Can the differences in the determinations of $V_{ub}$ and $V_{cb}$ be explained by New Physics?

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The precise determination of the CKM elements $V_{cb}$ and $V_{ub}$ is crucial for any analysis of new physics in the quark flavour sector (see [1] for a review). In the standard model (SM) with its gauge group $SU(3)C \times SU(2)L \times U(1)Y$ the tree-level $W$ coupling is purely left-handed and all charged-current processes are mediated by the $W$ boson only. This property is used to extract CKM elements from tree-level decays, i.e. from exclusive leptonic and semi-leptonic decays as well as from the inclusive processes. While these processes (since they appear at tree-level) are in most analysis assumed to be free of new physics (NP), however, physics beyond the SM can in principle affect the determination $V_{cb}$ and $V_{ub}$. Furthermore, the impact of NP on the exclusive and inclusive determination can be different in general.

The current situation concerning the determination of $V_{cb}$ and $V_{ub}$ is the following: For the CKM element $V_{cb}$ there has been a persistent discrepancy between the exclusive and inclusive determination for many years now which was slightly more than $2\,\sigma$. Recently, new results for the $B \to D^{*}\ell\nu$ form-factors have been obtained on the lattice [2] increasing the discrepancy between the inclusive and exclusive determination above the $3\,\sigma$ level. Concerning $V_{ub}$, the difference between the inclusive and exclusive determination increased some time ago because of the NNLO QCD corrections [3].

In detail, the current situation for the determination of $V_{cb}$ is the following:

\begin{align}
|V_{cb}| &= (4.242 \pm 0.086) \times 10^{-2} \quad \text{(inclusive)}, \\
|V_{cb}| &= (3.904 \pm 0.075) \times 10^{-2} \quad \text{(B \to D^{*}\ell\nu)}, \\
|V_{cb}| &= (3.850 \pm 0.191) \times 10^{-2} \quad \text{(B \to D\ell\nu)}. 
\end{align}

Here we added all errors in quadrature. The inclusive determination in taken from Ref. [4]. For the experimental input for $B \to D^{*}\ell\nu$ we used the HFAG average [3] and for the form-factor used for the $V_{cb}$ extraction from $B \to D\ell\nu$ we used the preliminary results of Ref. [6]. Note that now both exclusive values of $V_{cb}$ are below the inclusive determinations which disfavors right-handed currents as an explanation as we will see later in detail.

Concerning the CKM element $V_{ub}$ the situation for the different determinations is similar

\begin{align}
|V_{ub}| &= (4.41^{+0.21}_{-0.23}) \times 10^{-3} \quad \text{(inclusive)}, \\
|V_{ub}| &= (3.40^{+0.38}_{-0.33}) \times 10^{-3} \quad \text{(B \to \pi\ell\nu)}, \\
|V_{ub}| &= (4.3 \pm 0.6) \times 10^{-3} \quad \text{(B \to \tau\ell\nu)}, \\
|V_{ub}| &= (3.4 \pm 0.3) \times 10^{-3} \quad \text{(B \to \rho\ell\nu)}, 
\end{align}

meaning that again the semi-leptonic exclusive determinations are below the inclusive ones. The latter agrees with the determinations from $B \to \tau\ell\nu$. The inclusive result was calculated in Ref. [3] and for $B \to \rho\ell\nu$ one used the central value of Ref. [7] and doubled the error in order to be conservative [8]. The value of $V_{ub}$ from $B \to \pi\ell\nu$ is taken from the fit performed by HFAG [3].

Note that the problem in the inclusive and exclusive determination of $V_{cb}$ is not directly related to the $B \to D^{(*)}\tau\ell\nu$ problem where also a deviation from the SM of more than $3\,\sigma$ is observed by BABAR [8]. There, one considers the ratios $Br[B \to D^{(*)}\tau\ell\nu]/Br[B \to D^{(*)}\ell\nu]$ in which the dependence on the CKM elements drops out. However, due to the heavy tau lepton involved these observables [9, 10] are sensitive to NP contributions [11, 12], especially to charged Higgs contributions [11, 13, 14] which is also true for $B \to \tau\ell\nu$ [15, 16] and the
determination of $V_{ub}$ from this decay.

The question we want to address in this article is if these deviations among the different determination in the values of $V_{cb}$ and $V_{ub}$ can be explained by physics beyond the SM. For this purpose we will first study the general effect of additional effective operators (determined at the $B$ meson scale) in the next section and consider the phenomenological implication and connection to dimension-6 gauge-invariant operators in section [11]. Finally we conclude.

