Scheme for implementing quantum information sharing via tripartite entangled state in cavity QED

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We investigate economic protocol to securely distribute and reconstruct a single-qubit quantum state between two users via a tripartite entangled state in cavity QED. Our scheme is insensitive to both the cavity decay and the thermal field.

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I. INTRODUCTION

Entanglement is one of the most counterintuitive features in quantum mechanics; assisted with entangled state one can complete many impossible tasks within the classical world. One of the most striking applications of entanglement is quantum secret sharing (QSS) [1]. It is a process to distribute private key among three [1] or multiply parties [1, 2] securely. If and only if when they cooperate, they can get complete information about the message. Meanwhile, if one of them is dishonest, the honest players may keep the dishonest one from doing any damage. But, up to now, most existing QSS schemes only focused on creating a private key or splitting a classical secret among many parties. Recently, due to its promising applications in quantum secure communication, it attracts many attentions. However, many applications in quantum information theory require the distribution of quantum states. Cleve et al. [3] proposed a protocol providing robust and secure distribution of quantum states between nodes, which Lance et al. [4] termed as quantum state sharing to differentiate from the QSS of classical information. As quantum state carries quantum information, it is also called quantum information sharing (QIS). Quantum state sharing is a protocol where perfect reconstruction of quantum state is achieved with partial information of a multipartite quantum network. It also allows for secure communication in a quantum network where partial information is lost or acquired by disloyal parties.

Recently, Lance et al. demonstrated schemes for encoding a secret coherent state into a tripartite entangled state and distributing it to two players both theoretically [4] and experimentally [5]. Deng et al. [6] proposed a scheme for multiparty quantum state sharing of an arbitrary two-particle entanglement with EPR pairs. Li et al. [7] investigated protocol for multiparty secret sharing of single-qubit quantum information. In the previous schemes, Bell state measurements (in Ref. [4]) or even multiparticle joint measurements (in Ref. [7]), which is still under extensive research among many ambitious scientists, are widely employed. Zhang et al. [8], proposed a scheme for quantum information sharing of a single-qubit state based on entanglement swapping in cavity QED where the effects of cavity decay and thermal field are both eliminated. However, in quantum information theory, entangled states are precious resource, thus one should employ them with great deliberation. In Ref. [8], they utilize an EPR and tripartite GHZ state (five atomic qubits) to finish the task of a single-qubit state sharing between two parties while four qubits (two EPR states) can complete the task [9]. Furthermore Ref. [1] showed that three qubits (a pure tripartite entangled GHZ type state) is sufficient.

It is well known that multipartite qubits can be entangled in different inequivalent ways. For tripartite entangled quantum system, it falls into two classes of irreducible entanglements [9], that is, GHZ and W class state. The motivation of classifying entangled state is that, if the entanglement of two states is equivalent, then the two states can be used to perform the same task, although the probability of successful performance of the task may depend on the amount of entanglement of the state. But, in the branch of quantum state sharing, as well as in QSS, most of the previous schemes [1, 2, 3, 4, 5, 6, 7, 8] utilize the GHZ class of entangled state. W class state is also a promising candidate in implementing quantum communication and other tasks in the realm of quantum information processing. Recently, Joo et al. [10] presented a novel scheme for secure quantum communication via W state, where they proposed three different protocols for secure quantum communication, that is, quantum key distribution, probabilistic quantum secret sharing of classical information and their synthesis. In order to extensively investigate the applications of W class states in quantum communication, we engage ourselves in the work of implementing quantum state sharing via W class state in this paper. Here, we first investigate a physical scheme for Ref. [1] in cavity QED, which shares single-qubit quantum state via a tripartite GHZ state. Then we will argue that W class state can also fulfill the task probabilistically. The distinct advantage of the scheme is that during the passage of the atoms through the cavity field, a strong classical field is accompanied, thus our scheme is insensitive to both the cavity decay and the thermal field.

