Mathematical modelling of physical processes in a synchronous generator with permanent magnets for output voltage stabilization circuits

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Abstract. When some consumers are powered from independent sources of electric energy, there is needed stabilization of output voltage on the generator in a wide range of load current changes. The use of traditional stabilization methods in the power supply systems with electric machines does not always provide the required level of the supply voltage. The use of synchronous magnetoelectric generators in voltage stabilization circuits is of particular interest in this regard. For determining the possibility of their use, the operating conditions of electric machines were analyzed, the main factors affecting their external characteristics when varying the load level were identified. The studies were based on the mathematical model that ultimately made it possible to obtain dependences convenient for use in engineer.

1. Introduction

At present, a large number of AC electric machines have been developed, the physical processes in which are calculated by methods that are in turn based on a variety of approaches [1-8]. Moreover, the magnitude of the error introduced into the calculations always depends on the assumptions made in the description of mathematical models of machines.

It is known that the highest energy performance (the efficiency and the power factor), a sizeable specific moment (the moment to mass ratio) and minimum dimensions can provide a generator with magnetoelectric excitation from permanent magnets (MEG). However, its use in output voltage stabilization systems with a frequency of 400 Hz is minimal since:

- at the maximum load, the generator must develop the EMF at which the output voltage is equal to the rated value. When the load decreases, the generator is "overexcited", and its output voltage is higher than the permissible value. For eliminating this negative phenomenon in power supply systems, converters are used, the large mass and dimensions of which negate all the advantages of MEG;

- in many power supply systems, reliable operation of the protection is achieved by developing, within 5-7 seconds, the current three times exceeding its rated value. In generators, where the main magnetic flux is formed due to the winding, this requirement is realized due to the multiple (often 8-10 times) increase in the excitation current. In magnetoelectric generators, the rotor of which contains ferromagnetic structural elements, it is practically impossible to provide the required current value ($I_{sc} = E_1 / x_d$).
In this work, there are presented the results of analyzing the limit possibilities of using MEG in the systems of output voltage stabilization.

2. Physical processes and principal mathematical relations

If there are no ferromagnetic elements in the design of the MEG rotor, the inductive resistances along the longitudinal and transverse axes of the machine are the same. If we neglect the differential resistance of scattering, then they can be conditionally divided into three components: inductive resistance determined by the flow passing through the air gap and rotor; inductive resistance of slot scattering; frontal scattering inductance.

To evaluate the first one, we neglect the losses of the three-phase winding MDF in the stator magnetic circuit. Then, with the number of pole pairs \( p = 2 \) and the angle \( \alpha = \pi/4 \) on the stator surface, the law of the scalar magnetic potentials difference changing can be represented as follows:

\[
\Delta \varphi_m(\alpha) = \frac{2m\sqrt{2}}{\pi} \left( \frac{w_1 k_{win1}}{p} \right) I \sin p\alpha, \tag{1}
\]

where \( m \) is the number of phases; \( I \) is the current acting value in the phase; \( w_1 \) is the turning number in the phase, \( k_{win1} \) is the phase factor.

As numerical calculations show (Fig. 1 and 2), the configuration of magnetic field lines emerging from the stator at points corresponding to the \( \alpha \) angle does not change significantly with the varying number of slots \((z_1)\) and the bore diameter \((D_a)\) in the \( 116 \leq D_a \leq 175 \) mm range.

\[
\frac{1}{l_m} = \frac{1}{D_a} (0,91 + 6,31e^{-3.98\alpha}) . \tag{2}
\]

Then, the law of changing the normal component of the magnetic induction component on the inner surface of the stator can be written in the form:

\[
B(\alpha) = \frac{\mu_0 \Delta \varphi_m}{l_m} = \frac{2\sqrt{2}m\mu_0 w_1 k_{win1} I (0,91 + 6,31e^{-3.98\alpha})}{\pi p D_a} \sin p\alpha . \tag{3}
\]

Figure 1. Magnetic field force line of stator scattering at: (a) \( z_1=72 \) and \( D_a=150 \) mm; (b) – scattering at \( z_1=60 \) and \( D_a=150 \) mm.

