Wall stabilization of the rigid ballooning \(m = 1\) mode in a paraxial mirror trap

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The prospect of stabilization of the \(m = 1\) “rigid” ballooning regime in an open axially symmetric trap with the help of a conducting lateral wall surrounding a column of isotropic plasma is studied. It is found that for effective wall stabilization, the beta parameter must exceed 70\%. The dependence of the critical value of beta on the mirror ratio, the radial pressure profile, and the axial profile of the vacuum magnet has been studied. It is shown that when a conductive lateral wall is combined with conductive ends simulating the attachment of end MHD stabilizers to the central cell of an open trap, there are two critical beta values and two stability zones that can merge, making stable the entire range of allowable beta values \(0 < \beta < 1\).

I. INTRODUCTION

Continuing the study of ballooning instability started in the article \([1]\), in this paper we present the results of calculating the critical beta (\(\beta\), the ratio of the plasma pressure to the magnetic field pressure) for \(m = 1\) ballooning perturbations with azimuth number \(m = 1\) in a mirror trap (also called open trap). A proper ballooning equation for a plasma with diffuse plasma pressure radial profile was derived by Lynda Lodestro \([2]\) in 1986, but, in fact, neither she nor anyone else ever used it. Probably oblivion for many years of Lodestro’s work is due to the early termination of the TMX (Tandem Mirror eXperiment) and MFTF-B (Mirror Fusion Test Facility B) projects in the USA in the same 1986 \([3]\). However, achievement of a high electron temperature and high beta in the Gas-Dynamic Trap (GDT) at the Budker Institute of Nuclear Physics in Novosibirsk \([4–12]\), emergence of new ideas \([13]\) and new projects \([14, 15]\) makes us rethink old results.

Unlike many previous works \([16–26]\), whose authors studied the stability of the rigid ballooning mode in a plasma model with a radial pressure profile shaped as a step or a ring with sharp boundaries (sharp-boundary or staircase models), Lodestro derived an equation for a plasma with a diffuse radial profile of pressure in the paraxial (also called long-thin) approximation. It is suitable for describing both isotropic and anisotropic plasmas.

In this paper, the Lodestro equation is used to calculate the critical value of the parameter beta, above which an isotropic plasma will be stabilized by a lateral perfectly conductive wall surrounding the plasma column. The calculations are made for four different radial plasma profiles and many axial profiles of the vacuum magnetic field, some of which we previously used, studying the stability of small-scale ballooning perturbations \([1]\). In addition, we study the effect of wall stabilization in combination with the action of conductive ends, which imitate stabilization by the end magnetohydrodynamic (MHD) anchors.

The isotropic plasma approximation does not quite adequately describe the plasma state in open traps, except for traps with a very large mirror ratio. Calculations show that the critical beta in a plasma with an anisotropic pressure can be much smaller than that found by the isotropic plasma approximation. In other words, wall stabilization of an anisotropic plasma is more efficient than wall stabilization of an isotropic plasma. The results of the study of ballooning instability in an anisotropic plasma will be reported in a subsequent paper. Numerical solution of the Lodestro equation in the case of an anisotropic plasma is much more complicated and time-consuming. This circumstance, as well as the abundance of new results even for the case of isotropic plasma, motivated our decision to separate the case of isotropic plasma into a distinct article in order to describe the calculation technique in more detail here.

To avoid possible misunderstandings (and objections), it should be clarified that ballooning modes are usually understood as z-dependent pressure-gradient-driven modes with \(m \gg 1\). Some authors object to the use of the term “ballooning” for the \(m = 1\) mode, in which the internal deformations in the cross section of the plasma column are frozen due to the effects of Finite Larmor Radius (FLR effects). These authors propose to call such modes “rigid” or “global”. Nevertheless, referring to the \(m = 1\) rigid mode as a ballooning mode did not stop, therefore we prefer to call the oscillations studied in this paper as “rigid ballooning mode”, although such a name does not seem quite satisfactory to us.

In what follows, we will adhere to the following plan of presentation. In the next section II a review of the publications on the stability of the \(m = 1\) ballooning mode that preceded Lodestro’s paper is given; after this article, publications on ballooning instability in open traps practically ceased. In section III, the Lodestro equation is written and the necessary notation is introduced. Section IV presents the results of calculating the critical beta in the limit when a perfectly conducting wall surrounding the plasma column almost closely adjoins the lateral boundary of the column, but does not touch it. In this limit, the Lodestro ordinary differential equation reduces...
to an integral over the $z$ coordinate along the trap axis; the integral vanishes at the critical value of beta. Section V describes the solution of the Lodestro equation by the shooting method and presents the results of calculations for several model pressure and magnetic field profiles. In section VI, the shooting method is again used to solve the Lodestro equation with different boundary conditions that model the effect of conductive end plates placed in magnetic mirrors. Final section VII summarizes our results and conclusions.

II. LITERATURE REVIEW

There are a number of publications, in one way or another, related to the stability of ballooning MHD perturbations in mirror traps. Most of them were published in the 1980s. In the next decades, interest in the problem of ballooning instability in mirror traps significantly weakened (in contrast to what is happening in tokamaks, see e.g. [27–29]), which was a consequence of the termination of the TMX and MFTF-B projects in the USA in 1986 [3] as mentioned in Section I. A review of publications devoted to the stability of small-scale ballooning oscillations with a large azimuthal number $m \gg 1$ was made in our recent paper [1]. We will not repeat it here and immediately turn to works on the stability of ballooning oscillations in this article.

1. According to modern views, small-scale flute and ballooning oscillations with a large azimuthal number $m \gg 1$ must be stabilized due to the effects of a finite Larmor radius. This conclusion follows from the fundamental work of Rosenbluth, Kroll and Rostocker [30], where the role of FLR effects is revealed using the kinetic equation; see also [31, 32], where it is proved that the FLR effects can be included into equations of magnetohydrodynamics if the viscous stress tensor is preserved. In paraxial open traps, FLR effects can in principle stabilize all modes, except for oscillations with an azimuthal number $m = 1$. A more accurate estimate of the number of azimuthal modes that the effects of FLR stabilize is obtained in the article [33]. The FLR effects impose on oscillations with azimuthal number $m = 1$ the form of a rigid ("solid-state") displacement without deformation of the plasma interior in each cross section. However, even in this case, the displacement of the plasma column from the axis varies in different sections. As a result, the plasma column is bent. The bend is most noticeable in the region of the so-called unfavorable curvature near the minimum value of the magnetic field in the central section of the axially symmetric open trap. It is these (balloon) oscillations that we study in this article.

2. Kaiser, Nevins, and Pearlstein in a 1983 paper [16] investigated the stability of the $m = 1$ rigid mode in a quadrupole open trap in the paraxial approximation for a low-pressure plasma at $\beta \to 0$. These authors did not assume presence of a conductive wall around the plasma, but actually took into account the effects of FLR, since they considered only rigid displacements of the plasma.

3. Berk et al in 1984 showed with a kinetic treatment that a perfectly conducting wall located near the lateral plasma surface in case of large beta has a strong stabilization effect on the $m = 1$ mode in an axisymmetric mirror, which cannot otherwise be stabilized by FLR effects [17, 18]. These authors argued (in our opinion, not quite objectively) that in previous works the effect of wall stabilization was overlooked. They wrote: "Previous analyses either did not take boundary conditions into account properly [16] or were for isotropic pressure [19, 20], where beta of order unity is needed for stability." Berk et al analyzed the effect of wall stabilization and fast electron ring on curvature-driven modes, drift modes, anisotropy-driven modes such as AIC, however ballooning modes are not explicitly mentioned. The source of stability is the image currents generated by placing the wall (or properly shaped conductors) in close proximity of a high-beta axially localized plasma.

4. The MHD approach to the study of wall stabilization historically preceded Berk’s theory. Later it was inherited and supplemented by several authors [19–23]. For simplicity, Refs. [18–20, 22] assumed a sharp boundary pressure profile. In particular, Haas and Wesson in 1967 considered the hydromagnetic stability of a theta-pinch with a sharp boundary [19]. They allowed for the magnetic field to vary along the pinch axis, so that, in fact, they analyzed stability of a mirror trap. They found that the necessary condition for stability of the $m = 1$ mode is $\beta > \beta_{\text{crit}} = 1/[1 + (a/r_w)^2]$, where $a$ and $r_w$ are the radii of the plasma and conducting wall. In other words, $\beta > 50\%$ is required in the limit $a \to r_w$ in agreement with later publications.

