MOLECULAR HYDROGEN KINEMATICS IN CEPHEUS A

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ABSTRACT

We present the radial velocity structure of the molecular hydrogen outflows associated with the star-forming region Cepheus A. This structure is derived from the Doppler shift of the H$_2$ v = 1–0 S(1) emission line obtained by Fabry-Pérot spectroscopy. The east and west regions of emission, called Cep A (E) and Cep A (W), show radial velocities in the range of −20 to 0 km s$^{-1}$ with respect to the molecular cloud. Cep A (W) shows an increasing velocity with position offset from the core, indicating the existence of a possible accelerating mechanism. Cep A (E) has an almost constant mean radial velocity of −18 km s$^{-1}$ along the region, although with a large dispersion in velocity, indicating the possibility of a turbulent outflow. A detailed analysis of the Cep A (E) region shows evidence for the presence of a Mach disk on that outflow. In addition, we argue that the presence of a velocity gradient in Cep A (W) is indicative of a C-shock in this region. Following Riera and coworkers, we analyzed the data using wavelet analysis to study the line width and central radial velocity distributions. We found that both outflows have complex spatial and velocity structure characteristic of a turbulent flow.

Key words: infrared: ISM — ISM: individual (Cepheus A) — ISM: jets and outflows — ISM: kinematics and dynamics — ISM: molecules — turbulence

1. INTRODUCTION

Cepheus A is the densest core within the Cepheus OB3 molecular cloud complex (Sargent 1977) and a massive star-forming region. It contains a deeply embedded infrared source, which generates a total luminosity of ~2.4 × 10$^4$ L$_{\odot}$ (Koppenaal et al. 1979).

Two main regions of ionized and molecular gas about 2$^\circ$ apart and oriented roughly in the east-west direction have been detected, Cepheus A east (Bally & Lane 1982) and Cepheus A west (Simon & Joyce 1983; Garay et al. 1996). The first molecular hydrogen map of both regions using Fabry-Pérot spectroscopy was presented by Doyon & Nadeau (1988). Later, Hartigan et al. (1996) obtained high-resolution images in the v = 1–0 S(1) line emission of H$_2$ from Cep A west. The two Cep A regions with molecular hydrogen emission show quite different compositions.

The eastern region [hereafter Cep A (E)] hosts one of the first detected CO bipolar molecular outflows (Rodriguez et al. 1980). High-resolution observations show a more complex outflow of a quadrupolar nature (Torrelles et al. 1993). Torrelles et al. (1993) suggested that the source Cep A East:HW 2 (Hughes & Wouterloot 1984) is powering the P.A. = 45$^\circ$ outflow, but it is not clear if the powering source of the P.A. = 115$^\circ$ outflow is Cep A East:HW 3 or another source (L. F. Rodríguez 2004, private communication). Observations of $^{12}$CO, CS, and CO give evidence of multiple episodes of outflow activity (Narayanan & Walker 1996). Observations by Codella et al. (2003) of H$_2$S and SO$_2$ confirm the presence of multiple outflows. Highly variable H$_2$O and OH masers, commonly associated with young stellar objects, are surrounded by very dense NH$_3$ condensations that probably redirect the outflow into a quadrupolar structure (Torrelles et al. 1993; Narayanan & Walker 1996).

The western region [hereafter Cep A (W)] contains several radio continuum sources at 3 cm (Garay et al. 1996). Hartigan et al. (1986) identified a region of several Herbig-Haro objects known as HH 168 (= GGD 37) with large radial velocities and line widths. A bipolar outflow of CO with overlapping red- and blueshifted lobes is associated with this region (Bally & Lane 1982; Narayanan & Walker 1996). It should be pointed out that the energy source of Cep A (W) remains elusive (Raines et al. 2000; Garay et al. 1996; Torrelles et al. 1993; Hartigan & Lada 1985).

Although the two regions Cep A (E) and Cep A (W) could constitute a single large-structure outflow, several authors have presented evidence that suggests that Cep A (W) may be an independent region of activity, distinct from Cep A (E) (Raines et al. 2000; Garay et al. 1996; Hartigan & Lada 1985).

