SNR study on Fourier single-pixel imaging

Rui Li1, Jiaying Hong1, Xi Zhou1, Chengming Wang2, Zhengyu Chen1, Bin He1, Zhangwei Hu1, Ning Zhang1, Qin Li1, Ping Xue2 and Xiao Zhang3,4, ∗

1 School of Life Science, Beijing Institute of Technology, Beijing 100081, People’s Republic of China
2 State Key Laboratory of Low-dimensional Quantum Physics and Center for Atomic and Molecular Nanoscience, Department of Physics, Tsinghua University and Collaborative Innovation Center of Quantum Matter, Beijing 100084, People’s Republic of China
3 National Engineering Laboratory for Forensic Science, Institute of Forensic Science, Ministry of Public Security, Beijing 100038, People’s Republic of China
4 Author to whom any correspondence should be addressed.
E-mail: zhangx@bit.edu.cn

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Abstract
According to the properties of Fourier transform, Fourier single-pixel imaging uses the illumination lights with cosine distributions to obtain the Fourier spectrum of the object, and then apply the inverse Fourier transform to reconstruct the spatial information of the object. This technique does not require detector arrays, such as charge coupled device and has proven to be insensitive to distortion, which is a great improvement over traditional photography techniques. In this manuscript, we present a detailed analysis and discussion on the signal-to-noise ratio (SNR) of Fourier single-pixel imaging. Compared with conventional imaging whose SNR is independent of pixel number \(N\), Fourier single-pixel imaging achieves an improved SNR which is up to \(N\) times as high as the dynamic range of detection. Furthermore, this SNR benefit is further confirmed experimentally, in cases of one dimension and two dimensions.

1. Introduction

Single-pixel imaging uses a series of spatially distributed lights to illuminate an object, and then obtains the powers of transmitted/reflected lights from the object by single-pixel detector [1–6]. With the correlation between the spatially distributed lighting patterns and the light powers, the image of object could be reconstructed. According to Helmholtz reciprocity, single-pixel imaging using spatially modulated illumination and single-pixel detection is essentially equivalent to conventional imaging using single illumination and spatially resolved detector array, such as charge coupled device (CCD). The earliest single-pixel imaging technology dates back to the flying-spot camera proposed in 1884 [7, 8]. This camera applies a perforated disk (also known as Nipkow disk) to modulate a light source for scanning light spot across an object and achieves the object image with single-pixel detection in a point-by-point way.

So far, many single-pixel imaging techniques, including ghost imaging [9–12], computational ghost imaging [13, 14] and single-pixel camera [15], have been proposed. Using the spatial modulation of random patterns for illumination light field or detected light field, these techniques could recover the object image with the correlated algorithm [16–18] or compressed sensing [8, 13, 19]. Compared with conventional camera, single-pixel imaging has the advantages of low lighting power requirement [20] and insensitivity to distortion introduced by turbid [21–24], scattering [25] and dispersive/nonlinear media [26, 27]. Furthermore, single-pixel techniques could capture scenes by indirect measurements, where only indirect light is sampled by the detector. In other words, the detector does not require a direct line of sight to the target in single-pixel imaging [28–31]. Additionally, it is difficult for conventional approach to image at some wavebands, such as THz, where multi-pixel detector is unavailable or expensive. Single-pixel imaging is almost the best way to address this issue and has been demonstrated in THz band [32–36]. Recently, single-pixel technique has been applied for phosphorescence and fluorescence imaging [37, 38].
As an important way of single-pixel imaging, Fourier single-pixel imaging has proved to be a powerful imaging technique that can both provide high image quality and high efficiency of imaging [39, 40]. Moreover, Fourier single-pixel imaging has developed a series of imaging technologies from 2D image to 3D imaging, from monochrome to color imaging, from static imaging to dynamic imaging, from macroscopic to microscopic imaging, and from mono-modality imaging to multi-modal imaging [41–45]. Till now, SNRs of many types of single-pixel techniques have been studied in detail [46, 47]. However, to the best of our knowledge, there is no experimental study and theoretical discussion about the SNR of Fourier single-pixel imaging.

In this manuscript, we study the SNR of Fourier single-pixel imaging theoretically and experimentally, in cases of one dimension and two dimensions. Under the same condition, the SNR of Fourier single-pixel imaging is $N$ times as high as that of raster-scanning imaging, where $N$ is the total pixel number. Therefore, the upper limit of this SNR is up to $N$ times as high as the dynamic range of detection. In addition, the theoretical predictions agree with experimental results very well.

