Orientation, flow, and clogging in a two-dimensional hopper: Ellipses vs. disks

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received 21 October 2015; accepted in final form 11 May 2016
published online 31 May 2016

PACS 47.57.Gc – Granular flow
PACS 81.05.Rm – Porous materials; granular materials
PACS 78.20.Fm – Birefringence

Abstract – Two-dimensional (2D) hopper flow of disks has been extensively studied. Here, we investigate hopper flow of ellipses with aspect ratio $\alpha = 2$, and we contrast that behavior to the flow of disks. We use a quasi-2D hopper containing photoelastic particles to obtain stress/force information. We simultaneously measure the particle motion and stress. We determine several properties, including discharge rates, clogging probabilities, and the number of particles in clogging arches. For both particle types, the size of the opening, $D$, relative to the size of particles, $\ell$, is an important dimensionless measure. The orientation of the ellipses plays an important role in flow rheology and clogging. The alignment of contacting ellipses enhances the probability of forming stable arches. This study offers insights into applications involving the flow of granular materials consisting of ellipsoidal shapes, and possibly other non-spherical shapes.

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Hopper flows of granular materials involve dynamical granular states with important industrial applications [1,2]. Time-averaged granular flow models, often using hopper flow as a test case, have progressed from continuum mechanics models to mesoscopic models [2–6]. Fluctuations and clogging are also important characteristics for hopper flow. Various researchers [7–13] have sought to understand clogging in hopper flow from different perspectives, such as by probabilistic methods or by insights from stress-velocity correlations. Recent studies [14,15] involved an innovative titled hopper geometry, and concluded that there is no sharp clogging transition. However, the effect of particle shape on hopper flow is usually not a primary focus. In reality, particle shapes are often not spherical; rice and M&M candies are roughly ellipsoids; sand particles have irregular shapes. Thus, it is scientifically and industrially relevant to explore how particle shape affects flow rheology and clogging mechanisms.

A simple way to explore particle shape effects is to contrast 2D hopper flows of disks and ellipses for the time-averaged discharge rate, $\dot{N}$, and clogging probability. Discharge rates of hopper flow often follow the well-established Beverloo equation [16], which relates $\dot{N}$ to the hopper opening size, $D$: $\dot{N} \propto (D - kd_{avg})^{(n-1/2)}$, where $n$ is the spatial dimension (e.g. $n = 2$ or $3$ for two- or three-dimensional systems). $D$ is reduced by $kd_{avg}$ due to boundary effects, where $d_{avg}$ characterizes the grain size, and $k$ is an order-one constant [2]. Recent studies [17,18] have provided micromechanical insights into this equation with a coarse-grain technique. An important open question concerns the relevance of this relation for non-spherical particles. Several studies have used DEM simulation methods to understand how the aspect ratio of an ellipse could affect the discharge rate [19–25]. Two studies [22,23] reported that elliptical particles discharge faster than spherical particles. But, a more recent study by Liu et al. [26], utilizing both DEM and experiments, reported that ellipsoidal particles discharge slower than spherical particles, and proposed a modified Beverloo equation for these flows. These studies differ in their simulation models and in the assumed grain properties, such as Young’s modulus and friction. To our knowledge, there is still no clear physical understanding of how the shape of individual particles affects flow in a simple hopper geometry. This experimental work provides insights into this issue.

In this paper we experimentally test the applicability of the Beverloo equation to elliptical particles in quasi-2D hopper flows. We also measure the probability of clogging for these particles. We contrast these quantities to results for bi-disperse disks in the same hopper. To apply the
Beverloo equation, an issue arises for elongated particles such as ellipses: since there are two lengths for the particles, the major and minor principal axis lengths, \(d_{\text{maj}}\) and \(d_{\text{min}}\), respectively, which of them is relevant? Thus, there are two dimensionless length ratios, which can be taken as \(D/d_{\text{min}}\) and the ellipse aspect ratio, \(\alpha = d_{\text{maj}}/d_{\text{min}}\). In addition, the organization of elliptical grains, and the stable structures that they form when a jammed state occurs following a clog are important.

