Strange stars as persistent sources of gravitational waves

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ABSTRACT
We investigate the relevance of the gravitational-wave driven r-mode instability for strange stars. We find that the unstable r-modes affect strange stars in a way that is quite distinct from the neutron star case. For accreting strange stars we show that the onset of r-mode instability does not lead to the thermo-gravitational runaway that is likely to occur in neutron stars. Instead, the strange star evolves towards a quasi-equilibrium state on a timescale of about a year. This mechanism could thus explain the clustering of spin-frequencies inferred from kHz QPO data in Low-mass X-ray binaries. For young strange stars we show that the r-mode driven spin-evolution is also distinct from the neutron star case. In a young strange star the r-mode undergoes short cycles of instability during the first few months. This is followed by a quasi-adiabatic phase where the r-mode remains at a small, roughly constant, amplitude for thousands of years. Another distinguishing feature from the neutron star case is that the r-modes in a strange star never grow to large amplitudes. Our results suggest that the r-modes in strange star emit a persistent gravitational-wave signal that should be detectable with large-scale interferometers given an observation time of a few months. If detected, these signals would provide unique evidence for the existence of strange stars, which would put useful constraints on the parameters of QCD.

Key words: accretion - radiation mechanisms: non-thermal - relativity - stars: neutron - stars: rotation

1 INTRODUCTION
Quite plausibly, the most stable form of matter at supranuclear densities is a conglomerate of deconfined strange, up and down quarks. This could have important consequences for compact stars, with central densities several time the nuclear saturation density, that are born following supernova explosions (Witten 1984). It has, in fact, been suggested that if the strange matter hypothesis is correct then all the observed neutron stars may be strange stars (Alcock et al 1986; Colpi & Miller 1992). If it could be established that strange stars exist in the Universe it would have obvious implications for our understanding of pulsars. In addition it would put severe constraints on the QCD parameters [e.g. the bag constant in the MIT model (Farhi & Jaffe 1984)]. It is thus important to establish whether observations can distinguish a strange star from a neutron star. Unfortunately, this turns out to be a delicate problem.

Strange stars are held together by both the strong nuclear interaction and gravity, and the corresponding equation of state is quite accurately described by uniform density models (see Madsen (1998b) for an exhaustive review). A consequence of this is that, in contrast to the neutron star case, very small strange “dwarfs” (with mass $M \sim R^3$) can form. However, for the canonical mass of 1.4$M_\odot$ gravity dominates the strong interaction which leads to strange stars and neutron stars being similar in size (Alcock et al 1986). In other words, it is not clear that one would be able to distinguish between the two cases given observed masses and radii.

One consequence of the extreme stiffness of the strange matter equation of state is that strange stars can reach higher rotation frequencies than typical neutron stars. However, to date the fastest observed pulsar is the 1.56 ms PSR1937+21 which spins at a rate significantly below the breakup limit for most proposed supranuclear equation of state. In order to indicate the presence of a spinning strange star the period would have to be significantly shorter than 1 ms. Thus the claimed observation of a 0.5 ms pulsar in the remnant of SN1987A caused considerable excitement but, of course, it turned out to be flawed. Despite several searches [see for example (Edwards, Straten & Bailes 2001)] no sub-millisecond pulsars have yet been discovered.

It has been argued that strange stars might be distinguished by the fact that they cool faster than a neutron star would, but it is now accepted that the direct Urca mechanism may be operating [either by the proton fraction rising above $\sim 10\%$ (Lattimer et al 1994) or by hyperons being present (Page et al 2000)] in the core of a neutron star. This
would cool the neutron star very rapidly, which would make it difficult to identify a strange star from cooling data.

Another possible indication of a strange star would be thermonuclear flashes from the surface as accreted material is converted into strange matter. This mechanism would, however, only operate if the strange star was bare. It is generally anticipated that strange stars will be covered by a thin crust of "normal" nuclear matter (Alcock et al 1986). This crust is suspended above the strange matter core by the electromagnetic field. This means that the crust density cannot exceed that of neutron drip (Glendenning & Weber 1992). As neutrons begin to drip out of the crust nuclei they will migrate into the core and be converted into strange matter. The presence of such a crust would allow accretion to proceed without necessarily leading to flashes from the surface.

The sole observational indication against strange stars is provided by the glitching pulsars (Alpar 1987). The standard model for explaining the large glitches in, for example, the Vela pulsar relies on the transfer of angular momentum between the neutron star crust and a superfluid component. It is not easy to see how to construct an analogous model for glitches in a strange star.

Following the discovery that the r-modes in a rotating perfect fluid star would be driven unstable via the emission of gravitational waves [see (Andersson & Kokkotas 2001; Lindblom 2001) for detailed reviews], Madsen (1998a; 2000) suggested that this mechanism would provide the means for distinguishing between neutron stars and strange stars. The gist of Madsen's argument is that the bulk viscosity of strange matter is several orders of magnitude stronger than that of normal neutron star matter, which essentially shifts the window in which the r-modes are unstable to significantly lower temperatures. As a consequence one would not expect a newly born strange star to be immediately affected by the r-mode instability, while a neutron star might spin down significantly in the first few months. An observation of a newly born pulsar spinning near the Kepler limit might thus provide evidence for the presence of a strange star.

