Triangle singularities in $B^- \to D*0\pi^-\pi^0\eta$ and $B^- \to D*0\pi^-\pi^+\pi^-$

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I. INTRODUCTION

Hadron spectroscopy is a way to investigate Quantum Chromodynamics (QCD), which is the basic theory of the strong interaction. The success of the quark model in the low-lying hadron spectrum gives us an interpretation of the baryons as a composite of three quarks, and the mesons as that of quark and anti-quark [1, 2]. Meanwhile, the possibility of non conventional hadrons called exotics, which are not prohibited by QCD, have been intensively studied. One example is the $A(1405)$: the quark model predicts a mass at higher energy than the observed peak, and a $KN$ ($I = 0$) molecular state seems to give a better description as originally studied in Ref. [3] followed by many studies which are summarized in Refs. [4, 5]. The spectrum of the low-lying scalar mesons, such as $f_0(980)$ or $a_0(980)$ mesons, is also discussed in this picture [6, 8], while the possible explanation as tetrakauron states is also discussed in Refs. [9, 10]. These days, in the heavy sector, the $XYZ$ [11] and the $P_c$ [12, 13] were discovered, which cannot be associated with the states predicted by the quark model. Another sort of non conventional hadrons are the molecular states of other hadrons, which have been often invoked to describe many existing states (see recent review in Ref. [14]). Besides ordinary hadrons, molecular states or multiquark states, triangle singularities can generate peaks, but these peaks appear from a simple kinematical effect. These singularities were pointed out by Landau [15], and the Coleman-Norton theorem says that the singularity appears when the process has a classical counterpart [16]: in the decay process of a particle 1 into the particles 2 and 3, the particle 1 decays first into particles A and B, followed by the decay of A into the particles 2 and C, and finally the particles BC merge into the particle 3. The particles A, B, and C are the intermediate particles, and the singularity appears if the momenta of these intermediate particles can take on-shell values. A novel way to understand this process is proposed in Ref. [17].

For the decay of $\eta(1405)$ into $\pi^0\pi^0\eta$ via $\pi^0a_0$ and $\pi^0\pi^+\pi^-$ via $\pi^0f_0$, the triangle mechanism gives a good explanation [18, 20]. The $K^*KK$ loop generates the triangle singularity in this process, and the anomalously large branching fraction of the isospin-violating $\pi^0f_0$ channel reported by BESIII [21] is well explained with the mechanism.

The peak associated with this singularity can be misidentified with a resonance state. For example, the studies in Refs. [22, 24] suggest the possible explanation of $Z_c(3900)$ with the triangle mechanism. Similarly, a peak seen in the $\pi f_0(980)$ mass distribution, identified as the "$a_1(1420)$" meson by the COMPASS collaboration [23], is shown to be a manifestation of the triangle mechanism as studied in Refs. [22, 26, 27]. In particular, many $XYZ$ states are discovered as a peak of the invariant mass distribution in the heavy hadron decay. Then, the thorough study on the role of the triangle singularities in the heavy hadron decay is important to clarify the properties of the reported $XYZ$ states. In the $B^- \to K^-\pi^-D^0_s(2317)\ (K^-\pi^-D^*_s(2460))$ process, a peak can be generated by the triangle mechanism around 2800 MeV (2950 MeV) in the $\pi^-D^0_s(\pi^-D^*_s)$ invariant mass spectrum, which is driven by the $K^*DK$ ($K^*D^*K$) loop, and gives a sizable branching fraction into the channel [28]. The $D^0_s$ and $D^*_s$ in the final state are dynamically generated by the $DK$ and $D^*K$, and have large coupling with these states [29, 31]. Because the process of the triangle mechanism contains a fusion of two hadrons, the existence of a hadronic molecular state plays an important role in having a measurable strength. Then, the study of the singularity is also a useful tool to study the hadronic molecular states. Regarding the $P_c$ peak, discovered in the $J/\psi p$ invariant
Weak decays of heavy hadrons are turning into a good model of how the $K^0$ and $B^0$ mesons appear as the dynamically generated states of the mechanism proposed here, without the indication of the $K^*K$ or $K^*\eta$ vertex. Hence, we are able to form a realistic example of a physical process where this can occur, which also allows us to perform a quantitative calculation of the triangle singularity, and finally find the branching fraction $\text{Br}(B^- \to D^{*0}K^0) = \frac{1}{6}$ and $\text{Br}(B^- \to D^{*0}f_0) = \frac{2}{6}$. The complete Feynman diagram for these decays, with the triangle mechanism through the $a_0$ and $f_0$ mesons, is shown in Fig. 2.

