Soft interactions at high energies: QCD motivated approach

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Abstract: We propose a QCD motivated theoretical approach to high energy soft interactions, which successfully describes the experimental data on total, elastic and diffraction cross sections. We predict that the survival probability for the diffractive Higgs production at the LHC energy is small (less than 1%), and investigate the influence of suggested corrections e.g. threshold effects and semi-enhanced diagrams, on this value.

Keywords: Soft Pomeron, BFKL Pomeron, Diffractive Cross Sections, Survival Probability.

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1. Introduction

For the past three decades the physics of soft interactions has been considered an unsavory topic. Indeed, the relation between high energy phenomenology and the microscopic theory: QCD, has been neglected, as there has been no progress in solving the problem of the confinement of quarks and gluons. The goal of this letter is to present an attempt to construct a self consistent theoretical approach to strong interactions at high energy, which provides a natural bridge to the perturbative QCD description at short distances.

Following on the results of our paper [1], we question the widely held prejudice that high energy scattering stems from long distances. We believe, that the only dimensional scale that governs the high energy interaction, is the saturation momentum \( Q_s \), which appears both in pQCD at high parton density [2, 3, 5, 6], and in the non-perturbative approach to QCD based on the Colour Glass Condensate [4, 7]. The saturation momentum increases at high energies and, therefore, we believe that the higher the energy, the shorter are the distances involved in the interaction. The key argument supporting our hypothesis is the fact that in fits to data, the slope of the Pomeron trajectory \( \alpha'_P \) is very small [1, 8, 9]. At first sight a small \( \alpha'_P \) contradicts one of the principle results of Regge theory, i.e. that all resonances lie on Regge trajectories. However, the first theoretical analysis of the Pomeron’s structure in a theory in which we can treat the region of strong couplings (the strongly coupled N=4 super Yang-Mills theory with AdS/CFT correspondence), given in Ref. [10], shows that \( \alpha'_P \rightarrow 0 \) for \( t < 0 \). This result is compatible with our approach. In the same theory \( \alpha'_P > 0 \) for \( t > 0 \), which is in reasonable agreement with Regge theory and high energy phenomenology (see Fig. 1 which was taken from Ref. [10]). Therefore, the small value of \( \alpha'_P \) in the scattering region is not only possible, but is a requisite, so as to be consistent with a strong coupling theory.
In perturbative QCD $\alpha''_P$, for a BFKL Pomeron [11], which determines the high energy scattering amplitude, is equal to zero, since $\alpha''_P \propto 1/Q_s^2 \rightarrow 0$. $W = \sqrt{s}$ is the energy in the c.m. frame. Therefore, the assumption that $\alpha''_P \rightarrow 0$, provides the necessary condition so as to link the strong interactions with the hard interactions that are described by pQCD.

The only microscopic explanation of Regge theory is provided by the partonic approach [13, 14], in which the slope of the Pomeron trajectory is related to the mean transverse momentum of the exchanged partons, $\alpha''_P \propto 1/\langle p_t \rangle$. $\langle p_t \rangle$ is the mean parton momentum. The commonly held view from high energy phenomenology, is that $\alpha''_P = 0$. The phenomenological value of $\alpha''_P$ stems from the supposition, that the shadowing corrections are small, and the shrinkage of the forward elastic peak is induced by the slope of the Pomeron trajectory. The above supposition is not compatible with the high value of $\alpha''_P(0)$ implied by the ISR-Tevatron cross section data. In Ref. [1] an alternative option was suggested in which eikonal shadowing corrections are significant yielding a diminishing $\alpha''_P = 0.01 \text{GeV}^{-2}$ value. This result is not affected by a more complicated form of the Pomeron interactions. A similar result was obtained, also, in Ref. [8] in which the experimental elastic data was reproduced with $\alpha''_P = 0^*$. We conclude that a small $\alpha''_P$ is a reasonable, data compatible, option which induces a large typical parton momentum. The resulting running QCD coupling $\alpha_S(\approx b/\ln \left(\langle p_t^2 \rangle /\Lambda_{QCD}^2 \right)) \ll 1$. We shall consider it as the small parameter in the pQCD estimates of the Pomeron-Pomeron interaction vertices. We wish to stress that, regardless of its intuitive appeal, the parton model does not provide a reliable approach since it presumes a short range interaction in rapidity, while the exchange of gluons in QCD generate long range rapidity interactions. Nevertheless, the model convincingly illustrates, that the smallness of $\alpha''_P$, implies that the typical interaction distances are short. As we have discussed, we estimate that they are of the order of $1/Q_s$.

