Vector and fermion fields on a bouncing brane with a decreasing warp factor in a string-like defect

L. J. S. Sousa\textsuperscript{a,b}, C. A. S. Silva\textsuperscript{c}, D. M. Dantas\textsuperscript{a}, C. A. S. Almeida\textsuperscript{a}

\textsuperscript{a}Departamento de Física - Universidade Federal do Ceará - UFC
C.P. 6030, 60455-760 Fortaleza - Ceará - Brazil

\textsuperscript{b}Instituto Federal de Educação Ciência e Tecnologia do Ceará (IFCE) - Campus de Canindé
62700-000 Canindé - Ceará - Brazil

\textsuperscript{c}Instituto Federal de Educação Ciência e Tecnologia da Paraíba (IFPB) - Campus Campina Grande
Rua Tranquilino Coelho Lemos, 671, Jardim Dinâmérica, Campina Grande - Paraíba - Brazil

Abstract

In a recent work, a model has been proposed where a brane is made of a scalar field with bounce-type configurations and embedded in a bulk with a string-like metric. This model produces an AdS scenario where the components of the energy momentum tensor are finite and have its positivity ensured by a suitable choice of the bounce configurations. In the present work, we study the issue of gauge and fermion field localization in this scenario. In contrast with the five dimensional case here the gauge field is localized without the dilaton contribution. Nevertheless, it is remarkable that the localization of the fermion field depends on the introduction of a minimal coupling with the angular component of the gauge field, which differs clearly from five dimensional scenarios. Furthermore, we perform a qualitative analysis of the fermionic massive modes and conclude that only left handed fermions could be localized in the brane.

1. Introduction

Thick branes have been proposed as a smooth generalization of the Randall-Sundrum scenario \cite{1,2,3,4}. In this model, five-dimensional grav-
ity is coupled to scalar fields. Thick brane models consist in a more realistic scenario than the Randal-Sundrum one, since no singularities appear due to the form of the scalar potential functions.

As a matter of fact, thick brane models have been comprehensively used in the task of localization of physical fields on the brane. The importance of this subject stays in the fact that the introduction of extra dimensions affects both gravitational interactions and particle physics phenomenology, and leads to modifications in the standard cosmology. If the extra dimensions indeed exist, it will inevitably change our ideas about the universe. The quest of field localization can guide us to which kind of brane structure is more acceptable phenomenologically [5].

In this context, gravitons and fermions, as well as gauge fields can be localized on the brane in thick brane models. Gauge fields, in particular, are localized only with the help of the dilaton field. The Kalb-Ramond field localization in this scenario was also studied by [6]. There the use of the dilaton was again necessary in order to localize the Kalb-Ramond field on the brane.

On the other stand point, scenarios have been proposed where thick brane solutions are extended to spacetimes with dimension more than five [3]. Among these works, we have some where branes are embedded in a bulk with a string-like metric. The mainly motivation to study branes in the presence of a string-like bulk comes from the fact that most of the Standard Model fields are localized on a string-like defect. For example, spin-0, spin-1, spin-2, spin-1/2 and spin-3/2 fields are all localized on a string-like structure. Particularly, the bosonic fields are localized with exponentially decreasing warp factor, and the fermionic fields are localized on the defect with increasing warp factor [7]. Even more interesting is the fact that spin-1 vector [7], as well as the Kalb-Ramond field [5], which are not localized on a domain wall in Randal-Sundrum model, can be localized in the string-like defect.

However, most of the thick brane models in six dimensional scenarios, proposed so far, have been suffering from some drawbacks. The first difficult is related with the introduction of scalar fields as a matter-energy source in the equations. In this case it is very difficult to find analytical solution to the scalar field and to the warp-factor as well. Koley and Kar [8] have suggested a model where analytical solutions can be found in a six dimensional scenario, however they run into a second difficult. This difficult is related with the positivity of the components of the energy-momentum tensor and has been found by other authors also [3,8,9]. Finally, field localization are not possible for any field in the thick branes defined in Ref. [9], at least for physically
acceptable solutions.

