Introduction

Mechanics, as in classical physics, is usually taught as the first topic in most physics curricula as it includes concepts that serve as a foundation for other complex areas of physics, for example, relativity. Pre-service physics teachers or physics students, as highlighted in several research studies in science education (e.g. Clement, 1982; McDermott, 2008) experience conceptual difficulties in both mechanics and advanced topics such as electromagnetism (Dega, Kriek, & Mogese, 2013a; Govender, 2015). In mechanics, students are introduced to motion of everyday visible macroscopic objects such as balls, swings and vehicles. The important concepts in mechanics such as distance, speed and force, can be related to students’ everyday experiences as these conceptions exhibit a high degree of coherence with reality but when abstract concepts such as momentum and conservation laws are introduced, conceptual difficulties arise (Ergul, 2013). In conservation laws, it is thought that several concepts are interrelated in a given system and their understanding requires higher levels of thinking.

There have been only a few studies on students’ understanding of vectors and these showed that physics students face difficulties in vector addition, magnitude and direction in all introductory general physics courses (Knight, 1995; Nguyen & Meltzer, 2003). Knight (1995) conducted a survey of students’ knowledge of vectors at the beginning of an introductory mechanics course at a tertiary institution and found that about half of the students in a calculus-based introductory course enter with no knowledge of vectors and only about one-third have sufficient vector knowledge to proceed with mechanics. Similarly, Nguyen and Meltzer (2003) investigated pre- and post-tests of students’ knowledge of vector properties and vector addition of students in introductory mechanics courses. They concluded that over one-quarter of the students completing a calculus-based mechanics course and one-half of the students completing the algebra-based course were unable to add vectors in two dimensions.

These studies confirmed that physics students still have significant conceptual difficulties regarding vector-kinematics. Even though students have intuitive knowledge of vectors and vector operations, obtained, to some extent, by the study of mechanics, some are nevertheless unable to apply their knowledge in a precise and meaningful manner (Flores, Kanim, & Kautz, 2010).
2004). They lack a clear understanding of what is meant by vector direction and are confused about the tip-to-tail and parallelogram addition rules (Nguyen & Meltzer, 2003). In addition, many pre-service physics teachers have confusion with sign conventions in vectors, especially in the transition between one- and two-dimensional motion, and in differentiating scalars from vectors. For example, in a one-dimensional Cartesian system, one Cartesian component is taken which can have a negative or positive sign which students sometimes wrongly regard as a vector (Govender, 2007). Saltiel and Malgrange (1980) found that the graphical representations of motion can easily be misleading. In the case of decomposition of a velocity vector into its vector components, this may lead to a figure which can be confused with a velocity composition used to illustrate the change of velocity from one frame to another. Peters (1982) also observed misrepresentation in the graphical aspect of vectors. The author notes that once one has introduced one- and two-dimensional motion with their graphical representations, students may confuse the two motions. In drawing a displacement-time (s-t) graph for one-dimensional motion, the students in Peters' study assigned special significance to the direction of the tangent vector to the curve, including components and unit vectors in the time direction. Some of these students, using the Pythagoras theorem, also computed \((x^2 + (t^2)^2)^{1/2}\) to find the magnitude of the displacement vector, and some said that the distance travelled is the length on the displacement-time graph. At the level of a calculus-based physics course, students had difficulty in recognising the relationship between the concept of area under the curve and its corresponding kinematical quantity, and its application in physics problems (Nguyen & Rebello, 2011). Govender (1999) found that preservice teachers could not differentiate scalar and vector equations in solving incline-plane problems.

Goldberg and Anderson (1989) showed that the concept of ‘direction in a vector quantity’ was not clearly understood by many college physics students, especially with representations of negative values of velocity. University students experience difficulties comprehending concepts of velocity and acceleration and their directions, especially in the case of vertical projectile motion, and students face difficulties relating vector-kinematics to dynamic quantities (Flores et al., 2004). Flores et al. (2004) investigated and characterised those difficulties, and suggested improvement of instruction by making modifications to the content of the course and to the sequence of topics to improve student understanding of vectors. The content modification involved mainly a sequence of teaching ‘forces’ first using vector addition followed by vector subtraction. ‘Forces’ were also introduced in a constant velocity context and compared to ‘forces’ in dynamic acceleration contexts. Although the degree of modification and the results have varied from course to course, none of these modifications has been more than moderately successful. Flores et al. (2004) emphasised that there were still many alternative conceptions evident in students’ use of vector-kinematics and in their responses to conceptual questions involving vectors, despite the modifications they introduced. Thus, ongoing research of fundamental concepts in mechanics, together with current technology, is still necessary to understand students’ transition from high school physics to tertiary physics.

Theoretical Framework

This study is based on David Ausubel's theory of meaningful learning (Ausubel, 1968; Ausubel, Novak, & Hanesian, 1978). The fundamental idea of this theory is that learning takes place by means of the assimilation of new concepts and propositions into existing concepts and propositional frameworks held by the learner. Ausubel's believes that learning begins with students' recognition of their already known concepts and construction of a network of concepts linking to these prior concepts or cognitive structures. According to Ausubel's theory, to learn meaningfully, individuals must relate new knowledge to relevant concepts they already know; meaningful learning results when an individual consciously and explicitly links new knowledge to relevant concepts or propositions they already possess.

