ABSTRACT

Background: Methodological ground for compromise – rent strategy of loan debt management was settled by N.E. Egorova and A.M. Smulov (Businesses and banks: interaction, economic analysis, modeling, 2005). Article takes point of view on compromise – rent strategy as a time-sensitive approach to reduce the volume of bad loans.

Aims: We aimed to define the best approach to bad debt management in the volatile economic conditions.

Study Design: Analytical review.

Place and Duration of Study: CEMI Russian Academy of Sciences, 2013-2015.

Methodology: Methodology is based on complex approach which includes analysis of current condition of the banking system, fuzzy sets implementation in possible solutions of bad debt retirement.

Results: The authors proposed methodological principles and Economics and Mathematics bad debt management tools on the basis of the approach of the theory of fuzzy numbers, established a set of mathematical equations to reflect the balance between time of the debt retirement, quality of...
the debt and payment flows.

**Conclusion:** Issues of bad debt prediction are among the most topical at the present stage of development of the global banking system. The complexity of these issues is largely due to the significant uncertainty factors that accompany the processes of crisis and instability in the financial world system.

**Keywords:** Bad debt; banking system; compromise – rent strategy; debt retirement.

1. **INTRODUCTION**

In many ways, high-risk nature of banking determines the problem of interaction between the state, banks and enterprises, which is rather a difficult task.

The financial market is subject to strong influence from investment and speculative movement of transnational capital. External borrowings market disruption leads to inaccessibility of new cheap cash resources. At the time of economic stagnation and political instability there is a reduction of volume of investment and the loan portfolio, primarily due to the decrease of contributions to the industrial sector of the economy and fall of its quality - there is an increase of overdue debt. This is accompanied by cash outflow of deposits from both individuals and legal entities. Due to the lack of liquidity, the volume of investments in securities and shares is reduced.

Banking organizations due to the loss of liquidity are not able to refinance the company with bad debt (BD) and carry out the necessary credit and investment policy to generate profits, and companies are not able to generate the necessary cash flow at a level sufficient to debt servicing. In this regard, the task of BD managing is relevant for the present stage of the banking system [1].

Being in the high-risk area credit and investment activities of banks essentially depend on the structure of the investment portfolio that is placed in a variety of projects with different risk level [2].

Companies are to make loan repayments according to the contractual terms, regardless of its financial and economic situation. Sometimes it occurs that some investments can be completely or partially lost.

In today's economic climate, there is a rapid decrease in quality of loan portfolio generated during the period of accessible and cheap money. The result was companies and banking institutions BD increase [3]. Bad debt recovery becomes the issue that intimidate banking systems of different countries. According to Bloomberg, 92 of the largest US banks with assets more than 10 billion USD have BD 1.19% of the total portfolio at September 30, 2015; whereas 105 largest banks in Europe (excluding Switzerland) at the end of June had share of BD 5.6% [4]. In Russia, the share of overdue loans of individuals in the total loan portfolio reached even higher numbers; according to the Central Bank, it reached 8.3%. BD on corporate portfolio grew in January 2015 up to 12.1%. The share of overdue debt on loans to businesses rose during the same period from 6.2 to 6.8% [5].

BD dynamics depends not only on the development of general economic conditions, but also largely on the investment banks selected credit activity. Under strategy of loan debt management, we understand a set of common principles and rules as well as models ensuring this strategy. On the basis of the strategy banks accept credit and investment solutions to achieve the desired objectives in the long term. It appeared to be an urgent task to develop methods of strategic planning and management of investment and lending activities of banks, including BD recovery scheme [3].

Currently, economic and mathematical methods of strategic planning and management of bank investment activities continues to improve. However, along with the progress made in the development of these methods, some aspects of this problem are studied insufficiently. For example, the strategy of credit and investment resources of banks in the situation of banking crisis and difficult economic situation is not completely studied; procedure for finding the optimal structure of the total amount of loans and selection of the method of loan debt management is insufficiently formalized.

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1 Bad debt (BD) refers to three categories of adversely classified loans: substandard, which are unduly risky and, if unimproved, may be a future hazard; doubtful, whose collection is highly questionable and improbable; and loss, which are considered noncollectable.
Thus, the task of developing methods of strategic planning and management of investment activity of banks remains insufficiently studied.

Formation of optimal BD management strategies is based on evaluation of its effectiveness, which is characterized by the index equal to the difference between the reached (or forecasted) results on BD reduction, and the cost of operation of the control strategy for a certain time interval. The optimal management strategy corresponds to a set of solutions that provides maximum value of performance indicator.

2. TARGETING

Compromise-rent strategy is one widely adopted and successfully applied in practice strategy that allows determining the amount of the optimal rental payment for the debtor and the bank-creditor [6,7]. It is based on the separation of BD (referred to hereinafter S_3) into two parts. The first of these parts is S_1 that comprises portion of debt that cannot be paid (loss) and should be sold. It is believed that there is some strategic investor who trades S_1 in securities on quasiequivalence conditions. For the second part S_2 payment flow is formed on the new conditions, taking into account the current financial and economic condition of the debtor company, as well as the forecasted development of the economic situation. Thus S_1 + S_2 = S_3.

