Analysis of Mathematical Model on Impact of Unemployment Growth to Crime Rates

A R Soemarsono¹, I Fitria¹, K Nugraheni¹ and N Hanifa¹

¹Department of Mathematics, Institut Teknologi Kalimantan, Balikpapan, Indonesia

*Corresponding Email: annisarahmitas@lecturer.itk.ac.id

Abstract. Unemployment is a condition describes a person doesn’t have a job. People who are available to work but they’ve not found a job is called unemployed. The high unemployment rate is due to the growth of new workforce that is greater than the growth of job openings. The high unemployment rate triggered an increase in crime. Efforts to lower the criminality figures caused by high unemployment are closely related to government policy. The provision of control of government policy is applied to mathematical models of unemployment growth. The impact of unemployment growth against the crime rate can be identified by constructing a new model. Construction of models was done through the addition of a variable described the number of criminals into the initial model of unemployment that has been given control. Stability analysis of the construction of new models is done using Routh-Hurwitz stability criterion. Based on the results of the stability analysis, it is presented that the mathematical model on impact of unemployment growth to crime rates is an asymptotically stable model with the conditions of the parameters fulfilled.

1. Introduction
Every country has many problems. One of the biggest problems is about unemployment. Unemployment is a social and economic problem with specific treatment. Unemployment problem is closely related to labor force influenced by population growth. The difference between demand and supply in the labor market supported a rapidly population growth is a main factor in increasing unemployment rates. Unemployment problem describes crucial social conditions. The community struggles to maintain the level of welfare and minimum consumption [1]. Unemployment affects mental of person. This problem is not limited to the particular person but it has an impact to the whole family and the whole country, especially young generation [2].

The long of unemployment problem can reduce individual efficiency at work. This condition triggered an increase in the number of unemployment. As consequence of this condition, unemployed persons like job seekers and several similar communities become frustrated. They will be motivated to commit crimes either directly or indirectly. Unemployed persons have a big chance to make contact with criminals [3]. Crime is one of the most threatening actions affecting the economy, prosperity, development, social structure, and politics of developed and developing countries. This is a fact presents the relationship between economic conditions, unemployment, and crime [4].

Effort in solving unemployment problem must be encouraged. It needs an intervention from government. Some policies have been implemented to push unemployment growth. This condition is
presented in a mathematical model for optimal control of unemployment problem. In this paper, we start the first step of study with an initial model for unemployment problem from Misra and Singh [5]. In 2016, Munoli and Gani [6] constructed the initial model of Misra and Singh by adding some control variables. Representation of the control variables are opening new job vacancies and providing job vacancies for unemployed. Based on the fact about correlation between unemployment growth and crime rates, we continue the study by defining a new variable on Munoli and Gani’s model. The variable describes crime rates. Then, we analyze stability of the new model using Routh-Hurwitz stability criterion.

2. Methods
In this part, we show a model proposed by Munoli and Gani [6]. By analyzing the model, we can identify the role of government to push unemployment growth through some variable controls added to the model. Furthermore, we present some methods to investigate stability of the model.

2.1. Mathematical Model for Unemployment
Munoli and Gani [6] defined a nonlinear mathematical model for unemployment problem by considering a control from government. In the process of making model, they assumed that all entrants of the category unemployment are fully qualified to do any job at any time \( t \). They described the model as follow.

\[
\begin{align*}
\frac{dU}{dt} &= \Lambda - (1 - u_1)kUV - \alpha_1 U + \gamma E, \\
\frac{dE}{dt} &= (1 - u_1)kUV - \alpha_2 E - \gamma E, \\
\frac{dV}{dt} &= \alpha_2 E + \gamma E - \delta V + u_2 \phi U,
\end{align*}
\]

which

\[
U = \text{Number of unemployed persons at time } t \ (U(t) \geq 0),
\]

\[
E = \text{Number of employed persons at time } t \ (E(t) \geq 0),
\]

\[
V = \text{Number of job vacancies at time } t \ (V(t) \geq 0),
\]

\[
\Lambda = \text{Rate of increase the number of unemployed persons } (\Lambda > 0),
\]

\[
k = \text{Rate of unemployed persons become employed } (k > 0),
\]

\[
\alpha_1 = \text{Rate of migration and death of unemployed person } (\alpha_1 > 0),
\]

\[
\alpha_2 = \text{Rate of retirement and death of employed person } (\alpha_2 > 0),
\]

\[
\gamma = \text{Rate of employed persons become unemployed due to firing } (\gamma > 0),
\]

\[
\phi = \text{Rate of creation of new job vacancies } (\phi > 0),
\]

\[
\delta = \text{Diminution rate of job vacancies due to lack of funds } (\delta > 0),
\]

\[
u_1 = \text{Control representing the provision of job for the unemployed},
\]

\[
u_2 = \text{Control representing the opening of new job vacancies}.
\]