II. NEW PHYSICS EFFECTS IN (SEMI-) LEPTONIC $B$ DECAYS

In an effective field theory approach we can parametrize the effect of NP in a model independent way. Here we consider the most general operator basis up to dimension 6 given at the $B$ meson scale. There are two different ways how NP contributions can affect the determination of the CKM elements from tree-level $B$ decays: Through additional four-fermion operators (which can be generated at tree-level) and operators which modify the $W$ coupling to quarks (via loop-effects) and therefore the charged current after integrating out the $W$ boson.

A. 4-fermion operators

Let us first consider the four-fermion operators which can already be generated by integrating out heavy degrees of freedom at the tree-level. Here we consider the effective Hamiltonian $H_{\text{eff}} = \sum_i C_i O_i$ with the additional operators

$$O_R^S = \bar{\ell} P_L \nu \bar{q} P_R b, \quad O_L^S = \bar{\ell} P_L \nu \bar{q} P_L b, \quad O_T^L = \bar{\ell} \sigma_{\mu\nu} P_L \nu \bar{q} \sigma^{\mu\nu} P_L b,$$

where $q = u, c$ and the Wilson coefficients include only the effect of NP and $D_{q_L}$ is the QCD covariant derivative. Using the results of [20] we find the following (approximate) NP contribution to the determination of $V_{cb}$ and $V_{ub}$ from exclusive semi-leptonic modes:

$$V_{cb} = \frac{V_{cb}^{\text{SM}}}{1 + c_L^{cb} - c_R^{cb} - 1.6 \text{GeV}(d_R^{cb} + d_L^{cb}) + 5.5 \text{GeV}(g_R^{cb} + g_L^{cb})} (B \to D \ell \nu),$$

$$V_{cb} = \frac{V_{cb}^{\text{SM}}}{1 + c_L^{cb} - c_R^{cb} + 3.3 \text{GeV}(d_R^{cb} - d_L^{cb})} (B \to D^* \ell \nu),$$

with $q = u, c$ and assuming the absence (i.e. heaviness) of right-handed neutrinos. We postpone the discussion of a possible vector operator since these effects can also be induced by a modified $W$-$qb$ coupling. At zero recoil (i.e. maximal momentum transfer) where the CKM elements are extracted (and neglecting small lepton masses) there is no interference of scalar and tensor operators with the SM contribution both in exclusive and inclusive semi-leptonic modes and the relative importance of the operators is the same in all decay modes: the contribution to all decays from the tensor operator is simply proportional to $|C^T_L|^2$ while for $B \to D(\pi)\ell\nu$ the scalar contribution is proportional to $|C^S_R + C^S_L|^2$ and for $B \to D^*(\rho)\ell\nu$ the additional contribution scales like $|C^S_R - C^S_L|^2$ while in the inclusive decay (in the limit of vanishing lepton and charm (up) masses) we have $|C^S_R|^2 + |C^S_L|^2$. Therefore, these operator cannot explain why both exclusive determinations of $V_{ub}$ and of $V_{cb}$ are below the inclusive ones. The only exception is $B \to \tau \nu$. Here it is well know that the scalar operator generated by a charged Higgs affects the branching ratio [17, 30]. We turn now to the effect of a modified $W$ couplings.

B. Effects of modified $W$ couplings

The impact of NP via higher dimensional operators modifying the $W$-$qb$ quark coupling has been calculated for the inclusive decay in Ref. [19] and for the exclusive modes in Ref. [20]. Assuming again the absence of (light) right-handed neutrinos we can parametrize the NP contributions via the effective Hamiltonian

$$H_{\text{eff}} = \frac{4G_F V_{qb}}{\sqrt{2}} \bar{\ell} \gamma^\mu P_L \nu \left(1 + c_L^{qb}\bar{q}\gamma^\mu P_L b + g_L^{qb}\bar{q}i\not{D}\mu P_L b + d_L^{qb}i\partial^\nu (\bar{q}i\sigma_{\mu\nu} P_L b) + L \to R \right),$$

where $q = u, c$ and the Wilson coefficients include only the effect of NP and $D_{q_L}$ is the QCD covariant derivative.
For the inclusive determination of $V_{cb}$ the NP effects were calculated in Ref. [19]:

$$V_{cb} = \frac{V_{ub}^{SM}}{1 + c_{R}^{cb} - c_{R}^{ub} + 4.5\text{GeV}(d_{L}^{cb} - d_{R}^{ub})} \quad (B \to \tau \nu).$$