Students often come to science classrooms with pre-conceptions that are inconsistent with the scientific concepts they are about to be taught and these pre-conceptions form barriers to their new learning. To effect conceptual change, one requires the identification of students' existing conceptions (Vosniadou & Brewer, 1987), which constitute both valid and invalid knowledge in terms of current scientific knowledge. The epistemological and ontological view of knowledge is that knowledge is based on individual experiences and is constituted by individual conceptions (Ouit & Treadust, 2003). This means that from the epistemological perspective, students' conceptions are viewed based on how they describe the concepts being investigated; while from the ontological perspective, students' conceptions are viewed based on how they view the nature of the concepts being investigated. For example, in electromagnetism (Dega et al., 2013a), students may describe electric potential in terms of the charge and its position (V=Es), which shows their epistemological description of electric potential; on the
other hand, they may consider electric potential as a process or as a material body, which is how they may view the nature of electric potential. We refer to this as an ontological description of electric potential.

In this case, conceptual structures such as concept maps constitute knowledge when individuals regard them as viable in relationship to their experiences. Concept mapping is mainly based on the theory of meaningful learning. This theory has been advanced from the early studies beginning in the 80’s (Novak & Cañas, 2008; Yager, 1991) and is still a relevant theory for this study.

**Framework Categories of Students’ Conceptions**

Students’ pre-existing and pre-instruction understanding influences their new learning (Sewell, 2002). In this study, students’ conception is a combined term including both students’ alternative conceptions and conceptual (scientific) knowledge in a given domain of science. Alternative conceptions refers to students’ conceptions that are inconsistent with the scientific conceptions (Gilbert & Watts, 1983; Pinarbasi, Canpolat, Bayrakceken, & Geban, 2006) while conceptual knowledge is students’ conceptions that are consistent with the scientific conception, and is an explicit understanding of a concept and/or of the interrelations between concepts in a field of knowledge (Streveler et al., 2011).

Framework categories of students’ conceptions are classifications that are depicted on the basis of their epistemological descriptions and/or ontological considerations of concepts. This classification includes categories of both alternative conceptions and conceptual knowledge. The framework categories of conceptual knowledge in physics can be categorized as hierarchical (HCK) and relational (RCK) (Stoica, Moraru, & Miron, 2011), whereas the framework categories of alternative conceptions can be separated into three categories, namely, naïve physics (NP), lateral alternative conception (LAC), and ontological alternative conception (OAC). To elaborate, in science, the concepts ‘substance’ and ‘process’ are the two main ontological domains for descriptions of physical quantities. For example, in the domain of ‘process’, students may interchange, unknowingly, their understanding of energy as ‘energy as momentum’ or ‘momentum as energy’. This conception can be classified as LAC. If students consider energy as a ‘substance or material body’ instead of as a process, this conception can be classified as OAC (Dega, Kriek, & Mogese, 2013b). These framework categories of students’ conceptions are discussed in further detail below.

**Framework Categories of Conceptual Knowledge**

Hierarchical conceptual knowledge (HCK) is the classification and categorisation of concepts into correct ontological and/or lateral categories. For example, the concept of electric current is incorrectly characterised as ‘substance’ while electric current as the rate of flow of electric charge is correctly characterised as a ‘process’. Students’ alternative conceptions can arise due to failure in understanding such ontological and/or lateral categorisation. The hierarchical conceptual knowledge of students can be measured in relation to their ability to categorise concepts under a correct ontology. Hierarchical conceptual knowledge also involves an organization of concepts from more to less inclusive concepts, such as learning from an inclusive concept of ‘energy’ to ‘mechanical energy’ then to ‘potential and kinetic energies’ (Dega, 2012; Graham & Berry, 1996, 1997; Touger, Dufresne, Gerace, & Mestre, 1987).

Relational conceptual knowledge (RCK) is considered as the construction of relationships between pieces of information (Adey, 2005). Relational conceptual knowledge is also attributed to valid qualitative relationships between two or more concepts, such as the relationship between electric potential and charge (Dega, 2012).  

**Framework Categories of Alternative Conceptions**

Naïve physics (NP) is intuitive physics (DiSessa, 1982; McCloskey, 1983) defined as a simplified and less-organised theoretical view of a concept such as the Aristotelian ‘force implies velocity’.

Lateral alternative conception (LAC) is identified when students have difficulty describing concepts within an ontological category (Chi, 2008). For example, students may incorrectly describe electric potential with some of the properties of electric force because the formula for Coulomb’s force embeds the formula for electric potential (Dega et al., 2013a).
Ontological alternative conception (OAC) is identified as an incorrect categorisation of science concepts within an overarching science domain by the students. For example, several students fail to describe the nature of energy as a process and incorrectly describe it using the properties of a ‘substance’ (Dega et al., 2013a).

Framework Categories of Mixed and Loose Ideas

Mixed conception is an alternative conception that is viewed to have characteristic features of at least two of the predetermined categories (Dega et al., 2013a). Loose ideas are students’ descriptions that are believed to not be attached to any of the predetermined categories. The conceptions in this theme have characteristics of flexibility which means that they cannot be definitely categorised into either the predetermined or the mixed category (Dega et al., 2013a).

Concept maps can provide identification and classification of the framework categories discussed, as shown in analysis of the data below.