The realisation that the r-modes in a spinning compact star are generically unstable has led to a considerable effort aimed at understanding the possible astrophysical relevance of this mechanism. In the last two years several crucial issues have been investigated. Key results concern the interaction between oscillations in the core fluid and the crust (Bildsten & Ushomirsky 2000; Andersson et al 2000; Lindblom, Owen & Ushomirsky 2000), the role of the magnetic field (Spruit 1999; Tezgolu, Lamb & Shapiro 2000; Mendell 2001), superfluidity (Lindblom & Mendell 2000; Andersson & Comer 2001), and the effect of exotic particles that are thought to exist in the deep neutron star core (Jones 2001; Lindblom & Owen 2001). Of similar importance has been attempts to understand the nonlinear saturation of an unstable mode via hydrodynamical simulations (Stergioulas & Font 2000; Lindblom, T"ohline & Vallisneri 2000). For a more complete set of references, see recent review articles (Andersson & Kokkotas 2001; Lindblom 2001).

In parallel with the various attempts to implement more detailed physics into the r-mode model, there have been discussions regarding possible ways that the instability may manifest itself in current observational data. The original r-mode spin-evolution models led to results that accord well with the inferred initial spin period of about 20 ms for the Crab pulsar (Lindblom, Owen & Morsink 1998; Andersson, Kokkotas & Schutz 1999). However, the discovery of the 16 ms PSR 0537-6910 that is thought to have been born spinning significantly faster (at 6-9 ms assuming a reasonable braking index) indicates that the naive r-mode model needs refinement. Furthermore, one can readily show that external agents like the fallback of supernova debris and an acting magnetic propeller torque are of utmost importance in determining the spin of a newly born neutron star (Watts & Andersson 2001).

Adopting a scenario first suggested by Papaloizou & Pringle (1978) and Wagoner (1984), it has been suggested that the r-mode instability may be active in mature accreting neutron stars (Andersson, Kokkotas & Stergioulas 1999). This could provide an explanation for the apparent clustering of spin rates (in the range 260-590 Hz) inferred from kHz QPO data for Low-Mass X-ray Binaries (LMXB). The original ideas were based on the notion that the angular momentum lost through gravitational waves radiated by the r-modes would balance the accretion torque and prevent the star from spinning up. This would lead to a persistent gravitational-wave signal. However, as was pointed out by Spruit (1999) and Levin (1999), the onset of instability is likely to trigger a thermo-gravitational runaway. This leads to a brief period (a few months) of r-mode spin-down followed by a long period (millions of years) of accretion driven spin-up. Even though the r-mode instability would still be able to cause the observed clustering of LMXBs (Andersson et al 2000), the associated gravitational waves would likely not be detected (as the event rate would be too low).

As we will show in the following, the r-mode instability acts rather differently in a strange star. We will argue that unstable r-modes in an accreting strange star in a LMXB may, in fact, provide a persistent source of gravitational radiation. We will also consider the evolution of young strange stars. We will show how the r-mode driven spin-evolution of a strange star differs significantly from the neutron star case. Most significantly, we find that the r-modes never grow to large amplitudes in a strange star. The veracity of our results should be testable with large scale laser interferometers such as LIGO, VIRGO and GEO600 that are about to come on-line. We thus propose that gravitational-wave astronomy might soon be able to establish whether the accreting compact stars in LMXBs are, indeed, strange and perhaps even able to cause the observed clustering of LMXBs (Andersson et al 2000), the associated gravitational waves would likely not be detected (as the event rate would be too low).

We need to model the evolution of the spin rate, temperature and r-mode amplitude in an accreting strange star. The essential elements of our solution to this problem are discussed in sections 2-3, and we present our results for recycled strange stars in Section 4. In section 5 we discuss the evolution of young strange stars. Our conclusions are in Section 6. In an Appendix, we discuss possible refinements to the model. In particular, we consider the deconfinement of the strange star crust as the spin of the star changes and how nonlinear contributions to a strong viscous dissipation may serve to saturate an unstable r-mode.
2 MODELLING UNSTABLE R-MODES IN STRANGE STARS

The aim of this section is to collect results from the literature that will enable us to assess the relevance of the r-mode instability for, possibly accreting, strange stars. We have attempted to parameterise all relevant relations in order to make the dependence on the stellar (and QCD) parameters clear. The various parameterisations are summarised in Table 1.

2.1 R-mode estimates

In the rotating frame of reference the r-modes have frequency

$$\omega_r = \frac{2m\Omega}{l(l+1)}$$

(1)

where $\Omega$ is the rotation frequency of the star.

For simplicity we will assume that the star is described as an $n = 1$ polytrope. The main reason for this is that the corresponding r-mode timescales have already been summarised by Andersson & Kokkotas (2001). A more realistic mode spin-down.

Due to the emission of gravitational waves the $l = m = 2$ r-mode (which is expected to lead to the strongest instability) grows on a timescale

$$t_g \approx -47 M_{1.4}^{-1} R_{10}^{-4} P_{-3}^{-6} \text{ s}$$

(2)

(the minus sign indicates that the mode is unstable). We include only the contribution from the leading current multipole here.

