Upper limits on gravitational waves emission in association with the Dec 27 2004 giant flare of SGR1806-20

L. Baggio1, M. Bignotto2, M. Bonaldi3, M. Cerdonio2, L. Conti2, M. De Rosa4, P. Falferi3, P. Fortini3, M. Inguscio4, N. Liguori2, F. Marin4, R. Mezzena7, A. Mion7, A. Ortolan7, G.A. Prodi7, S. Poggi7, F. Salemi16, G. Soranzo9, L. Taffarello9, G. Vedovato9, A. Vinante7, S. Vitale7 and J.P. Zendri9

(1) Institute for Cosmic Ray Research, Univ. of Tokyo, 5-1-5 Kashiwanoha, Kashiwa, Chiba, 277-8582, Japan
(2) INFN Padova Section and Department of Physics, University of Padova, I-35131 Padova, Italy
(3) Istituto di Fotonica e Nanotecnologie CNR-ITC and INFN Gruppo Collegato di Trento, Padova Section, I-38050 Povo (Trento), Italy
(4) INOA I-80078 Pozzuoli (Napoli), Italy and INFN Firenze Section, I-50121 Firenze, Italy
(5) Physics Department, University of Ferrara and INFN Ferrara Section, I-44100 Ferrara, Italy
(6) LENS and Physics Department, University of Firenze and INFN Firenze Section, I-50121 Firenze, Italy
(7) Physics Department, University of Trento and INFN Gruppo Collegato di Trento, Padova Section, I-38050 Povo (Trento), Italy
(8) INFN, Laboratori Nazionali di Legnaro, I-35020 Legnaro (Pordenone) Italy
(9) INFN Padova Section, I-35100 Padova, Italy and
(10) http://www.auriga.lnl.infn.it

(Dated: *Corresponding author: cerdonio@pd.infn.it)

At the time when the giant flare of SGR1806-20 occurred, the AURIGA “bar” gw detector was on the air with a noise performance close to stationary gaussian. This allows to set relevant upper limits, at a number of frequencies in the vicinities of 900 Hz, on the amplitude of the damped gw wave trains, which, according to current models, could have been emitted, due to the excitation of normal modes of the star associated with the peak in X-rays luminosity.

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On 27 December 2004 the Soft Gamma-ray Repeater SGR1806-20 gave a giant flare, which was observed by a number of instruments [1].

The fluence, if the emission is assumed isotropic, at the distance of $d \sim 15$ kpc would imply an energy some hundred times larger than any other known giant flare [2,3]. Soft gamma-ray repeaters are thought to be magnetars (see [2] and refs. therein). It has been suggested [2,4] that the extreme energy event of 27 December 2004 is due to a catastrophic instability involving global crustal failure and magnetic reconnection [3]. Observations by CLUSTER and TC-2, in combination of data from GEOTAIL, gave evidence that the steep initial rise contains two exponential phases, of e-folding times 4.9 ms and 67 ms respectively, which covered the 24 ms before the time of the peak intensity $t_p$; all the timescales support the notion of a sudden reconfiguration of the stars magnetic field, producing large fractures in the crust [3]. In particular these authors remark that the intermediate $\approx 5$ ms time is naturally explained if the rising time is limited by the propagation of a triggering fracture of size $\approx 5$ km, as it would be predicted by the theory of reference [3].

According to a few somewhat different models, as a consequence of crustal cracking [3] or reconfiguration of the moment of inertia tensor [3], non-radial kHz oscillation modes of the neutron star would be excited, giving emission of gravitational waves (gw), possibly at frequencies where the gw bar detector AURIGA [2] is sensitive (see insert of Fig. 1). Both the above quoted models predict gw emission, starting very close to $t_p$, which involves kHz non radial modes of oscillation of a neutron star with few hundred ms damping time. The expected waveforms can be approximately parametrized as $h(t) = h_0 \exp(-t/\tau_s) \sin(2\pi f_s t)$, where $h_0$ is the maximum gw amplitude, $f_s$ and $\tau_s$ are the frequencies and damping times of normal modes; the polarization of the wave is not known. The frequencies of the various modes are still under study and depend on a variety of factors as EOS, temperature, density, age, rotational state of the star, etc. [10] so that we are unable to anticipate with any confidence what specific set of gw emission frequencies could be the one expected for a magnetar ready to undergo a supergiant flare. Still the lowest lying modes, $g_-$, $f_-$ and, marginally, $p - modes$, could well be in the frequency range $500 \div 1500$ Hz, depending on the status of the star.

