Stimulated Raman backscattering of laser radiation in deep plasma channels

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Stimulated Raman backscattering (RBS) of intense laser radiation confined by a single-mode plasma channel with a radial variation of plasma frequency greater than a homogeneous-plasma RBS bandwidth is characterized by a strong transverse localization of resonantly-driven electron plasma waves (EPW). The EPW localization reduces the peak growth rate of RBS and increases the amplification bandwidth. The continuum of non-bound modes of backscattered radiation shrinks the transverse field profile in a channel and increases the RBS growth rate. Solution of the initial-value problem shows that an electromagnetic pulse amplified by the RBS in the single-mode deep plasma channel has a group velocity higher than in the case of homogeneous-plasma Raman amplification. Implications to the design of an RBS pulse compressor in a plasma channel are discussed.

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I. INTRODUCTION

Stimulated Raman backscattering (RBS) of laser radiation in plasmas [1] is a parametric process in which a laser beam (pump wave) is backscattered off the electron plasma density fluctuations. These density perturbations are driven and amplified by the ponderomotive beatwave of pump and scattered electromagnetic (EM) waves. Under certain phase matching conditions a positive feedback loop develops that results in the onset of a temporal or spatio-temporal instability [2]. The RBS in transversely homogeneous plasmas has been extensively studied since early 1970s [1, 2], when it first came to the fore in the context of fast electron generation and target pre-heat in laser confinement fusion. The basic treatment of the RBS in homogeneous plasmas is now a classic that can be found in a number of textbooks [3].

Although RBS of long laser beams has been studied for at least three decades, the short-pulse regimes of this instability have only recently attracted attention due to the advances in generation and amplification of sub-picosecond multi-terawatt laser pulses [4]. The RBS of such pulses in rarefied homogeneous plasmas ($\omega_p \ll \omega_0$, where $\omega_0$ is a fundamental frequency of laser, $\omega_p = \sqrt{4\pi e^2 n_0/m_e}$ is an electron Langmuir plasma frequency, $-|e|$ is an electron charge, and $n_0$ is an electron plasma density) was explored in detail in both experiment [5, 6, 7, 8, 9, 10] and theory [11, 12, 13, 14, 15, 16, 17]. The recent upsurge of interest in the RBS has been specifically caused by theoretical discovery of sub-picosecond multi-terawatt laser pulses [18, 19] and laser particle acceleration.

Certain applications of short-pulse lasers, such as novel X-ray source development [21], generation of high harmonics of laser radiation [22] and laser particle acceleration [23] in tenuous plasmas, benefit from a long interaction distance. In a homogeneous plasma the region of high intensity interaction is confined to approximately one Rayleigh diffraction length $Z_R = k_0 r_0^2/2$, where $k_0$ is a wavenumber, and $r_0$ is a radius of a focal spot of a laser beam. Propagation over longer distances requires some form of laser guiding. Guiding by a plasma channel is the most promising experimental approach [23, 24, 25, 26, 27, 28] for high laser intensities. Excitation of relativistic plasma waves in channels was analyzed [29, 30, 31, 32, 33] for particle acceleration.

The guided laser pulses are not immune to parametric instabilities, such as forward and near-forward stimulated Raman scattering (SRS) in parabolic [34, 35, 36, 37, 38, 39], tapered [40], leaky [41], and single-mode flat channels [42], or large-angle SRS in plasma-filled capillaries [43]. Commonly, the energy losses of a laser pulse and excessive plasma heating due to the large-angle SRS (including RBS) are undesirable for some applications [44], and uncovering new physical mechanisms that enable the RBS suppression is up to date. For some applications, however, the RBS can be useful. E.g., the laser pulse leading front depletion by the RBS may seed either forward SRS or resonant modulational instability [13, 14]. Novel schemes of short-pulse amplification in a plasma [18, 19] are based on the backscattering of a long moderately intense pump laser beam into a short counter-propagating signal pulse: the energy of the pump is absorbed by the signal as it is amplified and compressed. The Raman compression could be a viable path to obtaining high-power single-cycle pulses. For transversely homogeneous plasmas, one of the challenges of Raman amplification is to ensure the uniformity of plasma along the interaction axis so that the signal be downshifted from the pump by almost exactly the plasma frequency $\omega_p$. In the present paper we discover novel features of the RBS in plasma channels, which are favorable for realization of Raman amplification (to anticipate, the RBS process in a deep plasma channel can be broadband enough to make exact resonant detuning of pump and signal un-
necessary.

To enable analytic progress and to facilitate qualitative understanding, we focus on plasma channels that support a single confined hf EM mode, referred to as a fundamental mode of channel (FMC). The laser pulse confined in a channel is assumed to be the FMC. The RBS in a single-mode channel proceeds differently than in a homogeneous plasma and can be characterized by the following novel features: (a) reduction of the RBS peak temporal growth rate; (b) broadening of the RBS amplification bandwidth; (c) modification of the transverse profile of the scattered mode from that of FMC. Depending on the parameters of channel (such as on-axis plasma density and density depression) and pump laser (such as frequency and intensity), those modifications can be either more or less prominent.

