Pressure determination of the water baffle shift in the capillary

I I Maksimov¹ and V V Alekseev²

¹Professor, Mari State University, Yoshkar-Ola, Russia
²Associate Professor, Cheboksary Cooperative Institute of the Russian University of Cooperation, Cheboksary, Russia
E-mail: av77@list.ru

Abstract. The ability of soils to pass water characterizes water permeability, which is an important characteristic, depending on the type of soil, porosity and pore size distribution. It is estimated using the filtration coefficient k. To have information on the value of water permeability is necessary when creating irrigation systems, reservoirs, canals, dams and many other cases. Water permeability allows calculating water losses from these structures. The use of such technique as the creation of multi-layer thin-layer shields is effective to prevent the loss of water during its filtration from irrigating canals. Dissimilar materials, which leak in case of single laying-down, can dramatically decrease the loss of water being multiply used. The pore sizes in the soils vary greatly, which leads to the appearance of ‘Jarman’s chains’ - alternating thin holes with thickenings. Water is trapped in the narrowest places of the capillary and is located in separate areas. Therefore, this kind of capillaries can be a gas-waterproof screen. The menisci layers formed at the contact, which exert considerable resistance to water filtration. Entrapped air bubbles are surrounded by water baffles. The article, according to the theory of the capillary potential (potential is considered as work on transferring a unit of mass of liquid from a capillary into a free liquid at the same temperature and at the same altitude level). The theory allow study the pressure that needs to be used to such a baffle to make it move.

1. Introduction
Efficiency and productivity of irritating systems are mostly defined by the loss of water during its filtration from irrigating channels. One of the techniques to prevent filtration is the creation of multi-layer thin-layer shields on the surface of the canal or the groundmass that forms the canal bed. Materials, which leak in case of single laying-down, being put layer by layer, form menisci that prevent the water filtration [1, 2]. Entrapped air bubbles, which cover the capillary, influence the movement of liquid in the capillaries. In most cases, these bubbles are divided by water baffles. This research is devoted to the determination, according to the theory of the capillary potential, of the pressure that needs to be used to such a baffle to make it move.

The height to which the liquid rises in a capillary is connected with its radius according to the equation of J. Jurin, which states that the thinner is a capillary the more the water would rise. Although examination of the distinct grounds shows that this equation is true only to a certain extent. If the diameter of capillaries is less than $10^{-8}$ m, Jurin’s equation cannot be applied.
In such thin capillaries, there is no space for the capillary water because all the space is filled with the funicular and adsorbed water [3, 4]. That is why the height to which the liquid rises in the capillaries from the layer of the ground waters is bigger in clays than in heavy clays (see Table 1). Table 1 also includes the radii of the effective capillaries and the heights determined according to the equation of J. Jurin. Moreover, if we simply take the soil pore space for the cylindrical one, the capillary in soils always has a variable diameter. In the interchange of thin holes with thickenings the water, in case of partial pore filling, will be only in the narrowest parts of the capillary in the form of the so-called ‘Jarman’s chains’. It is rather hard to take the water out of such a capillary.

| Soil/Ground      | Height of the capillary water rise, m | Radius of the effective capillary, 10^{-5}m |
|------------------|--------------------------------------|------------------------------------------|
| Sands            | 0.4–0.8                               | 1.88–3.75                                |
| Sandy loams      | 0.8–1.2                               | 1.25–1.88                                |
| Sandy clays      | 1.2–3.5                               | 0.42–1.25                                |
| Clays            | 3.5–6.0                               | 0.25–0.42                                |
| Heavy clays      | up to 4.5                             | up to 0.33                               |

If we apply pressure to one of the ends of a capillary, the menisci, which by their concavity face the pressure, will begin to sag more while the meniscus water holding forces will increase. The liquid is detained in such ‘Jarman’s chains’. That is why such capillaries can be gas- and waterproof shields [5–7].

In reclamation for water accumulation and tabulation to sandy soils of arid regions, in the construction of canals and reservoirs, the almost impermeable confining system is obtained by alternately stacking layers of dry sandy and loamy soils.

2. Experimental

Let’s examine the capillary potential as a work of transferring of a liquid mass unit from the capillary to a free liquid at a constant temperature and at the same level of height. In this case, specific volume energy of the free liquid can make the zero level, which can be used to count the specific energy in the capillary.

Let’s examine the system that consists of a dry capillary and some free liquid (Figure 1).

The total energy of such a system is defined by the surface energy of the capillary because the energy of the free liquid is equal to zero.

\[ E_0 = 2\pi rl\sigma_{sg}, \]

where \( r \) is the radius of the capillary, \( l \) is the length of the capillary, and \( \sigma_{sg} \) is the free energy of the area unit of the dry capillary (i.e. solid – gas interfaces).

After the contact of the capillary with the liquid, the energy of the system can be expressed by the equation
\[ E = 2\pi r l \sigma_{sl} + 2\pi (r^2 + h^2) \sigma_{lg} + A, \]  
\[ \text{where } 2\pi r l \sigma_{sl} \text{ is the surface energy of the solid – gas interface, } \]
\[ 2\pi (r^2 + h^2) \sigma_{lg} \text{ is the surface energy of the liquid – gas interface (} h \text{ is height of the spherical segment), and } A \text{ is a load of the introsusception of the liquid into the capillary, equal to } \pi r l \sigma_{lg}. \]

As the liquid is not free anymore, the “decrease” of its energy during the introsusception into the capillary is defined by the difference of \( E_0 - E \), i.e. the equation:
\[ \Delta E = 2\pi r l \sigma_{sl} - 2\pi (r^2 + h^2) \sigma_{lg} - \pi r l \sigma_{lg}, \]  
or, if applying the famous equation of Neiman,
\[ \Delta E = 2\pi r l \sigma_{sl} - 2\pi (r^2 + h^2) \sigma_{lg} - \pi r l \sigma_{lg}. \]

