Cost Effective Indoor HVAC Energy Efficiency Monitoring based on Intelligent Decision Support System under Fermatean Fuzzy Framework

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Abstract

The heating, ventilation, and air conditioning (HVAC) control system is in charge of the building’s energy efficiency. Indoor energy consumption trends can be intelligently monitored and minimized. Occupancy data is essential for saving a significant amount of energy. This energy footprint can play an important part in modern smart buildings to improve indoor green environments while lowering costs. Traditional energy monitoring and control systems can be enhanced by installing an occupancy monitoring system, which consists of a network of sensors and cameras. In this paper, we offer a novel and innovative convolutional neural network (CNN) based on real-time camera occupancy detection and recognition algorithms across various types of sensors, which provides realistic low-cost energy-saving solutions with robust graphical processing units (GPUs). Decision-making tools can be used to select the appropriate occupancy detection and recognition alternative for indoor environment and energy monitoring and management. In this research work, we develop the “Fermatean fuzzy prioritized weighted average (FFPWA) operator and Fermatean fuzzy prioritized weighted geometric (FFPWG) operator”. In the end, we give an algorithm for an intelligent decision support system (IDSS) using proposed AOs to compare our CNN based method with other existing sensors techniques.

Keywords: Fermatean fuzzy numbers, Aggregation operators, Video Processing, Convolutional Neural Network, Human Recognition and Detection.

1 Introduction

The concept of smart buildings opens up possibilities for achieving the goals of an integrated power management network and, possibly, an interior sustainable environment system. Because to technological advancements, per capita energy consumption has more than doubled. The output gap has narrowed, influencing
the country's lifestyle and economic progress. Energy generation research focuses on energy conservation and management efficiency enhancement. The energy and monitoring and management system can save energy while maintaining the quality of the green environment [1]. Energy is basic component to household, industry, agriculture, transport, information, infrastructure, technology development for any nation in the world. The growth of economy and the improved standard of living are dependent on energy [2]. Therefore, per capita energy consumption should be maximum for national evolution. It is a direct relation between energy use and economic growth of industrialized nations. Energy need is rising day by day consistently. Due to the nature of demand the availability of energy are getting insecure for the whole world included industrialized countries [3]. The energy use of domestic and industrial building appliances has gone up dramatically. But from the other hand, there was a decrease in the share of the electric power industry. Energy Conservation has developed with two strategies; green energy and green environment. In this current situation, renewable energy and a green climate are the main global tasks. In order to raise people's living conditions, a large amount of money has been expended in renewable energy and the green climate. Since fossil fuels have played a significant role in deteriorating both the Earth's ecosystem and weather patterns. After the industrial revolution, we have depleted fossil fuel supplies. 85% of power generation has been shown to be from fossil fuel resources. This critical share of fossil fuel are causing the global warming, therefore renewable energy and energy conservation strategies are getting special attention to explore and to get solution at national level [4]. The building energy automation system can be viable solution to save energy from HVAC system. Intelligent energy monitoring is very balanced approach for energy conservation and energy management against any user of energy. It assess to get exact energy inputs, energy penetration and energy consumption at every energy facility. Automatic energy monitoring system is also able to calculate, categorize and highlight energy consumption behavior and cost saving measures for energy use.

CNNs are a highly efficient model for the classification of image information, offering state-of-the-art image segmentation, recognition, identification and recovery results. The CNN approach achieves almost real-time high detection efficiency on the embedded device. A high-resolution camera can classify the object and count the number of individuals and the precise location of the objects. Real-time object detection plays a crucial role in the energy management of buildings. Automatic intelligent system works with the information of sensing algorithm. It operates with HVAC control system and provides multiple predictive outcomes. Because it is integrated of sensing technology with optimization and computation strategies conserving the performance of energy appliances. Occupancy detection and estimation can provide real time information which can be used in HVAC control system limiting energy consumption. It can be more affordable and efficient solution. It can also save cost. Therefore, efficient and economical intelligent HVAC monitoring and control system can be viable and sophisticated solution with the implementation of occupancy detection technology.

From a technical perspective, existing research studies have confirmed that the continuation of the decision-making support structure further than the classical optimization approach of a single objective function is widely accepted, as demonstrated in a collection of effective solutions [5]. This module makes it easier to address multi-criteria challenges by focusing on finding a solution that meets many, sometimes contradictory, goals. The presence of numerous criteria, the ambiguity of criteria, and the decision-making problem's complicated, subjective, and poorly articulated character provide a model for multi-criteria decision assistance [6]. It's also on these foundational principles that a range of IDSS have been introduced [7]. The present incarnation in this field indicates the fact the realistic and efficient handling of the problem referred to above
by the IDSS strategies. The literature review demonstrates the significance of the use of IDSS approaches in energy decision-making. The IDSS strategies are used as realistic and useful techniques to tackle energy policy decision-making problems or to pick and test RES technologies[8]. This can easily be verified by overview and meta-analysis papers such as Martín-Gamboa et al.[9], Løken[10], Arce et al.[11], and Wang et al.[12], which show the multitude of functional issues in the area of energy policy and IDSS techniques used as a formal context for the models created. The literature review also demonstrated the widespread application of fuzzy evolution (following the notion of fuzzy sets) of IDSS approaches in climate issues [13]. This ideology is significant because it is founded on the notion that fuzzy number theory models express the uncertainty of individual viewpoints in order to create more sensitive, concrete, and practical modelling outcomes. Main objectives of this article are:

• Fermatean fuzzy numbers (FFNs) are suitable to address the complexity issues practically with membership degree (MSD) and a non-membership degree (N-MSD). Taking advantage of FFNs, new aggregation operators (AOs) are proposed.

• We develop the "Fermatean fuzzy prioritized weighted average (FFPWA) operator" and "Fermatean fuzzy prioritized weighted geometric (FFPWG) operator".

• These aggregation operators (AOs) are a new way to modelling decision-making difficulties.

• Finally, we present an algorithm for an intelligent decision support system (IDSS) based on the proposed AOs, and we compare our CNN-based method to other existing sensor methodologies.

The rest of this paper is organized as follows. Section 2 basic definition regarding to Fermatean fuzzy set and in Section 3 consists on proposed AOs. We give an algorithm in Section 4 to solve complex decision problems based on proposed AOs. In Section 5 we present our intelligent energy management system. In Section 6 application of our proposed AOs given and in Section 7 we give concluding remarks of this research.

**MCDM based uncertain data modeling**

IDSS is a major component of decision science, with the primary purpose of determining the optimal options given a set of different attributes. In it, an expert or group of specialists evaluates each choice based on a variety of criteria, and their conclusions are expressed as either a crisp value or a linguistic value. However, in today's complicated and tough world, ambiguity is highly common and plays a crucial part in almost every decision-making process. As a result, inconsistencies in the analysis must be accounted for. Uncertainties play a crucial part when dealing with real-world problems such as clustering and classification, decision-making, supplier assessment, and so on, and it is impossible for DMs to obtain accurate results without dealing with imprecise, unclear, or uncertain data. According to Zadeh [14], the perception of a fuzzy set (FS) is a generalisation of a classical set. The membership function computes the degrees of membership of elements in a fuzzy set. In the literature, Zadeh's fuzzy set is discussed from a variety of angles in MCDM concerns. Atanassov [15] extended FS by introducing the idea of intuitionistic fuzzy set (IFS), which is defined as having a membership degree (MSD) and a non-membership degree (N-MSD) that is less than or equal to 1. This IFS characteristic is important for many practitioners who deal with real life issues in an IFS framework. IFS is not beneficial for solving complex decision-making problems since the sum of MSD and N-MSD must be equal to unity. Yager's
Pythagorean fuzzy set (PFS) [16, 17] is a significant generalization of IFS in which the sum of squares of MD and N-MSD is bounded by 1. Furthermore, it has a wide range of possible applications in a variety of fields, including decision-making and so on. However, the PFS technique could not be acknowledged in a number of cases. Consider the following scenario: an expert team was called to provide feedback on training institutes, and they were divided into two groups. The degree of MSD is expressed by the first group of experts as 0.92, whereas the degree of N-MSD is expressed by the second group of experts as 0.84. As can be observed, the square sum of MSD and N-MSD is more than one. The IFS and PFS were unable to depict this circumstance. Senapati and Yager [18] developed the concept of Fermatean fuzzy sets (FFSs) as a logical extension of the IFSs and PFSs to address this complication. In an FFS, the cubic sum of an object’s MSD and N-MSD values bounded by 1. FFS theory is currently playing an important role in a variety of disciplines since it is a solid notion for dealing with ambiguous and imprecise information in a Fermatean fuzzy context. Senapati and Yager [19] also proposed score, accuracy function and Fermatean fuzzy weighted product model (FF-WPM).

