Abstract. We review recent theoretical and phenomenological results on both the perturbative and non-perturbative Odderon. The HERA type of experiments constitutes a direct probe of the Odderon.
1 Introduction

The concept of Odderon was introduced in 1973 [1]. The Odderon is a J-plane singularity near \( J=1 \) in the odd-under-crossing amplitude \( F_- \). It was first formulated in the framework of asymptotic theorems, i.e. rigorous results derived from general principles - analyticity, unitarity and positivity. Its invention was stimulated by the experimental discovery, at ISR in 1972, of increasing total cross-sections.

The Odderon was longtime considered as an heretical concept because of the belief that \( F_- \) is dominated, at high energies, by \( J = 1/2 \) singularities (\( \rho \) and \( \omega \) Regge poles). However, it was rediscovered in 1980 in QCD, where it appears as a compound state of 3 reggeized gluons [2]. From 1990, there were important theoretical developments in perturbative QCD. There are also detailed phenomenological studies of the non-perturbative Odderon [3]. Finally, there is a running experiment at HERA, constituting a direct probe of the Odderon [4].

Of course, the Odderon has a long history. I will discuss here only recent results, some of them presented at the Heidelberg workshop “Pomeron and Odderon in Theory and Experiment” [5].

2 Calculation of the intercept of the perturbative Odderon in QCD

There are two lines of research in this framework:

1. The equivalence, in the multi-colour limit, with the hamiltonian of the completely integrable one-dimensional Heisenberg magnet [6].

2. Variational methods combined with conformal invariant techniques.

In the first line of research, the basic idea is that the Odderon is a C-odd state of 3 reggeized gluons which interact pairwise with a well-defined potential. The problem is to find an operator \( \hat{q}_3 \),

\[
\hat{q}_3^2 = -r_{12}^2 r_{23}^2 r_{31}^2 p_1^2 p_2^2 p_3^2,
\]

which commutes with the Odderon hamiltonian \( H \),

\[
[\hat{q}_3^2, H] = 0
\]

and has a much simpler form than \( H \).
Recently, Janik and Wosiek [7] did find the exact solution of the problem, formulated in terms of the eigenequation
\[ \hat{q}_3 f = q_3 f, \]  
where
\[ f = \left( \frac{\rho_{12} \rho_{13} \rho_{23}}{\rho_{10} \rho_{20} \rho_{30}} \right)^{\mu} \Phi(z), \]  
with \( \mu = h/3 \), \( h \) being the conformal weight. The function \( \Phi \) satisfies a third order linear equation
\[ a(z) \frac{d^3}{dz^3} \Phi(z) + b(z) \frac{d^2}{dz^2} \Phi(z) + c(z) \frac{d}{dz} \Phi(z) + d(z) \Phi(z) = 0, \]  
where
\[ a(z) = z^3(1 - z)^3, \]  
\[ b(z) = 2z^2(1 - z)^2(1 - 2z), \]  
\[ c(z) = z(z - 1) \left( z(z - 1)(3\mu + 2)(\mu - 1) + 3\mu^2 - \mu \right), \]  
\[ d(z) = \mu^2(1 - \mu)(z + 1)(z - 2)(2z - 1) - i\gamma_3 z(1 - z), \]  
\[ z = \frac{\rho_{12} \rho_{30}}{\rho_{10} \rho_{32}}, \rho_k = x_k + iy_k (k = 1, 2, 3), \rho_{ij} = \rho_i - \rho_j. \]

Janik and Wosiek did find the solution
\[ q_3 = -0.20526 i, \]  
 corresponding to the Odderon energy
\[ \epsilon = 0.16478. \]  

The relation between the Odderon energy \( \epsilon \) and the Odderon intercept \( \alpha_O(0) \) is given by the equation
\[ \alpha_O(0) = 1 - (9\alpha_s/2\pi)\epsilon. \]  

For realistic values of \( \alpha_s (\alpha_s \simeq 0.19) \) one gets
The Odderon energy is defined as
\[ \epsilon = E/D. \] (15)

