Nucleon Decay Matrix Elements with $N_f = 0$ and 2 Domain-Wall Quarks
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The nucleon decay matrix elements of three-quark operators are calculated with domain-wall fermions. Operators are renormalized non-perturbatively to match the $\overline{\text{MS}}$ (NDR) scheme at NLO. Quenched simulation studies involve both direct measurement of the matrix elements and the chiral Lagrangian parameters, $\alpha$ and $\beta$. We also report on the dynamical quark effects on these parameters.

1. INTRODUCTION

Nucleon decay \cite{1}, once observed, is the hallmark of the physics beyond the standard model. On-going deep mine experiments, though yet to observe an event, are pushing up the lower bound of the lifetime of the nucleon \cite{2}, excluding GUT models which allow nucleons decay more frequently.

The dominant decay mode of the nucleon is to a pseudoscalar meson and a lepton, where low energy hadronic interactions are important. The factorization technique, commonly used for the hadron decays, leads to the decay amplitude written in terms of the Wilson coefficients and the low-energy matrix elements of the dimension-six operator consisting of three quarks and one lepton. Various QCD model calculations had estimated the hadronic part of the matrix element with results varying over an order of magnitude (see a compilation in \cite{3}). The amplitude must be squared in the width or lifetime, meaning two orders of magnitude difference from different estimations. The initial lattice calculations show smaller but, still large deviation (see a compilation in \cite{3}).

Recent lattice calculations concentrate on how to remove the systematic uncertainties, especially the discretization error, by either taking the continuum limit \cite{4} of the Wilson fermion \cite{5} or employing the domain-wall fermion (DWF) \cite{6,7}. All these calculations, however, have been done within the quenched approximation.

We report here the updated results of the nucleon decay matrix elements with DWF with quenched approximation as well as the estimate of the low energy parameters with $N_f = 2$ dynamical DWF. Detailed calculations will be given in the forthcoming publication \cite{8}.

2. QUENCHED SIMULATION

The parameters of our quenched simulation are $\beta = 0.87$ for the gauge coupling of the DBW2 gauge action, domain-wall height $M_5 = 1.8$, fifth dimension $L_s = 12$, the lattice volume $16^3 \times 32$ where latter is the temporal direction. The inverse lattice spacing $a^{-1} \simeq 1.3$ GeV is obtained by the $m_\rho = 0.77$ GeV input at the chiral limit, which gives the physical spatial lattice size as $L \simeq 2.4$ fm. While we only have one lattice spacing, the scaling violation is expected to be small for DWF as the other hadronic observables have shown \cite{9}.

The renormalization of the operator is done in combination of the non-perturbative renormalization (NPR) with the continuum perturbation theory in NLO. The NPR scheme \cite{10} is employed for the lattice operator to match the MOM scheme. The one-loop matching factor that we have calculated converts it to the $\overline{\text{MS}}$ in NDR scheme. Then, using the two-loop anomalous dimension \cite{11}, the operators are run to $\mu = 2$ GeV.

We demonstrated the NPR work well last year \cite{7} reusing the data for the quark bilinear renormalization at $L_s = 16$. We have performed NPR calculation at $L_s = 12$, where the matrix elements are obtained as well, to get the proper renormal-
Figure 1. Summary of $W_0$ (Eq. 1) with direct and indirect method in quenched calculation. Operators are renormalized at $\mu = 2$ GeV.

As we have changed the analyses details to get effective statistics have been increased. Further Green functions to be averaged. In this way configurations, we make use of discrete symmetries are used for NPR.

While the number of independent gauge configurations analyzed in this study is unchanged (100 configurations), we make use of discrete symmetry transformation properties which relate the different Green functions to be averaged. In this way the effective statistics have been increased. Further we have changed the analyses details to get more robust results. All these have changed the results slightly and helped shrinking the error.

