Multiple layer Phase Shift
Linear Space-time Block Code for
High-speed Visible Light Communications

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Abstract

In this letter, we consider intensity modulation/direct detection (IM/DD) channel in the visible light communication (VLC) systems with multiple transmitter phosphor-based white light emitting diodes (LED) and single receiver avalanche photo diode (APD). We proposed a Multiple Layer Phase Shift Linear Space-time Block Code (MLPS-LSTBC). We show that our proposed code for VLC has the following main features: (a) The symbol transmission rate is \( \frac{N}{N+M-1} \), where \( N \) is the number of transmitter LED and \( M \) denotes the number of shift intervals contained by a single codeword per layer; (b) zero-forcing receiver can transform the virtual MIMO matrix channel into parallel sub-channels even without channel state information at the receiver side (CSIR); (c) Our MLPS-LSTBC can asymptotically enhance the spectral efficiency by \( \min(M,N) \), which is attractive for LED-based VLC with limited electrical modulation bandwidth. By simulations, we achieve the record data rate of 1.5 Gb/s with the bit error rate performance below the FEC limit of \( 2 \times 10^{-3} \) via multiple 100-MBaud transmission of OOK signal.

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I. Introduction

With sharply increasing wireless data demand and the saturation of radio frequency (RF) spectrum, VLC integrated with illumination has become a promising candidate to complement conventional RF communication. White LEDs used for VLC and simultaneous illumination can be categorized into two main types: red-green-blue (RGB) emitters and phosphor-based emitters. The phosphor-based white LED is more attractive for VLC due to its lower complexity and commercial availability. However, the intrinsic modulation bandwidth of typical phosphor-based white LED is limited to the several MHz range for the long response time of phosphor. So analog equalization is utilized to compensate for the insufficient frequency response. These experiments require filtering the low-speed phosphorescent component to increase the bandwidth of the VLC system.

To make the best of the limited bandwidth, spectrally efficient techniques have been introduced to VLC, such as discrete multitone (DMT), wavelength division multiplexing (WDM) and orthogonal frequency division multiplexing (OFDM) etc. Actually, DMT and WDM require complex circuitries. Since a directly-modulated LED is an intensity modulator, OFDM requires searching for an optimum DC operating point and optimal transmission signal power to modulate the LED intensity by reducing peak-to-average power ration (PARP). The above-mentioned works focus on the utilization of the single LED’s electrical bandwidth. Actually, at the transmitting end, each white light bulb is an array of LEDs. When the receiver is also equipped with multiple LEDs, a promising solution to boost the data rate without any bandwidth or power expansion is using MIMO techniques, where spatial multiplexing technique with ZF detection is considered. Usually, the optical MIMO channel is rank-deficient and requires the large enough size of the imaging lens and the proper angle of incidence of the lightwave which is not fit for practical design of portable VLC system.

Actually, When an array of LEDs is simultaneously communicating with a single APD, it is multi-input single output (MISO) system with multiple LEDs as effective transmitter antennas and the APD as a single receiver. MISO is a rank-deficient version of MIMO fit for VLC. Instead
of focusing on the modulation bandwidth of single LED and compensating for the slow decay of the phosphor or create a new format of LED, our goal is to design linear space-time block codes to increase the data rate that can be achieved. Motivated by the tapped-delay-line phenomenon and linear toeplitz space time block codes [8], we propose a Multiple layer Phase Shift Linear Space-time Block Code (MLPS-LSTBC) with multiple transmitter LEDs and a single high-rate APD receiver asymptotically enhancing the spectral efficiency by 

$$\min(M,N),$$

where here $N$ is the number of transmitter LED and $M$ denotes the number of shift intervals contained by a single codeword per layer and $N+M-1$ is the number of channel uses for $N$ transmitted symbols.

### II. Multiple Layer Phase Shift Linear Space-time Block Code (MLPS-LSTBC)

First, we introduce the proposed codewords of MLPS-LSTBC

$$\mathcal{T}(s,M,N) = [s_1g_1(t), s_2g_2(t), \cdots, s_Ng_N(t)]^T$$

(1)

Where $g_i(t) = \begin{cases} 
1, & \text{if } (i-1)T_s \leq t \leq (M+i-1)T_s \\
0, & \text{otherwise.}
\end{cases}$ and $T_s$ is the sample interval. From (1), we notice that the $i (i = 1, \cdots, N)$th layer of the codeword is essentially a single information-bearing codeword within $M$ time slots. The $(i+1)$th layer is phase-shifted behind the $i$th layer by a single time slot. Until time slot $(M+i-1)$, $s_i$ is fully fed to $i$th transmitter LED for transmission.

