Electron fractionalization into spinons and chargeons plays a crucial role in 2D models of strongly correlated electrons. In this paper we show that spin-charge separation is not a phenomenon confined to lower dimensions but, rather, we present a field-theoretic model in which it is realized in 3D. The model involves two gauge fields, a standard one and a two-form gauge field. The physical picture is that of a two-fluid model of chargeons and spinons interacting by the topological BF term. When a Higgs mechanism of the second kind for the two-form gauge field takes place, chargeons and spinons are bound together into a charge 1 particle with spin 1/2. The mechanism is the same one that gives spin to quarks bound into mesons in non-critical string theories and involves the self-intersection number of surfaces in 4D space-time. A state with free chargeons and spinons is a topological insulator. When chargeons condense, the system becomes a topological superconductor; a condensate of spinons, instead realizes $U(1)$ charge confinement.

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I. INTRODUCTION

The concept of spin-charge separation is one of the guiding principles of the modern approach to strongly correlated, low-dimensional systems [1]. The idea is that, in specific ground states, the electron is fractionalized into two "constituent" quasi-particles, the chargeon (holon) carrying only the charge degree of freedom and the spinon, carrying only the spin degree of freedom. The two quasi-particles interact via emergent gauge fields: the electron is reconstituted when the gauge interaction becomes strong enough to cause confinement.

The idea originates from the work of Tomonaga and Luttinger and was shown by Haldane [2] to be a generic feature of 1D metallic systems. Moreover, the idea of electron fractionalization is thought to play a crucial role in the physics of the high-$T_c$ cuprates. Indeed, spin-charge separation seems to be an unavoidable characteristic of the 2D $tJ$ model [3, 4] of the doped Mott insulators, capturing the essential physics of high-$T_c$ superconductivity [5]. There is an abundance of evidence that electron fractionalization in these models leads to new quantum orders not characterized by symmetry breaking [6].

A key ingredient of the spin-charge separation idea in 2D is the representation of chargeons and spinons as a two-fluid model with mutual Chern-Simons interactions [5], a picture that can be analytically derived from the $tJ$ model [3, 4]. Mixed Chern-Simons fluids as representations of condensed matter systems where first introduced in [7], where it was shown that they capture all the essential physics of 2D Josephson junction arrays. In the same paper it was also pointed out that this two-fluid construction can be generalized to 3D, the topological interaction being encoded in what is known as the BF term [8].

Based on this construction we proposed a new, topological mechanism for superconductivity based on the condensation of topological defects in a BF model [9], no symmetry breaking being involved. A pure BF theory has also been recently shown [10] to represent the long-distance physics of 3D topological insulators [11]. The electric permittivity and magnetic permeability of the material govern quantum phase transitions from topological insulators to topological superconductors and possibly to a $U(1)$ charge confinement phase [12].

Spin-charge separation is mostly believed to be a phenomenon confined to lower space dimensions (1D and 2D), where it is typically derived from fermionic models like, e.g., the Hubbard model. In this paper we show that this is not so. The topological excitations driving the topological insulator-topological superconductor and topological insulator-confinement quantum phase transitions in 3D are indeed chargeons and spinons. We show that the presence of a specific combination of relevant terms in the action leads to a generalized Higgs mechanism of the second kind for the two-form gauge field. When this happens, chargeons and spinons are bound together and the resulting excitation describes a charge 1 particle with spin 1/2. The spin arises in a subtle way: particle-antiparticle fluctuations about the symmetry-broken ground state are connected by a string whose only action is a topological term measuring the self-intersection number of the world-surface it sweeps in 4D (Euclidean) space-time. As was shown in [13], this factor is equivalent to the spin factor for fermionic particles in the path integral formalism [14]. Indeed, this is
the same mechanism giving spin to the quarks bound together in mesons in non-critical string theories, the only difference being that in the present case the string tensions and all other curvature terms vanish, so that the particles are not confined and the string carries only the spin information. This way we derive the fermionization of a bosonic two-fluid model rather then the other way around.

The idea is to represent chargeons and spinons as topological excitations of charge and spin gauge fields. These topological excitations represent quasi-particles that arise due to the compactness of the corresponding gauge groups, the charge and spin fields mediating the emergent gauge interactions between these quasi-particles.

A massive spin 1/2 particle in 3D is characterized by three degrees of freedom, a scalar charge degree of freedom and two degrees of freedom for the spin. Representing the spin as a separate bosonic entity, the spin field, requires thus a massive vector with two degrees of freedom. It is well known that, in 3D, massless vectors (photons) carry two degrees of freedom, called helicities, while massive vectors obtained from spontaneous symmetry breaking and described by the Proca Lagrangian carry three degrees of freedom. This is the reason why it is mostly believed that spin-charge separation is impossible in 3D. It is, however, not widely appreciated that the second is the famed Chern-Simons term which is possible non-relativistic effects would not alter the main conclusions.