2. One-dimensional Fourier single-pixel imaging

2.1. Principle

Generally, there are two typical configurations of single-pixel imaging for capturing the transmission or reflection function of object, as shown in figures 1(a) and (b) respectively. The LCD/projector displays/projects a series of illumination patterns before/on the object. The total powers of transmission/reflection lights from the object are measured by the single-pixel detector. Finally, the object image is reconstructed by particular algorithm. In principle, these two configurations are equivalent.

Let us consider the mathematical expression of SNR for Fourier single-pixel imaging with these two configurations. For simplicity, the one-dimensional $N$-pixel case of Fourier single-pixel imaging is analysed first. We implement the four-step phase-shifting method, which obtains each complex Fourier coefficient by utilizing the corresponding response of four illumination patterns [39]. The normalized one-dimensional spatial distribution of illumination pattern is expressed as:

$$D_{\phi}(j) = \frac{[1 + \cos(2\pi f_k j + \phi)]}{2},$$

where $\phi$ denotes initial phase, spatial position $j = 0, 1, \ldots, N - 1$ and spatial frequency $f_k = k/N$, $k = 0, 1, \ldots, N - 1$. The units of $j$ and $f_k$ are pixel and pixel$^{-1}$ respectively.

The photocurrent of the single-pixel detector is given by:

$$I_{\phi}(f_k) = i_0 \cdot \sum_{j=0}^{N-1} g(j) \cdot D_{\phi}(j),$$

where $i_0$ is a constant and $g(j)$ the transmission function (or reflection function) of the object. To get the real and imaginary parts for the Fourier spectrum of object at a frequency point, four photocurrent values $I_0(f_k), I_{\pi/2}(f_k), I_{\pi}(f_k)$ and $I_{3\pi/2}(f_k)$ have to be recorded. These photocurrents correspond to four illumination patterns $D_0, D_{\pi/2}, D_{\pi}$ and $D_{3\pi/2}$, which have same spatial frequency and constant phase shift.

Figure 1. (a) Configuration of single-pixel imaging for capturing the transmission function of object. LCD: liquid crystal display. (b) Configuration of single-pixel imaging for capturing the reflection function of object.
\(\pi/2\) between two adjacent patterns. Then according to equation (2), the Fourier spectrum of object \(S(f_k)\) is calculated.

\[
S(f_k) = I_0(f_k) - I_{\pi}(f_k) + i[I_{\pi/2}(f_k) - I_{3\pi/2}(f_k)]
\]

\[= i_0 \sum_{j=0}^{N-1} g(j)e^{-2\pi i j f_k}, \tag{3}\]

where \(i\) denotes the imaginary unit. Equation (3) describes the differential measurement, which effectively eliminates the DC term. Furthermore, the low-bandwidth common-mode noise in measurements, such as the change of ambient light caused by solar motion (bandwidth \(\gg 1\) Hz), could also be cancelled by differential measurement. Applying IFT to equation (3), we get the object image:

\[
O(j) = F^{-1}[S(f_k)] = \sum_{k=0}^{N-1} S(f_k) \cdot e^{2\pi i j f_k}, \tag{4}\]

where \(F^{-1}\) denotes the IFT operator. As known to all, the transmission/reflection functions of any objects could be regarded as a linear combination of delta functions. Therefore, we let \(g(j)\) be a delta function for simplicity:

\[
g(j) = \delta(j) = \begin{cases} 1, & j = 0 \\ 0, & j = 1, 2, \ldots, N - 1 \end{cases}. \tag{5}\]

With equations (3)–(5), the reconstructed object image could be further rewritten as:

\[
O(j) = \begin{cases} N i_0, & j = 0 \\ 0, & j = 1, 2, \ldots, N - 1 \end{cases}. \tag{6}\]

In the shot noise limit, the noise at each frequency point is uncorrelated and its variance is independent of frequency, i.e. \(\sigma^2(f_k) = \sigma_0^2\). Therefore, the noise variances add incoherently in the inverse discrete Fourier summation:

\[
\sigma^2(j) = N \sigma_0^2. \tag{7}\]

Note that \(\sigma_0^2\) is determined by detection parameters, such as detection bandwidth. According to equations (6) and (7), the SNR of Fourier single-pixel imaging is expressed as:

\[
\text{SNR}_{\text{Fourier}} = N i_0^2 / \sigma_0^2. \tag{8}\]

If using the same detector to image in raster-scanning way at the same condition, the SNR of image is given by:

\[
\text{SNR}_{\text{raster-scanning}} = i_0^2 / \sigma_0^2. \tag{9}\]

Comparing equations (8) and (9), their relationship is given by:

\[
\text{SNR}_{\text{Fourier}} = N \cdot \text{SNR}_{\text{raster-scanning}}. \tag{10}\]