It was suggested earlier in the context of Raman forward scattering (RFS) that plasma frequency variation across the channel may significantly reduce the peak growth rate. The effect is resulted from a strong localization of a scattering electron plasma wave (EPW) near the channel axis. However, due to the complicated hybrid EM nature of a relativistic EPW in a channel, only approximate results were obtained in Ref. 42. Specifically, it was assumed that the scattered radiation was in the fundamental channel mode. For the RBS, the EM component of the short-wavelength EPW may be neglected, enabling us to account for the transverse profile modification of the backscattered radiation. This is accomplished by expanding the transverse profile of the scattered field into the channel eigenmodes, i.e., the FMC plus continuum of the non-bound hf EM modes. These continuum modes of channel (CMC) do not exponentially decay outside of the channel (as the FMC does) and exhibit a cosine-like behavior at infinity, which makes them similar to the transverse Fourier harmonics of scattered radiation in a homogeneous plasma. The problem is complicated by the fact that the FMC and CMC are not independent. Radial shear of the plasma density couples them to each other not only creating the field structure different from that of FMC but also affecting spectral features of the instability. One of our goals is to evaluate the continuum-mode contribution to the RBS growth rate. We put emphasis on the regime of strong plasma wave localization (SPWL). Spectral features of this regime are markedly different from those of the homogeneous-plasma RBS. The SPWL occurs when the parameter \( \eta = (\Delta \omega_p/\omega_{p1})^2/(2\gamma_{hom}/\omega_{p1}) \) is large compared to unity [here and elsewhere, \( \gamma_{hom} = \sqrt{|a_0|^2 \omega_{hom} \omega_{p1}}/4 \) is the maximum increment of the weakly coupled RBS in a homogeneous plasma \( \approx 2, \omega_{p1} = \omega_p(x = 0) \) is a plasma frequency on axis, \( \Delta \omega_p = \omega_{p1} \) is the channel depth, and \( a_0 = cE_0/(m_0 \omega_0\gamma) \) is a normalized amplitude of electric field of the pump wave], which physically means that the plasma frequency depression in a channel exceeds the RBS bandwidth in a homogeneous plasma. In the SPWL regime, the maximum temporal increment is shown to scale as \( \gamma_{hom}/\eta \). Hence, plasma wave localization suppresses the instability. On the other hand, the transverse shear of the plasma frequency results in a broader instability bandwidth. Spectral maximum of the backscattered light is found to be red-shifted from the pump frequency by more than \( \omega_{p1} \), and the spectrum itself extends on the red side far beyond the frequency bandwidth of the homogeneous-plasma RBS. The bandwidth increase is due to the backscattering off the channel regions with higher local plasma frequencies and, therefore, higher red-shifts of scattered light. In the SPWL regime, the CMC contribution transversely shrinks the fastest-growing mode of scattered radiation and concentrates the scattered field near the channel axis. This effect is followed by enhancement of the RBS (in the numerical examples presented in the paper, the continuum modes add from 25% to 50% to the peak value of temporal increment).

The CMC contribution to the RBS process may be neglected if the pump amplitude is sufficiently small (or the channel is deep) to satisfy the inequality \( u_0 \ll (\Delta \omega_p/\omega_{p1})^2/(\omega_{p1}/\omega_{p1})^{3/4} \). This regime is referred to as single-mode, because the scattered field is fairly well represented solely by FMC. That only three unstable waves (two bound EM modes and a localized EPW) participate the single-mode RBS makes it the channel analog of the three-wave (i.e., weakly coupled) RBS in a homogeneous plasma despite the vast difference in spectral features. In the single-mode regime an analytic solution of the initial-value problem has been found, which describes the evolution of the single-mode backscattered EM signal in the field of a single-mode pump. It is shown that the maximum of the wave packet moves with the velocity \( 2c/3 \), which is higher than a group velocity of radiation \( c/2 \) of the weakly coupled RBS in a homogeneous plasma. High group velocity of the amplified pulse and its broad bandwidth produced by the transverse shear of plasma density profitably distinguish the single-mode Raman amplification in a plasma channel from its homogeneous-plasma counterpart [14].

The paper is organized as follows. In Sec. III the single-mode planar plasma channel is introduced, and basic equations governing the nonlinear plasma response to the combined pump and scattered radiation are derived. These equations are solved in Sec. IV by the mode expansion, and the generalized dispersion relation is derived. This dispersion relation allows for the coupling between the FMC and CMC of scattered radiation. In Sec. V the dispersion equation is solved in the most interesting and novel regime of SPWL. The criterion which allows to neglect the CMC contribution to the RBS process is proposed. The spectral bandwidth and peak temporal growth rate of the single-mode regime are derived. In Sec. VI the initial value problem describing the linear evolution of the EM signal is formulated and solved, and the group velocity of the signal is evaluated. Summary of the results is given in Sec. VII. The Appendix reviews some properties of the associated Legendre functions with an imaginary order which describe the continuous spectrum
II. BASIC EQUATIONS

To begin with, we define the unperturbed state of a plasma and laser radiation, that is, the background on which the instability grows. In order to make analytic progress, a planar plasma channel is chosen with the density profile given by

\[ \omega_p^2(x) = \omega_p^2 - \left( \Delta \omega_p^2 / 2 \right) \cosh^{-2}(x / \sigma). \]  

(1)

Plasma frequency in the channel varies between \( \omega_p \) and laser radiation, that is, the background on arbitrary polarization is axis is held fixed, and the channel depth is varied. The solution of the eigenvalue problem for the pump field is

\[ \text{The laser electric field in the channel is the solution of } \] 

in the center and

\[ \text{Through the rest of the paper, electron density at the channel axis is held fixed, and the channel depth is varied. The normalized } \]

\[ \text{The perturbed hf electric field in the channel is the solution of the eigenvalue problem } \]

\[ \mathcal{L}_0 a_0 \equiv \left[ -\partial^2 / \partial x^2 + k_p^2(x) \right] a_0 = \lambda_0 a_0 \]

(3)

with the boundary condition \( a_0(\infty) \rightarrow 0 \) here, \( \lambda_0 \equiv (\omega_p / c)^2 - k_p^2 \). \( k_p(x) \equiv \omega_p(x) / c \), and \( a_0 = [a_0] \).

In Eq. (3), we assume \( \lambda_0 \sim O(1) \), \( k_p \sim \omega_p / c \), and \( a_0 \ll 1 \) and neglect a relativistic correction to the mass of electron oscillating in the pump field. Thus, relativistic self-focusing of a laser beam is excluded. Relativistic self-guiding effects will be addressed in future publications. We require that the eigenvalue problem has the unique solution decaying at \( |x| \rightarrow \infty \), which gives the transverse profile of the FMC

\[ a_0(x) = u_0 \cosh^{-1}(x / \sigma) \equiv u_0 \psi_0 \]  

(4)

and a relation between the plasma channel depth and width

\[ \Delta \omega_p \sigma / c = 2. \]  

(5)

Also, the eigenvalue equation gives the dispersion relation for the pump field

\[ \lambda_0 = \omega_p^2 / c^2 - k_0^2 = k_p^2 - \sigma^{-2}, \]  

(6)

where \( k_p = \omega_p / c \).

