Spin current driven by magnetization dynamics

Kazuhiro HOSONO, Akihito TAKEUCHI, Gen TATARA
Department of Physics, Tokyo Metropolitan University, Tokyo, Japan
E-mail: hosono-kazuhiro@ed.tmu.ac.jp

Abstract. The spin current induced by the magnetization dynamics in metallic systems is studied theoretically. The s-d exchange interaction between the conduction electron and the local spin is treated perturbatively.

1. Introduction
The spin current, flow of the spins in a solids, plays a key role in the spintronics. Creation of the spin current by electrical means is possible by use of spin-orbit interaction, the mechanism known as the spin Hall effect. The spin Hall effect thus provides a new manipulation technique for inducing the spin current. Detection of spin current was carried out first by obtaining the resulting spin accumulation at the edge of the sample[1][2]. Recently, electrical detection of spin current was carried out in metallic system by use of the inverse spin Hall effect[3][4][5]. In Ref. [3], in a bilayer film comprised of a ferromagnetic NiFe layer and a nonmagnetic Pt layer, the spin current pumped from the magnetization dynamics in ferromagnetic layer was converted into charge current by the spin-orbit interaction in Pt layer. Therefore, the spin current is possible by observing the charge current. The ferromagnetic material is used for pumping spin current stemmed form magnetization dynamics. The inverse spin Hall effect has been studied theoretically[6][7][8][9].

So far the spin current induced from the magnetization dynamics was studied based on phenomenological theories[10][11]. In this paper, we calculate microscopically the spin current arising from the magnetization dynamics.

2. Model
Our model Hamiltonian is

\[ H(t) = \sum_{k,\sigma} \epsilon_k c_{k\sigma}^{\dagger} c_{k\sigma} + H_{\text{ex}}(t) + H_{\text{imp}}, \]

where \( c_{k\sigma}^{\dagger} (c_{k\sigma}) \) is creation (annihilation) operator of an electron with spin \( \sigma = \pm 1 \) for spin up and spin down states, respectively defined in the momentum space. The first term is free electron, \( \epsilon_k \equiv \frac{\hbar^2 k^2}{2m} - \epsilon_F \) is the free electron energy. \( H_{\text{ex}}(t) \) represents the exchange interaction with the time dependent magnetization, \( M_{k-k'}(t) \),

\[ H_{\text{ex}} = J_{\text{ex}} \sum_{k,k',\sigma\sigma'} M_{k-k'}(t) \cdot c_{k'\sigma'}^{\dagger} c_{k\sigma}. \]

Where, \( J_{\text{ex}} \) is the coupling constant. The magnetization structure here is arbitrary, the effect of elastic electron scattering by impurities is described by

\[ H_{\text{imp}} = \frac{1}{V} \sum_{k,k'/\sigma} V_{k-k'} c_{k\sigma}^{\dagger} c_{k\sigma}. \]
where $V_{k' - k} = u_0 \sum_{r_i} N_i e^{-i(k-k') \cdot R_i}$, and $u_0$ is a coupling constant, $R_i$ is position of random impurities, $N_i$ is the number of impurities, and $V$ is the volume of the system. $H_{imp}$ gives rise to a lifetime of the electron. We define $\eta = \frac{h}{2\pi}$, $\tau = \frac{2\pi N(0) n_{imp} g_{imp}^2}{\hbar^2}$ in the Born approximation. We assume a metallic case $\frac{\hbar}{e\tau} \ll 1$, where $N(0)$ is the density of states at fermi level and $n_{imp}$ is the concentration of impurities as $n_{imp} \equiv \frac{N}{V}$. For the perturbative expansion, we assume $\frac{J_{ex} \tau}{\hbar} \ll 1$. Since we are not interested in the effect of the spin-orbit interaction to the spin current, we don’t assume the spin-orbit interaction here.

3. Method

The spin current is defined as $j_{\mu}^\sigma(x, t) = i \frac{e}{2m} \overline{\langle \psi(x, t) | \sigma^\nu \partial_\mu \psi(x, t) - (\partial_\mu \psi(x, t)) \sigma^\nu \psi(x, t) \rangle}$ Using the Green’s function, it is written as

$$j_{\mu}^\sigma(x, t) = i \frac{e\hbar^2}{mV} \sum_{k, q} k_{\mu} \lim_{t' \to t} \text{Tr}[\sigma^\nu G_{k-\frac{q}{2} + k} (t, t')] e^{-i q \cdot x},$$

where $\text{Tr}[\ldots]$ means trace over the spin indices, and $G_{k, k'}(t, t')$ is the lesser component of the contour-ordered Green’s function, defined as $G_{k, k'}(t, t') \equiv \frac{1}{i\hbar} \langle c_{k}(t) c_{k'}(t) \rangle$.

We define the contour ordered Green’s function as $G_{k, k'}(t, t') = \frac{1}{i\hbar} \langle T_{c}[c_{k}(t) c_{k'}(t')] \rangle$ for taking the lesser Green’s function. The Dyson equation on a complex contour $c$ is given by

$$G_{k, k'}(t, t') = g_k(t - t') \delta_{k, k'} + J_{ex} \sum_{p} \int_{c} dt_1 g_k(t - t_1) \sigma_{\alpha, \beta}^\sigma M_p^\alpha_{-k} (t_1) G_{p, k'} (t_1, t'),$$

where $g_k(t - t') \equiv \frac{1}{i\hbar} \langle T_{c}[c_{k}(t) c_{k'}(t')] \rangle_{0_{imp}}$ is impurity averaged Green’s function. $(\ldots)_{0_{imp}}$ represents the expectation value by use of free Hamiltonian $H_0$ and to take the average of random impurities. The Dyson equation can be solved by iteration, assuming the weak coupling condition $\frac{J_{ex} \tau}{\hbar} \ll 1$.

