Ranks, Subdegrees and Suborbital graphs of the product action of Affine Groups

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Abstract

The action of affine groups on Galois field has been studied. For instance, [3] studied the action of Aff (q) on Galois field GF(q) for q a power of prime p. In this paper, the rank and subdegree of the direct product of affine groups over Galois field acting on the cartesian product of Galois field is determined. The application of the definition of the product action is used to achieve this. The ranks and subdegrees are used in determination of suborbital graph, the non-trivial suborbital graphs that correspond to this action have been constructed using Sims procedure and were found to have a girth of 0, 3, 4 and 6.

Keywords: Ranks, Subdegrees, Transitivity and Suborbital graphs

1. Introduction

Affine group Aff(q) over Galois field GF(q) is a group of all transformations of the form ax + b, where a, b ∈ GF(q) and a ≠ 0, these elements can be viewed as \((a \ b)\). Let a group G act transitively on a set Ω and \(G_a\) the stabilizer of a in G for a ∈ Ω, then Orb_G(a) is the Orbit of a in G. The \(G_a\)-orbits are called suborbital of G denoted by \(Δ_a\) and the number of them is the rank while the length of the suborbital is called the subdegrees of G on Ω.

The concept of suborbital graphs of non-trivial suborbital of a permutation group G was introduced by Sims [2]. Investigated some properties of the action of the stabilizer of \(∞ \in Γ(modulargroup)\) acting on the set of integers is transitive and imprimitive. Moreover, if x, y ∈ Z with x ≠ y then the suborbital \(O(o,x)\) and \(O(o,y)\) are disjoint. The suborbital graph \(Γ(0,x)\) has \(|x|\) components and it is paired with \(Γ(x,0)\). [4] studied the action of \(PSL(2,q)\) on the cosets of its cyclic subgroup \(C_{0−1}\), where \(k = (q−1,2)\). The action was found to be imprimitive and the number of the self-paired suborbital is \(q + 2q + 3\) and \(q + 1\), for \(p = 2, q \equiv 1mod4\) and \(q \equiv 1mod4\) respectively.

2. Preliminary Notes

Definitions 2.1 A group G is said to be transitive on a set Ω if for each pair of points a, b ∈ Ω, there correspond \(g \in G\) such that \(ga = b\). In other words the action is transitive if it has only one orbit.

Definition 2.2 Let G act transitively on Ω. Then G also acts on \(Ω × Ω\) by, \(g(a, y) = (ga, gy), g \in G, a, y \in Ω\). The \(G_a\)-orbits resulting from this action are referred to as suborbital of G on Ω denoted by \(O(a,y)\).

Definition 2.3 A graph \(Γ(V,E)\) is said to be bipartite if V can be partitioned into two subsets \(V_1\) and \(V_2\) such that the edges join the two vertices from different subsets and no edge joining vertices in the same subsets.

Theorem 2.3 [2] Let \(G_1, G_2, \ldots, G_n\) be groups with respect to the binary operations \((\ast_1, \ast_2, \ldots, \ast_n)\). Then the external direct product \(G_1 \times G_2 \times \ldots \times G_n\) is defined as:

\[
G_1 \times G_2 \times \ldots \times G_n = \{g_1 g_2 g_3 \ldots g_n : g_i \in G_i\} \quad \text{and} \quad (g_1, g_2, \ldots, g_n) \ast (h_1, h_2, \ldots, h_n) = (g_1 \ast_1 h_1, g_2 \ast_2 h_2, \ldots, g_n \ast_n h_n).
\]

The order of the group \(G_1 \times G_2 \times G_3 \times \ldots \times G_n\) is given by \(|G_1| \times |G_2| \times |G_3| \times \ldots \times |G_n|\). Suppose \(G_1\) acts on \(Ω_1\), \(G_2\) acts on \(Ω_2\) and \(G_3\) acts on \(Ω_3\). Then the product action of \(G_1 \times G_2 \times G_3\) on \(Ω_1 \times Ω_2 \times Ω_3\) is given by the rule,

\[
\{(g_1, g_2, g_3) (a, b, c) = (g_1(a), g_2(b), g_2(c))\}.
\]

