Collective Coordinate Quantization: Relativistic and Gauge Symmetric Aspects

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Abstract

The introduction and quantization of a center-of-mass coordinate is demonstrated for the one-soliton sector of nonlinear field theories in (1+1) dimensions. The present approach strongly emphasizes the gauge and BRST-symmetry aspects of collective coordinate quantization. A gauge is presented which is independent of any approximation scheme and which allows to interpret the new degree of freedom as the quantized center of mass coordinate of a soliton. Lorentz invariance is used from the beginning to introduce fluctuations of the collective coordinate in the rest frame of the moving soliton. It turns out that due to the extended nature of the soliton retardation effects lead to differences in the quantum mechanics of the soliton as compared to a point-like particle. Finally, the results of the semiclassical expansion are used to analyse effective soliton-meson vertices and the coupling to an external source. Such a coupling in general causes acceleration as well as internal excitation of the soliton.

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1 Introduction

It is a well known fact that the choice of a ground state violating certain symmetries of the underlying theory leads to the appearance of zero modes after quantization. Prominent examples are mean field systems like in nuclear physics [1] or soliton systems [2]. In these cases the ground state, represented by the mean field respectively the classical soliton, violates certain symmetries, among them translational symmetry. The associated zero energy modes are by no means unphysical: their excitation, which costs no energy, corresponds to infinitesimal translations and eventually leads to a spread of the wave function over the whole space such that translational symmetry is restored. In the ideal case one recovers an eigenstate of the momentum operator in analogy to plane waves.

Why does one bother about these zero modes? One reason is that they cause problems in numerical calculations due to their singular behaviour in the infrared limit. Another reason is a physical one: Whereas in the mathematical description these zero modes are just modes like all others, except for their zero energy eigenvalue, they are well separated from the other, non-zero modes in physical pictures; the zero modes correspond to collective motion, whereas the other modes are identified with intrinsic excitations. As such the zero modes are of quite different nature. Therefore, it appears desirable to represent the zero modes by suitable collective coordinates in contrast to the intrinsic modes. Once this is achieved one also has a better handle on the intrinsic modes eventually free of infrared problems.

In the following I am concentrating on nonlinear field theories in (1+1) dimensions possessing soliton solutions like the Sine-Gordon or $\phi^4$-model. There, the only zero mode is due to translational motion. Soon after the first attempts on quantizing the soliton sector dating back to the mid-seventies [3, 4, 5] (for a review see also [2]) a lot of attention has been attracted by the zero-mode problem [5, 6, 7, 8, 9, 10]. One particular route approaching this problem [5, 6, 7] is based mainly on the collective coordinate method in many-body theory [11]. Another topic addressed in connection with translational motion is Lorentz covariance. Using either an explicitly Lorentz covariant approach or summing up an infinite series of Feynman diagrams Lorentz covariance has been demonstrated [6, 9, 12].

Also in the midseventies it has been realized that there exists a con-
connection between the introduction of collective coordinates and gauge theories \[\text{[13]}\]. Only within the last few years this connection has been investigated closer and finally led to a BRST-symmetric treatment of collective coordinate quantization \[\text{[14]}\]. The method presented in the following strongly emphasizes the gauge- and BRST-symmetry aspects of collective coordinate quantization. In contrast to most other approaches I am using a Lagrangian formulation based on a functional integral representation of the time evolution operator. The principles of the present approach are described extensively in a work by J. Alfaro and P.H. Damgaard \[\text{[15]}\]. The following treatment of the center-of-mass motion of an extended particle can be regarded as an application of their ideas.

Apart from these more technical aspects of collective coordinate quantization special attention will be paid to fluctuations of the collective coordinate. These fluctuations are responsible for a diffusion of the soliton’s position, \textit{i.e.} the probability to find the soliton at a certain position spreads over the whole space. At first glance the relativistic covariant expressions for the soliton’s momentum and energy suggest the picture of a relativistic point-like particle. However, such a picture certainly contradicts causality; as an extended object the soliton must be subject to retardation effects as soon as it becomes accelerated. Fluctuations of the center-of-mass coordinate just provide such an acceleration. Related criticism in this direction has been presented in \[\text{[16]}\]. It will turn out that precisely due to retardation effects fluctuations of the collective coordinate of a soliton behave different as those of a point-like particle.

Before proceeding with the details let me present an outline of what follows. Section 2 gives a short review about soliton quantization as far as it concerns the present subject. In section 3 I explain the introduction of a collective coordinate and its relation to gauge theories. The rôle of BRST-symmetry in connection with approximation schemes is demonstrated with the example of a naïve semiclassical expansion. Section 4 describes a slightly different treatment of the collective fluctuations requiring a formal solution of the gauge constraint. Within a tree approximation the difference to the quantum mechanics of a relativistic point-particle is shown. In section 5 I address the question of soliton-meson vertices and the effect of an external source. Finally, section 6 contains a summary, a discussion of the results and an outlook concerning an extension to (3+1) dimensions.
2 Soliton quantization: A short review

The starting point will be the matrix element of the time evolution operator between states $\langle f \vert$ and $\vert i \rangle$ using the functional integral representation:

$$Z[j] = \langle f \vert T \exp \left( -i \int_{-T'/2}^{T'/2} dt \hat{H}(t) \right) \vert i \rangle = \int \mathcal{D}[\phi] \exp \left( i \int dxdt \mathcal{L}_g(x, t) \right)$$

Here, $\hat{H}(t)$ is the Hamilton operator related to the Lagrangian

$$\mathcal{L}_g = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + gj \phi$$

with $\phi(x, t)$ a dimensionless scalar field. Functional integration is to be taken over all configurations $\phi(x, t)$ with boundary conditions in time given by $\langle f \vert$ and $\vert i \rangle$. The dimensionless coupling constant $g$ measures the strength of a nonlinear self-interaction $U(\phi, g)$. Examples are the $\phi^4$-theory or the Sine-Gordon-theory,

$$U_{\phi^4}(\phi, g) = m^2 \phi^2 (g^2 \phi^2 - 1)/2 \ , \ U_{SG}(\phi, g) = m^2 (1 - \cos g\phi)/g^2 \ ,$$

both exhibiting soliton solutions. It turns out useful to rescale $\phi \rightarrow \phi/g$ thus explicitly demonstrating that $g^2$ plays the same rôle as $\hbar$; therefore, the latter can be absorbed into $g^2$. This leads to

$$Z[j] = \int \mathcal{D}[\phi] \exp \left( \frac{i}{g^2} \int dxdt \mathcal{L}(x, t) \right)$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + gj \phi$$

with $V(\phi) = U(\phi, g = 1)$. The external source $j(x, t)$ can be viewed as inducing weak perturbations thus probing the response of the system.

