Stangeness in Compact Stars and Signal of Deconfinement

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Phase transitions in compact stars are discussed including hyperonization, deconfinement and crystalline phases. Reasons why kaon condensation is unlikely is reviewed. Particular emphasis is placed on the evolution of internal structure with spin down of pulsars. A signature of a first order phase transition in the timing structure of pulsars which is strong and easy to measure, is identified.

1 Introduction

During their study of supernovae, Baade and Zwicky suggested in 1934 that the enormous energy that is released so suddenly as to make even distant objects visible in daylight and for weeks thereafter must originate in the transition of the core of an ordinary star to a neutron star—a star consisting of closely packed neutrons.

By an elementary calculation of the type one learns in potential theory, one can estimate the gravitational energy of a star of closely packed neutrons and find that a nucleon in a neutron star is bound 10 times more strongly by the weakest force, gravity, than a nucleon is bound in a nucleus by the strong force [1]. The implication of this statement is immediate. A star, when it is exploded by the binding energy of the newly formed neutron star releases in a few seconds about 10 times as much energy as the luminous star emitted in its lifetime of a few million years.

As was expected, and confirmed in observations made on SN1987A, about 99 % of the binding energy is carried off by neutrinos emitted over about 10 seconds from the hot protoneutron star. Of the remaining 1 % almost all is carried by the kinetic energy of the stellar remnant as it is hurled into space at a velocity of 12,000 km/s. The remaining fraction of the 1 % is in visible light.

These are wondrous thoughts and they inform us that the star that is formed at the death of a luminous star is made of matter under extreme conditions. In the year 1047 the Chinese court astronomer of the Sung Dynasty “observed the apparition of a guest star .... its color an iridescent yellow...” This was a part of the 1/10 % of energy that appeared as visible light. It was the Crab supernova visible today, much expanded and accelerating. The acceleration is powered by the rotational energy of the Crab pulsar, the neutron star formed at the end of the 10 million year life of the presupernova star. From observations on the nebula, it can be inferred that the power input by the pulsar is $10^{44}$ MeV/s. From this one compute the moment of inertia and then the rotational energy as $10^{55}$ MeV. The Crab pulsar, rotating now
Figure 1: Evolution of the Crab pulsar’s period according to the measured braking index.

30 times a second, will be rotating in a million years once a second (see Fig. 1). And still have some rotational energy left. We will exploit this slow rate of spin-down to develop a diagnostic for the deconfinement phase transition.

What is implied by the above observations about the internal structure and composition of neutron stars? I have already noted that gravitational binding is 10 times nuclear binding (see Fig. 2). The average density of neutron stars is several times nuclear density. This we can infer by balancing gravity against the centrifugal force at the surface of a millisecond pulsar. So nucleons reside within the repulsive interaction of their neighbors; gravity binds the star. The nuclear force resists gravity but fails; neutron stars exist. In crushing nucleons to densities where nuclear repulsion is dominant (see Fig. 3), gravity brings the Pauli principle fully into play in distributing baryon charge over as many species such as hyperons and quarks as is energetically favorable. The name neutron star therefore has to be understood as generic.

Stars rotate. We know this from the doppler broadening of the spectral lines of luminous stars. We infer it also from the periodic radiation from pulsars. If the periodicity were caused by vibration, the amplitude would decrease with time and the frequency remain constant. What is seen is the converse. The conclusion that what we see from pulsars is the beamed radiation of a rotating source makes eminent physical sense. The amplitude of vibrations would soon be damped by viscosity. Rotation is damped very weakly by coupling to the electromagnetic field and not at all by viscosity. This accords with the observed small rates of change of pulsar periods. A pulsar could be seen for only a short time in the first case; for millions of years in the second. If the pulsar signal were caused by vibration the observed pulsar population would imply the existence of an enormous number of silent neutron stars and their number would be irreconcilable with supernova rates.

