Study on the extrusion load during equal channel angular pressing

D T Zhang 1, Y X Tong 1, L Li 1 and Y F Zheng 2

1 Institute of Materials Processing and Intelligent Manufacturing, College of Materials Science and Chemical Engineering, Harbin Engineering University, HRB 150001, China
2 Department of Materials Science and Engineering, College of Engineering, Peking University, BJ 100871, China

E-mail: zhangdiantao@hrbeu.edu.cn

Abstract. The equal channel angular pressing or extrusion technique, ECAP/ECAE, is a severe plastic deformation process employed to produce bulk ultra-fine grained materials. In this work, the extrusion load for performing the ECAP process is studied. The resistance includes deformation resistance and frictional resistance during ECAP processing. Base on the slip line method, upper bound solution and finite element analysis, the formula of extrusion load is proposed. The calculated results of extrusion load are in a good agreement with the experimentally measured curves. According to the formula, the influencing factors on the extrusion load are the length of the samples, friction coefficient, geometric parameters of die, yield strength and work hardening coefficient of the materials.

1. Introduction
Ultra-fine grained (UFG) materials with outstanding mechanical properties and special physical property has attracted much attention of researchers in the field of materials science in recent years [1]. The equal channel angular pressing or extrusion technique, ECAP/ECAE, is a severe plastic deformation process employed to produce bulk ultra-fine grained materials [2]. During ECAP processing, a well-lubricated billet is forced to pass through two intersecting channels with a constant cross-sectional area. The billet undergoes a large amount of plastic strain by simple shear within the deformation zone located at the intersection of the die channels [3].

The extrusion load used to make billet plastic deformation is one of the most important parameters in the ECAP process. At present, the main methods to study the extrusion load include the slip line field theory [4], the upper-bound analysis [5] and FEM-based simulations [6]. However, the frictional resistance caused by the lateral expansion of the billet during the ECAP process was ignored in most of the studies. Undoubtedly, the frictional resistance is the main reason why it is difficult to produce specimens with a high length to diameter ratio. In the present paper, formulas of the extrusion load were derived by force analysis. The influence of the main factors was considered comprehensively, such as the length-diameter ratio of the billet, friction coefficient, geometric parameters of die, yield strength and work hardening coefficient of the materials.
2. Analysis

In this study, analytical modeling of the ECAP die with a circular cross-section and an external angle $\psi=\pi-\Phi$ was established, as shown in figure 1. As demonstrated by Zhang et al. [7], there were three zones during the ECAP process, namely, a small deformation zone, a main deformation zone and a non-deformation zone. The ECAP system reaches a steady force-balance when the plunger descends with a constant speed, the solution for calculating the pressing force $F$ is given by equation (1):

$$ F = f_1 + f_{II} + f_{III} $$

where $F_1$ denotes the resistance of shear deformation and $f_{II}, f_{III}, f_{III}$ denote the frictional resistance in a different zone.

According to Coulomb law of friction and the von Mises criteria, the frictional resistance is given by equation (2):

$$ f = \mu F_n = \frac{A \sigma_s}{\sqrt{3}} $$

where $\mu$ denotes the friction coefficient, $A$ denotes the contact area and $\sigma_s$ denotes the flow stress. Considering the geometry size of die and the movement of the sample, the frictional resistance in a different zone is given by equation (3):

$$ F_1 = \mu \pi D_1 \frac{\sigma_s}{\sqrt{3}} \\
F_{II} = \mu (\pi D_1^3) \frac{(180-\Phi)}{360} \frac{\sigma_s}{\sqrt{3}} \\
F_{III} = \mu \pi D_{III} \frac{\sigma_{III}}{\sqrt{3}} $$

where $D$ denotes the diameter of channel cross-section, $\Phi$ denotes the internal angle, $l_1, l_{II}, l_{III}$ denote the length of the sample in a different zone, $\sigma_{II}, \sigma_{III}, \sigma_{III}$ denote the flow stress in a different zone.

Base on the study of Segal et al. [8], the ECAP force is calculated using one shear plane and without consideration of the friction as follows:

$$ F = k A \gamma $$

where $k$ denotes the shear stress, $A$ denotes the cross-section area of the channel and $\gamma$ denotes the shear strain.

