Study on Freeway Vehicle Route Selection Based on Cumulative Prospect Theory

Liuqing Ding¹, Xianyu Wu¹*, Qingyu Xiao²

¹ School of Traffic and Transportation, Beijing Jiaotong University, Beijing, 100044, China
² Hunan Institute of Communications Sciences, Changsha, Hunan, 410007, China

*Corresponding author’s e-mail: 180120783@bjtu.edu.cn

Abstract. Taking the freeway network as the research object, taking the vehicle route selection with toll road as the main research content, referring to the idea of cumulative prospect theory, selecting the travel time and cost as the reference points, taking the prospect value based on value function and cumulative decision function as the decision basis, the route selection model is constructed. In the part of example analysis, a topological model is constructed according to the freeway network of a province, and the actual toll standard of the road network is taken as the initial parameter value to verify the proposed route selection model. The results show that the setting of parameters has an important impact on the prospect value, and the proposed route selection method for toll roads is effective and feasible. In addition, the influence of rate on route selection is analyzed, and it is concluded that on the route with low travel risk, truck 1 and truck 6 are greatly affected by the toll rate, which can provide decision-making reference for the transportation management department.

1. Introduction

The uneven distribution of freeway traffic volume in time and space leads to the waste of resources in some roads, and the pressure of road congestion in adjacent provinces is great. The cost and time of passage are the main factors affecting the passage of vehicles. Most scholars generally apply the traditional route selection model in the calculation model of road traffic volume, which mainly focuses on the expected utility theory, assuming that the road traveler is completely rational and always optimizes the benefit. However, in the process of travel in an uncertain environment, the route selection made by travelers is related to their own travel habits, risk awareness, their own preference experience and the travel stage. Therefore, it is of great significance to analyze and study the practical route selection behavior for road network space-time scheduling, road network flow travel guidance and balance adjustment.

Behavioral economics shows that people's travel decisions under uncertain conditions are characterized by bounded rationality. CPT uses people in reality as the research object to reveal the actual decision-making mechanism of individuals. Many scholars apply prospect theory to research, Fang Chengwu et al. apply CPT to the hesitant fuzzy TOPSIS multi-attribute decision-making method, and verify the feasibility of the method in the decision-making process[1]. Wang Tao et al. introduced the reward and punishment inferior operator and gray based on CPT[2]. The correlation coefficient is used to process the evaluation indicators of multiple retailers. The results of the calculation examples show that the comprehensive prospect value can better reflect the real evaluation of users. Hua Chengxu
applied the cumulative prospect theory to the prediction of college students’ travel mode selection, and concluded that the cumulative prospect theory is more suitable for the prediction of college students’ travel mode[3]. Related research shows that CPT has a certain degree of adaptability in the application of various fields. The choice of traffic travel path is a typical uncertainty and multi-attribute decision-making problem, so it is considered to be extended to the study of freeway path selection.

Vehicle routing behavior has an important impact on the measurement of road flow, but there are relatively few studies on freeway vehicle routing in uncertain environments. Considering that travel time and toll fees have a greater impact on route selection, the CPT model is extended to two reference points, and the sensitivity analysis of the model parameters is carried out, and the route selection of different types of trucks at different rates affecting road sections is explored.

2. Model building

The content of this section is mainly based on the principle and content of CPT to construct a freeway vehicle routing model. CPT is mainly divided into two stages, the editing stage and the evaluation stage. The editing stage is to determine a reference point according to the actual situation; the evaluation stage is to evaluate the gains and losses of the editing stage.

2.1. Determination of the value function

The relative value $\Delta(x)$ of the results of different choices $x_i$ relative to the reference point $x_0$ is used as the utility index when calculating the value function. Define the result that is better than the reference point as income ($\Delta(x) \geq 0$), and the result that is worse than the reference point as the income loss ($\Delta(x) < 0$) is the two parts of the traveller’s perceived value, referring to the definition of the value function [4] proposed by Tversky and Kahneman, in this article, the value function $g(\Delta(x))$ is defined as:

$$
\Delta(x) = x_i - x_0, \quad \text{(1)}
$$

$$
g(\Delta(x)) = \begin{cases}

\frac{(-\Delta(x))^\alpha}{\lambda} & \text{if } \Delta(x) \geq 0, \\
-\frac{\Delta(x)^\beta}{\lambda} & \text{if } \Delta(x) < 0,
\end{cases} \quad \text{(2)}
$$

In the formula: parameters $\alpha$ and $\beta$ are risk attitude coefficients. The larger their values indicate that decision makers tend to take risks ($0 < \alpha, \beta < 1$), where $\alpha$ is the gain sensitivity coefficient, $\beta$ is the loss sensitivity coefficient, and $\lambda$ is the loss avoidance coefficient. In the calculation of the transit time value function, the transit time is converted into a cost through a conversion factor. Usually $\alpha = \beta = 0.88, \lambda = 2.25$.

