Heterogeneous believes, segregation and extremism in the making of public opinions

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Abstract

The connection between contradictory public opinions, heterogeneous believes and the emergence of democratic or dictatorial extremism is studied extending our former two state dynamic opinion model. Agents are attached to a social-cultural class. At each step they are distributed randomly in different groups within their respective class to evolve locally by majority rule. In case of a tie the group adopts either one opinion with respective probabilities $k$ and $(1 - k)$. The value of $k$ accounts for the average of individual biases driven by the existence of heterogeneous believes within the corresponding class. It may vary from class to class. The process leads to extremism with a full polarization of each class along either one opinions. For homogeneous classes the extremism can be along the initial minority making it dictatorial. At contrast heterogeneous classes exhibit a more balanced dynamics which results in a democratic extremism. Segregation among subclasses may produce a coexistence of opinions at the class level thus averting global extremism. The existence of contradictory public opinions in similar social-cultural neighborhoods is given a new light.

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1 Introduction

In recent years the study of opinion dynamics has become a main stream of research in Physics [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Initiated long time ago [18, 19, 20] it is part of Sociophysics [21].

Outside physics, research has concentrated on analyzing the complicated psycho-sociological mechanisms involved in the process of opinion forming. In particular focusing on those by which a huge majority of people gives up to an initial minority view [22, 23]. The main ingredient being, for instance in the case of a reform proposal, that the prospect to loose definite advantages is much more energizing than the corresponding gains which by nature are hypothetical.

Such an approach is certainly realistic in view of the very active nature of minorities involved in a large spectrum of social situations. However we have shown [10, 16, 17] that even in the case of non-active minorities, public opinion obeys some internal threshold dynamics which breaks its democratic character. Although each agent does have an opinion, they may find themselves into local unstable doubting collective state while discussing with others in small groups. In such a case, we postulated that all group members adopt the same opinion, the one which is consistent with the common believes of the group. Examples of such common believes may be substantiated by saying like There is no smoke without a fire or In case of a doubt, better do nothing. Such a possibility of occurrence of local doubts results in highly unbalanced conditions for the competition of opinions within a given population even when each opinion has an identical convincing weight [5, 10].

In the present work we introduce social-cultural classes to include all agents which may meet together to discuss an eventual public issue. Each class is characterized by some common believes that result from the class average of all heterogeneous individual biases. Accordingly, in case of a tie in a local group, the resulting choice is either one opinion with respective probabilities $k$ and $(1 − k)$ where $k$ accounts for the corresponding common belief of the social-cultural class. The value of $k$ is constant within each class with $0 ≤ k ≤ 1$ and may vary from class to class.

Considering a class with a common belief $k$ and groups of size 4, $O$ denoting an opponent to the issue at stake and $S$ a supporter, our update rules writes,

$$SSSS \rightarrow SSSS; \quad OOOS \rightarrow OOOS,$$
Figure 1: Opinion flow diagram with the two attractors at 0 and 1 and the separator at a non symmetric value 65%. To survive a public debate one opinion must start at an initial support of more than fifteen percent while the other one needs an initial minority support. The associated extremism is dictatorial.

$$OOSS \rightarrow \begin{cases} \text{OOOO with probability } k \\ \text{SSSS with probability } (1-k) \end{cases},$$

where all permutations are allowed. In our earlier works $k = 1$ or $k = 0$ except in [17] and in an application to cancerous tumor growth [24]. For $0 \leq k \leq 1$ the opinion flow dynamics still converges towards a full opinion polarization but the separator location is now a function of $k$ and the group size distribution. It may vary from 0% to 100% as shown in Fig. 1.

The flow is fast and monotonous. When the initial majority makes the final opinion, the associated extremism is democratic, otherwise it is more or less dictatorial depending on the value of the initial minority which eventually
spreads over the all class. Various extreme social situations can be thus be
described. In case of an homogeneous high risk aversion sharing ($k = 0$) it is
found that a reform proposal may need an initial support of more than 90% of
the population to survive a public debate [10]. On the opposite, a shared
high prejudice against some ethnic or religious groups ($k = 1$) can make an
initial false rumor shared by only a few percent of the population to spread
over the whole population [5]. Moreover very small fluctuations in the initial
conditions may lead to opposite extremism.

