Theia: science cases and mission profiles for high precision astrometry in the future

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ABSTRACT

High-precision astrometry well beyond the capacities of Gaia will provide a unique way to achieve astrophysical breakthroughs, in particular on the nature of dark matter, and a complete survey of nearby habitable exoplanets. In this contribution, we present the scientific cases that require a flexibly-pointing instrument capable of high astrometric accuracy and we review the best mission profiles that can achieve such observations with the current space technology as well as within the boundary conditions defined by space agencies. We also describe the way the differential astrometric measurement is made using reference stars within the field. We show that the ultimate accuracy can be met without drastic constrains on the telescope stability.

Keywords: astronomy, astrophysics, dark matter, exoplanet, astrometry, differential, visible, high precision, space mission

1. INTRODUCTION

ESA’s \textit{Hipparcos} and \textit{Gaia} global astrometry scanning missions have revolutionized our view of the Solar Neighborhood and Milky Way, and provided crucial, new foundations for many disciplines of astronomy. The topic of high precision astrometry arose from the \textit{Space Interferometry Mission (SIM)} in the late 2000’s in order to detect Earth-like exoplanets but keeping the methodology to achieve absolute astrometry. Following the dismissal of SIM by NASA in 2010, a small team decided to propose another concept to address high precision astrometry called \textit{Nearby Earth astrometry Telescope (NEAT)}. NEAT consisting of a single off-axis mirror sending the light onto a detector either in a formation flight configuration\textsuperscript{1} was presented for the M3 call for missions at ESA. Then the \textit{Theia} concept with a single spacecraft carrying a Korsch three-mirror anastigmatic (TMA) telescope, a single focal plane and instrument metrology subsystems has been studied and submitted without success to M4, M5 and M7 calls for medium missions at ESA. We present here a summary of the current concept status for \textit{Theia} as it has been submitted for the phase 1 call for M7 call for missions.

The paper will quickly review the Science case, then present the mission and the management structure before exploring in more details how the most accurate measurements can be achieved using reference stars.

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Differential astrometry can push the boundaries of our knowledge further, as with this approach it is possible to achieve a degree of relative positional precision in small fields (typically $\leq 1^\circ$) vastly exceeding the levels obtained by a 10-yr Gaia mission. This opens the door to a) the identification of amplitudes of astrometric variability phenomena in bright stars and b) to the determination of relative proper motions for faint stars with a precision entirely out of the reach of state-of-the-art absolute astrometry.

Ultra-high precision relative astrometry is identified as one of the themes for a possible M-size mission in the final report of the senior committee of ESA’s Voyage 2050 long-term scientific plan. The Theia micro-arsecond astrometric observatory is poised to achieve unprecedented relative astrometric precision in small fields$^*$ that will bring breakthroughs on some of the most critical questions of cosmology, exoplanetary science, and particle physics. The details on the science cases have been described in the White Paper published$^2$ for the Voyage 2050 report.

### 2.1 Dark Matter

While the mass density of the Universe is dominated by dark matter, we know little of its nature, especially because dark matter particles have not yet been observed in large accelerators (LHC). Theia has been designed to test the standard picture of collisionless, cold dark matter, in the following ways:

- **Test the cold nature of DM particles.** Cosmological simulations of cold DM indicate that galaxy halos are full of subhalos$^3$. Extrapolating the observed trends of $M_{\text{dyn}}/L$ vs. $L$ to lower mass, subhalos below $10^8 M_\odot$ should be very or completely dark. N-body simulations indicate that subhalos passing through the disk affect the kinematics of the stars in the neighborhood out to several kpc, leaving waves that persist for up to several hundred Myr$^2$.$^4$. The proper motions measured by Theia at a dozen locations, located just above and below the galactic disk, should allow the first detection of kinematic imprints for several dark subhalos$^2$, which would be a major breakthrough.