2.2. Determination of subjective probability function

This article adopts the form of subjective probability function[5]:

$$
w^+(p) = \frac{p^\gamma}{\left[ p^\gamma - (1 - p^\gamma) \right]^\gamma}, \quad \text{(3)}
$$

$$
w^-(p) = \frac{p^\delta}{\left[ p^\delta - (1 - p^\delta) \right]^\delta}, \quad \text{(4)}
$$

In the formula: $p$ is the true probability value; $w(p)$ is the subjective probability weight; $\gamma(0 < \gamma < 1)$ is the parameter. Normally $\gamma = 0.61$ and $\delta = 0.69$. 

2.3. Determination of reference points

The endogenous method is used to determine the reference point [6], that is, the minimum value of the OD budget for the travel time (cost) of all paths. Under the environment where the reliability level is $\rho$, it is assumed that the path’s perceived travel time (toll) follows a normal distribution.

$\pi^{\alpha}_{k,g} \min = \min \{ \pi^{\alpha}_{k,g} | \pi^{\alpha}_{k,g} \leq \pi^{\alpha}_{k,g0} \} \geq \rho, \forall k \in K$,

$\pi^{\alpha}_{k,g0} = \min \{ \pi^{\alpha}_{k,g \ min} \}$,

The calculation process of $\pi^{\alpha}_{k,g \ min}$ is as follows:

$\pi^{\alpha}_{k,g \ min} = \min \{ \phi^{-1}(p) \times \sigma(\pi^{\alpha}_{k,g}) + E(\pi^{\alpha}_{k,g}) \}$,

In the formula: $E(\pi^{\alpha}_{k,g})$ is the mean value of the transit time (toll) of the path $k$, $\sigma(\pi^{\alpha}_{k,g})$ is the standard deviation, and $\pi^{\alpha}_{k,g0}$ is the reference point of the transit time (toll).

2.4. Prospect value

2.4.1. Calculation of the prospect value of passage time

The transit time of the freeway network segment adopts the BPR (Bureau of Public Road) function of the FBI, namely:

$t_{ag}(x_a, y_{ag}) = t_{a0}^0 \left[ 1 + \alpha \left( \frac{x_a}{C_a} \right)^\beta \right], \forall a \in A$,

In the formula: $t_{a0}^0$ is the transit time under the condition of free flow of the road section; $x_a$ is the flow of the road section; $C_a$ is the actual capacity of the road section; $\alpha$ and $\beta$ are parameters, usually $\alpha = 0.15$ and $\beta = 4$ are used. Considering the degradation of the road network, the capacity of the road section will attenuate with the change of road conditions, so $C_a$ is a variable. Assuming on $[\theta, \overline{C_a}, \underline{C_a}]$ that $C_a$ obeys a uniform distribution, its upper bound is the design capacity $\overline{C_a}$ of the section $a$, and the lower bound is the product of the design capacity $\underline{C_a}$ of the section $a$ and the degradation $\theta$ factor. Based on the above assumptions, the mean value and variance of the travel time of vehicle $g$ on road section $a$ can be calculated and derived, respectively:

$E(t_a) = t_{a0}^0 + \alpha \overline{C_a} x_a = \left( \frac{1 - \theta^{1-2\beta}}{\overline{C_a} (1 - \theta)(1 - \beta)} \right), \forall a \in A$,

$\text{var}(t_a) = \alpha^2 t_{a0}^2 \overline{C_a^2} \left( \frac{1 - \theta^{1-2\beta}}{\overline{C_a} (1 - \theta)(1 - 2\beta)} \right) - \left( \frac{1 - \theta^{1-\beta}}{\overline{C_a} (1 - \theta)(1 - \beta)} \right), \forall a \in A$,

Assuming that there are $k$ paths between OD pair $rs$ and $m + n + 1$ possible travel time $\pi^{\alpha}_{k,g-m} < \cdots < \pi^{\alpha}_{k,g0} < \cdots < \pi^{\alpha}_{k,g+n}$, the probability of their occurrence is $p_m, \cdots, p_n$ and denoted by $x_i = (\pi^{\alpha}_{k,g-m}, \cdots, \pi^{\alpha}_{k,g+n})$ and $p_i = (p_m, \cdots, p_n)$ respectively.

