On the RS2 realization of unparticles

Alexander Friedland, Maurizio Giannotti, Michael Graesser

Theoretical Division, MS B0285, Los Alamos National Laboratory, Los Alamos, NM 87544, USA

Abstract

To facilitate the study of the unparticle scenario it is very desirable to have a treatable model realizing it in four dimensions. Motivated by the general idea of the AdS/CFT correspondence, we consider a simple construction: the Randall-Sundrum 2 (single brane) setup with the Standard Model fields on the brane and a massive vector field in the warped bulk. We show that in this model the known properties of vector unparticles – the nontrivial phase of the CFT propagator, the necessity and dominance of contact interactions, the unitarity constraint on the conformal dimension of the operator, and the tensor structure dictated by conformal symmetry – follow by simple inspection of the brane-to-brane propagator. The phase has a physical interpretation as controlling the rate of escape of unparticles into the extra dimension. Requiring the correct sign for the imaginary part of the longitudinal polarization of the propagator, we obtain the unitarity condition $m_5^2 \geq 0$, which, unlike in the scalar case, is unchanged from flat space. This condition results in the unitarity bound $d_V \geq 3$, or, more generally, $d_V \geq D - 1$ for a vector unparticle in $D$-dimensional space. It is instructive to consider the RS 2 propagator in (Euclidean) position space: at large distances it behaves as a pure CFT propagator, while at short distances it turns into the 5d flat space propagator. The latter is softer than the former, thus regulating the would-be divergences of the spectral integral and turning the “contact” terms seen at low energies into finite-range interactions. Upon Fourier transforming to momentum space, one finds that at low momenta the CFT piece is subdominant to the “contact” interactions.

✩ preprint LA-UR-08-3550
1. Introduction and motivation

In anticipation of the LHC, one would like to explore the broadest range of possible scenarios. Motivated by this, Georgi considered [1, 2] a class of models with a hidden conformal field theory (CFT) sector, which couples to the Standard Model (SM) fields in the ultraviolet (UV). At low energies, this coupling gives rise to effective interactions between the SM and the CFT. As an example, a vector current in the SM could be coupled to a vector operator $O_\mu$ in the CFT via

$$\frac{c_0}{\Lambda^{d-1}} j_\mu^{SM} O_\mu,$$

where $c_0$ is a dimensionless constant and $d$ is the conformal dimension of $O_\mu$, not necessarily an integer.

Models of this type have rather unusual experimental signatures, in that they do not predict a discrete set of new particles, as in commonly considered scenarios of TeV-scale extra dimensions or supersymmetry. Indeed, CFTs do not even have “in” or “out” states. Rather, the new physics comes in the form of something less intuitive, dubbed “unparticle stuff” by Georgi [1, 3].

How can one think about unparticles? Georgi originally envisioned a CFT sector composed of a nonabelian gauge group with a set of new fermion fields. The coupling constants in this sector were assumed to flow into a nontrivial infrared (IR) fixed point, *a la* Banks-Zaks [4], at some scale $\Lambda_{\text{trans}}$, which is above the energies of the experiments, but below the mass scale $M$ of the messenger fields coupling the sector to the SM. This theory shares many features with QCD [5]. As the hidden sector “partons” are produced in a collision of SM particles, they undergo a QCD-like showering process, in which particles eventually hadronize if the theory confines at some scale in the far infrared (IR) [6]. In the limit of exact conformal invariance in the IR, the showering process never stops, hence the theory does not have non-interacting “out” states.

Many properties of unparticles can be inferred from conformal invariance alone, without going into the details of the complicated dynamics. Using such arguments, unparticles were shown to possess several curious properties. When produced in the final state, they behave as a noninteger number of massless particles [1]. When they mediate interactions between SM particles, the corresponding propagator has a nontrivial phase [2]. Both of these properties follow from the spectral representation of the “unparticle propagator”,
which, as argued in [2], by scale invariance has to have the form\(^1\)

\[
\langle \mathcal{O}(p)\mathcal{O}(-p) \rangle \propto \int_0^\infty dM^2 \frac{(M^2)^{d-2}}{p^2 - M^2 + i\epsilon} = \frac{\pi}{\sin d\pi} (p^2)^{d-2} e^{-i(d-2)\pi}. \quad (2)
\]

The integral in this equation converges only when \(1 < d < 2\).

Several other crucial features of unparticles have been pointed out more recently by Grinstein, Intriligator, and Rothstein (GIR) [7]. Using a combination of arguments based on conformal symmetry and on the properties of the Banks-Zaks model (which is at weak 't Hooft coupling), GIR found that:

- The value of \(d\) is limited from below by unitarity [8]. (In the context of unparticle physics, this was also noted in [9].) In particular, for primary, gauge invariant vector CFT operators, one must have \(d_V \geq 3\). Thus, for such operators the interval \(1 < d < 2\) on which the integral in Eq. (2) converges is completely excluded by unitarity.

- The value of \(d\) is not limited from above. For \(d \geq 2\), the unparticle scenario must additionally contain contact interactions between the SM fields. These contact interactions are necessary to cure the divergence in the spectral integral.

- These contact interactions are very important phenomenologically, as they dominate over the unparticles in SM-SM scattering processes.

- The tensor structure of the propagator is fixed by the conformal group [10]. The propagator in general (for \(d_V \neq 3\)) is not transverse, as that would be incompatible with conformal symmetry. This affects the rates for certain processes involving unparticles.

The picture of “unparticle stuff” as an “infinite shower” provides important physical intuition, but the resulting complicated set of QCD-like diagrams is not trivial to deal with, especially at strong coupling. In particular, the properties laid out above look rather mysterious in this framework. Does a noninteger number of massless particles represent something physical in the infinite shower? Since the presence of an imaginary part in a propagator usually indicates some intermediate states go on-shell, what goes on-shell in

\(^1\)For simplicity, we consider the scalar case here. The vector case, originally considered in [2], involves important subtleties, as we shall see later.
the CFT sector, which does not have discrete states of definite mass? Can the unitarity bound on \( d \) and the dominance of the contact terms be understood in simple, intuitive terms? In fact, we note that it took some time to realize the crucial features of unparticles given by GIR.

