Top quark pair + jet production at next-to-leading order: 
NLO QCD corrections to $gg \rightarrow t\bar{t}g$

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Abstract
The reaction $pp/p\bar{p} \rightarrow t\bar{t} + \text{jet} + X$ is an important background process for Higgs boson searches in the mass range below 200 GeV. Apart from that it is also an ideal laboratory for precision measurements in the top quark sector. Both applications require a solid theoretical prediction, which can be achieved only through a full next-to-leading order (NLO) calculation. In this work we describe the NLO computation of the subprocess $gg \rightarrow t\bar{t}g$. 
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1. Introduction

The main objective of the Large Hadron Collider (LHC) at CERN is the discovery of the Higgs boson and the measurement of its mass and couplings. To achieve this important goal a solid knowledge of the production mechanisms and the corresponding backgrounds is mandatory. In the Standard Model a light Higgs boson is currently favoured by the available data. Using the recently updated top mass of $m_t = 178.0^{\pm 4.3}$ GeV [1] the electroweak fits yield an upper bound of 251 GeV (at 95% C.L.) and a central value of $m_H = 117$ GeV [2]. To achieve a high signal significance in the Higgs searches in general, different production and decay mechanisms are combined. In the range up to 200 GeV the so-called weak boson fusion (WBF) process with the subsequent decay of the Higgs into a W-boson pair plays a dominant role. The most important background for the WBF process comes from the $t\bar{t} + \text{jet}$ process [3]. A very precise knowledge of this process is thus mandatory for the discovery of the Higgs boson. It is obvious that for precise measurements of the couplings a precise background determination is equally important. For example, it has been shown in Ref. [4] that even if one assumes only a 10% uncertainty of the $t\bar{t} + \text{jet}$ cross section it is still the dominant theoretical uncertainty in the measurement of $\sigma_H = \sigma_{\text{WBF}} \times B(H \to WW)$. As also pointed out in Ref. [5] this accuracy might be achievable only through a full next-to-leading order (NLO) calculation. In a recent analysis [5], the possibility to extract the background from extrapolation of experimental data has been studied. In this analysis it was found that a background determination with 5–10% accuracy might be possible. This is a very promising result. On the other hand — given the significance of the precise background determination — we believe that a cross-check with a full NLO QCD prediction is important. At the very end — having a good understanding of both results — both methods/results could and should be used as complementary.

In fact the $t\bar{t} + \text{jet}$ reaction is not only important as background for Higgs searches, it is also an important signal process on its own. It is well known that top quark physics allows a test of the Standard Model at high scale. As far as top quark production at hadron colliders is concerned, the state of the art is as follows. The differential cross section for top quark pair production is known to next-to-leading order accuracy in QCD [7,8,9,10]. In addition the resummation of logarithmically enhanced contributions has been studied in detail in Refs. [11,12,13,14,15,16,17,18]. Recently also the spin correlations between top quark and antitop quark were calculated at NLO in QCD [19,20]. Since single top quark production provides an excellent opportunity to test the charged-
current weak interaction of the top quark, it has also attracted a lot of interest in the past. In particular NLO corrections were studied in Refs. [21,22,23,24]. In Ref. [24] the NLO corrections for the fully differential cross section are given, keeping also the spin information of the top quark.

In that context the natural next step is the calculation of the NLO corrections for \( t\bar{t} + \text{jet} \) production. As far as top quark physics is concerned, interesting observables to study are those that vanish if there is no additional jet. Such observables allow for a direct test of the dynamics in the top quark sector. For example the asymmetry \( A(y) \)

\[
A(y) = \frac{N(\ell, \text{forward}) - N(\ell, \text{backward})}{N(\ell, \text{forward}) + N(\ell, \text{backward})}
\]  

(1)

is such an observable. Here \( N(\ell, \text{forward/backward}) \) denotes the number of forward/backward-going leptons as a function of the rapidity \( y \) of the additional jet. The measurement of this asymmetry allows for a direct test of the production and decay mechanisms. Using similar observables one can search for example for anomalous top–gluon couplings.

2. Outline of the calculation

In this section we briefly summarize the calculation of the NLO corrections for the subprocess \( gg \rightarrow t\bar{t}g \). In view of the number of external legs and the top mass as additional parameter it is obvious that even partial results are in general quite lengthy. In the following we will restrict our attention only to those parts of the calculation where special care is needed to construct a numerically stable program.

2.1. Virtual corrections

The calculation of the virtual corrections proceeds via the following steps:

1. Generation of the Feynman diagrams using for example Feynarts [26] or QGRAF [27].
2. Reduction of the tensor integrals to scalar one-loop integrals.
3. Reduction of the amplitudes to standard matrix elements.
4. Numerical phase-space integration of the squared matrix elements, including appropriate phase-space cuts.

