Finite time consensus control for nonlinear heterogeneous multi-agent systems with disturbances

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Abstract This paper discusses the finite time consensus (FTC) issue of nonlinear heterogeneous multi-agent systems (HMASs) by combining integral sliding-mode control (SMC), event-triggered control (ETC) and pinning control methods. The SMC is constructed separately for first-order and second-order agents to assure that the system is not interfered by the nonlinearities and disturbances when the state trajectory of the system moves on the sliding surface. To stimulate the FTC of system, the novel control protocols and the fully distributed ETCs with adjustable trigger frequency only rely on the local information are designed. Moreover, the Zeno behavior is eliminated and the range of pinning gain of each agent under directed weakly connected topology is determined. Meanwhile, we also give the conditional criteria for the system to achieve FTC. Eventually, the correctness of the obtained conclusion is illustrated by several simulation examples.

Keywords Finite time consensus · Sliding-mode control · Event-triggered control · Pinning control · Heterogeneous multi-agent systems

1 Introduction

Recently, the coordinated control of multi-agent systems (MASs) has evoked widespread concerns in engineering applications, such as UAV cooperative control [1], sensor network [2], robotic control [3] and distributed satellite formation [4]. Thereinto, the consensus problem of MASs plays a significant part on the research of coordinated control. For achieving the goal of consensus, the distributed controller needs to be designed to interactive information.

Early phase, numerous momentous achievements have been acquired in the study of linear MASs, such as [5,6]. However, linear system is an ideal state. In practical engineering, the system will be inevitably affected by the unknown factors, such as nonlinearities and external disturbance. This will affect the performance of the system seriously, as a result, the system cannot complete the designated tasks. In order to ensure that the system can still work in the complex environ-
ment, it is significant to study the nonlinear system. The work [7] adopted the Lipschitz condition to avoid nonlinear interference. The works [8, 9] used the properties of radial basis function neural networks to neutralize the nonlinear factors in the system. By adopting disturbance observer, the uncertainties and external disturbances are estimated in [10–12]. However, compared to the above methods, SMC is especially suitable for nonlinear systems owing to its low control cost, better robustness and anti-interference ability. At present, many crucial achievements on SMC have been published, such as [13–15] and the references therein. In [13,15], the continuous SMC and discontinuous SMC tracking protocols for second-order MASs are studied. A new decoupling distributed SMC is proposed in [14], which can eliminate the influence of singularity and weaken the influence of chattering. By employing distributed integral SMC based on ETC in [16], the state of nonlinear second-order MASs approximates the integral sliding mode surface in finite time.

It should be noted that the above works on SMC are all about homogeneous nonlinear MASs, which means that the dynamic models of the system possess the same properties. In actual systems, the dynamics of agents may be diverse due to various constraints or different task division in complex environments. For example, in a robot football game, some robots need to attack and some robots need to defend. Compared with homogeneous MASs, HMASs are applied for practical application appropriately on account of its distinct composition. The output consensus of HMASs is studied in [17, 18, 26]. The chain nonholonomic MASs are transformed into two sub-systems in [19]. A distributed protocol is designed by means of output regulation and state feedback. Aiming HMASs composed of linear and nonlinear agents [20], a nonlinear consensus protocol is proposed. For the HMASs contained the first-order and the second-order agents, the case that the velocity measurement information cannot be received is studied in [21,22]. However, it is worth noting that these works adopted period sampling control. In period sampling control, the communication cycle needs to be determined. When the communication cycle arrives, all agents will communicate with each other, which means that they will touch off inessential consumptions of communication resources.

For saving communication resources and decreasing controller update frequency, ETC is proposed by scholars to deal with the resource control of the system. In ETC, the condition of state error between the agents will be defined. When the state error threshold between the agents exceeds the defined condition, the controller of the agent will be automatically triggered to update and determine the next state of the agent. However, some distributed ETCs in certain works depend on the global information, for instance, the quantity of the agents [23,24], the eigenvalues of Laplacian matrix [25] associated with the system topology and others. However, the fully distributed ETC [26–28] is able to avoid continuous monitoring of measurement error. A distributed event-triggered consensus algorithm using an open-loop estimate and neighboring information is proposed in [26], and the control input of each agent is dependent on the estimate and information at discrete instants. A hybrid ETC mechanism is proposed in [27], which adopted time regularization to compel the minimum trigger interval time to be positive, it means Zeno behavior is excluded. Noting that the convergence time is omitted in these works, they only guarantee the asymptotic consensus of the system, even the best convergence rate is exponential consensus. In other words, the range of consensus convergence time is uncertain.

With the in-depth study of HMASs tracking control problem, the convergence rate is required strictly. For example, the higher accurate control is desired in some practical applications. As a result, the system is required to achieve consensus in the finite time. The superiority of FTC is not only faster convergence velocity, but also the better anti-interference ability. Therefore, more scholars have considered the FTC problem [29–36]. The work [29] considered the FTC of a group of MASs described by fully driven Euler–Lagrange dynamics. The integrated SMC is employed in [32], and the established ETC emerges the characteristic of fully continuous communication free. In [33] and [34], the FTC of HMASs with external disturbances that consist of the first-order agents and the second-order agents is discussed, but they utilized periodic sampling control, resulting in unnecessary waste of communication resources. The FTC of nonlinear HMASs under uncertainty and disturbance is investigated using ETC and FTC in [35], whose ETC is designed to be centralized and distributed, respectively. However, the finite time homogeneity theorem adopted cannot obtain the convergence time of the system.

As known that the topology of the system plays a key role in the coordinated control of MASs. Noting that most of the network topology investigated are directed
strongly connected graph [6,27], directed graphs with leaders [7,10,13,16], undirected graphs [32–35], and so on. However, in practical applications, the communication link may be limited in the system, such as some agents have no adjacent agent to communicate. Therefore, the directed weakly connected topology can be applied to severe environments and lessen communication overhead. In order to deal with the more complicated system environment and the limitation of communication link, the pinning control strategy [37] is an utterly effective means. The principle of pinning control is actually to select a target value for the states of the agents who cannot acquire neighbor information. Whenever the controller is updated, the states of the agents will be close to the target value.

