Local quantum Fisher information and local quantum uncertainty in two-qubit Heisenberg XYZ chain with Dzyaloshinskii–Moriya interactions

Soroush Haseli

Faculty of Physics, Urmia University of Technology, Urmia, Iran

E-mail: soroush.haseli@uut.ac.ir

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Abstract
Quantum correlations play an important role in quantum information theory. Quantum Fisher information (QFI) via local observables and via local measurements (i.e. local QFI (LQFI)) is a fundamental concept in quantum estimation and quantum metrology. LQFI captures the quantumness of correlations in a multicomponent quantum system. This new discord-like measure is very similar to the quantum correlations measure called local quantum uncertainty (LQU). In this work we will study the LQFI and the LQU in the two-qubit Heisenberg XYZ spin chain model with Dzyaloshinskii–Moriya (DM) interaction in the $z$-direction. Here we show that the DM interaction and spin interactions along the $x$- and $z$-axes can increase and maintain LQFI and LQU. It is also shown that because of thermal fluctuations LQFI and LQU are decreased by increasing temperature. They are equal to one at very low temperature and start to decay only after a threshold temperature.

Keywords: local quantum fisher information, local quantum uncertainty, heisenberg XYZ model

1. Introduction
Quantum entanglement is a special type of quantum correlation that plays an important role in quantum information theory. Quantum entanglement has a wide range of applications in quantum information tasks, such as quantum computing [1], quantum communications [2, 3] and quantum key distribution [4]. Much recent work has focussed on investigating and quantifying quantum entanglement for multipartite closed and open quantum systems [5–8]. Recent studies also showed that quantum entanglement is not the only quantum correlation in quantum information theory [9–12]. It has been shown that there are some separable quantum states that have quantum correlation despite the lack of quantum entanglement [10, 11]. So, introducing a suitable criterion for determining quantum correlations beyond entanglement has been the subject of many efforts. In reference [12] Ollivier and Zurek, have introduced quantum discord as the quantum correlation beyond entanglement. In reference [13], it is shown that classical means cannot communicate the measurement results completely if a measurement apparatus is in a nonclassical state. This means that information loss occurs even when the measuring device is not entangled with the system, and this lost information is the quantum discord. It has also been shown that quantum discord is more robust than quantum entanglement in dissipative systems [14]. Entropic quantum discord is introduced as the difference between quantum mutual information and classical information. Obtaining an analytical expression for quantum discord is only possible for certain classes of states, and the situation is somewhat complicated for the general state. The difficulty in calculating quantum discord is due to the optimization process over all local generalized measurements. This difficulty in calculating led to an alternative definition for quantum discord that is called
geometrics measure of quantum discord [15]. It is introduced as the minimum distance between the given state and the zero discord state. Analytically, calculating this geometric criterion requires a simpler optimization process than entropic quantum discord. Despite this advantage of geometric quantum discord, this criterion cannot be a suitable criterion for showing nonclassical correlations [16].

To address these difficulties, some methods and tools have been proposed to identify nonclassical correlations. In reference [17], the authors have introduced the notion of local quantum uncertainty (LQU) as a discord-like measure of nonclassical correlation. It is defined as the minimum uncertainty induced by applying local measurements on one part of quantum state using the concept of Wigner–Yanase skew information [18]. This measure meets all the conditions required for a measure of quantum correlations. In addition, LQU is associated to quantum Fisher information (QFI). It has been shown that in the unitary evolution of the density matrix i.e. \( \rho_\theta = e^{-iH\theta} \rho e^{iH\theta} \), the QFI associated with the phase parameter, majorizes the skew information. In reference [19], it has been shown that local QFI (LQFI) can be used to describe quantum correlations based on QFI. This measure is defined based on the optimizations over the observables related to one of the subsystems. In addition, LQFI provides a tool for understanding the role of quantum correlations beyond entanglement in improved accuracy and efficiency of quantum metrology protocols. Given that quantum Fisher’s information and local quantum uncertainty are both based on the concept of quantum uncertainty and they quantify nonclassical correlations, it is important to study these concepts in multipartite quantum systems. In references [20, 21], the authors study the LQFI and LQFI in the Heisenberg XYZ spin model. They show that LQFI and LQFI depend on the temperature and the coupling parameter in the anisotropic XY model. They show that for high temperatures, the quantum correlation decreases and reaches zero. In reference [22], the author studies the QFI of a 3 \( \times \) 3 bound entangled state and its relation with geometric discord.

