Traffic Measurements on Signalized Arterials from Vehicle Trajectories

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Probe data provide rich information of vehicle trajectories which include driving modes (e.g., acceleration/deceleration) and reflect traffic conditions. However, most of the applications only use probe data to measure travel times. This paper presents two alternative applications of vehicle trajectories on signalized arterials: one is a traffic signal timing estimation and the other is a traffic volume estimation. Both applications are based on a simple methodology that combines vehicle trajectories and traffic engineering concept shock wave. By testing the methodology using real world data, we demonstrate that probe data have more potential for traffic measurements.

KEYWORDS: probe data, vehicle trajectory, shock wave, signal timing estimation, traffic volume estimation

1. Introduction

For monitoring traffic states on roads, various sensors are installed on road networks. Fixed detector is the most widely used sensor for traffic measurements at a specific location on a road section. This type of sensors can directly measure the basic traffic state variables, such as traffic volume (flow) and occupancy (similar to traffic density), that are used for managing traffic congestion (e.g., traffic signal control). However, the fixed detector system covering area is limited to subsets of roads (e.g., major arterials) due to financial constraints for road authorities.

Alternatively, GPS equipped probe vehicle has received much attention for its potential as a new sensing technology. Probe sensor measures vehicle positions and velocities in a short time interval, which enables us to track the vehicle trajectory over roads. Thus, probe vehicles are sampled from all vehicles but have a great advantage over fixed sensor in terms of spatial region of measurement, i.e., we can measure some traffic states on minor roads as long as probe vehicles exist on these roads. Furthermore, since vehicle trajectory data have rich information which include driving modes (e.g., acceleration/deceleration, stop, and free driving) and reflect traffic conditions, probe data is used for estimating traffic state (e.g., Nanthawichit et al. [1]; Herrera and Bayen [2]; Yuan et al. [3]; Deng et al. [4]) and reconstructing vehicle trajectories (e.g., Coifman [5]; Claudel and Bayen [6, 7]; Mehran et al. [8]) by the fusion of various types of data, such as fixed data and signal timing data.

However, when it comes to the use of only probe vehicle data, most applications (in practice) only use the data to obtain directly measurable information such as travel times. This means that almost all the information of vehicle trajectory is not utilized. However, considering the fact that the vehicle trajectory reflects traffic conditions, we should be able to extract various information from trajectory data by exploiting knowledge of traffic flow characteristics.

This paper presents two alternative applications of vehicle trajectories on signalized arterials for estimating traffic variables: one is a traffic signal timing estimation and the other is a traffic volume estimation. Both applications are based on simple methods that combine vehicle trajectories and traffic engineering concept shock wave. By testing the methods using real world data, we demonstrate that probe data have more potential for traffic measurements.

This paper is organized as follows. Section 2 briefly overviews the kinematic wave theory (shock wave theory) which has been commonly used to describe and analyze traffic flow dynamics. We then give a more detailed explanation of shock waves on signalized arterials. In the following two sections, we show two applications of vehicle trajectories on signalized arterials for estimating traffic variables. Section 3 shows a traffic signal timing estimation method and Section 4 shows a traffic volume estimation method. For both applications, we test the performance of the methods by real world data. Section 5 concludes the paper.

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2. Shock Waves on Signalized Arterials

Kinematic wave theory (shock wave theory) for traffic flows was first advocated by Lighthill and Whitham [9] and Richards [10] and is commonly used for analyzing traffic dynamics. This theory consists of the conservation law of traffic flows and the Fundamental Diagram (FD) \( q(t, x) = Q(k(t, x), x) \), which relates traffic density \( k(t, x) \) (the number of vehicles per unit length) to traffic flow \( q(t, x) \) (the number of vehicles per unit time) at a specific location \( x \) on a road (Fig. 1); it describes the evolution of the macroscopic traffic variables (i.e., flow, density) in the time-space domain \((t, x)\). Note that according to the fundamental relationship among macroscopic traffic variables \( q = kv \), (space-mean) velocity \( v_A \) of vehicles for a traffic state \( A \) can be depicted in the FD as the slope from the origin to point \( A \).

