The Supersymmetric Particle Spectrum in Orbifold Compactifications of String Theory

A. Love and P. Stadler*

Department of Physics
Royal Holloway
University of London
Egham, Surrey, TW20 0EX, UK.

July 1997

Abstract

The supersymmetry breaking parameters and the resulting supersymmetric particle spectrum are studied in orbifold compactifications of string theory under the assumption that unification of gauge coupling constants at about $10^{16}$ GeV is a consequence of large moduli dependent string loop threshold corrections. The effect on the spectrum of various assumptions as to the modular weights of the states, the values of the Green-Schwarz parameter, $\delta_{GS}$, the origin of the $\mu$ parameter and the moduli dependence of Yukawa couplings is discussed. The effect of radiative corrections to the effective potential is also considered.

1 Introduction

In a generic supergravity theory, the soft supersymmetry breaking scalar masses, gaugino masses and $A$ and $B$ terms are free parameters. On the other hand if the supergravity theory is the low energy limit of an orbifold compactification of the heterotic string then these parameters are calculable in principle [1, 2, 3, 4, 5] since string theory has only

*P.Stadler@rhbnc.ac.uk
one free parameter, namely the string scale. However, the soft supersymmetry breaking parameters depend on the moduli of the orbifold model (including the dilaton expectation value) and the values of the moduli cannot be determined without a detailed knowledge of the non-perturbative superpotential probably responsible for supersymmetry breaking. One possible approach \cite{4} is to accept for the time being that we lack this detailed knowledge and to absorb this uncertainty into an angle, \( \theta \), which is a measure of the relative size of the auxiliary fields for the dilaton and the overall \( T \) modulus (with the vacuum energy taken to be zero).

In such an approach \cite{4} the supersymmetric particle spectrum has been derived as a function of this angle \( \theta \) when unification of gauge coupling constants at \( 2 \times 10^{16} \) GeV is due to large moduli dependent string loop corrections and also when it is due to extra matter states close to the unification scale. Here, we shall explore the robustness of the qualitative features of the spectrum obtained in the former case when various assumptions about the orbifold model are varied. In particular, we shall consider the effect of making one or more of the following changes to the assumptions in Brignole \textit{et al.} \cite{4}:

a. Using the modular weights allowed \cite{1, 6} for states in the twisted sectors of those abelian orbifolds which possess three \( N = 2 \) moduli, \( T_i \). Then it is possible to adopt a single overall modulus model with \( T_1 = T_2 = T_3 = T \) as in ref.\cite{4} with all three moduli on the same footing if, as is the case in gaugino condensate models, only the \( N = 2 \) moduli occur in the non-perturbative superpotential. The possible choices of modular weights are then further restricted by requiring that the string loop threshold corrections to the gauge coupling constants allow unification of all three observable sector gauge coupling constants at a single energy scale. The value of the overall modulus \( T \) is determined by requiring that this energy scale is \( 2 \times 10^{16} \) GeV. This is an alternative to choosing the simplest set of modular weights \cite{4} which will achieve the gauge coupling unification without reference to any particular orbifold.

b. Adopting the values for the Green-Schwarz parameters, \( \delta_{GS} \), suggested by the above orbifold models.

c. Taking account of the possible moduli dependence of the Yukawa couplings when all three states are in twisted sectors of the orbifold.

d. In the case that the \( \mu \) parameter originates from Kähler potential mixing, using the moduli dependence of \( \mu \) suggested by the discussion of ref.\cite{7} rather than taking \( \mu \) to be moduli independent, and in addition,

e. taking account of the radiative corrections to the tree level effective potential in calculating the Higgs scalar expectaton values \( v_1 \) and \( v_2 \) which affect the supersymmetric particle spectrum and also in calculating the Higgs scalar masses \cite{8, 9}.
We do not consider the effect of more than one independent modulus expectation value which has been considered elsewhere [5], nor do we consider the M-theory regime of strong ten dimensional string coupling [10] for which gauge coupling constant unification at the ‘observed’ energy scale may occur without large string loop threshold corrections if there is a large eleventh dimension [11].

The organisation of the paper is as follows. In section 2 all possible choices of modular weights for the standard model states in abelian orbifold compactifications with three \( N = 2 \) moduli \( T_i \) are obtained. The choice is restricted by demanding consistency with gauge coupling constant unification with \( T_1 = T_2 = T_3 = T \). The corresponding value of \( T \) is also given. In section 3 the soft supersymmetry breaking terms are presented as functions of the overall modulus \( T \) and the angle \( \theta \) introduced in ref.[4]. In section 4 the relevant renormalisation group equations for the running of the coupling constants and soft supersymmetry breaking parameters from the unification scale to the electroweak scale are displayed and the strategy for choosing the various string theoretic parameters and ensuring the correct electroweak breaking scale is discussed. In section 5 the resulting supersymmetric particle spectrum is explored, including the effect of radiative corrections to the effective potential. Finally, in section 6 we present our conclusions and make comparisons with the work of ref.[4].

2 Choices of modular weights

As will be seen in section 3, the values of the soft supersymmetry breaking parameters at the string scale depend on the modular weights of the matter states [1, 2, 3, 4, 5]. Let us first establish our conventions. In general we shall write the Kähler potential \( K \) to quadratic order in the matter fields in the form

\[
K = -\ln Y - \sum_i \ln (T_i + \bar{T}_i) + \sum_\alpha \tilde{K}_\alpha |\phi_\alpha|^2 + (Z\phi_1\phi_2 + h.c.)
\]  

(1)

with

\[
Y = S + \bar{S} - \sum_i \delta_i \ln (T_i + \bar{T}_i)
\]

(2)

\[
\delta_i = \frac{\delta_{GS}}{8\pi^2}
\]

(3)

and

\[
\tilde{K}_\alpha = \prod_i (T_i + \bar{T}_i)^{n_\alpha}
\]

(4)
In (1)-(4) any $U$ moduli associated with $Z_2$ planes are included as additional $T_i$ moduli, $\delta_{GS}^i$ are Green-Schwarz parameters, $\phi_\alpha$ are matter fields and the $Z\phi_1\phi_2$ term is present when the orbifold has a $Z_2$ plane (ie. when the action of the point group in that plane is as $Z_2$). The matter fields $\phi_1$ and $\phi_2$ are untwisted states associated with the $T$ and $U$ moduli for the $Z_2$ plane. The powers $n_\alpha^i$ are the modular weights for the matter fields $\phi_\alpha$.