For the convenience of calculations, we will consider such a free liquid that is equal in volume to the volume of a sphere, whose radius is equal to the radius of the capillary. In the case of an incompressible liquid, taking into consideration the immutability of the volume of liquid in the process of being drawn into the capillary, we can write:
\[ \frac{3}{4} \pi r^3 = \frac{2}{6} \pi h (3r^2 + h^2). \]

Now let’s define the length of the wetted part of the capillary through the height of the spherical segment \( h \).
\[ l = \frac{4r^3 + 3r^2 h + h^3}{3r^2}. \]

Let’s apply \( l \) from the equation (6) to the equation (4) and take the derivative \( \frac{d(\Delta E)}{dh} \). While equating this derivative to zero, we will find out \( h \) that determines the maximum of the system’s energy loss, i.e. the system remains in the condition of the energy development:
\[ h = -\frac{4r \pm \sqrt{16r^2 - 4r^2}}{2r}. \]

Taking into consideration that \( h \leq l \), we get:
\[ h = (2 - \sqrt{3})r = Cr, \]
where \( C \) is the derivative.

All the expressed ideas are based on the theory that a liquid can wet a capillary rather substantially, i.e. the interface angle of wetting \( \theta \) while its general determination is equal to zero.

In real conditions \( \theta = 0 \) only for very clean and non-adsorbed envelopes. Usually, the interface angle is not equal to zero. That is why to determine \( h \) you need to consider the angle \( \theta \).

From the simple geometrical relationships, we can show that \( h = Br \), where
\[ B = \frac{1 - \sqrt{1 - \cos^2 \theta}}{\cos \theta}. \]

Generalizing the equations (8) and (9), we can eventually get
\[ h = CBr. \]

From the geometrical relationships, it is clear that:
\[ \cos \theta_1 = \frac{2h}{1 + C^2 B^2}, \quad (11) \]

That allows finding out the interface angle \( \theta \) of a baffle in case of its full wetting. For the water in a glass capillary \( \theta_1 = 60^\circ \).

Let’s find the capillary potential for a baffle
\[ \varphi = \frac{\Delta E}{m} = \frac{\Delta E}{V \rho}, \quad (12) \]

where \( m \) is the mass of the liquid; \( V \) is the volume of the liquid; \( \rho \) is liquid density.

Let’s apply the theory of dimensions.
\[ \frac{J}{kg} = \frac{N \cdot m}{kg} = \frac{N}{m^2} \quad (13) \]

thus,
\[ \varphi = \frac{\Delta P}{\rho}. \]

Equating (12) to (13), we will get
\[ \frac{\Delta E}{V \rho} = \frac{\Delta P}{\rho} \quad \text{or} \quad \Delta P = \frac{\Delta E}{V}, \quad (14) \]

where \( \Delta P \) is the pressure change in the liquid in the capillary relative to the pressure in the free liquid at the same altitude level. From the equation (4) and (14) we can derive that:
\[ \Delta P = \frac{2\pi r l \sigma_{gl}}{V} - \frac{2\pi (r^2 + h^2) \sigma_{gl}}{V} - \frac{\pi r l \sigma_{gl}}{V} = \Delta P_1 - \Delta P_2 - \Delta P_3 \quad , \quad (15) \]

where \( \Delta P_1 \) is the internal pressure created by forces of interaction of liquid molecules with a solid wall that does not change, at least until the beginning of the movement of the baffle; \( \Delta P_2 \) is internal negative pressure, the "tension" of the liquid generated by the liquid-gas interface, which can be assumed unchanged up to the beginning of the movement; \( \Delta P_3 \) is internal negative pressure associated with the retraction of the liquid into the capillary, which may change if you change the shape of the baffle under the unilateral external pressure.

The condition of the begging of the baffle’s movement – its extreme points – is Equation
\[ |\Delta P_3| = \Delta P', \quad (16) \]

where \( \Delta P' \) is external gas pressure on the baffle, on the one hand, under which the extreme points of the baffle start to move or "shear" pressure.

Thus, the required "shear" pressure
\[ \Delta P' = \frac{\pi r l \sigma_{gl}}{V}, \quad (17) \]

where \( V = \pi r^2 l - \frac{2}{3} \pi h (3r^2 + h^2) \) is the amount of liquid in the baffle.

Taking \( h \) from (10) to the equation (17), after the transformations we will get
\[ \Delta P' = \frac{3l}{3l - r (3CB + C^3B^3)} \frac{\sigma_{gl}}{r} \]  

(18)

3. Results and Considerations

Table 2 shows the results of calculations of shear pressure at the effective radius of the capillaries in the soil that are given in Table 1. As can be seen from Table 2, for the used in hydraulic engineering and reclamation alternately stacking of a couple of layers of dry sandy and loamy soils the difference in pressure is in the range from 2 to 15.5 kPa, i.e. even one interface between the layers can keep the pressure of the water column from 0.2 to 1.5 m.

| Soil/Ground        | Shear pressure, Pa |
|--------------------|--------------------|
| Sands              | 1947–3883          |
| Sandy loams        | 3883–5840          |
| Sandy clays        | 5840–17381         |
| Clays              | 17381–29200        |
| Heavy clays        | 22121              |

4. Summary

There was proposed and tested the equation which allows determining the pressure that must be applied to the liquid baffle to move it through the capillary. The results can be used in hydraulic engineering and land reclamation in the design of the objects of water accumulation and inlet, because depending on the technical parameters of the object you can pick up both types of soil for stacking of layers and the required number of layers. Filtration losses, in this case, are minimized and the calculation of multi-layer thin-layer shields gets a theoretical basis.

References

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