Optimal reinsurance based on compound Poisson distribution

Anna Chadidjah *, Lienda Noviyanti, and Achmad Zanbar Soleh
Statistics Department, Faculty of Mathematics and Science, State University of Padjadjaran, Indonesia

*anna.chadidjah@unpad.ac.id

Abstract. BPJS Kesehatan (BPJS-Health) is a public agency for health, established to implement the social security program. The health insurance prepared through BPJS is in the form of health protection for participants to get health care benefits and protection to meet the basic needs of health care. It is provided to every person who has paid fees or whose contributions are paid by the government. Claims estimation in BPJS-Health is required to increase the optimal service to participants. Imprecision in determining such estimates could destabilize the BPJS-Health. In this research, estimated total health service claims submitted by partnered health facility to BPJS based on Buhlmann-Straub Credibility. We also use the compound Poisson distribution to illustrate the optimal retentions in a collective risk model.

1. Introduction
BPJS-Health is a public agency for health, established to implement the social security program. Claims estimation in BPJS-Health is required to increase the optimal service to participants. Imprecision in determining such estimates could destabilize the BPJS-Health. This allows a Health Insurance participant in a year to obtain health services with a large claim exceeding the total fee paid, as can be seen in Table 1.

| Month   | Case | Total Insurer Fee Paid (IDR) | Total Claim Paid by BPJS-Health (IDR) |
|---------|------|------------------------------|---------------------------------------|
| January | 6.430| 2.042.187.380                | 41.599.950.466                        |
| February| 9.042| 7.786.333.312                | 63.739.509.517                        |
| Mart    | 12.151| 14.064.021.720              | 70.216.977.916                        |
| April   | 10.330| 21.014.941.763              | 72.915.383.607                        |
| Mei     | 10.921| 27.245.201.242              | 75.256.461.099                        |
| June    | 6.887| 34.390.034.824              | 69.217.659.897                        |

Table 1. Family Care Facility Claim on BPJS-Health January - June 2014

Source: BPJS-Health KCU Bandung, data processed

The above situation is very potential to cause bankruptcy risk experienced by BPJS-Health due to total claims that exceed the total fee paid from insured. In this research will be discussed how BPJS-Health can take advantage of reinsurance services in overcoming the lack of payment of total health claims funds to Family Care Facility without disturbing the existing National Health Insurance...
Program (PJKN) reserve fund. This is supported by the Government Regulation of the Republic of
Indonesia number 73 of 1992 on the Implementation of Insurance Business that any insurance cover
that the amount of sum insured exceeds its own retention should get reinsurance support.

The insurer’s risk can be reduced by the use of reinsurance, in which another insurer (the reinsurer)
acts as an insurer of the insurance company (the ceding company) originally covering the risk [1].
Reinsurance helps protect insurers against unforeseen or extraordinary losses by allowing them to
spread their risks. Because reinsurer chargers premium to the insurance company, a properly
calculated optimal retention would be nearly as high as the insurer financial ability. The insurance
company has to set a retention limit which is the amount below which this company will retain
the reinsurance and above which it will purchase reinsurance coverage from another (the reinsuring) company.

Reinsurance Stop-Loss is a type of non-proportional reinsurance that provides protection against an
increase in the number of claims in both the number of claim and claim amount, otherwise it is
suitable for collective risk types and does not cost much in terms of administration [2-6]. Soleh AZ
et al. [7] consider compound Poisson-Lognormal distribution in determining the retention for stop-
loss reinsurance and Noviyanti L and Soleh AZ [8] use lognormal distribution to illustrate the optimal
retention in an individual risk model.

Based on The Indonesian Financial Services Authority (OJK) No.31/SEOJK.05/2015, the OJK
issued the new rule regarding self-retention and domestic reinsurance. Insurance companies are
obliged to have and implement self-retention for each risk in accordance with the self-retention
limits. Insurance companies must submit their first reinsurance support strategy to the OJK and
also must develop and implement a reinsurance support strategy to ensure that they have sufficient
capacity to meet their liabilities.

