Inflation in the early universe is now a part of the standard cosmology to solve the horizon and flatness problem as well as to account for the origin of density/curvature fluctuations. It is most commonly driven by a scalar field dubbed as inflaton, and the research on inflationary cosmology has long been focused on the shape of the inflaton potential in the particle physics context. Its underlying physics is now being probed using precision observations of the cosmic microwave background and large scale structure which are sensitive only to the dynamical nature of the inflaton. Reflecting this situation, a number of novel inflation models have been proposed extending the structure of the kinetic function, such as k-inflation, ghost condensate, and DBI inflation. In this Letter, we propose a new class of inflation model, G-inflation, which has a Galileon-like nonlinear derivative interaction of the form \( G(\phi, (\nabla\phi)^2)\phi \) in the Lagrangian with the resultant equations of motion being of second order. It is shown that (almost) scale-invariant curvature fluctuations can be generated even in the exactly de Sitter background and that the tensor-to-scalar ratio can take a significantly larger value than in the standard inflation models, violating the standard consistency relation. Furthermore, violation of the null energy condition can occur without any instabilities. As a result, the spectral index of tensor modes can be blue, which makes it easier to observe quantum gravitational waves from inflation by the planned gravitational-wave experiments such as LISA and DECIGO as well as by the upcoming CMB experiments such as Planck and CMBpol.

Now let us start investigating our model in detail. Assuming that \( \phi \) is minimally coupled to gravity, the total action is given by

\[
S = \int d^4x \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R + L_\phi \right].
\]

The energy-momentum tensor \( T_{\mu\nu} \) derived from the action reads

\[
T_{\mu\nu} = K_X \nabla_\mu \phi \nabla_\nu \phi + K g_{\mu\nu} - 2\nabla_\mu (G_{\nu\lambda} \phi) + g_{\mu\nu} \nabla_\lambda G \nabla_\lambda \phi - G_X \nabla_\phi \nabla_\mu \phi \nabla_\nu \phi.
\]

The equation of motion of the scalar field is equivalent to \( \nabla_\mu T_{\mu\nu} = 0 \). Here and hereafter we use the notation \( K_X \) for \( \partial K / \partial X \) etc.

Taking the homogeneous and isotropic background, \( ds^2 = -dt^2 + a^2(t)dx^2 \), \( \phi = \phi(t) \), let us study inflation driven by the Galileon-like scalar field, which we call “G-inflation.” The energy-momentum tensor has the form \( T_{\mu\nu}^G = \text{diag}(-\rho, p, p, p) \) with

\[
\rho = 2K_X X - K + 3G_X H \dot{\phi}^2 - 2G_\phi X,
\]

\[
p = K - 2(G_\phi + G_X \phi) X.
\]

Here, \( \rho \) has an explicit dependence on the Hubble rate \( H \). The gravitational field equations are thus given by

\[
3M_{Pl}^2 H^2 = \rho, \quad -M_{Pl}^2 \left( 3H^2 + 2\dot{H} \right) = \rho,
\]

and the scalar field equation of motion reads

\[
K_X \left( \ddot{\phi} + 3H \dot{\phi} \right) + 2K_X X \ddot{X} + 2K_X \phi X - K \phi
- 2(G_\phi - G_X \phi X) \left( \phi + 3H \dot{\phi} \right) + 6G_X \left[ (HX)' + 3H^2 X \right]
- 4G_X \phi \ddot{X} - 2G_\phi \phi X + 6HG_{XX} X \ddot{X} = 0.
\]

These three equations constitute two independent evolution equations for the background. Note that the appearance of the terms proportional to the Hubble parameter...
in Eqs. (1) and (7) reflects the fact that the Galileon symmetry is broken in the curved spacetime even if we constrain our functional form of the Lagrangian which possesses its symmetry in the Minkowski spacetime.