Type of transverse boundary conditions on the model is fixed-final time and free-final state system. The initial condition of model is \( U(0) = U_0, E(0) = E_0, V(0) = V_0 \). It means that \( U_0, E_0, V_0 \) presents an initial condition at time \( t = 0 \) while the condition on final time denoted by \( \lambda_U, \lambda_E, \lambda_V \) with \( \lambda_U(t_f) = \lambda_E(t_f) = \lambda_V(t_f) = 0 \). After presenting the mathematical model for unemployment, we show some steps below to analyze stability of system. The first step is finding an equilibrium point.

2.2. Equilibrium Point
The equilibrium point is a fixed point. It does not change based on the time. Perko [7] described that a point $\bar{x} \in \mathbb{R}^n$ is an equilibrium point of system if $f(\bar{x}) = 0$. Before we identify the stability of system, we need to make the unemployment model be a linear model.

2.3. Linearization

Linearization is the process of transforming a nonlinear system into a linear system. Linearization is carried out to determine the behavior of solution around the equilibrium point. Define the linear system $\dot{x} = f(x,u)$ which $f(x,u)$ is continuously differentiable in a domain $D_x \times D_u \subseteq \mathbb{R}^n \times \mathbb{R}^m$. The purpose of linearization is to approximate the nonlinear system by a linear model $\dot{x} = Ax + Bu$ with $A, B$ are matrices of dimension $n \times n$ and $n \times m$, respectively. The linear model is an accurate approximation to the nonlinear system only in a neighborhood of the point around which the linearization took place. Matrice $A, B$ formed as $A = \frac{\partial f}{\partial x}$ and $B = \frac{\partial f}{\partial u}$. Assume that the pair $(A, B)$ is stabilizable, then there exists a matrice $K \in \mathbb{R}^{m \times n}$ such that the eigenvalues of $A + BK$ are located strictly in the left-half complex plane. The formula is generalized using Jacobian matrice

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial u} \end{bmatrix}. \tag{4}$$

Jacobian matrice can be used to identify the stability of nonlinear system around the equilibrium point $\bar{x}$. Based on Jacobian matrice, we observe dynamic behavior of system by the eigen value.

$$|\lambda I - J| = 0,$$

$$\left| \begin{array}{cc} \lambda - \frac{\partial f_1}{\partial x} & -\frac{\partial f_1}{\partial u} \\ -\frac{\partial f_2}{\partial x} & \lambda - \frac{\partial f_2}{\partial u} \end{array} \right| = 0. \tag{5}$$

System $\frac{dx}{dt} = J(f(\bar{x}))x$ is a linearization on $\bar{x}$ [8]. Then, we analyze stability of system using stability theorem sample characterization equipment was principally used for three types of analysis.

2.4. Stability Analysis

Stability analysis is a process to analyze the transformation of local stability around the equilibrium point. The transformation of local stability can be observed by looking for an approximation around the equilibrium point. Using a theorem presented by Khalil [9], we identify a local stability for unemployment model. Given a differential equation $\dot{x} = Ax$ which $A \in \mathbb{R}^{n \times n}$. The matrice has different characteristic value $\lambda_1, ..., \lambda_k \ (k \leq n)$.

1. The initial point $x = 0$ is asymptotically stable if and only if $\text{Re}(\lambda_i) < 0$ for all $i = 1, ..., k$.
2. The initial point $x = 0$ is stable if and only if $\text{Re}(\lambda_i) \leq 0$ for $i = 1, ..., k$ and for all $\lambda_i$ with $\text{Re}(\lambda_i) = 0$.
3. The initial point $x = 0$ is unstable if only if $\text{Re}(\lambda_i) > 0$ for some $i = 1, ..., k$ or there exist $\lambda_i$ with $\text{Re}(\lambda_i) = 0$.

From the theorem, we need to determine number of system roots. Therefore, we use Descart’s rule.

2.5. Descart’s Rule

If a system has $n$th order, then the characteristic equation denoted by

$$P(\lambda) = \lambda^n + a_1\lambda^{n-1} + \cdots + a_n = 0 \tag{5}$$

which coefficient $a_i = 0, 1, ..., n$ has a real value. Assumed that $a_n \neq 0$. If $a_n = 0$, then $\lambda = 0$ is a solution. Let $N$ is number of sign conversion on coefficient sequence $\{a_n, a_{n-1}, ..., a_0\}$, with ignoring the coefficient’s zero. On Descart’s rule, the maximum number of system roots in equation (5) is $N$.
The system roots have real and positive value. While, the number of positive real roots are $N - 2, N - 4, \ldots$ In case $\omega = -\lambda$ and apply it to Descart's rule, the explanation of negative real roots will be obtained [10]. There is a condition where the real roots of equation are difficult to find. Hence, we implement Routh-Hurwitz criterion to investigate the stability of system.