Concept Maps Using Concept Map Tools

Concept maps are two-dimensional graphical representations of individuals’ knowledge structure of a particular conceptual topic (Novak & Cañas, 2008). The theoretical basis for concept maps is the theory of meaningful learning (Novak & Gowin, 1984) that considers concept and propositional learning as the basis on which individuals construct their own idiosyncratic meanings. Concept maps of physics concepts can be displayed as a relational and/or as a hierarchical structure (Novak & Gowin, 1984; Van Zele, Lenaerts, & Wieme, 2004; Zoller, 1990). The latter, possibly, reflects a hierarchical ordering of concepts in physics learning. However, some physicists may not view some physics topics as hierarchical (analysed by their epistemological nature) in the ways that classificatory areas of knowledge as in biology, are hierarchical and chemistry knowledge, are mostly cyclic rather than hierarchical. The authors in this study support the view that vector-kinematical concepts can be hierarchically ordered in learning physics and that concept maps can reveal this ordering. CmapTools facilitates such a process of ordering concepts during learning. CmapTools is built on Novak’s hierarchical arguments, therefore assuming hierarchical maps and may be criticised as directing students to trace only hierarchical maps. To avoid this claim, participants in this study drew their basic concept maps structure on paper first and then used CmapTools to capture their maps and extend these if necessary into complex maps.

Concept maps can be used for several purposes. They can be used to visualise and measure the depth, breadth and organization of an individual’s knowledge (Huer, 2005), to identify prior knowledge (Stoddart, Abrams, Gasper, & Canaday, 2000) and to reveal alternative conceptions (Mistades, 2009). Thus, concept maps can probe students’ conceptual structure, providing an efficient tool for assessment. The current concept map tool technology may be more powerful than pencil-and-paper concept mapping in the way that it facilitates the well-organised construction of concept maps and enables better management of large explicit, relational and hierarchical representations of concepts. This study elaborates one such concept map tool technology, copyrighted as IHMC CmapTools (Cmap, 2014).

CmapTools is software developed at the Institute for Human and Machine Cognition (IHMC) that helps learners, individually or collaboratively, to represent their knowledge using concept maps and promotes the sharing of knowledge (Novak & Cañas, 2008). CmapTools facilitates the construction of concept maps and permits better management of large representations for complex domains. In other words, this software brings together the strengths of concept mapping with the power of technology. The software is easy for users of all ages to construct and modify concept maps. It provides many capabilities for the creation of novel concept maps, including editing and an ability to attach links to resources and other concepts (Novak & Cañas, 2008). This study engaged pre-service teachers in the construction of their individual concept maps in vectors-kinematics using CmapTools. This was an innovative experience for them. Such a tool suited a framework categories analysis more easily than paper-pencil drawn concept maps in terms of legibility and provided a flexible resource if concepts were chosen to be displayed in hierarchical relationships.

Research in science has shown that one of the consistent problems encountered in physics education is that of teachers’ alternative conceptions (Helm, 1978; Oh, 2014; Segal & Cosgrove, 1992). In particular, pre-service teachers’ alternative conceptions in mechanics are linked to many other topics in physics and can constrain their teaching and learning of the more complex concepts. Fundamental topics in physics such as vector-kinematics in introductory university physics courses are often taught in only a few lectures and students often do not integrate
kinematics with dynamics (Flores et al., 2004; Govender, 1999). Thus, the difficulties which students have with dynamic concepts are compounded by difficulties with kinematics (Lawson, 1984).

Shaffer and McDermott (2005) studied the difficulties that vector-kinematics presents not only to introductory students, but also to teachers and graduate students. They found that in many instances the difficulties are mainly conceptual rather than mathematical, and, in some cases, the inability to perform basic vector operations is the problem. Many university physics students lack an understanding of the relationships between kinematical quantities and their graphic representations, such as the concept of area under the curve of a graph, and their application in physics problems (Nguyen & Rebello, 2011). These studies showed that standard instruction did not help students to apply the formalism that they were taught. Thus, it is supposed that many pre-service teachers embark on their teaching career before having developed an adequate conceptual understanding of core concepts of physics such as vector-kinematics, in spite of exposure to the topic at university.

To effect changes in teaching and learning, therefore, prior knowledge of pre-service teachers’ conceptions is necessary. In this regard, concept maps using currently available computerised technological tools, for example CmapTools (Novak & Cañas, 2008), can serve as a quick overview to physics educators of how their pre-service teachers make sense of and connections with basic concepts in creating propositions in a topic (Vanides, Yin, Tomita, & Ruiz-Primo, 2005). Thus, the research questions in this study are:

1. What are the epistemological (how they describe) and ontological (how they view) categories of pre-service teachers’ vector-kinematical concepts as represented by their concept maps using CmapTools technology?
2. Is there any category of the pre-service teachers’ conceptions predominantly revealed in their representations in their concept maps using CmapTools technology?

Research Methodology

General Background

Pre-service teachers were in their third-year of study in a four-year Bachelor of Education degree at a South African university, specializing in the subject Physical Science. Their complete course consists of three content modules in physics and chemistry and three method modules in physical sciences. Teachers had completed their first non-calculus mechanics module in their first or second year of study. Many of these pre-service teachers enrolled in physics at this university come from rural schools where poor teaching of physical sciences prevails and where limited resources and poor qualifications of teachers may have impacted on their learning. Their poor performance may also be due to language difficulties as for many of them their home language is isiZulu. They will have been taught in this medium up until Grade 5 but from then onwards will have been taught in English and sometimes code-switching may have taken place in their classes. All their assessments, including external national examinations in school and internal examinations at university would have been conducted in English. In schools they were taught one-dimensional vector-kinematics in physics and in the mechanics module at university they would have been introduced to two-dimensional vector-kinematics. The data was collected from pre-service teachers enrolled in their Method II module which covers Physical Science teaching.

