Improved empirical parametrizations of the $\gamma^*N \to N(1535)$ transition amplitudes and the Siegert’s theorem

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Abstract

Some empirical parametrizations of the $\gamma^*N \to N(1535)$ transition amplitudes violate the Siegert’s theorem, that relates the longitudinal and the transverse amplitudes in the pseudo-threshold limit (nucleon and resonance at rest). In the case of the electromagnetic transition from the nucleon (mass $M$) to the resonance $N(1525)$ (mass $M_R$), the Siegert’s theorem is sometimes expressed by the relation $|q|A_{1/2} = \lambda S_{1/2}$ in the pseudo-threshold limit, when the photon momentum $|q|$ vanishes, and $\lambda = \sqrt{2(M_R - M)}$. In this article, we argue that the Siegert’s theorem should be expressed by the relation $A_{1/2} = \lambda S_{1/2}/|q|$, in the limit $|q| \to 0$. This result is a consequence of the relation $S_{1/2} \propto |q|$, when $|q| \to 0$, as suggested by the analysis of the transition form factors and by the orthogonality between the nucleon and $N(1535)$ states. We propose then new empirical parametrizations for the $\gamma^*N \to N(1535)$ helicity amplitudes, that are consistent with the data and the Siegert’s theorem. The proposed parametrizations follow closely the MAID2007 parametrization, except for a small deviation in the amplitudes $A_{1/2}$ and $S_{1/2}$ when $Q^2 < 1.5$ GeV$^2$.

1. Introduction

The information relative to the structure of the electromagnetic transitions between the nucleon and the nucleon excitations ($\gamma^*N \to N^*$) has been parametrized using different forms [1, 2]. The representations in terms of helicity amplitudes, longitudinal and transverse, can be defined independently of the proprieties of the resonances. Alternatively, one can use a representation in terms of structure form factors, that emphasize precisely the symmetries associated with the nucleon resonances. The helicity amplitudes and the structure form factors are functions of the transition four-momentum transfer ($q$) squared, $q^2$, but are often represented in terms of $Q^2 = -q^2$, particularly in nucleon electroexcitation reactions ($Q^2 > 0$). In general the different helicity amplitudes are independent functions, except in some specific limits. The same holds for the form factors.

Taking the case of the nucleon as example: the electric and the magnetic form factors, $G_E$ and $G_M$, are independent functions, except in the threshold limit, $Q^2 = -4M^2$, where $G_E = G_M$ (threshold of the $\gamma^* \to NN$ reaction). In the case of the $\gamma^*N \to N^*$ transitions, there are constraints between helicity amplitudes, or between form factors, at the pseudo-threshold limit. The pseudo-threshold limit is the limit where the photon momentum $|q|$ vanishes, and both particles, the nucleon ($N$) and the resonance, labeled here in general as $R$, are at rest. In the pseudo-threshold $Q^2 = Q^2_{PS} = -(M_R - M)^2$ [3, 4].

The condition that expresses the relation between different amplitudes (or form factors) at the pseudothreshold is usually referred as the Siegert’s theorem. The Siegert’s theorem was introduced first in studies related with nuclear physics [3, 4] and was later used in pion electroproduction reactions [6, 7, 8, 9].

In this work, we study in particular the constraints of the Siegert’s theorem in the $\gamma^*N \to N(1535)$ transition, where $N(1535)$ is a spin $\frac{1}{2}$ state with negative parity ($J^P = \frac{1}{2}^-$). We will show in particular that some parametrizations of the $\gamma^*N \to N(1535)$ transition amplitudes, like the MAID2007 parametrization [8, 9], are not consistent with the Siegert’s theorem. In order to grant that the Siegert’s theorem is valid, one needs to ensure that $S_{1/2} \propto |q|$, near $|q| = 0$. In the present article, we propose then new parametrizations for the amplitudes $A_{1/2}$ and $S_{1/2}$, that are consistent with both, the empirical data and the Siegert’s theorem.

The consequences of the Siegert’s theorem for the $\gamma^*N \to \Delta(1232)$ and $\gamma^*N \to N(1520)$ helicity amplitudes are discussed in a separate article [11].

