Supplementary material for

Mate limitation and sex ratio evolution

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1) Data for Fig. 1: Empirical sex ratios for one foundress conditions

**Table S1.** Mean sex ratio (proportion male) produced by single foundresses. Data are from publications presented in West et al. (2005; Appendix) as focusing on the role of foundress number under Local Mate Competition. Ploidy: HD = haplodiploid; PA = Pseudo-arrhenotokous. When multiple treatments existed at single foundress conditions, the group sex ratio were averaged.

| Reference                        | Species                        | Ploidy | Proportion male with 1 foundress |
|----------------------------------|--------------------------------|--------|----------------------------------|
| Cremer and Heinze 2002           | *Cardiocondyla obscurior*      | HD     | 0.265                            |
| Peer and Taborsky 2004           | *Xylosandrus germanus*         | HD     | 0.06                             |
| Borsa and Kjellberg 1996         | *Hypothenemus hampei*         | PA     | 0.08                             |
| Roeder et al. 1996               | *Tetranychus urticae*          | HD     | 0.264                            |
| Roeder 1992                      | *Tetranychus urticae*          | HD     | 0.227                            |
| Nagelkerke and Sabelis 1998      | *Typhlodromus occidentalis*    | PA     | 0.322                            |
| Nagelkerke and Sabelis 1998      | *Phytoseiulus persimilis*      | PA     | 0.114                            |
| Moore et al. 2002                | *Liporrhopalum tentacularis*   | HD     | 0.11                             |
| Frank 1985                       | *Pegascapus assuetus*          | HD     | 0.09                             |
| Herre 1985                       | *Pegascapus (Blastophaga) sp. 2* | HD | 0.14                             |
| Herre 1985                       | *Pegascapus (Blastophaga) sp. 1* | HD | 0.19                             |
| Herre 1985                       | *Tetrapus costaricensis*       | HD     | 0.22                             |
| Sagarra et al. 2000              | *Anagyrus kamali*              | HD     |                                  |
| Jackson 1996                     | *Caraphractus cinctus*         | HD     | 0.23                             |
| Abe et al. 2003                  | *Melittobia australica*        | HD     | 0.01                             |
| Cooperband et al. 2003           | *Melittobia digitata*          | HD     | 0.05                             |
| King and Seidl 1993              | *Muscidifurax raptor*          | HD     | 0.37                             |
| King and Skinner 1991            | *Nasonia giraulti*             | HD     | 0.06                             |
| King and Skinner 1991            | *Nasonia vitripennis*          | HD     | 0.16                             |
| Author                  | Species                  | HD  | HD  |
|-------------------------|--------------------------|-----|-----|
| Werren 1980             | *Nasonia vitripennis*    |     | 0.087 |
| Werren 1983             | *Nasonia vitripennis*    |     | 0.15 |
| Shuker and West 2004    | *Nasonia vitripennis*    |     | 0.2 |
| Molbo and Parker        | *Nasonia vitripennis*    |     | 0.112 |
| Takagi 1986             | *Pteromalus puparum*     |     | 0.11 |
| King 1989               | *Spalangia cameroni*     |     | 0.51 |
| Rabinovich 2000         | *Telenomus fariae*       |     | 0.13 |
| van Welzen and Waage 1987 | *Telenomus remus*       |     | 0.11 |
| Schwartz and Gerling 1974 | *Telenomus remus*       |     | 0.271 |
| Waage and Lane 1984     | *Trichogramma evanescens* |     | 0.18 |
| Salt 1937               | *Trichogramma evanescens* |     | 0.224 |
| Luck et al. 2001        | *Trichogramma pretiosum* |     | 0.282 |
2) **Derivation of the mating function** \( f_2(z) = 1 - e^{-\alpha \frac{z}{1-z}} = 1 - e^{-aq} \) (table 2)

Assume that each male can successfully engage in \( a \) matings on a given patch, in the time available for matings. If males mate indiscriminately, so that each female can be mated multiple times, and a male does not avoid re-mating with the same female, then:

i) The total number of matings is \( azl \), where \( l \) is the number of total number of individuals in the patch (i.e. all the offspring of both sexes of all foundresses).

ii) Because there are \((1 - z)l\) females, the probability of a given female being mated in a given mating event is \( \frac{1}{(1-z)l} \), and the probability of not being mated in this event is \( 1 - \frac{1}{(1-z)l} \).