A common feature of solitonic systems is a degeneracy of (classical) vacua; for the Sine-Gordon model in the rescaled version (3) these vacua correspond to constant field configurations $\phi_0$ being integer multiples of $2\pi$. In $\phi^4$-theory there are only two such degenerate vacua at $\phi_0 = \pm 1$. In any case these vacua can be classified by integers. A soliton is described by a field configuration which interpolates between different vacua at spatial infinity, i.e. at $x = -\infty$ and $x = +\infty$. The
basic soliton is the one interpolating between neighboring vacua. The associated topological charge, a winding number, is a conserved quantity. For the basic soliton its value is one. Within the Sine-Gordon model, according to the bosonization rules [17], this topological charge has to be identified with fermion number. Being a conserved quantity, the topological charge provides a sort of superselection rule with the consequence that the total Fock space is decomposed into distinct, disconnected Fock spaces with fixed topological charge. Therefore, \( \mathcal{Z}[j] \) is properly defined after specifying not only the boundary values of the field in time but also the boundary values in space, i.e. the topological charge.

A semiclassical expansion \( \phi(x,t) = \phi_0(x,t) + g\zeta(x,t) \) is based on a classical solution \( \phi_0(x,t) \). In the topologically trivial sector this classical solution is one of the previously discussed constants \( \phi_0 \). Expanding the Lagrangian in (4) to second order in the fluctuating field \( \zeta(x,t) \) one recovers a Klein-Gordon-type theory with a mass \( \mu^2 = V^{(2)}(\phi_0) \) given by the second derivative of the potential \( V(\phi) \).

In a topologically nontrivial sector the classical solution is no longer a constant. In the simplest case it is a static solution \( \phi_0(x) \) obeying the equation

\[
-\partial_x^2 \phi_0(x) + V^{(1)}(\phi_0(x)) = 0 .
\]

(5)

Here \( V^{(1)}(\phi) \) denotes the first derivative of \( V(\phi) \) with respect to \( \phi \). The fluctuations \( \zeta(x,t) \) on top of such a soliton can be diagonalized if expanded with respect to a basis of eigenfunctions \( \zeta_n(x) \),

\[
\zeta(x,t) = a_0(t)\zeta_0(x) + \sum_{n=1}^N a_n(t)\zeta_n(x) ,
\]

(6)

with eigenfunctions \( \zeta_n(x) \) and their eigenvalues \( \Omega_n \) defined by

\[
-\partial_x^2 \zeta_n(x) + V^{(2)}(\phi_0(x))\zeta_n(x) = \Omega_n^2 \zeta_n(x) .
\]

(7)

Usually, there is, in addition to possible bound states, a continuum of scattering states. For these states the summation over \( n \) has to be considered a symbolic notation whose meaning is integration. Due to the translationally noninvariant soliton solution there is also a zero mode \( \zeta_0(x) \sim \partial_x \phi_0(x) \). It is this mode which restores the spontaneously broken symmetry.

The physical spectrum of the soliton sector has been discussed in [4] using methods developed for many-body theories [18]. The picture
emerging from the semiclassical approximation is the following (see also \[2\]): the ground state where all fluctuation modes are carrying out just zero point motion describes a soliton at rest. Excitation of the zero mode leads to translational displacement. It is obvious that this mode has to be treated different simply because its excitation costs no energy and thus a semiclassical expansion makes little sense. The excitation of a bound state solution corresponds to an excited soliton whereas the excitation of a scattering solution corresponds to a soliton-plus-meson state. If the excited state can decay into soliton-plus-meson states it represents a resonance.

The usual procedure in connection with (1+1) dimensional soliton systems introduces a collective coordinate as a time dependent center of mass coordinate of the soliton field \[5, 6, 7\]. Proper quantization of the enlarged system requires a constraint in order to remove the spurious degree of freedom. Such a constraint is nothing else but a gauge. The physics is completely independent of the choice of gauge; however, the choice of gauge defines the part of physics described by the “collective” coordinate. If one can solve a theory exactly the introduction of a collective coordinate would be nothing else but an academic game. However, the majority of theories requires an approximate solution. In these cases it can make a big difference which representation one chooses for the theory. As such a suitable gauge may give access to completely new approximation schemes.

In connection with soliton models an often used constraint is to remove the zero mode from the fluctuating field, the so-called “rigid” gauge. The disadvantage of this gauge is that it is formulated in terms of the soliton configuration \emph{at rest}. As such relativistic effects like Lorentz contraction are higher order effects with the consequence that an expansion in the coupling constant produces two corrections: Quantum corrections to the soliton mass and relativistic corrections. Nevertheless, relativistic covariance is not lost but just hidden. It requires a summation of infinitely many Feynman diagrams to show \emph{e.g.} the relativistic relation between energy and momentum. On the other hand, for fixed momentum the soliton is moving with constant velocity; this \emph{classical} part of the collective motion can be accounted for by a Lorentz-boost to the rest frame of the soliton \(\text{(see} \ e.g. \ \text{the} \ \text{first reference of} \ [3])\). Therefore, as long as fluctuations of the collective coordinate are neglected the soliton behaves as a point-like particle. The question is whether this picture persists if collective fluctuations
are included. As will be shown below this is not the case.

Another subject discussed in connection with solitonic theories was the question of meson-soliton vertices. By construction, in a naive semiclassical expansion there is no linear coupling of the mesonic fluctuations to the soliton. This was interpreted as the absence of “Yukawa”-terms, a problem in particular for the Skyrme model [19] in (3+1) dimensions which should describe the nucleon as an extended particle [20]. This problem was investigated and solved using the collective coordinate method [21], also in (1+1) dimensions [22]. A few years ago, a modification of the “rigid” gauge was presented, the so-called “nonrigid” gauge [23], especially designed for the treatment of meson-soliton scattering and the extraction of meson-soliton vertices.

This nonrigid gauge may serve as an example how a suitable choice of gauge may simplify calculations. Let me now present a more general approach making use of the gauge theoretic aspects of collective coordinate quantization.

### 3 Introduction of a collective coordinate

In the following I shall consider the generating functional (4) in the soliton sector. Thus,

$$Z[j] = \langle X_f|\mathcal{T} \exp \left( -i \int_{-T'/2}^{T'/2} dt \hat{H} \right)|X_i\rangle$$

(8)

describes the matrix element of the time evolution operator in the presence of an external source, now between states $\langle X_f|$ and $|X_i\rangle$. These states specify the ground state in the soliton sector with the center of mass of the soliton at $X_i$ respectively at $X_f$. Translational invariance implies that the matrix element (8) depends only on the difference $X_f - X_i$. The length $T'$ of the time interval will finally be taken infinite.