5. D’Ippolito and Hafizi in 1981 employed a simplified model of an axisymmetric tandem mirror to study the stability of low-$m$ ballooning modes in isotropic plasma with sharp-boundary surrounded by a perfectly conducting wall [20]. The scaling of the critical beta $\beta_{\text{crit}}$ with mode number $m$, connection length $L_c$ to the end plug, and the wall to plasma radius ratio $r_w/a$ was computed for several magnetic field profiles. Important simplification was that the end plugs of the tandem mirror are not explicitly modeled but are replaced by a boundary condition on the perturbation, viz., that the field lines are “tied” at some distance $L_c$ outside the central cell. The authors found a second zone of stability at large beta, but in the case of isotropic plasma they considered, the second zone arose only when beta was very close to unity.

6. D’Ippolito and Myra in 1984 numerically analyzed stability of the $m = 1$ rigid ballooning mode in an axisymmetric tandem mirror with inverted pressure profile [21]. Included in the analysis are the stabilizing effects of an externally applied force, such as the rf-induced ponderomotive force, and of a perfectly conducting lateral wall. The authors assumed isotropic plasma with a hollow stepwise pressure profile and studied wall stabilization. They found two zones of stability at low and high
betas. These two zones merge in case when the conducting wall is located sufficiently close to the plasma lateral boundary and the radial pressure profile has the shape of thin annular.

7. Kaiser and Pearlstein in their 1985’s paper \[22\] wrote equations for studying the stability of the \( m = 1 \) mode in an axially symmetric trap. They took into account the FLR effects and conductive wall in a plasma model with an arbitrary beta, but with a stepwise radial profile. These authors start by writing out Eq. ((11))\(^1\), which they say can be “synthesized” from three papers \[34–36\]. This equation contains a term that takes into account the FLR effects. Equation ((1)) is then used to derive equation ((11)), which was later used by Li, Kesner and Lane to analyze the stability of the rigid mode in \[24\], where it appears under the number ((28)). For the case when the conducting wall of the vacuum chamber is located as close as possible to the lateral surface of the plasma, equation ((11)) is simplified to equation ((19)), which was later studied by other authors, including Kesner in \[23\].

7. In the same year 1985, Kesner in Ref. \[23\] discussed the possibility of an axisymmetric tandem mirror in which stability accrues from wall stabilization. The author used an anisotropic plasma model with a sharp boundary with reference to the above-cited work by Kaiser and Pearlstein \[22\], and he studied the case when the chamber walls were located extremely close to the lateral surface of the plasma. In this case, as shown in \[22\], the ballooning mode becomes almost fluted. It was also assumed that the vacuum magnetic field has a parabolic profile up to magnetic mirrors. The axial distribution of the plasma pressure was given by two versions of the function \( p_\perp (B) \). One distribution corresponded to the pressure maximum in the median plane of the trap, the other distribution described the plasma with sloshing ions, when the pressure maximum was reached in the gap between the median plane and the magnetic mirrors. The author begins his analysis with equation ((1)), which is identical to equation ((19)) in paper \[22\] by Kaiser and Pearlstein.

It was shown that plasma stability is achieved if the parameter \( \beta \) exceeds a certain limiting value \( \beta_{\text{crit}} \), which depends on the degree of plasma anisotropy. The limiting value is the smaller, the stronger the anisotropy. For sufficiently large anisotropy, \( \beta_{\text{crit}} \) decreases to 0.4. For an isotropic plasma, this value increases to 0.8.

8. In paper \[24\], also published in 1985, Li, Kesner and Lane used the MHD energy principle to examine the stabilization effect of a conducting wall located near the plasma lateral surface. It was assumed that conducting wall is the only stabilization mechanism.

The calculation starts with equation ((28)), which coincides with equation ((11)) from the cited above article \[22\] by Kaiser and Pearlstein. In contrast to their own equation ((19)) in equation ((11)) the assumption has not yet been made that the conducting wall is located close to the lateral surface of the plasma, that is, the parameter \( \Lambda = (r_w^2 + a^2)/(r_w^2 - a^2) \) is not equal to infinity. However, then the authors only analyze the limit when \( \Lambda = \infty \). In this case, the plasma displacement \( \xi_n \) turns out to be quasiflute, i.e. \( aB_0\xi_n \approx \text{const} \) (where \( B_0 \) is the magnetic field in the vacuum gap), and the parameter \( \Lambda \) is knocked out of the equation by integrating it with respect to the variable \( z \) along the axis of the system under the condition that at the ends of the integration interval the displacement is not frozen into the ends (free-end boundary condition), i.e. \( aB_0\xi_n \approx 0 \) (where the prime ‘ stands for derivative over coordinate \( z \) along the trap axis), while, as the authors prove, normal to boundary of the magnetic field perturbation component \( \delta B_n \) vanishes.

Although the intermediate formulas are written for an anisotropic plasma, the final analysis is limited to the case of an isotropic plasma. For an isotropic pressure component, it is found that a hollow profile has better stability than a uniform pressure when the integral of the radial pressure profile is fixed.

9. Same authors in a latter paper \[25\], published in 1987, discussed the wall stabilization by partially enclosed wall using \( m = 1 \) model and stepwise radial profile of isotropic plasma. The stabilizing wall extends axially only over a part of the distance between the trap midplane and the mirror throat. The wall is located near the plasma surface in the bad curvature region and distant from the plasma in the good curvature region. A variational method is used to solve the equations for both regions, with the authors solving equation ((1)), which is the same as equation ((13)) from paper \[22\] cited above. At the ends of the plasma column, the boundary condition \( (B_0\xi_n/\sqrt{B})' = 0 \) was used, which for a plasma with a sharp boundary is equivalent to the boundary condition \( (aB_0\xi_n)' = 0 \). For the connection of the regions of close and distant plasma-wall proximity, a jump condition is used. The variational calculation is performed with a simple trial function (the choice of the trial function is substantiated with an exact numerical solution). The results show that (a) the removal of the conducting wall in the good curvature region does not significantly degrade plasma stability, (b) the acceptable ratio of the radius of the conducting wall to the plasma radius is about 1.1, and (c) for cases with a low mirror ratio, more conducting wall is needed for stability than for cases with a high mirror ratio.

10. In the next paper \[26\] of 1987, Li, Kesner and LoDestro have shown that a simple axisymmetric magnetic mirror may be MHD stable, provided that (i) a certain length of magnetic field has a series of ripples in it, (ii) with isotropic pressure the plasma beta value is higher than 50%, and (3) the conducting wall is very close to the plasma surface. The theory and its physical picture are discussed, and a Sturm-Liouville form is presented as well as numerical results that illuminate

\(^1\) We use double brackets to denote numbers of equations in cited papers.
the requirements of field structure and anisotropy. The authors numerically solve the equation ((1)), which is derived in the article [22] for plasma with a sharp boundary and has the number ((13)). The calculation is performed for plasma with isotropic pressure and for an anisotropic plasma, in which the transverse pressure varies in a magnetic field according to the law $p_\perp \propto B_{\text{max}}^2 - B^2$ or $p_\perp \propto (B/B_{\text{max}})^2(1 - B/B_{\text{max}})^{n-1}$. It is shown that anisotropy reduces the beta required for stability, particularly at low mirror ratio. It seems that this is the first work in which the critical beta is calculated with a finite value of the $\Lambda$ parameter, i.e. at a nonzero thickness of the vacuum gap between the plasma and the side conducting wall of the chamber.

11. LoDestro in the paper [2] of 1986 derived a ballooning equation for the $m = 1$ mode in an axisymmetric plasma with a diffuse pressure profile within the framework of the arbitrary-$\beta$, dominant-finite Larmor-radius analysis of Kaiser and Pearlstein [22]. According to the author, diffuse profiles preserve the sharp-boundary result that only the vacuum curvature appears in the drive. It is shown that the diffuse profile reduces the volume-averaged pressure, which is necessary for the stability of an isotropic plasma in the limit when the conducting wall approaches the plasma/vacuum interface and the mirror ratio approaches unity.

