HOT BUBBLES IN COOLING FLOW CLUSTERS

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ABSTRACT

As more cooling flow clusters of galaxies with central radio sources are observed with the Chandra and XMM-Newton X-ray observatories, more examples of “bubbles” (low-emission regions in the X-ray coincident with radio emission) are being found. These bubbles are surrounded by bright shells of X-ray emission, and no evidence of current strong shocks has yet been found. Using an analytic approach and some simplifying assumptions, we derive expressions relating the size and location of a bubble, as well as the density contrast between the bubble and the ambient medium, with the shock history of the bubble. These can be applied straightforwardly to new observations. We find that existing observations are consistent with a mild shock occurring in the past and with the bulk of the cool material in the X-ray shells being cooled at the cluster center and then pushed outward by the radio source. Strong shocks are generally ruled out. We also discuss Rayleigh-Taylor instabilities, as well as the case of a bubble expanding into an older bubble produced from a previous cycle of radio activity.

Subject headings: cooling flows — galaxies: clusters: general — intergalactic medium — radio continuum: galaxies — X-rays: galaxies: clusters

1. INTRODUCTION

Chandra high spatial resolution observations of clusters of galaxies reveal the presence of X-ray-deficient bubbles in the inner regions of many cooling flow clusters (Perseus [Abell 426], Fabian et al. 2000, 2002; Hydra A, McNamara et al. 2000; Abell 2052, Blanton et al. 2001, hereafter BSMW; Abell 496, Dupke & White 2002; Abell 2199, Johnstone et al. 2002; MKW 3s, Mazzotta et al. 2002; Abell 2597, McNamara et al. 2001; RBS 797, Schindler et al. 2000; Abell 85, Kempner, Sarazin, & Ricker 2002; Abell 133, Fujita et al. 2002; Abell 4059, Heinz et al. 2002; Virgo 2001; Abell 85, Kempner, Sarazin, & Ricker 2002; Abell 133, Fujita et al. 2002; Abell 4059, Heinz et al. 2002; Virgo 2001; BSMW). The presence of bubbles that do not coincide with strong radio emission (known as “ghost bubbles”) are being found. These bubbles are characterized by low X-ray emissivity, implying low density. In most cases, the bubbles are sites of strong radio emission. There are clusters where the bubbles are less well defined, although there may be hints of their existence (Abell 1795, Fabian et al. 2001; 3C 295, Allen et al. 2001b). The absence of evidence of shocks suggests that the bubbles are expanding and moving at subsonic or mildly transonic velocities (Fabian et al. 2000; McNamara et al. 2000; BSMW). The presence of bubbles that do not coincide with strong radio emission (known as “ghost bubbles” or “ghost cavities”), located farther from the centers of the clusters in Perseus (Fabian et al. 2000), MKW 3s (Mazzotta et al. 2002), and Abell 2597 (McNamara et al. 2001), suggests that the bubbles rise buoyantly.

The discovery of bubbles and their detailed observational study has stimulated theoretical studies of X-ray bubble formation and evolution (Churazov et al. 2001; Nulsen et al. 2002, hereafter N2002; Fabian et al. 2002). N2002 study the origin of the cool gas in the rims of enhanced X-ray emission in Hydra A and particularly address the role of magnetic fields. Although the current paper addresses some questions similar to those addressed by N2002, our analysis and results have only a small overlap with those of N2002. Our discussion is not specific to an individual cluster. We aim to provide simple analytical expressions that can be used for different conditions and evolutionary stages in a variety of cooling flow clusters. We illustrate our results by applying them to several specific clusters. A number of recent papers provide detailed numerical simulations of bubbles (Rizza et al. 2000; Churazov et al. 2001; Brüggen et al. 2002; Quilis, Bower, & Balogh 2001; Saxton, Sutherland, & Bicknell 2001; Reynolds, Heinz, & Begelman 2001; Brüggen & Kaiser 2001). However, it is difficult to generalize the results of these simulations and apply them to other observed cooling flow clusters.

In § 2, we briefly discuss the thermal evolution of shocked gas near the cluster center. Simple expressions for the properties of a bubble inflated by hot plasma (e.g., from a radio jet) are derived in § 3. In § 4, we show that a bubble is stable to Rayleigh-Taylor modes at early stages, becoming unstable only at late stages and in the outer (away from the cluster center) portion. Some properties of “ghost” bubbles (i.e., bubbles at relatively large radii with weak or no radio emission) are discussed in § 5. In § 6, we consider the interaction between bubbles, which we speculate may explain the X-ray and radio structure in the eastern radio “ear” in M87. We discuss and summarize our main results in § 7.

2. SHOCK COMPRESSION AND COOLING

In several of the radio bubbles that have been observed with Chandra, the bubbles are surrounded by relatively thin shells of dense, cool, X-ray-emitting thermal gas. The best cases are probably Perseus (Fabian et al. 2000, 2002) and
Abell 2052 (BSMW). What mechanism has produced these geometrically thin, cool, and dense shells? Below, we argue that the cool, dense gas may have come from farther in, toward the center of the cooling flow. However, simply slowly lifting (since there is no indication for strong shocks) dense material from the inner regions will result in thin shells and filaments having a sharp boundary with their ambient medium, but rather in a continuous, shallow density gradient. Instead, we examine the compression of the shell by a mild shock. For example, weak shock heating has been seen in the gas near the nucleus of the galaxy NGC 4636, which seems to have been heated by a shock expanding from the nucleus (Jones et al. 2002). We assume that the origin of the shock is energy deposited by a central active galactic nucleus (AGN), probably through the action of a jet. We assume that gas shocks and undergoes subsequent expansion as it equilibrates with the ambient gas pressure and rises from the cooling flow center to regions of lower gas pressure. At the same time, the gas cools radiatively.