2.6. Routh-Hurwitz criterion

According to Ogata [11], Routh-Hurwitz stability criterion is implemented to investigate the existence of unstable roots in a polynomial equation. By this method, we don't need to solve the polynomial equation. This stability criterion applies to polynomials with only a finite number of terms. When the criterion is applied to a control system, we can directly find the absolute stability from the coefficients of the characteristic equation. The steps on implementing Routh-Hurwitz stability criterion described as follow:

1. Write the polynomial in $s$ in the following form

$$p(\lambda) = a_0\lambda^n + a_1\lambda^{n-1} + \cdots + a_{n-1}\lambda + a_n = 0 \quad (6)$$

which the coefficients are real quantities. Assumed that $a_n \neq 0$; that is, any zero root has been removed.

2. If any of the coefficients are zero or negative in the presence of at least one positive coefficient, there exist a root or roots that are imaginary or that have positive real parts. Therefore, in such a case, the system is not stable. The necessary but not sufficient condition for stability is that the coefficients of equation $(6)$ all be present and all have a positive sign. If all $a$’s are negative, they can be made positive by multiplying both sides of the equation by $-1$.

3. If all coefficients are positive, arrange the coefficients of the polynomial in rows and columns according to the following pattern:

$$
\begin{array}{c|cccccc}
\lambda^n & a_0 & a_2 & a_4 & a_6 & \cdots \\
\lambda^{n-1} & a_1 & a_3 & a_5 & a_7 & \cdots \\
\lambda^{n-2} & b_1 & b_2 & b_3 & b_4 & \cdots \\
\lambda^{n-3} & c_1 & c_2 & c_3 & c_4 & \cdots \\
\lambda^{n-4} & d_1 & d_2 & d_3 & d_4 & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\
\lambda^2 & e_1 & e_2 & & & & \\
\lambda^1 & f_1 & & & & & \\
\lambda^0 & g_1 & & & & & \\
\end{array}
$$

with $b_1, b_2, b_3, \ldots, c_1, c_2, c_3, \ldots, d_1, d_2, \ldots$ and $g_1$ recursively obtained from

$$
\begin{align*}
b_1 &= \frac{a_1a_2 - a_0a_3}{a_1}, & b_2 &= \frac{a_1a_4 - a_0a_5}{a_1}, & b_3 &= \frac{a_1a_6 - a_0a_7}{a_1}, \\
c_1 &= \frac{b_1a_2 - b_2a_1}{b_1}, & c_2 &= \frac{b_1a_4 - b_2a_1}{b_1}, & c_3 &= \frac{b_1a_6 - b_2a_1}{b_1}, \\
d_1 &= \frac{c_1b_2 - c_2b_1}{c_1}, & d_2 &= \frac{c_1b_4 - c_2b_1}{c_1},
\end{align*}
$$

3. Results and Discussion

The main goal of this paper is doing an investigation to observe the stability of mathematical model on impact of unemployment growth to crime rates. Before we analyze the stability of model. We need to construct the initial model for unemployment described by Munoli and Gani [6]. We add a variable
represents the number of criminals at time $t$. Consequently, we have four main variables in the new model. The behavior of each variable is explained below.

3.1. Construction of Dynamic System Model

The impact of unemployment can reduce productivity and income of the community. Hence, it can cause poverty and other social problems that can lead to crime. Considering the hypothesis, the addition of compartment $C$ adopted from the model written by Sundar [4].

The rate of change in the number of criminals ($C$) can increase or decrease. Increase the number of criminals denoted by $q$. The increase is due to the interaction between unemployed persons and criminals. As consequence, the persons become criminals. This condition is expressed as $\beta UC$. The number of criminals can decrease because the criminals begin to looking for a job. The situation is written as $\varphi C$. Decrease the number of criminals caused by death of criminals. This condition is noted as $\alpha_3 C$. The rate of change in the number of criminals systematically presented by

$$\frac{dC}{dt} = q + \beta UC - \alpha_3 C - \varphi C$$  \hfill (7)

We add equation (7) to Munoli and Gani’s model. Furthermore, we make a compartment diagram describes mathematical model on impact of unemployment growth to crime rates.

