Construction fuzzy logic with curve shoulder in inference system mamdani

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Abstract. Fuzzy Logic is a popular methodology and widely used at cybernetics in developing software programs. Fuzzy logic is a logic which the truth values of variables any real number between 0 and 1 both are inclusive with the truth value shown through the membership function (curve). It is used to deal with the concept of partial truth, where the truth value may range between completely true or completely false. In general, fuzzy logic provides an inference structure that capable to match human reasoning which called an inference system. Fuzzy inference system Mamdani is the process of decision making based on fuzzy sets, fuzzy rules and fuzzy logic with Mamdani method (min-max). On Mamdani method, the fuzzy sets are obtained by taking the minimum value of the rule, then use it to modify the fuzzy region and apply it to the output with the operator OR (union). This paper concerns fuzzy logic construction with curve shoulder in inference system Mamdani. The result shows five steps that used to construct inference system Mamdani which are fuzzification, operator fuzzy logic, implication, aggregation, and defuzzification.

1. Introduction
Fuzzy Logic is a popular methodology and widely used at cybernetics in developing software programs. Fuzzy logic is a suitable technique since the solution is understandable with the human operators and it applies their skill with automatic operations in constructing the controller [1]. Several applications have been developed using fuzzy logic in air conditioner [2], transport planning [3], and medical diagnosis [4,5,6].

Fuzzy logic defines as a logic where the truth values somewhere in the range of 0 and 1 are inclusive with the truth value shown through the membership function (curve). It is an expansion of Boolean logic where the variables have the truth values of true or false explicitly. Membership function is a curve that represents the membership degree of every input variable into the interval [0,1]. Several membership functions that may be applied include triangular, trapezoidal and Gaussian [7]. A membership function only clarifies one fuzzy set. Membership function that is more than one usually give a description of an input variable and this membership function called curve shoulder.

The formulation process from an input to output with fuzzy logic is known as Fuzzy Inference [6]. Generally, three methods may be used in fuzzy inference system and Mamdani method is often applied in fuzzy models. It provides understandable results with a clear structure, also for the basic nature of rules that are intuitive and explainable [8]. On Mamdani method, the fuzzy sets are acquired by taking the minimum value of the rule, afterward modify the fuzzy region with the minimum value.
and operate it to the output with the operator OR. Therefore, in this article we construct the fuzzy logic with curve shoulder in inference system Mamdani.

2. Research Method
This research is a theoretical research conducted by analyzing several related literatures such as fuzzy logic, fuzzy inference system Mamdani. The results of this research were linked and ordered to disclose the relations among the theories. The procedures that applicable to this research are shown below.
1) Conduct theoretical study of some related materials.
2) Scrutinize the results of theoretical studies and analyzing fuzzy inference system Mamdani.
3) Conduct the conclusion.

3. Results & Discussion

3.1. Fuzzy Logic
Fuzzy logic was first established by Professor Lotfi A Zadeh from the University of California in 1965 [2,9]. It is an extension of infinite-valued logics and it aims to provide a basis for approximate reasoning with fuzzy set theory as the principal tool [10]. Basically, fuzzy logic proved to be a powerful tool for dealing this unсertainty to fuzzy systems which could support decisions in several fields, such as traffic control [11,12,13], mobile robot [14], FLPP [15] and environmental conditions [16].

Fuzzy logic defines as a logic where the truth values somewhere in the range of 0 and 1 are inclusive. Unlike Boolean logic where the variables have the truth values of true or false explicitly that expressed by 1 or 0. The truth values in fuzzy logic have a range between fully true and fully false.

3.1.1. Fuzzy Sets. Fuzzy logic begins with the theory of a fuzzy set which is a set with no crisp boundary. It may include elements with just partial membership degree [17]. A fuzzy set is defined by a membership function \( \mu \) that maps the variables in the interval \([0, 1]\) [6].

Reference [18], let \( X \) be the collection of element \( x \). Then a fuzzy set \( A \) in \( X \) is a set

\[
A = \{(x, \mu_A(x)) | x \in X\},
\]

where \( \mu_A(x) \) is the membership function of fuzzy set \( A \).

Fuzzy set has two attributes, which are linguistic and numeric [2]. Linguistic is naming a group that represents the state of natural language (slow, normal, and fast). Numeric is a value that indicates the size of a variable, for example, 5, 12, and 38.

3.1.2. Membership function. Membership function is a curve in where every point of the input is mapped to a membership value (or membership degree) inside the range of 0 and 1 [14]. Reference [19], the membership function \( \mu_A \) of a fuzzy set \( A \) is written as

\[
\mu_A : X \rightarrow [0,1],
\]

where all \( x \in X \) has a membership value \( \mu_A(x) \in [0,1] \).

