Heavy Ion Collisions: The Big Picture, and the Big Questions

Wit Busza, Krishna Rajagopal, and Wilke van der Schee

1Laboratory for Nuclear Science and Department of Physics, MIT, 77 Massachusetts Avenue, Cambridge, MA 02139, USA
2Center for Theoretical Physics, Massachusetts Institute of Technology, 77 Massachusetts Avenue, Cambridge, MA 02139, USA
3Institute for Theoretical Physics and Center for Extreme Matter and Emergent Phenomena, Utrecht University, Leuvenlaan 4, 3584 CE Utrecht, The Netherlands

Keywords
quark-gluon plasma, heavy-ion collisions, relativistic hydrodynamics, jets, multiparticle production

Abstract
Heavy ion collisions quickly form a droplet of quark-gluon plasma (QGP) with a remarkably small viscosity. We give an accessible introduction to how to study this smallest and hottest droplet of liquid made on earth and why it is so interesting. The physics of heavy ions ranges from highly energetic quarks and gluons described by perturbative QCD to a bath of strongly interacting gluons at lower energy scales. These gluons quickly thermalize and form QGP, while the energetic partons traverse this plasma and end in a shower of particles called jets. Analyzing the final particles in a variety of different ways allows us to study the properties of QGP and the complex dynamics of multi-scale processes in QCD which govern its formation and evolution, providing what is perhaps the simplest form of complex quantum matter that we know of. Much remains to be understood, and throughout the review big open questions will be encountered.
1. Introduction

In the past 50 years, as beams of ultra relativistic protons and nuclei have become available, the collisions of protons with nuclei and nuclei with nuclei, at higher and higher relative velocities (or total collision energy), have been studied in greater and greater detail. This review provides some answers to the questions: “Why do such studies?” and “What have we learned from them so far?” and “What are the big questions that they may illuminate in future?”.

We start with a qualitative description, in the center of mass frame (the “lab frame” at a collider), of the sequence of events that occur when two ultra relativistic nuclei collide, head on. This picture follows from the observed phenomenology (summarized in Section 3), relativity, and our understanding of the workings of QCD.\(^1\)

Each incident nucleus is a Lorentz contracted disc. For large nuclei such as Pb or Au, the diameter of the disc is about 14 fm (femtometer, or Fermi) and its thickness is about \(14/\gamma\) fm, where, at the highest beam energies attainable at RHIC and LHC, the relativistic \(\gamma\) factors are approximately 100 and 2500 respectively, corresponding to beam rapidities of \(\gamma = 5.3\) and 8.5. Each disc includes many colored quarks and antiquarks, with three more quarks than antiquarks per nucleon in the incident nuclei and with \(q \bar{q}\) pairs coming from quantum fluctuations in the initial state wave functions that are “almost real”, as a consequence of time dilation. These quarks and antiquarks are, in turn, sources of strong, almost completely transverse, color fields and corresponding field quanta, the gluons which also carry color.

The area density of quarks, antiquarks and gluons, partons for short in the language of Feynman, increases with the velocity of the nuclei. It is not uniform across the area of the disc, and fluctuates from nucleus to nucleus. The spatial variation of the partons primarily

\(^1\)For the reader interested in a more comprehensive introduction into heavy ion collisions we refer to the books (1, 2, 3). Note also that because we are required to limit the number of citations, throughout this review where possible we will cite articles in which more citations can be found.
reflects the instantaneous distribution of the nucleons inside the nuclei and of the partons inside the nucleons. Over all, the incident nuclei are highly complex systems of partons with a longitudinal momentum distribution (referred to as a structure function) that is close to being a superposition of that in the individual nucleons but with small modifications coming from the proximity, and motion, of nucleons in nuclei.

When the two discs, each a tiny fraction of a fm thick, overlap or collide, most of the incident partons lose some energy but are not kicked by any large angle. Most of these interactions are “soft”, meaning that they involve little transverse momentum transfer. These strong interactions can be described in terms of interacting fields or slabs of energy. In the language of fields and particles, as the two discs of strongly interacting transverse color fields and associated color charges collide, some color charge exchange occurs between the discs, and longitudinal color fields are produced, which fill the space between the receding two discs, reducing the energy in the discs themselves, and then gradually decay into $q\bar{q}$ pairs and gluons. A small fraction of the incident partons suffer hard perturbative interactions as the discs overlap initially which, as we will discuss later, lead to a relatively improbable but very important production of particles with high transverse momentum.

In high energy heavy ion interactions, the maximum energy density occurs just as the two highly Lorentz contracted nuclei collide. Clearly this system is very far from equilibrium, and its very high energy density is really just a consequence of Lorentz contraction. It is much more interesting to ask what we can say in a generic way about the average energy density say $1 \text{ fm}/c$ after the collision, by which time the two discs are $2 \text{ fm}$ apart. The expanding high energy density system produced around the midpoint between the two discs, where the collision occurred, has an energy density at that time that is still far in excess of $500 \text{ MeV}/\text{fm}^3$, the energy density inside a typical hadron. A rough estimate can be obtained from the available data for head-on LHC collisions with $\sqrt{s_{NN}} = 2.76 \text{ TeV}$ (corresponding to $\gamma = 1400$ and $y = 8.0$) by noting that the total transverse energy in particles with pseudorapidity between $-0.5$ and $0.5$ (so longitudinal velocity $-0.46 < v < 0.46$) is measured to be $1.65\pm0.1 \text{ TeV}$ (4), meaning that the average energy density $1 \text{ fm}/c$ after the collision is greater than $1.65 \text{ TeV}/(\pi(7 \text{ fm})^2(0.92 \text{ fm})) = 12 \text{ GeV}/\text{fm}^3$, about twenty times the energy density of a hadron. The entropy produced in these collisions is also enormous; to get a sense of this note that before the collision the entropy of the two incident nuclei is essentially zero whereas the final state after the collision can contain as many as 30,000 particles, and hence has a very large entropy. We shall return to this later, and in particular we shall see that most of this entropy is produced quickly, in the initial moments after the collision. To get a further sense of the magnitude of the average energy density $1 \text{ fm}/c$ after the collision, note that, as we shall see in Section 3, lattice calculations of QCD thermodynamics show that matter in thermal equilibrium at a temperature of $300 \text{ MeV}$ has an energy density $\approx 12 T^4 = 12.7 \text{ GeV}/\text{fm}^3$. Thus, the quarks and gluons produced in the collision cannot be described as a collection of distinct individual hadrons. Nevertheless, the quarks and gluons

$\sqrt{s_{NN}}$: is the total collision energy per nucleon-nucleon pair in the center of mass frame.

Pseudorapidity:
$\eta \equiv -\log \left[ \tan \left( \frac{\theta}{2} \right) \right]$, where $\theta$ is the polar angle in momentum space relative to the beam direction. $\eta$ is a standard proxy for rapidity $y$ because $\eta = y$ for massless particles.
in this high energy density matter are far from independent. They are so strongly coupled to each other that they form a collective medium that expands and flows as a relativistic hydrodynamic fluid with a remarkably low viscosity to entropy density ratio $\eta/s \approx 1/4\pi$ (5, 6), in units with $\hbar = k_B = 1$, within a time that can be shorter than or of order 1 fm/c in the rest frame of the fluid. This form of matter has been named Quark-Gluon Plasma, or QGP for short. Even if the transverse velocity of the fluid is small initially, say 1 fm/c after the collision, the pressure-driven hydrodynamic expansion rapidly builds up transverse velocities of order half the speed of light. As the discs recede from each other and the QGP produced between them is expanding and cooling, at the same time new QGP is continually forming in the wake of each receding disc, see Fig. 1. This happens because the quarks and gluons produced at high rapidity are moving at almost the speed of light in one of the beam directions, meaning that when enough time has passed in their frame for them to form QGP a long time has passed in the lab frame, around 330 fm/c for rapidity $y = 6.5$. Throughout this QGP production process, each disc gradually loses energy as partons with higher and higher rapidity separate from it and form QGP. In contrast, the occasional high transverse momentum particles seen in some collisions are produced by large-angle scattering at very early times, when the incident nuclei collide.

The process ends once QGP has formed at the rapidities where most of the baryon number from the incident nuclei ends up, which is expected to be about 2 units of rapidity less than that of the incident nuclei, based upon measurements made in lower energy proton-nucleus collisions (9). So, the discs lose about 85% of their energy while varying amounts

---

**Figure 1**

(Left) Space-time picture of a heavy ion collision, whereby the color gives an indication of the temperature of the plasma formed. Dynamics takes place as a function of proper time (blue curves), which is why plasma forms later at higher rapidities. (Right) Snapshots of a central 2.76 TeV PbPb collision at different times (different horizontal slices of the space-time picture on the left) with hadrons (blue and grey spheres) as well as QGP (red). In both figures, at a given time the hottest regions can be found at high rapidity close to the outgoing remnants of the nuclei and the red lines indicate the approximate longitudinal location of particles with rapidity $y = 0$, $y = 1$, and $y = 6$. (Figs. adapted from (7, 8).)
of QGP form at varying rapidities over a range that extends between $y = -6.5$ and $y = 6.5$ in collisions at the LHC. A good way to visualize the QGP production process described above is to consider the production of each volume element of QGP in its own local rest frame, where the two colliding nuclei have an asymmetric rapidity and energy, and then boost this volume of QGP back to the lab frame.

After production, each elemental volume of QGP expands in all directions. Looked at overall, the droplet of fluid flows hydrodynamically, as its initial high pressure drives fluid motion, expansion, and consequent cooling. This picture holds until the energy density at a given location in the fluid drops below that within an individual hadron, at which point the fluid falls apart into a mist of hadrons that scatter off each other a few times and then stream away freely. This mechanism of particle production, via an intermediate epoch during which a hydrodynamic fluid forms and expands, is quite different from the current understanding of particle production in elementary collisions in which only a few new particles are created.

Meanwhile, remnants of the original nuclei (excited nuclear matter compressed by about a factor of 5-10) progress in the forward and backward directions. This high baryon density system then expands and hydrodynamizes, forming hot quark-rich QGP after a time of order 1 fm/c in its own rest frame, corresponding to a time of about 330 fm/c in the lab frame. After further expansion, it subsequently falls apart into hadrons. Unfortunately, none of the LHC detectors are adequately instrumented around $y = 6.5$, an almost impossible task, meaning that the debris formed from this hot, high baryon density, QGP has not yet been studied.

So far we have considered head-on (“central”) collisions. How about non-central collisions? In the overlap region, the process is the same as described above, except that the droplet of QGP is formed with an initial approximately lenticular shape in the transverse plane. In reality, because nuclei are made of individual nucleons the energy density of the QGP that forms is lumpy in the transverse plane, making it neither perfectly circular in head-on collisions nor perfectly lenticular in non-central collisions. Deviations from circular symmetry in the initial shape of the QGP, whether due to off-center collisions or the lumpiness and fluctuations of the incident nuclei, result in anisotropies in the pressure of the hydrodynamic fluid, which in turn drive anisotropies in the expansion velocity and hence in the azimuthal momentum distribution of the finally produced particles.

In an off-center (non-central) collision, the parts of the incident nuclei that do not collide are referred to as spectators. At very early times, they create a magnetic field in the collision zone whose possible effects are the subject of much discussion that we do not have space to review (11). Later, they fragment into excited nuclei and hadrons, moving with almost the full rapidity of the incident nuclei. The extreme limit of an off-center collision is one in which the nuclei themselves miss each other but the Lorentz-contracted disc of electromagnetic fields around them do interact. These ultraperipheral collisions give rise to

---

Central and off-central: Ions colliding head-on are called central collisions, whereas if the ions only partially overlap the collision is non-central or peripheral.
and $\gamma A$ interactions (12), which we shall not pursue except to note here that they dominate the total nucleus-nucleus interaction cross-section.

Finally, a word about the hard collisions between two partons in the incident nuclei. Such collisions, especially where particles with very large transverse momenta (say, greater than a few tens of GeV/c) are produced, are rare but very important. They lead to the production, essentially from a point at the earliest times of the overall collision process, of high-energy parton pairs and electroweak bosons. The high-energy partons evolve, decay, radiate and finally produce a cone-shaped spray or “jet” of hadrons and/or high-energy photons, leptons or heavy $Q\bar{Q}$ pairs, all while traversing a region where QGP is in the process of being produced and evolving. They thus contain a wealth of information about the produced medium (in essence they “X-ray” the medium), and on how partons lose energy or disturb the medium as they interact with it. We shall return to this in Section 7.

As is evident from the above description, collisions of ultrarelativistic nuclei are complex, consisting of several distinct stages, each probing different aspects of QCD. What makes them interesting is that the regimes of QCD that they give us a means to explore are places where, because of the strength of the QCD interactions, we would not even have a zeroth order understanding of the properties of the matter that QCD describes, let alone the dynamical phenomena, without having seen what happens in these collisions. Heavy ion collisions are a laboratory that is rich with unique ways to probe fundamental aspects of QCD empirically, with some control over varying conditions. Our description of a relativistic heavy ion collision brings us directly to a set of questions: Do we have, at least qualitatively, an understanding of all the stages of a heavy ion collision? Have any fundamentally new phenomena been seen? Any unexplained phenomena? What new insights have we obtained, or can we obtain, about the workings of QCD from analysis of heavy ion collision data and corresponding theoretical calculations? How, and how well, do we understand the initial stages of the collision process, up to the creation of QGP? Which aspects involve weakly coupled dynamics, and which strongly coupled? What are the properties of QGP? From the study of jets and high momentum particles, what have we learned about the properties of strongly interacting matter, and about the dynamics of fast particles as they traverse strongly coupled matter? What new insights have we obtained about the formation of hadrons? Beginning in Section 3, we will attempt to answer some of these questions. But first, in the next Section we shall expand our perspective.

2. Why do we study ultrarelativistic heavy ion collisions?

The overarching answer with which we ended Section 1, albeit formulated as a suite of questions, is that studying ultrarelativistic heavy ion collisions may give us a path to a more complete understanding of how particles are produced in high energy collisions in QCD. This is a fundamental question that in fact long predates QCD: Heisenberg and Heitler wrestled with it in the 1930s and 1940s (13, 14), Fermi and Landau did so in the
1940s and 1950s (15, 16, 2), and Feynman tried his hand in the 1960s (17). We can now gain new purchase on these old questions by studying high energy collisions in a new regime in which experimenters have new knobs to dial including the size of each of the colliding nuclei, (proxies for) the impact parameter, the final state multiplicity, and more.

In this Section, we shall formulate variants of the "why are we doing this?" question that take us beyond the subject of ultrarelativistic heavy ion collisions per se. Are there insights that we hope to gain from studying these collisions that go beyond understanding the dynamics of these collisions, or even of ultrarelativistic collisions in QCD more generally? The affirmative answers to this question, which we shall divide into three groups below, motivate much of the experimental and theoretical efforts that we describe in this review.

1. QCD in Cosmology. Heavy ion collisions recreate droplets of the matter that filled the universe a microsecond or so after the Big Bang. And, it has been understood since the mid 1970s (23, 24) that when the universe was only a few microseconds old it was filled with matter at temperatures above $\Lambda_{\text{QCD}}$ (the fundamental energy scale in QCD, of order a few hundred MeV, which is best thought of as the inverse of the size of a hadron in QCD) and was too hot for protons, neutrons, or any hadrons to have formed. This direct and tangible connection to the earliest moments of the universe, together with the insight that the primordial matter found at these temperatures had to be some new form of matter not made of hadrons, provides two powerful motivations for studying ultrarelativistic heavy ion collisions. Historically, these were the motivations that provided much of the initial impetus for the field.

In the 1980s, a fair amount of work was done on possible observable consequences in cosmology of a first order phase transition between the hot primordial matter and ordinary hadronic matter. These all relied upon presuming a strong first order phase transition that occurred via the nucleation of widely separated bubbles of the low temperature hadronic phase. As the walls of these putative bubbles plowed through the microseconds-old universe over distances as long as centimeters or meters, they would have left behind matter that was inhomogeneous over these long length scales (25, 26). If this had happened, it would have modified the synthesis of light nuclei that occurred when the universe was minutes old. Starting in the late 1990s, and culminating in classic work in the 2000s (27, 28), it became clear from first-principles lattice QCD calculations of the pressure and energy density of hot QCD matter containing equal densities of quarks and antiquarks that the transition from primordial hot QCD matter to hadronic matter in the first few microseconds after the big bang proceeded via a continuous crossover, not a first order phase transition. This is in turn consistent with the modern understanding of big bang nucleosynthesis, which is in accord with cosmological observations without any of the disruption that a strong first order phase transition would have introduced (29). A continuous crossover does not introduce any fluctuations on length scales much longer than the fm-scale natural
Lattice QCD calculations of the pressure $p$, energy density $\varepsilon$ and entropy density $s$ of hot QCD matter in thermal equilibrium at temperature $T$ \cite{18, 19} show a continuous crossover around $T \sim 150$ MeV, from a hadron resonance gas (HRG) at lower temperatures to QGP at higher temperatures. Because QCD is asymptotically free, thermodynamic quantities will reach the Stefan-Boltzmann limit (weakly coupled quarks and gluons; the non-interacting limit is marked in the figure) at extremely high temperature. At the range shown, however, they are around 20\% below their Stefan-Boltzmann values, which is consistent with simple estimates for strongly coupled plasma based on holography \cite{20}.