In this work, we will study the local quantum uncertainty and QFI in the two-qubit Heisenberg XYZ chain with Dzyaloshinskii–Moriya (DM) interactions [23, 24]. We will show that DM can enhance the value of LQFI and LQFI. The work is organized as follows. In section 2, we will review the concepts of QFI and LQFI as the measures of quantum correlations. In section 3, we introduce the two-qubit Heisenberg XYZ spin system with DM interaction only in the \( z \)-direction. We also study the QFI and LQFI for the two-qubit Heisenberg XYZ spin system with DM interaction in this section. In section 4, we summarize the results.

2. Quantum uncertainty and quantum correlation

2.1. Local quantum Fisher information

QFI is a practical quantity for describing optimal accuracy in parameter estimation protocols [25–27]. Many attempts have been made to investigate the evolution of QFI to determine the relationship between quantum entanglement and quantum metrology [30, 31]. In references [28, 29], it has been shown that in the unitary evolution, quantum entanglement leads to a significant improvement in the efficiency and accuracy of parameter estimation. For a desired parametric state that depends on \( \theta \), the QFI is defined as follows

\[
F^2(\rho_\theta) = \frac{1}{4} \text{tr}(\rho_\theta L_\theta^2),
\]

where \( L_\theta \) is the symmetric logarithmic derivative operator. \( L_\theta \) is characterized as the solution of the equation

\[
\frac{\partial \rho_\theta}{\partial \theta} = \frac{1}{2} (L_\theta \rho_\theta + \rho_\theta L_\theta).
\]

The parametric state \( \rho_\theta \) can be obtained by the effect of the unitary evolution \( U_\theta = e^{iH\theta} \) on \( \rho_0 = U_\theta \rho U_\theta^\dagger \). For a given quantum state \( \rho = \sum \lambda_m |m\rangle \langle m| \) with \( \lambda_m \geq 0 \) and \( \sum \lambda_m = 1 \), the QFI \( F^2(\rho_\theta) \), that we denote by \( F^2(\rho, H) \), is obtained as

\[
F^2(\rho, H) = \frac{1}{2} \sum_{m \neq n} \frac{(\lambda_m - \lambda_n)^2}{\lambda_m + \lambda_n} |\langle m | H | n \rangle|^2.
\]

Let us consider the \( M \times N \) bipartite quantum state \( \rho = \sum_m \lambda_m |m\rangle \langle m| \) in the Hilbert space \( \mathcal{H}_A \otimes \mathcal{H}_B \). We supposed that the dynamics of the first part is described with the Hamiltonian \( H_A = H_A \otimes I_B \). In this case the LQFI can be written as [32].

\[
F^2(\rho, H_A) = \text{tr} \left( \rho H_A^2 \right) - \sum_{m \neq n} \frac{2\lambda_m \lambda_n}{\lambda_m + \lambda_n} |\langle m | H_A | n \rangle|^2.
\]

LQFI is used to characterize nonclassical correlations [19]. This quantity has special properties that any suitable correlation quantifier must have. It is possible to define a quantum correlation quantifier based on LQFI by minimizing LQFI over all local Hamiltonians \( H_A \)

\[
Q_A^2 = \min_{H_A} F^2(\rho, H_A),
\]

\( Q_A^2 \) vanishes for classical-quantum and classical-classical states [32]. \( Q_A^2 \) can be obtained easily for a bipartite quantum state with \( 2 \times N \) dimension. The overall shape of the local Hamilton is \( H_A = \sigma_i \hat{r} \), where \( |\hat{r}\rangle = 1 \) and \( \sigma = (\sigma_1, \sigma_2, \sigma_3) \) are usual Pauli matrices. It can be seen that the first term in equation (4) is equal to one, i.e. \( \text{tr} \left( \rho H_A^2 \right) = 1 \) and the second term is

\[
\sum_{m \neq n} \frac{2\lambda_m \lambda_n}{\lambda_m + \lambda_n} |\langle m | H_A | n \rangle|^2 = \sum_{i,j=1}^{3} \sum_{m \neq n} \frac{2\lambda_m \lambda_n}{\lambda_m + \lambda_n} |\langle m | \sigma_i \otimes I | n \rangle| |\langle n | \sigma_j \otimes I | m \rangle|.
\]

