Pulse-width voltage adjustment

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Abstract. In recent years there has been significant development of the element base of power electronics. The new circuit and design solutions have appeared, and the capabilities of the power supply systems of electrotechnological, radio-electronic and consumer devices have changed significantly. The current paper analyzes the methods of voltage regulation. The paper shows the method of obtaining an adjustable AC voltage, as well as the device for its implementation. The proposed method allows achieving a significant reduction in weight and size indicators of power supply units of various devices. The implementation of the concept is possible based on the method of modulating the output voltage with high-frequency pulses with varying duty cycle by means of pulsed electronics. Equations for calculating the harmonic components of the Fourier series of a pulse sequence for a half-sine curve are given.

The most dynamically developing direction of power electronics is considered, which is connected with solving the problem of voltage regulation in controlled rectifiers and inverter devices.

1. Introduction

The change in the root mean square value of an AC voltage is conveniently based on the use of a number of classical methods for voltage regulation [1–13]. In most cases, it is possible to regulate the output voltage with the help of an electric autotransformer, the lack of which is the presence of a galvanic connection between the input and output, as well as significant weight and size properties exceeding the electronic version 2 ... 4 times.

In the pulse-width mode (PWM-pulse-width modulation), the regulation of the alternating voltage has the disadvantage that part of the sinusoid of the alternating voltage is excluded from the operation, which leads to the appearance of high-frequency harmonics.

In the case of modulation of a sinusoidal or another form of voltage by modulating high-frequency pulses, its average or effective value may change [1-3].

With PWM, the formation of an output voltage curve can be determined by a sequence of pulses of different duration and polarity. The duration and frequency of the component pulses change in such a way that their average or effective value should change according to a given law. In [3], the equations are given for the expansion in a Fourier series of a voltage composed of n (four) pulses of the same duration on the positive half-period of an alternating voltage curve (Figure 1), taking into account that the angle is the center of the component pulses.
The amplitude of the half-wave of an alternating voltage composed of four pulses can be calculated by the equation (1) [3]:

\[
U_{H(n)\text{max}} = \frac{4E}{n\pi} (\cos n\beta_1 \cos n\frac{\lambda}{2} - \sin n\beta_1 \sin n\frac{\lambda}{2} + \cos n\beta_2 \cos n\frac{\lambda}{2} - \sin n\beta_2 \sin n\frac{\lambda}{2}) = \\
= \frac{8E}{n\pi} (\sin n\beta_1 + \sin n\beta_2) \sin n\frac{\lambda}{2}
\]

In the general case, with \(n\) unipolar pulses on a half-period of the output voltage waveform, the amplitude of the \(n\)-th harmonic is found from the equation (2):

\[
U_{u(n)\text{max}} = \frac{8E}{n\pi} \sin n\frac{\lambda}{2} \sum_{k=1}^{m} \sin n\beta_k \ , \ k=1,2,3...m
\]

By equating to zero the expression under the sign of the sum in (2), one can find the angles \(\beta\), corresponding to the centers of the constituent pulses, at which the most significant harmonics are excluded.

The ability to control and transform the voltage without changing its shape is to modulate the voltage with high-frequency pulses with varying duty cycle [4, 5]. As a result, the root mean square value of voltage \(U_{out}\) at the output of the transformer is estimated as (3):

\[
U_{\text{out}} = \frac{0.707U_{in\text{max}}}{kQ}
\]

where \(U_{\text{out}}\) – output root mean square voltage value; \(U_{in\text{max}}\) – maximum value of the input voltage; \(Q\) – modulating duty cycle ratio; \(k\) – transformation ratio.

The purpose of the current paper is to study the possibility of regulating the output voltage in electronic converters of AC and DC voltage (DC-DC, DC-AC, AC-AC) when modulating by high-frequency pulses with varying duty cycle, as well as by pulses of various durations.

2. Pulsed method of obtaining an adjustable voltage

In the pulsed method of forming a DC or AC voltage, first, high-frequency control pulses of the power switching elements, for example, IGBT transistors, are formed. Then the input voltage shape \(U_{in}\) is replaced by high-frequency components (Figure 2). By the means of the commutation of the power switches, a mean (for direct current) or RMS (for alternating current) voltage value is obtained from a sequence of power pulses.
3. The implementation of the method of obtaining an adjustable AC voltage

The device hereinafter referred to as an electronic converter (EC), is made on the basis of the above method for producing a controlled voltage by means of pulsed electronics. Given the various options for circuit design with the use of electronic key elements, the article provides only a functional diagram of the device.