The perturbed hf electric field in a plasma is

\[ \tilde{a}(x, z, t) = \tilde{a}_0 + \text{Re} \left[ a_s(x, z, t) e^{-i \omega_s t + i k_p z} \right], \]  

(7)

where \( a_s = eE_s / (\omega \psi_0 c) \) is a complex amplitude of the normalized electric field of backscattered radiation, \( \omega_s = \omega_p - \omega, k_s = -k_0 + k_p \). In the case of rarefied plasma \( \omega_p \ll \omega_0 \), which is considered here, the RBS is a resonant process in which only the Stokes component of scattered radiation is involved [see Eq. (4)]. The amplitudes \( a_s \) are slowly varying in time and space on the scales \( \omega_p^{-1} \) and \( k_0^{-1} \), respectively. We shall consider the weakly coupled RBS, whose temporal increment is smaller than the electron Langmuir frequency (which is valid at \( u_0 < \sqrt{\omega_p / \omega_0} \)). Hence, the envelope of scattered radiation is slowly varying in time and in the direction of propagation \( z \): \( \partial a_s / \partial t \ll \omega_p^2 |a_s| \), \( \partial a_s / \partial z \ll k_p^2 |a_s| \).

We neglect the ion density perturbation produced by the laser and scattered radiation. We consider ions to be fixed neutralizing positive background in the form of a channel with a density profile given by Eq. (1). This assumption is adequate for laser pulses shorter than an ion plasma period \( (\tau_L \ll 2 \pi / \omega_p) \). In the opposite limit of long pulses, various parametric instabilities have been analyzed previously. Ponderomotive force due to the interference of incident and scattered radiation excites perturbations of electron density, \( \delta n_e = n_e - n_0 \).

\[ \delta n_e(x, z, t) = \text{Re} \left[ \delta n_e(x, z, t) e^{-i \omega_p t + i k_p z} \right] \]  

(8)

where \( k_p = k_0 - k_s \). In a rarefied plasma, the amplitude \( \delta n_e \) of the scattering EPW varies in space slowly on the scale \( \omega_p^{-1} \). Moreover, for the regime of weak coupling, this amplitude is slowly varying on plasma temporal and spatial periods: \( \partial \delta n_e / \partial t \ll \omega_p^2 |\delta n_e|, \partial \delta n_e / \partial z \ll k_p^2 |\delta n_e| \).

The amplitudes of scattered EM wave and scattering EPW obey the coupled-modes equations, which follow from the equations of nonrelativistic hydrodynamics of electron fluid in the hf EM field and Maxwell’s equations for the scattered radiation,

\[ \left[ \frac{\partial}{\partial t} - c \frac{\partial}{\partial z} + i k_p^2 \left( \mathcal{L}_0 - \lambda_s \right) \right] a_s = \omega_p G_1 \psi_0 \nu, \]  

(9a)

\[ \left[ \frac{\partial}{\partial t} - i \omega_p \left( \Delta \omega_p / 2 \right) \left( 1 - \psi_0^2 \right) \right] \nu = \omega_p G_2 \psi_0 \psi_s, \]  

(9b)

where \( \nu = \omega_p / \omega_0 \) \( |\delta n_e| / n_0(x) \) \( G_1 = u_0 / (4 i \omega_0) \), \( G_2 = i \omega_p^2 u_0 / n_0 \), \( \tilde{\omega}_s = \omega_s / \psi_0 \), \( \bar{\omega}_s = \omega_p / \psi_0 \), and \( \lambda_s = (\omega_0 / c)^2 - k_0^2 \). The difference between \( \lambda_0 \) and \( \lambda_s \) is eliminated from all the further equations by the substitution \( a_s, \nu \ll \exp(icz(\lambda_0 - \lambda_s) / 2 \omega_s) \). The set (9) is valid under assumption that the short-wavelength scattering plasma wave \( k \approx 2 k_0 \gg k_p(1, 2) \) is predominantly electrostatic, with the EM component of the plasma wake neglected. We scale time and space to \( \omega_p^{-1} \) and \( k_p^{-1} \), respectively, and introduce the dimensionless variables \( \bar{t} = \omega_p t, \{ \bar{z}, \bar{x} \} = k_p \{ z, x \} \) (all the overbarred quantities which appear in the equations below are normalized in this way). Eqs. (9) recast now as

\[ \left[ \frac{\partial}{\partial \bar{t}} - \frac{\partial}{\partial \bar{z}} + i k_p^2 \left( \mathcal{L}_0 - \lambda_0 \right) \right] a_s = G_1 \psi_0 \nu, \]  

(10a)

\[ \left[ \frac{\partial}{\partial \bar{t}} - i (\Delta \omega_p / 2) \left( 1 - \nu^2 \right) \right] \nu = G_2 \psi_0 \psi_s. \]  

(10b)
In Sec. III we derive from Eqs. (10) the dispersion relation of RBS in the channel (11).

### III. GENERAL DISPERSION RELATION

Eqs. (10) can be solved by using the Fourier-Laplace transform of the envelopes, \(a_s, \nu \propto \exp(i\omega t - ikz)\). It is convenient to introduce a new variable \(y = \tan(x/\sigma)\), and express \(\psi = \sqrt{1 - y^2}\), \(\omega_p = 1 + C^2 y^2\), where \(C^2 = \Delta \omega_p^2 / 2 = 2/\sigma^2\). Plasma occupies the area \(-1 < y < 1\). Expressing \(\nu\) through \(a_s\) from Eq. (10b), inserting it into Eq. (10a), and using Eqs. (10a), (6), we obtain the equation of Eq. (11) comes from the equation for the associated Legendre functions [47].

with \(s = \mu = 1\). The degree \(s = 1\) given, the spectrum of Eq. (12) consists of one discrete level with \(\mu = 1\) and a continuum of modes with \(\mu^2 = - q^2, q\) real. Hence, the full set of solutions of Eq. (11) is composed of the single FMC \(P_1^1(y) = \psi_0\) and the set of CMC

\[
P_k^{\pm i q}(y) = \frac{\tanh(x/\sigma) \mp i q}{(1 \pm i q) \Gamma(1 \pm i q)} e^{\pm i q y / \sigma} = \frac{1}{1 \mp i q} \frac{1 + y}{1 - y}^{\pm i q / 2}. \tag{13}
\]

