Unbiased Self-Play

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Abstract

We present a general optimization framework for emergent belief-state representation without any supervision. We employed the common configuration of multiagent reinforcement learning and communication to improve exploration coverage over an environment by leveraging the knowledge of each agent. In this paper, we obtained that recurrent neural nets (RNNs) with shared weights are highly biased in partially observable environments because of their noncooperativeness. To address this, we designated an unbiased version of self-play via mechanism design, also known as reverse game theory, to clarify unbiased knowledge at the Bayesian Nash equilibrium. The key idea is to add imaginary rewards using the peer prediction mechanism (Miller et al., 2005), i.e., a mechanism for mutually criticizing information in a decentralized environment. Numerical analyses, including StarCraft exploration tasks with up to 20 agents and off-the-shelf RNNs, demonstrate the state-of-the-art performance.

1 Introduction

Evolving culture generates a high-level abstract representation without any bias in supervision. Symbols and languages are reasonable encoding frameworks for transferring knowledge over tiny/noisy channels and also for preventing neural nets from converging to local optima (Bengio, 2012). Communicative paradigms have been widely used in gradient-based optimization such as unsupervised learning (Goodfellow et al., 2014), reinforcement learning (Foerster et al., 2016; Sukhbaatar et al., 2016), and metalearning (Finn et al., 2017), to transfer an internal knowledge of the environment stored in the working memory to quickly adapt to the new environment with the knowledge stored in the working memory and learn accurate behaviors. The difference between supervised and evolutionary learning is the absence of oracles/forewarnings.

A partially observable environment with noncooperative agents, however, disables an agent from honestly sharing unbiased internal state information resulting in the agent taking actions such as concealing information and deceiving other agents at equilibrium (Singh et al., 2019). As a result, agents cannot fully observe the state of the environment; thus, they do not have sufficient knowledge to verify the information provided by other agents. Furthermore, neural networks are vulnerable to adversarial examples (Szegedy et al., 2014) and are likely to induce erroneous behavior with small perturbations. Discriminative models for information accuracy such as generative adversarial networks (GANs) (Goodfellow et al., 2014; Radford et al., 2016) and curriculum learning (Lowe et al., 2020), which assume accurate samples are obtained by supervision, cannot be applied to a partially observable environment where the distribution is not stable.

We generalize self-play (Tesauro, 1995; Silver et al., 2017) to noncooperative partially observable environments via mechanism design (Myerson, 1983; Miller et al., 2005), which is also known as reverse game theory. The key idea is to add imaginary rewards using the peer prediction method (Miller et al., 2005), which is a mechanism for evaluating the validity of information exchanged

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among agents in a decentralized environment, which is calculated based on social influence on the signals. We show that the imaginary reward enables us to reflect the bias of state representation on the gradient without oracles.

As the first contribution, we propose unbiased self-play (USP) and analytically demonstrate convergence to the global optimum (Section 3). We propose the imaginary reward based on the peer prediction method (Miller et al., 2005) and apply it to self-play. The mechanism affects the gradient of the local optima but not the global optimum. We propose to use the actions of the agents as feedback to verify the received signal from every other agent instead of the true state input and intent, which the agents cannot fully observe. Although USP only requires a modification of the baseline function for self-play, it drastically improves the convergence to the global optimum in any gradient-based learning algorithms.

As the second contribution, we report that the USP achieved state-of-the-art performance for various multiagent tasks up to 20 agents based on the results of numerical experiments (Section 5). Using predator prey (Barrett et al., 2011), traffic junction (Sukhbaatar et al., 2016; Singh et al., 2019), and StarCraft (Synnaeve et al., 2016) environments, which are typically used in the signaling games research, we compared the performances of USP with the current neural nets, including the state-of-the-art method, the LSTM, CommNet (Sukhbaatar et al., 2016), and IC3Net (Singh et al., 2019). We report that the model with IC3Net optimized by USP has the best performance.

**Notation:** Vectors are columns. Let \([n] := \{1, \ldots, n\}\). \(\mathbb{R}\) is a set of real numbers. \(i\) is the imaginary unit. \(\Re u\) and \(\Im u\) are real and an imaginary part of complex number \(u\), respectively. \(n\)-tuple are written as boldface of the original variables \(\mathbf{a} := (a_1, \ldots, a_n)\). Let \(\mathbf{1} := (1, \ldots, 1)^T\). Matrices are shown in uppercase letters \(\mathbf{Z} := (z_{ij}), i,j \in [n]\). \(E\) is the identity matrix. The set of probability distributions based on the support \(\mathcal{X}\) is described as \(\mathcal{P}(\mathcal{X})\).

## 2 Problem Definition

We assume \(n\) individuals share a learnable prior \(\theta\) with an encoder \(f_\theta\), a policy \(\pi_\theta\), and a loss \(\mathcal{L}_\theta\), in an environment where the state \(s_t \in \mathcal{S}\) evolves over time \(1 \leq t \leq T\) with actions \(a_{t,i} \in \mathcal{A}\), observations \(x_{t,i} \in \mathcal{X}\), a state transition probability \(T \in \mathcal{P}(\mathcal{S} \times \mathcal{A} \times \mathcal{S})\), an observation probability \(\mathcal{P} \in \mathcal{P}(\mathcal{S} \times \mathcal{X}^n)\), and reward function \(\mathcal{R} : \mathcal{S} \times \mathcal{A} \times \mathcal{X}^n \rightarrow \mathbb{R}^n\). The reward \(r_{t,i} := \mathcal{R}(s_t, a_t, x_t)\) is distributed based on the hidden state \(s_t\) and the joint action \(a_t\). The goal is to maximize the social welfare *i.e.*, the total cumulative reward defined as

\[
J(\theta; \mathcal{R}) := \mathbb{E}_{\pi_\theta} \left[ \sum_{t=1}^{T} \sum_{i=1}^{n} r_{t,i} \right].
\]

At every step \(t\), the agent infers the current belief state \(p(s_t | \theta, \xi_{t-1,i}, x_{t,i})\) with the encoder \(f_\theta(z_{t,i} | \xi_{t-1,i}, x_{t,i})\) given the previous history \(\xi_{t-1,i}\) and the current observation \(x_{t,i}\). The belief state model is transferred as \(n\) samples \(z_{t,i} := (z_{t,ij}, \ldots, z_{t,in})\) in the \(n\)-dimensional representation space \(\mathcal{Z} := \mathbb{R}^n\) to the next layer and other \((n-1)\) agents. Hence, we denote the signals sent at that time to a matrix \(\mathbf{Z}_t \in \mathbb{Z}^{n \times n}\), which contains \(n^2\) samples in total. The loss function \(\mathcal{L}_\theta\) figures out whether they choose the learning algorithm, e.g., Q-learning with replay memory (Mnih et al., 2015) or policy gradients (Schulman et al., 2015; Williams, 1992). For example, in REINFORCE (Williams, 1992), the loss function can be defined as follows:

\[
\mathcal{L}^\text{PG}_\theta(\xi_{t,i}, r_{t,i}) = -r_{t,i} \left[ \log \pi_\theta(a_{t,i} | z_{t,i}) + \log f_\theta(z_{t,i} | \xi_{t-1,i}, x_{t,i}) \right],
\]

where \(z_{t,i}\) and \(z_{t,i} := (z_{t,1i}, \ldots, z_{t,ni})\) are sent and received samples by \(i\), respectively. We name the framework as signaling games.

