The evanescent wavefield part of a cylindrical vector beam

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Abstract: The evanescent wave of the cylindrical vector field is analyzed using the vector angular spectrum of the electromagnetic beam. Comparison between the contributions of the TE and TM terms of both the propagating and the evanescent waves associated with the cylindrical vector field in free space is demonstrated. The physical pictures of the evanescent wave and the propagating wave are well illustrated from the vectorial structure, which provides a new approach to manipulating laser beams by choosing the states of polarization in the cross-section of the field.

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1. Introduction

Recently the subject of vector optical field has attracted much interest in linear and nonlinear optics realms due to its unique and novel properties and potential applications [1–3]. In contrast to most of the past research based on spatially homogeneous polarization states such as linear, elliptical, and circular polarization, the vector optical fields deal with inhomogeneous states of polarization in the field cross section. The vector optical fields have been studied intensively. For instance, focusing a radially polarized beam can lead to the smallest possible spot size [4, 5], and the angular optical momentum from the vector optical field has been demonstrated [6]. Evanescent waves, also known as inhomogeneous waves [7], have a long history and have been the subject of many studies and appear in many branches of physics [8]. The evanescent waves of nonparaxial beams have been extensively studied [8–13], due to the novel properties and potential applications in many areas, such as nanophotonics, guided optics, internal reflection spectroscopy, and optical microscopy [8].

Generation and applications of evanescent wave with different systems and instruments have been reported [14–16]. R. Merlin and his colleagues described the contribution of pure evanescent waves to focused fields for generating a small focal spot beyond diffraction limit [17, 18]. More recently, the evanescent waves of radially polarized and azimuthally polarized optical fields have been demonstrated [19–21].

In this paper, we study the evanescent wave of an arbitrary vector optical field with inhomogeneous states of polarization in the cross section of the field. The TE and TM terms (i.e. transverse electric mode (TE) term and transverse magnetic mode (TM) term in term of its vector angular spectrum) of the evanescent wave and the propagating wave of the cylindrical vector optical field are derived by the vector angular spectrum method. Propagation of the wavefield in z direction in free space is described by the propagation of the angular spectrum in the near field as well as in the far field since the angular spectrum propagation method is a solution of the Helmholtz equation [22, 23]. Two other ways to characterize such propagation are in terms of the Fresnel and the Fraunhofer approximation, but they are valid only under certain constraints. Recently, the focused azimuthal-variant vector beams have been studied under the nonparaxial and paraxial approximations by using the vectorial Rayleigh-Sommerfeld formulae [24] which can also be derived from the angular method [23].

Here in this paper, the ratio of the evanescent wave and the propagating wave of a cylindrical vector optical field as a function of the propagation distance in the near field is investigated. The intensity (squared modulus) distributions of the TE and TM terms of the propagating and the evanescent waves are analyzed in detail. These results, therefore, provide
useful information on how to spatially manipulate the evanescent waves of a vector cylindrical optical field in the near field by choosing appropriate states of polarization in the cross-section of the field.

2. Theoretical formulation

In the Cartesian coordinate system, the z-axis is taken to be the direction of wave propagation. A cylindrical optical vector field can be described as [1–3]

\[ E(r, \theta) = A(r) \mathbf{P}(\theta) = A(r)[\cos(m\theta + \theta_0) \hat{e}_x + \sin(m\theta + \theta_0) \hat{e}_y], \]

where \( r = \sqrt{x^2 + y^2} \) and \( \theta = \arctan(y/x) \) are the polar radius and the azimuthal angle in the polar coordinate system, respectively. \( \mathbf{P}(\theta) \) is the unit vector describing the distribution of the states of polarization of the vector field. \( m \) is the topological charge, and \( \theta_0 \) is the initial phase. \( \hat{e}_x \) and \( \hat{e}_y \) are the unit vectors in x and y directions respectively in the Cartesian coordinate system. \( A(r) \) represents the amplitude distribution in the cross-section of the cylindrical vector field. The time dependent factor \( \exp(-i\omega t) \) is omitted in Eq. (1) (\( \omega \) is the circular frequency).

