Hysteretic ac loss in a coated superconductor subjected to oscillating magnetic fields: ferromagnetic effect and frequency dependence

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Abstract

Numerical simulations of the hysteretic ac loss in a coated superconductor with a more realistic version of the architecture were performed via the finite-element technique in the presence of an oscillating magnetic field. The coated superconductor was electromagnetically modeled by resorting to the quasistatic approximation of a vector potential approach in conjunction with nonlinear descriptions of the superconducting layer and the ferromagnetic substrate therein by a power-law model and the Langevin equation, respectively. A diverse effect of the ferromagnetic substrate on the hysteretic ac loss, depending on the strength of the applied magnetic field, was displayed, and its underlying cause was identified. The dependence of the hysteretic ac loss on the applied frequency is found to be related to a critical amplitude of the applied magnetic field, and the eddy-current loss dissipated in the metal coatings becomes prominent as the frequency increases only at high applied magnetic fields.

Keywords: coated superconductor, hysteretic ac loss, ferromagnetic effect, frequency dependence, finite-element methods

(Some figures may appear in colour only in the online journal)

1. Introduction

Coated superconductors have been focused for years towards a wide range of applications [1–7], due to the persistent advance in the related materials technology [8]. One of the most critical properties regarding their use is the hysteretic ac loss caused by exotic stimulations, namely, imposed ac transport currents and/or applied oscillating magnetic fields. As for the topic of applying magnetic fields from a theoretical point of view, a number of studies have been carried out by means of either analytical calculations [9–11] or numerical analysis [10, 12–20], and several results in terms of the hysteretic ac loss with ferromagnetic effect or its dependence upon the applied frequency have been achieved. However fundamental these results may be, further investigations at a more realistic level remain promising, as a surplus adaption or hypothesis of the geometrical and material characteristics has been made in the existing models: examples are the thickness of the superconducting layer being scaled due to the intractable aspect ratio of width/thickness of a real coated superconductor [10, 14]; the ferromagnetic substrate being supposed as a linear medium with constant or even infinite permeability [9, 14, 17]; the field-dependent feature of the critical current, which is found to be tangible at high applied magnetic fields [21], being neglected or unclear in the literature [9, 12–14, 17]; and the superconductor being described by exploiting the magnetostatic–electrostatic analogs in order to adapt to the commercial software ANSYS [17].

Motivated by the situation described above, this paper is dedicated to taking a comprehensive examination of the
ferromagnetic effect and frequency dependence on the hysteretic ac loss of a coated superconductor, with a pristine geometry and containing all electromagnetically critical constituents of a real one rather than the simplified version, i.e., a superconductor strip over a substrate [9, 12, 13, 15–20], by making use of the quasistatic approximation of a vector potential approach in conjunction with an elaborate description of the nonlinear behavior in both the superconducting layer and the ferromagnetic substrate.

2. Mathematical model

As is the case for the commercially attainable product from SuperPower Inc. [22], the coated superconductor addressed in this work is supposed to be made up of four constituents to denote a more realistic version of material architecture, namely, a superconducting layer as the central element, a silver overlayer, a metallic substrate, and copper stabilizers, all being infinitely extended in the z-axis direction of a Cartesian coordinate system x, y, z, as demonstrated in figure 1. The frequency of the applied magnetic field covered by this work is limited to be in a low range (1–500 Hz), rendering the quasistatic approximation of the Maxwell equations valid and permitting the omission of the frequency-related characterization of the magnetic permeability and electrical conductivity as well.

Given a premise of this sort, the electromagnetic master equation for the coated superconductor as well as the surrounding coolant is established in terms of the magnetic vector potential A by using Ampère’s law within the quasistatic approximation as the state equation,

$$
\nabla \times \left( \frac{1}{\mu} \nabla \times \mathbf{A} \right) = -\sigma \frac{\partial \mathbf{A}}{\partial t},
$$

(1)

where the magnetic permeability \(\mu\) in the ferromagnetic substrate and the electrical conductivity \(\sigma\) in the superconducting layer are nonlinear. The two-dimensional (2D) reduced form of equation (1) in different materials has been presented in detail elsewhere [23].

