SUBDOMINANT INTERACTIONS AND $H_{c2}$ IN UBe$_{13}$.

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Abstract

We discuss a model based on a field-induced mixture of two odd-parity irreducible representations to explain the unusual features of $H_{c2}(T)$ in the heavy fermion compound UBe$_{13}$. We compare its predictions with recent pressure measurements as well as with the most prominent theoretical models which have been proposed up to now.

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I. INTRODUCTION

Heavy-fermion compounds are well-known candidates for a search and investigation of unconventional superconductivity. In these three dimensional systems, the symmetry analysis of the possible superconducting state, depending on the crystalline lattice of the compound \[1\], sets the frame for the identification of the unconventional superconducting phase. This analysis gives a complete list of possible superconducting phases together with their properties determined by symmetry, including the type and order of nodes in the superconducting gap. In spite of a favorable theoretical situation and numerous experimental work, there doesn’t exist yet a firm and widely accepted identification of these superconducting phases (comparable to the results on superfluid \(^3\)He) in any heavy fermion superconductor.

In this paper, we examine the upper critical field \(H_{c2}\) of the heavy fermion superconductor UBe\(_{13}\) as a probe of the symmetry of its superconducting phase. The upper critical field is usually not very sensitive to gap nodes (see for example the quantitative studies for various scenarios in UPd\(_2\)Al\(_3\) \[2\]). But it is sensitive to the spin state of the Cooper pairs or more generally to the parity of the order parameter, because in heavy fermion systems, the orbital limitation is so large that \(H_{c2}\) may be governed by the paramagnetic limitation.

UBe\(_{13}\) is a cubic compound with a \(T_c\) of order 1K, and it is a non-magnetic superconductor. Since the first measurements of its upper critical field \[3\], it is known that it presents two very unusual and intriguing features. First, \(H_{c2}(T)\) has a strong negative curvature close to \(T_c\) which changes sign at intermediate fields. Then, taking account of a realistic value of the conduction electrons gyromagnetic \(g - factor\), the paramagnetic limit at \(T=0\) is exceeded several times, while the strong negative curvature close to \(T_c\) shows that there exist a pronounced effect of the paramagnetic limitation.

Numerous explanations have been proposed since the first precise measurements of \(H_{c2}(T)\) in this compound \[3\]. Some have relied on additional hypothetic magnetic phase transitions \[4\], or on the field dependence of the normal state properties \[5-8\]. But none of these phenomenological interpretations have found a firm basis in other measurements.
or theoretical developments. Another hypothesis, much closer to the point of view adopted here, relies on two different superconducting order parameters with a weak Pauli limitation \[9\]. It has not been carried out quantitatively, but experimental support has been sought through the detection of a possible second phase transition already in zero field below the main superconducting transition \[9,10\]. To our opinion, such a second phase transition is not supported by the data, which only show a weak and smeared maximum in the specific heat or minimum in the thermal expansion. The model that we propose here involves a mixture of two different irreducible representations, but does not rely on nor predict such a second phase transition.

At present, the only competing quantitative explanation relies on a simple strong-coupling model \[11\]. It has been also successful in describing the evolution of both features up to pressures of 20 kbars \[12\]: the complete temperature and pressure dependence of \( H_{c2} \) comes out from a straightforward strong-coupling calculation with a single even parity state. The conflict with the paramagnetic limit at \( T=0 \) is resolved by its enhancement due both to direct strong-coupling effects (increase of the ratio \( \Delta/T_c \)) and to the (parameter free) inclusion of the formation of a spatially modulated superconducting state (FFLO), induced by the dominance of the paramagnetic limitation. Very good agreement with the data of Ref. \[12\] is provided in the whole pressure range, for \( g \) being close to its free electron value, by fitting the strong coupling constant \( \lambda \). The pressure dependence of \( \lambda \) agrees with that of the effective masses (as indicated by the Sommerfeld coefficient or the slope of \( H_{c2} \) at \( T_c \)), but it also turns out to be exceptionally large: \( \lambda \approx 15 \) at \( p=0 \). For all other known superconductors where the strong coupling regime due to electron-phonon interaction is well characterized, \( \lambda \) does not exceed 5.

The model in this paper is based on a field-induced mixture of two odd-parity irreducible representations of the symmetry group (\( O_h \)) of the normal phase of UBe\(_{13} \). This possibility was already evoked in the literature \[13,14\], but it has not been worked out to the extent that it could justify the particular choice of the order parameters and could be compared directly with the experimental data. Here, the choice of the two representations is unique,
thus providing a firm identification scheme for the superconducting state of UBe\textsubscript{13}. Within this scheme, the dominant component of the order parameter is analogous to the B-phase of the superfluid \textsuperscript{3}He. We make a quantitative comparison of the theoretical predictions with the data of Ref. [12] and propose an experimental check of the suggested scheme.

