Value-Added Service Investment and Pricing Strategies of a Multilateral Distribution Platform Considering User-Homing in a Duopoly

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ABSTRACT In reality, there are many multilateral distribution platforms that compete with each other. There are differences in user-homing: some users belong to only one platform, while some belong to multiple platforms. In addition, to attract more consumers, these platforms often invest in value-added services (VASs). In this paper, combining utility theory and Hotelling model, we investigate the VAS investment and pricing strategies of a multilateral distribution platform with considerations of user-homing and cross-group network externalities in a duopoly. We compare two universal situations: situation $S$ (all consumers, advertisers and delivery staff are single-homing) and situation $M$ (the consumers are single-homing while the advertisers and the delivery staff are partial multi-homing). The main results are as follows: 1) the optimal transaction fee under situation $M$ is higher than that under situation $S$; 2) the optimal delivery fee and advertising fee under situation $S$ are higher than those under situation $M$; 3) there is a threshold of the marginal VAS investment cost in both two situations, below which no VAS investing is the optimal choice; and 4) the optimal profits under situation $M$ are higher than those under situation $S$.

INDEX TERMS Multilateral distribution platform, value-added service investment, pricing strategy, user-homing, Hotelling model.

I. INTRODUCTION

In recent years, with the rapid development of the Internet and the platform economy, various platforms have mushroomed in the market. These platforms can be divided into bilateral platforms and multilateral platforms according to the number of participants. A bilateral platform, such as eBay, Xbox360, etc., is an intermediate platform that is required for two groups of participants in the market to conduct transactions. The income of one side of participants on the platform depends on the number of participants on the other side of the platform, and the platform provides services through proper pricing strategies that enable them to successfully trade on the platform [1]. A multilateral platform is a platform that brings together three or more distinct but interdependent user groups [2]. For example, “Uber eats” is a multilateral platform, and its users include consumers, “Uber” drivers (delivery staff) and advertisers.

The similarities between bilateral platforms and multilateral platforms are as follows: (1) they are trading venues, (2) these platforms themselves do not produce products, and (3) they can facilitate transactions between user groups by charging appropriate fees or earning the difference. The differences are as follows: (1) because there are multiple user groups interacting with each other, the external network effects among user groups are more diverse [3]; (2) in terms of a multilateral platform’s design, it is necessary to take care of each user group’s interests and the overall platform interests since there are many trade-offs; and (3) for a multilateral platform, the pricing structure is more complicated.

In practice, as a typical multilateral platform, multilateral distribution platform exists widely, but the relevant academic studies on pricing strategies of them are not sufficient. Therefore, pricing strategies of multilateral distribution platforms deserve attention [4]. Especially in the competitive market environment, bad or blind pricing strategy (e.g., vicious competition, etc.) will cause huge losses to enterprises. For example, in 1977, Amazon (http://www.amazon.com) carried out crazy price war relying on its stable financing channels.

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after listing in America, and even lost 5 to 10 dollars per online transaction, completely ignoring the product’s use value, which directly led to the continuous loss for 7 years, with a total loss of about 3 billion dollars.

To date, pricing for the multilateral distribution platform is based on each independent user group (e.g., cost-based pricing) and ignores the interactions among user groups, which make the pricing strategy somewhat unreasonable. Therefore, how to obtain pricing strategies for such a distribution platform by establishing a pricing model that fully reflects the network externalities among user groups is the first issue that we have to solve.

Meanwhile, competition among these platforms is also becoming fiercer. To attract and stabilize user scale, these platforms not only adopt price strategies, such as free registration fee and price subsidy, but also adopt non-price strategies, such as value-added services (VASs), to improve users’ stickiness to the platform. Providing VASs has become one of the important strategies for e-commerce platform to improve the competitive advantage of enterprises [5]. For example, “Eleme” (https://www.ele.me/home/), as a catering distribution platform that connects consumers, delivery staff and advertisers, has introduced attractive and interesting modules, such as “playing games to snatch a red envelope”, “I want to borrow” and so on, to attract more consumers. These additional modules are called VASs. Investing and developing VASs can facilitate the implementation of an agreement or provide benefits that are independent of the buyer-seller relationship [6], while simultaneously generating VAS costs, such as technical staff compensation, server resource occupancy, module maintenance cost and so on. If the VAS investment has little effect but the costs are huge, it will reduce the platform’s profits and even bankrupt the platform. How to balance investment costs and investment returns and develop an appropriate VAS investing strategy is the second issue that we have to handle.

In this paper, we investigate the VAS investment and pricing strategies of a multilateral distribution platform with consideration of user-homing and cross-group network externalities in a duopoly. “User-homing” can be explained as follows. Assume that there are multiple platforms providing similar but not identical services in a competitive market. If a user can only join one of the platforms, the user is called single-homing; if a user can join multiple platforms simultaneously, the user is called multi-homing (For example, one can hold credit cards from more than one bank at a time); if a user group of the platform consists of both users who only belong to the platform self and users who are from other platforms, the user group is called partial multi-homing [7] [8]. We compare two universal situations: situation S (all consumers, advertisers and delivery staff are single-homing) and situation M (the consumers are single-homing while the advertisers and the delivery staff are partial multi-homing). The specific difference between models in the situation S and the situation M is: in the situation S, the two platforms have their own delivery staff and advertisers, while in the situation M, some delivery staff and advertisers are shared by the two platforms.

At present, the existing multilateral platform pricing methods include dynamic pricing (suitable for platform pricing with peak periods) [9], Stackelberg pricing (suitable for platform pricing with master-slave relationship) [10], etc. Obviously, above methods are not applicable to the research object of this paper, i.e., two multilateral distribution platforms with equal status in the duopoly market. However, the Hotelling model can concisely and intuitively reflect the competition between two similar platforms. Therefore, first, by combining utility theory and Hotelling model, optimization models that maximize the platform’s profits in different situations are formulated. Subsequently, optimal pricing and VAS investment strategies are obtained by solving the optimization models. Finally, numerical analyses are carried out to investigate the influence of the relevant parameters on the optimal strategies and the maximum platform profits in the two situations, and derive managerial insights for decision makers.

The major contributions of this paper are as follows. (1) Our study fills the research gap on multilateral distribution platform pricing. We extend the bilateral platform pricing model to the multilateral platform pricing model by combining utility theory and Hotelling model; (2) By introducing VAS investment decision-making problem into the pricing model of multilateral distribution platform, we derive the optimal VAS investment amount for the platform, which effectively guides the VAS investment decisions; (3) We compare the optimal decisions and profits in two universal situations (i.e., situation S and situation M), and obtain the impacts of relevant parameters on the optimal decisions and profits in the two situations, which provides decision guidance for platforms to maximize their profits.

The rest of the paper is organized as follows. Section 2 reviews the relevant literature. Section 3 defines the notations and describes the problem. Section 4 establishes the multilateral platform model. Section 5 obtains the optimal pricing and VAS investment strategies by solving the model. Section 6 conducts the numerical analyses of specific examples. Finally, Section 7 contains some conclusions and suggested future work. All proofs are relegated to the Appendix.

II. LITERATURE REVIEW

Our work is related to two bodies of literature. The first body of literature studies the pricing strategies for a platform. The second body focuses on VAS investment strategies of a platform. We discuss them separately below.

A. PRICING STRATEGIES FOR A PLATFORM

Many scholars have studied the pricing of platforms from the perspective of bilateral markets. Armstrong (2006) [8] first proposed the concept of the bilateral market and studied the pricing strategy of bilateral platforms. He found that the attributes of the consumers, the platform’s fee structure and the network externalities impact the pricing strategies of bilateral platforms. Hagiu and Halaburda [11] studied the effect...
of different levels of information on a two-sided platform’s profits. Dietl et al. [12] investigated the advertising pricing models of a bilateral platform under different charging regulations. Chao and Derdenger [13] analysed the impact of mixed bundling in two-sided markets with base effects on pricing, social welfare, and the platform’s profits, and validated their theoretical predictions using empirical methods. Hao et al. [14] studied the issue of profit distribution and advertising pricing for APP (i.e., Application) developers and a mobile platform with a split structure. Lin and Wu [15] developed a model of the cross-platform and within-platform competition of sharing economy platforms with two-sided networks as infomediaries and studied the pricing strategies of the platforms under the conditions of single user ownership and multiple user ownership. Gal-Or et al. (2018) [16] established a two-stage game model of competitive advertising platforms based on bilateral market theory and studied the effect of user privacy concerns on the competition between online advertising platforms; Cachon et al. (2017) [9] studied the dynamic pricing of services platforms, and they found the optimal contract substantially increases the platform’s profit relative to contracts that have a fixed price or fixed wage (or both), and although surge pricing is not optimal, it generally achieves nearly the optimal profit.

The above studies comprehensively analyse the pricing of bilateral markets and provide references for this paper. However, few studies have extended bilateral market theory to multilateral platforms and their related studies. For example, Seamans and Zhu [17] empirically studied the impacts of multilateral platform entry strategies on the pricing and profits of the platform. Hagiu and Wright [2] redefined the multilateral market, studied the pricing problems under the two different business models, the multilateral market and enterprise vertical integration, and gave the conditions for the choice of the business model. Guijarro et al. [18] studied the pricing of a multilateral platform that consists of users, APP developers and web content providers, and analysed the impacts of the relevant parameters on the optimal pricing, social welfare and profits of the platform; Evans and Schmalensee [19] extended bilateral market to multilateral market, and they argued that the basic features of multilateral market are that there are two or more different user groups, there are externalities among groups, and the externalities among groups cannot be internalized. In addition, bilateral market is a special case of multilateral market. Jullien [10] studied multilateral market competition, platform compatibility and discriminatory pricing with the Stackelberg model, and found that leader platforms prefer compatibility measures to weaken market price competition.

