Mechanical behaviors of a novel auxetic honeycomb characterized by re-entrant combined-wall hierarchical substructures

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Abstract

The current focus of the metamaterials is to further improve their performance by the unit cell innovation, while for the auxetic metamaterials, the compromise between the mechanical properties and auxetic effect still needs more efforts. Given this issue, here we developed a novel auxetic honeycomb, named re-entrant combined-wall (RCW) honeycomb, by introducing four hierarchical substructures to the RE cell. Analytical expressions were derived and used to study the in-plane elastic constants of the RCW honeycomb, which were well confirmed by the established finite element model. Further, we investigated its crushing behaviors under large deformation by the explicit numerical method, and the quasi-static crushing experiments were also carried out by the 3D-printed specimens. Results show that the properties of the proposed RCW honeycomb have a high degree of orthogonality and tunability. Compared with the traditional RE honeycomb, the Young’s modulus of the RCW honeycomb in the y direction increases by more than 120%, and the Poisson’s ratio decreases by about 43%. Besides, behaviors of the cell wall contact induced by the adding substructure can lead to an interesting stress enhancement phenomenon under large deformation, which significantly increases its crushing strength, up to 140%, compared with the RE honeycomb. Therefore, the results in this work effectively demonstrate the improved mechanical properties and auxetic performance of the proposed RCW honeycomb. Besides, the adopted design strategy of hierarchical substructure also exhibits great potential for developing novel and excellent auxetic mechanical metamaterials.

1. Introduction

Mechanical metamaterials refer to a category of artificial structures with counterintuitive mechanical properties originating in their unit cells [1–6]. As one of the most widely studied mechanical metamaterials, auxetic metamaterial defines a kind of materials with special compressive properties, i.e., transverse expansion under uniaxial tension, which is also well known as the negative Poisson’s ratio (NPR) property [7–9]. Since the first fabrication of NPR foam in 1987 [10], many ordered and disordered auxetic materials/structures have been developed [11], and various NPR mechanical mechanisms, such as re-entrant polygon models [12, 13], chiral mechanism [14], rotating polygon models [15] et al have been explored. It has been proved that, owing to the NPR property, auxetic structures exhibit many superior properties, including increased indentation resistance [16], higher shear modulus [17, 18], enhanced impact resistance [19] and better energy absorption [20], compared with conventional materials. The mechanical properties can be conveniently assigned for the ordered auxetic structures through the adjustment of cell topologies and geometric parameters [21, 22]. Therefore, considering the requirement of the problems in various engineering applications, an ongoing challenge for the auxetic materials is to explore more novel cell topologies with excellent properties and to explain the relationship between the unit cells and the macroscopic mechanical properties, which has attracted extensive attention from scholars [23–34].
Re-entrant honeycomb, adapted from the traditional hexagonal honeycomb, is one of the most thoroughly studied auxetic materials owing to its straightforward NPR mechanisms and has occupied an important place in many engineering areas [35–37]. For its potential restrictions exposed in the applications, such as relatively low stiffness and strength, a series of cells have been developed to improve its performance [24, 38–41]. For instance, some scholars proposed to embed additional cell walls such as rhombic ligament [39], horizontal ligament [42] and vertical ligament [43] into the traditional RE cell. Besides, some hybrid cell configurations are also proposed [28, 29, 31], which usually consist of different cells, to improve the deformation modes and crushing strength of the single RE cell. Hierarchical design is an effective method to improve the mechanical metamaterials, which has been widely used in the conventional hexagonal honeycomb to improve its out-of-plane and in-plane properties [44–48]. Its main characteristic is to replace some parts, such as vertices or cell walls of the original cell, with smaller substructures. When undertaking crushing loads, those additional substructures will optimize the failure modes of the honeycombs, and significantly improve the crushing strength as well as the energy absorption [46, 49]. Therefore, for the auxetic mechanical metamaterials, the hierarchical design has also attracted much attention from scholars. Sun et al [50] proposed a novel hierarchical tube with auxetic property, and derived its equivalent elastic parameters by the Euler beam theory. Tan et al [46] proposed two hierarchical RE honeycombs by replacing the cell walls with the hexagon substructure and triangle substructures, and found that the two designs improved the crushing force of the original structure. Besides, the hierarchical design also shows great potential at the microscale. Considering the huge prospect in many applications such as medical equipment and flexible electronics, it is considered to be one of the most promising directions in this field [51–54]. Dudek et al [55] developed and fabricated 2D and 3D hierarchical mechanical metamaterials at the microscale by the 3D microprinted polymers, and found that the proposed designs have great capability of shape morphing. Cho et al [56] proposed a novel strategy to construct multilevel hierarchical metamaterials with programmable geometric shapes by the fractal cutting strategy, and confirmed it at both the macroscale and microscale. Nevertheless, for some innovations in the traditional auxetic honeycomb, the added additional walls may limit or weaken the response and the NPR effect of the RE honeycomb [32, 43, 57]. The compromise between the auxetic performance and the mechanical properties is still an important issue regarding the further application of the RE honeycomb. Further, for the hierarchical honeycombs, the current studies mainly used the triangle, regular hexagon or self-similar substructures, and proposing some novel substructures and embedding methods still has significance.

The innovative design of the re-entrant cell wall geometry may be another effective approach to make improvements. This approach well retains the relative position of the re-entrant walls, and therefore, the NPR behaviors associated with the bending deformation of the re-entrant walls will not be weakened and may even be strengthened. Zhang et al [58] proposed a vertical strut combined re-entrant auxetic structure, in which the straight inclined cell wall of the RE cell is replaced with a vertical ligament and an inclined ligament. They found that the novel design can achieve enhanced auxetic performance and a high degree of orthotropic characteristics. Similarly, another group also proposed a modified re-entrant honeycomb structure [39], which adopted an innovative re-entrant wall containing three ligaments in series, and found that the novel structure has an improved tuning capability of the NPR properties. However, it can be found that, in the previous studies, the modified re-entrant wall is usually a single ligament or multi-folded structure [60–62], and a few researches involve the hierarchical substructure, i.e., replacing the single wall with a closed substructure. The latter may be of great significance considering that, through this approach, the advantages of the hierarchical design in enhancing mechanical properties can be well utilized, while the NPR effect can be well retained.

Clarifying the relationship between the unit cell and the macroscopic mechanical properties is helpful for the engineering application of the auxetic structures [63–66]. In-plane elastic analytical expressions of the traditional or modified auxetic honeycombs have been widely discussed in the existing researches, in which the bending mechanism, or more other mechanisms, is considered to calculate the Young’s modulus and Poisson’s ratio [24, 67]. Although the elastic properties are usually evaluated within a linear range, the deformations of auxetic honeycombs often involve nonlinear behaviors, such as plastic deformation and cell wall extrusion, considering that it is often used in applications requiring large deformation. Therefore, the evaluation of crushing responses for the novel cell, including its strength and the deformation modes, has gotten more and more attention from scholars [23, 27, 29, 30]. While the majority of the relevant studies are carried out numerically, analytical models and experimental analyses have also been effectively explored, in which the additive manufacturing (AM) technology provides a powerful tool for the rapid verification of novel honeycomb concepts [23, 28, 68].