The second assumption underlying our approach is that only the triple Pomeron interaction is essential at high energy. This does not precludes multi Pomeron vertices which are estimated by fan diagrams. As such these vertices are not local in rapidity. This has been proven in pQCD (see Refs. [17,18]). This selection rule (considering only the triple Pomeron vertex) postulates that in the leading log $x$ approximation of pQCD, with a large number of colours ($N_c \gg 1$), all other possible interactions are suppressed. It is well known that at high energy, and for $N_c \gg 1$, the correct degrees of freedom are the colourless dipoles [19]. In terms of these dipoles, the selection rule reads, that only the decay of one dipole to two dipoles contributes to the high energy amplitude. This implies that a colourless dipole does not interact with the target via

*In their last five papers (see Refs. [9]) the Durham group has considered a diminishing, non zero, $\alpha''_P$. 

![Figure 1: The behaviour of the Pomeron trajectory in N=4 SYM accordingly to Ref. [10]. The figure is taken from Ref. [10]](image)
Figure 2: The different contributions to Pomeron Green’s function (enhanced diagrams of Fig. 2-a) and the semi-enhanced diagrams that has not been taken into account in the Pomeron Green’s function.

multi Glauber-type rescatterings, but only through the splitting of one dipole into two dipoles and the merging of two dipoles to one dipole. This postulate provides a natural bridge between our theoretical approach to high energy scattering, and the corresponding pQCD high energy scattering amplitude.

In this paper we present a theoretical approach for soft interactions at high energies, incorporating the following assumptions:

1. \( \alpha'_{IP} = 0 \).
2. The Pomeron interactions reduce via fan diagrams to triple Pomeron vertices.
3. \( \alpha'_{IR} \neq 0 \) and its slope, as well as other parameters of the Reggeons, should be determined from experiment.
4. A Pomeron - hadron interaction is treated phenomenologically using the eikonal formulae, in the framework of a two channel formalism, based on the Good-Walker mechanism [20].

The first two assumptions allow us to build a theory of interacting Pomeron, while the last two points are phenomenological. Details of the two channel (Good-Walker mechanism, see Ref. [20]) that we use for describing the elastic and diffractive scattering eigenstates are given in Ref. [1].

2. The Pomeron interaction (Pomeron Green’s function)

Our key idea is to use the general results of pQCD relating to high energy scattering [12]. Consequently:

1) In the leading order approximation of pQCD, only one Pomerons splitting into two Pomerons, and two Pomerons merging into one Pomeron are taken into account [17, 18]. All other Pomerons vertices do not explicitly appear in the leading log \( x \) approximation of pQCD. We, therefore, restrict ourselves to sum only
Pomeron diagrams containing triple Pomeron vertices.

2) We consider a theory with \( \alpha'_{P} = 0 \).

3) Using our approach, we can predict the anticipated values of all Pomeron interactions: the intercept of the Pomeron above unity \( \Delta_{I P} \) is \( \propto \alpha_{S} \), and the triple Pomeron vertex coupling \( g_{3P} \propto \alpha_{S}^{2} \).

The theory which includes all the above ingredients can be formulated in terms of a generating function [19, 26]

\[
Z(y, u) = \sum_{n} P_{n}(y) u^{n},
\]

where, \( P_{n}(y) \) is the probability to find \( n \)-Pomerons (dipoles) at rapidity \( y \). At rapidity \( y = Y = \ln(s/s_{0}) \) we can impose an arbitrary initial condition. For example, demanding that there is only one fastest parton (dipole), which is \( P_{1}(y = Y) = 1 \), while \( P_{n>1}(y = Y) = 0 \). In this case we have the following initial condition for the generating function

\[
Z(y = Y) = u.
\]

At \( u = 1 \)

\[
Z(y, u = 1) = 1,
\]

which follows from the physical meaning of \( P_{n} \) as a probability. The solution, with these two conditions, will give us the sum of enhanced diagrams (see Fig. 2-a).

For the function \( Z(u) \) the following simple equation can be written (see Ref. [1] and references therein)

\[
-\frac{\partial Z(y, u)}{\partial Y} = -\Gamma(1 \to 2) u(1 - u) \frac{\partial Z(y, u)}{\partial u} + \Gamma(2 \to 1) u(1 - u) \frac{\partial^2 Z(y, u)}{\partial^2 u},
\]

where, \( \Gamma(1 \to 2) \) describes the decay of one Pomeron (dipole) into two Pomerons (dipoles), while \( \Gamma(2 \to 1) \) relates to the merging of two Pomerons (dipoles) into one Pomeron (dipole).