Inspired by the deficiencies of the related works, including linear system, convergence time, periodic sampling control and topological constraints, we ponder over the SMC on the study of FTC for nonlinear HMASs. Then, ETC and pinning control are combined to avert topology constraints and economize resources. The primary contributions of this paper can be epitomized as:

1. On the basis of these homogeneous MASs in [13–16,29,30], we consider the HMASs contained the first-order and the second-order agents via integral SMC in this paper. This improves the ability of the nonlinear system under SMC to work in different task division. The difficulty lies in how to design SMC and ensure sliding surface can be stable in finite time.

2. In order to avoid that the event-triggered condition contains global information, and considering the special properties of HMASs, the design of ETC is the difficult part. Compared with the resembled works in [20,22,33,34], the fully distributed ETC proposed does not contain any global information and the traffic between agents is decreased. Thereinto, Zeno behavior is repelled.

3. FTC possesses better control precision than these works in [17,18,21,26,28] owing to the convergence time of the system can be known. Although FTC is utilized in the works [22,35], the convergence time cannot be received due to different methods. Considering that HMASs do not satisfy any law, the design of controller is a breakthrough direction. A new control protocol is proposed in this paper to complete FTC of HMASs and assist us in obtaining the convergence time.

4. Yet, most topology of the resemble works like [6,7,10,13,16,27,32–35] cannot be wielded to the case where the communication link is limited. The pinning control method is applied to achieve the consensus tracking in the directed weakly connected graph. Where HMASs need to meet the law of FTC and the agents in the system have different properties, so it is a challenge to calculate the pinning gain range of each agent. Eventually, this paper obtains the pinning gain range of all agents.

The rest of this paper is arranged as below. In Section 2, some fundamental concepts of graph theory and the dynamics of the HMASs are introduced. Section 3 narrates the work content based on ETC and pinning gain. In Section 4, the validity of the results is verified by simulations. In Section 5, the conclusion is presented.

2 Preliminaries

Algebraic graph theory is especially profitable in the study of MASs. Some lemmas, assumptions and definitions are presented in this section.

2.1 Description of algebraic graph theory

The communication flow of $N$ agents can be expressed by a directed communication topology $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ which composed of the node set $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, edge set $\mathcal{E} = \{v_i, v_j\} \subseteq \mathcal{V} \times \mathcal{V}$, and adjacency matrix $\mathcal{A} = (a_{ij})_{N \times N} \in \mathbb{R}^{N \times N}$. If the agent $i$ and agent $j$ are neighbors, $a_{ij} > 0$, otherwise $a_{ij} = 0$. In this paper, graph $\mathcal{G}$ does not contain rings, it means that $a_{ii} = 0$. Let $N_i = \{ j \in \mathcal{V} | a_{ij} > 0 \}$ to represent the neighbor set of the node $i$. $\mathcal{D} = \text{diag} \{D_1, D_2, \ldots, D_N\}, i \in I_N$ indicates the in-degree matrix, where $D_i$ demonstrates the in-degree of the $i$th node. The Laplacian matrix $L = \mathcal{D} - \mathcal{A}$.

2.2 Dynamic of HMASs

Considering the nonlinear HMASs composed of $M$ first-order agents and $N - M \ (N > M)$ second-order
agents. Thus, the dynamics of the HMASs can be indicated as:

\[
\begin{aligned}
\dot{x}_i(t) &= u_i(t) + f_i(x_i(t), t) + d_i(t), \quad i \in I_M \\
\dot{v}_i(t) &= u_i(t) + f_i(x_i(t), v_i(t), t) + d_i(t), \quad i \in I_{M-1}.
\end{aligned}
\]  

(1)

where \( I_M = \{1, 2, \ldots, M\} \), \( I_{M-1} = \{M + 1, M + 2, \ldots, N\} \), \( I_N = I_{M-1} \cup I_M \) and \( I_{M-1} \cap I_M = \emptyset \).

\( x_i(t), v_i(t), u_i(t) \in \mathbb{R} \) denote the position state, velocity state and control input of the agent \( i \), respectively. \( f_i(x_i(t), t) \) and \( f_i(x_i(t), v_i(t), t) \) indicate the nonlinear functions of the first-order agent and the second-order agent, respectively. \( d_i(t) \) indicates the external disturbance of each agent. HMASs has received extensive attention in many works, but in order to improve the performance of HMASs, we introduced the combination of SMC, FTC, ETC and pinning control. Since HMASs contains the agents with different properties and there is no certain law, the design of these control methods is critical, which is also a major challenge of this paper. The following contents of this paper will explain the design of these control methods in detail.

Definition 1 For any stochastic state of the agents, there exists a setting time \( T \), and the conditions for HMASs (1) to accomplish consensus in finite time are as follows:

\[
\begin{aligned}
\lim_{t \to T} ||x_i(t) - x_\delta|| &= 0, \quad i \in I_N \\
\lim_{t \to T} ||v_i(t)|| &= 0, \quad i \in I_{M-1}
\end{aligned}
\]

where the target value \( x_\delta \) is the desired position state by pinning control. We can obtain the different convergence position state of the agents by adjusting the value of \( x_\delta \). This is utterly convenient in practical application.

Assumption 1 For the HMASs (1), we consider that the nonlinear function and the external disturbance are bounded. And the upper bounds are limited by two positive constants, i.e., \( ||f_i(x_i(t), t)|| < \sigma_1 \), \( ||f_i(x_i(t), v_i(t), t)|| < \sigma_1 \), \( ||d_i(t)|| < \sigma_2 \) and \( \sigma_1 > 0 \), \( \sigma_2 > 0 \).

