Abstract

The standard model picture of flavor and CP violation is now experimentally verified, hence strong bounds on the flavor structure of new physics follow. We begin by discussing in detail the unique way that flavor conversion and CP violation arise in the standard model. The description provided is based on a spurion, symmetry oriented, analysis, and a covariant basis for describing flavor transition processes is introduced, in order to make the discussion transparent for non-experts. We show how to derive model independent bounds on generic new physics models. Furthermore, we demonstrate, using the covariant basis, how recent data and LHC projections can be applied to constrain models with an arbitrary mechanism of alignment. Next, we discuss the various limits of the minimal flavor violation framework and their phenomenological aspects, as well as the implications to the underlying microscopic origin of the framework. We also briefly discuss aspects of supersymmetry and warped extra dimension flavor violation. Finally we speculate on the possible role of flavor physics in the LHC era.
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1 Introduction

Flavors are replications of states with identical quantum numbers. The standard model (SM) consists of three such replications of the five fermionic representations of the SM gauge group. Flavor physics describes the non-trivial spectrum and interactions of the flavor sector. What makes this field particularly interesting is that the SM flavor sector is rather unique, and its special characteristics make it testable and predictive.\(^1\) Let us list few of the SM unique flavor predictions:

- It contains a single CP violating parameter.\(^2\)
- Flavor conversion is driven by three mixing angles.
- To leading order, flavor conversion proceeds through weak charged current interactions.
- To leading order, flavor conversion involves left handed (LH) currents.
- CP violating processes must involve all three generations.
- The dominant flavor breaking is due to the top Yukawa coupling, hence the SM possesses a large approximate global flavor symmetry (as shown below, technically it is given by \(U(2)_Q \times U(2)_U \times U(1)_L \times U(3)_D\)).

In the last four decades or so, a huge effort was invested towards testing the SM predictions related to its flavor sector. Recently, due to the success of the B factories, the field of flavor physics has made a dramatic progress, culminated in Kobayashi and Maskawa winning the Nobel prize. It is now established that the SM contributions drive the observed flavor and CP violation (CPV) in nature, via the Cabibbo-Kobayashi-Maskawa (CKM)\(^1\)\(^2\) description. To verify that this is indeed the case, one can allow new physics (NP) to contribute to various clean observables, which can be calculated precisely within the SM. Analyses of the data before and after the B factories data have matured\(^3\)\(^4\)\(^5\)\(^6\), demonstrating that the NP contributions to these clean processes cannot be bigger than \(\mathcal{O}(30\%)\) of the SM contributions\(^7\)\(^8\).

Very recently, the SM passed another non-trivial test. The neutral \(D\) meson system (for formalism see \(\text{e.g.}\)\(^9\)\(^10\)\(^11\)\(^12\)\(^13\) and refs. therein) bears two unique aspects among the four neutral meson system (\(K, D, B, B_s\)): (i) The long distance contributions to the mixing are orders of magnitude above the SM short distance ones\(^14\)\(^15\), thus making it difficult to theoretically predict the width and mass splitting. (ii) The SM contribution to the CP violation in the mixing amplitude is expected to be below the permil level\(^16\), hence \(D^0 - \bar{D}^0\) mixing can unambiguously signal new physics if CPV is observed. Present data\(^17\)\(^18\)\(^19\)\(^20\)\(^21\)\(^22\)\(^23\)\(^24\) implies that generic CPV contributions can be only of \(\mathcal{O}(20\%)\) of the total (un-calculable) contributions to the mixing amplitudes, again consistent with the SM null prediction.

We have just given rather solid arguments for the validity of the SM flavor description. What else is there to say then? Could this be the end of the story? We have several important reasons to think that flavor physics will continue to play a significant role in our understanding of microscopic physics at and beyond the reach of current colliders. Let us first mention a few examples that demonstrate the role that flavor precision tests played in the past:

\(^1\)This set of lectures discusses the quark sector only. Many of the concepts that are explained here can be directly applied to the lepton sector.

\(^2\)The SM contains an additional flavor diagonal CP violating parameter, namely the strong CP phase. However, experimental data constrains it to be smaller than \(\mathcal{O}(10^{-10})\), hence negligibly small.
• The smallness of $\Gamma(K_L \to \mu^+\mu^-)/\Gamma(K^+ \to \mu^+\nu)$ led to predicting a fourth quark (the charm) via the discovery of the GIM mechanism [25].

• The size of the mass difference in the neutral Kaon system, $\Delta m_K$, led to a successful prediction of the charm mass [26].

• The size of $\Delta m_B$ led to a successful prediction of the top mass (for a review see [27] and refs. therein).

This partial list demonstrates the power of flavor precision tests in terms of being sensitive to short distance dynamics. Even in view of the SM successful flavor story, it is likely that there are missing experimental and theoretical ingredients, as follows:

• Within the SM, as mentioned, there is a single CP violating parameter. We shall see that the unique structure of the SM flavor sector implies that CP violating phenomena are highly suppressed. Baryogenesis, which requires a sizable CP violating source, therefore cannot be accounted for by the SM CKM phase. Measurements of CPV in flavor changing processes might provide evidence for additional sources coming from short distance physics.

• The SM flavor parameters are hierarchical, and most of them are small (excluding the top Yukawa and the CKM phase), which is denoted as the flavor puzzle. This peculiarity might stem from unknown flavor dynamics. Though it might be related to very short distance physics, we can still get indirect information about its nature via combinations of flavor precision and high $p_T$ measurements.

• The SM fine tuning problem, which is related to the quadratic divergence of the Higgs mass, generically requires new physics at, or below, the TeV scale. If such new physics has a generic flavor structure, it would contribute to flavor changing neutral current (FCNC) processes orders of magnitude above the observed rates. Putting it differently, the flavor scale at which NP is allowed to have a generic flavor structure is required to be larger than $\mathcal{O}(10^5)$ TeV, in order to be consistent with flavor precision tests. Since this is well above the electroweak symmetry breaking scale, it implies an “intermediate” hierarchy puzzle (cf. the little hierarchy [28, 29] problem). We use the term “puzzle” and not “problem” since in general, the smallness of the flavor parameters, even within NP models, implies the presence of approximate symmetries. One can imagine, for instance, a situation where the suppression of the NP contributions to FCNC processes is linked with the SM small mixing angles and small light quark Yukawas [4, 5]. In such a case, this intermediate hierarchy is resolved in a technically natural way, or radiatively stable manner, and no fine tuning is required.\footnote{Unlike, say, the case of the $S$ electroweak parameter, where in general one cannot associate an approximate symmetry with the limit of small NP contributions to $S$.}

2 The standard model flavor sector

The SM quarks furnish three representations of the SM gauge group, $SU(3) \times SU(2) \times U(1)$: $Q(3,2,\frac{1}{6}) \times U(3,1,\frac{2}{3}) \times D(3,1,\frac{1}{3})$, where $Q,U,D$ stand for $SU(2)$ weak doublet, up type and down type singlet quarks, respectively. Flavor physics is related to the fact that the SM consists of...
three replications/generations/flavors of these three representations. The flavor sector of the SM is described via the following part of the SM Lagrangian

\[ \mathcal{L}^F = \bar{q} D^\mu q \delta_{ij} + (Y_U)_{ij} \bar{Q} U^j H_U + (Y_D)_{ij} \bar{Q} D^j H_D + \text{h.c.}, \]  

where \( D^\mu \equiv D_\mu \gamma^\mu \) with \( D_\mu \) being a covariant derivative, \( q = Q, U, D \), within the SM with a single Higgs \( H_U = i \sigma_2 H_D^* \) (however, the reader should keep in mind that at present, the nature and content of the SM Higgs sector is unknown) and \( i, j = 1, 2, 3 \) are flavor indices.

If we switch off the Yukawa interactions, the SM would possess a large global flavor symmetry, \( G_{\text{SM}}^F \)

\[ G_{\text{SM}}^F = U(3)_Q \times U(3)_U \times U(3)_D. \]

Inspecting Eq. (1) shows that the only non-trivial flavor dependence in the Lagrangian is in the form of the Yukawa interactions. It is encoded in a pair of \( 3 \times 3 \) complex matrices, \( Y_{U,D} \).

2.1 The SM quark flavor parameters

Naively one might think that the number of the SM flavor parameters is given by \( 2 \times 9 = 18 \) real numbers and \( 2 \times 9 = 18 \) imaginary ones, the elements of \( Y_{U,D} \). However, some of the parameters which appear in the Yukawa matrices are unphysical. A simple way to see that (see e.g. [30, 31, 32] and refs. therein) is to use the fact that a flavor basis transformation,

\[ Q \rightarrow V_Q Q, \quad U \rightarrow V_U U, \quad D \rightarrow V_D D, \]

leaves the SM Lagrangian invariant, apart from redefinition of the Yukawas,

\[ Y_U \rightarrow V_Q Y_U V_U^\dagger, \quad Y_D \rightarrow V_Q Y_D V_D^\dagger, \]

where \( V_i \) is a \( 3 \times 3 \) unitary rotation matrix. Each of the three rotation matrices \( V_{Q,U,D} \) contains three real parameters and six imaginary ones (the former ones correspond to the three generators of the \( SO(3) \) group, and the latter correspond to the remaining six generators of the \( U(3) \) group). We know, however, that physical observables do not depend on our choice of basis. Hence, we can use these rotations to eliminate unphysical flavor parameters from \( Y_{U,D} \). Out of the \( 18 \) real parameters, we can remove 9 \( (3 \times 3) \) ones. Out of the 18 imaginary parameters, we can remove 17 \( (3 \times 6 - 1) \) ones. We cannot remove all the imaginary parameters, due to the fact that the SM Lagrangian conserves a \( U(1)_B \) symmetry\(^4\). Thus, there is a linear combination of the diagonal generators of \( G_{\text{SM}}^F \) which is unbroken even in the presence of the Yukawa matrices, and hence cannot be used in order to remove the extra imaginary parameter.

An explicit calculation shows that the 9 real parameters correspond to 6 masses and 3 CKM mixing angles, while the imaginary parameter corresponds to the CKM celebrated CPV phase. To see that, we can define a mass basis where \( Y_{U,D} \) are both diagonal. This can be achieved by applying a bi-unitary transformation on each of the Yukawas:

\[ Q^{u,d} \rightarrow V_{Q^{u,d}} Q^{u,d}, \quad U \rightarrow V_U U, \quad D \rightarrow V_D D, \]

\(^4\)At the quantum level, a linear combination of the diagonal \( U(1)'s \) inside the \( U(3)'s \), which corresponds to the axial current, is anomalous.

\(^5\)More precisely, only the combination \( U(1)_{B-L} \) is non-anomalous.
which leaves the SM Lagrangian invariant, apart from redefinition of the Yukawas,

\[ Y_U \rightarrow V_Q u Y_U V_L^\dagger, \quad Y_D \rightarrow V_Q d Y_D V_L^\dagger. \]  

(6)

The difference between the transformations used in Eqs. (3) and (4) and the ones above (5,6), is in the fact that each component of the SU(2) weak doublets (denoted as \( Q^u \equiv U_L \) and \( Q^d \equiv D_L \) ) transforms independently. This manifestly breaks the SU(2) gauge invariance, hence such a transformation makes sense only for a theory in which the electroweak symmetry is broken. This is precisely the case for the SM, where the masses are induced by spontaneous electroweak symmetry breaking via the Higgs mechanism. Applying the above transformation amounts to “moving” to the mass basis. The SM flavor Lagrangian, in the mass basis, is given by (in a unitary gauge),

\[
\mathcal{L}_m^F = \left( q^i \not{\partial} q^i \delta_{ij} \right)_{\text{NC}} + \left( u_L^C, t_L^C \right) \begin{pmatrix} y_u & 0 & 0 \\ 0 & y_c & 0 \\ 0 & 0 & y_t \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix} (v + h) + (u, c, t) \leftrightarrow (d, s, b)
\]  

(7)

where the subscript NC stands for neutral current interaction for the gluons, the photon and the Z gauge bosons, \( W^\pm \) stands for the charged electroweak gauge bosons, \( h \) is the physical Higgs field, \( v \sim 176 \text{ GeV} \), \( m_i = y_i v \) and \( V^{\text{CKM}} \) is the CKM matrix

\[ V^{\text{CKM}} = V_Q V_Q^\dagger. \]  

(8)

In general, the CKM is a 3 \times 3 unitary matrix, with 6 imaginary parameters. However, as evident from Eq. (7), the charged current interactions are the only terms which are not invariant under individual quark vectorial \( U(1)_6 \) field redefinitions,

\[ u_i, d_j \rightarrow e^{i\theta_{u_i,d_j}}. \]  

(9)

The diagonal part of this transformation corresponds to the classically conserved baryon current, while the non-diagonal, \( U(1)_5 \), part of the transformation can be used to remove 5 out of the 6 phases, leaving the CKM matrix with a single physical phase. Notice also that a possible permutation ambiguity for ordering the CKM entries is removed, given that we have ordered the fields in Eq. (7) according to their masses, light fields first. This exercise of explicitly identifying the mass basis rotation is quite instructive, and we have already learned several important issues regarding how flavor is broken within the SM (we shall derive the same conclusions using a spurion analysis in a symmetry oriented manner in Sec. 3):

- Flavor conversions only proceed via the three CKM mixing angles.
- Flavor conversion is mediated via the charged current electroweak interactions.
- The charge current interactions only involve LH fields.

Even after removing all the unphysical parameters, there are various possible forms for the CKM matrix. For example, a parameterization used by the particle data group [33], is given by

\[
V^{\text{CKM}} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta_{\text{CKM}}} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta_{\text{CKM}}} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta_{\text{CKM}}} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta_{\text{CKM}}} & -c_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta_{\text{CKM}}} & s_{23} c_{13} e^{i\delta_{\text{CKM}}} \end{pmatrix},
\]  

(10)

where \( c_{ij} \equiv \cos \theta_{ij} \) and \( s_{ij} \equiv \sin \theta_{ij} \). The three sin \( \theta_{ij} \) are the three real mixing parameters, while \( \delta_{\text{CKM}} \) is the Kobayashi-Maskawa phase.
2.2 CP violation

The SM predictive power picks up once CPV is considered. We have already proven that the SM flavor sector contains a single CP violating parameter. Once presented with a SM Lagrangian where the Yukawa matrices are given in a generic basis, it is not trivial to determine whether CP is violated or not. This is even more challenging when discussing beyond the SM dynamics, where new CP violating sources might be present. A brute force way to establish that CP is violated would be to show that no field redefinitions would render a Lagrangian real. For example, consider a Lagrangian with a single Yukawa matrix,

$$\mathcal{L}^Y = Y_{ij} \bar{\psi}_L^i \phi \psi_R^j + Y_{ij}^* \bar{\psi}_R^j \phi^\dagger \psi_L^i,$$

(11)

where \(\phi\) is a scalar and \(\psi_X\) is a fermion field. A CP transformation exchanges the operators

$$\bar{\psi}_L^i \phi \psi_R^j \leftrightarrow \bar{\psi}_R^j \phi^\dagger \psi_L^i,$$

(12)

but leaves their coefficients, \(Y_{ij}\) and \(Y_{ij}^*\), unchanged, since CP is a linear unitary non-anomalous transformation. This means that CP is conserved if

$$Y_{ij} = Y_{ij}^*. \quad (13)$$

This is, however, not a basis independent statement. Since physical observables do not depend on a specific basis choice, it is enough to find a basis in which the above relation holds.

Sometimes the brute force way is tedious and might be rather complicated. A more systematic approach would be to identify a phase reparameterization invariant or basis independent quantity, that vanishes in the CP conserving limit. As discovered in [34, 35], for the SM case one can define the following quantity

$$C_{SM}^{SM} = \det[Y_D Y_D^\dagger, Y_U Y_U^\dagger], \quad (14)$$

and the SM is CP violating if and only if

$$\text{Im}(C_{SM}^{SM}) \neq 0. \quad (15)$$

It is trivial to prove that only if the number of generations is three or more, then CP is violated. Hence, within the SM, where CP is broken explicitly in the flavor sector, any CP violating process must involve all three generations. This is an important condition, which implies strong predictive power. Furthermore, all the CPV observables are correlated, since they are all proportional to a single CP violating parameter, \(\delta_{\text{KM}}\). Finally, it is worth mentioning that CPV observables are related to interference between different processes, and hence are measurements of amplitude ratios. Thus, in various known cases, they turn out to be cleaner and easier to interpret theoretically.

2.3 The flavor puzzle

Now that we have precisely identified the SM physical flavor parameters, it is interesting to ask what is their experimental value (using MS) [33]:

$$m_u = 1.5$$.3 MeV, \(m_d = 3.5$.6 MeV, \(m_s = 150^{+30}_{-40}\) MeV, 
\(m_c = 1.3\) GeV, \(m_b = 4.2\) GeV, \(m_t = 170\) GeV, 
$$|V_{ud}^{\text{CKM}}| = 0.97, \ |V_{us}^{\text{CKM}}| = 0.23, \ |V_{ub}^{\text{CKM}}| = 3.9 \times 10^{-3}, \$$
$$|V_{cd}^{\text{CKM}}| = 0.23, \ |V_{cs}^{\text{CKM}}| = 1.0, \ |V_{cb}^{\text{CKM}}| = 41 \times 10^{-3}, \$$
$$|V_{td}^{\text{CKM}}| = 8.1 \times 10^{-3}, \ |V_{ts}^{\text{CKM}}| = 39 \times 10^{-3}, \ |V_{tb}^{\text{CKM}}| = 1, \ \delta_{\text{KM}} = 77^\circ,$$

\(6\)It is easy to show that in this example, in fact, CP is not violated for any number of generations.
where $V_{ij}^{\text{CKM}}$ corresponds to the magnitude of the $ij$ entry in the CKM matrix, $\delta^{\text{KM}}$ is the CKM phase, only uncertainties bigger than 10% are shown, numbers are shown to a 2-digit precision and the $V_{ij}^{\text{CKM}}$ entries involve indirect information (a detailed description and refs. can be found in [33]).

Inspecting the actual numerical values for the flavor parameters given in Eq. (16), shows a peculiar structure. Most of the parameters, apart from the top mass and the CKM phase, are small and hierarchical. The amount of hierarchy can be characterized by looking at two different classes of observables:

- Hierarchies between the masses, which are not related to flavor converting processes – as a measure of these hierarchies, we can just estimate what is the size of the product of the Yukawa coupling square differences (in the mass basis)
  \[
  \sum_{i,j,k,l} (m_i^2 - m_j^2)(m_k^2 - m_l^2) = \mathcal{O}(10^{-17}) .
  \]
- Hierarchies in the mixing which mediate flavor conversion – this is related to the tiny misalignment between the up and down Yukawas; one can quantify this effect in a basis independent fashion as follows. A CP violating quantity, associated with $V_{ij}^{\text{CKM}}$, that is independent of parametrization [34,35], $J_{\text{KM}}$, is defined through
  \[
  \text{Im} \left[ V_{ij}^{\text{CKM}} V_{kl}^{\text{CKM}} (V_{il}^{\text{CKM}})^* (V_{kj}^{\text{CKM}})^* \right] = J_{\text{KM}} \varepsilon_{ikm} \varepsilon_{jln} = 
  \]
  \[
  = c_{12}c_{23}s_{12}s_{23}s_{13} \sin \delta^{\text{KM}} \simeq \lambda^2 A^2 \eta = \mathcal{O}(10^{-5}) ,
  \]
where $i,j,k,l = 1,2,3$. We see that even though $\delta^{\text{KM}}$ is of order unity, the resulting CP violating parameter is small, as it is “screened” by small mixing angles. If any of the mixing angles is a multiple of $\pi/2$, then the SM Lagrangian becomes real. Another explicit way to see that $Y_U$ and $Y_D$ are quasi aligned is via the Wolfenstein parametrization of the CKM matrix, where the four mixing parameters are $(\lambda, A, \rho, \eta)$, with $\lambda = |V_{us}| = 0.23$ playing the role of an expansion parameter [36]:

\[
V^{\text{CKM}} = \begin{pmatrix}
1 - \frac{\Lambda}{2} & \lambda \Lambda (\rho - i \eta) \\
-\lambda & 1 - \frac{\Lambda}{2} \Lambda^2 \\
\Lambda^3 (1 - \rho - i \eta) & -\Lambda^2 \\
\end{pmatrix} + \mathcal{O}(\lambda^4).
\]

Basically, to zeroth order, the CKM matrix is just a unit matrix!

As we shall discuss further below, both kinds of hierarchies described in the bullets lead to suppression of CPV. Thus, a nice way to quantify the amount of hierarchies, both in masses and mixing angles, is to compute the value of the reparametrization invariant measure of CPV introduced in Eq. (14)

\[
C^{\text{SM}} = J_{\text{KM}} \sum_{i,j,k,l} (m_i^2 - m_j^2)(m_k^2 - m_l^2) = \mathcal{O}(10^{-22}).
\]

This tiny value of $C^{\text{SM}}$ that characterizes the flavor hierarchy in nature would be of order 10% in theories where $Y_{U,D}$ are generic order one complex matrices. The smallness of $C^{\text{SM}}$ is something that many flavor models beyond the SM try to address. Furthermore, SM extensions that have new sources of CPV tend not to have the SM built-in CP screening mechanism. As a result, they give too large contributions to the various observables that are sensitive to CP breaking. Therefore, these models are usually excluded by the data, which is, as mentioned, consistent with the SM predictions.
3 Spurion analysis of the SM flavor sector

In this part we shall try to be more systematic in understanding the way flavor is broken within the SM. We shall develop a spurion, symmetry-oriented description for the SM flavor structure, and also generalize it to NP models with similar flavor structure, that goes under the name minimal flavor violation (MFV).

3.1 Understanding the SM flavor breaking

It is clear that if we set the Yukawa couplings of the SM to zero, we restore the full global flavor group, \( G_{SM} = U(3)_Q \times U(3)_U \times U(3)_D \). In order to be able to better understand the nature of flavor and CPV within the SM, in the presence of the Yukawa terms, we can use a spurion analysis as follows. Let us formally promote the Yukawa matrices to spurion fields, which transform under \( G_{SM} \) in a manner that makes the SM invariant under the full flavor group (see e.g. [37] and refs. therein). From the flavor transformation given in Eqs. (3,4), we can read the representation of the various fields under \( G_{SM} \) (see illustration in Fig. 1)

\[
\text{Fields : } Q(3, 1, 1), \ U(1, 3, 1), \ D(1, 1, 3) ; \\
\text{Spurions : } Y_U(3, \bar{3}, 1), \ Y_D(3, 1, \bar{3}).
\]

The flavor group is broken by the “background” value of the spurions \( Y_{U,D} \), which are bi-fundamentals of \( G_{SM} \). It is instructive to consider the breaking of the different flavor groups separately (since \( Y_{U,D} \) are bi-fundamentals, the breaking of quark doublet and singlet flavor groups are linked together, so this analysis only gives partial information to be completed below). Consider the quark singlet flavor group, \( U(3)_U \times U(3)_D \), first. We can construct a polynomial of the Yukawas with simple transformation properties under the flavor group. For instance, consider the objects

\[
A_{U,D} = Y_{U,D}^\dagger Y_{U,D} - \frac{1}{3} \text{tr} \left(Y_{U,D}^\dagger Y_{U,D}\right)1_3.
\]

Under the flavor group \( A_{U,D} \) transform as

\[
A_{U,D} \rightarrow V_{U,D} A_{U,D} V_{U,D}^\dagger.
\]

Thus, \( A_{U,D} \) are adjoints of \( U(3)_{U,D} \) and singlets of the rest of the flavor group [while \( \text{tr}(Y_{U,D}^\dagger Y_{U,D}) \) are flavor singlets]. Via similarity transformation, we can bring \( A_{U,D} \) to a diagonal form, simultaneously. Thus, we learn that the background value of each of the Yukawa matrices separately breaks the \( U(3)_{U,D} \) down to a residual \( U(1)^3_{U,D} \) group, as illustrated in Fig. 2.

![Figure 1: The SM flavor symmetry breaking by the Yukawa matrices.](image-url)
Let us now discuss the breaking of the LH flavor group. We can, in principle, apply the same analysis for the LH flavor group, $U(3)_Q$, via defining the adjoints (in this case we have two independent ones),

$$A_{Q^u,Q^d} \equiv Y_{U,D} Y_{U,D}^\dagger - \frac{1}{3} \text{tr} \left( Y_{U,D} Y_{U,D}^\dagger \right) I_3 .$$

(23)

However, in this case the breaking is more involved, since $A_{Q^u,Q^d}$ are adjoints of the same flavor group. This is a direct consequence of the $SU(2)$ weak gauge interaction, which relates the two components of the $SU(2)$ doublets. This actually motivates one to extend the global flavor group as follows. If we switch off the electroweak interactions, the SM global flavor group is actually enlarged to

$$G_{\text{weakless}}^{\text{SM}} = U(6)_Q \times U(3)_U \times U(3)_D ,$$

(24)

since now each $SU(2)$ doublet, $Q_i$, can be split into two independent flavors, $Q^u_i, Q^d_i$, with identical $SU(3) \times U(1)$ gauge quantum numbers. This limit, however, is not very illuminating, since it does not allow for flavor violation at all. To make a progress, it is instructive to distinguish the $W^3$ neutral current interactions from the $W^\pm$ charged current ones, as follows: The $W^3$ couplings are flavor universal, which, however, couple up and down quarks separately. The $W^\pm$ couplings, $g^\pm_2$, link between the up and down LH quarks. In the presence of only $W^3$ couplings, the residual flavor group is given by

$$G_{\text{exten}}^{\text{SM}} = U(3)_{Q^u} \times U(3)_{Q^d} \times U(3)_U \times U(3)_D .$$

(25)

In this limit, even in the presence of the Yukawa matrices, flavor conversion is forbidden. We have already seen explicitly that only the charged currents link between different flavors (see Eq. (7)). It is thus evident that to formally characterize flavor violation, we can extend the flavor group from $G_{\text{SM}}^{\text{SM}} \to G_{\text{exten}}^{\text{SM}}$, where now we break the quark doublets to their isospin components, $U_L, D_L$, and add another spurion, $g^\pm_2$, to get to this limit formally, one can think of a model where the Higgs field is an adjoint of $SU(2)$ and a singlet of color and hypercharge. In this case the Higgs vacuum expectation value (VEV) preserves a $U(1)$ gauge symmetry, and the $W^3$ would therefore remain massless. However, the $W^\pm$ will acquire masses of the order of the Higgs VEV, and therefore charged current interactions would be suppressed.

