Pinning enhancement upon the magnetic flux trapping in the clusters of a normal phase with fractal boundaries

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The magnetic flux trapping in type-II superconductor containing fractal clusters of a normal phase, which act as pinning centers, is considered. The critical current distribution for an arbitrary fractal dimension of the boundaries of the normal phase clusters is obtained. It is revealed that the fractality of the cluster boundaries intensifies the pinning and thereby raises the critical current of a superconductor. The pinning gain coefficient as a function of critical current for different fractal dimensions is calculated.

Isolated clusters of a normal phase in superconductors can act as effective pinning centers [1]-[4]. The presence of such clusters may significantly affect the vortex dynamics and transport properties, especially when the clusters have fractal boundaries [5]-[7].

Let us consider a superconducting film containing columnar inclusions of a normal phase, which are oriented transversely to the film surface. When the specimen is cooled below the critical temperature in the magnetic field directed along the axis of the normal phase inclusions, the magnetic flux will be frozen in the isolated clusters of a normal phase so the space distribution of the trapped magnetic field will be two-dimensional. Let us suppose that the film surface fraction covered by the normal phase is below the percolation threshold for the transfer of magnetic flux. Then there is a superconducting percolation cluster in the plane of the film where a transport current can flow. When the current increases the trapped flux remains unchanged until the vortices start to break away from the clusters of pinning force weaker than the Lorentz force created by the transport current. As this happens, the vortices have to cross the surrounding superconducting space and they will first do that through the weak links, which connect the adjacent normal phase clusters between themselves. In high-temperature superconductors (HTS), which are characterized by an extremely short coherence length, such weak links form readily in sites of various structural defects [8]-[11]. Moreover, a magnetic field further reduces the coherence length [12], thus resulting in more easy weak links formation. In conventional low-temperature superconductors, which are characterized by a large coherence length, weak links can be formed due to the proximity effects in sites of minimum distance between the next normal phase clusters.

According to the local configuration of weak links each normal phase cluster has its own value of the critical current, which contributes to the overall statistical distribution. By the critical current of the cluster we mean the current of depinning, that is to say, such a current at which the magnetic flux ceases to be held inside the cluster of a normal phase. When a transport current is gradually increases, the vortices will break away first from clusters of small pinning force and, therefore, of small critical current. Thus the change in the trapped magnetic flux \( \Delta \Phi \) caused by the action of the transport current \( I \) is proportional to the total number of the normal phase clusters of critical currents less than a preset value \( I \). Therefore, the relative change in the magnetic flux can be expressed with the cumulative probability function \( F = F(I) \) for the distribution of the critical currents

\[
\frac{\Delta \Phi}{\Phi} = F(I), \quad F(I) = \Pr \{ \forall I_j < I \}, \quad (1)
\]

where \( \Pr \{ \forall I_j < I \} \) is the probability that any \( j \)th cluster has a critical current \( I_j \) less than a given upper bound \( I \).

At the same time, the larger the normal phase cluster size, the more weak links are along its border with the surrounding superconducting space, and therefore, the smaller is the critical current of this cluster. If the concentration of entry points into weak links per unit perimeter length is constant for all clusters regardless of their size, the critical current \( I \) is inversely proportional to the cluster perimeter length \( P \): \( I \propto 1/P \). Taking into account that the magnetic flux trapped in a cluster is proportional to its area \( A \), the relative change in the total trapped flux can be expressed with the cumulative probability function \( W = W(A) \) describing the distribution of the areas of the normal phase clusters:

\[
\frac{\Delta \Phi}{\Phi} = 1 - W(A), \quad W(A) = \Pr \{ \forall A_j < A \}, \quad (2)
\]
where \( \Pr \{\forall A_j < A \} \) is the probability that the area \( A_j \) of an arbitrary \( j \)th cluster does not exceed a given upper bound \( A \). The distribution function \( W = W(A) \) of the cluster areas can be found by a geometric probability analysis of electron photomicrographs of superconducting films [13]. Thus in the practically important case of YBCO films with columnar defects [14] the exponential distribution of the cluster areas can be realized:

\[
W(A) = 1 - \exp \left( -\frac{A}{\overline{A}} \right)
\]

(3)

where \( \overline{A} \) is the mean area of the cluster.

