The Sliding-singlet Mechanism Revived* 

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Abstract 

It is shown, using a modification of an idea of Sen, that completely realistic supersymmetric grand-unified theories based on $SU(6)$ or larger unitary groups can be constructed using the sliding-singlet mechanism. These models have a simple structure, preserve the successful prediction of $\sin^2 \theta_W$, and can suppress Higgsino-mediated proton decay to an acceptable level in a simple way. 

1 Introduction 

The impressive unification of gauge couplings\textsuperscript{1} at a scale of $10^{16}$ GeV in the supersymmetric standard model has led to renewed interest in the idea of supersymmetric grand unification. The main theoretical difficulty with grand unified theories has always been the gauge hierarchy problem,\textsuperscript{2} of which a key aspect is the so-called “doublet-triplet splitting problem”.\textsuperscript{3} This refers to the fact that in grand-unified theories the color-triplet scalar that is in a unified multiplet with the Higgs doublet of the Standard Model must be superheavy to avoid rapid proton decay, while the Higgs doublet itself must have a mass near the Weak-interaction scale. 

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Four interesting and elegant ways to achieve natural doublet-triplet splitting have been proposed. These are the “sliding-singlet mechanism”\(^4\), the “missing partner mechanism”\(^5\), the “Dimopoulos-Wilczek mechanism”\(^6\) (also called the “missing-vacuum-expectation-value mechanism”), and the “GIFT mechanism”\(^7\). Each of these ideas has notable strengths and weaknesses. The missing partner mechanism is the only one which works in \(SU(5)\), the smallest unified gauge group, but requires in \(SU(5)\) the existence of Higgs fields in the high-rank tensor representations \(50, \overline{50}, \) and \(75\).\(^5\) The same mechanism works very elegantly in the flipped \(SU(5) \times U(1)\) group,\(^8\) but as this group is not fully unified the sharp prediction of gauge-coupling unification is lost.

The Dimopoulos-Wilczek mechanism is the only one which works in \(SO(10)\), regarded by many as the most attractive candidate for the grand-unified group. However, for such models to be fully realistic it seems that the Higgs sector must be somewhat involved.\(^9\) The “GIFT” mechanism (in which the Higgs doublet is light because it is a pseudo-goldstone field) solves the doublet-triplet problem in a very simple way in the group \(SU(6)\), but has the disadvantage that the quarks and leptons must get mass in a somewhat complicated fashion.\(^10\)

The first idea mentioned, the sliding-singlet mechanism, is perhaps the prettiest of all, but was shown to have a serious difficulty that prevents it from working in \(SU(5)\). In particular, the gauge hierarchy is destroyed by radiative corrections after supersymmetry breaks.\(^11\) Later A. Sen\(^12\) showed that the sliding-singlet mechanism can work in the group \(SU(6)\) to give a stable hierarchy. A shortcoming of his model, however, in the light of later, more precise measurements of the gauge couplings is that it introduces an intermediate scale into the sequence of gauge-group breaking. \(SU(6)\) breaks to \(SU(3) \times SU(3) \times U(1)\) at a scale of order \(10^{17}\) GeV, and then to \(SU(3)_c \times SU(2)_L \times U(1)_Y\) at an intermediate scale of order \(10^{10}\) GeV. This gives a value of \(\sin^2 \theta_W\) which is \(0.211 \pm 0.003\), or seven standard deviations from the presently measured value of \(0.2324 \pm 0.0003\), and in fact slightly worse than the minimal non-supersymmetric \(SU(5)\) model.

In this letter we show that a simple twist on Sen’s idea allows a fully realistic implementation of the sliding singlet mechanism in \(SU(6)\) and larger unitary groups. Before describing this improvement we will briefly explain the main ideas in the previous development of the sliding-singlet mechanism.
2 The Sliding-singlet Mechanism

(1) The basic idea.

