The optimal aircraft gas turbine engine control in low gas mode in the conditions of external additive noise

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Abstract. The article is devoted to algorithm development to select an optimal control law of a gas turbine engine in the conditions of external additive noise and incomplete information regarding the values of control parameters. A step-by-step algorithm for problem solving is given. The effectiveness of the algorithm is proved by numerical experiments.

1. Introduction

Currently, mathematical and computer modelling are the basis for the design of complex technical systems, which include aircraft structures, of course [1-2].

Generally speaking, computer modelling has gained widespread today as an example of this can be found in numerous papers devoted to various topics: security [3], queuing problems [4], pattern recognition [5,6], research and use of genetic algorithms [7-9], as well as solving applied problems based on modelling of the processes under study [10,11].

Direct tasks for calculating the strength, flight, dynamic and other aircraft structures characteristics are widely described in the literature, methods for their solution are known [12-14].

In aircraft engineering the inverse problem solution is primarily connected with strength characteristic calculation of aircraft structures [15,16] and heat transfer problems [17]. The inverse problems also include aircraft engines modes controlling tasks [18,19]. Due to the fuel combustion disturbance, the engine is exposed to random disturbances in the form of Gaussian white noises in real live operation. At the bench test stage, not all system feedbacks can be accurately known. Thus, direct problems describing the engine operation are represented as stochastic differential equations systems with unknown parameters and additive white noises in the right-hand side.

For such mathematical models, the inverse problem solution with respect to control parameters is made difficult by the need to estimate unknown parameters that can be realized in real conditions by measuring the phase characteristics of the system [20].

2. Problem formulation

Let the original linear dynamical system be described by differential equation system

\[
\dot{x} = qx + Ab \quad (t_0 \leq t \leq t_1);
\]

\[x(t_0) = x_0;\quad (1)\]
Here \( t \) — is the time, \([0, t_1]\) — the measurement interval, \( x = \left(x_1, ..., x_n\right)\) — state vector system (1) with components \( x_i = x_i(t); \ x_0 = \left(x_{01}, ..., x_{0n}\right)\) — given initial values vector of state vector, 
\( \varphi = \|\varphi_{ij}(t)\|_{n \times n}, \ A = \|a_{ij}(t)\|_{m \times s} \) — given function matrices; \( b = (b_1, ..., b_s) \), — system parameter vector.

It is necessary to calculate such parameter values \( b \) of system (1), so that the phase coordinates \( x = \left(x_1, ..., x_n\right) \) is minimally different from some given reference state, as a result of the direct problem solution \( x^* = \left(x_{1}^*, ..., x_{n}^*\right), \ x_i^* = x_i^*(t). \)

By random disturbances, system (1) is transformed into a stochastic:
\[
\dot{X} = \varphi X + Ab + \Delta \quad (t_0 \leq t \leq t_1);
X(t_0) = X^0;
\]
(2)

Here \( \Delta = \left(\Delta_1, ..., \Delta_n\right) \) — is random uncorrelated noise column vector \( \Delta_i = \Delta_i(t) \), affecting the system with zero expectation and given autocorrelation matrix \( K_\Delta(t,t') = \|K_{\Delta ij}(t,t')\|_{n \times n} \), \( X^0 = (X_{10}, ..., X_{n0}) \) — random initial state vector with expected value \( x_0 \) and a given correlation matrix \( K_0 \).

So, it is necessary to create a computational procedure for solving the inverse problem for the stochastic system (2) with respect to the parameters \( b \). We organize the solution procedure in the form of an iterative step-by-step process.

The mathematical formulation of the inverse problem can be formulated as the following unconstrained optimization problem:
\[
J = \int_{t_0}^{t_1} (x^k - x^*)^T G(x^k - x^*) dt \Rightarrow \min
\]
where \( x^k \) — phase state of the system at the \( k \)-th iteration of the solution, \( G \)-symmetric matrix of positive coefficients.