Apart from topological charge there are two other conserved quantities: total energy and momentum. For the ground state the total momentum is also the soliton’s momentum. For given total momentum the center-of-mass coordinate of the soliton will therefore classically describe a trajectory with constant velocity $\beta$. For the matrix
element (8) this implies that
\[ \beta = \frac{X_f - X_i}{T}. \]  \hfill (9)

Let me account for this situation right from the beginning by choosing a suitable comoving frame. Here, the external source can serve as a reference; instead of \( j(x, t) \) its arguments are taken to be
\[ j = j(\bar{x}; \bar{t}) , \]  \hfill (10)
with
\[ \bar{x} = \gamma x + \gamma \beta t + X_0 , \quad \bar{t} = \gamma t + \gamma \beta x , \quad X_0 = \frac{X_f + X_i}{2} . \]  \hfill (11)

This implies
\[ x = \gamma(\bar{x} - X_0 - \beta \bar{t}) , \quad t = \gamma(\bar{t} - \beta \bar{x} + \beta X_0) , \]  \hfill (12)
indicating that the field \( \phi(x, t) \) is measured with respect to a frame moving with velocity \( \beta \). As is standard relativistic notation, \( \gamma = 1/\sqrt{1 - \beta^2} \).

In order to have a more concrete picture for what \( Z[j] \) describes recall that in the Sine-Gordon model one can identify the soliton with a fermion according to the bosonization rules \( [17] \); in this case the external source can be viewed as an electric field. Even though this is a theory in just one space dimension this interpretation provides a more familiar physical picture.

### 3.1 Relation to gauge theories

The introduction of a collective coordinate follows a method described in \( [15] \). The first step consists in a transformation of the field \( \phi(x, t) \):
\[ \phi(x, t) = \chi(x - R(t), t) \]  \hfill (13)
At this stage, \( R(t) \) plays the rôle of a parameter, local in time, describing an arbitrary moving frame. After a transformation of coordinates from \((x, t)\) to \((y, t)\) with
\[ y = x - R(t) \]  \hfill (14)
the generating functional \( Z[j] \) reads

\[
Z[j] = \int \mathcal{D}[\chi] \exp \left( \frac{i}{g^2} \int dy dt \mathcal{L}^\prime(y, t) \right)
\]

\[
\mathcal{L}^\prime = \frac{1}{2} \dot{\chi}^2 - \ddot{R} \dot{\chi} \dot{\chi} - \frac{1}{2} (1 - \ddot{R}^2) \chi^2 - V(\chi) + jR \chi
\]

(15)

Here, I introduced the general notation

\[
\dot{\chi} = \frac{\partial}{\partial t} \chi(y, t) \quad , \quad \chi' = \frac{\partial}{\partial y} \chi(y, t) \quad , \quad \ddot{R} = \frac{\partial}{\partial t} R(t)
\]

(16)

i.e. partial derivatives with respect to the first and second argument.

The source which distinguishes between rest frame and moving frame now depends on \((y, t)\) and \(R(t)\) as

\[
j_R = j(\gamma(y + R) + \gamma \beta t + X_0, \gamma t + \gamma \beta(y + R))
\]

(17)

In deriving the expression (15) I have assumed a regularization which preserves translational and Lorentz invariance; otherwise there would be an additional contribution from the Jacobian of the transformation (13). Another subtlety arises for e.g. lattice regularization due to different ordering prescriptions; an example is Weyl ordering in operator formalism corresponding to mid-point prescription in the discretized functional integral. In this case one might get even two-loop corrections from the measure [24]. I will not go beyond 1-loop, so the latter problem plays no rôle here. A third point to care about is a possible change of limits of integration in going from \(x\) to \(y\) if \(R(t)\) reaches infinity somewhere in the time interval. Later on, \(R(t)\) will be quantized allowing, in principle, highly irregular configurations. At least within a semiclassical expansion, where a smooth classical path modified by small fluctuations dominates, these irregular configurations play no rôle. Then, one only has to look at \(R(t)\) at the initial and final times. These are boundary values and will not be subject to quantum fluctuations. The Lorentz transformation to the frame moving with velocity \(\beta\) is designed to take care of these boundary values. This means that for an appropriate choice of \(\beta\) any \(R(t)\) obeys trivial boundary conditions \(R(t = -T/2) = R(t = +T/2) = 0\) where \(T = T'/\gamma\) is the length of the time interval as measured in the comoving frame.

The second step towards a quantization of \(R(t)\) is to declare it a dynamical variable, i.e. it becomes a variable of functional integration.
This leads to a trivial local gauge symmetry:

\[
\begin{align*}
\chi(y, t) &\rightarrow \chi(y - \alpha(t), t) \\
R(t) &\rightarrow R(t) - \alpha(t)
\end{align*}
\]

(18)

The first transformation is equivalent to (13) except that \( R(t) \) is now replaced by \( R(t) + \alpha(t) \). Having introduced functional integration with respect to \( R(t) \) the latter can be shifted by \( -\alpha(t) \) precisely compensating the first transformation in (18). Obviously, there appear no vector gauge bosons; only the pure gauge degrees of freedom are involved.

Actually, the generating functional \( Z[j] \) is completely independent of \( R(t) \) because the latter was introduced by a transformation of functional integration variables. This independence allows one to derive Ward identities, in the present case being related to the conservation of total momentum. For the integrated functional \( \int \mathcal{D}[R] \ Z[j] \) this simply implies an additional infinite volume factor \( \int \mathcal{D}[R] \). This volume factor has to be cancelled by a suitable constraint, in order to recover the original generating functional. This constraint can be regarded as a gauge fixing term. Its choice is almost completely arbitrary, because it does not influence the physics. It does, however, influence the amount of physical information carried by \( R(t) \). The simplest case would be the insertion of a \( \delta \)-functional requiring \( R(t) = 0 \). This immediately yields the original representation (4); the variable \( R(t) \) would be completely unphysical, namely zero.

Obviously, one has to find a gauge which allows to identify \( R(t) \) with the center-of-mass coordinate of a quantum soliton. It is at this point where the equivalence to gauge theories shows its power for the first time. Recall, that, in principle, any gauge-variant object can serve as a gauge fixing function. Therefore, the task is to find a suitable gauge-variant object. At this point, physical reasoning gives the answer. Let me consider the conserved quantity associated with translations, \( i.e. \) the total momentum \( P(t) \) as measured in the comoving frame. Due to the presence of \( R(t) \) it becomes modified as

\[
P(t) = -\frac{1}{g^2} \int dx \, \dot{\phi}(x, t) \dot{\phi}(x, t) - \frac{1}{g^2} \int dy \, \dot{\chi}(y, t) \dot{\chi}(y, t) + \dot{R}(t) \int dy \, \chi^2(y, t)
\]

(19)

As a physical quantity \( P(t) \) is invariant under the gauge transformation (18) as can be checked explicitly. However, the individual terms
in the last expression are gauge-variant. The first term of the last
expression can be interpreted as the momentum of the field $\chi$ mea-
sured with respect to an additional moving frame now defined by $R(t)$.
Then, the last term is the momentum due to the motion of this frame
itself; it is this term which I want to identify with the total momentum
$P(t)$. The gauge, therefore, has to be

$$
G(t) = \frac{1}{g^2} \int dy \chi'(y, t) \dot{\chi}(y, t) = 0
$$

which is nothing else but the definition of the center of mass frame.
Note, that this gauge does not refer to any approximate solution of
the underlying field theory.

