Electromagnetic interactions of massive neutrinos and neutrino oscillations

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Abstract. An interplay between electromagnetic properties of massive neutrinos and neutrino oscillation phenomena is discussed. The role of neutrino oscillations in the search of neutrino magnetic moments in experiments on low-energy elastic neutrino-electron scattering is emphasized. A general treatment of neutrino spin-flavor oscillations in a magnetic field is presented. An impact of neutrino electromagnetic interactions on Majorana neutrino fluxes from supernovae is pointed out.

1. Introduction

Neutrinos are generally believed to be electrically neutral particles. At the same time, the fact that neutrinos have non-zero masses opens the door for neutrino electromagnetic interactions [1–3]. Such interactions can be induced by, for example, nonvanishing anomalous magnetic moments of massive neutrinos [4,5]. The effects of neutrino electromagnetic properties are searched both in astrophysics and in laboratory measurements. In particular, one might expect manifestations of these effects in astrophysical sources of neutrinos such as magnetars, neutron stars, supernovae, etc., where strong magnetic fields are known to exist. The coupling of neutrino magnetic moments to an external magnetic field can give rise to the phenomenon of neutrino spin-flavor oscillations [6–10] and, hence, can influence the neutrino fluxes from these sources. In laboratory experiments, a very sensitive tool for searching neutrino electromagnetic interactions is provided by the direct measurement of low-energy elastic neutrino-electron scattering. Deviations of the cross sections measured in such scattering experiments from the values predicted by the standard model of electroweak interaction could bring about an evidence for non-zero electromagnetic properties of massive neutrinos. For interpreting and analyzing the data of these experiments one must take into account the effect of oscillations of neutrinos traveling from the source to the detector [11].

In the present work we focus on the interplay between neutrino electromagnetic interactions and neutrino oscillation processes. The paper is organized as follows. Section 2 gives a brief
account of electromagnetic properties of massive neutrinos. Section 3 is devoted to the magnetic contribution to low-energy elastic neutrino-electron scattering. In section 4, an effective equation for treating neutrino spin-flavor oscillations in a magnetic field is formulated. In section 5, the influence of neutrino electromagnetic interactions on oscillations of Majorana neutrinos from supernovae is outlined.

2. Electromagnetic interactions of massive neutrinos

The effective electromagnetic interaction Hamiltonian for massive neutrino fields can be presented as

$$H_{EM} = \sum_{j,k} \nu_j \Lambda_{jk}^\mu \nu_k A^\mu,$$

where possible transitions between different massive neutrinos are also taken into account. The effective electromagnetic vertex in momentum-space representation depends only on the four-momentum $q = p_j - p_k$ transferred to the photon and can be expressed as follows:

$$\Lambda_{\mu}(q) = (\gamma_{\mu} - q_{\mu} q^2) \left[ f_Q(q^2) + f_A(q^2) q^2 \gamma_5 \right] - i\sigma_{\mu\alpha} q^\alpha \left[ f_M(q^2) + i f_E(q^2) \gamma_5 \right].$$

Here $\Lambda_{\mu}(q)$ is a $3 \times 3$ matrix in the space of massive neutrinos expressed in terms of the four Hermitian $3 \times 3$ matrices of form factors

$$f_Q = f_Q^\dagger, \quad f_M = f_M^\dagger, \quad f_E = f_E^\dagger, \quad f_A = f_A^\dagger,$$

where $Q, M, E, A$ refer respectively to the real charge, magnetic, electric, and anapole neutrino form factors. The Lorentz-invariant form of the vertex function (2) is also consistent with electromagnetic gauge invariance that implies four-current conservation.

For the coupling with a real photon in vacuum ($q^2 = 0$) one has

$$f_{j,k}^Q(0) = e_{j,k}, \quad f_{j,k}^M(0) = \mu_{j,k}, \quad f_{j,k}^E(0) = \epsilon_{j,k}, \quad f_{j,k}^A(0) = a_{j,k},$$

where $e_{j,k}$, $\mu_{j,k}$, $\epsilon_{j,k}$ and $a_{j,k}$ are, respectively, the neutrino charge, magnetic moment, electric moment and anapole moment of diagonal ($j = k$) and transition ($j \neq k$) types.

