Dual adaptive robust control for uncertain nonlinear active suspension systems actuated by asymmetric electrohydraulic actuators

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Abstract
This study investigates the vibration control issue of active suspension systems. Unlike previous results that neglect the actuator dynamics or consider the impractical symmetrical hydraulic cylinder model, this paper incorporates more reasonable asymmetric electrohydraulic actuator into active suspension system and derives its dynamic model. However, whether active suspension or electrohydraulic actuator suffers from nonlinearities (e.g. nonlinear spring, nonlinear damper and nonlinear actuator dynamics) and parameters uncertainties (e.g. the variations of sprung mass and hydraulic fluid’s bulk modulus as well as hydraulic cylinder original control volumes), which were rarely synthetically considered in the existing researches. To address these issues, we develop a novel dual adaptive robust controller (ARC). An ARC is firstly designed for main-loop system for stabilizing the car body and improving ride comfort in the presence of nonlinearities and parameter uncertainties as well as road disturbances. In order to meet the constraints requirements of suspension system, the tunable parameters in main-loop control law are optimized by solving linear matrix inequality with kidney-inspired algorithm. Another ARC is further synthesized for sub-loop system to deal with the nonlinear and uncertain dynamics in electrohydraulic actuator for ensuring the force tracking performance. Meanwhile, the uncertain parameters are estimated online to compensate the model deviation. The terminal control law is able to guarantee the asymptotic stability of close-loop system within Lyapunov framework. Finally, the effectiveness and robustness of the proposed controller are demonstrated via excessive simulation experiments over different road conditions.

Keywords
Dual adaptive robust control, $H_\infty$ optimization, active suspension, asymmetric electrohydraulic actuator, nonlinearity, uncertainty

Introduction
The vehicle suspension systems have attracted a great deal of attention of both academics and engineers over the past few years, due to their great contributions to ride comfort, suspension packaging and road holding.¹⁻³ It is well recognized that passive suspension and semi-active suspension cannot satisfy the drivers’ and passengers’ tight requirements under different road conditions. Therefore, active suspension is paid considerable interest, which has the ability to add and dissipate energy from system by using a force actuator, so as to achieve a remarkable vibration isolation effect. Since control strategies exert significant influence on the performance of...
active suspension system, many diverse control methods are developed, such as optimal control, fuzzy control, neural network control, adaptive and backstepping control, sliding mode control and so on. Specifically, a constrained $H_{\infty}$ control scheme was applied to quarter-car active suspension system with consideration of output and control constraints. An adaptive sliding mode controller based on the Takagi–Sugeno fuzzy approach was proposed for active suspension system, which is able to deal with the unknown nonlinearities and uncertainties, as well as to enhance the suspension performance. Recently, a robust $L_2$ gain state derivative feedback controller was designed for uncertain active suspension system, successfully improving the ride comfort without deterioration of road holding ability.

However, most of the existing literatures have focused on the main-loop controller design, i.e. figuring out the desired control force that can suppress the vibration of suspension system effectively. In these studies, active actuators are treated as ideal force generators and the complicated dynamics of the sub-loop system are neglected. Unfortunately, as one of the most widely used actuators in active suspension systems, electrohydraulic actuators (sub-loop system) are subjected to highly nonlinear dynamics, such as the nonlinear function between fluid flow and pressure difference, discontinuous nonlinearity caused by the direction change of valve opening. Aside from the severe nonlinearities, some parameters (e.g. bulk modulus of hydraulic fluid, hydraulic cylinder original control volumes) are intrinsically uncertain in electrohydraulic systems. The complex dynamic characteristics make it difficult to implement the desired control law; therefore, the neglect of the actuator dynamics in aforementioned studies will cause a huge gap between theoretical research and practical application. In order to realize the desired force, several recent researches on control of active suspension system have considered the nonlinear and uncertain behaviours in sub-loop system. Namely, incorporate the dynamics of hydraulic actuator that can generate the required active suspension force into the controller design. For example, Sun et al. proposed an adaptive sliding controller (ASC) for active suspension system, where the complex nonlinearities and some time-varying uncertainties in practical hydraulic systems were addressed via function approximation approach. A two-loop control architecture, applying a linear quadratic Gaussian method to main-loop system and a modular adaptive robust control (ARC) approach to force-loop system, was designed for quarter-car active suspension equipped with electrohydraulic actuator. In order to further improve the suspension performance while tackling the nonlinear properties and parametric uncertainties in sub-loop system, Sun et al. combined $H_{\infty}$ control with ARC scheme, and Kilicaslan utilized state-dependent Riccati equation and approximating sequence of Riccati equation techniques. Although the complex dynamics of sub-loop electrohydraulic actuator are considered and carefully addressed in these works, the main-loop active suspension system is assumed to be with linear dynamics and determinate parameters. As a matter of fact, main-loop system also suffers from inherent nonlinearities (e.g. nonlinear characteristics of suspension springs and dampers) and uncertainties (e.g. sprung mass variation with the change of passenger number or payload). Such an assumption is at variance with the real physical system. Therefore, an approximation-free control strategy with lighter computational burden was proposed for unknown nonlinear active suspensions equipped with hydraulic actuator. A proportional-integral-derivative controller using evolutionary algorithms was synthesized to deal with the nonlinear issue in quarter electrohydraulic vehicle suspensions. In addition, a recursive derivative nonsingular higher order terminal sliding mode controller based on high gain observer was developed for active suspension control. Note that the intrinsic nonlinearities and inevitable uncertainties exist not only in active suspension dynamics but also in electrohydraulic actuator, which should be addressed reasonably in control design.

The ARC technique, combining the advantages of adaptive and robust control methods while avoiding their drawbacks, has proven to be responsible for both parametric uncertainties and uncertain nonlinearities. With the use of backstepping method, the ARC was applied to hydraulic actuator, which has the ability to cope with the effects of parameters change and hard-to-model nonlinearities. In Chantranuwathana and Peng and Sun et al., active suspension systems were investigated with electrohydraulic actuator dynamics, where ARC was introduced to deal with the nonlinear behaviour and uncertain parameters only in sub-loop system. While in Sun et al., a nominal ARC controller was designed only for main-loop system with sprung mass variation, at the same time, the actuator dynamics are not taken into account. Therefore, it would be an interesting question whether we can apply ARC not only to main-loop but also to sub-loop of uncertain nonlinear active suspension system equipped with electrohydraulic actuator. Meanwhile, the relationship between the two-loop systems should be addressed reasonably thus to obtain a remarkable suspension performance.

Furthermore, taking a close examination of the above studies concerning active suspension systems actuated by electrohydraulic actuators, we get an important discovery. That is, the actuator dynamics in these works originate from the symmetrical hydraulic cylinder model derived by Merritt. Note the fact that the geometry size of a symmetrical hydraulic cylinder is bigger than an asymmetric hydraulic cylinder, whereas the chassis space of a real
vehicle is very limited. It is unreasonable and unrealistic to install a symmetrical hydraulic into the automobile chassis in practical application. In this sense, previous control methods based on the symmetrical hydraulic cylinder models may not guarantee the performance of the actual suspension systems equipped with asymmetric hydraulic cylinders.

Motivated by the above discussions, in this study, we devote to proposing a Dual-ARC controller for quarter-car active suspension system equipped with asymmetric electrohydraulic actuator, by taking the nonlinear characteristics and uncertain parameters not only in main-loop system but also in sub-loop system into account. To develop the Dual-ARC controller, we analyze the dynamics of active suspension and asymmetric electrohydraulic actuator. In addition to the nonlinearities of spring, damper and actuator, the developed models also suffer from uncertainties arising from the variation of sprung mass, hydraulic fluid bulk modulus, as well as hydraulic cylinder original volumes. Here, an ARC controller is firstly designed for main-loop system to stabilize the vehicle body under nonlinear dynamics and variable road disturbances, simultaneously increasing the robustness against sprung mass fluctuation. In order to guarantee the constraint requirements of the suspension system, the tunable parameters in main-loop control law are optimized through solving the linear matrix inequality (LMI) with kidney-inspired algorithm (KA). Further, another ARC controller is developed to address the nonlinear and uncertain dynamics in sub-loop asymmetric actuator, so as to achieve an accurate tracking performance for the main-loop control law. Meanwhile, the uncertain parameters are estimated online to compensate the model deviation arising from the parameters variation. The proposed control approach is able to ensure rigorous close-loop stability within the Lyapunov framework. Finally, the effectiveness and robustness of the proposed Dual-ARC controller is confirmed via numerous simulation analyses over different road conditions. The main contributions of this study can be listed as follows:

1. The active suspension system, which incorporates with asymmetric electrohydraulic actuator rather than unreasonable symmetric electrohydraulic actuator, is affected not only by spring, damper and actuator nonlinearities but also by the uncertainties of sprung mass, hydraulic fluid bulk modulus, as well as hydraulic cylinder original control volumes.
2. The ARC controllers are developed not only for main-loop system but also for sub-loop system, and both of them suffer from nonlinearities and uncertain parameters. In addition, the bounded stabilities for the two systems are established respectively.
3. In order to ensure the constraint performances, the main-loop controller parameters are optimized by $H_\infty$ optimization scheme based on LMI and KA approaches.
4. The adaptive control laws for main-loop system, sub-loop system, as well as the uncertain parameters estimation are all given based on the Lyapunov functions, thence the close-loop system stability under the proposed Dual-ARC controller is guaranteed.

The rest of this paper is organized as follows: The nonlinear system models and the problem statement are presented in ‘System modelling and problem statement’ section. The Dual-ARC controller is systematically designed in ‘Dual-ARC controller design’ section. Comparative numerical results are provided in ‘Simulations’ section and conclusions are drawn in ‘Conclusions’ section.

