Research Article

Dynamics of Terrorism in Contemporary Society for Effective Management

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In the contemporary world, the effect of faith in religion cannot be underestimated or overemphasized. In the olden days, traditional religion/faith of a particular locality was the only practice obtainable; however, new faiths emerged and are being absorbed in recent times. Extremism in the newly absorbed faith began to cause the indigenous religion to collapse and increase violence against innocent ones. This paper investigated the interaction between the extremism of faith leading to the act of terror and a susceptible individual (members of the society) to guide the policymakers and decision implementers to embrace the proposed model for counterterrorism for effective management of the insurgency. Mathematical modelling of epidemiology was conceptualized for the model formulation, and the resulting autonomous differential equations were critically analyzed with the Lipschitz condition, next generation matrix, and Bellman and Cooke’s criteria for the management of insurgency in the society. Thresholds were obtained to curtail recruitment into the fanatical groups, and the results of the simulated proposed model identified critical factors (parameters) to be considered for the complete eradication of violence in human society.

1. Introduction

The society’s aims to wage a war against terrorism and counterterror measures may range from invading certain territories for assassinations of terrorists to freezing assets of organizations with links to potential terrorists [1–3]. The dynamics of bicycles and space shuttles evolve according to Newton’s law, but people’s movements back and forth between different states of activity are driven by far more complex influences [4]. One might think it is impossible to model systems in which humans are the key moving parts, but that misinterprets the definition of modelling [5, 6]. Model is a simplified representation of reality [7]. Models of human systems simplified by omitting many factors may not produce the required result, and there may be more judgment involved in determining what constitutes a reliable model when analyzing human systems (complexity in modelling real-life events), but that makes systems like terrorism better not a worse topic for a research. The inherent complexity of human systems ironically implies that there is particular wisdom in keeping the model simple enough that one can understand intuitively “what makes the model work” [5, 7, 8]. Therefore, it is of interest to model the cause of the act of terrorism in human population using a system of ordinary differential equation (ODE) for effective management. The study builds on the work of Professor Emeritus Castillo-Chavez and Banks in [9], where fanatics are treated as infection, and here, fanatics progress to cause terrorism. This act of terrorism could be international or domestic in nature [10]. Extremism in faith causing terrorism for purposes such as self-protection, politically motivated, personal gain, or ideology towards mobilizing for political violence [11–14] is here considered. Many researchers have used the classical fourth-order Runge–Kutta (RK45) method due to the method’s efficiency and accuracy in solving ODEs and optimal control problems [15–18].
Hence, this study presents the mathematical dynamical model of terrorism and simulates using RK45 in Maple 18 to numerically picture the analytical solutions of the model.

2. Model Formulation and Assumptions

2.1. Model Formulation. A mathematical dynamics model of terrorism is formulated using the approach of mathematical epidemiology. The entire population is referred to as noncore (typically large), denoted as \( G(t) \), while the core population is a subset of \( G(t) \), which is further subdivided into three, as shown in Figure 1, and in the following hierarchical order: the naive (susceptible/vulnerable) subpopulation, which includes individuals who are yet to embrace ideology, denoted as \( S(t) \), semi-fanatic (exposed to fanatics) subgroup, \( E(t) \), and the extreme fanatic subpopulation, \( F(t) \), with assumption that only fanatics commit terror; example of such fanaticism is the case of religious parents that refused blood transfusion and almost caused violence against their child (the details of the story are given in [19, 20] and the court injunction is given in [13]). The mathematical model is presented in Figure 2. The formulation is based on the work of Castillio-Chavez and Banks [9].

2.2. Model Assumptions. The following are the assumptions made in formulating the dynamical model:

1. It is universally agreed that the government refers to and declares any group of faith (extremist) that causes violence against innocent members of the community as a terrorist group [21].
2. A general compartment, \( G(t) \), is assumed to be a very large population that does not discriminate her members in any form, be it religion, ethnicity, or tribalism.
3. It is assumed that the introduction of an extremist in the general population generates discrimination.
4. Repentance from the vulnerable class \( (S(t)) \) may return to their normal way of life, possibly, due to government policy on extremism or community norms, culture, and value, e.g., US Federal Bureau of Investigation partners with other operatives to help dismantle extremist network worldwide [10].
5. The model considered that the susceptible and the semi-fanatics classes return (repent) to noncore class, perhaps, due to self-relection or sanction or strictness of government policy about terrorism.
6. The model further assumes that individuals with extreme ideology will only be removed, either by natural death or extreme ideology (induced death due to their beliefs or terror). However, law gives freedom to practice religion [13] of choice, provided the person is in his/her rightful senses.
7. The model indeed considers no permanent resident in the terror compartment. Although extremist moves out from the fanatical compartment to commit terror (e.g., Boko Haram) and some return afterward, the government tags this set of people extremists or terrorists. The model considered Boko Haram as a fanatical group and agreed that they move out of their hideout to commit terror.
8. The general population can never be zero and has no age barrier.

