INTRODUCTION

Lattice QCD is a powerful tool to understand the strong interaction of hadrons from the first principles of QCD for quarks and gluons with the aid of numerical simulations. Physical quantities calculated with the method range from the spectrum of light hadrons to electroweak matrix elements. Systematic errors such as finite lattice volume and spacing, and the use of the quenched approximation are gradually being reduced thanks to development of computer power as well as simulation algorithms. Among these systematics, a current main concern is the effect of light dynamical quarks.

As a member of CP-PACS and JLQCD collaborations in Japan, I have been working on large scale lattice QCD simulations for many years. In this talk I report recent results of our collaborations in lattice QCD simulations with light dynamical quarks.

Let me first explain the necessity of dynamical quark effects, by presenting the quenched light hadron spectrum in the left panel of Fig. 1. These results have been obtained by the CP-PACS collaboration after taking the continuum limit. In this calculation the experimental $\rho$ and $\pi$ meson masses are used to fix the lattice spacing $a$ and the light quark mass $m_l$, where up and down quark masses are assumed to be equal ($m_u = m_d = m_l$). For the strange quark mass, two choices are compared, one employing the $K$ meson mass (filled symbols ; $K$-input) and other with the $\phi$ meson mass (open symbols; $\phi$-input). Experimental values are given by horizontal lines. This figure shows an overall agreement of the light hadron spectrum in the quenched lattice QCD at a 5–10% level. However, it is also clear that there are systematic deviations between the quenched spectrum and experiments beyond statistical errors of 2–3%. In particular, the hyperfine splitting between the $\phi$, $K^*$ meson masses and the $K$ meson mass is smaller than the experimental one. This indicates that full QCD simulations are indeed necessary for more accurate results. We then performed a large scale 2 flavor full QCD simulations. The right panel of Fig. 1 shows the $\phi$ meson mass from the $K$ input as a function of the lattice spacing $a$. As can be seen from the figure, the
deviation from an experimental value in quenched QCD is much reduced in the $N_f = 2$ full QCD after the continuum extrapolation. The effect of dynamical sea quarks is really important for reproducing the correct spectrum.

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N_f = 2 + 1 \text{ FULL QCD SIMULATIONS}
\]

The success of 2 flavor full QCD simulations motivates us to perform more accurate calculations, $N_f = 2 + 1$ flavor full QCD simulations, in order to remove the systematic error associated with the absence of the dynamical strange quark.

\[
N_f = 2 + 1 \text{ full QCD project}
\]

We have started 2+1 full QCD simulations as a joint project of CP-PACS and JLQCD collaborations [3, 4, 5], employing the RG improved gauge action and the Wilson-type quark action. In order to reduce the effect of the explicit chiral symmetry violation in the Wilson quark, we introduce the non-perturbative $O(a)$ improvement. A necessary parameter $c_{SW}$ has already been determined by our collaborations [6], prior to large scale simulations. We employ the standard Hybrid Monte-Carlo (HMC) algorithm to simulate degenerate up and down quarks, while polynomial HMC algorithm for the dynamical strange quark. The latter algorithm has been developed by us to simulate odd number of dynamical quarks [7].

**Simulations and analyses**

We take 3 values of lattice spacing, $a \simeq 0.07, 0.10, 0.12 \text{ fm}$, equally spaced in $a^2$, in order to perform the continuum extrapolation, with the $(2 \text{ fm})^3$ spatial volume. We accumulate more than 5,000 HMC trajectories at each lattice spacing. We take 5 values
of the degenerate up and down quark mass ranging between \( m_{PS}/m_V \simeq 0.6 \) and 0.78, where \( m_{PS} \) and \( m_V \) are the pseudo-scalar meson mass and the vector meson mass, respectively. For the strange quark mass, we take 2 values around \( m_{PS}/m_V \simeq 0.7 \). Note that our light quark mass is much heavier than the experimental value, \( m_{PS}/m_V = 0.18 \), while the strange quark mass is close to the value, \( m_{PS}/m_V \simeq 0.68 \), estimated by the 1-loop chiral perturbation theory.

For the chiral extrapolation of meson masses, we have used polynomial functions in quark masses, including up to quadratic terms with an interchange symmetry among 3 sea quarks and that among 2 valence quarks. Chiral fits are made for light-light(LL), light-strange(LS) and strange-strange(SS) mesons simultaneously. Polynomial functions describe quark mass dependences of data very well[5].

In order to estimate the systematics of the polynomial chiral extrapolation, we also employ another fit function, obtained by the Wilson chiral perturbation theory (WChPT)[9, 10, 11], which contains both chiral loop and finite lattice spacing effects. No difference between the WChPT fit and the polynomial fit is observed for PS meson masses, while a slight difference is detected in the small quark mass region for V meson masses[5]. This analysis suggests that the effect of chiral log is small in the region of the light quark mass employed in our simulations. Note however that our light quark mass may be too heavy to apply the NLO formula. Further analysis including data with lighter quark mass will be required for the definite conclusion on the effect of the chiral log to meson masses.

### Continuum extrapolation

Now let me consider the continuum extrapolation of some quantities.
Light meson spectra

Left two panels of Fig[2] show vector (φ and K*) meson masses as the function of $a^2$ with the strange quark mass from the K input, while right panels are K* and K meson masses with the strange quark mass from the φ input. Circles represent 2+1 flavor full QCD results, while 2 flavor and quenched results are given for comparison by squares and triangles, respectively. 2+1 flavor results are consistent with experimental values within 2% statistical errors after the continuum extrapolation. This agreement is encouraging. It is difficult, however, to pin down the effect of the dynamical strange quark on meson spectra, since 2% errors are much larger than those of 2 flavor results.