Theorem 2.4 [1] The \(G_1 \times G_2 \times G_3\) – orbit containing \((a, b, c) \in Ω_1 \times Ω_2 \times Ω_3\) is given by \(Orb_{G_1}(a) \times Orb_{G_2}(b) \times Orb_{G_3}(c)\) and the stabilizer of \((a, b, c)\) in \(G_1 \times G_2 \times G_3\) is given by \(Stab_{G_1}(a) \times Stab_{G_2}(b) \times Stab_{G_3}(c)\).
Theorem 2.5
Let $G$ be group acting on set $\Omega$. Then,

$$|Orb_{G}(a)| = \frac{|G|}{|Stab_{G}(a)|} \text{ for all } a \in \Omega$$

Theorem 2.6 [3] The suborbits of $G$ are of the form $\Delta_0 = \{0\}$, $\Delta_1 = \{a \in \Omega | a \neq 0\}$.

Corollary 2.7 [3] Let $G$ act on $\Omega$. The subdegrees are of the form $1$ and $(q - 1)$; thus the rank of $G$ is two.

Theorem 2.8 Let $G$ act transitively on $\Omega$ and let suborbit $\Delta_i (i = 0, 1, \ldots, s - 1)$ correspond to suborbital $O_i$. Then the suborbital graph $\Gamma_i$ is undirected if $\Delta_i$ is self-paired and directed if $\Delta_i$ is not self-paired.

Theorem 2.9 A transitive action is said to be primitive if and only if each of the non-trivial suborbital graph is connected.

Theorem 2.10 The chromatic number of a bipartite graph is $2$.

Theorem 2.11 Let $\Gamma$ be a connected undirected graph. Then $\Gamma$ is Eulerean if and only if every vertex is of even degree.

Theorem 2.12 [5] Let $\Gamma'$ be a suborbital graph of a transitive action. Then all disconnected components of $\Gamma'$ are isomorphic.

3. Main Results

Lemma 3.1 Let the group $G = Aff(q)$ act on the set $\Omega = \{0, a, a^2, a^3, \ldots, a^{(q-1)}\}$. Then $Stab_{G}(0)$ is a cyclic group $C_{(q-1)}$.

Proof. Let $Stab_{G}(0) = \{ ax + b : a(0) + b = 0, a \neq 0, a, b \in G(q)\}$

$$= \{(ax + b : b = 0, a \neq 0), \forall a, b \in G(q)\}$$

$$= \{(ax : a \in GF(q)^*)\} = C_{(q-1)}.$$ (2.1)

This can also be proved by a method applied by [3].

Proposition 3.2 Let $Aff(q_1) \times Aff(q_2)$ acts on $\Omega = \Omega_1 \times \Omega_2 = GF(q_1) \times GF(q_2)$. Then $Stab_{G}(0,0)$ is isomorphic to $C_{(q_1-1)} \times C_{(q_2-1)}$.

Proof. By Lemma 3.1, $Stab_{G_1}(0)$ and $Stab_{G_2}(0)$ are cyclic groups $C_{(q_1-1)}$ and $C_{(q_2-1)}$. It follows from theorem [7] that,

$$Stab_{G}(0) = Stab_{G_1}(0) \times Stab_{G_2}(0) = C_{(q_1-1)} \times C_{(q_2-1)}.$$ (2.2)

Theorem 3.3 Suppose that $G = G_1 \times G_2 = Aff(q_1) \times Aff(q_2)$ acts on $\Omega = \Omega_1 \times \Omega_2 = GF(q_1) \times GF(q_2)$. Then the rank is $4$ and subdegree are $1$, $(q_1 - 1)$, $(q_2 - 1)$ and $(q_1 - 1)(q_2 - 1)$.

Proof. Using Theorem [7] we have $Stab_{G}(0,0) = Stab_{G_1}(0) \times Stab_{G_2}(0) = (q_1 - 1) \times (q_2 - 1)$. By Theorem 2.5, we have,

$$|Orb_{G_2}(0,0)| = \frac{|G_1|}{|Stab_{G_1}(0,0)|} = \frac{q_1 - 1}{(q_1 - 1)(q_2 - 1)} = q_1 \times q_2.$$ \(= \Omega.\) Thus transitive.