iii) The probability of a given female not being mated in any of the \( azl \) matings is 

\[
\left(1 - \frac{1}{(1-z)l}\right)^{azl}.
\]

iv) If \( l \) is reasonably large, then 

\[
\left(1 - \frac{1}{(1-z)l}\right)^{azl} \approx \lim_{l \to \infty} \left(1 - \frac{1}{(1-z)l}\right)^{azl} = \left(e^{-\frac{1}{(1-z)}}\right)^{az} = e^{-\frac{az}{(1-z)}} = e^{-aq}
\]

which follows from the limit definition of \( e \). Note that \( q = \frac{z}{1-z} \) is the proportion of males to females.

v) Therefore, the probability of a given female being mated is approximately \( 1 - e^{-aq} \).

vi) The same would follow from assuming that the number of matings per female is Poisson distributed with mean 

\[
\frac{azl}{(1-z)l} = \alpha \frac{z}{1-z} = aq.
\]

Using the Poisson distribution is similarly justified when \( l \) is reasonably large, because then a) the number of ‘trials’ \( (azl) \), becomes large and b) the probability of success in each trial \( \left(\frac{1}{(1-z)l}\right) \) becomes small.

vii) Assuming that \( l \) is large, the same function can be found with a similar derivation even if a given pair never mates twice, but both males and females can mate multiply.
3) **Derivation of the mating function** \( f_3(z) = \min(aq, 1) = \min\left( a \frac{z}{1-z}, 1 \right) \) (table 2)

Again assume that a single male is capable of \( a \) matings, but females only mate once, after which they leave the pool of available mates. Now the total number of potential matings \( alz \) may exceed the actual matings, being limited by the number of females \( l(1-z) \). Therefore the number of matings is \( \min[alz, l(1-z)] \), and the proportion of mated females is

\[
\frac{\min[alz, l(1-z)]}{l(1-z)}
\]

so that

\[
f(z) = \min(\frac{az}{1-z}, 1) \quad \text{or} \quad f(z) = \min(aq, 1), \quad \text{where} \quad q = \frac{z}{1-z}
\]

4) **Stability analysis of equation (8)**

To be an ESS (Eshel et al. 1997), the candidate trait value must satisfy the criterion

\[
\left. \frac{d^2w}{dx^2} \right|_{x=y=x^*} < 0
\]

and to be convergence stable (Eshel et al. 1997), the criterion

\[
\left. \frac{d^2w}{dx^2} \right|_{x=y=x^*} + \left. \frac{d^2w}{dx dy} \right|_{x=y=x^*} < 0
\]

With the first mating function we find after simplification

\[
\left. \frac{d^2w}{dx^2} \right|_{x=y=x^*} = -\frac{2a}{n^2(1+ax^*)^3} \left[ 2n + (n + 1)a + (n - 1)x^*(a^2 + 2a) \right]
\]

and

\[
\left. \frac{d^2w}{dx^2} \right|_{x=y=x^*} + \left. \frac{d^2w}{dx dy} \right|_{x=y=x^*} = -\frac{a}{n(1+ax^*)^3} \left[ 2(n + 1) + a(n + 3) + (n - 1)(a^2x^* + 2ax^*) \right]
\]

In both cases it is easy to see that the components in square brackets are positive when \( x^* \geq 0, a \geq 0 \) and \( n \geq 1 \); therefore equation (8) is an ESS and convergence stable.
5) Stability analysis of equation (9)

With the mating function $f_2(x)$ we are restricted to numerical solutions for the equilibria. Therefore we also take a partly numerical approach to stability analysis.

First we derive

$$
\frac{d^2w}{dx^2}\bigg|_{x=y=x^*} =
\frac{a x^*}{2e^{-1+x^*}(-1+e^{-1-x^*})(-1+x^*)^3+3 x^*-3 x^* x^*+3 a x^* x^* x^*-2 a x^* x^* x^*+n(-1+x^*)(-1-(-2+a) x^*+(-1+2a) x^*)^2}
$$

and

$$
\frac{d^2w}{dx^2}\bigg|_{x=y=x^*} + \frac{d^2w}{dxdy}\bigg|_{x=y=x^*} =
\frac{e^{-1+x^*}(-1+e^{-1-x^*})(-1+x^*)^3+3 x^*-3 x^* x^*+3 a x^* x^* x^*-2 a x^* x^* x^*+n(-1+x^*)(-1-(-2+a) x^*+(-1+2a) x^*)^2}
$$

Next we numerically solve $x^*$ for any combination of $a$ and $n$ from equation (9), plug these values of $a$, $n$ and $x^*$ into the two equations above and check their sign for a range of parameter values we are interested in. In both cases, all combinations in the range $1 \leq a \leq 1000$ and $1 \leq n \leq 1000$ resulted in negative values for both criteria. Therefore, the results with the mating function $f_2(z)$ are evolutionarily stable and convergence stable in this (very large) parameter range.
Derivation and stability analysis of equation (10)

