Soliton dynamics in 1D quantum antiferromagnets

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The problem of dissipative motion of solitons in the case of tetrathymethyl ammonium manganese chloride (TMMC) is studied as a function of the external magnetic field and the temperature. Two specific situations are analyzed separately: the first, above the transition temperature $T_N$, in which the classical motion of the spin degree of freedom is described by a sine-Gordon (SG) equation of motion and, the second, below $T_N$, in which the system is described by a double sine-Gordon (2SG) equation of motion. The existence of a dissipative regime for the soliton motion and its influence on the dynamical structure factor - which might be experimentally detected - are reported.

I. INTRODUCTION

In the past few decades it has become well-established that the physical properties of some magnetic materials, TMMC, CsNiF$_3$ (cesium nickel fluorite) and CuCl$_2$ 2NC$_5$H$_5$ (diloro-bis-piridine copper II), for instance, have essentially one dimensional character above their transition temperatures. In those kind of materials the distance between magnetic ions along a given direction (magnetic chain direction) is shorter than in the other directions. In such an arrangement the intrachain coupling constant is typically more than two orders of magnitude stronger than the interchain coupling constant. Therefore, the system can be considered as a set of weakly interacting magnetic chains. Due to the relative simplicity of obtaining solitonic or solitary-wave solutions in 1D systems, these quasi-one-dimensional magnets turn out to be the paradigm for the study of the influence of the non-liner modes (solitons) in the dynamical properties of such systems at finite temperatures. Although all real magnetic materials investigated are not perfectly one dimensional, the assumption of the 1D behavior is shown to be in good agreement with the experimental results (see Ref. 1 and the references therein).

In magnetic materials solitons or solitary-waves can be regarded as ‘kinks’ or ‘twists’ in the spin space moving with constant speed and carrying a constant topological charge defined by the values of the spin variables at infinity. For low enough temperatures, when the linear modes (spin-waves) are not excited, the magnetic system can be represented in first approximation by a gas of non-interacting solitons. Using this idea, Mikeska calculated the soliton contribution to the dynamical structure factor of the classical one-dimensional magnets. From both works we learn that the assumption of ballistic motion for solitons is the origin of the ‘central peak’ behavior observed in neutron scattering experiments. A different situation could be found from the quantum field theory point of view when the temperature is raised. In this case the spin-wave (SW) modes are excited, therefore not all of the degrees of freedom of the system contribute to the soliton formation and a residual interaction (which couples the center of mass of the soliton to the spin-wave modes) shows up. In practice, the specific form of this kind of interaction is obtained via the collective coordinate method in the quantization process of the classical hamiltonian.

The soliton-SW coupling may result in a dissipative regime to the soliton motion depending on the form of the potential generated by the presence of the non-linear excitation. As it is known, the equation of motion for the spin variable in the TMMC below and above $T_N$ are a 2SG and a SG equation respectively. This fact makes the TMMC a suitable probe to investigate the appearance of a dissipative regime in the soliton motion provided that, below and above a certain Neél transition temperature $T_N$, the classical equation of motion for the spin variables are substantially different. The main purpose of this work will be therefore the analysis of the magnetic soliton motion above and below $T_N$ and the possible influence of the dissipative regime, found for $T < T_N$, on the solitonic contribution to the dynamical structure factor. In doing that, we will use the method developed in Ref. 9 for the analysis of the dissipative dynamics of solitons. As pointed out before, this formalism is based on the collective coordinate method and allows us to transform the original hamiltonian of the spin degree of freedom into one of a particle (the soliton) coupled to an infinite set of linear-modes (SW).

Starting from the interacting soliton-SW hamiltonian it is possible to obtain a Brownian-like equation of motion for the soliton center of mass via the Feynman-Vernon formalism. This effective equation of motion is written in terms of a damping constant that depends on the phase shifts of the scattering problem that emerges by the presence of the non-linear excitation coupled to the
SW. Therefore, the analysis of the scattering properties of the 2SG potential (for $T < T_N$) and the SG potential (for $T > T_N$) allows us to calculate the mobility as a function of the temperature and the external magnetic field. As it will be shown, above $T_N$ the SG solitons have infinite mobility in agreement with the ballistic motion used to understand the neutron scattering experiment for $H/T \geq 10kOe/K$. On the other hand, for low temperatures or low magnetic field, when the spin equation of motion for TMMC have a 2SG form, the soliton mobility is finite, changing the form of the dynamical structure factor considerably.