When \( m = 1 \) with \( \theta_0 = 0 \) and \( \pi/2 \), the vector fields correspond to the radially and azimuthally polarized vector fields, respectively. When \( m = 0 \), Eq. (1) degenerates to the horizontal (for \( \theta_0 = 0 \)) and vertical (for \( \theta_0 = \pi/2 \)) linearly polarized fields, respectively.

The transverse components of the vector angular spectrum \( A_x(\rho \cos \phi, \rho \sin \phi) \) and \( A_y(\rho \cos \phi, \rho \sin \phi) \) are given by the Fourier transform of the input field:

\[ \begin{align*}
A_x(\rho \cos \phi, \rho \sin \phi) &= \frac{k^2}{2\pi} \int_0^\infty \int_0^{2\pi} A(r) \cos(m\theta + \theta_0) \exp[-ikr \rho \cos(\theta - \phi) - ik \xi z] r \theta dr d\theta, \\
A_y(\rho \cos \phi, \rho \sin \phi) &= \frac{k^2}{2\pi} \int_0^\infty \int_0^{2\pi} A(r) \sin(m\theta + \theta_0) \exp[-ikr \rho \cos(\theta - \phi) - ik \xi z] r \theta dr d\theta,
\end{align*} \]

where \( k \) is the wavenumber. \( \rho \) and \( \phi \) are related to the transverse Fourier-transform variables \( u, v \) and \( \xi \), and we have

\[ u = \rho \cos \phi, \quad v = \rho \sin \phi, \quad \xi = \sqrt{1 - \rho^2}. \]

For Gaussian distribution \( A(r) = \exp(-r^2 / w^2) \) with an arbitrary polarized electromagnetic field as Eq. (1) in the source plane \( z = 0 \), where \( w \) indicates the waist of Gaussian beam, the angular spectrum is

\[ A(\rho \cos \phi, \rho \sin \phi) = \frac{k^2}{2\pi} \frac{\sqrt{\pi} kw^3 \rho}{8} \exp(-k^2 w^2 \rho^2 / 8) \left[ I_{(m-1)/2}(k^2 w^2 \rho^2 / 8) - I_{(m+1)/2}(k^2 w^2 \rho^2 / 8) \right] \]

\[ \times \left[ \cos(m\phi + \theta_0) \hat{e}_x + \sin(m\phi + \theta_0) \hat{e}_y - \cos(m\phi - \phi_0) \rho / \xi \hat{e}_z \right]. \]

where \( I(.) \) are the Bessel functions of the second kind and \( \hat{e}_z \) is the unit vector in z direction. The electric field component of the vector cylindrical optical field in the z plane can be represented as:
\[
E(r) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} A(\rho \cos \phi, \rho \sin \phi) \exp\left[ i k (ux + vy + \xi z) \right] du dv
\]
\[
= \frac{i^m k^3 w^3}{16\sqrt{\pi}} \int_{0}^{\pi} e^{-\frac{k^2 w^2 \rho^2}{8}} J_{m}(-kr \rho) \left[ I_{(m-1)/2} \left( \frac{k^2 w^2 \rho^2}{8} \right) - I_{(m+1)/2} \left( \frac{k^2 w^2 \rho^2}{8} \right) \right] \sin(m \theta + \theta_0) \cos(m \theta - \theta_0) \rho \exp \left( i k \xi z \right) \exp \left( i k \xi \right) \rho^2 d \rho.
\]

The electromagnetic field component of the vector cylindrical optical field in the \( z \) plane can be decomposed accordingly into the TE and TM terms [25–30]:
\[
E(\vec{r}) = E_{TE}(\vec{r}) + E_{TM}(\vec{r}).
\]

