Next-to-leading prediction of $\epsilon'/\epsilon$: an upgraded analysis

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Abstract

We present an updated theoretical prediction of $\epsilon'/\epsilon$, using the next-to-leading $\Delta S = 1$ effective hamiltonian and lattice QCD matrix elements. The CP violating phase is constrained by using both the experimental values of $\epsilon$ and $x_d$, assuming the theoretical determination of $f_B$. Predictions of $\cos \delta$ and $\sin 2\beta$ are also obtained in this way. For $\epsilon'/\epsilon$, our estimate is $\epsilon'/\epsilon = (2.8 \pm 2.4) \times 10^{-4}$.

1 The CP violating phase $\delta$ of the CKM matrix is constrained by comparing the theoretical prediction for $\epsilon$ with its experimental value. To this purpose, the relevant formula is

$$|\epsilon|_{\epsilon=0} = C_{\epsilon} B_K A^2 \lambda^6 \sigma \sin \delta \{ F(x_c, x_t) + F(x_t) \} \left[ A^2 \lambda^4(1 - \sigma \cos \delta) - F(x_c) \right],$$

where $x_q = m^2_q / M^2_W$ and the functions $F(x_i)$ and $F(x_i, x_j)$ are the so-called Inami-Lim functions [3], obtained from the calculation of the basic box-diagram and including QCD corrections. $F(x_i)$ is known at the next-to-leading order, which has been included in our calculation [4]. In eq. (1),

$$C_{\epsilon} = \frac{G_F^2 f_K^2 m_K \sqrt{2}}{6\sqrt{2}\pi^2 \Delta M_K},$$

where $\Delta M_K$ is the mass difference between the two neutral kaon mass eigenstates. Moreover, $\rho = \sigma \cos \delta$ and $\eta = \sigma \sin \delta$, where $\lambda$, $A$, $\rho$ and $\eta$ are the parameters of the CKM matrix in the Wolfenstein parametrization [5]. Finally, $B_K$ is the renormalization group invariant $B$-factor [6].

2 The theoretical estimate of the $B$-meson coupling constant $f_B$ is used to further constrain $\delta$, by comparing the theoretical prediction of the $B$ mixing parameter $x_d$ with its experimental value. From the $\Delta B = 2$ effective hamiltonian, one can derive

$$x_d = \frac{\Delta M_1}{\Gamma} = C_B \frac{f_B^2}{M_B} B A^2 \lambda^6 \left( 1 + \frac{\sigma^2 - 2\sigma \cos \delta}{M_B^2} \right) F(x_t),$$

where $B_B$ is the $B$-parameter relevant for $B - \bar{B}$ matrix element

$$\langle B_d | (\bar{d} \gamma^5 b) | B_d \rangle = 8/3 f_B^2 M_B^2 B_B.$$

Notice that $f_B B_B^{1/2}$ must be known, for the experimental value of $x_d$ to give a constraint on $\delta$.

3 From the $\Delta S = 1$ effective hamiltonian, one can calculate the expression of $\epsilon'$ in terms of CKM matrix elements, Wilson coefficients and local operator matrix elements. One has

$$\epsilon' = \frac{e^{i\pi/4}}{\sqrt{2} \sqrt{\omega} \omega} \left( \omega^{-1}(\text{Im} A_2) - (1 - \Omega_{IB}) \text{Im} A_0 \right),$$

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where \( \text{Im}A_0 \) are given by

\[
\text{Im}A_0 = -G_F Im\left( V_{ts}V_{td} \right) \left\{ \left( C_6 B_0 + \frac{1}{3} C_5 B_5 \right) Z + \left( C_4 B_4 + \frac{1}{3} C_3 B_3 \right) X + C_7 B_7^{1/2} \left( \frac{2 Y}{3} + \frac{Z}{6} + \frac{X}{2} \right) + \right. \\
\left. C_6 B_6^{1/2} \left( \frac{2 Y}{3} + \frac{Z}{6} + \frac{X}{2} \right) - C_9 B_9^{1/2} \frac{X}{3} + \left( \frac{C_1 B_1^c}{3} + C_2 B_2^c \right) X \right\},
\]

and

\[
(\text{Im}A_2)' = -G_F Im\left( V_{ts}^* V_{td} \right) \left\{ C_7 B_7^{3/2} \left( \frac{Y}{3} - \frac{X}{2} \right) + C_6 B_6^{3/2} \left( Y - \frac{X}{6} \right) + C_9 B_9^{3/2} \frac{2 X}{3} \right\}.
\]

Here \( \omega = \text{Re}A_2/\text{Re}A_0 \) and we have introduced \((\text{Im}A_2)'\) defined as

\[
\text{Im}A_2 = (\text{Im}A_2)' + \Omega_B (\omega \text{Im}A_0).
\]

