Research and Analysis of Maximum Likelihood Parameter Identification Based on The Recursion of Longdongnan Economic Model

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Abstract. By establishing the regional economic model of southeast Gansu Economic Model as the maximum likelihood parameter identification, the maximum likelihood identification is an estimation method based on probability and statistics theory, by constructing the likelihood function with independent variables and maximizing the likelihood function, the model has good statistical properties and is widely used in many fields. However, the research of maximum likelihood identification method is still limited. Therefore, it is of theoretical significance and academic value to study the maximum likelihood identification method for linear systems and Hammerstein nonlinear systems by combining data filtering technique, hierarchical identification principle and multi-innovation identification theory. In order to improve the identification accuracy, maximum likelihood gradient iteration and maximum likelihood gradient iteration based on finite data window are proposed for output error moving average system. This paper holds that the pattern conforms to the requirements of economic development of Southeast Longnan and has a strong reference for other regions. The simulation results verify the feasibility and validity of the identification modeling method.

Keywords: Longdongnan economic model, Parameter estimation, Maximum likelihood, Linear System, nonlinear system, Recursive identification.

1. Introduction

In this paper, a maximum likelihood parameter identification model is used to construct an observed likelihood function with unknown parameters as independent variables, and the maximum likelihood function is used to obtain the estimated parameters [1-3]. The maximum likelihood gradient iterative algorithm based on finite data window has the ability of identifying time-varying parameters [4-6].

Maximum likelihood identification (ML) is an estimation method based on probability and statistics theory. It has good statistical properties by constructing the likelihood functions of observed data and unknown parameters as independent variables, and maximizing the likelihood functions to...
obtain the estimated values of parameters, it is widely used in many fields [7-9]. However, the research of maximum likelihood identification method is still limited. Therefore, it is of theoretical significance and academic value to study the maximum likelihood identification method for linear systems and Hammerstein nonlinear systems by combining data filtering technique, hierarchical identification principle and multi-innovation identification theory. In order to improve the identification accuracy, maximum likelihood gradient iteration and maximum likelihood gradient iteration based on finite data window are proposed for output error moving average system. ITERATIVE identification can make full use of the system data and improve the identification accuracy. The maximum likelihood gradient iterative algorithm based on finite data window has the ability of identifying time-varying parameters [10-12].

Based on the identification and analysis of the economic development pattern in the southeast of Longnan, this paper holds that the pattern meets the requirements of the economic development in the southeast of Longnan, it has a strong reference significance to other areas. The simulation results verify the feasibility and validity of the identification modeling method.

2. Principle and method of maximum likelihood recursive algorithm

Based on the identification and analysis of the economic development pattern in the southeast of Longnan, this paper holds that the pattern meets the requirements of the economic development in the southeast of Longnan, it has a strong reference significance to other areas.

In fact, the Newton-Raphson method is a recursive algorithm which can be used for on-line identification. However, this method needs to collect a batch of \( L \) data each time, that is, each batch of data needs \( L \) observations, and then carries out a recursion according to \( L \) observations. This section discusses an algorithm that recursively computes parameter estimates for each observation. Essentially, it's an approximation of maximum likelihood.

Consider the following model: 

\[ A(z^{-1})z(k) = B(z^{-1})u(k) + D(z^{-1})v(k) \]  

(1)

In the formula: \( u(k) \) and \( z(k) \) are the inputs and outputs of the system, and \( \{v(k)\} \) is an uncorrelated random noise sequence with zero mean and zero variance \( \sigma_v^2 \). And:

\[
\begin{align*}
A(z^{-1}) & = 1 + a_1 z^{-1} + a_2 z^{-2} + \cdots + a_n z^{-n} \\
B(z^{-1}) & = b_1 z^{-1} + b_2 z^{-2} + \cdots + b_n z^{-n} \\
D(z^{-1}) & = 1 + d_1 z^{-1} + d_2 z^{-2} + \cdots + d_n z^{-n}
\end{align*}
\]  

(2)

Supposed \( \theta = [a_1; a_2; \cdots; a_n; b_1; b_2; \cdots; b_n; d_1; d_2; \cdots; d_n]^T \)  

So, the vector, let's call it

\[
\begin{align*}
z_j(k) & = z(k) - \hat{d}_j z(j-k) - \cdots - \hat{d}_{n_j} z(k-n_j) \\
u_j(k) & = u(k) - \hat{d}_j u(j-k) - \cdots - \hat{d}_{n_j} u(k-n_j) \\
v_j(k) & = v(k) - \hat{d}_j v(j-k) - \cdots - \hat{d}_{n_j} v(k-n_j)
\end{align*}
\]  

(4)
If $\hat{\Theta} = \Theta$ and $\theta = \hat{\theta}$ are fixed, minimize $\hat{\Theta}(k-1)$, and according to
\[
\hat{\Theta}(k-1)_{\theta = \hat{\theta}} = r(k) = K(k) \cdot r(k)
\] (6)

The sequence is divided into several levels according to the size of $r(k) = K(k) \cdot r(k)$. Potential factors, which belong to the lower level, are disadvantages or constraints.