The function \( f_3(z) = \min(a \frac{z}{1-z}, 1) \) is continuous, but not differentiable at \( z = \frac{1}{1+a} \), as can be seen in this example with \( a=2 \):

\[ f_3(z) = \begin{cases} a \frac{z}{1-z}, & \text{if } z < \frac{1}{1+a} \\ 1, & \text{if } z \geq \frac{1}{1+a} \end{cases} \]

The piecewise nature of this function complicates the analysis in some ways, and the resulting ESS is also piecewise defined.

Firstly, we know from classic LMC theory that when all females are fertilized, the ESS is \( x^* = \frac{n-1}{2n} \) (Hamilton 1967; West 2009). This will be the case if \( \frac{n-1}{2n} \geq \frac{1}{1+a} \), and we do not need to check the stability of this well-known solution.

However, if \( \frac{n-1}{2n} < \frac{1}{1+a} \), we seek the ESS with \( f_3(x) = a \frac{x}{1-z} \). Plugging this into equation (4) (main text), we find that \( \frac{dw}{dx}\bigg|_{x=y=x^*} = 0 \) when \( x^* = \frac{n+1}{2n} \). However, this solution is only relevant to us if \( \frac{n+1}{2n} \) falls in the interval \([0, \frac{1}{1+a}]\) (see figure above). It is easy to check that this can only be true if \( a < \frac{n-1}{n+1} < 1 \). Such scenarios where a single male cannot fertilize even one female are not realistic for our purposes, and hence the solution \( x^* = \frac{n+1}{2n} \) is in itself not
relevant for the purposes of this article.

Instead, whenever fertilization is incomplete, selection drives the sex ratio upwards until all females are fertilized, which happens at \( x^* = \frac{1}{1+\alpha} \). If \( \frac{n-1}{2n} < \frac{1}{1+\alpha} \), selection ceases here, and if \( \frac{n-1}{2n} > \frac{1}{1+\alpha} \), selection continues until \( x^* = \frac{n-1}{2n} \) is reached.

Now it still remains to be formally checked whether the candidate ESS \( x^* = \frac{1}{1+\alpha} \) is stable when \( \frac{n-1}{2n} < \frac{1}{1+\alpha} \). Fitness is not differentiable at this point, but instead of a second derivative analysis, we can show that \( \frac{dw}{dx} \bigg|_{y=x^*} \) changes sign from + to – as \( x \) passes \( x^* = \frac{1}{1+\alpha} \). This shows that deviant mutants cannot invade, and hence \( x^* = \frac{1}{1+\alpha} \) is an ESS.

To check this, we need only compute \( \frac{dw}{dx} \) for the left side of \( x = x^* = \frac{1}{1+\alpha} \). This is because the right side is in the regime of standard LMC. We assume \( \frac{n-1}{2n} < \frac{1}{1+\alpha} \), and therefore in this region we already know that selection and the derivative are negative.

For the left side we use \( f_3(x) = \alpha \cdot \frac{z}{1-z} \) and find \( \frac{dw}{dx} = \frac{a(2(x-y)^2+n(-1+4x-4y)(-1+y)+n^2(1-3y+2y^2))}{(x+n(-1+y)-y)^2} \), which can be shown to be positive when \( x < \frac{1}{1+\alpha} \) and \( y = x^* = \frac{1}{1+\alpha} < \frac{1}{2} \).

For convergence stability, we must show that \( \frac{dw}{dx} \bigg|_{x=y} \) similarly changes sign from + to – as \( y \) passes \( x^* = \frac{1}{1+\alpha} \). This shows that if the population is perturbed from the ESS, it will return to
it, and hence $x^* = \frac{1}{1+a}$ is convergence stable.

Again, with similar justification as above, we need only check the left side; on the right side, the derivative is known to be negative due to the stability of the standard LMC result.

For the left side we find that $\frac{dw}{dx} |_{x=y} = \frac{a(1+n-2ny)}{n-ny} = \frac{\frac{1}{n+1} - \frac{2y}{1-y}}{\frac{1}{n} + \frac{1}{n+1} - \frac{2y}{1-y}}$. Now, because $y < x^* = \frac{1}{1+a} \leq \frac{1}{2}$, both the numerator and denominator are necessarily positive, and hence the derivative is positive.

Therefore equation (7) in the main text is evolutionarily stable and convergence stable.
References for supplementary material

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