Tests of Loop Quantum Gravity from the EHT Results of Sgr A* 

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ABSTRACT

The Event Horizon Telescope (EHT) collaboration’s image of the compact object at the galactic centre is the first direct evidence of the supermassive black hole Sgr A*. The shadow of Sgr A* has an angular diameter $d_{sh} = 48.7 \pm 7 \mu\text{as}$ with fractional deviation from the Schwarzschild black hole shadow diameter $\delta = -0.08^{+0.09}_{-0.09}, -0.04^{+0.09}_{-0.10}$ (for the VLTI and Keck mass-to-distance ratios). Sgr A*’s shadow size is within 10% of the Kerr predictions, equipping us with yet another tool to analyze the gravity in the strong-field regime, including testing loop quantum gravity (LQG). We use Sgr A*’s shadow to constrain the metrics of two well-motivated LQG-inspired rotating black holes (LIRBHs) models characterized by an additional deviation parameter $L_q$, which recover the Kerr spacetime in the absence of quantum effects ($L_q \rightarrow 0$). When increasing the quantum effects through $L_q$, the black hole shadow size increases monotonically, while the shape gets more distorted, allowing us to constrain the fundamental parameter $L_q$. We use the astrophysical observables shadow area $A$ and oblateness $D$ to estimate the black hole parameters. It may be useful in extracting additional information about LIRBHs. While the EHT observational results completely rule out the wormhole region in the second LIRBH, a substantial parameter region of the generic black holes in both models agree with the EHT results. We find upper bounds on $L_q$ from the shadow of Sgr A*: $L_q \lesssim 0.0423$ and $L_q \lesssim 0.0821$ for the two LIRBHs respectively provide more stringent constraints than those got with the EHT image of M87*. 

Keywords: Astrophysical black holes (98); Black hole physics (159); Galactic center (565); Gravitation (661); Gravitational lensing (670)

1. INTRODUCTION

The theory of general relativity (GR), though being a widely tested standard model of gravity with remarkable consequences, such as the existence of black holes (Schwarzschild 1916) and gravitational waves, is nevertheless not free of pathologies, which has called for modifications and alternatives to it (Nojiri et al. 2017). The necessary extension of GR at quantum scale had been emphasized by Einstein (1916) himself, and thus far there have been several efforts in the direction (see Addazi et al. 2022, for a recent review), with the most promising candidate quantum gravity models being provided by string theory and loop quantum gravity (LQG, see Rovelli 1998, for a review).

Black holes are among the many fascinating objects in the Universe, enveloped by matter under extreme conditions in a regime of strong spacetime curvature. Studying these black holes can lead to a greater insight into their nature, their circumstances, and their significance for fundamental theories like GR as well as other theories like loop quantum gravity (LQG). The no-hair theorem states that an isolated and stationary black hole in GR is described by three parameters, viz. Mass $M$, spin $J$, and electric charge $Q$. Thus, they are defined by the Kerr-Newman metric (Newman et al. 1965), which goes to the Kerr metric (Kerr 1963) that are charged neutral. The Kerr back hole describes the astrophysical black holes because any residual electric charge is expected to rapidly neutralize (Israel 1967, 1968; Carter 1971, 1999; Hawking 1972). Indeed, direct evidence of this is still inconclusive, and it may be difficult to rule out non-Kerr black holes (Ryan 1995; Will 2006). Also, the no-hair theorem’s mathematical status is not without controversy, principally concerning the assumption (Chrusciel et al. 2012). One can test the no-hair theorem by calculating potential deviations from Kerr metrics like the LQG motivated rotating black holes. The celebrated theorem does not hold for the modified theories of gravity, like LQG, that admit non-Kerr black holes. The images of the supermassive black holes M87* (Akiyama et al. 2019a,b,c) and Sgr A*
(Akiyama et al. 2022a,b) observed by the Event Horizon Telescope (EHT) collaboration led us into an untouched stage of black hole physics, which offer a direct visualization of M87* and Sgr A*, as well as their surrounding environment. With this way of investigating the most extreme objects, one can explore the fundamental physics from the knowledge we get from the EHT observations. Indeed the EHT collaboration, in 2019, released the first horizon-scale image of the M87* supermassive black holes (Akiyama et al. 2019a,b,c) and black holes are now a physical truth. Utilizing the distance of M87* from the earth $D = 16.8$ Mpc and estimated mass $M = (6.5 \pm 0.7) \times 10^9 M_\odot$, puts bounds on the compact emission region size with angular diameter $d_{sh} = 42 \pm 3 \mu$as and circularity deviation $\Delta C \leq 0.10$. Recently, the EHT collaboration, in 2022, posted the Sgr A* black hole shadow results (Akiyama et al. 2022a,c,d,e,f,b); considering a black hole of mass $M = 4.0^{+1.1}_{-1.6} \times 10^6 M_\odot$ and distance $D = 8$ kpc from earth, the EHT shows that the Sgr A* shadow has an angular diameter $d_{sh} = 48.7 \pm 7 \mu$as with fractional deviation from the Schwarzschild black hole shadow diameter $\delta = -0.08^{+0.09}_{-0.09}, -0.04^{+0.09}_{-0.10}$ (for the VLTI and Keck mass-to-distance ratios respectively) and the images are consistent with the expected appearance of a Kerr black holes (Akiyama et al. 2022a,b). Compared with the EHT results for M87*, it reveals consistency with the predictions of GR (Akiyama et al. 2022).