Based on Figure 1, we have a new model for unemployment. The model is described below.

$$\frac{dU}{dt} = \Lambda - u_1 kUV - \alpha_1 U + \gamma E - \beta UC + \varphi C,$$  \hfill (8)

$$\frac{dE}{dt} = u_1 kUV - (\alpha_2 + \gamma) E,$$  \hfill (9)

$$\frac{dV}{dt} = (\alpha_2 + \gamma) E - \delta V + u_2 \varphi U,$$  \hfill (10)

$$\frac{dC}{dt} = q + \beta UC - \alpha_3 C - \varphi C.$$  \hfill (11)

Basically, the variables and parameters on the new model represents the same condition with the variables and parameters proposed by Munoli and Gani on equation (1), equation (2), and equation (3). On the new model presented by equation (8) to equation (11), we add a variable ($C$) and some parameters ($q, \beta, \alpha_3, \varphi$) which
\( C \) = Number of criminals at time \( t \) (\( C(t) \geq 0 \)),

\( q \) = Rate of increase the number of criminals (\( q > 0 \)),

\( \beta \) = Rate of unemployed persons have interaction with criminals (\( \beta > 0 \)),

\( \alpha_3 \) = Rate of criminals’s death (\( \alpha_3 > 0 \)),

\( \varphi \) = Rate of persons leaving a crime (\( \varphi > 0 \)).

After constructing the new model of unemployment problem related to crime rates, we find an equilibrium point of the new model.

3.2. Equilibrium Point of New Mathematical Model for Unemployment

Point \( \bar{x} = (U^*, E^*, V^*, C^*) \) is an equilibrium point of model in equation (8) to equation (11) if the conditions below are fulfilled.

\[
\begin{align*}
\Lambda - u_1 k UV - \alpha_1 U + \gamma E - \beta U C + \varphi C &= 0, \quad (12) \\
u_1 k UV - (\alpha_2 + \gamma) E &= 0, \quad (13) \\
(\alpha_2 + \gamma) E - \delta V + u_2 \phi U &= 0, \quad (14) \\
q + \beta U C - \alpha_3 C - \varphi C &= 0. \quad (15)
\end{align*}
\]

Then, we investigate the value of \( \bar{x} \). First, we use equation (15) to obtain the value of \( C \).

\[
C = \frac{q}{\alpha_3 + \varphi - \beta U} \quad (16)
\]

for \( U > 0, \alpha_3 + \varphi - \beta U \neq 0 \) and \( \alpha_3 + \varphi > \beta U \). After that, we determine the value of \( V \) using the sum of equation (13) and equation (14).

\[
V = \frac{u_2 \phi U}{\delta - u_1 k U} \quad (17)
\]

for \( \delta - u_1 k U \neq 0 \) and \( \delta > u_1 k U \). By substituting equation (17) into equation (13), we have the value of \( E \).

\[
E = \frac{u_1 k u_2 \phi U^2}{\alpha_2 \delta + \gamma \delta - u_1 k \alpha_2 U - u_1 k \gamma U} \quad (18)
\]

for \( \alpha_2 \delta + \gamma \delta - u_1 k \alpha_2 U + u_1 k \gamma U \neq 0 \) and \( \alpha_2 \delta + \gamma \delta > u_1 k \alpha_2 U + u_1 k \gamma U \). From equation (12), equation (13), and equation (15), we find the value of \( E \). Sum of the three equations presented as follow.

\[
\Lambda + q - \alpha_1 U - \alpha_2 E - \alpha_3 C = 0 \quad (19)
\]

The next step is substituting equation (16) to equation (18) into equation (19). Therefore, we get

\[
-A_1 U^3 + A_2 U^2 - A_3 U + A_4 = 0 \quad (20)
\]

which
\[
\begin{align*}
A_1 &= \alpha_1 u_1 k \alpha_2 \beta + \alpha_1 \beta u_1 k \gamma - \alpha_2 \beta u_1 k u_2 \phi, \\
A_2 &= \alpha_1 u_1 k \alpha_2 \varphi + \alpha_1 u_1 k \gamma \phi + \alpha_1 \alpha_2 \beta \delta + \alpha_1 \beta \gamma \delta + \alpha_1 u_1 k \alpha_2 \alpha_3 + \alpha_1 \alpha_3 u_1 k \gamma + qu_1 k \alpha_2 \beta
\end{align*}
\]
+\alpha_1 k_\beta y + \Lambda u_1 k_\alpha_2 \beta + \Lambda u_1 k_\beta y - \alpha_2 \alpha_3 u_1 k_\beta y - \alpha_2 \phi u_1 k_\beta y.

