Numerical investigation of convective heat transfer in fluid flow past a tandem of triangular and square cylinders in channel

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Abstract. A numerical study of a two-dimensional laminar flow and heat transfer characteristics in a horizontal channel containing two in-line obstacles of different shapes, namely an upstream triangular cylinder and a downstream square cylinder, was carried out. It is based on a coupling between the Lattice Boltzmann Method (LBM) and the Finite Difference Method (FDM). The airflow (Pr = 0.71) is assumed to be laminar and incompressible. All physical properties of the fluid, except its density, are supposed to be constant. The two cylinders are kept at a constant hot temperature, while the incoming flow is at a cold temperature. The Reynolds number and the horizontal separation distance between the cylinders are varied to investigate their influences on fluid flow and heat transfer.

1. Introduction
Flow around cylinders has been a subject of research for many scientists for a long time due to its practical applications. The majority of numerical or experimental investigation has focused on the effects of the form, number, and position of cylinders, Reynolds number, and the boundary of the computational domain. Many applications can be found in heat exchangers, buildings, bridges, mechanical systems, and electronic devices. Looking at the literature, one can conclude that the majority of numerical papers are about the flow passes on bluff bodies [1-15] such as circular cylinders [1-4] square cylinders [5-8] rectangular cylinders [9-12], and triangular cylinders [12-16]. All these works are done with a variety of numerical methods. From the above, it can be concluded that very limited attention has been focused on problems associated with the flow around two cylinders composed by the square cylinder and triangular cylinders or used triangular cylinders as disturber cylinders in front of square cylinders. For this reason, this study concentrates on the effects of Reynolds number and horizontal distance separation in a channel containing two in-line obstacles of different shapes, namely a triangular cylinder upstream and a square cylinder downstream.

2. Problem and numerical details

2.1. Physical model
Fig. 1 illustrates the physical model of the plane channel with the two obstacles. The air flow (Pr = 0.71) is considered laminar and incompressible. With the exception of its density, all the physical properties of the fluid are considered invariable. The L/d ratio is fixed at 30.75 and the blockage ratio
is $d/H=1/4$. The triangular cylinder is located at a distance $X/d = 8$ downstream of the inlet section of the channel. Both cylinders are maintained at a constant hot temperature $h = 0.5$, whereas the incoming flow is at a cold temperature $\theta = -0.5$. The input velocity profile is parabolic. The boundary conditions implemented on the top and bottom walls of the channel are $u = v = 0$, for velocity, and adiabatic for temperature. For the boundary conditions, the rebound scheme and a spatial quadratic interpolation \cite{17, 18} are used.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Schema of computational domain.}
\end{figure}

3. Numerical simulation

To simulate the fluid flow, the numerical method used is the multi relaxation time (MRT) lattice Boltzmann method \cite{19} which can be formulated as follows:

$$f(x+e,t+1) - f(x,t) = -M^{-1}S\left(m(x,t) - m^\omega(x,t)\right)$$  \hspace{1cm} (1)

Where $M$ is the $9\times9$ transformation matrix such as $m = Me$, and $S$ is the relaxation matrix where is a diagonal matrix, i.e., $S = \text{diag}(0, s_1, s_2, 0, s_4, s_5, s_6, s_7, s_8)$.

We consider a D2Q9 model and the particle speed $e_i$ are given by:

$$e_i = \begin{cases}
(0,0), & i = 0 \\
\cos((i-1)\pi/2), \sin((i-1)\pi/2), c, & 1 \leq i \leq 4 \\
\sqrt{2}\left(\cos((2i-9)\pi/4), \sin((2i-9)\pi/4)\right), c, & 5 \leq i \leq 8
\end{cases}$$  \hspace{1cm} (2)

Where $c = dx/dt$ is the lattice speed, and $dx$ and $dt$ are the lattice width and time step, respectively. Here, $dt$ is chosen to be equal to $dx$, thus $c=1$.