2. Siegert’s theorem

The parametrization of the current associated with a transition between the nucleon (state $J^P = \frac{1}{2}^-$) and a
$J^p = \frac{1}{2}^-$ resonance can be represented in terms of two form factors, $h_1$ and $h_3$ according with Ref. [4]. At the pseudo-threshold those form factors are related by the condition [4]

$$h_3(Q_{FS}^2) = \frac{M_R - M}{2M_R} h_1(Q_{FS}^2). \quad (1)$$

The functions $h_1, h_3$ can be related with the helicity amplitudes by $h_1 = -\sqrt{2} S_{1/2}/(|q| b)$ and $h_3 = -A_{1/2}/(M_R b)$, where $b = e \sqrt{(M_R + M + Q^2)/(8M_R M_R^2 - M^2)}$ and $e$ is the elementary electric charge. The helicity amplitudes $A_{1/2}$ (transverse) and $S_{1/2}$ (longitudinal) will be defined precisely later [see Eqs. (11)–(12)].

A direct consequence of the Eq. (1) is

$$A_{1/2} = \lambda \frac{S_{1/2}}{\langle q \rangle} \quad (|q| \to 0), \quad (2)$$

where we define

$$\lambda = \sqrt{2}(M_R - M). \quad (3)$$

Note, that, we chose to include the ratio $S_{1/2}/\langle q \rangle$ in the previous relation. In the case $\langle q \rangle = 0$, the factor $S_{1/2}/\langle q \rangle$ is interpreted as the limit $\langle q \rangle \to 0$. This point is important, since it is assumed that $A_{1/2}$ and $S_{1/2}/\langle q \rangle$ have the same order in $\langle q \rangle$, for small values of $\langle q \rangle$. The consequence of this observation is that if $A_{1/2} = O(1)$, meaning that $A_{1/2}$ converges to a constant in the pseudo-threshold limit, one can write also $S_{1/2} = O(\langle q \rangle)$, near $\langle q \rangle = 0$.

In this article, we will assume then, that, the amplitudes $A_{1/2}$ and $S_{1/2}$ behave, near the pseudo-threshold, as

$$A_{1/2} = O(1), \quad S_{1/2} = O(\langle q \rangle). \quad (4)$$

The structure given by Eqs. (4), near the pseudo-threshold can be derived from the analysis of the multipole transition amplitudes $\frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3}$.

In order to understand the meaning of the second relation in (4), we look for the charge density operator, $J^0$ (zero component of the transition current), in the pseudo-threshold limit. The charge operator can be defined in terms of the Dirac ($F_1$) and Pauli ($F_2$) form factors [see Eq. (10)]. When $J^0$ is projected into the spin states, which we represent by $\langle J^0 \rangle$, at the resonance rest frame, one obtains

$$\langle J^0 \rangle = \bar{F}_1 (\bar{u} \gamma_5 u), \quad (5)$$

where $\bar{u} u$ is the Dirac spinor of the resonance (nucleon) and

$$\bar{F}_1 = F_1 + \eta F_2, \quad (6)$$

with

$$\eta = \frac{M_R - M}{M_R + M} \quad (7)$$

In the case where the initial and final state have the same spin projection, we can conclude, that, in the pseudo-threshold limit at the $R$ rest frame: $(\bar{u} R \gamma_5 u) \propto \langle q \rangle$. Thus

$$\langle J^0 \rangle \propto \bar{F}_1 |q| \quad (8)$$

The previous condition defines the orthogonality between the nucleon and the resonance states when $\langle J^0 \rangle \to 0$, which implies that $\bar{F}_1 = O(1)$, $(\bar{F}_1 \to$ constant) or that $\bar{F}_1$ scales with some power of $|q|$, in the pseudo-threshold limit. The orthogonality between states at the pseudo-threshold generalizes the nonrelativistic definition of orthogonality between states with different masses when the recoil (and the mass difference) is neglected ($Q^2 = -q^2 = 0$).