With high-speed sampling process at the receiver APD, each layer is transformed into $M$ sampled values. With all the layers sampled, the codeword is transformed into an $N \times (N+M-1)$ matrix made up of the $NM$ sample values. Alternatively, $\mathcal{T}(s,M,N)$ can be explicitly written out as

$$[\mathcal{T}(s,M,N)]_{i,j} = \begin{cases} 
 s_{ij}, & \text{if } 0 \leq i-j \leq M, \\
0, & \text{otherwise.}
\end{cases}$$

(2)

Actually, $[s_{i1}, s_{i2}, \cdots, s_{iM}]$ is the vector made up of $M$ sampled value of $s_i$ and any $s_{ij}$ is a duplicate of $s_i$. Without confusion, we still let $s_i$ denote single $s_{ij}$ value for notational simplicity in the following. As shown in (2), $\mathcal{T}(s,M,N)$ is kind of commonly used space-time codeword. This is why we name our proposed scheme MLPS-LSTBC.

At the receiver, the received sampled values can be written as a vector such that

$$\mathbf{r} = \mathcal{T}^T(h,M,N) \mathbf{s} + \mathbf{n}$$

(3)
where \( \mathbf{r} = [r_1, r_2, \cdots, r_{N+M-1}]^T \), \( \mathbf{h} = [h_1, h_2, \cdots, h_M]^T \), \( \mathbf{n} = [n_1, n_2, \cdots, n_{N+M-1}]^T \). By some mathematical operations, (3) can be rewritten as

\[
\mathbf{r} = T(e_M, M, N) \text{diag}(h_1, h_2, \cdots, h_N) \mathbf{s} + \mathbf{n}
\]

Where \( e_M = [1, 1, \cdots, 1]_M^T \).

Comments:

1) Obviously, we can conclude that the symbol transmission rate of our MLPS-LSTBC is \( R = N/(N+M-1) \). Therefore, for a fixed \( M \), the transmission rate \( R \) can approach one if the transmitter LED number \( N \) is sufficiently large. With the intended repetitions of \( M \) time slots, the actual symbol rate of the \( i \)th substream for the \( i \)th transmitter LED is decreased by \( 1/M \). Despite the in-fact low symbol transmission rate, the receiver symbol rate can be high enough all the same. If the symbol transmission rate is identically as high as other schemes, the receiver symbol rate can be increased by a factor of \( M \). So the spectral efficiency is \( MN/(M + N - 1) \) approaching \( \min(M, N) \) with large enough \( M \) or \( N \).

2) From (3), we notice that the original MISO channel is transformed into a visual MIMO channel. When \( \mathbf{h} = [1, 1, \cdots, 1]_M^T \) and \( MN \) input signal symbols at the instance \( i \) are transmitted, the received signal is

\[
\mathbf{r}_i^{m_N+M-1} = e_M \otimes [h_i \otimes s_i, h_{i+1} \otimes s_{i+1}, \cdots, h_{i+m} \otimes s_{i+m}] + n_{i,m_N+M-1}^i
\]

where \( h_i \otimes s_i = [h_{i,1}s_{i,1}, h_{i,2}s_{i,2}, \cdots, h_{i,N}s_{i,N}] \) and \( \otimes \) denotes the convolution operation. Such a channel is a special artificial convolutive channel and hence, we can utilize the efficient Viterbi algorithm to detect the signal \( s \) with perfect CSIR. The vector \( e_M \) is actually the coefficient of the artificial tapped-delay-line and can be deliberately designed with proper optimization such waterfilling allocation form of limited transmission resources. Actually, \( e_M \) is an equivalent low pass filter before transmission resulting in a compressed spectral domain by a multiplication factor of \( 1/M \). With the actual surprising high-speed transmission beyond the limited electrical width, nevertheless, it appears to VLC channel that the modulated signal width can be mysteriously within the cut-off frequency.