Sample Selection

The entire Physical Science Method II module consisted of 35 students but only 28 volunteered for the study as some pre-service teachers while participating in the coursework did not want their data to be used in the study. The volunteers selected signed a letter of consent to participate in the study.

Instrument and Procedures

A quantitative research method which included a descriptive and interpretive qualitative research approach was followed. Pre-service teachers’ initial hand-drawn concept maps and their final computer generated CmapTools concept maps were the sources of data.

At the beginning of the module, an article on the introduction and construction of concept mapping by Vanides et al. (2005) provided the basis for developing generic concept mapping skills in a lecture session. Eight...
vector-kinematics concepts, namely, vector, scalar, direction, magnitude, speed, distance, time and velocity were initially given to afford pre-service teachers a practice session in articulating the cognitive demands of the task by seeking relationships in completing a concept map on paper. The authors played a supportive administrative role only, clarifying tasks to be accomplished and not interfering in the participants' drawing of concept maps. The initial concept maps in vector-kinematics were drawn at the beginning of the Physical Science Method II module.

The participants were then taken to a university Local Area Network (LAN) where computers were already preloaded with CmapTools software and were introduced to the skills of using CmapTools software to draw concept maps arising from their paper attempts. Participants were given instructions to display their network concept maps in vector-kinematics choosing their own structure and concepts as they interpreted how the network should be constructed. After a few practice sessions, they drew their final concept maps in vector-kinematics on CmapTools software building on the basic concepts provided. Data in the form of concept maps was collected from all 28 participants. The data was saved and collected immediately by the researchers for analysis. Students' names were not identified in the study and only numbers were assigned to individual concept maps, thus making them anonymous. The data collected was stored for safe-keeping. The limitations include the smaller class size (28 volunteered of 35) which is typical of physical science method classes as fewer students take physical science teaching than biology, limited computers, constraints of reservation time in university LAN and the tedious process of transcriptions from the 28 concept maps into propositions for framework analysis.

Data Analysis

The data was transcribed and then categorised based on the students' epistemological and ontological descriptions of vector-kinematical concepts. The two researchers in the study analysed the concept maps separately at first and then reached consensus on the emerged categories of the conceptions after some deliberations to ensure inter-rater reliability.

In response to the first research question, the framework analysis method (Rabiee, 2004; Ritchie & Spencer, 2002) was used for concept maps qualitative data analyses. The reason for using this method was its characteristic feature that allows themes to develop both from the literature and from the research participants' concept maps (Rabiee, 2004). From pre-service teachers' concept maps drawn on CmapTools (see example in Figure 1), transcribed statements were obtained from two linked concepts, for example, 'vectors' and 'direction and magnitude' together with their label or linking word, for example 'have', formed a proposition and when written out as 'Vectors have both magnitude and direction' denoted the transcribed statement as shown in Table 1. The transcribed statements generated from each participant's concept maps were classified into alternative conceptions and conceptual knowledge. Then, the emerged themes of the participants' conceptions were considered as the framework of the categories of their understanding in the context of their epistemological descriptions and ontological considerations of vector-kinematics. The scientifically valid and invalid responses that appeared on every participant's concept maps as propositions, hierarchies, cross-links and examples, were used in the organization of the themes.

Figure 1: A pre-service teachers' concept map in vector-kinematics using CmapTools.
The frequency of pre-service teachers’ conceptions and the extensiveness of the conceptions in the categories were analysed to study how often their conceptions were revealed and how extensive were their conceptions in the categories. In this case, frequency represented the number of independently described conceptions in the concept maps. The maximum frequency of a conception could be equal to the number of students involved in the concept mapping while its minimum was when only one student described a conception.

In response to the second research question, the extensiveness of the pre-service physics teachers’ conceptions in each category was analysed by grouping similar conceptions into a category. The number of items in each category was counted to provide a frequency count. The extensiveness in each category is represented by the total frequency of conceptions. This extensiveness mainly helped to show the disparities of concentration of the conceptions in the categories.

**Results of Research**

In response to the first research question, the findings revealed two categories of conceptual knowledge, and three categories of alternative conceptions, Mixed Conceptions (MC) and Loose Ideas (LI). The two categories of conceptual knowledge included hierarchical conceptual knowledge (HCK) and relational conceptual knowledge (RCK), which are discussed further and elaborated in Tables 1 and 2 respectively. Among the categories of alternative conceptions, the three identified were the presupposed categories used in science education literature (Dega et al., 2013a), namely, the Naïve Physics (NP), the Lateral Alternative Conceptions (LAC), and the Ontological Alternative Conceptions (OAC), which are elaborated upon in Tables 3-5. The Mixed Conceptions and Loose Ideas are respectively shown in Tables 6 and 7. The details of these categories are discussed below.

