Higgs Decay to Top Quarks at Hadron Colliders

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Abstract
Higgs bosons which decay principally to top quarks, such as in the minimal supersymmetric model, produce a peak-dip structure in the $gg \rightarrow t\bar{t}$ invariant-mass spectrum. This structure is potentially observable at the CERN Large Hadron Collider.
In the standard model of the electroweak interaction, the electroweak symmetry is broken by a scalar $SU(2)_L$ doublet Higgs field which acquires a vacuum-expectation value. This mechanism provides masses for the weak vector bosons, as well as for the quarks and leptons. However, the mechanism which is responsible for the quark and lepton masses need not be as closely linked to the mechanism which generates the weak-vector-boson masses as it is in the standard model. Since fermion masses violate the $SU(2)_L$ symmetry, whatever provides their masses must break this symmetry, and contribute to the weak-vector-boson masses. However, this contribution may be only a small component of the weak-vector-boson masses, which may arise dominantly via another mechanism [1].

An explicit realization of this scenario is provided by a two-Higgs-doublet model. One doublet may acquire a much larger vacuum-expectation value than the other doublet, and thereby provide the dominant contribution to the weak-vector-boson masses. However, this doublet need not couple to fermions, in which case the fermion masses arise solely from the Higgs doublet with a small vacuum-expectation value [2].

A two-Higgs-doublet model produces a spectrum of physical particles which consists of two neutral Higgs scalars, one neutral pseudoscalar, and a charged Higgs boson [2, 3]. The Higgs scalar most closely associated with the doublet with small vacuum-expectation value couples very weakly to the weak vector bosons, but couples with enhanced strength to the fermions (with respect to the standard-model coupling). The pseudoscalar Higgs particle does not couple to the weak vector bosons at all, but couples to the fermions with enhanced strength.

In this letter we consider searching for a Higgs boson, with zero or suppressed coupling to the weak vector bosons, via its decay to $t\bar{t}$. This particle is copiously produced at a hadron collider through gluon-gluon collisions, via a virtual top-quark loop [4], as shown in Fig. 1(a). However, at a hadron collider there is a large irreducible background from the QCD production of top quarks, shown in Fig. 1(b). We will show that the signal, $gg \rightarrow H \rightarrow t\bar{t}$,
and the background, $gg \rightarrow t\bar{t}$, interfere, generically resulting in a peak-dip structure at the Higgs mass. This phenomenon was first recognized in Ref. [5]. In some cases the dip dominates, such that the signal for the presence of the Higgs boson is a small \textit{deficit} in the production of $t\bar{t}$ pairs of invariant mass near the Higgs mass.

Another model which yields Higgs bosons with suppressed coupling to weak vector bosons is the Higgs sector of the minimal supersymmetric model [3]. This model necessarily has two Higgs doublets, one which couples to fermions with $T_{3L} = +1/2$ and one which couples to fermions with $T_{3L} = -1/2$. As usual, the pseudoscalar Higgs boson does not couple to the weak vector bosons. If this pseudoscalar is relatively heavy (greater than about 100-150 GeV, depending on the ratio of the vacuum-expectation values of the two Higgs doublets), the heavier Higgs scalar (whose mass is then close to that of the pseudoscalar) couples only weakly to the weak vector bosons. This is true regardless of the ratio of vacuum-expectation values of the two Higgs doublets.

For definiteness, we consider the Higgs sector of the minimal supersymmetric model, with the ratio of the vacuum-expectation values of the two Higgs doublets close to unity. The heavier Higgs scalar, $H$, nearly decouples from the weak vector bosons if its mass is greater than about 150 GeV, and its coupling to top quarks is close to standard-model strength. Our qualitative results are generic to the scenario where the dominant decay of the Higgs boson is to top quarks. If the ratio of vacuum-expectation values is much larger than unity, the coupling of $H$ to the top quark is suppressed and its coupling to the bottom quark enhanced, such that the production of the Higgs via a bottom-quark loop, and its decay to $b\bar{b}$, become non-negligible. In the case of a non-supersymmetric scalar Higgs boson from a two-doublet model, the decay of the Higgs to weak vector bosons may also be non-negligible. Both of these scenarios decrease the branching ratio of $H \rightarrow t\bar{t}$, and hence decrease the size of the effect of the Higgs boson on top-quark production.

