Generalized Gordon Identities, Hara Theorem and Weak Radiative Hyperon Decays

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Abstract

It is shown that an alternative form of the parity-nonconserving (PNC) transition electromagnetic current resolves partly a puzzle with the Hara theorem. New formulation of it has allowed PNC weak radiative hyperon transitions of the charged hyperons $\Sigma^+ \Rightarrow p + \gamma$ and $\Xi^- \Rightarrow \Sigma^- + \gamma$ revealing hitherto unseen transition toroid dipole moment.

1 Introduction

The weak radiative decays seem to have been first analyzed theoretically in \cite{1}. Unitary symmetry arrived, a theorem was proved by Hara that decay asymmetry of the charged hyperons vanished in the exact $SU(3)_f$ \cite{2}. Since experimental discovery of a large negative asymmetry in the radiative decay $\Sigma^+ \Rightarrow p + \gamma$ \cite{3}, confirmed later \cite{4} (see Table 1), the explanation of the net contradiction between experimental results and the Hara theorem prediction “has constitute a constant challenge to theorists” \cite{5}. The Hara theorem was formulated at the hadron level. But quark models while more or less succeeding in describing experimental data on branching ratios and asymmetry parameters (see e.g. \cite{6}) did not nevertheless reproduce the Hara claim without making vanish all asymmetry parameters in the $SU(3)_f$ symmetry limit. The origin of this discrepancy is not clear up to now although many authors have investigated this problem thoroughly \cite{7}, \cite{8}, \cite{9}, \cite{10} and is a real puzzle as similar calculations say of baryon magnetic moments are known to be rather
consistent at the quark and hadron level. Also $SU(3)_f$ symmetry breaking effects hardly can be so large due to the well-known Ademollo-Gatto theorem \cite{10} to be able to account for this puzzle.

We shall try to show that the discrepancy between the Hara theorem predictions and quark model result may be overcome due to possibility of the alternative multipole parametrization of the parity-nonconserving transition electromagnetic current which include not only dipole transition moment but also contribution of the toroid dipole moment \cite{11}, \cite{12}. Toroid dipole moment naturally arrives in the parity-violating (PV) part of the transition radiative matrix element and leads to reformulation of the Hara theorem. We shall also show that Vasant result as to the single-quark radiative transition \cite{13} is reproduced in our scheme while going from hadron to quark level allowing at the same time any sign of the asymmetry parameter.

\section{PV electromagnetic transition current and toroid dipole moment}

Let us consider PV electromagnetic transition current of the two particles with spin and parity $1/2^+$. Its possible form is not unique as the most general expression can be written in terms of 5 Lorentz structures $\gamma_\mu \gamma_5$, $P_\mu \gamma_5$, $k_\mu \gamma_5$, $\sigma_{\mu\nu} k^\nu \gamma_5$ and $i \epsilon_{\mu\nu\rho\lambda} \gamma_\nu P_\rho k_\lambda$, where $P_\mu = (p_1 + p_2)_\mu$, $k_\mu = (p_1 - p_2)_\mu$, $\sigma_{\mu\nu} = (i/2) \{ \gamma_\mu, \gamma_\nu \}$. But due to electromagnetic current conservation and generalized Gordon identities (see, e.g., \cite{11})

\begin{equation}
\pi_2 \left\{ i \epsilon_{\mu\nu\rho\lambda} P_\nu k_\lambda \gamma_\sigma \gamma_5 - i \Delta m \sigma_{\mu\nu} k_\nu + (k_\lambda^2 \gamma_\mu - \hat{k} k_\mu) \right\} \gamma_5 u_1 = 0
\end{equation}

\begin{equation}
\pi_2 \left\{ -ik_\lambda^2 \sigma_{\mu\nu} k_\nu + \Delta m (k_\lambda^2 \gamma_\mu - \hat{k} k_\mu) + \left[ k_\lambda^2 P_\mu - (k_\nu P_\nu) k_\mu \right] \right\} \gamma_5 u_1 = 0
\end{equation}

where $\Delta m = m_1 - m_2$ and $u_1, u_2$ are Dirac spinors of the baryons with masses $m_{1,2}$, this transition current can be reduced to say one of the following forms \cite{11}, \cite{12}