Obviously, in the process of solving an inverse problem, the researcher may have varying degrees of awareness regarding the parameters values. Also, not all parameters should be adjusted in the process of solving the inverse problem. To reflect this circumstance, we present the parameter vector \( b \) in the form: \( b = (u, v, w, \theta) \), and the system (1) at the \( k \)-th stage of solving the inverse problem is represented as:
\[
\begin{align*}
\dot{x}^k &= \varphi x^k + U u^k + V v^k + W w^k + \Theta \theta \quad (t_0 \leq t \leq t_1); \\
X^k(t_0) &= x_0 \quad (k = 0, 1, ...,);
\end{align*}
\]
(4)

Here the index \( k \) corresponds to the iteration number, \( \varphi = \|\varphi_{ij}(t)\|_{n \times n}, \ U = \|U_{ij}(t)\|_{n \times m}, \ V = \|V_{ij}(t)\|_{n \times q}, \ W = \|W_{ij}(t)\|_{n \times q}, \ \Theta = \|\Theta_{ij}(t)\|_{n \times l} \) — given function matrices \( \varphi_{ij}(t), \ U_{ij}(t), \ V_{ij}(t), \ W_{ij}(t), \ \Theta_{ij}(t) \); \( u^k = (u_1^k, ..., u_m^k) \) — adjustable parameter vector, the values \( u^0 \) of which are known at the initial \( (k = 0) \) stage of the corrections, and the final values are unknown; \( v^k = (v_1^k, ..., v_r^k) \) — adjustable parameter vector with unknown initial \( v^0 \) and final values;
Determining the measurement completeness 
\( t \), determining the measurement completeness 
\( \varphi \). (5) This estimate can be obtained on the basis of system 
\( k \), \( \ldots, w_0 \) initial values 
\( t \), \( \varphi \), \( \ldots, \mu \) — unknown and uncorrectable parameter vector.

Then the behavior of a real system with random disturbance affecting it at the \( k \)th stage the problem solution can be represented as:

\[
\begin{align*}
X^k &= \varphi X^k + U^k + V^k + W^k + \Theta \varphi + \Delta^k \\
X^k(t_0) &= X^{k0}, \quad (k = 0, 1, \ldots);
\end{align*}
\]

For direct calculations and minimization of the function (3), it is necessary to have an estimate of the initial values of those parameters which values were unknown before the calculations, so we have to have vector estimation \( \theta^0 = (v^0, w^0, \varphi) \). (5) This estimate can be obtained on the basis of system phase coordinates measurements (5). The measurement equation (meter) phase coordinates can be written as:

\[
Z^k(t) = HX^k(t) + Y^k(t)
\]  

Here: \( Z^k = [Z_1^k, \ldots, Z_{\nu}^k, \ldots, Z_{\rho}^k] \) — measurement results column vector with components \( Z^k_i(t) \), \( H = [H_{ij}(t)]_{\rho \times n} \) — given matrix with elements \( H_{ij}(t) \), determining the measurement completeness of the state vector \( X^k(t) \); \( Y^k = [Y_1^k, \ldots, Y_\rho^k] \) — random noise meter column vector with a given autocorrelation matrix \( K_Y(t, t') \).

So, it is required to calculate (correct) the parameters of the linear dynamic system (4) on the basis of measurements (6) of the stochastic dynamic system (5) from functional minimum condition (3).

If \( f \) correction steps are required to achieve the functional minimum (3), then the state \( x^f \) will correspond to the values of the parameters \( (u^f, v^f, w^*, \varphi) \).