3) From (4), the effective channel matrix consists of diagonal matrix with diagonal entries \( h_{i,i} = 1, 2, \cdots, N \) and full-rank Toeplitz matrix \( T(e, M, N) \), i.e.,

\[
(T^T(\alpha, M, N) T(\alpha, M, N))^{-1} T^T(\alpha, M, N) T(h, M, N) = \text{diag}(h_1, h_2, \cdots, h_N)
\]
So, with generalized inverse matrix of $T(\alpha,M,N)$ as the zero-forcing receiver, the effective channel is transformed into $\text{diag}(h_1,h_2,\cdots,h_N)$ with $N$ parallel eigen-beam directions even without any CSIR. The spatial-temporal power allocation across these beams can add an array gain by exploiting the CSIT (Channel State Information at the Transmitter) to precode the transmitted signals or CSIR to decode the received signals. It helps to reduce receiver complexity for higher spatial-rates by allowing parallel channel transmissions. This is one of the main advantages of the MLPS-LSTBC.

III. DIVERSITY ANALYSIS OF ZF RECEIVER FOR MLPS-LSTBCs

In this section, we will show that our MLPS-LSTBCs can provide full diversity even for the linear ZF receiver. In the following, and the derivation procedures is motivated by [8].

Lemma 1: [8] There exists a positive definite constant $\tilde{C}$ such that for any nonzero vector $\alpha$, the inequality holds. $\tilde{C} \|\alpha\|^2K \leq \det (T^H (\alpha,L,K) T (\alpha,L,K)) \leq \|\alpha\|^2K$. [\[\|\alpha\|^2\]

For notional simplicity, let $P=T^H (h,M,N) T (h,M,N)$. By Lemma 1, it is immediate to get

$\tilde{C} \|h\|^{2N}, [P^{-1}]_{kk} \geq C \|h\|^2$, where $C=\tilde{C} \prod_{i=1}^{N} |h_i|^2/\|h\|^{2N}$ and $\tilde{C}=(M/N)^N$.

Theorem 1: The proposed MLPS-LSTBC provides full diversity for zero-forcing receiver when QAM signal is transmitted.

Proof: First we need to derive the expressions of symbol error probabilities for square QAM modulation when the ZF receiver is employed. The SEP of a ZF receiver for the square QAM signal $s_k$ is given by Lemma 2

$P_{QAM}(h,s_k)=4 \left(1-\frac{1}{\sqrt{D}}\right) Q \left(2 \left(1-\frac{1}{\sqrt{D}}\right) \int_0^{\pi/4} \exp \left(-\frac{3E_s}{2(D-1)\sigma^2|h_{kk}|^2 \sin^2 \theta} \right) d\theta \right.$

$+ \left. \frac{4}{\pi \sqrt{D}} \left(1-\frac{1}{\sqrt{D}}\right) \int_{\pi/4}^{\pi/2} \exp \left(-\frac{3E_s}{2(D-1)\sigma^2|h_{kk}|^2 \sin^2 \theta} \right) d\theta \right)\)$. It is convent to use the following alternative expressions for the $Q$ function and the $Q^2$ function. We obtain $P_{QAM}(h,s_k) \leq \frac{4}{\pi \sqrt{D}} \left(1-\frac{1}{\sqrt{D}}\right) \int_{\pi/4}^{\pi/2} \exp \left(-\frac{3E_s}{2(D-1)\sigma^2|h_{kk}|^2 \sin^2 \theta} \right) d\theta \)$. This can be upper bounded by $P_{QAM}(h,s_k) \leq 4 \left(1-\frac{1}{\sqrt{D}}\right) \int_0^{\pi/4} \exp \left(-\frac{3E_s}{2(D-1)\sigma^2|h_{kk}|^2 \sin^2 \theta} \right) d\theta$. Taking an average over the random vector $h$ yields

\[\frac{1}{D} \sum_{k=1}^{D} P_{PSK}(h,s_k) \leq \frac{1}{D} \sum_{k=1}^{D} \frac{1}{(D-1) \exp (-a \|h\|^2) \frac{(D-1)}{D} \exp (-aC \|h\|^2)},\]

where, $a=3E_s/(2(D-1)\sigma^2)$. Taking an average over the random vector $h$ yields

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$$E \left[ P_{CAP}^{ZF} (h) \right] \leq \frac{(D-1)}{D} \det \left( I + aCM \Sigma \right)^{-1} \leq \frac{(D-1)}{D} \det \left( CM \Sigma \right)^{-1} a^{-N}.$$ This completes the proof of Theorem 1. □

The proposed MLPS-LSTBC provides full diversity for zero-forcing receiver for BPSK signal as an example of QAM. In IM/DD channel, the phase information is physically ignored by the intensity modulator and direction detector [9]. The only difference between OOK and BPSK is that the phase of the former is continual and the latter is not. Without consideration of the phase difference, OOK is also with the same full diversity for zero-forcing receiver as BPSK. By Theorem 1, it is immediate to get

Corollary 2: With zero-forcing receiver, the proposed MLPS-LSTBC provides full diversity for OOK signal as an example of QAM.

