Wear Prediction Model Based on the Fractal Contact Characteristics

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Abstract. A new fractal wear model is proposed. The new model is based on the multi-scale fractal contact model. Studying the elastoplastic and plastic state of contact asperities, the new wear model is established. The relationship between the amount of contact surface wear and the fractal dimension $D$, the characteristic coefficient $G$ and the material property constant $C$ is studied. Besides, wear coefficient of the rough contact surface is predicted. It can be seen from the results that the influence of the scale effect on contact surface must be considered, because of the asperities with the same deformation $\omega$ will be in different contact state. The relationship between amount of wear in the new model and amount of wear in existing model has the same trend. When $D = 1.45$, the amount of wear is the lowest. With the increasing of $G$ reflecting the roughness of contact surface, the wear amount is increasing. The material constant also has a certain influence on the wear amount, and the material with high hardness has low wear. The contributions of the amount of asperity wear (elastoplastic and plastic) have the same effect to the total wear amount. Besides, the result of new fractal wear coefficient and the result of classical have the same trend, however, the new wear model is more precise than the classical wear model and closer to the actual state.

Keywords. Rough surfaces, contact model, wear model, multi-level.

1. Introduction
Contact structures play an important role in mechanical products, especially in precision instruments. Due to the action of vibration and impact during the working process, the contact surface will suffer wear under the contact pressure, and the reliability and life of the components are greatly affected. Therefore, research on the asperity contact and asperity wear of contact surfaces is particularly important. Based on this, many scholars home and abroad have carried out a lot of research.

Majumdar and Bhushan [1] proposed a rough surface contact model (MB model) based on fractal geometry, which laid the foundation for microscopic fractal contact theory. However, the MB model only considered the elastic and plastic contact state of the asperities when establishing, but not considered the elastoplastic contact state during the modeling process. Subsequently, Kogut and Etsion [2] based on the MB model and applied the finite element theory to study the contact law of the asperity in the elastoplastic stage. With the development of contact theory, a large number of studies had found that scale parameters have certain impact on the contact model. Morag [3] studied the correlation between scale parameters and critical contact parameters based on fractal contact model and Hertz contact theory. Jackson [4] proposed a multi-scale model in which imaginary small-sized asperity were distributed over large-sized asperities. Wriggers [5] used finite element theory to study the contact...
parameters of different scales on the contact surface. Sun [6] and Shi [7] studied the contact parameters of different scale characteristics by using the finite element and wavelet decomposition theory. Miao [8], Ding [9], Cheng [10, 11] and Yun [12] studied the relationship between the critical contact parameters of asperities and the scale level n. Though above working, the fractal contact model was further improved.

Based on the fractal contact model and the Archard wear model [13], a large number of scholars have studied the wear of microscopic contact surfaces. Zhou [14, 15] predicted the wear process based on the MB fractal model. Ge [16] combined fractal theory with tribology theory and studied the relationship between fractal parameters and surface topography. Fei [17] verified the rationality of fractal wear model in engineering through experiments. Jiang [18] studied the wear process of the fractal contact surface. Wang [19], Ding [20] studied the relationship between the contact wear rate and the fractal parameters, and obtained the fractal parameters corresponding to the lowest wear rate. Su [21] simulated the microscopic wear of the contact surface by removing the volume of the asperity. In addition, more and more products in the engineering project began to apply the fractal wear model [22, 23] for performance evaluation.

In summary, the application of fractal theory to study the wear of contact surfaces provides a new research idea for micro-interface wear prediction and provides a new guidance method for engineering applications. However, the fractal wear model above is established based on the classical single-scale fractal contact theory. With the development of the multi-scale fractal contact model, the calculation accuracy and method of the contact model have been further improved, so the existing single-scale fractal wear model needs to be modified. In this paper, a new wear prediction model is established based on the multi-scale fractal contact model. The theoretical derivation is more reasonable and the prediction results are closer to reality.

2. Contact Model

2.1. Geometric Model

The contact surface has self-affine and multi-scale characteristics. Majumdar [1] used the W-M fractal function to characterize the 2D contour curve of the rough surface. The expression is as follows:

\[ z(x) = G^{D-1} \sum_{n=1}^{n_{\text{min}}} \frac{\cos(2\pi \gamma^n x)}{\gamma^{(2-D)n}} \quad (1 < D < 2) \]

where \( z(x) \) represents the height of the rough surface contour curve. \( x \) is the contour displacement coordinate. \( D \) is the contour fractal dimension. \( G \) is the contour feature coefficient. \( \gamma \) is a constant greater than 1, and \( \gamma \) is often taken \( \gamma = 1.5 \) [1, 3]. \( n_{\text{min}} \) is the lowest cutoff frequency corresponding to the contour profile. The contour profile curve is determined by \( D, G \) and \( n \), and the frequency order \( n \) is related to the sample length, and \( D \) and \( G \) can be obtained from the power spectrum of the two-dimensional WM function. As shown in figure 1, at different magnifications, the outer contour profile curve of the WM function exhibits self-affinity.

