Towards Determination of the Initial Flavor Composition of Ultrahigh-energy Neutrino Fluxes with Neutrino Telescopes

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Abstract

We propose a simple but useful parametrization of the flavor composition of ultrahigh-energy neutrino fluxes produced from distant astrophysical sources: \( \phi_e : \phi_\mu : \phi_\tau = \sin^2 \xi \cos^2 \zeta : \cos^2 \xi \cos^2 \zeta : \sin^2 \zeta \). We show that it is possible to determine or constrain \( \xi \) and \( \zeta \) by observing two independent neutrino flux ratios at the second-generation neutrino telescopes, provided three neutrino mixing angles and the Dirac CP-violating phase have been well measured in neutrino oscillations. Any deviation of \( \zeta \) from zero will signify the existence of cosmic \( \nu_\tau \) and \( \bar{\nu}_\tau \) neutrinos at the source, and an accurate value of \( \xi \) can be used to test both the conventional mechanism and the postulated scenarios for cosmic neutrino production.

PACS number(s): 14.60.Lm, 14.60.Pq, 95.85.Ry
I. INTRODUCTION

High-energy neutrino telescopes are going to open a new window on the Universe, since they can be used to probe and characterize very distant astrophysical sources [1]. The promising IceCube neutrino telescope [2], which has a kilometer-scale detector, is now under construction. If the relative fluxes of ultrahigh-energy $\nu_e (\bar{\nu}_e)$, $\nu_\mu (\bar{\nu}_\mu)$ and $\nu_\tau (\bar{\nu}_\tau)$ neutrinos are successfully measured at IceCube and other neutrino telescopes, it will be possible to diagnose the relevant cosmic accelerators (e.g., their locations and characteristics) and examine the properties of neutrinos themselves (e.g., neutrino mixing and leptonic CP violation).

Indeed, robust evidence for neutrino masses and lepton flavor mixing has been achieved from the recent solar [3], atmospheric [4], reactor [5] and accelerator [6] neutrino oscillation experiments. Due to neutrino oscillations, the neutrino fluxes observed at the detector $\Phi^D = \{\phi^D_e, \phi^D_\mu, \phi^D_\tau\}$ are in general different from the source fluxes $\Phi = \{\phi_e, \phi_\mu, \phi_\tau\}$. Note that our notation is $\phi^{(D)}_\alpha \equiv \phi^{(D)}_{\nu_\alpha} + \phi^{(D)}_{\bar{\nu}_\alpha}$ (for $\alpha = e, \mu, \tau$), where $\phi^{(D)}_{\nu_\alpha}$ and $\phi^{(D)}_{\bar{\nu}_\alpha}$ denote the $\nu_\alpha$-neutrino and $\nu_\alpha$-antineutrino fluxes, respectively. The relation between $\phi_{\nu_\alpha}$ (or $\phi_{\bar{\nu}_\alpha}$) and $\phi^{D}_{\nu_\beta}$ (or $\phi^{D}_{\bar{\nu}_\beta}$) is given by

$$\phi^{D}_{\nu_\beta} = \sum_\alpha \left(\phi_{\nu_\alpha} P_{\alpha\beta}\right),$$
$$\phi^{D}_{\bar{\nu}_\beta} = \sum_\alpha \left(\phi_{\bar{\nu}_\alpha} \bar{P}_{\alpha\beta}\right),$$

where $P_{\alpha\beta}$ and $\bar{P}_{\alpha\beta}$ stand respectively for the oscillation probabilities $P(\nu_\alpha \rightarrow \nu_\beta)$ and $P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta)$. As the Galactic distances far exceed the observed neutrino oscillation lengths, $P_{\alpha\beta}$ and $\bar{P}_{\alpha\beta}$ are actually averaged over many oscillations and take a very simple form:

$$P_{\alpha\beta} = \bar{P}_{\alpha\beta} = \sum_{i=1}^{3} |V_{\alpha i}|^2 |V_{\beta i}|^2,$$

where $V_{\alpha i}$ and $V_{\beta i}$ (for $\alpha, \beta = e, \mu, \tau$ and $i = 1, 2, 3$) are just the elements of the $3 \times 3$ neutrino mixing matrix $V$. Eqs. (1) and (2) lead us to a straightforward relation between $\phi_{\nu_\beta}$ and $\phi^{D}_{\nu_\beta}$:

$$\phi^{D}_{\nu_\beta} = \sum_\alpha \left(\phi_{\nu_\alpha} P_{\alpha\beta}\right).$$

