The realistic models of neutron stars are considered for simple $R + \alpha R^2$ gravity and equivalent Brance-Dicke theory with dilaton field in Einstein frame. For negative values of $\alpha$ we have no acceptable results from astrophysical viewpoint: the resulting solution for spherical stars doesn’t coincide with Schwarzschild solution on spatial infinity. The mass of star from viewpoint of distant observer tends to very large values. For $\alpha > 0$ it is possible to obtain solutions with required asymptotics and well-defined star mass. The mass confined by stellar surface decreases with increasing of $\alpha$ but we have some contribution to mass from gravitational sphere appearing outside the star. The resulting effect is increasing of gravitational mass from viewpoint of distant observer. But another interpretation take place in a case of equivalent Brance-Dicke theory with massless dilaton field in Einstein frame. The mass of star increases due to contribution of dilaton field inside the star. We also considered the possible constraints on $R^2$ gravity from GW 170817 data. According to results of Bauswein et al. the lower limit on threshold mass is $2.74^{+0.04}_{-0.01} M_\odot$. This allows to exclude some equations of state for dense matter. But in $R^2$ gravity the threshold mass increases for given equation of state with increasing of $\alpha$. In principle it can helps in future discriminate between General Relativity and square gravity (of course one need to know equation of state with more accuracy rather than now).

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I. INTRODUCTION

The general theory of relativity (GTR) is one of the most carefully checked physical theories to date. Within the framework of the GTR, it was possible to describe phenomena that can’t be explained using Newton’s theory of gravity (perihelion precession of planets, gravitational lensing), to predict a number of effects, to explain the existence of an upper limit to the mass of neutron stars. However, at the present moment the GTR is facing the problem of “dark energy” in cosmology. Observational data of type Ia supernovae and baryon acoustic oscillations in the cosmic microwave background provide strong evidence that the universe is expanding with acceleration. If one assumes that the universe is filled with just matter and radiation, such cosmological dynamics can’t be explained. The accelerating expansion can be caused by a substance with negative pressure (“dark energy”) but its physical nature is unclear. It is also worthwhile to mention the old problem of “dark matter”. Possible candidates for constituting “dark matter” (so called WIMPs — weakly interacting massive particles) are still not found using the Large Hadron Collider.

Accepted by the majority in the scientific community, the ΛCDM-model assumes that dark energy is nothing else but the non-zero vacuum energy. The cosmological constant idea was introduced already by A. Einstein with the purpose of constructing a stationary solution for the cosmological equations. It turns out the cosmological constant can also lead to the accelerating expansion of the universe (exponentially with time). From a phenomenological point of view, this model adequately describes observational data but has some fundamental disadvantages. The quantum field theory approach of describing gravity leads to the theoretical vacuum energy value being greater than the observational one by 120 orders of magnitude.

The discovery of the accelerating expansion of the universe stimulated the search for models of gravity that can be used to describe the cosmological evolution consistent with observations without “dark” components (ref. [1-4]). Models of modified gravity are interesting in that it is possible to offer a unified description of the cosmological acceleration and the early inflation within their framework [5-8]. Observational data of type Ia supernovae or the cosmic microwave background anisotropy can also be explained using these models [7] - [11].

It is not possible to confirm or to disprove such theories using only cosmological observational data. However, if the GTR just approximately describes the real gravity, one can hope that deviations from the GTR can be found in strong gravitational fields [12]. Nature has that kind of unique “laboratories” where our ideas about gravity can be tested. These objects are neutron stars. The matter inside them is compressed to the densities of $\sim 10^{15}$ g/cm$^3$, and the “escape velocity” near the surface is close to the speed of light. Alternative theories of gravity must at least not contradict the very fact of the existence of relativistic stars. The more detailed analysis includes examinations of the
mass–radius relations, the inertial characteristics and the rotation in these theories for neutron stars. Comparison of the obtained results with the GTR results allows to determine the possible ways of testing theories alternative to the GTR.