- **Test the collisionless nature of Dark Matter.** Cosmological simulations of DM particles indicate that structures (halos) have steep (“cuspy”) inner density profiles$^5$. This has never been verified in galaxies, whose intermittent outflows from supernovae explosions and active galactic nuclei shake the inner gravitational potential (where the gas dominates), causing violent relaxation that rapidly leads the DM particles to settle to a more homogeneous inner density profile$^6$. Dwarf spheroidal galaxies (dSphs) around the Milky Way (except the three most massive) are highly dominated by DM and should thus have cuspy density profiles. Analysis of proper motions adds two dimensions to the phase space probed by only using redshifts. Tests on mock data shows that this dramatically improves the measure of the inner slope of the density profile of dSphs$^2$.$^7$. If Theia found cuspy inner density profiles in the targeted dSphs, this would confirm that DM is collisionless. Conversely, if Theia found cores in the dSphs, this would provide the first measurement on the cross-section of self-interaction, a major breakthrough in particle physics.

- **Determine the shape of the outer halo of the Milky Way.** Cosmological simulations of cold DM indicate$^8$ that galaxy halos have prolate outer shapes (while the stellar disks are oblate). Theia will measure the proper motions of several distant known hyper-velocity stars thought to have received huge velocity kicks in three-body interactions involving the supermassive black hole at the center of the Milky Way$^9$. These measurements combined with redshifts should recover the 3D directions of the motion$^2$ allowing to determine the halo axis ratios to 5%, which is beyond the scope of Gaia.

These breakthroughs on dark matter will open up new directions in Astrophysics, thus helping us understand the origin and composition of the Universe.

$^*$None of the scientific aims listed below require absolute astrometry.
2.2 Early Universe

Theia’s precise astrometry will allow to probe the primordial Universe in complementary ways.

- **Test inflationary models and primordial black holes.** The power spectrum of primordial density fluctuations, which carries imprints from the initial inflation phase, is now well known on intermediate to large scales, thanks to the ESA’s Planck cosmic microwave background mission, as well as large-scale surveys of galaxies. But it is poorly known at small scales (below 2 Mpc). Very high peaks from non-gaussian density fluctuations produced at phase transitions (e.g., QCD) or by features in the inflation potential collapse into primordial black holes (PBHs), while other high peaks can produce ultra-compact mini-halos (UCMHs). PBHs have been proposed as an alternative to an unknown particle to explain dark matter. They could have masses down to \(10^{-11} M_\odot\) (otherwise they would have evaporated by Hawking radiation). The discovery of PBHs and/or UCMHs would constitute a major breakthrough, both for the existence of these objects and for constraints on inflationary models. Conversely, the absence of UCMHs would establish upper bounds on the amplitude of the primordial power spectrum on small scales. PBHs and UCMHs, if ubiquitous, could be detected by Theia by astrometric microlensing in the \(\sim 20\) deep, regularly cadenced Theia fields (5 deg\(^2\) in total). Other astronomical objects may mimic as PBHs or UCMHs, but stars and brown dwarfs should be directly observable by launch date out to 1 kpc, drastically reducing the false detections.

- **Stochastic background of gravitational waves.** The distortions to spacetime from gravitational waves (GWs) cause coherent distortions in the motions of stars in our Galaxy at locations that vary in time at the speed of light (0.3 pc per year). The QCD transition produces a stochastic background of GWs whose low frequencies are inaccessible to LISA or successors of LIGO, but well matched to Theia and Gaia. Theia will be complementary to Gaia in searching for coherent proper motions: Gaia will have 700 times more sky coverage than Theia at galactic latitudes \(|b| < 5^\circ\), but the detectability of coherent motions scales as proper motion precision over the square root of number of stars, and the 23 times higher proper motion precision of Theia’s Deep Fields, will make it competitive with Gaia 10-year. Both Theia and Gaia should be more competitive than planned pulsar timing timing arrays (EPTA, etc.).

2.3 Black holes and neutron stars

While the detection by LIGO of gravitational waves caused by the mergers of stellar-mass black holes (BHs) with other BHs or neutron stars has opened an important new window on this class of compact objects, there is much to be learned on the structure of neutron stars and the many roles of BHs. Theia will strongly enhance our understanding of compact objects in several ways.