4. Result

The spin current at the linear order in $J_{ex}$ (Fig.1(a)) is obtained as

$$j_{\mu}^{\nu(1)}(x, t) = i \frac{e\hbar^2}{mV} J_{ex} \text{Tr}[\sigma^\nu \sigma^\alpha] \sum_{k, q} k_{\mu} M_q^\alpha (\Omega) e^{i \Omega \cdot x} \left\{ \int \frac{d\omega}{2\pi} g_k - \frac{q}{2} \omega \frac{g_k}{g_k^*} + \frac{q}{2} \omega + \Omega \right\} <.$$

The lesser Green’s function is calculated using the Langreth theorem, $\left\{ \int \frac{d\omega}{2\pi} g_k - \frac{q}{2} \omega \frac{g_k}{g_k^*} + \frac{q}{2} \omega + \Omega \right\} < = \int \frac{d\omega}{2\pi} g_k - \frac{q}{2} \omega \frac{g_k}{g_k^*} + \frac{q}{2} \omega + \Omega$. Free lesser component is obtained as $g_{k, \omega}^{\alpha} = \frac{f(\omega)(g_k^* - g_k^* \omega)}{\hbar \omega - \xi_k + i\eta}$, where $g_k^{\alpha} = \frac{1}{\hbar \omega - \xi_k + i\eta} = (g_k^*)^\alpha$. Considering the case of the smooth and
slow magnetization structures \((\Omega \tau \ll 1, q l \ll 1(l \text{ is the electron mean free path})\), we obtain the spin current density in linear order in \(J_{\text{ex}}\) as

\[
j^{\nu(1)}_{\mu}(x, t) = \frac{2e\hbar^2 J_{\text{ex}} N(0) \epsilon_F}{3mV_\eta^2} \frac{\partial}{\partial x^\mu} \dot{M}^\nu(x, t).
\]

In the same way, the second order contribution (Fig.1(b)) is obtained as

\[
j^{\nu(2)}_{\mu}(x, t) = \frac{e\hbar^2 J_{\text{ex}}^2 N(0) \epsilon_F}{3mV_\eta^3} \{ \frac{\partial}{\partial x^\mu} (M(x, t) \times \dot{M}(x, t))^\nu + (M(x, t) \times \frac{\partial}{\partial x^\mu} M(x, t))^\nu \}.
\]

5. Discussion and Conclusion

The result of the spin current to the second order is therefore obtained as

\[
j^{\nu}_{\mu} = \frac{2e\hbar^2 J_{\text{ex}} N(0) \epsilon_F}{3mV_\eta^2} \left[ \frac{\partial}{\partial x^\mu} \dot{M}^\nu + \frac{J_{\text{ex}}}{2\eta} \{ \frac{\partial}{\partial x^\mu} (M \times \dot{M})^\nu + (M \times \frac{\partial}{\partial x^\mu} M)^\nu \} \right].
\] (1)

We see that there are two total derivative terms, \(\frac{\partial}{\partial x^\mu} \dot{M}^\nu\) and \(\frac{\partial}{\partial x^\mu} (M \times \dot{M})^\nu\). These terms look different, but are closely related if we use the Landau-Lifshitz-Gilbert (LLG) equation, \(\dot{M}^\nu = -\gamma(M \times B_{\text{eff}})^\nu + \alpha(M \times \dot{M})^\nu\), where \(\gamma\) and \(\alpha\) are the gyromagnetic ratio and the Gilbert damping constant, and \(B_{\text{eff}}\) is the total effective magnetic field. Using the LLG equation, rewritten as

\[
j^{\nu}_{\mu} = \frac{2e\hbar^2 J_{\text{ex}} N(0) \epsilon_F}{3mV_\eta^2} \left( (\alpha + \frac{J_{\text{ex}}}{2\eta}) \frac{\partial}{\partial x^\mu} (M \times \dot{M})^\nu - \gamma \frac{\partial}{\partial x^\mu} (M \times B_{\text{eff}})^\nu + \frac{J_{\text{ex}}}{2\eta} (M \times \frac{\partial}{\partial x^\mu} M)^\nu \right).\] (2)

The sign of the first term of Eq. (2), be controlled by the sign and the magnitude of \(J_{\text{ex}}\) (\(\alpha\) is positive.) In Ref. [9], it was shown that this spin current is converted into the charge current by the spin-orbit interaction.

The spin current across the interface between a ferromagnet and non-magnet was discussed in Ref. [10][11], and the spin current was obtained as \(j_s = aM + b(M \times \dot{M})\) where \(a\) and \(b\) are phenomenological constants describing the transmission. In contrast, our result, Eq. (1), is a general expression applicable to any slow magnetization profile. The result can be applied to the ferromagnetic and non-magnetic interface (assuming the magnetization gradually changes within a length scale of \(l\)), yielding \(j_s = aM + b(M \times \dot{M})\) with \(a = \frac{2e\hbar^2 J_{\text{ex}} N(0) \epsilon_F}{3mV_\eta^2}\) and \(b = \frac{J_{\text{ex}}}{2\eta} a\), and \(M\)’s represent the magnetization near the interface.

In conclusion, we have derived the spin current from the dynamical magnetization by Keldysh method in the case of smooth and slow magnetizations.

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