Optimization method of mechanical and electrical product quality consistency based on renewable cost contribution rate

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Abstract. Uncertainty in the manufacturing process is the main factor affecting the quality consistency and application reliability of electromechanical products. How to improve the quality consistency of their batches through tolerance design technology has been the focus of domestic and foreign electromechanical product manufacturers. Although the traditional tolerance design method can improve the consistency of batch products, from the perspective of engineering applications, this optimization method may not be optimal, and it will cause unnecessary cost waste. In order to improve the quality consistency to the optimization goal and control the cost increase to the minimum, this paper proposes a tolerance design method based on the renewable cost contribution rate, which comprehensively considers the improvement of consistency when the tolerance values decrease and costs increase in a non-linear form, and tolerance optimization strategies are formulated. Finally, by applying it to the quality optimization of electromechanical products, the correctness of the proposed method is verified.

1. Introduction

In the research and manufacturing process of equipment and systems, there are unavoidable effects and contradictions between the quality fluctuations and manufacturing costs of batch products [1]. The quality loss occurred when the quality characteristic value is out the customer’s tolerance limits [2]. Meanwhile, the manufacturing cost is necessary to ensure that the manufacturing capacity is in the producer’s tolerance limits [3]. It is obvious that the smaller the quality loss is, the higher the manufacturing costs are. If the dispersion of the batch product is too large, the loss of substandard products will lead to waste of manufacturing costs, and if the tolerance is lowered to improve product consistency, it will also lead to increased manufacturing costs [4,5]. Therefore, under the current conditions of product quality and manufacturing level, how to improve consistency with the minimum cost increase and judge whether it is necessary to optimize the quality of products through the improvement of manufacturing capability, has important application value.

Obviously, the smaller the quality loss, the higher the manufacturing cost [6]. Therefore, Taguchi [7] defined the total loss as the sum of quality loss and manufacturing cost, and pointed out that for the optimization scheme [8], the reduction in quality loss should be greater than the increase in manufacturing costs. Based on this, a robust tolerance design (RTD) method is proposed to reduce the total loss [9]. Although it can improve robustness, its optimization results may not be the
optimal solution [10]. In previous research on the robust design of electromechanical products, quality fluctuations were optimized only without considering cost changes. However, as tolerances change, the contribution factor must change accordingly. To express the quality loss during service and extend the connotations of Taguchi quality loss, Terán et al. [11] proposed a present worth model by transforming the quality loss during a certain service life to the present worth on the time node before the product leaves the factory. On this basis, Zhao et al. [12] established the present worth model based on service life distribution and defined product quality loss as the present worth of a loss caused by product obsolescence to reflect an actual loss that a product imparts on society after the product is put into service. Liu et al.[13] proposed several functions, such as exponential function, reciprocal power function, and polynomial function, to estimate the manufacturing cost for statistical tolerance allocation. Natarajan et al.proposed a two-objective (quality loss cost and total cost) tolerance allocation for an interchangeable assembly under diverse manufacturing environment in the shaft–hole production [14]. Khodaygan introduced an interactive method for the optimal design of process tolerances [15]. According to this method, the optimal process tolerances can be designed by concurrently optimizing the process capability function and the overall manufacturing cost. Then the service quality loss model is used to reduce the total loss by prolonging the product service life [17]. Khodaygan [16] proposed a tolerance synthesis method combined Shannon’s entropy-based TOPSIS algorithm to find the best asymmetric tolerances from Pareto solutions, and optimized the tolerance synthesis of mechanical assemblies with degrading components. Although the above references consider both quality loss and manufacturing cost to reduce the total loss, the important practical application value of quality fluctuation reaching the acceptable range is not paid special attention in the RTD process. Only by optimizing the total loss on the premise of ensuring that the quality fluctuation is reduced to the acceptable range can it really improve the product quality and avoid the waste of cost.