\( \Omega_B \) accounts for the isospin breaking contribution, see for example ref. [3]. The Wilson coefficients \( C_i \) have been evaluated at the next-to-leading order for \( \mu = 2 \text{ GeV} \), using the anomalous dimension matrices given in refs. [6, 7] and the initial conditions computed in refs. [10, 11] (given for HV in ref. [12]). Concerning the local operator matrix elements, their values are given by a set \( \{ B_i \} \) of \( B \)-parameters multiplied by the vacuum insertion approximation results. In turn, these can be written in terms of the three quantities (see eq. (4) and eq. (6))

\[
X = f_\pi (M_K^2 - M_\pi^2), \\
Y = f_\pi \left( \frac{M_K^2}{m_s(\mu) + m_d(\mu)} \right)^2, \\
\sim 12 X \left( \frac{0.15 \text{ GeV}^2}{m_s(\mu)} \right)^2, \\
Z = 4 \left( \frac{f_K}{f_\pi} - 1 \right) Y.
\]

The numerical values of the \( B \)-parameters have been taken from lattice calculations [13]. For those \( B \)-factors which have not yet been computed on the lattice, we have used educated guesses, see ref. [13].

More details on this analysis can be found in ref. [13, 14]. Compared to our previous work [2], the main improvements are the following:

1. The distribution for \( \cos \delta \), obtained by comparing the experimental value of \( \epsilon \) to its theoretical

| Parameters | Value |
|------------|-------|
| \( f_B B_{d, s}^{1/2} \) | \((200 \pm 40) \text{ MeV} \) |
| \( m_s(2 \text{GeV}) \) | \((128 \pm 18) \text{ MeV} \) |
| \( N_{QCD} = 4 \) | \((330 \pm 100) \text{ MeV} \) |
| \( \tau_B = 1.49 \pm 0.12 \times 10^{-12} \) | \( \Omega_B = 0.25 \pm 0.10 \) |
| \( V_{us}/V_{cb} = \lambda \sigma = 0.880 \pm 0.015 \) | \( m_t = (174 \pm 17) \text{ GeV} \) |

| Table 1. Values of the fluctuating parameters used in the numerical analysis. |

2. Updated values of the experimental parameters entering in the phenomenological analysis, such as the B meson lifetime \( \tau_B \), the \( B_0^0 - B_0^0 \) mixing parameter \( x_d \), the CKM matrix elements, \( |V_{cb}|, |V_{ub}|/|V_{cb}| \), etc., have been used.

3. The value of the strange quark mass \( m_s \) has been taken from lattice calculations [14], thus making a more consistent use of lattice results for the \( B \)-parameters of the relevant penguin operators.

4. All the results are presented with an estimate of the corresponding errors. These errors come from the limited precision of measured quantities, e.g. \( \tau_B \), and from theoretical uncertainties, e.g. the values of hadronic matrix elements.

The results of our analysis have been obtained by varying the experimental quantities, e.g. the value of the top mass \( m_t \), \( \tau_B \), etc. and the theoretical parameters, e.g. the \( B \)-parameters, the strange quark mass \( m_s(\mu) \), etc., according to their errors. Values and errors of the input quantities used in the following are reported in tables [14]. We assume a gaussian distribution for the experimental quantities and a flat distribution (with a width of 2\( \sigma \)) for the theoretical ones. The only exception is \( m_s(\mu) \), taken from quenched lattice QCD calculations, for which we have assumed a gaussian distribution, according to the results of ref. [14].

The theoretical predictions \( \cos \delta, \sin 2 \beta, \epsilon'/\epsilon \), etc. depend on several fluctuating parameters. We have obtained numerically their distributions, from which we have calculated the central values and the errors reported below.

Using the values given in the tables and the formulae given previously, we have obtained the following results:

1. The distribution for \( \cos \delta \), obtained by comparing the experimental value of \( \epsilon \) to its theoretical
Figure 1. Distributions of values for $\cos \delta$, $\sin 2\beta$ and $\epsilon'/\epsilon$, using the values of the parameters given in the tables. The solid histograms are obtained without using the $x_d$ constraint. The dashed ones use this constraint, assuming that $f_B B^{1/2}_B = 200 \pm 40$ MeV. The contour-plot of the event distribution in the $\rho - \eta$ plane is also shown.

| Constants                  | $f_p = 1.16634 \times 10^{-6}$ GeV$^{-2}$ | $f_\pi = 132$ MeV | $f_K = 160$ MeV |
|---------------------------|------------------------------------------|--------------------|-----------------|
| $m_c = 1.5$ GeV           | $\lambda = \sin \theta_c = 0.221$      | $\epsilon_{exp} = 2.268 \times 10^{-3}$ |
| $m_b = 4.5$ GeV           | $M_W = 80.6$ GeV                        | $R_\Lambda = 2.7 \times 10^{-7}$ GeV |
| $M_H = 140$ MeV           | $\omega = 0.045$                        | $\mu = 2$ GeV     |
| $M_K = 490$ MeV           |                                          |                   |
| $M_B = 5.278$ GeV         |                                          |                   |
| $\Delta M_K = 3.521 \times 10^{-12}$ MeV |

Table 2. Values of the constants used in the numerical analysis.