At \(|x| \to \infty\), the CMC [18] reveal the cosine-like behavior [see Eqs. (A2)]. Unlike the pump field given in the form of the FMC, the scattered radiation in a channel is not necessarily represented by FMC only. The CMC [18] describe the discrepancy between the true transverse field profile and the FMC. Therefore, we express the general solution of Eq. (11) as a mode expansion

\[
a_s(y) = \int_{-\infty}^{+\infty} P_1^1(y) a_s(q) dq. \tag{14}
\]

Knowing that \(L_{1,1}^1 P_1^1 = 0, L_{1,1}^2 P_1^1(y) = -(1 + q^2)(1 - y^2)^{-1} P_1^q(y)\), we arrive at the equation

\[
\int_{-\infty}^{+\infty} P_1^q(y) \left( \frac{1 + y^2}{1 - y^2} a_s(q) dq \right) = -\bar{a}^2 \left( \frac{p^2}{1 - y^2} + \frac{G}{\omega} G(\bar{\omega}, y) \right) \left( P_1^1(y) + \int_{-\infty}^{+\infty} P_1^q(y) a_s(q) dq \right). \tag{15}
\]

We multiply Eq. (15) by \(P_1^1(y)\) and integrate it over \(y\). Having in mind that \(\int_{-1}^{1} P_1^q(y)(1 - y^2)^{-1/2} dy = 0\) [formula (7.132.1) of Ref. [41]], we arrive at the dispersion equation

\[
p^2 + \frac{G}{2\omega} \tilde{Q}(\bar{\omega}) = -\frac{G}{2\omega} \int_{-\infty}^{+\infty} F(\bar{\omega}, q) a_s(q) dq, \tag{16}
\]

We multiply Eq. (15) by \(P_1^1(y)\) and integrate it over \(y\). Having in mind that \(\int_{-1}^{1} P_1^q(y)(1 - y^2)^{-1/2} dy = 0\) [formula (7.132.1) of Ref. [41]], we arrive at the dispersion equation

\[
\tilde{Q}(\bar{\omega}) = \int_{-1}^{1} (1 - y^2) G(\bar{\omega}, y) dy = \frac{2}{B^2} \left( 1 - \frac{2C^2}{3} + \frac{C^2}{B^2} \right) - \frac{(B^2 + C^2)(1 - B^2)}{B^4} \ln \left( \frac{1 + B}{1 - B} \right). \tag{18}
\]

It has been shown previously [42] that the function (18) describes a purely electrostatic response of a plasma in the limit of a broad shallow channel, that is \(k_x \sigma \gg 1\). In the case of RBS \(k_x \approx 2k_0\). Thus, any channel with \(\sigma >
$k_0^{-1}$ is wide, and the electrostatic description is always applicable to the plasma response in the RBS process. In order to find a closed-form dispersion equation, we have to determine explicitly the CMC amplitudes $a_s(q)$ in Eq. (10). We multiply Eq. (15) by $P^{-iq}(y)$ and integrate it over $y$. Using the orthogonality condition (41) and the identity $\Gamma(1+iq)\Gamma(1-iq) = \piq/\sinh(\piq)$, we obtain the expression

$$a_s(q) = -\tilde{\sigma}^2 \left( \frac{G}{2\omega} \right) \frac{q/\sinh(\piq)}{1 + (\tilde{\sigma}p)^2 + q^2}$$

$$\times \left[ F(\tilde{\omega}, -iq) + \int_{-\infty}^{+\infty} F(\tilde{\omega}, q, q')a_s(q')dq' \right],$$

where the kernel $F(\tilde{\omega}, y)P^{-iq}(y)dy$ describes the coupling between the modes of continuum with different indices $q$. Further, we take into account only the coupling between the FMC and CMC, and drop the last term in brackets in the RHS of Eq. (19). The resulting dispersion relation reads

$$p^2 + \frac{G}{2\omega} \tilde{Q}(\tilde{\omega}) = \tilde{\sigma}^2 \left( \frac{G}{2\omega} \right)^2 \Phi(\tilde{\omega}, p),$$

$$\Phi(\tilde{\omega}, p) = \int_{-\infty}^{+\infty} \frac{F(\tilde{\omega}, iq)F(\tilde{\omega}, -iq)}{1 + (\tilde{\sigma}p)^2 + q^2} \frac{q dq}{\sinh(\piq)}.$$  

Eq. (20a) describes the spatio-temporal evolution of initial perturbations of field and electron density $\tilde{\omega}(k)$. Real and imaginary parts of the complex frequency $\tilde{\omega}$ as a function of a real wavenumber shift $k$ determine the temporal evolution of spatial Fourier harmonics of the signal, and can be used to solve the initial-value problem. In the next Section, the complex solution $\tilde{\omega}(k)$ of the dispersion equation (20a) (with $k$ real) will be found in the limit of strong plasma wave localization (SPWL). The SPWL is achieved when the channel depth $\Delta\tilde{\omega}_p^2$ is much larger than the instability bandwidth, i.e. $\Delta\tilde{\omega}_p^2 \gg |\tilde{\omega}|$, or $|B|^2 \gg 1$. The unstable EPW will be found confined in the near-axis area with a transverse extent of about $|B|^{-1}$. For this regime, the contribution of CMC to the growth rate can be important.