Assumption 2 Suppose there are two vectors \( \xi_1 = [\xi_{11}, \xi_{12}, \ldots, \xi_{1p}]^T \), \( \xi_2 = [\xi_{21}, \xi_{22}, \ldots, \xi_{2p}]^T \) and a diagonal matrix \( \rho = \text{diag}(\rho_1, \rho_2, \ldots, \rho_p) \), \( \rho_i < 0 \), \( i \in \{1, 2, \ldots, p\} \), there exists a positive definite matrix \( H \) with suitable dimension satisfying \( \sum_{i=1}^{p} \xi_{1i} \rho_i \xi_{2i} = \xi_1^T H \xi_2 \).

\[ \tag{2} \]

Assumption 3 Suppose the communication topology of the nonlinear HMASs (1) is a directed weakly connected graph.

Lemma 1 (see [5]) \( \frac{dx^{|1+q}}{dt} = \frac{(1+\eta)\sin(x)^q dx}{dt} \), for \( x \in \mathbb{R} \) and \( \eta \in \mathbb{R} \).

Lemma 2 (see [38]) For arbitrary \( x_i \in \mathbb{R}, i = 1, 2, \ldots, n \), \( 0 < q < 1 \), it satisfies

\[
\left( \sum_{i=1}^{n} |x_i| \right)^q \leq \sum_{i=1}^{n} |x_i|^q \leq n^{1-q} \left( \sum_{i=1}^{n} |x_i| \right)^q.
\]

Lemma 3 (see [39]) Considering a system of dynamics \( \dot{x} = F(x) \) and \( F(0) = 0, x(0) = x_0, x \in \mathbb{R}^N \), suppose that there possesses a positive continuous function \( V(x) \) defined in the neighborhood of the origin. If the inequality \( \dot{V}(x) + c(V(x))^\eta \leq 0 \) is contented with the conditions \( c > 0, 0 < \eta < 1 \), then the system is stable in finite time and the setting time \( T \) satisfies that:

\[
T \leq \frac{V(x_0)}{c(1-\eta)}.
\]

3 Main results

In this section, an integral SMC for nonlinear HMASs (1) is designed firstly. Then, a novel control protocol is employed. On the basis of the combination of ETC and pinning control, we implement FTC of HMASs. In addition, the Zeno behavior is ruled out.

3.1 Controller design based on integral SMC and fully distributed ETC

To ensure that the HMASs (1) can converge in finite time, we design a novel distributed auxiliary function \( u_{il}(t) \) based on ETC moment for the agent \( i \). The triggering time sequence is \( \{t_0^i, t_1^i, \ldots, t_k^i, \ldots\} \):

\[
\begin{aligned}
u_{il}(t) &= -\alpha \text{sign} \left( \sum_{j \in N_l^i} a_{ij}(x_i(t_k^i) - x_j(t_k^i)) \right) \\
&+ c_i(x_i(t_k^i) - x_\delta) \eta, i \in I_M \\
u_{il}(t) &= -\beta \text{sign} \left( \sum_{j \in N_l^i} a_{ij}(x_i(t_k^i) - x_j(t_k^i)) + \gamma v_i(t_k^i) \right) \\
&+ c_i(x_i(t_k^i) - x_\delta) \eta - v_i(t), i \in I_{M-1}
\end{aligned}
\]  

(2)

where \( \alpha > 0, \beta > 0, \gamma > 0, \eta \in (0, 1), t \in [t_k^i, t_{k+1}^i] \), \( c_i \) is the pinning gain. From the auxiliary function...
The integral sliding-mode surfaces are designed as
\[ S_i(t) = x_i(t) + x_i(0) - \int_0^t u_{ii}(\tau) d\tau, \quad i \in I_M \]  
and
\[ S_i(t) = v_i(t) + v_i(0) - \int_0^t u_{ii}(\tau) d\tau, \quad i \in I_{N-M}. \]  

**Remark 1** Being aware of that the integral SMCs are designed for the first-order agents and the second-order agents, respectively. In compared with the works [13–16,29,30], it is obvious that we extend SMC from homogeneous MASs to HMASs.

When the HMASs remain on the integral sliding-mode surface, we can get \( S_i(t) = \dot{S}_i(t) = 0 \). Then, we complete the following dynamics of sliding-mode:
\[ \dot{x}_i(t) = u_{ii}(t), \quad i \in I_M \]  
and
\[ \dot{v}_i(t) = u_{ii}(t), \quad i \in I_{N-M}. \]  

We design a novel controller based on the auxiliary function \( u_{ii}(t) \) and sliding-mode observer \( u_{is}(t) \) to make the sliding surface stable in finite time. The system can always maintain on the sliding surface without any external interference. According to the state error, the event-triggered condition is proposed. \( t_{k+1}^i \) is the event-triggered moment of the agent \( i \) caused by the event-triggered condition. The control protocol changes instantly at \( t_{k+1}^i \) and remains constant until the next event-triggered moment \( t_{k+1}^{i+1} \).

The controller protocol is presented as:
\[ u_i(t) = u_{ii}(t) + u_{is}(t), \quad i \in I_N \]  
and
\[ u_{is}(t) = -(k_1 + k_2)\text{sgn}(S_i(t_k^i)), \quad i \in I_N, \]  
where \( k_1 \) and \( k_2 \) are all positive constants to be determined later.

In the following work, some sufficient conditions are proposed to ensure the FTC of nonlinear HMASs. For any agent \( i \), the position measurement error is determined as
\[ e_{ix}(t) = x_i(t_k^i) - x_i(t), \quad i \in I_N, \]  
and the velocity measurement error is determined as
\[ e_{iv}(t) = v_i(t_k^i) - v_i(t), \quad i \in I_{N-M}, \]  
where \( t \in [t_k^i,t_{k+1}^i] \). Following this, the consensus error of the system is
\[ E_i(t) = x_i(t) - x_\bar{x}, \quad i \in I_N. \]  