The gauge fixing procedure is completed once the corresponding
Faddeev-Popov determinant \[25\] is found. Here, it is given by

$$
det \frac{\delta G(t)}{\delta \alpha(t')} = det \left( \frac{1}{g^2} \int dy \chi'^2(y, t) \partial_t \delta(t - t') \right)
$$

The gauge fixing terms \[20\] and \[21\] give rise to additional terms in
the Lagrangian once an auxiliary field $b(t)$, i.e. a Lagrange-multiplier
for the gauge constraint, and anticommuting ghost fields $c(t), \bar{c}(t)$ for
the Faddeev-Popov determinant are introduced \[15\]. The generating
functional (4) finally reads

$$
Z[j] = \int D[\chi] D[R] D[b] D[c, \bar{c}] \exp \left( \frac{i}{g^2} \int dy dt \mathcal{L}''(y, t) \right)
$$

$$
\mathcal{L}'' = \mathcal{L}' + b(t) \chi'(y, t) \dot{\chi}(y, t) + \bar{c}(t) \chi'^2(y, t) \dot{c}(t)
$$

The gauge symmetry \[18\] of the enlarged system is now broken by
the gauge fixing terms. Instead, the functional \[22\] has a global BRST-
symmetry. The corresponding BRST-transformations are given by

$$
\delta_{\text{BRST}} \chi(y, t) = -c(t) \chi'(y, t)
$$
$$
\delta_{\text{BRST}} R(t) = -c(t)
$$
$$
\delta_{\text{BRST}} b(t) = 0
$$
$$
\delta_{\text{BRST}} c(t) = 0
$$
$$
\delta_{\text{BRST}} \bar{c}(t) = b(t)
$$

This BRST-symmetry is the second advantage of this approach.
It can be used to derive a series of Ward-identities relating certain
n-point-functions of the full theory. This allows to consistently ensure
the constraint in any approximation which is governed by an expansion parameter. Let me consider the relevance of BRST-invariance in a more detailed manner: The additional gauge fixing terms in (22) can be written as a BRST-variation

\[ \int dy \; L'' = \int dy \; L' + \delta_{BRST}(\bar{c}G) \quad (24) \]

Now, let me choose a slightly different gauge \( G(t) + \Delta G(t) \). Gauge invariance of any n-point function to be calculated with the functional integral (22) means

\[ \Delta \langle F_n \rangle = i \langle \int dt \; \delta_{BRST}(\bar{c}\Delta G) \; F_n \rangle = i \langle \int dt \; (\bar{c}\Delta G) \; \delta_{BRST}(F_n) \rangle = 0 \quad . \]

(25)

Here, \( \langle \ldots \rangle \) denotes averaging with respect to the functional integral (22). In order to fulfill equation (25) for arbitrary \( \Delta G \) one therefore has the requirement of BRST-invariance of \( F_n \). In the present case this is trivial if physical quantities are defined in terms of the original \( \phi(x,t) \). This automatically leads to gauge and BRST-invariant n-point-functions.

However, this is true only for the full theory. What happens if one approximates the full theory? What about gauge invariance? The answer can be found repeating the same analysis as in (25) but now assuming an approximate Lagrangian \( L_{app} \). When performing a “partial integration” of the BRST-variation as in (25) one now receives, in general, a contribution from the approximate Lagrangian. For the full theory this was absent because \( L'' \) was BRST-invariant. For an approximate Lagrangian to some order \( k \) of an expansion one gets instead

\[ \langle \int dt \; (\bar{c}\Delta G) \left( \delta_{BRST}F_n + i \left[ \int dydt \; \delta_{BRST}L_{app}^{(k)} \right] F_n \right) \rangle = 0 \quad . \]

(26)

Gauge invariance to the order \( k \) of approximation is now certainly ensured if

\[ \delta_{BRST}F_n + i \left[ \int dydt \; \delta_{BRST}L_{app}^{(k)} \right] F_n = \mathcal{O}(k+1) \quad . \]

(27)

Now, \( F_n \) can be an arbitrary n-point function and one is mainly interested in BRST-invariant n-point functions. A general requirement for a consistent approximation therefore reads

\[ \delta_{BRST}L_{app}^{(k)} = \mathcal{O}(k+1) \quad , \]

(28)
i.e. the approximate Lagrangian should be BRST-invariant up to the order of approximation it governs.

The previous discussion shows that, given a certain approximation scheme, BRST-invariance guarantees gauge invariant results at any order. However, this is no statement about the quality of the approximation. Indeed, in the next subsection I will present a naive semiclassical expansion also covering the zero-mode. Such a naive treatment neglects completely the opportunities provided by the introduction of a collective coordinate. Indeed, one of the great advantages of an explicit collective coordinate is the access to new approximation schemes. A first attempt in this direction is presented in section 4.

Finally a comment on the gauge fixing procedure: the careful reader may have noticed that the gauge $G(t)$ does not fix all possible modes; time independent gauge transformations are not gauge fixed and the Faddeev-Popov determinant would thus be singular due to the constant mode of the ghost field $c(t)$. However, a constant $R(t) = x_0$ is just a translation and thus a symmetry of our theory, suppose we neglect the external source $j(x,t)$ or treat it as a perturbation. As a consequence, $R(t)$ is only defined modulo constants. We can therefore restrict the paths $R(t)$ to nonconstant configurations. The same applies to the gauge transformations which involve only nonconstant $\alpha(t)$. Due to BRST variations this means that also the ghost $c(t)$ is restricted to nonconstant modes. Keeping this in mind the gauge fixing procedure is complete and no singularities should arise.

### 3.2 A naive semiclassical approximation

In order to illustrate the use of BRST-symmetry let me now perform a semiclassical expansion analogous to the one in section 2. The field $\chi(y,t)$ is divided into a classical and a fluctuating part, the latter being proportional to $g$ (which is equivalent to $\sqrt{\hbar}$):

$$\chi(y,t) = \chi_0(y,t) + g\eta(y,t)$$

The fluctuating field obeys trivial boundary conditions, i.e. it vanishes at spatial infinity as well as at initial and final time. A rescaling of the auxiliary fields,

$$b(t) \rightarrow gb(t)$$

$$\bar{c}(t) \rightarrow g\bar{c}(t)$$
c(t) \rightarrow gc(t) \,, \quad (30)

turns out to be useful. Similarly as for the field \( \chi(y, t) \) we separate \( R(t) \) into a classical part and a fluctuating part:

\[
R(t) = R_0(t) + gq(t) \quad (31)
\]

The BRST transformations of the quantum fields now read:

\[
\begin{align*}
\delta_{\text{BRST}} \eta(y, t) &= -c(t)\chi'_0(y, t) - gc(t)\eta'(y, t) \\
\delta_{\text{BRST}} q(t) &= -c(t) \\
\delta_{\text{BRST}} b(t) &= 0 \\
\delta_{\text{BRST}} c(t) &= 0 \\
\delta_{\text{BRST}} \bar{c}(t) &= b(t) 
\end{align*} \quad (32)
\]

Only the variation of \( \eta(y, t) \) has a \( g \)-dependent part.