The cumulative prospect value of travel time is:

$V^{\alpha}_{k,g} = \sum_{N=m}^{n} \delta_N g(\pi^{\alpha}_{k,g}) + \sum_{N=0}^{n} \delta_N g(\pi^{\alpha}_{k,g})$, 

where $\delta_N g(\pi^{\alpha}_{k,g})$ represents the prospect value of travel time $\pi^{\alpha}_{k,g}$.
Where: $v_k$ is the cumulative prospect value of $k$ path; $\delta_a$ is the subjective probability weight value of the corresponding possible outcome ($\delta_a^-$ is loss, $\delta_a^+$ is profit); $r_{k,g}$ is the travel time of vehicle $g$ on path $k$ between $rs$ OD pair.

$$
\delta_N^- = w^-(p_{a+1} + \cdots + p_N), 0 \leq N \leq n, \quad (12)
$$

$$
\delta_N^+ = w^+(p_{a-1} + \cdots + p_a), -m \leq N < 0, \quad (13)
$$

### 2.4.2. Calculation of the prospect value of tolls

The prospect value of tolls ($v_{k,g}^{rs}$) can be expressed as:

$$
v_{k,g}^{rs} = w(g)\left(\sigma_{k,g}^{rs}\right), \quad (14)
$$

In the formula: $v_{k,g}^{rs}$ is the prospect value of route $k$; $p_g^{rs}$ is the vehicle composition ratio between OD pair $rs$; $w(g)$ is the subjective probability weight value of the corresponding possible outcome ($w^-(p_g^r)$ is loss, $w^+(p_g^r)$ is profit); $\sigma_{k,g}^{rs}$ is the toll of vehicle $g$ on path $k$ between OD pair $rs$.

### 2.4.3. Comprehensive prospect value calculation

Introducing the risk preference parameter, the comprehensive prospect value of the path can be expressed as:

$$
v_{k,g}^{rs} = \chi_g v_{k,g}^{rs} + \left(1 - \chi_g\right)v_{k,g}^{rs}, \quad (15)
$$

In the formula, the value range of parameter $\chi_g$ is $0 \leq \chi_g \leq 1$, and the larger $\chi_g$ indicates that users pay more attention to transit time.

### 3. Example Analysis

Construct a topology model based on a provincial freeway network, and use the actual toll standard of the road network as the initial parameter value. Take the truck on the road network as an example to verify the vehicle routing model. The road network topology model is shown in figure 1.

![Figure 1. The simulation road network.](image)

Define the initial parameters of the road section, as shown in Table 1. To simplify the calculation, assuming that the toll rates are the same on all road sections, the toll rates (yuan) of each vehicle type are respectively $\phi_{a1} = 0.4$, $\phi_{a2} = 0.57$, $\phi_{a3} = 1.15$, $\phi_{a4} = 1.25$, $\phi_{a5} = 1.35$, $\phi_{a6} = 1.45$. Select OD pair (6, 16) as the research object, there are 4 feasible paths, path 1 is 6-12-13-16, path 2 is 6-11-14-16, path 3 is 6-11-12-13-16, path 4 is 6-12-11-14-16.

| Section | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---------|---|---|---|---|---|---|---|
| Starting point | 6-12 | 6-11 | 11-12 | 12-13 | 13-16 | 14-16 | 11-14 |
| Section length $l_s$/km | 210.90 | 169.60 | 110.10 | 94.30 | 197.20 | 170.30 | 183.90 |
Free flow time $t_0^0$/h 2.11 1.70 1.10 0.94 1.97 1.70 1.84

Assuming $\alpha = \beta = 0.88$, $\lambda = 2.25$ in the value function; $\gamma = 0.61$, $\delta = 0.69$ when calculating the weight function; the traveller’s requirement for travel reliability $\rho = 0.6$; the capacity degradation coefficient $\theta = 0.6$; the confidence level of the travel time in a certain interval is $p = 98\%$. Divide the confidence interval into the small areas of $K = 10$. The preference $\chi_g = 0.5$ of transit time and toll cost, the calculation result of the prospect value is shown in table 2.

