An Attempt to Describe Frequency Correlations among kHz QPOs and HBOs by Two-Armed Nearly Vertical Oscillations

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Abstract

We examine whether the two-armed \((m = 2)\) vertical p-mode oscillations trapped in the innermost region of magnetized accretion disks with finite disk thickness can describe kHz quasi-periodic oscillations (QPOs) and horizontal branch oscillations (HBOs) in low-mass X-ray binaries (LMXBs). First, we derive the frequency–frequency correlation of the two basic oscillations (both are fundamental modes in the vertical direction, but one is the fundamental and the other the first overtone in the radial direction), and compare it with the observed frequency correlation of twin kHz QPOs. Results show that the calculated frequency correlation can well describe the observed frequency one with reasonable values of the parameters. Second, we examine whether the observed frequency correlation between kHz QPOs and HBO can be described by regarding HBO as the first overtone oscillation in the vertical direction (and the fundamental in the radial direction). The results suggest that (i) the innermost parts of disks on the horizontal branch are strongly diminished in their vertical thickness (presumably by hot coronae) and (ii) the branch is roughly a sequence of variations of magnetic fields or disk temperature.

**Key words:** accretion, accretion disks — quasi-periodic oscillations — stars: neutron — two-armed disk oscillations — X-rays: stars

1. Introduction

In neutron-star low-mass X-ray binaries (NS LMXBs), kilo-hertz quasi-periodic oscillations (kHz QPOs) are often observed. In many cases they appear in pairs, and temporal changes of their frequencies are correlated. In Z-sources of LMXBs, horizontal branch oscillations (HBOs) are also observed, and their frequency change correlates with those of kHz QPOs (Psaltis et al. 1999). Studies on the origins of kHz QPOs and HBOs and on the cause of their correlations are of importance, since they will bring about a deep understanding of the innermost part of disks surrounding compact relativistic sources, and of the mass and spin of these sources.

There are many models of kHz QPOs and HBOs, but there is still no common consensus concerning their origins. One of the possible origins of kHz QPOs is trapped disk-oscillations excited in the inner region of the disks. Among many trapped-oscillation modes in disks, two-armed vertical p-mode oscillations are a likely candidate for kHz QPOs (Kato 2011b, hereafter referred to as Paper I). This is because in the case where disks are threatened by toroidal magnetic fields, (i) these oscillations have frequencies on the order of kHz QPOs, and (ii) can describe the correlated-time variation of observed twin kHz QPOs, if the strength of the toroidal magnetic fields varies with time.\(^1\)

To reinforce the above-mentioned model of kHz QPOs, however, some issues remain to be examined. That is, we should study how the time change of the disk structure other than the time change of the toroidal magnetic fields affects the correlation. In other words, we must examine how the correlation curve calculated in Paper I is changed, or modified, if the time variations of the disk structure due to other causes are considered.

As some time changes of disk structure other than a change of toroidal magnetic fields, two possibilities are conceivable. One is a time variation of the disk temperature. It is simply expected, since a change of the mass-accretion rate brings about a change of the disk temperature. Another possibility is a time change of the disk thickness. The cool, geometrically thin disks in NS LMXBs will be surrounded by hot corona, and the transition height to the corona will be subject to a change of the disk state. Hence, it will be of importance to examine whether or not and how the correlation curve calculated in Paper I is modified by a time change of the disk temperature or transition height. The first purpose of this paper is to examine this issue.

The second purpose is to examine whether the observed correlation between kHz QPOs and HBOs can also be described within the framework of vertical p-mode oscillations, by regarding HBOs as some higher modes of the oscillations. The results of an examination suggest that this possibility may not be unrealistic, if the vertical thickness of the innermost part of disks is strongly diminished in the state where they are on the horizontal branch (HB).

