Improved contact approaches for irregular polygonal or polyhedral blocks and their applications

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Abstract. Correct mathematical description and mechanical solution towards contact of polygonal or polyhedral blocks are fundamental issues in discontinuous deformation analysis of fractured rock masses. Encountering irregular polygonal or polyhedral shapes (such as blocks with small edges or tiny faces), traditional contact detection and solution approaches may predict incorrect or inaccurate results due to the inappropriate choice of contact parameters or algorithmic robustness problems. Aimed to solve the above disadvantages of conventional contact detection and solution approaches in treating irregular polygonal or polyhedral block shapes, we developed several improved approaches, including (1) the new cover-based contact detection approach to establish inequality constraints for irregular polygons, and (2) the improved potential-based penalty function approaches to compute contact forces for irregular blocks. These improved approaches were proposed based on the contact theory proposed by Shi and the penalty function approach proposed by Munjiza. Several new algorithms, such as the contact trace tracking algorithm, block overlap examination algorithm, intersection block construction algorithm, contact force integration algorithm, were developed. The correctness of proposed approaches was first verified by several benchmarking tests. Several specially designed cases showed that these improved approaches can correctly treat the contact scenarios involving irregular blocks while previous approaches may fail or produce inaccurate results. The proposed approaches were further applied to several application cases which involves a large number of blocks. The proposed approaches and several embedded algorithms enriched the algorithm database for discontinuum-based computational methods.

1. Introduction
From the scale that rock engineering projects are designed or rock avalanches occur the object of rock mass is usually regarded as a structure with the distribution of discontinuities such as bedding planes, rock joints and faults [1]. A rock block system can be generated by the intersection of different joints sets and the domain boundaries. Meanwhile, the crystalline rocks own grain structures and the interaction of mineral grains affect the mechanical properties of rock material [2]. From mathematical perspective, the above-mentioned rock mass structure or rock material can be abstracted to a geometrical system with polygons (in two-dimension) or polyhedrons (in three-dimension) as the basic components. As the contact states of discrete bodies may change among open, locked, and sliding, the mechanical response of a block/particle system under various external loading conditions is a nonlinear process. Thus, correct simulating the contact interaction of blocks or grain with their realistic shapes is the fundamental issue in predicting the deformation and failure behaviour of a block system or a rock specimen with grain structures.
Several numerical methods have been proposed to solve the contact interaction problems, such as
discrete element method \[^{[3]}\], discontinuous deformation analysis \[^{[4]}\], and numerical manifold method \[^{[5]}\].
Based on the solution types, these methods can be classified into the explicit contact force-based
solution ones and the implicit contact constraint-based solution ones. The explicit solution approaches
usually compute the contact forces of two bodies at specific instant and solve the physical process with
Newton’s second law (or some other equivalent format) \[^{[3,6,7]}\]. By comparison, the implicit solution
approaches solve the kinematic governing equations considering the inequality constraints with the
penalty function methods \[^{[5]}\], cone programming methods \[^{[8]}\], and others \[^{[9]}\]. Focuses on the contact
detection problems, the explicit approaches focus on the block contact states of an instant, while the
implicit approaches focus the converged block contact states in a process.

The block/particle shape is an important factor that influences both the mechanical response of a
block/particle system and the accuracy and robustness of specific contact solution algorithms. In
practice, the blocks with tiny edges, sharp angles or small faces can be encountered. For some
traditional contact detection and contact force solution approaches, these kind of irregular shape
properties sometimes may lead to inaccurate or incorrect mechanical responses with inappropriate
usage of contact geometrical or physical parameters. For examples, replication or incorrect vertex-to-
vertex or vertex-edge contact types and thus inappropriate contact constraints can be established when
the conventional contact type-based contact detection approach is used for polygonal block system
which evolves polygons with tiny edges; inaccurate response may occur when the contact potential-
based force computation approach is applied without accurate definition of contact potential field for
polygons with tiny edges.