![Figure 1. Contour profile curve of fractal function.](image)

2.2. Surface Contact Model

The contact process of two rough surfaces can be equivalent to the process in which a rigid plane will contact with a rough plane. The model is established based on the following assumptions [1]: (1) the
anisotropic properties of the asperity is not considered; (2) The hardening effect of the material during the deformation of the asperities is not considered; (3) the interaction between the asperities during the contact process is neglected; (4) the effect of the normal pressure is considered regardless of the influence of the frictional force.

2.2.1. Single Asperity Contact Model. The deformation of a single asperity is shown in figure 2, where $L$ is the size diameter of the asperity, $\delta$ is the height amplitude of the asperity, $\omega$ is the amount of deformation applied by the rigid plane to the asperity, which satisfies $0 < \omega \leq \delta$, and $d$ represents the distance between the rigid plane and the contact surface, which value is given in Ref. [1]. The dotted line in the figure represents the initial contour of the asperity, and the solid line represents the contour of the asperity after being deformed by external force.

![Figure 2. Contact model of single asperity.](image)

Under the action of a certain contact force, the radius of curvature $R$, height $\delta$ and deformation $\omega$ of the asperity surface are as follows:

$$R = \frac{L^0}{d^2z/dx^2}_{x=0} = \frac{L^0}{\pi^2G^{D-1}}$$

(2)

$$\delta = z(0) = G^{D-1} \cdot L^{2-D}$$

(3)

$$\omega = \delta - d$$

(4)

As the contact force increasing, $d$ gradually decreases and $\omega$ gradually increases. From the classical Hertz contact theory [24] and Kogut-Etsion contact theory [2], it can be learned that the deformation of the asperity body is divided into four stages: elastic, first elastoplastic, second elastoplastic and plastic deformation stage. In the following, each contact phase is discussed.

(1) Elastic Deformation Stage

According to the Hertz contact theory [24], the critical elastic contact of asperities is as follows:

$$\omega_c = \left( \frac{\pi KH}{2E} \right)^2 R = \left( \frac{\pi K\phi}{2} \right)^2 R$$

(5)

where $K = 0.454 + 0.41\nu$, $H$ is the hardness of the softer contact surface, $E$ is the equivalent elastic modulus, and $\nu$ is the Poisson's ratio. When $R \gg \omega$, the contact area and contact force of the asperities are as follows:

$$\begin{cases}
a = \pi Re \omega \\
P_s(a) = \frac{4}{3} \sqrt{\pi} \cdot E \cdot G^{D-1} \cdot a^{\frac{3-D}{2}}
\end{cases}$$

(6)

(2) Elastoplastic Deformation Stage

In Ref. [2], Kogut and Etsion used finite element analysis to analyze the elastoplastic deformation of the spherical surface in contact with the rigid plane, and divided the process into two stages: the first
elastoplastic stage \((\omega_\varepsilon^\prime < \omega \leq 6\omega_\varepsilon^\prime)\) and the second elastoplastic stage \((6\omega_\varepsilon^\prime \leq \omega < 110\omega_\varepsilon^\prime)\). The contact area and contact force of the asperities in the elastoplastic stage are as below:

\[
\begin{align*}
 a_{ep1} &= a_{ce}^* \cdot 0.93 \left( \frac{\omega}{\omega_{ce}} \right)^{1.136} \\
 P_{ep1}(a) &= P_{ce}^* \cdot 1.1282 \left( \frac{a}{a_{ce}} \right)^{1.136} \\
 a_{ep2} &= a_{ce}^* \cdot 0.94 \left( \frac{\omega}{\omega_{ce}} \right)^{1.146} \\
 P_{ep2}(a) &= P_{ce}^* \cdot 1.4988 \left( \frac{a}{a_{ce}} \right)^{1.146}
\end{align*}
\]

(7)

(8)

where \(a_{ce}^*\) and \(P_{ce}^*\) are the critical elastic contact area and critical elastic contact force obtained by (from) equation (6) when the deformation amount of the asperity is \(\omega_\varepsilon^\prime\), respectively.

(3) Plastic Deformation Stage

When \(a > a_{ep2}\) or \(\omega > 110\omega_\varepsilon^\prime\), the asperity is plastically deformed. The contact area and contact force [24] are as follows:

\[
\begin{align*}
 a &= 2\pi R a \\
 P_\varepsilon(a) &= H a
\end{align*}
\]

(9)

2.2.2. Contact Model of Rough Surface. According to the foregoing analysis, the rough surface is formed by stacking a series of asperity bodies of different scales. As shown in figure 3, when the rough contact surface produces a certain contact displacement, the contact states of the different asperity bodies with the same deformation amount are different, and the contact state of the asperity has a certain relationship with the scale level \(n\).