Parameters of neutron stars (mass, radius, moment of inertia etc) depend decisively from equation of state (EoS). The (EoS) for extreme dense matter in neutron stars is one of the puzzle of modern astrophysics. The measurement of neutron stars masses can be done with high precision by using post–Keplerian parameters. From recent observations [13, 14] it follows only that maximal mass of neutron stars is around \(2M_\odot\). This constraint on maximal mass ruled out many soft EoS of nuclear matter mainly with hyperons. Unfortunately the determination of neutron stars radii is more complicated task. Its values can be obtained from X-ray observation emitted by the atmosphere of star. A large number of unknown parameters determine process of emission and therefore estimations of neutron star radius from such observations are different from each to other (see [15] - [24]). Finally one need to point that there are no well-defined simultaneous measurements of mass and radius for neutron stars. Therefore current observations give only weak constraints on properties of neutron stars. Although many EoS with maximal mass limit \(M_{\text{max}} < 2M_\odot\) are considered now as unrealistic inaccuracy in the knowledge of exact dependence between mass and radius (\(M - R\) diagram) is very large for discrimination between dozens of another realistic EoS.

The simple \(R\)-squared theory of gravity was considered as viable alternative to GTR for description of neutron stars in many paper. Initially the perturbative approach was used. The scalar curvature \(R\) is defined by Einstein equations at zeroth order on the small parameter, i.e. \(R \sim T\), where \(T\) is the trace of energy–momentum tensor. This approach is applied to constructing of neutron star models in \(f(R) = R + \alpha R^2 + \beta R^3\) and \(f(R) = R + \alpha R^2(1 + \gamma \ln R)\) gravity also in [25] - [28].

In modified \(f(R)\) gravity model with cubic and quadratic terms, it is possible to obtain neutron stars with \(M \sim 2M_\odot\) for simple hyperon equations of state (EoS) although the soft hyperon equation of state is usually treated as non-realistic in the standard General Relativity [29]. The possible signatures of modified gravity in neutron star astrophysics also can include existence of neutron stars with extremely magnetic fields [30, 32].

The paper is organized as follows. We considered realistic models for simple \(R + \alpha R^2\) gravity and equivalent Brance-Dicke theory with dilaton field in Einsein frame. Basic equations and numerical scheme are presented in Section 2. The results of calculation including mass profile, mass–radius diagram are given in the next section. One can found that negative values of \(\alpha\) there is no acceptable result. The gravitational mass infinitely grows with distance. For positive \(\alpha\) it is possible to obtain models with well-defined star mass. The mass confined by stellar surface decreases with increasing of \(\alpha\) but we have some contribution to mass from gravitational sphere appearing outside the star. The resulting effect is increasing of gravitational mass from viewpoint of distant observer. Section 4 is devoted to possible discrimination between \(R^2\) gravity and General Relativity in light of recent detection of gravitational waves from merging neutron stars (object GW170817). We use results from Section 4 but in another interpretation namely in frames of equivalent Brance-Dicke theory with massless dilaton field in Einstein frame. In such theory the mass of star increases due to contribution of dilaton field inside the star. Such interpretation is more clear for merging of neutron stars. We can assume that merging of neutron stars can be considered in frames of General Relativity, and therefore apply results for threshold masses obtained in GTR. For given EoS the parameters of neutron stars in modified gravity change in comparison with GTR and as consequence the threshold mass increases with increasing of \(\alpha\).

II. MODIFIED TOLMAN-OPPENHEIMER-VOLKOFF EQUATIONS FOR \(f(R) = R + \alpha R^2\) GRAVITY

The action for a simple \(f(R)\)-gravity model can be written as (from now on, we use system of units \(G = c = 1\)):

\[
S = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left[ R + f(R) \right],
\]

where the Einstein-Hilbert action, which is proportional to the scalar curvature \(R\), was explicitly expressed. Here \(f(R)\) is a real differentiable function of the scalar curvature.