Along with above related studies, many scholars put their eyes on distribution platforms, which mainly focus on the platform design, the distribution route selection and the logistics mode, etc. For example, Arslan et al. [20] studied a peer-to-peer platform that automatically creates matches between parcel delivery tasks and ad hoc drivers and operates a fleet of backup vehicles to serve the tasks that cannot be served by the ad hoc drivers, and Bai et al. [21] considered an on-demand service platform that leverages revenue-sensitive independent providers to serve their time- and price-sensitive customers. Their results show that as the number of providers increases and the rate of the increase in customer demand grows, the platform should lower the pay-out rate; Kung and Zhong [4] conducted a comparative analysis of a bilateral distribution platform based on three pricing strategies, membership-based pricing, transaction-based pricing, and cross subsidization, and obtained the optimal pricing of the platform under different pricing strategies.

However, there are relatively few studies on the pricing of multilateral distribution platform. The same as two-sided platforms, multilateral platform exhibit cross-market network externalities [22] [23], which lead to correlation and mutual influence of any two sides, and the strategy for one side has a great influence on the strategy for the other [24] [25]. Therefore, this paper studies the pricing strategies for multilateral distribution platform, which is different from existing studies.

B. VAS INVESTMENT STRATEGIES OF A PLATFORM

In terms of VAS investments in platforms, Dou et al. [5] studied the investment strategy and pricing strategies of bilateral platforms with unilateral VAS investments and found that the invested user side will always be priced higher, and the price increment for the user side without investment could be larger than that for the invested user side if both sides are priced higher after the investment. Subsequently, Dou and He (2017) [26] considered the situation where the investment resources are limited and investigated the VAS investment and pricing strategies of a two-sided platform that can simultaneously attract the users on both sides through VAS investments. Anderson et al. [27] examined the bilateral platform performance investment and content investment issues with network externalities and gave the conditions for adopting different portfolio strategies; Considering the resource constraint, Dou et al. [28] studied how to make VAS investment strategy considering the negative intra-group network externality on the seller side from the perspective of a two-sided platform.

Based on the above studies, we can find that studies on the VAS investment strategy of distribution platforms in a duopoly are even rare, and almost all of them focus on bilateral platforms (e.g., Dou et al. [5]). However, most of the distribution platforms in reality are multilateral platforms, and making VAS investment strategies for them is more complex in the competitive market environment. Studies on VAS investment strategy for the multilateral distribution platform are of great significance.

Our paper makes a combination of exploring VAS investment strategies and pricing strategies for multilateral distribution platforms in a duopoly. We consider not only the user-homing of each user group, but also the cross-group network externalities from the perspective of multilateral markets. Besides, the prices for both sides and VAS investment amount are taken as decision variables. These considerations
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TABLE 1. Notations.

| Notation | Description |
|----------|-------------|
| $i$      | Platform $i$ ($i = 1, 2$). |
| $j$      | Situation $j$ ($j = S, M$). |
| $u_{ci}$ | The gross utility of a consumer on platform $i$. |
| $u_{ai}$ | The gross utility of an advertiser on platform $i$. |
| $u_{si}$ | The gross utility of a delivery staff person on platform $i$. |
| $n_{ci}$ | The demand of a consumer single-homing platform $i$ ($0 \leq n_{ci} \leq 1$). |
| $n_{ai}$ | The demand of an advertiser single-homing platform $i$ ($0 \leq n_{ai} \leq 1$). |
| $n_{si}$ | The demand of a delivery staff person single-homing platform $i$ ($0 \leq n_{si} \leq 1$). |
| $N_{ci}$ | The demand of a delivery staff person on platform $i$ ($0 \leq N_{ci} \leq 1$). |
| $N_{ai}$ | The demand of advertisers on platform $i$ ($0 \leq N_{ai} \leq 1$). |
| $\alpha_c$ | Delivery staff’s external network strength to consumers ($0 \leq \alpha_c \leq 1$). |
| $\alpha_m$ | Advertisers’ external network strength to consumers ($0 \leq \alpha_m \leq 1$). |
| $\alpha_s$ | Consumers’ external network strength to delivery staff ($0 \leq \alpha_s \leq 1$). |
| $\alpha_a$ | Consumers’ network external network strength to advertisers ($0 \leq \alpha_a \leq 1$). |
| $p_{ci}$ | Consumer’s transaction fee on platform $i$ (decision variable). |
| $p_{ai}$ | The delivery fee on platform $i$ (decision variable). |
| $x_i$ | The amount of value-added service (VAS) investments on platform $i$ (decision variable, $0 \leq x_i \leq 1$). |
| $v$ | The amount of platform’s fundamental investment ($0 \leq v \leq 1$). |
| $\beta$ | VAS investment coefficient ($0 \leq \beta \leq 1$), i.e., each unit of VAS investment contributes utility $\beta$ to consumers. |
| $\theta$ | Perceived value coefficient of consumers’ fundamental investments (uniformly distributed over the interval $[0, 1]$), i.e., the intrinsic usage benefit of each consumer from using the fundamental service of a platform. |
| $f$ | The distance from a delivery staff person to platform 1 (uniformly distributed over the interval $[0, 1]$). |
| $\eta$ | The distance from an advertiser to platform 1 (uniformly distributed over the interval $[0, 1]$). |
| $l_a$ | The advertising costs (or transport costs) per distance for the advertisers. |
| $l_s$ | The delivery costs (or transport costs) per distance for the delivery staff. |
| $c$ | Marginal VAS investment costs ($0 \leq c \leq 1$), i.e., the investment costs brought by each unit of VAS investment. |
| $\pi_i$ | The profits of the platform $i$. |
| $p_{cij}$ | The optimal consumer’s transaction fee in situation $j$. |
| $p_{aij}$ | The optimal delivery fee in situation $j$. |
| $p_{aj}$ | The optimal advertising fee in situation $j$. |
| $x_i^*$ | The optimal VAS investment amount in situation $j$. |
| $\pi_j^*$ | The optimal platform profits in situation $j$. |

together enable this work to differentiate itself from the existing research.

III. NOTATIONS AND THE PROBLEM
Notations are defined and explained as Table 1.

There are two multilateral distribution platforms, i.e., platform 1 and platform 2, in a duopoly market. Both the multilateral distribution platforms connect consumers, advertisers and delivery staff. The cross-group network externalities exist in the two platforms, i.e., the utility of one user group that is obtained from a platform is related to the number of users of the other user group on the platform. For consumers, the more delivery staff that join a platform, the higher the delivery service quality and efficiency will be, and the greater the consumer utility that will be obtained, i.e., the delivery staff have a positive cross-group external network effect on consumers. Advertisers with advertisements on the distribution platform, such as advertising bars or ad windows that pop up onto the interface, can irritate the consumers, i.e., advertisers have a negative cross-group external network effects on consumers. For advertisers, the more consumers that a platform has, the more attractive the platform will be to advertisers, i.e., consumers have a positive cross-group external network effect on advertisers. For delivery staff, the more consumers that a platform has, the more delivery orders that they will receive, which will bring more profits to them, i.e., consumers have a positive cross-group external network effect on delivery staff. The interactions among the three user groups are shown in Fig. 1. To widen user participation and earn more profits, multilateral distribution platforms tend to invest in VASs for users. However, investing in VASs will increase the costs. Moreover, if the investment costs are too large but the benefits are too small, the investment may decrease the platform’s profits and even lead to bankruptcy. Hence, how to develop the optimal VAS investment strategy for multilateral distribution platforms based on the practice and the optimal prices for each respective participant is the problem that we will solve through constructing and solving the model.
Before formulating the model, the following assumptions are made.

**Assumption 1:** The cross-group external network strength of the delivery staff to the consumers is greater than that of the consumers to the delivery staff ($\alpha_c > \alpha_s$).

**Assumption 2:** The cross-group external network strength of the consumers to the advertisers is greater than that of the advertisers to the consumers ($\alpha_a > \alpha_m$).

### IV. THE MODEL

In this paper, we study two multilateral distribution platforms connecting consumers, advertisers and delivery staff in a duopoly (e.g., “Eleme” and “Meituan”), and establish profit-maximizing models for two universal situations in practice: (1) all three users are single-homing (i.e., situation $S$), and (2) the consumers are single-homing while the advertisers and the delivery staff are partial multi-homing (i.e., situation $M$).

We give the theoretical bases of our model setting from two perspectives below: utility theory and Hotelling model.

**Utility Theory:** The demand of each user in each platform can be calculated through the utility function, because utility represents the satisfaction degree of users from the given consumption, and it is the standard for users to judge whether they participate in the transaction or not. For example, in a competitive environment, if the user gets more utility from they participate in the transaction or not. For example, in a consumption, and it is the standard for users to judge whether it can be calculated through the utility function, because utility.

**Hotelling Model:** The Hotelling model is a concise and intuitive model of competition between two companies, also known as the Hotelling line model [29]. Assume that platform 1 and platform 2 are located at the two endpoints of the Hotelling line, and their users (including consumers, delivery staff and advertisers) are uniformly distributed along the Hotelling line, as shown in Fig. 2. Both of the two platforms face users distributing in the Hotelling line, i.e., the total market demand of the two platforms is the whole Hotelling line. The distance from a user to an endpoint of the Hotelling line can represent either the physical distance from the user to the platform or the psychological distance (i.e., the degree of preference or loyalty of the user to the platform). A user spends a certain amount of transport cost per unit distance on the Hotelling line, therefore, it costs less to transact to the platform that is closer to the user. When the sales strategies of the two platforms are the same, both of the two platforms occupy their user market share from the midpoint of the Hotelling line to their own endpoints.