**Hierarchical Conceptual Knowledge of Vector-Kinematics**

Nine propositions of hierarchical conceptual knowledge of the pre-service teachers' were revealed in the vector-kinematics concepts. Most pre-service teachers frequently mentioned simple and basic conceptual knowledge, for example, a description of vectors as quantities having magnitude and direction (see Table 1).

| Conceptions                                                                 | Frequency |
|-----------------------------------------------------------------------------|-----------|
| 3. Vectors have both magnitude and direction                               | 21        |
| 4. Vectors include velocity, displacement, acceleration, and force          | 17        |
| 5. Vector operations include head-tail addition of vectors and parallelogram methods | 10        |
| 6. Vectors can be calculated using algebra and graphic representation        | 5         |
| 7. Direction can be shown using compass and angle. Angles are a form of direction | 2         |
| 8. Vectors include frame of reference                                       | 2         |
| 9. Vector operations include component methods and polar forms using unit vectors | 1         |
| 10. Force is an example of vectors and includes normal force, applied force and gravitational force | 1         |
| 11. Vector operation involves dot and cross products, component methods and polar forms | 1         |

As depicted from the concept maps of the 28 pre-service teachers’ and shown in Table 1, 75% of them described vectors as quantities having magnitude and direction, 61% mentioned examples of vectors, and 54% mentioned addition and subtraction operations in vectors. Only 4% (one student) mentioned vector operations as dot and cross products and attempted to give examples as work and torque, respectively.

**Relational Conceptual Knowledge in Vector-Kinematics**

As illustrated in Table 2, eight propositions of relational conceptual knowledge were depicted. The pre-service teachers considered velocity, acceleration and displacement as examples of vectors. A few (10%) mentioned addition...
of vectors in a qualitative form and only 4% tried to show multiplications of vectors, like scalar and dot products. A few (7%) were able to relate kinematics to dynamics like acceleration and force using Newton’s Second Law, and to describe the concept frame of reference in one-dimensional context as a set of coordinate axes to specify position. This shows that the pre-service teachers’ relational conceptual knowledge of vector-kinematics is shallow with little in-depth understanding of the concepts.

Table 2. Relational Conceptual Knowledge (RCK) in Vector-Kinematics.

| Conceptions                                                                 | Frequency |
|-----------------------------------------------------------------------------|-----------|
| 1. Velocity is time rate of change of displacement                          | 3         |
| 2. Displacement is change in position                                        | 3         |
| 3. Acceleration is rate of change of velocity                               | 2         |
| 4. Instantaneous velocity is the measure of velocity of an object at a particular moment | 1         |
| 5. Vectors are added or subtracted to give vector sum                        | 3         |
| 6. Dot product is used in calculation of work                                 | 1         |
| 7. Vector product is used in calculation of torque and magnetic force        | 1         |
| 8. Force is the product of acceleration and mass                             | 1         |
| **Extensiveness**                                                           | **15**    |

Naïve Physics in Vector-Kinematics

About two-fifths of the pre-service teachers (39%) described vectors only in a one-dimensional context as they depicted vectors as having either vertical or horizontal directions (Table 3). Their other alternative conceptions were mainly driven by this one-dimensional context of constructs of vectors like, for example, ‘scalars with no direction are vectors’ in which they gave distance as an example. This naïve physics seemed to arise from their experience of one-dimensional motion, in which speed is magnitude of velocity and distance is magnitude of displacement.

Table 3. Naïve Physics (NP) in Vector-Kinematics.

| Conceptions                                                                 | Frequency |
|-----------------------------------------------------------------------------|-----------|
| 1. Direction of vectors is either in horizontal or vertical axes, that is, vectors are studied in one-dimensional motion | 11        |
| 2. Vectors focus on velocity                                                 | 2         |
| 3. Motion in one dimension uses frame of reference                          | 1         |
| 4. Motion in one dimension describes average velocity, average acceleration and average speed | 1         |
| 5. Addition/subtraction of vectors is for one dimension                      | 1         |
| 6. Different angle vectors are added using head-to-tail method               | 1         |
| 7. Acceleration involves speed up and speed down                             | 1         |
| 8. Vectors are used to describe position                                     | 1         |
| 9. Vectors have magnitude                                                    | 1         |
| **Extensiveness**                                                           | **20**    |

Besides the aforementioned, most of the alternative conceptions from the pre-service teachers’ concept maps (in Table 3) were infrequently revealed by 7% and fewer of the sample, for example, ‘vectors focus on velocity’. Such a conception is probably due to the pre-service teachers’ common-sense experiences in life of describing vectors. Most high school teachers very often taught to use velocity first as an example of vectors.

One pre-service teacher (4%) incorrectly described average velocity and average acceleration in a one-dimensional motion. This might be due to the misleading equations and terminologies in some physics textbooks, like $v = \frac{d}{t}$ for average velocity and $a = \frac{v}{t}$ for average acceleration. Introductory physics textbooks use shortcuts, omissions
and simplifications which are meant to reduce complexity but are not, in fact, conducive to understanding, and make meaningful learning difficult. Instead, the textbooks lead pre-service teachers to a reduced understanding of the concepts of vector-kinematics and force them to memorise the algebraic equations and procedures that may lead to correct numerical answers to some end-of-chapter problems. Such presentations are misleading and incorrect in fundamental ways.

**Lateral Alternative Conceptions in Vector-Kinematics**

In total, thirteen lateral alternative conceptions were inconsistently described, with, eleven being depicted once and three being depicted twice. These conceptions included consideration of force as momentum, and velocity as force and energy (see Table 4). Seven percent of the pre-service teachers described acceleration and final velocity as parts of velocity. This showed their failure to understand velocity as the ratio of change of position to time elapsed. They perceived velocity as a primitive sense of relation between position and time or change of position or increase of position but not as a ratio. They also described acceleration as speeding up or speeding down but not as a ratio of change of velocity to the elapsed time.

