Impact of field ionization on the velocity of an ionization front induced by an electron beam propagating in a solid insulator

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**Abstract.** The propagation of an intense electron beam through a solid density insulator is analysed. The asymptotic behaviour of the front velocity is found for large beam densities. We show that at high beam densities the front velocity starts to fall when the induced $E$-field becomes strong enough to neutralize itself (through field ionization followed by self-consistent charge neutralization). We also show that the effect of the polarization current (caused by the $E$-field removing electrons from atoms) on the magnitude of the ionization front velocity, although noticeable, does not change the qualitative picture derived when neglecting it.

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1. Introduction

A high energy electron beam is generated when a high intensity laser pulse interacts with matter. As a result of the interaction of this beam with a thin foil an energetic proton beam can be produced [1]–[4] which can be utilized in many applications ranging from the Fast Ignition scheme of the Inertial Confinement Fusion [5, 6] to medicine. The quality of the proton beam is determined by the electron beam quality which can be affected by the foil material. The material of the foil is important because the propagation of an intense electron beam through the initially cold matter of the foil is influenced by the collisional and field ionization processes. The latter is especially important in insulators.

The dynamics of the laser generated electron beam and the associated ionization front were studied in a number of works [7]–[14]. In the experiment [9] glass targets were used to observe the time and space evolution of the fast electron beam propagating through a solid target. Two distinct types of fast electron propagation in a glass target were reported: the collisional electron cloud expansion accompanied by a highly collimated electron jet. In [11] it was shown that the jet observed in the experiment [9] could be explained by the presence of field ionization causing a filamentation type instability. This instability has been previously observed in a number of experiments [8, 12] when a dielectric target was used. The use of a metal target in the same experiments [8, 12] produced no sign of the instability. In this paper we study in detail the high density electron beam propagation in solid density insulators, extending the line of analysis started in [11] to the cases of high beam density and all possible values of the beam density profile parameter $p$ introduced in [11]. Finally, the impact of the polarization current induced by field ionization is discussed.

The expression for the ionization front velocity was obtained in [11] for small values of parameter $p$ determining the smoothness of the beam density profile $n_b = \bar{n}_b \cdot (e\varphi / W_b)^p$, where $\bar{n}_b$ is the beam density far from the beam head; $\varphi$ is the electrostatic potential; $W_b$ is the maximum beam electron energy in the front frame. The corresponding energy distribution (the integral over energy gives density) is $f(E_{tot}) = \bar{n}_b p/((1 - E_{tot}/W_b)^{1-p}W_b)$, where $E_{tot}$ is the total electron energy (note that the singularity at $E_{tot} = W_b$ is integrable). The reason for using such an unusual distribution is to keep beam electrons from spreading ahead of the beam. The electron energy distribution corresponding to such a beam density profile has no infinite super-thermal tail as seen in the Maxwellian distribution. The real beam distribution function in experiments is not well known; however, in [11] it was shown that the front velocity behaviour is not very sensitive to the beam distribution function and, therefore, a reasonably simple distribution can be used. That also means that for the qualitative description the choice of distribution parameter $p$ is not important. Also for simplicity we use constant cross-section for collisional ionization since the variations in the front velocity are mainly due to large variations in the field ionization rate and, therefore, the velocity general behaviour is not affected much by changes in the collisional ionization rate.

In [11] it was shown that the front velocity $V_f$ is the monotonically decreasing function of $\bar{n}_b$ until the energy density of the beam reaches about 1% of the atomic $E$-field energy density $E_a^2/8\pi$ ($E_a$ is the atomic $E$-field [11]) when the field ionization kicks in and the front velocity starts rapidly growing with $\bar{n}_b$. The numerical simulation in [11] has also shown that if the value of $\bar{n}_b$ is increased even further the front velocity eventually reaches its maximum and then slowly decreases. In this paper we derive the expression for the front velocity at high beam densities.
(for an arbitrary value of beam smoothness parameter \( p \) generalizing small \( p \) approximation used in [11]) and then use this expression to explain the slowing down of the front at high beam densities observed in [11]. In section 3 we use the newly derived expression for velocity to evaluate the impact of the polarization current.

