Constraints on the unified dark energy–dark matter model from latest observational data

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Abstract. The generalized Chaplygin gas (GCG) is studied in this paper by using the latest observational data including 182 gold sample type Ia supernovae (Sne Ia) data, the ESSENCE Sne Ia data, the distance ratio from \( z = 0.35 \) to 1089 (the redshift of decoupling), the cosmic microwave background shift parameter and the Hubble parameter data. Our results rule out the standard Chaplygin gas model (\( \alpha = 1 \)) at the 99.7% confidence level, but allow for the \( \Lambda CDM \) model (\( \alpha = 0 \)) at the 68.3% confidence level. At a 95.4% confidence level, we obtain \( w = -0.74^{+0.10}_{-0.09} \) and \( \alpha = -0.14^{+0.30}_{-0.19} \). In addition, we find that the phase transition from deceleration to acceleration occurs at redshift \( z_{q=0} \sim 0.78 - 0.89 \) at a 1σ confidence level for the GCG model.

Keywords: dark energy theory, supernova type Ia

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1. Introduction

The generalized Chaplygin gas (GCG) model [1] has been proposed as a candidate for the dark energy which presumably drives the observed current cosmic accelerating expansion. A unique feature of this model is that it has an exotic equation of state

\[ w(z) = \frac{p_{gcg}}{\rho_{gcg}} = -\frac{A}{\rho_{gcg}^{\alpha+1}}, \]  

(1)

where \( \rho_{gcg} \) and \( p_{gcg} \) are the energy density and pressure of the GCG respectively, \( A \) and \( \alpha \) are two model parameters and \( z \) is the redshift. The case of \( \alpha = 1 \) corresponds to the standard Chaplygin gas model [2]. Using the above expression one can solve the conservation equation of the GCG energy in a Robertson–Walker metric to obtain

\[ \rho_{gcg} = [A + B(1 + z)^{3(1+\alpha)}]^{1/(1+\alpha)}. \]  

(2)

Here \( B \) is an integration constant. It is interesting to note that the GCG smoothly interpolates between a non-relativistic matter phase in the past and a de Sitter phase at late times.

As a result, the GCG has been suggested as a model of unified dark matter and dark energy (UDME) [1], and has thus attracted a great deal of interest and many works have been done on this model [3]–[29]. It was claimed that this model produces an exponential blow-up matter power spectrum [4]. Let us note, however, that this problem can be resolved by admitting a unique decomposition of the GCG into dark energy and dark matter components [5]. Currently many observational constraints have been placed on this model, including those from the Sne Ia [6]–[15], [27], the CMBR [15]–[18], gamma-ray bursts [19], the gravitational lensing [6,14], [20]–[23], the x-ray gas mass fraction of clusters [12]–[14], the large scale structure [15,24,25], the Hubble parameter versus redshift data [27] and the age of high redshift objects [26]. However the results from different data are not always consistent with one another.

The aim of this paper is to investigate what new constraints can be obtained on the GCG using the latest observation data sets and to see whether or not the results from these data are consistent with previously obtained ones. A spatially flat universe is assumed in our discussion. The data sets used in this paper include the newly released gold + HST sample supernova (Sne Ia) data [30] and ESSENCE Sne Ia sample [31]. In addition
the combinations of these new supernova data with the Hubble parameter data \cite{32}, the CMB shift parameter \cite{33} and the distance ratio from $z = 0.35$ to 1089 (the redshift of decoupling) measured from the baryonic acoustic oscillations (BAOs) from Sloan Digital Sky Survey (SDSS) \cite{34} are analysed.