We begin with constructing an exactly de Sitter background, taking $K$ and $G$ as

$$K(\phi, X) = K(X), \quad G(\phi, X) = g(\phi)X. \quad (8)$$

In this case, inflation is driven purely kinematically, although G-inflation does not preclude a potential-driven inflationary solution with an explicit $\phi$-dependence in $K(\phi, X)$ in general; see Eq. (1). If $g(\phi) = \text{const}$, i.e., the Lagrangian has a shift symmetry $\phi \to \phi + \text{const}$, we have an exactly de Sitter solution satisfying $\phi = \text{const}$,

$$3M_p^2H^2 = -K, \quad D := K_X + 3gH\dot{\phi} = 0. \quad (9)$$

Let us now provide a simple example:

$$K = -X + \frac{X^2}{2M^3\mu}, \quad g = \frac{1}{M^3}, \quad (10)$$

where $M$ and $\mu$ are parameters having dimension of mass. The de Sitter solution is given by

$$X = M^3\mu x, \quad H^2 = \frac{M^3(1 - x)^2}{18\mu}x, \quad (11)$$

where $x \ (0 < x < 1)$ is a constant satisfying $(1 - x)/x\sqrt{1 - x}/2 = \sqrt{\frac{\mu}{M_p}}$. For $\mu \ll M_p$, it can be seen that $x \simeq 1 - \sqrt{3}\mu/M_p$ and hence the Hubble rate during inflation is given in terms of $M$ and $\mu$ as $H^2 \simeq M^3\mu/(6M_p^2)$. As the first term in $K(X)$ has the “wrong” sign, one may worry about ghost-like instabilities. However, as we will see shortly, this model is free from ghost and any other instabilities.

By allowing for a tilt of the function $g(\phi)$ we can find quasi-de Sitter inflation, which also induces a tilt in the spectral index of curvature fluctuations; see Eq. (26) below. To investigate this possibility we define

$$\epsilon := -\frac{\dot{H}}{H^2}, \quad \eta := -\frac{\ddot{\phi}}{H\dot{\phi}}, \quad \epsilon_g := \frac{M_p\dot{g}_\phi}{g}. \quad (12)$$

The slow-roll conditions are given by $|\epsilon| \ll 1$ and $|\eta| \ll 1$, one of which can be replaced by $|\dot{g}/Hg| \ll 1$ as long as $XK_{XX}/K_X = O(1)$. The slow-roll condition $|\dot{g}/Hg| \ll 1$ can also be written in terms of $\epsilon_g$ as $|\epsilon_g\sqrt{X/(-K)}| \ll 1$. In order to have $\rho \simeq -K$ we require $|XD/K| \ll 1$. Thus, the “slow-roll” equations are $3M_p^2H^2 \simeq -K$ and $D \simeq 0$. From the first equation and its time derivative, $6M_p^2H\dot{H} \simeq -K_X\dot{X}$, we find that $\epsilon$ and $\eta$ are actually related as $\epsilon \simeq 2\eta XK_X/K$.

For a toy model with $\epsilon_g = \text{const}$, namely, $g(\phi) = e^{\epsilon_g\phi}/M^3$, and various $K(X)$, we have solved numerically the relevant equations, and confirmed that the quasi-de Sitter solution is an attractor.

Inflation can be terminated by incorporating the $\phi$-dependence of the linear term in the kinetic function,

$$K(\phi, X) = -A(\phi)X + \Delta K, \quad (13)$$

to flip the sign of $A$ to the “normal” one ($A = \text{const} < 0$) due to the nontrivial evolution of $A(\phi)$ [$\simeq A(\phi)$] in the final stage of inflation, while $G(\phi, X)$ may still be of the form $G = g(\phi)X$ with $g \simeq \text{const}$. As an explicit example, one can take $K = -A(\phi)X^2/2M^3\mu$ and $G = X/M^3$ where $A(\phi) = \tanh[\lambda(\phi_{\text{end}} - \phi)/M_p]$ with $\lambda = O(1)$. Our numerical solution shows that soon after $\phi$ crosses $\phi_{\text{end}}$ to change the sign of $A$, it stalls and all the higher-order terms $\Delta K$ as well as terms from $G\square \phi$ become negligibly small within one $e$-fold. As a result, $\phi$ behaves as a massless canonical field, so that the energy density of the scalar field is diluted as rapidly as $\rho \propto a^{-6}$.