- **Merging supermassive black holes.** Merging of the most massive supermassive black holes also produce low-frequency GWs inaccessible by LISA. While these events are rare, they are the key signatures of the build up of the most massive galaxies and their active galactic nuclei, and understanding whether the presence of supermassive black holes at very high redshifts \((z \sim 8)\) is caused by massive PBHs or by the buildup of non-PBH black holes. Just as for the case of the stochastic GW background, Theia should be complementary to Gaia for detecting merging supermassive BHs with a similar overall sensitivity, which should be superior to that of pulsar timing arrays.

- **Intermediate-mass black holes.** Intermediate-mass black holes (IMBHs) are the fundamental link between stellar-mass BHs and supermassive ones lying in the cores of massive galaxies. But the kinematic evidence for IMBHs is sparse and debated, even in nearby globular clusters, with HST and Gaia, because candidate IMBHs may instead be small, dark subclusters of compact stars or stellar-mass BHs. Theia’s unprecedentedly precise proper motions (and parallaxes) will allow for many more central stars included in the mass modeling, which will allow a much more reliable distinction between IMBHs and dark subclusters. Observing with Theia at intermediate depth (200 total hours each) the half-dozen globular clusters with the greatest ratios of velocity dispersion to distance will provide the long awaited statistical view of the inner cores of globular clusters as a function of their mass, orbit around the Milky Way and core-collapse status.
• **Black Hole Formation & Demographics.** Astrometry offers a unique window into the BH population. *Theia* could identify BHs by astrometric microlensing in the 20 deep fields aimed for other targets. *Theia* will also dedicate ~ 15% of its total observing time to follow up on alerts. These could be BH candidates from *Roman*, because *Theia*'s higher temporal sampling compared to *Roman* is required to make precision mass estimates, through the combination of the magnification time series and the astrometry. The same data will provide excellent distance and space velocity information on the BHs.

In X-ray binaries (XRBs), systematics in estimating binary inclination angles and distances limit the quality of BH mass and spin estimates. *Theia* will measure the wobble of inclination angles as well as the parallaxes for ~ 50 XRBs (most often too distant for *Gaia*). These wobble measurements also yield the position angles of the orbits. This will allow testing if the jet directions (from VLBI) are perpendicular to the binary orbital planes, and hence determine if the BHs were produced in bona fide supernovae with asymmetric mass loss, or by prompt collapse.

• **Constraining the equation of state of neutron stars.** Astrometric measurements of XRBs can yield a set of precise neutron star (NS) masses and constrain the NS equation of state (one of the core questions in nuclear physics) in three ways: (1) The maximum mass of a NS is set by the equation of state, meaning that finding heavier neutron stars eliminates certain equations of state. (2) X-ray pulse profile fitting is primarily sensitive to $M/R$ and distance, so that better masses and distances will provide more accurate radii. (3) The lowest mass NSs are likely formed from electron capture supernovae at the Chandrasekhar mass, thus allowing an estimate of the NS binding energy at the Chandrasekhar mass, and it is expected that these neutron stars should be in known classes of wind-fed XRBs.

### 2.4 Earth-like planets around nearby sun-like stars

The detection and atmospheric characterization of temperate, potentially habitable telluric planets orbiting the nearest Sun-like stars sits high amongst the key science themes both in the final recommendations of the senior Scientific Committee for ESA’s long-term scientific plan Voyage 2050 and in the long-term outlook of the ASTRONET roadmap. *Theia*’s surgical single-measurement positional precision in pointed, differential astrometric mode (< 1 μas) will enable detection and high-confidence ($\geq 3\sigma$) true mass determination for Earths and Super-Earths ($M = 1$–5 $M_\oplus$) in the Habitable Zone (HZ) of the ~ 60 nearest solar-type stars, using high-cadence observations (≈ 100 visits) of each target and ≥ 3 reference stars.