Within a factor 10 in gw amplitude, AURIGA is sensitive to gws from $\sim 800$ to 1050 Hz. Here we limit the analysis to the most sensitive part of the band, namely between 850 and 950 Hz (see the insert in Fig. 1), where the detector sensitivity varies no more than a factor 4 in amplitude. Since the last upgrading of the suspensions on Dec 2nd 2004 the detector is well behaved in the sense that performs stationary gaussian, after epochs of environmental disturbances are vetoed by means of auxiliary channels (i.e. signals at frequencies where the detector is gw insensitive). During nights and week-ends the vetoed epochs becomes less frequent and shorter, so that the...
noise is driven by a zero mean stochastic gaussian process with a stationary correlation function. For what concern the directional sensitivity, the orientation of AURIGA in respect to the direction of SGR1806-20 was such that the antenna pattern, averaged over polarizations, gave maximal sensitivity at the time of the giant flare. Then we have a unique opportunity to search in our data for gravitational waves emitted at the peak time of the giant flare. We take the peak time \( t_p \) to be 21:30:26.68 UT of 27 December 2004 after taking into account the time difference between the arrival time of the flare X of SGR1806-20 at the AURIGA site. This time corresponds also to the peak position of the CLUSTER data which show, after the last exponential rise, the evident start of a phase in which damping occurs until the signal gets below 1/10 of the peak value, \( \sim 300 \) ms after the peak. Following the models quoted above, in both cases we can assume the peak time \( t_p \) as the start of the gw excitation and \( \tau_s = 100 \) ms, that is 1/3 of 300 ms, as the corresponding damping time. In order to extract the signal power first we reconstruct the gw amplitude \( h_v(t) \) at input through the detector transfer function. Then we slice the gw sensitive frequency band of AU-RIGA in contiguous and non-overlapping sub-bands \( f_j \) of constant width \( \Delta f \), and centered in \( f_j = f_j + \Delta f/2 \), by means of digital top-hat filters in the frequency domain \( T_j(f) = \vartheta((|f| - f_j) - \vartheta(|f| - f_j - \Delta f) \). Within each sub-band, we compute the the equivalent input signal power over a time span \( \Delta t \)

\[
E_j \equiv \int_{\Delta t} T_j * h_v^2(t + k\Delta t) \, dt ,
\]

where \( * \) stands for time convolution. The \( E_j(t) \) is sampled every \( \Delta t \) to construct the time series \( E_j(k) \) with \( k \) integer. We decided \( a \ priori \) a fixed partition of the time frequency plane: \( \Delta f = 1/(2\tau_s) = 5 \) Hz and \( \Delta t = 201.5 \) ms \( \approx 2\tau_s \). For each sub-band \( f_j \), we analyzed the resulting time series of \( E_j(k) \) over a time span of \( \pm 100 \) s around the peak time \( t_p \) to check the “off source” noise statistics. The \( E_j(k) \) sample including the peak time \( t_p \) is then compared to the measured noise statistics, looking for any evidence of excess power. To be more precise, the \( a \ priori \) choice of our sampling time made \( t_p \) to fall 120 ms after the beginning of the integration time \( \Delta t \) of the “on source” sample. Fig. 1 shows how \( E_j \) fluctuates on the time spans of \( \pm 5 \) s around the time of the flare \( t_p \) for the sub-band \( f_j = 930 \) Hz. A gw emission at frequency \( f_s \) would give an excess power in the band \( \Delta f \) centered at the \( f_j \) such that \( |f_s - f_j| < \Delta f/2 \). The released energy would be maximum in the “on source” sample. The excess signal power in each sub-band \( \Delta f \) can be easily calculated from the expected waveform and reads

\[
E_s \simeq \left( h_v^2/4 \right) \left\{ \left[ 1 + \frac{1}{\pi} \tan^{-1}\left( \frac{2x}{y} \right) \right] + O\left( \frac{1}{f_s\tau_s} \right) \right\} ,
\]

where \( x = (2\pi\tau_s)^{-1}/\Delta f \) and \( \delta \equiv |f_s - f_j|/\Delta f \) are the ratios between the signal bandwidth (\( \equiv 1/(2\tau_s) \)) and the detuning of the signal frequency and the bandwidth \( \Delta f \), respectively. With our choice of parameters for the analysis, the excess signal power is approximately (within a few % error)

\[
E_s \approx \frac{h_v^2\tau_s}{6} \left[ 1 - \left( \frac{f_s - f_j}{\Delta f_{eff}} \right)^2 \right] ,
\]

