IDENTIFYING STUDENTS UNDERSTANDING THROUGH PROBLEM POSING

N.B. Nasution¹
¹Pendidikan Matematika, Universitas Pekalongan, Pekalongan
e-mail: nurbaiti.nasution@unikal.ac.id

ABSTRAK

Salah satu konsep yang paling sering muncul dalam matematika adalah konsep mengenai fungsi. Sudah seharusnya jika mahasiswa calon guru memiliki pemahaman menyeluruh mengenai konsep fungsi khususnya fungsi linear dan kuadrat. Artikel ini bertujuan untuk mendeskripsikan suatu pembelajaran menggunakan model problem posing yang dapat digunakan sebagai cara untuk mengidentifikasi pemahaman mahasiswa mengenai konsep fungsi linear dan kuadrat. Selain mendeskripsikan pembelajaran, artikel ini juga membahas mengenai kesalahan konsep mahasiswa yang ditemui pada pembelajaran tersebut. Penelitian ini merupakan penelitian dengan jenis deskriptif kualitatif yang melibatkan sebanyak 27 mahasiswa. Teknik pengumpulan data dilakukan dengan cara observasi dan dokumentasi jawaban mahasiswa. Analisis data dilakukan secara kualitatif. Hasil analisis menunjukkan bahwa dari pembelajaran yang terjadi diketahui bahwa masih terdapat mahasiswa yang miskonsepsi mengenai definisi fungsi dan mengenai beberapa istilah-istilah yang sering muncul dalam pembelajaran konsep fungsi.

Kata kunci: problem posing, fungsi linear dan kuadrat, analisis kesalahan, deskripsi fungsi

Abstract

It is obvious that pre service mathematics teacher should have comprehensive understanding in any mathematics for school concept, including linear and quadratic function. The aim of this article is to describe a classroom activity using problem posing model that can be used as a way to identify students understanding about linear and quadratic function. Moreover, we also discussed about misconceptions found in the process. This research involved as many as 27 students. The data were collected using observation and documentation of students answer sheets. The data were analyzed qualitatively. The results show that there are students who have misconceptions about the definition of the function and about some terms that often appear in the learning.

Keywords: problem posing, quadratic and linear function, error analysis, description of function

BACKGROUND

One of the biggest obstacle in developing education’s quality is the professional competence of the teachers. The professional competence is a competence related to the teacher knowledge about the teaching materials. This competence can be sharpened when they attended the teacher program. Or in other word, this competence can be sharpened when they were still pre-service mathematics teachers. However, until today, many pre-service mathematics teacher still have misconceptions about one or two topics (Ball et al., 2001; Tanisli et al., 2013; Žilková, 2015). In fact, some of the misconceptions are still there when they are graduated. This can lead to another misconceptions for the students (Abdullah et al., 2015; Ang & Shahrill, 2014; Mctavish & Larusson, 2014).

As we already know that teacher's knowledge has a major influence on student achievement (Baumert et al., 2010; Fauth et al., 2019; Kunter et al., 2013). Therefore, any teacher program has a big responsibility to produce a high qualified mathematics teacher. The standard for the mathematics teacher education program must be set high (Alfaro & Joutsenlahti, 2020; Ayalew, 2017). This also applies to Department of Mathematics Education of Universitas Pekalongan. However, Nasution found that many pre-service mathematics teacher in Universitas Pekalongan are still having
difficulties in simple topics such as differentiation and integral of functions (Nasution, 2017).

From the effort of investigating from where the misconceptions begun (Nasution, 2018), it was found that some of the students are incapable of performing a simple manipulation in algebra. Facing this fact, it is necessary to go all the way back and identify students understanding even for the simplest concept like the linear and quadratic function. However, to identify such things, the administration process were not all easy. It was needed a certain time and effort to conduct the test (Lane et al., 2014). Therefore, we tried to design certain activity that can be done in the classroom which can also be used to identify student’s understanding about a certain simple concepts.

The activity was inspired by problem posing model and a quiz show that was once famous called “Apa Ini Apa Itu”. Problem posing is a learning model in which students are obliged to pose problems to their friends about the learning material (Singer et al., 2017). There are many research found that this model can be used to identify students understanding even (Siswono, 2004, 2005). However, in those research, teacher still needs to conduct a certain test or pretest to know students ability. Using the concept of problem posing and the quiz show in this article, we can identify students understanding without conducting a test.

In “Apa Ini Apa Itu” quiz show, the participants are asked to guess a certain thing using 3 hints. The first hint only reveal a little information about the problem. The second hint reveal more informations but still leaves uncertainty about the answer. The last hint (combined with the first two hints) reveals the answer. In our activity, the mystery thing were the formulas of certain linear and quadratic functions. In this article, there are 2 research questions that need to be answered. First is how was the process of the learning activity. Second is what is the identification results.