Technically the most complicated part is the evaluation of the pentagon-diagrams. Two sample diagrams are shown in Fig. 1. Let us first address the evaluation of the scalar 5-point integrals. To calculate these, we use two different methods. One calculation is based on the method given in Refs. [28,29]. The basic idea of this method is that finite 5-point integrals can be expressed in terms of 4-point integrals (see for example [30,31,32]). To apply this observation also to soft- and mass-singular integrals they are rewritten according to Refs. [28,29] in the following way:

\[
E^d = E^d_{\text{sing.}} + \left[ E^{(\text{mass},d=4)} - E^{(\text{mass},d=4)}_{\text{sing.}} \right].
\]

(2)

Here \( E^d \) denotes the original 5-point integral in \( d \) dimensions while \( E^{(\text{mass},d=4)} \) is obtained from the original integral by dressing the massless propagators with a small mass \( \lambda \). The subtraction term \( E^{(\text{mass},d=4)}_{\text{sing.}} \), which has the same singular structure as the 5-point integral \( E^{(\text{mass},d=4)} \) in the limit \( \lambda \rightarrow 0 \), is obtained by studying the soft and collinear behaviour of \( E^{(\text{mass},d=4)} \) and can be expressed in terms of 3-point functions [33]. Rewriting now the finite integral \( E^{(\text{mass},d=4)} \) in terms of 4-point integrals we thus succeeded in expressing the original 5-point integral in terms of 3- and 4-point functions. A more detailed discussion can be found in Ref. [29]. The sec-
ond method we used to calculate the five-point integrals is based on the fact that, even for divergent integrals, it is possible to obtain a representation as linear combination of 4-point integrals (see for example Ref. [32]). Expressing the 4-point function for \( d = 4 - 2\epsilon \) in terms of the finite 4-point function in 6 dimensions plus a combination of 3-point integrals allows us also to shift all the divergences to the 3-point integrals. Defining the 5-point functions through a linear combination of 4-point integrals (see for example [32]), it is possible to obtain a representation as

\[
E^d(p_0, p_1, p_2, p_3, p_4, m_0, m_1, m_2, m_3, m_4) = \frac{1}{i\epsilon^2} \int d^d \ell \prod_{j=1}^4 \frac{1}{(\ell + p_j)^2 - m_j^2 + i\epsilon},
\]

we obtain for example

\[
E_0(0, p_1, p_1 - p_3, p_4 - p_2, -p_2, m_t, m_t, 0, 0, m_t)|_{\text{sing}}.
\]

\[
= P(t_{13}) P(s_{45}) C_0(p_1 - p_3, p_4 - p_2, p_1, 0, 0, m_t)
+ P(t_{24}) P(s_{35}) C_0(p_4 - p_2, -p_2, p_1 - p_3, 0, m_t, 0)
- (t_{13} - t_{24})^2 P(t_{13}) P(t_{24}) P(s_{35}) P(s_{45})
\times C_0(0, p_1 - p_3, p_4 - p_2, m_t, 0, 0),
\]

(4)

with \( P(x) = 1/(x - m_t^2) \) and \( s_{ij} = (p_i + p_j)^2, t_{ij} = (p_i - p_j)^2 \). The parton momenta are assigned according to \( g(p_1) g(p_2) \rightarrow t(p_3) \bar{t}(p_4) g(p_5) \). For the cases at hand it is possible to solve all the required box integrals in 6 dimensions. We checked that the two methods yield the same results for the 5-point integrals \( E^d \).

Having solved the scalar integrals, the next step is the reduction of the 5-point tensor integrals to scalar one-loop integrals. In principle one could attack this problem using the standard Passarino–Veltman approach [34]. This method leads to spurious singularities in individual terms at the phase-space boundary due to vanishing Gram determinants in the denominator. These spurious singularities create numerical instabilities when doing the phase-space integration. Note that the spurious singularities cancel if one combines the individual terms analytically before doing the numerical integration. One solution of this problem is a time-consuming extrapolation technique, as was used for example in Ref. [29]. As an alternative to the extrapolation technique a different reduction procedure [35] was also used in Ref. [29]. In this work we follow the method developed in Ref. [35]. Essentially the same technique to reduce scalar 5-point integrals to scalar 4-point integrals is also applied to the tensor integrals. In this way the 5-point tensor integrals are directly reduced to 4-point tensor integrals. The explicit calculation shows that in this way the spurious singularities in individual terms, due to vanishing Gram determinants depending on 4 external momenta, are avoided [35].