In the case of a single overall modulus

$$T = T_1 = T_2 = T_3$$

(5)

these expressions reduce to

$$K = -\ln Y - 3\ln (T + \bar{T}) + \sum_\alpha \tilde{K}_\alpha |\phi_\alpha|^2 + (Z\phi_1\phi_2 + h.c.)$$

(6)

with

$$Y = S + \bar{S} - \tilde{\delta}_{GS} \ln (T + \bar{T})$$

(7)

where

$$\delta_{GS} = \sum_i \delta_{GS}^i$$

(8)

$$\tilde{\delta}_{GS} = \frac{\delta_{GS}}{8\pi^2}$$

(9)

and

$$\tilde{K}_\alpha = (T + \bar{T})^{n_\alpha}$$

(10)

with overall modular weights

$$n_\alpha = \sum_i n_\alpha^i$$

(11)

The only abelian orbifolds that possess three $N = 2$ moduli $T_i$ are $Z_2 \times Z_6$ and $Z_3 \times Z_6$, the former orbifold having in addition a single $U$ modulus. All possible modular weights for massless matter states in the twisted (and untwisted) sectors of abelian orbifolds can be determined using the approach of refs. \cite{2} and \cite{6}. For $Z_2 \times Z_6$ the allowed modular weights are

$$(Q, u, e) : n_\alpha = 0, -1, -2$$

(12)

and

$$(L, d, H) : n_\alpha = +1, 0, -1, -2, -3$$

(13)

where $Q$, $L$, and $H$ denote quark, lepton and Higgs $SU(2)_L$ doublets and $u$, $d$ and $e$ denote quark and lepton singlets. For $Z_3 \times Z_6$, the possible modular weights are

$$(Q, u, e) : n_\alpha = 0, -1, -2$$

(14)
For a single overall modulus $T$ the conditions for unification of the $SU(3)_C \times SU(2)_L \times U(1)$ gauge coupling constants $g_3, g_2$ and $\tilde{g}_1$ at a scale less than $10^{18}$ GeV may be taken to be $[12, 13, 2]$

$$\frac{b'_3 - b'_2}{b_3 - b_2} < 0$$  \hspace{1cm} (16)

and

$$\frac{b'_3 - b'_1}{b'_3 - b'_1} = \frac{5}{12}$$  \hspace{1cm} (17)

where the standard model renormalisation group coefficients are

$$b_3 = -3 \ , \ b_2 = 1 \ , \ b_1 = \frac{33}{5}$$  \hspace{1cm} (18)

and the $b'_i \ , \ i = 1, 2, 3$ which occur in the string loop threshold corrections $[12, 14, 15]$ are given by

$$b'_3 = 9 + 2 \sum_{g=1}^{3} (n_{Q(g)} + \frac{1}{2} n_{u(g)} + \frac{1}{2} n_{d(g)})$$  \hspace{1cm} (19)

$$b'_2 = 15 + \sum_{g=1}^{3} (3n_{Q(g)} + n_{L(g)}) + n_{H_1} + n_{H_2}$$  \hspace{1cm} (20)

and

$$\tilde{b}'_1 = \frac{99}{5} + \frac{1}{5} \sum_{g=1}^{3} (n_{Q(g)} + 8n_{u(g)} + 2n_{d(g)} + 3n_{L(g)} + 6n_{e(g)}) + \frac{3}{5} (n_{H_1} + n_{H_2})$$  \hspace{1cm} (21)

where the sum over $g$ is a sum over generations. Here the $U(1)$ coupling constant $\tilde{g}_1$ is normalised so that all three coupling constants are equal at the unification scale.

Assuming generation universality to avoid flavour changing neutral currents,

$$n_{Q(1)} = n_{Q(2)} = n_{Q(3)} = n_Q$$  \hspace{1cm} (22)

and similarly for the modular weights of the other states, the solutions of (16) and (17) with modular weights given by (12) and (13) or (14) and (15) are given in table 1 with

$$M_{\text{string}} \approx 0.53 \times g_{\text{string}} \times 10^{18}\text{GeV}$$  \hspace{1cm} (23)

and

$$g_{\text{string}} \approx 0.7 \ .$$  \hspace{1cm} (24)

The corresponding value of $T$ for which unification takes place at

$$M_X \approx 2 \times 10^{16}\text{GeV}$$  \hspace{1cm} (25)
Table 1: Modular weights

| $n_{QL}$ | $n_{UR}$ | $n_{DR}$ | $n_{LL}$ | $n_{ER}$ | $n_{H_1}$ | $n_{H_2}$ |
|----------|----------|----------|----------|----------|-----------|-----------|
| 0        | -2       | 1        | -3       | -1       | -1        | -1        |
| 0        | -1       | 0        | -3       | -2       | -1        | -1        |
| 0        | -1       | 1        | -2       | -2       | -1        | -1        |
| 0        | -2       | 0        | -4       | -1       | -1        | -1        |
| 0        | -1       | -1       | -4       | -2       | -1        | -1        |

is given by [12, 13, 2]

$$M_X / M_{string} = [(T + \bar{T})|\eta(T)|^4]^{(b'_3-b'_2)/2(b_3-b_2)}$$

and we find $T = 14.5$ is suitable for all choices of modular weights of table [I], and also gives the gauge couplings as $\alpha_s(m_Z) = 0.115$ and $\sin^2 \theta_W(m_Z) = 0.2315$. We have restricted $n_{H_1}$ and $n_{H_2}$ to take the value $-1$ for consistency with the two mechanisms for generating the $\mu$ parameter that we shall discuss in the next section, both of which require the Higgs fields to be untwisted sector states.

3 Soft supersymmetry breaking terms at the string scale

The soft supersymmetry scalar masses, gaugino masses and $A$ and $B$ terms which occur in a supergravity theory may be calculated from the low energy limit of an orbifold compactification of the heterotic string given the Kähler potential, the superpotential and the gauge kinetic function derived from the string theory [I, 2, 3, 4, 5]. In view of our current lack of detailed knowledge of the non-perturbative superpotential responsible for supersymmetry breaking, a possible approach [4] is to absorb this uncertainty into an angle $\theta$ which measures the relative contributions of the dilaton and $T$ modulus auxiliary fields to supersymmetry breaking. In this section we summarize the resulting formulae [4] for the soft supersymmetry breaking terms and discuss the choice of values for the Green-Schwarz parameters, the moduli dependence of the Yukawa couplings that occur in the $A$ term and the $\mu$ parameter that occurs in the $B$ term. The expressions for the soft supersymmetry breaking terms are expressed in terms of the angle $\theta$ defined by [4]

$$F^S - \delta_{GS}(T + \bar{T})^{-1}F^T = \sqrt{3} CY m_{3/2} \sin \theta$$

(27)
and
\[
\left( Y - \frac{\delta_{GS}}{3} \right) \frac{1}{2} F^T = C(T + \bar{T}) m_{3/2} \cos \theta \tag{28}
\]
where the auxiliary fields \( F^S \) and \( F^T \) for the dilaton and the overall \( T \) modulus are given in terms of \( G \equiv K + \ln |W|^2 \) by
\[
F^S - \delta_{GS}(T + \bar{T})^{-1} F^T = Y^2 m_{3/2} \frac{\partial G}{\partial S} \tag{29}
\]
and
\[
F^T = \frac{(T + \bar{T})^2 Y m_{3/2}}{3 \left( Y - \frac{\delta_{GS}}{3} \right)} \left( \frac{\partial G}{\partial T} + \delta_{GS}(T + \bar{T})^{-1} \frac{\partial G}{\partial S} \right). \tag{30}
\]
It has been assumed that all three moduli \( T_i, \ i = 1, 2, 3, \) are on the same footing in the Kähler potential and superpotential and possible (CP-violating) phases have been dropped for present purposes. The vacuum energy \( V_0 \) is given by
\[
V_0 = 3(C^2 - 1)m_{3/2}^2 \tag{31}
\]
where
\[
m_{3/2} = e^{G/2} \tag{32}
\]
at the minimum of the effective potential. Thus, if the vacuum energy is identified with the cosmological constant we should take \( C = 1 \). This we shall do throughout.