For simplicity, we assume a planar shock and a $\gamma = 5/3$ gas (neglecting any magnetic field pressure), so that the density and pressure enhancement factors at the shock are

$$\eta \equiv \frac{\rho_1}{\rho_i} = \frac{4.\mathcal{M}^2}{\mathcal{M}^2 + 3}$$

and

$$\alpha \equiv \frac{P_1}{P_i} = \frac{5.\mathcal{M}^2 - 1}{4},$$

respectively. Here, $\rho$ is the gas mass density, $P$ is the pressure, $\mathcal{M}$ is the Mach number of the shock, and the subscripts $i$ and $1$ stand for initial and postshock quantities, respectively, calculated at the initial location at which the gas is shocked, not its present location. The radiative cooling rate, which is assumed to be dominated by thermal bremsstrahlung, is taken to be

$$L = \kappa \rho^2 T^{1/2}.$$  (3)

We define the initial cooling time as

$$\tau_c = \frac{5 P_i}{2 L_i}.$$  (4)

The postshock gas cools radiatively, and its entropy decreases according to

$$\frac{3}{2} P \frac{d}{dt} \ln \left( P P_i^{-5/3} \right) = -L.$$  (5)

We assume that a time $t_1$ has transpired since the gas was shocked and that the gas has been cooling radiatively for this period.

After the shock, the gas pressure drops continuously, as the gas adjusts to the local ambient pressure, and then decreases further as the gas bubble rises out of the cooling flow center into regions of lower ambient pressure. Let $P_a \leq P_i$ be the final ambient pressure of the gas. A variety of different time histories for the gas pressure might be possible. We consider two extreme cases, which maximize or minimize the role of radiative cooling.

We first consider a case that maximizes the importance of radiative cooling. We assume that the postshock gas cools radiatively at a constant pressure $P_1 = \alpha P_i$ for the entire time $t_1$ and then undergoes a very rapid adiabatic expansion to its present pressure $P_a$, on a very short timescale, during which no radiative cooling occurs. This gives the maximum possible cooling, since the gas spends most of its time at the highest density and temperature values, at which radiative cooling is most efficient. This maximal cooling manifests a simple analytical solution of equation (5). The density at the end of time $t_1$, just before the adiabatic expansion, is

$$\rho_f = \left( 1 - \frac{3}{2 \alpha^{1/3}} \right)^{-2/3} \rho_1.$$  (6)

Note that the gas will have cooled completely if $t_1 \geq \tau_{\text{min}}$, where the minimum cooling time is

$$\tau_{\text{min}} = \frac{2 \alpha^{1/2}}{3 \eta^{3/2}} \tau_c.$$  (7)

After the rapid adiabatic cooling (during which we neglect radiative cooling), the ratio of the final shell density $\rho_f$ to that of its present ambient medium $\rho_a$ is

$$\frac{\rho_f}{\rho_a} = \left( \frac{\eta}{\alpha^{3/5}} \right) \left( 1 - \frac{3 \eta^{1/2}}{2 \alpha^{1/3}} \tau_c \right)^{-2/3} \left( \frac{P_i}{P_a} \right)^{-3/5}.$$  (8)

The opposite extreme, which minimizes the effect of cooling, occurs if rapid adiabatic expansion occurs just after the gas is shocked, and then the gas cools radiatively at a pressure of $P_a$. In this case, the gas will cool completely if $t_1 \geq \tau_{\text{max}}$, where

$$\tau_{\text{max}} = \frac{2 \alpha^{9/10}}{3 \eta^{3/2}} \left( \frac{P_i}{P_a} \right)^{2/5} \tau_c,$$

and the ratio of the final shell density to the present ambient density is

$$\frac{\rho_f}{\rho_a} = \left( \frac{\eta}{\alpha^{3/5}} \right) \left[ 1 - \frac{3 \eta^{1/2}}{2 \alpha^{9/10}} \left( \frac{P_i}{P_a} \right)^{-2/5} \left( \frac{P_i}{P_a} \right) \frac{t_1}{\tau_c} \right]^{-2/3} \times \left( \frac{\rho_i}{\rho_a} \right) \left( \frac{P_i}{P_a} \right)^{-3/5}.$$  (9)

As an example, we consider the conditions that might produce the density in the shells around the radio bubbles in Abell 2052, where the ratio of densities is about $\rho_f/\rho_a \approx 2$ (BSMW). The synchrotron age of the radio source in Abell 2052 is estimated to be $t_1 \approx 9 \times 10^8$ yr (Zhao et al. 1993). We first consider the possible role of cooling in creating the dense shells. The Chandra image and spectra imply that the present-day integrated isobaric cooling time is $t_{\text{cool}} \approx 2.6 \times 10^8$ yr (BSMW). However, the theoretical expressions given above are given in terms of the initial instantaneous cooling time $\tau_c$. The integrated isobaric cooling time (the time to cool to zero temperature at constant pressure) for bremsstrahlung cooling is $2 \tau_c/3$. This is related to the present-day integrated isobaric cooling time by

$$\frac{2}{3} \tau_c = t_{\text{cool}} \left( \frac{P_i}{P_a} \right)^{1/2} \left( \frac{\rho_f}{\rho_i} \right)^{3/2}.$$  (11)

Assuming maximal cooling, the cooling time that enters into the cooling term (second term on right-hand side) of equation (8) is the integrated isobaric cooling time after the
shock compression, which is
\[ \tau_{\text{min}} = \frac{2 \alpha^{1/2}}{3 \eta^{1/2}} t_c = t_{\text{cool}} \left( \frac{P_1}{P_a} \right)^{1/2} \left( \frac{\rho_1}{\rho_i} \right)^{3/2}. \] (12)

Using equation (8), this gives
\[ \tau_{\text{min}} = t_{\text{cool}} \left( \frac{P_1}{P_a} \right)^{-2/5} \left( 1 - \frac{t_1}{\tau_{\text{min}}} \right)^{-1}. \] (13)

This equation can be solved to give
\[ \tau_{\text{min}} = t_1 + t_{\text{cool}} \left( \frac{P_1}{P_a} \right)^{-2/5}. \] (14)