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To get to this limit formally, one can think of a model where the Higgs field is an adjoint of $SU(2)$ and a singlet of color and hypercharge. In this case the Higgs vacuum expectation value (VEV) preserves a $U(1)$ gauge symmetry, and the $W^3$ would therefore remain massless. However, the $W^\pm$ will acquire masses of the order of the Higgs VEV, and therefore charged current interactions would be suppressed.
can identify two bases where $g_2^{\pm}$ has an interesting background value: The weak interaction basis, in which the background value of $g_2^{\pm}$ is simply a unit matrix
\[
(g_2^{\pm})_{\text{int}} \propto 1_3,
\]
and the mass basis, where (after removing all unphysical parameters) the background value of $g_2^{\pm}$ is the CKM matrix
\[
(g_2^{\pm})_{\text{mass}} \propto V^{\text{CKM}}.
\]

Now we are in a position to understand the way flavor conversion is obtained in the SM. Three spurions must participate in the breaking: $Y_{U,D}$ and $g_2^{\pm}$. Since $g_2^{\pm}$ is involved, it is clear that generation transitions must involve LH charged current interactions. These transitions are mediated by the spurion backgrounds, $A_{Q^u,Q^d}$ (see Eq. (23)), which characterize the breaking of the individual LH flavor symmetries,
\[
U(3)_{Q^u} \times U(3)_{Q^d} \rightarrow U(1)^3_{Q^u} \times U(1)^3_{Q^d}.
\]

Flavor conversion occurs because of the fact that in general we cannot diagonalize simultaneously $A_{Q^u,Q^d}$ and $g_2^\pm$, where the misalignment between $A_{Q^u}$ and $A_{Q^d}$ is precisely characterized by the CKM matrix. This is illustrated in Fig. 3, where it is shown that the flavor breaking within the SM goes through collective breaking – a term often used in the context of little Higgs models (see e.g. [41] and refs. therein). We can now combine the LH and RH quark flavor symmetry breaking to obtain the complete picture of how flavor is broken within the SM. As we saw, the breaking of the quark singlet groups is rather trivial. It is, however, linked to the more involved LH flavor breaking, since the Yukawa matrices are bi-fundamentals – the LH and RH flavor breaking are tied together. The full breaking is illustrated in Fig. 4.

3.2 A comment on description of flavor conversion in physical processes

The above spurion structure allows us to describe SM flavor converting processes. However, the reader might be confused, since we have argued above that flavor converting processes must involve

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8Note that the interaction basis is not unique, given that $g_2^{\pm}$ is invariant under a flavor transformation where $Q^u$ and $Q^d$ are rotated by the same amount – see more in the following.
the three spurions, $A_{Q^u,d}$ and $g_2^\pm$. It is well known that the rates for charge current processes, which are described via conversion of down quark to an up one (and vice versa), say like beta decay or $b \to u$ transitions, are only suppressed by the corresponding CKM entry, or $g_2^\pm$. What happened to the dependence on $A_{Q^u,d}$? The key point here is that in a typical flavor precision measurement, the experimentalists produce mass eigenstate (for example a neutron or a $B$ meson), and thus the fields involved are chosen to be in the mass basis. For example, a $b \to c$ process is characterized by producing a $B$ meson which decays into a charmed one. Hence, both $A_{Q^u}$ and $A_{Q^d}$ participate, being forced to be diagonal, but in a nonlinear way. Physically, we can characterize it by writing an operator

$$O_{b\to c} = \bar{c}_{\text{mass}} \left( g_2^\pm \right)_{\text{mass}}^{cb} b_{\text{mass}},$$

where both the $b_{\text{mass}}$ and $c_{\text{mass}}$ quarks are mass eigenstate. Note that this is consistent with the transformation rules for the extended gauge group, $G^\text{SM_{exten}}$, given in Eqs. (25) and (26), where the fields involved belong to different representations of the extended flavor group.

The situation is different when FCNC processes are considered. In such a case, a typical measurement involves mass eigenstate quarks belonging to the same representation of $G^\text{SM_{exten}}$. For example, processes that mediate $B^0_d \to \overline{B}^0_d$ oscillation due to the tiny mass difference $\Delta m_{B_d}$ between the two mass eigenstates (which was first measured by the ARGUS experiment [42]), are described via the following operator, omitting the spurion structure for simplicity,

$$O_{\Delta m_{B_d}} = (\bar{b_{\text{mass}}} d_{\text{mass}})^2 .$$

Obviously, this operator cannot be generated by SM processes, as it violates the $G^\text{SM_{exten}}$ symmetry explicitly. Since it involves flavor conversion (it violates $b$ number by two units, hence denoted as $\Delta b = 2$ and belongs to $\Delta F = 2$ class of FCNC processes), it must have some power of $g_2^\pm$. A single power of $g_2^\pm$ connects a LH down quark to a LH up one, so the leading contribution should go like $D^i_L \left( g_2^\pm \right)^{ik} \left( g_2^\pm \right)^{kj} D^j_L \quad (i,k,j = 1,2,3)$. Hence, as expected, this process is mediated at least via one loop. This would not work as well, since we can always rotate the down quark fields into the mass basis, and simultaneously rotate also the up type quarks (away from their mass

Figure 4: The schematic structure of the various ingredients that mediate flavor breaking within the SM.
basis) so that $g^\pm_2 \propto 1_3$. These manipulations define the interaction basis, which is not unique (see Eq. (27)). Therefore, the leading flavor invariant spurion that mediates FCNC transition would have to involve the up type Yukawa spurion as well. A naive guess would be

$$O_{\Delta m_{B_d}} \propto \left[ \bar{b}_{\text{mass}} \left( g_2^\pm \frac{b}{m_{\text{mass}}} A_{Q_u}^{kl} (g_2^\pm)^{ld} d_{\text{mass}} \right) \right]^2,$$

where it is understood that $(A_{Q_u})^{kl}$ is evaluated in the down quark mass basis (tiny corrections of order $m_u^2$ are neglected in the above). This expression captures the right flavor structure, and is correct for a sizeable class of SM extensions. However, it is actually incorrect in the SM case. The reason is that within the SM, the flavor symmetries are strongly broken by the large top quark mass [40]. The SM corresponding amplitude consists of a rather non-trivial and non-linear function of $A_{Q_u}$, instead of the above naive expression (see e.g. [43] and refs. therein), which assumes only the simplest polynomial dependence of the spurions. The SM amplitude for $\Delta m_{B_d}$ is described via a box diagram, and two out of the four powers of masses are canceled, since they appear in the propagators.

### 3.3 The SM approximate symmetry structure

In the above we have considered the most general breaking pattern. However, as discussed, the essence of the flavor puzzle is the large hierarchies in the quark masses, the eigenvalues of $Y_{U,D}$ and their approximate alignment. Going back to the spurions that mediate the SM flavor conversions defined in Eqs. (21) and (23), we can write them as

$$A_{U,D} = \text{diag} \left( 0, 0, \frac{y_{t,b}^2}{3} 1_3 + \mathcal{O} \left( \frac{m_{c,s}^2}{m_{t,b}^2} \right) \right),$$

$$A_{Q_u,Q_d} = \text{diag} \left( 0, 0, \frac{y_{t,b}^2}{3} 1_3 + \mathcal{O} \left( \frac{m_{c,s}^2}{m_{t,b}^2} \right) + \mathcal{O} \left( \lambda^2 \right) \right),$$

where in the above we took advantage of the fact that $m_{c,s}^2/m_{t,b}^2$, $\lambda^2 = \mathcal{O} (10^{-5}, -4, -2)$ are small. The hierarchies in the quark masses are translated to an approximate residual RH $U(2)_U \times U(2)_D$ flavor group (see Fig. 5), implying that RH currents which involve light quarks are very small.

We have so far only briefly discussed the role of FCNCs. In the above we have argued, both based on an explicit calculation and in terms of a spurion analysis, that at tree level there are no flavor violating neutral currents, since they must be mediated through the $W^\pm$ couplings or $g_2^\pm$. In fact, this situation, which is nothing but the celebrated GIM mechanism [25], goes beyond the SM to all models in which all LH quarks are $SU(2)$ doublets and all RH ones are singlets. The $Z$ boson might have flavor changing couplings in models where this is not the case.

Can we guess what is the leading spurion structure that induces FCNC within the SM, say which mediates the $b \to d\nu\bar{\nu}$ decay process via an operator $O_{b \to d\nu\bar{\nu}}$? The process changes $b$ quark number by one unit (belongs to $\Delta F = 1$ class of FCNC transitions). It clearly has to contain down type LH quark fields (let us ignore the lepton current, which is flavor-trivial; for effects related to neutrino masses and lepton number breaking in this class of models see e.g. [44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54]). Therefore, using the argument presented when discussing $\Delta m_{B_d}$ (see Eq. (32)), the leading flavor invariant spurion that mediates FCNC would have to involve the up
Figure 5: The approximate flavor symmetry breaking pattern. Note that there is also a residual $U(1)_Q$ symmetry, as explained in Sec. 4.2.

type Yukawa spurion as well

$$O_{b \rightarrow d \nu} \propto \bar{d}^{i} g_{2}^{+} (A_{Q^{u}_{i}})_{k \ell} g_{2}^{+} \bar{D}^{j}_{L} \times \bar{\nu}_{\nu}.$$  (34)

The above considerations demonstrate how the GIM mechanism removes the SM divergencies from various one loop FCNC processes, which are naively expected to be log divergent. The reason is that the insertion of $A_{Q^{u}_{i}}$ is translated to quark mass difference insertion. It means that the relevant one loop diagram has to be proportional to $m_{i}^{2} - m_{j}^{2}$ ($i \neq j$). Thus, the superficial degree of divergency is lowered by two units, which renders the amplitude finite. Furthermore, as explained above (see also Eq. (37)), we can use the fact that the top contribution dominates the flavor violation to simplify the form of $O_{b \rightarrow d \nu}$

$$O_{b \rightarrow d \nu} \sim \frac{g_{2}^{4}}{16 \pi^{2} M_{W}^{2}} \bar{b}_{L} V_{tb}^{\text{CKM}} (V_{td}^{\text{CKM}})^{\ast} d_{L} \times \bar{\nu}_{\nu},$$  (35)

where we have added a one loop suppression factor and an expected weak scale suppression. This rough estimation actually reproduces the SM result up to a factor of about 1.5 (see e.g. [43, 55, 56, 57]).

We thus find that down quark FCNC amplitudes are expected to be highly suppressed due to the smallness of the top off-diagonal entries of the CKM matrix. Parameterically, we find the following suppression factor for transition between the $i$th and $j$th generations:

$$b \rightarrow s \propto |V_{tb}^{\text{CKM}} V_{ts}^{\text{CKM}}| \sim \lambda^{2},$$

$$b \rightarrow d \propto |V_{tb}^{\text{CKM}} V_{td}^{\text{CKM}}| \sim \lambda^{3},$$

$$s \rightarrow d \propto |V_{td}^{\text{CKM}} V_{ts}^{\text{CKM}}| \sim \lambda^{5},$$  (36)

where for the $\Delta F = 2$ case one needs to simply square the parametric suppression factors. This simple exercise illustrates how powerful is the SM FCNC suppression mechanism. The gist of it is that the rate of SM FCNC processes is small, since they occur at one loop, and more importantly

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9For simplicity, we only consider cases with hard GIM, in which the dependence on mass differences is polynomial. There is a large class of amplitudes, for example processes that are mediated via penguin diagrams with gluon or photon lines, where the quark mass dependence is more complicated, and may involve logarithms. The suppression of the corresponding amplitudes goes under the name soft GIM [43].
due to the fact that they are suppressed by the top CKM off-diagonal entries, which are very small. Furthermore, since
\[ |V_{ts,td}^{\text{CKM}}| \gg \frac{m_{c,u}^2}{m_t^2}, \] (37)
in most cases the dominant flavor conversion effects are expected to be mediated via the top Yukawa coupling.\(^{10}\)

We can now understand how the SM uniqueness related to suppression of flavor converting processes arises:

- RH currents for light quarks are suppressed due to their small Yukawa couplings (them being light).
- Flavor transition occurs to leading order only via LH charged current interactions.
- To leading order, flavor conversion is only due to the large top Yukawa coupling.

### 4 Covariant description of flavor violation

The spurion language discussed in the previous section is useful in understanding the flavor structure of the SM. In the current section we present a covariant formalism, based on this language, that enables to express physical observables in an explicitly basis independent form. This formalism, introduced in \(^{58,59}\), can be later used to analyze NP contributions to such observables, and obtain model independent bounds based on experimental data. We focus only on the LH sector.

#### 4.1 Two generations

We start with the simpler two generation case, which is actually very useful in constraining new physics, as a result of the richer experimental precision data. Any hermitian traceless $2 \times 2$ matrix can be expressed as a linear combination of the Pauli matrices $\sigma_i$. This combination can be naturally interpreted as a vector in three dimensional real space, which applies to $A_Q^d$ and $A_Q^u$.

We can then define a length of such a vector, a scalar product, a cross product and an angle between two vectors, all of which are basis independent\(^{11}\):

\[
|\vec{A}| \equiv \sqrt{\frac{1}{2}\text{tr}(A^2)}, \quad \vec{A} \cdot \vec{B} \equiv \frac{1}{2}\text{tr}(AB), \quad \vec{A} \times \vec{B} \equiv -\frac{i}{2}[A,B],
\]

\[
\cos(\theta_{AB}) \equiv \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} = \frac{\text{tr}(AB)}{\sqrt{\text{tr}(A^2)\text{tr}(B^2)}}.
\] (38)

These definitions allow for an intuitive understanding of the flavor and CP violation induced by a new physics source, based on simple geometric terms. Consider a dimension six $SU(2)_L$-invariant

\(^{10}\)This is definitely correct for CP violating processes, or any ones which involve the third generation quarks. It also generically holds for new physics MFV models. Within the SM, for CP conserving processes which involve only the first two generations, one can find exceptions, for instance when considering the Kaon and $D$ meson mass differences, $\Delta m_{D,K}$.

\(^{11}\)The factor of $-i/2$ in the cross product is required in order to have the standard geometrical interpretation $|\vec{A} \times \vec{B}| = |\vec{A}||\vec{B}| \sin \theta_{AB}$, with $\theta_{AB}$ defined through the scalar product as in Eq. (38).
operator, involving only quark doublets,
\[
\frac{C_1}{\Lambda_{NP}^2} O_1 = \frac{1}{\Lambda_{NP}^2} \left[ \bar{Q}_i (X_Q)_{ij} \gamma_\mu Q_j \right] \left[ \bar{Q}_i (X_Q)_{ij} \gamma_\mu Q_j \right] ,
\]
(39)
where \( \Lambda_{NP} \) is some high energy scale. \( X_Q \) is a traceless hermitian matrix, transforming as an adjoint of \( SU(3)_Q \) (or \( SU(2)_Q \) for two generations), so it “lives” in the same space as \( A_{Q^d} \) and \( A_{Q^u} \). In the down sector for example, the operator above is relevant for flavor violation through \( K^- - K^0 \) mixing. To analyze its contribution, we define a covariant orthonormal basis for each sector, with the following unit vectors
\[
\hat{A}_{Q^u,Q^d} = \frac{A_{Q^u} \times A_{Q^d}}{|A_{Q^u} \times A_{Q^d}|} , \quad \hat{J} = \frac{A_{Q^d} \times A_{Q^u}}{|A_{Q^d} \times A_{Q^u}|} , \quad \hat{J}_{u,d} \equiv \hat{A}_{Q^u,Q^d} \times \hat{J} .
\]
(40)
Then the contribution of the operator in Eq. (39) to \( \Delta c, s = 2 \) processes is given by the misalignment between \( X_Q \) and \( A_{Q^u,Q^d} \), which is equal to
\[
|C_{1,K}^{D,K}| = \left| X_Q \times \hat{A}_{Q^u,Q^d} \right|^2 .
\]
(41)
This result is manifestly invariant under a change of basis. The meaning of Eq. (41) can be understood as follows: We can choose an explicit basis, for example the down mass basis, where \( A_{Q^d} \) is proportional to \( \sigma_3 \). \( \Delta s = 2 \) transitions are induced by the off-diagonal element of \( X_Q \), so that \( |C_1^K| = |(X_Q)_{12}|^2 \). Furthermore, \( |(X_Q)_{12}| \) is simply the combined size of the \( \sigma_1 \) and \( \sigma_2 \) components of \( X_Q \). Its size is given by the length of \( X_Q \) times the sine of the angle between \( X_Q \) and \( A_{Q^d} \) (see Fig. 6). This is exactly what Eq. (41) describes.

Figure 6: The contribution of \( X_Q \) to \( K^0 - \bar{K}^0 \) mixing, \( \Delta m_K \), given by the solid blue line. In the down mass basis, \( \hat{A}_{Q^d} \) corresponds to \( \sigma_3 \), \( \hat{J} \) is \( \sigma_2 \) and \( \hat{J}_d \) is \( \sigma_1 \). The figure is taken from [59].

Next we discuss CPV, which is given by
\[
\text{Im} \left( C_{1,K}^{D,K} \right) = 2 \left( X_Q \cdot \hat{J} \right) \left( X_Q \cdot \hat{J}_{u,d} \right) .
\]
(42)

\footnote{This use of effective field theory to describe NP contributions will be explained in detail in the next section. Note also that we employ here a slightly different notation, more suitable for the current needs, than in the next section.}
The above expression is easy to understand in the down basis, for instance. In addition to diagonalizing $A_{Q^d}$, we can also choose $A_{Q^u}$ to reside in the $\sigma_1 - \sigma_3$ plane (Fig. 7) without loss of generality, since there is no CPV in the SM for two generations. As a result, all of the potential CPV originates from $X_Q$ in this basis. $C_1^K$ is the square of the off-diagonal element in $X_Q$, $(X_Q)_{12}$, thus $\text{Im}(C_1^K)$ is simply twice the real part ($\sigma_1$ component) times the imaginary part ($\sigma_2$ component). In this basis we have $\hat{J} \propto \sigma_1$ and $\hat{J}_d \propto \sigma_2$, this proves the validity of Eq. (42).

Figure 7: CP violation in the Kaon system induced by $X_Q$. $\text{Im}(C_1^K)$ is twice the product of the two solid orange lines. Note that the angle between $A_{Q^d}$ and $A_{Q^u}$ is twice the Cabibbo angle, $\theta_C$. The figure is taken from [59].

An interesting conclusion can be inferred from the analysis above: In addition to the known necessary condition for CPV in two generation [23]

$$X^J \propto \text{tr} \left( X_Q \left[ A_{Q^d}, A_{Q^u} \right] \right) \neq 0,$$

we identify a second necessary condition, exclusive for $\Delta F = 2$ processes:

$$X^{J_{u,d}} \propto \text{tr} \left( X_Q \left[ A_{Q^u,Q^d}, \left[ A_{Q^d}, A_{Q^u} \right] \right] \right) \neq 0,$$

These conditions are physically transparent and involve only observables.

4.2 Three generations

4.2.1 Approximate $U(2)_Q$ limit of massless light quarks

For three generations, a simple 3D geometric interpretation does not naturally emerge anymore, as the relevant space is characterized by the eight Gell-Mann matrices\textsuperscript{13}. A useful approximation appropriate for third generation flavor violation is to neglect the masses of the first two generation quarks, where the breaking of the flavor symmetry is characterized by $[U(3)/U(2)]^2$ [10]. This description is especially suitable for the LHC, where it would be difficult to distinguish between light quark jets of different flavor. In this limit, the 1-2 rotation and the phase of the CKM matrix become unphysical, and we can, for instance, further apply a $U(2)$ rotation to the first two

\textsuperscript{13}We denote the Gell-Mann matrices by $\Lambda_i$, where $\text{tr}(\Lambda_i \Lambda_j) = 2 \delta_{ij}$. Choosing this convention allows us to keep the definitions of Eq. (38).
generations to “undo” the 1-3 rotation. Therefore, the CKM matrix is effectively reduced to a real matrix with a single rotation angle, \( \theta \), between an active light flavor (say, the 2nd one) and the 3rd generation,

\[
\theta \simeq \sqrt{\theta_{13}^2 + \theta_{23}^2},
\]

where \( \theta_{13} \) and \( \theta_{23} \) are the corresponding CKM mixing angles. The other generation (the first one) decouples, and is protected by a residual \( U(1)_Q \) symmetry. This can be easily seen when writing \( A_{Q_d} \) and \( A_{Q_u} \) in, say, the down mass basis

\[
A_{Q_d} = \frac{y_d^2}{3} \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad A_{Q_u} = y_t^2 \begin{pmatrix} ♠ & 0 & 0 \\ 0 & ♠ & ♠ \\ 0 & ♠ & ♠ \end{pmatrix},
\]

where ♠ stands for a non-zero real entry. The resulting flavor symmetry breaking scheme is depicted in Fig. 5, where we now focus only on the LH sector.

An interesting consequence of this approximation is that a complete basis cannot be defined covariantly, since \( A_{Q_{u,Q_d}} \) in Eq. (46) clearly span only a part of the eight dimensional space. More concretely, we can identify four directions in this space: \( \hat{J} \) and \( \hat{J}_{u,d} \) from Eq. (40) and either one of the two orthogonal pairs

\[
\hat{A}_{Q_{u,Q_d}} \quad \text{and} \quad \hat{C}_{u,d} \equiv 2\hat{J} \times \hat{J}_{u,d} - \sqrt{3}\hat{A}_{Q_{u,Q_d}},
\]

or

\[
\hat{A}_{Q_{u,Q_d}}' \equiv \hat{J} \times \hat{J}_{u,d} \quad \text{and} \quad \hat{J}_Q \equiv \sqrt{3}\hat{J} \times \hat{J}_{u,d} - 2\hat{A}_{Q_{u,Q_d}}'.
\]

Note that \( \hat{J}_Q \) corresponds to the conserved \( U(1)_Q \) generator, so it commutes with both \( A_{Q_d} \) and \( A_{Q_u} \), and takes the same form in both bases.\(^{14}\) There are four additional directions, collectively denoted as \( \hat{D} \), which transform as a doublet under the CKM (2-3) rotation, and do not mix with the other generators. The fact that these cannot be written as combinations of \( A_{Q_{u,Q_d}} \) stems from the approximation introduced above of neglecting light quark masses. Without this assumption, it is possible to span the entire space using the Yukawa matrices \([60, 61, 62]\). Despite the fact that this can be done in several ways, in the next subsection we focus on a realization for which the basis elements have a clear physical meaning.

It is interesting to notice that a given traceless adjoint object \( X \) in three generations flavor space has an inherent \( SU(2) \) symmetry (that is, two identical eigenvalues) if and only if it satisfies

\[
[\text{tr} \left( X^2 \right)]^{3/2} = \sqrt{6} \text{tr} \left( X^3 \right).
\]

In this case it must be a unitary rotation of either \( \Lambda_8 \) or its permutations \( (\Lambda_8 \pm \sqrt{3}\Lambda_3)/2 \), which form an equilateral triangle in the \( \Lambda_3 - \Lambda_8 \) plane (see Fig. (8)).

As before, we wish to characterize the flavor violation induced by \( X_Q \) in a basis independent form. The simplest observable we can construct is the overall flavor violation of the third generation quark, that is, its decay to any quark of the first two generations. This can be written as

\[
\frac{2}{\sqrt{3}} \left| X_Q \times \hat{A}_{Q_{u,Q_d}} \right|,
\]

\(^{14}\)The meaning of these basis elements can be understood from the following: In the down mass basis we have \( \hat{A}_{Q_d} = -\Lambda_8 \), \( \hat{J} = \Lambda_7 \), \( \hat{J}_d = \Lambda_6 \) and \( \hat{C}_d = \Lambda_3 \). The alternative diagonal generators from Eq. (48) are \( \hat{A}'_{Q_d} = (\Lambda_3 - \sqrt{3}\Lambda_8)/2 = \text{diag}(0, -1, 1) \) and \( \hat{J}_Q = (\sqrt{3}\Lambda_3 + \Lambda_8)/2 = \text{diag}(2, -1, -1)/\sqrt{3} \). It is then easy to see that \( \hat{J}_Q \) commutes with the effective CKM matrix, which is just a 2-3 rotation, and that it corresponds to the \( U(1)_Q \) generator, \( \text{diag}(1, 0, 0) \), after trace subtraction and proper normalization.
Figure 8: The three unit-length diagonal traceless matrices with an inherent $SU(2)$ symmetry. $\hat{A}_{Q^d}$ and $\hat{A}_{Q^u}$ were schematically added (their angle to the $\Lambda_8$ axis is actually much smaller than what appears in the plot). The figure is taken from [59].

which extracts $\sqrt{|(X_Q)_{13}|^2 + |(X_Q)_{20}|^2}$ in each basis.

4.2.2 No $U(2)_Q$ limit – complete covariant basis

It is sufficient to restore the masses of the second generation quarks in order to describe the full flavor space. A simplifying step to accomplish this is to define the following object: We take the $n$-th power of $\left(Y_D Y_D^\dagger\right)$, remove the trace, normalize and take the limit $n \to \infty$. This is denoted by $\hat{A}_{Q^d}^n$:

$$\hat{A}_{Q^d}^n \equiv \lim_{n \to \infty} \left\{ \left(Y_D Y_D^\dagger\right)^n - \frac{1}{3} \text{tr} \left[ \left(Y_D Y_D^\dagger\right)^n / 3 \right] \right\}, \quad (51)$$

and we similarly define $\hat{A}_{Q^u}^n$. Once we take the limit $n \to \infty$, the small eigenvalues of $\hat{A}_{Q^u}^n, \hat{A}_{Q^d}^n$ go to zero, and the approximation assumed before is formally reproduced. As before, we compose the following basis elements:

$$\hat{J}^n \equiv \frac{\hat{A}_{Q^d}^n \times \hat{A}_{Q^u}^n}{|\hat{A}_{Q^d}^n \times \hat{A}_{Q^u}^n|}, \quad \hat{J}_d^n \equiv \frac{\hat{A}_{Q^d}^n \times \hat{J}_d^n}{|\hat{A}_{Q^d}^n \times \hat{J}_d^n|}, \quad \hat{C}_d^n \equiv 2\hat{J}_d^n \times \hat{J}_d^n - \sqrt{3} \hat{A}_{Q^d}^n, \quad (52)$$

which are again identical to the previous case. The important observation for this case is that the $U(1)_Q$ symmetry is now broken. Consequently, the $U(1)_Q$ generator, $J_Q$, does not commute with $A_{Q^d}$ and $A_{Q^u}$ anymore (nor does $\hat{C}_d^n$, which is different from $J_Q$ only by normalization and a shift by $A_{Q^d}$, see Eqs. (47) and (48)). It is thus expected that the commutation relation $[A_{Q^d}, \hat{C}_d^n]$ (where $A_{Q^d}$ now contains also the strange quark mass) would point to a new direction, which could not be obtained in the approximation used before. Further commutations with the existing basis elements should complete the description of the flavor space.

We thus define

$$\hat{D}_2 \equiv \frac{\hat{A}_{Q^d}^n \times \hat{C}_d^n}{|\hat{A}_{Q^d}^n \times \hat{C}_d^n|}. \quad (53)$$
In order to understand the physical interpretation, note that $\hat{D}_2$ does not commute with $A_{Q^d}$, so it must induce flavor violation, yet it does commute with $\hat{A}_{Q^d}^n$. The latter can be identified as a generator of a $U(1)$ symmetry for the bottom quark (it is proportional to $\text{diag}(0,0,1)$ in its diagonal form, without removing the trace), so this fact means that $\hat{D}_2$ preserves this symmetry. Therefore, it must represent a transition between the first two generations of the down sector.

We further define
\begin{equation}
\hat{D}_1 \equiv \hat{A}_{Q^d} \times \hat{D}_2 \left/ A_{Q^d} \times \hat{D}_2 \right., \quad \hat{D}_4 \equiv \hat{j}_d^n \times \hat{D}_2 \left/ \hat{j}_d^n \times \hat{D}_2 \right., \quad \hat{D}_5 \equiv \hat{j}_n \times \hat{D}_2 \left/ \hat{j}_n \times \hat{D}_2 \right.,
\end{equation}
which complete the basis. All of these do not commute with $A_{Q^d}$, thus producing down flavor violation. $\hat{D}_1$ commutes with $\hat{A}_{Q^d}^n$, so it is of the same status as $\hat{D}_2$. The last two elements, $\hat{D}_{4,5}$, are responsible for third generation decays, similarly to $\hat{j}_n$ and $\hat{j}_d^n$. More concretely, the latter two involve transitions between the third generation and what was previously referred to as the “active” generation (a linear combination of the first two), while $\hat{D}_{4,5}$ mediate transitions to the orthogonal combination. It is of course possible to define linear combinations of these four basis elements, such that the decays to the strange and the down mass eigenstates are separated, but we do not proceed with this derivation. It is also important to note that this basis is not completely orthogonal.