Stars have magnetic fields. This we know from the Zeeman effect. Flux conserva-
tion implies a very high field for the collapsed remnant. So the periodic signal from a pulsar is caused by the rotation of a magnetized neutron star. Like the magnetic field, the angular velocity is scaled up in the collapse, sometimes to tens of times per second. They slow down very slowly, taking 10 million years to do so. During the stage of rapid rotation they are centrifugally flattened. As they spin down due to the radiation their central density rises and they become more spherical. Through the Pauli principle, the increase of density with time induces continuous changes in internal composition. Like us, their faces change continuously. Like us they also may become more interesting with age.

I will tell you about some of the many ways in which the Pauli principle can act to create the many faces of neutron stars and the way in which spin-down of a pulsar may be used to detect some of the faces.

### 1.1 Charge Neutrality

The first thing I should say about stars—any star—is that to a very high degree they are charge neutral. This follows at once if we examine the balance between gravitational attraction on a charged particle at the surface of a star and the Coulomb repulsion acting on it. We find for a proton that

$$\frac{Z_{\text{net}}}{A} < \left(\frac{m_e}{e}\right)^2 \sim 10^{-36}. $$
This does not mean that no charged particles above this small number can exist in the star. It only means that the number of positive and negatively charged particles must be nearly equal. Nevertheless it is a very stringent condition on the nature of stellar matter as we shall see. Charge neutrality of a star (more precisely, its isospin asymmetry) also gives rise to a most amazing phenomenon—the formation of a Coulomb lattice in the region of a star occupied by two phases in equilibrium—the so-called mixed phase of a first order phase transition.

1.2 Degenerate Matter

Compact stars cool very rapidly after birth, falling from temperatures of several tens of MeV to an MeV or less in a few seconds as the neutrinos carry the bulk of the binding energy off. Thereafter they cool more slowly, but they are cold on the nuclear scale in any observational time-frame. Therefore compact stars are degenerate Fermi systems. The smearing at the Fermi surface due to temperature is extremely small compared to the Fermi energy. The Fermions fill all lowest momentum states up to the Fermi level. These momentum states are not the momentum states of free particles because of the interactions. But for purposes of discussion we may think of them as free momentum states of energy and density

\[ E_p = \sqrt{m^2 + p^2}, \quad \rho = \frac{1}{(2\pi)^3} \int_0^{p_F} p^2 d^3p. \]

2 Hyperon Stars

Originally, compact stars were thought of as closely packed neutrons. Even to-day some researchers, especially those who seek to solve the Brueckner-Bethe many-body theory with so-called realistic forces treat neutron stars as purely neutron. However this is very unrealistic. First we know from the valley of beta stability, that nuclear matter prefers to be isospin symmetric. Gravity does not fully permit this as noted above since excess charge would simply be blown away. There is of course another principle, and that is that a fully developed star should be in the ground state of cold matter. So the matter of a compact star will therefore be in the lowest energy state—at each density—consistent with charge neutrality.

What does the above conclusion mean for the composition of neutron star matter in first approximation? It means that a compact star cannot be purely made of neutrons. Such a configuration would be neutral but not in the lowest energy state. Some neutrons at the top of the Fermi sea would have energy sufficient to inverse beta decay to a proton, electron and neutrino. The neutrino, having no mass or very small mass, will have the escape velocity when it diffuses to the surface of the star. Lepton number is therefore not conserved and the star’s energy is lowered by their loss.

How many neutrons will decay and to what? It depends on the density so that the composition of neutron star matter is a continuously changing function of density. Let us see how this works. At low density, neutrons will decay to protons and electrons.
until the energy of the system cannot be further lowered. This situation is referred to as beta equilibrium and is attained when the Fermi energies satisfy the relation

\[ \mu_n = \mu_p + \mu_e. \]

This equation states that the momentum eigenstates are filled in such a way that no energy can be gained or lost by a reaction in either direction.