According to Spuskanyuk et al. [4], the shear stress can be calculated from a stress-strain relationship. From the analysis of Iwahashi et al. [9], the shear strain $\gamma=\psi$ is attained when the external angles $\psi=\pi-\Phi$, therefore the deformation resistance is given by equation (5):

$$ F_1 = \frac{\pi D_1^2}{4} \frac{\pi \sigma_s}{\sqrt{3}} = \frac{\pi D_1^2 \psi}{4\sqrt{3}} $$

From the upper-bound theorem, Luri et al. [10] analyzed the pressing force in the ECAP process for a die with circular cross-section, friction conditions. The pressing force is given by equation (6):

$$ F = \frac{\pi D_1^2}{2} \frac{(\pi - \Phi)}{\sin(\frac{\Phi}{2} + \frac{\sigma_s}{2})} + \mu \frac{\pi D_{III}}{\sqrt{3}} $$
The same equation of the deformation resistance can be obtained considering die structure and frictionless.

The commercial finite element software Deform v10.2 was used to carry out the simulations. Three dimensional finite element simulations of single pass ECAP were carried out with different internal angles $\Phi = 90^\circ$, $100^\circ$, $110^\circ$, $120^\circ$, external angles $\psi = \pi - \Phi$ and coefficient of friction $\mu = 0$, without considering friction resistance. These simulations were carried out with the constant yield strength $\sigma_s = 100$ MPa without considering work hardening. As shown in figure 2, cylindrical billet with diameter $d = 12$ mm and length $l = 60$ mm was considered for simulations. All simulations were performed with a speed of 2 mm/s.

Figure 3 shows the simulated pressing force versus ram time step during the ECAP process. It should be noted that three stages of the ECAP force can be distinguished in this particular case. At the first and second stages, the bent front part of the billet does not entirely fill the channel, therefore the force of the third stage represents the deformation resistance.

It can be stated that the error between the analytical and FEM results is lower than 10%, which means that a high level of agreement exists between both analyses, as shown in table 1. The difference between the results could be due to the gap between the billet and channel [11] and inhomogeneous deformation of the billet [12].

### Table 1. FEM results and calculated results of deformation resistance for dies with different internal angle.

| Internal angle $\Phi$ | FEM results | Calculated results | Error |
|-----------------------|-------------|--------------------|-------|
| $90^\circ$            | 27894N      | 30770N             | 9.3%  |
| $100^\circ$           | 25005N      | 27351N             | 8.6%  |
| $110^\circ$           | 21961N      | 23932N             | 8.2%  |
| $120^\circ$           | 18965N      | 20513N             | 7.5%  |

3. **Experiment**

In order to study the factors that affect the extrusion load during ECAP process, some experimental measurements were performed. To perform the experimental testing, MTS 311 series electro-hydraulic servo fatigue testing machine shown in figure 4(a) was employed. By using the split die that can be observed in figure 4(b), a cylindrical TA2 billet with size 12 mm in diameter and from 15 to 65 mm in length was extruded, as shown in figure 4(c). The billets were coated with molybdenum disulfide (MoS$_2$) lubricant before ECAP. All billets were processed at 400 °C with constant velocity ($v = 2$ mm/s).

In order to analyze the influence of material properties on the extrusion load, the compression properties of TA2 billets were tested at 400 °C by using a Gleeble 3500 thermal-mechanical simulation tester. The true stress-strain curve was obtained by compression tests with the strain rate 0.1/s.
4. Results and discussion

In the ECAP process, it is more useful to use the extrusion load instead of the required force. By substituting equation (3) and equation (5) to equation (1), the extrusion load $P$ can be obtained by equation (7):

$$ P = \frac{4\mu\sigma_{II}}{\sqrt{3}D} + \frac{2\mu\sigma_{III}}{\sqrt{3}} + \frac{4\mu\sigma_{III}}{\sqrt{3}D} + \frac{\sigma_{III}}{\sqrt{3}} $$

The formula shows that the extrusion load is related with length-diameter ratio of the billet, friction coefficient, geometric parameters of die and material properties.

As shown in figure 5, the true stress rises with the increase of true strain during the compression process because of work hardening which also occurs during ECAP process. From the analysis of Iwahashi et al. [9] and the fourth strength theory, the relation between the equivalent strain and the true strain is given by equation (8):

$$ \varepsilon_{eq} = \gamma / \sqrt{3} = -\frac{2}{\sqrt{3}} \ln(1 - \delta) = \frac{2}{\sqrt{3}} \varepsilon $$

where $\varepsilon_{eq}$ denotes the equivalent strain, $\delta$ denotes compression ratio and $\varepsilon$ denotes the true strain. According to the geometry of the model, the true strain corresponding to the maximum equivalent strain is 0.523.

| $\sigma_{II}$ (MPa) | $\sigma_{III}$ (MPa) | $\sigma_{III}$ (MPa) |
|---------------------|---------------------|---------------------|
| 233                 | 431                 | 547                 |

**Table 2.** Flow stress of TA2 in different zones.

![Figure 5](image_url). The true stress-strain compression curve of TA2 specimen at 400 °C.

