Development of a behavioural model of the MEMS accelerometer

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Abstract. This article presents a method for deriving the transfer function for the developed design of a micromechanical accelerometer. A behavioural model of the MEMS accelerometer has been developed taking into account its dynamic and static properties. The obtained simulation results confirm the reliability of the constructed model. For example, under the influence of 5g acceleration, the movement of the sensitive element is 0.9 μm. Reducing the natural frequency increases the stroke of the sensitive element by 0.5 μm.

1. Introduction

Currently, microelectromechanical systems (MEMS) are considered to be an increasingly promising area. Micromechanical devices are widely used in household and computer technology, medicine, automotive, and military industries, motion capture systems, and many other areas [1].

The high demand for new devices requires time-consuming research and development work, the use of new materials, the improvement of geometry, and the use of new physical principles. All this is done to meet the continuously growing demands of customers to increase the sensitivity and range of sensitivity [2].

As a rule, this is a very demanding and profitable market, so the use of computer systems for modelling and simulation at the development stage of MEMS devices will make it possible to detect design flaws in the device under development long before the first prototypes are created, which will allow you to achieve a competitive advantage.

Modern modelling and CAD tools used in conjunction with rapidly developing high-performance computing systems provide an excellent environment for numerical calculations of highly detailed device models.

The research purpose is to improve modern methods of modelling microstructures. In particular, micromechanical gyroscopes and accelerometers.

The main contribution of this paper is the derivation of the transfer function. And the creation of a behavioural model of a micromechanical accelerometer based on the transfer function.

2. Design of MEMS Gyroscope-Accelerometer

In this work, we developed an integrated micromechanical linear acceleration and rotation sensor with three sensitivity’s axis. Figure 1 shows a parametrizable geometric model of a gyroscope-accelerometer. Figure 2 shows a finite element model of a gyroscope-accelerometer.

The developed MEMS gyroscope-accelerometer has the ability to measure the values of angular velocities and linear acceleration along the X- and Y-axis located perpendicular to each other in the substrate plane, and the Z-axis directed perpendicular to the substrate plane.
Further in the text, we will consider the operation of the proposed micromechanical device in the accelerometer-only mode.

3. Development of the MEMS accelerometer transfer function
We derive the motion equations of the sensing element (SE) of the micromechanical accelerometer based on the Lagrange equations of the second kind [3]:

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} = Q_j, \ (j = 1, 2, \ldots)$$

(1)
where $T$ - the kinetic energy of the system; $Q_j$ - generalized forces; $q_j$ - generalized coordinates, $\dot{q}_j$ - generalized speed, $j$ - degree of freedom system.

Equations of motion of the developed micromechanical accelerometer with two degrees of freedom:

$$
\begin{align*}
\frac{d^2x}{dt^2} + 2\delta \frac{dx}{dt} + \omega_0^2 x &= (F_x + F_{el1})m^{-1} + \Omega(\dot{y} + Gx) \\
\frac{d^2y}{dt^2} + 2\delta \frac{dy}{dt} + \omega_0^2 y &= (F_y + F_{el2})m^{-1} - \Omega(\dot{x} - Gy)
\end{align*}
$$

(2)

where $x$, $y$ - move the sensing element along the X- and Y-axis; $m$ - mass of the sensing element; $\delta$ - the damping ratio; $\alpha_0, \omega_0$ - frequencies of the sensing element along the X- and Y-axis; $F_x, F_y$ - the force of inertia; $F_{el1}, F_{el2}$ - electrostatic force; $\Omega$ - angular velocity, $G$ - the stiffness of the elastic suspension.

To get the transfer function, we substitute all the variables in the equation of motion (2):

$$
\begin{align*}
\frac{d^2x}{dt^2} + 2\delta \frac{dx}{dt} + x(\omega_0^2 - 2N_sU_{dc}U_{AC1})\frac{\varepsilon \varepsilon_0 h}{d_m} &= ma_x + 2N_sU_{dc}U_{AC1}\frac{\varepsilon \varepsilon_0 h}{d_m} + \Omega(\dot{y} + Gx) \\
\frac{d^2y}{dt^2} + 2\delta \frac{dy}{dt} + y(\omega_0^2 - 2N_sU_{dc}U_{AC2})\frac{\varepsilon \varepsilon_0 h}{d_m} &= ma_y + 2N_sU_{dc}U_{AC2}\frac{\varepsilon \varepsilon_0 h}{d_m} - \Omega(\dot{x} - Gy)
\end{align*}
$$

(3)

where $\omega_0, \omega_0$ - natural frequencies of the sensing element along the X- and Y-axis; $N_s$ - the number of fingers of the movable comb electrode of the electrostatic drive; $U_{dc}$ - constant voltage; $U_{AC1}, U_{AC2}$ - variable voltages; $\varepsilon$ - relative permittivity of the air gap; $\varepsilon_0$ - vacuum permittivity; $h$ - thickness of the structural layer; $d_m$ - the gap between the fingers of the comb of the movable and stationary electrodes of the electrostatic drive; $a_x, a_y$ - linear accelerations along the X- and Y-axis.