3. Algorithm for solving the inverse problem

Extensive research was carried out to solve the problem [18,19], [21-24]. As a result, the following algorithm for solving the inverse problem was developed to select the dynamic system parameters:

1. For the initial form system (4) with measurements according to (5) - (6), the transition state matrix \( \psi(t, t_0) \) is determined, which satisfies the differential equation system in the form:

\[
Z^k(t) = HX^k(t) + Y^k(t)
\]

2. The functional matrix values are determined \( U^*(t), V^*(t), W^*(t), \Theta^*(t) \) according to the formulas:

\[
\begin{align*}
U^* &= \int_{t_0}^t \psi(t, \tau)U(\tau)d\tau, \quad V^* = \int_{t_0}^t \psi(t, \tau)V(\tau)d\tau \\
W^* &= \int_{t_0}^t \psi(t, \tau)W(\tau)dt, \quad \Theta^* = \int_{t_0}^t \psi(t, \tau)\Theta(\tau)dt
\end{align*}
\]

3. The matrix-valued function \( A(t) \) and the matrix \( \overline{A} \) are calculated where is
\[
A = (U^* V^*), \quad \overline{A} = \int_{t_0}^{t_1} A^T \Gamma A dt
\]

\(\Gamma\) — symmetric matrix of positive coefficients.

4. The functional matrix elements are determined \(V^{**}(t), W^{**}(t), \Theta^{**}(t)\) according to formulas:
\[
V^{**} = HV^*, \quad W^{**} = HW^*, \quad \Theta^{**} = H\Theta^*
\]

5. The matrix-valued function is formed \(B(t)\):
\[
B = (V^{**}, W^{**}, \Theta^{**})
\]

6. Disturbance process \(\Delta_k = \Delta_k(t)\) correlation matrix \(K^{*}_\Delta(t, \tau)\) is calculated by using the known correlation matrix \(K_0\) and \(K^{*}_\Delta(t', t^*)\) and defined by the expression:
\[
K^{*}_\Delta(t, \tau) = \psi(t, t_0)K_0\psi^T(\tau, t_0) + \int_{t_0}^{t_1} \int_{t_0}^{t_1} \psi(t, t')K^{*}_\Delta(t', t^*)\psi^T(\tau, t^*)dt'dt^*
\]

7. The correlation matrix \(K_{f\tau}\) of disturbance meter process is calculated \(Y_k(t)\):
\[
K_{f\tau} = H(t)K^{*}_\Delta(t, \tau)H^T(\tau) + KY(t, \tau)
\]

8. The integral equations system is solved
\[
\int_{t_0}^{t_1} f^*(t)Bdt = E_{g\times g} - \Psi_t \int_{t_0}^{t_1} f^*(\tau)K_{f\tau}d\tau = B^T
\]

relatively to unknown functions \(f^*(t)\) for effective, unbiased and stable unknown parameters values vector estimation \(\theta^0 = (\nu^0, w^0, \vartheta)\).

9. Based on the obtained measurements \(Z^k(t)\), the unknown parameters vector \(\theta^0 = (\nu^0, w^0, \vartheta)\) is calculated in the 0-th approximation:
\[
(\nu^{0k-1}, w^{0k-1}, \vartheta^{k-1}) = \int_{t_0}^{t_1} f^* Z^{k-1} dt
\]

\[
Z^{k-1} = \frac{1}{k} \sum_{j=0}^{k-1} Z^j^*.
\]

As a result, when \(k = 1\), we obtain the estimates \((\nu^{00}, w^{00}, \vartheta^{00})\).

10. Estimates of \((\tilde{u}^f, \tilde{v}^f, \tilde{\vartheta}^f)\) adjusted parameters \(u^f, v^f, \vartheta^f\) are figured out in accordance with the formula:
\[
(\tilde{u}^f, \tilde{v}^f, \tilde{\vartheta}^f) = \overline{A}^{-1} \int_{t_0}^{t_1} A^T \Gamma (\overline{A} - \psi(t, t_0)\lambda_0 - W^*w^* - \Theta^*\vartheta^{k-1}) dt
\]

If \(k = 1\), estimates \((\tilde{u}^f, \tilde{v}^f, \tilde{\vartheta}^f)\) are determined in the zero approximation.