The basic idea of the sliding singlet mechanism as first proposed in $SU(5)$ is based on the existence of terms of the following kind in the Higgs super-potential:

$$W_{2/3} = \mathcal{H} \cdot (\Sigma + S) \cdot H.$$  \hspace{1cm} (1)

Here, $\Sigma$ is an adjoint Higgs field ($SU(24)$, $\mathcal{H}$ and $H$ are an anti-fundamental and fundamental ($SU(5)+5$), and $S$ is a singlet. It is assumed that some other set of terms, $W(\Sigma)$, in the superpotential (there are many possibilities for these terms) gives $\langle \Sigma \rangle = \text{diag}(-\frac{2}{3} \Sigma_0, -\frac{2}{3} \Sigma_0, -\frac{2}{3} \Sigma_0, \Sigma_0, \Sigma_0)$, which breaks $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$. Then the equation, $F_{\mathcal{H}} = \partial W/\partial \mathcal{H} = 0$, which is valid at a supersymmetric minimum, gives $(\langle \Sigma \rangle - \langle S \rangle) \cdot \langle H \rangle = 0$. Since the $SU(2)$-doublets in $\mathcal{H}$ and $H$ are supposed to do the $SU(2) \times U(1)$ breaking of the Standard Model, they have vacuum expectation values which are non-vanishing: $v_1$ and $v_2$, where $|v_1|^2 + |v_2|^2 \equiv v^2 = (246 \text{GeV})^2$. This means that the $F_{\mathcal{H}} = 0$ equation implies that $\langle S \rangle = -\Sigma_0$ and therefore

$$\langle \Sigma \rangle + \langle S \rangle = \text{diag}(-\frac{5}{3} \Sigma_0, -\frac{5}{3} \Sigma_0, -\frac{5}{3} \Sigma_0, 0, 0).$$  \hspace{1cm} (2)

The $F_H = 0$ equation gives the same result. The mechanism receives its name from the fact that the singlet slides to cancel off the expectation value of the adjoint in the $SU(2)$ block. As a result of the form in Eq. (2), the terms in Eq. (1) give superheavy ($\Sigma_0 \approx 10^{16} \text{ GeV}$) mass to the color triplets in $\mathcal{H}$ and $H$, while leaving the doublets massless — the desired $2/3$ splitting.

(2) The stability of the hierarchy

This mechanism breaks down when account is taken of the breaking of supersymmetry.\textsuperscript{11} The potential that makes the singlet “slide” is weak: the terms $|F_{\mathcal{H}}|^2 + |F_H|^2$ give only a mass of order $v^2$ to $S$. The supersymmetry-breaking contributions to the potential of $S$ are of the same order as this and therefore disrupt the cancellation between $\Sigma$ and $S$. More specifically, because $S$ couples to the superheavy triplets in $\mathcal{H}$ and $H$, one-loop tadpole graphs\textsuperscript{11} which have these triplets running around the loop induce in the low-energy effective theory two terms that destroy the gauge hierarchy. These
are \( T_1 = O(m_g^2 M_G)S + H.c. \) and \( T_2 = O(m_g M_G)F_S + H.c. \), where \( M_G \) is the unification scale, and \( m_g \) the gravitino mass, which is of order the weak scale.

The term \( T_1 \), when added to the supersymmetric piece of the potential for \( S \), \( (|\langle \overline{\Pi} \rangle|^2 + |\langle H \rangle|^2) |S + \Sigma_0|^2 \), will evidently shift the expectation value of \( S \) from its supersymmetric minimum at \(-\Sigma_0\) by an amount of order \( m_g^2 M_G / v^2 \sim M_G \), and thus the term in Eq. (1) will give the doublets in \( \overline{\Pi} \) and \( H \) superheavy mass. Moreover, the term \( T_2 \) will (after eliminating the auxiliary field \( F_S \)) give a potential to the doublets of the form \( |\overline{\Pi} H + O(m_g M_G)|^2 \), which also is evidently incompatible with the gauge hierarchy.

