Effective Higgs-to-Light-Quark Coupling
Induced by Heavy Quark Loops

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We study the coupling arising via perturbative QCD of zero 4-momentum Higgs bosons to light quarks inside the nucleon. Qualitative comparison with the results obtained from one-loop order low energy theorems for the Higgs-nucleon interaction suggests the existence of a dynamical light-quark mass which falls off with momentum. Quantitative comparison leads to an estimate of $\alpha_s$ at very low $q^2$ quite near unity.

1 Introduction

The coupling of the zero 4-momentum Higgs to nucleons is characterized by a mass of order $210\text{MeV}$. Obviously, a direct coupling of the Higgs to a light quark inside nucleon is not sufficient to generate this value. The effective interaction of the Higgs with nucleons is known to be enhanced via coupling of the nucleon to a heavy quark triangle [Fig.1] through gluon exchanges. A Higgs boson low energy theorem has been applied in order to estimate the effective coupling for this interaction. This theorem basically relates the matrix elements $M(A \rightarrow B)$ and $M(A \rightarrow B + \text{Higgs})$

$$\lim_{p_{\text{Higgs}} \rightarrow 0} M(A \rightarrow B + \text{Higgs}) = \frac{N_h\alpha_s^2}{3\pi\langle\phi\rangle} \frac{\partial}{\partial \alpha_s} M(A \rightarrow B),$$

(1)

where $N_h$ is the number of heavy quarks. If $A$ and $B$ are both identified with the nucleon, then the physical matrix element $M(A \rightarrow B)$ is the nucleon mass, which in the chiral limit is proportional to $\Lambda_{QCD}$. Since $\alpha_s = (4\pi)/\left[9\ln(p^2/\Lambda_{QCD}^2)\right]$ to one loop order, we find that

$$\lim_{p_{\text{Higgs}} \rightarrow 0} M(q \rightarrow q + \text{Higgs}) \equiv g_{\text{HNN}}|_{\text{induced}} = \frac{2N_hm_N}{27\langle\phi\rangle},$$

(2)

leading to $N_h = 3$ an effective interaction characterized by a mass of about $210\text{MeV}$. This same argument can be applied to the (constituent-) quarks

$^a$Deceased
within the nucleon. In the chiral limit, the constituent-quark mass becomes the dynamical mass associated with the chiral noninvariance of the QCD vacuum. The matrix element \( M(A \rightarrow B) \) is just this dynamical mass, \( m_{\text{dyn}} \), which (like the nucleon mass \( m_N \) in the chiral limit) is necessarily proportional to \( \Lambda_{\text{QCD}} \). Consequently, one can repeat the derivation leading to (2) for the constituent-quark Yukawa interactions, and find the same result except for the replacement of \( m_N \) with \( m_{\text{dyn}} \), corresponding to a Yukawa-interaction mass of order 70 MeV.

\[
q=0 \quad k-p \quad k-p \quad k-p
\]

**Figure 1:** The heavy quark triangle.

In this work we would like to estimate the Higgs coupling to the nucleon by performing an equivalent lowest order perturbative QCD calculation. We study this coupling by coupling directly the heavy quark triangle to the light quarks inside the nucleon [Fig.2] and we incorporate all non-perturbative QCD effects into a dynamical mass function for the light quark. Therefore in order to determine the effective coupling we need to evaluate the two loop diagram in Fig.2. As a first approximation, we use a constant dynamical mass for the light quark. We subsequently utilize a more realistic function of the light quark momentum.