Since the shift symmetry of the original Lagrangian prevents direct interaction between $\phi$ and standard-model contents, reheating proceeds only through gravitational particle production as discussed by Ford [10], who has shown that at the end of inflation ($a = 1$) there exists radiation with its energy density corresponding at least to the Hawking temperature:

$$\rho_r|_{\text{end}} = \frac{\pi^2}{30}g_*T_H^4, \quad T_H = \frac{H_{\text{inf}}}{2\pi}. \quad (14)$$

Since $\rho_r = 3M_p^2H^2_{\text{inf}}(g_*/1440\pi^2)(H_{\text{inf}}/M_p)^2 < 3M_p^2H^2_{\text{inf}}$, the radiation component does not affect the cosmic history at that moment. Subsequently, the radiation energy density decays as $\rho_r \propto a^{-4}$, while the energy density of $\phi$ is diluted more rapidly as $\rho_\phi \propto a^{-6}$. Finally one finds $\rho_\phi \approx \rho_r$ at $a = \sqrt{3}M_pH_{\text{inf}}(\rho_r|_{\text{end}})^{-1/2}$.

Defining the reheating temperature by $\rho_r = \rho_\phi = (\pi^2/30)g_*T_H^4$, one can estimate

$$T_R \approx 0.01\frac{H_{\text{inf}}}{M_p} \approx 10^4\text{GeV}\left(\frac{H_{\text{inf}}}{10^{12}\text{GeV}}\right)^2. \quad (15)$$

Let us briefly comment on baryogenesis through leptogenesis [11]. If the mass of the lightest right handed neutrinos $N_1$ is smaller than the Hawking temperature $T_H$, they are copiously produced at the end of inflation. Then, their out-of equilibrium decay can lead to lepton asymmetry, which is converted to baryon asymmetry through the sphaleron effects. In our scenario, the decay parameter, which is defined as the ratio of the decay rate of $N_1$ to the Hubble parameter when $N_1$ becomes non-relativistic, is significantly suppressed compared to the standard thermal leptogenesis scenario [11] because the universe is dominated not by radiations but by the inflaton. Thus, the wash out processes are significantly suppressed and sufficient baryon asymmetry is produced.

The phase space diagram of G-inflation is depicted in Fig. 1. Note that in G-inflation the null energy condition (NEC) may be violated, i.e., $2M_p^2H = -(\rho + p) > 0$. 
In this gauge we have \( \beta \) and momentum constraint equations to eliminate \( b \) and \( \gamma \), and then substituting the Hamiltonian Sasaki-Mukhanov equation.

The NEC violation can occur stably \([8,12]\), in the sense that the squared sound speed (to be defined shortly) is positive and there are not any ghosts. (This is the condition for the stability concerning the short wavelength perturbations.) We stress that this can never occur in a healthy manner in k-inflation \([8]\). Moreover, NEC violating k-inflation cannot end, as argued in \([12]\).

We now move on to study scalar perturbations in this model using the unitary gauge with \( c_s^2 = 0 \) and

\[
\begin{align*}
\delta T^0_i = -G_X \phi^3 \partial_i \alpha \quad &\text{and hence this gauge does not coincide with the comoving gauge } \delta T^0_i = 0. \\
\text{Consequently, } R_\phi \text{ in general differs from the comoving curvature perturbation } R. \quad &\text{This point highlights the difference between the present model and the standard k-inflationary model described simply by } L_\phi = K(\phi, X) [14]. \quad \text{It will turn out that the variable } R_\phi \text{ is subject to an analogous wave equation to the familiar Sasaki-Mukhanov equation.}
\end{align*}
\]