In the limit of critical saturation, \(i_0\) is the maximum response photocurrent of detector. Consequently, \(i_0^2/\sigma_0^2\) is the dynamic range of the detector, i.e. the maximum SNR of raster-scanning imaging. As a result, the upper limit of \(\text{SNR}_{\text{Fourier}}\) is

\[
(\text{SNR}_{\text{Fourier}})_{\text{max}} = N \cdot \text{DR}, \tag{11}\]

where DR denotes the dynamic range of the detector, defined as the SNR for the maximum possible signal of detection, i.e. the ratio of the power of maximum possible signal versus the noise power. Equation (11) indicates that Fourier single-pixel imaging could provide an improved SNR, which is up to \(N\) times as high as the dynamic range of detection.

2.2. Experimental results

To confirm the theoretical conclusion, we measure SNRs for Fourier single-pixel imaging and raster-scanning imaging with the configuration shown in figure 1(a). In experiment, the illumination patterns are successively displayed by LCD. The LCD switches the illumination patterns every 0.25 s and the detector samples synchronically. The object used here is a one-dimensional spatial filter with rectangular transmission function, placed 2 mm from LCD. To introduce ambient noise, a random flashing light is employed as an interference source. With \(N = 32, 64, 128, 256\) and 512, the measured spectrum \(S(f_k)\) obtained by the four-step phase-shifting method are shown in figure 2. The noise intensities in these \(S(f_k)\) data are at the same level, because these data are measured under the same detection parameters.
Figure 2. The measured spectrum $S(f_k)$ in Fourier single-pixel imaging, when $N = 32, 64, 128, 256$ and $512$.

Figure 3. In one-dimensional case, the typical result of raster-scanning imaging and the recovered results of Fourier single-pixel imaging when $N = 32, 64, 128, 256$ and $512$ respectively.

For a fair comparison of SNRs, we apply the same experimental setup to capture the object image by conventional imaging approach. However, unlike the CCD used in camera, the single-pixel detector cannot offer the spatial resolution. Therefore, we make use of a scanning beam to detect different locations of the object in a point-by-point manner, i.e. one-dimensional raster-scanning imaging. Thus, the object image could be collected by single-pixel detection in sequence. Differential measurement is also employed in raster-scanning imaging. Consequently, the one-dimensional spatial distribution of illumination is actually a moving bright/dark point in grey background. The $l$th normalized illuminations in differential measurement $D_{l,+}(j)$ and $D_{l,-}(j)$ are expressed as:

$$D_{l,+}(j) = [1 \pm \delta(j-l)]/2, \quad l = 0, 1, \ldots, N-1.$$ (12)

The object image obtained by raster-scanning imaging is given by:

$$O(l) = I_+(l) - I_- (l),$$ (13)

where $I_+(l)$ and $I_- (l)$ represent measured photocurrents of the single-pixel detector, corresponding to illuminations of $D_{l,+}$ and $D_{l,-}$.

In experiment, the recovered one-dimensional object images are shown in figure 3, obtained by raster-scanning imaging and Fourier single-pixel imaging with $N = 32, 64, 128, 256$ and $512$ respectively.
Figure 4. In one-dimensional case, the theoretical prediction and the experimental SNR comparison between raster-scanning imaging and Fourier single-pixel imaging with $N = 32, 64, 128, 256$ and $512$. Error bars indicate standard deviation from 10 runs of the experiment.

Strong noise could be observed in the image achieved by raster-scanning imaging. The SNRs of raster-scanning imaging are experimentally measured with different $N$ and shown as the blue square in figure 4. The theoretical prediction line for $\text{SNR}_{\text{Fourier}}$ is obtained by equation (10) and shown in figure 4 in black. The experimental SNRs of Fourier single-pixel imaging with different $N$ (red circles) are compared in figure 4. These experimental results agree with the theoretical prediction very well. The SNR of Fourier single-pixel imaging is nearly $N$ times as high as the SNR of raster-scanning imaging under the same condition.

3. Two-dimensional Fourier single-pixel imaging

Next, let us consider the SNR of two-dimensional case for Fourier single-pixel imaging. Then equation (10) is rewritten as

$$\text{SNR}_{\text{Fourier}} = M^2 \cdot \text{SNR}_{\text{raster-scanning}} = N \cdot \text{SNR}_{\text{raster-scanning}},$$

where $M$ is the pixel number corresponding to the sides of square illumination. Thus the total pixel number of illumination/reconstruction $N$ equals $M^2$.