Based on the estimated age of the radio source and the observed cooling time in Abell 2052, the observed cooling time in the shells is much greater than the age of the source, \( t_{\text{cool}} \gg t_1 \). Thus, either \( (P_1/P_a) \gg 1 \) or \( \tau_{\text{min}} \sim t_{\text{cool}} \gg t_1 \). In the latter case, the cooling term in equation (8) (second term on right-hand side) is very close to unity, and cooling is not important. Alternatively, if \( (P_1/P_a) \gg 1 \), then we can have \( t_{\text{cool}} \sim t_1 \), and cooling might be important. This implies that the gas underwent a very strong shock, which greatly increased its pressure, and has since undergone a very large adiabatic expansion, which increased the cooling time to the large observed value. For the observed values in Abell 2052, \( t_{\text{cool}} \approx 30 t_1 \). Thus, for cooling to be important, we require that \( (P_1/P_a)^{2/5} \gtrsim 30 \) or \( (P_1/P_a) \gtrsim 4000 \). This pressure ratio can also be written as \( (P_1/P_a) = (P_1/P) (P_1/P_a) \). The second term represents the ratio of the ambient pressure at the initial location of the gas to the ambient pressure at its present location. One expects the radial pressure gradient in a cooling flow to nearly balance the gravitational potential; if the shocked gas was lifted to a larger radius by the expansion of the radio source or by buoyancy, then one expects \( P_i > P_a \). However, unless the gas originated at a much smaller radius (which is unlikely, given the substantial mass of the gas in the shells in Abell 2052; see BSMW and E. L. Blanton et al. 2002, in preparation), one would only expect that \( P_i \leq 2P_a \). Thus, cooling can only be important if the shock increased the pressure on the gas by a large factor, \( (P_1/P) \approx \alpha \gg 2000 \). This requires a large shock Mach number, \( \mathcal{M} \geq 40 \).

To drive a shock with such a large Mach number through the large mass of observed shell gas \( (\sim 5 \times 10^{10} M_\odot) \) in Abell 2052; E. L. Blanton et al. 2002, in preparation), the total energy of the “explosion” must be large \((\gtrsim 10^{62} \text{ ergs})\). If released in less than \( 10^7 \) yr, it implies a power of \( \gtrsim 10^{48} \) ergs s\(^{-1}\), which we consider unrealistically high. In addition, as the shock propagates to outer, lower density regions, it heats these regions to very high temperatures. These lower density regions will not have time to cool; hence, this scenario predicts large regions with very high temperatures just outside the dense shells, contrary to the observations. It would also require a coincidence to have the very strong initial shock and the subsequent radiative cooling nearly cancel one another, leaving the shell gas with densities and temperatures that are close to the ambient values at present. We therefore rule out the very high Mach number (strong shock) scenario.

If cooling is not important, then the high density of the shell material must be the result of shock compression and adiabatic expansion. For either equation (8) or equation (10), the first term on the right-hand side is related to the entropy jump in the shock to a power of \(-3/5\); hence, it is \( \lesssim 1 \). For Mach numbers of \( \mathcal{M} = 2.5, 5, \text{and} 10 \), its value is 0.80, 0.46, and 0.21, respectively. For both equations, the third term on the right-hand side is \( \gg 1 \), since the bubble will expand and rise buoyantly outward to regions of lower density in the cooling flow. However, it is possible that increasing the initial density of the gas will also increase its initial pressure, which would make the fourth term, which is \( \lesssim 1 \), smaller. Note that the third and fourth terms are related to the outward entropy jump to a power of \(-3/5\). Observed cooling flows always have convectively stable (i.e., increasing) entropy gradients, and thus the combination of terms three and four is always \( \gg 1 \). For a very weak shock and a shallow inner pressure profile, the lifted gas will have a density larger than its ambient value by a factor of \( \rho_i/\rho_a \). However, as mentioned above, to form a clear shell or filaments, we require a shock with, say, \( \mathcal{M} = 5 \). This means that the first term on the right-hand side will be \( \approx 1 \). Hence, we require that in Abell 2052, the cool material had an initial density ratio of \( \rho_i/\rho_a \gtrsim 4 \). For a density profile of \( \rho \propto r^{-1} \), this implies that the gas at \( \sim 14 \) kpc was lifted from \( \sim 3.5 \) kpc.

One concern with this simple model is whether there was enough gas at smaller radius to form the observed shells. Using the observed present-day gas density distribution in Abell 2052 (eq. [16] below) and extrapolating into smaller radii than those observed, we find that the total mass of gas within \( 7 \) kpc of the cluster center, where the density is \( \sim 4 \) times higher than on the upper portion of the bubbles, is only about \( 10^{10} M_\odot \), which is smaller than the observed total mass of the two shells in this cluster, \( \sim 5 \times 10^{10} M_\odot \) (E. L. Blanton et al. 2002, in preparation). This discrepancy suggests that there are other factors that have not been taken into account by our simple scenario. One possibility is that the medium was highly inhomogeneous. Inhomogeneity in the cooling flow medium is inferred from other arguments (Fabian 1994). We note in \S 4 below that the interface between the dense shells and the surrounding ambient gas is unstable, and mixing is expected. The mixing of cooler with hotter gas on small scales can lead to heat conduction. The cooler gas is radiating more efficiently, cooling the hotter gas. Therefore, cooling may be more important than what was assumed here for a homogeneous medium. This idea is supported by the strong correlation between the dense X-ray shells and the H\textalpha filaments (BSMW). Also, in an inhomogeneous medium the high-density phase emits more efficiently. Hence, the total mass expected in the shells can be overestimated. This by itself cannot erase the discrepancy we find above, but it can reduce it. Another possibility is that the density profile in the inner regions was much steeper before the inflation of the bubbles. This adds more available mass, and the high-density shell could have been lifted from farther out. If, for example, the dense gas was lifted from \( \sim 10 \) kpc, instead of from farther in, the mass within \( 10 \) kpc of the center would be then \( \sim 3 \times 10^{10} M_\odot \). Considering other uncertainties, a slightly steeper density gradient before the inflation of the bubbles may account for the discrepancy. Another interesting possibility is that the age of the bubbles is larger than the age inferred from the radio emission. However, it cannot be much longer than the buoyant rising time of \( \sim 2 \times 10^7 \) yr (BSMW). It is quite plausible that a few of the factors discussed above act together, accounting
for the mass discrepancy between the observed shells’ mass and the estimate from our simple model.