The electronic converter of alternating current is similar in properties to an electric regulated transformer, but with galvanic isolation of windings, with preservation of the input voltage form and improved weight and size properties.

The functional diagram of the device is shown in Figure 3 and contains a source of alternating current 1, a rectifier 2, a high-frequency switch 3, a load 7, a switching voltage filter 4, an inverter 5, a transformer 6, a key control unit 8 and an inverter 9, the controller 10, feedback block 11.

Figure 2. Half-sine curve, substituted by ten pulses with a duty cycle Q = 2.5.

Figure 3. A functional diagram of the electronic converter (EC).

The EC contains in its power circuit an alternating current power source 1 (Figure 3) with the given initial values of voltage and frequency of the input current. In order to regulate the output voltage without changing its shape, the device is equipped with a switch 3, which is placed between the rectifier 2 and the filter-limiter of the switching voltages 4, as well as an inverter 5 connected between the filter-limiter of the switching voltages 4 and the transformer 6.

In addition, in the functional diagram, in order to control the high-frequency key 3, the generator 8 is introduced, the pulses frequency and duty cycle of which are proportional to the output voltage at the load 7.

Rectifier 2 converts the alternating voltage of source 1 into a direct voltage; high-frequency switch 3 modulates the rectifier output voltage with high-frequency pulses; filter voltage limiter 4 limits switching overvoltages when the key is switched off; the inverter 5 generates an alternating voltage of the corresponding frequency at the output of the device; transformer 6 is required to obtain a given voltage across the load 7. The switch control unit 8 is designed to generate high-frequency pulses with a given frequency and duty cycle. The control unit 9 controls the inverter 5 and generates the frequency of the output voltage. The controller 10 sets the frequency parameters for the key and...
inverter control units. The feedback unit 11 generates signals to correct the parameters of the output voltage.

The device works as follows: when the voltage is applied, the controller 10 is turned on, the pulses of which are fed to the control unit 8 (which controls the switch 3) with a frequency of 10 ... 50 kHz and to the control unit 9 (which controls the inverter 5) with the frequency of the output voltage. High-frequency switch 3 generates a voltage with the waveform shown in Figure 1. Due to the fact that the rate of change of current at the output of the switch 3 with filter inductance is high, the switching voltage limiter 4 is installed.

The effective value of the voltage at the output of the transformer 6 depends on the duty ratio of the pulses [5–7].

The synchronization of the operation of the inverter 5 is carried out by the control unit 9 connected to the controller 10 in accordance with the frequency of the current in the load 7. The voltage is stabilized at the load by sending a signal from the feedback unit 11 to the controller 10, which changes the duty cycle of the pulses generated by the high-frequency switch 3. The principle of regulating the output voltage of the electronic signature is based on the modulation of the rectified voltage by high-frequency pulses with varying duty cycle. The mean value of the output voltage can be estimated at each interval (Figure 2) from equation (4):

\[
U_{\text{mean},i} = \frac{U_m (\sin \omega t_{2,i-1} + \sin \omega t_{2,i})}{2}
\]  

(4)

To clarify the principle of regulation, shown in Figure 2, introduced notations: \( t_{\text{imp}} \) – pulse duration time, \( t_{\text{p}} \) – pause time, \( U_m \sin \omega t \) – frequency-modulating pulse envelope with amplitude \( U_m \), \( x = \omega t \) – variable current moment coordinate.

The output voltage \( U_{\text{out}} \) represents the sum of the voltages \( U_{\text{mean},i} \), related to the number \( n \) of pulse portions as shown in (5):

\[
U_{\text{out}} = \frac{\sum_{i=1}^{n} U_{\text{mean},i}}{n}
\]  

(5)

The presented device relates to AC-to-AC converters. However, such a conversion approach can be used in devices for obtaining alternating voltage from the direct voltage in DC-AC converters and other converters in electrical and radio engineering. The device can also be effectively used in AC voltage regulators with reduced weight and size parameters by 30 ... 50%.