(14)

For the contribution of $c_{R}^{cb}$ in Eq. (14) we used the result of Ref. [21] where a global fit to all hadronic and leptonic moments was performed. We also used the result of Ref. [21] to estimate the total impact of $d_{L,R}^{cb}$ and $g_{L,R}^{cb}$ on $V_{ub}$ given the various hadronic and leptonic moments in Ref. [19]. This is possible since the relative effect on the moment of $d_{L,R}^{cb}$ and $g_{L,R}^{cb}$ is very similar to $c_{R}^{cb}$. Since the inclusive $V_{cb}$ mode is not very sensitive to $d_{L,R}^{cb}$ and $g_{L,R}^{cb}$ this approximation suffices for our purpose. Concerning the inclusive determination of $V_{ab}$ the impact of NP is expected to be even smaller because of the much smaller up-quark mass and we neglect this effect. Concerning $B \to \tau \nu$ the quantities $d_{R}^{cb}$ and $d_{L}^{cb}$ have an important effect:

$$V_{ab} = \frac{V_{ub}^{SM}}{1 + \frac{m_{u}^{2} - m_{b}^{2}}{m_{b}}(d_{R}^{cb} - d_{L}^{cb})} \quad (B \to \tau \nu).$$

(15)

### III. PHENOMENOLOGICAL ANALYSIS AND RESULTS

We are now in a position to examine whether the difference between the different determinations on the CKM elements can be due to NP effects. As noted in the last section, four fermion operators cannot bring the inclusive and exclusive determinations into agreement so we only consider the effect of a modified W coupling here. First note that any NP contained in $c_{L}$ only amounts to an overall scaling of all CKM elements. The simplest possibility to explain differences between the inclusive and exclusive determinations would be a right-handed charged currents generating $c_{R}$ (first studied in the context of left-right symmetric models [22]) both for $V_{cb}$ [20,23,26] and $V_{ub}$ [25,27,40] which can be even generated in the MSSM [25]. However, this is not favored anymore by the current data since all exclusive determinations are below the inclusive one. We show the effect of $c_{R}$ on the different determination of $V_{ab}$ and $V_{cb}$ in Fig. 1.

From Eq. (13) we can see immediately, that $d_{L}^{cb}$ cannot bring all determinations into agreement with the current data since the effect in $B \to D\ell\nu$ and $B \to D^{*}\ell\nu$ ($B \to \pi\ell\nu$ and $B \to \rho\ell\nu$) is opposite. Also $g_{L,R}^{cb}$ ($g_{L,R}^{cb}$) alone is not sufficient since it affects only $B \to D^{*}\ell\nu$ ($B \to \rho\ell\nu$). This means that we are left with $d_{R}^{cb}$. The effect of $d_{L}^{cb}$ and $d_{R}^{cb}$ on the determination of $V_{cb}$ and $V_{ab}$ is shown in Fig. 2. We can see that for $V_{cb}$ all different determination can be brought into agreement. For $V_{ab}$ also the exclusive semi-leptonic results can be brought into agreement with the inclusive one but a tension with $B \to \tau\nu$ is generated. Since $B \to \tau\nu$ is the only process under consideration involving a heavy tau lepton, one could in principle argue that additional operators (most likely scalar ones for example induced by a charged Higgs) affect this decay and bring all determinations into agreement. However, as we discuss it below, it is not realistic to expect a NP contribution to be responsible for the required value of the $d_{R}^{cb}$.

The operators considered so far were only invariant under the gauge group $U(1)$ of the electromagnetic interactions but not under the complete SM gauge group SU(3)$_{C} \times$ SU(2)$_{L} \times U(1)$_{Y} which any model of NP with particles above the EW scale should respect. The complete set of gauge-independent operators up to dimension-6 was derived in Ref. [28] and reduced to a minimal set in Ref. [29]. Following the notation of Ref. [28] the operators corresponding to $d_{L,R}$ are

$$Q_{dW}^{ij} = \frac{1}{\Lambda^{2}}(\bar{q}_{i}\sigma^{\mu\nu}q_{j})\tau^{I}\phi_{\mu\nu},$$

$$Q_{dW}^{ij} = \frac{1}{\Lambda^{2}}(\bar{q}_{i}\sigma^{\mu\nu}d_{j})\tau^{I}\phi_{\mu\nu},$$