2. Disk Models and Parameters Describing Disks

To describe disks and their oscillations we adopt a Newtonian formulation, except that the effects of general relativity are taken into account in the radial distributions of

\(^1\) In this paper we show that the cause of the correlated-time variation is not necessarily time variation of magnetic fields, but temperature variation and time variation of the disk thickness due to truncation by corona can also describe the correlation.
\[ \Omega(r), \kappa(r), \text{ and } \Omega_{\perp}(r), \text{ where } \Omega(r) \text{ is the angular velocity of disk rotation; } \kappa(r) \text{ and } \Omega_{\perp}(r) \text{ are, respectively, the epicyclic frequencies in the radial and vertical directions [see Ökazaki et al. (1987) for } \kappa(r), \text{ and Aliiev & Galtov (1981) and Kato (1990) for } \Omega_{\perp}(r)]. \] 

\[ \Omega(r) \text{ is approximated to be the angular velocity of the Keplerian rotation, } \Omega_{K}(r), \text{ since we are considering geometrically thin disks. Here, } r \text{ is the radial coordinate of the cylindrical coordinates } (r, \varphi, z) \text{ whose } z \text{ axis is perpendicular to the unperturbed disk plane and the origin is at the disk center.} \]

The unperturbed disks are axisymmetric with purely toroidal magnetic fields, \( B_0(r, z) \):

\[ B_0(r, z) = [0, B_0(r, z), 0]. \]

The disks are assumed to be isothermal in the vertical direction, and \( B_0(r, z) \) is distributed in the vertical direction in such a way that the Alfvén speed, \( c_\Lambda \), is constant in the vertical direction, i.e., \( (B_0^2/4\pi \rho_0)^{1/2} = \text{const.} \), where \( \rho_0(r, z) \) is the density in the unperturbed disks. Then, the hydrostatic balance in the vertical direction gives that \( \rho_0(r, z) \) and \( B_0(r, z) \) are distributed in the vertical direction as (e.g., Kato et al. 1998)

\[ \rho_0(r, z) = \rho_{00}(r) \exp \left( -\frac{z^2}{2H_0^2} \right), \]

and

\[ B_0(r, z) = B_{00}(r) \exp \left( -\frac{z^2}{4H_0^2} \right), \]

where the scale height, \( H(r) \), is related to \( c_s, c_\Lambda, \text{ and } \Omega_{\perp} \) by

\[ H^2(r) = \frac{c_s^2 + (c_\Lambda^2/2)}{\Omega_{\perp}^2}, \]

\( c_s(r) \) being the isothermal acoustic speed.

The disk described above is assumed to be terminated at a certain height, \( z_o \), by the presence of a hot, low-density corona. The transition height, \( z_o(r) \), will be determined by considering thermal and hydrostatic balances (and mass balance). Determining the height is, however, beyond the scope of this paper. Thus the height is taken as a parameter.

The above disk model that we adopt hereafter thus has three parameters to specify the disk structure, in addition to the mass, \( M \), and the spin parameter, \( a_+ \), of the central source. They are \( c_s^2(r), c_\Lambda^2(r), \text{ and the height of transition, } z_o(r) \). Instead of these parameters, which have dimensions, we introduce here three dimensionless parameters. One is \( c_s^2/c_\Lambda^2 \), and the second is \( z_o/r \); the other is \( c_s^2/(c_\Lambda^2)_0 \), where \( (c_\Lambda^2)_0 \) is a reference value of the square of the acoustic speed; we adopt the following form, based on the standard value of the Shakura–Sunyaev disks.

That is, in the case of non-magnetized standard Shakura–Sunyaev disks in which the gas pressure dominates over the radiation pressure, and the opacity mainly comes from the free–free processes, we have (e.g., Kato et al. 2008)

\[ c_s^2(r) = 1.83 \times 10^{16} (am)^{-1/5} m^{2/5} (r/r_g)^{-9/10} \text{ cm}^2 \text{ s}^{-2}, \]

where \( a \) is the conventional viscosity parameter, \( r_g \) is the Schwarzschild radius defined by \( r_g = 2GM/c^2 \), \( m \equiv M/M_\odot \), \( m = M/M_{\text{crit}}, M_{\text{crit}} \) being the critical mass-accretion rate defined by the Eddington luminosity. For the reference value, \( (c_s^2)_0 \), we adopt the value of equation (5) in the case of \( a = 0.1 \) and \( m = 0.3 \). That is, we define \( (c_s^2)_0 \) by

\[ (c_s^2)_0 = 1.79 \times 10^{16} m^{2/5} (r/r_g)^{-9/10} \text{ cm}^2 \text{ s}^{-2}. \]