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{Diagram for the decay of $B^-$ into $D^{*0}, \pi^-$ and $R$, where $R = a_0(980)$ or $f_0(980)$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig2.png}
\caption{Diagram for the decay of $B^- \to D^{*0}f_0 \pi^-(\pi^+\pi^-)$.

At first, we evaluate the $B^- \to D^*\pi R$ ($R = a_0, f_0$). This then produces the triangle diagram shown in Fig. 1. The $T$ matrix $t_{B \to D^*\pi R}$ will have the following form,

\begin{equation}
-i t_{B \to D^*\pi R} = \sum_{\text{pol.}} \int d^4q \left\{ \frac{i t_{B^* \to K^0\pi} K^0}{(2\pi)^4} \right\} \frac{e^{i m_{B^*}^2 - m_{K^0}^2 + i\epsilon}}{(P - q)^2 - m_{K^0}^2 + i\epsilon} \frac{e^{i m_{K^0}^2 - m_{K^+}^2 + i\epsilon}}{(P - q - k)^2 - m_{K^+}^2 + i\epsilon}.
\end{equation}

The amplitude in Eq. (1) is evaluated in the center-of-mass (CM) frame of $\pi R$. Now we need to calcu-
the quark level, the Cabibbo-allowed vertex is formed through an internal emission of a $W$ boson \[38\] (as can be seen in Fig. 3, producing a $c\bar{u}$ that forms the $D^{*0}$, with the remaining $d\bar{d}$ quarks hadronizing and producing the $K^-$ and $K^{*0}$ mesons with the selection of the $s\bar{s}$ pair from a created vacuum $u\bar{u} + d\bar{d} + s\bar{s}$ state. Since both $D^{*0}$ and $K^{*0}$ have $J^P = 1^-$, the interaction in the $B^- \to D^{*0}K^-K^{*0}$ vertex can proceed via $s$-wave and we take the amplitude of the form, \[t_{B^-\to D^{*0}K^-K^{*0}} = C \epsilon_\mu(K^+)\epsilon^\mu(D^*) \quad \text{(II.2)}\]

Given that we know that the branching ratio of this decay is \[\text{Br}(B^- \to D^{*0}K^-K^{*0}) = 1.5 \times 10^{-3} \quad \text{[11, 37]},\] we can determine the constant $C$ by calculating the width of this decay,

\[
\frac{d\Gamma_{B^-\to D^{*0}K^-K^{*0}}}{dM_{\text{inv}}(K^*D^*)} = \frac{1}{(2\pi)^3} \frac{|\vec{p}_{K^-}|}{4M_B^2} \sum \sum |t_{B^-\to D^{*0}K^-K^{*0}}|^2, \quad \text{(II.3)}
\]

where $|\vec{p}_{K^-}|$ is the momentum of $K^-$ in the $B^-$ rest frame, and $|\vec{p}_{K^{*0}}|$ is the momentum of $K^{*0}$ in the $K^{*0}D^{*0}$ CM frame. The absolute values of both momenta are given by

\[
|\vec{p}_{K^-}| = \frac{\lambda^{1/2}(M_B^2, M_{K^-}^2, M_{inv}(K^*D^*))}{2M_B}, \quad \text{(II.4a)}
\]

\[
|\vec{p}_{K^{*0}}| = \frac{\lambda^{1/2}(M_{inv}(K^*D^*), m_{K^*}^2, m_{D^*}^2)}{2M_{inv}(K^*D^*)}, \quad \text{(II.4b)}
\]

with $\lambda(x, y, z)$ the ordinary Kähler function.