As we shall see, our choice of order parameters does not allow for a consistent interpretation of all data in UBe\textsubscript{13}: in particular, it does not predict any nodes of the gap, whereas both old [16] and new [17] thermodynamic experiments hint at the presence of point nodes. We consider this approach as a first step which may help to clarify at least one aspect of the properties of UBe\textsubscript{13}, bearing in mind that the complexity of the physics of heavy fermion systems rarely allows for fully satisfying explanations...

II. CHOICE OF THE REPRESENTATIONS.

Rotational symmetry of the crystalline lattice of UBe\textsubscript{13} is described by the $O_h$ group. Following the list of Ref. [18] for strong spin orbit coupling, we have to consider 10 irreducible representations: $A_{1g,u}$; $A_{2g,u}$; $E_{g,u}$; $F_{1g,u}$; $F_{2g,u}$. The subscripts g and u denote correspondingly even or odd symmetry of the particular representation with respect to the inversion. The even representations are the analogs of the spin singlet pairing and the odd - of the spin triplet. The order parameter for an even representation is a scalar and for the odd - a vector function of the direction in the momentum space $\vec{d}(\vec{k})$. The capital letters denote A - one-dimensional, E - two-dimensional and F - three-dimensional representations. To determine which of the representations is realized as the order parameter in a particular compound, one has to know the projections of the scattering amplitude $V_{\alpha\beta\lambda\mu}(\vec{k}, \vec{k}')$ of quasiparticles in the normal phase on the basis functions of these representations. The scattering amplitude is then represented in the form:

$$V_{\alpha\beta\lambda\mu}(\vec{k}, \vec{k}') = \frac{1}{2} \sum_{\Gamma \sigma} g_{\beta\alpha} g_{\lambda\mu} \hat{V}_{\beta\alpha} \sum_{i=1}^{d_\Gamma} \Psi_{\Gamma i}^\Gamma(\vec{k}) \Psi_{\Gamma i}^{\dagger\Gamma}(\vec{k}')$$

$$+ \frac{1}{2} \sum_{\Gamma \mu} \hat{V}_{\beta\alpha} \sum_{j=1}^{d_\Gamma} (\vec{\Psi}_{\Gamma j}^{\dagger}\vec{g}_{\beta\alpha}(\vec{k}) \vec{\Psi}_{\Gamma j}(\vec{k}')) (\vec{\Psi}_{\Gamma j}^{\dagger}(\vec{k}) \vec{g}_{\lambda\mu})$$

(1)
The spin matrices are defined as \( \vec{g}_{\alpha\beta} = (i\sigma^y)_{\alpha\beta} \), \( g_{\alpha\beta} = i\sigma^y_{\alpha\beta} \) where \( \sigma^x, \sigma^y, \sigma^z \), are Pauli matrices. The sums are taken over all even and odd representations respectively. The greatest of the positive amplitudes \( V_{\Gamma_0} \) (the dominant one) determines the temperature of the real superconducting transition \( T_{c0} \) as prescribed by the BCS-theory

\[
T_{c0} = \frac{2\gamma}{\pi} \epsilon_{\Gamma} \exp \left( -\frac{1}{N(0)V_{\Gamma_0}} \right)
\]

where \( \ln \gamma \) is Euler’s constant, \( \epsilon_{\Gamma} \) a cut-off parameter and \( N(0) \) the density of states at the Fermi level. All other amplitudes are subdominant. The transformation properties of the order parameter not too far from \( T_{c0} \) are described by the representation \( \Gamma_0 \). A magnetic field \( (H) \) changes the symmetry of the system and the classification of the representations. If only one, dominating term \( V_{\Gamma_0} \) is kept in the sum (1) and if \( \Gamma_0 \) is a multidimensional representation, the magnetic field splits it. Several branches \( H_{c2}(T) \) start in that case in the plane \( (T,H) \) from the point \( (T_{c0},0) \). According to Ref. [13], if the direction of the field \( H \) coincides with the symmetry axis of the lattice, the different branches are classified by two quantum numbers. The first is \( N=m+n \), where \( m \) is the projection of angular momentum on the direction of field and \( n \) is the number of the Landau level, which describes the spatial dependence of the order parameter for the nucleating superconducting region. The second quantum number is the parity \( \sigma = \pm 1 \) with respect to reflection in the plane perpendicular to the direction of \( H \). The branch with the highest \( H \) for a given \( T \) determines the true \( H_{c2}(T) \) [1].