In order to give a sense of the physical interpretation of the different basis elements, it is helpful to see their decomposition in terms of Gell-Mann matrices, in the down mass basis (writing only the dependence of the leading terms on $\lambda$ and $\eta$, and omitting for simplicity $O(1)$ factors such as the Wolfenstein parameter $A$). This is given by
\begin{equation}
\begin{aligned}
\hat{D}_1 &\sim \{-1, \eta, 0, 0, 0, 0, 0, 0\}, \\
\hat{D}_2 &\sim \{-\eta, -1, 0, 0, 0, 0, 0, 0\}, \\
\hat{C}_n^d &\sim \{2\lambda, -2\eta\lambda, 1, 0, 0, 0, 0, 0\}, \\
\hat{D}_4 &\sim \{0, 0, 0, -1, \eta, -\lambda, -\eta\lambda^3, 0\}, \\
\hat{D}_5 &\sim \{0, 0, 0, -\eta, -1, \eta\lambda^3, -\lambda, 0\}, \\
\hat{j}_d^n &\sim \{0, 0, 0, -\eta\lambda, 1, \eta\lambda^2, 0\}, \\
\hat{j}_n &\sim \{0, 0, 0, -\eta\lambda, -\lambda, -\eta\lambda^2, 1, 0\}, \\
\hat{A}_{Q^d}^n &\sim \{0, 0, 0, 0, 0, 0, 0, -1\},
\end{aligned}
\end{equation}
where the values in each set of curly brackets stand for the $\Lambda_1, \ldots, \Lambda_8$ components. This shows which part of an object each basis element extracts under a dot product, relative to the down sector. For instance, the leading term in $\hat{D}_1$ is $\Lambda_1$, therefore it represents the real part of a $2 \to 1$ transition.

Similarly, it is also useful to see the leading term decomposition of $A_{Q^u}$ in the down mass basis,
\begin{equation}
A_{Q^u} \sim \left\{-\lambda y_c^2 - \lambda^5 y_t^2, \eta \lambda^5 y_t^2, -(y_c^2 + \lambda^4 y_t^2)/2, \lambda^3 y_t^2, -\eta \lambda^3 y_t^2, -\lambda^2 y_t^2, -\eta \lambda^4 y_t^2, -y_t^2 / \sqrt{3} \right\},
\end{equation}
ignoring the mass of the up quark.
Finally, an instructive exercise is to decompose $A_{Q^u}$ in this covariant “down” basis, since $A_{Q^u}$ is a flavor violating source within the SM. Focusing again only on leading terms, we have

$$A_{Q^u} \cdot \left\{ \hat{D}_1, \hat{D}_2, \hat{C}_d^n, \hat{D}_4, \hat{D}_5, \hat{J}_d^n, \hat{J}_u^n, \hat{A}_{Q_d}^n \right\} \sim \left\{ \lambda y_t^2 + \lambda^5 y_t^2, \lambda y_c^2, (y_c^2 + \lambda^4 y_t^2)/2, \lambda^3 y_c^2, \lambda^3 y_t^2, \lambda^2 y_t^2, 0, y_t^2/\sqrt{3} \right\}.$$  

This shows the different types of flavor violation in the down sector within the SM. It should be mentioned that the $\hat{D}_2$ and $\hat{D}_5$ projections of $A_{Q^u}$ vanish when the CKM phase is taken to zero, and also when either of the CKM mixing angles is zero or $\pi/2$. Therefore these basis elements can be interpreted as CP violating, together with $\hat{J}_u^n$.

In order to derive model independent bounds in the next section, we use the simpler description based on the approximate $U(2)_Q$ symmetry, rather than the full basis.

5 Model independent bounds

In order to describe NP effects in flavor physics, we can follow two main strategies: (i) build an explicit ultraviolet completion of the model, and specify which are the new fields beyond the SM, or (ii) analyze the NP effects using a generic effective theory approach, by integrating out the new heavy fields. The first approach is more predictive, but also more model dependent. We follow this approach in Secs. 7 and 8 in two well-motivated SM extensions. In this and the next section we adopt the second strategy, which is less predictive but also more general.

Assuming the new degrees of freedom to be heavier than SM fields, we can integrate them out and describe NP effects by means of a generalization of the Fermi Theory. The SM Lagrangian becomes the renormalizable part of a more general local Lagrangian. This Lagrangian includes an infinite tower of operators with dimension $d > 4$, constructed in terms of SM fields and suppressed by inverse powers of an effective scale $\Lambda > M_W$:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{C_i^{(d)}}{\Lambda^{(d-4)}} O_i^{(d)}(\text{SM fields}).$$  

This general bottom-up approach allows us to analyze all realistic extensions of the SM in terms of a limited number of parameters (the coefficients of the higher dimensional operators). The drawback of this method is the impossibility to establish correlations of NP effects at low and high energies – the scale $\Lambda$ defines the cutoff of the effective theory. However, correlations among different low energy processes can still be established implementing specific symmetry properties, such as the MFV hypothesis (Sec. 6). The experimental tests of such correlations allow us to test/establish general features of the new theory, which hold independently of the dynamical details of the model. In particular, $B$, $D$ and $K$ decays are extremely useful in determining the flavor symmetry breaking pattern of the NP model.

5.1 $\Delta F = 2$ transitions

The starting point for this analysis is the observation that in several realistic NP models, we can neglect non-standard effects in all cases where the corresponding effective operator is generated at tree level within the SM. This general assumption implies that the experimental determination of the CKM matrix via tree level processes is free from the contamination of NP contributions. Using
this determination, we can unambiguously predict meson-antimeson mixing and FCNC amplitudes within the SM and compare it with data, constraining the couplings of the $\Delta F = 2$ operators in Eq. (58).

5.1.1 From short distance physics to observables

In order to derive bounds on the microscopic dynamics, one needs to take into account the fact that the experimental input is usually given at the energy scale in which the measurement is performed, while the bound is presented at some other scale (say 1 TeV). Moreover, the contributing higher dimension operators mix, in general. Finally, all such processes include long distance contributions (that is, interactions at the hadronic level) in actual experiments. Therefore, a careful treatment of all these effects is required. For completeness, we include here all the necessary information needed in order to take the above into account.

A complete set of four quark operators relevant for $\Delta F = 2$ transitions is given by

\[
\begin{align*}
Q_1^{q_iq_j} & = \bar{q}_{\alpha jL} \gamma_\mu q_{\beta iL} \bar{q}_{jL} \gamma_\mu q_{iL}, \\
Q_2^{q_iq_j} & = \bar{q}_{\alpha jL} q_{\beta iL} \bar{q}_{jL} q_{iL}, \\
Q_3^{q_iq_j} & = \bar{q}_{\alpha jR} q_{\beta iL} \bar{q}_{\beta iL} q_{jR}, \\
Q_4^{q_iq_j} & = \bar{q}_{\alpha jR} q_{\beta iL} \bar{q}_{\beta jR} q_{iL}, \\
Q_5^{q_iq_j} & = \bar{q}_{\alpha jR} q_{\beta iL} \bar{q}_{\beta jL} q_{iR},
\end{align*}
\]

(59)

where $i,j$ are generation indices and $\alpha, \beta$ are color indices. There are also operators $\tilde{Q}_1^{q_iq_j}$, which are obtained from $Q_1^{q_iq_j}$ by the exchange $L \leftrightarrow R$, and the results given for the latter apply to the former as well.

The Wilson coefficients of the above operators, $C_i(\Lambda)$, are obtained in principle by integrating out all new particles at the NP scale. Then they have to be evolved down to the hadronic scales $\mu_b = m_b = 4.6$ GeV for bottom mesons, $\mu_D = 2.8$ GeV for charmed mesons and $\mu_K = 2$ GeV for Kaons. We denote the Wilson coefficients at the relevant hadronic scale, which are the measured observables, as $\langle M | L_{\text{eff}} | M \rangle_i$, where $M$ represents a meson (note that $\langle M | L_{\text{eff}} | M \rangle$ has dimension of $[\text{mass}]$). These should be functions of the Wilson coefficients at the NP scale, $C_i(\Lambda)$, the running of $\alpha_s$ between the NP and the hadronic scales and the hadronic matrix elements of the meson, $\langle M | Q^{q_iq_j} | M \rangle$ (here $q_iq_j$ stand for the quarks that compose the meson $M$). For bottom and charmed mesons, the analytic formula that describes this relation is given by

\[
\langle M | L_{\text{eff}} | M \rangle_i = \sum_{j=1}^{5} \sum_{r=1}^{5} \left( b_j^{(r,i)} + \eta c_j^{(r,i)} \right) a_j \eta a_j \frac{C_i(\Lambda)}{\Lambda^2} \langle M | Q^{q_iq_j} | M \rangle,
\]

(60)

where $\eta \equiv \alpha_s(\Lambda)/\alpha_s(m_t)$ and $b_j^{(r,i)}$ and $c_j^{(r,i)}$ are called “magic numbers”.

\footnote{note that the operator $Q_1$ has actually already been defined in Eq. (39) in the previous section, using a slightly different notation.}

\footnote{When a bound is written in terms of an energy scale, the running should start from this scale, which is not known \textit{a priori}. This is done in an iterative process, which converges quickly due to the very slow running of $\alpha_s$ at high scales.}
For both types of bottom mesons, the non-vanishing magic numbers are given by

\[ a_i = (0.286, -0.692, 0.787, -1.143, 0.143), \]

\[ b_i^{(11)} = (0.865, 0, 0, 0, 0), \]
\[ b_i^{(22)} = (0, 1.879, 0.012, 0, 0), \]
\[ b_i^{(23)} = (0, -0.493, 0.18, 0, 0), \]
\[ b_i^{(32)} = (0, -0.044, 0.035, 0, 0), \]
\[ b_i^{(33)} = (0, 0.011, 0.54, 0, 0), \]
\[ b_i^{(44)} = (0, 0, 2.87, 0), \]
\[ b_i^{(45)} = (0, 0, 0.961, -0.22), \]
\[ b_i^{(54)} = (0, 0, 0.09, 0), \]
\[ b_i^{(55)} = (0, 0, 0, 0.029, 0.863), \]
\[ c_i^{(11)} = (-0.017, 0, 0, 0, 0), \]
\[ c_i^{(22)} = (0, -0.18, -0.003, 0, 0), \]
\[ c_i^{(23)} = (0, -0.014, 0.008, 0, 0), \]
\[ c_i^{(32)} = (0, 0.005, -0.012, 0, 0), \]
\[ c_i^{(33)} = (0, 0, 0.028, 0, 0), \]
\[ c_i^{(44)} = (0, 0, 0, -0.48, 0.005), \]
\[ c_i^{(45)} = (0, 0, 0, -0.25, -0.006), \]
\[ c_i^{(54)} = (0, 0, 0, -0.013, -0.016), \]
\[ c_i^{(55)} = (0, 0, 0, -0.007, 0.019). \]

The hadronic matrix elements are

\[ \langle B_q | Q_1^{bq} | B_q \rangle = \frac{1}{3} m_{B_q} f_{B_q}^2 B_1^B, \]
\[ \langle B_q | Q_2^{bq} | B_q \rangle = -\frac{5}{24} \left( \frac{m_{B_q}}{m_b + m_q} \right)^2 m_{B_q} f_{B_q}^2 B_2^B, \]
\[ \langle B_q | Q_3^{bq} | B_q \rangle = \frac{1}{24} \left( \frac{m_{B_q}}{m_b + m_q} \right)^2 m_{B_q} f_{B_q}^2 B_3^B, \]
\[ \langle B_q | Q_4^{bq} | B_q \rangle = \frac{1}{4} \left( \frac{m_{B_q}}{m_b + m_q} \right)^2 m_{B_q} f_{B_q}^2 B_4^B, \]
\[ \langle B_q | Q_5^{bq} | B_q \rangle = \frac{1}{12} \left( \frac{m_{B_q}}{m_b + m_q} \right)^2 m_{B_q} f_{B_q}^2 B_5^B, \]

where \( q = d, s \), and the other inputs needed here are \[7, 33\]

\[ m_{B_d} = 5.279 \text{ GeV}, \quad f_{B_d} = 0.2 \text{ GeV}, \quad m_{B_s} = 5.366 \text{ GeV}, \quad f_{B_s} = 0.262 \text{ GeV}, \quad m_b = 4.237 \text{ GeV}, \]
\[ B_1^B = 0.88, \quad B_2^B = 0.82, \quad B_3^B = 1.02, \quad B_4^B = 1.15, \quad B_5^B = 1.99. \]

(63)

For the \( D \) meson, the \( a_i \) magic numbers are as in Eq. \((61)\), while the others are given by \[7\]

\[ b_i^{(11)} = (0.837, 0, 0, 0, 0), \]
\[ c_i^{(11)} = (-0.016, 0, 0, 0, 0), \]
\[ b_i^{(22)} = (0, 2.163, 0.012, 0, 0), \]
\[ c_i^{(22)} = (0, -0.20, -0.002, 0, 0), \]
\[ b_i^{(23)} = (0, -0.567, 0.176, 0, 0), \]
\[ c_i^{(23)} = (0, -0.016, 0.006, 0, 0), \]
\[ b_i^{(32)} = (0, -0.032, 0.031, 0, 0), \]
\[ c_i^{(32)} = (0, 0.004, -0.010, 0, 0), \]
\[ b_i^{(33)} = (0, 0.008, 0.474, 0, 0), \]
\[ c_i^{(33)} = (0, 0, 0.025, 0, 0), \]
\[ b_i^{(44)} = (0, 0, 3.63, 0), \]
\[ c_i^{(44)} = (0, 0, 0, -0.56, 0.006), \]
\[ b_i^{(45)} = (0, 0, 1.21, -0.19), \]
\[ c_i^{(45)} = (0, 0, 0, -0.29, -0.006), \]
\[ b_i^{(54)} = (0, 0, 0, 0.14, 0), \]
\[ c_i^{(54)} = (0, 0, 0, -0.019, -0.016), \]
\[ b_i^{(55)} = (0, 0, 0, 0.045, 0.839), \]
\[ c_i^{(55)} = (0, 0, 0, -0.009, 0.018). \]
The $D$ hadronic matrix elements are

\[
\langle D|Q^a_{1,2,3,4,5}\rangle = \frac{1}{3}m_D f_D^2 B_1^D, \\
\langle D|Q^a_{1,2,3,4,5}\rangle = -\frac{5}{24} \left( \frac{m_D}{m_c + m_u} \right)^2 m_D f_D^2 B_2^D, \\
\langle D|Q^a_{1,2,3,4,5}\rangle = \frac{1}{24} \left( \frac{m_D}{m_c + m_u} \right)^2 m_D f_D^2 B_3^D, \\
\langle D|Q^a_{1,2,3,4,5}\rangle = \frac{1}{4} \left( \frac{m_D}{m_c + m_u} \right)^2 m_D f_D^2 B_4^D, \\
\langle D|Q^a_{1,2,3,4,5}\rangle = \frac{1}{12} \left( \frac{m_D}{m_c + m_u} \right)^2 m_D f_D^2 B_5^D, \\
\]

and we also need to use

\[
m_D = 1.864 \text{ GeV}, \quad f_D = 0.2 \text{ GeV}, \quad m_c = 1.3 \text{ GeV}, \\
B_1^D = 0.865, \quad B_2^D = 0.82, \quad B_3^D = 1.07, \quad B_4^D = 1.08, \quad B_5^D = 1.455. \tag{66}
\]

Finally, for Kaons we use a slightly different formula \[7\]

\[
\langle K|\mathcal{L}_{\text{eff}}|K\rangle = \sum_{j=1}^{5} \sum_{r=1}^{5} \left( b_j^{(r,j)} + \eta c_j^{(r,j)} \right) \eta^{\alpha_j} \frac{C_i^{(\Lambda)}}{\Lambda^2} R_r \langle K|Q^{\alpha_j}_{1}\rangle. \tag{67}
\]

The magic numbers are \[64\]

\[
a_j = (0.29, -0.69, 0.79, -1.1, 0.14), \\
b_j^{(11)} = (0.82, 0, 0, 0, 0), \quad c_j^{(11)} = (-0.016, 0, 0, 0, 0), \\
b_j^{(22)} = (0.24, 0.011, 0, 0), \quad c_j^{(22)} = (0, -0.23, -0.002, 0, 0), \\
b_j^{(23)} = (0, -0.63, 0.17, 0, 0), \quad c_j^{(23)} = (0, -0.018, 0.0049, 0, 0), \\
b_j^{(32)} = (0, -0.019, 0.028, 0, 0), \quad c_j^{(32)} = (0, 0.0028, -0.0093, 0, 0), \\
b_j^{(33)} = (0, 0.0049, 0.43, 0, 0), \quad c_j^{(33)} = (0.00021, 0.023, 0, 0), \\
b_j^{(14)} = (0, 0, 0, 4.4, 0), \quad c_j^{(14)} = (0, 0, 0, -0.68, 0.0055), \\
b_j^{(15)} = (0, 0, 0, 1.5, -0.17), \quad c_j^{(15)} = (0, 0, 0, -0.35, -0.0062), \\
b_j^{(54)} = (0, 0, 0, 0.18, 0), \quad c_j^{(54)} = (0, 0, 0, -0.026, -0.016), \\
b_j^{(55)} = (0, 0, 0, 0.061, 0.82), \quad c_j^{(55)} = (0, 0, 0, -0.013, 0.018). \tag{68}
\]

We use here only the first (SM) hadronic matrix element,

\[
\langle K|Q^{\alpha}_{1}\rangle = \frac{1}{3}m_K f_K^2 B_1^K, \tag{69}
\]

and the others are related to this one by the ratios $R_r$. The other necessary inputs are thus \[7\]

\[
m_K = 0.498 \text{ GeV}, \quad f_K = 0.16 \text{ GeV}, \quad B_1^K = 0.6, \\
R_1 = 1, \quad R_2 = -12.9, \quad R_3 = 3.98, \quad R_4 = 20.8, \quad R_5 = 5.2. \tag{70}
\]
5.1.2 Generic bounds from meson mixing

We now move to the actual derivation of bounds on new physics from $\Delta F = 2$ transitions. It is interesting to note that only fairly recently has the data begun to disfavor models with only LH currents, but with new sources of flavor and CPV [3, 4, 5], characterized by a CKM-like suppression [65, 66, 67]. In fact, this is precisely the way that one can test the success of the Kobayashi-Maskawa mechanism for flavor and CP violation [4, 5, 6, 7, 8, 68, 69, 70, 71, 72, 73, 74, 75].

We start with the $B_d$ system, where the recent improvement in measurements has been particularly dramatic, as an example. The NP contributions to $B_d^0$ mixing can be expressed in terms of two parameters, $h_d$ and $\sigma_d$, defined by

$$M_{12}^{d} = (1 + h_d e^{2i\sigma_d})M_{12}^{d, SM},$$

(71)

where $M_{12}^{d, SM}$ is the dispersive part of the $B_d^0 - \bar{B_d}^0$ mixing amplitude in the SM.

In order to constrain deviations from the SM in these processes, one can use measurements which are directly proportional to $M_{12}^d$ (magnitude and phase). The relevant observables in this case are $\Delta m_{B_d}$ and the CPV in decay with and without mixing in $B_d^0 \rightarrow \psi K, S_{\psi K}$. These processes are characterized by hard GIM suppression, and proceed, within the SM, via one loop (see Eqs. (35) and (36)). In the presence of NP, they can be written as (see e.g. [30, 31, 32]):

$$\Delta m_{B_d} = \Delta m_{B_d}^{SM} |1 + h_d e^{2i\sigma_d}|,$$

$$S_{\psi K} = \sin [2\beta + \arg (1 + h_d e^{2i\sigma_d})].$$

(72)

The fact that the SM contribution to these processes involve the CKM elements which are not measured directly prevents one from independently constraining the NP contributions. Yet the situation was dramatically improved when BaBar and Belle experiments managed to measure CPV processes which, within the SM, are mediated via tree level amplitudes. The information extracted from these CP asymmetries in $B^\pm \rightarrow D K^\pm$ and $B \rightarrow \rho \rho$ is probably hardly affected by new physics. The most recent bounds (ignoring 2$\sigma$ anomaly in $B \rightarrow \tau \nu$) are [76, 77]

$$h_d \lesssim 0.3 \quad \text{and} \quad \pi \lesssim 2\sigma_d \lesssim 2\pi.$$  

(73)

Another example where recent progress has been achieved is in measurements of CPV in $D^0 - \bar{D}^0$ mixing, which led to an important improvement of the NP constraints. However, in this case the SM contributions are unknown [14, 15], and the only robust SM prediction is the absence of CPV [16]. The three relevant physical quantities related to the mixing can be defined as

$$y_{12} \equiv |\Gamma_{12}|/\Gamma, \quad x_{12} \equiv 2|M_{12}|/\Gamma, \quad \phi_{12} \equiv \arg (M_{12}/\Gamma_{12}),$$

(74)

where $M_{12}, \Gamma_{12}$ are the total dispersive and absorptive part of the $D^0 - \bar{D}^0$ amplitude, respectively. Fig. 9 shows (in grey) the allowed region in the $x_{12}^{NP}/x - \sin \phi_{12}^{NP}$ plane. $x_{12}^{NP}$ corresponds to the NP contributions and $x \equiv (m_2 - m_1)/\Gamma$, with $m_i, \Gamma$ being the neutral $D$ meson mass eigenstates and average width, respectively. The pink and yellow regions correspond to the ranges predicted by, respectively, the linear MFV and general MFV classes of models [18] (see Sec. 6 for details). We see that the absence of observed CP violation removes a sizable fraction of the possible NP parameter space, in spite of the fact that the magnitude of the SM contributions cannot be computed!

An updated analysis of $\Delta F = 2$ constraints has been presented in [7]. The main conclusions drawn from this analysis can be summarized as follows:

*Updated content*
Figure 9: The allowed region, shown in grey, in the $x_{12}^{\text{NP}} / x_{12} - \sin \phi_{12}^{\text{NP}}$ plane. The pink and yellow regions correspond to the ranges predicted by, respectively, the linear MFV and general MFV classes of models [18].

(i) In all the three accessible short distance amplitudes ($K^0\overline{K^0}$, $B_d\overline{B}_d$, and $B_s\overline{B}_s$) the magnitude of the NP amplitude cannot exceed the SM short distance contribution. The latter is suppressed by both the GIM mechanism and the hierarchical structure of the CKM matrix,

$$A_{\Delta F=2}^{\text{SM}} \approx \frac{G_F^2 m_t^2}{16 \pi^2} \left[ (V_{ti}^{\text{CKM}})^* V_{tj}^{\text{CKM}} \right]^2 \times \langle |(Q_{Li} \gamma^\mu Q_{Lj})|^2 |M| \rangle \times F \left( \frac{M_W^2}{m_t^2} \right),$$

where $F$ is a loop function of $\mathcal{O}(1)$. As a result, NP models with TeV scale flavored degrees of freedom and $\mathcal{O}(1)$ effective flavor mixing couplings are ruled out. To set explicit bounds, let us consider for instance the LH $\Delta F=2$ operator $Q_1$ from Eq. (59), and rewrite it as

$$\sum_{i \neq j} c_{ij} \Lambda^2 (Q_{Li} \gamma^\mu Q_{Lj})^2,$$

where the $c_{ij}$ are dimensionless couplings. The condition $|A_{\Delta F=2}^{\text{NP}}| < |A_{\Delta F=2}^{\text{SM}}|$ implies [78]

$$\Lambda > \frac{4.4 \text{ TeV}}{|(V_{ti}^{\text{CKM}})^* V_{tj}^{\text{CKM}}|/|c_{ij}|^{1/2}} \sim \begin{cases} 1.3 \times 10^4 \text{ TeV} \times |c_{sd}|^{1/2} \\ 5.1 \times 10^2 \text{ TeV} \times |c_{bd}|^{1/2} \\ 1.1 \times 10^2 \text{ TeV} \times |c_{bs}|^{1/2} \end{cases}$$

The strong bounds on $\Lambda$ for generic $c_{ij}$ of order 1 is a manifestation of what in many specific frameworks (supersymmetry, technicolor, etc.) goes by the name of the flavor problem: if we insist that the new physics emerges in the TeV region, then it must possess a highly non-generic flavor structure.

(ii) In the case of $B_d\overline{B}_d$ and $K^0\overline{K^0}$ mixing, where both CP conserving and CP violating observables are measured with excellent accuracy, there is still room for a sizable NP contribution (relative to the SM one), provided that it is, to a good extent, aligned in phase with the SM amplitude ($\mathcal{O}(0.01)$ for the $K$ system and $\mathcal{O}(0.3)$ for the $B_d$ system). This is because the theoretical errors in the observables used to constrain the phases, $S_{B_d\rightarrow \psi K}$ and $\epsilon_K$, are smaller with respect to the theoretical uncertainties in $\Delta m_{B_d}$ and $\Delta m_K$, which constrain the magnitude of the mixing amplitudes.
Table 1: Bounds on representative dimension six $\Delta F = 2$ operators (taken from [78], and the last line is from [58, 59]). Bounds on $\Lambda$ are quoted assuming an effective coupling $1/\Lambda^2$; or, alternatively, the bounds on the respective $c_{ij}$'s assuming $\Lambda = 1$ TeV. Observables related to CPV are separated from the CP conserving ones with semicolons. In the $B_s$ system we only quote a bound on the modulo of the NP amplitude derived from $\Delta m_{B_s}$ (see text). For the definition of the CPV observables in the $D$ system see Ref. [10].

(iii) In the case of $B_s$-$B_s$ mixing, the precise determination of $\Delta m_{B_s}$ does not allow large deviations in modulo with respect to the SM. The constraint is particularly severe if we consider the ratio $\Delta m_{B_s}/\Delta m_{B_s}$, where hadronic uncertainties cancel to a large extent. However, the constraint on the CPV phase is quite poor. Present data from CDF [79] and D0 [80] indicate a large central value for this phase, contrary to the SM expectation. The errors are, however, still large, and the disagreement with the SM is at about the 2$\sigma$ level. If the disagreement persists, and becomes statistically significant, this would not only signal the presence of physics beyond the SM, but would also rule out a whole subclass of MFV models (see Sec. 6).

(iv) The resulting constraints in the $D$ system discussed above are only second to those from $\epsilon_K$, and unlike the case of $\epsilon_K$, they are controlled by experimental statistics, and could possibly be significantly improved in the near future.