Thus, in order to clear up how the transport current acts on the trapped magnetic flux, it is necessary to find out the relationship between the distribution of the critical currents [Eq. (1)] and areas [Eq. (2)] of the normal phase clusters. As was first found in Ref. [7], clusters of a normal phase can have fractal boundaries, and this feature significantly affects the dynamics of the trapped magnetic flux. For fractal clusters the perimeter-area relation has the form

\[ P \propto A^{D/2} \]

(4)

which leads us to the following expression for the critical current of the cluster: \( I = \alpha A^{-D/2} \), where \( \alpha \) is the form factor and \( D \) is the fractal dimension of the cluster boundary.

The relation of Eq. (4) is consistent with the generalized Euclid theorem [15], which states that the ratios of the corresponding measures are equal when reduced to the same dimension. So it means that \( P^{1/D} \propto A^{1/2} \), which is valid both for Euclidean \( (D = 1) \) and fractal \( (D > 1) \) geometric objects. Inasmuch as the cluster boundary is a fractal, it is the statistical distribution of the cluster areas, rather than their perimeters, that is fundamental for finding the critical current distribution. The topological dimension of the cluster perimeter (equal to unity) does not coincide with its Hausdorff-Bezikovich dimension, which strictly exceeds unity. Therefore the perimeter length of a fractal cluster is not well defined, because its value depends on the resolution capacity of the cluster geometric size measurement. At the same time, the topological dimension of the cluster area is the same as the Hausdorff-Bezikovich one (both are equal to 2). Thus, the area restricted by the fractal curve is a well-defined finite quantity.

Speaking of the geometric characteristics of the normal phase clusters, we are considering the cross-sections of the extended objects, which indeed the normal phase clusters are, by the plane carrying a transport current. Therefore, though a normal phase cluster represents a self-affine fractal [16], we will analyze its geometric probability properties in the planar section only, where the boundary of the cluster is statistically self-similar.

In accordance with starting formulas of Eq. (1) and Eq. (2) the exponential distribution of the cluster areas of Eq. (3) gives rise to an exponential-hyperbolic distribution of critical currents:

\[
F(i) = \exp \left[ -\left( \frac{2 + D}{2} \right)^{2/D+1} i^{-2/D} \right]
\]

(5)

where \( i \equiv I/I_c \) is the dimensionless transport current and \( I_c = (2/(2+D))^{(2+D)/2} \alpha (\overline{A})^{-D/2} \) is the critical current of the transition into a resistive state.

The distribution function of Eq. (5) allows us to fully describe the effect of the transport current on the trapped magnetic flux (see Fig. 1). Knowing the cumulative probability function, the probability density \( f(i) \equiv dF/di \) for the critical current distribution can be readily derived:

\[
f(i) = \frac{2}{D} \left( \frac{2 + D}{2} \right)^{2/D+1} i^{-2/D-1} \exp \left[ -\left( \frac{2 + D}{2} \right)^{2/D+1} i^{-2/D} \right]
\]

The relative change in the trapped magnetic flux \( \Delta \Phi/\Phi \), which can be found from Eq. (5), is proportional to the density of vortices \( n \) broken away from the pinning centers by the current \( i \):

\[
n(i) = \frac{B}{\Phi_0} \int_0^i f(i') \, di' = \frac{B}{\Phi_0} \frac{\Delta \Phi}{\Phi},
\]

where \( B \) is the magnetic field and \( \Phi_0 \equiv hc/(2e) \) is the magnetic flux quantum \( (h \) is Planck’s constant, \( c \) is the velocity of light, and \( e \) is the electron charge). As may be seen from Fig. 1, breaking of the vortices away from the pinning centers becomes significant only for \( i > 1 \), i.e. after a resistive transition. Up to this point the trapped flux remains virtually unchanged because the Lorentz force created by such a small current is weaker than the pinning force.