Finally, we note that the derivation of equation (8) assumes pressure equilibrium between the dense, X-ray-bright shell and the ambient gas. Therefore, a density ratio of ~2 implies a temperature ratio of ~0.5, meaning that the mildly shocked gas (\(n \sim 5\)) is cooler than the ambient gas, as observed.

3. FORMATION OF A BUBBLE

In this section we estimate the size of the bubble inflated by the injection of energy from the central AGN. Recent numerical simulations of bubble evolution can be found in Churazov et al. (2001), Brüggen et al. (2002), Quilis et al. (2001), Saxton et al. (2001), and Brüggen & Kaiser (2001), while relevant recent numerical simulations of jets were conducted by, e.g., Clarke, Harris, & Carilli (1997), Rizza et al. (2000), and Reynolds et al. (2001). The general formation of bubbles from AGNs was studied previously via numerical simulations (see, e.g., Wiita 1978 and Norman et al. 1981), and a detailed analytical study of bubble and jet formation was conducted by Smith et al. (1983). Here we provide approximate analytic expressions (see also Churazov et al. 2000). Although these may be less accurate in any individual case than specific numerical simulations of that system, they are more general and can be applied more easily to new observations.

Although the derivations in this section are general, we scale quantities according to the cooling flow cluster Abell 2052. For the radial variation of the gas pressure and density in Abell 2052, we approximate the results of BSMW (their Fig. 2) by

\[
P(r) \approx 1.7 \times 10^{-10} \left( \frac{r}{30 \text{ kpc}} \right)^{-1.0} \text{ dyn cm}^{-2} \tag{15}
\]

and

\[
\rho(r) \approx 4.2 \times 10^{-26} \left( \frac{r}{30 \text{ kpc}} \right)^{-1.1} \text{ g cm}^{-3}, \tag{16}
\]

where \(r\) is the distance from the cluster center and \(r > 30 \text{ kpc}\). Within 30 kpc, the observed gas structure in Abell 2052 is strongly affected by the bubbles. However, we assume that the same profiles also applied to the interior regions of the cooling flow prior to its disruption by the AGN.

A lower limit on the energy blown into a bubble of radius \(R_b\) is the work it has done on the surrounding medium, \(E_{\text{min}} = \gamma/(\gamma - 1) (4\pi/3) R_b^3 P_e\), where \(P_e\) is the external pressure. Accounting both for the work done by the bubble and its present internal energy will give ~2 times as much energy. For example, if we take the centers of the bubbles in Abell 2052 to be at \(r_{\text{kpc}} = 14\), we crudely estimate \(E_b \approx 3 \times 10^{50}\) ergs.

In the specific numerical examples we give below, we assume that the main pressure support and internal energy in the bubble are due to very hot, nonrelativistic thermal gas, rather than to magnetic fields or relativistic particles. The results are essentially unchanged if the bubbles are supported by relativistic gas, by tangled magnetic fields, or by some mix of relativistic gas, nonrelativistic gas, and magnetic fields. The dependence of the bubble size on other parameters is identical, and the numerical values are changed by less than 8%. If the bubbles are supported by relativistic gas, the bubbles are a factor of \((11/16)^{1/3} = 0.93\) smaller, while if the bubbles are supported by magnetic fields, they are a factor of \((33/28)^{1/3} = 1.03\) larger. In Abell 2052 and Perseus, the minimum radio pressure in the bubble is about a factor of 10 smaller than the external pressure (Fabian et al. 2000; BSMW). This suggests that magnetic fields and relativistic electrons may not be the main source of pressure. The Chandra spectra of the bubbles in Abell 2052 are consistent with hot gas with a temperature \(\gtrsim 5\) keV providing the pressure in the bubbles (E. L. Blanton et al. 2002, in preparation). We assume that the very hot gas inside the bubbles comes from the central AGN, via jets, and does not contain much shocked intracluster gas. The high temperature is assumed to result from a strong termination shock at the end of the radio jet.

For simplicity, we divide the evolution of the bubble into two phases: an energy injection phase, lasting a time of \(\tau_i\), and a later phase, when the energy injection rate is much lower and is assumed to be zero. Let \(r\) be the total lifetime of the bubble from the start of the energy injection phase. We assume that the bubble undergoes spherical expansion into a constant-density medium. We take this constant density \(\rho_c\) to be the value in the undisturbed cooling flow at the distance \(r_c\) from the center of the cooling flow to the center of the bubble, \(\rho_c = \rho(r_c)\). This might be a good approximation if the radius of the bubble \(R_b\) were a small fraction of \(r_c\), and if the bubble were not buoyant. Most of the observed bubbles have \(R_b \sim r_c\). As a result, the gas density in the ambient medium should vary with position around the bubble. Also, the bubble will both expand and rise buoyantly outward, and this also should cause a variation in the ambient density around the bubble. As the bubble rises and expands, it will lift dense material from the very inner regions of the cooling flow (~1 kpc); the shell will not consist solely of material originating from its present-day center at \(r_c \sim 14\) kpc. The other major approximation we make is to ignore the thermal pressure of the ambient medium exterior to the bubble. This may be a good assumption at early stages, but not at late stages, when the Mach number of expansion of the bubble is small. Small Mach numbers at the present time are implied by the lack of clear shock structures surrounding the observed bubbles (Fabian et al. 2000; BSMW).