This circumstance allows simplifying the calculations significantly and presenting a value inversely proportional to the length of the magnetic field force line \( l_m \) (m), by such a dependence, as, for example, in expression (2):

\[
1/l_m = \frac{1}{D_a} (0,91 + 6,31e^{-3.98\alpha}) .
\]

\[
B(\alpha) = \frac{\mu_0 \Delta \varphi_m}{l_m} = \frac{2\sqrt{2}m\mu_0 w_1 k_{win1} I (0,91 + 6,31e^{-3.98\alpha})}{\pi p D_a} \sin p\alpha .
\]
To determine one of the components of inductance \( L_δ \) caused by the presence of magnetic flux in the air gap and rotor \( \Phi_m \), we find:

\[
\Phi_m = 2 \int B(r) dr = \mu_0 \frac{2\sqrt{2m_1 w_{k_{win}} L_{win}} \pi^4}{\pi p} \left[ (0.91 + 6.31e^{-3.98\alpha}) \sin p\alpha + \frac{6.31e^{-3.98\alpha}}{p} \right] 0.91 \sin p\alpha + \frac{6.31e^{-3.98\alpha}}{p} + \frac{1.01A_1}{15.856 + p^2} + \frac{6.31pA_1}{15.856 + p^2},
\]

(4)

where \( l_a \) is the stator pack length, m; \( A_1 = \frac{\mu_0 2\sqrt{2m_1 w_{k_{win}} L_{win}} \pi^4}{\pi p} \).

Since the magnetic force line approximation is determined under the condition of \( p = 2 \), the number of pole pairs in (4) is a constant, and the magnetic flux components are equal:

\[
\Phi_m = A_1(0.455 - 0.051 + 0.636) = 1.041 A_1.
\]

Then, dividing the flux linkage by the current, we obtain the final form of the expressions for inductance and resistance determined by the fluxes closing through the regions occupied by the rotor:

\[
L_δ = 2.08 \frac{m_1 \mu_0 (w_{k_{win}})_{win}^2 l_a}{\pi p} \quad \text{and} \quad X_δ = 4.16 f_1 \frac{m_1 \mu_0 (w_{k_{win}})_{win}^2 l_a}{p},
\]

(5)

where \( f_1 \) is the rotor rotation frequency.

In case of considering a generator with a rotor having permanent magnets with relative permeability \( \mu_r \), the values obtained from (5) must be multiplied by the correction coefficient \( [1 + \alpha_δ(\mu_r - 1)] \), taking into account the presence of pole overlap \( \alpha_δ \) and non-magnetic structural elements.

As a rule, for high-frequency generators, the number of turns in the coil is taken to be equal to unity. Therefore, the stator slot is open, less often half-open. Figure 2 shows the main geometric parameters taken into account when calculating the relative conductivity of the slot scattering.

![Figure 2. The machine stator slot configuration](image)

The expression determines the slot conductivity:

\[
\lambda_{\text{cond}} = \frac{h_3}{4b_n} + \frac{h_4 - h_3}{3b_n} \frac{k_p}{h_k} + \frac{h_k + h + h_2}{b_n} k_β',
\]

(6)

where \( k_p \approx 0.25 + 0.75\beta; \) \( k_β \approx 0.25 + 0.75k_p; \) \( \beta \) is the winding pitch shortening; \( h_2 \) is the distance from the upper conductor to the wedge. This empirical expression is hugely inconvenient for the subsequent analysis, so we try to simplify it. We neglect the first term, which gives an error of no more than 6% of the conductivity value and limit the number of variables by the width and depth of the slot \( h_{a1} \). In further calculations, we use the simplified expression of conductivity:

\[
\lambda_{\text{cond}} \approx \frac{h_{a1}}{3b_n},
\]

