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The essences of the weak $CP$ violation the quark and lepton Jarlskog invariants are determined toward future model buildings beyond the Standard Model (SM). Satisfying the unitarity condition, we obtain the CKM and MNS matrices from the experimental data. The Jarlskog determinant $J$ in the quark sector is found to be $\sim 1.36 \times 10^{-3} |\sin \alpha_{KS}|$ while $J$ in the leptonic sector is $\sim 3.34 \times 10^{-2} |\sin \alpha_{\ell KS}|$.

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I. INTRODUCTION

Ever since the discovery of weak $CP$ violation [1], its origin and cosmological implication has been a mystery. Ideas such as a tiny $CP$ violation effect in the strong interaction sector or scalar mediated weak $CP$ violation had not been considered any more as leading ones after Kobayashi and Maskawa(KM) found that three left-handed(L-handed) charged currents lead to weak $CP$ violation effects [2]. In early 1960’s, $CP$ violation had been an interesting topic even to laymen [3]. In late 1960’s, $CP$ violation had been considered as an indispensable ingredient in baryogenesis, creating baryons out of a bayonless Universe [4].

$CP$ violation can arise in three varieties, (1) the strong $CP$ violation [5], (2) the weak $CP$ violation, and (3) $CP$ violation by singlets beyond the Standard Model (BSM). The strong $CP$ problem has led to the so-called axion physics which is one of the leading candidates for dark matter in the Universe [6] but short of explaining the current baryon asymmetry in the Universe. Out of the remaining two, the leading candidate toward the baryon asymmetry is the $CP$ violation by the BSM singlets. On the other hand, the weak $CP$ violation hints a crucial information on a fundamental theory of elementary particles. The reason is the following. Because the strangeness changing neutral current effects are strongly suppressed [7], the flavor changing effects are dominated by flavor changing charged currents. In the SM, the L-handed doublets encode this information. The observation of the weak $CP$ violation [1] requires three or more L-handed SM doublets. If we supersymmetrize the SM and requires asymptotic freedom above the TeV scale, four or more L-handed doublets are forbidden. Thus, three L-handed doublets are unique. This observation leads to the following flavor puzzles.

The flavor puzzle in the SM constitutes in two parts: (i) “Why are there three chiral families?”, and (ii) “Why is the Cabibbo–Kobayashi–Maskawa(CKM) matrix [2][8] almost diagonal while it is not so in the Maki–Nakagawa– Sakata(MNS) matrix [9]?”. Here, we suggest the usefulness of Jarlskog determinant [10] answering the second flavor problem if three families are given. Because string theory has been believed to be sufficiently restrictive below the string scale, works on three families from string compactification exploded under the phrases ‘standard-like models in string compactification’ [11][27] and ‘SUSY GUTs from string’ [28][31]. All these models attempted to realize three chiral families. But, the more difficult problem is (ii) on the CKM matrix. Because of the reduction of the number of Yukawa couplings in GUTs compared to the standard-like models, an anti-SU(5) has been attempted for the flavor problem [32]. For a successful phenomenology, not only the Yukawa couplings but also the discrete symmetry $Z_{4R}$ [33] and a mechanism for SUSY breaking at an intermediate scale [34] are needed. GUTs help analyzing these issues also [35][36].

In this paper, in particular, we draw the attention that the Jarlskog determinant is useful in analyzing the actual data. In fact, using the Jarlskog determinant, we obtain numerical $3 \times 3$ CKM and MNS matrices at our best capability, which can be easily applied in the future model buildings.
II. A USEFUL PARAMETRIZATION

Because the flavor changing neutral current effects are almost absent, the structure of three L-handed doublets of quarks and leptons are enough for the flavor study in the SM,

\[
\begin{pmatrix}
\ell^\alpha_L & \ell^\alpha_R
\end{pmatrix}, \quad
\begin{pmatrix}
s^\alpha_L & b^\alpha_R
\end{pmatrix}, \quad
\begin{pmatrix}
\nu^\alpha_L & \nu^\alpha_R
\end{pmatrix}, \quad
\begin{pmatrix}
\nu^\alpha_L & \nu^\alpha_R
\end{pmatrix}, \quad
\begin{pmatrix}
\nu^\alpha_L & \nu^\alpha_R
\end{pmatrix},
\]

(1)

In Eqs. (1) and (2), we choose bases such that the lower components are the mass eigenstates while the upper components are mixed.