The first two terms in this action are purely topological terms: the first is called generally the BF term and represents a generalization to 3D of the mutual Chern-Simons terms in 2D. It preserves the P and T symmetries if the two-form gauge field is a pseudotensor. The second is the famed $\theta$-term of axion electrodynamics. The parameter $\theta$ is an angle variable with periodicity $2\pi$, the partition function being invariant under the shift $\theta \to \theta + 2\pi$. The $\theta$-term breaks generically the P and T symmetries: these are however restored when $\theta$ is quantized: $\theta = n\pi, n \in \mathbb{Z}$. Thus there are only two possible $\theta$ values compatible with the P and T symmetries: $\theta = 0$ and $\theta = \pi$. The third term in the action has the form of a standard Maxwell term for the effective gauge field $a_\mu$.

The physical interpretation is that

$$j_\mu = \frac{k}{2\pi} \epsilon_{\mu\nu\alpha\beta} \partial_\nu b_{\alpha\beta} , \quad \phi_\mu = \frac{k}{2\pi} \epsilon_{\mu\nu\alpha\beta} \partial_\alpha a_{\beta} ,$$

(4)

are the conserved charge and spin currents representing the low-energy fluctuations about a topologically ordered state. When the two Abelian gauge symmetries are $U(1)$ compact symmetries, the dual field strengths contain singularities

$$J_\mu = \int_C d\tau \frac{dx_\mu(\tau)}{d\tau} \delta^4 (x - x(\tau)) ,$$

$$\Phi_{\mu\nu} = \frac{1}{2} \int_S d^2\sigma X_{\mu\nu}(\sigma) \delta^4 (x - x(\sigma)) ,$$

$$X_{\mu\nu} = \epsilon^{ab} \frac{\partial x_\mu}{\partial \sigma^a} \frac{\partial x_\nu}{\partial \sigma^b} ,$$

(5)

where C and S are closed curves and compact surfaces parametrized by $x(\tau)$ and $x(\sigma)$ respectively. These have standard couplings to the charge and spin gauge fields: $ik\partial_\mu J_\mu$ and $ikb_{\mu\nu} \Phi_{\mu\nu}$ respectively and describe chargeon and spinon quasi-particle fluctuations about the ground state. Since in 3D the spinon is a vector with two degrees of freedom, the allowed polarizations are transverse when it is moving, exactly as in the case of a standard photon. Since, however, it is a massive vector particle, it can point in any space direction in its rest frame. So, in this case, it
is the direction of movement rather than the polarization
that is restricted: a spinon is a massive vector particle
that moves always perpendicularly to the direction in
which it is pointing. It has thus "quantum Hall-type"
responses to external fields.

In order to make computations tractable we shall add,
as a regulator, an infrared-irrelevant but gauge invariant
kinetic term for the charge degree of freedom,
\[ S \rightarrow S + S_{\text{reg}}, \quad S_{\text{reg}} = \frac{1}{12\Lambda^2} \int d^4x \ h_{\mu\nu\rho} h_{\mu\nu\rho}, \quad (6) \]
where \( h_{\mu\nu\rho} = \partial_{[\mu} b_{\nu\rho]} + \partial_{\nu} b_{\rho\mu} + \partial_{\rho} b_{\mu\nu} \) is the field strength
associated with the two-form gauge field \( b_{\mu\nu} \) and \( \Lambda \) is
a mass parameter of the order of the ultraviolet cutoff
\( \Lambda_0 \). This regulator term makes the quadratic kernels well
defined by inducing a mass \( m = e\Lambda k/\pi \) for all fields.
This is the anticipated topological, gauge invariant mass
[16] that is the 3D analogue of the famed Chern-Simons
topological mass [15]. This mass represents the gap for
the topologically-ordered state: it sets the energy scale
for charge- and spin-wave excitations and the thickness
scale for chargeon and spinon quasi-particle excitations.
It thus plays the same role as the inverse magnetic length
in the quantum Hall effect. The mass can be removed
again after integrations by letting \( \Lambda \rightarrow \infty \): in this case
the gap becomes infinite and only point-like chargeons
and spinons quasi-particles survive.

