Supersymmetric grandunification and fermion masses

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Abstract
A short review of the status of supersymmetric grand unified theories and their relation to the issue of fermion masses and mixings is given.

1 Why Grandunification?
There are essentially three reasons for trying to build grand unified theories (GUTs) beyond the standard model (SM).

• why should strong, weak and electromagnetic couplings in the SM be so different despite all corresponding to gauge symmetries?

• there are many disconnected matter representations in the SM (3 families of $L, e^c, Q, u^c, d^c$)

• quantization of electric charge (in the SM model there are two possible explanations - anomaly cancellation and existence of magnetic monopoles - both are naturally embodied in GUTs)

2 How to check a GUT?
I will present here a very short review of some generic features, predictions and drawbacks of GUTs. Details of some topics will be given in the next section.
2.1 Gauge coupling unification

This is of course a necessary condition for any GUT to work. As is well known, the SM field content plus the desert assumption do not lead to the unification of the three gauge couplings. However, the idea of low-energy supersymmetry (susy), i.e. the minimal supersymmetric standard model (MSSM) instead of the SM at around TeV and again the assumption of the desert gives a quite precise unification of gauge couplings at $M_{GUT} \approx 10^{16}$ GeV [1]. Clearly there is no a-priori reason for three functions to cross in one point, so this fact is a strong argument for supersymmetry. One gets two bonuses for free in this case. First, the hierarchy problem gets stabilized, although not really solved, since the famous doublet-triplet (DT) problem still remains. Secondly, at least in principle one can get an insight into the reasons for the electroweak symmetry breaking: why the Higgs (other bosons in MSSM) mass squared is negative (positive) at low energy [2].

2.2 Fermion masses and mixings

Although GUTs are not theories of flavour, they bring constraints on the possible Yukawas. In the MSSM the Yukawa sector is given by

$$W_Y = HQ^T Y_U u^c + \overline{Q}^T Y_D d^c + \overline{L}^T Y_E e^c ,$$

and the complex $3 \times 3$ generation matrices $Y_{U,D,E}$ are arbitrary. However, in a GUT the matter fields $Q$, $L$, $u^c$, $d^c$, $e^c$ fields live together in bigger representations, so one expects relations between quark and lepton Yukawa matrices.

Take for example the SO(10) GUT. All the MSSM matter fields of each generation live in the same representation, the 16 dimensional spinor representation, which contains and thus predicts also the right-handed neutrino. At the same time the minimal Higgs representation, the 10 dimensional representation contains both doublets $H$ and $\overline{H}$ of the MSSM (plus one color triplet and one antitriplet). The only renormalizable SO(10) invariant one can write down is thus

$$W_Y = 10_H 16_Y 16 ,$$

which is however too restrictive, since it gives on top of the well satisfied (for large $\tan \beta$) relation $y_b = y_{\tau} = y_t$ for the third generation, the much worse predictions for the first two generations ($y_s = y_{\mu} = y_c$ and $y_d = y_e = y_u$) and no mixing ($\theta_c = 0$) at all.
How to improve the fit? Let us mention two possibilities:

(1) Introduce new Higgs representations: although another $10_H$ can help with the mixing, the experimentally wrong relations $m_d = m_e$ and $m_s = m_{\mu}$ still occur, because the two bi-doublets in the two $10_H$ leave invariant the Pati-Salam $SO(6)=SU(4)_C$, so the leptons and quarks are still treated on the same footing. So the idea to pursue is to introduce bidoublets which transform nontrivially under the Pati-Salam $SU(4)$ color. This can be done for example by introducing a $\mathbf{126}_H$, which couples to matter as $\Delta W_Y = \mathbf{126}_H 16 Y_{126} 16$ and which gets a nontrivial vev in the $(2, 2, 15)_H$ $SU(3)$ color singlet direction [3, 4].

(2) Another possibility is to include the effects of nonrenormalizable operators. These operators can cure the problem and at the same time ease the proton decay constraints. The drawback is the loss of predictivity.