In this paper, we use the Kim–Seo(KS) form for the CKM matrix [37],

\[
V^{KS} = \begin{pmatrix}
c_1, & s_1 c_3, & s_1 s_3 \\
-c_2 s_1, & c_1 c_2 c_3 + s_2 s_3 e^{-i\alpha_{KS}}, & c_1 c_2 s_3 - s_2 c_3 e^{-i\alpha_{KS}} \\
-s_1 s_2 e^{i\alpha_{KS}}, & -c_2 s_3 + c_1 s_2 c_3 e^{i\alpha_{KS}}, & c_2 c_3 + c_1 s_2 s_3 e^{i\alpha_{KS}}
\end{pmatrix},
\]

(3)

where \(c_i\) and \(s_i\) are cosines and sines of three real angles \(\theta_i (i = 1, 2, 3)\) and \(\alpha_{KS}\) is a CP phase. The KS form is written such that the elements in the 1st row are all real, which makes it easy to draw the Jarlskog triangle with one side sitting on the horizontal axis. Furthermore, the (21) element is real and hence it is possible to determine three real angles from the experimental values of the first family related elements, (11), (12), and (21) entries. For the MNS matrix of Eq. (2), we use another four parameter set, \(\Theta_i\) (giving corresponding \(C_i\) and \(S_i\)) and \(\alpha_{KS}\).

The situation with the other forms of parametrization is the following. The original KM form [2] gives a complex CP violation, and in this case we define \(J = \text{Im} (V_{22} V_{31}^*) (V_{23} V_{31}^*) / |V_{21} V_{32}|^2 + |V_{22} V_{31}|^2\). The CP violation is encoded in the imaginary part and in this case we define \(J = \text{Im} (V_{22} V_{21}^*) (V_{32} V_{31}^*)\). If we take \(V_{22} V_{21}^*\) and \(V_{32} V_{31}^*\) as two sides of a triangle, they are elements of \(V_{i2} V_{i1}^*\) for \(i = 2\) and \(3\). So, three sides of a Jarlskog triangle corresponds to three values for \(i = 1, 2, 3\), and these three complex numbers make up a triangle in the complex plane because \(\sum_{i} V_{i2} V_{i1}^* = 0\) due to the unitarity of \(V\). As we choose 2 and 1 from the column entries, there are three ways to make column-triangles. Similarly, one can make three row-triangles which we do not use in this paper. The physical magnitude of weak CP violation is given by the Jarlskog determinant \(J\) which is twice of the area of the Jarlskog triangle shown in Fig. I where \(\alpha, \beta, \gamma\) are the angles determined from hadron phenomena [40].

But a simple form, readable from the \(3 \times 3\) CKM matrix itself, is given by [43]

\[
J = |\text{Im} V_{31} V_{22} V_{13}|, \quad \text{after making Det.} V \text{ real},
\]

(4)

If the determinant is real as required, there is no imaginary part in

\[
-\sum_{i,j,k} \epsilon_{ijk} (\text{Im} V_{i1} V_{j2} V_{k3}) = 0,
\]

(5)

\[\text{1 A global fit using 4 real data points for three real numbers can reduce error bars a bit, however.}\]
i.e. the permutations of \( \{i, j, k\} = \{1, 2, 3\} \) add up the imaginary parts to zero, implying any set of \( \{i, j, k\} \) has the same magnitude. Any set out of 6 can be used as \( J \), and there can be a consistency check on the determination of the CKM parameters by calculating \( J \) from these 6 sets, i.e.

\[
J = |\epsilon_{ijk} (\text{Im} V_{1i} V_{j2} V_{k3})|, \text{ with } i, j, k \text{ not summed.} \tag{6}
\]

As emphasized here, \( J \) makes sense only if we use a unitary matrix \( V \). It is useful to check the six terms independently so that the unitarity condition is satisfied. Since \( J \) turns out to be order \( 10^{-3} \), with an approximate form it is not warranted for the triangle to close and the error on \( J \) is \( O(10^{-3}) \).