The heavy quark triangle \( \delta \) [Fig.1] for heavy-quark mass \( M \) and gluon-momentum \( p \) couples the zero 4-momentum Higgs to two gluons, and is given by the following tensor

\[
I_{\mu \nu}^{ab} = \frac{2 \alpha_s}{\pi \langle \phi \rangle} \delta^{ab} (p_\mu p_\nu - p^2 g_{\mu \nu}) \left[ \frac{M^2}{p^2} + \frac{2M^4}{p^2 \sqrt{1 - 4M^2/p^2}} \ln \left| \frac{\tau_+ - \tau_-}{\tau_+ + \tau_-} \right| \right], \tag{3}
\]

where \( \tau_+ = 1 + \sqrt{1 - 4M^2/p^2} \), \( \alpha_s = g_s^2/(4\pi) \), and \( \langle \phi \rangle \) is the vacuum expectation value of the Higgs field. \( I_{\mu \nu}^{ab} \) is completely transverse which ensures gauge-parameter independence of light-quark Yukawa couplings induced via Fig.2. The expression in square brackets in (3) goes to \(-1/6\) in the heavy-quark limit \( (M^2 \gg p^2) \). For very large gluon momenta \( (p^2) \) the expression
in brackets vanishes, demonstrating that the heavy quark triangle serves as a cut-off in the integrals over the gluon momenta of Fig.2. This property ensures that the triangle of Fig.1 will not lead to divergent renormalization-dependent results when incorporated into Fig.2.

2 The Effective Coupling to a Light Quark with Constant Dynamical Mass

The Higgs coupling to light quarks induced via Fig.2 may be expressed as

\[ \Sigma_{\text{ind}}(k^2) = \Sigma_0(k^2) + \kappa \Sigma_1(k^2). \]  

(4)

To estimate \( \Sigma_0(k^2) \) and \( \Sigma_1(k^2) \), we first evaluate the two loop diagram in Fig.2 with a constant dynamical mass. We find it convenient to express the heavy quark triangle \( \mathcal{B} \) in the following form:

\[ I_{\mu\nu}^{ab}(p) = \frac{4\alpha_s}{\pi^2\langle \phi \rangle} \delta^{ab}(p_{\mu}p_{\nu} - p^2 g_{\mu\nu}) \int_0^1 dy \frac{y}{p^2 - \frac{M^2}{y(1-y)}}. \]  

(5)

We find that

\[ \Sigma_0(k^2) = \frac{4\alpha_s^2 m_{\text{dyn}}}{\pi^2\langle \phi \rangle} \int_0^1 y dy \int_0^1 dx_1 \int_0^{1-x_1} dx_2 L_0^{-1}, \]  

(6)

\[ \Sigma_1(k^2) = \frac{4\alpha_s^2}{\pi^2\langle \phi \rangle} \int_0^1 y dy \int_0^1 dx_1 \int_0^{1-x_1} dx_2 (1 - x_1 - x_2) \times \left[(3x_2 - 1)L_0^{-1} + k^2x_2^2(1 - x_2)M^{-2}L_0^{-2}\right], \]  

(7)
where
\[
L_0 = x_1 [y(1 - y)]^{-1} + k^2 M^{-2} x_2 (x_2 - 1) + m^2 M^{-2} x_2.
\]  
(8)

In the limit where \( m_{\text{dyn}} \ll M \)
\[
\Sigma_0(0) \to 2\alpha_s^2 m_{\text{dyn}} \left[ \ln(M/m_{\text{dyn}}) + 5/6 \right] / (3\pi^2 \langle \phi \rangle),
\]
(9)
\[
\Sigma_1(0) \to -\alpha_s^2 / (18\pi^2 \langle \phi \rangle),
\]
(10)
which indicates that \( \Sigma_0(0) \) in the limit of very heavy masses in the triangle depends logarithmically on the heavy quark mass. On the other hand \( \Sigma_1(0) \) is independent of the heavy quark mass in this limit. These properties also apply when the light quark has nonzero momentum \( k \) [Fig.’s 3 and 4]. Heavy flavours have different contributions in \( \Sigma_0 \), but have almost the same contributions in \( \Sigma_1 \). For a dynamical mass of \( 300\,\text{MeV} \), we have \( \Sigma_0(0) \approx 268\alpha_s^2 \text{MeV}/\langle \phi \rangle \) and \( \Sigma_1(0) \approx -0.05\alpha_s^2/\langle \phi \rangle \), and if we identify the \( k \) term with the dynamical mass, we have \( \Sigma_{\text{ind}}(0) \approx 253\alpha_s^2 \text{MeV}/\langle \phi \rangle \). This result corresponds to an induced Yukawa-coupling mass of \( 70\,\text{MeV} \), consistent with the one-loop low-energy theorem result (2), provided \( \alpha_s \approx 0.53 \). However, the result clearly is dependent through (9) upon the heavy quark mass \( M \), inconsistent with the low-energy-theorem result (2).