The progression from \( G(t) \) to \( S(t) \) is \( q \), i.e., \( q \) is the successful contact for recruitment of members to the susceptible class. \( \beta \) is the contact probability with the core group member, and this lies between zero and one (0 \( \leq \beta \leq 1 \)). \( \rho \) is treated as the conversion rate of susceptible class to \( E(t) \). The natural death rate is \( \mu \), and it is assumed equal for all compartments and parameter value in a case study of Nigeria, in which its inverse is an average lifespan and there exists death due to extreme fanatics \( (d \ or \ \delta) \) in this study, and both parameters will be used interchangeably.

It is also assumed that after a while, due to personal reasoning or phobia of sanction, \( S(t) \) and \( E(t) \) return to \( G(t) \) at the rates \( \alpha \) and \( \gamma \), respectively. The proportion of \( E(t) \) to \( F(t) \) is \( \sigma \), and this is treated with standard incidence function because of complex interaction in humans. The term \( \eta \) is used to denote the proportion of \( F(t) \) that commits terror, and \( y \) is the proportion of return fanatics after committing terror.

In view of the formulation and assumptions above, we present Figure 2, the block diagram of dynamics of the model, followed by model equations.

2.3. Model Diagram and Equations. Model equations:

\[
\begin{align*}
\frac{dG}{dt} &= \Lambda - \frac{q\beta G C}{N} + aS + yE - \mu G, \\
\frac{dS}{dt} &= \frac{q\beta G C}{N} - \frac{\rho S(E + F)}{C} - (\mu + a)S, \\
\frac{dE}{dt} &= \frac{\rho S(E + F)}{C} - \frac{\sigma E F}{C} - (\mu + \gamma)E, \\
\frac{dF}{dt} &= \frac{\sigma EF}{C} - (\mu + d + \eta)F + yT, \\
\frac{dT}{dt} &= \eta F - (\mu + y + d)T,
\end{align*}
\]

with initial condition
3. Properties and Analysis of the Model

It is essential to investigate the behaviour of model formulated before recommending it for use. The properties of the proposed model to be examined are positive invariant, regions of feasibility, reproductive number (recruitment threshold into extremism), equilibria states, and stability analysis (dynamics of human behaviour and criteria for management). All these properties and analyses were applied in many studies [18, 22, 23].

**Theorem 1.** Region of feasibility and positivity of solution: if $S(t), E(t), F(t), T(t), G(t)$ have nonnegative initial condition, then there exists a region

$$\Omega = \left\{ (S_F(t), E_F(t), F(t), T(t), G(t)) \in R^5_+ : 0 < S_F(t) + E_F(t) + F(t) + T(t) + G(t) \leq \frac{\Lambda}{\mu} \right\},$$

for all $t, S_F(t) \geq 0, E_F(t) \geq 0, F(t) \geq 0, T(t) \geq 0, G(t) \geq 0$.

**Proof.** The model concerns humans; therefore, parameters of the models are assumed to be nonnegative for all time $t$. Furthermore, we consider

$$N(t) = S_F(t) + E_F(t) + F(t) + T(t) + G(t)$$

and this implies

$$N(t) = \frac{\Lambda}{\mu} (1 - e^{-\mu t}) + N_0 e^{-\mu t}.$$

3.1. Interpretation. The result obtained in equation (6) showed that the population cannot be zero. Hence, validate the integral claim of assumption nine (8).
3.2. Existence and Uniqueness of the Solution. In mathematics, posing a problem is not enough if a solution does not exist and the uniqueness of such a solution is essential in some instances. Here, the existence and uniqueness of the model shall be tested using the Lipschitz condition.

Theorem 2. Suppose (1) is written in compact form as \( F_i(t, x), x(t_0) = x_0, i = 1, 2, \ldots, 5 \), and there exists a unique solution; then, (1) satisfies the Lipschitz condition.

Proof. Rewrite model equation (1) as

\[
F_1(t, G) = \Lambda - \frac{qBG}{N} + yE_F + aS_F - \mu G,
\]
\[
F_2(t, S_F) = \frac{qBG}{N} - \frac{\rho S_F (E_F + F)}{C} - (\mu + \alpha) S_F,
\]
\[
F_3(t, E_F) = \frac{\rho S_F (E_F + F)}{C} - \frac{\sigma E_F}{C} - (\mu + \gamma) E_F,
\]
\[
F_4(t, F) = \frac{\sigma E_F}{C} - (\eta + \mu + d) F + yT,
\]
\[
F_5(t, T) = \eta F - (\mu + d + y)T,
\]

and define equation (1) in the form

\[
X'_i = F_i(t, x), x(t_0) = x_0, i = 1, 2, \ldots, 5; \text{ then,}
\]