where \( \Delta f_{eff} = 4 \) Hz. To check the statistics of the “off source” samples, we histogram each time series \( E_j(k) \) and compare them with the predicted probability density functions assuming gaussian noise, by fitting for the variance separately in each sub-band. The fitting probability density function is a \( \chi^2 \) distribution with \( \alpha \) effective degrees of freedom

\[
p(E; \sigma^2) = 2^{-\alpha/2} (\pi/\sigma^2)^{\alpha/2-1} \exp(-E/2\sigma^2)\Gamma(\alpha/2)/\sigma^2 ,
\]

where \( \sigma^2 \) is the variance of the underlying gaussian stochastic process. We show in Fig. 2 the close agreement with prediction of the data for the frequency bin \( f_j = 930 \) Hz, over
drawn from the estimated noise probability distribution $p$ take advantage of the classical theory of hypothesis test-timescales of few minutes, is shown by the constancy in $p$-level distribution of the same fit for the histograms of of Fig. 2 and Tab.1). In Table 1 we report the parameter for all the sub-bands are consistent with a uniform dis-
have been checked by a $E$ ensures non-uniform coverage greater or equal to 90%. The simplicity the index of the sub-band). Its corresponding $\chi$-levels is very weak and, within the statistical $e$ which en-
take advantage of the classical theory of hypothesis testing to establish if the samples $E_{tp}$ corresponding to the arrival time $t_p$ are affected by the presence of a gw signal. To test the null hypothesis $H_0$, i.e. that the sample is drawn from the estimated noise probability distribution in absence of signals, we set a threshold $E_{cr}$ correspond-
ing to a confidence level (C.L.) $p(E < E_{cr}) \geq 1 - p_{cr}$. The threshold for $1 - p_{cr} = 95 \%$ C.L. corresponds to $E_{cr} = 8.8 \times \sigma^2_j$. Thus one sees from Table 1 that no excess of gw power is found at $t_p$ and therefore we have to set up upper limits. We set conservative confidence intervals for $E_s$ using a confidence belt construction which ensures non-uniform coverage greater or equal to 90%. The confidence belt construction proceeds as follows. Assume that the signal magnitude is $E_s$. The measured $E$ in each sub-band (Eq. 1) obeys a non-central $\chi^2$ distribution with central parameter equal to $E_s/\sigma^2$ (here we drop for simplicity the index of the sub-band). Its corresponding probability density function can be written as:

$$p(E_s, \sigma) = \frac{1}{2\sigma^2} \exp \left( - \frac{E_s + \sigma^2}{2\sigma^2} \right) \left( \frac{E_s}{\sigma^2} \right)^{(\alpha - 2)/4} \times I_{\alpha/2-1} \left( \sqrt{E_s/\sigma^2} \right) ,$$

where $I_k(x)$ are the modified Bessel functions of the first kind of order $k$. The $q$-quantile of this distribution, $E_q(E_s, \sigma)$, is implicitly defined by $q = \int_0^{E_s} p(E_s, \sigma) dE_s$. For each value of the unknown $E_s$ we define the 95% confidence belt boundaries $E_{hi}$ and $E_{low}$ as

$$E_{hi}(E_s, \sigma) = \begin{cases} 0 & \text{if } E_s < E_{cr} \sigma \\ E_{95\%}(E_s, \sigma) & \text{otherwise} \end{cases}$$

$$E_{low}(E_s, \sigma) = E_{95\%}(E_s, \sigma)$$

where $E_{cr}$ is implicitly defined by $E_{95\%}(E_s, \sigma) = E_{95\%}(0, \sigma)$. This confidence belt defines a set of confidence intervals on $E_s$, whose frequentist coverage is by construction - 90% for $E_s > E_{cr}$, and 95% for $E_s \leq E_{cr}$. In other words, for every value of $E_{tp}$ from each sub-band, if $E_{tp} < E_{95\%}(0, \sigma)$ we set an upper limit equal to $E_s$.