Several studies have been conducted to date that are only based on FFSs. When there are numerous options for a particular problem, the concept of AOs plays a critical role in determining the best solution. Mesiar and Pap [20] proposed the concept of aggregation of infinite sequences. Senapati and Yager [21] suggested fundamental AOs (geometric and averaging) for the FFS and their application to multi-criteria decision-making (MCDM). Rani and Mishra [22] introduced the MULTIMOORA technique for selecting electric car charging stations using Fermatean fuzzy Einstein AOs. Jeeveraj presented an interval-valued Fermatean fuzzy set idea as well as a novel total ordering theory for interval-valued FFS [23]. Garg et al. [24] proposed Yager AOs with Application in COVID-19 Using FFS. The concept of Hamacher interactive hybrid Averaging AOs for FFS was proposed by Shazadi et al. [25]. Riaz et al. [26] introduced novel concept of bipolar picture fuzzy. Jana et al. [27] proposed bipolar fuzzy Dombi AOs for MCDM. Sitara et al. [28] introduced q-rung picture fuzzy graph structures. Riaz and Hashmi [29] considerably tested the restrictions related with MSDs and NMSDs in the structures of FS, IFS, PFS, and FFS in 2019, and those barriers were provided quantitatively. To overcome these challenges, they developed the linear Diophantine fuzzy set (LDFS) by incorporating reference parameters to the nature of IFS. They claim that the concept of LDFS will eliminate the limits in present approaches for other sets and allow for the free selection of data in practise. Iampan et al. [30] introduced linear Diophantine fuzzy Einstein AOs and their application towards medical. Riaz et al. [31] developed prioritized AOs for linear Diophantine fuzzy numbers (LDFNs) with application to third-party logistic provider selection. Ashraf and Abdullah [32] proposed emergency decision support modeling for COVID-19 and also introduced spherical AOs and their application in MCDM [33]. Saha et al. [34] introduced new hybrid hesitant fuzzy weighted AOs depend on Archimedian and Dombi operations. Farid and Riaz [35] also introduced Einstein interactive geometric AOs with application to MCDM for q-ROFNs. Jana et al. introduced trapezoidal neutrosophic AOs [36], bipolar fuzzy Dombi prioritized [37], neutrosophic Dombi power AOs [38], trapezoidal neutrosophic normal AOs [39]. Yang et al. [40] presented continuous ordered weighted averaging AOs for interval-valued q-ROF information and their use in quality assessment of smartwatch aesthetic design. Chen et al. [41] introduced the notion of enhanced ordered weighted averaging AOs and their application towards MCDM. Chen et al. [42] also proposed power-average AOs for a proportional hesitant fuzzy linguistic term set, with an implementation to an online product recommender system for consumer decision-making. Jana and pal proposed dynamical hybrid method to design decision making process based on GRA approach [43].

Yager [44] developed several prioritized AOs. According to Yager, in situations where we choose a child’s
bike based on safety and cost qualities, we should not allow the cost-related benefit to compensate for the lack of safety. Then we have a type of priority connection between these two traits, with security being more important. This condition is known as an aggregation problem since the attributes have a priority relationship. Because we wish to take into account the satisfaction of higher priority criteria, such as safety in the example above, it is no longer practicable for the AOs in question, such as the average AO and the geometric AO. Yager offered the prioritised AOs in this situation by modelling attribute prioritising in terms of the weights associated with the attributes based on the satisfaction of the higher priority criteria.

2 Preliminaries

We briefly add a few concepts involved in the remaining work to construct this work self-sufficiently.

Definition 2.1. [18] A Fermatean fuzzy set $\tilde{\sigma}$ in $\mathfrak{F}$ is given as

$$\tilde{\sigma} = \langle \tilde{\mathfrak{F}}, \mu_{\tilde{\sigma}}^{1}(\tilde{\mathfrak{F}}), \nu_{\tilde{\sigma}}^{1}(\tilde{\mathfrak{F}}) \rangle : \tilde{\mathfrak{F}} \in \mathfrak{F}$$

where $\mu_{\tilde{\sigma}}^{1}, \nu_{\tilde{\sigma}}^{1} : \mathfrak{F} \rightarrow [0, 1]$ defines the MSD and N-MSD of the alternative $\tilde{\mathfrak{F}} \in \mathfrak{F}$ and for every $\tilde{\mathfrak{F}}$ we have

$$0 \leq \mu_{\tilde{\sigma}}^{1}(\tilde{\mathfrak{F}}) + \nu_{\tilde{\sigma}}^{1}(\tilde{\mathfrak{F}}) \leq 1.$$  

In addition, $\pi_{\tilde{\sigma}}(\mathfrak{F}) = \sqrt{1 - \mu_{\tilde{\sigma}}^{3}(\mathfrak{F}) - \nu_{\tilde{\sigma}}^{3}(\mathfrak{F})}$ is given as the indeterminacy degree of $\mathfrak{F}$ to $\tilde{\sigma}$.

Definition 2.2. [18] Let $\mathfrak{F}_1 = \langle \mu_{1}, \nu_{1} \rangle$ and $\mathfrak{F}_2 = \langle \mu_{2}, \nu_{2} \rangle$ be FFNs. Then

1. $\mathfrak{F}_1 \sqcup \mathfrak{F}_2 = \langle \max(\mu_{1}, \nu_{1}), \min(\mu_{2}, \nu_{2}) \rangle$
2. $\mathfrak{F}_1 \sqcap \mathfrak{F}_2 = \langle \min(\mu_{1}, \nu_{1}), \max(\mu_{2}, \nu_{2}) \rangle$
3. $\mathfrak{F}_1 \oslash \mathfrak{F}_2 = \langle \sqrt{\mu_{1}^{3} + \mu_{2}^{3} - \mu_{1}^{3}\mu_{2}^{3}}, \nu_{1}^{3} \rangle$
4. $\mathfrak{F}_1 \oslash \mathfrak{F}_2 = \langle \mu_{1}^{3}\mu_{2}^{3}, \sqrt{\nu_{1}^{3} + \nu_{2}^{3} - \nu_{1}^{3}\nu_{2}^{3}} \rangle$
5. $\sigma^{-1}(\mathfrak{F}) = \langle \sqrt{1 - (1 - \mu_{1}^{3})\nu_{1}^{3}}, \nu_{1}^{3} \rangle$
6. $\sigma \mathfrak{F}_1 = \langle \mu_{1}^{3}, \sqrt{1 - (1 - \nu_{1}^{3})\nu_{1}^{3}} \rangle$

Definition 2.3. [19] Suppose $\mathfrak{F} = \langle \mu, \nu \rangle$ is a FFN, then a score function $\mathfrak{S}$ and accuracy function $\mathfrak{R}$ of $\mathfrak{F}$ are defined as respectively,

$$\mathfrak{S}(\mathfrak{F}) = \mu^{3} - \nu^{3}, \quad \mathfrak{S}(\mathfrak{F}) \in [-1, 1]$$

$$\mathfrak{R}(\mathfrak{F}) = \mu^{3} + \nu^{3}, \quad \mathfrak{R}(\mathfrak{F}) \in [0, 1]$$

Definition 2.4. [19] Let $\mathfrak{F}_1 = \langle \mu_{1}, \nu_{1} \rangle$ and $\mathfrak{F}_2 = \langle \mu_{2}, \nu_{2} \rangle$ are any two FFN, and $\mathfrak{S}(\mathfrak{F}_1), \mathfrak{S}(\mathfrak{F}_2)$ are the score function of $\mathfrak{F}_1$ and $\mathfrak{F}_2$, and $\mathfrak{R}(\mathfrak{F}_1), \mathfrak{R}(\mathfrak{F}_2)$ are the accuracy function of $\mathfrak{F}_1$ and $\mathfrak{F}_2$, respectively, then

1. If $\mathfrak{S}(\mathfrak{F}_1) > \mathfrak{S}(\mathfrak{F}_2)$, then $\mathfrak{F}_1 > \mathfrak{F}_2$
2. If $\mathfrak{S}(\mathfrak{F}_1) = \mathfrak{S}(\mathfrak{F}_2)$, then

if $\mathfrak{R}(\mathfrak{F}_1) > \mathfrak{R}(\mathfrak{F}_2)$ then $\mathfrak{F}_1 > \mathfrak{F}_2$,
if $\mathfrak{R}(\mathfrak{F}_1) = \mathfrak{R}(\mathfrak{F}_2)$, then $\mathfrak{F}_1 = \mathfrak{F}_2$. 

5
It must always be remembered that the score function’s value is between -1 and 1. To help the following study, we add a further score function, \( \tilde{O}(\tau) = \frac{1 - \mu_{\tau}^3 + \nu_{\tau}^3}{2} \). We can see that \( 0 \leq \tilde{O}(\tau) \leq 1 \).

## 3 Fermatean Fuzzy Prioritized Aggregation Operators

Here, we present the notion of Fermatean fuzzy prioritized weighted average (FFPWA) operator and Fermatean fuzzy prioritized weighted geometric (FFPWG) operator. Then we discussed other properties of proposed operators like boundary, monotonicity and Idempotency.

### 3.1 FFPWA operator

**Definition 3.1.** Assume that \( \gamma_j = \langle \mu_{\tau}^j, \nu_{\tau}^j \rangle \) is the collection of FFNs, and FFPWA : \( \Lambda^n \rightarrow \Lambda \), is a mapping. if

\[
\text{FFPWA}(\gamma_1, \gamma_2, \ldots, \gamma_n) = \frac{\kappa_1^2}{\sum_{j=1}^{n} \kappa_j^2} \gamma_1 \oplus \frac{\kappa_2^2}{\sum_{j=1}^{n} \kappa_j^2} \gamma_2 \oplus \ldots \oplus \frac{\kappa_n^2}{\sum_{j=1}^{n} \kappa_j^2} \gamma_n
\]

then the mapping FFPWA is called Fermatean fuzzy prioritized weighted averaging (FFPWA) operator, where \( \kappa_j \) presented the score of \( k \)th FFN.

We may also take into account FFPWA by the following theorem depending on FFN’s operational rules.

**Theorem 3.2.** Assume that \( \gamma_j = \langle \mu_{\tau}^j, \nu_{\tau}^j \rangle \) is the collection of FFNs, we can evaluate FFPWA by

\[
\text{FFPWA}(\gamma_1, \gamma_2, \ldots, \gamma_n) = \left\langle \sqrt[n]{1 - \prod_{j=1}^{n} (1 - \mu_{\tau}^j \nu_{\tau}^j)}, \prod_{j=1}^{n} \nu_{\tau}^j \right\rangle
\]

**Proof.**

\[
\text{FFPWA}(\gamma_1, \gamma_2, \ldots, \gamma_n)
= \left\langle \sqrt[n]{1 - \prod_{j=1}^{n} (1 - \mu_{\tau}^j \nu_{\tau}^j)}, \prod_{j=1}^{n} \nu_{\tau}^j \right\rangle
= \left\langle \sqrt[n]{1 - \prod_{j=1}^{n} (1 - \mu_{\tau}^j \nu_{\tau}^j)}, \prod_{j=1}^{n} \nu_{\tau}^j \right\rangle
\]

We utilize mathematical induction to proof this theorem.

