Azimuthal emission patterns of proton in $^{58}\text{Ni} + ^{58}\text{Ni}$ collision at intermediate energy by using a quantum molecular dynamics model

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Abstract. The direct flow of a proton in the $^{58}\text{Ni} + ^{58}\text{Ni}$ collisions at incident energy 1.93 A GeV was computed by using a quantum molecular dynamics model (QMD). The mean transverse momentum projected on the reaction plane as a function normalized rapidity for the proton ($\langle p_x/m_p \rangle (Y(0))$) from $^{58}\text{Ni} + ^{58}\text{Ni}$ collisions with impact parameter ($b$) < 4 fm was studied. Furthermore, the direct flow of the proton as a function of transverse momentum ($v_1(p_t)$) at the incident energy 1.93 A GeV with $b < 2.5$ fm was investigated by taking into account the nuclear equation of state (hard and soft EoS). The theoretical results were compared with KaoS experiment and BUU model. The calculated results show that the theoretical calculations of $\langle p_x/m_p \rangle (Y(0))$, a soft EoS is consistent with experimental data. In addition, the $v_1(p_t)$ is confirmed by the soft EoS that is good agreement with the experimental data at the $p_t < 0.6$. Consequently, this work indicates that $\langle p_x/m_p \rangle (Y(0))$ and $v_1(p_t)$ are sensitive observable to probe the nuclear equation of state in dense nuclear matter.

1. Introduction

Relativistic heavy ion collisions at intermediate energies provides an exclusive tool for studying a nuclear equation of state (EoS) at high temperature, around 100 MeV and high density about 2-3 times the normal nuclear density ($\rho_0$) [1-2]. For example, in the reference [3-7]. Moreover, these experiments and knowledges are not only interested in themselves, but also helpful to understand the physics phenomena such as the evolution of supernova, neutron star formation, the evolution of the early universe and creation of elements in stellar nucleosynthesis [8].

According to the previous research [9], studied the direct flow and elliptic flow in $^{197}\text{Au} + ^{197}\text{Au}$ at energy between 40 and 150 A MeV, measured by using the INDRA 4-\pi multi-detector. For semi-central collisions, the elliptic flow of $Z \leq 2$ the particles switched from the flow in the plane to the outer plane about 100 A MeV, which was a good agreement with the results reported with FOPI Collaboration. The direct flow changed the energy from 50 to 60 A MeV and remained negative when energy was low. The angle of the discharge patterns of $K^\pm$ mesons by $^{58}\text{Ni} + ^{58}\text{Ni}$...
collisions with the FOPI spectrometer at the kinetic energy of the beam energy at 1.91 A GeV was investigated. The theoretical calculation of HSD and IQMD models, the results showed that the data exhibited different propagation patterns of $K^\pm$ mesons in the compressed and heated nuclear medium and favoured the existence of a kaon-nucleon in-medium potential, repulsive for $K^+$ mesons and attractive for $K^-$ mesons [10]. Another research studied about the rapidity-odd direct flow for measuring pion, proton and antiproton near mid-rapidity and reported in $^{197}$Au + $^{197}$Au collision at 7.7, 11.5, 14.5, 19.6, 27, 39, 62.4 and 200 GeV. At the intermediate impact parameter, proton and net-proton slopes ($d\bar{v}_1/dx$) showed a minimum between 11.5 and 19.6 GeV, the $d\bar{v}_1/dx$ changed significantly between 7.7 and 39 GeV. The proton and net-proton results qualitatively resemble predicted by the hydrodynamic model [11]. From the researches were mentioned, the direct flow had been interested and studied the behaviour of matters at high density in heavy ion collisions.

In this paper, we investigated the proton production integrated the mean transverse momentum projected on the reaction plane as a function normalized rapidity ($\langle p_x/m_p \rangle (Y(0))$) at impact parameter ($b$) less than 4 fm and the direct flow of proton as a function of the transverse momentum ($v_1(p_t)$) in $^{58}$Ni + $^{58}$Ni collisions at $b < 2.5$ fm, included the nuclear equation of state by using a quantum molecular dynamics model (QMD). The theoretical calculation by using the QMD model was compared with the KaoS experimental and BUU model [12].