In this research, estimated total health service claims submitted by partnered health facility to BPJS
based on Buhlmann-Straub Credibility with Stop-Loss reinsurance. We also use the compound
Poisson distribution to illustrate the optimal retentions in a collective risk model.

2. Total Claim
Let $S(t)$ be random variable of the aggregate claim for one year. There are two commonly used risk
models, individual risk models and collective risk models [9]. The collective risk model for one time
period is defined as follows

$$S(t) = C_1 + C_2 + \ldots + C_{N(t)}$$

(1)

$N(t)$ is a random variable denoted the number of claims occurring of the claim amount. We assume
that $C_1, C_2, \ldots, C_N$ are independent random variables with identical distribution and $N, C_1, C_2$ are
independent random variable.

Total Claim models can be seen in research conducted by [10-12]. The following steps to estimate the amount of retention are as follows

2.1. Estimation of mean and standard deviations of claim amount ($C$)
Assume $C$ be a Log-Normal distributed with parameters ($\mu$) and ($\sigma$). The parameter estimator as
follows:

$$E[C] = exp\left(\mu + \frac{\sigma^2}{2}\right) \text{ and } Var(C) = exp(2\mu + \sigma^2) \cdot (exp(\sigma^2) - 1)$$

(2)

2.2. Estimation of mean and standard deviations of number of claim.
Let $N(t)$ denote the random variable of the number of claims until the time to $t$ with $t \geq 0$. The case in
health insurance is a case where claims always occur so the probability of non-occurrence of a claim is
zero. The expectation and variance of the number claims of the Truncated Poisson distribution can be
calculated by the following equation:
\[ E[N(t)] = \frac{\lambda(t)e^{X(t)}}{e^{X(t)-1}} \quad \text{and} \quad \text{Var}[N(t)] = \frac{\lambda(t)e^{X(t)}}{e^{X(t)-1}} \left(1 - \frac{\lambda(t)}{e^{X(t)-1}}\right) \]  

(3)

The parameter estimate \( \lambda(t) \) can be obtained by a means of converting the Non Homogeneous Poisson Process (NHPP) to Homogenous Poisson Process so that the intensity function of the parameter \( \lambda(t) \) obtained from the multiplication of the average population may be claimed and the hazard function of the inter-arrival time of claim. Let \( \lambda(t) \) denote the arrival intensity function.

\[ \lambda(t) = \int_{t_1}^{t_2} N_p(t) h(t) dt \]  

(4)

The notation of \( h(t) \) is hazard function of inter-arrival time, \( t_1 \) is the last working day in certain month, and \( t_2 \) is the last working day in certain month.

2.3. Estimation of mean and standard deviation of total claim depend on Truncated Compound Poisson

The total claims of hospitalization \( \{s(t), t \geq 0\} \) are called compound Poisson processes where

\[ S(t) = f(N(t), Ci) \]

Assume the total claims of \( S(t) \) depend on the mutually independent \( N(t) \) and \( Ci \) distributions. Given the invariant of the maximum likelihood estimator, the expectation and variance of total claims can be calculated by the following formula

\[ (\hat{E}[S(t)]) = (\hat{E}[Ci])(\hat{E}[N(t)]) \]

\[ (\hat{\text{Var}}[S(t)]) = (\hat{\text{Var}}[Ci])(\hat{E}[N(t)]) + (\hat{E}[Ci])^2(\hat{\text{Var}}[N(t)]) \]

(5)

2.4. Estimation mean of total claims based on Buhlmann-Straub Credibility.

The Buhlmann-Straub model is a common model for the Buhlmann model with exposure changing for each period. The total claim estimate with Buhlmann-Straub’s credibility approach uses the following equation:

\[ E[S] = Z.E[S(t)] + (1 - Z).E[S(t)] \]

(6)

\( E[S(t)] \) being the mean of total claim of observation period, \( E[S(t)]^* \) is the average total claims in a month based on previous claims data, and \( Z \) is the Buhlmann-Straub credibility factor formulated as follows.

\[ Z = \frac{m}{m + K} \]

(7)

Total exposure (\( m \)) replaces Buhlmann’s credibility formula and parameter \( K \) is formulated as usual, with each equation that is

\[ m = \sum_{t+1}^{N} m_t; \quad K = \frac{EV_P}{VHM} = \frac{E[\text{Var}[S(t)]]}{\text{Var}[E[S(t)]]} \]