The differential cross section for $gg \rightarrow t\bar{t}$, including the squares of the scalar-Higgs amplitude and the continuum QCD amplitude, as well as the interference of the two amplitudes,
\[
\frac{d\sigma}{dz} = \frac{\alpha_s^2 G_F^2 m_t^2 s^2}{1536\pi^3} \frac{N(s/m_t^2)}{s - m_H^2 + im_H \Gamma_H(s)} \left| \frac{N(s/m_t^2)}{s - m_H^2 + im_H \Gamma_H(s)} \right|^2
- \frac{\alpha_s^2 G_F m_t^2 s}{384\pi\sqrt{2}} \beta^3 \left( \frac{1}{p_1 \cdot p_3} + \frac{1}{p_2 \cdot p_3} \right) \text{Re} \left[ \frac{N(s/m_t^2)}{s - m_H^2 + im_H \Gamma_H(s)} \right]
+ \frac{d\sigma_{QCD}}{dz} (1)
\]

where \(p_{1,2}\) are the momenta of the incoming gluons, \(p_{3,4}\) are the outgoing top-quark and top-antiquark momenta, \(z\) is the cosine of the scattering angle between an incoming gluon and the top quark, \(m\) is the top-quark mass, and \(\beta \equiv (1 - 4m^2/s)^{1/2}\) is the velocity of the top quark and top antiquark in the center-of-momentum frame. The dot product of the four momenta are

\[
\begin{align*}
    p_1 \cdot p_3 &= \frac{s}{4} (1 - \beta z) \\
    p_2 \cdot p_3 &= \frac{s}{4} (1 + \beta z). 
\end{align*}
\]

The function associated with the virtual top-quark loop is

\[
N(s/m_t^2) = \frac{3}{2} \frac{m_t^2}{s} \left[ 4 - \left( 1 - \frac{4m_t^2}{s} \right) I(s/m_t^2) \right]
\]

where

\[
I(s/m_t^2) = \left[ \ln \frac{1 + \beta}{1 - \beta} - i\pi \right]^2 (s > 4m_t^2).
\]

The energy-dependent Higgs width is

\[
m_H \Gamma_H(s) = \frac{3G_F m_t^2 s}{4\pi\sqrt{2}} \beta^3.
\]

The cross section for the continuum QCD production of \(gg \to t\bar{t}\) is

\[
\frac{d\sigma_{QCD}}{dz} = \frac{\pi\alpha_s^2}{12s} \beta \left( \frac{s^2}{(p_1 \cdot p_3)^2} - 9 \right) \left[ \frac{(p_1 \cdot p_3)^2}{s^2} + \frac{(p_2 \cdot p_3)^2}{s^2} + \frac{m_t^2}{s} - \frac{m_t^4}{4p_1 \cdot p_3 p_2 \cdot p_3} \right].
\]

\(^1\)The total cross section may be obtained from this expression by integrating over \(z\), the cosine of the scattering angle between a gluon and the top quark. The resulting expression agrees with that of Eq. 7 of Ref. 5, except the interference term, which is a factor of 2 too large in that paper. The resulting cross section, Fig. 2 in that paper, should display a large peak, as shown, followed by a small dip, rather than an equally-large peak and dip.
We show in Fig. 2 the cross section for $gg \rightarrow t\bar{t}$ as a function of the $t\bar{t}$ invariant mass, $\sqrt{s}$, for $m_t = 170$ GeV (the approximate central value from precision electroweak experiments and $m_H = 400, 500, 600, 700, \text{ and } 800$ GeV. For $m_H = 400$ GeV, the Higgs boson produces a narrow peak with a width of about 3.3 GeV. For $m_H = 500$ and 600 GeV, the presence of the Higgs boson produces a peak, followed by a dip, near the Higgs mass. For larger Higgs masses the peak is absent, and the Higgs reveals itself as a dip in the $t\bar{t}$ invariant-mass spectrum. For $m_H = 500$ GeV, the total top-quark cross section differs little from the cross section with no Higgs present, due to the cancellation between the peak and the dip. For larger Higgs masses, the presence of the Higgs results in a small decrease in the total top-quark cross section.

