α-vacuum and inflationary bispectrum

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Abstract

In this paper, we discuss the non-Gaussianity originated from the α-vacuum on the CMB anisotropy. For α-vacuum, there exist correlation between points in the acausal two patches of de Sitter spacetime. Such kind of correlation can lead to large local form non-Gaussianity in α-vacuum. For the single field slow-roll inflationary scenario, the spacetime is in a quasi-de Sitter phase during the inflation. We will show that the α-vacuum in this case will lead to non-Gaussianity with distinguished feature, of a large local form and a very different shape.
1 Introduction

In the standard hot big bang cosmology, there are several tough problems, including the flatness, isotropy and homogeneity, horizon and topological defects problems. The hot big bang theory is unable to answer these problem in a natural way. Inflation, as an add-on, is remarkably successful in solving these problems. It gives a natural initial condition for our observed universe \cite{1, 2, 3}. Furthermore the quantum fluctuations during inflation seed wrinkles in the Cosmic Microwave Background (CMB), and today’s large scale structure \cite{1, 5, 6, 7, 8}. As a result, inflation predicts a nearly scale invariant Gaussian CMB spectrum, which has been confirmed very well in the experiments\cite{10}.

However, inflation as a successful scenario in the very early universe has its own difficulties. One of the problems with inflation is that there are too many inflationary models, which cannot be distinguished by the scalar power spectrum and power spectrum index from the CMB observation. It is essential to find more powerful signatures which could distinguish various models from each other. Moreover, inflation has some conceptual problems, cosmological singularity problem and Trans-Planckian physics being two of them. In a sense, inflation scenario is not really a fundamental theory. The trouble with it mainly comes from our ignorance of the physics of very early universe, which should be governed by a quantum gravity theory.

The rapid development of precise experiment cosmology opens new windows to very early universe. The scalar spectral index and its running, the gravitational wave, non-Gaussianity and the isocurvature perturbation in the CMB \cite{10} are among the important probes. These probes will not only constrain a large amount of inflation models and make the paradigm more clear, but also shed light on various other issues beyond usual inflation scenario. These issues include the initial conditions of inflation model, Trans-Planckian physics and alternative models to inflation, etc..

Among the probes, non-Gaussianity is one of the most important ones. It contains much information: magnitude, shape, sign, and even running. In principle, it could distinguish various inflation models. The deviation from the Gaussian distribution of the CMB in the WMAP observation is parameterized by $f_{NL}$ \cite{11},

$$\zeta = \zeta_g + \frac{3}{5} f_{NL} \zeta_g^2,$$

(1)
where $\zeta$ is the curvature perturbation in the uniform density slices, and the subscript $g$ denotes the Gaussian distribution. In the WMAP5 year data [10], two kinds of non-Gaussianity, local form and equilateral form, have been analysed

$$- 9 < f_{NL}^{\text{local}} < 111(95\% \ CL) , \quad -151 < f_{NL}^{\text{equil}} < 253(95\% \ CL) .$$

(2)

The central value of the local form non-Gaussianity is 51. If the value of the local form non-Gaussianity is confirmed by the future experiments, such as Planck satellite, then it will be a great challenge to many inflation models, including the most well-studied single field inflation models.

In fact, the non-Gaussianity in CMB spectrum may come from various sources during the evolution. The temperature fluctuation $\Delta T/T$ is the observable in CMB observation. During the inflation, the quantum fluctuation of the inflaton $\delta \phi$ are generated, and the modes of the fluctuation grow with the exponentially expanding universe. After the fluctuations leave the horizon, the decoherence effect makes the quantum fluctuations to be the classical ones. In the large scale, the physical freedom of scalar perturbation is curvature perturbation $\zeta$. If we follow a mode,

$$\text{Initial Condition (Vacuum)} \rightarrow \delta \phi \rightarrow \zeta \rightarrow \frac{\Delta T}{T}.$$ 

(3)

All the transformations are linear at first order, thus the temperature fluctuations are Gaussian. Meanwhile, any deviation from linearity in these transformations and the changes in the initial condition will influence the final observable.

- Let us first consider the last stage of the transformation $\zeta \rightarrow \frac{\Delta T}{T}$. The fluctuations in the gravitational potential on the last scattering surface result in temperature fluctuations in the CMB, which is known as Sachs-Wolfe effect. The nonlinear Sachs-Wolfe effect generate $f_{NL}$ of order one.

- The curvature perturbation $\zeta$ is conserved in the single field inflation, while in the multiple field case, the entropy perturbation change the evolution of $\zeta$. It will suppress the perturbation conversion factor for $\zeta$ in this process. That is why in the curvaton mechanism [38] and new ekpyrotic models [39] the large local form non-Gaussianity is possible. (The other important reason for ekpyrotic models generating large non-Gaussianity is that the slow-roll condition breaks down.)
The primordial non-Gaussianity, which resides on the quantum fluctuation of the scalar field \([51]\), can be from the microphysics deep in the horizon. Since the observation requires the potential of the inflaton to be slow-roll, the interaction of the inflaton is weak, and non-Gaussianity is only the order of the slow roll parameter \(f_{NL} \sim \mathcal{O}(\epsilon, \eta)\). The picture will change when the modified gravity and non-canonical action are considered, such as in ghost inflation \([40]\), DBI inflation \([41]\), and k-inflation \([42]\). The non-linearity in the action can produce large equilateral form non-Gaussianity in the CMB. On the other hand, the back reaction argument \([51]\) explains why microphysics in the horizon cannot have large local form non-Gaussianity.

The initial condition could be another important source of non-Gaussianity. One attempt is to consider the thermal vacuum in the inflationary cosmology \([49]\), in which the equilateral and local form non-Gaussianity are both \(\gtrsim \mathcal{O}(1)\) in some cases. In this paper, we will consider one parameter family of vacuum states, called \(\alpha\)-vacuum, in de Sitter spacetime \([14, 15]\) and quasi-de Sitter spacetime in inflation. In these vacua, there are correlations between points in the acausal two patches of de Sitter spacetime. We will show that \(\alpha\)-vacuum can induce large local form non-Gaussianity.

As we know, the standard treatment in scalar driven inflation scenario is based on semi-classical gravity, in which the background is described by classical Einstein Gravity and the perturbations are quantized in the background. During the slow roll period, since the evolution of the background is in a quasi-de-Sitter phase, the perturbations could be treated as the quantum field in a de-Sitter spacetime. In the expanding background of inflation, the quantum modes are stretching across the horizon. The modes observed in the CMB could be in the trans-Planckian region at the very early time. If the inflation lasts about 70 e-foldings, the perturbation which we observe in our Horizon, at that time, is deep inside the horizon. And the wavelength of these perturbation is smaller than the Planck scale. The semi-classical description is not applicable for the perturbation. It is necessary to consider stringy effect or some other quantum gravity effects on inflation. The trans-Planckian physics in inflation was first raised in \([16]\). As in black hole physics, an efficient way to count
the Trans-Planckian effect is to modify the dispersion relations. Various dispersion relations and their physical implications have been widely studied\cite{17}. Another way to discuss the trans-Planckian physics is based on the space-time uncertainty from quantum gravity, such as string theory. This noncommutative effect will impact the power spectrum and gravitational waves of CMB \cite{35, 36} and also modify the non-Gaussianity\cite{37}.

There is another attempt to address the trans-Planckian physics in inflation, firstly suggested by Danielsson\cite{19}. The new important ingredient in the discussion is the introduction of one parameter family of \(\alpha\)-vacuum state in the inflationary background. In de Sitter spacetime, the Bunch-Davies vacuum is the standard vacuum which is invariant under de Sitter space isometry group. However, due to the absence of globally defined time-like Killing vector, the vacuum in de-Sitter spacetime cannot be defined uniquely. Similarly, the choice of the vacuum in the inflationary background is subtle and may induce observable signature on CMB data. It was argued that the effective field theory and semi-classical gravity are applicable from the length of new physical scale cutoff to the large scale of the universe. And it was also assumed that the modes were generated one by one at the Planck scale or new physics scale \(\Lambda\) such that the initial conditions are imposed at a mode-dependent time instead of in the infinite past. This is the motivation to introduction of \(\alpha\)-vacuum in inflation. Its physical implications on inflation have been intensely studied. The order of the correction to the power spectrum has been discussed in \cite{20, 21, 22, 23, 24, 25, 26, 27, 28, 29}. For example, in \cite{26} using the method of effective field theory the authors found that the correction was \(\sim O(\frac{\mathcal{H}^2}{\Lambda^2})\), and in \cite{25} the authors calculated the correction of power spectrum when the modes are initially created by adiabatic vacuum state, and found that the correction was \(\sim O(\frac{\mathcal{H}^3}{\Lambda^3})\). The careful analysis of these different corrections can be seen in \cite{28}. For the non-Gaussianity from the trans-Planckian physics it was first roughly analyzed in \cite{53}, and in \cite{30} its folded form was analyzed in the effective field theory. In this paper we follow the treatments in \cite{19, 24}. In \cite{24} the authors evaluated the \(\alpha\)-vacuum effect in general single field inflation background, and found an oscillating dependence on the wavenumber \(k\) for the power spectrum. The reason is that during inflation Hubble scale is not constant, and the coefficient for the \(\alpha\)-vacuum state sensitively relies on \(k\). We will show that this \(k\)-dependence lead to a
distinguished feature of non-Gaussianity.