12. Close and Lichtenberg in the paper [37] of 1989 reported results of experiment on MMX device at Berkeley. High-beta ballooning modes are studied in an axisymmetric multiple mirror which is made average-minimum $B$ with end cusps. Electric and magnetic field measurements in the plasma characterize the predominant azimuthal mode number as $m = 1$. The ballooning character of the mode is determined by measuring the ratio of the mode amplitude near the center of the plasma column from the axis, and confirmed by measurement of perturbed perpendicular magnetic fields. Theoretical growth rates are calculated numerically using ideal and resistive magnetohydrodynamic equations for the rigid $m = 1$ ballooning mode. Within experimental error it is found that the $m = 1$ resistive ballooning growth rate scales with the resistivity $\zeta(z) = \zeta_n(z)/\sqrt{2\psi_a}$ (2) and the derivative $d/dz$ in the second line acts on all factors to the right of $d/dz$. Required function

$$\phi(z) = a(z)B_v(z)\zeta_n(z)/\sqrt{2\psi_a}$$

depends on one coordinate $z$ along the axis and is expressed in terms of the plasma boundary radius $a = a(z)$, the vacuum magnetic field $B_v = B_v(z)$, and the small displacement $\zeta_n(z)$ of the plasma column from the axis. The normalizing denominator $\sqrt{2\psi_a}$ in Eq. (2) is present in LoDestro’s article, but we omit it below. It doesn’t make much sense, since $\psi_a$ is a constant, $\psi_a = \text{const}$. The parameter $\psi_a$ has the meaning of the reduced (i.e. divided by $2\pi$) magnetic flux through the plasma cross section $\pi a^2$. It is related to the plasma radius $a = a(z)$ by the equation

$$a^2/2 = \int_0^{\psi_a} d\psi / B.$$  (3)

The magnetic flux $\psi$ through an arbitrary section of the plasma and the radial coordinate $r$ relates the equation

$$r^2/2 = \int_0^\psi d\psi / B.$$  (4)

The magnetic field $B = B(\psi, z)$ in the paraxial approximation (i.e. with a small curvature of field lines) is related to the vacuum magnetic field $B_v = B_v(z)$ by the transverse equilibrium equation

$$B^2 = B_v^2 - \langle 4\pi \rangle_{|z|}^2 p_\perp.$$  (5)

The factor $[4\pi]$ in Eqs. (1) and (5) arises in the Gaussian system of units; LoDestro and other authors do not write it. Further, we also omit this factor. The kinetic theory predicts (see, for example, [35]) that the longitudinal and transverse plasma pressures can be considered as functions of $B$ and $\psi$, i.e. $p_\perp = p_\perp(B, \psi), p_\parallel = p_\parallel(B, \psi)$. In Eq. (1), one must assume that the magnetic field $B$...
is already expressed in terms of $\psi$ and $z$, and therefore $p_\perp = p_\perp(\psi, z)$, $p_\parallel = p_\parallel(\psi, z)$. In what follows, we will also use the notation

$$p = \frac{p_\perp + p_\parallel}{2}.$$  \hfill (6)

One should distinguish between the actual plasma radius $a = a(\psi_a, z)$ and the vacuum plasma radius

$$a_v(z) = \sqrt{\frac{2\psi_a}{B_v(z)}}.$$  \hfill (7)

It enters Eq. (1) as the ratio $a''_v/a_v$, where the prime denotes the derivative of $d/dz$ with respect to the coordinate $z$. LoDestro draws the reader’s attention to the fact that only the vacuum field line curvature $a''_v$ enters into the equation, but in fact the curvature $a''$ of the plasma boundary arises when calculating the derivative in the second line of the equation. We also point out that $\rho$ is the plasma mass density, and $\omega$ is the oscillation frequency.

The angle brackets in the equation (1) denote the average

$$\langle g \rangle = \frac{\int_0^\psi g/B \cdot d\psi}{\int_0^\psi d\psi/B} = \frac{2}{a^2} \int_0^\psi \frac{d\psi}{B} g$$  \hfill (8)

de an arbitrary function $g(\psi, z)$ over the plasma cross section. Parameter

$$\Lambda = \frac{r_w^2 + a^2}{r_w^2 - a^2}$$  \hfill (9)

is expressed in terms of the actual radius of the plasma/vacuum boundary $a = a(z)$ and the radius of the conducting cylinder $r_w = r_w(z)$, which surrounds the plasma column. The parameter $\Lambda = \Lambda(z)$ is generally a variable function of the z coordinate, but in the next three sections we assume that $\Lambda$ is a constant. The larger the $\Lambda$ value, the closer the conducting cylinder is to the plasma boundary. The $\Lambda \to \infty$ limit corresponds to the case when the conducting side wall is close to the plasma boundary, repeating its shape, but does not touch the plasma. The limit $\Lambda \to 1$ means that the lateral wall is removed to infinity.

We repeated the derivation of the LoDestro equation and now we are sure that it is correct, although there are typos in a pair of intermediate formulas in Ref. [2].

The boundary conditions for Eq. (1) and similar equations in the study of ballooning instability are traditionally set at the ends of the plasma column at the magnetic field maxima $B_r = B_m$, where $B''_r = 0$ and $p_\perp = p_\parallel = 0$. In accordance with the geometry of actually existing open traps, it is usually assumed that the magnetic field is symmetrical with respect to the $z = 0$ plane, and the magnetic mirrors (i.e., field maxima) are located at $z = \pm L$. Traditionally, two types of boundary conditions are considered. In the presence of conductive ends directly in magnetic mirrors, it is required that the boundary condition be satisfied

$$\phi = 0$$  \hfill (10)

at $z = \pm L$. A similar boundary condition is usually used in studying the stability of small-scale ballooning disturbances, thereby modeling the presence of a stabilizing cell behind a magnetic mirror (see, for example, [1]). If the plasma ends are isolated, the boundary condition is applied

$$\phi' = 0.$$  \hfill (11)

As a rule, it implies that other methods of MHD stabilization in addition to stabilization by a conducting lateral wall are not used. It is this boundary condition (11) that was used earlier in the works on the stability of the $m = 1$ ballooning mode.

**IV. ZERO VACUUM GAP LIMIT**

For this section, Eq. (1) is reduced for the limit $\Lambda \to \infty$ as the lateral conducting wall approaches the plasma/vacuum boundary, where it produces its maximum stabilizing effect. In this limit it is also possible to make analytic progress in assessing the effects of a diffuse profile.

Stabilization of the rigid ballooning mode by a conducting wall in the $\Lambda \to \infty$ limit was previously studied by Kesner [23], Li, Kesner and Lane [24] in the case of a plasma with a radial profile in the form of a step with a sharp border. They showed that the plasma displacement $\xi_n$ in this limit turns out to be quasistable, i.e. $\phi = aB_c\xi_n \approx \text{const}$, and the parameter $\Lambda$ is knocked out from an equation like (1) by integrating it over the coordinate $z$ along the axis of the mirror trap, if the boundary condition (11) is allowed, which means that at the ends of the integration interval, the displacement is not frozen into the ends (free-end boundary condition, insulating boundary condition).

For $\Lambda \to \infty$ the first term in Eq. (1) is formally greater than all the others, so the derivative $d\phi/dz$ must tend to zero in proportion to $1/\Lambda$, i.e.

$$\phi = \text{const} + \delta \phi(z),$$  \hfill (12)

moreover, $\delta \phi(z) = O(1/\Lambda)$, and the constant const can be considered equal to one due to the linearity of the equation (1) with respect to the function $\phi$. Substitute $\phi = 1$ into the second term in the equation (1) (that’s the whole square bracket on three lines) and integrate from $z = -L$ to $z = L$. For , the first term (with large $\Lambda$) will drop out if we use the boundary condition $\phi' = \delta \phi' = 0$ at the ends of the integration interval at $z = \pm L$. Performing the indicated procedure yields the integral
where \( \beta = 2p_0/B_v^2(0) = 2p_0 \), (17) so that
\[
B = \sqrt{B_v^2 - \beta f_k},
\]
\[
a^2 = \frac{1}{2} \int_0^1 d\psi, \quad \frac{a^2}{2} = \frac{1}{B_v},
\]
\[
(\bar{\rho}) = \frac{\beta}{a^2} \int_0^1 \frac{f_k}{B} d\psi.
\]
The critical value of the parameter \( \beta \) corresponding to the marginal stability \( \omega^2 = 0 \) is defined as the root of the equation
\[
W(\beta_{\text{crit}}) = 0, \quad (22)
\]
where
\[
W(\beta) = \int_{-1}^1 \left[ 2(\bar{\rho}) a'' + \frac{1}{2} \left( \frac{B'_u}{B_v} + \frac{2a'}{a} \right)^2 \left( 1 - \frac{\bar{\rho}}{B_v^2} \right) \right] \, dz.
\]
The calculations were performed in the Wolfram Mathematica® system for radial pressure profiles of the form
\[
f_k(\psi) = 1 - \psi^k \quad (0 \leq \psi \leq 1)
\]
for four index values \( k = \{1, 2, 4, \infty\} \).\(^2\) Function \( f_1 \) describes the most gentle pressure profile. For \( \beta/B_v^2 \ll 1 \) it approximately gives the parabolic dependence of the pressure \( p \) on the coordinate \( r \). The larger the index \( k \), the more table-like distribution \( p \) near the axis of the plasma column and the steeper it is near the boundary of the column. Index \( k = \infty \) corresponds to a pressure profile in the form of a step with a sharp boundary. In this case, the radial profile can be written in terms of a \( \theta \)-function such that \( \theta(x) = 0 \) for \( x < 0 \) and \( \theta(x) = 1 \) for \( x > 0 \):
\[
f_\infty(\psi) = \theta(1 - \psi).
\]
\(^2\) Details of the calculations are recorded in the file Global Mode Isotropic.nb.
\[ \frac{a_2^2}{2} = \frac{1}{\sqrt{B_v^2 - \beta}}; \quad (26a) \]
\[ \frac{a_2^2}{2} \left( \langle \beta \rangle_1 \right) = -\frac{2B_v^3 + 2B_v^2 \sqrt{B_v^2 - \beta} + \beta \sqrt{B_v^2 - \beta}}{3\beta}; \quad (27a) \]
\[ \frac{a_2^2}{2} \left( \langle \beta \rangle_2 \right) = \frac{1}{4} \left( \frac{(\beta + B_v^2)}{\sqrt{\beta}} \coth^{-1} \left( \frac{B_v/\sqrt{\beta}}{1 - B_v} \right) - B_v \right); \quad (27b) \]
\[ \frac{a_2^2}{2} \left( \langle \beta \rangle_4 \right) = \frac{1}{6} \left( \frac{(2\beta + B_v^2)}{\sqrt{B_v^2 - \beta}} \right) \frac{2}{\beta}; \quad (27c) \]
\[ \frac{a_2^2}{2} \left( \langle \beta \rangle_\infty \right) = \frac{2}{\beta \sqrt{B_v^2 - \beta}}; \quad (27d) \]