In Fig. 1 we show the results of the relevant form factor $W_0$, which, for example in the $p \to \pi^0/\pi^+$ decay, defined through Eq. 1,

$$\langle \pi; \bar{p} | O_{R/L,L,L}^{\beta} | p, 0 \rangle = P_L | W_0 - i\eta | u_p \rangle,$$

where $O_{R/L,L,L}^{\beta} = \epsilon^{ijk}(u^T C P_{R/L} d^j)P_L u^k$, $q_\mu$ is the momentum transfer from proton ($k_\mu$) to pion ($p_\mu$), $q_\mu = k_\mu - p_\mu$, $u_p$ is the proton spinor. The results both with direct and indirect calculations are shown. The former involves various two- and three-point functions with many different parameter values to allow extrapolation to the physical kinematics point, thus is expensive. The latter is obtained with help of tree-level chiral perturbation theory [12,14] where the low energy parameters $\alpha$ and $\beta$, defined as

$$\alpha P_L u_p = \langle 0 | O_{R/L,L,L}^{\beta} | p \rangle, \quad \beta P_L u_p = \langle 0 | O_{L,L,L}^{\beta} | p \rangle,$$

are calculated on the lattice. The other low energy parameters are taken from the experiment [7]. The indirect method uses only a few two point functions, thus less computational effort is required than the direct method. The SU(2) flavor symmetry of $u$ and $d$ quarks relates the different matrix elements, e.g.,

$$\langle \pi^+ | O_{R/L,L,L}^{\beta} | p \rangle = \sqrt{2}\langle \pi^0 | O_{R/L,L,L}^{\beta} | p \rangle.$$  

(3)

Every other possible matrix element is identical to one of those in Fig. 1 up to sign factor.

The final $\pi$ state matrix elements has apparent deviation between the direct and indirect calculations. This may simply mean the limit of the tree-level chPT, as the pion has large momentum. Indeed in the soft pion limit ($p_\mu \to 0$) the expected relation ($W_0 - i\eta|W_0\rangle_{R/L} = \alpha/\sqrt{2f_\pi}$ holds numerically within the error. We note that the final $K$ state matrix elements are estimated better than those for the $\pi$ by the indirect method.

The values of individual matrix elements are different from those obtained for $a^{-1} \simeq 2.3$ GeV with Wilson fermion [14], while ratios of the matrix elements are similar. We consider our DWF results are closer to the continuum limit. Indeed it is the case for the low energy parameters.

The results of $\alpha$ and $\beta$ are summarized in the Table 1. They have a different relative sign and the relation $\alpha + \beta = 0$ approximately holds within $1.3\sigma$. Recent quenched calculation results [5] with Wilson fermion in the continuum limit are also shown. There the operators are renormalized by an improved lattice perturbation theory. We remark that the DWF results at coarser lattice are consistent with them.

## 3. DYNAMICAL QUARK EFFECTS

To investigate the quenching error we calculate the $\alpha$ and $\beta$ parameters on the $N_f = 2$ dynamical DWF configurations [15]. The simulation parameters are the same as quenched, except for gauge coupling $\beta = 0.8$, which gives $a^{-1} \simeq 1.7$ GeV, $L \simeq 1.9$ fm by $m_\rho$ in the chiral limit. We have three dynamical quark masses ($m_{\text{dyn}}$) spanning
\[ m_s/2 \lesssim m_{\text{dyn}} \lesssim m_s, \] where \( m_s \) is the strange quark mass. The spatial lattice size divided by the pion Compton wave length is \( Lm_\pi = 4.7 \) at the smallest. We expect the systematic error from the finite volume effects is subdominant given the large statistical error of the matrix elements. The operator renormalization is done in the same way as quenched calculation. The number of configurations used is \( \sim 100 \) for matrix elements or \( \sim 40 \) for NPR for each dynamical mass.

The measurements are done at the valence masses equal to the dynamical masses. The pion mass dependence of \(|\alpha|\) for both quenched and dynamical calculations are shown in Fig. 2. The dynamical result has stronger \( m_\pi \) dependence than quenched. After rather long extrapolation to the chiral limit with linear function of quark mass we obtain the \( \alpha \) and \( \beta \) parameters as shown in Table 1. The relation \( \alpha + \beta = 0 \) holds within the error. The values are consistent with the quenched ones. We note that these estimates from lattice QCD lie in the middle between the smallest (0.003 GeV\(^3\)) and largest (0.03 GeV\(^3\)) estimates of QCD model calculations.

### 4. OUTLOOK

The present dynamical DWF simulation with quark masses \( m_{\text{dyn}} \gtrsim m_s/2 \) has not revealed the unquenching effect on the \( \alpha \) and \( \beta \) parameters in the chiral limit with 20\% statistical error. To reduce the error to 10\% level, much lighter dynamical quark region needs to be explored. As to this direction, the direct calculation of the matrix elements becomes more important. Furthermore, test of scaling to the continuum limit will also be needed.

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