2.3 Proton decay

This issue is connected to

(1) R-parity. It is needed to avoid fast proton decay. At the nonrenormalizable level one could for example have terms leading to R-parity violation of the type $16^3 16_H / M_{Pl}$. For this reason it is preferable to use the $126_H$ representation instead of the $16_H$. It is possible to show that such a $SO(10)$ with $126_H$ has an exact R-parity [5] at all energies without the introduction of further symmetries.

(2) DT splitting problem: Higgs $SU(2)_L$ doublets and $SU(3)_C$ triplets live usually in the same GUT multiplet; but while the $SU(2)_L$ doublets are light ($\approx M_W \ll M_{GUT}$), the $SU(3)_C$ triplets should be very heavy ($\geq M_{GUT}$) to avoid a too fast proton decay. For example, the proton lifetime in susy is proportional to $M_T^2$, which can give a lower limit to the triplet mass [6], although this limit depends on the yet unknown supersymmetry breaking sector [7].

The solutions to the DT problem depend on the gauge group considered, but in general models that solve it are not minimal and necessitate of additional Higgs sectors. For example the missing partner mechanism [8] in SU(5) needs at least additional $75_H$, $50_H$ and $\overline{50}_H$ representations. The same is true for the missing vev mechanism in SO(10) [9], where the $45_H$ and extra $10_H$ Higgses must be introduced. Also the nice idea of GIFT (Goldstones Instead of Fine Tuning) [10] can be implemented only by complicated models, while discrete symmetries for this purpose can be used with success only in connection with non-simple gauge groups [11]. Of course, although
not very natural, any GUT can "solve" the problem phenomenologically, i.e. simply fine-tuning the model parameters.

Clearly, whatever is the solution to the DT problem, the proton lifetime depends in a crucial (powerlike) way on the triplet mass. And this mass can be difficult to determine from the gauge coupling unification condition even in specific models because of the unknown model parameters \[12\] or use of high representations \[13\]. On top of this there can be large uncertainties in the triplet Yukawa couplings \[14\]. All this, together with the phenomenologically completely unknown soft susy breaking sector, makes unfortunately proton decay not a very neat probe of supersymmetric grandunification \[7\]. Of course, if for some reason the DT mechanism is so efficient to make the \(d = 6\) operators dominant (for a recent analysis in some string-inspired models see \[15\]), then the situation could be simpler to analyse \[16\], although many uncertainties due to fermion mixing matrices still exist in realistic nonminimal models \[17\]. Unfortunately there is little hope to detect proton decay in this case, unless \(M_{GUT}\) is lower than usual \[18\].

### 2.4 Magnetic monopoles

Since magnetic monopoles are too heavy to be produced in colliders, the only hope is to find them as relics from the cosmological GUT phase transitions. Their density however strongly depends on the cosmological model considered. Unfortunately, the Rubakov-Callan effect \[19\] leads to the nonobservability of GUT monopoles, at least in any foreseeable future. Namely, these monopoles are captured by neutron stars and the resulting astrophysical analyses \[20\] limits the monopole flux at earth twelve orders of magnitude below the MACRO limit \[21\].

This is very different from the situation in the Pati-Salam (PS) theory. Even in the minimal version the PS scale can be much lower than the GUT scale \[22\], as low as \(10^{10}\) GeV. the resulting monopoles are then too light to be captured by neutron stars and their flux is not limited due to the Rubakov-Callan effect. Furthermore, MACRO results are not applicable for such light monopoles \[21\].

### 2.5 Low energy tests

There are many different possible tests at low-energy, like for example the flavour changing neutral currents (see for example \[23\]) or the electric dipole moments \[24\]. In the latter case the exact value of the triplet mass is
much less important than in proton decay, but the uncertainties due to the susy breaking sector are still present. In some of these tests like neutron-antineutron oscillation we can get positive signatures only for specific models due to very high dimensional operators involved [25].

3 Fermion masses and mixings

The regular pattern of 3 generations suggests some sort of flavour symmetry.