Many deep learning algorithms are described using the communicative framework. For example, the cross-entropy loss of the zero-sum game of generative adversarial nets (Goodfellow et al., 2014) can be written as,

\[
\mathcal{L}^\text{GAN}_\theta(x_t, r_t) = -r_t \left[ \log \pi^D_\theta(a_t | z_t) + \log f^G_\theta(z_t | x_t) \right],
\]

where \(a_t, r_t \in \{-1, 1\}\) are an action and a feedback, respectively, which are used to check if the discriminator \(\pi^D_\theta\) successfully finds out the real or fake signal sent from the generator \(f^G_\theta\). In comparison of Eq (2) and (3), sample \(x_t\) is drawn from a stable distribution \(\mathcal{P}(x_t)\). The supervision
knows whether \( z_t \) is real or fake, while in the real situation, the sample \( x_t \) is drawn from the temporal distribution \( \mathcal{P}(x_t|s_{t-1}, a_{t-1}) = \int \mathcal{P}(x_t|s_t) dT(s_t|s_{t-1}, a_{t-1}) \). Hence, the agent should quickly adapt to the new distribution with prior \( \theta \) and the history \( \xi_{ti} \). In some cases, the “discriminators” are completely fooled by the adversarial agents as some information are completely new to the agents.

Gradient based optimization algorithms e.g., backpropagation, converges to the Bayesian Nash equilibrium (BNE) (Fudenberg, 1993), a local optima \( \theta^* \) such that \( \nabla_\theta J(\theta; \mathcal{R})|_{\theta=\theta^*} = 0 \) in such signaling games. Although the global optimality of BNE in GAN is proved in the Goodfellow et al. (2014)’s original paper, the global optimality does not hold with the environments in which agents have information asymmetry. We denote \( \theta_1 \geq_G \theta_2 \) iff \( J(\theta_1; \mathcal{R}) \geq J(\theta_2; \mathcal{R}) \) holds in a signaling game \( \mathcal{G} \), namely \( \theta_1 \) dominates \( \theta_2 \). We want to find the \( \theta \) which dominates all the parameters.

In mechanism design, a truthful game (Vickrey, 1961) is a game in which all agents make an honest reporting in the BNE \( \theta^* \). We name signals sent to themselves \( h_{ti} := z_{ti} \), and signals received from all the agents \( z_{ti} \) to agent \( i \)’s prestate and poststate, respectively. The truthfulness is achieved if all the poststate signals are equal to the prestate \( z_i = h_i \), i.e., all the agents share a complete information.

**Proposition 2.1.** (unbiased priors) If a signaling game \( \mathcal{G} \) guarantees truthful, and \( \mathcal{R}_i \) is symmetric for any permutation of \( i \in [n] \), then \( \theta^* \geq_G \theta \) holds for any \( \theta \), and the value is given by,

\[
J(\theta^*; \mathcal{R}) = \sup_{\theta} \int nT \mathcal{R}_1(s_t, a_t) d\pi_0^\theta(a_t|h_t) d f_0^\theta(h_t|x_t) d \mathcal{P}(x_t|s_t) dp(s_t). \tag{4}
\]

**Proof.** Refer Section A for the proofs.

The bias among the agents is quantitatively measured as average cross-entropy between the pair of individual beliefs among all the agents \( q_0(s_t|\xi_{ti}) := p(s_t|\theta, \xi_{ti}) \), expressed as,

\[
D_\theta(Z_t) := \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \mathbb{E}_{q_0(s_t|\hat{z}_{ti})} \left[ - \log q_0(s_t|\hat{z}_{ti}) \right]
= \frac{1}{n} \sum_{i=1}^n D_{\text{KL}}(q_0(s_t|\hat{z}_{ti}) \parallel q_0(s_t|z_{ti})) + \frac{1}{n} \sum_{i=1}^n \mathbb{I}[q_0(s_t|\hat{z}_{ti})]. \tag{5}
\]

The first term represents the bias among agents, and the second represents the entropy of knowledge each agent has about the environment. \( D_\theta \) has a lower bound on the amount of true information the environment has \( \mathbb{I}[p(s_t)] \). Since achieving truthfulness is essentially the same problem as minimizing \( D_\theta, \theta^* \) dominates all the parameters.

An obvious way to guarantee truthfulness is by adding \( D_\theta \) as a penalty term of the objective. We have two obstacles to this approach. One is that the new regularization term, which adds a bias to the social welfare \( J \). The second is that \( D_\theta \) contains the agent’s private state \( z_{ti} \). So, the exact amount cannot be measured by the design of the learner. If \( q_0(s_t|\hat{z}_{ti}) \) are reported correctly, then \( q_0(s_t|h_{ti}) \) should also be reported honestly; thus, unbiased learning is achieved. Therefore, we cannot assume any models \( q_0 \) to design optimization algorithms.

### 3 Proposed Framework

Our framework, unbiased self-play (USP) introduces peer prediction method (Miller et al., 2005), a truthful mechanism to encourage honest reporting based solely on observable variables. In the following, each of them is described separately and we verify that the proposed framework converges to the global optimum. We show the whole procedure in Algorithm 1.

#### 3.1 Peer Prediction

The peer prediction (Miller et al., 2005) is a task derived from the field of mechanism design using proper scoring rules (Gneiting & Raftery, 2007), which aims to encourage verifiers to honestly report their beliefs by assigning a reward measure score to their responses when predicting probabilistic events. These mechanisms assume at least two agents; a reporter and verifier.

The general scoring rule can be expressed as \( F(z||s) \) where \( z \in [0, 1] \) is the probability of occurrence reported by the reporter for the event \( s \), and \( F(z||s) \) is the score to be obtained if the reported event \( s \)
Algorithm 1 The unbiased self-play (USP), a model-agnostic optimizer for unbiased priors. \( z_{ti} \) and \( z_{ti} \) are sent and received state representations by \( i \), respectively.

**Require:** \( n, \theta, \alpha, \beta \).

1: \textbf{while} training loss has not converged \textbf{do}
2: \( \xi_{0i} \leftarrow \{ \}, \forall i \in [n] \).
3: \textbf{for} \( t = 1 \) to \( T \) \textbf{do}
4: \textbf{Self-play,}
5: \begin{align*}
\xi_t & \sim \mathcal{P}(\cdot | s_t), \\
z_{ti} & \sim f_\theta(\cdot | \xi_{t-1,i}, x_{ti}), \quad \forall i \in [n], \\
\alpha_{ti} & \sim \pi_\theta(\cdot | z_{ti}), \quad \forall i \in [n], \\
r_i & \leftarrow R(s_i, a_i), \\
\xi_{ti} & \leftarrow \{ \xi_{t-1,i}, x_{ti}, z_{ti}, \alpha_{ti}, a_{ti} \}, \quad \forall i \in [n].
\end{align*}
6: Compute the imaginary reward with peer prediction (\( \rightarrow \) Section 3.1),
7: \begin{align*}
u_{tij} & \leftarrow \log \pi_\theta(a_{tj} | z_{tij}), \quad \forall i, j \in [n], \\
y_i & \leftarrow (E - 11^T/n)U_t 1.
\end{align*}
8: Update the prior by stochastic gradient descent,
9: \( \theta \leftarrow \theta - \alpha \nabla_\theta \left( \frac{1}{n} \sum_{i=1}^{n} L_\theta(\xi_{ti}, r_{ti} + \beta y_{ti}) \right). \) \hfill (6)
10: \textbf{end for}
11: \textbf{end while}
12: \textbf{return} \( \theta^* \leftarrow \theta \).