This paper is organized as follows: in Sec. 2 we first discuss the vacuum states in de Sitter space, the relation between Euclidean vacuum and $\alpha$-vacuum. And then we introduce different correlation functions, and explain the property of the antipodal correlation in de Sitter space. In Sec. 3 and Sec. 4, we review the lagrangian formalism to compute the power spectrum and three point correlation of the curvature perturbation. Sec. 5 is the main result of this paper. We evaluate the local form and equilateral form non-Gaussianity for Euclidean vacuum and $\alpha$-vacuum in both de Sitter spacetime and inflationary backgrounds. We also draw the shapes of non-Gaussianity in each cases. Finally, we conclude in Sec. 6.

2 Vacuum state in de Sitter space

The spacetime of inflation is a quasi-de Sitter spacetime, which can be conventionally described by the FRW coordinate,

$$ds^2 = dt^2 - e^{2Ht}d^2x,$$  \hspace{1cm} (4)

where $H$ is the Hubble scale. In this section, to illustrate the feature of $\alpha$-vacuum clearly, we mainly discuss the de Sitter space in which $H$ is simply a constant. Note that the metric (4) actually covers only half of de Sitter spacetime.

The equation of motion for a scalar field in the background takes the form,

$$\ddot{\delta \phi} + 3H \dot{\delta \phi} - \nabla^2 \delta \phi + \frac{\partial V}{\partial \delta \phi} = 0.$$  \hspace{1cm} (5)

The scalar field could be inflaton, for which the mass of the scalar field is much less than the Hubble scale $m \ll H$. The complete solution of (5) can be expressed in momentum space [12],

$$\delta \phi(\tau, x)_k = \frac{1}{2} \sqrt{\frac{9}{4} - 12 \frac{m^2}{H^2}} \pi^{1/2} H(-\tau)^{1/2} e^{i k \tau} \left[ c_1(k) \nu^{(1)}(-k \tau) + c_2(k) \nu^{(2)}(-k \tau) \right],$$  \hspace{1cm} (6)

where $\tau = -\frac{1}{aH}$ is the conformal time in de Sitter space,

$$\nu = \sqrt{\frac{9}{4} - 12 \frac{m^2}{H^2}},$$  \hspace{1cm} (7)
and $H^{(1)}_{\nu}$ are Hankel functions of the first and second kind. In the limit of $\eta \to -\infty$ for a fixed $k$, which means that a mode of the scalar field is deep in the Hubble horizon,

$$H^{(1)}(-k\eta) \to \left(-\frac{2}{\pi k \eta}\right)^{1/2} e^{(-\pi i k \eta)}, \quad (8)$$

up to some constant phase factor.

When a mode is deep in the horizon, the spatial scale is much smaller than the Hubble scale so that the curvature effect is negligible and the scalar field is well described by quantum field theory in Minkowski space. As $\eta \to -\infty$, we could choose the vacuum as in the flat space and positive frequency modes from Hankel function of the second kind $H^{(2)}_{\nu}$, i.e. $c_2(k) = 0$. This is called thermal vacuum or Euclidean vacuum in de Sitter spacetime.

The scalar field is quantized by the canonical method,

$$\delta \phi = \sum_n (\delta \phi_n a_n + \delta \phi_n^* a_n^\dagger), \quad (9)$$

where $n$ denotes all the quantum numbers of the modes, and $\{a_n, a_n^\dagger\}$ are the annihilation and creation operators satisfying the commutative relation

$$[a_n, a_m^\dagger] = (2\pi)^3 \delta_{mn}. \quad (10)$$

And the Euclidean vacuum state is defined to be

$$a_n|\Omega\rangle = 0. \quad (11)$$

Let’s consider a mode $k$ as $\eta \to -\infty$. The physical wavelength of the modes is smaller than the Planck scale. It is natural to set a physical cutoff for momentum $p_c$. When $p > p_c$, one has to consider the trans-Planckian effect. The influence of new degrees of freedom and new physical law could be effectively encoded in the change of dispersion relation \[17\], or space-time noncommutativity \[35, 36\] or some other ways. When $p < p_c$, the solution of the Klein-Gorden equation is reliable, the solution of the scalar field is the linear combination of $\delta \phi(\eta, x)^\pm_k$ in \[6\]. A new set of modes for trans-Plankian effect is expressed as the combination of the Euclidean modes by a Bogoliubov transformation \[13, 14\] (Mottola-Allen transform),

$$\tilde{\delta} \phi_n = N_\alpha (\delta \phi_n + e^\alpha \delta \phi_n^*), \quad (12)$$

$$N_\alpha = \frac{1}{\sqrt{1 - e^{\alpha + \alpha^*}}},$$
where $\alpha$ is a complex number with $\text{Re} \alpha < 0$ to denote the rotation of field space, and $N_\alpha$ is derived from the Wronskian condition or the rule of Bogoliubov transformation. Since
\[
\delta \phi = \sum_n (\delta \phi_n a_n + \delta \phi^*_n a_n^\dagger) = \sum_n (\bar{\delta} \phi_n \bar{a}_n + \bar{\delta} \phi^*_n \bar{a}_n^\dagger),
\]
so the equation of $\bar{a}_n$ can be expressed as
\[
\bar{a}_n = N_\alpha (a_n - e^{\alpha^*} a_n^\dagger).
\]
Thus the new vacuum called $\alpha$-vacuum is defined as follows,
\[
|\alpha\rangle = 0,
\]
where the $\alpha$-vacuum is still de Sitter invariant, just like Euclidean one. The Bogoliubov transform can be implemented by a unitary transform $[15]$,
\[
\bar{a}_n = S a_n S^\dagger,
\]
where
\[
S = \exp \left\{ \sum_n c(a_n^\dagger)^2 - \bar{c}(a_n)^2 \right\}, \quad c(\alpha) = \frac{1}{4} \left( \ln \tanh \frac{-\text{Re} \alpha}{2} \right) e^{-i \text{Im} \alpha}.
\]
The relation between the two vacua is
\[
|\alpha\rangle = S |\Omega\rangle.
\]
Let us turn to the Green functions which are useful studying the power spectrum and non-Gaussianity. There are several kinds of Green functions, but all of them can be expressed by the Wightman function.

In the Euclidean vacuum, the Wightman function can be expressed as
\[
G_E(x, x') = \langle \Omega | \delta \phi(x) \delta \phi(x') | \Omega \rangle = \sum_n \delta \phi_n(x) \delta \phi^*_n(x')
\]
For the distance much smaller than the Hubble scale, $G_E$ takes the form in Minkowski spacetime,
\[
G_E(x, x') \sim \frac{1}{(t - t' - i \epsilon)^2 - |\vec{x} - \vec{x}'|^2}
\]
Near the light cone, the Green function is divergent.
The metric (14) only covers one half of the whole de Sitter spacetime. To illustrate the character of Wightman function for $\alpha$-vacuum, we must extend the analysis to the whole de Sitter spacetime. The de Sitter spacetime can be constructed as a hyperboloid in the five-dimensional flat spacetime,

$$-(X_0)^2 + (X_1)^2 + (X_2)^2 + (X_3)^2 + (X_4)^2 = \frac{1}{H^2},$$

where $X^i$ is the coordinate in the flat five-dimensional spacetime. Thus de Sitter spacetime is a maximal symmetric spacetime with constant curvature, and its symmetry group is $O(1,4)$. Define the antipodal point of $X$ as $X_A = -X$. In the Euclidean vacuum, the modes of scalar field can be chosen to obey the rule [13, 14],

$$\delta\phi_n(x_A) = \delta\phi_n^*(x),$$

(21)

where $x_A$ denotes the antipodal point to $x$ in de Sitter space as in fig. 1.

