Competing superfluid orders in spin-orbit coupled fermionic cold atom optical lattices

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The Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) phase, a superconducting state with non-zero total momentum Cooper pairs in a large magnetic field, was first predicted about 50 years ago, and since then became an important concept in many branches of physics. Despite intensive search in various materials, unambiguous experimental evidence for the FFLO phase is still lacking in experiments. In this Letter, we show that both FF (uniform order parameter with plane-wave phase) and LO phase (spatially varying order parameter amplitude) can be observed using fermionic cold atoms in spin-orbit coupled optical lattices. The increasing spin-orbit coupling enhances the FF phase over the LO phase. The coexistence of superfluid and magnetic orders is also found in the normal BCS phase. The pairing mechanism for different phases is understood by visualizing superfluid pairing densities in different spin-orbit bands. We also discuss the possibility of observing similar physics using spin-orbit coupled superconducting ultra-thin films.

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The interplay between magnetism and superconductivity leads to various interesting phenomena, which have been intensively studied in many different materials. The physics from such interplay can become even richer and more important when there exists strong spin-orbit (SO) coupling in underlying physical systems, as evidenced by the recent impressive progress on the search for Majorana fermions using superconductor-semiconductor nanowire (or thin film) heterostructures. Another well-known physics originating from the interplay between magnetism and superconductivity is the Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) superconducting phase, where electrons form Cooper pairings with non-zero center-of-mass momentum in the presence of a large Zeeman field, leading to non-zero magnetism coexisting with the superconducting order. In the past five decades, intensive experimental search for FFLO phases has been done in different materials, such as heavy-fermion superconductors CeCoIn$_3$, organic superconductors, iron pnictide superconductors, two-dimensional (2D) electron gases at LaAlO$_3$/SrTiO$_3$ interfaces, and one-dimensional cold Fermi gases. However, unambiguous experimental evidences for FFLO states are still elusive in experiments due the complexity of the underlying materials.

Ultracold degenerate Fermi gases may provide an ideal platform for exploring FFLO physics because of their intrinsic advantages such as the lack of orbital effects, free of disorder, as well as highly controllable experimental parameters. While previously FFLO phases have been widely studied in spin-imbalanced Fermi gases, the recent experimental realization of SO coupling for cold atoms provides a completely new route for the experimental observation of FFLO phases. Note that there are two different types of FFLO phases: FF state with uniform amplitude but spatially oscillating phase, and LO state with spatially oscillating amplitude but uniform phase of the order parameter.

In this Letter, we show that both FF phase and a generalized LO phase may be observed in SO coupled fermionic cold atom optical lattices. Here the superfluid order parameters are obtained by self-consistently solving the real space Bogoliubov-de-Gennes (BdG) equation. Without SO coupling, it is well known that LO states emerge in lattices with a large Zeeman field. With SO coupling, FF phases have been proposed in free space without lattices, where the superfluid gap equation is solved in the momentum space (thus the spatial oscillation of the LO phase cannot be found). Our real space BdG equation can capture both FF and LO phases, and we show that there is a competition between them in a SO coupled optical lattice. Generally SO coupling enhances the FF phase while suppresses the LO phase. The generalized LO phase has no spatially node points in the order parameter and magnetization, which are very different from traditional LO states in spin-imbalanced Fermi gases. The BCS pairing order also possesses finite magnetization, showing that the coexistence of magnetism and superfluid does not necessarily indicate the existence of FFLO phases. The pairing mechanism for FF and LO phases is understood by visualizing the superfluid pairing densities in different SO bands. Finally, we discuss the possibility of observing similar physics using SO coupled superconducting ultra-thin films (e.g., Pb).