Actually occurred. The scoring rule is proper if an honest declaration is consistent with the beliefs of the verifier, and the reported \( z \) maximizes the expected value of the earned score. — Note that it is strictly proper if it is the only option for maximizing the expected value. A representative example that is strictly proper is the logarithmic scoring rule \( F(z\|s) = \log z \), where the expected value for a 1-bit signal is the negative cross-entropy \( \mathbb{E}_{q(s)}[\log z] = q(s) \log z + (1 - q(s)) \log(1 - z) \) for belief \( q(s) \). One can find that \( z = q(s) \) is the only report that maximizes the score.

Since the proper scoring rule assumes that events \( s \) are observable, it is not applicable to problems such as partial observation environments where the true value is hidden. Miller et al. (2005), who first presented a peer prediction, posed scoring to a posterior of the verifiers that are updated by the signal, rather than the event. This concept is formulated by a model that assumes that an event \( s \) emits a signal \( z \) stochastically and infers the type of \( s \) using the signal of the reporters who receive it. The peer prediction (Miller et al., 2005) is denoted by \( F(p(s\|z)\|s) \) under the assumption that (1) is the type of event \( s \), and the signal \( z \) emitted by each type follow a given prior \( \theta \), (2) the prior is shared knowledge among verifiers, and (3) the posterior is updated according to the reports.

We apply the mechanism to our problem, i.e., the problem of predicting the agent’s optimal behavior \( a_{ti} \sim \pi_\theta(s_t) \) for the true state \( s_t \in S \). In self-play, the conditions of 1 and 2 are satisfied because the prior \( \pi_\theta \) is shared among the agents. Furthermore, the signal corresponds to 3, so that the peer prediction can be applied to the problem of predicting agent’s behavior. To summarize the above discussion. We see that one can distributes a cross-validation matrix \( U_t := (u_{tij})_{i,j \in [n]} \) as follows,
\begin{equation}
u_{tij} := F(\pi_\theta(a_{tj} | z_{tij})\|a_{tj}) = \log \pi_\theta(a_{tj} | z_{tij}), \end{equation}
which is an \( n \)-order square matrix representing the score from \( i \) to \( j \). The best response of \( i \) to maximize \( \nu_{tij} \) is to perform the honest reporting \( \xi_{tij} = h_{tij} \), because
\begin{equation}
\argmax_{z_{tij} \in \mathcal{Z}} \mathbb{E}_{\pi_\theta(a_{tj} | s_t)}[\log \pi_\theta(a_{tj} | z_{tij})] = \argmax_{z_{tij} \in \mathcal{Z}} \mathbb{E}_{\pi_\theta(s_{tj} | h_{tij})}[\log \pi_\theta(a_{tj} | z_{tij})] = h_{tij}.
\end{equation}

### 3.2 Imaginary Reward

*Imaginary rewards* are virtual rewards passed from an agent with different basis \( i \) from rewards passed from the environment, with the characteristic that they sum to zero. Since the most of RL environments, including the signaling games, are of no other entities than agent and environment,
two-dimensional structure are sufficient to describe them comprehensively if we wish to distinguish the sender of the reward. To keep the social welfare a real value, the system must be designed so that the sum of the imaginary rewards, i.e., imaginary part of the social welfare zero. In other words, it is not observed macroscopically and affects only the relative expected rewards of agents. The real and imaginary parts of the complex rewards are ordered by the mass parameter $\beta > 0$ during training, which allows the weights of the network to maintain a real structure.

The whole imaginary reward is denoted by $iY = (iy_{ij})_{i,j\in[n]}$ where $iy_{ij}$ is the imaginary reward passed from $i$ to $j$. The complex reward for the whole game is $R^+ := R + iY$ where $R$ is a diagonal matrix with the environmental reward $r_i$ as $(i, i)$-th entry. We write $G[iY]$ as the structure in which this framework is introduced. In this case, the following proposition holds.

**Proposition 3.1.** For any signaling games $G$, if $G[iY]$ is truthful and $R^+$ is a Hermitian matrix, while the BNE in $G[iY]$ dominates all the parameters in $G$.

**Proof.** Since $G[iY]$ is truthful, $\theta^* \succeq_{G[iY]} \theta$ holds from Proposition 2.1. Further, $R^+$ is Hermitian simply means that $iy_{ij} = -iy_{ji}$. Thus, $\Im sup_\theta J(\theta; R + iY) = 0$ holds, where $Y := Y_1$. Therefore, $\sup_\theta J(\theta; R + iY) = \sup_\theta J(\theta; R)$ holds. □

This indicates that $J(\theta^*; R + iY) \geq J(\theta^*; R)$, indicating that the BNE could be improved by introducing imaginary rewards. Also, since $\sum_i \sum_j iy_{ij} = 0$ from the Hermiticity of $R^+$, the imaginary rewards do not affect the social welfare of the system, which is a macroscopic objective, but only the expected rewards of each agent. The baseline function in policy gradient (Williams, 1992) is an example of a function that does not affect the objective when the mean is zero. However, the baseline is a quantity that is determined based on the value function of a single agent, whereas the imaginary reward is different in that (1) it affects the value of each agent, (2) it is a meaningful quantity only when $n \geq 2$ and is not observed when $n = 1$.

### 3.3 The Unbiased Self-Play

In our framework, a truthful mechanism is constructed by introducing the proper prediction mechanism into imaginary rewards. We perform zero-averaging by subtracting the mean of the scores from each element of the matrix, thereby making the sum to zero. This can be expressed as follows using the graph Laplacian $\Delta := E - \mathbf{1}\mathbf{1}^T/n$. $Y_t = \Delta U_t$ to get

$$R^+_t = R_t + i\Delta U_t,$$

which is the formula that connects meta-learning and mechanism design.

**Theorem 3.1.** (global optimality) For any signaling games $G$, USP converges to the global optimum if the following convergence condition holds,

$$\sup_\theta \left| \frac{d\Re J(\theta; R + iY)}{d\Im J(\theta; R + iY)} \right| < \beta,$$

where $\beta \in (0, \infty)$ is the (bounded) mass parameter.

