Small-q electron-phonon scattering and linear DC resistivity in high-$T_c$ oxides

G. Varelogiannis and E.N. Economou
Institute of Electronic Structure and Laser, FORTH, P.O. Box 1527, Heraklion Crete 71110, Greece

We examine the effect on the DC resistivity of small-q electron-phonon scattering, in a system with the electronic topology of the high-$T_c$ oxides. Some features are similar to that of the oxides yet the resistivity could be linear down to temperatures as low as 10 K, even if electrons are scattered also by optical phonons of about 500 K as reported by Raman measurements.

One of the most puzzling experimental facts in the optimally doped oxides is the linear temperature dependence of their DC resistivity for very low temperatures. In $Bi_2(Sr_{0.97}Pr_{0.03})_2CuO_6$ the DC resistivity is linear from 10K up to several hundreds of Kelvins [1]. This has been considered as a sign of absence of electron phonon scattering. In fact, even though a linear $T$ dependence of the DC resistivity is also possible in the case of electron-phonon scattering, this happens for temperatures that are higher than about one fourth of the characteristic phonon energies. From this perspective, the data of Ref. [1] indicate a complete absence of electron-phonon scattering for frequencies above 50K, while the optical phonons in the cuprates extend up to about 1000K.

To explain the linear $T$-dependence of the DC resistivity, several scenarios avoiding electron-phonon scattering have been proposed. A first approach was the so called Marginal Fermi Liquid [3], where the linearity was evidence of deviation from the classical Fermi liquid behavior in the case of some special collective excitation scattering. A second type of approach was that of gauge theory models where the linearity is a signature of a complete breakdown of Landau Fermi Liquid theory replaced by a quasi-one dimensional Luttinger liquid behavior [3].

A third type of approach was that of a nearly antiferromagnetic Fermi liquid [3], in which case there is singular electron-paramagnon scattering supposed to be also at the origin of d-wave superconductivity. Finally a linear $T$-dependent resistivity has also been associated with the presence of van Hove singularities in the vicinity of the Fermi surface and simple Coulomb scattering [3]. The relevance of the last two approaches has been questioned recently by Hlubina and Rice [3].

Although in all the previous scenarios electron-phonon scattering is neglected, there is strong evidence from Raman experiments that there is significant coupling of the electrons with the optical oxygen vibrations [4], and the absence of signature of this coupling in the DC resistivity is an unresolved puzzle. It also appears rather unphysical that phonons could be irrelevant for the $T$-dependence of the DC resistivity at so high temperatures.

We will propose here an alternative scenario that could reconcile the Raman and DC transport results. In this scenario the electron-phonon scattering is the dominant $T$-dependent resistive mechanism, the phonon spectrum and electronic topology are similar to that of the oxides yet the resistivity could be linear down to temperatures as low as 10K in optimally doped materials. A basic assumption of our approach is the dominance of small momentum transfer process in the electron-phonon scattering. This assumption, although it has not been based on solid physical arguments or computations, nevertheless has been successful in interpreting many puzzling features of the high-$T_c$ superconductors. Some features are the peak in the microwave conductivity [4], the momentum dependence of the anomalous dip above the gap and the enhancement of the anisotropy close to $T_c$ reported by ARPES on $BiSr_2CaCu_2O_8$ [4], as well as the presence of different gap symmetries in different oxides and even variations of the gap symmetry with doping [4].

An effectively similar hypothesis of small-$q$ scattering in the oxides is actually investigated by many authors in different contexts [6]. The dominance of forward scattering in the electron-phonon interaction could result from strong Coulomb correlations of the carriers that may break the electronic system in the vicinity of a phase separation instability [6].

One can briefly sketch our approach as follows. When the electron-phonon scattering is limited to small momentum transfer processes, then the contribution of the acoustic branch of the phonons is energetically separated from that of the optical branches. The acoustic branch extends up to an energy of the order $Ω_A = q_s v_s$ where $v_s$ is the sound velocity and $q_s$ a characteristic momentum cut-off of the scattering. Analyzing the phenomenology of $BiSr_2CaCu_2O_8$ we obtained $q_s \approx k_F/10$ [16]. Taking $v_s \approx 10^{-2} v_F$ and an average Fermi velocity $v_F \approx 2 \times 10^5 cms^{-1}$ in agreement with infrared measurements [4] we obtain $Ω_A \approx 40 K$. The energetic cut-off for the acoustic branch, which is the effective “transport” Debye frequency, being not larger than 50K, the resistivity can be linear for temperatures as small as 10K. The optical phonons, for which there is evidence from Raman that contribute dominantly to the electron-phonon scattering in the region of frequencies between 300 and 600 Kelvins, in principle should also dominate
the transport. However there is a significant difference between dominantly forward scattering of electrons with optical and acoustic phonons. In the first case we exchange small-momenta but large energies while in the second case we exchange small momenta and small energies and the phase space for these two types of processes is very different depending on how close the flat band regions are to the Fermi surface. In fact in order to have contribution from small momentum transfer processes to the transport, and avoid the well known $1 - \cos \theta$ coefficient from the Boltzmann equation, it is necessary to scatter between points with opposite Fermi velocities. In the case of small momentum transfer, this happens principally because of the “ondulations” of the bands in the flat region, and also because of the umpklapp processes. In both cases the flat regions are concerned, and depending on the distance of these regions from the Fermi surface, the acoustic branches can dominantly contribute to the transport. When the acoustic branch gives a dominant contribution, the resistivity is linear down to very low temperatures since the effective Debye frequency is the cut-off of the acoustic branch not larger than few tenths of Kelvins if the scattering is dominantly forward.