In the theory, when traffic states change from one state to another, a discontinuity of traffic density (or flow) occurs at a boundary between these states. This boundary is called a shock wave. The speed of a shock wave is derived as a direct consequence of the conservation law. To demonstrate this intuitively, let us consider a homogeneous road section where the traffic states change from stationary traffic state \( A \) to stationary traffic state \( B \) shown in the Fig. 2(a). The FD of the road is shown in Fig. 2(b). We also consider an observer who moves along the shock wave boundary (red line in Fig. 2(a)), which lies between states \( A \) and \( B \), with velocity \( \omega_{AB} \). The relative flow \( r_A \) in state \( A \), which is observed by the moving observer, is given by the following equation:

\[
r_A = (\text{relative speed}) \times (\text{density}) = (v_A - \omega_{AB})k_A.
\]  

(2.1)

where \( v_A \) and \( k_A \) are velocity and density of state \( A \), respectively. Similarly, the relative flow \( r_B \) in state \( B \) is given by

\[
r_B = (v_B - \omega_{AB})k_B.
\]  

(2.2)

Since no vehicles are destroyed nor created (i.e., the conservation law), the value of \( r_A \) must be equal to the value of \( r_B \) at the shock wave boundary. Consequently, we obtain the shock wave speed \( \omega_{AB} \) as

\[
\omega_{AB} = \frac{q_A - q_B}{k_A - k_B}.
\]  

(2.3)

\[\text{Fig. 1.} \quad \text{Fundamental diagram (}q_{\text{max}}:\text{ saturation flow rate, }k_j:\text{ jam density).}\]

\[\text{Fig. 2.} \quad \text{Graphical representation of a shock wave: (a) time-space diagram; (b) fundamental diagram.}\]

*The relative flow can be interpreted as the instantaneous flow passing the moving observer.*
From this equation, we find that the shock wave speed $\omega_{AB}$ is equal to the slope of the red line connecting state $A$ and $B$ in the FD.

Since shock waves represent one of the most characteristic phenomena in traffic flows, such as front and tail of queues, many applications of shock waves exist in the field of traffic engineering (see, for example, May [11]). Among them, a typical application is a shock wave analysis at signalized intersections, as shown in Fig. 3. The stop line to the signalized intersection is located at green and red horizontal lines in the time-space diagram; each color indicates whether the time is in a green or red phase. In this example and later sections, we employ a triangular fundamental diagram (Fig. 3(b)) adopted in many recent traffic flow studies [8].

Figure 3 shows two major shock waves that are occurred at signalized intersections: stopping shock wave and starting shock wave. The stopping shock wave (i.e., red lines in Fig. 3) is formed by the interaction between the arrival traffic (state $A$) and stopped traffic (state $B$). The stopping shock wave emanates from the stop line at the start of the red phase and moves upstream of the intersection with speed

$$
\omega_{AB} = \frac{q_A}{k_A - k_j}.
$$

The starting shock wave (i.e., blue lines in Fig. 3) is formed when vehicles begin to discharge at saturation flow rate $q_{\text{max}}$ (assume there is no queue-spillback) at the beginning of the (effective) green phase. The speed of the starting shock wave is given as the backward wave speed of the FD:

$$
\omega_{BC} = \frac{q_{\text{max}}}{k_c - k_j} = -w,
$$

where $k_c$ is the critical density at which the flow is maximized.

The characteristic of the above two shock waves is that, except the arrival traffic state (or demand) $(q_A, k_A)$, the all parameters (i.e., $k_j$ and $-w$) of determining these speeds are (approximately) known if we appropriately estimate the FD. This characteristic plays an important role of our applications of vehicle trajectories presented in the following sections.

3. Signal Timing Estimation

In this section, we show an estimation method of a signal timing that is one of signal control parameters. This method only requires a probe trajectory, a stop line at an intersection, and a starting shock wave $-w$, i.e., the information of fixed detectors is not required. For applying and verifying the estimation method, we use probe trajectories and red signal periods observed at the same time period.

3.1 Method

Figure 4 shows the time-space diagram at an intersection. The black continuous line represents the trajectory of probe vehicle on a target road section. The red line represents the red signal period at the intersection. From this figure, we see that stop and go movements of probe vehicle due to the signal red period.

Our signal timing estimation method is based on the geometrical relationships among these stop and go movements of vehicle trajectory, the stop line at the intersection, and the starting shockwave in the time-space diagram. In this estimation, we assume that the stop line position at the intersection and characteristics and structures of the target road.
section (i.e., FD) are known in advance. The procedure of the estimation method is shown below:

**Step 0.** Calibrate the FD of the target road section from the road structures or observations. Extract the stop and start positions of probe trajectories driving along the target road section. The vehicle start time and position in the time-space diagram are denoted by \( (t_p, X_p) \).

**Step 1.** Draw a starting shock wave with velocity \(-w\) from vehicle’s start position to the stop line position in the time-space diagram.

**Step 2.** Calculate the red signal ending time \( t_e \) by the following equation:

\[
 t_e = t_p - \frac{L}{w} \quad \text{where} \quad L = X_s - X_p,
\]

where \( X_s \) is the stop line position.

By applying the same procedure to the vehicle trajectory driving along the crossroad at the same time period, the beginning of the red phase of main road \( t_s \) can be estimated. Therefore we can, in principle, estimate not only the red ending time but also the red signal period if a probe vehicle exists on the crossroad at the target time period.