Figure 1: The Penrose diagram of de Sitter space, where the two blue point are antipodal points in the space. The left upper part of the diagram is the spatial flat patch for a observer on the left-hand boundary.

In $\alpha$-vacuum, the Wightman function takes the form,

$$G_{\alpha}(x, x') = \langle \alpha | \delta\phi(x) \delta\phi(x') | \alpha \rangle = \sum_n \tilde{\delta}\phi_n(x) \tilde{\delta}\phi_n^*(x').$$

(22)

With the equation (18), (21), $G_{\alpha}$ is of the form

$$G_{\alpha}(x, x') = N_\alpha^2 [G_E(x, x') + e^{\alpha + \alpha^*}G_E(x', x) + e^{\alpha^*}G_E(x, x_A) + e^\alpha G_E(x_A, x')] .$$

(23)
The Wightman function has some special properties. Firstly, the Green function contains singularity at antipodal points \( \{ x_A, x \} \). The singularity can not be observed because of the separation by the horizon. Secondly, the correlation is not acausal, because when one calculate the retarded (advanced) Green function below, the correlation from acausal patches does not exist. Finally, the Wightman function contains the correlation between points in the two patches of de Sitter spacetime. It brings correction to the power spectrum. And most importantly, it influences the shape of non-Gaussianity, which makes it much different from the Euclidean vacuum.

In order to calculate the power spectrum and non-Gaussianity from \( \alpha \)-vacuum, it is better to use the Green functions in momentum space. The power spectrum of scalar field can be read from the two-point correlator in momentum space,

\[
\langle \alpha | \delta \phi(k, \eta) \delta \phi(k', \eta') | \alpha \rangle = (2\pi)^3 \delta(k - k') \delta \phi_k(\eta) \delta \phi_k^*(\eta')
\]

\[
= (2\pi)^3 \delta(k - k') N_\alpha^2 [\delta \phi_k(\eta) + e^\alpha \delta \phi_k^*(\eta)] [\delta \phi_k^*(\eta') + e^{\alpha^*} \delta \phi_k(\eta')]
\]

where \( \delta \phi_k \) is the modes of scalar field in momentum field. In the Euclidean vacuum, for the leading order approximation

\[
\delta \phi_k(\eta) = (-H \eta)(1 - i \frac{k \eta}{k^2}) e^{-i k \eta} .
\]

When the modes cross the horizon, the quantum fluctuation is transformed to classical one, and the curvature perturbation is conserved for large scale. Thus we take the time \( \eta \) at the horizon crossing time, which is a good approximation to calculate power spectrum in single field case. For \( k \eta \ll 1 \),

\[
\langle \alpha | \delta \phi(k, \eta) \delta \phi(k', \eta') | \alpha \rangle \sim (2\pi)^3 \delta(k - k') \frac{H^2}{2k^3} N_\alpha^2 (1 + e^{\alpha + \alpha^*} - e^\alpha - e^{\alpha^*})
\]

\[
= (2\pi)^3 \delta(k - k') \frac{2\pi^2}{k^3} P(k) .
\]

Thus the power spectrum of the scalar field can be written as

\[
P(k) = \frac{H^2}{(2\pi)^2} \frac{1 + e^{\alpha + \alpha^*} - 2 \text{Re} e^\alpha}{1 - e^{\alpha + \alpha^*}}
\]

The leading order correction of power spectrum comes from the \( \text{Re} e^\alpha \). From the Wightman function in coordinate space, it is easy to see that the contribution is from \( G_E(x, x_A') \). Meanwhile, if we use a physical cutoff to set an initial condition of the
modes, the power spectrum should depend on the cutoff scale \( \Lambda \), which could be the string scale, Planck scale, or others.

Next, let us analyze the retarded Green function to prove that the antipodal point does not break the causality. The retarded Green function is defined as

\[
G_R(x, x') = i\theta(t - t')\langle \text{VAC}|[\delta\phi(x), \delta\phi(x')]|\text{VAC}\rangle, \tag{28}
\]

where

\[
\theta(t) = \begin{cases} 
1 & t > 0 \\
0 & t < 0 
\end{cases}.
\]

In the Euclidean vacuum, the retarded Green function in momentum space takes the form,

\[
G_{RE}(\eta, \tau) = i(2\pi)^3 \delta(k - k') \begin{cases} 
0 & \eta < \tau \\\n\delta\phi_k^*(\tau)\delta\phi_k(\eta) - \delta\phi_k(\tau)\delta\phi_k^*(\eta) & \eta > \tau
\end{cases}, \tag{30}
\]

And in the \( \alpha \)-vacuum, the retarded Green function takes the form,

\[
G_{Ra}(\eta, \tau) = i(2\pi)^3 \delta(k - k') \begin{cases} 
0 & \eta < \tau \\\n\tilde{\delta}\phi_k^*(\tau)\tilde{\delta}\phi_k(\eta) - \tilde{\delta}\phi_k(\tau)\tilde{\delta}\phi_k^*(\eta) & \eta > \tau
\end{cases}. \tag{31}
\]

According to the equation (12), the Green function can be expressed by the one in the Euclidean vacuum,

\[
G_{Ra}(\eta, \tau) = N_\alpha^2[G_{RE}(\eta, \tau) + e^{\alpha + \alpha^*}G_{RE}^*(\eta, \tau)] . \tag{32}
\]

From the above equation, the retarded Green function in \( \alpha \)-vacuum does not contain the correlation from the two patches of de Sitter spacetime, so the causality is kept in the vacuum.

3 ADM formalism and curvature perturbation in inflationary background

During inflation, the Hubble radius is changing slowly and the spacetime is not exactly a de Sitter spacetime. As usual, we have slow roll parameters,

\[
\epsilon \equiv -\frac{\dot{H}}{H^2} = \frac{\dot{\phi}^2}{2H^2} \simeq \frac{1}{2} \left(\frac{V'}{V}\right)^2, \\
\eta = -\frac{\ddot{\phi}}{H\dot{\phi}} + \frac{1}{2} \frac{\dot{\phi}^2}{H^2} \simeq \frac{V''}{V} = \frac{V''}{3H^2}. \tag{33}
\]
The condition $\epsilon, \eta \ll 1$ indicates that the velocity and the acceleration of inflaton is quite small. Despite the small deviation from pure de Sitter spacetime, one can still define $\alpha$-vacuum. However, the physical implications of $\alpha$ vacuum to CMB spectrum are very different. For example, in de Sitter space the power spectrum has a constant correction with a magnitude of $\mathcal{O}(H/\Lambda)$ \cite{19}, where $\Lambda$ is the scale of the new physics, while in inflationary background the correction is dependent of wavenumber $k$, so that the power spectrum oscillates with $k$ \cite{24}.

For simplicity, we just analyze the single field inflationary models with canonical action. If there exists only one scalar field in quasi-de Sitter spacetime, then there is just one perturbation freedom $\delta \phi$ from scalar field. However, there are also four scalar perturbation freedom from the metric $\delta g_{\mu \nu}$. $\delta \phi$ and $\delta g_{\mu \nu}$ do not decouple for scalar perturbation. The gauge invariance removes two of the scalar degrees of freedom by time and spatial reparametrisaions $x_i \rightarrow x_i + \partial_i \epsilon(t, x)$ and $t \rightarrow t + \epsilon(t, x)$ \cite{9}. The constraints in the action remove two other freedom. Thus there is only one scalar degree of freedom left. Thus, we should choose a convenient gauge and discuss the only physical freedom in single field inflationary model.

In general, the spacetime can be decomposed using ADM formalism \cite{50}, and the metric takes the form

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N^i dt)(dx^j + N^j dt),$$

(34)

where $h_{ij}$ is the metric of three dimensional spacial slices. The lapse $N$ and the shift vector $N_i$ contain the freedom of the scalar perturbation, such as time reparametrization and spatial reparametrization. With the $3 + 1$ decomposition, the extrinsic curvature of the spacial slice is

$$K_{ij} = N \Gamma^0_{ij} = \frac{1}{2N}(\dot{h}_{ij} - \nabla_i N_j - \nabla_j N_i),$$

(35)

where $\Gamma^0_{ij}$ is the Christoffel symbol in four dimension spacetime. And the intrinsic curvature of the spacial slices takes the form as

$$R^{(3)} = R - K_{ij}K^{ij} - K^2,$$

(36)

where

$$K = K^i_i.$$
To simplify the action, we introduce another parameter $E_{ij} \equiv NK_{ij}$, so the standard Einstein-Hilbert action can be written as

$$S = \frac{1}{2} \int d^4x \sqrt{h} \left[ NR^{(3)} - 2NV + N^{-1}(E_{ij}E^{ij} - E^2) + N^{-1}(\dot{\phi} - N^i\partial_i\phi)^2 - Nh_{ij}\partial_i\phi\partial_j\phi \right],$$

where $h = \det h_{ij}$. And in the action, there is no time derivative of $N$ or $N_i$, so they are lagrangian multipliers which can be solved directly as the constraint equations.