(8)

The credibility parameter (\( K \)) is estimated by Expected Value of Process Variance (EPV) and Variance of Hypothetical Means (VHM) of \( S(t) \)

2.5. Estimated Retention value through Re-Insurance Stop-Loss

Once the distribution of total claims is known, then the retention determination using \( \text{VaR} \) can be performed. The determination of retention \( (R^*) \) using \( \text{VaR} \) is based on the concept of minimizing the Total Risk \( (T) \) that the insurance company will encounter which is the sum of total claims paid by the insurer \( (X_i) \) and reinsurance premiums \( \delta(R) \) formulated as follows:

\[ T = X_i + \delta(R) \]
\[= X_t + (1 + \rho)E(X_R)\]

The Retension \((R^*)\) is obtained by minimizing \(VaR_t(R, \alpha)\) by the following objective function 

Optimization-\(VaR\): \(VaR_t(R^*, \alpha) = \min_{R \geq 0} \{VaR_t(R^*, \alpha)\} \) and 

\[
VaR_t(R^*, \alpha) = \begin{cases} 
R + \delta(R), & 0 < R \leq S^{-1}_X(\alpha) \\
S^{-1}_X(\alpha) + \delta(R), & R > S^{-1}_X(\alpha)
\end{cases}
\]

The Retension is 

\[R = S^{-1}_X(\rho^*)\]

if 

\[\rho^* < S_X(0) \text{ so } S^{-1}_X(\rho^*) > 0 \text{ and if } \alpha < \rho^* \text{ so } S^{-1}_X(\rho^*) < S^{-1}_X(\alpha)\]

3. Numerical Example

Hospitals are categorized into four classes based on the type of health services provided. Class A hospitals provide extensive Specialist and Subspecialist medical services so often referred to as Central Hospitals, Class B Hospitals provide Specialist Medical Services and 4 Limited Subspecialties, Class C Hospitals provide limited Specialized Medical Services, and D Class Hospital is only capable of holding one medical services such as Leprosy Hospital, Lung Hospital, and so on.

Based on the claim amount that proposed by hospitals (Class A, B, C and D) to BPJS-Health KC Bandung from January to June 2014 will be built a system of total payment claim effective PJKN, through statistical actuarial approach based on Re-Insurance Stop-Loss and Buhlmann-Straub Credibility of the total estimated PJKN claims

3.1. Estimation of Mean and Standard Deviation of Inpatient Claim Ammount

The first step is to divide the claim amount into Experience Data (ED) i.e. claim amount from January to May 2014 and Observation Data (OD) i.e claim amount during June 2014. Next it will be investigated whether the ED and OD distribution are assumed Log-Normal supported by real data.

Furthermore, based on Table 2, it is known that the mean of hospital class A claim per month during January to May 2014 was Rp.6,951,260,673 and increased to Rp.20,872,019,910 during June 2014. Similarly, standard deviations from hospital class A, claims per month during January to May 2014 are lower than in June 2014. This indicates high levels of heterogeneity of claims in June 2014

3.2. Estimation of Mean and Standard Deviations of the Number of Inpatient Claims

Table 3 provides information of the process to obtain the intensity function of the inter arrival time of inpatient claims in hospital for one month. This intensity function is obtained from inter- arrival time distribution of inpatient claims following the Weibull distribution. The intensity function of inter-arrival time claims in Class A hospital per month on observations from January to May 2014 fell from 5.99 to 1.72 in June 2014.