(7)
Permitting to calculate $\lambda_n$ (7) with the error not exceeding 4% of the value determined by (6). Specific conductivity and frontal scattering inductance are determined using the recommendations:

$$\lambda_{\text{cond}} = 0.34 \frac{q}{l_a} (l_{p1} - 0.64\beta_1), \quad (8)$$

where $q$ is the number of slots per a pole and phase; $\tau_1$ is the pole division; $l_{p1}$ is the winding frontal part length on the side of the machine. Accepting that the conductors in the winding frontal part have the shape of an arc with the diameter $\frac{\pi D_a\beta}{2p}$, we transform (8) to a more convenient form:

$$\lambda_{\text{cond}} = 0.158 \frac{q\pi D_a\beta}{pl_a}. \quad (9)$$

Then the inductive resistances caused by the flows of the slot $X_s$ and the frontal $X_f$ scattering can be calculated as follows:

$$X_s = 0.527 f_1 w_1^2 \frac{l_m b_{1m}}{pq b_n \cdot 10^5} \quad \text{and} \quad X_f = 0.784 f_1 w_1^2 \frac{D_a \beta}{p^2 \cdot 10^5}. \quad (9)$$

3. Experimental Investigation

The analysis takes into account the following factors:

1. The centrifugal force acting on the magnet mounts

$$F_k = \gamma_{\text{mag}} \omega^2 r_3 (r_{\text{ext}} - r_{\text{int}}) \alpha_0 l_a / 3,$$

where $\gamma_{\text{mag}}$ is the magnet specific density; $\omega_p$ is the angular frequency of the rotor rotation; $r_{\text{ext}}$ and $r_{\text{int}}$ are the magnets outer and inner radiuses.

2. The type of permanent magnet (there were considered the options using a samarium-cobalt magnet KS25DC-240 and a neodymium-based magnet 45SH).

3. Magnetic characteristics of 49k2F and 49KF steels used in manufacturing the stator and rotor bushings.

4. The overload coefficient $k_o$ (voltage safety margin), which is the ratio of the EMF in the idle mode to the rated voltage in the calculations, was taken to be $k_o = E_{\text{idle}} / U_n = 160 / 115 = 1.39$.

Table 1 presents the results of calculations according to (5), (9), as well as short-circuit current $I_{sc}$ and double load current $2I_{\text{nom}}$ in the $D_a$ function. All the calculations were carried out under the condition that the length of the stator pack was 0.1 m, and the magnets on the rotor were made of KS25DC-240/ high-temperature grade 45SH magnets.

| $D_a$, mm | 90   | 110  | 130  | 150  | 180  |
|----------|------|------|------|------|------|
| $B_n$, tesla | 0.67/0.82 | 0.73/0.91 | 0.80/0.97 | 0.88/1.04 | 0.97/1.13 |
| $q$ | 9.00/7.36 | 6.75/5.43 | 5.22/4.31 | 4.11/3.48 | 3.10/2.67 |
| $X_s$, ohms | 0.2946/0.1864 | 0.1654/0.1014 | 0.0989/0.0639 | 0.0613/0.0417 | 0.0348/0.0254 |
| $X_{sc}$, ohms | 0.1317/0.1610 | 0.0526/0.0654 | 0.0198/0.0240 | 0.0117/0.0138 | 0.0055/0.0064 |
| $X_{fsc}$, ohms | 0.0823/0.0550 | 0.0525/0.0340 | 0.0367/0.0250 | 0.0286/0.0205 | 0.0195/0.0145 |
| $X_s$, ohms | 0.5086/0.4024 | 0.2705/0.2008 | 0.1554/0.1129 | 0.1016/0.0760 | 0.0598/0.0454 |
| $I_{sc}$, amp | 314.6/397.6 | 591.5/796.8 | 1029.6/1417.2 | 1574.8/2105.3 | 2675.6/3524.2 |
| $2I_{\text{nom}}$, amp | 121.7/153.8 | 228.8/308.3 | 398.3/548.3 | 609.2/814.5 | 1035/1363 |

Considering that $I_{sc}$ in all the calculations exceeds $3I_{\text{nom}}$, the value equal to half the value presented in the last row of Tables 1, 2 is accepted as the rated current. Below, Figure 3 shows the dependences of the total generator power $S_n$ (kVA) and its specific weight per unit power $G/S_n$ (kg/kVA) as a function of the bore diameter.
As verification calculations show, using special computer programs and experimental studies of two prototype generators, the error in the parameters given in the Tables and Figure 3 does not exceed 10-12%. Therefore, in further analysis, we consider them reliable.