11. Corrective additives are calculated \(\vartheta^0, \vartheta^{00}, \vartheta^{01}\):
\[
\vartheta^{k-1} = \tilde{u}^f - u^0, \vartheta^{k-1} = \tilde{v}^f - v^0, \vartheta^{k-1} = \tilde{\vartheta}^f - \vartheta^0.
\]
12. The system is corrected when \( k = 1 \), due to the addition increment correction \( \dot{u}^0, \dot{v}^0 \) and unknown parameter estimations \( v^0 \) and \( w^0 \):

\[
u^k = u^0 + \delta u^{k-1}, v^k = v^0 + \delta v^{k-1}, w^k = w^0 + \delta w^{k-1}
\] (20)

13. Based on previous process state estimation \( \chi^k \), corresponding to parameter values \( u^k, v^k, w^k \), paragraphs 9-13 are repeated when \( k = k + 1 \) and subsequent approximation to the adjusted state is determined \( \chi^f \), corresponding to the adjusted parameters \( u^f, v^f, w^f, \theta \).

This process is repeated until the desired corrected state is obtained with a given accuracy.

4. Computing experiment
As an object of regulation, we consider a gas turbine engine (GTE), described by the equations [25]:

\[
\begin{align*}
\dot{G} &= K_1(u_G - m_2 - m_1 n), \\
\dot{F} &= K_2(u_F - m_3 - m_4 F), \\
\dot{n} &= b_1 G + b_2 F - a_1 n, \\
\dot{r} &= b_3 G + a_2 n - a_3 r
\end{align*}
\] (21 – 24)

Here: \( G \) — fuel supply, \( F \) — nozzle area, \( n \) — turbine speed, \( r \) — exhaust gas temperature, control system gain coefficient, \( a_1, a_2, b_1, b_2, b_3 \) — known engine parameters, \( m_1, m_2, m_3, m_4 \) — feedback coefficients, some of which are unknown, \( u_G, u_F \) — input control signals that change the fuel flow and output nozzle area.

We will consider the bench mode operation of the GTE in low gas mode within three seconds. It is necessary to adjust the control engine signals so as to transfer it within a given time period \( 0;3 \) from one mode to another, providing the given initial:

\[
\begin{align*}
n(t_0) &= n_0, \quad \dot{n}(t_0) = 0, \quad \tau(t_0) = \tau_0, \quad \dot{\tau}(t_0) = 0
\end{align*}
\] (25)

and (to avoid overshooting) the final:

\[
\begin{align*}
n(t_k) &= n_k, \quad \dot{n}(t_k) = 0, \quad \tau(t_k) = \tau_k, \quad \dot{\tau}(t_k) = 0
\end{align*}
\] (26)

turbine speed values \( n \) and gas temperatures \( r \). Here \( n_0, \tau_0, n_k, \tau_k \) — predefined parameter values.

It is required to determine the control actions \( u_G, u_F \) to ensure the given values of turbine speed \( n \) and outgoing gases temperature \( r \) for a given time interval of control based on the results of fuel supply and gas turbine engine nozzle area measurements. As a result of erosive fuel burning, random processes with known characteristics affect the engine. External disturbances also affect measuring devices.

According to the selection function algorithm that provides the required system motion (25) - (26) [25], the functions, determining the turbine speed and outgoing gases temperature, are as follows:

\[
\begin{align*}
n &= 0.1 + 0.1 t^2 - 0.022223 t^3 \\
r &= 0.15 + 0.11667 \alpha^2 - 0.02592 \alpha^3
\end{align*}
\] (27 – 28)

As the initial state of the system, the following condition was taken: \( n_0 = 0.1, \tau_0 = 0.15 \), and the final one is: \( n_k = 0.4, \tau_k = 0.5 \).