We also consider the effect of the pseudoscalar Higgs boson, $A$, on the $t\bar{t}$ invariant-mass spectrum. This particle does not couple to the weak vector bosons, and couples to the top quark with standard-model strength if the ratio of the vacuum-expectation values of the two Higgs doublets is close to unity. For larger values of this ratio, the pseudoscalar Higgs coupling to top quarks is suppressed, so its width is narrower. The production of the pseudoscalar Higgs via a bottom-quark loop and the decay to $b\bar{b}$ may also become significant, reducing the branching ratio to $t\bar{t}$ and the effect of the Higgs on top-quark production. However, the pseudoscalar Higgs does not couple to weak vector bosons in a generic two-Higgs-doublet model, so there is no competition from the decay to weak vector bosons.

The differential cross section for $gg \rightarrow t\bar{t}$, including the squares of the pseudoscalar-Higgs amplitude and the continuum QCD amplitude, as well as the interference of the two amplitudes, is

\[
\frac{d\sigma}{dz} = \frac{3\alpha_s^2 G_F^2 m_T^2 s^2}{2048\pi^3} \beta \left| \frac{P(s/m^2)}{s - m_T^2 + im_T\Gamma_T(s)} \right|^2 \\
- \frac{\alpha_s^2 G_F^2 m_T^2 s}{256\pi\sqrt{2}} \beta \left( \frac{1}{p_1 \cdot p_3} + \frac{1}{p_2 \cdot p_3} \right) \text{Re} \left[ \frac{P(s/m^2)}{s - m_T^2 + im_T\Gamma_T(s)} \right] \\
+ \frac{d\sigma_{QCD}}{dz}.
\]

\( \text{(7)} \)

The cross section in the vicinity of the Higgs mass depends only on the ratio $m_t/m_H$, so the qualitative results for other top-quark masses can be obtained by scaling the Higgs mass, modulo the effect of the change in the Higgs width.
There is no interference between the scalar-Higgs and the pseudoscalar-Higgs amplitudes. The function associated with the virtual top-quark loop is

\[ P(s/m^2) = -\frac{m^2}{s}I(s/m^2) \]  

(8)

where \( I(s/m^2) \) is given above, and the energy-dependent Higgs width is

\[ m_A \Gamma_A(s) = \frac{3G_F m^2 s}{4\pi \sqrt{2} \beta} \]  

(9)

The resulting cross sections, given in Fig. 3, are qualitatively similar to the cross sections in Fig. 2 for the scalar Higgs. The pseudoscalar-Higgs width is suppressed by \( \beta \), rather than \( \beta^3 \) as is the scalar-Higgs width, so the pseudoscalar Higgs boson is noticeably wider than the scalar Higgs boson for \( m_{H,A} = 400 \) and \( 500 \) GeV.

The peak-dip structure in the \( t\bar{t} \) invariant-mass spectrum due to the scalar or pseudoscalar Higgs bosons can be viewed as due to a final-state interaction of the \( t\bar{t} \) pair \[3, 4, 5, 6\]. If one cuts vertically through the top-quark loop in Fig. 1(a), putting the cut propagators on shell, the amplitude factorizes into the product of the amplitudes for the QCD production of \( t\bar{t} \) (Fig. 1(b)) and the elastic scattering of the \( t\bar{t} \) pair via the Higgs resonance, times a phase-space factor \( \beta \). The elastic \( t\bar{t} \) partial-wave scattering amplitude may be represented by a unitary phase-shift expression,

\[ \frac{1}{\beta} \frac{-m_H \Gamma_H(s)}{s - m_H^2 + im_H \Gamma_H(s)} = \frac{1}{\beta} e^{i\delta} \sin \delta \]  