**Table 4. Lateral Alternative Conceptions (LAC) in Vector-Kinematics.**

| Conceptions                                                                 | Frequency |
|----------------------------------------------------------------------------|-----------|
| 1. Velocity includes force and final velocity (velocity as a force)        | 2         |
| 2. Vectors have distance                                                   | 1         |
| 3. Velocity includes acceleration                                          | 1         |
| 4. Vectors can be energy                                                   | 2         |
| 5. Energy can be kinetic, mechanical and potential                         | 1         |
| 6. Frame of reference is sign convention, positive or negative             | 1         |
| 7. Vectors are known as scalar and vector quantities (vectors as inclusive physical quantities) | 1         |
| 8. Total distance is added using parallelogram method (distance as displacement) | 1         |
| 9. Magnitude of vector is total distance (magnitude of vector as scalar)   | 1         |
| 10. Force consists of mass and velocity (force as momentum)               | 1         |
| 11. ‘Mechanics’ as a vector has direction given by the sign convention (+ or -) | 2         |
| 12. Vectors have displacement                                              | 1         |
| 13. Velocity is the quantity that has magnitude and direction (velocity as inclusive concept) | 1         |
| **Extensiveness**                                                         | **16**    |

**Ontological Alternative Conceptions in Vector-Kinematics**

There were only three ontological alternative conceptions described which was insignificant comparatively to all the conceptions depicted. An example of this was a pre-service teacher’s consideration of vectors as a moving object rather than a characteristic property of a moving object (Table 5).

**Table 5. Ontological Alternative Conceptions (OAC) in Vector-Kinematics.**

| Conceptions                               | Frequency |
|-------------------------------------------|-----------|
| 1. Vectors move in a straight line (linear motion) | 1         |
| 2. Velocity measures how fast position of a moving body is moving          | 1         |
| 3. Direction represents motion of an object                                  | 1         |
| **Extensiveness**                                                               | **3**     |
Mixed Conceptions in Vector-Kinematics

Five mixed conceptions were analysed from the pre-service teachers’ depictions of conceptions on their concept maps. These were a mix of categories from conceptual knowledge and alternative conceptions or a mix of alternative conceptions from different categories (Table 6).

Table 6. Mixed Conceptions (MC) in Vector-Kinematics.

| Conceptions                                                                 | Frequency |
|-----------------------------------------------------------------------------|-----------|
| 1. Vectors have magnitude, direction and mass (HCK+OAC)                      | 1         |
| 2. Vectors fall under mechanics (HCK+NP)                                    | 1         |
| 3. Vectors have magnitude, direction and force (HCK+LAC)                    | 1         |
| 4. Examples of vectors are velocity, acceleration, conservation of mechanical energy, displacement, mechanical energy, kinetic energy (HCK+LAC) | 1         |
| 5. Vectors have magnitude and direction, like force, velocity and weight (HCK+LAC) | 1         |
| **Extensiveness**                                                           | **5**     |

Loose Ideas in Vector-Kinematics

Seven loose ideas were identified from the concept maps of the pre-service teachers (Table 7).

Table 7. Loose Ideas (LI) in Vector-Kinematics.

| Conceptions                                                                 | Frequency |
|-----------------------------------------------------------------------------|-----------|
| 1. One dimensional motion consists of force, displacement and relative velocity | 1         |
| 2. Forces are described by Newton’s laws                                     | 1         |
| 3. Vectors can represent motion                                              | 1         |
| 4. Vectors can be used to solve various problems in motion and forces         | 1         |
| 5. Vectors are to be taught with a fair balance of theory and practice       | 1         |
| 6. Vectors have many real applications                                       | 1         |
| 7. Vectors teach specific skills                                             | 1         |
| **Extensiveness**                                                           | **7**     |

Response to the second research question, extensiveness of the categories of the pre-service teachers’ conceptions in vector-kinematics, is shown in Table 8. The number in the table is meant to represent the number of independently-depicted conceptions in a category while the extensiveness, as stated in the Research Methodology, is associated with the total frequency of the independently-described conceptions in a category. The number was used to represent the number of conceptions while the extensiveness was used to show the relative concentration of conceptions in a framework category.
Table 8. Extensiveness of the pre-service physics teachers’ conceptions in Vector-Kinematics.

| Vector-Kinematics Conceptions | Number of conceptions | Percentage of number of conceptions | Extensiveness | Percentage of Extensiveness |
|-------------------------------|-----------------------|------------------------------------|---------------|-----------------------------|
| Conceptual knowledge          |                       |                                    |               |                             |
| HCK                           | 9                     | 17                                 | 60            | 48                          |
| RCK                           | 8                     | 15                                 | 15            | 12                          |
| Alternative conceptions       |                       |                                    |               |                             |
| NP                            | 9                     | 17                                 | 20            | 16                          |
| LAC                           | 13                    | 24                                 | 16            | 13                          |
| OAC                           | 3                     | 6                                  | 3             | 2                           |
| Mixed and Loose               |                       |                                    |               |                             |
| MC                            | 5                     | 9                                  | 5             | 4                           |
| LI                            | 7                     | 13                                 | 7             | 5                           |
| Total                         | 54                    | 100                                | 126           | 100                         |

As shown in Table 8, from the total number of 54 independent conceptions of the pre-service teachers depicted, 32% were in the categories of conceptual knowledge, 47% were in the categories of alternative conceptions and the rest (21%) were in the mixed conceptions and loose ideas. This showed that the number of pre-service teachers’ alternative conceptions was predominant compared to the scientific conceptions.

