Optimization of metal belt continuously variable transmission pulley based on multi-island genetic algorithm

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Abstract. The efficiency of metal belt CVT is an important index to evaluate its transmission performance, and the greater the axial deformation of the pulley, the greater the efficiency loss. In this paper, the deformation of pulley under different passive pressures is measured by experiment, and the reliability of the optimized 3D CAD model is verified by comparing with the data of the simulation model. And based on multi Island genetic algorithm (MIGA), using ISIGHT to integrate CATIA and ABAQUS, and the maximum stress is used as the constraint condition, penalty function integrate pulley deformation, mass and moment of inertia of the smallest to construct single objective function to optimize a CAD model, finally the test sample analysis verify the reliability of the optimization results. Research results show that, the optimized pulley mass was reduced by 16.96% and the inertia was reduced by 11.7%. Comparing the axial deformation of different speed ratios of the pulley, the deformation is smaller than that before the optimization, laid the foundation for the study of pulley transmission efficiency.

Keywords: Pulley deformation; MIGA; test analysis; structural optimization.

1. Introduction

Metal belt continuously variable transmission can realize the continuous adjustment of vehicle speed ratio, meet the increasingly high requirements of the international community for fuel economy and emission of vehicles, meet the needs of different loads of vehicles, and keep the engine and transmission always in the best transmission efficiency. The metal belt continuously variable transmission adopts friction transmission. The belt pulley is extruded by hydraulic clamping force to realize speed ratio change, but the clamping force causes the deformation of the metal belt pulley, and the transmission loss caused by the deviation of the running track of the metal belt accounts for more than 90% of the total loss[1], resulting in the loss of transmission efficiency and speed ratio of CVT[2-3]. Domestic scholars have also obtained the mathematical model of maximum deformation and maximum
deformation position distribution of the pulley, but they do not made quantitative analysis on the optimization of the specific design size of the metal belt continuously variable transmission pulley to obtain a better design structure of the pulley. In this paper, with the help of multi-island genetic algorithm, the design size of a domestic CVT driven pulley is optimized. The penalty function, taking the axial deformation, mass and rotational inertia of the pulley as the single objective function, obtains the optimal solution of the design parameters of the pulley, which minimizes the deformation of pulley at different speed ratios. The sensitivity of the optimized parameters is analyzed by the Latin square sample, the axial deformation, moment of inertia and mass of the pulley under different speed ratios before and after optimization are compared to verify the reliability of the optimized results. It lays a research foundation for the optimization of transmission efficiency and speed ratio of metal belt continuously variable transmission.

2. Theoretical modeling

The transmission mechanism of metal belt continuously variable transmission is composed of the main drive pulley, the driven pulley and the metal belt in Fig. 1. The fixed cone disc and the movable cone disc constitute the basic structure of the main drive pulley and the driven pulley.

![Fig 1. CVT transmission mechanism](image)

![Figure 2. Deformation model of pulley](image)
When CVT change speed, the clamping force of the hydraulic cylinder acting on the back of the movable conical disc makes the metal belt at different radius of the conical surface, which realize the infinite speed change. The clamping force can effectively ensure the transmission of the metal belt torque, however, due to the interaction between the metal belt and the pulley, the taper disc of the pulley undergoes axial deformation when changing speed [7], Fig. 2 shows the schematic diagram of the axial deformation of CVT pulley. The reaction force of the metal belt causes the axial deformation of the pulley and changes the angle between the fixed cone and the movable cone, which results in the change of the working position of the metal belt on the pulley from the ideal working condition to the actual working condition, the working radius from \( R_1 \) to \( R \), thus affecting the working efficiency and speed ratio of CVT. CVT speed ratio varies with the working radius of the metal belt on the taper disc of the pulley, so the position of the reaction force on the pulley varies with the speed ratio. Therefore, under the same structure, speed ratio is also an important factor leading to the axial deformation of the pulley. It can be seen that the greater the CVT speed ratio and axial force, the greater the axial deformation of the pulley, the transmission efficiency and work efficiency loss of CVT are greater.

3. Mechanical model of pulley

3.1. CVT geometric relation model
The geometric speed ratio of CVT is determined by the actual position of the metal belt on the conical radius of the driving wheel. The length of CVT metal belt is defined as

\[
L = R_1 (\pi + 2\beta) + R_2 (\pi - 2\beta) + 2a \cos \beta
\]

(1)

In the upper formula, \( L \) is the length of the metal belt, \( a \) is the center distance of the driven wheel, \( R_1 \) is the radius of the driving wheel and \( R_2 \) is the radius of the driven wheel.