In summary, we specify our disk models by three dimensionless parameters:

\[ \beta \equiv \frac{c_s^2}{(c_\Lambda^2)_0}, \quad \frac{c_\Lambda^2}{c_s^2}, \text{ and } \frac{z_o}{H}, \]

Generally speaking, these parameters can be taken to be slowly varying functions of \( r \) in our analyses. Except in subsection 4.2, however, they are taken to be constant, independent
of the radius, since in the oscillations that would be related to kHz QPOs, the trapped regions are narrow (see figures 7 and 8 in Kato 2012, and also Kato 2011a). In subsection 4.2, radial variations of $\beta$ and $c_A^2/c_s^2$ are briefly considered in relation to HBOs, since in the oscillations that would be related to HBOs, the trapped regions are not always narrow (see also figures 7 and 8 in Kato 2012).

The purpose of this paper is to examine how the frequencies of some basic oscillation modes of the two-armed vertical p-mode oscillations vary as the above parameters change, and whether or not correlated-frequency changes of these oscillation modes can describe the observed-frequency correlations among kHz QPOs and HBOs.

3. Radially Trapped Two-Armed Vertical p-Mode Oscillations

Disk oscillation modes in geometrically thin non-magnetized disks are usually classified by the node number in the vertical direction, $n$, and the frequency in the corotating frame (see, e.g., Kato 2001; Kato et al. 2008). The oscillations whose radial displacement vector, $\xi(r,t)$, has no node in the vertical direction, i.e., $n=0$, are called p-mode. Oscillations with $n \geq 1$, except for particular ones (c-mode oscillations), are classified into g-mode and vertical p-mode oscillations. Those whose frequencies in the corotating frame are lower are called g-mode, while those with higher frequencies are vertical p-mode.

Vertical p-mode oscillations of $n = 1, 2, 3, \ldots$ are further divided by any difference of the azimuthal wavenumber, $m$, and the node number in the radial direction, $n_r$. That is, the vertical p-mode oscillations are now classified by the set of $(m, n, n_r)$, where $n$ starts from 1 as $n = 1, 2, 3, \ldots$, while $n_r$ does from 0 as $n_r = 0, 1, 2, \ldots$.

Boundary conditions also introduce a more variety of solutions. In this paper, however, we consider only the case where the inner boundary can be taken to be a free boundary and outside of the outer boundary the trapped-region oscillations are evanescent (see Kato 2011a).

In this paper, among many vertical p-mode oscillations we focus on oscillations of two-armed ($m = 2$) vertical p-modes, since when $m = 2$ there are oscillation modes that are radially trapped in the innermost region of disks with moderate frequencies (Kato 2010). That is, two oscillations of $n_r = 0$ and $n_r = 1$ (both are fundamental in the vertical direction, i.e., $n = 1$) have frequencies comparable to those of kHz QPOs when the disk has a moderate amount of toroidal magnetic fields (Kato 2011a). Oscillations of $n = 2$, even if they have $n_r = 0$, have much lower frequencies compared with those of $n_r = 1$ and $n = 1$ (and thus those of kHz QPOs). Their frequencies can become comparable with those of HBOs. Considering these situations, we particularly pay our attention to two-armed ($m = 2$) vertical p-mode oscillations with the set of $(n, n_r)$ being (1, 0), (1, 1), and (2, 0). It is noted that oscillations with larger $n$ and $n_r$ are not interesting observationally, since they will not be observed with large amplitude due to phase mixing.

Eigen-functions of the above-mentioned trapped oscillations have been examined in cases where the disks are infinitely extended isothermal ones (Kato 2011a), and ones which are terminated at certain heights (Kato 2012). In the next section, we examine whether parameter dependences of these oscillations can describe the observed-frequency correlation of twin kHz QPOs and the correlation between kHz QPOs and HBOs.

