Contractor selection for construction project, with the use of fuzzy preference relation

Nabi Ibadova

*The Institute of Building Engineering, The Faculty of Civil Engineering, Warsaw University of Technology, Al. Armii Ludowej 16, 00-637 Warszawa, Poland

Abstract

During the phase of investment planning, choice of contractor is one of the most important decisions. One of the methods for assessing competences of contractors applying for the contract is their pre-selection. In the article, author describes algorithm for choice (selection) of contractor. The algorithm is based on fuzzy preference relation. In mathematical point of view, it is based on ordering theory and fuzzy sets theory. The article provides an example of the algorithm used for selection of contractor for construction works. The choice was made basing on criteria such as: reputation, technical capabilities, financial situation and organizational skills.

Keywords: Contractor selection; Contractor selection criteria; Fuzzy preference relation; Construction project planning

1. Introduction

The key stage for every construction project is its implementation. That stage is connected with the selection of contractor company. Choice of the right contractor strongly affects successful completion of construction works. In Poland, public investors are able to use one of the few selection (tendering) procedures (specified by national law). These procedures differ in regard to degree and scope of contractors’ competences evaluation. On the other hand, private clients are able to use any selection procedure they want. One of the popular procedures (used in many countries) is prequalification [7], [8].

* Corresponding author. Tel.: +48 22 234 65 15; fax: +48 22 825 74 15.
E-mail address: n.ibadov@il.pw.edu.pl
When deciding to use prequalification, the investor has to take into consideration many decisions regarding the course of the procedure and rules of contractors evaluation. Among the most important decisions are: setting criteria for contractors’ competences evaluation, deciding on the source of information and documents used during evaluation process, setting weight for different criteria [5], [7].

The choice of the right selection (assessment) criteria is crucial. The analysis of the problem published in professional journals and prequalification procedures, shows that different authors suggest different selection criteria [1], [5], [7], [8], [9], [10], [11]. However most of these criteria can be grouped into several categories, like: technical competences and financial capabilities of the contractor.

Technical competences include: qualifications and competence of personnel (for example in regard to design, implementation and management); experience gained in similar projects; quality management and assurance; effective project controlling; H&S management; environmental aspects; etc.

Financial capabilities include: budget and finances management for the project; reliable financing source and structure; innovative methods of financing; financing capacity and risk prevention skills; financial guarantees; etc.

Of course, the scope of criteria depends on project’s specification and complexity, as well as, on specific decision situation [2]. The criteria should describe the situation in the best possible manner, however their scope character and components may slightly differ [3].

Due to the uncertainty of decision-making situation, the decision maker may not have well-defined preferences [4]. In such case, author suggests use of fuzzy preference relation for both assessing contractors in terms of single criterion, and a set of criteria.

2. Fuzzy preference relation – basic concepts

It is presumed for the fuzzy preference relations that they comply with the conditions of transitivity, consistency and reflexivity, as well as, with other characteristics associated with these conditions and all the operations carried out on fuzzy sets [12].

Normal (non-fuzzy) binary relation R on the set X is a subset of Cartesian product $X \times X$ [6]. Binary relations are used for specifying relations between elements of X. For example, if for a pair $(x, y)$ of elements belonging to set $X$, $(x, y) \in R$, than we say that there is a relation R for pair $(x, y)$. Sometimes, in such case we can write it down like this: $xRy$. The classic example of binary relation is relation “not less, than” on the set of real numbers, for which: $(x, y) \in R \iff x \geq y$.

Fuzzy relation $\tilde{R}$ on the set X is called fuzzy subset of Cartesian product $X \times X$, which is characterized by membership function $\mu_{\tilde{R}} : X \times X \to [0,1]$. The operation of intersection of fuzzy binary relations $\tilde{R}_1$ and $\tilde{R}_2$ on the set X is called fuzzy binary relation $\tilde{R}$ with the membership function [6]:

$$\mu_{\tilde{R}}(x, y) = \sup_{z \in X} \min[\mu_{\tilde{R}_1}(x, z); \mu_{\tilde{R}_2}(z, y)]. \quad (1)$$

Fuzzy binary relation $\tilde{R}$ is symmetric, if for any $x, y \in X$, $\mu_{\tilde{R}}(x, y) = \mu_{\tilde{R}}(y, x)$ and antisymmetric, if $\mu_{\tilde{R}}(x, y) > 0$, than $\mu_{\tilde{R}}(y, x) = 0$. An example of fuzzy relation symmetry is the equivalence relation – alternative indifference relation.