The rise in $\varepsilon/T^4$ and $s/T^3$ seen in the figure is a direct manifestation of the crossover from a hadron gas to QGP which has more thermodynamic degrees of freedom because color is deconfined. Using experimental data to constrain $\varepsilon/T^4$ remains an outstanding challenge: comparing hydrodynamic calculations to various experimental measurements gives us information about $\varepsilon$ versus time but, although some information about $T$ can be obtained by analyzing measurements of photons, electrons and muons from heavy ion collisions \cite{21}, at present $T$ cannot be determined with sufficient accuracy to constrain $\varepsilon/T^4$ well enough to see the rise in the number of degrees of freedom in QGP.

(Figs. from \cite{19, 22})

A central goal of ultrarelativistic heavy ion collisions, then, is to use these experiments to recreate droplets of Big Bang matter in the laboratory -- where we can learn about its material properties as well as about its phase diagram in ways that we will never be able to do via observations made with telescopes or satellites. What can we learn from such studies? What have we learned so far? One of the most important discoveries made...
via studying ultrarelativistic heavy ion collisions is that matter that is a few trillions of
degrees hot is a liquid. The early ideas that motivated the field turned out to be half right:
primordial matter at these temperatures is not made of hadrons, as anticipated; however, at
the temperatures that have been achieved in heavy ion collisions to date, it is not a weakly
coupled plasma of quarks and gluons as originally expected. Instead, when the hadrons
that make up ordinary nuclei are heated to these extraordinary temperatures, the matter
that results is better thought of as a soup of quarks and gluons, in which there are no
hadrons to be found but in which every quark and gluon is always strongly coupled to its
neighbors, with no quasiparticles that can travel long distances between discrete scatterings.
We shall describe how this insight was obtained in Section 4. The material property that
quantifies the liquidness of a liquid made up of ultrarelativistic constituents is the ratio of
its shear viscosity $\eta$ to its entropy density $s$. The ratio $\eta/s$ is dimensionless in units
in which $\hbar$ and $k_B$ have been set to 1. This ratio plays a central role in the equations of
hydrodynamics where it governs the amount of entropy produced within the fluid as a sound
wave propagates through it, or more generally as it flows in any nontrivial way. It is the
natural dimensionless measure of the effects of shear viscosity in a relativistic fluid, and we
shall refer to it as the “specific viscosity”. In Section 4 we shall sketch how the combination
of data from ultrarelativistic collisions and hydrodynamic calculations are being used to
constrain $\eta/s$, and even its temperature dependence. We shall also see that $\eta/s$ for the
liquid of quarks and gluons produced in heavy ion collisions is close to the value $1/4\pi$.
Although because of its extraordinarily high temperature this liquid has extraordinarily
large values of both $\eta$ and $s$ relative to those of any quotient fluid, its specific viscosity $\eta/s$
is smaller than that of any other known fluid. Interestingly, $1/4\pi$ is the value of the ratio $\eta/s$
in the plasma of infinitely strongly coupled gauge theories that are cousins of QCD that have
dual gravitational description in terms of a black hole horizon in 4+1-dimensional Anti-de
Sitter space (30, 20), a horizon whose undulations encode the hydrodynamic motion of the
plasma (31). This connection between the properties of the primordial matter recreated in
heavy ion collisions, via a duality first discovered in string theory, to properties of black
hole horizons certainly provides strong motivation for pushing the determination of $\eta/s$,
and recently also the bulk viscosity, of quark-gluon plasma to higher accuracy.

The zeroth order input that a hydrodynamic calculation needs to get from the micro-
scopic theory of whatever hydrodynamic fluid it is seeking to describe is the equation of
state, relating the pressure of the fluid to its energy density. In QCD the equation of state,
and any other thermodynamic property of a static volume of quark-gluon plasma in thermal
equilibrium, can be calculated reliably via implementing the standard path integral formulation
of thermodynamics on a discretized lattice of points in space and Euclidean time,
and doing so for a series of lattices with smaller and smaller spacing between the points,
thus taking the “continuum limit” (32). Among many other conclusions, these lattice cal-
culations have taught us that the transition from the hot, liquid, quark-gluon plasma with

---

**Shear viscosity**: The larger the shear viscosity $\eta$ the more easily momentum
can be exchanged between distant fluid cells and, consequently, the faster a gradient in
fluid velocity (or a sound wave) dissipates into heat.

**Specific viscosity**: is the ratio of the shear viscosity to
the entropy density, $\eta/s$. It is the natural
dimensionless
measure of the
effects of shear viscosity in a
relativistic fluid.
zero baryon number to a gas of hadrons is a crossover (28), as in Fig. 2, with no further transitions anticipated as quark-gluon plasma gradually goes from liquid-like to gas-like at higher and higher temperatures (33). Because lattice calculations are built upon the Euclidean formulation of equilibrium thermodynamics, it is much more challenging to use them to gain information about transport coefficients including the shear and bulk viscosities, which describe the time-dependent processes via which infinitesimal perturbations away from equilibrium relax, producing entropy. Pioneering attempts in these directions have been made (34). Lattice calculations of more dramatically time-dependent phenomena, including the quenching of jets in the liquid plasma or the initial formation of the plasma from a far-from-equilibrium collision, are beyond the horizon.

The big picture that has emerged over the past 15 years, namely that the hot matter produced in ultrarelativistic heavy ion collisions rapidly forms a strongly coupled hydrodynamic liquid with a strikingly small value of $\frac{\eta}{s}$, has posed new, open, questions that motivate much experimental and theoretical investigation today. For example, how, and how quickly, does the hydrodynamic liquid form from the non-hydrodynamic initial conditions at the moment of the collision? Or, how does a hydrodynamic liquid emerge at its natural length scales (of order $1/T$ and longer with $T$ the temperature) in an asymptotically free gauge theory in which all matter, when resolved at short length scales, must be made of weakly coupled quarks and gluons? Or, what is the smallest droplet of this stuff that can sensibly be described using the language of hydrodynamics? We will return to the first and second of these big questions later, in Section 6 for the first and in Section 7 for the second. As to the third big question, it is currently the subject of intense investigation, both experimental and theoretical, having been put squarely on the agenda for the field by measurements made in $pPb$ and $pp$ collisions at the LHC and $dAu$ collisions at RHIC which indicate that even proton-sized droplets of hot QCD matter can exhibit liquid-like behavior. In response to this discovery, theorists have shown that, in the cousins of QCD with a dual gravitational description, the dynamics of a droplet of strongly coupled plasma with a temperature $T$ that is $\sim 1/T$ in size or larger can be described hydrodynamically (35, 36, 37). This suggests that hydrodynamic behavior should not persist in proton-nucleus collisions at lower collision energies and hence lower multiplicity. Noting that this question is at the center of a different article in this volume (38), we shall keep our discussion brief.

2. Phase diagram of QCD. Among the most important reasons for studying ultra relativistic collisions is the expectation that doing so will teach us about the phase diagram of hot QCD matter, in thermal equilibrium, as a function of both temperature and baryon doping (see Fig. 2). By baryon doping (or net baryon number density) we mean the excess of quarks over antiquarks in the hot matter. The standard parameter used to characterize the degree of baryon doping is the baryon number chemical potential $\mu_B$. To this point, we have set $\mu_B$ to zero, describing matter with equal densities of quarks and antiquarks. This is a very
good approximation for the matter produced at mid-rapidity in the highest energy heavy ion collisions at RHIC, an even better approximation at the LHC, and an exceedingly good approximation in the early universe. In all these cases, ordinary hadronic matter forms via a continuous crossover as the liquid QGP expands and cools. However, matter with $\mu_B = 0$ and varying temperature is only one edge of a phase diagram. A substantial component of understanding the nature of any complex material in condensed matter physics is mapping its full phase diagram, and the same is true in QCD. One way to study QGP doped with a significant excess of quarks over antiquarks would be to study the debris produced at very high rapidity in the highest energy heavy ion collisions, the rapidities where QGP forms from the compressed remnants of the incident nuclei. Neither RHIC nor the LHC feature detectors that can do this, at present. Instead, we can scan a region of the phase diagram of QCD by looking at heavy ion collisions with lower and lower collision energies in which the initial baryon number found in the incident nuclei makes a larger and larger contribution to the matter formed in the collisions: decreasing the collision energy increases $\mu_B$, scanning the phase diagram. Lower energy AA studies are underway at the SPS (39) and in the RHIC Beam Energy Scan (40)(BES), where tantalizing early results are in hand from a first phase of the BES program with relatively low statistics per collision energy (22). There is a second high statistics phase of the BES program planned for 2019-2020. Extensions of this program to even lower collision energies (and hence even higher $\mu_B$, albeit at lower temperature) are planned at the FAIR facility in Darmstadt, Germany (41) and at the NICA facility in Dubna, Russia (42). One of the central questions that these experiments aim to answer is whether the continuous crossover between liquid QGP and hadronic matter turns into a first order phase transition above some nonzero, critical, value of $\mu_B$, meaning in heavy ion collisions below some collision energy. There are many models for QCD in which the phase diagram features a critical point like this (43). (In QCD with two massless quarks – the “chiral limit” – the crossover at $\mu_B = 0$ becomes a sharp second order phase transition at which chiral symmetry is restored and a point at $\mu_B > 0$ where the transition becomes first order is a tricritical point (44, 43).) Furthermore, a critical point has also been seen in some pioneering efforts to explore physics at nonzero $\mu_B$ using lattice techniques, although because lattice calculations at nonzero $\mu_B$ suffer from a “sign problem” these calculations typically require small $\mu_B/T$ and to date it has not been possible to take the continuum limit (45). There are also tantalizing indications of increased non-Gaussian fluctuations (46, 47, 48) in exactly the observable that has been predicted to be most sensitive to critical fluctuations in RHIC collisions near the low end of the beam energy scan, but these indications are inconclusive given the presently available statistics. Do we know whether there is a critical point in the phase diagram at nonzero baryon doping? No. Are there strong motivations for the experimental program that aims to answer this question within the next few years? Yes. We have been relatively brief here, anticipating that the data and analyses coming soon will warrant a focused review of their
High baryon density at low temperatures in the cosmos

Pushing to very high baryon doping while staying at low temperature (aka squeezing nuclei without heating them) takes us into another interesting region of the QCD phase diagram. Matter that is sufficiently dense cannot be made of well-separated nucleons, even at low temperatures: the nucleons are crushed into one another. Because quarks attract each other, cold, dense matter in which quarks fill momentum space up to some high Fermi momentum is a color superconductor in which a condensate of correlated Cooper pairs of quarks creates a superfluid and yields the QCD-analogue of a Meissner effect. Extensive theoretical analyses of the phase diagram and consequent properties of color superconducting quark matter have been performed; they are well understood at asymptotic densities, but at densities of order 10 times that of nuclei they turn out to be sensitive to the ratio of the strange quark mass to the Fermi momentum as well as to the strength of the Cooper pairing, making them hard to pin down quantitatively. Experimental data is sorely needed. Unfortunately, the only place in the universe where cold dense quark matter may be found is in the centers of neutron stars. Remarkably, the first collision between two neutron stars has just been observed by the LIGO and VIRGO collaborations, via the gravitational waves it produced! Although the gravitational waves from this discovery event seen by LIGO only reveal the inspiraling incident neutron stars, with coming improvements to LIGO’s sensitivity future events will give us a view of the collision itself, making it possible to learn about the compactness and density profile of the incident neutron stars and, conceivably, whether or not they feature dense quark matter cores. If they do, present constraints on heat transport in neutron stars coming from X-ray observations of how they cool will turn into constraints on the transport properties of cold dense quark matter.

Asymptotic freedom: means that the QCD coupling \( \alpha_s \) weakens for interactions between quarks that are close together (or scatter at high energy), and is strong and nonperturbative at length (or energy) scales of order the (inverse) of the size of a hadron. In gas-like QGP at asymptotically high temperatures, the interaction energy is small compared to the kinetic energy.

3. Emergence of complex quantum matter. In the history of the universe, liquid quark-gluon plasma was the earliest complex form of matter to form. At much earlier times, when the temperature was a few orders of magnitude hotter than those of interest to us here, the matter that filled the universe was a weakly coupled plasma of quarks and gluons. We know this because QCD is asymptotically free, meaning that quarks and gluons interact with each other only weakly when they scatter off each other with large enough momentum transfer. Not only was liquid QGP the earliest complex matter to form, there is also a sense in which it is the simplest form of complex matter that we know of, namely the complex matter that is “closest”, most directly connected to, the fundamental laws that govern all matter in the universe, in this case the fundamental theory of QCD. Again because QCD is asymptotically free, we know that if we could hold a droplet of the liquid QGP with temperature \( T \) in place and study its microscopic structure with a spatial resolution that is much finer than \( 1/T \), for example via scattering high energy electrons off it in this thought experiment, what we would see is weakly coupled quarks and gluons. This is the genesis of

Busza, Rajagopal and van der Schee
the strongest motivation for developing experimental techniques for probing the structure of the liquid QGP on varying length scales. We know that at the shortest length scales we must see weakly coupled quarks and gluons. We also know that at length scales of order \(1/T\) and longer we see a liquid in which neighboring “unit cells” are tightly coupled to each other, meaning that the liquid flows hydrodynamically with a small \(\eta/s\). If we can probe both these length scales and scales in between, for example via studying how jets, (which are intrinsically multiscale probes), or heavy quarks with varying mass, or tightly bound quarkonium mesons with varying sizes, “see” the plasma and how the plasma responds to their passage through it, we have a chance to probe, and maybe even understand, how the simplest form of complex matter that we know emerges from weakly coupled, asymptotically free, constituents at short length scales. The question of how the almost infinite variety of complex forms of matter that we see in the world around us emerge from laws of nature that are so simple that they can easily fit on a T-shirt is one of the great quests of modern physics. If we can answer it for the case of liquid quark-gluon plasma, which we have a chance to do by virtue of this simple form of complex matter being so close to its laws-of-nature underpinnings, maybe we have a chance of shedding light on the larger more general question.

3. Phenomenology of heavy ion collisions

In the study of heavy ion collisions, experimenters have only two quantities under their direct control: which two nuclei they collide and at what energies. The energies are known to high precision. However, knowing the colliding nuclei is not the same as knowing the colliding systems. Neither the impact parameter \(b\) (the transverse distance between the center of masses of the two nuclei) nor the location and motion of the nucleons in the nuclei, let alone that of the quarks and gluons in the nucleons, are measurable quantities. They have to be inferred, as best as possible or as needed, event by event, from the observed outcome of the collision. This then makes it possible, after the fact, to select an ensemble of collisions with a relatively narrow distribution of impact parameters.

Based on nuclear and particle physics studies we know that, from the point of view of relativistic heavy ion collision studies, the nuclei can instantaneously be reasonably well approximated by a collection of nucleons, distributed on average according to a well-determined three-dimensional distribution. We also know the average quark and gluon content of the nucleons in the nuclei in terms of parton distribution functions or PDFs, and find that the PDFs in nuclei differ only mildly from those describing free nucleons (51). Furthermore we can use the measured energy dependent total inelastic \(pp\) cross-sections \(\sigma_{pp}(\sqrt{s})\) (52) to model the nucleons in the nucleus as hard spheres with a radius that depends on energy.

It will turn out to be useful to do a “gedanken experiment” where we imagine the colliding nuclei to be composed of \(A\) (transparent) spheres of radius \(\sqrt{\sigma_{pp}/4\pi}\), where \(A_{L,R}\)
Figure 3
(left) An example of a PbPb collision at LHC with impact parameter $b \approx 7$ fm. Number of participants (solid) are counted by nucleons that collide with any nucleon, whereas the number of binary collisions count all overlapping blue/red nucleon pairs. Spectators (dashed) do not collide. (Fig. from (53).) (right) Rapidity distributions of charged hadrons, in the rest frame of one of the nuclei, for AuAu collisions at 19.6 and 200 GeV (converted from pseudorapidity from (54) using a simplified Jacobian) and for PbPb collisions at 5.02 TeV (55).