### A. THE MODEL UNDER SITUATION $S$

Under situation $S$, all consumers, advertisers and delivery staff are single-homing. Platform $i$ charges $p_{ci}$ and $p_{ai}$ to the consumers and advertisers, respectively, and pays $p_{si}$ to the delivery staff. The demands of the consumers, advertisers and delivery staff on platform $i$ are $n_{ci}$, $n_{ai}$ and $n_{si}$, respectively. In addition, the amount that platform $i$ invests in VAS is $x_i$, and the costs of conducting the VAS investment are $\frac{cx_i^2}{2}$ [30]. Therefore, the profits of the distribution platform $i$ are

$$\pi_i = p_{ci}n_{ci} + p_{ai}n_{ai} - p_{si}n_{si} - \frac{cx_i^2}{2}. \tag{1}$$

The perceived utility coefficient of consumers joining platform 1 is $\beta$, and the amount of fundamental investment of this platform is $v$. According to the Hotelling model, the perceived utility of consumers joining the platform 1 is $\beta v$, while the perceived utility of consumers joining the platform 2 is $(1 - \beta) v$. Let $\beta$ denote the VAS investment coefficient, which represents the increment of consumers’ utility per unit of VAS investment, and, therefore, the consumers’ utility from the VAS investment of platform $i$ is $\beta x_i$. The consumers’ utility when delivery staff join platform $i$ is $\alpha_n n_{si}$, and the negative utility of the consumers from the advertisers is $\alpha_m n_{ai}$.

The gross utilities associated with the consumers on platform 1 and platform 2 are respectively

$$u_{c1} = \theta v + \alpha_c n_{c1} + \beta x_1 - p_{c1} - \alpha_m n_{a1}, \quad \text{and} \quad \tag{2}$$

$$u_{c2} = (1 - \theta) v + \alpha_c n_{c2} + \beta x_2 - p_{c2} - \alpha_m n_{a2}. \quad \tag{3}$$

A delivery staff person provides delivery services for consumers on platform $i$, and then he will receive $p_{si}$ in return. The consumers’ external network strength to delivery staff is $\alpha_s$. Therefore, the utility of the delivery staff from the external network effect is $\alpha_s n_{si}$. Assume that the distance of the delivery staff from platform 1 is $f$, and the distance of the delivery staff from platform 2 is $(1 - f)$, where $f$ is uniformly distributed in the interval $[0, 1]$. According to the Hotelling model, let $t_i$ represent the differentiation of the delivery staff (or the unit transport costs), and then $t_i f$ and $t_i (1 - f)$ are the transport costs of the delivery staff to platform 1 and platform 2, respectively. The gross utilities associated with a delivery staff person on platform 1 and platform 2 are
respectively
\[
\begin{align*}
u_{a1} &= \alpha_s n_{c1} + p_{s1} - t_a \eta, \quad \text{and} \quad (4) \\
u_{a2} &= \alpha_s n_{c2} + p_{s2} - t_a (1 - \eta). \quad (5)
\end{align*}
\]

Advertisers advertise on platform 1, and the utility they obtain from consumers is \(\alpha_s n_{c1}\). The advertising fee that an advertiser pays to the platform 1 is \(p_{s1}\). Assume that the distance of the advertiser from platform 1 is \(\eta\) and the distance of the advertiser from platform 2 is \((1 - \eta)\), where \(\eta\) is uniformly distributed in the interval \([0, 1]\). According to the Hotelling model, let \(t_a\) represent the differentiation of the advertisers (or the unit transport costs), and let \(t_a \eta\) and then \(t_a (1 - \eta)\) are the transport costs of the advertiser to platform 1 and platform 2, respectively. The gross utilities associated with an advertiser on platform 1 and platform 2 are respectively
\[
\begin{align*}
u_{a1} &= \alpha_s n_{c1} - p_{s1} - t_a \eta, \quad \text{and} \quad (6) \\
u_{a2} &= \alpha_s n_{c2} - p_{s2} - t_a (1 - \eta). \quad (7)
\end{align*}
\]

**B. THE MODEL UNDER SITUATION M**

Under this situation, the consumers are single-homing, while the advertisers and the delivery staff are partial multi-homing. The demands of the consumers, advertisers and delivery staff single-homing on platform 1 are \(n_{c1}\), \(n_{si}\) and \(n_{pi}\), respectively. The total demands of the advertisers and delivery staff on platform 1 are \(N_{ai}\) and \(N_{si}\), respectively. Similarly, the profits of the distribution platform 1 are
\[
\pi_i = p_{ai} N_{ai} + p_{ci} n_{ci} - p_{si} N_{si} - \frac{c x_i^2}{2}. \quad (8)
\]

The consumers’ utility from delivery staff joining the platform 1 is \(\alpha_i N_{si}\), and the negative utility of the consumers from the advertiser is \(\alpha_i N_{ai}\). The perceived utility coefficient of consumers joining platform 1 is \(\theta\), and the amount of fundamental investment of this platform is \(v\). According to the Hotelling model, the perceived utility of consumers joining the platform 1 is \(\theta v\), while the perceived utility of consumers joining the platform 2 is \((1 - \theta) v\). The gross utilities associated with the consumer on platform 1 and platform 2 are respectively
\[
\begin{align*}
u_{c1} &= \theta v + \alpha_i N_{si} + \beta x_1 - p_{c1} - \alpha_m N_{ai}, \quad \text{and} \quad (9) \\
u_{c2} &= (1 - \theta) v + \alpha_i N_{si} + \beta x_2 - p_{c2} - \alpha_m N_{ai}. \quad (10)
\end{align*}
\]

Assume that the distance of the delivery staff who single-homes on platform 1 from platform 1 is \(f_1\), and the distance of the delivery staff who belongs to platform 1 (single-homing or multi-homing) from platform 1 is \(f_2\), where \(f_1\) and \(f_2\) are uniformly distributed in the interval \([0, 1]\). According to the Hotelling model, let \(t_f\) represent the differentiation of the delivery staff (or the unit transport costs), and then \(t_f f_1\) and \(t_f (1 - f_2)\) are the transport costs of the advertiser to platform 1 and platform 2, respectively. The gross utilities associated with a delivery staff person on platform 1, platform 2 and both platforms 1 and 2 are respectively
\[
\begin{align*}
u_{s1} &= \alpha_s n_{c1} + p_{s1} - t_f f_1, \quad (11) \\
u_{s2} &= \alpha_s n_{c2} + p_{s2} - t_f (1 - f_2), \quad (12) \\
u_{s12} &= \alpha_s + p_{s1} + p_{s2} - t_f.
\end{align*}
\]

Assume that the distance of the advertiser who single-homes on platform 1 from platform 1 is \(\eta_1\), and the distance of the advertiser who belongs to platform 1 (single-homing or multi-homing) from platform 1 is \(\eta_2\), where \(\eta_1\) and \(\eta_2\) are uniformly distributed in the interval \([0, 1]\). According to the Hotelling model, let \(t_\eta\) represent the differentiation of the advertisers (or the unit transport costs), and then \(t_\eta \eta_1\) and \(t_\eta (1 - \eta_2)\) are the transport costs of the advertiser to platform 1 and platform 2, respectively. The gross utilities associated with an advertiser on platform 1, platform 2 and both platforms 1 and 2 are respectively
\[
\begin{align*}
u_{a1} &= \alpha_a n_{c1} - p_{a1} - t_\eta \eta_1, \quad (14) \\
u_{a2} &= \alpha_a n_{c2} - p_{a2} - t_\eta (1 - \eta_2), \quad (15) \\
u_{a12} &= \alpha_a - p_{a1} - p_{a2} - t_\eta.
\end{align*}
\]

**V. SOLVING THE MODELS AND THE RESULTS**

**A. SOLVING THE MODEL UNDER SITUATION S**

If the consumers joining the platform 1 and 2 have equal utilities, i.e., \(u_{1} = u_{2}\), then consumers will choose to join either platform. Let \(u_{1}(\theta^*) = u_{2}(\theta^*)\). Then, \(\theta^* = \frac{\alpha_m + \alpha_m (n_{a1} - n_{a2}) - \alpha_c (n_{a1} - n_{a2}) - \beta (x_1 - x_2) + p_{c1} - p_{c2}}{2v}\). If \(\theta > \theta^*, i.e., u_{1}(\theta) > u_{2}(\theta)\), then the consumers will join platform 1. If \(\theta \leq \theta^*, i.e., u_{1}(\theta) \leq u_{2}(\theta)\), then the consumers will join platform 2. According to the presupposition that \(\theta\) is uniformly distributed in the interval \([0, 1]\), the demand of consumers on platform 2 is
\[
n_{c2} = \int_{0}^{\theta^*} dy = \frac{1}{2v} \left[ \alpha_m (n_{a1} - n_{a2}) - \alpha_c (n_{a1} - n_{a2}) - \beta (x_1 - x_2) + p_{c1} - p_{c2} + v \right]. \quad (17)
\]

Likewise, let \(u_{1}(f^*) = u_{2}(f^*)\) and \(u_{1}(\eta^*) = u_{2}(\eta^*)\). Then, \(f^* = \frac{1}{2t_f} (\alpha_s n_{c1} - \alpha_s n_{c2} + p_{s1} - p_{s2} + t_f)\) and \(\eta^* = \frac{1}{2t_\eta} (\alpha_a n_{c1} - \alpha_a n_{c2} - p_{a1} + p_{a2} + t_\eta)\). If \(f \leq f^*\), i.e., \(u_{1}(f^*) \geq u_{2}(f^*)\), then the delivery staff will choose platform 1. If \(f > f^*\), i.e., \(u_{1}(f^*) < u_{2}(f^*)\), then the delivery staff will choose platform 2. If \(\eta \leq \eta^*\), i.e., \(u_{1}(\eta^*) \geq u_{2}(\eta^*)\), then the advertisers will choose platform 1. If \(\eta > \eta^*\), i.e., \(u_{1}(\eta^*) < u_{2}(\eta^*)\), then the advertisers will choose platform 2. Considering that both \(f\) and \(\eta\) are uniformly distributed in the interval \([0, 1]\), the demands of the delivery staff and the advertisers on platform 1, \(n_{a1}\) and \(n_{a1}\), are respectively
\[
\begin{align*}
n_{a1} &= \int_{0}^{f^*} dy = \frac{1}{2t_f} (\alpha_s n_{c1} - \alpha_s n_{c2} + p_{s1} - p_{s2} + t_f), \quad (18) \\
n_{a1} &= \int_{0}^{\eta^*} dy = \frac{1}{2t_\eta} (\alpha_a n_{c1} - \alpha_a n_{c2} - p_{a1} + p_{a2} + t_\eta). \quad (19)
\end{align*}
\]
All of the consumers, advertisers and delivery staff are single-homing. According to Hotelling’s specification, we have

\[ n_{s1} + n_{s2} = 1, \quad (20) \]
\[ n_{s1} + n_{s2} = 1, \quad (21) \]
\[ n_{a1} + n_{a2} = 1. \quad (22) \]