In the existing literatures, only the objective function of total loss is constructed during the entire tolerance optimization process, and the contribution coefficient calculated from the original tolerance is used as the weight loss parameter weight. When the tolerance changes in the optimization process, the objective function and manufacturing cost will be constant, which makes the optimization process difficult to be convincing. Only by recalculating the contribution coefficient and updating the objective function at each step of tolerance change can the correctness and rationality of the next optimization process be guaranteed. Based on the above ideas, a method for progressive optimization of RTD based on renewable cost contribution rate is proposed. The total loss objective function and the tolerance step control model can be updated based on manufacturing costs and total losses to determine the tolerance change for each iteration. Therefore, when the quality fluctuation is reduced to the optimization target, the increase in manufacturing cost can be minimized and the total loss can be minimized. The electromagnetic relay RTD is used as an example to verify the effectiveness of the method.

2. Mathematical model theory for quality optimization
Tolerance design is a process of balancing quality loss and manufacturing cost by studying the relationship between tolerance range and quality loss and manufacturing cost. Quality loss due to quality fluctuations will decrease as tolerances decrease; on the contrary, manufacturing costs will increase as tolerances decrease, and the increase in manufacturing costs will vary depending on the amount of tolerance reduction. It is easy to reduce only the quality fluctuation, and there will be many tolerance designs. However, it is a complex optimization problem to reduce the quality fluctuation to the optimization target with the minimum increase of manufacturing cost. Therefore, the best tolerance allocation scheme can be determined only by considering the internal relations between tolerance and manufacturing cost, tolerance and quality fluctuation and its loss in the process of tolerance design.
2.1. Manufacturing cost model

In order to establish the mathematical expression of the relationship between cost and tolerance, it is necessary to establish a unified model based on the actual process data. Wu et al. [18] presented five functions namely reciprocal, reciprocal squared, Sutherland-Roth, exponential and Michael-Siddall. This paper uses the reciprocal function to establish the functional relationship between manufacturing cost $C_i$ and tolerance $t_i$ as:

$$C_i(t_i) = \alpha_{i1} + \frac{\alpha_{i2}}{t_i} \times e^{\alpha_{i3}}$$

(1)

Where $\alpha_{i1} - \alpha_{i3}$ are constant coefficients, determined by converting the actual manufacturing cost corresponding to the tolerance into a single product. If a product has $n$ design parameters, the total manufacturing cost of the product can be expressed as:

$$C_M = \sum_{i=1}^{n} C_i(t_i)$$

(2)

2.2. Quality loss model

Taguchi said that even if the product is qualified, if it does not reach the optimal design value, it will cause a certain quality loss, and this loss is caused by changes in parameter tolerances. Park and Antony [16] summarized the characteristics of the quality loss function, constructed a mathematical model between the sample variance (standard deviation) and the average quality loss, and realized a quantitative assessment of the quality loss of the batch product. Among them, the expression of the single product quality loss function is as follows.

$$L(y) = k(y - m)^2$$

(3)

Where $y$ is the output response, $m$ is the optimal design value, and $k$ is the quality loss coefficient, which can be calculated from the pass threshold $\pm \Delta$ and the economic loss of the unqualified product $A$, expressed as $A = k\Delta^2$, and then the average quality loss can be expressed as:

$$L_Q = \frac{1}{N} \sum_{i=1}^{N} L(y_i) = k \frac{1}{N} \sum_{i=1}^{N} (y_i - m)^2 = k\sigma^2(y) = \frac{A}{\Delta^2}\sigma^2(y)$$

(4)

Where $\sigma(y)$ is the standard deviation of the output response distribution. Based on the $3\sigma$ principle, let $\Delta_{obj} = 3\sigma_{obj}$, where $\sigma_{obj}$ is the standard deviation of the consistent optimization goal. The average quality loss model can then be expressed as:

$$L_Q = \frac{A}{9}\left(\frac{\sigma(y)}{\sigma_{obj}}\right)^2 = \frac{A}{9}\delta^2$$

(5)

Where $\delta = \sigma(y)/\sigma_{obj}$ is the magnitude of the current quality fluctuation beyond the optimization objective.

