Dynamic Simulation of Helical Gears with Crack Propagating In to Gear Body*

Hanjun Jiang1, *, Fuhao Liu1, Manlin Wang2

1College of Automobile Engineering. Yancheng Institute of Technology. Yancheng, China
2School of management. Zhejiang University. Hanzhou, China

*Corresponding author e-mail: hanjunjiang@mvrlab.com

Abstract. The change of mesh stiffness induced by tooth crack will change the vibration characteristic of gear system. In this paper, an analytical model is proposed to investigate the helical gear mesh stiffness with tooth crack propagating into gear body. The developed model is validated by comparison with the FEA method. Hereafter, the mesh stiffness of helical gears with different crack depths is obtained and the effect of the crack depth on the mesh stiffness is studied. Afterwards, a 6 DOF helical gear dynamic model is established and the effects of crack depth on the dynamic response are investigated. The results indicated that the mesh stiffness of the helical gears reduced significantly with the present of tooth crack, the amplitude of periodical impulses increased as the crack depth increased, the rotate frequency and its harmonic frequencies are sensitive to the tooth crack.

1. Introduction (Heading 1)

The gear transmission system is widely used in automotive, aerospace, robotics, energy and marine equipment. Due to the expanding application of gear, numerous scientific studies have been conducted on gear health monitoring and early fault detection [1-3]. As is known, the occurrence of tooth crack will lead to the vibration and noise of the transmission system and even the presence of the failure of the whole system.

The severity of tooth damage is usually assessed by the reduction of the stiffness. Plenty of work has been carried out to investigate gear mesh stiffness. Ian Howard et al. [4] applied FEA method to calculate gear mesh stiffness with tooth root crack considering the effect of friction force. Parey and Tandon [5] introduced a nonlinear 6 DOF gear dynamic model with tooth root crack. Chen et al. [6] proposed an analytical mesh stiffness model of spur gear with tooth root crack propagating along both tooth width and crack depth. Yu et al. [7] developed an expression for the mesh stiffness with spatial crack. Liang et al. [8] investigated mesh stiffness models of cracked planetary gear with different root circle sizes. Cui et al. [9] calculated the meshing stiffness of cracked gears by using a general tooth profile equation. Jiang et al. [10] proposed analytical models of mesh stiffness for cracked spur gears considering gear body deflection.

Based on the discussion above, the published paper mostly concentrated on the mesh stiffness of spur gear with tooth crack. Little work has been done on the mesh stiffness of the helical gears with tooth crack. In addition, the methods referred in the literature only considered the influence of crack...
on the tooth deflection. While, the influence of crack on the gear body deflection should be considered as the crack propagating into the gear body. This paper formulated a new analytical approach to investigate the helical gear mesh stiffness with tooth crack propagating into gear body. Hereafter, the analytical results are compared with the FEM results in Ref. [11]. Then a 6 DOF gear dynamic is established and the effects of crack depth on the gear dynamics are simulated and investigated. Ease of Use

2. Modeling of gear mesh stiffness with crack

The mesh stiffness of a single helical gear can be defined as the force applied on the contact line over the deflection along the contact line. In order to obtain the mesh stiffness of a single helical gear, a mesh stiffness model is proposed to cut the helical gear into infinite slices along the tooth width as showed in Fig. 1. By applying this model, each slice can be considered as a spur gear and the stiffness of each slice can be calculated by energy method [6, 7, and 10].

![Fig. 1 Mesh stiffness model](image)

The deflections of a helical gear slice can be considered as a non-uniform cantilever beam with infinitesimal width dx and effective length d. The bending, shear and axial compressive energy can be expressed by the following equations [6, 7, and 10].

\[
U_b = \frac{F^2}{2k_b}, \quad U_s = \frac{F^2}{2k_s}, \quad U_a = \frac{F^2}{2k_a}
\]  

(1)

Where \(k_b, k_s, k_a\) represents the bending, shear and axial compressive stiffness in the same direction of the force \(F\).

