Spectral Decorrelation of Nuclear Levels in the Presence of Continuum Decay

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Abstract

The fluctuation properties of nuclear giant resonance spectra are studied in the presence of continuum decay. The subspace of quasi-bound states is specified by one-particle one-hole and two-particle two-hole excitations and the continuum coupling is generated by a scattering ensemble. It is found that, with increasing number of open channels, the real parts of the complex eigenvalues quickly decorrelate. This appears to be related to the transition from power-law to exponential time behavior of the survival probability of an initially non-stationary state.

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Level fluctuations, measured in terms of the nearest-neighbor-spacing-distribution (NNSD) and the $\Delta_3$-statistics, provide a commonly accepted tool for studying the quantum interplay between regular and chaotic dynamics. The standard treatment is restricted to bound states while, in many cases, the excited states are resonances embedded in the continuum. Already a generalization of the standard two-level repulsion theorem [1] to resonances [2] shows that this may significantly modify the correlations between the states. Generically, chaotic dynamics leads to level repulsion but the presence of the continuum (open system), is expected [3] to wash out the repulsion between the resonance energies. On the other hand, the lack of correlations between levels is normally interpreted as a manifestation of regular dynamics. It thus seems necessary to explore, on a fully quantitative level, what is the nature of the weakening of the repulsion due to openness and how it modifies the fluctuation characteristics.

The most practical way for describing an irreversible decay into the continuum is based on a scattering ensemble of non-hermitian random matrices [4]. Such a treatment follows naturally from the projection-operator technique [5] in which the subspace of asymptotically decaying states is formally eliminated. The resulting non-hermitian Hamiltonian

$$\mathcal{H} = H - \frac{i}{2} W$$  \hspace{1cm} (1)$$

acts in the space of quasi-bound states and the coupling to the continuum is accounted for by the anti-hermitian operator $W$. Unitarity of the scattering matrix imposes on $W$ the following factorization condition:

$$W = A A^T.$$  \hspace{1cm} (2)$$

For an open quantum system with $N$ quasi-bound states, $|i\rangle$, ($i = 1, ..., N$) which decay into $k$ open channels $a$ ($a = 1, ..., k$), the $N \times k$ matrix $A \equiv \{ A^a_i \}$ denotes the amplitudes for connecting the states $|i\rangle$ to the reaction channels $a$. The diagonalization of $\mathcal{H}$ in the basis $|i\rangle$ yields $N$ quasi-stationary states with complex eigenvalues $\mathcal{E}_j = E_j - i \Gamma_j / 2$, whose imaginary parts correspond to the ‘escape width’. The factorization of $W$ guarantees that $\Gamma_j \geq 0$. An
interesting effect – due to the separable form of $W$ – is that, in the strong-coupling limit ($W \gg H$), one observes a segregation of the states: $k$ states accumulate most the total width, $\Gamma = \sum_j \Gamma_j$, while the remaining $N - k$ states have nearly vanishing widths (they become 'enslaved').

For systems, such as the atomic nucleus, whose dynamics is expected to be classically chaotic, it is natural to consider the hermitian- and the anti-hermitian parts of $H$ to be statistically independent. Furthermore, the real and symmetric $N \times N$ matrix $H$ can be modeled as a member of the Gaussian orthogonal ensemble (GOE) of random matrices. For large $N$ the matrix elements of $H$ obey the following pair contraction formula:

$$\langle H_{ii'}H_{jj'} \rangle = \frac{a^2}{4N}(\delta_{ij}\delta_{i'j'} + \delta_{ij'}\delta_{i'j})$$

in the sense of GOE averaging. The constant $a$ is related to the mean level spacing, $D = 2a/N$.

For a general Gaussian ensemble of complex random matrices $\mathcal{H}$ an analogous contraction formula for $\langle \mathcal{H}_{ij}\mathcal{H}_{i'j'} \rangle$ is obtained which implies that the real and imaginary parts of $\mathcal{H}$ commute on average. Consequently, the two hypersurfaces, representing the real and imaginary parts of the energy lie in orthogonal subspaces. This, for sufficiently large $N$, may produce decorrelated spectra as seen from either the real or imaginary axes, in spite of a cubic repulsion on the complex plane.

However, this general Gaussian ensemble of complex random matrices is not applicable in the present case because of $S$-matrix unitarity. Instead, the anti-hermitian part of $\mathcal{H}$ is determined by the amplitudes $A_i^a$ via Eq. (2). Based on the GOE character of internal dynamics and orthogonal invariance arguments the amplitudes $A_i^a$ can be assumed to be Gaussian distributed. The corresponding correlator reads:

$$\langle A_i^aA_j^b \rangle = \frac{1}{N}\gamma^a\gamma^b\delta_{ij}, \quad \langle A_i^a \rangle = 0$$