2.3. Total loss model and cost contribution rate model

Taguchi defines total loss as the sum of quality loss and manufacturing cost, so total loss can be expressed as:

$$T_L = C_M + L = \sum_{i=1}^{n} C_i(t_i) + A \left(\frac{\sigma(y)}{\sigma_{obj}}\right)^2$$

(6)
As tolerance values decrease, cost increases must be considered at the same time. Therefore, if the cost is sensitive to tolerance changes, the tolerance reduction should be reasonably reduced during the optimization process. In this paper, the cost contribution rate \( \lambda_i \) is proposed with reference to the form of contribution rate \( \rho_i \) in the traditional tolerance design method to quantify the impact of tolerance \( t_i \) on the total manufacturing cost \( C_M \). Therefore, it can be defined as the percentage of the cost increase \( \Delta C_i \) caused by the reduction of tolerance \( \Delta t_i \) in the total cost increase \( \Delta C_M \) caused by all the reduction of tolerance, which can be expressed as:

\[
\lambda_i(t_i) = \frac{\Delta C_i}{\Delta C_M} = \frac{C_i(t_i + \Delta t_i) - C_i(t_i)}{\sum_{i=1}^{n}[C_i(t_i + \Delta t_i) - C_i(t_i)]}
\]  

(7)

2.4. Tolerance iteration model

In the process of optimizing quality, the amount of change in tolerance \( \Delta t_i \) directly determines the speed and accuracy of the optimization. If the tolerance change is too small, it will reduce the optimization efficiency; if it is too large, it will reduce the optimization accuracy. Therefore, it is necessary to establish an adaptive variable step tolerance control model. In this way, the total loss objective function can be updated during each iteration to obtain the best tolerance design.

In the traditional RTD, the contribution coefficient of each controllable factor reflects the degree of its significant influence on the output characteristics. Based on this, the contribution coefficient \( \rho_i \) in each iteration is set as the weight of the tolerance step control function. By using the most commonly used three-level orthogonal design (where level 2 is the center value \( x_i \) of the input variable, and level 1 and level 3 are the lower limits \( x_i - t \) and upper limits \( x_i + t \)), the contribution coefficient \( \rho_i \) of each parameter is calculated as:

\[
\rho_i = \rho_{ii} + \rho_{iq}
\]

(8)

Where \( \rho_{ii} \) and \( \rho_{iq} \) are the primary contribution rate and secondary contribution rate of the contribution factor \( x_i \).

\[
\rho_{ii} = \frac{S_{ii} - V_i}{S_T}
\]

(9)

\[
\rho_{iq} = \frac{S_{iq} - V_i}{S_T}
\]

(10)

Where \( S_{ii} \) and \( S_{iq} \) are the sum of the squares of the first fluctuation and the sum of the squares of the second fluctuation of the factor \( x_i \), and \( V_i \) are the error variances. \( S_T \) is the sum of the squared total deviations of the output characteristics \( y_j \).

In the initial stage of tolerance optimization, the tolerance step should be automatically set to a larger value to speed up the convergence of the optimization. In the final stage of optimization, the tolerance step size should be automatically set to a smaller value to achieve higher optimization accuracy.

Therefore, in the control function, based on the difference between the quality fluctuation range and the target range \( \sigma_{obj} \), an acceleration coefficient \( \beta \) is introduced to control the tolerance change step, that is:
\[ \beta = \frac{\delta^3}{1000} \]  \hspace{1cm} (11)

Then based on the traditional tolerance design method and the variable cost contribution rate proposed in this paper, a model can be derived that changes the tolerance with the number of iterations as:

\[ \Delta t_{i+1} = \frac{\rho_i}{\lambda_i(t_i)} \beta t_i \]  \hspace{1cm} (12)

