Possible Method to Reconstruct the Cosmic Equation of State from Strong Gravitational Lensing Systems

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A possible method to reconstruct the cosmic equation of state using strong gravitational lensing systems is proposed. The feasibility of the method is investigated by carrying out the reconstruction on the basis of a simple Monte-Carlo simulation. We show that the method can work and that the cosmic equation of state $w(z)$ can be determined within errors of $\Delta w \sim \pm 0.1 - \pm 0.2$ when a sufficiently large number of lensing systems ($N \sim 20$) for $z \lesssim 1$ are precisely measured. Statistics of lensed sources in a wide and deep survey like the SDSS are also briefly discussed.

§1. Introduction

Because of recent progress in the capability of observational facilities, gravitational lensing phenomena in the high-redshift universe have come to play a very important role in the fields of cosmology and astrophysics.\textsuperscript{1} Gravitational lensing phenomena are useful not only to probe the nature of dark matter in the universe but also to test cosmological models. Recent reports on the measurement of the cosmic shear field demonstrate this usefulness.\textsuperscript{1,2,3,4,5,6} It is also well recognized that strong lensing systems at high redshift are useful to constrain cosmological models, in particular, to test the cosmological constant in the universe.\textsuperscript{7,8,9}

Recent observations of cosmic microwave background anisotropies and distant supernovae favor a spatially flat universe whose expansion is currently accelerating. The cosmological model that includes a cosmological constant is the simplest model that explains these observations. Motivated by those observational results, variants of the cosmological constant model have been proposed in the framework of the cold dark matter (CDM) cosmological model. For example, quintessence has been recently investigated. A model that includes quintessence has the attractive feature of explaining the ‘coincidence problem’ – the near coincidence of the density of matter and the dark energy component at present.\textsuperscript{10,11,12}

Such generalized cosmological constants are regarded as a dark energy component in addition to the dark matter component in the universe. The dark energy component can be characterized by the equation of state $w = p_Q/\rho_Q$, where $p_Q$ is the pressure and $\rho_Q$ is the energy density. If the dark energy is the cosmological

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constant, then $w = -1$. For the quintessence model, $w$ takes an arbitrary value satisfying $-1 < w \leq 0$, and is a function of the redshift in general. Thus constraints on the equation of state are quite important for models of the dark energy component. Predictions of the CDM models with the dark energy component have been tested by comparison with various cosmological observations. An interesting approach to understanding the dark energy component is reconstruction of the cosmic equation of state or the quintessential potential.

Several authors have investigated gravitational lens systems as probes of the cosmic equation of state. Recently Futamase and Yoshida proposed a possible method to probe the dark energy component using strong gravitational lensing systems. They have pointed out that even one lensing system might give a strong constraint on the time variation of the dark energy component. Here, one might ask whether it is possible to reconstruct the cosmic equation of state or the quintessential potential using many strong lens systems, which may be detected using wide and deep surveys, e.g., the SDSS. In the present paper, motivated by the investigation in Ref. 24), we investigate the feasibility of a scheme for reconstructing the redshift evolution of the cosmic equation of state.

This paper is organized as follows. In §2, we briefly review theoretical formulas of the lensing model and a background cosmological model. In §3, a reconstruction scheme is worked out. In §4, the statistics of lensed sources detected in a wide and deep survey like the SDSS are briefly discussed. §5 is devoted to summary and conclusions. Throughout this paper we use units in which the velocity of light, $c$, equals 1.

§2. Lens equation

We restrict ourselves to a spatially flat FRW universe whose line element is written as

$$ds^2 = a(\eta)^2(-d\eta^2 + dr^2 + r^2d\Omega^2_{(2)}),$$

(1)

where $d\Omega^2_{(2)}$ is the line element of a unit two-sphere, $\eta$ is the conformal time and $a(\eta)$ is the scale factor normalized as $a = 1$ at present. We use a quintessential cosmological model consisting of a scalar field slowly rolling down its effect potential. Using the pressure $p_Q$ and the energy density $\rho_Q$ of the dark energy component, the effective equation of state of the dark energy component is written $w = p_Q/\rho_Q$, which is a function of the redshift in general quintessential models. Assuming the equation of motion

$$d(\rho_Qa^3) + p_Qd(a^3) = 0,$$

(2)

with the effective equation of state $w(z)$, we have the solution

$$\frac{\rho_Q(z)}{\rho_Q(z = 0)} = (1 + z)^3 \exp\left[3 \int_0^z \frac{w(z')}{1 + z'}dz'\right] \equiv f(z).$$