With the KS form, the Jarlskog triangle is shown in Fig. 2. Note that the horizontal axis is the number \( V_{11} V_{13}^* \) which is real and hence it is sitting on the \( x \)-axis. Namely, in the KS form, one side of any Jarlskog triangle is sitting on the \( x \)-axis. Out of three numbers from \( V_{1i} V_{i3}^* \), if we take \( i = 3 \) and using \( V_{31} \approx 1, V_{31} \) with the phase \( \alpha_{KS} \) is the angle at the origin. This invariant angle appears in any Jarlskog triangle. If we consider the Jarlskog triangle \( V_{12} V_{11}^* \), we note \( |V_{12} V_{11}^*| \approx \lambda \) and \( |V_{22} V_{21}^*| \approx \lambda \) and we have a shape shown in Fig. 3. Since \( J = O(\lambda^5) \), the small side has a length at most \( O(\lambda^4) \), which implies that two angles are close to 90°. Namely, from trigonometry, if we have two long sides of length \( \lambda \) and \( \lambda + O(\lambda^3) \), the angle between them, \( \epsilon \), is given for \( \lambda \approx 0.225 \) by

\[
\cos \epsilon = 1 - O(\lambda^3) + \cdots \approx 0.9886 \rightarrow \epsilon \approx 8.65^\circ. \tag{7}
\]
One among $\alpha, \beta, \gamma$ must be $\alpha_{KS}$, and $\epsilon$ cannot be $\alpha_{KS}$ since there is no angle close to 0 among angles $\alpha, \beta, \gamma$ of Fig. 1. The shape of this thin triangle is shown in Fig. 3 with an exaggerated $\epsilon$.

Our form of the CKM and MNS matrices are based on\(^2\)

\begin{align*}
V_{\text{CKM}} &= (U^{(u)})(U^{(d)})^\dagger \\
V_{\text{MNS}} &= (U^{(\nu)})(U^{(e)})^\dagger
\end{align*}

where $U^{(u,d)}$ and $U^{(\nu,\nu)}$ are diagonalizing unitary matrices of L-handed quark and lepton fields, respectively.

\section*{IV. DATA ON FLAVOR PHYSICS}

\subsection*{A. CKM matrix}

The CKM data in the PDG book is present by fitting to an approximate unitary matrix \[^{39}\text{PDG}\], which is not adequate in calculating the Jarlskog determinant because they did not use an exact CKM matrix. As we will show $J$ is $O(10^{-3})$. If one uses an approximae one, he is in error of not closing the triangle at $O(\lambda^4)$ which is $O(2 \times 10^{-3})$ and obtaining $J$ from the approximate expression is not reliable. So, we must use a parametrization where at least three elements do not contain the phase such that firstly three real angles are determined. We determine them, using the unitary KS form \[^{37}\text{PDG}\], using the (11), (12), and (21) elements of the PDG book \[^{40}\text{PDG}\],

\begin{align*}
(11) &= 0.97420 \pm 0.00021, \quad (12) = 0.2243 \pm 0.0005, \quad (21) = -0.218 \pm 0.004.
\end{align*}

Let us express the angles and the errors as

\begin{align*}
\theta_1 &= \bar{\theta}_1 + \delta_1, \quad \theta_2 = \bar{\theta}_2 + \delta_2, \quad \theta_3 = \bar{\theta}_3 + \delta_3, \quad \alpha_{KS} = \bar{\alpha}_{KS} + \delta_{KS}.
\end{align*}

Then, we obtain the following from Eqs. (3) and (10),

\begin{align*}
\bar{\theta}_1 &= 13.0432^\circ, \quad \bar{\theta}_2 = 14.9964^\circ, \quad \bar{\theta}_3 = 6.3541^\circ, \\
\delta_1 &= \pm 0.0533^\circ, \quad \delta_2 = \pm 4.017^\circ, \quad \delta_3 = \pm 2.364^\circ.
\end{align*}