The EHT observation of M87* and Sgr A* presents a new powerful technique to test the black-hole metric gravitationally in the strong-field regime and also provides an exceptional way to constrain the various black hole parameters and to test the underlying associated theories of gravity (Kocherlakota et al. 2021; Akiyama et al. 2022b; Ghosh et al. 2021; Afrin et al. 2021; Walia et al. 2022; Kumar et al. 2022; Islam et al. 2022; Sengo et al. 2022). Therefore, the EHT results offer a considerable recent complement to the set of observations that probe the strong-field regime of gravity. The supermassive black holes M87* and Sgr A* can be most acceptable prospects for testing LQG. LQG, being a non-perturbative approach to quantum gravity, goes beyond GR to resolve classical spacetime singularities in the black hole spacetimes (Ashtekar et al. 2006, 2007; Vandersloot 2007). Because of the inherent hardship in solving the complete system, the emphasis has been on spherically symmetric black hole spacetimes. In the semi-classical regimes, within the framework of LQG, it turns out that several spherical symmetric black holes models exists such that singularity occurring in the GR is now substituted by a transition regular surface (Ashtekar & Bojowald 2006; Boehmer & Vandersloot 2007; Modesto 2010; Perez 2017; Gambini & Pullin 2008, 2013; Corichi & Singh 2016; Olmedo et al. 2017; Ashtekar et al. 2018a,b; Bodendorfer et al. 2019a,b; Arruga et al. 2020; Assanioussi et al. 2020; Ben Achour et al. 2020; Gambini et al. 2020; Bodendorfer et al. 2021a,b; Blanchette et al. 2021; Assanioussi & Mickel 2021; Chen 2022).

We obtain the LQG-inspired rotating black holes (LIRBHs) via the revised Newman-Janis generating method (Liu et al. 2020; Brahma et al. 2021), which works quite well in generating rotating metrics starting with their non-rotating seed metrics arising in the modified gravities, including LQG (Azreg-Aïnou 2014; Brahma et al. 2021; Liu et al. 2020; Chen 2022; Modesto 2010). The LIRBHs or Kerr-like black holes which has an additional parameter ($L_q$) coming from the quantum effects, apart from mass ($M$) and rotation parameter ($a$) can be appropriately tested with astrophysical observations. We also show that it is possible, in

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**Figure 1.** Parameter space of (a) LIRBH-1: region I represents generic black holes with Cauchy and event horizons while region II is a no-horizon spacetime (left) (Liu et al. 2020) and LIRBH-2 (right) (Brahma et al. 2021) (a) LIRBH-2: region I represents generic black holes with Cauchy and event horizons, region II is a black hole with a single horizon and region III a wormhole which we shall show to be ruled out by EHT observations.
principle, to constrain the LQG parameter $L_q$ using the Event Horizon Telescope (EHT) observed shadow image cast by the M87* and Sgr A*. Further, we aim to investigate if the shadow images of Sgr A* can help us better question whether the two Kerr-like LQG-motivated black holes can be suitable candidates for astrophysical black holes. We also examine whether the EHT bounds for Sgr A* can provide more stringent constraints on the LQG black hole parameter than previously obtained with the bounds for M87* observations (Brahma et al. 2021).

The paper is organised as follows: in Section 2, we study the effect of the parameter $L_q$ on the photon geodesics and shadow silhouettes of LIRBHs. The Section 3 is dedicated to the estimation of the black hole parameters $L_q$ and $D$ utilizing the shadow observables $A$ and $D$. In Section 4, we constrain $L_q$ from the EHT deduced bound on the shadow observables $d_{sh}$ and $\delta$. Finally, we conclude in Section 5.

We work with geometrized units $8\pi G = c = 1$ throughout this paper, unless units are specifically defined.

2. METHODOLOGICAL FRAMEWORK

Here, we examine two well-motivated models, viz., LIRBH-1 (Modesto 2010; Liu et al. 2020) and LIRBH-2 (Brahma et al. 2021; Yang et al. 2022). The shadows of these two models have received considerable attention, and their shape and size are considerably different from those of the Kerr black hole shadows (Brahma et al. 2021; Yang et al. 2022; Liu et al. 2020; Devi et al. 2021; Walia 2022). The LIRBHs, in question, belong to a family of prototype non-Kerr black hole metrics with an additional deviation parameter of $L_q$ related to the quantum effects, besides $a$ and $M$ of Kerr black hole, which is included as a particular case of vanishing quantum effects $L_q \to 0$, and both LIRBHs provide singularity resolution of Kerr black holes. The black hole shadow (Bardeen 1973; Lumistet 1979), a purely geometry-dependent strong field construct, can in principle be used to determine the properties of the black hole spacetime, e.g., computation of parameters (Kumar & Ghosh 2020a). Hence, we examine the shadows of LIRBH (Liu et al. 2020; Brahma et al. 2021), whose line element in Boyer–Lindquist coordinates $(t, r, \theta, \phi)$ can be cast in a Kerr-like form (Azreg-Aïnou 2014)

$$
\text{\begin{equation}}
\begin{aligned}
&ds^2 = -\frac{\Psi}{\rho^2} \left[ \frac{\Delta}{\rho^4} (dt - a \sin^2 \theta d\phi)^2 - \frac{\rho^2}{\Delta} dr^2 \\
&\quad - \rho^2 \sin^2 \theta \left[ \frac{\Delta}{\rho^2} \{adt - (\omega(r) + a^2) d\phi\}^2 \right] \right],
\end{aligned}
\end{equation}
\text{\end{equation}}
$$

where $\rho^2 = \omega(r) + a^2 \cos^2 \theta$, and $\omega(r)$ and $\Delta(r)$ are model dependent metric functions; the Kerr-like form of metric (1) brings out the geodesic equations of motion significantly.