In the present experiments, we use a 2D wedge-shaped hopper, as in our previous studies [8,27]. The apparatus consists of two transparent Plexiglas sheets separated by aluminum spacers and aluminum hopper walls. The system is divided into upper and lower regions, both of which are hoppers. Approximately 5000 bi-disperse circular particles or 3000 identical elliptical particles are initially placed horizontally in the upper hopper section. Then we gently flip the hopper vertically to allow the particles to naturally settle under gravity. During flow, the packing fraction \(\phi\), above the free-flow zone (about height \(D\) above the opening line \([2,8,17,18]\)), stays around 0.75 for disks and 0.73 for ellipses, and decreases nearly linearly towards the opening. Very near the opening, the density falls substantially, perhaps as low as \(\phi \approx 0.50\). For ellipses, \(d_{\text{maj}} = 10\) mm, and \(d_{\text{min}} = 5\) mm. For disks, we have big particles (diameter \(d = 6\) mm) and small particles (diameter \(d = 5\) mm) with relative fraction small to large of \(2 : 1\). So the average diameter of the bi-disperse circular particles (disks) is 5.3 mm. The variance of the disk sizes, normalized by the mean disk size is 0.09. Thus, the polydispersity of the disks introduces a possible length scale ratio of \(\sim 1.1\). By comparison, the length scale ratio for ellipses set by the aspect ratio is 2. Thus, length scale effects associated with the polydispersity of disks are likely to have a small effect on flow properties, compared to the \(O(1)\) effect of the ellipse aspect ratio. Two sliding Teflon bars initiate or stop the flow. The upper hopper can be Reloaded with the hopper wall angle \(\theta = 30^\circ\) (\(\theta_w\) is half the full opening angle of the hopper). The particles are made of photoelastic materials (Vishay, PSM); when they are placed between crossed polarizers, transmitted light produces images with fringe-like patterns that depend on the forces acting on each particle [28]. The repose angle for the disks was roughly \(20^\circ\); for the ellipses, the repose angle was highly variable, and could be between \(\sim 25^\circ\) and \(45^\circ\). All material was cut from Vishay PSM material and had a Young’s modulus of about 5 MPa. We use the photoelastic response to measure grain-scale forces. The square of the image intensity gradient provides a measure of the local force, which we calibrate by applying known static loads to systems of particles [29]. Note that the contact forces between particles can also be accurately calculated based on photoelastic principles, for high-resolution images [30]. This is not possible in the present experiments that rely on high-speed images with modest frame sizes.

In order to simultaneously observe particles positions and the forces acting on them, we use two synchronized high-speed cameras (frame rate: 500 fps), one to take pictures for particle tracking (fig. 2, top) and the other for photoelastic measurements (fig. 2, middle). As in fig. 1, a beam splitter steers polarized light from the experiment into the two cameras. One camera has a polarizer that is crossed with respect to the original light polarization. The other lacks a second polarizer, and only registers the direct images of the particles. The images from the cameras are aligned through registration techniques to produce a composite image that details the location and orientation of particles, and the photoelastic response. Note that photoelastic images alone cannot be used to locate particles, since particles and/or their boundaries are often invisible. Figure 2 shows two original (direct and polarized) images from the two cameras, and the resulting overlapped image. This figure shows the force chains corresponding to strongly stressed particles. Below, we use this dual information to understand key differences between the flow and clogging of ellipses vs. disks.

We start by comparing the time-averaged discharge rate for disks and ellipses. We measured \(\dot{N}\) as the ratio of the total number of particles to the total time taken to empty the hopper. If a jam occurred, we re-initiated the flow by controlled taps with a small hammer located outside the Plexiglas. The total time to empty the material is the sum of the consecutive times during which the particles flowed. During the flow, intermittent force chains form and break, and the velocity fluctuates around a stable mean, similar to the observation of \([13,31]\). In fig. 3, we show the discharge rate raised to the \(2/3\) power, \(\dot{N}^{2/3}\), vs. the opening size, \(D\). If the Beverloo equation holds, there should be a linear relation (in 2D): \(N^{2/3} = (C \rho_B \sqrt{g/m})^{2/3} (D - kd_{\text{avg}}) = SD + Q\) [16]. Here, \(m\) represents the average mass per particle and \(\rho_B\) represents the bulk density. Note that \(\rho_B = \phi \ast (\text{material density})\).

The data for both disks and ellipses are consistent with such a relation, and hence the Beverloo equation is satisfied. The fitting parameters \(C\) and \(k\) appear in fig. 3 (We choose \(d_{\text{avg}} = 0.53\) cm for disks, \(d_{\text{avg}} = 0.5\) cm for ellipses and \(\phi \approx 0.6\) near the opening to calculate \(\rho_B\), \(C\) and \(k\)). They are comparable to results from 3D experimental work [26]. The slope \(S\) is around 25 for disks.
and around 12 for ellipses. In physical units, the ellipses flow more slowly than the disks at the same opening size; \( \dot{N} \) data in fig. 3 for the ellipses have a slope that is close to 1/2 that for the disks. Where might such a difference arise? Also, what is a reasonable way to compare these two data sets, given that the minor axis of the ellipses is comparable to the diameter of the disks, but the major axis is roughly twice the disk diameter? The Beverloo equation does not provide insight (except in a rough way through the boundary layer term, \( kd_{avg} \)) into the role of particle shape.