The first term on the r.h.s. describes two processes: the term proportional to \( u \) can be viewed as a probability of a dipole annihilation in the rapidity range \( (y \text{ to } y - dy) \) (death term). The term with factor \( u^{2} \) is a probability to create one extra dipole (birth term). The second term on the r.h.s. describes the same type of processes for Pomeron (dipole) merging.

The description of the parton system given by Eq. (2.4), is equivalent to the path integral formulation of the Pomeron interaction (see Ref. [1]). The path integral formulation of the Pomeron interaction leads to the automatic inclusion of \( t \)-channel unitarity constraints, while in the generating function formulation, \( s \)-channel unitarity has been taken into account.

Using the functional \( Z \), we find the scattering amplitude [6, 26], using the following formula:

\[
N(Y) \equiv \text{Im} A_{el}(Y) = \sum_{n=1}^{\infty} \frac{(-1)^{n}}{n!} \frac{\partial^{n} Z(y, u)}{\partial^{n} u} \bigg|_{u=1} \gamma_{n}(Y = Y_{0}, b),
\]

where, \( \gamma_{n}(Y = Y_{0}, b) \) is the scattering amplitude of \( n \)-partons (dipoles) at low energy. These amplitudes depend on the impact parameters which are the same for all \( n \) partons since \( \alpha'_{P} = 0 \). Consequently,
we neglect the diffusion of partons in impact parameter space. Eq. (2.5) corresponds to the partonic approach [13,14], in which a high energy scattering process can be viewed as a two stage sequence. The first stage is the development of the partonic wave function, which we consider by introducing the generating function \( Z \). The second stage is given by Eq. (2.5) corresponds to the interaction of the lowest energy partons (‘wee’ partons) with the target, which is described by the amplitudes \( \gamma_n(Y = Y_0, b) \). Assuming that there are no correlations between the interacting partons (dipoles) at low energy, we take \( \gamma_n(Y = Y_0, b) = \gamma_n^1(Y = Y_0, b) \) [6].

The generating function approach given by Eq. (2.1), Eq. (2.4) and Eq. (2.5), has the advantage that it can be solved analytically (see Ref. [29]). This solution leads to a constant cross section at high energy, while the interaction without the four Pomeron term, decreases at high energy [27](see also Refs. [25, 28]). The problem is that the inclusion of the four Pomeron term contradicts our QCD motivated approach. This highlights the importance of \( s \)-channel unitarity studies of the asymptotic high energy behaviour of the scattering amplitude. The above problem is not relevant to our present investigation as we restrict the energy range where we can trust our approach, to energies at which the four Pomeron term is not yet significant. As discussed in our paper [1] this range is \( W \leq 10^5 GeV \).

We wish to emphasise that the form of the equation, as well as the functional formulation of the Pomeron interaction, is the same as that for interacting dipoles in QCD. The only difference is that in QCD the size of the dipole can be changed due to an interaction, while in our case the size of the dipole is fixed. The idea that the transition from short distances to long distances goes through the stage of freezing the typical size of the interacting dipoles, has been around for a long time, and has both theoretical and phenomenological justification (see, for example, Refs. [21–24]). This gives us further confidence that our approach provides a natural link with the perturbative QCD approach.

Unfortunately, the exact solution to Eq. (2.4) with arbitrary initial and boundary conditions, is not very transparent. In a number of papers [1, 30,32–34, 37], it has been shown that, using Mueller-Patel-Salam-Iancu approximation, one can simplify the solution, and reduce it to a simple analytical form. This approximation yields a solution with an accuracy of the order of \( \gamma \exp (-\Delta P Y) \propto \alpha_s^2 \exp (-\alpha_s Y) \ll 1 \).

In ref. [1], and in this letter, we only calculate the Green’s function of the Pomeron, neglecting for example, diagrams of the Fig.2b type. The reason for this procedure is that the interaction of the Pomeron described by the exact Green’s function, will be simpler than the interaction of the bare Pomerons (see Ref. [35] in which this approach was suggested).

