Discrete $R$ Symmetries and Low Energy Supersymmetry

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Abstract

If nature exhibits low energy supersymmetry, discrete (non-$Z_2$) $R$ symmetries may well play an important role. In this paper, we explore such symmetries. We generalize gaugino condensation, constructing large classes of models which are classically scale invariant, and which spontaneously break discrete $R$ symmetries (but not supersymmetry). The order parameters for the breaking include chiral singlets. These simplify the construction of models with metastable dynamical supersymmetry breaking. We explain that in gauge mediation, the problem of the cosmological constant makes “retrofitting” particularly natural – almost imperative. We describe new classes of models, with interesting scales for supersymmetry breaking, and which allow simple solutions of the $\mu$ problem. We argue that models exhibiting such $R$ symmetries can readily solve not only the problem of dimension four operators and proton decay, but also dimension five operators. On the other hand, in theories of “gravity mediation,” the breaking of an $R$ symmetry is typically of order $M_p$; $R$ parity is required to suppress dimension four $B$ and $L$ violating operators, and dimension five operators remain problematic.
1 What Makes $R$ Symmetries Special

While it has long been argued that theories which incorporate general relativity cannot exhibit exact global continuous symmetries, discrete symmetries are another matter. When studying classical solutions of critical string theories, one often encounters such symmetries on submanifolds of the full moduli space of solutions $[1]$; these are believed, in general, to be discrete gauge transformations. Within spaces of supersymmetric solutions, a particularly interesting set of symmetries are the discrete $R$ symmetries. These can often be thought of as unbroken subgroups of the rotation group in higher dimensions; in such cases, the fact that they transform the supercharges is immediate.

We will reserve the term $R$ symmetry, in this paper, for symmetries other than $Z_2$ which rotate the supercharges. Any symmetry which multiplies all of the supercharges by $-1$ can be redefined by adding a rotation by $2\pi$, leaving an ordinary (non-$R$) $Z_2$. Conventional $R$ parity in this sense, is not an $R$ symmetry.

There are several reasons to think that, if supersymmetry plays some role in low energy physics, discrete $R$ symmetries might be relevant:

1. Perhaps the most important comes from the question of the cosmological constant (c.c.). In order that the c.c. be small, it is necessary that any constant in the superpotential be far smaller than $M_p^3$. The only type of symmetry which can suppress such a constant is an $R$ symmetry.$^1$

2. In gauge-mediated models, supergravity effects should be unimportant for understanding SUSY breaking (we will make this statement more precise shortly), and the theorem of Nelson and Seiberg $[2]$ requires a global $R$ symmetry in order to obtain supersymmetry breaking (in a generic fashion); correspondingly, an approximate global $R$ symmetry seems to be a requirement for metastable supersymmetry breaking $[3]$. Discrete $R$ symmetries are a particularly plausible way in which to account for such approximate continuous symmetries.

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$^1$Supersymmetric critical string theories have vanishing c.c. classically, and the superpotential is protected in higher orders of perturbation theory by non-renormalization theorems. In many cases, the vanishing of $W$ appears an accident, from the point of view of the low energy theory (it is not accounted for by symmetries). In flux vacua, vanishing of the cosmological constant is not typical, requiring $R$ symmetries, or accidental cancelations. More generally, as Banks has repeatedly stressed, the only Minkowski space gravity theories of which we can claim any complete understanding exhibit supersymmetry and $R$ symmetries. Similar remarks apply to would-be $\mu$ terms.
3. Discrete symmetries have long been considered in supersymmetric theories to suppress proton decay \cite{4} and $R$ symmetries might be particularly effective in suppressing dimension four – and five – operators which violate $B$ and $L$ \cite{5}.