Given the above considerations, this work aims to propose a novel auxetic metamaterial, named re-entrant combined-wall (RCW) honeycomb, by replacing each re-entrant wall of the traditional RE honeycomb with a hierarchical substructure to improve the mechanical properties of the traditional structure. An analytical model is derived to predict the elastic constants of the proposed RCW honeycomb, and the corresponding finite element model is established to confirm its effectiveness. Further, the nonlinear crushing responses of the RCW
honeycomb are investigated by an explicit numerical model and the corresponding quasi-static compression test. Through the analytical, numerical and experimental methods, the mechanical properties of the proposed RCW honeycomb are deeply investigated to provide a constructive guidance for its further research and application.

2. Models and methods

2.1. Geometry

The geometric configurations of the RE cell and the proposed RCW cell are shown in Figure 1. As shown in figure 1(b), the RE cell consists of eight cell walls, including four inclined walls, length $l_1$, two horizontal walls, length $l_0$, and two connected walls, length $l_2$. Besides, the RE cell also involves the following parameters: the included angle $\theta_0$, the cell length $s_1$, the cell height $h$, and the wall thickness $t$. As shown in figure 1(d), the RCW cell can be characterized by replacing the inclined wall of each RE cell with a hexagonal substructure shown in figure 1(a). The length of the walls of the added substructure is $l_3$, $l_4$, and $l_2$ respectively, and the angle $\theta_1$ is the included angle between the horizontal wall and the inclined wall of the substructure. Besides, the width of the RCW honeycomb is set to $b$, as shown in figure 1(e). To simplify the description, nondimensional parameters $\alpha = 2l_2/h$, $\beta = l_1/(l_0 \cos \theta_0)$ are defined, and the parameter $l_1$ is fixed as $2l_0$. Accordingly, the proposed RCW cell can be determined by the variables $h$, $\theta_0$, $\alpha$, $\beta$ and $t$, and other geometric parameters can be expressed as: $\theta_1 = \arctan [\tan \theta_0 (1 - \alpha) / (1 - \beta)]$, $l_0 = h / 2 \sin \theta_0$, $l_1 = h / \sin \theta_0$, $l_2 = \alpha h / 2$, $l_3 = \beta h_0 \cos \theta_0$, $l_4 = (h - \alpha h) / (2 \sin \theta_1)$ and $s = (2 - \cos \theta_0) h / \sin \theta_0$.

Relative density is usually used to describe the ratio of the actual cell wall volume of the honeycomb structure to its total cell space. According to the above description, the relative density of the RCW cell can be written as follows:

$$\rho_r = \frac{t(2h + 8l_2 + 8l_4)}{sh} = \frac{t (2 + 4 \alpha \sin \theta_0 + 4(1 - \alpha) \sin \theta_0 / \sin \theta_1)}{h (2 - \cos \theta_0)}$$

(1)

2.2. Analytical model of elastic constants

To explore the in-plane elastic properties of the proposed RCW honeycomb, an analytical model was established based on Castigliano’s second theorem [58, 65]. In this model, bending and stretching deformation mechanisms of the cell wall are involved, and the potential shear mechanism is ignored due to its small contribution to the cell
wall deformation \[67\]. The deformation is assumed within the linear range, and the analytical model of the simplified cell, as shown in figure 2(a), is assumed to be from an infinite honeycomb structure. This analytic model mainly focuses on the Young’s modulus and Poisson’s ratio of the structure under horizontal and vertical loading, and the cases of loading in other in-plane and out-of-plane directions are not discussed here. Therefore, the established analytical model in this work is an idealized model, in which all cells have a consistent deformation and a symmetrical boundary condition.

As shown in figures 2(b), (c), a 1/4 model of the RCW cell is adopted for theoretical derivation considering the symmetry of the structure. Besides, a simplified model with two core beams is extracted to calculate the point displacement, where point C is constrained with all freedoms, and point A is freely in the x and y directions under the force \(P\), force \(F\) and the moment \(M\). Assuming the cell walls as Euler-Bernoulli beams, the displacement or rotation of point A under the single force or moment can be described as:

\[
\begin{align*}
\delta_{MM} &= \frac{l_2 + l_4}{E_s I} , \quad \delta_{MP} = \delta_{PM} = -\frac{l_4^2 \cos \theta_1}{2E_s I} , \quad \delta_{PP} = -\frac{l_4^3 \cos^2 \theta_1}{3E_s I} + \frac{l_2 + l_4 \sin^2 \theta_1}{E_s A}, \\
\delta_{MF} &= \delta_{FM} = \frac{l_2^2 + l_4^2 \sin \theta_1 + 2l_2l_4}{2E_s I}, \quad \delta_{FP} = \delta_{PF} = -\frac{l_4^3 \cos \theta_1 \sin \theta_1}{3E_s I} , \\
\delta_{FF} &= \frac{l_2^3 + l_4^3 \sin^2 \theta_1}{3E_s I} + \frac{\sin \theta_1 l_2l_4^2 + l_2^2 l_4}{E_s I} + \frac{l_4 \cos^2 \theta_1}{E_s A}.
\end{align*}
\]

Where, \(\delta_{ij}\) means the displacement or rotation along \(j\)-direction caused by the force or moment along \(i\)-direction; \(E_s\) is the Young’s modulus of the matrix material; \(I\) is the second moment of area, which can be expressed as \(I = b t^3 / 12\); \(A\) is the cross section area, which can be expressed as \(A = b t\); \(b\) and \(t\) are width and thickness of the cell walls, respectively.

Figure 2(b) shows the case of loading along the \(y\) direction, where \(\sigma_y\) is the applied stress parallel to the \(y\) direction. In this case, the force \(P\) can be expressed as follows:

\[
P = \frac{\sigma_y b (l_1 - l_0 \cos \theta_0)}{2} = \sigma_y \gamma_1, \quad \gamma_1 = \frac{b (l_1 - l_0 \cos \theta_0)}{2}
\]

Considering the rotation of point A is constrained, the following expression can be given:

\[
\delta_{MM} M_1 + \delta_{PM} P = 0
\]

Accordingly, the moment \(M_1\) can be obtained as follows:

\[
M_1 = \frac{pl_4^2 \cos \theta_1}{2(l_2 + l_4)} = P \gamma_2, \quad \gamma_2 = \frac{l_4 \cos \theta_1}{2(l_2 + l_4)}
\]
Therefore, the total displacement of the point A along the x and y directions can be expressed as:

\[ u_y = \delta_{MP} M_1 + \delta_{FP} P = (\delta_{MP} \gamma_2 + \delta_{FP}) \gamma_1 \sigma_y, \]
\[ u_x = \delta_{MP} M_1 + \delta_{FP} P = (\delta_{MP} \gamma_2 + \delta_{FP}) \gamma_1 \sigma_y \]

