Quatetion representations of stiffness and momentum of the forces, acting in vibration isolating systems with stiffness compensators

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Abstract. This research is devoted to development of the spatial vibration isolation devices. The description of the vibration isolation systems has been presented through quaternions of the forces, momentums, and stiffness. The considered method allows taking into account the stochastic vibrations and describes it with the help of the hypercomplex numbers. The theory suggests the development of the vibration isolation devices, which have traction characteristics with zero stiffness area. To obtain such area in traction characteristic, a spatial vibration isolator is presented as a resilient element and the stiffness compensator, which is connected in parallel with it.

1. Introduction

Nowadays one of the most important engineering problems is the control of noise and vibration, which are made by working machines and mechanisms. This problem is acute in all areas of the machining, transport, industry. The bad impact of the vibration consists in destruction of machines and buildings, violation of readings of control devices and measurement tools. Noise and vibration negatively influence the activity and reliability of the devices. But the most detrimental effect is on the human body, causing various diseases.

Today there are many ways to decrease the vibration such as passive vibration isolators (springs, dampers), active vibration isolation systems, dynamic balancing of engines, and etcetera. However neither of these methods fully meets the modern requirements of the spatial vibration isolation, as most of them decrease the vibration only relatively one axis of the space, or the efficiency of the vibration level reduction does not meet sanitary standards.

In connection with the facts mentioned above this research is directed at the development of the spatial vibration isolation devices (SREDI with the support of the scholarship of the Russian Federation president, order №184 of the 10th of March 2015), which will best of all meet the modern requirements of the space vibration isolation, based on the effect of zero stiffness.

In [1, 2] a hypercomplex method of the description of the spatial mechanic oscillations was suggested in order to more precisely register and exclude them with the help of this vibration isolation device. In many foreign and domestic researches the theory of protection against vibration is considered relatively each axis of the space separately. In [2], the suggested technique supposes the spatial vibration isolation with the use of a mathematical apparatus of the quaternions and a notion of the vector space. Hypercomplex numbers allow forming and describing the conditions and requirements of the ideal vibration isolation in space.
2. Calculation and simulations

The ideal vibration isolation in space of a perfectly rigid body against random oscillations is provided (see Figure 1) if at any moment of time the sum of the forces and momentums, influencing the body, will be balanced, that is, the sum vector and sum moment will be equal to zero at any moment of time:

\[
\sum F_i = 0, \quad \sum M(F_i) = 0
\]

(1)

where \( F_i \) - forces, influencing the body (a protected object). They include: forces, transmitted through a protected device to a vibrating one \( (F_t) \); forces of the elastic interaction of the vibrating and protected objects \( (F_m) \); dissipative forces of the objects interaction \( (F_q) \); inertia forces of the intermediaries, connecting vibrating and protected objects \( (F_y) \). \( M(F_i) \) is the moment of the forces affecting the body (a protected object).

Each of the constituents of conditions (1) represents a sum of force projections on random coordinate axes and a sum of their moments with respect to these axes.

This theory assumes the development of the triaxial vibration isolation devices, which have traction characteristics with zero stiffness area. To obtain such area on the traction characteristic the spatial vibration isolator is represented as a resilient element and connected in parallel to the stiffness compensator.

The traction characteristic of the resilient element has a positive coefficient of stiffness relatively all three axes, and a compensator has a negative coefficient of stiffness and equal in modulus of rigidity of the resilient element relatively each spatial axis. To provide the floating point of the zero stiffness at changing of the operation loads the compensator is supplied with the rearranging system, tracking the relative position of the vibrating and protected objects and holding the working elements of the compensator in area of the working travel.

For description of the vibration isolation theory by means of hypercomplex numbers it is necessary to present mathematical dependencies of stiffness and forces moment in the vibration isolation systems with stiffness compensators.

From theoretical mechanics it is known that stiffness can be described as:

\[
C = \frac{F}{l},
\]

(2)

where \( l \) is the movement.

Also from the theory of mechanics, we know that:

\[
M = F \cdot l.
\]

(3)
Let us represent the force and the movement under random spatial oscillations with the help of quaternions:

\[ l = l_0 + x \cdot \vec{i} + y \cdot \vec{j} + z \cdot \vec{k}, \]  
(4)

\[ F = F_0 + F_x \cdot \vec{i} + F_y \cdot \vec{j} + F_z \cdot \vec{k}, \]  
(5)

where \( \vec{i}, \vec{j}, \vec{k} \) are unit vectors (orts).

Figure 2 shows the graphical example of one of the forces, acting in the vibration isolation system.

![Figure 2. The graphic representation of the force vector in the space.](image)

A hypercomplex description of stiffness and momentum of forces is the following:

\[ C = C_0 - C_x \cdot \vec{i} - C_y \cdot \vec{j} - C_z \cdot \vec{k}, \]  
(6)

\[ M = M_0 + M_x \cdot \vec{i} + M_y \cdot \vec{j} + M_z \cdot \vec{k}. \]  
(7)

Having substituted expressions (4) and (5) into dependence (2) and using algebra properties of quaternions, we will obtain:

\[ C = \frac{F_x \cdot x + F_y \cdot y + F_z \cdot z}{x^2 + y^2 + z^2} + \frac{F_x \cdot y - F_y \cdot z}{x^2 + y^2 + z^2} \cdot \vec{i} + \frac{F_x \cdot z - F_z \cdot x}{x^2 + y^2 + z^2} \cdot \vec{j} + \frac{F_y \cdot x - F_x \cdot y}{x^2 + y^2 + z^2} \cdot \vec{k}. \]  
(8)

Having substituting expressions (4) and (5) into dependence (3) and using algebra properties of hypercomplex numbers, we will obtain:

\[ M = -\left(F_x \cdot x + F_y \cdot y + F_z \cdot z\right) + \left(F_y \cdot z - F_z \cdot y\right) \cdot \vec{i} + \left(F_z \cdot x - F_x \cdot z\right) \cdot \vec{j} + \left(F_x \cdot y - F_y \cdot x\right) \cdot \vec{k}. \]  
(9)

The obtained expressions (8) and (9) allow representation of all forces, stiffness and forces momentum of the vibration isolation system with stiffness compensators by means of the quaternions mathematical apparatus relatively all spatial axes simultaneously.

3. Conclusion

The mathematical representation of the forces, stiffness and momentum of the forces in vibration isolation systems with the help of the quaternions sufficiently simplifies the process of description of similar systems in the space. This description will be taken into account when developing the calculations methods and the design of vibration isolation devices with stiffness correctors, for example, such as electromagnetic and supermagnetic stiffness compensators [3]. The development of such devices is supported by the scholarship for young scientists of the Russian Federation president by order №184 of the 10th of March 2015 for three years.
References
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