From Eqs. (17) – (22), we can solve the demand of the consumers on platform 2 \( n_{s2} \), the demand of the delivery staff on platform 1 \( n_{a1} \) and the demand of the advertisers on platform 1 \( n_{a1} \):

\[ n_{s2} = \begin{cases} \frac{\alpha_{m} t_{s} (\alpha_{a} - p_{a1} + p_{a2} + t_{s})}{2 (\alpha_{a} t_{s} - \alpha_{c} a_{s} t_{a} + t_{a} t_{s})}, \\ \frac{\alpha_{a} c_{x} (\alpha_{c} - p_{a1} + p_{a2} + t_{s})}{2 (\alpha_{a} t_{s} - \alpha_{c} a_{s} t_{a} + t_{a} t_{s})} \end{cases} \quad (23) \]
\[ n_{s1} = \begin{cases} \frac{\alpha_{m} t_{s} (\alpha_{a} - p_{a1} + p_{a2} + t_{s})}{2 (\alpha_{a} t_{s} - \alpha_{c} a_{s} t_{a} + t_{a} t_{s})}, \\ \frac{\alpha_{a} c_{x} (\alpha_{c} - p_{a1} + p_{a2} + t_{s})}{2 (\alpha_{a} t_{s} - \alpha_{c} a_{s} t_{a} + t_{a} t_{s})} \end{cases} \quad (24) \]
\[ n_{a1} = \begin{cases} \frac{\alpha_{m} t_{s} (\alpha_{a} - p_{a1} + p_{a2} + t_{s})}{2 (\alpha_{a} t_{s} - \alpha_{c} a_{s} t_{a} + t_{a} t_{s})}, \\ \frac{\alpha_{a} c_{x} (\alpha_{c} - p_{a1} + p_{a2} + t_{s})}{2 (\alpha_{a} t_{s} - \alpha_{c} a_{s} t_{a} + t_{a} t_{s})} \end{cases} \quad (25) \]

Platforms 1 and 2 maximize their profits according to the following equations:

\[ \max \pi_{1} (p_{c1}, p_{a1}, p_{a2}, x_{1}) = p_{a1} n_{s1} + p_{c1} (1 - n_{s2}) - p_{s1} n_{s2} - \frac{c x_{1}^{2}}{2}, \quad (26) \]
\[ \max \pi_{2} (p_{c1}, p_{a1}, p_{a2}, x_{1}) = p_{a2} (1 - n_{s1}) + p_{c2} n_{s2} - p_{s2} (1 - n_{s1}) - \frac{c x_{2}^{2}}{2}. \quad (27) \]

By maximizing the profits in (26) and (27) with (23), (24), and (25), we can derive the optimal investment and pricing strategies, which are summarized in the following theorem.

**Theorem 1**: Let \( c_{0} = \frac{\beta}{2} \) and

\[ c_{1} = \sqrt{\frac{\beta^{2} t_{a} t_{s}}{4 t_{a} t_{s} v - t_{a} (\alpha_{c} + \alpha_{s})^{2}}}. \]

If \( v \geq t_{a} (\alpha_{s} + \alpha_{s}^{2} t_{a} (\alpha_{c} + \alpha_{s})^{2})/4 t_{a} t_{s} \), the optimal VAS investment amount of platforms 1 and 2 under situation S can be given as follows:

(1) For \( \forall c \in [0, 1] \), the optimal transaction fees, delivery fees and advertising fees of platforms 1 and 2 are \( p_{s1}^{*} = p_{s2}^{*} = v - \alpha_{a} - \alpha_{s} \), \( p_{s3}^{*} = p_{s4}^{*} = v - \alpha_{c} - \alpha_{s} \) and \( p_{s5}^{*} = p_{s6}^{*} = v - \alpha_{c} - \alpha_{s} \), respectively; and the optimal demands of the consumers, advertisers and delivery staff on platforms 1 and 2 are \( n_{s1}^{*} = n_{s2}^{*} = \frac{1}{2} \), \( n_{a1}^{*} = n_{a2}^{*} = \frac{1}{2} \) and \( n_{s1}^{*} = n_{s2}^{*} = \frac{1}{2} \), respectively.

(2) If \( c \geq c_{1} \), the optimal VAS investment amount of platforms 1 and 2 are \( x_{s1}^{*} = x_{s2}^{*} = \frac{\beta}{2} \), \( c \geq c_{0} \), and the optimal profits of platforms 1 and 2 are

\[ \pi_{S}^{*} = \pi_{1}^{*} = \pi_{2}^{*} = \begin{cases} v + t_{a} + t_{s} - \alpha_{a} - \alpha_{c} + \alpha_{m} - \alpha_{s} - \frac{\beta^{2}}{8 c}, & c \geq c_{0} \\ v + t_{a} + \frac{t_{s}}{2} - \alpha_{a} - \alpha_{c} + \alpha_{m} - \alpha_{s} - \frac{\beta^{2}}{2 c} & , c < c_{0}; \end{cases} \]

(3) If \( c < c_{1} \), the optimal VAS investment amount of platforms 1 and 2 are \( x_{s1}^{*} = x_{s2}^{*} = 0 \), and the optimal profits of platforms 1 and 2 are \( \pi_{S}^{*} = \pi_{1}^{*} = \pi_{2}^{*} = \frac{\beta}{2} \).

**Proof**: See Appendix A.

**B. SOLVING THE MODEL UNDER SITUATION M**

Likewise, if the consumers joining platforms 1 and 2 have equal utilities, i.e., \( u_{c1} (\theta^{*}) = u_{c2} (\theta^{*}) \) where \( \theta^{*} = \frac{1}{2} [v + \alpha_{m} (N_{a1} - N_{a2}) - \alpha_{c} (N_{s1} - N_{s2}) - \beta (x_{1} - x_{2}) + p_{c1} - p_{c2}] \), then consumers will choose to either platform. If \( \theta > \theta^{*}, \) i.e., \( u_{c1} (\theta) > u_{c2} (\theta) \), then the consumers will join platform 1. If \( \theta \leq \theta^{*} \), i.e., \( u_{c1} (\theta) \leq u_{c2} (\theta) \), then the consumers will join platform 2. According to the presupposition that \( \theta \) is uniformly distributed in the interval \([0, 1] \), the demand of the consumers on platform 2 \( n_{s2} \) is

\[ n_{s2} = \int_{0}^{\theta^{*}} dy \]

\[ \int_{0}^{\frac{\beta}{2}} \left[ v + \alpha_{m} (N_{a1} - N_{a2}) - \alpha_{c} (N_{s1} - N_{s2}) \right] dy. \]

Let \( u_{s1} (f_{1}^{*}) = u_{s2} (f_{2}^{*}) = u_{s12} \) and \( u_{a1} (\eta_{1}^{*}) = u_{a2} (\eta_{2}^{*}) = u_{a12} \). Then \( f_{1}^{*} = \frac{1}{t_{a}} \left( \alpha_{a}, n_{c1} - \alpha_{s} - p_{s2} + t_{s} \right), f_{2}^{*} = \frac{1}{t_{a}} (-\alpha_{a} n_{c2} + \alpha_{s} + p_{s1} + t_{s}), \eta_{1}^{*} = \frac{1}{t_{a}} \left( \alpha_{a}, n_{c1} - \alpha_{s} - p_{s2} + t_{s} \right) \) and \( \eta_{2}^{*} = \frac{1}{t_{a}} (-\alpha_{a} n_{c2} + \alpha_{s} - p_{s1}). \) If \( f \leq f_{1}^{*} \), then the delivery staff will choose platform 1. If \( f_{1}^{*} < f < f_{2}^{*} \), then the delivery staff will choose platforms 1 and 2 equally. If \( f \geq f_{2}^{*} \), then the delivery staff will choose platform 1. Likewise, if \( \eta \leq \eta_{1}^{*} \), then the advertisers will choose platform 1. If \( \eta_{1}^{*} < \eta \leq \eta_{2}^{*} \), then the advertisers will choose platforms 1 and 2 equally. If \( \eta \geq \eta_{2}^{*} \), then the advertisers will choose platform 2.