**Participant:** Nucleon that collides with at least one other nucleon.

**Spectator:** Nucleon that does not collide and hence keeps on moving along the beam direction.

**Binary collisions:** Total number of nucleon pairs that collide, assuming transparency of the collision.

is the number of nucleons inside the left- and right-moving nuclei. We then call those nucleons that do not encounter any nucleon from the other nucleus spectators (dashed in Fig. 3). These nucleons continue traveling down the beam pipe, and the number of spectators $N_{\text{spec}}$ can hence in principle be measured directly, although in practice this is usually hard. In the gedanken experiment, all other ‘wounded’ or participating nucleons collide with at least one other nucleon and make up the number $N_{\text{part}}$ (solid in Fig. 3, by definition $N_{\text{spec}} + N_{\text{part}} = A_L + A_R$). It is unfortunate that $N_{\text{spec}}$ is not measurable in practice, since if it were then $N_{\text{part}}$ could be determined directly. Lastly, if we imagine the spheres as transparent we can also count the total number of encounters between left- and right-moving nucleons, which we will call the number of binary collisions $N_{\text{coll}}$. For example, if one "nucleus" consists of 7 nucleons lined up in a row and it collides head-on with a “nucleus” consisting of 4 nucleons in a row, $N_{\text{part}} = 11$, $N_{\text{coll}} = 28$, $N_{\text{spec}} = 0$ and the impact parameter $b = 0$. In a real central heavy ion collision a nucleon at the center of one nucleus will on average hit about 12 nucleons from the other, but less if it is located at the edge of the collision. So, $N_{\text{coll}}$ will then be much larger than $N_{\text{part}}$, and even more so for the more central collisions.

In a $pA$ collision, the probability of the proton hitting another nucleon is given by the ratio $\sigma_{pp}/\sigma_{pA}$ of inelastic scattering cross-sections. This makes it possible to determine that on average $N_{\text{coll}} = N_{\text{part}} - 1 = A \sigma_{pp}/\sigma_{pA}$, which can be measured directly. Experimental data on $pA$ collisions with widely varying $A$ and collision energies (going back to the 1970s (56)) show that the number of particles produced in such collisions is proportional to $N_{\text{part}}$ to a good approximation. Although in $AA$ collisions $N_{\text{part}}$ and $N_{\text{coll}}$ cannot be determined
directly from measured cross-sections, there is a well-defined theoretical procedure (57) (called a “Glauber Model Calculation”) for determining these abstract measures, at least on average within “centrality classes”. This procedure generates many configurations with different $b$, as illustrated in Fig. 3, and thereby generates Monte Carlo distributions of $N_{\text{part}}$ (as well as $N_{\text{coll}}$). It is then assumed that there is a monotonic relation between the number (or energy) of the produced particles and $N_{\text{part}}$. For example, it is assumed that events, in which the number (or energy) of particles falls into the highest 5% class, correspond to the 5% most central collisions, with $N_{\text{part}}$ or $N_{\text{coll}}$ (from the Glauber Model Calculation) in the highest 5% category. The bases for this prescription are, first, the experience from $pA$ collisions that we mentioned above and, second, the observation that the shape of the measured probability distribution for the number (or energy) of particles in an ensemble of $AA$ events is similar to the probability distribution for $N_{\text{part}}$ obtained from a Glauber Model calculation. Most important, the participant scaling observed for collisions of nuclei with widely varying $A$ (discussed below) provides a strong indication that these abstract measures in some way reflect a physical reality.

We now describe, as a function of energy, $N_{\text{part}}$ and $N_{\text{coll}}$, the most general features observed when two heavy ions collide at relativistic velocities. In order to give the “big picture”, in this discussion we shall ignore small differences and subtle effects. For useful summaries of and references to RHIC and LHC data we refer the reader to (58, 59, 60, 61) and (62, 63, 64). See also recent proceedings of Quark Matter conferences and (22, 65) for an overview of theoretical and experimental work.

**Hard collisions** High-$p_T$ $\gamma$ and $Z^0$ production have been studied in $pp$ and $AA$ collisions (66, 67, 68). The measured $pp$ cross-sections are well understood. They are in excellent agreement with predictions based on the known PDF’s and perturbative QCD theory (pQCD). The measured $AA$ production rates, in turn, are in excellent agreement with the product of $N_{\text{coll}}$ and the yield in a single $pp$ collision, taking into account the measured modifications of the PDF’s of nucleons inside nuclei and uncertainties in the determination of $N_{\text{coll}}$. Since the hard gammas and $Z^0$’s are not affected by the post-collision $AA$ environment (both are colorless), these results show that we have a good understanding of the initial hard (large $p_T$) parton-parton interactions in $AA$ collisions and of the determination of $N_{\text{coll}}$, meaning that these results provide independent confirmation from experimental data of the Glauber Model Calculations described above. This, in turn, implies that the $AA - pp$ comparisons of outgoing strongly interacting (colored) hard probes (jets, high-$p_T$ hadrons, heavy quarks, etc) can be used to give us valuable information about the nature of the medium produced in heavy ion collisions and on how the colored hard probes themselves are modified as they traverse the medium produced in $AA$ collisions. Experimental measurements of jets themselves, as well as of high-$p_T$ hadrons and heavy quarks which come from jets (69), all show that jets lose considerable energy as they propagate through
QGP, with the “lost” energy ending up as many soft particles ($\lesssim 3 \text{ GeV/c}$) moving at large angles relative to the original jet direction (71), suggesting that the jet leaves a wake behind in the liquid QGP. This suite of results and phenomena, collectively referred to as “jet quenching”, are very important since they give us direct evidence of very strong interactions occurring after the collision, strong interactions between the jet and the liquid as well as strong interactions within the perturbed liquid; we shall return to them in Section 7.

**Baryon stopping power:** It has long been known that in lower energy $pp$ collisions the longitudinal momentum of forward going protons in the final state have a flat distribution, evenly distributed between 0 and the incident energy (72, 73). This implies that, on average, a proton loses half its energy, which is about one unit of rapidity. In $pA$ and $AA$ collisions, the energy lost by the incident nucleons is higher on average, and more narrowly distributed. On average, in $AA$ collisions, each participant loses about two units of rapidity (9), which is to say 85% of its energy goes into the creation and kinetic energy of a very large number of particles, up to 30,000 in central PbPb collisions at the LHC. The net proton rapidity distribution in $AA$ collisions has a double hump structure (74), each consisting of hot baryonic matter moving at a speed of about two units of rapidity below that of the incident beam, and having a net baryon density of about 5-10 times that of normal nuclear matter (10). (At LHC the beam rapidity is at $y = 8.5$. As seen in the frame in which $y = 6.5$ is at rest, an incident disc that is Lorentz contracted by a factor of about $\cosh(2)$ is hit by a disc that is Lorentz contracted by about a factor of $\cosh(15)$ and brought approximately to rest, compressed by roughly $2 \cosh(2) \approx 7.5$.)

A further consequence is that the maximum value of the net baryon density at mid-rapidity is produced when heavy ions collide with $\sqrt{s_{NN}} \approx 7$ GeV. Above this collision energy, the mid-rapidity net baryon density, and so also the baryon chemical potential in the QGP produced at mid-rapidity, decreases with energy. By top RHIC collision energies, and even more so for LHC energies, both are essentially zero (75).

**Energy and centrality dependence of multi-particle production:** For practical reasons we have most information about charged particles, which corresponds to about 2/3 of all the produced particles. However there is no reason to doubt that the picture obtained from the charged particles is anything other than the whole picture!

From the lowest energies measured (76), through RHIC (54) to LHC (55) energies, for all $AA$, $pA$, $\pi A$ and $KA$ collisions, the total number of charged particles produced is approximately proportional to the number of participants. This is known as “participant scaling” and is not well understood. Even more surprisingly, provided that one takes into account the fraction of energy that is taken away by the forward going baryons and not available for particle production (mentioned above and see (72)), the total number of produced charged particles per participant in $AA$ collisions is the same as that in $pp$ and $e^+e^-$
collisions (77). However this arises, it suggests that on average in AA collisions most of the entropy production, which is proportional to the number of produced particles\(^2\) and hence the number of participants, occurs early in the collision and that there is little, if any, late stage entropy production. We shall later see that there are powerful arguments in support of this conclusion: after the early stage of the collision when entropy production is copious, a hydrodynamic fluid forms, and because this fluid has low specific viscosity, little entropy is produced subsequently, as the liquid flows.

For a given number of participants, the total number of produced particles \(N\) increases with energy (as \(N \propto s_{NN}^{0.15}\log(s_{NN})\))(79). Except for close to the receding discs, the longitudinal rapidity distributions look approximately like wide Gaussians with a width which increases as \(\log(s_{NN})\) (i.e. as the beam rapidity or longitudinal phase space, see Fig. 3) (55), and increases weakly from central to peripheral collisions (80). The produced particle density \(dN/dy\) has hence no boost invariant plateau and the maximum at mid-rapidity increases with energy as \((dN/dy \propto s_{NN}^{0.15})\)(81). The simplicity of these empirical facts at first glance seems to be at odds with the complex sequence of stages that precedes particle production. In the rest frame of the produced particles, the finally observed particle density \(dN/dy\) is the result of a local history which includes the initial impact of the nuclei, followed by the creation, expansion and flow of a hot medium, and its eventual hadronization into particles. At one level, the simplicity of the empirical facts can be explained by noting that the number of particles in the final state is proportional to its entropy and concluding that at any rapidity most of the entropy is produced very early in the collision, making \(dN/dy\) insensitive to all that happens later. However, these facts are nevertheless not fully understood. For example, the energy dependence and centrality dependence are surprisingly independent of each other from the lowest to the highest energies studied (54, 82, 83). This means that the naively expected increase with energy resulting from the increase of hard (proportional to \(N_{\text{coll}}\)) relative to soft (proportional to \(N_{\text{part}}\)) processes does not play a leading role in determining the number of produced particles.

Another interesting observation is the so-called extended longitudinal scaling. If the rapidity of one nucleus is kept constant and that of the other is gradually increased, we see that at first \(dN/dy\) increases, but it then reaches a limiting value (see Fig. 3). Thinking of the second nucleus as a wall of gluons, boosting these gluons more and more seems to have no effect on particle production in the collision around the rapidity of the first nucleus. This phenomenon has been observed for all systems studied (84) and is direct evidence that a kind of saturation occurs in the fast nucleus (85).

\(^2\)At the chemical freeze-out temperature, the multiplicity of each of the hadron species present in QCD is given to a good approximation by a thermal distribution. (We shall discuss this further below and in Section 4.) This makes a direct connection between the entropy at this moment and the number of charged particles possible. The contribution from any single species in a thermal distribution to \(N_{\text{ch}}\) is proportional to \(S\), with a proportionality constant that decreases with increasing mass. Adding up all the known species of hadrons yields \(N_{\text{ch}} \approx S/7.25\) at freeze-out (78).
Figure 4
(left) This event display (86) shows energy deposited in the CMS calorimeter in a heavy ion collision as a function of azimuthal angle $\phi$ and pseudorapidity $\eta$, a proxy for rapidity which is more easily measured. Two jets of very different energies are apparent, suggesting that one jet lost more energy as it traversed the droplet of QGP. (right) CMS event displays showing azimuthal distribution of charged tracks (green) and energy in the electromagnetic and hadronic calorimeters (red and blue respectively) from four heavy ion collision events as seen by the CMS detector. The azimuthal anisotropies are apparent, with the upper-right and lower-left events showing marked ellipticity and the bottom-right event showing a substantial anisotropy in a higher harmonic. It is remarkable that the strongly coupled character (left) and the liquid nature (right) of the QGP formed in these collisions can be seen so clearly in individual events.

Finally, we point out that all these facts do not support Landau’s (16, 2) and Fermi’s (15) early models, in which they postulated that the two colliding systems completely stop each other and then (in the case of Landau after a period of hydrodynamic expansion from rest) break up into particles according to thermodynamic laws. And, they are also inconsistent with Feynman’s intuition (17) that $dN/dy$ at mid-rapidity would not increase with increasing collision energy. Feynman expected the rapidity-distribution of the produced particles to broaden with increasing collision energy; this does happen, but, because of the rapid rise of the gluon PDF which Feynman did not anticipate, the total particle production increases fast enough that $dN/dy$ at mid-rapidity nevertheless increases.

**Particle correlations:** Strong correlations are observed between particles produced with momenta in different directions. They are much stronger than expected from the superposition of independent $pp$ collisions, and are evidence that the products of the initial collision act collectively.

Correlations between particles that are widely separated in rapidity are observed (87, 88). By causality, they must have their origin in early times and thus give information about correlations present at the earliest stages of the collision of the two nuclei. Azimuthal correlations as in Fig. 4 (right), in particular, have a very pronounced and rich structure and have been extensively studied as a function of the centrality of the collision, produced...
particle type, rapidity, transverse momentum, and expected event-by-event geometrical fluctuations of the nuclei (5, 89). As discussed in detail in Section 4 they can be remarkably well explained by relativistic hydrodynamics, if one assumes that in high energy heavy ion collisions, before the final production of free streaming particles, some kind of a relativistic liquid is formed, which expands, flows radially at about half the speed of light, and in which pressure-driven anisotropies in the flow velocity form and persist because the liquid has an incredibly low viscosity to entropy ratio, in fact lower than that of any other known liquid. It is for this reason that we know that QGP is a strongly coupled liquid.

A good way to see that the medium produced in a heavy ion collision indeed behaves like a low viscosity hydrodynamic liquid is to note the following. Like all nuclei, those that collide in heavy ion collisions are lumpy, meaning that the energy density of the matter produced in the earliest moments of the collision must also be lumpy. If that matter were a tenuous gas-like plasma, made of lots of particles that fly around while interacting only rarely with each other, the initial lumpiness would quickly disappear as the particles fly around in random directions, and at the end of the day all you would see is an isotropic explosion of particles, with just as many particles going in any direction as in any other. If, instead, the matter that is produced is a liquid whose motion is governed by hydrodynamics, the initial lumpiness will mean initial pressure gradients, and these pressure gradients will drive anisotropic flow in the liquid. If the viscosity of the liquid is large, these anisotropic flows will damp out. Instead, what is seen in heavy ion collisions is substantial anisotropies in the azimuthal distribution of particles in the final state (as in Fig. 4), which reflect azimuthal anisotropies in the geometry of the overlap region of the colliding nuclei. This means that the matter produced in the collisions must be a fluid with low specific viscosity.

**Medium properties:** Azimuthal correlations give information about the relativistic hydrodynamic nature of the medium, its transport coefficients, and about the fluctuations in the initial state from which it forms (5, 6, 90). Jet-quenching studies (91, 92, 69) give us a wealth of information on both how the medium responds when a high energy quark or gluon jet produced in an initial hard scattering traverses it, and how a fast quark or gluon jet are modified by the medium as they pass through it. As mentioned earlier, it is jet quenching that shows that QGP is extremely strongly interacting and is giving us much insight into the workings of QCD. So we return to this important topic later, in Section 7. There are no known measurements that give us direct and unambiguous information about what is the nature of the produced low viscosity fluid, is it in equilibrium, how does it form and how does it equilibrate, what is its equation of state and phase diagram, what are the best degrees of freedom for its description, how many thermodynamic degrees of freedom does it have compared to a hadron gas, and is it a liquid of deconfined quarks and gluons. However there are indirect measurements that give us insight into these questions and, together with theoretical studies, particularly lattice gauge calculations, a consistent
picture of the nature of QGP is emerging.

For example, near mid-rapidity, in RHIC or LHC central AA collisions, the ratios of the hadrons containing the lighter $u$, $d$, and even the $s$ valence quarks (93) are well represented by a system in chemical equilibrium at a temperature of about 155 MeV (94, 95). (Note that the number densities of charm and bottom quarks do not reach chemical equilibrium because the temperature is not high enough, meaning that their multiplicities retain memory of their initial production. Top quarks are not relevant here because of their short lifetimes.) On the other hand, the transverse momentum spectra are consistent with a system in equilibrium with a temperature of about 95 MeV and substantial radial flow (75, 96). Consistent with QGP being a strongly coupled liquid which behaves hydrodynamically as it expands and cools, there are no indications of any abnormal production of very low momentum pions (wavelength $\sim$ size of QGP droplet) (97), for example from the formation of a region of disoriented chiral condensate (98).