We use the expression given by Castor, McCray, & Weaver (1975) for the expansion of an interstellar bubble; Bicknell & Begelman (1996) also applied this expression to study the bubble formed by the radio jet in M87, and Churazov et al. (2000) used it to derive the parameters of bubbles in the Perseus Cluster. Scaling to a constant density \(\rho_c\) appropriate for \(r_c \sim 5\) kpc in Abell 2052, the radius of the bubble under these assumptions is given by

\[
R_b \simeq 7.8 \left( \frac{r}{10^7 \text{ yr}} \right)^{3/5} \left( \frac{\dot{E}}{10^{44} \text{ ergs s}^{-1}} \right)^{1/5} \left( \frac{\rho_c}{10^{-25} \text{ g cm}^{-3}} \right)^{-1/5} \text{kpc}. \tag{17}
\]

The energy injection rate used here is similar to that determined for the Perseus Cluster, where Fabian et al. (2002) constrain the jet power to be between \(10^{44}\) and \(10^{45}\) ergs s\(^{-1}\). The calculation above neglects the effect of the ambient pressure on the bubble; therefore, it overestimates the radius at late evolutionary stages. Comparing
with the results of Brüggen et al. (2002), we find that we overestimate the radius of the bubble by 70% for the case \( E = 4.4 \times 10^{41} \) ergs s\(^{-1}\) at \( t = 8.56 \times 10^{6} \) yr (their Fig. 2), and by 45% for the case \( E = 3.8 \times 10^{42} \) ergs s\(^{-1}\) at \( t = 1.25 \times 10^{7} \) yr (their Fig. 4). For the energetic case of \( E = 10^{44} \) ergs s\(^{-1}\) at \( t = 5 \times 10^{6} \) yr, on the other hand, equation (17) gives the same radius as their simulation (their Fig. 5). Thus, equation (17) is a good approximation within the expected limit.

In the subsequent expressions for the bubble evolution, it is useful to replace the energy deposition rate \( E \) with \( E_b/\tau_l \) and to replace the age of the bubble \( t \) with its radius \( R_b \). The expansion velocity of the bubble surface as a function of its radius is given by

\[
v_b(R_b) \approx 570\left(\frac{\tau_l}{10^7 \text{ yr}}\right)^{-1/3}\left(\frac{E_b}{10^{59} \text{ ergs}}\right)^{1/3}
\times\left(\frac{\rho_c}{10^{-25} \text{ g cm}^{-3}}\right)^{-1/3}\left(\frac{R_b}{10 \text{ kpc}}\right)^{-2/3} \text{ km s}^{-1}.
\]

(18)

At the end of the injection phase, the bubble radius is

\[
R_b(\tau_l) \approx 9.8\left(\frac{\tau_l}{10^7 \text{ yr}}\right)^{2/5}\left(\frac{E_b}{10^{59} \text{ ergs}}\right)^{1/5}
\times\left(\frac{\rho_c}{10^{-25} \text{ g cm}^{-3}}\right)^{-1/5} \text{ kpc ,}
\]

(19)

and its surface expansion velocity at the end of the energy injection phase is

\[
v_b(\tau_l) \approx 580\left(\frac{\tau_l}{10^7 \text{ yr}}\right)^{-3/5}\left(\frac{E_b}{10^{59} \text{ ergs}}\right)^{1/5}
\times\left(\frac{\rho_c}{10^{-25} \text{ g cm}^{-3}}\right)^{-1/5} \text{ km s}^{-1}.
\]

(20)

The expressions we use for the bubble expansion require that it be highly supersonic, which implies \( v_b \gg c_L \approx 820 \text{ km s}^{-1} \) for a temperature of \( 3 \times 10^7 \) K. Thus, the expressions we give will become inaccurate when \( R_b \gtrsim 10 \text{ kpc} \).

4. RAYLEIGH-TAYLOR INSTABILITY AND BUBBLES

The deceleration of the bubble surface for \( t \leq \tau_l \) is given by \( a_b \equiv v_b = - (6/25) R_b \tau_l \). Written as a function of time (for \( t \leq \tau_l \)), the deceleration is

\[
a_b \approx - 7.3 \times 10^{-8}\left(\frac{\tau_l}{10^7 \text{ yr}}\right)^{-1/5}\left(\frac{E_b}{10^{59} \text{ ergs}}\right)^{1/5}
\times\left(\frac{\rho_c}{10^{-25} \text{ g cm}^{-3}}\right)^{-1/5}\left(\frac{t}{10^7 \text{ yr}}\right)^{-7/5} \text{ cm s}^{-1}.
\]

(21)

while as a function of the bubble radius, it is

\[
a_b \approx - 7.7 \times 10^{-8}\left(\frac{\tau_l}{10^7 \text{ yr}}\right)^{-2/3}\left(\frac{E_b}{10^{59} \text{ ergs}}\right)^{2/3}
\times\left(\frac{\rho_c}{10^{-25} \text{ g cm}^{-3}}\right)^{-2/3}\left(\frac{R_b}{10 \text{ kpc}}\right)^{-7/3} \text{ cm s}^{-1}.
\]

(22)

This deceleration (or negative acceleration) is equivalent to a gravitational acceleration in the outward radial direction (or a positive gravitational acceleration \( g \)). We determine the gravitational acceleration from the assumption of hydrostatic equilibrium,

\[
g = \frac{1}{\rho(r)} \frac{dP(r)}{dr}.
\]

(23)

For the pressure and density profiles for the gas in Abell 2052 (eqs. [15] and [16]), the acceleration is

\[
g \simeq - P/\rho \simeq - 1.2 \times 10^{-7}\left(\frac{r}{10 \text{ kpc}}\right)^{-0.9} \text{ cm s}^{-2}.
\]

(24)

For bubbles inflated about a center that is not the cluster center, the least stable portion of the bubble is likely to be its outermost region. Here the dense shell lies above the low-density interior of the bubble. Let \( r_c \) be the radius of the center of the bubble; this might correspond to the point at which a jet from the center was stopped in a termination shock. As before, \( R_b \) is the radius of the bubble (measured from \( r_c \)). Then the radius of the outermost part of the bubble is \( r = r_c + R_b \). If we use equation (16) for the density profile in Abell 2052, we find that for

\[
r = r_c + R_b \lesssim 15\left(\frac{\tau_l}{10^7 \text{ yr}}\right)^{0.4}\left(\frac{E_b}{10^{59} \text{ ergs}}\right)^{-0.4}
\times\left(\frac{R_b}{10 \text{ kpc}}\right)^{1.4} \text{ kpc ,}
\]

(25)

the magnitude of the gravitational acceleration exceeds that of the deceleration, and the interaction between the low-density bubble and the dense, X-ray–bright shell is Rayleigh-Taylor (RT) unstable. With the scaling used above, we find that the expansion of the bubble is RT stable as long as the bubble radius is \( R_b \lesssim 4, 6, \) or \( 8 \) kpc, for \( r_c = 0, 1, \) and \( 3 \) kpc, respectively. For a larger energy \( E_b \), the shell remains stable to later times; for example, for \( E_b = 2 \times 10^{59} \) ergs and \( r_c = 0 \), the shell is stable as long as \( R_b \lesssim 8 \) kpc. At late stages, the shell becomes unstable. However, in our treatment of the bubble expansion, we neglected the ambient pressure (we assumed highly supersonic expansion). At late stages, the ambient pressure will increase the deceleration of the shell, making it more stable against RT modes. We conclude that the shell becomes RT unstable only when it is large and only on its outermost segment. Along other segments, the gravitational acceleration component perpendicular to the interface is lower, and the expansion is RT stable until the bubble almost stops expanding.