One may naturally wonder if there exists a different way of thinking about unparticles that is (i) more intuitive and (ii) more treatable than a Yang-Mills hidden sector at strong coupling. A natural candidate to examine are models based on warped extra dimensions. The possibility of describing features of the CFT sector in this framework is strongly suggested by the celebrated AdS/CFT correspondence \([11, 12]\).

The idea has been discussed by several authors, to a varying degree of detail. Already in \([1]\) it is mentioned that scenarios with infinite extra dimensions, as introduced by Randall and Sundrum \([13]\), can have “unparticle-like behavior” (without elaboration). Ref. \([14]\) proposes to realize a scenario of deconstructed unparticles in the two-brane setup. Similar ideas are also discussed in \([15]\). Finally, a recent work by Cacciapaglia, Marandella, and Terning (CMT) \([16]\), containing the most detailed analysis of the problem to date, arrives at several important results, as described later.

There seems to be no consensus, however, on whether such “holographic” (i.e., based on the AdS/CFT correspondence) constructions should be regarded as genuine models of unparticle physics. For example, Ref. \([5]\), which appeared after \([14]\), remarks that “to date no explicit model has been constructed that would exhibit unparticle behavior”. Refs. \([17, 18]\) also do not use AdS constructions to model unparticles. On the other hand, Refs. \([15, 16]\) do refer to their respective AdS-based constructions as unparticle models. Also, Ref. \([18]\) notes that the soft-wall models in the Randall-Sundrum \([13]\) limit should describe unparticle physics. Note that the issue does not reduce to questioning the validity of the AdS/CFT correspondence: a model of unparticles should incorporate not only the scale-invariant sector, but also the breaking of scale invariance at the dimensional transmutation scale and the coupling of the UV theory to the SM \([3]\).

What basic checks can one perform? As a true model, an AdS-based construction must reproduce all of the known properties of unparticles. As an approximate description, on the other hand, it could reproduce some of the properties, but fail on others (\( a \ la \) AdS/QCD \([19]\)). Examining the AdS/unparticle literature, one does not get a clear confirmation that all unparticle properties are present. While, as mentioned, the contact terms are seen on the AdS side (as shown by CMT), the unitarity bounds on the op-
erator dimensions are not seen. Moreover, some discussions seem to imply an upper bound on $d$, in contrast to GIR. Even the phase of the propagator, which uniquely follows from scale invariance [2], does not explicitly appear. Going beyond the unparticle literature, one can find many important results in the earlier studies of the Randall-Sundrum (RS) models and their connection to the AdS/CFT correspondence, e.g., [20, 21, 22, 23, 24, 25]. In particular, properties such as the contact terms [25] and the imaginary part of the propagator [24, 25] were noted. Other questions, however, e.g., the unitarity properties of the RS models do not seem to have been investigated.

In what follows, we consider a very simple scenario: the Randall-Sundrum 2 (RS 2) model where a single positive tension Minkowski brane is sandwiched between two anti-deSitter 5d regions. The SM fields are localized on the brane and a single massive vector field can propagate in the bulk. When considered at scales much below the AdS curvature $\kappa$, this model will be seen to satisfy all of the properties of unparticles listed above: the phase of the CFT propagator, the contact terms and cancellations, the unitarity bounds, and the tensor structure. Moreover, we will see that the model resolves the singularities of the “contact terms”: they correspond to finite range interactions and disappear above the scale $\kappa$. Our analysis also almost trivially leads to the unitarity bounds on the conformal dimension of CFT vector operators in $D$ spacetime dimensions. These and other results are discussed after all of the properties of unparticles are confirmed.

In the interests of clarity, this paper focuses on conceptual points. Detailed derivations will be given in the companion paper [26].

2. Properties of unparticles from the RS 2 realization

As mentioned, we consider a model with a single vector field in the RS 2 bulk and SM fields on the brane. The physical meaning of the unparticle propagator is simple: it is the brane-to-brane propagator of the bulk vector.

The propagator is the Green’s function of the following equation [26] ($a = e^{-\kappa|x_5|}$ denotes the warp factor in the metric, $g_{MN} = diag(a^2 \eta_{\mu\nu}, -1)$):

$$ - (p^2 \eta_{\mu\nu} - p_{\mu}p_{\nu}) A^\mu - \partial_5 \left( a^2 \left[ \eta_{\mu\nu} - \frac{p_{\mu}p_{\nu}}{p^2 - m^2 a^2} \right] \partial_5 A^\mu \right) + m_5^2 a^2 A_\nu = 0 . \quad (3) $$

Two crucial physical ingredients are as follows.

- First, the field has a bulk mass, $m_5$. In AdS/CFT, this mass is known to control the conformal dimension of the resulting vector CFT operator.
With \( m_5 \neq 0 \), the vector field has four degrees of freedom. From the 4-dimensional point of view, it has three transverse polarizations \( (\epsilon_\mu \alpha \rho \sigma \mu = 0) \) and one longitudinal \( (\epsilon_\mu \parallel \rho \sigma) \). The longitudinal component is often omitted in the literature. It is important to keep it: first, it is essential for obtaining the correct tensor structure that respects conformal symmetry; second, it is crucial for our unitarity arguments.

- Second, in deriving the propagator (Green’s function), we need to fix the boundary conditions away from the brane. Physically, collisions of the SM particles on the brane result in outgoing waves of the bulk vector. The corresponding boundary condition is known as the Hartle-Hawking or radiative \([22, 24]\). It is also obtained if one rotates the modified Bessel function solutions giving finite action \([12]\) from Euclidean to Minkowski space.