To summarize this discussion, a detailed list of constraints derived from $\Delta F = 2$ observables is shown in Table 1, where we quote the bounds for two representative sets of dimension six operators – the left-left operators (present also in the SM) and operators with a different chirality, which arise in specific SM extensions ($Q_1$ and $Q_4$ from Eq. [59], respectively). The bounds on the latter are stronger, especially in the Kaon case, because of the larger hadronic matrix elements and enhanced renormalization group evolution (RGE) contributions. The constraints related to CPV correspond to maximal phases, and are subject to the requirement that the NP contributions are smaller than 30% (60%) of the total contributions [4, 5] in the $B_d$ ($K$) system (see Eq. [73]). Since the experimental status of CP violation in the $B_s$ system is not yet settled, we simply require that the NP contributions would be smaller than the observed value of $\Delta m_{B_s}$ (for less naive treatments see e.g. [7, 81], and for a different type of $\Delta F = 2$ analysis see [82]).
5.2 Robust bounds immune to alignment mechanisms

There are two interesting features for models that can provide flavor-related suppression factors: degeneracy and alignment. The former means that the operators generated by the NP are flavor-universal, that is diagonal in any basis, thus producing no flavor violation. Alignment, on the other hand, occurs when the NP contributions are diagonal in the corresponding quark mass basis. In general, low energy measurements can only constrain the product of these two factors. An interesting exception occurs, however, for the left-left (LL) operators of the type defined in Eq. (39), where there is an independent constraint on the level of degeneracy [23]. The crucial point is that operators involving only quark doublets cannot be simultaneously aligned with both the down and the up mass bases. For example, we can take $X_Q$ from Eq. (39) to be proportional to $A_{Q^u}$. Then it would be diagonal in the down mass basis, but it would induce flavor violation in the up sector. Hence, these types of theories can still be constrained by measurements. The “best” alignment is obtained by choosing the NP contribution such that it would minimize the bounds from both sectors. The strength of the resulting constraint, which is the weakest possible one, is that it is unavoidable in the context of theories with only one set of quark doublets. Here we briefly discuss this issue, and demonstrate how to obtain such bounds.

5.2.1 Two generation $\Delta F = 2$ transitions

As mentioned before, the strongest experimental constraints involve transitions between the first two generations. When studying NP effects, ignoring the third generation is often a good approximation to the physics at hand. Indeed, even when the third generation does play a role, a two generations framework is applicable, as long as there are no strong cancelations with contributions related to the third generation. Hence, for this analysis we can use the formalism of Sec. (4.1).

The operator defined in Eq. (39), when restricted to the first two generations, induces mixing in the $K$ and $D$ systems, and possibly also CP violation. We can use the covariant bases defined in Eq. (40) to parameterize $X_Q$,

$$X_Q = L \left( X^{u,d} \hat{A}_{Q^u,Q^d} + X^J \hat{J} + X^{J_{u,d}} \hat{J}_{u,d} \right),$$

(78)

and the two bases are related through

$$X^u = \cos 2\theta C X^d - \sin 2\theta C X^{J_d}, \quad X^{J_u} = -\sin 2\theta C X^d - \cos 2\theta C X^{J_d},$$

(79)

while $X^J$ remains invariant. We choose the $X'$ coefficients to be normalized,

$$(X^d)^2 + (X^J)^2 + (X^{J_d})^2 = (X^u)^2 + (X^J)^2 + (X^{J_u})^2 = 1,$$

(80)

such that $L$ signifies the “length” of $X_Q$ under the definitions in Eq. (38),

$$L = |X_Q| = (X^d_Q - X^u_Q) / 2,$$

(81)

where $X^1,2_Q$ are the eigenvalues of $X_Q$ before removing the trace.

Plugging Eqs. (78) and (79) into Eq. (41), we obtain expressions for the contribution of $X_Q$ to $\Delta m_K$ and $\Delta m_D$, without CPV,

$$C^K_1 = L^2 \left[ (X^J)^2 + (X^{J_d})^2 \right],$$

(82)

$$C^D_1 = L^2 / 2 \left[ 2(X^J)^2 + (X^d)^2 + (X^{J_d})^2 + \left((X^{J_d})^2 - (X^d)^2\right) \cos(4\theta C) + 2X^dX^{J_d} \sin(4\theta C) \right].$$

28
In order to minimize both contributions, we first need to set $X^J = 0$. Next we define

$$\tan \alpha \equiv \frac{X^J_d}{X^d}, \quad r_{KD} \equiv \sqrt{\frac{(C^K_1)_{\text{exp}}}{(C^D_1)_{\text{exp}}}},$$

where the experimental constraints $(C^K_1)_{\text{exp}}$ and $(C^D_1)_{\text{exp}}$ can be extracted from Table I. Then the weakest bound is obtained for

$$\tan \alpha = \frac{r_{KD} \sin(2\theta_C)}{1 + r_{KD} \cos(2\theta_C)},$$

and is given by

$$L \leq 3.8 \times 10^{-3} \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right).$$

(83)

(84)

(85)

A similar process can be carried out for the CPV in $K$ and $D$ mixing, by plugging Eqs. (78) and (79) into Eq (42). Now we do not set $X^J = 0$, otherwise there would be no CPV (since $X_Q$ would reside in the same plane as $A_{Qd}$ and $A_{Qu}$). Moreover, there are many types of models in which we can tweak the alignment, but we do not control the phase (we do not expect the NP to be CP-invariant), hence they might give rise to CPV. The weakest bound in this case, as a function of $X^J$, is given by

$$L \leq \frac{3.4 \times 10^{-4}}{[(X^J)^2 - (X^J)^4]^{1/4}} \left( \frac{\Lambda_{\text{NP}}}{1 \text{ TeV}} \right).$$

(86)

The combination of the above two bounds is presented in Fig. 10.

Figure 10: The weakest upper bound on $L$ coming from flavor and CPV in the $K$ and $D$ systems, as a function of the CP violating parameter $X^J$, assuming $\Lambda_{\text{NP}} = 1$ TeV. The figure is taken from [23].

We should note that $L$ is simply the difference between the eigenvalues of $X_Q$ (see Eq. (81)), thus the bounds above put limits on the degeneracy of the NP contribution.
5.2.2 Third generation $\Delta F = 1$ transitions

Similar to the analysis of the previous subsection, we can use other types of processes to obtain model independent constrains on new physics. Here we consider flavor violating decays of third generation quarks in both sectors, utilizing the three generations framework discussed in Sec. 4.2. Since the existing bound on top decay is rather weak, we use the projection for the LHC bound, assuming that no positive signal is obtained.

We focus on the following operator

$$O_{LL}^h = i \left[ \overline{Q}_i \gamma^\mu (X_Q)_{ij} Q_j \right] \left[ H^\dagger \tilde{D}_\mu H \right] + \text{h.c.},$$

(87)

which contributes at tree level to both top and bottom decays [83]. We omit an additional operator for quark doublets, $O_{u,LL}^u = i \left[ \overline{Q}^3 \tilde{H} \left[ (\not{D} \tilde{H})^\dagger Q^2 \right] - i \left[ \overline{Q}^3 (\not{D} \tilde{H}) \right] \tilde{H}^\dagger Q^2 \right]$, which induces bottom decays only at one loop, but in principle it should be included in a more detailed analysis.

The experimental constraints we use are [84, 85, 86]

\[
\begin{align*}
\text{Br}(B \rightarrow X_s \ell^+ \ell^-) &< 1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2 = (1.61 \pm 0.51) \times 10^{-6}, \\
\text{Br}(t \rightarrow (c,u)Z) &< 5.5 \times 10^{-5},
\end{align*}
\]

(88)

where the latter corresponds to the prospect of the LHC bound in the absence of signal for 100 fb$^{-1}$ at a center of mass energy of 14 TeV. We adopt the weakest limits on the coefficient of the operator in Eq. (87), $C_{LL}^h$, derived in [83]:

\[
\begin{align*}
\text{Br}(B \rightarrow X_s \ell^+ \ell^-) &\rightarrow |C_{LL}^h|_b < 0.018 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2, \\
\text{Br}(t \rightarrow (c,u)Z) &\rightarrow |C_{LL}^h|_t < 0.18 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2,
\end{align*}
\]

(89)

and define $r_t^b \equiv |C_{LL}^h|_t / |C_{LL}^h|_b$.

The NP contribution can be decomposed in the covariant bases as

$$X_Q = L \left( X^{u,d} A_{Q^u,Q^d} + X^J \tilde{J} + X^{J_u,d} \tilde{J}_{u,d} + X^{J_q} \tilde{J}_q + X^{\tilde{D}\tilde{D}} \right),$$

(90)

where again the coefficients are normalized such that $L = |X_Q|$. The contribution of $X_Q$ to third generation decays is given by Eq. (50). The weakest bound for a fixed $L$ is obtained, as before, by finding a direction of $X_Q$ that minimizes the contributions to $|C_{LL}^h|_t$ and $|C_{LL}^h|_b$, thus constituting the “best” alignment. However, since $\tilde{J}_Q$ commutes with $A_{Q^u,Q^d}$, as discussed above, it does not contribute to third generation decay in neither sectors. In other words, $X_Q \propto \tilde{J}_Q$ is not constrained by such a process. On the other hand, any component of $X_Q$ may also generate flavor violation among the first two generations (when their masses are switched back on), which is more strongly constrained. Specifically, the bound that stems from the case of $X_Q \propto \tilde{J}_Q$ is [59]

$$L < 0.59 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2; \quad \Lambda_{NP} > 1.7 \text{ TeV},$$

(91)

where the latter is for $L = 1$. This is stronger than the limit given below for other forms of $X_Q$, hence this does not constitute optimal alignment. To conclude this issue, all directions that
contribute to first two generations flavor and CPV at $\mathcal{O}(\lambda)$, that is $\hat{J}_Q$, $\hat{D}$ and $\hat{A}'_{Q_u, Q_d}$, are not favorable in terms of alignment [59].

The induced third generation flavor violation, after removing these contributions, is then given by

$$4 \left| X_Q \times \hat{A}_{Q_u, Q_d} \right|^2 = (X^J)^2 + (X^{J_u, d})^2,$$

and in order to see this in a common basis, we express $X^{J_u}$ as

$$X^{J_u} = \cos 2\theta X^{J_d} + \sin 2\theta X'_{d},$$

with $\theta$ as defined in Eq. (45). From this it is clear that $X^J$ contributes the same to both the top and the bottom decay rates, so it should be set to zero for optimal alignment. Thus the best alignment is obtained by varying $\alpha$, defined as before by

$$\tan \alpha \equiv \frac{X^{J_d}}{X'^d}.$$ (94)

Here we use $X'^d$, which is the coefficient of $\hat{A}_{Q_d}$, instead of $X''_d$, since the former does not produce flavor violation among the first two generations to leading order (up to $\mathcal{O}(\lambda^5)$).

We now consider two possibilities: (i) complete alignment with the down sector; (ii) the best alignment satisfying the bounds of Eq. (89), which gives the weakest unavoidable limit. Note that we can also consider up alignment, but it would give a stronger bound than down alignment, as a result of the stronger experimental constraints in the down sector. The bounds for these cases are [58] [59]

(i) $\alpha = 0, \quad L < 2.5 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2; \quad \Lambda_{NP} > 0.6 \ (7.9) \ \text{TeV},$

(ii) $\alpha = \frac{\sqrt{3} \theta}{1 + r_{tb}}, \quad L < 2.8 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2; \quad \Lambda_{NP} > 0.6 \ (7.6) \ \text{TeV},$ 

as shown in Fig. 11, where in parentheses we give the strong coupling bound, in which the coefficient of the operators in Eqs. (39) and (87) is assumed to be $16\pi^2$. Note that these are weaker than the bound in Eq. (91).

It is important to mention that the optimized form of $X_Q$ generates also $c \rightarrow u$ decay at higher order in $\lambda$, which might yield stronger constraints than the top decay. However, the resulting bound from the former is actually much weaker than the one from the top [59]. Therefore, the LHC is indeed expected to strengthen the model independent constraints.

5.2.3 Third generation $\Delta F = 2$ transitions

Finally, we analyze $\Delta F = 2$ transitions involving the bottom and the top. For simplicity, we only consider complete alignment with the down sector

$$X_Q = L \hat{A}_{Q_d},$$ (96)

as the constraints from this sector are much stronger. This generates in the up sector top flavor violation, and also $D^0 - \bar{D}^0$ mixing at higher order. Yet there is no top meson, as the top quark decays too rapidly to hadronize. Instead, we analyze the process $pp \rightarrow tt$ (related to mixing by
crossing symmetry), which is most appropriate for the LHC. It should be emphasized, however, that in this case the parton distribution functions of the proton strongly break the approximate $U(2)_Q$ symmetry of the first two generations. The simple covariant basis introduced in Sec. 4.2.1, which is based on this approximate symmetry, cannot be used as a result. Furthermore, this LHC process is dominated by $uu \rightarrow tt$, so we focus only on the operator involving up (and not charm) quarks.

The bound that would stem from this process at the LHC was evaluated in [58, 59] to be

$$C_{tt}^1 < 7.1 \times 10^{-3} \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right)^2,$$

for 100 fb$^{-1}$ at 14 TeV. Since the form of $X_Q$ that we consider also contributes to transitions between the first two generations, we should additionally take into account the experimental constraints in the $D$ system, given in Table 1 (we use the CPV observable).

The contribution of $X_Q$ to these processes is calculated by applying a CKM rotation to Eq. (96). CPV in the $D$ system is then given by $\text{Im} \left[ (X_Q)_{12}^2 \right]$, and $\left| (X_Q)_{13} \right|^2$ describes $uu \rightarrow tt$. Note that we have

$$\begin{align*}
(X_Q)_{12} & \approx -\sqrt{3} LV_{ub}^{\text{CKM}} (V_{cb}^{\text{CKM}})^* , \\
(X_Q)_{13} & \approx -\sqrt{3} LV_{ub}^{\text{CKM}} (V_{tb}^{\text{CKM}})^* .
\end{align*}$$

The resulting bounds are

$$L < 12 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right) ; \quad \Lambda_{NP} > 0.08 (1.0) \text{ TeV} ,$$

for $uu \rightarrow tt$ and

$$L < 1.8 \left( \frac{\Lambda_{NP}}{1 \text{ TeV}} \right) ; \quad \Lambda_{NP} > 0.57 (7.2) \text{ TeV} ,$$
for $D$ mixing.

The limits in Eqs. (99) and (100) can be further weakened by optimizing the alignment between the down and the up sectors, as in the previous subsection. Since this would only yield a marginal improvement of about 10%, we do not analyze this case in detail.

To conclude, we learn that for $\Delta F = 2$ processes, the existing bound is stronger than the one which will be obtained at the LHC for top quarks, as opposed to $\Delta F = 1$ case considered above.

6 Minimal flavor violation

As we have seen above, SM extensions with general flavor structure are strongly constrained by measurements. This is a consequence of the fact that, within the SM, flavor conversion and CP violation arise in a unique and suppressed manner. It is therefore valuable to investigate beyond the SM theories, where the breaking of the global flavor symmetries is induced by the same source as in the SM. In such models, which go under the name of minimal flavor violation (MFV), flavor violating interactions are only generated by the SM Yukawa couplings (see e.g. [37, 87, 88, 89, 90, 91]). Although we only consider here the quark sector, the notion of MFV can be extended also to the lepton sector. However, there is no unique way to define the minimal sources of flavor symmetry breaking, if one wants to keep track of non-vanishing neutrino masses [92, 93, 94].

In addition to the suppression of FCNCs, there are two important aspects of the MFV framework: First, low energy flavor conversion processes can be described by a small set of operators in an effective Lagrangian, without reference to a specific model. Furthermore, MFV arises naturally as a low energy limit of a sizable class of models, in which the flavor hierarchy is generated at a scale much higher than other dynamical scales. Examples of microscopic theories which flow to MFV at the IR are supersymmetric models with gauge or anomaly mediation [95, 96, 97, 98] and a certain class of warped extra dimension models [99, 100, 101, 102, 103, 104].

The basic idea can be described in the language of effective field theory, without the need of referring to a specific framework. MFV models can have a very different microscopical dynamics, yet by definition they all have a common origin of flavor breaking—the SM Yukawa matrices. After integrating out the NP degrees of freedom, we expect to obtain a low energy effective theory which involves only the SM fields and a bunch of higher dimension, Lorentz and gauge invariant, operators suppressed by the NP scale $\Lambda_{MFV}$. Since flavor is broken only via the SM Yukawas, we can study the flavor breaking of the MFV framework by the simple following prescription: We should construct the most general set of higher dimensional operators, which in addition of being Lorentz and gauge invariant, they are required also to be flavor invariant, using the spurion analysis that we have introduced above. A simple example for such an operator is the one from Eq. (39), where the matrix that mixes the generations is a combination of the appropriate Yukawa matrices,

$$O_{1}^{MFV} = \frac{1}{(\Lambda_{MFV})^2} \left[ \bar{Q}_i \left( a_u A_{Q^u} + a_d A_{Q^d} + \ldots \right)_{ij} \gamma_\mu Q_j \right]^2. \quad (101)$$

Here the dots represent higher order terms in $A_{Q^u, Q^d}$.

It is important to realize that quite often, in models which exhibit MFV-like behavior, the Yukawa couplings are associated with constant factors, such that they appear as $x_U Y_U, x_D Y_D$. These factors might come from loop suppression, RGE etc. In general they are not necessarily small, since for example large Logs from RGE flow might compensate for the loop suppression. We should thus consider these “effective” Yukawa couplings $x_i Y_i$, rather than just $Y_i$, as operators would usually involve $f(x_U Y_U, x_D Y_D)$.
To get a further insight on the structure of MFV models, it is useful to classify the framework according to the strength of breaking of the individual flavor group components. Since, within the SM, and also as suggested by the data, the only source of CP breaking is the CKM phase, it is also useful to extend $G_{SM}^C$ of Eq. (2) to include CP as a discrete group:

$$G_{SM}^C = U(3)_Q \times U(3)_U \times U(3)_D \times CP.$$  \hspace{1cm} (102)

The low energy phenomenology of MFV models can be divided as follows (see below for details):

(i) Small effective Yukawas – The SM flavor group, $G_{SM}^C$, is approximately preserved by the NP dynamics, in the sense that all the effective Yukawa couplings are small

$$|x_U y_U^i|, |x_D y_D^i| \ll 1.$$  \hspace{1cm} (103)

(ii) Large effective top Yukawa – The effective down type Yukawas are still small, but the top coupling is $O(1)$.

(iii) Large third generation Yukawas – Both the top and the bottom effective Yukawa couplings are large. This can happen for instance in two Higgs doublet models (see e.g. [105, 106, 107, 108]) with large $tan \beta$, but also in theories with only one Higgs doublet, but a large $x_D$ factor. However, in this case CP is only broken by the up and down Yukawa matrices, hence no extra sources of flavor diagonal phases are present in the microscopic theory.

(iv) Large effective Yukawas and flavor diagonal phases – This is the most general case, where both the top and bottom Yukawa couplings are large and new flavor diagonal CP violating phases are present. It is thus denoted as general MFV (GMFV) [40].

Obviously, within the MFV framework, a built-in approximate $U(2)$ symmetry for the first two generation is guaranteed for the low energy phenomenology [40]. As we shall see, the models which belong to the MFV class, especially in the cases of (i)-(iii), enjoy much of the protection against large flavor violation that we have found to exist in the SM case, and therefore tend to be consistent with current flavor precision measurements (for reviews see e.g. [55, 56, 57] and refs. therein).

6.1 MFV with small effective bottom Yukawa

Here we deal with the first two cases (i)+(ii), where $x_D y_D \ll 1$. In the following we absorb the $x$ factors into the Yukawa for simplicity of notation.

6.1.1 Small effective top Yukawa

If we are interested in SM processes where the typical energy scale is much lower than $\Lambda_{MFV}$ and the NP is not strongly coupled, then we expect that the dominant non-SM flavor violation would arise from the lowest order higher dimension operators. For processes involving quark fields, the leading operators are of dimension six. Consider, for instance, the following $\Delta F = 2$ MFV Lagrangian:

$$L_{MFV}^{\Delta F=2} = \frac{1}{(\Lambda_{MFV})^2} \left[ \overline{Q}_i \left( a_u A_{Qu} + a_d A_{Qd} \right)_{ij} Q_j \right]^2 + \frac{1}{(\Lambda_{MFV})^2} \left[ \overline{Q}_i \left( b A_{Qu} Y_D \right)_{ij} D_j \right]^2 + \ldots,$$  \hspace{1cm} (104)
where we write a LL operator and a LR operator for down quarks, and we assume that they are both suppressed by the same MFV scale\(^\text{17}\). We can immediately reach two conclusions: First, the LR operator is subdominant, since its lowest order flavor violating contribution contains three Yukawa matrices, compared to two for the LL operator (note that a term of the form \(\bar{Q}_i (Y_D)_{ij} D_j\) does not induce down flavor conversion). Next, we only need to take into account the leading terms, as a result of the small effective Yukawas. Therefore, this case can be named linear MFV (LMFV) \(^{40}\).

Let us, for instance, focus on flavor violation in the down sector, which is more severely constrained. We want to estimate what is the size of flavor violation which is mediated by \(L_{\Delta F=2}^{\text{MFV}}\), restricting ourselves to the first operator for now. The experimental information is obtained by looking at the dynamics (masses, mass differences, decay, time evolution etc.) of down type mesons, hence we can just look at the form that \(L_{\Delta F=2}^{\text{MFV}}\) takes in the down quark mass basis. By definition \(A_{Q^d}\) is then diagonal, and does not mediate flavor violation, but \(A_{Q^u}\) is not diagonal and is given by

\[
(A_{Q^u})_{\text{down}} = V^{\text{CKM}} \text{diag}(0, 0, y_t^2) (V^{\text{CKM}})^\dagger - \frac{y_t^2}{3} \mathbb{1}_3 + \mathcal{O}\left(\frac{m_c^2}{m_t^2}\right) \approx y_t^2 V^{\text{CKM}}_{ti} (V^{\text{CKM}}_{tj})^* ,
\]

where we take advantage of the approximate \(U(2)\) symmetry discussed before. As expected, we find that within the MFV framework, FCNC processes are suppressed by roughly the same amount as the SM processes, and therefore are typically consistent with present data, at least to leading order.

Within the LMFV framework, several of the constraints used to determine the CKM matrix (and in particular the unitarity triangle) are not affected by NP \(^{90, 91}\). In this context, NP effects are negligible not only in tree level processes, but also in a few clean observables sensitive to loop effects, such as the time dependent CP asymmetry in \(B_d \to \psi K_{L,S}\). Indeed the structure of the basic flavor changing coupling which results from Eqs. (104) and (105) implies that the weak CPV phase of the operator \((\bar{b}d)^2\), related to \(B_d - \bar{B}_d\) mixing, is \(\arg\left[ (V^{\text{CKM}}_{td} (V^{\text{CKM}}_{tb})^*)^2 \right] \), exactly as in the SM. This construction provides a natural (\textit{a posteriori}) justification of why no NP effects have been observed in the quark sector: By construction, most of the clean observables measured at \(B\) factories are insensitive to NP effects in the LMFV framework.

In Table 2, we report a few representative examples of bounds on higher dimensional operators in the LMFV framework. For simplicity, only leading spurion dependence is shown on the left handed column. The built-in CKM suppression leads to bounds on the effective NP scale not far from the TeV region. These bounds are very similar to the bounds on flavor conserving operators derived by precision electroweak tests. This observation reinforces the conclusion that a deeper study of rare decays is definitely needed in order to clarify the flavor problem: The experimental precision on the clean FCNC observables required to obtain bounds more stringent than those derived from precision electroweak tests (and possibly discover new physics) is typically in the \(1 - 10\%\) range. Table 3 demonstrates that discriminating between the SM and a theory with LMFV behavior is expected to be a difficult task.

### 6.1.2 Large effective top Yukawa

The consequence of a large effective top Yukawa, \(x_U y_t \gtrsim 1\), is the need to take into account higher order terms in the up Yukawa matrix, and resum all these terms to a single effective contribution.

\(^{17}\text{Strictly speaking, this does not have to be the case, as these operators might be generated by different processes in the underlying theory.}\)
Table 2: Bounds on the NP scale (at 95% C.L.) for some representative $\Delta F = 1$ [109] and $\Delta F = 2$ [7] MFV operators (assuming effective coupling $\pm 1/\Lambda^2$), and corresponding observables used to set the bounds.

| Operator | Bound on $\Lambda_{\text{MFV}}$ | Observables |
|----------|----------------------------------|-------------|
| $H^1 \left( \overline{D}_R Y^+_{D} A Q^u \sigma_{\mu \nu} Q_L \right) (e F_{\mu \nu})$ | 6.1 TeV | $B \to X_s \gamma, B \to X_s \ell^+ \ell^-$ |
| $\frac{1}{2} (\overline{Q}_L A Q^u \gamma_{\mu} Q_L)^2$ | 5.9 TeV | $\epsilon_K, \Delta m_{B_d}, \Delta m_{B_s}$ |
| $H^1_D \left( \overline{D}_R Y^+_{D} A Q^u \sigma_{\mu \nu} T^a Q_L \right) (g_s G^a_{\mu \nu})$ | 3.4 TeV | $B \to X_s \gamma, B \to X_s \ell^+ \ell^-$ |
| $(\overline{Q}_L A Q^u \gamma_{\mu} Q_L) (E_R \gamma_{\mu} E_R)$ | 2.7 TeV | $B \to X_s \ell^+ \ell^-, B_s \to \mu^+ \mu^-$ |
| $i (\overline{Q}_L A Q^u \gamma_{\mu} Q_L) H^0 D_{\mu} H_U$ | 2.3 TeV | $B \to X_s \ell^+ \ell^-, B_s \to \mu^+ \mu^-$ |
| $(\overline{Q}_L A Q^u \gamma_{\mu} Q_L) (\overline{L}_L \gamma_{\mu} L_L)$ | 1.7 TeV | $B \to X_s \ell^+ \ell^-, B_s \to \mu^+ \mu^-$ |
| $(\overline{Q}_L A Q^u \gamma_{\mu} Q_L) (e D_{\mu} F_{\mu \nu})$ | 1.5 TeV | $B \to X_s \ell^+ \ell^-$ |

Table 3: Some predictions derived in the LMFV framework, compared to the SM [78].

| Observable | Experiment | LMFV prediction | SM prediction |
|------------|------------|----------------|--------------|
| $\beta_s$ from $\mathcal{A}_{\text{CP}} (B_s \to \psi \phi)$ | [0.10, 1.44] @ 95% CL | 0.04(5) | 0.04(2) |
| $\mathcal{A}_{CP} (B \to X_s \gamma)$ | < 6% @ 95% CL | < 0.02 | < 0.01 |
| $B (B_d \to \mu^+ \mu^-)$ | $< 1.8 \times 10^{-8}$ | $< 1.2 \times 10^{-9}$ | $1.3(3) \times 10^{-10}$ |
| $B (B \to X_s \tau^+ \tau^-)$ | $< 5 \times 10^{-7}$ | $1.6(5) \times 10^{-7}$ |
| $B (K_L \to \pi^0 \nu \bar{\nu})$ | $< 2.6 \times 10^{-8}$ @ 90% CL | $< 2.9 \times 10^{-10}$ | $2.9(5) \times 10^{-11}$ |

However, the results derived for the LMFV case are in principle still valid for a large effective top Yukawa.

Yet, one subtlety does arise in this case: Contributions to $1 \to 2$ transitions which proceed through the charm and the top are correlated within LMFV, but are independent in the current case (see Sec. 6.3 below, and specifically the discussion around Eqs. (123) and (124)). Distinguishing between these cases can be achieved by comparing $K^+ \to \pi^+ \nu \bar{\nu}$ and the CPV decay $K_L \to \pi^0 \nu \bar{\nu}$, or via $\epsilon_K$. This needs to be accomplished both theoretically and experimentally to the level of $O(m_t^2/m_s^2)$. Unfortunately, the smallness of this difference prevents tests of the first in the near future, while the second is masked by long distance contributions at the level of a few percents [110]. Nevertheless, the ability to discriminate between these two cases is of high theoretical importance, since it yields information about short distance physics (such as the mediation scale of supersymmetry breaking via the Log’s size or anomalous dimensions) well beyond the direct reach of near future experiments.