The exact expression for the Pomeron Green’s function has the form

\[
G_P(Y) = 1 - \exp \left( \frac{1}{T(Y)} \right) \frac{1}{T(Y)} \Gamma \left( 0, \frac{1}{T(Y)} \right), \tag{2.6}
\]

where \( \Gamma (0, x) \) is the incomplete gamma function (see 8.350 - 8.359 in Ref. [38]) and

\[
T(Y) = \gamma e^{\Delta P Y}. \tag{2.7}
\]

where, \( \gamma \) has a simple meaning of being the amplitude of of the two dipoles interaction at low energy. The derivation of Eq. (2.6) as well as derivation of the formulae for the cross sections of the single and double diffraction production, are given in Ref. [1].
3. The results of the fit

The pertinent details of our fit to the experimental data, and our determination of the relevant parameters of the model, needed to describe the soft interactions, are contained in [1]. In this section we will only mention the salient features, and results of the fit.

Our fit is based on 55 experimental data points, which includes the \( p-p \) and \( \bar{p}-p \) total cross sections, integrated elastic cross sections, integrated single and double diffraction cross sections, and the forward slope of the elastic cross section in the ISR-Tevatron energy range. The model gives a good reproduction of the data, with a \( \chi^2/d.o.f. \approx 1.25 \). A significant contribution to \( \chi^2/d.o.f. \) stems from the uncertainty for the value of two single diffraction cross sections, and of the total cross section at the Tevatron, without these three points we achieve a \( \chi^2/d.o.f. = 1.0 \). In addition to the quantities contained in the data base, we obtain a good description of the CDF [39] differential elastic cross sections and the single diffractive mass distribution at \( t = 0.05 \ GeV^2 \). An important advantage of our approach, is that the model provides a very good reproduction of the double diffractive (DD) data points. Other attempts to describe the DD data e.g. (see Refs. [9]), were not successful in reproducing the DD experimental results over the whole energy range.

The predicted energy behaviour of the above mentioned cross sections, as well as the forward slope of the elastic cross section, are given in detail in Ref. [1]. We mention here only our results for the LHC energy, which are of particular interest i.e. at \( W=14 \ TeV \) we have \( \sigma_{tot} = 92.1 \ mb, \sigma_{el} = 20.9 \ mb, \sigma_{sd} = 11.8 \ mb, \sigma_{dd} = 6.08 \ mb \) and \( B_{el} = 20.6 \ GeV^{-2} \).

We note that the lowest energy data point in our fit [1] to the total cross section was at \( W = 19.2 \ GeV \). In Fig. 3, we show the results of our calculations in which we have extrapolated our model to lower energies. As can be seen, it provides a fair description of the data even at energies as low as \( W = 7 \ GeV \), where we underestimate the experimental numbers just by about 10%.

4. Survival probability for diffractive production of the Higgs boson.

In our approach, the survival probability for the diffractive production of the Higgs boson, can be written
Table 1: Comparison of the GLMM (this paper) and KMR [8] models. The numbers in parenthesis are the estimates for the contribution of the semi-enhanced diagrams to the survival probability.

As a product of two factors,

\[
\langle | S^2 | \rangle = \langle | S^2_{enh} | \rangle \text{(enhanced diagrams)} \times \langle | S^2_{2ch} | \rangle \text{(Good-Walker mechanism)}.
\] (4.1)

The second factor has been determined in a number of papers, and all the groups obtain approximately the same values. The first factor is new and was first evaluated in Ref. [1], where the exact formula for \( \langle | S^2_{enh} | \rangle \) was derived. It has the form

\[
\langle | S^2_{enh} (MPSI) | \rangle (Y) = S \left( T(Y,Y') \right) = 2 e^{-\Delta_F(Y-Y')} S_1 \left( T(Y,Y') \right) + e^{-2\Delta_F(Y-Y')} S_2 \left( T(Y,Y') \right);
\] (4.2)

\[
S(T) = \frac{1}{T^3} \left\{ -T + e^\frac{T}{2} (1 + T) e^\frac{T}{2} \Gamma \left( 0, \frac{1}{T} \right) \right\};
\] (4.3)

\[
S_1(T) = \frac{1}{T^3} \left\{ -T(1 + T) + (1 + 2T) e^\frac{T}{2} \Gamma \left( 0, \frac{1}{T} \right) \right\};
\] (4.4)

\[
S_2(T) = \frac{1}{T^3} \left\{ T \left[ (T-1)^2 - 2 \right] + (1 + 3T) e^\frac{T}{2} \Gamma \left( 0, \frac{1}{T} \right) \right\},
\] (4.5)

where,

\[
T \left( Y,Y' \right) = \gamma \left( e^{\Delta_F(Y-Y')} - 1 \right) \left( e^{\Delta_F Y'} - 1 \right).
\] (4.6)

This factor diminishes the value of the survival probability, and causes a decrease in its magnitude by a factor of 5-6, in going from Tevatron to LHC energies (see Table 1).
concerned the so called, threshold effect, which suggests that one can only use the one Pomeron exchange approximation at sufficiently large rapidities \(Y > 2\). In our approach there is no need for an additional cutoff in rapidity, since the scattering amplitude is described by the sum of the contributions of both Pomerons and Reggeons.