Figure 1 shows how the fractality of the cluster boundary affects the magnetic flux trapping. Graph (a) corresponds to extreme case of Euclidean clusters \( (D = 1) \), while graph (c) relates to the clusters of boundaries with maximum fractality \( (D = 2) \); as an example, the Peano curves have such a fractal dimension.) Whatever the geometry and
morphology of the clusters may be, their graphs \(\Delta \Phi / \Phi \text{ vs } i\) will always fall within the region between these two limiting curves. As an example, graph (b) for the case of fractal dimension \(D = 1.5\) is shown.

Figure 1 demonstrates an important consequence of Eq. (5), according to which the fractality of normal phase clusters hinders the breaking away of the vortices and thereby strengthens pinning. As may be clearly seen from the inset of Fig. 1, the bell–shaped curve of the critical current distribution broadens out, moving towards greater magnitudes of current as the fractal dimension increases. The pinning amplification due to the fractality can be characterized by the pinning gain factor

\[
k_\Phi \equiv 20 \log \frac{\Delta \Phi (D = 1)}{\Delta \Phi \text{ (current value of } D)} \text{ dB}
\]

which is equal to the relative decrease in the fraction of magnetic flux broken away from fractal clusters of the dimension \(D\) compared to the case of Euclidean ones \((D = 1)\). This quantity can be calculated from the following formula:

\[
k_\Phi = \frac{20}{\ln 10} \left( \left( \frac{2 + D}{2} \right)^{2/D+1} i^{-2/D} - \frac{3.375}{i^2} \right)
\]

The dependences of the pinning gain on the transport current as well as on the fractal dimension are given in Fig. 2. The highest amplification is reached when the clusters have the maximum fractality \((D = 2)\):

\[
\max_D k_\Phi = \frac{20}{\ln 10} \left( \frac{4i - 3.375}{i^2} \right)
\]

Let us note that the pinning gain characterizes the transport properties of a superconductor in the range of the transport currents corresponding to a resistive state \((i < 1)\). At smaller currents the total trapped flux remains unchanged (see Fig. 1) for a lack of pinning centers of such small critical currents, so the breaking away of the vortices has not started yet. When the vortices leave the normal phase clusters and start to move, their motion induces an electric field, which, in turn, creates a voltage drop across the sample. Thus, at a transport current greater than the current of the resistive transition some finite resistance appears, so that the passage of electric current is accompanied by energy dissipation. As for any hard superconductor (type–II, with pinning centers) this dissipation does not mean the destruction of phase coherence yet. Some dissipation always accompanies any motion of a magnetic flux that can happen in a hard superconductor even at low transport current. Therefore the critical current in such materials cannot be specified as the greatest non-dissipative current. The superconducting state collapses only when the growth of dissipation becomes avalanche-like as a result of thermomagnetic instability.

Thus, the fractality of normal phase cluster boundaries strengthens the magnetic flux pinning. This phenomenon provides principally new possibilities for increasing the current-carrying capability of composite superconductors by optimizing their geometric morphological properties with no changes of the chemical constitution.

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FIG. 1. Effect of a transport current on trapped magnetic flux at different values of the fractal dimension. Curve (a) corresponds to the case of $D = 1$; curve (b) of $D = 1.5$; curve (c) of $D = 2$. The inset shows the corresponding critical current distributions.
FIG. 2. Pinning gain as a function of transport current at different values of the fractal dimension: $D = 1.5$ (b) and $D = 2$ (c). The inset shows the dependence of the pinning gain on the fractal dimension at fixed values of a transport current: $i = 1$ (curve (1)) and $i = 1.6875$ (here the pinning gain has a maximum, curve (2)).