Quark mass

Quark masses are determined for the $\overline{\text{MS}}$ scheme at the scale $\mu = 2$ GeV. Lattice results are translated to the $\overline{\text{MS}}$ scheme at $\mu = a^{-1}$ using tadpole-improved one-loop matching factor [8], and then evolved to $\mu = 2$ GeV using the four-loop RG-equation. Quark mass results are shown in Fig.3 As already observed in $N_f = 2$ QCD [2], values of the strange quark mass determined for either the K- or the φ-inputs, while different at finite lattice spacings, extrapolate to a common value in the continuum limit. Therefore the quark masses in the continuum limit is estimated from a combined fit to data with the K- and the φ-inputs. We finally obtain[5]

\[
\begin{align*}
    m_{\overline{\text{MS}}}^{\text{ud}}(\mu = 2 \text{ GeV}) &= 3.50(14)(^{+26}_{-15}) \text{ MeV}, \\
    m_{\overline{\text{MS}}}^{\text{s}}(\mu = 2 \text{ GeV}) &= 91.8(3.9)(^{+6.8}_{-4.1}) \text{ MeV}.
\end{align*}
\]  

(1)

Dynamical up and down quarks reduce significantly the quark masses. The effect of strange quark is less dramatic, and we do not see deviations from the $N_f = 2$ results, $m_{\text{ud}} = 3.44^{+0.14}_{-0.22}$ Mev, $m_{\text{s}} = 88^{+6}_{-6}$ Mev (K input)[2] beyond statistical errors.
Pseudo-scalar meson decay constant

PS meson decay constants are estimated using matching factor determined by tadpole-improved one-loop perturbation theory. The results with the $K$-input are

$$f_\pi = 140.7(9.3) \text{ MeV}, \quad f_K = 160.9(9.1) \text{ MeV}, \quad f_K/f_\pi = 1.142(17). \quad (2)$$

We recall that in our $N_f = 2$ QCD calculation, the magnitude of scaling violation was so large that we were not able to estimate values in the continuum limit [2]. The situation is much better in the present case and $f_\pi$ and $f_K$ turn out to be consistent with experiment. The errors are large, however. Furthermore, the ratio $f_K/f_\pi$ differs significantly from the experimental value, $1.223(12)$. A long chiral extrapolation is a possible cause of the discrepancy.

Flavor singlet mesons

In this subsection, I briefly present a preliminary result on the flavor singlet meson mass[12]. In Fig. 4 we present PS meson masses including $\eta'$ and $\eta$ as a function of the light quark mass ($1/K_{ud}$ where $K$ is the hopping parameter) for 2 values of the strange quark mass ($1/K_s$) at $a \simeq 0.12 \text{ fm}$. Small solid circles denote LL and LS meson results while small open circles correspond to SS meson results without disconnected diagrams. Once we correctly include contributions from disconnected diagrams, the SS state is mixed with the flavor singlet state, so that the mass of the SS state becomes a little lighter, as shown by large open circles in the figure. The singlet $\eta'$, denoted by large solid circles in the figure, appears much heavier than other PS mesons.
By the polynomial chiral extrapolation to the physical point, we obtain $m_\eta = 545(16)$ MeV, consistent with the experimental value (550 MeV), while $m_{\eta'} = 871(46)$ MeV, which is much larger than octet PS meson masses and is smaller than the experimental value (960 MeV) only by 100 MeV (2 $\sigma$). The U(1) problem seems to be solved. More studies at two other lattice spacings, however, will be required for the final conclusion.

**SUMMARY AND OUTLOOK**

**Summary**

CP-PACS and JLQCD collaborations has performed the 2+1 full QCD project, using the RG improved gauge action and non-perturbatively $O(a)$ improved clover quark action. Configuration generations have already been completed and the analyses are now being finalized. Light meson masses agree with experimental values after the continuum extrapolation assuming that the $a^2$ contribution dominates the scaling violation. Values of the up-down quark mass and the strange quark mass are determined in the continuum limit. We observe that the dynamical strange quark effect is much small than that of the up-down quarks.

Currently there are several on-going analyses, which include the non-perturbative determination of renormalization factors to remove an ambiguity of 1-loop estimates, the flavor singlet meson mass as presented, and heavy quark quantities using a relativistic heavy quark action.

**PACS-CS project**

We have just started a new project, PACS-CS project, which uses a new cluster PACS-CS. The PACS-CS starts operating this July with the peak speed of 14.3 Tflops[13]. In order to remove the most serious ambiguity due to the chiral extrapolation, the PACS-CS collaboration wishes to go down to lighter up-down quark masses with the clover fermion, employing the domain decomposed HMC algorithm proposed by Lüscher[14]. Our preliminary test study indicates that we can go down to as small as 15 MeV quark mass[15].

**Nucleon force**

Last but not least, I briefly introduce an interesting application of lattice QCD technique to the nucleon force (the potential between two nucleons). Recently we try to extract this $NN$ potential on the lattice from the Bethe-Salpeter wave function $\phi$ and the effective Schrödinger equation as

$$V(r) = E + \frac{1}{m_N} \nabla^2 \phi(r) \phi(r).$$
Fig. 5 gives the $N\,N$ potential for the $(J^P,I) = (0^+,1)$ channel, obtained in quenched QCD at $a \simeq 0.14$ fm and $m_\pi \simeq 880$ MeV\textsuperscript{[16]}. We clearly observe the strong repulsive force at the short distances. Although errors are still too large to see an expected attractive force at the intermediate distance, this method seems promising. Currently we investigate systematics of this method.

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