The soft supersymmetry breaking scalar masses, \( m_\alpha \), are given by
\[
m_\alpha^2 = (3C^2 - 2)m_{3/2}^2 + n_\alpha C^2 m_{3/2}^2 \frac{Y}{Y - \frac{\delta_{GS}}{3}} \cos^2 \theta \tag{33}
\]
with overall modular weights \( n_\alpha \) as in (11). The gaugino masses \( M_a \) are given by
\[
2m_{3/2}^{-1} \langle \text{Re} f_a \rangle M_a = \sqrt{3} C Y \sin \theta + \left( \frac{Y}{Y - \frac{\delta_{GS}}{3}} \right)^{1/2} C(T + \bar{T}) \cos \theta \frac{(b'_a - \delta_{GS})}{16\pi^3} \hat{G}_2(T, \bar{T}) \tag{34}
\]
where
\[
\hat{G}_2(T, \bar{T}) = G_2(T) - 2\pi(T + \bar{T})^{-1} \tag{35}
\]
\[
G_2(T) = -4\pi \frac{d \ln \eta}{dT} \tag{36}
\]
where \( \eta(T) \) is the Dedekind function and, for the standard model, \( b'_a, \ a = 1, 2, 3, \) are given by (19)-(21). The real part of the gauge kinetic function \( \text{Re} f_a \) is given by
\[
\text{Re} f_a = g_a^{-2}(M_{\text{string}}) \tag{37}
\]
and is determined from the gauge coupling constant $g_a(m_Z)$ at the electroweak scale $Q = m_Z$ by

$$g_a^{-2}(M_{\text{string}}) - g_a^{-2}(m_Z) = \frac{b_a}{16\pi^2} \ln\left( \frac{m_Z^2}{M_{\text{string}}^2} \right)$$  \hspace{1cm} (38)$$

with $M_{\text{string}}$ as in [23].

The soft supersymmetry breaking $A$ terms $A_{\alpha\beta\gamma}$ are given by

$$A_{\alpha\beta\gamma} = -\sqrt{3} C m_{3/2} \sin \theta - C m_{3/2} \left( \frac{Y}{Y - \delta_{GS}} \right)^{1/2} \cos \theta \omega_{\alpha\beta\gamma}(T)$$  \hspace{1cm} (39)$$

where

$$\omega_{\alpha\beta\gamma}(T) = 3 + n_\alpha + n_\beta + n_\gamma - (T + \bar{T}) \frac{\partial \ln h_{\alpha\beta\gamma}}{\partial T}$$  \hspace{1cm} (40)$$

and the trilinear term $\tilde{W}_3$ in the perturbative superpotential has been written as

$$\tilde{W}_3 = h_{\alpha\beta\gamma} \phi_\alpha \phi_\beta \phi_\gamma.$$  \hspace{1cm} (41)$$

The modular weights $n_\alpha$, $n_\beta$, and $n_\gamma$ are chosen to correspond to one of the solutions for gauge coupling constant unification (for the $Z_2 \times Z_6$ or $Z_3 \times Z_6$ orbifold) discussed in the previous section. Since we are assuming large values of Re$T$ in order to reduce the unification scale to $2 \times 10^{16}$ GeV, we shall use the asymptotic form of $h_{\alpha\beta\gamma}$ valid for large Re$T$,

$$h_{\alpha\beta\gamma} \sim \exp\left( -\frac{\pi}{3} \lambda_{\alpha\beta\gamma} T \right)$$  \hspace{1cm} (42)$$

where $\lambda_{\alpha\beta\gamma}$ is an integer in the range 0 to 4 for the $Z_2 \times Z_6$ orbifold and in the range 0 to 10 for the $Z_3 \times Z_6$ orbifold [16]. The constant of proportionality in (12) is expected to be of order $g_{\text{string}}$. Here and in (33) and (34) we shall use the Green-Schwarz parameter obtained by inserting $\delta_{\text{GS}_i}$, $i = 1, 2, 3$, for the $Z_2 \times Z_6$ or $Z_3 \times Z_6$ orbifold in (8) and (9). Although, in general, the Green-Schwarz parameters $\delta_{\text{GS}_i}$ have different values for the different complex planes, contradicting the assumption that all three complex planes are on the same footing in $G$, this better approximates the situation than neglecting the Green-Schwarz parameters. A simple model is to take a pure gauge hidden sector with $E_8$ gauge group. Then [12, 17]

$$\delta_{\text{GS}_i} = \frac{b_a}{3} \left( 1 - \frac{2 |G_i|}{|G|} \right)$$  \hspace{1cm} (43)$$

where $a$ now refers to the hidden sector gauge group and the $i$th complex plane is left unrotated by the subgroup $G_i$ of the point group $G$. With $b_a = -90$ for $E_8$ we have

$$\delta_{\text{GS}} = -40 \text{ or } -50$$  \hspace{1cm} (44)
for $Z_2 \times Z_6$ or $Z_3 \times Z_6$ respectively.

The soft supersymmetry breaking $B$ term is more model dependent because of different possible origins for the $\mu$ parameter. If the $\mu$ term is generated non-perturbatively as an explicit superpotential term $\mu W \phi_1 \phi_2$, where $\phi_1$ and $\phi_2$ are the superfields for the Higgs scalars $H_1$ and $H_2$, then the $B$ term, which in this case we denote by $B_W$, is given by

$$m_{3/2}^{-1} B_W = -1 - \sqrt{3} C \sin \theta \left( 1 - Y \frac{\partial \ln \mu W}{\partial S} \right) - \left( \frac{Y}{Y - \delta_{GS}} \right)^{1/2} C \cos \theta \left( 3 + n_{H_1} + n_{H_2} - (T + \bar{T}) \frac{\partial \ln \mu W}{\partial T} - \delta_{GS} \frac{\partial \ln \mu W}{\partial S} \right).$$

(45)

If the $\mu$ parameter is gaugino condensate induced then

$$\mu_W \propto W_{np} \frac{\partial \ln \eta(T_3)}{\partial T_3} \frac{\partial \ln \eta(U_3)}{\partial U_3}$$

(46)

where $W_{np}$ is the non-perturbative superpotential and the orbifold is assumed to possess a $Z_2$ plane, taken to be the third complex plane with associated moduli $T_3$ and $U_3$ and untwisted matter fields $\phi_1$ and $\phi_2$. Such a mechanism is possible for the $Z_2 \times Z_6$ orbifold though not for the $Z_3 \times Z_6$ orbifold which does not possess a $Z_2$ plane. In the case of $Z_3 \times Z_6$ we take $\mu_W$ constant as in ref.[7]. Because $\phi_1$ and $\phi_2$ are then necessarily untwisted states the modular weights $n_{H_1}$ and $n_{H_2}$ should be taken to be $-1$. It is somewhat problematic to employ this mechanism in the context of the simple model with only a single overall modulus $T$ being considered here. However if we neglect the auxiliary field for $U_3$, or equivalently assume that $U_3$ does not contribute significantly to the supersymmetry breaking, then (45) is correct when $T_1$, $T_2$ and $T_3$ are on the same footing. There is also the difficulty that gaugino condensate models in general produce a negative vacuum energy $V_0$ rather than zero vacuum energy, as we have assumed after (32). Nonetheless, we think it worthwhile to study this mechanism to obtain a flavour of the effect on the supersymmetric particle spectrum of the kind of moduli dependence of the $\mu$ parameter that can occur in physically motivated models. After evaluating $\frac{\partial \ln \mu W}{\partial S}$ and $\frac{\partial \ln \mu W}{\partial T}$ we obtain

$$m_{3/2}^{-1} B_W = 3C^2 - 1 - \left( \frac{Y}{Y - \delta_{GS}} \right)^{1/2} C \cos \theta \left( n_{H_1} + n_{H_2} - (T + \bar{T}) \left( \frac{\partial \ln \eta(T)}{\partial T} \right)^{-1} \frac{\partial^2 \ln \eta(T)}{\partial T^2} \right).$$

(47)

Here $\frac{\partial \ln \mu W}{\partial S}$ and $\frac{\partial \ln \mu W}{\partial T}$ have been written in terms of $F_T$ and $F_S$ and so in terms of $\theta$ using (27)-(30) and because $T_i, i = 1, 2, 3$, are not on the same footing in (46), $(T + \bar{T}) \frac{\partial \ln \mu W}{\partial T}$ has been interpreted as $(T_3 + \bar{T}_3) \frac{\partial \ln \mu W}{\partial T_3}$ evaluated at $T_3 = T$. 