On the other hand, among the 126 extensiveness of the pre-service teachers’ conceptions, 31% were alternative conceptions, 4% were mixed conceptions, and 5% were loose ideas. Their conceptual knowledge constituted three-fifths (60%) of their conceptions (Table 8). This showed that the extensiveness of the categories of the pre-service teachers’ conceptual knowledge (60%) was greater than that of the categories of their alternative conceptions (31%). In other words, the pre-service teachers’ conceptions were more concentrated on their conceptual knowledge than their alternative conceptions, which showed that the pre-service teachers’ had more common conceptual knowledge than alternative conceptions in vector-kinematics.

Hierarchical conceptual knowledge was the most extensively described category of conceptions with an extensiveness of 60 (48%) while the extensiveness of relational conceptual knowledge was only 15 (12%), as shown in Table 8. This means that the concentration of the pre-service teachers’ conceptions on hierarchical conceptual knowledge was far greater than on their relational conceptual knowledge.

Naïve physics was the second most extensive category (16%) of the pre-service teachers’ conceptions, after hierarchical conceptual knowledge (Table 8). Naïve physics was the most extensive (about 52%) among the alternative conceptions.

Thus, the analysis of individual concept maps indicate that the pre-service teachers’ understanding of vector-kinematics concepts is complex and shows that their paths of learning and creating of propositions at different hierarchical levels are not coherent, but chaotic, idiosyncratic and diverse. Their hierarchical conceptual knowledge was more extensive than their relational conceptual knowledge and other categories of alternative conceptions.

Discussion

Analysis of the data in this research study produced seven framework categories of the participants’ conceptions in vector-kinematics (see Tables 1-7) with differing extensiveness (summarised in Table 8). Two categories were conceptual knowledge (hierarchical and relational), three were alternative conceptions (naïve, lateral, ontological) and two were mixed conceptions and loose ideas.

Inclusive hierarchical description of vectors in conceptual category in Hierarchical Conceptual Knowledge (HCK) and their examples (Table 1) were described most frequently (60 conceptions) resulting in high percentages in extensiveness (48%) of this category in relation to all other categories (see Table 8). The depicted examples obtained from their concept maps shown in Table 1 reflect previous and meaningful knowledge (in terms of Ausubel’s work) which included memorised pieces of definitions, for example, ‘vectors are quantities that have direction and magnitude’ and differentiated terms such as ‘scalars have no direction compared to vectors’ was evident. Table 8 reveals that 41% of alternative concepts were Naïve Physics (NP) and Lateral Alternative Conceptions (LAC) with a 29% extensiveness of distribution. These alternative concepts form conceptual anchors for pre-service teachers foundational knowledge and while incorrect, are also at a lower-order cognitive level as described in Bloom’s tax-
onomy of knowledge (Bloom et al., 1956). This is a cause of concern. Studies confirm that kinematics is a difficult topic in mechanics but significant as from the motions of objects, forces are understood. Studies also confirm that instruction in kinematics is far from adequate as the Baseline test (Hestenes & Wells, 1992) documents the general weakness of kinematics instruction. The test also reveals widespread deficiencies in the qualitative understanding of acceleration which relies on vector-kinematics notions. Even higher level concepts of vector-kinematics are required for university physics learning, as for vector operations of addition and subtraction, dot and cross products and component methods using polar forms and unit vectors. These were rarely evident in the concept maps of pre-service teachers in this study. Also, mixed conceptions in Table 6 reveal that they do not differentiate vectors from other distinct concepts such as ‘vector is acceleration’ and ‘vector is energy’. The pre-service teachers also confused algebraic formalism with vector addition. For example, ‘distance can be added using parallelogram method’ in which they failed to differentiate between algebraic sum and vector addition. In addition, the pre-service teachers have such naïve conceptions of vector addition or subtraction that they realise them as simple algebraic sum or difference.

The results in this study confirm that while 60% of the pre-service teachers seemed to have a coherent understanding of the inclusive and basic hierarchical concepts, there was also inconsistent, chaotic or absent understanding of the more advanced concepts.

Most of the pre-service teachers’ concept maps represented vector-kinematics in a one-dimensional context – none of them described two-dimensional motions such as circular and projectile motions. The reason for this could be that most school-based teaching of vector-kinematics starts with the representation of vectors in one dimension, and their mathematical thinking of addition and subtraction of vectors is still evident in an intuitive way even though they were previously taught linear and circular motion in two dimensions in the mechanics module. This means they still understand change of velocity as change of its magnitude only, as in the case of linear motion. This shows their failure to understand that change of velocity can also happen due to change of direction. It seems that most students generalise the one-dimensional vector addition and subtraction to velocity only, and fail to extend their knowledge to the concept of change of velocity in a uniform circular motion, which causes centripetal acceleration. This naïve understanding is due to their generalization of a one-dimensional specific case of vector addition and subtraction to all physical situations (Govender, 2007). Knowledge of circular motion was hardly evident in their concept maps. This is a complex concept involving a combination of calculation, diagram, and kinematics. Baseline tests (Hestenes & Wells, 1992) with physics university students showed scores 30% and below, confirming difficulties in geometry in kinematics where parallelogram laws, scalar and vector analysis involving tangential and radial components need to be understood.