Therefore the gravitational equations can be written as:

\[
R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{1}{1 + f'_R} \left[ -8\pi T_{\mu\nu} - \nabla_\mu \nabla_\nu f_R \right. \\
+ g_{\mu\nu} \nabla^\alpha \nabla_\alpha f_R + \frac{1}{2} \left( f(R) - R f_R \right) g_{\mu\nu}].
\]

Here \(f_R \equiv \frac{df(R)}{dR}\), and \(T_{\mu\nu}\) are the components of the energy–momentum tensor.
As applied to compact non-rotating stars, the solution of the obtained equations should be sought in terms of the spherically symmetric metric:

$$ds^2 = B(r)\,dt^2 - A(r)\,dr^2 - r^2(d\theta^2 + \sin^2 \theta \,d\phi^2).$$

(3)

For unknown functions of the radial coordinate $A$ and $B$, and for the scalar curvature $R$, we have the equations (4 - 6) on the assumption that the energy-momentum tensor is diagonal $T^\mu_\nu = \text{diag}(-\rho, p, p, p)$:

$$A' = \frac{2rA}{3(1 + f_R)} \left[8\pi A(\rho + 3p) + \frac{A}{2}R - \frac{3B'}{2rB} + Af(R) - f_R \left(\frac{3}{2} + \frac{3B'}{2rB}\right)f_{2R} R'\right],$$

(4)

$$B'' = \frac{B'}{2} \left(\frac{A'}{A} + \frac{B'}{B}\right) + \frac{2A'B}{rA} + \frac{2B}{(1 + f_R)}\left[-8\pi A\rho - \frac{A}{2}R + \left(\frac{3}{2} + \frac{3B'}{2rB}\right)f_{2R} R'\right],$$

(5)

$$R'' = R'\left(\frac{A'}{2A} - \frac{B'}{2B} - \frac{2}{r}\right) - \frac{A}{3f_{2R}}\left[8\pi(\rho - 3p) - (1 - f_R)R - 2f(R)\right] - \frac{f_{4R}}{f_{2R}} R'^2.$$  

(6)

Here $\rho$ and $p$ are the density and the pressure of the matter respectively, and the functions $f_{2R}$ and $f_{3R}$ are the second and the third derivatives of the function $f(R)$ with respect to the scalar curvature. The relationship between the pressure and the density is given by the chosen equation of state. The other equation can be derived from the Bianchi identity:

$$p' = -\frac{\rho + p}{2} B'.$$

(7)

The resulting system of the differential equations can be solved if the boundary conditions (at the centre of the star and at infinity) are specified. Further, we will consider in detail the simple case when $f(R) = \alpha R^2$, where $\alpha$ is a parameter.

We also give description of our task in terms of scalar-tensor theory [33, 34]. One can consider the equivalent Brans-Dicke theory with following coupling between scalar field $\Phi = 1 + df(R)/dR$ and curvature $R$:

$$S_g = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(\Phi R - U(\Phi)\right).$$

(8)

Here $U(\Phi) = Rf'(R) - f(R)$ is potential of scalar field. In Einstein frame under conformal transformation $\tilde{g}_{\mu\nu} = \Phi g_{\mu\nu}$ we have

$$S_g = \frac{1}{16\pi} \int d^4x \sqrt{-\tilde{g}} \left(\tilde{R} - 2\tilde{g}^{\mu\nu}\partial_\mu \phi \partial_\nu \phi - 4V(\phi)\right).$$

(9)

Here redefined scalar field $\phi = \sqrt{3}\Phi/2$ and potential in $V(\phi) = \Phi^{-2}(\phi)U(\Phi(\phi))/4$ are introduced.

For spacetime metric we use the same form as in the case of $f(R)$ gravity (with redefined metric functions $\tilde{A}$, $\tilde{B}$):

$$ds^2 = \Phi ds^2 = \tilde{B}^2(\tilde{r}) dt^2 - \tilde{A}^2(\tilde{r}) d\tilde{r}^2 - \tilde{r}^2 d\Omega^2,$$

(10)

where $\tilde{r}^2 = \Phi r^2$, $\tilde{B}^2 = \Phi B^2$. From relation

$$\Phi A^2 d\tilde{r}^2 = \tilde{A}^2 d\tilde{r}^2$$

one can obtain the following link between $\tilde{A}$ and $A$ (for $f(R) = R + \alpha R^2$ gravity):

$$\tilde{A}(r) = A(r) \left(1 + \frac{r}{2} \frac{d\ln \Phi}{dr}\right)^{-1} = A(r) \left(1 + \alpha \frac{dR(r)}{1 + 2\alpha R(r) \frac{dR(r)}{dr}}\right)^{-1}.$$
Therefore the mass parameter $\tilde{m}(r)$ is

$$
\tilde{m}(r) = \frac{r}{2} \left( 1 - A^{-1} \left( 1 + \frac{\alpha r}{1 + 2\alpha R} \frac{dR}{dr} \right)^2 \right)
$$