To confirm equation (14), we measure SNRs of two-dimensional Fourier single-pixel imaging and raster-scanning imaging with the configuration in figure 1(b). In experiment, two-dimensional illumination patterns are successively projected onto the object by a commercial digital projector (XQ-13, XianQi Technology Co., Ltd.) and the light powers are measured by a silicon photodiode power sensor (S130C, Thorlabs Inc.). To measure image SNR conveniently, the object used here is a 5 mm-diameter convex mirror on a black background, located at a distance of 45 cm from the projector. When $N = 64, 256, 1024$ and 4096, experimental SNRs of Fourier single-pixel imaging $\text{SNR}_{\text{Fourier}}$ are shown in figure 5 as red circles. In addition, we capture the object image by raster-scanning imaging using the same configuration. Similar to the one-dimensional case of raster-scanning imaging, the object image is obtained by the single-pixel detector point by point via differential measurement, using a scanning beam from projector. Then $\text{SNR}_{\text{raster-scanning}}$ could be calculated with experimental data and shown as the blue square in figure 5. According to equation (14), the theoretical prediction line for $\text{SNR}_{\text{Fourier}}$ is achieved and shown in figure 5 in black. There is a good agreement between experimental and theoretical results, indicating that the analysis above is correct. When $N = 4096$, the results of raster-scanning imaging and Fourier single-pixel imaging are presented in figure 6. Compared with raster-scanning imaging, Fourier single-pixel technique could enhance image quality significantly under the same condition.

Equation (14) is derived when the transmission/reflection function of the object is a delta function. Actually, it could be further expanded to arbitrary object. Generally, the transmission/reflection function of arbitrary two-dimensional object $g_{2D}(j, p)$ can be written as a linear combination of delta functions...
modulated by $g_{2D}(j, p)$:

$$g_{2D}(j, p) = \sum_{f=1}^{M} \sum_{p' = 1}^{M} g_{2D}(f', p') \delta_{2D}(j - f', p - p'), \tag{15}$$

where $j$ and $p$ denote two-dimensional spatial indices of image pixels. $\delta_{2D}(j, p)$ is two-dimensional delta function, which is equal to 1 as $j = p = 0$, and 0 as $j \neq 0$ or $p \neq 0$.

The imaging SNR expression for object with delta-like transmission/reflection function, i.e. equation (14), is also applicable to the case of arbitrary two-dimensional object for the following two reasons. (1) According to equation (15), the transmission/reflection function of arbitrary object could be expressed as a linear combination of delta functions corresponding to image pixels. (2) Single-pixel imaging system is a linear system. Therefore SNR at each image pixel is actually described by the delta-function model. As a result, the discussion and conclusion drawn from equation (14) are applicable for imaging of arbitrary two-dimensional object.

4. Discussion

4.1. SNR comparison with other single-pixel imaging

For comparison purposes, the SNR expressions of conventional ghost imaging (CGI) and differential ghost imaging (DGI), which have been confirmed by experiments [18, 48], are listed below.

$$\text{SNR}_{\text{CGI}} = \frac{m}{N_{\text{speckle}}} \frac{\Delta T_{\text{min}}^2}{T^2}, \tag{16}$$
and

$$\text{SNR}_{\text{DGI}} = \frac{\text{SNR}_{\text{CGI}}}{N_{\text{speckle}}} \cdot \frac{\Delta T_{\text{min}}^2}{\delta T^2}, \quad \text{(17)}$$

where $m$ denotes the number of measurements, $N_{\text{speckle}}$ the number of speckles in the beam, i.e. the pixel number of two-dimensional illumination, $\Delta T_{\text{min}}$ the minimum variation of the object transmission/reflection function to be detected. The average quadratic transmission/reflection function of the object is given by

$$\overline{T}^2 \approx \int_{\text{beam}} T^2(x)dx/A_{\text{beam}}, \quad \text{(18)}$$

where $T(x)$ denotes the transmission/reflection function of the object, $A_{\text{beam}}$ the illumination beam area, $x$ spatial coordinate. The variance of $T(x)$ is defined as

$$\overline{\Delta T^2} = \overline{T^2} - (\overline{T})^2, \quad \text{(19)}$$

where $\overline{T}$ is the average transmittance of the object.