Calculating the coefficient

\[ A = \left( \frac{B_v'}{B_v} + \frac{2a'}{a} \right); \quad (28) \]

reveals that it is proportional to the derivative of the vacuum magnetic field:

\[ A_1 = \left( \frac{1}{B_v} - \frac{1}{\sqrt{B_v^2 - \beta}} \right) B_v'; \quad (29a) \]
\[ A_2 = \left( \frac{1}{B_v} - \frac{\sqrt{\beta}}{(B_v^2 - \beta) \csch^{-1} \left( \sqrt{B_v^2/\beta - 1} \right)} \right) B_v'; \quad (29b) \]
\[ A_4 = \frac{1}{2} \left( \frac{B_v^2 - 2\beta}{B_v^2 - \beta} \right) \frac{B_v'}{B_v} - \frac{1}{\sqrt{B_v^2 - \beta}} \frac{2}{\beta}; \quad (29c) \]
\[ A_\infty = \frac{\beta B_v'}{\beta B_v' - B_v'}; \quad (29d) \]

The authors of the publications cited above used various axial profiles of the vacuum magnetic field \( B_v \) in their calculations. For example, D’Ippolito and Myra [21] studied plasma stabilization by some external force by modeling the magnetic field in a tandem trap with an interpolation function that approximately replicated the real vacuum field. Kesner [23] studied wall stabilization in the \( \Lambda \to \infty \) limit by simulating the magnetic field with a parabola. Li, Kesner and Lane [24, 25] modeled the magnetic field as a sum of a constant and a cosine. Li, KesnerLo and LoDestro [26] did about the same. Binding to only one specific field model in these works does not allow one to find out how the axial profile of the magnetic field should be modified in order to lower the critical value of beta and thereby simplify the transition to a stable plasma confinement regime.

To investigate the dependence of critical beta on the axial profile of the magnetic field, we performed calcula-
tions for the family of functions

\[ B_\nu(z) = \left[ 1 - \left( 1 - K^{-\nu/2} \right) |z|^{\nu} \right]^{-2/\nu} \]  \hspace{1cm} (30)

with different values of the parameters \( K = \{20, 16, 12, 8, 4\} \), \( \mu = \{1, 2, 4, 6\} \) and \( \nu = \{0.5, 2, 6\} \). Previously, such a family was used by Mirnov and Bushkova [45], as well as by ourselves in a recent paper [1]. The parameter \( K \) defines the mirror ratio \( B_\nu(\pm1)/B_\nu(0) = K \). The “steepness” of magnetic mirrors depends on the parameters \( \mu \) and \( \nu \). Figure 2 shows the profiles of the vacuum magnetic field for different values of \( \mu \) and \( \nu \). It is easy to see that both with an increase in \( \mu \) and with an increase in \( \nu \), the profile steepens near the magnetic mirrors while the quasi-homogeneous region at the center of the trap expands.

The functions (30) have the peculiarity that the derivative \( B'_\nu \) does not vanish at the ends of the integration interval \( z = \pm1 \), as was assumed when deriving Eq. (23). According to the physics of the matter, this should mean that magnetic coils of such a small size are installed in the mirror throats that, on the scale under consideration, the coils can be considered “point”. On the size of a point coil, the derivative \( B'_\nu(\pm1) \) abruptly changes to zero. Accordingly, the second derivative \( B''_\nu(\pm1) \) of any function (30) must be supplemented with the delta function \( \delta(z \mp 1) \). We write this rule in symbolic form:

\[ B''_\nu(z) \Rightarrow B''_\nu(1) \delta(z - 1) + B''_\nu(-1) \delta(z + 1). \]  \hspace{1cm} (31)

In addition, it should be noted that the function (30) for \( \mu = 1 \) has a break at \( z = 0 \), since the derivative

\[ |z|' = 1 + 2\theta(z). \]  \hspace{1cm} (32)

experiences a jump there. Second derivative

\[ |z|'' = 2\delta(z) \]  \hspace{1cm} (33)

to zero. Accordingly, the second derivative

\[ B''_\nu(z) \Rightarrow B''_\nu(1) \delta(z - 1) + B''_\nu(-1) \delta(z + 1). \]  \hspace{1cm} (31)

In addition, it should be noted that the function (30) for \( \mu = 1 \) has a break at \( z = 0 \), since the derivative

\[ |z|^\prime = 1 + 2\theta(z). \]  \hspace{1cm} (32)

enters Eq. (23) under the integral sign through the vacuum curvature \( a'_\nu \) and also contains the delta function \( \delta(z) \). If \( \mu > 1 \), this delta function enters the integrand, being multiplied by \( |z|^{\mu-1} \), so it makes a zero contribution to the integral, but in the case of \( \mu = 1 \) it is necessary special care in calculating the integral. The combination \( \mu = 1, \nu = 2 \) is remarkable in that the function corresponding to it minimizes the absolute value of the integral in the Rosenbluth-Longmire criterion [43], which determines the stability condition for flute oscillations in open traps (see [44]). Section VI will describe two options for smoothing the function \( |z| \), which were used in modeling plasma stabilization by conducting plates at the ends.

The results of the numerical solution of the equation (23) are collected in tables I–VI. The search for the roots of the equation was carried out using the FindRoot utility built into the Wolfram Mathematica® system. It searches for the first root near the initial guess \( \beta_{\text{start}} \) passed to it. The right side of Eq. (23) as a function of \( \beta \) tends to infinity at \( \beta \rightarrow 1 \), forming a peak, especially narrow at large values of the parameters \( \mu = \{4, 6\} \).

| k | \( \nu \backslash \mu \) | 0.5 | 1 | 2 | 4 | 6 |
|---|---|---|---|---|---|---|
| 1 | | 0.969 118 | N/F | N/F | N/F | N/F |
| 2 | | 0.988 454 | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |
| 0.5 | 0.988 454 | N/F | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |
| 0.5 | 0.988 454 | N/F | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |
| 0.5 | 0.988 454 | N/F | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |

Table I. \( \beta_{\text{crit}} \) for an isotropic plasma in a magnetic field (30) at \( K = 20 \) and \( \Lambda = \infty \). Minimum value \( \beta_{\text{min}} = 0.767 052 \) is achieved for \( k = \infty, \mu = 1, \nu = 0.5 \).

| k | \( \nu \backslash \mu \) | 0.5 | 1 | 2 | 4 | 6 |
|---|---|---|---|---|---|---|
| 1 | | 0.969 118 | N/F | N/F | N/F | N/F |
| 2 | | 0.988 454 | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |
| 0.5 | 0.988 454 | N/F | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |
| 0.5 | 0.988 454 | N/F | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |

Table II. \( \beta_{\text{crit}} \) for an isotropic plasma in a magnetic field (30) at \( K = 16 \) and \( \Lambda = \infty \). Minimum value \( \beta_{\text{min}} = 0.766 592 \) is achieved for \( k = \infty, \mu = 1, \nu = 0.5 \).

