Mode conversion using optical analogy of shortcut to adiabatic passage in engineered multimode waveguides

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Abstract: A shortcut to adiabatic mode conversion in multimode waveguides using optical analogy of stimulated Raman adiabatic passage is investigated. The design of mode converters using the shortcut scheme is discussed. Computer-generated planar holograms are used to mimic the shaped pulses used to speed up adiabatic passage in quantum systems based on the transitionless quantum driving algorithm. The mode coupling properties are analyzed using the coupled mode theory and beam propagation simulations. We show reduced device length using the shortcut scheme as compared to the common adiabatic scheme. Modal evolution in the shortened device indeed follows the adiabatic eigenmode exactly amid the violation of adiabatic criterion.

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1. Introduction

Mode-division multiplexing (MDM) is a promising technique where multiple optical modes are used as independent data channels to transmit optical data [1]. Several basic building blocks for MDM, such as the mode add/drop multiplexer [2] and the mode generator [3], have been proposed. Mode converter is another important building block for MDM systems. Two major routes to realize mode conversion in integrated optics devices are resonant coupling and adiabatic coupling. By designing the coupling region of a resonant mode converter to be half the beat length, light is converted from one mode to the other [4]. While resonant mode converter can be made very short; the difficulty is to determine the exact beat length due to device parameter variations from fabrication. Recently, due to the analogies between quantum mechanics and wave optics, light propagation in waveguide structures has been exploited as a tool to visualize or investigate many adiabatic coherent quantum phenomena which may otherwise be difficult to observe. In particular, coupled waveguide system has proven to be a useful tool for such realizations. Examples include optical analogy of stimulated Raman adiabatic passage (STIRAP) [5, 6], optical analogies of multi-level STIRAP [7], and etc [8]. Device applications include broadband beam splitter [9], directional coupler [10], and others. In these devices, light passage among coupled single mode waveguides resembles population transfer in quantum systems, and the waveguides are curved such that the coupling rates resemble that of the coupling pulses in STIRAP. However, due to restrictions of the planar lightwave circuit (PLC) technology, only adjacent waveguides can be efficiently coupled, and it is difficult to implement more elaborate coupling schemes with PLC waveguide arrays. Other interesting variations of STIRAP, such as STIRAP via a continuum [11, 12] and tripod STIRAP [13–15], involve more complex coupling schemes; to realize these analogies with coupling schemes other than nearest-neighbor coupling, single mode waveguides in 3D arrangement is needed [16, 17]. We have previously proposed adiabatic mode conversion/splitting devices using computer-generated planar holograms (CGPHs) in multimode waveguides based on STIRAP [18] and multistate STIRAP [19]; in which multiplexed long-period gratings with coupling coefficient variations along
the propagation direction are used to mimic the delayed laser pulses in STIRAP, and the light passage among the guided modes resembles population transfer in quantum systems. These devices are based on the PLC technology, but the multimode nature of the waveguides makes it possible to implement arbitrary coupling scheme among the guided modes.

In atomic and molecular physics, STIRAP refers to the adiabatic transfer of population between two energy levels in a three-level system via two delayed optical pulses [20], and the system is characterized by its robustness to pulse parameter variations. This is of particular importance to waveguide devices based on optical analogies of STIRAP. Such devices have good fabrication tolerance because they do not require a precise definition of coupling length and coupling strength due to the adiabatic nature. On the other hand, they need to be sufficiently long to satisfy the adiabatic condition. Otherwise, unwanted coupling among the adiabatic modes would deteriorate the device efficiency. However, long device length reduces device density and induces more transmission losses. It is desirable to combine the compactness of resonant devices and the robustness of adiabatic devices.