Theoretical model: Consider fermionic cold atoms confined in a two-dimensional (2D) Rashba SO coupled optical lattice and subject to an in-plane Zeeman field. The system can be described by a Fermi-Hubbard Hamiltonian

$$H = H_0 + H_{SO} + H_Z,$$  \hspace{1cm} (1)

where $H_0 = -t \sum_{\langle ij \rangle \sigma} \hat{c}_{i \sigma}^\dagger \hat{c}_{j \sigma} - g \sum_{i \sigma} \hat{n}_{i \uparrow} \hat{n}_{i \downarrow}$ is the usual single particle Hamiltonian with the on-site interaction, $\hat{c}_{i \sigma}$ is the atom annihilation operator at the...
\(i\)-th site with spin \(\sigma\), and \(\hat{n}_{i\sigma}\) is the particle number operator. \(t, \mu, \text{ and } U\) are hopping strength, chemical potential, and on-site interaction strength, respectively.

\[H_{SO} = -i\alpha \sum_{\langle ij \rangle} \hat{c}^\dagger_i (\mathbf{d}_{ij} \times \hat{\mathbf{s}} \cdot \mathbf{e}_j) \hat{c}_j\]

is the Rashba SO coupling with \(\hat{c}_i = (\hat{c}_{i\uparrow}, \hat{c}_{i\downarrow})\), \(\hat{\mathbf{s}}\) is the Pauli matrix, and \(\mathbf{d}_{ij}\)

is the unit bond vector between the nearest neighboring sites \(i\) and \(j\). \(H_Z = h \sum_i (\hat{c}_{i\uparrow}^\dagger \hat{c}_{i\uparrow} + \hat{c}_{i\downarrow}^\dagger \hat{c}_{i\downarrow})\) is the in-plane Zeeman field. The Fermi surface is not symmetric around \(k = 0\) because of the in-plane Zeeman field (see the single-particle spectrum in Fig. 4a), making it possible to observe Cooper pairing with finite momentum.

The superfluid phases can be studied under the standard mean-field approximation, where the on-site interaction can be decomposed as \(-U \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} \approx \Delta_i \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\downarrow} + \Delta_i \hat{c}_{i\downarrow}^\dagger \hat{c}_{i\uparrow} + |\Delta_i|^2 / U - U \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - U \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - U \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + U (\hat{n}_{i\uparrow} \hat{n}_{i\downarrow} + \hat{n}_{i\downarrow} \hat{n}_{i\uparrow})\) is the in-plane Zeeman field. The Fermi surface is not symmetric around \(|\Delta_i| = 1\), making it possible to distinguish different phases, we consider the amplitude of \(\langle \hat{n}_{i\uparrow} \hat{n}_{i\downarrow}\rangle\) for different initial inputs, we compare the energy inputs of \(\Delta_i\)

and self-consistently solve the BdG Eq.

\[
\sum_j \left( \frac{H_{ij}}{\Delta^*_{ij}} - \sigma_{iy} H^*_{ijy} \right) \Phi^a_j = E_n \Phi^a_j,
\]

(2)

where \(N\) is the total number of sites, \(H_{ij}\) is a \(2 \times 2\) matrix with components \(H_{ij}(\sigma) = -i\delta_{ij} + \sigma_{iy} \mu_{i\sigma}\). \(H_{ij}(\uparrow\downarrow) = (h - n^i_{yx}) \delta_{ij} + \alpha (\delta_{ix} + \delta_{yx} - \delta_{ix} - \delta_{yx}) - i\delta_{ix} \delta_{iy} - i\delta_{yx} \delta_{iy} - \delta_{ix} \delta_{iy} + \delta_{ix} \delta_{iy} + \delta_{ix} \delta_{iy} + \delta_{ix} \delta_{iy})\), and \(H(\uparrow\downarrow) = H(\uparrow\downarrow)\).

\([\delta_{ij}]\) is equal to one only when \(i\) and \(j\) sites are the nearest neighbors, and \(\mu_{i\sigma} = \mu + U (\hat{n}_{i\sigma})\). \(\hat{n}_{i\sigma}\) and \(\hat{c}_{i\sigma}\) denote the \(xy\) coordinates of the site \(i\). The quasiparticle wavefunction \(\Phi^a_j = \begin{pmatrix} u^n_{j\uparrow}, u^n_{j\downarrow}, v^n_{j\downarrow}, v^n_{j\uparrow} \end{pmatrix}^T\).