and by illustration,

\[
|F_1(t, G_2) - F_1(t, G_1)| = \left| \left( \frac{qBG}{N} + \mu \right) G_2 - \left( \frac{qBG}{N} + \mu \right) G_1 \right| \leq \left| \left( \frac{qBG}{N} + \mu \right) \right| |G_2 - G_1|,
\]
\[
|F_2(t, S_{F2}) - F_2(t, S_{F1})| = \left| -\left( -\frac{\rho (E_F + F)}{C} + (\mu + \alpha) \right) S_{F2} - \left( -\left( \frac{\rho (E_F + F)}{C} + \mu + \alpha \right) S_{F1} \right) \right| \leq \left| -\left( -\frac{\rho (E_F + F)}{C} + (\mu + \alpha) \right) \right| |S_{F2} - S_{F1}|,
\]
\[
|F_3(t, E_{F2}) - F_3(t, E_{F1})| = \left| -\frac{\rho S_F - \sigma F}{C} - (\mu + \gamma) \right| E_{F2} - \left( -\frac{\rho S_F - \sigma F}{C} - (\mu + \gamma) \right) E_{F1} \right| \leq \left| -\frac{\rho S_F - \sigma F}{C} - (\mu + \gamma) \right| |E_{F2} - E_{F1}|. \]
Remark 1.

(i) A system of ODEs is said to exist and is said to be unique in the solution if it satisfies the Lipschitz condition.

(ii) Lipschitz conditions cater for continuity and boundedness. According to [24], first-order ODE is said to exist and is said to be unique if the partial derivatives of the function exist. This is stated in the next theorem relevant to the proposed model. Hence, the proposed model is said to be mathematically well-posed.

Theorem 3. Let \( \Omega \) be denoted the region stated in Theorem 1 and bounded in \( 0 \leq R < \infty \). It suffices to show that \(|\partial f_i(t, x)/\partial x|, i = 1 \) to \( 5 \), \( x = G, S_F, E_F, F, T \) are continuous in \( \Omega \).

Proof. Rewrite equation (1) as

\[
\begin{align*}
F_1(t, G, S_F, E_F, F, T) &= \Lambda - \frac{qBG}{N} + \gamma E_F + \alpha S_F - \mu G, \\
F_2(t, G, S_F, E_F, F, T) &= \frac{qBG}{N} - \rho S_F \left( \frac{E_F + F}{C} \right) - (\mu + \alpha)S_F, \\
F_3(t, G, S_F, E_F, F, T) &= \rho S_F \left( \frac{E_F + F}{C} \right) - \sigma E_F \left( \frac{E_F}{C} \right) - (\mu + \alpha)E_F, \\
F_4(t, G, S_F, E_F, F, T) &= \sigma E_F \left( \frac{E_F}{C} \right) - (\eta + \mu + d)F + yT, \\
F_5(t, G, S_F, E_F, F, T) &= \eta F - (\mu + d + y)T,
\end{align*}
\]

(15)

\[
\begin{align*}
\frac{\partial F_1}{\partial G} &= \left| \frac{qBG}{N} - \mu \right| < \infty, \\
\frac{\partial F_1}{\partial S_F} &= |\alpha| < \infty, \\
\frac{\partial F_1}{\partial E_F} &= |\gamma| < \infty, \\
\frac{\partial F_1}{\partial T} &= |\alpha| < \infty, \\
\frac{\partial F_2}{\partial G} &= \left| \frac{qBG}{N} \right| < \infty, \\
\frac{\partial F_2}{\partial S_F} &= \left| \rho \left( \frac{E_F + F}{C} \right) + (\mu + \alpha) \right| < \infty, \\
\frac{\partial F_2}{\partial E_F} &= \left| \rho S_F \left( \frac{E_F}{C} \right) \right| < \infty, \\
\frac{\partial F_2}{\partial T} &= \left| \rho S_F \left( \frac{E_F}{C} \right) \right| < \infty, \\
\frac{\partial F_3}{\partial F} &= \left| \rho S_F \left( \frac{E_F}{C} \right) \right| < \infty, \\
\frac{\partial F_3}{\partial T} &= \left| \rho S_F \left( \frac{E_F}{C} \right) \right| < \infty.
\end{align*}
\]

Hence, \(|\partial f_i/\partial x| \in R < \infty\), for \( i = 1, 2, \ldots, 5 \) and \( x = G, S_F, E_F, F, T \) exist and have solution in feasible region. This completes the proof.

3.2.1. Interpretation of the Result. The result of existence and uniqueness is that the problem of extremism is in the human population and the solution also lies therein.

3.3. Threshold for Recruitment into the Ideology. The long-term sustainability of the core subpopulation is analyzed by using the concept of basic reproduction number (BRN) in epidemiology [25]. With this BRN, one can deduce that for certain parameter values, the model predicts the extinction
of the core population \( c(t) \), and effective recruitment shows that the core population will persist (endemic in epidemiology) [16]. Next theorem gives the thresholds for recruitment into the ideology.

**Theorem 4**

\[
\frac{dC}{dt} = q\beta (1 - C)C - (\mu + \alpha)S - (\mu + \gamma)E - (\mu + \delta)F - (\mu + \delta)T,
\]

\[
= (q\beta (1 - C))C - (\mu + n)C - \delta (F + T),
\]

where \( n = \alpha, \gamma \leq (q\beta (1 - C) - (\mu + n))C, \]

\[
= -(\mu + n)C \left( 1 - \frac{q\beta}{\mu + n} \right) < 0,
\]

and by the hypothesis of the theorem,

\[
R_0 = \frac{q\beta}{\mu + n}
\]

Hence, \( dC/dt = -(\mu + n)C(1 - R_0) < 0 \).