For \( n = 2 \)

\[
\frac{\kappa_1^2}{\sum_{j=1}^{2} \kappa_j^2} \gamma_1 = \left\langle \sqrt[2]{1 - (1 - \mu_{\tau}^1 \nu_{\tau}^1) (1 - \mu_{\tau}^2 \nu_{\tau}^2)}, \nu_{\tau}^2 \right\rangle
\]
This shows that the Equation 2 is valid for $n = 2$, now let that the Equation 2 holds for $n = k$, i.e.,

\[
\text{FFPWA}(\mathbb{v}_1, \mathbb{v}_2, \ldots, \mathbb{v}_k) = \left\langle \sqrt{1 - \prod_{j=1}^k (1 - \mu_j^{3\mathbb{v}_j^{2\mathbb{v}_j}})} \prod_{j=1}^k \mathbb{v}_j^{2\mathbb{v}_j^{2\mathbb{v}_j}} \right\rangle
\]

For $n = k + 1$, we have,
Theorem 3.3. (Idempotency) Assume that $\nu_j = (\mu_j, v_j)$ is the assemblage of FFNs, where $\kappa_j = \prod_{k=1}^{j-1} \tilde{\nu}(\nu_k)$ $(j = 2, \ldots, n)$, $\kappa_1 = 1$ and $\tilde{\nu}(\nu_k)$ is the score of $k^{th}$ FFN. If all $\nu_j$ are equal, i.e., $\nu_j = \nu$ for all $j$, then

$$\text{FFPWA}(\nu_1, \nu_2, \ldots, \nu_n) = \nu$$

Proof. From Definition 3.1, we have

$$\text{FFPWA}(\nu_1, \nu_2, \ldots, \nu_n) = \frac{\kappa_1^2}{\sum_{j=1}^{n} \kappa_j^2} \nu_1 \oplus \frac{\kappa_2^2}{\sum_{j=1}^{n} \kappa_j^2} \nu_2 \oplus \ldots \oplus \frac{\kappa_n^2}{\sum_{j=1}^{n} \kappa_j^2} \nu_n$$

$$= \frac{\kappa_1^2}{\sum_{j=1}^{n} \kappa_j^2} \nu \oplus \frac{\kappa_2^2}{\sum_{j=1}^{n} \kappa_j^2} \nu \oplus \ldots \oplus \frac{\kappa_n^2}{\sum_{j=1}^{n} \kappa_j^2} \nu$$

$$= \frac{\sum_{j=1}^{n} \kappa_j^2}{\sum_{j=1}^{n} \kappa_j^2} \nu$$

$$= \nu$$

Corollary 3.4. If $\tilde{\nu}_j = (\mu_j, v_j)$ is the assemblage of largest FFNs, i.e., $\nu_j = (1, 0)$ for all $j$, then

$$\text{FFPWA}(\tilde{\nu}_1, \tilde{\nu}_2, \ldots, \tilde{\nu}_n) = (1, 0)$$
Proof. One can easily prove this corollary in similar way to the Theorem 3.3. □

Corollary 3.5. (Non-compensatory) If $\overline{\nu}_1 = \langle \mu^2, v_1 \rangle$ is the smallest FFN, i.e., $\overline{\nu}_1 = (0, 1)$, then

$$FFPWA(\overline{\nu}_1, \overline{\nu}_2, \ldots, \overline{\nu}_n) = (0, 1)$$

Proof. Here, $\overline{\nu}_1 = (0, 1)$ then by definition of the score function, we have,

$$\hat{\kappa}(\overline{\nu}_1) = 0$$

Since,

$$\kappa^2 = \prod_{k=1}^{j-1} \hat{\kappa}(\overline{\nu}_k) \quad (j = 2, \ldots, n), \quad \text{and} \quad \kappa^2_1 = 1$$

$\hat{\kappa}(\overline{\nu}_k)$ is the score of $k^{th}$ FFN.

We have,

$$\kappa^2 = \prod_{k=1}^{j-1} \hat{\kappa}(\overline{\nu}_k) = \hat{\kappa}(\overline{\nu}_1) \times \hat{\kappa}(\overline{\nu}_2) \times \ldots \times \hat{\kappa}(\overline{\nu}_{j-1}) = 0 \times \hat{\kappa}(\overline{\nu}_2) \times \ldots \times \hat{\kappa}(\overline{\nu}_{j-1}) \quad (j = 2, \ldots, n)$$

$$\prod_{k=1}^{j} \kappa^2 = 1$$

From Definition 3.1, we have

$$FFPWA(\overline{\nu}_1, \overline{\nu}_2, \ldots, \overline{\nu}_n) = \frac{\kappa^2_1}{\sum_{j=1}^{\kappa^2_1}} \overline{\nu}_1 \oplus \frac{\kappa^2_2}{\sum_{j=1}^{\kappa^2_2}} \overline{\nu}_2 \oplus \ldots \oplus \frac{\kappa^2_n}{\sum_{j=1}^{\kappa^2_n}} \overline{\nu}_n$$

$$= \frac{1}{\overline{\nu}_1} \oplus \frac{0}{\overline{\nu}_2} \oplus \ldots \oplus \frac{0}{\overline{\nu}_n}$$

$$= \overline{\nu}_1 = (0, 1)$$

□

Corollary 3.5 implies that incentives would not be obtained by anyone although the higher importance conditions had been satisfied by the smallest FFN.

Theorem 3.6. (Monotonicity) Assume that $\overline{\nu}_j = \langle \mu^2_j, v^2_j \rangle$ and $\overline{\nu}_j' = \langle \mu^2_j', v^2_j' \rangle$ are the assemblages of FFNs, where $\kappa^2_j = \prod_{k=1}^{j-1} \hat{\kappa}(\overline{\nu}_k)$, $\kappa^2_j' = \prod_{k=1}^{j-1} \hat{\kappa}(\overline{\nu}_k)'$ ($j = 2, \ldots, n$), $\kappa^2_1 = 1$, $\kappa^2_1' = 1$, $\hat{\kappa}(\overline{\nu}_k)$ is the score of $\overline{\nu}_k$ FFN, and $\hat{\kappa}(\overline{\nu}_k)'$ is the score of $\overline{\nu}_k'$ FFN. If $\mu^2_j \geq \mu^2_j'$ and $v^2_j \leq v^2_j'$ for all $j$, then

$$FFPWA(\overline{\nu}_1, \overline{\nu}_2, \ldots, \overline{\nu}_n) \leq FFPWA(\overline{\nu}_1', \overline{\nu}_2', \ldots, \overline{\nu}_n')$$

Proof. Here, $\mu^2_j \geq \mu^2_j$ and $v^2_j \leq v^2_j$ for all $j$, If $\mu^2_j \geq \mu^2_j'$,

$\Rightarrow (\mu^2_j)^3 \geq (\mu^2_j')^3$ $\Rightarrow \sqrt[3]{(\mu^2_j)^3} \geq \sqrt[3]{(\mu^2_j')^3} \Rightarrow \sqrt[3]{1 - (\mu^2_j)^3} \leq \sqrt[3]{1 - (\mu^2_j')^3}$

$\Rightarrow \sqrt[3]{\prod_{j=1}^{3} (1 - (\mu^2_j)^3)} \leq \sqrt[3]{\prod_{j=1}^{3} (1 - (\mu^2_j')^3)}$

$\Rightarrow \sqrt[3]{\prod_{j=1}^{3} (1 - (\mu^2_j)^3)} \leq \sqrt[3]{\prod_{j=1}^{3} (1 - (\mu^2_j')^3)}$
\[
\sqrt[3]{1 - \prod_{j=1}^{n}(1 - (\mu_j^3)^{\kappa_j^3})} \leq \sqrt[3]{1 - \prod_{j=1}^{n}(1 - (\mu_j^3)^{\bar{\kappa}_j^3})}
\]

Now, \(\nu_j^3 \leq \nu_j^3\).
\[
(\nu_j^3)^{\kappa_j^3} \leq (\nu_j^3)^{\bar{\kappa}_j^3}
\]
\[
\prod_{j=1}^{n}(\nu_j^3)^{\kappa_j^3} \leq \prod_{j=1}^{n}(\nu_j^3)^{\bar{\kappa}_j^3}
\]

Let
\[
\bar{\nu} = \text{FFPWA}(\bar{\nu}_1, \bar{\nu}_2, \ldots, \bar{\nu}_n)
\]
and
\[
\bar{\nu}^* = \text{FFPWA}(\bar{\nu}_1^*, \bar{\nu}_2^*, \ldots, \bar{\nu}_n^*)
\]
We get that \(\bar{\nu}^* \geq \bar{\nu}\). So,
\[
\text{FFPWA}(\bar{\nu}_1, \bar{\nu}_2, \ldots, \bar{\nu}_n) \leq \text{FFPWA}(\bar{\nu}_1^*, \bar{\nu}_2^*, \ldots, \bar{\nu}_n^*)
\]

\[\text{Theorem 3.7. (Boundary) Assume that } \bar{\nu}_j = (\mu_j^3, \nu_j^3) \text{ is the assemblage of FFNs, and}
\]
\[\bar{\nu}^- = (\min_j(\mu_j^3), \max_j(\nu_j^3)) \text{ and } \bar{\nu}^+ = (\max_j(\mu_j^3), \min_j(\nu_j^3))
\]
\[\text{Then, } \bar{\nu}^- \leq \text{FFPWA}(\bar{\nu}_1, \bar{\nu}_2, \ldots, \bar{\nu}_n) \leq \bar{\nu}^+
\]
where \(\kappa_j^3 = \prod_{k=1}^{j-1} \delta(\bar{\nu}_k) \) (\(j = 2, \ldots, n\)), \(\kappa_1^3 = 1\) and \(\delta(\bar{\nu}_k)\) is the score of \(k^{th}\) FFN.

\[\text{Proof. Since,}
\]
\[\min_j(\mu_j^3) \leq \mu_j^3 \leq \max_j(\mu_j^3)
\]
and
\[\min_j(\nu_j^3) \leq \nu_j^3 \leq \max_j(\nu_j^3)
\]

