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We discuss a QCD model with one colorless scalar boson, whose renormalization group equations can be approximately solved near the QCD Landau pole, leading to an instability of the effective potential. This model has been discussed by us in the case of QCD with a nonperturbative infrared fixed point which may restore the potential stability. A fixed point appears when the QCD coupling constant is determined in an all order calculation in a particular sum of diagrams. This coupling can be parameterized in terms of an effective dynamical gluon mass which is determined through Schwinger-Dyson equations, whose solutions are compatible with QCD lattice simulations. Phenomenological studies indicate that this coupling freezes in the infrared region at moderate values. We then compute the SM scalar boson Higgs coupling (λ) evolution considering this frozen coupling constant. The existence of such nonperturbative infrared fixed point moves the λ evolution towards stability. For the phenomenologically preferred infrared value of the QCD coupling constant the SM Higgs potential is stable up to the Planck scale.

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I. INTRODUCTION

The Standard Model (SM) experimental data can only be confronted with the theory when quantum corrections are taken into account. The discovery of a Higgs-like boson at the LHC with a mass $M_h \approx 126$ GeV \cite{1,2}, was quite puzzling because the determination of this mass at the loop level in terms of all other known masses of the model indicate the following bound $M_h > (129.1 \pm 1.5)$ GeV \cite{3}. This result is perhaps the most remarkable evidence that we may need some new physics or interactions that could solve other puzzles, for example, the existence of neutrino masses, the hierarchy problem, the dark matter and dark energy, etc... The study of new physics requires high energy scales, up to the order of the Planck scale $\Lambda_{Pl} \sim 10^{19}$ GeV, and the observed Higgs boson mass allows the computation of the scalar self-coupling (λ) evolution, with the result that this coupling turns out to be negative before the scale $\Lambda_{Pl}$, when only the known SM particles are considered. This result indicates a possible SM instability if no new physics effects appear in the quantum corrections.

The importance of the quantum corrections, and in particular of QCD corrections, to the evolution of the scalar self-coupling was evidenced by Chetyrkin and Zoller recently \cite{4,5}. The SM stability up to the Planck scale will be in danger when $\lambda \leq 0$, where $\lambda$ appears in the the classical Higgs potential

$$V(\phi) = m^2 \phi^{\dagger} \phi + \lambda (\phi^{\dagger} \phi)^2,$$

and the authors of Ref.\cite{4,5} have shown that the use of 3-loop $\beta$ functions change the scale where the scalar self-coupling (λ) vanishes by more than one order of magnitude when compared to the scale obtained at 1-loop. It is worth mentioning that these authors particularly call attention to the importance of the QCD contributions to the λ evolution, and we naturally may think what would happen to this problem if the strong coupling constant could be summed at several orders.

The SM stability (or near critical behavior) has been discussed many times with different points of view, but in one particular model, QCD with one colorless scalar proposed by Meissner and Nicolai \cite{6}, it is quite clear to see that the ultraviolet (UV) instability of the effective potential may be associated to the infrared (IR) QCD Landau pole. The renormalization group (RG) equations for this model can be approximately solved near the Landau pole, where it is possible to verify that the large scale scalar self coupling evolution is directly related to the QCD IR behavior. This quasi-realistic model is going to be reviewed in Section II, since, in our opinion, this result may not be widely known. Therefore, the QCD corrections are important to the scalar self coupling evolution \cite{4,5}, and the UV effective potential instability may be related to the IR QCD Landau pole \cite{6}. These facts will lead us to the next

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question: If there is no such Landau pole, and the QCD coupling freezes at low energy? For instance, the IR QCD coupling freezing appears in the AdS/QCD analysis of Ref.[7], and is a condition for QCD analyticity as discussed in the Analytic Perturbation Theory proposal of Ref.[8]. This possibility will affect the effective potential calculation if the strong coupling constant ($\alpha_s$) has a fixed point, and freezes at a relatively small IR value.

Recently some of us calculated the model of Ref.[6] in the presence of a QCD nonperturbative fixed point [9]. It was verified that not only the barrier of instability found previously [6] is changed but the local minimum of the potential is also changed. This nonperturbative fixed point is consistent with a strong coupling constant that freezes in the IR due to the presence of an effective dynamical gluon mass. The existence of this dynamical mass has been observed in Schwinger-Dyson equations [10–14] and is consistent with lattice simulations [15]. In Section III we will discuss this solution for the QCD coupling constant, and present several phenomenological results indicating that its IR value is moderately small, i.e. even smaller than the top quark Yukawa coupling. This fact justify the use of the RG equations to compute the scalar self coupling evolution.