Suppose that $A = Stab_{G_1}(0)$, $B = Stab_{G_2}(0)$, $\alpha$ be a primitive element in $\Omega_1$ and $\beta$ be a primitive element in $\Omega_2$.

Then $A$–orbits in $\Omega_1$ are $\{0\}$ and $\{a\}$ while $B$–orbits in $\Omega_2$ are $\{0\}$ and $\{b\}$, using Theorem [5] the suborbits of $G$ on $\Omega$ are:

$$\Delta_0 = Orb_{G}(0,0) = \{0\} \times \{0\} = \{(0,0)\}.$$ (3.1)

Therefore, $|\Delta_0| = 1$.

$$\Delta_1 = Orb_{G}(0,\beta) = \{0\} \times \{\beta, \beta^2, \beta^3, \ldots, \beta^{(q-2)}\}$$
\[\begin{align*}
\Delta_1 &= \{(0,\beta), (0, \beta^2), (0, \beta^3), \ldots, (0, \beta^{q_2-1})\}. \\
\text{Therefore, } |\Delta_1| &= (q_2 - 1). \\
\Delta_2 &= Orb_0(\alpha, 0) = \{\alpha, \alpha^2, \alpha^3, \ldots, \alpha^{(q_2-1)}\} \times \{0\} \\
&= \{(0, \alpha), (\alpha^2, 0), (\alpha^3, 0), \ldots, (\alpha^{(q_2-1)}, 0)\}. \\
\text{Therefore, } |\Delta_2| &= (q_1 - 1). \\
\Delta_3 &= Orb_0(\alpha, \beta) = \{\alpha^2, \alpha^3, \ldots, \alpha^{(q_2-1)}\} \times \{(\beta, \beta^2, \beta^3, \ldots, \beta^{(q_2-1)})\} \\
\text{Therefore, } |\Delta_3| &= (q_1 - 1)(q_2 - 1). \\
\text{Hence rank is } 4 \text{ and subdegree are } 1, (q_1 - 1), (q_2 - 1) \text{ and } (q_1 - 1)(q_2 - 1). \\
\end{align*}\]

Suppose that \(Stab_{G_1 \times G_2 \times G_3}(a, b, c)\) acts on \(\Omega_1 \times \Omega_2 \times \Omega_3\) and \(\Delta\) is an orbit of \(Stab_{G_1 \times G_2 \times G_3}(a, b, c)\) on \(\Omega_1 \times \Omega_2 \times \Omega_3\). The suborbital \(O\) corresponding to \(\Delta\) is given by,

\[O = \{(g_1, g_2, g_3)(a, b, c) : (g_1, g_2, g_3) \in \Delta, (g_1, g_2, g_3) \in G_1 \times G_2 \times G_3\} = \{(g_1(a), g_2(b), g_3(c)), (g_1(a), g_2(\beta), g_3(\gamma)) : (\alpha, \beta, \gamma) \in \Delta, (g_1, g_2, g_3) \in G_1 \times G_2 \times G_3\}.\] (3.5)

The suborbital graph \(I_i\) corresponding to suborbital \(O\) is formed by taking \(\Omega_1 \times \Omega_2 \times \Omega_3\) as the vertex set and directed edges from \((p, q, r)\) to \((s, t, u)\), if and only if \((p, q, (p, q, r), (s, t, u)) \in O\). The suborbital graph corresponding to \(\Delta_0\) is the null graph.

The construction of suborbital graphs corresponding to the action of \(Aff(q_1) \times Aff(q_2)\) on \(GF(q_1) \times GF(q_2)\) is as follows:

By Equations (3.1), (3.2), (3.3) and (3.4), it follows that \((0, 0) \in \Delta_0, (0, \beta) \in \Delta_1, (\alpha, 0) \in \Delta_2\) and \((\alpha, \beta) \in \Delta_3\). By application of Equation (3.5), we obtain,

\[O_1 = \{((g_1, g_2)(0, 0), (g_1, g_2)(0, \beta) : g_1, g_2 \in G \times G\} = \{((g_1(0), g_2(0)), (g_1(0), g_2(\beta))\} = \{((p, q), (s, t)) : p = s, q = t\}.\] (3.6)

\[O_2 = \{((g_1, g_2)(0, 0), (g_1, g_2)(a, 0)) = \{((g_1(0), g_2(0)), (g_1(a), g_2(0))\} = \{((p, q), (s, t)) : p 6 = s, q = t\}.\] (3.7)

\[O_3 = \{((g_1, g_2)(0, 0), (g_1, g_2)(\alpha, \beta)) = \{((g_1(0), g_2(0)), (g_1(\alpha), g_2(\beta))\} = \{((p, q), (s, t)) : p 6 = s, q = t\}.\] (3.8)