Now the LQFI can be obtained as

\[
Q_A^2 = 1 - \lambda_{\text{max}}^W,
\]

where \( \lambda_{\text{max}}^W \) is the largest eigenvalue of the real symmetric matrix \( W \) with the elements

\[
[W]_{ij} = \sum_{m \neq n} \frac{2\lambda_m \lambda_n}{\lambda_m + \lambda_n} |\langle m | \sigma_i \otimes I | n \rangle| |\langle n | \sigma_j \otimes I | m \rangle|.
\]
2.2. Local quantum uncertainty

The uncertainty principle sets a bound on our ability to predict the measurement outcomes of two incompatible observables with arbitrary precision, simultaneously. In general, the uncertainty of measuring a single observable \( K \) on a quantum state \( \rho \) is defined by variance as

\[
\text{Var}(\rho, H) = \text{tr} \left[ \rho K^2 \right] - (\text{tr}[\rho K])^2. \tag{9}
\]

This uncertainty may include the contributions of a classical and quantum nature. In order to determine the quantum part of variance, the concept of skew information is introduced by Wigner and Yanase as [18]

\[
I(\rho, K) = \frac{1}{2} \text{tr} \left[ \sqrt{\rho} K^2 \sqrt{\rho} \right], \tag{10}
\]

where \([\ldots]\) denotes the commutator. It is important to note that unlike variance the Wigner–Yanase skew information (WYSI) is not affected by classical mixing. Using the notion of WYSI Girolami et al introduced a measure for quantum correlations [17]. This measure is called LQU and it is defined by minimizing the WYSI over the local observable as

\[
\mathcal{U}(\rho) = \min_{K_A} \mathcal{I}(\rho, K_A \otimes I_B), \tag{11}
\]

where \( K_A \) is an observable acting on subsystem \( A \). The explicit form of LQU is defined as

\[
\mathcal{U}(\rho) = 1 - \max[\lambda_1, \lambda_2, \lambda_3], \tag{12}
\]

where \( \lambda_i \)'s are eigenvalues of the \( 3 \times 3 \) matrix \( M \) with the elements

\[
[M]_{ij} \equiv \text{tr} \left\{ \sqrt{\rho} (\sigma_i \otimes I_B) \sqrt{\rho} (\sigma_j \otimes I_B) \right\}, \tag{13}
\]

where \( i, j = 1, 2, 3 \) and \( \sigma_i \)'s are Pauli matrices.

3. Model

Let us consider the bipartite system consisting of a two-spin anisotropic Heisenberg XYZ chain in the presence of the Dzyaloshinskii–Moriya (DM) interaction. The Hamiltonian of the model is defined as [33, 34]

\[
H = J_x \sigma^1_1 \sigma^1_2 + J_y \sigma^2_1 \sigma^2_2 + J_z \sigma^3_1 \sigma^3_2 + D \cdot (\sigma_1 \times \sigma_2), \tag{14}
\]

where \( J_k \)'s \((k = x, y, z)\) are the spin-spin interaction coupling, \( D \) is the strength of DM interaction and \( \sigma_i \)'s \((i = x, y, z)\) are Pauli matrices of \( K \)th spin. If the coupling constant \( J_k > 0 \) then the system is antiferromagnetic and if \( J_k < 0 \) then the system is ferromagnetic. In this work we consider the DM interaction in the \( z \) direction. The Hamiltonian of the XYZ model with DM interaction in the \( z \) direction is defined as

\[
H = J_x \sigma^1_1 \sigma^1_2 + J_y \sigma^2_1 \sigma^2_2 + J_z \sigma^3_1 \sigma^3_2 + D_z (\sigma^3_1 \sigma^3_2 - \sigma^3_1 \sigma^3_2), \tag{15}
\]

Let us consider \(|0\rangle \) and \(|1\rangle \) as the ground and excited state of a two level particle, respectively. In computational basis \( \{|00\rangle, |01\rangle, |10\rangle, |11\rangle\} \), the Hamiltonian can be written in the following matrix form

\[
H = \begin{pmatrix}
J_z & 0 & 0 & J_z - J_y \\
0 & -J_z & J_x + J_y + 2iD_z & 0 \\
0 & J_x + J_y - 2iD_z & -J_z & 0 \\
J_z - J_y & 0 & 0 & J_z \\
\end{pmatrix}. \tag{16}
\]

