Synthesizing of a sliding control law for a hydraulic automatic stabilization system

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Abstract. The sliding mode control has the property of control independence from the characteristics of the unchangeable part of the automatic stabilization system (ASS). In the article, this mode is proposed to be applied to ASS to stabilize the horizontal working plane using two hydraulic cylinders. A model of this ASS has been developed. The paper synthesizes a sliding control law for ASS and simulates ASS operation on MATLAB-SIMULINK. The simulation results indicate that ASS is a stable system and works quite efficiently.

1. Introduction

Automatic stabilization systems (ASS) are widely used in many branches of science, techniques and industry. ASS is practically used on all vehicles: cars, ships, airplanes, etc. These vehicles always have various working equipment that must be protected from vehicle vibrations and noise. ASS are quite successful in solving such problems. Hydraulic ASS are of particular interest for research. Hydraulic ASS have the following advantages [1]: compact structure, high performance, fast response, reliable operation in combination with a sliding mode controller with a wide range of regulation, high noise immunity, the control signal is calculated accurately [2, 3].

The purpose of this article is synthesizing of a sliding control law for the automatic stabilization system (ASS) of a horizontal working plane located on two hydraulic cylinders. To achieve this goal, a mathematical model of the ASS is developed and the sliding control law for the ASS is synthesized on its basis. Testing of the sliding mode control of the ASS was performed in the MATLAB SIMULINK software environment [5, 6].

2. Model of ASS with two electro-hydraulic cylinders

The model of ASS is depicted in Figure 1, where the OAB plane is the working plane (the plane on which the working equipment M with weight P should be placed). Each side of a rectangular plane has lengths $a$ and $b$, respectively (see Figure. 1). The point O on the working plane is the origin of the Oxyz coordinate system. The O’A’B’ plane is the floor surface of a moving vehicle (for example, the floor plane of a car). The initial distance between the floor plane of the car and the working plane is $h$. 
The working plane is connected to the floor plane of the vehicle at points O, A, B through a vertical rack OO' and two hydraulic cylinders AA' and BB' (2 hydraulic cylinders have initial lengths $\ell_1, \ell_2$). The OAB plane can rotate around the Ox and Oy axes, but cannot rotate around the Oz axis, couplings A, B, A', B' are ball joints that can freely rotate around the Ox, Oy, Oz axes.

In the Oxyz coordinate system, the Ox and Oy axes are parallel to the floor, and the Oz axis is perpendicular to the floor. The model works on the principle of changing the length of the hydraulic cylinders $\ell_1, \ell_2$ so that the OAB plane rotates around the Oy and Ox axes.

![Figure 1. Geometric model of a balancing system with two electro-hydraulic cylinders.](image)

Let $\theta_1, \theta_2$ be the angles of rotation of the plane OAB around the axis Oy and Ox, respectively, in the coordinate system Oxyz, we have the following transformation matrix [7]:

$$
\begin{align*}
R_Y(\theta_1) &= \begin{bmatrix}
\cos \theta_1 & 0 & -\sin \theta_1 \\
0 & 1 & 0 \\
\sin \theta_1 & 0 & \cos \theta_1
\end{bmatrix}, &
R_X(\theta_2) &= \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos \theta_2 & \sin \theta_2 \\
0 & -\sin \theta_2 & \cos \theta_2
\end{bmatrix},
\end{align*}
$$

(1)

where $R_Y(\theta_1)$ is the transformation matrix when the OAB plane is rotated around the Oy axis by an angle $\theta_1$, $R_X(\theta_2)$ is the transformation matrix when the OAB plane is rotated around the Oy axis by an angle $\theta_2$.

Let $R$ be the transformation matrix when rotating OAB around the axis Oy and Ox by the corresponding angles $\theta_1, \theta_2$, then:

$$
R = R_X(\theta_2)R_Y(\theta_1).
$$

(2)

Substituting (1) into (2), we get:

$$
R = \begin{bmatrix}
\cos \theta_1 & 0 & -\sin \theta_1 \\
\sin \theta_1 \sin \theta_2 & \cos \theta_2 & \cos \theta_1 \sin \theta_2 \\
\cos \theta_1 \sin \theta_2 & -\sin \theta_2 & \cos \theta_1 \cos \theta_2
\end{bmatrix}
$$

(3)

Let $A_1, B_1$ be the corresponding coordinates of $A$ and $B$ after rotating OAB plane around the axis Oy and Ox, the corresponding angles are $\theta_1, \theta_2$, then:

$$
A_f = RA; \quad B_f = RB.
$$

(4)
where:

\[ A = \left[ a, \frac{b}{2}, \theta \right]^T, \quad B = \left[ a, \frac{-b}{2}, \theta \right]^T. \]  