Since the amplitude $S_{1/2}$ can also be defined by $J^0$, assuming current conservation [4], in the cases where the spin projections are conserved (photon with zero spin projection), we can also write $\langle J^0 \rangle \propto S_{1/2}$. Combined this result with the result (8), we conclude, that, the orthogonality between the states, defined at the pseudo-threshold, implies

$$S_{1/2} \propto \bar{F}_1 |q|. \quad (9)$$

In the following, we will also show that the first condition in (4), $A_{1/2} = O(1)$, implies that $\bar{F}_1 = O(1)$. Therefore, the combination of the result (9) and $A_{1/2} \propto \bar{F}_1$, is compatible with the Siegert’s theorem (2), apart from normalization factors. To prove the relation (9), we need to look for the explicit parametrization of the amplitudes $A_{1/2}$ and $S_{1/2}$.

We introduce next the formalism associated with the electromagnetic transition current, the electromagnetic form factors and the helicity amplitudes in the $\gamma N \to N(1535)$ transition. Later, we discuss the implications of the Siegert’s theorem in the structure of the transition form factors.

3. $\gamma^* N \to N(1535)$ transition

The $\gamma^* N \to N(1535)$ transition can be represented, omitting the asymptotic states, in the units of the elementary electric charge $e$, as $\frac{2}{3} \frac{2}{3} \frac{2}{3} \frac{2}{3}$.

$$J^\mu = \bar{F}_1 (Q^2) \left( \gamma^\mu - \frac{\partial q^\mu}{q^2} \right) \gamma_5 + F_2(Q^2) \frac{ig_{\mu\nu} q_\nu}{M_R + M} \gamma_5. \quad (10)$$

\[1\]$In the case of current conservation, the amplitude $S_{1/2}$ can be calculated using the operator $(\bar{e}_i \cdot J) |q|/Q$, as in Eq. (12), or using the operator $J^\mu$.\]
where \( F_1 \) and \( F_2 \) are respectively the Dirac and Pauli form factors, as mentioned before. Given the structure of Eq. (10), we can ensure, that, both components of the current, the Dirac and the Pauli terms, are conserved separately.

### 3.1. Helicity amplitudes (at the R rest frame)

Since the transition \( \gamma^* N \rightarrow N(1535) \) correspond to a transition between two states with spin \( \frac{1}{2} \) (transition \( \frac{1}{2}^+ \rightarrow \frac{1}{2}^- \)), there are only two helicity amplitudes to be considered, the transverse \( (A_{1/2}) \) and the longitudinal \( (S_{1/2}) \) amplitudes. Those amplitudes are defined, at the resonance rest frame, as follows [2]:

\[
A_{1/2} = \sqrt{2\pi\alpha K} \langle R, +\frac{1}{2} | e_+ \cdot J | N, -\frac{1}{2} \rangle,
\]

\[
S_{1/2} = \sqrt{2\pi\alpha K} \langle R, +\frac{1}{2} | e_0 \cdot J | N, +\frac{1}{2} \rangle \langle q | O \rangle Q,
\]

where \( Q = \sqrt{Q^2} \) (assuming that \( Q^2 > 0 \)), as before \( |q| \) is the photon (and nucleon) momentum, and \( e_\lambda \) \((\lambda = 0, +)\) is the photon polarization vector. The momentum \( |q| \) is determined by

\[
|q| = \sqrt{Q^2 + Q^2} = \frac{\sqrt{Q^2 \cdot Q^2}}{2MR},
\]

where \( Q^2 = (M_R \pm M)^2 + Q^2 \).

Based on the current (10), we can write the amplitudes [2][12][13][14], as

\[
A_{1/2}(Q^2) = 2b\tilde{F}_1(Q^2),
\]

\[
S_{1/2}(Q^2) = -\sqrt{2b} (M_R - M) \frac{|q|}{Q^2} \times \left[ \tilde{F}_1(Q^2) - \frac{4M^2_J |q|^2}{(M_R - M)^2 Q^2} F_2(Q^2) \right],
\]

where \( b = e \sqrt{\frac{Q^2}{3M^2_R - M^2}} \), as before, and \( \tilde{F}_1 \) is defined by Eq. (6). The factor \( e \) appears because the current \( J^\mu \) is defined in units of the elementary electric charge.

In Eq. (14) we decompose the amplitude \( S_{1/2} \) into a \( F_1 \) term and a term in \( |q|^2 \), in order to facilitate the following discussion.