Since a Majorana field has half the degrees of freedom of a Dirac field, its electromagnetic properties are also reduced, namely in the Majorana case the charge, magnetic and electric form-factor matrices are antisymmetric and the anapole form-factor matrix is symmetric. Therefore, a Majorana neutrino does not have diagonal charge and dipole magnetic and electric form factors. It can only have a diagonal anapole form factor. On the other hand, Majorana neutrinos can have as many transition form factors as Dirac neutrinos.

3. Magnetic $\nu - e^-$ scattering and the role of neutrino oscillations

In the scattering experiments searching for neutrino magnetic moments one typically studies the cross section, which is differential in the energy transfer $T$. This differential cross section can be presented as

$$\frac{d\sigma}{dT} = \frac{d\sigma_{(w)}}{dT} + \frac{d\sigma_{(\mu)}}{dT},$$

where $d\sigma_{(w)}/dT$ is the usual, weak-interaction cross section, and that due to the neutrino magnetic moments in the free-electron approximation is given by (the analysis and discussion of the effects beyond the free-electron approximation can be found, for instance, in Refs. [12–16])

$$\frac{d\sigma_{(\mu)}}{dT} = \frac{\pi \alpha^2}{m_e^2} |\mu_\nu(L, E_\nu)|^2 \left( \frac{1}{T} - \frac{1}{E_\nu} \right).$$
Here $E_\nu$ is the incident neutrino energy, $L$ is the source-detector distance, and $\mu_\nu(L, E_\nu)$ is an effective magnetic moment defined as $[2, 11]$

$$|\mu_\nu(L, E_\nu)|^2 = \sum_{j=1}^{3} \left| \sum_{k=1}^{3} U_{\ell k}^* e^{i \Delta m_{jk}^2 L/2E_\nu (\mu_{jk} - i \epsilon_{jk})} \right|^2,$$

where $U_{\ell k}$ is the neutrino mixing matrix element and $\Delta m_{jk}^2 = m_j^2 - m_k^2$ is the neutrino square mass difference. The effective magnetic moment depends on the neutrino flavor $\ell$ in the source and on the $L/E_\nu$ ratio. In the short-baseline experiments such as GEMMA $[17]$, where $L \ll L_{jk} = 4\pi E_\nu/|\Delta m_{jk}^2|$, the effect of neutrino oscillations is insignificant, so that

$$|\mu_\nu(L, E_\nu)|^2 = \sum_{j=1}^{3} \left| \sum_{k=1}^{3} U_{\ell k}^* (\mu_{jk} - i \epsilon_{jk}) \right|^2.$$

On the contrary, in the long-baseline experiments, where $L \gg L_{jk}$, neutrinos can change their flavor many times when traveling from the source to the detector. Taking into account the effect of decoherence in the neutrino oscillation process, one obtains

$$|\mu_\nu(L, E_\nu)|^2 = \sum_{j,k=1}^{3} |U_{\ell k}|^2 |(\mu_{jk} - i \epsilon_{jk})|^2.$$

In both cases the effective magnetic moment does not depend on the neutrino energy and on the source-detector distance.

4. Neutrino oscillations in a magnetic field

The Hamiltonian of neutrino interaction with a magnetic field $\vec{B}$ is given by

$$H_B = -\sum_{j,k} \mu_{jk} \bar{\nu}_j (\vec{\Sigma} \cdot \vec{B}) \nu_k + h.c., \quad \vec{\Sigma} = \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix},$$

where $\vec{\sigma}$ is the Pauli matrix. In what follows, we limit ourselves to the case of two Dirac neutrino physical states, $\nu_1$ and $\nu_2$, with masses $m_1$ and $m_2$. For treating neutrino evolution in the presence of a uniform magnetic field in the ultrarelativistic limit, we employ a four-component basis of the helicity states $\nu_{1,s=\pm 1}$ and $\nu_{2,s=\pm 1}$. The Schrödinger-like evolution equation is then given by