These facts, combined with the observed azimuthal correlations, participant scaling and jet quenching, are consistent with the following interpretation. Very early in the collision of the two Lorentz contracted nuclei, a thin cylindrical volume of QGP liquid is formed, with an entropy that is determined early, before the fluid hydrodynamizes. At first this liquid has a non-uniform energy density and temperature distribution determined by the lumpiness of the colliding nuclei. It expands and cools in accordance with relativistic hydrodynamics, and because its specific viscosity is so small it does so almost isentropically. When the temperature of the system locally falls below about 155 MeV, the QGP goes through a crossover phase transition and hadronizes. It is not known whether the hadrons are produced in chemical equilibrium or chemically equilibrate quickly, after the phase transition. All this is the so-called chemical freeze-out. The produced hadronic system then continues to interact, expand and cool until the temperature falls to about 95 MeV when thermal freeze out occurs. After thermal freeze out, the hadrons stream outwards freely, eventually reaching the detectors. At the thermal freeze out time, in addition to thermal motion the hadrons have radial and anisotropic velocities inherited from the flow of the expanding liquid that came before.

Measurements of quarkonia ($J/\psi$ and $\Upsilon$ mesons made from moderately heavy 1.3 GeV charm and heavy 4.2 GeV bottom quarks respectively) production in heavy ion collisions compared to that in $pp$ collisions (64) provide further information about the properties of the QGP medium, in two different ways. Consider first the case in which the production of a heavy $Q\bar{Q}$ pair in the hard collisions at the very beginning of the collision process is rare, for example as for $b\bar{b}$ pairs in LHC collisions, ideally meaning that in each heavy ion collision there are zero or one $b\bar{b}$ pairs. The $b\bar{b}$ pair finds itself immersed in the QGP medium which, via Debye screening, weakens the attractive force between the pair. The smallest, most tightly bound, $\Upsilon(1S)$ state has a size comparable to or even smaller than the Debye length of QGP, meaning that the $b$ and $\bar{b}$ may be close enough together to remain bound even when
The dimuon invariant mass distribution shows the different \( \Upsilon \) states, whereby the red dashed line shows the \( pp \) result added to the PbPb background and normalized to the \( \Upsilon(1S) \) state (99). Clearly the \( \Upsilon(2S) \) and \( \Upsilon(3S) \) states in PbPb collisions are much less pronounced, which is interpreted as the melting of these larger and less strongly bound \( b\bar{b} \) states when they find themselves immersed in QGP.

\[ \text{Immersed in QGP. The } \Upsilon(3S), \text{ on the other hand, is comparable in size to ordinary hadrons meaning that a } b \text{ and } \bar{b} \text{ with this separation do not attract each other when screened by QGP, and drift apart. The } \Upsilon(2S) \text{ is an intermediate case. Figure 5 is a beautiful example of data which shows that } \Upsilon \text{ states with different sizes and binding strengths do indeed have different probabilities of surviving in QGP, supporting this picture.} \]

\( J/\psi \) production in LHC heavy ion collisions is interestingly different (64). These collisions are sufficiently energetic that, on average, about 30 \( c\bar{c} \) pairs are produced in each heavy ion collision (100). In \( N_{\text{coll}} \) independent \( pp \) collisions, in which the same number of \( c\bar{c} \) pairs are produced, any \( J/\psi \)'s that form originate from the \( c \) and \( \bar{c} \) produced in a single hard scattering. In a heavy ion collision, those primordial \( J/\psi \) are expected to fall apart in QGP, as above. However, it now becomes possible for a \( J/\psi \) to form via a new process in which a \( c \) and \( \bar{c} \) from different initial hard scatterings thermalize in, and diffuse through, the QGP formed in the collision and then happen to find each other at the time of hadronization. \( c\bar{c} \) production is so copious at LHC energies that there are more \( J/\psi \)'s produced in heavy ion collisions via this recombination process than are produced in the standard fashion in \( N_{\text{coll}} \) independent \( pp \) collisions. This confirms that the \( c \) and \( \bar{c} \) quarks produced in heavy ion collisions wander independently of each other, and is thus a direct confirmation that quarks in QGP are not confined within hadrons.

**Comparing \( AA \) collisions with \( pp \) and \( pA \):** Unlike in \( AA \) collisions, the jet quenching phenomenon is not seen in \( pA \) collisions: at mid-rapidity the number of jets seen is just what
one would expect from $N_{coll}$ $pp$ collisions (101). (At large forward and backward rapidities there are deviations from this, deviations that are understood as coming from differences between nuclear and nucleon initial states (101).) This absence of jet quenching came as no surprise since $pA$ collisions produce an energetic final state that is small in transverse extent and because in ultrarelativistic collisions the incident nucleus is highly contracted in the longitudinal direction the nascent jets are quickly outside the energetic final state and cannot encounter the spectators from the incident nucleus. What did come as a surprise is how many other phenomena are similar in $AA$ and $pA$ collisions, and even in $pp$ collisions, in particular when the comparison is done between collisions in different systems with the same final state particle density $dN/dy$. Examples include the rapidity distribution (102), particle spectra (103), particle ratios including those involving strangeness (104), and, most significantly, the azimuthal anisotropies (105, 106) encoded in multiparticle correlations that were once thought to be unique to $AA$ collisions. Although these similarities are not yet well understood and are currently topics of intense debate, it is tempting to interpret them as indicating that proton-sized droplets of QGP can be formed in those $pp$ and $pA$ collisions that produce final states with sufficiently large $dN/dy$. This has prompted a recent theoretical focus on the question of how small the smallest droplet of QGP that can be described hydrodynamically can be, and the realization that in the case of a strongly coupled liquid-like QGP the answer seems to be around $1/T$ (36, 37, 35). This makes it plausible after the fact that some sufficiently energetic $pp$ or $pA$ collisions can make droplets of QGP with temperatures well in excess of the inverse of the proton size. A full discussion of the “heavy ion” features seen in small collision systems can be found in a companion article in this volume (38).

4. A hydrodynamic fluid

A crucial feature in our description of a heavy ion collision and the interpretation of the observed facts is that shortly after the initial impact of the heavy ions and before the hadronization process, the system (QGP) is in the form of a near perfect (extremely low specific viscosity) liquid. We now address in more detail and rigor and to the extent current understanding allows questions such as: how and to what extent do we understand the state of this system?; is it in equilibrium?; is it hydrodynamized and locally isotropic?; what do we know about its transport properties?

As explained earlier, we know that at its peak the energy density of the system is far in excess of that of hadrons, let alone nuclei. There is no way that the system could be a tightly packed collection of hadrons. Instead, it has to be described in terms of the quarks and gluons themselves. The interplay between two crucial features of QCD determines the nature of this state of matter. First, because of asymptotic freedom and the high energies probed at RHIC and the LHC it could be that the interactions between the quarks and gluons are so weak that an equilibrium thermal state of matter would never be reached.
A peripheral heavy ion collision produces an approximately elliptical collision region (shaded red). A gas of weakly interacting particles would give a more or less isotropic distribution of final particles (red), whereas a fluid would give rise to an anisotropic distribution (blue), due to the difference in pressure gradients in the transverse directions. In a hydrodynamic model with several temperature-dependent parametrizations of $\eta/s$ (see paper), it is compared with ALICE measurements of the anisotropy, as obtained by the integrated Fourier coefficients $v_n$ ($n = 2$ to $4$ from top to bottom), for $\sqrt{s_{NN}} = 2.76$ TeV collisions as a function of the centrality class (0% being head-on collisions). For more off-central collisions there is an increasing and large $v_2$, giving a hint into the importance of hydrodynamic evolution.

We show event-by-event distributions of the $v_2$ distribution for off-central collisions compared to ATLAS measurements. In this Section, we shall discuss the comparison between precise measurements of the anisotropy and increasingly sophisticated hydrodynamic calculations, as in the middle and right figures.

Second, at energy scales within an order of magnitude of the confinement/deconfinement energy scale, QCD is strongly coupled. The implication of this was not fully realized before experiments at RHIC began, as the most common expectation was the formation of an equilibrated gas of quarks and gluons with a temperature somewhat above the confinement/deconfinement scale. We now realize that in this temperature range QCD describes a relativistic fluid consisting of quarks and gluons that are so strongly coupled to their neighbors that the resulting liquid cannot even be described in terms of quasi-particles. The weak coupling picture must be correct at early times in collisions with exceedingly high energy; even in these collisions, the strong coupling picture would become applicable later after a hydrodynamic fluid has formed. The question of for how long during the initial moments of a RHIC or LHC collision a weakly coupled picture can be applied remains open.

The crucial distinction between both scenarios can be found by measuring the anisotropy of particles produced in heavy ion collisions. Qualitatively this is easy to understand, as we saw in Section 3: in the case of weakly interacting gas of particles, scatterings are rare, the directions of the momenta of the gas particles are random, the initial spatial anisotropy in the collision zone is washed out by random motion, and the azimuthal distribution of particles in the final state ends up isotropic. In this case, the measured two-particle correlations are trivial, coming only from effects like momentum conservation in late-time decays of hadrons. Alternatively, if the quarks and gluons form a strongly coupled liquid soon enough, while the distribution of energy density produced in the collision remains anisotropic, this non-circular and lumpy drop of fluid will expand in a hydrodynamic fashion, yielding faster expansion in the direction of larger gradients: hydrodynamics converts...
spatial anisotropies into momentum anisotropy. For perfectly circular collisions this would not lead to an interesting distinction, but in the hydrodynamic picture we would expect anisotropy arising because the incident nuclei are made of nucleons and hence lumpy as well as an increasing anisotropy in the particle spectrum as we probe less central, less circular, collisions.\(^3\)

To quantify the measurement of the azimuthal momentum anisotropy, we perform a Fourier transformation on the angular distribution of (charged) hadrons in the final state of the collision (115), which results in the anisotropic flow coefficients \(\tilde{v}_n\), defined from

\[
\frac{d\tilde{N}}{d\varphi} = \frac{\tilde{N}}{2\pi} \left( 1 + 2 \sum_{n=1}^{\infty} \tilde{v}_n \cos(n(\varphi - \tilde{\Psi}_n)) \right),
\]

where \(\varphi\) is the angle in the transverse plane, \(\tilde{\Psi}_n\) are the event plane angles (the first angle where the \(n\)th harmonic component has its maximum multiplicity), and \(\tilde{N}\) is the average number of particles of interest per event. All these observables can in principle be measured as a function of rapidity, centrality, transverse momentum and, around mid-rapidity (in collider experiments), also differentially for different particle species. The second to fourth harmonics are shown as a function of centrality in Fig. 6 (middle), as extracted from the 2-particle correlator with particles separated by a large gap in rapidity\(^4\). (We shall come back to the hydrodynamic curves shortly.)

As anticipated, the system before hadronization indeed requires a full hydrodynamic simulation in order to generate the sizable anisotropies found. Hydrodynamics is a gradient expansion, assuming that a fluid is everywhere close to thermal equilibrium, but allowing for small gradients in both temperature and velocity field. In ideal (0th order) hydrodynamics these gradients are ignored, which by assumption gives an isotropic plasma in the plasma’s local rest frame. For viscous (first order) hydrodynamics the gradients lead to an anisotropic

\(^3\)It is also worth noting that when people have modeled the bulk dynamics of the matter produced in heavy ion collisions via a system of colliding particles, fitting such models to empirical observations inevitably requires unphysically large scattering cross-sections (for example parton-parton inelastic scattering cross-sections 15 times larger than in perturbative QCD (112) or values of \(\alpha_s\) as large as 0.6 (113) or unphysically short mean free paths). For example, in both the BAMPS (113) and AMPT (114) approaches, the particles in the model have mean free paths that are much shorter than their de Broglie wavelengths. Although these approaches differ from hydrodynamics in detail, at a qualitative level what is happening in these models is that interactions in a particulate model are being dialed up to a sufficient degree that the model describes a fluid with low specific viscosity. (This has been shown explicitly for BAMPS (113).)

\(^4\)There are several ways to measure the \(v_n\) found in Eq. (1), most notably via measuring correlations among 4, 6, 8 or more particles, or via analyzing particles separated in rapidity. Both techniques are designed to exclude ‘jet-like’ correlations between nearby particles that come from the same jet shower or nearly back-to-back correlations from pairs of jets. We shall not review the by now quite sophisticated methods for extracting the \(v_n\) (116). We shall also not review the dependence of the \(v_n\) on transverse momentum or on hadron species (60), even though their dependence on particle momentum and mass provide important evidence in support of their origin from a single hydrodynamic fluid with a common flow velocity, or their distribution around their average value in each centrality class, which also support a consistent picture. (See e.g. (117, 118, 63))
stress tensor $T_{\mu\nu}$ according to

$$T_{\mu\nu} = \varepsilon u_{\mu} u_{\nu} + p[\varepsilon] \Delta_{\mu\nu} - \eta[\varepsilon] \sigma_{\mu\nu} - \zeta[\varepsilon] \Delta_{\mu\nu} \nabla_{\mu} u^\nu + O(\partial^2),$$

where $\varepsilon$ is the energy density and $u_{\mu}$ the velocity field, both depending on the full space-time coordinates. In the local fluid rest frame where $u_{\mu}^{\text{LRF}} = (1, 0, 0, 0)$ the projector is given by $
abla_{\mu} u_{\nu} = \Delta_{\mu\nu} = \text{diag}(0, 1, 1, 1)$, and in any frame $\Delta_{\mu\nu} u_{\mu} = \Delta_{\mu\nu} u_{\nu} = 0$. The first two terms in (2) are just ideal hydrodynamics, whereby the stress-energy tensor is given by an isotropic fluid with energy density $\varepsilon$ that is boosted with a velocity $u_{\mu}$. This fluid has a pressure that is given by the equation of state $p[\varepsilon]$, which is an input into hydrodynamics that depends on the microscopic properties of the theory under consideration. For heavy ion collisions this is the QCD equation of state, which is usually obtained from lattice calculations like those of Fig. 2 (119) (see however (120)). Lattice calculations are also used to relate the energy density to the temperature.

Beyond ideal hydrodynamics one needs to include corrections proportional to gradients and consistent with the symmetries present. For scale invariant viscous relativistic hydrodynamics it turns out that the only transport coefficient possible at first order in gradients is the shear viscosity $\eta[\varepsilon]$, which accompanies the $\sigma_{\mu\nu}$ of (3), containing first derivatives of the fluid velocity. Close to the deconfinement/confinement transition, QCD is definitely not scale invariant, and there it is also necessary to include the term proportional to the bulk viscosity $\zeta[\varepsilon]$. Just like $p[\varepsilon]$, the viscosities depend on the microscopic properties of the theory, but these transport properties are notoriously difficult to determine from a lattice calculation because they describe the (time-dependent) process by which small deviations from equilibrium relax whereas what is calculated directly on the lattice is (time-independent) derivatives of the equilibrium partition function. We will return to the determination of transport properties shortly.

Hydrodynamic evolution follows from the conservation of the stress-energy tensor after specifying the equation of state, the transport coefficients and the energy and velocity profiles at an initial time. In the hydrodynamic evolution equations, $\nabla_{\mu} T^{\mu\nu} = 0$, the

\[\text{In practice, solving the equations of viscous hydrodynamics is a bit more involved since when they are discretized they contain modes with wavelengths of order the discretization scale that propagate faster than light. These modes are unphysical and are outside the regime of the hydrodynamic gradient expansion, but because they are acausal they make the numerical scheme unstable. This makes it necessary in practice to solve a version of 2nd order hydrodynamics and verify that the choice of 2nd order terms does not much affect the final results, as must be the case if the gradient expansion is under control. (See for example (121, 5, 6)) We also note that we shall only review the application of hydrodynamics to collisions at LHC and top RHIC energies and, at these energies, for production of QGP far from the fragmentation regions. Extending such calculations outside these regions, as relevant for the exploration of the QCD phase diagram via the RHIC Beam...}\]
shear viscosity arises in the combination \( \frac{\eta}{(\varepsilon + p)} = \frac{\eta}{(Ts)} \), which is proportional to the length scale over which momentum can be transported in the fluid (6). At weak coupling, when the hydrodynamic fluid is made up of quasiparticles with a well-defined mean free path \( \lambda_{\text{mfp}} \), it can be shown that \( \eta/(\varepsilon + p) \propto \lambda_{\text{mfp}} \) meaning that \( \eta/s \propto T\lambda_{\text{mfp}} \) (123, 20, 6).

