Sloshing induced damping in vertically vibrating systems

J J De Courcy, L Constantin, B Titurus, T C S Rendall and J E Cooper
Department of Aerospace Engineering, University of Bristol,
Queens Building, University Walk, Bristol, BS8 1TR, England, UK
E-mail: j.decourcy@bristol.ac.uk

Abstract.
All aircraft are subject to a range of loading throughout ground and flight operations, which ultimately define the sizing and weight of the aircraft structure. Active and passive loads alleviation technologies provide an approach to reduce dynamic loads arising from atmospheric gusts and turbulence, leading to more fuel-efficient aircraft designs. Within the H2020 SLOWD project, fuel sloshing is being considered as a method for alleviating loads in aircraft wings via an increase in effective damping. Recent work has considered the transient response of a vertically vibrating, single degree of freedom system coupled to a rectangular liquid-filled tank. This research revealed identifiable dissipation regions in the free vibration responses characterised by their own distinct equivalent damping ratio values. In this work, free surface displacement has been extracted from high-speed camera footage during the chosen sloshing regimes, which are representative of a decaying parametrically excited fluid. These results are compared against a fluid-structure coupled numerical model based upon smoothed particle hydrodynamics, previously shown to have good agreement with the experimental damping response. Further analysis of the free-surface response of the numerical solution notes a presence of an undesired travelling longitudinal wave. The analysis of this discrepancy between the model and experiment is then used to improve the numerical formulation, showing a requirement for modelling surface tension.

1. Introduction
All aircraft are subject to a range of loading throughout ground and flight operations, which ultimately define the sizing and weight of the aircraft structure [1]. Active and passive load alleviation technologies provide an approach to reduce dynamic loads arising from atmospheric gusts and turbulence, leading to a more fuel-efficient aircraft design. Conventional active techniques include deployment of control and spoiler surfaces [2], whereas passive approaches such as aeroelastic tailoring and folding wingtips [3] are less widely implemented. Within the H2020 SLOWD project, fuel sloshing is being considered as a method for alleviating loads in aircraft wings via an increase in effective damping. The project aims to define a holistic approach, encompassing both experimental and numerical approaches to quantifying energy dissipation mechanisms from sloshing fuel.

Liquid sloshing has seen many cases of use within the civil engineering field, for instance in the form of tuned dampers to reduce the lateral response of tall buildings to wind or earthquakes [4]. Within civil aircraft, tuned mass dampers are used to reduce noise and vibration of engines...
or aeroelastic response, but these tend to consist of mechanical systems rather than sloshing fluid. Vertically excited systems containing liquids have also been studied previously [5], with applications such as liquid propellant tanks. Relatively little work has considered the use of fuel sloshing for loads alleviation in aircraft structures, with preliminary studies focusing on numerical investigation of fuel sloshing on transonic aerofoil pitch-heave motion [6]. There is therefore a need to develop validated numerical tools to couple the wing and fuel sloshing models, to enable exploitation of the potential added damping effects arising from sloshing in novel wing tank designs.

This paper builds upon previous work [7, 8] investigating the transient sloshing response of a purely vertically excited tank. Experimental studies were performed to provide deep understanding of the sloshing problem, with identification of the fluids free surface behaviour from video footage. Additionally, fluid-structure coupled numerical tools were developed, in particular the use of smoothed particle hydrodynamics (SPH) and equivalent mechanical models (EMM) provided a good representation of the fluid induced damping within the initial violent sloshing response. Within this work, emphasis is placed on the regions that follow this violent stage, which are dominated by the symmetric sloshing motion of the fluid under parametric excitation. Comparison of the SPH simulated free surface motion is made against the experimentally identified response, and the inclusion of surface-tension is shown to significantly improve the accuracy of the modelled free surface behaviour.

2. Experimental setup

Previous work considered the damping effect of liquid sloshing in a single degree of freedom structure undergoing transient vertical vibration [7]. Vertical motion was isolated to represent the dominant wing tank motion from atmospheric excitation. Additionally, the structural dry response required linear stiffness characteristics and minimal damping to isolate the fluid-induced behaviour.

To achieve these aims, the ‘T-Beam’ structure was developed as shown in figure 1a. This configuration is composed of two steel strips joined in a T shape with a liquid filled tank placed at the midpoint of the horizontal beam. Inclusion of the joint between the two strips allows the connection point to translate in-plane and rotate, enabling vertical deflections up to 14 times the beam thickness while maintaining constant transversal stiffness. Within this work, all results shown consider the case of step release from the maximum displacement of 14 mm, with either a dry tank or filled to 50% filling level which is known to maximise fluid-induced damping [7]. The rectangular tank included has dimensions 60x20x30 mm in length, height and width, respectively. At 50% fill the liquid has a mass of 18g, which is thus added to the dry case to maintain a constant mass and frequency, to ensure dynamic comparability.