**METHOD**

This was a descriptive qualitative study that was held in Universitas Pekalongan. The subject of this study is the students of English For Mathematics course in the 2018/2019 academic year. The data was collected by observation and documentation of students answer sheets. The observation was done when implementing the classroom activity, and the documentation was done by collecting the students answer sheets.

There were 3 kinds of answer sheet. They were the first sheet (S1) was filled by the description of the function from each student (more about this are explained below), the second sheet (S2) was filled by the formula of the function, and third sheet (S3) that was filled by the guess of his/her friend description. All of this sheet were graded and the score were compared. The criteria of scoring are written in Table 1.

| Scoring Criteria For | The Criteria                                                                 | Score |
|----------------------|-----------------------------------------------------------------------------|-------|
| **S1**               | All three sentences are formed a well defined formula of the function and the formula fits with S2. | 1     |
|                      | The descriptions are not well defined                                          | 0     |
| **S2**               | The formula fits the description                                              | 1     |
|                      | The formula does not fit the description                                      | 0     |
| **S3**               | He/she can guess his/her friend description correctly.                        | 1     |
|                      | He/she cannot guess his/her friend description                                 | 0     |
The activity was held in English For Mathematics course (Bahasa Inggris Matematika). It is a specific course of which the objective is to teach students about how to get familiar with text about mathematics that was written in English. The course is considered suitable for this study because in the course, students are given many basic concepts about mathematics but using English as the language of instruction.

RESULT AND DISCUSSION

The activity that is created in this study consist of two steps. The first step is asking the student to describe some linear and quadratic functions. The students were given some limitation about the description such as how many sentences they can use, like the ones in “Apa Ini Apa Itu” Quiz. Then, the second step is asking the other student to finding the fo rmula of the function described. This part is the problem posing part. This design was inspired by the classroom activity created by some reserchers (Kullberg et al., 2017; Swan, 2008). It was told teacher can see how much students understanding about a concept when they are forced to create an example about it (Hatisaru, 2020; Telegina et al., 2019). Here, we provide the details about the idea of the classroom activity.

**The first step: Describing the function** In this step, the lecturer asked students to pick any linear and quadratic function. Then, the students are asked to describe both of the functions each in three sentences. Therefore, each students needs to make 6 sentences to describe 2 functions. The characteristics of the sentences used in the description is that the combination of those three sentences must refer to a specific linear or quadratic function. An example of such description for linear function is given in Table 2.

| The Sentence                  |
|-------------------------------|
| First I am a linear function. |
| Second My gradient is 4.       |
| Third My graph passes the point (3,5) |

From Table 2, it can be seen that each sentence can refer to many linear functions. However, if the three of them are put together, they can only refer to a specific linear function which is \( y = 4x - 7 \). In order to get a comprehensive understanding, the students are asked to pay attention from a sentence to a sentence.

From the first sentence, we can see that the function that we are trying to find is a linear function. Hence, we can use formula

\[ y = mx + c \]  \( (1) \)

with \( m \) is the gradient and \( c \) is the constant. Next, from the second sentence, we can see that the gradient is 4. Thus, we can replace the value of \( m \) in \( (1) \) with 4 and become

\[ y = 4x + c \]  \( (2) \)

Last, we use the third sentence to find the value of \( c \). Here we need an easy calculation that gives \(-7\) as the value of \( c \). Therefore, \( (2) \) can be written by

\[ y = 4x - 7 \]

The formula in \( (3) \) shows us the formula of the function we are looking for.

Note that the order of the sentences does play a significant role here. If we only consider the first sentence, there are an infinite number of function that satisfy the description. But if we combine the first and the second sentences, the result will be different. Eventhough, there are still an infinite number of function satisfy the description but the number is definitely smaller then the ones only from the first sentence. Moreover, by putting together the first, second, and the third sentences, particularly in that order, we get a specific
linear function which is \( y = 4x - 7 \). Therefore, the description of the function must consist of three sentences which if we put them together in a certain order, they will refer to a specific function.

Why the description should be limited to 3 sentences? It is because the lecturer wants to challenge the students to create a set of sentences that can describe a function with limited number of sentences. If students were aware that they were being limited to 3 sentences, then they should carefully pick such sentences that can describe the function. Moreover, the students would eventually think about the sufficiency of the description they made. It is hoped that using this way students would enhance their problem solving ability, since one step that is needed in solving problem is knowing the sufficiency of the information given in the problems which can also be interpreted as “understanding the problem” (Schoenfeld, 1987).

**The second step : Finding the formula of the function.** After the students specified the description of the functions, they need to write them down on 2 sheets of paper. The first sheet (S1) contains only the description of the functions and the second sheet (S2) contains only the formula. Next, the students exchange S1 to each other and trying to find the formula from the S1’s they get. The steps they needed to find the formulas and the formulas are written in another different sheet (S3). The S3’s are returned to the owner of the description.