One way, and the most ambitious one, is to consider the flavour symmetry group as part (subgroup) of the grand unified gauge symmetry (described by a simple group). In such an approach all three generations come from the same GUT multiplet. For example, in SU(8) the 216 dimensional representation gets decomposed under its SU(5) subgroup into three copies (generations) of \( \overline{5} \) and 10 with additional SU(5) multiplets. Similarly, in the SO(18) GUT, the 256 dimensional spinorial representation is nothing else than 8 generations of \((16 + \overline{16})\) in the SO(10) language. The problem in all these theories is what to do with all the extra light particles [26].

Another possibility is to consider the product of the flavour (or, in general, extra) symmetry with the GUT symmetry (simple) group. In the context of SO(10) GUTs most of them use small representations for the Higgses, like \( 16_H, \overline{16}_H \), and \( 45_H \). The philosophy is to consider all terms allowed by symmetry, also nonrenormalizable. The DT problem can be naturally solved by some version of the missing vev mechanism, which however means that many multiplets are usually needed. Such models are quite successfull [27], although the assumed symmetries are a little bit ad-hoc. There is also a huge number of different models with almost arbitrary flavour symmetry group, but unfortunately there is no room to describe them here (see for example the recent review [28]).

What we will consider in the following is instead a SO(10) GUT with no extra symmetry at all. We want to see how far we can go with just the grand unified gauge symmetry alone. To ensure automatic R-parity, we are forced not to use the \( 16_H \) and \( \overline{16}_H \) Higgses, but instead a pair of \( 126_H \) and \( \overline{126}_H \) (5 index antisymmetric representations, one self-dual, the other anti-self-dual; both of them are needed in order not to break susy at a large scale). In fact under R-parity the bosons of 16 are odd, while those of 126 are even, since

\[
R = (-1)^{3(B-L)+2S}
\]

[29], and the relevant vev in the SU(5) singlet directions have \( B - L = 1 \) for
\[16_H \ (\nu^c), \text{ while it has } B - L = 2 \text{ for } 126 \ (\text{the mass of } \nu^c).

So the rules of the game are: stick to renormalizable operators only, consider SO(10) as the only symmetry of the model, take the minimal number of multiplets (it does not mean the minimal number of fields!) that is able to give the correct symmetry breaking pattern SO(10) \rightarrow SU(3) \times SU(2) \times U(1).

Such a theory is given by [30] (see however [31] for a similar approach): on top of the usual three generations of 16 dimensional matter fields, it contains four Higgs representations: 10_H, 126_H, \( \overline{126}_H \) and 210_H (4 index antisymmetric). It has been shown recently [32] that this theory is also the minimal GUT, i.e. it has the minimal number of model parameters, being still perfectly realistic (not in contradiction with any experiment).

As we have seen, the \( \overline{126}_H \) multiplet is needed both to help the 10_H multiplet in fitting the fermion masses and mixings, and for giving the mass to the right-handed neutrino without explicitly breaking R-parity. Let us now see, why the 210_H representation is needed.

The Yukawa sector is given by

\[ W_Y = 10_H 16 Y_{10} 16 + \overline{126}_H 16 Y_{126} 16. \quad (4) \]

The fields decompose under the SU(2)_L \times SU(2)_R \times SU(4)_C subgroup as

\[ 10_H = (2, 2, 1) + (1, 1, 6), \quad (5) \]
\[ 16 = (2, 1, 4) + (1, 2, \overline{3}), \quad (6) \]
\[ \overline{126}_H = (1, 3, 10) + (3, 1, \overline{10}) + (2, 2, 15) + (1, 1, 6). \quad (7) \]

The right-handed neutrino \( \nu^c \) lives in \((1, 2, \overline{3})\) of 16, so it can get a large mass only through the second term in (4):

\[ M_{\nu_R} = \langle (1, 3, 10)_{\overline{126}} \rangle Y_{126}, \quad (8) \]

where \( \langle (1, 3, 10) \rangle \) is the scale of the SU(2)_R symmetry breaking \( M_R \), which we assume to be large, \( O(M_{\text{GUT}}) \).