4. Frequency Correlation and Comparison with Observations

We consider two problems separately. First, we examine the frequency correlation between the $n_r = 0$ and $n_r = 1$ oscillations, both with $n = 1$. This is an extension of Paper I. Second, correlations between the $n = 1$ and $n = 2$ oscillations both with $n_r = 0$ are considered.

4.1. Correlation between $n_r = 0$ (with $n = 1$) and $n_r = 1$ (with $n = 1$) Oscillations

We take the standpoint that the fundamental ($n_r = 0$) and first overtone ($n_r = 1$) oscillations in the radial direction (both fundamental in the vertical direction, i.e., $n = 1$) are the upper and lower kHz QPOs, respectively. To check this possibility,
we examine how a set of frequencies of these two oscillations moves on a frequency–frequency diagram when parameters specifying the disk structure change, and compare this correlation curve on the diagram with the observed frequency–frequency plots of the twin kHz QPOs.

This comparison has already been done in Paper I in the case where the disk is non-terminated isothermal ones ($\eta_0 = \infty$) with $\beta = \frac{c_A^2}{c_s^2} = 1.0$ (figure 3 in Paper I) and $\beta = 3.0$ (figure 5 in Paper I) by changing the strength of toroidal magnetic fields in the range of from $c_A^2/c_s^2 = 0$ to 100. Here, the oscillations with these effects included are more systematically compared with observations by considering the cases where other disk parameters are changed.

First, we focus on disks with $M = 1.4 M_\odot$ and $a_*$ = 0, and consider the case where the disk is non-terminated isothermal ($\eta_0 = \infty$). In this case we find that the calculated correlation curve is almost universal in the sense that it is almost a part of the unique curve for a moderate set of $\beta$ and $c_A^2/c_s^2$. To demonstrate this, let us first consider the case where $\beta$ is fixed at $\beta = 3.0$, and $c_A^2/c_s^2$ is changed from 0 to 64, which is shown in figure 1 (in figure 5 of Paper I, $c_A^2/c_s^2$ is changed in the range of from 0 to 100). When $c_A^2/c_s^2 = 0$, the point representing the set of frequencies of the $n_t = 0$ (with $n = 1$) and $n_t = 1$ (with $n = 1$) oscillations on the frequency–frequency diagram is the right end of the curve in figure 1. In another end, $c_A^2/c_s^2 = 64$, the point is the left end of the curve. As the value of $c_A^2/c_s^2$ changes in the range of from 0 to 64, the point on the diagram moves along the curve from the right end to the left end. Next, let us consider the case where $\beta = 10.0$, and $c_A^2/c_s^2$ is changed in the range of from 0 to 64, which is shown in figure 2. A comparison of figures 1 and 2 shows that an increase of $\beta$, while keeping other parameter values unchanged, decreases the frequencies of both the $n_t = 0$ and $n_t = 1$ oscillations (see Kato 2011a). Hence, in the case of figure 2, the correlation curve on the frequency–frequency diagram shifts toward the left-bottom corner compared with the case of figure 1. In the overlapping part, however, the curves in figures 1 and 2 are almost the same.

Next, let us consider the case where $c_A^2/c_s^2$ is fixed and $\beta$ is changed in the range of 1.0 to 10.0, which is shown in figure 3 for $c_A^2/c_s^2 = 4.0$ and in figure 4 for $c_A^2/c_s^2 = 16.0$. In both figures, the left-end points of the curves are for $\beta = 10.0$. This is because if the value of $\beta$ increases while keeping other parameter values unchanged, the frequencies of both oscillations decrease (Kato 2011a). As $\beta$ decreases, the point representing the set of the frequencies moves along the correlation curve, and reaches to the right-upper end of the curve when $\beta = 1.0$. In the case of $c_A^2/c_s^2 = 16.0$, the frequencies of both oscillations are low compared with those in the case of $c_A^2/c_s^2 = 4.0$ if the other parameters are fixed. Hence, in figure 4 the calculated-correlation curve shifts toward the left-lower end compared with the case of $c_A^2/c_s^2 = 4.0$. However, in the region where the frequencies overlap, the correlation curve is almost the same as that in figure 3. That is, as mentioned above, the correlation curve is almost unique. Of course, if the mass of the central source differs from $M = 1.4 M_\odot$, the correlation curves on the frequency–frequency diagram do not overlap (see below).

