We study the evolution of the universe by assuming an integrated model, which involves interacting dark energy and holographic principle with Hubble scale as IR cutoff. First we determined the interaction rate at which relativistic matter is converting to dark energy. In the next step, we evaluated the equation of state parameter which describes the nature of dark energy. Our result predicts that the present state of the universe is phantom dominated which is highly likely by observational data. Again our analysis successfully addresses the problem of present accelerated expansion of the universe. We also found that the universe was previously undergoing a decelerated expansion and transition from deceleration to acceleration occurs at a time of \( t_q = 0 \approx 0.798 t_0 \), where \( t_0 \) is the present age of the universe.

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I. INTRODUCTION

One of the most intriguing discovery of modern day cosmology is the recent accelerated expansion of the Universe, which was first predicted by the observations of Supernova Ia \[1, 2\] and subsequently confirmed by the observation of Cosmic Microwave Background Radiation \[3, 4\], Weak Lensing \[5\], Large-scale Structure \[6–8\], WMAP Data \[9\] and Planck Data \[10\]. As a possible theoretical explanation of this major break through, it is considered that the most part of the present Universe is made up of a form of energy that exerts a negative pressure and drives the acceleration. The unknown nature of such energy brings it a name Dark Energy and recent observational data \[11\] suggest that 68.3 percentage of the present Universe is this unknown form of energy. On the other hand, the rest content of the universe is gravitating matter with a large part is non-baryonic and is called Dark matter again due to its nature.

Due to lack of any concrete knowledge about the nature of the dark energy, there are a number of proposed candidates and their number is increasing day by day. Among them the natural and simplest candidate is cosmological constant. But it suffers from fine-tuning problem: mismatching in the magnitude of cosmological constant as predicted by field theory and present observation by 123 order. So dynamical dark energy models such as Quintessence \[11–13\] and K-essence \[14\] are proposed. Again there are discussion on an exotic form of dark energy, named as phantom energy \[15, 16\], which violates strong energy condition. There exist also modified matter dynamical dark energy models like Chaplygin gas \[17, 18\] and also many modified theory of gravity like \( f(R) \) gravity \[19, 20\]. But most of them are artificially constructed in the sense that it introduces too many free parameters to able to fit with observational data or not able to explain all features of the Universe, like for example coincidence problem: why the observed values of the cold dark matter density and dark energy density are of the same order of magnitude today although they differently evolve during the expansion of the Universe.

A new alternative to the solution of dark energy problem may be found in the Holographic Principle \[21, 22\]. According to the Holographic principle, the number of degree of freedom in a bound system should scales with its boundary area not with its volume. By applying this principle to Cosmology, Cohen et al. \[23\] found an upper bound on the entropy contained of the Universe. For a system with size L and Ultra Violet (UV) cut-off without decaying into black hole, it is required that the total energy in a region of size L should not exceed the mass of a black hole of the same size, thus \( L^3 \rho_A \leq L M^2_{pl} \). The largest allowed L is the one saturating this inequality, so \( \rho_A = 3c^2 M^2_{pl} L^{-2} \), where c is a constant, \( \rho_A \) is the quantum zero point energy density and \( M_{pl} \) is the Planck mass. It just means a duality between UV cut-off and Infrared (IR) cut-off, where UV cut-off is related to vacuum energy and IR cut-off is related...
to the large scale of the Universe. In literature [25–27], it is considered that this holographic dark energy interacts with matter during the evolution of the Universe with Hubble scale, particle horizon or event horizon as IR cut-off.

In this work, we use an interacting holographic dark energy model with Hubble scale as IR cut-off and study the evolution of the universe. First we calculate the equation of state parameter of the dark energy and then find deceleration parameter. We next evaluate the transition time from decelerated to present accelerated expansion. We found that our analysis successfully addresses the problem of present accelerated expansion of the universe and determination of crossover value. Also it predicts that present universe is phantom dominated which has handsome theoretical [28] and observational [29] support.

II. INTERACTING DARK ENERGY MODEL

For a spatially flat ($k = 0$) FRW universe with scale factor $a$ and filled with dust and dark energy, the Friedman equations take the form

$$\left(\frac{\dot{a}}{a}\right)^2 = \left(\frac{8\pi G}{3}\right)(\rho_m + \rho_x),$$

(1)

$$2\left(\frac{\ddot{a}}{a}\right) + \left(\frac{\dot{a}}{a}\right)^2 = -8\pi Gp_x$$

(2)

and the energy conservation equation becomes

$$(\dot{\rho}_x + \dot{\rho}_m) + 3H(\rho_m + \rho_x + p_x) = 0$$

(3)

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter, $\rho_x$ = dark energy density, $\rho_m$ = matter energy density, and $p_x$ = pressure of the dark energy.