In a strongly coupled fluid, \( \eta/s \) is well-defined and small, but quasiparticles with mean free paths cannot be defined since attempting to do so would result in a \( \lambda_{\text{mfp}} \) comparable to or smaller than the de Broglie wavelength \( 1/T \). Whether the fluid is weakly or strongly coupled, \( \eta \) arises in the hydrodynamic equations in this combination and it is the specific viscosity \( \eta/s \) that controls how rapidly sound waves, shear stress, or gradients of any sort introduced in the initial conditions are dissipated into heat, meaning that it is this quantity that is ultimately constrained by comparing hydrodynamic calculations to data. To proceed further, it is necessary to model the initial energy and velocity profile at some proper time \( \tau_0 \). Fortunately we find that the insights we present do not depend strongly on simplifying assumptions that we have to make to solve our equations. A simple model used is to take two discs of heavily Lorentz contracted nuclei to collide at some impact parameter \( b \) and from this construct an initial energy profile that follows the overlap of the two discs (the Glauber model) with an overall amplitude as a free parameter. The velocity profile is often taken to be zero in the transverse plane and, in the longitudinal direction, the evolution is assumed to be boost invariant around the collision point at \( t = z = 0 \). Since this assumption implies that the longitudinal velocity is given by \( v_z = z/t \), this gives a simple and convenient model for an expanding plasma where all physics just depends on proper time \( \tau = \sqrt{t^2 - z^2} \) and the transverse coordinates. In state of the art hydrodynamic calculations which do not assume boost invariance, \( v_z = z/t \) remains a good approximation but the initial distribution of energy density does depend on rapidity.

Having specified the initial conditions and the hydrodynamic equations, the latter via choosing \( \eta/s \) and taking \( p[\varepsilon] \) and \( T[\varepsilon] \) from lattice calculations, it is possible to start a simulation of the hydrodynamic evolution of this putative hydrodynamic quark-gluon plasma. This simulation evolves the hydrodynamic variables describing an expanding and cooling droplet of matter forward in time up to a ‘freeze-out’ hypersurface in space-time where the fluid temperature has dropped to a specified value of order the temperature \( T \), where the crossover from QGP to hadrons occurs. At the freeze-out hypersurface, the fluid is converted into a thermal distribution of hadrons, conserving energy and momentum (124). Subsequent evolution is described via a gas of hadrons, which interact with each other as further expansion and cooling occurs until all scattering ceases at a lower ‘kinetic freeze-out’ temperature. The resultant ratios between the numbers of different hadron species, single

---

Energy Scan mentioned in Section 2, requires extending (2) to incorporate the time evolution of the conserved baryon number current. It is well-known how to do this (122), but less is known about the QCD equation of state and transport coefficients at nonzero baryon chemical potential. There is also an additional complication in that when the Lorentz contraction of the incident nuclei is only moderate the dynamics is intrinsically 3-dimensional.
particle spectra for various hadron species, and anisotropy coefficients $v_n$ can all be directly compared with experimental data.

In this model, in order to generate as much transverse flow (both isotropic, or radial, flow and anisotropic flow as described by the $v_n$) as seen in data it is necessary to take $\tau_0$ smaller than 1.0 fm/c, in some calculations as small as 0.2 fm/c. (In more advanced models that include the growth of the transverse velocity before $\tau_0$, this constraint can be somewhat weaker (125).) The amplitude of the initial energy density profile for central collisions ($b = 0$) is fitted to give the observed total particle multiplicity per unit rapidity. The multiplicity as a function of impact parameter $b$ is then a prediction of the model, which can be compared with the experimental results and used to obtain a constraint on $\eta/s$.

The precise magnitude of the anisotropies $v_n$ then depends quite sensitively on the viscosity of the plasma. Already from the relatively straightforward simulation with smooth initial conditions described above, it can be estimated that $\eta/s \sim 0.08 - 0.20$ during the hydrodynamic phase in heavy ion collisions at RHIC energies (126) (see (127) for a full 3D hydrodynamic simulation). This is one of the greatest discoveries of the heavy ion programs at RHIC and LHC: the experimental data is well described by the hydrodynamic evolution of a droplet of quark gluon plasma with a specific viscosity smaller than that of any other fluid known in nature. Quark-gluon plasma is hence sometimes referred to as the most perfect liquid.

As an example, and to give a sense of how well the system is understood currently, in Figure 6 (middle) a more precise comparison between experimental data and one particular hydrodynamic calculation is made. A crucial ingredient in this computation is the initial condition for the (lumpy, fluctuating) transverse profile, which is taken from a Monte Carlo Glauber model, with fluctuating positions of individual protons and neutrons, convolved with fluctuations of the energy density within a single nucleon that is based upon a saturation model in which color fields are large in magnitude but weakly coupled. Fluctuations in the initial state are necessary to obtain agreement with current, precise, data including in particular the $v_3$ anisotropies in head-on collisions and the odd harmonics such as $v_5$ in Figure 6 (middle). Without such fluctuations, the collisions would be perfectly symmetric under parity in the $\vec{b}$ direction, which would imply that all harmonics $v_n$ with $n$ odd would vanish. In actual collisions $v_3$ is in fact larger than the higher harmonics, showing that fluctuations break this parity symmetry (128). The authors of Ref. (107) evolved hydrodynamics with five different assumptions for $\eta/s$ as a function of temperature, see Figure 6 (middle). Current analyses of data are beginning to yield some constraints on this temperature-dependence. It is also possible to introduce a bulk viscosity at temperatures near the QCD phase transition (129), where it is expected to be important. The bulk viscosity in (129) is needed to get an accurate fit of the transverse momentum spectrum. The authors of (129) find that introducing bulk viscosity improves the fit to single particle
transverse momentum spectra without spoiling the quality of the fit for the elliptic flow $v_2$, but the optimal value of $\eta/s$ for the matter produced in LHC heavy ion collisions changes from 0.16 to 0.095.

In addition to providing a more accurate description of the systematic dependences of the measured flow coefficients upon averaging, an event-by-event analysis of a large ensemble of events with fluctuating initial conditions also makes it possible to compare hydrodynamic calculations of the distributions of the $v_n$ coefficients to experimental distributions. It then turns out that the distribution $\delta v_n \equiv (v_n - \langle v_n \rangle) / \langle v_n \rangle$ is largely independent of the hydrodynamic transport coefficients but is instead sensitive to the initial shape of the energy density (130) (Fig. 6, right), including its lumpiness. Hence, these distributions are an excellent way to constrain the hydrodynamic initial conditions, after which other observables can then be used with greater confidence to constrain transport coefficients such as $\eta/s$. The correlations between different event plane angles $\Psi_n$ also turn out to be a useful event-by-event observable. These correlations are not only sensitive to the average $\eta/s$ during the hydrodynamic evolution, but can also begin to constrain different hypotheses for the temperature dependence of $\eta/s$ (107).

It is an essential question how the conclusions about the shear viscosity depend on the model used, especially considering the uncertainty in the initial profiles as well as in the bulk viscosity and the temperature dependence of the shear viscosity. It has recently become possible to study the model dependence more systematically by doing a Bayesian analysis over a space of model parameters that include most models available, with recent estimates obtained via fitting to many different kinds of data from both RHIC and the LHC giving $\eta/s \approx (0.07^{+0.05}_{-0.04}) + c(T - T_c)$ (131) for temperatures $T > T_c = 154$ MeV with $T_c$ corresponding to the crossover between QGP and hadrons, but where the constant $c$ is at present only constrained to be between 0 and 1.58/GeV.

The small value for the dimensionless shear viscosity ratio $\eta/s$ is especially interesting. At weak coupling, this ratio is proportional to the ratio of the quasiparticle mean free path to the mean spacing between quasiparticles. A larger value of the ratio means that momentum can more easily be transported over significant distances, which is what is required in order to dissipate shear stress into heat. And, weaker coupling means larger values of this ratio. (For a weakly coupled gas of gluons, in fact $\eta/s \sim 1/g^4 \log(1/g)(132)$, where $g$ is the QCD coupling, namely the QCD analogue of $e$ in electromagnetism.) At strong coupling, on the other hand, each volume element of the QGP fluid is so strongly coupled to its neighbors that very little (net) momentum can be transferred to nearby fluid elements, meaning that velocity gradients remain, shear stress does not dissipate, and the specific viscosity is small. The measured value of $\eta/s$ for QGP turns out to be so small, however, that the fluid cannot be described in terms of quasiparticles with mean free paths since to do so would require mean free paths that are smaller than $1/T$. Strikingly, for certain infinitely strongly coupled quantum theories with a large number of degrees of freedom that are described by
a holographic dual gravitation theory, it can be computed that $\eta/s = 1/4\pi \approx 0.08$ (30), which is conspicuously close to the (average) viscosity found in hydrodynamic calculations used to model the dynamics of droplets of QGP produced in heavy ion collisions. Although QCD itself is not known to have a holographic dual, this motivates using gauge theories which do have dual gravitational descriptions to model dynamics in heavy ion collisions, as we shall elaborate later.

5. Thermalization, Hydrodynamization and Isotropization

The success of the hydrodynamic paradigm begs the question: when, why and how does the colliding debris begin to be accurately described by hydrodynamics, which is to say hydrodynamize? We would like to understand the underlying physics behind these questions, preferably from a QCD point of view. A related question is whether, if hydrodynamics is applicable for most of the evolution, this implies that the QGP formed in a heavy ion collision also thermalizes fast. The answer need not be yes, since a thermal state, by definition, is locally isotropic and free of gradients whereas hydrodynamics can work well when contributions that are first order in gradients are significant while those coming from higher order gradients are small. (Also note recent work (133, 6, 134) that succeeded in resumming all orders in gradients in a simplified geometry.) We hence have to ask how the pressure anisotropy evolves during the hydrodynamic evolution.

The droplet of QGP formed in a heavy ion collision is expanding rapidly, and even though it has the smallest dimensionless specific viscosity $\eta/s$ found in nature, the gradients present due to the fast expansion (initially mostly in the longitudinal direction) imply that the viscous corrections, which are first order in gradients, are in fact sizable. Indeed, when
\( v_z = z/t \), as for a boost-invariant velocity profile, it is clear that at early times the gradients of the velocity field are large. When extracting the pressure anisotropy as in Fig. 7 it can be seen that the gradient corrections are important for proper times between 0.3 and 6 fm/c and the plasma only becomes approximately isotropic after a proper time of roughly \( \tau = 6 \) fm/c (125).

So even if hydrodynamics is indeed a good description around times as early as 0.5 fm/c, because of the significant initial gradients and the smallness of the specific viscosity, the fluid does not fully isotropize and hence thermalize before a much later time of around 6 fm. We say that the fluid hydrodynamizes rapidly, within a proper time of around 0.2 – 0.6 fm/c or 0.4 – 1.0 fm/c at LHC or RHIC energies respectively (appropriately, these are typical starting times used in the hydrodynamic simulations described in Section 4), with this hydrodynamization followed subsequently by an extended period of hydrodynamic evolution with significant gradients in the fluid, before isotropization and complete thermalization at a substantially later time.

The estimates of the hydrodynamization time that we have quoted are often based upon assuming that when QGP hydrodynamizes in a heavy ion collision it does so without any initial transverse fluid velocity. This extra assumption is, however, unnecessary and in fact we will see shortly that all theoretical frameworks would predict the generation of some transverse flow already during the far-from-equilibrium, pre-hydrodynamization, stage of the collision, which can hence resemble hydrodynamic evolution. The question of when the QGP formed in a heavy ion collision hydrodynamizes is hence intrinsically linked to how hydrodynamics becomes applicable, and in particular how much the far-from-equilibrium pre-hydrodynamic dynamics resembles hydrodynamics itself. We will return to this in the next section.

6. Initial stage

The hydrodynamic model described above works well, perhaps surprisingly well, explaining many features of the particle spectra and the anisotropic flow coefficients. This poses three urgent questions: How does the debris left after a heavy ion collision evolve into an almost perfect hydrodynamic fluid so fast? And, how should this initial non-hydrodynamic stage in the dynamical evolution be described and in what initial conditions for the hydrodynamic stage does this result? We shall sketch the present understanding of both questions. The third question is how is entropy produced? This question provides a further reason for interest in the initial stage because almost all of the entropy produced in a heavy ion collision is produced before hydrodynamization: because the specific viscosity of the hydrodynamic liquid is so small, very little additional entropy is produced during the later, longer, hydrodynamic expansion. This means that the multiplicity of particles produced in the final state of a heavy ion collision is controlled by the dynamics occurring during its initial stage.
We provide a cartoon of how the pressure anisotropy (vertical axis) and $f$, the typical occupation number of modes with momentum $\sim Q_s$ (see text) in the gluon wave function, evolve during the initial stages of a heavy ion collision if one assumes that this can be described entirely at weak coupling. The weakly coupled dynamics can be described in terms of classical Yang-Mills fields if $f \gg 1$ and in terms of kinetic theory if $f \ll 1/\alpha_s$, meaning that as long as $\alpha_s$ is small enough there is a regime in which both descriptions are valid. The evolution begins with classical gluon fields at high occupancy $f \sim 1/\alpha_s$ and moderate pressure anisotropy, proceeds to kinetic theory at low occupancy and large pressure anisotropy, after which the matter thermalizes, meaning that the pressures become isotropic and the occupancy of modes in the kinetic theory reaches $f \sim 1$. The plot shows the energy density as a function of time and longitudinal coordinate $z$ in units of $\mu$ for a collision of two highly contracted parallel sheets of energy in strongly coupled SYM theory colliding head-on along the $z$-direction at $t = z = 0$. Here, $N_c^2\mu^3/2\pi^2$ is the energy per transverse area of the incident sheets of energy, with $N_c$ the number of colors in the theory. The energy density contains far-from-equilibrium regions, even including regions where energy density is negative and a restframe cannot even be defined (135). After the red dashed line the evolution of the plasma (in green) is hydrodynamic within 5% accuracy (136) (Figs. from (137, 138)).

From a purely QCD point of view these questions are unfortunately hard to answer because non-perturbative real time dynamics cannot be studied on the lattice. During the initial stage, and in particular during its earliest moments, many of the important scattering processes involve high transverse momentum transfer and hence can be described using pQCD. Soft, strongly coupled interactions are also important, in particular later during the initial stage as the matter hydrodynamizes, namely as it is becoming a strongly coupled fluid. It is therefore reasonable that various authors have developed entirely weak coupling descriptions of the initial stage while at the same time various authors have modeled far-from-equilibrium dynamics non-perturbatively using holography. We consider the two approaches in turn.

In the context of pQCD, the starting point for the description of the initial stages involves the phenomenon of “saturation” in the gluon wave function of the incident nuclei (139). When colliding ions at higher and higher energies, the gluons that collide and end up near mid-rapidity after the collision are gluons from the parton distribution function (PDF) of the incident nuclei with smaller and smaller momentum fraction $x = p_z/P$, defined with respect to the momentum of the nucleon $P$. In a perturbative analysis, at large $Y = \log(1/x)$, the gluon PDF increases rapidly with increasing $Y$. At mid-rapidity in
collisions at higher and higher energy, meaning smaller and smaller $x$, there will be more and more gluons. Until, that is, above some gluon density, gluon merging becomes as important as gluon splitting as $x$ is decreased further. The occupation number of gluon modes in momentum space with this value of $x$ and below is of order $1/\alpha_s$ and this component of the wave function of the nucleus is referred to as saturated. The typical transverse momentum of these saturated gluons is referred to as the saturation scale $Q_s$ and the number density of these gluons per unit area in the transverse plane is then given by $Q_s^2/\alpha_s$. The premise of the quantitative version of this analysis is that $\sqrt{\alpha_s(Q_s)}$ (where $\alpha_s$ is the running QCD coupling constant which becomes small at high momentum transfer) is small when evaluated at the scale $Q_s$. It is at the saturation scale where we find the low-$x$ gluons which dominate the interaction in an ultrarelativistic heavy ion collision. (See (140) for an accessible introduction.)