Further, the effective strain along the x and y directions can be expressed as

\[ \varepsilon_y = \frac{u_y}{l_0 \sin \theta_0} = (\delta_{MP} \gamma_2 + \delta_{FP}) \gamma_1 \sigma_y, \]
\[ \varepsilon_x = \frac{u_x}{l_0 \cos \theta_0} = (\delta_{MP} \gamma_2 + \delta_{FP}) \gamma_1 \sigma_y \]

Accordingly, the elastic constants of the RCW honeycomb along the y direction, including its Young’s modulus \( E_y \) and Poisson’s ratio \( \nu_{yx} \), can be obtained as follows:

\[ E_y = \frac{\sigma_y}{\varepsilon_y} = \frac{l_0 \sin \theta_0}{(\delta_{MP} \gamma_2 + \delta_{FP}) \gamma_1} \]
\[ \nu_{yx} = -\frac{\varepsilon_x}{\varepsilon_y} = \frac{(\delta_{MP} \gamma_2 + \delta_{FP}) l_0 \sin \theta_0}{(\delta_{MP} \gamma_2 + \delta_{FP})(l_1 - l_0 \cos \theta_0)} \]

Figure 2(c) shows the case of loading along the x direction, where \( \sigma_x \) is the applied stress parallel to the x direction. In this case, the force \( F \) and the moment \( M_2 \) can be expressed as follows:

\[ F = \frac{\sigma_x b l_0 \sin \theta_0}{2} = \sigma_x \gamma_3, \quad \gamma_3 = \frac{b l_0 \sin \theta_0}{2} \]
\[ M_2 = -F (l_2^2 + l_4^2 \sin \theta_1 + 2l_2 l_4) = F \gamma_4, \quad \gamma_4 = \frac{-l_2^2 + l_4^2 \sin \theta_1 + 2l_2 l_4}{2(l_2 + l_4)} \]

Accordingly, the elastic constants along the x direction, including its Young’s modulus \( E_x \) and Poisson’s ratio \( \nu_{xy} \), can be obtained by a similar method:

\[ E_x = \frac{\sigma_x}{\varepsilon_x} = \frac{l_1 - l_0 \cos \theta_0}{(\delta_{MP} \gamma_4 + \delta_{FP}) \gamma_3} \]
\[ \nu_{xy} = -\frac{\varepsilon_y}{\varepsilon_x} = \frac{(\delta_{MP} \gamma_4 + \delta_{FP}) (l_1 - l_0 \cos \theta_0)}{(\delta_{MP} \gamma_4 + \delta_{FP}) l_0 \sin \theta_0} \]

### 2.3. Finite element models

In this work, the numerical models, including the static finite element (FE) model and the explicit dynamic FE model, were established by the commercial software ABAQUS. First, for the static FE model, as shown in figures 3(a), (b), a 4 \( \times \) 4 RCW honeycomb (four cells in both x- and y-direction) is adopted for simulations. Four-node curved shell elements (S4R elements) are used to characterize the cell wall of the honeycomb, and the element size is set to 0.1 mm under the cell height of 5 mm. The density \( \rho \), Young’s modulus \( E \), and Poisson’s ratio \( \nu \) of the matrix material are set to 2700 kg m\(^{-3}\), 69 GPa and 0.3 respectively referring to aluminum alloy, and the potential plastic behavior is ignored. The detailed boundary conditions of the static FE model have been given in figures 3(a), (b). As shown in figure 3(a), the loading case along the y direction is modeled by the displacement loads along the y direction. Especially, the nodes at the lower boundary are fixed along the y direction and free along the x direction, and the nodes at the upper boundary are imposed with a uniform displacement along the y direction. The nodes at the left and right boundaries are coupled in the x direction. Therefore, the equivalent Young’s modulus of the structure can be calculated according to the reaction force and displacement of the nodes at the upper boundary, and the equivalent Poisson’s ratio can be calculated according to the displacements of the nodes at the upper, left and right boundaries. Similarly, the boundary condition loading along the x direction can be easily obtained from figure 3(b). It should be noted that the current setting is idealized to simulate a periodic honeycomb boundary [40, 65], which does not exist in practical tensile or compression tests. The purpose of establishing this FE model is to verify the established analytical model.

Further, an explicit FE model has been established to explore the crushing behaviors of the RCW honeycomb. As shown in figure 3(c), the honeycomb specimen, with nine cells in y-direction and seven cells in x-direction, is freely placed between two rigid plates. The lower plate is fixed, and the upper plate moves at a uniform speed. A surface-to-surface contact algorithm with a friction coefficient of 0.2 [68] is employed to describe the contact between the specimen and the plates. For the case loading along the x-direction, the simulation is performed with similar conditions, where the specimen is with twelve cells in y-direction and five cells in x-direction, as shown in figure 3(d). To obtain acceptable computational efficiency and relatively accurate quasi-static response, a speed boundary of 2 m s\(^{-1}\) was adopted, by which the ratio of kinetic energy to internal energy has been confirmed to be less than 5%. The size of the shell elements was set to 0.3 mm under the cell height of 5 mm according to the additional mesh convergence analysis. A self-contact algorithm without
friction was used to prevent cell penetration. Aluminum alloy is assumed as an elastic perfect-plastic material with the yield strength $\sigma_y = 76$ MPa.

### 2.4. Specimen and experimental set-up

To further study the crushing behaviors of the proposed RCW honeycomb, four groups of specimens have been fabricated by the process of multi-jet fusion (MJF), a kind of 3D printing process, with nylon material (HP3DHPR-PA12, provided by Hewlett-Packard), and the in-plane quasi-static compression tests have been carried to capture the deformation modes and the crushing stresses of the specimens. The geometric parameters of the specimens are set as $h = 15$ mm, $a = 0.5$, $b = 0.5$, $t = 0.8$ mm, and $\theta_0$ is set to $45^\circ$ and $60^\circ$, respectively. Besides, to avoid the potential tilt phenomenon during the compression process, the length of the specimen is set to be larger than its height, as shown in figures 4(e)–(h). A group of tensile samples has been printed through the same process to determine the mechanical properties of the selected nylon, as shown in figures 4(a), (b). The tensile force and displacement were recorded by a high precision force sensor and an extensometer, and five independent tests have been carried out to reduce accidental test errors. Tensile test results are shown in figure 4(c), and accordingly, the Young’s modulus is set to 1.1 GPa, and the plastic properties after yielding can be characterized by a series of discrete stress-strain sampling points. As shown in figure 4(d), quasi-static in-plane compression tests were performed on an electronic universal testing machine of 20 kN capacity, and the compressive speed was set as 2 mm min$^{-1}$. The reaction force and the complete deformation process of the specimens were recorded by a force sensor and a digital camera for further analysis.

