Numerical solution explore of rigid flexible coupling multi-body system dynamic model

Sun Qian¹, Mingyu Lu²

¹Traffic Engineering College Anhui Sanlian University, Hefei, Anhui, 230601, China
²Traffic Engineering College Anhui Sanlian University, Hefei, Anhui, 230601, China
281066959@qq.com

Abstract This paper analyzes and discusses the problems related to the numerical solution of the rigid flexible coupling multibody system dynamic model. In order to solve the problems of high frequency oscillation and large calculation amount in the solution of the rigid ordinary differential equations, it tries to improve the explicit Runge Kutta method, and applies the Gill method to solve the dynamic differential equations. And confirmed by a case study: In the process of numerical solution of rigid-flexible coupling multi-body system dynamics model, the improved GILL method has small truncation error, fast integration speed and small computation amount. It can provide important basis for the subsequent algorithm improvement and optimization, which deserves the attention of the professional.

1. Rigid-flexible coupling multi-body system dynamics
In the engineering field, mechanical systems such as vehicles and machines are composed of multiple interconnected translational and rotating components. In the actual operation of the mechanical system, if there is elastic deformation at the same time when the component undergoes large displacement movement, the mechanical system is defined as a flexible multi-body system. Multi-rigid body system dynamics is a new branch of discipline formed based on classical mechanics. The system is composed of multiple rigid bodies in a certain way, and the multi-rigid body system is processed by traditional classical mechanics methods. However, with the increase in the number of rigid bodies and the complicated development of connection conditions and restraint methods, in order to adapt to this trend, the relevant personnel tried to introduce graph theory into the dynamics of multi-rigid body systems, and realized system structure and kinematics and dynamics with the help of graph theory tools. Combination of calculation formulas[1]. The current research on this problem mainly focuses on solving the following four problems: The first is to describe the kinematics characteristics of the system through mathematical tools, and the correspondence between the various kinematics feature vectors is given; the second is to give the construction system. The method of dynamic equations facilitates the programmed processing of computer solutions and increases the speed of calculation; the third is to form a special working program to solve the algorithmic problems in the numerical calculation of the model; the fourth is to realize the personalized processing of general theories.

2. Stiff differential equations
In the process of analyzing the dynamic model of the flexible multi-body system under the condition of rigid-flexible coupling, the mechanical system mass matrix can be defined as M, the stiffness matrix can be defined as K, and the Jacobian matrix corresponding to the constraint equation can be
defined as $C_q$, the generalized external force is defined as $Q_c$, the velocity quadratic vector is defined as $Q_v$, the generalized coordinates are defined as $q$, and the generalized acceleration is defined as $\ddot{q}$.

The dynamic model of the rigid-flexible coupling multi-body system can be described by dynamic differential equations and constraint equation differential/algebraic equations, as shown in the following equation (1):

$$
\begin{align*}
M\ddot{q} + Kq + C_q^T\lambda &= Q_c + Q_v \\
C(q, t) &= 0
\end{align*}
$$

As shown in the above equation (1), the equations have typical rigidity, high-order nonlinearity, and matrix ill-conditioned characteristics. The integral variables show obvious high-frequency oscillation characteristics. In order to effectively solve the equations, first try to simplify formula (1) to form a system of ordinary differential equations with variable coefficients, which is simplified to the initial value problem, as shown in the following formula (2):

$$
y' = f(t, y), \quad y(t_0) = Z
$$

Regarding the above equation (2) as a system of linear ordinary differential equations, the right end of the equation can be simplified, as shown in the following equation (3):

$$
f(t, y) = Ay(t) + h(t)
$$

In formula (3), $y(t)$ is the m-dimensional function corresponding to the $t$ value to be solved, and $A$ is the $m \times m$ order matrix. Assuming that the standard type of matrix $A$ is a diagonal matrix, its eigenvalues can be described in the following formula (4):

$$
\lambda_k = T_k + iU_k, \quad (k=1,2,\ldots, m)
$$

From this, a definition can be made: In the analysis of the dynamic model of a rigid-flexible coupled multi-body system, the Lambert linear system can be collectively referred to as a rigid equation set, and the following conditions are satisfied

1. $\text{Re} \lambda_k < 0, \quad (k=1,2,\ldots, m)$
2. $r = \frac{\max |\text{Re} \lambda_k|}{\min |\text{Re} \lambda_k|}$

The ratio $r$ can be defined as the rigidity ratio (when the rigidity ratio is of the order of 10, it is immediately defined as the critical rigidity ratio).

For nonlinear systems, assuming that the above formula (2) satisfies the initial conditions, the exact solution is described as $\tilde{y}(t) \in [a, b]$. Then on $\tilde{y}(t)$, within the range of the solution value adjacent to the threshold, the exact solution characteristics of equation (2) are analyzed to form a perturbation equation relative to this equation, which is defined as the following equation (5):

$$
dy/dt = J(t)y + [f(t, \tilde{y}(t) - \tilde{y}(t))]
$$

Combining the above equations (2) and (5), there is a perturbation relationship, and it can be found that for nonlinear systems, the stiffness ratio can essentially be described as the ratio of the maximum eigenvalue to the minimum eigenvalue of $J(t)$ in the Jacobi matrix, $r$ There is a functional relationship with time $t$.