By combining (2), (9), (10) and (11), we update (2) to
\[ u_{ii}(t) = -\alpha \text{sgn}(\sum_{j \in N_i} a_{ij} (E_i(t) - E_j(t)) + e_{ix}(t) - e_{ix}(t) + c_i e_{ix}(t) + c_i E_i(t) + \gamma e_{ix}(t) + \gamma v_i(t) + c_i e_{ix}(t) + c_j E_j(t)) + \gamma v_i(t), \quad i \in I_{N-M} \]  
where \( \Phi_i(t) \) is called the event-triggered function and can be represented as
\[ \Phi_i(t) = |(d_{ij} + c_i) e_{ix}(t)|^\eta + \left| \sum_{j \in N_i} E_j(t) + e_{ix}(t) \right|^\eta - |\theta_1 E_i(t)|^\eta, \quad i \in I_M. \]
and
\[ \Phi_i(t) = |(l_{ii} + e_i)e_i(t) + \gamma e_i(t)|^\eta \]
\[ + \sum_{j \in I_i} E_j(t) + e_j(t) |^\eta, \quad (15) \]
\[ - |\theta_2(E_i(t) + v_i(t))| ^\eta, \quad i \in I_{N-M} \]
where \( l_{ii} \) is the in-degree of the agent \( i \).

**Remark 2** According to the event-triggered function (14) and (15), they are fully distributed event-triggered conditions. The discrepancy from the works [20, 22, 33, 34] is that the update of controller is only related to its own and the neighbors information, which does not contain any global information. Parameters \( \theta_1 \) and \( \theta_2 \) are adopted to adjust the update frequency of ETC. While \( \theta_1 \) and \( \theta_2 \) are large, the update frequency is reduced. But decreasing the \( \theta_1 \) and \( \theta_2 \), the convergence curve will be smoother. In practical application, \( \theta_1 \) and \( \theta_2 \) can be modified to adapt to the realistic situation.

**Theorem 1** Under the Assumption 1 and the distributed control protocol (7) based on the integral SMC, the consensus tracking issue of the HMASs (1) will slide to the sliding surface (3) and (4) in finite time when \( k_1 > \sigma_1 \) and \( k_2 > \sigma_2 \). Furthermore, the boundary of setting time \( T_1 \) is
\[ T_1 \leq \frac{2\sqrt{V_{total}(0)}}{\omega_1}, \quad (16) \]
where \( \omega_1 = \sqrt{2(k_1 + k_2 - \sigma_1 - \sigma_2)} \) and \( V_{total}(t) = \frac{1}{2} \sum_{i=1}^{N} S_i^2(t) \).

**Proof** Firstly, taking the derivative of integral SMC (3) and (4) and utilizing the control protocol (7) we will get
\[ \begin{cases} S_i(t) = f_i(x_i(t), t) + d_i(t) \\ \quad - (k_1 + k_2)sgn(S_i(t_i^k)), \quad i \in I_M \end{cases} \]
\[ \begin{cases} S_i(t) = f_i(x_i(t), v_i(t), t) + d_i(t) \\ \quad - (k_1 + k_2)sgn(S_i(t_i^k)), \quad i \in I_{N-M} \end{cases} \quad (17) \]
Select the Lyapunov function as
\[ V_{total}(t) = V_1(t) + V_2(t), \quad (18) \]
where \( V_1(t) = \frac{1}{2} \sum_{i=1}^{M} S_i^2(t) \) and \( V_2(t) = \frac{1}{2} \sum_{i=M+1}^{N} S_i^2(t) \). Based on SMC (3), one has
\[ \dot{V}_1(t) = \sum_{i=1}^{M} S_i(t)\dot{S}_i(t) \]
\[ = \sum_{i=1}^{M} S_i(t)(\dot{x}_i(t) - u_{i1}(t)) \]
\[ = \sum_{i=1}^{M} S_i(t)(u_{i1}(t) + f_i(x_i(t), t) + d_i(t) - u_{i1}(t)) \]
\[ \leq -\sigma_1 \sum_{i=1}^{M} |S_i(t)| \]
\[ = -\sqrt{2}(k_1 + k_2 - \sigma_1 - \sigma_2)V_1(t)^\frac{1}{2} \]

In the same way, based on SMC (4), we have
\[ \dot{V}_2(t) = \sum_{i=M+1}^{N} S_i(t)\dot{S}_i(t) \]
\[ \leq -\sigma_2 \sum_{i=M+1}^{N} |S_i(t)| \]
\[ = -\sqrt{2}(k_1 + k_2 - \sigma_1 - \sigma_2)V_2(t)^\frac{1}{2} \]
Let \( \omega_1 = \sqrt{2(k_1 + k_2 - \sigma_1 - \sigma_2)} \), we obtain \( \dot{V}_1(t) \leq -\omega_1 V_1(t)^\frac{1}{2} \) and \( \dot{V}_2(t) \leq -\omega_1 V_2(t)^\frac{1}{2} \). From Lemma 2, we get
\[ \dot{V}_{total}(t) = \dot{V}_1(t) + \dot{V}_2(t) \]
\[ \leq -\omega_1 (V_1(t) + V_2(t))^\frac{1}{2}, \quad (22) \]
\[ \leq -\omega_1 V_{total}(t)^\frac{1}{2} \]
With the conditions \( k_1 > \sigma_1 \) and \( k_2 > \sigma_2 \), we are cognizant of \( \omega_1 > 0 \), which implies that \( \dot{V}_{total}(t) < 0 \). Therefore, it follows the requirements of Lemma 3 and
each $S_i(t)$ is stable in finite time. The setting time can be calculated as $T_1 \leq \frac{2\sqrt{V_{\text{total}}(0)}}{\omega_1}$. That completes the proof.

**Remark 3** It implies that $f_i + d_i - (k_1 + k_2)\text{sgn}(0) = 0$ when $S_i(t) = \dot{S}_i(t) = 0$. According to LaSalle invariance theorem, that can also be regarded as an observer: $f_i + d_i = (k_1 + k_2)\text{sgn}(S_i)$. By using the sliding-mode observer $u_{1i}(t)$ (8), the effective observation of nonlinearity and disturbance is carried out to realize compensation.