3. **Maximum likelihood parameter identification**

Modelling steps
\[
J(\theta, k) \approx J(\theta, k-1) + \frac{1}{2} v^2(k)
\] (7)

Obviously, $\eta^*(k)$ is greater than zero, so we know from formula (7) that if the parameter estimate $\theta(k)$ at time $k$ makes
\[
J(\theta, k) \approx \frac{1}{2} \left[ \theta - \hat{\theta}(k-1) \right]^T \cdot P^{-1}(k-1) \left[ \theta - \hat{\theta}(k-1) \right] + \frac{1}{2} v^2(k) + \frac{1}{2} \eta(k)
\] (8)

Then, we can rewrite the above Equation (8) to
\[
v(k) = \left[ D(z^{-1}) \right]^{-1} \cdot \left[ A(z^{-1}) \cdot z(k) - B(z^{-1}) \cdot u(k) \right]
\] (9)

After estimation, it can be obtained by Equation (10),
\[
v(k) \approx v(k) + \frac{\partial v(k)}{\partial \theta} \cdot (\theta - \hat{\theta})_{\theta = \hat{\theta}} \cdot J(\theta)_{\theta = \hat{\theta}} \cdot \frac{1}{2} \sum_{k=1}^{L} v^2(k)_{\hat{\theta}} = \min
\] (10)

Available by least squares
\[
J^*(\theta, k) = 2J(\theta, k)
\] (11)

\[
J^*(\theta, k) = \left[ \hat{\theta}(k-1) - r(k) \right]^T \cdot P^{-1}(k) \left[ \hat{\theta}(k-1) - r(k) \right] \cdot \eta^*(k)
\] (12)

Then $J^*(\theta, k)$ gets the minimum. Using Matrix inversion formula
\[
(A + BC)^{-1} = A^{-1} - A^{-1}B(I + CA^{-1})^{-1}CA^{-1}
\] (13)
Then from Equation (12), the first formula can be deduced

$$P(k) = P(k-1)h_f(k)\left[1 + h_f^T(k)P(k-1)h_f(k)\right]^{-1}$$

(14)

In addition, similar to the derivation of the least squares, the recurrence formula of the gain Matrix can be obtained

$$P(k) = P(k-1) - \frac{P(k-1)h_f(k)h_f^T(k)P(k-1)}{1 + h_f^T(k)P(k-1)h_f(k)}$$

(15)

Therefore, the recursive maximum likelihood parameter estimation KML is described as formula (15).

4. An example of rural economic identification in southeastern Yunnan

The net income of per capita income and various sources of income in rural areas were selected as the objects of analysis. The data were derived from the survey of household basic conditions in the four counties in the southeast of Longnan from 2006 to 2018. The sample was 32 questionnaires in Wushan County, Tianshui City, and 33 in Hui County, Weinan City. There are 30 counties in Zhuanglang County in Pingliang City and 35 counties in Qingyang City. (In order to group the Gini coefficient, 120 survey reports were actually used and 30 were in counties, totaling 120 households). To identify the rural economy in southeastern Yunnan, the data list is shown in Table 1:

| Table 1 | Rural household income in Longdong region, 2006-2018. |
|---------|---------------------------------------------------|
|         | Average income per household / yuan                |
| year    | Southeast | Hui County | Wushan County | Huan County | Zhuanglang County |
| 2006    | 7353.67 | 7370.82 | 6176 | 6673.22 | 13573.33 |
| 2009    | 9788.44 | 9615.70 | 8947 | 8567.47 | 17822.66 |
| 2012    | 12212.97 | 13352.74 | 11449 | 10199.99 | 20734.66 |
| 2015    | 17208.43 | 17423.13 | 17239 | 15934.08 | 26262.66 |
| 2018    | 24193.51 | 25104.39 | 21757 | 22761.32 | 39110 |

From 2006 to 2018, the overall income of rural households in the southeast of Longdong showed a significant upward trend. From Table 1, it can be seen that the per capita income of Zhuanglang County is always higher than the per capita income of southeastern Yunnan Province, especially in 2012. It can be seen from this that Zhuang Lang has the highest economic level and the fastest development in the southeastern region of Longdong, while the development in the other three places is basically the same, and there is no big floating. It coordinates development within the region, reduces the income gap between regions, and promotes the overall development of the southeastern region. Longdong region household per capita, as is shown in Fig.1. Net income per rural household per capita, Average income per household, as are shown in Fig.2 and 3.
In Figure 1, 2, 3, based on the identification and analysis of the economic development pattern in the southeast of Longnan, this paper holds that the pattern meets the requirements of the economic development in the southeast of Longnan, it has a strong reference significance to other areas. The simulation results verify the feasibility and validity of the identification modeling method.

**5. Summary**

In order to improve the identification accuracy, maximum likelihood gradient iteration and maximum likelihood gradient iteration based on finite data window are proposed for output error moving average system. ITERATIVE identification can make full use of the system data and improve the identification accuracy. The maximum likelihood gradient iterative algorithm based on finite data window has the
ability of identifying time-varying parameters. On the basis of identifying and analyzing the economic development pattern of southeast Longnan, this paper holds that the pattern conforms to the requirements of economic development of Southeast Longnan and has a strong reference for other regions. The simulation results verify the feasibility and validity of the identification modeling method.

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