A_3 = \begin{align*}
\Delta u_1 k_\alpha_2 \alpha_3 + \Delta u_1 k_\alpha_3 \gamma + \Delta u_1 k\gamma \phi + \Delta u_1 k\gamma \phi + \Delta \alpha_2 \beta \delta + \Delta \beta \gamma \delta + \Delta \phi \gamma \delta.
\alpha_1 k_\alpha_2 \beta + \alpha_1 k_\alpha_3 \gamma + \alpha_1 \gamma \delta + \alpha_1 \gamma \delta + \alpha_1 \gamma \phi + \alpha_1 \gamma \phi.
\end{align*}

A_4 = \begin{align*}
\Delta \alpha_2 \alpha_3 \delta + \Delta \alpha_3 \gamma \delta + \Delta \alpha_2 \beta \delta + \Lambda \gamma \delta \phi + \Delta \alpha_2 \beta \delta + \gamma \delta \phi.
\end{align*}

It is difficult to determine the solution of equation (20). However, we can obtain the number of positive solutions on equation (20) using Descart’s rule. By Descart’s rule, equation (20) has one or three positive solutions for $U$. Consequently, we have a nonnegative equilibrium point of the new model on equation (8) to equation (11):

$$E^* = \frac{u_1 k_\beta y U^2}{\alpha_2 \delta + \gamma \delta - u_1 k_\alpha_2 U^* - u_1 k_\gamma U^*},$$

with $\alpha_2 \delta + \gamma \delta - u_1 k_\alpha_2 U + u_1 k_\gamma U \neq 0$ and $\alpha_2 \delta + \gamma \delta > u_1 k_\alpha_2 U + u_1 k_\gamma U$,

$$V^* = \frac{u_1 k_\beta y U^*}{\delta - u_1 k_\gamma U^*},$$

with $\delta - u_1 k_\gamma U \neq 0$ and $\delta > u_1 k_\gamma U$,

$$C^* = \frac{q}{\alpha_3 + \phi - \beta U^*}$$

with $\alpha_3 + \phi - \beta U \neq 0$ and $\alpha_3 + \phi > \beta U$.

We denote $x = (U^*, E^*, V^*, C^*)$ is a nonnegative solution of the new model for unemployment on equation (8) to equation (11). After that, we need to linearize the new model. From equation (8) to equation (11), we observe that the equations represent nonlinear model.

### 3.3. Linearization of New Mathematical Model for Unemployment

The purpose of linearization is obtain a characteristic equation of the new model for unemployment. We implement Taylor series expansion to linearize equation (8) to equation (11). Taylor series expansion is used around the equilibrium point $\tilde{x}$ until we have a Jacobian matrix described below.

$$J = \begin{bmatrix}
-u_1 kV^* + \alpha_x - \beta U^* & \gamma & -u_1 kU^* & -\beta U^* + \phi \\
u_1 kV^* & -\alpha_x - \gamma & u_1 kU^* & 0 \\
u_2 \phi & \alpha_x + \gamma & -\delta & 0 \\
\beta \phi & 0 & 0 & \beta U^* - \alpha_3 - \phi
\end{bmatrix}.$$ (21)

Let

$$d_1 = u_1 kV^*,$$
$$d_2 = u_1 kU^*,$$
$$d_3 = \beta U^* - \phi,$$
$$d_4 = \alpha_x + \phi - \beta U^*,$$
$$e_1 = d_4 + \alpha_x + \beta C^*,$$
$$e_2 = u_1 k\phi,$$
$$e_3 = \beta C^*,$$
$$e_4 = \alpha_x + \gamma,$$

which $d_1, d_2, d_3, d_4, e_1, e_2, e_3, e_4 > 0$. The Jacobian matrix on equation (21) can be written as

$$J = \begin{bmatrix}
-e_2 & \gamma & -d_2 & -d_3 \\
-d_1 & -e_2 & d_2 & 0 \\
e_2 & e_4 & -\delta & 0 \\
e_3 & 0 & 0 & -d_4
\end{bmatrix}.$$ (22)
The behavior of dynamic system can be identified by eigen value of Jacobian matrice $J$ on equation (22).

$$J = \begin{bmatrix}
-e_1 - \lambda & \gamma & -d_2 & -d_3 \\
-e_4 - \lambda & d_2 & e_5 & 0 \\
e_4 & -\delta - \lambda & 0 & 0 \\
e_5 & 0 & 0 & -d_4 - \lambda
\end{bmatrix}.$$ (23)