An inverse representation of the power law [24], combined with Kim’s model [25], is adopted to characterize the nonlinear dependence of the supercurrent density on the local fields,

$$
\mathbf{J} = \frac{J_{c0}}{1 + |\mathbf{B}|/B_0} \left( \frac{E}{E_c} \right)^{1/n} \frac{\mathbf{J}}{|\mathbf{J}|},
$$

(2)

where \(J_{c0}\) is the zero-field critical current density depending on the prescribed criterion \(E_c\) and \(n\) is the creep exponent, whereas \(B_0\) represents the critical magnetic flux density for which the critical current density is halved. Assuming that the superconducting layer is made of yttrium–barium cuprate cooled with liquid nitrogen, the present analysis has used \(J_{c0} = 2.5 \times 10^{10} \text{ A m}^{-2}\) or \(I_{c0} = 100 \text{ A}[22]\), \(E_c = 1 \mu\text{V cm}^{-1}\), \(n = 21[26]\), and \(B_0 = 0.1 \text{ T}[27]\). A residual resistivity \(\rho_0 = 10^{-17} \Omega\text{m}\), to account for the flux creep due to the thermal activation in the superconductor as well as to ensure numerical stability around the zero electric field [28, 29], is assigned to the superconducting layer in this work.

If the ferromagnetic substrate is made of Ni-based alloy, whose \(B–H\) characteristic shows a minor loop that permits the neglect of coercivity [30, 31], calling upon a reversible-paramagnet approximation in the Langevin form [32],

$$
|\mathbf{B}| = \mu_0 \left\{ M_s \left( \coth \left( \frac{H}{H_0} \right) - \frac{H_0}{H} \right) + |\mathbf{H}| \right\},
$$

(3)

with the saturation magnetization \(M_s\) and the auxiliary magnetic field strength \(H_0\) linked to the magnetic susceptibility at zero field \(\chi_0\) by \(H_0 = M_s/3\chi_0\), is logically suggested. The hysteretic loss in the ferromagnetic substrate is therefore not taken into account here, to be self-consistent with such a hypothesis, though an empirical formula for it is already attainable [15, 19]. It is presumed that \(M_s = 7.5 \times 10^5 \text{ A m}^{-1}\) and \(\chi_0 = 250[23, 33]\) in this paper for a ferromagnetic substrate, unless stated otherwise.

The hysteretic ac loss per unit length in a full cycle of applied magnetic field is computed as

$$
U_{ac} = \int_{t_{inc}} dt \oint \mathbf{E} \cdot \mathbf{J} \, dx \, dy,
$$

(4)

where \(\Omega\) is the cross-sectional area of the respective domain of each constituent in the coated superconductor.

The electromagnetic master equation (1) is numerically discretized by means of Galerkin’s finite-element method [34] and Euler’s finite-difference scheme [35], respectively, in the spatial and temporal domain, and the generated nonlinear system of finite-element equations is solved by resorting to the Jacobian-free Newton–Krylov algorithm, an advanced approach founded on a synergistic combination of Newton-type methods for superlinearly convergent solutions of nonlinear equations and Krylov subspace methods for solving the Newton correction equations [36]. The oscillating magnetic field, with amplitude \(\mu_0 H_0\) and frequency \(\nu\), is imposed by attaching a Dirichlet condition on the outer bounds of the whole computational domain, a square having a side length of 20 times the width of the coated superconductor.

3. Results and discussion

With the above-described theoretical foundations, numerical simulations of appraising the effect of the ferromagnetic

Figure 1. Cross-sectional view of an infinitely extended coated superconductor with the superconducting layer covered by a silver cap and deposited upon a metallic substrate and then sandwiched by top and bottom copper stabilizers. The thickness of each constituent from top to bottom is respectively 20, 2, 1, 50, and 20 \(\mu\text{m}\), with an identical width of 4 mm, as in the SCS405 conductor from SuperPower Inc. [22]. The dimensions shown in this drawing are not to scale.
substrate on the hysteretic ac loss of a coated superconductor subjected to an oscillating magnetic field, together with its frequency dependence, were carried out by assigning the representative geometrical and material characteristics aforementioned to the coated superconductor of figure 1. These simulations also include the eddy-current loss dissipated in the metal coatings using the suggested cryogenic resistivity of pure Ag (0.27 $\mu\Omega$ cm), Ni (0.5 $\mu\Omega$ cm), and Cu (0.19 $\mu\Omega$ cm) to respectively represent the silver overlayer, the metallic substrate, and the copper stabilizers [37].