It was found that the diameter inflated from 12 mm to 12.24 mm by measuring the tail of the billets. The results show that small plastic deformation appeared in the zone I during the ECAP process. So the yield strength could be used to denoted the flow stress in the first zone, as shown in figure 5. The
billet accumulated a large amount of plastic strain by simple shear in the second zone. According to Kim [13], the strain and the external angle are linear increased in the zone II. The flow stress of the zone II is given by equation (9):

$$
\sigma_{II} = \int_{\varepsilon_1}^{\varepsilon_3} F(\varepsilon) d\varepsilon \cdot (\varepsilon_{III} - \varepsilon_1)^{-1}
$$

(9)

where \(\varepsilon_I\) denotes the true strain in zone I and \(\varepsilon_{III}\) denotes the true strain in zone III. In the third zone, the strain of the billet remained constant, therefore the flow stress of the zone III is the true stress corresponding to the true strain \(\varepsilon=0.523\), as shown in figure 5. In order to study the behavior of the extrusion load when material properties changes, table 2 shows a summary of the flow stress in different zone by using the compression curve shown in figure 5.

Figure 6 is the exclusion load versus ram displacement curve. The second stage on the load curve during the ECAP process is observed. Referring to figure 6, the extrusion load increases gradually with increasing the ram movement. The relationship between the exclusion load and the ram displacement shows approximately linear with two different slopes. The positions of the inflection point depend on the geometry of dies. In this study, the ram displacement of the inflection is about 12 mm according the experimental results. At the end of Stage I, the frictional resistance and deformation resistance of the zone II are the same, no matter how long the sample is, as shown in figure 1(a). The extrusion load at the inflection point is given by equation (10):

$$
P_I = \frac{4\mu \sigma_{II}}{\sqrt{3}D} + \frac{2\mu \sigma_{II} \psi}{\sqrt{3}} + \frac{\sigma_{II} \psi}{\sqrt{3}}
$$

(10)

Where \(L\) denotes the length of the billet. The peaking load of stage I had a linear relationship with the length of the billet, and the intercept and the slope of the fitting line were calculated by origin software, as shown in figure 7. Based on known conditions, the friction coefficient \(\mu=0.137\) was calculated by the slope of the fitting line.

![Figure 6. Relationship curve of experimental load and ram displacement for different lengths of billet processed at 400 °C.](image1)

![Figure 7. The fitting curve for peak load at the end of stage I.](image2)

The movement position of the billet at the end of stage II is shown in figure 1(c). Through the comparison and analysis of the peak load at the end of different stages, the slope of the second stage curve \(k_{II}\) can be given by equation (11):

$$
k_{II} = \frac{P_{II} - P_I}{L-12} = \frac{4\mu \sigma_{III} - \sigma_{II}}{\sqrt{3}D}
$$

(11)

The slopes of the fitting curves and the calculated results of the friction coefficient are shown in table 3. The average of the friction coefficient is 0.131±0.003, which is close to the research results of
Perez et al. [14]. By substituting the friction coefficient into equation (10), the frictional resistance of zone II is 68.3 MPa, the deformation resistance of the zone II is 260.6 MPa, the sum of frictional resistance and deformation resistance is 328.9 MPa. It can be stated that the error between calculated results and the intercept of the fitting curve is 1.8%, which means that the formulas are correct and practical. By using all the parameters in equation (7), the peak load of stage II is 1060 MPa, the error between calculated and experimental results is 6%, and it is shown that the analytical formulation predicts the extrusion load accurately.

Table 3. The calculated results of friction coefficient in the stage II.

| Length of the sample | Slope of the curve | Friction coefficient |
|----------------------|--------------------|---------------------|
| 65                   | 7.78               | 0.129               |
| 55                   | 7.61               | 0.126               |
| 45                   | 7.96               | 0.131               |

5. Conclusion
In this work, it has been found that the extrusion load consists of the deformation resistance and frictional resistance during the ECAP process. The same formula of the deformation resistance has been obtained by the slip line field solution and upper-bound solution, and the calculated results have been verified by FEM simulation. Based on the experimental results, the formulas of extrusion load have been analyzed by considering material properties, geometric parameters of die, length-diameter ratio and friction coefficient. The calculated load has been validated by measuring experimental results. The formulas will provide the support for the die design and the process optimization in the field of ECAP.

Acknowledgments
The authors acknowledge the support given by the Fundamental Research Funds for the Central Universities (HEUCMF180205) and the National Natural Science Foundation of China (51701049).

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