It could be seen from the above inequality that \( C(t) \) decays exponentially fast to zero. If \( R_0 < 1 \), it implies that the core population cannot be established, but from the assumptions of the model, the general population can never be extinct, so there will always be people vulnerable to fanatics. Reducing resistance to vulnerability increases the core population, or the longer the residence time \( 1/n \), the higher the possibility of increased recruitment (i.e., \( R_0 \geq 1 \)).

The approach of next generation matrix (NGM) is employed for determining effective threshold \( (R_e) \) for recruitment of fanatics. From (1), we extract \( F \) and \( V \), which, respectively, denote new infection and other transmissions in the model, i.e.,

\[
F = \begin{pmatrix}
\frac{\rho S_F}{C} & \frac{\rho S_F}{C} & 0 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

\[
V = \begin{pmatrix}
\gamma + \mu & 0 & 0 \\
0 & \beta + \mu + d & -\gamma \\
0 & -\beta & \mu + d + y
\end{pmatrix},
\]

\[
V^{-1} = \begin{pmatrix}
\frac{1}{\gamma + \mu} & 0 & 0 \\
0 & a & c \\
0 & b & d
\end{pmatrix},
\]

\[
(i) \text{ Assume } n = \min\{\alpha, \gamma\} \text{ and } R_{0,\min} = q\beta/\mu + n. \text{ Then,}
\lim_{n \to \infty} C(t) \to 0.
\]

\[\text{(ii) Effective reproductive ratio } R_e = \rho/\mu + \gamma.\]

**Proof.** Since \( C = S + E + F + T \), then

\[
\frac{dC}{dt} = (q\beta (1 - C))C - (\mu + n)C - \delta (F + T),
\]

where \( a = (\mu + \beta + d)/(\mu + \beta + d + \gamma + \beta + \delta), \)

\( b = (\mu + \beta + d + \gamma + \beta)/(\mu + \beta + d), \)

\( c = (\mu + \beta + d + \gamma)/(\mu + \beta + d + y), \)

and \( d = (\mu + d + y)/(\mu + d + y + \beta)/(\mu + d) \);

then, with little algebraic simplification, \( FV^{-1} \) is obtained to be

\[
FV^{-1} = \begin{pmatrix}
\frac{\rho}{\mu + \gamma} & a \rho & c \rho \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}.
\]

Hence, \( R_e \) is the spectral radius of NGM given by

\[
R_e = \rho(FV^{-1}) = \frac{\rho}{(\mu + \gamma)}.
\]

\[\square\]

3.4. Equilibria and Stability Analysis

3.4.1. Equilibrium State. Investigating long-time dynamics of the model, the process starts by equating (1) to zero (i.e., at equilibrium for the model):

\[
\Lambda - \frac{q\beta G E}{N} + \gamma E_F + \alpha S_F - G = 0,
\]

\[
\frac{\rho S_F (E_F + F)}{C} - \frac{\alpha E_F F}{C} - (\mu + \gamma)E_F = 0,
\]

\[
\frac{\rho S_F (E_F + F)}{C} - \frac{\alpha E_F F}{C} - (\mu + \gamma)E_F = 0,
\]

\[
\frac{\sigma E_F F}{C} - (\eta + \mu + d)F + y T = 0,
\]

\[
\eta F - (\mu + d + y)T = 0.
\]
Also, the biological interpretation of this is that for a society free of extremists, the following results were obtained:

\[ E_0^* = \{ (G, S_p, E_p, F, T) = \left( \frac{\Lambda}{\mu}, 0, 0, 0 \right) \}, \]

\[ E_1^* = \left\{ \frac{1}{R_0} \frac{\Lambda}{\mu} - R_0, 0, 0, 0 \right\}, \]

\[ E_2^* = \left\{ \frac{\Lambda}{\mu} - CR_0, CR_0, 0, 0, 0 \right\}, \]

\[ E_3^* = \left\{ \frac{\Lambda}{\mu} + \frac{C}{\mu + \alpha} \frac{(\mu + \alpha)(\alpha - \gamma)}{\mu} - R_0 R_e, C \frac{C}{\mu + \alpha} \frac{R_0 R_e - 1}{\rho}, 0, 0 \right\}, \]

\[ E_4^* = \left\{ \frac{\Lambda N}{\mu (CR_0 + N)} - \frac{\Lambda CR_0}{\mu (CR_0 + N)}, 0, 0, 0 \right\}, \]

\[ E_5^* = \left\{ \frac{N (C \mu (\alpha - \gamma) + \Lambda \rho)}{\mu R e (C q \beta + N (\mu + \gamma))} \frac{1}{R e}, C \frac{C (\Lambda q \beta \rho - \mu (\mu + \gamma) (C q \beta + N (\mu + \alpha)))}{\mu (C q \beta + N (\mu + \gamma))}, 0, 0, 0 \right\}. \] (23)