In the fuzzy logic, several classes of parameterized membership functions that may be applied are triangular, trapezoidal, Gaussian, bell shape and sigmoid [7]. The triangular membership function is mainly formed by two lines (linear), shown in Figure 1.
The membership function of triangular is written as

\[
\mu(x) = \begin{cases} 
0 & x \leq a \text{ or } x \geq c \\
\frac{x-a}{b-a} & a \leq x \leq b \\
\frac{c-x}{c-b} & b \leq x \leq c 
\end{cases}
\]

A membership function can only define one fuzzy set. Membership function that is more than one usually give a description of an input variable and this membership function called curve shoulder, shown in Figure 2.

In general, curve shoulder is the combination of decreasing linear, triangle and increasing linear. For instance, the speed has three levels which are ‘Slow’, ‘Normal’ and ‘Fast’, shown in Figure 3.
3.1.3. Operation Fuzzy. The membership function is clearly an important part of a fuzzy set. Therefore, the operations fuzzy are determined via their membership functions. Three main operations of fuzzy logic are union, intersection, and complement [10,20,21].

Union (operator OR): The membership function of the union \( A \cup B \) is defined for every \( u \in U \) by
\[
\mu_{A \cup B}(u) = \max\{\mu_A(u), \mu_B(u)\}.
\]

Intersection (operator AND): The membership function of the intersection \( A \cap B \) is defined for every \( u \in U \) by
\[
\mu_{A \cap B}(u) = \min\{\mu_A(u), \mu_B(u)\}.
\]

Complement: The membership function of the complement of a fuzzy set \( A \) is defined for every \( u \in U \) by
\[
\mu_{\overline{A}}(u) = 1 - \mu_A(u).
\]

3.1.4. Fuzzy Rules. The fuzzy rules are widely used in fuzzy logic system and are frequently implemented to capture the uncertainty lies in human reasoning [1]. Fuzzy rule, also known as fuzzy if-then rule, is shown as
\[
\text{if } x \text{ is } A \text{ then } y \text{ is } B,
\]
where \( A \) and \( B \) are linguistic values. The if-clause is called antecedent (premise), whereas the then-clause is called consequence (conclusion). For example, “if the road is slippery, then driving is dangerous” and “if a tomato is red, then it is ripe” [7].

3.2. Fuzzy inference system Mamdani
Fuzzy inference is a formulation process from an input to an output using fuzzy logic [6,17]. Fuzzy inference system is the decision-making process based on fuzzy sets, fuzzy rules, and fuzzy logic. This system is compatible with solving real-world problems which are complicated to obtain a mathematical model, whereas it is uncomplicated to describe the solution linguistically with if-then rules [22]. They have been capably helpful in fields including decision making [23] and business plans [24,25,26]. In general, three methods in fuzzy inference system may be used which are Mamdani method, Sugeno method, and Tsukamoto method.

Fuzzy inference system Mamdani is often applied in fuzzy models, mostly because it provides understandable results with a clear structure, also for the basic nature of rules that are intuitive and explainable [8]. Fuzzy inference system Mamdani, known also as Min-Max method, was formulated by Ebrahim Mamdani in 1975. On Mamdani fuzzy systems both antecedent and consequent are linguistic terms and the output corresponds to the superposition of individual outputs given by each rule [27]. An application of fuzzy logic in ship berthing [28] used fuzzy inference system Mamdani with curve shoulder. The structure of fuzzy inference system Mamdani shown in Figure 4, the source of Figure 4 is [27].

![Figure 4. The structure of fuzzy inference system Mamdani](image-url)
Fuzzy inference system Mamdani composed of five steps [29]:

Step 1: Fuzzification

Fuzzification is the transformation process of numerical value from the primary data to linguistic values (fuzzy values) [30,31]. The first step is to convert the input, which is always a crisp numerical value, into membership degrees according to appropriate fuzzy sets. The output of the fuzzification is a membership degree in a linguistic set with the interval [0,1]. The process of fuzzification forms a fuzzy set using membership functions (curves). There are different curves used in fuzzification and the most widely used is the curve shoulder.

Step 2: Operator fuzzy logic

If the fuzzy inference system consists of many input variables, which also mean the antecedent of the rule contains many parts, then the operator fuzzy logic is requested to combine the membership values and acquire a value that shows the result of the rule. The if-then rules that can be formed are obtained from the results of the number of each fuzzy set in the input variable [25]. The output to the operator fuzzy logic is a single truth value. The most popular operators of fuzzy logic are OR operator (function max) and AND operator (function min) [29].