Aiming to enhance the accuracy and robustness of the contact detection and force computation
approach for contact scenarios with irregular convex polygonal or polyhedral blocks, two types of
methods were presented in this paper. Method 1 is a contact cover-based contact detection approach to
eliminate the contact parameter sensitivity problem in establish correct contact constraint for the
implicit solution approaches. Method 2 is an improved contact potential-based force computation
approach that aims to obtain more accurate contact force for irregular block. Detailed procedure of the
two methods in associated solution frame will be shown and several examples are used to verify the
correctness and potential of engineering applications.

2. Methods

2.1 An improved implicit contact solution approach

2.1.1 Theoretical background

The concept of entrance block proposed by Shi \[^{[10]}\] enable us to equivalently transform the contact of
two polygons into the contact of a given point and an entrance block, while the contact geometrical
information can be found on the boundary of the entrance block, which consists of contact covers. For
two-dimensional polygons, two types of contact covers, \(C(0,1)\) and \(C(1,0)\), which mean the contact
covers formed by vertex from block A and edge from block B, and the contact covers formed by edge
from block A and vertex from block B, respectively, are used to form all contact constraints of two
polygons.

2.1.2 A cover-based contact detection algorithm

The cover-based contact detection algorithm includes mainly two procedures, (1) the establishment of
potential contact cover list, and (2) the determination of states of each contact cover \[^{[11]}\].
To determine whether a vertex and an edge forms a potential contact cover, two criteria are used to
filter unnecessary ones: the entrance filter criterion and the distance filter criterion. The entrance filter
criterion aims to filter the vertex-edge pairs which is impossible to contact in current step by using the
neighbouring geometry information. For a vertex-edge pair \(a_i \sim b_j \sim b_{j+1}\), all the unit vectors \(e_i\) along the
dge that joining the vertex \(a_i\) should point outwards of the edge \(b_j \sim b_{j+1}\). Considering the quasi-
parallel cases, the entrance filter criterion for \(a_i \sim b_j \sim b_{j+1}\) is represented by
\[ \mathbf{e}_i \cdot \mathbf{n}_j \geq \sin (\theta_{e}^{\text{tol}}) \]  \hspace{1cm} (1)

where \( \mathbf{n}_j \) represents the unit normal vector of edge \( \mathbf{b}_j \mathbf{b}_{j+1} \) that point outward of block \( B \), \( \theta_{e}^{\text{tol}} \) represents the allowed overlapping angle. The distance filter criterion is used to filter the vertex-edge pairs which are too far to contact in current step. The contact region \( \Omega_{j(j+1)}^{\text{C1}} \) of edge \( \mathbf{b}_j \mathbf{b}_{j+1} \) is the union of point \( (\mathbf{p}_i) \) from three sub-regions \( \Omega_{j(j+1)}^{\text{C1}}, \Omega_{j(j+1)}^{\text{C2}}, \) and \( \Omega_{j(j+1)}^{\text{C3}} \):

\[
\Omega_{j(j+1)}^{\text{C1}} = \{ \mathbf{p}_i | d_{ij}^n \leq d_{n}^{\text{tol}} \text{ and } r_{ij} \in [0,1] \} \]  \hspace{1cm} (2a)

\[
\Omega_{j(j+1)}^{\text{C2}} = \{ \mathbf{p}_i | d_{ij} \leq d_{e}^{\text{tol}} \} \]  \hspace{1cm} (2b)

\[
\Omega_{j(j+1)}^{\text{C3}} = \{ \mathbf{p}_i | d_{ij+1} \leq d_{n}^{\text{tol}} \} \]  \hspace{1cm} (2c)

where \( d_{n}^{\text{tol}} \) is the input contact tolerance to determine the valid contact region of an edge; \( d_{n}^j \) is the perpendicular distance of vertex \( \mathbf{a}_i \) to edge \( \mathbf{b}_j \mathbf{b}_{j+1} \); \( d_{ij} \) is the distance of vertex \( \mathbf{a}_i \) to edge nodes \( \mathbf{b}_j \).