9
On the other hand if the $\mu$ parameter is generated by a term of the form $(Z\phi_1\phi_2 + h.c.)$ in the Kähler potential [7] mixing the Higgs superfields then the tree level form of $Z$ is

\[ Z = (T_3 + \bar{T}_3)^{-1}(U_3 + \bar{U}_3)^{-1} \]  

(48)

if the third complex plane is the $Z_2$ plane with whose moduli, $T_3$ and $U_3$, the untwisted matter fields $\phi_1$ and $\phi_2$ are associated for this mechanism. The effective $\mu$ parameter $(\mu_Z)_{eff}$ derived from the Higgsino mass term is

\[ (\mu_Z)_{eff} = |W_{np}|Z(1 + C \cos \theta) \]  

(49)

and the final form of the $B$ term, which in this case we denote by $B_Z$ is given by

\[ m_{3/2}^{-1}B_Z = \frac{2(1 + C \cos \theta) - 3(C^2 - 1)}{(1 + C \cos \theta)}. \]

(50)

In particular, when $V_0$ is zero so that $C = 1$, as we are assuming throughout, $B_Z$ takes the constant value $2m_{3/2}$. This compares with $2(1 + \cos \theta)m_{3/2}$ in ref. [4], where $Z$ was taken to be a moduli independent constant. In arriving at (49) and (50) we have again assumed that there is no significant supersymmetry breaking due to the $U$ modulus, so as to be able to neglect the auxiliary field for the $U$ modulus. Also, here and elsewhere in this section the usual rescaling by a factor $e^{K/2} \frac{W_{ap}}{|W_{np}|}$ required to go from the supergravity theory derived from the orbifold compactification of the string theory to the globally supersymmetric theory has been carried out, together with normalisation of the matter fields (see, for example, ref. [4]).

4 Running of coupling constants and supersymmetry breaking parameters

The method for running coupling constants and supersymmetry breaking parameters from the unification scale $M_X$ to the electroweak scale is well known. (See, for example, refs. [18] and [19].) The relevant renormalisation group equations for our purposes are summarized in appendix A and the relevant solutions in appendix B, with the bottom quark and $\tau$ lepton Yukawa couplings, as well as the first and second generation Yukawa couplings, neglected but the effect of the $\mu$ parameter retained. The top Yukawa, $h_t$, and the $\mu$ parameter have been defined through the superpotential terms

\[ W = h_t Q_t t^c H_2 - \mu H_1 H_2 \]  

(51)
where $Q_t$ is the doublet $(t \, b)_L$ for the top and bottom quarks, $t^c$ is the corresponding singlet and the Higgs doublets are

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \tag{52}$$

where $H_1^0$ and $H_2^0$ have expectation values $v_1$ and $v_2$ respectively. In (51), $Q_t t^c H_2$ is shorthand for $Q_t^T i \tau^2 H_2 t^c$ and $\mu H_1 H_2$ for $\mu H_1^T i \tau^2 H_2$. The tree level Higgs scalar potential $V_{eff}$ in terms of the above expectation values is

$$V_{eff} = \mu_1^2 v_1^2 + \mu_2^2 v_2^2 - 2\mu_3^2 v_1 v_2 + \frac{1}{8} (g_1^2 + g_2^2) (v_1^2 - v_2^2)^2 \tag{53}$$

where

$$\mu_1^2 = m_1^2 + \mu^2, \quad \mu_2^2 = m_2^2 + \mu^2, \quad \mu_3^2 = -\mu B m_{3/2} \tag{54}$$

and $m_1$ and $m_2$ are the soft supersymmetry breaking masses for $H_1$ and $H_2$. Minimisation of the tree level effective potential gives

$$\omega^2 = \frac{\mu_1^2 + \frac{1}{2} m_Z^2}{\mu_2^2 + \frac{1}{2} m_Z^2} \tag{55}$$

and

$$\frac{\omega}{\omega^2 + 1} = \frac{\mu_3^2}{\mu_1^2 + \mu_2^2} \tag{56}$$

at the electroweak scale $Q = m_Z$ where

$$\omega^{-1} = \tan \tilde{\theta} = \frac{v_1}{v_2}. \tag{57}$$

Also the following inequalities must hold

$$\mu_1^2 + \mu_2^2 > 2|\mu_3|^2 \tag{58}$$

$$\mu_3^4 > \mu_1^2 \mu_2^2 \tag{59}$$

$$\mu_2^2 + m_{QL}^2 + m_{UR}^2 > m^2 (2|A_t| - 3) \tag{60}$$

as explained in ref. [18].

Our strategy for fixing some of the parameters in the models is as follows. Knowing the values $m_1^2(0)$, $m_2^2(0)$ and $B(0)$ of the soft supersymmetry breaking parameters at the gauge coupling constant unification scale $M_X$ (which differ little from their values at the string scale) and assuming values for the gravitino mass $m_{3/2}$ in the range 100 GeV to 10 TeV, (53) and (56) are a pair of equations that can be solved for $\mu(0)$ and $\omega$. Then

$$m_W^2 = \frac{g_2^2}{2} (v_1^2 + v_2^2) \tag{61}$$
determines $v_1$ and $v_2$. In addition
\[ m_t = h_t v_2 \]
fixes $h_t$ at the electroweak scale and eqn. (62) determines $h_t(0)$. We run all renormalisation group equations from the gauge coupling constant unification scale $M_X$ and ignore small effects due to the difference between $M_X$ and the string scale.

The supersymmetry breaking parameters at the string scale are calculated as in §3. Then the predictions for the supersymmetric particle spectrum to be discussed in the next section are parameterised by the angle $\theta$ which measures the relative contribution of the dilaton $S$ and the modulus $T$ to supersymmetry breaking and the gravitino mass (assuming zero vacuum energy $V_0$ so that $C = 1$). In addition, the outcome for the spectrum depends on the choice of modular weights $n_\alpha$ from amongst the sets allowed for the $Z_2 \times Z_6$ and $Z_3 \times Z_6$ orbifolds, as in table 1, and on the mechanism adopted to generate the $\mu$ parameter, which influences the form of the $B$ term. The choice of modular weights also fixes the value of $T$ from (26). The Green-Schwarz parameters $\delta_{GS}$ are taken from (44).