**LIRBH-1:** It is the rotating counterpart (Liu et al. 2020) of the semi-classical LQG inspired spherical solution (Modesto 2010). The LIRBH-1 is described by metric (1) (Liu et al. 2020) with

$$
\text{\begin{equation}}
\begin{aligned}
\Delta(r) &= \frac{(r - r_+)(r - r_-)r^2}{(r - r_+)^2}, \\
\omega &= \frac{r^4 + a^2}{(r + r_+)^2}, \quad \Psi(r) = \frac{r^4 + a^2}{r^2}.
\end{aligned}
\end{equation}
\text{\end{equation}}
$$

Here $r_+ = 2M/(1 + L_q)^2$, $r_- = 2ML_q^2/(1 + L_q)^2$, and $\rho = \sqrt{r^2 + r_+^2} = 2ML_q/(1 + L_q)^2$; $L_q = (\sqrt{1 + \gamma^2} - 1)/(\sqrt{1 + \gamma^2} + 1)$ is the polymeric function, where $\gamma$ is the Immirzi parameter and $\delta$ is the polymeric parameter such that $\gamma \delta < 1$. The ADM mass and black hole spin are denoted by $M$ and $a$, respectively. Also, the parameter $a_0 = A_{\min}/8\pi$ is related to the minimum area gap of LQG, $A_{\min} = 8\pi\ell_p^2\gamma \sqrt{s_m(s_m + 1)}$, where $\ell_p$ is the Planck length and $s_m$ is the smallest value of the representation on the edge of the spin network crossing the surface (Santos et al. 2016); considering SU(2) group representation we have, $s_m = 1/2$ and we further set $\gamma = 1$, and thus $a_0 = \sqrt{3}/2\ell_p^2$ (Santos et al. 2016).

Further, in the limit $L_q = 0 = a_0$, the metric (1) goes over to the Kerr solution. A root analysis of $\Delta = (r - r_+)(r - r_-)r^2/(r^2 + a^2) = 0$, yields a parameter space $(L_q, a/M)$ where two real roots corresponding to the horizons are obtained (region I in left panel of Fig. 1), as well as, parameter space corresponding no horizon regular spacetime where no real roots of $\Delta$ are obtained (region II in left panel of Fig. 1). We shall test the LIRBH-1 model with the EHT results of M87* and Sgr A*, which has not been done yet.

**LIRBH-2**—LIRBH-2 (Brahma et al. 2021; Yang et al. 2022) is derived using Newman–Janis algorithm with the spherical LQG inspired quantum extension of Schwarzschild spacetime (Bodendorfer et al. 2021a) as seed metric. Again, the LIRBH-2 is described by the metric (1) with metric functions

$$
\text{\begin{equation}}
\begin{aligned}
\Delta(r) &= 8L_qM_B^2a^2 + a^2, \\
\omega &= b^2, \quad \Psi(r) = \rho^2
\end{aligned}
\end{equation}
$$

with

$$
\text{\begin{equation}}
\begin{aligned}
b^2(x) &= \frac{L_q}{\sqrt{1 + x^2}} \frac{M_B^2(x + \sqrt{1 + x^2})^6 + M_B^2}{(x + \sqrt{1 + x^2})^3}, \\
a(x) &= \left(1 - \frac{1}{2L_q \sqrt{1 + x^2}} \right) \frac{1 + x^2}{b(x)^2}.
\end{aligned}
\end{equation}
$$

Here $x = r/(\sqrt{8L_qMB}) \in (-\infty, \infty)$; $L_q = (l_k/M_BM_W)^{2/3}/2 \geq 0$ is a dimensionless parameter, where the quantum parameter $l_k$ arises from holonomy modifications (Bodendorfer et al. 2021a,b) and it
is directly related to the minimum area gap of the LQG theory with the areal radius given by Eq. (6) (Brahma et al. 2021). Here, $M_B$ and $M_W$ correspond to the Dirac observables in the model with black hole of mass $M_B$ and white hole of mass $M_W$ (Bodendorfer et al. 2021a,b); we are interested in the case $M_B = M_W = M$, i.e., a symmetric bounce. Again, solving $\Delta = 0$ for real roots segregates the $(L_q, a/M)$ parameter space into three regions as shown in Fig. 1: (i) region I, a generic black hole with two horizons which is the most physically relevant region (ii) region II denoting a black hole with a single event horizon and (iii) region III corresponding to a horizon-less wormhole. We are interested only in black holes, i.e., regions I and II, as the wormhole region has been ruled out by observational results of M87* (Brahma et al. 2021). Also, we intend to constrain the parameter $L_q$ with observations of Sgr A* and estimate $L_q$ using black hole shadow.