It is convenient to choose the comoving gauge, in which the inflaton perturbation vanishes in the spatial slices,

$$\delta\phi = 0, \quad h_{ij} = a^2 e^{2\zeta}\delta_{ij}.$$

The spatial metric $h_{ij}$ is the nonperturbative form [51, 52], and the tensor perturbation is omitted for considering only scalar perturbation. In this gauge, $\zeta$ is the physical degree of freedom, which is constant outside the horizon in single field inflation.

In the comoving gauge, the constraint equation is

$$\nabla_i [N^{-1}(E^i_j - \delta^i_j E)] = 0,$$

$$R^{(3)} - 2V - N^{-2}(E_{ij}E^{ij} - E^2) - N^{-2}\dot{\phi}^2 = 0.$$

From these constraints and the equations of the background, the action can be expanded to second order, third order and even higher order of $\zeta$. In order to get the action order by order, we need to expand the $N$ and $N_i$ first. For the shift vector $N_i$, it can always be decomposed as

$$N_i = \partial_i\psi + \tilde{N}_i,$$

where $\partial_i\tilde{N}_i = 0$, and $\psi$ denotes the scalar perturbation of metric $g_{0i}$. These lagrangian multipliers can be decomposed in powers of $\zeta$,

$$N = 1 + \alpha_1 + \alpha_2 + \cdots,$$

$$\psi = \psi_1 + \psi_2 + \cdots,$$

$$\tilde{N}_i = \tilde{N}^{(1)}_i + \tilde{N}^{(2)}_i + \cdots,$$

where the subscript denotes the order, for example $\alpha_n \sim \mathcal{O}(\zeta^n)$. From the constraint equation on $N_i$ (40), we could get

$$\alpha_1 = \frac{\dot{\zeta}}{H} \quad \partial^2\tilde{N}_i^{(1)} = 0.$$
Using appropriate boundary condition, \( \tilde{N}_i \) can be set to 0. From the view point of inflationary perturbation, \( \tilde{N}_i \) represents the vector perturbation of the metric, which vanishes in the boundary. From the constraint equation on \( N \) (40), using the equation of \( R^{(3)} \)

\[
R^{(3)} = -2a^{-2}e^{-2\kappa}[(\partial \zeta)^2 + 2\partial^2 \zeta],
\]

and Friedmann equation of the background, the first order of \( \psi \) is given by

\[
\psi_1 = -\frac{\zeta}{H} + a^2 \frac{\dot{\phi}^2}{2H^2} \partial^{-2} \dot{\zeta}.
\]

To get the action to the quadratic order of \( \zeta \), it is enough to expand \( N \) and \( N_i \) to the first order of \( \zeta \), because the second order term in \( N \) and \( N_i \) will multiply the zero order of constraint equation which is zero. With the same reason, to get the cubic action for \( \zeta \), we do not need expand the \( N \) and \( N_i \) to the cubic order. And the second order expansion of \( N \) and \( N_i \) in the cubic action vanish or reduce to total derivatives. So the action to the quadratic and cubic order of \( \zeta \) can be obtained by substituting \( N, N_i \) to the first order into the action and then expanding the action to the second and the third order of \( \zeta \).

After integrating by parts, the quadratic action of \( \zeta \) takes the form as

\[
S_2 = \frac{1}{2} \int d^4x \frac{\dot{\phi}^2}{H^2} \left[ a^3 \dot{\zeta}^2 - a(\partial \zeta)^2 \right].
\]

To get the field equation and solve the curvature perturbation for different modes, a rescale field is defined as

\[
v \equiv z \zeta, \quad z \equiv a\sqrt{2\epsilon}.
\]

The equation of motion is

\[
v''_k + (k^2 - \frac{z''}{z})v_k = 0,
\]

where the prime \( ' \) denotes the derivative with the conformal time \( \tau \). Considering the slow roll condition the expression of the conformal time is somehow different from the value in de Sitter space,

\[
d\tau = \frac{d(aH)}{(aH)^2(1 - \epsilon)}.
\]

The conformal time can be written as

\[
\tau \simeq -\frac{1}{aH(1 - \epsilon)} \simeq \frac{1 + \epsilon}{aH}.
\]
For power-law inflation, the conformal time takes a form, \( \tau = -1/(aH)(1 - \epsilon) \).
In the field equation,
\[
\frac{z''}{z} \simeq 2a^2H^2(1 + \frac{5}{2}\epsilon - \frac{3}{2}\eta),
\]
so the solution of (48) is the form of Bessel functions,
\[
v_k = \frac{1}{2}(-\pi\tau)^{1/2}
\left[c_1(k)H_\nu^{(1)}(-k\tau) + c_2(k)H_\nu^{(2)}(-k\tau)\right],
\]
where
\[
\nu = \frac{4}{9} + \epsilon - 3\eta.
\]

The solution is similar to (6). In the case of the single field inflation, there is only one degree of freedom for the scalar perturbation, so using scalar field perturbation \( \delta\phi \) as the physical freedom or using the curvature perturbation, the two kinds of description is the same. On the large scale, the curvature perturbation \( \zeta \) is conserved in the single field case, while the scalar field decays to other fields at the end of inflation. Thus it is more clear to use \( \zeta \) as the physical freedom to describe the perturbation. The results of power spectrum from the two descriptions are the same up to a factor 2\( \epsilon \).

When \( \tau \rightarrow -\infty \), the Hubble radius is infinite relative to the modes \( k \), the term \( \frac{z''}{z} \) can be omitted and gravitational effect is negligible, so \( v_k \propto e^{-ik\tau} \) in the Euclidean vacuum. On the other hand, when \( k \rightarrow +\infty \), the contribution to the \( v_k \) comes from both the negative and positive frequency in \( \alpha \)-vacuum,
\[
v_k = c_1 \frac{e^{-ik\tau}}{\sqrt{2k}} + c_2 \frac{e^{+ik\tau}}{\sqrt{2k}}.
\]

We may have the question why the equation of motion is still effective as \( k \rightarrow +\infty \). At scale \( \Lambda \), the new physics and some new freedom will emerge. The new physics scale is set to the Planck scale, string scale or any other ones. In this paper, we assume that the new physics scale \( \Lambda > H \), and \( \Lambda \) is constant. Thus at least on large scales, the field equation (48) is reliable. There are two functions \( c_1 \) and \( c_2 \), which depends on the new physics. What’s the most important, we can determine the \( c_1(k), c_2(k) \) from the boundary condition at \( k/a = \Lambda \). In other words, we can effectively choose the appropriate boundary condition to take into account new physics, even without knowing its nature. Whatever the new physics, it is in the short distance. On the large scale, the solution just contains the new variable \( c_1 \) and \( c_2 \). The information
of the new physics will give different value of $c_1$ and $c_2$ \cite{18}. This change of initial condition will eventually show up in the CMB anisotropy.

Considering the modes with wavelength larger than the new physics scale, but smaller than the Hubble radius,

$$v'_k = -i \sqrt{\frac{k}{2}} c_1 e^{-i k \tau} + i \sqrt{\frac{k}{2}} c_2 e^{i k \tau} . \quad (55)$$

With the limit value of $v_k$ \cite{54} and $v'_k$, and the boundary condition at $k/a = p_c = \Lambda$, $c_1$ and $c_2$ can be determined,

$$c_1 = \frac{\sqrt{2k}}{2} e^{i k \tau_c} [v_k(\tau_c) + \frac{i}{k} v'_k(\tau_c)]$$

$$c_2 = \frac{\sqrt{2k}}{2} e^{-i k \tau_c} [v_k(\tau_c) - \frac{i}{k} v'_k(\tau_c)] , \quad (56)$$

where $\tau_c$ is the time when the $k$ modes is at the boundary, thus it depends on $k$, which can be solved by $k/a = p_c = \Lambda$. As in \cite{19, 24} we set the boundary condition as the wave function at the scale $\Lambda$ only containing emergent wave, and at momentum $p = p_c$, the quantum fluctuation of scalar field takes the form as

$$\frac{d\delta \phi}{dt} = -i p_c \delta \phi . \quad (57)$$

The calculations of $\delta \phi$ and $\zeta$ in single field inflation are similar, so at the boundary

$$\frac{1}{a} \frac{d(v_k/a)}{d\tau} = -i \frac{k}{a^2} v_k . \quad (58)$$

With the relation (58) and the expression for $c_1$ and $c_2$ (56), we could obtain

$$c_1 = \frac{1}{2} [2 + i (\frac{Ha}{k})_c \sqrt{2k e^{i k \tau_c} v_k(\tau_c)}]$$

$$c_2 = -i \frac{1}{2} (\frac{Ha}{k})_c \sqrt{2k e^{-i k \tau_c} v_k(\tau_c)} , \quad (59)$$

where $(Ha/k)_c = H_c/p_c = H_c/\Lambda$. Note that in de Sitter space the Hubble scale $H$ is constant, while in slow roll inflation the Hubble radius is changing $H = H_0 a^{-\epsilon}$. Thus in slow roll inflation $c_1$ and $c_2$ is dependent of $k$.