Viscous dissipation tends to suppress the growth of an unstable mode, and we find

$$t_s \approx 7.4 \times 10^7 \alpha_{0.1}^{0.3} M_{1.4}^{5/9} R_{10}^{-1/3} T_{9}^{-5/3} \text{ s}$$

(3)

for the shear viscosity. For the bulk viscosity (which is due to the change in concentration of down and strange quarks caused by the mode oscillation) the viscosity coefficient takes the form

$$\zeta = \frac{\hat{\alpha} T^2}{\omega^2 + \beta T^4}$$

(4)

where the coefficients $\hat{\alpha}$ and $\beta$ are given by Madsen (2000).

From this we see that the bulk viscosity gets weaker at both very low and very high temperatures, cf. Fig. 1 of Madsen (2000). For low temperatures we find

$$t_b^{\text{low}} \approx 7.9 M_{1.4}^{2} R_{10}^{-4} P_{-3}^{-2} T_{9}^{-2} m_{100}^{-1} \text{ s}$$

(5)

Meanwhile, at high temperatures we cannot readily write down a parameterised expression for the bulk viscosity timescale. The relevant timescale has to be calculated numerically for each given stellar model. However, this has little impact on our present study since the star would likely not remain in the high temperature part of the instability window long enough for an unstable mode to grow significantly. Hence, we do not consider the high-temperature case (above say $10^9$ K) further in this paper.

Given the above estimates one can readily construct a critical curve in the $\Omega - T$ plane above which the r-mode will be unstable. The mode is unstable whenever

$$\tau^{-1} = t_g^{-1} + t_s^{-1} + t_b^{-1}$$

(6)

where $t_{g,b,s}$ are the timescales given in Eqns. 2, 3 and 4, is negative. If we assume canonical values for the various parameters we obtain the results shown in Figure 1 (cf. Fig. 1 of Madsen 2000).

From this figure we immediately see that the r-mode instability would not become active (apart from a brief initial period expected to last a few tens of seconds when the star is significantly hotter than $10^9$ K) until the star has cooled to a temperature of a few times $10^8$ K. A young strange star would therefore not be spun down significantly by the r-mode instability during its first few months of existence. But after a year or so the star will enter the “low-temperature” instability window and undergo a phase of r-mode spin-down.

2.2 Temperature evolution

Given that the viscous damping timescales are strongly dependent on the temperature we obviously need to model the thermal evolution of the star. To do this we first of all assume that the stellar core is isothermal.

For a strange star, one can estimate that the (integrated) heat capacity is

$$C_v = \frac{1}{2} M_0^2 \frac{1}{4} \frac{1}{3} T_{9}^{-2} \text{ MeV}$$

(7)

where $M_0$ is the mass of the star and $T_9$ is the core temperature in units of $10^9$ K.

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where $T_{\text{min}}$ is the temperature. The relevant differential equation is:

$$E_{\text{neutrino}} = 3.77 \times 10^{37} R_{10}^3 T_8^0 \text{erg s}^{-1}.$$  (9)

We can define a cooling rate for an isolated star as

$$t \approx \frac{2E_{\text{thermal}}}{E_{\text{neutrino}}}$$  (10)

(with the factor of 2 included since the thermal energy is quadratic in the temperature). Combining equations $\text{(9)}$ and $\text{(8)}$, we obtain

$$t \approx 4.1 \times 10^8 M_{1.4}^2 R_{10}^{-4} T_8^{-4} \text{ s}$$  (11)

(11)

where we have assumed that the initial temperature is high enough to make its contribution to the formula insignificant. From this formula we see that it would take the star roughly 10 years to cool down to a temperature of $10^8$ K. A canonical strange star spinning at the Kepler limit enters the main r-mode instability window in Figure 2 at $T \approx 6 \times 10^8$ K. Given the above cooling timescale, this would happen roughly three days after the star was born. The star then evolves through the instability phase for the next $10^7$ years or so, until the core temperature has fallen below $10^7$ K.

If the star is accreting we must also account for heating due to the accreted material. In the case of strange stars, the main heating source is likely to be due to the conversion of accreted nucleons into strange matter. Alcock et al. (1986) estimate a release of $\epsilon = 20$ MeV of heat per nucleon due to this conversion. This then gives

$$\dot{E}_{\text{accretion}} = \dot{n} \epsilon,$$  (12)

where $\dot{n}$ is the rate at which nucleons enter the strange matter core. As we will discuss in the Appendix this is not necessarily the same as the accretion rate of nucleons onto the star. In the case where all accreted nucleons are immediately converted we get

$$\dot{E}_{\text{accretion}} = 1.19 \times 10^{37} M_{-8} \epsilon_{20} \text{ erg s}^{-1}.$$  (13)

Finally, a detailed study should also account for the fact that the shear viscosity will contribute to the heating of the fluid. We estimate the associated heating rate as

$$\dot{E}_{\text{viscosity}} = 1.1 \times 10^{45} \alpha^2 M_{1.4}^4 R_{10}^{13/9} P_{-3}^{-0.1} T_8^{-5/3},$$  (14)

where $\alpha$ represents the amplitude of the r-mode (see below).