IV. SOLUTION OF DISPERSION RELATION IN THE SPWL REGIME

A. Temporal increment of instability

The parameter area for the SPWL regime is prescribed by the inequality $|B|^2 > 1$, or $|\tilde{\omega}| < (\Delta\tilde{\omega}_p/2)^2$. Taking the estimate $|\tilde{\omega}| \sim \tilde{\omega}_{\text{hom}} = \left(\gamma_{\text{hom}}/2\right)|\omega|$, we establish the limitation on the laser amplitude from above

$$u_0 < U_0 \equiv \Delta\tilde{\omega}_p^2/(2\sqrt{\tilde{\omega}}).$$

Eq. (21) provides the same scaling in $\tilde{\omega}$ as the condition of weak coupling $u_0 < 1/\sqrt{\tilde{\omega}}$ for the RBS in homogeneous plasmas $\tilde{\omega}$. In the SPWL regime, the unstable electron density perturbations are localized stronger than the EM waves. The size of the area of localization is $\tilde{\omega} \sim 2\tilde{\sigma}/|B|$. To evaluate the RHS of the dispersion equation (20a) in the limit $|B|^2 \gg 1$, we note that the integral (17) that determines the kernel of Eq. (20a) can be represented as $F(\tilde{\omega}, iq) = \int_{-\infty}^{+\infty} F(y, iq)(1 - B^2y^2)^{-1} dy$, where $F(y, iq) = (1+C^2y^2)P_{-iq}(y)$ is independent of $B$. At $|B|^2 \gg 1$, only $y$ values from the close vicinity of channel axis, that is, $|y| < |B|^{-1}$, will contribute to the integral. Therefore, effective coupling between the FMC and CMC occurs near the channel axis in the area with a transverse extent of order $|B|^{-1}$. In this case, the RHS of Eq. (20a) can be expanded in powers of $B^{-1}$. To evaluate the lowest-order term of the expansion, the on-axis value of $F(y, iq)$ is taken, i.e., $F(0, iq) = -iq/[(1 - iq)\Gamma(1 - iq)]$. Then, the integral (20a) becomes

$$\Phi(1 + |B|^2 \gg 1) \approx \frac{1}{\pi} \int_{-1}^{1} \frac{dy}{1-B^2y^2} \int_{-\infty}^{+\infty} \frac{q dq}{1 + (\tilde{\sigma}p)^2 + q^2} \ln \left( \frac{1 + B}{B} \right) \approx \frac{\pi^2}{2} \frac{1 + (\tilde{\sigma}B)^2}{(\tilde{\sigma}B)^2}.$$ 

Evaluating the plasma response function in the limit of large $|B|$ as $\tilde{Q}(\tilde{\omega}) \approx i\tilde{\omega}/B$, we reduce Eq. (20a) to a relatively simple form:

$$p^2 + \frac{G}{2\omega} \frac{i\tilde{\omega}}{B} = \left( \frac{G}{2\omega} \right)^2 \left( \frac{\pi}{(\tilde{\sigma}B)^2} \right)^2 \left( 1 - \sqrt{1 + (\tilde{\sigma}p)^2} \right).$$

The RHS of this equation represents the contribution from the CMC which cannot be $\text{a priori}$ neglected. The generic SPWL regime can be subdivided into the single- and multi-mode sub-regimes. Below, we find the boundary between them and their spectral features.

Under the condition $|\tilde{\sigma}p|^2 \sim 2\omega_0/|B|^2 < 1$, the CMC contribution may be eliminated from Eq. (20a):

$$p^2 + \left( \frac{G}{2\omega} \right)^2 (\tilde{\omega}/B) = 0.$$  

Eq. (21) describes the interaction of the pump wave (FMC) and the single FMC of scattered light via the
strongly localized EPW. Therefore, the single-mode sub-regime admits an analogy with the three-wave RBS in a homogeneous plasma (these processes, however, have quite different spectral properties). Eq. (25) yields the solution with a maximum imaginary part,
\[ \tilde{\omega}(k = 0) = e^{i\pi/3} \sqrt{9\pi^2/(2\eta)\gamma_{\text{hom}}}, \] (25)
where the parameter \( \eta = \Delta\tilde{\omega}_p^2/(2\gamma_{\text{hom}}) > 1 \) is of the order of \( |B|^2 \). Real part of the solution (25) gives the red-shift of the spectral maximum \( \Delta\tilde{\omega} = \sqrt{\pi^2/(2\eta)\gamma_{\text{hom}}}/2 \). Eq. (21) also predicts a blue-side limitation of the RBS bandwidth at \( \text{Re}\tilde{\omega}_c = -\sqrt{\pi^2/ \eta(\gamma_{\text{hom}})/2} \). From the red side (\( \text{Re}\tilde{\omega} > 0 \)), Eq. (24) gives no limitation, and the spectrum has a tail far extended in this area.

The validity condition \( |B|^2 > 2\tilde{\omega}_0 \gg 1 \) for Eq. (24) separates the SPWL sub-regimes. It is more restrictive than \( |B|^2 > 1 \) and may be written as \( |\tilde{\omega}| < \Delta\tilde{\omega}_p^2/(8\tilde{\omega}_0) \). Substituting the solution (25) into the latter inequality provides the parameter area for the single-mode sub-regime:
\[ u_0 < U_1 = U_0/|\sqrt{\pi^2/(2\tilde{\omega}_0)}|^{3/4} \ll U_0. \] (26)

The larger \( u_0 \), the stronger the instability is driven, and the larger is the population of CMC excited. When \( u_0 \) falls within the interval
\[ U_1 < u_0 < U_0, \] (27)
the CMC contribution is no more negligible, and the RBS process becomes essentially multi-mode. The RHS of Eq. (24) cannot be omitted then, and the dispersion equation (24) is solved numerically.

In Fig. 1 we present an example of the dispersion curves obtained for the RBS in both single- and multi-mode SPWL regimes. For the fixed values of normalized laser frequency \( \tilde{\omega}_0 = 10 \) and amplitude of electric field \( u_0 = 0.007 \) the increment \( \text{Im}\tilde{\omega} \) is found numerically versus \( \text{Re}\tilde{\omega} \) for two different values of the channel depth: (I) \( \Delta\tilde{\omega}_p = 2 \), or \( n_e(x = 0)/n_e(|x| = \infty) = 1/3 \), and (II) \( \Delta\tilde{\omega}_p = 1/\sqrt{2} \), or \( n_e(x = 0)/n_e(|x| = \infty) = 0.8 \). In Fig. 1 the solution of full Eq. (23) for the set of parameters (I) is plotted with the curve (1). The curves (2) and (3) are obtained via numerical solution of Eq. (24) for the set of parameters (II), with the CMC contribution deducted in the case of the curve (2). The reference spectrum of RBS in a homogeneous plasma is presented in Fig. 1 by the curve (4).