System modelling and problem statement

Nonlinear quarter-car model

Here, the structure of quarter-car active suspension system is shown in Figure 1. $m_s$ is the sprung mass that represents the car body mass, and $m_u$ is the unsprung mass that stands for the lumped mass of wheel assembly. The spring, damper and active actuator are installed in parallel between the sprung mass and the unsprung mass. $F_s$ and $F_d$ denote the forces produced by nonlinear spring and nonlinear damper, respectively. $F_t$ stands for the elasticity force generated by the tyre. $z_s$ and $z_u$ are the vertical displacements of sprung mass and unsprung mass, respectively. $z_r$ is the input of road displacement.

According to Newton’s law, the dynamic equations of quarter-car active suspension system are presented as follows

\begin{align}
    m_s \ddot{z}_s &= -F_s(z_s, z_u) - F_d(\dot{z}_s, \dot{z}_u) + F_s(z_s, z_u) + F_d(\dot{z}_s, \dot{z}_u) - F_t(z_u, z_r) \\
    m_u \ddot{z}_u &= F_s(z_s, z_u) + F_d(\dot{z}_s, \dot{z}_u) - F_t(z_u, z_r)
\end{align}

(1)
Considering the nonlinear characteristics of the spring and damper, while keeping the generality of the model, $F_s$ and $F_d$ are expressed as

$$
F_s(z_s, z_u) = k_1 (z_s - z_u) + f_s(z_s, z_u) \\
F_d(\dot{z}_s, \dot{z}_u) = c_1 (\dot{z}_s - \dot{z}_u) + f_d(\dot{z}_s, \dot{z}_u)
$$

where $k_1$ is the linear stiffness coefficient of the spring and $c_1$ is the linear damping coefficient of the damper. $f_s$ and $f_d$ denote the nonlinear dynamics in $F_s$ and $F_d$, respectively.

**Remark 1:** In some existing results, we can find that the nonlinear dynamics of suspension spring and damper are given in specific form, which are limited in practical application. For example, in Deshpande et al., $F_d$ consists of a linear part and a quadratic part, whereas in Sun et al., $F_d$ is represented by a piecewise linear formula and $F_s$ is denoted as a sum of a linear term and a cubic term. In order to make the designed controller more applicable in practice, $F_s$ and $F_d$ are composed of a linear part and a nonlinear function in the form of equation (2). Here $f_s$ and $f_d$ are not described with specific expressions so that the model can cover more realistic situations.

Additionally, the tyre is modelled as a linear spring with an equivalent stiffness coefficient $k_t$, so $F_t$ in equation (1) is given by

$$
F_t = k_t (z_u - z_r)
$$

To facilitate the control design, we define the state variable as

$$
x = [x_1 \ x_2 \ x_3 \ x_4]^T = [z_s \ \dot{z}_s \ z_u \ \dot{z}_u]^T
$$

In view of equations (1) to (4), there is

$$
x_1 = x_2 \\
x_2 = \theta_1 [-k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_s(x_1, x_3) - f_d(x_2, x_4) + F] \\
x_3 = x_4 \\
x_4 = \sigma [k_1(x_1 - x_3) + c_1(x_2 - x_4) + f_s(x_1, x_3) + f_d(x_2, x_4) - F - k_t(x_3 - z_r)]
$$

in which $\theta_1 = 1/m_s$, $\sigma = 1/m_u$.

**Asymmetric electrohydraulic actuator model**

The asymmetric electrohydraulic actuator, widely used in practical application, is shown in Figure 2. The structure of electrohydraulic actuator consists of an asymmetric hydraulic cylinder and an electrohydraulic servo valve.
As depicted in Figure 2, $P_S$ is the supply pressure of the hydraulic system and $P_T$ ($P_T \approx 0$) is the return pressure. $P_1$ and $P_2$ denote the pressure in non-rod chamber and rod chamber, respectively. $A_1$ and $A_2$ represent the equivalent areas of non-rod piston and rod piston, respectively. $Q_1$ denotes the flow size flowing into or flowing out of the non-rod chamber, and $Q_2$ denotes the flow size inflowing to or outflowing from the rod chamber. That is to say, $Q_1$ and $Q_2$ remain greater than or equal to zero at any time. What is more, $\dot{z}$ stands for the velocity of the piston rod, which is corresponding to $\dot{z}_s - \dot{z}_u$ in active suspension system. We define that $\dot{z}$ is positive when the piston rod is extending. $x_v$ stands for the spool displacement of electrohydraulic servo valve, whose positive direction is marked in Figure 2. Obviously, the numerical signs of $x_v$ and $\dot{z}$ are consistent. In addition, it should be noted that the supply pressure $P_S$ is always greater than the pressure in non-rod chamber or rod chamber.\textsuperscript{25,36}

The output force $F$ of the hydraulic cylinder can be easily obtained

$$F = P_1A_1 - P_2A_2$$  \hspace{1cm} (6)

The load pressure $P_L$ is defined as $P_L = F/A_1$. Besides, the flows through the electrohydraulic servo valve are written as follows\textsuperscript{30,37}

$$Q_1 = \begin{cases} 
C_d w x_v \sqrt{\frac{2(P_S - P_1)}{\rho} \text{sgn}(\dot{z})}, & x_v > 0 \\
C_d w x_v \sqrt{\frac{2P_1}{\rho} \text{sgn}(\dot{z})}, & x_v < 0 
\end{cases}$$

$$Q_2 = \begin{cases} 
C_d w x_v \sqrt{\frac{2P_2}{\rho} \text{sgn}(\dot{z})}, & x_v > 0 \\
C_d w x_v \sqrt{\frac{2(P_S - P_2)}{\rho} \text{sgn}(\dot{z})}, & x_v < 0 
\end{cases}$$  \hspace{1cm} (7)

where $C_d$ is the discharge coefficient, $w$ is the valve orifice area gradient and $\rho$ is the density of hydraulic fluid. Note that the flow equations are quite complicated nonlinear functions; meanwhile, a big difference appears in flows $Q_1$ and $Q_2$ when the piston is moving in positive and opposite direction. Fortunately, we found that no matter the piston rod extends or contracts a certain displacement, the total flow size of entering into the hydraulic cylinder plus flowing out from the hydraulic cylinder is the same. Therefore, we can define the load flow as\textsuperscript{38}

$$Q_L = \frac{Q_1 + Q_2}{2}$$  \hspace{1cm} (8)

According to the geometric relations, we can get that\textsuperscript{31,32}

$$Q_1 \approx A_1 \dot{z} \text{sgn}(\dot{z})$$
$$Q_2 \approx A_2 \dot{z} \text{sgn}(\dot{z})$$  \hspace{1cm} (9)
Based on equations (6) to (9), the load flow can be represented as
\[ Q_L = \tilde{Q}_L \text{sgn}(\dot{z}) = \Theta g(x_i)x_i \text{sgn}(\dot{z}) \]  
(10)
in which
\[ \Theta = C_d w \sqrt{\frac{2}{\rho(A_1^2 + A_2^2)}} \frac{A_1 + A_2}{2}, \quad g(x_i) = \sqrt{\frac{A_1 + A_2}{2} P_S - \left( \frac{A_2 - A_1}{2} P_S + A_1 P_L \right) \text{sgn}(x_i)}. \]

On the other hand, the flow continuity equations of non-rod chamber and rod chamber give \(^{31,39}\)
\[
Q_1 = \left[ \frac{A_1}{2} \dot{z} + C_i (P_1 - P_2) + C_e P_1 + \frac{V_{10} + A_1 z}{\beta_e} P_1 \right] \text{sgn}(\dot{z})
\]
\[
Q_2 = \left[ \frac{A_2}{2} \dot{z} + C_i (P_1 - P_2) - C_e P_2 - \frac{V_{20} - A_2 z}{\beta_e} P_2 \right] \text{sgn}(\dot{z})
\]  
(11)
where \( C_i \) and \( C_e \) are the internal and external leakage coefficients of asymmetric hydraulic cylinder, respectively. \( \beta_e \) is the bulk modulus of fluid. \( V_{10} \) and \( V_{20} \) are the initial volumes of non-rod chamber and rod chamber, respectively. Therefore, the load flow can also be expressed in the following form according to equation (8)
\[ Q_L = \left[ \frac{A_1}{2} \dot{z} + C_{ie} (P_1 - P_2) + \left( \frac{V_{10} + A_1 z}{2\beta_e} P_1 - \frac{V_{20} - A_2 z}{2\beta_e} P_2 \right) \right] \text{sgn}(\dot{z})
\]  
(12)
in which \( C_{ie} = C_i + C_e/2 \) is the total leakage coefficient. By replacing \( P_1 \) and \( P_2 \) with \( P_S \) and \( P_L \) according to the above equations, we can get
\[ Q_L = \left( \eta_1 \dot{z} + \eta_2 F + \eta_3 + \frac{F}{\theta_2} \right) \text{sgn}(\dot{z}) \]  
(13)
where
\[
\theta_2 = \frac{2(A_1^2 + A_2^2)\beta_e}{V_{10} A_1^2 + V_{20} A_2^2 + (A_1^2 - A_2^2)z}, \quad \eta_1 = \frac{A_1 + A_2}{2},
\]
\[
\eta_2 = \frac{C_{ie}(A_1^2 + A_2^2)}{A_1^2 + A_2^2}, \quad \eta_3 = \frac{C_{ie} P_S (A_2 - A_1) h}{A_1^2 + A_2^2} \quad \text{and} \quad h = \frac{A_2^2 + A_3^2}{2} + \frac{A_2^2 - A_1^2}{2} \text{sgn}(x_i).
\]

In terms of equations (10) and (13), we can obtain that
\[ \dot{F} = \theta_2 \left[ -\eta_1 \dot{z} - \eta_2 F - \eta_3 + \tilde{Q}_L \right] \]  
(14)
It is clearly obvious from equations (10) and (14) that the output force of the electrohydraulic actuator can be regulated by controlling the opening size and direction of the electrohydraulic servo valve.