On the other hand, in a recent work [10], a model was proposed where a brane is made of a scalar field with bounce-type configurations and embedded in a bulk with a string-like metric. This model produces a sound AdS scenario where none of the important physical quantities is infinite. Among these quantities are the components of the energy momentum tensor, which have its positivity ensured by a suitable choice of the bounce configurations. Another advantage of this model is that the warp factor can be obtained analytically from the equations of motion for the scalar field, obtaining as a result a thick brane configuration, in a six dimensional context. It has been shown that scalar field localization is suitable in the scenario proposed in Ref. [10], paving the way in the sense of localization of other fields. Therefore, in the present work we will study the possibility of localization of vector and fermion fields in these scenario, in order to test its applicability and robustness.

This paper is organized as follows. In section 2 we address the model introduced in Ref. [10], where a bulk scalar field with bounce-type configurations generates a brane which is embedded in a bulk with a string-like metric. In section 3 we address the vector field localization, and in section 4 the fermion field localization is established. On the other hand, section 5 cope with qualitatively analysis of the fermionic massive modes. Section 6 is devoted to remarks and conclusions.

2. The model

The use of bulk scalar fields to generate branes was introduced by Goldberger and Wise [11, 12], and has been largely studied in the literature [13, 14, 15, 16, 17, 18]. In the six-dimensional context, Koley and Kar [8] have built a scenario where the brane is made of scalar fields and analytical “thin brane” solutions have been found out. Several progress have been obtained in the work by Koley and Kar in the intend of construct brane solutions in six dimensions, as well as, in the task of localize physical fields. However, some troubles with the energy conditions (WEC, SEC, NEC) [19] were found. In this model, the energy momentum tensor violates all the energy conditions since its components are not positive defined.

Recently, an AdS type solution was found in a model which assumes a six dimensional action for a bulk scalar field in a double well $V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$ potential minimally coupled to gravity in the presence of a cosmological constant [10]. In this scenario, which will underly the present article, it is admitted that the scalar field equation possess bounce-like statics solutions
depending only on the radial extra dimension, where the simplest is \( \phi(r) = v \tanh(ar) \).

The model is described by the action

\[
S = \frac{1}{2\kappa_6^2} \int d^6x \sqrt{-g} \left[ (R - 2\Lambda) + g^{AB} \nabla_A \phi \nabla_B \phi - V(\phi) \right],
\]

where \( \kappa_6 \) is the 6-dimensional gravitational constant, and \( \Lambda \) is the bulk cosmological constant.

The fields live in a string-like scenario with the following metric

\[
ds^2 = g_{MN} dx^M dx^N = g_{\mu\nu} dx^\mu dx^\nu + \tilde{g}_{ab} dx^a dx^b = e^{-A(r)} \tilde{g}_{\mu\nu} dx^\mu dx^\nu + dr^2 + e^{B(r)} d\Omega^2_{(5)},
\]

where \( M, N, \ldots \) denote the 6-dimensional space-time indices, \( \mu, \nu, \ldots \), the 4-dimensional brane ones, and \( a, b, \ldots \) denote the 2-extra spatial dimension ones. Also \( d\Omega^2_{(5)} = R_5^2 d\theta^2 \), \( P = e^{-A(r)} \) and \( Q = R_0^2 e^{-B(r)} \).

In the same way of the model introduced by Koley and Kar, the one introduced in [10] has the advantages to be analytical. However, the introduction of the bounce-type configurations to the scalar field that generates the brane supports a way to solve the problems with the energy conditions since the energy density may be positive or negative on the brane depending on the choice of the bounce configurations.

Moreover, the finiteness of the relation between the four \( (M_p) \) and six \( (M_6) \) dimensional reduced Plank scale [20] is ensured by the form of the warp factor found out by the authors which is given by

\[
A(r) = \beta \ln \cosh(ar) + \frac{\beta}{2} \tanh^2(ar).
\]

In the expression above \( \beta = \frac{1}{3} \kappa_6^2 \nu^2 \). Moreover, the warp factor found out is equal to 1 at \( r = 0 \) which ensures that on the brane one has a 4D Minkowski space-time. Besides, as \( r \) goes to zero or infinity, the warp factor goes to 0.

In the reference [10], the authors also pointed the interesting possibility of localization of the standard model fields in this scenario. In this way, the scalar field localization has been implemented paving the way for the localization of other fields. An interesting result is that any non-gravitational trapping mechanism has not been necessary to localize scalar field in this model, which can been seen as an advantage when compared with results of Dzhunushaliev and Folomeev [9].
In the present work, we will deal with vector and fermionic fields localization. It is known that it is possible to localize chiral fermions in the “5D version of this model” [1]. However, to localize vector field in this setup, in five dimensions, we need to have a dilaton field present in the model forming a “bounce-gravity-dilaton system” [1]. We expect, in this work, to localize fields in this scenario that is more realistic than the RS model ones, without the necessity of the dilaton field.