Parameter sets (I) and (II) correspond to the SPWL regime as the condition \( u_0 \ll U_0 \) is very well satisfied for both. For the set (I) the inequality \( u_0 < U_1 \approx 0.019 \) holds, and the single-mode regime is the case. Contribution of the CMC is negligibly small: numerical solutions of the full [Eq. (23)] and single-mode [Eq. (24)] dispersion equations coincide within the thickness of the line (1). Such a good coincidence is not the case for the parameter set (II), which corresponds to the multi-mode regime \( (u_0 > U_1 \approx 0.0047) \). Comparison between the curves (2) and (3) demonstrates considerable enhancement of RBS due to the CMC contribution: the CMC correction to the peak increment amounts to about 25%. The basic characteristics of RBS in the multi-mode SPWL regime can be summarized as follows:

1. The peak growth rate is reduced if compared with the case of homogeneous plasma.
2. Contribution from the CMC enhances the scattering process.
3. The RBS bandwidth inside a channel is significantly larger than in a homogeneous plasma.
4. The frequency spectrum experiences overall red-shift from the Stokes frequency \( \omega_0 - \omega_p \).

The last two features are clear advantages of the SPWL regime for the Raman amplification of short pulses in plasmas. Due to the broadband nature of the process, exact tuning the signal frequency to \( \omega_0 - \omega_p \) is not necessary to get a considerable amplification rate in the linear regime. We can also estimate here the RBS growth rate modification for the parameters typical of a channel-guided laser driven accelerator. The plasma channel created in the experiment was capable of single-mode guiding of the laser pulse with a radius 8μm at the level \( e^{-2} \) in intensity, which under ansatz (4) gives \( \sigma = 4.65\mu \text{m} \). The electron density at the bottom of the channel \( n_1 = 4 \times 10^{18} \text{ cm}^{-3} \) gives \( k_{pl}\sigma = 1.75 \) and, according to the matching condition (5), provides the effective normalized electron density depression \( \Delta\tilde{\omega}_p^2 \approx 1.3 \).
(we have to mention that an actual channel shape is a plasma column with a density depression at the axis surrounded by the walls which are few laser wavelengths thick and approximately twice the bottom density; as the walls density is much below critical, we assume that the main contribution to the guiding effect is made from the near-axis density profile which admits the approximation in the form (with the effective channel depth just calculated). The laser wavelength \( \lambda_0 = 0.8 \mu m \) gives \( \omega_0 \approx 20 \). To fall within the SPWL regime of RBS the guided laser pulse must possess the intensity \( I < 4 \times 10^{14} \text{ W/cm}^2 \) to be in accordance with the limitation \( I \leq \omega_0 \approx 0.14 \) [see Eq. (29)]. For \( I \approx 3 \times 10^{14} \text{ W/cm}^2 \) \( \approx 0.012, \approx 0.028 \omega_0 \) RBS proceeds in the multi-mode SPWL regime with \( \eta \approx 24 \). The increment formally evaluated from Eq. (27) gives the peak increment \( \gamma \approx 0.5 \gamma_{\text{hom}} \) whereas taking account of the CMC contribution increases it to \( 0.8 \gamma_{\text{hom}} \). Therefore, in the regime considered, 20% reduction of the peak RBS increment can be expected in comparison with the case of homogeneous plasma, and the contribution from CMC to the peak increment amounts to 30% of \( \gamma_{\text{hom}} \).

**B. Transverse profile of scattered radiation in multi-mode SPWL regime**

Results of the previous Subsection show high sensitivity of the RBS to the transverse structure of scattered radiation in a channel: taking account of CMC increases the value of the growth rate. Here, we find the CMC-related correction to the transverse profile of scattered radiation. For the parameters of the case (II) [the curve (3) in Fig. 1] we evaluate the integral \( \int (|a_s(\bar{x})|^2) \) numerically for the fastest growing mode, and present the transverse distributions of intensity \( |a_s(\bar{x})|^2 \) in Fig. 2. The solid curve is given by Eq. (14). The reference intensity profile of the FMC is plotted with a dashed line. The coupling between the FMC and CMC transversely compresses the scattered radiation beam. To evaluate the effective compression numerically, we define the rms beam size as \( \bar{x}^2 = \left( \int_{-\infty}^{\infty} x^2 |a_s(\bar{x})|^2 \right) \). For the intensity profiles with and without CMC shown in Fig. 2 the ratio of the rms sizes is \( \sqrt{\langle x_{\text{FMC}}^2 \rangle / \langle x_{\text{FMC}}^2 \rangle} \approx 0.77 \). So, the numerical example shows 23% transverse compression of the intensity profile reached in the multi-mode regime. Simultaneously, for the parameters chosen, power of the scattered radiation remains almost unchanged: difference between the integrals \( \int_{-\infty}^{\infty} |a_s(\bar{x})|^2 \) calculated for the FMC and FMC + CMC is about 3%. Hence, in the multi-mode SPWL regime, coupling between FMC and CMC compresses the scattered radiation near channel axis where the pump field has maximum. As a consequence, increase in the peak increment occurs. For the estimates made at the end of previous Subsection the compression was about 30% and intensity enhancement on the axis about 40%.