**Theorem 3.4** All the non-trivial graphs corresponding to this action are undirected.

**Proof.**

By Equations (3.6) to (3.8), if \((p, q, (p, q, s, t, u)) \in O_i, (i = 1, 2, 3)\) then \((s, t, (p, q)) \in O_i\).

**Theorem 3.5** The suborbital graph \(I_3\) is Eulerian if \(q\) is odd.

**Proof.**

If \(q\) is odd, \(|\Delta_3| = q - 1\) and therefore by Theorem 2.11, \(I_3\) is Eulerian.

**Lemma 3.6** Let \(Comp_{I_i}(0, 0) (i = 1, 2, 3)\) be the set of all vertices in the component containing \((0, 0)\) in the suborbital graph \(I_i\). Then,

\[Comp_{I_1}(0, 0) = \{(0, x) : x \in GF(q_2)\} = \{(0, x) : x \in GF(q_1)\}\] (3.9)

\[Comp_{I_2}(0, 0) = \{(x, 0) : x \in GF(q_1)\}\] and

\[Comp_{I_3}(0, 0) = \{GF(q_1) \times GF(q_2)\}.\] (3.11)
Proof
Let \((a, b) \in \text{Comp}_1(0, 0)\). Then there exists a path \((0, 0) \rightarrow (a_1, b_1) \rightarrow \cdots \rightarrow (a_i, b_i) = (a, b)\). It follows from Equation (3.6) that, \(0 = a_1 = a_2 = \cdots = a\). Therefore,
\[(a, b) \in \{(0,x) : x \in GF(q_2)\}\]. This implies that,
\[\{\text{Comp}_1(0, 0) \subseteq (0,x) : x \in GF(q_2)\}\] (3.12)
Let \((0,b) \in \{(0,x) : x \in GF(q_2)\}\). If \(b = 0\), then \((0,b) = (0,0) \in \text{Comp}_1(0,0)\). If \(b \neq 0\), then by Equation (3.6) there exists an edge \((0,0) \rightarrow (0,b) \in \Gamma_1\). Therefore, \((0,b) \in \text{Comp}_1(0,0)\). This implies that, \[\{(0,x) : x \in GF(q_2) \subseteq \text{Comp}_1(0,0)\}\] (3.13)
From Equations (3.12) and (3.13) we get Equation (3.8). Equation (3.10) is proved in a similar way as Equation (3.9).

Let \((x,y) \in GF(q_1) \times GF(q_2)\). Consider the vertex \((a,b)\), where \(x \neq a \in GF(q) \setminus \{a\}\) and \(y \neq b \in GF(q) \setminus \{b\}\). By Equation (3.8), \((a,b)\) is adjacent to both \((0,0)\) and \((x,y)\). It follows that \((x,y) \in \text{Comp}_3(0,0)\), hence \[GF(q_1) \times GF(q_2) \subseteq \text{Comp}_3(0,0)\]. But \[GF(q_1) \times GF(q_2)\] is the set of all the vertices in \(\Gamma_3\), thus \[\text{Comp}_3(0,0) \subseteq GF(q_1) \times GF(q_2)\]. Thus we get Equation (3.12).

**Theorem 3.7**  The graphs \(\Gamma_1, \Gamma_2\) and \(\Gamma_3\) has \(q_1, q_2,\) and \(1\) components respectively, where \(q_1, q_2 > 2\).

**Proof.**
By Theorem [5], all the components in a suborbital graph are isomorphic. Applying Lemma 3.6,
\[|\text{Comp}_1(0,0)| = q_1, |\text{Comp}_3(0,0)| = q_2, |\text{Comp}_3(0,0)| = q_1 q_2\]. Since each of the graphs has \(q_1q_2\) vertices, there are \(q_1\) components for \(\Gamma_1\), \(q_2\) components for \(\Gamma_2\) and \(1\) component for \(\Gamma_3\).