It is easier to evolve the thermal energy $E_{\text{thermal}}$ rather than the temperature. The relevant differential equation is:

$$\dot{E}_{\text{thermal}} = \dot{E}_{\text{accretion}} + \dot{E}_{\text{viscosity}} - \dot{E}_{\text{neutrino}},$$  (15)

In the absence of an r-mode, the temperature is determined by a balance between accretion heating and neutrino cooling. Equations $\text{(13)}$ and $\text{(15)}$ then lead to

$$T = 8.25 \times 10^7 \left[ M_{-8} \epsilon_{20} R_{10}^{-3/2} \right]^{1/6} \text{ K}.$$  (16)

For temperatures of this order, shear viscosity dissipation is much smaller than bulk viscosity dissipation, so that the r-mode instability sets in when $|\tilde{g}| = |\tilde{\omega}|$. Equations $\text{(13)}$ and $\text{(14)}$ combine to give the corresponding critical rotation period:

$$P \approx 2T_8^{-1/2} M_{1.4}^{3/4} m_{100}^{-1} \text{ ms}.$$  (17)

It is illuminating to compare these predictions to the inferred clustering in the LMXB data. We predict that stars with accretion rates in the range $10^{-2} \leq M_{-8} \leq 1$ should be confined to spin frequencies in the range 300 – 450 Hz. Given the many uncertainties in our simple model, this compares quite favourably with the 260 – 590 Hz range suggested by observations. Conversely, it is clear that the current observational data cannot be used to rule out the possibility that the LMXBs harbour strange stars.

### 3 THE SPIN-EVOLUTION OF ACCRETING STRANGE STARS

In order to investigate the way in which a strange star evolves under influence of both accretion and an unstable r-mode we use a phenomenological spin evolution model similar to that of Owen et al. (1988). We assume that we can model the star’s angular momentum as the sum of the bulk angular momentum $J$ and the canonical momentum of the r-mode $J_c$. The total torque on the star $\dot{J}$ can then be written as

$$\dot{J} = \dot{I} \Omega + I \dot{\Omega} + \dot{J}_c,$$  (18)

where the dots indicate time-derivatives. In the following we will write the moment of inertia as $I = \tilde{I} M R^2$ with $\tilde{I} = 0.261$ for an $n = 1$ polytrope.

Note that, since we are modelling the star as an $n = 1$ polytrope, accretion leads to a change in the mass $M$ while the radius $R$ remains constant. This is a peculiarity associated with our particular stellar model, but it has no effect on the outcome of our evolutions. We have verified that this is the case by considering other polytropes, and also including the effect of the centrifugal flattening in our model. This more general study leads to results that are virtually identical to the ones obtained from the model described here so we will discuss only this simple case.

For the $l = m = 2$ r-mode one can show that the canonical angular momentum is

$$J_c = -\frac{3\Omega^2 \tilde{I} M R^2}{2}$$  (19)

where $\alpha$ is the mode amplitude (as defined by Owen et al) and $J = 1.635 \times 10^{-2}$ for an $n = 1$ polytrope.

The r-mode is driven by gravitational radiation and damped by viscosity. We thus assume that the canonical angular momentum evolves according to

$$\frac{\dot{J}_c}{2J_c} = -\frac{1}{\tau}$$  (20)

(the factor of 2 is included since $J_c$ is proportional to the square of the perturbation).

The total torque on the star is the sum of torques due to gravitational radiation and accretion. Throughout the main part of the paper we neglect magnetic dipole radiation. There are two simple reasons for this. Firstly, such
radiation is likely to be suppressed in an accretion environment, especially since the magnetic field of the LMXBs are unlikely to be stronger than \( \sim 10^9 \) G. Secondly, in the case of young isolated strange stars one can argue that the main r-mode spindown torque will dominate the magnetic dipole torque. We will comment on this further in Section 7. It is, of course, possible that the interplay between the r-mode and the interior magnetic field will affect the development of the instability (as suggested by, for example Spruit (1999) and Rezzolla et al. (2000)). We do not account for this possibility in our study.

The torque from the gravitational wave emission from the \( l = m = 2 \) current multipole is

\[
J_g = 3J \alpha^2 MR^2 t_g^{-1}
\]

while we assume accretion to lead to a torque

\[
\dot{J}_a = 6\sqrt{GMR} .
\]

This expression is, of course, likely to be a serious simplification of the true accretion torque, but it should be sufficient for the rather qualitative considerations of the present paper.

Equations (18)-(22) combine to give equations for the evolution of \( \alpha \) and \( \Omega \).

\[
\dot{\alpha} = -\alpha \left[ \frac{1}{t_g} + \left( 1 - 3 \alpha^2 J / 2I \right) \left( \frac{1}{t_s} + \frac{1}{t_b} \right) + \frac{\dot{M}}{2\Omega I} \left( G R^3 \right)^{1/2} \right] \tag{23}
\]

\[
\dot{\Omega} = \frac{\dot{M}}{M} \left( \frac{G}{MR^3} \right)^{1/2} - \frac{\dot{M}\Omega}{M} - 3\alpha^2 J / I \left( \frac{1}{t_s} + \frac{1}{t_b} \right) \tag{24}
\]

The differential equations (23) and (24) can be integrated to give trajectories in the \( \Omega - T \) plane along which an accreting strange star would be expected to evolve.

Note that (23) indicates that the r-mode is unstable, in the sense that \( \dot{\alpha} > 0 \), whenever

\[
\frac{1}{t_s} + \frac{\dot{\Omega}}{2\Omega} + \frac{\dot{M}}{2M} < 0 \tag{25}
\]

This is notably different from (1). The difference is, however, easily explained. Eqn (25) follows since \( J_t \) is conserved for an isolated star, cf. Ho \& Lai (2000). Eqn (23) is the appropriate criterion for onset of instability in a star that is spun up by accretion. In our evolutions the r-mode instability is consequently active whenever (25) holds.