2. Theories

2.1. Quantum Molecular Dynamics (QMD) Model

The baryon dynamics are narrated within framework of Quantum Molecular Dynamics (QMD). The nuclear equation of state describes the possibility of compressing nuclear matter. The quantum molecular dynamics [13] in which each nucleon is represented by a coherent state of the form

$$\psi(\mathbf{r}, \mathbf{p}, t) = \frac{1}{(2\pi L)^{3/4}} \exp \left\{ -\frac{(\mathbf{r} - \mathbf{r}_0)^2}{4L} \right\} \exp \{ i \mathbf{p} \cdot (\mathbf{r} - \mathbf{r}_0) \},$$

where $\mathbf{r}_0$ is the centre of Gaussian wave pocket and $L = 1.08$ fm is the width of the wave pocket. Consequently, the density of the system with $N$ nucleons in coordinate space is given as follows:

$$\rho(\mathbf{r}, t) = \sum_{i=1}^{N} \frac{1}{(2\pi L)^{3/2}} \exp \left\{ -\frac{(\mathbf{r} - \mathbf{r}_0)^2}{2L} \right\}. \tag{2}$$

The time evolution of the $N$ - body distribution is determined by the motion of the centroid of Gaussian $\{\mathbf{r}_0, \mathbf{p}_0\}$ which are propagated by the Poisson brackets,

$$\dot{\mathbf{r}}_0 = \{\mathbf{p}_0, H\} \tag{3}$$

and

$$\dot{\mathbf{p}}_0 = \{\mathbf{r}_0, H\} \tag{4}$$

with $H$ is the nuclear Hamiltonian

$$H = \sum_{i=1}^{N} \sqrt{\mathbf{p}_0^2 + m_i^2} + \sum_{i<j}^{N} \left( U_{ij}^{Str} + U_{ij}^{Coul} \right). \tag{5}$$

Here $U_{ij}^{Str}$ is a nuclear mean field, $U_{ij}^{Coul}$ is the Coulomb interaction. The compression of strong nuclear force is described by the value of constant (compressibility) defined as [14]:
\[ K = 9\rho^2 \frac{\partial^2}{\partial^2 \rho^2} \left( \frac{E}{A} \right). \] (6)

The energy per nucleon \((E/A)\) as a function of density is clarified in the equation (6) that is approximated by the standard momentum-dependent Skyrme parameterizations \((U)\) with a soft EoS is represented by a value of \(K = 200\) MeV, while a hard EoS is represented by a value of \(= 380\) MeV. The Skyrme parameterizations \((U)\) read

\[ U = \alpha \left( \frac{\rho}{\rho_0} \right) + \beta \left( \frac{\rho}{\rho_0} \right)^\gamma, \] (7)

where \(\rho\) is the nuclear density which is measured in units of the saturation density \((\rho_0)\) of cold nuclear matter \((\rho_0 \approx 0.16\) fm\(^{-3}\)). The relation of the \(\alpha\), \(\beta\) and \(\gamma\) is shown in table 1. The differential value of \(K\) constant is shown in figure 1.

**Table 1.** Parameters in the equations (6) and (7) for the soft and hard nuclear equation of state (EoS) [13].

| \(K\) (MeV) | \(\alpha\) | \(\beta\) | \(\gamma\) | EoS   |
|------------|----------|---------|---------|------|
| 200        | -356     | 303     | 7/6     | Soft |
| 380        | -124     | 70.5    | 2       | Hard |

**Figure 1.** The equations of state in our calculations. The density dependence of the energy per particle in nuclear matter at temperature \(T=0\) is displayed for the four different sets of parameters show in table 1[13].
2.2. Direct flow
The phenomenon of collective flow could be quantitatively described in terms of the anisotropies of the azimuthal emission pattern, expressed by a Fourier series [15]

\[ \frac{\mathrm{d}N}{\mathrm{d}\phi} (\phi) \propto 1 + 2v_1 \cos (\phi) + 2v_2 \cos (2\phi), \]

(8)

and

\[ v_1 = \left\langle \frac{p_x}{p_t} \right\rangle = \langle \cos (\phi) \rangle, \]

(9)

whereas \( \phi \) is the azimuthal angle of the outgoing particle with respect to the reaction plane. \( p_t = \sqrt{p_x^2 + p_y^2} \) is the transverse momentum. The first order Fourier coefficient, \( v_1 \) describes the collective sideward ‘direct flow’ as showed in figure 2 below:

Figure 2. Isotropic flow as the direct flow: if \( v_1 \) is large, the circle will move on the x-axis [3].

2.3. Rapidity
Rapidity, as its name implies, is related to velocity. It is a dimensionless variable, \( y \), describes the rate at which a particle is moving with respect to the chosen reference point situated on the line of motion. Mathematically, it is defined as [16]

\[ y = \frac{1}{2} \ln \left( \frac{E + p_z}{E - p_z} \right), \]

(10)

when \( p_z \) is the momentum variable parallel to z axis of proton and \( E \) is also total energy. The mid-rapidity (\( Y^{(0)} \)) is rapidity of proton that moves out from the central collision (\( y \)) to scaled by the rapidity of the beam energy (\( y_0 \)):

\[ Y^{(0)} = \frac{y}{y_p}, \]

(11)
3. Results and discussion

![Graph showing mean transverse momentum (⟨px/m_p⟩) on the reaction plane (Y(0)) for proton in 58Ni + 58Ni collisions at 1.93 A GeV with b < 4 fm. The theoretical calculations compare with the experimental data from KaoS [17].](image)

**Figure 3.** Mean transverse momentum (⟨px/m_p⟩) projects on the reaction plane (Y(0)) of the proton in 58Ni + 58Ni collisions at 1.93 A GeV with b < 4 fm. The theoretical calculations compare with the experimental data from KaoS [17].

