Diphoton signals in theories with large extra dimensions to NLO QCD at hadron colliders

M. C. Kumar\textsuperscript{a,b}, Prakash Mathews\textsuperscript{a}, V. Ravindran\textsuperscript{c}, Anurag Tripathi\textsuperscript{c}

\textsuperscript{a) Saha Institute of Nuclear Physics, 1/AF Bidhan Nagar, Kolkata 700 064, India.}
\textsuperscript{b) School of Physics, University of Hyderabad, Hyderabad 500 046, India.}
\textsuperscript{c) Regional Centre for Accelerator-based Particle Physics, Harish-Chandra Research Institute, Chhatnag Road, Jhunsi, Allahabad, India.}

Abstract

We present a full next-to-leading order (NLO) QCD corrections to diphoton production at the hadron colliders in both standard model and ADD model. The invariant mass and rapidity distributions of the diphotons are obtained using a semi-analytical two cut-off phase space slicing method which allows for a successful numerical implementation of various kinematical cuts used in the experiments. The fragmentation photons are systematically removed using smooth-cone-isolation cuts on the photons. The NLO QCD corrections not only stabilise the perturbative predictions but also enhance the production cross section significantly.

\textsuperscript{1}mc.kumar@saha.ac.in
\textsuperscript{2}prakash.mathews@saha.ac.in
\textsuperscript{3}ravindra@mri.ernet.in
\textsuperscript{4}anurag@mri.ernet.in
1 Introduction

The gauge hierarchy problem has been one of the main motivations to go beyond the standard model (SM). A novel idea that addresses this problem was put forward by Arkani-Hamed, Dimopoulos and Dvali (ADD) wherein they introduced extra spatial dimensions and allowed only gravity to propagate in the extra dimensions, keeping the SM fields confined to a 3-brane [1]. As the inverse square law behavior of gravity has so far been tested down to sub-millimeter length scales, the size of the extra dimensions, in this model, should be much smaller than sub-millimeter. The apparent weakness of gravity as compared to the other forces seen in nature, can now be accounted for through the volume of the extra dimensions. The relation between the fundamental scale $M_s$ at which the new physics sets in (above which the extra dimensions are dynamically accessible) and the Planck scale $M_P$ is given by

$$M_P^2 \sim M_S^{(d+2)} R^d,$$

where $d$ is the number of extra spatial dimensions and $R$, the size of the extra dimensions. Since $R$ is of order of a milli-meter, the scale $M_s$ can be as low as a few TeV, which circumvents the hierarchy problem. The propagation of a massless graviton in $4 + d$ dimensions, after compactifying the extra dimensions on a $d$-dimensional torus, manifests itself as an infinite tower of massive Kaluza-Klein (KK) modes on the 3-brane. Each KK mode couples with SM field through energy momentum tensor with a coupling proportional to $\kappa \sim 1/M_P$. However, the effective coupling after summing over all the KK modes is enhanced significantly due to large multiplicity of KK modes. In any typical scattering process at colliders, the gravity can enter through their KK propagator as well as through the real emission of KK states. These KK states are large in number. Hence the suppression resulting from coupling $\kappa$ is compensated by the large multiplicity factor resulting either from the sum of KK propagator $\mathcal{D}(Q^2)$ or from the phase space of large number of real KK states. For example, if the KK states enter through a propagator,
we find that any typical amplitude will be proportional to

\[ \kappa^2 D(Q^2) = \kappa^2 \sum_n \frac{1}{Q^2 - m_n^2 + i\epsilon}, \]

\[ = \frac{8\pi}{M_s^2} \left( \frac{Q}{M_s} \right)^{(d-2)} \left[ -i\pi + 2I(\Lambda/Q) \right], \tag{2} \]

where \( \Lambda = M_s \) is the explicit cut-off on the KK sum and the function \( I \) can be found in [2]. Thus, for \( M_s \sim \mathcal{O}(\text{TeV}) \), the gravity effects can become significant and hence the collider phenomenology associated with this model is very interesting [2]. To exemplify, the virtual effects of the KK modes could lead to the enhancement of the cross sections of pair productions in the processes like Drell-Yan, diphoton and dijet while the real emissions could lead to large missing \( E_T \) signals giving some new observable like mono-jet, mono-photon in an experiment. Owing to a very high centre-of-mass energy of \( \sqrt{S} = 14 \text{ TeV} \) and a large gluon flux at the large hadron collider (LHC), rich collider signals resulting from this model have been reported in the literature [2–7]. However, these results are based on leading order (LO) calculations. At the hadron colliders like LHC, the QCD effects are often considerably large and hence the quantum corrections can influence the predictions significantly. In the ADD model, QCD effects [8] have been shown to increase the di-lepton productions and also to stabilise the perturbative predictions. Hence in this paper we study the impact of the QCD corrections for the diphoton signal in the ADD model.