Considering that both \( f \) and \( \eta \) are uniformly distributed in the interval \([0, 1] \), we obtain that

\[ n_{s1} = \int_{0}^{f_{1}^{*}} dy = \frac{1}{t_{a}} \left( \alpha_{a}, n_{c1} - \alpha_{s} - p_{s2} + t_{s} \right), \]
\[ n_{s2} = \int_{0}^{1} dy = 1 = 1 - \frac{t_{a}}{t_{s}} (\alpha_{s} - \alpha_{a} n_{c1} + p_{s2} + t_{s}), \]
\[ n_{a1} = \int_{0}^{\eta_{1}^{*}} dy = \frac{1}{t_{a}} \left( \alpha_{a}, n_{c1} - \alpha_{s} - p_{s2} + t_{s} \right), \]
\[ n_{a2} = \int_{0}^{1} dy = 1 - \frac{t_{a}}{t_{s}} (\alpha_{a} - \alpha_{a} n_{c2} - p_{s1}). \]

The consumers are single-homing while the advertisers and the delivery staff are partial multi-homing. According to
Hotelling’s specification, we have

\[ N_{s1} + n_{s2} = 1, \]  
\[ n_{s1} + N_{s2} = 1, \]  
\[ n_{s1} + n_{s2} = 1, \]  
\[ N_{a1} + n_{a2} = 1, \]  
\[ n_{a1} + N_{a2} = 1. \]  

From Eqs. (28) – (37), we can obtain the following results:

\[ n_{c2} = \frac{[\alpha m_1 t_s (\alpha_1 p_{a2} - p_{a1}) - \alpha c t_0 (p_{a1} - p_{a2} + \alpha c)]}{2 (a_1 m_1 t_s - \alpha c t_0 t_v)}, \]  
\[ n_{s1} = \frac{[\alpha c m_1 t_s (-\alpha a - p_{a1}) + \alpha c a t_0 (p_{a1} - p_{a2} - t_v)]}{2 t_s (a_1 m_1 t_s - \alpha c a t_0 + t_a t_v)}, \]  
\[ n_{a1} = \frac{[\alpha m_1 t_s (p_{a1} - \alpha a) + \alpha a t_0 (p_{a1} - p_{a2} + t_v)]}{2 t_a (a_m t_s - \alpha a t_0 + t_a t_v)}, \]  
\[ n_{a2} = \frac{[\alpha a c t_0 (p_{a1} - \alpha a - t_a) + \alpha a m t_s (p_{a1} - p_{a2} + t_v)]}{2 t_a (a_a m t_s - \alpha c a t_0 + t_a t_v)}. \]

Platforms 1 and 2 maximize their profits using the following problems:

\[ \max \pi_1 (p_{c1}, p_{a1}, p_{s1}, x_1) = \left[ p_{a1} (1 - n_{a2}) + p_{c1} (1 - n_{c2}) - p_{s1} (1 - n_{s2}) - \frac{c^2}{2} \right], \]  
\[ \max \pi_2 (p_{a2}, p_{a2}, p_{s2}, x_2) = \left[ p_{a2} (1 - n_{a1}) + p_{c2} n_{c2} - p_{s2} (1 - n_{s1}) - \frac{c^2}{2} \right]. \]

By maximizing the profits in (43) and (44) with (38) – (42), we can derive the optimal investment and pricing strategies, which are summarized in the following theorem.

**Theorem 2**: Let \( c_0 = \frac{\beta}{2} \) and

\[ c_2 = \frac{2\beta^2 t_a t_s}{8 t_a t_v - t_a (\alpha c + \alpha s)^2 - t_s (\alpha a - \alpha m)^2}, \]

If \( v \geq t_a (\alpha c + \alpha s)^2 + t_s (\alpha a - \alpha m)^2 / 4 t_a t_s \), the optimal VAS investment and pricing strategies of platforms 1 and 2 under situation M can be given as follows:

1. For \( v \in [0, 1] \), the optimal transaction fees and advertising fees of platforms 1 and 2 are

   \[ p_c^* = p_{c2} = v - \frac{\alpha a (3 - 3 \alpha a)}{4 t_a} - \frac{\alpha c (3 - 3 \alpha c)}{4 t_a}, \]

   \[ p_m^* = p_{m2} = 1 \] for \( \alpha a < \alpha c \), otherwise.

2. For \( v \geq c_2 \), the optimal VAS investment amount of platforms 1 and 2 are

   \[ x_{s1}^* = x_{s2}^* = \frac{1}{2} \] for \( c \geq c_0 \), and the optimal profits of platforms 1 and 2 are

   \[ \pi_{s1}^* = \pi_{s2}^* = \pi_{s1}^* = \pi_{s2}^* = \frac{\beta^2}{8c}, \]

   \[ c \geq c_0; \]

   \[ \pi_{m1}^* = \pi_{m2}^* = \frac{\beta^2}{8c}, \]

   \[ c < c_0; \]

3. For \( c < c_2 \), the optimal VAS investment amount of platforms 1 and 2 are

   \[ x_{s1}^* = x_{s2}^* = \pi_{m1}^* = \pi_{m2}^* = \frac{\beta^2}{8c}, \]

   \[ Proof: \text{See Appendix B}. \]

**VI. NUMERICAL ANALYSES AND DISCUSSIONS**

To reflect the influences of the relevant parameters on the optimal VAS investment strategies, pricing strategies and profits of the distribution platforms in different situations, we conduct numerical analyses using MATHEMATICA 11.

We denote the situation in which “all three users are single-homing” as S and the situation in which “the consumers are single-homing while the advertisers and the delivery staff are partial multi-homing” as M.

**A. COMPARISON OF THE OPTIMAL TRANSACTION FEES UNDER THE TWO DIFFERENT SITUATIONS**

From Theorems 1 and 2, we can find that under the same situation, the pricing formula of the optimal transaction fee is the same regardless of the value of \( c \). The changes of the optimal transaction fee with the relevant parameters \( (\alpha_s, \alpha_c, \alpha_m, v, t_s, t_a) \) under situations S and M are shown
FIGURE 3. The changes of the optimal transaction fees $p_{cs}^*$ and $p_{cm}^*$.

TABLE 2. The assignment of the relevant parameters in Fig. 3.

| Subfigures in Fig. 3 | Assignments |
|----------------------|-------------|
| (a)                  | $\alpha_s$ 0.80 $\alpha_c$ 0.20 $\alpha_m$ 0.10 $v$ 0.96 $t_s$ 0.80 $t_a$ 0.50 |
| (b)                  | $\alpha_s$ 0.05 $\alpha_c$ 0.20 $\alpha_m$ 0.10 $v$ 0.90 $t_s$ 0.80 $t_a$ 0.50 |
| (c)                  | $\alpha_s$ 0.20 $\alpha_c$ 0.60 $\alpha_m$ 0.10 $v$ 0.96 $t_s$ 0.80 $t_a$ 0.50 |
| (d)                  | $\alpha_s$ 0.30 $\alpha_c$ 0.40 $\alpha_m$ 0.51 $v$ 0.96 $t_s$ 0.70 $t_a$ 0.80 |
| (e)                  | $\alpha_s$ 0.30 $\alpha_c$ 0.50 $\alpha_m$ 0.40 $v$ 0.20 $t_s$ 0.80 $t_a$ 0.50 |
| (f)                  | $\alpha_s$ 0.30 $\alpha_c$ 0.50 $\alpha_m$ 0.40 $v$ 0.20 $t_s$ 0.90 $t_a$ 0.50 |
| (g)                  | $\alpha_s$ 0.30 $\alpha_c$ 0.50 $\alpha_m$ 0.50 $v$ 0.90 $t_s$ 0.50 $t_a$ 0.50 |

TABLE 3. The assignment of the relevant parameters in Fig. 4.

| Subfigures in Fig. 4 | Assignments |
|----------------------|-------------|
| (a)                  | $\alpha_s$ 0.80 $\alpha_c$ 0.10 $t_a$ 0.50 |
| (b)                  | $\alpha_s$ 0.10 $\alpha_c$ 0.10 $t_a$ 0.50 |
| (c)                  | $\alpha_s$ 0.40 $\alpha_c$ 0.80 $t_a$ 0.50 |

in Figs. 3 (a) – (g). For each subfigure, the assignment of the relevant parameters is shown in Table 2.

From Fig. 3, we can obtain the following results. First, whether under situation $S$ or $M$, the optimal transaction fee decreases with the external network strengths $\alpha_s$ and $\alpha_m$, and the optimal transaction fee increases with the amount of fundamental investment $v$. Second, regardless of the value of $\alpha_c$, $\alpha_m$, $t_s$, or $t_a$, the optimal transaction fee stays unchanged under the single-homing situation $S$. Third, under the multi-homing situation, the optimal transaction fee decreases with $\alpha_s$ and $t_a$, while it increases with $\alpha_m$ and $t_s$.

Based on the observation of Fig. 3, we can derive that transaction fees are affected by various factors, mainly, $\alpha_s$, $\alpha_m$, and $v$. The increase of external network strength of consumers on delivery staff and advertisers increases the importance of consumers to them, and the platform will improve the satisfaction of consumers via reducing transaction fees. In addition, when the platform’s fundamental investment increases, the platform will inevitably increase the transaction fees to improve profits and control risks. The findings indicate that the platform can guarantee profits and control costs through adjusting transaction fees in two different competitive environments. For example, in the real world, platforms, such as “Eleme” etc., will adjust their transaction prices from time to time.

Meanwhile, the optimal transaction fee under situation $S$ is always lower than that under situation $M$. Under the multi-homing situation, delivery staff and advertisers can cooperate with several platforms at the same time, and the platform will have to improve the delivery staff’s and lower advertisers’ fees in order to maintain its the scale of the delivery staff and advertisers, which increases the platform’s operation cost. Therefore, the platform will increase transaction fees in order to guarantee profits and control risks. Through the above analysis, we suggest that the platform needs to set up their own distribution system, or high operating costs will make it lose its competitive edge in terms of its price.

B. COMPARISON OF THE OPTIMAL DELIVERY FEES UNDER THE TWO DIFFERENT SITUATIONS

From Theorems 1 and 2, we can find that under the same situation, the pricing expression of the optimal delivery fee is the same regardless of the value of $c$. The changes of the optimal delivery fee with the relevant parameters ($\alpha_s$, $\alpha_c$ and $t_s$) under situations $S$ and $M$ are shown in Figs. 4 (a) – (c). For each subfigure, the assignment of the relevant parameters is shown in Table 3.