3. Localization of the vector field

In this section we will address the issue of the vector field localization. As we will see, it is possible to localize the vector field in this scenario if one has an exponentially decreasing warp factor, without the necessity of the dilaton field, as in the case of the scalar field discussed in Ref. [10].

In order to deal with the vector field localization, let us introduce the action

$$S_m = -\frac{1}{4} \int d^D x \sqrt{-g} g^{MN} g^{RS} F_{MR} F_{NS},$$

where $F_{MN} = \partial_M A_N - \partial_N A_M$.

From the action above, one obtains the equation of motion

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} g^{RS} F_{NS}) = 0,$$

which results in

$$\eta^{\mu\nu} g^{MN} \partial_\mu F_{\nu N} + e^{A+\frac{B}{2}} \partial_r \left( e^{-\frac{A+B}{2}} g^{MN} F_{rN} \right) + R_0^{-1} e^{B-A} g^{MN} \partial_\theta F_{N \theta} = 0. \quad (6)$$

This equation can be written in terms of equations for the vector field components as follows

$$\left( \eta^{\mu\nu} \partial_\mu \partial_\nu + e^{B/2} \partial_r e^{-(A+B)} \partial_r + \frac{e^{A-B}}{R_0^2} \partial_\theta^2 \right) A_\lambda - e^{B/2} \left( \partial_r e^{-(A+B)} \partial_r \right) \partial_\lambda A_r = 0,$$

$$\left( \eta^{\mu\nu} \partial_\mu \partial_\nu + \frac{e^{B-A}}{R_0^2} \partial_\theta^2 \right) A_r = 0,$$

and

$$\partial_r \left( e^{-2A+\frac{B}{2}} \partial_\theta A_r \right) = 0. \quad (9)$$

As has been done in [7], if we choose the gauge condition $A_\theta = 0$ and assume the decomposition

$$A_\mu (x^M) = a_\mu (x^\nu) \sum \rho_m e^{i\theta},$$

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and
\[ A_r(x^M) = a_r(x^\mu) \sum \rho_m e^{i\theta}, \]  
we can show that there exist the s-wave (l = 0) constant solution \( \rho_m = \rho_0 = \text{constant} \) and \( a_r = \text{constant} \). Note that we assume that \( \partial_\mu a^\mu = \partial^\mu f_\mu = 0 \), where \( f_\mu \) is defined by \( f_\mu = \partial_\mu a_\nu - \partial_\nu a_\mu \).

By substituting the constant solution in the initial action, the resultant integral in the variable \( r \) is
\[ I_1 \propto \int_0^\infty dr e^{-\frac{B}{2}}. \]  

In order to have zero mode localization for the vector field in this model, we need that \( I_1 \) to be finite. It is clear that in the case for \( A = B \) and with \( A \) given by Eq.(3) the condition above is satisfied (as can be seen in figures (1) and (2)). It is interesting to note that, in the domain wall case, this term is not present. That is why in 5D domain wall it is not possible to localize the vector field only by means of the gravitational interaction.

For further reference we may say that it is possible to obtain the zero mode localization for the vector field without imposing the gauge condition \( A_\theta = 0 \). Indeed, if one consider a \( r \) dependence of \( A_\theta \), say, \( A_\theta = A_\theta(r) \), the system ((7) - (9)) will assume the form
\[
\bigg( \eta^{\mu\nu} \partial_\mu \partial_\nu + e^{B/2} \partial_r e^{-(A+B)/2} \partial_r + \frac{e^{B-A}}{R_0^2} \partial_\theta^2 \bigg) A_\lambda +
\]
\[-e^{B/2} \left( \partial_r e^{-(A+B)/2} \partial_r \right) \partial_\lambda A_r - \frac{e^{B-A}}{R_0^2} \partial_\theta \partial_r A_\theta = 0, \]  
\[
\bigg( \eta^{\mu\nu} \partial_\mu \partial_\nu + \frac{e^{B-A}}{R_0^2} \partial_\theta^2 \bigg) A_r - \frac{e^{B-A}}{R_0^2} \partial_\theta \partial_r A_\theta = 0, \]  
and
\[ \partial_r (e^{-2A+B/2} (\partial_r A_\theta - \partial_\theta A_r)) = 0. \]  