![Graph showing direct flow (v1) as a function of transverse momentum (p_t) for proton in 58Ni + 58Ni collisions at 1.93 A GeV with b < 2.5 fm. The theoretical calculation from the quantum molecular dynamic model is compared with the experimental data from KaoS [18].](image)

**Figure 4.** Shows the direct flow (v1) of the proton as a function of transverse momentum (p_t) in 58Ni + 58Ni collisions at 1.93 A GeV with b < 2.5 fm. The theoretical calculation from the quantum molecular dynamic model is compared with the experimental data from KaoS [18].
Figure 3 displays the $\langle p_x/m_p \rangle$ $(Y(0))$ of the proton in $^{58}$Ni + $^{58}$Ni collisions at 1.93 A GeV for the impact parameter less than 4 fm by taking into account the nuclear equation of state (EoS; soft and hard). The result from figure 3 shows that, the theoretical calculations of the $\langle p_x/m_p \rangle$ $(Y(0))$ of the proton, the soft EoS is consistent with experimental data [12]. It is seen clearly from this figure that the proton potential is a repulsive potential, the protons are pushed away from nucleons. Otherwise, the protons are accelerated during their propagations from the central of the collision. Consequently, the number of protons with a smaller momentum decrease, the protons are confined in the nucleon by the strong nuclear force, which are difficultly observed. The effect of the theoretical result has more fluctuations at low momentum. On the other hand, the number of protons with a larger momentum increase, lead to a good fit to the experimental data. Moreover, the similar research’s results are found in reference [3, 19-21]. In order to comparing the theoretical calculations with the experimental data further, we calculated the root mean square errors (RMSE) for each value by using the soft and hard EoS. The results are given in table 2. It is found that the result calculated by using the soft EoS has smallest RMSE. This mean that the soft EoS is the best one for describing the experimental data.

Figure 4 displays the $v_1(p_t)$ in $^{58}$Ni + $^{58}$Ni collisions at intermediate energy 1.93 A GeV for the impact parameter less than 2.5 fm by including the nuclear equation of state. The results showed that, the theoretical calculation of the soft EoS is consistent with experimental data at the $p_t < 0.6$. On the other hand, the soft EoS is not a good agreement with experimental data at $p_t$ increases higher than 0.6, we fail to describe the data here. (The previous research’s result is similar to our results and is found in reference [12].) The problem is ascribed to a momentum dependence of the nucleon mean field that is too hard at large momenta, which pushes the high-momentum of the protons too soon from the central of the system before the direct flow is measured [12,19].

| Results | Soft EoS | Hard EoS |
|---------|----------|----------|
| $\langle p_x/m_p \rangle$ | 0.1562 | 0.1884 |
| $v_1$ | 0.1748 | 0.2043 |

Table 2. The root mean square errors [22] (RMSE) for the calculated results of the mean transverse momentum projects on the reaction plane as a function normalized rapidity and the direct flow of the proton as a function of the transverse momentum in $^{58}$Ni + $^{58}$Ni collisions at 1.93 A GeV.

4. Conclusion

In summary, the azimuthal emission patterns of the proton in $^{58}$Ni + $^{58}$Ni collisions at intermediate energy 1.93 A GeV with the mean transverse momentum projects on the reaction plane as a function of rapidity in the impact parameter ($b < 4$ fm) and the direct flow of the proton as a function of transverse momentum in $b < 2.5$ fm are investigated within the QMD model. The theoretical calculations by using the QMD model is compared with the experimental data [12, 20-21]. We find that these observations can be described by calculating results with the soft EOS (the compression modulus $K$ is 200 MeV). The theoretical calculations of $\langle p_x/m_p \rangle$ $(Y(0))$ with $b < 4$ fm, the soft EoS is consistent with experimental data. In addition, the theoretical calculations of the $v_1(p_t)$ with $b < 2.5$ fm, the soft EoS is a good agreement with experimental data [12]. This work suggests that $\langle p_x/m_p \rangle$ $(Y(0))$ and $v_1(p_t)$ are sensitive observable to probe the nuclear equation of state in dense nuclear matter.
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