At the back of the ionization front (where the \( E \)-field has to vanish because a nonzero field in 1D can exist only inside of the front) we can assume the quasi-neutrality condition (i.e. the difference between the beam density \( n_b \) and the induced charge density is relatively small) to find the amplitude of the \( E \)-field there. Using the expressions for induced charge and the beam density profile (see equations (8) and (10) in [11]) we find that the normalized \( E \)-field at the rear of the ionization front is

\[
\varepsilon = \frac{[\bar{n}_b \phi^p V_f]}{[n_i(x) V_{E_a}]}, \tag{1.1}
\]

where \( V_{E_a} \) is the electron drift velocity in the \( E \)-field with amplitude \( E_a \); \( n_i \) is the ion density; \( \varepsilon = -E/E_a \) and \( \phi = e\varphi/W_b \) are the dimensionless \( E \)-field amplitude and potential respectively.

In 1D geometry the \( E \)-field can be written as the gradient of the potential, \( \varepsilon = d\phi/dx \cdot (W_b/eE_a) \), therefore, we can integrate equation (1.1) over the whole quasi-neutral region from \( x_1 \) (start of the region) to \(-\infty\) (or in terms of potential from \( \phi = \phi_1 \) to \( \phi = 1 \)) to find the ionization front velocity \( V_f \) as a function of unknown \( x_1 \). We find the function \( n_i(x) \) in this region by exploiting the fact that collisional ionization dominates, and, therefore, the continuity equation for ions simplifies to \( V_f \frac{dn_i}{dx} \approx v_cln_i \) (where \( v_cl = K_{cl}N_a \) is the collision ionization frequency; \( N_a \) is the atom density; \( K_{cl} \) is constants), and gives \( n_i = n_i(x_1) \exp(v_cl(x - x_1)/V_f) \).

Substituting \( n_i \) into equation (1.1) and integrating we obtain the ionization front velocity as a function of two unknowns: the potential \( \phi_1 \) and the ion density \( n_i(\phi_1) \) at the beginning of the quasi-neutral region

\[
V_f^2 = \frac{1 - \phi_1^{1-p} n_i(\phi_1) W_b v_cl}{1 - p} \frac{\bar{n}_b}{eE_a} V_{E_a}. \tag{1.2}
\]

The dependence of \( \phi \) and \( n_i(\phi_1) \) on the beam density, \( p \) and other parameters is discussed in the section 2. The change in the estimated front velocity due to introduction of polarization current into the model is discussed in section 3. We summarize the findings in section 4.

2. Non-neutral region impact on the ionization front velocity at high beam densities

The potential \( \phi_1 \) corresponding to the start of the quasi-neutral region and the ion density \( n_i(\phi_1) \) are determined by the preceding strongly non-neutral region where the beam charge causes the \( E \)-field to grow. The maximum value of the \( E \)-field, \( \varepsilon_m \), and the corresponding potential \( \phi_m(\phi_1 > \phi_m) \) in the case of arbitrary value of \( p \) are (the derivation is similar to that of equations (29) and (30) from [11])

\[
\phi_m = \left[ \frac{\varepsilon_m^2 E^2(\rho + 1)}{8\pi W_b \bar{n}_b} \right]^{1/(1+p)}, \quad \varepsilon_m^2 v_{El}(\varepsilon_m) = \frac{8\bar{n}_b \phi_m^{2p} V_f^2 e}{E_a V_{E_a} N_a}. \tag{2.1}
\]
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Figure 1. The qualitative structure of the $E$-field, beam and induced charge densities, and ionization rates for the case $\phi^\text{max}_m < \phi_m < 1$ ($n_b = 10^{18}$ cm$^{-3}$, $n_0 = 3 \times 10^{22}$ cm$^{-3}$, $p = 0.3$).

where $\nu_{EI}(\varepsilon_m) = \nu_a e^{-1/\varepsilon_m/\varepsilon_m}$ is the field ionization frequency; $\nu_a = 6m_e^2/\hbar^3$; \hbar is the Planck constant. In section 3 we are going to derive even more general expression for $\varepsilon_m$ by including the polarization current but that correction will be shown to be insignificant.