2.1. Shadow silhouette

The EHT images of supermassive black holes exhibit a dark brightness depression surrounded by a bright ring-like feature (Akiyama et al. 2019a, 2021). They are composed of an emission component that is mainly determined by the theory of agnostic and relatively uncertain radiative and accretion physics, plus a series of bright rings asymptotically spiralling and approaching the dark region (Johnson et al. 2020; Broderick et al. 2022). This boundary, the black hole shadow silhouette, though not fully resolved yet by the EHT, can be obtained analytically as the locus of gravitationally lensed photons travelling in close proximity to the black hole on the observer’s celestial plane. Besides, it is independent of the various astrophysical phenomena (Johnson et al. 2020). The black hole shadow can be employed as a tool to test modified theories of gravity, besides constraining the potential deviations from Kerr metric; it has thus eventuated in a comprehensive literature addressing shadows in both GR (Falcke et al. 2000; de Vries 2000; Shen et al. 2005; Yumoto et al. 2012; Atamurotov et al. 2013; Abdujabbarov et al. 2015; Johannsen et al. 2016; Cunha & Herdeiro 2018; Kumar & Ghosh 2020a; Afrin & Ghosh 2022a; Chael et al. 2021) and modified theories of gravity (Amarilla et al. 2010; Johannsen & Psaltis 2011; Amir et al. 2018; Singh & Ghosh 2018; Kumar et al. 2021; Mizuno et al. 2018; Allahyari et al. 2020; Papnoi et al. 2014; Kumar et al. 2020d, a,b, Kumar & Ghosh 2020b; Brahma et al. 2021; Ghosh et al. 2021; Afrin & Ghosh 2022b; Vagnozzi et al. 2022; Vagnozzi & Visinelli 2019; Afrin et al. 2021; Walia et al. 2022; Kumar et al. 2022; Islam et al. 2022; Sen go et al. 2022; Kuang et al. 2022; Junior et al. 2022). The fact that the silhouette of black hole shadows encode in them, the strong-field properties of the spacetime, suggests that, we can use them for performing strong-field gravitational tests (Johannsen & Psaltis 2010; Cunha & Herdeiro 2018; Baker et al. 2015). The lightlike geodesics in the LQG spacetime (1), just as in the Kerr spacetime, follow the Hamilton-Jacobi equation (Carter 1968),

$$\frac{\partial S}{\partial \lambda} = -\frac{1}{2}g^{\alpha \beta} \frac{\partial S}{\partial x^\alpha} \frac{\partial S}{\partial x^\beta},$$

(8)

where $\lambda$ is the affine parameter along the geodesics, and $S$ is the Jacobi action given by

$$S = -\mathcal{E}t + \mathcal{L}_z \phi + \mathcal{S}_r(r) + \mathcal{S}_\phi(\theta),$$

(9)

where the conserved photon energy $\mathcal{E} = -p_\nu \partial_\nu$ and axial angular momentum $\mathcal{L}_z = p_\phi \partial_\phi$, arise due to the two translational and rotational symmetry of the metric (1). Further, the axially symmetric metric (1) insinuates a fourth conserved quantity, the Carter’s constant $\mathcal{Q}$, which ensures the decoupling of $r$ and $\theta$ equations (Carter 1968). Following (Tsukamoto 2018; Kumar & Ghosh 2020a, 2021; Brahma et al. 2021; Liu et al. 2020), we find that the Hamilton-Jacobi equations are separable, and obtain null geodesics in the first-order differential form

$$\rho^2 \frac{dt}{d\lambda} = \frac{\omega(r) + a^2}{\Delta} [\mathcal{E}(\omega(r) + a^2) - a \mathcal{L}_z] - a(\mathcal{E} \sin^2 \theta - \mathcal{L}_z),$$

(10)

$$\rho^2 \frac{dr}{d\lambda} = \pm \sqrt{R(r)},$$

(11)

$$\rho^2 \frac{d\theta}{d\lambda} = \pm \sqrt{\Theta(\theta)},$$

(12)

$$\rho^2 \frac{d\phi}{d\lambda} = \frac{a}{\Delta} [\mathcal{E}(\omega(r) + a^2) - a \mathcal{L}_z] - \left(\mathcal{E} - \frac{\mathcal{L}_z}{\sin^2 \theta}\right),$$

(13)

where $R(r)$ and $\Theta(\theta)$ respectively refer to radial and polar effective potentials as follows

$$\mathcal{R}(r) = \left(\frac{\omega(r) + a^2}{\mathcal{E}} - a \mathcal{L}_z\right)^2 - \Delta(K + (a \mathcal{E} - \mathcal{L}_z)^2),$$

(14)

$$\Theta(\theta) = K - \left(\frac{\mathcal{L}_z^2}{\sin^2 \theta} - a^2 \mathcal{E}^2\right) \cos^2 \theta.$$  

(15)

Here the separability constant $K = Q - (a \mathcal{E} - \mathcal{L}_z)^2$ is related to the non-apparent symmetries of metric (1) through a quadratic Killing tensor (Hioki & Miyamoto 2008). Interestingly, Eqs. (10)-(15) have the same mathematical form as in the Kerr case and further, for $L_q = 0$ reduce exactly to those of the Kerr (Chandrasekhar 1985). The $Q$ is related to the $\theta$-velocity of the photon and for $Q = 0$ the photon motion is restricted to the equatorial plane; while the $\mathcal{L}_z$ controls the $\phi$-motion (Chandrasekhar 1985; Teo 2021). The black hole shadow silhouette is formed by the spherical photons (SPOs) that move on constant radii $r_p > r_+$, hitting the observer plane asymptotically far away, obtained by solving (Chandrasekhar 1985)

$$\mathcal{R}(r_p) = 0 = \mathcal{R}'(r_p)$$

(16)
Following (Tsukamoto 2018; Kumar & Ghosh 2020a, 2021; Brahma et al. 2021; Liu et al. 2020), to reduce the degrees of freedom from three to two, we introduce dimensionless quantities: $\xi \equiv L/E$, $\eta \equiv K/E^2$, in Eq. (14) – which uniquely determine each light path – and solve Eq. (16) to obtain the critical impact parameters ($\xi_c, \eta_c$) for the SPOs,

$$\xi_c = \frac{a^2 - \frac{2\Delta(r)(\omega'(r)+2r)}{\Delta'(r)} + \omega(r) + r^2}{\omega'} ,$$

$$\eta_c = -\frac{4\Delta(r) (\omega'(r) + 2r) (a^2\omega'(r) + 2a^2r + (\omega(r) + r^2) \Delta'(r)) - 4\Delta(r)^2 (\omega'(r) + 2r)^2 - (\omega(r) + r^2)^2 \Delta'(r)^2}{a^2 \Delta'(r)^2} ,$$