4. Calculation of the harmonic components of the Fourier series for an arbitrary partition of the modulated curve

The pulse sequence is shown in Figure 2, where the interval \( \Delta x \) corresponds to the sum of the pulse durations \( \omega t_{\text{imp}} = \omega t_2 - \omega t_1 \) and the pause time \( \omega t_{\text{p}} = \omega t_3 - \omega t_2 \), which for brevity are indicated as \( \tau_{\text{imp}} \) and \( \tau_{\text{p}} \) respectively:

\[
\Delta x = \omega \tau_{\text{imp}} + \omega \tau_{\text{p}} = \tau_{\text{imp}} + \tau_{\text{p}}
\]

If the function with the period of \( T = \pi \) is divided by \( m \) equal intervals \( \Delta x \), then \( \Delta x = \frac{\pi}{m} \). The pulse duty ratio \( Q \) by definition can be expressed as(6):

\[
Q = \frac{\tau_{\text{imp}} + \tau_{\text{p}}}{\tau_{\text{imp}}}
\]  

(6)

then for a pulse with duration \( \tau_{\text{imp}} \) the equation will be:

\[
\tau_{\text{imp}} = \frac{\pi}{mQ}
\]
Points \( kx \) located on the axis \( \omega t = x \) are given by the equation:

\[
x_k = \Delta t k = \frac{\pi k}{m}, \quad k = 0, 1, 2, ..., m-1, m
\]

The sequence of pulses describes the function \( u(x) \) (7):

\[
u(x) = \begin{cases} 
U_m \sin x, & x \in \bigcup_{i=0}^{m-1} [x_i - \tau, x_i], \\
0, & x \in \bigcup_{i=0}^{m-1} (x_i, x_{i+1} - \tau).
\end{cases}
\]

(7)

Since the function \( u(x) \) is piecewise continuous on the segment \([0, \pi]\), it can be decomposed into a Fourier series (8):

\[
u(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos 2nx + b_n \sin 2nx,
\]

(8)

Where the coefficients \( a_n \), \( b_n \) and \( a_0 \) can be found using the equations (9):

\[
a_n = \frac{2}{\pi} \int_0^\pi u(x)\cos 2nxdx = \frac{2U_m}{\pi} \sum_{k=1}^{m} \int_{x_k-\tau}^{x_k} \sin x \cos 2nxdx, \quad n = 0, 1, 2, ...
\]

\[
b_n = \frac{2}{\pi} \int_0^\pi u(x)\sin 2nxdx = \frac{2U_m}{\pi} \sum_{k=1}^{m} \int_{x_k-\tau}^{x_k} \sin x \sin 2nxdx, \quad n = 1, 2, 3, ...
\]

(9)

As a result of calculations, we obtain expressions for the coefficients of the Fourier series \( (a_0, a_n \) and \( b_n) \) (10):

\[
a_0 = \frac{2U_m}{\pi} \sin \frac{\pi}{2mQ} \cdot \frac{\cos \frac{\pi(Q-1)}{2mQ}}{\sin \frac{\pi}{2m}},
\]

\[
a_n = \frac{2U_m}{\pi} \left\{ \frac{\sin \frac{(2n+1)}{2mQ} \cdot \cos \frac{(2n+1)(Q-1)}{2mQ}}{(2n+1)\sin \frac{\pi(2n+1)}{2m}} - \frac{\sin \frac{(2n-1)}{2mQ} \cdot \cos \frac{(2n-1)(Q-1)}{2mQ}}{(2n-1)\sin \frac{\pi(2n-1)}{2m}} \right\},
\]

\[
b_n = \frac{2U_m}{\pi} \left\{ \frac{\sin \frac{(2n+1)}{2mQ} \cdot \sin \frac{(2n+1)(Q-1)}{2mQ}}{(2n+1)\sin \frac{\pi(2n+1)}{2m}} - \frac{\sin \frac{(2n-1)}{2mQ} \cdot \sin \frac{(2n-1)(Q-1)}{2mQ}}{(2n-1)\sin \frac{\pi(2n-1)}{2m}} \right\}.
\]

(10)

The calculation of the harmonic components of the Fourier series is carried out for the parameter values: \( Q = 2, \ m = 10 \). Figure 4 shows the half-sine 8, which is modulated by high-frequency pulses, the first five harmonics 1 ... 5 of decomposition and the constant component 6 of the modulated sinusoidal curve 8, and also the total curve of 7 five harmonics and a constant component.
Figure 4. Curves decomposition of the modulated sinusoidal curve into the harmonic series, and a DC component.

The total curve 7 shown in Fig. 4 is close in shape to the original modulated curve 8, and its amplitude with a duty cycle $Q = 2$ is half the size of the original one.