For $N$-pixel two-dimensional imaging, Fourier single-pixel imaging needs $4N$ measurements, thus we let measurement number $m = 4N$ and pixel number $N_{\text{speckle}} = N$ for a fair comparison. Then equations (16) and (17) are further written as

$$\text{SNR}_{\text{CGI}} = \frac{4\Delta T_{\text{min}}^2}{T^2}, \quad \text{(20)}$$

and

$$\text{SNR}_{\text{DGI}} = \frac{4\Delta T_{\text{min}}^2}{\delta T^2}, \quad \text{(21)}$$

Under this condition, the SNRs of CGI and DGI are both independent of pixel number, but determined by object transmission/reflection function. According to equations (14), (20) and (21), $\text{SNR}_{\text{Fourier}}/\text{SNR}_{\text{CGI}}$ and $\text{SNR}_{\text{Fourier}}/\text{SNR}_{\text{DGI}}$ are proportional to $N$, given by

$$\frac{\text{SNR}_{\text{Fourier}}}{\text{SNR}_{\text{CGI}}} = \frac{N \cdot \overline{T^2}}{4\Delta T_{\text{min}}^2} \cdot \text{SNR}_{\text{raster--scanning}} \propto N, \quad \text{(22)}$$

and

$$\frac{\text{SNR}_{\text{Fourier}}}{\text{SNR}_{\text{DGI}}} = \frac{N \cdot \delta T^2}{4\Delta T_{\text{min}}^2} \cdot \text{SNR}_{\text{raster--scanning}} \propto N. \quad \text{(23)}$$

With the typical parameter values of object [18, 48]:

$$\overline{T^2} = 0.0112, \quad \delta T^2 = 0.0038, \quad \Delta T_{\text{min}}^2 = 0.05, \quad \text{(24)}$$

we get $\text{SNR}_{\text{Fourier}}/\text{SNR}_{\text{CGI}} = 39.6$ dB and $\text{SNR}_{\text{Fourier}}/\text{SNR}_{\text{DGI}} = 34.9$ dB when $N = 4096$. Therefore, Fourier single-pixel imaging could provide a far higher image quality than CGI and DGI, because of the SNR enhancement factor of $N$. Experimentally, $\text{SNR}_{\text{Fourier}}$ is measured to be 38.2 dB with $N = 4096$ as indicated in figure 5, and 64.2 dB with $N = 60 025$ in reference [39]. However the experimental SNRs of CGI and DGI are low ($<\sim 10$ dB) even with $m = 100 000$ measurements [18].

As is stated above, Fourier single-pixel imaging could offer higher SNR with fewer measurement number than CGI and DGI. Compared with raster-scanning imaging, it needs the quadrupled time consumption for image measurement due to the phase shifting method. However, its significant SNR advantage is worth the additional time. In addition, the time consumption of Fourier single-pixel imaging could be reduced by sparse sampling [39] and random sampling [49] in Fourier domain.

### 4.2. The discussion for SNR enhancement

In principle, the image SNR of raster-scanning imaging is determined by detection parameters and independent of pixel number $N$. However, not only imaging range but also SNR are improved by a factor of $N$ in Fourier single-pixel imaging. To explain the relationship between SNR and $N$, let us consider the principle of Fourier single-pixel imaging in figure 7. The single-point detector measures the spectral information of object, which consists of signal and noise in frequency domain. Therefore, after IFT, the reconstruction result also consists of signal, i.e. the object image, and noise in spatial domain. Supposing a slit is used as the object, its transmission function is mathematically a delta-like function. Thus the measured signal in frequency domain is actually a very wide function. When IFT is performed, all of data points in frequency domain contributes to each point on the reconstruction result in spatial domain. Therefore, the sum of all measurements of signal in frequency domain equals the height of delta function in spatial domain. Because there are $N$ pixels in frequency domain, the amplitude of reconstruction result is $N$
times as high as the amplitude of measured signal, as illustrated in equation (6). Thus, the peak power of signal actually increases by $N^2$ times in spatial domain. Instead, the noise variances add incoherently in IFT. Therefore, the noise variances, i.e. noise power, only increases by $N$ times in spatial domain, as illustrated in equation (7). As a result, the SNR of Fourier single-pixel imaging is improved by pixel number $N$.

5. Conclusion

In summary, the SNR characteristics of Fourier single-pixel imaging is discussed in detail. Theoretical analysis indicates that Fourier single-pixel imaging has an SNR gain of $N$ over raster-scanning imaging, where $N$ is the pixel number of imaging. To demonstrate that this conclusion is independent of configuration type, the theoretical prediction is experimentally verified with two different configurations under one-dimensional and two-dimensional conditions respectively. We believe this study may shed light on the inherent properties of Fourier single-pixel imaging modality, which overcomes the fundamental bottlenecks in dynamic range of detection hardware and therefore has potential applications in biomedical microscope, military remote sensing and astronomical observation.

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ORCID iDs

Xiao Zhang © https://orcid.org/0000-0003-4886-7327

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