![Figure 3. Rough surface contact model.](image)

In Ref. [11], Cheng Yu studied the influence of the scale level \(n\) on the critical contact state, and concluded that the first six consecutive scale ordinals have an absolute influence on the contact parameters, which can represent the contact morphology of the rough surface, instead of \(n\) needed to tend to infinity in the W-M function. It gives the critical scale level \(n\) expression as follows:

\[
n_i = f(\nu, H, E, G, D)
\]

(10)

where \(\nu\) is the Poisson’s ratio, \(H\) is the softer contact surface hardness, \(E\) is the equivalent elastic modulus, and \(f\) is a functional relationship [11]. Based on this, Ref. [12] introduced the linear factor and the new
integral summation method to calculate the contact parameters, and further improved the multi-scale fractal contact model.

(1) Contact Area Distribution Function

In 1975, Mandelbrot [25] based on the distribution law of the area of the ocean island, studied the number of maximum contact points of the vertical contour line, which is as follows:

$$ N(A > a) = \left( \frac{a}{a_c} \right)^{D} $$

(11)

where $a_c$ is the maximum cross-sectional area in the contact surface. Based on this, the asperity cross-sectional area distribution function is as follows:

$$ n(a) = \frac{dN}{da} = \frac{D}{2} \cdot \frac{a^D}{a^{1+D}} $$

(12)

In addition, it is considered that there is a certain inclusion relationship in the maximum contact point as follows:

$$ N_e \subseteq N_n \subseteq N_{ep} \subseteq N_p $$

(13)

The cross-sectional area distribution function of the asperity in different contact states obtained by equations (11)-(13) are as follows:

$$ n_e(a) = \frac{D}{2} \cdot \frac{a^D}{a^{1+D}} $$

$$ n_{ep1}(a) = \frac{D}{2} \cdot \frac{1}{a^{1+D}} \left[ a_{cep1}^{D} - a_{ce}^{D} \right] $$

$$ n_{ep2}(a) = \frac{D}{2} \cdot \frac{1}{a^{1+D}} \left[ a_{cep2}^{D} - a_{cep1}^{D} \right] $$

$$ n_p(a) = \frac{D}{2} \cdot \frac{1}{a^{1+D}} \left[ a_c^{D} - a_{cep2}^{D} \right] $$

(14)

(2) Rough Surface Contact Parameters

The integral force of the distribution function of the single asperities in different contact states on the corresponding regions is obtained, and the contact force of the rough surface are showed as follows:

$$ P_e = \Phi_1 \cdot \int_{0}^{a_c} p_e(a) \cdot n_e(a) \cdot da $$

$$ P_{ep1} = \Phi_2 \cdot \int_{0}^{a_{cep1}} p_{ep1}(a) \cdot n_{ep1}(a) \cdot da $$

$$ P_{ep2} = \Phi_3 \cdot \int_{0}^{a_{cep2}} p_{ep2}(a) \cdot n_{ep2}(a) \cdot da $$

$$ P_p = \Phi_4 \cdot \int_{0}^{a_c} p_p(a) \cdot n_p(a) \cdot da $$

(15)

The contact area of the rough surface are as follows:
In equation (16), $\Phi_i$ is the corresponding correction coefficient [12] in different contact states, and $a_{el.}$, $a_{elp.}$, $a_{elpl.}$, and $a_{pl.}$ respectively represent the maximum cross-sectional area under different contact states of the rough surface.

The total contact parameter values of the rough contact surface are obtained from (15) and (16). The result is as follows:

$$
A_\ast = \frac{A_1 + A_{elp1} + A_{elp2} + A_p}{A_1}
$$

$$
F_\ast = \frac{P_1 + P_{elp1} + P_{elp2} + P_p}{E \cdot A_1}
$$

where $E$ is the modulus of elasticity and $A_1$ is the nominal contact area. Based on the multi-scale fractal contact model, an analysis of the wear process of the rough contact surface is showed below.

3. Wear Model

When the existing fractal wear model was established, only the contribution of the asperity wear amount in the plastic stage is considered. With the further improvement of the fractal contact theory, it is found that the scale characteristic will affect the change of the critical contact parameters, resulting in the change of different contact states’ proportion [12]. The change of the contact state of the asperity will further affect the wear process of the asperity, and the amount of wear will be different. Based on the above fractal contact theory, the wear of asperity in four kinds of contact states is considered in this paper: elastic, first-order elastoplastic, second-order elastoplastic and plastic. The variation of the wear amount of asperity in each contact state with the fractal parameters is analyzed. In addition, a quantitative characterization equation is given for the fractal wear coefficient of rough surface, and a new wear prediction model is established.