In supersymmetric, $R$ symmetric theories, because $W$ transforms under the symmetry, there is a close tie between the scale of supersymmetry breaking and the scale of $R$ breaking, if the c.c., $\Lambda$, is small. This is because

$$|\langle W \rangle| \sim |\langle F \rangle| M_p.$$  \hspace{1cm} (1)

We will see in this paper that in “gravity mediated” theories ($\sqrt{F} \sim 10^{11}$ GeV), except under special circumstances, this implies that the $R$ symmetry is broken by scalar field vevs of order $M_p$. As a result, the $R$ symmetry does not significantly constrain the low energy Lagrangian, and cannot account for proton stability or other phenomena. On the other hand, in gauge mediation, the contribution of the SUSY breaking interactions to $W$ is typically much smaller than eqn. (1). New interactions are required; the needed scale is precisely that which enters in retrofitted models \cite{6}. This suggests that retrofitting, rather than being a sort of Rube Goldberg contraption to implement (metastable) supersymmetry breaking, may be essential to understanding the smallness of the c.c.

These types of considerations lead us to consider more broadly a set of questions about discrete $R$ symmetries.

1. Spontaneous breaking of discrete $R$ symmetries is familiar in pure supersymmetric gauge theories (gaugino condensation), and in theories with massive matter fields. In section 2 we discuss a set of theories which generalize gaugino condensation. These models are quite close to a set of theories considered some time ago by Yanagida \cite{7}. This larger set of theories includes examples with gauge singlets and matter fields. These theories, like the pure gauge theory, are classically scale invariant, and exhibit intricate discrete symmetries.

2. In section 3 we explain why discrete $R$ symmetries are typically badly broken in gravity-mediated models.

3. In the framework of gauge mediation, discrete $R$ symmetries are distinctly more interesting. In section 4 we explain why, in gauge mediation, retrofitting is almost inevitable if

\footnote{This point was first stressed to one of the authors many years ago by T. Banks.}
one is to understand the smallness of the cosmological constant. We construct simple, gauge-mediated models in this framework.

4. In section 5, we revisit an earlier suggestion of Yanagida’s for solving the $\mu$ problem of gauge-mediated models, noting that (for similar reasons as in [7]) it is readily understood in retrofitted models with singlets, and argue that a large value of $\tan \beta$ is typical. (In gravity mediation, a source for $\mu$ with a similar flavor was proposed in [9]. In gauge mediation, the issue is more severe, since $B_\mu$ tends to be parametrically too large; our solution is particularly directed at this issue.)

5. While there have been studies of discrete $R$ symmetries and proton decay, almost all of these are within the context of intermediate scale supersymmetry breaking. While we argue that such symmetries are unlikely to play a significant role in gravity mediated models, in the context of gauge mediated models, discrete $R$ symmetries can be effective in suppressing dimension four and five contributions to proton decay. We explain in section 6 why one cannot apply anomaly constraints for this question, and give simple examples of symmetries which suppress some or all dangerous dimension four or dimension five operators.

6. Discrete $R$ symmetries might have cosmological relevance. They have been considered for models of inflation (e.g. [10]) and might well play a role in AD baryogenesis [11]. Discrete $R$ symmetries can lead to the suppression of higher dimension operators and thus account for very flat directions which might be relevant for these phenomena. The observations of this paper could well be relevant to to understanding Peccei-Quinn symmetries as well. We comment on these issues in the concluding section, leaving a more thorough investigation for future work.

2 Generalizations of Gaugino Condensation

Gaugino condensation is considered in many contexts, but its principal distinguishing feature is that it breaks a discrete $R$ symmetry without breaking supersymmetry. Many other models, such as supersymmetric QCD with massive quarks, dynamically break such symmetries, but in thinking about a variety of questions, it would be helpful to have models like pure susy gauge

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3Banks has put forth an alternative proposal, with the framework of Cosmological Supersymmetry Breaking [8], to understand the relation of these scales. In his proposal, discrete $R$ symmetries also play an important role, both in understanding the c.c. and in suppressing dangerous rare processes.
theory, with scales generated by dimensional transmutation. In this section, we develop a class of such models.

