Heavy-meson decay constants from QCD sum rules

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Abstract. We sketch a recent sum-rule extraction of the decay constants of the heavy pseudoscalar mesons $D$, $D_s$, $B$, and $B_s$ from the two-point correlator of heavy–light pseudoscalar currents [1]. Our main emphasis lies on the control over all the uncertainties in the decay constants, related both to the input QCD parameters and to the limited accuracy of the method of sum rules. Gaining this control has become possible by application of our new procedure of extracting hadron observables based on a dual threshold depending on the Borel parameter. For the charmed-meson decay constants, we find

$$f_D = (206.2 \pm 7.3_{\text{OPE}} \pm 5.1_{\text{syst}}) \text{ MeV},$$
$$f_{D_s} = (245.3 \pm 15.7_{\text{OPE}} \pm 4.5_{\text{syst}}) \text{ MeV}.$$

For the beauty mesons, the decay constants turn out to be extremely sensitive to the precise value of the $\overline{\text{MS}}$ mass of the $b$-quark, $m_b(m_b)$. By requiring our sum-rule estimate to match the average of the lattice determinations of $f_B$, we extract the rather accurate value

$$m_b(m_b) = (4.245 \pm 0.025) \text{ GeV}.$$

Feeding this parameter value into our sum-rule formalism leads to the beauty-meson decay constants

$$f_B = (193.4 \pm 12.3_{\text{OPE}} \pm 4.3_{\text{syst}}) \text{ MeV},$$
$$f_{B_s} = (232.5 \pm 18.6_{\text{OPE}} \pm 2.4_{\text{syst}}) \text{ MeV}.$$

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INTRODUCTION

The extraction of ground-state decay constants from Shifman–Vainshtein–Zakharov sum rules [2] is a complicated problem: First, one should construct a reliable operator product expansion (OPE) for the Borel-transformed correlation function $\Pi(\tau)$, where $\tau$ denotes the Borel parameter, of two pseudoscalar heavy–light currents. We make use of the OPE for this correlator to three-loop accuracy [3], reshuffled in terms of the $\overline{\text{MS}}$ heavy-quark masses, in which case the perturbative expansion exhibits a reasonable convergence [4].

Second, even if the parameters of this OPE are known precisely, the knowledge of the truncated OPE for a correlator allows one to extract the characteristics of the bound state only with some error which reflects the intrinsic uncertainty of the method of QCD sum rules. Acquiring control over this systematic uncertainty poses a very subtle problem [5].
Recently, we have formulated a novel approach for extracting ground-state parameters from some correlator [6]. Let us briefly recall the most essential features of our approach: As consequence of the assumption of quark–hadron duality the ground-state contribution and the OPE with a cut applied at some effective continuum threshold $s_{\text{eff}}$ are related by

$$f_2^2 M_Q^4 e^{-M_Q^2 \tau} = \Pi_{\text{dual}}(\tau, s_{\text{eff}}(\tau)) \equiv \int ds e^{-s \tau} \rho_{\text{pert}}(s) + \Pi_{\text{power}}(\tau).$$  

(1)

Here, $\rho_{\text{pert}}(s)$ is the perturbative spectral density known to order $\alpha_s^2$; $\Pi_{\text{power}}$ describes the series of power corrections expressed in terms of condensates of increasing dimensions.

Of course, in order to extract the decay constant one has to fix the effective continuum threshold $s_{\text{eff}}$. Moreover, as is obvious from (1), $s_{\text{eff}}$ should be a function of $\tau$. Otherwise the $\tau$-dependences of the l.h.s. and the r.h.s. of (1) do not match each other. However, the exact effective threshold, corresponding to employing on the l.h.s. of (1) the exact hadron mass and decay constant, is clearly not known. The extraction of hadron parameters from the sum rule consists therefore in attempting (i) to find a good approximation to the exact continuum threshold and (ii) to acquire control over the accuracy of this approximation.

Let us introduce the dual invariant mass $M_{\text{dual}}$ and the dual decay constant $f_{\text{dual}}$ by the definitions

$$M_{\text{dual}}^2(\tau) = -\frac{d}{d\tau} \log \Pi_{\text{dual}}(\tau, s_{\text{eff}}(\tau)),$$

(2)

$$f_{\text{dual}}^2(\tau) = M_Q^4 e^{M_Q^2 \tau} \Pi_{\text{dual}}(\tau, s_{\text{eff}}(\tau)).$$

(3)

If the mass of the ground state is known, any deviation of this dual mass from the actual ground-state mass yields an indication of the excited-state contributions picked up by the dual correlator. Assuming a specific functional form of the effective continuum threshold and requiring the least deviation of the dual mass (3) from the known ground-state mass in the Borel window leads to a variational solution for the effective threshold. As soon as this latter quantity has been fixed, one calculates the decay constant from (3). The (naïve) standard assumption for the effective threshold is that it is just a $\tau$-independent constant. In addition to this approximation, we have considered polynomials in $\tau$: the reproduction of the dual mass improves considerably for the $\tau$-dependent quantities, which means that a dual correlator with $\tau$-dependent threshold isolates the ground-state contribution much better and is less contaminated by excited states than a dual correlator with the traditional $\tau$-independent threshold. As consequence, the accuracy of extracted hadron observables improves considerably. Experience gained from the study of potential models shows that the band of values obtained from the linear, quadratic, and cubic Ansätze for the effective threshold encompasses the true value of the decay constant [6]. It was also shown that the extraction procedures in quantum theory and QCD are quantitatively very similar to each other [7]. This talk summarizes our recent findings [1] on heavy-meson decay constants.
DECAY CONSTANTS OF THE D AND D_s MESONS