To determine the CP phase $\alpha_{KS}$, we use the formula for the Jarlskog angle. If $|\sin \bar{\alpha}_{KS}|$ turns out to be greater than 1 from this formula, then we determine $\bar{\alpha}_{KS} = \pm 90^\circ$ depending on the sign of the equation. Since the shape of the Jarlskog triangle is the same in any parametrization, the CP violation error will be the same in any parametrization. So, for the error bars of $J$, i.e. $\delta_J$, we use the PDG value because the error in the shape of the Jarlskog triangle is reliable even if the PDG group uses the approximate form. We use the PDG value $\delta_J/J = 0.15/3.18 = 1/21 \times 2^\dagger$, even though the PDG value $J$ itself is off by a factor of $10^{-2}$. If the formula gives $|\sin \bar{\alpha}_{KS}| > 1$, we use $\bar{\alpha}_{KS} = 90^\circ$. Since $J \approx \frac{1}{2} \sin 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \sin \bar{\theta}_1 (\sin \bar{\alpha}_{KS})$, applying small errors in Eq. (13), we obtain $\delta (\sin \bar{\alpha}_{KS}) = -\frac{1}{21} \times (-0.04717)$ and $\delta (\cos \bar{\alpha}_{KS}) = 0.28402$, leading to $\delta \alpha_{KS} = 16.5^\circ$. Therefore, we obtain the following numerical data on $V_{\text{CKM}}$ using the error propagation method,

\begin{equation}
V_{\text{data}}^{\text{CKM}} = \begin{pmatrix}
0.97420 \pm 0.00021, & 0.22430 \pm 0.00137, & 0.02498 \pm 0.00026 \\
-0.2180 \pm 0.00419, & +0.93524 \pm 0.01653, & +0.10415 \pm 0.00321 \\
-i(0.02864 \pm 0.00144) \sin \alpha_{KS}, & +i(0.25717 \pm 0.05522) \sin \alpha_{KS}, & -i(0.25717 \pm 0.05522) \cos \alpha_{KS} \\
+(0.02864 \pm 0.00144) \cos \alpha_{KS}, & -i(0.25717 \pm 0.05522) \cos \alpha_{KS}, & +i(0.25717 \pm 0.05522) \cos \alpha_{KS} \\
-i(0.0584 \pm 0.01529) \sin \alpha_{KS}, & -0.10690 \pm 0.00339, & +0.96001 \pm 0.01709 \\
-(0.0584 \pm 0.01529) \cos \alpha_{KS}, & +i(0.25054 \pm 0.05234) \sin \alpha_{KS}, & +i(0.25054 \pm 0.05234) \cos \alpha_{KS} \\
+i(0.25054 \pm 0.05234) \sin \alpha_{KS}, & +i(0.25054 \pm 0.05234) \cos \alpha_{KS}, & +i(0.25054 \pm 0.05234) \cos \alpha_{KS}
\end{pmatrix}
\end{equation}

In addition, we determine $J = 1.364^{+0.599}_{-0.599} \times 10^{-3} |\sin \alpha_{KS}|$ which is considered to be an experimental value.

\[^2\text{The usual definition in Ceccucci et al. on the CKM matrix \[^{40}\text{PDG}\] is the same as ours but the definition on the MNS matrix in Nakamura and Petcov for the MNS matrix \[^{44}\text{PDG}\] is the opposite to ours.\]
B. Neutrino oscillation and MNS matrix

The 2018 fit [45], using the PDG parametrization of the MNS matrix obtained ±1σ mass squared differences in the normal (NH) and inverted (IH) hierarchies as [45],

\[ \Delta m^2_{21} = 0.755^{+0.020}_{-0.016} \times 10^{-4} \text{ eV}^2, \]

and

\[ \Delta m^2_{31} = 2.50^{+0.03}_{-0.02} \times 10^{-3} \text{ eV}^2, \]

\[ \Delta m^2_{31} = 2.42^{+0.03}_{-0.04} \times 10^{-3} \text{ eV}^2. \]