With these ingredients, one obtains the vector boson propagator

\[
\Delta_{\rho\sigma}(p^2) = \left(-\eta_{\rho\sigma} + \frac{p_\rho p_\sigma}{p^2}\right) \Delta_T(p^2) - \frac{p_\rho p_\sigma}{p^2} \Delta_L(p^2),
\]

(4)

\[
\Delta_T(p^2) = \frac{1}{2} \left[p \frac{H^{(1)}_{\nu-1}(p/\kappa)}{H^{(1)}_\nu(p/\kappa)} - \kappa(\nu - 1)\right]^{-1},
\]

(5)

\[
\Delta_L(p^2) = \frac{1}{2m_5^2} \left[p \frac{H^{(1)}_{\nu-1}(p/\kappa)}{H^{(1)}_\nu(p/\kappa)} - \kappa(\nu + 1)\right].
\]

(6)

where \( p \equiv \sqrt{p^2}, H^{(1)}_\nu(x) \) is the Hankel function of the first kind and

\[
\nu = \sqrt{1 + \frac{m_5^2}{\kappa^2}}.
\]

This propagator is easily obtained by taking the outgoing wave solutions of Eq. (3) in the bulk and imposing matching conditions on the brane (\textit{cf.}, \textit{e.g.}, \[24\]). For the transverse polarizations, the bulk solution is \( c(p)e^{i\nu(x_5)}H^{(1)}_\nu(p/\kappa)|x_5\rangle \), and \( c(p) \) is fixed from the condition \( \partial_5 \Delta_T(p^2, +\epsilon) - \partial_5 \Delta_T(p^2, -\epsilon) = 1 \). For the longitudinal mode, a similar bulk solution exists for the combination \( g(p^2, x_5) \equiv a^2 \partial_5 \Delta_L(p^2, x_5)/(p^2 - m_5^2 a^2) \), with the matching condition \( g(p^2, +\epsilon) - g(p^2, -\epsilon) = -1/m_5^2 \). See \[26\] for details.

The properties of unparticles due to Georgi and GIR listed in the introduction follow from this propagator by simple inspection.
Contact terms: Obviously, Eqs. (4,5,6) do not describe a pure CFT propagator. In a pure CFT, one expects $x^{-2d}$ everywhere, or in momentum space $p^{2d-4}$. The RS 2 propagator, in the $p \to 0$ limit, looks instead like a contact interaction, $\Delta^{(L)}(0) = -\kappa(1 + \nu)/2m_5^2$, $\Delta^{(T)}(0) = -1/2\kappa(\nu - 1)$. Expanding the propagator in series for small $p/k$, we get for the longitudinal part

$$
\Delta^{(L)}(p^2) \simeq \frac{\kappa}{2m_5^2} \left[ -(1 + \nu) + \frac{(p/\kappa)^2}{2(\nu - 1)} + \frac{(p/\kappa)^4}{8(\nu - 1)^2(\nu - 2)} \right.
+ \frac{(p/\kappa)^6}{16(\nu - 1)^3(\nu - 2)(\nu - 3)} + \frac{(5\nu - 11)(p/\kappa)^8}{128(\nu - 1)^4(\nu - 2)^2(\nu - 3)(\nu - 4)} + \cdots
+ \left. \frac{2\pi}{\Gamma(\nu)^2} (i - \cot \pi \nu) \left( \frac{p}{2\kappa} \right)^{2\nu} [1 + \cdots] \right].
$$

The transverse part has a very similar expansion, in the denominator.

The terms in the first and second lines have the structure of contact terms: const, $p^2$, $p^4$, etc. (Corresponding to the Fourier transform of $\delta(x)$, $\partial^2 \delta(x)$, $\partial^4 \delta(x)$, etc.) Assuming for the moment that $\nu > 1$, we see that the nonanalytic terms in the last line are subdominant. As we will see shortly, $\nu \geq 1$ is indeed required, by unitarity. Hence, the dominance of the contact terms is established.

Two important caveats must be mentioned at this point: First, the case $\nu = 1$ involves some subtleties that will be discussed in the companion paper [26]. Here, we will assume $\nu$ is not too close to 1. Second, although we just referred to the terms in the first two lines of Eq. (8) as “contact” terms, this is not strictly accurate. The whole series describes an interaction with a finite range, as will be discussed later.

CFT: We now turn to the last line of Eq. (8). The leading nonanalytic term in the expansion behaves as $p^{2\nu}$, exactly as expected for a CFT with

$$
d_{\nu} = \nu + 2.
$$

In fact, the following general result is well known [27, 28],

$$
(d - p)(d + p - D) = m_5^2/\kappa^2.
$$

For the vector field (a 1-form), $p = 1$; the spacetime dimension on the brane is $D = 4$, indeed yielding Eq. (9), with $\nu$ as in Eq. (7).
To find the leading nonanalytic piece for the transverse mode, we expand in series, assuming $\nu - 1$ is not too small, as already mentioned.

$$\Delta_T(p^2) \simeq \frac{1}{2\kappa} \left[-(\nu - 1) + ... + \frac{2\pi (i - \cot \pi \nu)}{\Gamma(\nu)^2} \left( \frac{p}{2\kappa} \right)^{2\nu} + ... \right]^{-1}$$

$$\simeq \frac{1}{2\kappa} \left[-\frac{1}{\nu - 1} + ... - \frac{1}{(\nu - 1)^2} \frac{2\pi (i - \cot \pi \nu)}{\Gamma(\nu)^2} \left( \frac{p}{2\kappa} \right)^{2\nu} + ... \right]. \quad (11)$$

The RS 2 propagator thus contains the CFT part, both in its transverse and longitudinal components, at a subleading order in $p/\kappa$.

The phase: Notice that while the analytical terms in the expansion of the propagator are purely real, the CFT piece does have both real and imaginary parts. Notice that $i - \cot \pi \nu = -\exp(-i\pi \nu)/\sin \pi \nu$. Since $d = \nu + 2$, the nonanalytic terms, both longitudinal and transverse, have exactly the phase of the unparticle propagator, Eq. (2).