**Remark 4** Due to the influence of the factors such as the time delay of the signal function and the inertia link, the SMC will produce chattering behavior. Utilizing saturation function $sat(s)$ instead of signal function $\text{sgn}(s)$ can exclude chattering behavior. Here, 

$$sat(s) = \begin{cases} \Delta, s > \Delta \\ s, |s| \leq \Delta \\ -\Delta, s < -\Delta \end{cases}$$

where $\Delta$ can be a constant $-\Delta, s < -\Delta$ or a variable for adaptive adjustment. This method actually utilizes continuous function $sat(s)$ to convert the system to a continuous system.

**Remark 5** We know that $k_1$ and $k_2$ are the upper bounds of nonlinear functions and external disturbances, respectively. If the values of $k_1$ and $k_2$ are larger, the setting time $T_1$ for the consensus of the sliding-mode face will be shortened, but the result is that the convergence curve fluctuates; if the values of $k_1$ and $k_2$ are smaller, $T_1$ will be extended, but the convergence curve is smoother, and each $S_i$ is closer to zero. Hence, $k_1$ and $k_2$ can be adjusted according to the actual situation.

After entering the sliding-mode surface, it is conscious of $S_i(t) = \dot{S}_i(t) = 0$. From $\dot{S}_i(t) = 0$, we can transform the sliding-mode dynamics of the HMASs in (1) into the following forms:

$$\begin{align*}
\dot{x}_i(t) &= -\alpha \text{sgn}(\sum_{j \in N_i} a_{ij}(E_i(t) - E_j(t) + e_{ix}(t) - e_{jx}(t))) \\
&\quad + c_i e_{ix}(t) + c_i E_i(t))^{\eta}, \quad i \in I_M. \\
\dot{v}_i(t) &= -\beta \text{sgn}(\sum_{j \in N_i} a_{ij}(E_i(t) - E_j(t) + e_{ix}(t) - e_{jx}(t))) \\
&\quad + \gamma e_{iv}(t) + \gamma v_i(t) + c_i e_{ix}(t) + c_i E_i(t))^{\eta} \\
&\quad - v_i(t), \quad i \in I_{N-M}. 
\end{align*}$$

(23)

### 3.2 System stability analysis

On the basis of the controller (7), we achieve a sufficient condition to solve the consensus of nonlinear HMASs (1).

**Theorem 2** In the case of Assumption 2, Assumption 3, sliding-mode dynamics (23), (24) and the fully distributed ETCs (14), (15), the FTC of the nonlinear HMASs (1) can be implemented if the following conditions hold:

1. $\alpha > 0$, $\beta > 0$, $0 < \eta < 1$
2. $c_i > -l_{ii} - \theta_1, i \in I_M$
3. $-l_{ii} - \theta_2 < c_i \leq \gamma - l_{ii}, i \in I_{N-M}$.

Meanwhile, the setting time $T_2$ we obtained is:

$$T_2 \leq \frac{(1 + \eta)V_{2\text{total}}(0)^{1-\eta}}{(1-\eta)\min(\omega_1, \omega_2)},$$

(25)

where $V_{2\text{total}}(t) = V_3(t) + V_4(t)$ will be determined later.

**Proof** We select $V_{2\text{total}}(t)$ as:

$$V_{2\text{total}}(t) = V_3(t) + V_4(t),$$

(26)

where $V_4(t) = \frac{\beta}{1+\eta} \sum_{i=M+1}^{N} |\rho_i (E_i(t) + v_i(t))|^{1+\eta},$

$$V_3(t) = \frac{\alpha}{1+\eta} \sum_{i=1}^{M} |\rho_i E_i(t)|^{1+\eta} \text{ and } \rho_i < 0, i \in I_N.$$

Using Lemma 1 to consider the derivative of $V_3(t)$, it has:

$$\begin{align*}
\dot{V}_3(t) &= \alpha \sum_{i=1}^{M} \text{sgn}(\rho_i E_i(t))^{\eta} \rho_i \dot{E}_i(t) \\
&= -\alpha^2 \sum_{i=1}^{M} \text{sgn}(\rho_i E_i(t))^{\eta} \rho_i \text{sgn}(l_{ii} + c_i) E_i(t) \\
&\quad + (l_{ii} + c_i) e_{ix}(t) - \left( \sum_{j \in N_i} E_j(t) \right)^{\eta} \\
&\quad + \sum_{j \in N_i} e_{jx}(t)) \right)^{\eta}
\end{align*}$$

(27)

As

$$\text{sgn}(\rho_i E_i(t))^{\eta} \leq |\rho_i E_i(t)|^{\eta}$$

(28)
and
\[
sig((l_{ii} + c_i) E_i(t) + (l_{ii} + c_i) e_{ix}(t))
\leq \left| \sum_{j \in N_i} E_j(t) + \sum_{j \in N_i} e_{jx}(t) \right|^\eta
\leq \left| \sum_{j \in N_i} E_j(t) + \sum_{j \in N_i} e_{jx}(t) \right|^\eta
\]

Based on ETC (14), we have
\[
\dot{V}_3(t) \leq -\alpha^2 \sum_{i=1}^{M} |\rho_i(E_i(t))|^\eta \rho_i(|(l_{ii} + c_i)|^\eta + |\theta_1|^\eta)
\]
\[
|E_i(t)|^\eta.
\]

According to Assumption 2,
\[
\dot{V}_3(t) \leq -\alpha^2 (\rho(E(t)))^T H_1 (|(l + c)|^\eta + |\theta_1|^\eta)
\]
\[
|E(t)|^\eta,
\]

where \(l = [l_{11}, l_{22}, \ldots, l_{MM}]^T, c = [c_1, c_2, \ldots, c_M]^T, E(t) = [E_1(t), E_2(t), \ldots, E_M(t)]^T\) and \(\rho = \text{diag}(\rho_1, \rho_2, \ldots, \rho_M), H_1 \in \mathbb{R}^{M \times M}\) is a positive definite matrix.