(3) The SU(6) Model of Sen

In 1984 A. Sen made the clever observation\(^{12}\) that the sliding-singlet mechanism can be made stable to supersymmetry-breaking radiative effects in groups of larger rank, like SU(6). The essential point is that the expectation values of \( \overline{\Pi} \) and \( H \) that force the singlet to slide can now be those which break SU(6) down to SU(5), which are very large compared to the supersymmetry-breaking scale, rather than those which break SU(2)_L \times U(1)_Y. Thus the supersymmetric part of the potential for the sliding singlet is made more rigid and less subject to disruption by supersymmetry-breaking effects.

The relevant terms have the same form as in Eq. (1), with now, of course the adjoint \( \Sigma \) being a 35 and the fundamentals \( \overline{H} \) and \( H \) being 6 + 6.

Suppose that some terms in the superpotential cause \( \langle \Sigma \rangle \) to have the form \( \langle \Sigma \rangle = \text{diag}(-\Sigma_0, -\Sigma_0, -\Sigma_0, +\Sigma_0, +\Sigma_0, +\Sigma_0) \). Let the VEVs of the standard-model-singlet components of \( \overline{H} \) and \( H \) be \( \langle \overline{H}_6 \rangle = \langle H^6 \rangle = H_0 \gg M_W \). Then, in the supersymmetric limit, the equations \( F_{\overline{H}} = 0 \) and \( F_{\overline{H}} = 0 \) yield the same condition \( \langle S \rangle = -\Sigma_0 \) as before.

With supersymmetry breaking, the addition of the term \( T_1 \) to the potential for \( S \) gives \( V(S) = 2 H_0^2 |S + \Sigma_0|^2 + (O(m_g^2 M_G)S + H.c.) \). This leads to a shift of the VEV of \( S \) from the value \(-\Sigma_0\) by an amount of order \( m_g^2 M_G / H_0^2 \). So that this may be no larger than the Weak scale it is only necessary that \( H_0 \gg \sqrt{m_g M_G} \sim 10^9 \text{ GeV} \). Similarly, the term \( T_2 \) will lead to a contribution to the potential of the form \( |\overline{\Pi} H + O(m_g M_G)|^2 \), as already noted above. This
by itself would induce a VEV for $H$ and $\overline{H}$ of order $\sqrt{m_G}$. But since the ‘6’ components of these fields are assumed to have VEVs this large anyway, the term $T_2$ poses no problem for the gauge hierarchy.

There are, however, three apparent problems that the sliding singlet mechanism faces in $SU(6)$, and it is instructive to see how they are resolved in Sen’s scheme. (a) From Eq. (1) it appears that the $F_S = 0$ equation forces $\langle \overline{H} \rangle = \langle H \rangle = 0$. (b) While the form of $\langle (\Sigma + S) \rangle$ means that it does not contribute to the masses of the $SU(2)$ doublets, the VEVs $\langle H^6 \rangle$ and $\langle \overline{H}_6 \rangle$ do. In particular, one has from Eq. (1): $\overline{H}_i \Sigma_6 \langle H^6 \rangle$ and $\langle H_6 \rangle \Sigma_i H^i$, where $i = 1, 2$ are the $SU(2)$ indices. (c) The term of Eq.(1) makes a contribution to $F_\Sigma \equiv \partial W/\partial \Sigma$ of $\langle H \overline{H} \rangle = \text{diag}(0, 0, 0, 0, 0, H_0^2)$. This creates the danger that the form of the VEV of $\langle \Sigma \rangle$ necessary for the sliding-singlet mechanism to work (specifically, that $\langle \Sigma_i \rangle = \langle \Sigma_6 \rangle$) would be destabilized.

As for (a), in Sen’s model, the $F_S = 0$ equation does indeed imply that $\langle \overline{H} \rangle = \langle H \rangle = 0$ is the correct vacuum in the supersymmetric limit. But, as already mentioned, including supersymmetry-breaking effects gives just the term $|\overline{H}H + O(m_G)|^2$, and leads to $H_0 \sim \sqrt{m_G}$. As we have seen this is large enough for the hierarchy not to be destabilized.