3.1. Ferromagnetic effect

The hysteretic ac loss of a coated superconductor with different ferromagnetic substrates, and that of a reference with nonmagnetic substrate, were calculated as a function of the amplitude of an applied transverse oscillating magnetic field with $\nu = 50$ Hz and the normalized results in the electromagnetic steady state (all established since the second cycle) were plotted; see figure 2. Through observing this figure, whereas the general trend of any property of the substrate is uniform, the effect of the ferromagnetic substrate on the normalized hysteretic ac loss $U_{ac}/(\mu_0 H_a)^2$, suffered by the entire coated superconductor, is evident, and it exhibits three distinct characters depending on the amplitude of the applied magnetic field $\mu_0 H_a$. For small values of $\mu_0 H_a$, the ferromagnetic effect creates an increase of the hysteretic ac loss in comparison with the reference, being particularly pronounced as the ferromagnetic property of the substrate, controlled by the value of $M_s$, strengthens and the value of $\mu_0 H_a$ abates. Conversely, the hysteretic ac loss for moderate values of $\mu_0 H_a$ is suppressed due to the ferromagnetic effect, first building up and then trailing off as the value of $\mu_0 H_a$ increases. Eventually, the hysteretic ac loss of all cases asymptotically converges for large values of $\mu_0 H_a$, the ferromagnetic effect becoming insignificant because of the magnetization saturation of a practical ferromagnet. The set of curves given as an inset, representing the variation of the normalized hysteretic ac loss in the superconducting portion only, behaves similarly in both tendency and magnitude for small and moderate values of $\mu_0 H_a$ to those of the entire coated superconductor, implying that (i) the ferromagnetic effect mostly acts on the superconducting layer and (ii) the eddy-current loss dissipated in the coatings only become tangible for large values of $\mu_0 H_a$, given the geometrical and material characteristics in this work.

It is worth noting that the increase of the hysteretic ac loss for small values of $\mu_0 H_a$ in the presence of a ferromagnetic substrate is likely to be attributed to the edge effect due to the magnetic concentration of the ferromagnet as a slight extension of the width of the substrate, by only a factor of 1.1, to create a wider substrate, will lead to a substantial decrease of the hysteretic ac loss, being completely below that of the nonmagnetic case, as clearly demonstrated in figure 2. However, widening the substrate will accordingly enhance the portion of eddy-current loss due to the increase of the conducting volume, and as a result will cause the hysteretic ac loss of the entire coated superconductor to slightly higher than others for large values of $\mu_0 H_a$, as figure 2 displays. This unfavorable effect can be dramatically mitigated as the cryogenic resistivity of the substrate increases, which has been proven by using a higher resistivity, for Ni–5% W substrate at 77 K [38]. These findings, revealed on a theoretical version of a coated superconductor with all critical constituents present and relying on the finite-element technique with the power-law current–voltage model and nonlinear permeability, alongside the previous predictions of a superconductor strip on a ferromagnetic substrate using an analytic model with critical state model and infinite permeability [9], perhaps
suggest that the ferromagnetic effect is capable of reducing the hysteretic ac loss in a coated superconductor exposed to an oscillating magnetic field, at least for small and moderate values of $\mu_0 H_a$, by carefully tuning the width of the substrate. (An adjunctive calculation of the scenario addressed by the analytical model [9] and experiments [39], putting the present investigations into an established perspective, was also carried through, and the related results can be found in the appendix.)