In Fig. 3 we show the scattering cross section at rather low energies. Our parametrization provides a good description of the experimental data also at low energies. Since in our parametrization we include the contribution of the secondary Reggeons, there is no need for an additional cutoff. The second question concerns the semi-enhanced diagrams of the Fig. 2-b type. Indeed, in our fit since \(g_1\) and \(g_2\) turn out to be large, we could anticipate a large contribution. A self consistent approach which includes all Pomeron diagrams will be published in a separate paper, which is presently being prepared [36]. In the present paper we only calculate the exclusive contribution of the diagrams of type Fig. 2-b, given by the following formula (the proof of this formula will be given in Ref. [36])

\[
\langle S^2 \rangle = \frac{1}{\int d^2b a_H^2(b)} \int d^2b \left\{ \int d^2b' \frac{a_H^2(b)}{\left(1 + g_i \Gamma(b'; m_i) T(\gamma, \Delta, y) + g_k \Gamma(|\tilde{b} - \tilde{b}'|; m_i) T(\gamma, \Delta, y)\right)^2} \right\}
\times \left(1 - \exp\left(\int d^2b' \frac{g_i g_k \Gamma(b'; m_i) \Gamma(|\tilde{b} - \tilde{b}'|; m_i) T(\gamma, \Delta, y) + g_k \Gamma(|\tilde{b} - \tilde{b}'|; m_i) T(\gamma, \Delta, y)}{1 + g_i \Gamma(b'; m_i) T(\gamma, \Delta, y) + g_k \Gamma(|\tilde{b} - \tilde{b}'|; m_i) T(\gamma, \Delta, y)}\right)\right).
\]

(4.6)

In Eq. (4.6) \(T(\gamma, \Delta, y)\) is given by Eq. (2.7), and

\[
\Gamma(b, m_i) = \frac{m_i^3 b}{4\pi} K_1(m_i, b)
\]

(4.7)

where \(K_1\) is the modified Bessel function of the second kind. The vertices \(g_i\), masses \(m_i\) as well as the values of the parameters \(\gamma\) and \(\Delta\) and the form of the hard amplitude \(A_H\) were taken from Ref. [1].

The numerical estimates, in which the semi-enhanced contribution (Eqn. (4.6)) was exclusively included, are given in parenthesis in Table 1. The semi-enhanced diagrams were neglected in [1]. Our present calculation, with only the semi-enhanced diagrams, serves only as a reminder that our calculation in Ref. [1] is incomplete. We expect, though, \(S^2(\text{semi-enhanced})\) to considerably reduce \(\langle |S^2_{\text{ch}}| \rangle\) (Good-Walker mechanism), especially at the LHC energy. Estimates which include both semi-enhanced and enhanced diagrams will be published soon [36].

In spite of the fact that we do not need to take the threshold effect into account, we estimate its influence on the value of the survival probability. We assume that we can use Pomeron exchange only for \(Y > y_0\), in this case the first enhanced diagram shown in Fig. 4 has the following contribution (see a detailed discussion in Refs. [33, 34])

\[
A(\text{Fig. 4}) = \frac{g^2 \gamma^2}{\Delta^2} e^{-2 \Delta(Y - y_0)} + O(\gamma e^{-\Delta Y}).
\]

(4.8)

Therefore, in order to introduce the threshold correction we need to change \(\gamma \to \gamma \exp(\Delta y_0)\), and multiply the Pomeron contribution by the step function \(\Theta(Y - y_0)\). The first observation means that in order to
maintain the quality of the experimental data description obtained without the threshold effect, we need to increase the value of $\gamma$ in our parametrization by a factor $\exp(\Delta y_0)$. We, then, need to take into account $\Theta(Y-y_0)$. Doing so we obtain the following values for $\langle |S_{enh}^2| \rangle$ at $y_0 = 0, 1.5, 2.3$: for the Tevatron energy we obtain values of 0.285, 0.7, 0.99 respectively, and for the LHC energy the corresponding values are 0.06, 0.12, 0.19 respectively. Our conclusion is that the threshold effect is not needed in our approach. Regardless, its inclusion does not change the fact that the value of $\langle |S_{enh}^2| \rangle$ decreases by a factor of 5-6 in going from the Tevatron to the LHC energy. It should be stressed that since $Y-y_1 \approx y_2 - 0 \approx 1/\Delta \approx 3$, the theoretical accuracy associated with this calculation is low.