In Eq.(15) D is a normalization constant and the energy functional is given by
\[ E = \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} d\nu \epsilon_n(\nu) |\alpha_n(\nu)|^2, \] (16)
where
\[ \epsilon_n(\nu) = 2\text{Re}[\psi(1 + |n| + i\nu) - \psi(1)], \] (17)
\[ \alpha_n(\nu) = \int_0^\infty dr r^{-2-2i\nu} \int_0^{2\pi} d\phi e^{-i\nu} \left( i\nu + \frac{n + 1}{2} + r e^{i\phi} (h - i\nu - \frac{n - 1}{2}) \right) (i\nu - \frac{n}{2}) \]
\[ (-\tilde{h} + i\nu - \frac{n}{2}) Z(r, \phi), \] (18)
\[ h = 1/2 + n/2 - i\nu, \tilde{h} = 1/2 - n/2 + i\nu, \]
\[ -\infty < \nu < \infty, n = \ldots, -1, 0, 1, \ldots, \] (19)
\[ Z(z, z^*) = |z(1-z)|^{2h/3} \Psi(z, z^*). \] (20)

The function \( \Psi \) in Eq.(20) is invariant under the transformations \( z \rightarrow 1-z \) and \( z \rightarrow 1/z \) (Bose symmetry in the 3 gluons).

The following trial function \( \Psi \) was used in Ref.9:
\[ \Psi = \sum_{k=1}^{N-N_1} c_k a^{k/2-1/6} + \sum_{k=1}^{N_1} d_k a^{k-1/6} \ln a, \] (21)
where
\[ \alpha_O(0) = 0.94. \] (14)
\[ a = \frac{r^2 r_0^2}{(1 + r^2)(1 + r_0^2)(r^2 + r_0^2)}. \]  

(22)

The result is

\[ \epsilon = 0.22269, \]  

(23)

a value which has to be compared with Eq.(12). The corresponding Odderon intercept is

\[ \alpha_O(0) = 0.96. \]  

(24)

By comparing the values (23) and (14), one sees that there is only a 2% difference between the “exact” result and the variational one.

We draw from this section the conclusion that the Odderon intercept is very close to 1, i.e. is much higher than the 1/2 (\(\rho, \omega\)) intercept. We therefore expect important Odderon effects at high energy.

3 Recent phenomenological studies of the non-perturbative Odderon

Kilian and Nachtmann [10] recently shown that the Odderon could induce spectacular effects in the pseudoscalar meson production in ep scattering at high energies, e.g. in the \(p_\perp\) and rapidity distributions for pion production in the photoproduction region. The model used in Ref.10 - a simple Regge pole contribution - is nice as a toy model but is highly unrealistic: a detailed study of \(\rho\) values (\(\rho = ReF/ImF\)) at low and medium energies [3] shows beyond doubt that the Odderon Regge pole has only a very small contribution, at least in the forward direction. The forward and non-forward data in a huge energy range favor the maximal Odderon [3]. On a strictly theoretical level, the Regge singularity in the perturbative QCD is much more complicated than a simple Regge pole and it very difficult to imagine the dynamical mechanism through which such a singularity could be resolved in just a simple Regge pole in the non-perturbative region.

Another recent phenomenological study of the Odderon originates from the Stochastic Vacuum Model (SVM) of Dosch and Simonov [11, 12]. A very interesting connection
can be established in SVM between the Y-shape of the baryon and the coupling of the Odderon. The particular diquark-quark structure of the baryon corresponds to the suppression of the Odderon as a simple Regge pole which, as we already discussed, is not favored by the data. Fortunately, the authors of Ref. 12 are able to make one prediction which is quasi-independent on the singularity structure of the Odderon: a high cross-section (of the order of 300 nb) in the photoproduction of pions with single dissociation (breakup of the target proton). The HERA type of experiments could check this interesting prediction.