3.2. Frequency dependence

Figure 3 shows the normalized hysteretic ac loss $\frac{U_{ac}}{(\mu_0 H_a)^2}$, suffered by the entire coated superconductor and the superconducting layer therein in the electromagnetic steady state (also established since the second cycle), against the amplitude of an applied transverse oscillating magnetic field $\mu_0 H_a$, addressing a series of values of frequency $\nu$ within a limited band (1–500 Hz). A general feature revealed by this figure, irrespective of the uniform tendency in terms of the amplitude $\mu_0 H_a$ for any value of the frequency $\nu$, is that there exists a critical value of $\mu_0 H_a$ above and below which the dependence of the hysteretic ac loss on the frequency is quite distinct. A growing increase with the frequency of the hysteretic ac loss in the entire coated superconductor is seen to occur as the value of $\mu_0 H_a$ increases from the critical point, whereas a decreasing trend emerges for values of $\mu_0 H_a$ below the critical point, excluding the exceptions at small values of $\mu_0 H_a$, where the dependence becomes irregular. In contrast, the hysteretic ac loss in the superconducting portion, given as an inset in figure 3, displays a regular variation with increasing frequency, being respectively elevated and degraded at values of $\mu_0 H_a$ above and below the critical point. Providing the geometrical and material characteristics of the coated superconductor chosen in this work, the threshold value of $\mu_0 H_a$, distinguishing the distinct variations of the hysteretic ac loss with the amplitude of the applied magnetic field, is estimated to be slightly less than 25 mT for the case of the entire coated superconductor, while for the case of the superconducting layer it is slightly higher than 25 mT, according to the numerical interval for $\mu_0 H_a$ in the present simulations.

Data extracted from figure 3 to clearly display the variation of the hysteretic ac loss with frequency at a certain value of $\mu_0 H_a$ is presented in figure 4 for the superconducting layer (left), the copper stabilizers (middle), and the entire coated superconductor (right). The hysteretic ac loss as a function of frequency, $U_{ac}(\nu)$, was normalized by that of $\nu_0 = 1$ Hz in each case of this figure. The value of $U_{ac}(\nu_0)$ for normalization is therefore different for the three graphs in figure 4. The left figure reveals that the above-described decrease and increase of the hysteretic ac loss in the superconducting layer with increasing frequency, respectively below and above the critical point, both develop as an exponential dependence that becomes more pronounced as the value of $\mu_0 H_a$ approaches the extremes, whereas at a value of $\mu_0 H_a$ around the critical point, being slightly higher than 25 mT, the variation of the frequency only gives rise to a tiny change in the hysteretic ac loss. The middle figure shows that the eddy-current loss in the copper stabilizers increases linearly with increasing frequency, but the slope is distorted as compared to the theoretically expected value of 1 [38], weakly at the extremes of $\mu_0 H_a$ but rather markedly at intermediate values of $\mu_0 H_a$. These findings, to some extent, are consistent with the previous studies [10]. The dependence of the total loss in the entire coated superconductor, illustrated in the right figure, indicates that the hysteretic ac loss in the superconducting layer is dominant at small values of $\mu_0 H_a$, where an exponential decrease of the total loss with increasing frequency is displayed, whereas at large values of $\mu_0 H_a$ a quasilinear increase in the hysteretic ac loss emerges, implying that the contribution of the eddy-current loss in the metal coatings becomes prominent.

Figure 3. Normalized hysteretic ac loss, suffered by the entire coated superconductor and by the superconducting layer (inset) per cycle, as a function of the amplitude of an applied transverse oscillating magnetic field with different frequency (1–500 Hz) in the electromagnetic steady state (all established since the second cycle).
Figure 4. Normalized hysteretic ac loss, suffered by the superconducting layer (left), the copper stabilizers (middle), and the entire coated superconductor (right) per cycle, as a function of the normalized frequency of an applied transverse oscillating magnetic field with different amplitudes $\mu_0 H_0$ in the electromagnetic steady state (all established since the second cycle). The value of frequency for normalization $\nu_0$ is 1 Hz, and the value used for normalization, $U_{ac}(\nu_0)$, is different for the three graphs.

Figure 5. Normalized hysteretic ac loss, suffered by the silver overlayer (left), the ferromagnetic substrate (middle), and the copper stabilizers (right) per cycle, as a function of the amplitude of an applied transverse oscillating magnetic field with $\nu = 1, 10, 25, 50, 100, 250, and 500 \, \text{Hz}$ (from the lower curve up) in the electromagnetic steady state (all established since the second cycle).

