T_\odot = 5760$ K are the black body radiation temperatures of Earth and Sun, respectively. Solar energy is dissipated and entropy increased through processes such as absorption, heating and evaporation. In a steady state, $\dot{\sigma}$ balances the production of entropy $\dot{\sigma}_p$, i.e., $\dot{\sigma} \approx \dot{\sigma}_p$. However, non-equilibrium geological (fossils), atmospheric (chemical composition of the atmosphere), and biological (DNA replication, protein synthesis) structures and processes, which have been preserved over geological time $\Delta \tau \approx 4 \times 10^9$ years, suggest that $\dot{\sigma}_p$ has been smaller than $\dot{\sigma}$. Therefore, Earth likely has gone through periods where exergy $S$ has been accumulated in the system. This stored exergy would be spontaneously released as additional entropy if the Earth would fail to support its peculiar non-equilibrium state, for example through a temporary excess in internal or external dissipative forces. Such destructive forces can be triggered by impacts of extraterrestrial massive objects (asteroids, comets) and radiation (supernovae, gamma-ray bursts), or internally by rapid technological advance (industrial exploitation of global resources) and warfare (weapons of mass destruction). Both types of events have the potential to increase Earth’s entropy load almost instantaneously.

To establish an upper bound for $S$, it is assumed that Eq. (1) is valid throughout geological time, yielding $S \leq S_{\text{max}} = \int \dot{\sigma} \text{d}A \text{d}t \approx \dot{\sigma} A_E \Delta \tau/(k_B \log 2) = 5 \times 10^{54}$ bits, where $k_B$ is Boltzmann’s constant and $A_E = 4\pi R_E^2$ is Earth’s surface with radius $R_E = 6 \times 10^6$ m. This assumption includes that radiation from the Sun was roughly
Figure 1: Schematic representation of a hypothetical increase in exergy $S$ over geological time away from equilibrium. After $\Delta \tau \sim \text{Gyr}$ exergy $S \lesssim S_{\text{max}}$ becomes comparable to $S_{\text{BH}}^{3/4}$. Upon total loss of exergy, entropy increases spontaneously and planetary black hole formation occurs (solid red line over grey box). In a second scenario, sustained accumulation of exergy slowly continues without large and potentially catastrophic fluctuations in entropy (dashed red line). Constant during $\Delta \tau$, originating from thermonuclear burning of hydrogen in a main sequence star. The upper bound $S = S_{\text{max}}$ represents an extreme situation where the entire exergy influx is stored, although, more realistically, dissipative processes will prevent $S$ to saturate the limit. It is next shown that a comparable limit on $S$ can be imposed by considering gravitational entropy bounds.

In his seminal work \cite{Bekenstein1981}, Bekenstein introduced a general entropy bound for any space region $O$ which extends to a radius $R$ and contains energy $E$. He argued that if the total entropy $S$ associated with $O$ is larger than the corresponding entropy $S_{\text{BH}}$ of a black hole with energy $E_{\text{BH}} > E$ and Schwarzschild radius $R_{\text{BH}} > R$,

$$S_{\text{BH}} = \frac{2\pi ER}{\hbar c \log 2},$$

then $O$ itself cannot be a black hole, but can be made into one by an adiabatic process which slowly increases $E$ until a black hole is formed ($\hbar = h/2\pi$ is Planck’s constant and $c$ is the vacuum speed of light.) Because the resulting object has entropy $S_{\text{BH}}$ and entropy could not have been reduced during the adiabatic formation process, it follows that $S$ should be less or equal $S_{\text{BH}}$ in the first place, i.e., $S \leq S_{\text{BH}}$.

The Bekenstein bound can be further tightened if entropy is assumed to be an extensive function of volume $V \sim R^3$ and energy $E$, which means that $S$ is a varying function $S(E, V)$. Under this assumption the Bekenstein bound necessarily follows a different scaling \cite{Gour2003},

$$S < S_{\text{BH}}^{3/4},$$

which results in a stricter bound if $S_{\text{BH}} \gg 1$. Eq. (3) represents the so-called extensive entropy bound. Remarkably, the same scaling derives from the Stephan-Boltzmann law \cite{Gour2003}, which further indicates that thermodynamics of black holes and black bodies are related. This relationship can be made explicit for planet Earth as follows.