In order to get realistic masses we need

\[ \langle (2, 2, 1)_{10} \rangle = \begin{pmatrix} v^d_{10} & 0 \\ 0 & v^u_{10} \end{pmatrix} \neq 0, \quad (9) \]
\[ \langle (2, 2, 15)_{\overline{126}} \rangle = \begin{pmatrix} v^d_{126} & 0 \\ 0 & v^u_{126} \end{pmatrix} \neq 0, \quad (10) \]
which contribute to the light fermion masses as

\begin{align}
M_U &= v_{10}^u Y_{10} + v_{126}^u Y_{126}, \\
M_D &= v_{10}^d Y_{10} + v_{126}^d Y_{126}, \\
M_{\nu_D} &= v_{10}^u Y_{10} - 3 v_{126}^u Y_{126}, \\
M_E &= v_{10}^d Y_{10} - 3 v_{126}^d Y_{126}.
\end{align}

(11)  (12)  (13)  (14)

The factor of $-3$ for leptons in the contribution from $126_H$ comes automatically from the fact that the SU(3)$_C$ singlet in the adjoint 15 of SU(4)$_C$ is in the $B - L$ direction $\text{diag}(1,1,1,-3)$. This is clearly absent in the contribution from $10_H$, which is a singlet under the full SU(4)$_C$.

The light neutrino mass comes through the famous see-saw mechanism [33]. From

\begin{equation}
W = \frac{1}{2} \nu^c M_{\nu_R} \nu^c + \nu^c M_{\nu_D} \nu_L + \ldots
\end{equation}

one can integrate out the heavy right-handed neutrino $\nu^c$ obtaining the effective mass term for the light neutrino states $M_N = -M_{\nu_D} M_{\nu_R}^{-1} M_{\nu_D}$. As we will now see, there is another contribution in our minimal model.

(1) We saw that both $\langle (2, 2, 1)_{10} \rangle$ and $\langle (2, 2, 15)_{126} \rangle$ need to be nonzero and obviously $O(M_W)$. With $10_H$, $126_H$ and $\overline{126}_H$ Higgses one can write only two renormalizable invariants:

\begin{equation}
W_H = \frac{1}{2} M_{10}^2 10_H^2 + M_{126} 126_H \overline{126}_H,
\end{equation}

where $M_{10}, M_{126} \approx O(M_{GUT})$ or larger due to proton decay constraints. So the mass term looks like

\begin{equation}
\frac{1}{2} (10_H, 126_H, \overline{126}_H) \begin{pmatrix} M_{10} & 0 & 0 \\
0 & 0 & M_{126} \\
0 & M_{126} & 0 \end{pmatrix} \begin{pmatrix} 10_H \\
126_H \\
\overline{126}_H \end{pmatrix}.
\end{equation}

Clearly all the doublets have a large positive mass, so their vev must be zero. Even fine-tuning cannot solve the DT problem in this case. So the idea to overcome this obstacle is to mix in some way $10_H$ with $\overline{126}_H$ ($126_H$), and after that fine-tune to zero one combination of doublet masses. So the new mass matrix should look something like
\[
\begin{pmatrix}
M_{10} & x & y \\
x & 0 & M_{126} \\
y & M_{126} & 0
\end{pmatrix}
\]

(18)

with \(x, y\) denoting such mixing. The light Higgs doublets will thus be linear combinations of the fields in \((2,2,1)_{10}\) and \((2,2,15)_{126H,126h}\) and this will get a nonzero vev after including the soft susy breaking masses.

(2) The minimal representation that can mix 10 and 126 is 210, as can be seen from \(10 \times 126 = 210 + 1050\). 210 is a 4 index antisymmetric SO(10) representation, which decomposes under the Pati-Salam subgroup as

\[210 = (1,1,1)+(1,1,15)+(1,3,15)+(3,1,15)+(2,2,6)+(2,2,10)+(2,2,10)\].