(17)
\[ \{X, Y\} = \{-\xi_c \csc \theta_0, \pm \sqrt{\eta_c + a^2 \cos^2 \theta_0 - \xi_c^2 \cot^2 \theta_0}\}. \]

The shadow coordinates \(\{X, Y\}\) for the LQG black holes (1), casted in Kerr-like form, have the same functional form as of the Kerr case (Tsukamoto 2018; Kumar & Ghosh 2020a; Kumar et al. 2020c; Afrin et al. 2021; Afrin & Ghosh 2022b; Ghosh & Afrin 2022); this makes the shadow analysis substantially simplified.

The shadow silhouette is constructed by plotting \((X, Y)\) in parametric form as a function of \(r_p\). From Fig. 2, it turns out that, the shadows become smaller in size and get more distinctly distorted with an increase in the quantum effects, \(L_q\); this shows that the LQG parameter, that is expected to have significance only at Planck scale, in reality has non-negligible, rather profound effect on observable effects like that of the shadow shape and size. We exploit this visible effect to see whether these imprint of the \(L_q\) in the shadow can be exploited to extract and also constrain the parameter \(L_q\) analytically. Additionally, there is a horizontal shift in shadow centre along the \(X\)-axis, with increase \(a\), due to the frame dragging effect.

3. PARAMETER ESTIMATION

The black hole shadow, which is seen to encode the nature of background spacetime in its characteristic shape and size (cf. Fig. 2), can serve as a tool to not only test the underlying theory of gravity but also to constrain the deviations from GR (Cunha et al. 2019; Banerjee et al. 2020; Allahyari et al. 2020; Yan et al. 2020; Vagnozzi et al. 2020; Khodadi et al. 2020; Jusufi et al. 2022b, a; Okyay & Övgün 2022; Roy et al. 2022; Chen et al. 2022; Pantig & Övgün 2022; Khodadi & Lambiase 2022; Odintsov & Oikonomou 2022). We aim to get more information about the LIRBHs, one of the most important step in which is to extract the parameter \(L_q\) observationally, which has not been done yet in the framework of LQG. Thus we outline a simple method of black hole parameter estimation using the shadow observables – shadow area \(A\).
and oblateness $D$ — which is robust in the sense that it can be employed to a haphazard shadow shape utilizing minimal symmetry (Kumar & Ghosh 2020a). To estimate the LQG parameter $L_q$ besides the black hole spin $a$, associated with the LIRBHs under consideration, we define the area enclosed within the shadow silhouette as (Abduljabbarov et al. 2015; Kumar & Ghosh 2020a)

\[ A = 2 \int Y(r_p) dX(r_p) \]
\[ = 2 \int_{r_p^+}^{r_p^-} \left( Y(r_p) \frac{dX(r_p)}{dr_p} \right) dr_p, \quad \text{(19)} \]

where $r_p^+$ are respectively the prograde and retrograde SPO radii obtained as the smallest and largest real roots of $\eta_c = 0$, $\xi_c(r_p^+) \gtrless 0$, outside the event horizon (Teo 2021). Next, we quantify the deformation in shadow shape — induced by $L_q$ and $a$ — from a perfect circle, with the shadow oblateness ($D$) observable, which can be written as (Tsupko 2017; Kumar & Ghosh 2020a),

\[ D = \frac{X_r - X_l}{Y_t - Y_b} \quad \text{(20)} \]

where the subscripts $l$, $r$, $t$ and $b$ stand for the left and right ends of the shadow silhouette, where $Y(r_p) = 0$, 

Figure 4. Constraints from EHT results of angular shadow diameter $d_{sh}$: modelling M87* as LIRBH-1 (left) and LIRBH-2 (right) at $17^\circ$ (top) and $90^\circ$ (bottom) inclinations. The solid and dashed curves correspond respectively to the $1\sigma$ and $2\sigma$ bounds of the measured Schwarzschild deviation, $\delta = -0.01 \pm 0.17$ of M87*, as reported by the EHT.
Figure 5. Constraints from EHT results of angular shadow diameter $d_{sh}$: modelling Sgr A* as LIRBH-1 (left) and LIRBH-2 (right) black holes at 50°(top) and 90°(bottom) inclinations. The solid and dashed curves correspond respectively to the 1σ and 2σ bounds of the measured shadow diameter, $d_{sh} = 48.7 \pm 7 \mu\text{as}$ of Sgr A*, as reported by the EHT.

for positive $a$, and the top and bottom points, where $Y''(r_p) = 0$ respectively (Hioki & Maeda 2009). For spherically symmetric black hole it is straightforward to understand that $D = 1$, however, for the rotating black holes with extra deviation parameters, characteristically $D \neq 1$; in the Kerr case $1 \leq D \leq \sqrt{3}/2$ (Tsupko 2017; Kumar & Ghosh 2020a; Afrin et al. 2021). Note also that $D$ is closely related to other measures of oblatness, e.g. the deviation from circularity studied in Bambi et al. (2019) (see also Vagnozzi & Visinelli (2019)).