(11)

For functions $\tilde{A}(\tilde{r})$ and $\tilde{B}(\tilde{r})$ we have the following equations (the tildes are omitted for simplicity):

$$
\frac{1}{r^2} \frac{d}{dr} \left[ r \left( 1 - A^{-1} \right) \right] = 8\pi e^{-4\phi/\sqrt{3}} \rho + e^{-2\lambda} \left( \frac{d\phi}{dr} \right)^2 + V(\phi),
$$

(12)

$$
\frac{1}{r} \left[ A^{-1} \frac{dB}{B} - \frac{1}{r} \left( 1 - A^{-1} \right) \right] = 8\pi e^{-4\phi/\sqrt{3}} p + e^{-2\lambda} \left( \frac{d\phi}{dr} \right)^2 - V(\phi),
$$

(13)

These equations are nothing else than ordinary TOV equations with redefined density and pressure and additional density and pressure of scalar dilaton field.

The hydrostatic equilibrium condition is

$$
\frac{dp}{dr} = -\frac{\rho + p}{2} \left( \frac{dB}{dr} - \frac{2}{\sqrt{3}} \frac{d\phi}{dr} \right).
$$

(14)

For scalar dilaton field we have equation equivalent to equation for scalar curvature in $f(R)$ theory:

$$
\Box \phi + \frac{dV(\phi)}{d\phi} = -\frac{4\pi}{\sqrt{3}} e^{-4\phi/\sqrt{3}} (\rho - 3p).
$$

(15)

Here $\Box$ is D’Alamber operator in metric (10). The potential of scalar field in considered case of $R^2$-gravity is

$$
V(\phi) = \frac{1}{4\alpha} \left( 1 - e^{-2\phi/\sqrt{3}} \right)^2.
$$

(16)

The unknown parameters (the mass and the radius) are dependent on the density and the pressure in the centre of a neutron star $p(0) = p_c$, $\rho(0) = \rho_c$. We considered a representative set of equation of states. Firstly the well-known APR EoS [35] is obtained from three-nucleon potential and Argonne 18 potential with UIX potential. For completeness, we have also considered the SLy EoS [36], [37] obtained from many body calculations with simple two-nucleon potential. As an example of EoS based on relativistic mean-field (RMF) calculations we take the GM1 model firstly considered by Glendenning and Moszkowski [38]. We included into consideration the realistic EoS proposed recently by [39]. The maximal mass for GM1 EoS is below than limit established of observations (for General Relativity) but for $\alpha \sim 10$ we found that maximal mass is around two solar.

For the metric function $A(r)$, the following condition must be met at large distances:

$$
A(r) = \left( 1 - \frac{2M}{r} \right)^{-1},
$$

where $M$ is the gravitational mass of an object. According to this, we choose $A(0) = 1$ as the condition in the centre [40]. The function $B(r)$ is included in the equations as $B' / B$ and $B'' / B$, so the solution of the system does not depend on the value of $B$ in the centre of a star. The condition $B(r) \to 1$ must be met at an infinite distance from the star ($\to \infty$). One can solve the equations for an arbitrary value of $B(0)$, find the solution of $B(r)$ for sufficiently large distances, extrapolate the asymptotics of the solution for $r \to \infty$, and hence obtain the value of $B_\infty$. Thus, the solution has the required asymptotics $B \to 1$, if the initial value is given by $B(0) = B(0) / B_\infty$.

The regularity of the equations at $r = 0$ demands that for the derivative $B'(r)$ at $r = 0$ the condition $B'(0) = 0$ is met. The algorithm for finding condition for the scalar curvature in the centre was developed using the bisection method. When the surface of the star is reached, the function $R(r)$ must have a positive value and then decrease rapidly while asymptotically approaching zero. The values of $R(0)$ were being found in the range $-100R_0 < R < 100R_0$, where $R_0 = 8\pi (\rho(0) - 3p(0))$ is the value of the curvature in the GTR.