### 3.2 Application

In order to verify the method, we show an estimation result using real world data. The study site is Komazawa Street in Tokyo, Japan shown in Fig. 5. This road section is a single lane facility and includes 8 signalized intersections. Analysis time period is 30 minutes (from 7:30 to 8:00) during morning rush on 1 December, 2012. The used data are 4 observed probe vehicle trajectories and red signal periods at each intersection. The latter data is used for verification. The probe data included the time and position at 1 second intervals.

In the present analysis, we use the triangular FD with parameters that are summarized in Table 1. We apply this FD to all the intersections, since the target road section is regarded as homogeneous (i.e., the number of lanes, width of lane, and traffic condition are almost constant). The forward wave speed was determined by the mean speed of probe vehicles between two consecutive intersections. The backward wave speed and jam density were determined by considering the structure of the target road sections. The saturation flow rate is calculated using the above three parameters.

Figure 6 shows the starting shock waves from probe vehicles’ start positions. Each red horizontal line and each black line represent the true red signal period and the starting shock wave, respectively. Note that we exclude the start position data that is expected to be influenced by queue-spillbacks or vehicles waiting for right turning.
Figure 7 shows the results of the red signal ending time estimation. For all the data, the root-mean-square error (RMSE) between true and estimated red signal ending times is 4.9 seconds. Since the average cycle length of all the intersections is about 120 seconds, our result seems to be good as a whole. This result may be explained by the fact that the starting shock waves appeared clearly due to heavy congestion. However, there is a large error at intersection 2. One possible reason for such error would be that the vehicle’s start position is far from the stop line of intersection 2.

### 4. Traffic Volume Estimation

Traffic volume (or traffic flow) is one of the basic traffic state variables, which is utilized for various schemes of traffic engineering, such as travel delay estimations, traffic signal controls, origin-destination estimations, and environmental impact estimations. However, in many countries, observable points of traffic volumes are limited to major arterials where sensing infrastructures (i.e., fixed detectors) are installed. Considering that the installation and maintenance costs of such infrastructures are not negligible, it would be worthwhile for area-wide traffic monitoring including suburban areas and minor streets if traffic volumes can be estimated by using only probe vehicle data.

As an example of such a method, this section shows a simple traffic volume estimation method by using vehicle trajectories, which is also based on the shock waves at signalized intersections. In contrast to the previous section in which the starting shock waves are given, this method estimates the traffic volume approximately by estimating the shock waves from the relationship between vehicle trajectories and signal timing.
4.1 Method

The main assumptions of the method presented in this section are summarized as follows:

- The FD of the target road section is triangular and known.
- The stop line positions and signal timings of target intersections are known.
- We ignore the following two phenomena: the starting loss time which is required to reach free flow speed when the signal turns green and the vehicle platoon dispersion between consecutive intersections.
- We also ignore the incoming/outgoing vehicles from/to the crossroad at the intersection.

For the second assumption, we may utilize the signal timing estimation method presented in the previous section. According to the first and third assumptions, and the kinematic wave theory, all the vehicle trajectories are piecewise linear with velocities 0 or constant. Although the third and fourth assumptions are rather strong, these provide a starting point to construct the simple traffic volume estimation method as described below.

As mentioned in Section 2, the characteristic of the starting and stopping shock waves is that the states on one side of these shock waves (i.e., stopped traffic) are known in advance. This implies that the states of the other side, including arrival traffic demand, can be estimated by using shock wave speeds. The traffic volume estimation method presented in this section relies on this simple idea. Specifically, the proposed traffic volume estimation method firstly estimates the starting and stopping shock waves from the relationship between vehicle trajectories and signal timing. We then identify a time-space region in which the traffic state (i.e., flow and density) is expected to be unchanged by considering the consecutive shockwaves at different intersections. The traffic state in identified region is obtained by adopting the shock wave speeds, which are the boundary of the region, to the FD.

The detailed procedure of the estimation method is shown below:

**Step. 0** Calibrate the FD of the target road section from the road structures or observations. Extract the stop and start positions of probe trajectories driving along the target road section.

**Step. 1** By drawing the backward wave from a vehicle’s start position in the time-space diagram, identify the signal red period that brings the vehicle to a stop. Also, determine the starting shock wave for the red period by connecting the start position and the red signal ending time by a straight line.

**Step. 2** Determine the stopping shock wave for the red period by connecting the vehicle’s stop position and the red starting time by a straight line.

**Step. 3** After determining all the shock waves from all the probe vehicle trajectories, identify a region in which the traffic state (i.e., flow and density) is expected to be unchanged by considering the consecutive shockwaves at different intersections.

**Step. 4** For each intersection, the traffic volume during a period is estimated by summing up the traffic volumes of the identified regions.