The Fermi momentum of the neutron will increase with increasing density toward the interior of a star \( (k_F \propto \rho^{1/3}) \). Eventually, the neutron Fermi energy will be so high that the neutron can decay also to other baryon species besides the proton. It can decay for example to the \( \Lambda \) or \( \Sigma^- \), etc. You may object that strangeness is changed by such decays. However it is only the strong interaction that conserves strangeness. The weak interaction violates it. For example,

\[ \mu^- + K^+ \longrightarrow \mu^- + \mu^+ + \nu \longrightarrow 2\gamma + \nu. \]

The gammas and neutrinos diffuse out of the star lowering its energy. Reactions become irreversible. Net strangeness is built up through reactions like the strong followed by weak interaction

\[ N + N + \mu^- \longrightarrow N + \Sigma^- + K^+ \mu^- \longrightarrow N + \Sigma^- + 2\gamma + \nu, \]

or by the weak interaction (see Fig. 4)

\[ N + N + \mu^- \longrightarrow N + \Sigma^- + 2\gamma + \nu \]

The strong reaction followed by the weak can occur when the protoneutron star is

\[ \text{Figure 4: Weak interactions such as the one depicted are of little importance in vacuum because of their strong suppression but carry a neutron star to its final equilibrium state in the confined phase when there is insufficient energy to produce an associated kaon.} \]

far from equilibrium, and the direct weak conversion of a nucleon to hyperon will carry the system to final equilibrium when there is insufficient energy to produce
the intermediate state containing the kaon. You see at each point the Pauli principle drives neutral matter from a predominance of neutrons at low density to an ever richer mixture of baryon species, especially hyperons at high density where the increasing Fermi energy exceeds the thresholds for conversion of neutrons to other more massive baryon species. The conclusion is robust since it rests principally on the exclusion principle.

**3 Neutron Star Matter**

I discussed hyperonization and kaon condensation in 1985 and found that hyperonization has a strong effect on limiting the maximum mass that a neutron star can have, and that kaon condensation was most likely prevented by hyperonization [2]. I will recall the reasons for this in a latter section.

The covariant Lagrangian that I generalized to include hyperons (i.e., the entire baryon octet) from earlier work of Johnson and Teller [3], Duerr [4], in the mid 1950’s and and Walecka [5] in 1974 is

\[
\mathcal{L} = \sum_B \overline{\psi}_B (i\gamma_\mu \partial^\mu - m_B + g_{\sigma B} \sigma - g_\omega B \gamma_\mu \omega^\mu - \frac{1}{2} g_\rho B \gamma_\mu \tau \cdot \rho^\mu) \psi_B \\
+ \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) - \frac{1}{4} \omega_{\mu\nu} \omega^{\mu\nu} + \frac{1}{2} m_\omega^2 \omega_{\mu} \omega^{\mu} \\
- \frac{1}{4} \mathbf{P}_{\mu\nu} \cdot \mathbf{P}^{\mu\nu} + \frac{1}{2} m_\rho^2 \mathbf{P}_{\mu} \cdot \mathbf{P}^{\mu} - \frac{1}{3} b m_n (g_{\sigma} \sigma)^3 - \frac{1}{4} c (g_{\sigma} \sigma)^4 \\
+ \sum_{e^-,\mu^-} \overline{\psi}_\lambda (i\gamma_\mu \partial^\mu - m_\lambda) \psi_\lambda.
\]

I certainly do not want to discuss the theory in any detail. First of all I think it should be emphasized that we have very little knowledge of matter at high density and it seems to me that there is a limit to how much effort can be invested meaningfully in elaborating theories of dense matter. I choose instead a simple theory guided by the following principles:

1. Compact stars are relativistic and should be described in the framework of General Relativity.
2. Matter is causal and should be described by a relativistically covariant theory.
3. Stellar matter at any density obeys the condition of microscopic stability (i.e. Chatelier’s principle).
4. At high density (or momentum transfer) quarks are asymptotically free.
5. Theory of dense matter should be constrained by what is known of nuclear matter at saturation (binding, saturation density, compressibility, symmetry energy, nucleon effective mass).
6. Since nuclei provide no information on the hyperon sector, constraints on hyperon couplings should be obtained from data on hypernuclei and the binding of the Λ in nuclear matter.

I believe that what is allowed by the laws of nature is quite possibly realized somewhere in the universe. It is in that spirit that I study compact stars and their many faces—hyperon stars, hybrid stars, strange matter stars..., constrained by the above principles and data.

