High energy pA collisions 
in the color glass condensate approach  
II. Quark production

Jean-Paul Blaizot(1), François Gelis(1), Raju Venugopalan(2)

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1. Service de Physique Théorique
   Bât. 774, CEA/DSM/Saclay
   91191, Gif-sur-Yvette Cedex, France
2. Physics Department
   Brookhaven National Laboratory
   Upton, NY 11973, USA

Abstract
We compute the production of quark-antiquark pairs in high energy collisions between a small and a large projectile, as in proton-nucleus collisions, in the framework of the Color Glass Condensate. We derive a general expression for quark pair-production, which is not $k_\perp$-factorizable. However, $k_\perp$-factorization is recovered in the limit of large mass pairs or large quark–anti-quark momenta. Our results are amenable to a simple interpretation and suggest how multi-parton correlations at small $x$ can be quantified in high-energy proton/deuteron-nucleus collisions.

1 Introduction

In an accompanying paper [1], heretofore referred to as I, we developed the Color Glass Condensate (CGC) framework [2–4] to address the case where the parton densities in the projectile are small while those of the target are large. This may be applied for instance to the proton/deuteron-nucleus collisions that are currently being studied at Brookhaven’s Relativistic Heavy Ion Collider (RHIC) and will be further studied at the Large Hadron Collider at CERN in the near future. In our previous paper [1], we solved the Yang-Mills equations, for two light-cone sources, to first order in the density of the dilute projectile and to all orders in the density of the target. Our computations were performed in

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the Lorenz/covariant gauge $\partial_\mu A^\mu = 0$. Paper I focused on developing a general formalism for gluon production at high energies and on phenomenological consequences such as the Cronin effect. In I, we derived a compact expression of the gauge fields produced in proton-nucleus collisions. In this paper, we will use this result to study the production of quark–anti-quark pairs in high energy proton-nucleus collisions.

Quark pair production in high energy hadronic collisions has previously been studied in the frameworks of collinear factorization [7–9] and $k_T$-factorization [10–12]. A comprehensive list of references for the particular applications of these formalisms to proton-nucleus collisions can be found in ref. [13]. At high energies, where the intrinsic transverse momenta of the partons is not negligible and may be on the order of the saturation scale [12], $k_T \sim Q_s$, $k_T$-factorization is a convenient formalism to discuss particle production. Phenomenological studies for hadron colliders have been discussed in refs. [14,15] and for heavy-ion collisions in ref. [16].

The issue of $k_T$-factorization for quark production in the CGC approach was addressed by two of us in [17]. We showed that, in the CGC language, the $k_T$-factorization of Collins and Ellis [10] is recovered exactly when the parton densities of the projectile and the target are not too large, namely, when $\rho_p/k_T^2, \rho_A/k_T^2 \ll 1$. Here $\rho_p, \rho_A$ denote the number of parton sources per unit area in the projectile and target respectively and $k_T$ is a typical momentum of the produced parton. Our results also suggested however that $k_T$-factorization is not robust and would fail when either or both $\rho_p, \rho_A \sim k_T^2$. High energy proton-nucleus collisions are ideal to test this conjecture explicitly since the proton source may be taken to be dilute ($\rho_p/k_T^2 \ll 1$) while the nuclear target is dense ($\rho_A/k_T^2 \sim 1$). For gluon production, we (and others previously) showed that $k_T$-factorization remains valid even in proton-nucleus collisions, although one has to use an “unintegrated nuclear gluon distribution” which is not the canonical one. However, we will show in this paper that $k_T$-factorization fails unequivocally for pair production in pA-collisions. Nevertheless, our final result (eq. 48) can be expressed in terms of a small number of distribution functions which have a simple interpretation.

Unlike the unintegrated distribution discussed in the case of gluon production, these distributions cannot all be expressed in terms of 2-point correlators of adjoint Wilson lines. In addition to a term containing the correlator $\langle U(x)U^\dagger(y)\rangle$, one has a term proportional to $\langle U(x)U^\dagger(y)U^\dagger(y)\rangle$, i.e. a product of two Wilson lines in the fundamental representation (denoted $U$) and an adjoint Wilson line, and a third term proportional to the product of four fundamental Wilson lines: $\langle U(x)U^\dagger(y)U^\dagger(y)\rangle$. The transverse coordinates here represent those of the quark and the anti-quark in the pair production amplitude and in the complex conjugate amplitude.
large mass pairs and for large momenta of the pair, the transverse separation of the pair can be neglected, and one can show, through an identity, that the $k_\perp$-factorized result in terms of the two point function of adjoint Wilson lines is recovered.

All the above Wilson lines are computed in the presence of the classical color field of the nucleus and therefore depend on the source $\rho_A$. The brackets $\langle \cdots \rangle$ here denote the weighted quantum mechanical average of an operator in the background field of the proton and the nucleus, which can be expressed as

$$\langle O \rangle = \int [D\rho_p][D\rho_A]W_p[x_0^p, \rho_p]W_A[x_0^A, \rho_A]O[\rho_p, \rho_A].$$  \hspace{1cm} (1)

This averaging procedure is essential in order to restore gauge invariance since $O[\rho_p, \rho_A]$ is computed in a particular gauge. More importantly, it is also through $W_p$ and $W_A$ that quantum effects, due to evolution of the light cone wave functions of the target and projectile with $x$, are incorporated in the study of pair production at high energies. The arguments $x_0^p$ and $x_0^A$ denote the scale in $x$ separating the large-$x$ static sources from the small-$x$ dynamical fields. In the McLerran-Venugopalan model, the functional $W_A$ that describes the distribution of color sources in the nucleus is a Gaussian in $\rho_A$ [18–21]. In general, this Gaussian is best interpreted as the initial condition for a non-trivial evolution of $W_A[x_0^A, \rho_A]$ with $x_0^A$. This evolution is described by a Wilson renormalization group equation (often called the JIMWLK equation [22–33]). This evolution equation has been solved recently numerically by Rummukainen and Weigert [34]. One is therefore in a position to make predictions for high energy pair-production in hadronic collisions that are sensitive to not only the “unintegrated gluon distribution” but to more subtle correlations among partons in hadrons at high energies. Conversely, comparisons with pair production experiments at high energies can test, with a high degree of precision, whether the JIMWLK equations, or equivalently the Color Glass Condensate, is the right effective theory of high energy QCD.

This paper is organized as follows. In section 2, we briefly review the solution to the Yang-Mills equations derived in I and re-express it in a way convenient for our discussion. We next proceed to the computation of the quark pair production amplitude in section 3. This amplitude consists of two sets of terms which we name “regular” and “singular” terms. The former correspond to the following cases: a) the gluon produces the pair before interacting with the nucleus — the pair therefore scatters on the nucleus on its way out, and b) the gluon interacts with the nucleus and produces the pair after the encounter. The singular case corresponds to the situation where the pair is produced inside the nucleus and re-interacts on the way out. Naively, one would not expect this term to contribute at very high energies, due to the brevity of the encounter. Nevertheless, in the Lorenz gauge that we choose to work in, such a term does exist and is important in obtaining our final result. Adding both sets of terms together, we obtain the time-ordered amplitude for pair production. For completeness, and to confirm our result for the time ordered amplitude, we also compute the retarded pair production amplitude, which, to this order, carries an identical
physical content albeit a different interpretation. The computation is presented in appendix B. In section 4, we compute the pair production cross-section. Our final result (eq. 48) is not $k_\perp$-factorizable in the same manner as the result for the gluon production cross-section is. The various terms in our result however have a simple interpretation and for large mass or large momentum pairs, one does recover the $k_\perp$-factorized result - as expected from our derivation in [17]. We finally address the case of the single quark production cross-section in section 5. Single quark distributions were recently studied by Tuchin [35] in the context of a Glauber model of independent scatterings. In contrast to our momentum space approach, Tuchin’s results are formulated entirely in coordinate space and therefore do not address the issue of $k_\perp$-factorization. Further, our results are more general, being valid for non-Gaussian correlations as well. For the case of a Gaussian distribution of color sources, we provide in appendix A explicit expressions for all the correlators that appear in our results for both pair production and inclusive single quark production. We end with a summary of our results and an outlook on future work on these questions.

2 The Yang-Mills gauge field produced in pA collisions

Our first step in calculating the pair production amplitude is to find solutions of the classical Yang-Mills equations in the presence of proton and nuclear sources of color charge. Since we want to compute quark production at the lowest order in the source $\rho_p$ that describes the proton and to all orders in the source $\rho_A$ that describes the nucleus, we need to compute the gauge field up to the same order. The Yang-Mills equations, to this required order, were solved in covariant gauge in I. We will here just quote the results and re-express them in a manner convenient for the computation of the pair production amplitude in the next section. Following I, we denote $A_\mu^A$ the field of the nucleus alone, $A_\mu^p$ the field of the proton alone and $A_\mu$ (without any subscript) the total gauge field at order 1 in the proton source and to all orders in the nuclear source.

The gauge field created by the nucleus alone is given in coordinate space by the expression

$$A_\mu^A(x) = -g\delta^{\mu\nu}\delta(x^+) \frac{1}{\nabla_\perp} \rho_A(x_\perp).$$  \hspace{1cm}(2)$$

Its Fourier transform, which will appear repeatedly as we proceed, is

$$A_\mu^A(q) = 2\pi g\delta^{\mu\nu}\delta(q^+) \frac{\rho_A(q_\perp)}{q_\perp}.$$  \hspace{1cm}(3)$$

Note that this field is linear in the source $\rho_A$: it is indeed a well known fact that, in covariant gauge, the classical color field created by a single projectile, no matter how dense, is linear in the classical color source. We denote as $A_\mu$ the first correction to this field due to the proton source, which is of order one
in \( \rho_p \). This correction was derived in the covariant gauge in I (see section 3.5 of I).

Our expression for \( A^\mu \) has both singular terms (proportional to \( \delta(x^+) \)) and regular terms. It will be convenient for our later discussion to group them accordingly, and express \( A^\mu \) as

\[
A^\mu(q) = A^\mu_{\text{reg}}(q) + \delta^\mu - A^-_{\text{sing}}(q) .
\]

Our expression for \( A^\mu_{\text{reg}} \) is

\[
A^\mu_{\text{reg}}(q) = A^\mu_p(q) + \frac{ig}{q^2 + iq^+ \epsilon} \int \frac{d^2k_{\perp}}{(2\pi)^2} \left\{ C^\mu_U(q, k_{\perp}) \left[ U(k_{2\perp}) - (2\pi)^2 \delta(k_{2\perp}) \right] + C^\mu_{V', \text{reg}}(q) \left[ V(k_{2\perp}) - (2\pi)^2 \delta(k_{2\perp}) \right] \right\} \frac{\rho_\nu(k_{1\perp})}{k_{1\perp}^2} .
\]

In this formula, the first term is the color field of the proton alone, given by eq. (3) with \( \rho_A \) replaced by \( \rho_p \). In the second term, \( k_{1\perp} \) is the momentum coming from the proton and \( k_{2\perp} \), defined as \( k_{2\perp} \equiv q_{\perp} - k_{1\perp} \), is the momentum coming from the nucleus. The 4-vectors \( C^\mu_U(q, k_{\perp}) \) and \( C^\mu_{V', \text{reg}}(q) \) are given by the following relations:

\[
C^+_{V,U}(q, k_{1\perp}) = -\frac{k_{1\perp}^2}{q^+ + iq^+ \epsilon} \ , \quad C^-_{V,U}(q, k_{1\perp}) = \frac{k_{1\perp}^2 - q_{\perp}^2}{q^+} \ , \quad C_{V',\text{reg}}^1(q, k_{1\perp}) = -2k_1^i , \\
C^+_{V',\text{reg}}(q) = 2q^+ \ , \quad C^-_{V',\text{reg}}(q) = 2q^- - \frac{q_{\perp}^2}{q^+} \ , \quad C_{V',\text{reg}}^i(q) = 2q^i .
\]

\( U \) and \( V \) are the Fourier transforms of Wilson lines in the adjoint representation of \( SU(N) \):

\[
U(k_{\perp}) = \int d^2 x_{\perp} e^{ik_{\perp} \cdot x_{\perp}} U(x_{\perp}) , \quad V(k_{\perp}) = \int d^2 x_{\perp} e^{ik_{\perp} \cdot x_{\perp}} V(x_{\perp}) ,
\]

with

\[
U(x_{\perp}) \equiv \mathcal{P}_+ \exp \left[ ig \int_{-\infty}^{+\infty} dz^+ A_-(z^+, x_{\perp}) \cdot T \right] , \\
V(x_{\perp}) \equiv \mathcal{P}_+ \exp \left[ \frac{g}{2} \int_{-\infty}^{+\infty} dz^+ A_-(z^+, x_{\perp}) \cdot T \right] ,
\]

where the \( T^a \) are the generators of the adjoint representation of \( SU(N) \) and \( \mathcal{P}_+ \) denotes a “time ordering” along the \( z^+ \) axis. The subscript ‘reg’ on \( C^\mu_{V'}(q) \) indicates that the corresponding term of \( A^\mu \) does not contain any \( \delta(x^+) \) when expressed in coordinate space. The relationship\(^4\) between \( C^\mu_{V',\text{reg}}(q) \) and the vector \( C^\mu_V(q) \)