Such reference transition paths are illustrated in Figure 1. Thus, optimal control \( u_G, u_F \) must match the reference paths. (27). The initial conditions for (21) and (22) are not given, but they can definitely be determined from expressions (23) - (24). So, we get:

\[
\begin{align*}
G_0 &= 0.253, \quad F_0 = 0.103
\end{align*}
\] (29)
Substituting expressions (27) - (28) into relations (21) - (22), we obtain expressions for the fuel supply and nozzle area, which the optimal control must satisfy:

\[ G = 0.253 + 1.52\tau - 0.32\tau^2 - 0.04\tau^3 \]  
\[ F = 0.103 - 1.005\tau + 0.49\tau^2 - 0.03\tau^3 \]  

The reference processes trajectories are presented in Figure 2.

**Figure 1.** GTE transition reference trajectories by \(n\) and \(\tau\) in the control process without overshoot.

**Figure 2.** Reference transition paths in \(G\) and \(F\), corresponding to the transition in \(n\) and \(\tau\) without overshoot.

Then the GTE regulation task will be as follows: it is required to correct unknown parameters of control systems \(u_G\) and \(u_F\) (21) - (22) with initial conditions (29) on the interval (0;3) with the target vector functions (30) - (31). In this case, the resulting control will provide the optimal trajectory for GTE transition with respect to turbine speed \(n\) and outgoing gases temperature \(\tau\).

To carry out a computational experiment, we used author software that performs calculations according to Algorithm 1. The program generates random processes as random system disturbances and a meter—uncorrelated white Gaussian noises with intensities: \(G^X=0.06\) — noise intensity disturbing the system, \(G^Z=0.05\) — noise intensity acting on the meter.

The engine parameters corresponding to the low gas mode during bench tests have the following values [25]:

\[ a_{11} = 3.03, \quad a_{21} = 0.113, \quad a_{22} = 0.333, \quad b_1 = 0.79, \quad b_2 = 1, \quad b_{21} = 0.153. \]

Control system gain coefficient \(K_1=100, K_2=100\). Feedbacks \(m_1, m_3\) are unknown, \(m_2 = 0.0647, m_4 = 0.1655\).

4.1. Experiment №1: constant control signal control

Let the control be exercised by a constant control signal, so \(u_G\) and \(u_F\) are constants.

Let’s show expressions (21) – (22) in the form (4):

\[ (\dot{x}_1, \dot{x}_2)^T = \varphi \times (x_1, x_2)^T + V \times (v_1, v_2)^T + \Theta \times (\varphi_1, \varphi_2)^T, \]
\[ t \in [0,3], \quad (x_1(0), x_2(0))^T = (0.253, 0.103)^T \]

Here \((x_1, x_2)^T = (G; F)^T, (v_1, v_2)^T = (u_G; u_F)^T, (\varphi_1, \varphi_2)^T = (m_1; m_2)^T, \varphi = \begin{bmatrix} -K_1m_2 & 0 \\ 0 & -K_2m_4 \end{bmatrix}\)

\[ V = \begin{bmatrix} K_1 & 0 \\ 0 & K_2 \end{bmatrix}, \quad \Theta(t) = \begin{bmatrix} -K_1n & 0 \\ 0 & -K_2\tau \end{bmatrix}. \]

The inverse problem is solved for the system of differential equations (32) with random...
perturbations in the initial state (29) to determine the optimal parameters \( \left( v_1, v_2 \right)^T \) with the reference phase characteristics (30), (31) on the interval \((0; 3)\)

As a stop criterion of the algorithm, the increment of the required parameters was adopted by less than 0.5%. The following results were obtained:
1. The adjusted state was reached in 4 iterations.
2. The initial values of the control parameters \( u_G, u_F \) are defined as \( u_G = 0.1110; u_F = -1.0010 \)
3. The optimal values of the control parameters \( u_G^f, u_F^f \) are defined as: \( u_G^f = 0.23505, u_F^f = -0.6361 \)
4. The unknown feedback values are determined: \( m_1 = 0.7258, m_3 = -2.0567 \)

The optimal values of the control parameters made it possible to achieve a measure of proximity between the real and reference states about 0.9.