The above discussion neglects radiative corrections to the effective potential. When these are included [8, 9] the strategy for obtaining the expectation values $v_1$ and $v_2$ has to be amended. Those supersymmetric particle masses that depend on $v_1$ and $v_2$ are then modified as well as the Higgs scalar masses. We will discuss these points in detail in the next section. We have not considered the radiative corrections to (58)-(60) which may exclude some values of $\theta$, in particular the dilaton dominated case [20].

5 The supersymmetric particle spectrum

The expressions for the masses of the supersymmetric partners of standard model states in terms of the soft supersymmetry breaking parameters are well known. For the first two generations of quarks and leptons the Yukawa couplings and $A$ terms are negligible and the corresponding squark and slepton mass terms are simply the soft supersymmetry breaking scalar masses. For the third generation it is necessary to allow for a non-negligible top Yukawa coupling and the top squark masses are given by
\[ m_{\tilde{t}_{i_u}}^2 = m_t^2 + \frac{1}{2} \left( m_Q^2 + m_U^2 \pm \left( (m_Q^2 - m_U^2)^2 + 4 m_t^2 (A_t m_3/2 + \mu \omega^{-1}) \right)^{1/2} \right) \] (63)
where $m_Q$ and $m_U$ refer to the scalar partners of the quark doublet and one of the quark singlets for the third generation respectively, and the D term has been neglected.
The gluino mass is given by the Majorana mass term. However, the Wino and Zino mix with the Higgsinos. The chargino mass matrix has eigenvalues \( m_{c,h,l} \) given by

\[
2m_{c,h,l}^2 = M_2^2 + \mu^2 + 2m_W^2 \pm \Delta^{1/2}
\]

where

\[
\Delta = (M_2^2 - \mu^2)^2 + 4m_W^2(M_2^2 + \mu^2 + 2M_2\mu \sin 2\theta) + 4m_W^4 \cos^2 2\theta
\]

The neutralino mass matrix has the form

\[
i\bar{W}^3 & i\tilde{B} & \tilde{h}_0^0 & \tilde{h}_1^0 \\
-M_2 & 0 & -\frac{\mu v_2}{\sqrt{2}} & -\frac{\mu v_1}{\sqrt{2}} \\
0 & -M_1 & \frac{\mu v_2}{\sqrt{2}} & -\frac{\mu v_1}{\sqrt{2}} \\
-\frac{\mu v_2}{\sqrt{2}} & \frac{\mu v_1}{\sqrt{2}} & 0 & \mu \\
-\frac{\mu v_1}{\sqrt{2}} & -\frac{\mu v_2}{\sqrt{2}} & \mu & 0 \\
\]

+ h.c. \quad (66)

In addition the charged Higgs has mass

\[
m_{H^\pm}^2 = m_W^2 + \mu_1^2 + \mu_2^2
\]

and the neutral Higgses have masses

\[
m_c^2 = \mu_1^2 + \mu_2^2
\]

and

\[
m_{a,b}^2 = \frac{1}{2} \left( m_c^2 + m_Z^2 \pm \left[ (m_c^2 + m_Z^2)^2 - 4m_c^2m_Z^2 \cos^2 2\theta \right]^{1/2} \right) . \quad (69)
\]

In our detailed calculations, the mass \( m_b \) given by (69) is generically lower than the experimental bound. However, the one loop radiative corrections to the Higgs scalar masses are substantial \cite{8,9} and we shall use the one loop Higgs scalar effective potential in what follows. The one loop corrected formulae for the Higgs masses can be found in ref.\cite{9}. When the one loop corrections to the effective potential are included the minimisation conditions (55) and (56) for the expectation values \( v_1 \) and \( v_2 \) are modified with the result that

\[
\omega^2 = \frac{2\mu_1^2 + M_Z^2 + \frac{1}{v_1} \frac{\partial \Delta V_1}{\partial v_1} - \frac{v_2}{v_1} \frac{\partial \Delta V_1}{\partial v_2}}{2\mu_2^2 + m_Z^2}
\]

and

\[
\frac{\omega}{\omega^2 + 1} = \frac{2\mu_2^2}{2\mu_1^2 + m_Z^2} + \frac{2\mu_2^2}{2\mu_1^2 + m_Z^2} + \frac{1}{v_1} \frac{\partial \Delta V_1}{\partial v_1} + \frac{1}{v_2} \frac{\partial \Delta V_1}{\partial v_2}
\]

where \( \Delta V_1 \) is the one loop correction to the effective potential evaluated at \( m_Z \) and

\[
\frac{\partial \Delta V_1}{\partial v_i} = \frac{3}{16\pi^2} \left[ m_i^2 \frac{\partial m_i^2}{\partial v_i} \left( \ln \frac{m_i^2}{m_Z^2} - 1 \right) + m_i^2 \frac{\partial m_i^2}{\partial v_i} \left( \ln \frac{m_i^2}{m_Z^2} - 1 \right) + 2m_i^2 \frac{\partial m_i^2}{\partial v_i} \left( \ln \frac{m_i^2}{m_Z^2} - 1 \right) \right], \quad i = 1, 2 . \quad (72)
\]
The strategy for fixing the parameters in the models is essentially that described in §4 except that (70) and (71) should now be regarded as a pair of equations for \( v_1, v_2 \) and \( \mu(0) \) given the soft supersymmetry breaking parameters \( m^2_Q, m^2_U \) and \( A_t \) at the string scale and given values for \( m_t \) and \( m_{3/2} \), rather than as a pair of equations that can be solved for \( \mu(0) \) and \( \omega \).

In deriving the possible supersymmetric particle spectrum we have insisted on no negative squared masses at the string scale to avoid high scale symmetry breaking in the standard model. We have also insisted on the following experimental constraints. From LEP1.5 data, there are no charged or coloured sparticles with masses less than 65 GeV, the lightest Higgs is heavier than 65 GeV and the lower bound on the charginos is 80 GeV. Tevatron data indicates that the gluino mass is above 175 GeV, but should not exceed 1.5 TeV (to avoid reintroducing the hierarchy problem). The top quark mass is known to be 175±6 GeV. The vev of the Higgs responsible for the top quark mass has a maximum value given by

\[
v_1^2 + v_2^2 = \frac{2m_Z^2}{(g^2 + g'^2)}
\]

with \( v_1^2 = 0 \) implying \( v_2(\text{max}) = 173.3 \) GeV. Since \( m_t = h_t v_2 \) this puts a lower limit on \( h_t \) if \( m_t = 175 \pm 6 \) GeV is to be obtained. Specifically the value at \( M_X \) is \( h_t(\text{min}) = 0.52 \) and so it is appropriate to set \( \lambda = 0 \) in (12) for the top Yukawa coupling. One loop minimisation conditions have been used throughout and the Higgs masses are one loop corrected. The parameter \( \omega \) is found to be never greater than 6, justifying the neglect of the b-quark contribution. The D terms have been included in the mass of the lightest sleptons, the right selectron and the left sneutrino. In the figures the following notation is used for the masses:

- \( c_h, c_l \): heavy and light charginos
- \( t_h, t_l \): heavy and light stops
- \( H_a, H_b \): heavy and light CP-even Higgses respectively
- \( m_t \): top quark
- \( E_R, V_L \): right selectron and left sneutrino respectively
- \( N1 \): lightest neutralino
- \( g \): gluino

Particles not displayed are the three neutralinos which are degenerate with the charginos, the charged and CP-odd Higgses which are degenerate with \( H_a \), the remaining squarks which are all only slightly less massive than the gluino, and the left selectron which is always heavier than \( V_L \).