Now, if we square the $T$ matrix in \[\text{(II.2)}\] and sum over the polarizations, we get

\[
\sum \sum |t_{B^-\to D^{*0}K^-K^{*0}}|^2 \quad \text{(II.5)}
\]

\[
\frac{C^2}{\Gamma_{B^-}} = \frac{\text{Br}(B^- \to D^{*0}K^-K^{*0})}{\int dM_{\text{inv}}(K^*D^*) \frac{2}{(2\pi)^3} |\vec{p}_{K^-}| |\vec{p}_{K^{*0}}| \left(2 + \frac{(M_{inv}(K^*D^*) - m_{K^*}^2 - m_{D^*}^2)^2}{4m_{K^*}^2 m_{D^*}^2}\right)}, \quad \text{(II.8)}
\]

where we used the fact that $(p_{K^*} + p_{D^*})^2 = M_{inv}(K^*D^*)^2$, \textit{i.e.}, $p_{K^*} \cdot p_{D^*} = \frac{1}{2} (M_{inv}(K^*D^*)^2 - m_{K^*}^2 - m_{D^*}^2)$. Then, using this last equation in Eq. \[\text{(II.3)}\], we get

\[
\frac{C^2}{\Gamma_{B^-}} = C^2 \sum_{\text{pol}} \epsilon_\mu(K^+)\epsilon^\mu(D^*) \quad \text{(II.6)}
\]

\[
= C^2 \left(2 + \frac{(p_{K^*} \cdot p_{D^*})^2}{m_{K^*}^2 m_{D^*}^2}\right) \quad \text{(II.7)}
\]

\[
= C^2 \left(2 + \frac{(M_{inv}(K^*D^*) - m_{K^*}^2 - m_{D^*}^2)^2}{4m_{K^*}^2 m_{D^*}^2}\right), \quad \text{(II.8)}
\]

where the integral has the limits $M_{inv}(K^*D^*)|_{\text{min}} = m_{D^*} + m_{K^*}$ and $M_{inv}(K^*D^*)|_{\text{max}} = M_B - m_{K^*}$.

Now we calculate the contribution of the vertex $K^{*0} \to \pi^- K^+$. For that we will use the chiral in-

\[
\mathcal{L}_{\text{VPF}} = -ig \left(V^\mu [P, \partial_\mu P]\right), \quad \text{(II.10)}
\]
where the \( VPP \) subscript refers to the fact that we have a vertex with a vector and two pseudoscalar hadrons. The symbol \( \langle \ldots \rangle \) stands for the trace over the \( SU(3) \) flavour matrices, and \( g = m_V/2f_\pi \) is the coupling of the local hidden gauge, with \( m_V = 800 \text{ MeV} \) and \( f_\pi = 93 \text{ MeV} \). The \( SU(3) \) matrices for the pseudoscalar and vector octet mesons \( P \) and \( V^\mu \) are given by

\[
V^\mu = \begin{pmatrix}
\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega^\mu \\
\rho^-_\mu \\
K^0 \\
\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega^\mu \\
k^{\mu+} \\
k^{\mu0} \\
\phi^0 
\end{pmatrix}, \tag{II.11}
\]

\[
P = \begin{pmatrix}
\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta \\
\pi^- \\
K^- \\
\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta \\
K^+ \\
K^0 \\
-\sqrt{\frac{2}{3}}\eta
\end{pmatrix}. \tag{II.12}
\]

Performing the matrix operations and the trace we get

\[
\mathcal{L}_{K^*K} = -igK^{*0\mu}(K^-\partial_\mu\pi^+ - \pi^+\partial_\mu K^-). \tag{II.13}
\]