As an example, consider an SU($N$) gauge theory with $N_f < N$ massless flavors, and, in addition, $N_f^2$ singlets, $S_{f,f'}$. Include a superpotential:

$$W = y S_{f,f'} Q_{f'} Q_f - \frac{1}{3} \gamma \text{Tr} S^3$$

where the last term denotes a general cubic coupling of the $S_{f,f'}$ fields. For convenience, we have taken the superpotential to respect an SU($N_f$) symmetry; $\gamma$ and $y$ can be taken real, by field redefinitions. This model possesses a discrete $\mathbb{Z}_{2(3N-N_f)}$ R symmetry, which is free of anomalies (this remains true away from the SU($N_f$) symmetric limit). Calling

$$\alpha = e^{\frac{2\pi i}{3N-2N_f}}$$

we can take the transformation laws of the various fields to be:

$$\lambda \to \alpha^{3/2} \lambda \quad S_{f,f'} \to \alpha S_{f,f'} \quad (Q, \overline{Q}) \to \alpha (Q, \overline{Q}).$$

For $N_f < N$, treating $\gamma$ and $y$ as small, we can analyze the system by including the familiar non-perturbative superpotential of SU($N$) QCD with $N_f$ flavors

$$W_{\text{dyn}} = (N - N_f) \Lambda^{\frac{3N-N_f}{N-N_f}} \det(\overline{Q}Q)^{-\frac{1}{N-N_f}}.$$  

In the SU($N_f$) symmetric limit, the $\frac{\partial W}{\partial Q}, \frac{\partial W}{\partial S} = 0$ equations admit solutions of the form

$$S_{f,f'} = s \delta_{f,f'} \quad Q_f \overline{Q}_{f'} = v^2 \delta_{f,f'}.$$  

with

$$v^2 = \left( \frac{\gamma}{y^3} \right)^{\frac{N-N_f}{N-2N_f}} \alpha^{2k} \Lambda; \quad s = \left( \frac{y^{N_f}}{\gamma N} \right)^{\frac{N-N_f}{N-2N_f}} \alpha^{2k} \Lambda.$$  

Perturbing away from the symmetric limit, one can then check that there is no qualitative change in the solutions (e.g. the number is unchanged). For $N_f \geq N$, the theory has baryonic flat directions, and does not have a discrete set of supersymmetric ground states. Adding additional singlets and suitable (non-renormalizable) couplings, one can again spontaneously break the discrete symmetries. One can also consider generalizations to other gauge groups and to different matter content. These and related matters will be thoroughly explored in reference \[13\].

\[4\] This model has been considered previously in \[7\], who find the solutions obtained in eqn. \[5\].
3 Discrete $R$ Symmetries in Gravity Mediation

Supergravity models, without $R$ symmetries, seem to have a virtue with respect to the cosmological constant. If one has a pseudo-modulus, $Z$, with, say, a constant superpotential, $W_0$, then if the field $Z$ varies over the Planck scale, the positive and negative terms in the potential are of a similar order of magnitude. This is different than the case of gauge mediation, where additional interactions, or a rather peculiar value for $W_0$, are required to account for a small cosmological constant.

In a hidden sector supergravity model, with an underlying $R$ symmetry, one might hope to break the $R$ symmetry through hidden sector dynamics. More precisely, one might hope that the same dynamics which is responsible for supersymmetry breaking would spontaneously break the $R$ symmetry, generating a superpotential of a size suitable to cancel the cosmological constant.

However, in such theories, quite generally, the $R$ symmetry breaking is large (broken by Planck-scale vevs). The difficulty is associated with the cosmological constant. In supergravity theories, one requires that $W \sim F M_p$. In a theory with an $R$ symmetry at the scale of the hidden sector dynamics, a constant in the superpotential is forbidden, so one might expect that $W \sim F Z$, for some field, $Z$, transforming non-trivially under the $R$ symmetry. Then one would have $Z \sim M_p$.

One can almost make this rigorous. Consider, first, O’Raifeartaigh (OR) type models, with a continuous $R$ symmetry. Assuming that all fields are small compared to $M_p$, we can limit our considerations to renormalizable theories. In this case, one can show that, at tree level, there is a chiral field, $Z$, whose fermionic component is the Goldstino, and whose scalar components are a (tree level) modulus and a massless pseudoscalar [14, 15]. The effective superpotential of the theory is:

$$W = Z F.$$  \hspace{1cm} (8)

One can readily prove, in such models, a bound [16]:

$$|\langle W \rangle| \leq \frac{1}{2} f_r |F|$$  \hspace{1cm} (9)

where $f_r$ is the r-axion decay constant. In order to cancel the cosmological constant, one needs $f_r \sim M_p$. The inequality, eqn. 9, appears rather general [16].
If supersymmetry is dynamically broken in the hidden sector in a theory without flat directions, there are no large field vevs, and the c.c. is large and positive. In theories with metastable breaking, one typically has, at most, approximate moduli, so again the c.c. is positive.