Different scenarios for the development of the economic situation are influenced by: 1) refinancing interest rate - for the banking organization; and 2) interest rate - for the financed enterprises, companies. The payments flow is determined for each period (usually a quarter) generated by the strategy used [8].

To generate the required BD management strategy is an important to take into account initial distribution of its volume in the S_1 and S_2. The value of the indicator of economic efficiency of this strategy, depending on the adopted BD structure will be different due to the different scope and quality of loans in each of the parts. For example, a larger volume and lower quality of S_2 require more time for its elimination; on the other hand, lower quality and more significant amount of S_1 leads to a corresponding increase in the number of different securities received in the course of their purchase, realization of which also require significant costs [9].

Thus, there is a balance between parts S_1 and S_2, when the index of economic efficiency will be on the maximum level. This ratio will be optimal in the Pareto sense, ie, through reallocation, improvements cannot be made to at least one criteria (the ratio of the volume and quality) without reducing any other (at least one) criteria [10,11]. The indicator of the cost-effectiveness of the strategy will also depend on the initial state of the BD and the external economic environment. [6].

In this paper, the authors proposed to take a decision on BD retirement based on a compromise-rent strategy [7] in the synthesis of the theory of fuzzy sets [11]. Compromise-rent strategy is a highly effective tool for BD managing, but its use is hampered by uncertainties, because you cannot possibly know the impact of forecast economic and financial conditions on credit worthiness of the company in the future. However, with reasonable certainty, for example, based on expert judgment, we can assume that the values of BD will be in a given range, and some of these ranges in some sense will be preferable to others. Fuzzy numbers or a probability distribution [12] are commonly used for the mathematical description of these parameters. The use of probabilistic approach in this case is difficult because of the impossibility to obtain representative statistical observations of random variable BD behavior.

A fuzzy number is characterized by its membership function - \( \psi(P) \), which domain is so-called universal set, which includes all default values of the analyzed parameter; and a range of values is the unit interval \([0,1]\). This membership function defines the degree of membership of P element in fuzzy set. The higher the value of the membership function, the higher the degree of membership estimated universal set element to the fuzzy set [11,12]. In this case, we consider the fuzzy number with a convex membership function. Such arbitrary fuzzy number \( \tilde{P} \) - is shown on Fig. 1 with its membership function.

On Fig. 1 the ordinate value in interval \([0,1]\), membership function \( \psi \) of fuzzy numbers is called the level of membership \( \alpha \). At the same time abscissa points with identical ordinate values of the membership function gives two meanings. The lower and upper value of the fuzzy number on the level \( \alpha \), these meanings are also called the borders of credibility interval on the \( \alpha \) level, for example \([P_01, P_02]\) have credibility interval on the \( \alpha \) level, and \([P_01, P_02]\) at zero level.
Fig. 1. Membership function of arbitrary fuzzy number

Fig. 2 shows commonly used in practice fuzzy number described by their membership functions: π-like, trapezoidal, triangular, sigmoid, Gaussian [12], the choice of which is determined by the economic nature of the object.

For the case the most characteristic is π-like, and trapezoidal membership functions of fuzzy number, which characterizes the variable BD. This is due to the presence of restrictions (regulations) in banking which do not allow increasing BD above specified limit. This limit is defined with the help of as minimum two values: either as a regulator formed by the parent organization, either as intra-planning regulator (if it is defined more strictly than the external controller). However, further in order to simplify we use triangular membership function [13].

3. ESTIMATION OF PERSPECTIVES

Thus, since the structure of the BD and its parameters have a certain character; the problem of finding the optimal BD management strategy is formulated in terms of the theory of fuzzy sets:

\[ \tilde{S}_1 + \tilde{S}_2 = \tilde{S}_0 \]  \hspace{1cm} (1)

\[ 0 \leq \tilde{S}_1 \leq S_{1\text{max}} \]  \hspace{1cm} (2)

\[ 0 \leq \tilde{S}_2 \leq S_{2\text{max}} \]  \hspace{1cm} (3)

\[ \tilde{P}( \tilde{S}_1, \tilde{S}_2, T ) \rightarrow \text{max} \]  \hspace{1cm} (4)

where \( \tilde{S}_1, \tilde{S}_2 \) and \( \tilde{S}_0 \) - fuzzy numbers, the sign ~ identifies fuzzy variable.

The maximum value of BD, intended for the exchange for securities - \( S_{1\text{max}} \) is given by banking institution, taking into account the external economic environment and the quality of the asset. The maximum value of the BD - \( S_{2\text{max}} \) to form payment flow in the new conditions is given by the banking organization, taking into account the external economic environment and the financial capacity of the debtor company and taking into account the bank's claims with a maturity of credit - T.