Based on Eqs. (14)-(15), we can conclude that the term \( |q|^2 F_2 \) can be dropped in comparison with \( \tilde{F}_1 \), we obtain immediately the Siegert’s theorem condition, since

\[
A_{1/2} = 2b\tilde{F}_1, \quad S_{1/2} = \sqrt{2b} \frac{|q|}{M_R - M} \tilde{F}_1,
\]

in the pseudo-threshold limit, \( Q^2 \rightarrow -(M_R - M)^2 \).

We look now for the results of the MAID2007 parametrization. The results for the amplitude \( S_{1/2} \) and \( A_{1/2} |q|/\lambda \) are presented in the Fig. 1. One can note in the figure, that \( |q|A_{1/2} \neq \lambda S_{1/2} \), since the functions differ at the pseudo-threshold, \( Q^2 = Q^2_{PS} \approx -0.36 \text{ GeV}^2 \), when we start to draw the lines.

From Eqs. (14)-(15), we can conjecture, that, the deviation from the Siegert’s theorem condition (2) in the MAID2007 parametrization, may be a consequence of the dependence on \( |q| \) of the function \( F_2 \), when \( |q| \rightarrow 0 \). Since we know from Eq. (14), that \( \tilde{F}_1 = O(1) \) (because \( \tilde{F}_1 \) goes to a constant), when \( |q| \rightarrow 0 \), we may conjecture that \( F_2 \propto 1/|q|^3 \), in order to obtain \( S_{1/2} = O(1) \) in the MAID2007 parametrization. In the conditions of the Siegert’s theorem (2), however, we expect \( F_2 = O(1/|q|^{3-n}) \) with \( n \geq 1 \).

### 3.2. Form factors

We turn now for the analysis of the transition form factors. The transition form factors \( F_1 \) and \( F_2 \) can be determined inverting Eqs. (13)-(15). The results are

\[
F_1 = \frac{1}{2b} \frac{(M_R - M)^2 Q^2}{4M^2} \left[ A_{1/2} - \lambda \frac{S_{1/2}}{|q|} \right] + \frac{1}{2b} \left[ A_{1/2} - \lambda \frac{S_{1/2}}{|q|} \right],
\]

\[
\eta F_2 = -\frac{1}{2b} \frac{(M_R - M)^2 Q^2}{4M^2} \left[ A_{1/2} - \lambda \frac{S_{1/2}}{|q|} \right] + \frac{1}{2b} \lambda \frac{S_{1/2}}{|q|}.
\]

For the convenience of the discussion we multiply \( F_2 \) by \( \eta \), given by Eq. (7).

From Eqs. (17)-(18), we can conclude, that, in the sum \( \tilde{F}_1 = F_1 + \eta F_2 \), all terms cancel, except for the term \( A_{1/2} / (2b) \), as expected from Eq. (14). From the
equations, we can also conclude that if the factor $\mathcal{R} = A_{1/2} - \Delta S_{1/2}/|q|$ does not vanish ($\mathcal{R} \neq 0$), or it does not vanish fast enough with $|q|$ when $|q| \to 0$, then the form factors $F_1$ and $\eta F_2$ diverge in the limit $|q| \to 0$.

Considering the MAID2007 parametrization, where $\mathcal{R} = O(1/|q|)$, since $A_{1/2}, S_{1/2} = O(1)$, we conclude that when $|q| \to 0$, $F_1, -\eta F_2 = O(1/|q|^n)$ (dominance of the term in $S_{1/2}$). These results are consistent with the previous estimate of $F_2$ for the MAID2007 parametrization. We checked numerically the divergence of the form factors $F_1, F_2$ in the MAID2007 parametrization.

If, however, the Siegert’s theorem \cite{2} is valid, and $\mathcal{R} = O(|q|^n)$ with $n \geq 2$, we conclude that $F_1, -\eta F_2 = O(1/|q|^{2-n})$. In the simplest case, when $n = 1$, we obtain $F_1, -\eta F_2 = O(1/|q|)$. It is interesting to note, that, even in the conditions of the Siegert's theorem, the form factors $F_1, F_2$ may diverge in the pseudo-threshold limit.