One can attempt to require that the $R$ symmetry is broken at some scale higher than the intermediate scale, by some other dynamics. For example, there could be additional gauge interactions at a scale $\Lambda^3 = F M_p[17]$, as might be expected in retrofitted models. But the hidden sector necessarily possesses a field(s), $Z$, neutral under the $R$ symmetry, to generate the needed $F$ term; it couples to $W^2$ through a term of the form $W_Z = Z W^2 / M_p$. However, because $Z$ is neutral, the coupling

$$Z^2 W^2 / M_p^2$$

is also allowed, generating a tadpole for $Z$, and yielding an expectation value of order $M_p$.

More intricate constructions, for example using singlet fields as in the models of section 2, and introducing additional symmetries, might achieve the desired structure, but the required models are clearly complicated. We conclude from this that theories with gravity-mediated supersymmetry breaking will typically break $R$ symmetries by large amounts. Such symmetries will not suppress proton decay, or play other interesting roles in low energy dynamics.

A particularly interesting proposal to understand a small $W$ in theories with discrete (and approximate continuous) $R$ symmetries has been put forward in [18]. These authors generate a small constant in the superpotential in theories with a Fayet-Iliopoulos term, in heterotic orbifold models. This provides a small parameter. In the low energy theory, an expectation value for $W$ arises at very high order in this small parameter as a consequence of discrete symmetries. As in our previous examples, this corresponds to the breaking of the $R$ symmetry at energy scales well above the scale of supersymmetry breaking.

4 Discrete $R$ Symmetries and Retrofitted Models

Having established that $R$ symmetries are not of interest in gravity mediation, our focus will be on theories with dynamical supersymmetry breaking and gauge mediation. One simple class of gauge-mediated models begins with retrofitted O’Raifeartaigh (OR) models [6]. At first sight, these models seem rather contrived, with singlets and additional gauge interactions added just so. But when one thinks about the cosmological constant, retrofitting takes on an aspect of inevitability. One of the disturbing features of gauge mediated models, as opposed to models
with supersymmetry broken at an intermediate scale, is the need for dynamics at some much larger scale to account for the vanishing of the c.c. (or, alternatively, for a constant in the superpotential with some very large scale, unrelated to any other scale seemingly required for the models). Retrofitted models offer a possible solution to this puzzle. Consider the simplest of retrofitted models:

\[ W = \frac{c}{32\pi^2} \frac{Z W^2}{M_p} + Z A^2 + MYA. \]  

(11)

Calling

\[ \langle \lambda \lambda \rangle = N \Lambda^3 e^{-\frac{cZ}{M_p}} \approx W_0 - \mu^2 Z \quad \mu^2 = \frac{c\Lambda^3}{M_p} \quad W_0 = N\Lambda^3, \]  

(12)

the low energy effective superpotential is (for \( Z \ll M_p \)):

\[ W = W_0 + Z(A^2 - \mu^2) + ZA^2 + MYA, \]  

(13)

a simple OR model, with a constant of suitable order of magnitude to give a small c.c. We see that \( \langle W \rangle \) is automatically of the correct order of magnitude to cancel the c.c.\(^5\) The goldstino decay constant, \( F \approx \mu^2 \). In this model, one would still like to account for the scale \( M \). One can consider the possibilities that \( M \gg \mu \), or \( M \sim \mu \). The scale relevant to low energy soft breaking terms is

\[ \Lambda_m^2 = \frac{|F|^2}{M^2} \]  

(14)

so the case of \( M \sim \mu \) corresponds to relatively low scale breaking (say \( 10^5 \) GeV), while \( M \gg \mu \) corresponds to high scale gauge mediation. In ref. \[20\], an alternative scaling was suggested:

\[ \mu^2 \sim \frac{W_0^4}{M_p^4} \sim M^2 \]  

(15)

but the critical relation required for canceling the cosmological constant, \( W \sim FM_p \), is spoiled; the scale of the new interactions is much too large. In the next subsection, we write models with a hierarchy between \( M^2 \) and \( F \). In subsection 4.2 we develop models with \( M^2 \sim F \), exploiting the models of section 2 to avoid the problematic relations of ref [20].

