**QCD Factorization and Re-scattering in Proton-Nucleus Collisions**

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**Abstract**  
The extension of the factorization theorems of perturbative QCD to power corrections associated with re-scattering in nuclear collisions is reviewed. The importance of hadron-nucleus collisions is discussed.

Perturbative QCD (pQCD) has been very successful in interpreting and predicting hadronic scattering processes in high energy collisions, even though the physics associated with individual hadron’s wave function is nonperturbative. It is the QCD factorization theorem [1] that provides prescriptions to separate long- from short-distance effects in hadronic cross sections. The leading power contributions to a general hadronic cross section involve only one hard collision between two partons from two incoming hadrons of momenta $p_A$ and $p_B$. The momentum scale of the hard collision is set by producing either a heavy particle (like $W/Z$ or virtual photon in Drell-Yan production) or an energetic third parton, which fragments into either a jet or a hadron $h$ of momentum $p'$. The cross section can be factorized as [1]

$$E_h \frac{d\sigma_{AB\rightarrow h(p')}}{d^3p'} = \sum_{ijk} \int dx' f_{j/B}(x') \int dx f_{i/A}(x) \int dz D_{h/k}(z) E_h \frac{d\hat{\sigma}_{ij\rightarrow k}}{d^3p'}(xp_A, x'p_B, p'/z),$$

(1)

where $\sum_{ijk}$ runs over all parton flavors and all scale dependence is implicit. The $f_{i/A}$ are twist-2 distributions of parton type $i$ in hadron $A$, and the $D_{h/k}$ are fragmentation functions for a parton of type $k$ to produce a hadron $h$. For jet production, the fragmentation from a parton to a jet, suitably defined, is calculable in perturbation theory, and may be absorbed into the hard partonic part $\hat{\sigma}$. For heavy particle production, the fragmentation function is replaced by $\delta(1 - z)$.

The factorized formula in Eq. (1) illustrates the general leading power collinear factorization theorem [11]. It consistently separates perturbatively calculable short-distance physics into $\hat{\sigma}$, and isolates long-distance effects into universal nonperturbative matrix elements (or distributions), such as $f_{i/A}$ or $D_{h/k}$, associated with each observed hadron. Quantum interference between long- and short-distance physics is power-suppressed, by the large energy exchange of the collisions. Predictions of pQCD follow when processes with different hard scatterings but the same set of parton distributions and/or fragmentation functions are compared [2].

With the vast data available, the parton distributions of a free nucleon are well determined by the QCD global analysis [3, 4]. With recent effort, a number of sets of fragmentation functions to light hadrons are becoming available though the precision is no way near that of parton distributions due to the limited data [5, 6].

Studies of hard processes at the LHC will cover a very large range of momentum fraction $x$ of parton distributions: $x \geq x_T e^y/(2 - x_T e^{-y})$ with $x_T = 2p_T/\sqrt{s}$ for inclusive jet production in Eq. (1), where $y$ and $p_T$ are the rapidity and transverse momentum of the produced jets, respectively. For the most forward or backward jets, or low $p_T$ Drell-Yan dileptons, the $x$ can be as small as $10^{-6}$ at $\sqrt{s} = 14$ TeV. The number of gluons having such a small longitudinal momentum fraction $x$ and transverse size $\Delta r_\perp \propto 1/p_T$ may be so large that gluons appear more like a collective wave than individual particles, and a new nonperturbative regime of QCD, such as the gluon saturation or color glass condensate [7], might be reached.

The use of heavy ion beams allows one to enhance the coherent effects because of a larger number of gluons. In the small $x$ regime of hadron-nucleus collision, a hard collision of a parton of the projectile nucleon with a parton of the nucleus occurs coherently with all the nucleons at a given impact parameter.
The coherence length (~ 0.1/x fm) by far exceeds the nuclear size. To distinguish parton-nucleus multiple scattering from partonic dynamics internal to the nucleus, we classify the multiple scattering in the following three categories: (a) initial-state interactions internal to the nucleus, (b) initial-state parton-nucleus interactions (ISI), and (c) final-state parton-nucleus interactions (FSI), as shown in Fig. 1[2].

Initial-state interactions internal to the nucleus change the parton distributions of the nucleus, as shown in Fig. 1a. As a result, the effective parton distributions of a large nucleus are different from a simple sum of individual nucleon’s parton distributions. Since only a single parton from the nucleus participates the hard collision to leading power, the effect of the initial-state interactions internal to the nucleus does not change the hard collisions between two incoming partons, and preserves the factorized single scattering formula in Eq. (1)[3], except that the twist-2 parton distributions $f_i/A$ are replaced by corresponding effective nuclear parton distributions, which are defined in terms of the same operators but on a nuclear state. Such effective nuclear parton distributions include the “EMC” effect and other nuclear effects so that they differ from the parton distributions of a free nucleon. But, they are still twist-2 distribution functions by the definition of the operators of the matrix elements; and are still universal.