| k | \( \nu \backslash \mu \) | 0.5 | 1 | 2 | 4 | 6 |
|---|---|---|---|---|---|---|
| 1 | | 0.969 118 | N/F | N/F | N/F | N/F |
| 2 | | 0.988 454 | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |
| 0.5 | 0.988 454 | N/F | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |
| 0.5 | 0.988 454 | N/F | N/F | N/F | N/F | N/F |
| 6 | | 0.997 185 | 0.999 002 | N/F | N/F | N/F |

Table III. \( \beta_{\text{crit}} \) for an isotropic plasma in a magnetic field (30) at \( K = 12 \) and \( \Lambda = \infty \). Minimum value \( \beta_{\text{min}} = 0.765 656 \) is achieved for \( k = \infty, \mu = 1, \nu = 0.5 \).
Table IV. \( \beta_{\text{exit}} \) for an isotropic plasma in a magnetic field (30) at \( K = 8 \) and \( \Lambda = \infty \). Minimum value \( \beta_{\min} = 0.763 \, 185 \) is achieved for \( k = \infty \), \( \mu = 1 \), \( \nu = 0.5 \).

| k \( \nu \backslash \mu \) | 1          | 2          | 4          | 6          |
|---------------------------|------------|------------|------------|------------|
| 1                         | 0.968 \, 295 N/F | N/F        | N/F        | N/F        |
| 2                         | 0.986 \, 970 N/F | N/F        | N/F        | N/F        |
| 6                         | 1.0        | N/F        | N/F        | N/F        |
| 2                         | 0.884 \, 018 0.974 \, 017 0.996 \, 725 0.999 \, 421 |
| 2                         | 0.922 \, 754 0.990 \, 435 0.999 \, 758 0.999 \, 996 |
| 6                         | 0.994 \, 532 0.999 \, 886 N/F | N/F        |
| 4                         | 0.828 \, 410 0.937 \, 536 0.976 \, 714 0.986 \, 174 |
| 2                         | 0.876 \, 563 0.967 \, 413 0.992 \, 044 0.996 \, 504 |
| 6                         | 0.985 \, 300 0.999 \, 478 0.999 \, 996 1.0        |
| \( \infty \)             | 0.763 \, 185 0.887 \, 935 0.940 \, 394 0.955 \, 414 |
| 2                         | 0.821 \, 129 0.932 \, 068 0.970 \, 801 0.980 \, 570 |
| 6                         | 0.972 \, 340 0.997 \, 444 0.999 \, 857 0.999 \, 979 |

Table V. \( \beta_{\text{exit}} \) for an isotropic plasma in a magnetic field (30) at \( K = 4 \) and \( \Lambda = \infty \). Minimum value \( \beta_{\min} = 0.751 \, 161 \) is achieved for \( k = \infty \), \( \mu = 1 \), \( \nu = 0.5 \).

| k \( \nu \backslash \mu \) | 1          | 2          | 4          | 6          |
|---------------------------|------------|------------|------------|------------|
| 1                         | 0.964 \, 113 N/F | N/F        | N/F        | N/F        |
| 2                         | 0.981 \, 067 N/F | N/F        | N/F        | N/F        |
| 6                         | 0.999 \, 894 N/F | N/F        | N/F        | N/F        |
| 2                         | 0.876 \, 139 0.965 \, 939 0.992 \, 846 0.997 \, 602 |
| 2                         | 0.908 \, 158 0.982 \, 414 0.998 \, 320 0.999 \, 791 |
| 6                         | 0.987 \, 788 0.999 \, 600 1.0        | N/F        |
| 4                         | 0.818 \, 959 0.924 \, 308 0.965 \, 448 0.976 \, 498 |
| 2                         | 0.857 \, 282 0.950 \, 967 0.982 \, 075 0.989 \, 404 |
| 6                         | 0.958 \, 880 0.995 \, 161 0.999 \, 671 0.999 \, 949 |
| \( \infty \)             | 0.751 \, 161 0.868 \, 862 0.920 \, 574 0.936 \, 167 |
| 2                         | 0.796 \, 373 0.905 \, 503 0.948 \, 568 0.960 \, 802 |
| 6                         | 0.929 \, 436 0.983 \, 598 0.996 \, 047 0.998 \, 187 |

Table VI. \( \beta_{\text{exit}} \) for an isotropic plasma in a magnetic field (30) at \( K = 2 \) and \( \Lambda = \infty \). Minimum value \( \beta_{\min} = 0.704 \, 229 \) is achieved for \( k = \infty \), \( \mu = 1 \), \( \nu = 0.5 \).

| k \( \nu \backslash \mu \) | 1          | 2          | 4          | 6          |
|---------------------------|------------|------------|------------|------------|
| 1                         | 0.944 \, 517 0.997 \, 926 N/F | N/F        | N/F        |
| 2                         | 0.957 \, 531 0.999 \, 963 N/F | N/F        | N/F        |
| 6                         | 0.987 \, 054 N/F | N/F        | N/F        | N/F        |
| 2                         | 0.842 \, 901 0.925 \, 434 0.959 \, 601 0.969 \, 620 |
| 2                         | 0.862 \, 057 0.939 \, 763 0.970 \, 102 0.978 \, 678 |
| 6                         | 0.916 \, 647 0.974 \, 858 0.992 \, 704 0.996 \, 555 |
| 4                         | 0.778 \, 691 0.866 \, 595 0.905 \, 872 0.918 \, 198 |
| 2                         | 0.799 \, 898 0.884 \, 208 0.920 \, 471 0.931 \, 682 |
| 6                         | 0.863 \, 613 0.932 \, 931 0.959 \, 835 0.967 \, 726 |
| \( \infty \)             | 0.704 \, 229 0.793 \, 394 0.834 \, 532 0.847 \, 731 |
| 2                         | 0.726 \, 839 0.813 \, 216 0.851 \, 734 0.869 \, 953 |
| 6                         | 0.797 \, 425 0.871 \, 675 0.902 \, 472 0.912 \, 002 |

Figure 2. Axial profiles of the magnetic field corresponding to different values of the parameters \( \mu \) and \( \nu \) (shown in the figures) for the same mirror ratio \( K = 10 \).

and \( \nu = \{6\} \). To avoid the algorithm reaching the edges of the physically admissible interval \( 0 < \beta < 1 \), where the integrand in Eq. (23) could have singularities, the search for roots was performed on a narrower interval \( 0 < \beta_{\min} < \beta < \beta_{\max} < 1 \).

Success or failure of the search for roots by \texttt{FindRoot} depends very much on the luck in choosing an initial approximation \( \beta_{\text{start}} \). In the first calculations, the al-
and the problem of lost roots disappeared. As a result of which each individual term became real.

In the final version of our numerical code, the root found by the `FindRoot` utility was passed to the `FindRoot` utility as an initial guess. Occasionally it happened that one of the two utilities listed did not find the root, while the other found the root. Such cases have been carefully investigated in order to improve the code.

In the course of one such study, it became clear that Mathematica® can express the result of taking the integrals (19) and (21) through different functions depending on the Wolfram version Mathematica® and integration utility options. (This fact showed itself when adapting the program code to the case of an anisotropic plasma.) Integrals in symbolic form are calculated by the `Integrate` utility. In the newest versions of Wolfram Mathematica®, this utility by default outputs the result of integration as a list of formulas with an indication of their range of applicability for all conceivable parameter relationships on which the integral depends. The time of such integration is sometimes too long, so we use the `Integrate` utility with the `GenerateConditions` -> `False` option to disable the generation of conditions. We then usually simplify the resulting expression using `Simplify` or `FullSimplify` utilities with a set of conditions such as $0 < \beta < 1$, $B_0^2 > \beta$, $B_\nu > 1$, etc. Strictly speaking, we are not sure that this method always gives the correct result, but so far we have not noticed obvious errors.

When calculating the integral (21), the simplification `Simplify` was not done. At the stage of calculations for an isotropic plasma, this omission did not manifest itself in any way. However, when porting the code to a program for an anisotropic plasma, the `FindRoot` utility repeatedly lost the roots of the (23) equation for some chaotic values of the anisotropy parameter. The study showed that the integral (21) for $k = 2$ contained 7 terms, 3 of which gave complex values. In this case, the sum of all terms was approximately real (with an accuracy of the order of $10^{-16}$). However, sometimes the `FindRoot` utility seemed to hit a dead end. Near the critical value $\beta$, the real part of the (21) integral should tend to zero, but the imaginary part did not become less than the real part due to insufficient accuracy in calculating the sum of terms.