Step 3: Implication

A fuzzy set shaped by a membership function is called a consequent. In implication, the consequent is reformed by a function that related to the antecedent. The output of the implication is a fuzzy set. The function of this step is the Min method to obtain \( \alpha \)-predicate of each rule. In general, it can be written:

\[
\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x)),
\]

where

- \( \mu_{A \cap B}(x) \): membership value results of \( A \cap B \)
- \( \mu_A(x) \): membership value of fuzzy set \( A \)
- \( \mu_B(x) \): membership value of fuzzy set \( B \)

Step 4: Aggregation

For the fuzzy set solutions, the results of the implication are aggregated. The aggregation is used to merge the fuzzy sets to show the outputs of rules become one fuzzy set for making a decision [29]. The input of the aggregation is a group of the output functions, and the output is one output fuzzy set. There are three methods that can be used in making the compositions of rules, namely max (maximum), additive, and probabilistic OR [9]. Max method is often applied in fuzzy inference system, by taking the maximum value rules. Max method may be written:

\[
\mu(x_i) = \max\left(\mu_{sf}(x_i), \mu_{kf}(x_i)\right),
\]

where

- \( \mu(x_i) \): fuzzy set solutions
- \( \mu_{sf}(x_i) \): membership degree of fuzzy solutions to \( i \)-th rule
- \( \mu_{kf}(x_i) \): membership degree of consequent fuzzy to \( i \)-th rule

Step 5: Defuzzification

Defuzzification defined as a mapping from an output of fuzzy system into a non-fuzzy (crisp) output [32]. In other words, defuzzification is the transformation process in the output from fuzzy inference system into a single numerical value. Five methods may be used for defuzzifying a fuzzy set which are Bisector of Area (BOA), Center of Area (COA), Mean of Maximum (MOM), Smallest of Maximum (SOM), and Largest of Maximum (LOM) [7]. COA method and MOM method are often used than the other defuzzification methods.

The COA method is defined as
\[ z_{COA} = \frac{\int z \mu_A(z) \cdot z \, dz}{\int \mu_A(z) \, dz}, \]

where \( \mu_A(z) \) is the membership function of the aggregated output \( z \).

The MOM method is the mean of maximization \( z \) where the membership function reaches maximum \( \mu^* \), defined as

\[ z_{MOM} = \frac{\int_{Z'} z \, dz}{\int_{Z'} 1 \, dz}, \]

where \( Z' = \{ z | \mu_A(z) = \mu^* \} \).

Figure 5 shows the fuzzy inference system Mamdani with two inputs and a single output, in which each input-output variable have three fuzzy sets.

**Figure 5.** Fuzzy inference system Mamdani

4. Conclusion

Fuzzy inference system Mamdani is the decision-making process based on fuzzy sets, fuzzy rules and fuzzy logic with Mamdani method (min-max). Curve shoulder has more than one membership function that usually describes an input variable. On Mamdani method, five steps may construct an inference system with the curve shoulder. First, fuzzification in which to transform the crisp numerical value into fuzzy sets. Second, apply operator fuzzy logic with intersection or union. Third, apply implication method with minimum method. Fourth, apply aggregation method with maximum method. Fifth, the last step, defuzzification which is transforming the output back from inference system into crisp value.
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References
[1] Nagarajan D, Lathamaheswari M, Kavikumar J and Deenadayalan E 2019 Interval Type-2 Fuzzy Logic Washing Machine. International Journal of Fuzzy Logic and Intelligent Systems (IJFIS), 19(4), 223-33.
[2] Saepullah A and Wahono R S 2015 Comparative Analysis of Mamdani, Sugeno and Tsukamoto Method of Fuzzy Inference System for Air Conditioner Energy Saving. Journal of Intelligent System, 1(2), 143-7.
[3] Sarkar A, Sahoo G and Sahoo U C 2012 Application of Fuzzy Logic in Transport Planning. International Journal on Soft Computing (IJSC), 3(2), 1-21.
[4] Gorgulu O and Akilli A 2016 Use of Fuzzy Logic Based Decision Support Systems in Medicine. Ethno Med, 10(4), 393-403.
[5] Casalino G, Castellano G, Castiello C, Pasquadibisceglie V and Zaza G 2019 A Fuzzy Rule-Based Decision Support System for Cardiovascular Risk Assessment. International Workshop on Fuzzy Logic and Applications, 11291, 97-108.
[6] Casalino G, Castellano G, Pasquadibisceglie V and Zaza G 2018 Contact-Less Real-Time Monitoring of Cardiovascular Risk Using Video Imaging and Fuzzy Inference Rules. Information, 10(1), 9.
[7] Jang J R, Sun C and Mizutani E 1997 Neuro-Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence. Prentice Hall, USA.
[8] Kovac P, Rodic D, Pucovsky V, Savkovic B and Gostimirovic M 2014 Multi-Output Fuzzy Inference System for Modeling Cutting Temperature and Tool Life in Face Milling. Journal of Mechanical Science and Technology, 28(10), 4247-56.
[9] Pamuji A 2017 Fuzzy Logic Inference System for Determining The Quality Assessment of Student’s Learning ICT. Scientific Journal of Informatics, 4(1), 57-65.
[10] Klir G J and Folger T A 1988 Fuzzy Sets, Uncertainty, and Information. Prentice Hall, Englewood Cliffs, New Jersey, 1-64.
[11] Azim M A and Huda M N 2010 Fuzzy Traffic Control System. Term Paper Based on Case Study and Implementation of a Fuzzy Application, Queen’s University.
[12] Yulianto B 2003 Application of Fuzzy Logic to Traffic Signal Control Under Mixed Traffic Conditions. Traffic Engineering and Control, 44(9), 332-5.
[13] Sumiati, Sigit H T and Kapuji A 2014 Mamdani Fuzzy Inference System Application Setting for Traffic Lights. International Journal of Application or Innovation in Engineering & Management (IJAIEM), 3(10), 56-62.
[14] Khosravi H and Ghorshi S 2018 Effective Redirecting of The Mobile Robot in a Messed Environment Based On The Fuzzy Logic. International Journal of Fuzzy Logic Systems (IJFIS), 8(3), 1-13.
[15] Bhattacharyya S and Majumdar P 2019 On New Centroid Based Techniques for Solving Fuzzy Linear Programming Problems. International Journal of Fuzzy Logic and Intelligent Systems (IJFIS), 19(4), 299-306.
[16] Umam C, Sutan S M, and Hendrawan Y 2019 Fuzzy Logic in Determining The Control Temperature and Humidity in Plant Factory for Cultivation of Pak Choy (Brassica Chinensis L.) Hydroponics. The Indonesian Green Technology Journal, 8(1), 9-14.
[17] The MathWorks Inc. Fuzzy Logic Toolbox User’s Guide, 4th ed., USA.
[18] Zimmerman H J 2001 Fuzzy Set Theory and It's Applications, 4th ed, Springer Science & Business Media, New York.
[19] Driankov D, Hellendoorn H and Reinfrank M 1996 An Introduction to Fuzzy Control, 2nd ed., Springer Science & Business Media, New York.

