Comparative Study on Two Solutions of 3-RPS Parallel Mechanism Pose

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Abstract. In this paper, the performance of the different methods for solving the pose of parallel mechanism is different in the amount and precision of calculation. Taking typical 3-RPS parallel mechanism as the analysis object, kinematics models of the mechanism were established with quaternion and Rodriguez parameter method respectively, and the bar length of the maximum reachable space was solved by genetic algorithm, so as to compare the advantages and disadvantages of the solution of the model. Finally, numerical simulation is used to compare the amplitude and fluctuation degree of the curve according to the smooth spline interpolation in both calculation and precision. The results show that the Rodriguez parameter method is superior to the quaternion method in the accuracy of pose resolution of 3-RPS parallel mechanism, while the quaternion method is superior in the real-time performance of the solution. This result provides a theoretical basis for the pose control of parallel mechanism.

1. Introduction
Parallel mechanism is widely used in industrial fields due to its high rigidity, strong bearing capacity and lack of input error, and has become a hotspot in robotics and mechanism research [1]. The pose description method of the parallel mechanism dynamic platform is of great significance for the kinematics research of the parallel mechanism. The commonly used pose description methods include Rodriguez parameter method [2], Euler angle method [3], quaternion method [4], etc. Over the years, many scholars have used these methods to conduct a large number of researches and have achieved fruitful results. Wu Yuyao uses the Rodriguez parameter method to establish a spacecraft model and perform pose control so that the closed-loop system can complete the "stationary to stationary" attitude maneuver [5]. Tian Xin, Ge Xinsheng et al. used the Rodriguez parameter method to estimate the pose dynamics of the 3D rigid body pendulum and constructed the Lyapunov function of the system [6]. Wang Shun used the quaternion method to interpolate the pose space and solved the continuous multi-position interpolation problem of the angular velocity of the 6-3 platform parallel machine tool [7]. Yukio- Takeda, Daisuke Matsuura et al. used the Euler angle to analyze the kinematics of the 3-RPSR parallel mechanism, clarified the relationship between design parameters and orientation capability, and designed a parallel mechanism with high orientation capability [8]. However, Euler's law has singularity in mathematics [9], so this paper only describes the pose of Rodriguez parameter method and quaternion method.

What differences exist between these two kinds of pose descriptions in attitude description of parallel mechanism and what differences exist in their solutions? Based on this, this paper uses the Rodriguez parameter method and the quaternion method to analyze the difference of the position of the 3-RPS parallel mechanism by using the typical 3-DOS parallel mechanism with less degrees of freedom.
2. Organizational structure description

Figure 1 shows the structure of the 3-RPS parallel mechanism. The moving platforms are all equilateral triangles, which are hinged by three spatially symmetric RPS branches. The lengths of the regular triangles on the dynamic platform are denoted as L and Lm respectively, and the introduction of the scale factor μ is L = μLm. The structural parameters of the moving platform can be determined by Lm. The coordinate system {O1} and {O2} of the moving platform are established, and the coordinate parameters under the two coordinate systems of each apex of the mechanism are shown in Table 1. In this organization, the three-branched mobile pair is the active pair and is described and adopted as

\[ L_{A1A2}, L_{B1B2}, L_{C1C2}. \]

![Figure 1. 3-RPS structure diagram](image)

Figure 1. 3-RPS structure diagram

![Figure 2. branch vector](image)

Figure 2. branch vector

| Table 1. Coordinate parameters of fixed and fixed platforms |
|-----------------|-----------------|-----------------|
|                | A_{1,2}         | B_{1,2}         | C_{1,2}         |
| Relative to {O1} | (0, \sqrt{3}/3 \mu L_m, 0) | (\mu L_m/2, \sqrt{3}/6 \mu L_m, 0) | (-\mu L_m/2, \sqrt{3}/6 \mu L_m, 0) |
| Relative to {O2} | (O, Lm/\sqrt{3}, O) | (Lm/2, Lm/2\sqrt{3}, O) | (-Lm/2, Lm/2\sqrt{3}, O) |

3. Institutional pose inverse model Rodriguez parameter method modeling

3.1 Basic definition of Rodriguez parameter method

The rigid body having a fixed point O is rotated about the unit vector n. There is a certain point vector r on the rigid body, and the angle θ is rotated around the n-axis to the position of r'. In 1840, the French mathematician Rodriguez proposed [10][11], introducing parameters:

\[ \xi = \tan\frac{\theta}{2} n \] (1)