To begin with, in Sec. II we review the models currently applied to the spin dynamics of the TMMC compound, above and below its transition temperature, and also the corresponding classical equations of motion. In Sec. III we summarize the obtainment of the quantized soliton-SW hamiltonian and the damping parameter of the temperature and the external magnetic field. Sec. IV is devoted to the study of the influence of the soliton Brownian motion is calculated as a function of material parameters will be used: $J_\parallel = 13.4 K$, $S = 5/2$, $A/J_\parallel = 0.01 - 0.02$, $J_\perp/J_\parallel = 1.5 \cdot 10^{-5}$ and $y = 2.01$.

In order to start the classical description of the spin dynamics it is convenient to look at two main different situations, namely, temperatures below and above the transition temperature. For temperatures below $T_N$ the system described by (1) displays a long range magnetic order, therefore the staggered spontaneous magnetization is not zero and the system can be described in the mean field approximation as a set of non-interacting antiferromagnetic chains with an additional spontaneous magnetization in the $y$ direction. Explicitly,

$$H = \sum_i \left\{ J_\parallel S_i S_{i+1} + A(S_i^z)^2 - g\mu_B BS_i^z - g\mu_B B_\perp^{MF} (-1)^i S_i^y \right\},$$

(3)

where

$$B_\perp^{MF} = \eta J_\perp ((-1)^i S_i^y)/g\mu_B,$$

(4)

and $\eta$ accounts for the presence of neighbouring chains in the model. In the specific case of TMMC, $\eta = 6$. The intrachain mean field $B_\perp^{MF}$ is usually replaced by its saturation value $B_\perp^{sat} \approx 22.3$ Oe which results from the substitution of $S_i^y$ in (4) by its maximum value.

At this point, we can carry on the classical description of the spin dynamics. In order to do that it is convenient to change the spin variables to the following form

$$S_{e,o} = \pm S [\sin(\Theta \pm \theta) \cos(\Phi \pm \varphi), \sin(\Theta \pm \theta) \sin(\Phi \pm \varphi), \cos(\Theta \pm \theta)],$$

(5)

where $e$ and $o$ stands for even and odd sites within a chain.

Using the representation (6) a $\Phi$-dependent part of the hamiltonian (3) can be obtained (see Ref. 3 for details). Explicitly,

$$H^\Phi = \frac{1}{2} J_\parallel S^2 \int dz \left[ \frac{1}{c^2} (\partial_\parallel \Phi)^2 + (\partial_\perp \Phi)^2 - \frac{1}{4} b_\perp^2 \sin^2 \Phi - 2b_\perp \sin \Phi \right],$$

(6)

where

$$c^2 = 4 + \frac{2A}{J_\parallel}, \quad b = \frac{g\mu_B B}{J_\parallel S}, \quad b_\perp = \frac{g\mu_B B_\perp^{MF}}{J_\parallel S}. $$

(7)

It should be stressed that the hamiltonian (3) is an approximated description of the real TMMC system. To reproduce the experimental results, magnon-mass and solitonic energy, for instance, quantum effects and the out of plane component of the magnetization must be taken into account. To go on with the classical description of the $\Phi$-dependent part of the original hamiltonian (3) the equation of motion associated to (6),

$$\frac{1}{c^2} \partial_t \Phi = \partial_\perp \Phi - \frac{b_\perp^2}{8} \sin 2\Phi - b_\perp \sin \Phi$$

(8)

has to be solved. Equation (8) is not completely integrable, however, it has solitonic solutions in the form of $2\pi$-kinks(antikinks) moving with velocity $u$. Explicitly,

$$\cos \Phi = \frac{\pm 2 \sqrt{\alpha}}{1 + \alpha \sin^2 y} \sinh y,$$

(9)
\[
\sin \Phi = 1 - \frac{2}{1 + \alpha \sinh^2 y},
\]

where
\[
\alpha = \frac{b_\perp}{b_\perp + b^2/4}, \quad y = (z - z(t)) \sqrt{\frac{b_\perp + b^2/4}{1 - u^2/c^2}},
\]

and the position of the soliton center of mass \(z(t)\) is given by
\[
z(t) = z_0 + ut.
\]