The adjustment curve of the average voltage value depicted in Fig. 5 and shows the change in the value of the constant component of the duty ratio $Q$. So, in the absence of pauses, that is, $Q = 1$, the average voltage value corresponds to 0.63, and for $Q = 2$ - 0.315. With a further increase of $Q$, for example, $Q = 6.65$, the average value decreases to 0.1 of the amplitude for a full-wave single-phase rectifier.

Figure 6 shows the envelope of amplitude values $U_{i\text{ max}}$ of harmonics when modulating a half-sine sine ten pulses with a duty cycle $Q = 2$. It should be noted that the largest component of the frequency spectrum is a multiple of the frequency of the envelope splits (in this case divided into $m = 10$ intervals with $Q = 2$). Elimination of high-frequency components requires the installation of appropriate filters. The higher the modulation frequency, the easier it is to eliminate the high harmonics that make up the spectrum at the output of the converter.

Figure 5. Adjustment curve of the average value of the full-wave voltage single-phase rectifier depending on the pulse duty ratio.

Figure 6. The envelope of the amplitude values of the harmonics at $Q = 2$ and $m = 10$.

On the envelope of the initial phases of the harmonics (Figure 7), the phase angle decreases with an increase in the order of harmonics to the tenth, and then again increases and decreases to the 30th
harmonic. Zero levels of the initial phases are observed at the maximum amplitudes, that is, at 10, 30, 50th.

In the proposed method of controlling the change in the duty cycle of the pulses, an adjusting steeply dipping characteristic is obtained; therefore, the development of control circuits for an alternating and constant voltage impulse controller with an adjusting characteristic close to linear is of great interest. The control circuit contains a square wave generator and a triangular pulse generator.

Unlike the control pulse formers from [3], the control pulses of the DC-DC converters are formed in such a way that the output voltage from the PWM is generated in the form of pulses of variable duration modulated according to a linear or sinusoidal law. Voltage regulation is carried out by varying the pulse duration according to the chosen modulation law. A device for voltage modulation can be performed according to the scheme shown in Figure 8.

The timing diagram of the device is shown in Figure 9.

The converter power switches are controlled by generating of the control pulses in the first section of duration π, and then in the second section of the same duration, but with opposite polarity of the pulses (Figure 9). Formation of voltage by changing the amplitude of the control voltage \( U_y \), for example, with sinusoidal control, is reflected by the modulation factor \( \mu \) - the ratio of the output voltage value to the maximum value (amplitude) of the input voltage:

\[
\mu = \frac{U_{\text{out}}}{U_m}
\]

When regulating the DC voltage, the value of the DC voltage is taken as \( U_m \).

![Figure 7. Adjustment curve of the average value of the full-wave voltage single-phase rectifier depending on the pulse duty ratio.](image)

![Figure 8. The envelope of the amplitude values of the harmonics at Q = 2 and m = 10.](image)
Consider the case of regulation of alternating voltage (Figure 9, right). The shape of the adjustable alternating voltage \( u(\omega t) = U_m \sin \omega t \) of duration \( \pi \) is modulated by triangular unipolar pulses in accordance with the following condition: the intersection points of triangular pulses with a sinusoidal curve the alternating voltage \( u(\omega t) = U_m \sin \omega t \) is divided into a number of sections with or lack of voltage. In this case, the presence of voltage will be in those areas where the sinusoid curve \( u(\omega t) = U_s \sin \omega t \) lies above the triangular pulses. For the effective value of the regulated alternating voltage, we obtain the dependence shown in Figure 10.

The block diagram of the control device and the power section of the circuit shown in Figure 8 were simulated on a computer in the software product MatLab (Simulink) (Figure 11).

As a result of modeling, an adjustment characteristic was obtained, which coincides with the calculated one.

5. Conclusions
As a result of the research, a method for obtaining controlled alternating voltage when modulated by high-frequency pulses with varying duty cycle is presented. The formulas for calculating the harmonic
components of the Fourier series when modulating a sinusoidal curve are given. The graphs of the first five harmonics and the constant component, as well as the curve of their addition with a duty cycle equal to two, are presented. It is noted that the form of the output voltage remains constant while simultaneously changing its effective value.

Devices have been developed that implement a method for obtaining adjustable ac and dc voltages, which make it possible to achieve a substantial reduction in the weight and size indications of power supply units of various devices. The device refers to AC-to-AC converters and DC-to-DC ones. Similar transformations can also be used in devices for producing an alternating voltage from DC-AC and in other devices of electrical and radio engineering.

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