In order to compare with the situations of de Sitter space, we requires $c_2/c_1 = e^\alpha$ in $\alpha$-vacuum . The solution of field equation \cite{18} takes the form as \cite{52} up to an unimportant overall phase factor. Thus the parameter $e^\alpha$ is,

$$e^\alpha = \frac{c_2}{c_1} = -e^{-2i \kappa \tau_c} \frac{i}{2 \frac{\Lambda}{H} + i} , \quad (60)$$
from which we know that in de Sitter space $e^\alpha$ is independent of $k$, while in slow roll inflation $e^\alpha$ is dependent of $k$. The magnitude of $e^\alpha$ is

$$|e^\alpha| = \sqrt{\frac{1}{4\Lambda^2 + 1}} \sim \frac{H}{2\Lambda}. \quad (61)$$

The scalar power spectrum is

$$P_\zeta = \frac{k^3}{2\pi^2} \left| \frac{v_k}{z} \right|^2 \approx \frac{1}{8\pi^2} \frac{H^2}{\epsilon} \frac{1 + e^{\alpha + \alpha^*} - 2\text{Re} e^\alpha}{1 - e^{\alpha + \alpha^*}} (-k\tau)^{3-2\nu}, \quad (62)$$

where $\nu = \frac{3}{2} + 3\epsilon - \eta$. If $\Lambda \gg H$

$$\text{Re} e^\alpha = -\frac{1}{4\Lambda^2 + 1} \sin[2\frac{\Lambda}{H(1 - \epsilon)}] + \frac{2\Lambda}{4\Lambda^2 + 1} \cos[2\frac{\Lambda}{H(1 - \epsilon)}] \approx \frac{H}{2\Lambda} \cos[2\frac{\Lambda}{H(1 - \epsilon)}], \quad (63)$$

where $H$ is the value when the momentum of modes $k$ is $p_c$. It is clear that $\frac{\Lambda}{H}$ depends on $k$. The exact relation can be derived from

$$\Lambda = p_c = \frac{k}{a(\tau_c)}, \quad H = H_0 a^{-\epsilon}, \quad (64)$$

then

$$\frac{\Lambda}{H} \propto k^\epsilon. \quad (65)$$

Therefore, in slow roll inflation, the correction to the power spectrum will oscillate with the variable wavenumber $k$. The correction is $O(\frac{H}{\Lambda})$, and the correction is likely to be observed in the future experiments.

### 4 Three-point correlator in Euclidean vacuum

As we discuss in the last section, to get the cubic action of $\zeta$ we only need to expand the lagrangian multipliers $N$ and $N_i$ to the first order of $\zeta$. The cubic action could be obtained by substituting the $N$ and $N_i$ to the ADM action, and expanding the action to the third order of $\zeta$. Then by integrating by parts many times and using some technique such as field redefinition, the action can be further simplified. From the cubic action, the three point correlator is simply obtained by using path integral formalism at the tree level.
Integrating by parts and dropping the total derivatives, the cubic action can be written as

\[
S_3 = \frac{1}{2} \int d^4x a^3 \left[ \frac{2}{a^2} \frac{H^2}{H^2} \zeta (\partial \zeta)^2 - \frac{\dot{\zeta}^2}{H^3} - \frac{4}{a^4} \partial^2 \psi_1 \partial_i \zeta \partial_i \psi_1 - \frac{3}{a^4} \zeta \partial^2 \psi_1 \partial^2 \psi_1 \right. \\
+ \left. \frac{1}{a^4} \frac{\dot{\zeta}}{H} \partial^2 \psi_1 \partial^2 \psi_1 + \frac{3}{a^4} \zeta \partial_i \partial_j \psi_1 \partial_i \partial_j \psi_1 - \frac{1}{a^4} \frac{\dot{\zeta}}{H} \partial_i \partial_j \psi_1 \partial_i \partial_j \psi_1 \right]. 
\]

At first glance, the leading order term in the cubic action is \(O(\epsilon^0)\), but after careful integration by parts, all the terms \(O(\epsilon^0)\) and \(O(\epsilon^1)\) will cancel out, so that the leading order of the cubic action is \(O(\epsilon^2)\). If substituting the equation of \(\psi_1\) (45), after integration by parts, the action has terms like \(\ddot{\zeta}\). It is convenient to use the field equation from the quadratic action,

\[
\frac{\delta L}{\delta \zeta} \bigg|_1 = a \left( \frac{d \partial^2 \chi}{dt} + H \partial^2 \chi - \epsilon \partial^2 \zeta \right),
\]

where

\[
\partial^2 \chi \equiv a^2 \epsilon \dot{\zeta},
\]

\(\chi\) is the second term in \(\psi_1\) (45). The final result of the cubic action is

\[
S_3 = \int d^4x \left[ 4a^5 \epsilon^2 H \dot{\zeta}^2 \partial^{-2} \dot{\zeta} + 2f(\zeta) \frac{\delta L}{\delta \zeta} \bigg|_1 \right],
\]

where

\[
f(\zeta) = -\frac{2\eta + 3\epsilon}{4} \zeta^2 + \frac{1}{2} \epsilon \partial^{-2}(\zeta \partial^2 \zeta) + \cdots.
\]

Here we omit the terms in \(f(\zeta)\) which contains the derivative of \(\zeta\) because \(\zeta\) is conserved on the large scale, and any derivative of \(\zeta\) has no contribution. In the cubic action, the contribution from the \(f(\zeta)\) terms is obtained from the redefinition of \(\zeta \rightarrow \zeta_n + f(\zeta_n)\). With the redefinition

\[
S_2[\zeta] \rightarrow S_2[\zeta_n] - \int d^4x 2f(\zeta_n) \frac{\delta L}{\delta \zeta} \bigg|_1,
\]

the second terms in the cubic action is canceled. When we calculate the three-point correlator, both the contributions coming from the cubic \(\zeta\) and the contributions from the redefinition should be taken into account.
The three point correlator could be computed using the path integral formalism in the interaction picture,

$$\langle \zeta(t, k_1) \zeta(t, k_2) \zeta(t, k_3) \rangle_{\text{tree}} = i \int_{t_0}^{t} dt' \langle [\zeta(t, k_1) \zeta(t, k_2) \zeta(t, k_3), L_3(t')] \rangle ,$$  \hspace{1cm} (72)

where \( t_0 \) is some time that the modes is deep inside the horizon. The integration can be divided into three parts. The first part is from the period during which the modes are deep inside the horizon. In this range, the modes oscillate rapidly, so the contribution is simply zero. In Euclidean vacuum the value of \( t_0 \) is set to \(-\infty\), while in \( \alpha \)-vacuum the value of \( t_0 \) is at the boundary for new physics. We assume that \( \Lambda \gg H \), so that \( t_0 \) in the \( \alpha \)-vacuum is also deep inside the horizon. In both situations, the contribution from this part is zero. The second part is the region well outside the horizon. Because the value of \( \zeta \) is constant, the contribution only contains the redefinition of \( \zeta \). The third region is near the horizon, where we use the solution of field equation (52) and compute the three point correlator from the path integral.