On the other hand, for temperatures above \(T_N\) the value of the \(b_\perp\) is very small. In fact, in this situation \(b_\perp\) can be set equal to zero and, a well-known solitonic solution for equation (13) can be found: the \(\pi\)-kink(antikink) solution for the SG equation
\[
\sin \Phi_s(z,t) = \pm \tanh \left[ (1 - u^2/c^2)^{-1/2}(z - z(t))b/2 \right].
\]

As it can be seen, the model for TMMC in the continuum approximation leads us to different kinds of solitonic solutions depending on the temperature. A 2SG soliton solution given by (3) and (10) for \(T < T_N\) and, a SG solution (13) for temperatures above \(T_N\). As it was already mentioned, from the classical point of view, these soliton solutions will move with constant velocity throughout the sample. However, looking at the soliton dynamics from the quantum field theory perspective, the interaction with the spin waves can transform this ballistic regime into a dissipative one. In the next section we shall be aiming at the investigation of the mobility of the two types of solitons, below and above \(T_N\).

### III. SOLITON MOBILITY

The quantum dynamics of our spin system (1) can be analyzed by studying the quantum mechanics of the field theory described by the action
\[
S[\Phi] = J_s S^2 \int \left\{ \frac{1}{2c^2} (\partial_t \Phi)^2 - \frac{1}{2} (\partial_z \Phi)^2 + U(\Phi) \right\} dt dz,
\]

where
\[
U(\Phi) = \frac{b^2}{8} \sin 2\Phi + b_\perp \sin \Phi.
\]

To quantize the system described by (14) we need to evaluate
\[
G(t) = \text{tr} \int D\Phi \exp \left\{ \frac{i}{\hbar} S[\Phi] \right\}
\]

where the functional integral has the same initial and final configurations and \(\text{tr}\) means to evaluate it over all such configurations. As the functional integral in (10) is impossible to be evaluated for a potential energy density as in (13) we must choose an approximation to do it. Since the magnetic moments at the manganese sites in the TMMC are large (5/2), the semi-classical limit will be chosen as the appropriate one in our case. Within the functional integral formalism of quantum mechanics, the semi-classical limit is simply the stationary phase method applied to (13) around the solitonic solutions (3), (10) or (13) in which we are interested. When this is done we are left with an eigenvalue problem that reads
\[
\left\{ \frac{d^2}{dz^2} + U''(\Phi_s) \right\} \psi_n(z - z_0) = k_n^2 \psi_n(z - z_0),
\]

where \(\Phi_s\) is denoting the soliton-like solution around which we are expanding \(\Phi(z,t)\) and \(\psi_n(z - z_0)\) are the spin wave modes in the presence of the soliton.

Now one can easily show that \(d\Phi_s/dz\) is a solution of (13) with \(k_n = 0\). The existence of this mode is related to the translation invariance of the system and causes the divergence of the functional integral in (16) in the semi-classical limit (Gaussian approximation). The way out of this problem is the so-called collective coordinate method. This method consists basically in expanding the field configurations about \(\Phi_s(z)\) as
\[
\Phi(z,t) = \Phi_s(z - z_0(t)) + \sum_{n=1}^{\infty} c_n \psi_n(z - z_0(t)),
\]

but regarding the c-number \(z_0\) as a position operator. Using expansion (15), the second quantized version of (1) can be written as
\[
H = \frac{1}{2M_s} \left( P - \sum_{mn} \hbar g_{mn} b_m^+ b_m \right)^2 + \sum \hbar \Omega_n b_m^+ b_m,
\]

where \(\Omega_n = ck_n\).

In the hamiltonian (19), \(P\) stands for the momentum canonically conjugated to \(z_0\).

\[
M_s = \frac{2J_s S^2 a}{c^2} \int_{-\infty}^{\infty} dz U(\Phi_s(z))
\]

is the soliton mass and the coupling constant \(g_{mn}\) is given by
\[
g_{mn} = \frac{1}{2i} \left[ \sqrt{\frac{\Omega_m}{\Omega_n}} + \sqrt{\frac{\Omega_n}{\Omega_m}} \right] \int dz \psi_m(z) \frac{d\psi_n(z)}{dz}.
\]

The operators \(b_m^+\) and \(b_m\) are respectively the creation and annihilation operators of the excitations of the magnetic system (magnons) in the presence of the soliton. In fact, the term
\[
\sum_{mn} \hbar g_{mn} b_m^+ b_m,
\]
can be interpreted as the total linear momentum of the magnons of the system and therefore, we are left with a problem in which the momentum associated to the soliton is now coupled to the magnons’ momenta. This effective model suggests that, as the population of magnons is a temperature-dependent quantity, the mobility of the soliton will be strongly related to the temperature of the system and its dynamics (determined by $U''$) will be non trivial.