The growth time for RT instabilities (the \( \epsilon \)-folding time) is given by \( \tau_{RT} = |g_e k|^{-1/2} \), where \( \lambda \) and \( k = 2\pi/\lambda \) are the wavelength and wavenumber of the mode being considered. The effective gravitational acceleration, \( g_e \), at the outer part of the shell is \( g_e = |g| - |a_b| \). For disruption of the entire bubble, we consider a large-scale mode with \( \lambda = 2 R_b \). The timescale for the growth of this mode is

\[
\tau_{RT} = 1 \times 10^7\left(\frac{R_b}{10 \text{ kpc}}\right)^{1/2}\left(\frac{g_e}{1 \times 10^7 \text{ cm s}^{-2}}\right)^{-1/2}\text{ yr}.
\]

(26)

The growth time for RT instabilities, as given above, is comparable to the estimated age of the bubbles; for ex-
ample, in Abell 2052 the estimated age of the radio bubbles is about $9 \times 10^6$ yr (BSMW). This suggests that in most cases the bubble will be stable during its inflation phase. In other cases the bubbles will start to be fragmented, but only in the outer regions. This is in reasonable agreement with the fact that the bubbles in Abell 2052 are fairly complete, except for possible gaps at their outer edges (the north side for the north bubble and the south side for the south bubble). Considering that the timescale given by equation (26) is only for the outer segments of the bubbles, and that magnetic fields can suppress the growth of RT instabilities (particularly at small scales), a bubble may be able to remain relatively intact for a longer period of time. Evidence that bubbles do survive for $\sim 10^8$ yr comes from the “ghost bubbles” seen in Perseus (Fabian et al. 2000) and Abell 2597 (McNamara et al. 2001). It may be that these ghost bubbles were produced under different circumstances (e.g., stronger magnetic fields) than the bubbles in Abell 2052, which appear to be starting to be disrupted. However, one still might expect prominent structures due to RT instabilities to appear at $\sim (1-3) \times 10^7$ yr ($\S$ 5).

At the outer edge of the dense shell, there is a significant drop in the density, and this interface would be unstable when the inner discontinuity is stable. However, at early times, when the bubbles are small, they are expected to expand supersonically (eq. [18]), and the outer density discontinuity is the result of a shock, rather than merely a contact discontinuity.

5. GHOST BUBBLES

There are several cases of outer, radio-faint, isolated bubbles, or “ghost bubbles,” including those in the Perseus Cluster (Fabian et al. 2000, 2002), Abell 2597 (McNamara et al. 2001), and possibly MKW 3s (Mazzotta et al. 2002). These bubbles are believed to have risen outward buoyantly. The bubbles are X-ray–faint, and no trail of bright X-ray material is observed behind them. The two outer bubbles in Perseus are isolated and have well-defined shapes, which are elongated in the azimuthal direction. For the northwestern and southern bubbles, the distances from the center of the cluster to the center of the bubbles are 1.5 and 2 times their widths, respectively. We assume that the line connecting the bubble centers to the center of the cluster is inclined by $\sim 45^\circ$ to our line of sight, so that the bubbles are actually at distances that are $\sim 3$ times their diameter.

Churazov et al. (2001) give the rising velocity of a buoyant spherical bubble as $v_\nu \simeq (8\rho R_b/3C)^{1/2}$, where the drag coefficient is $C \simeq 0.75$. Multiplying this velocity by the RT e-folding time, we find that the distance $D$ for which a bubble will be buoyant before starting to be disrupted in an observationally noticeable way is $r \simeq R_b$. Hence, a bubble will not rise much before being disrupted by RT instability modes. If the bubbles in Perseus were inflated to their diameters, as in Abell 2052, and then rose outward because of buoyancy to a total distance of $\sim 3$ times their diameter, their age must be $\sim 4-5$ times the growth time of large-scale RT modes. Thus, we would expect these bubbles to have undergone considerable disruption. Indeed, carefully examining the available images of the pair of outer bubbles in Perseus (Fabian et al. 2000, 2002) we identify what seems to be an RT protrusion in the northwest bubble and some other irregularities in the southern bubble. We attribute these to developed RT instability modes. We also attribute the elongation of the bubbles in the azimuthal direction to RT instabilities; this cannot be a result of the bubble reaching a radius at which its density equals the ambient density, since in that case the interior of the bubbles would not be X-ray–faint.

The presence of bubbles even farther from the centers of clusters can be explained in two ways. First, magnetic fields may suppress RT instabilities. Fabian et al. (2002) argued that magnetic suppression of RT instabilities was needed to explain the sharp edges in the outer bubbles in Perseus. However, the suppression is efficient only for short wavelengths, and not for long-wavelength modes that disrupt the bubble. Second, it is possible that the bubbles were formed farther from the center, by a jet expanding outward. An example of this process in action might be Cygnus A (Smith et al. 2002). However, Cygnus A is somewhat unusual, as it is the most luminous radio source in the nearby universe. In most cases, radio jets do not propagate to large distances in cooling flow clusters. Thus, we do not expect to find large bubbles beyond a radius of $\sim 50$ kpc.