Equations (23) and (24) only apply as long as \( \alpha \) remains “small”. As an unstable r-mode grows exponentially one would expect the equations to remain valid only for a very limited period of time. Intuitively, one would expect to growth of the mode to be halted once nonlinear effects become relevant. Recent attempts to model the evolution of large amplitude unstable modes in numerical hydrodynamical simulations suggest that the r-modes may not saturate until \( \alpha \) has grown to values of order unity (Stergioulas \& Font 2000; Lindblom, Tolhine \& Vallisneri 2000). As we will demonstrate below, the r-modes are unlikely to ever grow to such amplitudes in a strange star. Thus we assume that the r-mode does not saturate due to nonlinear effects, and consider equations (23) and (24) to be relevant throughout our evolutions.

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As long as the r-mode remains stable an accreting star will spin up, and its core temperature should be well approximated by (13). As is clear from Figure 1 this means that the star will enter the r-mode instability window on the branch where bulk viscosity provides the main damping mechanism. The subsequent evolution will therefore be manifestly different from the neutron star case where the star would undergo a thermo-gravitational runaway once it reached the instability limit. We shall now show that in the case of strange stars the subsequent evolution may well proceed extremely slowly (on the accretion timescale) and lead to emission of persistent gravitational waves.

As soon as the star enters the instability window the r-mode will grow and lead to both shear viscosity heating and a spin-down torque. Intuitively, one would expect the star to continue spinning up until the r-mode amplitude becomes sufficiently large to balance the accretion torque. However, the situation will not be stable unless this balance occurs for a star residing on the instability curve, i.e. for which we have \( \dot{\alpha} = 0 \). This situation is well approximated by \( |t_s| \approx t_b \).

Use equation (23) and assume that i) the mode amplitude is small (such that \( 1 >> 3\alpha^2 J / 2I \)) and ii) the temperature is high enough that the shear viscosity dissipation can be neglected.

In order for the situation to correspond to a quasiequilibrium the mode-amplitude required to balance the accretion torque is equal to:

\[
\alpha^2 \approx 6.7 \times 10^{-11} M_{7.8} M_{1.4}^{15/4} R_{10}^{-11/2} m_{100}^{-7/2} \tag{26}
\]

while the mode amplitude required for the shear viscosity heating to prevent the star from cooling down is equal to:

\[
\alpha^2 \approx 3.6 \times 10^{-8} M_{7.8}^{-1/18} m_{100}^{5/3} R_{10}^{7/3} M_{1.4}^{-2} T_{8}^{7/3} \times \left[ 3.77 R_{10}^2 T_{8} - 1.19 M_{7.8} c_{10} \right] \tag{27}
\]

In obtaining these relations we have made use of (13) to express \( P \) in terms of \( T \).

Now we could in principle equate the two expressions (26) and (27) and solve for the “equilibrium” temperature. However, because this leads to a nonlinear equation it is non-trivial. In order to get an algebraic answer we instead assume that the star remains close to the equilibrium temperature (14). Linearising the equation with \( T = T_{eq} + \delta T \) it is easy to estimate that the required solution corresponds to

\[
\frac{\delta T}{T_{eq}} = \frac{1}{1 + 942 T_8^{23/6} R_{10}^{43/6} m_{100}^{1/3} T_{10}^{137/36}} \tag{28}
\]

From this we can deduce that once the star enters the instability regime it never evolves far away from the equilibrium temperature given by (14). In other words, gravitational waves halt the accretion spin-up once the star reaches the r-mode instability curve.

To complete this picture it is necessary to demonstrate that the equilibrium described above is a stable one, i.e. that for small departures from the equilibrium values \( (\alpha, T, P) \) the star returns to the original state. This is straightforwardly achieved by assembling our three differential equations for these quantities, obtaining linearised equations for the departure from equilibrium, and writing the time dependence of this small departure as \( e^{\omega t} \). The solution to the
eigenvalue problem so obtained shows that the configuration is indeed stable (the real part of \( w \) is negative), with departures from equilibrium being damped on a timescale intermediate between the temperature evolution timescale and the gravitational radiation reaction one. This result will be of interest should such wandering behaviour ever be seen in an accreting system.

Numerical results supporting these analytic estimates are shown in Figure 2. As is readily apparent, after a brief interval in which \( \alpha \) and \( T \) vary sharply on a timescale close to the thermal one, the system settles down to a steady state. All subsequent evolution occurs on the accretion timescale.