The above perturbative analysis implies that just after a heavy ion collision one ends up at mid-rapidity with gluon modes with transverse momenta up to $\sim Q_s$ that are over-occupied. Making this analysis more quantitative leads to the conclusion that $Q_s$ is of order 1 or 3 GeV for collisions at RHIC or the LHC, not so high as to make the assumptions of the perturbative treatment incontrovertible. Next, these gluons with transverse momenta of order $Q_s$ radiate softer gluons and scatter with the growing bath of softer gluons until hydrodynamization is achieved (141, 142). These processes are somewhat involved, and can be described via weakly coupled classical field theory or an effective kinetic theory of weakly coupled partons in overlapping regions of parameter space. (The first can be used when there are modes with occupation numbers that are $\gg 1$; the second works for occupation numbers that are smaller than $1/\alpha_s$. ) Plasma instabilities can play a role in the classical approach, although to leading order the classical evolution is self-similar due to the rapid longitudinal expansion (143). This expansion also drives the occupation numbers down, though, and at later times during the pre-hydrodynamic stage the effective kinetic theory must be used. (See Fig. 8.) The earliest analyses of these processes yielded the conclusion that in the limit of very weak coupling the parametric dependence of the hydrodynamization time is $\tau_{\text{hydro}} \gtrsim \alpha_s^{-13/5} Q_s^{-1}$ (141, 142). As numerical analyses of both the classical and the kinetic evolution have advanced, the currently most quantitative estimate is that, if one assumes $\alpha_s = 0.3$, the kinetic theory description of the energy density, transverse pressure, and longitudinal pressure hydrodynamizes after a time that is about, or even a little less than, 1 fm/c (144).

We know the specific viscosity is small and the coupling strong in the hydrodynamic liquid. This motivates exploring strongly coupled analyses of hydrodynamization as an alternative path to insights. The option that has been pursued most successfully is to analyze the complete far-from-equilibrium initial stage assuming that the dynamics is strongly coupled throughout using holography, which provides a dual gravitational description for certain gauge theories around infinitely strong coupling (20). This duality is truly remark-

\[ 't Hooft coupling: \]
\begin{align*}
\text{The smallness of } \alpha_s & \text{ controls perturbative corrections in QCD at weak coupling while the largeness of the } 't \text{ Hooft coupling } \\
\lambda & \equiv 4\pi \alpha_s N_c \text{ controls finite coupling corrections in holography.} \\
\alpha_s = 0.3 & \text{ corresponds to } \lambda \approx 11, \text{ meaning that this coupling is neither close to nor far from both 0 (perturbative methods) and } \infty \text{ (holography).}
\end{align*}
Holography

started with a seminal paper by Maldacena (149), which provides an exact equivalence between certain string theories and certain (supersymmetric) gauge theories. In one direction, this exact equivalence has led to a much better understanding of quantum gravity by using gauge theory dynamics. To use the equivalence in the other direction, it is also possible to take the limit where string theory becomes a theory of ordinary classical gravity in a curved space-time with a negative cosmological constant and one extra dimension. In that case, the gauge theory has many colors and is infinitely strongly coupled. The equivalence can then provide reliable insights into complex dynamical questions in a strongly coupled gauge theory. Position in the extra dimension encodes the length scale of excitations in the gauge theory. For example, the position of a horizon in the gravitational spacetime corresponds to \(1/T\) in the gauge theory, with \(T\) the temperature of the strongly coupled plasma with \(\eta/s = 1/4\pi\) (30, 20). Because all aspects of a one-higher-dimensional gravitational theory are encoded in features of the gauge theory, the mapping is referred to as a holographic duality.

able, as it maps intractable real-time far-from-equilibrium non-perturbative QFT problems onto equivalent, but tractable, computations within classical general relativity in Anti-de Sitter space, a (4+1)-dimensional space-time with a negative cosmological constant. Due to the strong interactions, the hydrodynamization time can be much shorter than at weak coupling. An early hint of this was the discovery that small perturbations around an equilibrium thermal state (equivalent to exciting quasi-normal modes of the dual black hole horizon) relax exponentially with characteristic time \(\tau \sim 1/\pi T\) (145). Computations of the relaxation of many far-from-equilibrium disturbances to boost-invariant expanding flows (146, 147) have shown that hydrodynamization occurs within a time \(T_{\text{hydro}} \sim 0.7/T_{\text{hydro}}\), where \(T_{\text{hydro}}\) is the temperature at which hydrodynamization occurs, and furthermore show a remarkably wide applicability of the quasi-normal mode analysis (148).

More advanced calculations permit the complete and rigorous simulation of the collision of sheets or discs of energy density in the infinitely strongly coupled super-Yang-Mills theory that is a cousin of QCD with a dual holographic description (150, 138) from the moment of collision through hydrodynamization and subsequent hydrodynamic expansion and cooling, including the development of radial and elliptic flow (151, 35). This allows for direct and quantitative analyses of the hydrodynamization process after a collision, analyses which yield an affirmation of the hydrodynamic picture sketched above. In this context, the most important conclusion is that a system that begins with an ultrarelativistic collision can become hydrodynamic quickly, with a collisions starting from a wide range of initial conditions yielding values for \(T_{\text{hydro}}\) between 1/4 and 1, as well as a hydrodynamic fluid that is initially strongly anisotropic, with significant gradients. For the hydrodynamic
calculation of Fig. 7, solving the equation $\tau T(\tau) = 1$ leads to $T_{\text{hydro}} \approx 0.35$ fm/c, whereby at that time $T_{\text{hydro}} \approx 560$ MeV. Hydrodynamization may occur at an even earlier time and hotter temperature if $T_{\text{hydro}} T_{\text{hydro}} \lesssim 1$.

These calculations yield other qualitative insights about the pre-hydrodynamic stage in a collision. For example, they show that the far-from-equilibrium dynamics of the collision yields a hydrodynamic fluid whose longitudinal velocity profile is to a very good approximation boost invariant but whose energy (and entropy) density profile is far from boost invariant, taking on a shape that is approximately Gaussian in rapidity with a width of 0.98 (136). This is qualitatively in line with what is seen empirically, but is too narrow. Calculations have also been performed that follow the collision of strongly coupled sheets of energy density that carry “baryon number” (a conserved quantum number introduced by hand in the holographic gauge theory) showing that after the collision the “baryon number” distribution is also centered on mid-rapidity (152), rather than losing only a few units of rapidity as in QCD. This, and the narrowness of the energy/entropy distribution, are almost certainly consequences of the fact that the gauge theory used in these calculations is not asymptotically free. The fact that in QCD the coupling is weak at the earliest moments of the collision is indeed important. This provides strong motivation for recent developments in the holographic framework, including collisions in theories that are not scale invariant (153) and that feature weaker-than-infinitely-strong coupling (154, 155) which give a shear viscosity that is larger than canonical, a nonzero bulk viscosity, and somewhat larger hydrodynamization times. It will be quite interesting to see how the distributions of energy, entropy and “baryon number” change in these collisions. As a final example, these calculations permit the assessment of how much radial transverse flow develops already before hydrodynamization. While hydrodynamic gradients of course generate this flow later, early far-from-equilibrium evolution can do so too, and in fact at strong coupling it is found that more pre-hydrodynamic flow is generated than would arise if this earliest epoch were instead hydrodynamic (35, 156). Similar results have also been obtained in an equivalent study at weak coupling (157).

A question that holographic calculations have not yet addressed (because to date they have not included any representation of the fact that in QCD the Lorentz contracted incident nuclei are made of nucleons) is how the lumpiness of the energy density is distributed over the transverse plane at the start of the hydrodynamic stage in a heavy ion collision. In order to make comparisons to the increasingly precise measurements of azimuthal anisotropies and correlations described in Section 3, the fluctuations in the energy density across the transverse plane must be included. All phenomenological modeling includes the lumpiness coming from the initial positions of the participating (or “wounded”) nucleons inside the colliding nuclei, via the Monte Carlo Glauber model described in Section 3. The simplest models just assume that each wounded nucleon contributes a Gaussian blob of energy density, but the precision of present data is sufficient that fluctuations that are somewhat
smaller than a nucleon must be included in order to optimize model predictions. Refined models translate the density of wounded nucleons into a locally varying saturation scale, and then use this as a guide to placing fluctuating color sources in the transverse plane, sources which in turn drive the numerical evolution of classical Yang-Mills fields whose stress-energy tensor is then used to initialize hydrodynamics (158, 159). Much remains to be done, including implementing an intermediate kinetic theory description, introducing lumpiness into the holographic calculations to provide a strong coupling benchmark, and in the long run testing the predictions of saturation calculations for the gluon distribution across the transverse plane in the incident nuclei against measurements at a future electron-ion collider (160). However, the best available calculations that begin with an initially lumpy energy density and follow its hydrodynamic evolution give an excellent simultaneous fit for RHIC and LHC to the probability distributions of the \( v_n \)’s even for very off-central collisions (107).

7. Jets in quark-gluon plasma

In occasional heavy ion collisions, partons from the incident nuclei scatter off each other at very large momentum transfer, creating two or more quarks or gauge bosons with very high transverse momentum (many tens of GeV at RHIC; as high as 100 or even 1000 GeV at the LHC). When such a hard scattering occurs in a proton-proton collision, each hard parton that is produced showers into a spray of softer partons within some irregular cone in momentum space, called a jet. Jet production and showering in vacuum is well described by perturbative QCD (161). When a jet is produced in a heavy ion collision, the partons in the shower must plow through the droplet of QGP produced in the same collision. As this happens, the jet partons: (i) lose energy and forward momentum, (ii) pick up momentum transverse to their original direction, and (iii) deposit energy and momentum into the droplet of QGP, creating a wake. The first of these phenomena is well-established experimentally (162, 163) and there are strong indications of the third (71, 70). The second, which is referred to as momentum broadening since it can broaden the shape of a jet in momentum space, is apparent in all theoretical approaches but has not yet been seen experimentally (162, 164).

In the longer term, and in particular once we have high statistics jet data at RHIC from the future SPHENIX detector (165) and from higher luminosity running at the LHC in the early 2020s, the motivation for precision analyses of how jets are modified via their passage through QGP is that this may teach us about the inner workings of QGP. This is the closest we can ever come to probing QGP by doing a scattering experiment and, as we discussed in Section 2, this is the best possible path toward addressing one of the big open questions in the field: how does a strongly coupled liquid emerge from an asymptotically free gauge theory? When the short-distance structure of QGP is resolved, it must consist of weakly coupled quarks and gluons. And yet, at length scales of order \( 1/T \) and longer they
become so strongly correlated as to form a liquid. Just as Rutherford found nuclei within atoms and Friedman, Kendall and Taylor found quarks within protons by doing scattering experiments, in the longer term experimentalists hope to see the short-distance particulate structure of QGP by seeing rare events in which a jet parton resolves, and scatters off, a parton in a droplet of QGP.

There are many physics questions (involving larger or more common effects) that are very interesting in their own right that must be understood quantitatively before realizing the vision of using jets as microscopes trained upon a droplet of QGP. This program is well underway, and could easily be the subject of an entire review of its own (see e.g. (91, 92, 69)). The most basic observation is that jets lose a substantial amount of energy, often 10 GeV or more, as they traverse a droplet of QGP. Noting that losing this amount of energy over only a few fm of distance corresponds to an enormous $dE/dx$, this provides a direct, and completely independent, confirmation that the matter produced in a heavy ion collision is strongly coupled. This energy loss can be seen in many ways including for example just by counting the number of jets with a given (high) transverse momentum: it is suppressed in heavy ion collisions relative to what would be seen in $N_{\text{coll}}pp$ collisions, which is to say relative to the expected number of jets if there would be no interaction with the medium as explained in Section 3. This is quantified by the nuclear modification factor (166)

$$R_{AA}(p_T) = \frac{dN^{AA}/dp_T}{(N_{\text{coll}})dN^{pp}/dp_T},$$

with $dN^{xx}/dp_T$ the number of jets (or in other contexts particles of a specified type) produced in $AA$ or $pp$ collisions. Indeed, Fig. 9 shows a large suppression of these jets, especially for central collisions in which the droplet of QGP that the jets need to traverse is the largest. A crucial check of this procedure is the fact that high $p_T$ colorless probes, such as $\gamma$’s or Z-bosons are indeed found to have $R_{AA} = 1$, as expected since they do not interact with QGP.

Throughout the study of jets it is important to realize that jets with a high transverse momentum $p_T$ are produced with a probability that drops very rapidly with increasing $p_T$. The production probability for jets produced at mid-rapidity with values of $p_T$ that are not within an order of magnitude of the beam energy scales roughly as $p_T^{-6}$ (167). The steepness of the energy spectrum implies that a small fractional jet energy loss corresponds to a large suppression in $R_{AA}$ for jets. (As a contrafactual example, if we imagine that all jets lose 10% of their energy, i.e. jets with 100 GeV started as 110 GeV jets, then since 110 GeV jets are approximately $(100/110)^6 \approx 56\%$ rarer than 100 GeV jets it follows that we would observe a nuclear modification factor of $R_{AA} \approx 0.56$.) In reality, different jets with the same initial energy lose very different amounts of energy as we shall describe below, meaning that this argument must be made at the ensemble level, but the conclusion is the same: because of the steepness of the jet energy spectrum the suppression in $R_{AA}$ for jets is

---

Nuclear modification factor: Ratio of the number of some countable objects (e.g. jets defined via a specified reconstruction procedure with a given $p_T$, hadrons of a specified type with a given $p_T$, etc.) found in nuclear collisions divided by the (theoretical) value that would be expected from an analogous number of proton-proton collisions without the presence of a medium.
On the left we show the nuclear modification factor $R_{AA}$ for jets for three different centralities as a function of jet transverse momentum $p_T$ (163). On the right we show the dijet asymmetry $A_J$ for $pp$ collisions and for peripheral (left) and central (right) heavy ion collisions. The PYTHIA+HYDJET distribution shows the expected asymmetry if no nuclear effects were present (162).

Figure 9 (right) illustrates another way of seeing that jets lose energy, and also provides direct evidence that in a given event some jets lose more energy than others. This arises for two reasons. First, the characteristics of jets with a given energy vary quite considerably and there are now a variety of theoretical arguments (at both weak and strong coupling) that indicate that a jet that fills a cone with a wide opening angle (and at weak coupling contains many partons) loses much more energy than a narrower jet with the same energy carried by fewer harder partons (168, 169, 170, 164). Because of the steepness of the jet spectrum described above, the ensemble of jets that comes out of the droplet of QGP will be dominated by those jets that lost relatively little energy, meaning that the jets that survive in a heavy ion collision with a given energy are likely to be those that started out the narrowest and are on average narrower than typical jets with the same energy in proton-proton collisions. There is some evidence for this effect in measured jet shapes (171). Note that measuring $R_{AA}$ for high-$p_T$ hadrons is quite different: in both $pp$ and $AA$ collisions, a high-$p_T$ hadron is statistically likely to come from a specific, unusual, type of jet that contains one very hard parton and is very narrow; selecting (‘triggering on’) hadrons therefore constitutes selecting an unusual sample of jets that lose less energy, and this selection effect becomes stronger at higher $p_T$. This is one reason that $R_{AA}$ for hadrons rises at the highest $p_T$ even though $R_{AA}$ for jets remains comparably suppressed.

The second reason why some jets lose more energy than others is that when two or more jets are produced in a collision they each traverse different lengths of QGP. There is evidence for this effect in measurements of a $v_2$-like anisotropy for particles with high transverse momentum that originate in jets (172): these jets typically lose less energy when moving along the short axis, as measured by the event-plane angle of the $v_2$ at low $p_T$.
(described in Section 4). All of this is to say that parton energy loss is a dominant effect contributing to the modification of many jet observables in heavy ion collisions as compared to proton-proton collisions. $dE/dx$, the rate of parton energy loss in plasma, is parametrized in different ways for partons that are assumed to be traversing a weakly coupled plasma versus for those which are assumed to be traversing a strongly coupled plasma that behaves as it would in a holographic gauge theory (173, 169, 174, 175). In either case, present data is being used to constrain the magnitude of $dE/dx$ and in the near-future, as the precision of the data improves further, it should become possible to differentiate between different choices for the $T$, $x$- and $E$-dependence of $dE/dx$.