With $R = R_E$ and $M = M_E = 6 \times 10^{27}$ g it is $S_{\text{BH}} = 9 \times 10^{74}$ bits, a number that is 20 orders of magnitude larger than the exergy limit $S_{\text{max}} = 5 \times 10^{54}$ bits. However, if it is reasonably assumed that Earth’s entropy is an extensive function of volume and energy, then Eq. (3) gives

$$S_{\text{ext}} = 2 \times 10^{56} \text{bits} \approx S_{\text{max}}.$$  

Thus, Earth’s extensive entropy bound $S_{\text{ext}} = S_{\text{BH}}^{3/4}$ and maximum exergy $S_{\text{max}}$ are roughly within the same order of magnitude. It suggests that planetary and black hole physics may not be independent.

Because the extensive entropy bound is saturated for black holes, a possible interpretation of Eqs. (3) and (4) adheres that an Earth-like planet can turn into a black hole if it spontaneously releases enough accumulated exergy to reach the extensive entropy bound (Figure 1). Although, in the case of the Earth, it is uncertain what fraction of the incoming exergy flux has been stored in the system over time, the similarity between $S_{\text{ext}}$ and $S_{\text{max}}$ draws a potentially
catastrophic scenario: a collapse into a planetary black hole, where Earth’s final physical state becomes a frozen low temperature quasi-equilibrium, with a Hawking-Bekenstein temperature $T_{BH} \approx 0.03$ K. The resulting state is then cooled slowly by radiation from the Sun and from the cosmic background until solar energy is consumed and until the Universe has reached temperatures $T_B$ below $T_{BH}$.

If similar physical conditions exist on other planets, then the formation of planetary black holes after crossing extensive entropy limits could result in testable predictions. Thus far, the main theoretical candidates for planetary mass ($M_{BH} \lesssim 10^{30}$g) black holes have been primordial black holes [Carr(2005)], which could have emerged from strong density fluctuations in the early Universe. Because the young Universe has cooled rapidly and strong density fluctuations have vanished, the total population of primordial black holes could not have increased ever since (Also, processes such as Hawking radiation and black hole mergers would lead to a smaller population.) In contrast, Earth-like planets in stellar systems may reach their extensive entropy bounds and become black holes after gigayears of stellar evolution. This implies that the population of planetary black holes should increase during cosmological time. It is notable hereby that the astronomical detection of planetary mass black holes is within reach: black hole abundances in the mass range $M_{BH} \gtrsim 10^{27}$g and $M_{BH} \lesssim 10^{17}$g have been experimentally constrained from microlensing and from measurements of the gamma-ray galactic background [Carr(2005)].

Another aspect of planetary black holes regards the so-called Fermi’s paradox. It says that the vast spatial and temporal dimensions of the Universe seem incompatible with the absolute lack of evidence for extraterrestrial civilizations (“Where is everybody?”). Such lack of evidence may hint that planetary black holes form very efficiently before civilizations can escape their host planets through advanced space travel. Alternatively, technological progress itself may trigger those dissipative forces that lead to a formation of a planetary black hole. If this is the case, intelligent life forms may exist in the Universe which deliberately keep technological progress and industrial advancement at a slow pace, so to not perturb catastrophically their habitat’s non-equilibrium state.

References

[Bekenstein(1981)] Bekenstein, J. D. 1981, Phys. Rev. D, 23, 287
[Carr(2005)] Carr, B. J. 2005, astro-ph/0504034
[Gour(2003)] Gour, G. 2003, Phys. Rev. D, 67, 127501, gr-qc/0212087
[Hawking(1994)] Hawking, S. W. 1994, Public Lectures - Life in the Universe. http://www.hawking.org.uk/text/public/life.html
[Kleidon & Lorenz(2004)] Kleidon, A. & Lorenz, R. 2004, in Non-equilibrium thermodynamics and the production of entropy: Life, Earth, and beyond (Springer Verlag), 1–16

\footnote{A catastrophic “instability” as a threat to civilization has been mentioned in [Hawking(1994)].}