IV. SIMULATIONS AND DISCUSSIONS

The performances of MLPS-LSTBC and DCO-OFDM systems with LED clipping distortion are analyzed through Monte Carlo simulations. The average electrical signal power is varied from 0 dBm to 60 dBm with AWGN power -20 dBm. As a result, the simulated electrical SNR range is from 20 dB to 80 dB, within the reported SNR values for indoor VLC systems [10]. We use the one-order betterworth low-pass filter to represent the frequency-selective characteristics of IM/DD channel. In all analyses, a channel bandwidth of $B = 100$ MHz reported by [2], a number of subcarriers of $N_s = 1024$ for QAM, and a number of guard interval subcarriers of $N_g = 4$ are considered [11]. The clipping level [12] is at 13 dB equal to 43 dBm. In the legends of the Fig.1, as an example, $16 \times 32$ denotes $N = 16$ and $M = 32$. We just simulate the performance of MLPS-LSTBC with $M = N$. From Fig.1, the following trends can be observed:

1) The performance enhancement for DCO-OFDM and MLPS-LSTBC, when increasing the power from 0 dBm to the optimum value, can be explained by the increase in the SNR and the absence of signal clipping at low amplitudes.

2) OOK is almost the most practical modulation scheme with relatively low complexity of detection. Clipped OOK is still in the form of OOK with the signal power constant equal the clipped level; so the BER curves of MPLS-LSTBC in the high power regimes tends to show the "floor" phenomenon. The OFDM signal exhibits a high PAPR. The high PAPR in OFDM stems from the superposition of a large number of usually statistically independent sub-channels that can constructively sum up to high signal peaks in the time
domain. So the performance of clipped DCO-OFDM becomes deteriorated sharply at the clipping threshold of LED non-linearity effect. High modulation orders, such as 256-QAM, are more sensitive to clipping distortions. So the MLPS-LSTBC is of better performance against the clipping effect of LED. Therefore, system design should consider LED clipping effects and should optimize the OFDM signal power and the considered modulation order.

3) Within the BER margin of $2 \times 10^{-3}$ given that such error rate can still be successfully handled by the FEC mechanism, we demonstrated transmission at 1500 Mbit/s gross data rate by simulations. DCO-OFDM [12] may achieve the equal rate equipped with extremely high modulation order $2^{15}=32768$. 256-QAM is beyond the tolerance of total BER. Say nothing of 32768-QAM which is inevitably degraded sharply. The reported record data rate 1.1 Gps [3] is achieved 220-MBaud for 32-CAP signals by using optical blue filter, precompensation and DFE to improve electrical modulation bandwidth of LED. Our record data rate is 1.5Gps by collaborating an array of LED spatially and temporally. Our 1.5Gps requires almost 128-CAP modulation ($2^{1500/220(>7)} \rightarrow 2^8 = 256$). It is reported in [3] that the maximum symbol rate to achieve the BER of $\leq 10^{-3}$ for 128-CAP is 110-MBaud. 256-CAP may be of no better performance than that of 128-CAP. Moreover, CAP has the same spectral characteristics and theoretical performance as QAM [13].
V. CONCLUSION AND FUTURE WORK

In this letter, we designed MLPS-LSTBC with full diversity and high the spectral efficiency for IM/DD wireless VLC systems. We proved that our proposed codes can enhance the spectral efficiency by $\min(M,N)$ with one of $M$ and $N$ is large enough without using optical blue filter, precompensation and DFE. MLPS-LSTBC requires a sample rate as high as the expected available symbol rate. For OOK modulation, G-bit transmission requires APD with several G-Hz sample rate. The requirement of high-speed sample can be relaxed by using more spectral-efficient modulation techniques. So the future work may focus on the combination of MLPS-LSTBC with CAP or proposing new carrierless power- and spectral-efficient modulation techniques for unipolar intensity modulator of VLC. This issue is now under our investigation.

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