The energy and momentum ‘lost’ by a jet in a heavy ion collision is, of course, not lost. We now know from experiment that it ends up shared among many soft hadrons in the final state of the collision that are spread out over a wide range of angles, up to 60 or even 120 degrees, around the jet direction (71, 70). This is certainly qualitatively consistent with a picture in which the jet excites a wake in the droplet of QGP, namely a region of moving and perhaps heated plasma behind the jet that carries the momentum in the jet direction ‘lost’ by the jet. Like the unperturbed plasma, this wake becomes many soft hadrons after the droplet of QGP falls apart into hadrons. Because they carry net momentum in the jet direction, some of the hadrons from the wake must end up within what experimentalists see as the jet (164, 176). This means that a quantitative understanding of the wake is a prerequisite to a quantitative understanding of the soft component of jets reconstructed in heavy ion collisions. Quantitative studies of the hydrodynamics of these wakes are now being done (177) and theorists should soon be able to do large-scale Monte Carlo calculations which track jet production in a hard scattering, jet showering, jet quenching, and the hydrodynamics of the specific wake produced by each specific jet (see i.e. (178)). Although full-scale calculations remain to be done, there are preliminary indications in some calculations (164, 179) that the wakes made by jets shooting through the plasma do not have time to fully hydrodynamize, as they yield more 2-4 GeV hadrons and fewer 0-2 GeV hadrons than they would have if they had completely hydrodynamized (164). This is exciting as it raises the prospect of using jets, specifically their wakes, to obtain experimental access to the physics of hydrodynamization. In this way, analysis of jets in heavy ion collisions may yield insights into how QGP forms as a function of time in addition to, in the longer term, revealing how QGP emerges as we coarsen the resolution scale of the microscope with which we probe it. Achieving this longer term goal will require having a quantitative understanding built upon precise data of energy loss (which results in an ensemble of narrower jets), jet wakes (which make jets as observed wider), and the accumulation of transverse momentum by the jet partons via their soft interactions with the liquid QGP. Only then will it be possible to look for the rare (power-law-rare, not exponentially rare) hard scattering events in which a parton within a jet (or, even more rarely, a jet itself) gets kicked by a detectable angle as it resolves, and scatters off, a parton.
within the liquid.

It is at present an unfortunate aspect in the studies of hard probes that experimentally only the final particles can be measured: it is in general not possible to directly compare a probe before and after passing through QGP. Recently this situation has been improved by selecting events with an energetic photon or Z-boson and one or more energetic jets (67), which have the advantage that the photon or Z-boson is unperturbed by the plasma and hence gives some probabilistic information about the energy of the jet or jets produced in the same event. Nevertheless, measuring the photon yields little information about the width of the jet, which plays an important and perhaps dominant role in determining how much energy it loses. In this context, it is exciting that experimentalists have recently begun to measure a host of different jet substructure observables, beyond the traditional jet shape, jet width, jet fragmentation function and jet mass, that are constructed in a variety of different ways via grooming jets and obtaining operational measures of their substructure (180, 181, 182). Although this has not yet been realized, it may be possible to identify an observable that is (relatively) unmodified by the passage of a jet through QGP and that in pp collisions is in one-to-one correspondence with the width of the jet. If this potential is realized, by measuring other observables that are sensitive to energy loss as a function of this observable it will be possible to study the quenching of jets for which we have some information about what their widths would have been in the absence of quenching.

8. Summary and big questions

SUMMARY POINTS

1. We study heavy ion collision to gain insight into perhaps the simplest form of complex matter, described by the fundamental laws of QCD. This super hot liquid filled the microseconds old universe, making it the first complex matter to form as well as the source of all protons and neutrons. Heavy ion collisions are little bangs, recreating droplets of big bang matter.

2. Within a time of order of 1 fm/c, the matter and entropy produced in a heavy ion collision form a droplet of strongly coupled QGP, evolving according to relativistic hydrodynamics with very small specific viscosity.

3. QGP is neither a collection of hadrons nor a nearly free gas of quarks and gluons. The colored quarks are free to diffuse and are not confined, but at the same time they are always very strongly coupled with their neighbors in the liquid.

4. Hydrodynamics converts spatial anisotropies into momentum anisotropy, giving us a direct experimental probe of both the spatial geometry, which is the source of the anisotropies, and the viscosity, which seeks to dissipate them. The QGP is very lumpy when it forms. As it expands and cools hydrodynamically, as the lumps
smooth out the resulting momentum anisotropies persist because the specific viscosity of QGP is small.

5. The strongly coupled nature of QGP is also seen and probed at a broad range of length scales by jet quenching: the rapid loss of energy by highly energetic partons traversing QGP.

6. There is a wealth of experimental data, from longitudinal rapidity and transverse momentum distributions to quark flavor, and from two particle to multiparticle correlations, which are surprisingly similar across a variety of colliding systems spanning three orders of magnitude in both volume and energy.

BIG QUESTIONS

1. How does QGP form and hydrodynamize within 1 fm/c? What are the qualitative differences, if any, between the description of hydrodynamization in a heavy ion collision obtained by assuming a weakly coupled initial stage versus a strongly coupled holographic calculation? Note that perturbative calculations typically treat $\alpha_s = 0.3$ as small while holographic calculations treat the corresponding 't Hooft coupling $\lambda \approx 11$ as large. What can we learn about the timescales and dynamics of hydrodynamization, and hence QGP formation, by analyzing the wakes that jets leave behind as they traverse a droplet of QGP?

2. What are the limits of the applicability of hydrodynamics? Can it be applied even to systems of size a fermi or less? What is the smallest droplet of QGP that behaves hydrodynamically, and how does the answer to this question change at very high temperatures where $\eta/s > 1$ and QGP is no longer a strongly coupled liquid?

3. How does a strongly coupled liquid emerge when QGP is analyzed with a spatial resolution of order $1/T$ or coarser, given that because QCD is asymptotically free what you will see at much finer resolution is weakly coupled quarks and gluons? How can we use jets to see the inner workings of QGP and answer this question? If we can understand how QGP emerges from an asymptotically free gauge theory, can we use this understanding to obtain general lessons about how complex forms of matter emerge from simple underlying laws?

4. How can we relate measurements of the gluon distribution in nuclei made at a future Electron Ion Collider to the distribution of the energy density across the transverse plane immediately after a heavy ion collision — quantitatively?

5. Can we obtain an experimental determination, even indirectly, of the temperature of the matter produced in a heavy ion collision at a time at which we can also determine its energy density? If we could, we could obtain an experimental determination of
the number of thermodynamic degrees of freedom, the quantity whose increase reflects the liberation of color above the crossover in the QCD phase diagram.

6. How do the hydrodynamics of QGP and the thermodynamics of its transition to hadronic matter as it cools change as QGP is doped with an excess of quarks over antiquarks? Is there a critical point in the region of the QCD phase diagram that heavy ion collisions can explore, or do all collisions that make QGP only explore a crossover in the phase diagram?

7. Can we explain the distribution of energy and entropy (particle multiplicity) as a function of rapidity in heavy ion collisions over a wide range of collision energies from first principle computations? Ditto for hadronization, and in particular can we explain why hadronization produces hadrons in chemical equilibrium? More generally, why are many bulk phenomena so similar for $AA, pA, pp, \pi A$ and in some cases even $e^+e^-$ collisions, over an enormous range of collision energies?

8. Is there color superconducting quark matter at the centers of some or all neutron stars? This important question about the phase diagram of QCD cannot be addressed by heavy ion collisions; we hope that observations of binary neutron stars colliding and merging will help.

Acknowledgments

We are pleased to acknowledge helpful comments from Gian Michele Innocenti, Guilherme Milhano, Greg Ridgway, Raju Venugopalan, Jing Wang, Ryan Weller and Bill Zajc. This work is supported by the U.S. Department of Energy under grant Contract Number DE-SC0011090. WS is supported by VENI grant 680-47-458 from the Netherlands Organisation for Scientific Research (NWO).

References

1. K. Yagi, T. Hatsuda & Y. Miake, Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol. 23, 1 (2005).
2. W. Florkowski, “Phenomenology of Ultra-Relativistic Heavy-Ion Collisions”.
3. “Quark-Gluon Plasma 5”, World Scientific (2016), New Jersey.
4. ALICE Collaboration, A. Toia, J.Phys. G38, 124007 (2011), arXiv:1107.1973.
5. U. Heinz & R. Snellings, Ann. Rev. Nucl. Part. Sci. 63, 123 (2013), arXiv:1301.2826.
6. P. Romatschke & U. Romatschke, arXiv:1712.05815.
7. W. van der Schee, arXiv:1407.1849.
8. Y.-J. Lee, A. S. Yoon & W. Busza, http://web.mit.edu/mithig/movies/LHCanmation.mov.
9. W. Busza & R. Ledoux, Ann. Rev. Nucl. Part. Sci. 38, 119 (1988).
10. W. Busza & A. S. Goldhaber, Phys. Lett. 139B, 235 (1984).
11. D. E. Kharzeev, J. Liao, S. A. Voloshin & G. Wang, Prog. Part. Nucl. Phys. 88, 1 (2016), arXiv:1511.04050.
12. A. J. Baltz et al., Phys. Rept. 458, 1 (2008), arXiv:0706.3356.
13. W. Heisenberg, Nature 164, 65 (1949).
14. J. Hamilton, W. Heitler & H. W. Peng, Phys. Rev. 64, 78 (1943).
15. E. Fermi, Prog. Theor. Phys. 5, 570 (1950).
16. L. Landau, Izv. Akad. Nauk Ser. Fiz. 17, 51 (1953).
17. R. P. Feynman, Phys. Rev. Lett. 23, 1415 (1969).
18. S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg & K. K. Szabo, Phys. Lett. B730, 99 (2014), arXiv:1309.5258.
19. HotQCD Collaboration, A. Bazavov et al., Phys. Rev. D90, 094503 (2014), arXiv:1407.6387.
20. J. Casalderrey-Solana, H. Liu, D. Mateos, K. Rajagopal & U. A. Wiedemann, “Gauge/String Duality, Hot QCD and Heavy Ion Collisions”, Cambridge University Press (2014), Cambridge, UK, hep-th/1101.0618.
21. J.-F. Paquet, C. Shen, G. S. Denicol, M. Luzum, B. Schenke, S. Jeon & C. Gale, Phys. Rev. C93, 044906 (2016), arXiv:1509.06738.
22. http://science.energy.gov/˜/media/np/nsac/pdf/2015LRP/2015_LRPNS_091815.pdf.
23. J. C. Collins & M. J. Perry, Phys. Rev. Lett. 34, 1353 (1975).
24. A. D. Linde, Rept. Prog. Phys. 42, 389 (1979).
25. E. Witten, Phys. Rev. D30, 272 (1984).
26. J. H. Applegate & C. J. Hogan, Phys. Rev. D31, 3037 (1985).
27. F. Karsch, Nucl. Phys. A698, 199 (2002), hep-ph/0103314, in “Quark matter 2001. Proceedings, 15th International Conference on Ultrarelativistic nucleus nucleus collisions, QM 2001, Stony Brook, USA, January 15-20, 2001”, p. 199-208.
28. Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz & K. K. Szabo, Nature 443, 675 (2006), hep-lat/0611014.
29. D. Thomas, D. N. Schramm, K. A. Olive, G. J. Mathews, B. S. Meyer & B. D. Fields, Astrophys. J. 430, 291 (1994), astro-ph/9308026.
30. G. Policastro, D. Son & A. Starinets, Phys. Rev. D87, 081601 (2001), hep-th/0104066.
31. V. E. Hubeny, S. Minwalla & M. Rangamani, arXiv:1107.5780.
32. H.-W. Lin & H. B. Meyer, “Lattice QCD for Nuclear Physics”, Springer (2015).
33. S. Borsanyi, G. Endrodi, Z. Fodor, S. D. Katz & K. K. Szabo, JHEP 1207, 056 (2012), arXiv:1204.6184.
34. H. B. Meyer, Eur. Phys. J. A47, 86 (2011), arXiv:1104.3708.
35. W. van der Schee, Phys. Rev. D87, 061901 (2013), arXiv:1211.2218.
36. P. M. Chesler, Phys. Rev. Lett. 115, 241602 (2015), arXiv:1506.02209.
37. P. M. Chesler, JHEP 1603, 146 (2016), arXiv:1601.01583.
38. J. L. Nagle & W. A. Zajc, arXiv:1801.03477.
39. NA61/SHINE Collaboration, A. Aduszkiewicz et al., Eur. Phys. J. C77, 671 (2017), arXiv:1705.02467.
40. X. Luo, Nucl. Phys. A956, 75 (2016), arXiv:1512.09215, in “Proceedings, 25th International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions (Quark Matter 2015): Kobe, Japan, September 27-October 3, 2015”, p. 75-82.
41. CBM Collaboration, J. M. Heuser, EPJ Web Conf. 13, 03001 (2011), in “Proceedings, International Workshop on Hot and Cold Baryonic Matter (HCBM 2010): Budapest, Hungary, August 15-20, 2010”, p. 03001.
42. V. Toneev, PoS CPOD07, 057 (2007), arXiv:0709.1459, in “Proceedings, 4th International Workshop on Critical point and onset of deconfinement (CPOD07): Darmstadt, Germany, July 9-13, 2007”, p. 057.
43. M. A. Stephanov, Prog. Theor. Phys. Suppl. 153, 139 (2004), hep-ph/0402115, in “Non-perturbative quantum chromodynamics. Proceedings, 8th Workshop, Paris, France, June 7-11, 2004”, p. 139-156, [Int. J. Mod. Phys.A20,4387(2005)].
44. K. Rajagopal & F. Wilczek, In *Shifman, M. (ed.): At the frontier of particle physics, vol. 3* 2061-2151, 2061 (2000), hep-ph/0011333, in “At the frontier of particle physics. Handbook of QCD. Vol. 1-3”, ed: M. Shifman & B. Ioffe, p. 2061-2151.
45. P. de Forcrand, PoS LAT2009, 010 (2009), arXiv:1005.0539, in “Proceedings, 27th International Symposium on Lattice field theory (Lattice 2009): Beijing, P.R. China, July 26-31, 2009”, p. 010.
46. M. A. Stephanov, Phys. Rev. Lett. 102, 032301 (2009), arXiv:0809.3450.
47. C. Athanasiou, K. Rajagopal & M. Stephanov, Phys. Rev. D82, 074008 (2010), arXiv:1006.4636.
48. STAR Collaboration, X. Luo, PoS CPOD2014, 019 (2015), arXiv:1503.02558, in “Proceedings, 9th International Workshop on Critical Point and Onset of Deconfinement (CPOD 2014): Bielefeld, Germany, November 17-21, 2014”, p. 019.
49. M. G. Alford, A. Schmitt, K. Rajagopal & T. Schafer, Rev. Mod. Phys. 80, 1455 (2008), arXiv:0709.4635.
50. Virgo, LIGO Scientific Collaboration, B. Abbott et al., Phys. Rev. Lett. 119, 161101 (2017), arXiv:1710.05832.
51. K. J. Eskola, P. Paakkinen, H. Paukkunen & C. A. Salgado, Eur. Phys. J. C77, 163 (2017), arXiv:1612.05741.
52. Particle Data Group Collaboration, C. Patrignani et al., Chin. Phys. C40, 100001 (2016).
53. B. Alver, M. Baker, C. Loizides & P. Steinberg, arXiv:0805.4411.
54. PHOBOS Collaboration, B. Alver et al., Phys. Rev. C83, 024913 (2011), arXiv:1011.1940.
55. ALICE Collaboration, J. Adam et al., Phys. Lett. B772, 567 (2017),
arXiv:1612.08966.

56. W. Busza, Acta Phys. Polon. B8, 333 (1977), in “Tutzing Conf. 1976: 543”, p. 333, http://www.actaphys.uj.edu.pl/fulltext?series=Reg&vol=8&page=333.

57. M. L. Miller, K. Reygers, S. J. Sanders & P. Steinberg, Ann. Rev. Nucl. Part. Sci. 57, 205 (2007), nucl-ex/0701025.

58. BRAHMS Collaboration, I. Arsene et al., Nucl. Phys. A757, 1 (2005), nucl-ex/0410020.

59. PHOBOS Collaboration, B. Back, M. Baker, M. Ballintijn, D. Barton, B. Becker et al., Nucl. Phys. A757, 28 (2005), nucl-ex/0410022.

60. STAR Collaboration, J. Adams et al., Nucl. Phys. A757, 102 (2005), nucl-ex/0501009.

61. PHENIX Collaboration, K. Adcox et al., Nucl. Phys. A757, 184 (2005), nucl-ex/0410003.

62. N. Armesto & E. Scomparin, Eur. Phys. J. Plus 131, 52 (2016), arXiv:1511.02151.

63. P. Foka & M. A. Janik, Rev. Phys. 1, 154 (2016), arXiv:1702.07233.

64. P. Foka & M. A. Janik, Rev. Phys. 1, 172 (2016), arXiv:1702.07231.

65. http://www.esf.org/fileadmin/user_upload/esf/Nupecc-LRP2017.pdf.