Several models will now be presented that are representative of the variety of the
supersymmetric particle spectra that can occur.

In figure 1 the resulting mass spectrum is shown for the $Z_3 \times Z_6$ orbifold, characterised here by $\delta_{GS} = -50$, with $m_{3/2} = 100$ GeV, $h_t = 0.7$, modular weights as in line 2 of table 1 and with the $B$ term given by $B_W$ with $\mu W$ constant. The only allowed region is bounded on the left by $E_R$ acquiring a too low mass, and on the right by $c_l$ becoming too light while $m_t$ is always in the vicinity of 175 GeV. Further, the acceptable part of the spectrum is limited by the requirement of positive squared scalar masses at the string scale which confines it to the regions between the two pairs of vertical lines shown in the figure, centred on the dilaton dominated limits ($\theta = \frac{\pi}{2}, \frac{3\pi}{2}$). The part of the spectrum around $\theta = \frac{\pi}{2}$ is ruled out due to $m_t$ being unacceptably low and the electroweak symmetry is unbroken there.

As in ref.[4], there is a clear division of the spectrum into a heavy and a light group although now there is a far greater variation in masses as $\theta$ is varied than was the case in ref.[4]. This latter effect is attributable directly to the magnitude of $\delta_{GS}$ and the effect it has on the gaugino masses which feed through to all sparticles. At the dilaton dominated limit the spectrum is qualitatively similar to that in ref.[4] but away from this limit we see that the gluino (and squarks) are often heavier than the heavy stop (in [4] the gluino mass was fixed). Particularly noticeable, and attributable to $|\delta_{GS}|$, is the mass of the lightest neutralino (which is mostly $M_1$) which often exceeds 100 GeV. This is worth noting because, as seen in figure 1, on the left hand limit of the allowed region its role as the ‘lightest supersymmetric particle’ is jeopardised in favour of the right selectron. This is why the D term has been included in $E_R$ (it can add 10 GeV or more). The lightest Higgs, $H_b$, is also in the region of 100 GeV and the light chargino can be lighter than the sleptons.

A change in $m_{3/2}$ will scale all masses (except $m_t$). Decreasing $m_{3/2}$ narrows the allowed region by virtue of $E_R$ and $c_l$ becoming too light on the right hand edge, giving an approximate effective minimum of $m_{3/2} \approx 70$ GeV, below which the dilaton dominated limit is unreachable. Increasing $m_{3/2}$ rapidly increases the gluino mass to way above the limit 1.5 TeV. For $m_{3/2} \geq 250$ GeV even the dilaton dominated limit is excluded. The allowed regions may not be extended to the right significantly, even with a high gravitino mass because $c_l$ remains too light there. A variation in $h_t$ affects all masses due to its appearance in the one-loop effects and an increase in $h_t$ will increase all masses slightly. However adjustment of $h_t$ is restricted by $m_t$ and it may not deviate far from 0.7 without pushing $m_t$ out of the experimental bounds.

A different choice of modular weights from table 1 will not change the spectrum very
Figure 1: $\delta_{GS} = -50$, $m_{3/2} = 100$ GeV, $T = 14.5$, $h_t = 0.7$, $B \equiv B_W$, modular weights as in line 2 of table [1].

Figure 2: $\delta_{GS} = -40$, $m_{3/2} = 100$ GeV, $T = 14.5$, $h_t = 0.9$, $B \equiv B_W$, modular weights as in line 2 of table [1].
Figure 3: $\delta_{GS} = -40$, $m_{3/2} = 100$ GeV, $T = 14.5$, $h_t = 0.9$, $B \equiv B_Z$, modular weights as in line 2 of table I.

much other than by changing the acceptable width at the string scale. The modular weights from line 3 of table I are the least restrictive at the string scale, the ‘heaviest’ weight being $-2$, and so the acceptable part of the spectrum is widened slightly, while lines 4 and 5 from table I have the opposite effect. Thus the spectrum displayed in figure I is very typical and deviations from it are small.

Figure 2 shows that spectrum obtained for a $Z_2 \times Z_6$ orbifold ($\delta_{GS} = -40$) with $\mu_W$ as in (16) has two valid regions. The inclusion of the derivatives of $\mu_W$ in $B_W$ is instrumental in obtaining this result, were they to be neglected we would obtain only one valid region similar to figure I. Each region is qualitatively similar to that shown in figure I, although both regions are bounded on the right by $m_t$ becoming too low. The dilaton dominated limit is not reachable at $\theta = \frac{3\pi}{2}$ for this reason, while that at $\theta = \frac{\pi}{2}$ is reachable. Note that here $h_t$ cannot deviate far from 0.9 without $m_t$ being pushed outside the experimental bounds. Conversely, obtaining an acceptable $m_t$ beyond the displayed regions would require an unacceptably high value of $h_t$. We also find $70 < m_{3/2} < 250$ GeV for an acceptable spectrum, as before.

Concerning the other form of the $B$ term, $B_Z$, which is only valid for the $Z_2 \times Z_6$ orbifold because it requires a $Z_2$ plane, an example is shown in figure I. Comparison with
figure 2 shows some differences. In the right hand region the $H_a-H_b$ splitting is increased and in both regions the $t_h-t_l$ splitting is reduced. There is near degeneracy between $c_h$, $t_l$ and $H_a$ in the right hand region. The light group remains relatively unaffected, although the top quark is, on average, heavier in the left hand region than the right hand region. To obtain central values of $m_t$ the left and right regions require $h_t = 0.8, 1.0$ respectively.

The dilaton dominated limits at $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ are respectively included and excluded as in figure 2.

6 Conclusions

We have studied the supersymmetric particle spectrum in orbifold compactifications of string theory where unification of gauge coupling constants at $2 \times 10^{16}$ GeV is due to large moduli dependent string loop threshold corrections, making a number of changes to the assumptions in Brignole et al. [4] in order to explore the robustness of the qualitative features of the spectrum obtained. The specific orbifold models considered here show that the inclusion of the derivatives of $\mu_W$ in $B_W$ is important in obtaining acceptable spectra in two separate regions and that it is also necessary for $h_t \approx 0.9$ to obtain correct values for the top quark mass. For $Z_2 \times Z_6$ orbifold models there is then little resultant difference between the $B_Z$ and $B_W$ mechanisms. For both mechanisms the gravitino mass is restricted approximately to the range 70-250 GeV in order to satisfy the upper and lower mass limits imposed in §5. It is also apparent that the effects of the Green-Schwarz coefficient are not negligible. In the examples presented here, $|\delta_{GS}|$ is substantial enough to shift the spectrum away from the dilaton dominated limit and partly out of the regions allowed at the string scale resulting in a considerably narrower acceptable range for $\theta$. The lightest neutralino is often heavier than usually assumed ($\sim 100$ GeV as opposed to $\sim 50$ GeV [4]), and can be of similar mass to (or heavier than) the light Higgs and the right selectron. In addition $|\delta_{GS}|$ induces a large variation in the masses as $\theta$ is varied, particularly for the heavy group. In principle this should make the goldstino angle, $\theta$, easier to determine if sparticles are eventually discovered.

Acknowledgments

We are grateful to George Kraniotis for helpful discussions. This research was supported in part by PPARC and P.S. was supported by a Royal Holloway studentship.
A Renormalisation group equations

The one loop renormalisation group equations for the various coupling constants and soft supersymmetry breaking parameters defined in the text, including the contribution of the $\mu$ parameter are as follows. In all cases the bottom quark and $\tau$ lepton Yukawa coupling have been neglected.