From Equation 3 we have,
\[ \min_j (\mu_j^2) \leq \mu_j^2 \leq \max_j (\mu_j^2) \]

\[ \sqrt[3]{\min_j (\mu_j^2)^3} \leq \sqrt[3]{(\mu_j^2)^3} \leq \sqrt[3]{\max_j (\mu_j^2)^3} \]

\[ \sqrt[3]{1 - \max_j (\mu_j^2)^3} \leq \sqrt[3]{1 - (\mu_j^2)^3} \leq \sqrt[3]{1 - \min_j (\mu_j^2)^3} \]

\[ \sqrt[3]{\left(1 - \max_j (\mu_j^2)^3\right)^{\sum_{j=1}^{\nu} x_j}} \leq \sqrt[3]{\left(1 - (\mu_j^2)^3\right)^{\sum_{j=1}^{\nu} x_j}} \leq \sqrt[3]{\left(1 - \min_j (\mu_j^2)^3\right)^{\sum_{j=1}^{\nu} x_j}} \]

From Equation 4 we have,
\[
\min_j(v^2_j) \leq v^2_j \leq \max_j(v^2_j) \Rightarrow \min_j(v^3_j) \leq \max_j(v^3_j) \leq \max_j(v^2_j) \\
\Rightarrow \prod_{j=1}^n \min_j(v^3_j) \leq \prod_{j=1}^n (v^3_j) \leq \prod_{j=1}^n \max_j(v^3_j) \\
\Rightarrow \min_j(v^2_j) \leq \prod_{j=1}^n (v^2_j) \leq \max_j(v^2_j)
\]

Let

\[ FFPWA(\bar{\nu}_1, \bar{\nu}_2, \ldots, \bar{\nu}_n) = \bar{\nu} = (\mu^2, v^2) \]

Then, \( \bar{\nu}(\nu) = \mu^3 - v^3 \leq \max_j(\mu^2) - \min_j(v^2) = \bar{\nu}(\nu_{\text{max}}) \) So, \( \bar{\nu}(\nu) \leq \bar{\nu}(\nu_{\text{max}}) \).

Again, \( \bar{\nu}(\nu) = \mu^3 - v^3 \geq \min_j(\mu^2) - \max_j(v^2) = \bar{\nu}(\nu_{\text{min}}) \) So, \( \bar{\nu}(\nu) \geq \bar{\nu}(\nu_{\text{min}}) \).

If, \( \bar{\nu}(\nu) \leq \bar{\nu}(\nu_{\text{max}}) \) and \( \bar{\nu}(\nu) \geq \bar{\nu}(\nu_{\text{min}}) \), then

\[ \nu_{\text{min}} \leq FFPWA(\bar{\nu}_1, \bar{\nu}_2, \ldots, \bar{\nu}_n) \leq \nu_{\text{max}} \tag{5} \]

If \( \bar{\nu}(\nu) = \bar{\nu}(\nu_{\text{max}}) \), then \( \mu^3 - v^3 = \max_j(\mu^2) - \min_j(v^3) \)

\[ \Rightarrow \mu^3 - v^3 = \max_j(\mu^2) - \min_j(v^3) \]

\[ \Rightarrow \mu^3 = \max_j(\mu^2), \quad v^3 = \min_j(v^3) \]

Now, \( H(\nu) = \mu^3 + v^3 = \max_j(\mu^2) + \min_j(v^3) = H(\nu_{\text{max}}) \)

\[ FFPWA(\bar{\nu}_1, \bar{\nu}_2, \ldots, \bar{\nu}_n) = \nu_{\text{max}} \tag{6} \]

If \( \bar{\nu}(\nu) = \bar{\nu}(\nu_{\text{min}}) \), then \( \mu^3 - v^3 = \min_j(\mu^2) - \max_j(v^3) \)

\[ \Rightarrow \mu^3 - v^3 = \min_j(\mu^2) - \max_j(v^3) \]

\[ \Rightarrow \mu^3 = \min_j(\mu^2), \quad v^3 = \max_j(v^3) \]

Now, \( H(\nu) = \mu^3 + v^3 = \min_j(\mu^2) + \max_j(v^3) = H(\nu_{\text{min}}) \)

\[ FFPWA(\bar{\nu}_1, \bar{\nu}_2, \ldots, \bar{\nu}_n) = \nu_{\text{min}} \tag{7} \]
Thus, from Equations 5, 6 and 7, we get

\[
\tilde{\tau}^- \leq \text{FFPWA}(\tilde{\tau}_1, \tilde{\tau}_2, \ldots, \tilde{\tau}_n) \leq \tilde{\tau}^+
\]

\]

3.2 FFPWG operator

**Definition 3.8.** Assume that \( \tilde{\tau}_j = \langle \mu_j^2, \nu_j^2 \rangle \) is the collection of FFNs, and FFPWG : \( \Lambda^n \rightarrow \Lambda \), is a mapping, if

\[
\text{FFPWG}(\tilde{\tau}_1, \tilde{\tau}_2, \ldots, \tilde{\tau}_n) = \tilde{\tau}_1 \frac{\sum_{j=1}^{\nu_1}}{\nu_1} \otimes \tilde{\tau}_2 \frac{\sum_{j=1}^{\nu_2}}{\nu_2} \otimes \ldots \otimes \tilde{\tau}_n \frac{\sum_{j=1}^{\nu_n}}{\nu_n}
\]

(8)

then this mapping FFPWG is called FFPWG operator, where \( \kappa_j = \prod_{k=1}^{j-1} \tilde{\nu}(\tilde{\tau}_k) \) (\( j = 2, \ldots, n \)), \( \kappa_1 = 1 \) and \( \tilde{\nu}(\tilde{\tau}_k) \) is the score of \( k \)th FFN.

**Theorem 3.9.** Assume that \( \tilde{\tau}_j = \langle \mu_j^2, \nu_j^2 \rangle \) is the collection of FFNs, we can find FFPWG by

\[
\text{FFPWG}(\tilde{\tau}_1, \tilde{\tau}_2, \ldots, \tilde{\tau}_n) = \left\langle \prod_{j=1}^{n} \mu_j^2 \frac{\sum_{j=1}^{\nu_j}}{\nu_j} \right\rangle \sqrt{1 - \prod_{j=1}^{n} (1 - \nu_j^2) \frac{\sum_{j=1}^{\nu_j}}{\nu_j} \frac{\sum_{j=1}^{\nu_j}}{\nu_j}}
\]

(9)

**Proof.**

\[
\text{FFPWG}(\tilde{\tau}_1, \tilde{\tau}_2, \ldots, \tilde{\tau}_n) = \left\langle \prod_{j=1}^{n} \mu_j^2 \frac{\sum_{j=1}^{\nu_j}}{\nu_j} \right\rangle \sqrt{1 - \prod_{j=1}^{n} (1 - \nu_j^2) \frac{\sum_{j=1}^{\nu_j}}{\nu_j} \frac{\sum_{j=1}^{\nu_j}}{\nu_j}}
\]

We utilize mathematical induction to proof this theorem.

For \( n = 2 \)

\[
\tilde{\tau}_1 \frac{\sum_{j=1}^{\nu_1}}{\nu_1} = \left\langle \mu_1^2 \frac{\sum_{j=1}^{\nu_1}}{\nu_1} \right\rangle \sqrt{1 - \nu_1^2} \frac{\sum_{j=1}^{\nu_1}}{\nu_1}
\]

\[
\tilde{\tau}_2 \frac{\sum_{j=1}^{\nu_2}}{\nu_2} = \left\langle \mu_2^2 \frac{\sum_{j=1}^{\nu_2}}{\nu_2} \right\rangle \sqrt{1 - \nu_2^2} \frac{\sum_{j=1}^{\nu_2}}{\nu_2}
\]

Then
Now this shows that the Equation 9 is valid for $g$, we have,

$$\sum_{n} j = \kappa_1 g_1 = \kappa_2 g_2$$

Thus, the Equation 9 is valid for $n = 2$, now assume that the Equation 9 holds for $n = k$, i.e.,

$$\text{FFPWG}(\gamma_1, \gamma_2, \ldots \gamma_k) = \left(\prod_{j=1}^{k} \mu_j^{\gamma_j - 1}, \sqrt{1 - \prod_{j=1}^{k}(1 - v_2^j \gamma_j)}\right)$$

Now $n = k + 1$, we have,
Theorem 3.10. Below we define some of FFPWG operator’s appealing properties.

\[ \text{FFPWG}(\overline{\gamma}_1, \overline{\gamma}_2, \ldots, \overline{\gamma}_{k+1}) = \text{FFPWG}(\overline{\gamma}_1, \overline{\gamma}_2, \ldots, \overline{\gamma}_k) \circ \overline{\gamma}_{k+1} \]

\[
= \left( \prod_{j=1}^{k+1} \mu^j \frac{\gamma_j^2}{\mu_j^2 \gamma_j^2} \right)^{3 \left( \frac{1}{1 - \prod_{j=1}^{k+1} (1 - \gamma_j^2) \frac{\gamma_j^2}{\mu_j^2 \gamma_j^2}} \right)} \\
\quad \left( 1 - \prod_{j=1}^{k+1} (1 - \gamma_j^2) \frac{\gamma_j^2}{\mu_j^2 \gamma_j^2} \right) \left( 1 - (1 - \gamma_j^2) \frac{\gamma_j^2}{\mu_j^2 \gamma_j^2} \right) \\
\quad \left( 1 - (1 - \gamma_j^2) \frac{\gamma_j^2}{\mu_j^2 \gamma_j^2} \right) \\
\quad \left( 1 - (1 - \gamma_j^2) \frac{\gamma_j^2}{\mu_j^2 \gamma_j^2} \right)
\]

This shows that for \( n = k + 1 \), Equation 2 holds. Then,

\[
\text{FFPWG}(\overline{\gamma}_1, \overline{\gamma}_2, \ldots, \overline{\gamma}_n) = \left( \prod_{j=1}^{k+1} \mu^j \frac{\gamma_j^2}{\mu_j^2 \gamma_j^2} \right)^{3 \left( \frac{1}{1 - \prod_{j=1}^{k+1} (1 - \gamma_j^2) \frac{\gamma_j^2}{\mu_j^2 \gamma_j^2}} \right)} (1 - \prod_{j=1}^{k+1} (1 - \gamma_j^2) \frac{\gamma_j^2}{\mu_j^2 \gamma_j^2})
\]

Below we define some of FFPWG operator’s appealing properties.