**Proof (in summary.)** $R_t + i\Delta U_t$ is Hermitian; and if Eq (14) holds, then $G[i\Delta U_t]$ is truthful from Proposition 2.1 and 3.1, the BNE $\theta^*$ dominates any $\theta$ in $G$ because

$$J(\theta^*; R + iY) = \sup_\theta J(\theta; R + iY) = \sup_\theta J(\theta; R).$$

Therefore, convergence to the global optima is achieved. □

### 4 Related Work

Self-play is known as *evolutionary learning* in the deep learning community (Bengio, 2012) mainly as an approach to the emerging representations without supervision (Bansal et al., 2018; Balduzzi et al., 2019). TD-Gammon (Tesauro, 1995) introduced self-play as a framework to train neural nets with TD($\lambda$) (Sutton & Barto, 1998) and achieve professional-grade levels in backgammon. AlphaGo (Silver et al., 2017) defeated the Go champion by combining supervised learning with professional game records and self-play. AlphaZero (Silver et al., 2018) successfully learnt beyond its
Table 1: Loss in five the signaling games tasks. PP-n: Predator prey and TJ-n: Traffic junction. The experiment was repeated thrice. The average and standard deviation are listed. **Bold** is the highest score. The models listed in the top three rows show the models optimized by self-play, and the bottom row shows IC3Net (Singh et al., 2019) optimized by unbiased self-play (ours).

|       | PP-3       | PP-5       | TJ-5       | TJ-10      | TJ-20      |
|-------|------------|------------|------------|------------|------------|
| SP LSTM | 1.92 ± 0.35 | 4.97 ± 1.33 | 22.32 ± 1.04 | 47.91 ± 41.2 | 819.97 ± 438.7 |
| CommNet | 1.54 ± 0.33 | 4.30 ± 1.14 | 6.86 ± 6.43 | 26.63 ± 4.56 | 463.91 ± 460.8 |
| IC3Net | 1.03 ± 0.06 | 2.44 ± 0.18 | 4.35 ± 0.72 | 17.54 ± 6.44 | 216.31 ± 131.7 |
| USP IC3Net | **0.69 ± 0.14** | **2.34 ± 0.21** | **3.93 ± 1.46** | **12.83 ± 2.50** | **132.60 ± 17.91** |

Figure 1: Learning curves in three-agent predator prey (PP-3). (a) Bias rate, the fraction of steps that an agent shares a false prestate ($z_{tij} \neq \hat{h}_{tij}$), (b) the real part of the reward, and (c) the imaginary part.

own performance entirely based on self-play. Bansal et al. (2018) show that competitive environments contribute to emerging diversity and complexity. Rich generative models such as GANs (Goodfellow et al., 2014; Radford et al., 2016) are frameworks for acquiring an environmental model by using competitive settings. It is worthy to note that RNNs such as world models (Ha & Schmidhuber, 2018; Eslami et al., 2018) are capable of more comprehensive ranges of exploration in partially observable environments and generation of symbols and languages (Bengio, 2017; Gupta et al., 2019; Chevalier et al., 2019). The difference between evolutionary learning and supervised learning is the absence of human knowledge and oracles.

Communication has gained attention in the field of multiagent reinforcement learning (MARL) such as RIAL/DIAL (Foerster et al., 2016), which optimizes discrete signals using Q-learning for communication, and CommNet (Sukhbaatar et al., 2016) and IC3Net (Singh et al., 2019), which optimizes continuous signals by backpropagation. One method of MARL is to employ independent RLs (InRLs), which learn each policy independently. However, InRL leads to non-Markovian problems, that is, many optimization algorithms (Williams, 1992; Sutton & Barto, 1998) lose their assumed Markov property and converge to a bad local optimum. The bad local optimum may cause problems such as a reduction in the generalization performance due to overfitting with the opponent’s policy (Heinrich & Silver, 2016; Lanctot et al., 2017).

There are two branches to deal with the non-Markovian problems. One approach focuses on agents, while the other focuses on the environment. Examples of the former approach are the tit-for-tat policy (Leibo et al., 2017) and semi-supervised learning (Lowe et al., 2020) by designers, but the range of this approach is significantly limited. The latter approaches include counterfactual rewards such as QUICR learning (Agogino & Tumer, 2006) and COMA (Foerster et al., 2018) and intrinsic rewards such as curiosity (Pathak et al., 2017; Houthooft et al., 2016). Observe that the the counterfactual reward cannot be applied to continuous actions but discrete. Besides, curiosity has a noisy TV problem (Azar et al., 2019), wherein information cannot be verified since the method can only measure the diversity of information between an agent’s knowledge and observations. To the best of our knowledge, this work is the first attempt to apply mechanism design to evolutionary learning.

5 Numerical Experiment

In this section, we established the convergence of USP through the results of numerical experiments with deep neural nets. We considered three environments for our analysis and experiments (→ Fig. 2). (a) a predator prey environment (PP) in which predators with limited vision look for a prey on a
Figure 2: Experimental environments. a: Predator Prey (PP-$n$) (Barrett et al., 2011). Each agent receives $r_{ti} = -0.05$ at each time step. After reaching the prey, they receive $r_{ti} = 1/m$, where $m$ is the number of predators that have reached the prey. b: Traffic Junction (TJ-$n$) (Sukhbaatar et al., 2016; Singh et al., 2019). $n$ cars with limited sight inform the other cars of each position to avoid a collision. The cars continue to receive a reward of $r_{ti} = -0.05$ at each time as an incentive to run faster. If cars collide, each car involved in the collision will receive $r_{ti} = -1$. c: Combat task in StarCraft: Blood Wars (SC) (Synnaeve et al., 2016).

Table 2: StarCraft Results: USP beats the baselines. The experiment was repeated thrice. All the scores are negatives. The average and standard deviation are listed. **Bold** is the highest score.

| StarCraft task         | CommNet          | IC3Net          | IC3Net w/ ours |
|------------------------|------------------|-----------------|----------------|
| Explore-10Medic 50x50  | 1.592 ± 0.169    | 2.276 ± 1.427   | **1.364 ± 0.107** |
| Combat-10Mv3Ze 50x50   | 3.202 ± 1.382    | 3.815 ± 0.392   | **1.717 ± 1.548** |

(a) a traffic junction environment (TJ) similar to Sukhbaatar et al. (2016) in which agents with limited vision learn to signal in order to avoid collisions. (c) StarCraft: Brood Wars (SC) explore and combat tasks which test control on multiple agents in various scenarios where agent needs to understand and decouple observations for multiple opposing units.

We compare the performances of USP with self-play (SP) and SP with curiosity (Houthooft et al., 2016) using three tasks belonging to the signaling games, comprising up to 20 agents. The hyper-parameters are listed in the appendix. With regard RNNs, three models namely LSTM, CommNet (Sukhbaatar et al., 2016), and IC3Net (Singh et al., 2019), were compared. The empirical mean of the social welfare $J$ was used as a measure for the comparison. IC3Net is an improvement over CommNet, which is a continuous communication method based on LSTM. Actor-critic and value functions were added to the baselines in all the frameworks. We performed 2,000 epochs of experiment with 500 steps, each using 120 CPUs; the experiment was conducted over a period of three days.

**PP and TJ:** Table 1 lists the experimental results for each task. We can see that IC3Net with USP outperforms the one with SP for all tasks. Fig. 1 (a) shows that USP elicits truthful information, (b) confirms that the social welfare of USP exceeds that of the SPs, and (c) confirms that the average of the imaginary part is zero. From these experimental results, we conclude that the USP successfully obtained truthful learning and state-of-the-art in tasks comprising three to twenty agents.