So, for the \( t \) matrix we get,

\[
-t_{K^*K^+\pi^-} = -ig\epsilon_{K^*}^\mu(p_{K^+} - p_\pi)_\mu \tag{II.14}
\]

\[
\simeq -ig\epsilon_{K^*} \cdot (\vec{p}_\pi - \vec{p}_{K^+}), \tag{II.15}
\]

Finally we only need to calculate \( t_{K^+K^-R} \), before we can analyse the triangle diagram. The coupling of \( R \) with \( \pi^0\eta \) or \( \pi^+\pi^- \) proceeds in \( s \)-wave. Then, the vertex is written simply as a constant,

\[
t_{K^+K^-R} = g_{K^-K^+,R}. \tag{II.16}
\]

We can now analyse the effect of the triangle singularity on the \( B^- \to D^*\pi^{-}R \) decay. Substituting Eqs. (II.2), (II.15) and (II.16) for Eq. (II.11), the decay amplitude \( t_{B^-\to D^*\pi^{-}R} \) is written as

\[
t_{B^-\to D^*\pi^{-}R} = -ig_{K^-K^+,R}gC \sum_{\text{pol. of } K^*} \int d^3q \frac{\epsilon_{D^*} \cdot \epsilon_{K^*} \cdot (\vec{p}_{K^+} - \vec{p}_\pi)}{\left(2\pi\right)^4 q^2 - m_{K^*}^2 + i\epsilon (P - q)^2 - m_{K^*}^2 + i\epsilon (P - q - k)^2 - m_{K^*}^2 + i\epsilon}, \tag{II.17}
\]

where for \( t_{B^-\to D^*\pi^{-}K^-K^+} \) we have also the spatial components of the polarization vectors, and \( \vec{p}_{K^+}, \vec{p}_{K^+} \) are taken in the CM frame of \( \pi R \). As we have mentioned below Eq. (II.15), the momentum of the \( K^{*0} \) around the triangle peak is small compared with the mass, and we can omit the zeroth component of the polarization vector of the \( K^{*0} \).

Now we only need to calculate the width \( \Gamma \) associated

\[
t_{B^-\to D^*\pi^{-}R} = g_{K^-K^+,R}gCi \int d^3q \frac{\epsilon_{D^*} \cdot (\vec{p}_{K^+} - \vec{p}_\pi)}{\left(2\pi\right)^4 q^2 - m_{K^*}^2 + i\epsilon (P - q)^2 - m_{K^*}^2 + i\epsilon (P - q - k)^2 - m_{K^*}^2 + i\epsilon}, \tag{II.19}
\]

where \( \vec{p}_{K^+} = \vec{P} - \vec{q} - \vec{k} = -(\vec{q} + \vec{k}) \) and \( \vec{p}_\pi = \vec{k} \).

Defining \( f(\vec{q}, \vec{k}) \) as a product of the three propagators
in Eq. (II.19), we can use the formula,
\[ \int d^3q_i f(q_i, \vec{k}) = k_i \int d^3q \frac{\vec{q} \cdot \vec{k}}{|\vec{k}|^2} f(q, \vec{k}), \]
which follows from the fact that the $\vec{k}$ is the only vector not integrated in the integrand of Eq. (II.19). Then, Eq. (II.19) becomes
\[ t_{B^+ \to D^+ \pi R} = -\tilde{c}_D \cdot \vec{k} g_{K^- K^+, R} g_C t_T, \]

Squaring and summing over the polarizations of $D^*$, Eq. (II.20) becomes
\[ \sum_{\text{pol}} |t_{B^+ \to D^+ \pi R}|^2 = |\vec{k}|^2 g_{K^- K^+, R}^2 g_C^2 |t_T|^2, \text{ (II.22)} \]

\[ t_T = \int \frac{d^3q}{(2\pi)^3} \left( 2 + \frac{\vec{q} \cdot \vec{k}}{|\vec{k}|^2} \right) \frac{1}{8 \omega^* \omega' k_0 - \omega - \omega^*} \frac{1}{P^0 + \omega + \omega' - k_0} \frac{1}{P^0 - \omega - \omega' - k_0 + i\epsilon} \times \frac{1}{P^0 - \omega^* - \omega + i\epsilon} \]