As for (b), in Sen’s model, there are indeed mass terms of order $H_0$ connecting the doublets in the fundamentals with the doublets in the adjoint. But there are also mass terms of order $\Sigma_0 \sim M_G$ connecting the doublets in the adjoints to themselves. Thus by a “see-saw mechanism”, there are doublets which are eigenstates with mass of order $H_0^2/\Sigma_0 \sim m_g \sim M_W$. For details, readers are referred to Ref. (12).

Finally, as for (c), in Sen’s model the contribution of $\langle H \overline{H} \rangle$ to $F_\Sigma$ is only of order $m_g M_G$, and so the form of $\langle \Sigma \rangle$ required for the sliding singlet mechanism to work is only shifted by $O(m_g)$, preserving the hierarchy.

From the foregoing, it is clear that the vacuum expectation values of the $H$ and $\overline{H}$ being at the intermediate scale $m_g M_G \sim 10^9$ GeV rather than at the GUT scale is crucial in the model of Sen. What this means is that at the grand unification scale $SU(6)$ breaks to $SU(3) \times SU(3) \times U(1)$, which then breaks to the standard model group at $10^9$ GeV. The consequence is that $\sin^2 \theta_W$ is predicted to be $0.211 \pm 0.003$, which as noted in the Introduction is far from the presently observed value.
3 A Satisfactory Sliding-singlet Mechanism

We shall now describe an implementation of the sliding-singlet mechanism in $SU(6)$, which incorporates the essential idea of Sen, but in which the unified group breaks all the way to the Standard Model at the unification scale of $10^{16}$ GeV. This idea can be generalized to all $SU(N)$, with $N \geq 6$.

Let the Higgs superpotential of an $SU(6)$ model have the form

$$W = W(\Sigma) + W(\mathcal{H}_A, H_A) + \sum_{A=1,2} \lambda_A \mathcal{H}_A (\Sigma + S_A) h_A + \sum_{A=1,2} \lambda_A h_A (\Sigma + \mathcal{S}_A) H_A.$$  

(3)

$W(\Sigma)$ is some set of terms that has as one of its discrete set of minima

$$\langle \Sigma \rangle = \text{diag}(-\Sigma_0, -\Sigma_0, -\Sigma_0, +\Sigma_0, +\Sigma_0, +\Sigma_0).$$

One possibility is

$$W(\Sigma) = \Sigma^3 + M \lambda \Sigma^2 + X (\Sigma^2 - m^2),$$

where $X$ is a singlet. Another possibility, which is simpler but uses a higher-dimensional operator is

$$W(\Sigma) = \Sigma^4 - M^2 \Sigma^2.$$ 

$W(\mathcal{H}_A, H_A)$ is such as to give the fields $\mathcal{H}_A$ and $H_A$ vacuum expectation values of order $M_G$ pointing in the ‘6’ direction. These fundamentals together with the adjoint break $SU(6)$ all the way down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ at the scale $M_G \sim 10^{16}$ GeV.

The equations $F_{h_A} = 0$ and $F_{\mathcal{H}_A} = 0$ force the singlets $S_A$ and $\mathcal{S}_A$ to slide so that

$$\langle S_A \rangle = \langle \mathcal{S}_A \rangle = -\Sigma_0,$$

as before.

A critical point is that the fields $\mathcal{H}_A$ and $h_A$, which are in the representations $\overline{6} + 6$, have vanishing expectation values in the supersymmetric limit. (As we shall see, they will get expectation values of order $m_g$ when supersymmetry breaks.) This allows a simple resolution of the three potential difficulties mentioned in the last section.

(a) The equation $F_{S_A} = 0$ implies that $\langle \mathcal{H}_A h_A \rangle = 0$ in the supersymmetric limit. But in contrast to Sen’s model, this is here not at all inconsistent with $\mathcal{H}_A$ having an expectation value of order $M_G$, since it can be satisfied by $\langle h_A \rangle = 0$. When supersymmetry-breaking effects are included one has

$$V(S) = | \mathcal{H}_A h_A + O(m_g M_G) |^2.$$ 

This merely induces an expectation value of order $m_g$ in the ‘6’ component of $h_A$. (It also induces a mass-squared term of order $m_g^2 M_G$ connecting the scalar doublets in $\mathcal{H}$ and $h$, but since $\mathcal{H}$ has a mass of order $M_G$, as will be seen shortly, this only gives a contribution of order $m_g^2$ to the light doublet, which turns out to be in $h$.) The same discussion applies, of course to the $F_{\mathcal{H}_A} = 0$ equation.