The idealised SDOF system is shown within figure 1b. A frequency of 10.05 Hz and stiffness of 1100 N/m was recorded experimentally, leading to a SDOF equivalent mass of 276 g. Figure 2 shows the acceleration response of the dry structure, as the grey signal envelope. A lightly damped response is observed, with damping ratios between 0.23% and 0.34% of critical damping observed.

In contrast to this, the wet response at 50% fill is also shown within figure 2 in blue. Observing the acceleration envelope on a logarithmic scale shows the emergence of three distinct linear damping regions, which will herein be referred to as R1, R2 and R3. Each region is characterised by a distinct flow regime within the fluid, and the relevant damping ratio reflecting this.

- **R1** - Turbulent flow with complex free surface behaviour. Maximum damping ratio, with energy dissipated as a result of strong fluid impacting with the top and bottom tank surfaces. Studied in depth experimentally and numerically in previous work [7].
- **R2** - Symmetric lateral sloshing motion, with significant damping but less than R1.
R3 - Small free surface motion and return to the structural damping.

Within this work free surface motion is analysed during the R2 and R3 damping regions. A procedure for identifying free surface motion from experimental analysis is presented and used in understanding fluid behaviour, and in addition provides a comparison metric for analysing the numerical response using SPH. For this a multi-step process designed to identify the sloshing surface from video footage was developed, aiming to filter out the three-dimensional effects and robustly extract the 2D surface displacement. A brief summary will be given here, however, for a thorough description and analysis see [8].

High speed footage was recorded at 480 frames per second, initially the moving tank frame of reference must be identified from the fixed camera viewpoint. This is done by searching each frame for the point of maximum cross-correlation with a common feature present at all time instances. Following the accurate determination of the tank location, a series of image filters are applied to each frame in order to isolate the free surface:

(i) The temporal frame is binarised to identify contrasting regions.
(ii) Boundaries of all strongly contrasting regions (nominally the surface) are identified using a
Moore-Neighbour tracing algorithm.

(iii) Geometric filtering is applied to remove tank boundary points and any small point clusters.
(iv) A fourth degree polynomial is fitted to the remaining boundary points, and outliers removed from a defined tolerance from the fit. This provides a more distinct set of surface points.
(v) The mean value at each longitudinal station is computed and stored to mitigate 3D effects.
(vi) A Savitzky-Golay polynomial filter is used to smooth the data. Any x stations which do not possess data inherit points from the previous time instance to preserve temporal and spatial continuity.
(vii) A further polynomial is fitted, with tighter tolerances for removing outliers.
(viii) A final Savitzky-Golay filter is applied, resulting in an accurate set of points defining the free surface.

3. SPH numerical formulation

Smoothed particle hydrodynamics is the chosen numerical technique for modelling the sloshing response within this work. As a Lagrangian technique, the formulation is based on discretising the continuum equations defining fluid flow onto a set of particles, rather than onto a mesh as in the alternative finite-volume techniques. In doing so, a localised interpolation is constructed to evaluate an arbitrary field \( f \) and its derivatives at the position of a particle \( x_i \). These interpolations take the form of convolution sums

\[
 f(x_i) \approx \sum_j f(x_j) W(x_i - x_j, h)V_j \quad (1)
 \]

\[
 \nabla f(x_i) \approx -\sum_j f(x_j) \nabla W(x_i - x_j, h)V_j \quad (2)
 \]

Here \( W(x_i - x_j, h) \) is a compact kernel function approximating the Dirac-delta function, which smooths the field over a length scale \( h \) termed the ‘smoothing length’. This interpolation acts to reconstruct the continuous field from a discrete neighbourhood of points, where each particle is designated a mass \( m_j \), density \( \rho_j \) and hence volume \( V_j \). For further details of the process and equivalent operators for vector fields see [9] and the complete SPH formulation used within this work becomes

\[
 \frac{Du_i}{Dt} = -\sum_j m_j \left( \frac{P_j}{\rho_j^2} + \frac{P_i}{\rho_i^2} + \Pi_{ij} \right) \nabla W_{ij} + \sum_j m_j \rho_j \frac{\rho_i}{\rho_j} \frac{x_{ij} \cdot \nabla W_{ij}}{|x_{ij}|^2 + 0.001h^2} u_{ij} + a_{ST,i} + g \quad (3)
 \]

\[
 \frac{D\rho_i}{Dt} = \sum_j m_j u_{ij} \cdot \nabla W_{ij} + \delta h c_0 D_i \quad (4)
 \]

\[
 P = \frac{\rho_0 c_0^2}{\gamma} \left[ \left( \frac{\rho}{\rho_0} \right)^\gamma - 1 \right] \quad (5)
 \]

\[
 W_{ij} = \alpha_d \left( 1 - \frac{|x_{ij}|}{2h} \right)^4 + \left( \frac{2|x_{ij}|}{h} + 1 \right) \quad (6)
 \]