The semiclassical expansion of the Lagrangian of (22) reads

\[
\mathcal{L}'' = \mathcal{L}_0 + g\mathcal{L}_1 + g^2\mathcal{L}_2 + g^3\mathcal{L}_3 + \ldots \quad (33)
\]

The classical part is given by \( \mathcal{L}_0 \):

\[
\mathcal{L}_0 = \frac{1}{2} \dot{\chi}_0^2 - \ddot{R}_0 \dot{\chi}_0 - \frac{1}{2}(1 - \dot{R}_0^2)\chi_0^2 - V(\chi_0) \quad (34)
\]

The next term, \( \mathcal{L}_1 \), is linear in the quantum fluctuations. Requiring that the linear terms vanish leaves us with

\[
\mathcal{L}_1 = jR_0\chi_0 \quad (35)
\]

and defines the field equation,

\[
- \ddot{\chi}_0 + \dddot{R}_0\chi_0' + 2\ddot{R}_0\chi_0' + (1 - \dot{R}_0^2)\chi_0'' - V^{(1)}(\chi_0) = 0 \quad , \quad (36)
\]

as well as the constraint on the classical field \( \chi_0(y, t) \):

\[
\int dy \dot{\chi}_0\dot{\chi}_0 = 0 \quad (37)
\]

Note that this set of equations is equivalent to the original field equation in terms of \( \phi(x, t) \); the appearance of an additional degree of freedom \( R_0(t) \) in (34) is compensated for by the constraint (37). From
the term linear in \( q(t) \) one can derive another equation describing the conservation of the total momentum:

\[
\partial_t \left[ \dot{R}_0 \int dy \chi_0^2 \right] = 0
\]  

This equation is, however, not independent of (36) and (37).

The last term in the semiclassical expansion I am taking into account is the one bilinear in the fluctuating fields:

\[
\mathcal{L}_2 = -\frac{1}{2} \dot{\eta}^2 - \dot{R}_0 \dot{\eta} \dot{\chi}_0 - \frac{1}{2} \eta^2 \mathcal{L}(\chi_0) \eta^2 + jR_0 \eta + b \dot{\eta} \dot{\chi}_0 + c \dot{\chi}_0^2 \dot{\chi}_0 + \frac{1}{2} \dot{\chi}_0^2 \chi_0^2 - \dot{q} \dot{\eta} \chi_0 \dot{\chi}_0 + \dot{q} jR_0 \chi_0 \]  

Higher order terms in this expansion can be derived in a systematic manner.

This completes the list of terms which are needed in the following. The solution of the field equation (36) and the constraint (37) as well as the diagonalization of \( \mathcal{L}_2 \) yielding the fluctuation normal modes can now be performed analytically. It remains to be shown that the approximation by \( \mathcal{L}_2 \) is BRST-invariant up to this order. This is easily done. Taking into account the field equation (36) and the constraint (37) the BRST-variation of \( \mathcal{L}_2 \) is of order \( \mathcal{O}(g) \) and thus negligible in this approximation. This guarantees gauge invariance up to \( \mathcal{O}(g) \).

The leading order gauge constraint (37) is easily fulfilled by a static configuration

\[
\chi_0(y, t) = \chi_0(y) \ .
\]

This ansatz and equation (36) imply a constant velocity \( \dot{R}_0 \). Keeping in mind that the moving frame was chosen such that \( R(t) \), and therefore also \( R_0(t) \), satisfies trivial boundary conditions one can take \( \dot{R}_0 = 0 \) and, due to translational invariance, also \( R_0 = 0 \). The equation for \( \chi_0(y) \) now reads

\[
\chi_0''(y) - V^{(1)}(\chi_0(y)) = 0
\]

This equation is just the classical field equation for a soliton. Its solution therefore is

\[
\chi_0(y) = \phi_0(y) \ .
\]
In the reference frame, $\chi_0(y) = \phi_0(\gamma(x - X_0 - \beta t))$ appears as a Lorentz-boosted classical soliton. Its rest mass is given by

$$M_0 = \int dy \chi_0'^2(y) \quad . \quad (43)$$

Let me now turn to the fluctuation modes. Their Lagrangian $L_2$, taking into account the properties of the classical solutions $R_0(t)$ and $\chi_0(y)$, reads:

$$L_2 = \frac{1}{2} \dot{\eta}^2 - \frac{1}{2} \dot{\eta}'^2 - \frac{1}{2} V^{(2)}(\phi_0) \eta^2 + j \eta \quad + b j \dot{\phi}_0' + \bar{c} \phi_0'^2 \dot{c} \quad + \frac{1}{2} \dot{\phi}_0'^2 - \dot{q} \dot{\phi}_0' - q \phi_0' \quad (44)$$

As in (6) one expands $\eta(y,t)$ in a basis of eigenfunctions $\zeta_n(y)$ of the hermitian operator $-\partial_y^2 + V^{(2)}(\phi_0(y))$, with coefficients $a_n(t)$. The functions $\zeta_n(y)$ are precisely the eigenfunctions of (7), now in the rest frame of the moving soliton $\phi_0(y)$. The zero mode wave function $\zeta_0(y)$ is related to the classical soliton solution $\phi_0(y)$ through

$$\zeta_0(y) = \frac{\phi_0'(y)}{\sqrt{M_0}} \quad . \quad (45)$$

The Lagrange function $L_2 = \int dy L_2$ describes a set of oscillators with coordinates $a_n(t), n \geq 1$, the collective fluctuation mode $q(t)$ and a set of spurious zero energy modes:

$$L_2 = \frac{1}{2} \sum_{n \neq 0} \left( \dot{a}_n^2 - \Omega_n^2 a_n^2 \right) + \sum_{n \neq 0} a_n \int dy j \zeta_n \quad + \frac{M_0 q^2}{2} - \sqrt{M_0} \dot{a}_0 - q \sqrt{M_0} \int dy j \zeta_0 \quad + \frac{1}{2} \dot{a}_0^2 + \sqrt{M_0} \dot{\phi}_0 + M_0 \dot{c} \quad (46)$$

The coupling to $b$ essentially sets $\dot{a}_0 = 0$ in any n-point function which does not depend on $b$. Constant $a_0$ should be excluded as a quantum degree of freedom because it is related to translational symmetry. We can therefore neglect $a_0$ and find for the physically relevant part the usual fluctuation modes except that now $a_0$ is replaced by $\sqrt{M_0} q$.

