Opinion Dynamic with agents immigration

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Abstract

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I. INTRODUCTION

Recent years, a large class of interdisciplinary problems has been successfully studied with statistical physics methods. Statistical physics establishes the bridge from microscopic characteristics to macroscopic behaviors, for systems containing a large number of interacting components. Using both analytical and numerical tools, it has contributed greatly to our understanding of various complex systems. In this paper, we are motivated by the statistical physics of a sociological problem, namely, opinion dynamics.

As one of the classical and traditional research areas in both social science and theory physics, opinion dynamics has attracted much attention. A lot of models concerning the process of opinion formation, such as voter model, bounded confidence model, have been proposed previously. Meanwhile, some of recent studies discussed and described the opinion dynamics on both common conditions and various complex networks.

The issue of individual mobility has become increasingly fundamental due to the Human migration and human dynamic. The issue is also important in other contexts such as the emergence of Cooperation among individuals [20] and species coexistence in cyclic competing games [21]. Recently, some empirical data of human movements have been collected and analyzed [22,23]. From the standpoint for dynamic of complex systems, when individuals (nodes, agents) are mobile, the edges in the topological structures are no longer fixed, yielding more different results on that than before.

In our paper, we try to propose a new model combining conventional opinion dynamics with agents immigration according to information transmission and evolution. In our simulation, we finally find a series of results reflecting special and different features of opinion dynamics with immigration. By introducing a parameter $\alpha$ to control the weight of influence of individual opinions, according to a recent study considering weight influence, we find that there also exist an optimal value of $\alpha$ leading to the shortest consensus time for all individuals on an isotropic plane we concern. After presenting the results of simulation in different situations, we also analysis the results of our model in mathematical way, which could be denoted as follow (yang han xing)

II. MODEL

As previous classical model focusing on the material process of spread and formation of opinions, we spend more efforts on finding special results when individuals carry opinions with immigration. To focus on a more efficient situation, we just confine our discussion on an isotropic plane, without special network effects. On the other hand, the individuals we concern are just holding two kinds of opinion, the positive opinion $\psi^+ > 0$ and the negative opinion $\psi^- < 0$. According to one model on opinion dynamics proposed before (.), we introduce the weight exponent to control the weight of influence of each individual. We describe that all of the opinions of individuals evolve simultaneously completely rely on its neighbors’ opinion and neighbors’ weight. Here we describe the evolution process of the whole individuals on the plane in mathematical way, which could be denoted as follow (yang han xing)
the step as this opinion, the individual we concern evolves its opinion at each step. After changing their opinions, there are \(N_+\) positive ones and \(N_-\) negative ones. Here we simulate the in-

\[ p_+ = \frac{\sum_i \omega_i^u}{\sum_i \omega_i^v + \sum_j \omega_j^u}, \tag{1} \]

\[ p_- = \frac{\sum_i \omega_i^v}{\sum_i \omega_i^u + \sum_j \omega_j^v} \tag{2} \]

where \(p_+\) and \(p_-\) denote the probability of choosing positive opinion and negative opinion, and the number of neighbors holding positive opinion is \(u\) while the number of negative ones is \(v\). Here the model considers the agents with weight impact \(\omega\), which is controlled by weight exponent \(\alpha\). If the probability \(p_+\) is larger than \(p_-\), the agent we concern will choose positive opinion at the next step. And it will be same as choosing negative opinion.

In this model, the individual we concern evolves its opinion at \(t+1\) according to its neighbors’ opinion in its view radius \(r\), which is shown in Figure 1. In Figure 1, the red agents hold positive opinions and black agents hold negative opinions. There are \(u+v\) neighbors in the view range of individual we concern, while here are \(u\) individuals hold positive opinion and \(v\) individuals hold negative opinion at \(t\) step. After comparing the weight of positive opinion and negative opinion, the individual we concern evolves its opinion at \(t+1\) step as this

\[ \psi_i^{(t+1)} = \sum_j \psi_j^{(t)} \tag{3} \]