implying that the average trace is $\langle TrW \rangle = \Sigma^N_a \gamma^a$. The diagonal elements $W_{ii} = \Sigma^k_a (A_i^a)^2$ are then positive, statistically independent and obey a $\chi_k$-square distribution.
Unlike the amplitudes $A^a_i$ the matrix elements of $W$ are not statistically independent, however. The number of independent random parameters, $Nk - \frac{1}{2}k(k-1)$ for $k \leq N$, is reduced by the second term as a consequence of the rotational invariance of $W_{ij} = \sum^k_{a=1} A^a_i A^a_j$ (the scalar product between $N$ $k$-dimensional vectors $A_i$ in the channel space). Only for $k = N$ the correlations in $W$ are specified by $\frac{1}{2}N(N-1)$ parameters, as for the GOE. Thus a decorrelation of the projected spectra may result. In most realistic cases, however, the number of open channels $k$ is smaller than $N$. To assess the dependence on the number of open channels we perform a systematic numerical study of the spectral correlations as a function of $k$.

Since the nuclear interaction is predominantly two body in nature, the matrix representation of the nuclear Hamiltonian should be related to the so-called 'embedded' Gaussian orthogonal ensemble (EGOE) [7] rather than the GOE. Therefore, to make our study realistic from the nuclear physics point of view, we generate the hermitian part of $\mathcal{H}$ from the model in ref. [11] instead of using a GOE random ensemble. The Hamiltonian includes a mean-field part and a zero-range and density-dependent two-body interaction. The matrix representation of $H$ is expressed in the basis of one-particle one-hole (1p1h) and two-particle two-hole (2p2h) excitations generated by the mean-field part and by discretizing the continuum [11]. The spectral fluctuations of the corresponding real eigenvalues, measured in terms of the NNSD and $\Delta_3$, coincide with those of the GOE [11], even though significant deviations from the Gaussian distribution of the matrix elements are found [12,13].

Because of time-reversal invariance the anti-hermitian part of $\mathcal{H}$ is generated by a Gaussian ensemble of real amplitudes $A^a_i$ with correlator (4), where $\gamma^a = 1$, i.e. we assume that all channels are equivalent and the strength of the external coupling is comparable to the internal one. In the specific calculations presented below, we select quadrupole excitations ($J^\pi = 2^+$) in $^{40}$Ca. To ensure acceptable statistics, in the quasi-bound-state space all 1p1h and 2p2h states up to an excitation energy of 40 MeV are included. This yields a $1661 \times 1661$ Hamiltonian matrix. Fig. 1 shows the resulting eigenvalue distribution on the complex energy plane for an increasing number $k$ of open channels. For $k = 10$ the majority
of the energies lie very close to the real axis and only a few states acquire a significant width which is a trace of the 'collective synchronization' discussed in ref. [4]. Increasing $k$, the distribution becomes more uniform and the width $\Delta_g$ of the empty strip between the cloud of eigenvalues and the real axis widens. This is understandable as $\Delta_g$ is equal to the 'correlation width' which describes the asymptotic behavior of the decay process [14].

The NNSD on the plane can be determined by calculating the normalized distances $s_i = d_i \rho_n(\mathcal{E}_i)^{1/2}$, where $d_i$ stands for the Euclidean distance between the eigenvalue $\mathcal{E}_i$ and its nearest neighbor, and $\rho_n(\mathcal{E}_i)$ for the local density of eigenvalues determined from $n$ nearest neighbors of $\mathcal{E}_i$. Similarly as in ref. [15], the choice $n = 10$ turns out satisfactory and guarantees stability. The numerical results are compared to the Poisson distribution $P(s) = (\pi/2)s \exp(-\pi s^2/4)$ (dashed lines in the $\text{rh}$ column of Fig. 1), which shows linear repulsion on the plane, and to the $P(s) = (81\pi^2/128)s^3 \exp(-9\pi s^2/16)$ with cubic repulsion (solid lines). The latter gives a good description for the NNSD of symmetric Gaussian random matrices [12] [16] and, for a large number of open channels, also fit our numerical results nicely. For a few open channels (upper right part of Fig 1.) we see a weaker then cubic repulsion, however.

Now we come to the central point namely the fluctuation properties of the real parts $E_i$ of the energy eigenvalues. The corresponding NNSD and $\Delta_3$-statistics are shown in Fig. 2. It is well known that, without coupling the continuum, the spectra show GOE characteristics for both measures [11]. However, for many open channels a decorrelation takes place. In fact, for large $k$ the results are well reproduced by a Poissonian shape of the NNSD (lower left part of Fig. 2). Quite surprisingly, this even holds for $k/N$ of a few percent (middle left part of Fig. 2). Already for ten open channels ($k/N = 6 \times 10^{-1}$), there is a visible deviation from the Wigner distribution (upper left part of Fig. 2). These numerical observations lead to the conclusion that the appropriate way of describing these deviations is to consider superpositions of Wigner and Poisson distributions rather than Wigner and Gaussian [3].