The parameters of the background theory of LQG – treated as intrinsic parameters of the model black holes – can be extracted from the shadow observables (Hioki & Maeda 2009; Kumar & Ghosh 2020a; Afrin et al. 2021; Afrin & Ghosh 2022b) if the extrinsic parameters viz., $\theta_o$ and $d$ can be measured independently. The one-to-one correspondence between the shadow characteristics, i.e., $A$ and $D$, and the black hole parameters $L_q$ and $a$ is evident from Fig. 2 (Kumar & Ghosh 2020a; Afrin & Ghosh 2022b). The maximum deformation in shadow shape, a deviation from perfect circle, is observed only at a high inclina-
Figure 6. Constraints from EHT results of Schwarzschild shadow deviation $\delta$: modelling Sgr A* as LIRBH-1 (left) and LIRBH-2 (right) black holes at 50° (top) and 90° (bottom) inclinations. The blue and red solid contours correspond respectively to the 1σ bounds of the measured Schwarzschild deviation $\delta = -0.08^{+0.09}_{-0.09}$ (VLTI), $-0.04^{+0.09}_{-0.10}$ (Keck) of Sgr A*, as reported by the EHT. The dashed lines correspond to the respective 2σ bounds.

We set up now, a framework, for directly translating the bounds from the EHT results of M87* and Sgr A* to the LIRBHs, utilizing the characteristic shadow observables – angular shadow diameter ($d_{sh}$) and Schwarzschild shadow deviation ($\delta$) – that capture the details of the background theory of gravity. The angular diameter of shadow, for an observer at distance $d$ from the black hole, is defined as $A/M^2$ and $D$ for any $L_q$ and $a/M$ are found to intersect at unique points (cf. Fig. 3) and the coordinates of the intersections uniquely determine the two black hole parameters $L_q$ and $a/M$. We tabulate selective estimated parameters of the LIRBHs in Table 1.

4. CONSTRAINING WITH EHT OBSERVATIONS
Table 1. Estimated parameters of the two LGQ model black holes

| Model  | Shadow observables | Estimated parameters |
|--------|--------------------|----------------------|
|        | A/M²               | D                    | L₉       | a/M      |
| LIRBH-1| 6                  | 0.999                | 0.0802   | 0.0049   |
|        | 8                  | 0.996                | 0.6393   | 0.0319   |
|        | 20                 | 0.980                | 0.1844   | 0.3226   |
|        | 45                 | 0.956                | 0.1240   | 0.4751   |
|        | 80                 | 0.940                | 0.0023   | 0.8520   |
| LIRBH-2| 26                 | 0.999                | 0.4816   | 0.0075   |
|        | 45.33              | 0.996                | 0.3323   | 0.1873   |
|        | 60.80              | 0.980                | 0.1991   | 0.4710   |
|        | 68.53              | 0.956                | 0.1174   | 0.7107   |
|        | 76.27              | 0.920                | 0.0276   | 0.9105   |

2021; Afrin & Ghosh 2022b)

\[ d_{sh} = 2 \frac{R_a}{d}, \quad R_a = \sqrt{\frac{A}{\pi}}, \quad (21) \]

where \( R_a \) is the areal shadow radius. Apart from distance \( d \), the \( d_{sh} \) implicitly depends on the mass \( M \) and parameter \( L_9 \) of the black holes (1) besides the observation angle \( \theta_o \). Using EHT considered mass and distance of M87* and Sgr A*, we calculate the angular diameter of the shadows for the two LIRBHs in question.

The EHT images of both the M87* and Sgr A* exhibit a luminous thick ring of emission, with diameters 42 ± 3 μas and 51.8 ± 2.3 μas respectively – consistent with the expectations from a central supermassive black hole (Akiyama et al. 2019a, 2022f) – surrounding a brightness depression, namely the black hole shadow (Akiyama et al. 2019a, 2022b). To quantify the difference between the model shadow diameter (\( \tilde{d}_{\text{metric}} \)) and the Schwarzschild shadow diameter \( 6\sqrt{3}M \), we introduce Schwarzschild shadow deviation (\( \delta \)) given by (Akiyama et al. 2022f,b),

\[ \delta = \frac{\tilde{d}_{\text{metric}}}{6\sqrt{3}} - 1. \quad (22) \]

Here \( \tilde{d}_{\text{metric}} = 2R_a \) where \( R_a \) is given by Eq. (21). In case of Kerr black holes, \( \delta \) is \(-0.075, 0\) (Akiyama et al. 2022b) with the variations \( a \in [0, M] \) and \( \theta_0 \in [0, \pi/2] \), and thus for any black hole to cast shadows consistent with those of Kerr black holes’ should be within this range. Thus theories of gravity predicting shadows smaller than \( \delta < -0.075 \) and larger than \( \delta > 0 \) Kerr black holes can, aided by the \( \delta \) observable, be tested. Interestingly, the LQG black hole models that we consider, cast shadows that are distinctly smaller and more distorted than the corresponding Kerr shadows, and can thus be tested and constrained with EHT results. With the mass and distance of M87* and Sgr A* as considered by EHT (Akiyama et al. 2019a, 2022f,b), we calculate the Schwarzschild deviation of the shadows cast by the LIRBHs.

There are some caveats to the present analysis, some of which are directly related to the observational appearance of M87* and Sgr A* itself viz., the uncertainties induced by the different telescopes in the sparse array as well as more fundamental ones owing to the still uncertain radiative and accretion physics (Gralla 2021) that obfuscate the actual predictions of the EHT; any analytical testing of the theories of gravity would certainly be subject to these (Afrin & Ghosh 2022b). Despite these uncertainties, the theoretical analysis, utilizing the EHT observational bounds, can serve as an initial probe of LQG which would call for further scrutiny with future more precise observations.