Based on Jacobian matrice $J$ on Eq.23, we have a characteristic equation:

$$A = \lambda^4 + e_1 \lambda^3 + e_4 \lambda^3 + \delta \lambda^3 + d_4 e_5 \lambda^2 - d_2 e_5 \lambda^2 + e_4 e_5 \lambda^2 + e_1 \delta \lambda^2 + d_4 e_5 \lambda^2 + d_2 e_5 \lambda^2 - d_1 \gamma \lambda^2 + d_3 d_4 e_5 \lambda + d_3 e_5 \delta \lambda
-d_1 \gamma - d_4 e_5 \delta \lambda + d_1 e_5 \lambda + d_3 e_5 \lambda + d_4 e_5 \lambda + d_4 e_5 \lambda
+d_2 d_3 e_5 \lambda - d_1 d_2 \gamma - d_1 d_2 \gamma - d_1 d_4 e_5 \lambda + d_1 e_5 \lambda + d_2 d_4 e_5 \lambda - d_2 d_3 e_5 \lambda + d_1 d_2 d_4 e_5 \lambda + d_3 d_5 e_5 \lambda.$$ (24)

The equation above can be noted as follow.

$$a_0 \lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0$$

which

$$\begin{align*}
a_0 &= 1, \\
a_1 &= d_4 + \delta + e_1 + e_4, \\
a_2 &= d_2 e_2 + d_3 e_5 + e_1 \delta + e_4 \delta + e_1 e_4 + d_4 \delta + e_4 e_1 + e_4 e_4 - d_1 \gamma - d_2 e_4, \\
a_3 &= d_1 d_2 e_4 + d_2 e_2 e_4 + d_3 e_3 e_4 + e_1 e_4 \delta + d_4 e_1 \delta + d_2 d_4 e_2 + e_1 e_4 \delta + d_4 e_1 e_4 \\
&\quad - d_1 \delta - d_2 e_2 \gamma - d_3 d_4 \gamma - d_2 d_4 e_4 - d_2 e_4 e_4, \\
a_4 &= d_3 e_3 e_4 \delta + d_1 d_2 d_4 e_4 + d_2 d_4 e_2 e_4 + e_4 e_1 e_4 \delta - d_2 d_3 e_3 e_4 - d_3 d_4 e_2 \gamma - d_3 d_4 \gamma \\
&\quad - d_2 d_4 e_4 e_4.
\end{align*}$$

After finding a characteristic equation described by equation (24), we then apply Routh-Hurwitz criterion to determine the stability of new model for unemployment.

### 3.4. Stability Analysis of New Mathematical Model for Unemployment

The condition we need for applying Routh-Hurwitz criterion is each coefficient of the first column has a positive value. A system is unstable if there exist one or more coefficients have negative value. The number of positive roots of characteristic equation is directly proportional to the number of sign change (positive sign change into negative sign) on the first column. Routh-Hurwitz criterion can be implemented to analyze the stability of system using coefficient $a_i$ without calculating the roots of polynomial. Implementation of Routh-Hurwitz criterion for equation (24) is figured below.

$$\begin{array}{c|cccc}
\lambda^4 & a_0 & a_2 & a_4 & 0 \\
\lambda^3 & a_1 & a_3 & 0 & 0 \\
\lambda^2 & b_1 & b_2 & 0 & 0 \\
\lambda^1 & c_1 & 0 & 0 & 0 \\
\lambda^0 & f_1 & 0 & 0 & 0
\end{array}$$

with

$$\begin{align*}
b_1 &= \frac{a_1 a_2 - a_0 a_3}{a_1}, \quad a_1 \neq 0, \\
c_1 &= \frac{b_1 a_3 - b_2 a_1}{b_1}, \quad b_1 \neq 0, \\
f_1 &= \frac{c_1 b_2 - c_2 b_1}{c_1}, \quad c_1 \neq 0.
\end{align*}$$
The equilibrium point of mathematical model on impact of unemployment growth to crime rates is stable if the conditions $a_0, a_1, a_2, a_3, a_4 > 0$, $a_1d_2 > a_0a_3$, and $a_3a_2a_3 > a_3^2 + a_1^2a_4$ are fulfilled. Moreover, each coefficient of the first column must be positive. Using the representation value of $a_0, a_1, a_2, a_3, a_4$ on equation (24), we get:

1. **Equilibrium point condition for $a_1, a_2, a_3, a_4 > 0$.**
   - Coefficient $a_1$ has a positive value ($a_1 > 0$) if $d_4 + \delta + e_1 + e_4 > 0$.
   - Coefficient $a_2$ has a positive value ($a_2 > 0$) if
     \[
     d_2e_2 + d_3e_3 + e_1\delta + e_4\delta + e_1e_4 + d_4\delta + d_4e_1 + d_4e_4 > d_1\gamma + d_2e_4.
     \]
   - Coefficient $a_3$ has a positive value ($a_3 > 0$) if
     \[
     d_1d_2e_4 + d_1d_4\gamma + d_2d_4e_4 + d_2e_2e_4 + d_3e_3\delta + e_1e_4\delta + e_1e_4 + d_4\delta + d_4e_1 + d_4e_4 > d_1\gamma + d_2e_4.
     \]
   - Coefficient $a_4$ has a positive value ($a_4 > 0$) if
     \[
     d_3e_3e_4\delta + d_1d_2d_4e_4 + d_2d_4e_2e_4 + d_4e_1e_4\delta
     > -d_2d_3e_3e_4 - d_2d_4e_2\gamma - d_1d_4\delta - d_2d_4e_4.
     \]