The longer-range correlations (spectral rigidity) expressed by the $\Delta_3$-statistics show a similar tendency, although the transition is somewhat slower. In addition, as is seen in
Fig. 2, the transition region $L_{\text{max}}$ from GOE to Poissonian characteristics is restricted to about 10 normalized distance units. This appears to be consistent with the findings in [18] for hermitian separable problems, where $L_{\text{max}}$ increases with increasing length of the string of eigenvalues. In the present case the string is comparatively short. On a more formal level [19], the $\Delta_3$-statistics is known to be non-universal above a certain $L_{\text{max}}$. For systems with a known classical limit, $L_{\text{max}}$ is determined by the inverse of the period of the shortest periodic orbits. We wish to mention, without showing the results explicitly, that an analogous analysis for the imaginary parts of $E_i$ show Poissonian fluctuations for any number of the open channels. This asymmetry in the statistical properties of $E_j$ and $\Gamma_j$ is related to the different properties of the real and imaginary parts of $H$, especially for smaller values of $k$.

Another way of understanding the decorrelation of the resonance energies due to the presence of continuum decay comes from the relation between the wave-packet dynamics and the stationary states [20]. The latter can be obtained via the Fourier transform of the time evolution of a generic wave packet. For a bound-state problem such a wave packet resides in the interaction region forever and thus, the structure of the corresponding phase space can be resolved with arbitrary accuracy. Consequently, for a chaotic system, the whole complexity (delocalization, random nodal pattern, scars, etc.) of stationary states can be reproduced. Coupling to the continuum, sets a limit for this process, however. As time progresses, the wave packet will leak out of the interaction region and makes it impossible to resolve all details of the dynamics. As a result the wave functions, projected onto the interaction region, look more regular than their counterparts in a closed system. The leakage is expected to occur faster with increasing $k$. A quantitative measure of the speed is the survival probability $P(t)$ of a randomly chosen wave packet $|F\rangle$, initially localized in the interaction region. As a convenient and experimentally motivated choice we consider a state excited by the isovector quadrupole operator ($|F\rangle = \hat{F}|0\rangle$). When expanded $|F\rangle$ involves all the eigenstates $|\chi_i\rangle$ of $H$ and
\[ P(t) = |\langle F(0)|F(t)\rangle|^2 = \left| \sum_{j=1}^{N} \langle 0|\hat{F}|\chi_j\rangle\langle \chi_j|\hat{F}|0\rangle e^{i\varepsilon_j t/\hbar} \right|^2 \]  

(for a complex symmetric matrix the left and right eigenvectors are the same). In the absence of continuum coupling, \( P(t) \) remains constant (on average) after a rapid initial dephasing due to the non-stationarity of \( |F\rangle \). For an open system, on the other hand, a decay of \( P(t) \) is to be expected. The most interesting feature is the dependence of the decay law on the number of open channels: For a small \( k \) the decay is very slow and well represented by a power-law \( (P(t) \sim t^{-z}) \). For \( k = 1 \) we find \( z \approx -1/2 \), in reasonable agreement with the estimates of ref. [22]. As \( k \) increases \( z \) grows very fast and, for \( k > 100 \), \( P(t) \) drops exponentially on long time scales, \( \text{i.e.} \ P(t) \sim \exp(-\eta t) \), with the decay constant \( \eta \) growing rapidly with \( k \) (Fig. 3). These observations go in parallel with the classical picture of open phase space phenomena such as a chaotic scattering [23]. For a small number of the open channels the decay is governed by a power-law. This is associated with larger fractal dimensions of the set of singularities generating chaotic behavior than for many open channel cases which lead to an exponential decay.

In summary, the numerical analysis presented in this work shows that GOE correlated spectra of quasi-bound states become fully decorrelated in the presence of continuum coupling and when the number of open channels is large. This transition is accompanied by a change of the decay properties of the average survival probability of a non-stationary wave packet, turning from power-law to exponential. This appears to be consistent with the semiclassical relation [24] between the time-dependence of \( P(t) \) and the structure of the resonances. An exponential behavior of \( P(t) \) corresponds to the region of strongly overlapping resonances (Ericson fluctuations [25]), while the power-law decay, with small power indices \( z \) [26], corresponds to isolated resonances, and it is this isolation which preserves the original fluctuations.

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Figure Captions

Figure 1: Left column: The eigenvalue distribution of the non-hermitian Hamiltonian $\mathcal{H}$ defined in Eq. (1) for different number $k$ of open channels. The hermitian part $H$ is chosen as the Hamiltonian of [12] while the anti-hermitian part $W$ is given by Eq. (2) taking the amplitudes $A$ as members of the Gaussian ensemble [4]. Right column: the corresponding NNSD on the complex plane.

Figure 2: The NNSD (lhs) and the $\Delta_3$ statistics (rhs) of the real parts $E_i$ for energy eigenvalues of $\mathcal{H}$ and different number $k$ of open channels.

Figure 3: The time dependence of the survival probability $P(t)$ of a wave packet, initialized by the isovector quadrupole operator, for various numbers of open channels.