The moving platform coordinate system Ox_1y_1z_1 coincides with the ground coordinate system Ox_1y_1z_1, and the component of \( \vec{P} \) on Ox_2y_2z_2 is denoted as \( r = [x_2, y_2, z_2]^T \), when Ox_2y_2z_2 is equivalent to Ox_1y_1z_1 rotation. When it reaches a new position, the component of \( \vec{P} \) is unchanged in Ox_2y_2z_2, and its component on Ox_1y_1z_1 is \( r' = [x_1, y_1, z_1]^T \), and the triple-depletion formula is applied according to formula [12] can be re-displayed as:

\[ r' = r + \frac{2}{1 + \xi^2} [\xi \times r + \xi \times (\xi \times r)] = Ar \] (2)

The transformation matrix A can be represented by the Rodriguez parameter:

\[ A = \frac{1}{1 + \xi_1^2 + \xi_2^2 + \xi_3^2} \begin{bmatrix} 1 + \xi_1^2 - \xi_2^2 - \xi_3^2 & 2(\xi_1 \xi_2 - \xi_3) & 2(\xi_1 \xi_3 + \xi_2) \\ 2(\xi_1 \xi_2 + \xi_3) & 1 - \xi_1^2 + \xi_2^2 - \xi_3^2 & 2(\xi_1 \xi_2 + \xi_3) \\ 2(\xi_1 \xi_3 - \xi_2) & 2(\xi_2 \xi_3 + \xi_1) & 1 - \xi_1^2 - \xi_2^2 + \xi_3^2 \end{bmatrix} \] (3)
3.2 Rodriguez parameter inverse solution model
With the Rodriguez parameter transformation matrix $A_1$ and the slave branch vector Figure 2, the vector of the dynamic coordinate system is transformed into a fixed coordinate system by coordinate transformation, and the coordinate parameters of each point in Table 1 are used to obtain three driver rod length vectors at this time. $L_i (i=1,2,3)$ can be expressed in a fixed coordinate system as:

$$L_i = b_i - B_i \ i = 1,2,3$$  \hspace{1cm} (4)

Solve to get three rod length expressions:

$$L^2_{A_1A_2} = \left( \frac{\sqrt{3}L_m}{3\lambda_0^2} (1 - 2q_1^2 + 2q_2^2) - \frac{\sqrt{3}}{3} \mu L_m \right)^2 + \left( \frac{2\sqrt{3}L_m q_1}{3\lambda_0^2} + z_2 \right)^2$$  \hspace{1cm} (5)

$$L^2_{B_1B_2} = \left( \frac{L_m}{\lambda_0^2} \left( \frac{1 + q_1^2 - q_2^2}{2} - \sqrt{3} \xi_0 q_2 \right) - \frac{\mu L_m}{2} \right)^2 + \left( \frac{L_m}{\lambda_0^2} \left( \frac{\sqrt{3}(1 + q_1^2 - q_2^2)}{6} \right) + \frac{\sqrt{3}}{6} \mu L_m \right)^2$$  \hspace{1cm} (6)

$$L^2_{C_1C_2} = \left( \frac{L_m}{\lambda_0^2} \left( \frac{1 + q_1^2 - q_2^2}{2} - \sqrt{3} \xi_0 q_2 \right) + \frac{\mu L_m}{2} \right)^2 + \left( \frac{L_m}{\lambda_0^2} \left( \frac{\sqrt{3}(1 + q_1^2 - q_2^2)}{6} \right) + \frac{\sqrt{3}}{6} \mu L_m \right)^2$$  \hspace{1cm} (7)

among them: $\lambda_0^2 = 1 + q_1^2 + q_2^2 + q_3^2$

The above three equations are the inverse pose model of the kinematic Rodriguez parameter of the 3-RPS symmetric parallel mechanism.

3.3 Quaternion Modeling
$Q = \frac{\cos \theta}{2} + \sin \theta \mu = [q_0, q_1, q_2, q_3]^T = \left[ \frac{\cos \theta}{2} \ \lambda \sin \theta \ \mu \ \sin \theta \right]^T$ describes the direction of the equivalent rotation of the rigid body and the angle of the rotation. $u$ is a unit vector of the rotation direction, $\theta$ is a rotation angle, and $l$, $m$, and $n$ are direction cosines of $u$. Thus, the quaternion $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$ constitutes a unit four-dimensional supersphere[13].

