Branching ratio of Type Ib/c supernovae into GRB-supernovae

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ABSTRACT

We study centered and decentered nucleation of black holes in core-collapse of massive stars in binaries. By Bekenstein’s gravitational-radiation recoil mechanism, a newly nucleated black hole typically leaves the central core prematurely. With low probability, the black hole remains centered and matures to a high-mass black hole which spins rapidly if the binary is compact. GRB030329/SN2003dh demonstrates that Type Ib/c supernovae are the parent population of long GRBs, whose branching ratio is $R = (2 - 4) \times 10^{-3}$. We identify $R$ with the low probability of centered nucleation in compact binaries. Decentered events are predicted to produce a single short burst in gravitational radiation. Centered events are predicted to produce a second, long-burst in gravitational radiation powered by a luminous black hole. These signatures are of interest to LIGO, VIRGO and TAMA.

1. Introduction

GRB030329/SN2003dh (Stanek et al. 2003; Hjorth et al. 2003) and GRB980425/SN1998bw (Galama et al. 1998) show that Type Ib/c supernovae are the parent population of long GRBs. Type Ib/c SNe are believed to represent core-collapse events of massive stars in compact binaries (Woosley 1993; Paczyński 1998; Brown et al. 2000; Bethe, Brown & Lee 2003). They are probably part of a continuous sequence adjacent to Type II SNe, ordered by increasing compactness of the binary in which the hydrogen (Ib/c) and the helium (Ic) envelope are removed in a common envelope phase (Nomoto, Iwamoto & Suzuki 1995; Turatto 2003). The remaining naked star rotates rapidly at the orbital period by tidal spin-up. As the inactive iron-core succumbs to its own weight and that of the surrounding He-envelope, a rotating black hole nucleates during core-collapse (Bethe, Brown & Lee 2003). Some of the binding energy liberated during gravitational collapse will be channeled to eject matter, producing an accompanying hydrogen (and helium) deficient Type Ib (Type Ic) supernova (MacFadyen 2003).

The branching ratio of Type Ib/c SNe to GRB-SNe can be calculated from the ratio $(1 - 2) \times 10^{-6}$ of observed GRBs-to-Type II supernovae (Porciani & Madau 2001), a beaming
factor of 450 (van Putten & Regimbau 2003) to 500 (Frail et al. 2001) and a rate of about 0.2 of Type Ib/c-to-Type II supernovae (Cappellaro, Barbon & Turatto 2003), giving

\[ R_{[\text{Ib/c} \to \text{GRB}]} = \frac{N(\text{GRB-SNe})}{N(\text{Type Ib/c})} \approx (2 - 4) \times 10^{-3}. \] (1)

This ratio is remarkably small, suggesting a higher-order down-selection process.

The small branching ratio (1) can be attributed to various factors in the process of creating GRBs in Type Ib/c supernovae (Podsiadlowski et al. 2004), e.g., not all baryon poor jets successfully punch through the remnant stellar envelope (MacFadyen & Woosley 1999), and not all massive progenitors making Type Ib/c supernovae nucleate rapidly rotating black holes. It is unlikely that either one of these down-selection processes by itself accounts for the smallness of \( R \). Rather, a combination of these might effectively contribute to a small branching ratio.