Six equilibrium states were obtained. The first expression in (23) is interpreted as a state whereby the society will be completely free of terrorism, and the general population alone exists. In summary of result (23), for all dynamical states, the model proved that the act of terrorism can be completely eradicated through the threshold parameters \( R_0 \) and \( R_0 < R_e \).

\[ \lambda_1 = -\mu, \lambda_2 = -(\mu + d), \lambda_3 = -(\eta + \mu + d + y), \lambda_4 = -(q \beta S_F + \alpha + \mu), \lambda_5 = \rho - (\mu + y) \] Clearly, the first four eigenvalues of (24) are found to be negative and the last eigenvalue is found to be negative, \( \lambda_5 = \rho - (\mu + y) < 0 \), which implies that the removal/recovery from ideology must be higher than recruitment; otherwise, the society will not enjoy yearning peace. This is the Routh–Hurwitz criterion for the stability of FFE of the Jacobian matrix \( J \).

For fanatic persistent equilibrium (FPE),
and for simplicity, several assumptions were made on $N$ and $C$ in line with the equilibria stated above. To analyze the stability of persistence of the extremist in the society, the concept of Bellman and Cooke is employed. Next is the statement of the result by Bellman and Cooke cited in [28, 29].

**Theorem 5** (Bellman and Cooke, 1963). Let $H(z) = P(z, e^\lambda)$, where $P(z, w)$ is a polynomial with principal term.

Suppose $H(iy), y \in \mathbb{R}$, is separated into real and imaginary parts:

$$H(iy) = F(y) + iG(y).$$  \hspace{1cm} (26)

If the zeros of $H(y)$ have negative real parts, then the zeros of $F(y)$ and $G(y)$ are real, simple, and alternate, and inequality (24) is satisfied.

$$F(0)G'(0) - G'(0)G(0) > 0, \quad \forall y \in \mathbb{R}. \hspace{1cm} (27)$$

Conversely, all zeros of $H(z)$ will be in the left half plane provided that either of the following conditions is satisfied:

(i) The zeros of $F(y)$ and $G(y)$ are real, simple, and alternate, and inequality (24) is satisfied at least for one $y$.

(ii) All zeros of $F(y)$ are real, and for each zero, relation (24) is satisfied.

(iii) All zeros of $G(y)$ are real, and for each zero, relation (24) is satisfied.

Proof. Since (1) has Jacobian matrix in (22), which has the characteristic polynomial that can be separated into real and imaginary (23), by the hypothesis of the theorem,

$$H(i\lambda) = F(\lambda) + iG(\lambda),$$  \hspace{1cm} (28)

where

$$F(\lambda) = \lambda^3 + \left(\begin{array}{cccc}
F \sigma w - S_t p x - a q \beta + a a q \beta_2 + b a \beta + c a \beta - d q \beta - \mu q \beta - q y \beta + a b + a c & -b + \sqrt{b^2 - 4ac} \\
2a - 2a \mu - a y - b c + b d + 2b \mu + b y + \eta y + c d + 2c \mu + c y - d \mu - \mu^2 - \mu y & \lambda^3
end{array}\right).$$

$$+ (-F S t p q \rho x \beta - F S t d \rho \sigma x - 2 F S t \mu \rho x - F S t p \rho x y + F a \sigma \omega \beta - F q \sigma \omega \beta + F d \rho \omega \beta)$$

$$+ F d \rho \omega \beta + F \mu \sigma \omega \beta + F q \sigma \omega \beta + S t c q \rho x \beta - S t d q \rho x \beta - S t \mu q \rho x \beta$$

$$- S t q \rho x \beta + F a d \sigma w + 2 F a \mu \omega + F a \sigma w y + F d \mu \sigma w + F \mu ^2 \sigma w + F \mu \sigma w y + S t \eta \rho x y$$

$$+ S t c d \rho x + 2 S t c p \rho x - S t p \rho x y - S t d \mu \rho x - S t \mu ^2 \rho x - S t \mu \rho x y - a b \sigma \beta + a b d \rho \beta$$

$$+ a b \mu \beta + a b q \beta + a b q \beta + a d q \beta + a c \mu \beta + a c q \beta + a b q \beta - a d d q \beta.$$
\[
\begin{aligned}
- bc\mu\beta - bcqy\beta - c\eta qx\beta + d\eta qx\beta + \gamma\mu qx\beta + \gamma qxy\beta - ab\eta y - abc d - 2abc \\
- abc y + ab d\mu + abu^2 + abxy + ab\eta y + ac d\mu + acu^2 + ac\mu y - b\eta\mu y - bc d\mu - bc\mu^2 - bc\mu y)\lambda,
\end{aligned}
\]

\[
G(\lambda) = (-\eta \beta - a + b + c - d - 2 - \mu - y)\lambda^4
\]