One approach is to seek non-dimensional rescaled representations of the data for \( \dot{N} \) and \( D \). The former, has dimensions of inverse time. The time scale must come from \( g \) and a length scale related to the particle size. Similarly, the scale for \( D \) involves a particle-scale length. For simplicity, we assume that the same measure, \( \ell_i \), which depends on the particle species (\( i = d \) for disks or \( i = e \) for ellipses) applies for both \( \dot{N} \) and \( D \). We write \( \dot{N} = (g/\ell_i)^{1/2} \dot{N}' \) and \( D = \ell_i D' \), where the primed quantities are dimensionless. Since \( \dot{N}^{2/3} \propto D \), for dimensionless quantities: \( \dot{N}'^{2/3} = \ell_i^{1/3} \dot{N}^{1/3} \ell_i D' \).

There is a universal expression \( \dot{N}'^{2/3} = S'D' + Q' \) if \( \ell_i^{1/3} S_e/\ell_d^{1/3} S_d = 1 \). The measured slopes for ellipses and disks satisfy \( S_d/S_e \approx 2 \), which yields \( \ell_i^{1/3}/\ell_d^{1/3} \approx 2 \). This is roughly satisfied if \( \ell_d = d \) and \( \ell_e \approx d_{maj} \). Then, \( \ell_e^{1/3}/\ell_d^{1/3} = (d_{maj}/d)^{1/3} = 2.3 \). This is reasonably consistent with the ratio of the slopes from the fitting lines in fig. 3, \( S_d/S_e = 2.0 \pm 0.2 \). Figure 4 shows data for the dimensionless flow rate \( \dot{N}' \) vs. dimensionless hopper opening, \( D' \), for \( \ell_e = 1.0 \text{ cm} = d_{maj} \) as the length scale.
Fig. 5: (Color online) Semi-log plot of $\tau N$ vs. opening size for both disks and ellipses (hopper wall angle $\theta_w = 30^\circ$).

Fig. 7: (Color online) Histogram of the number of particles in the blocking arch for disks and ellipses.

factor, and a best fit $\ell_e = 0.88$ cm. For the latter choice of $\ell_e$, the collapse of the disk and ellipse flow rate data is complete, within the scatter.

Previously [27], we showed for our disks that flowing/clogging can be described as a Poisson process, which can also be seen from [7,12,15]. If the probability of flow without a clog in time $dt$ is $dt/\tau_e$, then the probability that the flow persists without clogging until time $t$ is $P = \exp(-t/\tau_e)$, where the survival time $\tau_e$ reflects the inverse of the clogging probability of hopper flow. For ellipse we also have observed this exponential characteristics. In fig. 5, we compare the average survival time of a system of disks and a system of ellipses, where we use $\tau N$ (at a given $D$), i.e., the average number of discharged grains before clogging, similar to the term "n" used in [12]. For both particle types, $\tau N$ grows strongly with $D$, consistent with exponential dependence, i.e., $\ln(\tau N) = AD + B$. Although the experimentally accessible ranges of $D$ for the two data types do not overlap, extrapolation suggests that ellipse flows jam more readily than disk flows at the same $D$. Like the flow rate, it is interesting to rescale the physical quantities in fig. 5. The vertical axis is already dimensionless. If we rescale $D$ by the mean diameter of the disks and by the major diameter of the ellipses, we obtain a good collapse of the disk and ellipse data for $\tau N$, as shown in fig. 6 ($\ln(\tau N) = A'D' + B'$), where the primes refer to fits to the scaled dimensionless data. The fact that our data shows an exponential growth of $\tau$ with $D$ indicates that we do not have evidence of a critical opening size, beyond which there is no clogging.

As discussed previously [9,10,28], the formation of transient force chains during hopper flow is related to the stick-slip events of hopper flow, which, in turn, control the flow rate and rheology. Hence, we expect that a larger probability of forming long-lived force chains (e.g. near the opening) will be correlated with a lower discharge rate.