It is not surprising that up to this order the Lagrange function (46) coincides with expressions using different gauges, as for example the
rigid gauge \[3\]. Using the same semiclassical expansion and requiring gauge invariance should lead to the same results. Indeed, the effective collective Lagrange function in the rest frame of the soliton reads

\[ L_{\text{eff}} = -\frac{1}{g^2}M_0 - \sum_{n \neq 0} \frac{\Omega_n}{2} + \frac{M_0}{2} q^2 + \mathcal{O}(g^2) \quad . \] (47)

This is the usual expansion to be found in e.g. \[3, 4\]. It is clear that counting \(q(t)\) as of \(\mathcal{O}(g)\) requires the inclusion of terms of order \(\mathcal{O}(g^2)\) in order to recover the equivalence of rest mass and kinetic mass up to \(\mathcal{O}(1)\). This is another hint that the collective coordinate and therefore also the zero mode has to be treated different.

4 A modified semiclassical expansion

The semiclassical approximation of the previous subsection has to be modified such that \(q(t)\) no longer counts as \(\mathcal{O}(g)\). A glance at the BRST-transformations \[32\] tells us that also \(a_0(t)\) has to be treated different. Actually, one has to solve the gauge constraint for \(a_0(t)\). For that purpose let me write down the gauge \(G(t)\) in terms of the \(a_n(t)\) separating the terms containing \(a_0(t)\):

\[
G(t) = \hat{a}_0\sqrt{M_0} + g\hat{a}_0 \sum_{n \neq 0} a_n \{\zeta_0\zeta_n\} - ga_0 \sum_{n \neq 0} \hat{a}_n \{\zeta_0\zeta_n\} + g \sum_{m, n \neq 0} \hat{a}_m a_n \{\zeta_m\zeta_n\} = (\sqrt{M_0} \partial_t + \mathcal{O}(g)) a_0 + \mathcal{O}(g) \quad . \] (48)

Here, I have introduced the notation

\[ \{abc\} = \int dy \, a(y)b(y)c(y) \quad . \] (49)

From the last line of (48) written in compact notation it is obvious that one can solve the gauge constraint for \(a_0\), at least formally. First of all, it turns out that \(a_0 = \mathcal{O}(g)\). Secondly, in order to avoid additional factors in the measure when integrating out \(b\) and \(a_0\) one has to transform

\[ \bar{c} \rightarrow \bar{c} \left(\sqrt{M_0} \partial_t + \mathcal{O}(g)\right) \quad . \] (50)
Performing the integration over $b$ and $a_0$ then leads to the following Lagrange function:

$$L' = \frac{-M_0}{g^2} + \frac{1}{2} \sum_{n \neq 0} \left( \dot{a}_n^2 - \Omega_n^2 a_n^2 \right) + \sum_{n \neq 0} a_n \left\{ j \zeta_n \right\} + \sqrt{M_0} \dot{\bar{c}} c$$

$$+ \frac{1}{2g^2} \dot{q}^2 \left( M_0 + 2g \sqrt{M_0} \sum_{n \neq 0} a_n \left\{ \zeta_n' \zeta_0 \right\} + O(g^2) \right) + \frac{1}{g} \left\{ j_q \phi_0 \right\}$$

$$+ O(g) \tag{51}$$

Here, the factor $1/g^2$ in front of the action in $(22)$ is included in $(51)$. I have written down all terms relevant for the construction of an effective collective Lagrange function at leading order tree level. Here, tree level is meant only with respect to the $a_n$-modes with $n \neq 0$, i.e. I consider the interaction of $q(t)$ with the $a_n(t)$ excluding loops. Only the leading order terms in $g$ are considered.

Let me proceed with such a construction. The only term which has to be accounted for is the coupling of $\dot{q}^2$ to the $a_n$. Including this term one gets

$$L_{eff} = \frac{-M_0}{g^2} + \frac{M_0}{2g^2} \dot{q}^2(t)$$

$$- \frac{M_0}{2g^2} \dot{q}^2(t) \int dt' \dot{q}^2(t') \sum_{n \neq 0} \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{\omega^2 - \Omega_n^2} \left\{ \zeta_0 \zeta_n' \right\}^2$$

$$+ \frac{1}{g} \left\{ j_q \phi_0 \right\} + O(q^6, g^0) \tag{52}$$

This is a nonlocal expression. However, recall that $q(t)$ describes the fluctuations around the classical trajectory. There are certainly cases where these fluctuations can be regarded small. This means that an expansion in time derivatives makes sense. Therefore, expanding in $\omega^2/\Omega_n^2$ and using the relation (see second reference of [8])

$$\left\{ \zeta_0 \zeta_n' \right\} = \frac{\Omega_n^2}{2} \left\{ \zeta_0 \zeta_n \right\} \tag{53}$$

one arrives at

$$L_{eff} = \frac{-M_0}{g^2} + \frac{M_0}{2g^2} \dot{q}^2(t) + \frac{M_0}{8g^2} \dot{q}^4(t) + \frac{M_0}{2g^2 r_{ms}} \dot{q}^2(t) \dot{q}^2(t) + \ldots + \frac{1}{g} \left\{ j_q \phi_0 \right\} \tag{54}$$
Here, the dots denote terms of $\mathcal{O}(q^0)$ or of higher time derivatives. The expression $r^2_{ms}$ denotes the mean square radius of the mass distribution of the soliton, i.e.

$$r^2_{ms} = \int dy \, y^2 \zeta_0^2(y) .$$  \hfill (55)

The first three terms in (54) can be interpreted as resulting from a Taylor expansion of $\sqrt{1 - \dot{q}^2}$. Indeed, if we consider the action of a relativistic point-like particle with mass $M$,

$$S = - \int ds \, M \sqrt{\dot{x}_\mu(s) \dot{x}^\mu(s)} ,$$  \hfill (56)

and introduce the fluctuation $q(t)$ in the moving frame as

$$x^0 = \gamma t + \gamma \beta q(t) , \quad x^1 = \gamma \beta t + \gamma q(t)$$  \hfill (57)

we find for the first three terms

$$S \simeq \int dt \, \left( - M + M^2 \ddot{q}^2(t) + \frac{M}{8} \dot{q}^4(t) + \mathcal{O}(\dot{q}^6) \right) .$$  \hfill (58)

This expansion looks like a nonrelativistic expansion. It should be noted, however, that $t$ and $q(t)$ denote the time respectively the fluctuation around the classical straight line path as measured in its rest frame.