3.1. Wear Model of the Single Asperity

As shown in figure 4, the wear profile of a single asperity on the contact surface has a broad classification: ball head type and ball stage type. Due to the contact force, the tip of the asperity produces a certain contact deformation, as shown in figure 2, when the wear of the asperity of the contact surface is spherical table type wear. The research in this paper is limited to the wear after the contact deformation occurs, so the ball-type wear is only considered in the model and the influence of the ball-type wear is ignored.

![Figure 4. Wear morphology.](image-url)
The asperity spheroid type wear volume is approximated by the volume of the round table as follows:

\[
dV = \frac{1}{3} (a_{\text{down}} + a_{\text{up}} + \sqrt{a_{\text{down}} \cdot a_{\text{up}}^3}) \cdot dh
\]  

(18)

where \(a_{\text{down}}\) and \(a_{\text{up}}\) are the areas of the lower and upper sections respectively, and their sizes can be solved by equations (5)-(9) and \(\Theta\).

3.2. Wear Model of the Rough Surfaces

The wear of the contact surface is characterized in the microscopic morphology as the spalling of the asperities, which is macroscopically characterized by the change in the distance between the two contact planes, so that the wear process can be represented by the distance between the two contact faces. As shown in figure 5, the wear depth is divided into minute stages by \(\Delta h\), and the amount of wear can be regarded as the sum of the volume of the asperity peeling in each minute stage \(\Delta h\). The rough contact surface contains asperities of different sizes. When the contact surface produces a small amount of wear depth \(\Delta h\), different asperities will produce different amounts of wear.

By calculating equations (14) and (18), the wear volume of the asperity body under different contact conditions of the rough contact surface is determined by the following equations.

\[
V_i = \int n_i(a) \cdot dV
\]

(19)

It is considered the wear effects of asperities in different contact states in equation (19), and respectively integrates the wear amounts of the four-contact-state of the asperities, and obtains the wear prediction model of the asperities with different contact states on the rough contact surface.

For the rough contact surfaces, the total wear is as follows

\[
sum(V) = V_e + V_{ep1} + V_{ep2} + V_p
\]

(20)

In equation (20), \(V_e\), \(V_{ep1}\), \(V_{ep2}\), and \(V_p\) are the wear amount of the asperity in the elastic, first-order elastoplastic, second-order elastoplastic, and plastic contact states, respectively, which are solved by the corresponding integral domain by equation (19).

3.3. Fractal Wear Coefficient

Referring to the classical wear model [13], the wear model of the different contact states of the rough surfaces, which solved by equations (15) and (19) is as follows:

\[
\frac{dV_i}{ds} = K_i P_i
\]

(21)

where \(ds\) is the relative slip amplitude of wear and \(K\) is the local wear coefficient, which is the contact force under different contact states, where \(i\) takes \(e, ep1, ep2, p\).

The wear relative slip amplitude \(ds\) is approximated by the average slip amplitude, and the results is as follows:

\[
ds \approx \Delta x = \frac{\sum \int n_i(a) \cdot l \cdot da}{\sum \int n_i(a) \cdot da}
\]

(22)

where \(n_i(a)\) is the distribution function of the cross-sectional area of the asperity in different contact states, according to equation (14), where \(l = a^{1/2}\).

Equations (15), (19), (21), and (22) are solved in tandem to obtain the wear coefficient of the new wear model as follows:

\[
K_{\text{fractal}} = \frac{\sum V_i}{\Delta x \cdot \sum P_i}
\]

(23)
The adhesive wear coefficient based on the fractal theory is essentially calculating the ratio of the abrasive grain volume generated by the surface of contact asperities to the total asperity volume, or the probability of contact with the asperities to produce abrasive grains. Therefore, the fractal wear coefficient and the Archard wear coefficient are substantially comparable. Compared to the existing wear model, the new model considers the wear of the asperities in four contact states.

Figure 5. Simplified model of rough surface.

4. Verification of Simulation Results
In order to make the rough surface wear prediction result more convincing, the fractal contact model is verified firstly, and the rationality and credibility of the fractal contact model are verified to support the rough surface wear prediction model. In order to facilitate the comparative verification during quantitative analysis, the selection of each parameter value in the calculation should be consistent with the relevant parameters in Refs. [11, 19-21, 26, 27], and all use dimensionless parameters. The wear amount is \( V' = \frac{V}{\sqrt{A'}} \).

4.1. Verification of the Contact Model
The new contact model was compared with the classical GW model [26], MB model [1] and Bhushan experimental data [27]. Take the same parameter value: \( D = 1.49, Y/E = 0.05 \) and \( G = 10^{-16} \) m. The calculation results are shown in figure 6.