3 The Effective Coupling to a Light Quark with a Dynamical Mass Function

We now consider a more realistic case in which the light quark has a momentum-dependent dynamical mass, and is therefore more sensitive to infrared dynamics corresponding to the low energy region for \( \alpha_s \). We use the dynamical mass function proposed by Holdom (5)
\[
\Sigma_{\text{QCD}}(p^2) = (A + 1)\Lambda^3 / (A\Lambda^2 - p^2),
\]
(11)
where \( A \) is a constant and \( \Lambda \) has the dimensions of energy and is the low energy mass scale. This expression has been successfully tested with different physical parameters at low energies. \( \Sigma_{\text{QCD}}(k^2) \) We find that
\[
\Sigma_0(k^2) = \frac{4\alpha_s^2 M^2}{3i\pi^4 \langle \phi \rangle} \int_0^1 y \, dy \int d^4 p \times \frac{\gamma_\mu (A + 1) \Lambda^3 \left[ A\Lambda^2 - (p - k)^2 \right] \gamma_\nu (p^\mu p^\nu - p^2 g^\mu\nu)}{\left[ (p - k)^2 [A\Lambda^2 - (p - k)^2]^2 - (A + 1)^2 \Lambda^6 \right] p^4 \left( p^2 - \frac{M^2}{y(1-y)} \right)},
\]
(12)
Figure 3: Contributions of top, bottom, and charm quarks to $\Sigma_0(k^2)$ in units of $\alpha_s^2/\langle \phi \rangle$ versus $(k^2)^{1/2}$ with a constant dynamical mass of 300 MeV for the light quark. The top quark has the largest contribution, followed by bottom and charm quark contributions respectively.

Figure 4: Contributions of top, bottom, and charm quarks to $\Sigma_1(k^2)$ in units of $\alpha_s^2/\langle \phi \rangle$ versus $(k^2)^{1/2}$ with a constant dynamical mass of 300 MeV for the light quark. The top quark has the largest contribution, followed by bottom and charm quark contributions respectively.
\[ k \Sigma_1(k^2) = \frac{4 \alpha_s^2 M^2}{3 \pi^2 \langle \phi \rangle} \int_0^1 y dy \int d^4 p \]

\[ \times \frac{\gamma_\mu (\not k - \not k)}{(p - k)^2 [AA^2 - (p - k)^2] - (A + 1)^2 \Lambda^6} p^\mu \left( p^2 - \frac{M^2}{y(1 - y)} \right). \]  

(13)

For a nonzero light-quark momentum \( k \), these loop integrals are quite complicated. However, for \( k^2 = 0 \), the quark's Lagrangian-mass shell in the chiral limit, we find that

\[ \Sigma_0(0) = 4 \alpha_s^2 M^2 (A + 1) \Lambda^3 \int_0^1 y dy \int_0^\infty dx (AA^2 + x) \]

\[ \times \left[ [x(AA^2 + x)^2 + (A + 1)^2 \Lambda^6] (x + M^2/[y(1 - y)]) \right]^{-1}, \]  