\begin{equation}
J^{(A)}_\mu(k_\nu) = \frac{e_\eta}{(2\pi)^3 \sqrt{1 - k_\lambda^2 / M^2}} \pi_2 \left[ \frac{1}{M^2} (k_\lambda^2 \gamma_\mu - \hat{k} k_\mu) G^{PV}_1 (k_\lambda^2) + \frac{1}{M} \sigma_{\mu\nu} k_\nu G^{PV}_2 (k_\lambda^2) \right] \gamma_5 u_1, \tag{2.3}
\end{equation}

\begin{equation}
J^{(A)}_\mu(k_\nu) = \frac{e_\eta}{(2\pi)^3 \sqrt{1 - k_\lambda^2 / M^2}} \pi_2 \left[ \frac{1}{M} \sigma_{\mu\nu} k_\mu G^{(d)} (0) + \frac{k_\lambda^2 P_\mu - (k_\nu P_\nu) k_\mu}{M(k_\lambda^2 - \Delta m^2)} \right] \left[ G^{(d)} (k_\lambda^2) - G^{(d)} (0) \right] + \frac{i}{M^2} \epsilon_{\mu\nu\rho\lambda} P_\nu k_\lambda \gamma_\sigma \gamma_5 G^{(T)} (k_\lambda^2) \gamma_5 u_1, \tag{2.4}
\end{equation}
where $\eta = \sqrt{1 - \Delta m^2/M^2}$, $M = m_1 + m_2$.

(The corresponding parity-conserving (PC) current can be obtained from Eq.(2.3) just by multiplying every structure to $\gamma_5$ and changing the superscript PV to PC.) In terms of $G_{1,2}^{PC,PV}$ the decay asymmetry is written as

$$\alpha = \frac{2Re(G_2^{PV*}(0)G_2^{PC}(0))}{|G_2^{PV}(0)|^2 + |G_2^{PC}(0)|^2}. \tag{2.5}$$

But the formfactors introduced by Eq.(2.3) do not correspond to the well-defined multipole expansion of currents [11], [12], [14], [15], [16]. That is why we would like to base our discussion on the Eq.(2.4) which as has been shown explicitly in [14] does correspond to the definite multipole expansion in a properly chosen reference system, where $k_\mu^2 = \Delta m^2 - k^2$. This reference system is given by the equality of the kinetic energies (e.k.e.) of the both baryons involved and enables us to write the nonrelativistic reduction in the form [14]

$$G^{(d)}(k_\mu^2)\pi_2 i\sigma_{\mu\nu}k_\nu\gamma_5 u_1 A_\mu \rightarrow G^{(d)}_{e.k.e.}(k^2)\phi_2^+\sigma\phi_1 [E + i\Delta mA]$$

$$\rightarrow dE - dA, \tag{2.6}$$

$$G^{(T)}(k_\mu^2)\pi_2 i\epsilon_{\mu\nu\rho\lambda}\gamma_\nu P_\rho k_\lambda u_1 A_\mu \rightarrow G^{(T)}_{e.k.e.}(k^2)k \times [k \times \sigma] A$$

$$\rightarrow G^{(T)}_{e.k.e.}(k^2)\phi_2^+\sigma\phi_1 \nabla \times B. \tag{2.7}$$

Here $d$ is the transition dipole moment, $E$ and $B$ are the electric and magnetic fields, respectively, $\phi_{1,2}$ are Pauli spinors of the baryons involved.

One can see that indeed the parametrization given by the Eq.(2.4) is a multipole one where the dipole transition and toroid dipole transition moments are given, respectively by

$$d = \frac{e}{M}G^{(d)}(0) = \frac{e}{M}G^{(d)}_{e.k.e.}(\Delta m^2), \tag{2.8}$$

$$T = \frac{e}{M^2}G^{(T)}(0) = \frac{e}{M^2}G^{(T)}_{e.k.e.}(\Delta m^2). \tag{2.9}$$

The derivatives of the formfactors $G^{(d)}(k_\lambda^2)$ and $G^{(T)}(k_\lambda^2)$ define the corresponding transition averaged radii. As

$$G_2^{PV}(k_\lambda^2) = G^{(d)}(0) + \frac{k_\lambda^2 - \Delta m^2}{M\Delta m}G^{(T)}(k_\lambda^2) \tag{2.10}$$

we obtain that

$$\frac{e}{M}G_2^{V}(0) = d - \Delta m T. \tag{2.11}$$

Note that $\Delta m$ here has pure kinematical origin, that is with $\Delta m = 0$ the decay discussed would not go. This formula partly resolves a puzzle with the Hara theorem. Indeed in the $SU(3)_f$ limit:
• The dipole transition moments of the charged hyperon decays should vanish and presumably stay small due to Ademollo-Gatto theorem [10] even in the presence of the $SU(3)_f$ breaking terms;

• The toroid transition dipole moments defined by the Eq.(2.9) need not to be zero for these decays as their contributions decouples automatically in the limit $\Delta m = 0$.