Expanding the action \([8]\) to second order in the perturbation variables and then substituting the Hamiltonian and momentum constraint equations to eliminate \( \alpha \) and \( \beta \), we obtain the following quadratic action for \( R_\phi \):

\[
\mathcal{S}^{(2)} = \frac{1}{2} \int d\tau \left[ G(R_\phi')^2 - \mathcal{F}(\nabla^2 R_\phi)^2 \right], \quad (17)
\]

where

\[
\begin{align*}
z := \frac{a \dot{\phi}}{H - G_X \phi^3 / 2M_{Pl}^2},
\mathcal{F} &:= K_X + 2G_X \left( \phi + 2H \dot{\phi} \right)^2 - 2\frac{G_X^2}{M_{Pl}^2} X^2 \\
+ 2G_X X \dot{\phi} - 2 \left( G_\phi - X G_{\phi X} \right),
\mathcal{G} &:= K_X + 2XK_{XX} + 6G_X H \dot{\phi} + 6\frac{G_X}{M_{Pl}^2} X^2 \\
-2 \left( G_\phi + X G_{\phi X} \right) + 6G_X X H \dot{\phi},
\end{align*}
\]

and the prime represents differentiation with respect to the conformal time \( \tau \). The squared sound speed is therefore \( c_s^2 = \mathcal{F} / \mathcal{G} \). To avoid ghost and gradient instabilities we require the conditions \( \mathcal{F} > 0 \) and \( \mathcal{G} > 0 \). One should note that the above equations have been derived without assuming any specific form of \( K(\phi, X) \) and \( G(\phi, X) \).

It is now easy to check whether a given G-inflation model is stable or not. In the simplest class of models \([8]\), we have

\[
\mathcal{F} = -\frac{K_X}{3} + \frac{XK_X^2}{3K}, \quad \mathcal{G} = -K_X + 2XK_{XX} - \frac{XK_X^2}{K} \quad (21)
\]

where the “slow-roll” suppressed terms are ignored. For the previous toy model \([10]\) one obtains \( \mathcal{F} = x(1-x)/6(1-x/2) \) and \( \mathcal{G} = 1-x+(1-x/2)^{-1} \). Since \( 0 < x < 1 \), both \( \mathcal{F} \) and \( \mathcal{G} \) are positive. In this model, the sound speed is smaller than the speed of light: \( c_s^2 \leq (4\sqrt{2} - 5)/21 \approx 0.031 < 1 \).

In the superhorizon regime where \( O(\nabla^2) \) terms can be neglected, the two independent solutions to the perturbation equation that follows from the action \([10]\) are

\[
R_\phi = \text{const}, \quad \int \frac{\text{d} \tau'}{z^2 \mathcal{G}}. \quad (22)
\]

The latter is a decaying mode in the inflationary stage and in the subsequent reheating stage in our model, and hence can be neglected. In this limit one can show that \( R_\phi \) coincides with the comoving curvature perturbation.

The power spectrum of \( R_\phi \) generated during G-inflation can be evaluated by writing the perturbation equation in the Fourier space as

\[
\frac{d^2 u_k}{d\tilde{y}^2} + \left( k^2 - \frac{\tilde{z} \cdot \tilde{u}}{\tilde{z}} \right) u_k = 0, \quad (23)
\]

where \( dy = c_s \text{d} \tau, \quad \tilde{z} := (\mathcal{F})^{1/4} z, \) and \( u_k := \tilde{z} R_\phi \cdot k \).

Let us again focus on the class of models \([8]\). Note that the sound speed \( c_s \) may vary rapidly in the present case, and hence one cannot neglect \( \epsilon_s := c_s^2 / H c_s \) even when working in leading order in “slow-roll.” Indeed, one finds \( \epsilon_s \approx \eta X(G_X / \mathcal{G} - F_X / \mathcal{F}) \). With some manipulation, one obtains \( \tilde{z}_{yy} - \tilde{z} \tilde{z} \approx (-y)^{-2}[2 + 3 \epsilon(X)] \) with

\[
\mathcal{C}(X) := \frac{K}{K_X} \frac{Q_X}{Q}, \quad Q(X) := \frac{(K - XK_X)^2}{18M_{Pl}^4 X c_s^2 \sqrt{\mathcal{F} \mathcal{G}}} \quad (24)
\]
It should be emphasized that scalar fluctuations are generated even from exactly de Sitter inflation. This is because, as mentioned before, the Galileon symmetry is broken in the de Sitter background, which is manifest from $\dot{\phi} = \text{const}$. This situation is in stark contrast with other inflation models: scalar fluctuations cannot be generated from the de Sitter background with $\dot{\phi} = 0$ in usual potential-driven inflation, while the exactly de Sitter background cannot be realized in k-inflation.