(14)

\[ \Sigma_1(0) = \frac{2 \alpha_s^2 M^2 (A + 1)^2 \Lambda^6}{\pi^2 \langle \phi \rangle} \int_0^1 y dy \int_0^\infty dx (3x^2 + 4AA^2 x + A^2 \Lambda^4) \]

\[ \times \left[ [x(AA^2 + x)^2 + (A + 1)^2 \Lambda^6]^{-2} (x + M^2/[y(1 - y)]) \right]^{-1}. \]  

(15)

Choosing \( A = 2 \) and \( \Lambda = 317 MeV \), we find that \( \Sigma_0(0) = 62 \alpha_s^2 MeV/\langle \phi \rangle \) and \( \Sigma_1(0) = 0.051 \alpha_s^2/\langle \phi \rangle \), in which case \( \Sigma_{ind} \approx (62 \rightarrow 77) MeV \) depending upon whether we consider \( k \) on the dynamical \( O(300 MeV) \) mass shell or assign \( k \) a value of zero (consistent with the vanishing Lagrangian mass in the chiral limit). In either case, however, we obtain a result that is independent of the heavy quark mass, as predicted by the low energy theorem. Quantitative agreement with (2) is obtained provided \( \alpha_s \approx 0.95 \rightarrow 1.06 \). The near unity value of \( \alpha_s \) is consistent with infrared expectations, particularly criticality arguments for chiral symmetry breaking as well as the anticipated freezeout of the strong coupling near unity discussed by Mattingly and Stevenson.

The momentum dependence of the effective Yukawa interaction can be studied via the following approximation in the denominator of the momentum integrals (12) and (13)

\[ \Sigma_{QCD}(k - p)^2 \rightarrow \Sigma_{QCD}(0), \]  

(16)

an approximation providing a lower bound for the effective Yukawa interaction. Making this substitution, we find that

\[ \Sigma_0(k^2) = \frac{-4(A + 1) \alpha_s^2 M^2}{\Lambda^2 \langle \phi \rangle} \int_0^1 y dy \int_0^1 dx_1 \int_0^{1 - x_1} dx_2 \int_0^{1 - x_1 - x_2} dx_3 L^{-2}, \]  

(17)
\[ \Sigma_1(k^2) = \frac{8\alpha^2 M^2}{N^2 \pi^2 < \phi> \int_0^1 y dy \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \int_0^{1-x_1-x_2} dx_3} \times (1 - x_1 - x_2 - x_3) \left[ (1/3)F_0 A^{-4} L^{-3} - (1/3)F_1 A^{-2} L^{-2} + F_2 L^{-1} \right], \quad (18) \]

where
\[ L = \frac{M^2 x_3}{A^2 y(1-y)} - \frac{k^2}{A^2} (x_1 + x_2)(1 - x_1 - x_2) + \left[ \frac{A + 1}{A} \right]^2 x_1 + A x_2, \quad (19) \]
\[ F_0 = 3k^2(x_1 + x_2)^2 \left[ k^2 - A A^2 + 3k^2(x_1 + x_2)(x_1 + x_2 - 1) + A A^2 (x_1 + x_2) - k^2(x_1 + x_2)^3 \right], \quad (20) \]
\[ F_1 = -\frac{3}{2}(A A^2 - k^2) + (x_1 + x_2) \left[ (9/2)(AA^2 - k^2) - 12(x_1 + x_2)^2 k^2 + 21(x_1 + x_2)k^2 - 6k^2 \right], \quad (21) \]
\[ F_2 = -6(x_1 + x_2) + 3. \quad (22) \]

In this “lower-bound” approximation, the momentum dependence of \( \Sigma_0(k^2) \) and \( \Sigma_1(k^2) \) [Fig.’s 5 and 6] confirm the heavy flavour independence of the induced coupling in the heavy quark limit (the contributions of top and bottom quarks lead to almost identical curves). As evident from these figures, there is a large enhancement in the coupling around \( k^2 \approx (500MeV)^2 \). This is due to the onset of branch cut singularity beginning when \( k^2 = [\Sigma_{QCD}(k^2)]^2 \) which occurs when \( \sqrt{k^2} \approx 530MeV \). However, caution must be used in attempting a physical interpretation for this branch cut, because the form chosen for \( \Sigma_{QCD} \) is not likely to be applicable for Minkowskian \( k^2 \) near or past the singularity at \( k^2 = AA^2 \).