All vertex-edge pairs that passes the two criteria will form the potential contact cover list, which means the chosen contact pair have the potential to contact in current analysing step. Then, only the contact cover which is recognized as active will be used to formulate the contact penalty terms. The activity of contact covers is determined by two cases. (1) At the beginning of each step, the active contact covers will be inherited from previous step if there are closed contact covers in previous step; otherwise, the contact covers which corresponds to the largest perpendicular distance will be set to active. (2) In the open-close iteration cycles, the moving path of each vertex-edge contact cover will be tracked and for covers that the vertex moving trajectory and the edge trajectory intersects from open to closed status will be set to active.

\subsection*{2.1.3 Contact detection procedure}

During the computation process using discontinuous deformation analysis method, the cover-based approaches are implemented into two phases: the contact detection phase and the contact states adjustment phase in open-close interaction cycles. For contact detection phase, two loops, i.e., the vertex(A)-edge(B) loop and edge(A)-vertex(B) loop, are cycled and in each loop, the contact cover that passes the distance filter criterion and entrance filter criterion are added to the potential contact cover list. Then, the initial active cover determination rule is used to determine the initial active contact covers. For the open-close iteration cycles, the activity and associated open-locked-sliding states of each contact cover will be updated according to predefined criterion. Then, the contact detection algorithm is implemented into the conventional implicit discontinuous deformation analysis framework to solve the mechanical response of discrete bodies.

\subsection*{2.2 An improved explicit contact force solution approach}

\subsection*{2.2.1 Theoretical background}

For explicit contact solution approach, the key issue is to compute the contact force based on the penetrated geometrical condition of two bodies. Depending whether the penetration depth or penetrated volume are used, there are different contact force computation format. The contact potential-based penalty function approach proposed by Munjiza \cite{11} set up a mathematical framework for accurate description of normal contact force of two bodies. The normal contact force can be obtained by integrating the gradient of two potential fields in overlapping areas (in 2D) or volumes (in 3D). in Triangle and tetrahedron are initially used as examples, while the irregular polygon or polyhedron shapes are rarely noticed.

\subsection*{2.2.2 An improved potential-based contact solution approach}

In this improved potential-based contact solution approach, the new definition of potential field function for arbitrarily-shaped polygons are given \cite{13, 14}. The potential of a point \( p \) in a polygon \( \Omega_b \) is defined as
\[ \varphi(p) = \begin{cases} 0 & , p \notin \Omega_b \\ \frac{d_{\text{exit}}}{H} & , p \in \Omega_b \end{cases} \]  

(3)

where \( k_n \) is the normal penalty parameter; \( H \) is a unit length value that can be set to average length of all polygon edges of the assembly; \( d_{\text{exit}} \) is the shortest distance for point \( p \) to exit polygon \( \Omega_b \). Specially for 3D convex polyhedron, the potential for a point \( p \) in a convex polyhedron \( \Omega_h \) is defined as follows:

\[ \varphi(p) = \begin{cases} 0 & , p \notin \Omega_h \\ k_n \cdot \min \left( \frac{\left| d_{pf} \right|}{H} \right) & , p \in \Omega_h \end{cases} \]  

(4)

where \( k_n \) is the normal penalty parameter; \( H \) is a unit length value that can be set to average length of all block edges; \( d_{pf} \) is the distance of the point \( p \) to a surface of polyhedron,

\[ d_{pf} = \mathbf{n}_f \cdot (\mathbf{x}_p - \mathbf{x}_f) \]  

(5)

where \( \mathbf{x}_p \) and \( \mathbf{x}_f \) are the vectors of coordinates of point \( p \) and a point on the face \( f \); \( \mathbf{n}_f \) is the unit normal vector of face \( f \) which points outwards of the polyhedron.

Contact detection in this approach include the overlap examination and construction of the body overlaps. For overlap examination, the concept of entrance block \(^{[10]}\) is used. For convex polygons, the entrance block is a convex polygon and the boundaries consists of several contact covers formed by vertex-edge pairs. Thus, the entrance filter criterion is used to establish the contact covers (entrance block boundaries) and the conventional point-in-polygon judgment algorithm is used to determine whether two bodies are penetrated or not. For construction of overlapping areas or volumes, a step-by-step block splitting procedure is used by recognizing the boundary of one penetrated body as cutting line (in 2D) and cutting plane (in 3D).

