Renormalization Group in Six-derivative Quantum Gravity

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Alternative Gravities and Fundamental Cosmology
ALTECOSMOFUN’21, Virtual Session

7th of September 2021, via Zoom
This talk is based on collaboration with
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and based on the paper ArXiv:hep-th/2104.13980
Motivation:
Let’s first quantize matter, put it on curved spacetime background, only later quantize gravitation (Utiyama, De Witt, Shapiro)

Observation:
1-loop off-shell divergences of standard matter theory (with two derivatives) are proportional to $R^2$ and $C^2$ on a curved spacetime background. Counterterms needed to be added to the divergent matter effective action are of these types $R^2$ and $C^2$ (in $d = 4$) even if the gravitational theory was Einstein-Hilbert Quantum Gravity with $R$ in the action

Conclusion:
These counterterms contain higher derivatives of the background metric. Higher derivatives are inevitable!
Higher Derivative Quantum Gravity

Four-derivative theory (Stelle ‘77)

\[ S_{\text{QG}} = \int d^4 x \sqrt{|g|} \left( \omega_\kappa R + \theta_R R^2 + \theta_C C^2 \right) \]

General higher-derivative theory (Asorey, Lopez, Shapiro ‘96)

\[ S_{\text{QG}} = \int d^4 x \sqrt{|g|} \left( \omega_\Lambda + \omega_\kappa R + \sum_{n=0}^{N} \omega_{R,n} R \Box^n R + \sum_{n=0}^{N} \omega_{C,n} C \Box^n C + O \left( R^3 \right) \right) \]

6-derivative theories

Here we consider the case \( N = 1 \).
We quantize the theory and study RG flow at one-loop level
We generalize Stelle’s gravity and quantum results from it
Why 6-derivative Quantum Gravity?

Theoretical motivations

- quantum super-renormalizability
- possibility of UV-finiteness
- exact and unambiguous expressions for $\beta$-functions of running coupling parameters of the theory
- gauge- and scheme-independence of UV-divergences
- possibility of Lee-Wick (LW) pair of complex conjugate poles of the propagator (to ameliorate the problem of unitarity in HD QG)
- *amazingly* simple and analytic final results for $\beta$-functions
- very good theoretical laboratory for study RG flows in QG

6-der QG

is better behaved on the quantum level than 4-der HD QG of Stelle!
The theory

Classical theory

Action:

\[ S_{\text{QG}} = \int d^4x \sqrt{|g|} \mathcal{L}, \]

Lagrangian (density):

\[ \mathcal{L} = \omega_C C_{\mu\nu\rho\sigma} \Box C^{\mu\nu\rho\sigma} + \omega_R R \Box R + \theta_C C^2 + \theta_R R^2 + \theta_{\text{GB}} E_4 + \omega_\kappa R + \omega_\Lambda \]

(some) GR invariant scalar terms:

\[ C^2 = C_{\mu\nu\rho\sigma}^2 = R_{\mu\nu\rho\sigma}^2 - 2R_{\mu\nu}^2 + \frac{1}{3} R^2, \]

\[ E_4 = R_{\mu\nu\rho\sigma}^2 - 4R_{\mu\nu}^2 + R^2. \]

Fundamental ratio of the theory:

\[ x = \frac{\omega_C}{\omega_R} \]
Super-renormalizable Quantum Gravity

Propagator of all quantum modes in UV regime (monomial asymptotics \( \Box^3 \))

\[ \Pi \sim k^{-6} \]

Superficial degree of divergence \( \Delta \) of \( L \)-loop graph \( G \)

\[ \Delta = 4L + V[\text{vertex}] - I[\text{propagator}] \]

Graviton \( h_{\mu\nu} \) and FP ghost fields \( C_\mu \) are dimensionless \( \Rightarrow \) the same maximal number of derivatives in vertices as in propagators in UV

\[ [\text{vertex}] = -[\text{propagator}] = k^6 \]

Bound on \( \Delta \)

\[ \Delta \leq 4 - 2(L - 1); \quad \text{for } L \geq 4 \quad \Delta < 0 \]

\( \Rightarrow \) no loop divergences for higher loops (quantum corrections are finite)
Consequences of power counting of UV divergences