(3)
In the case that the equation of state is written as $w(z) = w_0(1 + z)^\nu$, we have

$$f(z) = (1 + z)^3 \exp \left[ 3w_0 \frac{(1 + z)^\nu - 1}{\nu} \right].$$

(4)

In the limit of a constant equation of state, $\nu = 0$, the dark energy density evolves as $\rho_Q(z) \propto f(z) = (1 + z)^3(1 + w_0)$. We denote the density parameters of the matter component and the dark energy at present by $\Omega_0$ and $\Omega_Q(= 1 - \Omega_0)$, respectively.

Then, from the Friedman equation, we have the formula for the angular diameter distance

$$D_A(z_1, z_2) = \frac{1}{H_0(1 + z_2)} \int_{z_1}^{z_2} \frac{dz'}{[\Omega_0(1 + z')^3 + \Omega_Q f(z')]^{1/2}},$$

(5)

where we assumed $z_1 < z_2$, and $H_0 = 100h \text{ km/s/Mpc}$ is the Hubble constant.

Following previous investigations, we here consider the lens equation with a lens potential given by an isothermal ellipsoid model as

$$\Phi = \frac{4\pi\sigma_v^2 D_{LS}}{D_S} \sqrt{(1 - \epsilon)\theta_1^2 + (1 + \epsilon)\theta_2^2},$$

(6)

where $\sigma_v$ is the one-dimensional velocity dispersion, $D_{LS}$ is the angular diameter distance between lens and source objects, and $D_S$ is the distance between the source and the observer. The ratio $\epsilon$ of the minor axis to the major axis is related to the ellipticity $\epsilon$ by $\epsilon = \sqrt{(1 + \epsilon)/(1 - \epsilon)}$. Thus the lens equation gives the elliptical image of the Einstein ring with minor axis and major axis as

$$\theta_\pm = \theta_E \sqrt{1 \pm \epsilon},$$

(7)

where $\theta_E = 4\pi\sigma_v^2 D_{LS}/D_S$. Denoting the redshifts of the lens and the source by $z_l$ and $z_s$, respectively, we can write $D_{LS} = D_A(z_l, z_s)$ and $D_S = D_A(0, z_s)$.

§3. Reconstruction scheme

We assume a set of lensing systems whose observable quantities $z_l$ and $z_s$ and $\sigma_v$, $\epsilon$ and $\theta_E$ are determined for each system. If $\sigma_v$, $\epsilon$ and $\theta_E$ are measured for a lensing system, $D_S/D_{LS}$ is determined from Eq. (6). Thus the measurement of $\sigma_v$, $\epsilon$ and $\theta_E$ determines the ratio of distances $D_S/D_{LS}$. In our reconstruction procedure, we assume that a set of observable quantities $z_l$, $z_s$ and $R \equiv D_S/D_{LS}$, instead of $\sigma_v$, $\epsilon$ and $\theta_E$ are determined. To be specific, we construct the set of $N$ assumed lensing systems in a simulation as follows. First, we choose $z_s$ through a homogeneous random process in the range of redshifts $z_s^{\text{min}} \leq z_s \leq z_s^{\text{max}}$. Throughout this section we set $z_s^{\text{min}} = 0.5$ and $z_s^{\text{max}} = 1$. In a similar way, we choose $z_l$ in the range $z_l^{\text{min}} \leq z_l \leq z_l^{\text{max}}$, where we set $z_l^{\text{min}} = 0.1$ and $z_l^{\text{max}} = z_s^{\text{max}} - 0.2$ in the present paper. Using a cosmological model, we then compute $D_S/D_{LS}$ using the distance-formula (5). Observational errors will cause an observed value of $D_S/D_{LS}$ to differ from the theoretical prediction. We assume that the ellipticity and the angle of the
Einstein ring are well determined for each lensing system and that the observational noise is dominated by an error in \( \sigma_v \). Then we set
\[
R = \frac{D_S(z_l, z_s)}{D_{LS}(z_l, z_s)} \times (1 + p)^2,
\] (8)
where \( p \) is a random variable that is characterized by a Gaussian probability function with mean 0 and the variance \( \sigma^2 \).