### 3. Results and discussion

#### 3.1. Linear elastic properties

##### 3.1.1. Analytical model validation

To validate the established analytical model, a comparative analysis of analytical and numerical results was performed, in which the elastic constants of the RCW honeycomb are obtained from equations (8), (9) and (11), (12). Besides, a comparative analysis of traditional RE cell is also performed, and its analytical elastic constants are obtained from the existing publication [35]. Figure 5 shows the comparison results of the Young’s modulus and Poisson’s ratio of the RE honeycomb and the RCW honeycomb with $\theta_0$ from $45^\circ$ to $70^\circ$. It can be found
Figure 4. Specimen fabrication and experimental setup of the honeycomb quasi-static crushing test: (a) tensile test of the nylon material; (b) geometric configuration of the tensile sample; (c) stress-strain curves of the tensile tests; (d) compression test of the honeycomb specimen; (e)–(f) RCW honeycomb specimen with $\theta_0 = 45^\circ$ in x- and y-directions; (g)–(h) RCW honeycomb specimen with $\theta_0 = 60^\circ$ in x- and y-directions.

Figure 5. Comparison results of the elastic constants from the analytical and numerical models: (a) Young’s modulus of RE honeycomb; (b) Young’s modulus of RCW honeycomb; (c) Poisson’s ratio of RE honeycomb; (d) Poisson’s ratio of RCW honeycomb.
Table 1. Elastic constants of the RE and RCW honeycombs from the numerical and analytical models.

| Groups | r (mm) | \( \theta_y (\degree) \) | \( \alpha_2/\beta \) | \( E_y \) (MPa)  | \( E_x \) (MPa)  | \( \nu_{xy} \) | \( \nu_{xy} \) |
|--------|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|        | Num    | Ana             | Error (%)       | Num             | Ana             | Error (%)       | Num             | Ana             | Error (%)       |
| RE     | 0.1    | 45°             | —               | 1.72            | 1.71            | 0.48            | 5.70            | 5.71            | 0.12            | -0.55           | -0.55           | 0.16            | -1.81           | -1.83           | 0.76            |
|        | 0.15   | 50°             | —               | 9.12            | 9.15            | 0.03            | 19.91           | 20.23           | 1.58            | -0.67           | -0.67           | 0.51            | -1.46           | -1.49           | 1.70            |
|        | 0.2    | 60°             | —               | 51.73           | 52.99           | 2.37            | 51.15           | 52.99           | 3.99            | -0.98           | -1.00           | 1.90            | -0.97           | -1.00           | 3.01            |
|        | 0.25   | 70°             | —               | 254.47          | 277.40          | 9.01            | 107.71          | 114.40          | 6.21            | -1.45           | -1.56           | 7.41            | -0.62           | -0.64           | 4.65            |
| RCW    | 0.1    | 45°             | 0.2/0.2         | 4.62            | 4.59            | 0.57            | 12.91           | 12.99           | 0.59            | -0.59           | -0.59           | 0.04            | -1.64           | -1.66           | 1.22            |
|        | 0.15   | 50°             | 0.8/0.8         | 597.55          | 613.67          | 2.70            | 46.41           | 47.43           | 2.20            | -2.16           | -2.21           | 2.51            | -0.17           | -0.17           | 2.01            |
|        | 0.2    | 60°             | 0.2/0.8         | 1716.10         | 1733.10         | 0.99            | 115.51          | 121.60          | 5.27            | -3.18           | -3.17           | 0.28            | 0.24            | -0.22           | 3.94            |
|        | 0.25   | 70°             | 0.8/0.2         | 725.25          | 771.27          | 6.34            | 178.81          | 192.68          | 7.76            | -1.24           | -1.30           | 4.73            | 0.31            | -0.32           | 6.12            |

Note: Error = \(|(\text{Num} - \text{Ana})/\text{Num}| \times 100\%; \text{Num} and \text{Ana} are the results from the numerical and analytical models, respectively.
from figure 5 that the analytic elastic constants in two directions of the honeycombs have a good agreement with the numerical results, with a maximum error below 2%. Table 1 shows the statistical results of comparison results, in which the geometric parameters \( t, \theta_0, \alpha \) and \( \beta \) have been given in the table, and the height \( h \) is fixed at 5 mm. As seen in table 1, the errors of the Young’s modulus of the RE honeycomb are at least 0.12% and at most 9.01%, and the errors of its Poisson’s ratio are from 0.16% to 7.41%. For the RCW honeycomb, the errors of the Young’s modulus vary between the limits of 0.57% to 7.76%, and the errors of the Poisson’s modulus vary between the limits of 0.04% to 6.12%. Therefore, it can be concluded from the above analysis that the established analytic model has high effectiveness for further analysis. However, it still needs to be explained that the usefulness of the analytical model and the simulation model is very limited, which can only describe the idealized deformation process. Our aim is to use the confirmed analytical model to illustrate the relationship between the geometric configurations and the linear mechanical behaviors under the ideal deformation. When the analytical model is considered for practical application, more complex deformation mechanisms and boundary conditions still need to be considered.

3.1.2. Parameter influence analysis
Based on the confirmed analytical models of the RCW honeycomb, a detailed parametric influence analysis involving the parameters \( \theta_0, \alpha \) and \( \beta \) has been carried out, in which the baseline settings of the parameters \( h, \theta_0, \alpha \) and \( \beta \) are 5 mm, 60°, 0.5 and 0.5, respectively. Figure 6 shows the elastic constants of the RCW honeycombs with variable \( \theta_0 \) and two typical geometric configurations of the RCW cell with specific \( \theta_0 = 45^\circ \) and \( \theta_0 = 70^\circ \).

Figure 6. (a) Young’s modulus and (b) Poisson’s ratio of the RCW honeycomb with variable \( \theta_0 \); (c) Configurations of RCW cell with specific \( \theta_0 = 45^\circ \) and \( \theta_0 = 70^\circ \).