II. THE CA VITY MODEL

We consider two identical two-level atoms simultaneously interacting with a single-mode cavity and driven by a classi-
The interaction between the single-mode cavity and the atoms can be described, in the rotating-wave approximation, as

$$H = \omega_0 S_Z + \omega_a a^+ a + \sum_{j=1}^{2} [g(a^+ S_j^- + a S_j^+)] + \Omega(S_j^+ e^{-i\omega t} + S_j^- e^{i\omega t})$$

(1)

where $S_Z = \frac{1}{2} \sum_{j=1}^{2} |e\rangle_j \langle e| - |g\rangle_j \langle g|$, $S_j^+ = |e\rangle_j \langle g|$, $S_j^- = |g\rangle_j \langle e|$, and $|e\rangle_j$ and $|g\rangle_j$ are the excited and ground states of $j$th atom, respectively. $a^+$ and $a$ are the creation and annihilation operators for the cavity mode, respectively. $\omega$ is the coupling constant between atomic system and the cavity, $\Omega$ is the Rabi frequency, $\omega_0$, $\omega_a$ and $\omega$ are atomic transition frequency, cavity frequency and the frequency of the driven classical field, respectively.

While in the case of $\omega_0 = \omega$, in the interaction picture, the interaction Hamiltonian is

$$H_I = \Omega \sum_{j=1}^{2} (S_j^+ + S_j^-) + g \sum_{j=1}^{2} (e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+),$$

(2)

where $\delta$ is the detuning between atomic transition frequency $\omega_0$ and the cavity frequency $\omega_a$. We define the new atomic basis

$$|+\rangle_j = \frac{1}{\sqrt{2}}(|g\rangle_j + |e\rangle_j), |\rangle_j = \frac{1}{\sqrt{2}}(|g\rangle_j - |e\rangle_j).$$

(3)

Then we can rewrite $H_I$ as

$$H_I = \frac{2}{\sqrt{2}} \sum_{j=1}^{2} [2\Omega \sigma_{z,j} + ge^{-i\delta t} a^+ (\sigma_{z,j} + \frac{1}{2} \sigma_j^+ - \frac{1}{2} \sigma_j^-) + ge^{i\delta t} a (\sigma_{z,j} + \frac{1}{2} \sigma_j^- - \frac{1}{2} \sigma_j^+)],$$

(4)

where $\sigma_{z,j} = \frac{1}{2} (|+\rangle_j \langle +| - |\rangle_j \langle -|)$, $\sigma_j^+ = |+\rangle_j \langle -|$, and $\sigma_j^- = |\rangle_j \langle +|$. The time evolution of this system is decided by Schrödinger equation

$$i d[\psi(t)]/d t = H_I |\psi(t)\rangle.$$  

We perform the unitary transformation $|\psi(t)\rangle = e^{iH_0 t} |\psi(t)'\rangle$ with $H_0 = 2\Omega \sum_{j=1}^{2} \sigma_{z,j} = \sum_{j=1}^{2} \Omega (S_j^+ + S_j^-)$, then we obtain

$$i d[\psi(t)']/d t = H'_I |\psi(t)\rangle,$$

(6)

with

$$H'_I = \frac{2}{\sqrt{2}} \sum_{j=1}^{2} [ge^{-i\delta t} a^+ (\sigma_{z,j} + \frac{1}{2} \sigma_j^+ e^{2i\delta t} - \frac{1}{2} \sigma_j^- e^{-2i\delta t}) + ge^{i\delta t} a (\sigma_{z,j} + \frac{1}{2} \sigma_j^- e^{-2i\delta t} - \frac{1}{2} \sigma_j^+ e^{2i\delta t})].$$

(7)

In the strong driving regime ($2\Omega \gg \delta, g$), we can realize a rotating-wave approximation, i.e., eliminate the terms oscillating fast, which induce Stark shifts. By doing so one just introduce a ignorable imperfection of the generated state. Then the interaction Hamiltonian reduces to

$$H_{I'} = \frac{2}{\sqrt{2}} \sum_{j=1}^{2} (e^{-i\delta t} a^+ + e^{i\delta t} a) \sigma_{z,j} = \frac{2}{\sqrt{2}} \sum_{j=1}^{2} (e^{-i\delta t} a^+ + e^{i\delta t} a) (S_j^+ + S_j^-).$$

(8)