So the toroid transition dipole moment of the $\Sigma^+ \Rightarrow p + \gamma$ may be in the origin of the large asymmetry observed [4].

3 The extension of the Hara theorem

In order to state our result in another way we write the PV part of the radiative transition matrix element with the Lorentz structure $O_T^\mu = i\epsilon_{\mu\nu\lambda\rho}P_{\nu}k_{\lambda}\gamma_{\rho}$ in the framework of the $SU(3)_f$ symmetry approach following strictly [2] as

$$M = J_{\mu}^{(T)}\epsilon_{\mu} + H.C. = \{a^T(\overline{B}_3^2O_B^T B_1^1 + \overline{B}_1^3O_B^T B_3^2 + \overline{B}_1^1O_B^T B_2^3) + c^T(\overline{B}_1^3O_B^T B_2^1 + \overline{B}_1^1O_B^T B_3^1 + \overline{B}_2^1O_B^T B_3^2 + \overline{B}_3^1O_B^T B_2^3)\}\epsilon_{\mu} \quad (3.1)$$

where $B_3^2$ is the $SU(3)_f$ baryon octet, $B_1^1 = p$, $B_2^3 = \Sigma^+$ etc., and $a^T$ and $c^T$ are up to a factor the toroid dipole moments of the neutral and charged hyperon radiative transitions, respectively.

Here positive signs in front of every baryon bilinear combination arrive due to Hermitian properties of the relevant Lorentz structure. Now all 6 PV radiative transitions are open in contrast to the Hara result

$$M = J_{\mu}^{(d)}\epsilon_{\mu} + H.C. = \{a^d(\overline{B}_3^2O_B^d B_1^1 + \overline{B}_2^3O_B^d B_3^2 - \overline{B}_3^1O_B^d B_2^3) - \overline{B}_1^3O_B^d B_3^1\}\epsilon_{\mu} \quad (3.2)$$

based on another Lorentz structure form $i\sigma_{\mu\nu}k_{\nu}\gamma_5$ [2] which in turn comes to hadron-dynamics from QED. We display in the Table 2 the results of Eq.(3.1) and [2] together with the result of a traditional single-quark radiative transition which we have taken from [4]. The parameter $c^T \sim T$ in the 3rd column of the Table 2 opens a possibility to account for large nonzero asymmetry in the charged hyperon radiative decays even in the $SU(3)_f$ symmetry limit for the corresponding coupling constants.

4 New derivation of the Vasanti formula

Radiative hyperon decays were analyzed in [13] also at the quark level upon taking into account chiral invariance considerations. We shall try to rederive the main result of [13], namely, that the PV single-quark radiative transition $s \rightarrow d + \gamma$
is proportional to \((m_s - m_d)\) , using the Lorentz structure \(O_T\). At the quark level we write for the \(s \Rightarrow d + \gamma\) transition matrix element

\[
M = \bar{d}\gamma_5(A + B\gamma_5)i\epsilon_{\mu\nu\lambda\rho}P_{\nu}k_\lambda\gamma_\rho\epsilon_\mu\gamma_5s
\]

(4.1)

and upon using Eq.(2.1), where now all quark quantities are assumed, arrive at

\[
M = \bar{d}[A(m_s + m_d) + B(m_s - m_d)\gamma_5]i\sigma_{\mu\nu}k_\nu\epsilon_\mu s,
\]

(4.2)

that is in fact the main Vasanti result [13] is reproduced. The factors \((m_s \pm m_d)\) arrive due to the generalized Gordon identities. The relative signs of \(A\) and \(B\) are not fixed here so it is possible to obtain negative value of the asymmetry parameter. With the chiral invariance induced one gets exactly the Vasanti formula [13] as then \(A = B\). Note that Eq.(4.2) (with \(A = B\)) was obtained in [13] upon assuming (i) chiral invariance, (ii) validity of the original Hara theorem. We have proved in fact that the introduction of the toroid structure at the quark level is in some way equivalent to the chiral invariance approach of [13] and to the diagram approach result of [17].