Inclusion angle \( \beta \) of metal belt on pulley

\[
\sin \frac{\beta - 180}{2} = \frac{(R_1 - R_2)}{a}
\]

(2)

Geometric Velocity Ratio of CVT

\[
i = \frac{R_2}{R_1}
\]

(3)

3.2. The direction of the wheel shaft force distribution model
Micro element CVT is selected for cone mechanical analysis [4-5], which shows in figure 3. A conical force can be obtained from a tangential equilibrium equation for metal belt

\[
(F + dF) \cos \frac{d\theta}{2} - F \cos \frac{d\theta}{2} - 2\mu \cos \psi F_N \, ds = 0
\]

(4)

In formula: \( F \) is the friction force on the conical surface of the pulley; \( F_N \) is the positive pressure of the metal belt on the pulley; \( d\theta \) is the unit contact angle between the metal belt and the pulley; \( \mu \) is the friction factor between the metal belt and the pulley; \( d\psi \) is the unit length of the metal belt; \( F_N \) is the axial thrust of the driven pulley; \( \alpha \) is the angle between the conical surface of the main and slave pulleys of CVT; \( \psi \) is the friction angle between the metal belt and the conical surface of the pulley.
Assuming that the enclosure angle of the metal strip element is infinitely small, therefore

\[ \cos \frac{d\theta}{2} = 1 \] (5)

By the formula (4) can be obtained with friction force per unit length in the meta

\[ dF = 2\mu \cos \psi F_N ds \] (6)

The balance equation for micro axial direction

\[ F_N ds + \mu \sin \psi \sin \frac{\alpha}{2} F_N ds - \cos \frac{\alpha}{2} F_N ds = 0 \] (7)

According to formulas (6) and (7), the axial force acting on the unit length of the metal strip can be obtained as follows

\[ F_N ds = \frac{dF}{2\mu \cos \psi} \left( \cos \frac{\alpha}{2} - \mu \sin \frac{\alpha}{2} \sin \psi \right) \] (8)

Formula (8) shows the transmission torque and friction angle affecting the distribution of the axial force.

4. Simulation model and verification

4.1. FEM Simulation of Pulley
Simplify the actual model of driven pulley to build three-dimensional parametric model, and make the following assumptions for research: (1) The metal strip composed of metal sheets is an integral body with uniform performance and infinite rigidity in both longitudinal and transverse directions; (2) the deformation of the two cones of the pulley is uniform; (3) the metal strip does not slide actively at different speed ratios.

In order to ensure the accuracy of the calculation results of the pulley model after parametric modeling, the hexahedral mesh is used in the model partition, and the meshed model is shown in Figure 4. Because the different speed ratio determines the different working radius, wrapping angle and the contact width between the metal belt and the taper surface of the pulley, under the different speed ratio, the nodes in the corresponding working area are selected, and the axial force and friction force are applied to the nodes. By the reference [4], under the different speed ratio, the deformation of the pulley increases with the increase of the speed ratio. Fig. 5 shows a finite element loading model with velocity
ratio $i = 2.43$ (maximum velocity ratio), The stress nephogram shown in Fig. 6, which is output by setting up ABAQUS.

From the cloud image, it can be seen that the stress concentration area occurs at the position where the fixed cone disc and the axis are connected, while the axial deformation is larger near the working area of the metal strip and smaller far away from the working area of the metal strip. The different thickness of the pulley in the loading area at different speed ratios leads to different stiffness in the corresponding area of the pulley. The conical stiffness near the pulley shaft is large and the axial deformation is small. Therefore, the reasonable structure size significant to reduce the axial deformation of the pulley. According to the engineering practice, this paper optimizes the pulley with the objective of minimizing the axial deformation of the pulley and the energy required for the rotation of the pulley, improving the transmission efficiency and work efficiency of CVT.

Figure 6. Stress nephogram model at maximum velocity ratio

Fig 4. Finite element mesh of driven pulley

Fig 5. Loading model of driven pulley
4.2. Axial Verification Test of Belt Wheel

Using the fatigue test bench, the test system consist of the purchased GT2-P12 written test sensor and its kit, GT2-H32 universal sensor and its kit, the computer installed with sensor data processing software and the self-designed bracket used to fix the sensor on the rear case of the transmission.