In the case of large detuning ($2\delta \gg g$), there is no energy exchange between the atomic system and the cavity. The resonant transitions are $|e\rangle_j \langle g|k\rangle n \leftrightarrow |g\rangle_j \langle e|k\rangle n$ and $|e\rangle_j \langle e|k\rangle n \leftrightarrow |g\rangle_j \langle g|k\rangle n$. The transition $|e\rangle_j \langle g|k\rangle n \leftrightarrow |g\rangle_j \langle e|k\rangle n$ is mediated by $|e\rangle_j \langle e|k\rangle n \pm 1$ and $|g\rangle_j \langle g|k\rangle n \pm 1$. The contributions of $|e\rangle_j \langle e|k\rangle n \pm 1$ are equal to those induced by $|g\rangle_j \langle g|k\rangle n$, and the corresponding Rabi frequency is $g^2/2\delta$. Since the transition paths interfere destructively, the Rabi frequency is independent of the photon number of the cavity mode. The Rabi frequency for $|e\rangle_j \langle g|k\rangle n \leftrightarrow |g\rangle_j \langle g|k\rangle n$, mediated by $|e\rangle_j \langle e|k\rangle n \pm 1$ and $|g\rangle_j \langle e|k\rangle n \pm 1$, is also $g^2/2\delta$. The Stark shift for the states $|e\rangle_j$ and $|g\rangle_j$ are both equal to $g^2/4\delta$. The photon-number-dependent Stark shifts $g(e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+)$ induced by the strong classical field are negative to those induced by $g(e^{-i\delta t} a^+ S_j^- + e^{i\delta t} a S_j^+)$, thus the photon-number-dependent Stark shifts are also canceled. Then the effective interaction Hamiltonian can be described as

$$H_e = \frac{\lambda}{2} \sum_{j=1}^{2} (|e\rangle_j \langle g| + |g\rangle_j \langle e|) + \sum_{j,k=1; j \neq k}^{2} (S_j^+ S_k^- + S_j^- S_k^+) + H.c.),$$

(9)

where $\lambda = g^2/2\delta$. The distinct feature of the effective Hamiltonian is that it is independent of the photon number of the cavity field, that is, allowing it to be in a thermal state. Our scheme is based on such kind of cavity, so it is insensitive to both the cavity decay and the thermal field. The time evolution operator of the system is

$$U(t) = e^{-iH_0 t} e^{-iH_e t}.$$
atom, in addition, individual users could not do any damage to the process. Here, we assume that communication over a classical channel is insecure, which means we can’t resort to the simplest method of teleportation [13]. One could also securely complete the task using standard quantum cryptography [14], which requires more resource and measurements [1]. Initially, Alice possesses three atoms and they are prepared in the GHZ state

\[|\psi\rangle_{2,3,4} = \frac{1}{\sqrt{2}}(|eee\rangle + i|ggg\rangle)_{2,3,4} \] (12)

The combined state of the four atoms is

\[|\psi\rangle = \frac{1}{\sqrt{2}}(|\alpha|e\rangle_1 + \beta|g\rangle_1) \otimes (|eee\rangle + i|ggg\rangle)_{2,3,4} \] (13)

Alice simultaneously sends atoms 1 and 2 into a single-mode cavity meanwhile they are driven by a classical field, the time evolution of the system is governed by Eq. (10). Choosing to adjust the interaction time \(\lambda t = \pi/4\) and modulate the driving field \(\Omega t = \pi\), lead the quantum state of the four atoms system, after interaction, to

\[|\psi\rangle = \frac{1}{2}(|\alpha|e\rangle_{1,2} + \beta|gg\rangle_{1,2} + i|ge\rangle_{1,2} - \beta|ee\rangle_{3,4} \] (14)

Alice then sends atoms 3 and 4 to Bob and Charlie, respectively. After Alice confirms that Bob and Charlie both receive one atom, then she operates two measurements on atoms 1 and 2 in the basis \{\(|e\rangle, |g\rangle\}\), respectively. Then the state of atoms 3 and 4 collapse to one of the following unnormalized states

\[|\varphi\rangle_{3,4} = \frac{1}{2}(\alpha|ee\rangle_{3,4} + \beta|gg\rangle_{3,4}), \] (15a)

\[|\varphi\rangle_{3,4} = \frac{1}{2}(\alpha|ee\rangle_{3,4} - \beta|gg\rangle_{3,4}), \] (15b)

\[|\varphi\rangle_{3,4} = \frac{1}{2}(\alpha|gg\rangle_{3,4} + \beta|ee\rangle_{3,4}), \] (15c)

\[|\varphi\rangle_{3,4} = \frac{1}{2}(\alpha|gg\rangle_{3,4} - \beta|ee\rangle_{3,4}). \] (15d)