[20] Lee C C 1990 Fuzzy Logic in Control Systems: Fuzzy Logic Controller Part I. IEEE Transactions on Systems, Man, and Cybernetics, 20(2), 404-18.

[21] Klir G J and Yuan B 1995 Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice Hall, USA.

[22] Castellano G, Castiello C, Pasquadibisceglie V and Zaza G 2017 FISDet: Fuzzy Inference System Development Tool. International Journal of Computational Intelligence Systems, 10(1), 13-22.

[23] Harliana P and Rahim R 2017 Comparative Analysis of Membership Function on Mamdani Fuzzy Inference System for Decision Making. International Conference on Information and Communication Technology (IconICT), 930.

[24] Morim A, Fortes E S, Reis P, Cosenza C, Doria F and Gonçalves A 2017 Think Fuzzy System: Developing New Pricing Strategy Methods for Consumer Goods Using Fuzzy Logic. International Journal of Fuzzy Logic Systems (IJFLS), 7(1), 1-15.

[25] Fibriayora A A I D, Gandhiadi G K, Tastrawati N K T and Kencana E N 2019 Application of Mamdani Fuzzy Method to Determine Round Bread Production at PT Vanessa Bakery. E-Jurnal Matematika, 8(3), 204-10.

[26] Dairo A O and Szücs K 2019 Towards Fuzzy Analytics for Digital Video Advertising Campaign Effectiveness and Customer Experience. International Journal of Fuzzy Logic and Intelligent Systems (IJFIS), 19(4), 332-41.

[27] Espitia H, Soriano J, Machón I and López H 2019 Design Methodology for the Implementation of Fuzzy Inference Systems Based on Boolean Relations. Electronics, 8, 1243.

[28] Nguyen V S and Im N K 2019 Automatic Ship Berthing Based on Fuzzy Logic. International Journal of Fuzzy Logic and Intelligent Systems (IJFIS), 19(3), 163-71.

[29] Wang C 2015 A Study of Membership Functions on Mamdani-Type Fuzzy Inference System for Industrial Decision-Making. Theses and Dissertations. Paper 1665.

[30] Jamal Z and Rusydi M 2016 Development of Fuzzy Logic Controller for Trainer Kit Based on Microcontroller. 2nd International Conferences on Information Technology and Business (ICITB), 36-41.

[31] Khalifa A B and Frigui H 2015 Multiple Instance Mamdani Fuzzy Inference. International Journal of Fuzzy Logic and Intelligent Systems (IJFIS), 15(4), 217-31.

[32] Lee C C 1990 Fuzzy Logic in Control Systems: Fuzzy Logic Controller Part II. IEEE Transactions on Systems, Man, and Cybernetics, 20(2), 419-35.