From Fig. 4 (a), we can find that the optimal delivery fee decreases with the external network strength $\alpha_s$ under situation $M$, but it is not affected by $\alpha_s$ under situation $S$. From Fig. 4 (b), the optimal delivery fee increases with the external network strength $\alpha_c$ under either situation $S$ or $M$. From Fig. 4 (c), the optimal delivery fee decreases with $t_s$ under situation $S$, but it is not affected by $t_s$ under situation $M$. 
Via the observation of Fig. 4, we obtain that the delivery fees are mainly affected by $\alpha_s$ and $\alpha_c$. When the consumer scale increases under situation $M$, the platform needs to appropriately reduce the delivery fee to promote the re-consumption of consumers. Furthermore, when the delivery staff only belong to a delivery platform (i.e., situation $S$), the platform generally does not change the delivery fee standard due to the stability of the delivery cost and the scale of consumers. In addition, when the number of delivery staff increases (i.e. $\alpha_c$ increases), the costs associated with the platform’s payment of delivery staff will certainly increase, which leads to a corresponding increase in the delivery fee charged by the platform. The findings indicate that the platform needs to fully consider its competitive environment and the scale of its delivery staff before making decisions on the delivery fee. Only in this way can the platform make decisions on the delivery fee that are satisfactory to consumers, delivery staff and the platform.

In addition, the optimal delivery fee under situation $S$ is always greater than that under situation $M$. Under situation $S$, the delivery staff’s orders are smaller than orders of situation $M$. Hence, high operating costs make the platform improve the delivery fees. The findings show that when the platform builds its own distribution system, only as much as possible to improve consumer scale and reduce the operating costs of the distribution system to make delivery costs be at a reasonable price level.

C. COMPARISON OF THE OPTIMAL ADVERTISING FEES UNDER THE TWO DIFFERENT SITUATIONS

From Theorems 1 and 2, we can find that under the same situation, the pricing expression of the optimal advertising fee is the same regardless of the value of $c$. The changes of the optimal advertising fee with the relevant parameters ($\alpha_m$, $\alpha_a$ and $t_a$) under situations $S$ and $M$ are shown in Figs. 5 (a) – (c). For each subfigure, the assignment of the relevant parameters is shown in Table 4.

From Fig. 5 (a), we can find that the optimal advertising fee increases with the external network strength $\alpha_m$ under either situation $S$ or $M$. From Fig. 5 (b), the optimal advertising fee increases with the external network strength $\alpha_a$ under situation $M$, but it is not affected by $\alpha_a$ under situation $S$. From Fig. 5 (c), the optimal advertising fee increases with $t_a$ under situation $S$, but it is not affected by $t_a$ under situation $M$.

Through the observation of Fig. 5, we derive that when the advertising putting on the platform increases, in the different market environment, the platform will increase the charge of advertising. The main reason is that the increase in the number of advertising makes consumers produce aversion emotion to platform, which directly results in the decrease of consumer’s scale. Therefore, the platform will increase advertising fee to make up for the loss of overmuch advertisement. This indicates that the platform negotiating with advertisers must work out a reasonable advertising number, which minimizes the negative effects of excessive advertising.
Alternatively, the platform can also adjust the number of advertisements via changing advertising fees.

In addition, the optimal advertising fee under situation $S$ is always greater than that under situation $M$. When the advertisers are in single-homing situation, the advertisers can only be forced to accept the high advertising fees charged by the platform, however, when the advertisers cooperate with multiple platforms (i.e., situation $M$), the advertisers can negotiate lower advertising fees with them. Accordingly, advertising fees under situation $S$ are higher. Via the above analyses, we suggest that advertisers need to invest across multiple platforms, not only to reduce their investment costs, but also to deliver better advertising effect.

D. COMPARISON OF THE OPTIMAL VAS INVESTMENT UNDER THE TWO DIFFERENT SITUATIONS

Let $\beta = 0.15$, $v = 0.9$, $t_d = 0.4$, $t_a = 0.1$, $\alpha_d = 0.5$, $\alpha_c = 0.2$, $\alpha_m = 0.6$, and $\alpha_t = 0.4$. Then, we conduct sensitivity analyses on $c$ ($c \in [0, 1]$). According to Theorems 1 and 2, the threshold of the marginal VAS investment costs under situations $S$ and $M$ are $c_0 = 0.075$, $c_1 = 0.1125$ and $c_2 = \frac{120}{277}$, respectively. The analyses results are shown in Fig. 6.

E. COMPARISON OF THE OPTIMAL PROFITS UNDER THE TWO DIFFERENT SITUATIONS

The changes of the optimal profits with the relevant parameters ($\beta$, $v$, $c$, $\alpha_d$, $\alpha_c$, $\alpha_m$, $\alpha_t$, $t_d$, and $t_a$) under situations $S$ and $M$ are shown in Figs. 7 (a) – (g). For each subfigure, the assignment of the relevant parameters is shown in Table 5.

From Fig. 7, we can obtain the following results. First, whether under situation $S$ or $M$, the optimal profits decrease with the external network strengths $\alpha_d$, $\alpha_c$, and $\alpha_m$, and the optimal profits increase with $\alpha_t$ and $t_a$. Second, under situation $S$, the optimal profits increase with $t_d$. Third, under situation $M$, the optimal profits decrease with $t_a$.

Based on the observation of Fig. 7, we can clearly derive that the optimal platform profits are greatly affected by $\alpha_c$ and $\alpha_m$. When the number of delivery staff increases, the platform operating costs will increase correspondingly, while the consumer’s scale remains unchanged, this leads to the reduction of platform profits. In addition, when the number of advertisers increases, it will bring considerable advertising revenue to the platform, thus increasing the platform profits.

| TABLE 5. The assignment of the relevant parameters in Fig. 7. |
|---------------------------------------------------------------|
| Assignments        | $\beta$ | $v$ | $c$ | $\alpha_d$ | $\alpha_c$ | $\alpha_m$ | $\alpha_t$ | $t_d$ | $t_a$ |
|----------------- |------- |----- |----- |----------- |----------- |----------- |----------- |------- |------- |
| (a)             | 0.30  | 0.90 | 0.40 | 0.40      | 0.10      | 0.40      | 0.20      | 0.10   |       |
| (b)             | 0.30  | 0.90 | 0.80 | 0.20      | 0.10      | 0.40      | 0.25      | 0.10   |       |
| (c)             | 0.30  | 0.90 | 0.40 | 0.20      | 0.30      | 0.60      | 0.20      | 0.14   |       |
| (d)             | 0.30  | 0.90 | 0.40 | 0.20      | 0.30      | 0.10      | 0.20      | 0.14   |       |
| (e)             | 0.30  | 0.90 | 0.40 | 0.20      | 0.40      | 0.10      | 0.20      | 0.10   |       |
| (f)             | 0.30  | 0.90 | 0.95 | 0.20      | 0.40      | 0.10      | 0.50      | 0.20   |       |

**FIGURE 6.** The impact of the marginal VAS investment costs $c$ on the optimal VAS investment amount $x^*$.
These findings indicate that it is necessary to control the reasonable delivery staff’s number and strive for more advertising investment in different market environments, which have a great impact on platform profits. Meanwhile, the optimal profits under situation S are always lower than those under situation M. In situation M, delivery staff and advertisers will cooperate with multiple platforms, under which fees paying delivery staff can be reduced, and the number of advertisers will increase. Therefore, comparing to situation S, the platform profits will be higher. Different from the observation of Fig. 3, the findings suggest that the platform must be sure to carefully consider factors, such as delivery costs and advertising fees, and seek appropriate operation mode when conducting distribution system construction and advertising investment bidding.

VII. CONCLUSIONS

In this paper, we investigated the optimal VAS investment and pricing strategies of a multilateral distribution platform with considerations of user-homing and cross-group network externalities in a duopoly. First, we established the profit-maximizing model under the two universal situations in practice. Subsequently, we obtained the optimal prices and the optimal amount of VAS investment by solving the model. Finally, we studied the impacts of the related parameters on the optimal strategies through numerical simulation.

Unlike existing studies, our study focuses on competitive multilateral distribution platforms, which makes up for the limitation that only the pricing and VAS investment strategies of bilateral platforms are discussed.

Our work has limitations that provide avenues for future research. First, the impacts of intra-group network externalities on competitive distribution platforms can be considered, which makes the problem closer to reality. Second, the influence of the consumer charging mode on pricing can be considered. For example, a consumer may not only pay the transaction fee but also the membership fee, which makes the consumer get additional discounts. Third, we can extend our static models to dynamic models.

APPENDIX A

PROOF OF THEOREM 1

Proof: Taking the first-order derivatives of Eq. (26) with regard to \( p_{c1}, p_{a1}, p_{a1} \) and \( x_1 \), we can obtain

\[
\frac{\partial \pi_1}{\partial p_{c1}} = \frac{t_s [a_m (a_d + p_{a1} - p_{a2}) - \alpha_d p_{a1}] + t_a [p_{c1}(a_c + \alpha_s) - \alpha_c (p_{c2} + \alpha_s)] + t_s (p_{c2} - 2p_{c1} + v + \alpha x_1 - \alpha x_2)}{2t_s (a_d a_m + vt_a) - 2t_a \alpha_c \alpha_s}, \quad (A1)
\]

\[
\frac{\partial \pi_1}{\partial p_{a1}} = \frac{a_d (p_{c1} - p_{a2}) + \alpha_d p_{a1} (p_{c2} - p_{c1}) - \alpha_d a_m p_{a1} + \alpha_m a_s (a_d + p_{c1}) + p_{a1} (2a_c \alpha_s - 2vt_a) + \alpha a d t_2 (x_1 - x_2) + (vt_a - \alpha x_1) (t_a + p_{a2})}{2t_s (a_d a_m + vt_a) - 2t_a \alpha_c \alpha_s}, \quad (A2)
\]