(19)

Of course one can now add other renormalizable terms to (16), and all such new terms are (in a symbolic notation)

\[\Delta W_H = 210^3_H + 210^2_H + 210_H 126_H 126_H + 210_H 10_H 126_H + 210_H 10_H 126_H\].

(20)

The last two terms are exactly the ones needed for the mixings between 10\(_H\) and 126\(_H\) (126\(_H\)), i.e. contributions to \(x, y\) in (18). It is possible to show that \(W_H + \Delta W_H\) are just enough for SO(10)\(\rightarrow\)SM. In the case of single-step breaking one thus has

\[\langle (1,1,1)_{210} \rangle \approx \langle (1,1,15)_{210} \rangle \approx \langle (1,3,15)_{210} \rangle \approx \langle (1,3,10)_{126} \rangle \approx M_{GUT}\].

(21)

(3) Now however there are five bidoublets that mix, since \((2,2,10)\) and \((2,2,10)\) from 210\(_H\) also contribute. To be honest, there is only one neutral component in each of these last two bidoublets, since their \(B-L\) equals \pm 2, so the final mass matrix for the Higgs doublets is 4 \times 4. Only one eigenvalue of this matrix needs to be zero, and this can be achieved by fine-tuning. Each of the two Higgs doublets of the MSSM is thus a linear combination of 4 doublets, each of which has in general a vev of order \(O(M_W)\):

\[\langle (2,2,1)_{10} \rangle \approx \langle (2,2,15)_{126} \rangle \approx M_W \approx \langle (2,2,10)_{210} \rangle \approx \langle (2,2,10)_{210} \rangle\].

(22)

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This mixing is nothing else than the susy version of \([3, 4]\).

(4) Due to all these bidoublet vevs, a SU(2)_L triplet will also get a tiny but nonzero vev. Applying the susy constraint \(F_{(3,1,10)_{126}} = 0\) to

\[
W = M_{126}(3, 1, 10)_{126} (3, 1, \overline{10})_{126} + (2, 2, 1)_{10} (2, 2, \overline{10})_{210} (3, 1, 10)_{126} + \ldots
\]

one immediately gets

\[
\langle (3, 1, \overline{10})_{126} \rangle \approx \frac{\langle (2, 2, 1)_{10} \rangle \langle (2, 2, \overline{10})_{210} \rangle}{M_{126}} \approx \frac{M_W^2}{M_{GUT}} \neq 0 .
\]

This effect is just the susy version of \([34]\).

(5) Since \(\nu\) lives in \((2, 1, 4)_{16}\), the second term in (4) gives among others also a term \((3, 1, \overline{10})_{126} (2, 1, 4)_{16} Y_{126}(2, 1, 4)_{16}\), which contributes to the light neutrino mass. So all together one gets for the light neutrino mass (\(c\) is a model dependent dimensionless parameter)

\[
M_N = -M_{\nu_D}^T M_{\nu_D}^{-1} M_{\nu_D} + c \frac{M_W^2}{M_{GUT}} Y_{126} .
\]

The first term is called the type I (or canonical) see-saw and is mediated by the SU(2)_L singlet \(\nu^c\), while the second is the type II (or non-canonical) see-saw, and is mediated by the SU(2)_L triplet.

Equations (11), (12), (13), (14), (8) and (25) are all we need in the fit of known fermion masses and mixings and predictions of the unknown ones. A possible procedure is first to trade the matrices \(Y_{10}\) and \(Y_{126}\) for \(M_U\) and \(M_D\). The remaining freedom in \(M_U\) and \(M_D\) is still enough to fit \(M_E\). But then some predictions in the neutrino sector are possible. For this sector we need to reproduce the experimental results \((\theta_{\ell})_{12,23} \gg (\theta_q)_{12,23}\) and \((\theta_{\ell})_{13}\) small. The degree of predictivity of the model however depends on the assumptions regarding the see-saw and on the CP phases.