The fit [45] gives almost the same MNS angles for the NH and IH mass squared differences, and hence these MNS matrices are taken to be the same here. The leptonic data is not accurate enough in particular on the phase [46], i.e. \[ \delta_{\text{PDG}} = 1.32^{+0.21}_{-0.15} \] [PDG/o = 238^{+38}_{-27} \] for NH, and \[ \delta_{\text{PDG}} = 1.56^{+0.13}_{-0.12} \] [PDG/o = 281^{+23}_{-20} \] for IH. Three real angles in the PDG parametrization are given by \[ \theta_{12}/^o = 34.5^{+1.2}_{-1.0}, \quad \theta_{23}/^o = 47.7^{+1.2}_{-1.1}, \quad \text{and} \quad \theta_{13}/^o = 8.45^{+0.16}_{-0.14}. \] The Jarlskog determinant is about \[ 3.3 \times 10^{-2} \sin \delta_{\text{MNS}} \] in both cases, which ranges \[ [-3.5, +0.3] \times 10^{-2} \sin \delta_{\text{PDG}} \] for NH and \[ [-3.5, -1.6] \times 10^{-2} \sin \delta_{\text{PDG}} \] for IH. Thus, the neutrino data is not accurate enough to pinpoint the value \( \delta_{\text{MNS}} \). To use the KS parametrization, we need real (11), (12), and (21) elements, but to our best knowledge we should obtain \( \Theta_i \) from the neutrino oscillation data presented in the \( \Theta_{ij} \) form [44]. So, we convert \( V^\text{PDG}_{\text{MNS}} \) to \( V^\text{KS}_{\text{MNS}} \) in the following way, satisfying the unitarity of \( V^\text{KS}_{\text{MNS}} \) with a real determinant,

\[ V^\text{PDG}_{\text{MNS}} \rightarrow V^\text{KS}_{\text{MNS}} = L V^\text{PDG}_{\text{MNS}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_{\text{PDG}}} \end{pmatrix} \]

where

\[ L = \begin{pmatrix} 1 & \frac{C_{23}S_{23} + C_{12}S_{13}S_{23}e^{-i\delta_{\text{PDG}}}}{\sqrt{2}C_{12}C_{23}S_{13}S_{23}S_{23}C_{23}S_{12}S_{12}C_{12}S_{12} + C_{12}S_{12}C_{12}S_{12}C_{23}S_{23}S_{23}C_{23}S_{12}} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{2}C_{12}C_{23}S_{23}S_{23}S_{23}S_{23}C_{23}S_{13}S_{12}C_{23}S_{23}S_{23}S_{23}C_{23}S_{13}S_{12} + 2C_{12}C_{23}S_{13}S_{23}S_{23}S_{23}C_{23}S_{12}S_{12}C_{23}S_{12}S_{12}}{C_{12}S_{13}S_{23}S_{12}S_{12}C_{23}S_{12}S_{12}C_{23}S_{12}S_{12}C_{23}S_{12}S_{12}C_{23}S_{12}S_{12}C_{23}S_{12}S_{12}} } \end{pmatrix}. \]

Then, from the values of [45] [46], we obtain,\(^3\)

\[ V_{\text{MNS}(11)}^\text{KS} = C_{12}C_{13} = 0.81518^{+0.00174}_{-0.00097}, \]

\[ V_{\text{MNS}(12)}^\text{KS} = C_{13}S_{12} = 0.56026^{+0.01707}_{-0.01423}, \]

\[ V_{\text{MNS}(21)}^\text{KS} = -\sqrt{C_{23}^2S_{12}^2 + C_{12}^2S_{13}^2S_{23}^2 + 2C_{12}C_{23}S_{13}S_{23}S_{12}C_{23}S_{12}C_{23}S_{12}C_{23}S_{12}} \cos \delta_{\text{PDG}} \]

\[ = -0.39158^{+0.00171}_{-0.00147} - 0.08720^{+0.00214}_{-0.00167} \cos \delta_{\text{PDG}}. \]