Extreme-precision, sub-m/s Doppler techniques from the ground are expected to provide a global census of temperate terrestrial planets around nearby late-type M dwarfs within the next 10–15 years. However, for solar-type primaries the Doppler method might be limited by stellar activity and ultimately miss any HZ Earth-mass companions whose orbits are not close to edge-on. *Theia*’s astrometric sensitivity will allow reaching three key goals in exoplanetary science:

1. *Theia* astrometry will allow determining the true mass function of temperate 1–5 $M_\oplus$ rocky planets around solar-type stars, which is today completely unknown;

2. by measuring the true masses and full three-dimensional architecture in multiple systems *Theia* will allow studying for the first time the full demographics of planetary systems in the presence of temperate telluric planets around the nearest Sun-like stars, in high synergy with *Gaia* and Doppler surveys;

3. the temperate telluric planets detected by *Theia* will constitute the fundamental input target list for direct-imaging / spectroscopic missions aimed at searching for atmospheric biomarkers.

It will be key for such ambitious space observatories either in the optical / near-infrared (e.g., NASA’s proposed flagship missions HabEx, LUVOIR) or in the thermal infrared (e.g., the LIFE concept for an ESA’s L-class mission) providing their target list and avoiding their research phase (about 50% of their mission time) so that they can invest all their precious observing time performing spectroscopy of the atmospheres knowing exactly where to look. Prior knowledge of the true masses will also crucially help in interpreting any molecular detections in their atmospheres.
Updated estimates of *Theia* sensitivity\(^{17}\) indicate that the median detectable mass across the full HZ for the *Theia* stellar sample is \(\simeq 1.1 \, M_\oplus\). If present-day extrapolations of the occurrence rate of true Earth-like planets\(^{18}\) at \(37^{+48}_{-21}\%\) are accurate, we then expect to detect between 9 and 57 such planets. Furthermore, the number of HZ Earths per star can be larger than unity, since a typical HZ can dynamically sustain more than one planet.

### 2.5 Basic scientific requirements

*Theia* is a single field, visible-wavelength (400–900 nm) differential astrometry mission, meaning that the derived astrometric parameters for the target stars in a field will have position, parallax and proper motion relative to a local reference frame tied to a global one. At the time of the *Theia* mission, the most accurate and complete optical reference frame will be that of the *Gaia* catalog. By using *Gaia* global astrometry parameters as priors, the astrometric solution of all the stars observed by *Theia* will be automatically tied to the *Gaia* frame, without the need of forcing physical priors on sources such as quasars or remote giant stars. To cover the science cases described above, two main observing modes are necessary:

- **Faint Star Mode** (FSM) required by the Dark Matter and Compact Objects science cases, with 5 μas/yr end-of-mission precision on proper motions for up to \(> 10^4\) science targets and references down to \(R \simeq 20\), observable within \(\sim 0.5^\circ\) fields. The FSM uses the majority (exact trade-off to be determined) of a nominal 4-yr mission;

- **Bright Star Mode** (BSM) required by the Exoplanets science case, with 1 μas single-measurement precision in 1-hr integration for the science targets and up to \(10^2\) bright references with \(R \leq 13\), observable within \(\sim 0.5^\circ\) fields. The BSM utilizes the remainder of the observing time a nominal 4-yr mission not devoted to the FSM.

These characteristics make *Theia* much superior to all the competition. The deep fields will achieve 23 times the proper motion precision of *Gaia* (10 years)\(^{1}\) and 14 times that of *HST* \(^{16}\) (10 years). In the bright-star regime\(^2\), *Theia*’s 1-μas precision in 1-hr integration at the reference value \(R = 10\) mag exceeds that of *Gaia* \(^{19},\) *Roman* \(^{20},\) and *VLTI/GRAVITY*\(^{21}\) by factors of \(\sim 40 – 50,\) \(\sim 10 – 20,\) and \(\sim 30 – 100\) respectively, and other missions/instruments (e.g., *HST, JWST, ELT/MICADO, Rubin*) by even larger factors.

### 3. MISSION CONFIGURATION

We describe quickly here the requirements, mission profile and spacecraft design, resources and communications. We also assess the technology readiness level (TRL) of the different components, the development plans, fall-back scenarios, impact on science.