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\(^5\)While the orders of magnitude are correct, it is still necessary to tune the constants in the lagrangian to obtain the high degree of cancelation required in nature. As written, this can be achieved by tuning the constant \( c \). Whether this might be explained through Weinberg’s argument [19], or in some other way, we will not address here.
4.1 Models with a Hierarchy of Scales

The ability to construct models with gauge singlet fields which break discrete $R$ symmetries opens up a wide range of model-building possibilities. We have stressed that the cosmological constant points in the direction of retrofitted OR, i.e. OR models in which the dimensionful parameters of the theory are determined by some high scale dynamics. Model building with OR models is challenging in and of itself. Shih [21] studied O’Raifeartaigh models quite generally, showing that in theories with fields with only (continuous) $R$ charges 0 or 2, the $R$ symmetry is not spontaneously broken. He also exhibited simple models which violate this condition, and do break the $R$ symmetry. Perhaps the simplest example is provided by a theory with fields $\phi_{\pm 1}, \phi_3, X_2$, where the subscripts denote the $R$ charge, and with superpotential:

$$W = X_2(\phi_1 \phi_{-1} - \mu^2) + M_1 \phi_1 \phi_1 + M_2 \phi_{-1} \phi_3. \quad (16)$$

Motivated by this model, consider a theory with fields $X_0, S_{2/3}, \phi_0, \phi_{2/3}, \phi_{4/3}$, where the subscript denotes the discrete $R$ charge ($\phi_q \rightarrow \alpha^q \phi_q$, where $\alpha$ is some appropriate root of unity). $S_{2/3}$ is a field with a large mass and an $R$ symmetry breaking vev, presumed to arise from some high scale dynamics as in the models of section 2. The superpotential is

$$W = \frac{1}{M_p} X_0 S_{2/3}^2 + y X_0 \phi_{2/3} \phi_{4/3} + \lambda_1 S_{2/3} \phi_{2/3} \phi_{2/3} + \lambda_2 S_{2/3} \phi_{4/3} \phi_0 \quad (17)$$

(up to terms involving higher dimension operators). The expectation value of $S$ leads to mass terms for the $\phi$ fields; the resulting low energy effective theory is that of eqn. (16) with

$$M_1 = \lambda_1 S_{2/3} \quad M_2 = \lambda_2 S_{2/3} \quad \mu^2 = -\frac{S_{2/3}^2}{M_p}. \quad (18)$$

Below the scale $S_{2/3}$, the theory possesses an accidental, (approximate) continuous $R$ symmetry which is spontaneously broken.

Note that the mass terms are large compared to the effective $F$ term, $M^2 \gg \mu^2$. Assuming $X$ couples to some messenger fields, with coupling $W_{mess} = X \bar{M} M$, the scale of the low energy soft terms is set by $F/M$

$$\Lambda_m = \frac{S^2}{M_p}. \quad (19)$$

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6In order that this superpotential be the most general consistent with symmetries, it is necessary, beyond the $R$ symmetry, to impose a $Z_2$ under which the $\phi$’s are even and the other fields are odd, and an additional symmetry to forbid a mass term for $\phi_{4/3} \phi_{2/3}$. Without a mass term, the model possesses a $U(1)$ symmetry, under which the fields have charges:

$$X : -3; S : 1; \phi_{2/3} : -1/2; \phi_{4/3} : 7/2; \phi_0 : -9/2.$$  

A discrete subgroup is enough to eliminate the unwanted term.
If $\Lambda_m \sim 10^5 \text{ GeV}$, for example, then we have $S \sim 10^{11.5} \text{ GeV}$.