This relation indicates that the observation of $\Phi^D$ at a neutrino telescope can at least help

- to determine or constrain the flavor composition of cosmic neutrino fluxes [8,9], if three neutrino mixing angles and the Dirac CP-violating phase hidden in $P_{\alpha\beta}$ have been measured to a good degree of accuracy (e.g., a precision of 10% or smaller relative error bars [10]);

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1One may also use neutrino telescopes to test the stability of neutrinos [7,8], possible violation of CPT symmetry [8] and other exotic scenarios of particle physics and cosmology.
or

• to determine or constrain one or two of three neutrino mixing angles and the Dirac CP-violating phase [11,12], provided the production mechanism of ultrahigh-energy neutrinos at an astrophysical source (e.g., the conventional source to be mentioned below) is really understood.

Hence neutrino telescopes will serve as a very useful tool to probe both the high-energy astrophysical processes and the intrinsic properties of massive neutrinos.

This paper aims at a determination of the flavor composition of cosmic neutrino fluxes at the source with the help of neutrino telescopes. Different from the previous works (see, e.g., Refs. [7–9,11,12]), our present study starts from a generic parametrization of the initial neutrino fluxes:

\[
\{ \phi_e, \phi_\mu, \phi_\tau \} = \{ \sin^2 \xi \cos^2 \zeta, \cos^2 \xi \cos^2 \zeta, \sin^2 \zeta \} \phi_0, \quad (4)
\]

where \( \xi \in [0, \pi/2] \) and \( \zeta \in [0, \pi/2] \), and \( \phi_0 \) denotes the total flux (i.e., the sum of three neutrino fluxes). Provided ultrahigh-energy neutrinos are produced by certain astrophysical sources (e.g., Active Galactic Nuclei or AGN) via the decay of pions created from \( pp \) and \( p\gamma \) collisions, for instance, their flavor content is expected to be

\[
\{ \phi_e : \phi_\mu : \phi_\tau \} = \left\{ \frac{1}{3} : \frac{2}{3} : 0 \right\}. \quad (5)
\]

This “standard” neutrino flux ratio corresponds to \( \zeta = 0 \) and \( \tan \xi = 1/\sqrt{2} \) (or equivalently \( \xi \approx 35.3^\circ \)) in our parametrization. It turns out that any small departure of \( \zeta \) from zero will measure the existence of cosmic \( \nu_\tau \) and \( \bar{\nu}_\tau \) neutrinos, which might come from the decays of \( D_s \) and \( B\bar{B} \) mesons produced at the source [13]. On the other hand, any small deviation of \( \tan^2 \xi \) from \( 1/2 \) will imply that the conventional mechanism for ultrahigh-energy neutrino production from the AGN has to be modified. Similar arguments can be put forward for the neutrino fluxes from some other astrophysical sources, such as the postulated neutron beam source [14] with

\[
\{ \phi_e : \phi_\mu : \phi_\tau \} = \{1 : 0 : 0\} \quad (6)
\]

(or equivalently \( \{\xi, \zeta\} = \{\pi/2, 0\} \)) and the possible muon-damped source [15] with

\[
\{ \phi_e : \phi_\mu : \phi_\tau \} = \{0 : 1 : 0\} \quad (7)
\]

(or equivalently \( \{\xi, \zeta\} = \{0, 0\} \)). Therefore, we are well motivated to investigate how the true values of \( \xi \) and \( \zeta \) for a specific astrophysical source can be determined or constrained by use of the second-generation neutrino telescopes and with the help of more precise data from the upcoming long-baseline neutrino oscillation experiments. This goal is indeed reachable, as we shall explicitly demonstrate in the remaining part of this paper.