Here, we consider the case where the disks are terminated in the vertical direction. To demonstrate rather an extreme case, we adopt $\eta_0 = 1.5$. The correlation curve is calculated by changing the value of $c_A^2/c_s^2 = 4.0$ from 1 to 10, while fixing the value of $c_A^2/c_s^2$. The case where $c_A^2/c_s^2 = 4.0$ is shown in figure 5, and the case of $c_A^2/c_s^2 = 16.0$ is in figure 6. The results show that a decrease of the disk thickness brings about no change of the essential characteristics of the correlation curve.

Next, we show the effects of the mass and the spin of the central source on the correlation curve (see also Paper I). Figure 7 is for three cases of $a_*$ = 0, 0.2, and 0.4 with $M = 1.4 M_\odot$. Non-truncated disks with $\beta = 3.0$ are adopted and $c_A^2/c_s^2$ is changed from $c_A^2/c_s^2 = 0$ to 64. Figure 8 is the same as figure 7, except that $M = 1.8 M_\odot$ is adopted.\footnote{Figures 7 and 8 are close to figure 3 in Paper I, but the parameter values considered are a little different.}

4 Figures 7 and 8 are close to figure 3 in Paper I, but the parameter values considered are a little different.
concentrated in the inner region of the disks (figure 8 in Kato 2012). Hence, the correlation curve shifts in the right-upper direction on the frequency–frequency diagram as \( a_s \) increases, as shown in figures 7 and 8 (see also figures 3 and 4 in Paper I). An increase in the mass of the central source shifts the correlation curve in the left-lower direction on the diagram, as also understood easily from the fact that the increase in mass decreases the frequencies of oscillations. A comparison between figure 7 and figure 8 shows, for example, that if the masses of typical LMXBs presented in figures are \( 1.4 M_\odot \), their spin will be around \( a_s \sim 0 \), and at most \( a_s < 0.2 \). If their masses are \( 1.8 M_\odot \), their spin is larger and \( 0.2 < a_s < 0.4 \).

4.2. Correlation between the \( n = 1 \) (with \( n_r = 0 \)) and \( n = 2 \) (with \( n_r = 0 \)) Oscillations

In the previous subsection, we show that the correlation between the \( n_l = 0 \) and \( n_r = 1 \) oscillations (both with \( n = 1 \)) is almost unchanged on the frequency–frequency diagram with changes of \( \beta, c_\Lambda^2/c_s^2 \), and \( \eta_s \), although the range of the curve depends on the parameter ranges. Unlike the above results, the correlation between the \( n = 1 \) and \( n = 2 \) oscillations depends strongly on the disk structure. Here, we demonstrate in detail how strong this dependence is.

Let us first consider the case where \( \beta \) and \( \eta_s \) are fixed at some values, and \( c_\Lambda^2/c_s^2 \) is changed in the range of from 0 to 64, which is shown in figure 9. The mass of the central source is \( M = 1.4 M_\odot \) with no spin, \( a_s = 0 \). The frequency of the \( n = 1 \) (with \( n_r = 0 \)) oscillations is taken on the abscissa, and that of the \( n = 2 \) (with \( n_r = 0 \)) oscillation is on the ordinate. In this figure the scales of the ordinate and the abscissa differ from those in the previous subsection in order to superpose the calculated-correlation curves on the frequency–frequency diagram plotting the observational data (figure 2.9 in a review paper by van der Klis 2004). The purpose here is to examine whether the \( n = 2 \) (with \( n_r = 0 \)) oscillations can describe the observed HBOs in Z-sources.\(^5\)

\(^5\) HBO harmonic and Sub HBO are of no concern here, since their amplitudes are small, and they will be subsidiary.