According to the beam theory, the potential energy stored in a meshed slice can be represented by

\[
U_b = \int_0^d U_b \cos \theta d = \frac{M^2}{2EI}
\]

(2)

\[
U_s = \int_0^d \cos \theta d \frac{1.2F^2}{2GA}
\]

(3)

\[
U_a = \int_0^d \cos \theta d \frac{F^2}{2EA}
\]

(4)

Where \(U_b, U_s, U_a\) are the bending, shear and axial compressive potential energy stored in the meshed slice under the action of the force \(F\). The expressions of \(F, F_a, F_s\) and \(M\) are obtained as
\[ F_a = F \sin \alpha, \quad F_b = F \cos \alpha \]  
\[ M = \begin{cases}  F_a x - F_b h, & r_0 \cos \alpha_2 \leq x \leq r_f \cos \alpha_2 + d \\ F_b (d + r_0 \cos \alpha_2 - x) - F_a h, & 0 \leq x \leq r_f \cos \alpha_2 \end{cases} \]  

Based on Eqs. (4)-(7), the bending stiffness \( k_b \) can be calculated by,
\[ \frac{1}{k_b} = \int_{r_0 \cos \alpha_2}^{r_f \cos \alpha_2 + d} \frac{(\cos \alpha x - \sin \alpha h)^2}{E_A} dx \quad r_0 \cos \alpha_2 \leq x \leq r_f \cos \alpha_2 + d \]  

The shear stiffness \( k_s \) and axial compressive stiffness \( k_c \) can be obtained as,
\[ \frac{1}{k_s} = \int_{0}^{r_f \cos \alpha_2 + d} \frac{1}{2 \cos \alpha} dx \]  
\[ \frac{1}{k_c} = \int_{0}^{r_f \cos \alpha_2 + d} \frac{\sin \alpha}{E_A} dx \]  

In the formulas (2)-(8), \( h, d, x, dx \) are shown in Fig.1 (b). \( E \) denotes the Young modulus. \( G \) denotes the shear modulus. \( I_x \) and \( A_x \) denote the area moment of inertia and area of the section where the distance between the section and the acting point of the applied force is \( x \), which are showed as follows.
\[ I_x = \begin{cases} \frac{1}{12} (hx + hx)^2 dw & hx \leq hc \\ \frac{1}{12} (hc + hx)^2 dw & hx > hc \end{cases} \]  
\[ A_x = \begin{cases} (hx + hx) dw & hx \leq hc \\ (hx + hc) dw & hx > hc \end{cases} \]  

Here, \( hx \) denotes the half height of the section where the distance between the section and the acting point of the applied force is \( x \). The expression of \( hx \) can be represented by
\[ hx = \begin{cases} r_0 (\alpha_1 + \alpha_2) \cos \alpha_1 - r_f \sin \alpha_1 - \sqrt{r_f^2 - (x - r_f \cos \alpha_2)^2} & r_0 \cos \alpha_2 \leq x \leq r_f \cos \alpha_2 + d \\ r_0 \cos \alpha_1 - \sqrt{r_0^2 - (x + r_0 \cos \alpha_2)^2} & r_f \cos \alpha_2 \leq x \leq r_0 \cos \alpha_2 \\
\sqrt{r_0^2 - x^2} - \sqrt{r_f^2 - x^2} & x \leq r_f \cos \alpha_2 \\
\sqrt{r_f^2 - x^2} - \sqrt{r_0^2 - x^2} & 0 \leq x \leq r_0 \cos \alpha_2 \\
\end{cases} \]  

Besides, according to the research by Yang and Sun [12], the contact stiffness of a single slice can be represented by
The total equivalent mesh stiffness of one slice pair in mesh can be calculated as:

\[ K_s = \frac{\pi Edw}{4(1-t^2)\cos \beta_s} \]  

(13)

The total equivalent mesh stiffness of one slice pair in mesh can be calculated as:

\[ K_s = \frac{1}{K_{s_1} + K_{s_2} + \frac{1}{K_{s_1}} + \frac{1}{K_{s_2}}} \]  

(14)

By integrating the mesh stiffness of each sliced tooth pair along the contact line \( l \), the total equivalent mesh stiffness of helical gear pair in mesh can be expressed as:

\[ K_e = \int K_s \]  

(15)

3. Effect of tooth crack on gear mesh stiffness

3.1. Comparisons with FEA results

In order to verify the proposed analytical model, FEA results from Ref. [11] are chosen for comparison. The main parameters of the helical gear pair are listed in Table I. The mesh stiffness calculated from the developed analytical model and the FEA results in Ref. [11] are plotted in Fig. 2. It can be found that the result from the proposed analytical model closely match the FEA result.