The spectral analysis leads to the following spectrum for the Hamiltonian

\[
E_{1,2} = J_z \pm (J_z - J_y), \quad E_{3,4} = -J_z \pm \kappa, \tag{17}
\]

with

\[
\kappa = \sqrt{4D^2_z + (J_x + J_y)^2}. \tag{18}
\]

The eigenstates of the Hamiltonian are

\[
|\Phi_{1,2}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}, \quad |\Phi_{3,4}\rangle = \frac{|01\rangle \pm e^{i\theta}|10\rangle}{\sqrt{2}}, \tag{19}
\]

where

\[
\cos \theta = \frac{J_x + J_y}{\sqrt{4D^2_z + (J_x + J_y)^2}}. \tag{20}
\]

Here we want to investigate the temperature dependence of the LQU and LQFI in the two qubit Heisenberg XYZ model. When the typical solid state system (two-qubit system) is in
the thermal equilibrium at temperature $T$, the density operator can be defined as

$$\rho(T) = Z^{-1} e^{-\beta H} = Z^{-1} \sum_{i=1}^{4} e^{-\beta E_i} |\Phi_i\rangle \langle \Phi_i|, \quad (21)$$

where $Z = tr(e^{-\beta H})$ is the partition function of the system, and $eta = 1/k_B T$ for which $k_B$ is the Boltzmann constant (considered as $k_B = 1$ henceforth for simplicity). Now, the density matrix of the system in thermal equilibrium can be obtained as

$$\rho_z(T) = \begin{pmatrix} r & 0 & 0 & s \\ 0 & u & v & 0 \\ 0 & v^* & u & 0 \\ s & 0 & 0 & r \end{pmatrix} \quad (22)$$

where

$$r = \frac{e^{-J_z/T}}{Z} \cosh \left( \frac{J_z - J_x}{T} \right),$$

$$u = \frac{e^{J_x/T}}{Z} \cosh \left( \frac{\kappa}{T} \right),$$

$$v = -\frac{e^{J_y/T}}{Z} \sinh \left( \frac{\kappa}{T} \right) \left( 2iD_z + J_x + J_y \right),$$

$$s = \frac{e^{-J_y/T}}{Z} \sinh \left( \frac{J_z - J_y}{T} \right). \quad (23)$$

The partition function of the system can be written as

$$Z = 2e^{-J_z/T} \cosh \left( \frac{J_z - J_x}{T} \right) + 2e^{J_z/T} \cosh \left( \frac{\kappa}{T} \right). \quad (24)$$

In order to obtain LQFI, the matrix elements of the matrix $W$ must be specified. According to equation (8), it can be easily shown that the off-diagonal elements become zero and the diagonal elements are given by

$$W_{11} = \frac{4(r-s)(u-|v|)}{(u-|v|)+(r-s)} + \frac{4(r+s)(u+|v|)}{(u+|v|)+(r+s)},$$

$$W_{22} = \frac{4(r+s)(u-|v|)}{(u-|v|)+(r+s)} + \frac{4(r-s)(u+|v|)}{(u+|v|)+(r-s)},$$

and

$$W_{33} = \frac{4(r-s)(v-|u|)}{(v-|u|)+(r-s)} + \frac{4(r+s)(v+|u|)}{(v+|u|)+(r+s)},$$

$$W_{44} = \frac{4(r+s)(v-|u|)}{(v-|u|)+(r+s)} + \frac{4(r-s)(v+|u|)}{(v+|u|)+(r-s)}.$$
Figure 4. LQU for the XYZ model with DM interaction oriented along the $z$-direction. (a) $J_x = -1, J_y = -0.5$ and $D_z = 1$. (b) $J_x = -1, J_y = 0.2$ and $D_z = 1$. (c) $J_x = -1, J_y = -0.5$ and $D_z = 1$. (d) $J_x = -1, J_y = -1$ and $J_z = 0.2$.