At this point we are ready to study the mobility of the wall because we have been able to map that problem into the hamiltonian (19), which on its turn has been recently used to study the mobility of polarons in 1D environments and skyrmions in 2D electronic systems. The main result obtained in those calculations can be summarized as follows. The damping function $\gamma(t)$ (basically the inverse of the mobility) is given by

$$\gamma(t) = \frac{\hbar}{2M} \int_0^\infty d\omega d\omega' \{S(\omega, \omega')(\omega - \omega') \times [n(\omega) - n(\omega')]\cos(\omega - \omega')t\},$$

where

$$n(\omega) = \frac{1}{e^{\beta\hbar\omega} - 1}$$

is the Bose function and,

$$S(\omega, \omega') = \sum_{mn} |g_{mn}|^2 \delta(\omega - \Omega_n)\delta(\omega' - \Omega_m)$$

is the so-called scattering function.

In the long time limit $\gamma(t)$ can, to a good approximation, be written as

$$\gamma(t) \approx \bar{\gamma}(t)\delta(t),$$

where $\delta(t)$ is the Dirac delta function and $\bar{\gamma}(T)$ is given by

$$\bar{\gamma}(T) = \frac{1}{2\pi M_s} \int_0^\infty dE R(E) \frac{\beta E \rho \beta E}{(e^{\beta E} - 1)^2}.$$  

In (27), $R(E)$ is the reflection coefficient of the “potential” $U''(\Phi_s)$ in the Schrödinger-like equation (17). For simplicity we will express the reflection coefficient $R(E)$ in terms of the even and odd scattering phase shifts $\delta^e(k)$ and $\delta^o(k)$ as

$$R(k) = \sin^2(\delta^e(k) - \delta^o(k)).$$

At this point we can perform the calculation of the soliton mobility in TMMC for temperatures above and below $T_N$.

A. Soliton mobility for $T < T_N$

To calculate the soliton mobility below $T_N$ we need the explicit form of the potential $U''(\Phi_s)$ involved in (27).

For the case of the 2SG soliton this potential can be written as

$$U''(\Phi_s^{2SG}) = \frac{1}{\lambda^2} \left[ 1 - 2\text{sech}^2\left(\frac{z}{\lambda} + \rho\right) - 2\text{sech}^2\left(\frac{z}{\lambda} - \rho\right) \right. + 2\text{sech}\left(\frac{z}{\lambda} + \rho\right)\text{sech}\left(\frac{z}{\lambda} - \rho\right),$$

where

$$\lambda = \frac{1}{b_1 + b_2/4}, \quad \cos \rho = \frac{1}{\sqrt{\alpha}}.$$  

The second and third terms in the r.h.s. of (29) are the potentials of the noninteracting $\pi$-solitons located at $z/\lambda = \pm \rho$ whereas the last term describes the interaction of the two $\pi$-solitons at $z/\lambda = \pm \rho$ respectively.

For all finite values of $\lambda$ and $\rho$, the system is translationally invariant and, consequently, the potential (29) has a zero energy state that is given by

$$\psi_0 \propto \text{sech}(\frac{z}{\lambda} + \rho) + \text{sech}(\frac{z}{\lambda} - \rho),$$

which is nothing but the Goldstone mode of the $2\pi$-soliton for finite transverse magnetization and finite external field.