The normalized mode is given in terms of the Hankel function as

$$u_k = \frac{\sqrt{\pi}}{2} \sqrt{-y} H^{(1)}_\nu(-ky), \quad \nu := \frac{3}{2} + \epsilon \mathcal{C},$$

from which it is straightforward to obtain the power spectrum and the spectral index:

$$\mathcal{P}_T = \frac{Q}{4\pi^2} \bigg|_{c_s, k=1/(-\nu)}, \quad n_s - 1 = -2\epsilon \mathcal{C}. \quad (26)$$

The behavior of tensor perturbations in G-inflation is basically the same as in the usual inflation models and is completely determined geometrically. Therefore, the power spectrum and the spectral index of primordial gravitational waves are given by $\mathcal{P}_T = (8/M^2_\text{Pl})(H/2\pi)^2$ and $n_T = -2\epsilon$. However, it would be interesting to point out that the tensor spectrum can be blue in G-inflation with possible violation of the NEC. The positive tensor spectral index not only is compatible with current observational data, but also broadens the limits on cosmological parameters [13]. Moreover, the amplitude of tensor fluctuation with such a blue spectral index is relatively enhanced for large frequencies, which makes its direct detection easier.

As a concrete example, let us come back again to the previous toy model [10], in which the tensor-to-scalar ratio is given by

$$r \simeq \frac{16\sqrt{6}}{3} \left( \frac{\sqrt{3\mu}}{M_{\text{Pl}}} \right)^{3/2} \quad \text{for} \quad \mu \ll M_{\text{Pl}}. \quad (27)$$

With the properly normalized scalar perturbation, $\mathcal{P}_T = 2.4 \times 10^{-9}$, we can easily realize large $r$ to saturate the current observational bound, exceeding the predictions of the chaotic inflation models [10]. For example, for $M = 0.00425 \times M_{\text{Pl}}$ and $\mu = 0.032 \times M_{\text{Pl}}$ we find $r = 0.17$, which is large enough to be probed by the PLANCK satellite [18]. Note that neither the standard consistency relation, $r = -8n_T$, nor the k-inflation-type consistency relation, $r = -8c_s n_T$, holds in our model.

In summary, we have proposed a novel inflationary mechanism driven by the Galileon-like scalar field. Our model — G-inflation — is a new class of inflation models with the term proportional to $\Box \phi$ in the Lagrangian, which opens a new branch of inflation model building. Contrary to the most naive expectation, the interaction of the form $G(\phi, (\nabla \phi)^2) \Box \phi$ gives rise to derivatives no higher than two in the field equations [8]. In this sense, G-inflation is distinct also from ghost condensation [4] and B-inflation [19]. After G-inflation, the universe is reheated through the gravitational particle production with successful thermal leptogenesis. We have also shown that G-inflation can generate (almost) scale-invariant density perturbations, possibly together with a large amplitude of primordial gravitational waves. These facts have great impacts on the planned and ongoing gravitational wave experiments and CMB observations. In a forthcoming paper we shall compute the non-Gaussianity of the curvature perturbation from G-inflation [20], which would be a powerful discriminant of the scenario in addition to the violation of the standard consistency relation.

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**Note added** While this paper was being completed, Ref. [8] appeared, in which the quadratic action for cosmological perturbations is given independently, though it is investigated in a different context, that is, dark energy.

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1 Another interesting inflation model with the enhanced tensor-to-scalar ratio has been proposed in [18], which relies on a sound speed greater than the speed of light.

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