In Section IV we present our main result. We compute the SM Higgs boson self coupling evolution and find that it does not cross the line $\lambda = 0$ up to $\Lambda_{Pl}$, indicating, under the conditions described in this work, that the SM is stable up to the Planck scale, although the $\lambda$ value is near the criticality region as we approach $\Lambda_{Pl}$, but on the safe side. The SM RG group equations are solved at one-loop level with the very same initial conditions of Ref.[4, 5]. We obtain the same resut when using the perturbative $\beta$ functions, but obtain a positive $\lambda$ up to $\Lambda_{Pl}$ with a nonperturbative $\beta$ function. The result is essentially modified by the fact that we are using an all order result for $\alpha_s$ in a particular sum of diagrams! Section V contains our conclusions.

II. QCD WITH ONE COLORLESS SCALAR

In this section we basically review the model of Ref.[6, 9]. This model mimics the SM in the fact that it contains QCD with only one quark coupled to the scalar field. We could think of this model as the SM one with gluons, the top quark and a Higgs potential, given by the following Lagrangian:

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{q} \gamma^\mu D_\mu q - \frac{1}{2} \partial_\mu \phi \partial^\mu \phi + g_Y \phi \bar{q} q - \frac{\lambda}{4} \phi^4,$$

(2)

where $q$ is the quark field, $\phi$ is a colorless scalar and $F^{\mu\nu}$ is the usual QCD field tensor. The three different coupling constants are: a) The gauge coupling $g_s$ that appears in $F_{\mu\nu}$ and in the covariant derivative $D_\mu$, which, for convenience, will be redefined in the following as

$$z \equiv \frac{g_s^2}{4\pi^2} = \frac{\alpha_s}{\pi},$$

(3)

b) the Yukawa coupling $g_Y$, written as

$$x \equiv \frac{g_Y^2}{4\pi^2},$$

(4)

and c) the scalar self-coupling $\lambda$, which will be referred as

$$y \equiv \frac{\lambda}{4\pi^2}.$$  

(5)

These couplings obey the following RG equations at one-loop

$$2 \frac{dy}{dL} = a_1 y^2 + a_2 y x - a_3 x^2,$$

(6)

$$2 \frac{dx}{dL} = b_1 x^2 - b_2 x z,$$

(7)

$$2 \frac{dz}{dL} = -cz^2,$$

(8)

with the scalar anomalous dimension given by $\gamma(x, y, z) = -hx$, where the values of the different parameters appearing in these equations are $a_1 = 6$, $a_2 = 3$, $a_3 = 3/2$, $b_1 = 9/4$, $b_2 = 4$, $c = 7/2$ and $h = 3/4$ [16]. In the above equations

$$L \equiv \ln \frac{\Phi^2}{v^2},$$

(9)
and \( v \) is some renormalization mass scale that can be associated to the SM vacuum expectation value.

The solutions for \( z(L) \) and \( x(L) \) are well known

\[
\begin{align*}
z(L) &= \frac{z_0}{1 + cz_0 L/2}, \\
x(L) &= \frac{b_2 - c}{b_1 - K z(L)^{1-b_2/c}} z(L),
\end{align*}
\]

where \( K \) is chosen to satisfy \( x(0) = x_0 \). The parameter \( z_0 \) is found in order to obtain a coupling constant compatible with physical values of the QCD coupling. For instance, typical values in [6] were assumed to be \( x_0 = 0.120 \) and \( z_0 = 0.249 \). The \( x(L) \) can be also written as

\[
x(L) = \frac{b_2 - c}{b_1} z(L) \left( 1 - \frac{K}{b_1} z(L)^{1-b_2/c} \right)^{-1} = \frac{b_2 - c}{b_1} z(L) \left( 1 + \frac{K}{b_1} z(L)^{1-b_2/c} + \ldots \right),
\]

and since \( b_2 > c \) the power of \( z(L) \) in the denominator will be negative and at the weak coupling limit \( x(L) \) has the following behavior

\[
x(L) \to \delta z(L),
\]

where

\[
\delta = \frac{b_2 - c}{b_1}.
\]

Note that in the IR limit \( z(L) \to \infty \), i.e. \( L \to \ln(\Lambda^2_{QCD}/v^2) \), where \( \Lambda_{IR}/v = \exp(-1/cz_0) \). With the \( x(L) \) solution and the above results plugged into the differential equation (6), we obtain \( y(L) \) as

\[
y(L) \to \rho z(L),
\]

where \( \rho \) is a constant [6].