The remaining part of this paper is organized as follows. In section II, we derive the analytical relations between the neutrino flavor parameters (\( \xi \) and \( \zeta \)) at an astrophysical source and the typical observables of neutrino fluxes at a terrestrial detector. Section III is devoted to a detailed numerical analysis of the dependence of those observables on \( \xi \) and \( \zeta \). A brief summary of our main results is given in section IV.
II. OBSERVABLES

Because of neutrino oscillations and the $\nu_\tau$ “regeneration” in the Earth [2], it is especially important to detect all three flavors of the cosmic neutrinos at a neutrino telescope. The sum of $\phi^D_e$, $\phi^D_\mu$ and $\phi^D_\tau$ is equal to that of $\phi_e$, $\phi_\mu$ and $\phi_\tau$,

$$\phi_0 \equiv \phi_e + \phi_\mu + \phi_\tau = \phi^D_e + \phi^D_\mu + \phi^D_\tau,$$

as one may easily see from Eqs. (2) and (3). A measurement of $\phi_0$ may involve large systematic uncertainties, but the latter can be largely cancelled out in the ratio of two neutrino fluxes. Therefore, let us follow Ref. [12] to define

$$\left\{ R_e, R_\mu, R_\tau \right\} \equiv \left\{ \frac{\phi^D_e}{\phi^D_\mu + \phi^D_\tau}, \frac{\phi^D_\mu}{\phi^D_\mu + \phi^D_e}, \frac{\phi^D_\tau}{\phi^D_e + \phi^D_\mu} \right\}$$

(9)

as our working observables. We remark that these ratios are largely free from the systematic uncertainties associated with the measurements of $\phi^D_e$, $\phi^D_\mu$ and $\phi^D_\tau$. In particular, it is relatively easy to extract $R_\mu$ from the ratio of muon tracks to showers at IceCube [16], even if those electron and tau events may not well be disentangled. Since $R_e$, $R_\mu$ and $R_\tau$ satisfy

$$\frac{R_e}{1 + R_e} + \frac{R_\mu}{1 + R_\mu} + \frac{R_\tau}{1 + R_\tau} = 1,$$

(10)

only two of them are independent.

Note that Eqs. (6) and (7) represent two peculiar (non-standard) scenarios of cosmic neutrino production. In principle, one may also assume an exotic astrophysical source which only produces $\nu_\tau$ and $\bar{\nu}_\tau$ neutrinos; i.e., $\{ \phi_e : \phi_\mu : \phi_\tau \} = \{ 0 : 0 : 1 \}$ or equivalently $\zeta = \pi/2$ with unspecified $\xi$ in our parametrization. The expression of $R_\alpha$ (for $\alpha = e, \mu, \tau$) can then be simplified in such special cases:

$$R_\alpha = \begin{cases} \frac{P_{e\alpha}}{1 - P_{e\alpha}}, & \text{for } \{ \xi, \zeta \} = \{ \pi/2, 0 \}, \\ \frac{P_{\mu\alpha}}{1 - P_{\mu\alpha}}, & \text{for } \{ \xi, \zeta \} = \{ 0, 0 \}, \\ \frac{P_{\tau\alpha}}{1 - P_{\tau\alpha}}, & \text{for } \{ \xi, \zeta \} = \{ \ast, \pi/2 \}. \end{cases}$$

(11)

Even if the third case is completely unrealistic, it could serve as an example to illustrate the salient feature of $R_\alpha$ defined above.

Without loss of generality, we choose $R_e$ and $R_\mu$ as two typical observables and derive their explicit relations with $\xi$ and $\zeta$. By using Eqs. (4), (8) and (9), we obtain

$$R_e = \frac{P_{ee} \sin^2 \xi + P_{\mu e} \cos^2 \xi + P_{\tau e} \tan^2 \zeta}{\sec^2 \zeta - \left[ P_{ee} \sin^2 \xi + P_{\mu e} \cos^2 \xi + P_{\tau e} \tan^2 \zeta \right]},$$

$$R_\mu = \frac{P_{e\mu} \sin^2 \xi + P_{\mu \mu} \cos^2 \xi + P_{\tau \mu} \tan^2 \zeta}{\sec^2 \zeta - \left[ P_{e\mu} \sin^2 \xi + P_{\mu \mu} \cos^2 \xi + P_{\tau \mu} \tan^2 \zeta \right]}.$$