If the fields \( A_\lambda \) and \( A_r \) are decomposed as in Eq.(10) and Eq.(11) it is possible to obtain the constant solution \( \rho_m = \rho_0 = \text{constant} \) and \( a_r = \text{constant} \) for the zero mode and s-wave. But in this case the function \( A_\theta \) has to satisfy the equation
\[ A_\theta''(r) + \left( -2A' + \frac{B'}{2} \right) A_\theta'(r) = 0, \]  
(16)
whose general solution may be expressed as follows

\[ A_\theta(r) = K_1 \int_r^\infty e^{2A(r) - \frac{1}{2}B(r)}dr + K_2, \]  

(17)

where \( K_1, K_2 \) are integration constants. Obviously this equation admits a more simple solution as \( A_\theta = \text{constant} \). For this specific case the results obtained for the vector field localization would be the same found above for \( A_\theta = 0 \). However, as will be seen this constant solution will be relevant for the localization of the fermion field in the next section. It is worthwhile to mention here that other solutions for \( A_\theta \) are possible, since they are finite for all \( r \) and \( B(r) > 4A(r) \). However the simplest solution to achieve fermion confinement is \( A_\theta = \text{constant} \).

![Figure 1: Sketch of the integrand in eq.(12).](image)

We can verify that this is a case of a smooth linear decreasing exponential. Here \( \beta = a = 1 \).

![Figure 2: Proof of convergence of \( I_1 \) (Eq.(12)).](image)

The function \( I_1 \) has a form similar to a linear decreasing exponential. After integration \( I_1 \) worth approximately 1.243 for \( \beta = a = 1 \).

4. Localization of the fermionic field

In this section, we will address the issue of fermionic field localization in a string-like defect scenario. At first, we will following the procedure of the reference [7], where fermionic fields are localized on a string-like defect. However, as will be shown, it is not possible to localize the fermionic field in this scenario if one requires an exponentially decreasing warp factor, which is possible in the case of scalar and vector fields as we have seen.

On another standpoint there exist an approach developed by Liu and collaborators [21], where it is possible to localize the fermionic field in the case of a decreasing warp factor. In this approach, one modifies the covariant
derivative by adding a minimal coupling between the fields. On other hand, here we propose a mechanism in order to localize the fermion field using the minimal coupling in the bouncing brane keeping an decreasing warp factor in scenarios of codimension two brane.

To begin with, let us proceed as in the reference [7]. We have that the fermionic field action can be written in a scenario with six dimensions as

\[ S = \int d^6x \sqrt{-g} \bar{\Psi} i \Gamma^M D_M \Psi, \]  

(18)

and the equation of motion related with this action is

\[ \left( \Gamma^\mu D_\mu + \Gamma^r D_r + \Gamma^\theta D_\theta \right) \Psi(x^M) = 0, \]  

(19)

where the matrices $\Gamma^M$ represent the Dirac matrices in a curved spacetime. These matrices are related with the Dirac matrices in the flat spacetime as

\[ \Gamma^M = h^M_M \gamma, \]  

(20)

where the vielbein $h^M_M$ is given by the relation

\[ g_{MN} = \eta_{\bar{M} \bar{N}} h^\bar{M}_M h^\bar{N}_N. \]  

(21)

The covariant derivative has the standard form

\[ D_M = \partial_M + \frac{1}{4} \Omega^\bar{M}_M \gamma \gamma, \]  

(22)

where the spin connection $\Omega^\bar{M}_M$ is defined by

\[ \Omega^\bar{M}_M = \frac{1}{2} h^\bar{M}_N \left( \partial_M h^\bar{N}_N - \partial_N h^\bar{N}_M \right) + \frac{1}{2} h^\bar{N}_N \left( \partial_M h^\bar{M}_N - \partial_N h^\bar{M}_M \right) - \frac{1}{2} h^\bar{P} \bar{M} h^\bar{Q} \bar{N} h^\bar{R} \bar{M} \left( \partial_P h_{QR} - \partial_Q h_{PR} \right). \]  

(23)