The gauge coupling constants $g_i, i = 1, 2, 3$, obey

$$ \frac{dg_i^2}{d\ln Q} = \frac{b_i}{8\pi^2} g_i^4, \ i = 1, 2, 3 $$  \hspace{1cm} (74)

with normalisation of the $U(1)$ coupling constant such that

$$ g_3^2(M_X) = g_2^2(m_X) = \frac{5}{3} g_1^2(M_X) $$  \hspace{1cm} (75)

at the unification scale $M_X$, then

$$ b_3 = -3, \ b_2 = 1, \ b_1 = 11 $$  \hspace{1cm} (76)

and the corresponding gaugino masses obey

$$ \frac{dM_i}{d\ln Q} = \frac{b_i}{8\pi^2} g_i^2 M_i . $$  \hspace{1cm} (77)

The renormalisation group equation for the top quark Yukawa coupling $h_t$ is

$$ \frac{dY_t}{dt} = \frac{d\ln E}{dt} Y_t - 6Y_t^2 $$  \hspace{1cm} (78)

where

$$ t \equiv \ln \left( \frac{M_X^2}{Q^2} \right) $$  \hspace{1cm} (79)

$$ Y_t \equiv \frac{h_t^2}{16\pi^2} $$  \hspace{1cm} (80)

and

$$ E(t) = (1 + \beta_2 t)^{\frac{16\pi^2}{3\beta_2}} (1 + \beta_2 t)^{\frac{16\pi^2}{3\beta_1}} (1 + \beta_1 t)^{\frac{16\pi^2}{3\beta_1}} $$  \hspace{1cm} (81)

with

$$ \beta_i = b_i \tilde{\alpha}_i(0) $$  \hspace{1cm} (82)

and

$$ \tilde{\alpha}_i(t) = \frac{\alpha_i(t)}{4\pi} = \frac{g_i^2(t)}{16\pi^2} . $$  \hspace{1cm} (83)

The $\mu$ parameter obeys

$$ \frac{d\mu^2}{dt} = (3\tilde{\alpha}_2 + \tilde{\alpha}_1 - 3Y_t) \mu^2 . $$  \hspace{1cm} (84)
The soft supersymmetry breaking $A_t$ and $B$ parameters obey

$$\frac{dA_t}{dt} = m_{3/2}^{-1}\left(\frac{16}{3}\tilde{\alpha}_3 M_3 + 3\tilde{\alpha}_2 M_2 + \frac{13}{9}\tilde{\alpha}_1 M_1\right) - 6Y_t A_t \quad (85)$$

$$\frac{dB}{dt} = m_{3/2}^{-1}(3\tilde{\alpha}_2 M_2 + \tilde{\alpha}_1 M_1) - 3Y_t A_t \quad . \quad (86)$$

The corresponding equations for all other $A$ terms are obtained by deleting the $Y_t$ term in (85). Scalars $\phi_\alpha$ that are the supersymmetric partners of the first and second generation quarks and leptons have soft supersymmetry breaking masses $m_\alpha$ obeying

$$\frac{dm_\alpha^2}{dt} = 4\sum_{i=1}^{3} C^\alpha_i \tilde{\alpha}_i M_i^2 \quad (87)$$

where the group theory factors $C^\alpha_i$ have the values $C^\alpha_3 = \frac{4}{3}$ for an $SU(3)_C$ triplet, $C^\alpha_2 = \frac{3}{4}$ for an $SU(2)_L$ doublet, and $C^\alpha_1 = Y^2$ for a state with weak hypercharge $Y$.

The renormalisation group equations for the masses $\mu_1$ and $\mu_3$ in the Higgs scalar potential are

$$\frac{d\mu_1^2}{dt} = (3\tilde{\alpha}_2 M_2^2 + \tilde{\alpha}_1 M_1^2) + (3\tilde{\alpha}_2 + \tilde{\alpha}_1 - 3Y_t)\mu_1^2 \quad (88)$$

and

$$\frac{d\mu_3^2}{dt} = \left(\frac{3}{2}\tilde{\alpha}_2 + \frac{1}{2}\tilde{\alpha}_1 - \frac{3}{2}Y_t\right)\mu_3^2 + 3\mu m_{3/2} Y_t A_t - \mu(3\tilde{\alpha}_2 M_2 + \tilde{\alpha}_1 M_1) \quad . \quad (89)$$

The renormalisation group equations for the masses of the scalars which are the supersymmetric partners of the third generation quarks and leptons are expressed conveniently [18] in terms of the quantities

$$m_4^2 = m_D^2 + m_U^2 - 2m_Q^2$$

$$m_5^2 = \frac{2}{3}(\mu_2^2 - \mu_1^2) - m_U^2$$

$$m_6^2 = \frac{3}{2}m_D^2 + m_L^2 - (\mu_1^2 - \mu_2^2)$$

$$m_7^2 = m_L^2 - \frac{1}{2}m_E^2$$

(90)

where $m_Q$ and $m_L$ refer to the scalar partners of the quark and lepton doublets and $m_U$, $m_D$ and $m_E$ refer to the scalar partners of the quark and lepton singlets for the third generation. Then the remaining renormalisation group equations for these masses and for the mass $\mu_2$ in the Higgs scalar potential are

$$\frac{dm_4^2}{dt} = -6\tilde{\alpha}_2 M_2^2 + 2\tilde{\alpha}_1 M_1^2$$

$$\frac{dm_5^2}{dt} = -\frac{16}{3}\tilde{\alpha}_3 M_3^2 + 2\tilde{\alpha}_2 M_2^2 - \frac{10}{9}\tilde{\alpha}_1 M_1^2$$

$$\frac{dm_6^2}{dt} = -\frac{16}{3}\tilde{\alpha}_3 M_3^2 + 2\tilde{\alpha}_2 M_2^2 - \frac{10}{9}\tilde{\alpha}_1 M_1^2$$

20
\[
\frac{dm_6^2}{dt} = 8\tilde{\alpha}_3 M_3^2 + \frac{2}{3} \tilde{\alpha}_1 M_1^2
\]
\[
\frac{dm_7^2}{dt} = 3\tilde{\alpha}_2 M_2^2 - \tilde{\alpha}_1 M_1^2
\]
\[
\frac{dm_D^2}{dt} = \frac{16}{3} \tilde{\alpha}_3 M_3^2 + \frac{4}{9} \tilde{\alpha}_1 M_1^2
\]  \hspace{1cm} (91)

and
\[
\frac{d\mu_2^2}{dt} = 3\tilde{\alpha}_2 M_2^2 + \tilde{\alpha}_1 M_1^2 + (3\tilde{\alpha}_2 + \tilde{\alpha}_1) \mu^2 - 6Y_t \mu_2^2 - 3Y_t \left( A_t^2 m_{3/2}^2 - \mu^2 + \frac{1}{2} m_D^2 - \frac{1}{2} m_2^2 - \frac{3}{2} m_3^2 \right). \tag{92}
\]

## B Solutions of the renormalisation group equations

Analytic solutions of the equations of appendix A may be obtained along the lines of refs. [18] and [19]. Including the contribution of the $\mu$ parameter they are as follows.