The “lower-bound” approximation indicates only a soft dependence of \( \Sigma_0 \) on \( k^2 \) near the origin [Fig. 5]. This can be tested by considering the Taylor series for \( \Sigma_0(k^2) \) explicitly:
\[ \Sigma_0(k^2) = \Sigma_0(0) + \frac{1}{2} \left[ \frac{\partial^2}{\partial k^2} \Sigma_0(k^2) \right]_{k=0} k^2 + O(k^2)^2 \]

\[ \approx \Sigma_0(0) + \frac{4\alpha^2 M^2 (A + 1) A^3}{\pi^2 <\phi>} k^2 \int_0^1 y dy \int_0^\infty dx \left[ -x^6 - 2A A^2 x^5 - 3A^2 A^4 x^4 + A^6 x^3 [7(1 + A)^2 - A^3] + 12A(1 + A) A^2 A^8 x^2 + 6A^2(1 + A)^2 A^{10} x + (1 + A)^2 \right] \times \left[ \Sigma_1(k^2) \right]_{k=0} \left[ x + M^2/|y(1-y)| \right] \left[ x(A A^2 + x^2 + A^6(1 + A)^2)^3 \right]^{-1} \quad (23) \]
Figure 5: Lower-bound-approximation contributions of top, bottom, and charm quarks to \( \Sigma_0(k^2) \) in units of \( \alpha_s^2/\langle \phi \rangle \) versus \((k^2)^{1/2}\) with the dynamical mass function \( \Sigma_{QCD} \) for the light quark. The top quark has the largest contribution, followed by bottom and charm quark contributions respectively.

Figure 6: Lower-bound-approximation contributions of top, bottom, and charm quarks to \( \Sigma_1(k^2) \) in units of \( \alpha_s^2/\langle \phi \rangle \) versus \((k^2)^{1/2}\) with the dynamical mass function \( \Sigma_{QCD} \) for the light quark. The top quark has the largest contribution, followed by bottom and charm quark contributions respectively.
The next-to-leading correction in (23) leads to a linear approximation to $\Sigma_0(k^2)$ whose slope is consistent with the lower bound approximation as $k^2 \to 0$, as shown in Fig.7. Finally, we note that the induced coupling on the dynamical Euclidean mass shell of the light quark at $k^2 = -\Lambda^2$ is found from (17) and (18) to be $\Sigma_{ind}(-\Lambda^2) \geq 48\alpha_s^2\text{MeV}/\langle \phi \rangle$, a result in agreement with the low energy theorem (1) provided $\alpha_s \leq 1.2$.

4 Discussion

By using a realistic dynamical mass function (11), we obtain a heavy-quark-induced Yukawa interaction with nucleonic constituent quarks that is independent of the heavy quark mass, as predicted by the Higgs low-energy theorem. Quantitative agreement with the low-energy theorem result is obtained provided the infrared value of $\alpha_s$ is $0.95 \to 1.06$, a range anticipated from criticality arguments for chiral symmetry breaking and from the expected freezeout of the strong coupling near unity.

We reiterate that the low-energy theorem follows from 1-loop corrections to the gluon propagator, motivating our explicit comparison to the lowest order graph [Fig.2] in perturbative QCD. The large size of $\alpha_s$ that follows from such a comparison suggests a need to consider three-loop diagrams as well. However, there are reasons to believe that the relevant expansion pa-
rameter near criticality is $\alpha_s N_c/(4\pi)$, perhaps providing some suppression of higher-order contributions.

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