The nine moments with the associated distribution functions are classified in this order:

$$m = (\rho, e, j_x, j_y, j_z, q_x, q_y, q_z, p_{xx}, p_{yy})^T$$

$$f = (f_0, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8)^T$$

\begin{pmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -4 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\
1 & 4 & -2 & -2 & -2 & 2 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & -1 & 0 & -1 & -1 & -1 \\
1 & 0 & 2 & 0 & 2 & 0 & 1 & -1 & -1 \\
1 & 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 \\
1 & 0 & 0 & 2 & 0 & 2 & 1 & 1 & -1 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1
\end{pmatrix}

\begin{pmatrix}
\rho \\
e \\
\epsilon \\
j_x \\
q_x \\
j_y \\
q_y \\
p_{xx}
\end{pmatrix}

\begin{pmatrix}
f_0 \\
f_1 \\
f_2 \\
f_3 \\
f_4 \\
f_5 \\
f_6 \\
f_7
\end{pmatrix}$
The equilibria of the moments \( m^{eq} \) at the equilibrium are:
\[
\begin{align*}
\frac{e^{eq}}{e^{eq}} &= -2\rho + 3(j_x^2 + j_y^2) \\
\frac{e^{eq}}{e^{eq}} &= \rho - 3(j_x^2 + j_y^2) / \rho \\
q_x^{eq} &= -j_x \\
q_y^{eq} &= -j_y \\
p_{xx}^{eq} &= (j_x^2 - j_y^2) / \rho \\
p_{yy}^{eq} &= j_x j_y / \rho
\end{align*}
\]

Where:
\[
\frac{m^{eq}}{m^{eq}} = (0, e^{eq}, e^{eq}, 0, q_x^{eq}, 0, q_y^{eq}, p_{xx}^{eq}, p_{yy}^{eq})^T
\]  
(4)

The macroscopic fluid density and moment flux are calculated by:
\[
\rho(x,t) = \sum_i f_i(x,t)
\]
\[
J(j_x, j_y) = \sum_i f_i(x,t)e_i
\]  
(6)

The finite difference method [19, 20] is used to solve the macroscopic temperature. An explicit coupling between both schemes MRT and FDM is carried out. The energy equation is given by:
\[
\partial_t \theta + U \cdot \nabla \theta = \alpha \Delta \theta
\]  
(7)

The above equation is resolved explicitly by using first-order direct time difference scheme and the second-order central difference scheme for space discretization, with the same grid points as for the LBM scheme.

4. Results and discussion

The numerical code (HTLBM) has been validated in the previous work (see [21-25]). A preliminary work was performed to identify the optimum grid. Various simulations were carried out for \( w= 4d, \ Re = 100 \), and for different uniform grids \( (Nx \times Ny) \) in order to examine the grid independence. To optimize grid refinement with simulations efficiency, the grid \( 1567 \times 203 \) was selected for all the further calculations.

The local Nusselt number (Nu) based on square cylinder height is expressed by the following relationship:
\[
Nu = -d \frac{\partial \theta}{\partial n}\bigg|_{\text{block surface}}
\]  
(8)

Figure 2 depicts the instantaneous velocity plots for \( w= 4d, \ Pr = 0.71 \), and for four values of the Reynolds number \( (Re=20, 40, 80, \ and \ 100) \). In general, when a cylinder is positioned in the wake of another cylinder in the transverse flow, its unsteady charge becoming dependent not only on the flow characteristics in its wake but also on those in the wake of the upstream cylinder. For \( Re=20 \), It is apparent that behind each cylinder, two symmetrical vortex zones appear on either side of the wake which turns in opposite senses. The velocities vectors present a perfect symmetry in the channel. The increase of the Reynolds number leads to an increase in the magnitude and strength of these recirculation zones. For \( Re = 80 \), the symmetrical behaviour observed behind each cylinder disappeared. The pattern of the fluid flow changes and the wake loses its original symmetry. An unsteady separation flow over each cylinder is observed where the fluid flow oscillates with a repetitive state of swirling vortices. The von Karman vortex streets are visualized behind the rear face.
of the triangular and square cylinder. The oscillations in the wake grow in magnitude for Re=100. The unsteadiness in the flow increases with increasing Reynolds number.