(12)
The source flavor parameters $\xi$ and $\zeta$ can then be figured out in terms of $R_e$ and $R_\mu$:

$$\sin^2 \xi = \frac{r_e \left(P_{\tau \mu} - P_{\mu \mu}\right) - r_\mu \left(P_{\tau e} - P_{\mu e}\right) + \left(P_{\mu \mu} P_{\tau e} - P_{\mu e} P_{\tau \mu}\right)}{(r_e - P_{\tau e}) \left(P_{e \mu} - P_{\mu \mu}\right) - (r_\mu - P_{\tau \mu}) \left(P_{e e} - P_{\mu e}\right)},$$

$$\tan^2 \zeta = \frac{r_e \left(P_{\mu \mu} - P_{e \mu}\right) - r_\mu \left(P_{\mu e} - P_{e e}\right) + \left(P_{e \mu} P_{\mu e} - P_{e e} P_{\mu \mu}\right)}{(r_e - P_{\tau e}) \left(P_{e \mu} - P_{\mu \mu}\right) - (r_\mu - P_{\tau \mu}) \left(P_{e e} - P_{\mu e}\right)},$$

(13)

where the notations $r_e \equiv R_e/(1 + R_e)$ and $r_\mu \equiv R_\mu/(1 + R_\mu)$ have been used to simplify the expressions. Indeed, $r_e = \phi_e^D/\phi_0$ and $r_\mu = \phi_\mu^D/\phi_0$ hold. One may in principle choose either $(R_e, R_\mu)$ or $(r_e, r_\mu)$ as a set of working observables to inversely determine $\xi$ and $\zeta$. The first set has been chosen by a few authors (see, e.g., Refs. [10,12]) and will also be adopted in this paper. Note that the averaged neutrino oscillation probabilities $P_{\alpha \beta}$ depend on three mixing angles ($\theta_{12}, \theta_{23}, \theta_{13}$) and the Dirac CP-violating phase ($\delta$) in the standard parametrization of the $3 \times 3$ neutrino mixing matrix $V$ [17]. Taking account of $\theta_{13} < 10^\circ$ [18] but $\theta_{12} \approx 34^\circ$ and $\theta_{23} \approx 45^\circ$ [19], we express $P_{\alpha \beta}$ as the first-order expansion of the small parameter $\sin \theta_{13}$:

$$P_{ee} = 1 - \frac{1}{2} \sin^2 2\theta_{12},$$

$$P_{e\mu} = \frac{1}{2} \sin^2 2\theta_{12} \cos^2 \theta_{23} + \frac{1}{4} \sin 4\theta_{12} \sin 2\theta_{23} \sin \theta_{13} \cos \delta,$$

$$P_{e\tau} = \frac{1}{2} \sin^2 2\theta_{12} \sin^2 \theta_{23} - \frac{1}{4} \sin 4\theta_{12} \sin 2\theta_{23} \sin \theta_{13} \cos \delta,$$

$$P_{\mu\mu} = 1 - \frac{1}{2} \sin^2 2\theta_{23} - \frac{1}{2} \sin^2 2\theta_{12} \cos^4 \theta_{23} - \frac{1}{2} \sin 4\theta_{12} \sin 2\theta_{23} \cos^2 \theta_{23} \sin \theta_{13} \cos \delta,$$

$$P_{\mu\tau} = \frac{1}{2} \sin^2 2\theta_{23} - \frac{1}{8} \sin^2 2\theta_{12} \sin^2 2\theta_{23} + \frac{1}{8} \sin 4\theta_{12} \sin 4\theta_{23} \sin \theta_{13} \cos \delta,$$

$$P_{\tau\tau} = 1 - \frac{1}{2} \sin^2 2\theta_{23} - \frac{1}{2} \sin^2 2\theta_{12} \sin^4 \theta_{23} + \frac{1}{2} \sin 4\theta_{12} \sin 2\theta_{23} \sin^2 \theta_{23} \sin \theta_{13} \cos \delta,$$

(14)

in which the higher-order terms, such as $O(\sin^2 \theta_{13}) < 3\%$, have been safely neglected. One can see that the sensitivity of $P_{\alpha \beta}$ to $\delta$ is suppressed due to the smallness of $\sin \theta_{13}$. Hence the dependence of $R_\alpha$ on $\delta$ is expected to be insignificant. In addition, it is hard to distinguish between the cases of $\theta_{13} = 0^\circ$ and $\delta = 90^\circ$, because it is the product of $\sin \theta_{13}$ and $\cos \delta$ that appears in the analytical approximations of $P_{\alpha \beta}$.