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Fig. 7. Same as figure 1, except for cases where the spin parameter, \( a_s \), is non-zero are considered. Among three correlation curves, the upper one is for \( a_s = 0.4 \), the middle one is for \( a_s = 0.2 \). The lower one is for \( a_s = 0 \), and the same as that of figure 1. (Color online)

Fig. 8. Same as figure 7, except that the mass of the central source is 1.8 \( M_\odot \). (Color online)

Except for a nearly vertical curve, five curves are shown in figure 9. Let us now account for the meaning of the curves, from upper to lower, in the order of the left-end points of these curves (the third and fourth curves are close each other). The uppermost (red) curve is for \( \eta_s = \infty \) and \( \beta = 3.0 \), drawn by changing \( c_\Lambda^2/c_s^2 \) in the range of from 4.0 to 64.0. The right-upper end of this curve is for \( c_\Lambda^2/c_s^2 = 4.0 \) and the left-lower end is for \( c_\Lambda^2/c_s^2 = 64.0 \). Sets of \((\eta_s, \beta)\) for the fourth (green) and fifth (blue) curves are, respectively, \((2.0, 3.0)\) and \((1.8, 3.0)\). A set of \((\eta_s, \beta)\) for the nearly vertical (magenta) curve on the right-hand side is \((1.5, 1.0)\). [The second (cyan) and third (sienna) curves are mentioned later.]

The above-mentioned curves show that in a non-truncated disk (\( \eta_s = \infty \)) the correlation curve runs above the observed plots on the frequency–frequency diagram. As disks become thin in the vertical direction, the correlation curve moves downward on the diagram. However, the gradients of the curves become sharper as \( \eta_s \) decreases, and in an extreme case of \( \eta_s = 1.5 \), the curve (magenta one) is almost vertical.\(^6\) That is, these curves cannot well describe observations.

One of the possible reasons for this discrepancy between the calculated curves and the observations might be that in the above calculations, \( c_\Lambda^2/c_s^2 \) has been taken to be constant, independent of \( r \). In real disks the magnetic field may decrease outward. If so, in considering the frequency of the \( n = 2 \) oscillations this should be taken into account, since the trapped region of the oscillations is rather wide. (In trapped \( n = 1 \) oscillations, however, the trapped region is narrow and the effects of the radial decrease of \( c_\Lambda^2/c_s^2 \) are minor.) Considering this situation, we made a tentative attempt; we have calculated the frequencies of the \( n = 2 \) oscillations on the assumption that the value of \( c_\Lambda^2/c_s^2 \) is constant as far as \( r = 4r_g \), and outside of \( 4r_g \) it decreases as \( c_\Lambda^2/c_s^2 = (c_\Lambda^2/c_s^2)_0 (4r_g/r)^2 \), where \( (c_\Lambda^2/c_s^2)_0 \) is the constant value of \( c_\Lambda^2/c_s^2 \) inside of \( 4r_g \). The correlation curves obtained by using this \( c_\Lambda^2/c_s^2 \) are shown as the second (cyan) and third (sienna) curves for \((\eta_s, \beta)\) being

\(^6\) In the nearly vertical curve, the upper-right end shifts rightward compared with the other curves. This is because \( \beta = 1.0 \) is adopted, different from the other curves where \( \beta = 3.0 \) is adopted.
Next, let us consider the case where $\eta_s = 1.8$ and $\beta = 3.0$ with the above-modified $c_\lambda^2/c_s^2$ better describes observations compared with the other cases, although it is not yet satisfactory.