As a palliative remedy to the problem, we forced the `FindRoot` utility to solve only the real part of Eq. (23). Then the missing roots were found. After all, we simplified the integration result in Eq. (21) using the `FullSimplify` utility with a suitable set of conditions, as a result of which each individual term became real and the problem of lost roots disappeared.

Let’s proceed to the analysis of the results presented in the tables. First of all, it is useful to check that the calculated values of $\beta_{\text{crit}}$ do indeed indicate the lower bound of the stability zone. To do this, it suffices to study the dependence of the integral (23) on $\beta$. An example of such a dependence is shown in Figure 3. It proves that there is only one stability zone $W > 0$ and that it is located in the region $\beta > \beta_{\text{crit}}$, where $\beta_{\text{crit}}$ is the root of Eq. (22).

Each table is made for one fixed value of the mirror ratio $K$. Within each individual table, it is not difficult to detect a trend towards a decrease in the critical beta with an increase in the steepness of the radial pressure profile as the index $k$ increases from $k = 1$ to $k = \infty$ for a fixed pair of indices $\mu$ (horizontally from 1 to 6) and $\nu$ (vertically from top to bottom from 0.5 to 6). The abbreviation N/F instead of a number says that the root was not found. This can mean both that the root does not exist, or that it exists but is less than 1 by less than $10^{-6}$. From a practical point of view, it’s all the same: it’s hard to imagine that in a real experiment one can get so close to the theoretical limit $\beta = 1$.

Further, we see that the critical beta increases both as the index $\mu$ increases and as the index $\nu$ increases. In other words, stabilization of the rigid ballooning mode is more problematic in traps with short and steep magnetic mirrors. The smallest value of critical beta is reached at $k = \infty$, $\mu = 1$ and $\nu = 0.5$. It changes slightly within from $\beta_{\text{min}} = 0.767052$ for $K = 20$ up to $\beta_{\text{min}} = 0.704229$ for $K = 2$. Comparison of the critical beta values in different tables with the same pairs of indices $\mu$ and $\nu$ also shows that the value of the mirror ratio $K$ has very little effect on the result of calculations in the interval of sufficiently large values of $K$, but begins to decrease more noticeably for $K < 4$. However, this fact can hardly be of practical importance, since it is difficult to imagine how an isotropic plasma can be confined in a trap with a small mirror ratio. Some reduction in the set of combinations of indices $k$, $\mu$, $\nu$, for which no solution has been found, with a decrease in $K$, in general, is also only of academic
Table VII. \( \beta_{\text{crit}} \) for an isotropic plasma in a magnetic field (34) at \( \Lambda = \infty \). Minimum value \( \beta_{\text{min}} = 0.728335 \) is achieved for \( k = \infty \), \( q = 2 \), \( K = 2 \).

interest.

To be able to compare our calculations with the results of other authors, we also calculated \( \beta_{\text{crit}} \) for the magnetic field, which is given by the functions

\[
B_v(z) = 1 + (K - 1) \sin^q(\pi z/2) \tag{34}
\]

with three index values \( q = \{2, 4, 8\} \). The variant \( q = 2 \) occurs in several works, in particular, it was used by Li, Kesner and Lane [24]. The results are presented in Table VII. The minimum value is reached at \( q = 2 \). It weakly depends on the mirror ratio in the interval from \( K = 20 \) to \( K = 4 \), but decreases more noticeably with a further decrease in \( K \). In particular, \( \beta_{\text{min}} = 0.728335 \) for \( K = 2 \).

Unlike the field of the form (30), function (34) is smooth everywhere and has no breaks. But even in the absence of such a kink on the vacuum field profile \( B_v(z) \), on the profile of the plasma boundary \( a(z) \) near the median plane \( z = 0 \), a “swell” is formed in the form of a “thorn” with a large curvature on spearhead. An example of such a “spike” for \( q = 2 \) is shown in 4(a). At \( q = 8 \) the “thorn” expands, forming a diamagnetic “bubble” named after Beklemishev [13], as in Figure 4(c). The plots of the plasma boundary \( a(z) \) in Figure 4 are plotted for different values of \( k \) and different values of \( \beta_{\text{crit}} \) corresponding to them, but with the same value of magnetic flux \( \psi = \psi_a = 1 \) captured in plasma. Interestingly, such plots \( a(z) \) almost coincide, although the values of \( \beta_{\text{crit}} \) for radial pressure profiles with different \( k \) differ quite significantly.

The displacement profile of the plasma column \( \xi_a(z) \) is shown in Figure 5 for the same value \( \beta = 0.9 \) for all radial pressure profiles \( k \). We emphasize that the displacement is not constant, although, as mentioned above, \( \phi(z) = \text{const} \) for \( \Lambda \to \infty \). At the critical values of beta indicated in the table VII and in the figure 4, the displacement profiles would be practically the same for all \( k \), since the profiles of the plasma boundary \( a(z) \) practically coincide in the figure 4.

The next section describes the method and results of solving the LoDestro equation (1) with a finite value of the parameter \( \Lambda \). We used the tables I–VI to check convergence of the method we used for large values of the parameter \( \Lambda \).

\[ \begin{array}{|c|cccc|}
\hline
k & q/K & 16 & 8 & 4 & 2 \\
\hline
1 & 2 & 0.999120 & 0.999425 & 0.995482 & 0.974971 \\
 & 4 & N/F & N/F & N/F & N/F \\
 & 8 & N/F & N/F & N/F & 0.999212 \\
\hline
2 & 2 & 0.936669 & 0.932510 & 0.920411 & 0.874111 \\
 & 4 & 0.975089 & 0.966600 & 0.955448 & 0.90739 \\
 & 8 & 0.987452 & 0.982331 & 0.968520 & 0.913126 \\
\hline
4 & 2 & 0.884435 & 0.878896 & 0.863139 & 0.807071 \\
 & 4 & 0.93456 & 0.924951 & 0.904090 & 0.835497 \\
 & 8 & 0.952088 & 0.942790 & 0.920539 & 0.847408 \\
\hline
\infty & 2 & 0.819595 & 0.812656 & 0.793243 & 0.728335 \\
 & 4 & 0.876482 & 0.865035 & 0.837831 & 0.756216 \\
 & 8 & 0.899629 & 0.886476 & 0.856387 & 0.767997 \\
\hline
\end{array} \]

Figure 4. The profile of the plasma boundary in the field in the form (34) for different values of the parameter \( q \) (shown in the figures) and critical values of beta for different pressure profiles with different parameters \( k \) (shown in the figures). The area occupied by plasma at \( \beta = 0 \) is shaded.

V. FINITE VACUUM GAP CASE

We used the built-in utility \texttt{ParametricNDSolveValue} to find a solution to the ordinary differential equation (1) in the Wolfram \texttt{Mathematica} \textcopyright{} system. It returns a reference \( pf \) to the interpolation function of the \( z \) coor-
Due to the symmetry of the magnetic field noted above, the desired function \( \phi(z) \) must be even, hence \( \phi'(0) = 0 \). As for the point \( z = 1 \), according to (11), the equality \( \phi'(1) = 0 \) must also take place there. However, due to the presence of delta functions in the coefficients of the equation, the derivatives \( \phi'(0+) \) and \( \phi'(1-) \) will no longer be equal to zero.

The values of these derivatives \( \phi'(0+) = BC0 \) and \( \phi'(1-) = BC1 \) can be calculated using the built-in utility Coefficient. It easily extracts the coefficient of \( \delta(z) \) on the right side of Eq. (1). All terms in this coefficient come from the second pair of square brackets. Let’s denote this coefficient as \( N0 \), and the coefficient at \( \phi'' \) (that is, the sum of terms inside the first pair of square brackets) as \( D0 \). Both coefficients must be calculated by substituting \( z = 0 \). The desired value of the derivative \( \phi'(0+) = BC0 \) is found by the formula \( BC0 = -N0/2D0 \). Since \( N0 \) contains the factor \( \phi(0) \), this boundary condition links \( \phi'(0+) \) and \( \phi(0) = \phi(0+) \).

In a similar way, one can find the boundary condition \( \phi'(1-) = BC1 \) on the right boundary if, before substituting function \( B_v(z) \) from Eq. (30) into the coefficients of Eq. (1) make the substitution \( |z| \to 1 - |1 - z| \). Then terms appear in the second square bracket that contain the delta function \( \delta(z - 1) \). The coefficient of this delta function will be denoted as \( N1 \), and the coefficient of \( \phi'' \) will be denoted as \( D1 \). Both these coefficients should be taken at \( z = 1 \). The desired value of the derivative \( \phi'(1-) = BC1 \) is found by the formula \( BC1 = N1/2D1 \). Since \( N1 \) contains the factor \( \phi(1) \), this boundary condition links \( \phi'(1-) \) and \( \phi(1) = \phi(1-) \).