\[
+ \left( -FS_Fro\sigma x + Fq\omega\beta - S_Fq\rho x\beta + Fa\sigma w \\
+ Fd\sigma w + 2F\mu\sigma w + F\omega wy + S_F\epsilon rpx - S_Fd\mu px - S_F\eta rpx y + abq\beta + acq\beta \\
- adq\beta + a\mu q\beta - aqqy\beta - abq\beta - accq\beta + a dq\beta + a\mu q\beta + aqy\beta - bcaq\beta \\
+ bdaq\beta + b\eta q\beta + b\eta y + c\eta q\beta + c\eta q\beta + c\eta y\beta - abc + abd \\
+ 2abu + aby + ab\eta y + acd + 2acu + acy - a d\mu - a\mu^2 - a\mu y - b\eta y + bcd - 2bc\mu \\
- bcy + b\mu d + b\mu^2 + b\mu y + \eta\mu + cd\mu + c\mu^2 + c\mu y
\right)
\]

\[
\lambda^2 + FS_Fdq\rho x\beta + FS_F\epsilon rpx\beta + FS_F\mu rpx\eta + FS_F\eta rpx - F\mu q\omega\beta - F\mu q\omega\beta \\
- Fa\sigma q\omega\beta + Fa d\eta q\omega - F\mu q\omega\beta + Fa\sigma q\omega\beta - S_Fd\rho x\beta
\]

The characteristic polynomial could simply be written as

\[H(\lambda) = a_0\lambda^5 + a_1\lambda^4 + a_2\lambda^3 + a_3\lambda^2 + a_4\lambda + a_5.\]  \hspace{1cm} (30)

It can be separated into real and imaginary parts as

\[F(\lambda) = H(\lambda) + iG(\lambda),\]  \hspace{1cm} (31)

where \(a_1, a_3,\) and \(a_5\) are the coefficients of \(F(t)\) and \(a_0, a_2,\) and \(a_4\) are the coefficients of \(G(t)\).

Then,

\[F(0)G'(0) - F'(0)G(0) = a_4a_5 > 0,\]  \hspace{1cm} (32)

where

\[
\begin{aligned}
a_4 &= -FS_F\eta q\omega\beta - FS_Fd\mu px - 2FS_F\mu rpx - FS_F\eta rpx y + Fa\sigma q\omega\beta + Fa\sigma q\omega\beta + F d\mu s\omega \\
+ F\mu q\omega\beta + F\omega q\omega y + S_F\epsilon rpx - S_Fd\mu q\eta\beta - S_Fd\rho q\eta x\beta + Fa d\eta s\omega \\
+ 2F\mu s\omega + F\omega w + F d\mu s\omega + F d\mu \sigma w + F\mu \sigma w + F\omega w + S_F\epsilon rpx y + S_Fd\mu px + 2S_F\epsilon rpx \\
+ S_F\epsilon rpx y - S_Fd\mu px - S_F\mu px y - abcq\beta + ab d\mu q\beta + ab\mu q\beta + abqy\beta + abcqy\beta \\
+ ac dq\beta + ac\mu q\beta + acqy\beta + abcq\beta - ab dq\beta + ab\mu q\beta + abqy\beta - abcqy\beta - ac d\mu q\beta \\
- ac\mu q\beta - acqy\beta - b\beta qy\beta - bc d\mu q - bc\mu q\beta - bc\mu q\beta - c\eta qx\beta + d\eta qx\beta + \gamma\mu qx\beta \\
+ ac\mu y - b\eta q\beta - bc d\mu - bc\mu^2 - bc\mu y,
\end{aligned}
\]
and the characteristic equation is

\[ |J - \lambda I| = \lambda^3 + 1.437291938\lambda^2 + 0.7149792438\lambda^3 \\
+ 0.1624877869\lambda^2 + 0.01713983038\lambda \\
+ 0.000676661147 = 0. \]

By the hypothesis of the theorem and in comparison, \( a_4 = 0.000676661147 \) and \( a_5 = 0.01713983038 \). We established that

\[ a_4 \ast a_5 = 1.159786 E - 5 > 0. \]

Also, the persistence equilibrium is stable.

4. Discussion of Results and Conclusion

This paper presents the novel model dynamics of fanatical group progress on terrorism, and it improved the work Chavez and Song [25]. The model conceptualized the mathematical modelling of infectious disease by fanatics, in which extremism is considered as infection of the susceptible group. The model was checked by some mathematical properties, e.g., the existence and uniqueness of solution, positivity, and region of feasibility of solution. This is necessary for merging real-life events with the abstraction of mathematics. The proposed model is analyzed and synthesized using simple algebra and numerical
Table 1: Description of parameters and variables of the model and value used for numerical simulation.