**V. SPATIO-TEMPORAL EVOLUTION OF BACKSCATTERED PULSE IN THE SINGLE-MODE SPWL REGIME**

**A. Group velocity of scattered radiation**

In this Section we address the linear Raman amplification of EM wave packet in a plasma channel. First, we evaluate the group velocity of backscattered light, i.e., \( v_g \equiv c \left( \text{Re} \omega / \partial k \right) \) at \( \text{Im} \omega / \partial k = 0 \). By definition, thus calculated group velocity determines the speed of the pulse peak. On substituting \( \omega = \Omega + i \gamma \) with \( \Omega \) and \( \gamma \) real into the dispersion relation \( \omega = \sqrt{\Omega^2 + k^2 \omega^2} \), we separate real and imaginary parts of the equation, exclude \( \gamma \), and obtain the algebraic relation which defines implicitly the dispersion function \( \Omega(k) \):

\[
\tilde{k}^2 + 4 \Omega + 3 \Omega^2 - \frac{\Omega^2 + 2 \tilde{k} \Omega^2 + \tilde{k}^2 \Omega + b}{3 \Omega + 2 \tilde{k}} = 0
\]

Differentiating Eq. (28) with respect to \( \tilde{k} \), and plugging into the resulting equation \( \tilde{k} = 0 \) and \( \Omega(0) = \text{Re} \omega \) from Eq. (29) (i.e., the wave number and real part of frequency corresponding to the peak growth rate), we find the group velocity \( v_g = -2c/3 \) (the minus sign means that the amplified pulse moves in the backward direction).

The absolute value of the group velocity in a channel is higher than in a homogeneous plasma, where \( v_{g\text{hom}} = -c/2 \). To get a qualitative interpretation of
this effect we return to the basic equations \([10]\). Taking the single-mode approximation for the scattered light envelope \(a_s(\bar{x}, \bar{z}, \bar{t}) \approx A_s(\bar{x}, \bar{t})v_0(\bar{x})\), initial condition \(v(\bar{t}=0) \equiv 0\), and the plasma response function \(Q(\bar{\omega}) \equiv \pi i/B(\bar{\omega}) = (2\pi/\Delta \omega_p)\sqrt{\bar{\omega}}\), we reduce the set \([10]\) to the single equation \((\partial_t - \partial_z)A_s \approx u_0[\Delta \omega_p/(16 \omega_0)](\bar{N}(\bar{t}))\), where the overlap integral between the scattering electron density perturbation and transverse intensity profile of the FMC \((\bar{N}(\bar{t})) \equiv \int_{-\infty}^{\infty} \psi_0^2(\delta n^*_s/n_0) \, dx = u_0\sqrt{\pi\bar{\omega}}[2\omega_0/\Delta \omega_p]^2 \int_{-i}^{i} A_s(\tau)/\sqrt{t - \tau} \, d\tau\) describes the effective plasma response. The transverse shear of the plasma density produces an effective decay of the plasma response expressed in terms of the convolutions \((\bar{N}(\bar{t})) \propto A_s(\bar{t}) \ast \bar{F}^{-1/2}\) [compare with the nondamped case of homogeneous plasma, where \((\partial_t - \partial_z)a_s = \gamma^2_{\text{hom}} \int_{-i}^{i} a_s(\tau) \, d\tau \equiv \gamma^2_{\text{hom}} a_s(\bar{t}) + 1\)]. Therefore, the tail of the amplified signal experiences the growth rate reduction according to \(A_s(\bar{t}) \propto \bar{N}(\bar{t}) \propto A_s(\bar{t}) \ast \bar{F}^{-1/2}\), and, consequently, the signal maximum moves closer to the signal leading front than in a homogeneous-plasma case. This argument qualitatively explains the increase in the group velocity of scattered light in a channel.

### B. Evolution of EM wave packet

We solve here the initial-value problem for the equations \([10]\)

\begin{align}
    a_s(\bar{x}, \bar{z}, \bar{t} = 0) &= a_{s0}(\bar{z})v_0(\bar{x}), \\
    \bar{v}(\bar{x}, \bar{z}, \bar{t} = 0) &= 0.
\end{align}  

(29a-b)

which specifies the EM wave packet initially matched with an unperturbed plasma channel. We naturally take account of these initial conditions by introducing a new dependent variable \(a^1_s(\bar{x}, \bar{z}, \bar{t}) = a_s(\bar{x}, \bar{z}, \bar{t})H(\bar{t})\) [where \(H(\bar{t})\) is the Heaviside step-function]. In the characteristic variables \(\theta = -\bar{z}\) and \(\eta = -\bar{z} - \bar{t}\), the set \([10]\) reads

\[
\begin{align*}
    \left[ \frac{\partial}{\partial \theta} + \frac{i}{2\bar{\omega}_s} (\bar{L}_0 - \bar{\lambda}_0) \right] a^1_s - G_1\psi_0 \nu &= a_{s0}(\theta)\psi_0 \delta(\theta - \eta), \\
    \left[ \frac{\partial}{\partial \eta} - i\frac{\Delta \omega^2}{4\nu} (1 - \psi_0^2) \right] \nu + G_2\psi_0 a^1_s &= 0,
\end{align*}
\]

where \(\delta(t)\) is the Dirac delta-function. We apply the Fourier transform with respect to the co-moving “spatial” variable \(\eta\), assuming \(\delta(\theta - \eta) = (2\pi)^{-1} \int_{-\infty}^{\infty} \exp[ik(\eta - \theta)] \, dk\), and then exclude the Fourier image \(\nu(\bar{x}, \theta, k)\). Multiplying the resulting equation for the Fourier image \(a^1_s(\bar{x}, \theta, k)\) by \(\psi_0(\bar{x})\) and then integrating over \(\bar{x}\) we obtain the averaged equation

\[
\begin{align*}
    \frac{1}{\langle \psi_0^2 \rangle} \left\{ \langle \psi_0 \partial^1 a^1_s / \partial \theta \rangle + \frac{i}{2\bar{\omega}_s} \left( \langle \psi_0 (\bar{L}_0 - \bar{\lambda}_0) a^1_s \rangle \right) \right. \\
    - G_\nu \left[ \frac{1 + C^2(1 - \psi_0^2)}{k [1 - B^2(1 - \psi_0^2)]} a^1_s \right] \biggr\} = a_{s0}(\theta)e^{-i\theta^2},
\end{align*}
\]

(30)

where \(B^2 \equiv -\Delta \omega^2/(4k)\). In a generic case, transverse profile of the signal \(a^1_s(\bar{x})\) is a superposition of the FMC and CMC. In this Section, we consider the single-mode SPWL regime of Raman amplification and, for all instants of time, take the wave packet in the form of FMC [hence, \((\bar{L}_0 - \lambda_0)a^1_s = 0\)]. We substitute \(a^1_s(\bar{x}, \theta, k) = A(\theta, k)v_0(\bar{x})\) into Eq. (30) and get finally

\[
\frac{\partial A}{\partial \theta} + \gamma^2_{\text{hom}} \left( \frac{Q(k)}{2i} \right) A = a_{s0}(\theta)e^{-ik\theta}. 
\]