Additionally, if we set \( A_1 = A_2 = A_r, \quad V_{10} = V_{20} = \frac{V}{2} \), then equation (14) can be written as
\[ \dot{F} = \frac{4\beta_e A_r}{V_{r}} \left[ -A_r (\dot{z}_s - \dot{z}_a) - C_e \frac{F}{A_r} + \tilde{Q}_L \right] \]  
(15)
where \( V_r \) is the total actuator volume, and \( \tilde{Q}_L = C_d w \sqrt{\frac{P_S - P_L}{\rho}} \text{sgn}(x_i) \). Equation (15) is the same as the symmetric electrohydraulic actuator model existing in many active suspension studies. \(^{20-24}\) Therefore, we can conclude that the symmetric actuator model is a special case of asymmetric actuator dynamics that we have derived in the above.
Remark 2. Model (14) reflects some important nonlinear characteristics of asymmetric electrohydraulic actuator due to the nonlinearities of $\theta_2$, $Q_L$, and $h$ in $q_1$. In the practical hydraulic system, $\beta_c$, $A_1$ and $A_2$ are all positive. Additionally, since $z$ is the displacement of piston rod, it is clear that $V_{10} + A_1z > 0$ and $V_{20} - A_2z > 0$. Therefore, $\theta_2$ is positive.

Remark 3: Apart from the nonlinearities, the dynamics of active suspension and electrohydraulic actuator concerning models (5) and (14) contain some uncertain parameters. Specifically, $\theta_1$ is uncertain because the sprung mass $m_s$ may vary with the passenger number or payload. The fluid bulk modulus $\beta_c$ is variable due to the flexibility of tubes, leakage, trapped air and so on. And the original control volumes $V_{10}$ and $V_{20}$ are also unknown. Due to the change of piston rod displacement $z$, parameter $\theta_2$ is uncertain. Moreover, in a practical hydraulic system, these parameters are all bounded. Here, we assume that $m_{s\text{min}} < m_s < m_{s\text{max}}$ and $\beta_{e\text{min}} < \beta_e < \beta_{e\text{max}}$, thence there is $\theta_{1\text{min}} < \theta_1 < \theta_{1\text{max}}$ and $\theta_{2\text{min}} < \theta_2 < \theta_{2\text{max}}$, where $\theta_{1\text{min}} = \frac{1}{m_{s\text{max}}}$, $\theta_{1\text{max}} = \frac{1}{m_{s\text{min}}}$, $\theta_{2\text{min}} = \frac{2(4^3 + A_1^3)\rho_{\text{min}}}{(24^3 - A_1^3)L}$, $\theta_{2\text{max}} = \frac{2(4^3 + A_1^3)\rho_{\text{max}}}{A_2^2L}$ and $L$ is the stroke of the hydraulic cylinder.

**Problem statement**

The performance requirements of vehicle suspension systems mainly include the following three aspects.\textsuperscript{13,40}

**Ride comfort:** As one of the main tasks of suspension system, it is devoted to stabilizing the vertical motion of vehicle body and isolating vibrations transmitted to passengers or cargos as much as possible.

**Suspension stroke constraint:** Once exceeding the allowable maximums, suspension will hit its limit block, which will deteriorate the ride comfort and even damage the suspension structure. Therefore, it should ensure that

$$|z_s - z_u| < z_{\text{max}}$$

where $z_{\text{max}}$ is the maximum deflection of suspension system.

**Road holding:** In order to suppress the hop of wheels and ensure the driving safety of vehicles, the tyres should keep firm uninterrupted contact with the road surface. Thus the dynamic tyre load should not exceed the static tyre load, that is

$$F_I(z_u, z_t) < (m_s + m_u)g$$

where $g$ denotes the gravitational constant.

It should be emphasized that the above performance requirements are inherently in conflict,\textsuperscript{25,40} and a controller should be designed to achieve a good ride comfort and a good road holding capacity as well as to ensure the suspension stroke constraint despite in the presence of the complicated dynamics of asymmetric electrohydraulic actuator. Based on the aforementioned statements, this study tries to deal with the following problem.

**Problem 1.** Given the active suspension system actuated by asymmetric electrohydraulic actuator, synthesize the adaptive robust control law $u$ for spool displacement to stabilize the vertical motion of suspension system while to address the parametric uncertainties and high nonlinearity in both main-loop system and sub-loop system. Meanwhile, the constraints requirements in equations (16) and (17) as well as the close-loop system stability are guaranteed.

**Dual-ARC controller design**

In this section, we will synthesize a Dual-ARC controller for active suspension system equipped with asymmetric electrohydraulic actuator. The controller structure diagram is shown in Figure 3. The ARC controller is firstly designed for main-loop system to attenuate the suspension vibration in face of spring, damper nonlinearities and sprung mass uncertainties. In order to guarantee the suspension deflection and dynamic tyre load constraints, the tunable parameters in main-loop ARC control law are optimized by $H_{\infty}$ optimization based on LMI and KA approaches. The sub-loop ARC controller is further designed to deal with the nonlinear and uncertain dynamics in asymmetric electrohydraulic actuator. Besides, the uncertain parameters are estimated online while the close-loop system asymptotical stability is ensured. We will present the detailed controller design process in the following text.
comfort. The ideal suspension state is to make the passengers unaware of the vibrations arising from the road.

First we will develop an ARC controller for main-loop system (1) without consideration of the actuator dynamics, a desired ARC force $F_r$ is synthesized for $F$ to stabilize the vertical motion of vehicle body while improving ride comfort. The ideal suspension state is to make the passengers unaware of the vibrations arising from the road roughness. Instead of only focusing on the vibration state of sprung mass like Liu et al., we assume the ideal trajectories of sprung mass and unsprung mass are both in zero. That is $x_{1r} = 0, x_{2r} = 0, x_{3r} = 0, x_{4r} = 0$. In this manner, the suspension is at its best state; meanwhile, we can regulate suspension to meet the constraint requirements in next procedure.

The error vector between actual states and reference trajectories is given as

$$\psi = \begin{bmatrix} e_1 & e_2 & e_3 & e_4 \end{bmatrix}^T = \begin{bmatrix} x_1 - x_{1r} & x_2 - x_{2r} & x_3 - x_{3r} & x_4 - x_{4r} \end{bmatrix}^T$$

The switch function $s$ is defined by a combination of the error variables $e_1, e_2, e_3$ and $e_4$

$$s = \Lambda \psi$$

in which $\Lambda = \begin{bmatrix} \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \end{bmatrix}$ and $\lambda_i > 0 \ (i = 1, 2, 3, 4)$

By differentiating the switch function $s$ based on the nonlinear and uncertain dynamics of system (5), one can obtain that

$$s = \lambda_1 (x_1 - x_{1r}) + \lambda_2 (x_2 - x_{2r}) + \lambda_3 (x_3 - x_{3r}) + \lambda_4 (x_4 - x_{4r})$$

$$= \lambda_1 (x_2 - x_{2r}) + \lambda_3 (x_4 - x_{4r}) + \lambda_2 x_2 - \lambda_4 x_4$$

$$+ \lambda_4 \tilde{\theta}_1 + \tilde{\theta}_1 \left[ -k_1 (x_1 - x_3) - c_1 (x_2 - x_4) - f (x_1, x_3) - f (x_2, x_4) + F \right]$$

$$+ \lambda_4 \tilde{\theta} \left[ k_1 (x_1 - x_3) + c_1 (x_2 - x_4) + f (x_1, x_3) + f (x_2, x_4) - F - k_l (x_3 - z) \right]$$

where $\tilde{\theta}_1$ is the estimation value of uncertain parameter $\theta_1$, and the estimate error between $\theta_1$ and $\tilde{\theta}_1$ is defined as $\tilde{\theta}_1 = \theta_1 - \tilde{\theta}_1$. According to equation (19), the adaptive model compensation term depending on online parameter...
adaptation is designed as
\[
F_{ar} = k_1(x_1 - x_3) + c_1(x_2 - x_4) + f_1(x_1, x_3) + f_2(x_2, x_4) + \frac{1}{\lambda_2} \left[ -\dot{\lambda}_1(x_2 - x_2r) - \lambda_3(x_4 - x_4r) + \lambda_2 \dot{x}_2 + \lambda_4 \dot{x}_4 + \lambda_4 \sigma k_i \right]
\]

(20)

and the adaptive law of \( \dot{\theta}_1 \) will be synthesized later. Meanwhile, we need to synthesize a robust term \( F_{sr} \) for \( F_r \) due to the existence of uncertain parameter and road disturbances. In this manner, \( F_r = F_{ar} + F_{sr} \) and \( F_{sr} = F_{sr1} + F_{sr2} \).