Fuzzy relation is called transitive, if $\tilde{R} \circ \tilde{R} \subseteq \tilde{R}$. Transitivity is one of the basic characteristics of rational preference relation.

While making a decision, expert/decision-maker may not have specific preference in regard to all or some alternatives. In such case, we are dealing with fuzzy preference relation, which membership function for each pair $x, y \in X$ determines the genuineness (authenticity) $\mu_{\tilde{R}}(x, y) \in [0,1]$ of the fact, that alternative x is “not worse” than
alternative $y$. Fuzzy preference relation needs to be reflexive, so that each alternative is genuinely not worse than itself. Basing on the fuzzy relation $\sim R$ one can establish fuzzy relation of strong preference $\sim \text{sp}$ $R$. Membership function for strong preference fuzzy relation can be defined as follows [6]:

\[
\mu_{\text{sp}}(x, y) = \begin{cases} 
\mu_{\sim R}(x, y) - \mu_{\sim R}(y, x), & \text{if} \quad \mu_{\sim R}(x, y) \geq \mu_{\sim R}(y, x), \\
0, & \text{in other case}
\end{cases} 
\]

(2)

Fuzzy preference relation allows for comparison of mutual preferability of alternatives. If on the set of alternatives, there is established a fuzzy preference relation $\sim R$, than during process of decision making, it is determined which of the alternatives is best, from decision maker's point of view. Such alternatives are being called: non-dominated (admissible).

If we define the set of alternatives by $X$ and its fuzzy preference relations’ membership functions by $\mu_{R}$, than fuzzy subset of non-dominated alternatives for set $(X, \mu_{R})$ can be described by following membership function [6]:

\[
\mu_{n}^{\text{nd}} = 1 - \sup_{x,y \in X} (\mu_{R}(y,x) - \mu_{R}(x,y)).
\]

(3)

With the use of these formulas and operations on fuzzy sets presented in [12] paper (union, intersection, and complementation), one can solve the task of contractor selection for construction project and create hierarchy of alternatives, that takes into account several evaluation criteria.

3. Description of algorithm for solving contractor selection problem

Let’s assume that on the set of alternatives (contractors) $X = \{x_i\}$, fuzzy preference relations $R_1, R_2, \ldots R_m$ are established with the adequate membership functions $\mu_{R_i}(x_i, x_j)$ and weights $w_k$ for adequate relations, which are resulting from importance of evaluation criteria. One needs to find the best alternative (contractor) from the set $\{X, R_k\}$.

The solution for such problem can be found, by following the procedure below:

1. Create $n \times n$ matrices for relations $R_1, R_2, \ldots R_m$ along with adequate membership functions $\mu_{R_i}(x_i, x_j)$ with the use of formula (4):

\[
\mu_{R_i}(x_i, x_j) = \begin{cases} 
1, & \text{if} \quad x_i \geq x_j \text{ or } x_j \approx x_j \\
0, & \text{if} \quad x_i < x_j
\end{cases}.
\]

(4)

2. Create fuzzy relation $P_1 = R_1 \cap R_2 \cap \ldots \cap R_m$. It is a matrix $n \times n$, which elements $\mu_{P_1}(x_i, x_j)$ are described by the formula (5):

\[
\mu_{P_1}(x_i, x_j) = \min\{\mu_{1}(x_i, x_j), \ldots, \mu_{m}(x_i, x_j)\}.
\]

(5)

3. Define subset of non-dominated (admissible) alternatives $x_i$ in the set $\{X, P_1\}$ according to the formula (6):

\[
\mu_{P_1}^{\text{nd}}(x_i) = 1 - \sup_{x_j} [\mu_{P_1}(x_j, x_i) - \mu_{P_1}(x_i, x_j)].
\]

(6)
4. Create \( n \times n \) matrix for fuzzy relation \( P_2 \), taking into consideration criteria weights \( w_k \), which elements are described by formula (7):

\[
\mu_p(x_i, x_j) = \sum_{k=1}^{m} w_k \mu_k(x_i, x_j).
\]

5. Define subset of non-dominated (admissible) alternatives \( x_i \) in the set \( \{X, P_2^1\} \) according to the formula (8):

\[
\mu_{P_2^m}(x_i) = 1 - \sup_{x_j} [\mu_{P_2^1}(x_j, x_i) - \mu_{P_2^1}(x_i, x_j)].
\]

This function will order alternatives in regard to their degree of admissibility.