**Figure 5:** Baryon populations in a hyperon star [6].

**Figure 6:** Stellar sequences labelled by the completeness of the description of hadronic matter [1]. Reprinted with permission of Springer–Verlag New York; copyright 1997.

### 4 Composition of Hyperon Stars

The result of incorporating hyperons into a description of neutron stars is illustrated in Fig. 1. Neutrons remain the majority at 60% but protons and strange baryons each make up about 20% in the maximum mass star [3].

Near the edge of the star, neutrons are the dominant species as can be seen from Fig. 3, but at a depth of several kilometers, the Σ− is populated, and at all higher densities, the electron population is suppressed. This is because baryon number in the star is conserved but not lepton number (neutrino diffusion). Consequently, with rising density, charge neutrality can be achieved among charged particles carrying baryon charge with little need for electrons. Therefore the electron population saturates and falls with increasing density. (See Fig. 2 in Ref. [2] and the discussion of...
kaon condensation of section III.) Electron saturation has an important consequence for kaons, as was explained already in 1985 [2], kaons would be energetically favorable substitutes for electrons if the electron chemical potential (Fermi energy) were to exceed the kaon mass in the medium. In such an event, kaons, being bosons, could all occupy the lowest energy state (having zero momentum) whereas electrons, being Fermions, cost the Fermi energy. However, when charge neutrality can be achieved among the baryons, whose number is fixed at the birth of the star, not even the cost of a reduced kaon effective mass has to be paid. Deconfinement has the same effect on the electron chemical potential as charged baryons and for the same reason.

Three stellar sequences are shown in Fig. 6 corresponding to different degrees of completeness of the calculation with respect to beta equilibrium. Some of the possible consequences of this revised picture of a “neutron” star are:

1. The maximum mass that the Fermi pressure and the repulsion of the nuclear force can sustain against gravity will be reduced, because the transformation of nucleons at the top of their Fermi sea to hyperons reduces the pressure, that is to say, softens the equation of state. This effect, first discussed in Ref. [2], was refined in Ref. [6]. The estimated limiting neutron star mass lies in the range 1.5 to 2$M_\odot$ and its radius lies in the range $\sim 10.5$ to 12 km. However, as stated above, theory is only constrained by general principles at high density, and the quoted numbers, as with any others, have to be understood in that context.

2. It is usually thought that at the end of its life a star will either collapse entirely to a black hole equal to its mass, say 10 to 50 $M_\odot$ or a small fraction of the binding energy of the core will be transmitted by neutrinos to the infalling star and explode it leaving behind a neutron star. There is another possibility suggested by the Fig. 6. If the baryon number of the collapsing core produces a compact star whose mass lies above the limiting mass of a hyperon star but below that of a neutron star (which we may use to approximate a protoneutron star before it has cooled and deleptonized), the core will hover in a hot bloated state for a few seconds while many of the neutrinos escape, blowing off the rest of the star as they do so, and then the core will collapse to a low-mass black hole of 1.5 to 2 $M_\odot$ [7, 8, 9].

3. We expect therefore two populations of black holes (1) massive ones of tens of solar masses that swallow up the entire progenitor star, neutrinos included (no supernova) (2) Low-mass black holes accompanied by a neutrino burst and a SN that distributes enriched material to the cosmos.

4. Transport properties like electrical conductivity, superfluity, and cooling of the star will be effected by the hyperon populations.

5 Hybrid Stars

There is another possibility, not exclusive of the hyperon nature of dense neutral matter. If densities are attained in the centers of compact stars that are high enough
to crush nucleons into their quark constituents then quark matter, which existed in
the early universe as very hot matter, may exist in a cold state in the cores of hyperon
stars. I call these hybrid stars \[10, 11\].

The possibility of quark matter interiors was discussed already in 1976 by Baym
and others, and was investigated right up to the present. However I discovered in
1990 that a very important aspect of phase transitions was overlooked in all of the
early work, including my own.