Thus,
\[
\dot{V}_3(t) \leq -\lambda_1 \alpha^2 \sum_{i=1}^{M} |E_i(t)|^\eta ((l_{ii} + c_i)|^\eta + |\theta_1|^\eta)
\]
\[
\times |E_i(t)|^\eta
\]
\[
= -\lambda_1 \alpha^2 \sum_{i=1}^{M} (\frac{1+\eta}{\alpha|\Omega_1|^{1+\eta}})^{\frac{2\eta}{1+\eta}} ((l_{ii} + c_i)|^\eta + |\theta_1|^\eta)
\]
\[
\times (V_3(t))^{\frac{2\eta}{1+\eta}}
\]

where \(\lambda_1 > 0\) is the minimum eigenvalue of the matrix \(H_1, \Omega_1\) is the minimum value in \(|\rho_1|^\eta, |\rho_2|^\eta, \ldots, |\rho_M|^\eta|\).

Let \(\omega_2 = \lambda_1 \alpha^2 \sum_{i=1}^{M} (\frac{1+\eta}{\alpha|\Omega_1|^{1+\eta}})^{\frac{2\eta}{1+\eta}} ((l_{ii} + c_i)|^\eta + |\theta_1|^\eta)\), one can get
\[
\dot{V}_3(t) \leq -\omega_2 (V_3(t))^{\frac{2\eta}{1+\eta}}.
\]

In the same way, taking the derivative of \(V_4(t)\), it has:
\[
\dot{V}_4(t) = \beta \sum_{i=M+1}^{N} \|\rho_i(E_i(t) + v_i(t))\|^\eta \rho_i(\dot{E}_i(t) + \dot{v}_i(t))
\]
\[
= -\beta^2 \sum_{i=M+1}^{N} \|\rho_i(E_i(t) + v_i(t))\|^\eta \rho_i \rho_i(|(l_{ii} + c_i)|^\eta)
\]
\[
\times E_i(t) + (l_{ii} + c_i) e_{ix}(t) - \left( \sum_{j \in N_i} E_j(t) + \sum_{j \in N_i} e_{jx}(t) \right)^\eta
\]
\[
+ \gamma |e_{ix}(t) + \gamma v_i(t)|^\eta
\]
\[
\leq -\beta^2 \sum_{i=M+1}^{N} \|\rho_i(E_i(t) + v_i(t))\|^\eta \rho_i(|(l_{ii} + c_i)|^\eta)
\]
\[
+ \gamma v_i(t)|^\eta + |(l_{ii} + c_i) e_{ix}(t) + \gamma e_{ix}(t)|^\eta
\]
\[
+ \sum_{j \in N_i} E_j(t) + \sum_{j \in N_i} e_{jx}(t)
\]

By using ETC (15), then
\[
\dot{V}_4(t) \leq -\beta^2 \sum_{i=M+1}^{N} \|\rho_i(E_i(t) + v_i(t))\|^\eta \rho_i(|(l_{ii} + c_i)|^\eta)
\]
\[
+ \gamma v_i(t)|^\eta + |\theta_2(E_i(t) + v_i(t))|^\eta
\]
\[
(35)
\]

According to Assumption 2, we have
\[
\dot{V}_4(t) \leq -\beta^2 (\rho(E(t) + v(t)))^T H_2 ((l + c) E(t))
\]
\[
+ \gamma |v(t)|^\eta + |\theta_2(E(t) + v(t))|^\eta)
\]
\[
(36)
\]

where \(E(t) = [E_{M+1}(t), E_{M+2}(t), \ldots, E_N(t)]^T, l = [l_{M+1}, l_{M+2}, l_N]^T, c = [c_{M+1}, c_{M+2}, \ldots, c_N]^T\), \(\rho = \text{diag}(\rho_{M+1}, \rho_{M+1}, \ldots, \rho_N), H_2 \in \mathbb{R}^{(N-M) \times (N-M)}\) is a positive definite matrix.

Then,
\[
\dot{V}_4(t) \leq -\lambda_2 \alpha^2 \sum_{i=M+1}^{N} |E_i(t) + v_i(t)|^\eta
\]
\[
\times ((l_{ii} + c_i) E_i(t) + \gamma v_i(t)|^\eta
\]
\[
+ |\theta_2(E_i(t) + v_i(t))|^\eta)
\]
\[
(37)
\]

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we select $\gamma \geq l_{ii} + c_i$, then (37) can be simplified as

$$
\dot{V}_4(t) \leq -\lambda_2 \min_{i=M+1}^{N} \beta^2 \Omega_2 \sum_{i=M+1}^{N} |E_i(t) + v_i(t)|^2
\times ((l_{ii} + c_i)^\eta + \theta_2^\eta) |E_i(t) + v_i(t)|^\eta,
$$

(38)

$$
= -\lambda_2 \min_{i=M+1}^{N} \beta^2 \Omega_2 \left( \frac{1 + \eta}{\beta |\Omega_2|^{1+\eta}} \right)^{2n/t_\gamma} \times ((l_{ii} + c_i)^\eta + \theta_2^\eta) V_4(t)^{2n/t_\gamma},
$$

where $\lambda_2 \min > 0$ is the minimum eigenvalue of the matrix $H_2$, $\Omega_2$ is the minimum value in $|\rho_{M+1}|^\eta$, $|\rho_{M+1}|^\eta$, $\ldots$, $|\rho_N|^\eta$.