A global analysis of current neutrino oscillation data [19] yields

$$30^\circ < \theta_{12} < 38^\circ,$$

$$36^\circ < \theta_{23} < 54^\circ,$$

$$0^\circ \leq \theta_{13} < 10^\circ,$$

(15)

at the 99% confidence level. The best-fit values of three neutrino mixing angles are $\theta_{12} \approx 34^\circ$, $\theta_{23} \approx 45^\circ$ and $\theta_{13} \approx 0^\circ$ [19], but the CP-violating phase $\delta$ is entirely unrestricted. Although the present experimental data remain unsatisfactory, they can be used to constrain the correlation between the parameters ($\xi, \zeta$) and the observables ($R_\alpha, R_\beta$). We shall illustrate our analytical results by taking a few typical numerical examples in the subsequent section.
III. ILLUSTRATION

First of all, let us follow a rather conservative strategy to scan the reasonable ranges of \((\theta_{12}, \theta_{23}, \theta_{13}, \delta)\) and \((\xi, \zeta)\) so as to examine the sensitivities of \((R_e, R_\mu, R_\tau)\) to these six parameters. We take \(\delta \in [0^\circ, 180^\circ]\) in addition to the generous intervals of three mixing angles given in Eq. (15), and allow \(\xi\) to vary in the region \(\xi \in [0^\circ, 90^\circ]\). As the amount of \(\nu_\tau\) and \(\bar{\nu}_\tau\) neutrinos produced at those realistic astrophysical sources is expected to be very small or even vanishing, we restrict the parameter \(\zeta\) to a very narrow domain \(\zeta \in [0^\circ, 18^\circ]\), which corresponds to \(\sin^2 \zeta \lesssim 0.1\) [13]. It should be noted that we do the numerical computation by using the exact expressions of \(P_{\alpha\beta}\) instead of Eq. (14). Our results are shown in FIG. 1. Two comments are in order:

1. The source parameter \(\xi\) is quite sensitive to the values of the neutrino flux ratios \(R_\alpha\)
   (for \(\alpha = e, \mu, \tau\)). Even if three neutrino mixing angles involve a lot of uncertainties
   and the Dirac CP-violating phase is entirely unknown, a combined measurement of
   \((R_e, R_\mu)\) or \((R_e, R_\tau)\) can constrain the value of \(\xi\) to an acceptable degree of accuracy.
   This encouraging observation assures that the second-generation neutrino telescopes
   can really be used to probe the initial flavor composition of ultrahigh-energy neutrino fluxes.
   Note that the standard pion-decay source \(\{\phi_e : \phi_\mu : \phi_\tau\} = \{1/3 : 2/3 : 0\}\) will produce
   \(\{\phi_e^D : \phi_\mu^D : \phi_\tau^D\} \simeq \{1/3 : 1/3 : 1/3\}\) or equivalently \(R_e \approx R_\mu \approx R_\tau \approx 0.5\) at the
   detector, as shown in FIG. 1, if \(\theta_{23} \approx 45^\circ\) is fixed and \(\theta_{13} < 10^\circ\) is taken. Such an
   expectation cannot be true, however, when \(\xi\) deviates from its given value \(\xi = 35.3^\circ\)
   and (or) when \(\theta_{23}\) departs from its best-fit value \(\theta_{23} = 45^\circ\). More precise neutrino
   oscillation data will greatly help to narrow down the \((R_\alpha, \xi)\) parameter space.