**Theorem 3.10. (Idempotency)** Assume that \( \overline{\gamma}_j = (\mu_j^2, \nu_j^2) \) is the assemblage of FFNs, where \( \nu_j^2 = \prod_{k=1}^{j-1} \hat{\beta}(\overline{\gamma}_k) \) (\( j = 2, \ldots, n \), \( \nu_1^2 = 1 \) and \( \hat{\beta}(\overline{\gamma}_k) \) is the score of \( k^{th} \) FFN). If all \( \overline{\gamma}_j \) are equal, i.e., \( \overline{\gamma}_j = \overline{\gamma} \) for all \( j \), then

\[
\text{FFPWG}(\overline{\gamma}_1, \overline{\gamma}_2, \ldots, \overline{\gamma}_n) = \overline{\gamma}
\]

**Proof.** From Definition 3.1, we have

\[
\text{FFPWG}(\overline{\gamma}_1, \overline{\gamma}_2, \ldots, \overline{\gamma}_n) = \overline{\gamma}_1^2 \overline{\gamma}_2 \overline{\gamma}_3 \ldots \overline{\gamma}_n \overline{\gamma}_j \overline{\gamma}_j \overline{\gamma}_j
\]

\[
\quad = \overline{\gamma} \overline{\gamma} \overline{\gamma} \ldots \overline{\gamma} \overline{\gamma} \overline{\gamma}
\]

\[
\quad = \overline{\gamma}
\]

\[\square\]
Corollary 3.11. If $\gamma_j = (\mu_j, \nu_j^2)$ for $j = (1, 2, \ldots, n)$ is the assemblage of largest FFNs, i.e., $\gamma_j = (1, 0)$ for all $j$, then $FPWG(\gamma_1, \gamma_2, \ldots, \gamma_n) = (1, 0)$

Proof. We can easily obtain Corollary similar to the Theorem 3.3.

Corollary 3.12. (Non-compensatory) If $\gamma_1 = (\mu_1, \nu_1^2)$ is the smallest FFN, i.e., $\gamma_1 = (0, 1)$, then $FPWG(\gamma_1, \gamma_2, \ldots, \gamma_n) = (0, 1)$

Proof. Here, $\gamma_1 = (0, 1)$ then by definition of the score function, we have, $\bar{\kappa}(\gamma_1) = 0$

Since,

$\kappa_j = \prod_{k=1}^{j-1} \bar{\kappa}(\gamma_k) \quad (j = 2, \ldots, n)$, and $\kappa_1 = 1$

$\bar{\kappa}(\gamma_k)$ is the score of $k^{th}$ FFN.

We have,

$\kappa_j = \prod_{k=1}^{j-1} \bar{\kappa}(\gamma_k) = \bar{\kappa}(\gamma_1) \times \bar{\kappa}(\gamma_2) \times \cdots \times \bar{\kappa}(\gamma_{j-1}) = 0 \times \bar{\kappa}(\gamma_2) \times \cdots \times \bar{\kappa}(\gamma_{j-1}) \quad (j = 2, \ldots, n)$

$\prod_{k=1}^{j} \kappa_j = 1$

From Definition 3.1, we have

$FPWG(\gamma_1, \gamma_2, \ldots, \gamma_n) = \prod_{j=1}^{n} \frac{\nu_j^2}{\nu_j^2 - \nu_j^2} \otimes \prod_{j=1}^{n} \frac{\nu_j^2}{\nu_j^2 - \nu_j^2} \otimes \cdots \otimes \prod_{j=1}^{n} \frac{\nu_j^2}{\nu_j^2 - \nu_j^2}$

$= \prod_{j=1}^{n} \frac{\nu_j^2}{\nu_j^2 - \nu_j^2} \otimes \cdots \otimes \frac{\nu_j^2}{\nu_j^2 - \nu_j^2} = \gamma_1 = (0, 1)$

Corollary 3.12 implies that incentives would not be obtained by anyone although the higher importance conditions had been satisfied by the smallest FFN.

Theorem 3.13. (Monotonicity) Assume that $\gamma_j = (\mu_j, \nu_j^2)$ and $\gamma^*_j = (\mu_j^*, \nu_j^2)$ are the assemblages of FFNs, where $\kappa_j = \prod_{k=1}^{j-1} \bar{\kappa}(\gamma_k)$, $\kappa^*_j = \prod_{k=1}^{j-1} \bar{\kappa}(\gamma^*_k)$ ($j = 2, \ldots, n$), $\kappa_1 = 1$, $\kappa^*_1 = 1$, $\bar{\kappa}(\gamma_k)$ is the score of $\gamma_k$ FFN, and $\bar{\kappa}(\gamma_k)$ is the score of $\gamma^*_k$ FFN. If $\mu_j \geq \mu_j^*$ and $\nu_j^2 \leq \nu_j^2$ for all $j$, then $FPWG(\gamma_1, \gamma_2, \ldots, \gamma_n) \leq FPGW(\gamma_1^*, \gamma_2^*, \ldots, \gamma_n^*)$

Proof. Here, $\nu_j^2 \geq \nu_j^2$ and $\mu_j \leq \mu_j^*$ for all $j$. If $\nu_j^2 \geq \nu_j^2$,

$\nu_j^2 \geq (\nu_j^2)^3 \Rightarrow (\nu_j^2)^3 \leq (\nu_j^2)^3 \Rightarrow (\nu_j^2)^3 \leq 1 - (\nu_j^2)^3 \Rightarrow 1 - (\nu_j^2)^3 \leq 1 - (\nu_j^2)^3$

$\Rightarrow \sqrt[3]{1 - (\nu_j^2)^3} \leq \sqrt[3]{1 - (\nu_j^2)^3}$

$\Rightarrow \sqrt[3]{1 - (\nu_j^2)^3} \leq \sqrt[3]{1 - (\nu_j^2)^3}$

\[\Rightarrow \sqrt[3]{1 - (\nu_j^2)^3} \leq \sqrt[3]{1 - (\nu_j^2)^3}\]
\[ \prod_{j=1}^{n}(1-(\nu_j^3 \rho_j^2)^{1/\rho_j^2}) \leq \prod_{j=1}^{n}(1-(\nu_j^3 \rho_j^2)^{1/\rho_j^2}) \]

Now,
\[ \mu_j^2 \leq \mu_j \]
\[ (\mu_j^2)^{1/\rho_j} \leq (\mu_j)^{1/\rho_j} \]
\[ \prod_{j=1}^{n}(\mu_j^2)^{1/\rho_j} \leq \prod_{j=1}^{n}(\mu_j)^{1/\rho_j} \]

Let
\[ \mathcal{V} = \text{FFPWG}(\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_n) \]
and
\[ \mathcal{V}' = \text{FFPWG}(\mathcal{V}_1^*, \mathcal{V}_2^*, \ldots, \mathcal{V}_n^*) \]

We get that \( \mathcal{V}' \geq \mathcal{V} \). So,
\[ \text{FFPWG}(\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_n) \leq \text{FFPWG}(\mathcal{V}_1^*, \mathcal{V}_2^*, \ldots, \mathcal{V}_n^*) \]

\[ \text{Theorem 3.14. (Boundary) Assume that } \mathcal{V}_j = (\mu_j^2, \nu_j^3) \text{ is the assemblage of FFNs, and } \]
\[ \mathcal{V}^- = (\min_j(\mu_j^2), \max_j(\nu_j^3)) \text{ and } \mathcal{V}^+ = (\max_j(\mu_j^2), \min_j(\nu_j^3)) \]

Then,
\[ \mathcal{V}^- \leq \text{FFPWG}(\mathcal{V}_1, \mathcal{V}_2, \ldots, \mathcal{V}_n) \leq \mathcal{V}^+ \]

where \( \kappa_j = \sum_{k=1}^{j-1} \hat{\Omega}(\mathcal{V}_k) \) (\( j = 2, \ldots, n \)), \( \kappa_1 = 1 \) and \( \hat{\Omega}(\mathcal{V}_k) \) is the score of \( k^{th} \) FFN.

\[ \text{Proof. The proof of this theorem is same as Theorem 3.7.} \]

\[ \square \]

4 Methodology for IDSS using proposed AOs

Let \( \Pi = (\Pi_1, \Pi_2, \ldots, \Pi_m) \) be the collection of alternatives and \( \eta^k = (\eta_1^k, \eta_2^k, \ldots, \eta_n^k) \) is the assemblage of criteria, priorities are given here between the criteria presented by the linear pattern. \( \eta_1^k > \eta_2^k > \eta_3^k \ldots \eta_n^k \) indicates criteria \( \eta_j^k \) has a higher priority than \( \eta_j^k \) if \( j > i \). \( R = (r_1, r_2, \ldots, r_p) \) is a team of decision-makers (DMs) and DMs are not given the same priority. Prioritization is provided by a linear pattern between the DMs given as, \( \mathcal{Y}_1 > \mathcal{Y}_2 > \mathcal{Y}_3 \ldots \mathcal{Y}_p \) indicates DM \( \mathcal{Y}_i \) has a higher priority than \( \mathcal{Y}_j \) if \( i > j \). DMs provide a matrix of their own opinion \( D^{(p)} = (\rho_{ij}^{(p)})_{m \times n} \), where \( \rho_{ij}^{(p)} \) is given for the alternatives \( \Pi_i \in \Pi \) with respect to the criteria.
Step 3: Equation 10

The suggested operators will be implemented to the MCGDM, which will require the preceding steps.

\[

d_j \in \eta_i \text{ by } \bar{Y}_p \text{ decision maker in the form of FFNs. If all Criterions are the same types, there is no need for normalization, but there are two types of Criterions (benefit type attributes } \tau_b \text{ and cost type attributes } \tau_c \text{) in MCGDM, in this case using the normalization formula the matrix } D^{(p)} \text{ has been changed into normalize matrix } Y^{(p)} = (\mathcal{B}^{(p)})_{m \times n}, \]

\[
(\mathcal{B}^{(p)})_{m \times n} = \begin{cases} 
(\mathcal{B}^{(p)})_j; & j \in \tau_c \\
(\mathcal{B}^{(p)})_j^C; & j \in \tau_b 
\end{cases} 
\]

where \((\mathcal{B}^{(p)})_j^C\) show the compliment of \(\mathcal{B}^{(p)}_j\).