In QCD, the infra-red safe observable exhibit a feature called factorisation, according to which collinear singularities can be factored out from the partonic cross sections in a process independent way and then they are either absorbed into the bare parton distribution functions (PDF) if they originate from initial state partons or into fragmentation functions if they are from final state partons. This procedure introduces a scale called factorization scale \( \mu_F \), which is arbitrary. In addition, ultra-violate renormalisation introduces renormalisation scale \( \mu_R \) which is again arbitrary. The truncated perturbative expansion leaves our theoretical predictions \( \mu_F \) and \( \mu_R \) dependent, these scale dependence will go down as we include more and more terms in the expansion. In addition,
the fitted PDFs are usually not fully constrained due to insufficient experimental data. Hence, predictions beyond LO are often more reliable than LO ones.

Diphoton production process is an important probe for the Higgs boson search at the LHC. NLO QCD corrections to this process in the SM are available in the literature [9–12] and hence the diphoton signal has been a useful tool for precision studies. This process has also been used to search for the physics beyond the standard model, such as extra dimensional models, super symmetry and the unparticle physics. Di-photon production [5] at Tevatron has set stringent constraints on the parameters of the ADD model [13]. It will also play an important role at LHC. The DØcollaboration [13] assumed a K-factor for their analysis but a full NLO QCD calculation for the ADD model does not exist for the diphoton production. In this paper, we have systematically computed all the QCD effects to NLO in perturbation theory to various important observable in diphoton production that are sensitive to the ADD model. Quantitative estimates of QCD corrections to these observable are presented and our predictions are expected to be less sensitive to the factorisation scale.

2 The Diphoton Production

In the SM, at leading order (LO), diphoton production proceeds via quark anti-quark annihilation subprocess \( q + \bar{q} \rightarrow \gamma + \gamma \).\(^5\) In the ADD model, the SM fields couple to KK modes through the energy-momentum tensor of the SM fields with a strength denoted by \( \kappa \). Hence, diphotons are produced in (i) quark antiquark annihilation \( q + \bar{q} \rightarrow \gamma + \gamma \) and (ii) gluon fusion process \( g + g \rightarrow \gamma + \gamma \) via the exchange of KK modes. A comprehensive phenomenology taking into account all the above LO processes has been done in [5]. It was observed that unitarity restricts the maximum value of the invariant mass \( Q \) of the diphotons. Following [5], we restrict the invariant mass \( Q \) to \( Q < 0.9 \, M_s \).

At NLO, the SM as well as ADD leading order quark antiquark annihilation processes

\(^5\)The gluon-gluon fusion process through quark loop, though of order \( \alpha_s^2 \), is comparable to the LO for studies of photon pairs having small invariant masses, \( M_{\gamma\gamma} \). As it falls off rapidly as \( M_{\gamma\gamma} \) increases, it no longer enjoys the status of LO process for our study on the production of large invariant mass photon pairs in the context of ADD model and is truly a NNLO contribution.
get $O(\alpha_s)$ QCD radiative corrections through virtual gluons in $q+\bar{q} \rightarrow \gamma+\gamma+\gamma$ one loop and real gluon emissions in $q+\bar{q} \rightarrow \gamma+\gamma+g$ processes. To this order, $q(\bar{q})+g \rightarrow q(\bar{q})+\gamma+\gamma$ process also shows up in both SM and ADD. The LO gluon fusion process in the ADD model gets NLO QCD corrections to order $\alpha_s$ through $g+g \rightarrow \gamma+\gamma+\gamma$ one loop and $g+g \rightarrow \gamma+\gamma+g$ processes. Since KK modes appear at the propagator level, the LO SM (ADD) processes interfere with the corresponding NLO ADD (SM) processes giving order $\alpha_s$ NLO QCD corrections. We have incorporated all these NLO QCD corrections in this article for the study that follows.