| Parameter/variable | Description                                                                 | Value   | Source |
|--------------------|-----------------------------------------------------------------------------|---------|--------|
| \(\Lambda\)       | Recruitment into the general population (birth/immigration)                  | 500     | Assumed|
| \(\beta\)         | Contact rate of core member with \(G(t)\)                                   | 0–1.0   | Randomness|
| \(\sigma\)        | Per capita recovery rate of \(S(t)\) to \(G(t)\)                           | 0.1     | Assumed|
| \(\rho\)          | The force of embrace of ideology to \(E(t)\), the semi-fanatic class (conversion rate to \(E(t)\)) | 0.2     | Assumed|
| \(\mu\)           | Natural death rate (assumed to be constant)                                | 0.0181  | [30]    |
| \(\sigma\)        | Rate of progression of \(E(t)\) to \(F(t)\)                               | 0.3     | Assumed|
| \(\gamma\)        | Measure of recovery of semi-fanatics to \(G(t)\)                          | 0.1     | Assumed|
| \(q\)             | Successful contact rate to recruitment of members to the class of core group (susceptible) | 0.5     | Estimated|
| \(y\)             | The proportion of survived fanatics returning to their settlement (fanatics) after committing terror | 0.35    | Estimated|
| \(\eta\)          | Proportion of extremists \(F(t)\) that move to commit terror               | 0.4     | Estimated|
| \(d\) or \(\delta\)| Induced death rate due to extremism of ideology                           | 0.05    | Assumed|
| \(G\)             | Noncore group                                                              | 500     | Estimated|
| \(S_F\)           | The naive (susceptible/vulnerable) population to fanatic group             | 85      | Assumed|
| \(F\)             | Individuals who have completely internalized themselves in the ideology (fanatic) | 40      | Assumed|
| \(E\)             | Semi-fanatic group                                                         | 50      | Assumed|
| \(T\)             | Group of extremists who commit terror acts                                 | 60      | Assumed|
| \(N\)             | General population                                                         | 735     | Estimated from [31] |

Figure 3: Continued.
concepts for understanding the model dynamics. The important results obtained include equilibrium points (i.e., equation 23), threshold parameter (equation 18), and stability analyses in equations 24 and 32. All these results are essential for the society to coexist peacefully and help the decision maker gain good insight on the dynamics of extremism for effective management. (result in Section 3.5.1 and Table 2). Figures 3 and 4 discuss the findings from the simulation. Figure 3(a) presents the noncore human population of the model. The variation in the plots is because of introducing at least a fanatic into the population, who interacts and tries to recruit members into the ideology. The percentage increase in the efforts of fanatics ($\beta_2 = 0(0.21)$), which does not yield positive recruitment into fanatics ($\rho = 0$), is shown in Figure 3(a) while Figure 3(b) illustrates the dynamics in the noncore population with the successful effort of fanatics for recruitment ($\beta_2 = \rho = 0(0.21)$) and its effect on the general population. Consequently, there is recruitment into fanatics in proportion to the successful contact made. Thus, this is
alarming and if proactive measures are not taken, the entire world may be in chaos. Figures 4(a)–4(d) show the dynamics of the consequence of successful recruitment into the core population from the general population at different percentages.

Findings revealed the following:

(i) The model of abstraction of the real-life event (terrorism) agrees with the mathematical sense by the mathematical properties investigated.
Six (6) equilibrium points were obtained, which tells the complexity of handling acts of terrorism. (ii) Threshold parameter \( R_0 \) of the core ideological group and effective recruitment \( R_e \) of fanatics were obtained and sensitive for the contemporary society to enjoy peace (equations (16) and (17)).

The model is stable and close to equilibrium points (Table 2).

Numerical simulation supports the analytical solution of the threshold parameters, meaning that for the society to be terrorism free, there is a need to cut the chain or channel of recruitment into extremism.

In conclusion, the study proved that fanatics are the source of terrorism, specifically when the efforts of the fanatics gained successful sympathy of masses and recruitment. Thus, for the contemporary society to enjoy peace, serious effort is required on the class of fanatics to prevent excessiveness. Hence, an appropriate measure and the class of population for which the implementation is required for curbing excessiveness are subjects for further study.

4.1. Recommendation. The model formulation and assumptions in the work are related to Nigeria, and further study should investigate appropriateness of the model for other countries.

It is worthy of recommendation for decision makers and policy implementers to consider and carefully be attentive and proactively respond to any form(s) of innovation in policy implementers to consider and carefully be attentive to the class of population for which the implementation is required for curbing excessiveness are subjects for further study.