Rewrite equation (3) by grouping and combining the variables:

$$
\begin{align*}
\frac{d^2x}{dt^2} + 2\delta \frac{dx}{dt} + xG\Omega &= 4N_sU_{dc}U_{AC1}\frac{\varepsilon \varepsilon_0 h}{d_m} + ma_x + \dot{y}\Omega \\
\frac{d^2y}{dt^2} + 2\delta \frac{dy}{dt} + yG\Omega &= 4N_sU_{dc}U_{AC2}\frac{\varepsilon \varepsilon_0 h}{d_m} + ma_y - \dot{x}\Omega
\end{align*}
$$

(4)

Since the product of $\varepsilon \varepsilon_0 h$ has a very small dimension, the entire argument can be equated to 0. We can also neglect the influence of $\dot{y}$ for the first equation of the system. Similarly, you can do $\dot{x}$ for the second equation of the system. Therefore, equation (4) will have the form:

$$
\begin{align*}
\frac{d^2x}{dt^2} + 2\delta \frac{dx}{dt} + xG\Omega &= ma_x \\
\frac{d^2y}{dt^2} + 2\delta \frac{dy}{dt} + yG\Omega &= ma_y
\end{align*}
$$

(5)

We translate the equations of motion (5) into the operator form:

$$
\begin{align*}
mp^2x(p) + 2\delta px(p) + G\Omega x(p) &= ma_x(p) \\
mp^2y(p) + 2\delta py(p) + G\Omega y(p) &= ma_y(p)
\end{align*}
$$

(6)

where $x(p)$ - the Laplace image of the inertial mass displacement along the X-axis; $y(p)$ - the Laplace image of the inertial mass displacement along the Y-axis; $a_x(p), a_y(p)$ - the Laplace image of the object acceleration along the X- and Y-axis.
Since the input signal for this system is acceleration, and the output signal is the value of the movement of the sensitive element, the transfer functions of the system are defined by the following expressions:

\[
W_x(p) = \frac{x(p)}{a_x} = \frac{m}{mp^2 + 2\delta p + G\Omega}
\]  

(7)

\[
W_y(p) = \frac{y(p)}{a_y} = \frac{m}{mp^2 + 2\delta p + G\Omega}
\]  

(8)

Due to the fact that the equations of the transfer functions of the movement of the SE (7) and (8) along the X- and Y-axis turned out to be the same, in the future we will use the transfer function along the X axis for simplicity.

If enter new coefficients \( K_x = 1/G_x \), \( T_x = \sqrt{m/G_x} \), \( \xi = \delta/2\sqrt{mG_x} \), then the resulting expression will be a transfer function along the channel the effect of an external force-the movement of the SE:

\[
W(p) = \frac{x(p)}{F_x} = \frac{K_x}{T_x^2 p^2 + 2\xi T_x p + 1}
\]  

(9)

where \( T_x \) - the SE time constant; \( K_x \) - the transfer coefficient; \( \xi \) - relative damping coefficient. The resulting expression will be used to get a behavioral model.

4. Behavioral model of a micromechanical accelerometer

After calculating the coefficients of the resulting transfer function (9), we will implement it in the mathematical model of the micromechanical accelerometer. Figure 3 shows a behavioral model of a micromechanical accelerometer with a transfer function in the Simulink environment of the MATLAB software package.

![Figure 3](image-url)

**Figure 3.** Behavioural model of a micromechanical gyroscope-accelerometer with a transfer function.
The model structurally consists of two conditional parts. The first part is the determination of constants and input effects, and the implementation of equation (4). The second part is the correction of the results obtained through the transfer function.

The resulting behavioural model should be checked for validity. To do this, we will evaluate how the model reacts to changes in key parameters.

We will simulate the movement of the sensor element under the influence of acceleration of $1g$, $5g$, $10g$. Figure 4 shows the simulation results.

**Figure 4.** Movement of the micro-mechanical accelerometer at accelerations of $1g$, $5g$, $10g$.

We will conduct a simulation of the movement of the sensor element at different values of natural frequencies. The simulation results are shown in Figure 5. The acceleration value is $1g$, and the values of the natural frequencies are $\omega_{01}=7726$ Hz and $\omega_{02}=5726$ Hz.

**Figure 5.** Results of simulation of the movement of a micromechanical accelerometer at $1g$ acceleration and different values of natural frequency.
We will simulate the movement of the sensor element at different values of the damping coefficient. Figure 6 shows the simulation results. The acceleration value is 1g, and the damping coefficient values are $\delta = 7.9 \times 10^{-6}$ and $\delta = 7.9 \times 10^{-8}$.

![Figure 6. Results of modeling the movement of the sensor element at an acceleration of 1g and different values of the stiffness coefficient.](image)

5. Conclusion
This article presents the topology and design of an integrated micromechanical gyroscope-accelerometer. The transfer function is also derived and a behavioural model is constructed for the developed design in the accelerometer mode. Modelling of the behavioural model showed the following results. Under the influence of acceleration, the movement of the sensor element is units of $\mu m$, as shown in Figure 4. Figure 5 shows that a decrease in the natural frequency value leads to an increase in the travel of the sensor element by 0.5 $\mu m$. An increase in the damping coefficient leads to a change in the stroke of the sensor element and the appearance of noise, as shown in Figure 6. The results obtained correspond to the actual behaviour of the micromechanical accelerometer.

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References
[1] Lysenko I E, Ezhova O A, Tkachenko A V, Naumenko D V, Guha K and Rao K S 2019 The results of modelling of a micromechanical accelerometer J. Phys.: Conf. Ser. 1410 012226
[2] Lysenko I E, Naumenko D V and Ezhova O A 2020 Design and simulation high aspect ratio torsion suspension of MEMS z-axis accelerometer Book of Abstracts of 7th International School and Conference «Saint Petersburg OPEN 2020» on Optoelectronics, Photonics, Engineering and Nanostructures (St. Petersburg: St. Petersburg Academic University) pp 478-479
[3] Raspopov V Ja 2007 Micromechanical devices (Tula: Tula state university)