2. **Equilibrium point condition for $a_1, a_2 > a_0, a_3$.**
   \[
   (d_4 + \delta + e_1 + e_4)(d_2e_2 + d_3e_3 + e_1\delta + e_4\delta + e_1e_4 + d_4\delta + d_4e_1 + d_4e_4 - d_1\gamma - d_2e_4).
   \]

3. **Equilibrium point condition for $a_1, a_2 > a_3^2 + a_1^2a_4$.**
   \[
   -d_2e_4)(d_1d_2e_4 + d_2e_2e_4 + d_3e_3\delta + d_1e_4\delta + e_1e_4 + d_4\delta + d_4e_1 + d_4e_4 - d_1\gamma - d_2e_4 - d_2d_4e_4 - d_2e_2e_4).
   \]

4. **Coefficient $b_1 > 0$.**
   The value of $b_1$ is obtained from $b_1 = \frac{a_1a_2^2 - a_0a_3}{a_4}$ which $a_1 > 0$ and $a_1a_2 > a_0a_3$. Therefore, we have $b_1 > 0$.

5. **Coefficient $c_1 > 0$.**
   The value of $c_1$ is obtained from $c_1 = \frac{b_1a_2^2 - b_2a_1}{b_1}$ which $b_1 > 0$ and $b_1a_3 > b_2a_1$. Then, the value of $b_2$ is obtained from $b_2 = \frac{a_1a_2^2 - a_0a_3}{a_4}$ which $a_1 > 0$ and $a_1a_4 > a_0a_5$. Based on Routh-Hurwitz criterion, we have $a_5 > 0$. Hence, we get $a_1a_4 > 0$. Consequently, the condition $b_2 > 0$ is fulfilled. Therefore, we have $c_1 > 0$.

6. **Coefficient $f_1 > 0$.**
   The value of $f_1$ is obtained from $f_1 = \frac{c_1b_2 - c_2b_1}{c_1}$ which $c_1 > 0$ and $c_1b_2 > c_2b_1$. Then, the value of $c_2$ is obtained from $c_2 = \frac{b_1a_2^2 - b_2a_1}{b_1}$ which $b_1 > 0$ and $b_1a_5 > b_2a_1$. Based on Routh-Hurwitz criterion, we have $a_5 > 0$ and $b_3 = 0$. Moreover, we find that $c_2 = 0$. Therefore, we have $c_1b_2 > 0$. Consequently, coefficient $f_1 > 0$.

The value of $a_0, a_1, a_2, a_3, a_4$ is positive. It means that all coefficients on equation (24) is also positive. We have determined that $a_1a_2 > a_0a_3$ and $a_1a_2a_3 > a_3^2 + a_1^2a_4$. In addition, each element of the first column has a positive value. As consequence of the condition, the equilibrium point $\bar{x} = (U^*, E^*, V^*, C^*)$ is asymptotically stable based on Routh-Hurwitz criterion.

Finally, we have presented that the new model on impact of unemployment growth to crime rates is an asymptotically stable model. In the part below, we show some simulation results represents stability of the new model.

### 3.5. Simulation of New Mathematical Model for Unemployment

We do some simulations to clearly see the stability of new mathematical model using MATLAB. We determine the initial values of each variable $U, E, V, C$ noted by $U(0), E(0), V(0), C(0)$. We also
give the value for parameters based on the study of Misra and Singh [5] along Munoli and Gani [6]. Some assumptions we need to establish the stability of new model.

**Table 1.** Parameters of new mathematical model for unemployment

| Parameter | Value     | Reference       | Parameter | Value     | Reference       |
|-----------|-----------|-----------------|-----------|-----------|-----------------|
| \( \Lambda \) | 5000      | Misra and Singh [5] | \( U(0) \) | 10,000    | Munoli and Gani [6] |
| \( \alpha_1 \) | 0.04      | Misra and Singh [5] | \( E(0) \) | 1.000     | Munoli and Gani [6] |
| \( \gamma \) | 0.001     | Munoli and Gani [6] | \( V(0) \) | 100       | Munoli and Gani [6] |
| \( \alpha_2 \) | 0.04      | Misra and Singh [5] | \( C(0) \) | 500       | Assumption     |
| \( \phi \) | 0.007     | Munoli and Gani [6] | \( k \)   | \( 9 \times 10^{-5} \) | Assumption |
| \( \delta \) | 0.05      | Munoli and Gani [6] | \( \alpha_3 \) | 0.04     | Assumption     |
| \( \beta \) | \( 2.2 \times 10^{-6} \) | Assumption | \( q \) | 15        | Assumption     |
|            |           |                  | \( \varphi \) | \( 5 \times 10^{-5} \) | Assumption |