The suggested operators will be implemented to the MCGDM, which will require the preceding steps.

Algorithm

Step 1:

Obtain the decision matrix \(D^{(p)} = (\mathcal{B}^{(p)})_{m \times n}\) in the format of FFNs from DMs.

\[
\begin{align*}
\bar{Y}_1 & \quad \Pi_1 \quad \eta_1^c \\
& \quad (\mu_{11}^{c1}, \nu_{11}^{c1}) \quad (\mu_{12}^{c1}, \nu_{12}^{c1}) \quad \cdots \quad (\mu_{1n}^{c1}, \nu_{1n}^{c1}) \\
& \quad (\mu_{21}^{c1}, \nu_{21}^{c1}) \quad (\mu_{22}^{c1}, \nu_{22}^{c1}) \quad \cdots \quad (\mu_{2n}^{c1}, \nu_{2n}^{c1}) \\
& \quad \vdots \quad \vdots \quad \ddots \quad \vdots \\
& \quad (\mu_{m1}^{c1}, \nu_{m1}^{c1}) \quad (\mu_{m2}^{c1}, \nu_{m2}^{c1}) \quad \cdots \quad (\mu_{mn}^{c1}, \nu_{mn}^{c1}) \\
\bar{Y}_2 & \quad \Pi_2 \quad \eta_2^c \\
& \quad (\mu_{11}^{c2}, \nu_{11}^{c2}) \quad (\mu_{12}^{c2}, \nu_{12}^{c2}) \quad \cdots \quad (\mu_{1n}^{c2}, \nu_{1n}^{c2}) \\
& \quad (\mu_{21}^{c2}, \nu_{21}^{c2}) \quad (\mu_{22}^{c2}, \nu_{22}^{c2}) \quad \cdots \quad (\mu_{2n}^{c2}, \nu_{2n}^{c2}) \\
& \quad \vdots \quad \vdots \quad \ddots \quad \vdots \\
& \quad (\mu_{m1}^{c2}, \nu_{m1}^{c2}) \quad (\mu_{m2}^{c2}, \nu_{m2}^{c2}) \quad \cdots \quad (\mu_{mn}^{c2}, \nu_{mn}^{c2}) \\
\bar{Y}_p & \quad \Pi_m \quad \eta_m^c \\
& \quad (\mu_{11}^{cP}, \nu_{11}^{cP}) \quad (\mu_{12}^{cP}, \nu_{12}^{cP}) \quad \cdots \quad (\mu_{1n}^{cP}, \nu_{1n}^{cP}) \\
& \quad (\mu_{21}^{cP}, \nu_{21}^{cP}) \quad (\mu_{22}^{cP}, \nu_{22}^{cP}) \quad \cdots \quad (\mu_{2n}^{cP}, \nu_{2n}^{cP}) \\
& \quad \vdots \quad \vdots \quad \ddots \quad \vdots \\
& \quad (\mu_{m1}^{cP}, \nu_{m1}^{cP}) \quad (\mu_{m2}^{cP}, \nu_{m2}^{cP}) \quad \cdots \quad (\mu_{mn}^{cP}, \nu_{mn}^{cP})
\end{align*}
\]

Step 2:

There are two sorts of criterion in the decision matrix: \((\tau^{aPc})\) cost type performance indicators and \((\tau^{aPb})\) benefit type performance indicators. There is no need for normalisation if all performance indicators are of the identical type; nevertheless, in MCDM, there may be two types of performance indicators. In this scenario, the matrix was modified to the transforming response matrix \(Y^{(p)} = (\mathcal{B}^{(p)})_{m \times n}\) using the normalization formula Equation 10

Step 3:

Evaluate the values of \(\kappa_j^{(p)}\) by given formula.

\[
\kappa_j^{(p)} = \prod_{k=1}^{p-1} \hat{f}(\mathcal{B}^{(k)})_{ij} \quad (p = 2, \ldots, n),
\]
\[ \kappa^{(1)}_{ij} = 1 \]

**Step 4:**

Using one of provided AOs to combine all of the independent Farmatian decision matrices \( Y^{(p)} = (\mathcal{B}^{(p)})_{m \times n} \), into one combined evaluation matrix of the alternatives \( W^{(p)} = (\mathcal{X}^{(p)})_{m \times n} \).

\[ \mathcal{X}^{ij} = FFPWA(\mathcal{B}^{(1)}_{ij}, \mathcal{B}^{(2)}_{ij}, ..., \mathcal{B}^{(p)}_{ij}) = \left\langle \frac{\sqrt{\prod_{j=1}^{n}(1 - \prod_{i=1}^{m}(1 - ((\mu_{ij}^3)^{3/2})^{\frac{1}{\kappa_{ij}}}, \prod_{j=1}^{n}(1 - ((\nu_{ij}^3)^{3/2})^{\frac{1}{\kappa_{ij}}})} \right\rangle (12) \]

or

\[ \mathcal{X}^{ij} = FFPWA(\mathcal{B}^{(1)}_{ij}, \mathcal{B}^{(2)}_{ij}, ..., \mathcal{B}^{(p)}_{ij}) = \left\langle \prod_{i=1}^{m}(1 - \prod_{j=1}^{n}(1 - ((\mu_{ij}^3)^{3/2})^{\frac{1}{\kappa_{ij}}}, \prod_{j=1}^{n}(1 - ((\nu_{ij}^3)^{3/2})^{\frac{1}{\kappa_{ij}}}) \right\rangle (13) \]

**Step 5:**

Calculate the values of \( \kappa^{3}_{ij} \) by following formula.

\[ \kappa^{3}_{ij} = \prod_{k=1}^{j-1} \hat{O}(\chi^{3}_{ik}) \quad (j = 2, ..., n), \]

\[ \kappa^{3}_{i1} = 1 \]

**Step 6:**

Aggregate the q-ROF values \( \chi^{3}_{ij} \) for each alternative \( \Pi_i \) by the FFPWA (or FFPWG) operator:

\[ \chi^{3}_{ij} = FFPWA(\mathcal{P}_{i1}, \mathcal{P}_{i2}, ..., \mathcal{P}_{in}) = \left\langle \frac{\sqrt{\prod_{j=1}^{n}(1 - \prod_{i=1}^{m}(1 - ((\mu_{ij}^3)^{3/2})^{\frac{1}{\kappa_{ij}}}, \prod_{j=1}^{n}(1 - ((\nu_{ij}^3)^{3/2})^{\frac{1}{\kappa_{ij}}})} \right\rangle (15) \]

or

\[ \chi^{3}_{ij} = FFPWG(\mathcal{P}_{i1}, \mathcal{P}_{i2}, ..., \mathcal{P}_{in}) = \left\langle \prod_{i=1}^{m}(1 - \prod_{j=1}^{n}(1 - ((\mu_{ij}^3)^{3/2})^{\frac{1}{\kappa_{ij}}}, \prod_{j=1}^{n}(1 - ((\nu_{ij}^3)^{3/2})^{\frac{1}{\kappa_{ij}}}) \right\rangle (16) \]

**Step 7:**

Analyze the score for all cumulative alternative assessments.

**Step 8:**

The alternatives were classified by the score function and, eventually, the most suitable alternative was selected.

### 5 Convolutional Neural Network Object Detection Model

Here, the theoretical previous literature is discussed for understanding the related techniques of machine learning, sensors networks, deep learning, neural networks and automatic energy management systems. The algorithm of various conventional techniques are also argued.
5.1 Machine learning

The methodology of our proposed model explains all the stages of human recognition and detection as shown in the Figure 5.1.

![Figure 1: Video Based Human Detection Model](image)

5.1.1 Video recording and processing

In our proposed model, a camera was installed to monitor and record the human activities in video form and video was uploaded to create frames of images. So it was used to interpret 2D images in the frames. The operation of real time object recognition system was based on the sampling video interval of 10 minutes. Duration of each small sample represents 1 min video clip to study the behavior of real time object.

5.1.2 Single shot detector (SSD) model

This model is used for real time object detection and to eliminate the region of application to enhance the speed of the overall process. This model uses 59 frames of images per second (FPS) to recognize and detect an object in real time. It computes multiple classes for extraction of features with small convolution filters. It uses multi-scale feature and boxes to recuperate the drop-in accuracy [45]. The step by step features extraction for this model is shown in Figure 5.1.2.
5.1.3 Object Recognition

In this proposed research work a novel convolutional neural network (CNN) technique is used for object recognition. It has used an image of 190 × 123 pixel (height × width) as an input for object recognition. It observes noises and distortion due to a more height of image and it creates problems for object recognition. Therefore, size normalization technique uses for the adjustment of size. Our proposed model is encompassed of occupancy recognition, activity level classification and validation of the recognition system [46].