with $\omega^*(\vec{q}) = \sqrt{m_{K^+}^2 + |\vec{q}|^2}$, $\omega(\vec{q}) = \sqrt{m_{K^0}^2 + |\vec{q}|^2}$. To regularize the integral in Eq. (II.23) we use the same cutoff of the meson loop that will be used to calculate $t_{K^+ K^- \to \eta \eta}$ and $t_{K^+ K^- \to \pi^+ \pi^-}$ (Eq. (II.36)), $\theta(\eta_{\text{max}} - |q^*|)$, where $q^*$ is the $K^-$ momentum in the $R$ rest frame [17].

In Ref. [17] it was found that there is a singularity associated with this type of loop functions when Eq. (18) of Ref. [17] is satisfied. From that equation we can determine that the singularity will appear around $M_{\text{inv}}(\pi R) = 1418$ MeV.

To be completely correct in our analysis we have to use the width of $K^{*0}$. We implement that replacing $\omega^* \to \omega^* - i \frac{\Gamma}{\omega^*}$ in Eq. (II.23), which will reduce the singularity to a peak [17].

For the three body decay of $B^- \to D^{*0} \pi^- R$ in Fig. 1 the mass distribution is given by
\[ \frac{d\Gamma}{dM_{\text{inv}}(\pi R)} = \frac{1}{(2\pi)^3} \frac{|\vec{p}_{D^*}||\vec{p}_\pi|}{4M_B^2} \sum_{\text{pol}} |t_{B^- \to D^+ \pi R}|^2, \text{ (II.24)} \]

with
\[ |\vec{p}_{D^*}| = \frac{\lambda^{1/2}(M_B^2, m_{D^*}^2, M_{\text{inv}}^2(\pi R))}{2M_B}, \text{ (II.25a)} \]
\[ |\vec{p}_\pi| = |\vec{k}| = \frac{\lambda^{1/2}(M_{\text{inv}}(\pi R), m_{\pi}^2, M_B^2)}{2M_{\text{inv}}(\pi R)}, \text{ (II.25b)} \]

With Eq. (II.20) and a factor $1/\Gamma_{B^-}$, the mass distribution of $B^-$ decaying into $D^* \pi R$ is written as

\[ \frac{1}{\Gamma_{B^-}} \frac{d\Gamma}{dM_{\text{inv}}(\pi R)} = C^2 \frac{g^2}{(2\pi)^3} \frac{|\vec{p}_{D^*}| |\vec{k}|^2}{4M_B^2} |t_T| \times g_{K^+ K^+, R}^2, \text{ (II.26)} \]

where $C^2$ is given in Eq. (II.9). However, the problem here is that the $a_0$ and $f_0$ are
not stable particles, but resonances that have a width and decay to $\pi^0\eta$ and $\pi^+\pi^-$, respectively. To solve this without having to consider $R$ a virtual particle and having a four body decay, we can consider the resonance as a normal particle but we add a mass distribution to the decay width in Eq. (II.24),

$$
\frac{d\Gamma}{dM_{\text{inv}}(\pi R)} = \frac{1}{(2\pi)^3} \int dM_{\text{inv}}^2(R) \left( -\frac{1}{\pi} \text{Im}D \right) |g_{K-K^+,R}|^2 \frac{|\vec{p}_{D^*}||\vec{p}_\pi|}{4M_B^2} \sum \sum |\tilde{t}_{B^-,D^*\pi R}|^2 ,
$$

with

$$
D = \frac{1}{M_{\text{inv}}^2(R) - M_R^2 + iM_R \Gamma_R},
$$

where $M_{\text{inv}}(R)$ stands for $M_{\text{inv}}(\pi^0\eta)$ and $M_{\text{inv}}(\pi^+\pi^-)$ for $R = a_0$ and $f_0$, respectively, and $\tilde{t}_{B^-,D^*\pi R} = t_{B^-,D^*\pi R}/g_{K-K^+,R}$. What Eq. (II.27) is accomplish-

ing is a convolution of Eq. (II.24) with the mass distribution of the $R$ resonance given by its spectral function.