\[
\frac{\partial \pi_1}{\partial p_{a1}} = \frac{t_s [\alpha_c (a_d + p_{c1} - p_{c2}) - \beta x_1 + \beta x_2] + \alpha d (p_{c2} - p_{c1}) + 2p_{c1} (a_d a_m + vt_a) - \alpha d a_m t_2 - vt_a a_s + 2a_d a_m t_2 + (t_a + p_{a1}) a c t_2 + (t_a + p_{a1}) a c t_2}{2t_s (a_d a_m + vt_a) - 2t_a \alpha_c \alpha_s}, \quad (A3)
\]

\[
\frac{\partial \pi_1}{\partial x_1} = \frac{t_s [\beta p_{c1} a _c + p_{c1} a _c] - 2c a x_1 (vt_a + \alpha d a_m)}{2t_s (vt_a + \alpha_d a_m) - 2t_a \alpha_c \alpha_s}. \quad (A4)
\]
Taking the first-order derivatives of Eq. (27) with regard to $p_{c2}$, $p_{d2}$, $p_{s2}$ and $x_2$, we can obtain
\[
\frac{\partial \pi_2}{\partial p_{c2}} = \begin{cases} 
\frac{t_s[\alpha_m(\alpha_a - p_{a1} + p_{a2}) - \alpha_d p_{a2}]}{2t_s(\alpha_c\alpha_m + vt_a) - 2t_a\alpha_c\alpha_s} 
+ t_a \left[ \frac{p_{a2}(\alpha_a + \alpha_c) - \alpha_c(p_{a1} + \alpha_s)}{+t_s(p_{c1} - 2p_{c2} + v - \beta x_1 + \beta x_2)} \right], 
\end{cases}
\]
(A5)

\[
\frac{\partial \pi_2}{\partial p_{d2}} = \begin{cases} 
\frac{\alpha_s\alpha_c(p_{a2} - p_{a1}) + \alpha_d t_s(p_{a1} - p_{c2}) + \alpha_t(p_{a2} - \alpha_m p_{d2}) + (vt_a - \alpha_c)(t_a + p_{a1}) + \beta\alpha_t s}{2t_s(\alpha_c\alpha_m + vt_a) - 2t_a\alpha_c\alpha_s}, 
\end{cases}
\]
(A6)

\[
\frac{\partial \pi_2}{\partial p_{s2}} = \begin{cases} 
\frac{2t_s(\alpha_c\alpha_m + vt_a) - 2t_a\alpha_c\alpha_s}{-2p_d(\alpha_c\alpha_m + vt_a) - 2\alpha_c\alpha_m t_s}, 
\end{cases}
\]
(A7)

\[
\frac{\partial \pi_2}{\partial x_2} = \begin{cases} 
\frac{2t_s(\alpha_c\alpha_m + vt_a) - 2t_a\alpha_c\alpha_s}{-2t_s(\alpha_c\alpha_m + vt_a) - 2t_a\alpha_c\alpha_s}. 
\end{cases}
\]
(A8)

Then, we obtain the Hessian matrix $H_1$ and $H_2$ from Eq. (26) and (27). Let $N = \alpha_a\alpha_m t_a - \alpha_c\alpha_t s + t_a t_s v$, then we obtain
\[
H_1 = \begin{pmatrix} 
\frac{t_a t_s}{N} & \alpha s t_a + \alpha c t_a & \frac{\alpha_m t_s - \alpha a t_s}{2N} & \beta t_a t_s \\
\frac{\alpha s t_a + \alpha c t_a}{N} & \alpha a t_s + \alpha c t_a & \frac{\alpha s t_a + \alpha c t_a}{N} & \frac{\beta t_a t_s}{2N} \\
\frac{\alpha m t_s - \alpha a t_s}{2N} & \frac{\alpha s t_a + \alpha c t_a}{N} & \frac{\alpha a t_s + \alpha c t_a}{2N} & \frac{\beta t_a t_s}{2N} \\
\frac{\beta t_a t_s}{2N} & \frac{\beta t_a t_s}{2N} & \frac{\beta t_a t_s}{2N} & \frac{\beta t_a t_s}{2N} 
\end{pmatrix},
\]
(A9)

and $H_1 = H_2$.

(A10)

The first-order sequential principal minor of Hessian matrix $H_1$ is $-\frac{t_a t_s}{N} = -\frac{t_a t_s}{N} < 0$. The second-order sequential principal minor of Hessian matrix $H_1$ is
\[
\left| \begin{array}{ccc}
\frac{t_a t_s}{N} & \alpha s t_a + \alpha c t_a & \frac{\alpha m t_s - \alpha a t_s}{2N} \\
\frac{\alpha s t_a + \alpha c t_a}{N} & \alpha a t_s + \alpha c t_a & \frac{\alpha s t_a + \alpha c t_a}{N} \\
\frac{\alpha m t_s - \alpha a t_s}{2N} & \frac{\alpha s t_a + \alpha c t_a}{N} & \frac{\alpha a t_s + \alpha c t_a}{2N} \\
\frac{\alpha m t_s - \alpha a t_s}{2N} & \frac{\alpha s t_a + \alpha c t_a}{N} & \frac{\alpha a t_s + \alpha c t_a}{2N} \\
\end{array} \right| = \frac{t_a t_s[4t_s(\alpha_c\alpha_m + t_a v) - t_a(\alpha_c + \alpha_s)^2]}{4N^2} > 0.
\]

The third-order sequential principal minor of Hessian matrix $H_1$ is $|t_a^2(\alpha_a + \alpha_s)^2(t_v v - \alpha_c - \alpha_s) - t_a t_s(\alpha_a - \alpha_m)(\alpha_c + \alpha_s)(\alpha_a + \alpha_s)(\alpha_a + \alpha_m) + 4t_a t_s(\alpha_c\alpha_m + t_a v)(\alpha_c t_s - t_v) + t_a t_s(\alpha_a - \alpha_m t_a)^2 + t_a^2(\alpha_a - \alpha_m)^2(\alpha_a t_s + t_a v)]/4N^3 < 0$.

(1) If $|H_1| \geq 0$, i.e., $c \geq \frac{\beta^2 t_a t_s}{4t_a t_s v - t_a(\alpha_c + \alpha_s)^2 - t_a(\alpha_a - \alpha_m)^2}$, then the Hessian matrix $H_1$ is negative definite. Through the first-order conditions $\partial \pi_1/\partial p_{c1} = 0$, $\partial \pi_1/\partial p_{a1} = 0$, $\partial \pi_1/\partial p_{s1} = 0$ and $\partial \pi_1/\partial x_1 = 0$, the optimal solutions are obtained, i.e., $p_{c1}^* = p_{d1}^* = v - \alpha_a - \alpha_s$, $p_{a1}^* = p_{s1}^* = \alpha_c - t_s$, $p_{c2}^* = p_{d2}^* = \alpha_m + t_a$, $x_1^* = x_2^* = \frac{\beta}{2c}$, $n_{c1}^* = n_{s1}^* = \frac{1}{2}$. And the optimal profit is $\pi_1^* = \pi_2^* = \frac{v + t_a + t_s - \alpha_a - \alpha_c - \alpha_m - \alpha_s - \alpha_c - \alpha_a}{c} - \frac{\beta^2}{8c}$.

However, if $x_1^* = x_2^* > 1$, i.e., $c < \frac{\beta}{2c}$ (For convenience, let $c_0 = \frac{\beta}{2c}$), $x_1$ and $x_2$ are not at their boundary. Hence, in this situation, $x_1^* = x_2^* = 1$, and we derive the responding optimal profit, i.e., $\pi_1^* = \pi_2^* = \frac{v + t_a + t_s - \alpha_a - \alpha_c - \alpha_m - \alpha_s - \alpha_c - \alpha_a}{2c} - \frac{\beta^2}{8c}$.

In summary, if $c \geq c_1$, the optimal VAS investment amount of platforms 1 and 2 are
\[
x_1^* = x_2^* = \begin{cases} 
\frac{\beta}{2c}, & c \geq c_0 \\
1, & c < c_0, 
\end{cases}
\]
and the optimal profits of platforms 1 and 2 are
\[
\pi_1^* = \pi_2^* = \frac{v + t_a + t_s - \alpha_a - \alpha_c - \alpha_m - \alpha_s - \alpha_c - \alpha_a}{2c} - \frac{\beta^2}{8c}, \quad c \geq c_0,
\]
\[
\pi_1^* = \pi_2^* = \frac{v + t_a + t_s - \alpha_a - \alpha_c - \alpha_m - \alpha_s - \alpha_c - \alpha_a}{2c} - \frac{\beta^2}{8c}, \quad c < c_0.
\]

(2) If $|H_1| < 0$, i.e., $c < c_1$, then the Hessian matrix $H_1$ is neither positive definite nor negative definite. Since the profit functions are continuous and bounded, and the stationary points of the profit functions are unique, then the optimal solutions are arrived when $x_1$ and $x_2$ are at their boundary, therefore, the optimal solutions can be obtained by comparing the profits of boundary solutions.

Given $x_1^* = x_2^* = 1$, by maximizing the formula (26), the optimal solutions with no VAS investment can be derived as $p_{c1}^* = p_{d1}^* = v - \alpha_a - \alpha_s$, $p_{s1}^* = p_{d2}^* = \alpha_c - t_s$, $p_{c1}^* = p_{d2}^* = \alpha_m + t_a$, $n_{c1}^* = n_{s2}^* = \frac{1}{2}$, $n_{s1}^* = n_{s2}^* = \frac{1}{2}$, and the optimal profit of the platform is $\pi_1^* = \pi_2^* = \frac{v + t_a + t_s - \alpha_a - \alpha_c - \alpha_m - \alpha_s}{2c} - \frac{\beta^2}{8c}$.