It is obvious that the action of a point-like particle depends only on the velocity of the fluctuations. The fourth term in (54) uncovers a substantial difference in the quantum mechanics of a soliton as compared to the one of a point-like particle. This is almost evident due the appearance of the mean-square radius $r^2_{ms}$. The origin of this term lies in the interaction of the collective fluctuation $q(t)$ with the intrinsic field $\chi(y, t)$. Indeed, treating the acceleration $\dddot{q}(t)$ as perturbatively small, the classical field equation now including the $q(t)$-terms has the soliton solution

$$\chi_0(y, t) = \phi_0(\gamma(t)y)$$  \hfill (59)

with

$$\gamma(t) = \frac{1}{\sqrt{1 - \dot{q}^2(t)}} .$$  \hfill (60)

Note that this solution obeys the constraint,

$$\int dy \, \dot{\chi}_0(y, t) \chi'_0(y, t) = \gamma^2(t) \ddot{q}(t) \dddot{q}(t) \int dz \, z \phi''(z) .$$  \hfill (61)
suppose \( \phi_0^2(z) \) is an even function of \( z \). This is the case for a soliton solution centered at \( z = 0 \).

With the field (59) the classical action reads

\[
S_{cl} = \frac{1}{g^2} \int dt dy \left( \frac{1}{2} \chi_0^2 - \frac{1}{2} (1 - \dot{q}^2) \chi_0' - V(\chi_0) \right)
\]

\[
= \int dt \left( \frac{M_0}{2g^2 r_{ms}} \gamma^3(t) \dot{q}^2(t) \ddot{q}^2(t) - \sqrt{1 - \dot{q}^2(t)} \frac{M_0}{g^2} \right) .
\] (62)

This derivation demonstrates that in an adiabatic limit treating the velocity \( \dot{q}(t) \) as approximately constant the correct Lorentz covariant expression for the action of a point-like particle arises. The corrections to this picture are due to nonadiabatic terms. In particular, the kinetic term \( \dot{\chi}_0^2 \) is the origin of the additional term depending on \( \ddot{q}(t) \). As such this term can be understood as a feedback of the intrinsic field \( \chi_0(y, t) \) due to the accelerated collective motion along the path \( q(t) \). In the adiabatic approximation this feedback is a time dependent Lorentz-contraction.

What is the consequence of this feedback? Consider the situation where a weak external force induces periodic motion \( q(t) \) with a frequency \( \omega \). The response \( q(t) \) to such a force is given by the propagator or Green’s function of \( q(t) \). Already from the classical action (62) it is clear that accelerated motion gets an additional suppression from the \( \ddot{q} \)-term as compared to a point-like particle. Indeed, the feedback of the intrinsic field produces a sort of frequency cutoff; for periodic acceleration with a frequency above this cutoff the soliton may eventually respond weaker than a point-like particle. Of course, there may also be a resonance at higher frequencies. Let me exclude this possibility for the moment.

I now present a rough estimate of such a cutoff: For that purpose I keep only the term quadratic in \( \dot{q}(t) \) and the term containing \( \ddot{q} \); the latter being fourth order in \( q(t) \) is treated in a Hartree-type approximation. The Green’s function for \( q(t) \) now reads

\[
\langle q(t)q(t') \rangle = i \int \frac{d\omega}{2\pi} \frac{e^{-i\omega(t-t')}}{\omega^2 + 2r_{ms}^2 \langle \dot{q}^2 \rangle \omega^4} .
\] (63)

Instead of performing a selfconsistent approximation let me assume for convenience that \( \langle \dot{q}^2 \rangle = 1/2 \); this accounts for the fact that we are dealing with a relativistic theory and therefore \( \langle \dot{q}^2 \rangle \) should be less
than one. In contrast to the Green’s function of a point-like particle\footnote{In the present approximation a nonrelativistic particle.} an additional factor $(1+r^2_{ms}/\omega^2)$ appears in the denominator leading to a $1/\omega^4$ behaviour at high frequencies. The transition from the $1/\omega^2$- to the $1/\omega^4$-behaviour takes place at

$$\omega^2 r^2_{ms} \sim 1.$$  \hspace{2cm} (64)

The cutoff is thus given roughly by the condition that the corresponding time period of the external perturbation becomes smaller than the time needed to transfer information across the soliton. As such this additional suppression of the response at high frequencies is a typical retardation effect and probably related to form factors for elastic scattering.

This derivation of an estimate of the intrinsic cutoff should not be taken too seriously because higher order effects have been neglected. Nevertheless, the fact that the internal structure of the soliton has an effect on its quantized motion is beyond any doubt. Also the time scale of these effects must be related to the extension of the soliton, simply due to the familiar uncertainty relation.

5 Soliton-meson vertices and the coupling to an external source

As a last topic I consider the question of whether there is e.g. a sort of Yukawa coupling, \textit{i.e.} a soliton-meson-soliton vertex. This question originates from skyrmion physics\footnote{\textit{In the present approximation a nonrelativistic particle.}}: the classical skyrmion, a soliton in (3+1) dimensions, has, by definition, no linear coupling to the fluctuation modes, similar as in the present (1+1)-dimensional case. This was interpreted as the absence of a Yukawa coupling. The resolution of this problem is intimately connected to the treatment of collective coordinates\footnote{By now present another explanation of this problem.}

Recall that the spectrum in the one-soliton sector was discovered considering the \textit{ground state} expectation value of the time evolution operator in this sector. This ground state consists of a soliton moving with constant velocity with no mesonic fluctuation modes excited. A Yukawa coupling cannot show up in first order perturbation theory for

\begin{align*}
\end{align*}
the ground state expectation value, simply because it changes meson number. It will, however, show up in first order perturbation theory for an off-diagonal matrix element where either the initial or the final state contains one meson. The corresponding form factors have been discussed by Goldstone and Jackiw \[4\] and were found to be given by the normal mode wave functions \(\zeta_n(x)\).

How could one extract such a coupling in the present approach? It is clear that one has to prepare an initial or final state containing one meson. This meson will propagate forward respectively backward in time until it will be absorbed due to the meson field operator in the Yukawa coupling. In the present case, the meson field reads

\[
\phi(\bar{x}, \bar{t}) = \phi_0(\gamma\bar{x} - \gamma X_0 - \gamma\beta\bar{t}) + g \sum_{n \neq 0} a_n(\gamma\bar{t} - \gamma\beta\bar{x} + \gamma\beta X_0)\zeta_n(\gamma\bar{x} - \gamma X_0 - \gamma\beta\bar{t}) + \mathcal{O}(g^2),
\]

expressed in terms of the laboratory frame coordinates \((11)\) and using the solution of the field equation. Note that the constraint allows to replace the zero mode \(a_0(t)\) by terms of order \(\mathcal{O}(g)\). A meson can be absorbed or emitted by this field with an amplitude given by \(\zeta_n(\gamma\bar{x} - \gamma X_0 - \gamma\beta\bar{t})\). The corresponding form factor in the Breit frame is then given by

\[
G_n(p) = \frac{1}{\sqrt{2\Omega_n}} \int dX_0 e^{-ipX_0} \zeta_n(-X_0). \tag{66}
\]

Here, the Breit frame is essentially the rest frame of the soliton. The factor \(1/\sqrt{2\Omega_n}\) is just the coefficient which appears when the “coordinate” \(a_n(t)\) is expressed in terms of creation and annihilation operators, respectively their corresponding c-number analogues in the functional integral. Result \(66\) corresponds to the expression derived in \([4]\).