5. Conclusions

In this letter we present a consistent theoretical approach that (i) provides a natural bridge both with a N=4 SYM Pomeron with $\alpha'_P = 0$, and with the pQCD approach; (ii) gives an analytical expression for summing the Pomeron interactions; (iii) leads to a very good ($\chi^2/d.o.f = 1$) description of all available data on strong interactions at high energy and (iv) predicts survival probability to be less than 1% at the LHC.

Within the framework of our approach, we still need to solve the problems of 1) accounting for the semi-enhanced diagrams summed together with the enhanced ones (see Ref. [36]); 2) to develop a theoretical approach for the Pomeron-hadron interactions; and 3) to formulate the Pomeron-Reggeon interaction on a more sound theoretical basis.

To illustrate our achievements and problems, we compare our approach with the work of the Durham group (KMR), whose results were presented in six papers recently published [8,9]. Both approaches consider $\alpha'_P$ as being small.

In both programs the Pomeron interaction was taken into account. The difference between the two approaches is that KMR made an ad hoc “reasonable” assumption, that the multi-Pomeron vertices have the following form, for the transition of $n$ Pomerons to $m$ Pomerons

$$ g_m^n = n m \lambda^{n+m-2} g_N/2 = n m \lambda^{n+m-3} g_3/2. \quad (5.1) $$

No theoretical arguments or theoretical models were offered in support of this assumption, which certainly contradicts the pQCD approach [17,18]. We view this assumption as problematic, since it eliminates the hope of building an approach that matches with pQCD. In paper 4 of Ref. [9], the system of two equations (Eq.(26) and Eq.(27) of this paper) has been suggested for the opacity $\Omega_P^P$, and it is stated that they follow from the parton model. Since there is no proof of this claim, and while we are inclined to believe the authors, that the equations could indeed be proved, we should still bear in mind, that the parton approach (see Refs. [13,14]) is based on the assumption that there is only a short range rapidity interaction between partons, while we, due to exchange of gluons in QCD, have a long range rapidity interaction.
KMR numerically solved Eq.(5.1) using an iteration method. The accuracy of the solution and its stability were not discussed. Since our approach is a particular case of the KMR approach with only the triple Pomeron interaction, we are surprised that the interaction method works, since our Pomeron Green’s function $G^P \propto \sum_n (-1)^n n! (\gamma \exp(\Delta Y))^n$. Such a series cannot not be reproduced by an iterative procedure. Unfortunately, KMR do not provide any details of their numerical procedure.

For single diffraction in the region of high mass, KMR encounter the same problem as we have in this paper, since they do not include the full set of semi-enhanced diagrams (an example of a neglected diagram is the second diagram of Fig. 2-b).

Although we consider the KMR approach to be much less theoretically reliable than ours, the numerical results regarding the elastic and total cross sections are very close. We consider this fact as confirmation that most properties of the high energy interaction are related to the Pomeron interaction, but not to the details of the interaction.

We disagree with KMR, regarding the predicted value of the survival probability for central exclusive diffractive Higgs production. In Ref. [8] the effect of enhanced and semi-enhanced diagrams have been neglected, with some supportive arguments of why it is reasonable approximation, however, in the last five papers [9] some of the diagrams have been included. The main difference is that KMR consider the threshold effect as being very important, and by choosing $Y > 2.3$, they obtain a suppression factor 0.83 for the Tevatron energy, and 0.67 for the LHC.

We have explained why in our approach, it is not necessary to include the threshold effect separately. We have shown that the threshold effect does not change the fact, that the value of the survival probability decreases by a larger factor in going from the Tevatron to the LHC energy. We are of the opinion that, KMR should, first, systematically introduce the Reggeon contributions, and only after that discuss the threshold effect, both in the calculation of the survival probability and in the scattering amplitude, especially for the case of single diffraction production.

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