Similarly, figure 7 shows the elastic constants of the RCW honeycombs with variable parameters \( \alpha \). As shown in figure 7(a), the parameter \( \alpha \) shows a weak influence on the Young’s modulus, where neither \( E_y \) nor \( E_x \)
changed much. Meanwhile, with $\alpha$ increasing, the values of $n_{yx}$ and $n_{xy}$ vary in a limited range, from values about $-1.9$ and $-0.5$ to values about $-1.4$ and $-0.2$, respectively, as shown in figure 7(b). Besides, it can be found that when the value of $\alpha$ varies from 0 to 1, $n_{yx}$ is always lower than $n_{xy}$, which demonstrates a greater auxetic effect in y-direction. Further, figure 8 shows the influence analysis of the parameter $b$. As shown in figure 8(a), $b$ shows a great effect on the elastic modulus of the y-direction, with an increase of more than two orders of magnitude, while its effect on that of the x-direction is not obvious. It can be inferred that with $b$ increasing, the inclination angle of the inclined cell wall also gradually increases, as shown in figure 8(c), which makes the carrying capacity of the cell wall in y-direction greatly enhanced. Besides, the parameter $b$ also shows a significant influence on the Poisson’s ratio in y-direction, with the change from about $-0.8$ to a minimum of about $-13.5$. Further, it can be seen from figures 6–8 that the parameter $t$ has a dominant effect on the Young’s moduli of both x and y directions, up to two orders of magnitude, which is accessible considering the direct effect of $t$ on the relative density of the honeycomb structure. However, its effect on the Poisson’s ratio in both directions is greatly smaller, which is relatively obvious only when the value of $\beta$ larger than 0.8, as shown in figure 8(b). This point may be of significance considering that the auxetic effect can be only affected a little when adjusting the relative density by the wall thickness.

Further, the comparative analysis of the static constants between the RE honeycomb and the RCW honeycombs and their orthogonality has also been performed, as shown in figure 9. Especially, the properties in the y direction are mainly concerned considering its huge superiority as discussed in figures 6–8. The heights of both two structures here are uniformly set as 5 mm, and the relative densities are uniformly set as 0.1. As shown in figure 9(a), the parameter $\alpha$ has a small influence on the value of $E_{I_y}$, where with $\beta = 0.5$, $\alpha = 0.2 \sim 0.8$, the $E_{I_y}$ of the RCW honeycomb is more than 120% larger than that of the RE honeycomb. Similarly, the Poisson’s ratio of the former is also about 43% smaller than that of the latter, as shown in figure 9(c). The parameter $\beta$ has a significant effect on the elastic constants, as discussed in figure 8, and therefore, adjusting its value can obtain a sharp contrast with those of the RE honeycomb, as shown in figures 9(b)–(d). Figures 9(e)–(f) shows the variation of the orthotropic characteristics of the RE and RCW honeycomb with $\alpha$, $\beta$ and $\theta_0$. It can be found that with the $\theta_0$ increasing, the values of $E_{I_y} / E_{I_x}$ of both two honeycombs increase significantly, which

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**Figure 7.** (a) Young’s modulus and (b) Poisson’s ratio of the RCW honeycomb with variable $\alpha$; (c) Configurations of RCW cell with $\alpha = 0.2$ and $\alpha = 0.8$. 

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reflects the greater stiffness of the honeycombs in y-direction. For the proposed RCW honeycomb, however, the value of $E_l/I_{xy}$ has a wider range of changes, which is greatly dependent on the value of the $\alpha$ and $\beta$, where with the $\alpha$ and $\beta$ increasing, the orthotropic characteristics of the RCW honeycomb have a significant enhancement. Especially, when $\beta = 0.8$, the value of $E_l/I_{xy}$ can be up to about 17.4 when $\theta_0 = 60^\circ$, almost 17 times that of the RE honeycomb. Therefore, it can be inferred that, by tuning the multi geometric parameters, the RCW honeycomb has great tunability in two orthogonal directions.

3.2. Analysis of crushing behaviors

3.2.1. Experiment analysis

Figure 10 shows the comparison of the experimental and numerical results about the crushing behaviors of the RCW honeycomb in y-direction, where three typical stages are selected to describe its crushing evolution. When the deformation of the specimen with $\theta_0 = 45^\circ$ begins, as shown in figure 10(a), crushing force rises rapidly, which just corresponds to the elastic deformation. Further crushed, the force curves gradually flatten, and the deformations begin to enter the plastic stage. As shown in figure 10(c), clear rotation occurs in the inclined walls of the substructure at stage 1, and there were no obvious contacts or disturbances between the cell walls. At stage 2, with partial inclined walls contacting with the horizontal cell walls, the hexagonal substructures begin to transform into a quadrangular shape, and accordingly, the vertical cell walls of the substructure start to bend and rotate. Meanwhile, it can be seen that the force values shown in figure 10(a) start to increase from stage 2. It should be noted that this force enhancement behavior here is not due to the densification of cellular structures, but due to the generation of new plastic hinges related to the cell contact. With further crushing, the volume of the substructure is further reduced, and the force curves rise sharply at stage 3, as shown in figure 10(a). For the RCW honeycomb with $\theta_0 = 60^\circ$, its force plateau seems to be longer than that of the honeycomb with $\theta_0 = 45^\circ$. The deformation of the former is mainly distributed in the center of the structure, and the substructure does not form an obvious volume to provide more support, as shown in figure 10(d). On the contrary, the deformation of the honeycomb with $\theta_0 = 45^\circ$ is more uniform, which is reflected in the internal volume and distribution of the

Figure 8. (a) Young’s modulus and (b) Poisson’s ratio of the RCW honeycomb with variable $\beta$; (c) Configurations of RCW cell with $\beta = 0.2$ and $\beta = 0.8$. 
deformed substructures, and it is the gradual crushing behavior of the substructure that makes the force gradually increase.

Further, the crushing behaviors of the RCW honeycomb in x-direction are also investigated, as shown in figure 11. It can be found that the crushing plateau in x-direction, with the value of about 150 N shown as shown in figures 11(a), (b), is significantly lower than that in y-direction, with the value about 500 N as shown in figures 10(a), (b). The deformation in x-direction shows a layer-by-layer collapse behavior, in which the vertical cell walls on the same layer rotate regularly, and the substructure seems to have only little deformation in the prophase of deformation as shown in figures 11(c), (d). Therefore, under a similar structural scale, the plastic hinges of x-direction loading condition are significantly less than those of y-direction loading condition at the early stage of crushing, which leads to a relatively lower force of the former. Besides, when loading in x-direction, a similar force increment is also observed, as shown in figures 11(a), (b), and its essence still belongs to the further crushing of the substructures. Further, by comparing the two structures loading x-direction, the force curves of the honeycomb with smaller $\theta_0$ seem to end the force plateau and start the force enhancement earlier, where the transitional point of the specimen with $\theta_0 = 45^\circ$ is at stage 2, and that of another is at stage 3. This phenomenon may be because the volume of the substructure in the former specimen is larger than that of the latter, and the earlier contact and extrusion accordingly occur in the former structure.

Combining the results in figures 10 and 11, it can be observed that the trend of the force-displacement curves of the numerical model is greatly consistent with that of the experiment. Potential errors mainly come from the
fracture failure of the 3D-printing specimens during the crushing process. Besides, the typical deformation modes captured from the experiment are also well reflected by the numerical simulation. Therefore, by the synthesis of experiment and simulation, it can be believed that the mechanical behaviors of the RCW honeycomb have good interpretability and reproducibility, and the validity of the numerical model can also be well verified.