The consequences that follow from a correct treatment of the phase transition are
quite astonishing. We find that a remarkably intricate crystalline lattice of nuclear
and quark matter is formed, that changes in size and form in various regions of the
star according to the ambient pressure.

I will not go into a full description of the theorems that I proved. But I will briefly
state them and physically motivate them. I recall to you first what is very familiar.
You know that when ice begins to melt, as the day warms, the temperature of the
mixture of ice and water remains constant until the ice is completely transformed.
Then the water can warm.

What may not be so familiar is that not only does the temperature of water and
ice remain constant but so do all their other properties. The density of ice and the
density of water remain constant, for example, until the ice is completely melted. If
the same experiment were performed at constant temperature but varying pressure,
it would be found that the pressure remained constant until the transformation was
complete. This is all very familiar; we know it from our courses in thermodynamics
and statistical physics. What is not familiar is that the above description of a first
order phase transition is a very special case. It pertains to substances that have a
single independent component (or conserved charge)—in the above example, H\(^2\)O.

The general case of a first order phase transition in a (complex) substance of more
than one independent component is very different. I proved the following theorems
\[11\]:

1. Any first order phase transition in a complex substance is characterized by a
variation of the common pressure, and the variation of every property of the
two phases in equilibrium as the proportion of the phases changes. This is the
exact antithesis of the phase transition in the single-component substance that
I just described.

2. If one of the independent components is electric charge, then the two phases
will in general have opposite charges and the energy of the mixed phase will
be minimized when the rare phase arranges itself on a lattice immersed in the
dominant phase.

3. Because of theorem 1, the lattice will vary in form and size with proportion of
the phases.

Words cannot describe wonders as well as images. Let me give you an image. If
water were of such a nature as described, ie. had two instead of one independent
component and one of them was electric charge, then a lake would not freeze over
starting with a sheet of ice on top, but ice spheres would form throughout the volume of the lake, of slightly different size and spacing at top and bottom, because of the different pressure. That is a valid image for neutron star matter when it is in the pressure gradient of the star and it is of such a density that quark matter and nuclear matter are in equilibrium. If I may say so, I find this a remarkable revelation. I am

Figure 7: Showing the quark gas core, surrounding crystalline region, hyperon liquid and thin nuclear crust. Geometric phases are denoted a q(uark) drops, etc. [1] Reprinted with permission of Springer–Verlag New York; copyright 1997.

Figure 8: For a slightly less massive star than depicted in Fig. 7 the interior structure is vastly different. The Coulomb lattice extends to the center, but only several geometrical phases are present [1] Reprinted with permission of Springer–Verlag New York; copyright 1997.

astonished that so many researchers before me missed it. That in fact it took me five years to arrive at this understanding which now seems so clear and obvious.

I have proven the above theorems elsewhere, though I have not always laid them out as theorems [11, 12, 13, 14]. Here I want to provide only the physical understanding, which is just as good as a mathematical proof, and usually more illuminating. Nuclei become unbound around $A \sim 250$, and no other bound configuration of nucleons exists until $A$ is so large that gravity binds them. But neutron star matter must be neutral to be bound by gravity. So while charge neutral nuclear matter is
highly isospin asymmetric and unfavored by the strong force, such a configuration for a large number of baryons is nonetheless favored by the balance of forces since gravity dominates.

The symmetry driving forces in nuclear matter consist of the nuclear force itself (say the coupling of the rho meson to the nucleon isospin current) which favors symmetry, and a Fermi energy contribution which favors distribution of baryon charge equally over nucleon and proton (and at higher density over as many species as are energetically available). In quark matter, which we think of as approximately free, only the Fermi contribution exists and even at moderate density there are three Fermi seas available, those of the light quarks. Consequently, when the density is raised to the point that some neutron star matter is converted to quark matter, the repulsive isospin restoring force in the nuclear matter can be relieved by exchanging charge and possibly strangeness (mediated by the intermediate vector bosons) between regions of neutron star matter and those of quark matter in equilibrium with it. The degree to which the isospin repulsion can be reduced obviously depends on the proportion of matter in each phase in equilibrium and is restricted by the conservation laws. Therefore the energy is not a linear function of the proportion of phases in equilibrium but has the form

$$E = (1 - \chi)E_H(\chi) + \chi E_Q(\chi)$$

where $\chi = V_Q/V$. Consequently the pressure is not a constant in the mixed phase (recall $P = -\partial E/\partial V$). Fig. 9 shows how the charge on the two phases changes continuously as a function of the proportion $\chi$ of quark matter in the coexistence phase. Likewise all other properties vary. This schematic proof can be made rigorous [11].