By this principle, the pose transformation matrix of the moving platform can be expressed as B:

$$B = \begin{bmatrix}
q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2q_1q_2 - 2q_0q_3 & 2q_1q_3 + 2q_0q_2 \\
2q_1q_2 + 2q_0q_3 & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2q_2q_3 - 2q_0q_1 \\
2q_1q_3 - 2q_0q_2 & 2q_2q_3 + 2q_0q_1 & q_0^2 - q_1^2 - q_2^2 + q_3^2
\end{bmatrix}$$ \hspace{1cm} (8)

Combining the structural size parameters of Table 1 with Equation 3, the closed-vector method is used to obtain the quaternion length expression of the mechanism[14]:

$$L^2_{A_1A_2} = \left( \frac{\sqrt{3}L_m}{3} (q_0^2 - 2q_1^2 + 2q_2^2) - \frac{\sqrt{3}}{3} \mu L_m \right)^2 + \left( \frac{2\sqrt{3}L_m q_1}{3} + z_2 \right)^2$$  \hspace{1cm} (9)

$$L^2_{B_1B_2} = \left( L_m \left( \frac{(q_0^2 + q_1^2 - q_2^2)}{2} - \sqrt{3} q_0 q_2 \right) - \frac{\mu L_m}{2} \right)^2 + \left( L_m (q_0 q_2 \frac{\sqrt{3}(q_0^2 + q_1^2 - q_2^2)}{6}) + \frac{\sqrt{3}}{6} \mu L_m \right)^2$$  \hspace{1cm} (10)

$$L^2_{C_1C_2} = \left( L_m \left( \frac{(q_0^2 + q_1^2 - q_2^2)}{2} - \sqrt{3} q_0 q_2 \right) + \frac{\mu L_m}{2} \right)^2 + \left( L_m (q_0 q_2 \frac{\sqrt{3}(q_0^2 + q_1^2 - q_2^2)}{6}) + \frac{\sqrt{3}}{6} \mu L_m \right)^2$$  \hspace{1cm} (11)
The above three equations are the inverse pose model of the kinematics represented by the kinematics of the 3-RPS symmetric parallel mechanism.

4. Model comparison and numerical simulation

4.1 Model Comparison
The initial parameters of the 3-RPS parallel mechanism are defined as follows: the distance between the three connection points A2, B2, and C2 of the moving platform is \( L_m = 50\sqrt{3} \text{ mm} \), and the \( \mu \) is 1.5, that is, the distance \( L_B = 75\sqrt{3} \text{ mm} \) between the three connection points A1, B1, and C1 on the fixed platform. Since the 3-RPS mechanism has three output parameters, the moving platform is defined to have an angle of rotation of -90° to 90° around the X-axis, a rotation angle of -90° to 90° around the Y-axis, and a displacement distance of 110 mm to 150 mm in the Z-axis direction.

Solving the maximum reachable space of the 3-RPS mechanism according to the set parameters can be equivalent to the optimal solution of the inverse length of the model. The genetic algorithm is used to solve the problem. The number of iterations is different due to the difference of the mathematical model. As shown in Fig. 3, the model solved by the quaternion method needs to be iterated 85 times to achieve stability and the fitness value is 18, and the result is 174.6652 mm, 104.3462 mm, and 153.7322 mm. The iteration required by the parametric model reached 131 times, and the fitness value was 20, the result was 174.6653 mm, 104.3462 mm, and 153.7321 mm. In the case that the results of the two methods are similar, the model established by the quaternion method is better than the model established by the Rodriguez parameter, and the quaternion method is more efficient.

4.2 Numerical simulation
By setting the angle parameter of the output Y-axis to 90°, the distance moved on the Z-axis is 130 mm, and the X-axis is rotated for one week. The two models solve the required calculation of the rod length (see Table 2) and efficiency. Since the result of a single solution operation is too small to show that it is not obvious enough, the solution is obtained multiple times and obtained by linear interpolation. As shown in the Fig. 4, the quaternion method requires less time to solve, and the result is also in line with the calculation of the algorithm.