At contrast heterogeneous populations ($k = \frac{1}{2}$) are found to exhibit a
more balanced dynamics. Extremism is turned democratic. At odd, seg-
regation among subclasses leads opposite extremism within each subclass,
which in turn stabilizes the associated overall class opinion at a permanent
coeexistence of both opinions thus averting extremism.

Earlier version of a threshold dynamical process can be found in the
study of voting in democratic hierarchical systems [19, 25]. There, groups
of agents vote for a representative to the higher level using a local majority
rule. Going up the hierarchy turns out to be exactly identical to an opinion
forming process in terms of equations and dynamics. Instead of voting, agents
update their opinions. The probability of electing a representative at some
hierarchy level $n$ is equal to the proportion of opinions sharing an opinion
after $n$ updates [17, 19]. However within all these earlier studies the value of
$k$ is always taken equal to one except in [17] where it was a function of the
ration majority/minority within some finite size neighborhood. The existence
of threshold dynamics in social phenomena was advocated long time ago in
qualitative studies of some social phenomena [26, 27].

The rest of the paper is organized as follows. The model is defined in the
next Section. It is then solved exactly within the simple case of groups of size
three in Section 3. The counter intuitive case of groups of size 4 is studied
in Section 4. In Section 5 small fluctuations are shown to sometimes lead
to contradictory public opinions. It sheds a new light on the fact that very
similar areas in terms of their respective believes can hold opposite view,
for instance about the feeling of safety. Segregation effects are studied in
Section 6 to find they can drive either democratic extremism or coexistence
of opinions thus avoiding global extremism. Including a size distribution for
local groups is presented in Section 7. Last section contains some discussion.
2 Setting the problem

We start partitioning a given population among different social-cultural classes. Then we consider each class independently to study the dynamics of opinion forming among its $N$ individual members facing some issue. It may be a reform proposal, a behavior change like stopping smoking, a foreign policy decision or the belief in some rumor. The process is held separately within each class of agents.

We discriminate between two levels in the process of formation of the global opinion, an external level and an internal one. The first one is the net result from the global information available to every one, the private information some persons may have and the influence of mass media. The second level concerns the internal dynamics driven by people discussing freely among themselves. Both levels are interpenetrated but here we decoupled them to study specifically the laws governing the internal dynamics.

Accordingly, choosing a class of agents at a time $t$ prior to the public debate the issue at stake is given a support by $N_s(t)$ individuals (denoted S) and an opposition from $N_o(t)$ agents (denoted O). Each person is supposed to have an opinion with $N_s(t) + N_o(t) = N$. Associated individual probabilities to be in favor or against the proposal at time $t$ are,

$$p_{s,o}(t) \equiv \frac{N_{s,o}(t)}{N},$$

with,

$$p_s(t) + p_o(t) = 1.$$

From this initial configuration, people start discussing the project. However they don’t meet all the time and all together at once. Gatherings are shaped by the geometry of social life within physical spaces like offices, houses, bars, restaurants and others. This geometry determines the number of people, which meet at a given place. Usually it is of the order of just a few. Groups may be larger but in these cases spontaneous splitting always occurs with people discussing in smaller subgroups.

To emphasize the mechanism at work in the dynamics which arises from local interactions no advantage is given to the minority with neither lobbying nor organized strategy. People discuss in small groups. To implement the psychological process of collective mind driven update, a local majority rule is used within each group. Moreover an identical individual persuasive power is assumed for both sides with the principle “one person - one argument”.
On this basis all members of a group adopt the opinion which had the initial majority. In case there exists no majority, i.e. at a tie in a group of even size, all members yet adopt the same opinion, but now either one with respective probabilities $k$ and $(1 - k)$ where $k$ is a function of the class.

3 The intuitive case: the group of size three

We first consider the case of update groups with the same size three. Accordingly to our local majority rule groups with either 3 S or 2 S ends up with 3 S. Otherwise it is 3 O. The probability to find one supporter S after $n$ successive updates is,

$$p_s(t + n) = p_s(t + n - 1)^3 + 3p_s(t + n - 1)^2(1 - p_s(t + n - 1)),$$  

where $p_s(t + n - 1)$ is the proportion of supporters S at a distance of $(n - 1)$ updates from the initial time $t$.