The step-wise visualization with PP-3 is shown in Fig. 3 and 4. Our framework catches up more quickly with the environmental information than the conventional approach (Fig 4 (a)), whereas the single-agent entropy of the self-play is less than USP. The phenomena suggests that an agent trained under the vanilla self-play “overfits” the environment due to lack of variant samples. A gap between the solid and dotted lines indicates the communication bias i.e., KL-divergence between each model, which shows that our framework deal with the implicit bias yielded by self-play. Both of $D_{KL}$’s converges to very similar value as agents eventually reached to the goals. Fig 4 (b) shows that as IC3Net closes the communication gate after the agent reached the goals (as pointed out by Singh et al. (2019)), the variance of MI from the middle of the episodes to the end. USP opens the gate by incentivizing to share the true information, and hence the information gain increases in the same interval. Fig 4 (c) shows that although the uncertainty of USP is greater than self-play at the end, the deficit is covered by the communication process (compare a and c).
Figure 3: Domain adaptation results. The recurrent agent with unbiased prior $\theta$ learns the background distribution $p(s_t)$ with few noisy observations. The 1,000 poststates are embedded into two-dimensional space by t-SNE algorithms (Van der Maaten & Hinton, 2008). Colors correspond to the number of steps. Average representation is shown in a black line. The 95% confidence area in the last ten steps is shown as a dotted circle.

Figure 4: 

a: Cross-entropy of the poststate $D_{\theta}(Z_t)$ (solid lines) along with the entropy $\mathbb{H}[q_{\theta}(s_t|h_{t+1})]$ (dotted lines).

b: Mutual information between pre and poststates.

c: Cross-entropy of the prestate.

StarCraft: Table 2 shows a comparison of social welfare in the exploration and combat tasks in StarCraft. (i) In the search task, 10 Medics find one enemy medic on a 50×50-cell grid; like PP, the reward is a competitive task where the reward is divided by the number of medics found. (ii) In the combat task, 10 Marines versus 3 Zealots fight on a 50×50 cell grid. The maximum step of the episode is set at 60. We find that IC3Net, with its information-hiding gate, performs less than CommNet but performs better when trained in USP due to the truthful mechanism.

6 Concluding Remark

Our objective was to construct a general framework for emergent unbiased state representation without any supervision. Firstly, we proposed the USP and theoretically clarified its convergence to the global optimum in the general case. Secondly, we performed experiments involving up to 20 agents and achieved the state-of-the-art performance for all the tasks. Herein, we summarize the advantages of our framework.

1. **Strong convergence**: USP guarantees convergence to the global optimum theoretically and experimentally; self-play cannot provide such a guarantee. Besides, the imaginary reward $i\Delta U_t$ satisfies the baseline condition.

2. **Simple solution**: The only modification required for USP is that $\beta \Delta U_t 1$ should be added to the baseline to easily implement it for deep learning software libraries such as TensorFlow and PyTorch.

3. **Broad coverage**: USP is a general framework, the same as self-play. Since USP is independent of both agents and environments and supports both discrete and continuous control, it can be applied to a wide range of domains. No supervision is required.

To the best of our knowledge, introducing mechanism design to meta-learning is a new direction for the deep-learning community. We expect that many other frameworks will be developed using the methodology employed in this study.
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A Theory

"...a human brain can learn such high-level abstractions if guided by the messages produced by other humans, which act as hints or indirect supervision for these high-level abstractions; and, language and the recombination and optimization of mental concepts provide an efficient evolutionary recombination operator, and this gives rise to rapid search in the space of communicable ideas that help humans build up better high-level internal representations of their world." (Bengio, 2012)

A.1 Proof of Proposition 2.1

Proposition A.1. (unbiased priors) If a signaling game $G$ guarantees truthful, and $R_i$ is symmetric for any permutation of $i \in [n]$, then $\theta^* \succeq_G \theta$ holds for any $\theta$, and the value is given by,

$$J(\theta^*; R) = \sup_\theta \int nT R_1(s_t, a_t) \, d\pi_\theta^p(a_t|\tilde{h}_t) \, df_\theta^p(h_t|x_t) \, dP(x_t|s_t) \, dp(s_t). \quad (12)$$

Proof. Suppose that $J(\hat{\theta}; R) = \sup_\theta J(\theta; R) > J(\theta^*; R)$ and $f_\hat{\theta}(z_{i tj}|x_{i tj}) \neq f_\theta^*(h_{i tj}|x_{i tj})$.

From Oliehoek et al. (2008), $\hat{\theta}$ minimizes $D_{KL}(p || q_\theta(s|z_{i tj}))$. As $q_\theta(s|z_{i tj}) \neq q_\theta^*(s|h_{i tj})$, $D_{KL}(p || q_\theta(s|z_{i tj})) > D_{KL}(p || q_\theta^*(s|h_{i tj}))$ holds. That contradicts that $D_{KL}(p || q_\theta(s|z_{i tj}))$ is the minimum value. Therefore, $\theta^*$ must be the global optimum solution.

Since $f_{\theta^*}(z_{i tj}) = f_{\theta^*}(h_{i tj})$ holds, from the symmetry of $R$, the objective are written as follows.

$$J(\theta^*; R) = \sup_\theta J(\theta; R) = \sup_\theta \int \sum_{i=1}^{n} \sum_{t=1}^{T} R_i(s_t, a_t) \left[ \prod_{i=1}^{T} \, d\pi_\theta(a_t|\tilde{s}_{i tj}) \, dq_\theta(\tilde{s}_{i tj}|z_{i tj}) \right] \, df_\theta(h_t|x_t) \, dP(x_t|s_t) \, dp(s_t)$$

$$= \sup_\theta \int \sum_{i=1}^{n} \sum_{t=1}^{T} R_i(s_t, a_t) \left[ \prod_{i=1}^{T} \, d\pi_\theta(a_t|\tilde{s}_{i tj}) \, dq_\theta(\tilde{s}_{i tj}|h_t) \right] \, df_\theta(h_t|x_t) \, dP(x_t|s_t) \, dp(s_t)$$

$$= \sup_\theta \int nT R_1(s_t, a_t) \, d\pi_\theta^p(a_t|\tilde{s}_t) \, df_\theta^p(h_t|x_t) \, dP(x_t|s_t) \, dp(s_t).$$

A.2 Proof of Theorem 3.1

Theorem A.1. (global optimality) For any signaling games $G$, USP converges to the global optimum if the following convergence condition holds.

$$\sup_\theta \left| \frac{d \text{Re} \, J(\theta; R + iY)}{d \text{Im} \, J(\theta; R + iY)} \right| < \beta,$$  \quad (14)

where $\beta \in (0, \infty)$ is the (bounded) mass parameter.

Proof.