At Step 1, if the backward wave reaches a red signal ending time, the corresponding red period is identified as the cause of the vehicle’s stop straightforwardly. However, the backward wave from the observed probe vehicles does not often reach any red signal ending time exactly. In that cases, we employ the following rules. If the backward wave does not cross red periods (Fig. 8(a)), we suppose that the starting shock wave emanates from the left side nearest red signal ending time from the backward wave. This means that the discharge flow rate is less than the saturation flow rate. If the backward wave reaches a point within a red period (Fig. 8(b)), we suppose that the starting shock wave emanates from the corresponding red signal ending time with the backward wave speed \(-\frac{w}{C_0}\). In this case, the vehicle trajectory is regarded as having an observation error; its the starting point is ignored.

At Step 2, we suppose that the stopping shock wave emanates from the red signal period that identified at Step 1. This means that the proposed method excludes the situation the vehicle is being stopped completely when the signal turns green. We also suppose that vehicles uniformly arrive at each intersection, which corresponds to the procedure that the stopping shock wave is a straight line.

Under the assumptions mentioned above, the traffic state between two consecutive shockwaves at different intersections should be unchanged. At Step 3, we modify the downstream stopping shock wave so that the speed of the

![Fig. 8. Illustrations of Step 1.](image)
shock wave is equal to that of the upstream starting shock wave. This is because, in general, the accuracy of the starting shock wave is better than that of the stopping shock wave that is determined by traffic demand. Figure 9 illustrates the procedure of Step 3. The dotted lines represent the estimated stopping shock waves, and the solid lines represent the modified stopping shockwaves.

Figure 10 shows an example of the estimation of traffic states after Step 3. Each colored region represents the time-space region in which the traffic state is unchanged. Since the traffic state in each region is obtained by adopting the shock wave speeds to the FD, the traffic volume for each intersection is estimated by summing up the traffic volumes of the estimated regions. It should be noted that the estimated time period for each intersection varies depending on the length of shockwaves lie upstream and downstream of the intersection.

4.2 Application

The proposed method is tested by using real world data which is adopted in Section 3.2. Analysis time period was chosen during the morning peak period from 6:30 to 9:00 on 1 December, 2012. For the test, we choose 4 intersections (No. 4, 5, 6, 7) where many stop-and-go behaviors were observed. The used data are 13 observed probe vehicle trajectories, red signal periods at each intersection, and the traffic volume of each intersection. The third data is used for verification. We also adopt the FD shown in Section 3.2.

The estimation results are presented in Figs. 11 and 12. The number of estimations are 52 (= 13 vehicles × 4 intersections). Figure 11 shows the scatter plot of true and estimated traffic volumes per hour. The (true) average traffic volume of the target road section is 652.77 (veh/hour). Each point in the figure represents the estimation result by each probe vehicle at each intersection. Note that the estimation periods among points are different. For all the data, the RMSE between the true and estimated traffic volumes is 203.03. From the figure, the tendencies of overestimation and underestimation of proposed method cannot be seen.

Figure 12 shows the scatter plot of the estimated time periods and the differences between true and estimated traffic volumes. The maximum estimated time period is 280 seconds; the minimum estimated time period is 16 seconds; the average estimated time period is 128 seconds. From the figure, it is found that there is not strong correlation between

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1If we omit this procedure, we cannot uniquely determine the traffic state between two consecutive shockwaves at different intersections.
estimated time periods and the differences. It is also found that there is no big difference among the intersections regarding the estimation accuracy.

While the results of the proposed method indicates the possibility of traffic volume estimation using probe vehicle data, it is necessary to improve the estimation accuracy. One possible direction for improving is to calibrate the FD more appropriately. However, even if such an improvement is conducted, there would remain limitations of the method in terms of the length of estimated time period. Therefore, a traffic volume estimation method which utilizes multiple probe trajectories is being investigated by authors.

5. Conclusion

This paper presented two new applications of vehicle trajectories on signalized arterials for estimating traffic variables: one is a traffic signal timing estimation and the other is a traffic volume estimation. Both applications are based on simple methods that combine vehicle trajectories and traffic engineering concept shock wave; fixed detector data is not required. The estimation results of the proposed methods using real world data indicate the possibilities of traffic measurements by probe vehicle data only. The accuracy of estimation methods will improve as the number of probe vehicles increases because shock waves can be estimated more accurately if multiple probe vehicles exist in one cycle. However, since the paper is the first step towards building such a method, further investigations of the proposed methods are needed using more real world data. In particular, the sensitivity analysis of the FD would be important for evaluating the robustness of the estimation methods. Also, to effectively utilize probe vehicle data, it would be worthwhile and interesting to extend the proposed methods to the framework that incorporates not only the current probe vehicle data but also the historical data.
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