Because of the twist-2 nature of the nuclear parton distributions, their nuclear dependence can only come from the $A$-dependence of the nonperturbative input parton distributions at a low momentum scale $Q_0$ and the $A$-dependent modifications to the DGLAP evolution equations from $Q_0$ to $Q$ due to the interactions between partons from different nucleons in the nucleus [9][10]. The nonperturbative nuclear dependence in the input distributions can be either parameterized with the parameters fixed by fitting experimental data [11][12], or calculated with some theoretical inputs [13][14].

Let hard probes be those whose cross sections are dominated by the leading power contributions in Eq. (1). That is, nuclear dependence of the hard probes comes entirely from that of nuclear parton distributions and is universal. Because of the wide range of $x$ and $Q$ covered, the hard probes in hadron-nucleus collisions at the LHC directly detect the partonic dynamics internal to the nucleus and provide excellent information on nuclear parton distributions. The knowledge of these distributions is very useful for understanding the gluon saturation, a new nonperturbative regime of QCD [14][15].

On the other hand, the hadronic cross sections receive the power-suppressed corrections to Eq. (1) [16][17][18]. These corrections can come from several different sources, including the effect of partons’ non-collinear momentum components and effect of non-vanish invariant mass of the fragmenting parton $k$, as well as the effects of interactions involving more than one partons from each hadron, as shown in Fig. 1b and 1c. Although such multiple coherent scattering is formally a higher-twist effect and
suppressed by powers of the large momentum scale of the hard collision, the corrections to the leading power factorized formula in Eq. (1) can be substantial due to a large density of soft gluons available to the ISI at the same impact parameter of the hard collision in a large nucleus; and the large densities of soft partons available to the FSI, which are either from the initial wave functions of the colliding nuclei or produced in the long-range soft parton interactions along with the hard collision.

Consider the scattering of an elementary particle or a parton (a quark or a physically polarized gluon) of momentum $xp$ in nuclear matter, as shown in Fig. 2. A hard-scattering with momentum transfer $Q$ can resolve states whose lifetimes are as short as $1/Q$. The off-shellness of the scattered parton increases with the momentum transfer simply because the number of available states increases with increasing momentum; and typically, the scattered parton (say, of momentum $p_1$) is off-shell by order $m_J \leq Q$, with $m_J$ as the invariant mass of the jet into which parton fragments. Further interactions of the off-shell parton are suppressed by an overall factor of $1/m_J^2 \sim 1/Q^2$, since the effective size of the scattered parton decreases with momentum transfer; and by the strong coupling evaluated at scale $m_J \sim Q$. That is, the re-scattering in nuclear collisions is suppressed by $\alpha_s(Q)/Q^2$ compared to single scattering.

The counting of available states ensures that $m_J \geq \sum_h \langle N_h \rangle m_h \gg \Lambda_{QCD}$, where $\sum_h$ runs over all hadron types in the jet and $\langle N_h \rangle$ is corresponding multiplicity. On the other hand, if we are to recognize the jet, we must have $m_J \ll E_J = p_0^J$, with $E_J$ being energy of the jet. In the rest frame of the nucleus, the scattered parton has a lifetime, $\Delta t \sim 1/m_J^2 \left( \frac{E_J}{m_J} \right)$. Thus, at high enough jet energy, $\Delta t > R_A$, the lifetime of the scattered parton will exceed the size of nuclear matter, even though the parton itself is far off the mass shell. That is, the interactions of the scattered off-shell parton with nuclear matter may be treated by the formalism of pQCD.

In order to consistently treat the power suppressed multiple scattering, we need a factorization theorem for higher-twist (i.e., power suppressed) contributions to hadronic hard scattering. It was shown in Ref. [16] that the first power-suppressed contribution to the hadronic cross section can be factorized into the form

$$ E_h \frac{d\sigma_{AB \rightarrow h(p')}^{(4)}}{d^3p'} = \sum_{(i'i')jk} \int dx' f_{j/B}(x') \int dz D_{h/k}(z) $$

$$ \times \int dx_1 dx_2 dx_3 T_{(i'i')/A}(x_1, x_2, x_3) E_h \frac{d\hat{\sigma}_{(i'i')→k}^{(4)}}{d^3p'}(x_i p_A, x_i' p_B, p_z), $$

where the partonic hard part $\hat{\sigma}_{(i'i')→k}$ is infrared safe and depends on the identities and momentum fractions of the incoming partons, but is otherwise independent of the structure — in particular the size — of the hadron and/or heavy ion beams. In Eq. (2), the correlation functions $T_{(i'i')/A}$ are defined in terms of matrix elements of twist-4 operators made of two-pairs of parton fields of flavor $i$ and $i'$, respectively; and the superscript “(4)” indicates the dependence on twist-4 operators.