We can show however, that, if we represent any of the functions $A_{1/2}$ and $S_{1/2}/|q|$, by a non-singular function $F$ of $Q^2$, we can write $\mathcal{R} = O(|q|^n)$ with $n \geq 2$, since in the expansion of a function $F(Q^2)$ in powers of $|q|$, near $|q| = 0$, the first term vanishes. This result is the consequence of the relation $\frac{dF}{dq} = \frac{M F(q)}{M^2 + Q^2 + Q^2} \frac{dF}{dQ^2}$, where $\frac{dF}{dq}$ vanishes in the pseudo-threshold, unless $\frac{dF}{dQ^2}$ diverges. The implication of the previous result is that if $\mathcal{R} = O(|q|^2)$, one obtains, according with the previous estimate, $F_1, -\eta F_2 = O(1)$. As consequence, both form factors $F_1$ and $F_2$ are finite at the pseudo-threshold. We present next a parametrization of the amplitudes $A_{1/2}$, $S_{1/2}$ consistent with the result $\mathcal{R} = O(|q|^2)$.

### 4. Modified MAID parametrization

We consider now parametrizations of the $\gamma^*N \to N(1535)$ helicity amplitudes, that differs from the MAID2007 parametrization. Since the proposed parametrization is based in the form of the MAID2007 parametrization, but is also compatible with the Siegert’s theorem, we label it as MAID-SG parametrization (SG holds for Siegert). In the MAID-SG parametrization one uses

\begin{align}
A_{1/2} &= a_0 \left(1 + a_1 Q^2\right) e^{-a_2 Q^2}, \\
S_{1/2} &= \frac{2M_R |q|}{Q^2} s_0' \left(1 + s_1 Q^2 + s_2 Q^4\right) e^{-s_3 Q^2},
\end{align}

where the $a_0, a_1, a_2, s_1, s_2$ and $s_3$ are adjustable parameters and $s_0'$ will be fixed by the Siegert’s theorem condition \cite{3}. Comparatively to the original MAID2007 parametrization \cite{8, 10}, we replaced $s_0 \to (2M_R |q|) s_0'/Q^2$ and add an extra term in $Q^2$ for $S_{1/2}$.

The extra term ($s_2 Q^2$) is important in order to obtain a parametrization based on small coefficients (between $10^{-3}$ and $10^3$), in the spirit of the previous MAID parametrizations. The factor $(2M_R |q|)/Q^2$ is included to give the correct behavior (proportional to $|q|$) near $|q| = 0$, and preserve the high $Q^2$ behavior of the parametrization, since $2M_R |q|/Q^2 = \sqrt{Q^2/Q} \to 1$, for very large $Q^2$.

Note that, using Eqs. (19) and (20), one has $A_{1/2} = O(1)$ and $S_{1/2} = O(|q|)$, when $|q| \to 0$. However, to ensure the Siegert’s theorem, we still need to constrain the value of $s_0'$ by Eq. (2). We fit all the coefficients to the MAID data \cite{10}. Since the MAID analysis gives negligible error bars for the amplitude $S_{1/2}$ when $Q^2 > 1.5$ GeV$^2$, for the propose of the fit we use an error of $0.01 \times 10^{-3}$ GeV$^{-1/2}$. The coefficients determined by the best fit are presented in Table 1.

![Table 1](image)

| Amplitude | $A_{1/2}$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ |
|-----------|-----------|-------|-------|-------|-------|
| MAID2007  | 66.40     | 1.61  | –     | –     | 0.70  |
| MAID-SG   | 54.99     | 2.09  | –     | –     | 0.70  |

Amplitude $S_{1/2}$

| Amplitude | $S_{1/2}$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
|-----------|-----------|-------|-------|-------|-------|
| MAID2007  | –2.00     | 23.90 | –     | –     | 0.81  |
| MAID-SG   | –9.46     | 11.57 | 0.172 | 0.93  |

Although we could impose the Siegert’s theorem re-fitting only the amplitude $S_{1/2}$, for a question of consistence one chose to fit both amplitudes simultaneously. The coefficients associated with the new fit based on Eqs. (19)-(20) are presented in Table 1, in comparison with the MAID2007 parametrization, which violates the Siegert’s theorem. To facilitate the comparison with MAID2007, we replace $s_0'$ by $(2M_R |q|)/Q^2$.