The presented fuzzy model in the form of the optimization problem (1-4) allows you to create optimum BD management strategy in fuzzy forecast economic conditions, and to find the optimal ratio between \( S_1 \) and \( S_2 \).

The next step is to find a compromise solution for the exchange of \( S_1 \) for securities. A private entrepreneur or government organization can act as a strategic investor acquiring debts. In this regard, the proposed methodological approaches can be used in the implementation of public-private partnership (PPP). Typically, the state as a strategic investor assumes the liabilities of socially important enterprises. After state acts as a financial intermediary in the interaction with the business for the purpose of rehabilitation and development of the national economy priority projects [14].

In order to solve the problems of debt \( S_2 \), it is necessary to form a payment flow, which is determined for a fixed period of time (usually a quarter). This sets a new maturity of the debt equal to T quarters and forms the necessary quarterly equal payment flow - loan payments.
until maturity. From the equality of the entire volume of debt and the amount of payments for time $T$, taking into account the interest rate, the equal fuzzy payment flow is determined by the equation:

$$\tilde{S} = \tilde{r}_k \tilde{S}_0 (1 + \tilde{r}_k)^T / ((1 + \tilde{r}_k)^T - 1)$$  \hspace{1cm} (5)$$

Here $\tilde{S}$ is the current loan payments; $\tilde{S}_0$ - initial BD; $T$ - number of periods (quarters); $\tilde{r}_k$ - interest rate. Here the interest rate as a forecasted parameter is also uncertain and appears in fuzzy number.

If the company due to its financial and economic capacity cannot carry out the current quarterly payment more than $\tilde{S}_{\text{max}}$ (when investment and reproduction processes become impossible, then time of the debt retirement $T$ at the same payout equal to $\tilde{S}$ is found in the next power equation:

$$(1 + \tilde{r}_k)^T = \tilde{S} / (\tilde{r}_k \tilde{S}_0),$$

$$\tilde{S} \leq \tilde{S}_{\text{max}}.$$  \hspace{1cm} (6)$$

Bank losses in the form of interest arising from the non-repayment of funds on loans refer to the operating costs of the strategy. Current (quarterly) cost value is determined from the equation:

$$\tilde{z}_i = \tilde{S} [(1 + \tilde{r}_k)^i - 1].$$  \hspace{1cm} (7)$$

Then the cost-effectiveness index of the selected strategy $P_r$ for each stage of the time $\tau$ in the interval $[1, T]$ is found in the equation:

$$P_r = \left( \sum_{i=1}^{\tau} \frac{\tilde{S}_i}{(1 + \tilde{r}_k)^i} \right) - \left( \sum_{i=1}^{\tau} \frac{\tilde{z}_i}{(1 + \tilde{r}_k)^i} \right).$$  \hspace{1cm} (8)$$

$\tilde{S}_i$ is determined by the equation (5) for each $T = i$, or selected according to the inequality (6), and the current value $\tilde{z}_i$ is calculated by equation (7). The BD decrease generated by management strategy that provides payments $S_i$ at the time $t = i$ and the costs of operation of the strategy - $\tilde{z}_i$ at each time step are given to the initial moment of time by discounting procedure using a discount rate, which is as forecasted parameter, is uncertain and is also given in fuzzy number.

If we consider that part of BD intended for the exchange for securities, is liquidated at a known cost in the first period (quarter), than for the first quarter in the equation (8) to the quarterly payment is added value $\tilde{S}_1$, and to the cost - the known losses during its liquidation.

Thus, the indicator of economic efficiency is equal to the discounted cash received by bank from the borrower for a time $\tau$ taking into account interest rates due to non-repayment of funds on loans, as well as costs of the strategy functioning.

4. CONCLUSIONS

Issues of BD prediction are among the most topical at the present stage of development of the global banking system. The complexity of these issues is largely due to the significant uncertainty factors that accompany the processes of crisis and instability in the financial world system [15].

In these circumstances, banks should not only follow the dynamics of the BD and overcome its steady growth, but also control the BD structure.

Deterministic approaches to solving this problem are usually ineffective. Indeed, now it is difficult to predict what part of the loans given to enterprises is transformed into BD. It is difficult to predict if there will be a strategic investor, ready to buy up debt, and what are the conditions under which the transaction will be carried out; how to implement the debt restructuring of those enterprises whose condition approached the verge of bankruptcy, etc.

The authors proposed methodological principles and Economics and Mathematics BD management tools on the basis of the approach of the theory of fuzzy numbers. It allows formulating the optimal control strategy of loan debt, based on a comprehensive assessment of the economic efficiency of its two parts. The first part is BD to be sold to a strategic investor, and the second one is a debt restructured under compromise - rent principle, taking into account
the interests of both the bank and the real possibility of the company to repay the debt. Methodological approaches formulated in the article can be widely used, inter alia, in the implementation of PPPs mechanisms, matching the economic interests of the state, private business and the banks.

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COMPETING INTERESTS

Authors have declared that no competing interests exist.

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