Phenomenological aspects of flavor changing processes are considered in the context of the supergravity model. Various flavor changing neutral current processes in B and K decays are calculated in such models. For lepton flavor violating processes the $\mu^+ \to e\gamma$ branching ratio and the T odd triple vector correlation for the $\mu^+ \to e^+e^+e^-$ process are investigated in the SU(5) SUSY GUT. Possibility to find SUSY effects through these Flavor changing processes in future experiments are also discussed.

1 Introduction

Present understanding of the elementary particle physics is based on the gauge theory of quarks and leptons which is called the Standard Model (SM). Although the SM describes experimental results very well up to the present available energy scale, it is possible that new physics appears just above this energy scale. One of the most promising candidates of physics beyond the SM is unified theory based on supersymmetry (SUSY), therefore SUSY particle and SUSY Higgs boson searches are the most important targets of the present and future collider experiments.

In order to explore SUSY indirect search experiments are also important. For example, through flavor changing neutral current (FCNC) processes and CP violation in B and K meson decays it may be possible to identify new physics effects. Also processes like proton decay, lepton flavor

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violation (LFV) such as $\mu \to e\gamma$ and neutron and electron electric dipole moments (EDM) are important because these are either forbidden or strongly suppressed within the SM.

In this talk we discuss FCNC processes in B and K decays such as $b \to s\gamma$, $B^0 - \bar{B}^0$ mixing, the CP violating neutral kaon mixing parameter $\epsilon_K$ and $K \to \pi\nu\bar{\nu}$ in the supergravity model and LFV processes in the SUSY GUT. In the context of SUSY models flavor physics has important implications. Since these processes depend on the structure of the squark and slepton mass matrices we may be able to get some insight on the SUSY breaking mechanism. In fact general SUSY breaking terms tend to induce too large FCNC and LFV. In the minimal supergravity model we assume that the scalar mass terms have universal structure at the Planck scale and therefore there are no FCNC effects nor LFV from the squark and slepton sector at this scale. The physical squark and slepton masses are determined taking account of the renormalization effects from the Planck to the weak scale. This will induce sizable SUSY contributions to various FCNC processes in the B and K decays [1]. Also if there is interaction which breaks lepton flavor conservation between the Planck and the weak scales, the LFV effects can be induced in the slepton mass matrices.

In this talk after short introduction to the flavor problem in the SUSY model we discusses the results of numerical analysis for FCNC and LFV processes within the context of the supergravity model.

## 2 Flavor Problem in SUSY Models

As we mentioned in Introduction the squark and the slepton mass matrices becomes new sources of flavor mixings in the SUSY model and generic mass matrices would induce too large FCNC and LFV effects if the superpartners’ masses are in the 100 GeV region. If we assume that the SUSY contribution to the $K^0 - \bar{K}^0$ mixing is suppressed because of the cancellation among the squark contributions of different generations, the squarks with the same $SU(3) \times SU(2) \times U(1)$ quantum numbers have to be highly degenerate in masses. When the squark mixing angle is in a similar magnitude to the Cabibbo angle the requirement on degeneracy becomes as

$$\frac{\Delta m_{\tilde{q}}^2}{m_{\tilde{q}}^2} \lesssim 10^{-2} \left( \frac{m_{\tilde{q}}}{100 \text{GeV}} \right)$$  \hspace{1cm} (1)
for at least the first and second generation squarks. Similarly, the \( \mu^+ \rightarrow e^+\gamma \) process puts a strong constraint on the flavor off-diagonal terms for slepton mass matrices which is roughly given by

\[
\frac{\Delta m_{\tilde{\mu}\tilde{e}}^2}{m_{\tilde{l}}^2} \lesssim 10^{-3} \left( \frac{m_{\tilde{l}}}{100\text{GeV}} \right)^2.
\]  

(2)

In the SUSY model based on the supergravity these flavor problems can be avoided by setting SUSY breaking mass terms universal at the very high energy scale. In fact all the scalar fields are assumed to have the same SUSY breaking mass at the Planck scale in the minimal supergravity model and therefore there are no FCNC and LFV at this scale. Physical squark and slepton masses are, however, defined at the weak scale and these masses are determined through the renormalization group equations (RGE). As a result we can derive:

1. Squarks for the first and second generations remain highly degenerate so that the constraint from the \( K^0 - \bar{K}^0 \) mixing can be safely satisfied.
2. Due to the effect of large top Yukawa coupling constant the stop and the sbottom can be significantly lighter than other squarks. This will induce sizable contributions to FCNC processes such as \( b \rightarrow s\gamma \), \( b \rightarrow sl^+l^- \), \( \Delta M_B \), \( \epsilon_K \) and \( K \rightarrow \pi\nu\bar{\nu} \).
3. In the SUSY GUT the large top Yukawa coupling constant also induces the flavor mixing in the slepton sector so that LFV processes such as \( \mu^+ \rightarrow e^+\gamma \), \( \mu^+ \rightarrow e^+e^-\gamma \) and \( \mu^- - e^- \) conversion in atoms receive large SUSY contributions.