The leading order contribution to the tree point correlator in Euclidean vacuum is as follows:

- **Contribution from \( \dot{\zeta}^2 \partial^{-2} \dot{\zeta} \).** We choose \( t_0 = -\infty \) and \( t = 0 \), which will not influence the final results in Euclidean and \( \alpha \)-vacuum.

$$\langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = -i(2\pi)^3 \delta(\sum_i k_i) \zeta_{k_1}(0) \zeta_{k_2}(0) \zeta_{k_3}(0)$$

$$\int_{-\infty}^{0} d\tau \; g \; \frac{d}{d\tau} \zeta^{*}_{k_1}(\tau) \frac{d}{d\tau} \zeta^{*}_{k_2}(\tau) \partial^{-2} \frac{d}{d\tau} \zeta_{k_3}(\tau)$$

$$+ \text{perms} + \text{c.c.} ,$$  \hspace{1cm} (73)

where \( K = k_1 + k_2 + k_3 \), \text{perms} denotes exchanging \( k_1, k_2, k_3 \), and \text{c.c.} represents the complex conjugate of the preceding terms. The prefactor \( g = 4a^3 \varepsilon^2 H \). The three point correlator from \( \dot{\zeta}^2 \partial^{-2} \dot{\zeta} \) is

$$\langle \dot{\zeta}(k_1) \dot{\zeta}(k_2) \dot{\zeta}(k_3) \rangle = -i(2\pi)^3 \delta(\sum_i k_i) \frac{H^4}{24 \varepsilon} \frac{1}{\prod_i k_i^3} \left( \frac{k_1^2 k_2^2}{K} \right) + \text{perms} + \text{c.c.}$$

$$= (2\pi)^3 \delta(\sum_i k_i) (P_\zeta)^2 \frac{1}{\prod_i k_i^3} \varepsilon \left( \frac{1}{K} \sum_{i>j} k_i^2 k_j^2 \right) .$$  \hspace{1cm} (74)

- **The redefinition \( \zeta \mapsto \zeta_n + (-2\eta + 3\varepsilon/4) \zeta_n^2 \).**
The three point correlator from this contribution is

\[(2\pi)^7 \delta \left( \sum_i k_i \right) (P_\zeta)^2 \frac{1}{\prod_i k_i^3} \frac{-2\eta + 3\epsilon}{8} \sum_i k_i^3 \] (75)

- The redefinition \( \zeta \mapsto \zeta + (\epsilon/2)\partial^{-2}(\zeta\partial^2\zeta) \).

The three point correlator from this contribution is

\[ (2\pi)^3 \delta \left( \sum_i k_i \right) \frac{H^4}{2^4 \epsilon^2} \prod_i k_i^3 k_i^2 k_i^3 + \text{perms} \]
\[ = (2\pi)^7 \delta \left( \sum_i k_i \right) (P_\zeta)^2 \frac{1}{\prod_i k_i^3} \frac{\epsilon}{8} \sum_{i\neq j} k_i k_j^2 \] (76)

Finally, taking all the contributions into account, we have the three-point correlator in Euclidean vacuum [51],

\[ \langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle = (2\pi)^7 \delta \left( \sum_i k_i \right) (P_\zeta)^2 \frac{1}{\prod_i k_i^3} A, \] (77)

where

\[ A = \epsilon \frac{1}{K} \sum_{i>j} k_i^2 k_j^2 + \frac{-2\eta + 3\epsilon}{8} \sum_i k_i^3 + \frac{\epsilon}{8} \sum_{i\neq j} k_i k_j^2. \] (78)

5 Non-Gaussianity in \( \alpha \)-vacuum

In this section, we analyze the shape of non-Gaussianity and especially its local form in \( \alpha \)-vacuum. We consider both the de Sitter spacetime and quasi-de Sitter spacetime and show that the local form non-Gaussianity in quasi-de Sitter case has distinctive feature.

The power spectrum and bispectrum are defined as

\[ \langle \zeta(k_1) \zeta(k_2) \rangle \equiv (2\pi)^3 \delta(k_1 + k_2) \frac{2\pi^2}{k_1^3} P_\zeta(k_1), \] (79)
\[ \langle \zeta(k_1) \zeta(k_2) \zeta(k_3) \rangle \equiv (2\pi)^3 \delta(k_1 + k_2 + k_3) B_\zeta(k_1, k_2, k_3). \] (80)

Non-Gaussianity measures the deviation of CMB power spectrum from Gaussian distribution,

\[ \zeta = \zeta_0 + \frac{3}{5} f_{\text{NL}} (\zeta_0^2 - \langle \zeta_0^2 \rangle), \] (81)
in which \( f_{NL} \) characterize the size of non-Gaussianity:

\[
\frac{6}{5} f_{NL} = \prod_i k_i^3 \frac{B_\zeta}{\sum_i k_i^4 4\pi^4 P_\zeta^2} .
\] (82)

In Euclidean vacuum, the three-point correlator is given by (77), (78), so that the parameter of non-Gaussianity is given by

\[
f_{NL} = 10^{-3} \frac{1}{\sum_i k_i^3} A .
\] (83)

The calculation of the three point correlator in the above section can be extended to the \( \alpha \)-vacuum. One can simply plug the value of \( \zeta_k \) for the \( \alpha \)-vacuum into the equation (73) to evaluate the three-point correlator. To distinguish \( \zeta \)'s in different vacua, we use \( \tilde{\zeta}_k \) to denote its value in the \( \alpha \)-vacuum,

\[
\langle \tilde{\zeta}(k_1)\tilde{\zeta}(k_2)\tilde{\zeta}(k_3) \rangle = (2\pi)^7 \delta(\sum_i k_i)(P_\zeta')^2 \frac{1}{\prod_i k_i^4} A' .
\] (84)

Note that \( P_\zeta \) is different in the two backgrounds, and it will give small correction to \( A' \). And \( A' \) contains two parts. In de Sitter space,

\[
A'(ds) = N_\alpha^6 (1 + 4\text{Re}(e^\alpha))A(k_1, k_2, k_3) + \tilde{A}'(ds)
\]

\[
\tilde{A}'(ds) = N_\alpha^3 \text{Re}(e^\alpha)[A(-k_1, k_2, k_3) + A(k_1, -k_2, k_3)
\]

\[
+ A(k_1, k_2, -k_3) - 3A(k_1, k_2, k_3)] ,
\] (85)

where \( A(k_1, k_2, k_3) \) is defined in equation (78) and we neglect the higher order contribution from slow roll parameter and \( \text{Re} e^\alpha \). In de Sitter space, \( \text{Re} e^\alpha \) is independent of \( k \), and \( N_\alpha = 1/\sqrt{1 - e^{\alpha + \alpha^2}} \sim 1 \). In the discussion below the value of \( N_\alpha \) is set to 1, and \( 4\text{Re}(e^\alpha)A(k_1, k_2, k_3) \) is the next order contribution to the first part of \( A' \), so we will also neglect it. In quasi-de Sitter spacetime,

\[
A'(q) = A(k_1, k_2, k_3) + \tilde{A}'(q)
\]

\[
\tilde{A}'(q) = \left[ \text{Re}(e_1^\alpha)A(-k_1, k_2, k_3) + \text{Re}(e_2^\alpha)A(k_1, -k_2, k_3) + \text{Re}(e_3^\alpha)A(k_1, k_2, -k_3)
\]

\[
- \text{Re}(e_1^\alpha) + \text{Re}(e_2^\alpha) + \text{Re}(e_3^\alpha)]A(k_1, k_2, k_3) \right] ,
\] (86)

where the index \( k_1, k_2, k_3 \) of \( e^\alpha \) denote its dependence of wavenumber and the upper index \( (q) \) denotes quasi-de Sitter for short. The value of \( \text{Re}(e^\alpha) \) (63) sensitively depends on the variable \( k \), since \( \Lambda/H \gg 1 \).
There are two forms of non-Gaussianity, which are of particular importance in the data analysis of WMAP. One is the equilateral form and the other is local form non-Gaussianity. The equilateral form requires $k_1 \sim k_2 \sim k_3$, and the three momentum vector compose an equilateral triangle. In this case, in Euclidean vacuum and $\alpha$-vacuum $f_{\text{NL}}$ are the same in the leading order approximation,

$$f_{\text{NL}}^{\text{equil}} \simeq \frac{10}{9} \left(\frac{23}{8}\epsilon - \frac{3}{4}\eta \right). \quad (87)$$

In both case, the equilateral form non-Gaussianity $f_{\text{NL}} \sim O(\epsilon, \eta)$.