The first approach was to consider models in which type I dominates. It was shown that such models predict a small atmospheric neutrino mixing angle \(\theta_{atm} = (\theta_{\ell})_{23}\) if the CP phases are assumend to be small \([4, 35]\). On the other hand, a large atmospheric neutrino mixing angle can be also large, if one allows for arbitrary CP phases and fine-tune them appropriately \([36]\).

A completely different picture emerges if one assumes that type II see-saw dominates. In this case even without CP violation one can naturally have a large atmospheric neutrino mixing angle, as has been first emphasized for the approximate case of second and third generations only in \([37, 38]\).
In the three generation case the same result has been confirmed \[39\]. On top of this, a large solar neutrino mixing angle and a prediction of $U_{e3} \approx 0.15 \pm 0.01$ (close to the upper experimental limit) have been obtained \[39\]. Even allowing for general CP violation does not invalidate the above results: although the error bars are larger, the general picture of large atmospheric and solar neutrino mixing angles and small $U_{e3}$ still remains valid \[40\].

It is possible to understand why type II see-saw gives so naturally a large atmospheric mixing angle. In type II the light neutrino mass matrix \(25\) is proportional to $Y_{126}$. From \(12\) and \(14\) one can easily find out, that $Y_{126} \propto M_D - M_E$, from which one gets \[41\]

\[
M_N \propto M_D - M_E . \tag{26}
\]

As a warm-up let us take the approximations of just (a) two generations, the second and the third, (b) neglect the masses of the second generation with respect of the third ($m_{s,\mu} \ll m_{b,\tau}$) and (c) assume that $M_D$ and $M_E$ has small mixings (this amounts to say; that in the basis of diagonal charged lepton mass, the smallness of the $(\theta_\ell)_{23} = \theta_{cb}$ is not caused by accidental cancellation of two large numbers). In this approximate set-up one gets

\[
M_N \propto \begin{pmatrix} 0 & 0 \\ 0 & m_b - m_\tau \end{pmatrix} . \tag{27}
\]

This is, in type II see-saw there is a correspondence between the large atmospheric angle and $b - \tau$ unification \[38\].

Remember here that $b - \tau$ Yukawa unification is no more automatic, since we have also $126_H$ Higgs on top of the usual $10_H$. It is however quite well satisfied phenomenologically.

One can do better: still take $m_{s,\mu} \approx 0$, but allow a nonzero quark mixing. In this case the atmospheric mixing angle is

\[
\tan 2\theta_{atm} = \frac{\sin 2\theta_{cb}}{2 \sin^2 \theta_{cb} - \frac{m_b - m_\tau}{m_b}} . \tag{28}
\]

Since $\theta_{cb} \approx \mathcal{O}(10^{-2})$, one again finds out the correlation between the large atmospheric mixing angle and $b - \tau$ unification at the GUT scale.

The result can be confirmed of course also for finite $m_{s,\mu,c}$, although not in a so simple and elegant way.

Of course there are many other models that predict and/or explain a large atmospheric mixing angle (for a recent review see for example \[42\]).
What is surprising here is, however, that no other symmetry except the
gauge SO(10) is needed whatsoever.

4 The minimal model

As we saw in the previous section, one can correctly fit the known masses
and mixings, get some understanding of the light neutrino mass matrix, and
obtain some new predictions. What we would like to show here is that the
model considered above has less number of model parameters than any other
GUT, and can be then called the minimal realistic supersymmetric grand
unified theory (even more minimal than SU(5)!) [32].

The Higgs sector described by (16) and (20) contains 10 real parameters
(7 complex parameters minus four phase redefinitions due to the four com-
plex Higgs multiplets involved). The Yukawa sector (4) has two complex
symmetric matrices, one of which can be always made diagonal and real by
a unitary transformation of 16 in generation space. So what remains are
15 real parameters. There is on top of this also the gauge coupling, so all
together 26 real parameters in the supersymmetric sector of renormalizable
SO(10) GUT with three copies of matter 16 and Higgses in the representa-
tions 10_H, 126_H, \( \overline{126}_H \) and 210_H. We will not consider the susy breaking
sector, since this is present in all supersymmetric theories, GUTs or not.