\(^4\)The terms responsible for a \( \delta(x^+) \) in \( A^\mu(x) \) are those for which \( C^\mu_{V'}(q)/q^2 \) does not go to zero when \( q^- \to \infty \). It is immediate to verify that this happens only for \( C^-_{V'} \). \( C^\mu_{V',\text{reg}} \) is obtained from \( C^-_{V'} \) the piece responsible for this non-zero limit.
introduced in the section 3.5 of I is\footnote{\(C_U\) and \(C_V\) are related to the well known Lipatov vertex \cite{36,37} via the relation \(C_U^\mu = C_U^\mu + \frac{1}{2} C_V^\mu\). The properties of these are explored further in paper I.}:

\[ C_{\nu, \text{reg}}(q) = C_{\nu}(q) + \delta^\mu q^2 \quad . \tag{9} \]

The “singular” term reads:

\[ A_{\text{sing}}^{-}(q) = -ig \int \frac{d^2 k_{\perp}}{(2\pi)^2} \left[ V(k_{2\perp}) - (2\pi)^2 \delta(k_{2\perp}) \right] \frac{\rho_p(k_{1\perp})}{k_{1\perp}^2} \quad . \tag{10} \]

It will in fact be more convenient later to have this part of the field in coordinate space:

\[ A_{\text{sing}}^{-}(x) = i g^2 (A_{-}^{-}(x) \cdot T) V(x^+, -\infty; x_\perp) \theta(x^-) \frac{1}{\nabla_\perp} \rho_p(x_\perp) \quad , \tag{11} \]

where \(V(x^+, -\infty; x_\perp)\) denotes an incomplete Wilson line:

\[ V(x^+, -\infty; x_\perp) \equiv \mathcal{P}_+ \exp \left[ i g^2 \int_{-\infty}^{x^+} dz^+ A_{-}^{-}(z^+, x_\perp) \cdot T \right] \quad . \tag{12} \]

In the case of gluon production \(I\), we have seen that the Wilson line \(V\) drops out of the gluon production amplitude. This result was to be expected from the fact that there are gauges in which such a Wilson line, characterized by an unusual factor \(1/2\) in the exponent, does not appear at all. Naturally, we expect the same for quark-antiquark production, and one of our tasks in the next section will be to exhibit the mechanism by which this Wilson line drops out of the pair production amplitude.

## 3 Pair production amplitude

### 3.1 Generalities

Once we know the time-dependent classical gauge field created in the collision of the two projectiles, it is straightforward to calculate the production of quark-antiquark pairs in the collision. One can simply forget about the projectiles themselves, and consider only the classical field. The probability to produce exactly one \(q\bar{q}\) pair\footnote{If one uses instead the retarded quark-propagator, one obtains the average number \(\overline{N_{q\bar{q}}}\) of produced pairs in the collision. At leading order in \(\rho_p\), at most one pair can be produced per collision, so that one expects \(P_1 = \overline{N_{q\bar{q}}}\). We check this property explicitly in appendix B.} in the collision can be expressed in terms of the time-ordered quark propagator in the presence of the classical field\footnote{This formula for \(P_1\) is correct at leading order in \(\rho_p\), but is incomplete in general. Indeed, it should also contain the prefactor \(|\langle 0_{\text{in}} | 0_{\text{out}} \rangle|^2\), which is mandatory for unitarity to be preserved (see \cite{38} for more details). This prefactor only affects higher order corrections in \(\rho_p\).} as

\[ P_1(\rho_p, \rho_A) = \int \frac{d^3 q}{(2\pi)^3 2E_q} \int \frac{d^3 p}{(2\pi)^3 2E_p} |\mathcal{M}(q,p)|^2 \quad , \tag{13} \]

\[ P_1(\rho_p, \rho_A) = \int \frac{d^3 q}{(2\pi)^3 2E_q} \int \frac{d^3 p}{(2\pi)^3 2E_p} |\mathcal{M}(q,p)|^2 \quad , \tag{13} \]
where the argument \([\rho_p, \rho_A]\) indicates that this is the production probability in one particular configuration of the color sources. In order to turn this probability into a cross-section, one must average over the initial classical sources \(\rho_p\) and \(\rho_A\) with the weight \(W_p[\rho_p]W_A[\rho_A]\) and one must integrate over all the impact parameters \(b\), to obtain,

\[
\sigma_1 = \int d^2 b \int [D\rho_p][D\rho_A]W_p[\rho_p]W_A[\rho_A]P_1[\rho_p, \rho_A].
\]

In eq. (13), the time-ordered amplitude \(M_F\) is related to the quark propagator as follows:

\[
M_F(q, p) \equiv \overline{\psi}(q)T_F(q, -p)\psi(p),
\]

where \(T_F(q, -p)\) is the interacting part of the Feynman quark propagator in the presence of the classical field, with its external legs amputated. The arguments \(q\) and \(-p\) are respectively the outgoing and the incoming 4-momenta on this quark propagator.

### 3.2 Regular terms

Since we want the pair production amplitude to first order in the source \(\rho_p\) and to all orders in \(\rho_A\), we must insert the total field \(A^\mu\) exactly once on the quark propagator, and we can insert the field \(A^\mu\) an arbitrary number of times since it does not change the power counting in \(\rho_p\). At this point, it is convenient to study separately the terms in \(A^\mu\) which do not contain a \(\delta(x^+)\) from those that contain a \(\delta(x^+)\).

Let us begin with the former. The terms that do not contain a \(\delta(x^+)\) are the ones in eq. (5) that we called \(A_{\text{reg}}^\mu\). Physically, these are terms for which the pair is produced outside the nucleus; more precisely, the point where \(A^\mu\) is inserted on the quark line must be outside the nucleus. Regular terms are represented by the four diagrams of figure 1. In order to calculate these diagrams, we need

![Figure 1: Regular terms in the time-ordered pair production amplitude. The gluon line terminated by a cross denotes a classical field insertion. The black dot denotes multiple insertions of the field \(A_\Lambda^\mu\) (with at least one insertion).](image-url)
the expression for the multiple insertions of the field $A^a_\mu$ on the time-ordered quark propagator, denoted by a black dot in the diagrams. This reads [38,39]:

\[
\frac{p^+}{p \to k} \left[ \frac{p + k}{k} \right] = 2\pi \delta(k^+) \gamma^+ \text{sign}(p^+) \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \left[ \bar{U}(x_\perp) \gamma^+ (x_\perp) - 1 \right], \tag{16}
\]

where $\bar{U}$ is a Wilson line in the fundamental representation of $SU(N)$, defined by

\[
\bar{U}(x_\perp) \equiv \mathcal{P}_+ \exp \left[ i g \int_{-\infty}^{+\infty} dz A_\mu^+(z^+, x_\perp) \cdot t \right]. \tag{17}
\]

The $t^a$ are the generators of the fundamental representation of $SU(N)$. The contributions of the diagrams of figure 1 to the time-ordered pair production amplitude are obtained using the free Feynman (time-ordered) quark propagator in the intermediate quark lines:

\[
S^0_\mu(p) \equiv i \frac{q^\mu + m}{p^2 - m^2 + i\epsilon}. \tag{18}
\]

The contribution of the diagram (a) to $\mathcal{M}_p$ is simply given by

\[
\mathcal{M}^{(a)}_p(q, p) = \left\{ -ig A^{\mu a}_\text{reg}(p + q) \right\} [\gamma^\mu t^av(p)]. \tag{19}
\]

Since the first term of eq. (5) brings a $\delta(p^- + q^-)$ with it, this term cannot contribute to the production of a pair on shell, for which we must have $p^- > 0$ and $q^- > 0$. (This first term would correspond to the production of a pair from the proton alone, which is kinematically forbidden). Using also $\bar{\mu}(q)[\gamma^+ v(p) = 0$, we can write the contribution from diagram (a) more explicitly as

\[
\mathcal{M}^{(a)}_p(q, p) = g^2 \int d^2 k_\perp \frac{\rho_{p,a}(k_1^\perp)}{(2\pi)^2 k_\perp^2} \int d^2 x_\perp e^{i(k_1^\perp + q - k_\perp) \cdot x_\perp} \left\{ \left( \frac{\bar{\mu}(q)C_{\mu}(p + q, k_1^\perp)\gamma^+ t^av(p)}{(p + q)^2} - \frac{\bar{\mu}(q)\gamma^+ t^av(p)}{p^+ + q^+} \right) \left[ U(x_\perp) - 1 \right] - \frac{\bar{\mu}(q)\gamma^+ t^av(p)}{p^+ + q^+} \left[ V(x_\perp) - 1 \right] \right\}. \tag{20}
\]

The contribution of the diagram (b) reads:

\[
\mathcal{M}^{(b)}_p(q, p) = -\int \frac{d^4k}{(2\pi)^4} 2\pi \delta(k^+) \left\{ -ig A^{\mu a}_\text{reg}(p + q - k) \right\} \int d^2 x_\perp e^{ik_\perp \cdot x_\perp} \left\{ \bar{\mu}(q)\gamma^+ t^av(p) \gamma^+ \left[ U(x_\perp) - 1 \right] \right\} \tag{21}
\]

where we have already used the fact that the quark propagator on which the eikonal interaction with the nucleus is inserted carries a negative energy $-p^+ < 0$. The $k^+$ integral is trivial because of the $\delta(k^+)$. The $k^-$ integral can be performed by using the theorem of residues. The propagator $S^0_\mu(-p + k)$ has a pole above the real axis in the $k^-$ plane. So do the denominators $(p + q - k)^2$.
Using the variable momentum flowing from the nucleus to the quark line, and

Similarly for the diagram (c), the poles in the variable $k^-$ are such that only the first term in $A_{\text{reg}}$ contributes. We obtain

\begin{equation}
M_{\nu}^{(c)}(q, p) = -g^2 \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{\rho_{p,a}(k_{1\perp})}{k_{1\perp}} \int d^2x_{\perp} e^{i(p_{\perp} + q_{\perp} - k_{1\perp}) \cdot x_{\perp}} \times \frac{\mp(q)\gamma^+ [\bar{U}(x_{\perp}) - 1](-\not{q} - \not{p} + m)\gamma^- t^a v(p)}{2p^+ q^- + (q_{\perp} - k_{1\perp})^2 + m^2} \tag{23}
\end{equation}

By the same method, we can write the contribution of the diagram (d) as follows:

\begin{equation}
M_{\nu}^{(d)}(q, p) = g^2 \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{d^2k_{2\perp}}{(2\pi)^2} \frac{\rho_{p,a}(k_{1\perp})}{k_{1\perp}} \int d^2x_{\perp} d^2x_{\perp} \ e^{i(p_{\perp} + q_{\perp} - k_{1\perp}) \cdot x_{\perp}} \times \frac{\mp(q)\gamma^+ [\bar{U}(x_{\perp}) - 1](-\not{q} - \not{p} + m)\gamma^- t^a v(p)}{2p^+ [(q_{\perp} - k_{1\perp})^2 + m^2] + 2q^+ [(q_{\perp} - k_{1\perp} - k_{1\perp})^2 + m^2]} \tag{24}
\end{equation}

In this expression, $x_{\perp}$ and $y_{\perp}$ are the transverse coordinates of the quark and the antiquark respectively, $k_{1\perp}$ is the momentum flowing from the proton, $k_{2\perp}$ the momentum flowing from the nucleus to the quark line, and $p_{\perp} + q_{\perp} - k_{1\perp} - k_{1\perp}$ the momentum flowing from the nucleus on the antiquark line.

One can see at this point that the sum of the regular terms\(^8\) contains the Wilson line $V$. As we shall see shortly, it is cancelled by the contribution coming from the singular terms.

### 3.3 Singular terms

We must now evaluate the contribution to the pair production amplitude coming from the field $A_{\nu}^{\text{sing}}$. Since this field is proportional to $\delta(x^+)$ in coordinate space, its contribution is a term where the pair is produced inside the nucleus. Technically, this means that in order to correctly compute this term, we must

\(^8\)For this conclusion to be valid, it was crucial to have isolated the singular terms in the gauge field from the regular ones. Indeed, the singular terms have a large $k^-$ behavior which prevent one from closing the integration axis by the contour at infinity.