The dependence \( H_{c2}(T) \) for p-wave pairing has been analyzed in detail by Scharnberg and Klemm [19]. No change of sign of the curvature was found, which is natural in a problem with only one parameter \( V_{\Gamma_0} \). But if all terms are retained in the sum (1), the magnetic field has an additional effect: it can mix basis functions having the same quantum number

\[1\] Strictly speaking even for conventional s-wave pairing, there are many branches \( H_{c2}(T) \) starting from \( T_c \) and corresponding to different \( n \), but one knows that the highest branch corresponds to \( n=0 \).
N, which originally belonged to different representations. Substantial change of the original basis functions can take place for fields of order $\mu_B H \approx T_c$. A change of the basis functions will change the projections $V_{\alpha\beta\lambda\mu}(\vec{k}, \vec{k}')$ on these functions, including the projection which determines the superconducting transition. A substantial admixture of the subdominant coefficients $V_T$ may therefore also take place on energy scales $\approx T_c$, imitating an extreme strong coupling effect. As regard the field induced change of the interaction potential itself, it is of the order of $\mu_B H/\epsilon_F$ and can be neglected.

The minimum model which can include these effects must contain two representations well mixed by the magnetic field. An advantage of the minimum model is that it remains tractable and contains only one additional parameter. Whether or not the two-representations model is sufficiently accurate for a description of the magnetic properties of a particular superconductor depends on the values of coefficients in the sum (1) for this material, and on the temperature interval in which one expects to reproduce these properties. In what follows, we apply the two-representation model to reproduce the unusual temperature dependence of $H_{c2}$ in UBe$_{13}$. We use the above mentioned characteristic features of this dependence as a guidance for the choice of two dominating representations.

The critical magnetic field can exceed the paramagnetic limit if the order parameter in the high field region is predominantly of the odd-parity type (i.e. a vector-function $\vec{d}(\vec{k})$), and if the projection of $\vec{d}(\vec{k})$ on the magnetic field is small or absent. To provide a good mixing of the two participating representations by the magnetic field we assume that both representations are of the odd-parity type. The pronounced paramagnetic effect in the low field region indicates that in that region the order parameter is dominated by a one-dimensional representation. In that case it is not possible to eliminate completely one projection of $\vec{d}(\vec{k})$. Such an elimination becomes possible in strong fields when the admixture of the second representation is appreciable. An inspection of the representation table for the $O_h$ group with the corresponding basis functions [20] suggests the following scheme: the superconducting phase which appears at $H=0$ and $T=T_{c0}$ belongs to the $A_{1u}$ representation with the basis function
The unit vectors \( \hat{x}, \hat{y}, \hat{z} \) are directed along three mutually perpendicular four-fold axes of the cube. The order parameter of the form (3) corresponds to a state of spin \( S=1 \), orbital momentum \( L=1 \) and total angular momentum \( J=0 \). This is the most symmetric odd-parity state, which is analogous to the B-phase of superfluid \(^3\)He. The superconducting gap for that state has the full symmetry of the \( O_h \) group and it does not have nodes required by symmetry.

We assume in what follows that the magnetic field is oriented along the \( z \)-direction. Only one branch of \( H_{c2}(T) \) starts from \( T_{c0} \) (ignoring higher Landau levels) and this branch is characterized by the quantum numbers \( N = 0 \) and \( \sigma = +1 \). The second representation is \( E_u \). For the present discussion it is convenient to choose the basis functions of \( E_u \) in the following form:

\[
\Psi_1(\vec{k}) = 1 + \frac{\epsilon}{\sqrt{2}}(\hat{x}k_x + \hat{y}k_y - 2\hat{z}k_z),
\]

\[
\Psi_2(\vec{k}) = 1 - \frac{\epsilon^2}{\sqrt{2}}(\hat{y}k_y - \hat{x}k_x),
\]

where \( \epsilon = e^{2\pi i/3} \) is a cubic root of 1. \( \Psi_1(\vec{k}) \) transforms as a function with \( m=0 \), and \( \Psi_2(\vec{k}) \) with \( m = \pm 2 \). One can see immediately that the \( z \)-component of the combination

\[
\Psi_\infty(\vec{k}) = \sqrt{2}(1 + \epsilon)\Psi_0 + \Psi_1,
\]

is zero and the paramagnetic limitation is absent for such an order parameter. The elimination of the \( z \)-component would not be possible for the \( A_{2u} \) representation, since as a function of \( \vec{k} \) it is orthogonal to \( z \)-components of the basis functions of all other odd representations. So, we assume that the pairing potential contains contributions of only two representations \( A_{1u} \) and \( E_u \).