The NLO partonic cross sections are often ill-defined due to soft and collinear singularities that result from the presence of zero momentum gluons and mass-less partons. In addition to these singularities, we encounter collinear (QED) singularities that originate when the photon in the final state becomes collinear to the quark or the anti-quark emitting it. These (QED) singularities go away if we also include the diphoton production channels resulting from the fragmentation of partons. This involves introduction of non-perturbative fragmentation functions. These functions are poorly constrained. Hence, in our study we do not include fragmentation photons but consider only direct photons. Alternatively, we can suppress QED collinear singularities using the smooth-cone-isolation prescription proposed by Frixione [14]. In the rapidity–azimuthal angle $(y, \phi)$ plane the amount of transverse hadronic energy $E_T$ in any cone of radius $r = \sqrt{(\Delta y)^2 + (\Delta \phi)^2}$ with $r < r_0$ centered around the photon must satisfy

$$E_T \leq E_{iso}^{T} \left( \frac{1 - \cos(r)}{1 - \cos(r_0)} \right)^n.$$ (3)

The above prescription safely removes all the photons from the fragmentation processes without disturbing soft and collinear partons.

An analytical computation incorporating smooth-cone-isolation and other kinematical constraints at NLO level is hard to achieve. Hence, we resort to a semi-analytical approach called two cutoff phase space slicing method [15]. In this method, two small slicing parameters $\delta_s$ and $\delta_c$ are introduced to isolate the cross sections that are sensitive
to soft and collinear singularities. The remaining part of the cross section denoted by \( d\hat{\sigma}^{\text{fin}}(\delta_s, \delta_c) \) is soft and collinear free. The soft divergences come from virtual as well as real gluons when their momenta become zero. On the other hand the collinear singularities arise due to mass less nature of the partons. We compute these soft and collinear sensitive cross sections (they are singular in 4 dimensions) analytically in \( 4 + \varepsilon \) dimensions which regulate these singularities. The soft singularities cancel between virtual and real gluons when their contributions are added appropriately. The remaining collinear singular terms which appear as poles in \( \varepsilon \) are systematically removed by collinear counter terms in \( \overline{MS} \) factorization scheme. This is usually done at an arbitrary scale \( \mu_F \). Hence we will end up with a finite cross section coming from (a) soft and collinear sensitive regions denoted by \( d\hat{\sigma}^{\text{sc,fin}}(\delta_s, \delta_c) \) (sc denotes soft and collinear) and (b) \( d\hat{\sigma}^{\text{fin}}(\delta_s, \delta_c) \) part of the cross section. Their sum, ie. \((a) + (b)\), is expected to be free of choice of the slicing parameters. This is an essential prerequisites for the implementation of the phase space slicing method.

3 Numerical Results

In this section, we present our results for invariant mass (Q) and rapidity (Y) distributions of the photon pair at LHC. We have employed the kinematical cuts given by ATLAS collaboration [16]: the transverse momentum \( p_T \gamma > 40 \text{ GeV} \) for the harder photons, \( p_T \gamma > 25 \text{ GeV} \) for the softer photon, and the rapidity \( |y_\gamma| < 2.5 \) for each photon.

In addition, the photons are isolated from hadronic activity according to Eq. (3), with \( n = 2, r_0 = 0.4, E_T^{\text{iso}} = 15 \text{ GeV} \). The minimum separation between the two photons is taken to be \( r_{\gamma\gamma} = 0.4 \). For the LO, we have used CTEQ6L PDFs and CTEQ6M for NLO studies [17], with the corresponding value of \( \alpha_s(M_Z) = 0.118 \) and 5 light quark flavours. The factorisation and renormalisation scales are taken to be \( Q \), the invariant mass of the diphoton pair. The electromagnetic coupling constant is chosen to be \( \alpha = 1/128 \). For our numerical analysis we have chosen the ADD parameters, \( M_s = 2 \text{ TeV} \) and \( \Lambda = M_s \) for the number of extra spatial dimensions \( d = 3 \). This choice of \( M_s = 2 \) is consistent
with the limits from [13].