\(^9\)In fact, only the diagram (a) contains the Wilson line $V$. For the diagrams (b), (c) and (d), only the term $A_{\nu}^{\text{pri}}$ of the total field $A_{\nu}$ contributes, for kinematical reasons.
go to coordinate space and regularize the $\delta(x^+)$ by giving a small width to the nucleus,

$$\delta(x^+) \rightarrow \delta_\epsilon(x^+) ,$$  \hspace{1cm} (25)

where $\delta_\epsilon(x^+)$ is a regular function (whose support is $[0, \epsilon]$), which becomes a $\delta(x^+)$ when $\epsilon$ goes to zero. We need not specify this function further because the results do not depend on the precise choice of the regularization. The singular contribution is depicted in figure 2. The field $A^\mu_{\text{sing}}$ is inserted on the quark line at the ‘time’ $x^+$; the quark and the anti-quark then rescatter on the field $A^\mu_\Lambda$ of the nucleus between $x^+$ and $\epsilon$. This contribution to the amplitude can be written in coordinate space as,

$$M^\text{sing}_F(q,p) = \int d^4xe^{i(p+q) \cdot x} \tilde{U}(+\infty, x^+; x_\perp) \times [ig\gamma^+ t^a A_{\text{sing}}^a(x)] \tilde{U}^\dagger(+\infty, x^+; x_\perp)v(p) ,$$  \hspace{1cm} (26)

where we have introduced the incomplete Wilson line

$$\tilde{U}(+\infty, x^+; x_\perp) \equiv P_+ \exp \left[ ig \int_{x^+}^{+\infty} dz^+ A^-_\Lambda(z^+, x_\perp) \cdot t \right] .$$  \hspace{1cm} (27)

Note that here the fundamental Wilson line and its complex conjugate are evaluated at the same point $x_\perp$. This is because in the limit $\epsilon \rightarrow 0$, at the end of the interaction, the components of the pair will not have had sufficient time to separate from each other. Note further that the four possible contributions (analogue to the four regular terms) are already summed here by replacing $\tilde{U}\tilde{U}^\dagger$ by $\tilde{U}$ and $\tilde{U}\tilde{U}^\dagger$ by $\tilde{U}^\dagger$.

In order to simplify this expression, we need the following algebraic identity:

$$\tilde{U}(+\infty, x^+; x_\perp)t^a \tilde{U}^\dagger(+\infty, x^+; x_\perp) = t^b U^{ba}(+\infty, x^+; x_\perp) .$$  \hspace{1cm} (28)

Using also the explicit form eq. (11) of the singular field, we can write the
singular contribution to the amplitude as\(^ {10}\),

\[
\mathcal{M}_F^{\text{sing}}(q, p) = g^2 \int dx^- d^2 x_\perp e^{i(p^+ + q^+) x^-} e^{-i(p_\perp + q_\perp) \cdot x_\perp}
\]

\[
\times \left\{ \frac{g}{2} \int_0^{+\infty} + \int_{-\infty}^{+\infty} dx_\perp \left[ U(x^+, \pm \infty; x_\perp) (A^{-}_{\lambda}(x) \cdot T) V(x^+,-\infty; x_\perp) \right]_{ab} \right\}
\]

\[
\times \pi(q) \gamma^+ t^a \theta(x^-) \frac{1}{\nabla^2_{\perp}} \rho_{p,b}(x_\perp) v(p) .
\]

(29)

Finally we need the formula

\[
\frac{g}{2} \int_{-\infty}^{+\infty} dx^+ U(x^+, \pm \infty; x_\perp) (A^{-}_{\lambda}(x) \cdot T) V(x^+,-\infty; x_\perp) = U(x_\perp) - V(x_\perp),
\]

(30)

which we encountered previously in I. The remaining Fourier integral in eq. (29) can be performed explicitly, and we obtain

\[
\mathcal{M}_F^{\text{sing}}(q, p) = g^2 \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{\rho_{p,a}(k_\perp)}{k_\perp^2} \int d^2 x_\perp e^{i(p_\perp + q_\perp - k_\perp) \cdot x_\perp}
\]

\[
\times \pi(q) \gamma^+ t^b v(p) \left[ V(x_\perp) - U(x_\perp) \right]_{ba} .
\]

(31)

We see that, as anticipated, the term in \( V \) in \( \mathcal{M}_F^{\text{sing}} \) cancels the term in \( V \) we found previously in \( \mathcal{M}_F^{(a)} \).

### 3.4 Complete time-ordered amplitude

We must now combine the terms \( \mathcal{M}_F^{(a,b,c,d)} \) and \( \mathcal{M}_F^{\text{sing}} \). In order to do so, we shall use the following identities:

\[
\pi(q) \gamma^- = \frac{1}{2q^+} \pi(q) \gamma^+ (\gamma - m) \gamma^- ,
\]

\[
\gamma^- v(p) = \frac{1}{2p^+} \gamma^- (\gamma - m) \gamma^+ v(p) ,
\]

(32)

which are simple consequences of the Dirac equation obeyed by the free spinors.

The second trick is to introduce dummy variables \( k_\perp \) and \( y_\perp \) via the identity

\[
1 = \int \frac{d^2 k_\perp}{(2\pi)^2} \int d^2 y_\perp e^{ik_\perp \cdot y_\perp} ,
\]

(33)

\(^{10}\)We have used the fact that the support of the \( x^+ \) integral is \( 0 < x^+ < \epsilon \) in order to approximate \( \exp(i(p^+ + q^-) x^+) \approx 1 \).
so that all the terms have the same integration variables. Putting all these terms together, we obtain the following expression for the complete amplitude\footnote{In order to obtain this expression, we have also used a cancellation between the $-t^a$ terms in $U^a U^a - t^a$ and in $t^a [U - 1]_{ba}$.}:

\[
\mathcal{M}_F(q,p) = g^2 \int \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{d^2 k_{\perp}}{(2\pi)^2} \frac{d^2 k_{\perp}}{k_{\perp}^2} \rho_{p,a}(k_{\perp}) \int d^2 x_{\perp} d^2 y_{\perp} e^{i k_{\perp} \cdot x_{\perp}} e^{i (p_{\perp} + q_{\perp} - k_{\perp}) \cdot y_{\perp}} \]

\[
\times \left\{ \begin{array}{l}
\mp(q) \gamma^+(q - \bar{q} + m) \gamma^-(q - \bar{q} + m) \gamma^+[\bar{U}(x_{\perp}) t^a \bar{U}(y_{\perp})] v(p) \\
2p^+([q_{\perp} - k_{\perp}]^2 + m^2) + 2q^+([q_{\perp} - k_{\perp} - k_{\perp}]^2 + m^2)
\end{array} \right\} + \mp(q) \left[ \frac{C_U(p + q, k_{\perp})}{(p + q)^2} - \frac{\gamma^+}{p^+ + q^+} \right] t^b v(p) U^b a(x_{\perp}),
\]

(34)

This is our final formula for the time ordered pair production amplitude.

### 3.5 Physical interpretation and limits of eq. (34)

The interpretation of the two terms in this amplitude can be made more transparent if we note that

\[
\mp(q) \left[ \frac{C_U(p + q, k_{\perp})}{(p + q)^2} - \frac{\gamma^+}{p^+ + q^+} \right] t^b v(p) = \mp(q) \frac{C_{\perp}(p + q, k_{\perp})}{(p + q)^2} t^b v(p),
\]

(35)

where $C_{\perp}(p + q, k_{\perp})$ is the effective Lipatov vertex for the production of a gluon of momentum $p + q$ via the fusion of two gluons of momenta $k_1$ and $p + q - k_1$.

We can then interpret the first term of eq. (34) as the production of a $q\bar{q}$ pair by a gluon from the proton before it goes through the nucleus. It is therefore the $q\bar{q}$ state that interacts with the nucleus, and hence the two Wilson lines in the fundamental representation at two different transverse coordinates. The second term in eq. (34) corresponds to a gluon from the proton going through the nucleus before producing the $q\bar{q}$ pair. We recognize the Lipatov vertex that is known to appear in the gluon production amplitude in pA collisions (see the discussion in 1) and the adjoint Wilson line $U$ describes the rescatterings of this gluon while it goes through the nucleus before it finally produces the pair outside the nucleus.

It is straightforward to verify that eq. (34) agrees with the expression that we obtained in [17] in the limit of low nuclear density (see eqs. (15), (19) and (20) of [17]), namely, when we expand all the Wilson lines of eq. (34) to first order in the source $\rho_a$. The amplitude in eq. (34) satisfies another important property: there is a cancellation between the two terms of eq. (34) in the limit $k_{\perp} \to 0$. Indeed, for $k_{\perp} = 0$, the curly bracket of eq. (34) becomes

\[
\left\{ \cdots \right\} = \mp(q) \frac{\gamma^+[\bar{U}(x_{\perp}) t^a \bar{U}(y_{\perp})] v(p)}{p^+ + q^+} - \frac{\mp(q) \gamma^+[t^b U^b a(x_{\perp})] v(p)}{p^+ + q^+}.
\]

(36)
Since this expression does not depend on $k_\perp$ any more, the integration over $k_\perp$ in eq. (34) produces a $\delta(x_\perp - y_\perp)$. We can therefore replace $U(x_\perp)\ell^a\bar{U}^\dagger(y_\perp)$ in the first term by $t^bU^{ba}(x_\perp)$ (thanks to eq. (28)) – which proves the announced cancellation. This property ensures collinear factorizability on the proton side\textsuperscript{12}, by making the $k_\perp$ integral at most logarithmically singular.

4 Pair production cross-section

4.1 Average over the color sources

Before we square the amplitude in order to obtain the pair production cross-section, it is convenient to write it in a more compact form by introducing some shorthand notation. We shall write the amplitude as

$$M_F(q,p) = g^2 \int \frac{d^2 k_1}{(2\pi)^2} \frac{d^2 k_\perp}{(2\pi)^2} \rho_{p,a}(k_1, k_\perp) \int \frac{d^2 x_\perp d^2 y_\perp}{(2\pi)^2} e^{ik_\perp \cdot x_\perp} e^{i(p_\perp + q_\perp - k_\perp - k_1 \perp) \cdot y_\perp}$$

$$\times \mathfrak{M}(q, p) \left\{ T_{q\bar{q}}(k_1, k_\perp) |\bar{U}(x_\perp)\ell^aU^\dagger(y_\perp)| + T_g(k_1) |t^bU^{ba}(x_\perp)| \right\} v(p),$$

(37)

where we denote\textsuperscript{13}

$$T_{q\bar{q}}(k_1, k_\perp) \equiv \frac{\gamma^+(\not q - \not k + m)\gamma^-(\not q - \not k - \not k_1 + m)\gamma^+}{2p^+[(q_\perp - k_\perp)^2 + m^2] + 2q^+[(q_\perp - k_\perp - k_1 \perp)^2 + m^2]}$$

$$T_g(k_1) \equiv \frac{\not q\cdot(p + q)k_{1\perp}}{(p + q)^2}.$$

(38)

Squaring the amplitude, and averaging over the configurations of the color sources $\rho_p$ and $\rho_A$, we have the following expression for the differential proba-

\textsuperscript{12}This property has been assumed and exploited in several recent studies of pA collisions [40–44].