(31)

Here, the plasma response function [compare to Eq. (18)] reads

\[
Q(k) = \int_{-1}^{1} \frac{1 + C^2 y^2}{1 - B^2 k^2(1 - y^2)} \, dy.
\]

(32)

In the SPWL regime, relatively large wave number shifts, \(|k| \gg (\Delta \omega_p/2)\) (that is, \(|Bk| \gg 1\)), contribute to the signal evolution. Hence, the main contribution to the integral \([32]\) is made by the integrand values in the vicinity of \(y = 0\) (or \(|y| \ll |Bk|^{-1}\)). Approximating the integrand as \((1 - B^2 k^2)^{-1}\) we find the approximate response function \(Q(k) \approx -2\pi \bar{F}/\Delta \omega_p\) which allows to present Eq. (31) in the form

\[
\left( \frac{\partial}{\partial \theta} + i\bar{F} k^{-1/2} \right) A = a_{s0}(\theta)e^{-ik\theta},
\]

(33)

where \(\bar{F} = \pi \gamma_{\text{hom}} \sqrt{\gamma_{\text{hom}}/(2\eta)}\). Solution of Eq. (33) reads

\[
A(\theta, k) = \int_{0}^{\infty} a_{s0}(\theta_1 - \theta)e^{-ik(\theta_1 - \theta)} - i\bar{F} \theta_1 / \sqrt{k} \, d\theta_1. 
\]

(34)

Inverting the Fourier transform \([43]\) and returning to the lab-frame variables, we find the longitudinal evolution of the signal, \(A(\bar{x}, \bar{t}) = \int_{0}^{\infty} a_{s0}(\theta_1 + \bar{z}) \bar{D}(\theta_1 - \theta_1)d\theta_1\), where the the Green function of RBS in the single-mode SPWL regime, \(\bar{D}(\theta_1 - \theta) = (2\pi)^{-1} \int_{-\infty}^{\infty} \exp[ik(\theta_1 - \theta) - i\bar{F} \theta_1 / \sqrt{k}]d\theta_1\), can be found explicitly in terms of a generalized hypergeometric function \(1F_3\) \([44]\):

\[
\bar{D}(\theta_1 - \theta) = \delta(\theta_1 - \theta) + H(t(\theta_1)) (F \bar{F})^2 \times 1F_3 \left[ \frac{1}{2}; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; k (F \bar{F})^2(t(\theta_1)/4) \right].
\]

(35)

Solution of the initial-value problem expressed explicitly in terms of the Green function \([45]\) reads
VI. CONCLUSION

We have investigated the RBS of laser radiation in the regime of strong plasma wave localization (SPWL) in a plane plasma channel, which can support only one trapped EM mode (the single-mode channel). For the SPWL regime, transverse variation of the plasma frequency exceeds the RBS bandwidth calculated for a homogeneous plasma with an on-axis electron density, and the scattering plasma wave is localized stronger than a driving beatwave of pump and scattered radiation. In this case, the transverse profile of the scattered radiation is a superposition of the fundamental mode of a channel (FMC) and a continuum of non-bound modes of channel (CMC). Depending on the plasma and laser parameters, the RBS with SPWL can proceed either in a single- or multi-mode regime. In the single-mode case, excitation of the CMC is suppressed almost completely, and only the FMC of scattered radiation is involved in the process. This allows for a physical analogy between the single-mode SPWL regime of RBS in a plasma channel and a three-wave RBS in a homogeneous plasma. In the multi-mode SPWL regime, the CMC play an essential role. The CMC contribution transversely shrinks the scattered radiation beam, and increases the growth rate of the instability. Spectral features of both sub-regimes are qualitatively similar. The temporal growth rate is slower, and amplification bandwidth is greater than in the case of homogeneous plasma, and the frequency spectrum experiences an overall red-shift. The group velocity of scattered radiation, \( v_g \approx 2c/3 \), is increased versus its homogeneous-plasma value, \( v_{g \text{hom}} = c/2 \). All these features are the consequences of the transverse shear of the electron plasma density. The broadband nature of RBS in the SPWL regime and relatively high group velocity of scattered radiation are the features profitable for the Raman amplification of short pulses in deep plasma channels.

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APPENDIX A: PROPERTIES OF ASSOCIATED LEGENDRE FUNCTIONS WITH IMAGINARY ORDER

The associated Legendre functions \( P^\pm iq(y) \), where \( q \) is a positive real number, are the solutions of Eq. \((12)\) with \( s = 1 \) and \( \mu = \pm iq \). The orthogonality condition for \( P^\pm iq(y) \) reads

\[
\frac{1}{2\pi} \int_{-1}^{1} P_1^iq(y)P_1^{-iq'}(y) \frac{dy}{1-y^2} = \frac{\sinh(\pi q)}{\pi q} \delta(q' - q),
\]

which is evaluated using the substitution \( y = \tan(\varphi/\sigma) \), formulas \((3.982)\), \((3.987.1)\), and \((3.981.6)\).
The cosine function can be constructed of $P_1^{\pm i q}(x)$ by summing up the even complex conjugate solutions

$$P_1^{\pm i q}(x) = \frac{1}{2}(P_1^{i q}(x) + P_1^{i q}(-x))$$

and have the asymptotic

$$P_1^{i q} \left( \frac{x}{\sigma} \to +\infty \right) \sim \frac{e^{ixq/\sigma}}{\Gamma(1 - iq)},$$

(A2a)

$$P_1^{i q} \left( \frac{x}{\sigma} \to -\infty \right) \sim \left( \frac{iq + 1}{iq - 1} \right) e^{-ix|q|/\sigma} \frac{1}{\Gamma(1 - iq)}.$$  (A2b)

The cosine function can be constructed of $P_1^{\pm i q}(x)$ by summing up the even complex conjugate solutions $P_1^{\pm i q}(x)$.

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