All details about the above calculation can be found in Ref.[6, 9], but the main point is the result of Eq. (13), meaning that in this limit the scalar self coupling evolution is governed by the IR QCD behavior. In general, depending on the initial values of the coupling constants we may have instability of the effective potential associated to the IR behavior of the QCD coupling constant [6]. At some extent these equations may explain the results of Ref. [4, 5] indicating that the SM \( \lambda \) evolution depend strongly on the QCD corrections, and the big change in scales as we go from 1 to 3 loops. The long and short distance scales are related due to the different behaviors (signal) of the \( \beta \) functions, but not necessarily in all cases the effective potential in one give scale is affected by the behavior of some coupling in a different scale (as one example see the scalar QED case in Ref.[9]), but if there is a mixing of couplings and an even mild dependence between different scales their effect may modify the stability conditions of an effective potential.

### III. A NONPERTURBATIVE QCD COUPLING

It is the purpose of this work to verify how the stability of the SM Higgs potential can be affected by a possible nonperturbative fixed point or freezing of the QCD coupling constant. The possibility that QCD develops an effective dynamically generated mass for the gluons and this one modify the IR coupling constant behavior has been discussed in the realm of Schwinger-Dyson equations for a long time. A review of this effect can be seen in Ref. [12], and some of the lattice simulations that helped to confirm these results can be found in Ref. [15]. We will skip the discussion about the nature of these solutions and just use expressions for the strong coupling constant in the full range of momenta, which is all that is necessary to compute the evolution of the SM scalar self coupling. The most complete expression for the nonperturbative strong coupling constant as a function of the dynamical gluon mass is [14]

\[
\alpha_s(k^2) = \left[ 4\pi \tilde{b} \ln \left( \frac{k^2 + f(k^2,m^2(k^2))}{\Lambda^2_{QCD}} \right) \right]^{-1},
\]
where the function $f(k^2, m^2(k^2))$ is given by

\[
f(k^2, m^2(k^2)) = \rho_1 m_2(k^2) + \rho_2 \frac{m_4(k^2)}{k^2 + m^2(k^2)} + \rho_3 \frac{m_6(k^2)}{(k^2 + m^2(k^2))^2},
\]

where $\Lambda_{QCD}$ is the QCD characteristic scale, $\rho_1, \rho_2, \rho_3$ are fitting parameters and the function $m^2(k^2)$ represents a running dynamical gluon mass that is given by

\[
m^2(k^2) = \frac{m^4_g}{k^2 + m^2} \ln \left( \frac{k^2 + \rho m^2_g}{\Lambda_{QCD}^2} \right)^{\gamma_2 - 1},
\]

where $m_g$ is an effective dynamical gluon mass IR value and the remaining parameters can be obtained in Ref.\[14\].

Eq.\[14\] is a very detailed fit of $\alpha_s(k^2)$ but for practical calculations it is well mapped and can be replaced by the simple expression determined by Cornwall \[11\]

\[
g^2(k^2) = \frac{1}{\beta_0 \ln \left( \frac{k^2 + 4m^2}{\Lambda_{QCD}^2} \right) } = 4\pi \alpha_s(k^2),
\]

where $\beta_0 = (11N - 2n_f)/4\pi^2$ with $n_f$ quark flavors and $N = 3$. Note that we are neglecting the running $m_g$ behavior in the above expression. The factor 4 appearing in Eq.\[17\] has now been substituted by a smaller factor \[14\], but we can use Eq.\[17\] because our calculation will mostly depend on the IR value of the coupling, i.e. $\alpha_s(0)$, and these ones will be established by the phenomenological data that we shall describe in the sequence. Furthermore, we advance that the data that we are going to present is consistent with the values obtained in several phenomenological calculations (see, for instance, \[17\] \[20\]) where Eq.\[17\] was used. As a last comment we must stress that Eq.\[17\] maps into the one loop perturbative QCD coupling constant, because above a few GeV the $m_g$ effect is negligible.

If we want to solve the SM RG equations using a nonperturbative QCD coupling it surely cannot be large, or at least it must be smaller or of the order of the top quark Yukawa coupling that is the one that gives an important (and problematic) contribution to the effective potential. However this is exactly what is going to happen, and most of the phenomenological models trying to extract the $\alpha_s$ IR coupling value seems to indicate a small number. For instance, the description of jet shapes observables require $\alpha_s(0)$ to be of the order 0.63 \[21\], the famous models of quarkonium potential calculations of Ref.\[22\] use an IR coupling of order $\alpha_s(0) \approx 0.6$, the ratio $R_{e^+e^-}$ computed by Mattingly and Stevenson \[23\] can fit the data with $\alpha_s(0) / \pi \approx 0.26$, analysis of $e^+e^-$ annihilation, as well as bottomonium and charmonium fine structure in the framework of the background perturbation theory may lead to a frozen value of the coupling constant as low as $\alpha_s(0) \approx 0.4$ \[24\]. Recent analysis of experimental data on the unpolarized structure function of the proton indicates that \[20\]:

\[
0.40 \leq \alpha_s,_{NLO}(Q^2 \to 0) \leq 0.56,
\]

what is also consistent with $\alpha_s$ values extracted from the GDM sum rule \[25\].