where \(\psi_j^{(t)}\) denotes the opinion state of the \(j\)th neighbor of the \(i\)th agent we concern at the \(t\) step. After changing their opinion in the way above, all the individuals immigrate on the plane. All the agents would be confined in the plane by periodic boundary condition. The velocity and direction angle of each agent are randomly distributed, which are kept by each agent all the time. After enough period of time, number of the individuals holding positive opinions \(N_+\) and the number of other individuals holding negative ones \(N_-\) reach a plateaus and dynamic equilibrium. At that certain point, we believe that the process of opinion dynamics would be terminated. And we could find that if all of the individuals enter the plateaus, the total number of individuals who hold positive opinion at \(t\) step \(\psi_i'\) would be approximately equal to the number of individuals who also hold that at \(t+1\) step \(\psi_i^{(t)}\). And we could carry on this description with mathematical language,

\[ \sum_i \psi_i^{(t+1)} = \sum_i \psi_i^{(t)} \tag{4} \]

The total time steps the system took could be defined as \(T_c\) for convergent time.

### A. Results and analysis

In the following discussion and simulation, we confines our individuals on an isotropic plane \((L \times L)\). The length of the boundary of this plane \(L\) is 20, and the total number of individuals on the plane would be \(N\). Here we simulate the individuals have their initial velocity under Gauss distribution, which would be more rational and close to facts. Each individual hold their opinions (positive one or negative one) and their fixed weight of opinion with random probability. The distribution of agents’ weight was established in a random way at the beginning of evolution. The exponent \(\alpha\) in equations (1) and (2) controls the evolution process. And here we define

**FIG. 2:** (Color online) Cumulative payoff distribution for different values of \(\alpha\). The distribution is obtained after the cooperation density becomes stable. The multiplication factor is set to be \(r = 1.6\). Solid curves are theoretical predictions from Eq. (2).

**FIG. 3:** (Color online) Times series of cooperator density in hubs’ neighborhoods for (a) The multiplication factor is \(r = 1.2\) and each data point is obtained by averaging over 50 runs.
\( \rho \) and \( \Delta \rho \) to describe the changing process of the individuals holding positive opinion. They are denoted in equations as follow

\[
\rho_c = \frac{N_+}{N} \quad \text{(5)}
\]

\[
\Delta \rho = \frac{\Delta N_+}{N} \quad \text{(6)}
\]

In Figure 2, we show \( \rho_c \) as function of evolution time \( t \) for different view radius \( r \), both \( r=1.2 \) and \( r=1.5 \). The most interesting thing we could find in this figure is that when evolution time \( t \) is around 6500, the value of \( \Delta \rho \) plummets obviously, which finally reach the level under 0.1. In fact, when changes of \( \Delta \rho \) has lower amplitude of variation, it also means that the individuals holding positive opinion enter the period of dynamic equilibrium.

In this situation, we could find the consensus time in Figure 3. If \( t < 4000 \) or \( t > 6500 \), the \( \Delta \rho \) changes in a very small range which would also be shown in the figure. But the consensus time is not directly impacted by only view radius \( r \) of each individual. In Figure 4, here converge time \( T_c \) is taken as a function of average velocity \( \alpha \), while the view radius \( r \) is equal to 1.2. Interestingly, we show that would reach a minimum value when \( \alpha \) is around 2 under different values of total number of individuals on the plane \( N \). Here we present that consensus time \( T_c \) changes with \( \alpha \) in a "smile curve". And certainly the value of would be higher if the \( N \) is more. In fact, it is obvious to explain that when there are more individuals holding different opinions, they would take more time to reach consensus or dynamic equilibrium. In that we show that \( N \) and weight exponent \( \alpha \) could both directly determine the consensus time \( T_c \). The more cogept demonstration is shown in Figure 2, which presents as a function of \( \alpha \). Here \( \rho_c \) is the density of individuals holding positive opinions at the consensus time.

In the Figure 2, we present that \( \rho_c \) will reach a maximum when is around 2. Meanwhile, the value of is greater if view radius is lager. To show the result in a more intuitive way, we present the distribution map in Figure 6. In Figure 6, the red points are the ones holding positive opinions while the black points present negative ones. In this figure, we present the specific distribution.