3. Tolerance optimization process
The optimization process of tolerance allocation is shown in Figure 1. First, design parameters and their tolerances are set as initial conditions, and acceptability thresholds are used as consistency optimization goals. The experimental plan will then be determined through orthogonal experimental design. After the output characteristics have been calculated, the contribution factor \(\rho_i\) and cost contribution rate \(\lambda_i\) of each design parameter will be determined through contribution analysis. New tolerances in the iteration will then be obtained according to the tolerance steps. If the quality fluctuation range \(\sigma_{j+1}\) of the current tolerance is still greater than the target range \(\sigma_{\text{obj}}\), the solution will be set to the known conditions of the next iteration, and the cost contribution rate \(\lambda_i\) of the design parameters will be updated, and then the above process is performed to reduce the parameter tolerance until The quality fluctuation range of the output characteristics meets the requirements.

4. Case study
The balanced force electromagnetic relay is a typical electromagnetic actuator and widely used in load control systems of aerospace and military equipment. Its holding torque, which is a very important output performance for vibration resistance and product reliability, is chose as the optimization target of RTD to verify the validity and correctness of the method proposed in this paper.
The electromagnetic system of balanced force relay is described in Figure 2, consisting of the coil, iron core, yokes, armatures, magnet and shaft. The armature assembly is the rotation mechanism to implement the working process of pickup and release. When the armature assembly is at pickup position as Figure 2 shows, the holding torque is generated by the interaction of permanent magnet and coil. And the holding torque is affected by the design parameters of all parts in the electromagnetic system.

Therefore, 11 input variables were selected for robust design optimization. As shown in Table 1, their tolerance values and lower limits are determined based on the actual manufacturing process.

![Figure 2. Structure of electromagnetic part at pickup position.](image)

| Input variables                      | Central values and tolerances | Lower limits |
|--------------------------------------|------------------------------|-------------|
| A - Armature assembly position (mm)  | 9±0.4                        | ±0.05       |
| B - Yoke assembly position (mm)      | 9±0.3                        | ±0.05       |
| C - Yoke polar diameter (mm)         | 70±0.5                       | ±0.05       |
| D - Core diameter (mm)               | 22±0.3                       | ±0.05       |
| E - Armature polar diameter (mm)     | 18±0.5                       | ±0.05       |
| F - Armature outer diameter (mm)     | 20±0.5                       | ±0.05       |
| G - Iron core outer diameter (mm)    | 25±0.4                       | ±0.05       |
| H - Contact pressure/2* (N)          | 6±0.3                        | ±0.05       |
| I - Reaction spring preload/2* (N)   | 5±0.4                        | ±0.05       |
| J - Coil resistance/10* (Ω)          | 5±0.4                        | ±0.05       |
| K - Rebound spring preload/2* (N)    | 5±0.4                        | ±0.05       |

*Note: The values are divided to make the tolerances in the same order of magnitude to other variables.

4.1. Establishment of cost model

The input variable manufacturing cost model is established by the unified model of equation (1), and its coefficients and initial manufacturing costs are shown in Table 2. As the design parameter tolerance decreases, the manufacturing cost increases non-linearly, as shown in Figure 3.
Table 2. Manufacturing cost model coefficients.

| Coefficients | $a_1$ | $a_2$ | $a_3$ | $C_i$(CNY) |
|--------------|-------|-------|-------|------------|
| A            | -51.067 | 44.6 | 0.083 | 70.083     |
| B            | -31.5  | 28.567| 0.035 | 67.114     |
| C            | -24.1  | 14.867| 0.019 | 6.204      |
| D            | -31.033| 17.933| 0.114 | 35.963     |
| E            | -35.533| 24.6  | 0.094 | 18.516     |
| F            | -35.7  | 19.933| 0.021 | 5.013      |
| G            | -27.333| 26.07 | 0.017 | 5.809      |
| H            | -22.767| 16.367| 0.011 | 32.392     |
| I            | -5.4333| 16.233| 0.069 | 38.049     |
| J            | -5.7333| 17.767| 0.078 | 42.287     |
| K            | -10.5  | 17.767| 0.050 | 36.194     |

Figure 3. Manufacturing cost model with input parameters.