\[
V_{\alpha\beta\lambda\mu}(\vec{k}, \vec{k}') = \frac{1}{2}V_0(\Psi_0(\vec{k}) \cdot \vec{g}_{\beta\alpha})(\Psi_0^*(\vec{k}) \cdot \vec{g}_{\lambda\mu}) + \frac{1}{2}V_1 \sum_{s=1,2} (\Psi_s(\vec{k}) \cdot \vec{g}_{\beta\alpha})(\Psi_s^*(\vec{k}) \cdot \vec{g}_{\lambda\mu})
\]

(7)
with two independent coupling constants $V_0$ and $V_1$ or, formally, two BCS transition temperatures $T_{c0}$ and $T_{c1}$. Then the order parameter in the vicinity of the transition line $H_{c2}(T)$ is a linear combination of the basis functions $\vec{\Psi}_0, \vec{\Psi}_1, \vec{\Psi}_2$:

$$\vec{d}(\vec{k}, \vec{R}) = \Delta_0(\vec{R})\vec{\Psi}_0(\vec{k}) + \sum_{s=1,2} \Delta_s(\vec{R})\vec{\Psi}_s(\vec{k})$$

where $\Delta_j(\vec{R})$ $j=0,1,2$ are functions of the coordinates.

III. CRITICAL FIELD $H_{C2}(T)$

The dependence of $H_{c2}(T)$ for our model can be found as in Ref. [19]. As a starting point we use Eq.(3) of Ref. [13]. With the interaction potential given by Eq. (7) and the order parameter of Eq. (8), the equations for the functions $\Delta_j(\vec{R})$ are:

$$\frac{1}{N(0)V_j} \Delta_j(\vec{R}) = 2\pi T \sum_{\omega_n} \frac{1}{4\pi} \int d\Omega' \sum_{j'=0}^2 \int_0^\infty ds \left\{ e^{-s L_{op}} \left[ 1 - \left[ 1 - \cos(2sgH) \right] \hat{n} \hat{n}^{tr} \right] \right\} \Delta_{j'}(\vec{R})\vec{\Psi}_{j'}(\vec{k})$$

where

$$L_{op} = 2|\omega_n| - i \text{sgn}(\omega_n) \vec{v}_F(k_F).(i\vec{\nabla} + 2e\vec{A}(\vec{R}))$$

$j = 0,1,2$; $\vec{A}(\vec{R})$ is the vector potential for the magnetic field $H$, $g$ is an effective gyromagnetic ratio. It need not be equal to its free electron value $g = 2$; besides the usual renormalization due to spin orbit coupling, it includes here also possible Fermi-liquid corrections. In what follows, $g$ will be used as a fitting parameter. The unit vector $\hat{n}$ is parallel to the $H$ direction (thereafter also called $z$-direction). This is a linear system of differential equations of infinite order. Its solutions are given by the eigenfunctions $f_n(\vec{R})$ for the Landau levels of an electron under magnetic field:

$$\Delta_j(\vec{R}) = \sum_n \eta_{jn} f_n(\vec{R})$$

Now we use the fact that the combination (8) corresponds to the quantum number $N=0$. For the functions $\vec{\Psi}_0$ and $\vec{\Psi}_1$ $m=0$, while for $\vec{\Psi}_2$ $m = \pm 2$. This selects $n=0$ for $j=0,1,$ and
n=2 for j=2. Since the spatial dependence of \(f_n(\vec{R})\) is known, we suppress the second index in the notation \(\eta_{jn}\) and after transformations following that of Ref. [19], we arrive at a linear algebraic system for the amplitudes \(\eta_j\):

\[
(F_{00} + P - \ln \sqrt{h})\eta_0 + \sqrt{2}(1 + \epsilon)(F_{01} - P)\eta_1 + \frac{1 - \epsilon^2}{\sqrt{2}}F_{02}\eta_2 = 0, \tag{12}
\]

\[
\sqrt{2}(1 + \epsilon^2)(F_{01} - P)\eta_0 + (F_{00} - F_{01} + 2P)
- \ln \sqrt{h} - \ln q)\eta_1 + \frac{1 - \epsilon}{2}F_{02}\eta_2 = 0, \tag{13}
\]

\[
\frac{1 - \epsilon}{\sqrt{2}}F_{02}\eta_0 + \frac{1 - \epsilon^2}{2}F_{02}\eta_1
+ (F_{00} + F_{01} - \ln \sqrt{h} - \ln q)\eta_2 = 0. \tag{14}
\]