Consider the following Lyapunov function
\[
V_1 = \frac{1}{2} s^2
\]

(21)

Taking the time derivative of \( V_1 \) along with equations (19) and (20), we have
\[
V_1 = s \left\{ \left( \dot{\lambda}_2 \dot{\theta}_1 - \dot{\lambda}_4 \sigma \right) F_{ar} + \lambda_4 \sigma k_i z_r + \dot{\lambda}_2 \dot{\theta}_1 \left[ -k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_1(x_1, x_3) - f_2(x_2, x_4) + F_i \right] \right\}
\]

(22)

Then the robust term \( F_{sr1} \) against road interferences is designed as
\[
F_{sr1} = \frac{1}{\lambda_2} \left[ -\kappa_1 s - \delta_1 \tanh(\Gamma_1 s) \right], \quad (|\lambda_4 \sigma k_i| < \delta_1)
\]

(23)

in which \( \kappa_1 \) is a positive tunable parameter. \( \tanh(\Gamma_1 s) \) is a differentiable function that can be made arbitrarily close to \( \text{sgn}(s) \) by setting \( \Gamma_1 \) to a larger value. Note that \( F_{sr1} \) is dissipating in nature, which has an ability to reduce the impact of road disturbances. By substituting \( F_{sr1} \) into equation (22), the time derivative of \( V_1 \) leads to
\[
V_1 = s \left\{ -\kappa_1 s - \delta_1 \tanh(\Gamma_1 s) - \lambda_4 \sigma k_i z_r + \dot{\lambda}_2 \dot{\theta}_1 \left[ -k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_1(x_1, x_3) - f_2(x_2, x_4) + F_i \right] \right\}
\]

(24)

In what follows, the robust control law \( F_{sr2} \) is chosen to satisfy the following conditions

1) \( s \left\{ \left( \dot{\lambda}_2 \dot{\theta}_1 - \dot{\lambda}_4 \sigma \right) F_{sr2} + \dot{\lambda}_2 \dot{\theta}_1 \left[ -k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_1(x_1, x_3) - f_2(x_2, x_4) + F_i \right] \right\} \leq \varepsilon_1
\)

2) \( s \left\{ \dot{\lambda}_2 \dot{\theta}_1 - \dot{\lambda}_4 \sigma \right\} F_{sr2} \leq 0
\)

(25)

where \( \varepsilon_1 \) is a positive number that can be arbitrarily small. \( \lambda_2 \) and \( \lambda_4 \) are selected to satisfy \( \lambda_2 \dot{\theta}_1 - \dot{\lambda}_4 \sigma > 0 \). One smooth robust control law \( F_{sr2} \) satisfying equation (25) is given in the following way
\[
F_{sr2} = -\frac{1}{4(\lambda_2^2 \theta_{\min} - \dot{\lambda}_4 \sigma)} m_1 s
\]

(26)

in which \( m_1 \) can be any smooth function satisfying
\[
m_1 \geq \frac{1}{\varepsilon_1} \left( \theta_{\text{max}} - \theta_{\text{min}} \right)^2 \lambda_2^2 \left[ -k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_1(x_1, x_3) - f_2(x_2, x_4) + F_{ar} + F_{sr1} \right]^2
\]

In terms of the robust control law \( F_{sr2} \) and equation (24), one can obtain that
\[
V_1 \leq s \left\{ -\kappa_1 s - \frac{\dot{\lambda}_2 \dot{\theta}_1 - \dot{\lambda}_4 \sigma}{4(\lambda_2^2 \theta_{\min} - \dot{\lambda}_4 \sigma)} m_1 s + \dot{\lambda}_2 \dot{\theta}_1 \left[ -k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_1(x_1, x_3) - f_2(x_2, x_4) + F_i \right] \right\}
\]

(27)

\[
\leq -\kappa_1 s^2 + \varepsilon_1 = -2\kappa_1 V_1 + \varepsilon_1
\]
which results in

\[ V_1(t) \leq V_1(0)e^{-2\kappa_1 t} + \frac{\varepsilon_1}{2\kappa_1}(1 - e^{-2\kappa_1 t}) \quad (28) \]

Inequality (28) indicates that all state trajectories tracking errors \( e_1, e_2, e_3, e_4 \) will converge to bounded ranges as \( t \to \infty \). Thence, the control law composed of the sum of \( F_{a1}, F_{a2} \) and \( F_{a3} \) is able to guarantee the bounded stability of active suspension main-loop system in spite of the existence of nonlinearities and uncertainties.

**Parameters optimization for main-loop ARC control law**

Obviously, the main-loop ARC control law presented above can improve the vehicle body stability while guaranteeing the bounded stability of trajectories tracking error. However, the suspension constraint requirements in equations (16) and (17) are not taken into consideration. As observed from the ARC laws in equations (20), (23) and (26), we can find that the parameters \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \kappa_1 \) will greatly affect the control force \( F_r \), thence to influence the control effect of active suspension system. In order to respect the constraint conditions of suspension system, a \( \mathcal{H}_{\infty} \) optimization method, which combined LMI with KA, \(^{41,42}\) is employed to obtain appropriate tunable parameters of \( F_r \).

For system (1) in the form of the equation (5), the active suspension system can be described as

\[ \dot{x} = Ax + BF + w_1 \quad (29) \]

in which

\[ A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -k_1 & -c_1 & k_1 & c_1 \\ m_s & m_s & m_s & m_s \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ m_s \\ 0 \end{bmatrix}, \quad w_1 = \begin{bmatrix} 0 \\ -f_s - f_d \\ m_s \\ f_s + f_d + k_1 z_r \end{bmatrix} \]

Considering the change of sprung mass, we assume \( m_{sn} \) is the nominal value of \( m_s \). Let \( A = A_m + \Delta A, B = B_m + \Delta B \), where \( A_m \) and \( B_m \) are the nominal matrices of \( A \) and \( B \). \( \Delta A \) and \( \Delta B \) denote the bounded perturbations with respect to \( A \) and \( B \), respectively. Then equation (29) can be written as

\[ \dot{x} = A_m x + B_m F + w_2 \quad (30) \]

in which \( w_2 = \Delta Ax + \Delta BF + w_1 \). With the control force \( F_r \) synthesized in ‘Main-loop ARC controller design’ section, equation (30) is converted to the following form

\[ \dot{x} = A_w x + w_3 \quad (31) \]

in which

\[ w_3 = \Delta Ax + \Delta BF + w' \]

\[ w' = \begin{bmatrix} F_{a2} + z \chi - \rho_1 m_{sn} \delta_1 \tanh(\Gamma_1 s) \\ m_{sn} \\ -F_{a2} - z \chi + \rho_1 m_{sn} \delta_1 \tanh(\Gamma_1 s) + k_1 z_r \end{bmatrix}^T \]

\[ A_w = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \rho_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & \rho_2 & k_1 \lambda_1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\kappa_1 \lambda_1 & -(\lambda_1 + \kappa_1 \lambda_2) & \lambda_4 + k_1 \lambda_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \lambda_4 + k_1 \lambda_4 \\ \lambda_3 + \kappa_1 \lambda_4 \\ \lambda_3 + \kappa_1 \lambda_4 \end{bmatrix} \]
Specifically,
\[ \rho_1 = \frac{1}{\lambda_2 - \lambda_4 \frac{m_n}{m_2}}, \quad \rho_2 = \frac{m_{sn}/m_n}{\lambda_2 - \lambda_4 \frac{m_n}{m_2}}, \quad \alpha = \frac{1}{\lambda_2 \lambda_4 - \lambda_4 \sigma} \]

\[ \chi = [\lambda_4 \sigma k_1 x_3 + \lambda_2 \check{x}_{2r} + \lambda_4 \check{x}_{4r} - \lambda_1 (x_2 - x_{2r}) - \lambda_3 (x_4 - x_{4r}) - \kappa_1 s - \delta_1 \tanh(\Gamma_1 s)] \]

In view of the constraint requirements in equations (16) and (17), the control output vector is defined as
\[ z = \begin{bmatrix} z_1 (z_s - z_u)/z_{\text{max}} & z_{3s} (z_u - z_r)/g \end{bmatrix}^T. \]
where \( z_1, z_2 > 0 \) are the corresponding weight values. Meanwhile, the disturbance vector gives \( w = [w_3 \ z_r]^T. \) Thus the close-loop system can be expressed as
\[
\begin{align*}
\dot{x} &= A_w x + B_w w \\
z &= C_w x + D_w w
\end{align*}
\] (32)

in which \( A_w \) is presented above, and
\[
B_w = [I_{4 \times 4} \ 0], \\
C_w = \begin{bmatrix} \frac{z_1}{z_{\text{max}}} & 0 & -\frac{z_1}{z_{\text{max}}} & 0 \\
0 & 0 & \frac{z_{3s} k_1}{g} & 0 \\
0 & 0 & \frac{z_{3s} k_1}{(m_s + m_u) g} & 0 \\
0 & 0 & 0 & -\frac{z_{3s} k_1}{(m_s + m_u) g} \end{bmatrix}, \\
D_w = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -\frac{z_{3s} k_1}{(m_s + m_u) g} & 0 & 0 & 0 \end{bmatrix}
\]

It can be seen from equation (32) that the unknown parameters in matrix \( A_w \) will influence the control output \( z, \) which should be optimized by a feasible and effective method. \( H_\infty \) norm \( \|\Gamma_{wz}\|_\infty \) from disturbance \( w \) to performance output \( z \) reflects the disturbance suppression ability of system (32), which is selected as the evaluation metrics. Our aim is to obtain the optimal parameters \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \kappa_1 \) that can minimize \( \|\Gamma_{wz}\|_\infty. \)

**Lemma 1.** In terms of the close-loop system (32), for a given scalar \( \gamma > 0, \) the \( H_\infty \) performance \( \|\Gamma_{wz}\|_\infty < \gamma \) can be achieved, if there exist \( P = P^T > 0 \) and positive numbers \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \kappa_1 \) satisfying the following inequality
\[
\begin{bmatrix} A_w^T P + PA_w^T & PB_w & C_w^T \\
* & -\gamma I & D_w^T \\
* & * & -\gamma I \end{bmatrix} < 0
\] (33)

The proof for lemma 1 can be found in many works,\(^{43,44} \) which is omitted here. Based on the above lemma, solving the minimum of \( \|\Gamma_{wz}\|_\infty \) can be converted to the following LMI optimization problem
\[
\begin{align*}
\min \gamma \\
\text{subject to LMI (32)}
\end{align*}
\] (34)

Since the two unknown matrices \( A_w \) and \( P \) appear in equation (33) in a nonlinear form, equation (34) cannot be directly solved by employing traditional convex optimization algorithms. While if the parameters \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \kappa_1 \) are given in advance, the inequality equation (33) can be efficiently solved through Matlab LMI Toolbox. Therefore, an approach combining stochastic search algorithm KA with LMI solving is proposed to address the problem (34).