Structure of divergences

- The only possible divergent structures are $C^2$, $R^2$, $E_4$, $R$ and $\Lambda$
- The divergences $C^2$, $R^2$, $E_4$ receive contributions only at one-loop level ($L = 1$)
- The $R$ (Newton’s gravitational constant) divergence receive contributions also at $L = 2$ level
- The $\Lambda$ (cosmological constant) divergence receive contributions also at $L = 2, 3$ levels
- From 4-loop level the theory is finite
- Terms $\theta_C$, $\theta_R$, $\theta_{\text{GB}}$, $\omega_\kappa$, and $\omega_\Lambda$ do not affect the counterterms $C^2$, $R^2$, $E_4$
- Terms $O(R^3)$ may affect above $\Rightarrow$ possibility of complete UV-finiteness of the model

Here we concentrate on the most difficult to get counterterms $C^2$, $R^2$ and $E_4$
Minimal model

Minimal working model

\[ S_{\text{min}} = \int d^4 x \sqrt{|g|} \left\{ \omega_C \, C_{\mu\nu\rho\sigma} \Box C^{\mu\nu\rho\sigma} + \omega_R \, R \Box R \right\} \]

The expected form of exact one-loop divergences

\[ S_{\text{div}} = \int d^4 x \sqrt{|g|} \left\{ c_C \, C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma} + c_R \, R^2 + c_{GB} \, E_4 \right\} \]

Expected dependence
due to dimensional reasons

\[ [c_C] = [c_R] = [c_{GB}] = E^0, \quad [x] = \left[ \frac{\omega_C}{\omega_R} \right] = E^0 \]

\[ c_C = c_C(x), \quad c_R = c_R(x), \quad c_{GB} = c_{GB}(x) \]
Universality of UV-divergences in effective action $\Gamma$

**Power counting for $L = 1$, $\Delta = 4$**

- counterterm action $S_{\text{div}}$ contains up to four derivatives on the metric
- classical minimal action $S_{\text{min}}$ contains precisely six derivatives on the metric $\implies$ classical EOM $\varepsilon^{\mu\nu}$ are with six derivatives

**Parametrization independence theorem (Kallosh, Tyutin, Tarasov)**

$$\Gamma(\alpha_i) - \Gamma(\alpha_i^0) = \int d^4x \sqrt{|g|} \varepsilon^{\mu\nu} f_{\mu\nu} \quad \text{with} \quad f_{\mu\nu} = f_{\mu\nu}(g_{\kappa\lambda}, \alpha_i, \alpha_i^0)$$

**Independence of $S_{\text{div}}$**

- of gauge choices
- of gauge-fixing choices
- of parametrization ambiguities for quantum field
- of scheme choice for renormalization
One-loop computation

Method of computation

- covariant Barvinsky-Vilkovisky trace technology (generalized Schwinger-DeWitt method)
- quantum variable $h_{\mu\nu} = g_{\mu\nu} - \bar{g}_{\mu\nu}$
- based on the simple one-loop formula
  \[ \Gamma^{(1)} = \frac{i}{2} \text{Tr} \ln \hat{H}, \quad \text{with} \quad \hat{H} = \frac{\delta^2 S}{\delta \phi^2} \]
- minimal gauge fixing choice for gauge parameters
- simplified contributions from Faddeev-Popov and third ghosts quantum fields
  \[ \Gamma^{(1)} = \frac{i}{2} \text{Tr} \ln \hat{H} - i \text{Tr} \ln \hat{M} - \frac{i}{2} \text{Tr} \ln \hat{C} \]
- very difficult computation (needed to be done using Mathematica xTensor package for symbolic tensor algebra)
- various checks on it were successfully performed
Quantum results

Results in 6-derivative gravitational theory in \( d = 4 \)

\[
\Gamma_{\text{div}}^{(1)C,R,E} = -\frac{1}{2\epsilon(4\pi)^2} \int d^4x \sqrt{|g|} \left\{ \left( \frac{2x}{9} + \frac{397}{40} \right) C^2 - \frac{7}{36} R^2 + \frac{1387}{180} E_4 \right\}
\]

with \( \epsilon = \frac{4-d}{2} \) as a parameter of DIMREG scheme and the fundamental ratio \( x = \frac{\omega_C}{\omega_R} \)