![Mesh stiffness comparison](image)

(a) Analytical model    (b) FEA results from Ref. [11]

Fig. 2 Mesh stiffness comparison

3.2. Effect of crack depth on gear mesh stiffness

The tooth root crack is assumed to be a straight curve going through the whole tooth and propagating into the gear body. The crack depth \( q \) remain constant through the tooth width. By defining the crack angle \( \alpha_c = 60^\circ \), and the crack depth \( q \) increases from 0 mm to 3 mm, the mesh stiffness of the helical gear pair with different crack depths are obtained, which are plotted in Fig. 3.

![Mesh stiffness comparison](image)

Tab. 1 Main parameters of the helical gears [11]

|                  | Pinion | Gear |
|------------------|--------|------|
| Tooth number     | 25     | 31   |
| Normal modulus(mm)| 2.95   |      |
| Transverse pressure angle(°) | 20     |      |
| Normal tip coefficient | 0.236  |      |
| Normal addendum coefficient | 1.02   |      |
| Young modulus(Pa) | 2×10^{11} |      |
| Tooth width(mm)  | 30     |      |
| Helical angle(°) | 21.5   |      |
| Passion’s ratio  | 0.3    |      |
The results indicate that by introducing tooth root crack, mesh stiffness of the helical gear pair decreases significantly. And the reduction of the mesh stiffness increases with the growth of the crack depth.

4. Dynamic simulation of helical gear system with tooth root crack

4.1. Helical gear dynamic model

Based on the mesh stiffness of helical gear pair with tooth crack, a 6 DOF gear dynamic model is established to investigate the vibration characteristics of helical gear transmission system with and without tooth root crack. In this paper, the effect of the friction force is neglected. The dynamic model is shown in Fig. 4 and the equations are displayed as follows.

\[
\begin{align*}
J_p \ddot{\theta}_p &= T_p - N \cdot R_{pb} \\
J_g \ddot{\theta}_g &= -T_g - N \cdot R_{gb} \\
m_p x_p &= N \cdot \tan \beta_b - K_{pbx} x_p - C_{pbx} x_p \\
m_g x_g &= -N \cdot \tan \beta_b - K_{gbx} x_g - C_{gbx} x_g \\
m_p y_p &= -K_{pbx} y_p - C_{pbx} y_p \\
m_g y_g &= N - K_{gbx} y_g - C_{gbx} y_g
\end{align*}
\]

(16)
Where $J_p / J_g$ is the inertial moment of pinion/gear; $m_p / m_g$ is the mass of pinion/gear; $T_e / T_e$ is the external torque; $K_{p_{bx}} / K_{g_{bx}}, K_{p_{by}} / K_{g_{by}}, C_{p_{bx}} / C_{g_{bx}}, C_{p_{by}} / C_{g_{by}}$ is the stiffness and damping in $x$ and $y$ direction of the supporting bearings. $\beta$ is the helix angle of the base circle of pinion/gear; $R_{pb} / R_{gb}$ is the radius of the base circle of pinion/gear. $N$ is the mesh force and it can be expressed as

$$N = K_m \cdot DTE + C_m \cdot \dot{DTE} \quad (17)$$

In equation (16), DTE denotes dynamic transmission error of the helical gear system. $K_m$ denotes total equivalent mesh stiffness of helical gear pair in mesh and $C_m$ denotes damping between mesh teeth [6]. The parameters for the gear system are given in Table II.

| Tab. 2 Parameters of the helical gear system |
|---------------------------------------------|
| Mass(kg) | Pinion | Gear |
|---------|--------|------|
| 1.1668 | 1.7941 |
| Moment of inertia(kg m²) | 0.919×10⁻³ | 2.2×10⁻³ |
| Radial stiffness of the bearing(N/m) | 6.56×10⁸ | 6.56×10⁸ |
| Damping of the bearing(Ns/m) | 1.8×10⁻¹ | 1.8×10⁻¹ |
| Damping between the meshing gear(Ns/m) | 67 |
| Torque of pinion (N·m) | 20 |
| Rotating of pinion (rpm) | 955 |

4.2. Dynamic responses of helical gear system with different crack depths

ODE45 subroutine is applied to solve the dynamic equations and dynamic responses of helical gear system with different crack depths are investigated. The DTE of time domain and frequency domain under different crack depths are plotted in Fig. 5 and Fig. 6.