**Lemma 3.8**  For the action of \(\text{Aff}(2) \times \text{Aff}(q_2)\) on \(GF(2) \times GF(q_2)\), when \(q_2 > 3\), \(\Gamma_3\) is bipartite.

**Proof.**
Let \(V (\Gamma_3)\) be the vertices corresponding to \(\Gamma_3\), then the vertices of \(G\) can be partitioned into two subsets \(V_1\) and \(V_2\) such that, \(V_1 = \{(0,b) : b \in GF(q_2)\}\) and \(V_2 = \{(1,b) : b \in GF(q_2)\}\). Thus from Equation (3.8), the edges move from a vertex in \(V_1\) to a vertex in \(V_2\) and no edge in \(\Gamma_3\) joining vertices in the same subset. Hence from Definition 2.3, \(\Gamma_3\) is bipartite.

**Corollary 3.9**  The chromatic number of a connected suborbital graph \(\Gamma_3\) of \(\text{Aff}(2) \times \text{Aff}(q_2)\) acting on \(GF(2) \times GF(q_2)\), where \(q_2 > 3\) is 2.

**Proof.**
By Lemma 3.8, \(\Gamma_3\) is bipartite and using Theorem 2.10, chromatic number of a bipartite graph is 2.

**Theorem 3.10**  The suborbital graph \(\Gamma_3\) corresponding to the action of \(\text{Aff}(2) \times \text{Aff}(q_2)\) on \(GF(2) \times GF(q_2)\), where \(q_2 > 3\), has a girth of either 4 or 6.

**Proof.**
Let \(\Gamma_3\) be bipartite and \(a\) be a primitive element in \(GF(2)\), if \(q_2 > 3\), \((0,0) \rightarrow (1,a) \rightarrow (0,a^2) \rightarrow (1,a^3) \rightarrow (0,0)\) and \((0,0) \rightarrow (1,a^2) \rightarrow (0,a^3) \rightarrow (1,a^4) \rightarrow (0,a^5) \rightarrow (1,a^6) \rightarrow (0,0)\), are all circuits of even length by Equation (3.8). Thus the girth is 4 and 6 respectively since 4 and 6 are the smallest length of a cycle respectively. Note that if \(q_2 = 3\), the result is as shown in Fig 3.6.

**Theorem 3.11**  The suborbital graph \(\Gamma_1\) and \(\Gamma_2\) corresponding to the action of \(\text{Aff}(q_1) \times \text{Aff}(q_2)\) on \(GF(q_1) \times GF(q_2)\), has a girth of 0 when \(q_1 = q_2 = 2\) and a girth of 3 when \(q_1 = q_2 > 2\).

**Proof.**
Note that when \(q_1 = q_2 = 2\), the results follows from Fig. 1, Fig. 2 and Fig. 3. If \(q_1 = q_2 > 2\),
\[(0,0) \rightarrow (0,b) \rightarrow (0,b^2) \rightarrow (0,0) \in \Gamma_1\].
Using Equations (3.6) to (3.8). Thus the girth is 3 since 3 is the smallest length of a cycle.

**Corollary 3.12** The action of $\text{Aff}(q_1) \times \text{Aff}(q_2)$ on $GF(q_1) \times GF(q_2)$ is imprimitive.

**Proof.**

From the Theorem 3.7, there are some non-trivial suborbital graphs which are disconnected. Thus from Theorem 2.9 the action is imprimitive.

**Example 3.1** Let $G = \text{Aff}(2) \times \text{Aff}(2)$ act on $\Omega = GF(2) \times GF(2)$. The suborbital $O_1, O_2$ and $O_3$ corresponding to the suborbit $\Delta_1, \Delta_2$ and $\Delta_3$ is,