Results in 4-derivative Stelle theory in \( d = 4 \)

\[
\Gamma_{\text{div}}^{(1)C,R,E} = -\frac{1}{2\epsilon(4\pi)^2} \int d^4x \sqrt{|g|} \left\{ -\frac{133}{20} C^2 + \left( -\frac{5}{2} x'^2 + \frac{5}{2} x' - \frac{5}{36} \right) R^2 + \frac{196}{45} E_4 \right\}
\]

with \( x' = \frac{\theta_R}{\theta_C} \)
**System of $\beta$ functions**

$$
\beta_C = \mu \frac{d \theta_C}{d \mu} = \frac{1}{(4\pi)^2} \left( \frac{2}{9} \frac{\omega_C}{\omega_R} + \frac{397}{40} \right), \quad \text{exact}
$$

$$
\beta_R = \mu \frac{d \theta_R}{d \mu} = -\frac{1}{(4\pi)^2} \frac{7}{36}, \quad \text{exact}
$$

$$
\beta_{GB} = \mu \frac{d \theta_{GB}}{d \mu} = \frac{1}{(4\pi)^2} \frac{1387}{180}, \quad \text{exact}
$$

$$
\beta_\kappa = \mu \frac{d \omega_\kappa}{d \mu} = -\frac{1}{(4\pi)^2} \left[ \frac{5\theta_C}{6\omega_C} + \frac{\theta_R}{2\omega_R} - \frac{5\theta_R}{\omega_C} \right],
$$

$$
\beta_\lambda = \mu \frac{d \omega_\lambda}{d \mu} = \frac{1}{(4\pi)^2} \left[ \frac{5\omega_\kappa}{2\omega_C} - \frac{\omega_\kappa}{6\omega_R} - \frac{5}{2} \left( \frac{\theta_C}{\omega_C} \right)^2 - \frac{1}{2} \left( \frac{\theta_R}{\omega_R} \right)^2 \right].
$$
Solutions for RG flows

\[ \theta_C(t) = \theta_C(0) + \beta_C t = \theta_C(0) + \frac{1}{(4\pi)^2} \left( \frac{2x}{9} + \frac{397}{40} \right) t, \]

\[ \theta_R(t) = \theta_R(0) + \beta_R t = \theta_R(0) - \frac{1}{(4\pi)^2} \frac{7}{36} t, \]

\[ \theta_{GB}(t) = \theta_{GB}(0) + \beta_{GB} t = \theta_{GB}(0) + \frac{1}{(4\pi)^2} \frac{1387}{180} t, \]

\[ \omega_\kappa(t) = \omega_\kappa(0) + a_\kappa t + b_\kappa t^2, \]

\[ \omega_\Lambda(t) = \omega_\Lambda(0) + a_\Lambda t + b_\Lambda t^2 + c_\Lambda t^3. \]

Observation

For \( t \to +\infty \) couplings \( \theta_R \) and \( \theta_{GB} \) tend to \(-\infty\) and \(+\infty\) respectively (we have asymptotic freedom in them); the coupling \( \theta_C \) tends also to AF in UV, if not the special value of ratio \( x \): \( x_* = -\frac{3573}{80} = -44.6625 \).

For \( x = x_* \) the coupling \( \theta_C \) sits at the non-trivial FP (asymptotic safety).
Conclusions

Six-derivative Gravity

- super-renormalizability and options for UV-finiteness
- exact and universal beta functions for $\theta_C$, $\theta_R$ and $\theta_{GB}$ couplings
- gauge- and parametrization-independence of UV divergences
- exact RG flows and asymptotic freedom in UV

Further developments

- conditions for AF in UV and AS
- dominance of free propagation over interactions
- rescaling of the graviton field (like Fradkin, Tseytlin)
- quantum stability of the Lee-Wick complex conjugate pairs
- addition of terms $O(R^3)$ for UV-finiteness
- spectrum around flat Minkowski and around (A)dS spacetimes
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Dziękuję!

Thank you!

Obrigado!