\textsuperscript{13}The momenta $p$ and $q$ of the produced particles have not been listed among the arguments of these objects in order to make the equations more compact.
The integration over the probability to produce a pair in a collision at impact parameter $b$:

$$\frac{dP}{d^2p_y dq_y db_y} = \frac{g^4}{(16\pi^3)^2} \int_{k_{1\perp},k_{1\perp},k_{\perp},k'_{\perp}} \left< \rho_{p,a}(k_{1\perp}) \rho_{p,a}^*(k'_{1\perp}) \right>$$

$$\times \left\{ \text{tr}_d \left[ (\bar{q} + m) T_{qq}(q - m) \gamma^0 T_{qq}^{tr} \gamma^0 \right] \text{tr}_c \left( \tilde{U}(x_{\perp}) t^a \tilde{U}^\dagger(y_{\perp}) t^b \tilde{U}^\dagger(x'_{\perp}) \right) 
+ \text{tr}_c \left( \tilde{U}(x_{\perp}) t^a \tilde{U}^\dagger(y_{\perp}) t^b \tilde{U}^\dagger(x'_{\perp}) \right) 
+ \text{tr}_c \left( \tilde{U}(x_{\perp}) t^a \tilde{U}^\dagger(y_{\perp}) t^b \tilde{U}^\dagger(x'_{\perp}) \right) 
+ \text{tr}_c \left( \tilde{U}(x_{\perp}) t^a \tilde{U}^\dagger(y_{\perp}) t^b \tilde{U}^\dagger(x'_{\perp}) \right) \right\}. \quad (39)$$

In this formula, $\text{tr}_d$ denotes a trace over the Dirac indices (we sum over all spin states of the produced fermions), $\text{tr}_c$ is a trace over the color indices (we sum over the color of the produced quark and antiquark). We have omitted the arguments in $T_{qq}$ and $T_g$, and $T_{qq}^{tr}$, $T_g^{tr}$ denote the same quantities with $k_{1\perp}, k_{\perp}$ replaced by $k'_{1\perp}, k'_\perp$. We can already see that, in general, this probability depends on 2-, 3- and 4-point correlators of Wilson lines. As in I, we can introduce the proton unintegrated gluon distribution as follows:

$$g^2 \left< \rho_{p,a}(k_{1\perp}) \rho_{p,a}^*(k'_{1\perp}) \right> = \frac{\delta^{a'a'}_{\perp}}{\pi d_A} \left[ \frac{k_{1\perp} + k'_{1\perp}}{2} \right]^2 \int_{X_{\perp}} e^{i(k_{1\perp} - k'_{1\perp}) \cdot X_{\perp}} \frac{d\varphi_p}{d^2 X_{\perp}} \frac{d\varphi_p(k_{1\perp} + k'_{1\perp})}{d X_{\perp}}. \quad (40)$$

The integration over $X_{\perp}$ runs over the transverse profile of the proton. $d_A \equiv N^2 - 1$ is the dimension of the adjoint representation of $SU(N)$, and $d\varphi_p / d^2 X_{\perp}$ is the number of gluons per unit area and unit of transverse momentum in the proton (namely, the proton non-integrated gluon distribution per unit area). As long as we are interested in transverse momenta which are large compared to the inverse proton size, we can approximate $k_{1\perp} \approx k'_{1\perp}$ and write

$$g^2 \left< \rho_{p,a}(k_{1\perp}) \rho_{p,a}^*(k'_{1\perp}) \right> = \frac{\delta^{a'a'}_{\perp}}{\pi d_A} k_{1\perp}^2 \int_{X_{\perp}} e^{i(k_{1\perp} - k'_{1\perp}) \cdot X_{\perp}} \frac{d\varphi_p(k_{1\perp})}{d^2 X_{\perp}}. \quad (41)$$

We will also neglect the difference $k_{1\perp} - k'_{1\perp}$ in the other factors of eq. (39).

Let us now turn to the various correlators of Wilson lines that appear in eq. (39). In eq. (39), the coordinates $x_{\perp}, x'_{\perp}, y_{\perp}, y'_{\perp}$ are relative to the center of the proton. Shifting all of them by the impact parameter $b$ makes them relative to the center of the nucleus, at the expense of introducing an extra factor $\exp(i(k_{1\perp} - k'_{1\perp}) \cdot b)$ in eq. (39). A generic property of these correlators
of Wilson lines is that, for a large nucleus, they are approximately invariant by translation in the transverse plane: the violations of translation invariance arise when the separation between two coordinates become comparable to the nuclear radius.

The simplest term in eq. (39) is the fourth one, involving a correlator between two Wilson lines in the adjoint representation. By analogy with eq. (41), we can define an unintegrated gluon distribution $d\phi_{g,g}^A/d^2X_\perp$ for the nucleus as follows:

$$d\phi_{g,g}^A \left( \frac{l_\perp | X_\perp \rangle}{2\pi l_\perp^2} \right) = g^2 N \int \frac{d\phi_{q\bar{q}}^A(\mathbf{k}_\perp, l_\perp - \mathbf{k}_\perp; l_\perp | X_\perp)}{d^2X_\perp}.$$

(42)

Again, we have neglected $l_\perp - l'_\perp$ in these manipulations, because this difference is of the order of the inverse of the radius of the nucleus or smaller. This equation can be seen as a definition of the unintegrated gluon distribution in the nucleus, that agrees at leading order in the source $\rho_A$ with what we have used for the proton. It is a natural object to use in this problem because it absorbs all the rescattering effects of the gluon on the nucleus. In fact, the same quantity already appeared for gluon production in pA collisions. Note that this is not the expectation value of the canonical number operator (except at leading order in the source $\rho_A$). Up to some trivial factors, this object is the square of the scattering amplitude of a gluon on the nucleus.

In order to deal with the 3-point correlator, it is convenient to define a function $d\phi_{q\bar{q}}^A/d^2X_\perp$ such that:

$$d\phi_{q\bar{q}}^A \left( \frac{k_\perp \cdot x_\perp - t'_\perp \cdot x'_\perp}{2\pi l_\perp^2} \right) \frac{d\phi_{q\bar{q}}^A(\mathbf{k}_\perp, l_\perp - \mathbf{k}_\perp; l_\perp | X_\perp)}{d^2X_\perp}.$$

(43)

Note that by construction, we have

$$\int d\phi_{q\bar{q}}^A(\mathbf{k}_\perp, l_\perp - \mathbf{k}_\perp; l_\perp | X_\perp) = \frac{d\phi_{q\bar{q}}^A(\mathbf{l}_\perp | X_\perp)}{d^2X_\perp}.$$

(44)

Indeed, integrating over $\mathbf{k}_\perp$ forces the coordinates $x_\perp$ and $y_\perp$ of the components of the $q\bar{q}$ pair to become equal. When this occurs, the $q\bar{q}$ pair in an octet state $\phi_A^g$ is identical to the $\phi_A^g$ introduced in section 4.1 of I. The superscript $g,g$ has been introduced for reasons that will become obvious later.
is effectively indistinguishable from a gluon. Mathematically, this statement is a consequence of the identity in eq. (28). One can similarly write the 4-point correlator as follows

\[
\alpha^a e^{i(k_{\perp} \cdot x_{\perp} - k_{\perp}' \cdot x_{\perp}')} e^{i(l_{\perp} \cdot k_{\perp})} e^{-i(l_{\perp} \cdot k_{\perp}')} \begin{tr}\left( \bar{U}(x_{\perp}) e^{a} \bar{U}^{-1}(y_{\perp}) e^{a'} \bar{U}^{-1}(x_{\perp}') \right) \end{tr}
\]

\[
\equiv \frac{g^2 N}{2 \pi l_T^2} \int_{x_{\perp}} e^{i(l_{\perp} - l_{\perp}')} x_{\perp} \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} = \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} .
\]  

(45)

Note further that this definition is such that

\[
\int_{k_{\perp},k_{\perp}'} \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} = \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} = \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} .
\]  

(46)

In other words, collapsing one \( q\bar{q} \) pair to a single point restores the 3-point correlator, and collapsing each pair to a point restores the 2-point correlator.

In terms of the functions we have just introduced, we can write the differential production probability as follows:

\[
\frac{dP_{1}(b)}{d^2 p_{\perp}^2 q_{\perp}^2 dq_{\perp}^2 dq_{\perp}^2} = \frac{\alpha^2 N}{8 \pi^4 d_{A}} \int_{k_{\perp},k_{\perp}';k_{\perp}''} \delta(p_{\perp} + q_{\perp} - k_{\perp} - k_{\perp}')
\]

\[
\times \int_{x_{\perp},y_{\perp}} e^{i(k_{\perp} - k_{\perp}') \cdot (x_{\perp} - y_{\perp} + b)} \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} \left\{ \begin{array}{l}
\text{tr} \left[ (q + m) T_{\bar{q}q} (q - m) \gamma^{0} T_{\bar{q}q}^{0} \right] \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} \\
\text{tr} \left[ (q + m) T_{\bar{q}q} (q - m) \gamma^{0} T_{\bar{q}q}^{0} \right] \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} \\
\text{tr} \left[ (q + m) T_{\bar{q}q} (q - m) \gamma^{0} T_{\bar{q}q}^{0} \right] \frac{d\varphi_{\perp} \cdot \varphi(q)}{d^2 X_{\perp}} \\
\end{array} \right\} .
\]  

(47)

At this point, it is straightforward to integrate over the impact parameter in order to turn the probability \( P_{1}(b) \) into a cross-section. This integration produces
a $\delta(k_{1\perp} - k'_{1\perp})$, which makes the integration over $k'_{1\perp}$ trivial. Then, integrating over the coordinates $X_\perp$ and $Y_\perp$ can be done independently in the proton and in the nucleus, which restores the $\varphi_p$ and the $\varphi_A$ functions for the whole projectiles rather than per unit area. Our final expression for the differential cross-section of quark pairs produced in proton-nucleus collisions is

$$
\frac{d\sigma}{d^2p_{\perp}d^2q_{\perp}dy_ddy_q} = \frac{\alpha_s^2N}{8\pi^4d_\perp} \int \frac{\delta(p_{\perp} + q_{\perp} - k_{1\perp} - k_{2\perp})}{k_{1\perp}^2k_{2\perp}^2} \times \left\{ \right.
\left. \int \frac{d\tau_d}{k_{1\perp} k_{1\perp}'} \left[ (\bar{q} + m)T_g \bar{q} \gamma^\perp \gamma^\perp \gamma^\perp \gamma^\perp \varphi^{\perp \perp}_{A}(k_{1\perp}, k_{2\perp} - k_{1\perp}; k_{1\perp} - k_{1\perp}') \right]
+ \int \frac{d\tau_d}{k_{1\perp} k_{1\perp}'} \left[ (\bar{q} + m)T_g \bar{q} \gamma^\perp \gamma^\perp \gamma^\perp \gamma^\perp \varphi^{\perp \perp}_{A}(k_{1\perp}, k_{2\perp} - k_{1\perp}; k_{2\perp} - k_{2\perp}') \right]
+ \int \frac{d\tau_d}{k_{1\perp} k_{1\perp}'} \left[ (\bar{q} + m)T_g \bar{q} \gamma^\perp \gamma^\perp \gamma^\perp \gamma^\perp \varphi^{\perp \perp}_{A}(k_{2\perp}, k_{2\perp} - k_{2\perp}; k_{2\perp}') \right]
+ \int \frac{d\tau_d}{k_{1\perp} k_{1\perp}'} \left[ (\bar{q} + m)T_g \bar{q} \gamma^\perp \gamma^\perp \gamma^\perp \gamma^\perp \varphi^{\perp \perp}_{A}(k_{1\perp}, k_{2\perp} - k_{1\perp}; k_{1\perp} - k_{1\perp}') \right]
\right\} \varphi_p(k_{1\perp}). \tag{48}
$$

This expression for pair production is the main result of this paper. Note that since $k_{1\perp} = k'_{1\perp}$, the abbreviations $T_{q\bar{q}}, T_{q\bar{q}}, T_g$ are related to eqs. (38) as follows:

$$
T_{q\bar{q}} \equiv T_{q\bar{q}}(k_{1\perp}, k_{1\perp}) , \quad T_{q\bar{q}}' \equiv T_{q\bar{q}}(k_{1\perp}, k_{1\perp}') ,
T_g \equiv T_g(k_{1\perp}). \tag{49}
$$

The Dirac traces in eq. 48 can be computed quite straightforwardly using a symbolic manipulation program like FORM. We will give explicit expressions for these in a future paper where we explore the phenomenological consequences of our result.

### 4.2 Leading twist approximation

The unintegrated distributions that we encountered in the previous section contain contributions from all twists and are in general quite complicated. As discussed briefly in the introduction, they can be obtained from numerical solutions of the JIMWLK equations [34]. For Gaussian correlations, these can be computed analytically, and an explicit derivation is given in appendix A. Here, we will discuss only the leading twist approximation, which has a simple physical interpretation and is useful in order to compare eq. (48) with the result obtained for $pp$ collisions.