M87* bounds—Using an extensive library of ray-traced general-relativistic magnetohydrodynamic (GRMHD) simulations of black holes, the EHT has inferred a central compact mass of M87*, \( M_{\text{M87}*} = 6.5 \times 10^6 M_\odot \) which is consistent with the previous stellar dynamical measurements, and distance of \( d_{\text{M87}*} = 16.8 \) Mpc from earth (Akiyama et al. 2019a,b,c). For simplicity, we do not consider in our analysis, the possible uncertainties in the mass and distance measurements of the target black hole, as the EHT results already take into consideration the various uncertainties, to obtain the bounds on the observables. The characteristic features and dimensions of the observed image of the M87* is consistent with the expected appearance of Kerr black hole in GR, still the current uncertainty in the measurement of spin, inclination angle and the relative deviation of quadrupole moments do not completely rule out Kerr-like black holes in modified gravities (Akiyama et al. 2019a,b,c; Cardoso & Pani 2019) including those in LQG. But to be consistent with the dimensional expectations of the corresponding Kerr black hole’s shadow at a given spin \( a \), the results of the 2019 EHT drive can put constraints on the LQG parameter \( L_q \), as we shall explore here. Previously, the parameter space of the LIRBH-2 has been constrained with the shadow diameter of M87* at \( \theta_0 = 17^\circ \) and the wormhole region III has been ruled out (Brahma et al. 2021). We obtain the numerical value of the constraints with M87* so as to be able to compare with the constraints obtained with results of Sgr A*, at both \( \theta_0 = 90^\circ, 17^\circ \). Meanwhile, we also reaffirm the results of the earlier work (Brahma et al. 2021). Calibrating the size of the shadow of M87* with the ring diameter that the EHT has measured, yields the 1σ bound \( \delta = -0.01 \pm 0.17 \) (Akiyama et al. 2019c; Psaltis et al. 2020; Kocherlakota et al. 2021). The constant 1σ and 2σ contours delimits a finite parameter space of the LIRBHs, as can be seen from Fig. 4, and upper limits can be placed on \( L_q \). For LIRBH-1: \( L_q \in [0, 0.1687] \) within 2σ, \( L_q \in [0, 0.0686] \) within 1σ confidence levels at
0.1643) within $2\sigma$, $L_q \in [0.0.0643)$ within 1σ confidence levels at $\theta_0 = 17^\circ$. For LIRBH-2: $L_q \in [0.0.3904)$ within $2\sigma$, $L_q \in [0.0.1633)$ within 1σ confidence levels at $\theta_0 = 90^\circ$ and $L_q \in [0.0.3350)$ within $2\sigma$, $L_q \in [0.0.1253)$ within 1σ confidence levels at $\theta_0 = 17^\circ$. Thus we infer a conservative constraint $L_q \in [0.0.0686)$ for LIRBH-1 and $L_q \in [0.0.1253)$ for LIRBH-2 from the EHT results of M87*.

**Sgr A* bounds**—For both the LIRBHs, the parameter $L_q$ is yet to be constrained astrophysically via black hole shadows, which we intend to do here with the observational predictions of both M87* and Sgr A*. Though the M87* already provides a ground to constrain the LQG theories, the observations of Sgr A* would offer independent tests in a much higher curvature regime, as a consequence of $\mathcal{O}(M_{\text{Sgr-A*}}) \sim 10^6 M_\odot$ being smaller than $\mathcal{O}(M_{\text{M87}}) \sim 10^9 M_\odot$ by several orders (Vagnozzi et al. 2022); thus we can leverage the varied range of conditions that can be probed with the two different target black holes (Akiyama et al. 2022b).

Besides, for Sgr A*, the ratio between the mass $M_{\text{Sgr-A*}} = 4.0 \times 10^6 M_\odot$ and the predicted size of the bright emission is approximately $d_{\text{Sgr-A*}} = 8 \text{kpc}$ (Akiyama et al. 2022b), which could be used as priors from independent observations stellar dynamic observations of S0-2 star’s orbits by Keck telescopes and Very Large Telescope Interferometer (VLTI) (Do et al. 2019; Gravity Collaboration et al. 2019, 2021, 2022; Akiyama et al. 2022b) and the predicted size of Kerr shadow can be directly compared to the observations (Akiyama et al. 2022b).