Integration of the equation system was performed using the Runge–Kutta–Merson fourth-order method.

It should be noted that when the surface of the star is reached (that is, any of the conditions $\rho(r) < 0$ or $p(r) < 0$ is met) the system is simplified to the three equations (4), (5), (6) with zero density and pressure.
III. RESULTS OF CALCULATIONS FOR $R^2$-GRAVITY

Let us further consider negative and positive values of $\alpha$.

1) $\alpha < 0$. In this case, the scalar curvature outside the star oscillates (Fig.1). The value of the scalar curvature in the general theory of relativity $R(0) = 8\pi(\rho(0) - 3p(0))$ was chosen as the boundary condition for the scalar curvature in the centre of the star. Analysis shows that the qualitative form of the solution for the scalar curvature outside the star depends weakly on the value of $R(0)$.

![Figure 1](image)

Figure 1: The scalar curvature $R$ as a function of the radial coordinate for the star with the central density $\rho = 3 \cdot 10^{14}$ g/cm$^3$, obtained for APR EoS. The scalar curvature $R$ hereafter is given in units of $r_g^{-2}$ and parameter $\alpha$ in $r_g^2$, where $r_g = \sqrt{G M / c^2}$.

On the assumption that the solution for the functions $A(r)$ and $B(r)$ at large distances tends to the Schwarzschild solution, one can expect that

$$A(r) \to (1 - 2M(r)/r)^{-1}, \quad r \to \infty.$$  

However, the function $A(r)$ at large distances oscillates under the law (see Fig.3 and Fig.2)

$$A(r) \approx 1 + A_0(r) \sin \left( \sqrt{6|\alpha|} \cdot r \right), \quad A_0(r) << 1,$$

where $A_0(r)$ is a decreasing amplitude of the oscillation. This makes it impossible to determine the gravitational mass using the form of the solution for $A(r)$. One can use the solution for $B(r)$. The function $B(r)$ approaches some constant value from below as $r \to \infty$ so that $B(r) < B_\infty(r)$.

![Figure 2](image)

Figure 2: The function $A(r)$ as a function of the radial coordinate in narrow interval for parameters as on previous figure.

The main results of our calculations consist of the following:

1. For the large negative values $\alpha$, a significant increase of gravitational mass is observed outside the star.
2. For the small negative values of $\alpha$, the mass function undergoes a small smooth growth at large (in comparison with the size of the star) distances. But in any case the gravitational mass grows infinitely.

We depicted the mass-radius relation for negative values of $\alpha$ for case where integration of equations was performed to the distance $10^5$ km and $2.5 \times 10^5$ km (see Fig. 4, Fig. 5).

![Figure 3: The function $A(r)$ as a function of the radial coordinate for parameters as on Fig. 1.](image)

![Figure 4: Mass-radius relation for APR EoS. On the left: $\alpha = -0.025$. On the right: $\alpha = -0.0025$. The integration was performed to the distance of $10^5$ km. Hereafter $r_s$ is a star radius.](image)

As a result of the interpolation of the gravitational mass function to infinity ($r \to \infty$), the value of the function tends to infinity. Thus it can be argued that the solution of the gravitational equations can not asymptotically approach the Schwarzschild solution.

Although the mass function has an apparent kink and plateau for the given equation of state with any initial values of the pressure and the density (taken from the specified interval) in the centre of the star, an increase in the mass with the distance from the star is observed for the values of the parameter close to $\alpha = -0.025$. It is possible to reduce this increase by decreasing the value of the parameter $\alpha$, yet it is not possible to achieve a parallel alignment of the plateau for any values of the parameter (see Fig. 6, Fig. 7).

Similar results have been obtained for other equations of state. This allows to conclude that the gravitational mass of an object measured by a distant observer has an enormous value. This obviously does not represent the observations.

As for the boundary condition for the scalar curvature $R$ in the centre of the star, its changing does not lead to any qualitative change in the behaviour of the system solution.