Next, let us consider the case where $c_\lambda^2/c_s^2$ and $\eta_s$ are fixed at some values, and $\beta$ is changed in the range of from $\beta = 1.0$ to $\beta = 10$. The mass of the central source is again $M = 1.4M_\odot$ with no spin, $a_\ast = 0$. Among the six curves in figure 10, the uppermost (red) curve is for a non-truncated disk ($\eta_s = \infty$) with $c_\lambda^2/c_s^2 = 4.0$. This curve is above the observed sequence of HBOs on the diagram. As in figure 9, the curve moves downward as the disk becomes thin in the vertical direction. That is, sets of parameters ($\eta_s$, $c_\lambda^2/c_s^2$) in the second (blue), third (green), fifth (magenta), and sixth (cyan) curves are (1.8, 4.0), (2.0, 16.0), (1.8, 16.0), and (1.5, 4.0). [In the next paragraph we discuss the fourth (sienna) curve.] It is noted that in the two cases of the same $\eta_s$, the correlation curve calculated by a larger value of $c_\lambda^2/c_s^2$ runs below that calculated by a smaller $c_\lambda^2/c_s^2$. The correlation curve mentioned above cannot well describe observations, since they have larger gradients on the diagram compared with those of observations, as in the case of figure 9. Here, however, we should remember again, as in the case of figure 9, that for the $n = 2$ oscillations the trapped region is wide, and thus a radial change in $c_\lambda^2/c_s^2$ should be taken into account in calculating their frequencies. Here, we consider a case where $c_\lambda^2/c_s^2$ decreases outside of $4r_g$ as $c_\lambda^2/c_s^2 = (c_\lambda^2/c_s^2)_0(4r_g/r)^{\beta}$, where $(c_\lambda^2/c_s^2)_0$ is the constant value of $c_\lambda^2/c_s^2$ inside of $r = 4r_g$. The correlation curve in the case of $\eta_s = 1.8$ and $(c_\lambda^2/c_s^2)_0 = 16.0$ is shown as the fourth (sienna) curve. This curve seems to better describe the observations.

### 5. Summary and Discussion

In this paper we have examined the possibility that the two-armed ($m = 2$) vertical p-mode oscillations trapped in...
the innermost region of magnetized disks are the origin of kHz QPOs and HBO in LMXBs. The disks are assumed to be isothermal in the vertical direction, but truncated at a certain height. The magnetic fields are assumed to be toroidal. More concretely speaking, we suggested that (i) two oscillations that are both fundamental in the vertical direction (i.e., \( n = 1 \)), but the fundamental \((n_t = 0)\) and first overtone \((n_t = 1)\) in the radial direction are twin kHz QPOs and (ii) the oscillation that is the first overtone in the vertical direction (i.e., \( n = 2 \)), but the fundamental in the radial direction (i.e., \( n_t = 0 \)) is an HBO. To examine this possibility we compared (i) the calculated-frequency correlation between the \( n_t = 0 \) and \( n_t = 1 \) oscillations (both with \( n = 1 \)) with plots of the observed twin kHz QPOs on the frequency–frequency diagram, and (ii) the calculated-frequency correlation between the \( n = 1 \) and \( n = 2 \) oscillations (both with \( n_t = 0 \)) with the observed correlation between kHz QPOs and HBO on the frequency–frequency diagram.

We found that the frequency correlation between the \( n_t = 0 \) and \( n_t = 1 \) oscillations (both with \( n = 1 \)) can correctly describe the observed correlation of twin kHz QPOs. That is, in the present disk model, there are three parameters specifying the disk structure, i.e., \( \beta \equiv c_{\text{obs}}^2/c_{\text{obs}}^2 \), \( \gamma \equiv c_{\text{obs}}^2/c_{\text{obs}}^2 \), and \( \eta_r \equiv z_r/H \). Changes of these disk parameters lead to a change in the frequency of two oscillations of \( n_t = 0 \) and \( n_t = 1 \) (with \( n = 1 \)), and we have a correlation curve on the frequency–frequency diagram. We find that the correlation curve is plotted almost along a unique curve, independent of the disk parameters (see figures 1 to 6), as long as \( M \) and \( a_s \) are fixed. It is noted, however, that the range of the correlation curve depends on the range of the changes of the disk parameters (see figures 1 to 6).