The equations are written in the dimensionless units of Ref. [19]: \(h = \frac{2H}{H_0}, H_0 = \frac{\Phi_0}{\pi \xi_0^2}, \xi_0 = \frac{\hbar v_F}{2\pi T_c}, \Phi_0 = \frac{\pi hc}{2}, t = \frac{T}{T_c}, q = \frac{T_c}{T_c} 1.\) For the calculation of angular averages, we assume a spherically symmetric Fermi surface so that \(\vec{v}_F \) is parallel to \(\vec{k}_F\). Then the coefficients \(F_{00}, F_{01}, F_{02}\) and \(P\) are given by the following expressions:

\[
F_{00} = F_{00} \left( \frac{t}{\sqrt{h}} \right) = \int_0^\infty \ln \left[ \frac{\sqrt{h}}{t} \tanh \left( \frac{\rho t}{2\sqrt{h}} \right) \right] \left( \int_0^1 (1 - x^2) e^{-\frac{x^2}{2} (1-x^2)} dx \right) \frac{\rho d\rho}{2}, \tag{15}
\]

\[
F_{01} = F_{01} \left( \frac{t}{\sqrt{h}} \right) = \int_0^\infty \ln \left[ \frac{\sqrt{h}}{t} \tanh \left( \frac{\rho t}{2\sqrt{h}} \right) \right] \left( \int_0^1 \frac{(1 - x^2)(1 - 3x^2)}{2} e^{-\frac{x^2}{4} (1-x^2)} dx \right) \frac{\rho d\rho}{2}, \tag{16}
\]

\[
F_{02} = F_{02} \left( \frac{t}{\sqrt{h}} \right) = \frac{t}{4\sqrt{2}h} \int_0^\infty \frac{\rho^2 d\rho}{\sinh(\frac{\rho\lambda}{\sqrt{h}})} \left( \int_0^1 (1 - x^2) e^{-\frac{x^2}{4} (1-x^2)} dx \right), \tag{17}
\]

\[
P = P \left( \frac{t}{\sqrt{h}}, \lambda \sqrt{h} \right) = -\frac{t}{\sqrt{h}} \int_0^\infty \frac{1 - \cos(\rho \lambda \sqrt{h})}{\sinh(\frac{\rho\lambda}{\sqrt{h}})} \left( \int_0^1 x e^{-\frac{x^2}{4} (1-x^2)} dx \right) d\rho \tag{18}
\]

with \(\lambda = \frac{\hbar v_F H}{2\pi T_c}\). \(H_{c2}(T)\) is found from the condition of compatibility of Eqns. (12)-(14). With the shorthand notations
\[ G = F_{00} - \ln \sqrt{h} - \frac{2}{3}Q, \quad \text{where} \quad Q = \ln q. \]

this condition has the form:

\[
(G + \frac{2Q}{3})(G - \frac{Q}{3})^2 - F_0^2(3G + 2F_0) + \\
3P[(G + F_0)^2 - \frac{Q^2}{9}] + \frac{9}{4}F_{02}^2(2F_0 - 3P - G) = 0 \quad (19)
\]

which gives an implicit equation for \( h \).

IV. DISCUSSION

Analytic solution of Eq. (19) is possible only in some limiting cases (cf. Appendix). In order to compare the predictions of Eq. (19) to the data, we have performed straightforward numerical calculations. Fig. 1 shows \( H_{c2}(T) \) for three different irreducible representations together with the data on UBe\(_{13}\). A pure \( A_{1u} \) representation has a paramagnetic limitation 3 times higher than a pure (even) \( A_{1g} \), leading to a good fit of the low field part of \( H_{c2}(T) \) for a reasonnable value of the effective \( g - f \) actor: \( g = 1.2 \). But the admixture of the \( E_u \) representation is essential to reproduce the upturn of \( H_{c2}(T) \) below \( T_c/2 \).

The surprise has been that this large influence of the admixture has been found for a parameter \( T_{c1}/T_{c0} \) of only 0.12. This comes from the fact that the admixture of the representations along the \( H_{c2}(T) \) curve is controled by the applied field, and it starts much above \( T_{c1} \). In any case, the corresponding admixture of \( E_u \) components in the order parameter (defined in Eq. (11)) remains less than 10% down to \( T=0 \): see Fig. 2 where the coefficients \( \eta_1 \) and \( \eta_2 \) of definition (8)-(11) have been reported normalized to \( \sum \eta_i^2 = 1 \). So one is still far from the limit of complete suppression of the paramagnetic limitation (see Appendix).

One can note on Fig. 4 that most of the admixture of the \( E_u \) representation comes from the \( \tilde{\Psi}_1 \) function (Eq. (4)), which compensates the \( m_z = 0 \) component of the \( A_{1u} \) representation with a Landau level \( n = 0 \) instead of \( n = 2 \) for the \( \tilde{\Psi}_2 \) function (Eq. (5)).