The results for the amplitudes $A_{1/2}$ and $S_{1/2}$ in the MAID-SG parametrization are presented in Fig. 2 (solid line), and are compared with the result from MAID2007 (dashed line). It is interesting to see that the two parametrizations are almost undistinguishable for $Q^2 > 1.5$ GeV$^2$. From the figure, we conclude, that, the constraints of the Siegert’s theorem, can be included in the parametrization of the $\gamma^*N \to N(1535)$ helicity amplitudes, without a significant loss of accuracy.

The results for the amplitudes are consistent with the Siegert’s theorem expressed in the form of Eq. (16), combined with $F_1 = O(1)$. Using the new parametrization for the amplitudes $A_{1/2}$ and $S_{1/2}$, it is possible now to look to the form factors $F_1$ and $F_2$ based on Eqs. (17).
significantly for larger values of $Q^2$, dominated by the form factors $A$ and $S$. Then in the limit $Q^2 \to \infty$, the functions $\tilde{F}_{i/2}$ and $\tilde{F}_{i+1/2}$ are determined by the MAID-SG parametrization. In Fig. 3, one can also see, that the function $\tilde{F}_{1/2}$ is the resonance $\Delta(1232)$ transition. In this transition, the electric ($G_E$) and the Coulomb ($G_M$) quadrupole form factors, are related at the pseudo-threshold limit, by the condition: $G_E = k G_C$, where $k = \frac{M_R - M}{2M_R}$, and $M_\Delta$ is the $\Delta$ mass. When applied to the helicity amplitudes, one obtains the condition $E/|q| = \lambda_3 S_{1/2}/|q|^2$, where $E \equiv A_{1/2} - A_{3/2}/\sqrt{3}$ is the electric amplitude and $\lambda_3 = \sqrt{2}(M_\Delta - M_R)$. One can show, that the previous condition for the amplitudes is violated by the MAID2007 parametrization. Although the MAID2007 verify $E = \lambda_3 S_{1/2}/|q|$. The constraints of the Siegert’s theorem have implications also in the helicity amplitudes associated with other $\gamma^*N \to N^*$ transitions. In particular, the parametrization proposed here, can be used in the study of the $\gamma^*N \to N(1650)$ transition, since it is also a $\frac{1}{2}^+ \to \frac{1}{2}^+$ transition.

In the case of the $\gamma^*N \to N(1520)$ transition the Siegert’s theorem implies that $4E = \lambda_3 S_{1/2}/|q|$, where $E \equiv -\lambda_3 A_{1/2}/|q|^2$, is the electric amplitude in the transition, and $\lambda_3 = \sqrt{2}(M_R - M)$ (one can see then, that apart the factor $1/2$ at the l.h.s., and the replacement $A_{1/2} \to E$, the condition is the same as for the $\gamma^*N \to N(1535)$ transition.

Another interesting case is the $\gamma^*N \to \Delta(1232)$ transition. In this transition, the electric ($G_E$) and the Coulomb ($G_M$) quadrupole form factors, are related at the pseudo-threshold limit, by the condition: $G_E = k G_C$, where $k = \frac{M_\Delta - M}{2M_\Delta}$, and $M_\Delta$ is the $\Delta$ mass. When applied to the helicity amplitudes, one obtains the condition $E/|q| = \lambda_3 S_{1/2}/|q|^2$, where $E \equiv A_{1/2} - A_{3/2}/\sqrt{3}$ is the electric amplitude and $\lambda_3 = \sqrt{2}(M_\Delta - M)$. One can show, that the previous condition for the amplitudes is violated by the MAID2007 parametrization. Although the MAID2007 verify $E = \lambda_3 S_{1/2}/|q|$. The results for the form factors are presented in the Fig. 3. In the figure, it is clear, that $F_1$ and $F_2$ are finite at the pseudo-threshold, as one expects from the dependence $R = O(q^6)$, discussed previously.