In the random-walk model of To et al. [11,12], the probability of stable blocking arches near the opening depends largely on the number of particles in the arch. If we assume that $D$ relative to particle size is the only relevant factor, we might argue that for the same hopper opening size, the hopper flow of ellipses needs fewer particles to form the blocking arch than the hopper flow of disks, since for $\dot{\tau}_c$, the microscopic length scale for the ellipses is double that for disks. Alternatively, if a flow of ellipses where the opening size $D_1$ has the same clogging probability as a flow of disks where the opening is $D_2$, one might expect that there should be similar number of particles in the blocking arches for ellipses than for disks. However, this is not the case. Figure 7 shows statistics for the number of particles in blocking arches for disks and ellipses, where the $D$’s were chosen (differently) so that the two systems have similar clogging probabilities ($\tau N = 1097$ for disks and 1186 for ellipses, $D = 2.9$ cm for disks and 5.5 cm for ellipses). Figure 7 shows that for the ellipses, the number of particles forming the blocking arch has a wider distribution and sometimes can be as large as 18 particles. On average, the blocking arch consists of more particles for ellipses than for disks: 12 ellipses and 8 disks. Hence the particle size is not the only differentiating factor between ellipse flow and disk flow.
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Fig. 8: (Color online) 2D histogram of the probability distribution of \((\theta_1, \theta_2)\) for stressed ellipses during flow. Inset: illustration of the angles \((\theta_1, \theta_2)\) that characterize the orientations of contacting ellipses. The red line represents the approximate force chain direction.

Where does the additional effective stability for ellipses compared to disks come from? We address this question using synchronized particle tracking and photoelastic stress measurements. Figure 2(c) is a snapshot, and a typical time-averaged stress profile can be found in [8]. The overall time-averaged stress profile does not differ obviously for disks vs. ellipses. Stress are strongest near the hopper wall, extending with a chain-like structure towards the central region of the hopper.

The source of the difference lies in the fact that ellipses have a coupling between their orientation that affects their mechanical stability and local density. Particle tracking results (a complete supplementary video is available at the web site in ref. [32]) show that ellipse rotation during the flow affects the force chain structure and stability. This rotation is not simply random.

Specifically, we find a systematic correlation between particle orientation and force chain orientation, defined below. Force chains tend to lie along lines corresponding to the local major principal stress. Using image registration and photoelastic techniques, we determine this direction by connecting lines through the centers of the ellipses that experience strong forces. We determine the mean force acting on a particle from the gradient-squared measure discussed above and in previous papers [29,33]. We characterize the orientations of contacting ellipses relative to their contact line, using the angles \((\theta_1, \theta_2)\) illustrated in fig. 8, inset. Since \((\theta_1, \theta_2)\) and \((\theta_2, \theta_1)\) correspond to similar cases, we define \(\theta_1 < \theta_2\). We collect all angle pairs for force chain particles (defined to be at or above the mean force) from approximately 1000 image sequences during a flow, and plot the resulting probability distribution in fig. 8. There exists an orientation preference: neighboring ellipses forming force chains tend to align with \((\theta_1 > 50^\circ, \theta_2 > 50^\circ)\) to the contact line (i.e. the local direction of the force chain).

This orientation preference is even stronger if we limit the analysis to the force chains that jam the hopper at the outlet. These chains are usually among the strongest. They form a much smaller data set, but we show results below for about 100 cases of clogging arches. Figure 9(a) shows the corresponding \((\theta_1, \theta_2)\) probability distribution, and fig. 9(b) shows the distribution along the line \(\theta_1 = \theta_2\). There is clearly a strong preference towards \((\theta_1 > 50^\circ, \theta_2 > 50^\circ)\) for neighboring particles that aligned parallel in the clogging force chains.

This preferred orientation in strong force chains is a natural consequence of stability. Two neighboring particles differing significantly from this orientation, will typically rotate to a more stable, denser configuration where they are more parallel [34]. Simple stability analysis shows that approximate lines of particles with a parallel configuration can be stable, even without surrounding particles. These calculations are supported by a simple test: when multiple ellipses (we have tried up to 10 particles) are placed on a smooth surface in a parallel configuration, it is possible to compress the line of particles without buckling. Such configurations are hypostatic and a similar test with disks shows that even a very small number of particles in a line will buckle under uni-axial compression. This suggests a starting point for a more detailed quantitative approach to describing the relation between force networks and ellipse orientation. More experimental data, such as similar measurements of ellipses with other aspect ratios and statistics of \((\theta_1, \theta_2)\) with other hopper opening sizes, will be helpful for building theoretical models.
In this paper, we investigated the effect of particle shape on hopper flow by comparing flow properties of ellipses to those of disks. By comparing the discharge rate and clogging probability of ellipses to disks, we find that simple scaling laws allow us to map the flow rates and clogging probabilities of our elliptical particles onto those for disks. For both of these properties, the relevant particle length scale is close to the major diameter of the ellipses. Analysis of the synchronized particle-tracking and stress data shows that the strongly stressed elliptical particles that form the strong force chains, tend to align parallel to their neighbors and transverse to the direction of the force chains. This effect produces more stable force chains.

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This work was supported by IFPRI, by NSF grant DMR-1206351, and by NASA grants NNX10AU01G and NNX15AD38G.

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