During installation, we should ensure the normal performance of gearbox and transmission mechanism, adjust gear to N gear, active pressure to 0 MPA, Table 1 shows the applied passive pressure, and test the deformation of the cone surface of the passive belt axle under the actual working condition by using the probe on the sensor. The experimental data are shown in table 1:

| stress(MPa) | 0   | 1   | 1.5 | 2   | 2.5 | 3   | 3.5 | 4   | 4.5 | 5   | 5.5 | 6   |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Displacement(m) | 0.01 | 0.037 | 0.05 | 0.096 | 0.10 | 0.117 | 0.170 | 0.171 | 0.190 | 5   | 5.5 | 6   |
|             | 5   | 5   | 4   | 5   | 5   | 5   | 5   | 5   | 5   | 5   | 2   | 0   |

Notes: 1. the deformation of OMPa is caused by the vibration of the bench; 2. the actual deformation under various pressures = the real-time measured data - the deformation of OMPa

In the finite element analysis software, the same load is applied in the same area of the model, the data of the experimental loading and simulation of the axial deformation of the pulley are obtained. The maximum error is less than 2%. Therefore, the performance of the model is consistent with that of the real model, and it is reliable as the object of optimization analysis.

5. Establishment of optimization model

5.1. Mathematical Model of Parameter Optimization

Using CAD software to model driven pulley, considering engineering and production practice, the optimized parameters are shown in Fig. 7.

![Figure 7](image)

**Figure 7.** Optimal design variables for pulleys

![Figure 8](image)

**Figure 8.** Sectional view of pulley

(1)Design variables

In order to ensure more efficient start-up and transmission of CVT, it is necessary to minimize the mass and inertia of the pulley. The main design parameters are 11 design variables as shown in Figure 7:

\[
Z = Z_{\text{var}} = [A_1, A_2, L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8, L_9]'
\]  

(2)Constraint condition

Based on the maximum stress constraint of the pulley, a certain life of the pulley can be guaranteed. According to the production and research experience of a domestic CVT company for many years, in order to ensure the reliability of the pulley, the maximum stress should be satisfied: \( \sigma \leq 380 \text{MPa} \).
(3) Mathematical optimization model

In order to improve the transmission efficiency of CVT, minimize the energy loss and make pulley not easy to fail, the deformation, mass and moment of inertia of the pulley are optimized as a single objective function under the constraint of stress. The mathematical model is constructed in formula (10):

$$\min \sum_{i=1}^{n} k_i O_i, n = 3$$

$$Z = [A_1, A_2, L_1, L_2, L_3, L_4, L_5, L_6, L_7, L_8]^T$$

$$\text{s.t.} \quad \sigma_{max} \leq 380 \text{MPa}, m = 2$$

$$Z \subseteq D$$

In the model, $k_i$ is the weight of the $i$-th optimization objective, $O_i$ is the corresponding value of the $i$-th optimization objective, $\sigma_i$ is the corresponding maximum stress value of the $i$-th component, and $D$ is the range of values of the design variables. The range of values of each variable is as follows:

| Design variables (units) | Current value | Lower limit of value | Upper limit of value |
|--------------------------|---------------|----------------------|----------------------|
| $A_1 (\degree)$         | 120           | 91                   | 130                  |
| $A_2 (\degree)$         | 101           | 90                   | 109                  |
| $L_1 (\text{mm})$       | 4.2           | 3                    | 6                    |
| $L_2 (\text{mm})$       | 33            | 0                    | 33                   |
| $L_3 (\text{mm})$       | 5.5           | 4                    | 6                    |
| $L_4 (\text{mm})$       | 4.5           | 3                    | 6                    |
| $L_5 (\text{mm})$       | 3.75          | 3                    | 5                    |
| $L_6 (\text{mm})$       | 19            | 18                   | 25                   |
| $L_7 (\text{mm})$       | 14            | 13                   | 19                   |
| $L_8 (\text{mm})$       | 18            | 16                   | 20                   |
| $L_9 (\text{mm})$       | 5             | 3                    | 6                    |

5.2. Multi-Island Genetic Algorithms

Genetic algorithm (GA) simulates the process of natural selection and population evolution of organisms in nature and adapts the global optimization probability. Multi-island genetic algorithm (MIGA) is derived from genetic algorithm, which places each population in multiple interoperable regions to form sub-populations, it is also known as "islands", which is the biggest difference from traditional genetic algorithm. Individuals of each sub-population select, cross and mutate on the island, excellent individuals on each island migrate regularly to other islands for the operation of traditional genetic algorithm. The migration interval and migration rate are two more parameters of multi-island genetic algorithm than traditional genetic algorithm. The migration interval is the algebra of each migration. The migration rate determines the migration of individuals on each island during a migration and the percentage of the number of subpopulations. Many engineering problems are easy to fall into local optimum solution. The migration operation of multi-island genetic algorithm makes the initial value converge after optimizing operation, and then uses genetic variation to migrate to new subgroups to re-inherit with new initial points, which effectively avoid falling into local optimum solution and improving the chance of containing local optimum solution [9].
5.3. Optimization process

The optimization process is shown in Fig. 9. From Fig. 9[10], it can be seen that before optimization, the optimization parameters and optimization objectives need to be determined. In this paper, the optimization objects and parameters are the thickness of the pulley (figs. 7 and 8).