**Fig. 5** DTE with different crack depths in time domain
In Fig. 5, the time range of the dynamic simulation is chosen to be 0.1s to 0.3s. It is obvious to observe from the figure that the amplitude of the periodical impulse increases as the crack depth grows. And the time interval between two adjacent impulses is equivalent to 60/955=0.628s exactly.

Under the given rotating speed of the helical pinion, the rotate frequency of pinion is 955/60=15.9Hz and the mesh frequency is 397.5Hz. In Fig. 6, the frequency range of the dynamic simulation is chosen to be 0 to 1000. The ever increasing rotating frequency and its harmonics can be easily observed from the figure that the amplitude of the rotating frequency as well as its harmonics increase as the crack depth grows.

5. Conclusion
An analytical model is proposed to investigate the mesh stiffness of helical gear transmission system with crack propagating into gear body in this paper, and the calculated results of the developed model are validated by comparison with the FEA results from Ref. [11]. Meanwhile, the effect of tooth crack depth on helical gear mesh stiffness is investigated as well as a 6 DOF gear dynamic model is established to study the dynamic response of the helical gear system with different crack depths. Mesh stiffness of the helical gear pair decreases significantly with the present of tooth crack, and the reduction of the mesh stiffness increases with the growth of the crack depth. As the crack depth increases, the amplitude of periodical impulses increase in time domain, the rotate frequency and its harmonics become more and more apparently.

References
[1] Wu S, Zuo MJ, Parey A. Simulation of spur gear dynamics and estimation of fault growth. J Sound Vib 2008; 317; 608-24.
[2] Loutridis SJ. Instantaneous energy density as a feature for gear fault detection. Mech Syst Signal Process 2006; 20(5); 1239-53.
[3] Ma, H. , Pang, X. , Feng, R. , Zeng, J. , & Wen, B. Improved time-varying mesh stiffness model of cracked spur gears. Engineering Failure Analysis, 2015, 55, 271-287.
[4] Ian Howard, Shengxiang Jia, etc, The dynamic modeling of a spur gear in mesh including friction and a crack. Mechanical Systems and Signal Processing 2001; 15(5), 831-853.
[5] Parey A, Tandon N. Spur gear dynamic models including defects—a review. Shock Vib Dig 2003;35(6): 465–78.
[6] Chen Z, Shao Y. Dynamic simulation of spur gear with tooth root crack propagating along width and crack depth. Eng Fail Anal 2011;18(8):2149–64.

[7] Yu, W., Shao, Y., & Mecheške, C. K. The effects of spur gear tooth spatial crack propagation on gear mesh stiffness. Engineering Failure Analysis, 2015, 54, 103-119.

[8] Liang, X., Zuo, M. J., & Pandey, M. Analytically evaluating the influence of crack on the mesh stiffness of a planetary gear set. Mechanism and Machine Theory, 2014, 76, 20-38.

[9] Cui, L., Zhai, H., & Zhang, F. Research on the meshing stiffness and vibration response of cracked gears based on the universal equation of gear profile. Mechanism & Machine Theory, 2015, 94, 80-95.

[10] Jiang, H. & Liu, F. Analytical models of mesh stiffness for cracked spur gears considering gear body deflection and dynamic simulation, Meccanica, 2019, 54: 1889–1909.

[11] He, S., R. Gunda and R. Singh, Inclusion of Sliding Friction in Contact Dynamics Model for Helical Gears. Journal of Mechanical Design, 2007. 129(1): 48-57.

[12] Yang DCH, Sun ZS. A rotary model for spur gear dynamics. ASME J Mech Transm Autom Des 1985;107(4):529–535.