6.2 Large bottom Yukawa

The effects of a large effective bottom Yukawa usually appear in two Higgs doublet models (such as supersymmetry), but they can also be found in other NP frameworks without an extended Higgs sector, where $x_D y_b$ is of order one due to a large value of $x_D$. In any case, we can still assume that the Yukawa couplings are the only irreducible breaking sources of the flavor group.

For concreteness, we analyze the case of a two Higgs doublet model, which is described by the Lagrangian in Eq. (1) (focusing only on the quark sector) with independent $H_U$ and $H_D$. This Lagrangian is invariant under an extra $U(1)$ symmetry with respect to the one Higgs case – a symmetry under which the only charged fields are $D$ (charge +1) and $H_D$ (charge −1). This
symmetry, denoted $U(1)_{PQ}$, prevents tree level FCNCs, and implies that $Y_{U,D}$ are the only sources of flavor breaking appearing in the Yukawa interaction (similar to the one Higgs doublet scenario). By assumption, this also holds for all the low energy effective operators. This is sufficient to ensure that flavor mixing is still governed by the CKM matrix, and naturally guarantees a good agreement with present data in the $\Delta F = 2$ sector. However, the extra symmetry of the Yukawa interaction allows us to change the overall normalization of $Y^{U,D}$ with interesting phenomenological consequences in specific rare modes.

The normalization of the Yukawa couplings is controlled by the ratio of the vacuum expectation values of the two Higgs fields, or by the parameter

$$\tan \beta = \langle H_U \rangle / \langle H_D \rangle.$$  \hspace{1cm} (106)

For $\tan \beta \gg 1$, the smallness of the $b$ quark (and $\tau$ lepton) mass can be attributed to the smallness of $1/\tan \beta$, rather than to the corresponding Yukawa coupling. As a result, for $\tan \beta \gg 1$ we cannot anymore neglect the down type Yukawa coupling. Moreover, the $U(1)_{PQ}$ symmetry cannot be exact – it has to be broken at least in the scalar potential in order to avoid the presence of a massless pseudoscalar Higgs. Even if the breaking of $U(1)_{PQ}$ and $G_{PQ}^{SM}$ are decoupled, the presence of $U(1)_{PQ}$ breaking sources can have important implications on the structure of the Yukawa interaction, especially if $\tan \beta$ is large \cite{37, 111, 112, 113}.

Since the $b$ quark Yukawa coupling becomes $O(1)$, the large $\tan \beta$ regime is particularly interesting for helicity-suppressed observables in $B$ physics. One of the clearest phenomenological consequences is a suppression (typically in the $10 - 50\%$ range) of the $B \to \ell \nu$ decay rate with respect to its SM expectation \cite{114, 115, 116}. Potentially measurable effects in the $10 - 30\%$ range are expected also in $B \to X_{s}\gamma$ \cite{117, 118, 119} and $\Delta M_{B_s}$ \cite{120, 121}. Given the present measurements of $B \to \ell \nu, B \to X_{s}\gamma$ and $\Delta M_{B_s}$, none of these effects seems to be favored by data. However, present errors are still sizable compared to the estimated NP effects.

The most striking signature could arise from the rare decays $B_{s,d} \to \ell^+ \ell^-$, whose rates could be enhanced over the SM expectations by more than one order of magnitude \cite{122, 123, 124}. An enhancement of both $B_s \to \ell^+ \ell^-$ and $B_d \to \ell^+ \ell^-$ respecting the MFV relation $\Gamma(B_s \to \ell^+ \ell^-)/\Gamma(B_d \to \ell^+ \ell^-) \approx |V_{ts}^{CKM}/V_{td}^{CKM}|^2$ would be an unambiguous signature of MFV at large $\tan \beta$ \cite{109}.

Dramatic effects are also possible in the up sector. The leading contribution of the LL operator to $D - \bar{D}$ mixing is given by

$$C_{1}^{SM} \propto \left[ y_s^2 \left( V_{cs}^{CKM} \right)^* V_{us}^{CKM} + (1 + r_{GMFV}) y_b^2 \left( V_{cb}^{CKM} \right)^* V_{ub}^{CKM} \right]^2 \sim 3 \times 10^{-8} \zeta_1,$$  \hspace{1cm} (107)

for $\tan \beta \sim m_t/m_b$, where $r_{GMFV}$ accounts for the necessary resummation of the down Yukawa, and is expected to be an order one number. In such a case, the simple relation between the contribution from the strange and bottom quarks does not apply \cite{40}. We thus have

$$\zeta_1 = e^{2i\gamma} + 2 r_{sb} e^{i\gamma} + r_{sb}^2 \sim 1.7 i + r_{GMFV} \left[ 2.4i - 1 - 0.7 r_{GMFV} (1 + i) \right],$$  \hspace{1cm} (108)

$$r_{sb} \equiv \frac{y_s^2}{y_b^2} \left| \frac{V_{us}^{CKM} V_{cb}^{CKM}}{V_{us}^{CKM} V_{cb}^{CKM}} \right| \sim 0.5,$$

where $\gamma \approx 67^\circ$ is the relevant phase of the unitarity triangle. We thus learn that MFV models with two Higgs doublets can contribute to $D - \bar{D}$ mixing up to $O(0.1)$ for very large $\tan \beta$, assuming a TeV NP scale. Moreover, the CPV part of these contributions is not suppressed compared to the CP conserving part, and can provide a measurable signal. In Fig. 9 we show in pink (yellow) the
range predicted by the LMFV (GMFV) class of models. The GMFV yellow band is obtained by scanning the range \( r_{\text{GMFV}} \in (-1, +1) \) (but keeping the magnitude of \( C_1^\text{lin} \) fixed for simplicity).

Sizeable contributions to top FCNC can also emerge for large \( \tan \beta \). For a MFV scale of \( \sim 1 \) TeV, this can lead to \( \text{Br}(t \rightarrow cX) \sim \mathcal{O}(10^{-5}) \) \(^{[40]}\), which may be within the reach of the LHC.

### 6.3 General MFV

The breaking of the \( G^\text{SM} \) flavor group and the breaking of the discrete CP symmetry are not necessarily related, and we can add flavor diagonal CPV phases to generic MFV models \(^{[60, 61, 125]}\). Because of the experimental constraints on electric dipole moments (EDMs), which are generally sensitive to such flavor diagonal phases \(^{[61]}\), in this more general case the bounds on the NP scale are substantially higher with respect to the “minimal” case, where the Yukawa couplings are assumed to be the only breaking sources of both symmetries \(^{[37]}\).

If \( \tan \beta \) is large, the inclusion of flavor diagonal phases has interesting effects also in flavor changing processes \(^{[126, 127, 128]}\). The main consequences, derived in a model independent manner, can be summarized as follows \(^{[40]}\): (i) extra CPV can only arise from flavor diagonal CPV sources in the UV theory; (ii) the extra CP phases in \( B_s - \bar{B}_s \) mixing provide an upper bound on the amount of CPV in \( B_d - \bar{B}_d \) mixing; (iii) if operators containing RH light quarks are sub-dominant, then the extra CPV is equal in the two systems, and is negligible in \( 2 \rightarrow 1 \) transitions. Conversely, these operators can break the correlation between CPV in the \( B_s \) and \( B_d \) systems, and can induce significant new CPV in \( \epsilon_K \).

We now analyze in detail this general MFV case, where both top and bottom effective Yukawas are large and flavor diagonal phases are present, to prove the above conclusions. We emphasize the differences between the LMFV case and the non-linear MFV (NLMFV) one. It is shown below that even in the general scenario, there is a systematic expansion in small quantities, \( V^\text{CKM}_{\text{td}}, V^\text{CKM}_{\text{ts}} \), and light quark masses, while resumming in \( y_t \) and \( y_b \). This is achieved via a parametrization borrowed from non-linear \( \sigma \)-models\(^{18}\). Namely, in the limit of vanishing weak gauge coupling (or \( m_W \rightarrow \infty \)), \( U(3)_Q \) is enhanced to \( U(2)_Q \times U(1)_{Q^d} \), as discussed in Sec. \( \text{[3]} \). The two groups are broken down to \( U(2) \times U(1) \) by large third generation eigenvalues in \( A_{Q^u, Q^d} \), so that the low energy theory is described by a \([U(3)/U(2) \times U(1)]^2\) non-linear \( \sigma \)-model. Flavor violation arises due to the misalignment of \( Y_U \) and \( Y_D \), given by \( V^\text{CKM}_{\text{td}} \) and \( V^\text{CKM}_{\text{ts}} \), once the weak interaction is turned on. It should be stressed that while below we implicitly assume a two Higgs doublet model to allow for a large bottom Yukawa coupling, this assumption is not necessary, and the analysis is essentially model independent.

As discussed in Sec. \( \text{[3, 3]} \), the breaking of the flavor group is dominated by the top and bottom Yukawa couplings. Yet here we also assume that the relevant off-diagonal elements of \( V^\text{CKM} \) are small, so the residual approximate symmetry is \( \mathcal{H}^\text{SM} = U(2)_Q \times U(2)_U \times U(2)_D \times U(1)_{Q^d} \) (\( U(1)_Q \) is enhanced to \( U(2)_Q \), and there is also a \( U(1)_{Q^d} \) symmetry for the third generation). The broken symmetry generators live in \( G^\text{SM}/\mathcal{H}^\text{SM} \) cosets. It is useful to factor them out of the Yukawa matrices, so we parameterize

\[
Y_{U,D} = e^{i\hat{\rho}_{Q}} e^{\pm i\hat{\chi}/2} \tilde{Y}_{U,D} e^{-i\hat{\rho}_{U,D}},
\]

where the reduced Yukawa spurions, \( \tilde{Y}_{U,D} \), are

\[
\tilde{Y}_{U,D} = \begin{pmatrix} \phi_{U,D} & 0 \\ 0 & y_{t,b} \end{pmatrix}.
\]

\(^{18}\)Another non-linear parameterization of MFV was presented in \(^{[129]}\).
Here $\phi_{U,D}$ are $2 \times 2$ complex spurions, while $\chi$ and $\hat{\rho}_i$, $i = Q, U, D$, are the $3 \times 3$ matrices spanned by the broken generators. Explicitly,

$$\chi = \begin{pmatrix} 0 & \chi \end{pmatrix}, \quad \hat{\rho}_i = \begin{pmatrix} 0 & \hat{\rho}_i \end{pmatrix}, \quad i = Q, U, D,$$

(111)

where $\chi$ and $\rho_i$ are two dimensional vectors. The $\hat{\rho}_i$ shift under the broken generators, and therefore play the role of spurion “Goldstone bosons”. Thus the $\rho_i$ have no physical significance. On the other hand, $\chi$ parameterizes the misalignment of the up and down Yukawa couplings, and therefore corresponds to $V_{td}^{\text{CKM}}$ and $V_{ts}^{\text{CKM}}$ in the low energy effective theory (see Eq. (119)).

Under the flavor group, the above spurions transform as,

$$e^{i\phi L} = V_i e^{i\rho_i} U_i^\dagger, \quad e^{i\phi U} = U_Q e^{i\tilde{\chi} Q} U_Q^\dagger, \quad Y_i' = U_Q Y_i U_i^\dagger.$$

(112)

Here $U_i = U_i(V_i, \hat{\rho}_i)$ are (reducible) unitary representations of the unbroken flavor subgroup $U(2)_i \times U(1)_3$.

$$U_i = \begin{pmatrix} U_i^{2 \times 2} & 0 \\ 0 & e^{i\varphi_i} \end{pmatrix}, \quad i = Q, U, D.$$

(113)

For $V_i \in \mathcal{H}^{\text{SM}}$, $U_i = V_i$. Otherwise the $U_i$ depend on the broken generators and $\hat{\rho}_i$. They form a nonlinear realization of the full flavor group. In particular, Eq. (112) defines $U_i(V_i, \hat{\rho}_i)$ by requiring that $\hat{\rho}_i'$ is of the same form as $\hat{\rho}_i$, Eq. (111). Consequently $\hat{\rho}_i$ is shifted under $\mathcal{G}^{\text{SM}}/\mathcal{H}^{\text{SM}}$, and can be set to a convenient value as discussed below. Under $\mathcal{H}^{\text{SM}}$, $\chi [\rho_i]$ are fundamentals of $U(2)_Q [U(2)_i]$ carrying charge $-1$ under the $U(1)_3$, while $\phi_{U,D}$ are bi-fundamentals of $U(2)_Q \times U(2)_{U,D}$.

As a final step we also redefine the quark fields by modding out the “Goldstone spurions”,

$$\tilde{u}_L = e^{-i\tilde{\chi}/2} e^{-i\tilde{\rho}_Q} u_L, \quad \tilde{d}_L = e^{i\tilde{\chi}/2} e^{-i\tilde{\rho}_Q} d_L, \quad \tilde{u}_R = e^{-i\tilde{\rho}_Q} u_R, \quad \tilde{d}_R = e^{-i\tilde{\rho}_Q} d_R.$$

(114)

The latter form reducible representations of $\mathcal{H}^{\text{SM}}$. Concentrating here and below on the down sector, we therefore define $\tilde{d}_{L,R} = (\tilde{d}_{L,R}^{(2)}, 0) + (0, \tilde{b}_{L,R})$. Under flavor transformations $\tilde{d}_L^{(2)} = U_Q^{2 \times 2} \tilde{d}_L^{(2)}$ and $\tilde{b}_L' = \exp(i\varphi_Q) \tilde{b}_L$. A similar definition can be made for the up quarks.

With the redefinitions above, invariance under the full flavor group is captured by the invariance under the unbroken flavor subgroup $\mathcal{H}^{\text{SM}}$ (see e.g. [130]). Thus, GMFPV can be described without loss of generality as a formally $\mathcal{H}^{\text{SM}}$ invariant expansion in $\phi_{U,D}, \chi$. This is a straightforward generalization of the known effective field theory description of spontaneous symmetry breaking [130]. The only difference in our case is that $Y_{U,D}$ are not aligned, as manifested by $\chi \neq 0$. Since the background field values of the relevant spurions are small, we can expand in them.

We are now in a position to write down the flavor structure of quark bilinears from which low energy flavor observables can be constructed. We work to leading order in the spurions that break $\mathcal{H}^{\text{SM}}$, but to all orders in the top and bottom Yukawa couplings. Beginning with the LL bilinears, to second order in $\chi$ and $\phi_{U,D}$, one finds (omitting gauge and Lorentz indices)

$$\bar{\tilde{b}}_L \tilde{b}_L, \quad \bar{\tilde{d}}_L^{(2)} \tilde{d}_L^{(2)}, \quad \bar{\tilde{d}}_L^{(2)} \phi_{U} \phi_{U} \tilde{d}_L^{(2)}, \quad \bar{\tilde{d}}_L^{(2)} \chi \tilde{b}_L, \quad \bar{\tilde{d}}_L^{(2)} \chi \chi \tilde{d}_L^{(2)}.$$

(116)

The first two bilinears in Eq. (116) are diagonal in the down quark mass basis, and do not induce flavor violation. In this basis, the Yukawa couplings take the form

$$Y_U = (V^{\text{CKM}})^\dagger \text{diag}(m_u, m_c, m_t), \quad Y_D = \text{diag}(m_d, m_s, m_b).$$

(118)
This corresponds to spurions taking the background values $\rho_Q = \chi/2$, $\rho_{U,D} = 0$ and $\phi_D = \text{diag}(m_d, m_s)/m_b$, while flavor violation is induced via

$$\chi_i = i(V_{td}^{\text{CKM}}, V_{ts}^{\text{CKM}}), \quad \phi_U = (V_2^{\text{CKM}})^\dagger \text{diag} \left( \frac{m_u}{m_t}, \frac{m_c}{m_t} \right). \quad (119)$$

$V_2^{\text{CKM}}$ stands for a two generation CKM matrix. In terms of the Wolfenstein parameter $\lambda$, the flavor violating spurions scale as $\chi \sim (\lambda^3, \lambda^2)$, $(\phi_U)_2 \sim \lambda^5$. Note that the redefined down quark fields, Eqs. (114,115), coincide with the mass eigenstate basis, $\tilde{d}_{L,R} = d_{L,R}$, for the above choice of spurion background values.

The LR and RR bilinears which contribute to flavor mixing are in turn (at leading order in $\chi$ and $\phi_{U,D}$ spurions),

$$\tilde{d}_L^\dagger \chi \tilde{b}_R, \quad \tilde{d}_L^\dagger \chi^\dagger \phi_D \tilde{d}_R^{(2)}, \quad \tilde{b}_L \chi^\dagger \phi_D \tilde{d}_R^{(2)}; \quad \tilde{d}_R^\dagger \phi_D \chi \tilde{b}_R, \quad \tilde{d}_R^\dagger \phi_D^\dagger \chi^\dagger \phi_D \tilde{d}_R^{(2)}. \quad (120)$$

To make contact with the more familiar MFV notation, consider down quark flavor violation from LL bilinears. We can then expand in the Yukawa couplings,

$$\mathcal{Q} \left[ a_1 Y_U Y_U^\dagger + a_2 \left( Y_U Y_U^\dagger \right)^2 \right] + \left[ b_2 \mathcal{Q} Y_U Y_U^\dagger Y_D Y_D^\dagger Q + \text{h.c.} \right] + \ldots, \quad (122)$$

with $a_{1,2} = \mathcal{O}(x_u^2, x_d^2)$, $b_2 = \mathcal{O}(x_d^2, x_d^2)$. Note that the LMFV limit corresponds to $a_1 \gg a_2, b_2$, and the NLMFV limit to $a_1 \sim a_2 \sim b_2$. While $a_{1,2}$ are real, the third operator in Eq. (122) is not Hermitian and $b_2$ can be complex [60], introducing a new CP violating phase beyond the SM phase. The leading flavor violating terms in Eq. (122) for the down quarks are

$$\tilde{d}_L^\dagger \left[ (a_1 + a_2 y_i^2) \xi_{ij}^l + a_1 \xi_{ij}^c \right] d_L^l + \left[ b_2 y_b^2 d_L^l \xi_{ib} b_L + \text{h.c.} \right] =$$

$$c_b \left( \tilde{d}_L^\dagger \chi \tilde{b}_L + \text{h.c.} \right) + c_c \tilde{d}_L^\dagger \chi^\dagger \tilde{d}_L^{(2)} + c_c \tilde{d}_L^\dagger \phi_U \phi_U^\dagger \tilde{d}_L^{(2)}, \quad (123)$$

where $\xi_{ij}^k = y_k^2 (V_{ki}^{\text{CKM}})^* V_{kj}^{\text{CKM}}$ with $i \neq j$. On the right hand side we have used the general parameterization in Eqs. (116,117) with

$$c_b \simeq (a_1 y_t^2 + a_2 y_1^4 + b_2 y_b^2), \quad c_c \simeq (a_1 y_t^2 + a_2 y_1^4) \quad \text{and} \quad c_{\tilde{c}} \simeq a_1, \quad (124)$$

to leading order. The contribution of the $c_b$ bilinear in flavor changing transitions is $\mathcal{O}(1)$, compared to the $c_t$ bilinear, and can thus be neglected in practice.

A novel feature of NLMFV is the potential for observable CPV from RH currents, to which we return below. Other important distinctions can be readily understood from Eq. (123). In NLMFV (with large tan $\beta$) the extra flavor diagonal CPV phase $\text{Im}(c_b)$ can be large, leading to observable deviations in the $B_{d,s} - \bar{B}_{d,s}$ mixing phases, but none in MFV. Another example is $b \to s \nu \bar{\nu}$ and $s \to dv \bar{\nu}$ transitions, which receive contributions only from a single operator in Eq. (123) multiplied by the neutrino currents. Thus, new contributions to $B \to X_d \nu \bar{\nu}$, $B \to K \nu \bar{\nu}$ vs. $K_L \to \pi^0 \nu \bar{\nu}$, $K^+ \to \pi^+ \nu \bar{\nu}$ are correlated in LMFV ($c_b \simeq c_t$), see e.g. [109,131,132], but are independent in NLMFV with large tan $\beta$. $\mathcal{O}(1)$ effects in the rates would correspond to an effective scale $\Lambda_{\text{MFV}} \sim 3$ TeV in the four fermion operators, with smaller effects scaling like $1/\Lambda_{\text{MFV}}$ due to interference with the SM contributions. Other interesting NLMFV effects involving the third generation, such as
large deviations in $\text{Br}(B_{d,s} \to \mu^+\mu^-)$ and $b \to s\gamma$, arise in the minimal supersymmetric standard model (MSSM) at large $\tan \beta$, where resummation is required \([117, 118, 120, 133]\).

Assuming MFV, new CPV effects can be significant if and only if the UV theory contains new flavor diagonal CP sources. The proof is as follows. If no flavor diagonal phases are present, CPV only arises from the CKM phase. In the exact $U(2)_Q$ limit, the CKM phase can be removed and the theory becomes CP invariant (at all scales). The only spurions that break the $U(2)_Q$ flavor symmetry are $\phi_{U,D}$ and $\chi$. CPV in operators linear in $\chi$ is directly proportional to the CKM phase (see Eq. \([123]\)). Any additional contributions are suppressed by at least $[\phi_{U}^i\phi_U^i, \phi_D^i\phi_D^i] \sim (m_s/m_b)^2 (m_c/m_t)^2 \sin \theta_C \sim 10^{-9}$, and are therefore negligible.

Flavor diagonal weak phases in NLMFV can lead to new CPV effects in $3 \to 1$ and $3 \to 2$ decays. An example is $\Delta B = 1$ electromagnetic and chromomagnetic dipole operators constructed from the first bilinear in Eq. \([120]\). The operators are not Hermitian, hence their Wilson coefficients can contain new CPV phases. Without new phases, the untagged direct CP asymmetry in $B \to X_{d,s}\gamma$ would essentially vanish due to the residual $U(2)$ symmetry, as in the SM \([134]\), and the $B \to X_s\gamma$ asymmetry would be less than a percent. However, in the NLMFV limit (large $y_b$), non-vanishing phases can yield significant CPV in untagged and $B \to X_s\gamma$ decays, and the new CPV in $B \to X_s\gamma$ and $B \to X_d\gamma$ would be strongly correlated. Supersymmetric examples of this kind were studied in \([135, 136, 137]\), where new phases were discussed.

Next, consider the NLMFV $\Delta b = 2$ effective operators. They are not Hermitian, hence their Wilson coefficients $C_i/\Lambda_{\text{MFV}}^2$ can also contain new CP violating phases. The operators can be divided into two classes: class-1, which does not contain light RH quarks \([d_L^2 \chi b_L, R]^2, \ldots\]; and class-2, which does \([d_R^2 \phi_D^i \chi b_L, \phi_D^i \chi b_R, \ldots]\). Class-2 only contributes to $B_s - \bar{B}_s$ mixing, up to $m_d/m_s$ corrections. Taking into account that $SU(3)_F$ (approximate $u-d$-s flavor symmetry of the strong interaction) breaking in the bag parameters of the $B_s - \bar{B}_s$ vs. $B_d - \bar{B}_d$ mixing matrix elements is only at the few percent level in lattice QCD \([138, 139]\), we conclude that class-1 yields the same weak phase shift in $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixing relative to the SM. The class-1 contribution would dominate if $\Lambda_{\text{MFV}}$ is comparable for all the operators. For example, in the limit of equal Wilson coefficients $C_i/\Lambda_{\text{MFV}}^2$, the class-2 contribution to $B_s - \bar{B}_s$ mixing would be $\approx 5\%$ of class-1. The maximal allowed magnitude of CPV in the $B_d$ system is smaller than roughly 20%. Quantitatively, for $\text{Im}(C_i) \approx 1$, this corresponds to $\Lambda_{\text{MFV}} \approx 18$ TeV for the leading class-1 operator, which applies to the $B_s$ system as well. Thus, sizable CPV in the $B_s$ system would require class-2 contributions, with $O(1)$ CPV corresponding to $\Lambda_{\text{MFV}} \approx 1.5$ TeV for the leading class-2 operator. Conversely, barring cancelations, within NLMFV models NP CPV in $B_s - \bar{B}_s$ mixing provides an upper bound on NP CPV in $B_d - \bar{B}_d$ mixing.

For $2 \to 1$ transitions, the new CPV phases come suppressed by powers of $m_{d,s}/m_b$. All the $2 \to 1$ bilinears in Eqs. \([116, 117, 120, 121]\) are Hermitian, with the exception of $\bar{d}_L^2 \chi^i \phi_D^i a_R^{(2)}$. This provides the leading contribution to $\epsilon_K$ from a non-SM phase, coming from the operator $O_{LR} = (\bar{d}_L^2 \chi^i \phi_D^i a_R^{(2)})^2$. Its contribution is $\approx 2\%$ of the SM operator $O_{LL} = (\bar{d}_L^2 \chi^i \bar{d}_L^{(2)})^2$ for comparable Wilson coefficients $C_{LR,LL}/\Lambda_{\text{MFV}}^2$. For $C_{LL}$, $\text{Im}(C_{LR}) \approx 1$, a new contribution to $\epsilon_K$ that is 50% of the measured value would correspond to $\Lambda_{\text{MFV}} \approx 5$ TeV for $O_{LL}$ and $\Lambda_{\text{MFV}} \approx 0.8$ TeV for $O_{LR}$.

Note that the above new CPV effects can only be sizable in the large $\tan \beta$ limit. They arise from non-Hermitian operators (such as the second operator in \([122]\)), and are therefore of higher order in the $Y_D$ expansion. Whereas we have been working in the large $\tan \beta$ limit, it is straightforward to incorporate the small $\tan \beta$ limit (discussed above in Sec. \([6.1.2]\)) into our
formalism. In that case the flavor group is broken down to \(U(2)_Q \times U(2)_U \times U(1)_t \times U(3)_D\), and the expansion in Eq. (109) no longer holds. In particular, resummation over \(y_b\) is not required. Flavor violation is described by linearly expanding in the down type Yukawa couplings, from which it follows that contributions proportional to the bottom Yukawa are further suppressed beyond the SM CKM suppression.

It should also be pointed out that NLMFV differs from the next-to-MFV framework \([4, 5]\), since the latter exhibits additional spurions at low energy.

### 6.4 MFV in covariant language

The covariant formalism described in Sec. 4 enables us to offer further insight on the MFV framework. In the LMFV case, the NP source \(X_Q\) from Eq. (39) or Eq. (87) is a linear combination of the \(A_{Q^d}\) and \(A_{Q^u}\) “vectors”, naturally with \(O(1)\) coefficients at most. Hence we can immediately infer that no new CPV sources exist, as all vectors are on the same plane, and that the induced flavor violation is small (recall that the angle between \(A_{Q^u}\) and \(A_{Q^d}\) is small – \(O(\lambda^2)\)). These conclusions are of course already known, but they emerge naturally when using the covariant language.

In the GMFV scenario, \(X_Q\) is a general function of \(A_{Q^u}\) and \(A_{Q^d}\). We can alternatively express it in terms of the covariant basis introduced in Sec. 4.2.2, since this basis is constructed using only \(A_{Q^u}\) and \(A_{Q^d}\). Then, it is easy to see that an arbitrary function of the Yukawa matrices could produce any kind of flavor and CP violation \([60, 61, 62]\). However, the directions denoted by \(\vec{D}\) require higher powers of the Yukawas, so their contribution is generically much smaller (in [60] it was noticed that some directions, which we identify as \(\vec{D}\), are not generated via RGE flow). Therefore, the induced flavor and CP violation tend to be restricted to the submanifold which corresponds to the \(U(2)_Q\) limit (that is, the directions denoted by \(A_{Q^u,Q^d}, \vec{J}, \vec{J}_{u,d}\) and \(\vec{C}_{u,d}\)).