The local form non-Gaussianity requires that one of $k_i \ll$ the other two $k$. For instance, $-k_1 + k_2 + k_3 \sim k_3$. The three momentum vectors compose an isosceles triangle, $k_1 = k_2 \gg k_3$. The modes $k_3$ exits horizon much earlier than the other two modes. In Euclidean vacuum, if we take the limit $k_1 = k_2 \gg k_3$, then

$$f_{\text{NL}}^{\text{local}} = \frac{5}{3}(3\epsilon - \eta) = \frac{5}{6}(1 - n_s), \quad (88)$$

where $n_s$ is the spectral index,

$$n_s - 1 = \frac{d\ln P_\zeta}{d\ln k}. \quad (89)$$

In Euclidean vacuum, $P_\zeta \simeq H^2/8\pi^2\epsilon$, and $n_s - 1 = 2\eta - 6\epsilon$. This means that the local form non-Gaussianity in Euclidean vacuum is the same order of $\epsilon$ and $\eta$.

The local form non-Gaussianity in Euclidean vacuum can be estimated by the back-reaction method[51]. The modes $k_3$ leaves the horizon earlier than the other two modes. The effect from the modes $k_3$ is rescaling the background spacetime. The scale factor changes $a(t) \rightarrow a(t)e^{\zeta_3} \sim a(t)(1 + \zeta_3)$, so that the coordinates change accordingly $\delta x = \zeta_3 x$, where $\zeta_3$ is the amplitude of the $k_3$ modes. The back reaction of the background will impact the modes deep in the horizon. The wavenumber decreases $\delta k = -\zeta_3 k$, and the modes will leave the horizon earlier,

$$\delta k = \delta a \cdot H = aH\delta t H. \quad (90)$$

According to the equation $\delta a = aH\delta t$, we get the relation $\delta t = -\zeta_3/H$. From the definition of $f_{\text{NL}}$ (81), the local form non-Gaussianity is

$$f_{\text{NL}}^{\text{local}} = \frac{5}{3} \frac{\Delta \zeta}{\zeta_g^2} = \frac{5}{3} \frac{\Delta t \frac{d\zeta}{dt}}{\zeta_g^2} = \frac{5}{3} \frac{\zeta_3}{\zeta_g^2} \frac{d\zeta}{d\ln k} H \simeq \frac{5}{6}(1 - n_s), \quad (91)$$
where the last equation uses the definition of power index $n_s$. The back-reaction method gives the same result as the one obtained in the direct way, but this method could be applied to other models. It indicates that the microphysics from inflaton cannot have large local form non-Gaussianity in Euclidean vacuum. Even if the action is in non-canonical form, such as DBI inflation \cite{41} and K-inflation \cite{42}, the local form non-Gaussianity is still $1 - n_s$ up to a order one constant.

However, it is a totally different story in $\alpha$-vacuum. Firstly, we consider the situation of de Sitter spacetime, where $\text{Re} e^{\alpha}$ is independent of $k$. The non-Gaussianity parameter $f_{NL}$ contains the contributions from $A$ and $\tilde{A}^{(ds)}$. The contributions from $A$ is the same as in the Euclidean vacuum, which is omitted for conciseness. Thus $f_{NL}$ from $\tilde{A}^{(ds)}$ is

$$f_{NL} = \text{Re}(e^{\alpha}) \frac{10}{3} \sum k_i^3 \left[ -\frac{2\eta + 3\epsilon}{8} \sum k_i^3 - \frac{\epsilon}{8} \sum_{i \neq j} k_i k_j^2 \right] + \epsilon \sum_{i < j} \frac{k_i^2 k_j^2}{-k_1 + k_2 + k_3 + k_1 - k_2 + k_3 + k_1 + k_2 - k_3 - k_1 + k_2 + k_3} \right].$$ (92)

If we take the local form limit $k_3 \ll k_1 \sim k_2$, $-k_1 + k_2 + k_3 \sim k_3$, then

$$f_{NL}^{\text{local}} \approx \frac{10}{3} \text{Re}(e^{\alpha}) \frac{\epsilon}{k_2} \frac{k_3}{k_3}. \quad (93)$$

Although the prefactor $\text{Re}(e^{\alpha})$ is a small quantity $\sim \mathcal{O}(\epsilon^2)$, $k_2/k_3$ can be a huge number in the CMB window, say $k_{\text{max}}/k_{\text{min}} \sim 10^6$ for WMAP data, so that $f_{NL}^{\text{local}}$ could be of order one or even larger in $\alpha$ vacuum.

Secondly, we consider the local form non-Gaussianity in inflationary background, where $e^{\alpha}$ strongly depends on $k$. The local form non-Gaussianity takes the form as

$$f_{NL}^{\text{local}} \approx \frac{5}{3} \text{Re}(e^{\alpha}_k) + \text{Re}(e^{\alpha}_k) \frac{\epsilon}{k_3} \frac{k_3}{k_3} \approx \frac{10}{3} \text{Re}(e^{\alpha}_k) \frac{\epsilon}{k_3} \frac{k_3}{k_3} \approx \frac{5}{3} \Lambda \cos[2 \frac{\Lambda}{H(1 - \epsilon)}] \frac{\epsilon}{k_3} \frac{k_3}{k_3}. \quad (94)$$

Similar to de Sitter spacetime, $f_{NL}^{\text{local}}$ is linear in $\epsilon \frac{k_2}{k_3}$ such that a large local form non-Gaussianity is possible. However since $\text{Re}(e^{\alpha})$ is $k$-dependent, $f_{NL}^{\text{local}}$ has distinctive feature.

As a example, take $\Lambda$ to be $10^{17}$Gev which is phenomenological string scale, $H(k_3) = 10^{15}$Gev, the slow roll parameter $\epsilon = 0.01$, $k_3 = 1$ (with unit $0.002Mpc^{-1}$), and $1 \leq k_2 \lesssim 10^6$. In Fig\[2\] we draw the local form non-Gaussianity: the red line
Figure 2: The red line represents $f_{NL}^{local}$ in de Sitter space and the blue line represents $f_{NL}^{local}$ quasi-de Sitter space. The possible largeness of local form non-Gaussianity in $\alpha$-vacuum seems to violate the bound set by the back-reaction argument. Why the back reaction method is not applicable in $\alpha$-vacuum? Generally speaking, the correlation from two patches of de Sitter spacetime makes the back reaction cannot give the whole effect of non-Gaussianity. In $\alpha$-vacuum the FRW metric cannot describe the physics completely, and we must extend the space to the whole de Sitter space. It is clearer to see the correlation in space coordinates. For instance, one point $x_1$ is far outside the horizon, and the other two points $x_2, x_3$ are deep in the horizon. In the whole de Sitter space, the antipodal point $x_{1A}$ is approaching the points $x_2, x_3$ deep in the horizon. When $x_{1A}$ is near the light cone of $x_2$ or $x_3$, there is divergence in tree-level three-point correlator.

If we carefully analyze $f_{NL}$ in $\alpha$-vacuum (92), we will find that there is also divergence for folded form non-Gaussianity.\footnote{The folded non-Gaussianity in $\alpha$-vacuum is also discussed in [53], in which they deal with...} Therefore, it is interesting to determine...
the shape of non-Gaussianity, which has potential to distinguish different inflationary models if data analysis is accurate enough.\footnote{We would like to thank Xingang Chen for his careful explanation and discussion about the shape of non-Gaussianity.}

The definition of the shape is

$$\frac{A}{k_1 k_2 k_3}.$$  \hspace{1cm} (95)

We divide $A$ into several parts. We use subscript $\epsilon$ and $\eta$ to denote the parts proportional to $\epsilon$ or $\eta$. In any case, we always have the contribution without including the modifications from $\alpha$-vacuum:

$$\mathcal{A}_\epsilon = \epsilon \left( \frac{1}{K} \sum_{i>j} k_i^2 k_j^2 + \frac{3}{8} \sum_i k_i^3 + \frac{1}{8} \sum_{i \neq j} k_i k_j^2 \right),$$ \hspace{1cm} (96)

$$\mathcal{A}_\eta = \eta \left( -\frac{1}{4} \sum_i k_i^3 \right),$$ \hspace{1cm} (97)
which has been discussed in [51]. The modifications in de Sitter spacetime are

\[
\tilde{A}^{(ds)}_\epsilon = \epsilon \text{Re}(e^{\alpha}) \left[ -\frac{3}{8} \sum_i k_i^3 - \frac{1}{8} \sum_{i \neq j} k_i k_j^2 
+ \sum_{i < j} k_i^2 k_j^2 \left( \frac{1}{k_1 + k_2 + k_3} + \frac{1}{k_1 - k_2 + k_3} \right)
+ \frac{1}{k_1 + k_2 - k_3} - \frac{3}{k_1 + k_2 + k_3} \right]
\]