KA is a nature-inspired algorithm firstly proposed by Jaddi et al.\(^{41} \) and successfully used by Du et al.,\(^{42} \) which imitates the working process of kidney in human body and includes filtration, reabsorption and excretion. It has proven to be superior to other state-of-the-art algorithms such as firefly algorithm, genetic algorithm, bat algorithm, particle swarm optimization, in terms of search ability and convergence speed when solving global
optimization problem. Hence KA is employed to find the feasible solutions of equation (33), and the optimized parameters are represented by a vector $q = \lambda_1, \lambda_2, \lambda_3, \lambda_4, \kappa_1$. In the algorithm solving process, $q_{ij}$ denotes the $i$th ($j = 1, 2, \cdots, N$) solution vector at $j$th ($j = 1, 2, \cdots, M$) generation, and $G_j$ represents the population including $N$ solutions in $j$th iteration. In addition, the objective value is the minimal $\gamma_{ij}$ for equation (34) under individual $q_{ij}$. The flow diagram of KA is shown in Figure 4 and the algorithm process is briefly explained as follows.

Step 1: Population initialization. The initial population $G_0$, including $N$ solutions, is randomly generated within a suitable search space. Since not all the generated solution vectors are valid, a feasibility analysis is performed for each individual by solving LMI of (33). If the individual generated in initial population or subsequent procedures are infeasible for (33), it will be regenerated until the value is valid so as to improve the search efficiency. Then problem (34) is solved based on all feasible solutions $q_{ij0}$, and the solution with smallest objective value is set as $q_{best,0}$.

Step 2: Generating new population $G_j$. In order to improve the population quality, new population $G_j$ is produced by moving the solution from last iteration towards the best solution found so far. The movement is formulated as

$$q_{ij} = q_{ij-1} + \text{rand}(q_{best,j-1} - q_{ij-1})$$

(35)

$\text{rand}(q_{best,j-1} - q_{ij-1})$ represents a random number between 0 and the value of $(q_{best,j-1} - q_{ij-1})$.

Step 3: Objective evaluation. Objective value $\gamma_{ij}$ in terms of each solution $q_{ij}$ in $G_j$ is calculated by solving problem (34), and the solution with smallest objective value is set as $q_{best,j}$.

Step 4: Filtration. The solutions in $G_j$ are filtered by applying a filtration rate $fil_j$, so as to obtain the better solutions while reducing the iteration number. The filtration rate $fil_j$ is calculated based on the objective values of

![Figure 4. Flow diagram of KA.](image-url)
all solutions in $G_j$

$$\text{fil}_j = \frac{\sum_{i=1}^{N} \gamma_{i,j}}{N}$$  \hspace{1cm} (36)

Solutions with smaller objective values than $\text{fil}_j$ are assigned to the filtered blood (FB), while the rest are put into waste (W).

Step 5: Reabsorption. For enhancing the algorithm robustness, the solution that has been transferred into W is replaced with a new one by using the following movement operator

$$q_{i,j} = q_{i,j} + \text{ran}(q_{\text{best},j} - q_{i,j})$$  \hspace{1cm} (37)

The new solution with smaller objective value than $\text{fil}_j$ still has a chance to become a part of FB.

Step 6: Excretion and insertion. In order to avoid the local convergence phenomenon, the solutions are excreted if they cannot enter FB after giving them a chance of reabsorption. Then the excreted one is inserted with a new solution to improve the diversity of solutions.

Step 7: Mergence. The solutions that have been assigned to FB and W are merged together, and the $q_{\text{best},j}$ is updated.

Step 8: Termination judgement. If the stopping criterion is met, then output the optimal solution $q_{\text{best}}$ and the corresponding objective value, otherwise go back to Step 2.

Sub-loop ARC controller design

The purpose of this section is to design an ARC controller for system (14), so that the actual output force $F$ of asymmetric electrohydraulic actuator can accurately follow the desired optimized one $F_r$. At the same time, the nonlinear dynamics and parametric uncertainties of the whole system are considered comprehensively. In the actuator system (14), the load flow $\hat{Q}_L$ reflecting the physical sense is regarded as the control input, which is also adopted in works by Sun et al. and Yao et al.\textsuperscript{20,29} Similar to the main-loop controller, the ARC control law of sub-loop is synthesized in the following form

$$\hat{Q}_L = \hat{Q}_{L,a} + \hat{Q}_{L,r1} + \hat{Q}_{L,r2}$$  \hspace{1cm} (38)

where $\hat{Q}_{L,a}$ is an adaptive model compensation term, $\hat{Q}_{L,r1}$ and $\hat{Q}_{L,r2}$ are the robust terms, which are used to attenuate the influences of rough road and uncertain parameters, respectively.

The control force discrepancy between $F$ and $F_r$ is written as $\zeta = F - F_r$, and the differential of $\zeta$ gives

$$\dot{\zeta} = \dot{F} - \dot{F}_r$$  \hspace{1cm} (39)

From equations (20), (23) and (26), we can get that

$$\dot{F}_r = \frac{\partial F_r}{\partial x_1} \dot{x}_2 + \frac{\partial F_r}{\partial x_2} \dot{x}_2 + \frac{\partial F_r}{\partial x_3} \dot{x}_4 + \frac{\partial F_r}{\partial x_4} \dot{x}_4 + \frac{\partial F_r}{\partial \theta_1} \dot{\theta}_1$$  \hspace{1cm} (40)

In view of the active suspension system with actuator dynamics, a Lyapunov function is chosen based on the trajectories tracking errors and force tracking error

$$V_2 = \frac{1}{2} s^2 + \frac{1}{2} \zeta^2$$  \hspace{1cm} (41)

From equations (14), (27) and (39), we can obtain

$$\dot{V}_2 \leq (\lambda_2 \theta_1 - \lambda_4 \sigma)s \cdot \zeta - \kappa_1 s^2 + e_1 + \zeta \left\{ (\hat{\theta}_2 + \tilde{\theta}_2) [-\eta_1 (x_2 - x_4) - \eta_2 F - \eta_3 + \hat{Q}_L] - \frac{\partial F_r}{\partial x_1} \dot{x}_2 - \frac{\partial F_r}{\partial x_3} \dot{x}_4 - \frac{\partial F_r}{\partial \theta_1} \dot{\theta}_1 ight. \\
- \left. \frac{\partial F_r}{\partial x_2} \left[ -k_1 (x_1 - x_3) - c_1 (x_2 - x_4) + f_s (x_1, x_3) + f_d (x_2, x_4) + \ddot{F} \right] \\
- \frac{\partial F_r}{\partial x_4} \sigma [k_1 (x_1 - x_3) + c_1 (x_2 - x_4) + f_s (x_1, x_3) + f_d (x_2, x_4) - F - k_i (x_3 - z_r)] \right\}$$  \hspace{1cm} (42)
in which $\hat{\theta}_2$ is the online estimate value of uncertain parameter $\theta_2$ and the estimate error is defined as $\tilde{\theta}_2 = \theta_2 - \hat{\theta}_2$. We will derive the adaptive law of $\hat{\theta}_2$ in next section.

According to equation (42), we can derive the adaptive model compensation term as

$$\tilde{Q}_{La} = \eta_1(x_2 - x_4) + \eta_2 F + \eta_3 + \frac{1}{\theta_2} \left\{ - (\lambda_2 \hat{\theta}_1 - \lambda_4 \sigma) s + \frac{\partial F_r}{\partial x_1} x_2 + \frac{\partial F_r}{\partial x_3} x_4 + \frac{\partial F_r}{\partial \hat{\theta}_1} \hat{\theta}_1 \right. \left. + \frac{\partial F_r}{\partial x_2} \hat{\theta}_1 [-k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_s(x_1, x_3) - f_d(x_2, x_4) + F] + \frac{\partial F_r}{\partial x_4} \sigma [k_1(x_1 - x_3) + c_1(x_2 - x_4) + f_s(x_1, x_3) + f_d(x_2, x_4) - F - k_1 x_3] \right\}$$

(43)

By substituting $\tilde{Q}_{La}$ into equation (42), we have

$$\dot{V}_2 \leq -\kappa_2 s^2 + e_1 + \lambda_2 \hat{\theta}_1 s \cdot \zeta + \zeta \left\{ \tilde{\theta}_2 \left( \tilde{Q}_{Ls1} + \tilde{Q}_{Ls2} \right) + \tilde{\theta}_2 [-\eta_1(x_2 - x_4) - \eta_2 F - \eta_3 + \tilde{Q}_L] \right. \left. - \frac{\partial F_r}{\partial x_2} \hat{\theta}_1 [-k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_s(x_1, x_3) - f_d(x_2, x_4) + F] - \frac{\partial F_r}{\partial x_4} \sigma k_1 z_r \right\}$$

(44)

Using the same technique as equation (23), the robust term for road disturbances is designed as

$$\tilde{Q}_{Ls1} = \frac{1}{\theta_2} \left[ -\kappa_2 \zeta - \delta_2 \tanh(\Gamma_2 \zeta) \right], \quad \left( \left| \frac{\partial F_r}{\partial x_4} \sigma k_1 z_r \right| \leq \delta_2 \right)$$

(45)

where $\kappa_2$ and $\Gamma_2$ are both positive constants, and $\tanh(\Gamma_2 \zeta)$ can be made arbitrarily close to function $\text{sgn}(s)$. Note that $\zeta \left( \tilde{\theta}_2 \tilde{Q}_{Ls2} - \frac{\partial F_r}{\partial x_4} \sigma k_1 z_r \right) \leq -\kappa_2 \zeta^2$, thence there is

$$\dot{V}_2 \leq -\kappa_1 s^2 + e_1 + \kappa_2 s^2 + \zeta \left\{ \tilde{\theta}_2 \tilde{Q}_{Ls2} + \tilde{\theta}_1 \lambda_2 s - \frac{\partial F_r}{\partial x_2} [-k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_s(x_1, x_3) - f_d(x_2, x_4) + F] + \frac{\partial F_r}{\partial x_4} [-\eta_1(x_2 - x_4) - \eta_2 F - \eta_3 + \tilde{Q}_L] \right\}$$

(46)