To determine the effect of machine parameters on the rigidity of its characteristics, we perform some transformations. In formulas (5) and (9), we represent the number of turns in the phase as

$$w_1 = 2pq,$$

and the ratio

$$\frac{h_{a1}}{b_n}$$

is expressed in terms of the cross-sectional area of the slot $S_{n1}$ and the inner diameter of the stator. Then:

$$b_n = \frac{\pi D_{a} k_1}{2mpq} \quad \text{and} \quad \frac{h_{a1}}{b_n} = \frac{S_{n1}}{b_n^2} = \frac{4m^2 p^2 q^2 S_{n1}}{\pi^2 D_{a}^2 k_1^2}.$$

Assuming that the induction in the gap $B_\delta$ is distributed according to the sinusoidal law, we’l divide each inductive resistance by the EMF value $E_1 = 8.88f q k_{w1} D_{a} k_{D} B_\delta$, thereby determining the components of the objective function $F$, which affects the rigidity of the external characteristic.

The expression for $F$ that is inversely proportional to the short circuit current has the form:

$$F = 1.41 \cdot 10^{-5} \frac{k_{w1} q}{D_{a} B_\delta} + 3.46 \cdot 10^{-5} \frac{k_{1} k_{w1} D_{a} B_\delta}{B_{s}} + 0.353 \cdot 10^{-5} \frac{\beta q}{l_{a} k_{w1} B_\delta}.$$

(10)

Here, four parameters can be considered variables: the bore diameter $D_{a}$, the slot area $S_{n1}$, the induction in the air gap $B_\delta$ and the number of slots per pole (and phase) $q$.

The inner diameter of the stator is, as a rule, limited by the strength parameters of individual elements of the rotor design, since the tensile stress (compression) increases in proportion to the square of the angular frequency of the rotor rotation $\omega_r^2$ and $(D_{a})^3$.

The value of the cross-sectional area of the slot is firmly connected with the values of the rated current and current density selected according to the cooling conditions. The length of the stator pack and the number of slots per pole (and phase) is determined by the required values of the EMF and the output voltage. Note that if $D_{a} = \text{const}$, then $F(q)$ – the function monotonically increases and has no extrema. There is a conclusion: for the external characteristic to be extremely rigid, the parameter $q$ should be selected as the minimum value.

Now let’s consider the values of the weighting coefficients in (10). The first and the third coefficients have the most significant effect on the function under study, and with decreasing $q$ the first term decreases more intensely than the third one. It means the following:

1. With small bore diameters in the studied area, the armature reaction flow passing through the areas occupied by the rotor $Q_{o1}$ has the most significant effect on the output power of the generator.

![Figure 3. Generator full power $S_n$ (kVA) and its specific weight $G/S_n$ (kg/kVA) dependences in the function of the bore diameter](image)
The combined effect of the slot and frontal scattering fluxes on the output power is 3.2 – 3.8 times lower.

2. As the bore diameter increases, the specific weight of the components that form the objective function is levelled.

3. Further increasing the rigidity of the external characteristic can be achieved in two ways: the first, obvious solution, is associated with the use of new materials with improved magnetic and strength characteristics; the second, less obvious solution, involves finding ways to reduce $x_s$ and $x_a$.

Here, for example, one can consider installing copper screens in the stator slots, as it is done in shock generators [4].

Let’s consider the effect of the buffer stage generation system included between the generator and the consumer on the specific weight [5]. The buffer stage allows adjusting the output voltage, thereby stabilizing the voltage at the load, which in some cases is a prerequisite for using this type of generator. Stabilization of the output voltage can be achieved in two ways: by parallel or series connection of the buffer stage (Fig. 4).