(98)

\[
\tilde{A}^{(ds)}_\eta = \eta \text{Re}(e^{\alpha}) \left( \frac{1}{4} \sum_i k_i^3 \right). 
\]

(99)

And the modifications in quasi-de-Sitter case are

\[
\tilde{A}^{(q)}_\epsilon = \epsilon \left[ \frac{1}{3} (\text{Re}(e_{k_1}^\alpha) + \text{Re}(e_{k_2}^\alpha) + \text{Re}(e_{k_3}^\alpha)) \right] \left[ -\frac{3}{8} \sum_i k_i^3 - \frac{1}{8} \sum_{i \neq j} k_i k_j^2 
+ \sum_{i < j} k_i^2 k_j^2 \left( \frac{\text{Re}(e_{k_1}^\alpha)}{k_1 + k_2 + k_3} + \frac{\text{Re}(e_{k_2}^\alpha)}{k_1 - k_2 + k_3} + \frac{\text{Re}(e_{k_3}^\alpha)}{k_1 + k_2 - k_3} \right)
- \frac{\text{Re}(e_{k_1}^\alpha) + \text{Re}(e_{k_2}^\alpha) + \text{Re}(e_{k_3}^\alpha)}{k_1 + k_2 + k_3} \right]
\]

(100)

\[
\tilde{A}^{(q)}_\eta = \eta \left[ \text{Re}(e_{k_1}^\alpha) + \text{Re}(e_{k_2}^\alpha) + \text{Re}(e_{k_3}^\alpha) \right] \left( \frac{1}{12} \sum_i k_i^3 \right),
\]

(101)

which are very different from de Sitter spacetime case.

It is more illustrative to draw the shapes of non-Gaussianity in various cases. Since \( \tilde{A}^{(ds)}_\epsilon \) and \( \tilde{A}^{(q)}_\epsilon \) are small number relative to \( \mathcal{A}_\eta \), we will not draw their shapes. In Fig. 4 and 5, we draw the shapes of non-Gaussianity in Euclidean vacuum. In Fig. 6, we draw the shape of \( \tilde{A}^{(ds)}_\epsilon \), and in Fig. 7 and 8, we draw the shape of \( \tilde{A}^{(q)}_\epsilon \). In all the figures, we use the following convention: \( k_3 = 1 \), the x-axis is \( k_1/k_3 \), the y-axis is \( k_2/k_3 \), and the value of z-axis is \( \mathcal{A}/k_1 k_2 k_3 \) up to a slow roll parameter. The \( x - y \) plane diagonal from \((0, 1)\) to \((1, 0)\) denotes the folded form, i.e. \( k_1 + k_2 = k_3 \). From the shapes of non-Gaussianity in Euclidean vacuum, the folded form is finite, except the points \((0, 1)\) and \((1, 0)\). On contrast, the shapes of non-Gaussianity in \( \alpha \)-vacuum, the folded form is divergent. What’s more, the \( f_{NL} \) for the folded form in \( \alpha \)-vacuum is divergent as shown in Equation (92).

The local form non-Gaussianity in the shape is near the point \((0, 1)\) and \((1, 0)\). Fig. 7 reflects the oscillating character in inflationary background.
Figure 4: $\mathcal{A}_\epsilon/k_1k_2k_3$. This is the shape for Euclidean vacuum proportional to $\epsilon$. With the shape of $A_\eta/k_1k_2k_3$, they give the leading contribution for $\alpha$-vacuum.

Figure 5: $|A_\eta|/k_1k_2k_3$. This is the shape for Euclidean vacuum proportional to $\eta$. 

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Figure 6: $\tilde{A}^{(ds)}/k_1k_2k_3$. This is the local $\alpha$-vacuum shape for de Sitter space. The local form non-Gaussianity near the points (1,0) and (0,1) is large.

Figure 7: $\tilde{A}^{(q)}/k_1k_2k_3$. This is the local $\alpha$-vacuum shape proportional to $\epsilon$ for inflationary space. The local form non-Gaussianity near the points (1,0) and (0,1) is large, and it reflects some oscillating character.
6 Discussion

In this paper, we studied the physical implication of $\alpha$-vacuum on the CMB non-Gaussianity. We found that the $\alpha$-vacuum may lead to large local form non-Gaussianity, and its signature is distinct. In de-Sitter spacetime, the local form is large which is proportional to $k_2/k_3$, and in the inflationary background, the local form could still be large but there is oscillating character for $f_{NL}$. Another distinctive feature of $\alpha$-vacuum is that it leads to a divergent folded form non-Gaussianity. To illustrate the picture more clearly, we drew the shapes for different vacua and different backgrounds. We found that all of these figures have dramatically different feature. For the de Sitter spacetime the local form is large, and for the inflationary spacetime the local form not only large, but also oscillating. These are different from standard slow-roll inflation, where the local form is proportional to spectral index of scalar perturbation.

The observational feature from our study is different from \cite{53} which only considered the $\alpha$-vacuum correction from the de Sitter background. And even though

Figure 8: $|\tilde{A}_n^{(q)}|/k_1 k_2 k_3$. This is the local $\alpha$-vacuum shape proportional to $\eta$ for inflationary space. The local form is large and oscillating.
our study has some overlaps with the one in [30], our method and main results are different. In [30], they used the effective field theory (EFT) method during inflation. Since the energy scale of inflation is very high, from the point of view of EFT, some higher order derivative terms will be not negligible, and the correction to the non-Gaussianity origins from the stronger interaction at the beginning of the inflation. Comparing with trans-Planckian physics from $\alpha$-vacua, EFT method could be another branch in inflationary cosmology. Our treatment focused on the influence of the fluctuation vacuum, while EFT emphasized the action. From the view of experiments, due to different motivation, the final prediction of observation is not the same. [30] analyzed the folded form non-Gaussianities, which has not been analyzed in WMAP5 yet. In our study, we concluded that the trans-Planckian effect would enhance the local form non-Gaussianities, which is the essential part of the WMAP data analysis.

There are various interesting issues to address on the implication of $\alpha$-vacuum on inflation. First of all, the loop effect in the $\alpha$-vacuum is still an open question. Steven Weinberg has given a wonderful discussion about the loop effect in inflationary correlators [54, 55, 56]. In $\alpha$-vacuum the problem is focused on how to renormalize the scalar perturbation in the loop diagram. In some papers, it has been argued that the $\alpha$-vacuum is not well defined in de Sitter space due to the divergence [31, 32]. However in [33] using the Schwinger-Keldysh formalism a consistent renormalization method has been constructed to deal with the divergence. The other discussion on this issue can be found in [34]. It would be interesting to understand the loop effect and its physical implications in $\alpha$-vacuum.

Secondly, the non-Gaussianity in $\alpha$-vacuum is sensitively dependent of the initial condition. In the case of single field inflation with higher derivative terms, the sound speed $c_s$ is not 1. For example in the DBI inflation [41] and K-inflation [42], the lagrangian is not canonical, and the sound speed $c_s \ll 1$ in some situations. The sound horizon $c_s H^{-1}$ may be smaller than the length scale of new physics. In this case, the initial condition at the new physics scale is chosen in the place larger than sound Hubble scale. From the calculation of the non-Gaussianity, we know that in the path integral formalism the near Horizon crossing region impacts the results of non-Gaussianity. The divergence of the non-Gaussianity in the $\alpha$-vacuum may not
exist as pointed out in the paper [53]. This situation also appear in some special trans-Planckian physics, such as noncommutative inflation [36], in the IR region the effective string scale is smaller than the Hubble scale.

Thirdly, the different initial condition of $\alpha$-vacuum will dramatically change the correction of power spectrum [25, 26], thus it predicts different non-Gaussianity. Meanwhile, the trans-Planckian dispersion relation and noncommutative geometry will also give different prediction for non-Gaussianity.

Fourthly, it is also interesting to consider the physical implication of the $\alpha$-vacuum in other inflationary models. One class of them is the multiple field inflation [43]. It has some very different signatures from the single field inflation: it has non-negligible gravitational wave and large local form non-Gaussianity [44, 45, 46, 47]. Another class of inflation models is inspired by string theory. In particular, DBI inflation is a very remarkable scenario. It may give large equilateral non-Gaussianity but no local form non-Gaussianity. It is worthwhile to discuss the $\alpha$-vacuum effect in these inflationary models.

Finally, it could be expected that the tri-spectrum of CMB from $\alpha$-vacuum is different from the one in Euclidean vacuum. A more careful investigation would be valuable.

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