The BdG equation should be solved self-consistently with \(\langle \hat{n}_{i\sigma}\rangle = \sum_{n=1}^{2N} \left[ |u^n_{i\sigma}|^2 f(E_n) + |\sigma_{iy} f(E_n)\right]\),

\[n_{ix} = \sum_{n=1}^{2N} \left[ u^n_{ix}^* u^n_{i\downarrow} (1 - f(E_n)) - u^n_{ix}^* u^n_{i\downarrow} f(E_n)\right],
\]

\[\Delta_{ij} = \sum_{n=1}^{2N} \left[ u^n_{ix} u^n_{i\downarrow} (1 - f(E_n)) - u^n_{ix} u^n_{i\downarrow} f(E_n)\right].
\]

Here the Fermi-Dirac distribution \(f(E_n) = 1 / (1 + \exp (E_n/k_B T))\).

In our numerical simulation, we choose different initial inputs of \(\Delta_i\) and self-consistently solve the BdG Eq. [2] with a periodic boundary condition to obtain the superfluid order parameters. If different phases are obtained for different initial inputs, we compare the energies of these phases to determine the ground state. To distinguish different phases, we consider the amplitude variation \(\sigma_1 = \sqrt{\sum_i (|\Delta_i| - |\Delta|^2 / N)}\) and phase variation \(\sigma_2 = \sqrt{\sum_i (\Delta_i - \Delta / N)}\) around the spatial average \(\Delta = \sum_i \Delta_i / N\) of the order parameter. The BCS superfluid phase is characterized by \(|\Delta| \neq 0, \sigma_1 = \sigma_2 = 0\), the LO phase \(|\Delta| \neq 0, \sigma_1 \neq 0, \sigma_2 = 0\), while the FF phase \(|\Delta| \neq 0, \sigma_1 = 0, \sigma_2 \neq 0\).

Phase diagram: In Fig. 1, we plot the zero-temperature mean-field phase diagram in the \((\mu, h)\) plane for \(U = 4t\) without and with SO coupling. The phase diagram with and without HFC are similar and here we only show that without HFC. The phase diagram is symmetric about \(\mu = 0\) due to the particle-hole symmetry. Since the Zeeman field is along the \(x\) direction, we can define the particle-hole operator, \(C (\hat{c}_{i\uparrow}, \hat{c}_{i\downarrow}) C^{-1} = e^{i\pi R} \left( \hat{c}_{i\downarrow}^\dagger - \hat{c}_{i\uparrow}^\dagger \right)\) and \(C^2 = 1\). Under this transformation, \(CH(\mu)C^{-1} = H(-\mu)\), leading to the symmetric phases about \(\mu = 0\) observed in the numerical calculations.

From Fig. 1.b, we see that without SO coupling, there is a large area of LO phase for a large \(h\), and the BCS phase dominates at small \(h\). There is no FF phase. These results are consistent with previous investigation [23]. In the presence of SO coupling, the FF phase emerges. We see the FF parameter region can come from the LO phase, the normal phase, as well as the BCS phase regions. This result shows that the SO coupling enhances \(h_{\text{FF}}\) for the transition to normal phase [23, 24], and reduces \(h_{\text{FF}}\) from BCS to FF phases. The parameter region for the LO phase is reduced to a small part around \(\mu = 0\) (half filling) as shown in Fig. 1(b). There is a clear competition between FF and LO phases in the presence of SO coupling.