5.1.4 Feature Extraction and Classification

Our proposed research work used five convolutional layers and three fully connected layers in CNN architecture [47]. In convolutional layer, output feature of an image can be calculated as:

\[ x(out) = \frac{x(in) - k + 2p}{s} + 1 \]  

(17)

\( x(in) = \) input image feature map (height or width)
\( p = \) padding size
\( k = \) filter size
\( s = \) stride size
\( x(out) = \) output feature map of an image (height or width)
Therefore, feature map size in 1st convolutional layer is $93 \times 60 \times 96$, and the output of the 1st convolutional layer is calculated as; $x(output) = \lceil(190 + 2(0) - 13)/2\rceil + 1 = 90$. It has used the maximum size of filter to avoid invalid noise inside feature. CNN based algorithm works on neurons activity. Therefore, the neuron’s outcome is known as function $f$. The function of input image $x$ is derived as:

$$f(x) = \tanh(x)$$

(18)

or

$$f(x) = (1 + e^{-x})^{-1}$$

(19)

The non-saturating nonlinearity works faster in training than saturating nonlinearities.

$$f(x) = \max(0,x)$$

(20)

Training process of information is faster in Rectified Linear Unit (ReLU) than in tanh. ReLU is also used for the removal of vanishing gradient. Therefore, it increases the processing speed. After that, learning has initiated after ReLU based activity. Generalization is processed with local normalization scheme and it expresses in form of following equation;

$$b_{k,x,y}^i = \frac{a_{k,x,y}^i}{i + a \left\{ \sum_{j=\max(0,k-n/2)}^{\min(N-1,k+n/2)} (a_{k,x,y}^j)^2 \right\}^\beta}$$

(21)

In the above mentioned equation, $a_{k,x,y}^i$ is denoted the neuron activity by kernel $k$ at the position level $(x,y)$ and after that it is applied the ReLU non-linearity. $n$ is the sum for kernel maps at exact spatial location and $N$ denotes the total number of kernel used in the convolutional layer. The other constant $i, a$ and $\beta$ are the hyper-parameters. To design the robust and noiseless CNN approach, the feature map attained through the local response normalization channel for max pooling layer. After the 1st convolutional layer, the 2nd layer is advanced 128 filters spending the size of $7 \times 7 \times 96$, stride of $1 \times 1$ pixels and the padding of $2 \times 2$ pixels in the horizontal and vertical directions. In the structure of CNN the application of first two layers are to focus on weak level image for feature extraction in form of blobs or edges. The next three layers are applied for advanced level extraction of feature. Therefore, the convolution of five layers attains the results of feature maps and pixels in the form of 128 feature maps taking the size of $11 \times 7$ pixels. After that these are fed to next additional fully connected layers respectively. This approach is finally achieved the human detection and subtract the background. At the end, softmax function is applied in the last fully connected layer, which is called output layer. The function of softmax is given as:

$$\sigma(s)_j = \frac{e^{s_j}}{\sum_{m=1}^{k} s_{mn}}$$

(22)

Therefore, CNN based human detection system may have the concern of over-fitting. But it can be decreased with the implementation of augmentation and dropout technique. The proposed model works on the algorithm for the real time human recognition and detection. The detection of occupancy is done with video processing which works on per frame optimization technique. The accuracy level of algorithm has achieved better results. Real time human recognition and detection system SSD model is used for extraction of feature map and has applied convolutional layers for object detection in video frames. Human detection is achieved with the isolation of backgrounds. Human can be identified and tracked in video surveillance system. This system may have multiple applications. Because it has also capacity to implement in biometric verification process with human face identification. This model has achieved better results of classification as compared to conventional methods. It works at very low cost. CNN model shows better recognition and detection results.
6 IDSS to Energy Consumption pattern

In this section we compare our proposed CNN model with other existing models under the Farmatian fuzzy environment. Other models are energy consumption pattern with passive infra-red (PIR) sensors, Energy consumption pattern with ultrasonic sensor, energy Consumption pattern with sensors fusion, energy consumption pattern with radio frequency (RF) signals and energy Consumption pattern with WLAN, WiFi and Bluetooth. (cite) Planned indoor energy monitoring and management systems are assessed according to the criterions which are given as

- \( \eta_1 \): Environmental protection
- \( \eta_2 \): Cost
- \( \eta_3 \): Total occupancy count (number of people)
- \( \eta_4 \): Heating, ventilation, and air condition (Watts)
- \( \eta_5 \): Profit
- \( \eta_6 \): Organizational practices that are socially positive

Consider a set of alternatives \( \Pi = \{ \Pi_1, \Pi_2, \Pi_3, \Pi_4, \Pi_5 \} \), where \( \Pi_1 \) = energy Consumption pattern with Camera based CNN Technique, \( \Pi_2 \) = energy consumption pattern with passive infra-red (PIR) sensors, \( \Pi_3 \) = energy consumption pattern with ultrasonic sensor, \( \Pi_4 \) = energy consumption pattern with radio frequency (RF) signals, \( \Pi_5 \) = energy Consumption pattern with WLAN, WiFi and Bluetooth and \( \eta = \{ \eta_1, \eta_2, \eta_3, \eta_4, \eta_5, \eta_6 \} \) is the collection of criterions given above, Priorities shall be defined between all the criteria set out in the linear pattern as \( \eta_1 > \eta_2 > \eta_3 > \eta_4 > \eta_5 > \eta_6 \). DMs are providing a matrix about their own judgment \( D(p) = (B(p))_{m \times n} \), where \( B(p) \) is given for the alternatives \( \Pi_i \in \Pi \) with respect to the criteria \( \eta_j \), \( \eta_k \) by \( \eta_p \) DM in the form of FFNs.

### Step 1:

Obtain the decision matrix \( D(p) = (B(p))_{m \times n} \) in the format of FFNs by the DMs, given in Table 1, Table 2 and Table 3.

|      | \( \eta_1 \)      | \( \eta_2 \)      | \( \eta_3 \)      | \( \eta_4 \)      | \( \eta_5 \)      | \( \eta_6 \)      |
|------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
| \( \Pi_1 \) | (0.90,0.00)       | (0.65,0.35)       | (0.75,0.15)       | (0.95,0.15)       | (0.75,0.00)       | (0.45,0.25)       |
| \( \Pi_2 \) | (0.95,0.25)       | (0.80,0.30)       | (0.55,0.25)       | (0.75,0.15)       | (0.45,0.45)       | (0.35,0.15)       |
| \( \Pi_3 \) | (0.85,0.15)       | (0.35,0.55)       | (0.75,0.25)       | (0.55,0.00)       | (0.65,0.35)       | (0.45,0.00)       |
| \( \Pi_4 \) | (0.75,0.35)       | (0.81,0.25)       | (0.65,0.15)       | (0.35,0.25)       | (0.75,0.25)       | (0.35,0.75)       |
| \( \Pi_5 \) | (0.80,0.25)       | (0.60,0.00)       | (0.25,0.15)       | (0.15,0.65)       | (0.65,0.15)       | (0.25,0.65)       |
Table 2: Farmatian fuzzy decision matrix from $\bar{\gamma}_2$.

| $\eta_i$ | $\eta_i^1$ | $\eta_i^2$ | $\eta_i^3$ | $\eta_i^4$ | $\eta_i^5$ | $\eta_i^6$ |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\bar{\Pi}_1$ | (0.75,0.25) | (0.55,0.30) | (0.85,0.15) | (0.95,0.15) | (0.80,0.25) | (0.90,0.15) |
| $\bar{\Pi}_2$ | (0.55,0.15) | (0.60,0.35) | (0.45,0.15) | (0.75,0.35) | (0.65,0.30) | (0.75,0.00) |
| $\bar{\Pi}_3$ | (0.90,0.60) | (0.65,0.20) | (0.25,0.55) | (0.65,0.55) | (0.15,0.25) | (0.70,0.30) |
| $\bar{\Pi}_4$ | (0.50,0.00) | (0.55,0.40) | (0.15,0.10) | (0.50,0.60) | (0.10,0.15) | (0.60,0.35) |
| $\bar{\Pi}_5$ | (0.85,0.35) | (0.70,0.30) | (0.65,0.55) | (0.25,0.50) | (0.50,0.30) | (0.50,0.25) |

Table 3: Farmatian fuzzy decision matrix from $\bar{\gamma}_3$.

| $\eta_i$ | $\eta_i^1$ | $\eta_i^2$ | $\eta_i^3$ | $\eta_i^4$ | $\eta_i^5$ | $\eta_i^6$ |
|----------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\bar{\Pi}_1$ | (0.90,0.15) | (0.85,0.25) | (0.80,0.00) | (0.70,0.35) | (0.80,0.20) | (0.70,0.30) |
| $\bar{\Pi}_2$ | (0.80,0.25) | (0.55,0.15) | (0.60,0.25) | (0.50,0.30) | (0.60,0.30) | (0.60,0.30) |
| $\bar{\Pi}_3$ | (0.75,0.15) | (0.65,0.25) | (0.35,0.00) | (0.50,0.35) | (0.75,0.30) | (0.35,0.25) |
| $\bar{\Pi}_4$ | (0.35,0.35) | (0.50,0.35) | (0.45,0.25) | (0.55,0.45) | (0.25,0.25) | (0.65,0.00) |
| $\bar{\Pi}_5$ | (0.65,0.25) | (0.65,0.25) | (0.60,0.15) | (0.65,0.25) | (0.65,0.55) | (0.45,0.40) |

Step 2:
Using Equation 10, normalize decision matrixes gained by DMs. $\eta^c$ is cost type criteria and others are benefit type criterions. Normalized Farmatian fuzzy decision matrices are given in Table 4, Table 5 and Table 6.