This result dictates the insertion of the factor \((m_s - m_d)\) into the parameter \(c\) (see the 2nd column of the Table 2, and single-quark transition terms in [7], [8] and other works cited in [3]) to assure the correct behaviour of the corresponding quark PV transition amplitudes. And \textit{vice versa} the results of [13] and [17] together with the generalized Gordon identities have just shown that at the quark level it is a toroid dipole moment which is generated with its characteristic Lorentz structure \(O_T = i\epsilon_{\mu\nu\lambda\rho}P_{\nu}k_\lambda\gamma_\rho\).

5 Summary and Conclusion

In order to resolve a contradiction between the experiments claiming large negative asymmetry in \(\Sigma^+ \Rightarrow p + \gamma\), the Hara theorem, predicting zero asymmetry for \(\Sigma^+ \Rightarrow p + \gamma\) and \(\Xi^- \Rightarrow \Sigma^- + \gamma\) in the exact \(SU(3)_f\) symmetry and quark models which cannot reproduce the Hara theorem results without making vanish all asymmetry parameters in the \(SU(3)_f\) symmetry limit, we have considered a parity-violating part of the transition electromagnetic current in the alternative form allowing well-defined multipole expansion. Part of it which is connected with the Lorentz structure \(i\epsilon_{\mu\nu\lambda\rho}P_{\nu}k_\lambda\gamma_\rho\) enables as to reformulate the Hara theorem thus opening a possibility of nonzero asymmetry parameters for all 6 weak radiative hyperon decays and revealing hitherto unseen transition toroid dipole moments. Our result is consistent with the traditional results of the single-quark transition models (see column 2 of the Table 2) if the relevant parameter has an intrinsic kinematical factor \(\Delta m\). We also have reproduced Vasanti formula at the quark level.

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Table 1. Hyperon radiative transitions, experiment

| Transition   | BR     | Asymmetry   |
|--------------|--------|-------------|
| $\Sigma^+ \to p\gamma$ | 1.23 ± 0.06 | −0.76 ± 0.08 |
| $\Sigma^0 \to n\gamma$ | −       | −           |
| $\Lambda^0 \to n\gamma$ | 1.63 ± 0.14 | −           |
| $\Xi^0 \to \Lambda\gamma$ | 1.06 ± 0.16 | +0.44 ± 0.44 |
| $\Xi^0 \to \Sigma^0\gamma$ | 3.56 ± 0.43 | +0.20 ± 0.32 |
| $\Xi^- \to \Sigma^-\gamma$ | 0.128 ± 0.023 | +1.0 ± 1.3 |

Table 2. Hyperon radiative transitions, theory

| Transition PNC amplitude | in [7] | in [2] | from Eqs.(3.1) |
|--------------------------|--------|--------|----------------|
| $\Sigma^+ \to p\gamma$  | $-\frac{4}{3}b$ | 0      | $c^d$          |
| $\Sigma^0 \to n\gamma$  | $\frac{1}{3\sqrt{2}}b$ | $\frac{1}{\sqrt{2}}a^d$ | $\frac{1}{\sqrt{2}}a^d$ |
| $\Lambda^0 \to n\gamma$ | $\frac{3}{\sqrt{6}}b$ | $\frac{1}{\sqrt{6}}a^d$ | $-\frac{1}{\sqrt{6}}a^d$ |
| $\Xi^0 \to \Lambda\gamma$ | $\frac{1}{\sqrt{6}}b$ | $-\frac{1}{\sqrt{6}}a^d$ | $\frac{1}{\sqrt{6}}a^d$ |
| $\Xi^0 \to \Sigma^0\gamma$ | $-\frac{1}{3\sqrt{2}}b$ | $\frac{1}{\sqrt{2}}a^d$ | $-\frac{1}{\sqrt{2}}a^d$ |
| $\Xi^- \to \Sigma^-\gamma$ | $\frac{2}{3}b$ | 0      | $c^d$          |