In this paper we present the radial velocity structure of the Cep A molecular hydrogen outflows obtained from the H$_2$ v = 1–0 S(1) Doppler-shifted emission line at 2.122 mm measured by scanning Fabry-Pérot spectroscopy. Because of the complexity found in the velocity structures, we decided to study the kinematics by using an asymmetric wavelet analysis following Riera et al. (2003), who used this method to study H$_2$O Fabry-Pérot observations of the HH 100 jet.

Our results show that the two regions represent turbulent H$_2$ outflows with significant differences from a kinematic point of view. A detailed analysis of the Cep A (E) region provides evidence for the presence of a Mach disk near the tip of the outflow.

In § 2 we describe the observations. From these data, in § 3 we generate a Doppler-shifted H$_2$ image, as well as radial velocity, velocity gradient, and line width maps, and study the flux-velocity diagrams (Salas & Cruz-González 2002). By using the
asymmetric wavelet transform, the clumpy structures of both regions of Cep A are kinetically analyzed and discussed in § 4. The conclusions are then summarized and presented in § 5.

2. OBSERVATIONS

On 1998 October 5, we observed the Cepheus A region with the 2.1 m telescope of the Observatorio Astronómico Nacional at San Pedro Mártir (SPM; Mexico).

The measurements were obtained with the CAMILA near-infrared camera/spectrograph (Cruz-González et al. 1994) with the addition of a cooled tunable Fabry-Pérot interferometer (located in the collimated beam of the cooled optical bench) and a 2.12 μm interference filter. A detailed description for the infrared-scanning Fabry-Pérot instrumental setup is presented by Salas et al. (1999).

The Fabry-Pérot setup has a spectral resolution of 24 km s⁻¹, and to restrict the spectral range for the ν = 1–0 S(1) H₂ line emission, an interference filter (2.122 μm with Δλ = 0.02 μm) was used. The bandwidth of this filter allows 11 orders of interference. Only one of these orders contains the 2.122 μm line, and the remaining orders contribute to the observed continuum. The spatial resolution of the instrumental array is ~0.86 pixel⁻¹. The field of view covers a 3.67 × 3.67 region, which corresponds to 0.7 × 0.7 pc² at the adopted distance of 725 pc (Johnson 1957). With this field of view, one set of images was required for the eastern region and another for the western region of Cep A.

Fig. 1.—Velocity channel maps of the H₂ ν = 1–0 S(1) line emission in Cep A, showing the radial velocity structure of the two regions Cep A (E) and Cep A (W). The velocity varies from ~42.1 km s⁻¹ in channel 8 (top) to ~2.9 km s⁻¹ in channel 12 (bottom), and the channel width is 9.8 km s⁻¹. The scale of the region is shown in the top panel. The contour intervals indicate the emission intensity in the range of 0.54–1.5 counts s⁻¹ and have the same increment values of 0.2 counts s⁻¹. The cross indicates the position of the 6 cm peak of HW 2 at R.A. = 22°45′17″ and decl. = +62°14′01″ (J2000; see Hughes & Wouterloot 1984). Offsets are referred to this reference position.
Images of each region of interest were obtained at 26 etalon positions, corresponding to increments of 9.82 km s\(^{-1}\). The observing sequence consists of tuning the etalon to a new position and imaging the source, followed by a sky exposure at an offset of 5° south from the source. The integration time of 60 s per frame was short enough to cancel the atmospheric line variations at each etalon position but long enough to obtain a good signal-to-noise ratio (S/N). Images were taken under photometric conditions with a FWHM of 1.6.

Spectral calibration was obtained by observing the line at 2.1332885 μm of the argon lamp at each position of the etalon, giving a velocity uncertainty of 1 km s\(^{-1}\) in the wavelength fit. A set of high- and low-illumination sky flats were obtained for flat-fielding purposes.

We reduced the data to obtain the velocity channel images using the software and the data reduction technique described in Salas et al. (1999).

3. RESULTS

3.1. \(\text{H}_2\) Velocity Maps

Velocity channel images were individually obtained for Cep A (E) and Cep A (W) from the position-velocity cube data. For each pixel on the image, we subtracted a continuum intensity level calculated from the median of the channels with no \(\text{H}_2\) emission. \(\text{H}_2\) \(v = 1-0\ S(1)\) line emission was detected in velocity channels \(-40\) to 0 km s\(^{-1}\) in Cep A (E) and in channels \(-40\) to 10 km s\(^{-1}\) in Cep A (W). The two sets of maps were pasted together to create velocity channel maps of the complete region. Figure 1 shows five of these maps (8–12) covering local standard of rest (LSR) velocities from \(-42\) to \(-3\) km s\(^{-1}\). Cep A (E) shows \(\text{H}_2\) emission in six separated clumps of emission (Fig. 1, B–G), whereas in Cep A (W) the emission can be distinguished in six regions (Fig. 1, H–M).