In order to illustrate how realistic are the previous arguments for the high-$T_c$’s, we consider a tight binding fit to the ARPES reported Fermi surface and dispersion of $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$. The most characteristic feature is the presence of extended van Hove singularities that cover about 30% of the Brillouin zone, and in order to produce such extended van Hove singularities in tight binding, hoping terms up to the fifth nearest neighbors were found to be necessary:

$$E_k = t_1 (\cos k_x + \cos k_y) + t_2 \cos k_x \cos k_y +$$

$$+ t_3 \frac{1}{2} (\cos 2k_x + \cos 2k_y) + t_4 \frac{1}{2} (\cos 2k_x \cos k_y + \cos k_x \cos 2k_y) + t_5 \cos 2k_x \cos 2k_y$$ (1)

The dispersion we consider is not very different from that considered in Ref. 17 and corresponds to the set of parameters: $t_1 = -0.525$, $t_2 = 0.0337$, $t_3 = 0.0287$, $t_4 = -0.175$ and $t_5 = 0.0175$. This dispersion fits well the ARPES results 18 and especially the extended van Hove singularities taken exactly at $E_F$. We show in figure (1a) a quarter of the Brillouin zone and the corresponding electron dispersion given by (1). The white area is a region of 50K around the Fermi surface. One can see that when the extended van Hove singularities are sufficiently close to the Fermi surface, the ondulations of the band in the flat region creates effective branches of Fermi surface around the points $(\pi, 0)$ and $(0, \pi)$ in addition to the principle branch around $(\pi, \pi)$. The scattering from one branch to the other is associated with a reversal of the Fermi velocity. These ondulations together with the umpklapp processes imply that even small momentum transfer processes give a significant contribution to the transport. Although the flat band fluctuates over few tenths of $\text{meV}$ the electronic density of states has a very sharp peak characteristic of extended saddle points. We show in Figure 2 the DOS for the dispersion shown in figure 1a where the van Hove peak lies exactly at $E_F$.

The phase space available for transport efficient electron phonon scattering can be measured considering the following definition of the transport Eliashberg function

$$\alpha_{tr}^2 F_{tr}(\Omega) \approx \sum_{k,k'} \frac{\tilde{A}_{kk'}}{2 |v_{kk'}|} \frac{(\Omega_{k-k'} - E_k + E_{k'})}{} \delta(\Omega_{k-k'} - E_k + E_{k'})$$ (2)

where an electron scatters from the occupied state $E_k$ to the empty state $E_{k'}$. The velocities are defined by $v_k = \nabla_k E_k$ and in the case of elastic scattering in an isotropic system we have $(v_{kk'})^2 \approx \frac{E_k}{2|v_k||v_{k'}|} \approx 1 - \cos \theta$ where $\theta$ is the angle between the velocities $v_k$ and $v_{k'}$. The coefficients $\tilde{A}_{kk'}$ are scattering amplitude matrix elements which are too complicated for explicit evaluation. Thus we shall use the following arbitrary but simple analytical form, which satisfies our basic requirement that $\tilde{A}_{kk'}$ should become negligible for $|\vec{k} - \vec{k}'| > q_c$:

$$\tilde{A}_{kk'} \approx -g_{A,O}^2 \left( 1 + 2 \frac{-\cos(k_x - k'_x) - \cos(k_y - k'_y)}{g_e^2} \right)^{-1}$$ (3)

where $g_{A,O}^2 \approx \frac{g_A^2}{g_e^2} \frac{2 - \cos(k_x - k'_x) - \cos(k_y - k'_y)}{}$ is proportional to the scattering cross section with the acoustic and $g_{A,O}^2 \approx g_O^2$ to the scattering cross section with the optical phonons. The scattering is therefore dominated by the processes in which the exchanged momentum is smaller than $q_c$ that plays the role of a smooth momentum cut-off. The exact form of the momentum cut-off is irrelevant for our qualitative arguments.