By tidal interaction with the companion star, the naked star is not spherical prior to collapse. Black holes nucleated in nonspherical collapse possess recoil by Bekenstein’s gravitational radiation recoil mechanism (Bekenstein 1973). Tidal deformation produces a systematic recoil velocity, which may combine with random multipole mass-moments to produce a distribution in recoil velocities. Some of the black holes will leave the central high-density core prematurely, before completion of the stellar collapse process. These events are decentered. Other holes will remain centered and surge into a high-mass object surrounded by a high-density accretion disk or torus. These events are centered. Centered black holes becomes luminous in a state of suspended accretion, if they rotate rapidly. They spin down against emissions in gravitational radiation and other radiation channels (van Putten & Levinson 2003). The latter includes a burst in high-energy radiation from torus winds which radiatively drives a supernova (van Putten et al. 2004), closely related to (Bethe, Brown & Lee 2003). Here, we quantify the various stages in the nucleation of black holes in stellar collapse. We favor an association with binaries (Nomoto, Iwamoto & Suzuki 1995; Turatto 2003) based on the Type II/Ib event SN1993J (Maund et al. 2004) and the proposed association of GRB-supernovae remnants with soft X-ray transients (Bethe, Brown & Lee 2003). We shall identify a branching ratio of core-collapse events producing centered nucleation of black holes with the probability of low kick velocities based on the Bekenstein recoil mechanism.

A related but different mechanism for explaining the small branching ratio based on kick velocities in core-collapse poses fragmentation into two or more objects (Davies et al. 2002). In this scenario, GRBs are associated with the formation of a fireball in the merger of binaries possessing small kick velocities. It is motivated, in part, in the search for delay
mechanisms in creating a GRB, after the onset of the supernova on the basis of X-ray line-emissions in GRB011211. However, X-ray line-emissions produced in radiatively powered supernovae allow the same time-of-onset of the GRB and the supernova, obviating the need for any delay mechanism (van Putten et al. 2004).

2. Centered Nucleation

Rotating black holes are described by Kerr (Kerr 1963). In core-collapse of massive stars, rotating black holes nucleate by accumulation of mass and angular momentum from infalling matter. The Kerr solution describes the constraint

$$J_H \leq GM^2/c$$

for a black hole of mass $M$ and angular momentum $J_H$, where $G$ is Newton’s constant and $c$ is the velocity of light. Table I summarizes the key quantities of Kerr black holes. Quite generally, initial collapse of a rotating core produces a torus (Rees, Ruffini & Wheeler 1974; Duez, Shapiro & Yo 2004), which initially satisfies $J_T > GM_T^2/c$. Thus, the nucleation of black holes takes place through a first-order phase-transition: a torus forms of increasing mass by accumulation of matter, diluting its angular momentum until it satisfies (2) and collapses into an extremal black hole. The alternative of a second-order phase transition which initially forms a sub-solar mass black hole, requires rapid shedding of excess angular momentum by gravitational radiation. However, limited mass-densities in core-collapse probably render this mechanism ineffective in competition with mixing on the free-fall timescale of the core. Nevertheless, gravitational radiation emitted from a non-axisymmetric torus prior to the nucleation of the black hole is potentially interesting (Rees, Ruffini & Wheeler 1974; Duez, Shapiro & Yo 2004).

Gravitational radiation in the formation of black holes through a first-order phase transition is important in non-spherical collapse, even when its energy emissions are small relative to the initial mass of the black hole. The Bekenstein gravitational radiation-recoil mechanism operates already in the presence of initial asphericities of about $10^{-3}$, producing a recoil of 300km/s or less. The radius of the accretion disk or torus around a newly formed stellar mass black hole is $R_T \sim 10^7$cm. A torus of a few tenths of a solar mass forms by accumulation of matter spiralling in, compressed by a factor $\sim (r/r_{ISCO})^4$ as it stalls against the angular momentum barrier outside the inner most stable circular orbit (ISCO). The time-of-collapse of stellar matter from a radius $r$ is about the free-fall timescale,

$$t_{ff} \approx 30 \text{ s } \left( \frac{M_{He}}{10M_\odot} \right)^{-1/2} \left( \frac{r}{10^{10}\text{cm}} \right)^{3/2},$$

(3)
where $M_{\text{He}}$ denotes the mass of the progenitor He-star. The newly formed low-mass black hole is typically kicked out of the central high-density region into surrounding lower-density regions before core-collapse is completed. It continues to grow off-center by accretion of relatively low-density matter – a high-density accretion disk never forms. With low but non-zero probability, the black hole has small recoil, remains centered and surges into a high mass black hole surrounded by a high-density torus.