### 7 Supersymmetry

Supersymmetric models provide, in general, new sources of flavor violation, for both the quark and the lepton sectors. The main new sources are the supersymmetry breaking soft mass terms for squarks and sleptons and the trilinear couplings of a Higgs field with a squark-antisquark or slepton-antislepton pairs. Let us focus on the squark sector. The new sources of flavor violation are most commonly analyzed in the basis in which the corresponding (down or up) quark mass

\[\tilde{q}_{Mi}(M_{\tilde{q}}^2)^{MN}_ij \tilde{q}_{Nj} = (\tilde{q}_{Li} \tilde{q}_{Rk}^*) \left( \left[ \begin{array}{cc} (M_{\tilde{q}}^2)^{Lij}_q & A_{ij}^q v_q \\ A_{jk}^q v_q & (M_{\tilde{q}}^2)^{Rij}_q \end{array} \right] \right) \left( \tilde{q}_{Lj} \tilde{q}_{Rk}^* \right), \]  

where \(M, N = L, R\) label chirality, and \(i, j, k, l = 1, 2, 3\) are generation indices. \((M_{\tilde{q}}^2)^L\) and \((M_{\tilde{q}}^2)^R\) are the supersymmetry breaking squark masses-squared. The \(A_q^q\) parameters enter in the trilinear scalar couplings \(A_{ij}^q H_q \tilde{q}_{Li} \tilde{q}_{Rj}\), where \(H_q (q = u, d)\) is the \(q\)-type Higgs boson and \(v_q = \langle H_q \rangle\).

In this basis, flavor violation takes place through one or more squark mass insertion. Each mass insertion brings with it a factor of \((\delta_{ij}^q)_{MN} \equiv (M_{\tilde{q}}^2)^{MN}_ij / \tilde{m}_q^2\), where \(\tilde{m}_q^2\) is a representative \(q\)-squark mass scale. Physical processes therefore constrain

\[[(\delta_{ij}^q)_{MN}]_{\text{eff}} \sim \max[(\delta_{ij}^q)_{MN}, (\delta_{ik}^q)_{MP}(\delta_{kj}^q)_{PN}, \ldots, (i \leftrightarrow j)]. \]
For example,

\[
[(\delta_{12}^d)_{LR}]_{\text{eff}} \sim \max[A_{12}^d v_d / \tilde{m}_d^2, (M_{d}^2)_{LLk} A_{k2}^d v_d / \tilde{m}_d^4, A_{1k}^d v_d (M_{d}^2)_{RK2} / \tilde{m}_d^4, \ldots, (1 \leftrightarrow 2)].
\] (127)

Note that the contributions with two or more insertions may be less suppressed than those with only one.

In terms of mass basis parameters, the \((\delta_{ij}^q)_{MM}'s\) stand for a combination of mass splittings and mixing angles:

\[
(\delta_{ij}^q)_{MM} = \frac{1}{m_q^2} \sum_{\alpha} (K_M^q)_{i\alpha} (K_M^q)^*_{j\alpha} \Delta \tilde{m}_{q_i q_j}^2,
\] (128)

where \(K_M^q\) is the mixing matrix in the coupling of the gluino (and similarly for the bino and neutral wino) to \(q_L - \tilde{q}_{Ma}\); \(\tilde{m}_q^2 = \frac{1}{3} \sum_{\alpha=1}^{3} m_{q_{Ma}}^2\) is the average squark mass squared, and \(\Delta \tilde{m}_{q_i q_j}^2 = m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2\).

Things simplify considerably when the two following conditions are satisfied \[140, 141\], which means that a two generation effective framework can be used (for simplicity, we omit here the chirality index):

\[
|K_{ik} K_{jk}^*| \ll |K_{ij} K_{ij}^*|, \quad |K_{ik} K_{jk}^* \Delta \tilde{m}_{q_i q_j}^2| \ll |K_{ij} K_{ij}^* \Delta \tilde{m}_{q_i q_j}^2|,
\] (129)

where there is no summation over \(i, j, k\) and where \(\Delta \tilde{m}_{q_i q_j}^2 = m_{\tilde{q}_i}^2 - m_{\tilde{q}_j}^2\). Then, the contribution of the intermediate \(\tilde{q}_k\) can be neglected, and furthermore, to a good approximation, \(K_{ik} K_{ik}^* + K_{ij} K_{ij}^* = 0\). For these cases, we obtain a simpler expression for the mass insertion term

\[
(\delta_{ij}^q)_{MM} = \frac{\Delta \tilde{m}_{q_i q_j}^2}{m_q^2} (K_M^q)_{ij} (K_M^q)^*_{jj},
\] (130)

In the non-degenerate case, in particular relevant for alignment models, it is useful to take instead of \(m_q\) the mass scale \(\tilde{m}_{q_{ij}}^q = \frac{1}{2}(m_{\tilde{q}_i} + m_{\tilde{q}_j})[142]\), which better approximates the full expression. We also define

\[
\langle \delta_{ij}^q \rangle = \sqrt{(\delta_{ij}^q)_{LL} (\delta_{ij}^q)_{RR}}.
\] (131)

The new sources of flavor and CP violation contribute to FCNC processes via loop diagrams involving squarks and gluinos (or electroweak gauginos, or higgsinos). If the scale of the soft supersymmetry breaking is below TeV, and if the new flavor violation is of order one, and/or if the phases are of order one, then these contributions could be orders of magnitude above the experimental bounds. Imposing that the supersymmetric contributions do not exceed the phenomenological constraints leads to constraints of the form \((\delta_{ij}^q)_{MM} \ll 1\). Such constraints imply that either quasi-degeneracy (\(\Delta \tilde{m}_{q_i q_j}^2 \ll (\tilde{m}_{q_i}^2)^2\)) or alignment (\(|K_{ij}^q| \ll 1\)) or a combination of the two mechanisms is at work.

Table 4 presents the constraints obtained in Refs. [17, 18, 143, 144] as appear in [140]. Whenever relevant, a phase suppression of order 0.3 in the mixing amplitude is allowed, namely we quote the stronger between the bounds on Re(\(\delta_{ij}^q\)) and 3Im(\(\delta_{ij}^q\)). The dependence of these bounds on the average squark mass \(\tilde{m}_q\), the ratio \(x \equiv m_{\tilde{g}}^2 / \tilde{m}_q^2\) as well as the effect of arbitrary strong CP violating phases can be found in [140].

For large \(\tan \beta\), some constraints are modified from those in Table 4. For instance, the effects of neutral Higgs exchange in \(B_s\) and \(B_d\) mixing give, for \(\tan \beta = 30\) and \(x = 1\) (see [140, 143, 146] and refs. therein for details):

\[
\langle \delta_{13}^d \rangle < 0.01 \left( \frac{M_{A^0}}{200 \text{ GeV}} \right), \quad \langle \delta_{23}^d \rangle < 0.04 \left( \frac{M_{A^0}}{200 \text{ GeV}} \right),
\] (132)
Table 4: The phenomenological upper bounds on \((\delta^q_{ij})_{MM}\) and on \(\langle \delta^q_{ij} \rangle\), where \(q = u,d\) and \(M = L,R\). The constraints are given for \(\tilde{m}_q = 1\) TeV and \(x \equiv m_{\tilde{g}}/\bar{m}_q^2 = 1\). We assume that the phases could suppress the imaginary parts by a factor \(\sim 0.3\). The bound on \((\delta^d_{23})_{RR}\) is about 3 times weaker than that on \((\delta^d_{23})_{LL}\) (given in table). The constraints on \((\delta^d_{12,13})_{MM}, (\delta^u_{12})_{MM}\) and \((\delta^d_{23})_{MM}\) are based on, respectively, Refs. [143], [17] and [144].

\[
\begin{array}{|c|c|c|c|}
\hline
q & ij & (\delta^q_{ij})_{MM} & \langle \delta^q_{ij} \rangle \\
\hline
\hline
d & 12 & 0.03 & 0.002 \\
d & 13 & 0.2 & 0.07 \\
d & 23 & 0.6 & 0.2 \\
u & 12 & 0.1 & 0.008 \\
\hline
\end{array}
\]

Table 5: The phenomenological upper bounds on chirality-mixing \((\delta^q_{ij})_{LR}\), where \(q = u,d\). The constraints are given for \(\tilde{m}_q = 1\) TeV and \(x \equiv m_{\tilde{g}}/\bar{m}_q^2 = 1\). The constraints on \(\delta^d_{12,13}, \delta^u_{12}, \delta^d_{23}\) and \(\delta^q_{ii}\) are based on, respectively, Refs. [143], [17], [144] and [147] (with the relation between the neutron and quark EDMs as in [148]).

\[
\begin{array}{|c|c|c|}
\hline
q & ij & (\delta^q_{ij})_{LR} \\
\hline
\hline
d & 12 & 2 \times 10^{-4} \\
d & 13 & 0.08 \\
d & 23 & 0.01 \\
d & 11 & 4.7 \times 10^{-6} \\
u & 11 & 9.3 \times 10^{-6} \\
u & 12 & 0.02 \\
\hline
\end{array}
\]

where \(M_{A^0}\) denotes the pseudoscalar Higgs mass, and the above bounds scale roughly as \((30/\tan \beta)^2\).

The experimental constraints on the \((\delta^q_{ij})_{LR}\) parameters in the quark-squark sector are presented in Table 5. The bounds are the same for \((\delta^q_{ij})_{LR}\) and \((\delta^q_{ij})_{RL}\), except for \((\delta^d_{12})_{MN}\), where the bound for \(MN = LR\) is 10 times weaker. Very strong constraints apply for the phase of \((\delta^q_{11})_{LR}\) from EDMs. For \(x = 4\) and a phase smaller than 0.1, the EDM constraints on \((\delta^u,d,\ell)_{LR}\) are weakened by a factor \(\sim 6\).

While, in general, the low energy flavor measurements constrain only the combinations of the suppression factors from degeneracy and from alignment, such as Eq. (130), an interesting exception occurs when combining the measurements of \(K^0 - \bar{K}^0\) and \(D^0 - \bar{D}^0\) mixing to test the first two generation squark doublets (based on the analysis in Sec. 5.2.1). Here, for masses below the TeV scale, some level of degeneracy is unavoidable [23]:

\[
\frac{m_{\tilde{Q}_2} - m_{\tilde{Q}_1}}{m_{\tilde{Q}_2} + m_{\tilde{Q}_1}} \leq \begin{cases} 
0.034 & \text{maximal phases} \\
0.27 & \text{vanishing phases}
\end{cases}
\]

(133)

Similarly, using \(\Delta F = 1\) processes involving the third generation (Sec. 5.2.2), the following bound is obtained [59]

\[
\frac{m_{\tilde{Q}_2}^2 - m_{\tilde{Q}_3}^2}{2m_{\tilde{Q}_2}^2 + m_{\tilde{Q}_3}^2} < 20 \left( \frac{\bar{m}_Q}{100 \text{GeV}} \right)^2,
\]

(134)
which is rather weak and insignificant in practice. The bound that stems from $\Delta F = 2$ third generation processes (Sec. 5.2.3) is $\left| m_{Q_1}^2 - m_{Q_3}^2 \right| < 0.45 \left( \frac{\tilde{m}_Q}{100 \text{ GeV}} \right)^2$.

Note that the latter limit is actually determined by CPV in $D$ mixing (see discussion in Sec. 5.2.3). It should be mentioned that by carefully tuning the squark and gluino masses, one finds a region in parameter space where the above bounds can be ameliorated [149].

The strong constraints in Tables 4 and 5 can be satisfied if the mediation of supersymmetry breaking to the MSSM is MFV. In particular, if at the scale of mediation, the supersymmetry breaking squark masses are universal, and the $A$-terms (couplings of squarks to the Higgs bosons) vanish or are proportional to the Yukawa couplings, then the model is phenomenologically safe. Indeed, there are several known mechanisms of mediation that are MFV (see, e.g. [150]). In particular, gauge-mediation [95, 96, 151, 152], anomaly-mediation [97, 98], and gaugino-mediation [153] are such mechanisms. (The renormalization group flow in the MSSM with generic MFV soft-breaking terms at some high scale has recently been discussed in Refs. [60, 154].) On the other hand, we do not expect gravity-mediation to be MFV, and it could provide subdominant, yet observable flavor and CP violating effects [155].

8 Extra Dimensions

Models of extra dimensions come in a large variety, and the corresponding phenomenology, including the implications for flavor physics, changes from one extra dimension framework to another. Yet, as in the supersymmetric case, one can classify the new sources of flavor violation which generically arise:

**Bulk masses** – If the SM fields propagate in the bulk of the extra dimensions, they can have bulk vector-like masses. These mass terms are of particular importance to flavor physics, since they induce fermion localization which may yield hierarchies in the low energy effective couplings. Furthermore, the bulk masses, which define the extra dimension interaction basis, do not need to commute with the Yukawa matrices, and hence might induce contributions to FCNC processes, similarly to the squark soft masses-squared in supersymmetry.

**Cutoff, UV physics** – Since, generically, higher dimensional field theories are non-renormalizable, they rely on unspecified microscopic dynamics to provide UV completion of the models. Hence, they can be viewed as effective field theories, and the impact of the UV physics is expected to be captured by a set of operators suppressed by the framework dependent cutoff scale. Without precise knowledge of the short distance dynamics, the additional operators are expected to carry generic flavor structure and contribute to FCNC processes. This is somewhat similar to “gravity mediated” contributions to supersymmetry breaking soft terms, which are generically expected to have an anarchic flavor structure, and are suppressed by the Planck scale.

**“Brane”-localized terms** – The extra dimensions have to be compact, and typically contain defects and boundaries of smaller dimensions [in order, for example, to yield a chiral low energy four dimension (4D) theory]. These special points might contain different microscopical degrees of freedom. Therefore, generically, one expects that a different and independent class of higher dimension operators may be localized to this singular region in the extra dimension manifold. (These are commonly denoted ‘brane terms’, even though, in most cases, they have very little to
do with string theory). The brane-localized terms can, in principle, be of anarchic flavor structure, thus providing new flavor and CP violating sources. One important class of such operators are brane kinetic terms: their impact is somewhat similar to that of non-canonical kinetic terms, which generically arise in supersymmetric flavor models.

We focus on flavor physics of five dimension (5D) models, with bulk SM fields, since most of the literature focuses on this class. Furthermore, the new flavor structure that arises in 5D models captures most of the known effects of extra dimension flavor models. Assuming a flat extra dimension, the energy range, $\Lambda_{5D} R$ (where $\Lambda_{5D}$ is the 5D effective cutoff scale and $R$ is the extra dimension radius with the extra dimension coordinate $y \in (0, \pi R)$), for which the 5D effective field theory holds, can be estimated as follows. Since gauge couplings in extra dimensional theories are dimensionful, i.e. $\alpha_{5D}$ has mass dimension $-1$, a rough guess (which is confirmed, up to order one corrections, by various naive dimensional analysis methods) is $\Lambda_{5D} \sim 4\pi/\alpha_{5D}$. Matching this 5D gauge coupling to a 4D coupling of the SM at leading order, $1/g^2 = \pi R/g_{5D}^2$, we obtain

$$\Lambda_{5D} R \sim \frac{4}{\alpha} \sim 30.$$  \hfill (136)

Generically, the mass of the lightest Kaluza-Klein (KK) states, $M_{KK}$, is of $\mathcal{O}(R^{-1})$. If the extra dimension theory is linked to the solution of the hierarchy problem and/or directly accessible to near future experiments, then $R^{-1} = \mathcal{O}(\text{TeV})$. This implies an upper bound on the 5D cutoff:

$$\Lambda_{5D} \lesssim 10^2 \text{TeV} \ll \Lambda_K \sim 2 \times 10^6 \text{TeV},$$  \hfill (137)

where $\Lambda_K$ is the scale required to suppress the generic contributions to $\epsilon_K$, discussed above (see Table 1).

The above discussion ignores the possibility of splitting the fermions in the extra dimension. In split fermion models, different bulk masses are assigned to different generations, which gives rise to different localizations of the fermions in the extra dimension. Consequently, they have different couplings to the Higgs, in a manner which may successfully address the SM flavor puzzle \[157\]. Separation in the extra dimension may suppress the contributions to $\epsilon_K$ from the higher dimension cutoff-induced operators. As shown in Table 1, the most dangerous operator is

$$O^4_K = \frac{1}{\Lambda_{5D}^2} (\bar{s}_L d_R) (\bar{s}_R d_L).$$  \hfill (138)

This operator contains $s$ and $d$ fields of both chiralities. As a result, in a large class of split fermion models, the overlap suppression would be similar to that accounting for the smallness of the down and strange 4D Yukawa couplings. The integration over the 5D profiles of the four quarks may yield a suppression factor of $\mathcal{O}(m_d m_s / v^2) \sim 10^{-9}$. Together with the naive scale suppression, $1/\Lambda_{5D}^2$, the coefficient of $O^4_K$ can be sufficiently suppressed to be consistent with the experimental bound.

In the absence of large brane kinetic terms, however, fermion localization generates order one non-universal couplings to the gauge KK fields \[158\] (the case with large brane kinetic terms is similar to the warped scenario discussed below). The fact that the bulk masses are, generically, not aligned with the 5D Yukawa couplings implies that KK gluon exchange processes induce, among others, the following operator in the low energy theory: $$[\langle D_L \rangle^{12}_L/6M_{KK}^2] (\bar{s}_L d_L)^2,$$ where $(D_L)_{12} \sim \lambda$ is the LH down quark rotation matrix from the 5D interaction basis to the mass basis. This structure provides only a mild suppression to the resulting operator. It implies that to satisfy the $\epsilon_K$ constraint, the KK and the inverse compactification scales have to be above $10^3$ TeV,
beyond the direct reach of near future experiments, and too high to be linked to a solution of the hierarchy problem. This problem can be solved by tuning the 5D flavor parameters, and imposing appropriate 5D flavor symmetries to make the tuning stable. Once the 5D bulk masses are aligned with the 5D Yukawa matrices, the KK gauge contributions vanish, and the configuration becomes radiatively stable.

The warped extra dimension [Randall Sundrum (RS)] framework [159] provides a solution to the hierarchy problem. Moreover, with SM fermions propagating in the bulk, both the SM and the NP flavor puzzles can be addressed. The light fermions can be localized away from the TeV brane [160], where the Higgs is localized. Such a configuration can generate the observed Yukawa hierarchy, and at the same time ensure that higher dimensional operators are suppressed by a high cutoff scale, associated with the location of the light fermions in the extra dimension [161, 162]. Furthermore, since the KK states are localized near the TeV brane, the couplings between the SM quarks and the gauge KK fields exhibit the hierarchical structure associated with SM masses and CKM mixings. This hierarchy in the couplings provides an extra protection against non-standard flavor violating effects [163], denoted as RS-GIM mechanism [66, 67] (see also [164, 165]). It is interesting to note that an analogous mechanism is at work in models with strong dynamics at the TeV scale, with large anomalous dimension and partial compositeness [166, 167, 168]. The link with the hierarchy problem. Moreover, with SM fermions propagating in the bulk, both the SM and the reduced Planck mass. If $c_{x_i}$ is the reduced Planck mass. If $c_{x_i} < 1/2$, then $f_{x_i}$ is exponentially suppressed. Hence, order one variations in the 5D masses yield large hierarchies in the 4D flavor parameters. We consider the cases where the Higgs VEV either propagates in the bulk or is localized on the IR brane. For a bulk Higgs case, the profile is given by $\tilde{v}(\beta, z) \approx \sqrt{k(1+\beta)} z^{2+\beta}/\epsilon$, where $z \in (\epsilon, 1)$ ($\tilde{z} = 1$ on the IR brane), and $\beta \geq 0$. The $\beta = 0$ case describes a Higgs maximally-spread into the bulk (saturating the AdS stability bound [174]). The relevant part of the effective 4D Lagrangian, which involves the zero modes and the first KK gauge states ($G^i$), can be approximated by [66, 67, 173]

$$L^{4D} \supset (\mathcal{V}_{U,D})_{ij} H_{U,D} Q_i f_{Q_i} (U, D)_{ij} f_{U_j, D_j} r_{00}^\phi (\beta, c_{Q_i}, c_{U_j, D_j}) + g_s G^i x_i x_i \left[ f_{x_i} r_{00}^g (c_{x_i}) - 1/\xi \right],$$

where $g_s$ stands for a generic effective gauge coupling and summation over $i, j$ is implied. The corrections for the couplings relative to the case of fully IR-localized Higgs and KK states are given by the functions $r_{00}^\phi$ [173] and $r_{00}^g$ [175, 176], respectively:

$$r_{00}^\phi (\beta, c_L, c_R) \approx \frac{\sqrt{2}(1+\beta)}{2 + \beta - c_L - c_R}, \quad r_{00}^g (c) \approx \frac{\sqrt{2}}{J_1(x_1) 6 - 4c} \left( 1 + e^{c/2} \right),$$

where $r_{00}^\phi (\beta, c_L, c_R) = 1$ for brane-localized Higgs and $x_1 \approx 2.4$ is the first root of the Bessel function, $J_0(x_1) = 0$.

In Table 6 we present an example of a set of $f_{x_i}$ values that, starting from anarchical 5D Yukawa couplings, reproduce the correct hierarchy of the flavor parameters. We assume for simplicity an
IR-localized Higgs. The values depend on two input parameters: \( f_{U_3} \), which has been determined assuming a maximally IR-localized \( t_R \) (\( c_{U_3} = -0.5 \)), and \( y_{5D} \), the overall scale of the 5D Yukawa couplings in units of \( k \), which has been fixed to its maximal value assuming three KK states. On general grounds, the value of \( y_{5D} \) is bounded from above, as a function of the number of KK levels, \( N_{KK} \), by the requirement that Yukawa interactions are perturbative below the cutoff of the theory, \( \Lambda_{5D} \). In addition, it is bounded from below in order to account for the large top mass. Hence the following range for \( y_{5D} \) is obtained (see e.g. \[104, 177\]):

\[
\frac{1}{2} \lesssim y_{5D} \lesssim \frac{2\pi}{N_{KK}} \quad \text{for brane Higgs} ; \quad \frac{1}{2} \lesssim y_{5D} \lesssim \frac{4\pi}{\sqrt{N_{KK}}} \quad \text{for bulk Higgs},
\]

where we use the rescaling \( y_{5D} \to y_{5D} \sqrt{1 + \beta} \), which produces the correct \( \beta \to \infty \) limit \[178\] and avoids subtleties in the \( \beta = 0 \) case.

With anarchical 5D Yukawa matrices, an RS residual little CP problem remains \[104\]: Too large contributions to the neutron EDM \[66, 67\] and sizable chirally enhanced contributions to \( \epsilon_K \) \[7, 65, 175, 179, 180\] are predicted. The RS leading contribution to \( \epsilon_K \) is generated by a tree level KK gluon exchange, which leads to an effective coupling for the chirality-flipping operator in Eq. \[138\] of the type \[65, 173, 175, 179, 180\]

\[
C_4^{KK} \simeq \frac{g_{ss}^2}{M_{KK}^2} f_Q f_Q f_d f_d r_{00}^q(c_Q) r_{00}^q(c_d) \sim \frac{g_{ss}^2}{M_{KK}^2} \frac{2m_d m_s}{(v y_{5D})^2} r_{00}^Q(\beta, c_Q, c_d) r_{00}^H(\beta, c_Q, c_d).
\]

The final expression is independent of the \( f_{x_i} \), so the bound in Table 1 can be translated into constraints in the \( y_{5D} - M_{KK} \) plane. The analogous effects in the \( D \) and \( B \) systems yield numerically weaker bounds. Another class of contributions, which involves only LH quarks, is also important to constrain the \( f_Q - M_{KK} \) parameter space.

In Table 6, we summarize the resulting constraints. For the purpose of a quantitative analysis we set \( g_{ss} = 3 \), as obtained by matching to the 4D coupling at one-loop \[177\] (for the impact of a smaller RS volume see \[181\]). The constraints related to CPV correspond to maximal phases, and are subject to the requirement that the RS contributions are smaller than 30% (60%) of the SM contributions \[1, 5\] in the \( B_d \) (\( K \)) system. The analytical expressions in the table have roughly a 10% accuracy over the relevant range of parameters. Contributions from scalar exchange, either Higgs \[178, 182\] or radion \[183\], are not included, since these are more model dependent and known to be weaker \[184\] in the IR-localized Higgs case.

Constraints from \( \epsilon'/\epsilon_K \) have a different parameter dependence than the \( \epsilon_K \) constraints. Explicitly, for \( \beta = 0 \), the \( \epsilon'/\epsilon_K \) bound reads \( M_{KK}^{\min} = 1.2 y_{5D} \) TeV. When combined with the \( \epsilon_K \) constraint, we find \( M_{KK}^{\min} = 5.5 \) TeV with a corresponding \( y_{5D}^{\min} = 4.5 \) \[173\].

The constraints summarized in Table 7 and the contributions to the neutron EDM which generically require \( M_{KK} > \mathcal{O}(10 \text{ TeV}) \) \[66, 67\] are a clear manifestation of the RS little CP

| Flavor | \( f_Q \) | \( f_U \) | \( f_D \) |
|--------|----------|----------|----------|
| 1      | \( A\lambda^3 f_{Q3} \sim 3 \times 10^{-3} \) | \( \frac{m_t}{m_t} f_{U3} \sim 1 \times 10^{-3} \) | \( \frac{m_s}{m_t} f_{D3} \sim 2 \times 10^{-3} \) |
| 2      | \( A\lambda^2 f_{Q3} \sim 1 \times 10^{-2} \) | \( \frac{m_t}{m_t} f_{U3} \sim 0.1 \) | \( \frac{m_s}{m_t} f_{D3} \sim 1 \times 10^{-2} \) |
| 3      | \( \frac{m_t}{v y_{5D} f_{U3}} \sim 0.3 \) | \( \frac{m_t}{m_t} f_{U3} \sim 2 \times 10^{-2} \) | \( \frac{m_s}{m_t} f_{D3} \sim 2 \times 10^{-2} \) |

Table 6: Values of the \( f_{x_i} \) parameters (Eq. \[139\]) which reproduce the observed quark masses and CKM mixing angles starting from anarchical 5D Yukawa couplings. We fix \( f_{U_3} = \sqrt{2} \) and \( y_{5D} = 2 \) (see text).
problem. The problem can be amended by various alignment mechanisms [101, 103, 104, 176, 185]. In this case, the bounds from the up sector, especially from CPV in the $D$ system [18, 23], become important. Constraints from $\Delta F = 1$ processes (in either the down sector [66, 67, 186, 187, 188] or $t \rightarrow cZ$ [189]) are not included here, since they are weaker in general, and furthermore, these contributions can be suppressed (see [186, 187, 188]) due to incorporation of a custodial symmetry [190].

It is interesting to combine measurements from the down and the up sector in order to obtain general bounds (as done for supersymmetry above). Using $K$ and $D$ mixing, Eq. (86), the constraint on the RS framework is [23]

$$m_{KK} > 2.1 f_{Q_3}^2 \text{ TeV},$$

for a maximal phase, where $f_{Q_3}$ is typically in the range of $0.4-\sqrt{2}$. We thus learn that the case where the third generation doublet is maximally localized on the IR brane (fully composite) is excluded, if we insist on $m_{KK} = 3$ TeV, as allowed by electroweak precision tests (see e.g. [191]). The bounds derived from $\Delta F = 1$ and $\Delta F = 2$ processes involving the third generation are [58, 59]

$$m_{KK} > 0.33 f_{Q_3}^2 \text{ TeV},$$  

$$m_{KK} > 0.4 f_{Q_3}^2 \text{ TeV},$$

respectively.