Other phenomenological calculations considering a finite IR QCD coupling can be found in Ref.\[17\], and a compilation of some results can be seen in Ref.\[18\]. We can add to these phenomenological computations the theoretical $\alpha_s(0)$ value obtained through the functional Schrödinger equation which is equal to 0.5 \[10\], and the quite extensive list of an IR finite effective coupling calculations based on the Schwinger-Dyson equations (SDE) within the Pinch Technique \[11\] \[13\], which also lead to a successful strong interaction phenomenology \[19\] \[20\].

The main point that we want to emphasize is that there are many theoretical and phenomenological evidences that the QCD coupling freezes in the IR, and more important: The IR value of this coupling is not even so large! The IR value of the dynamical gluon mass is approximately bounded to $m_g > 0.6 - 1.2 \Lambda_{QCD}$ in order to ensure positivity of the imaginary part of the gauge-boson propagator \[26\]. The preferred phenomenological value of this mass is associated to the ratio $m_g / \Lambda_{QCD} \approx 2$ \[11\] \[19\]. For a dynamical gluon mass $m_g = 1.5 \Lambda_{QCD}$ we obtain an IR QCD coupling of order of 0.8. Therefore, according to the many phenomenological determinations of the $\alpha_s(0)$ values described previously, and with the phenomenological determinations of the effective dynamical gluon mass \[19\] we will consider the following range for the IR value of this coupling

\[
0.4 \leq \alpha_s(0) \leq 0.8,
\]

what is in agreement with previous discussions about the IR value of the strong coupling \[13\]. Note that $\alpha_s(0) = 0.4$ corresponds to $m_g / \Lambda_{QCD} \approx 4.6$, and with the range of values shown in Eq.\[18\] we cover the many different determinations of $\alpha_s(0)$ and $m_g$ that can be found in the literature. At this point we have set up most of the information necessary to calculate the SM scalar self coupling evolution.
IV. SM SCALAR SELF COUPLING EVOLUTION

The SM stability is connected to the evolution of the scalar self coupling $\lambda$ appearing in the classical Higgs potential where the scalar field may acquire a vacuum expectation value $v \approx 246.2$ GeV. The evolution of any SM ordinary coupling $\alpha_i$ is governed by the respective $\beta$ function

$$\beta_i(\alpha_i) = \mu^2 \frac{d}{d\mu^2} \alpha_i(\mu), \quad (19)$$

where $\alpha_i$ represents $\lambda$ and any gauge or Yukawa SM couplings. The SM stability up to the Planck scale will be in danger when $\lambda \leq 0$, and the evolution of this coupling is determined solving the coupled system of differential equations given by Eq. (19).

From Eq. (17) we can easily obtain the respective $\beta$ function that we will use in the RG equations. This coupling is approximately constant in the IR and matches with the standard perturbative behavior above the scale $\Lambda_{QCD}$. Eq. (17) generates the following $\beta$ function

$$\beta(t) = -\beta_0 q^3 \frac{e^t}{e^t + a^2}, \quad (20)$$

with $a^2 = 4m^2 / \Lambda_{QCD}^2$ and $t = \ln(k^2 / \Lambda_{QCD}^2)$.

To compute Eq. (19) we will use the same one-loop SM $\beta$ functions used in Ref. [4, 5], except by the one-loop perturbative QCD $\beta$ function, that is substituted by Eq. (20), and will vary $m_t$ in order to cover the range described in Eq. (18). The reason for choosing Eq. (17) is that this coupling matches with the perturbative one in a continuous manner. Couplings like the one of Ref. [22] or the one of Ref. [17] may be well motivated in the IR, however to compute the RG equations we would need to perform a fit interpolating the IR values of these couplings with the perturbative tail, as discussed in Ref. [27], but introducing a large uncertainty in the calculation. Other couplings like the APT one [8] predicts an unphysical slope $d\alpha_s(q^2)/dq^2$ at $q^2 = 0$. Therefore, Eq. (17) will be quite appropriate to our purposes, and a reasonable representative of all IR frozen couplings with the advantage of showing a smooth transition from the IR to the perturbative ultraviolet region as observed in phenomenological analysis [25].