In Z-sources of LMXBs, HBOs are often observed, and their time variations correlate with those of kHz QPOs (Psaltis et al. 1999). In this paper, we have examined whether the observed correlation between kHz QPOs and HBOs can be described within the framework of two-armed vertical p-mode oscillations. We formed the picture that the oscillation which is the first overtone in the vertical direction (i.e., \( n = 2 \)) and the fundamental in the radial direction (i.e., \( n_t = 0 \)) is an HBO. Observations show that the amplitude of HBOs is much larger than that of kHz QPOs. This will be accounted for by this model, since as shown before (Kato 2011a, 2012), the mass-accretion rate was supposed to increase monotonically in the direction HB–NB (normal branch)–FB (flaring branch) (e.g., Priestely et al. 1986). Recent spectral studies by the extended ADC (accretion disk corona) model on GX 340+0 (Church et al. 2006) and GX 5–1 (Jackson et al. 2009), however, suggest the opposite trend (Church et al. 2006). That is, in Church et al. (2008)’s model, based on their spectral analyses, the soft apex between NB and FB is a state in which the mass-accretion rate, \( \dot{m} \), is low, and \( \dot{m} \) increases on NB to HB. In their model, due to strong radiation from the central NS, which results from a large \( \dot{m} \), the innermost part of accretion disks on the upper NB and HB are supposed to be disrupted. They also suppose that this disruption is the cause of a decrease of the frequency of kHz QPOs on HB apart from the hard apex and of the appearance of radio emission in the upper NB and HB (Penninx 1989).

The disk structure of HB suggested by our present results is similar to that suggested by Church et al. (2006) in the sense that optically thick disks on HB must have a high temperature, and be geometrically thin due to horizontal truncation (perhaps by the presence of corona). Concerning the direction of the increase of the mass-accretion rate on HB, our results are insufficient to say anything definitively, but suggest an increase of the mass-accretion rate along HB apart from the hard apex since, as mentioned before, in our results HB would be a sequence of a disk-temperature increase apart from the hard apex. It will be of importance to notice here that geometrically thin, high-temperature disks with strong magnetic fields are realized, as disks bridging between advection dominated accretion flows (ADAFs) and optically thick standard disks (or slim disks), when the mass-accretion rate is close to or above, the Eddington limit (Machida et al. 2004; Oda et al. 2007, 2009, 2010).

It is noted that we suppose there is no appreciative outward retreat of the inner edge of geometrically thin disks during the evolution along HB. We calculated the frequency changes of \( n_t = 0 \) and \( n_t = 1 \) (both with \( n = 1 \)) oscillations by changing the radius of the inner edge, although the results are not given in the text. The results show that their correlation curve on the frequency–frequency diagram has a sharper gradient (close to 3:2) than those of the observed twin kHz QPOs.

Excitation of the trapped-disk oscillations is a problem that remains to be examined. The most conceivable process is stochastic excitation by turbulence (Goldreich & Keely 1977a, 1977b). This process is now known to be the main cause of solar and stellar non-radial oscillations. Similar processes will be expected in disks, where much stronger turbulence is expected compared with the inside of the stellar
convection zone.

The results that the two-armed vertical p-mode oscillations can well describe the observed correlation of kHz QPOs encourage us to extend this oscillation model to the 3:2 twin high-frequency QPOs (HF QPOs) observed in black hole (BH) LMXBs. Model fittings of the observed spectra of BH LMXBs show that the spin parameter, $a_s$, of some BH sources is rather large, and close to the extreme value (see McClintock et al. 2011 for review). The set of $n_r = 0$ and $n_r = 1$ oscillations (with $n = 1$), however, cannot describe the twin HF QPOs of such high-spin sources, since the frequency of the $n_r = 0$ (with $n = 1$) oscillations becomes higher than that of the observed upper HF QPOs if such a high spin is adopted. In highly spinning sources the $n_r = 0$ (with $n = 1$) oscillations might be damped by spatial closeness of the corotation-resonant point from the propagation region, and higher order modes (i.e., lower frequency modes) might be observed predominantly. We will examine in the near future whether HF QPOs of BH sources can also be described within the framework of the two-armed p-mode oscillations.

*Note added in proof (2012 May 19):*  
In this paper we have calculated the frequency–frequency correlation curves, based on the approximation that the vertical p-mode oscillations are nearly vertical and thus the horizontal motions associated with them can be taken to be small perturbations over the vertical ones (Kato 2011a). In the limiting case of no magnetic field, however, we can calculate the frequencies of trapped oscillations and thus the frequency–frequency correlation without introducing the approximation. Studies on that case show that the results of the present paper that the two-armed vertical p-mode oscillations can well describe the observed frequency correlations are unchanged, but the magnetic field required to describe the observed correlation is less than those in this paper. This issue will be discussed in a subsequent paper.

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