We have first checked our numerical code by studying the dependence of observable on the slicing parameters, $\delta_s$ and $\delta_c$. In the left (right) panel of Fig. 1 we have plotted the order $\alpha_s$ contribution to the invariant mass distribution of diphotons in SM and ADD against the slicing parameter $\delta_s$ ($\delta_c$) in the range between $5 \times 10^{-2}$ and $10^{-5}$. For the $\delta_s$ variation (left panel) we have fixed $\delta_c = 10^{-5}$ and for the $\delta_c$ variation (right panel) we have fixed $\delta_s = 10^{-3}$. These plots show that our numerical results are least sensitive to the slicing parameters for a wide range. The percentage of uncertainty that results from the choice of slicing parameters is found to be around 6.7%. This study confirms the reliability of our code for further predictions. For our numerical predictions, we have chosen $\delta_c = 10^{-5}$ and $\delta_s = 10^{-3}$. Other important check on our code comes from a detailed comparison of our SM results against those given in the literature [9–12]. In particular, we find that our SM results are in very good agreement with those given

Figure 1: Stability of the order $\alpha_s$ contribution to the total (SM+ADD) cross section against the variation of the phase space slicing parameters $\delta_s$ (left) and $\delta_c$ (right) in the invariant mass distribution of the di-photon system with $M_s = 2$ TeV and $d = 3$ at $Q = 700$ GeV.
Invariant mass distribution LHC

\[ d \sigma / dQ \ (\text{pb/GeV}) \]

\( M_s = 2 \text{ TeV} \)

\( d = 3 \)

\( \delta_s = 10^{-3} \)

\( \delta_c = 10^{-5} \)

\( \text{SM } q \bar{q} \)

\( \text{SM } qg \)

\( \text{SM } gg \) (LO)

\( \text{ADD } q \bar{q} \)

\( (-1) \times \text{ADD } qg \)

\( \text{ADD } gg \)

\( \text{SM*ADD } q \bar{q} \)

\( \text{SM*ADD } qg \)

\( \text{SM*ADD } gg \)

\( \text{SM + ADD Total} \)

\[ 10^{-7} \]

\[ 10^{-6} \]

\[ 10^{-5} \]

\[ 10^{-4} \]

\[ 10^{-3} \]

\[ 10^{-2} \]

\[ 400 \ 600 \ 800 \ 1000 \]

Figure 2: Various subprocess contribution to the invariant mass distribution of the diphoton production with \( M_s = 2 \text{ TeV} \) and \( d = 3 \). The SM \( gg \) subprocess (lower solid line) is at \( \mathcal{O}(\alpha_s^2) \) while all other subprocess are at order \( \mathcal{O}(\alpha_s) \).

in [11] with their choice of parameters.

In Fig. 2, we have presented various subprocess contributions to the invariant mass distribution of the diphoton system for the range \( 400 \leq Q \leq 1100 \text{ GeV} \) where gravity (through KK modes) contribution dominates over the SM. Both \( q\bar{q} \) and \( gg \) initiated subprocesses in ADD give large positive contributions while the \( qg \) initiated subprocess gives a negative contribution. The interference of the SM with ADD (SM*ADD) from both \( q\bar{q} \) and \( gg \) subprocesses gives almost \( Q \) independent contribution, while the contribution from the \( gg \) subprocess falls steeply at higher values of \( Q \). Owing to the large gluon flux at the LHC, the \( gg \) initiated subprocesses in ADD give the dominant contribution over the rest, thus making the observable effects of ADD model clearly visible in the large \( Q \) region. We have also plotted the SM gluon-gluon fusion sub-process through quark loop contribution separately in the Fig. 2. It is clear from the plot that its contribution is negligible compared to SM quark anti-quark initiated processes and hence belongs to NNLO contributions. Hence, we have not included this in our study.
In Fig. 3, we have presented the invariant mass (left panel) and rapidity (right panel) distributions of the diphoton production in both SM and ADD model. We have plotted LO and NLO contributions separately to demonstrate the impact of QCD corrections. It is clear from the plots that the QCD corrections to both invariant mass and rapidity distributions in SM as well as in ADD model are large for the entire range of $Q$ considered. In the left panel we find that the contribution from ADD dominates over that of SM starting around $Q = 500$ GeV. The exact value where this happens depends crucially on the parameters of ADD model. For the rapidity distribution (right panel), we have considered $|Y| \leq 2.0$ and integrated over $Q$ in the range $600 \leq Q \leq 1100$ GeV where the KK effects are dominant. The cross section is found to be maximum at the central rapidity region both in SM and in ADD model, the later differing by more than an order of magnitude.