A general loss function $\ell_\psi : A \times Z \rightarrow \mathbb{R} \cup \{\infty\}$ for any strictly concave nonnegative function $\psi : \mathcal{P}(A) \rightarrow \mathbb{R} \cup \{\infty\}$ is defined as follows:

$$\ell_\psi(a, z) := \mathcal{D}_\psi(\pi_\theta(a|z)||\delta(a|\cdot)), \quad (15)$$

where $\delta(a|\cdot)$ is a point-wise probability that satisfies $\lim_{\epsilon \rightarrow 0} \int_{B(\epsilon; a)} \delta(a|\tilde{a}) \, d\tilde{a} = 1$ for an open ball $B(\epsilon; a)$, and $\mathcal{D}_\psi$ is Bregman divergence (Bregman, 1967) defined by the following equation.

$$\mathcal{D}_\psi(p||q) := \psi(p) - \psi(q) + \int \nabla \psi(q) \, d(p - q).$$
Sending a truthful signal is the best response to minimize the expectation of the general loss function. For example, KL-divergence is a special case of Bregman divergence when $\psi = -H[\cdot]$, and the following equation holds.

$$
\mathbb{E}_{\pi_0} [\ell_\psi] = \int D_\psi (\pi_0(a_j|z_i) \| \delta(a_j|\cdot)) \, d\pi_0(a_j|h_i) \\
= D_{KL} (\pi_0(a_i|z_i) \| \pi_0(a_i|h_i)) \geq 0. \quad (17)
$$

The equality holds if and only if $z_i = h_i$. Notice that $\pi_0(a_i|h_i) = \pi_0(a_i|z_i)$.

**Proposition A.2.** If $[C]$ is a truthful mechanism of $G$, and $\sum_i C_i = 0$, self-play with $G[C]$ converges to $\sup_\theta J(\theta; R)$.

**Proof.** Since $\sum_i C_i = 0$, $\mathbb{E}_{\pi_0} [C_i] = 0$ holds. Hence, for an arbitrary baseline $b$, $b + C_i$ also satisfies the baseline condition. Therefore, from the policy gradient theorem (Sutton & Barto, 1998), self-play converges to $J(\theta^*; R + C)$. Further, since $[C]$ is a truthful mechanism, $J(\theta^*; R + C) = \sup_\theta J(\theta; R + C) = \sup_\theta J(\theta; R)$ holds from Proposition 2.1. \( \square \)

**Proposition A.3.** (Bregman mechanism) For any signaling games, if $\sup_\theta \| dV^\pi_0 / d\beta Y_\psi \| < 1$, $[i \psi_\psi]$ is an unbiased truthful mechanism of $G \in \mathcal{B}$ for a general cost function:

$$
\gamma_\psi(a|z) := \Delta U_\psi(a|z) \mathbb{1} = \\
\begin{bmatrix}
    n - 1 & -1 & \cdots & -1 \\
    -1 & n - 1 & \cdots & -1 \\
    \vdots & \vdots & \ddots & \vdots \\
    -1 & -1 & \cdots & n - 1
\end{bmatrix}
\begin{bmatrix}
    \ell_\psi(a_1|z_1) & \cdots & \ell_\psi(a_n|z_1) \\
    \ell_\psi(a_1|z_2) & \cdots & \ell_\psi(a_n|z_2) \\
    \vdots & \ddots & \vdots \\
    \ell_\psi(a_1|z_n) & \cdots & \ell_\psi(a_n|z_n)
\end{bmatrix}
\begin{bmatrix}
    1 \\
    1 \\
    \vdots \\
    1
\end{bmatrix}, \quad (18)
$$

where $\Delta := nE - \mathbf{1}^T$ is a graph Laplacian.

**Proof.**

The problem we dealt with is designing a scoring rule for information that satisfies two properties: 1) regularity, the score should be finite if the information is sufficiently correct, and 2) properness, the score should be maximized if and only if the information is true. The well-known example of the scoring rule is mutual information (MI), which compares a pair of probabilistic distributions $p$ and $q$ in the logarithmic domain. However, MI cannot apply to continuous actions. Instead, we introduce a more general tool, the regular proper scoring rule as defined below.

**Definition A.1.** (Gneiting & Raftery, 2007) For a set $\Omega$, $\mathcal{F}(\cdot, \cdot) : \Psi(\Omega) \times \Omega \rightarrow \mathbb{R} \cup \{+\infty\}$ is a regular proper scoring rule if and only if there exists a strictly concave, real-valued function $f$ on $\Psi(\Omega)$ such that

$$
\mathcal{F}(p|x) = f(p) - \int_\Omega f^*(p(\omega), x) \, dp(\omega) + f^*(p, x) \quad (19)
$$

for $p \in \Psi(\Omega)$ and $x \in \Omega$, where $f^*$ is a subgradient of $f$ that satisfies the following property.

$$
f(q) \geq f(p) + \int_\Omega f^*(p, \omega) \, dq(p - q) (\omega) \quad (20)
$$

for $q \in \Psi(\Omega)$. We also define $\mathcal{F}$ for $q \in \Psi(\Omega)$ as $\mathcal{F}(p||q) := \int_\Omega \mathcal{F}(p|x) \, dq(x)$, and describe a set of regular proper scoring rules $\mathcal{B}$.

For $\mathcal{F}, \mathcal{F}_1, \mathcal{F}_2 \in \mathcal{B}$, the following property holds (Gneiting & Raftery, 2007).

1. Strict concavity: if $q \neq p$, then $\mathcal{F}(q||q) > \mathcal{F}(p||q)$.
2. $\mathcal{F}_1(p||q) + a \mathcal{F}_2(p||q) + f(q) \in \mathcal{B}$ where $a > 0$ and $f$ are not dependent on $p$.
3. $-D_\psi \in \mathcal{B}$, where $D_\psi$ is the Bregman divergence.

**Lemma A.1.** For any $\mathcal{F} \in \mathcal{B}$ and a function $\ell_{\mathcal{F}}$ defined as shown below, if $\sup_\theta \| dV^\pi_0 / d\beta \ell_\psi \| < 1$, then $[i \ell_{\mathcal{F}}]$ is a truthful mechanism of $G$.

$$
\ell_{\mathcal{F}}(a_j|z_i) := \begin{cases} 
- \mathcal{F}(\pi_0(a_j|z_i)||a_j) & (i \neq j) \\
0 & (i = j)
\end{cases} \quad (21)
$$
Hence, the objective varies in the range $0 \leq J(\theta; R_0) \leq 2c$. 

### A.3 Self-Play Converges to Local Optima

**Theorem A.2.** If $\mathcal{G}$ is non-truthful, self-play does not converge to the global optimum $\sup_0 J(\theta; R)$. 

**Proof.** 

**Example A.1.** (One-bit two-way communication game) Fig. 4 shows an example of a non-cooperative partially observable environment with 1-bit state. The reward structure is presented in Table 4. The sum of rewards is maximized when both agents report the correct state to the environment, 

$$1^T R_{\text{com}}(s, a) = \begin{cases} 2c & (a_1 = a_2 = s) \\ 0 & \text{(otherwise)} \end{cases}.$$ 

Hence, the objective varies in the range $0 \leq J(\theta; R_{\text{com}}) \leq 2c$. 

### A.3 Self-Play Converges to Local Optima

**Theorem A.2.** If $\mathcal{G}$ is non-truthful, self-play does not converge to the global optimum $\sup_0 J(\theta; R)$. 