Eq. (3) exhibits 3 fixed points $p_{s,0} = 0$, $p_{s,1} = 1$, and $p_{c,3} = \frac{1}{2}$. First two corresponds to a total opinion polarization along respectively a total opposition to the issue with zero supporters left and a total support. Both are attractors of the dynamics. The last point with a perfectly balanced opinion splitting is the separator of the dynamics as shown in Figs. (2) and (3).

To reach the attractor, the dynamics requires a sufficient number of updates. In solid terms each update means some real time measured in numbers of days whose evaluation is out the scope of the present work. An illustration is given starting from $p_s(t) = 0.45$. We get successively $p_s(t + 1) = 0.42$, $p_s(t + 2) = 0.39$, $p_s(t + 3) = 0.34$, $p_s(t + 4) = 0.26$, $p_s(t + 5) = 0.17$, $p_s(t + 6) = 0.08$ down to $p_s(t + 7) = 0.02$ and $p_s(t + 8) = 0.00$. Within 8 successive updates, 45% of the agents who were supporting the issue have shifted against it. The process has preserved and reinforced the initial majority making democratic the resulting extremism. The dynamics is perfectly symmetric with respect to both opinions as seen from Figs. (2) and (3). It is worth to stress that here the social-cultural character of the class is not activated since a local majority is always found.
4 The counter intuitive case: the group of size four

It is only when dealing with even groups that the common believes driven bias can be analyzed. There, the “one person - one argument” rule allows for the possibility of a tie with no local majority. In such a case participants are in a non-decisional state. They are doubting collectively, both opinions being supported by an equal number of arguments. Here we evoke a common belief “inertia principle” to lift the doubt. We state that at a tie the group eventually adopts the opinion O with a probability $k$ and the opinion S with the probability $(1 - k)$ where $k$ accounts for the collective bias produced by the common believes of the group members. Some specific situations are considered below.

To illustrate the model we consider groups of size 4. The probability to find one supporter S after $n$ successive updates becomes,

$$p_s(t + n) = p_s(t + n - 1)^4 + 4p_s(t + n - 1)^3(1 - p_s(t + n - 1))$$
Figure 3: Variation of $p_s(t)$ for groups of size 3 as function of repeated updates with three initial support $p_s(t) = 0.48, 0.50, 0.52$. The resulting extremism is democratic since it is along the initial majority.
Figure 4: Variation of $p_{c,4}$ as function of $k$. For a collective risk aversion ($k = 1$), $p_{c,4} \approx 0.77$ while for a collective novelty attraction ($k = 0$), $p_{c,4} \approx 0.23$. In case of no collective bias ($k = \frac{1}{2}$), $p_{c,4} = \frac{1}{2}$. An initial support of $p_s = 0.40$ is shown to lead to an extremism in favor ($p_S = 1$) for the range of bias $0 \leq k \leq 0.36$. At contrast the extremism is against ($p_S = 0$) for the whole range $0.36 \leq k \leq 1$.

\[ +6(1-k)p_s(t+n-1)^2\{1-p_s(t+n-1)^2\}, \quad (4) \]

where $p_s(t+n-1)$ is the proportion of supporters at a distance of $(n-1)$ updates from initial time $t$. Last term includes the tie case contribution (2S-2O) weighted with the probability $k$.