The discrepancy between the corrected and target trajectories is reflected mainly in the trajectory of the nozzle area of the GTE (Figure 3).

![Figure 3](image_url)

**Figure 3.** The discrepancy between the measurement results of the system \( (Z_1', Z_2') \) and the reference vector functions \( (G, F) \) at process iterations.

### 4.2. Experiment №2: control of a linear signal in time

It is proposed to control actions as linear functions with unknown coefficients:

\[
\begin{align*}
  u_G &= u_G^0 + u_G^1 t \\
  u_F &= u_F^0 + u_F^1 t
\end{align*}
\]  
(33)  
(34)

In this case corrected parameters are the coefficients of linear equations (33)-(34) \( u_G^0, u_G^1, u_F^0, u_F^1 \) and \( u_F^1 \).

So, expressions (21) - (22) are converted to the form (4) as follows (the dimension of the problem increases):

\[
\begin{align*}
  \left( \dot{x}_1, \dot{x}_2 \right)^T &= \varphi \times \left( x_1, x_2 \right)^T + V \times \left( v_1, v_2, v_3, v_4 \right)^T + \Theta \times \left( \vartheta_1, \vartheta_2 \right)^T, \\
  t &\in [0,3], \left( x_1(0), x_2(0) \right)^T = (0.25, 3.0, 10.3)^T
\end{align*}
\]  
(35)

Here: \( \left( x_1, x_2 \right)^T = (G; F)^T; \left( v_1; v_2; v_3; v_4 \right)^T = \left( u_G^0, u_G^1; u_F^0, u_F^1 \right)^T; \theta(t) = \begin{bmatrix} -K_1 & 0 & 0 & -K_2 \\ 0 & -K_1 & 0 & -K_2 \end{bmatrix} (\vartheta_1, \vartheta_2)^T = (m_1; m_2)^T; \)

\[
\varphi = \begin{bmatrix} -K_1 m_2 & 0 & 0 & -K_2 m_4 \\ 0 & -K_1 m_2 & 0 & 0 \end{bmatrix}; V = \begin{bmatrix} K_1 & K_1 t & 0 & 0 \\ 0 & 0 & K_2 & K_2 t \end{bmatrix}.
\]
The inverse problem is solved for differential equation system (35) with random disturbances in the initial state (29) to determine the optimal parameters $(v_1, v_2, v_3, v_4)^T$ with the reference phase characteristics (30), (31) in the interval $(0; 3)$. As a result of Algorithm 1 implementation, the following results are obtained:

1. The adjusted state was reached in 5 iterations.
2. The initial values of the control parameters are determined $u_G, u_F$: $u_G^0 = 0.1111; u_F^0 = -0.0002; u_G^1 = -1.0011; u_F^1 = 0.00013$. 
3. Optimal values of control parameters are determined: $u_G^{0f} = 0.2302; u_F^{0f} = -0.001; u_G^{1f} = -0.3907; u_F^{1f} = -0.1912$. 
4. The unknown feedback values are determined: $m_1 = 0.7258; m_2 = -2.0567$. 

Thus, expressions for GTE linear control are obtained (Figure 4.):

$$u_G = 0.2302 - 0.001 \cdot t$$

$$u_F = -0.3907 - 0.1912 \cdot t$$

The obtained expressions for control allowed us to achieve a measure of proximity between the real and reference trajectories about 0.01 (Figure 5). 

Let’s notice that in case of need control accuracy can be improved if the control is considered as a quadratic polynomial with unknown coefficients.

Figure 4. Linear GTE control.

Figure 5. The discrepancy between the results of measuring the system $(Z^1, Z^2)$ and the reference vector function $(G, F)$ at the iterations of the process for linear control.
5. Conclusions
The developed algorithm makes it possible to solve effectively inverse problems for dynamic stochastic systems with unknown parameters by measuring their phase characteristics. The performed computational experiments proved the applicability of the developed algorithm for control problems of complex technical systems in disturbance.

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