At leading order in the density $\mu_A^2$ that describes the nucleus, it is easy to derive a closed form expression for the functions $\varphi_{A}^{g,g}, \varphi_{A}^{g,q\bar{q}}$ and $\varphi_{A}^{q\bar{q},q\bar{q}}$ introduced in the previous section. The only ingredient needed for this leading order
calculation is the average of two $\rho_A$'s, namely,
\[
\langle \rho_{A,a}(x_\perp)\rho_{A,a'}(x'_{\perp}) \rangle = \delta_{aa'}\delta(x_\perp - x'_{\perp})\mu_A^2.
\] (50)

From the definitions given in the previous section, one then obtains the following relations:
\[
\varphi_{g,g}^A(k_\perp) \approx \pi R^2 \left[ \pi d_A g^2 \frac{\mu_A^2}{k_{2\perp}^2} \right],
\]
\[
\varphi_{g,q\bar{q}}^A(k_\perp, k_{2\perp} - k_{\perp}\vert k_{2\perp}) \approx \frac{\pi R^2}{2} \left[ \pi d_A g^2 \frac{\mu_A^2}{k_{2\perp}^2} \right] (2\pi)^2 \left[ \delta(k_{\perp}) + \delta(k_{\perp} - k_{2\perp}) \right]
\]
\[
\varphi_{q\bar{q},q\bar{q}}^A(k_{\perp}, k_{2\perp} - k_{\perp}\vert k'_{\perp}, k_{2\perp} - k'_{\perp}) \approx \pi R^2 \left[ \pi d_A g^2 \frac{\mu_A^2}{k_{2\perp}^2} \right]
\]
\[
\times (2\pi)^4 \left[ \frac{C_F}{N} \left( \delta(k_{\perp} - k_{2\perp})\delta(k'_{\perp} - k_{2\perp}) + \delta(k_{\perp})\delta(k'_{\perp}) \right) + \frac{1}{2N^2} \left( \delta(k_{\perp} - k_{2\perp})\delta(k'_{\perp}) + \delta(k_{\perp})\delta(k'_{\perp} - k_{2\perp}) \right) \right].
\] (51)

In the derivation of these relations, we have assumed that the nuclei have a large transverse area $\pi R^2$ and that their density is nearly uniform in the transverse plane. If one substitutes these leading twist approximations in eq. (48), it is straightforward to re-obtain the expression of the cross-section we derived in a previous paper for $pp$ collisions (see eqs. (42), (B5), (B6) and (B7) of ref. [17]).

Note also that the leading twist expressions for $\varphi_{g,q\bar{q}}^A$ and $\varphi_{q\bar{q},q\bar{q}}^A$ have a simple interpretation. At this order, the probe (gluon or $q\bar{q}$ pair) interacts with the nucleus by a single gluon exchange. The two delta functions in $\varphi_{g,q\bar{q}}^A$ correspond to the gluon being attached to the antiquark line (\(\delta(k_{\perp})\) – there is no momentum flow from the nucleus to the quark line) or to the quark line (\(\delta(k_{2\perp} - k_{\perp})\) – all the momentum from the nucleus flows on the quark line). An identical interpretation holds for the four terms of $\varphi_{q\bar{q},q\bar{q}}^A$.

4.3 \textit{k$_\perp$-factorization for pair production ?}

From eq. (48), one sees immediately that in general we cannot write the pair production cross-section for $pA$ collisions in a $k_{\perp}$-factorized manner, at least not in the usual sense. The closest to $k_{\perp}$-factorization we can achieve for $pA$ collisions is precisely eq. (48), where we see that we need three different functions in order to describe the nucleus. The physical reason for this is that a quark-antiquark pair is a composite object: the amplitude for producing a pair depends on how much momentum the nucleus gives to the quark and how much momentum it gives to the antiquark (in case the pair is formed before going through the nucleus), and not just on the total momentum taken to the nucleus. This also explains why a true $k_{\perp}$-factorization is possible when we treat the problem of pair production at leading order in the source $\rho_A$. Indeed, at this
order, the pair can only exchange one gluon with the nucleus, which is either attached to the quark or to the antiquark, and a single momentum variable completely characterizes this exchange.

From this discussion, one can also foresee in what limit a true \( k_\perp \) factorization would be possible. From the integral relations of eqs. (44) and (46), we indeed see that if we can neglect the dependence on \( k_\perp \) in \( T_{q\bar{q}}(k_{1\perp}, k_{2\perp}) \), then the integrations over \( k_\perp \) and \( k'_\perp \) act only on the functions \( \varphi_{q\bar{q}}^{q\bar{q}} \) and \( \varphi_{A}^{q\bar{q}} \) and simply give \( \varphi_{A}^{q\bar{q}} \) which we can now factor out. Such an approximation is valid when the momenta \( p_\perp, q_\perp \) of the produced quark and antiquark are large, or when the mass \( m \) of the produced quark is large. In this case, one effectively produces a pair which has a small transverse size and behaves like a single gluon.

5 Single quark cross-section

We shall now compute the single quark inclusive cross-section which is phenomenologically interesting in its own right. It is obtained from the pair production production cross-section given in eq. (48) by integrating over the transverse momentum \( p_\perp \) and rapidity \( y_p \) of the antiquark. Integrating over \( p_\perp \) is trivial thanks to the delta function \( \delta(p_\perp + q_\perp - k_{1\perp} - k_{2\perp}) \), and simply amounts to replacing \( p_\perp \to k_{1\perp} + k_{2\perp} - q_\perp \) in the rest of the expression.

There is an interesting simplification of the first term, involving the 4-point function \( \varphi_{A}^{q\bar{q}} \). One notices that \( T_{q\bar{q}} \) (see the first of eqs. (38)) does not depend on \( p_\perp \). Moreover, since \( T_{q\bar{q}} \) has a \( \gamma^+ \) on the right, the product \( T_{q\bar{q}}(p-m)\gamma^0 T_{q\bar{q}}\gamma^0 \) does not depend on \( p_\perp \) either (because \( \gamma^+(p-m)\gamma^+ = 2p^+\gamma^+ \) does not depend on \( p_\perp \)). Therefore, this object does not depend on \( k_{2\perp} \) after the substitution \( p_\perp \to k_{1\perp} + k_{2\perp} - q_\perp \). As a consequence, the integration over \( k_{2\perp} \) can be performed directly for this term, since we have

\[
\int \frac{d^2k_{2\perp}}{(2\pi)^2} \frac{1}{k_{2\perp}^2} \varphi_{A}^{q\bar{q}}(k_{1\perp}, k_{2\perp} - k_\perp; k'_\perp, k_{2\perp} - k'_\perp) = \frac{2\pi C_F}{g^2N_c} (2\pi)^2 \delta(k_\perp - k'_\perp) \int \mathop{\text{tr}} \langle \tilde{U}(x)\tilde{U}(x') \rangle \ . \ (52)
\]

Thus we see that the 4-point function reduces to a much simpler 2-point function. It is then natural to define, by analogy with eq. (42), the following function:

\[
\int \mathop{\text{tr}} \langle \tilde{U}(x)\tilde{U}(x') \rangle \equiv \frac{g^2}{2\pi k_{\perp}^2} \varphi_{A}^{q\bar{q}}(k_\perp) \ . \ (53)
\]

The normalization has been chosen such that this definition becomes equivalent to eq. (41) in the limit where the source \( p_A \) which describes the nucleus is a weak source. Because \( T_{q\bar{q}} \) depends explicitly on \( p_\perp \), it becomes \( k_{2\perp} \)-dependent after the substitution \( p_\perp \to k_{1\perp} + k_{2\perp} - q_\perp \). For this reason, no such simplification
occurs for the last three terms of eq. (48). We can now write the single quark production cross-section as follows:

\[
\frac{d\sigma_q}{d^2q_\perp dy_q} = \frac{\alpha_s^2 N}{8\pi^2 d_A} \int \frac{dp^+}{p^+} \int \frac{1}{k_{1\perp}^2 k_{2\perp}^2} \times \left\{ \text{tr}_{A} \left[ (\gamma^0 T^{\dagger}_{q\bar{q}})(k_{1\perp}, k_{2\perp})(\gamma^0 T^{\dagger}_{g})(k_{1\perp}, k_{2\perp}) \right] \phi_{\perp}^{q,q}(k_{2\perp}) \right\} + \left\{ \text{tr}_{A} \left[ (\gamma^0 T^{\dagger}_{g})(k_{1\perp}, k_{2\perp}) \right] \phi_{\perp}^{g,g}(k_{2\perp}) \right\} .
\]  

(54)

In the first term, we have re-labeled \( k_{\perp} \) as \( k_{2\perp} \) in order to be able to factor out the term \( 1/k_{2\perp}^2 \) \cite{35}. Note also that \( p_{\perp} \) should be understood as \( k_{1\perp} + k_{2\perp} - q_{\perp} \) in this formula. Obviously, we cannot factor out in eq. (54) a common function \( \phi_A \) that would describe the gluon content of the nucleus. This of course means that the single quark production cross-section, even if simpler than the pair production cross-section, is still not \( k_{\perp} \)-factorizable.

Recently, in ref. \cite{35}, Tuchin considered the inclusive single quark distribution in proton-nucleus collisions for a model where the collisions on the successive nucleons are independent. This is equivalent to the case of Gaussian correlations in the MV-model. Tuchin’s discussion is in the light-cone gauge \( A_+ = 0 \), and is expressed as the convolution of a) the square of the amplitude for the emission of a gluon \( g \) by a valence quark \( q_v \) of the proton b) the square of the amplitude for the splitting of this gluon into a \( q \bar{q} \) pair and c) the rescatterings of the \( q_v g q \bar{q} \) system on the nucleus. Since his results are expressed entirely in coordinate space, Tuchin does not address the issue of \( k_{\perp} \)-factorization. Further, since our results, in contrast, are entirely formulated in momentum space, a detailed comparison of his result with the Gaussian correlation limit of our general expression is not at present feasible. Nevertheless, several features of his result can be identified with terms appearing in our result.

6 Conclusions

We have computed in this paper the production of quark pairs in high energy proton-nucleus collisions in the Color Glass Condensate approach. We find that
the pair production cross-section, unlike the inclusive gluon production cross-section discussed in I, is not $k_{\perp}$-factorizable into a product of unintegrated distributions, because the quark and the anti-quark can separately interact with the nucleus after they are produced. The same is true also for the single quark distribution. In the limit of very large mass pairs, or when the quark and the anti-quark have large transverse momenta, $k_{\perp}$-factorization is recovered.

Though not $k_{\perp}$-factorizable in the usual sense, the pair production cross-section can still be expressed in terms of the product of the unintegrated gluon distribution of the proton times the sum of four terms containing four such unintegrated distributions describing the nucleus. One of these four terms is the 2-point correlator of two adjoint Wilson lines which already appeared in gluon production in pA collisions. This distribution corresponds to a gluon interacting with the nucleus in the amplitude and in the complex conjugate amplitude. Two of the other terms contain a 3-point correlator of two fundamental Wilson lines and an adjoint Wilson line. This corresponds to a gluon in the amplitude and a quark–anti-quark pair in the complex conjugate amplitude or vice versa. The fourth unintegrated distribution is related to the 4-point correlator of four fundamental Wilson lines; this of course corresponds to quark–anti-quark pairs in both the amplitude and the complex conjugate amplitude.

The single quark distribution is also not $k_{\perp}$-factorizable. A significant simplification nevertheless occurs because the correlator of 4 fundamental Wilson lines is replaced by the correlator of two fundamental Wilson lines. One therefore has only contributions from 2-point and 3-point correlators to the single quark distribution.

The three and four point correlators that appear in pair production cross-section probe non-trivial multi-parton correlations in addition to those involved in inclusive gluon production alone. The three point function that appears here is the same as one that appears in inelastic and diffractive deeply inelastic scattering at high energies [45,46]. These correlators of Wilson lines are therefore the relevant universal degrees of freedom at high energies, appearing in all high energy scattering reactions. The quantum evolution of these N-point correlators of Wilson lines at small $x$ is described by the JIMWLK equations. Measurements of the pair production and single quark distributions can therefore provide a sensitive test of such evolution equations.

In the McLerran-Venugopalan model of Gaussian correlations, these 2, 3 and 4-point correlators of Wilson lines can be computed analytically. The explicit expressions for these are derived in appendix B. The 2-point function has been known for a long time [31,47]. The 3-point function is, for Gaussian correlations, the $q \bar{q}g$-propagator and was computed in refs. [46,45,48,49]. For single quark production in the MV-model, as discussed here and by Tuchin previously, it is only these correlators that appear in the cross-section. For pair production, one needs the 4-point function as well. This is computed for Gaussian correlations in appendix B.

As we discussed extensively in I, the MV-model is a good model for high-energy scattering when $x$ is not too small – or quantum evolution is not important. When quantum evolution sets in, the initial condition of Gaussian
correlations changes quickly \[50,51\] with non-Gaussian correlations becoming increasingly important at smaller values of \(x\). The data at RHIC suggest that the MV-model might provide a good description of Deuteron-Gold collisions at central rapidities (when \(x \sim 10^{-2}\)) but is a bad description already at forward rapidities corresponding to \(x \sim 10^{-3}\). Computations of single quark and pair-production cross-section in both the MV-model and with the JIMWLK evolution equations will greatly help confirm the picture of D-Au collisions that is emerging from the RHIC data. We plan to pursue the phenomenological implications of our results in a future publication.