Depending on the value of $\phi_m$ the ionization front structure can be described by one of the two following cases. In the case $\phi_m \approx 1$ (see figure 1) the total ion density produced by field ionization in the head region is approximately $2n_i(\phi_m)$ due to the symmetry of the $E$-field profile near its maximum value (see equation (27) in [11]). Therefore, $n_i(\phi_1) \approx 2n_i(\phi_m)$, and we can use $\phi_1 \approx \phi_m$ because the relative width of the field ionization region is small (see discussion after equation (29) in [11]). Substituting $n_i(\phi_m)$ from equation (1.1) (since at $\phi_m$ the quasi-neutrality condition is also satisfied) we obtain

$$V_f \approx \frac{1 - \phi_1^{1-p} \phi_m^p 2W_b \nu_{EI}}{1 - p \varepsilon_m} eE_a.$$  (2.2)

Equation (2.2) shows that initially the front velocity increases with $n_b$ but eventually starts falling proportional to a small negative power $-p/(p + 1)$ of $n_b$ (see figure 2, region between data points marked ‘1’ and ‘2’). Therefore, the steeper the beam profiles the faster the front velocity falls, and that confirms the simulation results for different $p$ in [11]. The front velocity has its maximum value when $\phi_m$ is equal to $\phi_1^{\text{max} V_f} \approx p^{1/(1-p)}$.

For very large beam densities we have $\phi_m \ll 1$ ($\phi_m < \phi_m^{\text{max} V_f}$). The induced $E$-field is so strong that field ionization becomes dominant within some part of the quasi-neutral region at the beam back (see figure 3). Consider this part of the quasi-neutral region to be a separate region connecting the non-neutral region and the collision ionization dominated part of the quasi-neutral region. In this case the values of $\phi_1$ and $n_i(\phi_1)$ required for equation (1.2) are determined by field ionization (not collisional ionization as it was before), i.e. $V_f d n_i/d x \approx \nu_{EI}(\varepsilon)n_0$. Substituting
By introducing the polarization current into equations (12) and (13) from [11] and solving them numerically we found the velocity of the ionization front as a function of electron beam density $\bar{n}_b$ ($n_b = \bar{n}_b \phi^p$, $p = 0.3$) for different beam electron energies measured in the reference frame of the front. The dotted lines with triangles correspond to the front velocity calculated without the polarization current. Squares with numbers 1 and 2 refer to beam densities for which figures 1 and 3 are plotted, correspondingly.

From equation (1.1) (which is derived using the quasi-neutrality condition) and using the relationship $(d/dx) = eE_a/W_b \cdot (d/d\phi)$, we obtain the equation for the $E$-field in this new region:

$$\varepsilon = \frac{1}{\ln A_1}, \quad A_1 \equiv \frac{W_b \nu_a}{eE_a} n_0 V_{E_1} \left[ p \frac{\varepsilon}{\phi} - \frac{\partial \varepsilon}{\partial \phi} \right]^{-1}. \quad (2.3)$$

Now (when $\phi_m \ll 1$) the potential $\phi_1$ ($\phi_1 \gg \phi_m$), which is defined as the beginning of the quasi-neutral and collision ionization dominated region, corresponds to the point where the value of the field ionization rate drops below the collisional ionization rate $\nu_{EI}(\phi_1) N_a \approx \nu_\text{el} n_1(\phi_1)$. After some algebra we find

$$V_f \approx \frac{1 - \phi_1^{1-p}}{1 - p} \frac{\phi_1^p}{\varepsilon(\phi_1)} \frac{W_b \nu_a}{eE_a}, \quad \phi_1 = p^{l/(1-p)}. \quad (2.4)$$

Therefore $V_f$ exhibits only weak logarithmic dependence on $\bar{n}_b$ through $\varepsilon$. Note that the potential $\phi_m$ steadily falls with the beam density according to equation (2.1) while $\phi_1$ is fixed, and that leaves increasingly more space (from $\phi_m$ to $\phi_1$) for the field ionization dominated quasi-neutral region shown in figure 3. Finally, figure 4 illustrates how the front velocity depends on the beam density $n_L = n_b/\sqrt{1 - c_f^2}$ and energy $W_L/mc^2 = (1 + c_tc_b)/(\sqrt{1 - c_f^2})(1 - c_f^2) - 1$ in the laboratory frame, where $c_f = V_f/c$ and $c_b = \sqrt{1 - (1 + W_b/mc^2)^{-2}}$.
Figure 3. The structure of the $E$-field, beam and induced charge densities, and ionization rates for the case $\phi_m < \phi_{m}^{\text{max}}$ ($n_b = 10^{20} \text{ cm}^{-3}$, $n_0 = 3 \times 10^{22} \text{ cm}^{-3}$, $\rho = 0.3$). The green ellipse shows the new region (compared to figure 1) where the quasi-neutrality condition is satisfied and field ionization is dominant.