We created a color-coded velocity image from the three channel velocity maps with more copious emission (\(-32.3, -22.5,\) and \(-12.7\) km s\(^{-1}\)) as blue, green, and red, respectively. Figure 2 presents this color composite map. It should be noted that these velocities are somewhat bluer than the \(-11.2\) km s\(^{-1}\) systemic velocity found from millimeter-wavelength line observations of CO (e.g., Narayanan & Walker 1996), as had already been noted by Doyon & Nadeau (1988).

The velocity structure in both regions shows a very complex pattern. However, a slight systematic change from blue to red starting at the center of the image can be appreciated on the western outflow that is not present in the eastern region. On the other hand, the Cep A (E) region shows a large amount of small clumps with different velocities lying side by side.

We have calculated the centroid radial velocity for each pixel by taking only 10 velocity channels around the peak intensity. Figure 3 presents histograms of centroid radial velocity for the two \(\text{H}_2\) emission regions of the image in Figure 2. Regardless of the difference in morphology for the two regions, they have a...
similar fractional distribution of pixels for a given radial velocity. However, Cep A (W) has a wing that extends into positive radial velocities and a small peak in the negative velocities.

Figure 4 presents the centroid radial velocity as a function of displacement along the right ascension axis for both Cep A (E) and Cep A (W). All the channels with detected emission of H$_2$ from Figure 2 are shown. The radial velocity of Cep A (E) has an almost constant mean value of $\pm 18$ km s$^{-1}$ along the region, with a high dispersion around this value. Meanwhile, the radial velocity of Cep A (W) increases its value with offset position to the west. We have fitted a line to the position-velocity data for each region and found rms residual values of 8.3 and 10.2 km s$^{-1}$ for Cep A (W) and Cep A (E), respectively. The smaller velocity dispersion and the large number of small clumps with different velocities (see Fig. 2) in the eastern region indicate a more turbulent outflow in Cep A (E) compared with Cep A (W).

### 3.2. A Mach Disk in Cep A (E)

The Cep A (E) outflow culminates in an arc-shaped structure (Fig. 1, G) that resembles a bow shock. This region is amplified in Figure 5a. A bright spot can be seen in the center of the bow. The centroid velocities corresponding to this region are shown in Figure 5c. The highest blueshifted velocity of the region ($-40$ km s$^{-1}$) corresponds to a slightly elongated region (in the direction perpendicular to the outflow) that includes the bright spot and decreases toward the bow, as can also be seen in a position-velocity diagram in Figure 5d. This kinematic behavior is expected if the bright spot corresponds to the Mach disk of the jet, where the jet material interacts with previously swept material accumulating in front of the jet and behind the bow shock.

A few cases are known in which a Mach disk is observed in H$_2$, such as the case of Kumar et al. (2002) at the N1 outflow in S233IR (Porras et al. 2000), in which a flattened structure is seen in high-definition H$_2$ images, although it is not supported spectroscopically. In HH 7 Khanzadyan et al. (2003) detect a flattened [Fe ii] structure that coincides with a blueshifted knot in H$_2$. This case is similar to the present case for Cep A (E). Blueshifted velocity is expected, since the jet material flows in all directions upon interacting with the Mach disk, in particular toward the present direction of the observer, nearly perpendicular to the outflow axis. As noted by Kumar et al. (2002), a detection of a Mach disk has rather interesting implications: (1) the jet would be partially molecular, (2) the
velocity of the jet must be small enough to prevent dissociation of H$_2$ in the Mach disk, and (3) the jet should be heavy. In this case we can estimate the velocity of the molecular jet from the maximum observed velocity ($-40$ km s$^{-1}$) minus the rest velocity ($V_0 = -11.3$ km s$^{-1}$) to be around 29 km s$^{-1}$. The distance from the Mach disk to the bow shock apex is 500–800 ($D = 725$ pc). We regard this as the kinematic evidence of a C-shock, since the slow acceleration of material ahead of the shock is quite evident.