In order to evaluate the expression (27) for the damping constant we need the even and odd phase shifts associated to the potential (29). Unfortunately, their analytical evaluation is very complicated for all finite values of $\lambda$ and $\rho$, and in what follows we will only study the situation of weak external fields ($b_1 \gg b_2^2/2$). In this case ($\rho \ll 1$) the Schrödinger-like equation (17) can be written as

$$\left\{-\frac{\partial^2}{\partial z^2} + V(z)\right\} \psi_n(z) = \kappa_n^2 \psi_n(z),$$

where

$$\kappa_n^2 = k_n^2 - \frac{1}{\lambda^2} - \rho^2 - \frac{1}{\lambda^2},$$

and the potential (29) is now reduced to the sum of two contributions, one coming from the spontaneous staggered magnetization and, the other from the presence of the weak external field. Explicitly,

$$V(z) = V_0(z) + \left(\frac{\rho}{\lambda}\right)^2 V_1(z),$$

with

$$V_0(z) = -2\text{sech}^2\left(\frac{z}{\lambda}\right)$$

and

$$V_1(z) = -8 \tanh^2\left(\frac{z}{\lambda}\right) \text{sech}^2\left(\frac{z}{\lambda}\right).$$
The calculation of the even and odd phase shifts for a potential of the form (34)-(36) is reported in Ref. 10 and here we will only show the fundamental results of the numerical solution of the Schrödinger-like equation (32). Fig. 1 and Fig. 2 show the even and odd parity phase shifts for different values of the external field.

As it can be seen in Fig. 1 the even phase shift is $\pi/2$ at the origin. This behavior is in complete agreement with the existence of an even bound state corresponding to the Goldstone mode. On the other hand, the odd phase shift $\delta_o(0) = \pi$, indicates the presence of an odd bound state. This result was previously obtained by Kivshar et al.\[18\] in the study of the small-amplitude modes around the localized solution of the 2SG equation and shows that there is always an odd bound state in this kind of system. Therefore, the spectrum of (34) is composed by: i) the $\psi_0$ solution (31) corresponding to the translation mode of the soliton (Goldstone mode) ii) an internal mode which appears when the system is perturbed by the external magnetic field and iii) the $\psi_k$ solutions which constitute the continuum modes and correspond to magnons.

In order to find the damping coefficient we must compute the reflection coefficient $R(k)$. This can be done by inserting the numerical results of the even and odd phase shifts into the general expression (28). In Fig. 3, we have plotted $R(k)$ for different values of the perturbation parameter $\rho$ for the whole range of $k$. As it can be seen the major contribution for the reflection coefficient comes from the low energy states, in agreement with the well behaved potentials (35) and (36).

Having done that, one can immediately integrate the function $R(k)$ in expression (27) which finally allows us to describe the damping coefficient as a function of the temperature (see Fig. 4). It is important to notice that we have not considered the odd bound state of the potential (34) in computing the damping coefficient because in evaluating the scattering matrix (25), only elastic terms are taken into account (see for instance Ref. 9, Ref. 12 or Ref. 13).

As it can be seen, the damping coefficient is linear for high temperatures. This result can be obtained directly.
from (27). In fact, for $T$ high enough the damping constant can be approximated by

$$\tilde{\gamma}(T) \simeq \frac{1}{2\pi M_s \beta} \int_0^\infty dE \frac{R(E)}{E} \propto T$$

(38)

which is linear in $T$, independently of the explicit form of $R(E)$. In the low temperature regime we can write

$$\tilde{\gamma}(T) \simeq \frac{1}{2\pi M_s} \int_0^\infty dE \frac{\bar{R}(E) \beta E e^{-\beta E}}{E}$$

(39)

where $E$ always presents a gap determined by the presence of the magnetic field and/or the spontaneous staggered magnetization. Here we shall not attempt to write an approximate expression for (39) because the correct behavior of the reflection coefficient was only numerically determined. As it is shown in Fig. 4, for low enough temperatures, the damping coefficient drops exponentially to zero due to the existence of the gap. As the temperature increases the damping coefficient rises following a power law behavior until it becomes linear for high enough temperatures. This strong temperature dependence of the damping parameter, for $T$ below the transition temperature, will influence directly the correlation function between the magnetic solitons.