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A Gaussian averages of Wilson lines

A.1 Introduction
In this appendix, we derive the expression of the correlators of Wilson lines that appear in eq. (39). The most complicated one is the 4-point function \(C(x_\perp, y_\perp; u_\perp, v_\perp) \equiv \text{tr} \langle \bar{U}(x_\perp)t^a\bar{U}(y_\perp)U(u_\perp)t^aU(v_\perp) \rangle\), and it is trivial to verify that the 3-point and 2-point correlators that also appear in eq. (39) can be obtained as limits of the 4-point function:
\[
\text{tr} \langle \bar{U}(x_\perp)t^a\bar{U}(y_\perp)t^bU(v_\perp)U(u_\perp)^a U(u_\perp)^b U(v_\perp) \rangle = C(x_\perp, y_\perp; u_\perp, u_\perp).
\]

Thanks to these relations, it is sufficient to compute the 4-point function. Similar correlators have been studied in \[46\] and \[52\], although the 4-point function we need here is not derived explicitly in these papers. It is in general impossible to obtain closed expressions for these correlators. An exception is the case of a Gaussian weight function for the probability distribution of the source \(\rho_A\):

\[
W_A[\rho_A] = \exp \left[ -\int d^2x d^2x_\perp \rho_A(x_\perp) \rho_A(x_\perp) \frac{\bar{\rho}_A(x_\perp)\rho_A(x_\perp)}{2\mu^2_A(x_\perp)} \right].
\]

We denote\(^{16}\) \(\bar{\rho}_A(x_\perp, x_\perp) \equiv \delta_\perp(x_\perp)\rho_A(x_\perp)\), and \(\mu^2_A(x_\perp)\) is the density of color charges at a given \(x_\perp\) (this function is strongly peaked around \(x_\perp = 0\)). The most elementary correlator is

\[
\langle \rho_A(x_\perp, x_\perp) \rho_A(y_\perp, y_\perp) \rangle = \delta_\perp \delta(x_\perp - y_\perp) \delta(x_\perp - y_\perp) \mu^2_A(x_\perp),
\]

\(^{16}\)We must reintroduce the smearing in \(x_\perp\) here in order to make sense of the path ordering that appears in the Wilson lines. It will however not play any role in the final results.
which is local in $x^+$ and $x_\perp$. (The latter property is not essential in the following calculation, and can be relaxed if necessary.)

In the calculation of the 4-point function $C(x_\perp, y_\perp; u_\perp, v_\perp)$, the most complicated task is to deal with the $SU(N)$ color algebra. A very convenient way to do that is to use a semi-graphical method. We can represent pictorially the correlator that we need as

$$C(x_\perp, y_\perp; u_\perp, v_\perp) = \ldots$$

(58)

In this diagrammatic representation, the dotted lines represent Wilson lines in the fundamental representation with the horizontal axis the $x^+$ axis. Note that they therefore depend on the source $\rho_A$ to all orders. The arrows indicate the direction of the path-ordering, i.e. whether we have a $\tilde{U}$ of a $\tilde{U}^\dagger$. The thick black dots are color matrices $t^a$ in the fundamental representation, and the wavy line that connects them denotes a contraction of their color indices ($t^a \cdots t^a$). The Wilson lines are connected at $x^+ = \pm \infty$ by solid lines which merely indicate how the Wilson lines are multiplied (these solid lines do not contain any $\rho_A$ nor color matrices). The closed loop indicates that the product of Wilson lines is traced. An essential ingredient in the forthcoming calculation is the Fierz identity in $SU(N)$\footnote{See [53] for a detailed discussion of group algebra manipulations in non-abelian gauge theories.}:

$$t^a_{ij} t^a_{kl} = \frac{1}{2} \delta_{il} \delta_{jk} - \frac{1}{2N} \delta_{ij} \delta_{kl},$$

(59)

which has the following graphical representation:

$$\ldots = \frac{1}{2} \ldots - \frac{1}{2N} \ldots$$

(60)

Thus, the 4-point correlator $C$ simplifies into a sum of two simpler terms:

$$C(x_\perp, y_\perp; u_\perp, v_\perp) = \frac{1}{2} \ldots - \frac{1}{2N} \ldots$$

(61)

It is therefore sufficient to calculate the following object of $SU(N) \times SU(N)$:

$$\mathcal{M}_{ijkl}(x_\perp, y_\perp; u_\perp, v_\perp) \equiv \left\langle \left[ \tilde{U}^\dagger(y_\perp) \tilde{U}(x_\perp) \right]_{ij} \left[ \tilde{U}^\dagger(y_\perp) \tilde{U}(u_\perp) \right]_{kl} \right\rangle,$$

(62)
which can be represented as a diagram with open ends:

$$M_{ijkl}(x_\perp, y_\perp, u_\perp, v_\perp) = \cdots y \cdots \cdots \cdots x \cdots \cdots \cdots u \cdots \cdots \cdots v \cdots .$$

(63)

Once we know this object, the correlator $C$ is given by:

$$C(x_\perp, y_\perp; u_\perp, v_\perp) = \frac{1}{2} M_{ikkk}(x_\perp, y_\perp, u_\perp, v_\perp) - \frac{1}{2N} M_{ikkk}(x_\perp, y_\perp, u_\perp, v_\perp) .$$

(64)

### A.2 Tadpole corrections

Thanks to the Gaussian form of the functional $W[\rho_A]$, we can expand $M$ in powers of the elementary correlator $\langle \rho_A \rho_A \rangle$ using Wick’s theorem. This amounts to expanding the Wilson lines in powers of the source $\rho_A$ and to connecting the $\rho_A$’s pairwise. From the point of view of its color structure, such a pairing $\langle \rho_A \rho_A \rangle$ contains a contracted pair $t^a \cdots t^a$ of color matrices and can be represented by two black dots connected by a wavy link. Since $\langle \rho_A \rho_A \rangle$ is local in $x^+$, only links that have their endpoints at the same $x^+$ are allowed. Some of them are “tadpoles” that start and end on the same Wilson line, and can be represented as follows:

$$\cdots x \cdots \cdots \cdots \cdots \cdots \cdots u \cdots \cdots \cdots \cdots y \cdots .$$

(65)

In this diagrammatic representation, the black dots are color matrices\(^{18}\) $t^a$, and the solid line simply indicates in what order they are multiplied. A tadpole insertion on the Wilson line of transverse coordinate $x_\perp$ at the “time” $z^+$ brings a factor $-C_F \mu^2_A(z^+)L(x, x)$, with:

$$L(x, x) \equiv g^4 \int_{z_\perp} G_0(x_\perp - z_\perp)^2 ,$$

(66)

where $C_F \equiv (N^2 - 1)/2N$ is the quadratic Casimir in the fundamental representation of $SU(N)$ and where

$$G_0(x_\perp - z_\perp) \equiv \int \frac{d^2 k_\perp}{(2\pi)^2} \frac{e^{i k_\perp \cdot (x_\perp - z_\perp)}}{k_\perp^2} .$$

(67)

\(^{18}\)For tadpoles, there is one $t^a$ at each end of the wavy line, even if only one dot appears in the diagram.
is the 2-dimensional free massless propagator. Note that tadpole insertions are color singlet, and therefore commute with the rest of the expression. As a consequence, we can easily factor them out and resum them. This gives a factor\textsuperscript{19}:

$$T \equiv e^{-\frac{1}{2} C_{F} \mu_{A}^{2} [L(x,x) + L(y,y) + L(u,u) + L(v,v)]},$$ \hspace{1cm} (68)

where we denote the density integrated over the longitudinal coordinate:

$$\mu_{A}^{2} \equiv \int_{-\infty}^{+\infty} dz^{+} \mu_{A}^{2}(z^{+}),$$ \hspace{1cm} (69)

by the same symbol, but without an argument.

### A.3 Non-tadpole corrections

We can now write

$$\mathcal{M} = T \mathcal{N},$$ \hspace{1cm} (70)

where \( \mathcal{N} \) is the same as \( \mathcal{M} \), but containing only links that connect different Wilson lines. A typical term in \( \mathcal{N} \) with two links could be the following:

\begin{align*}
\begin{array}{c}
\text{\includegraphics[width=0.3\textwidth]{diagram1.png}}
\end{array}
\end{align*}

(71)

Note that since \( \langle \rho_{A} \rho_{A} \rangle \) is local in \( x^{+} \), links must connect points at the same \( x^{+} \). Therefore, diagrams with crossed links are not allowed. By a systematic use of the Fierz identity of eq. (60), we can completely remove the links. On the previous example, this would lead to the following four terms\textsuperscript{20}:

\begin{align*}
\begin{array}{c}
\text{\includegraphics[width=0.9\textwidth]{diagram2.png}}
\end{array}
\end{align*}

(72)

Note that the closed loop that appears in the second term can be evaluated directly and replaced by the trace of the unit matrix in the fundamental representation \( \text{tr}_{c}(1_{c}) = N \). Therefore, we see that all the terms that are generated

\textsuperscript{19}The factor \( 1/2 \) in the exponential is due to the fact that the two ends of a link are ordered in \( z^{+} \) if they belong to the same Wilson line.

\textsuperscript{20}The diagrams have been rearranged a bit in order to make them look nicer, without changing their topology.
in the process of eliminating the links with the Fierz identity are one of the following two terms:

\[ N^a \equiv \begin{array}{c}
  \text{x} \\
  \text{y}
\end{array} \quad N^b \equiv \begin{array}{c}
  \text{x} \\
  \text{y}
\end{array} . \quad (73) \]

This property, observed for the example of eq. (71), is in fact completely general.

Let us now denote \( N_n(z^+_1, \cdots, z^+_n) \) the piece of \( N \) that contains \( n \) links, at the “times” \( z^+_1 < \cdots < z^+_n \). We therefore have,

\[ N = \sum_{n=0}^{\infty} \int_{z^+_1 < \cdots < z^+_n} N_n(z^+_1, \cdots, z^+_n) . \quad (74) \]

Using the above remarks, we can write each \( N_n \) as a linear combination of \( N^a \) and \( N^b \):

\[ N_n \equiv a_n N^a + b_n N^b . \quad (75) \]

Note that the term with zero links is exactly \( N^a \), so that we have \( a_0 = 1 \) and \( b_0 = 0 \). Once we know \( N_{n-1} \), the next term in the expansion, \( N_n \), can be obtained by adding one link to \( N^a \) and \( N^b \) in all the possible ways:

\[ N_n = \mu^2(\gamma) a_{n-1} \left[ L(x, v) - L(x, u) + \cdots \right] + \mu^2(\gamma) b_{n-1} \left[ L(x, v) - L(x, u) + \cdots \right] , \quad (76) \]

where, generalizing eq. (66), we denote

\[ L(x, v) \equiv g^4 \int_{x} G_0(x_{\perp} - z_{\perp}) G_0(v_{\perp} - z_{\perp}) . \quad (77) \]

There are six possible ways to link four lines pairwise, and we have only represented two of them. Note that there is a \( - \) sign for links connecting two Wilson lines going in the same direction (i.e. two \( U \)'s or two \( U^\dagger \)'s). The next step is to use eq. (60) in order to remove all the links and to replace by \( N \) the closed loops that may appear in the process. Finally, identifying with \( N_n = a_n N^a + b_n N^b \), we obtain the following recursion relation:

\[ \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \mu^2(\gamma) M \begin{pmatrix} a_{n-1} \\ b_{n-1} \end{pmatrix} , \quad (78) \]
where $M$ is a $2 \times 2$ matrix given by

$$M \equiv \begin{pmatrix} \alpha C_F + \frac{1}{2N}(\beta - \gamma) & \frac{1}{2}(\alpha - \beta) \\ \frac{1}{2}(\gamma - \beta) & \gamma C_F + \frac{1}{2N}(\beta - \alpha) \end{pmatrix},$$  \hspace{1cm} (79)$$

with

$$\alpha \equiv L(x, v) + L(u, y),$$

$$\beta \equiv L(x, u) + L(v, y),$$

$$\gamma \equiv L(x, y) + L(v, u).$$  \hspace{1cm} (80)$$

Note that $M$ does not depend on $z_{n\pm}$. Given the initial condition $a_0 = 1, b_0 = 0$, it is now very easy to formally solve the recursion relation to obtain

$$\begin{pmatrix} a_n \\ b_n \end{pmatrix} = \begin{bmatrix} \prod_{i=1}^{n} \mu_A^2(z_i^+) \end{bmatrix} M^n \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \hspace{1cm} (81)$$