From the dashed distribution, we obtain

$$\sin 2\beta = 0.65 \pm 0.12.$$  \hspace{1cm} (11)

Figure 2 also contains the contour-plot of the event distribution in the $\rho - \eta$ plane, showing the effect of the $\epsilon$ and $x_d$ constraints, when the $f_B$-cut is imposed.

3 In figure 3, several informations on $\epsilon'/\epsilon$ are provided. Contour-plots of the distribution of the generated events in the $\epsilon'/\epsilon$-cos$\delta$ plane are shown, without and with the $f_B$-cut. One notices a very mild dependence of $\epsilon'/\epsilon$ on cos$\delta$. As a consequence one obtains approximatively the same prediction in the two cases (see also figure 1):

$$\epsilon'/\epsilon = (2.3 \pm 2.1) \times 10^{-4} \text{ no - cut,}$$  \hspace{1cm} (12)

and

$$\epsilon'/\epsilon = (2.8 \pm 2.4) \times 10^{-4} \text{ } f_B \text{- cut.}$$  \hspace{1cm} (13)

In figure 3, we also give $\epsilon'/\epsilon$ as a function of $m_t$. The band corresponds to the 2$\sigma$ prediction.

In spite of several differences, the bulk of our results overlap with those of ref. [19]. It is reassuring that theoretical predictions, obtained by using different approaches to evaluate the operator matrix elements, are in good agreement.
Figure 2. Above, contour-plots of the event distributions in the plane $\epsilon'/\epsilon - \cos \delta$ without and with the $f_B$-cut.

Below, $\epsilon'/\epsilon$ as a function of $m_\ell$.

| $B$-parameters | $B_{3/2} = 0.75 \pm 0.15$ | $B_{3/2}^{3/2} = 0.62 \pm 0.10$ |
|----------------|--------------------------|-------------------------------|
| $B_{1-2} = 0 - 0.15^\ast$ | $B_{3,4} = 1 - 6^\ast$ | $B_{2-8}^{(1/2)} = 1^\ast$ |
| $B_{0,6} = B_{2-8}^{(3/2)} = 1.0 \pm 0.2$ | | |

Table 3. Values of the $B$-parameters, for operators renormalized at the scale $\mu = 2$ GeV. The only exception is $B_K$, which is the RG invariant $B$-parameter. $B_{3/2}^{3/2}$ has been taken equal to $B_K$, at any scale. The value reported in the table is $B_{3/2}^{3/2}(\mu = 2\text{GeV})$. Entries with a $^\ast$ are educated guesses, the others are taken from lattice QCD calculations.

On the basis of the latest analyses, it seems very difficult for $\epsilon'/\epsilon$ to be larger than $10 \times 10^{-4}$. This may happen by taking the matrix elements of the dominant operators, $Q_6$ and $Q_8$, much different than usually assumed. One possibility, discussed in ref. [19], is to take $B_6 \sim 2$ and $B_8 \sim 1$, instead of the usual values $B_6 \sim B_8 \sim 1$. To our knowledge, no coherent theoretical approach can accomodate so large value of $B_6$.

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References

[1] M. Lusignoli et al., Nucl. Phys. B369 (1992) 139.
[2] M. Ciuchini et al., Phys. Lett. B301 (1993) 263.
[3] T. Inami, C.S. Lim, Prog. Th. Phys. 65 (1981) 297; Erratum 65 (1981) 1772.
[4] A.J. Buras, M. Jamin and P.H. Weisz, Nucl. Phys. B347 (1990) 491.
[5] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.
[6] A.J. Buras, W. Slominski and H. Steger, Nucl. Phys. B238 (1984) 529; ibid. B245 (1984) 369.
[7] A.J. Buras and J.-M. Gerard, Phys. Lett. B192 (1987) 156.
[8] A.J. Buras et al., Nucl. Phys. B400 (1993) 37; ibid. B400 (1993) 75.
[9] M. Ciuchini et al., Nucl. Phys. B415 (1994) 403.
[10] J.M. Flynn, L. Randall, Phys. Lett. B224 (1989) 221; Erratum B235 (1990) 412.
[11] G. Buchalla, A.J. Buras, M.K. Harlander, Nucl. Phys. B337 (1990) 313.
[12] A.J. Buras et al., Nucl. Phys. B370 (1992) 69; Addendum, ibid. B375 (1992) 501.
[13] M. Ciuchini et al., ROME prep. 94/1024.
[14] M. Ciuchini at al., to appear in the proceedings of the “1st Rencontres du Vietnam”, Hanoi (December 1993).
[15] see, e.g., ref. [14] and references therein.
[16] C.R. Alton et al., ROME prep. 94/1018, CERN-TH.7256/94 (June 1994).
[17] M. Schmidtler, K.R. Schubert, Zeit. Phys. C53 (1992) 25.
[18] A. Ali and D. London, CERN-TH.7248/94; CERN-TH.7398/94.
[19] A. Buras, M. Jamin, M.E. Lautenbacher, Nucl. Phys. B408 (1993) 209.
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