Figure 6. Comparison between the new and classic models.

As shown in the above figure: (1) When the contact force is small \( F' < 2.8 \), the new model and the classical MB model have similar contact area prediction trends. Under the same contact force, the contact area predicted by the new model is larger than the contact area predicted by the MB model, which is consistent with the Bhushan experimental data. This is reasonable because the MB model ignores the effects of asperity in the elastoplastic phase, while the new model considers the four contact states of the asperity at multiple scales. Therefore, the contact area of the new model is larger than that of the MB model. (2) When the contact load is large \( F' > 2.8 \), the deviation between the new model and the experimental data is getting smaller. When \( 5 < F' < 7 \), it almost coincides with the experimental data, and the relationship between contact area and contact are accurately predicted. (3) The contact area predicted by the new model is larger than that predicted by the GW model under the same contact force condition. This is because the GW model is based on statistic method, and its accuracy is subjective to
instrument and equipment so that it cannot reflects the contact characteristics of the contact surface completely. Besides it only considers the influence of a single scale, so it lacks precision.

4.2. Analysis of Influence Parameters of Wear Model

Based on the theoretical verification of the fractal contact model, the main influence parameters of the wear model are analyzed below.

4.2.1. Influence of Scale Levels \( n \).

The relationship between the critical contact areas \( a_p \), \( a_{cep1} \) and \( a_{cep2} \) of the asperities with the scale level \( n \) is shown in figure 7 by equations (5)-(8). In the semi-logarithmic coordinate system, a linear relationship appears, and as \( n \) increases, the critical contact area of the asperities decreases. The value of the critical contact parameters corresponding to the asperity in different scale \( n \) is different, which leads to a certain change in the proportion of asperity in different contact states [12]. It can be seen that the influence of scale parameters should be considered when establishing the contact model.

**Figure 7.** Relationship between scale level \( n \) and critical contact parameters.

4.2.2. Influence of the Fractal Dimension \( D \).

In order to find out the relationship between the fractal dimension \( D \) and the wear amount, the relationship between the wear amount of the elastic, first elastoplastic, second elastoplastic, plastic contact state’ asperities and the fractal dimension \( D \) is studied. Given the initial condition [20]: the fractal dimension \( D \) takes 1.1 to 1.9, the characteristic coefficient \( G=10^9 \) m, and the material constant \( C=0.0764 \) [11, 12, 20]. The simulation results are shown in figure 8.

**Figure 8.** Relationship between wear amount and fractal dimension \( D \).

It can be seen from figure 8 that the wear amount of the asperities in each contact state tends to decrease first and then increase with the increasing of the fractal dimension \( D \), and the optimal fractal dimension \( D \) could be found so that the wear amount of the asperities is minimized at this point. In addition, when the fractal dimension \( D \) is less than 1.64, the wear of the asperity body in the elastoplastic
stage accounts for the main contribution. When the fractal dimension D is greater than 1.64, on the contrary, the proportion of wear in the elastoplastic and plastic stages alternates. This is because the fractal dimension D reflects the fineness of the contour. When D is small, the number of asperities considered is less. The plastic phase asperities are difficult to distinguish from the elastoplastic phase asperities. The amount of convex wear is the main proportion. On the other hand, when D is large, the number of asperities considered increases, and the asperities of different contact states are distinct. So, the amount of wear of the asperities in the plastic phase is a major proportion, and should not be ignored.

According to the data in Table 1, the amount of wear of the asperity in the elastic and first elastoplastic stage has less contribution to the total wear, but the contribution of the wear of the asperity in the second elastoplastic and plastic stage is large. The amount of wear in the second elastoplastic stage and the amount of wear in the plastic stage have the same contribution to the total wear. The contribution of the asperity wear in the second elastoplastic stage should not be ignored. But for the elastic, first elastoplastic stage asperity, the amount of wear is negligible. In addition, the variation of the wear amount predicted by the new model is consistent with the prediction results in Refs. [19, 20].

Table 1. The ratio of the wear amount of each order asperity to the total wear amount.

| D     | R_e/% | R_ep1/% | R_ep2/% | R_p/% |
|-------|-------|---------|---------|-------|
| 1.1   | 2.25x10^{-4} | 1.50x10^{-4} | 51.79   | 48.20 |
| 1.3   | 1.07x10^{-5} | 7.53x10^{-3} | 67.61   | 32.38 |
| 1.5   | 8.52x10^{-7} | 2.29x10^{-2} | 71.72   | 28.24 |
| 1.7   | 3.26x10^{-7} | 2.50x10^{-2} | 28.05   | 71.92 |
| 1.9   | 5.58x10^{-8} | 1.47x10^{-3} | 0.9455  | 99.05 |

Where, \( R_i = V_i / V_{\text{total}} \).