Generally, the traditional s-wave BCS pairing does not support finite magnetization because of the equal contribution from both spins. The superfluid phases that support the coexistence of superfluidity and magnetism correspond to FFLO phases or Sarma phases [31, 32], which are usually gapless. However, in the presence of SO coupling and Zeeman field along the \(x\) direction, a BCS pairing with gapped excitations also has a finite magnetization as observed in Figs. 2.a-b. Figs. 2.a and 2.b show the phase transition from BCS states to FF states and from BCS states to LO states, respectively, as the Zeeman field increases. At the transition point, there are sudden jumps of the order parameter \(\Delta\), the average
magnetization $m = \sum_i \langle \hat{m}_i \rangle / N (\hat{m}_i = c_\uparrow^\dagger c_\downarrow + c_\downarrow^\dagger c_\uparrow)$ and the variations $\sigma_2$ and $\sigma_1$. Specifically, $\Delta$ decreases, while $m$, $\sigma_2$ and $\sigma_1$ increase at the phase transition point. The sudden jump of $\Delta$ and $m$ may come from the finite size effect. In FF phase, the order parameter only has the phase variation, which can be seen from the fact that $\sigma_2$ is equal to $\Delta$ in the FF phase. As the SO coupling $\alpha$ increases, either the transition from BCS to FF phases or from LO to FF phases can occur (Figs. 2(c) (d)). The later case shows the growth of FF phases and the suppression of LO phases by the SO coupling. Around the phase transition point from FF to LO phases, there is no clear sudden jump of the magnetization. The kink of $\Delta$ around $\alpha = 0.3$ in Fig. 2(d) (not at the phase transition point) is due to the change of periodicity of $\Delta$ in the LO phase.

**Real space signatures:** Previously the LO phase in lattice models is generally characterized by an inhomogeneous real order parameter and the existence of domain walls at the node points $\Delta_i = 0$ that contain the largest magnetization. These results are reproduced in our calculation without SO coupling. However, when SO coupling is included, $\Delta_i$ is no longer real in the LO phase and does not contain nodes, as clearly seen from Fig. 3. The real and imaginary parts of the order parameter have different phase and amplitude (Fig. 3a), indicating the order parameter has both phase and amplitude variations, which are very different from traditional FF or LO phases. Such non-zero $\Delta_i$ is caused by the imbalance between the pairings with momentum $\pm Q$, which will be explained in details in the next section. These arguments and Fig. 3b show that the generalized LO order parameters can be written as

$$\Delta_i = \Delta_0 \exp (i\mu Q_y) + \Delta_1 \exp (-i\mu Q_y + \phi),$$

where $\phi$ is the relative phase between two $\pm Q$ components. The magnetization $m$ also oscillates in space and reaches the maximum at the minimum of the absolute $|\Delta|$ (See Fig. 3b). In FF phase $\Delta_i = \Delta_0 \exp (i\mu Q_y)$, thus the phase varies, but the magnitude of the order parameter and the magnetization are uniform, as shown in Fig. 3(c),d. The Fourier transformation of the order parameters shows two peaks at $\pm Q$ for the LO phase, but one peak at Q for the FF phase, as expected.

**Pairing mechanism:** It is natural to ask why the combination of SO coupling and in-plane Zeeman field can enhance the parameter region for the FF pairing while suppress that for the LO pairing. This can be understood from different pairing densities for FF and LO phases. We find that in the FF phase, the pairing mainly occurs around the Fermi surface lying at the lower energy band in the helicity representation, while in the LO phase, the pairing occurs at both energy bands. Therefore there is no FF phase without SO coupling. For a typical FF phase with $\alpha = 0.75$, $\mu = -3.0$, and $h = 0.6$, the single particle band structure is shown in Fig. 4 with the Fermi surface plotted at the bottom of the box with green lines. The SO coupling and in-plane Zeeman field lead to asymmetric Fermi surface around the origin, i.e., the lower band (denoted as $-$) has the lowest energy state located at positive $k_y$ while the upper one (denoted as $+$) at negative $k_y$. Our calculations show that particles around the Fermi surface of the lower band with opposite spins contribute to most of pairings as shown in Fig. 4 because of its higher density of states comparing with the upper one. The pairing has finite center-of-mass momentum due to the deformation of the Fermi surface.