2. In contrast, the source parameter \(\zeta\) seems to be insensitive to \(R_\alpha\) (for \(\alpha = e, \mu, \tau\)).
   The reason for this insensitivity is two-fold: (a) the values of \(\zeta\) have been restricted
   to a very narrow range \((0^\circ \leq \zeta \leq 18^\circ)\); and (b) the numerical uncertainties of three
   neutrino mixing angles and the Dirac CP-violating phase are too large. Provided \(\theta_{12}, \theta_{23}, \theta_{13}\)
   and \(\delta\) are all measured to a high degree of accuracy in the near future, it will
   be possible to find out the definite dependence of \(R_\alpha\) on \(\zeta\) for a given value of \(\xi\).

To be more explicit, we are going to consider three typical scenarios of cosmic neutrino fluxes
and illustrate the sensitivities of \(R_e, R_\mu\) and \(R_\tau\) to the source parameters \((\xi, \zeta)\) and to the
unknown neutrino mixing parameters \((\theta_{13}, \delta)\).

We argue that the simple flavor content of ultrahigh-energy neutrino fluxes from the
standard pion-decay source could somehow be contaminated for certain reasons: e.g., a
small amount of \(\nu_e, \nu_\mu\) and \(\nu_\tau\) and their antiparticles might come from the decays of heavier
hadrons produced by \(pp\) and \(p\gamma\) collisions [13]. Similar arguments can also be made for
the postulated neutron beam source and the possible muon-damped source, as our present
knowledge about the mechanism of cosmic neutrino production remains very poor. Following
a phenomenological approach, we slightly modify the scenarios listed in Eqs. (5), (6) and (7)
by allowing the relevant \(\xi\) and \(\zeta\) parameters to fluctuate around their given values. Namely, we consider

- **Scenario A:** \(30^\circ \leq \xi \leq 40^\circ\) and \(0^\circ \leq \zeta \leq 18^\circ\), serving as a modified version of the
  standard pion-decay source (originally, \(\xi = 35.3^\circ\) and \(\zeta = 0^\circ\));
• **Scenario B:** $80 \leq \xi \leq 90^\circ$ and $0^\circ \leq \zeta \leq 18^\circ$, serving as a modified version of the postulated neutron beam source (originally, $\xi = 90^\circ$ and $\zeta = 0^\circ$);

• **Scenario C:** $0^\circ \leq \xi \leq 10^\circ$ and $0^\circ \leq \zeta \leq 18^\circ$, serving as a modified version of the possible muon-damped source (originally, $\xi = 0^\circ$ and $\zeta = 0^\circ$).

For simplicity, we fix $\theta_{12} = 34^\circ$ and $\theta_{23} = 45^\circ$ in our numerical analysis. We take four typical inputs for the unknown parameters $\theta_{13}$ and $\delta$: (a) $\theta_{13} = 0^\circ$ (in this case, $\delta$ is not well-defined and has no physical significance); (b) $\theta_{13} = 5^\circ$ and $\delta = 0^\circ$; (c) $\theta_{13} = 5^\circ$ and $\delta = 90^\circ$; and (d) $\theta_{13} = 5^\circ$ and $\delta = 180^\circ$. Our numerical results for the sensitivities of $R_e$, $R_\mu$, and $R_\tau$ to $\xi$ and $\zeta$ in scenarios A, B and C are shown in FIGs. 2, 3 and 4, respectively. Some discussions are in order.

1. **Scenario A in FIG. 2.** The neutrino flux ratios $R_e$, $R_\mu$, and $R_\tau$ are all sensitive to the small deviation of $\xi$ from its standard value $\xi = 35.3^\circ$. In contrast, the changes of three observables are very small when $\zeta$ varies from $0^\circ$ to $18^\circ$. This insensitivity is understandable: about half of the initial $\nu_\mu$ and $\bar{\nu}_\mu$ neutrinos oscillate into $\nu_\tau$ and $\bar{\nu}_\tau$ neutrinos, whose amount dominates over the survival amount of initial $\nu_\tau$ and $\bar{\nu}_\tau$ neutrinos at the detector. In other words, $\phi_D^p \gg \phi_\tau$ and $\phi_D^{-} \sim \phi_D^{-} \sim \phi_D^{-}$ hold, implying that $R_\alpha$ must be insensitive to small $\phi_\tau$ or equivalently to small $\zeta$. It is therefore difficult to pin down the value of $\zeta$ from this kind of astrophysical sources. As pointed out in the last section, the result of $R_e$ in the $\theta_{13} = 0^\circ$ case (solid curves) is almost indistinguishable from that in the $\delta = 90^\circ$ case (dotted curves). The sensitivity of $R_\alpha$ to $\delta$ is insignificant but distinguishable, if the value of $\theta_{13}$ is about $5^\circ$ or larger.