Before comparing with other GUTs, for example SU(5), let us count the
number of model parameters in MSSM. There are 6 quark masses, 3 quark
mixing angles, 1 quark CP phase, 6 lepton masses, 3 lepton mixing angles
and 3 lepton CP phases (assuming Majorana neutrinos). On top of this,
there are 3 gauge couplings and the real \( \mu \) parameter. Thus, all together,
again 26 real parameters. They are however distributed differently, so that
in the Yukawa sector there are only 15 parameters in our minimal SO(10)
GUT, which has to fit 22 MSSM (at least in principle) measurable low-energy
parameters. Although in this fitting also few vevs that contain parameters
from the Higgs and susy breaking sector play a role, the minimal SO(10) is
nevertheless predictive.

One can play with other SO(10) models: the renormalizable ones need
more representations and thus have more invariants, while the nonrenor-
malizable ones (those that use 16_H instead of 126_H) have a huge number of
invariants, some of which must be very small due to R-symmetry constraints.
Of course, with some extra discrete, global or local symmetry, one can forbid
these unpleasant and dangerous terms, remaining even with a small number
of parameters, but as we said, this is not allowed in our scheme, in which we want to obtain as much information as possible just from GUT gauge symmetry (and renormalizability).

The simplicity of the minimal renormalizable supersymmetric SU(5) looks as if the number of parameters here could be smaller than in our previous example. What however gives a large number of parameters is the fact, that SU(5) is not particularly suitable for the neutrino sector. In fact, one can play and find out, that the minimal SU(5) with nonzero neutrino masses is obtained adding the two index symmetric $15_H$ and $\bar{15}_H$, and the number of model parameters comes out to be 39, i.e. much more than in the minimal SO(10).

5 Conclusion

Before talking about flavour symmetries it is important first to know, what we can learn from just pure GUTs. The minimal GUT is a SO(10) gauge theory with representations $10_H$, $126_H$, $\bar{126}_H$, $210_H$ and three generations of $16$. Such a realistic theory is renormalizable and no extra symmetries are needed. It can fit the fermion masses and mixings, and can give an interesting relation between $b-\tau$ Yukawa unification and large atmospheric mixing angle. It has a testable prediction for $U_{e3}$. Due to the large representations involved, it is not asymptotically free, which means that it predicts some new physics below $M_{Pl}$.

There are many virtues of this minimal GUT. As in any SO(10) all fermions of one generation are in the same representation and the right-handed neutrino is included automatically, thus explaining the tiny neutrino masses by the see-saw mechanism. Employing $126_H$ instead of $16_H$ to break $B-L$ maintains R-symmetry exact at all energies. It is economical, it employs the minimal number of multiplets and parameters, and thus it is maximally predictive. It gives a good fit to available data and gives a framework to better understand the differences between the mixings in the quark and lepton sectors.

There are of course also some drawbacks. First, in order to maintain predictivity, one must believe in the principle of renormalizability, although the suppressing parameter in the expansion $M_{GUT}/M_{Planck}$ is not that small. Of course, in supersymmetry these terms can be small and stable, but this choice is not natural in the 't Hooft sense. Second, the DT splitting problem is here, and attempts to solve it require more fields [43]. Finally, usually
it is said that 126 dimensional representations are not easy to get from superstring theories, although we are probably far from a no-go theorem.

There are many open questions to study in the context of the minimal SO(10), let me mention just few of them. First, proton decay: although it is generically dangerous, it is probably still possible to fit the data with some fine-tuning of the model parameters as well as of soft susy breaking terms. An interesting question is whether the model is capable of telling us which type of see-saw dominates. If it is type I or mixed, can it still give some testable prediction for $U_{e3}$? Also, gauge coupling unification should be tested in some way, although large threshold corrections could be nasty [13]. And finally, is there some hope to solve in this context or minimal (but still predictive) extensions the doublet-triplet splitting problem?

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