First we can estimate the necessary size of the corresponding dimensionless Wilson coefficient $C_{dW}^{ij}$

$$C_{dW}^{ij} \approx V_{ij}V_{cb}^{3}\frac{\Lambda^{2}}{v},$$

(17)

which is at least of order one. Since this operator can only be induced at the loop-level, one would need non-perturbative NP interactions. Furthermore, from these expression we can see that any modification of the W coupling incorporated in $d_{L,R}$ would also lead to a modification of the Z coupling proportional to $d_{L}$ which after EW symmetry breaking only differs by a CKM rotation. Even for flavour-diagonal Z-quark couplings, in order not to violate bounds from FCNC processes, one would get a very large correction to $Z - bb$ coupling which is a factor $\cos(\theta_{W})/V_{cb}$ larger than the contribution to the W coupling. Clearly, this is not compatible with electroweak precision constraints.

Therefore, the current discrepancies between the inclusive and exclusive determinations $V_{ab}$ and $V_{cb}$ cannot
be explained by a model of NP respecting the SM gauge
symmetries. This means that they must be due to sta-
tistical fluctuations and/or underestimated errors. Since
the only operator which can bring the determination into
agreement (the one corresponding to $d_L$) is generated in
the SM via low energy QCD dynamics and has the struc-
ture of a magnetic form factor, one might speculate if
this hints towards underestimated theoretical errors in
the calculation of the form factors for the semi-leptonic
exclusive decays. In fact, already a change of only around
4% in the form factors for $B \to \pi \ell \nu$ and $B \to \rho \ell \nu$ could
bring the determinations of $V_{cb}$ into agreement. Problems
in the determinations of $V_{ub}$ from exclusive semileptonic
decays would also explain why the determination of $V_{ub}$
from $B \to \tau \nu$ agrees with the inclusive determination.
However, in this case one would have some tensions in
the global fit to the unitary triangle \[30, 31\] pointing to-
wards NP in $B_d - \overline{B}_d$ mixing (see for example Ref. \[32\]
for an review). Since the exclusive determinations agree
among each other it is of course also possible that un-
derestimated errors in the inclusive determinations cause
the current disagreement in the determinations of $V_{ub}$
and $V_{cb}$.

IV. CONCLUSIONS AND OUTLOOK

In this article we examined if NP can explain the dif-
fferences between the inclusive and exclusive determina-
tions of $V_{ub}$ and $V_{cb}$. Using an effective field theory
approach we found that there is only one operator ca-
pable of explaining the current experimental situation
which corresponds to a modified momentum dependent
$W$-$qb$ coupling. However, in an $SU(2)$ invariant theory
of physics beyond the SM the corresponding Wilson co-
efficient would need to be unacceptably large, violating
electroweak precision constraints on the $Z$-$bb$ coupling
which rules out a NP explanation.

Therefore, in case the current experimental situation
remains the differences between the inclusive and exclu-
sive determinations must be due to problems in the the-
ory predictions for the in- and/or exclusive decay modes.
Since the operator needed to generate agreement between
the different determinations resembles a factor one
might speculate that the inclusive determinations of the
CKM elements (and $V_{ub}$ from $B \to \tau \nu$) maybe closer
to the real value than the exclusive semi-leptonic ones.
However, using the inclusive value for $V_{ub}$ would generate
tensions in the global CKM fit which would point to NP
in $B_d - \overline{B}_d$ mixing or, after all, could suggest that the
exclusive determinations might be more accurate.

Clearly, the current situation requires close reexami-
nation of the theory predictions for all inclusive and exclu-
sive determination of the CKM elements $V_{ub}$ and $V_{cb}$.
Precise predictions for $V_{ub}$ are crucial to judge if there is
NP in $B \to \tau \nu$ and reducing the error in $V_{ub}$ is indispen-
sable for precision predictions for $B_s \to \mu^+ \mu^-$. In our
analysis we assumed the absence of light right-handed
neutrinos. Relaxing this assumption will enlarge the op-
erator basis and would require a separate analysis.

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FIG. 2: Left: $|V_{cb}|$ as a function of $d_{cb}^L$ extracted from different processes. Blue: inclusive decays. Red: $B \to D^* l \nu$. Yellow: $B \to D l \nu$. Right: $|V_{ub}|$ extracted from different processes. Blue: inclusive decays, red(gray): $B \to \rho l \nu$, yellow: $B \to \rho l \nu$, green: $B \to \tau \nu$. $d_{qb}^L$ is assumed to be real.

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