The impact of the above observation on neutron star structure is major. In the early work, the mixed phase which had constant pressure independent of the proportion, was absent from the monotonically varying pressure environment of stars and a large density discontinuity occured at the pressure corresponding to the mixed phase. These artificial aspects were consequences of treating the star as a purely neutron star, which is beta-unstable, or imposing local charge neutrality on the solution of theory for a beta-stable star. As proven in Ref. [11], the conditions of local neutrality in the mixed phase are incompatible with Gibbs conditions for equilibrium. Conservation laws are global and in non-uniform systems (like the mixed phase) must be imposed on the solution of ones theory only in a global sense. Each phase in equilibrium can be charged; only the total change must vanish. The early work on quark matter in neutron stars therefore missed the fascinating formation of spatial structure in the mixed phase that results from the competition of the Coulomb interaction and the surface interface energy.

The key concepts are encapsulated in the words “degree(s) of freedom” and “driving force(s)”. The degree of freedom here is that of exchanging charge to achieve an energetically more favorable concentration. The driving force is the isospin restoring force of nuclear matter, composed of contributions from the Fermi energy and explicitly through the interaction of baryons with the rho meson.
Figure 9: Charge density on regions of both phases in equilibrium and the uniform electron density as a function of proportion of quark phase [1] Reprinted with permission of Springer–Verlag New York; copyright 1997.

Figure 10: Diameter (D) and spacing (S) of geometrical phases as a function of proportion of quark phase.

6 Crystals in Stars

Now turn to the proof of the last two theorems. The symmetry restoring force causes the rearrangement of charge so that nuclear matter has positive charge and quark matter has the opposite, all in accord with the conservation of electric charge. The Coulomb force will tend to break regions of like charge into smaller ones. The surface interface energy will resist. The competition is resolved when the rare phase moves to lattice sites immersed in the dominant phase. Charge is thus shielded over long ranges. Fig. 10 shows how the geometric phases vary in form from drops to rods to slabs, and in size and spacing as a function of proportion of the quark phase. The dimensions are typically nuclear and the spacings vary widely as the boundaries of the mixed phase region are approached. Of course nature interpolates between these idealized geometries.

All of the above can be put into rigorous form. But I like the physical motivation better. Of course it must be put into mathematical form to preform an estimate of the crystalline structures and how they change in size, spacing and form as the proportion changes at various depths in the star [12].
Pulsar Spin-down and Internal Structure

Pulsars are rotating neutron stars with large magnetic fields. When the field is not aligned with the rotation axis, the rotation of the field stimulates electromagnetic radiation, sometimes over a very broad band of frequencies, along a cone with the magnet axis as center. A wind of electron-positron pairs is also created. These processes produce a torque on the pulsar. Rotational energy is slowly radiated. The rates at which pulsar periods change are in the range $10^{-20}$ to $10^{-14}$ s/s. It takes of the order of ten million years to radiate most of the rotational energy.

As the pulsar spins down, it evolves in shape from a spheroid to sphere. The interior density rises ever so slowly. As it rises the density thresholds for various of the transformations that I have discussed will be reached first at the center of the star and then moving outward to embrace a slowly expanding region of the star. We propose to use the timing structure of pulsar spin-down to detect these transformations, especially the deconfinement phase transition [15]. The coupling of rotation to the weak electromagnetic process of radiation is favorable for detection, since the expansion of the transformed region typically has a time-scale of $10^5$ years.