Efforts have been made to optimize the pulses used in STIRAP to minimize nonadiabatic coupling in the process and to speed up the passage [21, 22]. One interesting approach is the shortcut to adiabatic passage (SHAPE) [23] using the transitionless quantum driving algorithm [24]. For a system with a time-dependent Hamiltonian \( H_0(t) \) initially in an eigenstate \( |\Psi_0(0)\rangle \), the adiabatic theorem states that it will follow the same instantaneous eigenstate \( |\Psi_n(t)\rangle \) closely, as long as the evolution of the Hamiltonian is slow enough [25]. SHAPE is based on the reverse engineering approach that one can always find Hamiltonians \( H(\tau) \), associated with any chosen \( H_0(\tau) \), that drive the instantaneous eigenstates \( |\Psi_n(t)\rangle \) exactly. The Hamiltonian \( H(t) \) can be found as [23, 24]

\[
H(t) = H_0(t) + H_1(t), \tag{1}
\]

where

\[
H_1(t) = i\hbar \sum_n \left( \frac{\partial}{\partial t} |\Psi_n(t)\rangle \langle \Psi_n(t)| \right). \tag{2}
\]

Of course, prior knowledge of the instantaneous eigenstates is required to implement the SHAPE algorithm. Due to the potential complexity of \( H(t) \), elaborating coupling schemes might be required to realize optical analogies of such systems. In this paper, we propose an optical realization of the SHAPE scheme using CGPH in multimode waveguides. The designed mode converter could find applications in MDM systems.

2. Shortcut to adiabatic passage in multimode waveguides

In a step-index multimode waveguide supporting \( N \geq 3 \) forward-propagating modes, we consider three distinct modes \( |\Psi_1\rangle \), \( |\Psi_2\rangle \), and \( |\Psi_3\rangle \), coupled by a CGPH. The CGPHs are multiplexed long-period gratings which couple the guided modes depending on the grating shape and periodicity [26]. We can design the CGPH to mimic the STIRAP process, with modes \( |\Psi_1\rangle \) and \( |\Psi_2\rangle \) and modes \( |\Psi_2\rangle \) and \( |\Psi_3\rangle \) coupled by gratings \( \Lambda_{12} \) and \( \Lambda_{23} \) with coupling coefficients \( \kappa_{12} \) and \( \kappa_{23} \). Figure 1 shows the level scheme of STIRAP, the schematic of a CGPH loaded multimode waveguide, and amplitude profiles of the three guided modes. When only \( \Lambda_{12} \) and \( \Lambda_{23} \) are present, the evolution of mode amplitudes \( A_i (i = 1, 2, 3) \) obey the coupled mode equations

\[
\frac{d}{dz} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = \begin{bmatrix} 0 & \kappa_{12}(z) & 0 \\ \kappa_{21}(z) & 0 & \kappa_{23}(z) \\ 0 & \kappa_{32}(z) & 0 \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} = H_0(z) \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix}. \tag{3}
\]

Replacing the spatial variation \( z \) with the temporal variation \( \tau \), (3) is used to describe the probability amplitudes of a three-level atomic system driven by two laser pulses shown in Fig. 1(a)
using the Schrödinger equation ($\hbar = 1$) under the rotating-wave approximation, in which $A$ represents the probability amplitudes of the states being populated, $\kappa_{mn}$ is the Rabi frequency of the pulse coupling states $|\Psi_m\rangle$ and $|\Psi_n\rangle$. Solving for the eigenmodes $|\Psi_D\rangle$, $|\Psi_+\rangle$, and $|\Psi_-\rangle$, we can find a dark eigenmode as

$$|\Psi_D\rangle = \frac{1}{\sqrt{\kappa_{23}^2 + \kappa_{12}^2}} (\kappa_{23} |\Psi_1\rangle - \kappa_{12} |\Psi_3\rangle).$$

When the two spatially variable coupling coefficients $\kappa_{12}(z)$ and $\kappa_{23}(z)$ are applied in a counterintuitive scheme to mimic the laser pulses in STIRAP, $|\Psi_D\rangle$ can be used to convert $|\Psi_1\rangle$ to $|\Psi_3\rangle$. If the device length is not sufficiently long, unwanted couplings happen among $|\Psi_D\rangle$, $|\Psi_+\rangle$, and $|\Psi_-\rangle$, resulting in low conversion efficiency.