Up to now, the quantum information (atom 1) is encoded into the state of atoms 3 and 4, which is shared between Bob and Charlie, thus the distribution of quantum information is completed. Each of the two users can only obtain the amplitude information of atoms 1 by local operation and classical information available (the result of Alice’s measurements). If and only if they cooperate, one of them, and only one of them, can get the complete information of the atoms for the sake of no-cloning theorem [15]. Assuming the collapsed state is in the form of Eq. (15b) after Alice’s measurements and then she declares the measurement result to Bob and Charlie over a public channel. If Alice designates Charlie to reconstruct the quantum state and Bob would like to cooperate with him, then Charlie could obtain complete information of Alice’s atoms by local operation.

Next we will discuss the process in detail. Bob let his atom (atom 3) crosses a classical field tuned to the transitions \(|g\rangle \rightarrow |e\rangle\). Choose the amplitudes and phases of the classical fields appropriately so that atom 3 undergoes the transitions

\[|e\rangle \rightarrow \frac{1}{\sqrt{2}}(|e\rangle_3 + |g\rangle_3), \] (16)

which leads the state in Eq. (15b) into the unnormalized state

\[|\varphi\rangle_{3,4} = \frac{1}{2\sqrt{2}}(|g\rangle_3(\alpha|e\rangle_4 + \beta|g\rangle_4) + |e\rangle_3(\alpha|e\rangle_4 - \beta|g\rangle_4)) \] (17)

Bob performs a computational basis measurement on his atom and informs Charlie the measurement result then the state of Charlie’s atom is exactly the state of Alice’s state or relate it up to a corresponding unitary transformation. Charlie can choose the correct unitary transformation based on Bob’s computational measurement result. Without Bob’s cooperation, Charlie can only get the measurement results of Alice from the public channel and can only reconstruct the state with a successful probability of 1/2. Similarly, Bob can also reconstruct the state on his atom if Charlie chooses to cooperate with him. From Eq. (17) we can see that the successful probability of reconstruct the original state from Eq. (15b) is 1/4. There is four equal probable states in Eq. (15), so the total probability of successfully reconstructing the original state in both nodes reaches unit.

IV. QIS VIA W STATE

If Alice initially possesses three atoms prepared in the W class entangled state

\[|\psi\rangle_{2,3,4} = (a|gge\rangle + b|geg\rangle + ic|egg\rangle)_{2,3,4}, \] (18)

where \(|a|^2 + |b|^2 + |c|^2 = 1\), and we can assume \(|a| > |b| > |c|\) without loss of generality. So, the combined state for the four atoms is

\[|\psi\rangle = (a|e\rangle + b|g\rangle)_{1}(a|gge\rangle + b|geg\rangle + ic|egg\rangle)_{2,3,4}. (19)\]

Then, Alice simultaneously sends atoms 1 and 2 into a single-mode cavity meanwhile they are driven by a classical field, the time evolution of the system is governed by Eq. (10). Choosing to adjust the interaction time \(\lambda t = \pi/4\) and modulate the driving field \(\Omega t = \pi\), lead the quantum state of the four atoms system, after interaction, to

\[|\psi\rangle = \frac{1}{\sqrt{2}}(|g\rangle_{1,2}(\beta|a|e\rangle + \beta|b|g\rangle + \alpha|c|g\rangle)_{3,4}

\[-i|e\rangle_{1,2}(\beta|a|e\rangle + \beta|b|g\rangle - \alpha|c|g\rangle)_{3,4}

+|e\rangle_{1,2}(\alpha|a|e\rangle + \alpha|b|g\rangle + \beta|c|g\rangle)_{3,4}

-i|g\rangle_{1,2}(\alpha|a|e\rangle + \alpha|b|g\rangle - \beta|c|g\rangle)_{3,4}. \] (20)
Alice then sends atoms 3 and 4 to Bob and Charlie, respectively. After Alice confirms that Bob and Charlie both receive one atom, she operates two computational measurements on atoms 1 and 2, respectively. The state of atoms 3 and 4 collapse to one of the following unnormalized states

\[ \varphi_{3,4} = \frac{1}{\sqrt{2}}(\beta a|ge⟩ + \beta b|eg⟩ + \alpha c|gg⟩)_{3,4}, \text{(21a)} \]