Passing Eq. (1) to the ParametricNDSolveValue utility, we excluded the \( \delta(z) \) delta function from its right side. In the Wolfram Mathematica© this is done by the rule \( \delta(z) \to 0 \). In addition, we substituted \( \omega^2 = 0 \) because we wanted to calculate the critical (marginal) value of beta, and not the ballooning frequency for a given beta.

The boundary conditions for the ParametricNDSolveValue utility were specified on the left boundary as

\[
\phi[0] = 1, \quad \phi'[0] = BC0
\]  

(in the Wolfram Mathematica©, function arguments are written in square brackets). It would be a mistake to set the boundary condition \( \phi'[1] = BC1 \) on the right boundary, since for an arbitrary pair of values of the free parameters \( \beta \) and \( \Lambda \), a second-order ordinary differential equation with three boundary conditions, generally speaking, does not have solutions. Note also that the boundary condition \( \phi[0] = 1 \) is absolutely necessary, since the amplitude of the solution \( \phi[0] \) enters the boundary condition \( BC0 \). If \( \phi[0] \) or \( \phi[1] \) were not fixed, the ParametricNDSolveValue utility would find the null solution \( \phi[z] \equiv 0 \).

As mentioned above, the ParametricNDSolveValue utility returns a reference to an interpolation function, which is the solution of the equation passed to it with the specified boundary conditions. Denoting the refer-

![Figure 5. Displacement profile \( \xi_n(z) \) of the plasma column in the field (34) for \( \beta = 0.9 \), \( \Lambda \to \infty \) and various values of the parameter \( q \) and \( k \) (indicated in the figures).](image-url)
ence as \( pf \), we can calculate the value \( pf[\beta, \Lambda][z] \) of the solution at the point \( z \) given the numerical values of the parameters \( \beta \) and \( \Lambda \).

To calculate the critical value of \( \beta_{\text{crit}} \) for a given value of the \( \Lambda \) parameter, we essentially used the shooting method. In the classical implementation, this method consists in the fact that the differential equation is numerically integrated for a certain numerical value \( \beta \) and given boundary conditions on one boundary of the interval, and it is looked at where the found solution arrives at the other boundary: above or below the target. In the next step, the input value of \( \beta \) is adjusted with the intent to “hit the target”. This is where the name of the “shooting” method comes from.

In our implementation of the shooting method\(^3\) there is no need to re-integrate the differential equation with each new value of \( \beta \), since the \texttt{ParametricNDSolveValue} utility has already done everything. It is enough to pass the equation

\[
pf[\beta, \Lambda][1] = BC1
\]

(36)
to the \texttt{FindRoot} or \texttt{RootSearch} utility that we mentioned in the previous section. In fact, we used both of these utilities, calculating the root of Eq. (36) twice. We concluded that the root did not exist only if both utilities did not find a solution to Eq. (36).

The calculations were done for those combinations of parameters \( k \in \{1, 2, 4, \infty\} \), \( K \in \{20, 16, 12, 8, 4, 2\} \), \( \mu \in \{1, 2, 4, 6\} \), \( \nu \in \{0.5, 2, 6\} \), which are listed under section IV in tables I–VI, for discrete values of the parameter \( \Lambda \in \{1, 1.01, 1.02, \ldots, 500, 1000\} \). For \( \Lambda = 500 \), the calculated critical beta value differed from the value found in the previous section for \( \Lambda = \infty \) only in the fifth decimal place.

Figure 6 shows plots of \( \beta_{\text{crit}} \) versus \( \Lambda \) for the mirror ratio \( K = 8 \) at \( \nu = 2 \) and various values of \( \mu \) and \( k \). Comparison of figures 6(a)–(d) confirms the tendency noted in the previous section to increase critical value of beta as the magnetic mirrors steepen with increasing parameter \( \mu \). We also see that parabolic radial pressure profile \((k = 1)\) is unstable for all \( \Lambda \) if \( \mu \geq 2 \). For the next steepest profile \((k = 2)\), the stability zone disappears at \( \mu = 6 \) and \( \Lambda < 10 \), as shown in Fig. 6(d).

The main trends identified by our calculations are listed below:

- As expected, critical betas for the case \( \Lambda = 1 \), when conducting lateral wall is removed \((r_w/a = \infty)\), have not been found.
- Next to \( \Lambda = 1 \) checked value of \( \Lambda \) was 1.01 corresponding to very wide gap between plasma and conducting wall \((r_w/a = 14.1774)\). Unexpectedly, for such a wide gap, critical betas were found for some combinations of \( k, \mu, \nu \), and \( K \).
- The first found critical value of beta for a given combination of parameters \( k, \mu, \nu, q \), and \( K \) is as close as possible to unity at \( \Lambda = \Lambda_{\min} \). We did not set ourselves the goal of calculating \( \Lambda_{\min} \) exactly, but simply chose the minimum values of \( \Lambda \) from the available list of discrete values for which we calculated \( \beta_{\text{crit}} \).
- The stability zone is almost independent of the mirror ratio if \( K \lesssim 4 \).
- The minimum \( \beta_{\text{crit}} \) found for the studied set of radial and axial profiles is about 70%, which is slightly lower than the value 80% reported in earlier publications.

VI. WALL STABILIZATION COMBINED WITH CONDUCTIVE ENDS

Finally, it makes sense to perform calculations with the replacement of the boundary condition (11), which describes the insulating ends of the trap, with the boundary condition (10), which means that the plasma is frozen into the conductive ends. The assumption of freezing into the ends is traditionally used in the theory of small-scale ballooning oscillations, but it has not been used before in the study of the hard ballooning mode.

Drawing an analogy with the works of D’Ippolito and Hafizi [20] and D’Ippolito and Myra [21], it could be expected that when the plasma is stabilized by simultaneously conducting ends and conducting side walls, two stability limits can exist. In particular, D’Ippolito and Myra investigated the stability of low-\( m \) modes in an axisymmetric tandem mirror with an inverted (or hollow) steep-wise pressure profile under the action of external rf-force. They found two critical beta values, \( \beta_{\text{crit}} \) and \( \beta_{\text{crit2}} \): one at low beta due to the balancing of the ponderomotive force with the curvature drive, and one at high \( \beta \) due to the proximity of the conducting wall which

\(^3\) Of course, we are not the pioneers of this implementation, as it is described in the Wolfram \textit{Mathematica}\textsuperscript{©} documentation.
enables magnetic line bending to balance the curvature drive. Corresponding to two critical values of beta, there are two zones of stability. The first zone exists at low plasma pressure, at $0 < \beta < \beta_{\text{crit}1}$, and the second one exists at high pressure, at $\beta_{\text{crit}2} < \beta < 1$. As calculations by D’Ippolito and Myra have shown, these two zones can merge.

To test our expectations, we examined the solution of Eq. (1) with the boundary conditions (10) at $z = 1$ and (35) at $z = 0$. Preceding our calculations, we will show that under such boundary conditions there is indeed a stability zone at a small beta. To do this, we formulate a variational principle, that is, multiply Eq. (1) by $\phi(z)$ and integrate the result over the interval from $z = -1$ to $z = +1$. After integrating by parts in the first line of Eq. (1), taking into account the boundary conditions, we obtain the integral equation

$$\int_{-1}^{1} \left[ \Lambda + 1 - \frac{2 \langle \bar{\rho} \rangle}{B_v^2} \right] \left( \frac{d \phi}{dz} \right)^2 dz =$$

$$\int_{-1}^{1} \phi^2 \left[ - \frac{d}{dz} \left( \frac{B_v'}{B_v} + \frac{2a'}{a} \right) \left( 1 - \frac{\langle \bar{\rho} \rangle}{B_v^2} \right) + \omega^2 \rho \frac{B_v'}{B_v} - 2 \frac{\langle \bar{\rho} \rangle}{B_v^2} a_v' \right] dz.$$

In the $\beta \to 0$ limit, it becomes much simpler, since then $\langle \bar{\rho} \rangle = 0$, $(B_v'/B_v + 2a'/a) = 0$, and we get the equality

$$\int_{-1}^{1} \left[ \Lambda + 1 \right] \left( \frac{d \phi}{dz} \right)^2 dz = \omega^2 \int_{-1}^{1} \left[ \frac{\langle \rho \rangle}{B_v} \right] \phi^2 dz. \quad (38)$$

Under the above boundary conditions, the derivative $d\phi/dz$ cannot be equal to zero identically over the entire integration interval, so both integrals on the left and right sides of this equality are greater than zero. Therefore, the square of the frequency is also positive, $\omega^2 > 0$, which means stability. Roughly estimating the derivative $d\phi/dz \sim -1$ (that is, $d\phi/dz \sim -1/L$ in dimensional units), it is easy to see that $\omega$ corresponds to the frequency of Alfven oscillations, and as the gap between the plasma and the conducting wall decreases, it increases in proportion to $\sqrt{\Lambda}$.

