Invited Paper

Twenty years after “The Impact of Chaos”: what have been achieved and what should be answered

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Abstract: Twenty years has passed since the book of “The Impact of Chaos on Science and Society,” edited by Prof. Celso Grebogi and Prof. James A. Yorke, has been published. This book had influenced the researches held during the FIRST program and have been influencing the current researches following them. Thus, I would like to summarize how the questions posed in the book have been answered partly by our current generation and what questions and the other emerging questions should be considered by the next generation.

Key Words: deterministic chaos, nonlinear time series analysis, delay coordinates, universality, individuality, open questions

1. Introduction: The book

Deterministic chaos, found independently by Ueda [1] and Lorenz [2] and named by Li & Yorke [3], is a phenomenon within which a simple rule generates complex behavior. Since then, the idea of deterministic chaos has spread widely into various fields including neuroscience [4], biology [5], medicine [6], atmospheric science [7–9], seismology [10], social science [11] and economics [12]. In 1990, the influential paper by Sugihara and May [13] has been published in Nature for distinguishing chaos from noise based on time series data. Thus, at the time around 1990, chaos had been recognized as a way to explain the underlying dynamics for complicated phenomena in many areas. In 1991, a conference was organized by the United Nations University and the University of Tokyo for evaluating what kinds of impacts had been brought by the idea of deterministic chaos in natural and social sciences [14]. In the conference held in Tokyo, the researchers at the time, coming from various fields, discussed what impacts the deterministic chaos has brought until that time. This conference had been summarized in the book of Ref. [14]. Therefore, the book, containing 16 chapters, can be regarded as a good summary of how deterministic chaos influenced our thoughts until 1990s.

I noticed the existence of the book of Ref. [14] from around 10 years ago, when I found the book in our laboratory. Since I was interested in applying nonlinear time series analysis, it had become a good starting point to know what had been analyzed from the viewpoints of nonlinear dynamics before in a particular field. Sometime after, I do not remember when, the book had gone from the
bookshelf and I had lost its access.

In May 2017, I have happened to encounter again this book in a bookshop in Shibuya, Tokyo when I enjoyed a holiday looking for a book to read. After starting re-reading this book, I found that I had touched many of the problems included there. Thus, I wanted to summarize what questions the researchers had 20 years ago and how these questions had been solved, unsolved, or generating further questions.

1.1 Questions posed in the book
Based on the book of Ref. [14], I interpreted and summarized, in May 2017, that the following questions seem to have been posed at the conference held in 1991 for the future generation:

1. Machine learning should be used to find an optimal solution.
2. One needs to seek the applications of the “chaos theory”.
3. Question what biology can do for mathematics.
4. It was not known well whether chaos theory is effective in analyzing an economic phenomenon.
5. The impact of chaos theory to meteorology has been limited to its theory.
6. Applications of symbolic dynamics might be effective.

In the following section, I try to provide the current tentative answers for these questions.

2. Questions partly answered during the FIRST period
The “Funding Program for World Leading Innovative R&D on Science and Technology (FIRST Program) called the Aihara Innovative Mathematical Modelling Project, initiated by the Council for Science and Technology Policy (CSTP), supported by the Japan Society for the Promotion of Science (JSPS), had started in March 2010 until the end of March 2014. This project aimed at developing science and technology for complex systems, and applying them in various fields within which some solutions are necessary soon. Hence, here I mainly follow the researches spanning out from this project and try to answer the above questions.

2.1 Machine learning should be used to find an optimal solution
This question had been tried to be solved from various people since then. Because we had seen that computers defeated human professional players in board games including chess [15] and go [16] until now, this line of researches is getting reality. For example, in the FIRST program, Dr. Kamal and his co-workers considered how cars can be driven automatically [17].

Now we need to consider what future machine learning will bring us and how we can keep human welfare in such a society. In this line of research, I predicted how industrial structure will change in the future [18].

2.2 One needs to seek the applications of the “chaos theory”
I have multiple answers in this question. During the period of the FIRST program, two abstract terms had driven the applicational researches: individuality and universality [19, 20]. We need to observe individual systems closely for finding out what fundamental laws govern the systems. At the same time, we can extract what are common among the individual systems, which is universality. For this point, I would like to raise my own line of researches on point processes. It was about 13 years ago when I wanted to analyze spike trains generated by neurons from the viewpoint of dynamical systems theory. But, at that time, there were not good methods for this purpose. A spike train is a series of events, which are firings of a neuron. A series of events is more generally called a point process. There are lots of examples of point processes in natural and social sciences including, for example, neuronal firings, economic trades, earthquake activity, social network service, lightnings, and crimes. Hence, a series of events can be found universally, while there were not good ways for
analyzing them. Thus, we have created an approach [21–23], where we prepare a time window of fixed length. Second, we slide the time window by a fixed amount of time repeatedly to decide the positions of the time window. Third, we calculate the distance between every pair for the positions of the time window using distances for point processes [21, 24–26] and further analysis such as obtaining a recurrence plot [27, 28], estimating the maximal Lyapunov exponent [29, 30], and providing short-term prediction [31]. Moreover, by using this approach, we can combine a point process with a time series sampled with a fixed time interval seamlessly [32] for investigating the directional couplings between them [33], for instance. Therefore, by extracting individual characteristics for many systems and looking for their universality, one can use the “experience” gained by analyzing a system, for analyzing another system.