\[ \varphi_{3,4} = \frac{1}{\sqrt{2}}(\beta a|ge⟩ + \beta b|eg⟩ - \alpha c|gg⟩)_{3,4}, \text{(21b)} \]

\[ \varphi_{3,4} = \frac{1}{\sqrt{2}}(\alpha a|ge⟩ + \alpha b|eg⟩ + \beta c|gg⟩)_{3,4}, \text{(21c)} \]

\[ \varphi_{3,4} = \frac{1}{\sqrt{2}}(\alpha a|ge⟩ + \alpha b|eg⟩ - \beta c|gg⟩)_{3,4}. \text{(21d)} \]

Now the quantum information (atom 1) is encoded into the state of atoms 3 and 4. Neither of the two users could obtain complete information of Alice’s atom by local operation and classical information available. If and only if they cooperate, one of them, and only one of them, can get the quantum information. Without loss of generality, we assume the collapsed state is Eq. (21c) after Alice’s measurement, and then she declares the measurement result to Bob and Charlie over a public channel. It can be rewritten as

\[ \varphi_{3,4} = \frac{1}{\sqrt{2}}[|g⟩_3(\alpha a|e⟩_4 + \beta c|g⟩_4) + \alpha b|c⟩_3|g⟩_4]. \text{(22)} \]

If Alice designates Charlie to reconstruct the quantum state, meanwhile, Bob would like to cooperate (send the result of his measurement to Charlie) then Charlie could obtain complete information of Alice’s atom by local operation. Next we will discuss the process in detail.

Bob performs a computational basis measurement on his atom and informs Charlie the measurement result. If Bob’s measurement result is \(|g⟩_3\), then the state of Charlie’s atom is

\[ \varphi_4 = \frac{1}{\sqrt{2}}(\alpha a|e⟩_4 + \beta c|g⟩_4). \text{(23)} \]

To reconstruct the initial state on atom 4 in cavity QED, Charlie needs another single-mode high-Q optical cavity with the initial state \(|0⟩\) and a photon detector. According to the Jaynes-Cummings model, the Hamiltonian of the resonant \((\delta = \omega_0 - \omega = 0)\) interaction system between the atom and the cavity is

\[ H = \omega(a^+a + S_z) + g(aS_+ + a^+S_-). \text{(24)} \]

The time evolution of the interaction under the Hamiltonian in Eq. (24) are

\[ |g⟩|0⟩ \rightarrow |g⟩|0⟩, \quad \text{(25a)} \]

\[ |e⟩|0⟩ \rightarrow (\cos gt|e⟩|0⟩ - \sin gt|g⟩|1⟩). \quad \text{(25b)} \]

Sending atom 4 into the cavity and taking the interaction time \(t_1 = 1/g \arccos(|c|/|a|)\). According to Eq. (25) we can get the state of the quantum system after interaction as

\[ |\Psi(t_1)⟩_{4,C} = \frac{|c⟩}{\sqrt{2}}[(\alpha|e⟩_4 + \beta|g⟩_4)|0⟩ + \alpha a\sqrt{|a|^2 - |c|^2}|g⟩_4]|1⟩ \quad \text{(26)} \]

If Alice designates Bob to reconstruct the quantum state with the cooperation of Charlie, the state of Eq. (21c) should be rewritten as

\[ |\varphi⟩_{3,4} = \frac{1}{\sqrt{2}}[(\alpha b|e⟩_3 + \beta c|g⟩_3)|g⟩_4 + \alpha a|g⟩_3|e⟩_4]. \quad \text{(27)} \]

After been informed the computational basis measurement result, Bob sends his atom into the resonant cavity and choose the interaction time \(t_2 = 1/g \arccos(|c|/|b|)\), we can get the state of the quantum system after interaction as

\[ |\Psi(t_2)⟩_{3,C} = \frac{|c⟩}{\sqrt{2}}[(\alpha b|e⟩_3 + \beta c|g⟩_3)|0⟩ + \alpha a\sqrt{|b|^2 - |c|^2}|g⟩_3]|1⟩ \quad \text{(28)} \]

Finally, by detecting the cavity, the state of atom 3 (see Eq. (28)) or atom 4 (see Eq. (26) collapsed to the state of Alice’s qubit with a probability of \(1/2|c|^2\). For the rest three states in Eq. (21), the corresponding collapsed state will relate Alice’s state up to a corresponding unitary transformation. So, the total probability of successful of our scheme is \(P_s = 2|c|^2\), which is decided by the smallest superposition coefficients of the W state used as quantum channels. We also note that the probability is equal to the probability of successful teleportation with nonmaximally entangled W state \([16]\). So, the optimal probability for quantum state sharing with W state is \(P = 2/3\), and then the nonmaximally entangled W state will just be a maximally entangled state \((a = b = c = 1/\sqrt{3})\).