From Eq. (4) both attractors $p_{s,0} = 0$ and $p_{s,1} = 1$ are recovered. However the unstable fixed point $p_{c,4}$ has now departed from the symmetric value $\frac{1}{2}$ to the non-symmetric value,

\[ p_{c,4} = \frac{(6k-5) + \sqrt{13 - 36k + 36k^2}}{6(2k-1)}, \quad (5) \]

except at $k = \frac{1}{2}$ where $p_{c,4} = \frac{1}{2}$. Fig. (4) shows the variation of $p_{c,4}$ as a function of $k$. The effect of the common believes of the class in the formation
Figure 5: Variation of $p_s(t+1)$ as function of $p_s(t)$ for groups of size 4. On the left side $k = 1$ making $p_{c,3} \approx 0.77$. Arrows show the direction of the flow for an initial support $p_s(t) < 0.77$. On the right side $k = 0$ making $p_{c,3} \approx 0.23$. Arrows show the direction of the flow for an initial support $p_s(t) > 0.23$. 

of the associated public opinion is seen explicitly. For instance, an initial support of $p_s = 0.40$ leads to an extremism in favor the issue at stake for the whole range of bias $0 \leq k \leq 0.36$. At contrast the extremism is against it when $0.36 \leq k \leq 1$.

For instance, when the issue relates to some reform proposal and the class shares a high risk aversion, a tie supports the Status Quo, i.e., $k = 1$ which yields $p_{c,4} \approx 0.77$. The initial support for the reform to make the final public opinion has thus to start with more than 77% as shown in Figs. (5) and (6)). When it does, the corresponding extremism is democratic.

On the other hand, an issue within a context where novelty is preferred drives a non-decisional state to adopt the opinion S making $k = 0$ with in turn $p_{c,4} \approx 0.23$. An initial support of more than 23% is now sufficient to invade the whole population. In case it happens, the resulting extremism is dictatorial since it is along an initial minority view. Associated flow dynamics are shown on the right side of Figs. (5-6).

In the case of groups of size 4 the number of updates to reach a full polarization is smaller than is the case of size 3 as shown in Fig. (7) at $k = 1$ and $k = 0$. The number of required updates to have an extremism completed
Figure 6: Variation of $p_s(t)$ for groups of side 4 as function of repeated updates. On the left size $k = 1$ with three initial support $p_s(t) = 0.75, 0.77, 0.79$. To make the extremism, the initial support has to be larger than 77%. When it happens it is democratic. On the right side $k = 0$ with three initial support $p_s(t) = 0.21, 0.23, 0.25$. The resulting extremism is dictatorial when it corresponds to an initial minority.
can be evaluated as,

\[ n \simeq \frac{1}{\ln[\lambda]} \ln\left[ \frac{p_c - \nu_{SP}}{p_c - p_{+}(t)} \right], \tag{6} \]

where \( \lambda \) is the first derivative of \( p_s(t+1) \) with respect to \( p_s(t) \) taken at \( p_s(t) = p_c \). Moreover \( \nu_{SP} = 0 \) if \( p_{+}(t) < p_c \) while \( \nu_{SP} = 1 \) when \( p_{+}(t) > p_c \). The number of updates being an integer, its value is obtained from Eq. (7) rounding to an integer. The number of updates diverges at \( p_c \). The situation is symmetric with respect to \( k = 0 \) and \( k = 1 \) with the divergence at respectively \( p_c = 0.23 \) and \( p_c = 0.77 \). It occurs at \( p_c = 0.50 \) for \( k = 3 \).

For instance, starting as above with groups of size 3 from \( p_s(t) = 0.45 \) we get with \( k = 1 \) the series \( p_s(t+1) = 0.24, \ p_s(t+2) = 0.05 \) and \( p_s(t+3) = 0.00 \). Within 3 successive updates 45% of support has shifted their support to an opposition. Even an initial support above 50% with \( p_s(t) = 0.70 \) yields \( p_s(t+1) = 0.66, \ p_s(t+2) = 0.57, \ p_s(t+3) = 0.42, \ p_s(t+4) = 0.20, \ p_s(t+5) = 0.03, \) and \( p_s(t+6) = 0.00 \). Only 6 updates are enough to have 70% of supporters to shift their opinion.

5 Small fluctuations and contradictory public opinions in similar areas

We now discuss the effect of small differences of shared believes in the making of public opinion of neighboring groups. For instance we consider a city area and its suburb as shown in Fig. (8). One class covers the city with \( k = 0.49 \) and another one the suburb with \( k = 0.47 \). Such a minor difference is not explicitly felt while crossing from one area to another. Both area are perceived as identical. However the study of the dynamics of opinion starting from the same initial conditions within each area shows that sometimes huge differences can be driven by either minor differences in the initial conditions.