B. Soliton mobility for $T > T_N$

To perform the calculation of the $\pi$-soliton mobility for $T > T_N$ we simply set to zero the $b_\perp$ in the hamiltonian (11) and therefore, the equation of motion for the $\Phi$-dependent part of the spin degree of freedom (8) becomes a SG equation with solitonic solution (13). In this case the potential involved in the Schrödinger-like equation (17) which determines the fluctuations around the soliton solution have the form

$$U''(z) = \xi^2(1 - 2\text{sech}^2\xi z),$$

(40)

where $\xi = b/2$. The spectrum of (40) contains a bound state with zero energy

$$\psi_0 = \sqrt{\frac{\hbar}{2}}\text{sech}(\xi z), \quad k_0^2 = 0,$$

(41)

which constitutes the translation mode of the soliton (Goldstone mode), and a continuum of quasiparticles modes (magnons) given by

$$\psi_n(x) = \frac{1}{\sqrt{L}} \left[ \frac{k_n + i\xi \tanh(\xi z)}{k_n + i\xi} \right] e^{ik_n z},$$

(42)

where

$$k_n = \frac{2n\pi}{L} - \frac{\delta(k_n)}{L}, \quad \delta(k) = \arctan \left( \frac{2\xi k}{k^2 - \xi^2} \right).$$

(43)

As it was already mentioned, the reflection coefficient $R$ for a general symmetric potential can be expressed in terms of the corresponding even and odd phase shifts by the relation (23). Re-expressing (12) in terms of parity eigenstates it is easy to prove that the potential (10) belongs to the class of reflectionless potentials because its phase shifts are given by

$$\delta^{e,o}(k) = \arctan(\xi/k),$$

(44)

that do not distinguish between odd and even parities. Therefore no matter how high the temperature rises above $T_N$ the damping coefficient is always zero and as a direct consequence the ballistic regime for the soliton results.

As we have seen, the solitonic solutions in TMMC have different regimes for $T$ below and above $T_N$. Below the transition temperature the $2\pi$-solitons behave like a Brownian particle with a finite damping parameter. On the other hand, for $T$ above $T_N$ the $\pi$-solitons have infinite mobility corresponding to the ballistic regime. The next section is devoted to studying the influence of the changes of the solitonic solutions mobility in the dynamical properties of TMMC.

IV. DYNAMICAL STRUCTURE FACTOR

In this section we will investigate the dependence of the dynamical properties of TMMC with respect to the temperature and the magnetic field. For $T$ below the transition temperature, this will be done through the computation of the dynamical structure factor of a dilute gas of $2\pi$-solitons in a dissipative regime. With this result, we can analyze the main differences with the assumption of ballistic regime using by Holyst in the same situation.
In a general form, the longitudinal and transverse dynamical structure factors with respect to the external field $B$ can be defined as

$$S^{(\perp)} = \frac{1}{(2\pi)^{3/2}} \int dtdz e^{i(qz-\omega t)} \langle S^{x}(0,0) S^{y}(z,t) \rangle,$$

where $S^{x(y)}$ corresponds to the spin component in the $x(y)$ direction.

To begin with, let us recall the main results for the longitudinal dynamical structure factor reported in Ref. 3. Using the model of non-interacting $2\pi$-soliton gas in the ballistic regime $S^{(\parallel)}(q,\omega)$ can be written approximately as

$$S^{(\parallel)}(q,\omega) = n_{2\pi}S^2 |F_{2\pi}(q)|^{2} \frac{p(\omega/q)}{2\pi q},$$

where

$$p(\omega/q) = \sqrt{\frac{\beta E_{2\pi}}{2c^2q^2}} \exp(-\frac{\beta E_{2\pi}\omega^2}{2c^2q^2}),$$

$$E_{2\pi} = 2B\mu_{B}S \left[ \sqrt{1 + 4\beta N} \right]$$

$$+ 4\beta \frac{q}{b} \sinh^{-1} \left( b^{\perp} \frac{1/2}{2} \right)$$

and

$$F_{2\pi}(q) = \frac{i\pi d_{\pi}}{2} \frac{\sin(qd_{\pi}\sigma)}{\cosh(qd_{\pi}\sqrt{1-\alpha/8})},$$

$$\sigma = \sqrt{1-\alpha} \ln \left( \frac{2}{\alpha} - 1 + \frac{2}{\alpha} \sqrt{1-\alpha} \right), \quad d_{\pi} = \frac{8}{b}.$$ (50)

The correlations described by (44) are induced by single kinks moving from the origin to the position $z$ in a time interval $t$. As expected, the Maxellian velocity distribution used to describe the $2\pi$-kink gas is directly reflected in the Gaussian dependence of the longitudinal structure factor with the frequency.