2. **Scenario B in FIG. 3.** The neutrino flux ratio $R_e$ is sensitive to the small departures of $\xi$ and $\zeta$ from their given values $\xi = 90^\circ$ and $\zeta = 0^\circ$. This salient feature can be understood as follows. Since $\nu_\mu$ (or $\bar{\nu}_\mu$) and $\nu_\tau$ (or $\bar{\nu}_\tau$) neutrinos at the detector mainly come from the initial $\nu_e$ (or $\bar{\nu}_e$) neutrinos via the oscillation, the sum of $\phi_D^p$ and $\phi_D^{-}$ is expected to be smaller than or comparable with the survival $\nu_e$ (or $\bar{\nu}_e$) flux $\phi_D^e$. The roles of $\phi_e$ and $\phi_\tau$ are important in $\phi_D^e$ and $\phi_D^\mu + \phi_D^\tau$, respectively. It turns out that $R_e$ depends, in a relatively sensitive way, on $\xi$ through $\phi_D^e$ in its numerator and on $\zeta$ through $\phi_D^\mu + \phi_D^\tau$ in its denominator. Note also that the nearly degenerate results of $R_e$ for $\theta_{13} = 5^\circ$ and $\delta = (0^\circ, 90^\circ, 180^\circ)$ are primarily attributed to the fact that $P_{ee}$ in the numerator of $R_e$ is actually independent of $\delta$. On the other hand, the discrepancy between $R_e(\theta_{13} = 0^\circ)$ and $R_e(\theta_{13} = 5^\circ)$ in FIG. 3 results from the $\mathcal{O}(\sin^2 \theta_{13})$ terms of $P_{\alpha\beta}$, which have been neglected in Eq. (14). In comparison with $R_e$, the neutrino flux ratios $R_\mu$ and $R_\tau$ are not so sensitive to the small changes of $\xi$ and $\zeta$. But the measurement of $R_\mu$ and $R_\tau$ is as important as that of $R_e$, in order to determine the flavor composition of ultrahigh-energy neutrino fluxes at such an astrophysical source.

3. **Scenario C in FIG. 4.** In this case, in which $\phi_D^e$ and $\phi_D^\mu$ mainly come from the initial $\phi_\mu$ via $\nu_\mu \rightarrow \nu_e$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations, one can similarly understand the numerical behaviors of $R_e$, $R_\mu$ and $R_\tau$ changing with the small deviation of $\xi$ from its given value $\xi = 0^\circ$. To explain why three observables are almost independent of the fluctuation of $\zeta$, we take $\theta_{23} = 45^\circ$ and $\theta_{13} \rightarrow 0^\circ$, which guarantee $P_{\tau\alpha} = P_{\mu\alpha}$ (for $\alpha = e, \mu, \tau$) to
hold in the leading-order approximation. We can then simplify Eq. (12) in the $\xi \to 0^\circ$ limit and arrive at the following result:

$$R_{\alpha}|_{\xi \to 0^\circ} = \frac{P_{\mu\alpha} + P_{\tau\alpha} \tan^2 \zeta}{\sec^2 \zeta - \left( P_{\mu\alpha} + P_{\tau\alpha} \tan^2 \zeta \right)} = \frac{P_{\mu\alpha}}{1 - P_{\mu\alpha}},$$

(16)

where $\zeta$ is completely cancelled out. Therefore, the tiny dependence of $R_{\alpha}$ on $\zeta$ appearing in FIG. 4 is just a natural consequence of the $O(\sin \theta_{13})$ or $O(\sin^2 \theta_{13})$ corrections to Eq. (16).