2) $\alpha > 0$. In the case of $\alpha > 0$, the solution for the scalar curvature and for the function $A(r)$ behaves differently (Fig. 8, Fig. 9). The solution at large distances becomes asymptotic to the Schwarzschild solution for only one value
Figure 5: The same as on Fig. 4. On the left: $\alpha = -0.0025$, the integration was performed to the distance of $10^5$ km. On the right: $\alpha = -0.0025$, the integration was performed to the distance of $2.5 \times 10^5$ km.

Figure 6: The profiles of functions $B(r)$ and $M(r)$ for $\alpha = -0.025$. The integration was performed to the distance of $10^5$ km. The value of the density in the centre of the star is $\rho = 3 \cdot 10^{14}$ g/cm$^3$ of the scalar curvature in the centre. The mass of the star can be determined using $A(r)$:

$$M(r) = \left(1 - \frac{1}{A(r)}\right) \frac{r}{2}.$$ (17)
The mass-radius relation is given on Fig. 10.

Based on the obtained results, one can conclude that neutron stars have larger radii at the given stellar mass in the considered theory of gravity than in the GTR. However, the observed gravitational mass is a sum of the mass of the star itself and the mass of the "gravitational sphere" i.e. the region outside the star where the scalar curvature decreases rapidly while asymptotically approaching zero (see Fig. 11).

Considering the dependence of mass from radial coordinate in Brance-Dicke theory in Einstein frame we have more
Figure 10: The mass-radius relation for $\alpha > 0$ using APR EoS. On the left: $\alpha = 2$, the maximum mass is $2.20 \, M_\odot$. On the right: $\alpha = 5$, the maximum mass is $2.24 \, M_\odot$. The integration was performed to the distance of $50 \, \text{km}$.

Figure 11: The profile of $m(r)$ for various values of $\alpha$. The integration was performed to the distance of $50 \, \text{km}$. The value of the density in the centre of the star is $\rho = 3 \cdot 10^{14} \, \text{g/cm}^3$. The profile of mass slowly increases outside the star with distance.}

clear picture. The mass grows with $\alpha$ but the increasing of mass occurs within star. Outside the star mass doesn’t depend from radial coordinate.

IV. GRAVITATIONAL WAVES AND $r^2$ GRAVITY

Additional information about parameters of neutron stars can be extracted from gravitational waves generated by possible merging of neutron stars. First event of such type was detected recently (GW170817, see [41] - [44]). The measured chirp mass is $M_c = 1.118^{+0.004}_{-0.002} \, M_\odot$ and corresponding total mass is $M_{tot} = 2.74^{+0.04}_{-0.01} M_\odot$ with masses of components $M_a = 1.36 - 1.60 M_\odot$ and $M_b = 1.17 - 1.36 M_\odot$. From analysis of spectrum of gravitational waves performed by Abbott et al. [43] follows that amplitude of tidal effects is relatively small and therefore large radii of neutron stars are unacceptable.

From observations follows that electromagnetic radiation from GW170817 included a weak gamma-ray burst kilonova emission from the radioactive decay of the merger ejecta, and X-ray/radio emission. If maximum mass is sufficiently large supramassive neutron star appears after merging. This star should spin-down before gravitational collapse into a black hole. As result rotational energy is transferred into or Gamma Ray Burster (GRB) jet or kilonova (KN) ejecta. Therefore it is possible estimate the upper limit of neutron star with using gravitational wave signal and limits on energy of GRB and KN from electromagnetic signal. This idea was used in recent work [45]. According to this paper the remnant appeared after merging was not long lived, because of the relatively low energy of the ejecta inferred from electromagnetic wave data. This assumption gives upper limit on maximal mass of neutron star as...
2.17\,M_\odot with 90\% confidence. Then authors of [46] suggested a powerful method to constrain properties of neutron stars from total mass of GW170817. Assumption that merger did not result in a prompt collapse [47] allows to conclude that the radius \( R_{1.6} \) and \( R_{\text{max}} \) of nonrotating NSs with a mass of 1.6 \( M_\odot \) and maximal mass correspondingly should be larger than 10.68\,+0.08\,-0.04 km and 9.66\,+0.04\,-0.03 km. According to hydrodynamical simulations [48] the threshold mass for given \( R_{\text{max}} \) has a maximum for some mass. For larger radii the threshold mass grows. Assuming some limit on threshold mass it is possible to obtain upper limit on radius of star with maximal mass or 1.6\,M_\odot. Therefore combining various estimation one can obtain the acceptable interval for radii.