Table 2. Route prospect value.

| Truck | Route 1 | Route 2 | Route 3 | Route 4 |
|-------|---------|---------|---------|---------|
| 1     | 3.1052  | 1.0553  | -1.7975 | -13.0441|
| 2     | 3.0396  | 1.2726  | -1.4587 | -11.5901|
| 3     | 3.3031  | 1.8555  | -1.0315 | -10.6909|
| 4     | 4.5873  | 2.5241  | -1.5285 | -15.7261|
| 5     | 1.4770  | 0.7699  | -0.4758 | -4.0959 |
| 6     | 9.4492  | 4.5781  | -3.9678 | -37.3935|

Analysing the path prospect data in table 2 shows that the prospect values of different vehicles on different paths are different, the prospect values of path 3 and path 4 are lower than path 1 and path 2, truck 6 least willing to choose route 4. Different types of vehicles have different road tolls and different requirements for reliability, so they have different risk preferences, which are reflected in the different prospects of their route selection.

In order to verify the influence of the toll rates on the route selection, 6 different charging rates are selected, as shown in table 3. Figure 2 shows the changes in the prospect values of vehicles of different models in the case of tariff fluctuations.

Table 3. Different toll rates.

| $\varphi$/yuan | 1 | 2 | 3 | 4 | 5 | 6 |
|---------------|---|---|---|---|---|---|
| Truck 1       | 0.20 | 0.30 | 0.40 | 0.45 | 0.50 | 0.55 |
| Truck 2       | 0.37 | 0.47 | 0.57 | 0.62 | 0.67 | 0.72 |
| Truck 3       | 0.95 | 1.05 | 1.15 | 1.20 | 1.25 | 1.30 |
| Truck 4       | 1.05 | 1.15 | 1.25 | 1.30 | 1.35 | 1.40 |
| Truck 5       | 1.15 | 1.25 | 1.35 | 1.40 | 1.45 | 1.50 |
| Truck 6       | 1.25 | 1.35 | 1.45 | 1.50 | 1.55 | 1.60 |
Figure 2. Prospect value under fluctuating charge rate.

From the data in the figure 2, it can be seen that on the same route, truck 1 and truck 6 fluctuate greatly with the rate. Therefore, the above-mentioned two types of trucks should be considered when the toll rate adjustment is made, and can achieve the purpose of attracting vehicles.

4. Conclusion

Based on the cumulative prospect theory, this paper considers the decision-making behaviour of road users in an uncertain travel environment, establishes a freeway vehicle path selection model, and quantitatively analyzes the impact of toll rate on vehicle traffic behaviour, and finds the toll rate of routes with less travel risk the influence on the choice of vehicle route is higher than the route with higher travel risk. The research work in this article provides basic technical support for freeway management, and can be applied to freeway differentiated toll collection, traffic distribution and road network state prediction.

References

[1] Fang C W, Liang Y P, Wu F. (2019) Hesitant fuzzy TOPSIS multi-attribute decision making method based on cumulative prospect theory. Journal of Nanyang Institute of Technology, 22(05):11(06):20-23+30.

[2] Wang T, Gao Y. (2019) Multi-retailer service evaluation of smart grid based on cumulative prospect theory. J.University of Shanghai for Science and Technology, 41(06):599-604.

[3] Hua C X, Gan H C. (2019) Accumulated prospect theory applied to college student travel modes selection. Logistics Sci-Tech, 42(08):100-103+107.

[4] Tversky A, Kahneman D. (1992) Advances in prospect theory: Cumulative representation of uncertainty [J]. Journal of risk and uncertainty, 5(4):297-323.

[5] Sepehr G, Aref D, Zhang L. (2019) Modeling effects of travel time reliability on mode choice using cumulative prospect theory [J]. Transportation Research Part C, 108.

[6] SUN C, CHENG L, ZHU S L, et al. (2015) Multiclass stochastic user equilibrium model with elastic demand: considering systematic and accidental delays. Transportation research record: journal of the transportation research board, (2497):1-11.