After nucleation of the black hole, an accretion disk may form provided that the specific angular momentum $j_m$ of infalling matter exceeds that of the inner most stable circular orbit (ISCO),

$$j_m \geq l(a/M)GM/c$$

on the angular momentum $J_H$ of a black hole of mass $M$. Here, $l(a/M)$ denotes the dimensionless specific angular momentum of matter in circular orbits on the ISCO, where $a = J_H/M$ denotes the specific angular momentum of the black hole. Explicitly, we have (Bardeen 1970; Bardeen, Press & Teukolsky 1972)

$$l = \frac{2}{3\sqrt{3}} \left(1 + 2\sqrt{3}z - 2\right)$$

in terms of $z = r_{\text{ISCO}}/M = 3 + Z_2 - [(3 - Z_1)(3 + Z_1 + 2Z_2)]^{1/2}$ with $Z_1 = 1 + (1 - q)^{1/3} \left[(1 + q)^{1/3} + (1 - q)^{1/3}\right]$, $Z_2 = (3q^2 + Z_1^2)^{1/2}$ and $q = a/M$. Notice that $2\sqrt{3} \leq l \leq 2\sqrt{3}$, between an extremal black hole ($a = M$, $z = 1$) and a non-rotating black hole ($a = 0$, $z = 6$). The evolution of the newly nucleated black hole continues to be governed by angular momentum loss of the surrounding matter, until the inequality in (4) is reversed.

The black hole rapidly grows uninhibited, while the inequality (4) is reversed ($j_m < l(a/M)GM/c$). This surge continues until once again (4) holds. In dimensionless form, (4) becomes

$$l \left(\frac{\beta j(s)}{m^2(s)}\right) = \frac{k_1\beta s^2}{m(s)},$$

where

$$\beta = k_2 \frac{\omega c s_0}{G\rho_c} = 4.22k_2P_d^{-1}R_1^{-1}(M_{\text{He}}/10M_{\odot})^{-1/3}$$

in terms of the dimensionless integrals $j(s) = 4\pi \int_0^s \hat{\rho} s^4 ds$ and $m(s) = 4\pi \int_0^s \hat{\rho} s^2 ds$ of the normalized Lane-Emden density distribution with $\hat{\rho} = 1$ at the origin and the zero $\hat{\rho} = 0$ at $s_0 = 6.89685$. Here, $(k_1, k_2) = (1, 1)$ in cylindrical geometry for which $j_m = \omega r^2$, and $(k_1, k_2) = (5/3, 2/3)$ in spherical geometry for which $j_m = (2/3)\omega r^2$; $P_d$ denotes the binary period in days, $R_1$ denotes the radius in units of the solar radius $6.96 \times 10^{10}$ cm (Kippenhahn & Weigert 1990), and $M_{\text{He}}$ the mass of the progenitor He-star.

Fig. 1 shows the solutions as a function of dimensionless period $1/\beta$. The upper branch shows that rapidly spinning black holes plus accretion disk form in small-period binaries.
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(Bethe, Brown & Lee 2003; Lee, Brown & Wijers 2002), following a surge for periods beyond the bifurcation points

\[ \frac{a}{M} = 0.9541, \quad \frac{E_{\text{rot}}}{E_{\text{rot}}^{\text{max}}} = 0.6624, \quad \frac{M}{M_{\text{He}}} = 0.4051. \]  

\hspace{1cm} (7)

spherical geometry \( (\beta = 5.157) \):

\[ \frac{a}{M} = 0.7679, \quad \frac{E_{\text{rot}}}{E_{\text{rot}}^{\text{max}}} = 0.3220, \quad \frac{M}{M_{\text{He}}} = 0.3554. \]  

\hspace{1cm} (8)

The resulting mass and energy fractions as a function of \( 1/\beta \) are shown in Fig. 2. These two geometries serve to bound the range of values in more detailed calculations, e.g., through multi-dimensional numerical simulations.