The energy loss equation representing processes of multipolarity $n$ is of the form

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{1}{2}I\Omega^2\right) = -C\Omega^{n+1}$$

where, for magnetic dipole radiation, $C = \frac{2}{3}m^2\sin^2\alpha$, $n = 3$, $m$ is the magnetic dipole moment and $\alpha$ is the angle of inclination between magnetic moment and rotation axis. We shall refer to $n$ appearing in the energy-loss equation as the intrinsic index. In the pulsar community, the above equation is integrated to yield

$$\dot{\Omega} = -K\Omega^n \quad (K = C/I), \quad \text{if } I = \text{constant or } \Omega \ll 2I/I'.$$

and $n$ is known as the braking index. If the above equation held then the dimensionless measurable quantity

$$\frac{\Omega\dot{\Omega}}{\Omega^2} = n$$

would provide the value of the braking index, which for magnetic dipole radiation is 3, and which for the several pulsars for which it is measured has values around 2-2.5. The above two equations would hold rigorously for a rigidly rotating magnetized body.

However a rotating star is not a rigid body. At high angular velocity it is centrifugally deformed and becomes more spherical with time as the star radiates. Consequently the moment of inertia is not constant. Moreover as the central density rises with decreasing angular velocity, thresholds for various of the internal transformations spoken of above will occur. Such changes in internal structure, which effect the distribution of mass-energy in the star, bring about there own changes in the moment of inertia. The equation that follows from the energy loss equation is therefore

$$\dot{\Omega} = -\frac{C}{I(\Omega)} \left[1 + \frac{I'(\Omega)\Omega}{2I(\Omega)}\right]^{-1}\Omega^n$$
and the measurable dimensionless braking index is not a constant and not an integer

\[ n(\Omega) \equiv \frac{\Omega \dot{\Omega}}{\dot{\Omega}^2} = n - \frac{3I'\Omega + I''\Omega^2}{2I + I'\Omega}. \]

where \( I' = dI/d\Omega \). We will find that for a first order phase transition, the observable
index will be far removed from its canonical value of \( n \).

8 Effect of Phase Transitions on Rotation

The equation of state depends on the composition of matter, and hence the structure
of stars depends on their composition. As we suggested above, in the course of its
spin-down, thresholds for various transitions will be reached, first at the center and
then in an expanding region. Figure 11 shows the evolution of the radial boundaries
between different phases (on the x-axis) as the angular velocity decreases with time.
We must incorporate these effects into a calculation of the moment of inertia.

In classical mechanics and Newtonian gravity the moment of inertia involves a
simple and straightforward calculation. In General Relativity the calculation is quite
different because spacetime is warped, not only by the distribution of mass-energy, but
also by rotation. A particle dropped from great distance from a point on the equatorial
plane onto a rotating star would not drop toward its center but would acquire an ever
increasing angular velocity in the same direction as the angular velocity of the star.
The local inertial frames are set into rotation. The angular velocity \( \omega(r) \) of the local
frames is a function of distance from the star, being greatest at its center, and falling
off as \( 1/r^3 \) outside the star. We show its ratio to that of the angular velocity of the star
\( \Omega \) in Fig. 12. Of course this phenomenon effects the structure of the star itself. The
reason is that the centrifugal force acting on any fluid element in the star depends,
not on the angular velocity \( \Omega \) but on the angular velocity relative to that of the local
inertial frames, \( \Omega - \omega(r) \). Prior to our work in 1992 there was no expression available
for calculation of the moment of inertia except in the approximation that the star
was not rotating. The approximation is sometimes known as the slow approximation.
The angular momentum and moment of inertia are given by [16, 17]

\[ J_{\text{sph}} \Omega = \frac{8 \pi}{3} \int_0^R dr^4 r^4 \frac{\epsilon + P(\epsilon)}{\sqrt{1 - 2 m(r) / r}} [\Omega - \omega(r)] e^{-\Phi(r)} \]

where \( e^{2\Phi(r)} \) is the time metric function \( g_{tt} \) of Oppenheimer–Volkoff. This is not
adequate to our purpose because the distribution of mass, energy and pressure in the
star in the above expression refer to a nonrotating star. It lacks therefore the effect of
the dependence on the internal structure of the star—the way this structure changes
with \( \Omega \), as well as the more elementary centrifugal distortion of the star. The above
equation refers to a spherical Oppenheimer–Volkoff star.