Table 2. Comparison of calculations of two kinds of attitude algorithms

| Attitude algorithm | Multiplication and division operation times/times | Addition and subtraction operation times/times | Number of execution operations/times |
|-------------------|-----------------------------------------------|-----------------------------------------------|-------------------------------------|

Figure 3. Two model convergence diagram
Figure 4. Two ways to calculate time
Quaternion method 40 30 23
Rodriguez parameter method 55 40 26

The rod length parameter calculated by Euler angle is the true value and then the error can be expressed as:

\[ v_i = L_o - L_i \quad i = (1,2,3) \]  (12)

Where \( L_o \) is the true value and \( L_i \) is the calculated value.

| Table 3. Partial length of the Rodriguez parameter solution |
|---------------------------------|---------|---------|------------------|---|---|---|
| \( L_{A_1A_2} \) | \( 1 \) | \( 2 \) | \( 3 \) | \( \cdots \) | \( 138 \) | \( 139 \) | \( 140 \) |
| 119.75527 | 107.07241 | 93.826457 | \( \cdots \) | 89.434939 | 102.68271 | 115.62979 |
| 7673313 | 4911061 | 1360416 | \( \cdots \) | 0558068 | 9428576 | 2495938 |

| \( L_{B_1B_2} \) | \( 1 \) | \( 2 \) | \( 3 \) | \( \cdots \) | \( 138 \) | \( 139 \) | \( 140 \) |
| 132.84905 | 125.93441 | 119.20385 | \( \cdots \) | 95.152832 | 93.458492 | 91.482238 |
| 6736857 | 0356644 | 7948781 | \( \cdots \) | 0357940 | 6866975 | 1609401 |

| \( L_{C_1C_2} \) | \( 1 \) | \( 2 \) | \( 3 \) | \( \cdots \) | \( 138 \) | \( 139 \) | \( 140 \) |
| 192.68741 | 194.66050 | 195.69223 | \( \cdots \) | 143.16581 | 138.09276 | 134.15623 |
| 2098202 | 9839215 | 8704550 | \( \cdots \) | 0254032 | 5224305 | 5816980 |

| Table 4. Partial length of the quaternion solution |
|---------------------------------|---------|---------|------------------|---|---|---|
| \( L_{A_1A_2} \) | \( 1 \) | \( 2 \) | \( 3 \) | \( \cdots \) | \( 138 \) | \( 139 \) | \( 140 \) |
| 119.76234 | 107.07082 | 93.832142 | \( \cdots \) | 89.421736 | 102.68431 | 115.63123 |
| 2421414 | 3412327 | 9533476 | \( \cdots \) | 782165 | 4124000 | 2187432 |

| \( L_{B_1B_2} \) | \( 1 \) | \( 2 \) | \( 3 \) | \( \cdots \) | \( 138 \) | \( 139 \) | \( 140 \) |
| 132.85236 | 125.92792 | 119.21348 | \( \cdots \) | 95.162873 | 93.447234 | 91.502138 |
| 7234637 | 3128745 | 7071625 | \( \cdots \) | 4124647 | 8985432 | 5726234 |

| \( L_{C_1C_2} \) | \( 1 \) | \( 2 \) | \( 3 \) | \( \cdots \) | \( 138 \) | \( 139 \) | \( 140 \) |
| 192.69245 | 194.67999 | 195.70137 | \( \cdots \) | 143.17326 | 138.10312 | 134.15482 |
| 6521947 | 3214789 | 6471367 | \( \cdots \) | 9178400 | 6546812 | 1784986 |

According to the formula 12 and combined with Table 3, the data of Table 4 can obtain the error precision of the calculation results of the two methods. In combination with the statistical principle function, there are different nonlinearities in different places, or there are multiple extreme points, then the polynomial, especially the low-order polynomial, is used to complete the fitting. It is very inappropriate, so this article uses a smooth spline to fit the curve. The smooth spline is based on the sum of the square error[15]:
Figure 5, 6, 7 respectively after fitting of each rod long error curve, according to the method as shown in figure obtained by error and error quaternion method compare the fluctuation amplitude is small, and in the X axis rotate over the Angle of 40° to 40°, as a result of the 3 RPS parallel mechanism makes its own structure characteristics of analysis method is far less than the length of error quaternions is obtained. As a whole, the results obtained by parametric method are more stable than those obtained by quaternion method.

5. Conclusion
By analyzing the quaternion method and Rodriguez parameters described in 3-RPS parallel mechanism gesture differences, analyses the kinematics model established by the two methods in calculating, the results show that the quaternion method in terms of the amount of calculation and efficiency is superior to Rodriguez parameter method, but Rodriguez parameter method in computing precision and computing stability is better than the quaternion method.

For 3-RPS parallel mechanism, it is better to adopt Rodriguez parameter for attitude control if the range of rotation Angle of end-effector is small, and it is better to use quaternion method for control if the 3-RPS parallel mechanism is applied in the situation of high real-time performance. The results provide reference for attitude control and trajectory planning of 3-RPS parallel mechanism.

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