\[
\tilde{\alpha}_i(t) = \tilde{\alpha}_i(0)(1 + \beta_i t)^{-1} \tag{93}
\]
\[
\frac{M_i(t)}{M_i(0)} = \frac{\tilde{\alpha}_i(t)}{\tilde{\alpha}_i(0)} \tag{94}
\]
\[
Y_i(t) = \frac{Y_i(0) E(t)}{1 + 6Y_i(0) F(t)} \tag{95}
\]
where
\[
F(t) = \int_0^t E(t') dt' \tag{96}
\]

Also,
\[
\mu^2(t) = \mu^2(0) \left( \frac{1 + \beta_2 t}{1 + \beta_1 t} \right)^{\frac{1}{2}} \equiv \mu^2(0) q^2(t) \tag{97}
\]

\[
m_{3/2}(1 + 6Y_t(0) F(t)) A_t(t) - m_{3/2} A_t(0) = \left( 1 + 6Y_t(0) F(t) \right) \left( \frac{16}{3} \tilde{\alpha}_3(0) M_3(0) h_3(t) \right)
\]
\[
+ 3\tilde{\alpha}_2(0) M_2(0) h_2(t) + \frac{13}{9} \tilde{\alpha}_1(0) M_1(0) h_1(t) \right) \right)
\]
\[
- 6Y_t(0) I(t) \tag{98}
\]

where
\[
h_i(t) = \frac{t}{(1 + \beta_i t)} \tag{99}
\]

and
\[
I(t) = \int_0^t dt' t' (1 + \beta_3 t')^{\frac{16}{3\beta_3}} (1 + \beta_2 t')^{\frac{2}{2\beta_2}} (1 + \beta_1 t')^{\frac{13}{2\beta_1}}
\]
\[
\times \left[ \frac{16}{3} \tilde{\alpha}_3(0) M_3(0)}{(1 + \beta_3 t')} + \frac{3\tilde{\alpha}_2(0) M_2(0)}{(1 + \beta_2 t')} + \frac{13}{9} \tilde{\alpha}_1(0) M_1(0)}{(1 + \beta_1 t')} \right]. \tag{100}
\]
For the first and second generation supersymmetric partners of quarks and leptons

\[ B(t) - B(0) = \frac{1}{2}(A_t(t) - A_t(0)) \]
\[ + m_{3/2}^{-1} \left( -\frac{8}{3}\tilde{\alpha}_3(0)M_3(0)h_3(t) + \frac{3}{2}\tilde{\alpha}_2(0)M_2(0)h_2(t) + \frac{5}{18}\tilde{\alpha}_1(0)M_1(0)h_1(t) \right). \]

(101)

For the first and second generation supersymmetric partners of quarks and leptons

\[ m_{\alpha}^2(t) - m_{\alpha}^2(0) = 2\sum_{i=1}^{3} C_i^\alpha \tilde{\alpha}_i(0)M_i^2(0)f_i(t) \]

(102)

where

\[ f_i(t) = \beta_i^{-1}(1 - (1 + \beta_i t)^{-2}). \]

(103)

Also for the masses \(\mu_1\) and \(\mu_3\) in the Higgs scalar potential (the equation for \(\mu_2\) is integrated numerically),

\[ \mu_1^2(t) = \mu_1^2(0) - \mu^2(0) + \mu^2(0)q^2(t) + \frac{3}{2}\tilde{\alpha}_2(0)M_2^2(0)f_2(t) + \frac{1}{2}\tilde{\alpha}_1(0)M_1^2(0)f_1(t) \]

(104)

and

\[ \mu_3^2(t) = q(t)\mu_3^2(0) + \frac{3g(t)Y_t(0)}{1 + 6Y_t(0)F(t)}A_t(0)m_{3/2}\mu(0) - \mu(0)q(t)(3\tilde{\alpha}_2(0)M_2(0)h_2(t) + \tilde{\alpha}_1(0)M_1(0)h_1(t)) \]

(105)

\[ + \frac{3Y_t(0)\mu(0)q(t)I(t)}{1 + 6Y_t(0)F(t)}. \]

Finally for the masses of the supersymmetric partners of the third generation quarks and leptons

\[ m_3^2(t) - m_3^2(0) = -3\tilde{\alpha}_2(0)M_2^2(0)f_2(t) + \tilde{\alpha}_1(0)M_1^2(0)f_1(t) \]
\[ m_5^2(t) - m_5^2(0) = -\frac{8}{3}\tilde{\alpha}_3(0)M_3^2(0)f_3(t) + \tilde{\alpha}_2(0)M_2^2(0)f_2(t) - \frac{5}{9}\tilde{\alpha}_1(0)M_1^2(0)f_1(t) \]
\[ m_6^2(t) - m_6^2(0) = 4\tilde{\alpha}_3(0)M_3^2(0)f_3(t) + \frac{1}{3}\tilde{\alpha}_1(0)M_1^2(0)f_1(t) \]
\[ m_7^2(t) - m_7^2(0) = \frac{3}{2}\tilde{\alpha}_2(0)M_2^2(0)f_2(t) - \frac{1}{2}\tilde{\alpha}_1(0)M_1^2(0)f_1(t) \]
\[ m_8^2(t) - m_8^2(0) = \frac{8}{3}\tilde{\alpha}_3(0)M_3^2(0)f_3(t) + \frac{2}{9}\tilde{\alpha}_1(0)M_1^2(0)f_1(t). \]

(106)

References

[1] M.Cvetič, A.Font, L.E.Ibañez, D.Lüst and F.Quevedo, Nucl. Phys. B361(1991)194
[2] L.E. Ibañez and D. Lüst, Nucl. Phys. B382(1992)305
[3] B. de Carlos, J.A. Casas and C. Muñoz, Phys. Lett. B299(1993)234
[4] A. Brignole, L.E. Ibañez and C. Muñoz, Nucl. Phys. B422(1994)125
[5] A. Brignole, L.E. Ibañez, C. Muñoz and C. Scheich, preprint FTUAM 95/26, LBL-37564
[6] D. Bailin and A. Love, Phys. Lett. B288(1992)263
[7] I. Antoniadis, E. Gava, K.S. Narain and T.R. Taylor, Nucl. Phys. B432(1994)187
[8] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B257(1991)83
[9] J. Ellis, G. Ridolfi and F. Zwirner, Phys. Lett. B262(1991)477
[10] P. Horava and E. Witten, Nucl. Phys. B460(1996)506
[11] E. Witten, Nucl. Phys. B471(1996)135
[12] J.P. Derendinger, S. Ferrara, C. Kounnas and F. Zwirner, Nucl. Phys. B372(1992)145
[13] L.E. Ibañez, D. Lüst and G. G. Ross, Phys. Lett. B272(1991)251
[14] V.S. Kaplunovsky, Nucl. Phys. B307(1988)145
[15] L.J. Dixon, V.S. Kaplunovsky and J. Louis, Nucl. Phys. B355(1991)649
[16] D. Bailin, A. Love and W.A. Sabra, Nucl. Phys. B403(1993)265
[17] D. Bailin, A. Love, W.A. Sabra and S. Thomas, Mod. Phys. Lett. A9 (1994)2543
[18] L.E. Ibañez and C. Lopez, Nucl. Phys. B233(1984)511
[19] L.E. Ibañez, C. Lopez and C. Muñoz, Nucl. Phys. B256(1985)218
[20] J.A. Casas, A. Lleyda and C. Muñoz, Phys. Lett. B380(1996)59