Now it is easy to analyse the effect of an external source coupling to \(\phi(\bar{x}, \bar{t})\). According to \(65\) such a coupling creates and absorbs mesons leading to resonance or scattering states. In addition, there is a coupling of \(j(\bar{x}, \bar{t})\) to the classical soliton solution. It measures, the additional interaction energy. But there is still another effect. In the presence of an external source the soliton will be accelerated. This can be seen from \(65\) now also including the external source. The modified equation for \(R_0(t)\) is

\[
-M_0\ddot{R}_0(t) = \sqrt{M_0} \int dy j_{R_0} \zeta_0(y) \tag{67}
\]
where the source is treated as a small perturbation and changes in the mass of the soliton due to excitations have been neglected. The zero-mode wave function projects on that part of the external source which is responsible for an acceleration of the soliton, at least in a perturbative sense. A nonperturbative treatment requires a presumably numerical solution of the field equation and the constraint in the presence of an external source.

6 Summary, discussion and outlook

One aim of this paper has been to present another variation of introducing collective coordinates with special emphasis on gauge and BRST-symmetry aspects. I have presented a gauge based on a physical picture and without any reference to a classical soliton solution. The result has been a coupled set of equations for the classical collective coordinate and the soliton field. Lorentz covariance has been built in right from the beginning introducing the collective coordinate in a convenient moving frame. This procedure has been justified by the classical solution $\ddot{R}_0(t) = 0$ together with the boundary values implying $R_0(t) = 0$. The second aim was to emphasise the special rôle of the collective coordinate and to work out the relation to the quantum mechanics of a relativistic point-like particle. Deviations from this picture manifest themselves as additional terms with higher time derivatives in the effective theory of collective fluctuations. They represent retardation effects and imply an intrinsic cutoff in the collective response related to the dimension of the soliton, eventually also a resonant behaviour. Soliton-meson vertices have been discussed and corresponding form factors have been derived in accordance with ref. [4]. Finally, the effect of external sources has been considered. For a weak external source its effect to leading order has been shown to be twofold: acceleration of the whole system, i.e. collective excitation, and internal excitation.

The fact that one can extract collective coordinates analytically and thereby eventually remove infrared problems from the “intrinsic” system (which, of course, depends on the gauge) allows a combination of numerical and analytical methods, at least in principle. In cases, where a semiclassical expansion or any other analytical approach for the intrinsic system fails, one has to use numerical schemes like Monte-
Carlo simulations. It may turn out that one has to choose appropriate
gauges different from the present one; I have already mentioned that
there might be differences in connection with certain regularization
schemes.

I have chosen a soliton theory in (1+1) dimensions for the discus-
sion of collective coordinate quantization mainly because of its relative
simplicity. This does not mean that the physical situation is simple.
Indeed, it is precisely the interpretation of solitons as extended parti-
cles with internal structure which makes them so attractive and also
nontrivial. An application of the present approach to any field the-
ory which admits symmetry-breaking mean-field solutions seems to
be straightforward. I am not claiming that I have found the ultimate
gauge in connection with collective coordinate quantization even if
this gauge may appear very appealing to some readers. Depending on
the aims different choices may be more advantageous. After all, any
gauge is as good as the other as concerns the physics. The difference
is measured by the calculational effort.

It is worthwhile to discuss possible consequences of the results pre-
sented above. In particular, the retardation effects modifying the
behaviour of the collective fluctuations as compared to a point-like
particle may also play a rôle when calculating quantum corrections to
the soliton energy. The collective zero-point motion has to be taken
into account and may very well modify existing calculations.

The present approach also allows to calculate the effects of strong
external fields, at least numerically, and still distinguish between col-
llective and intrinsic phenomena. This is achieved already at the clas-
sical level due to a coupled set of equations for \( R_0(t) \) and \( \chi_0(y,t) \).

Let me finally consider implications of the previous analysis for
soliton models in (3+1) dimensions. As long as translational motion
is concerned one can expect a similar behaviour. This means that
the quantization of translational motion for a moving soliton can be
carried out in its rest frame. The results are then transformed to the
reference frame by a Lorentz-boost.

The fluctuations of the collective motion now also involve rota-
tions. Let us restrict considerations to the case where \( \vec{R}(t) \) always
points in the same direction. Then, possible differences may still
occur due to the presence of higher derivative terms as in \( e.g. \) the

\[ A \text{ standard situation in } e.g. \text{ nuclear physics.} \]
Skyrme-Lagrangian\textsuperscript{3}. As long as these higher derivative terms lead to just higher powers of $\ddot{R}$ the situation is similar to the present case, at least at tree level: Recall that due to Lorentz-invariance of the action all four-derivatives appear contracted and therefore, after introducing $\ddot{R}(t)$, spatial derivatives appear always in the combination

$$\vec{\nabla} F \vec{\nabla} G - (\dddot{\bar{R}} \vec{\nabla}) F (\dddot{\bar{R}} \vec{\nabla}) G,$$

with $F, G$ field dependent quantities. Therefore, one can repeat the same arguments leading to the effective action for the collective fluctuation $\bar{q}(t)$, in particular in the adiabatic limit.

One should also expect terms depending on $\dddot{\bar{q}}(t)$. Such terms can arise already before solving the field equations due to terms in the Lagrangian where the pion field appears with second time derivatives or even higher. The lesson of the present paper is that such terms indicate some intrinsic structure. The origin of these terms in the Lagrangian can be understood if one recalls that these Lagrangians are usually itself effective Lagrangians; this means that they are in some way derived from an underlying theory by integrating out short distance degrees of freedom, in an analogous way as for the derivation of the effective Lagrange function (54). In a gradient expansion the, in principle, nonlocal structure of the effective Lagrangian is traded with terms containing second and higher derivatives acting on the fields. There is already some intrinsic structure due to the short distance degrees of freedom which have been integrated out. The relevant length scale has to be smaller than the extension of the soliton; otherwise, the effective Lagrangian wouldn’t make sense.

The discussion of soliton-meson vertices seems to be applicable in (3+1) dimensions as well. Of course, the number of vertices will be much larger due to the richer set of internal quantum numbers like spin, isospin etc..

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\textsuperscript{3}This question was brought to my attention by R. Alkofer
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