Having established that the spin-up of an accreting strange star will be halted by the r-mode instability we can readily assess the detectability of the resultant gravitational waves. Following And"{e}rsson, Kokkotas and Stergioulas (1999) (see also Bildsten (1998)) we use

\[
h^2 \approx \frac{4G}{c^3} \left( \frac{1}{\omega_T} \right)^2 |\dot{E}| \tag{29}
\]

and \( \dot{E} = -\omega J_{\text{acc}}/m \), where \( \omega \) is the mode frequency measured in the inertial frame to give:

\[
h \approx 2.3 \times 10^{-27} P^{1/2} M_{1.4}^{1/2} R_{10}^{1/4} M_{-8}^{1/2} d_{10} \mathrm{km}\,
\]

Since the detectability of the source increases roughly as the square-root of the number of observed cycles we find that the effective amplitude is

\[
h_{\text{eff}} \approx \sqrt{\frac{\omega T_{\text{obs}}}{2\pi}} h \sim 10^{-21} \tag{31}
\]

after a few months of observation. In other words: this signal should easily be detected by a large scale interferometric detector.

\[\text{Figure 2. The evolution of the r-mode amplitude (upper frame), the temperature (middle frame) and the spin period (bottom frame) corresponding to the first } 10^{10} \text{ s following the initial onset of r-mode instability in an accreting strange star.}\]

\[\text{Figure 3. The evolution of a young strange star (with canonical parameters } M = 1.4M_\odot \text{ and } R = 10 \text{ km} \text{ under the influence of the r-mode instability. The thin solid line represents the relevant part of the instability window from Fig. 2. The first year of the stars evolution following the first onset of instability are shown as a thick solid line.}\]

5 THE SPIN-DOWN OF YOUNG STRANGE STARS

We next want to investigate whether the spin-evolution of a young strange star is also distinct from that of a nascent neutron star. To do this we simply assume that \( \dot{M} = 0 \) in the various evolution equations from Section 3 and initialize the integration with \( P = P_K \), i.e. the star is spinning at the break-up limit when it cools to the temperature at which the r-mode first goes unstable. A typical result of this calculation, corresponding to the first year of the evolution, is shown in Figure 4.

At first sight, the result shown in Figure 4 may be a bit surprising. A strange star clearly evolves in a way that is rather different from the familiar results for hot young neutron stars, cf. Owen et al (1998). In particular, the strange star never evolves far into the instability window for the \( \ell = m = 2 \) r-mode. This is essentially because the main r-mode instability phase does not begin until a strange star has cooled to a temperature below \( 10^9 \) K. At that temperature the cooling time is considerably longer than the spin-down time due to an r-mode of sizeable amplitude. This means that, once the r-mode begins to grow it efficiently spins the star down through the bulk viscosity part of the instability curve. Once stable, the mode dies away exponentially and cooling will again govern the evolution of the star. Eventually the mode is again unstable and the cycle is repeated. After a few years the system reaches a quasi-equilibrium where the mode amplitude evolves on the cooling time scale, and the star spins down along the bulk-viscosity branch on the instability window. That this would be the case has previously been suggested by Madsen (2000).

This result has considerable repercussions for the r-modes as a gravitational-wave source. In Figure 4 we show the evolution of the r-mode amplitude corresponding to the case shown in figure 3. Here it is interesting to note that the mode amplitude varies greatly during the initial phase of r-mode spindown. Furthermore, it is clear that the evolution never leads to \( \alpha \) reaching values of order unity. This is
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We will not attempt to assess the detectability of the gravitational waves. To do this we consider the data in Figure 4. From the figure which we infer that $h \sim 10^{-27}$ and $f \sim 900$ Hz at $t \sim 10^5$ s. Then, knowing that the effective amplitude improves roughly as the square-root of the number of detected cycles, i.e. that $h_{\text{eff}} \approx \sqrt{f_{\text{obs}} h}$, we find that one would need an integration time of $t_{\text{obs}} \sim 40$ years to achieve an effective amplitude of order $10^{-21}$. This is clearly not feasible, and we can conclude that the late part of the signal shown in Figure 4 is unlikely to be detected from sources in the Virgo cluster.

The usual rationale for considering sources in the Virgo cluster is that we need a reasonable event rate to make a strong case that a certain model leads to detectable gravitational waves. In the present case, it may be sufficient to focus on sources in our own Galaxy. After all, the gravitational-wave signal from a strange star lasts considerably longer than that from a young neutron star. From Figure 4 we can see that the signal remains above $h \sim 10^{-29}$ for more than 300 years. Taking the corresponding wave frequency to be 600 Hz (again extrapolated from the figure), we find that the effective amplitude would reach $10^{-21}$ after two weeks of integration if the source was located at a distance of 10 kpc, i.e. at the centre of the Galaxy. Finally combining the expected supernova rate on 3/yr/galaxy with a lifetime of the gravitational-wave signal of 300 years or so, we deduce that there could be as many as 10 or so systems active in the Galaxy at any given time. At least if all compact stars born in supernovae are strange.

The above results offer exciting possibilities given the fact that new large scale interferometers are about to come into operation. Our study suggests that these detectors should be able to test the idea that “all neutron stars are strange stars” in the near future.

Before we conclude our discussion of young strange stars, it is worthwhile making two remarks. Firstly, one might wonder whether the r-mode driven spin-down would have observational effects in addition to the generated gravitational waves. It could, for example, leave a distinct imprint on the electromagnetic pulsar signal. After all, the r-mode phase may last for thousands of years in a young strange star born spinning at, or near, the Kepler limit and it is possible that the presence of the gravitational-wave torque could lead to some anomalous spin-down behaviour. Unfortunately, one can argue that this is unlikely to be the case, unless the star is observed in the first 10 years or so. One can easily show that the r-mode spin-down torque is dominated by the standard contribution from an electromagnetic dipole,