In reality, the scattering amplitude coefficients are also frequency dependent especially for very anisotropic systems as the high-$T_c$ oxides. In fact the phonon system in the oxides is three dimensional but the electronic system is two dimensional, and obviously all phonons will not have the same probability to scatter with electrons. Phonon symmetry considerations also influence the probability of scattering. General calculations of such probabilities is a rather complex task 19, that we will avoid here focusing on our phase space arguments. We therefore adopt ad-hoc a frequency structure of the scattering that agrees with experiment. In fact the optical vibrations that are relevant to the scattering by the in plane oxygens are now well documented 20 and range between 25 and 50 $\text{meV}$. On the other hand there is evidence from Raman spectroscopy 21, as well from a comparison to a study of the spectral dependence of the gap ratio 20, that electrons couple strongly to these optical phonons. This spectral structure also explains the disagreement
between the infrared and the other gap measurements. We therefore consider a spectrum consisting by the acoustic branch that extends up to $\Omega_A \approx 50K$ and the optical branches that we take as a constant distribution between $\Omega_1 \approx 25meV$ and $\Omega_2 \approx 50meV$.

To obtain the transport parameters we follow the conventional approach and we suppose that to a first approximation, the transport scattering time has the same definition as the quasiparticle lifetime except that the normal Eliashberg function is replaced by the transport Eliashberg function. This assumption is common to many theoretical approaches to the linear resistivity problem.

For the quasiparticle lifetime $1/\tau = -2Im(\Sigma)$ only the lowest order contribution of phonons in the electronic self energy $\Sigma$ is taken in agreement with Migdal’s theorem leading to

$$\frac{1}{\tau} = \pi \int d\Omega \frac{\Omega^2}{F(\Omega)} \left[ 2 \coth \frac{\Omega}{2T} \right]$$

$$\tanh \frac{\omega + \Omega}{2T} + \tanh \frac{\omega - \Omega}{2T}$$

(C. 4)

Certainly, for a more accurate approach we should solve the Kubo problem for the specific anisotropic situation that we consider, but such treatment is beyond the scope of this manuscript. We focus here on the DC resistivity which corresponds to the $\omega \rightarrow 0$ limit. The transport scattering time for our spectrum is given by

$$\frac{1}{\tau} \approx 8\pi G_A \int_0^{\Omega_A} d\Omega \frac{\Omega^2}{\exp(\beta \Omega) - \exp(-\beta \Omega)} +$$

$$+ 4\pi G_O T \left[ \frac{1}{2} \ln \frac{0.5[\exp(\beta \Omega_2) + \exp(-\beta \Omega_2)] - 1}{0.5[\exp(\beta \Omega_2) + \exp(-\beta \Omega_2)] + 1} - \frac{1}{2} \ln \frac{0.5[\exp(\beta \Omega_1) + \exp(-\beta \Omega_1)] - 1}{0.5[\exp(\beta \Omega_1) + \exp(-\beta \Omega_1)] + 1} \right].$$

(C. 5)

Indeed we plot in figure 3 the resistivity in arbitrary units as a function of temperature when the van Hove peak in the electronic DOS lies about 30K below the Fermi level. We see the remarkable linearity from 10K up to 900K that reproduces qualitatively the results of Ref. [1].

Linear $T$ dependence of the resistivity do not necessarily means $G_A > G_O$. In fact, the relative influence of the acoustic branch to the slope of the $T$ dependence of the resistivity is much larger than indicated from the ratio $G_A/G_O$. One can understand this taking for example the high temperature expansion of equation (5) $1/\tau \approx 2\pi T \left[ G_A \Omega_A^2 + G_O \ln(\Omega_2/\Omega_1) \right]$. Both acoustic and optical phonons give a linear $T$ dependence in this regime. However the contribution of the optical branches to the slope depends logarithmically on the width of the optical part of the spectrum, while the acoustic branch contributes as the square of the number of Kelvins to which $\Omega_A$ corresponds. For our rather concentrated optical spectrum the acoustic branch determines the $T$-slope of the resistivity even for $G_A < G_O$. Therefore, if we have a linear $T$ dependence in the region $0.2\Omega_A < T < \Omega_1$ because of the acoustic branch, when at higher temperatures we enter the regime $T > \Omega_2$, contrary to what one might naively expect, there is not a significant variation of the slope corresponding to the entry in game of the optical phonons even if the electrons are strongly coupled to them. Notice also that concerning the pairing, the optical phonons could be dominant within our approach. However, since they are inefficient for the transport, the total coupling strength for superconductivity can be an order of magnitude higher than the effective total transport coupling strength, in agreement with the phenomenology of high-$T_c$’s.