In addition, $\tilde{Q}_{Ls2}$ is constructed to dominate the model uncertainties arising from uncertain parameters, which is any function meeting the following conditions

1) \( \zeta \left( \tilde{\theta}_2 \tilde{Q}_{Ls2} + \tilde{\theta}_1 \lambda_2 s - \frac{\partial F_r}{\partial x_2} [-k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_s(x_1, x_3) - f_d(x_2, x_4) + F] + \frac{\partial F_r}{\partial x_4} [-\eta_1(x_2 - x_4) - \eta_2 F - \eta_3 + \tilde{Q}_L] \right) \leq e_2 \)

2) \( \zeta \frac{\partial \tilde{Q}_{Ls2}}{\partial z_r} \leq 0 \)

(47)

where $e_2$ is an arbitrarily small positive number, and a smooth function $\tilde{Q}_{Ls2}$ satisfying the constraints with regard to equation(47) can be given as

$$\tilde{Q}_{Ls2} = -\frac{1}{4 \theta_{2\text{min}}} m_2 \zeta$$

(48)

in which $m_2$ is any smooth function satisfying

$$m_2 \geq \frac{1}{e_{2a}} \left( \theta_{1\text{max}} - \theta_{1\text{min}} \right)^2 \left\{ \lambda_2 s \left[ \frac{\partial F_r}{\partial x_2} [-k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_s(x_1, x_3) - f_d(x_2, x_4) + F] \right] \right)^2$$

$$+ \frac{1}{e_{2b}} \left( \theta_{2\text{max}} - \theta_{2\text{min}} \right)^2 \left[ \eta_1(x_2 - x_4) - \eta_2 F - \eta_3 + \tilde{Q}_L + \tilde{Q}_{Ls1} \right]^2$$

where $e_{2a}$ and $e_{2b}$ are any positive scalars such that $e_2 = e_{2a} + e_{2b}$. 


Further, one can derive the following result
\[ V_2' \leq -\kappa_1 s^2 - \kappa_2 \zeta^2 + e_1 + e_2 \]
\[ \leq -2\kappa V_2 + e \] (49)

where \( \kappa = \min(\kappa_1, \kappa_2) \) and \( e = e_1 + e_2 \).

Equation (49) shows that the Lyapunov function is bounded by
\[ V_2(t) \leq V_2(0)e^{-2\kappa t} + \frac{e}{2\kappa}(1 - e^{-2\kappa t}) \] (50)

Hence, the close-loop system is bounded stable. That is to say, for the uncertain and nonlinear active suspension system (5) actuated by an asymmetric electrohydraulic actuator (14), by employing the ARC control law of equations (43), (45) and (48), the actual output force \( F \) can follow the desired one \( F_r \) with a bounded tracing error.

In addition, based on the load flow and spool displacement equation (10) and ARC control law obtained above, the terminal ARC control law \( u = x \) can be obtained by
\[ u = \frac{\bar{Q}_L}{\Theta g(u, F)} \] (51)

**Parameters update laws and stability analysis**

We have proven the ultimate boundedness of the nonlinear uncertain active suspension system without and with actuator dynamics under ARC control law. Here, we will present the adaptive update law of \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \).

Choosing a Lyapunov function as
\[ V_3 = \frac{1}{2}s^2 + \frac{1}{2}\zeta^2 + \frac{1}{2r_1}\hat{\theta}_1^2 + \frac{1}{2r_2}\hat{\theta}_2^2 \] (52)

Differentiating \( V_3 \) along with equations (24) and (46), one can obtain the following result
\[ \dot{V}_3 = s \cdot \dot{s} + \zeta \cdot \dot{\zeta} - \frac{1}{r_1} \hat{\theta}_1 \dot{\theta}_1 - \frac{1}{r_2} \hat{\theta}_2 \dot{\theta}_2 = -\kappa_1 s^2 + (\lambda_2 \hat{\theta}_1 - \lambda_4 \sigma)F_{sr2} \cdot s \\
\quad + \lambda_2 \hat{\theta}_2[1 - k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_s(x_1, x_3) - f_a(x_2, x_4) + F_a] \\
\quad + F_{sr1}[s - k_2 \zeta^2 + \theta_2 \bar{Q}_{L2} \cdot \zeta] \\
\quad + \zeta \left\{ \lambda_2 \hat{\theta}_2 - \frac{\partial F_r}{\partial x_2}[-k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_s(x_1, x_3) - f_a(x_2, x_4) + F] \\
\quad + \frac{\partial F_r}{\partial x_1}[-\eta_1(x_2 - x_4) - \eta_2 F - \eta_3 + \bar{Q}_{L1} + \bar{Q}_{L1}] \right\} \\
\quad - \frac{1}{r_1} \hat{\theta}_1 \dot{\theta}_1 - \frac{1}{r_2} \hat{\theta}_2 \dot{\theta}_2 \] (53)

It is clear from equations (26) and (48) that \( (\lambda_2 \theta_1 - \lambda_4 \sigma)F_{sr2} \cdot s \leq 0 \) and \( \theta_2 \bar{Q}_{L2} \cdot \zeta \leq 0 \). Then, to make \( \dot{V}_3 \) non-positive, the adaptive control law of \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \) are constructed in the following discontinuous projection form, which is updated online and given as\(^{28}\)
\[ \hat{\theta}_j = \text{Proj}_{\hat{\theta}_j}(r_j \tau_j) = \begin{cases} 0 & \hat{\theta}_j = \theta_{j_{\max}} \text{ for } r_j \tau_j > 0 \\ 0 & \hat{\theta}_j = \theta_{j_{\min}} \text{ for } r_j \tau_j < 0 \\ r_j \tau_j & \text{otherwise} \end{cases} \] (53)

\[ \theta = \text{Proj}_\theta = \begin{cases} 0 & \theta_j = \theta_{j_{\max}} \text{ & } r_j \tau_j > 0 \\ 0 & \theta_j = \theta_{j_{\min}} \text{ & } r_j \tau_j < 0 \\ r_j \tau_j & \text{otherwise} \end{cases} \]
where \( j = 1, 2 \) and \( r_j > 0 \) are tunable gains. \( \tau_j \) are adaptive functions to be synthesized as follows

\[
\begin{align*}
\tau_1 &= \lambda_2 \left( -k_1(x_1 - x_3) - c_1(x_2 - x_4) - f_s(x_1, x_3) - f_d(x_2, x_4) + F_{a_r} + F_{a_1} \right) x + \lambda_2 s_i \\
\tau_2 &= \left[ -\eta_1(x_2 - x_4) - \eta_2 F - \eta_3 + Q_{La} + Q_{La_1} \right] \zeta
\end{align*}
\] (55)

Noting the projection mapping property of equation (54), there is

\[
\dot{V}_3 \leq -\kappa_1 s_i^2 - \kappa_2 s_i^2
\]

which implies that with the terminal ARC control law (51) and uncertain parameters update laws (54) and (55), the close-loop system is asymptotically stable, i.e. the trajectories tracking errors \( \psi \) and force tracking error \( \zeta \) asymptotically converge to zero as \( t \to \infty \), by using Barbalat’s Lemma.

### Simulations

In this section, numerical simulations are provided to illustrate the effectiveness and robustness of the proposed Dual-ARC controller. In the simulations, the unknown nonlinear terms \( f_s(x_1, x_3) \) and \( f_d(x_2, x_4) \) appearing in spring force \( F_s \) and damper force \( F_d \) are the same as those used in Liu et al.\(^\text{13}\) and Deshpande et al.,\(^\text{14,33}\) which is given by

\[
\begin{align*}
f_s &= k_2(x_1 - x_3)^3 \\
f_d &= c_2(x_2 - x_4)^2
\end{align*}
\] (57)

where \( k_2 \) and \( c_2 \) are the nonlinear stiffness and damping coefficient of spring and damper. We should mention that \( f_s(x_1, x_3) \) and \( f_d(x_2, x_4) \) must be differentiable according to the ARC laws of equations (43), (45) and (48). However, in some cases,\(^\text{11,23,35}\) the nonlinear term \( F_d \) may contain absolute function or piecewise function, which is non-differentiable at the discontinuous point. Fortunately, they can be equivalently substituted by a symbolic function \( \text{sgn}(x) \), which can be arbitrarily approached by a differentiable function \( \tanh(kx) \).\(^\text{29,30}\) What is more, the quarter-car model parameters are listed in Table 1.\(^\text{32}\)

### Effectiveness analysis

The effectiveness of the proposed control strategy (Dual-ARC) is evaluated on three different road profiles and compared with passive suspension system (Passive) and the existing results in Liu et al.\(^\text{13}\) about active suspension system with ASC in [13]. The simulation results are presented in the following. Herein, the relative suspension deflection (RSD) is defined as \( (z_i - z_a)/z_{max} \), and the relative tyre force (RTF) is defined as \( F_t(z_a, z_r)/(m_i + m_a)/g \). \( |RSD| < 1 \) and \( |RTF| < 1 \) should be maintained so as to satisfy the suspension constraint requirements in equations (16) and (17).