Now we use a dark energy model [30] which rests on following three assumptions:

(i) The dark energy density is derived using holographic principle and is given by

$$\rho_x = 3c^2M_P^2L^{-2},$$

(4)

where $c$ is a dimensionless constant of $O(1)$, $L$ is IR cut-off and $M_P$ is the Planck mass with $M_P^2 \sim \frac{1}{8\pi G}$.

(ii) IR cutoff is taken as the inverse of Hubble scale, i.e., $L = H^{-1}$. So we can write

$$\rho_x = 3c^2M_P^2H^2.$$  

(5)

(iii) Matter and dark energy do not conserve separately but they interact with each other and one may grow at the expense of the other.

So the energy conservation equation in the presence of dark energy can be written as

$$\dot{\rho}_m + 3H\rho_m = Q,$$

$$\dot{\rho}_x + 3H(1 + \alpha)\rho_x = -Q.$$  

(6)

Where $Q = \Gamma\rho_x$ with $\Gamma > 0$ is the reaction rate and $\alpha = \frac{\rho_x}{\rho_m}$ denotes the equation of state parameter for the dark energy. Equations (5) and (6) together lead to

$$\dot{\rho}_x = -\{\Gamma + 3H(1 + \alpha)\}\rho_x.$$  

(7)

Again from equations (4) and (5), one can get

$$\rho_m = \left(\frac{1}{c^2} - 1\right)\rho_x.$$  

(8)

Taking derivative of equation with respect to time and using equation (6), we get

$$\dot{\rho}_x = \left(\frac{\Gamma}{r} - 3H\right)\rho_x.$$  

(9)
where \( r = \frac{\rho_m}{\rho_x} \) is the ratio of matter and dark energy densities.

By comparing above equation (9) with equation (7), one can find an expression for equation of state parameter of dark energy as

\[
\alpha = -\frac{\Gamma}{3H} \left( \frac{1}{r} + 1 \right).
\]  

(10)

III. ESTIMATION OF \( \Gamma \)

Since in our model, we assume that the universe started with only relativistic matter and dark energy appeared due to its decay with \( \Gamma \) as the interaction rate between matter and dark energy, the value of dark energy density parameter for present day must be equal to \( \Gamma t_0 \), where \( t_0 \) is the present age of the universe. By considering observational data [10] that at present 68.3 percentage of the universe is filled with dark energy and the rest are matter which has been achieved in \( 13.82 \times 10^9 \) years through their interaction, one estimates that

\[
\Gamma \approx 4.94 \times 10^{-11} \text{(yr)}^{-1}.
\]  

(11)

IV. EQUATION OF STATE PARAMETER OF DARK ENERGY

According to our model, the Universe is filled with dark energy and relativistic matter. But standard model of cosmology, where dark energy is absent, assumes that the universe was radiation dominated in the early time and it becomes matter dominated later. Combining these two, here we discuss the evolution of dark energy in two different era separately.

In radiation dominated era, \( a(t) \propto t^{\frac{1}{2}} \). The percentage of dark energy is much smaller than relativistic matter, i.e. \( r \) is very large and hence bracketed term in the equation (10) can be taken as unity. So the equation of state parameter of dark energy becomes

\[
\alpha = -\frac{2}{3} \Gamma t.
\]  

(12)

For matter dominated era, \( a(t) \propto t^{\frac{2}{3}} \) and hence the equation of state parameter takes the form

\[
\alpha = -\frac{\Gamma t}{2} \left( \frac{1}{r} + 1 \right).
\]  

(13)

Putting the values of \( \Gamma \), \( t \) and \( r \) in above equation, we found that the present value of equation of state parameter of dark energy is \( \alpha_0 \approx -1.076 \). This implies the universe is presently phantom dominated.

The variation of \( r \) and equation of state parameter \( \alpha \) are shown in Figure 1 and Figure 2 respectively.

V. DECELERATION PARAMETER

Hubbles law explains the expansion of the universe. But whether the expansion is accelerating one or decelerating one, it can be determined by deceleration parameter. The decelerating parameter is defined as

\[
q = \frac{-\ddot{a}a}{a^2}.
\]  

(14)

From equation (2), one can get

\[
2q - 1 = \frac{8\pi G \rho_x}{H^2}
\]  

(15)
which on simplification gives
\[ q = \frac{1}{2} + \frac{3}{2} \left(1 + r\right)^{-\alpha}. \tag{16} \]

Putting present value of \( \alpha \) and \( r \), we find \( q \approx -0.602 \). This indicates presently expansion of the universe is accelerating one. For early universe, \( r \) is very large and \( q \approx 0.5 \). Which tells in early period of evolution, universe undergoes a decelerated expansion. Again transition from deceleration to acceleration
\( q = 0 \) occurs, only when

\[
\alpha = -\frac{1+r}{3}.
\]  