| Data Type       | Hospital Class | Mean (IDR)    | Standard Deviation (IDR) |
|-----------------|----------------|--------------|--------------------------|
| Experience      | A              | 6,951,260,673| 8,705,755,231            |
| (January to Mei 2014) | B              | 2,571,015,977| 2,529,379,602            |
|                 | C              | 811,743,969  | 644,092,060              |
|                 | D              | 313,627,437  | 369,665,393              |
|                 | A              | 20,872,019,910| 36,832,100,888          |
Table 3. Kolmogorov-Smirnov Goodness of fit of Inter-Arrival Time Claim Based on Weibull (α, β) and The Intensity of Inter Arrival Time

| Data Type               | Hospital Class | P-value | α̂  | β̂  | λ̂(1) |
|-------------------------|----------------|---------|-----|-----|-------|
| Experience              | A              | 0,7530  | 2,4651 | 55,888 | 5,99  |
| (January - Mei 2014)    | B              | 0,3606  | 2,4549 | 74,496 | 20,30 |
|                         | C              | 0,6420  | 2,2566 | 69,137 | 18,00 |
|                         | D              | 0,4853  | 2,3062 | 71,371 | 14,60 |
| Observation             | A              | 0,8251  | 6,695 | 105,930 | 1,72  |
| (June 2014)             | B              | 0,2876  | 44,089 | 116,260 | 11,20 |
|                         | C              | 0,4560  | 41,083 | 115,030 | 8,62  |
|                         | D              | 0,4507  | 24,099 | 115,160 | 7,39  |

Based on all the magnitudes in Table 3, the mean and standard deviation of the number of claim are presented in Table 4. Based on NHPP truncated at zero, we have mean of the number of hospitalization claims in class A hospital during January to May 2014 is 6 claims with standard deviation of 2 claims, while the mean of the number of hospitalization claims in Class A hospital during June 2014 are 3 claims with standard deviation of 1 claim. More information can be seen in Table 4

3.3. Estimation of Mean and Standard deviations of Total Inpatient Claims based on the Truncated Compound Poisson distribution

Estimated Inpatient Total Claims in hospital A class during January to May 2014 were Rp41,789,575,371 while the total estimated claims in hospital A class during June 2014 was Rp43,829,999,835. This indicates that the longer the BPJK-Health budget is to be raised to family care facility. This also occurs in the hospital for classes B, C, and D which can be seen in Table 5

Table 4. Mean and Standard Deviation of the Number of Inpatient Claim for One Month Depend on NHPP Truncated at Zero

| Data Type               | Hospital Class | Mean | Standard Deviation |
|-------------------------|----------------|------|--------------------|
| Experience              | A              | 6    | 2                  |
| (January to Mei 2014)   | B              | 20   | 4                  |
|                         | C              | 18   | 5                  |
|                         | D              | 14   | 4                  |
| Observation             | A              | 3    | 1                  |
| (June 2014)             | B              | 11   | 4                  |
|                         | C              | 8    | 3                  |
|                         | D              | 7    | 3                  |
Table 5. Mean and Standard Deviation of Inpatient Total Claim depend on NHPP Truncated at Zero

| Data Type               | Hospital Class | Mean (IDR)  | Standard Deviation (IDR) |
|-------------------------|----------------|-------------|--------------------------|
| Experience (January to Mei 2014) | A              | 41,789,575,371 | 27,236,683,058          |
|                         | B              | 52,283,374,429 | 16,264,201,526          |
|                         | C              | 14,661,512,766 | 4,403,906,135           |
|                         | D              | 4,601,003,345  | 1,856,804,461           |
| Observation (June 2014)  | A              | 43,829,999,835 | 59,122,877,969          |
|                         | B              | 22,450,052,489 | 8,865,152,605           |
|                         | C              | 9,839,577,814  | 4,288,031,313           |
|                         | D              | 3,019,897,947  | 1,825,382,521           |

3.4. Estimated Annual Rate of Inpatient Claims based on Buhlmann-Straub's Credibility

An estimated Total Claims for all class hospital for July 2014 as follow. Based on Experience Data (January to May 2014) and Observation Data (June 2014) with the factor credibility value (Z) as shown in Table 6. For class A in July 2014 is estimated that BPJS-Health will issue an inpatient claim fund of Rp. 43,829,645,118. More information can be seen in Table 6.

The estimated inpatient total claims to be paid by BPJS-Health to the Hospital during July 2014 is Rp79,149,726,707. The next issue is whether BPJS-Health will cover all of its own admissions claims or divide the risk of loss by reinsuranc, In order to answer the above question, a retention study is needed as will be explained in the sub-section below.