We next consider reconstructing the cosmic equation of state from the set of \( N \) lensing systems specified by \( z_{li}, z_{si} \) and \( R_i \), where the subscript \( i \) denotes the data of the \( i \)-th lensing system. Our method is a simple application of the reconstruction scheme from supernovae data proposed by Chiba and Nakamura. We adopt the following fitting formula for the angular diameter distance:
\[
D_A(z_1, z_2) = \frac{1}{H_0} \frac{1}{1 + z_2} \left( \eta_f(z_1) - \eta_f(z_2) \right),
\] (9)
with
\[
\eta_f(z) = 2\alpha[y^{-8} + \beta y^{-6} + \gamma y^{-4} + \delta y^{-2} + \zeta]^{-1/8},
\] (10)
and \( y = 1/\sqrt{1 + z} \). Here the subscript \( f \) refers to fitting formulas. We use the constraint equation from the Friedman equation:
\[
\alpha = \frac{[1 + \beta + \gamma + \delta + \zeta]^{9/8}}{1 + 3\beta/4 + \gamma/2 + \delta/4}.
\] (11)
Then we determine the parameters \( \beta, \gamma, \delta \) and \( \zeta \) so as to minimize
\[
F_A(\beta, \gamma, \delta, \zeta) = \sum_{i=1}^{N} \left[ \frac{(R_i - R_{fi}(z_{li}, z_{si}, \beta, \gamma, \delta, \zeta))^2}{R_i^2} \right],
\] (12)
where \( R_{fi} \) is defined by
\[
R_{fi}(z_{li}, z_{si}, \beta, \gamma, \delta, \zeta) = D_A(0, z_{si}) \frac{\eta_f(0) - \eta_f(z_{si})}{\eta_f(z_{li}) - \eta_f(z_{si})}.
\] (13)
Thus \( R_{fi} \) does not depend on \( \alpha \), so the relation (11) is needed to determine \( \alpha \) in our prescription. Finally, the cosmic equation of state is given by
\[
w_f(z) = \frac{-4yd\eta/ dy^2}{3(d\eta/ dy)(\Omega_0(d\eta/ dy)^2 - 4)}.
\] (14)

Figure 1 demonstrates the feasibility of the reconstruction scheme for cosmological models whose parameters are \( \Omega_0 = 0.3, w_0 = -0.5, \nu = 0.4 \) (left panels) and \( \Omega_0 = 0.3, w_0 = -0.7, \nu = -0.4 \) (right panels). In each panel the dashed curve

* Note that the variance \( \sigma^2 \) is distinct from \( \sigma_v^2 \), which represents the velocity dispersion for the lens model.
represents the theoretical curve \( w(z) = w_0(1 + z)^\nu \), while the solid curve and shaded region represent the mean and the 1-sigma variance of the ensemble average of the reconstructed curves. The upper panels display the results for which the parameters \( \beta, \gamma, \delta \) and \( \zeta \) are determined by minimizing the function \( F_A \). The middle panels correspond to the case in which the parameters are determined so as to minimize the function \( F_B \) instead of \( F_A \), where

\[
F_B(\beta, \gamma, \delta, \zeta) = \sum_{i=1}^{N} \left[ \frac{(R_{fi}(z_{li}, z_{si}, \beta, \gamma, \delta, \zeta) - R_{fi})^2}{R_{fi}(z_{li}, z_{si}, \beta, \gamma, \delta, \zeta)^2} \right].
\] (15)

The lower panels display the results for which the reconstructed curve of \( w(z) \) is obtained by averaging the two reconstructed curves obtained using \( F_A \) and \( F_B \). In Fig. 1 we assumed \( \sigma = 0.03 \) and \( N = 20 \).