CFT tensor structure: Combining the transverse and longitudinal nonanalytic parts, we see that the correct CFT tensor structure nontrivially emerges (cf. [10]). Explicitly, combining the leading nonanalytic parts from Eqs. (8) and (11) according to Eq. (4) and recalling that $m^2_5 = \kappa^2(\nu^2 - 1)$ from Eq. (7), we get

$$\Delta_{\mu\nu}^{\text{non-analyt}} = C \pi e^{-i\pi \nu} \frac{p^{2\nu}}{\sin \pi \nu} \left( -\eta_{\mu\nu} + \frac{2\nu}{\nu + 1} \frac{p_\mu p_\nu}{p^2} \right), \quad (12)$$

where the overall constant is $C \equiv [(\nu - 1)^2 \Gamma(\nu)^2 2^{2\nu} \kappa^{2\nu} + 1]^{-1}$.

The divergences: Note that $\Delta_{\mu\nu}^{\text{non-analyt}}$ in Eq. (12) becomes singular at integer values of $\nu$ (and hence $d$). On the other hand, the full propagator, being a combination of well defined Bessel functions, is perfectly well defined for any $\nu \geq 1$. Indeed, physically there is nothing special about the values of $m_5$ that yield integer $\nu$. This means the singularities in $\Delta_{\mu\nu}^{\text{non-analyt}}$ must be cancelled by the corresponding singularities in the analytic terms. Indeed, from Eq. (8), we explicitly see how this cancellation takes place: the residues of the poles of the cotangent match the corresponding residues of the poles of the analytic terms.

Thus, the cancellations between the singularities of the “CFT propagator” and the “contact terms” in the RS 2 picture reduce simply to a well known property of the expansion of the Bessel functions. One should not be alarmed that individual terms in the expansions become singular, after all the function
is being expanded around its branch point. We will return to the analytic properties of the propagator later.

Unitarity: The imaginary part of the propagator has a direct physical meaning in the RS 2 setup, as will be discussed later. Namely, it tells us about the width of the “escape into extra dimensions”. This width clearly has to be nonnegative, not to violate unitarity. This must hold for any choice of the wavefunctions that the propagator is sandwiched between. Therefore, it must hold separately for the transverse and longitudinal modes. The relative sign between them can only be correct if

\[ m^2 > 0, \]  

as follows from inspecting Eqs. (5) and (6). It can be shown that this condition is not only necessary, but also sufficient for unitarity \[ 26 \]. Notice that this seemingly trivial condition differs, for example, from the scalar case. A scalar field in the AdS\(_{D+1}\) background can in fact have a negative mass-squared, so long as \[ m^2 / \kappa^2 \geq -D^2 / 4 \] \[ 29, 12 \].

Substituting \( m^2 \) in the definition of \( d \), we find \( d_V \geq 3 \), exactly the condition derived by Mack \[ 8 \]. Eq. (13) in fact turns out to be more general, as discussed later.

3. Discussion

We now discuss several issues in unparticle physics, for which the RS 2 realization provides helpful insights.

Spectral representation as a sum over KK modes: Let us return to the spectral representation, Eq. (2). In the RS 2 model, this equation describes a physical sum over a continuous spectrum of single-particle states, the Kaluza-Klein (KK) modes of the bulk field. In this realization, the term “unparticle stuff” \[ 1, 3 \] gains a simple physical meaning: it is a tower of particles with a continuous spectrum and couplings that scale as a power of mass. The unparticle scenario can then be viewed as a case \[ 30 \] of a more general framework in which fields have continuously distributed mass \[ 31 \].

Notably, Ref. \[ 31 \] specifically proposed to realize a scalar field with continuously distributed mass as a scalar living in a flat five-dimensional spacetime coupling to the SM fields on the brane. The connection between flat extra dimensions and unparticles was also noted in, \( e.g., 32 \).

What the RS 2 construction brings to this picture is a way to control the relative couplings of different KK modes to fields on the brane. The physics
of this is most transparent upon transforming the field equation for the bulk field into the form of the Schrödinger equation. This transformation is discussed in the original RS 2 paper \[13\] and, e.g., in \[33\] (in the context of the RS 2 scenarios extended with additional compact dimensions \[34\]). In the Schrödinger description, the low-energy bulk KK modes have to tunnel through a potential \(V(s) = f s^{-2}\) to reach the brane, where \(f\) is a function of \(m_5/\kappa\). As can be easily seen \[33\], for this particular potential the wavefunctions on the brane are power-law, rather than exponentially, suppressed, leading to a CFT behavior. The power is determined by the function \(f\), and hence by the value of the bulk mass \(m_5\), yielding Eq. (7).

The phase: The presence of the imaginary part in the unparticle propagator also gains a physical meaning in the RS 2 realization. It is clear what goes on-shell in the propagator: collisions of the SM particles on the brane can excite bulk KK modes with the right mass. The unparticles “leak out” into the bulk because of the incomplete gravitational binding (or, equivalently tunnel under the volcano potential). The physical discussion of this escape is given in Refs. \[24, 35\]. This escape corresponds to the unparticle production. Even though in an experiment, it appears as the production of an integer or noninteger number of massless four-dimensional particles (“neutrinos”) \[1\], in the RS 2 model what is produced is a single KK plane wave, which is massive from the four-dimensional point of view. Notice that, once escaped, the bulk particles do not re-interact with the brane fields, unless the extra dimension is compactified (cf. \[14\]).

Notice also that the analytic (“contact”) terms in the expansion of the RS 2 propagator do not have imaginary parts. Physically, this is because they correspond to heavy degrees of freedom, as will be discussed shortly. These heavy degrees of freedom are exchanged far off-shell, without exciting asymptotic outgoing plane waves far from the brane.

Since the analytic terms dominate at low \(p\), the phase of the complete propagator is rather small. The phase factor multiplying the nonanalytic piece, \(e^{-i\pi(d-2)}\), by itself is not particularly meaningful: for example, the fact that it becomes real at integer \(d\) in no way implies that the unparticles do not escape from the brane in that case. All that happens is the \(\cot \pi \nu\) term in Eq. (8) blows up there; this divergence is, however, cancelled by the corresponding analytic term. On the other hand, the imaginary part of the nonanalytic piece is a meaningful quantity, telling us the escape rate \[24\].