Showing the factorization at the next-to-leading power is a beginning toward a unified discussion of the power-suppressed effects in a wide class of processes. A systematic treatment of double scattering in a nuclear medium is an immediate application of the generalized factorization theorem. Because of the infrared safe nature of the partonic hard part $\hat{\sigma}_{(i'i')→k}$ in Eq. (2), the nuclear dependence of the double scattering comes entirely from the correlation functions of two-pairs of parton fields, which can be linearly proportional to the $A^{1/3}$ (or nuclear size) [19]. Therefore, if the scattered off-shell parton has a lifetime longer than the nuclear size, the re-scattering receives an $A^{1/3}$ type enhancement factor from the medium size, and gets an overall suppression factor, $\frac{\alpha_s(Q)A^{1/3}λ^2}{Q^2}$, where the $\lambda$ with dimension of mass represents the nonperturbative scale of the twist-4 correlation functions. A semiclassical estimate gives $λ^2 \sim \frac{(fm)^2}{\pi} \langle F^{+α}F_α^+ \rangle$ [19].

For the ISI encountered by the incoming parton, the $λ^2$ is proportional to the average squared
transverse field strength, $\langle F^+ F^- \rangle$, inside the nucleus, which should be more or less universal. From the data on Drell-Yan transverse momentum broadening, it was found \cite{20} that $\lambda_{FSI}^2 \approx \lambda_{DY}^2 \sim 0.01 \text{GeV}^2$. However, the numerical value of $\lambda^2$ for the FSI does not have to be equal to the $\lambda_{FSI}^2$ due to extra soft gluons produced by the instantaneous soft interactions of long-range fields between the beams at the same time when the jets were produced by two hard partons, as shown in Fig. 3.

It was explicitly shown \cite{21, 22, 23} that the corrections to hadronic Drell-Yan cross section cannot be factorized beyond the next-to-leading power. However, it can be shown by using the technique developed in Ref. \cite{16} that the type of $A^{1/3}$-enhanced power corrections to hadronic cross sections in hadron-nucleus collisions can be factorized to all powers,

$$E_h \frac{d\sigma_{(2n)}^{AB \to h(p')}}{d^3p'} = \sum_{(i_n)j} f_{j/B}(x') \otimes D_{h/k}(z) \otimes T_{(i_n)/A}^{(2n)}(x_i) \otimes E_h \frac{d\sigma_{(2n)}^{(i_n)j \to k}}{d^3p'}(x_i p_A, x' p_B, p'_z), \quad (3)$$

where $\otimes$ represents convolutions in fractional momenta carried by the partons and $T_{(i_n)/A}^{(2n)}(x_i)$ represent the correlation functions of $n$-pairs of parton fields with parton flavors $i_n = 1, 2, \ldots, n$. In Eq. (3), there is no power corrections initiated by the high twist matrix elements of the incoming hadron $B$ because such contributions do not have the $A^{1/3}$-type enhancement in hadron-nucleus collisions. On the contrary, in nucleus-nucleus collisions, even the nuclear size enhanced power corrections cannot be formally factorized beyond the next-to-leading power.

Let semi-hard probes in hadronic collisions be those with a large momentum exchange as well as large power corrections. We expect that pQCD has a good predictive power to semi-hard observables in hadron-nucleus collisions, and the pQCD factorization approach does not work well for semi-hard observables in nucleus-nucleus collisions.

In conclusion, the factorized single scattering formula in Eq. (1) remains valid for hard probes in nuclear collisions, except that the parton distributions are replaced by corresponding effective nuclear parton distributions, which are independent of the hard scattering and universal. Hard probes in nuclear collisions at the LHC can provide excellent information on nuclear parton distributions and detect the partonic dynamics internal to the nucleus.

However, the power-suppressed corrections to the single scattering formula can be substantial and come from several different sources. The effect of the off-shellness of the fragmenting parton $k$ leads to a correction of the order $(m_j/p_T)^2$ with $m_j = \sum_h \langle N_h \rangle m_h$. Both the ISI and FSI double scattering give the power-suppressed corrections proportional to $\alpha_s A^{1/3} \lambda^2 / p_T^2$, with $\lambda_{FSI}^2$ relatively small and almost universal and $\lambda_{FSI}^2$ sensitive to the number of soft partons produced by the instantaneous collisions of long-range fields between nucleons of incoming beams.

Beyond double scattering (or next-to-leading power corrections), pQCD calculations might not be reliable due to the lack of factorization theorems at this level. However, pQCD factorization for the type of $A^{1/3}$-enhanced power corrections in hadron-nucleus collisions is likely to be valid at all powers.

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