Combining (3), (4) and (5), we get:

\[ A_1 = \begin{bmatrix} a \cos \theta_1 \\ \frac{b}{2} \cos \theta_2 + a \sin \theta_1 \sin \theta_2 \\ a \cos \theta_2 \sin \theta_1 - \frac{b}{2} \sin \theta_2 \end{bmatrix}, \quad B_1 = \begin{bmatrix} a \cos \theta_1 \\ a \sin \theta_1 \sin \theta_2 - \frac{b}{2} \cos \theta_2 \\ a \cos \theta_2 \sin \theta_1 + \frac{b}{2} \sin \theta_2 \end{bmatrix}. \]

After rotating the OAB plane, angles \( \theta_1, \theta_2 \) now the length of the hydraulic cylinders, respectively are:

\[ \ell_1 = A_1 A'; \quad \ell_2 = B_1 B'. \]

On the other hand:

\[ A' = \left[ a, \frac{b}{2}, h \right]^T; \quad B' = \left[ a, \frac{-b}{2}, h \right]^T. \]

Combining (6), (7) and (8), we get:

\[ \ell_1^2 = (a - a \cos \theta)^2 + \left( \frac{b}{2} \sin \theta_1 \sin \theta_2 + (h - a \sin \theta_1 \cos \theta_2 + \frac{b}{2} \sin \theta_2)^2 \right. \\
\ell_2^2 = (a - a \cos \theta)^2 + \left( \frac{b}{2} \sin \theta_1 \sin \theta_2 + (h - a \sin \theta_1 \cos \theta_2 - \frac{b}{2} \sin \theta_2)^2 \right. \]  

Thus, to rotate the plane OAB by the corresponding angles \( \theta_1, \theta_2 \) around the axes \( \text{Oy and Ox} \), the length of the hydraulic cylinders \( \ell_1, \ell_2 \) is determined according to the system of equations (9).

3. Model of hydraulic cylinder with servo valve

The hydraulic cylinder with servo valve model and its parameters are shown in Figure 2.

![Figure 2. Structural model of hydraulic cylinder with servo valve](image-url)
Table 1: Parameters of model.

| Description                          | Symbol | Description                          | Symbol |
|--------------------------------------|--------|--------------------------------------|--------|
| Mass of the load                     | $m$    | Oil drain coefficient of servo valve | $K_0$  |
| Piston surface area                  | $F_1$  | Spring stiffness                     | $C$   |
| Hydraulic pressure                   | $P_1$  | Flow loss coefficient                | $\lambda$ |
| Gain of servo valve                  | $K_v$  | Volume of oil in the cylinder chamber | $V$ |
| Control current of the servo valve   | $I$    | Modulus of elasticity of hydraulic oil | $B$ |
| Control voltage                      | $u(t)$ | Viscous coefficient oil friction     | $f$   |
| The position of the object $m$       | $y$    | Elasticity coefficient of the gas tank | $C_1$ |
| Flow into hydraulic cylinder         | $Q_1$  | Servo valve inlet pressure           | $P_S$ |
| Flow out of hydraulic cylinder       | $Q_2$  | Servo valve outlet pressure           | $P_T$ |

4. Synthesizing of a sliding control law for the hydraulic system model of reciprocating motion.

Considering the vertical moving hydraulic cylinder with mass $m$, taking into account the influence of gravity $P=m.g$, then we have the following system of equations:

$$
\begin{align*}
\dot{K_v}I - K_0P_1 &= F_1 \frac{dy}{dt} + \frac{V}{2B} \frac{dP_1}{dt} + \lambda P_1 \\
F_1P_1 &= m \frac{d^2y}{dt^2} + Cy + f \frac{dy}{dt} + mg
\end{align*}
\quad (10)
$$

Let us define the state variables of the hydraulic cylinder with servo valve [4 – 6]:

$$
x_1 = y; \quad x_2 = \frac{dy}{dt}; \quad x_3 = P_1.
$$

Then the system of equations (10) will be rewritten as follows:

$$
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -g - \frac{C}{m} x_1 - \frac{f}{m} x_2 + \frac{F_1}{m} x_3 + dt \\
\dot{x}_3 &= \frac{2BK_0}{V} - \frac{2B_k}{V} x_2 - \left(\frac{2Bk_0}{V} + \frac{2B\lambda}{V}\right) x_3
\end{align*}
\quad (11)
$$

The task is to determine the control signal $u(t)$ so that the output signal $y(t)$ corresponds to the input signal $y_{ref}(t)$. The equation of the sliding surface has the following form [9]:

$$
S(\dot{e}) = k_3 \dot{e} + k_2 \dot{\dot{e}} + k_1 \dot{\dot{\dot{e}}} + \ddot{e}^2
\quad (12)
$$

where: $\dot{e} = x_1 - y_{ref}$ (here an error is indicated by an "$e"), $k_1$, $k_2$, $k_3$ are constants chosen in such a way that the characteristic polynomial of the equation $S(\dot{e}) = 0$ satisfies the Hurwitz stability criterion [4 – 6].