In and Out states: Comparing the CFT and the AdS descriptions, one may notice the following apparent “paradox”. The CFT does not have “out”
states: what is produced in a SM-SM collision is a ball of hidden sector quarks and gluons that continues to expand as the shower develops. In contrast, in the RS 2 setup what is produced is a wavepacket propagating in the bulk, which corresponds to a notion of an “out” state. Notice, however, that just like one cannot reverse the development of the shower and use it as the “in” state for producing SM particles, the wavepacket in the RS 2 description, once emitted into the bulk, cannot be accessed from the brane.

Unitarity bounds in $D$ dimensions: Let us now elaborate on the unitarity condition derived in Sect. 2. The bound followed from the requirement that the imaginary part of the longitudinal component of the propagator have the right sign, corresponding to particles escaping from the brane. Observe that our argument at no point relied on having exactly four spacetime dimensions on the brane. This means that $m_5^2 > 0$ is in fact a more general unitarity condition, valid for vector CFT operators in a $D$-dimensional Minkowski spacetime. To convert it into a bound on the dimension of the conformal operator, recall that for general $D$ the relationship between $\nu$ and $m_5^2$ (or directly between $d$ and $m_5^2$) is given in Eq. (10). Therefore, $m_5^2 \geq 0$ gives

$$d_{\text{unit}} \geq D - p = D - 1.$$  \hspace{1cm} (14)

This agrees with the results of [36]. The RS 2 construction thus gives a remarkably simple derivation of this result.

Two more comments should be made. First, the unitarity bound is known to apply to gauge invariant, primary operators. Indeed, our massive vector field does not have any gauge degrees of freedom and is not a derivative of a scalar. Second, the bound on the CFT vector operator comes from imposing the positivity constraint on the coefficient of the correlator of the first descendent operator ($\langle \partial \mu \partial \nu O_\mu(x) \partial \nu O_\nu(0) \rangle$) [7]. This is consistent with the fact that our bound comes from the longitudinal part of the propagator.

$UV$ limit – flat space: Now, let us examine what happens in the limit $p \gg \kappa$. If we also assume $m_5^2 \gg \kappa$, the propagator takes the form [26]

$$\Delta_{\mu\nu}^{\text{flat}}(p^2) = \left( -\eta_{\mu\nu} + \frac{p_\mu p_\nu}{p_4^2} \right) \frac{1}{2} \frac{-i}{p_4^2 - m_5^2} \frac{p_\mu p_\nu}{2 m_5^2} \sqrt{p_4^2 - m_5^2} \quad (15)$$

$$\equiv \left( -\eta_{\mu\nu} + \frac{p_\mu p_\nu}{p_4^2} \right) \Delta_T^{\text{flat}}(p_4^2) - \frac{p_\mu p_\nu}{p_4^2} \Delta_L^{\text{flat}}(p_4^2) \quad \text{ (16)}$$

This is nothing but the propagator of the massive field in flat 5d space. Indeed, the tensor structure of the standard 5d vector propagator, $(-\eta_{MN} +
$P_M P_N (m_5^2)/(P^2 - m_5^2 + i\epsilon)$, can be decomposed into transverse and longitudinal parts according to $-\eta_{\mu\nu} + p_\mu p_\nu/m_5^2 = [-\eta_{\mu\nu} + p_\mu p_\nu/p_5^2] + (p_\mu p_\nu/p_5^2)(p_5^2 - m_5^2)/m_5^2$. Upon integrating over $p_5$, we find Eq. (15).

The propagator in Eq. (15) is seen to be purely imaginary. Physically, this means that in flat space the KK modes can freely escape into extra dimensions. At low momenta, $p \ll \kappa$, gravity provides (incomplete) confinement to the brane; for $p \gg \kappa$, this confinement is negligible.

The RS 2 model thus completes the unparticle scenario in the UV with 5d flat space. This is a different completion from the asymptotically free 4d Yang-Mills theory envisioned by Georgi. Nevertheless, it adequately regulates the theory, as we will see next.

More on flat space: unitarity. Consider now the limit $p \gg m_5$ of Eq. (15). The transverse propagator looks “unparticle-like”, with dimension 3/2, which is just the engineering dimension of the vector field in 5d. It is important that this does not violate the unitarity bound on vector operators in CFT: the bound only applies to gauge invariant operators and in this limit the longitudinal polarization becomes a gauge degree of freedom. (The longitudinal propagator is seen to blow up.) For finite $m_5$, all four polarizations on the brane are physical, but the theory in the flat space limit is not conformal. This example illustrates some sense in which it is possible to consider phenomenological signatures of “vector unparticles” with dimensions below the $d_V \geq 3$ bound: a vector field that can propagate in flat extra dimensions could be interpreted experimentally as a vector unparticle with the “wrong” dimension. A less trivial example of gauge-variant vector unparticles could be provided by models with additional warped compact dimensions [34], in which the “photon-unparticle” decay (photon escape into extra dimensions) is possible [35, 33].

Still more on flat space: no contact terms. One more important observation about the flat space limit needs to be made: the propagator in this limit has a cut, but no contact terms. Comparing with the $p \ll \kappa$ expansion, Eq. (8), we see that “the contact terms” seen at low energies disappear above the scale $\sim \kappa$. This implies that these terms are not fundamental point-like interactions, but in fact have finite range, $\gtrsim \kappa^{-1}$. This also suggests that the

\footnote{It must be mentioned that our treatment here is to the leading order. To better define the theory in the UV, one may want to replace the mass term by the Higgs mechanism and also to deconstruct the 5-dimensional theory (lattice the $x_5$ coordinate) on scales shorter than the AdS curvature.}
flat space UV regime of the RS 2 model softens and regularizes what would otherwise be a divergent behavior of the pure CFT. To understand the implications of this better, let us consider in turn the spectral representation of the RS 2 propagator and its behavior in position space.