2. The luminosity distance of the GCG model

Using the equations (1) and (2), it is easy to obtain that the present value of the equation of state for the GCG is

$$w = \frac{A}{A + B}.$$ (3)

For a flat universe containing the baryonic matter and the GCG energy as a unification of dark energy and dark matter, the Friedmann equation can be expressed as

$$H^2(z, H_0, w, \alpha) = H_0^2 E^2(z, w, \alpha),$$ (4)

where

$$E(z, w, \alpha) = \left[ \Omega_b(1 + z)^3 + (1 - \Omega_b)(1 + w)(1 + z)^{3(1+\alpha)} - w \right]^{1/(1+\alpha)}.$$ (5)

$\Omega_b$ is the present dimensionless density parameter of baryonic matter and $H_0 = 100h$ km s$^{-1}$ Mpc$^{-1}$ is the present Hubble constant. The Hubble Space Telescope key projects give $h = 0.72 \pm 0.08$ \cite{35} and the WMAP observations give $\Omega_b h^2 = 0.0233 \pm 0.0008$ \cite{36}. Apparently the case of $\alpha = 0$ corresponds to the scenario of the cosmological constant plus the dark matter in which the present dimensionless density parameter of the cosmological constant is $\Omega_\Lambda = -w(1 - \Omega_b)$. For a flat universe, the luminosity distance $d_L(z)$ can be expressed as

$$d_L(z, H_0, w, \alpha) = (1 + z) \int_0^z \frac{dz'}{H(z', H_0, w, \alpha)}.$$ (6)

3. Observational constraints

The Sne Ia data sets considered in this paper include the latest gold data set and ESSENCE data set. Recently Riess et al \cite{30} released the 182 gold Sne Ia data set with the MLCS2k2 method. The data set consists of 119 previously published data points \cite{37}, 16 points discovered recently by the Hubble Space Telescope (HST) and 47 points from the first-year release of the SNLS data set \cite{38}.

The ESSENCE programme (equation of state: supernovae trace cosmic expansion—an NOAO Survey programme) is designed to measure the history of cosmic expansion over the past five billion years. The four-year data were released in \cite{31}. By using the MLCS2k2 light curve fitting technique with the ‘glosz’ prior to measure luminosity distances, 60 Sne Ia points are obtained. Here the 105 Sne Ia points given in table 9 in \cite{31} are used which contain 45 nearby Sne Ia and 60 ESSENCE Sne Ia.

In order to break the degeneracy between $\alpha$ and $w$, external constraints are required. Here we use the distance ratio from $z = 0.35$ to 1089 (the redshift of decoupling), the
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Figure 1. The 68.3%, 95.4% and 99.7% confidence regions for $w$ versus $\alpha$. The solid lines, dashed lines, dotted lines and dot–dashed lines represent the results from the 182 gold sample, the distance ratio, the CMB shift parameter and the Hubble parameter data respectively. The coloured areas show the results from the combination of these four databases.

CMB shift parameter and the Hubble parameter data. The distance ratio $R_{0.35}$ measured from the SDSS BAOs from [34] is expressed as

$$R_{0.35}(w, \alpha) = \left(\frac{0.35}{E(0.35, w, \alpha)}\right)^{1/3} \left[\frac{\int_0^{0.35} dz/E(z, w, \alpha)}{\int_0^{1089} dz/E(z, w, \alpha)}\right]^{2/3}. \tag{7}$$

Observations impose the constraint $R_{0.35} = 0.0979 \pm 0.0036$.

The CMB shift parameter $R$ is also used to constrain the GCG model here and it can be expressed as [33]

$$R(w, \alpha) = \sqrt{\Omega_m} \int_0^{z_r} \frac{dz}{E(z, w, \alpha)}, \tag{8}$$

for a flat universe, where $z_r = 1089$ and $\Omega_m = \Omega_b + (1 - \Omega_b)(1 + w)^{1/(1+\alpha)}$ [13, 14, 28, 29] is the effective matter density parameter for the GCG as a UDME model. From the three-year WMAP results [39], the shift parameter is constrained to be $R = 1.70 \pm 0.03$ [40].