3.3. Cep A (W) Velocity Gradients

The western H$_2$ outflow in Cep A displays a series of wide arcs reminiscent of thin sections of shells. The outflow has been described as a hot bubble (Hartigan et al. 2000) that drives C-shocks into the surrounding medium. In some shock fronts along this outflow, they observed that H$_2$ emission leads to the optical [S ii] $\lambda6717$, which in turn leads H$\alpha$. This is taken as evidence in favor of a C-shock that slowly accelerates and heats the ambient medium ahead of the shock. We find further evidence of this from the H$_2$ kinematics. A velocity gradient from higher to lower velocities is observed in some of the individual arc structures, as is the case for the one labeled I in Figure 1. The spatial map of the centroid velocities in Cep A (W) is shown in Figure 6. A smooth velocity gradient covering from $-36$ to $-8$ km s$^{-1}$ is observed in the southwest border of the arc, in the direction indicated by the arrow. That is, there is a velocity gradient going from blueshifted to closer to the rest velocity of the molecular cloud ($-11.2$ km s$^{-1}$) in a region of 17", or 0.06 pc at $D = 725$ pc. We regard this as the kinematic evidence of a C-shock, since the slow acceleration of material ahead of the shock is quite evident.

3.4. Flux-Velocity Relation

We have calculated the flux-velocity diagrams separately for the Cep A (E) and Cep A (W) outflows, as described in Salas & Cruz-Gonzalez (2002). For every pixel with signal
above the detection threshold, we added the fluxes of all the pixels with centroid velocities in bins of observed centroid velocity $|v_{\text{obs}} - v_{\text{rest}}|$ with respect to the rest velocity of the region ($-11.2$ km s$^{-1}$). The flux-velocity diagrams so obtained are shown in Figures 7 and 8.

As shown by Salas & Cruz-González (2002), this procedure gives similar flux-velocity relations for a variety of outflows, consisting of a flat spectrum for low velocities followed by a power-law decrease above a certain break velocity. The power-law index is very similar for different outflows, as is the case for Cep A (E) and Cep A (W), in which it is $-2.6 \pm 0.3$ and $-2.7 \pm 0.9$, respectively (Figs. 7 and 8, solid lines). The break velocity, however, is a little different. The logarithm of $v_{\text{break}}$ (in km s$^{-1}$) takes values of $0.95 \pm 0.07$ and $1.13 \pm 0.13$, respectively, a difference of around 2 $\sigma$, which suggests that $v_{\text{break}}$ may be larger for Cep A (W). As was discussed in Salas & Cruz-González (2002), outflows of different lengths ($l$) show break velocities varying as $v_{\text{break}} \propto l^{0.4}$, a result that is taken to imply an evolutionary effect, similar to the case of CO outflows (Yu et al. 1999). However, in the case of Cep A (E) and Cep A (W) the outflow length is very similar, as might be the outflow age. Salas & Cruz-González also argue that another cause for a difference in break velocities could be the amount of turbulence in the outflow, which might be the case for Cep A, as is mentioned at the end of § 3.1. We next explore this possibility through the use of a wavelet analysis.

4. WAVELET ANALYSIS OF THE H$_2$ EMISSION FROM CEPHEUS A

4.1. Description

Both H$_2$ emission regions in Cep A show a rather complex velocity-position structure (see Fig. 2). We have carried out a wavelet analysis in an attempt to understand the relation between sizes of clumps, velocity, and velocity dispersion as a function of position along the outflows.

Fig. 6.—Velocity gradients in Cep A (W). The variations in the gray scale show velocity in the range $-40$ km s$^{-1}$ (black) to $-4$ km s$^{-1}$ (white).