In order to write explicitly the equation (19), we have to calculate the matrices $\Gamma^M$ as well as the covariant derivative. Using the metric (2) and the equation (20), we have that the relation between the Dirac matrices in a curved space-time and the Dirac matrices in the flat space-times is given by

\[ \Gamma^\mu = P^{-\frac{1}{2}} \gamma^\mu; \quad \Gamma^r = \gamma^r; \quad \Gamma^\theta = Q^{-\frac{1}{2}} \gamma^\theta. \]  

(24)

The nonvanishing components of the spin connection (23) are

\[ \Omega^\mu^\nu = \frac{1}{2} P^{-\frac{1}{2}} P^r \delta^\mu_r; \quad \Omega^\nu^\theta = -\frac{1}{2} Q^{-\frac{1}{2}} Q^r \delta^\theta_r. \]  

(25)
Moreover, one can explicitly write the covariant derivative components (22) as

\[ D_\mu \Psi = \left( \partial_\mu - \frac{1}{4} P' \Gamma_\mu \Gamma_\mu \right) \Psi; \ D_\theta \Psi = \left( \partial_\theta - \frac{1}{4} Q' \Gamma_\theta \Gamma_\theta \right) \Psi; \ D_r \Psi = \partial_r \Psi. \]  

(26)

In order to write the equations of motion to these fields, we have to set the way how the Dirac matrices act on the spinor \( \Psi \). This approach was presented by the references [7, 21, 22], which we follow closely. First, let us assume that the spinor can be written in two parts, the right part \( \Psi_R \) and the left part \( \Psi_L \) as

\[ \Psi(x^M) = \sum_l (\Psi_R^l + \Psi_L^l) e^{il\theta}. \]  

(27)

The \( \Gamma \) matrices act on these spinor as

\[ \Gamma_\mu \partial_\mu \Psi_R(x^\mu) = m \Psi_L(x^\mu); \quad \Gamma_\mu \partial_\mu \Psi_L(x^\mu) = m \Psi_R(x^\mu). \]  

(28)

Or yet in terms of the \( \gamma \) matrices in the flat spacetime, as

\[ \gamma_\mu \partial_\mu \Psi_R(x^\mu) = P^{-\frac{1}{2}} m \Psi_L(x^\mu); \quad \gamma_\mu \partial_\mu \Psi_L(x^\mu) = P^{-\frac{1}{2}} m \Psi_R(x^\mu). \]  

(29)

Naturally, for \( m = 0 \) one has

\[ \gamma_\mu \partial_\mu \Psi_R(x^\mu) = \gamma_\mu \partial_\mu \Psi_L(x^\mu) = 0. \]  

(30)

These equations can still be put in the form

\[ \gamma^r \Psi_R(x^r) = + \Psi_R(x^r); \quad \gamma^r \Psi_L(x^r) = - \Psi_L(x^r), \]  

\[ \gamma^\theta \Psi_R(x^\theta) = i \Psi_R(x^\theta); \quad \gamma^\theta \Psi_L(x^\theta) = i \Psi_L(x^\theta). \]  

(31)

(32)

We require here that \( \psi(x^\mu) \) must satisfies the Dirac equation on the brane, namely \( \gamma^\mu \psi_\mu = 0 \). Thus, taking into account the s-wave case, and the equations (24), (25), (27), (28), (31) and (32), we will have that the equations of motion (19) can be written as

\[ \left( \partial_r + \frac{P'}{P} + \frac{1}{4} \frac{Q'}{Q} \right) \alpha(r) = 0. \]  

(33)

The general solution to the equation above is given by

\[ \alpha(r) = c_2 P^{-1} Q^{-\frac{1}{4}}, \]  

(34)
where \( c_2 \) is a constant of integration.

It is still necessary to verify if the solution is normalizable. In order to do this, we need analyze the action (18) with the spinor \( \psi \) replaced by this solution and verify if the resultant integral in the variable \( r \) is finite. From this, the interesting integral for the case analyzed here assumes the following form

\[
I_{1/2} \propto \int_0^\infty dr P_3^2 Q_1^2 \alpha(r)^2. \tag{35}
\]

At last, let us replace the expression (34) for \( \alpha \) in the equation (35) in a way that, for the case where \( A = B \), we have

\[
I_{1/2} \propto \int_0^\infty dr P_3^2 P^{-2} \propto \int_0^\infty d\epsilon \epsilon^{1/2} A(\epsilon). \tag{36}
\]

We note that no localization of the fermionic fields is possible in the case of a smooth warp factor given by \( A(r) = \beta \ln \cosh^2(\alpha r) + \beta^2 \tanh^2(\alpha r) \). Even the possibility of assume \( \beta < 0 \) is not possible since \( \beta = \frac{1}{4}\nu^2 \). In consequence, in our case it is not possible to appeal to a growing warp factor.