In the following we discuss the results of numerical analysis for the processes listed in (2) and (3).

3 B and K decays and the Supergravity Model

In the minimal SM various FCNC processes and CP violation in B and K decays are determined by the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The constraints on the parameters in the CKM matrix elements can be conveniently expressed in terms of the unitarity triangle. With CP violation at B factory as well as rare K decay experiments we will be able to check consistency of the unitarity triangle and at the same time search for effects of physics beyond the SM. In order to distinguish possible new physics effects
it is important to identify how various models can modify the SM predictions. Although general SUSY models can change the lengths and the angles of the unitarity triangle in variety ways, the supergravity model predicts a specific pattern of the deviation from the SM. Namely, we can show that the SUSY loop contributions to FCNC amplitudes approximately have the same dependence on the CKM elements as the SM contributions. In particular, the complex phase of the $B^0 - \bar{B}^0$ mixing amplitude does not change even if we take into account the SUSY and the charged Higgs loop contributions. In terms of the unitarity triangle this means that the angle measurements through CP asymmetry in $B$ decays determine the CKM matrix elements as in the SM case. On the other hand the length of the unitarity triangle determined from $\Delta M_B$ and $\epsilon_K$ can be modified.

We have calculated the $\Delta M_B$, $\epsilon_K$ and branching ratios of $b \to s\gamma$, $b \to s l^+ l^-$, $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ in the SUSY model based on supergravity. In the calculation we have used updated results of various SUSY search experiments at LEP2 and Tevatron as well as the next-to-leading QCD corrections in the calculation of the $b \to s\gamma$ branching ratio. In Fig. 1 and Fig. 2 we present $\Delta M_{B_d}$ and $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ in the present model normalized by the same quantities calculated in the SM as the function of the $b \to s\gamma$ branching ratio. Note that these ratios are essentially independent of the CKM parameters because, as mentioned above, the SUSY and the charged Higgs boson loop contributions have the same dependence on the CKM parameters. Although we only present the results for $\Delta M_{B_d}$ and $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$, $\epsilon_K$ and $\text{Br}(K^+ \to \pi^+ \nu \bar{\nu})$ provide the same constraints on the SUSY parameters respectively because these quantities are almost equal if normalized by the SM prediction. We have calculated the SUSY particle spectrum based on two different assumptions on the initial conditions of RGE. The minimal case corresponds to the minimal supergravity where all scalar fields have a common SUSY breaking mass at the GUT scale. In the second case shown as “all” in the figures we enlarge the SUSY parameter space by relaxing the initial conditions for the SUSY breaking parameters, namely all squarks and sleptons have a common SUSY breaking mass whereas an independent SUSY breaking parameter is assigned for Higgs fields.

From Fig. 1 and Fig. 2 we can conclude that the $\Delta M_{B_d}$ (and $\epsilon_K$) can be enhanced by up to 40% and $\text{Br}(K_L \to \pi^0 \nu \bar{\nu})$ (and $\text{Br}(K_+ \to \pi^+ \nu \bar{\nu})$) is suppressed by up to 10% for extended parameter space and the corresponding numbers for the minimal case are 20% and 3%. The ratio of two Higgs
Figure 1: $\Delta M_{B_d}$ normalized by the SM value for $\tan \beta = 2$ as a function of $b \to s\gamma$ branching ratio. The square (dot) points correspond to the minimal (enlarged) parameter space of the supergravity model. The vertical lines correspond to the CLEO 95 upper and lower bounds.

vacuum expectation value, $\tan \beta$, is 2 for these figures and the deviation from the SM turns out to be smaller for large value of $\tan \beta$.

These deviations may be evident in future when B factory experiments provide additional information on the CKM parameters. It is expected that the one of the three angles of the unitarity triangle is determined well through the $B \to J/\psi K_S$ mode. Then assuming the SM, one more physical observable can determine the CKM parameters or $(\rho, \eta)$ in the Wolfenstein parametrization. New physics effects may appear as inconsistency in the determination of these parameters from different inputs. For example, the $\rho$ and $\eta$ parameters determined from CP asymmetry of B decay in other modes, $\frac{\Delta M_{B_d}}{\Delta M_{B_s}}$ and $|V_{ub}|$ can be considerably different from those determined through $\Delta M_{B_d} \epsilon_K$ and $\text{Br}(K \to \pi\nu\bar{\nu})$ because $\Delta M_{B_d} \epsilon_K$ are enhanced and $\text{Br}(K \to \pi\nu\bar{\nu})$’s are suppressed in the present model. The pattern of these deviations from the SM will be a key to distinguish various new physics effects. We also note from Fig. 1 and Fig. 2 that, although the new results reported at ICHEP98 ($2.0 \times 10^{-4} < \text{Br}(b \to s\gamma) < 4.5 \times 10^{-4}$) does not change the situation very
Figure 2: $\text{Br}(K_L \rightarrow \pi^0\nu\bar{\nu})$ normalized by the SM value for $\tan \beta = 2$ as a function of $b \rightarrow s\gamma$ branching ratio.