Therefore, a key idea in nonlinear time series analysis has stemmed from delay coordinates [34, 35], which can be constructed by making a vector using consecutive measurements from a dynamical system. If the dimension for the delay coordinates is more than twice than the dimension for the original dynamical system, then one can, in general, establish one-to-one correspondence between a state in the original dynamical system and its reconstruction by the delay coordinates [34, 35]. This theorem by Takens has been extended in several ways including forced systems by Stark [36, 37] and point processes by Sauer [38] and Huke and Broomhead [39]. This idea of delay coordinates can be used in various contexts where one needs to predict future values for a time series as well as identify directional couplings [33, 40].

Thus, the idea of delay coordinates had influenced in a great extent in our line of researches on mathematically modelling prostate cancer [41–45]. There, one usually has a one-dimensional observation of tumor marker called prostate specific antigen (PSA). Although we have the restricted observation, we had constructed a two or three dimensional system because from the observation of PSA, we can reconstruct the whole state space information.

In our prostate cancer work, another important influence from the “chaos” theory came from the “chaos” control including the OGY control [46] and the Pyragus control [47]. These examples of the chaos control had let us know that we can target a periodic orbit for improving the health condition under intermittent androgen suppression of prostate cancer [41]. Although we found later that we can target a periodic orbit for only a limited type of patients [48], the notion of “chaos” control gave the great influence for justifying the optimal treatment schedule for the intermittent androgen suppression.

2.3 Question what biology can do for mathematics

I can provide two stories here based on our researches. The first story stemmed from the analysis of a spike train. For analyzing a spike train, we found that combining a distance for a spike train [24] with a recurrence plot [27, 28] could provide us good standing points. Thus, since 2006, we started researches for defining distances for point processes [21, 25, 26, 49] as well as researches on recurrence plots [33, 50, 51]. A part of story has been given above. There is a side story. To justify the analysis using a recurrence plot, we showed that a recurrence plot can contain dynamical information sufficient enough for recovering the original time series [52, 53]. This technique was primarily used for reconstructing the driving forces. But, recently, the technique has gone back to biology again [54]: reconstruction of three-dimensional structure of chromosomes in a single cell from a molecular biological dataset of Hi-C data [55], where one records which parts of chromosomes are spatially neighbors for each other. Therefore, biology had influenced the formation of mathematics, which has given the feedback to biology already!

The second story is on the prostate cancer work. Although we found that the intermittent androgen suppression cannot always construct a periodic orbit as a controlled state [48], there is another story of partly successful control, within which we can delay the relapse, or the growth of PSA [56–58]. This is a method of control for an unstable system. In the field of medicine, delaying the relapse may be sufficient for improving the quality of life.
2.4 It was not known well whether chaos theory is effective in analyzing an economic phenomenon

By using a distance for a marked point process, we have shown that there is a time period when a foreign exchange market behaves as deterministic chaos [22]. Although the underlying dynamics of a foreign exchange market might evolve as time passes, I believe that the fundamental nature could be valid in the future. Thus, analyzing an economic phenomenon is becoming a matter of hedge funds rather than that of econophysics.

2.5 The impact of chaos theory to meteorology has been limited to its theory

The debate on whether or not there exists a climate or weather attractor has been discussed in 1980s and early 1990s [7–9]. Recently, we provided the results with more depths using surrogate data analysis combined with a statistic tuned for high-dimensional dynamics and short-term prediction that the Arctic Oscillation Index is consistent with deterministic chaos [59]. Thus, my current view is that climates and weathers are high-dimensional nonlinear systems, for which we might get benefits by using some methods developed in dynamical systems theory.

In addition, the chaos theory sees an opportunity for the recent meteorology, especially the applications for predicting renewable energy outputs [60–62]. For short-term predictions up to 2 hours, now it is believed that time series prediction may do better than numerical weather prediction [63]. In the future, these predictions will be combined soon to provide the better prediction in its quality and depth.

2.6 Applications of symbolic dynamics might be effective

At the stage when the conference was held in 1991, the best approach for using symbolic dynamics was with a generating partition [64, 65], which provides us one-to-one correspondence between a time series and its symbolic sequence. Although newer methods for estimating generating partitions have been proposed [66, 67] in 2000s, the practice of symbolic dynamics has been changed dramatically with permutations or ordinal patterns [68]. Especially, permutation entropy has enabled us to estimate the metric entropy without using a generating partition [68–70]. Recently, a new method has been proposed to improve the estimate by the permutation entropy [71]. In addition, permutations help us to distinguish deterministic time series from stochastic time series [72] as well as identify directional couplings based on multivariate time series [40, 73, 74].