Therefore, the only alternative that remains is to use the treatment introduced in the reference [21] which consists in modifies the covariant derivative (22) by adding a term of minimal coupling. If one does this, the new covariant derivative reads

\[
D_M = \partial_M + \frac{1}{4} \Omega_M^N \gamma_M \gamma_N - ie A_M, \tag{37}
\]

where \( A_M \) is a gauge field, \( e \) the electrical charge, and \( i \) is the imaginary unit. Such modification does not change the relation between the gamma matrices (24), neither the nonvanishing components of the spin connection (25). On the other hand, the covariant derivatives components in this case, will be changed to the following forms

\[
D_\mu \Psi = [\partial_\mu - (P'/8P)\Gamma_r \Gamma_\mu - ie A_\mu] \Psi, \tag{38}
\]

\[
D_r \Psi = (\partial_r - ie Ar) \Psi, \tag{39}
\]

\[
D_\theta \Psi = [\partial_\theta - (Q'/8Q)\Gamma_r \Gamma_\theta - ie A_\theta] \Psi.
\]

Using the conditions (27), (28), (31) and (32), taking into account only the zero mode of the field and for the s-wave, the equation of motion for the right mode reads

\[
\left( \partial_r + \frac{P'}{P} + \frac{1}{4} \frac{Q'}{Q} - ie A_\gamma(r) + eQ^{-1/2} A_\theta(r) \right) \alpha(r) = 0, \tag{40}
\]

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where we have assumed that the Dirac equation \( \gamma^\mu \partial_\mu \Psi(x^\mu) = 0 \) is valid on the brane, and the field \( A_M \) can be decomposed in its components \( A_\mu(x^\mu), A_r(r), \) and \( A_\theta(r) \). The solution of the equation (40) is given by

\[
\alpha(r) = c_3 P^{-\frac{1}{2}} \exp \left( \int^r (ieA_r - eQ^{-\frac{1}{2}} A_\theta) \, dr \right). \tag{41}
\]

Inserting this solution in the action (18), the integral in the variable \( r \) reads

\[
I_{\frac{1}{2}} \propto \int_0^\infty \left( dr P^{-\frac{1}{2}} \exp \left( -2e \int^r Q^{-\frac{1}{2}} A_\theta \right) \right) = \int_0^\infty \, dr \exp \left( \frac{1}{2} A(r) - 2e R_0^{-1} \int^r e^{\frac{1}{2} B(r)} A_\theta \right). \tag{42}
\]

The form of field \( A_\theta \) in the above integral is essential to ensure the zero mode localization, i.e., to ensure the finiteness of the integral. In this case it is sufficient that \( A_\theta = a_\theta = constant \). The integral (42) is naturally convergent for \( r \to 0 \), so we have to study it in the other limit, \( r \to \infty \). It is easy to see that in this limit the function \( A(r) \) is linear on \( r \) and, in this case, it is possible to write \( A(r) = B(r) \approx \beta r \). In this situation we can easily see that (42) converges. Indeed, it will assumes the simple form

\[
I_{\frac{1}{2}} \propto \int_0^\infty \, dr \exp \left( \frac{1}{2} \beta r - \frac{4a_0}{R_0 \beta} e^{\frac{1}{2} \beta r} \right). \tag{43}
\]

Now, setting \( a_0 \equiv \frac{R_0 \beta}{8} \) and since the exponential function always assumes values greater than values from a linear function, one sees that the integral above is convergent (as can be seen in figures (3) and (4)). Moreover, for left handed fermions we must set \( a_0 \to -a_0 \) in order to converge that integral.

5. Fermionic massive modes

In this section we treat the fermionic massive modes using a qualitative analysis. In order to do this, we put the equations of motion of the massive spinor in the form of the Schrödinger equation.