To test our hypothesis, we performed a series of calculations for the profile of the vacuum magnetic field given by the formula (34) for three values of the index $q = \{2, 4, 8\}$ and a mirror ratio from some set $K = \{24, 20, 16, 8, 4\}$. Figures 7 illustrate some of the results. Let’s formulate the main observations:

- In accordance with our expectations, two stability zones are found for moderate values of the $\Lambda$ parame-
ter and a sufficiently large mirror ratio $K$. The lower zone $\beta < \beta_{\text{crit}}$ exists even for $\Lambda = 1$.

- Contrary to expectations, with other things being equal, the instability zone is maximum for the steepest pressure profile ($k = \infty$). Recall that in section V it is for such a profile that the instability zone had the minimum dimensions.

- Contrary to expectations, it turned out that at a fixed $\Lambda$, the stability zones expand and can merge with a decrease in the mirror ratio $K$ and/or a decrease in the steepness of the radial pressure profile (a decrease in $k$).

- If an instability zone exists between two stability zones for some combinations of the parameters $k$, $q$, and $K$, then it disappears if $\Lambda > \Lambda_{\text{crit}} > 1$. The value of $\Lambda_{\text{crit}}$ is the smaller, the smaller $k$, $K$ and the larger $q$. The largest value $\Lambda_{\text{crit}} = 1.53$ ($r_w/a = 2.18485$) in our calculations was found at $K = 24$, $k = \infty$, $q = 2$.

- For a smooth magnetic field profile with the index $q = 2$, unstable zones are found for all investigated values of the mirror ratio $K = \{24, 20, 16, 8, 4\}$. For $q = 4$ and $q = 8$ unstable zones are found for $K = 8$ and larger. For $K = 4$ the unstable zone was found only for $q = 2$ and radial profiles $k = 4$, $k = \infty$.

When trying to repeat similar calculations for the magnetic field (30) our Wolfram Mathematica® code warned of possible problems with the $p_f$ interpolation function. We got rid of such diagnostic messages by smoothing function (30) near the point $z = 0$. One variant of smoothing was achieved by replacing

$$|z| \Rightarrow \frac{\sqrt{z^2 + \delta^2} - \delta}{\sqrt{1 + \delta^2} - \delta},$$

(39)

where it was assumed that the parameter $\delta$ is sufficiently small; in particular, the values $\delta = \{0.1, 0.05, 0.01\}$ were tested. These calculations showed that the smoothing of the magnetic field profile near the median plane (an increase in $\delta$) effectively contributes to the reduction of the instability zone.

As a result of the replacement (39), the derivative $B'_v$ vanishes smoothly as $z \to 0$, but not as $z \to 1$. Therefore, a second replacement was tested

$$|z| \Rightarrow \frac{1}{2} [\text{sn}((2z - 1)K(n))n + 1],$$

(40)

which smooths the derivative $B'_v$ both for $z \to 0$ and $z \to \pm 1$. This replacement includes Jacobi elliptic function $\text{sn}$ and complete elliptic integral of the first kind $K$. The parameter $n$ can vary widely, but suitable values lie in the range $-2 < n < -5$. With $n = -4$, our calculations gave approximately the same results as those obtained for the replacement (39) with $\delta = 0.05$. From this fact we concluded that the smoothing of the field near magnetic mirrors is not as significant as near midplane.

Summing up everything said in this section, we can state the following:

- With a not too large mirror ratio, a sufficiently steep magnetic field, and a sufficiently smooth pressure profile, the hard ballooning mode $m = 1$ can be stabilized.

Figure 7. $\beta_{\text{crit}}(\Lambda)$ (lower curve) and $\beta_{\text{crit}2}(\Lambda)$ (upper curve of the same color).
at any value of beta even if the side conducting wall is dismantled.

- If, on the other hand, the side wall is close enough to the plasma, then, in combination with end conductors, this mode can be stabilized for any axial magnetic field profile and any radial pressure profile.
- Smoothing the magnetic field profile at the center of the trap is an effective way to reduce and even eliminate the instability zone.

VII. CONCLUSIONS

In the present work, we have studied the wall stabilization of the \( m = 1 \) rigid ballooning mode in an open axially symmetric trap using a conducting cylindrical wall of the chamber surrounding the plasma column. To simplify the problem, we used the isotropic plasma approximation, planning in the next article to present the results of calculations for an anisotropic plasma. In contrast to the works of predecessors, who studied only the not quite realistic case of a plasma with a radial profile in the form of a step or a ring with a sharp boundary (sharp-boundary or staircase models), we considered four variants of a diffuse pressure profile with different degrees of steepness, specified by the index \( k \), as well as many variants of the axial profile of the vacuum magnetic field given by the functions (30) and (34) with different values of the indices \( \mu, \nu, q \) at different mirror ratios \( K \).

Stabilization by a conducting wall becomes possible if the plasma beta (that is, the dimensionless ratio of the plasma pressure to the magnetic field pressure) exceeds a certain critical value \( \beta_{\text{crit}} \). Therefore, our goal was to calculate this critical value and study its dependence on the radial pressure profile, the axial profile of the magnetic field, the mirror ratio, and the magnitude of the vacuum gap between the plasma and the conducting wall. For calculations, we developed a numerical code in the Wolfram Mathematica\textsuperscript{©} system, which solved the equation (1), previously derived by Linda LoDestro, using the shooting method.

On the whole, our calculations confirmed the assertion available in the literature that for wall stabilization of an isotropic plasma, the beta parameter must exceed 80%. However, we have found examples of plasma configurations with a critical beta of 70%. Investigating the dependence of \( \beta_{\text{crit}} \) on the parameters of the problem, we found that the mirror ratio has a relatively weak effect on the value of \( \beta_{\text{crit}} \). The dependence of \( \beta_{\text{crit}} \) on the parameters \( k \) is more significant (the larger \( k \), the steeper the radial profile, the smaller \( \beta_{\text{crit}} \)), \( \mu, \nu \) and \( q \) (the larger \( \mu, \nu \) and \( q \), the shorter the magnetic mirrors, the larger \( \beta_{\text{crit}} \)), as well as on the parameter \( \Lambda \) (the larger \( \Lambda \), the smaller the gap between the plasma and the conducting wall, the smaller \( \beta_{\text{crit}} \)). We were surprised by the fact that the stability zone can formally exist even at very wide vacuum gap between conducting wall and the plasma surface (when \( \Lambda \rightarrow 1 \)), although the width of such a stability zone tends to zero, since \( \beta_{\text{crit}} \rightarrow 1 \).

We also studied the stabilization of the rigid ballooning mode by a combination of the conductive lateral wall and conductive end plates, which simulate the attachment of MHD end stabilizers to the central cell of a mirror trap. Our calculations have shown the great efficiency of this method of stabilization. We found the presence of two zones of stability. The low beta zone is due to the curvature drive being balanced by the end plate effect, and the upper beta zone is due to the curvature drive being compensated by the proximity of the conductive lateral wall. These two zones merge, making the entire range of allowable values of beta \( 0 < \beta < 1 \) stable as the mirror ratio decreases or the vacuum gap between the plasma and the side wall decreases.

The key feature of an isotropic plasma is the constancy of pressure along the magnetic field lines. In an anisotropic plasma, the pressure depends on the magnitude of the magnetic field on the field line, usually decreasing towards the magnetic mirrors. This fact noticeably complicates the calculations, since the equation (5), generally speaking, cannot be solved with respect to \( B \) as simply as in the case of an isotropic plasma. In the next article, we will present the results of calculations for a fairly realistic dependence of \( p_\perp \) on \( B \) when the equation (5) is solved without using numerical methods.

Another continuation of studies on the stabilization of the rigid ballooning mode can be the rejection of the simplifying assumption \( \Lambda = \text{const} \). The constancy of \( \Lambda \) in the equation (1) implies that the radius \( r_w = r_w(z) \) of the conducting wall depends in a complex way on the plasma radius \( a(z) \) (and hence on \( \beta \)) from the radial pressure profile. A more realistic case is \( r_w = \text{const} \). In addition, as previously shown by Li, Kesner and Lane [25], special profiling of a conductive wall can expand the stability zone to some extent by lowering the value of \( \beta_{\text{crit}} \).

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