(10)

where \( \delta(s) \) is the phase shift, which passes through \( \pi/2 \) at \( s = m_H^2 \). Using the Cutkosky rules, the absorptive part of the color-singlet, zeroth-partial-wave Higgs amplitude is thus \( a_0^{abs} = ia_0^{QCD} e^{i\delta} \sin \delta \), where \( a_0^{QCD} \) is the color-singlet, zeroth partial wave of the QCD amplitude. The total color-singlet, zeroth-paritial-wave amplitude is the sum of the QCD amplitude and the Higgs amplitude,

\[ a_0 = a_0^{QCD} + ia_0^{QCD} e^{i\delta} \sin \delta + a_0^{disp} e^{i\delta} \sin \delta \]

\[ = e^{i\delta}[a_0^{QCD} \cos \delta + a_0^{disp} \sin \delta] \]

\[ = a_0^{QCD} \frac{s - m_H^2}{s - m_H^2 + im_H \Gamma_H(s)} + a_0^{disp} \frac{-m_H \Gamma_H(s)}{s - m_H^2 + im_H \Gamma_H(s)} \]  

(11)
where $\alpha_{0}^{disp}$ is the one-loop-induced coupling of the Higgs to the initial gluons due to the dispersive part of the top-quark loop. We see that the interference of the absorptive part of the loop diagram with the tree diagram yields an exact zero at the resonance energy, $s = m_{H}^{2}$ ($\delta = \pi/2$). The one-loop-induced coupling due to the dispersive part of the loop produces the usual resonant peak, so generically a peak-dip structure results. If the dispersive part of the loop is small compared with the absorptive part, the dip will be the dominant feature of the structure in the invariant-mass spectrum. This is what occurs for $m_{H,A} = 700$ and 800 GeV in Figs. 2 and 3.

Peak-dip structures due to final-state interactions are well-known in hadronic physics 9, 10, 11. An example of a dip is the effect of the $f_{0}(975)$ on the $\pi\pi$ invariant-mass spectrum in double-diffractive pion production 13. The argument in the preceding paragraph shows that a peak-dip structure in the $t\bar{t}$ invariant-mass spectrum results whenever there is a resonant final-state interaction of the $t\bar{t}$ pair, regardless of the physics which produces the resonant final-state interaction.\footnote{However, a spin-one resonance cannot effect the $t\bar{t}$ invariant-mass distribution at leading order, because the incoming gluons are forbidden to form a state of unit total angular momentum by Yang’s theorem 14.} It also shows that the peak-dip structure is not washed out by QCD corrections, since it depends only on the final-state interaction of the $t\bar{t}$ pair, whose production may in principle be calculated to any order in QCD.

At the Fermilab Tevatron, a top quark of mass greater than 150 GeV is predominantly produced via quark-antiquark annihilation. For example, for $m_{t} = 170$ GeV, the gluon-fusion contribution to the total top-quark cross section is about 20% 8. Thus any structure in the $gg \rightarrow t\bar{t}$ cross section due to a Higgs boson would be buried beneath the $q\bar{q} \rightarrow t\bar{t}$ continuum, and require very large statistics to uncover. In contrast, at a higher-energy hadron collider, such as the CERN Large Hadron Collider (LHC), the gluon-fusion process is the dominant source of top quarks. At a 4 TeV $p\bar{p}$ collider, the gluon-fusion process accounts for about half of the total top-quark cross section (for $m_{t} \approx 170$ GeV).

The $t\bar{t}$ invariant mass can be reconstructed at hadron colliders via $t\bar{t} \rightarrow b\bar{b}W^{+}W^{-}$, followed by hadronic decay of one $W$ boson and leptonic decay of the other. The presence of
two $W$ bosons and two top quarks constrains the kinematics sufficiently that the event can be fully reconstructed, in principle. We assume in the following that the $t\bar{t}$ invariant mass can be reconstructed with reasonable resolution.