#### 3.1 Payload and Platform Requirements

The baseline *Theia* Payload Module (PLM) uses the heritage knowledge of the consortium members for space mission concepts like *Gaia, HST/FGS, SIM, NEAT/M3, Theia/M4+M5 and Euclid*. Two different possible concepts can be adopted. A NEAT-like mission consisting of a single off-axis mirror sending the light onto a detector either in a formation flight configuration\(^3\) or on a deployable boom, or a *Euclid*-like mission with a Korsch three-mirror anastigmatic (TMA) telescope, a single focal plane and instrument metrology subsystems\(^2\). Both concepts consist in adopting a long focal length, diffraction limited, telescope and additional metrology control of the focal plane array. The baseline remains therefore the 0.8-m on-axis TMA telescope operating at visible wavelengths described in the *Theia/M5* proposal but with an update of the optical design where all optics are still coaxial but with a field of view whose center is shifted by 0.45 deg in order for the light beam to avoid the plane mirror after reflection on M3 (Fig. 1).

Compared to the proposed *Theia/M5* mission concept, another progress is the new CMOS detectors which allow up to \(10^9\) small-size (\(\sim 4\mu m\)) pixels with well-controlled systematics, capability of reading pre-determined windows around objects with pixel readout at \(\geq 1\)kHz rates to prevent saturation of bright stars. Existing

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\(^{1}\)https://www.cosmos.esa.int/web/gaia/science-performance#astrometric\%20performance

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examples include the Sony IMX411ALR sensor with 150 Mpxixels or the Pyxalis GigaPyx sensor that can reach 220 Mpxixels (with a 30k×30k px CMOs sensor foreseen is the near future by Pyxalis thanks to the “stitching technique”). Such detectors would considerably simplify the payload with a few or even a single detector and read-out electronics instead of 24 covering the required ∼ 0.5° field-of-view (FoV) in the focal plane array (FPA), but also shorten the focal length to ≃ 13 m.

The key requirement on sub-µas-level differential astrometric precision for Theia’s BSM implies control of all effects that impact the relative positions of the Nyquist-sampled the point-spread function (apparent size of 0.136′′ for a 0.8-m telescope in the visible). The precision of relative positions determination on the detector depends on the photon noise (limited by the reference stars), the geometrical stability of the focal plane array, the optical aberrations, and the variation of the detector response between pixels. This translates into a fundamental requirement of ∼ 1 × 10⁻⁵ pixel precision. This requirement can be relaxed by an order of magnitude for the FSM, dominated by photon noise.

To monitor a variety of sources of distortions of the FPA, and to allow the associated systematic errors to be corrected, Theia will rely as baseline on metrology laser feed optical fibers placed at the back of the nearest mirror to the detector(s).

In addition to measuring the FPA physical shape, the rest of the telescope needs monitoring to control time-variable aberrations at a certain level. The first estimation was at sub-µas level, but new developments show that it may have been too conservative (see Sect. 5). Indeed the telescope geometry is expected to vary, even at very stable environments such as L2 and therefore a baseline telescope metrology subsystem was based on a concept of linear displacement interferometers and piezo activators, independently on each linear element of the structure, greatly simplifying the system-level metrology in the Theia M5 proposal. In this baseline design, telescope and FPA are cooled to ∼ 130 K and ∼ 150 K, respectively, with a key requirement of ∼ 30 mK stability over the integration time and for mm-level stability, translating in a thermal control system coupling passive (V-Groove radiation shields) with active cooling solutions (e.g. JT coolers). However, the new method to derive the astrometry signal using the reference stars allows us to measure the telescope distortion described in Sect. 5 and therefore considerably relax the initial requirement put for M5 from several hours to fraction of a second (the frame exposure time is between 20 ms for the target star and 0.5 s for the reference stars).

We outline in Sect. 3.5 some of the strategies that can be adopted to cope with costs and Technology Readiness Levels (TRLs) of a few initial mission-critical elements, while safeguarding the main science objectives, during the assessment phase.