Table 6. Estimation Total Inpatient Total Claim (Buhlmann-Straub Credibility)

| Hospital Class | $R$  | $m$  | $Z$    | $E[S]$      |
|----------------|------|------|--------|-------------|
| A              | 1,8196 | 5901 | 0.999831 | 43,829,645,118 |
| B              | 0,1559 | 5150 | 0.999805 | 22,455,844,243 |
| C              | 0,1899 | 2118 | 0.999528 | 9,841,853,385  |
| D              | 0,3654 | 635  | 0.998428 | 3,022,383,961  |

3.5. Total Hospital Inpatient Claim Payment System based on Retention Using the VaR

Assuming $\alpha$ value of 1% -7% and loading of reinsuranc premiums used by 1%, 3%, 5%, and 7% respectively, the retention and reinsuranc of these reinsuranc scenarios for the total claims estimated in July 2014 amounting to Rp 79,149,726,707 are follow:

Table 7. Payment Total Inpatient Claims System

| $\rho$ | Retention (IDR) | Reinsuranc (IDR) |
|--------|-----------------|------------------|
| 1%     | 64,621,121,647  | 14,528,605,060   |
| 3%     | 67,341,466,769  | 11,808,259,938   |
| 5%     | 68,746,904,512  | 10,402,822,195   |
| 7%     | 69,729,565,444  | 9,420,161,263    |
If BPJS-Health tolerates the reinsurance loading of 1% then the total claims that can be paid on July 2014 is Rp64,621,121,647 so that need reinsured only Rp14,528,605,060. The greater the reinsurance loading tolerated by BPJS-Health, the greater the total claim that can be paid.

4. Conclusion

Developing an effective inpatient claims system involving retention and reinsurance is important when BPJS-Health intends to improve the principle of PJKN professionalism. In addition to spreading the risk of claims, BPJS-Health can stabilize the balance sheet and minimize the loss reserves so that it can be used to extend health services to BPJS-Health participants. This paper also provides information on estimating the total claim a month ahead that must be provided by BPJS-Health in anticipation of family care facility claims on PJKN Health services. Based on this information, BPJS-Health is expected to provide estimated loss reserves of claim to expedite payment of PJKN claims from family care facility.

References

[1] Panjer, H.H., dan Wilmot, G.E. 1992. Insurance Risk Models. Schaumburg, Ill, Society of Actuaries, USA.

[2] Borch, K. 1960. An Attempt to Determine the Optimum Amount of Stop-Loss Reinsurance. Transactions of the 16th International Congress of Actuaries, 2, 579-610.

[3] Kahn, P. 1961. Some Remarks on a Recent Paper by Borch. ASTIN Bulletin I, 265-71.

[4] Ohlin, J. 1969. On A Class of Measures of Dispersion with Application to Optimal Reinsurance. ASTIN Bulletin, 5, 249-266.

[5] Pesonen, M. 1984. Optimal Reinsurances. Scandinavian Actuarial Journal, 65-90.

[6] Wang, Shaun. 1995. Insurance Pricing and Increased Limits Ratemaking by Proportional Hazards Transforms. Insurance: Mathematics and Economics, 17: 43-54.

[7] Soleh AZ, Noviyanti L and Nurrahmawati I 2015 Retention for Stop-Loss Reinsurance to Minimize VaR in Compound Poisson-Lognormal Distribution. AIP Conf. Proc. 1692, 020026; http://dx.doi.org/10.1063/1.4936454

[8] Noviyanti L and Soleh AZ 2016 Optimal Retention Quota-Share Rate Insurance using Value at Risk Presented at International Actuarial Science and Statistics Institut Teknologi Bandung.

[9] Klugman, S., Panjer, H., & Wilmot, G. 2008. Loss Models: From Data to Decisions

[10] Meyers, G., dan Schenker, N. 1983. Parameter Uncertainty in The Collective Risk Model. PCAS, LXX, 11.

[11] Hogg, R.V., dan Klugman, S.A. 1984. Loss Distribution. New York; Wiley.

[12] Goovaerts, M., dan Kaas, R. 1988. Between The Individual and Collective Model for Total Claims. ASTIN Bulletin, 18, 169-17