6. Find intersection of membership functions \( \mu_{P_2^m} \) and \( \mu_{P_2^m} \) according to the formula (9):

\[
\mu_{P_2^m}(x_i) = \min\{\mu_{P_2^m}(x_i), \mu_{P_2^m}(x_i)\}, \quad (9)
\]

where:

\( \mu_{P_2^m}(x_i) \) – states the degree of admissibility for alternative \( x_i \). It means, that the higher the value \( \mu_{P_2^m}(x_i) \), the “better” the alternative \( x_i \) is.

7. The choice of the best alternative from the set \( X_{ad} \) (most adequate in regard to chosen principles of evaluation and criteria importance) is made by the use of formula (10):

\[
\mu_{P_2^m}(x) = \sup_{x \in X} \mu_{P_2^m}(x_i), \quad (10)
\]

Let’s use the presented algorithm in exemplary task of contractor selection.

4. Example of contractor selection

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Let’s assume that the client needs to make a choice. He has to select a contractor from the set \( D=(d_1, \ldots, d_n) \) of construction companies. The investor (decision-maker) uses following criteria: \( K_1 \) - reputation; \( K_2 \) - technical capabilities; \( K_3 \) - financial situation and \( K_4 \) - organizational skills.

According to expert’s evaluation, on the base of the above criteria, following preference relations are established on the set of alternative contractors \( D \):

\[
R_1 : d_1 \succ d_2, d_2 \approx d_3, d_3 \succ d_4, \\
R_2 : d_1 \approx d_2, d_3 \succ d_4, d_4 \approx d_1, \\
R_3 : d_1 \approx d_3, d_3 \approx d_4, d_4 \approx d_2, \\
R_4 : d_1 \approx d_4, d_2 \approx d_3, d_3 \approx d_2, \\
\]

It is necessary to find compromise on established criteria, with the use of preference relations composition \( P_1 , P_2 \). Weights for criteria were assumed as follows: \( w_1=0.2; w_2=0.3; w_3=0.3; w_4=0.2 \).

Assuming fulfillment of transitivity condition with the use of formula (4), we need to create relations’ matrices \( R_1, R_2, \ldots R_4 \), presented in tables 1-4:
Table 1. Matrix for preference relation $R_1$

| $d_i / d_j$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|-------------|-------|-------|-------|-------|
| $d_1$       | 1     | 1     | 1     | 1     |
| $d_2$       | 0     | 1     | 1     | 1     |
| $d_3$       | 0     | 1     | 1     | 1     |
| $d_4$       | 0     | 0     | 0     | 1     |

$\mu_{R_1}(d_i, d_j) = $

Table 2. Matrix for preference relation $R_2$

| $d_i / d_j$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|-------------|-------|-------|-------|-------|
| $d_1$       | 1     | 1     | 0     | 0     |
| $d_2$       | 1     | 1     | 0     | 0     |
| $d_3$       | 1     | 0     | 1     | 1     |
| $d_4$       | 1     | 1     | 0     | 1     |

$\mu_{R_2}(d_i, d_j) = $

Table 3. Matrix for preference relation $R_3$

| $d_i / d_j$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|-------------|-------|-------|-------|-------|
| $d_1$       | 1     | 1     | 1     | 1     |
| $d_2$       | 0     | 1     | 1     | 0     |
| $d_3$       | 0     | 1     | 1     | 0     |
| $d_4$       | 1     | 1     | 1     | 1     |

$\mu_{R_3}(d_i, d_j) = $

Table 4. Matrix for preference relation $R_4$

| $d_i / d_j$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|-------------|-------|-------|-------|-------|
| $d_1$       | 1     | 1     | 1     | 1     |
| $d_2$       | 0     | 1     | 1     | 0     |
| $d_3$       | 0     | 1     | 1     | 0     |
| $d_4$       | 1     | 1     | 1     | 1     |

$\mu_{R_4}(d_i, d_j) = $

In the next step, we create relation matrix $P_1$, which is an intersection of relations $R_1, R_2, \ldots R_4$ with adequate elements $\mu_{P_i}(d_i, d_j)$. Table 5 presents relation matrix $P_1$.