6. INTERACTION OF BUBBLES

The existence of ghost bubbles in clusters with more centrally located, radio-bright bubbles or lobes (Fabian et al. 2000, 2002; McNamara et al. 2001) suggests that the radio sources at the centers of cooling flows are episodic. Thus, we now consider the interaction of a new radio bubble with a previously inflated ghost bubble. Assume that a currently expanding bubble runs into a small, low-density cavity due to a ghost bubble. A small portion of solid angle, $\Omega \ll 4\pi$, of the bubble’s surface is assumed to enter the cavity (ghost bubble). We call the portion of the dense bubble’s shell that enters the cavity a “blob.” We further assume that the ghost bubble is elongated in the radial direction, such that the cross section stays constant; such a bubble is not like the bubbles observed in Perseus, but more like one left by a radio jet. We also neglect the pressure inside the cavity, and since we assume that the volume of the cavity is small, we neglect the decrease in the thermal pressure inside the current bubble due to the expansion of its thermal gas to the volume of the cavity. The effects we neglect here reduce the efficiency of the acceleration process. However, later we neglect effects that increase the efficiency of the proposed scenario.

At the collision time $t_c$, measured from the birth of the active bubble, the bubble radius is $R_c$. As in $\S$ 3, we assume a constant energy injection rate into the bubble, and the bubble expands inside a medium of constant density $\rho_c$. The radius of the bubble as a function of time is given by equation (17). The pressure $P$ just inside the bubble acts on its surface, such that the momentum changes according to $d(Mvb)/dt = 4\pi R_b^3 P$, where $M = 4\pi R_b^3 \rho_c/3$ is the mass of the dense shell on the surface of the bubble and $v_b = dR_b/dt = (3/5)R_b/t$ is its expansion velocity. The portion of the bubble’s surface that enters into the low-density cavity has a mass $M_{bl} = (\Omega/4\pi) M$, which stays constant for times $t > t_c$. The rate of change of the momentum of that mass is $d(M_{bl}v_{bl})/dt = \Omega R_b^2 P$. From the expression for the rate of change in momentum for the bubble and the blob (the portion running into the cavity), we find

$$M_{bl} \frac{dv_{bl}}{dt} = \frac{\Omega}{4\pi} \frac{d(Mvb)}{dt}. \tag{27}$$
From the expressions for $M$ and $R_b$, we find that $dM/dt = 4\pi R_b^2 v_b p_c$, and $dv_b/dt = -(6/25) R_b^2$. Substituting these and the expression for $M_{bl}$ in equation (27), we get

$$\frac{dv_{bl}}{dt} = \frac{21 R_b}{25 R_c^2},$$

with the initial condition $v_{bl} = v_c = (3/5) R_c/t_c$ at $t = t_c$.

Using the dependence of $R_b$ on $t$, the last equation can be integrated analytically to give

$$v_{bl} = \frac{9}{2} v_c \left[ 1 - \frac{7}{9} \left( \frac{t}{t_c} \right)^{-2/5} \right],$$

where $v_c = (3/5) R_c/t_c$ is the velocity of the shell and blob at the time of the collision, $t = t_c$. The last equation can be integrated analytically to give the distance of the blob from the center of the bubble,

$$\frac{R_{bl}}{R_c} = \frac{9}{5} \sqrt{\frac{7}{2}} \left( \frac{t}{t_c} \right)^{3/5} + \frac{27}{10 t_c} \quad t \geq t_c.$$

The idealized picture that the bubble continues to expand undisturbed breaks down as evolution proceeds. But for a time, $t \sim 2t_c$, we can assume that it is adequate, and we find $v_{bl} \sim 1.85 v_c = 2.4 R_b$, where $v_b$ is the expansion speed of the bubble surface at $t = 2t_c$. During that time, the bubble has expanded by a factor of $2^{3/5} = 1.52$, i.e., $R_b = 1.52 R_c$, while the blob is at a distance of $R_{bl} = 1.89 R_c = 1.25 R_b$ from the center of the bubble. The blob is accelerated quite efficiently and can reach a relatively high velocity. If the bubble surface velocity was originally $v_c = 400$ km s$^{-1}$, then the blob can have $v_{bl} \sim 800$ km s$^{-1}$.

Eventually, the accelerated blob will leave the cavity, enter a high-density region, and be decelerated as a result of the drag with its surroundings and gravity. We neglect the buoyant force and assume that the acceleration by the high-pressure bubble interior has ceased; both effects, if included, will lower the deceleration and make the blob reach larger distances. From Churazov et al. (2001), we find the gravitational acceleration in M87 to be $g = -5 \times 10^{-8} (r/10$ kpc $)^{-1/2}$ cm s$^{-2}$, where $r$ is the distance from the cluster center. The drag force can be written as $F_D = 0.5 S C S v_{bl}^2 p_a$, where $C \approx 0.75$ is the drag coefficient, $S$ the cross section of the blob, $v_{bl}$ its velocity, and $p_a$ the ambient density. We treat the blob as a cylinder of length $l$ and radius $R$ whose axis of symmetry is parallel to the direction of motion. Its density is $p_{bl}$, so that its mass is $M_{bl} = p_{bl} \pi R^2 l$. The drag deceleration is then $a_D = -F_D/M_{bl} \approx 0.4 (v_{bl}^2 / l) (p_a / p_{bl})$. Scaling the variables, the equation of motion for the blob becomes

$$\frac{d^2 r}{dt^2} \approx -5 \times 10^{-8} \left[ \left( \frac{r}{10 \text{ kpc}} \right)^{-1/2} + \left( \frac{l}{5 \text{ kpc}} \right)^{-1} \right] \times \left( \frac{v}{800 \text{ km s}^{-1}} \right) \left( \frac{p_a}{0.3 p_{bl}} \right) \text{ cm s}^{-2}.$$

We solved this equation numerically for a blob leaving the ghost bubble cavity and entering the high-density medium at $r = 10$ kpc. The outward velocity of the blob is 800 km s$^{-1}$ at $r = 10$ kpc. In Figure 1, we plot $r$ as function of time for three sets of parameters. The figure shows that a blob can reach relatively large distances from the cluster center in reasonably short times.