It is easy to establish that the model [3] describes a
topological insulator [11]. First of all, the charge degrees
of freedom, carried by the two-form gauge field \( b_{\mu\nu} \), have
no dynamics in the bulk of the material since the \( BF \)
term is a topological term. The only dynamic matter de-
grees of freedom are edge modes describing surface Dirac
fermions [10]. Secondly, let us consider the coupling of
the matter currents [4] to an electromagnetic field \( A_{\mu} \)
dictated by the form of the effective action [3],
\[ S \rightarrow S + i \int d^4x j_{\mu} A_{\mu}. \quad (7) \]
In presence of a non-vanishing angle \( \theta \), the spin field
carries vorticity. The \( BF \) coupling (contained in the defi-
nition of the current \( j_{\mu} \)) is the charge unit of chargeons
whereas \( \theta \) is the flux unit of spinons. Integrating out
all matter degrees of freedom one obtains, in the limit
\( \Lambda \rightarrow \infty \), the effective electromagnetic action
\[ S_{\text{eff}} = \int d^4x \ \frac{1}{4e^2} F_{\mu\nu} F_{\mu\nu} + \frac{i\theta}{16\pi^2} F_{\mu\nu} \tilde{F}_{\mu\nu}. \quad (8) \]
This describes a bulk dielectric material with an axion
electrodynamics term that characterizes strong topologi-
ical insulators when \( \theta = \pi \).

In [12] we have shown that a condensation of chargeons
turns the topological insulator into a topological super-
conductor, while the condensation of spinons leads to a
\( U(1) \) charge confinement regime. In what follows we will
concentrate on a different quantum phase transition.

It is well known that adding marginal terms to an ac-
tion can drive the system to a new fixed point, describing
an entirely different physics. In the present case there are
indeed three additional marginal terms that can be added
to the model [3]: \( b_{\mu\nu} f_{\mu\nu}, \) \( b_{\mu\nu} f_{\mu\nu} \) and \( b_{\mu\nu} \epsilon_{\mu\nu\alpha\beta} \partial_{\alpha} \Phi_{\beta} \). All
these terms, taken one by one, break the gauge invari-
ance [2] and introduce thus new, unwanted degrees of
freedom. There is, however one particular combination
of these three terms that can be added to the effective
action and that preserves both gauge invariances, albeit
[2] is realized in a different, more subtle way:
\[ S = \frac{i k^2}{2\theta} \int d^4x \ b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu} \epsilon_{\mu\nu\alpha\beta} \left( b_{\alpha\beta} + \frac{\theta}{4\pi k} f_{\alpha\beta} \right) + \frac{4\pi^2 k^2}{\epsilon^2 \theta^2} \int d^4x \ \left( b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu} \right) \left( b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu} \right), \quad (9) \]

Something very interesting happens when the addi-
tional marginal terms in the effective action combine
with the original ones to give [9]. Only the combina-
tion \( (b_{\mu\nu} + \frac{\theta}{4\pi k} f_{\mu\nu}) \) appears in the action and the tensor
gauge invariance [2] is preserved if it is combined with
a corresponding shift \( a_{\mu} \rightarrow a_{\mu} - \frac{\theta}{2k} g_{\mu} \). This combined
transformation can be exploited to entirely absorb \( f_{\mu\nu} \)
to \( b_{\mu\nu} \), giving the effective action
\[ S = \frac{i}{32\theta} \int d^4x \ b_{\mu\nu} \epsilon_{\mu\nu\alpha\beta} \partial_{\alpha} \Phi_{\beta} + \int d^4x \ \frac{\pi^2}{4\epsilon^2 \theta^2} b_{\mu\nu} b_{\mu\nu} + \frac{i}{16\pi} \int d^4x \ b_{\mu\nu} \epsilon_{\mu\nu\alpha\beta} \Phi_{\alpha\beta} + i \int d^4x \ b_{\mu\nu} \Phi_{\mu\nu}, \quad (10) \]
where we have rescaled, for notational simplicity, the ten-
sor gauge field by a factor 4 and we have included the
couplings to external electromagnetic fields and to quasi-
particle excitations. Note that the original \( BF \) coupling
\( k \) falls completely out of the action and is replaced by the
factor \( \theta/\pi \), that takes over the role of charge unit.

In this gauge-fixed form, the original gauge symme-
try [2] appears as broken. This is nothing else than a
Higgs mechanism of the second kind for the tensor gauge
symmetry [2]. This is also known as the St"uckelberg
mechanism [19], in which a scalar longitudinal polariza-
tion "eats up" two transverse polarizations to become a
massive vector. The St"uckelberg mechanism is the dual
of the standard Higgs mechanism and is thus responsible
for confinement [19], soldering in this case chargeons
to spinons.