$$i \frac{d}{dt} \begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix} = H_{eff} \begin{pmatrix} \nu_{1,s=1} \\ \nu_{1,s=-1} \\ \nu_{2,s=1} \\ \nu_{2,s=-1} \end{pmatrix},$$

where the effective Hamiltonian $H_{eff}$ consists of the vacuum and interaction parts,

$$H_{eff} = H_{vac} + H_B.$$

We transform to the flavor basis using the relations

$$\nu_e^{R,L} = \nu_{1,s=\pm 1} \cos \theta + \nu_{2,s=\pm 1} \sin \theta, \quad \nu_\mu^{R,L} = -\nu_{1,s=\pm 1} \sin \theta + \nu_{2,s=\pm 1} \cos \theta,$$

where

$$H_{vac} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad H_B = \begin{pmatrix} 0 & M_B \\ M_B & 0 \end{pmatrix},$$

and

$$M_B = \frac{\mu_0}{2} \frac{\ell}{m_e c} \frac{1}{L/E_\nu} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}.$$
where $\nu_e^{R,L}$ and $\nu_\mu^{R,L}$ are electron and muon neutrino chiral states. In Eq. (13) it is taken into account that in the discussed ultrarelativistic limit the chiral and helicity components practically coincide. In the flavor representation, the vacuum Hamiltonian acquires the form

$$H_{vac}^f = \omega \begin{pmatrix} -\cos 2\theta & 0 & \sin 2\theta & 0 \\ 0 & -\cos 2\theta & 0 & \sin 2\theta \\ \sin 2\theta & 0 & \cos 2\theta & 0 \\ 0 & \sin 2\theta & 0 & \cos 2\theta \end{pmatrix},$$

(14)

where $\omega = \Delta m^2_{21}/4E_\nu$. The Hamiltonian of the neutrino interaction with a magnetic field in the flavor representation can be presented as [18]

$$H_B^f = \begin{pmatrix} -\left(\frac{\mu}{\gamma}\right)_{ee} B_\parallel & \mu_{ee} B_\perp & -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_\parallel & \mu_{e\mu} B_\perp \\ \mu_{ee} B_\perp & \left(\frac{\mu}{\gamma}\right)_{ee} B_\parallel & \mu_{e\mu} B_\perp & -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_\parallel \\ -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_\parallel & \left(\frac{\mu}{\gamma}\right)_{e\mu} B_\parallel & \mu_{\mu\mu} B_\perp & -\left(\frac{\mu}{\gamma}\right)_{\mu\mu} B_\parallel \\ \mu_{e\mu} B_\perp & -\left(\frac{\mu}{\gamma}\right)_{e\mu} B_\parallel & \mu_{\mu\mu} B_\perp & \left(\frac{\mu}{\gamma}\right)_{\mu\mu} B_\parallel \end{pmatrix},$$

(15)

where $B_\parallel$ and $B_\perp$ are the parallel and transverse magnetic-field components with respect to the neutrino velocity, and the quantities $(\mu/\gamma)_{\ell\ell'}$ and $\mu_{\ell\ell'}$ ($\ell, \ell' = e, \mu$) are related to those in the mass representation $\mu_{jk}$ ($j, k = 1, 2$) as follows:

$$\mu_{ee} = \mu_{11} \cos^2 \theta + \mu_{22} \sin^2 \theta + \mu_{12} \sin 2\theta,$$

$$\mu_{e\mu} = \mu_{12} \cos 2\theta + \frac{1}{2} (\mu_{22} - \mu_{11}) \sin 2\theta,$$

$$\mu_{\mu\mu} = \mu_{11} \sin^2 \theta + \mu_{22} \cos^2 \theta - \mu_{12} \sin 2\theta,$$

and

$$\left(\frac{\mu}{\gamma}\right)_{ee} = \frac{\mu_{11}}{\gamma_1} \cos^2 \theta + \frac{\mu_{22}}{\gamma_2} \sin^2 \theta + \frac{\mu_{12}}{\gamma_2} \sin 2\theta,$$