The capability of the reconstruction scheme depends on the errors involved in measuring the lensing systems, which are incorporated by \( p \) (or \( \sigma \)) in Eq. (8) in our simulation. Figure 2 displays the variance of reconstructed equation of state \( \Delta w(z) \) at \( z = 0.5 \) as a function of \( \sigma \) for the same cosmological models as in Fig. 1, i.e., with \( \Omega_0 = 0.3, w_0 = -0.5, \nu = 0.4 \) (left panels) and \( \Omega_0 = 0.3, w_0 = -0.7, \nu = -0.4 \) (right panels). This shows that the equation of state can be reconstructed within errors of \( \Delta w \sim \pm 0.1 - \pm 0.2 \).

In our simulation, for a set of data obtained through a random process, the scheme fails in reconstruction and gives a non-realistic value of \( w(z) \). We removed such a case in our simulation and considered only the case that the reconstructed equation of state satisfies \(-3/2 \leq w(z) \leq 0\) for \( 0 \leq z \leq 1 \). 'Failed' reconstructions appear more frequently as \( \sigma \) becomes larger than 0.03. For example the ‘failed’ reconstruction appears at the rate of 3%, 20% and 50% for \( \sigma = 0.01, 0.03 \) and 0.05, respectively. Hence our reconstruction scheme might not be feasible for \( \sigma \gtrsim 0.03 \) for general models other than the quintessential model.

§ 4. Discussion – Lensing statistics

In the previous section we have assumed a set of \( N (= 20) \) lensing systems distributed in the range \( 0.5 \leq z_s \leq 1 \). In this section we give some grounds for this assumption. Lensing statistics have been thoroughly investigated, and following previous investigations, we can roughly estimate the number of lensed sources per redshift as (see the Appendix for a derivation)

\[
\frac{dN(z_s)}{dz_s} = 5.8 \times 10^7 (H_0 r(z_s))^5 \frac{dH_0 r(z_s)}{dz_s} \frac{n_{\text{gal}}(z_s)}{h^3 \text{Mpc}^{-3}} \times \left( \frac{n_{\text{halo}}}{10^{-2} h^3 \text{Mpc}^{-3}} \right) \left( \frac{\sigma_v}{250 \text{ km/s}} \right)^4 \text{str}^{-1},
\] (16)

where \( r(z) \) is the comoving distance at redshift \( z \), \( n_{\text{gal}}(z_s) \) denotes the comoving number density of galaxies, and \( n_{\text{halo}} \) denotes the comoving number density of the lensing halo, which is assumed to be constant throughout the universe. In deriving
Eq. (16), the singular isothermal sphere model is assumed as the lens model, and the probability that multiple images appear is computed.

Figure 3 displays the expected distribution of lensed sources $\frac{dN}{dz_s}$ for cosmological models with $\Omega_0 = 0.3$, $w_0 = -0.5$, $\nu = 0.4$ (left panels) and $\Omega_0 = 0.3$, $w_0 = -0.7$, $\nu = -0.4$ (right panels). Here we adopted a luminosity function of the APM galaxies fitted to the Schechter function

$$\phi(L) dL = \phi^* \left( \frac{L}{L^*} \right)^\kappa \exp \left( -\frac{L}{L^*} \right) \frac{dL}{L^*},$$

(17)

with $\phi^* = 1.40 \times 10^{-2} h^3$ Mpc$^{-3}$, $\kappa = -0.97$, and $M^* = -19.50 + 5 \log_{10} h$. Then the comoving number density of galaxies at $z$ which are brighter than the limiting magnitude $B_{\lim}$ is given by

$$n_{\text{gal}}(z, < B_{\lim}) = \int_{L(B_{\lim,z})}^\infty \phi(L) dL = \phi^* \Gamma[\kappa + 1, x(B_{\lim}, z)],$$

(18)

where $x(B_{\lim}, z) = L(B_{\lim}, z)/L^* = [d_L(z)/1 h^{-1} \text{ Mpc}]^{2.2 - 0.4B_{\lim}}$, $\Gamma[\kappa, x]$ is the incomplete Gamma function, and $d_L(z)$ is the luminosity distance. In each panel of Fig. 3, the three solid curves correspond to the cases $B_{\lim} = 21$, 22, and 22.5.