A complete comparison of the best fits obtained in our model with the results under pressure of Ref. [12] are presented on Fig. 3. Three parameters are used in the process of
fitting: $v_F$, which is directly found from the slope $\left( -\frac{dH_{c2}}{dT} \right)_{T=T_c}$, the effective $g-factor$, which is determined by the curvature of the low-field part of the data, and the ratio $\frac{T_{c1}}{T_{c0}}$, which is controled by the high-field region.

The values of the fitting parameters for different pressures are presented on Fig. 4. They suggest several comments. First of all, we note that the two-representations $(A_{1u}, E_u)$ model can reproduce the unusual features of $H_{c2}(T)$ in UBe$_{13}$. Good quantitative agreement is obtained everywhere except in high fields. At least two reasons can be invoked for the observed deviations: the effect of the omitted representations in equation (8) is expected to become stronger in the low temperature range, and the strong coupling effects, known to exist in this compound ([16,22]), have been neglected in our calculations and would also reinforce $H_{c2}(T)$ in this temperature range [12]. So these deviations can be ascribed to some oversimplifications of our model, without fundamentally questionning its validity.

The fitting parameters have realistic values. The pressure dependence of $v_F$ is in agreement with that of the Sommerfeld coefficient (deduced from Ref. [21]). This is not a surprise, as this feature is almost model independent and it has been already pointed out in Ref. [12]. The effective $g-factor$ increases slowly from 1.2 to 1.5 and is not too far from the free electron value. In the absence of any other probe of $g$, no additionnal cross-checking seems possible.

The more characteristic parameter of the model is the ratio $T_{c1}/T_{c0}$. Pronounced effect of the subdominant interaction via admixture of the second representation is observed even when the temperature of the subdominant transition $T_{c1}$ is much smaller than the real transition temperature $T_{c0}$. At ambient pressure $T_{c1}/T_{c0}$ is of the order of 0.1, whereas it grows linearly with pressure in the interval $0 < P < 20kbar$, up to a value of 0.7. Extrapolation of this dependence in a region of higher pressures predicts a crossover of the two temperatures at $P \approx 30kbar$. For $P > 30kbar$, $T_{c1}$ would be larger than $T_{c0}$ and the transition in zero magnetic field will take place in one of the three possible phases belonging to the $E_u$ representation [18]. An experimental observation of this crossover would be a good
check of the proposed two-representation scheme. The details of the crossover (if observed) could give support to the proposed choice of the two representations.

The observed qualitative changes (suppression of the upturn) of the curve $H_{c2}(T)$ with increasing pressure are easily understood within the proposed scheme. When $T_{c1}$ approaches $T_{c0}$, two representations can be considered as one three-dimensional representation, in which case the paramagnetic limitation can already be suppressed in low fields by a suitable combination of the basis functions, and no upturn will appear. A possibility to fit better the experimental curve when $T_{c1}$ (and $V_1$) is growing is also natural, since in that case the role of the omitted representation is getting relatively smaller.

Eventually, let us note the difference between our model and the two transition proposal of ref. [9]. In the latter one, it is proposed that a second transition is present below $\approx 0.6 T_c$, and that the upturn comes from the apparition of a new phase with a weaker Pauli limitation. The recent experimental effort reported in [1] strives to detect such a transition already in zero field. In our model, the second phase (the $E_u$ representation) does not appear in zero field-zero pressure. It is only the applied field which introduces the mixture of the representations at finite temperatures (see Fig. 2). A new phase transition is avoided precisely because the field allows such a mixture without additional symmetry breaking. The present scenario, in addition to providing quantitative predictions, also has the advantage of giving a realistic account of the smoothness of all features observed below $T_{c0}$ in UBe$_{13}$.

V. CONCLUSION

The proposed scheme satisfactorily explains the temperature dependence of $H_{c2}(T)$ in UBe$_{13}$, but it is in conflict with the observed power-law temperature dependencies of the specific heat, London penetration depth and of the longitudinal relaxation time in NMR-experiments [10] which reveal the existence of gap nodes: with the $A_{1u}$ order parameter, the nodes in the superconducting gap can only be accidental. The argument given above for the choice of the representations indeed leaves no other possibility than this $A_{1u} - E_u$
mixture among odd representations. There remains a possibility of a mixture of even and odd representations \[13,15\]. The choice of possible pairs of competing representations in that case is much bigger. We have not analyzed that possibility systematically though one would expect that in that case, because of a weaker coupling between the representations, the change in the curvature would be sharper. A promising route could also be the inclusion of strong coupling effects, which have recently received microscopic experimental confirmation \[22\]

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APPENDIX:

We consider now some limiting cases where the solution of Eq. (19) with respect to \( h \) can be found analytically. This analysis is useful both for a clarification of the underlying physical picture as well as for a check of the numerical calculations. Let us first consider the limit \( T \to 0 \). The limiting values of the functions of the variable \( t/\sqrt{h} F_{00}, F_{01}, F_{02} \) for \( t/\sqrt{h} \to 0 \) are:

\[
F_{00}(0) = \int_{0}^{\infty} \ln\left(\frac{\rho}{\sqrt{2}}\right) \frac{\rho d\rho}{2} \int_{0}^{1} (1 - x^2) e^{-\frac{x^2}{4}(1-x^2)} dx
= \ln\left(\frac{e}{2\sqrt{\gamma}}\right)
\]

\[
F_{01}(0) = \int_{0}^{\infty} \ln\left(\frac{\rho}{\sqrt{2}}\right) \frac{\rho d\rho}{4}
\int_{0}^{1} (1 - x^2)(1 - 3x^2) e^{-\frac{x^2}{4}(1-x^2)} dx
= -\frac{1}{6}
\]

\[
F_{02}(0) = \frac{1}{4\sqrt{2}} \int_{0}^{\infty} \rho d\rho \int_{0}^{1} (1 - x^2) e^{-\frac{x^2}{4}(1-x^2)} dx
\]

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\[ \frac{1}{3\sqrt{2}} \quad \text{(A3)} \]

\( P(\frac{t}{\sqrt{h}}, \lambda\sqrt{h}) \) in a limit \( t/\sqrt{h} \to 0 \) remains a function of \( \lambda\sqrt{h} \):

\[
P = P(0, \lambda\sqrt{h}) = -\int_0^\infty \frac{d\rho}{\rho} [1 - \cos(\rho\lambda\sqrt{h})] \int_0^1 x^2 e^{-u^2(1-x^2)} dx
\quad \text{(A4)}
\]

For a further simplification we consider the limit \( \lambda\sqrt{h} \to \infty \) as being relevant to the actual situation in UBe\textsubscript{13}. To evaluate the asymptotics of the integral in Eq. (A4) in that limit, let us split the interval of integration in two: \( (0, \rho_0) \) and \( (\rho_0, \infty) \) where \( \rho_0 \) is chosen to meet the following condition:

\[
\frac{1}{\lambda\sqrt{h}} \ll \rho_0 \ll 1 \quad \text{(A5)}
\]

In the first interval, one can then assume \( e^{-\frac{x^2}{4}(1-x^2)} \approx 1 \), and the contribution \( I_1 \) of this interval is evaluated straightforwardly:

\[
I_1 = \frac{1}{3} \int_0^{\rho_0} \frac{d\rho}{\rho} [1 - \cos(\rho\lambda\sqrt{h})] = \frac{1}{3} \ln(\gamma \lambda\sqrt{h}\rho_0), \\
\quad \text{(A6)}
\]

where \( \ln \gamma = C \) is Euler’s constant. When integrating over the second interval \( (\rho_0, \infty) \), one can drop the oscillating term \( \cos(\rho\lambda\sqrt{h}) \), yielding for the contribution \( I_2 \) of this interval:

\[
I_2 = \int_0^1 \frac{x^2}{2} dx \int_\infty^{\rho_0} \frac{du}{u} e^{-u}, \text{ where } u_0 = (1 - x^2) \frac{\rho_0^2}{4}
\]

Integrating over \( u \) by parts and taking into account that

\[
\int_0^\infty \ln u e^{-u} du = -C = -\ln \gamma
\quad \text{(A7)}
\]

we arrive at the following contribution of \( I_2 \):

\[
I_2 = -\frac{1}{3} \ln(\rho_0\sqrt{\gamma}) + \frac{4}{9}
\quad \text{(A8)}
\]

The sum of (A6) and (A8) gives the principal order terms in the asymptotics of \( P \) for \( \lambda\sqrt{h} \to \infty \):
\[ P = \frac{1}{3} \ln(\lambda \sqrt{\gamma h}) + \frac{4}{9} \]  \hspace{1cm} (A9)

Given the limiting values Eqs. (A1) - (A3) and the asymptotic (A9), one can find a limiting value of \( H_{c2} \) at \( T=0 \) and \( \lambda \sqrt{h} \to \infty \). According to (A9) \( P \to \infty \) when \( \lambda \sqrt{h} \to \infty \). Collecting in Eq. (A9) the terms proportional to \( P \) and setting them to zero we arrive at the following equation:

\[
(G + F_{01})^2 = \frac{Q^2}{9} + \frac{9}{4} F_{02}^2 \hspace{1cm} (A10)
\]

Using here the limiting values Eqs. (A1) - (A3) we solve this equation with respect to \( h \).