According to estimations the mass transferred into electromagnetic emission lies in interval 0.03\,M_\odot < M_{\text{em}} < 0.05\,M_\odot [49, 50]. For hypothetical prompt collapse one can put that threshold mass is

\[
M_{th} < M_{\text{tot}} = 2.74^{+0.04}_{-0.01}\,M_\odot.
\]

Therefore total binary mass can be considered for delayed collapse or no collapse scenario as lower limit for threshold mass. For threshold mass authors of [46] suggested following approximation:

\[
M_{th} = \left(-3.606 \frac{GM_{\text{max}}}{c^2 R_{1.6}} + 2.38\right) M_{\text{max}}
\]

where \( R_{1.6} \) is radius of nonrotating neutron star with mass of 1.6\,M_\odot and \( M_{\text{max}} \) means maximal value of NS mass for given equation of state. Another approximation for threshold mass can be defined via radius of stellar configuration with maximal mass:

\[
M_{th} = \left(-3.38 \frac{GM_{\text{max}}}{c^2 R_{\text{max}}} + 2.43\right) M_{\text{max}}
\]

For given equation of states (EoS) one can obtain the value of \( M_{th} \). These results can be applied for analysis of various models of gravity. In any case we propose that deviations from GR are sufficiently small and therefore the process of merging can be described in frames of post-newtonian formalism and we can use the approximation of threshold mass suggested in [46]. However the deviations from General Relativity affect on mass and radius of star for given density in center and then threshold mass for corresponding EoS should be vary.

As mentioned above for \( f(R) = R + \alpha R^2 \) gravity the mass confined by stellar surface decreases with increasing of \( \alpha \). The total gravitational mass increases due to contribution of gravitational sphere with nonzero scalar curvature appearing outside the star. The radii of sphere is around around tens km for realistic values of \( \alpha \). Therefore picture of merging is not clear. If we assume that part of gravitational mass outside the star doesn’t affect considerably on process of merging one can conclude that threshold mass decreases. In scalar-tensor theory in conformal gauge the situation is more easy. Dilaton sphere outside the star appears but its contribution to total gravitational mass is negligible. In principle one can consider that deviation from General Relativity is equivalent to some modification of equation of state for dense matter. Therefore we can use approximation (Fig.18), (Fig.19) for threshold mass.

We investigated the varying of threshold mass for EoS mentioned above in a case of scalar-tensor theory of gravity equivalent to \( f(R) = R + \alpha R^2 \) gravity. The main result is increasing of threshold mass with increasing of \( \alpha \). One note that the difference between values calculated by using Eqs. (18) and (19) slightly increases with \( \alpha \). However even for General Relativity (\( \alpha = 0 \)) this difference can exceed 0.05\,M_\odot (for SLy4 EoS). Increasing of threshold mass for \( \alpha = 10^2 \) is around of 0.12 − 0.14 \( M_\odot \) (according to (18)) in comparison with General Relativity. For approximation (19) we have more interesting picture: for SLy4 and APR EoS threshold mass increases by \( \sim 0.25 \, M_\odot \). In a case of MYN EoS increasing is 0.18 \( M_\odot \) and for GM1 we have again 0.14 \( M_\odot \).

Therefore from total mass of binary merger GW170817 we cannot in principle extract rigid constraints on parameter \( \alpha \). One can hope however that in future the upper bound on threshold mass will be established. For this one need to observe the event like GW170817 but with a higher chirp mass and with evidence of prompt collapse. In this case for any realistic EoS we can in principle estimate the upper limit of \( \alpha \).