The effect of the scalar and pseudoscalar Higgs bosons on the $t\bar{t}$ invariant-mass distribution is quite small, and will require large statistics to observe. For example, the peak and dip in the invariant-mass distribution in Fig. 2 due to a scalar Higgs of mass 500 GeV are each about a 3.5% effect, each contained in a bin of width about 25 GeV. The statistical significance of the peak and dip depend on the number of $t\bar{t}$ events in the mass bin containing the signal. We present in Fig. 4 the luminosity function of gluon-gluon collisions at hadron colliders of various energy. Multiplication of the $gg \to t\bar{t}$ cross section in Figs. 2 and 3 at a given invariant mass, $\sqrt{s} = M$, by the luminosity function at the same invariant mass yields the inclusive hadronic differential cross section, $d\sigma/dM$, for $pp, p\bar{p} \to t\bar{t} + X$. At the LHC, with 100 $fb^{-1}$ of integrated luminosity, the raw number of events in an invariant-mass bin of width 25 GeV at $M = 500$ GeV is about $4 \times 10^6$. Including a branching ratio for the one hadronic - one leptonic decay of the $W$ bosons of about 0.3, and a factor of 1/3 to account for acceptance and efficiencies, leaves approximately $4 \times 10^5$ events. This is far more than necessary to detect a 3.5% effect at the 5$\sigma$ level. For a scalar Higgs of mass 800 GeV, the dip corresponds to a 1.5% decrease in the number of events in a bin of width 50 GeV. The number of events, including branching ratios, acceptance, and efficiencies, is about $6 \times 10^4$, yielding a significance of about 3.5$\sigma$ for the signal, marginally significant. The results for the pseudoscalar Higgs are comparable.

Lower-energy machines also have the potential to observe a signal, depending on the luminosity and the Higgs mass. A 4 TeV $pp$ collider requires about 100 $fb^{-1}$ of integrated luminosity to detect a scalar Higgs of mass 500 GeV with a 5$\sigma$ significance. However, the Fermilab Tevatron would require an integrated luminosity far in excess of 100 $fb^{-1}$ to detect such a Higgs boson.

In contrast to the class of models being considered here, in the standard Higgs model the branching ratio of the Higgs boson to $t\bar{t}$ is much less than unity. For $m_t = 170$ GeV,
the branching ratio is at most 18% (this value occurs for $m_H \approx 500$ GeV). The Higgs boson produces only a tiny wiggle in the $t\bar{t}$ invariant-mass spectrum at the Higgs mass, too small to appear in Figs. 2 or 3.

We conclude that the peak-dip structure in the $gg \to t\bar{t}$ invariant mass distribution, due to a scalar or pseudoscalar Higgs boson decaying principally to top quarks, is potentially observable at the LHC, if the $t\bar{t}$ invariant-mass resolution is less than the width of the Higgs boson. Although the standard-model Higgs boson is not of this type, Higgs bosons which decay predominantly to top quarks are natural in two-Higgs-doublet models, in particular the heavy scalar Higgs, $H$, and the pseudoscalar Higgs, $A$, of the minimal supersymmetric model. The peak-dip structure is unfortunately not observable at the Fermilab Tevatron with 1 $fb^{-1}$ of integrated luminosity, due to the limited statistics and the relatively large rate of production of top quarks via quark-antiquark annihilation.

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Figure Captions

Fig. 1 - Feynman diagrams for $gg \rightarrow t\bar{t}$: (a) s-channel Higgs scalar ($H$) or pseudoscalar ($A$) exchange via a top-quark loop (crossed diagram not shown), (b) leading-order QCD ($u$- and s-channel diagrams not shown).

Fig. 2 - Total cross section for $gg \rightarrow t\bar{t}$, for $m_t = 170$ GeV, as a function of the $t\bar{t}$ invariant mass. The calculation includes the effects of the heavy Higgs scalar ($H$) of the minimal supersymmetric model (with the ratio of the vacuum-expectation values of the two Higgs doublets close to unity), as well as the continuum QCD production of $t\bar{t}$, for $m_H = 400, 500, 600, 700,$ and $800$ GeV.

Fig. 3 - Same as Fig. 2, but for the Higgs pseudoscalar ($A$).

Fig. 4 - Luminosity function for gluon-gluon collisions at hadron colliders of various energy. Multiplication of the $gg \rightarrow t\bar{t}$ cross section in Figs. 2 and 3 at a given invariant mass, $\sqrt{s} = M$, by the luminosity function at the same invariant mass yields the inclusive hadronic differential cross section, $d\sigma/dM$, for $pp, p\bar{p} \rightarrow t\bar{t} + X$. The gluon distribution functions have been evaluated at $\mu = M$. 
Figure 1
Figure 2
Figure 3

\[ hg \rightarrow t\bar{t} \]
\[ m_t = 170 \text{ GeV} \]

\[ \sigma \text{ (pb)} \]

\[ m_H \text{ (GeV)} \]

\[ \sqrt{s} \text{ (GeV)} \]
Figure 4