We propose that such a blob, produced by the interaction between bubbles, may explain the X-ray and radio structure in the eastern radio “ear” in M87 (Belsole et al. 2001; Churazov et al. 2001 and references therein). Unlike bubbles in other cooling flow clusters, there is a positive correlation between X-ray and radio emission on the long, jetlike feature connecting the center of M87 with the eastern radio ear (lobe; see, e.g., Churazov et al. 2001). Churazov et al. (2001; see also Reynolds, Heinz, & Begelman 2002) suggest that the eastern radio lobe of M87 is a buoyant bubble that drags dense material with it, giving the positive correlation between the X-ray and radio emission. Saxton et al. (2001) suggest a similar explanation for the northern middle lobe in the moderate cooling flow (Allen et al. 2001a) in the Centaurus Cluster (Abell 3526). Both these papers present gas-dynamical simulations of buoyant bubbles. Their model seems to be unable to explain all cases, e.g., the southwestern lobe in Hydra A (N2002). That model may have problems even in M87, its prime target. The general X-ray structure of M87, as revealed in Chandra archive data (Young et al. 2002; see also XMM-Newton observations by Belsole et al. 2001), does not resemble the structure Churazov et al. (2001) find in their numerical simulations. Indeed, in a recent paper, Brüggen et al. (2002) find in their numerical simulations that the X-ray emissivity along the simulated jet is lower than the ambient emissivity, which contradicts the explanation of Churazov et al. (2001) and Saxton et al. (2001). We here propose a tentative scenario, in which the dense X-ray material was ejected from the central region of M87 as a bullet. The jet, which formed the ear, formed the cavity through which dense X-ray material (the bullet) expanded outward. The distance of the ear from the cluster center is ~20 kpc, which can easily be reached by the bullet (see Fig. 1). The idealized bullet, in the calculation presented here, is a stream of dense X-ray material pushed by an inner bubble. Inspecting Chandra images (Young et
6. DISCUSSION AND SUMMARY

We derived simple expressions that can be used to analyze some properties of X-ray–deficient bubbles in clusters of galaxies. We demonstrated their application for Abell 2052 and Perseus. We also addressed the X-ray and radio structure to the east of the center of M87.

Our main results can be summarized as follows.

1. The cool material that forms the X-ray–bright shells around the X-ray–deficient radio bubbles in the cooling flow cluster Abell 2052, and very likely in other cooling flow clusters (e.g., Perseus, Fabian et al. 2002; Hydra A, N2002), was lifted from the very inner regions of the cooling flow, rather than being shocked and then cooled substantially because of shock compression. The material went through a shock, but a mild one, the main effect of which was to shape the material into a dense shell around the radio bubble. To achieve the present shell-to-ambient density ratio of $\sim 2$, the dense gas had to have an initial density $\sim 4$ times its present ambient density. This is quite plausible for dense gas lifted from $\sim 5$–10 kpc. One problem with this model is that the observed mass in the shells in Abell 2052 is somewhat larger than the expected mass in these inner regions, determined by extrapolating the observed gas density profile into the center. We suggest that a steeper density gradient in the inner region prior to the bubble inflation and/or inhomogeneities of the shell and ambient medium can account for this mass discrepancy in our simple model.

2. The presence of cool material in the X-ray shells surrounding the bubbles has implications for a possible strong AGN shock propagating from the cluster center at the onset of the radio activity. Such a shock is not observed presently in any cluster, but may have a duty cycle on a long timescale. Soker et al. (2001) proposed that such strong intermittent shocks, with shock velocities $\gtrsim 7000$ km s$^{-1}$, may result in much lower mass-cooling rates. Suppose, for example, the presently cool shell was at an initial temperature of $\sim 10^7$ K and then was hit by a strong shock with a speed of $\sim 7000$ km s$^{-1}$ (a Mach number \(M \sim 15\)). From equations (1), (2), and (8), we find that the gas was shocked to a temperature of $\sim 6 \times 10^8$ K, and then cooled adiabatically (the radiative cooling will be negligible during a time of $< 10^8$ yr) to $\sim 7 \times 10^7$ K. This is still too hot to match the presently observed dense shell. We therefore rule out such a scenario. Cases with even higher Mach numbers were ruled out in § 2. We conclude that the presence of massive dense and cool shells precludes that the radio activity was initiated by a very strong shock (if the age of the present radio activity is $\lesssim 10^8$ yr).

3. From observations, it appears that the inflation of bubbles by AGN activity in cooling flow clusters is a general phenomenon. Our estimate, via simple approximation of the bubble size, explains this as a result of the insensitivity of the bubble size (eqs. [17] and [19]) to the injection energy rate and ambient density. Therefore, a bubble will be formed with a size of several kpc for most reasonable levels of AGN activity.

4. During the early, inflated phase, i.e., while energy is injected by the AGN, the interface between the hot bubble and the dense shell is Rayleigh–Taylor (RT) stable, explaining the smooth-surface bubbles close to the center of cooling flow clusters (e.g., Perseus). Later, the upper segment of the shell becomes unstable, and the shell may break up there (e.g., the northern shell in Abell 2052). The interface between the dense shell and the outer, lower density medium is RT unstable, but the density ratio is close to unity, and, considering some stress and friction, the instability evolves slowly. It is quite possible that the dense shell will start to be broken up by these later RT–unstable modes. After these bubbles have been buoyant to about twice their diameter, the interface between the low-density bubbles and the outer regions becomes RT unstable, and large RT fingers with sizes not much smaller than the bubble size may be observed penetrating the bubbles. The outer bubbles in Perseus seem to be disrupted by RT instabilities.

5. We study the case of a bubble expanding into an older bubble. The portion of the still-active bubble that enters the low-density cavity formed by the older bubble will be accelerated to higher velocities, up to $\sim 2$ times the original expansion velocity of the bubble. These quickly moving blobs may travel out to radii of 20–30 kpc in reasonable times. We speculate that such high-density blobs may explain the high-density X-ray–emitting regions along the radio structures in M87. In M87, the cavity was formed by a radio jet, which formed the eastern ear, and the blobs created a stream of dense X-ray–emitting gas.

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