Although the numerical examples taken above can only serve for illustration, they do give us a ball-park feeling of the correlation between the source parameters ($\xi, \zeta$) and the neutrino mixing parameters ($\theta_{12}, \theta_{23}, \theta_{13}, \delta$) via the working observables ($R_e, R_\mu, R_\tau$) at a neutrino telescope. This observation is certainly encouraging and remarkable.

**IV. SUMMARY**

We have proposed a simple parametrization of the initial flavor composition of ultrahigh-energy neutrino fluxes generated from very distant astrophysical sources: $\phi_e : \phi_\mu : \phi_\tau = \sin^2 \xi \cos^2 \zeta : \cos^2 \xi \cos^2 \zeta : \sin^2 \zeta$. The conventional mechanism and the postulated scenarios for cosmic neutrino production can all be reproduced by taking the special values of ($\xi, \zeta$). Of course, such a parametrization is by no means unique. An alternative,

$$\phi_e : \phi_\mu : \phi_\tau = \frac{x}{1 + t} : \frac{1 - x}{1 + t} : \frac{t}{1 + t},$$

(17)

with $x \in [0, 1]$ and $t \in [0, \infty)$ in general, is also simple and useful. For a realistic astrophysical source, it should be more reasonable to take $0 \leq t \ll 1$. Comparing between Eqs. (4) and (17), one can immediately arrive at $x = \sin^2 \xi$ and $t = \tan^2 \zeta$. The ($x, t$) and ($\xi, \zeta$) languages are therefore equivalent to each other.

After defining three neutrino flux ratios $R_{\alpha}$ (for $\alpha = e, \mu, \tau$) as our working observables at a neutrino telescope, we have shown that the source parameters $\xi$ and $\zeta$ can in principle be determined by the measurement of two independent $R_{\alpha}$ and with the help of accurate neutrino oscillation data. The standard pion-decay source, the postulated neutrino beam source and the possible muon-damped source have been slightly modified to illustrate the sensitivities of $R_{\alpha}$ to small departures of $\xi$ and $\zeta$ from their given values. We have also examined the dependence of $R_{\alpha}$ upon the smallest neutrino mixing angle $\theta_{13}$ and upon the Dirac CP-violating phase $\delta$. Our numerical examples indicate that it is quite promising to determine or constrain the initial flavor content of ultrahigh-energy neutrino fluxes with the second-generation neutrino telescopes.

How to measure $R_{\alpha}$ to an acceptable degree of accuracy is certainly a big challenge to IceCube and other neutrino telescopes. A detailed analysis of the feasibility of our idea for a specific neutrino telescope is desirable, but it is beyond the scope of this paper. Here we remark that our present understanding of the production mechanism of cosmic neutrinos needs the observational and experimental support. We expect that neutrino telescopes may help us to attain this goal in the long run.
ACKNOWLEDGMENTS

One of us (Z.Z.X.) would like to thank X.D. Ji for his warm hospitality at the University of Maryland, where part of this work was done. He is also grateful to Z. Cao for useful discussions, and to R. McKeown and P.D. Serpico for helpful communications. Our research is supported in part by the National Natural Science Foundation of China.
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FIG. 1. Allowed regions of the neutrino flux ratios $R_\alpha$ ($\alpha = e, \mu, \tau$) versus the source parameters $\xi$ and $\zeta$, where we have scanned the 99% C.L. intervals of three neutrino mixing angles and taken the Dirac CP-violating phase $\delta \in [0^\circ, 180^\circ]$. The horizontal and vertical lines in the $(R_\alpha, \xi)$ plots correspond to $\xi = 35.3^\circ$ and $R_\alpha = 0.5$, respectively.
FIG. 2. Numerical illustration of **scenario A**, where $\theta_{12} = 34^\circ$ and $\theta_{23} = 45^\circ$ have typically been input in our calculations. The horizontal and vertical lines in the $(\xi, R_\alpha)$ plots correspond to $R_\alpha = 0.5$ and $\xi = 35.3^\circ$, respectively.
FIG. 3. Numerical illustration of scenario B, where $\theta_{12} = 34^\circ$ and $\theta_{23} = 45^\circ$ have typically been input in our calculations.
FIG. 4. Numerical illustration of scenario C, where $\theta_{12} = 34^\circ$ and $\theta_{23} = 45^\circ$ have typically been input in our calculations.