The Bardeen trajectory corresponds to continuing accretion beyond surge, wherein matter remaining in the remnant envelope forms an accretion disk outside the ISCO. At this point, magnetohydrodynamical stresses within the disk as well as disk winds may drive continuing accretion. Accretion from the ISCO onto the black hole further increases the black hole mass and spin according to \( zM^2 = \text{const.} \) (Bardeen 1970), generally causing spin-up towards an extremal state of the black hole. In Fig. 1 this is indicated by accretion upwards beyond the upper ISCO-branch.

Radiative spin-down corresponds to a long-duration burst of gravitational radiation emitted by a non-axisymmetric torus (van Putten 1999; van Putten et al. 2004), described by a frequency and energy

\[ E_{gw} = (4 \times 10^{53} \text{ erg}) M_T\eta_{0.1} \left( \frac{E_{\text{rot}}}{E_{\text{rot}}^{\text{max}}} \right), \quad f_{gw} = (500 \text{ Hz}) M_T\eta_{0.1}, \]  

\hspace{1cm} (9)

where \( M_T = M/7M_\odot \), and \( \mu = M_T/0.03M \) and \( \eta = \Omega_T/\Omega_H \) denote the relative mass and angular velocity of the torus. This takes place if the torus is uniformly magnetized with the remnant magnetic field of the progenitor star. In Fig. 1, this radiative spin-down is indicated by a transition downwards from the upper ISCO branch to the branch on which the angular velocities of the black hole and of matter at the ISCO match (\( \Omega_H = \Omega_{\text{ISCO}} \) and \( \eta = 1 \)). This radiative transition lasts for the lifetime of rapid spin of the black hole – a dissipative timescale of tens of seconds (van Putten & Levinson 2003). Additional matter accreted is either blown off the torus in its winds, or accumulates and accretes onto the black hole after spin-down.

3. Branching ratio

In what follows, we shall consider a two-dimensional Gaussian distribution of kick velocities in the equatorial plane associated with the the tidal deformation of the progenitor
star by its companion, assuming a velocity dispersion $\sigma_{kick} \simeq 100 \text{km/s}$ in Bekenstein’s recoil mechanism.

The probability of centered nucleation during $t_{ff} \simeq 30 \text{s}$ is that of a kick velocity $v_{kick} < v^* = 10 \text{km/s}$, i.e.:

$$P_c = P(v_{kick} < v^*) \simeq 0.5\% \left( \frac{v^*}{10 \text{km/s}} \right)^2 \left( \frac{\sigma_{kick}}{100 \text{km/s}} \right)^{-2}. \quad (10)$$

While the numerical value has some uncertainties, the selection mechanism by gravitational radiation-recoil effectively creates a small probability of centered nucleation. We identify the branching ratio of Type Ib/c SNe into GRBs with the probability of centered nucleation,

$$R_{[\text{Ib/c} \rightarrow \text{GRB}]} = P_c \simeq 0.5\%, \quad (11)$$

effectively creating a small, higher-order branching ratio.

4. Single and double bursts in gravitational radiation

The proposed centered and decentered core-collapse events predicts a differentiation in gravitational wave-signatures. These signatures are of interest to the newly commissioned gravitational wave-detectors LIGO, VIRGO and TAMA, both as burst sources and through their collective contributions to the stochastic background in gravitational radiation (van Putten & Levinson 2003).

The black hole nucleation process is accompanied by a short burst in gravitational radiation, specifically in response to non-axisymmetric toroidal structures and fragmentation (Bekenstein 1973; Rees, Ruffini & Wheeler 1974; Davies et al. 2002; Popov 2004; Duez, Shapiro & Yo 2004). Its gravitational radiation signature depends on details of the hydrodynamical collapse. Centered nucleation is followed by a long burst in gravitational radiation.