4.2. Calculation of fluctuation Contribution

In the orthogonal test design, the most commonly used three-level orthogonal test design is generally selected, and the fluctuation contribution coefficient $\rho_i$ of each design parameter is calculated by formula (8-10).

Table 3. Results of contribution rates in the first iteration.

| Input variables | $\rho_i$ (%) | $\lambda_i$ (%) | $\Delta t_i$ |
|-----------------|-------------|----------------|-------------|
| A               | 9.91        | 7.15           | 0.1398      |
| B               | 10.11       | 10.63          | 0.1112      |
| C               | 2.16        | 1.45           | 0.2665      |
| D               | 0           | 5.37           | 0.05        |
| E               | 7.12        | 4.59           | 0.1121      |
| F               | 2.12        | 1.5            | 0.3048      |
| G               | 14.57       | 22.8           | 0.0502      |
| H               | 13.06       | 18.7           | 0.0723      |
| I               | 0           | 9.23           | 0.05        |
| J               | 0           | 8.73           | 0.05        |
| K               | 9.9         | 9.84           | 0.696       |
4.3. Tolerance optimization process

During the tolerance optimization, a comparative analysis is performed on whether to consider the cost optimization tolerance tolerance results.

(1) Tolerance design without considering cost.

Figure 4. Changes in tolerance values during iteration.

Figure 5. Cost increase of each parameter.

Figure 6. Results of tolerance optimization process.

(2) Tolerance design considering cost.

Figure 7. Change in cost contribution rate during iteration.

Figure 8. Changes in tolerance values during iteration.
It can be seen from the comparison between Figure 4 and Figure 8 that in the optimization process without considering the cost factor, most of the design parameters have reached the tolerance extreme before the optimization is completed, so this optimization method is not optimal.

By comparing Figure 5 with Figure 9, it is obvious that the optimization method that takes the cost factor into consideration makes the cost increase smaller in the process of tolerance design. Comparing the results of the tolerance optimization process of the two methods, that is, Figure 6 and Figure 10, it can be seen intuitively that the optimization method considering the cost factor is better and more consistent with the goal of consistent optimization.

4.4. Verification
The calculation model between the design parameters and the holding force of the electromagnetic relay was established by the Kriging method. Set the number of samples to 1000 and use the Monte Carlo method to generate a virtual sample of the batch product. Then the quality characteristics of the tolerance scheme before and after the consistency optimization are analyzed and compared to verify the correctness of the improved RTD method. The distribution of holding force after robust optimization is shown in Figure 7. Its standard deviation is 1.4989, and the optimization target error is 0.073%. Taking the qualified threshold value of 111.8 ~ 120.8N, the inherent reliability of the optimized electromagnetic relay can be increased from 0.5889 to 0.9993, thereby greatly improving the application reliability of the product.

Figure 9. Cost increase of each parameter.
Figure 10. Results of tolerance optimization process.
Figure 11. Retention force distribution after robustness optimization.
5. Conclusion
In reducing quality fluctuations and minimizing total losses, changes in tolerances cause quality losses and manufacturing costs to change in opposite directions. In addition, the changed tolerances will have a new impact on both of them. This brings great difficulties to the total loss optimization. In this paper, an RTD method is proposed to integrate the optimization objectives of consistency and total loss for practical engineering applications.

The cost contribution rate coefficient \( \lambda \) and acceleration coefficient \( \beta \) were calculated through orthogonal experimental design to update the tolerance step size control model and the total loss objective function. Therefore, the optimal path of the total loss optimization is gradually determined through the control of the tolerance until the quality fluctuation approaches the target value.

Taking electromagnetic actuator as an example, the RTD results of the proposed method is compared to the two cases of optimization without considering manufacturing costs and with constant contribution rate. The results show that the proposed method can achieve better results than the conventional techniques.

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