4.2.3. Influence of the Characteristic Coefficient G. In order to study the influence of the characteristic coefficient G on the wear amount of each contact state asperity, the initial conditions are given: the fractal dimension D = 1.4, and the material constant C = 0.0764. The wear amount and total wear amount corresponding to the four contact state asperities under different characteristic coefficients G are as shown in figure 9.

![Figure 9. Relationship between wear amount and characteristic coefficient G.](image)

From figure 9, the characteristic coefficient G and the wear amount V are linear in the double logarithmic coordinate system. As the G increasing, the corresponding wear amount of the asperities in each contact state also increases. This is because the characteristic coefficient G reflects the roughness of the contact surface, and the larger the G, the coarser of the contact surface and the greater of the amount of wear, which is in line with objective facts. And its prediction results are the same as those predicted in the Refs. [19, 20]. In addition, when G < H, the contribution of the asperity wear amount in the elastoplastic phase is greater than the contribution of the asperity wear amount in the plastic phase; when G > H, the result is opposite. This alternation shows that the transformation process between the
elastoplastic asperities and the plastic asperities is related to the characteristic coefficient $G$. When studying the wear of the contact surface, the wear of the asperity in the plastic phase should not be considered only, and the contribution of the wear of the asperity in the elastoplastic phase should also be considered.

According to the data in table 2, Compared with the contribution of the second elastoplastic and plastic stage asperity wear, the elastic and first elastoplastic stage asperity wear has less contribution to the total wear amount and the order of magnitude vary widely. Obviously, the first two stages’ contribution can be ignored in engineering calculations. With the increase of the characteristic coefficient $G$, the proportion of the wear of the asperity in the elastoplastic phase decreases, and the proportion of the wear of the asperity in the plastic phase increases. There are the reasons: when the characteristic coefficient $G$ is small, the surface is relatively flat, and the number of asperities in the plastic contact phase is small, so the wear amount is relatively small. With the increase of the characteristic coefficient $G$, the roughness of the contact surface gradually increases, and the asperities in the plastic contact phase increase, so the wear amount of the asperities increases in the plastic contact state.

| $G$ (m) | $R_e$ (%) | $R_{ep1}$ (%) | $R_{ep2}$ (%) | $R_p$ (%) |
|---------|-----------|---------------|---------------|-----------|
| $10^{-16}$ | 5.05x10$^{-8}$ | 2.21x10$^{-2}$ | 99.47 | 0.51 |
| $10^{-12}$ | 1.39x10$^{-6}$ | 5.16x10$^{-2}$ | 91.42 | 8.51 |
| $10^{-9}$ | 4.33x10$^{-6}$ | 1.32x10$^{-2}$ | 56.71 | 43.26 |
| $10^{-4}$ | 9.09x10$^{-6}$ | 2.17x10$^{-3}$ | 9.12 | 90.87 |
| $10^0$ | 1.62x10$^{-5}$ | 3.28x10$^{-4}$ | 0.54 | 99.45 |

4.2.4. Influence of the Material Coefficient $C$. In order to analyze the influence of the material properties of the contact surface on the wear of the asperity body, the influence of the material constant ($C = E/H$) on the wear of the asperity in different contact states is studied. The fractal dimension $D = 1.4$, and the characteristic coefficient $G = 10^{-9}$ m, and the result is shown in figure 10.

![Figure 10. Relationship between wear amount and material coefficient C.](image)

It is concluded from figure 10, as the material constant $C$ increases in the semi-logarithmic coordinates, the amount of wear of the asperity bodies in each contact state tends to decrease. This is reasonable. The increase of the material constant $C$ means that the relative hardness of the material decreases, the material is softer and it is easier to deform when contact occurs, so the phenomenon of asperity wear and peeling is unlikely to occur, which is in line with the reality.

According to the data in table 3, under the specific material characteristics, the contribution of the asperity wear amount in the elastic and first elastoplastic stage is small, which is different from the
magnitude of the second elastoplastic and plastic stage asperity wear. It will be negligible in engineering calculations with a high probability. In addition, the second elastoplastic and plastic stage asperities wear in the same order of magnitude and the wear amount is similar. Therefore, the contribution of the amount of asperity wear in the second elastoplastic phase should not be ignored in the actual calculation.

| C   | $R_e$ (%) | $R_{ep1}$ (%) | $R_{ep2}$ (%) | $R_p$ (%) |
|-----|-----------|---------------|---------------|-----------|
| $10^4$ | 3.81x10$^4$ | 3.38x10$^2$  | 16.54         | 83.42     |
| $10^3$ | 3.30x10$^4$ | 3.25x10$^2$  | 27.71         | 72.26     |
| $10^2$ | 2.57x10$^4$ | 2.53x10$^2$  | 21.94         | 78.03     |
| $10^1$ | 2.58x10$^4$ | 2.52x10$^2$  | 21.89         | 78.08     |
| $10^0$ | 3.80x10$^4$ | 3.39x10$^2$  | 16.58         | 83.38     |