A short burst in gravitational radiation is hereby common to all Type Ib/c supernovae. This may apply to Type II events as well. Type II events are possibly associated with low spin-rates and could represent delayed core-collapse via an intermediate “nucleon” star (SN1987A, e.g. Bethe, Brown & Lee (2003)). Their gravitational wave-emissions are thereby essentially limited to that produced by kick (if any) and collapse of this nucleon star. For a recent review of the short-duration ($<< 1 \text{s}$) bursts of gravitational waves in core-bounce, more closely related to Type II supernovae, see (Fryer, Holz & Hughes 2004) and references therein.

On account of (7-8), core-collapse compact can produce high-mass, rapidly spinning black holes in centered nucleation, whose rotational energy can reach about one-half the
maximal spin-energy of a Kerr black hole. In a suspended accretion state, these black holes spin-down in the process of emitting a long-duration burst of tens of seconds in gravitational waves (van Putten et al. 2004). This long duration burst comes as a second burst, after the short burst of gravitational radiation in centered nucleation.

To conclude, Type Ib/c supernovae produce a short, single burst of gravitational radiation at birth of a low-mass black hole. The sub-population of GRB-supernovae produce a subsequent long burst of gravitational radiation representing spin-down of the black hole. The second burst takes place after a quiescent or subluminous (Mineshige 2002) surge of the black hole into a high-mass object.

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Table 1: Trigonometric parametrization of a Kerr black hole. Here, $M$ denotes the mass of the black hole, $a = J_H/M$ denotes the specific angular momentum, $E_{\text{rot}}$ the rotational energy and $M_{\text{irr}}$ denotes the irreducible mass.

| Symbol | Expression | Comment |
|--------|------------|---------|
| $\lambda$ | $\sin \lambda = a/M$ | |
| $\Omega_H$ | $\tan(\lambda/2)/2M$ | |
| $J_H$ | $M^2 \sin \lambda$ | |
| $E_{\text{rot}}$ | $2M \sin^2(\lambda/4)$ | $\leq 0.29M$ |
| $M_{\text{irr}}$ | $M \cos(\lambda/2)$ | $\geq 0.71M$ |
Fig. 1.— Centered nucleation of black holes in core-collapse of a uniformly rotating massive star: accumulated specific angular momentum of the central object (arbitrary units) versus dimensionless orbital period $1/\beta$. Arrows indicate the evolution as a function of time. Kerr black holes exist inside the outer curve (diamonds). A black hole nucleates following the formation and collapse of a torus, producing a short burst in gravitational radiation. In centered nucleation, the black hole surges to a high-mass object by direct infall of matter with relatively low specific angular momentum, up to the inner continuous curve (ISCO). At this point, the black hole either spins up by continuing accretion or spins down radiatively against gravitational radiation emitted by a surrounding non-axisymmetric torus. In this state, the black hole creates a baryon poor jet along an open “ergotube” as input to GRB-afterglow emissions. This state proceeds until the angular velocity of the black hole equals that of the torus (dot-dashed line). These curves are computed for a Lane-Emden mass-distribution with polytropic index $n = 3$ in the limit of conservative collapse, neglecting energy and angular momentum loss in radiation and winds. Shown are the results in cylindrical geometry. This scenario fails in decentered collapse, where the newly nucleated black hole leaves the high-density core prematurely, prohibiting the formation of a high mass black hole surrounded by a high-density torus in a state of suspended accretion. The probability of centered nucleation defines the branching ratio of Type Ib/c supernovae into long GRBs.
Fig. 2.— The black hole mass $M$ and rotational energy $E_{\text{rot}}$ of formed after surge in centered nucleation, expressed relative to the mass $M_{\text{He}}$ of the progenitor He-star. The results are shown in cylindrical geometry (continuous) and spherical geometric (dashed). Note the broad distribution of high-mass black holes with large rotational energies of $5 - 10\%$ (spherical to cylindrical) of $M_{\text{He}}c^2$. 