To support former results, we mainly focus on the results that positive opinions take dominant rate. In order to discuss the parameters reflecting immigration of individuals, we present consensus time \( T_c \) as a function of average velocity of individuals with different \( N \) in Figure 5. In this figure, we find that consensus time \( T_c \) would increase approximately in a linear way when \( v \) is less than 1.5. After that, it decreases in a certain range without sharp changes.

To discuss the model in a more reliable way, we try to analysis the process by founding up a series of equations for \( m \)
agents in total as follow. In equations, we define that the \( i \)th individual we concern has a view radius \( r \), and at the \( t \) step there are \( s_m \) individuals in its view range as its neighbors, and here we define that \( s_{ij} \) as the \( i \)th neighbor of the \( j \)th agent we concern. In that, it is \( j \)th opinion state of agent’s neighbor at \( t \) step that determine the opinion updating of this individuals at next time step \( t + 1 \). If \( \psi \) is positive, individuals who holding positive opinions would have greater weight than those who hold negative opinions. As a consequence, the equations could be formed as follow:

\[
\begin{align*}
\psi_{1}^{(t+1)} &= \psi^{(t)}(r, s_{11}) \cdot \omega_{s_{11}} + \ldots + \psi^{(t)}(r, s_{m-1,1}) \cdot \omega_{s_{m-1,1}} + \psi^{(t)}(r, s_{m,1}) \cdot \omega_{s_{m,1}} \\
\psi_{m}^{(t+1)} &= \psi^{(t)}(r, s_{1,m-1}) \cdot \omega_{s_{1,m-1}} + \ldots + \psi^{(t)}(r, s_{m-1,m-1}) \cdot \omega_{s_{m-1,m-1}} + \psi^{(t)}(r, s_{m,m-1}) \cdot \omega_{s_{m,m-1}} \\
\psi_{1}^{(t+1)} &= \psi^{(t)}(r, s_{1,1}) \cdot \omega_{s_{1,1}} + \ldots + \psi^{(t)}(r, s_{m-1,1}) \cdot \omega_{s_{m-1,1}} + \psi^{(t)}(r, s_{m,m}) \cdot \omega_{s_{m,m}}
\end{align*}
\]  

(7)

To describe the model in a simpler way, we try to apply linear algebra instead of these traditional equations. In order to write in that way, we also introduce a new parameter \( n_{ij}^{(t)} \) into this matrix description. Here \( n_{ij}^{(t)} \) reflects that the times of opinion exchanging or sharing of \( j \)th individual we concern at \( t \) step. In other word, \( n_{ij}^{(t)} \) is a standard that concerns how many times the \( i \)th individual impacts others opinion updating choice of next time step at \( t \) step. When the whole individuals get into the plateaus of dynamic equilibrium, we discussed in Sec.2, the opinions individuals holding would be described as where \( \omega(i) \) is the weight of the \( i \)th agent, which is

\[
\begin{align*}
\begin{pmatrix}
\psi_{1}^{(t+1)} \\
\psi_{2}^{(t+1)} \\
\vdots \\
\psi_{m}^{(t+1)}
\end{pmatrix} &=
\begin{pmatrix}
\psi_{1}^{(t)} & \psi_{2}^{(t)} & \ldots & \psi_{m}^{(t)}
\end{pmatrix}
\begin{pmatrix}
\omega_{1}^{(t)} \\
\omega_{2}^{(t)} \\
\vdots \\
\omega_{m}^{(t)}
\end{pmatrix}
\end{align*}
\]

(8)

III. CONCLUSION AND DISCUSSIONS

The new mode of opinion evolution with immigration is different from the conventional opinion dynamics. It presents that density of positive opinion agents would be maximum when the weight exponent \( \alpha \) is around 2. In summary, we found up a new model for opinion exchange and communication among agents with immigration. The state matrix we present for analysis and quantitative simulation could also be widely used for more complex situation. The opinion carried by agents represent a kind of state or parameter of agents in motion, so more application and analysis could be carried off with this model and method in future. Discussion we present above is not only to demonstrate our model, but also open up a new combination between opinion communication and agent-based motion. State consensus time is also a very important parameter to describe a system or a group of agents, which could also be one certain standard for different situations.
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