To illustrate our statement we consider two very similar initial conditions with an issue at stake having respectively 49% and 51% of support among both city and suburb populations. We then follow the dynamics of the corresponding public opinion flows.

In the first case, 51% of support to the issue results in both populations to a full support to the issue making both geographical areas identical as seen in Fig. (9). The process is completed within an estimate of ten updates. However a tiny decrease of 2% in the initial support down at 49% split the
Figure 7: Variation of the number of required updates to reach full extremism for groups of size 3 and 4 at $k = 0$. 
Figure 8: A city with $k = 0.49$ and its surrounding suburb with $k = 0.47$. 
two neighboring similar areas. The city is now fully opposed to the issue while the suburb stays unchanged with a full support to it (see Fig. (9)).

Above case may shed a new light on situations in which contradictory feelings or opinions are sustained in areas which are nevertheless very similar like for instance the feeling of safety. It shows how an insignificant change in either the initial support or the bias driven by the common believes, may yield drastic differences in the outcome of public opinion. Fig. (10) shows the variations of the number of updates needed to reach extremism for several values of the bias $k$. Here too, the same initial support is shown to lead to totally different outcomes as function of $k$. A value $p_S = 0.30$ leads to $p_S = 0$ for both $k = 0.50$ and $k = 0.70$ while it yields $p_S = 1$ for $k = 0.10$. For $p_S = 0.70$ all 3 cases lead to $p_S = 1$. 

Figure 9: Evolution of $p_s(t)$ as function of the updates for groups of size 4 with $k = 0.49$ and $k = 0.47$. Two initial supports are considered. For $p_s = 0.51$ both $k = 0.49$ and $k = 0.47$ leads to $p_S = 1$. But for $p_s = 0.49$ only $k = 0.47$ leads to $p_S = 1$ while $k = 0.49$ leads to $p_S = 0$. 

\[ P_S = 0.49 \]
\[ P_S = 0.51 \]
\[ k = 0.47 \]
\[ k = 0.49 \]
Figure 10: Number of required updates to reach extremism for several values of the local bias with $k = 0.10$, $k = 0.50$ and $k = 0.70$. Associated values are shown for an initial support of respectively 30% and 70%
6 Segregation, democratic extremism and co-existence

Up to now we have considered at a tie an average local bias \( k \) which results form a distribution of heterogeneous believes within a population. It means that all members of that population do mix together during the local group updates whatever is the individual respective believes. At this stage it is worth to note that different situations may arise in the distribution of the individual \( k_i \).

We discuss two cases for which either all \( k_i \) are equal, i.e. an homogeneous population or they are all distributed among two extreme values for instance 0 and 1. There the existence of subclasses as a result of individual segregation may turn instrumental in producing drastic changes in the final global public opinion of the corresponding class.

Consider first two different homogeneous classes A and B in two different areas with respectively \( k_i = 0 \) for all \( i \) in A and \( k_j = 1 \) for all \( j \) in B. From Eqs. (5-6) an initial support \( p_s = 0.25 \) yields \( p_S = 1 \) for A and \( p_S = 0 \) for B as seen in Fig. (11). For A the extremism in support of the issue is dictatorial since along the initial minority \( p_s = 0.25 \). At odd, in B the extremism is against the issue and democratic since along the initial majority of \( 1 - p_s = 0.75 \) against it.

Now consider above classes A and B but as subclasses of the same class within one unique area. Two situations may occur as illustrated in Fig. (12) where A individuals are represented by circles and B ones by squares. They are white when in favor and black if against. In the first situation (higher part of the Figure) people from each subclass do not mix together while updating their individual opinions. They are segregating each other yet sharing the same class within the same geographical area. As a result two opposite extremism for each subclass is obtained as above. However the novelty with respect to distinct geographical areas is that here the resulting public opinion of the global class which does include the whole population is no longer exhibiting any extremism. The dynamics of segregated updates of opinions has produced a stable coexistence of both initial opinions with thus a balance collective stand. A poll over the bias would reveal the average value \( k = 1/2 \).