3.2.2. Numerical analysis of crushing responses

By the confirmed numerical models, the crushing behaviors of the RCW honeycomb are further investigated and compared with the traditional RE honeycomb. Figure 12 shows the deformation processes of the RE honeycombs and RCW honeycombs under the compression loads, where, $\alpha$ and $\beta$ are uniformly set to 0.5, and $\theta_0$ is set to 60°. Besides, for comparing the two structures under the same mass, their relative densities are uniformly set to 0.1, and accordingly, their wall thicknesses are 0.13 mm and 0.19 mm, respectively. As shown in figure 12(a), the deformation of the RCW honeycomb in the $y$-direction at the strain of 0.2 mainly occurs in the middle of geometry and extends in an X-shaped mode. Clear lateral contraction is observed on the left and right sides of the honeycomb, which just demonstrates its NPR effect under large deformation. With further crushing,
more and more cells gather to the center of the structure at the strain of 0.4. Due to the cell wall contact, part of the cells enters a transitional phase, and the hexagonal substructure becomes a quadrilateral structure. At the strain of 0.6, the cells are further squeezed, and partial substructures completely lose their internal volumes, which indicates that the honeycomb starts to enter the densification stage. For the case of \( x \)-direction loading, the deformation of the RCW honeycomb takes place layer by layer, as shown in figure 12(b). The deformations of the substructures seem to be little in the prophase, and the rotation of the vertical cell wall is the main deformation mechanism. The collapse of substructures occurs from the later stage of the crushing process, as shown in stage 3 of figure 12(b). As shown in figure 12(c), for the deformation of the RE honeycomb under \( y \)-direction loading, the bending deformation of the inclined cell wall is the main deformation mechanism, which makes a relatively simplified deformation process, i.e., a gradual densification behavior from local to global. While its deformation under \( x \)-direction loading has an obvious transition state, i.e., the cells first become rhombic shape, and then further collapse.

Figure 13 shows the crushing stresses and dynamic Poisson’s ratios of the RE honeycomb and the RCW honeycomb. As shown in figure 13(a), stress curves of the RE honeycomb start with a rapid rise, and
subsequently, the curves go into a stable plateau, where the strength is mainly from the plastic deformation of cell walls. With the densification of the RE honeycomb, crushing stress rises sharply at the strain of about 0.7. It can be found that for the RE honeycomb, the stresses in two orthogonal directions seem to have a little difference. The stress curves of the RCW honeycombs exhibit different characteristics compared with the RE honeycomb. Specifically, its stress under the y-direction loading appears a similar initial and plateau stage, while the curve has an obvious increase at the strain of about 0.4, which is due to the fact that the contact and further collapse of the substructures produce more plastic energy dissipation, as shown in figure 12(a). The stress values

![Figure 12. Deformation process from the numerical models of (a) RCW honeycomb in y-direction, (b) RCW honeycomb in x-direction, (c) RE honeycomb in y-direction and (d) RE honeycomb in x-direction.](image)

![Figure 13. (a) Crushing stresses of the RE and RCW honeycomb; (b) Poisson's ratios of the RE and RCW honeycomb.](image)
of the RCW honeycomb at the strain of 0.3 and 0.6 is about 60% and 140% higher than those of the RE honeycomb, respectively, which just demonstrates the great superiority of the proposed RCW honeycomb in crushing strength under large deformation. This point may be of great significance considering that the enhanced strength is helpful in the areas of impact resistance and energy absorption [1]. However, the stress under x-direction loading has a clear decline after the initial peak and reaches the lowest point among the four curves, which exhibits a weak bearing capacity of the RCW honeycomb in the x direction. This may be because, the corresponding deformation gradually changes from overall behaviors to the layer-by-layer collapse, as shown in figure 12(b).

During the crushing process, the auxetic honeycomb will occur the transverse shrinkage due to its negative Poisson’s ratio. This shrinkage behavior is non-linear, which involves the geometric nonlinearity, plastic deformations and cell wall contact et al. Therefore, we used the dynamic Poisson’s ratio to reflect the transverse shrinkage at various times, which has been widely used in the studies of honeycomb crushing performance [27, 57]. The detailed calculation steps are given in the supplementary materials. As shown in figure 13(b), the curves of the dynamic Poisson’s ratio have a similar overall trend, i.e., decreasing rapidly within a short strain and subsequently rising until the end. It can be found that the Poisson’s ratios of both two honeycombs have a clear difference in the two directions. Generally, the Poisson’s ratio of the RE honeycomb in the x direction is significantly lower than that in the y direction, while for the RCW honeycomb, the auxetic performance in the y direction is more significant. By comparison, the Poisson’s ratio of the RCW honeycomb in y-direction is significantly higher than that of the RCW honeycomb in the same direction. While, due to the high orthogonality of the deformation modes shown in figures 12(a), (b), the auxetic effect of the RCW honeycomb in x-direction is lower than that of the RE honeycomb.

Further, we systematically investigated the parameter influence on the crushing behaviors of the RCW honeycomb. In the following analysis, in addition to a special description, the geometric parameters are set as \( \theta_0 = 60^\circ, \alpha = 0.5, \beta = 0.5 \). Figure 14 shows the crushing behaviors of the RCW honeycomb with \( \theta_0 = 45^\circ \) and \( \theta_0 = 70^\circ \), including the crushing stress, dynamic Poisson’s ratio, and deformation modes. It can be seen that, when \( \theta_0 = 45^\circ \), the difference of the stress-strain curves in two orthogonal directions seems to be not clear, which includes a similar trend of a former plateau and a gradual enhancement phase, as shown in figure 14(a). Besides, the trends of the dynamic Poisson’s ratio in both two directions are also similar, and the negative Poisson’s ratio in y-direction is greater than that in x-direction. From the deformation modes in figure 14(c) we can also find that, when crushing in y-direction, the hexagonal substructure of the RCW cell can form a quadrilateral volume due to the cell wall contact, and this behavior is evenly distributed in the honeycomb. Similarly, when crushing in x-direction, as shown in figure 14(e), the honeycomb also forms many hexagonal volumes owing to the extrusion and contact. Therefore, the stress enhancement shown in figure 14(a) can be well explained by the support of the substructure or its deformed structure. With \( \theta_0 = 70^\circ \), the stress-strain curves also show a clear stress enhancement, while its enhancement point, with the strain of about 0.64, seems to be later than that of the former honeycomb with the strain of about 0.45. This may be because the substructure of the RCW honeycomb with \( \theta_0 = 45^\circ \) has a larger initial volume, and accordingly, the substructure can happen contact and deformation behavior earlier. Besides, it may be also due to the fact that the deformation in y-direction of the honeycomb with \( \theta_0 = 70^\circ \) has a clear X-shaped mode, while the deformation of the honeycomb with \( \theta_0 = 45^\circ \) seems to be relatively uniform and consistent throughout the structure, as observed in the experiment shown in figures 10, 11.