66. ATLAS Collaboration, G. Aad et al., Phys. Rev. Lett. 110, 022301 (2013), arXiv:1210.6486.

67. CMS Collaboration, S. Chatrchyan et al., JHEP 1503, 022 (2015), arXiv:1410.4825.

68. ATLAS Collaboration, G. Aad et al., Phys. Rev. C93, 034914 (2016), arXiv:1506.08852.

69. M. Connors, C. Nattrass, R. Reed & S. Salur, arXiv:1705.01974.

70. CMS Collaboration, Submitted for publication, 2018.

71. CMS Collaboration, V. Khachatryan et al., JHEP 1601, 006 (2016), arXiv:1509.09029.

72. A. E. Brenner et al., Phys. Rev. D26, 1497 (1982).

73. D. S. Barton et al., Phys. Rev. D27, 2580 (1983).

74. BRAHMS Collaboration, I. C. Arsene et al., Phys. Lett. B677, 267 (2009), arXiv:0901.0872.

75. ALICE Collaboration, B. Abelev et al., Phys. Rev. C88, 044910 (2013), arXiv:1303.0737.

76. E178 Collaboration, J. E. Elias, W. Busza, C. Halliwell, D. Luckey, P. Swartz, L. Votta & C. Young, Phys. Rev. D22, 13 (1980).

77. PHOBOS Collaboration, B. Back et al., Phys. Rev. C74, 021902 (2006).

78. B. Muller & K. Rajagopal, Eur. Phys. J. C43, 15 (2005), hep-ph/0502174.

79. ALICE Collaboration, E. Abbas et al., Phys. Lett. B726, 610 (2013), arXiv:1304.0347.

80. PHOBOS Collaboration, B. B. Back et al., Phys. Rev. Lett. 91, 052303 (2003),
81. ALICE Collaboration, K. Aamodt et al., Phys. Rev. Lett. **105**, 252301 (2010), arXiv:1011.3916.

82. ATLAS Collaboration, G. Aad et al., Phys. Lett. **B710**, 363 (2012), arXiv:1108.6027.

83. ALICE Collaboration, J. Adam et al., Phys. Rev. Lett. **116**, 222302 (2016), arXiv:1512.06104.

84. W. Busza, Nucl. Phys. **A854**, 57 (2011), arXiv:1102.3921, in “Saturation the color glass condensate and the plasma: What have we learned from RHIC? Proceedings, Workshop, Upton, Brookhaven, USA, May 10-12, 2010”, p. 57-63.

85. F. Gelis, A. M. Stasto & R. Venugopalan, Eur. Phys. J. **C48**, 489 (2006), hep-ph/0605087.

86. CMS Collaboration, S. Chatrchyan et al., Phys.Rev. **C84**, 024906 (2011), arXiv:1102.1957.

87. CMS Collaboration, S. Chatrchyan et al., Eur. Phys. J. **C72**, 2012 (2012), arXiv:1201.3158.

88. ALICE Collaboration, K. Aamodt et al., Phys. Lett. **B708**, 249 (2012), arXiv:1109.2501.

89. W. Li, Nucl. Phys. **A967**, 59 (2017), arXiv:1704.03576, in “Proceedings, 26th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions (Quark Matter 2017): Chicago, Illinois, USA, February 5-11, 2017”, p. 59-66.

90. P. Braun-Munzinger, V. Koch, T. Schäfer & J. Stachel, Phys. Rept. **621**, 76 (2016), arXiv:1510.00442.

91. Y. Mehtar-Tani, J. G. Milhano & K. Tywoniuk, Int. J. Mod. Phys. **A28**, 1340013 (2013), arXiv:1302.2579.

92. G.-Y. Qin & X.-N. Wang, Int. J. Mod. Phys. **E24**, 1530014 (2015), arXiv:1511.00790, [309(2016)].

93. P. Koch, B. Müller & J. Rafelski, Int. J. Mod. Phys. **A32**, 1730024 (2017), arXiv:1708.08115.

94. F. Becattini, M. Bleicher, T. Kollegger, T. Schuster, J. Steinheimer & R. Stock, Phys. Rev. Lett. **111**, 082302 (2013), arXiv:1212.2431.

95. A. Andronic, P. Braun-Munzinger, K. Redlich & J. Stachel, arXiv:1710.09425.

96. STAR Collaboration, L. Adamczyk et al., Phys. Rev. **C96**, 044904 (2017), arXiv:1701.07065.

97. PHOBOS Collaboration, B. B. Back et al., Phys. Rev. **C70**, 051901 (2004), nucl-ex/0401006.

98. K. Rajagopal, hep-ph/9504310.

99. CMS Collaboration, A. M. Sirunyan et al., arXiv:1706.05984.

100. B.-W. Zhang, C.-M. Ko & W. Liu, Phys. Rev. **C77**, 024901 (2008), arXiv:0709.1684.

101. CMS Collaboration, V. Khachatryan et al., Eur. Phys. J. **C76**, 372 (2016),
102. W. Busza, Acta Phys. Polon. B35, 2873 (2004), nucl-ex/0410035, in “Theoretical physics. Proceedings, 44th Cracow School, Zakopane, Poland, May 28-June 6, 2004”, p. 2873-2894.

103. ALICE Collaboration, J. Adam et al., Phys. Lett. B760, 720 (2016), arXiv:1601.03658.

104. ALICE Collaboration, J. Adam et al., Nature Phys. 13, 535 (2017), arXiv:1606.07424.

105. P. Bozek & W. Broniowski, Phys. Rev. C88, 014903 (2013), arXiv:1304.3044.

106. ATLAS Collaboration, M. Abbott et al., Phys. Rev. C96, 024908 (2017), arXiv:1609.06213.

107. H. Niemi, K. J. Eskola & R. Paatelainen, Phys. Rev. C93, 024907 (2016), arXiv:1505.02677.

108. ALICE Collaboration, K. Aamodt et al., Phys. Rev. Lett. 107, 032301 (2011), arXiv:1105.3865.

109. ATLAS Collaboration, G. Aad et al., JHEP 1311, 183 (2013), arXiv:1305.2942.

110. D. Rischke & G. Levin, Nucl. Phys. A750, pp.1 (2005).

111. BRAHMS, PHOBOS, STAR & PHENIX, Nucl. Phys. A757, pp.1 (2005).

112. D. Molnar & M. Gyulassy, Nucl. Phys. A697, 495 (2002), nucl-th/0104073, [Erratum: Nucl. Phys.A703,893(2002)].

113. Z. Xu & C. Greiner, Phys. Rev. Lett. 100, 172301 (2008), arXiv:0710.5719.

114. Z.-W. Lin, C. M. Ko, B.-A. Li, B. Zhang & S. Pal, Phys. Rev. C72, 064901 (2005), nucl-th/0411110.

115. STAR Collaboration, K. H. Ackermann et al., Phys. Rev. Lett. 86, 402 (2001), nucl-ex/0009011.

116. J.-Y. Ollitrault, A. M. Poskanzer & S. A. Voloshin, Phys. Rev. C80, 014904 (2009), arXiv:0904.2315.

117. STAR Collaboration, J. Adams et al., Phys. Rev. Lett. 92, 052302 (2004), nucl-ex/0306007.

118. CMS Collaboration, S. Chatrchyan et al., Phys. Rev. C89, 044906 (2014), arXiv:1310.8651.

119. P. Huovinen & P. Petreczky, Nucl. Phys. A837, 26 (2010), arXiv:0912.2541.

120. S. Pratt, E. Sangaline, P. Sorensen & H. Wang, Phys. Rev. Lett. 114, 202301 (2015), arXiv:1501.04042.

121. R. Baier & P. Romatschke, Eur.Phys.J. C51, 677 (2007), nucl-th/0610108.

122. C. Shen, G. Denicol, C. Gale, S. Jeon, A. Monnai & B. Schenke, “A hybrid approach to relativistic heavy-ion collisions at the RHIC BES energies”, in “26th International Conference on Ultrarelativistic Nucleus-Nucleus Collisions (Quark Matter 2017) Chicago, Illinois, USA, February 6-11, 2017”.

123. M. A. York & G. D. Moore, Phys. Rev. D79, 054011 (2009), arXiv:0811.0729.
124. F. Cooper & G. Frye, Phys. Rev. **D10**, 186 (1974).
125. W. van der Schee, P. Romatschke & S. Pratt, Phys.Rev.Lett. **111**, 222302 (2013), arXiv:1307.2539.
126. P. Romatschke & U. Romatschke, Phys.Rev.Lett. **99**, 172301 (2007), arXiv:0706.1522.
127. B. Schenke, S. Jeon & C. Gale, Phys.Rev.Lett. **106**, 042301 (2011), arXiv:1009.3244.
128. B. Alver & G. Roland, Phys. Rev. **C81**, 054905 (2010), arXiv:1003.0194, [Erratum: Phys. Rev.C82,039903(2010)].
129. S. Ryu, J. F. Paquet, C. Shen, G. S. Denicol, B. Schenke, S. Jeon & C. Gale, Phys. Rev. Lett. **115**, 132301 (2015), arXiv:1502.01675.
130. H. Niemi, G. S. Denicol, H. Holopainen & P. Huovinen, Phys. Rev. **C87**, 054901 (2013), arXiv:1212.1008.
131. J. E. Bernhard, J. S. Moreland, S. A. Bass, J. Liu & U. Heinz, Phys. Rev. **C94**, 024907 (2016), arXiv:1605.03954.
132. P. B. Arnold, G. D. Moore & L. G. Yaffe, JHEP **0011**, 001 (2000), hep-ph/0010177.
133. M. P. Heller & M. Spalinski, Phys. Rev. Lett. **115**, 072501 (2015), arXiv:1503.07514.
134. G. S. Denicol & J. Noronha, arXiv:1711.01657.
135. P. Arnold, P. Romatschke & W. van der Schee, JHEP **1410**, 110 (2014), arXiv:1408.2518.
136. P. M. Chesler, N. Kilbertus & W. van der Schee, JHEP **1511**, 135 (2015), arXiv:1507.02548.
137. A. Kurkela, Nucl. Phys. **A956**, 136 (2016), arXiv:1601.03283, in “Proceedings, 25th International Conference on Ultra-Relativistic Nucleus-Nucleus Collisions (Quark Matter 2015): Kobe, Japan, September 27-October 3, 2015”, p. 136-143.
138. J. Casalderrey-Solana, M. P. Heller, D. Mateos & W. van der Schee, Phys. Rev. Lett. **111**, 181601 (2013), arXiv:1305.4919.
139. L. D. McLerran & R. Venugopalan, Phys.Rev. **D49**, 2233 (1994), hep-ph/9309289.
140. E. Iancu, “QCD in heavy ion collisions”, in “Proceedings, 2011 European School of High-Energy Physics (ESHEP 2011): Cheile Gradistei, Romania, September 7-20, 2011”, p. 197-266.
141. R. Baier, A. H. Mueller, D. Schiff & D. T. Son, Phys. Lett. **B502**, 51 (2001), hep-ph/0009237.
142. R. Baier, A. H. Mueller, D. Schiff & D. T. Son, Phys. Lett. **B539**, 46 (2002), hep-ph/0204211.
143. J. Berges, K. Boguslavski, S. Schlichting & R. Venugopalan, Phys. Rev. **D89**, 074011 (2014), arXiv:1303.5650.
144. A. Kurkela & Y. Zhu, Phys. Rev. Lett. **115**, 182301 (2015), arXiv:1506.06647.
145. G. T. Horowitz & V. E. Hubeny, Phys.Rev. **D62**, 024027 (2000), hep-th/9909056.
146. P. M. Chesler & L. G. Yaffe, Phys.Rev. **D82**, 026006 (2010), arXiv:0906.4426.
147. M. P. Heller, R. A. Janik & P. Witaszczyk, Phys.Rev.Lett. 108, 201602 (2012), arXiv:1103.3452.
148. M. P. Heller, D. Mateos, W. van der Schee & D. Trancanelli, Phys.Rev.Lett. 108, 191601 (2012), arXiv:1202.0981.
149. J. M. Maldacena, Int. J. Theor. Phys. 38, 1113 (1999), hep-th/9711200, [Adv. Theor. Math. Phys.2,231(1998)].
150. P. M. Chesler & L. G. Yaffe, Phys.Rev.Lett. 106, 021601 (2011), arXiv:1011.3562.
151. P. M. Chesler & L. G. Yaffe, JHEP 1510, 070 (2015), arXiv:1501.04644.
152. J. Casalderrey-Solana, D. Mateos, W. van der Schee & M. Triana, JHEP 1609, 108 (2016), arXiv:1607.05273.
153. M. Attems, J. Casalderrey-Solana, D. Mateos, D. Santos-Oliván, C. F. Sopuerta, M. Triana & M. Zilhão, arXiv:1604.06439.
154. S. Grozdanov & W. van der Schee, Phys. Rev. Lett. 119, 011601 (2017), arXiv:1610.08976.
155. S. Wauber, A. Schaefer, A. Vuorinen & L. G. Yaffe, JHEP 1511, 087 (2015), arXiv:1509.02983.
156. M. Habich, J. Nagle & P. Romatschke, arXiv:1409.0040.
157. L. Keegan, A. Kurkela, A. Mazeliauskas & D. Teaney, JHEP 1608, 171 (2016), arXiv:1605.04287.
158. B. Schenke, P. Tribedy & R. Venugopalan, Phys. Rev. Lett. 108, 252301 (2012), arXiv:1202.6646.
159. C. Gale, S. Jeon, B. Schenke, P. Tribedy & R. Venugopalan, Phys. Rev. Lett. 110, 012302 (2013), arXiv:1209.6330.
160. A. Accardi et al., Eur. Phys. J. A52, 268 (2016), arXiv:1212.1701.
161. T. Sjostrand, S. Mrenna & P. Z. Skands, Comput. Phys. Commun. 178, 852 (2008), arXiv:0710.3820.
162. CMS Collaboration, S. Chatrchyan et al., Phys. Lett. B712, 176 (2012), arXiv:1202.5022.
163. ATLAS Collaboration, , ATLAS-CONF-2017-009, http://inspirehep.net/record/1512658/files/ATLAS-CONF-2017-009.pdf.
164. J. Casalderrey-Solana, D. Gulhan, G. Milhano, D. Pablos & K. Rajagopal, JHEP 1703, 135 (2017), arXiv:1609.05842.
165. A. Adare et al., arXiv:1501.06197.
166. PHENIX Collaboration, K. Adcox et al., Phys. Rev. Lett. 88, 022301 (2002), nucl-ex/0109003.
167. M. Spousta & B. Cole, Eur. Phys. J. C76, 50 (2016), arXiv:1504.05169.
168. J. G. Milhano & K. C. Zapp, Eur. Phys. J. C76, 288 (2016), arXiv:1512.08107.
169. P. M. Chesler & K. Rajagopal, JHEP 1605, 098 (2016), arXiv:1511.07567.
170. K. Rajagopal, A. V. Sadofyev & W. van der Schee, Phys. Rev. Lett. 116, 211603
171. CMS Collaboration, V. Khachatryan et al., JHEP 1611, 055 (2016), arXiv:1609.02466.
172. CMS Collaboration, A. M. Sirunyan et al., Phys. Lett. B776, 195 (2018), arXiv:1702.00630.
173. P. M. Chesler & K. Rajagopal, Phys. Rev. D90, 025033 (2014), arXiv:1402.6756.
174. J. Casalderrey-Solana, D. C. Gulhan, J. G. Milhano, D. Pablos & K. Rajagopal, JHEP 1410, 019 (2014), arXiv:1405.3864, [Erratum: JHEP09,175(2015)].
175. J. Casalderrey-Solana, D. C. Gulhan, J. G. Milhano, D. Pablos & K. Rajagopal, JHEP 1603, 053 (2016), arXiv:1508.00815.
176. J. G. Milhano, U. A. Wiedemann & K. C. Zapp, arXiv:1707.04142.
177. W. Chen, S. Cao, T. Luo, L.-G. Pang & X.-N. Wang, Phys. Lett. B777, 86 (2018), arXiv:1704.03648.
178. JETSCAPE Collaboration, S. Cao et al., Phys. Rev. C96, 024909 (2017), arXiv:1705.00050.
179. Z. Hulcher, D. Pablos & K. Rajagopal, arXiv:1707.05245.
180. CMS Collaboration, A. M. Sirunyan et al., arXiv:1708.09429.
181. ALICE Collaboration, S. Acharya et al., Phys. Lett. B776, 249 (2018), arXiv:1702.00804.
182. STAR Collaboration, K. Kauder, Nucl. Phys. A967, 516 (2017), arXiv:1704.03046, in “Proceedings, 26th International Conference on Ultra-relativistic Nucleus-Nucleus Collisions (Quark Matter 2017): Chicago, USA, Feb 5-11, 2017”, p. 516-519.