From the coefficients $a_n, b_n$, the correlator $C(x_\perp, y_\perp; u_\perp, v_\perp)$ is then obtained by using eqs. (61), (70), (74) and (75). Using eq. (61) amounts to "closing the ends" of $N$. Multiplying also by the contribution of the tadpole insertions, we get:

$$C(x_\perp, y_\perp; u_\perp, v_\perp) = \frac{1}{2} \sum_{n=0}^{+\infty} \int \frac{dz_{1\perp} < \cdots < z_{n\perp}}{z_{1\perp} < \cdots < z_{n\perp}} a_n \left( N^2 - 1 \right) T. \hspace{1cm} (82)$$

In order to make this formula more explicit, we must integrate eq. (81) over the $z_{1\perp}^+$’s, which gives$^{21}$:

$$\int_{z_{1\perp}^+ < \cdots < z_{n\perp}^+} \begin{pmatrix} a_n \\ b_n \end{pmatrix} = \frac{1}{n!} \mu_A^{2n} M^n \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \hspace{1cm} (83)$$

The only remaining task is to calculate the $n$-th power of the constant matrix $M$. This can be done by finding its eigenvalues, which are easily shown to be

$$\lambda_{\pm} = \frac{1}{2} \left[ \frac{N}{2} (\alpha + \gamma) + \frac{1}{N} (\beta - \alpha - \gamma) \pm \sqrt{\frac{N^2}{4} (\alpha - \gamma)^2 + (\alpha - \beta)(\gamma - \beta)} \right].$$  \hspace{1cm} (84)$$

It is then clear that we can write

$$\sum_{n=0}^{+\infty} \int_{z_{1\perp}^+ < \cdots < z_{n\perp}^+} a_n = a_+ e^{\mu_A^2 \lambda_+} + a_- e^{\mu_A^2 \lambda_-},$$  \hspace{1cm} (85)$$

$^{21}$The ordering of the $z_{1\perp}^+$’s in eq. (74) can be trivially removed because the recursion matrix commutes with itself at different times. This gives a factor $1/n!$. 

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with $a_+$ and $a_-$ two numbers determined from the initial condition. At this point, it is a simple matter of algebra to derive our final expression for the 4-point function $C$:

$$
C(x_\perp, y_\perp; u_\perp, v_\perp) = NC e^{-\frac{N}{2}A [\Gamma(x-u)+\Gamma(y-v)]} \times e^{-\left(\frac{\Delta}{2}\right)\mu_A^2 \left[\Gamma(x-v)+\Gamma(x-y)+\Gamma(u-v)+\Gamma(u-y)\right]} \times \left\{ \frac{1 + \sqrt{\Delta}}{2\sqrt{\Delta}} e^{\frac{\Delta}{2} \mu_A^2 (\alpha-\gamma)} - \frac{1 - \sqrt{\Delta}}{2\sqrt{\Delta}} e^{-\frac{\Delta}{2} \mu_A^2 (\alpha-\gamma)} \right\},
$$

(86)

where we denote

$$
\Delta \equiv 1 + \frac{16}{N^2} \frac{\left(\alpha-\beta\right)\left(\gamma-\beta\right)}{\left(\alpha-\gamma\right)^2},
$$

(87)

and

$$
\Gamma(y-u) \equiv L(y,y) + L(u,u) - 2L(y,u).
$$

(88)

A comment is in order regarding this result. The quantity $L(x,y)$ defined in eq. (77) is strongly infrared divergent. Indeed, if one introduces by hand an upper cutoff in the integration over $z_\perp$ at $z_\perp \sim \Lambda_{QCD}^{-1}$, then $L(x,y) \sim \Lambda_{QCD}^{-2}$. However, the combination $\Gamma$ of these objects, defined in eq. (88), has a much milder infrared singularity which is logarithmic in the cutoff scale. This is also the case of the differences $\alpha-\gamma$, $\alpha-\beta$ and $\gamma-\beta$ which appear in eq. (86). Therefore, we conclude that the 4-point function studied here is not more infrared singular than the 2-point functions already discussed extensively in the literature. Physically, the reason why the quadratic divergence is softened to a logarithmic divergence is that we consider a correlator made of a singlet combination of Wilson lines. The reason why we have nevertheless a residual logarithmic divergence is that the color neutrality of the nucleus is not enforced in the functional of eq. (56). We have introduced by hand a cutoff at the scale $\Lambda_{QCD}$ which is equivalent to imposing color neutralization at the scale of the size of the nucleon.

### A.4 Large $N$ limit

Eq. (86) becomes substantially simpler in the large $N$ limit. Indeed, in this limit, we have $\Delta = 1$. We have therefore,

$$
C(x_\perp, y_\perp; u_\perp, v_\perp) \underset{N \to \infty}{=} \frac{N^2}{2} e^{-\frac{N}{2}A [\Gamma(x-u)+\Gamma(x-y)+\Gamma(u-v)+\Gamma(u-y)]} e^{\frac{N}{2}A (\alpha-\gamma)}.
$$

(89)

Using the explicit expressions of $\alpha$, $\gamma$ (eq. (80)), this simplifies greatly to read,

$$
C(x_\perp, y_\perp; u_\perp, v_\perp) \underset{N \to \infty}{=} \frac{N^2}{2} e^{-\frac{N}{2}A [\Gamma(x-v)+\Gamma(y-u)]}.
$$

(90)

This result could also have been obtained very simply from the recursion relation eq. (78) because the recursion matrix $M$ is diagonal in the large $N$ limit.
A.5 Special cases

There are a few degenerate cases in which the 4-point function we have calculated in this appendix simplifies into a simpler function. These special cases are useful as a check, because for these the result is already known in the literature.

A.5.1 $x_\perp = v_\perp$ or $y_\perp = u_\perp$

There is an obvious simplification in $C(x_\perp, y_\perp; u_\perp, v_\perp)$ if $x_\perp = v_\perp$ or if $y_\perp = u_\perp$. Indeed, any of these equalities makes $\beta = \gamma$, which implies $\Delta = 1$. For instance, in the case of $x_\perp = v_\perp$, we obtain from eq. (86) the following formula:

$$C(x_\perp, y_\perp; u_\perp, x_\perp) = N C_p e^{-\frac{\alpha}{4}\mu_\rho^2} \left[ \Gamma(x-u) + \Gamma(y-x) \right]$$

$$\times e^{-\left(\frac{\alpha}{4} - \frac{\gamma}{4}\right)\mu_A^2} \left[ \Gamma(x-y) + \Gamma(x-u) + \Gamma(y-u) \right] e^{\frac{\alpha}{4}\mu_\rho^2 (\alpha - \gamma)}$$

$$= N C_p e^{-\frac{\alpha}{4}\mu_A^2 \Gamma(y-u)}.$$  (91)

This simple formula is indeed what we already know (see [39] for instance) for this degenerate case, since we expect:

$$C(x_\perp, y_\perp; u_\perp, x_\perp) = C_F \text{tr} \langle \tilde{U}(y_\perp) \tilde{U}(u_\perp) \rangle.$$  (92)

A.5.2 $x_\perp = y_\perp$ or $u_\perp = v_\perp$

A slightly less trivial simplification is obtained when $x_\perp = y_\perp$ or $u_\perp = v_\perp$. Indeed, any of these equalities makes $\beta = \gamma$, which also implies $\Delta = 1$. When $u_\perp = v_\perp$, eq. (86) simplifies into:

$$C(x_\perp, y_\perp; u_\perp, u_\perp) = N C_p e^{-\frac{\alpha}{4}\mu_\rho^2 \left( \Gamma(x-u) + \Gamma(y-u) \right)}$$

$$\times e^{-\left(\frac{\alpha}{4} - \frac{\gamma}{4}\right)\mu_A^2 \left( \Gamma(x-y) + \Gamma(x-u) + \Gamma(y-u) \right) e^{\frac{\alpha}{4}\mu_\rho^2 (\alpha - \gamma)}}$$

$$= N C_p e^{-\frac{\alpha}{4}\mu_A^2 \left( \Gamma(x-u) + \Gamma(y-u) \right) e^{-\left(\frac{\alpha}{4} - \frac{\gamma}{4}\right)\mu_A^2 \Gamma(x-y)}}.$$  (93)

It is trivial to check that this formula is equivalent to eq. (C7) of [46] (the correspondence between our $\Gamma(x-y)$ and the $\nu(x-y)$ defined in [46] is $2\nu(x-y) = \mu_A^2 \Gamma(x-y)$).

B Retarded pair production amplitude

B.1 Introduction

In section 3, we have calculated the time-ordered pair production amplitude, which is directly related to the probability $P_1$ of producing a single quark-antiquark pair in a collision. We also conjectured that, since we are treating
the problem at leading order in the source \( \rho_p \) that describes the proton, at most one pair can be produced in a collision. This implies that the average number of pairs \( \mathcal{N}_{q\bar{q}} \) produced in a collision must be equal to the probability \( P_1 \), simply because the probabilities \( P_2, P_3, \cdots \) to produce 2 or more pairs in a collision are all zero at this order.

On the other hand, we can calculate directly the average pair multiplicity \( \mathcal{N}_{q\bar{q}} \) from the retarded pair production amplitude, as explained in [38], from the formula,

\[
\mathcal{N}_{q\bar{q}} = \int \frac{d^3q}{(2\pi)^3 2E_q} \int \frac{d^3p}{(2\pi)^3 2E_p} |\mathcal{M}_R(q,p)|^2,
\]

where \( \mathcal{M}_R(q,p) \) is the pair production amplitude calculated with retarded quark propagators (as opposed to Feynman quark propagators in the case of \( \mathcal{M}_F(q,p) \)). Since the calculation of the retarded amplitude presents a number of significant differences with respect to the time-ordered amplitude, its calculation provides a non-trivial cross-check of the results we have derived in section 3. Because of the expected equality \( P_1 = \mathcal{N}_{q\bar{q}} \), we expect the retarded and time-ordered amplitude to be identical “up to a phase”.

\[
\mathcal{M}_R = \mathcal{M}_F \tilde{U}(y_{\perp}),
\]

Figure 3: Top: the contributions to the Feynman amplitude. The shaded area represents the nucleus. A gluon is emitted by the proton, and splits into a pair: after the collisions with the nucleus (left), inside the nucleus (middle) or before the collision (right). Bottom: the same contributions for the retarded amplitude.

In fact, without doing any calculation, we can guess what this phase is. The contributions to the two amplitudes are sketched in figure 3, and one can see that the only difference between the two is that the antiquark line comes from the region of negative \( x^+ \) in the case of the retarded amplitude. This is so because the retarded quark propagator can only move forward in time. From this figure, we see that moving the antiquark line from the left of the diagram to the right of the diagram changes the amplitude by a factor which is precisely the scattering matrix of the antiquark on the nucleus. In a sense to be made more precise later, we expect the following relation:

\[
\mathcal{M}_R = \mathcal{M}_F \tilde{U}(y_{\perp}),
\]
where \( y_\perp \) is the transverse coordinate of the antiquark.

In order to calculate explicitly the retarded amplitude, we must use the free retarded quark propagator:

\[
S^0_R(p) \equiv i \frac{\not{p} + m}{p^2 - m^2 + ip^+ \epsilon},
\]

which differs from eq. (18) by the location of its poles in the complex energy plane. Note that the \( p^+ \epsilon \) in the denominator could as well have been \( p^- \epsilon \) or \( p_0 \epsilon \) because \( p^+, p^- \) and \( p_0 \) have the same sign at the pole. The other technical difference is the form of the scattering matrix of a quark/antiquark on the nucleus. In the retarded case, it is given by [38]:

\[
\begin{align*}
\pi & \rho \delta(k^+) \gamma^+ \int d^2 x_\perp e^{i k_\perp \cdot x_\perp} \left[ \tilde{U}(x_\perp) - 1 \right].
\end{align*}
\]

We see that it coincides with the time-ordered scattering operator given in eq. (16) when \( p^+ > 0 \) (as expected since the poles of the retarded and time-ordered free propagator are the same if \( p^+ > 0 \)), but differs from it when \( p^+ < 0 \).

### B.2 Singular terms

Let us start with the calculation of the term involving the field \( A_\text{sing}^\mu \), where the pair is produced inside the nucleus. It is trivial to reproduce the calculation of section 3.3 with the Feynman rules appropriate for the retarded propagator. We obtain:

\[
M_\text{sing}^R(q, p) = g^2 \int \frac{d^2 k_\perp}{(2\pi)^2} \rho_{p,a}(k_\perp) \int d^2 x_\perp e^{i(p_\perp + q_\perp - k_\perp) \cdot x_\perp}
\]

\[
\times \frac{\pi(q) \gamma^+ b^b \tilde{U}(x_\perp)v(p)}{p^+ + q^+} [V(x_\perp) - U(x_\perp)]_{ba}.
\]

This is indeed the same as eq. (31), up to the factor \( \tilde{U}(x_\perp) \) (the transverse coordinate of the antiquark is \( x_\perp \) in this expression).