Both the TE and TM components are perpendicular to the propagation axis. With reference to [13] and [20], they can be written in the following forms:
\[
E_{TE}(r) = \frac{1}{2} \left( \frac{k}{2\pi} \right)^2 \int_{0}^{\infty} \int_{0}^{\pi} \exp\left( -\frac{k^2 w^2 \rho^2}{8} \right) \left[ I_{(m-1)/2} \left( \frac{k^2 w^2 \rho^2}{8} \right) - I_{(m+1)/2} \left( \frac{k^2 w^2 \rho^2}{8} \right) \right] \exp\left( i k \xi \right) \rho d \rho \sin(m \theta + \theta_0) \cos(m \theta - \theta_0) \rho \left[ 2 \pi i^m J_{m}(-kr \rho) \cos(m \theta + \theta_0) + 2 \pi i^{m-2} J_{m-2}(-kr \rho) \cos((m-2) \theta + \theta_0) \right] \hat{e}_x
\]
\[
+ \left[ 2 \pi i^m J_{m}(-kr \rho) \sin(m \theta + \theta_0) - 2 \pi i^{m-2} J_{m-2}(-kr \rho) \sin((m-2) \theta + \theta_0) \right] \hat{e}_y.
\]

And the corresponding evanescent wave components can be obtained according to the calculations similar in Refs [19, 21]:
\[
E_{TE}^{ev}(r) = \frac{1}{2} \left( \frac{k}{2\pi} \right)^2 \int_{0}^{\infty} \int_{0}^{\pi} \exp\left( -\frac{k^2 w^2 \rho^2}{8} \right) \left[ I_{(m-1)/2} \left( \frac{k^2 w^2 \rho^2}{8} \right) - I_{(m+1)/2} \left( \frac{k^2 w^2 \rho^2}{8} \right) \right] \exp\left( i k \xi \right) \rho d \rho \sin(m \theta + \theta_0) \cos(m \theta - \theta_0) \rho \left[ 2 \pi i^m J_{m}(-kr \rho) \cos(m \theta + \theta_0) - 2 \pi i^{m-2} J_{m-2}(-kr \rho) \cos((m-2) \theta + \theta_0) \right] \hat{e}_x
\]
\[
+ \left[ 2 \pi i^m J_{m}(-kr \rho) \sin(m \theta + \theta_0) + 2 \pi i^{m-2} J_{m-2}(-kr \rho) \sin((m-2) \theta + \theta_0) \right] \hat{e}_y.
\]
\[ E_{\text{TM}}^v(r) = \left( \frac{k}{2} \right)^2 \left( \frac{\sqrt{\pi k w^2 \rho}}{8} \right) \exp(-k^2 w^2 \rho^2 / 8) \left[ I_{(m-1)/2}(k^2 w^2 \rho^2 / 8) - I_{(m+1)/2}(k^2 w^2 \rho^2 / 8) \right] \]

\[
\exp(i k \xi z) \rho d \rho \left\{ \frac{1}{2 \rho^2 - 1} \left[ 2 \pi i^2 J_m(-k \rho \cos(m \theta + \theta_0)) - 2 \pi i^2 J_{m-2}(-k \rho \cos((m-2) \theta + \theta_0)) \right] \hat{e}_x \right. \\
+ \frac{1}{2 \rho^2 - 1} \left[ 2 \pi i^2 J_m(-k \rho \sin(m \theta + \theta_0)) + 2 \pi i^2 J_{m-2}(-k \rho \sin((m-2) \theta + \theta_0)) \right] \hat{e}_y \\
- \frac{i \rho}{\sqrt{2 \rho^2 - 1}} \left[ 2 \pi i^2 J_{m-1}(-k \rho \cos((m-1) \theta + \theta_0)) \right] \hat{e}_z \right\},
\]

where \( J(.) \) are the Bessel functions of the first kind.

In order to compare the contributions of the propagating and the evanescent waves associated with the cylindrical vector field, the integrated intensity (squared modulus) of the propagating and the evanescent fields, \( I_{pr} \) and \( I_{ev} \), are calculated respectively [19, 21]:

\[
I_{pr} = \iint |E_{pr}|^2 dxdy = \iint_0^{2\pi} \left[ |a|^2 + |b|^2 \right] \rho d \rho d \phi, \quad \text{(9)}
\]

\[
I_{ev} = \iint |E_{ev}|^2 dxdy = \iint_0^{2\pi} \left[ |a|^2 + |b|^2 \right] \exp(-2kz\sqrt{\rho^2 - 1}) \rho d \rho d \phi, \quad \text{(10)}
\]