Let $\omega_3 = \lambda_2 \min_{i=M+1}^{N} \beta^2 \Omega_2 \left( \frac{1 + \eta}{\beta |\Omega_2|^{1+\eta}} \right)^{2n/t_\gamma} ((l_{ii} + c_i)^\eta + \theta_2^\eta)$, then

$$
\dot{V}_4(t) \leq -\omega_3 V_4(t)^{2n/t_\gamma}.
$$

(39)

From Lemma 2, by combining (33) and (39), those can be described as

$$
\dot{V}_{2total}(t) = \dot{V}_3(t) + \dot{V}_4(t)
\leq -\min{\omega_2, \omega_3} (V_3(t) + V_4(t))^{2n/t_\gamma}.
$$

(40)

$$
= -\min{\omega_2, \omega_3} V_{2total}(t)^{2n/t_\gamma}.
$$

According to Lyapunov stability theory, for the first-order agents, when $(l_{ii} + c_i)^\eta + \theta_2^\eta > 0, \dot{V}_3(t) < 0$; for the second-order agents, when $(l_{ii} + c_i)^\eta + \theta_2^\eta > 0, \dot{V}_4(t) < 0$. Thus, we affirm that $\dot{V}_{2total}(t) < 0$ and it satisfies Lemma 3, which deduces that the HMASs (1) can achieve FTC. Distinct from [22] and [35], we acquired the boundary of the setting time that is $T_2 \leq \frac{(1+\eta) V_{2total}(0)^{1/n}}{(1-\eta) \min{\omega_2, \omega_3}}$. Consequently, the stability time of the whole system is $T = T_1 + T_2$. That finishes the proof.

3.3 Event interval lower bounds

The trigger sequence $t^i_k \in \{0, 1, 2, \ldots, \infty\}$ is referred to as the time instant while the controller is updated. Zeno behavior is actually triggered many times in an exceedingly short time interval. To avoid Zeno behavior, we must ensure that the difference between the two trigger instants is positive, namely $t^i_{k+1} - t^i_k > 0$.

**Theorem 3** Considering the nonlinear HMASs (1) with the distributed controller (7) based on the integral SMCs (3), (4) and the ETCs (14), (15), the difference between any two trigger instants is positive for all agents.

**Proof** According to the trigger condition (14) of the first-order agents, we set

$$
\Gamma_i(t) = |(l_{ii} + c_i) e_{ix}(t)|^{\eta}, i \in I_M, t \in [t^i_k, t^i_{k+1}).
$$

(41)

then taking the derivation of (41), we can acquire

$$
\frac{d}{dt} \Gamma_i(t) = \eta \text{sign}((l_{ii} + c_i) e_{ix}(t))^{\eta-1} (l_{ii} + c_i) \frac{d}{dt} e_{ix}(t)
\leq (l_{ii} + c_i) \eta |(l_{ii} + c_i) e_{ix}(t)|^{\eta-1} |u_{ii}(t)|
+ k_1 + k_2 + \sigma_1 + \sigma_2)
$$

(42)

Let $\psi_1(t) = (l_{ii} + c_i) \eta |(l_{ii} + c_i) e_{ix}(t)|^{\eta-1} |u_{ii}(t)|
+ k_1 + k_2 + \sigma_1 + \sigma_2$ and $\xi^i_k = t^i_{k+1} - t^i_k$, if $l_{ii} + c_i > 0$, $\psi_1(t) > 0$. From (41), we can obtain $\Gamma_i(t^i_k) = 0, i \in I_M$ and

$$
\Gamma_i(t) = \frac{d}{dt} \Gamma_i(t) (t - t^i_k) < \psi_1(t) (t - t^i_k) < \psi_1(t) \xi^i_k.
$$

(43)

As we know that $\Gamma_i(t) > 0, \psi_1(t) > 0$, thus $\xi^i_k > \frac{\Gamma_i(t)}{\psi_1(t)} > 0$. So far, we exclude the Zeno behavior of first-order agents. And then it’s homologous to second-order agents, we set

$$
\Gamma_i(t) = |(l_{ii} + c_i) e_{ix}(t) + \gamma e_{iv}(t)|^{\eta}, i \in I_{N-M}, t \in [t^i_k, t^i_{k+1}),
$$

(44)

in the same way, we obtain

$$
\frac{d}{dt} \Gamma_i(t) = \eta \text{sign}((l_{ii} + c_i) e_{ix}(t) + \gamma e_{iv}(t))^{\eta-1} (l_{ii} + c_i) \frac{d}{dt} e_{ix}(t)
\times \left( l_{ii} + c_i \frac{d}{dt} e_{ix}(t) + \gamma \frac{d}{dt} e_{ix}(t) \right)
\leq \eta |(l_{ii} + c_i) e_{ix}(t) + \gamma e_{iv}(t)|^{\eta-1}
\times ((l_{ii} + c_i) |u_{ij}(t)|
+ \gamma |u_{ii}(t)| + \gamma (k_1 + k_2 + \sigma_1 + \sigma_2))
$$

(45)

Let $\psi_2(t) = \eta |(l_{ii} + c_i) e_{ix}(t) + \gamma e_{iv}(t)|^{\eta-1} ((l_{ii} + c_i) |u_{ij}(t)| + \gamma |u_{ii}(t)| + \gamma (k_1 + k_2 + \sigma_1 + \sigma_2))$, if $l_{ii} + c_i > 0, \psi_2(t) > 0$. we acquire

$$
\Gamma_i(t) = \frac{d}{dt} \Gamma_i(t) (t - t^i_k) < \psi_2(t) (t - t^i_k) < \psi_2(t) \xi^i_k.
$$

(46)

Following this, $\xi^i_k > \frac{\Gamma_i(t)}{\psi_2(t)} > 0$ can be proved. To sum up, the Zeno behavior of the entire agents is excluded.

**Remark 6** On the basis of [6, 7, 10, 13, 16, 27, 32–35], in order to achieve FTC of nonlinear HMASs and exclude the Zeno behavior. We can infer that $c_i > -l_{ii} - \theta_1, i \in I_M$.
$I_M$ and $-l_{ii} - \theta_2 < c_i \leq \gamma - l_{ii}, i \in I_{N-M}$ from the proof of Theorem 2 and $c_i > -l_{ii}, i \in I_N$ from the proof of Theorem 3. Therefore, we speculate on the range of $c_i$: $c_i > -l_{ii}, i \in I_M$ and $-l_{ii} < c_i \leq \gamma - l_{ii}, i \in I_{N-M}$. It is worth noting that if the agent $i$ with zero in-degree must be pinned, because it cannot receive neighbor information. On the contrary, the pinning gain $c_i$ of the agent without zero in-degree is 0, because it can communicate with its neighbors to confirm its movement trajectory.