This part of the activity is intended to practice student’s understanding about linear and quadratic function. Since it was made by their classmates, students can get various type of description. Hence, their understanding can be tested. Moreover, lecturer also warn if there were some description that is unsolveable, students should emphasize them on their own S3. In this way, students need to have thorough understanding about the topics. This part is also the significant part in problem posing model. The result of scoring criteria of S1, S2, and S3 are shown in Table 3. In Table 3, it was shown the numbers of students that get certain score in each sheet.

| Linear Function | Quadratic Function |
|-----------------|--------------------|
| S1 27 -         | 25 14              |
| S2 27 -         | 13 14              |
| S3 24 3         | 14 13              |

After the scoring, we analyzed students answer sheet and identify the students understanding based from the sheets. From the analysis result, it was known that all students made well-defined description about linear function. However, the sentences they used were all in the same kind like shown in Table 2. This is pretty disappointing, since the lecturer has taught them many different vocabularies such as x-intercept, y-intercept, slope, etc. Even so, they still use the “almost” same sentence like the one from Table 2. An example of the description from the students is shown in Figure 1.
There was one student who made different description compared to the example. He made the description so carefully that it needed several concepts to find the formula. The description are shown in Figure 2.

It can be seen that the sentences used are not well-structured since he/she used the word “intersects” instead of “intersection”. However, to find the formula of the function described in Figure 2, we need to know how to find the intersection between 2 straight lines (using system of two variables linear equation). Also, we need to know the concept if two lines are perpendicular and how to find the formula of a line that connects 2 points. Therefore, the student who made them has far better understanding about linear function compared to his/her classmates.

Some of the students are asked about why they use the same sentences like the example after the activity. Their response was that it was easier using that way. They did not need to think much and the calculations were considered easy. Another response was that it was easier to follow lecturer steps for finding the formula, but they needed to change the number involved so their description looked different from the one from the lecturer. From these response, we can see that it was not because their weakness in writing sentence in English that caused them writing those descriptions very much similar with the example. It was because they did not want to spend more effort to create description that was more challenging and sophisticated. From this, we can conclude that the students do not have confidence in their own ability. Thus, only from here, we can not conclude anything about their understanding yet.

Later on, we did find some mistakes by student from their S3. Since all of the description were similar (only different in the number used) then the description from other classmates were more like theirs. Hence, it is predictable if they could find the correct formula from others descriptions. However, there were 3 students that made mistakes about the calculations. This mistakes were showns in Figure 3.
Figure 3 shows us that the description made is already well defined. However, the calculation from the one who interpreted the description are wrong. The mistakes lays on the step when finding the constant of $y = 5x + c$. The one who interpreted the description did not understand the meaning of “a graph passes point (1,13)”. If a graph of a function $y = 5x + c$ passes point (1,13) then to find the constant, we should substitute $y$ with 13 and $x$ with 1 on the equation $y = 5x + c$. Student did not realize this substitution because from the example shown in Table 1, lecturer only use the $y$-intercept as the point which is passed by the function. Hence, when they need to find the constant, according to them, they only need to know the ordinat of the point.

From here, we know that students who got description that is different from the example were having trouble detecting whether the description is well defined or trouble finding the formula. The cause is because they did not fully understand the meaning of the terms “graph passes a point”. This is in line with The Newmann’s Error Analysis which says that the first error that can be made by students is understanding the term included in problems (Abdullah et al., 2015; Nasution, 2018).

Another finding was that it some students haven’t fully understand about the definition of function. This was shown in Figure 4.

On the left picture, we can see that the students describe the function by saying that the graph passes on (2,0) and (2,-2). This is clearly a sign that the student does not understand the definition of a function. If it is a function, then it should not have different ordinats for one absis. Then this description is not well defined.

The students who got the description could detect the mistakes made by their friends but it was not straightforward. The answer is shown in the right picture. From the picture, we can see that eventhough eventually the student knew that the description was wrong, he/she did not know it directly. He/she knew after he/she did calculations and found a contradiction, which was $-2 = 0$.

Thus, from here we can see that students still have misconceptions in their understanding about linear and quadratic function, especially about definition of a function. In Newmann’s Theory, this kind of error is included in the comprehension stage (Abdullah et al., 2015; Nasution, 2018).
CONCLUSION AND SUGGESTION

Based on the discussion above, it can be concluded that we can create a certain classroom activity from problem posing model which aims to identifying students understanding about linear and quadratic functions. Furthermore, the identification process shows us that student still have limited understanding about certain terms such as ‘graph passes a point’ and ‘intersects the y-axis’. Also, there are some students that haven’t fully understand about the definition of function.

The activity described above can be used when the teacher/lecturer wants to 1) identify students understanding about linear and quadratic function and 2) create some challenging task for students with a minimum effort since the teacher does not need to come up with the problem. However, from the founding of the study we need to emphasize the concept of linear and quadratic function first. Especially, when it comes to simple concept like the meaning of a graph passes a point and the definition of a function. Students are more likely do not care about something simple such as those concepts. Also, for getting better understanding, lecturer/teacher needs to come up with a lot of example of cases.

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