Table 5. Matrix for relation $P_1 = R_1 \cap R_2 \cap R_3 \cap R_4$

| $d_i / d_j$ | $d_1$ | $d_2$ | $d_3$ | $d_4$ |
|-------------|-------|-------|-------|-------|
| $d_1$       | 1     | 1     | 0     | 0     |
| $d_2$       | 0     | 1     | 0     | 0     |
| $d_3$       | 0     | 1     | 1     | 0     |
| $d_4$       | 0     | 0     | 0     | 1     |

$\mu_{P_1}(d_i, d_j) = $

We find subset of non-dominated (admissible) alternatives $d_i$ in the set $\{D, P_1\}$ according to the formula (6):

$$\mu_{P_1}^n(d_i) = 1 - \sup\{0 - 1; 0 - 0; 0 - 0\} = 1,$$

(12)
\[ \mu_{P_1}^{nd}(d_2) = 1 - \sup \{1 - 0; 0; 0; 0 - 0\} = 0, \]  
\[ \mu_{P_1}^{nd}(d_3) = 1 - \sup \{0 - 0; 0 - 0; 1 - 0\} = 1, \]  
\[ \mu_{P_1}^{nd}(d_4) = 1 - \sup \{0 - 0; 0 - 0; 0 - 0\} = 1, \]

thus we have:

\[ \mu_{P_1}^{nd}(d) = [1/d_1; 0/d_2; 1/d_3; 1/d_4]. \]

Taking into consideration criteria weights, \( n \times n \) matrix is created for fuzzy relation \( P_2 \) with the use of equation (7). The matrix is presented in table 6.

| Table 6. Matrix for relation \( P_2 \) |
|---------------------------------------|
| \( d_i \) | \( d_1 \) | \( d_2 \) | \( d_3 \) | \( d_4 \) |
|----------|-------|-------|-------|-------|
| \( d_1 \) | 1     | 1     | 0.7   | 0.7   |
| \( d_2 \) | 0.3   | 1     | 0.4   | 0.2   |
| \( d_3 \) | 0.6   | 1     | 1     | 0.8   |
| \( d_4 \) | 0.8   | 0.8   | 0.5   | 1     |

We find subset of non-dominated (admissible) alternatives \( d_i \) in the set \( \{D, P_2^\supseteq\} \) according to the formula (8):

\[ \mu_{P_2}^{nd}(d_1) = 1 - \sup \{0.3 - 1; 0.6 - 0.7; 0.8 - 0.7\} = 0.9, \]

\[ \mu_{P_2}^{nd}(d_2) = 1 - \sup \{1 - 0.3; 1 - 0.4; 0.8 - 0.2\} = 0.3, \]

\[ \mu_{P_2}^{nd}(d_3) = 1 - \sup \{0.7 - 0.6; 0.4 - 1; 0.5 - 0.8\} = 0.9, \]

\[ \mu_{P_2}^{nd}(d_4) = 1 - \sup \{0.7 - 0.8; 0.2 - 0.8; 0.8 - 0.5\} = 0.7, \]

thus we have:

\[ \mu_{P_2}^{nd}(d) = [0.9/d_1; 0.3/d_2; 0.9/d_3; 0.7/d_4]. \]

With the use of formula (9), we find intersection of membership functions \( \mu_{P_1}^{nd}(d) \) and \( \mu_{P_2}^{nd}(d) \) of non-dominated (admissible) alternatives \( P_1 \) and \( P_2 \):

\[ \mu^{nd}(d) = \min \{\mu_{P_1}^{nd}(d), \mu_{P_2}^{nd}(d)\} = [0.9/d_1; 0/d_2; 0.9/d_3; 0.7/d_4] \]

Thus, according to the formula (10) we can state, that the best contractors (alternatives) are \( d_1 \) and \( d_3 \).
5. Conclusions

According to the presented example, client should choose between contractors d₁ and d₃. In regard to the assumed weights and fuzzy preference relation, they are equally preferable. In such case final selection of the contractor may be made on the base of the bid amount (cost), as in Polish public procurement law it is the most important criterion. It is worth to note, that weight for criteria in this example was estimated only for the purpose of presenting described method.

Creation of fuzzy preference relation allows for hierarchization of all contractors (worst to best), so in case of withdrawal of the best construction company, client is able to replace it with the second best. In the provided example, if both d₁ and d₃ withdraw their offers, they can be replaced with d₄.

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