**Spectral representation: regularization by flat space.** In light of our observations about the flat space limit, let us reexamine the spectral representation of the unparticle propagator. As already mentioned, the integral in Eq. (2) in the RS 2 model becomes a sum over the KK states. The couplings of the states with masses below the curvature scale grow as \((M^2)^{d/2}\), but the growth is halted at the scale \(\kappa\). Those states instead behave as in the 5-dimensional flat space, meaning that they couple to the brane with equal strengths. Schematically, in the RS 2 model one can write (omitting factors),

\[
\langle O(\vec{p})O(-\vec{p})\rangle \sim \int_0^{\kappa^2} dM^2 \frac{(M^2)^{d/2}}{p^2 - M^2 + i\epsilon} + \int_{\kappa^2}^{\infty} dM^2 \frac{(\kappa^2)^{d/2}}{M^2} \frac{(\kappa^2)^{d/2}}{p^2 - M^2 + i\epsilon}.
\]

In the second integral, the measure of integration \(dM^2/M\) comes from \(dp^5\).

The second integral converges, yielding \(\sim \kappa^{d-2}\) for \(\kappa \gg p\). For \(d < 2\), this becomes infinitesimally small. The upper limit in the first integral can then be extended to infinity, recovering the spectral representation of Eq. (2). Physically, for \(1 < d < 2\) the interactions involving exchange of momentum \(p\) is dominated by modes with masses not much greater than \(\sim p\) and the contribution of the UV tail is negligible. For \(d \geq 2\), on the other hand, the contributions of the heavy states \((M \gg p)\) dominate the integral. The answer in that case is sensitive to the physics in the UV by construction \cite{16} and diverges as the upper integration limit is taken to infinity.

It is physically clear that the interactions dominated by short-distance modes has to look like contact terms at low energies \((p \ll \kappa)\). Indeed, it is easy to show that in the limit \(\kappa \to \infty\) the divergent part of the spectral function is localized to contact terms. For this, we observe that differentiating and integrating back with respect to \(p^2\) drops the \(\delta(x)\) counterterm (constant in momentum space). Differentiating Eq. (2) with respect to \(p^2\) yields an integral that converges for \(d < 3\). Thus, the divergence of the integral for \(2 < d < 3\) is in the additive constant. Similarly, differentiating twice extends the interval of convergence to \(d < 4\). Notice that the improved convergence of the spectral integral upon the subtraction of local terms was shown earlier by CMT \cite{16}.

It is important to stress that the dominant part of the spectral integral becomes local and divergent only in the limit of \(\kappa \to \infty\). For finite \(\kappa\), everything
is regular, as we have seen explicitly in studying the propagator, Eq. (18). The RS 2 model does not require fundamental contact counterterms.

Generalizing the convergence argument above, we can write

$$\int_0^{\kappa^2} \frac{(M^2)^{d-2}}{p^2 - M^2 + i\epsilon} = \frac{\pi}{\sin d\pi} (p^2)^{d-2} e^{-i(d-2)\pi} + a_0 + a_1p^2 + \ldots + a_{[d-2]}(p^2)^{[d-2]} + \cdots,$$

(18)

where \([d]\) denotes the greatest integer less than \(d\) and the coefficients \(a_n\) diverge as \(\kappa^{2([d-2]-n)}\) with the cut-off of the integral. The low-energy expansion of the RS 2 propagator, Eq. (8), has exactly this form.

The argument that for \(d > 2\) the spectral integral is dominated by short-distance physics is quite general and applies to any realization of the unparticle scenario, not just RS 2. All that is necessary is that the integral be somehow regularized above the transmutation scale in the UV. In fact, the argument generalizes to a broader set of models with a continuous spectrum of excitations, not only CFTs.

**Propagator in position space.** An important insight into the physics of the RS 2 model comes from considering the behavior of the propagator in (Euclidean) position space. Fourier transforming the vector propagator, we obtain an expression of the form \(D_{ij}(x) = a(x)\delta_{ij} + b(x)x_i x_j / x^2\). In Fig. 1 for illustration we plot \(a(x)\) for several values of \(\nu\) (top panel). The corresponding plot for \(b(x)\) is qualitatively similar [26]. We also show, for comparison, the position space correlator for a scalar field (bottom panel).

We see that, in both cases, the propagator at short distances takes the form expected in five-dimensional flat space. Moreover, importantly, at long distances it goes into the pure CFT regime. The two regimes appear as power laws (straight lines) in the Figure. The slopes of the lines (the conformal dimensions) at large distances depend on \(\nu\) as in Eq. (7), while at short distances all lines have the same slope, characteristic of the flat 5d space. Importantly, the curves become less steep at short distances: the flat space completion softens the propagator, as already noted.

The transition between CFT and flat space (“transmutation”) for generic \(\nu\) is seen to occur at the scale of the AdS curvature. When \(\nu\) is close to 1 for the vector or 2 for the scalar, i.e., when \(m_5 \ll \kappa\), the transition window becomes extended. The details of the transition are seen to differ for the vector and scalar cases. For the scalar, in the transition window the interaction is dominated by a mode bound to the brane, i.e., the interaction becomes four-
Figure 1: The Euclidean Green’s function of the vector (top) and scalar (bottom) fields in position space. For the vector field, we plot the function $a(x)$, defined by $D_{ij}(x) = a(x)\delta_{ij} + b(x)x_i x_j/x^2$. For simplicity, the AdS curvature $\kappa$ was set to 1, i.e., the distance $x$ is in units of $\kappa^{-1}$. Three different values of $\nu$ are considered, as labeled in the plots. In both cases, the functions exhibit two power law regimes: 5d flat space at short distances ($x < 1$) and the CFT at long distances ($x \gg 1$). The transition behavior is seen to differ for the two: the scalar has a pronounced regime where the localized mode dominates.
Here, we wish to discuss two qualitatively important – if seemingly paradoxical – features seen numerically in Fig. 1: (i) The pure CFT form of the propagator at long distances \( \propto (x^2)^{-2-\nu} \) suggests it is a Fourier transform of \( p^{2\nu} \). Yet, we have seen that in the momentum space expansion (cf. Eqs. (8, 11)) the \( p^{2\nu} \) term is subdominant. (ii) The leading terms in the low-momentum expansion \( (p^0, p^2, ...) \) have the form of contact interactions. Yet, contrary to some discussions in the literature, no contact interactions are seen in position space.