Messengers are readily coupled to this system. For example, a model with minimal gauge mediation is obtained by coupling $X_0$ to messengers $M$ and $\overline{M}$ transforming as a 5 and $\overline{5}$ of $SU(5)$. Models of general gauge mediation require more intricate hidden sectors.

### 4.2 O’Raifeartaigh Models with a Single Dimensionful Parameter

The simplest O’Raifeartaigh model

$$W = X(A^2 - \mu^2) + mYA$$

has two dimensionful parameters, one with dimensions of mass and one with dimensions of mass-squared. In order to obtain gauge mediation with a low scale of supersymmetry breaking, one wants $m^2 \sim \mu^2$. On the other hand, if the underlying scale is $M_p$, and if both parameters arise from gaugino condensation in some new gauge group, then

$$\mu^2 \sim \Lambda^3/M_p \quad m \sim \frac{\Lambda^3}{M_p^2}$$

i.e.

$$m^2 \sim \mu^2 \frac{\mu^2}{M_p^2}$$

To avoid this, in [20] it was suggested that $\mu^2$ might arise from a coupling to $W_4^4/M_p^4$. However, this is not compatible with our suggested explanation of the cancelation of the c.c.; the scale of $W_4^2$ is much too large. In this section, we consider an alternative type of O’Raifeartaigh model, in which the only dimensionful parameter has dimensions of mass-squared.

To illustrate some of the issues, consider, first, a model with three fields:

$$W = X(A^2 - \mu^2) + \lambda Y A^2.$$ 

This model formally breaks supersymmetry, but it is not attractive as one linear combination of $X$ and $Y$ decouples from $A$. So instead consider a model with additional fields:

$$W = yX(AB - \mu^2) + \lambda_1 Y A^2 + \lambda_2 ZB^2$$

This model breaks supersymmetry, and none of the fields decouple. The model, however, is not the most general consistent with symmetries, and the Coleman-Weinberg calculation leads to an unbroken $R$ symmetry.
We can avoid these difficulties, by again taking Shih’s model, and replacing both mass terms with a field $\Phi_0$:

$$W = X(\phi_- \phi_1 - \mu_X^2) + \lambda_1 \Phi_0 \phi_1 + \lambda_2 \Phi_0 \phi_- \phi_3 + \chi(\Phi_0^2 - \mu_\chi^2) + \epsilon X \Phi_0^2. \quad (25)$$

In order that this model be the most general consistent with symmetries, in addition to the $R$ symmetry we impose a $\mathbb{Z}_4$ symmetry under which

$$\Phi_0 \to -\Phi_0; \phi_1 \to i\phi_1; \phi_- \to -i\phi_-; \phi_3 \to -i\phi_3. \quad (26)$$

We have defined $X$ to be the field which couples to $\phi_1 \phi_- \phi_3$.

If we first set $\epsilon = 0$, and take $\mu_X^2 \gg \mu_\chi^2$, the model is quite simple to analyze. The fields $\Phi_0$ and $\chi$ are massive, and integrating them out yields the model of eqn. 16. There is, as in that model, a flat direction in the theory, which, for a range of parameters, is stabilized with a non-vanishing vev for $X$ (breaking the approximate, continuous $R$ symmetry of the low energy theory). Turning on a small, non-zero $\epsilon$ does not yield qualitative changes in the theory. The only light field, for $\epsilon = 0$, is the pseudomodulus $X$. For small $\epsilon$, there is still a pseudomodulus. The equations $\frac{\partial W}{\partial \Phi} = 0$ still have a one (complex) parameter set of solutions; the pseudomodulus is now a linear combination of $\chi$ and $X$. Similarly, there is no qualitative change if $\mu_X^2$ is comparable to, but slightly less than, $\mu_\chi^2$. Both of these parameters can arise through retrofitting as in 11 and satisfy the critical relation.