Language may also have limited the pre-service teachers’ construction of propositions in their concept maps, as English is a second language for most of them. Direct preposition words such as ‘to’ and ‘in’, amongst others, are generally expressed in other ways in African languages and may be one of the reasons for the difficulties that these teachers experienced in documenting their concepts using concept maps. This was evidenced by them repeatedly using the same or familiar label or linking words or phrases such as “can be” and “include” in the propositions in their concept maps. Similar language difficulties were also faced with third-year pre-service physics teachers of similar backgrounds when drawing concept maps for electromagnetism (Govender, 2015).

Hierarchical knowledge is about categorisation and classification into levels, whereas relational conceptual knowledge is about seeking meaningful relationships amongst the concepts within a hierarchy or across hierarchies (cross-links). The results in Table 1 of the study showed that the pre-service teachers were lacking higher order conceptual understanding of vector-kinematics as relational conceptions (only 12%). Relational conceptions are at a higher level of cognitive understanding than hierarchical description of concepts (48%). Relational conceptions may involve more semantic and integrated networking connections linked to language competency and hence their lower occurrence in this study.

The identified categories of conceptual knowledge and alternative conceptions of vector kinematics in this study are similar to the naïve physics, lateral alternative conceptions, ontological alternative conceptions, mixed conceptions, and loose ideas categories usually depicted in electromagnetism, except for the category of Phenomenological Primitives (p-primes) (Dega, 2012; Dega et al., 2013a) which were not identified. This may be due to concept maps being used as tools for data collection. According to diSessa (1993), phenomenological primitives are intuitive or spontaneous physics which are made up of smaller and more fragmented cognitive structures. From this study it appears that concept maps may have limitations in depicting smaller and fragmented pieces of cognitive structure as they are often used to connect two or more concepts using propositional words. The use of
probing and in-depth interviews is suggested to make explicit these smaller fine-grained cognitive structures.

The preservice-teachers concept maps (Figure 1-7) showed significant alternate concepts, demonstrating that framework categories can be used to support teaching and learning by identifying and rectifying basic conceptual errors. Also, evident from this study was the lack of new knowledge concepts in vector-kinematics even though these had already been taught in a previous university mechanics module and points to the way students learn physics. Novak's work with concept mapping was based on Ausubel's theory of meaningful learning, which is a constructivist learning theory arguing that new knowledge should be integrated into existing structures in order to be remembered and receive meaning. Concept mapping stimulates this process by making it explicit by requiring the student to pay attention to the relationships between concepts. Concept map use in mechanics modules is advised by Jonassen, Carr and Yeuh (1998) who argue that students show some of their best thinking when they try to represent something graphically, and thinking is a necessary condition for learning. Experiments have shown that subjects using concept mapping outperform non-concept mappers in longer term retention tests (Novak, Gowin, & Johansen, 1983) and is therefore a must for the teaching and learning process (Stoica et al., 2011).

Conclusions

Seven framework categories of the pre-service teachers' conceptions were identified, with differing extensive-ness, namely, two categories of conceptual knowledge (hierarchical and relational), three of alternative conceptions (naïve physics, lateral alternative conceptions, and ontological alternative conceptions) and two of mixed conceptions and loose ideas. The extensiveness of conceptual knowledge of the pre-service teachers was 60% while that of their alternative conceptions, mixed conceptions and loose ideas was 40%. The categories of conceptual knowledge and alternative conceptions showed a significant conceptual knowledge gap even after exposure to vector-kinematics in the high school curriculum, external national matriculation examinations and a first module in mechanics at university. The findings showed a lack of pre-service teachers' higher-level conceptual understanding and their frequent use of more intuitive, chaotic and inconsistent conceptions in vector-kinematics.

Furthermore, most of the pre-service teachers' descriptions of vector-kinematics were based on one-dimen-sional motion and the study showed they lacked knowledge of higher level mathematical operations for deeper understanding in mechanics such as dot and cross products.

Implications

The overarching values and details of the extensiveness of vector-kinematics summarised in Table 8 provide useful information and suggest ways that the first-year mechanics curriculum and instruction related to such concepts should be designed. It is evident that the pre-service teachers have some basic hierarchical and relational conceptual knowledge but this needs to be developed to a higher level of conceptual understanding. In addition, their naive physics needs to be developed to scientific conceptions, and their lateral and ontological alternative conceptions have to be shifted to correct lateral and ontological concepts.

At an introductory physics level, it is important to introduce pre-service teachers to a mathematical view of vectors as ordered pairs of numbers with the unit vectors notation in rectangular components which is currently not part of the module.

Instructional suggestions for use of concept maps and collaboration (Govender, 2015) supplemented by technology such as CmapTools may develop effective teaching strategies and may be a relatively fast and legible way of revealing and modifying pre-service teachers' conceptions in such a fundamental area of physics.

In general, based on the results of this study, it is proposed that a conceptual change supportive approach be incorporated into the physics curriculum with the aim of concept development and categorical correction. Concept development is meant for development of inclusive hierarchical and simple relational conceptual knowledge to in-depth and higher order conceptual knowledge relevant to the level. In addition, this conceptual development method could be used to develop the naïve intuitive conceptions to scientific conceptions. Categorical correction is proposed for changing the incorrect categorical alternative conceptions to correct conceptual knowledge, that is, lateral and ontological alternative conceptions to correct categorical conceptions.
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