4 Odderon and experimental data

A natural difficulty in detecting Odderon effects is the fact that in general the Odderon ($F_-$ amplitude) is mixed with the Pomeron ($F_+$ amplitude). Moreover, the experimental fact that the difference of antihadron-hadron and hadron-hadron total cross-sections $\Delta \sigma \propto \text{Im} F_-$ is much smaller than the total cross-section itself, $\sigma_T \propto \text{Im} F_+$, indicates that the coupling of the Odderon is much smaller than the coupling of the Pomeron. It is therefore not surprising that, till now, there are only two relatively clear experimental indications of the Odderon:

1) the experimental discovery at ISR [13] of a difference between $(d\sigma/dt)_{\bar{p}p}$ and $(d\sigma/dt)_{pp}$ in the dip-shoulder region ($|t| \simeq 1.3 \text{GeV}^2$). This difference is induced by an Odderon with an effective intercept close to 1, a fact which nicely fits the prediction [8, 9] of the perturbative QCD. Moreover, the extrapolation of the maximal Odderon at high $t$ nicely reproduces the existing data [1];

2) the experimental discovery of a bump at very small $t$ ($|t| \simeq 2 \cdot 10^{-3} \text{GeV}^2$) in the high precision $dN/dt \, \bar{p}p$ data [14] at $\sqrt{s} = 541 \text{GeV}$ can be interpreted in terms of oscillations of a very small period, which could be related to unitarity constraints. In their turn, these oscillations induce a high value of the semi-theoretical parameter $\rho$ at $t = 0$, compatible with an important Odderon contribution in the forward direction [15].

However, the most convincing evidence of the Odderon could come from the pseudoscalar meson production at HERA. The H1 experiment [4] offers an unique opportunity of exploring pure $C = -$ channels, without any Pomeron mixing: the Odderon is here in competition only with the photon. The results of the experiment will be available by the beginning of 1999.
5 Conclusions

There is now quite an intense activity concerning both the perturbative and the non-perturbative Odderon. QCD calculations show that the intercept of the perturbative Odderon is very close to 1. The non-perturbative Odderon has also to be visible in the HERA type of experiments.

Acknowledgements

I thank Dr. Pierre Gauron for very useful discussions.

References

[1] L.Lukaszuk and B.Nicolescu, *Nuovo Cimento Letters* 8, 405 (1973).

[2] J.Bartels, *Nucl. Phys.* B 175, 365 (1980); J.Kwiecinski and M.Praszalowicz, *Phys. Lett.* B 94, 413 (1980).

[3] P.Gauron, E.Leader and B.Nicolescu, *Phys. Lett.* B 238, 406 (1990).

[4] H1 Collab., S.Tapprogge et al. in Proc. of the Int. Conf. on the Structure and the Interactions of the Photon (Photon 92), ed.F.C.Erne and A.Bujis (World Scientific, Singapore, 1993).

[5] http://www.thphys.uni-heidelberg.de/ws/

[6] L.N.Lipatov, *JETP Lett.* 59, 571 (1994); L.D.Faddeev and G.P.Korchemsky, *Phys. Lett.* B 342, 311 (1994); G.P.Korchemsky, *Nucl. Phys.* B 443, 255 (1995); *Nucl. Phys.* B 462, 333 (1996).

[7] R.A.Janik and J.Wosiek, [hep-th/9802101](http://arxiv.org/abs/hep-th/9802101); see also L.N.Lipatov, talk given at IPN Orsay, France, March 1997.

[8] P.Gauron, L.N.Lipatov and B.Nicolescu, *Phys. Lett.* B 260, 407 (1991); *Phys. Lett.* B 304, 334 (1993).
[9] M.A.Braun, P.Gauron and B.Nicolescu, hep-ph/9809567.

[10] W.Kilian and O.Nachtmann, Eur.Phys.J. C 5, 317 (1998).

[11] H.G.Dosch and Y.A.Simonov, Phys. Lett. B 205, 339 (1988).

[12] M.Rueter and H.G.Dosch, Phys. Lett. B 380, 177 (1996); M.Rueter, H.G.Dosch and O.Nachtmann, hep-ph/9806342.

[13] A.Breakstone et al., Phys. Rev. Lett. 54, 2180 (1985); S.Erhan et al., Phys. Lett. B 152, 131 (1985).

[14] UA4/2 Coll., C.Augier et al., Phys. Lett. B 316, 448 (1993).

[15] P.Gauron, B.Nicolescu and O.V.Selyugin, Phys. Lett. B 397, 305 (1997).