6
The foregoing also resolves potential difficulty (c). The contribution of the last two terms in Eq. (3) to $F_\Sigma$ is $\sum_A \lambda_A h_A \bar{\Pi}_A + \sum_A \bar{\lambda}_A H_A \bar{h}_A$. But, as in Sen’s model, this is of order $m_g M_G$. The result is that the vacuum expectation value of $\Sigma$ is shifted by order $m_g$, leaving the gauge hierarchy intact.

Potential difficulty (b) was that the expectation values of $\bar{H}_A$ and $H_A$ would give superlarge mass terms that connect doublets in the adjoint $\Sigma$ to those in the fields $h_A$ and $\bar{h}_A$. However, as there is only one doublet with quantum numbers $(1, 2, -\frac{1}{2})$ in the adjoint, only one linear combination of the two doublets with $(1, 2, \frac{1}{2})$ that are contained in $h_1$ and $h_2$ is made superheavy, the orthogonal linear combination being the light Higgs multiplet; and similarly for the conjugate doublets. The situation is made clear by examining the full doublet mass matrix.

$$W_{\text{doublet mass}} = \left( \Sigma_i^6, (\bar{h}_1)_i, (\bar{h}_2)_i, (H_1)_i, (H_2)_i \right).$$

$$\begin{pmatrix}
M_\Sigma & \lambda_1 \langle H_1 \rangle & \lambda_2 \langle H_2 \rangle & 0 & 0 \\
\lambda_1 \langle H_1 \rangle & 0 & 0 & 0 & 0 \\
\lambda_2 \langle H_2 \rangle & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & c |\langle H_2 \rangle|^2 & -c \langle H_1 H_2^* \rangle \\
0 & 0 & 0 & -c \langle H_1^* H_2 \rangle & c |\langle H_1 \rangle|^2 \\
\end{pmatrix} \begin{pmatrix}
\Sigma_i^6 \\
(h_1)^i \\
(h_2)^i \\
(H_1)^i \\
(H_2)^i \\
\end{pmatrix}. \tag{4}$$

Here ‘i’ is an $SU(2)_L$ index. The parameters $M_\Sigma$ and $c$ depend on the details of $W(\Sigma)$ and $W(\bar{H}_A, H_A)$ respectively. $c$ has dimensions of inverse mass, and typically the mass of the $H$ and $\bar{H}$ fields goes as $c |\langle H \rangle|^2$. One sees from the form of the matrix that the goldstone doublets that are eaten in the breaking of $SU(6)$ down to the Standard Model are contained in the $\bar{H}_A$ and $H_A$, while the light doublets that are the Higgs of the Standard model are contained in the $\bar{h}_A$ and $h_A$. It should be noted that due to the shifts caused by supersymmetry-breaking, some of the zeros in Eq. (4) are really non-vanishing and of order $m_g$.

The mass matrix of the color-triplet Higgs is similar in form.
\[ W_{\text{doublet mass}} = \left( \Sigma_0^a, (\overline{h}_1)_a, (\overline{h}_2)_a, (\overline{H}_1)_a, (\overline{H}_2)_a \right). \]

\[
\begin{pmatrix}
0 & \lambda_1 \langle H_1 \rangle & \lambda_2 \langle H_2 \rangle & 0 & 0 \\
\Sigma_0 \langle H_1 \rangle & 0 & 0 & -2 \lambda_1 \Sigma_0 & 0 \\
\Sigma_0 \langle H_2 \rangle & 0 & 0 & 0 & -2 \lambda_2 \Sigma_0 \\
0 & -2 \lambda_1 \Sigma_0 & 0 & c |\langle H_2 \rangle|^2 & -c \langle H_1 H_2 \rangle \\
0 & 0 & -2 \lambda_2 \Sigma_0 & -c \langle H_1^* H_2 \rangle & c |\langle H_1 \rangle|^2 \\
\end{pmatrix}
\begin{pmatrix}
\Sigma_0^a \\
(h_1)^a \\
(h_2)^a \\
(H_1)^a \\
(H_2)^a \\
\end{pmatrix}.
\]