Note, \( x_{ij} = x_i - x_j \) and the equivalent for velocity \( u_{ij} \), additionally \( W(x_i - x_j, h) \) is herein referred to as \( W_{ij} \). Equation 3 is the momentum equation, including an artificial [10] and laminar viscosity [11], but also a surface tension term discussed later. Equation 4 defines continuity, where a diffusive term from the \( \delta \)-SPH scheme is introduced to remove high-frequency noise in the density and pressure fields [12]. A form of Tait’s equation of state is used to evaluate
particle pressure from density, this is generally known as the weakly compressible SPH (WCSPH) method. Wendland’s quintic kernel (equation 6) is used for the smoothing function, and temporal integration is performed using the Newmark-beta method. Finally, boundaries are enforced using dynamic boundary conditions [13].

With the sloshing response strongly coupled to the tank motion, the fluid-structure system must be solved entirely. Here, a partitioned approach is used, where the aforementioned SPH formulation solves the fluid and an independent SDOF oscillator solves for the structure. Solvers are coupled through the exchange of hydrodynamic force and structural kinematics using a ‘strong’ coupling methodology which iterates and synchronises the structural and fluid domains at every numerical time instance, ensuring energy is conserved within the system.

With the experimental sloshing analysis occurring at relatively small length scales, surface tension behaviour may not be negligible. Surface tension is thus included within the formulation, aiming to determine its importance in reproducing correct free-surface behaviour. For this a continuum surface force (CSF) model is used, which applies forces to surface particles acting to minimise surface area and energy. The formulation used here aims for robust computation of the surface normal vectors (to thus determine curvature), during violent sloshing and impact with solid boundaries. Normal vectors are computed via

\[ \mathbf{n}_i = -L_i \sum_j \nabla W_{ij} V_j \]  
\[ \tilde{\mathbf{n}}_i = \frac{\sum_j n_i W_{ij} V_j}{\sum_j W_{ij} V_j} \]

where the summation in equation 7 provides an estimate of the surface normal, and is corrected through a renormalisation matrix \(L_i\), accounting for any particle disorder which may arise during fluid impacting in WCSPH. Equation 8 performs a smoothing on the normals in the form of a Shepard interpolant, acting to attenuate any spurious vectors which may arise.

Finally, particles in the vicinity of solid boundaries have their normal prescribed according to the desired wetting behaviour. This step follows the procedure of [14] having the form \(\mathbf{n}^{\text{mod}}_i = f(\tilde{\mathbf{n}}_i, \theta, y^+)\), such that normals are prescribed by a contact angle \(\theta\) and the prescription smoothed into the fluid \((y^+)\) to avoid any sharp discontinuities. From here, curvature of the surface is calculated as,

\[ \kappa_i = \kappa_i / \mathcal{L}_i, \quad \kappa_i = \sum_j V_j (\tilde{\mathbf{n}}^{\text{mod}}_j - \tilde{\mathbf{n}}^{\text{mod}}_i) \cdot \nabla W_{ij}, \quad \mathcal{L}_i = \sum_j V_j W_{ij}. \]

Note, these calculations are only performed on particles in the vicinity of the free surface, hence the factor \(\mathcal{L}_i\) to correct for the truncation of the particle neighbourhood. To detect the surface, the minimum eigenvalue of the renormalisation matrix \(L_i\) is determined. This ranges from 0 for particles completely outside the fluid to 1 for particles completely submerged; particles with eigenvalues below 0.75 are deemed to lie on the surface. Finally, the resulting surface tension based acceleration acting on surface particles is calculated as

\[ a_{ST,i} = \frac{\sigma}{\rho_i} \kappa_i \tilde{\mathbf{n}}_i, \]

where \(\sigma\) is a coefficient defining the strength of the surface tension.