For the equation system (2), it has typical rigid differential characteristics. In a rigid-flexible coupling multi-body system, the rigidity of the equation system has many factors that may be related to the excessive integral variables, the strong nonlinearity of the equation coefficients, and the mechanical system. There is a strong coupling relationship between the large displacement movement of the component and the elastic deformation. The above factors are manifested in the slow or fast variables in the process of numerical integration, which limits the speed of numerical integration to a certain extent[2]. From this point of view, on the premise of ensuring accuracy, increasing the speed of numerical integration is the primary problem to be solved in the numerical solution of the dynamic model of the rigid-flexible coupling multi-body system.
3. Numerical solution

Under normal circumstances, the Gear algorithm package has good suitability for meeting the rigid stability threshold multi-step method. When the eigenvalues of the Jacobi matrix corresponding to the right end function of the above formula (2) are close to the imaginary axis, the reliability of this algorithm is not guaranteed, and even failures will occur. The main reason is: in addition to the value range of $2 \leq k \leq 6$, the k-step k-step algorithm does not have a stability, and the absolutely stable area $A(T)$ will show a rapid attenuation trend due to the increase of $k$ value, leading to it relatively limited in the process of numerical solution of dynamic model of rigid-flexible coupling multi-body system\[^3\].

The explicit Runge-Kutta method is widely used in the numerical solution of differential equations. It has good adaptability to equations with relatively low rigidity, and the effectiveness of this method is higher when the step length and accuracy are clearly restricted. However, when applied to the numerical solution of the dynamics model of a rigid-flexible coupled multi-body system, the step size of this method cannot be expanded flexibly, and the integration speed is relatively limited. In order to improve this problem, we try to improve the explicit Runge-Kutta method, and the Gill algorithm formed has good suitability and small truncation error. This algorithm has good stability. At the same time, DIRK has $A^0$stability in high-frequency oscillation problems, so it has more advantages than Gear algorithm\[^4\] in the numerical solution of the dynamic model of rigid-flexible coupled multi-body system.

4. Stability of numerical solution

The traditional numerical integration method for solving differential equations has certain limitations when solving stiff equations. The main reason is that the eigenvalues corresponding to the coefficient matrix $A$ in the above formula (3) contain a certain proportion of negative real parts, but Numerical calculation methods in the traditional sense have good applicability only to finite stable regions, and they are not ideal for stiff equations with negative real parts. Therefore, some researchers put forward the concept of stability for the coefficient matrix $A$, that is, for a certain numerical integral formula, when it is defined as $A$ stable, the stable region of the formula contains the compound left half plane. The numerical calculation is $A$ stable, which means that the numerical integration formula has good stability for arbitrarily large integration step length, without any restriction or influence on the step length. However, while introducing the concept of $A$ stability, there is also a restrictive result, that is, the explicit linear multi-step method cannot belong to the category of $A$ stability. At the same time, the order of $A$ stable implicit linear multi-step method should be less than 2, and among all $A$ stable second-order methods, the truncation error of the trapezoidal numerical integration formula is the smallest\[^5\]. On this basis, relevant researchers have further expanded the concept of $A(T)$ and $A^0$stability, in order to weaken the definition of $A(T)$ and improve the current numerical calculation method and $A(T)$ and $A(T)$. $A^0$The matching of stability requirements.

5. Application examples

Taking the common portal crane in the port machinery system as an example, the following figure (Figure 1) shows the simulation model of the four-link boom system of a certain type of portal crane. In Figure 1, bodies 1 to 3 are deformable bodies, and the other parts are rigid bodies. The Gill algorithm is used to perform dynamic simulation of the mechanical system. The following figure (Figure 2 and Figure 3) shows a schematic diagram of the dynamic simulation stress-time curve. Figure 3 shows that the dynamic simulation result is reliable. The algorithm is in the rigid-flexible coupling multi-body The system dynamics model has good applicability in numerical solution problem processing\[^6\].
6. Concluding remarks

In summary, the analysis and discussion of the related problems of the numerical solution of the dynamic model of the rigid-flexible coupling multi-body system are carried out, and the following conclusions are drawn: First, Gear is considered to solve the system of rigid ordinary differential equations. The preferred algorithm, but affected by the high-frequency oscillation factors of the integral variable, this solution does not have the A stability region, which causes the algorithm to fail; second, the implicit Runge-Kutta method contains the A stability region, but the dynamic differential
equations have changes. Coefficients and nonlinear characteristics, the difference method can only obtain the J(t) matrix, which requires a large amount of calculation. In order to solve this problem, try to improve the explicit Runge-Kutta method and apply the Gill method to solve the dynamic differential equations; third, apply the improved Gill algorithm to the process of numerical solution of the dynamic model of the rigid-flexible coupling multi-body system. It significantly improves the integration speed, reduces the calculation workload, and avoids the effect of the large truncation error on the numerical solution effect, which can be used as an important basis for subsequent algorithm improvement.

Acknowledgments
Fund Project: Anhui Provincial Department of education natural science research key project “Identification of black spots in urban road traffic accidents based on GIS kernel density analysis” (Grant NO. KJ2020A0808).

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