5 $R$ Symmetries and the $\mu$ Problem of Gauge Mediation

Retrofitting has been discussed as a solution to the $\mu$ problem [7, 22, 23]. If the source of the $\mu$ term is a coupling of the gaugino condensate responsible for the hidden sector $F$ term,

$$W_\mu = \frac{W^2}{M_p^2} H_U H_D \quad (27)$$

the resulting $\mu$ term is very small; it would seem necessary to introduce still another interaction, with a higher scale. Not only does this seem implausibly complicated, but it is once more problematic from the perspective of the c.c. Models with singlets, on the other hand, allow lower dimension couplings and larger $\mu$ terms [7]. For example, a model such as that of section 4.1 provides an interesting possible solution to the $\mu$ problem. If the product $H_U H_D$ has $R$ charge $2/3$, it can couple to $S^2/M_p$ with coupling $\lambda$. This gives a $\mu$ term whose order of magnitude is $\lambda \Lambda_m$ ($\Lambda_m$ is the low energy mass scale of eqn. 19),

$$W_\mu = \lambda \frac{S^2_{2/3}}{M_p} H_U H_D. \quad (28)$$
The $F$ component of $S$ is naturally of order $m^2_{3/2}$, so this does not generate an appreciable $B_\mu$ term; the $B_\mu$ term must be generated at one loop, or through the operator

$$W_{B\mu} = \frac{1}{M_p^2} X_0 S^2_{2/3} H_U H_D$$

which can lead to an appreciable $B_\mu$ term if $S \sim 10^{12}$ GeV. For $S < 10^{12}$, renormalization group evolution generates $B_\mu$. A rough calculation yields $\tan \beta \sim 30$. Alternative structures lead to different scaling relations; these, as well as a more detailed analysis of $\tan \beta$, will appear elsewhere [24].

We can similarly solve the $\mu$ problem in the single-scale models of section 4.2. Now, if $\frac{A^3}{M_p} \sim S^2 \sim 10^{15}$ GeV$^2$, say, then

$$W_\mu = \frac{S^2}{M_p} H_U H_D$$

yields a $\mu$ term of a suitable size.

6 Discrete $R$ Symmetries and Proton Decay

Most supersymmetric model building seeks to suppress dangerous dimension four lepton and baryon number violating operators by imposing $R$ parity. We have remarked that $R$ parity is not really an $R$ symmetry at all. Unlike the $R$ symmetries we are focussing on in this paper, there is no requirement that it be broken; this leads, most strikingly, to stable dark matter. While discrete $R$ symmetries might forbid dangerous dimension four and dimension five operators, these symmetries must be broken; the size of this breaking, and the transformation properties of the fields, will control the size of $B$ and $L$ violating effects [5].

In model building with discrete symmetries, one would seem to have a great deal of freedom in both the choice of symmetry group and in the transformation properties of the fields. Many authors, in attempting to use discrete symmetries ($R$ or non-$R$) to forbid proton decay, impose a variety of constraints. For example, the authors of [5], who focus, as we do, mainly on discrete $R$ symmetries, require:

1. Absence of anomalies.

2. $\mu$ term forbidden in the superpotential.
3. Kahler potential terms permitted which give rise to a \( \mu \) term of order the supersymmetry breaking scale.

The anomaly constraints cannot be imposed, however, without making very strong assumptions about the microscopic theory. Any \( R \) symmetry must be spontaneously broken, by a substantial amount, in order to account for the (near) vanishing of the c.c. From the perspective of anomaly cancelation, this means that there may be massive states, at the TeV scale of higher, which contribute to anomalies. Such couplings can also generate the \( \mu \) term. So, in fact, one has few ways of constraining the microscopic theory from low energy considerations.

Most discussions of the use of \( R \) symmetries to suppress proton decay are framed in the context of gravity mediation, and we have seen that once one requires a small cosmological constant, this is problematic. So our focus here will be on gauge mediated models.

6.1 \( R \) Symmetries in Gauge Mediation

It is easy to see that discrete symmetries can suppress all unwanted dimension four and five operators. To illustrate this point, suppose that the theory possesses conventional \( R \) parity, in addition to an \( R \) symmetry, under which all quark and lepton superfields are neutral, while the Higgs transform like the superpotential.\footnote{This particular assignment forbids dimension five operators which would generate a Majorana neutrino mass. Different choices can achieve this. Model building of this type can be restricted in interesting ways by requiring unification or cancelation of anomalies, but neither of these are required by general principles \cite{25}.} This forbids all dangerous dimension four and dimension five operators. Once \( R \) symmetry breaking is accounted for, dimension five operators may be generated, but they will be highly suppressed.