### 9 High $p_T$ Flavor Physics Beyond the SM

So far we have mostly focused on information that can be gathered from observables related to flavor conversion and in particular to low energy experiments, the exception being top flavor violation, which will be studied in great detail at the LHC. However, much insight can be obtained on short distance flavor dynamics, if one is to observe new degrees of freedom which couple to the SM flavor sector. This is why high $p_T$ collider analyses are also useful for flavor physics (see e.g. [155, 192, 193, 194, 195, 196, 197, 198, 199, 200]). Below we discuss implications of measurements related to both flavor diagonal information and flavor conversion transitions.
Most of the analysis discussed in the following is rather challenging to be done at the LHC for the quark sector, due to the difficulty in distinguishing between jets originated from first and second generation quarks. However, it is certainly possible to distinguish the third generation quarks from the other ones. Furthermore, even though not discussed in this review, the charged lepton sector, which possesses a similar approximate symmetry structure, allows for rather straightforward flavor tagging. Therefore, some of the analysis discussed below can be applied more directly to the lepton sector (see, e.g., [201, 202, 203, 204, 205]). For the quark sector, future progress in the frontier of charm tagging\footnote{Some progress has been recently achieved at the Tevatron in this direction [206], and one might expect that the LHC would perform at least as well, given that its detectors are better (we thank Gustaaf Brooijmans for bringing this point to our attention).} may play a crucial role in extracting further information regarding the breaking of the SM approximate symmetries.

In general, one may say that not much work has been done on the issues discussed below, and that there are many issues, both theoretical and experimental, to study on how to improve the treatment related to high $p_T$ flavor physics at the LHC era. While we do not attempt here to give a complete or even in depth description of the subject of flavor at the LHC, we at least try to touch upon a few of the relevant ingredients which may help the reader to understand the potential richness and importance of this topic.

\section{Flavor diagonal information}

Naively, one might think that flavor physics is related to flavor converting amplitudes, say when the sum of the flavor charges of the incoming particles is different from that of the outgoing particles. However, this is not entirely true, since (as we have discussed in detail in Sec. 4) any form of non-universality, if not aligned with the quark mass basis, would induce some form of flavor conversion. Furthermore, non-universal terms involving new states, which transform non-trivially under the $SU(2)_L$ gauge group and are gauge invariant (such as LH squark square masses), unavoidably induce flavor conversion at some level, since these cannot be simultaneously diagonalized in the up and down mass bases (see discussion in Sec. 5.2).

The information that can be extracted is most usefully expressed in terms of the manner that the SM flavor symmetry, $G^{SM}$, is broken by the NP flavor diagonal sources. Of particular importance is whether the approximate $U(2)$ symmetry, which acts on the light quarks, is broken, since in this case the data implies that a strong mechanism of alignment must be at work. Even if the $U(2)$ symmetry is respected by the new degrees of freedom, any non-universal information, related to the breaking of $G^{SM}$, would be also extremely useful. In general, this kind of experimental insight is linked to the microscopic nature of the new dynamics. Such knowledge is invaluable, and is typically related to scales well beyond the direct reach of near future experiments. As an example of flavor diagonal information that can be extracted at the LHC era, we discuss the spectrum of new degrees of freedom which transform under the SM flavor group and the coupling of a flavor singlet state to the SM quarks.

\subsection{Spectrum}

Among the first parameters that can be extracted once new degrees of freedom are found are their masses. The phenomenology changes quantitatively based on the representation of the new particles under the flavor group. However, the interesting experimental information that one would wish to extract is similar in essence. Suppose, for instance, that we have the new states, discovered
at the LHC, transforming as an irreducible representation of the $U(3)_U$ SM flavor group (this is a reasonable assumption, given that the top couplings yield the most severe hierarchy problem). If the masses of all new states are identical or universal, then not much flavor information could be extracted. Otherwise, it is useful to break the states according to their representation of the approximate $U(2)_U$ symmetry, obtained by setting the up and charm masses to zero. The simplest non-trivial case, which we now consider, is when the new states transform in the fundamental representation of the flavor group. The most celebrated example of this case is the up type squarks, but also the KK partner of the up type quarks in universal/warped extra dimension. Under the $U(2)_U$ approximate flavor group, the fundamental states would transform as a doublet and singlet. Thus, we can think of the following three possibilities listed by order of significance (regarding flavor physics):

(i) The spectrum is universal, and the $U(2)_U$ doublet and singlet are of identical masses. This implies a flavor blind underlying dynamics.

(ii) The spectrum exhibits an approximate $2 + 1$ structure, i.e. the doublet and singlet differ in mass. This spectrum is expected in a wide class of models, where the NP flavor dynamics preserve the SM approximate symmetry structure. Examples of this class are the MFV and next-to-MFV frameworks, which contain various classes of supersymmetry models, warped extra dimension models etc. There is highly non-trivial physical content in this case, since the $U(3)_U \to U(2)_U$ breaking of the new physics cannot be generic: New physics with such breaking, if not aligned with the SM up type Yukawa, induces top flavor violation (as we have discussed in Sec. 5.2 to be constrained at the LHC) and more importantly $c \to u$ transition contributing to $D - \bar{D}$ mixing. Furthermore, hints on the origin of the flavor puzzle and flavor mediation scale could be extracted.

(iii) The spectrum is anarchic, i.e. there is no approximate degeneracy between the new particles’ masses. This case is the most exciting in terms of flavor physics, since it suggests that some form of alignment mechanism is at work, to prevent too large contributions to various flavor violating processes. Thus, there is a potential that when combining the spectrum information with high $p_T$ and low energy measurements, information on the origin of the flavor hierarchies and flavor mediation scale could be extracted.

Let us also consider another case: Suppose that the newly discovered particles are in the adjoint representation of the $U(3)_{Q,U,D}$ flavor group. An example of this case is the KK excitation of a flavor gauge boson of extra dimension models [99, 101, 102, 104]. As discussed in Sec. 6.3 under the approximate $U(2)_{Q,U,D}$ flavor group, an adjoint consists of a doublet (which corresponds to the four broken generators), a triplet and a singlet (both correspond to the unbroken generators). Once again, there are three possible cases: A universal spectrum, an approximate $3 + 2 + 1$ structure or an anarchic spectrum with alignment. The case of a bi-fundamental representation has been recently discussed in [207].

9.1.2 Couplings

Another source of precious flavor diagonal information, which has not been widely studied, is the coupling of a flavor singlet object. Celebrated examples would be in the form of non-oblique and non-universal corrections to the coupling of the $Z$ to the bottom due to the top Yukawa, or just the predicted Higgs branching ratio into quarks, which favors third generation final states. A more exotic example is the quark coupling of a new gauge boson, such as the $Z'$ variety, supersymmetric
gauginos\textsuperscript{20} or KK gauge bosons in extra dimension models. In these cases, we can view the coupling as a spurion which either transforms under the fundamental representation of the flavor group (the Higgs case) or as an adjoint (the other cases). The approach would be therefore to characterize the flavor information according to the three items listed above. If the couplings are flavor universal, then there is not much to learn. If, however, the couplings obey the $2 + 1$ rule, it already tells us that the new interactions do not only follow the SM approximate symmetry structure, but are also quasi-aligned with the SM third generation direction. The case where the couplings are anarchical is the most exciting one, as it requires a strong alignment mechanism, and may lead to a new insight on the SM flavor puzzle.

As an example for the case of a $2 + 1$ structure, let us imagine that a color octet resonance\textsuperscript{21} is discovered at the LHC in the $t\bar{t}$ channel\textsuperscript{20,21}. One may suggest that this is an observation of a KK gluon state, yet other options are clearly possible as well (assuming that the particle’s spin is consistent with one). It would be a particularly convincing argument in favor of the anarchic warped extra dimension framework if one is to prove experimentally that the decay channels into the light quarks are much smaller than the $t\bar{t}$ one. The challenge in this measurement would be to compete against the continuous di-jet background. The ability to have charm tagging is obviously a major advantage in such a scenario. Not only that it would help to suppress the background, but also a bound on the deviation from universality could be translated to a bound on the warped extra dimension volume, and thus hint for the amount of hierarchy produced by the warping\textsuperscript{181,211}.

To conclude the subject of flavor diagonal information, we schematically show possible consequences in Figs. 12 and 13. The former presents different structures of the spectrum or coupling of newly discovered degrees of freedom, and the latter demonstrates how such a measurement at the LHC affects the NP parameter space, in addition to existing low energy bounds.

### 9.2 Flavor non-diagonal information

So far we have mostly considered flavor conversion at low energies. In the following we briefly mention possible signals in which new degrees of freedom are involved in flavor converting processes, hopefully to be discovered soon at the LHC. Clearly, more direct information regarding flavor physics would be obtained in case the new states induce some form of flavor breaking beyond non-universality. For concreteness, let us give a few examples for such a possibility:

- A sfermion, say squark, which decays to a gaugino and either of two different quark flavors, both with considerable rate\textsuperscript{196}.
- A gluino which decays to quark and squark of a different flavor with a sizable rate\textsuperscript{198}.
- A lifetime measurement of a long lived stop\textsuperscript{195,199}.
- A single stop production from the charm sea content due to large scharm-stop mixing.
- A $Z'$ state or a KK gauge boson which decay into two quarks of a different flavor.
- A charged higgs particle which decays to a top and a strange\textsuperscript{193}.

\textsuperscript{20}In the case of softly broken supersymmetry, it is most likely that the gauginos’ coupling will be characterized by a unitary matrix – a remnant of supersymmetric gauge invariance. In such a case, unless large flavor violation in the gauginos’ couplings is present, they are expected to exhibit universality.

\textsuperscript{21}A recent proposal to distinguish between a color octet resonance an singlet one can be found in\textsuperscript{208}.
As in the above, we separate the discussion to the case where the approximate $U(2)$ flavor symmetry is respected by the new dynamics and the one in which it is badly broken.

(i) $U(2)$ preserving – flavor conversion occurs between the third generation and a light one. The corresponding processes then contain an odd number of third generation quarks. Since ATLAS and CMS have a top and bottom tagging capability, this class of processes can be observed with a reasonable efficiency. In the absence of charm tagging, there is no practical way to differentiate between the first two generation (thus, the information that can be extracted is well described by the covariant formalism presented in Sec. 4.2.1). Recall that in the exact massless $U(2)$ limit, the first two generations are divided into an active state and a sterile non-interacting one. In the absence of CP violating observables at the LHC, the measurement of flavor conversion is directly translated to determination of the amount of the third-active transition strength, or the corresponding mediating generator denoted as $\hat{J}_u$ in Sec. 4.2.1.

(ii) In order to go beyond case (i), charm tagging is required, which would enable to observe flavor violation that differentiate between the first two generations at high $p_T$. Almost no work has been performed on this case, but the corresponding measurement would be equivalent to probing the “small” CP conserving generators denoted by $\hat{D}_{1,4}$ in Sec. 4.2.2.

Fig. 14 demonstrates how detecting a clear signal of flavor violation at the LHC affects the NP parameter space, in addition to flavor diagonal information (Fig. 13).
Figure 13: A schematic representation of bounds on the new physics parameter space, given by the mixing between two generations $\theta_{ij}$ and the difference in mass/coupling. Left: A typical present constraint arising from not observing deviations from the SM predictions (the allowed region is colored). Right: Adding a possible measurement of a mass/coupling difference at the LHC. This figure is inspired by a plot from [212].

Figure 14: A schematic representation of bounds on the new physics parameter space. Here we include, in addition to the low energy data and the mass/coupling difference measurement in Fig. 13, a positive signal of flavor violation at the LHC.
10 Conclusions

The field of flavor physics is now approaching a new era marked by the conclusion of the B-factories and the rise of the LHC experiments. In the last decade or so, huge progress has been achieved in precision flavor measurements. As of today, no evidence for deviation from the standard model (SM) predictions has been observed, and in particular it is established that the SM is the dominant source of CP violation phenomena in quark flavor conversion. Furthermore, strong bounds related to CP violation in the up sector were recently obtained, which provide another non-trivial test for the SM Kobayashi-Maskawa mechanism.

The unique way of the SM to induce flavor violation implies that the recent data is translated to stringent bounds on new microscopical dynamics. To put it differently, any new physics at the TeV scale, motivated by the hierarchy problem, cannot have a general flavor structure. As we have discussed in detail in these lectures, it is very likely that for a SM extension to be phenomenologically viable, it has to possess the SM approximate symmetry structure, characterized by the smallness of the first two generation masses and their mixing with third generation quarks.

In the LHC epoch, while continuous progress is expected in the low energy precision tests frontier, dramatic progress is foreseen in measurements related to top flavor changing neutral processes. Moreover, in the event of new physics discovery, a new arena for flavor physics tests would open, if the new degrees of freedom carry flavor quantum numbers. At the LHC high energy experiments, extraction of flavor information is somewhat limited by its hadronic nature. In particular, distinguishing between the first two generation quarks is extremely challenging. Nevertheless, the power of this information is in probing physics at scales well beyond the direct reach of near future experiments. Thus, we expect flavor physics to continue playing an important role in our understanding of nature at short distances.

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References

[1] N. Cabibbo, “Unitary Symmetry and Leptonic Decays,” Phys. Rev. Lett. 10 (1963) 531–533.
[2] M. Kobayashi and T. Maskawa, “CP Violation in the Renormalizable Theory of Weak Interaction,” Prog. Theor. Phys. 49 (1973) 652–657.
[3] Z. Ligeti, “The CKM matrix and CP violation,” Int. J. Mod. Phys. A20 (2005) 5105–5118, hep-ph/0408267.
[4] K. Agashe, M. Papucci, G. Perez, and D. Pirjol, “Next to minimal flavor violation,” hep-ph/0509117.
[5] Z. Ligeti, M. Papucci, and G. Perez, “Implications of the measurement of the $B_s^0 - \bar{B}_s^0$ mass difference,” Phys. Rev. Lett 97 (2006) 101801, hep-ph/0604112.
[6] A. J. Buras, “Testing the CKM Picture of Flavour and CP Violation in Rare K and B Decays and Particle-Antiparticle Mixing,” Prog. Theor. Phys. 122 (2009) 145–168, 0904.4917.

[7] UTfit Collaboration, M. Bona et al., “Model-independent constraints on $\Delta F=2$ operators and the scale of new physics,” JHEP 03 (2008) 049, 0707.0636.

[8] J. Charles, “Status of the CKM matrix and a simple new physics scenario,” Nucl. Phys. Proc. Suppl. 185 (2008) 17–21.

[9] G. Blaylock, A. Seiden, and Y. Nir, “The Role of CP violation in $D^0$ anti-$D^0$ mixing,” Phys. Lett. B355 (1995) 555–560, hep-ph/9504306.

[10] S. Bergmann, Y. Grossman, Z. Ligeti, Y. Nir, and A. A. Petrov, “Lessons from CLEO and FOCUS Measurements of $D^0$-anti-$D^0$ Mixing Parameters,” Phys. Lett. B486 (2000) 418–425, hep-ph/0005181.

[11] S. Bianco, F. L. Fabbri, D. Benson, and I. Bigi, “A Cicerone for the physics of charm,” Riv. Nuovo Cim. 26N7 (2003) 1–200, hep-ex/0309021.

[12] E. Golowich, S. Pakvasa, and A. A. Petrov, “New physics contributions to the lifetime difference in $D^0$ - anti-$D^0$ mixing,” Phys. Rev. Lett. 98 (2007) 181801, hep-ph/0610039.

[13] E. Golowich, J. Hewett, S. Pakvasa, and A. A. Petrov, “Implications of $D^0$ - $\bar{D}^0$ Mixing for New Physics,” Phys. Rev. D76 (2007) 095009, 0705.3650.

[14] A. F. Falk, Y. Grossman, Z. Ligeti, and A. A. Petrov, “SU(3) breaking and $D^0$ - anti-$D^0$ mixing,” Phys. Rev. D65 (2002) 054034, hep-ph/0110317.

[15] A. F. Falk, Y. Grossman, Z. Ligeti, Y. Nir, and A. A. Petrov, “The $D^0$ - anti-$D^0$ mass difference from a dispersion relation,” Phys. Rev. D69 (2004) 114021, hep-ph/0402204.

[16] Y. Grossman, A. L. Kagan, and Y. Nir, “New physics and CP violation in singly Cabibbo suppressed D decays,” Phys. Rev. D75 (2007) 036008, hep-ph/0609178.

[17] M. Ciuchini et al., “$D$ - $\bar{D}$ mixing and new physics: General considerations and constraints on the MSSM,” Phys. Lett. B655 (2007) 162–166, hep-ph/0703204.

[18] O. Gedalia, Y. Grossman, Y. Nir, and G. Perez, “Lessons from Recent Measurements of $D$ – $\bar{D}$ Mixing,” Phys. Rev. D80 (2009) 055024, 0906.1879.

[19] E. Golowich, J. Hewett, S. Pakvasa, and A. A. Petrov, “Relating D0-anti-D0 Mixing and D0 $\rightarrow$ 1+1- with New Physics,” Phys. Rev. D79 (2009) 114030, 0903.2830.

[20] I. I. Bigi, M. Blanke, A. J. Buras, and S. Recksiegel, “CP Violation in D0 - anti-D0 Oscillations: General Considerations and Applications to the Littlest Higgs Model with T-Parity,” JHEP 07 (2009) 097, 0904.1545.

[21] I. I. Bigi, “No Pain, No Gain – On the Challenges and Promises of Charm Studies,” 0907.2950.
[22] A. L. Kagan and M. D. Sokoloff, “On Indirect CP Violation and Implications for $D^0 - \bar{D}^0$ and $B_s - \bar{B}_s$ mixing,” Phys. Rev. D80 (2009) 076008, 0907.3917.

[23] K. Blum, Y. Grossman, Y. Nir, and G. Perez, “Combining $K - \bar{K}$ mixing and $D - \bar{D}$ mixing to constrain the flavor structure of new physics,” Phys. Rev. Lett. 102 (2009) 211802, 0903.2118.

[24] Y. Grossman, Y. Nir, and G. Perez, “Testing New Indirect CP Violation,” Phys. Rev. Lett. 103 (2009) 071602, 0904.0305.

[25] S. L. Glashow, J. Iliopoulos, and L. Maiani, “Weak Interactions with Lepton-Hadron Symmetry,” Phys. Rev. D2 (1970) 1285–1292.

[26] M. K. Gaillard and B. W. Lee, “Rare Decay Modes of the K-Mesons in Gauge Theories,” Phys. Rev. D10 (1974) 897.

[27] P. J. Franzini, “B anti-B Mixing: A Review of Recent Progress,” Phys. Rept. 173 (1989) 1.

[28] R. Barbieri and A. Strumia, “What is the limit on the Higgs mass?,” Phys. Lett. B462 (1999) 144–149, hep-ph/9905281.

[29] R. Barbieri and A. Strumia, “The `LEP paradox′,” hep-ph/0007265.

[30] Y. Nir, “CP violation: A New era,” hep-ph/0109090.

[31] Y. Nir, “CP violation in meson decays,” hep-ph/0510413.

[32] Y. Nir, “Probing new physics with flavor physics (and probing flavor physics with new physics),” 0708.1872.

[33] Particle Data Group Collaboration, C. Amsler et al., “Review of particle physics,” Phys. Lett. B667 (2008) 1.

[34] C. Jarlskog, “Commutator of the Quark Mass Matrices in the Standard Electroweak Model and a Measure of Maximal CP Violation,” Phys. Rev. Lett. 55 (1985) 1039.

[35] C. Jarlskog, “A Basis Independent Formulation of the Connection Between Quark Mass Matrices, CP Violation and Experiment,” Z. Phys. C29 (1985) 491–497.

[36] L. Wolfenstein, “Parametrization of the Kobayashi-Maskawa Matrix,” Phys. Rev. Lett. 51 (1983) 1945.

[37] G. D’Ambrosio, G. F. Giudice, G. Isidori, and A. Strumia, “Minimal flavour violation: An effective field theory approach,” Nucl. Phys. B645 (2002) 155–187, hep-ph/0207036.

[38] G. Perez, “Brief Introduction to Flavor Physics,” 0911.2092.

[39] R. Harnik, G. D. Kribs, and G. Perez, “A universe without weak interactions,” Phys. Rev. D74 (2006) 035006, hep-ph/0604027.

[40] A. L. Kagan, G. Perez, T. Volansky, and J. Zupan, “General Minimal Flavor Violation,” Phys. Rev. D80 (2009) 076002, 0903.1794.
[41] M. Schmaltz and D. Tucker-Smith, “Little Higgs Review,” Ann. Rev. Nucl. Part. Sci. 55 (2005) 229–270, [hep-ph/0502182](http://www.arXiv.org/abs/hep-ph/0502182).

[42] ARGUS Collaboration, H. Albrecht et al., “Observation of B0 - anti-B0 Mixing,” Phys. Lett. B192 (1987) 245.

[43] G. Buchalla, A. J. Buras, and M. E. Lautenbacher, “Weak decays beyond leading logarithms,” Rev. Mod. Phys. 68 (1996) 1125–1144, [hep-ph/9512380](http://www.arXiv.org/abs/hep-ph/9512380).

[44] L. S. Littenberg, “The CP Violating Decay K0(L) → pi0 Neutrino anti-neutrino,” Phys. Rev. D39 (1989) 3322–3324.

[45] G. Buchalla and A. J. Buras, “K → pi nu anti-nu and high precision determinations of the CKM matrix,” Phys. Rev. D54 (1996) 6782–6789, [hep-ph/9607447](http://www.arXiv.org/abs/hep-ph/9607447).

[46] Y. Grossman, Y. Nir, and R. Rattazzi, “CP violation beyond the standard model,” Adv. Ser. Direct. High Energy Phys. 15 (1998) 755–794, [hep-ph/9701231](http://www.arXiv.org/abs/hep-ph/9701231).

[47] Y. Grossman and Y. Nir, “K(L) → pi0 nu anti-nu beyond the standard model,” Phys. Lett. B398 (1997) 163–168, [hep-ph/9701313](http://www.arXiv.org/abs/hep-ph/9701313)

[48] Y. Nir and M. P. Worah, “Probing the flavor and CP structure of supersymmetric models with K → pi nu anti-nu decays,” Phys. Lett. B423 (1998) 319–326, [hep-ph/9711215](http://www.arXiv.org/abs/hep-ph/9711215).

[49] G. Buchalla and G. Isidori, “The CP conserving contribution to K(L) → pi0 nu anti-nu in the standard model,” Phys. Lett. B440 (1998) 170–178, [hep-ph/9806501](http://www.arXiv.org/abs/hep-ph/9806501).

[50] G. Perez, “Implications of neutrino masses on the K(L) → pi0 nu anti-nu decay,” JHEP 09 (1999) 019, [hep-ph/9907205](http://www.arXiv.org/abs/hep-ph/9907205).

[51] G. Perez, “The K(L) → pi0 nu anti-nu decay in models of extended scalar sector,” JHEP 02 (2000) 043, [hep-ph/0001037](http://www.arXiv.org/abs/hep-ph/0001037).

[52] Y. Grossman, G. Isidori, and H. Murayama, “Lepton flavor mixing and K → pi nu anti-nu decays,” Phys. Lett. B588 (2004) 74–80, [hep-ph/0311353](http://www.arXiv.org/abs/hep-ph/0311353).

[53] A. J. Buras, F. Schwab, and S. Uhlig, “Waiting for precise measurements of K+ → π+ν̅ν̅ and K_L → π^0ν̅ν̅,” Rev. Mod. Phys. 80 (2008) 965–1007, [hep-ph/0405132](http://www.arXiv.org/abs/hep-ph/0405132).

[54] A. J. Buras, T. Ewerth, S. Jager, and J. Rosiek, “K+ → pi+ nu anti-nu and K(L) → pi0 nu anti-nu decays in the general MSSM,” Nucl. Phys. B714 (2005) 103–136, [hep-ph/0408142](http://www.arXiv.org/abs/hep-ph/0408142).

[55] A. J. Buras, “Minimal flavor violation,” Acta Phys. Polon. B34 (2003) 5615–5668, [hep-ph/0310208](http://www.arXiv.org/abs/hep-ph/0310208).

[56] A. J. Buras, “Flavour physics and CP violation,” [hep-ph/0505175](http://www.arXiv.org/abs/hep-ph/0505175).

[57] G. Isidori, “Effective Theories for Flavour Physics beyond the Standard Model,” [0908.0404](http://www.arXiv.org/abs/0908.0404).

[58] O. Gedalia, L. Mannelli, and G. Perez, “Covariant Description of Flavor Violation at the LHC,” [1002.0778](http://www.arXiv.org/abs/1002.0778).
[60] G. Colangelo, E. Nikolidakis, and C. Smith, “Supersymmetric models with minimal flavour violation and their running,” *Eur. Phys. J.* **C59** (2009) 75–98, [0807.0801](https://arxiv.org/abs/0807.0801).

[61] L. Mercolli and C. Smith, “EDM constraints on flavored CP-violating phases,” *Nucl. Phys. B**817** (2009) 1–24, [0902.1949](https://arxiv.org/abs/0902.1949).

[62] J. Ellis, R. N. Hodgkinson, J. S. Lee, and A. Pilaftsis, “Flavour Geometry and Effective Yukawa Couplings in the MSSM,” *JHEP* **02** (2010) 016, [0911.3611](https://arxiv.org/abs/0911.3611).

[63] D. Becirevic et al., “$B_d - ar{B}_d$ mixing and the $B_d \to J/\psi K_s$ asymmetry in general SUSY models,” *Nucl. Phys. B**634** (2002) 105–119, [hep-ph/0112303](https://arxiv.org/abs/hep-ph/0112303).

[64] M. Ciuchini et al., “Delta M(K) and epsilon(K) in SUSY at the next-to-leading order,” *JHEP* **10** (1998) 008, [hep-ph/9808328](https://arxiv.org/abs/hep-ph/9808328).

[65] S. Davidson, G. Isidori, and S. Uhlig, “Solving the flavour problem with hierarchical fermion wave functions,” *Phys. Lett. B**663** (2008) 73–79, [0711.3376](https://arxiv.org/abs/0711.3376).

[66] K. Agashe, G. Perez, and A. Soni, “B-factory signals for a warped extra dimension,” *Phys. Rev. Lett.* **93** (2004) 201804, [hep-ph/0406101](https://arxiv.org/abs/hep-ph/0406101).

[67] K. Agashe, G. Perez, and A. Soni, “Flavor structure of warped extra dimension models,” *Phys. Rev. D**71** (2005) 016002, [hep-ph/0408134](https://arxiv.org/abs/hep-ph/0408134).

[68] J. P. Silva and L. Wolfenstein, “Detecting new physics from CP-violating phase measurements in B decays,” *Phys. Rev. D**55** (1997) 5331–5333, [hep-ph/9610208](https://arxiv.org/abs/hep-ph/9610208).

[69] Y. Grossman, Y. Nir, and M. P. Worah, “A model independent construction of the unitarity triangle,” *Phys. Lett. B**407** (1997) 307–313, [hep-ph/9704287](https://arxiv.org/abs/hep-ph/9704287).