### Table 1. Parameters of quarter-car active suspension with asymmetric electrohydraulic actuator.

| Parameter   | Value       | Unit | Parameter   | Value       | Unit |
|-------------|-------------|------|-------------|-------------|------|
| \( m_i \)   | 500         | kg   | \( \lambda_2 \) | 6.406 × 10⁻⁴ | m²   |
| \( m_u \)   | 40          | kg   | \( \nu_1 \)   | 1.884 × 10⁻⁴ | m³   |
| \( k_1 \)   | 16,021      | Nm⁻¹ | \( \nu_2 \)   | 9.600 × 10⁻⁵ | m³   |
| \( k_2 \)   | 180,000     | Nm⁻³ | \( \rho \)    | 21          | MPa  |
| \( \alpha_1 \)| 1.419       | Nsm⁻¹| \( \beta_e \) | 860         | kgm⁻³|
| \( \alpha_2 \)| 400         | Nsm⁻²| \( \beta_s \) | 700         | MPa  |
| \( k_t \)   | 240,000     | Nm⁻¹ | \( w \)      | 3.460 × 10⁻² | m    |
| \( z_{max} \)| 0.15        | m    | \( C_d \)    | 0.67        | –    |
| \( A_1 \)   | 1.256 × 10⁻³| m²   | \( C_w \)    | 2.450 × 10⁻¹⁰| –    |
**Isolated bump road profiles.** Firstly, an isolated bump road excitation considered in Huang et al.,\textsuperscript{11} Liu et al.\textsuperscript{13} and Sun et al.\textsuperscript{35} is chosen as the road disturbance input, which is described by

$$z_r = \begin{cases} 
\frac{h_1}{2} \left( 1 - \cos \frac{2\pi v}{L} t \right), & 0 \leq t \leq \frac{L}{v} \\
0, & \text{otherwise}
\end{cases}$$

where $h_1$ and $L$ are the height and length of the bump, and $v$ is the vehicle velocity, which are set as $h_1 = 0.1 \text{ m}$, $L = 5 \text{ m}$ and $v = 45 \text{ km/h}$.

Figure 5 shows the comparisons among passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC in terms of the time responses for sprung mass displacement, sprung mass acceleration, RSD and RTF under bump road input. As seen from Figure 5(a) to (d), the magnitudes of sprung mass displacement, sprung mass acceleration and RTF for the two controlled active suspensions are less than those of passive suspension; meanwhile, the Dual-ARC is apparently superior to the ASC in [13]. This indicates that the Dual-ARC can work better in stabilizing vehicle body, improving ride comfort and road holding.

Additionally, it is well recognized that the vehicle body attitude stability and ride comfort can be quantified by the root mean square (RMS) values of sprung mass displacement and sprung mass acceleration, respectively, and the suspension constraint performance is closely related to the peak values of [RSD] and [RTF]. Hence, the quantized values are given in Table 2, which reflect the percentages of performance improvement achieved by the Dual-ARC and ASC in [13] with respect to the passive system. As shown in Table 2, the vehicle body attitude

**Figure 5.** Bump road response comparisons among passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC: (a) sprung mass displacement; (b) sprung mass acceleration; (c) relative suspension deflection; and (d) relative tyre force.

ARC: adaptive robust control; ASC: adaptive sliding controller.
stability, ride comfort and road holding capacity of ASC in [13] increase by 66.33%, 57.90% and 35.15% compared to the passive suspension, whereas the improvements of Dual-ARC are 99.37%, 93.28% and 71.17%, respectively. Although the RSD peak values of ASC in [13] and Dual-ARC are slightly higher than those of passive suspension, they remain less than 1thence the suspension deflection constraint is guaranteed. Also, in view of these four response curves, the settle time of Dual-ARC is much less than that of ASC in [13] and passive suspension. These findings sufficiently reflect the superior control performance of the presented control approach when subjected to the bump road.

In addition, the time histories of the desired force and actual actuator output force for the proposed controller are plotted in Figure 6. It is clear from the figure that the real force can track the required force with extremely high accuracy, which demonstrates that the Dual-ARC is able to deal with the nonlinearities in asymmetric electrohydraulic actuator perfectly.

Periodic sinusoidal road profiles. We will further verify the improved performance of the proposed controller under periodic sinusoidal road profiles, and the corresponding road displacement is formulated as

\[
    z_r = 0.0508 \sin (2\pi t) + 0.010 \sin (10.5\pi t) + 0.002 \sin (21.5\pi t)
\]  

(59)

This road excitation is reasonable since it covers the low-frequency vibrations that are close to the vehicle body’s natural frequency (1 Hz), as well as the high-frequency sensitive vibrations.

The comparative performance curves for the three systems, i.e. passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC, are provided in Figure 7. It is clear from Figure 7(a) that the proposed Dual-ARC controller can acquire a much smoother vehicle body attitude than passive suspension and active suspension with ASC in [13]. After carefully observing the sprung mass acceleration curves in

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**Table 2.** Performance comparisons among passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC under bump road.

| Performance indexes | RMS $z_s$ | RMS $\dot{z}_s$ | Peak $|\text{RSD}|$ | Peak $|\text{RTF}|$
|---------------------|-----------|----------------|----------------|----------------|
| Passive             | 0.0300    | 1.3742         | 0.4764         | 0.4034         |
| ASC in [13]         | 0.0101 (66.33%) | 0.5785 (57.90%) | 0.5765 (21.01%) | 0.2616 (35.15%) |
| Dual-ARC            | 0.00019 (99.37%) | 0.0924 (93.28%) | 0.6818 (43.12%) | 0.1163 (71.17%) |

ARC: adaptive robust control; ASC: adaptive sliding controller; RMS: root mean square; RSD: relative suspension deflection; RTF: relative tyre force.
Figure 7(b), one can find that the magnitudes of Dual-ARC and ASC in [13] are apparently lower than that of passive suspension, and the decrease of Dual-ARC is more remarkable. These results indicate that an increased vehicle body stability and ride comfort can be obtained by using Dual-ARC controller. Figure 7(c) and (d) shows that the RSD and the RTF for the three suspension systems are all less than 1 and fall into their acceptable ranges. At the same time, the dynamic tyre load for ASC in [13] is better than the passive suspension, while worse than the Dual-ARC. Hence, the proposed Dual-ARC will result in more satisfactory road holding capacity on periodic sinusoidal road. Additionally, the force tracking effect is shown in Figure 8. As seen, the actual actuator output is capable to follow the desired force precisely. This is reasonable because the control law design of Dual-ARC has taken into account the nonlinear and uncertain actuator dynamics.

Table 3 shows the quantized performance comparisons among passive suspension, active suspension with ASC in [13] as well as active suspension with Dual-ARC concerning vehicle body attitude stability, ride comfort, suspension deflection constraint and road holding capacity. Clearly, the two active suspensions can considerably enhance the suspension performance with respect to the passive suspension, and the improvement of Dual-ARC is more significant. Specifically, the RMS values of $z_s$, $\dot{z}_s$ and the Peak values of $|\text{RSD}|$ and $|\text{RTF}|$ for Dual-ARC decrease by 99.77%, 86.02%, 51.22%, 56.66%, respectively, when compared with passive suspension. These reductions are greater than those seen for the ASC in [13]. These numerical comparisons further confirm the effectiveness of our designed controller.

**Random ISO D class road profiles.** In order to further validate the effectiveness of the proposed controller, a random road input approaching to the realistic road is employed. The road displacement is obtained by using white noise filtration method based on the ISO standards, which is described as $^{25,42}$
\[ z_r = -2\pi n_0 v_z + 2\pi \sqrt{G_q(n_0)v w_0(t)} \]  

where \( v \) is the vehicle longitudinal velocity, \( n_0 = 0.1 \) is the reference space frequency, \( G_q(n_0) \) is the road roughness coefficient and \( w_0(t) \) is a Gaussian white noise with a zero mean value. Here, we assume that the vehicle is traveling at a speed of \( v = 20 \text{ m/s} \) under D class road with \( G_q(n_0) = 1024 \times 10^{-6} \text{ m}^3 \).

Figure 9 compares the time response histories of sprung mass displacement, sprung mass acceleration, RSD and RTF for the three different suspension systems under random road. What is more, the quantized performance is presented in Table 4. From Figure 9(a) we can see that although the sprung mass displacement of ASC in [13] is substantially reduced when compared with passive suspension, the Dual-ARC has only minor fluctuations even in the presence of such a severe road condition. As shown in Table 4, the displacement RMS value of ASC in [13] reduce by 62.33% with respect to the passive suspension, but the decrease of Dual-ARC is up to 99.38%. This suggests that the proposed Dual-ARC can enhance the vehicle body stability more effectively. As can be seen in Figure 9(b), the magnitudes of sprung mass acceleration are significantly decreased by the two active suspension controller (Dual-ARC and ASC in [13]). In contrast to passive suspension, the acceleration RMS values of ASC in [13] and Dual-ARC are reduced by 28.56% and 66.26%, respectively, thence the Dual-ARC outperforms the ASC in [13] and the passive one in increasing ride comfort. Meanwhile, from Figure 9(c) and (d), we can observe that the Dual-ARC and ASC in [13] have almost no improvement in suspension deflection and dynamic tyre load with respect to the passive suspension. However, the absolute values of RSD and RTF for the three suspension systems are all strictly less than unity, which means that the suspension stroke constraint is guaranteed while the wheel will maintain uninterrupted contact with the random road surface. In addition, the force tracking effect for the Dual-ARC is given in Figure 10. We can see that the proposed controller is effective to conduct on the highly nonlinear active actuator and produce desired force in spite of the existence of a harsh road input.

### Table 3. Performance comparisons among passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC under periodic sinusoidal road.

| Performance indexes | RMS $z_s$ | RMS $\dot{z}_s$ | Peak | Peak | RSD | RTF |
|---------------------|-----------|-----------------|------|------|-----|-----|
| Passive             | 0.0738    | 3.0179          | 0.9141 | 0.7053 |
| ASC in [13]         | 0.0104 (85.91%) | 1.0333 (85.76%) | 0.4493 (150.85%) | 0.4433 (157.15%) |
| Dual-ARC            | 0.00017 (199.77%) | 0.4220 (186.02%) | 0.4459 (151.22%) | 0.3057 (156.66%) |

ARC: adaptive robust control; ASC: adaptive sliding controller; RMS: root mean square; RSD: relative suspension deflection; RTF: relative tyre force.
Robustness analysis

Furthermore, it is more reasonable to evaluate the robustness of the proposed controller against the uncertain parameters existing in main-loop system and sub-loop system. The following three parameter perturbation cases are implemented on passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC, respectively.