$$J_{\text{em}} = -\frac{2B^2R^6\Omega^3}{3c^3},$$

whenever

$$B > 1.7 \times 10^{15} \alpha M_{1.4} P_{-3}^{-4} G$$

Taking the spin-period to be 2 ms and assuming a magnetic field of $10^{12}$ G (fairly typical for a young pulsar) we find that the spin-down will be mainly electromagnetic once the mode-amplitude falls below $\alpha \sim 10^{-3}$. From Figure 4 we can see that this will happen after the first ten years or so. In other words, we should not necessarily expect the presence of a small amplitude r-mode to make an imprint on the electromagnetic signal from a young pulsar.
Secondly, we note a recent paper by Middleditch et al. (2000) which provides possible evidence for the existence of a 2.14 ms pulsar in the remnant of SN1987A. The observations were made over several years between 1992 and 1996, and the data indicate a surprisingly large, and highly variable spin-down rate. It is interesting to note that these suggestions agree qualitatively with the results of our simple spin-evolution model for a newly born strange star. In particular, it is clear that one would expect to find a highly variable $P$ during the phase where the $r$-mode amplitude oscillates on the cooling timescale. Should the Middleditch observations prove to be reliable, they may thus indicate that a strange star was born in SN1987A.

6 SUMMARY

In this paper we have studied the $r$-mode instability in the context of strange stars. We have shown that unstable $r$-modes affect strange stars in a way that is quite distinct from the neutron star case. For accreting strange stars, the onset of $r$-mode instability does not lead to a thermo-gravitational runaway. Instead, the strange star evolves towards a quasi-equilibrium on a timescale of about a year. This could explain the clustering of spin-frequencies inferred from kHz QPO data in Low-mass X-ray binaries. For young strange stars we showed that the $r$-mode driven spin-evolution is also distinct from the neutron star case. For a young strange star the $r$-mode undergoes short cycles of instability during the first few months. This is followed by a quasi-adiabatic evolution where the $r$-mode remains at a small, roughly constant, amplitude for perhaps as long as $10^5$ years. Another interesting feature of these evolutions is that the $r$-modes in a strange star never grow to large amplitudes. This could prove to be a crucially important observation since recent hydrodynamical simulations (Stergioulas & Font 2000; Lindblom, Tohline & Vallisneri 2000) indicate that the $r$-modes will not be significantly affected by nonlinear effects until at much larger amplitudes than those reached in our study.

The main conclusion of this study is that the $r$-modes in a strange star should emit a persistent gravitational-wave signal that ought to be detectable with large-scale interferometers given an observation time of weeks to months. If detected, these signals would provide unique evidence for the existence of strange stars in the Universe, which would put useful constraints on the parameters of QCD.

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APPENDIX: TWO COMPLICATING ISSUES

In this Appendix we discuss briefly two issues that could prove to be relevant for the spin-evolution of a strange star. A preliminary study shows that these effects will not alter the conclusions drawn in this paper qualitatively, but that they must be considered in any attempt to obtain reliable quantitative estimates.

6.1 Deconfinement of the crust

As mentioned in the Introduction, a strange star is likely to have a thin crust of normal nuclear matter. The inner edge of this crust is sharply defined by the nuclear drip density, since the crust relies on the electromagnetic field to suspend it above the strange matter core and save it from deconfinement. Neutrons are obviously not affected by the magnetic field and will fall into the strange core as soon as they begin to drip out of the nuclei. For a given equation of state, one can determine the maximum mass of the crust. This has been done by Glendenning and Weber (1992). Not very surprisingly, the maximum mass depends also on the rotation rate of the star. Yuan and Zhang (1999) have provided a simple analytic calculation of the maximum crust mass, which agrees reasonably well with the numerical results of Glendenning and Weber. For simplicity, we will use the analytic result here, i.e. we assume that

$$M_{\text{crust,max}} \approx \left[ 2.2 + 2.9 R_{10}^{-5} M_{1.4}^{-1} (\Omega/\Omega_K)^2 \right] \times 10^{-5} M_{\odot}. \quad (35)$$

Clearly, this estimate may have impact on a discussion of accreting strange stars.

Strange star accretion can be seen as a two-stage process. First, the accreted matter, of mass say $dM$, is added to the crust. If $M_{\text{crust}} < M_{\text{crust,max}}$ the increment $dM$ can be added to the crust without any matter migrating into the core and thus no significant accretion heating. This is particularly relevant since accretion leads to spin-up, and an increase in $M_{\text{crust,max}}$.

Conversely, in the case of a rapidly spinning down star (perhaps due to an unstable $r$-mode), a substantial portion of the crust will ‘fall’ into the core on the spindown timescale. This may then rapidly heat the core. Consider the case of spindown due to a saturated $r$-mode, for which

$$\dot{\Omega} \approx \frac{3\alpha^2 \Omega J}{R_g}. \quad (36)$$

Taking the time-derivative of (35) we have

$$\dot{M}_c \approx 5.85 \times 10^{-5} R_{10}^{-5} M_{1.4}^{-1} \frac{\Omega^2}{\Omega_K^2} M_{\odot}/s \quad (37)$$

or, after using (10),

$$\dot{M}_c = -1.5 \times 10^{-7} \alpha^2 R_{10}^{-3} M_{1.4}^{-1} P_{-3}^{-8} M_{\odot}/s \quad (38)$$

For large mode amplitudes and a spin-rate near the Kepler limit, this would correspond to a huge rate of deconfinement of the crust. The effect of this on the temperature of the star, and the evolution of the unstable $r$-mode could clearly be significant.