Recently, Simon et al [32] obtained 9 data points of $H(z)$ at redshift $z_i$ based on differential ages of passively evolving galaxies determined from the Gemini Deep Deep
Figure 2. The 68.3%, 95.4% and 99.7% confidence regions for \( w \) versus \( \alpha \). The solid lines, dashed lines, dotted lines and dot–dashed lines represent the results from the 105 ESSENCE Sne Ia data, the distance ratio, the CMB shift parameter and the Hubble parameter data respectively. The coloured areas show the results from the combination of these four databases.

Survey [41] and archival data [42] at redshift \( 0 \lesssim z \lesssim 1.8 \) [32], [43]–[45]. These estimated \( H(z_i) \) data have been used to constrain the cosmological models [27,32,44,45]. Here we also use these data to constrain the GCG model.

The constraints on the GCG model parameters \( \alpha \) and \( w \) can be obtained by minimizing

\[
\chi^2(H_0, w, \alpha) = \chi^2_{\text{Sne}}(H_0, w, \alpha) + \chi^2_{\text{dis}}(w, \alpha) + \chi^2_{\text{CMB}}(w, \alpha) + \chi^2_H(H_0, w, \alpha),
\]

where

\[
\chi^2_{\text{Sne}}(H_0, w, \alpha) = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i, H_0, w, \alpha)]^2}{\sigma^2_{\mu_i}},
\]

\[
\chi^2_{\text{dis}}(w, \alpha) = \frac{(R_0.35(w, \alpha) - 0.0979)^2}{0.0036^2},
\]

\[
\chi^2_{\text{CMB}}(w, \alpha) = \frac{(R(w, \alpha) - 1.70)^2}{0.03^2},
\]

and

\[
\chi^2_H(H_0, w, \alpha) = \sum_i \frac{[H_{\text{obs}}(z_i) - H_{\text{th}}(z_i, H_0, w, \alpha)]^2}{\sigma^2_{H_i}}.
\]
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Figure 3. The 68.3%, 95.4% and 99.7% confidence regions for \( w \) versus \( \alpha \). The solid lines, dashed lines, dotted lines and dot–dashed lines represent the results from the 182 gold sample plus 60 ESSENCE Sne Ia data, the distance ratio, the CMB shift parameter and the Hubble parameter data respectively. The coloured areas show the results from the combination of these five databases. The best fit happens at \( w = -0.74 \) and \( \alpha = -0.14 \).

Here the distance modulus \( \mu(z) = m(z) - M \). The parameters \( M \) and \( m \) are the absolute and apparent magnitude respectively. The theoretical apparent magnitude \( \mu_{\text{th}} \) is relative with the luminosity distance \( d_L \): \( \mu_{\text{th}} = 5 \log_{10}(d_L(z)) + 42.38 \). Since we are interested in the model parameter \( w \) and \( \alpha \), \( H_0 \) in the \( \chi^2 \) is a nuisance parameter, and we marginalize over it to get the probability distribution function of \( w \) and \( \alpha \):

\[
L(w, \alpha) = \int dH_0 P(H_0) e^{-\chi^2(H_0, w, \alpha)/2},
\]

where \( P(H_0) \) is the prior distribution function for the present Hubble constant. In this paper a Gaussian prior \( H_0 = 72 \pm 8 \) km S\(^{-1}\) Mpc\(^{-1}\) is considered.

The confidence contours of \( w \) and \( \alpha \) are shown in figures 1, 2, 3, in which solid, dashed, dotted and dot–dashed lines represent, respectively, the results from the Sne Ia, the distance ratio, the shift parameter and the Hubble parameter data. The results from the combination of these four data sets are given by the coloured regions in these figures. Figure 1 shows the constraints from the 182 gold Sne Ia sample + the distance ratio + the shift parameter + the Hubble parameter data. It is easy to see that a combination of these data sets rules out the \( \Lambda \)CDM at the 68% confidence level and the Chaplygin gas at the 99.7% confidence level.
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**Figure 4.** The evolution of the deceleration parameter $q(z)$ from fitting it to the gold and ESSENCE Sne Ia + the distance ratio + the CMB shift parameter + the Hubble parameter data. The thick black line is drawn by using the best fit parameters. The shaded area shows the $1\sigma$ error.