The expected number of lensed objects $N$ is obtained by integrating $dN/dz_s$. For example we have $N \simeq 410, 2800, 7000$ for $B_{\lim} = 21, 22, 22.5$, respectively, for the model with $\Omega_0 = 0.3$, $w_0 = -0.7$, $\nu = -0.4$ within a solid angle of $\pi$ str. This is almost the same result as that for the model with $\Omega_0 = 0.3$, $w_0 = -0.5$ and $\nu = 0.4$. We thus conclude the SDSS project in progress should detect many lensed objects. Of course, the above is an over-estimation because the formula contains the case in which the ratio of the magnitude of the two lensed images is infinite. The actual number of ‘clean’ lensed systems near the Einstein ring should be significantly smaller. If 1% of the above estimation are clean lensed systems, this number should be sufficient to carry out the reconstruction proposed in this paper.

§5. Conclusion

In this paper we have proposed a possible method to reconstruct the cosmic equation of state using strong gravitational lensing systems. We have investigated the feasibility by working out a reconstruction process based on a simple Monte-Carlo simulation. Our result shows that the method can work when a sufficiently large numbers of lensing systems ($N \sim 20$ at $z \lesssim 1$) are precisely observed. For example, if the velocity dispersion (the lens model) can be measured within errors smaller than $\sim 3\%$, the cosmic equation of state can be determined within errors $\Delta w \sim \pm 0.1 - \pm 0.2$. The accuracy required for the measurement of the velocity dispersion might be quite high. However, it should be noted that such accurate measurements have been made using the Keck-II 10m telescope.

* In the present paper we neglect the magnification bias, because galaxies in the range of rather small redshift, $0.5 \leq z_s \leq 1$, are considered as lensed sources. The magnification bias enhances the expected number of the lensed sources. However, this effect does not alter our conclusion qualitatively.
We have adopted a simple model for the redshift distribution function of lensed sources. Specifically, in the present paper we have assumed a homogeneous random distribution in the range $0.5 \leq z_s \leq 1$. The feasibility of the reconstruction scheme sensitively depends on the redshift distribution of the lensing system. Hence, further investigations of distributions of lensing systems are needed. In general, the capability of our reconstruction scheme becomes worse as the redshift of lensed sources becomes higher. This feature originates from the fact that the fraction of the dark energy component becomes smaller relative to the matter component as the redshift becomes higher. This makes $D_A(z_l, z_s)$ less sensitive to the cosmic equation of state $w(z)$ as $z_l$ and $z_s$ become larger.

Our investigation is based on some other idealistic assumptions. We have assumed a flat spatial geometry of the universe. The crucial assumption might be that the matter fraction in the universe $\Omega_0$ is determined precisely. The errors in the determination of $\Omega_0$ cause additional errors in reconstructing $w(z)$.

We have assumed a singular isothermal ellipsoid as a model of our lensing galaxies because of its simplicity. However, it is not known if the singular isothermal ellipsoid model is a good model for lensing galaxies. In fact, the modeling of lensing galaxies is not easy, because of many theoretical and observational ambiguities such as the existence of an anisotropic velocity dispersion. However, there has been some progress in this direction. Dynamical observations and models of lensing elliptical galaxies in the local universe have been studied by Rix et al.\cite{rix31}. Furthermore, the relation between the observed stellar velocity dispersion and the velocity dispersion associated with dark matter has been studied in the case of singular isothermal sphere model.\cite{rix32} It is expected that such studies will provide more accurate results in the near future, based on the large sample of galaxies in SDSS. Thus we hope that our method will arrow for realistic reconstruction of the cosmic equation of state in the near future.