much, future improvement on the $b \rightarrow s\gamma$ branching will give great impacts on constraining the size of possible deviation from the SM in FCNC processes.

4 LFV in the SU(5) SUSY GUT

Another interesting possibility to search for SUSY effects through flavor physics is to look for LFV process such as $\mu^+ \rightarrow e^+\gamma$, $\mu^+ \rightarrow e^+e^+e^-$ and $\mu^- - e^-$ conversion in atoms. The experimental upper bound on these processes quoted in PDG 98 are $\text{Br}(\mu^+ \rightarrow e^+\gamma) \leq 4.9 \times 10^{-11}$, $\text{Br}(\mu^+ \rightarrow e^+e^+e^-) \leq 1.0 \times 10^{-12}$ and $\frac{\sigma(\mu^- T_i \rightarrow e^+ T_i)}{\sigma(\mu^- T_i \rightarrow \text{capture})} \leq 4.3 \times 10^{-12}$. Recently there are considerable interests on these processes because predicted branching ratios turn out to be close to the upper bounds in the SUSY GUT.

As discussed in Section 2 no LFV is generated at the Planck scale in the context of supergravity model. In the SUSY GUT scenario, however, the LFV can be induced through renormalization effects on slepton mass matrix because the GUT interaction breaks lepton flavor conservation. In the minimal SUSY SU(5) GUT, the effect of the large top Yukawa coupling constant results in the LFV in the right-handed slepton sector. The numerical
calculation shows that there is unfortunate cancellation between different diagrams so that \( Br(\mu^+ \rightarrow e^+\gamma) \) is below \( 10^{-13} \) level for most of the parameter space\[11\]. This is in contrast with the SO(10) model where both left- and right-handed sleptons induce LFV and the predicted branching ratio is at least larger by two order of magnitudes\[12\].

We have calculated the \( Br(\mu^+ \rightarrow e^+\gamma) \) in the context of the SUSY SU(5) model and pointed out that the branching ratio can be enhanced for large value of \( \tan \beta \) once we take into account effects of higher dimensional operators to explain realistic fermion masses\[7\]. In the minimal case the Yukawa coupling is given by the superpotential \( W = (y_u)_{ij} T_i \cdot T_j \cdot H(5) + (y_d)_{ij} T_i \cdot \bar{F}_j \cdot \bar{H}(5) \) where \( T_i \) is 10 dimensional and \( \bar{F}_j \) is 5 dimensional representation of SU(5). It is well known that this superpotential alone cannot explain the lepton and quark mass ratios for the first and second generations although the \( m_b/m_\tau \) ratio is in reasonable agreement. One way to obtain realistic mass ratios are to introduce higher dimensional operators such as \( \frac{f_{ij}}{M_{Planck}} \Sigma (24) \cdot T_i \cdot \bar{F}_j \cdot \bar{H}(5) \). We investigated how inclusion of these terms changes prediction of the branching ratio. It turns out that the branching ratio is quite sensitive to the details of these higher dimensional operators. Firstly, once we include these terms the slepton mixing matrix elements \( \lambda_{\tau} \equiv V_{e31}^* V_{e32} \) which appear in the formula of the \( \mu^+ \rightarrow e^+\gamma \) amplitude is no longer related to the corresponding CKM matrix elements. More importantly, for large value of \( \tan \beta \), the left-handed slepton also induces the LFV and the predicted branching ratio becomes enhanced by two order of magnitudes as in the SO(10) case\[13\]. The destructive interference among the different diagrams also disappear. We show one example of such calculation in Fig.3 where \( Br(\mu^+ \rightarrow e^+\gamma) \) can be close to \( 10^{-11} \) level for large values of \( \tan \beta \) in the non-minimal case.