with

\[
a = \left( \frac{k}{2\pi} \right)^2 \frac{\sqrt{\pi k w^2 \rho}}{8} \exp(-k^2 w^2 \rho^2 / 8) \left[ I_{(m-1)/2}(k^2 w^2 \rho^2 / 8) - I_{(m+1)/2}(k^2 w^2 \rho^2 / 8) \right] \sin((1-m)\phi - \theta_0), \quad \text{(11)}
\]

\[
b = \left( \frac{k}{2\pi} \right)^2 \frac{\sqrt{\pi k w^2 \rho}}{8\sqrt{1-\rho^2}} \left\{ I_{(m-1)/2}(k^2 w^2 \rho^2 / 8) - I_{(m+1)/2}(k^2 w^2 \rho^2 / 8) \right\} \cos((1-m)\phi - \theta_0), \quad \text{(12)}
\]

\[
b_{ev} = -i \left( \frac{k}{2\pi} \right)^2 \frac{\sqrt{\pi k w^2 \rho}}{8\sqrt{2\rho^2 - 1}} \left\{ I_{(m-1)/2}(k^2 w^2 \rho^2 / 8) - I_{(m+1)/2}(k^2 w^2 \rho^2 / 8) \right\} \cos((1-m)\phi - \theta_0), \quad \text{(13)}
\]

Substituting Eqs. (11)-(13) into Eqs. (9) and (10), and performing integrations over \( \phi \) from zero to \( 2\pi \) yield a single integration of \( \rho \). Then, the corresponding results can be obtained by performing numerical integration over \( \rho \). The ratio \( \delta = (I_{pr} - I_{ev})/I_{pr} \) provides the direct information about the propagating and the evanescent components of the field.
The ratio $\delta = (I_{pr} - I_{ev})/I_{pr}$ for $w = 0.1\lambda$, (highly nonparaxial case) and $w = 0.5\lambda$ cases as a function of different distances $z$ from the initial plane $z = 0$ are shown in Fig. 1. It can be clearly seen that the evanescent field dominates near the source plane and the relative weight of $I_{ev}$ would drastically decrease with the increase of the propagation distance. It becomes negligible in the propagation distance $z = 0.5\lambda$, as shown in Fig. 1. Comparing Figs. 1(a) with 1(b), one can recognize that the relative weight of $I_{ev}$ would reduce when the waist size $w$ increases, i.e., the relative weight of the evanescent wave component is increasing with increasing nonparaxial feature as the waist size $w$ decreases. As it is expected, bigger values of the waist size $w$ leads to reduced nonparaxial behavior. The transverse and the longitudinal components of the evanescent wave become negligible with larger waist size $w$ and hence the relative weight of $I_{ev}$ would be smaller. Furthermore, it can also be found from Fig. 1 that the relative weight of $I_{ev}$ would increase with the increase of the topological charge. It can be explained by the fact that the field distribution will increasingly diverge and extend from the center of beam with the increasing topological charge. As a result, the relative weight of $I_{ev}$ would increase with the increasing topological charge under the same conditions and beam parameters. The calculation results indicate that the evanescent wave component with different initial phase $\theta_0$ for any topological charge (except $m = 1$) is the same as shown in Fig. 1. For $m = 1$ case with different initial phase $\theta_0$, the relative weight of $I_{ev}$ of the azimuthal polarization ($m = 1$, $\theta = \pi/2$) is the maximum and the relative weight of $I_{ev}$ of the radial polarization ($m = 1$, $\theta = 0$) is the minimum as shown in Fig. 1. The physical explanation for the exception $m = 1$ is that the values of the $z$-components of either the propagation wave or the evanescent wave are different for different initial phase $\theta_0$, and especially the values of the $z$-components of either the propagation wave or the evanescent wave are the maximum for $\theta_0 = \pi/2$ (i.e., radial polarization) whereas the $z$-components of either propagation wave or evanescent wave are zero for $\theta_0 = 0$ (i.e., azimuthal polarization), which can also be recognized from Eqs. (5)-(8).