Notice also that in the limit of $\Gamma_R \to 0$, $i\text{Im}D = -i\pi \delta(M_{\text{inv}}^2(R) - M_R^2)$ and we recover Eq. (II.24). Evaluating explicitly the imaginary part of $D$, Eq. (II.27) becomes

$$
\frac{d\Gamma}{dM_{\text{inv}}(\pi R)} = \frac{1}{(2\pi)^3} \int dM_{\text{inv}}^2(\pi R) \frac{1}{\pi} |g_{a_0\to\pi^0\eta}|^2 \frac{|\vec{p}_{D^*}||\vec{p}_\eta|}{2M_{\text{inv}}^2(\pi^0\eta)} \sum \sum |\tilde{t}_{B^-,D^*\pi R}|^2 \frac{1}{(M_{\text{inv}}^2(\pi^0\eta) - M_R^2)^2 + (M_R \Gamma_R)^2}.
$$

Then Eq. (II.29) becomes

$$
\frac{d\Gamma}{dM_{\text{inv}}(\pi a_0)} = \frac{1}{(2\pi)^3} \int dM_{\text{inv}}^2(\pi^0\eta) \frac{|\vec{p}_{D^*}||\vec{p}_\eta|}{4M_B^2} \sum \sum |\tilde{t}_{B^-,D^*\pi R}|^2 \frac{M_{a_0}|g_{a_0\to\pi^0\eta}|^2 |g_{K-K^+,a_0}|^2}{(M_{\text{inv}}^2(\pi^0\eta) - M_{a_0}^2)^2 + (M_{a_0} \Gamma_{a_0})^2} \frac{1}{8\pi^2 M_{\text{inv}}^2(\pi^0\eta)} |\tilde{q}_\eta| .
$$

But since for the resonance we have formally,

$$
\frac{|g_{a_0\to\pi^0\eta}|^2 |g_{K-K^+,a_0}|^2}{(M_{\text{inv}}^2(\pi^0\eta) - M_{a_0}^2)^2 + (M_{a_0} \Gamma_{a_0})^2} = |t_{K^+K^-,\pi^0\eta}|^2 ,
$$

Eq. (II.32) reduces to

$$
\frac{d\Gamma}{dM_{\text{inv}}(\pi a_0)} = \frac{1}{(2\pi)^3} \int dM_{\text{inv}}^2(\pi^0\eta) \frac{|\vec{p}_{D^*}||\vec{p}_\eta|}{2M_{\text{inv}}^2(\pi^0\eta)} \sum \sum |\tilde{t}_{B^-,D^*\pi R}|^2 \frac{1}{(M_{\text{inv}}^2(\pi^0\eta) - M_{a_0}^2)^2 + (M_{a_0} \Gamma_{a_0})^2} 8\pi^2 M_{\text{inv}}^2(\pi^0\eta) \frac{1}{8\pi^2 M_{\text{inv}}^2(\pi^0\eta)} |\tilde{q}_\eta| .
$$
where we approximated $M_{\text{inv}}(\pi^0\eta)$ as $M_R$. For the case of $f_0(980)$, $f_0 \rightarrow \pi^+\pi^-$ is not the only possible decay and as such $\Gamma_{f_0\rightarrow\pi^+\pi^-}$ will not be the same as the $\Gamma_{\pi^+\pi^-\pi^-}$ in Eq. (II.27). However, when we put $|t_{K+K^-\rightarrow\pi^+\pi^-}|^2$ in the end, we already select the $\pi\pi$ part of the $f_0$ decay. Thus, for the case of $f_0$ we just need to substitute, in Eq. (II.34), $t_{K+K^-\rightarrow\pi^+\pi^-}$, $M_{\text{inv}}(\pi a_0) \rightarrow M_{\text{inv}}(\pi f_0)$, $M_{\text{inv}}(\pi^0\eta) \rightarrow M_{\text{inv}}(\pi^+\pi^-)$, and $|\vec{q}_o| \rightarrow |\vec{q}_a|$, with

$$|\vec{q}_a| = \frac{\lambda^{1/2}(M^2_{\text{inv}}(\pi^+\pi^-), m^2_{\pi^+}, m^2_{\pi^-})}{2M_{\text{inv}}(\pi^+\pi^-)}.$$