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Appendix A

--- Lensing Statistics ---

Here we briefly review the estimation of lensing statistics.\cite{rix23,rix27} Introducing the cross section $\pi a_{cr}^2$, the optical depth due to the gravitational lensing can be written

$$d\tau = n_{halo}(z_l)(1 + z_l)^3 \pi a_{cr}^2 dt,$$

(A.1)
where \( n_{\text{halo}}(z_l) \) is the comoving number density of the halo at redshift \( z_l \). Adopting a singular isothermal sphere as the model of the lens halo, we have

\[
d\tau = 16\pi^3\sigma_v^4n_{\text{halo}}(z_l)\left(\frac{r(z_s) - r(z_l)}{r(z_l)}\right)^2 r(z_l)^2 dr,
\]

where \( r(z) \) is the comoving distance and \( \sigma_v \) is the velocity dispersion. In deriving Eq. (A.2), we have used the assumption of a spatially flat universe and the fact that \( a_{\text{cr}} \) is written as

\[
a_{\text{cr}} = (1 + z_l)^{-1}r(z_l)4\pi\sigma_v^2 D_{LS}\frac{D_S}{D_S},
\]

where \( a_{\text{cr}} \) is the critical radius for multiple images.

The probability function that a source at \( z_s \) becomes multiple images, which is defined by \( P(z_s) = \int_0^{z_s} d\tau \), can be written as

\[
P(z_s) = 2.2 \times 10^{-3}(H_0 r(z_s))^3 \left(\frac{n_{\text{halo}}}{10^{-2}h^3 \text{Mpc}^{-3}}\right) \left(\frac{\sigma_v}{250 \text{km/s}}\right)^4,
\]

where we have assumed the comoving number density is constant \( n_{\text{halo}}(z) = n_{\text{halo}} \). Then the expected number of lensed galaxies per unit solid angle is computed by

\[
N = \int_0^\infty dz_s n_{\text{gal}}(z_s) P(z_s) r(z_s)^2 \frac{d^{2}r(z_s)}{dz_s^2},
\]

which reduces to

\[
N = 5.8 \times 10^7 \int_0^\infty dz_s(H_0 r(z_s))^5 \frac{dH_0 r(z_s)}{dz_s} \frac{n_{\text{gal}}(z_s)}{h^3 \text{Mpc}^{-3}}
\times \left(\frac{n_{\text{halo}}}{10^{-2}h^3 \text{Mpc}^{-3}}\right) \left(\frac{\sigma_v}{250 \text{km/s}}\right)^4.
\]

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Fig. 1. Feasibility of the reconstruction scheme for cosmological models whose parameters are $\Omega_0 = 0.3$, $w_0 = -0.5$, $\nu = 0.4$ (left panels) and $\Omega_0 = 0.3$, $w_0 = -0.7$, $\nu = -0.4$ (right panels). Here we have assumed lensed sources ($N = 20$) distributed in the range $0.5 \leq z_s \leq 1$ and $\sigma = 0.03$. In each panel, the dashed curve is the theoretical curve, while the solid curve and shaded region represent the mean and the 1-sigma variance of the reconstructed curves. The upper (middle) panels display the results for which the parameters $\beta, \gamma, \delta$ and $\zeta$ are determined by minimizing the function $F_A (F_B)$. The lower panels display the result for which the reconstructed curve $w_f(z)$ is obtained by averaging the reconstructed curves obtained from both $F_A$ and $F_B$. 
Fig. 2. Variance of the reconstructed cosmic equation of state $\Delta w$ at $z = 0.5$ for cosmological models whose parameters are $\Omega_0 = 0.3$, $w_0 = -0.5$, $\nu = 0.4$ (left panels) and $\Omega_0 = 0.3$, $w_0 = -0.7$, $\nu = -0.4$ (right panels). Here we have assumed that the number of lensing sources $N = 20$ is distributed in the range $0.5 \leq z_s \leq 1$, as in Fig. 1.
Fig. 3. $dN/dz_s$ for cosmological models with $\Omega_0 = 0.3$, $w_0 = -0.5$, $\nu = 0.4$ (left panels) and $\Omega_0 = 0.3$, $w_0 = -0.7$, $\nu = -0.4$ (right panels). Here we have adopted the luminosity function for galaxies fitted to the Schechter function. In each panel, the solid curves correspond to the cases in which the limiting magnitude is $B_{\text{lim}} = 21$, 22 and 22.5, from bottom to top.