Fig. 7.—Flux-velocity relation of H$_2$ $v = 1$--0 S(1) line emission in Cep A (E). The long-dashed line shows the flat low velocity behavior up to a break velocity, whereas the solid line indicates a power-law decrease. The dashed lines show the range of possible values of the power-law index ($-2.6 \pm 0.3$) and of log($v_{\text{break}}$) $(0.95 \pm 0.07)$. 
regions, the main axis, is parallel to the bases of different sizes, one along and one across the main axis.

along the east-west direction. On these rotated images, we symmetry, we rotated the H2 image so that the long part of both structures) of the line center velocity and of the line widths obtained, the spatial averages (over the characteristic sizes of the two regions of the structures) of the line center velocity and of the line widths from total H2 flux images created by adding the continuum-characteristic sizes. The sizes of these clumps are determined from images created by adding the continuum-Gill & Henriksen (1990) to study turbulence in molecular clouds. Although their procedure is equivalent to the one followed by our case is similar to that of Riera et al. (2003) in the sense that overlap with their neighbors. Naturally, the biggest clumps identify the size of the smaller clumps and then the bigger ones. For each pair (a_x, a_y) with 1 pixel ≤ a_x ≤ 30 pixels and 1 pixel ≤ a_y ≤ 30 pixels for Cep A (E) and 1 pixel ≤ a_x ≤ 35 pixels and 1 pixel ≤ a_y ≤ 35 pixels for Cep A (W), with a resolution of 0.853 pixel⁻¹ in the x-direction and 0.848 pixel⁻¹ in the y-direction. The upper limiting values for a_x and a_y were selected to allow very little overlapping in the size of the regions in adjacent peaks. The values of the peaks y_k of the wavelet transform obtained for each position x along the region are shown in the bottom panel of Figure 9. Also shown as error bars are the values of

where \( r = [(x/a_x)^2 + (y/a_y)^2]^{1/2} \); \( a_x \) and \( a_y \) are the scale lengths of the wavelets along the x- and y-axes, respectively; and C = \( (a_x^2 + a_y^2)^{-1/2} \). This is a very commonly used wavelet, but we also choose to use it because it simplifies the detection of intensity peaks, better approaches the shape of the intensity peaks, and behaves well under fast Fourier transform (FFT) calculations.

To compute the wavelet transform, we have to calculate the convolutions

\[
T_{a_x, a_y}(x, y) = \int \int I(x', y') g(r'; a_x, a_y) dx' dy'
\]

for each pair \((a_x, a_y)\), where \( r' = \left[ \left( (x' - x)/a_x \right)^2 + \left( (y' - y)/a_y \right)^2 \right]^{1/2} \), \( I(x, y) \) is the intensity at pixel position \((x, y)\), and \( g(r'; a_x, a_y) \) is given by equation (1). These convolutions are calculated by using a FFT algorithm (Press et al. 1992).

The wavelet-transformed images \( T_{a_x, a_y}(x, y) \) correspond to smoothed versions of the intensity of the H2 image. We use these images to find the sizes of the structures in the H2 regions of Cep A. First, on the transformed image with \( a_x = a_y = 1 \), we fixed the position of \( x \) and found all the values of \( y \) where \( T_{a_x, a_y}(x, y) \) has a local maximum. Several maxima can be found for each position \( x \) that correspond to different structures observed across the regions. The maxima found with \( a_x = a_y = 1 \) also correspond to the local maxima of \( I(x, y) \).

For each pair \((x, y)\) where \( I(x, y) \) has a maximum, we determine \((a_x, a_y)\) in the \( a_x \) and \( a_y \) space, where the wavelet transform has a local maximum. The values of \( a_x \) and \( a_y \) determine the characteristic size of the clump with a maximum intensity at \((x, y)\). The \((a_x, a_y)\)-space is searched in such a way that we first identify the size of the smaller clumps and then the bigger ones. This progressive selection allows us to avoid choosing clumps that overlap with their neighbors. Naturally, the biggest clumps have a structure similar to the whole region.

4.2. Size of the H2 Clumps

The results obtained with the process described above are shown in Figure 9. This figure shows the two images of molecular hydrogen emission of Cep A, which have been rotated by 19° so the lengthwise dimension of the regions are more or less parallel to the x-axis. In the case of Cep A (E), the x-axis has been also inverted (i.e., west is to the right), so the x-values in both panels in Figure 9, although arbitrary in origin, are an estimate of the offset from the central region between the two regions. It has to be clarified that the x-coordinates are values from independent images, so there is really no correlation between them. However, the span in the vertical and horizontal axis are kept the same in both panels of Figure 9 to ease the size comparison for each region. Six large structures (B–G) were identified in Cep A (E) and six (H–M) for Cep A (W) (see Fig. 1). The spatial limits of these regions are shown in Table 1.