### 3.2 Mission Profile

Theia needs to point at all-sky directions. L2 is the selected option for the orbit, since it is very favorable for overall stability simplifying thermo-elastic design issues. The Theia spacecraft will be directly injected into a large Lissajous or Halo orbit at L2. To avoid parasitic light from the Sun onto the telescope and the detector, Theia spacecraft have baffles that protect them from Sun light at angle larger than ±45° from the Sun direction. The baseline launcher is Ariane 6.2. The launch strategy would consist in a unique burn of upper stage injecting...
The observing time baseline to properly investigate the science program of *Theia* is 4 years including time devoted to orbit maintenance. A total of approximately 6 months has been estimated for the orbit transfer including the spacecraft and instrument commissioning. From the total of $\sim 35000$ h dedicated for the scientific program, about 15 min per slew will be dedicated to reconfiguration and station-keeping. Fields containing science targets and reference stars will be observed in two modes (FSM, BSM, see Sect. 2.7). The pointing acquisition of science fields observed in either mode can be performed with a standard high accuracy star tracker and the attitude measurements performed with a fine guidance sensor (FGS).

### 3.3 Preliminary spacecraft design

The preliminary *Theia* M5 mission analyses allowed identification of a safe and robust mission architecture mostly relying on high TRL technologies. The proposed mission architecture adopts the 0.8-m Korsch TMA telescope accommodated vertically on top of a platform including all support subsystems.

At the initial stage, a high thermal stability of the telescope was thought to be necessary to ensure its performances. It was obtained through the use of a Sun Shield on which is accommodated the Solar Array and a vertical V-groove screen. The design of the satellite is based on the *Euclid* service module with a downscaled size to better suit to specific *Theia* needs. Similarly to the *Euclid* and *Herschel* satellites, the *Theia* Korsch telescope is placed on top of the service module in a vertical position.

### 3.4 Resources and Communications

The baseline M7 *Theia* satellite had a dry and wet mass of $\sim 1000$ kg and $\sim 1300$ kg, respectively, and a payload mass of $\sim 400$ kg. Power resources were estimated at $\sim 1400$ W for the S/C and $\sim 400$ W for the PLM. In terms of communication requirements, the amount of science data produced by the payload module was estimated to a total average of 95 Gbytes/day (135 Gbytes/day worst case), including a compression factor of 2.5 with a download data rate of $\sim 75$ Mbps with an ESA 35m ground station. Consequently, daily visibility...
periods of about 4 hours would be necessary, similar to *Euclid*. All the above requirements fit the constraints in the technical annex to the present Call. The foreseen simplified payload configuration will further relax these requirements.

### 3.5 TRL assessment, development plans, fall-back scenarios, impact on science

The current assessment of the TRL for the various key technologies within the proposed baseline design (and identified options) is summarised in Table 1. The key Theia PLM profits from a series of developments performed for past missions, but phase-A activities will be necessary to raise the TRL of the telescope and FPA metrology system and electronics. Phase-A activities will also be required to raise the TRL and space-qualify large-format CMOS detectors, to breadboard the FPA as an integrated system, and for straylight assessment.

Phase-A activities will be needed to demonstrate the achievement of the centroiding precision goal of $\sim 1 \times 10^{-5}$ pixels on CMOS detectors. Crouzier et al.\textsuperscript{22} and Nemati et al.\textsuperscript{23} have achieved $6 \times 10^{-5}$ pixel precision in the laboratory on a c2V CCD. Centroiding algorithms for differential astrometry on TESS science data have demonstrated in-flight uncertainties of $\leq 1 \times 10^{-5}$ pixels on single frames and $\sim 2 \times 10^{-6}$ as collective performance for an observing sequence\textsuperscript{24}.