![Buffer stage connection variants](image)

**Figure 4.** Buffer stage connection variants: (a) - with parallel connection; (b) – with series connection.

With parallel connection (Fig. 4a) stabilization of the output voltage is achieved by shunting the load by the buffer stage, as a result of which it selects a part of the energy generated by the generator. At the rated voltage on the load, no current flows through the buffer stage. Therefore, all the generated energy is consumed by the load. Reducing the load current leads to increasing the output voltage, which causes the connection of the buffer stage. In this case, the current generated by the generator is distributed between the load and the buffer, leading to unproductive energy consumption.

When the buffer stage (Fig. 4b) is connected in series, stabilization of the output voltage is achieved by changing the voltage drop across it. Such a switching circuit leads to a constant flow of the load current through the stage. When the load current is equal to the rated value, the voltage drop across the buffer stage depends on its internal resistance, which should be minimal. With decreasing the load current, the voltage drop across the buffer stage should increase.

Thus, in both cases, an active element should be used as a compensator for voltage changes changing its internal resistance depending on the output voltage. Moreover, its characteristic should not significantly affect the shape of the voltage produced by the generator.

The buffer stage can be implemented on a different element base that includes both active and passive elements. Various types of semiconductor switches act as active elements, and resistors, chokes, capacitors act as passive elements.

At low load currents, changing the internal resistance of the buffer stage is relatively easily controlled by shunting the resistor with a semiconductor device (for example, a transistor). The shape of the output voltage curve does not undergo significant distortions. At high load currents, this solution is unacceptable, since the semiconductor element should work in the key mode. The key mode of operation leads to distortions of the output voltage curve, which is accompanied by the appearance of harmonics and causes additional losses, both in the generator and in the load.
Comparison of the buffer stage connection schemes allows establishing that its power in the first variant (Fig. 4a) is comparable with the load power. In the second variant (Fig. 4b), it is significantly smaller and proportional to the power loss at the internal resistance of the generator. One of the options for the circuit solution of the buffer stage with a combined connection is shown in Figure 5. The output voltage applied to the load is changed due to the voltage on the secondary winding of the transformer directed towards the electromotive force of the generator.

![Figure 5. Transformer buffer stage.](image)

Preliminary calculations show that its specific weight per unit power with selecting corresponding materials of the magnetic circuit is at least 0.3 kg/kW.

4. Conclusion
According to the research results, it has been revealed the following: in voltage stabilization systems, when using a modern element base, a generator with excitation from permanent magnets and a buffer stage loses to an adjustable three-stage generator in the total weight and dimensions. At the same time, the high efficiency of the MEG and significantly increasing the technical capabilities of power electronics gives reason to consider the generation system extremely promising.

References
[1] Gemintern V I, Nakhamkin AM and Radina E V. 1990 Computer-aided design systems for electrical machines. (Moscov: Energoatomizdat) p 286. (in Russian)
[2] Kenio T. and Nagamori S. 1989 DC motors with permanent magnets (Moscow: Energoatomizdat) p 184. (in Russian)
[3] Stelting G and Beiss A 1991 Electric micromachines (Moscow: Energoatomizdat) p 229. (in Russian)
[4] Vlasov A I 2010 Magnetoelectric generators in the power supply system of new generation aircraft. (Chuvash State University). (in Russian)
[5] Bachurin P.A., Korobkov D.V., Kharitonov S.A., Khlebnikov A.S. 2016. DC power system with magnetoelectric generator Energy supply. Energy, Electrical audit, 3 146. (in Russian)
[6] Freeman E M 1974 Equivalent circuits from electromagnetic theory low-frequency induction devices Proceedings of IEEE 121(10) 1117-1121.
[7] Staton D, Popescu M, Hawkins D, Wu L J and Zhu Z Q 2012 Analytical modeling and analysis of open-circuit magnet loss in surface-mounted permanent-magnet machines. IEEE Transactions onMagnetics 48(3) 1234-1247.
[8] Lubin T, Mezani S, and Rezzoug A 2012 Two-dimensional analytical calculation of magnetic field and electromagnetic torque for surface-inset permanent-magnet motors IEEE Transactions onMagnetics 48(6) 2080-2091.