4.3. Verification of Fractal Wear Coefficient
Through the quantitative analysis of the main influence parameters of the wear model in Section 3.2, the contribution of the wear amount of the asperity body to the total wear amount in the second elastoplastic stage is as important as the contribution of the asperity wear amount in the shaping stage. The wear effect of the asperities in the elastoplastic contact state should be considered. Based on the above analysis, the wear coefficient of the contact surface is studied below. The relationship between the wear factor of new wear model and the wear factor in the existing wear model [21] is compared by quantitative calculation. The parameter values are the same: the fractal dimension $D = 1.24$ [1, 21], and the single-step wear depth $\Delta h = 10^6$ m [21]. The comparison was verified by three materials, and the material parameters are shown in table 4.

| Number | Material         | Poisson’s ratio $\nu$ | Elastic Modulus E (GPa) | Hardness coefficient H (GPa) |
|--------|------------------|------------------------|--------------------------|-----------------------------|
| 1      | ZrO$_2$/TiN      | 0.22                   | 187                      | 24.1                        |
| 2      | ZrO$_2$/CrC      | 0.27                   | 85                       | 15.2                        |
| 3      | AISI1095/AISI1020| 0.30                   | 115                      | 1.73                        |

The simulation results are shown in figure 11. In the figure, K1, K2 and K3 are the wear coefficients of the three kinds of contact materials based on the new wear model, which are shown in solid lines in the figure, and W1, W2 and W3 are based on three kinds of contact materials respectively. The wear factor of the existing wear model [21] is shown in broken line in figure 11.

As shown in figure 11, under a certain fractal dimension $D$, the fractal wear coefficient increases with the increase of the characteristic coefficient $G$, and has a certain nonlinear relationship. The predicted wear coefficient is the same in dimension and the same trend; the value of the wear coefficient $W$ in the existing wear model is smaller 0.6 to 1 times than the value of the wear coefficient $K$ in the new wear model.

Analysis: (1) Because only the contribution of the asperity spalling in the plastic contact state to the wear amount is considered, and the influence of the asperity wear in the elastoplastic phase is neglected in Ref. [21]. In the process of establishing new wear model, the wear amount of the asperity in the elastoplastic contact state is considered, so the calculation result is larger than the prediction result of the existing wear model which only considers the contribution of the wear amount of the asperity in the plastic contact state. (2) Under the same conditions, the proportion of the wear of the asperity in elastoplastic phase and the asperity in the plastic phase is same, and the wear coefficient $W$ is smaller than the $K$. (3) From the results of large number of non-lubricated wear tests conducted by metals and non-metals [28], $10^{-3}$ to $10^{-5}$ is an order of magnitude of the wear coefficient of most materials. In summary, the rationality of the prediction results of the wear coefficient of the new wear model is verified.
5. Conclusion

(1) The scale level \( n \) has a certain influence on the critical contact parameters, and has a linear relationship in the semi-logarithmic coordinates. Therefore, the influence of the scale effect must be considered when studying the asperity contact state. The fractal contact model considering multi-scale effect has improved prediction accuracy. Compared with the classical fractal contact model, the prediction results are closer to the Bhushan experimental data under large load conditions and small load conditions.

(2) Under the influence of material parameters, the wear of asperities in the elastoplastic phase accounts for about 20%. Under the influence of fractal parameters, the proportion of the wear of the asperity in the elastoplastic phase and the plastic phase alternates; therefore, the contribution of the wear of the asperity in the elastoplastic contact state to the total wear cannot be ignored. Due to the large difference in magnitude, the amount of wear of the elastic and first elastoplastic stage asperities is negligible in engineering calculations.

(3) Based on the multi-scale fractal contact theory, a new prediction method of wear coefficient is established. The prediction result is similar to the existing wear coefficient prediction result. Due to the influence of multi-scale and the influence of asperity wear under elastoplastic contact state, the wear coefficient prediction result of the new wear model is 0.4-1 times larger than the wear coefficient prediction result of the existing wear model.

(4) This paper adopts the two-dimensional contact model, and some simplifications are made in the contact modeling, which has a certain gap with the engineering reality. In addition, the prediction accuracy has certain disadvantages compared with the prediction accuracy of the three-dimensional contact model. However, the methods and ideas used in the establishment of the contact model are relatively novel, and the influence of the asperity in the elastoplastic stage is studied in the wear model with certain innovation. It is hoped that the theoretical ideas and prediction results of the research process can provide some guidance for subsequent engineering applications.