(17)

As expected, transition should occur when dark energy is dominant one i.e. \( r \leq 1 \). We construct the Table 1, which presents different value of transition times for different values of dark energy density parameter \( \Omega_x \) in terms of present age of the universe \( (t_0) \).

| \( \Omega_x \) | \( r \) | \( \alpha \) | Transition Time\( (t_{q=0}) \) |
|-------------|-------|-------|------------------|
| 0.50        | 1.000 | -0.667| 0.976t_0         |
| 0.51        | 0.960 | -0.653| 0.937t_0         |
| 0.52        | 0.923 | -0.641| 0.901t_0         |
| 0.53        | 0.887 | -0.629| 0.866t_0         |
| 0.54        | 0.852 | -0.617| 0.832t_0         |
| 0.55        | 0.818 | -0.606| 0.798t_0         |
| 0.56        | 0.786 | -0.595| 0.767t_0         |
| 0.57        | 0.754 | -0.587| 0.736t_0         |
| 0.58        | 0.724 | -0.575| 0.707t_0         |
| 0.59        | 0.695 | -0.565| 0.678t_0         |
| 0.60        | 0.667 | -0.556| 0.651t_0         |

Since in our interacting model we predict that dark energy is created at the cost of matter, at any time the dark energy density parameter must be equal to the product of interaction rate and age of the universe at that time. Now we construct Table 2, where we show the different values of the dark energy density parameter for different transition times that we get in Table 1.

| Transition Time\( (t_{q=0}) \) | \( \Omega_x = \Gamma t_{q=0} \) |
|------------------|-------|
| 0.976t_0         | 0.667 |
| 0.937t_0         | 0.640 |
| 0.901t_0         | 0.615 |
| 0.866t_0         | 0.591 |
| 0.832t_0         | 0.568 |
| 0.798t_0         | 0.545 |
| 0.767t_0         | 0.524 |
| 0.736t_0         | 0.502 |
| 0.707t_0         | 0.483 |
| 0.678t_0         | 0.462 |
| 0.651t_0         | 0.445 |

Comparing Table 1 and Table 2, we found that the transition time from deceleration to acceleration is \( t_{q=0} \approx 0.798t_0 \). The variation of deceleration parameter with time is shown in Figure 3.
FIG. 3: Variation of deceleration parameter with time

VI. COINCIDENCE PROBLEM

By definition, we know that \( r = \frac{\dot{\rho}_m}{\dot{\rho}_x} \), which gives
\[
\dot{r} = r \left( \frac{\dot{\rho}_m}{\rho_m} - \frac{\dot{\rho}_x}{\rho_x} \right). \tag{18}
\]

Using energy conservation equation (6), we get
\[
\dot{\frac{r}{r}} = 3H \left[ \alpha + \frac{\Gamma}{3H} \left( 1 + \frac{1}{r} \right) \right]. \tag{19}
\]

Putting current values of various parameters in the above equation (19), we find
\[
\left( \frac{\dot{r}}{r} \right)_0 = 1.287 \times 10^{-3} \times 3H_0. \tag{20}
\]

Thus \( r \) varies more slowly in this model than in the conventional \( \Lambda \)CDM model where \( \left( \frac{\dot{r}}{r} \right)_0 = 3H_0 \) and in Scalar-Tensor theory, where \( \left| \frac{\dot{r}}{r} \right|_0 = 2 \times 10^{-2} \times 3H_0. \)

VII. DISCUSSION AND CONCLUSION

In this study, we use interacting holographic dark energy model, where we take Hubble scale as IR cutoff. We assume that during the evolution of the universe, dark energy is created at the cost of relativistic matter. As a success of our work, we first determined the interaction rate at which relativistic matter is converting to dark energy. Again we calculate the equation of state parameter which describes the nature of dark energy. Our result predicts that the present state of the universe is phantom dominated whereas in the early period of evolution phantom energy was absent. Again our model is successful in explaining the present accelerated expansion of the universe. Our result tells that the universe was previously undergoing a decelerated expansion and transition from deceleration to acceleration occurs at
a time of $t_q = 0.798 t_0$, where $t_0$ is the present age of the universe. So in this case the transition is delayed in comparison with previous result [30] where scalar-tensor theory is used. It also considerably softens the coincidence problem.

Thus our integrated model involving interacting dark energy and holographic principle with Hubble scale as IR cutoff can accommodate present accelerated expansion of the universe. It is also successful in determining interaction rate between dark energy and relativistic matter and transition time from decelerated to accelerated expansion. Again it predicts that the present universe is phantom dominated which is highly likely by observational data [29] and softens the coincidence problem.

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