(II.35)

The amplitudes $t_{K+K^-\rightarrow\pi^+\pi^0\eta}$ and $t_{K+K^-\rightarrow\pi^+\pi^-\pi^-}$ themselves are calculated based on the chiral unitary approach, where the $a_0$ and $f_0$ appear as dynamically generated states $[33, 34]$. The cutoff parameter $q_{\text{max}}$ which appears for the regularization of the meson loop function in the Bethe-Salpeter equation,

$$t = [1 - V G]^{-1} V,$$

(II.36)

is determined as $q_{\text{max}} = 600$ MeV for the reproduction of the $a_0$ and $f_0$ peaks (around 980 MeV in invariant mass of $\pi^0\eta$ or $\pi^+\pi^-\pi^-$) $[42, 43]$. In Eq. (II.36), $t$, $V$, and $G$ are the meson amplitude, interaction kernel, and meson loop function, respectively.

Finally, we can substitute everything we have calculated into Eq. (II.34) and obtain,

$$\frac{1}{\Gamma_{B^-}} \frac{d^2\Gamma}{dM_{\text{inv}}(\pi R)dM_{\text{inv}}(R)} = \frac{g^2}{(2\pi)^5} \frac{[\vec{p}_D \cdot |\vec{q}_\pi||\vec{k}|^3]}{4M_B^2} |t_T \times t_{K+K^-\rightarrow\pi^0\eta(\pi^+\pi^-)}|^2 \frac{C^2}{\Gamma_{B^-}}.$$

(II.37)

### III. RESULTS

Let us begin by showing in Fig. 4 the contribution of the triangle loop (defined in Eq. (II.23)) to the total amplitude. We plot the real and imaginary parts of $t_T$, as well as the absolute value with $M_{\text{inv}}(R)$ fixed at 980 MeV. As can be observed, there is a peak around 1420 MeV, as predicted by Eq. (18) of Ref. [17].

In Figs. 5 and 6 we plot Eq. (II.37) for both $B^- \rightarrow D^{*0}\pi^-\eta\pi^0$ and $B^- \rightarrow D^{*0}\pi^-\pi^+\pi^-$, respectively, by fixing $M_{\text{inv}}(\pi R) = 1418$ MeV, which is the position of the triangle singularity, and varying $M_{\text{inv}}(R)$. We can see a strong peak around 980 MeV and consequently we see that most of the contribution to our width $\Gamma$ will come from $M_{\text{inv}}(R) = M_R$. For Fig. 5 the dispersion is bigger, we have strong contributions for $M_{\text{inv}}(\pi^0\eta) \in [880, 1080]$. However, for Fig. 6 most of the contribution comes from $M_{\text{inv}}(\pi^+\pi^-) \in [940, 1020]$. The conclusion is that when we calculate the mass distribution $\frac{d\Gamma}{dM_{\text{inv}}(\pi a_0)}$, we can restrict the integral in $M_{\text{inv}}(R)$ to the limits already mentioned.

When we integrate over $M_{\text{inv}}(R)$ we obtain

$$\frac{d\Gamma}{dM_{\text{inv}}(\pi R)}$$

which we show in Fig. 7. We see a clear peak of the distribution around 1420 MeV, for $f_0$ and $a_0$ production.