Applying our extraction procedures to the charmed mesons yields the following results:

\[ f_D = (206.2 \pm 7.3_{\text{OPE}} \pm 5.1_{\text{syst}}) \text{ MeV}, \] (4)

\[ f_{D_s} = (245.3 \pm 15.7_{\text{OPE}} \pm 4.5_{\text{syst}}) \text{ MeV}. \] (5)

The OPE error is obtained by the bootstrap allowing for variation of the QCD parameters (quark masses, \( \alpha_s \), condensates) in the relevant ranges. One observes a perfect agreement of our predictions with the lattice computations (Fig. 1). It should be emphasized that the \( \tau \)-dependent threshold constitutes a crucial ingredient for the successful extraction of the decay constant from the sum rule. Obviously, the standard \( \tau \)-independent approximation yields a much lower value for \( f_D \) which lies rather far from the data and the lattice results.

FIGURE 1. Our results for \( f_D \) and \( f_{D_s} \); the bootstrap analysis of the OPE uncertainties (a,b); comparison with both lattice determinations and experimental data (c,d). For a detailed list of references, see Ref. [1].
DECAY CONSTANTS OF THE $B$ AND $B_s$ MESONS

The values of the beauty-meson decay constants extracted from the sum rule (1) turn out to be extremely sensitive to the precise value of $m_b \equiv \overline{m}_b(m_b)$. Our results for these decay constants may be parameterized as follows:

$$f_B(m_b) = \left[ 193.4 - 37 \left( \frac{m_b - 4.245 \text{ GeV}}{0.1 \text{ GeV}} \right) \right] \text{MeV},$$  \hspace{1cm} (6)

$$f_{B_s}(m_b) = \left[ 232.5 - 43 \left( \frac{m_b - 4.245 \text{ GeV}}{0.1 \text{ GeV}} \right) \right] \text{MeV}. \hspace{1cm} (7)$$

Making use of the $b$-mass range $\overline{m}_b(m_b) = (4.163 \pm 0.016) \text{ GeV}$ [8] yields results barely compatible with the lattice determination (Fig. 2). Requiring the sum-rule result for $f_B$ to

![Graphs showing distributions of $f_B$ and $f_{B_s}$ values for different $m_b$ values with QCD-SR and Lattice data comparison.](image)

**FIGURE 2.** Our results for $f_B$ and $f_{B_s}$: the bootstrap analysis of the OPE uncertainties (a,b); comparison with both lattice determinations and experimental data (c,d). For a detailed list of references, see Ref. [1].
match the average of the lattice results yields a rather precise value of the $b$-quark mass:

$$\overline{m}_b(\overline{m}_b) = (4.245 \pm 0.025) \text{ GeV}. \quad (8)$$

For this value of $m_b$, our sum-rule estimates for the $B$- and $B_s$-meson decay constants are

$$f_B = (193.4 \pm 12.3_{(\text{OPE})} \pm 4.3_{(\text{syst})}) \text{ MeV}, \quad (9)$$

$$f_{B_s} = (232.5 \pm 18.6_{(\text{OPE})} \pm 2.4_{(\text{syst})}) \text{ MeV}. \quad (10)$$

**CONCLUSIONS**

We presented the results of our recent analysis of the heavy-meson decay constants from the correlator of pseudoscalar currents [1]. Our special emphasis was laid on the study of all uncertainties in the extracted value of the decay constant, viz., on the OPE uncertainty related to the not precisely known QCD parameters and on the intrinsic uncertainty of the method related to a limited accuracy of the extraction procedure. According to our recent findings, the accuracy of the sum-rule predictions may be considerably improved and the intrinsic uncertainties of hadron parameters may be probed by allowing for $\tau$-dependent Ansätze for the effective continuum threshold. The parameters of this effective threshold can be fixed by minimizing the deviation of the dual mass from the known meson mass in the window. This strategy has now been applied to the decay constants of heavy mesons.

Our main results are as follows:

1. The analysis of charm mesons unambiguously demonstrates that the application of the Borel-parameter-dependent effective threshold leads to two essential improvements:
   (i) the accuracy of decay constants extracted from the sum rule is considerably improved;
   (ii) it has become possible to derive realistic systematic uncertainties and to reduce their values to the level of a few percent. By application of our novel extraction procedures the results from QCD sum rules have been brought into a perfect agreement with the findings from lattice QCD and experiment.

2. The $B$ and $B_s$ decay constants are extremely sensitive to the precise value of $\overline{m}_b(\overline{m}_b)$. Therefore, no reasonable predictions for the decay constants may be extracted unless the $b$-quark mass is known with very high accuracy. On the other hand, the strong sensitivity of these decay constant to the $b$-quark mass opens the promising possibility to extract the $b$-quark mass if the decay constant is known. Following this line and matching the results from QCD sum rules for $f_B$ to the average of the lattice evaluations allows us to obtain a rather accurate estimate of the $b$-quark mass. Our value is in good agreement with several lattice results but, interestingly, it does not overlap with the recent accurate determination of $\overline{m}_b(\overline{m}_b)$ reported in [8] (see also [1]). Clearly, this issue requires further investigation.

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