The largest of the two roots gives \( H_{c2} \):

\[
h = \frac{1}{4 \gamma q^{4/3}} \exp \left[ \frac{5}{3} + 2 \sqrt{\frac{Q^2}{9} + \frac{1}{8}} \right] \hspace{1cm} (A11)
\]

For this value of \( h \) one obtains from Eqs. (12) - (14) the limiting values of the ratios of the coefficients in the definition (11):

\[
\frac{\eta_1}{\eta_0} = \frac{1 + \epsilon^2}{\sqrt{2}} \hspace{1cm} (A12)
\]

This ratio corresponds to a complete elimination of the \( z \)-projection of the order parameter (cf. Eq. (10)).

Another ratio is:

\[
\frac{\eta_1}{\eta_2} = \frac{2 \sqrt{2}}{3} (1 - \epsilon) \left[ \frac{Q}{3} + \sqrt{\frac{Q^2}{9} + \frac{1}{8}} \right] \hspace{1cm} (A13)
\]

The order parameter obtained with these coefficients has the form:

\[
\vec{d}(\vec{k}) = \sqrt{3} \hat{y} k_y \hspace{1cm} (A14)
\]

In the opposite limit when \( T \to T_c \) and \( H_{c2} \to 0 \) we can expand the functions entering Eq. (13) in powers of \( h \) and \( \tau = 1 - t \). These functions are defined as double integrals. For integration over \( \rho \) the convergence is provided by the exponential factor \( e^{-\frac{2t}{\sqrt{\rho}}} \) in a region \( \rho \sim \sqrt{h}/t \ll 1 \). It means that all other functions can be expanded in powers of \( \rho^2 \) which,
after integration over \( \rho \), gives an expansion over \( h/t^2 \). We keep only terms which are necessary for finding \( \tau(h) \) with an accuracy up to \( h^2 \):

\[
F_{00} - \ln \sqrt{t} = -\ln t - \frac{a}{t^2} + b \frac{h^2}{t^4}, \quad F_{01} = -f_1 \frac{h}{t^2},
\]

\[
F_{02} = f_2 \frac{h}{t^2}, \quad P = -a \lambda \frac{h^2}{t^2},
\]

with \( a = \frac{7}{12} \zeta(3), \ b = \frac{31}{40} \zeta(5), \ f_1 = \frac{7}{60} \zeta(3), \ f_2 = \frac{7}{15 \sqrt{2}} \zeta(3) \) where \( \zeta(z) \) is Riemann’s zeta function. Substitution of this expansion in Eq. (19) gives

\[
\tau(h) = ah + \left( \frac{3}{2} a^2 - b + a \lambda^2 - \frac{1}{Q} \left[ 2 f_1^2 + \frac{3}{2} f_2^2 \right] \right) h^2 \quad (A15)
\]

The term linear on \( h \) comes from \( F_{00} \) and is determined entirely by the \( A_{1u} \) representation, but the \( h^2 \) term is influenced by the \( E_u \) representation via the functions \( F_{01} \) and \( F_{02} \) (or the coefficients \( f_1 \) and \( f_2 \)). This influence is getting stronger when \( Q \) decreases, i.e. when the two transition temperatures are coming closer. The term proportional to \( 1/Q \) is definitely negative: it means that for sufficiently small \( Q \), the dependence of \( H_{c2}(T) \) will have positive curvature starting from \( T_c \).
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FIGURES

FIG. 1. Upper critical field of UBe$_{13}$ at zero pressure (from [1]), together with three calculations of $H_{c2}(T)$ for three different irreducible representations: $A_{1g}$, $A_{1u}$, and a mixture $A_{1u} - E_u$. The same values of the effective $g$ – factor (adjusted for the odd parity representations) and the Fermi velocity have been used for the three fits.

FIG. 2. Temperature dependence of the normalized components $\eta_1$ and $\eta_2$ of the respective basis functions $\tilde{\Psi}_1$ and $\tilde{\Psi}_2$ of the $E_u$ representation, along the $H_{c2}(T)$ fit of Fig. 1. Note that they both start to grow much above $T_{c1} = 0.12 T_{c0}$.

FIG. 3. Upper critical field of UBe$_{13}$ under pressure [1], and best fit of our $A_{1u} - E_u$ model (full lines) for each pressure: the main features are well reproduced, and deviations appear only at low temperatures (see discussion in the main text).

FIG. 4. Pressure ($p$) dependence of the three parameters of the fits of Fig. 3. The comparison of the $p$-dependence of the Fermi velocity $v_F$ with the Sommerfeld coefficient $\gamma(p)$ of ref. [21] is gratifying. Extrapolation of the linear increase of $\frac{T_{c1}}{T_{c0}}$ predicts that the $E_u$ representation will appear first in zero field above $p \approx 30 kbar$. 

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