![Diagram](image_url)

**FIG. 4**: Triangle amplitude $t_T$ for the decay $B^- \rightarrow D^{*0}\pi R$. We take $M_{\text{inv}}(R) = 980$ MeV.

However, we also see that the distribution stretches up to large values of $M_{\text{inv}}(\pi R)$ where the phase space of the reaction finishes. This is due to the $|\vec{k}|^3$ factor in Eq. (II.37) that contains a $|\vec{k}|$ factor from phase space and a $|\vec{k}|^2$ factor from the dynamics of the pro-
process, as we can see in Eq. \[^{122}\]. Yet, a clear peak in $M_{\text{inv}}(\pi R)$ can be seen for both the $B^- \to D^{*0}\pi^- f_0$ and $B^- \to D^{*0}\pi^- a_0$ reactions.

Integrating now $\frac{d\Gamma}{dM_{\text{inv}}(\pi a_0)}$ and $\frac{d\Gamma}{dM_{\text{inv}}(\pi f_0)}$ over the $M_{\text{inv}}(\pi a_0)$ ($M_{\text{inv}}(\pi f_0)$) masses in Fig. 7, we obtain the branching fractions

$$\text{Br}(B^- \to D^{*0}\pi^- a_0; a_0 \to \pi^0\eta) = 1.66 \times 10^{-6}, \quad (\text{III.1a})$$

$$\text{Br}(B^- \to D^{*0}\pi^- f_0; f_0 \to \pi^+\pi^-) = 2.82 \times 10^{-6}, \quad (\text{III.1b})$$

These numbers are within measurable range.

Note that we have assumed all the strength of $\pi^0\eta$ from 880 MeV to 1080 MeV to be part of the $a_0$ production, but in an experimental analysis one might associate part of this strength to a background. We note this in order to make proper comparison with these results when the experiment is performed.

IV. SUMMARY

We have performed the calculations for the reactions $B^- \to D^{*0}\pi^- a_0(980); a_0 \to \pi^0\eta$ and $B^- \to D^{*0}\pi^- f_0(980); f_0 \to \pi^+\pi^-$. The starting point is the reaction $B^- \to D^{*0}K^0K^-$, which is a Cabibbo favored process and for which the rates are tabulated in
the PDG\cite{PDG} and are relatively large. Then we allow the $K^{*0}$ to decay into $\pi^- K^+$ and the $K^+ K^-$ fuse to give the $a_0(980)$ or the $a_0(2000)$. Both of them are allowed, since the $K^{*0} K^-$ state does not have a particular isospin. The triangle diagram corresponding to this mechanism develops a triangle singularity at about 1420 MeV in the invariant masses of $\pi^- f_0$ or $\pi^- a_0$, and makes the process studied relatively large, having a prominent peak in those invariant mass distributions around 1420 MeV.

We evaluate $\frac{d^2\Gamma}{dM_{\text{inv}}(\pi^0 f_0) dM_{\text{inv}}(\pi^+ \pi^-)}$, and see clear peaks in the $M_{\text{inv}}(\pi^0 f_0)$, $M_{\text{inv}}(\pi^+ \pi^-)$ distributions, showing clearly the $a_0(980)$ and $f_0(980)$ shapes. Integrating over $M_{\text{inv}}(\pi^0 f_0)$ and $M_{\text{inv}}(\pi^+ \pi^-)$ we obtain $\frac{d\Gamma}{dM_{\text{inv}}(\pi a_0)}$ and $\frac{d\Gamma}{dM_{\text{inv}}(\pi f_0)}$, respectively, and these distributions show a clear peak for $M_{\text{inv}}(\pi a_0)$, $M_{\text{inv}}(\pi f_0)$ around 1420 MeV. This peak is a consequence of the triangle singularity, and in this sense the work done here should be a warning not to claim a new resonance when this peak is seen in a future experiment. On the other hand, the results make predictions for an interesting effect of a triangle singularity in an experiment that is feasible in present experimental facilities. The rates obtained are also within measurable range. Finding new cases of triangle singularities is of importance also, because their study will give incentives to update present analysis tools to take into account such possibility when peaks are observed experimentally, avoiding the natural tendency to associate those peaks to resonances.

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