Figure 15 shows the crushing behaviors of the RCW honeycomb with \( \alpha = 0.2 \) and \( \alpha = 0.8 \). As shown in figures 15(a), (b), the stress-strain curves show clear orthogonality, that is, the crushing stress in y-direction, with a plateau and subsequent rise phase, is greatly higher than that in x-direction, with a long downward phase, which is similar to the observation in figure 13(a). Therefore, it can be found that the orthotropic characteristics of the RCW honeycomb occur in a wide range of parameter \( \alpha \). The deformation modes of the two structures in y-direction show a similar X-shaped mode, as shown in figures 15(c), (d). Nevertheless, when \( \alpha = 0.8 \), the deformed honeycomb shows a slight tilt phenomenon, which may be related to the higher vertical cell wall. For the deformation in x-direction, as shown in figures 15(e), (f), the modes of the two configurations are generally similar, while the difference is that the deformed substructures of the configuration with larger \( \alpha \) have large volumes, which is helpful to make the enhancement point earlier, with the strain of about 0.52 to 0.45, as shown in figures 15(a), (b). Further, figure 16 shows the comparative analysis of the crushing behaviors between different \( \beta \). It can be found that with the \( \beta \) increasing, the Poisson’s ratio in y-direction has a significant decrease, with the minimum value from about 0.4 to about 1.9, as shown in figures 16(a), (b). Meanwhile, when \( \beta = 0.8 \), the difference in the crushing stress in two directions is also relatively significant, as shown in figure 16(b), where the plate stress in y-direction is about 0.2 MPa, while that in x-direction is only about 0.05 MPa. Combining the elastic analysis in figure 8 we can deduce that the parameter \( \beta \) has a marked influence on the orthotropic characteristics of the linear elastic and large deformation behaviors of the proposed RCW honeycomb. The deformation modes of the two configurations are generally similar, as shown in figures 16(c)–(f),
and the substructure of the configuration with $\beta = 0.8$ seems to be easier to crush and lose the inner volume in y-direction than that of the configuration with $\beta = 0.2$. Therefore, the stress of the latter starts to increases at an earlier strain about 0.35, and the growth rate is relatively slow, while the enhancement behavior of the former is relatively sharp at the strain about 0.52.

3.2.3. Analytical analysis of crushing strength

Considering the crushing strength of the RCW honeycomb in the y direction is higher and more stable than that in the other direction, an analytical model is derived here to predict the plateau stress for better guiding the application of the RCW honeycomb. The analytical model is according to the conservation of energy, that is, the work done by the external force is equal to the plastic dissipation energy of the cell walls. Besides, a quasi-static loading condition is assumed in this work, and therefore, the potential kinetic energy caused by the inertial effect is ignored.

Figure 17 gives the deformation modes of a representative RCW cell with $\alpha = 0.5$ and $\beta = 0.5$, and its 1/4 model has been adopted to describe its deformation process. Where, $H_0$ and $S_0$ are the height and length of the original cell, respectively; $H_1$ and $S_1$ are the height and length of the deformed cell, respectively. It can be found that with the rotation of the cell wall, point C gradually coincides with line AB, and the hexagon goes into a transitional configuration of the quadrangle at this moment. Considering the geometry of the quadrangle cannot be determined uniquely, two limit cases are discussed, i.e., the value of $\alpha$ closed to 0 and 1, respectively. As shown in figure 18, the numerical deformation processes of two RCW cells with $\alpha = 0.2$ and 0.8 are captured, and the corresponding analytical models are also given.
According to the geometry relationship shown in figures 18(c), (d), relative displacement $\mu_{y-1}$ and plastic hinges $\Delta_{\theta-1}$ of the cell with smaller $\alpha$ can be calculated as:

$$\mu_{y-1} = l_4 \sin \theta_1, \quad \Delta_{\theta-1} = 8\theta_1$$  \hspace{1cm} (13)

Similarly, as shown in figures 18(g)–(h), the relative displacement and plastic angles of the cell with larger $\alpha$ can be expressed as:

$$\mu_{y-2} = \frac{h}{2} - 2t, \quad \Delta_{\theta-2} = 4 \times \left[ \theta_1 + \left( \frac{\pi}{2} - \theta_1 \right) + \frac{\pi}{2} \right] = 4\pi$$  \hspace{1cm} (14)

Therefore, for a general RCW cell, its deformation is assumed as the combination of the above two modes, and its relative displacement and plastic angles can be written as:

$$\mu_y = (1 - \alpha)\mu_{y-1} + \alpha\mu_{y-2},$$

$$\Delta_{\theta} = (1 - \alpha)\Delta_{\theta-1} + \alpha\Delta_{\theta-2}$$  \hspace{1cm} (15)

Consequently, according to energy conservation, the analytical plateau stress $\sigma_0$ can be expressed as:

$$\sigma_0 = \frac{\Delta_{\theta}M_p}{S_0b\mu_y} = \frac{M_p[8(1 - \alpha)\theta_1 + 4\alpha\pi]}{sb \left[ (1 - \alpha)l_4 \sin \theta_1 + \alpha \left( \frac{h}{2} - 2t \right) \right]}$$  \hspace{1cm} (16)
Where, $S_0 = s = (2 - \cos \theta_0)h/\sin \theta_0$ is the original cell length; $b$ is the out-of-plane width of the cell wall; $M_p$ is the fully plastic bending moment, and equal to $\sigma_{pl}bt^2/4$ for rectangular beams.

Figure 19 shows the numerical and analytical analysis of the crushing strength of the RCW honeycomb under the $y$-direction loading, in which the parameters $t$, $\theta_0$, $\alpha$ and $\beta$ are involved in the comparison. As shown in figure 19(a), the parameter $t$ has a significantly positive effect on the plateau stress, with the analytical values of 0.13 MPa, 0.55 MPa and 1.31 MPa, respectively, which agree well with the numerical values of 0.12 MPa, 0.62 MPa and 1.35 MPa. Besides, the parameters $\theta_0$, $\alpha$ and $\beta$ also have a positive effect on the plateau stress, while their influence is smaller than that of the parameter $t$. As shown in figures 19(b)–(d), the analytic stresses have a good agreement with the numerical results, especially, the analytic model can well reflect the trend of the influence of the parameters. Therefore, it can be considered that the established analytical model is effective for predicting the crushing stress of the RCW honeycomb in $y$-direction.