The cross sections do depend on the isolation criterion. The $E_{T}^{iso}$ at the partonic level need not be the same as that of the hadrons at the detector level, which gives rise to
The dependency of the cross sections on $E_T^{\text{iso}}$. In the smooth cone isolation prescription discussed above, large logarithms of $E_T^{\text{iso}}$ often spoil the reliability of fixed order computation. We can study the effect of these logarithms by varying the function that appears in the isolation criterion. We present in Fig. 4 the dependency of our cross sections on the choices of $E_T^{\text{iso}}$ (varied between 5 GeV and 30 GeV), and $n$, (varied between 1 and 2). We find that the dependency is unnoticeable making our predictions reliable for experimental study.

Finally we consider the invariant mass distribution at the Tevatron for both the SM and ADD model to NLO QCD. We have used $M_s$ value which is consistent with the experimental bounds [13] for the di-electromagnetic signal which is the combined $e^+e^-$ and $\gamma\gamma$ final state. In this analysis we are hence interested only in gauging the impact of the QCD corrections to these studies. In Fig. 5 we plot the invariant mass distribution of the diphoton system in the range $100 < Q < 1000$ GeV at the Tevatron ($\sqrt{S} = 1.96$ GeV) for both the SM and including the ADD contribution at LO and NLO in QCD.
We have used the following kinematical cuts: (a) transverse momentum $p_T > 15\ (14)$ GeV for the harder (softer) photons, (b) rapidity $|y_{\gamma}| < 1.1$ for each photon, and (c) $r_0 = 0.4$ and $r_{\gamma\gamma} = 0.4$. In addition for the smooth-cone-isolation we use $E_T^{iso} = 2$ GeV and $n = 2$. The contributions of the various subprocess is shown in the right panel, for the range $400 < Q < 1000$ GeV. We have used the number of extra spacial dimensions $d = 4$ and $M_s = 2$ TeV. The impact of QCD corrections at the Tevatron is much mild compared to the LHC where the gluonic flux is overwhelming.

4 Conclusions

In this article, we have systematically computed NLO QCD corrections to the diphoton production process at the hadron colliders in SM as well as in ADD model. We use a semi-analytical two cut-off phase space slicing method to compute invariant mass as well as rapidity distributions of the diphotons system. We have applied the kinematical cuts used by the ATLAS detector collaboration for our study. A smooth-cone-isolation prescription
on the diphotons has been used to reject poorly known fragmentation photons. Our method takes care of all the soft and collinear singularities that appear at NLO level in QCD. We have explicitly shown that our NLO results are least sensitive to the slicing parameters $\delta_s$ and $\delta_c$. Our SM results are in good agreement with those given in the literature. Predictions for invariant mass distribution of diphotons in ADD model with $M_s = 2$ TeV are found to be large compared to those in SM for invariant mass $Q > 600$ GeV. This is due to large gluon flux at the LHC which enhances the gluon initiated production channels over the rest. In addition, the QCD corrections are significantly large both in the SM and in the ADD over the entire range of $Q$ considered. For the rapidity distribution, we have integrated $Q$ in the region $600 \leq Q \leq 1100$ GeV where the gravity (through KK modes) contributes significantly. We find that the QCD corrections are important throughout the region $|Y| \leq 2.0$. In addition, our results are expected to be less sensitive to the uncertainties coming from the choice of factorisation scale.

In summary, we have accomplished an important task of computing all the partonic contributions at NLO level in QCD to diphoton production at hadron colliders both in SM and ADD model. These QCD corrections for the ADD model and its interference with the SM are being presented for the first time, while to this order the SM results already exist in the literature. The NLO QCD effects are found to be large and they are expected to reduce theoretical uncertainties, thus providing an excellent opportunity to put stringent bounds on the parameters of the ADD model when the experimental results are available. Quantitative impact of the NLO QCD corrections to both the ADD and RS model would be addressed in a future publication [18].

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