From the observed image of Sgr A*, the EHT, besides obtaining diameter of the bright emission ring, has also measured the shadow diameter $d_{\text{sh}} = 48.7 \pm 7 \mu\text{as}$ and Schwarzschild shadow deviation $\delta = -0.08^{+0.09}_{-0.09}$ (VLTI), $-0.04^{+0.10}_{-0.10}$ (Keck) at 1σ confidence level; the image is consistent with the shadow of Kerr black hole in GR (Akiyama et al. 2022b). We model the Sgr A* as the LIRBHs respectively and impose the EHT inferred bounds on $d_{\text{sh}}$ (cf. Fig. 5) to find the constraints on the parameter $L_q$. We obtain the following constraints (i) for LIRBH-1: $L_q \in [0.0.1471)$ within $2\sigma$, $L_q \in [0.0.0761)$ within 1σ confidence levels at $\theta_0 = 90^\circ$, $L_q \in [0.0.1439)$ within $2\sigma$, $L_q \in [0.0.0690)$ within 1σ confidence levels at $\theta_0 = 90^\circ$ and $L_q \in [0.0.1415)$ within $2\sigma$, $L_q \in [0.0.0683)$ within 1σ confidence levels at $\theta_0 = 90^\circ$ and $\theta_0 = 0^\circ$ (and (ii) for LIRBH-2: $L_q \in [0.0.3027)$ within $2\sigma$, $L_q \in [0.0.1707)$ within 1σ confidence levels at $\theta_0 = 90^\circ$, $L_q \in [0.0.3374)$ within $2\sigma$, $L_q \in [0.0.1417)$ within 1σ confidence levels at $\theta_0 = 90^\circ$ and $L_q \in [0.0.2832)$ within $2\sigma$, $L_q \in [0.0.1260)$ within 1σ confidence levels at $\theta_0 = 0^\circ$. Next, we impose the bounds on $\delta$ (cf. Fig. 6) and obtain the EHT consistent parameter ranges as follows: (i) for LIRBH-1: $L_q \in [0.0.0968)$ within $2\sigma$, $L_q \in [0.0.0492)$ within 1σ confidence levels at $\theta_0 = 90^\circ$, $L_q \in [0.0.0913)$ within $2\sigma$, $L_q \in [0.0.0437)$ within 1σ confidence levels at $\theta_0 = 90^\circ$ and $L_q \in [0.0.0902)$ within $2\sigma$, $L_q \in [0.0.0423)$ within 1σ confidence levels at $\theta_0 = 0^\circ$ and (ii) for LIRBH-2: $L_q \in [0.0.2566)$ within $2\sigma$, $L_q \in [0.0.1170)$ within 1σ confidence levels at $\theta_0 = 90^\circ$, $L_q \in [0.0.2145)$ within $2\sigma$, $L_q \in [0.0.0955)$ within 1σ confidence levels at $\theta_0 = 90^\circ$ and $L_q \in [0.0.1834)$ within $2\sigma$, $L_q \in [0.0.0821)$ within 1σ confidence levels at $\theta_0 = 0^\circ$. We note that the bounds on the two observables $d_{\text{sh}}$ and $\delta$ are very similar, and thus comparable, as is evident from the obtained limits on the black hole parameters; comparing all the upper limits we infer the the upper bound on $L_q$ are: $L_q^{\text{max}} \in [0.0423, 0.0492]$ for LIRBH-1 and $L_q^{\text{max}} \in [0.0821, 0.1170]$ for LIRBH-2 as $\theta_0$ varies from 0° to 90°.

5. CONCLUSIONS

The EHT collaboration anticipated the shadow size of the supermassive black hole SgrA*, established on the basis of previous information on the mass-to-distance ratio of the black hole. The EHT results agree with the Kerr metric’s prediction, and there is no evidence for any violations of the theory of GR. The Sgr A* has the largest mass-to-distance ratio among available black holes, which makes Sgr A* the optimal target for testing the no-hair theorem.

We show the BH shadow of these LIRBHs are significantly different from those of Kerr black holes with the same spin and indicate the feasibility of testing the no-hair theorem by constraining the deviation parameter $L_q$ associated with LIRBHs, with EHT results of SgrA*. LIRBHs are modifications of the Kerr spacetime, e.g., of the null geodesic structure of the spacetime – most important for our purpose, and it leads to substantial changes in its properties that may be valuable to empirically test the no-hair theorem (Johannsen & Psaltis 2010). Indeed, we have considered two LIRBHs that resolve the singularity problem in GR and, in the absence of quantum effects ($L_q = 0$), go over to the Kerr metric. To construct the shadow, we solve the Hamilton-Jacobi equations and find that they are still separable and yield first-order photon geodesic equations.

Interestingly, the shadow silhouettes exhibit deviations in characteristic shape and size from the Kerr black hole shadows; the shadows are smaller and more distorted as $L_q$ increases. Further, there is a possibility of degeneracy between the shadow characteristics of LIRBHs with parameters $L_q$ and $\alpha$ and the shadows of Kerr black holes with some spin $\alpha_*$. We investigate this possibility further by constructing shadow observables $A$, $D$, $d_{\text{sh}}$ and $\delta$ that quantify the shadow dimensions and deformations. Using observables $A$ and $D$, we follow a simple contour intersection technique to estimate the quantum parameter $L_q$ and the black hole spin $\alpha$, which accord additional information about the quantum nature of gravity.

Further, modelling M87* as LIRBH-1 and LIRBH-2, and imposing the observational bounds on the $d_{\text{sh}}$ observable at different inclinations, we get the EHT con-
sistent range of the LQG parameters: $0 \leq L_q < 0.0686$ for LIRBH-1 and $0 \leq L_q < 0.1253$ for LIRBH-2. Intending to probe the LQG at a different curvature scale, we impose the observational bounds of Sgr A* on two of the shadow observables $d_{sh}$ and $\delta$, to find that the astrophysical allowed ranges of the LQG parameter become more constricted — $0 \leq L_q < 0.0423$ for LIRBH-1 and $0 \leq L_q < 0.0821$ for LIRBH-2 from the EHT results of Sgr A*. Thus with observational results of Sgr A*, we can put more stringent bounds on both the LIRBHs than those that we get from M87*.

That the Kerr black hole spacetime singularities represent a limitation of the classical theory of GR and are likely to be resolved in the LQG, e.g., LIRBHs are regular everywhere and go to the Kerr solution in the absence of quantum effects ($L_q = 0$). However, the LIRBH metrics do not result from a direct loop quantization of the Kerr spacetime, but these models furnish singularity resolution of Kerr black holes; thereby, LIRBHs can capture the effective regular spacetime description of LQG and hence can be suitable candidates for astrophysical black holes.

Many interesting avenues are amenable for future work; it will be intriguing to analyze accretion models in LIRBHs. Since we find that the LQG parameter profoundly influences shadow, it may have several astrophysical consequences, e.g., gravitational lensing. In the spirit of the no-hair theorem, one can consider a further detailed analysis of the two LIRBHs with different astronomical observations.

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