$$O_1 = \{(g_1, g_2)(0, 0), (g_1, g_2)(0, 1) : (g_1, g_2) \in G_1 \times G_2\}$$

$$= \{(g_1(0), g_2(0)), (g_1(0), g_2(1)) : (g_1, g_2) \in G_1 \times G_2\}$$

$$= (p, q), (s, t) : p = s, q \neq t). \quad (3.13)$$

$$O_2 = \{(g_1, g_2)(0, 0), (g_1, g_2)(1, 0) : (g_1, g_2) \in G_1 \times G_2\}$$

$$= \{(g_1(0), g_2(0)), (g_1(1), g_2(0)) : (g_1, g_2) \in G_1 \times G_2\}$$

$$= (p, q), (s, t) : p \neq s, q = t). \quad (3.14)$$

$$O_3 = \{(g_1, g_2)(0, 0), (g_1, g_2)(1, 1) : (g_1, g_2) \in G_1 \times G_2\}$$

$$= \{(g_1(0), g_2(0)), (g_1(1), g_2(1)) : (g_1, g_2) \in G_1 \times G_2\}$$

$$= (p, q), (s, t) : p \neq s, q \neq t). \quad (3.15)$$

Using Equations (3.13), (3.14) and (3.15), the suborbital graphs $\Gamma_1, \Gamma_2$ and $\Gamma_3$ will be constructed as in Fig. 1, Fig. 2 and Fig. 3 respectively.

**Figure 3.1:** Suborbital graph $\Gamma_1$ corresponding to the action of $\text{Aff}(2) \times \text{Aff}(2)$ on $GF(2) \times GF(2)$

**Figure 3.2:** Suborbital graph $\Gamma_2$ corresponding to the action of $\text{Aff}(2) \times \text{Aff}(2)$ on $GF(2) \times GF(2)$
The suborbital graph \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \) are all undirected, regular of degree 1, has a girth of 0 and they are disconnected each with 2 components and each component has diameter 1.

**Example 3.2** Let \( G = Aff(2) \times Aff(3) \) act on \( \Omega = GF(2) \times GF(3) \). The suborbital \( O_1, O_2 \) and \( O_3 \) corresponding to the suborbit \( \Delta_1, \Delta_2 \) and \( \Delta_3 \) is,

\[
O_1 = \{ ((g_1, g_2)(0,0), (g_1, g_2)(0,1)) : (g_1, g_2) \in G_1 \times G_2 \}
= \{ ((g_1(0), g_2(0)), (g_1(0), g_2(1)) : (g_1, g_2) \in G_1 \times G_2 \}
= (p,q),(s,t) : p = s, q \neq t). \tag{3.16}
\]

\[
O_2 = \{ ((g_1, g_2)(0,0), (g_1, g_2)(1,0)) : (g_1, g_2) \in G_1 \times G_2 \}
= \{ ((g_1(0), g_2(0)), (g_1(1), g_2(0)) : (g_1, g_2) \in G_1 \times G_2 \}
= (p,q),(s,t) : p \neq s, q = t). \tag{3.17}
\]

\[
O_3 = \{ ((g_1, g_2)(0,0), (g_1, g_2)(1,1)) : (g_1, g_2) \in G_1 \times G_2 \}
= \{ ((g_1(0), g_2(0)), (g_1(1), g_2(1)) : (g_1, g_2) \in G_1 \times G_2 \}
= (p,q),(s,t) : p \neq s, q \neq t). \tag{3.18}
\]

Using Equations (3.16), (3.17) and (3.18), the suborbital graphs \( \Gamma_1, \Gamma_2 \) and \( \Gamma_3 \) will be constructed as in Fig. 4, Fig. 5 and Fig. 6 respectively.

The suborbital graph \( \Gamma_1 \) is undirected, regular of degree 2, has a girth of 3 and it is disconnected. It has 2 connected components each with a diameter of 1.
The suborbital graph $\Gamma_1$ is undirected, regular of degree 2, has a girth of 3 and it is disconnected. It has 2 connected components each with a diameter of 1.

The suborbital graph $\Gamma_2$ is undirected, connected, regular of degree 2 hence Eurelian, has a girth of 6 and a diameter of 2.

**Conclusion**

The subdegrees of the product action of affine groups $\text{Aff}(q_1) \times \text{Aff}(q_2)$ acting on $GF(q_1) \times GF(q_2)$ is $1, (q_1 - 1), (q_2 - 1)$ and $(q_1 - 1)(q_2 - 1)$. The number of elements in all the four $G$ -orbit is, $1 + (q_1 - 1) + (q_2 - 1) + (q_1 - 1)(q_2 - 1) = q_1q_2 = |\Omega|$. Thus the rank is 4.

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