- **Case 1:** \( m_{sn} = 500 \text{ kg}, \, \beta_{en} = 700 \text{ MPa}, \, V_{10n} = 1.884 \times 10^{-4} \text{ m}^3, \, V_{20n} = 9.600 \times 10^{-5} \text{ m}^3 \) (Normal parameters).
- **Case 2:** \( m_{s} = (1 - 20\%) \times m_{sn}, \, \beta_{e} = (1 + 30\%) \times \beta_{en}, \, V_{10} = 2.512 \times 10^{-4} \text{ m}^3, \, V_{20} = 6.400 \times 10^{-5} \text{ m}^3 \).
- **Case 3:** \( m_{s} = (1 + 20\%) \times m_{sn}, \, \beta_{e} = (1 - 30\%) \times \beta_{en}, \, V_{10} = 1.256 \times 10^{-4} \text{ m}^3, \, V_{20} = 1.280 \times 10^{-5} \text{ m}^3 \).
Due to the strong robustness of the proposed control approach, the response curves for the three cases are very close, which are very difficult to distinguish. Therefore, the quantitative performance indexes are employed to evaluate the robustness of the designed controller. The quantized values for passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC are compared under the three parameters perturbation cases. Meanwhile, the performance improvement percentage for each active control case is calculated with respect to the corresponding passive suspension. The numerical comparisons under bump road are given in Table 5. It is easy to observe that the RMS values of $z_s$ and $\dot{z}_s$, and the peak value of RTF under ASC in [13] are lower than those of passive suspension, but higher than those of Dual-ARC, no matter the parameters change or not. In addition, the $|\text{RSD}|$ peak value of each case is far less than 1, hence the suspension stroke constraint is ensured. These results further verify the effectiveness of the proposed Dual-ARC controller. As seen in Table 5, the vehicle body stability, ride comfort and road holding capacity of ASC in [13] have improved about 65%, 57% and 33%, respectively, for all parameters perturbation cases, while the improvement of Dual-ARC are about 99%, 93%, 70%, respectively. There is no significant performance change for the two active suspension systems in spite of suffering from parameter variations. Therefore, we can conclude that ASC in [13] and Dual-ARC are both robust against the uncertain system parameters, whereas the control effect of Dual-ARC is superior to the ASC in [13].

Table 6 shows the quantized performance comparisons for the three suspension systems and the performance improvement percentages for the two active suspension systems in terms of different system parameters under

![Figure 10. Desired force and actual actuator output force of Dual-ARC for random road.](image-url)
periodic sinusoidal road. It is apparent that the Dual-ARC exhibits more excellent vibration isolation capability since it can reduce the RMS values of \( z_s \) and \( \dot{z}_s \), as well as the peak values of \( \text{RSD} \) and \( \text{RTF} \) more significantly than passive suspension and ASC in [13]. On the other hand, for ASC in [13] and Dual-ARC, the vehicle body stability, ride comfort and suspension hard constraints only have minor fluctuations when system parameters are changing. At the same time, for ASC in [13], the quantized value variation ranges concerning \( z_s \), \( \dot{z}_s \), \( \text{RSD} \) and \( \text{RTF} \) are 0.0034, 0.4229, 0.0176 and 0.1697, respectively, while for Dual-ARC, the variation ranges are 0.00001, 0.0852, 0.0065 and 0.1503, respectively. The fluctuation scopes of Dual-ARC are much smaller than those of ASC in [13]. It thus proves that the proposed control method is less sensitive to the parametric uncertainties and possesses stronger robustness.

The performance of passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC under different system parameters is compared on random road, and the corresponding quantitative values are presented in Table 7. It is easy to obtain that the Dual-ARC is more effective than passive suspension and ASC in [13] on random road, which is similar to the conclusions on bump and periodic sinusoidal roads. In contrast to passive suspension, the vehicle body attitude stability of Dual-ARC enhances up to 99% for all listed cases, and the improvement of ride comfort can be maintained at about 66%. While for ASC in [13], the performance improvement percentages of vehicle body attitude stability and ride comfort show more noticeable changes under the three parameters perturbation cases. The peak values of ASC in [13] and Dual-ARC are both lower than that of passive suspension and are insensitive to the parameter variations. Additionally, the road holding capacity (\( \text{RTF} \) peak value) of ASC in [13] is worse than passive suspension, but that of the Dual-ARC is better than passive suspension and the percentages are quite stable. These results sufficiently indicate that the Dual-ARC has stronger robustness in improving vehicle body stability and ride comfort while guaranteeing the hard constraints in the presence of parametric uncertainties. The reason for the strong robustness is owing to that the main uncertain parameters are estimated online in the controller design process.

### Table 6. Performance comparisons among passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC for parameters perturbation cases under periodic sinusoidal road.

| Performance indexes | RMS \( z_s \) | RMS \( \dot{z}_s \) | Peak \( \text{RSD} \) | Peak \( \text{RTF} \) |
|---------------------|--------------|----------------|----------------|----------------|
| Passive             |              |                |                |                |
| Case 1              | 0.0738       | 3.0179         | 0.9141         | 0.7053         |
| Case 2              | 0.0770       | 3.2728         | 0.7726         | 0.7803         |
| Case 3              | 0.0623       | 2.4966         | 0.9029         | 0.5626         |
| ASC in [13]         |              |                |                |                |
| Case 1              | 0.0104 \((85.91\%)\) | 1.0333 \((65.76\%)\) | 0.4493 \((50.85\%)\) | 0.4433 \((37.15\%)\) |
| Case 2              | 0.0124 \((83.90\%)\) | 1.2864 \((60.69\%)\) | 0.4381 \((43.30\%)\) | 0.5438 \((30.31\%)\) |
| Case 3              | 0.0090 \((85.55\%)\) | 0.8635 \((65.41\%)\) | 0.4557 \((49.53\%)\) | 0.3741 \((33.51\%)\) |
| Dual-ARC            |              |                |                |                |
| Case 1              | 0.00017 \((99.77\%)\) | 0.4220 \((86.02\%)\) | 0.4459 \((51.22\%)\) | 0.3057 \((56.66\%)\) |
| Case 2              | 0.00017 \((99.78\%)\) | 0.4671 \((85.73\%)\) | 0.4496 \((41.81\%)\) | 0.3822 \((51.02\%)\) |
| Case 3              | 0.00018 \((99.71\%)\) | 0.3818 \((84.71\%)\) | 0.4431 \((50.92\%)\) | 0.2319 \((58.78\%)\) |

ARC: adaptive robust control; ASC: adaptive sliding controller; RMS: root mean square; RSD: relative suspension deflection; RTF: relative tyre force.

### Table 7. Performance comparisons among passive suspension, active suspension with ASC in [13] and active suspension with Dual-ARC for parameters perturbation cases under random road.

| Performance indexes | RMS \( z_s \) | RMS \( \dot{z}_s \) | Peak \( \text{RSD} \) | Peak \( \text{RTF} \) |
|---------------------|--------------|----------------|----------------|----------------|
| Passive             |              |                |                |                |
| Case 1              | 0.0369       | 1.4056         | 0.5422         | 0.7055         |
| Case 2              | 0.0357       | 1.7018         | 0.5468         | 0.8609         |
| Case 3              | 0.0378       | 1.2169         | 0.5137         | 0.5870         |
| ASC in [13]         |              |                |                |                |
| Case 1              | 0.0139 \((62.33\%)\) | 1.0042 \((12.85\%)\) | 0.4623 \((14.74\%)\) | 0.8184 \((16.00\%)\) |
| Case 2              | 0.0157 \((56.02\%)\) | 1.2557 \((26.21\%)\) | 0.4508 \((17.56\%)\) | 1.0093 \((17.24\%)\) |
| Case 3              | 0.0125 \((66.93\%)\) | 0.8381 \((131.13\%)\) | 0.4700 \((8.51\%)\) | 0.6884 \((17.27\%)\) |
| Dual-ARC            |              |                |                |                |
| Case 1              | 0.00023 \((99.38\%)\) | 0.4743 \((16.26\%)\) | 0.5214 \((3.84\%)\) | 0.5962 \((15.49\%)\) |
| Case 2              | 0.00024 \((99.24\%)\) | 0.5271 \((66.51\%)\) | 0.5306 \((2.96\%)\) | 0.7648 \((11.16\%)\) |
| Case 3              | 0.00022 \((99.47\%)\) | 0.4367 \((166.73\%)\) | 0.4978 \((3.10\%)\) | 0.5678 \((13.27\%)\) |

ARC: adaptive robust control; ASC: adaptive sliding controller; RMS: root mean square; RSD: relative suspension deflection; RTF: relative tyre force.
Conclusions

This paper proposed a novel dual ARC controller for active suspension system equipped with more reasonable asymmetric electrohydraulic actuator, where the nonlinearities and uncertainties existing in main-loop system and sub-loop system are both taken into account. An ARC is firstly designed for main-loop system to obtain the desired control force in the presence of nonlinear spring, nonlinear damper as well as uncertain sprung mass and road disturbances. In order to satisfy the constraint requirements of suspension system, an $H_\infty$ optimization scheme based on LMI and KA is proposed to obtain the appropriate parameters in main-loop control law. Another ARC is further designed for nonlinear uncertain sub-loop system in order to compel the actual actuating force to precisely follow the optimized desired one. The control law no matter in main-loop system or in sub-loop system can guarantee the bounded stability of the corresponding system. Meanwhile, by selecting appropriate estimation law for uncertain parameters in suspension system, the asymptotic stability of the close-loop system can be achieved within the Lyapunov framework. Finally, excessive simulations are conducted on isolated bump road, periodic sinusoidal road as well as random road. The results sufficiently confirm that the proposed Dual-ARC is effective and robust in stabilizing body attitude, improving ride comfort while ensuring the suspension hard constraints.

Acknowledgements

This work was supported by the National Key R&D Program of China (Grant No. 2016YFC0802900), China.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) received no financial support for the research, authorship, and/or publication of this article.

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