We should consider what a non-generating partition can do [75]. Even if a partition is not generating, symbolic dynamics can provide more rigorous and faster calculations.

3. Discussions

We have already solved or is heading for the direction to solve almost all the questions posed in 20 years ago in Ref. [14]. Thus, I would like to raise questions for the future generation.

We should develop methods for analyzing stochastic nonlinear systems. We can classify dynamical systems into four classes using two axes (Fig. 1): either linear or nonlinear, and either deterministic or stochastic. As an example of a linear deterministic system, there is a model of electric circuits (Fig. 1A). In a class of linear stochastic systems (Fig. 1B), there exists the auto-regressive linear model [76]. Nonlinear deterministic systems (Fig. 1C) include deterministic chaos discussed above especially in Sections 2.2, 2.4 and 2.5. But, nonlinear stochastic systems (Fig. 1D) have not been considered seriously. In the past, researchers were attempting to distinguish deterministic nonlinear systems from stochastic linear systems [31]. In the future, we need to consider a stochastic nonlinear system [77–80]. I am feeling that a class of nonlinear stochastic systems might include dynamics in biology, medicine, geoscience (including weather, climate and earthquakes), and economies, although a rigorous method is necessary for identifying such nonlinear stochasticity.

One of the useful approaches for this purpose is that of symbolic dynamics (see Section 2.6). In the approach of symbolic dynamics, we can naturally use the idea in information theory in the practice of nonlinear time series analysis [81–83]. Furthermore, by dividing phase space by boxes, we can
approximate the underlying dynamics by a Markov model with which we can run some rigorous calculations [84].

Another promising direction is the analysis of nonstationary time series [85–87]. A key idea is probably to apply delay coordinates for forced systems [36, 37]. Although some applications have been proposed until now [33, 40, 52, 88, 89], we should get into this direction further to understand interactions among accessible and/or hidden components in the real world deeply. Permutations or ordinal patterns are easy to use and would be promising. I would like to pay attention how the researches of Refs. [74, 90–92] will develop in the future.

An important avenue is on the analysis of point processes. As I have raised previously, there are a lot of examples of point processes. But, methods for analyzing point processes are still limited. Probably, developing methods for analyzing point processes would generate other applications such as compression and communications, which are largely linked with practically engineering [93], and theoretically, biology especially neuroscience [94, 95] (see Section 2.3). Thus, I would like to seek a platform, namely a conference where researchers analyzing point processes in different fields can get together to exchange their ideas. The tentative name for the conference is “Current Opinions On Point Processes (COOPP)” . This is my homework to be done in the future.

To promote the researches in point processes, we may seek relationships between distances and kernels [49]. Distances show how different two corresponding times are, while kernels show how similar two corresponding instances are. Thus, generally, distances are related to kernels inverse-proportionally. Therefore, by introducing the techniques of kernel methods for analyzing of point processes, we may be able to prepare tools sufficient for understanding and/or modelling the underlying dynamics behind point processes. Hence, this direction of research will also enrich the researches in the machine learning community (see Section 2.1).

In addition, we need to ensure theoretically, by proofs, that methods for nonlinear dynamics work appropriately. For this sake, ordinal patterns [69], symbolic dynamics [67] and recurrence plots [53, 96–98] seem to have some advantages judging from the recent literature. For example, to identify directional couplings and slow driving forces from point processes, we need to combine two extensions.
of Takens’ theorems: one by Stark [36] and the other by Huke and Broomhead [39].

The other problem that should be considered intensively in the near future is how we can infer and predict using imperfect models [99–101] or partial models [102]. Takens’ embedding theorem [34] assumes that we only have some pieces of information for the underlying dynamics that are continuous and differentiable on an \( m \)-dimensional manifold. In the real world, we may have some extra pieces of information available for the underlying systems. One of the promising approaches would be to use the Bayesian theory [103]. But, the other approaches are definitely necessary.

We also have to seek other applications using and/or from the ideas of “chaos theory”. Our recent good example is the recurrence plot of recurrence plots [104], which coarse-grains a time series and roughly grasps “what’s going on” in a large time scale. In a similar way, we need to prepare other applications so that we can show various properties for the underlying dynamics.

Moreover, we should consider how to combine the “chaos theory” or more generally the dynamical systems theory, with the other fields including machine learning, statistics, signal processing, control theory, and information theory. Especially, combining multiple predictions [105] will provide new scopes for machine learning, statistics, and signal processing. Although we have used an idea of stabilizing a piecewise linear system and obtaining an optimal treatment schedule for prostate cancer, we need to extend this idea for a piecewise smooth (nonlinear) system [106]. In addition, but not least, applications of point processes will provide new problems for information theory. Thus, we need to consider real-world problems by removing the borders of the existing fields so that we can improve the quality of life, in general.

Otherwise, we should remind what we learned during the FIRST program: we should observe each system carefully and look for some common properties.

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