4. Results
4.1. Experimental results
Tank motion was tracked as part of the visual identification procedure in figure 3a, previous analysis [8] has shown the identified tank displacement signal is in good agreement with accelerometer data. Information can be drawn from the identified surface displacements taken
in the moving frame of reference. Surface identification begins approximately at 3 seconds, as this time marks the end of fluid impacting the top tank surface such that an entire continuous surface can be obtained. Initially, the midpoint of this surface is used as a point of reference as it oscillates with the largest amplitude over the identified tank displacements, as presented in figure 3a. The amplitude of the sloshing response is significantly larger than the tank within R2, and this amplitude drops off into R3. This reduction in fluid motion correlates well with the reduction in induced damping across regions within figure 2, \( \zeta_{R2} = 0.87\% \rightarrow \zeta_{R3} = 0.21 \).

Further analysis of the surface motion shows the dominant frequency is 5.18 Hz, which is very close to the half-frequency of the parametric excitation in this mode [15]. Additionally, the theoretical resonant frequency for this symmetric mode can be calculated for these tank dimensions [7], resulting in a value of 4.7 Hz. With the observed fluid oscillating close to its resonant frequency, it can be concluded that energy dissipation within R2 is a result of experiencing the high amplitude response of the fluid within one of its sloshing modes.

The full surface evolution is shown in figure 3, where the vertical axis shows position of the 2D surface along the tank length (x) and the colour map represents the corresponding surface displacement. An interesting feature is that despite the fluid moving through the violent R1 regime, which is highly anti-symmetric in its free surface motion, the fluid settles into an almost entirely symmetric pattern. This behaviour indicates an underlying mechanism which attenuates any asymmetric fluid motion not directly driven by the parametric excitation.

4.2. Numerical results
A series of numerical simulations was ran with and without surface tension, and the free-surface was extracted. The surface tension coefficient was calibrated against the experimental surface signal, with a surface tension coefficient (\( \sigma \)) of 0.2 N/m giving the best correlation. To provide a quantitative metric of surface correlation, a statistical indicator based upon the Modal Assurance Criterion (MAC) is calculated at every time instance. The MAC value of the two vectors defining free-surface amplitudes is computed, producing a value of unity for exactly matching surfaces and tending toward zero for dissimilar shapes.
Initially considering the surface evolution without surface tension, figure 5a, it is immediately noticeable there is a significant deviation from the symmetric sloshing pattern. Linear streaks moving up and down the figure show presence of waves moving longitudinally across the length of the tank. This response shows the emergence of the first asymmetric mode in the fluid, which acts at a quarter of the excitation $\sim 2.5 \text{ Hz}$. Note the underlying fluid motion still maintains a dominant 5Hz component, however, the superposition of these modes has the effect of inducing a ‘beating’ behaviour in the midpoint, as the point of maximum displacement is moved off-centre by the asymmetric wave. The highly nonlinear features present in the surface then result in the deviation from good correlation during R2, shown by the oscillatory MAC indicator within figure 4. In terms of the coupled response of the structure, a reduced damping ratio is observed during R2 ($\zeta_{R2} = 0.54\%$) partly due to the competition of the waves deteriorating the formation of the first symmetric sloshing mode, and resonant conditions not occurring in the fluid.

In contrast to this sub-optimal comparison, the numerical modelling with surface tension shows improved comparison with experiment, yielding a mostly symmetric pattern in figure 5b. An improved MAC correlation with time is also observed. The previous asymmetries stemming from the R1 regime, which take the form of highly curved surface waves, are now attenuated. The symmetric wave can now form un-interrupted, setting up the parametric resonant conditions in the fluid. An improvement in the second regime damping is therefore noted with an increase to 0.64% of the critical value. However, there remains some discrepancy with the experimental damping values despite having a similar surface motion. Therefore, despite providing a greatly improved surface response, there is further work to be done to ensure the numerical model can accurately resolve the damping response of the coupled fluid-structure system.

5. Conclusions
Sloshing fluid motion inside a transient vertically vibrating tank has been investigated, both experimentally and numerically. Three distinct damping regions are observed, each with a corresponding fluid response. An experimental free surface identification method was used to extract the fluid motion during the second symmetric sloshing stage, where energy is dissipated whilst driving the fluid under the conditions of parametric resonance. A fluid-structure coupled numerical model based upon smoothed particle hydrodynamics was developed, to complement the experimental investigations and to provide insight into the physics of the sloshing motion. Comparison of the numerical predictions without surface tension with the experimental surface showed decreased correlation due to the presence of an undesired travelling longitudinal wave interacting with the developing symmetric mode. However, inclusion of a robust surface tension model attenuated the highly curved surface features, allowing development of the symmetric sloshing mode, improving surface correlation and damped response under parametric excitation. The modelling of surface tension in sloshing problems at smaller tank scales potentially ensures
that full-scale flow regimes can be reproduced. However, further work is required to ensure damping response of the coupled system can be accurately resolved numerically.

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