\[
\begin{array}{|c|c|c|c|c|c|c|c|c|}
\hline
\alpha & 0 & 10 & 20 & 50 & \text{EoS} \\hline
\text{APR} & 2.23 & 10.01 & 11.29 & 2.23 & 10.46 & 11.47 & 2.25 & 10.60 & 11.58 & 2.29 & 10.86 & 11.73 & 2.31 & 10.96 & 11.87 \\hline
\text{GM1} & 1.93 & 12.13 & 13.66 & 1.96 & 12.17 & 13.62 & 1.97 & 12.27 & 13.68 & 2.00 & 12.44 & 13.81 & 2.03 & 12.56 & 13.93 \\hline
\text{SLy4} & 2.05 & 9.97 & 11.50 & 2.09 & 10.41 & 11.67 & 2.11 & 10.52 & 11.78 & 2.15 & 10.84 & 11.94 & 2.17 & 10.90 & 12.07 \\hline
\text{MYN} & 1.95 & 11.34 & 12.76 & 2.00 & 11.54 & 12.81 & 2.02 & 11.59 & 12.90 & 2.06 & 11.79 & 13.04 & 2.08 & 11.94 & 13.17 \\hline
\end{array}
\]

Table I: The parameters of neutron stars (maximal mass, radius for maximal mass configuration and radius \( R_{1.6} \) for star with \( M = 1.6\,M_\odot \)) for various EoS in GTR and scalar-tensor gravity with potential (16) for some \( \alpha \). The radii are given in km.
Figure 12: Threshold masses for various EoS calculated according to Eqs. (18) (solid) and (19) (dashed) as function of parameter $\alpha$.

| $\alpha$ | APR 2.97 (2.94) | GM1 3.14 (3.16) | SLy4 2.94 (2.88) | MYN 3.06 (3.07) |
|---------|-----------------|-----------------|-----------------|-----------------|
| 10      | 3.00 (3.05)     | 3.17 (3.19)     | 2.98 (2.99)     | 3.10 (3.13)     |
| 20      | 3.03 (3.09)     | 3.18 (3.21)     | 3.01 (3.02)     | 3.13 (3.15)     |
| 50      | 3.07 (3.16)     | 3.22 (3.26)     | 3.06 (3.10)     | 3.17 (3.21)     |
| 100     | 3.11 (3.19)     | 3.26 (3.30)     | 3.09 (3.12)     | 3.20 (3.25)     |

Table II: Threshold masses in GTR and for scalar-tensor theory calculated from Eqs. (18) and (19) (in brackets) correspondingly.

V. CONCLUSION

We considered the models of neutron stars in $f(R) = R + \alpha R^2$ gravity and its equivalent scalar-tensor theory in Einstein frame. For neutron star matter the realistic equations were used. In General Relativity for Tolman-Oppenheimer-Volkoff equations we have Schwarzschild solution outside the star but for modified gravity situation is more complex. One need to integrate equations outside the star with zero energy density and pressure and check consistency with asymptotic flatness on spatial infinity. As result the gravitational sphere surrounding star appears with some contribution to gravitational mass from viewpoint of distant observer. For negative values of $\alpha$ the careful consideration shows that gravitational mass grows infinitely with distance. Although we have for scalar curvature required asymptotic $R \to \infty$ at $r \to \infty$ but the metric function $B(r) \to 1$ more slowly then $r^{-1}$ and therefore function $m(r)$ grows with $r$. Another situation take place for $\alpha > 0$. The mass confined by stellar surface decreases in comparison with General Relativity with increasing of $\alpha$ (for given central density). But the net effect due to gravitational sphere is increasing of gravitational mass. This effect doesn’t depend from equation of state for dense matter. For equivalent scalar-tensor theory with massless dilaton scalar field we have another interpretation. Stellar mass grows with $\alpha$ but this occurs due to increasing mass within stellar surface. The contribution of dilaton sphere to gravitational mass is negligible. On spatial infinity the solutions for mass profiles coincide.

We considered the possible influence of deviation from general theory of relativity on coalescence of neutron stars. We assumed that effectively this influence can be regarded as modification of EoS for dense matter and parameters of neutron stars consequently. This assumption allows to conclude that in a case of scalar-tensor theory with potential $V(\phi) \sim (1 - e^{-2\phi/\sqrt{3}})^2$ threshold mass increases. In future one can hope that EoS for dense matter will be established with sufficient accuracy and therefore statistics of events like GW170817 can help us discriminate between GTR and
simple $R$-square gravity.

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