V. DISCUSSIONS

We have presented a two-party quantum state sharing scheme via a tripartite GHZ type entangled state, as well as a W type entangled state. Now we will discuss the security of our scheme.

(1) If there is an eavesdropper who has been able to entangle an ancilla with the quantum channel, and at some later time he can measure the ancilla to gain information about the measurement results of the legal users. However, Hillery et al. [11] showed that if this entanglement does not introduce any errors into the procedure, then the state of the system is a product of the entangled state and the ancilla. In other word, the eavesdropper could gain nothing about the measurements on the quantum channel from observing his ancilla. Conversely, if he does gain some information about the two legal users’ measurements, she must inevitably introduce errors in the procedure.
(2) If one of the users is the eavesdropper, i.e., Bob, who wants to obtain Alice’s information without the cooperation of the third party and without being detected. If Alice designates Bob to reconstruct the state and Charlie agrees to cooperate with Bob, Bob can eavesdrop the state with unit successful probability without being detected. Without the cooperation of Charlie, Bob can also eavesdrop the state with a successful probability of 0.5. If Alice designates Charlie to receive the state, he can also recover the state with the help of Bob. If Bob lies his measurement results to him then Bob gains nothing and Charlie can’t obtain the correct state deterministically.

(3) Bob can also get the qubit that Alice sends to Charlie, and sends Charlie a qubit that he has prepared before hand. He wants to reconstruct Alice’s state without the help of Charlie. By doing so, only when Alice designates him to reconstruct the state he can get the state without being detected. If Alice assigns Charlie to recover the state, then Bob’s eavesdropping behavior can be detected. Because Bob does not know Alice’s measurement result thus the qubit he sent to Charlie is not in the correct quantum state, which will lead to the difference between the state recovered by Charlie and the state Alice has sent. By checking a subset of the state with Charlie publicly, the eavesdropping behavior of Bob can be detected.

Next, we will give a brief analysis of the experimental feasibility of our scheme. For the interaction between the optical cavity and two two identical atoms, it is noted that the atomic state evolution is independent of the cavity field state, so, during the process, it does not require the transfer of quantum information between the atoms and cavity. In addition, with the help of a strong classical driving field the photon-number dependent parts in the evolution operator are canceled. Thus the scheme is insensitive to the thermal field and the cavity decay. So, the requirement on the quality factor of the cavities is greatly loosened. In our scheme, the two atoms must be simultaneously interaction with the cavity. But in real case, we can’t achieve simultaneousness in perfect precise. Calculation on the error [17] suggests that it only slightly affects the fidelity of the reconstruct state. Furthermore, the time required to complete the process should be much shorter than that of atom radiative time and cavity field decay. Osnaghi et al. [18] show that for the Rydberg atoms with principal quantum numbers 50 and 51, the interaction time is much shorter than the atomic radiative time. So our scheme is realizable by using cavity QED techniques presently available.

VI. CONCLUSION

In summary, we have investigated an economic and experimentally feasible scheme to securely distribute and reconstruct a single-qubit quantum state between two parties via a tripartite entangled state in cavity QED. If and only if when they cooperate with each other, they can reconstruct the quantum state. Any contempt to get complete information of the state without the cooperation of the third party can’t be succeed in a deterministic way. The distinct advantage of the scheme is that during the passage of the atoms through the cavity field, a strong classical field is companied, thus our scheme is insensitive to both the cavity decay and the thermal field. In addition, Our scheme only employs single-qubit computational basis measurements, thus it may offer a simple and easy way of demonstrating quantum state sharing experimentally in cavity QED with atomic qubits. Our scheme can also be generalized to multipartite case via multipartite entangled state in a straight forwards way.

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