Figure 5 shows the dependence of the normalized hysteretic ac loss $U_{ac}/(\mu_0 H_a)^2$ in the respective metal coating on the amplitude of an applied transverse magnetic field $\mu_0 H_a$ to demonstrate how the screening effect [40] due to the induced supercurrent in the superconducting layer affects these dependences at different frequencies. It can be seen from this figure that, at any value of the frequency, the screening effect mostly acts at small and moderate values of $\mu_0 H_a$, particularly in the case of silver, for which a roughly linear increase is found, whereas at large values of $\mu_0 H_a$ the normalized hysteretic ac loss remains nearly constant as it should [38], implying an insignificant screening effect.

Worthy of comment is that, though all the results presented above were obtained by applying a transverse oscillating magnetic field, the main observations were thought to hold with other field orientations, with only the peak of the curves in figures 2 and 3 being shifted, as those reported in [18], according to the results obtained for other orientations of the applied magnetic field.

4. Conclusion

In conclusion, the ferromagnetic effect on the hysteretic ac loss of a coated superconductor subjected to an oscillating magnetic field, together with the frequency-dependent features, has been examined by means of a numerical model in this paper. It is mainly observed that the ferromagnetic substrate mostly affects the superconducting layer, and its effect on the hysteretic ac loss is diverse, depending on the strength of the applied magnetic field, a phenomenon recalling the
measurement reported elsewhere [39, 41, 42]. An inverse dependence of the hysteretic ac loss on the applied frequency exists above and below a critical amplitude of the applied magnetic field, and as the applied frequency increases, the eddy-current loss dissipated in the metal coatings tends to be prominent only at high applied magnetic fields. The screening effect, due to the induced supercurrent in the superconducting layer, on the metal coatings is found to be significant only at small and moderate applied magnetic fields, irrespective of the applied frequency.

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Appendix. Superconductor strip on an idealized ferromagnetic substrate

Considering a bilayer heterostructure of a thin superconductor strip sitting on a ferromagnetic substrate and quoting the characteristics given in [9, 39], the hysteretic ac loss inside the superconductor strip was calculated as a function of the amplitude of an applied transverse oscillating magnetic field \(\mu_0 H_0\), or the related normalized value \(H_0/j_c d_s\), for a superconductor (SC) strip sitting on a nonmagnetic (NM) substrate (thin solid line), or on a ferromagnetic (FM) substrate with \(a_m/a_s = 1\) (thick solid line), or with \(a_m/a_s = 1.1\) (dashed line) in the electromagnetic steady state (established since the second cycle). The dimensions of \(a = a_s = 5\) mm, \(d_s = 2.3\) \(\mu\)m, and \(d_m = 25\) \(\mu\)m, together with the critical current density of \(j_c = 1.2 \times 10^{10}\) A m\(^{-2}\), as those in [9, 39], were adopted in the present calculations. The relative permeability of the ferromagnetic substrate here was assumed to be constant and as high as \(10^4\) to approach the infinite assumption made in [9].

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Figure A.1. Normalized imaginary part of the ac susceptibility \(\chi''/\pi a^2\) as a function of the amplitude of an applied transverse oscillating magnetic field \(\mu_0 H_0\), or the related normalized value \(H_0/j_c d_s\), for a superconductor (SC) strip sitting on a nonmagnetic (NM) substrate (thin solid line), or on a ferromagnetic (FM) substrate with \(a_m/a_s = 1\) (thick solid line), or with \(a_m/a_s = 1.1\) (dashed line) in the electromagnetic steady state (established since the second cycle). The dimensions of \(a = a_s = 5\) mm, \(d_s = 2.3\) \(\mu\)m, and \(d_m = 25\) \(\mu\)m, together with the critical current density of \(j_c = 1.2 \times 10^{10}\) A m\(^{-2}\), as those in [9, 39], were adopted in the present calculations. The relative permeability of the ferromagnetic substrate here was assumed to be constant and as high as \(10^4\) to approach the infinite assumption made in [9].