In the second situation (lower part of the Figure) people from each subclass do mix together while updating their individual opinions. As a result,
Figure 11: The dynamic of public opinions for two populations on different areas with respectively $k = 0$ and $k = 1$ at an initial support of $p_s = 0.25$ is shown on the upper and lower curves. In between the two populations are in the same area as subclasses of one unique class. The upper one has segregation and yields a coexistence of opinions. The lower one has mixing and reveals a democratic extremism.
at a tie with mix individuals, A people adopt the opinion in favor while B people go on against. With two sub-populations with more or less the same large enough size this process is equivalent on average to have a bias $k = 1/2$. The resulting extremism is democratic since it is along the initial global majority among the whole population. Mixing or segregation within the same situation may thus lead to drastically different public opinions as illustrated in Fig. (12).

7 Group size distribution

Most of our analysis dealt with groups of size four. In real life people meet and discuss in several group sizes, from two up to five, six. Such a generalization was treated in [10] in the case $k = 1$. Along the same line we can define the general update Equation,

$$p_s(t + n) = \sum_{i=1}^{L} a_i \sum_{j=N[\frac{i}{2}]+1}^{i} C_j^i p_s(t + n - 1)^j p_o(t + n - 1)^{(i-j)}$$

$$+ (1 - k_i) V(i) C_i^i p_s(t + n - 1)^{\frac{i}{2}} p_o(t + n - 1)^{\frac{i}{2}}$$

(7)

where $C_j^i \equiv \frac{i!}{(i-j)!j!}$, $N[\frac{i}{2}]+1 \equiv$ Integer Part of $(\frac{i}{2} + 1)$, $p_s(t + n - 1)$ is the proportion of supporters after $(n - 1)$ updates and $V(i) \equiv N[\frac{i}{2}] - N[\frac{i-1}{2}]$. It gives $V(i) = 1$ for $i$ even and $V(i) = 0$ for $i$ odd. We also have introduced the possibility of having the local bias $k_i$ to a function of the size $i$ of the group. Simultaneously, we have for the proportion of opponents, $p_o(t + n) = 1 - p_s(t + n)$. The proportion of groups of size is defined by the probability distribution $a_i$ with the constraint $\sum_{i=1}^{L} a_i = 1$ where $L$ is the largest group size and $i$ refers to the group size.

Clearly an infinite number of size distribution $\{a_i\}$ is possible. Also various value of $k$ can be considered depending on the size group as well as some agent dependence though the mixing of difference sub-classes. However the existence of local collective non-decisional state monitored by the occurrence of local ties in groups of even size will always occur. Such a feature whatever its amplitude is does produce an asymmetry in the polarization dynamics towards either one of the two competing opinions thus preserving the main result of the simple version of the model presented in this paper.
Figure 12: A population composed from two subclasses sharing opposite believes. The circles have $k = 0$ while the squares have $k = 1$. The initial proportion in favor (white) is identical for both of them at $p_s = 0.25$. Black color expressed an opinion against. In the upper series, peoples segregate while updating their opinions. The result is a perfect balance of the global public opinion with all circles against (blacks) and all squares in favor (whites). At contrast, in the lower series, circles and squares do mix together while updating. The result is a democratic extremism with all circles and squares in favor (white).
8 Conclusion

To conclude, we have presented a simple model which is able to reproduce some complexity of the social reality. It suggests that the direction of the inherent polarization effect in the formation of a public opinion driven by a democratic debate is biased from the existence of common believes within a population. Homogeneous versus heterogeneous situations were shown to result in different qualitative outcomes.

At this stage we did not address the difficult question on how to remedy this reversal opinion phenomenon with the natural establishment of dictatorial extremism. The first hint could be in avoiding the activation of common general background in the social representation of reality. However direct and immediate votes could be also rather misleading. Holding an immediate vote without a debate as soon as a new issue arises has other drawbacks. At this stage, the collaboration with psycho sociologists as well as political scientists would be welcome.

In addition, in real life not every person is open mind and changes opinion. Therefore it would be interesting to introduce stubborn agents in the model. The model may generalize to a large spectrum of social, economical and political phenomena that involve propagation effects. In particular it could shed a new light on both processes of fear propagation and rumors spreading.

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