Let us just mention at the end that there are also other methods to solve the scalar 5-point integrals and perform the reduction of the tensor integrals. For example one could also use the methods developed in Refs. [36,37,38]. (For Ref. [36] see also Walter Giele’s talk in these proceedings.)

2.2. Real corrections

The calculation of the required matrix elements is straightforward. A sample diagram for the reaction

\[ gg \rightarrow t\bar{t}gg \] is shown in Fig. 2. We used two different methods to obtain the required colour-ordered helicity amplitudes:

1. A Feynman-diagram-based approach where we evaluate all the diagrams contributing to one specific colour-ordered subamplitude.

2. Using the recurrence relations à la Berends and Giele [39].

We find complete agreement in the results of the two methods. Furthermore we also checked that our results agree with the ones obtained using Madgraph [40]. To extract the singularities from collinear or soft partons we use the dipole subtraction method.
The idea of the subtraction method is to add and subtract a term which, on the one hand, cancels pointwise the singularities of the matrix elements in the singular regions of the phase-space and is, on the other hand, easy enough to be integrated analytically. Schematically the NLO contribution is then obtained from the following formula:

$$\sigma_{NLO} = \int_{m+1} \left[ \sigma_{\text{real}} - \sigma_{\text{sub}} \right]_{\text{finite}} + \int_{m} \left[ \sigma_{\text{virt}} + \int_{1} \bar{\sigma}_{\text{sub}} \right]_{\text{finite}} + \int_{\text{finite}} dx \int_{m} \left[ \sigma_{\text{fact}}(x) + \bar{\sigma}_{\text{sub}}(x) \right] \cdot \left(3\right)$$

Here $\sigma_{\text{fact}}(x)$ denotes the contribution from the factorization of initial-state singularities due to the presence of coloured partons in the initial state. The contributions $\bar{\sigma}_{\text{sub}}^1, \bar{\sigma}_{\text{sub}}$ are obtained from $\sigma_{\text{sub}}$ by integrating out the ‘unresolved’ parton. The result is split into the two terms $\bar{\sigma}_{\text{sub}}, \bar{\sigma}_{\text{sub}}$ to render the last two integrals individually finite. A remarkable feature of the subtraction method is that the analytic integration of the subtraction has to be done only once and that in the whole procedure no approximation is made. This is made possible by the universality of soft and collinear factorization in QCD. The explicit expressions for $\sigma_{\text{sub}}, \sigma_{\text{sub}}^1$, and $\bar{\sigma}_{\text{sub}}$ can be obtained from the colour-ordered subamplitudes using the formulae given in Ref. [43]. In particular $\bar{\sigma}_{\text{sub}}$ is obtained from a sum over individual dipole contributions. In the case at hand we have to include the contribution from 36 individual dipoles. We do not consider the splitting $g \to t\bar{t}$ because the divergence is regulated by the quark masses. (For light quarks one could consider the corresponding dipoles to render the integration numerically more stable.) We have checked that the combination of the 36 dipoles indeed reproduces all the singular limits arising from single unresolved configurations.

3. Status and results

The current status of the project is as follows. Most of the separate contributions are implemented in the form of computer programs allowing the numerical evaluation of the cross sections. In Fig. 3 we show as an example, the result at the parton level for the virtual corrections (defined as the second term in Eq. 5) $(k_\perp > 20 \text{ GeV})$.}

![Figure 3. Result for the virtual corrections for the subprocess $gg \to t\bar{t}g$ as defined by the second term in Eq. 5 $(k_\perp > 20 \text{ GeV})$.](image)

different centre-of-mass energies. (Given that the separation shown in Eq. 5 involves some freedom, this individual contribution does not have a direct physical interpretation unless the remaining contributions are added — we just show it for illustrative purposes.) As can be seen from Fig. 3 the method we used for the treatment of the tensor integrals gives indeed numerically stable results. Furthermore we note that the inclusion of $d\bar{\sigma}_{\text{sub}}^1$ together with the renormalization of the coupling and the quark mass renders the second term in Eq. 5 finite, as it must be. This is an important cross-check. As mentioned earlier we also checked that the integrand for the first contribution in Eq. 5 is also finite for all single unresolved phase-space configurations. Given the complexity of the project we think it is very important to have independent cross-checks for every individual contribution. While most of the calculation is already cross-checked, we still work to finish also the remaining checks. Complete results will be presented elsewhere.

4. Conclusions

In this work we discuss the NLO calculation for the partonic reaction $gg \to t\bar{t}g$. Up to remaining cross-checks, the calculation is almost finished.
particular we have shown that the virtual corrections are stable using the reduction procedure discussed in Ref. [35].

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