Let:

$$
x_1 = \ddot{x}_2 = \frac{2BK_0 F_1 I}{mV} \frac{g + f}{m} \frac{d^2}{dt^2} + \frac{C_f}{m} \frac{d^2}{dt^2} x_1 + \frac{f^2}{m^2} - \frac{C}{m} x_2 - \left(\frac{2BK_0 F_1}{mV} + \frac{F_1 f}{m^2} + \frac{2B\lambda f}{m^2}\right) x_3
\quad (13)$$
From (11), (12) and (13) we have:

\[ S(e) = bu + a_1x_1 + a_2x_2 + a_3x_3 + a_4. \]  

(14)

where:

\[
\begin{align*}
b &= \frac{2BK_1F_1}{K_f m V^2}; \\
a_1 &= \frac{Cm}{m^2} + k_3 - \frac{C}{m}; \\
a_2 &= \frac{f^2}{m^2} - \frac{2F_1^2B}{m} - \frac{k_3}{m} + k_2; \\
a_3 &= \frac{k_3F_1}{m} - \frac{2BK_1F_1}{m^2} - \frac{F_1f}{m^2} - \frac{2Bk_3}{m^2}; \\
a_4 &= \frac{g f}{m} - k_1g - k_3y_{ref} - \frac{f}{m} dt + k_1 dt
\end{align*}
\]

\[
\dot{S}(e) = b\dot{u} + b_1\dot{u} + c_1x_1 + c_2x_2 + c_3x_3 - a_2(g - dt),
\]

(15)

where:

\[
\begin{align*}
b_1 &= \frac{2a_1BK_1}{K_f V}; \\
c_1 &= \frac{a_1C}{m}; \\
c_2 &= \frac{a_1f}{m} - \frac{2a_2F_1B}{V}; \\
c_3 &= \frac{a_3F_1}{m} - \frac{2a_3BK_0}{V} - \frac{2a_3Bk_3}{V}.
\end{align*}
\]

For error \( e \to 0 \) (\( y \to y_{ref} \)) then \( \dot{S}S < 0 \) or does \( S \) have a sign opposite to \( S \) [5, 6, 8]:

\[ S = -K \text{sign}(S). \]

(16)

From (15) and (16) we have:

\[ \dot{u} = -\frac{1}{b}(b_1\dot{u} + c_1x_1 + c_2x_2 + c_3x_3 - a_2g) + K\text{sign}(S). \]

(17)

### 5. Simulation results

| Symbol | Description | Value  | Unit   |
|--------|-------------|--------|--------|
| \( m \) | Mass of the load | 500    | kg     |
| \( F_1 \) | Piston surface area | 31.2   | cm²    |
| \( K_f \) | Gain of servo valve | 10     | -      |
| \( K_0 \) | Oil drain coefficient of servo valve | 2.58e-12 | m³s⁻¹Pa⁻¹ |
| \( C \) | Spring stiffness | 0.1    | Nm⁻¹   |
| \( \lambda \) | Flow loss coefficient | 5.0e-3 | -      |
| \( V \) | Volume of oil in the cylinder chamber | 652.8  | cm³    |
| \( B \) | Modulus of elasticity of hydraulic oil | 0.1    | -      |
| \( f \) | Coefficient of viscous friction of hydraulic oil | 588    | Nsm⁻¹  |

#### 5.1 Simulation results for reciprocating of the hydraulic cylinder using servo valves.

From the sliding surface equation (14) and the control voltage equation (17), we simulate in the MATLAB SIMULINK software [4] with coefficients \( k_1 = 21, k_2 = 298, k_3 = 968, K = 100|S| \). The initial position of an object with mass \( m \) is 15 cm.

From Figures 3, 4, 5 and 6, we see that the system converges quickly in the range from 1.5 to 2 seconds.
5.2 Simulation results for automatic stabilization system of the horizontal working plane using two hydraulic cylinders.

Based on equations (9), equations (14) and (17), the operation of stabilizing the horizontal plane of the ASS using two hydraulic cylinders is simulated with model parameters: length of the working plane \( a = 93.6 \text{ cm} \); width of the working plane \( b = 62.5 \text{ cm} \); height \( h = 72.268 \text{ cm} \).

At the input signal, the angles \( \theta_1, \theta_2 \) are equal to \( \theta_1 = 10^0 \) and \( \theta_2 = 15^0 \), respectively. From Figures 7, 8, we see that the system quickly stabilizes for a period from 1.5 to 2 seconds.
6. Conclusion
In the article, we designed an ASS of the horizontal working plane using two hydraulic cylinders. Synthesizing of a sliding control law for ASS and simulate ASS operation on MATLAB SIMULINK software. The simulation results in MATLAB SIMULINK show that the system stabilizes fairly quickly within 1.5-2 seconds. The output signal is stable and weakly affected by noise. The noted positive properties of the ASS can be explained by the choice of a sliding control mode and the fact that this control mode allows the hydraulic SAS to be endowed with the desired dynamic properties.

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