**Proof.** 

**Example A.1.** If $G$ is a linear combination of Bregman divergences, then $\ell_\theta$ is an unbiased posterior for $\theta$.

**Example A.2.** The scoring rule $\ell_\theta$ is unbiased for $\theta$.

**Example A.3.** The scoring rule $\ell_\theta$ is unbiased for $\theta$.

**Example A.4.** The scoring rule $\ell_\theta$ is unbiased for $\theta$.

**Example A.5.** The scoring rule $\ell_\theta$ is unbiased for $\theta$.

**Example A.6.** The scoring rule $\ell_\theta$ is unbiased for $\theta$.

**Example A.7.** The scoring rule $\ell_\theta$ is unbiased for $\theta$.
Figure 5: **Left:** One-bit two-way communication game $G_{\text{com}}^2$. $S = A = \{0, 1\}$, $X = S \cup \{\circ\}$, and $p(s) = 1/2$. Agents are given correct information from the environment with a probability of $\lambda > 0$: $p(s|x, z_j) = \lambda$, $p(\circ|x) = 1 - \lambda$. $q_1(s|x_1, z_j)$ follows a Bernoulli distribution and estimates the state $p(s)$ of the environment from observations and signals $z_j \in Z = X$. Let $h_i = x_i$. **Right:** An example of the non-cooperative solutions: $s = 1$, $x = \langle \circ, 1 \rangle$, $z = \langle \circ, 0 \rangle$, $u = \langle 0, 1 \rangle$, $r = \langle -1, 1 \rangle$ and $r_1 + r_2 = 0 < 2c$.

Table 4: $G_{\text{com}}^2$’s reward function $R^2(s, a), c > 0$ is a constant that determines the nature of the game.

| $(r_1, r_2)$ | $s$ | $1 - s$ |
|--------------|-----|---------|
| $a_1$        | $(c, c)$ | $(1, -1)$ |
| $1 - s$      | $(-1, 1)$ | $(0, 0)$ |

**Proposition A.4.** If $c < 1$, $J(\theta^*; R_{\text{com}}) < \sup_\theta J(\theta; R_{\text{com}})$ holds.

**Proof.**

Since $p(s) = 1/2$, we can assume $s = 1$ without loss of generality. Besides, we discuss only Agent 1 because of symmetry. From $Z = \{0, 1\}$, Agent 1’s signaling policy $f_1$ sends the correct information $x$ or false information $1 - x$ when it knows $x$. Hence, we can represent the policy by using parameter $\theta \in [0, 1]$ as follows.

$$f_\theta(z|x) = \begin{cases} \theta^*(1 - \theta)^{1 - z}, & (x = 1) \\ 1/2, & (x = \circ) \end{cases}$$ (25)

Differentiating $f_\theta$ with $\theta$ gives the following.

$$\frac{d f_\theta}{d \theta} = \begin{cases} 2z - 1, & (x = 1) \\ 0, & (x = \circ) \end{cases} G_{\text{com}}^2.$$ (26)

Therefore, if $\pi^*$ is the BNE, then the policy gradient for $\theta$ is as follows.

$$\frac{d}{d \theta} U(\pi^*, \theta) = \frac{d}{d \theta} \int V_1^* \, dq_1 \, dP \, d\theta = \int V_1^* \, dq_1 \frac{df_1}{d \theta} \, dP \, d\theta$$

$$= \lambda \int V_1^* \frac{1}{2} \, dz_1 \bigg|_{s=x_1=1}$$

$$= \lambda \int (2z_1 - 1) R_1 \, d\pi_1^* \, d\pi_2^* \, dq_2 \, dz_1 \bigg|_{s=x_1=1}$$

$$= \lambda(1 - \lambda) \int (2z_1 - 1) R_1 \, d\pi_1^* \, d\pi_2^* \, dz_1 \bigg|_{s=x_1=1, x_2=0}$$

$$= \lambda(1 - \lambda) \sum_{z_1=0}^1 (2z_1 - 1) R_1 (s, \langle x_1, z_1 \rangle) \bigg|_{s=x_1=1}$$

$$= \lambda(1 - \lambda) (R_1 (1, \langle 1, 1 \rangle) - R_1 (1, \langle 1, 0 \rangle))$$

$$= \lambda(1 - \lambda) (c - 1) < 0.$$ (27)
As the policy gradient is negative from the assumption of $c \in (0, 1)$, $\theta^* = 0$ gets the Nash equilibrium from $\theta \geq 0$, thereby resulting in always sending false information to the opposite:

$$f_{\theta^*}(z|x) = \begin{cases} 1 - z & (x = 1) \\ 1/2 & (x = \circ) \end{cases}.$$  \hspace{1cm} (28)

Let $J(x_1, x_2) := J|_{x=(x_1, x_2)}$. We can get $J^*$ and $\hat{J}$ as follows.

$$J^* = \int J^*(x_1, x_2) dP^2 dp$$

$$= J^*(1, 1)\lambda^2 + J^*(1, 0) \cdot 2\lambda(1 - \lambda) + J^*(0, 0)(1 - \lambda)^2$$

$$= 2c\lambda^2 + 0 + 2c/2^2 \cdot (1 - \lambda)^2$$

$$= 2c \left[ \lambda^2 + \frac{1}{4} (1 - \lambda)^2 \right],$$  \hspace{1cm} (29)

and

$$\hat{J} = \int \hat{J}(x_1, x_2) dP^2 dp$$

$$= \hat{J}(1, 1)\lambda^2 + \hat{J}(1, 0) \cdot 2\lambda(1 - \lambda) + \hat{J}(0, 0)(1 - \lambda)^2$$

$$= 2c\lambda^2 + 2c \cdot 2\lambda(1 - \lambda) + 2c/2^2 \cdot (1 - \lambda)^2$$

$$= 2c \left[ \lambda^2 + 2\lambda(1 - \lambda) + \frac{1}{4} (1 - \lambda)^2 \right],$$  \hspace{1cm} (30)

respectively. Therefore, $\hat{J} - J^* = 4c\lambda(1 - \lambda) > 0$, and $J^* < \hat{J}$ holds. \hspace{1cm} \Box

From Proposition A.3, $G = G^2_{com}$ is the counterexample that global optimality does not occur. \hspace{1cm} \Box

A.4  Zero-one mechanism solves $G^2_{com}$.

Proposition A.5. (zero-one mechanism) Let $\ell : A \times Z \to \{0, 1\}$ be a zero-one loss between an action and a message $\ell(a_i|z_j) := a_j(1 - z_i) + (1 - a_i)z_i$, and

$$\mathcal{Y}(a|z) := \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \ell(a_1|z_2) \ell(a_2|z_1) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \hspace{1cm} (31)$$

If $\beta > (1 - c)(1 - \lambda)/\lambda$, then $[i|\mathcal{V}]$ is an unbiased truthful mechanism of $G^2_{com}$, and self-play with $G^2_{com}[i|\mathcal{V}]$ converges to the global optimum $\sup_\theta J(\theta; R_{com}) = 2c[1 - 3/4 \cdot (1 - \lambda)^2]$.

Proof.