If the centroid goal is degraded by one order of magnitude, the core science objectives focused on faint-source astrometry (dark matter, cosmology, compact objects) will not be impacted, as they rely on photon-noise limited observations. The exoplanet science case, focused on the bright-star regime for which the ultimate systematic noise floor achievable is critical, could accommodate up to a factor of three in degradation of the centroiding precision goal. Relaxing the requirement would still allow to reach sensitivity to $1.0 - 1.2 \, M_\oplus$ HZ planets around $\sim 25\%$ of the nearest solar-type stars sample (while still achieving sensitivity to 2-5 $M_\oplus$ super-Earths for the 60-target sample). This would enable in turn detection of at least a few such companions (or at least a handful if higher multiplicity in the HZ is considered) in the event of their true occurrence rate matching the lower end of the estimate from Bryson et al.\textsuperscript{18}, but as many as $\geq 10$ true Earth-like planets (not considering higher multiplicity in the HZ) in case the true occurrence rate exceeds the median value determined by Bryson et al.\textsuperscript{18}.

Plans for laboratory work to perform the above mentioned Phase-A activities have been laid out. A detailed PLM design will be presented at the time of Phase-2 proposal submission, in which the trade-off between the baseline and the other configurations outlined here will be resolved. A possible back-up scenario will also be presented, which might include, e.g., an alternative configuration taking advantage of existing, high-TRL metrology solutions, or alternative strategies for centroiding on very bright objects (for example using diffraction spikes).

### 4. MANAGEMENT SCHEME

Over 100 researchers were originally part of the *Theia* M5 mission Consortium. Presently 11 European Union countries are represented. The core team includes members from Italy, France, Germany, Sweden, Spain, Austria, Denmark, Portugal, and we have additional contributions from Switzerland, The Netherlands, and Poland. Scientists from countries outside Europe (Israel, USA) are also involved. The management team of the *Theia* Consortium consists of the Co-PIs and Co-Is from each of the main contributing countries plus the central leadership of the Consortium: the PI is envisioned to be supported by Project Manager, Instrument Lead, Calibration and Operation Lead, Science Ground Segment Manager. The Co-PIs lead the national groups responsible for the major components of the payload, Italy, France, Germany & Sweden. All other countries contributing either to the science case, or to the payload development, or both, are led by a Co-I. We envision a relevant role for the *Theia* Science Team, that would advise ESA on all aspects of the mission potentially affecting its scientific performance, assisting the ESA Project Scientist in maximising the overall scientific return of the mission.

The *Theia* mission is proposed to be a fully European mission, led by ESA. In a preliminary iteration, the *Theia* Consortium has converged on a scenario for the distribution of responsibilities for providing the following Payload Module (PLM) systems: camera + FGS, telescope (optics, structure, thermal stabilization control), focal plane array, all metrology subsystems, on-board control software, overall PLM assembly, integration and verification (AIV).
Even if ESA did not select *Theia* for the M7 call, we continue the work to develop the key science and technical issues.

5. RECENT DEVELOPMENTS ON DIFFERENTIAL ASTROMETRY MEASUREMENTS

Recently, we have undertaken a study to better understand how to reach the accuracy of the measurements in the BSM case for exoplanets in order to derive the requirements on the telescope design both mechanical and optical. The principle of the measurement is summarized on Fig. 2. What is observed is a target in the middle of a 0.5 deg × 0.5 deg field where one can find many fainter reference stars. However the telescope will move the positions of the star with respect of an exact conjugation on the detector of the grid on the sky, due to the different aberrations / defects of the telescope, including the large field distortion. What is measured is the position on the detector \((X_D, Y_D)\) that needs to be related to the "true" position on the sky. For the reference stars, these positions \((X_G, Y_G)\) are known thanks to the Gaia Catalogue.

The mathematical transformation of the images of stars towards their "true" positions on the sky can be approximated by a 2D polynomial of order \(n\). Explicitly, if \(X\) and \(Y\) correspond to an approximation of the "true" coordinates, one reads:

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\begin{align*}
X &= P(A, X_D, Y_D) = \sum_{i+j=0}^{n} A_{ij} X_D^i Y_D^j \\
Y &= P(B, X_D, Y_D) = \sum_{i+j=0}^{n} B_{ij} X_D^i Y_D^j
\end{align*}
\]

with \(A\) and \(B\) the coefficients of this polynomial. There are therefore a total of \((n + 1)(n + 2)\) coefficients to determine. Using a classical Levenberg-Marquardt method\(^\text{25,26}\), the position of the reference stars can be fitted to their known \((X_G, Y_G)\) positions on the sky and therefore determine the values of these coefficients \(A\) and \(B\).