To understand point (i), consider the following mathematical property of Fourier transforms [37]. If a function is analytic everywhere on the real axis, its high frequency Fourier modes are suppressed exponentially. The exponential factor is determined by the distance from the closest singularity to the real axis. When the function has a point of nonanalyticity on the real axis, its high frequency Fourier modes are only power suppressed. Simply put, “sharp features” of the function (discontinuities, cusps, etc) carry the high frequency signal.

In light of this, consider the RS 2 propagator. Schematically, it is the sum of two pieces, one of which \( (p^{2\nu}) \) has a singularity on the real axis (at \( p^2 = 0 \)). This singularity dominates the Fourier transform at large “frequencies”, i.e., large position space distances. That \( p^{2\nu} \) occurs at a subleading order in the small-\( p^2 \) expansion does not change this conclusion.

Now consider point (ii). Observe that to find the small-distance behavior of the correlator we need its large-momentum behavior. Yet, the latter is not obvious from the first several terms of the expansion around \( p^2 = 0 \). Indeed, a series around the origin describes a function only inside a circle up to the nearest singularity. A simple illustration is provided by a massive scalar field in four dimensions: the interaction has a finite range, \( \sim m^{-1} \), even though transforming the small-\( p^2 \) expansion of \( [p^2 + m^2]^{-1} \) term-by-term one would get a series of contact terms \( \partial^{2n}\delta^{(4)}(x)/m^{2n+2} \). The “nonperturbative effect” \( e^{-mx} \) is missed in this series. In our case, the RS 2 propagator is being expanded around its branch point and hence the radius of convergence of the series is zero. The large-momentum behavior of the propagator is instead given in Eq. (15), from which we explicitly get no contact terms.

The position-space picture gives a simple prescription for describing the unparticle propagator in a generic realization. Take the propagator to be of the form \( (g_{\mu\nu} - 2x_\mu x_\nu/x^2)/x^{2d} \) at large distances. This respects the dimensional [24]. The vector behaves differently. These details are beyond the scope of this paper and will be discussed in [26].
conformal symmetry. Now, soften somehow the short-distance core, for $|x| < \Lambda^{-1}$, to make the Fourier transform possible. The resulting interaction at momentum scales $\ll \Lambda$ will be dominated by “contact” terms and will possess all the other unparticle properties discussed in the introduction.

It is curious to note that the behavior of the propagator at both low and high momenta is dictated by the short-distance part of the interaction. Indeed, we have established that the pure CFT interaction at long distances contribute subdominantly to low-energy scattering. We already encountered this while examining the spectral representation: the UV tail dominates the integral. Remarkably, “long distance physics” and “low energy physics” are not the same.

**Effective field theory view.** It is worth mentioning that the last point in no way contradicts the general principles of effective field theory. Indeed, regardless of the scale at which the different parts of the interaction arise, at low energies the interaction is described by a series of effective operators $\sim \Lambda^{-2} J_{SM} J_{SM}, \sim \Lambda^{-4} g_{SM} \partial^2 J_{SM}, ...$, plus the piece $\sim \Lambda^{-2d+2} g_{SM} (p^2)^{(d-2)} J_{SM}$ that comes from the low-mass modes that cannot be integrated out. For $d \geq 3$ it then follows that the contact terms coming from UV dominate in scattering. Notice that this expansion should be directly compared to Eqs. (18) and (8). The latter, in particular, fixes the relative coefficients of all the effective operators, as should happen in any concrete model realizing the unparticle scenario.

**On other realizations and generalizations.** In this paper, we have considered a vector field in the bulk and shown that it possesses all the properties of unparticles. The AdS/CFT conjecture is of course known to be more general, and we believe that our results generalize as well: bulk fields of other spins should also give unparticles of corresponding spin (cf. [16]). This RS 2/unparticle conjecture should be investigated further.

Other realizations of unparticles are certainly possible. Even within the warped extra dimensions framework, the UV regime can be different, as illustrated, e.g., by the Lykken-Randall [38] scenario treated in [26]. Depending on the realization, the relative coefficients between the low-energy effective operators will be different, but the general results following from the effective field theory analysis must be preserved. In particular, the dominance of the contact interactions should persist, by simple power counting.

We close with the following observation. Phenomenologically, one may be interested to consider a broader class of hidden sector models with a continuous or quasi-continuous spectrum. Even though the underlying theory may
not be a strict CFT, it may turn out to look approximately scale invariant in a range of energies accessible to a given experiment (cf. QCD, \[5\]). In such a case, many of the features of unparticles discussed here could still apply. This particularly refers to the properties following from Eq. (18). Whenever the spectral integral is UV-dominated, the “contact terms” will dominate in SM-SM scattering processes. The unparticle phase space and the phase of the CFT piece of the propagator are also derived from the spectral representation and, so long as the latter looks “sufficiently conformal” in the range of energies of the experiment, would follow their unparticle forms. In this sense, they are more robust features than the unitarity bounds, which, as we saw, could be avoided by relaxing strict conformal symmetry, or considering gauge-variant operators.

4. Conclusions

In summary, at energies much below the AdS curvature, the RS 2 model possesses the known properties of unparticle physics. This includes both the CFT features – the unitarity bounds and the tensor structure – and the features originating from the breaking of the CFT in the UV, particularly the dominant “contact” terms. We explicitly see, as observed in \[3\], that the unparticle physics scenario is not only about a scale invariant theory – the CFT breaking and coupling to the SM in the UV are its essential ingredients.

The UV regularization of the CFT by the flat five-dimensional space is different from the Banks-Zaks scenario envisioned in the original work \[1\], nevertheless it is sufficient to control the divergences of the spectral function and guarantee cancellations between the “contact” terms and the CFT parts. At the same time, in the infrared, the CFT is preserved, ensuring that the CFT tensor structure and unitarity bounds are preserved.