In figure 1, LQFI is plotted as a function of temperature for the XYZ model including the DM interaction in the $z$ direction. As can be seen, due to thermal fluctuations LQFI is decreased by increasing temperature. LQFI are equal to one at very low temperature and starts to decay only after a threshold temperature. This happens because thermal fluctuations only affect quantum correlations at temperatures above the characteristic temperature set by the gap energy, which is nonzero for a finite sized systems. We also see that the characteristic temperature at which the LQFI begins to decrease increases with increasing interaction parameter $J_z$. Figure 2(a) shows the changes of LQFI in terms of interaction parameter $J_z$. As the value of this parameter increases, the amount of LQFI increases and reaches a constant value of one both for the systems of a ferromagnetic and antiferromagnetic nature. It can also be seen that the LQFI is decreased by increasing temperature. In figure 2(b), the LQFI is plotted as a function of interaction parameter $J_z$. As can be seen, the LQFI decreases with increasing the value of $J_z$ for both ferromagnetic and antiferromagnetic systems. It reaches zero for the antiferromagnetic system. Figure 2(c) represents the LQFI in terms of interaction parameter $J_x$. As can be seen, for ferromagnetic systems LQFI increases and reaches to its maximum value of one without dependence on temperature, while for antiferromagnetic systems LQFI increases and reaches a fixed value. This fixed value varies with the different temperatures. In figure 2(d), the LQFI is plotted as a function of the strength of DM interaction $D_z$. As can be seen, LQFI increases with increasing $D_z$ and it always saturates to one. From figures 2(a) and (d) it can be seen that when $J_x$ and $D_z$ increase, the LQFI always saturates to one. This indicates that spin–spin coupling in the $x$- and $z$-directions and spin–orbit coupling in the $z$-direction can increase and maintain the value of the LQFI of the system.

In a similar way, to obtain LQU, the matrix elements of matrix $M$ must be defined. Considering equation (13), it can be easily shown that the off-diagonal elements equal to zero and the diagonal elements are given by

$$M_{11} = 2 \left( \sqrt{r - s} \sqrt{|u| + v} + \sqrt{r + s} \sqrt{|u| + v} \right),$$

$$M_{22} = 2 \left( \sqrt{r + s} \sqrt{|u| - v} + \sqrt{r - s} \sqrt{|u| - v} \right),$$

$$M_{33} = 2 \left( \sqrt{r - s} \sqrt{|u| + v} + \sqrt{r + s} \sqrt{|u| + v} \right).$$

So, the LQU is obtained as

$$U(\rho) = 1 - \max \{M_{11}, M_{22}, M_{33}\}.$$

In figure 3, LQU is plotted as a function of temperature for the XYZ model including the DM interaction in the $z$-direction. Figures 4(a)–(d) show the LQU in terms of interaction parameters $J_x$, $J_y$, $J_z$ and $D_z$, respectively. For LQU the results are quite similar to those for LQFI.

\[ W_{33} = \frac{2(u^2 - |v|^2)}{u} + \frac{2(r^2 - s^2)}{r}. \]
4. Conclusion

In conclusion, the LQFI plays an important role in evaluating quantum correlations. This is due to its relationship with the concept of local quantum uncertainty. In this work we have studied the LQFI and LQU in the two-qubit Heisenberg XYZ spin chain model with DM interaction along the $z$-direction. We have investigated the effect of temperature on LQFI and LQU for this model. It was shown that due to thermal fluctuations LQFI and LQU are decreased by increasing temperature. They are equal to one at very low temperature and start to decay only after a threshold temperature. This happens because thermal fluctuations only affect quantum correlations at temperatures above the characteristic temperature set by the gap energy, which is nonzero for a finite-sized system. We also see that the characteristic temperature at which the LQFI and LQU begin to decrease increases with increasing interaction parameter $J_z$. We have also shown that LQFI and LQU are increased by increasing interaction parameters. It was shown that when $J_z$ and $D$ increase, the LQFI and LQU always saturate to one. This indicates that spin–orbit coupling in the $x$- and $z$-directions and spin–orbit coupling in the $z$-direction can increase and maintain the value of the LQFI and LQU of the system. Most importantly, the present work shows that LQFI and LQU exhibit a similar variation.

ORCID iD

Soroush Haseli [https://orcid.org/0000-0003-1031-4815](https://orcid.org/0000-0003-1031-4815)

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