The H2 intensity maps have then been convolved with a set of wavelets \( g(r; a_x, a_y) \) with 1 pixel ≤ \( a_x \) ≤ 30 pixels and 1 pixel ≤ \( a_y \) ≤ 30 pixels for Cep A (E) and 1 pixel ≤ \( a_x \) ≤ 35 pixels and 1 pixel ≤ \( a_y \) ≤ 35 pixels for Cep A (W), with a resolution of 0.853 pixel⁻¹ in the x-direction and 0.848 pixel⁻¹ in the y-direction. The upper limiting values for \( a_x \) and \( a_y \) were selected to allow very little overlapping in the size of the regions in adjacent peaks.

The values of the peaks \( y_k \) of the wavelet transform obtained for each position \( x \) along the region are shown in the bottom panel of Figure 9. Also shown as error bars are the values of

\[
g(r; a_x, a_y) = C(2 - r^2)e^{-r^2/2}, \tag{1}
\]
Fig. 9.—Position and characteristic sizes of the structures (bottom) and contour plots of the H$_2$ image (top) obtained from the velocity channels for Cep A (E) (left) and Cep A (W) (right). Contour levels are shown from 0.0 to 1.5 counts s$^{-1}$ in 0.1 counts s$^{-1}$ increments for both plots. Crosses show the positions of the maximal $y_k$ of the wavelet transform obtained for the different values $x$ along the region. The characteristic sizes $a_{x,k}$ and $a_{y,k}$ of these maximals are shown as error bars centered on the positions of the maximal. We have separately rotated the map of each region by 19° so they are more or less parallel to the $x$-axis. The distances $y$ (across the regions) and $x$ (along the regions), in pixel units, are with respect to the rotated image and have an arbitrary origin. In the Cep A (E) case, the $x$-axis has been inverted (i.e., west is to the right) for analysis purposes (see text).
rms velocity dispersion of the structure of size $x$ position structures.

mate of the characteristic sizes along and across the observed $ax$ and $ay$ small (2 pixels) and large (25 pixels) structures located with to attain nature of these anisotropies is not clear. each region as a whole, with the eastern region being longer than it may be possible that the individual structures follow the pattern of Cep A (E), a property that has also been qualitatively observed. It western outflow, however, shows relatively wider structures than indicating structures that are longer than they are wide. The Table 1. We note that all the values are in general less than one, Figure 10 shows $\langle a_y \rangle$ as a function of position $x$ along the two Cep A regions. In both outflows we notice the presence of small (2 pixels) and large (25 pixels) structures located with no discernible order along the axial position. The values of $a_{x,k}$ as a function of $a_{x,k}$ are shown in Figure 11 for Cep A (E) and Cep A (W). These graphs represent a measure of the symmetry of the structures. Isotropic structures are expected to attain $a_{x,k} = a_{y,k}$ and should thus be located on a line of unitary slope. The values of the slope $m$ for each region are also given in Table 1. We note that all the values are in general less than one, indicating structures that are longer than they are wide. The western outflow, however, shows relatively wider structures than Cep A (E), a property that has also been qualitatively observed. It may be possible that the individual structures follow the pattern of each region as a whole, with the eastern region being longer than it is wide, whereas the western region is the opposite. However, the nature of these anisotropies is not clear.

4.3. Spatial Distributions of the Radial Velocities and the Line Widths

In this section we describe the spatial dependence of the kinetic properties of the two main $H_2$ emission regions of Cep A.

From the cube of position-velocity data, we calculate two moments of the line profiles for each pixel:

$V_c = \frac{\int v I_c dv}{\int I_c dv}$

(4)

and

$W^2 = \frac{\int (v - V_c)^2 I_c dv}{\int I_c dv}$.

(5)

In these equations, $v$ is the radial velocity and $I_c$ is the intensity at a fixed position $(x, y)$ of successive channel maps. The integrals are carried out over all the velocity channel maps. Here $V_c$ is the barycenter of the line profile (i.e., the "line center" radial velocity), and $W$ is a second-order moment that reflects the width of the line profile.