The above discussion does, however, come with a few caveats. The electromagnetically supported gap between the crust and core is incredibly thin: Alcock et al. (1986) suggest $10^{-13}$ m, which is of order $10^{-3}$ atomic diameters, or 10 nuclear diameters. This means that, unless the crust and core execute nearly exactly the same oscillation the two will overlap with part of the crust “dissolving” into the core. In
the limit where the crust is infinitely rigid and so does not partake in the oscillation, we can imagine the interior oscillating core dissolving the inner part of the crust, in a shell of thickness corresponding to the radial motion of the strange core. The crust would then presumably rapidly contract, with another shell being dissolved, and so on. Of course, in reality the crust is far from rigid. Being much thinner and less massive than a neutron star crust, its modulus of rigidity is very small. It follows that the crust may participate in a fluid oscillation mode to a considerable extent, as discussed in the neutron star case by Levin & Ushomirsky (2000). We have not attempted to quantify the extent to which the crust of a strange star participates in (say) a core r-mode, but it is an interesting question that may be worth further consideration.

Finally, the fact that the core-crust boundary corresponds to the density at which neutrons being to drip out of the nuclei implies that possible viscous Ekmans layers will be much less significant than in the neutron star case, cf. the comments by Madsen (2000). In view of this, we have not included effects due to viscous rubbing at the crust interface in our analysis.

6.2 Nonlinear viscosity saturation

The possibility that nonlinear contributions to the viscosity coefficients may saturate an unstable mode has been suggested by Reisenegger (private communication). However, in the case of a “normal” neutron star fluid the standard bulk viscosity is too weak for this effect to be relevant (the mode does saturate, but only at unphysical amplitudes). In the case of strange stars, the bulk viscosity is significantly stronger and thus it could well be that the nonlinear contributions to the viscosity turn out to be relevant. Incidentally, it may be worth pointing out that a similar effect may be highly relevant in the case of strong hyperon induced bulk viscosity (Jones 2001; Lindblom & Owen 2001).

We want to investigate the consequences of the fact that the low temperature bulk viscosity coefficient can be written (cf. eqn. (21) of Madsen 2000)

\[ \zeta = \frac{\hat{\alpha} T^2}{\omega^2} \left[ 1 + \frac{3}{16\pi^2} \frac{m^2 \Delta v}{3\mu_d v_0 T} \right]^2 \] (39)

In the estimate used earlier in the paper (eg. to create the data displayed in Figure 3) the \( \Delta v/v_0 \) term was neglected. The bulk viscosity calculation could, of course, be generalised in order to include this compression term but before doing this it is useful to estimate whether this contribution will ever be relevant. To do this we use

\[ \frac{\Delta v}{v_0} \approx \frac{\delta r}{R} \] (40)

where \( \delta r \) is the radial component of the r-mode displacement vector. The two terms in the bracket of (39) contribute equally when

\[ \frac{\delta r}{R} \approx 0.07\alpha \left( \frac{\Omega}{\Omega_K} \right)^2 \approx 7.5 \times 10^{-2} \mu_{\text{iso}} m_{100}^{-2} T_9 \] (41)

where we have used the estimate of the height of the r-mode at the surface of the star provided by Anderson and Kokkotas (2001). Parameterising to typical values we can use this result to get an estimate of the r-mode amplitude at which the nonlinear contribution to the bulk viscosity should not be neglected. We then get

\[ \alpha \approx 1.07T_9 \mu_{\text{iso}} m_{100}^{-2} \left( \frac{P}{P_K} \right)^2 \] (42)

from which we can deduce that the contribution may be important in stars cooler than (say) \( 10^9 \) K, i.e. in the main instability window of Figure 3.

We can also estimate the amplitude at which the r-mode saturates because of the nonlinear contribution to the viscosity. To do this (in a suitably simple way that avoids volume integration over eigenfunctions etc) we use

\[ \frac{3}{16\pi^2} \left( \frac{m^2}{3\mu_d T R} \frac{\delta r}{R} \right)^2 \approx 0.36\alpha^2 P_{-3}^{-4} m_{100}^2 \mu_{\text{iso}}^2 T_9^2 \] (43)

and modify the bulk viscosity timescale to

\[ \tau_{b,\text{new}} = \tau_b \left[ 1 + 0.36\alpha^2 P_{-3}^{-4} m_{100}^2 \mu_{\text{iso}}^2 T_9^2 \right]^{-1} \] (44)

Assuming that the mode amplitude is large (such that we can neglect the first term in the bracket of the denominator), and that the shear viscosity is suitably weak, we arrive at a saturation amplitude (by setting \( \dot{\alpha} = 0 \) the evolution equation for the mode amplitude)

\[ \alpha_s^2 \approx 19.4\mu_{\text{iso}}^2 P_{-3}^2 T_9^4 M_{1.4}^{-3} \] (45)

Although this is a rough estimate that may be far from the detailed answer it shows that the effect could, in principle, be relevant and that nonlinear viscosity saturation should be taken seriously for strange stars. However, as is clear from Figure 3 a young strange star never evolves into the regime where the nonlinear contribution to the bulk viscosity dominates. Hence, although it is conceptually interesting, we do not expect this effect to play a role in saturating an unstable r-mode in a young strange star that enters the instability window spinning at or near the breakup limit.
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