We compute the $\lambda$ evolution at one-loop order. Doing this we will not satisfy the Weyl consistency conditions, which imposes a suitable coupling constant counting scheme to solve the RG equations [28], introducing a small difference in the calculation of the stability scale. On the other hand we have the fact that a $\beta$ function like the one of Eq. (20) is nonperturbative, i.e. is an all order result in a particular sum of diagrams! Of course the main question here is to verify how the criticality of the SM changes in the presence of such coupling constant.

Our calculation follows closely the one of Chetyrkin and Zoller [4, 5], not only what concern the $\beta$ functions, but we use exactly the same initial conditions shown in Table 1 of Ref. [5] at $\mu = M_t$, where the top mass is $M_t = 172.9 \pm 0.6 \pm 0.9$ GeV, $M_H = 125.7$ GeV, $\alpha_s(M_Z) = 0.1185 \pm 0.0007$ and $n_q = 6$. This allow us to check the results comparing our calculation of the $\lambda$ evolution with the standard case, i.e. with the perturbative $\alpha_s$ coupling at 1 and 3-loop order, obtaining agreement with Ref. [5], whose curves can be seen in Fig. (1) indicated by 1-loop and 3-loop, corresponding respectively to the standard perturbative calculation of Ref. [5] at 1 and 3-orders. We than changed the perturbative QCD $\beta$ function at one-loop order by the one of Eq. (20).

The curves corresponding to the frozen strong couplings obtained using Eq. (20) and considering the IR range of the strong coupling given by Eq. (18) are described by the shaded area in Fig. (1). The lower curve in this area corresponds to the largest $\alpha_s(0)$ value, obtained when $m_t / \Lambda_{QCD} \approx 1.5$ whereas when we move to the upper area we go to smaller IR values of the coupling constant. Note that for a coupling constant up to $\alpha_s(0) = 0.8$ we can be confident that the solutions of the RG equations are still reliable. For $\alpha_s(0)$ in the range given by Eq. (18) the scalar coupling $\lambda$ do not cross the line $\lambda = 0$ up to $\Lambda_{Pl}$.

V. CONCLUSIONS

One of the points raised in this work is that in certain cases, as discussed in the case of a QCD model with a colorless scalar, the scalar self coupling evolution may have a large contribution from the QCD coupling due to its Landau pole. This fact was discussed for the first time by Meissner and Nicolai [6], which was reviewed in Section II, and may imply an instability barrier in the model. The absence of the Landau pole changes the position of this instability barrier [9]. The relation between the IR QCD coupling and the scalar coupling evolution may be one possible reason for the strong dependence on the QCD corrections found by Chetyrkin and Zoller [4, 5] in the calculation of the SM scalar self coupling evolution. In that case more and more QCD corrections help to push the coupling (or the effective potential) towards stability.
In Section III we discussed the nonperturbative QCD coupling determined in the solutions of the Schwinger-Dyson equations in a gauge invariant way through the use of the pinch technique [12, 14]. These equations are consistent with an IR finite gluon propagator and coupling constant, and the same result has been obtained through lattice QCD simulations [15]. A finite IR gluon propagator always imply in a nonperturbative IR fixed point [29], and this one may change the minimum of the effective potential of a scalar boson coupled to QCD [9]. The main points of that Section was to present an all order coupling in a particular sum of diagrams, and to show phenomenological evidences that the IR value of the QCD coupling may freeze at one moderately small value, which allows a reliable computation of the RG equations. This small value of the coupling is also obtained in several theoretical frameworks.

We have restricted our calculation of the scalar self coupling evolution to QCD IR coupling values smaller than 0.8, which is not even large if compared to the Yukawa top quark coupling, and is phenomenologically acceptable. Slightly larger $\alpha_s(0)$ values also indicate stability of the potential, but are not expected to be compatible with the low energy strong interaction phenomenology, and their evolution may not be reliably computed with the RG equations.

Considering a simple expression for the IR finite coupling constant or its $\beta$ function, that is consistent with the existence of an effective dynamical gluon mass and with a smooth transition between the perturbative and nonperturbative QCD regimes, we computed the RG evolution of the Standard Model scalar Higgs coupling ($\lambda$). If $\lambda \leq 0$ the Higgs potential can be metastable or unstable, however our calculation shows that this coupling does not cross the line $\lambda = 0$ up to $\Lambda_{Pl}$, indicating, under the conditions described in this work, that the SM is stable up to the Planck scale. According to this result the SM Higgs potential, more precisely the $\lambda$ value is near the critical region as we approach $\Lambda_{Pl}$, but on the safe side, although this certainly does not eliminate the need of new physics up to the Planck scale in order to solve the many SM puzzles.

Acknowledgments

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