Given $x_1^* = x_2^* = 1$, by maximizing the formula (26), the optimal solutions with maximal VAS investment can be derived as $p_{c1}^* = p_{d1}^* = v - \alpha_a - \alpha_s$, $p_{s1}^* = p_{d2}^* = \alpha_c - t_s$, $p_{c1}^* = p_{d2}^* = \alpha_m + t_a$, $n_{c1}^* = n_{s2}^* = \frac{1}{2}$, $n_{s1}^* = n_{s2}^* = \frac{1}{2}$, and the optimal profit of the platform is $\pi_1^* = \pi_2^* = \frac{v + t_a + t_s - \alpha_a - \alpha_c - \alpha_m - \alpha_s}{2c} - \frac{\beta^2}{8c}$.

Compare $\pi_{s1} = x_1 = 0$ and $\pi_{s2} = x_2 = 1$, and the minimal gap between the optimal profits can be calculated as $\pi_{s1} = x_1 = 1$, $\pi_{s2} = x_2 = 0$. Notice that $c > 0$, then $n_{s1} = 0$, $\pi_{s1} = 0$. Therefore, the optimal amount of VAS investment is $x_1^* = x_2^* = 0$. 

\[\square\]
\section*{Appendix B

Proof of Theorem 2}

Proof: Taking the first-order derivatives of Eq. (43) with regard to $p_{c1}$, $p_{a1}$, $p_{a2}$ and $x_1$, we can obtain

$$\frac{\partial \pi_1}{\partial p_{c1}} = \frac{\{ t_a [\alpha_m (\alpha_a + p_{a1} - p_{a2}) - \alpha_a p_{a2}] + t_a \} \cdot \{ p_{c2} (p_{c2} - 2 p_{c1} + v + \beta x_1 - \beta x_2) \}}{2 t_s [\alpha_a \alpha_m + v t_a]} - 2 t_a \alpha_a \alpha_c \alpha_s \quad \text{(A11)}$$

$$\frac{\partial \pi_1}{\partial p_{a1}} = \frac{\{ t_s [\alpha_a (2 p_{c1} + p_{c2})] \}}{2 t_s [t_a (v t_s - \alpha_a \alpha_c) + \alpha_a \alpha_m t_s]} \quad \text{(A12)}$$

$$\frac{\partial \pi_1}{\partial p_{a2}} = \frac{\{ t_s \} \{ \beta t_p p_{c1} - 2 c x_1 \}}{2 t_s \{ \alpha_a \alpha_m + v t_a \} + \beta t_p p_{c1}}$$

$$\frac{\partial \pi_1}{\partial x_1} = \frac{\{ t_s \} \{ \alpha_a \alpha_m t_s (\alpha_a + p_{a1} - p_{a2}) + t_s \} + \alpha_a \alpha_m t_s (\alpha_a + p_{a1} - p_{a2}) + t_s \alpha_a \alpha_c^2}{2 t_s \{ \alpha_a \alpha_m + v t_a \} - 2 t_s \alpha_a \alpha_c} \quad \text{(A14)}$$

Taking the first-order derivatives of Eq. (44) with regard to $p_{c2}$, $p_{a2}$, $p_{a2}$ and $x_2$, we can obtain

$$\frac{\partial \pi_2}{\partial p_{c2}} = \frac{\{ t_s [\alpha_m (\alpha_a + p_{a1} - p_{a2}) - \alpha_a p_{a2}] + t_a \} \cdot \{ p_{c2} (p_{c2} - 2 p_{c1} + v + \beta x_1 - \beta x_2) \}}{2 t_s [\alpha_a \alpha_m + v t_a]} - 2 t_a \alpha_a \alpha_c \alpha_s \quad \text{(A15)}$$

$$\frac{\partial \pi_2}{\partial p_{a2}} = \frac{\{ t_s \} \{ \alpha_a \alpha_m t_s (\alpha_a + p_{a1} - p_{a2}) + t_s \} + \alpha_a \alpha_m t_s (\alpha_a + p_{a1} - p_{a2}) + t_s \alpha_a \alpha_c^2}{2 t_s \{ \alpha_a \alpha_m + v t_a \} - 2 t_s \alpha_a \alpha_c} \quad \text{(A16)}$$

$$\frac{\partial \pi_2}{\partial x_2} = \frac{\{ t_s \} \{ \beta t_p p_{c1} - 2 c x_1 \}}{2 t_s \{ \alpha_a \alpha_m + v t_a \} + \beta t_p p_{c1}}$$

Then, we obtain the Hessian matrix $H_1$ and $H_2$ from Eq. (26) and (27). Let $N = \alpha_{a1} \alpha_{a2} m t_s - \alpha_a \alpha_c t_a + t_a f_a v$, then we obtain

$$H_1 = \begin{pmatrix}
\frac{t_a t_s}{N} & \frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a1} t_a}{N} \\
\frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a1} t_a}{N} \\
\frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N} \\
\frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N}
\end{pmatrix} \quad \text{(A19)}$$

and $H_1 = H_2$.

The first-order sequential principal minor of Hessian matrix $H_1$ is $-\frac{t_a t_s}{N} - \frac{t_a t_s}{N N} < 0$. The second-order sequential principal minor of Hessian matrix $H_1$ is

$$H_2 = \begin{pmatrix}
\frac{t_a t_s}{N} & \frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a1} t_a}{N} \\
\frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a1} t_a}{N} \\
\frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N} \\
\frac{\alpha_{a1} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N} & \frac{\alpha_{a2} t_a}{N}
\end{pmatrix} \quad \text{(A20)}$$

The third-order sequential principal minor of Hessian matrix $H_1$ is $(\frac{t_a t_s}{N} - \frac{t_a t_s}{N N} - \frac{t_a t_s}{N N}) < 0$. Then, the Hessian matrix $H_1$ is negative definite. Through the first-order conditions $\frac{\partial \pi_1}{\partial p_{c1}} = 0$, $\frac{\partial \pi_1}{\partial p_{a1}} = 0$, $\frac{\partial \pi_1}{\partial p_{a2}} = 0$ and $\frac{\partial \pi_1}{\partial x_1} = 0$, the solution is obtained. According to $p_{c1} = p_{c2} = v - \frac{\alpha_a (\alpha_a - \alpha_c)}{4 t_a}$, $P_1 = \frac{1}{4} (\alpha_a - \alpha_c)$, $P_2 = \frac{1}{4} (\alpha_a + \alpha_c)$, $x_1 = \frac{v}{2}$, $n_1 = n_2 = 1$, $n_2 = \frac{1}{4} (t_s + \frac{t_a}{4})$, the optimal profit is

$$\pi_1 = \pi_2 = v - \frac{t_{a1} t_{s1}}{2} + \frac{t_{a2} t_{s2}}{2} < 0 \quad \text{(Convenience)}$$

However, if $x_1 = x_2 = \frac{v}{2}$, $n_1 = n_2 = 1$, then, the optimal profit is $\pi_1 = \pi_2 = v - \frac{t_{a1} t_{s1}}{2} - \frac{t_{a2} t_{s2}}{2} < 0 \quad \text{(For convenience)}$. Hence, in this situation, $x_1 = x_2 = 1$, we derive the responding optimal profit, i.e., $\pi_1 = \pi_2 = \frac{v}{2} - \frac{t_{a1} t_{s1}}{2} - \frac{t_{a2} t_{s2}}{2} < 0 \quad \text{(For convenience)}$. Therefore, $x_1 = x_2 = 1$.

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In summary, if $c \geq c_2$, the optimal VAS investment amount of platforms 1 and 2 are

$$x_1^* = x_2^* = \begin{cases} \frac{\beta}{2c}, & c \geq c_0 \\ 1, & c < c_0, \end{cases}$$

and the optimal profits of platforms 1 and 2 are

$$\pi_1^* = \pi_2^* = \begin{cases} \frac{v - \frac{\alpha_2^2 + 2\alpha_2^2 - 6\alpha_2^2 m}{16t_2} - \frac{\beta^2}{8c}}{2}, & c \geq c_0 \\ \frac{v - \frac{\alpha_2^2 + 2\alpha_2^2 - 6\alpha_2^2 m}{16t_2} - \frac{c}{2}}{2}, & c < c_0. \end{cases}$$

(2) If $|H_1| < 0$, i.e., $c < c_2$, then the Hessian matrix $H_1$ is neither positive definite nor negative definite. Since the profit functions are continuous and bounded, and the stationary points of the profit functions are unique, then the optimal solutions are arrived when $x_1$ and $x_2$ are at their boundary, therefore, the optimal solutions can be obtained by comparing the profits of boundary solutions.

Given $x_1^* = x_2^* = 0$, by maximizing the formula (43), the optimal solutions with no VAS investment can be derived as

$$p_1 = p_2 = v - \frac{\alpha_2 (\alpha_2 - 3\alpha m)}{4t_2} - \frac{(\beta^2 + 4c^2)}{8c},$$

and the optimal profit of the platform is

$$\pi_1^* = \pi_2^* = \frac{v}{2} - \frac{\alpha_2^2 + 2\alpha_2^2 - 6\alpha_2^2 m}{16t_2} - \frac{\beta^2}{8c},$$

Given $x_1^* = x_2^* = 1$, by maximizing the formula (43), the optimal solutions with maximal VAS investment can be derived as

$$p_1 = p_2 = v - \frac{\alpha_2 (\alpha_2 - 3\alpha m)}{4t_2} - \frac{(\beta^2 + 4c^2)}{8c},$$

and the optimal profit of the platform is

$$\pi_1^* = \pi_2^* = \frac{v}{2} - \frac{\alpha_2^2 + 2\alpha_2^2 - 6\alpha_2^2 m}{16t_2} - \frac{\beta^2}{8c}.$$

Compare $\pi_{x_1=x_2=0}$ and $\pi_{x_1=x_2=1}$, and the minimal gap between the optimal profits can be calculated as $\pi_{x_1=x_2=1} - \pi_{x_1=x_2=0} = -\frac{c}{2}$. Notice that $c > 0$, then $\pi_{x_1=1} < \pi_{x_2=0}$. Therefore, the optimal amount of VAS investment is $x_1^* = x_2^* = 0.$

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