In figure 2 we give the results from the 105 ESSENCE Sne Ia data + the distance ratio + the shift parameter + the Hubble parameter data. This figure shows that a combination of them allows for the $\Lambda$CDM at the $68.3\%$ confidence level; however it rules out the Chaplygin gas at the $99.7\%$ confidence level.

Figure 3 displays the results from the 182 gold Sne Ia + 60 ESSENCE Sne Ia + the distance ratio + the shift parameter + the Hubble parameter data. Here in order to cancel the double counting of Sne Ia data, the 45 nearby Sne Ia data are discarded. The combination of these data sets rules out the Chaplygin gas model at the $99.7\%$ confidence level and allows for the $\Lambda$CDM at the $68.3\%$ confidence level. The degeneracy between $w$ and $\alpha$ is broken, and a very stringent constraint on GCG from these data sets, i.e., at a $95.4\%$ confidence level $w = -0.74^{+0.10}_{-0.09}$ and $\alpha = -0.14^{+0.30}_{-0.19}$, is obtained.

In addition the deceleration parameter $q$ is studied for the GCG model. The results are shown in figure 4. We obtain that the phase transition from deceleration to acceleration of our universe occurs at the redshift $z_{q=0} \sim 0.78 - 0.89$ at a $1\sigma$ confidence level, which is larger than that estimated using the 157 gold data in [37] ($z_{q=0} \sim 0.33 - 0.59$) and 182 gold data in [46] ($z_{q=0} \sim 0.28 - 0.59$), whereas it is comparable with that obtained in [47] from gold + SNLS Sne Ia data for DGP brane ($z_{q=0} \sim 0.8 - 0.93$). The present acceleration is also investigated; we obtain $-q_{z=0} \sim 0.50 - 0.61$ at a $1\sigma$ confidence level.
4. Conclusions and Discussion

Observations indicate that our universe now is dominated by two dark components: dark energy and dark matter. The GCG model has an interesting characteristic: it can unify the dark matter and dark energy. In this paper we mainly focus on the constraints on this UDME model from newly released observational data. The latest gold Sne Ia and ESSENCE Sne Ia data are used. The distance ratio from \( z = 0.35 \) to the redshift of decoupling (\( z = 1089 \)), the CMB shift parameter and the Hubble parameter data are also used as external constraints on this model. We find that the degeneracy between parameters \( w \) and \( \alpha \) is broken by a combination of these data sets. The joint analysis indicates that the Chaplygin gas model (\( \alpha = 1 \)) is ruled out at the 99.7\% confidence level. This is the same as what was obtained in [13, 17, 27] using other observation data. The scenario of cosmological constant plus the dark matter (\( \alpha = 0 \)) is allowed at a 1\% confidence level. At a 95.4\% confidence level we find \( w = -0.74^{+0.10}_{-0.09} \) and \( \alpha = -0.14^{+0.30}_{-0.19} \), which are comparable with what was obtained in [13, 17], where \( \alpha = -0.09^{+0.54}_{-0.33} \) is obtained from the x-ray gas mass fractions of galaxy clusters plus the dimensionless coordinate distance of Sne Ia and FRIIb radio galaxies [13], \( \alpha < 0.6 \) from the CMBR power spectrum measurements from BOOMERANG and Archeops plus the Sne Ia data [17], and \(-0.21 \leq \alpha \leq 0.42 \) from the Hubble parameter versus redshift data + the size of SDSS BAO + SNLS Sne Ia. By investigating the deceleration parameter, we find that for the GCG model the universe enters the acceleration era at the redshift \( z_{q=0} \sim 0.78 - 0.89 \) in a 1\% confidence level.

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