Indeed, corresponding to the merger of the spin and
charge gauge fields \( a_{\mu} \) and \( b_{\mu\nu} \) into a single tensor field
with 3 massive degrees of freedom, there is also a merger
of chargeons and spinons into a unique string-like quasi-
particle excitation with open world-sheets, describing
magnetic fluxes with charged dyons at their ends. The
closed boundaries of the open surfaces represent the
world-lines of dyon-antidyon fluctuations with charge \( \theta/\pi \)
and current \( J_{\mu} = (1/2\pi) \partial_{\nu} \Phi_{\mu\nu} \) as can be inferred from
the induced electromagnetic action
\[ S_{\text{eff}} = \int d^4x \left( \frac{1}{4e^2} F_{\mu\nu} F_{\mu\nu} + \frac{i \theta}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu} \right) + \ii \int d^4x \left( \frac{\theta/\pi}{2\pi} \partial_\mu \Phi_{\mu\nu} + \frac{4\pi}{e^2} F_{\mu\nu} \Phi_{\mu\nu} \right) . \] (11)

This is the same induced action as in (9) but the matter degrees of freedom have now dynamics in the bulk.

In order to gain more insight into the character of the resulting soldered quasi-particle let us compute its induced action by using the explicit form (5) for \( \Phi_{\mu\nu} \). The relevant and marginal terms in this action are
\[ S_{QP} = \frac{\Lambda^2}{4\pi} K_0 \left( \frac{m}{\Lambda_0} \right) \int_S d^2\sigma \sqrt{g} + \frac{\Lambda^2}{16\pi m^2} \int_S d^2\sigma \sqrt{g} R - \frac{\Lambda^2}{16\pi m^2} \int_S d^2\sigma \sqrt{g} \partial_{\mu} t_{\mu} \partial_{\nu} t_{\nu} - \frac{\theta}{\pi} \frac{\Lambda}{1 + \frac{\pi^2}{e^2}} \nu + \frac{e^2 m}{8\pi^2} f \left( \frac{m}{\Lambda_0} \right) \int_{\partial S} d\sigma \sqrt{g} \partial_{\mu} x_{\mu} \partial_{\nu} x_{\nu} , \] (12)

where \( m = e\Lambda/4\pi \), \( f(x) = \int_{-\infty}^x dz K_1(z)/z \) and \( K_0 \) and \( K_1 \) are Bessel functions of imaginary argument, with asymptotic behaviours \( K_0(x) \sim \exp(-x)/\sqrt{x} \) and \( f(x) \sim \exp(-x)/\sqrt{x} \) for large \( x \). The geometric quantities in this expression are defined in terms of the induced surface metric \( g_{ab} = \frac{\partial x_\alpha}{\partial \sigma^a} \frac{\partial x_\beta}{\partial \sigma^b} \) as \( g = \det g_{ab} = X_{\mu\nu}X^{\mu\nu}/2 \) and \( t_{\mu\nu} = X_{\mu\nu}/\sqrt{g} \). The quantity \( R \) is the scalar (intrinsic) curvature of the world-surface while
\[ \nu = \frac{1}{4\pi} \int d^2\sigma \sqrt{g} e_{\mu\nu\alpha\beta} g^{ab} \partial_{\mu} t_{\alpha} \partial_{\nu} t_{\beta} , \] (13)

represents the (signed) self-intersection number of the world-surface.

The important point is that, to avoid infinities and obtain a well-defined boundary term when we remove the UV cutoff, the renormalization flow implies \( e \to \infty \) as \( \Lambda_0 \to \infty \) (and \( \Lambda = \text{const} \Lambda_0 \to \infty \)). As a consequence of this running coupling constant, both the string tension and the curvature terms in (13) vanish at large distances and the induced quasi-particle action becomes
\[ S_{QP} = m_s \int_{\partial S} d\sigma \sqrt{\frac{dx_\mu}{d\tau} \frac{dx_\nu}{d\tau} - i\frac{\theta}{\pi} \nu} , \] (14)

where \( m_s \) is the renormalized mass of the particle. Note that the only remnant of the string at large distances is the topological self-intersection number. It has been shown [13] that at \( \theta/\pi = 1 \) this topological term is just another representation of the spin factor of a point particle with spin 1/2. This shows that, in the Higgs phase (of the second kind) chargeons and spinons recombine into a single particle with charge 1 and spin 1/2.

We thus conclude that spin-charge separation can occur in 3D when a non trivial axion term with \( \theta \)-angle \( \pi \) is generated in the electromagnetic action. The phase with free separated chargeons and spinons is a strong topological insulator. Topological superconductivity occurs when chargeons condense and an exotic \( U(1) \) confinement would be realized in presence of spinon condensate.

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