$$\left(\frac{\mu}{\gamma}\right)_{e\mu} = \frac{\mu_{12}}{\gamma_2} \cos 2\theta + \frac{1}{2} \left(\frac{\mu_{22}}{\gamma_1} - \frac{\mu_{11}}{\gamma_1}\right) \sin 2\theta,$$

$$\left(\frac{\mu}{\gamma}\right)_{\mu\mu} = \frac{\mu_{11}}{\gamma_1} \sin^2 \theta + \frac{\mu_{22}}{\gamma_2} \cos^2 \theta - \frac{\mu_{12}}{\gamma_2} \sin 2\theta.$$

Here $\gamma_1$ and $\gamma_2$ are the Lorenz factors of the massive neutrinos, and $\gamma_{12} = 2\gamma_1\gamma_2/\left(\gamma_1 + \gamma_2\right)$.

The evolution equation (11) in the flavor basis can be recast into a fourth-order homogeneous linear differential equation with constant coefficients, which is exactly solvable. This means that one can derive closed-form expressions for neutrino flavor and spin oscillation probabilities. In a general case these expressions appear to be very cumbersome. However, in some specific situations the oscillation probabilities can be well described with simple formulas. For example, these can be the cases when the neutrino interaction with the longitudinal magnetic-field component $B_\parallel$ is negligible and (i) $\omega \gg \mu_\nu B$ or (ii) $\omega \ll \mu_\nu B$, where $\mu_\nu$ is a putative magnetic moment. In the regime $\omega \gg \mu_\nu B$, assuming that at $t = 0$ the neutrino is in the $\nu_\ell^f$ state, the flavor-change and spin-flip probabilities, $P_{\nu_\ell^f \rightarrow \nu_\mu}$ and $P_{\nu_\nu \rightarrow \nu_\nu}$, depend on time according to

$$P_{\nu_\ell^f \rightarrow \nu_\mu}(t) = \left[1 - P_{\nu_\ell^f \rightarrow \nu_\mu}(t)\right] \sin^2 2\theta \sin^2 \omega t, \quad P_{\nu_\nu \rightarrow \nu_\nu}(t) = \sin^2 \omega B t,$$

(16)
where $\omega_B = \mu_\nu B$. In the regime $\omega \ll \mu_\nu B$, the discussed probabilities are well approximated by

$$P_{\nu^\ell \rightarrow \nu^\ell'}(t) = \cos^2 2\theta \sin^2 \omega_B t, \quad P_{\nu^\ell \rightarrow \nu^R}(t) = \frac{1}{2} (1 + \sin 2\theta) \sin^2 2\omega_B t.$$ (17)

The latter approximations become exact in the limit $\omega/\omega_B \to 0$.

5. Electromagnetic interactions and oscillations of supernova neutrinos

Although it was pointed out long time ago that the neutrino-neutrino refraction in the supernova environment may be very important for neutrino flavor conversions, the nonlinear evolution of neutrino flavors has recently been found to dramatically change the neutrino energy spectra [19]. Depending on the initial neutrino fluxes and energy spectra, a complete swap between neutrino spectra of electron and non-electron flavors can take place in the whole or a finite energy range, as a direct consequence of collective neutrino oscillations. Furthermore, the impact of nonzero transition magnetic moments for massive Majorana neutrinos on collective neutrino oscillations has been explored in Refs. [20, 21]. For a magnetic field of $10^{12}$ G and the transition magnetic moment at the level of $10^{-22} \mu_B$, which is just two orders of magnitude larger than the standard-model prediction corresponding to neutrino masses of the order of 0.1 eV, the pattern of spectral splits of supernova neutrinos may be observed in future experiments such as the Jiangmen Underground Neutrino Observatory (JUNO) [22, 23]. The identification of the spectral splits will allow probing values of the neutrino magnetic moments which are extremely small and impossible to detect in other terrestrial experiments.

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5