Table 4: Normalized Farmatian fuzzy decision matrix from $\bar{\gamma}_1$.

| $\eta^c_i$ | $\eta^c_i^1$ | $\eta^c_i^2$ | $\eta^c_i^3$ | $\eta^c_i^4$ | $\eta^c_i^5$ | $\eta^c_i^6$ |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\bar{\Pi}_1$ | (0.90,0.00) | (0.35,0.65) | (0.75,0.15) | (0.95,0.15) | (0.75,0.00) | (0.45,0.25) |
| $\bar{\Pi}_2$ | (0.95,0.25) | (0.30,0.80) | (0.55,0.25) | (0.75,0.15) | (0.45,0.45) | (0.35,0.15) |
| $\bar{\Pi}_3$ | (0.85,0.15) | (0.55,0.35) | (0.75,0.25) | (0.55,0.00) | (0.65,0.35) | (0.45,0.00) |
| $\bar{\Pi}_4$ | (0.75,0.35) | (0.25,0.81) | (0.65,0.15) | (0.35,0.25) | (0.75,0.25) | (0.35,0.75) |
| $\bar{\Pi}_5$ | (0.80,0.25) | (0.00,0.60) | (0.25,0.15) | (0.15,0.65) | (0.65,0.15) | (0.25,0.65) |

Table 5: Normalized Farmatian fuzzy decision matrix from $\bar{\gamma}_2$.

| $\eta^c_i$ | $\eta^c_i^1$ | $\eta^c_i^2$ | $\eta^c_i^3$ | $\eta^c_i^4$ | $\eta^c_i^5$ | $\eta^c_i^6$ |
|------------|-------------|-------------|-------------|-------------|-------------|-------------|
| $\bar{\Pi}_1$ | (0.75,0.25) | (0.30,0.55) | (0.85,0.15) | (0.95,0.15) | (0.80,0.25) | (0.90,0.15) |
| $\bar{\Pi}_2$ | (0.55,0.15) | (0.35,0.60) | (0.45,0.15) | (0.75,0.35) | (0.65,0.30) | (0.75,0.00) |
| $\bar{\Pi}_3$ | (0.90,0.60) | (0.20,0.65) | (0.25,0.55) | (0.65,0.55) | (0.15,0.25) | (0.70,0.30) |
| $\bar{\Pi}_4$ | (0.50,0.00) | (0.40,0.55) | (0.15,0.10) | (0.50,0.60) | (0.10,0.15) | (0.60,0.35) |
| $\bar{\Pi}_5$ | (0.85,0.35) | (0.30,0.70) | (0.65,0.55) | (0.25,0.50) | (0.50,0.30) | (0.50,0.25) |
Table 6: Normalized Farmatian fuzzy decision matrix from $\bar{Y}_3$.

|   | $\eta^*_{1}$ | $\eta^*_{2}$ | $\eta^*_{3}$ | $\eta^*_{4}$ | $\eta^*_{5}$ | $\eta^*_{6}$ |
|---|-------------|-------------|-------------|-------------|-------------|-------------|
| $\bar{\Pi}_1$ | (0.90,0.15) | (0.25,0.85) | (0.80,0.00) | (0.70,0.35) | (0.80,0.20) | (0.70,0.30) |
| $\bar{\Pi}_2$ | (0.80,0.25) | (0.15,0.55) | (0.60,0.25) | (0.50,0.30) | (0.60,0.30) | (0.60,0.30) |
| $\bar{\Pi}_3$ | (0.75,0.15) | (0.25,0.65) | (0.35,0.00) | (0.50,0.35) | (0.75,0.30) | (0.35,0.25) |
| $\bar{\Pi}_4$ | (0.35,0.35) | (0.35,0.50) | (0.45,0.25) | (0.55,0.45) | (0.25,0.25) | (0.65,0.00) |
| $\bar{\Pi}_5$ | (0.65,0.25) | (0.25,0.65) | (0.60,0.15) | (0.65,0.25) | (0.65,0.55) | (0.45,0.40) |

Step 3:
Determine the $\kappa^*(p)_{ij}$ values using the Equation 11.

$$\kappa^*(1)_{ij} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$\kappa^*(2)_{ij} = \begin{pmatrix} 0.8645 & 0.3841 & 0.7093 & 0.9270 & 0.7109 & 0.5378 \\ 0.9209 & 0.2575 & 0.5754 & 0.7093 & 0.5000 & 0.5198 \\ 0.8054 & 0.5618 & 0.7031 & 0.5832 & 0.6159 & 0.5456 \\ 0.6895 & 0.2421 & 0.6356 & 0.5136 & 0.7031 & 0.3105 \\ 0.7482 & 0.3920 & 0.5061 & 0.3644 & 0.6356 & 0.3705 \end{pmatrix}$$

$$\kappa^*(3)_{ij} = \begin{pmatrix} 0.6078 & 0.1653 & 0.5629 & 0.8593 & 0.5319 & 0.4640 \\ 0.5355 & 0.1065 & 0.3062 & 0.4891 & 0.3119 & 0.3695 \\ 0.6092 & 0.2060 & 0.2985 & 0.3232 & 0.3042 & 0.3590 \\ 0.3878 & 0.1087 & 0.3186 & 0.2334 & 0.3507 & 0.1821 \\ 0.5878 & 0.1341 & 0.2804 & 0.1623 & 0.3489 & 0.3526 \end{pmatrix}$$

Step 4:
Use FFPWA to aggregate all individual Farmatian fuzzy decision matrices $Y(p) = (\rho^*(p)_{ij})_{m \times n}$ into one cumulative assessment matrix of the alternatives $W(p) = (\chi^*(p)_{ij})_{m \times n}$ using Equation 12 given in Table 7.

Table 7: Collective Farmatian fuzzy decision matrix

|   | $\eta^*_{1}$ | $\eta^*_{2}$ | $\eta^*_{3}$ | $\eta^*_{4}$ | $\eta^*_{5}$ | $\eta^*_{6}$ |
|---|-------------|-------------|-------------|-------------|-------------|-------------|
| $\bar{\Pi}_1$ | (0.8648, 0.0000) | (0.3304, 0.6417) | (0.7997, 0.0000) | (0.9171, 0.1948) | (0.7796, 0.0000) | (0.7312, 0.2273) |
| $\bar{\Pi}_2$ | (0.8611, 0.2064) | (0.3044, 0.7359) | (0.5343, 0.2138) | (0.7153, 0.2300) | (0.5512, 0.3752) | (0.5835, 0.0000) |
| $\bar{\Pi}_3$ | (0.8600, 0.2382) | (0.3048, 0.7192) | (0.6288, 0.0000) | (0.5794, 0.0000) | (0.6081, 0.3474) | (0.5470, 0.0000) |
| $\bar{\Pi}_4$ | (0.6472, 0.0000) | (0.2988, 0.7269) | (0.5491, 0.1429) | (0.4394, 0.3498) | (0.6184, 0.2099) | (0.4825, 0.0000) |
| $\bar{\Pi}_5$ | (0.7936, 0.2784) | (0.2091, 0.6309) | (0.5051, 0.2167) | (0.3400, 0.5516) | (0.6124, 0.2353) | (0.3821, 0.4793) |
Step 5:
Determine the values of $\kappa_{ij}$ by using Equation 14.

$$\kappa_{ij} = \begin{bmatrix} 1 & 0.8234 & 0.3178 & 0.2402 & 0.2118 & 0.1561 \\ 1 & 0.8149 & 0.2565 & 0.1466 & 0.0992 & 0.0553 \\ 1 & 0.8113 & 0.2663 & 0.1663 & 0.0993 & 0.0587 \\ 1 & 0.6356 & 0.2042 & 0.1187 & 0.0618 & 0.0379 \\ 1 & 0.7391 & 0.2801 & 0.1567 & 0.0683 & 0.0415 \end{bmatrix}$$

Step 6:
Aggregate the Farmatian fuzzy values $\chi_{ij}$ for each alternative $\Pi_i$ by the FFPWA operator using Equation 15 given in Table 8.

| $\chi_{i1}$ | (0.7899, 0.0000) |
| $\chi_{i2}$ | (0.7319, 0.0000) |
| $\chi_{i3}$ | (0.7299, 0.0000) |
| $\chi_{i4}$ | (0.5614, 0.0000) |
| $\chi_{i5}$ | (0.6576, 0.3704) |

Step 7:
Compute the score for all Farmatian fuzzy aggregated values $\chi_{i}^3$.

$$\bar{\sigma}(\chi_{i1}^3) = 0.7464$$
$$\bar{\sigma}(\chi_{i2}^3) = 0.6960$$
$$\bar{\sigma}(\chi_{i3}^3) = 0.6944$$
$$\bar{\sigma}(\chi_{i4}^3) = 0.5885$$
$$\bar{\sigma}(\chi_{i5}^3) = 0.6168$$

Step 8:
Ranks according to score values.

$$\chi_{i1}^3 > \chi_{i2}^3 > \chi_{i3}^3 > \chi_{i5}^3 > \chi_{i4}^3$$

So,

$$\Pi_1 > \Pi_2 > \Pi_5 > \Pi_5 > \Pi_4$$

$\Pi_1$ is indicated that our proposed model is best among all the existing models.
6.1 Comparison with Fermatean Fuzzy WPM

The "weighted product model" (WPM) is a well-known and often used MCDM method for evaluating a set of possible choices in terms of a collection of preferences. By multiplying a number of ratios, one for each choice criterion, each decision alternative is compared to the others. Each ratio is raised to the power of the related criterion's relative weight. Fermatean fuzzy WPM is proposed by Senapati and Yager [19] to get optimum alternative in MCDM problems. We solve our given problem by the Fermatean fuzzy WPM and get the same optimal decision, the ranking obtained by the FF-WPM is $\Pi_1 > \Pi_3 > \Pi_2 > \Pi_4 > \Pi_5$. By this one can see the feasibility, authenticity, and superiority of the proposed method.

7 Conclusion

This study focused on occupancy detection models and algorithms for indoor building energy management using camera and sensor data. For energy monitoring and management within buildings, we used a new convolutional neural network-based human recognition system based on a camera. For human detection, the camera employs high-infrared and visible lights with greater accuracy. Traditional structures, on the other hand, feature sensors-based occupancy monitoring, which delivers fewer or restricted information. Sensors are limited in their ability to detect, recognize, and count humans. In longer-range applications, sensors are unable to identify and recognize humans. We concluded that camera-based (CNN) person detection can save more HVAC energy and costs while also promoting a clean atmosphere inside buildings. We offered some additional prioritized AOs under the Fermatean fuzzy framework to test the applicability of our notion. Some ISSD problem results may be influenced by a failure to consider the links of qualities to the uncertain environment. The FFNs are used to express the assessment of decision-makers, the vagueness and incompleteness of the information are effectively addressed. Meanwhile, we developed prioritized AOs named as "Fermatean fuzzy prioritized weighted average (FFPWA) operator and Fermatean fuzzy prioritized weighted geometric (FFPWG) operator". Various desirable properties of these AOs, as well as their interconnections, have been thoroughly investigated. In further research, considering the superiority of new FFNs, one can extend them to some AOs, such as power mean AOs, Dombi's aggregation operators, Bonferroni mean operators, Heronian mean operators, and so on. Furthermore, we intend to apply the suggested AOs and technique to a wide range of practical applications, including image processing, game theory, pattern recognition, cluster analysis, and uncertain programming.

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