Once the geometrical information of penetrated body is obtained, the contact force can be formulated by integrating the potential values among all boundaries of the penetrated body, a segment splitting algorithm and surface splitting algorithm are used for numerical integration of contact forces.

2.2.3 A general solution procedure

The general solution procedure starts from the given initial conditions with the block configuration and initial velocities. Then, the contact detection process follows and the normal contact forces can be obtained. On this basis, the acceleration terms can be obtained with the Newton’s second law and both the velocity and block configurations can be updated. Another computation cycle follows with the updated block frame and block velocities.

3. Application examples

3.1 Examples of the cover-based approach

In this example, two models with different joint spacing and thus different block numbers are established and the failure process of these block systems considering frictionless joints and the gravity effects are simulated. Model (a) consists of 229 blocks, with a minimum edge of 0.135 m. Model (b) consists of 826 blocks, with a minimum edge of 0.13 m. Contact tolerance ratio \( (r_{tol}) \) is defined as the ratio of contact tolerances \( d_{tol}^n \) used in the proposed cover-based approach and in conventional type-based approach. Different contact tolerance ratio ranging from 1 to 10 is used during the simulation, while the basic contact tolerances are set to 0.2 multiplied by the minimum edge length. The simulation results after 1.0s is shown in Figure 1 and Figure 2. In Figure 1 and Figure 2, (a) shows the result by conventional type-based approach with \( r_{tol} = 1 \); (b) and (c) show the results by the cover-based approach with \( r_{tol} = 1 \) and \( r_{tol} = 10 \), respectively. The simulation results with the cover-based contact detection approach are physically viable. The configuration difference when different contact tolerance values are used is possibly due to different numerical damping effects associated with the automatic time step strategy.
3.2 Examples of the potential-based approach

3.2.1 Runout of a polygonal block system
In this example, a rock slope model considering the intersection of two joints set is generated. In this model, polygons with small edges exists along the model boundaries. The potential failure mode and failure configuration with different friction angles (case 1: 15°; case 2: 30°) are investigated using the improved potential-based contact force solution approach. The physically viable numerical results in Figure 3 shows the simulation with the improved potential-based approach can correctly solve the contact of polygons with small edges. By comparison of the results with case 1 and case 2, the model failure process is faster with smaller friction angle.

(a) initial model
3.2.2 Failure of a polyhedral block system

In this example, the three-dimensional version of the potential-based contact force solution approach is applied. A rock column is cut by two sets of joints with different joint dip and strikes and two models are formed. Compared with the dimension in height (10m), the smallest edge lengths for the two models are 0.11m and 0.006m, respectively. Figure 4 shows the improved potential-base approach can correctly solve the polyhedral blocks system with small edges and small faces. The simulation results of model 1 shows symmetric properties along the symmetric plane.
Figure 4 Numerical simulation results of convex polyhedral block systems using the potential-based contact solution approach

4. Discussion and conclusions
Focusing on the robustness and efficiency issues in treating irregular convex polygons and polyhedra, this paper introduces a cover-based approach to detect and solve the contact constraints for convex polygons and an improved potential-based approach to compute the contact forces of both convex polygons and convex polyhedra.

The major theoretical improvement of cover-based contact detection approach is to apply a global detection and determination strategy instead of using the conventional contact-type based local detection strategies. Thus, the cover-based strategy can avoid the contact sensitivity issues occurs in conventional type-based approaches. The novelty of improved potential-based contact force approach lies in the new definition of the potential function for irregular polygon or polyhedral shapes with the shortest-path concept. The overlap examination based on entrance block concept, and the numerical integration scheme considering the irregular shapes also facilitate the accurate computation of contact force.

The potential of using the two contact solution approaches for mechanical analysis of complex polygonal or polyhedral block systems is verified by several examples. In the future, these basic contact detection and solution algorithms could be combined with more realistic contact models and parallel computing techniques to extend its application scenarios.

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