With knowledge of the eigenmodes, we can use the SHAPE scheme to obtain a new coupling matrix, such that the system will follow $|\Psi_D\rangle$ exactly. Replace $t$ with $z$ in (2), let $\hbar = 1$, and substitute $|\Psi_D\rangle$, $|\Psi_+\rangle$, and $|\Psi_-\rangle$ in to (2), we obtain

$$H_1(z) = \begin{bmatrix} 0 & 0 & i\kappa_{13}(z) \\ 0 & 0 & 0 \\ -i\kappa_{13}(z) & 0 & 0 \end{bmatrix},$$

with

$$\kappa_{13}(z) = \frac{\kappa_{12}(z)\kappa_{23}(z) - \kappa_{23}(z)\kappa_{12}(z)}{\kappa_{12}(z)^2 + \kappa_{23}(z)^2}.$$  

The shortcut here is thus to add a grating $\Lambda_{13}$ coupling $|\Psi_1\rangle$ and $|\Psi_3\rangle$ to the original CGPH implementing the STIRAP scheme $H_0(z)$ in (3) as shown in Fig. 1(b). We also note that grating $\Lambda_{13}$ is 90° out of phase with gratings $\Lambda_{12}$ and $\Lambda_{23}$ due to the presence of $i$ in (5). To implement $H(z) = H_0(z) + H_1(z)$ in a coupled PLC waveguide system would be difficult, because the waveguides representing $|\Psi_1\rangle$ and $|\Psi_3\rangle$ are not adjacent to each other. In a multimode waveguide, by designing a CGPH to implement $H(z)$ with the addition of a grating $\Lambda_{13}$ coupling modes $|\Psi_1\rangle$ and $|\Psi_3\rangle$ using the coupling coefficient in (6), we can realize optical analogy of the SHAPE scheme as shown in Fig 1(b). In the following, we use a numerical example to demonstrate SHAPE in an engineered multimode waveguide and compare it with STIRAP in the same system.
3. Numerical results

We consider a 3 \( \mu m \) wide, five-moded polymer waveguide similar to the one in [19] for mode conversion from \( |\Psi_1\rangle \) to \( |\Psi_3\rangle \) via \( |\Psi_2\rangle \). The CGPHs used to implement the SHAPE and STIRAP schemes are designed at the zero-detuning wavelength \( \lambda_0=1.55 \mu m \) and the TE polarization. The maximum effective index modulation is assumed to be \( \Delta n=0 \).

Figure 2 shows Gaussian shaped coupling coefficients \( \kappa_{12}(z)=k_{12}\exp\left[-(z-z_0/2)^2/c^2\right] \) and \( \kappa_{23}(z)=k_{23}\exp\left[-(z+z_0/2)^2/c^2\right] \), with \( k_{12} \) and \( k_{23} \) directly proportional to \( \Delta n \), chosen arbitrarily to mimic the counterintuitive optical pulses used in STIRAP. The parameter \( c \) is chosen to be the same as the delay \( z_0 \) to minimize nonadiabatic coupling [20], and we define it to be a function of the total device length \( L \) (mm) as \( c = z_0 = 3L/20 \). To implement SHAPE, the coupling coefficient \( \kappa_{13}(z) \) corresponding to the additional \( \Lambda_{13} \) is calculated using (6) and also shown in Fig. 2.