In conclusion, a small momentum cut-off of the electron-phonon scattering, first limits the acoustic branch to energies of few tenths of Kelvins and secondly for systems with the electronic topology of the oxides implies a significant relative contribution of the acoustic phonons to the transport. In that case the effectively relevant Debye energy for the transport is the cut-off of the acoustic branch and the resistivity can be linear for temperatures as small as 10K even though optical Raman active phonons of $\approx 500K$ couple strongly to electrons and could even be responsible for superconductivity.

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[1] D.M. King et al., Phys. Rev. Lett. 73, 3298 (1994)
[2] C.M. Varma et al., Phys. Rev. Lett. 63, 1996 (1989);
P.B. Littlewood and C.M. Varma, J. App. Phys. 69, 4979 (1991)
[3] P. Lee and N. Nagaosa, Phys. Rev. B 46, 5621 (1992)
[4] N.E. Bickers, D.J. Scalapino and S.R. White, Phys. Rev. Lett. 62, 961 (1989); P. Monthoux, A. Balatsky and D. Pines, Phys. Rev. Lett. 67, 3348 (1991)
[5] D.M. Newns et al., Comments Cond. Matter Phys. 15, 273 (1992)
[6] R. Hlubina and T.M. Rice, Phys. Rev. B 51, 9253 (1995)
[7] See e.g. D. Zech et al., Nature (London) 371, 681 (1994) and references therein
[8] R. Liu et al, Phys. Rev. B 37, 7971 (1988)
[9] G. Varelogiannis and L. Pietronero, Phys. Rev. B 52, R15753 (1995)
[10] G. Varelogiannis et al., Phys. Rev. B 54, R6877 (1996)
[11] G. Varelogiannis, preprint (cond-mat/9511139); G. Varelogiannis and M. Peter, Czech. J. of Phys. 46 (S2), 1047 (1996)
[12] S. Chakravarty et al., Science 261, 337 (1993); A.A. Abrikosov, Physica C 244, 243 (1995); J. Ruvalds et al., Phys. Rev. B 51, 3797 (1995); M. Weger et al., J. Low Temp. Phys. 95, 131 (1994); G. Santi et al., J. of Supercond. 8, 215 (1995)
[13] M. Kulic and R. Zeyher, Phys. Rev. B 49, 4395 (1994)
[14] M. Marder, N. Papanicolaou and G.C. Psaltakis, Phys. Rev. B 40, 6920 (1990)
[15] A.N. Andriotis et al., Phys. Rev. B 47, 9208 (1993)
[16] Z. Schlesinger et al., Phys. Rev. B 41, 11237 (1990); D. van der Marel et al., Physica C 176, 1 (1991); A.E. Azrak et al., Phys. Rev. B 49, 9846 (1994)
[17] R.J. Radtke and M. Norman, Phys. Rev. B 50, 9554 (1990)
[18] Z.-X. Shen and D.S. Dessau, Phys. Rep. 253, p. 1-162 (1995); J. Ma et al., Phys. Rev. B 51, 9271 (1995)
[19] See e.g. H. Krakauer, W.E. Pickett and R.E. Cohen, Phys. Rev. B 47, 1002 (1993) and references therein
[20] G. Varelogiannis, Phys. Rev. B 50, 15974 (1994); Physica C 249, p. 87-110 (1995); R. Combescot and G. Varelogiannis, J. of Low Temp. Phys. 102, 193 (1996)
[21] G.D. Mahan, Many particle physics, Plenum Press (New York and London) 1990

**Figure Captions**

**Figure 1:** (a) The dispersion of Equation 1 on a quarter of the Brillouin zone in $\pi/100$ units. The white region represents an energy window of 50K around the Fermi level. The extended van Hove singularities correspond to the plateaus centered at $(0, 100)$ and $(100, 0)$ ($\bar{M}$ points in BiSr$_2$CaCu$_2$O$_8$) which cover about one third of the Brillouin zone. (b) The corresponding electronic density of states (DOS) with the sharp peak at the Fermi level indicating the extended van Hove singularity.

**Figure 2:** The ratio $G_A/G_O$ as a function of the energetic position of the van Hove peak in the DOS with respect to the Fermi level.

**Figure 3:** The resistivity in arbitrary units as a function of temperature when the van Hove peak lies about 30K below $E_F$ for the spectrum described in the text where optical phonons in the range 25 – 50meV couple strongly to electrons. The linear behavior from 10K up to 900K reproduces qualitatively the results of [1].
ACOUSTIC EFFICIENCY

E van Hove (eV)