Data Availability

No data used for this study except parameters values picked from literature and have been adequately cited.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] K. Bentson, *An Epidemiological Approach to Terrorism*, Biblioscholar, India, 2006.
[2] J. P. Caulkins, D. Grass, G. Feichtinger, and G. Tragler, “Optimizing counter-terror operations: should one fight fire with “fire” or “water”?”, *Computers & Operations Research*, vol. 35, no. 6, pp. 1874–1885, 2008.
[3] A. V. Matveev, “perspective use of modeling for information counter-terrorism,” *The International Journal of the Humanities: Annual Review*, 2016.
[4] D. Grass, J. P. Caulkins, G. Feichtinger, G. Tragler, and D. A. Behrens, *Optim. Control Nonlinear Process. With Appl. Drugs, Corrupt. Terror*, Springer Verlag, Berlin/Heidelberg, Germany, pp. 1–551, 2008.
[5] M. J. Keeling and P. Rohani, *Modeling Infectious Diseases in Humans and Animals*, Princeton University Press, Priceton, NJ, USA, 2008.
[6] H. Weiss, *An Mathematical Introduction to Population Dynamics*, Google Scholar, IMPA, West Neuchada, Edge, 2010.
[7] C. Dym, *Principles of Mathematical Modeling - Google Books*, Elsevier, Amsterdam, Netherlands, 2004.
[8] M. Zhien, *Dynamical Modeling and Analysis of Epidemics*, World Scientific, Singapore, 2009.
[9] C. Castillo-Chavez and H. T. Banks, “Bioterrorism. Mathematical modeling applications in homeland security,” *SIAM Frontiers in Applied Mathematics*, vol. 4, no. 1, 2003.
[10] F. B. I. Fbi “Terrorism —: https://www.fbi.gov/investigate/terrorism.
[11] UNODC, “Counter-terrorism module 2 key issues: drivers of violent extremism,” 2018, https://www.unodc.org/e/sj/en/terrorism/module-2/key-issues/drivers-of-violent-extremism.html.
[12] T. J. Badey, “The role of religion in international terrorism on JSTOR,” 2002, https://www.jstor.org/stable/20832152.
[13] V. Nigeria Supreme Court, “Tega Esabunor and anor. V. FAWEY and OTHERS – judgements,” 2019, https://judgements.lawnigeria.com/2019/04/24/tega-esabunor-and-anor-v-faweya-and-others/.
[14] M. Juergensmeyer, “Does religion cause terrorism?” *Cambridge Companion to Relig. Terror*, Cambridge University Press, Cambridge, MA, USA, pp. 11–22, 2017.
[15] E. Jung, S. Lenhart, S. Lenhart, and Z. Feng, “Optimal control of treatments in a two-strain tuberculosis model,” *Discrete & Continuous Dynamical Systems - B*, vol. 2, no. 4, pp. 473–482, 2002.
[16] L. Nemaranzhe, *A Mathematical Modeling of Optimal Vaccination Strategies in Epidemiology*, UWC Scholar-ETD Repository, UWC Library and retrieved from http://hdl.handle.net/11394/3762, 2010.
[17] E. A. Bakare, A. Nwagwo, and E. Danso-Addo, “Optimal control analysis of an SIR epidemic model with constant recruitment,” *International Journal of Applied Mathematical Research*, vol. 3, no. 3, 2014.
[18] M. Zamir, G. Zaman, and A. S. Alshomrani, “Sensitivity analysis and optimal control of anthroponotic cutaneous leishmanía,” *PLoS One*, vol. 11, no. 8, Article ID e0160513, 2016.
[19] N. Opera, *How A Woman Fought over Blood Transfusion Administered to Her Sick Child - Opera News.
[20] TheGuardian, *When a Parent or One in Loco Parentis Refuses Blood Transfusion for Their Child, the Court Will Step in | the Guardian Nigeria News*, Nigeria and World News — Features — the Guardian Nigeria News – Nigeria World News, Santa Fe, New Mexico, 2019.
[21] BBC News, Nigeria: US "to Name Boko Haram as a Terrorist Group" -BBC News.
[22] E. L. Ochoa, J. F. Camacho, and C. V. D. León, “Qualitative stability analysis of an obesity epidemic model with social contagion,” *Discrete Dynamics in Nature and Society*, vol. 2017, Article ID 1084769, 12 pages, 2017.
[23] R. Resmawan and L. Yahya, “Sensitivity analysis of mathematical model of coronavirus disease (COVID-19) transmission,” *CAUCHY*, vol. 6, no. 2, p. 91, May 2020.
[24] R. Henry, *A Modern Introduction to Differential Equations*, Academic Press, Cambridge, MA, USA, 2021.

[25] C. C. Chavez and B. Song, "7. Models for the transmission dynamics of fanatic behaviors, Bioterrorism: Mathematical Modeling Applications in Homeland Security," in *Frontiers in Applied Mathematics*, Society for Industrial and Applied Mathematics, pp. 155–172, 2003.

[26] C. Okoye, O. C. Collins, and G. C. E. Mbah, "Mathematical approach to the analysis of terrorism dynamics," *Security Journal*, vol. 33, no. 3, pp. 427–438, 2020.

[27] G. Otieno, J. Koske, and J. Mutiso, "Cost effectiveness analysis of optimal malaria control strategies in Kenya," *Mathematics*, vol. 4, no. 1, p. 14, 2016.

[28] A. E. Lekan, A. A. Momoh, A Tahir, and U. M. Modibbo, "On the stability of endemic equilibrium state of HIV/AIDS model with irresponsible infective immigrants," *Pacific J. Sci. Technol.*, vol. 16, no. 2, 2015.

[29] M. Bawa, "Stability analysis of the endemic equilibrium state on the spread of malaria using bellman and cooke’s theorem," *IOSR Journal of Mathematics*, vol. 13, no. 2, pp. 26–33, 2017.

[30] W. Bank, "Life expectancy at birth, total (years)-Nigeria | Data," 2021, https://data.worldbank.org/indicator/SP.DYN.LE00.IN?locations=NG.

[31] MacroTrends, "Nigeria Population 1950-2021 | MacroTrends," 2021, https://www.macrotrends.net/countries/NGA/nigeria/population.

[32] Amnesty International, "Boko haram at a glance - Amnesty International," *Amnesty International*, 2015.

[33] BBC NEWS, *Nigeria’s Boko Haram Attacks in Numbers - as Lethal as Ever - BBC News*, 2018.