First, we consider the massive counterpart of the eq.(40). Indeed, with help of eq.(29) we arrive at

\[
[\partial_r + H_{R,L}(r)] \alpha_{R,L}(r) = \pm m P^{-\frac{1}{2}} \alpha_{L,R}(r), \tag{44}
\]

where \( H_{R,L}(r) = \frac{p^r}{p^r} + \frac{1}{4} Q\prime - ieA_r(r) \pm eQ^{-\frac{1}{2}} A_\theta(r) \). Note that the expression (44) is composed by two equations with coupled quiralities. Applying the
Figure 3: Sketch of the integrand in eq. (43). We can verify that this is a case of a smooth linear decreasing exponential. Here $\beta = a = 1$ and $a_0 = 2$.

Figure 4: Proof of convergence of $I_{\frac{1}{2}}$ (Eq. (43)). The function $I_{\frac{1}{2}}$ has a form similar to a linear decreasing exponential. After integration $I_{\frac{1}{2}}$ worth approximately 0.736 for $\beta = a = 1$ and $a_0 = 2$.

following change in the independent variable $\frac{dz}{dr} = P^{-\frac{1}{2}}(r)$ and decoupling the left mode and the right mode we have

$$[\partial_z + H_{R,L}(z)] [\partial_z + H_{L,R}(z)] \alpha_{R,L}(z) = -m^2 \alpha_{R,L}(z). \quad (45)$$

Through a new change of variable $\tilde{\alpha}_{R,L}(z) = \exp \left[ -\int_z H_{R,L}(z)dz \right] \alpha_{R,L}(z)$, we can put eq. (45) in the form of Schrödinger equation, namely

$$\left( -\partial_z^2 + \left[ -\partial_z \left( e^{A_\theta(z)} \sqrt{P(z)} \right) \pm \left( e^{A_\theta(z)} \sqrt{P(z)} \right)^2 \right] \right) \tilde{\alpha}_{R,L}(z) = m^2 \tilde{\alpha}_{R,L}(z). \quad (46)$$

Therefore our potential can be written in term of $r$ as

$$V_{R,L}(r) = \left[ -\frac{e}{R_0} \sqrt{P(r)} \partial_r \left( A_\theta(r) \sqrt{P(r)} \right) \mp \left( \frac{e A_\theta(r) \sqrt{P(r)}}{R_0} \right)^2 \right]. \quad (47)$$

Now we study the choice $A_\theta(r) = constant$ which it was convenient for confinement of the zero modes of the vectorial and spinorial fields. For this choice, we present a sketch of the potentials in Fig. (5).

We note from the fig. (5) that both potentials are gap free when $r \to \infty$. Also we note that only the left mode potential assumes a volcano type form. This feature indicates that only left handed fermions are confined in our brane. As a matter of fact, the potential for right handed fermions is always
attractive. Also it reaches their asymptotic value on the infinity under the axis, which would not guarantee the right mode localization [23].

The complete analysis of the massive modes must to include the calculation of the resonant modes. However this calculation requires the numerical solution of equation (46). We defer this numerical analysis for a future work.

Figure 5: Sketch of the left mode potential (thick line) and sketch of the right mode potential (dot-dashed line) of the eq.(47). Here $\beta = 1$ and $a = 10$.

6. Remarks and conclusions

This work adds results in studies about thick brane in codimension two spaces. Here we have implemented a mechanism in order to localize vector and fermion fields in the scenario introduced in Ref. [10], where a thick brane is generated from a scalar field on a string-like defect. We have found that both, vector and fermion fields, can be localized in this scenario only with the gravitational interaction, which confirms the applicability and robustness of this model.

It is worthwhile to mention that in order to localize fermion field we show that, for the first time treating a bouncing brane, a previous localization of the gauge field is required. Indeed, we must have a component of the gauge field in the direction of the angular extra coordinate to obtain a convergent result for the integral of localization. In the literature, this component is usually turns to be zero as a gauge choice [21].

In the case under analysis here it was not necessary to appeal to a growing warp factor in order to localize fermion fields, since no localization of these fields is possible in the case of the smooth warp factor [3]. Therefore, the only alternative in order to localize fermions in this context it is to modify the covariant derivative [22] by adding a term of minimal coupling [21], which
was never used for the bouncing brane. In this case, the fermionic field can be localized on the brane.

Finally, we perform a qualitative analysis of the fermionic massive modes. We conclude that only left handed fermions could be localized in the brane. A numerical analysis of the massive modes is deferred for a future work.

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