We derived (in another connection) the expression for the moment of inertia which
incorporates the effects that are missing in the above result. The expression depends,
not on the Schwarzschild metric, but on the metric of a static rotating spacetime.
The metrical functions are denoted by $\lambda(r, \Omega) \cdots \psi(r, \Omega)$ and are given in Ref. [18] and refer to $g_{tt} = e^{2\nu}$, $g_{\phi t} = \omega(r)e^{2\psi}$, etc. The expression is

$$J = I\Omega = 4\pi \int_0^{\pi/2} d\theta \int_0^{R(\theta)} dr \frac{e^{\lambda(r,\omega)} e^{\mu(r,\omega)} e^{\nu(r,\omega)} e^{\psi(r,\omega)}}{e^{2\nu(r,\omega)} - 2\psi(r,\omega)} \left[\epsilon + P(\epsilon)\right] [\Omega - \omega(r, \Omega)]^2.$$

The distribution of mass, energy density and pressure in this expression refer now to those of a rotating star rather than an Oppenheimer–Volkoff star.

Figure 13 shows how the moment of inertia changes with frequency (and hence time). What is particularly noteworthy are: (1) As the frequency decreases from high values, the moment of inertia decreases according to a particular trajectory, which is distinctly different when projected to low angular velocity from the trajectory actually followed. The two trajectories refer, so to speak to a star of two distinct structures. From Fig. 11 we can see that the high frequency (ie lower density) structure corresponds to a star with a mixed phase of confined and deconfined matter that extends from the center of the star to about 8 km. The lower frequency trajectory corresponds to a star (the same one but later in time) that has a quark matter core of 4-5 km radius. (2) During its evolution from one internal structure to the other, the star actually spins up. The singularities and changes of sign in the derivatives $I'$ and $I''$ in the transition region will produce a marked signal in the measurable braking index $n(\Omega)$ as can be seen by the presence of these derivatives in the index. In fact the dimensionless observable $\Omega \ddot{\Omega} / \dot{\Omega}^2$ is singular at both turning points in Fig. 13 and
Figure 13: Moment of inertia as a function of rotational frequency. The “bachbend” occurs during the epoch of phase transition. It is entirely analogous to backbending observed in nuclei [13].

Figure 14: Measurable braking index. The canonical value for magnetic dipole radiation is 3. The large departures, which last for $\sim 10^5$ years, are caused by the growing region in which nuclear matter is converted to quark matter [15].

is of opposite sign. The observable braking index is shown in Fig. 14. Its departure from the expected value of three during the phase transition epoch is spectacular. If observed, it would be a clear signal of a first order phase transition.

We should enquire whether it is easy to measure an anomalous braking index and how likely it is that an anomaly will be observed in the population of some 700 pulsars presently known in our part of the galaxy ($r < R_{\text{galaxy}}/3$). Using the frequency interval $\Delta \Omega$ in which $n(\Omega)$ is greater than 6 or less than -3, and a typical pulsar spin-down rate $\dot{\Omega}$ we find the duration of the transition epoch

$$\Delta T \approx -\Delta \Omega/\dot{\Omega} \sim 10^5 \text{ years}$$

Since the mean life of pulsars is $\sim 10^7$ years, 1/100 pulsars may exhibit the signal (i.e. 7 of those 700 presently known). Moreover, although for normal pulsars $\dot{\Omega}$ is very small and difficult to measure, for a pulsar in the phase transition epoch, this derivative is large—even infinite—at two times during the epoch. Consequently, the braking index should be easy to measure for pulsars in the transition epoch. In fact, difficulty in measuring the braking index could be used to de-select candidates.

For the reasons discussed above, the prognosis for discovering first order phase transitions in neutron stars appears to be excellent. The signal is very strong, is easy to measure and lasts for an appreciable fraction of a pulsar’s active lifetime, so that
roughly ten percent of pulsars will be passing through a phase transition epoch, if indeed the deconfinement transition occurs in neutron stars.

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