Figure 4. The ionization front velocity is plotted versus beam density $n_L$ ($\rho = 0.3$) in the laboratory frame for three different beam energy $W_L$ values in the laboratory frame 0.3, 1 and 3 MeV. The values of the velocity are obtained from numerical solution of equations (12) and (13) from [11] with and without addition of the polarization current.
3. Polarization current effect

In our analysis of the ionization front so far we have neglected the current induced by the field ionization itself due to the polarization of atoms. This current can inhibit the strong field formation and therefore may be worth considering. The amplitude of the current $j_{\text{polar}}$ can be derived from the balance of the work done by the $E$-field, $-e j_{\text{polar}} E$, and the energy input required by the field ionization, $I_{\text{El}} N_a$,

$$j_{\text{polar}} = -I_{\text{El}} N_a / [e E a]$$

where $I$ is the ionization energy. This field ionization induced current will have the largest amplitude at the point $\phi_m$ of maximum field strength $\varepsilon_m$. The new equation (modified equation (1.1)) for $E$-field $\varepsilon_m$ is

$$\bar{n}_b \phi_m^p V_t = n_i(\phi_m) V_E \varepsilon_m + j_{\text{polar}}(\phi_m).$$

The new expression for $n_i(\phi_m)$ can be found by expanding $\varepsilon$ near $\varepsilon_m$ and integrating $V_t \, dn_i/dx \approx v_{\text{El}}(\varepsilon) N_a$ from 0 to $\phi_m$:

$$n_i(\phi_m) = \sqrt{E_a N_a \varepsilon_m v_{\text{El}}(\varepsilon_m) / [8V_E a]}. \quad (3.3)$$

Solving equation (3.2) for $v_{\text{El}}(\varepsilon_m)$ we find

$$\sqrt{e^{1/\varepsilon_m}} = \left[ \frac{n_0 \nu_a I}{\bar{n}_b e E_a V_f \phi_m} \right]^{1/2} \left[ 1 + \sqrt{1 + A_2} \right], \quad A_2 \equiv \frac{4}{\pi} \frac{V_f}{V_{\text{Ed}} W_b} \, p + 1 \phi_m. \quad (3.4)$$

Equation (3.4) is the new equation for the maximum amplitude $\varepsilon_m$ of the $E$-field, which improves on equation (29) in [11] by accounting for the polarization current effect. We find that we can neglect the polarization current effect on the value of $\varepsilon_m$ if $A_2 \ll 1$ (i.e. for small beam density $\bar{n}_b$), in which case equation (3.2) reduces to equation (29) in [11]. At the other extreme (for very high beam densities) when $\phi_m \ll 1$ the polarization current effects also can be neglected due to the weak logarithmic dependence (see equations (2.3 and 2.4)) of $\varepsilon(\phi_1)$ on $\bar{n}_b$. For the intermediate case (when neither $A_2 \ll 1$ nor $\phi_m \ll 1$ is satisfied) the numerical solution of Poisson’s equation (equations (12) and (13) from [11] with the addition of the polarization current correction) has shown that the reduction of the ionization front velocity caused by the polarization current term does not exceed 15% ($p = 0.3, \, n_0 = 3 \cdot 10^{22}/cc$) and does not change the qualitative behaviour described above (see figure 2).

4. Summary

We analysed the propagation of an intense electron beam through a solid density insulator. We found that if the beam energy density is so large that $\phi_m < \phi_{\text{max}}^V I$ then the higher the beam density $\bar{n}_b$ the earlier the $E$-field ‘cut-off’ occurs along the beam length (due to the increasing number of electrons generated by the field ionization), resulting in the reduction of the field ionization contribution and the deceleration of the ionization front noticed in the simulation in [11]. For even higher beam densities ($\phi_m \ll 1$) the field ionization rate after the cut-off rises to become
comparable to the collisional ionization rate, and the velocity decline with the beam density becomes logarithmically weak. We found that the effect of the polarization current (caused by the $E$-field removing electrons from atoms) on the magnitude of the ionization front velocity may be noticeable (up to 15% decrease at the velocity maximum, see figure 2) but does not change the qualitative picture derived when neglecting it.

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