The right part of Fig. 2 shows in green color a grid of 13 × 13 stars around the telescope field center and in red
color the barycenters of the spot diagrams of Zemax ray tracing on the detector. The positions in magenta are
the result of the fitting of a polynomial with order \( n = 7 \) and 72 unknowns. In panel (c), the distance between
the detector position and the true position can be of the order of several hundreds of pixels, whereas in panel
(d) the distance of the fitted and true position is only a small fraction of a pixel (\( \approx 10^{-5} \) pixel).

Using a linear combination of the reference stellar positions, e.g. their barycenter with adequate weighting, a
reference point can be located close to the target image. With the polynomial fit, we can then estimate the sky
position of the target, and make the astrometric measurement between the target and this reference point. An
important issue is the accuracy of the fit. Figure 3 shows that the error on the position decreases exponentially
with \( n \) and goes ultimately below \( 10^{-5} \) pixels for \( n = 7 \). This accuracy does not change significantly with
the number of reference stars if the number of constrains \( N_{\text{stars}} \) is greater than half the corresponding number of
parameters, \( (n+1)(n+2)/2 \), as can be seen from the right panel of Fig. 3.

We have also re-computed the ray tracing and the fitting procedure with an optical configuration of the
telescope where the M2 mirror has been tilted by one arcsecond. The distorted positions are found far away
from the initial value (several hundreds of pixels), but the estimated position in the field center neighborhood
remains determined at the \( 10^{-5} \) pixel pixel precision, pointing out the capability of the fitting procedure to
correct for significant instrumental defects.

However, we need to make the 2D polynomial computation more robust at higher orders than \( n = 7 \). Indeed,
with \( 13 \times 13 - 1 = 168 \) reference stars, a polynomial with a maximum of 336 parameters should be determined
corresponding to a polynomial of order of \( n \leq 16 \) unless there are redundancies not yet understood.

We plan now to repeat our analysis using Fourier series as well as Zernike polynomials. A last question still
to be investigated is the best strategy to define a reference barycenter close to the center of the field where the
target star is located and with its actual field of view. Our plan is to use realistic reference stars and to take into
account both their positions in the field and the photon noise of their image. We are also going to investigate
crowded fields as the cores of globular clusters.

We conclude that the fitting procedure using polynomials can correctly take into account for the optical
distortion of the nominal TMA telescope. This can also compensate systematic astrometric errors caused by a
1 arcsec tilt to the M2 mirror (corresponding to 1 \( \mu \)m displacement of one side of the mirror structure), showing
that there is no need for a very precise metrology control of the telescope. In other words, the type of metrology
that we use for the telescope structure is not based on metrology lasers located on the mechanical structure itself
but is funded on the position on the detector of reference stars compared to their actual position on the sky as
determined by Gaia. Our measurements are built on what we can call a reference star metrology.

Figure 3. Measurement errors on the target position at the center of the field in function of the order of the polynomials and
number of reference stars (left panels). The right window shows how this accuracy changes with the number of reference
stars used with a 2D polynomial of order \( n = 7 \).
6. CONCLUSION
Theia is a project for an astrometric observatory based on high-precision differential astrometry measurements on a limited single field (0.5 deg×0.5 deg). The two main science cases are nature of dark matter and Earth-like planets around solar neighbourhood. Theia’s payload is a 0.8 m diameter diffraction limited TMA Korsch telescope. New very large format CMOS visible detectors are being investigated in order to cope with a diffraction-limited yet large field of view without too many detectors. Telescope stability impacts performances except if one is able to calibrate the telescope distortion using reference stars with Gaia positions. Initial results show that with 50 to 100 reference stars whose position is known with Gaia, one can calibrate the center of the field to an accuracy lower than the $10^{-5}$ pixel.

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