![Velocity profile for w=4d](image)

Figure 2. Velocity profile for w=4d (a) Re=20, (b) Re=40, (c) Re=80 (d) Re=100.

Figure 3 presents the isotherms behaviours for various Reynolds numbers. From major observations, it can be deduced that the increase of Re affects favourably the heat transfer from the hot cylinders to the flow. The increase of Re from 20 to 40 leads to the reduction of the thickness of the thermal layer near to the cylinders walls. For these values of Re, the flow is symmetrical and characterized by a vertical gradient of temperature through the channel. For Re=80 and 100, the flow structure is changed to be asymmetric. The shape of the temperature fields reflects the fluid motion. An indicative temperature gradient is generated around the cylinders reflecting the presence of a significant heat transfer in these zones.
In order to understand the influence of varying gap spacing on fluid flow and heat transfer, Figure 4 and 5 illustrate the instantaneous velocity profiles and isotherms at Re=100 and for various spacing ratios W=2d, 4d and 6d. Let us note that at low Reynolds number (Re=20), the flow presents a constant symmetry and the gap spacing don’t affect the behaviors of the fluid flow and the heat transfer. When Re=100, two modes are observed depending on the gap space between the two cylinders. When w=2d, the square cylinder is very close to the triangular, which prevents the appearance of vortex in the space between them. As w increases to w=4d and 6d, the flow pattern is qualitatively changed. An alternate vortex are created between the cylinders and in the downstream of the second cylinder. At w=2d there is no flow separation and the isotherms have diffusion-type profiles while they become more non-uniform at w=4d and 6d because of the flow separation.
Figure 4. Velocity profile for Re=100 (a) w=2d, (b) w=4d, (c) w=6d.

Figure 5. Isotherms for Re=100 (a) w=2d, (b) w=4d, (c) w=6d

Figure 6 shows the variation of the local Nusselt number (Nu) along the surfaces of the square cylinder for w=4d and different Re. An increase of Nu is observed on all faces of the cylinder with the increase of Re. Similarly, we observed that the Nusselt number on the frontal face, where the isotherms are tightly wrapped, is the highest of all cylinder faces; which signifies greater heat transfer. In the front face of the square cylinder, the Nusselt number increases from the stagnation point to achieve a maximum value on the borders. On the upper and lower faces of the square cylinder, the heat transfer decreases along the direction of flow owing the advection of the heat flow downstream.
by the fluid. The lowest value of the Nusselt number is found behind the obstacle where it is almost constant.

Figure 6. Variation of Nu in square cylinder with different Re for w=4d.

Figure 7. Variation of Nu in square cylinder with different w for Re=20 and Re=100.

Figure 7 illustrates the variation of the local Nusselt number (Nu) along the surfaces of the square cylinder for various values of gap spacing (w=2d, 4d, 6d) and for Re=20 and Re=100. For Re=20, the local Nusselt number present the same comportment for all the gap spaces studied. Whereas, when
Re=100, a higher fluid velocity results in a stronger temperature gradient, which means an increased heat transfer from the cylinder surface. On the front face, Nu increases as d increases, but decreases on both the side faces comparing w=2d and w=4d.

5. Conclusion

The 2D HTLBM was implemented to laminar flow (air: Pr=0.71) and the heat transfer in a channel containing a tandem of triangular cylinder and square cylinder for different Reynolds number and gap spacing. Based on the results, one can conclude that the flow structure is strongly influenced by the Reynolds number and the gap space. The heat transfer is clearly affected by varying gap spaces between the two cylinders, particularly for large Reynolds numbers.

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