References

[1] Majumdar A and Bhushan B 1991 Fractal Model of Elastic-Plastic Contact Between Rough Surfaces Journal of Tribology 113 (1) 1-11
[2] Kogut L and Etsion I 2002 Elastic-Plastic Contact Analysis of a Sphere and a Rigid Flat Journal of Applied Mechanics 69 (5) 657-662
[3] Morag Y and Etsion I 2007 Resolving the contradiction of asperities plastic to elastic mode transition in current contact models of fractal rough surfaces Wear 262 (5-6) 624-629
[4] Jackson R L and Streator J L 2006 A multi-scale model for contact between rough surfaces Wear 261 (11-12) 1337-1347
[5] Wriggers P and Nettingsmeier J 2007 Homogenization and Multi-Scale Approaches for Contact Problems Computational Contact Mechanics
[6] Sun W, Huang X, Sun Z Y and Huang W Q 2017 A Multiscale Calculation Method of Normal Contact Stiffness of Actual Interfaces Machinery Design and Manufacture (10) 1-4
[7] Shi J, Liu J, Ding X, Yang Z M and Gong H 2017 On the Multi-scale Contact Behavior of Metal Rough Surface Based on Deterministic Model Chinese Journal of Mechanical Engineering 53 (03) 111-120
[8] Miao X and Huang X 2014 A complete contact model of a fractal rough surface Wear 309 (1-2) 146-151
[9] Ding X X, Yan R Q and Jia Y 2014 Construction and Analysis of Fractal Contact Mechanics Model for Rough Surface based on base length Tribology 34 (4) 341-347
[10] Cheng Y, Yuan Y, Li G and Xu Y Q and Li W Z 2016 The Elastic-plastic Contact Mechanics Model Related Scale of Rough Surface Journal of Northwestern Polytechnical University 34 (3) 485-492
[11] Yuan Y, Cheng Y, Liu K and Li G 2017 A revised Majumdar and Bushan model of elastoplastic contact between rough surfaces Applied Surface Science 425
[12] Yun R D and Ding B 2019 New Fractal Contact Model Considered Multi-Scale Levels J. Chinese Journal of Mechanical Engineering 55 (09) 80-89
[13] Archard J F 1953 Contact and Rubbing of Flat Surfaces Journal of Applied Physics 24 (8) 981-988
[14] Zhou G Y, Leu M C and Blackmore D 1993 Fractal geometry model for wear prediction Wear 170 (1) 1-14
[15] Zhou G, Leu M and Blackmore D 1995 Fractal geometry modeling with applications in surface characterization and wear prediction International Journal of Machine Tools & Manufacture 35 (2) 203-209
[16] Ge S R and Zhu H 2005 Tribological fractal M. China Machine Press
[17] Fei B, Jiang Z D 1999 Research of Fractal Theory of Adhesive Wear of Engineering Rough Surface Tribology (01) 79-83
[18] Jiang S W, Jiang B, Li Y, Deng H, Zheng C Q and Yi G F 2003 Calculation of Fractal Dimension of Worn Surface Tribology (06) 533-536
[19] Wang X, Zhang S W and Fan Q Y 2002 Abrasive Wear Model Based on Fractal Theory Lubrication Engineering (3) 2-4
[20] Ding X, Zhang Z T, Ren Q C, Bai C H and Wang P X 2016 Abrasive Wear Prediction Model Based on Fractal Theory Journal of Gansu Sciences 28 (05) 84-88
[21] Su Y W, Cheng W, Zhu A B, Fan T F and Guo C X 2014 Contact and Wear Simulation Between Fractal Surfaces Journal of Xi’an Jiaotong University 47 (7) 52-56
[22] Zhang K F, Yuan H Q and Nie P 2014 Tool Wear Condition Monitoring Based on Generalized Fractal Dimensions Journal of Vibration and Shock 33 (01) 162-164+169
[23] Wang Z Q, Hu Gong, Fang F Z and Liu B 2016 Recognition of Wear Condition of Micro Milling Tool Based on Length Fractal Dimension Journal of Vibration, Measurement & Diagnosis 36 (03) 592-597+611
[24] Kompoupolous K and Yan W 1997 A Fractal Analysis of Stiction in Microelectromechanical System Journal of Tribology 119 (3) 391-400
[25] Mandelbrot B B and Wheeler J A 1983 The Fractal Geometry of Nature American Journal of Physics 51 (4) 468
[26] Greenwood J A and Williamson J B P 1966 Contact of Nominally Flat Surfaces Proceedings of the Royal Society of London 295 (1442) 300-319
[27] Bhushan B 1985 The Real Area of Contact in Polymeric Magnetic Media-part II: Experimental Data and Analysis ASLE Transactions 28 (2) 181-197
[28] Rabinowicz E and Tanner R I 1966 Friction and Wear of Materials Journal of Applied Mechanics 33 (2) 479