### Table I: List of fit parameter $\sigma^2$ of the histogrammed $E$ data samples $\pm 100$ s around $t_p$ tabulated as increasing sub-band frequencies; the data for the 870 Hz sub-band have been discarded $a$ priori as this band is contaminated by environmental noise. The “on source” value of $E$ including the trigger time $t_p$ is also reported as well as the computed upper limit with confidence $\geq 95\%$.

| $f_j$ | $\sigma_j^2 \times 10^{12}$ | $E_{tp} \times 10^{12}$ | $E_{95} \times 10^{15}$ |
|-------|-----------------|-----------------|-----------------|
| [Hz]  | [Hz$^{-1}$]      | [Hz$^{-1}$]      | [Hz$^{-1}$]      |
| 855   | 4.78            | 4.90            | 0.86            |
| 860   | 1.89            | 4.15            | 0.34            |
| 865   | 1.96            | 9.83            | 0.35            |
| 875   | 2.94            | 2.93            | 0.53            |
| 880   | 4.30            | 10.3            | 0.77            |
| 885   | 5.11            | 4.52            | 0.92            |
| 890   | 6.15            | 6.51            | 1.11            |
| 895   | 5.89            | 8.57            | 1.06            |
| 900   | 6.93            | 6.60            | 1.25            |
| 905   | 6.18            | 6.78            | 1.11            |
| 910   | 3.69            | 19.7            | 0.66            |
| 915   | 2.60            | 7.06            | 0.47            |
| 920   | 1.61            | 6.57            | 0.29            |
| 925   | 0.87            | 3.19            | 0.16            |
| 930   | 0.71            | 5.25            | 0.13            |
| 935   | 1.24            | 2.57            | 0.22            |
| 940   | 3.56            | 19.3            | 0.64            |
| 945   | 11.2            | 30.2            | 2.01            |
otherwise our procedure gives a two-sided confidence interval. In all sub-bands we obtain upper limits, which can be written as $\mathcal{E}_s^{\gamma} \approx 18 \times \sigma_j^2$. These limits range from $\mathcal{E}^{1/2} = 3.5 \times 10^{-21} \text{Hz}^{-1/2}$ to $\mathcal{E}^{1/2} = 1.4 \times 10^{-21} \text{Hz}^{-1/2}$, according to AURIGA sensitivity.

The initial amplitude of the neutron star normal modes $h_0$ is related to $\mathcal{E}$ by Eq. (3) that gives, for the best upper limit, $h_0 \leq 2.7 \times 10^{-20}$. We discuss now the upper limit in terms of the total gw energy $\epsilon_{gw} = E_{gw}/M_\odot c^2$ emitted by the normal modes excitation during the peak of the giant flare of SGR1806-20. The well known formula of the quadrupolar radiation, for the expected gw signal, can be written as $h_0 = (\epsilon_{gw} cR_S/(4\pi^2\tau_s)^{1/2})/(fs_d)$, where $R_S$ is the Swartzchild radius of one solar mass black hole. Thus the resultant upper limit on $\epsilon_{gw}$ reads

$$\epsilon_{gw} \leq 3 \times 10^{-6} \left( \frac{\mathcal{E}}{1.3 \times 10^{-41} \text{Hz}^{-1}} \right) \times \left( \frac{15 \text{ kpc}}{d} \right) \left( \frac{930 \text{ Hz}}{f_s} \right)^2 \frac{\tau_s}{0.1 \text{s}}, \quad (6)$$

We should notice that a gw bar detector has a polarization dependent sensitivity; hence, for an unpolarized or linearly polarized gw, the result in Eq. (6) should be multiplied by a factor 2 or $\cos^2(2\psi)$ respectively, where $\psi$ is the angle between the bar axis and the polarization of the wave. We conclude that, if the star ever emitted gws from excitation of its normal modes at any of the frequencies studied here, in the time span $\Delta t$ containing the flare time $t_p$, the gw amplitudes and energetics are limited as above. If the giant flare of SGR1806-20 on 27 December 2004 is indeed some 100 times more energetic (however see ref. [14]) and if the gw luminosity scales with the em luminosity, then, for the frequencies considered, our upper limits come close to the predictions of the models of refs. [7, 8] which give an energetics of the order of $\epsilon_{gw} \approx 5 \times 10^{-6}$. The method used here is of course sub-optimal and the upper limits are somewhat weaker than the “optimal” matched filter. In any case this work shows that, as there is the specific peak time $t_p$ to be used as external trigger, it is worth to make searches even with a single detector if its noise is well behaved. An extension of such searches involving the gw detectors on the air in a coincidence search, would also allow to use the information of the gw travel delays between the detectors to select against spurious, and would give the most exhaustive and efficient search, in terms of frequency coverage and confidence in improving the limits, if not to get a candidate detection.

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FIG. 3: Upper limits on the gw energy released at the source around the flare peak time $t_p$, expressed as a fraction of $M_\odot c^2$. 