From the value of variables and parameters defined on Table 1, we can observe the number of unemployed persons \( U \) will be stable. Based on Figure 2, the number of unemployed persons have the maximum value around time \( t = 30 \) until \( t = 40 \). The maximum value is over 80,000 persons. The behavior of stability appears after passing time \( t = 50 \). The number of unemployed persons begins to decrease when the graph towards time \( t = 50 \). The number of unemployed persons reaches around 20,000 persons on stable condition.

![Figure 2. Stability on the number of unemployed persons (U)](image)

Considering Figure 3, we have a result that the number of unemployed persons \( U \) is more than the number of employed persons \( E \). This is due to lack of job vacancies \( V \). By the initial condition
\( E(0) = 1000 \), we identify that the number of employed persons decreases near to 0. There are still employed persons after time \( t = 100 \) with minimum value near to 0. The number of employed persons will be stable after passing time \( t = 100 \).

![Figure 3. Stability on the number of employed persons (E)](image)

On Figure 4 below, we can observe that the number of job vacancies (\( V \)) reaches the maximum value in the middle of time \( t = 0 \) and \( t = 50 \). In the time, the number of job vacancies is about 370. The number of job vacancies begins to decrease after passing time \( t = 150 \). The number of employed job vacancies decreases near to 0. Nevertheless, there are still job vacancies after time \( t = 150 \) with minimum value near to 0. The number of job vacancies will be stable after passing time \( t = 150 \).

![Figure 4. Stability on the number of job vacancies (V)](image)
The stability on the number of criminals ($V$) is described by Figure 5. Based on the figure, we obtain that the number of criminals significantly increases until at time $t = 100$. The high number of unemployed persons can trigger an unemployed person to do a crime ($C$). This is one of the factors cause high crime rates. The number of criminals will be stable after passing time $t = 100$. But the number of criminals is still on the maximum value. It is more than 100,000 persons.

![Figure 5. Stability on the number of criminals ($C$)](image)

From the simulations above, we can state that each variables of the new model is stable.

4. Conclusion
In this paper, we have constructed a new mathematical model on impact of unemployment growth to crime rates. By adding a variable represents the number of criminals into the new model, we ensure that the new model is asymptotically stable. Based on the new model, we observe that there is an impact of unemployment growth to crime rates. Unemployment growth is directly proportional to crime rates. The impact of unemployment growth to crime rates can be reduced by optimizing the controls of system.

5. References
[1] Galindro A and Torres D F M 2018 A simple mathematical model for unemployment: A case study in Portugal with optimal control Stat. Optim. Inf. Comput. 6 116–129
[2] Pathan G and Bhathawala P H 2017 A Mathematical Model for Unemployment-Taking an Action without Delay Int. J. Math. its Appl. 12 41–48
[3] Box S and Hale C 1985 Unemployment, imprisonment and prison overcrowding Contemp. Crisis. 9 209-228
[4] Sundar S, Tripathi A, and Naresh R 2018 Does unemployment induce crime in society? a Mathematical study Am. J. Appl. Math. Stat. 6 44–53
[5] Misra A K and Singh A K 2013 A Delay Mathematical Model for the Control of Unemployment Differ. Equations Dyn. Syst. 21 291-307
[6] Munoli S B and Gani S 2016 Optimal control analysis of a mathematical model for unemployment Optim. Control Appl. Methods 37 798-806
[7] Lawrence P 2001 Differential Equations and Dynamical Systems Third Edition, ed J E Marsden,
L Sirovich, and M Golubitsky (New York: Springer) p 555

[8] Packard A, Poolla K, and Horowitz R 2002 Jacobian Linearizations, Equilibrium Points Dyn. Syst. Feed. (University of California, Berkeley) p 169–200

[9] Khalil H K 1996 Nonlinear Systems (New Jersey: Prentice-Hall) p 750

[10] Murray J D 2003 Mathematical Biology biomedical applications Third Edition, ed S S Antman, J E Marsden, L Sirovich, and S Wiggins (New York: Springer-Verlag) p 811

[11] Ogata K 2010 Modern Control Engineering Fifth Edition (New Jersey: Prentice Hall) p 894

Acknowledgments

We thank to Lembaga Penelitian dan Pengabdian kepada Masyarakat Institut Teknologi Kalimantan (LPPM ITK) for supporting our research.