The utility of having a concrete, treatable realization of the unparticle scenario is demonstrated by the ease with which the known properties of unparticles are obtained here, and, moreover, extended to strong coupling. We hope that further applications of RS-like models can shine light on other theoretical issues of unparticle physics. Additionally, phenomenology of unparticles continues to be an active area of research (see, e.g., \[39\] for a list of references). It is hoped that having the RS 2 realization will lead to further progress in this area as well.
Acknowledgments

It is a pleasure to acknowledge conversations with T. Bhattacharya, L. Randall, Yu. Shirman, S. Thomas, and L. Vecchi. Preliminary results of this work were presented at seminars at UC Berkeley and Caltech in May 2008 and in more complete form in November 2008 at the Brookhaven Forum 2008. This work was supported by the U.S. Department of Energy at Los Alamos National Laboratory under Contract No. DE-AC52-06NA25396.

References

[1] H. Georgi, Phys. Rev. Lett. 98, 221601 (2007) [arXiv:hep-ph/0703260].
[2] H. Georgi, Phys. Lett. B 650, 275 (2007) [arXiv:0704.2457 [hep-ph]].
[3] H. Georgi and Y. Kats, Phys. Rev. Lett. 101, 131603 (2008) [arXiv:0805.3953 [hep-ph]].
[4] T. Banks and A. Zaks, Nucl. Phys. B 196, 189 (1982).
[5] M. Neubert, Phys. Lett. B 660, 592 (2008) [arXiv:0708.0036 [hep-ph]].
[6] M. J. Strassler and K. M. Zurek, Phys. Lett. B 651, 374 (2007) [arXiv:hep-ph/0604261]; M. J. Strassler, [arXiv:0801.0629 [hep-ph]].
[7] B. Grinstein, K. Intriligator and I. Z. Rothstein, Phys. Lett. B 662, 367 (2008) [arXiv:0801.1140 [hep-ph]].
[8] G. Mack, Commun. Math. Phys. 55, 1 (1977).
[9] Y. Nakayama, Phys. Rev. D 76, 105009 (2007) [arXiv:0707.2451 [hep-ph]].
[10] W. Mueck and K. S. Viswanathan, Phys. Rev. D 58, 106006 (1998) [arXiv:hep-th/9805145].
[11] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/9711200]; S. S. Gubser, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B 428, 105 (1998) [arXiv:hep-th/9802109].
[12] E. Witten, Adv. Theor. Math. Phys. 2, 253 (1998) [arXiv:hep-th/9802150].

[13] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 4690 (1999) [arXiv:hep-th/9906064].

[14] M. A. Stephanov, Phys. Rev. D 76, 035008 (2007) [arXiv:0705.3049 [hep-ph]].

[15] J. P. Lee, arXiv:0710.2797 [hep-ph].

[16] G. Cacciapaglia, G. Marandella and J. Terning, arXiv:0804.0424 [hep-ph].

[17] H. Georgi, Harvard University Physics Colloquium, Apr 14, 2008; http://media.physics.harvard.edu/video/index.php?id=COLLOQ_GEORGI_041408.flv

[18] A. Falkowski and M. Perez-Victoria, JHEP 0812, 107 (2008) [arXiv:0806.1737 [hep-ph]].

[19] C. Csaki, M. Reece and J. Terning, arXiv:0811.3001 [hep-ph].

[20] J. Maldacena, 1999; E. Witten, remarks at ITP Santa Barbara conference, “New Dimensions in Field Theory and String Theory,” http://www.itp.ucsb.edu/online/susy_c99/discussion/

[21] S. S. Gubser, Phys. Rev. D 63, 084017 (2001) [arXiv:hep-th/9912001].

[22] S. B. Giddings, E. Katz and L. Randall, JHEP 0003, 023 (2000) [arXiv:hep-th/0002091].

[23] N. Arkani-Hamed, M. Porrati and L. Randall, JHEP 0108, 017 (2001) [arXiv:hep-th/0101248].

[24] S. L. Dubovsky, V. A. Rubakov and P. G. Tinyakov, Phys. Rev. D 62, 105011 (2000) [arXiv:hep-th/0006046].

[25] M. Perez-Victoria, JHEP 0105, 064 (2001) [arXiv:hep-th/0105048].

[26] A. Friedland, M. Giannotti, M. Graesser, in preparation.

[27] Y. Oz and J. Terning, Nucl. Phys. B 532, 163 (1998) [arXiv:hep-th/9803167].
[28] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. 323, 183 (2000) [arXiv:hep-th/9905111].

[29] P. Breitenlohner and D. Z. Freedman, Phys. Lett. B 115, 197 (1982); Annals Phys. 144, 249 (1982); L. Mezincescu and P. K. Townsend, Annals Phys. 160, 406 (1985).

[30] N. V. Krasnikov, Int. J. Mod. Phys. A 22, 5117 (2007) [arXiv:0707.1419 [hep-ph]].

[31] N. V. Krasnikov, Phys. Lett. B 325, 430 (1994).

[32] K. Cheung, W. Y. Keung and T. C. Yuan, Phys. Rev. D 76, 055003 (2007) [arXiv:0706.3155 [hep-ph]].

[33] A. Friedland and M. Giannotti, Phys. Rev. Lett. 100, 031602 (2008); arXiv:0709.2164 [hep-ph].

[34] A. G. Cohen and D. B. Kaplan, Phys. Lett. B 470, 52 (1999) [arXiv:hep-th/9910132]; R. Gregory, Phys. Rev. Lett. 84, 2564 (2000) [arXiv:hep-th/9911015]; T. Gherghetta and M. E. Shaposhnikov, Phys. Rev. Lett. 85, 240 (2000) [arXiv:hep-th/0004014].

[35] V. A. Rubakov, Phys. Usp. 44, 871 (2001) [Usp. Fiz. Nauk 171, 913 (2001)] [arXiv:hep-ph/0104152].

[36] S. Minwalla, Adv. Theor. Math. Phys. 2, 781 (1998) [arXiv:hep-th/9712074].

[37] A. B. Migdal, Qualitative Methods In Quantum Theory, Front. Phys. 48, 1 (1977).

[38] J. D. Lykken and L. Randall, JHEP 0006, 014 (2000) [arXiv:hep-th/9908076].

[39] K. Cheung, W. Y. Keung and T. C. Yuan, arXiv:0809.0995 [hep-ph].