With these values of $V_c$ and $W$ computed for all positions $(x, y)$ on the plane of the sky, we calculate the following spatial averages (Riera et al. 2003):

$\langle V_c \rangle = \frac{\int_{S_{a_i, a_j}} V_c(x', y') I(x', y') dx' dy'}{\int_{S_{a_i, a_j}} I(x', y') dx' dy'}$,

(6)

$\langle W^2 \rangle = \frac{\int_{S_{a_i, a_j}} W^2(x', y') I(x', y') dx' dy'}{\int_{S_{a_i, a_j}} I(x', y') dx' dy'}$,

(7)

$\langle \Delta v^2 \rangle = \frac{\int_{S_{a_i, a_j}} [V_c(x', y') - \langle V_c \rangle]^2 I(x', y') dx' dy'}{\int_{S_{a_i, a_j}} I(x', y') dx' dy'}$,

(8)

where $I(x', y')$ is the $H_2$ flux obtained from co-adding all the channel maps. These integrals are carried out over areas $S_{a_i, a_j}$, which are ellipses with central positions $(x, y)$ and major and minor axes, $a_x$ and $a_y$, corresponding to all the values that have been identified as peaks of the wavelet transform.

The value of $\langle W^2 \rangle(x, y)$ corresponds to the line width spatially averaged over the ellipse $S_{a_i, a_j}$ with a weight $I(x', y')$. The same weight spatial average is calculated for the line center velocities $\langle V_c \rangle$ within the ellipse, as well as the standard deviation $\langle \Delta v^2 \rangle^{1/2}$ of these velocities.

Figure 12 shows the line centers, line widths, and standard deviations as a function of position $x$ along the regions. (All of the points at different positions $y$ across the region and with different
Fig. 11.—Left: Values of $a_y$ plotted as a function of $a_x$ for the different regions along Cep A (E). The linear fits to the points are drawn as solid lines. The slope $m$ of the linear fit is listed in Table 1. Right: Same as the left panel, but for Cep A (W).
The line center or centroid velocity, displayed in the top panels, shows different behaviors for Cep A (E) and Cep A (W). As had been previously noted, Cep A (E) has a constant velocity along the outflow ($\sim -19$ km s$^{-1}$), whereas Cep A (W) shows a velocity gradient from about $-21$ to $-2$ km s$^{-1}$. A large dispersion in the line centers for Cep A (W) around positions 115 and 162 is probably due to insufficient S/N, since a similar increase in the dispersion velocity (Fig. 12, middle) is concurrent but lacks a corresponding increment of the line width (bottom). Other features present in these figures seem real. Most remarkably, the velocity dispersion for Cep A (E) (left middle) increases monotonically with distance from the source, going from around 4 to 10 km s$^{-1}$ in 150 pixels or 0.5 pc. No such increase is observed in the western outflow, for which we obtain a constant $3 \pm 2$ km s$^{-1}$ velocity dispersion. The line widths (bottom) seem dominated by the width of the instrumental profile of 22 km s$^{-1}$.

Hence, we conclude from this analysis that the eastern and western outflows seem intrinsically different. Whereas the eastern outflow shows a constant line center velocity and an increasing velocity dispersion, the western outflow behaves otherwise, showing a velocity gradient and a constant velocity dispersion with a lower value. This had been noted in our qualitative analysis of the observations. Now, if we take the velocity dispersion within each cell as a measure of turbulence, which seems reasonable, then the region Cep A (E) is more turbulent than Cep A (W), and it is notable that the turbulence increases with distance from the “central” source compared with a constant behavior in the western source.

4.4. Deviations of the Line Center Velocity and the Size of the Region

Figure 13 shows the deviations of the line center velocity (velocity dispersion), averaged over sizes chosen from the wavelet

![Figure 12](image_url)
spectrum, as a function of the size of each region. Such deviations of the line center velocity, averaged over sizes chosen from a wavelet spectrum, have been previously used by Gill & Henriksen (1990) to study turbulence in molecular clouds. In this figure we can appreciate a large dispersion of values. However, points tend to clump together around certain regions of the diagrams.

A closer examination of Figure 13 for Cep A (E) shows a cluster of points at position (20, 9) corresponding to the structure labeled G (see Fig. 9), whereas the points corresponding to structure C lie just below of them. Taken separately, each one of these groups seem to define a line in the diagram, offset vertically from one another but of comparable slopes. The slope of the lines, in fact, is also similar to the slope that would be obtained by fitting a line to all the data, \( C_1 \), with a slope \( \alpha \) of 0.2574 for Cep A (E) and 0.199 for Cep A (W).