4 Simulation examples

Several simulation examples will be applied to verify the effectiveness of FTC for nonlinear HMASs with external disturbances in this section.

Consider the communication topology of the nonlinear HMASs (1) shown as Fig. 1, in which the first-order agents are \{1, 2\}, and the second-order agents are \{3, 4, 5\}. Then, we select $a_{ij} = 1 (i \neq j)$, $\alpha = 1.6$, $\beta = 2.4$, $\gamma = 0.5$, $\theta_1 = 1.8$, $\theta_2 = 5.3$, $\eta = 0.9$, $k_1 = 0.29$, $k_2 = 0.45$, $x_3 = 1$. In addition, the initial position and velocity states of all agents are selected arbitrarily as $x(0) = \{5, -2.5, 1.5, -4, 2\}^T$, $v(0) = \{-3.5, -2.4\}^T$. The nonlinear functions of each agent are $f(x_1(t), t) = -0.2 \cos(x_1(t))$, $f(x_2(t), t) = 0.12 \cos(x_2(t))$, $f(x_3(t), v_3(t), t) = -0.18(\cos(x_3(t)) + \sin(v_3(t)))$, $f(x_4, v_4(t), t) = 0.14 (\cos(x_4) + \sin(v_4(t)))$, $f(x_5, v_5(t), t) = -0.06(\cos(x_5) + \sin(v_5(t)))$, and the external disturbances of each agent are $d_1(t) = 0.15 \sin(t)$, $d_2(t) = -0.2 \sin(t)$, $d_3(t) = -0.16 \sin(t)$, $d_4(t) = -0.1 \sin(t)$, $d_5(t) = 0.13 \sin(t)$.

Firstly, the validity of FTC of integral SMC is discussed. According to Remark 5, the appropriate values of $k_1$ and $k_2$ are determined. Then, the state trajectory of SMC is displayed in Fig. 2(a). It can be found that convergence curve is smooth and tends to 0. Therefore, the correction of Theorem 1 is guaranteed. In Fig. 2(b), we choose $k_1 = 3$ and $k_2 = 10$, which have large difference with $\sigma_1$ and $\sigma_2$. Thus, the tracking curve of SMC fluctuates. For all agents, we get that $\dot{S}_i(t)$ approaches 0. Namely, $\dot{x}_i(t)$ approaches $u_{ii}(t)$ for first-order agents and $\dot{v}_i(t)$ approaches $u_{ii}(t)$ for second-order agents. Then, the problem is transformed from nonlinear system to linear system. Hence, this simulation example verifies the anti-interference capacity of SMC to nonlinear functions and disturbances.

The discussion of FTC for the nonlinear HMASs triggered by a class of fully distributed ETC is divided into two cases:

**Case 1:** From Fig. 1, as a result of the agent 2 and agent 5 have zero in-degree, they should be selected to be pinned for the achievement of FTC. According to Remark 6, we determine $c_2 = 0.8$, $c_5 = 0.5$ and $c_i = 0 (i \in \{1, 3, 4\})$. Under the above conditions,
(c) Event-triggered times graph

Fig. 3 Trajectories and event times graph of agents

(b) velocity state

The events times of agents

(a) position state

Fig. 4 Trajectories and event times graph of agents without pinning control
the consensus tracking curve of HMASs is depicted in Fig. 3. In where, Fig. 3(a) and Fig. 3(b) are the state evolution curves of position and velocity. Fig. 3(c) depicts the event times of each agent. From these simulation examples, we can observe that the FTC of nonlinear HMASs (1) can be guaranteed under ETC and pinning control, and rule out the Zeno behavior.

Case 2: For the sake of testifying the effectiveness of the pinning control method, we set $c_i = 0 (i \in I_n)$. Namely, we do not pin any agent. The trajectories of the agents are shown in Fig. 4. It reveals that the FTC cannot be brought about.

Case 3: In order to verify that $\theta_1$ and $\theta_2$ are able to adjust the frequency of ETC, we determine $\theta_1 = 1.5$ and $\theta_2 = 1.6$. Simultaneously, the other parameters remain as is. Since $\theta_1$ and $\theta_2$ are smaller than the values in case 1, the trigger times will theoretically be reduced. The verification in Fig. 5 shows that the trigger times are indeed reduced, but the cost is that the convergence time is longer.

Remark 7 At the same time, the trigger times of each agent in fully distributed ETC sampling and period control sampling are depicted in Table I. It can be inferred that the trigger times of ETC are less than that of periodic sampling control, and the trigger times are reduced by about 46%. Moreover, from the comparison of Fig. 3(c) and Fig. 4(c) in case 1 and case 2, since the pinning control sets the target value of the state, the trigger times can be significantly reduced on the basis of ETC. In summary, ETC and pinning control methods can abate the times of communication effectively (Table 1).

5 Conclusion

In this work, an integral SMC is established for nonlinear HMASs comprised of first-order and second-order agents to address the nonlinearity and disturbance issues of the system. It adapts to general situation preferably rather than homogeneous MASs. The simulation results show that the consensus effect of integral SMC and setting time are related to the upper bound parameters of nonlinearities and external disturbances. By utilizing a fully distributed ETC scheme, it averts the continuous communication. Comparing with period sampling control, the trigger times are greatly reduced. Meanwhile, we excluded the Zeno behavior. Utilizing
Lyapunov stability theory, the range of the pinning gain under the directed weakly connected topology is that the agents without in-degree must be pinned. Then, we obtained the consensus setting time by combined ETC and pinning control, and the setting time is related to the eigenvalues of the adaptive positive definite matrices and other system parameters. Furthermore, we discovered that pinning control can assist ETC to further abate trigger times. Ultimately, three simulation results indicate the anti-interference of SMC to nonlinearity and disturbance, the validity of the FTC for the system and the pinning control. We will optimize ETC to further deplete the trigger times later on, such as dynamic ETC.

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**Data availability** The data that support the findings of this study are available from the corresponding author upon reasonable request.

**Declarations**

**Conflict of interest** The authors declare that they have no conflicts of interest.

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