Once we have discussed the appropriate ranges for the beam waist $w$ and the propagation distance $z$, we will intuitively show the contribution of the evanescent and the propagation waves to the optical field. Examples of the intensity distributions of the TE and TM components for the evanescent and the propagation waves are depicted in Figs. 2, 3, and 4. The value of $w$ is set to be $0.1\lambda$ for the highly nonparaxial case. The plane $z = 0.2\lambda$ is selected as the reference plane. $\theta$ is considered to be $\pi/4$ for $m = 0, 1$, and $2$, respectively. Apparently, the TE and TM terms are orthogonal to each other in the field. The intensity distribution profiles of the TE and TM components of the evanescent and the propagation terms are similar, but the magnitude of intensity for the propagation term is smaller than that of the evanescent term. This verifies that the evanescent field dominates near the source plane and the contribution of the evanescent wave to the cylindrical vector field is considerable in magnitude. From Fig. 2 and Fig. 4, for $m = 0$ and $m = 2$ with $w = 0.1\lambda$ and propagation
distance $z = 0.2\lambda$, it can be found that the intensity distribution profiles of both the evanescent and the propagation waves are composed of a pattern with two peaks and some pairs of side lobes. Moreover, the intensities of the side lobes are much lower than those of the peaks. In these cases, the magnitude of the TM term is greater than that of the TE term. As analyzed above, the magnitude of the intensities of the evanescent wave and its two terms (TE and TM) decrease with the increase of the waist width $w$ and the propagation distance $z$. The two peaks will superimpose as a Gaussian distribution. The numerical results indicate that the two peaks will superimpose as a Gaussian distribution for $m = 0$ and the propagation distance $z = 0.2\lambda$ when $w = 0.5\lambda$ as described in Ref [30]. For $m = 2$, it needs farther propagation distance or greater waist width (e.g., $w = 3\lambda$ for $z = 0.2\lambda$). Figure 3 illustrates the intensity distributions of the propagating and the evanescent components of the cylindrical vector beam when $m = 1$, $\theta_0 = \pi/4$, $w = 0.1\lambda$, $z = 0.2\lambda$. These patterns resemble the doughnut shape with a dark spot in the center and some bright rings around it. Such a field has important applications in optical trapping and manipulating of nano-objects [3]. We also present the contributions of the evanescent and the propagation waves to the radially and the azimuthally polarized optical fields in Fig. 5. The TE components of both the evanescent and the propagation waves of the radial polarization optical field are zero as the TM components of the azimuthal polarization.

Fig. 2. Intensity distribution of the propagating and evanescent components of the field as a function of the propagation distances with $m = 2$, $\theta_0 = \pi/4$, $w = 0.1\lambda$, $z = 0.2\lambda$. (a) TE term of the propagating component. The scale is in the order of $10^{-3}$; (b) TM term of the propagating wave; (c) propagating wave; (d) TE term of evanescent wave; (e) TM term of evanescent wave; (f) evanescent wave; (g) total field (both propagating and evanescent waves).

Fig. 3. Intensity distribution of the propagating and evanescent components of the field as a function of the propagation distances with $m = 1$, $\theta_0 = \pi/4$, $w = 0.1\lambda$, $z = 0.2\lambda$. (a) TE term of the propagating component. The scale is in the order of $10^{-3}$; (b) TM term of the propagating wave; (c) propagating wave; (d) TE term of the evanescent wave; (e) TM term of the evanescent wave; (f) evanescent wave; (g) total field (both propagating and evanescent waves).
3. Conclusion

By using the angular spectrum method, the Maxwell’s equation is resolved. The TE and TM terms of the propagating wave and the evanescent wave components of nonparaxial Gaussian beam with inhomogeneous polarization in the cross-section of the field are obtained in the integral form. The contribution of the evanescent wave to the vector optical field is analyzed with the numerical results. The relative weight of the evanescent wave would increase with the increasing topological charge but would reduce with the increasing waist size \( w \) and drastically decrease with the increasing propagation distance. The intensity distribution profiles of the evanescent term are similar to those of the propagating term, but the magnitude of the intensities of the propagating term is different from that of the evanescent term. The intensity distribution is different with the increasing waist width \( w \) or propagation distance \( z \).
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