In the western outflow it is more difficult to identify clumps of points, and we were not able to find a relation of velocity dispersion with position either. A least-squares fit of a line to the data gives a slope of 0.20. However, there appear to be two disperse clumps of points, a lower one and an upper one, each defining a line with a larger slope \( \alpha \) and parallel to each other. This latter value would be closer to a case in which the virial theorem applies.

The analysis of the individual H\(_2\) condensations (cf. Fig. 1) in the Cep A (E) and Cep A (W) regions is illustrated in Figures 14 and 15, respectively. We show the deviations for the line center velocity, averaged over sizes chosen from the wavelet spectrum for each condensation. For each H\(_2\) condensation, the values of the best-fit slope \( \alpha \), are presented in the last column of Table 1. The Cep A (E) clumps yield a mean \( \alpha \) of 0.21 ± 0.21, whereas the Cep A (W) yield a slightly higher value of 0.34 ± 0.20. However, some knots show poor correlations than others (see plots for knots B, E, H, and I). Using only knots C, D, F, and G for Cep A (E) and knots J, K, L, and M for Cep A (W) yields an \( \alpha \) of 0.30 ± 0.20 and 0.44 ± 0.08, respectively.

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**Fig. 13.**—Relation of the deviations of the line center velocity, \( \langle \Delta v^2 \rangle^{1/2} \) (or \( \sigma \)), averaged over regions chosen from the wavelet spectrum, and the size \( a \) of the region, \((a_x^2 + a_y^2)^{1/2}\), for the complete Cep A (E) (left) and Cep A (W) (right) regions. The solid line is the best fit to the points, with a slope \( \alpha \) of 0.2574 for Cep A (E) and 0.199 for Cep A (W).
Fig. 14.—Same as Fig. 13, but for regions B–G of Cep A (E). The solid line is the best fit to the points. The values of the slope $\alpha$ of the fits are given in the last column of Table 1.
Fig. 15.—Same as Fig. 14, but for regions H–M of Cep A (W).
respectively, which support a Kolmogorov case in the first and a more virialized region for the second case.

5. CONCLUSIONS

We have presented the velocity structure of the molecular hydrogen outflows from the regions Cep A (E) and Cep A (W) obtained from the H$_2$ $\nu = 1\rightarrow 0$ S(1) Doppler-shifted line emission at 2.12 μm. Both the velocity channel maps and the integrated H$_2$ image show a complex structure of 12 individual clumps along two separated structures oriented roughly in the east-west direction.

Given the complexity of these structures, we have carried out an anisotropic wavelet analysis of the H$_2$ image, which automatically detects the position and characteristic sizes (along and across the region axis) of the clumps.

1. There is evidence for a Mach disk in Cep A (E). The efflux point is located at the center of a bow shock structure, and we measure blueshifted velocities of 22–28 km s$^{-1}$. This observation implies that a molecular jet is driving the outflow.

2. Cep A (W), on the other hand, is consistent with a hot bubble in expansion driving C-shocks. We presented the kinematic gradient of one such shock as an example.

3. The H$_2$ flux-velocity relation is present in both outflows. The break velocity of the eastern outflow is lower than that of the western outflow, and we have argued that this is indicative of greater turbulence in Cep A (E).

4. The wavelet analysis has confirmed and quantified trends observed in the centroid velocity measurements: the eastern outflow shows a constant line center velocity and an increasing velocity dispersion, whereas the western outflow shows a velocity gradient and a constant velocity dispersion. The larger velocity dispersion and gradient in the eastern outflow is taken as indicative of turbulence and allows us to conclude also that turbulence increases with distance.

5. Suggestive propositions about the kind of turbulence present in both outflows are extracted from an analysis of the relation of the velocity dispersion as a function of the size of the structures (cells) identified as unities by the wavelet spectrum. Using only knots with a good correlation yields an $\alpha$ of 0.30 ± 0.20 for Cep A (E) and 0.44 ± 0.08 for Cep A (W), which support a Kolmogorov case in the first and a more virialized region for the second case.

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