3.3. Discussions

The topology of unit cell plays a key role in the mechanical properties of metamaterials. In recent years, focusing on improving the mechanical properties of the auxetic honeycomb under large deformation, a series of novel cells have been proposed, as mentioned in introduction section. Generally, when the new design is aimed at resisting impact or absorbing energy, some basic performance requirements have become the consensus of researchers, including higher crushing strength, increased auxetic effect and more stable deformation behaviors et al. However, those requirements are usually difficult to be met at the same time and often need to be
compromised. For example, the improvement of crushing strength may be accompanied by the degradation of auxetic effect or stability. The proposed design in this work may provide a novel solution aiming at these issues. Generally, in this manuscript, we developed a novel auxetic honeycomb to further improve the mechanical properties of the RE honeycomb, including its stiffness, Poisson’s ratio and crushing strength. By comparing with the existing publications in this field, the main innovation in this work can be summarized as proposing a novel hexagonal substructure and its embedding method, as shown in figure 1. According to the analyses from the analytical models, simulations and experiments, this design can bring the following advantages:

First of all, the hexagonal substructure and its embedding method are specially designed, which can lead the honeycomb cell to bend inward orderly. In particular, the inclined wall of the substructure is relatively easy to bend, which can cause a relatively stable and auxetic behavior under y-direction loading, as shown in figure 10. Even in the later stage of the compression process, the dynamic Poisson’s ratio of the RCW honeycomb is still significantly lower than the RE honeycomb, as shown in figure 13(b). For some existing designs, the auxetic...
effect may be weakened or completely disappear [32, 57], which may not be conducive for the structure to better resist impact using the NPR effect.

Further, the proposed hexagon substructure can provide a progressive stress enhancement during the later stage of the crushing process, as shown in figure 13(a). It should be noted that this effect is not caused by the structural densification, but by the change of deformation mode of the substructure. In similar studies, this mechanism, i.e., using the change of structural deformation mode to improve the crushing strength, is not rare [24, 57, 68]. However, a considerable part of those structures has a second stress plateau, which is accompanied by a significant stress peak. This secondary peak force may have potential damage to the protected structure object. For the proposed RCW honeycomb, the hexagonal substructure first transitions to a parallelogram, and then under the effect of cell wall contact, the parallelogram continues to be crushed, just as depicted in figure 17. This progressive and orderly deformation mode provides a progressive stress enhancement under y-direction loading, as shown in figures 15(a) and 16(a). Indeed, this enhancement may also become abrupt with the parameter $q_0$ decreasing, as shown in figure 14(a). It may be because the relatively large volume of the substructures restricts the cell wall to continue to deform along the original plastic hinges.

Besides, with the development of micromachining, constructing hierarchical auxetic honeycombs at the micro-scale has received extensive attention [55, 56]. Although the concept of the RCW honeycomb proposed in this manuscript is confirmed by the conventional 3D printing process, its further manufacturing and research at the microscale, such as by the micromprinting technology, is still of significance. For the proposed RCW honeycomb, the number of substructures is only four, which is greatly less than some existing structures, such as the cells constructed by filling walls with many triangles or hexagons [46]. Therefore, under certain processing accuracy, the proposed RCW cell, as shown in figure 1(d), may allow a smaller fabrication size compared with the existing designs. Meanwhile, with the advanced micro-scale manufacturing technology, the higher-order hierarchical structure, i.e., by replacing the walls of the RCW cell with smaller substructures, still has the potential to further improve the proposed design.

Due to the significant orthogonality, the crushing strength and the stability of the RCW honeycomb in x-direction are worse than those in y-direction, as shown in figures 10–11 and 13(a). This structural instability is mainly due to the plastic collapse of the vertical cell wall, as found in figure 11. Accordingly, the crushing stress has a significant decline or fluctuation, as shown in figures 13(a) and 11(b), which may not be conducive to energy absorption or structure protection. Therefore, in similar innovative designs to the RE honeycomb, researchers seem to prefer the analysis of y-direction loading [30, 35, 68]. In order to systematically study the

![Figure 19. Comparison results of the crushing stress by numerical and analytical models: (a) variable $t$; (b) variable $\theta_0$; (c) variable $\alpha$; (d) variable $\beta$.](image-url)
mechanical behaviors of the proposed honeycomb and to provide some theoretical reference for application, both two orthogonal directions are involved in this work. Besides, to further expand the comparison, the well-known double-arrow (DA) honeycomb structure is also involved in this work, which has a relatively simple geometry [69]. As shown in the supplementary materials, the deformation in both two orthogonal directions of the DA honeycomb is mainly caused by the inward rotation of the arrows, which is a relatively stable process. Therefore, it may be significant to add the arrow-head-shaped element to the design of the auxetic metamaterials. The advantage of the proposed RCW honeycomb is its significant superiority in the crushing stress of y-direction, and its value of specific energy absorption (SEA) is about 72% higher than that of the RE honeycomb and about 35% higher than that of the DA honeycomb. Therefore, the additional comparison further illustrates the effectiveness of the proposed hexagonal strategy in strength improvement. Meanwhile, based on the above analysis, we propose to take the y-direction as the main working direction of the RCW honeycomb to obtain a more stable crushing process and a higher SEA.

4. Conclusions

In this manuscript, a novel auxetic metamaterial, named re-entrant combined-wall (RCW) honeycomb, was developed by replacing the single inclined wall of the traditional RE honeycomb with a hierarchical substructure. Analytical, numerical and experimental methods were adopted to investigate the mechanical behaviors of the proposed novel structure systematically. Specifically, an analytical model was derived to calculate the equivalent elastic constants, including Young’s modulus and Poisson’s ratio. Then, we established the numerical models to further investigate the linear elastic properties as well as the crushing behaviors under large deformation. Quasi-static compression experiment was also carried out to verify the proposed RCW honeycomb concept and the established numerical models. Combining the confirmed analytical and numerical models, detailed analyses of the parameter effects and the orthotropic properties were performed. According to the results, some meaningful conclusions can be summarized as follows:

1. The proposed RCW honeycomb structure has great designability. Especially, adjusting the cell wall thickness $t$ can obtain a wide range of Young’s modulus, up to two orders of magnitude, with a little influence on Poisson’s ratio. Besides, adjusting the parameters $\theta_0$ and $\beta$ has a clear effect on the auxetic effect, and the parameter $\alpha$ has a direct impact on the large deformation modes of the substructure.

2. Compared with the traditional RE honeycomb, the elastic performance and the auxetic effect of the RCW honeycomb have great improvement. Quantitatively, the Young’s modulus in the y direction increases by more than 120%, and the Poisson’s ratio decreases by about 43%.

3. The contact and extrusion behaviors of the added substructure lead to a clear transformation of the honeycomb deformation modes, and accordingly, the crushing strength of the RCW honeycomb in y-direction has a significant enhancement, with a 140% improvement than that of the RE honeycomb, which may be of potential significance on the fields of impact resistance and energy absorption. However, due to the significant orthogonality, the crushing strength and the stability of the RCW honeycomb in x-direction are relatively worse. This point is helpful to guide the layout of RCW honeycomb in practical applications.

4. The design strategy adopted in this work, i.e., replacing the re-entrant inclined wall with a hierarchical substructure, has preliminarily shown its advantages by which mechanical properties of the honeycomb structure can be significantly improved while the auxetic effect can be well enhanced. Therefore, this design strategy is expected to obtain more attention in the future development of honeycomb metamaterials.

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Data availability statement

No new data were created or analysed in this study.
Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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