We can contemplate more interesting symmetries, which do not include \( R \) parity, and for which the Higgs, quarks and leptons have more intricate assignments under the \( R \) symmetry. In the absence of \( R \) parity, given that the \( R \) symmetry is necessarily broken, dangerous dimension four operators will be generated, and it is important that they be adequately suppressed. Consider, first, the case where the \( R \) symmetry is broken by a gaugino condensate in a pure gauge theory. Suppose that \( B \) and \( L \)-violating operators of the form

\[
\delta W_{b,l} \sim \frac{W_2^2}{M_p^3} \Phi \Phi
\]

are permitted by the symmetries. Even if \( \sqrt{F} \) is as large as \( 10^9 \text{ GeV}, W_2^2/M_p^3 \approx 10^{-18}, \) more than adequately suppressing proton decay.
In the presence of a singlet field such as $S$, the constraints are more severe, however. Even in the low gravitino mass case, the small parameter, $S/M_p$, is of order $10^{-9}$. So suppression of dangerous operators by a single factor of $S$ is not adequate. One requires that many operators be suppressed by two powers of $S$.

7 Conclusions

From this discussion, a coherent framework for electroweak symmetry breaking due to supersymmetry appears to emerge. Many of the problematic aspects of most susy model building are resolved in retrofitted models with $R$ symmetries. The elements of the framework are:

1. Gauge mediation: As in all gauge mediated models, the problem of flavor changing neutral currents is eliminated.

2. Discrete $R$ symmetries, spontaneously broken: such symmetries are likely to play a role in gauge-mediated models, e.g. to account for the approximate $R$ symmetries needed for metastable supersymmetry breaking. We have exhibited large classes of models which generalize gaugino condensation, in that they break discrete $R$ symmetries without breaking supersymmetry.

3. Metastable supersymmetry breaking: As illustrated by the ISS and retrofitted models, metastable supersymmetry breaking provides a rich setting in which to obtain dynamical supersymmetry breaking.

4. Retrofitting: In addition to providing a very simple realization of metastable supersymmetry breaking, retrofitting resolves the puzzle endemic to gauge-mediated models of the mismatch of scales required to cancel the cosmological constant. The most troubling feature of the retrofitted models – additional interactions introduced solely to account for the OR scale – automatically yields a term in the superpotential of the correct order of magnitude.

5. $\mu$ term: This is easily generated in this framework, without elaborate additional sets of fields and/or arbitrary scales. At the messenger scale, $B_\mu$ vanishes; renormalization group evolution generates $B_\mu$ and large (but not excessively large) $\tan \beta$.

6. The problem of dimension five operators and proton decay can be readily resolved in models with $R$ symmetries, provided that the scale of supersymmetry breaking is low, as
in gauge mediation.

There are many open questions in this framework, some of which we have indicated. The space of models which dynamically break discrete $R$ symmetries – the generalization of gaugino condensation – needs further exploration. More detailed analysis of the $B_\mu$ term, and its implications for $\tan \beta$, is warranted. Another area where discrete $R$ symmetries might play an important role is cosmology. Such symmetries can lead to flat or nearly flat directions, which might be important in inflation and/or baryogenesis. These questions will all be taken up in future work.

There are also problems we have not dealt with here, most notably the so-called little hierarchy. We have little to add on this question. The little hierarchy might be ameliorated by “squashing” of the spectrum, as is possible in models of General Gauge Mediation [26]. It is also possible that some sort of mild anthropic constraint might account for a small hierarchy. For example, in a given model of inflation and reheating, there will be constraints on the mass of the gravitino. This, in turn, might force a higher scale of supersymmetry breaking than naive tuning arguments would suggest. Another class of questions has to do with axions. These, too, might force a rather high scale of supersymmetry breaking [27].

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