[70] J. M. Soares and L. Wolfenstein, “CP violation in the decays $B_0 \to \Psi K(S)$ and $B_0 \to \pi^+ \pi^-$: A Probe for new physics,” *Phys. Rev. D**47** (1993) 1021–1025.

[71] N. G. Deshpande, B. Dutta, and S. Oh, “SUSY GUTs contributions and model independent extractions of CP phases,” *Phys. Rev. Lett.* **77** (1996) 4499–4502, [hep-ph/9608231](https://arxiv.org/abs/hep-ph/9608231).

[72] A. G. Cohen, D. B. Kaplan, F. Lepeintre, and A. E. Nelson, “B factory physics from effective supersymmetry,” *Phys. Rev. Lett.* **78** (1997) 2300–2303, [hep-ph/9610252](https://arxiv.org/abs/hep-ph/9610252).

[73] G. Barenboim, G. Eyal, and Y. Nir, “Constraining new physics with the CDF measurement of CP violation in $B \to \psi K_s$,” *Phys. Rev. Lett.* **83** (1999) 4486–4489, [hep-ph/9905397](https://arxiv.org/abs/hep-ph/9905397).

[74] G. Eyal, Y. Nir, and G. Perez, “Implications of a small CP asymmetry in $B \to \psi K_s$,” *JHEP* **08** (2000) 028, [hep-ph/0008009](https://arxiv.org/abs/hep-ph/0008009).

[75] S. Laplace, Z. Ligeti, Y. Nir, and G. Perez, “Implications of the CP asymmetry in semileptonic B decay,” *Phys. Rev. D**65** (2002) 094040, [hep-ph/0202010](https://arxiv.org/abs/hep-ph/0202010).

[76] V. Tisserand, “CKM fits as of winter 2009 and sensitivity to New Physics,” [0905.1572](https://arxiv.org/abs/0905.1572).
[77] M. Bona et al., “Status of the Unitarity Triangle Analysis,” 0909.5065.

[78] G. Isidori, Y. Nir, and G. Perez, “Flavor Physics Constraints for Physics Beyond the Standard Model,” 1002.0900.

[79] CDF Collaboration, T. Aaltonen et al., “First Flavor-Tagged Determination of Bounds on Mixing-Induced CP Violation in $B^0_s \rightarrow J/\psi\phi$ Decays,” Phys. Rev. Lett. 100 (2008) 161802, 0712.2397.

[80] D0 Collaboration, V. M. Abazov et al., “Measurement of $B^0_s$ mixing parameters from the flavor-tagged decay $B^0_s \rightarrow J/\psi\phi$,” Phys. Rev. Lett. 101 (2008) 241801, 0802.2255.

[81] CKMfitter Group Collaboration, J. Charles et al., “CP violation and the CKM matrix: Assessing the impact of the asymmetric $B$ factories,” Eur. Phys. J. C41 (2005) 1–131, hep-ph/0406184.

[82] D. Pirjol and J. Zupan, “Predictions for $b \rightarrow ss\bar{s}$, $d\bar{d}s\bar{s}$ decays in the SM and with new physics,” JHEP 02 (2010) 028, 0908.3150.

[83] P. J. Fox, Z. Ligeti, M. Papucci, G. Perez, and M. D. Schwartz, “Deciphering top flavor violation at the LHC with $B$ factories,” Phys. Rev. D78 (2008) 054008, 0704.1482.

[84] BABAR Collaboration, B. Aubert et al., “Measurement of the $B \rightarrow X_s\ell^+\ell^-$ branching fraction with a sum over exclusive modes,” Phys. Rev. Lett. 93 (2004) 081802, hep-ex/0404006.

[85] Belle Collaboration, M. Iwasaki et al., “Improved measurement of the electroweak penguin process $B \rightarrow X/s \ell^+\ell^-$,” Phys. Rev. D72 (2005) 092005, hep-ex/0503044.

[86] ATLAS Collaboration, J. Carvalho et al., “Study of ATLAS sensitivity to FCNC top decays,” Eur. Phys. J. C52 (2007) 999–1019, 0712.1127.

[87] R. S. Chivukula and H. Georgi, “Composite Technicolor Standard Model,” Phys. Lett. B188 (1987) 99.

[88] L. J. Hall and L. Randall, “Weak scale effective supersymmetry,” Phys. Rev. Lett. 65 (1990) 2939–2942.

[89] E. Gabrielli and G. F. Giudice, “Supersymmetric corrections to epsilon prime / epsilon at the leading order in QCD and QED,” Nucl. Phys. B433 (1995) 3–25, hep-lat/9407029.

[90] A. Ali and D. London, “Profiles of the unitarity triangle and CP-violating phases in the standard model and supersymmetric theories,” Eur. Phys. J. C9 (1999) 687–703, hep-ph/9903535.

[91] A. J. Buras, P. Gambino, M. Gorbahn, S. Jager, and L. Silvestrini, “Universal unitarity triangle and physics beyond the standard model,” Phys. Lett. B500 (2001) 161–167, hep-ph/0007085.

[92] V. Cirigliano, B. Grinstein, G. Isidori, and M. B. Wise, “Minimal flavor violation in the lepton sector,” Nucl. Phys. B728 (2005) 121–134, hep-ph/0507001.
[93] S. Davidson and F. Palorini, “Various definitions of minimal flavour violation for leptons,”
Phys. Lett. B642 (2006) 72–80, hep-ph/0607329.

[94] M. B. Gavela, T. Hambye, D. Hernandez, and P. Hernandez, “Minimal Flavour Seesaw Models,” JHEP 09 (2009) 038, 0906.1461.

[95] M. Dine, A. E. Nelson, and Y. Shirman, “Low-energy dynamical supersymmetry breaking simplified,” Phys. Rev. D51 (1995) 1362–1370, hep-ph/9408384.

[96] M. Dine, A. E. Nelson, Y. Nir, and Y. Shirman, “New tools for low-energy dynamical supersymmetry breaking,” Phys. Rev. D53 (1996) 2658–2669, hep-ph/9507378.

[97] L. Randall and R. Sundrum, “Out of this world supersymmetry breaking,” Nucl. Phys. B557 (1999) 79–118, hep-th/9810155.

[98] G. F. Giudice, M. A. Luty, H. Murayama, and R. Rattazzi, “Gaugino Mass without Singlets,” JHEP 12 (1998) 027, hep-ph/9810442.

[99] R. Rattazzi and A. Zaffaroni, “Comments on the holographic picture of the Randall-Sundrum model,” JHEP 04 (2001) 021, hep-th/0012248.

[100] G. Cacciapaglia et al., “A GIM Mechanism from Extra Dimensions,” JHEP 04 (2008) 006, 0709.1714.

[101] A. L. Fitzpatrick, G. Perez, and L. Randall, “Flavor from Minimal Flavor Violation & a Viable Randall- Sundrum Model,” 0710.1869.

[102] G. Perez and L. Randall, “Natural Neutrino Masses and Mixings from Warped Geometry,” JHEP 01 (2009) 077, 0805.4652.

[103] C. Csaki, A. Falkowski, and A. Weiler, “A Simple Flavor Protection for RS,” Phys. Rev. D80 (2009) 016001, 0806.3757.

[104] C. Csaki, G. Perez, Z. Surujon, and A. Weiler, “Flavor Alignment via Shining in RS,” 0907.0474.

[105] S. L. Glashow and S. Weinberg, “Natural Conservation Laws for Neutral Currents,” Phys. Rev. D15 (1977) 1958.

[106] G. G. Athanasiu and F. J. Gilman, “BOUNDS ON CHARGED HIGGS PROPERTIES FROM CP VIOLATION,” Phys. Lett. B153 (1985) 274.

[107] W.-S. Hou and R. S. Willey, “Effects of Charged Higgs Bosons on the Processes b → s Gamma, b → s g* and b → s Lepton+ Lepton-,” Phys. Lett. B202 (1988) 591.

[108] B. Grinstein, M. J. Savage, and M. B. Wise, “B → X(s) e+ e- in the Six Quark Model,” Nucl. Phys. B319 (1989) 271–290.

[109] T. Hurth, G. Isidori, J. F. Kamenik, and F. Mescia, “Constraints on New Physics in MFV models: A Model- independent analysis of Δ F = 1 processes,” Nucl. Phys. B808 (2009) 326–346, 0807.5039.
[110] A. J. Buras, D. Guadagnoli, and G. Isidori, “On $\epsilon_K$ beyond lowest order in the Operator Product Expansion,” [1002.3612](https://arxiv.org/abs/1002.3612).

[111] L. J. Hall, R. Rattazzi, and U. Sarid, “The Top quark mass in supersymmetric SO(10) unification,” *Phys. Rev.* **D50** (1994) 7048–7065, [hep-ph/9306309](https://arxiv.org/abs/hep-ph/9306309).

[112] T. Blazek, S. Raby, and S. Pokorski, “Finite supersymmetric threshold corrections to CKM matrix elements in the large tan Beta regime,” *Phys. Rev.* **D52** (1995) 4151–4158, [hep-ph/9504364](https://arxiv.org/abs/hep-ph/9504364).

[113] G. Isidori and A. Retico, “Scalar flavor changing neutral currents in the large tan beta limit,” *JHEP* **11** (2001) 001, [hep-ph/0110121](https://arxiv.org/abs/hep-ph/0110121).

[114] W.-S. Hou, “Enhanced charged Higgs boson effects in $B \rightarrow \tau$ anti-neutrino, mu anti-neutrino and $b \rightarrow \tau$ anti-neutrino + X,” *Phys. Rev.* **D48** (1993) 2342–2344.

[115] A. G. Akeroyd and S. Recksiegel, “The effect of $H^+$ on $B^+ \rightarrow \tau^+ \nu/\tau$ and $B^+ \rightarrow mu^+ \nu/mu$,” *J. Phys.* **G29** (2003) 2311–2317, [hep-ph/0306037](https://arxiv.org/abs/hep-ph/0306037).

[116] G. Isidori and P. Paradisi, “Hints of large tan(beta) in flavour physics,” *Phys. Lett.* **B639** (2006) 499–507, [hep-ph/0605012](https://arxiv.org/abs/hep-ph/0605012).

[117] M. S. Carena, D. Garcia, U. Nierste, and C. E. M. Wagner, “$b \rightarrow s$ gamma and supersymmetry with large tan(beta),” *Phys. Lett.* **B499** (2001) 141–146, [hep-ph/0010003](https://arxiv.org/abs/hep-ph/0010003).

[118] G. Degrassi, P. Gambino, and G. F. Giudice, “$B \rightarrow X/s$ gamma in supersymmetry: Large contributions beyond the leading order,” *JHEP* **12** (2000) 009, [hep-ph/0009337](https://arxiv.org/abs/hep-ph/0009337).

[119] M. S. Carena, D. Garcia, U. Nierste, and C. E. M. Wagner, “Effective Lagrangian for the $\bar{t}b H^+$ interaction in the MSSM and charged Higgs phenomenology,” *Nucl. Phys.* **B577** (2000) 88–120, [hep-ph/9912516](https://arxiv.org/abs/hep-ph/9912516).

[120] A. J. Buras, P. H. Chankowski, J. Rosiek, and L. Slawianowska, “$\Delta M_{d,s}$, $B^0 d$, $s \rightarrow \mu^+ \mu^-$ and $B \rightarrow X_s \gamma$ in supersymmetry at large tan $\beta$,” *Nucl. Phys.* **B659** (2003) 3, [hep-ph/0210145](https://arxiv.org/abs/hep-ph/0210145).

[121] A. J. Buras, P. H. Chankowski, J. Rosiek, and L. Slawianowska, “$\Delta M(s)$ / $\Delta M(d)$, sin 2 Beta and the angle $\gamma$ in the presence of new $\Delta F = 2$ operators,” *Nucl. Phys.* **B619** (2001) 434–466, [hep-ph/0107048](https://arxiv.org/abs/hep-ph/0107048).

[122] C. Hamzaoui, M. Pospelov, and M. Toharia, “Higgs-mediated FCNC in supersymmetric models with large tan(beta),” *Phys. Rev.* **D59** (1999) 095005, [hep-ph/9807350](https://arxiv.org/abs/hep-ph/9807350).

[123] S. R. Choudhury and N. Gaur, “Dileptonic decay of $B/s$ meson in SUSY models with large tan(beta),” *Phys. Lett.* **B451** (1999) 86–92, [hep-ph/9810307](https://arxiv.org/abs/hep-ph/9810307).

[124] K. S. Babu and C. F. Kolda, “Higgs mediated $B^0 \rightarrow \mu^+ \mu^-$ in minimal supersymmetry,” *Phys. Rev. Lett.* **84** (2000) 228–231, [hep-ph/9909476](https://arxiv.org/abs/hep-ph/9909476).

[125] J. R. Ellis, J. S. Lee, and A. Pilaftsis, “B-Meson Observables in the Maximally CP-Violating MSSM with Minimal Flavour Violation,” *Phys. Rev.* **D76** (2007) 115011, [0708.2079](https://arxiv.org/abs/0708.2079).
[126] M. Gorbahn, S. Jager, U. Nierste, and S. Trine, “The supersymmetric Higgs sector and $B^-\bar{B}$ mixing for large $\tan \beta$,” 0901.2065.

[127] L. Mercolli, “Revisiting CP-violation in Minimal Flavour Violation,” 0903.4633.

[128] P. Paradisi and D. M. Straub, “The SUSY CP Problem and the MFV Principle,” Phys. Lett. B684 (2010) 147–153, 0906.4551.

[129] T. Feldmann and T. Mannel, “Large Top Mass and Non-Linear Representation of Flavour Symmetry,” Phys. Rev. Lett. 100 (2008) 171601, 0801.1802.

[130] S. Weinberg, “The quantum theory of fields. Vol. 2: Modern applications,” Cambridge, UK: Univ. Pr. (1996) 489 p.

[131] S. Bergmann and G. Perez, “Constraining models of new physics in light of recent experimental results on $a(\psi K_S)$,” Phys. Rev. D64 (2001) 115009, hep-ph/0103299.

[132] C. Bobeth et al., “Upper bounds on rare K and B decays from minimal flavor violation,” Nucl. Phys. B726 (2005) 252–274, hep-ph/0505110.

[133] C. Bobeth, T. Ewerth, F. Kruger, and J. Urban, “Enhancement of $B(\text{anti-}B(d) \to \mu^+\mu^-)$ / $B(\text{anti-}B(s) \to \mu^+\mu^-)$ in the MSSM with minimal flavor violation and large $\tan \beta$,” Phys. Rev. D66 (2002) 074021, hep-ph/0204225.

[134] J. M. Soares, “CP violation in radiative $b$ decays,” Nucl. Phys. B367 (1991) 575–590.

[135] W. Altmannshofer, A. J. Buras, and P. Paradisi, “Low Energy Probes of CP Violation in a Flavor Blind MSSM,” Phys. Lett. B669 (2008) 239–245, 0808.0707.

[136] A. J. Buras, P. H. Chankowski, J. Rosiek, and L. Slawianowska, “Correlation between $\Delta M(s)$ and $B^0 (s, d) \to \mu^+\mu^-$ in supersymmetry at large $\tan \beta$,” Phys. Lett. B546 (2002) 96–107, hep-ph/0207241.

[137] T. Hurth, E. Lunghi, and W. Porod, “Untagged $B \to X/s+d$ gamma CP asymmetry as a probe for new physics,” Nucl. Phys. B704 (2005) 56–74, hep-ph/0312260.

[138] D. Becirevic, V. Gimenez, G. Martinelli, M. Papinutto, and J. Reyes, “$B$-parameters of the complete set of matrix elements of $\Delta(B) = 2$ operators from the lattice,” JHEP 04 (2002) 025, hep-lat/0110091.

[139] HPQCD Collaboration, E. Gamiz, C. T. H. Davies, G. P. Lepage, J. Shigemitsu, and M. Wingate, “Neutral $B$ Meson Mixing in Unquenched Lattice QCD,” Phys. Rev. D80 (2009) 014503, 0902.1815.

[140] G. Hiller, Y. Hochberg, and Y. Nir, “Flavor Changing Processes in Supersymmetric Models with Hybrid Gauge- and Gravity-Mediation,” JHEP 03 (2009) 115, 0812.0511.

[141] G. Hiller, Y. Hochberg, and Y. Nir, “Flavor in Supersymmetry: Anarchy versus Structure,” JHEP 03 (2010) 079, 1001.1513.

[142] G. Raz, “The mass insertion approximation without squark degeneracy,” Phys. Rev. D66 (2002) 037701, hep-ph/0205310.
[143] A. Masiero, S. K. Vempati, and O. Vives, “Flavour physics and grand unification,” 0711.2903.

[144] M. Artuso et al., “B, D and K decays,” Eur. Phys. J. C57 (2008) 309–492, 0801.1833.

[145] G. Isidori and A. Retico, “B_{s,d} \to \ell^+\ell^- and K_L \to \ell^+\ell^- in SUSY models with nonminimal sources of flavor mixing,” JHEP 09 (2002) 063, hep-ph/0208159.

[146] J. Foster, K.-i. Okumura, and L. Roszkowski, “New constraints on SUSY flavour mixing in light of recent measurements at the Tevatron,” Phys. Lett. B641 (2006) 452–460, hep-ph/0604121.

[147] M. Raidal et al., “Flavour physics of leptons and dipole moments,” Eur. Phys. J. C57 (2008) 13–182, 0801.1826.

[148] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, “A complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model,” Nucl. Phys. B477 (1996) 321–352, hep-ph/9604387.

[149] A. Crivellin and M. Davidkov, “Do squarks have to be degenerate? Constraining the mass splitting with Kaon and D mixing,” Phys. Rev. D81 (2010) 095004, 1002.2653.

[150] Y. Shadmi and Y. Shirman, “Dynamical supersymmetry breaking,” Rev. Mod. Phys. 72 (2000) 25–64, hep-th/9907225.

[151] M. Dine and A. E. Nelson, “Dynamical supersymmetry breaking at low-energies,” Phys. Rev. D48 (1993) 1277–1287, hep-ph/9303230.

[152] P. Meade, N. Seiberg, and D. Shih, “General Gauge Mediation,” Prog. Theor. Phys. Suppl. 177 (2009) 143–158, 0801.3278.

[153] Z. Chacko, M. A. Luty, A. E. Nelson, and E. Ponton, “Gaugino mediated supersymmetry breaking,” JHEP 01 (2000) 003, hep-ph/9911323.

[154] P. Paradisi, M. Ratz, R. Schieren, and C. Simonetto, “Running minimal flavor violation,” Phys. Lett. B668 (2008) 202–209, 0805.3989.

[155] J. L. Feng, C. G. Lester, Y. Nir, and Y. Shadmi, “The Standard Model and Supersymmetric Flavor Puzzles at the Large Hadron Collider,” Phys. Rev. D77 (2008) 076002, 0712.0674.

[156] G. D. Kribs, “Phenomenology of extra dimensions,” hep-ph/0605325.

[157] N. Arkani-Hamed and M. Schmaltz, “Hierarchies without symmetries from extra dimensions,” Phys. Rev. D61 (2000) 033005, hep-ph/9903417.

[158] A. Delgado, A. Pomarol, and M. Quiros, “Electroweak and flavor physics in extensions of the standard model with large extra dimensions,” JHEP 01 (2000) 030, hep-ph/9911252.

[159] L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension,” Phys. Rev. Lett. 83 (1999) 3370–3373, hep-ph/9905221.

[160] Y. Grossman and M. Neubert, “Neutrino masses and mixings in non-factorizable geometry,” Phys. Lett. B474 (2000) 361–371, hep-ph/9912408.
[161] T. Gherghetta and A. Pomarol, “Bulk fields and supersymmetry in a slice of AdS,” *Nucl. Phys. B586* (2000) 141–162, [hep-ph/0003129](http://arxiv.org/abs/hep-ph/0003129).

[162] S. J. Huber and Q. Shafi, “Fermion Masses, Mixings and Proton Decay in a Randall-Sundrum Model,” *Phys. Lett. B498* (2001) 256–262, [hep-ph/0010195](http://arxiv.org/abs/hep-ph/0010195).

[163] S. J. Huber, “Flavor violation and warped geometry,” *Nucl. Phys. B666* (2003) 269–288, [hep-ph/0303183](http://arxiv.org/abs/hep-ph/0303183).

[164] G. Burdman, “Constraints on the bulk standard model in the Randall-Sundrum scenario,” *Phys. Rev. D66* (2002) 076003, [hep-ph/0205329](http://arxiv.org/abs/hep-ph/0205329).

[165] G. Burdman, “Flavor violation in warped extra dimensions and CP asymmetries in B decays,” *Phys. Lett. B590* (2004) 86–94, [hep-ph/0310144](http://arxiv.org/abs/hep-ph/0310144).

[166] D. B. Kaplan and H. Georgi, “SU(2) x U(1) Breaking by Vacuum Misalignment,” *Phys. Lett. B136* (1984) 183.

[167] H. Georgi, D. B. Kaplan, and P. Galison, “CALCULATION OF THE COMPOSITE HIGGS MASS,” *Phys. Lett. B143* (1984) 152.

[168] H. Georgi and D. B. Kaplan, “Composite Higgs and Custodial SU(2),” *Phys. Lett. B145* (1984) 216.

[169] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” *Adv. Theor. Math. Phys. 2* (1998) 231–252, [hep-th/9711200](http://arxiv.org/abs/hep-th/9711200).

[170] E. Witten, “Anti-de Sitter space and holography,” *Adv. Theor. Math. Phys. 2* (1998) 253–291, [hep-th/9802150](http://arxiv.org/abs/hep-th/9802150).

[171] N. Arkani-Hamed, M. Porrati, and L. Randall, “Holography and phenomenology,” *JHEP 08* (2001) 017, [hep-th/0012148](http://arxiv.org/abs/hep-th/0012148).

[172] R. Contino, T. Kramer, M. Son, and R. Sundrum, “Warped/Composite Phenomenology Simplified,” *JHEP 05* (2007) 074, [hep-ph/0612180](http://arxiv.org/abs/hep-ph/0612180).

[173] O. Gedalia, G. Isidori, and G. Perez, “Combining Direct & Indirect Kaon CP Violation to Constrain the Warped KK Scale,” *Phys. Lett. B682* (2009) 200–206, [0905.3264](http://arxiv.org/abs/0905.3264).

[174] P. Breitenlohner and D. Z. Freedman, “Positive Energy in anti-De Sitter Backgrounds and Gauged Extended Supergravity,” *Phys. Lett. B115* (1982) 197.

[175] C. Csaki, A. Falkowski, and A. Weiler, “The Flavor of the Composite Pseudo-Goldstone Higgs,” *JHEP 09* (2008) 008, [0804.1954](http://arxiv.org/abs/0804.1954).

[176] C. Csaki and D. Curtin, “A Flavor Protection for Warped Higgsless Models,” *Phys. Rev. D80* (2009) 015027, [0904.2137](http://arxiv.org/abs/0904.2137).

[177] K. Agashe, A. Azatov, and L. Zhu, “Flavor Violation Tests of Warped/Composite SM in the Two-Site Approach,” *Phys. Rev. D79* (2009) 056006, [0810.1016](http://arxiv.org/abs/0810.1016).

[178] A. Azatov, M. Toharia, and L. Zhu, “Higgs Mediated FCNC’s in Warped Extra Dimensions,” *Phys. Rev. D80* (2009) 035016, [0906.1990](http://arxiv.org/abs/0906.1990).
[179] S. Casagrande, F. Goertz, U. Haisch, M. Neubert, and T. Pfoh, “Flavor Physics in the Randall-Sundrum Model: I. Theoretical Setup and Electroweak Precision Tests,” *JHEP* **10** (2008) 094, [0807.4937](https://arxiv.org/abs/0807.4937).

[180] M. Blanke, A. J. Buras, B. Duling, S. Gori, and A. Weiler, “$\Delta F=2$ Observables and Fine-Tuning in a Warped Extra Dimension with Custodial Protection,” *JHEP* **03** (2009) 001, [0809.1073](https://arxiv.org/abs/0809.1073).

[181] H. Davoudiasl, G. Perez, and A. Soni, “The Little Randall-Sundrum Model at the Large Hadron Collider,” *Phys. Lett.* **B665** (2008) 67–71, [0802.0203](https://arxiv.org/abs/0802.0203).

[182] K. Agashe and R. Contino, “Composite Higgs-Mediated FCNC,” *Phys. Rev.* **D80** (2009) 075016, [0906.1542](https://arxiv.org/abs/0906.1542).

[183] A. Azatov, M. Toharia, and L. Zhu, “Radion Mediated Flavor Changing Neutral Currents,” *Phys. Rev.* **D80** (2009) 031701, [0812.2489](https://arxiv.org/abs/0812.2489).

[184] B. Duling, “A Comparative Study of Contributions to $\epsilon_K$ in the RS Model,” [0912.4208](https://arxiv.org/abs/0912.4208).

[185] J. Santiago, “Minimal Flavor Protection: A New Flavor Paradigm in Warped Models,” *JHEP* **12** (2008) 046, [0806.1230](https://arxiv.org/abs/0806.1230).

[186] M. Blanke, A. J. Buras, B. Duling, K. Gemmler, and S. Gori, “Rare K and B Decays in a Warped Extra Dimension with Custodial Protection,” *JHEP* **03** (2009) 108, [0812.3803](https://arxiv.org/abs/0812.3803).

[187] A. J. Buras, B. Duling, and S. Gori, “The Impact of Kaluza-Klein Fermions on Standard Model Fermion Couplings in a RS Model with Custodial Protection,” *JHEP* **09** (2009) 076, [0905.2318](https://arxiv.org/abs/0905.2318).

[188] M. Bauer, S. Casagrande, U. Haisch, and M. Neubert, “Flavor Physics in the Randall-Sundrum Model: II. Tree- Level Weak-Interaction Processes,” [0912.1625](https://arxiv.org/abs/0912.1625).

[189] K. Agashe, G. Perez, and A. Soni, “Collider Signals of Top Quark Flavor Violation from a Warped Extra Dimension,” *Phys. Rev.* **D75** (2007) 015002, [hep-ph/0606293](https://arxiv.org/abs/hep-ph/0606293).

[190] K. Agashe, R. Contino, L. Da Rold, and A. Pomarol, “A custodial symmetry for $Z b$ anti-$b$,” *Phys. Lett.* **B641** (2006) 62–66, [hep-ph/0605341](https://arxiv.org/abs/hep-ph/0605341).

[191] H. Davoudiasl, S. Gopalakrishna, E. Ponton, and J. Santiago, “Warped 5-Dimensional Models: Phenomenological Status and Experimental Prospects,” [0908.1968](https://arxiv.org/abs/0908.1968).

[192] Y. Grossman, Y. Nir, J. Thaler, T. Volansky, and J. Zupan, “Probing Minimal Flavor Violation at the LHC,” *Phys. Rev.* **D76** (2007) 096006, [0706.1845](https://arxiv.org/abs/0706.1845).

[193] S. Dittmaier, G. Hiller, T. Plehn, and M. Spannowsky, “Charged-Higgs Collider Signals with or without Flavor,” *Phys. Rev.* **D77** (2008) 115001, [0708.0940](https://arxiv.org/abs/0708.0940).

[194] F. del Aguila *et al.*, “Collider aspects of flavour physics at high $Q$,” *Eur. Phys. J.* **C57** (2008) 183–308, [0801.1800](https://arxiv.org/abs/0801.1800).

[195] G. Hiller and Y. Nir, “Measuring Flavor Mixing with Minimal Flavor Violation at the LHC,” *JHEP* **03** (2008) 046, [0802.0916](https://arxiv.org/abs/0802.0916).