The following equation holds.

$$\frac{d}{d\theta} V_{\beta,1}^{*} dq_1 = \frac{d}{d\theta} \int (V_1^{*} - \beta \mathcal{Y}_1 d\pi_2^{\circ} dq_2) dq_1$$

$$= -\lambda(1 - \lambda)(1 - c) - \beta \int \mathcal{Y}_1 d\pi_2^{\circ} dq_1 dq_2 \frac{df_1}{d\theta} \frac{df_2}{d\theta} dP^2 dp$$

$$= -\lambda(1 - \lambda)(1 - c) - \beta \lambda \int \mathcal{Y}_1 d\pi_2^{\circ} dq_2 (2z_1 - 1) dz_1 df_2 dP \bigg|_{s=x_1=1}$$

$$= -\lambda(1 - \lambda)(1 - c) - \beta \lambda^2 \int (2z_1 - 1)\ell(a_2|z_1) d\pi_2^{\circ} dq_2 dz_1 \bigg|_{s=x_1=x_2=1}$$

$$= -\lambda(1 - \lambda)(1 - c) - \beta \lambda^2 \sum_{z_1=0}^{1} (2z_1 - 1)\ell(1|z_1)$$

$$= -\lambda(1 - \lambda)(1 - c) - \beta \lambda^2(\ell(1|1) - \ell(1|0))$$

$$= -\lambda(1 - \lambda)(1 - c) + \beta \lambda^2$$

$$= \lambda^2 \left[ \beta - (1 - c) \frac{1 - \lambda}{\lambda} \right].$$  \hspace{1cm} (32)
Therefore, if \( \beta > (1 - c)(1 - \lambda)/\lambda \), then \( \theta^* = 1 \) holds, and \( J^* = \hat{J} \) holds. The value of \( \hat{J} \) is clear from the proof of Lemma A.3. \( \square \)

\([i\mathcal{Y}]\) is also known as the peer prediction method (Miller et al., 2005), which is inspired by peer review. This process is illustrated in Fig. 6 Left, and the state-action value functions are listed in Fig. 6 Right.

### B Complexity Analysis

Although the computational complexity of \( \beta \mathcal{Y}_\psi \) per iteration is \( O(n^3) \) as it involves the multiplication of \( n \)-order square matrices, we can reduce it to \( O(n^2) \) to obtain \( \mathcal{Y}_\psi = nU_\psi - 1^T U_\psi 1 \). The spatial complexity is \( O(n^2) \), and the sample size is \( O(n) \).

### C Experimental Environments

In the experiment, we used two partial observation environments. This setting is the same as that adopted in existing studies (Sukhbaatar et al., 2016; Singh et al., 2019). Fig. 2 shows the environments.

#### C.1 Predator Prey (PP)

Predator-Prey (PP) is a widely used benchmark environment in MARL (Barrett et al., 2011; Sukhbaatar et al., 2016; Singh et al., 2019). Multiple predators search for prey in a randomly initialized location in the grid world with a limited field of view. The field of view of a predator is limited so that only a few blocks can be seen. Therefore, for the predator to reach the prey faster, it is necessary to inform other predators about the prey’s location and the locations already searched. Thus, the prey’s location is conveyed through communication among the predators, but predators can also send false messages to keep other predators away from the prey.

In this experiment, experiments are performed using PP-3 and PP-5, which have two difficulty levels. In PP-3, the range of the visual field is set to 0 in a \( 5 \times 5 \) environment. In PP-5, the field of view is set to 1 in a \( 10 \times 10 \) environment. The numbers represent the number of agents.

#### C.2 Traffic Junction (TJ)

Traffic Junction (TJ) is a simplified road intersection task. An \( n \) body agent with a limited field of view informs the other bodies of its location to avoid collisions. In this experiment, the difficulty level of TJ is changed to three. TJ-5 solves the task of crossing two direct one-way streets. For TJ-10, there are two lanes, and each body can not only go straight, but also turn left or right. For TJ-20, the two-lane road will comprise two parallel roads, for a total of four intersections. Each number corresponds to \( n \).

In the initial state, each vehicle is given a starting point and a destination and is trained to follow a determined path as fast as possible while avoiding collisions. The agent is in each body and takes two actions, i.e., accelerator and brake, in a single time step. It is crucial to ensure that other vehicles do
not approach each other to prevent collision while making good use of multiagent communication. That is similar to blinkers and brake lights.

C.3 StarCraft: Blood Wars (SC)

Explore: In order to complete the exploration task, the agent must be within a specific range (field of view) of the enemy unit. Once the agent is within the enemy unit’s field of view, it does not take any further action. The reward structure of the enemy unit is the same as the PP task, with the only difference being that the agent’s (absolute x, absolute y) and the enemy’s (relative x, relative y, visible), where visible is a visual range. If the enemy is not in exploration range, the relative x and relative y are zero. The agent has nine actions to choose from: eight basic directions and one stay action.

Combat: Agents make their own observations (absolute x, absolute y, health point + shield, weapon cooldown, previous action) and (relative x, relative y, visible, health point + shield, weapon cooldown). Relative x and y are only observed when the enemy is visible, corresponds to a visible flag. All observations are normalized to lie between (0, 1). The agent must choose from 9+M actions, including 9 basic actions and 1 action to attack M agents. The attack action is only effective if the enemy is within the agent’s view, otherwise it is a no-problem. In combat, the environment doesn’t compare to Starcraft’s predecessors. The setup is much more difficult, restrictive, new and different, and therefore not directly comparable.

In Combat task, we give a negative reward $r_{time} = -0.01$ at each time step to avoid delaying the enemy team’s detection. When an agent is not participating in a battle, at each time step, the agent is rewarded with (i) normalized health status at the current and previous time step, and (ii) normalized health status at the previous time step displays the time steps of the enemies you have attacked so far and the current time step. The final reward for each agent consists of (i) all remaining health * 3 as a negative reward and (ii) 5 * m + all remaining health * 3 as a positive reward if the agent wins. Give health*3 to all living enemies as a negative reward when you lose. In this task, a group of enemies is randomly initialized in half of the map. Thus the other half making communication-demanding tasks even more difficult.

D Hyperparameters

Table 5: Hyperparameters used in the experiment. $\beta$ are grid searched in space $\{0.1, 1, 10, 100\}$, and the best parameter is shown. The other parameters are not adjusted.

| Notation | Value |
|----------|-------|
| Agents   | $n$ \{3, 5, 10, 20\} |
| Observation | $x_{ti} \in \mathcal{X}$ $\mathbb{R}^9$ |
| Internal state | $h_{ti} \in \mathcal{Z}$ $\mathbb{R}^{64}$ |
| Message | $z_{ti} \in \mathcal{Z}$ $\mathbb{R}^{64}$ |
| Actions | $a_{ti} \in \mathcal{A}$ \{←,↑,↓,→,a_{stop}\} |
| True state | $s_t \in \mathcal{S}$ \{0, 1\} $^{25\sim400}$ |
| Episode length | $T$ 20 |
| Learning rate | $\alpha$ 0.001 |
| Mass parameter | $\beta$ 10 |
| Discount rate | $\gamma$ 1.0 |
| Metrics | $\psi$ $-\mathbb{H}\left[\cdot\right]$ |