Let us take the range \( \delta_{\text{PDG}} = [225^\circ, 315^\circ] \) and \([45^\circ, 135^\circ]\). Now, from these experimental (11), (12), and (21) elements of \( V_{\text{MNS}} \), we can determine three real angles using the KS parametrization

\[ \begin{cases} \Theta_1 = 35.3948^\circ \\ \Theta_2 = 39.1733 - 55.9735^\circ \text{ for } \delta_{\text{PDG}} = 225^\circ - 315^\circ \\ \Theta_3 = 14.6965^\circ \end{cases} \]

where we used \( \delta_{\text{PDG}} \) near \( \frac{3\pi}{2} \) and \( \frac{\pi}{2} \). Small errors of three real angles are determined by the error propagation method,\(^4\)

\[ \delta_1 = \begin{cases} +1.16133^\circ \\ -0.96745^\circ \end{cases}, \quad \delta_2(\Theta_2 = 47.4633^\circ) = \begin{cases} +2.37414^\circ \\ -2.34132^\circ \end{cases}, \quad \delta_3 = \begin{cases} +9.11772^\circ \\ -7.59831^\circ \end{cases}. \]

\(^3\) If we used the PDG parametrization, we must involve elements of 3rd family members, \textit{i.e.} (23) and (33) elements, to determine three real angles.

\(^4\) \( \delta_2 \) depends on the value of \( \Theta_2(\delta_{\text{PDG}}) \).
These are needed to compute the error bars for the MNS matrix with the KS parametrization. As an illustration, we use three real angles $\Theta_{1,2,3}$ determined from $\delta_{\text{PDG}}^{\ell} = \frac{3\pi}{2}$, i.e. for $\Theta_2$ we choose the median $\Theta_2 = 47.4633^\circ$. Thus, we obtain the following $V_{MNS}^{\ell}$,

$$
\begin{pmatrix}
0.81518^{+0.01174}_{-0.00978}, & 0.56026^{+0.01707}_{-0.01423}, & 0.14695^{+0.08925}_{-0.07438} \\
-0.39158^{+0.01371}_{-0.01475}, & +0.53308^{+0.02423}_{-0.02085}, & -i(0.186939^{+0.11121}_{-0.09203}) \sin \alpha_{KS}^\ell , \\
-i(0.42679^{+0.02029}_{-0.01895}) \sin \alpha_{KS}^\ell , & -0.17152^{+0.10217}_{-0.08463}, & +i(0.71274^{+0.03291}_{-0.02853}) \cos \alpha_{KS}^\ell , \\
-(0.42679^{+0.02029}_{-0.01895}) \cos \alpha_{KS}^\ell , & +i(0.58101^{+0.02637}_{-0.02268}) \sin \alpha_{KS}^\ell , & -i(0.71274^{+0.03291}_{-0.02853}) \cos \alpha_{KS}^\ell
\end{pmatrix}
$$

In this case, we obtain $J \simeq 3.34 \times 10^{-2} |\sin \alpha_{KS}^\ell|$ from any one out of 6 possible products of the form given in Eq. (6). Note that for $\cos \alpha_{KS}^\ell \simeq 0$, the absolute values of $V_{MNS}^{KS}$ is somewhat close to a tri-bimaximal form.

Comparing this KS value with the previous PDG value $3.5 \times 10^{-2} |\sin \delta_{\text{PDG}}|$, we have a relation

$$
|\sin \alpha_{KS}^\ell| \simeq 0.56 |\sin \delta_{\text{MNS}}|,
$$

from which we obtain $|\alpha_{KS}^\ell| \simeq 30.9^\circ$ if $|\sin \delta_{\text{MNS}}| = 1$. Thus, we estimate $J \lesssim 3.5 \times 10^{-2}$. If we take the initial central value of the T2K experimental range $|\delta_{\text{T2K}}| = 270^\circ$ [17], we can draw the leptonic Jarlskog triangle as shown in Fig. 4. This is because both $|\alpha_{KS}^\ell| \simeq 30.9^\circ$ and $|\delta_{\text{T2K}}| \simeq 90^\circ$ must belong to the corner angles of this triangle.

V. CONCLUSION

We attempted to present the approximate CKM and MNS matrices in the form of 3 × 3 matrices, Eqs. (14) and (22), by determining three real angles. In the final matrices, we included the least known phases $\alpha_{KS}$ and $\alpha_{KS}^\ell$ as free parameters. The Jarlskog invariants in the quark and lepton sectors are determined as $J_q \sim 1.36 \times 10^{-3} |\sin \alpha_{KS}|$ and $J_\ell \sim 3.34 \times 10^{-2} |\sin \alpha_{KS}^\ell|$, respectively, which are the essential information for future model buildings from string compactification.

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