Finally, we consider T violating asymmetry in the \( \mu^+ \rightarrow e^+e^+e^- \) decay. In the polarized muon decay we can define T odd triple vector correlation \( \langle \vec{\sigma} \cdot (\vec{p}_1 \times \vec{p}_2) \rangle \) where \( \vec{\sigma} \) is muon polarization and \( \vec{p}_1 \) and \( \vec{p}_2 \) are two independent momenta of decay particles\[14\]. We have investigated possibility of sizable T odd asymmetry in the SU(5) SUSY GUT\[8\]. In order to have this asymmetry we need to introduce a CP violating phase other than the KM phase. In this model the phase can be provided by the complex phases in the SUSY breaking terms, for example, the phase in the triple scalar coupling constant (A term). Since this phase also induces electron and neutron EDMs, we have calculates the T odd asymmetry in the \( \mu^+ \rightarrow e^+e^+e^- \) taking into account
Figure 3: Dependence of the branching ratio of $\mu \rightarrow e\gamma$ on $\tan \beta$ for the right-handed selectron mass 200GeV (dashed lines) and 300GeV (solid lines). The thick lines are for the non-minimal case that $V_{\tilde{e}}$ and $V_{l}$ are the same as $V_{KM}$, and the thin lines are for the minimal case in which $V_{\tilde{e}} = V_{KM}$ and $V_{l} = 1$. In this figure we choose the bino mass 60GeV, $a_0 = 0$, the higgsino mass positive. The long-dashed line is the experimental upper bound.
EDM constraints. By numerical calculation we show that the asymmetry up to 20% is possible. The branching ratio for $\mu^+ \rightarrow e^+e^+e^-$ turned out to crucially depend on the slepton mixing element $\lambda_\tau$ which is an unknown parameter once we take into account the higher dimensional operators for the Yukawa coupling constants. For $\lambda_\tau = 10^{-2}$ we can show that the branching ratio of $10^{-14}$ is possible with 10% asymmetry which can be reached in future experiment with sensitivity of $10^{-16}$ level.

5 Conclusions

We have considered various flavor changing processes in the supersymmetric standard model based on the supergravity. Flavor changing neutral current processes in B and K decays such as $B^0 - \bar{B}^0$ mixing, $\epsilon_K$ and branching ratio of $K \rightarrow \pi \nu \bar{\nu}$ are calculated and it is shown that the deviation from the SM becomes as large as 40 % for $B^0 - \bar{B}^0$ mixing and $\epsilon_K$ but somewhat smaller for $K \rightarrow \pi \nu \bar{\nu}$ processes. We also investigated the lepton flavor violation in the SU(5) SUSY GUT. It is pointed out that the $\mu \rightarrow e \gamma$ branching ratio can be enhanced for large $\tan \beta$ if we take into account the higher dimensional operators in the Yukawa coupling constants at the GUT scale. The T odd triple vector correlation is also calculated for the $\mu^+ \rightarrow e^+e^+e^-$ process and it is shown that the asymmetry up to 20% is possible due to the CP violating phases in the supersymmetry breaking terms. Experiments on B, K and LFV processes in near future, therefore, will provide very important opportunities to investigate into the structure of the SUSY breaking sector.

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References

[1] S. Bertolini, F. Borzumati, A. Masiero and G. Ridolfi, Nucl. Phys. B353, (1991) 591.

[2] T. Goto and Y. Okada, Prog. Theor. Phys. 94, (1995) 407 and references therein.
[3] A. Ali, G. Giudice and T. Mannel, Z. Phys. C 67, (1995) 417; P. Cho, M. Misiak and D. Wyler, Phys. Rev. D 54, (1996) 3329; T. Goto, Y. Okada, Y. Shimizu and M. Tanaka, Phys. Rev. D 55, (1997) 4273; J. Hewett, J.D. Wells, Phys. Rev. D 55, (1997) 5549.

[4] T. Goto, T. Nihei and Y. Okada, Phys. Rev. D 53, (1996) 5233; Erratum, ibid. D54, (1996) 5904.

[5] R. Barbieri and L.J. Hall, Phys. Lett. B 338, (1994) 212.

[6] T. Goto, Y. Okada, and Y. Shimizu, KEK-TH-567, hep-ph/9804294, to be published in Phys. Rev. D.

[7] J. Hisano, D. Nomura, Y. Okada, Y. Shimizu and M. Tanaka, KEK-TH-575, hep-ph/9805367, to be published in Phys. Rev. D.

[8] Y. Okada, K. Okumura and Y. Shimizu, Phys. Rev. D58, (1998) 051901.

[9] CLEO Collaboration, M.S. Alam et al., Phys. Rev. Lett. 74, (1995) 2885.

[10] CLEO Collaboration, CLEO CONF 98-17.

[11] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, Phys. Lett. B 391, (1997) 341; Erratum, ibid. B 397, (1997) 357.

[12] R. Barbieri, L. Hall, and A. Strumia, Nucl. Phys. B445, (1995) 219.

[13] N. Arkani-Hamed, H. Cheng, and L. Hall, Phys. Rev. D53, (1996) 413.

[14] S.B. Treiman, F. Wilczek and A. Zee, Phys. Rev. D 16, (1977) 152.