First, we demonstrate SHAPE (with the addition of \( H_1(z) \)) in a mode converter at 5 mm length using the beam propagation method (BPM), which solves the wave equation governing light propagation in the CGPH loaded multimode using a finite difference scheme. We design a CGPH using the method outlined in [26] to implement the coupling matrix \( H(z)=H_0(z)+H_1(z) \). Figure 3 shows the calculated CGPH pattern. The CGPH pattern is used as an effective index perturbation to the multimode waveguide. Figure 4 shows the calculated beam propagation using mode \( |\Psi_1\rangle \) as the input from the left hand side. According to the SHAPE scheme, \( |\Psi_1\rangle \) is converted to \( |\Psi_3\rangle \) at the output on the right hand side in a 5 mm device. As a comparison, the beam propagation in a 5 mm STIRAP mode converter designed according to \( H_0(z) \) in (3) is shown in Fig. 5. Complex mode coupling is evident, and the conversion fails at such a short distance. In Fig. 6(a), the corresponding modal power evolution along the propagation distance for different modes is shown for the SHAPE case. As shown in (4), when the system evolution follows the dark eigenmode \( |\Psi_D\rangle \), no component of \( |\Psi_2\rangle \) is excited. From Fig. 6(a), it is clear that \( |\Psi_2\rangle \) is not involved in the conversion, and the modal power evolution indeed follows what is expected from the STIRAP process when the adiabatic criterion is satisfied, but at a much shorter device length. The smooth conversion curve shows the good device length tolerance which is characteristic of adiabatic devices. For comparison, the modal power evolution for the STIRAP case is shown in Fig. 6(b). Coupling occurs among the eigenmodes due to the breakdown of adiabaticity, resulting in the excitation of \( |\Psi_2\rangle \) and lowered conversion efficiency at the output. Clearly, the phenomenon of SHAPE can be observed in an engineered...
multimode waveguide using BPM, and it provides a shortcut to the STIRAP scheme.

![Fig. 3. Calculated CGPH pattern to implement the SHAPE scheme for mode conversion. Dashed lines indicate the waveguide core.](image)

Next, we calculate the shortest converter length using both STIRAP and SHAPE to evaluate the effectiveness of the shortcut algorithm. The STIRAP case is considered first. We numerically solve (3) using $\kappa_{12}$ and $\kappa_{23}$ defined in Fig. 2 and plot the normalized modal power at the output of the converter as a function of the device length in Fig. 7(a). It is clear that as the device length decreases, adiabaticity breaks down, $\ket{\Psi_2}$ is excited, and the conversion efficiency deteriorates due to coupling among the eigenmodes. The excitation of $\ket{\Psi_2}$ indicates that the mode converter no longer follows the adiabatic pathway described in (4). To obtain a conversion efficiency $\geq 99\%$ using STIRAP, the minimum device length is 11.4 mm in this numerical example. Next, we consider the SHAPE case. We note that the maximum value of coupling coefficient corresponding to $\Delta n = 0.003$ is 1.594 mm$^{-1}$ for this polymer waveguide platform in our numerical example. So, we cap the maxima of $\kappa_{13}(z)$ at 1.594 mm$^{-1}$ in our simulation to account for physical realizability in fabrication and to avoid additional scattering loss resulting from large effective index modulation. In Fig. 7(b), we show the normalized modal power at the output of the converter as a function of the device length for SHAPE. Complete conversion can
be observed at shorter lengths then STIRAP because the system follows $|\Psi_D\rangle$ exactly without the excitation of $|\Psi_2\rangle$. For the same 99% conversion efficiency using SHAPE, the device length can be reduced to 3.9 mm in this numerical example, corresponding to a 65% reduction in device length as compared to STIRAP. As evidenced by the excitation of $|\Psi_2\rangle$, coupling among eigenmodes occurs at lengths below 3.9 mm because of the limitation we put on the maximum value of $\kappa_{13}$; otherwise, complete conversion can be achieved at arbitrarily shorter length.

4. Conclusion

In conclusion, an optical realization of a shortcut to the STIRAP scheme is proposed. The CGPH loaded multimode waveguide platform provides a viable tool for optical realization of adiabatic coherent quantum phenomena with coupling schemes other than nearest-neighbor coupling. The SHAPE scheme, originally developed to speed up adiabatic passage in laser coupled quantum systems, can be readily applied to the design of integrated optical mode converters. Shortened mode converter length has been confirmed by numerical solution of the coupled mode equations and by beam propagation simulations in an engineered multimode waveguide.
Fig. 7. Normalized modal power at the mode converter output for different device lengths. $|\Psi_1\rangle$ is used as the input. (a) STIRAP (b) SHAPE.

Modal power evolution in the shortened device indeed follows the dark eigenmode of STIRAP exactly amid the violation of adiabatic criterion.

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