When the chemical potential is increased, the defor-
The energy \[ E \sim 2KHz, \ U \sim \alpha \sim 1.5KHz \] in the FFLO phase, only a weak optical lattice is needed. For a typical set of parameters \[ t \sim 2KHz, \ U \sim 8KHz, \ \alpha \sim 1.5KHz \] in Pb experiments, a typical set of parameters is \[ \alpha = 0.75, \ h = 0.6, \ \mu = -3.0 \]. Since we are mainly interested in the superfluid phase, only a weak optical lattice is needed. For a typical set of parameters \[ t \sim 2KHz, \ U \sim \alpha \sim 1.5KHz \] the resulting pairing order \[ \Delta \sim 1KHz \sim 50nK \], which could be further enhanced by increasing the interaction through the Feshbach resonance. The critical Zeeman field is generally at the order of 0.5 KHz. Note that although our calculations are performed in 2D, the same physics should also occur for a 3D Fermi gas with a pancake geometry because the third dimension is not coupled with the SO coupling and Zeeman field. These parameters shows that the FFLO phases should be observable with reasonably low temperature and realistic experimental setup. In experiments, the magnetization and the pairing order strength can be observed in the standard spin-resolved time of flight image \[ [37] \]. While the finite center-of-mass momentum of the Cooper pairs may be observed using noise-correlation method \[ [38] \] or momentum-resolved radio-frequency spectroscopy \[ [39] \]. In previous spin-imbalanced Fermi gases, the observation of the coexistence of superfluid and magnetism is generally taken as a signature of the FFLO phase \[ [20] \]. However, it is not longer true in our case because a BCS phase also has finite magnetization. In order to observe the FFLO phase, one should detect the pairing momentum or the magnetization oscillation in the LO phase directly.

**Possible observation in SO coupled superconducting thin films:** Finally, we remark that the same physics may also be observed using SO coupled s-wave ultrathin superconducting films subject to an in-plane Zeeman field, which may be realized using an in-plane magnetic field or a magnetic semiconductor substrate. Recently, superconductivity in the extreme 2D limit (down to two atomic layers) has been observed in experiments for many materials \[ [27, 28] \]. In some of these thin films, such as Pb, strong Rashba spin-orbit coupling exists and can be tuned through a variable Schottky barrier \[ [29] \]. Furthermore, the Hc2 critical field for these materials can be extremely large for an in-plane magnetic field \[ [30] \]. Such spin-orbit coupled ultrathin superconducting films open the door for the possible observation of FFLO phases, similar as the role of semiconductor-superconductor nano-heterostructures for the recent search of Majorana fermions \[ [13, 14] \]. In Pb experiments, a typical set of parameters is \[ t \sim 40 \text{ meV}, \ \alpha \sim 17 \text{ meV} \]. However, the interaction \[ U \] is generally much weaker, leading to an experimentally observed s-wave order parameter \[ \Delta \sim 0.7 \text{ meV} \] (corresponds to the Pb thin film superconducting transition temperature \[ T_c = 6 \text{ K} [28] \]). The required Zeeman field for the phase transition \[ h \sim \Delta \], which requires a magnetic field \[ B \sim 5 \text{ KHz} \].
T for a small $g$-factor $g = 2$, which is below the upper critical magnetic field $B \sim 8$ T. In the thin films, the FFLO vector $Q$ is much smaller due to the small deformation of the Fermi surface by a small Zeeman field, and the resulting order parameter oscillation period should be much longer. In experiments, the local order parameter minima in the LO state can accommodate normal quasiparticles, which lead to nonzero differential conductance that can be detected through local tunneling measurement. In addition, Josephson junction between a FFLO superconductor and a conventional BCS superconductor [40] can also be used to detect the FFLO phases.

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