In the Optimization component, multi-island genetic algorithm is selected to optimize. Specific parameters are set as shown in Table 3.

5.4. Validation and analysis of optimization results

After iteration, three sets of optimal design parameters (number 9 is the optimal level to satisfy the constraints) are obtained, and the specific parameters of the optimal solution are shown in Table 4.
Table 4. Parameter tables before and after optimization

| Parameters (Units) | Optimization | Optimized | rate of change | (100%) |
|-------------------|--------------|-----------|----------------|--------|
| $A_1$ ($^\circ$)  | 120          | 102       | 15             |        |
| $A_2$ ($^\circ$)  | 101          | 94        | 6.9307         |        |
| $L_1$ (mm)        | 4.2          | 3.5961    | 14.3784        |        |
| $L_2$ (mm)        | 33           | 3         | 90.9091        |        |
| $L_3$ (mm)        | 5.5          | 4.8780    | 11.3085        |        |
| $L_4$ (mm)        | 4.5          | 4.8169    | 7.043          |        |
| $L_6$ (mm)        | 3.75         | 3.7077    | 1.1285         |        |
| $L_6$ (mm)        | 19           | 24        | 26.316         |        |
| $L_8$ (mm)        | 14           | 15        | 7.143          |        |
| $L_8$ (mm)        | 18           | 18        | 0              |        |
| $L_9$ (mm)        | 5            | 3         | 40             |        |
| Mass(kg)          | 4.797        | 4.235     | 11.7           |        |
| Inertia           | 7.4912       | 6.221     | 16.96          |        |

(1) Validation of optimization results

In ISIGHT, the optimum Latin square is used for sample analysis. Each design variable is selected within the corresponding range of values and 100 sample points are selected. After dimension unification of displacement, mass, moment of inertia, the weights are set at 0.6, 0.2 and 0.2 respectively, and the samples are analyzed. The sensitivity of parameters in sample analysis is shown in Figure 10. Blue color represents a positive effect parameter and red is a negative effect parameter. However, in sample analysis, the direction of influence of design variables is affected by the absence of stress as a constraint. From the sensitivity analysis chart, it can be seen that the design variables such as $L_6$, $A_1$, $A_2$, $L_1$, $L_2$, $L_3$, $L_7$, and so on, which have a great influence on the objective function. It is consistent with the optimization results, thus the correctness of the optimization results can be judged.

(2) Analysis of Optimized Model
Fig 11. Maximum deformation of pulley at different speed ratios before and after optimization

The following model is reconstructed in CATIA with the optimized design parameters in Table 4. The model is put into ABAQUS to analyze the axial deformation of the pulley at different speed ratios. In order to simulate the actual working condition of CVT, by loading with input torque of 100 and friction coefficient of 0.09, the maximum deformation curve before and after optimization is obtained, which shows in Figure 11.

6. Summary
a. In this paper, the deformation of three-dimensional model under different passive forces is simulated. The conical deformation experiment verifies that the CAD model of the pulley is consistent with the performance of the pulley entity. The model can be used for optimum design.

b. Based on multi-island genetic algorithm (MIGA), ISIGHT is used to integrate CATIA and ABAQUS to optimize the model, and the reliability of the optimization results of design variables is verified by sample analysis. Using the optimized variable design size to rebuild the three-dimensional model, it is found that the optimized mass and moment of inertia of the pulley have been greatly reduced. The deformation of the optimized pulley is compared with that of the non-optimized pulley at different speed ratios. It is found that with the increase of speed ratio, the reduction of the optimized deformation of the pulley increases, and the optimization result of the deformation of the pulley is obvious. The correctness of the optimization results is further verified, which provides a powerful basis for the study of the efficiency of belt pulley transmission.

Acknowledgements
The authors acknowledge the Special Projects of Changsha Zhuzhou Xiangtan National Independent Innovation Demonstration Zone (Grant: 2018XK2302, 2017XK2107).

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