(5)

Here ‘\( a \)’ is a color index. There is only one zero-eigenvalue of this matrix corresponding to the goldstone mode that is eaten in the breaking of SU(6) down to the Standard Model, namely \( (2 \Sigma_0, 0, \langle H_1 \rangle, \langle H_2 \rangle) \). Thus natural doublet-triplet splitting has been achieved.

It is interesting to see how the amplitude for Higgsino-mediated proton decay, which is generally a problem for supersymmetric grand unified theories, depends on the parameters of the model. From the matrices given in Eqs. (4) and (5) it is straightforward to derive that the propagator of the colored Higgsino that mediates proton decay is given by

\[
(M_3)^{-1} = -\frac{c}{4 \Sigma_0^2} \left[ \frac{(\sqrt{\lambda_2 \lambda_1 / \lambda_1 \lambda_1} |\langle H_1 \rangle|^2 - \sqrt{\lambda_1 \lambda_1 / \lambda_2 \lambda_2} |\langle H_2 \rangle|^2)^2}{(|\lambda_1 \langle H_1 \rangle|^2 + |\lambda_2 \langle H_2 \rangle|^2)^{1/2}} \right].
\]

(6)

This is to be compared to the value \((M_3)^{-1} \sim (\Sigma_0)^{-1}\) that one gets in the (fine-tuned) minimal supersymmetric SU(5) model. One sees that there is an extra factor in the Higgsino-mediated proton-decay amplitude which is of order \( M_H / \Sigma_0 \), where we recall that \( M_H \sim c |\langle H \rangle|^2 \). If this factor is of order \( 10^{-1} \) then the Higgsino-mediated proton-decay rate is comfortably suppressed below present bounds.

In this model there are, altogether, in the Higgs sector a \( 35 + 4(6 + \overline{6}) \). (This is compared to a \( 35 + 2(6 + \overline{6}) \) in Sen’s model.) In terms of multiplets of the SU(5) subgroup there are \( 24 + 5(5 + \overline{5}) \) as well as some singlets. One pair of \( 5 + \overline{5} \) gets eaten by the gauge bosons in SU(6)/SU(5) \( \times U(1) \). Thus, the model differs from minimal supersymmetric SU(5), as far as computing \( \sin^2 \theta_W \) is concerned, by the presence of three additional scalar multiplets and one additional gauge multiplet of \( (5 + \overline{5}) \). All of the components of
these extra multiplets are superheavy, and as they are small representations, they have only a minor effect on $\sin^2 \theta_W$. One can show that the shift from the prediction of (fine-tuned) minimal supersymmetric $SU(5)$ is $\Delta \sin^2 \theta_W \approx \frac{3\alpha(M_Z)}{10\pi} \left(5 \ln \left(\frac{\Sigma_0}{\langle H \rangle} \right) + \ln \left(\frac{\Sigma_0}{M_H} \right) \right)$.

If the expectation values of the adjoint and fundamental Higgs fields that break $SU(6)$ are within a factor of three of each other, then the first term in the parentheses gives a typical threshold correction of about $\pm 0.005$. The second term in parentheses is interesting since the argument of the logarithm is essentially the suppression factor of the Higgsino-mediated-proton-decay amplitude. Thus a suppression of the proton decay rate by a factor of $10^{-2}$ below the minimal supersymmetric $SU(5)$ level would give a shift of $\sin^2 \theta_W$ upward by about $0.002$, which is negligible.