### B.3 Regular terms

We must now calculate the four “regular” (in the sense that the pair is produced outside of the nucleus) diagrams of figure 1 in the case of the retarded amplitude. The diagram (a) does not contain any internal quark propagator and is therefore identical in the time-ordered and retarded cases:

\[
M_\text{R}^{(a)}(q, p) = M_\text{T}^{(a)}(q, p).
\]
Contrary to the time-ordered case, the pole structure of the diagram (d) is such that it does not contribute in the retarded case\textsuperscript{22}:

\[ M_{R}^{(d)}(q, p) = 0. \]  

(100)

The diagram (c) is also very simple: indeed, the only fermion propagator that interacts with the nucleus is the quark line which carries the momentum \( q^+ > 0 \). Therefore, this propagator is the same in the retarded and in the time-ordered case:

\[ M_{R}^{(c)}(q, p) = M_{O}^{(c)}(q, p). \]  

(101)

All the difficulties are concentrated in the diagram (b). Indeed, since this diagram has a fermion propagator carrying the momentum \(-p^+ < 0\), its pole structure is not the same in the retarded and in the time-ordered case. In addition, the interaction operator of the antiquark on the nucleus is different. The diagram (b) can be divided in three terms:

\[ M_{R}^{(b)} = M_{R,1}^{(b)} + M_{R,2}^{(b)} + M_{R,3}^{(b)}. \]  

(102)

\( M_{R,1}^{(b)} \) is obtained by keeping only the first line of the field in eq. (5) (namely replacing the solution of Yang-Mills equations by the field of the proton alone). This term can be written as follows:

\[ M_{R,1}^{(b)}(q, p) = g^2 \int \frac{d^2 k_{1\perp}}{(2\pi)^2} \frac{d^2 k_{\perp}}{(2\pi)^2} \rho_{p, a}(k_{1\perp}) \int d^2 x_{\perp} d^2 y_{\perp} e^{i k_{1\perp} x_{\perp}} e^{i(p_{\perp} + q_{\perp} - k_{\perp}) y_{\perp}} \]

\[ \times \frac{\pi(q) \gamma^+(\not q - \not k + m) \gamma^-(\not q - \not k_1 + m) \gamma^+[\tau^a - t^a U(y_{\perp})] \gamma^+(p)}{2p^+((q_{\perp} - k_{\perp})^2 + m^2) + 2q^+((q_{\perp} - k_{\perp} - k_{1\perp})^2 + m^2)}. \]  

(103)

The term we denote \( M_{R,2}^{(b)} \) is obtained from the diagram (b) by keeping only the piece proportional to \( C_{\nu, \text{reg}}^\mu \) in eq. (5). Using the fact that

\[ C_{\nu, \text{reg}}^\mu(p + q - k) = 2(p + q - k)^\nu - \delta^\mu_{\nu} \frac{(p + q - k)^2}{p^+ + q^+ - k^+}, \]  

(104)

we obtain:

\[ \int \frac{dk^-}{2\pi} \frac{\pi(q) t^b C_{\nu, \text{reg}}(p + q - k)((-p + \not k + m) \gamma^+[\tilde{U}(x_{\perp}) - 1] \gamma^+(p)}{[(p + q - k)^2 + i(p^+ + q^+ - k^+)\epsilon][(-p + k)^2 - m^2 + i(-p^+ + k^+)^\epsilon]} \]

\[ = \frac{\pi(q) \gamma^+ t^b [\tilde{U}(x_{\perp}) - 1] \gamma^+(p)}{p^+ + q^+}. \]  

(105)

With the help of this relation, we obtain easily:

\[ M_{R,2}^{(b)} = -g^2 \int \frac{d^2 k_{1\perp}}{(2\pi)^2} \frac{d^2 k_{\perp}}{(2\pi)^2} \rho_{p, a}(k_{1\perp}) \int d^2 x_{\perp} d^2 y_{\perp} e^{i(p_{\perp} + q_{\perp} - k_{\perp}) x_{\perp}} \]

\[ \times \frac{\pi(q) \gamma^+ t^b [\tilde{U}(x_{\perp}) - 1] \gamma^+(p)}{p^+ + q^+} [V(x_{\perp}) - 1]_{ba}. \]  

(106)
Finally, $\mathcal{M}^{(b)}_{b,3}$ is obtained from the diagram (b) by keeping only the piece proportional to $C^\mu_\nu$ in eq. (5). It reads:

$$
\mathcal{M}^{(b)}_{b,3} = ig^2 \int \frac{d^2k_{1\perp}}{(2\pi)^2} \frac{d^2k_{\perp}}{(2\pi)^2} \frac{d^2\rho_{p,a}(k_{1\perp})}{k_{1\perp}^2} \int d^2x_{\perp} \frac{d^2\gamma_{\perp} e^{ik_{\perp} \cdot y_{\perp}} e^{i(p_{\perp} + q_{\perp} - k_{\perp} - k_{1\perp}) \cdot x_{\perp}}}{k_{\perp}^2} \times \int \frac{dk}{2\pi} \frac{\pi(q)C_{V}(p + q - k, k_{1\perp})(\gamma^{+} \rho^{+} \gamma^{+} \rho^{+})[U(y_{\perp}) - 1]v(p)}{[(p + q - k)^2 + i\epsilon][(-p + k)^2 - m^2 - i\epsilon][U(x_{\perp}) - 1]_{ba}}.
$$

(107)

### B.4 Total retarded amplitude

The main difficulty at this point is that we must combine this term in $C^\mu_\nu(p + q - k, k_{1\perp})$ with a term in $C^\mu_\nu(p + q, k_{1\perp})$ contained in $\mathcal{M}^{(a)}_R$. In order to do this, one must notice the following relation:

$$
\pi(q)\rho^{+}C_{V}(p + q, k_{1\perp})v(p) = \frac{i}{(p + q)^2} \int \frac{dk}{2\pi} \frac{\pi(q)\rho^{+}C_{V}(p + q - k, k_{1\perp})(\gamma^{+} \rho^{+} \gamma^{+} \rho^{+})v(p)}{[(p + q - k)^2 + i\epsilon][(-p + k)^2 - m^2 - i\epsilon]}|_{k_{\perp}=0}.
$$

(108)

which is valid if $k_{\perp} = 0$ in the r.h.s. and introduce in $\mathcal{M}^{(a)}_R$ dummy variables $y_{\perp}$ and $k_{\perp}$ via a trivial factor (which ensures that $k_{\perp} = 0$):

$$
1 = \int \frac{d^2k_{\perp}}{(2\pi)^2} d^2y_{\perp} e^{ik_{\perp} \cdot y_{\perp}}.
$$

(109)

In order to simplify the notations, let us denote:

$$
\mathcal{F}(q_{\perp}, p_{\perp} - k_{\perp}, k_{1\perp}) \equiv i \int \frac{dk}{2\pi} \frac{\rho^{+}C_{V}(p + q - k, k_{1\perp})(-\gamma^{+} \gamma^{+} \gamma^{+} \gamma^{+})}{[(p + q - k)^2 + i\epsilon][(-p + k)^2 - m^2 - i\epsilon]},
$$

(110)

where we have only indicated the transverse momenta in the list of arguments. Note that this object depends only on the difference $p_{\perp} - k_{\perp}$ rather than $p_{\perp}$ and $k_{\perp}$ separately. Finally, we need to perform the transformation $k_{1\perp} \rightarrow p_{\perp} + q_{\perp} - k_{1\perp} - k_{\perp}$ for the terms containing $C_V$ (or equivalently the $\mathcal{F}$ defined above).

Combining all the contributions to the retarded pair production amplitude, we can write it as follows:

$$
\mathcal{M}_R(q, p) = \int d^2y_{\perp} e^{ip_{\perp} \cdot y_{\perp}} \pi(q)K(q_{\perp} ; y_{\perp})U(y_{\perp})\gamma^{+}v(p),
$$

(111)
with

\[ K(q_\perp; y_\perp) \equiv g^2 \int \frac{d^2 k_1\perp \, d^2 k_\perp \, \rho_{p,a}(k_1\perp)}{(2\pi)^2} \int \frac{d^2 x_\perp}{k_1\perp} e^{ik_\perp \cdot x_\perp} e^{i(q_\perp - k_\perp - k_1\perp) \cdot y_\perp} \]

\[
\times \left\{ \gamma^+(\bar{q} - \bar{k} + m) \gamma^-(\bar{q} - \bar{k} - \bar{k}_1 + m) \left[ \bar{U}(x_\perp) t^{a} \bar{U}^\dagger(y_\perp) - t^a \right] \\
+ \frac{\gamma^+(\bar{q} - \bar{k}_1 - q_\perp - k_1\perp)}{p^+ + q^+} \left[ U(x_\perp) - 1 \right]_{ba} \right\}. \tag{112}
\]

Note that the quantity \( K(q_\perp; y_\perp) \) does not depend on \( p_\perp \).

### B.5 Comparison with the time-ordered amplitude

Let us now go back to our original problem, namely to verify that we indeed have

\( P_1 = N_{q\bar{q}} \). In order to make this verification easy, the time-ordered amplitude given in eq. (34) needs some rewriting. By performing on the term in \( C_\nu \) the transformations described in the section B.4, we can write:

\[ \mathcal{M}_\nu(p, q) = \int d^2 y_\perp e^{ip_\perp \cdot y_\perp} \mathcal{T}(q) K(q_\perp; y_\perp) \gamma^+ v(p), \tag{113} \]

with the same \( K(q_\perp; y_\perp) \) as in eq. (112). We see that the time-ordered and the retarded amplitudes differ only by the unitary matrix \( \bar{U}(y_\perp) \) in the integrand: this is the precise meaning of “identical up to a phase” in this context.

From eq. (113), we can write the single quark spectrum as:

\[
\frac{dP_1}{dq_\perp dq_q} = \frac{1}{2\pi^2} \int \frac{d^2 p_\perp}{(2\pi)^2} \frac{dp^+}{p^+} \int d^2 y_\perp d^2 y'_\perp e^{ip_\perp \cdot (y_\perp - y'_\perp)} \\
\times \text{tr}_a \left[ (\bar{q} + m) K(q_\perp; y_\perp) \gamma^+ \gamma^0 K^\dagger(q_\perp, y_\perp) \gamma^0 \right]. \tag{114}
\]

Since \( \gamma^+(\bar{q} - \bar{m}) \gamma^+ = 2p^+ \gamma^+ \) does not depend on \( p_\perp \), nor does \( K \), it is trivial to perform the integration over \( p_\perp \), which gives a \( \delta(y_\perp - y'_\perp) \). Therefore, we can rewrite the above equation as follows:

\[
\frac{dP_1}{dq_\perp dq_q} = \frac{1}{32\pi^4} \int dp^+ d^2 y_\perp \text{tr}_a \left[ (\bar{q} + m) K(q_\perp; y_\perp) \gamma^+ \gamma^0 K^\dagger(q_\perp, y_\perp) \gamma^0 \right]. \tag{115}
\]

Similarly, one can write the average number of produced quarks as:

\[
\frac{dN_{q\bar{q}}}{dq_\perp dq_q} = \frac{1}{64\pi^4} \int \frac{d^2 p_\perp}{(2\pi)^2} \frac{dp^+}{p^+} \int d^2 y_\perp d^2 y'_\perp e^{ip_\perp \cdot (y_\perp - y'_\perp)} \\
\times \text{tr}_a \left[ (\bar{q} + m) K(q_\perp; y_\perp) \bar{U}(y_\perp) \gamma^+ (\bar{q} - m) \gamma^+ \bar{U}^\dagger(y'_\perp) \gamma^0 K^\dagger(q_\perp, y'_\perp) \gamma^0 \right]. \tag{116}
\]

Again, the integration over \( p_\perp \) produces a \( \delta(y_\perp - y'_\perp) \), and we see that the matrices \( \bar{U}(y_\perp) \) and \( \bar{U}^\dagger(y'_\perp) \) cancel each other to produce a unit matrix. Therefore,
we conclude that:
\[
\frac{dP_1}{d^2q_\perp dy_q} = \frac{dN_{q\bar{q}}}{d^2q_\perp dy_q},
\]
(117)
as expected.

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