We can calculate the explicit dependence of $R$ near the pseudo-threshold, using the functions $A, S$ defined by $A \equiv A_{1/2}$ and $S_{1/2} \equiv (2M_R|q|)^2/Q^4 S$. One obtains then

$$R = \frac{M_R}{M} \left[ A' - A \left( \frac{S'}{S} - \frac{1}{4M_R M} \right) \right]|q|^2,$$

(21)

neglecting terms in $O(|q|^6)$. In Eq. (21), $A, S$ and $A', S'$ represent respectively the functions and the derivatives in the limit $Q^2 = Q^2_{PS}$.

In Fig. 3, one can also see, that the function $\tilde{F}_1$ is dominated by the form factors $F_1$, for larger values of $Q^2$. It is also possible to observe that the form factor $F_3$ has large values for $Q^2 < 0.5$ GeV$^2$, but decreases significantly for larger values of $Q^2$, and is negligible for $Q^2 > 1.5$ GeV$^2$. A consequence of the result $F_2 \approx 0$, is that the amplitudes $A_{1/2}$ and $S_{1/2}$ are correlated by the relation $S_{1/2} = -\frac{\sqrt{2}M_R^2 - M^2}{2M_R M} A_{1/2}$, where $\tau = \frac{Q^2}{(M_R + M)^2}$, for $Q^2 > 1.5$ GeV$^2$. As discussed in Refs. [13, 15], the result $F_2 \approx 0$, suggests that there is a cancellation between the valence quark contributions and the meson cloud contributions.
at pseudo-threshold (the r.h.s. and the l.h.s. vanish both), this is not sufficient to ensure that \( G_E = \kappa G_C \).

In the Fig. 4 we compare at the top the form factors \( G_E \) and \( \kappa G_C \), given by the MAID2007 parametrization. It is clear in the graph, that, the Siegert’s theorem is violated. At the bottom, we consider an improved parametrization where the Siegert’s theorem is imposed and fitted to the \( G_E \) and \( G_C \) data (defining a new MAID-SG parametrization). In this case, one can see the convergence of \( G_E \) to \( \kappa G_C \) at the pseudo-threshold. The \( \gamma^*N \to \Delta(1232) \) transition form factors and their relation with the Siegert’s theorem are discussed in detail in Ref. [11].

6. Summary and conclusions

In the present article we discuss the implications of the constraints in the \( \gamma^*N \to N(1535) \) helicity amplitudes, when the nucleon and the resonance \( N(1535) \) are both at rest (pseudo-threshold limit). In this limit the transverse (\( A_{1/2} \)) and the longitudinal (\( S_{1/2} \)) amplitudes are related by the Siegert’s theorem (2). We concluded, that the Siegert’s theorem is the consequence of the orthogonality between the nucleon and resonance states.

From the analysis of the structure of the current and the transition form factors, we conclude also, that, the amplitudes \( A_{1/2} \) and \( S_{1/2} / |q| \) are both finite and non-zero in the pseudo-threshold limit [recall Eq. (16) with \( \bar{F}_1 = \mathcal{O}(1) \)]. Based on this result, we explain why the MAID2007 parametrization for the amplitudes \( A_{1/2} \) and \( S_{1/2} \) violates the Siegert’s theorem, and propose an alternative parametrization, consistent with both the Siegert’s theorem and the data. The new parametrization is similar to the MAID2007 parametrization for both amplitudes when \( Q^2 > 1.5 \text{ GeV}^2 \), but deviates from MAID2007 for smaller values of \( Q^2 \). In the new parametrization, the amplitude \( S_{1/2} \) differs more significantly from the MAID2007 parametrization for \( Q^2 < 0 \), and vanishes at the pseudo-threshold as expected (\( S_{1/2} \propto |q| \)).

We concluded also, that, the Dirac and Pauli form factors are free of singularities at the pseudo-threshold as expected from the Siegert’s theorem, expressed under the condition \( A_{1/2} - \Delta S_{1/2} / |q| = \mathcal{O}(|q|^2) \), near the pseudo-threshold.

The methods proposed in this article to study the structure of the helicity amplitudes and the structure of the transition form factors in the \( \gamma^*N \to N(1535) \) transition, can be extended for the transitions \( \gamma^*N \to \Delta(1232) \), \( \gamma^*N \to N(1520) \) [11] and others.

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