The quark and lepton masses can arise in a straightforward way. The down-type quarks and charged leptons can get mass from a $15 \bar{6} 6$ term: $\psi^\alpha \psi^\beta h_{\alpha \beta}$, where we have suppressed flavor indices. This is just the analogue of the $10 \bar{5} 5$ term in minimal $SU(5)$. The up-type quarks (if there is minimal quark and lepton content) get mass from the dimension-5 operator $\psi^{\alpha \beta} \psi^{\gamma \delta} h^{\alpha \beta \gamma \delta \zeta \eta} / M_G$, where again we have suppressed flavor indices. When the $H$ gets an expectation value of order $M_G$ in the ‘6’ direction, this term reduces to the ordinary $10 10 5$ coupling of minimal $SU(5)$. In other words, the quarks and leptons get mass as in a “minimal $SU(6)$ model”. The sliding-singlet mechanism in no way complicates the issue of light fermion masses as it does in the “GIFT” approach.

It can be shown that the gauge hierarchy can be made stable to the effect of higher-dimension operators in the Higgs sector. There are two kinds of operators that must be excluded from the Higgs superpotential. These are $h_A h_B$ terms that would directly give mass to the light doublets, and $\bar{H}_A (\Sigma$ or $S) H_B$ terms, which would create the difficulties (a)–(c) discussed in the last section. It is straightforward to invent discrete or continuous symmetries that forbid these kinds of terms to sufficiently high order in $1/M_P$.

As a final comment on $SU(6)$ it might be asked whether one could not modify Sen’s model in a different way to make it realistic, by simply adding another pair of fundamentals, $H' + \bar{H}'$, which do not couple to the sliding singlets and which have a superpotential that gives them VEVs of order $M_G$ that break $SU(6)$ to $SU(5)$, while the fundamentals that participate in
the sliding-singlet mechanism get VEVs of order $\sqrt{m_g M_G}$. While it may be possible to construct such models, they would face certain difficulties that would almost certainly make them more complicated than the model we have presented. In particular, if the fields $H'$ and $H'$ do not couple to the adjoint or the other fundamentals, there would be unwanted goldstone bosons, while if they do they would tend to destabilize the VEVs of the adjoint or make the VEVs of the other fundamental be of order $M_G$.

The sliding-singlet mechanism in the realistic form described above is immediately generalizable to any $SU(N)$ for $N > 6$.

4 Conclusions

The sliding-singlet is perhaps the most elegant solution to the doublet-triplet-splitting problem of grand-unified theories. We have shown that a perfectly realistic implementation of the mechanism in $SU(6)$ and larger unitary groups can be achieved by a variation on an old idea of A. Sen.

The sliding-singlet mechanism has certain advantages over other approaches that have been proposed. The missing-partner mechanism requires either large representations of Higgs to exist (in $SU(5)$) or an abandonment of the precise and successful prediction of $\sin^2 \theta_W$ (in flipped $SU(5) \times U(1)$). The “GIFT” mechanism makes it difficult to generate quark and lepton masses in a straightforward way. The Dimopoulos-Wilczek mechanism seems to require (at least in $SO(10)$) a somewhat involved Higgs sector (though it is the only mechanism that works in $SO(10)$, which may be the most promising group for grand unification from the point of view of understanding the pattern of quark and lepton masses).

Looked at as a whole, grand unified models based on the sliding-singlet mechanism as implemented here can claim to be the simplest in structure that exist. The Higgs sector requires only a single adjoint and a set of fundamentals and singlets. The prediction of $\sin^2 \theta_W$ is undisturbed by large threshold corrections at the GUT scale, the Higgsino-mediated proton-decay amplitude has automatically an extra factor compared to minimal $SU(5)$ that allows it to be suppressed to an acceptable level in a simple way. Both the Higgs sector and the Yukawa sector are simple in structure. And the hierarchy can be made stable to the effects of higher-dimension operators in straightforward ways.
Note Added: After this work was completed the author became aware of related work of G. Dvali, *Phys. Lett.* **324B**, 59 (1994). Dvali’s viewpoint is different and involves the idea that Higgs doublets are light because they are related to goldstone bosons by a custodial $SU(N)$ symmetry. His Higgs are therefore in $(6,N) + H.c.$ of $SU(6) \times SU(N)$. He is led, however, to a structure similar to Eq. (3) of this paper. (See sec. 7 of Dvali’s paper.)

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