On the other hand, to get a better idea of the changes in the dynamical properties when we cross the transition temperature, it is convenient to calculate the dynamical structure factor for $T$ above the transition temperature. As it was demonstrated before, above $T_{N}$, the $\pi$-solitons moves without dissipation and, therefore, the ballistic regime is valid. Using again the model of a dilute gas of solitons, the dynamical structure factor can be written as

$$S^{(\parallel)}(q,\omega) = \frac{S^2}{(2\pi)^{3/2}} \sqrt{\frac{E_{\pi}\beta}{c^2q^4}} |F^{(\parallel)}(q)|^{2} \exp\left( -\frac{E_{\pi}\beta\omega^2}{2c^2q^2} \right),$$

where

$$F^{(\parallel)}(q) = \frac{2\pi}{b} \text{sech}(q\pi/b)$$

and

$$E_{\pi} = B\mu_{B}S.$$ (52)

As it can be seen, the dependence with the frequency remains almost unchanged no matter what the temperature is. At the same time, it should be noticed that once the intensity of the central peak and the density of kinks in (47) and (48) are proportional, the intensity of the central peak for $T < T_{N}$ will be lower. This is a consequence of the smaller number of solitons for temperatures below the transition temperature.

With the previous results for $T$ above and below $T_{N}$ in mind, we can go further on and study the influence of the dissipative regime in the dynamical properties of the $2\pi$-kink gas. As it was shown before, below the transition temperature the $2\pi$-solitons move in a dissipative regime. Therefore, the position of the center of mass for those kind of excitations as a function of time can be written as

$$z(t) = z_{0} + \frac{v_{0}}{\gamma(T)} (1 - \exp(-\gamma(T)t)),$$ (53)

where $z_{0}$ is the initial position, $v_{0}$ is the initial velocity and $\gamma(T)$ is the temperature-dependent damping parameter. Now, to calculate the dynamical structure factor we will use the $2\pi$-soliton solutions (9)-(11) with $z(t)$ given by (44). Following the same procedure that led to equation (46) and, after some calculations, $S^{(\parallel)}(q,\omega)$ can be written as

$$S^{(\parallel)}(q,\omega) = \frac{2n_{2\pi}S^2}{\pi} |F_{2\pi}(q)|^{2} \Gamma(q,\omega),$$ (54)

where

$$\Gamma = \sum_{n=0}^{\infty} \frac{(-1)^{n}}{n!} \left( \frac{\alpha^2}{2\beta E_{2\pi}\gamma^2} \right)^{n} \sum_{m=0}^{2n} (-1)^{m} C^{2n}_{m} \frac{m\gamma}{m!} \frac{2m\gamma}{(2n-m)!},$$ (55)

and

$$C^{2n}_{m} = \frac{(2n)!}{m!(2n-m)!}.$$ (56)

As it can be seen, the dissipative regime for the magnetic solitons in the case in which $T < T_{N}$, changes considerably the behavior of the longitudinal dynamical structure factor. Although the expression (44) is valid for all finite values of $q$, we couldn’t perform the entire sum to get a closed expression. Therefore, it is helpful to study the behavior of (44)-(48) for small momentum in order to compare it with the ballistic behavior result (44). Assuming that

$$q \ll \frac{\gamma}{c} \sqrt{2\beta E_{2\pi}},$$ (57)

the dynamical structure factor $S^{(\parallel)}(q,\omega)$ can be written as

$$S^{(\parallel)}(q,\omega) = \frac{2\alpha}{\pi} |F_{2\pi}(q)|^{2} \Lambda(q,\omega)$$ (58)
where

\[ A(q, \omega) = 2\pi \delta(\omega) \exp - \frac{q^2 c^2}{\beta E_2 \gamma^2} + \]
\[ \frac{q^2 c^2}{\beta E_2 \gamma^2} \left[ \frac{\gamma}{\gamma^2 + \omega^2} - \frac{\gamma}{4\gamma^2 + \omega^2} \right]. \]

(59)

Within the approximation of small momentum, the behavior of \( S(l)(q, \omega) \) with the frequency, changes from the ‘Gaussian’ central peak to a ‘Lorentzian’ dependence. Therefore, as the temperature is lowered below \( T_N \), the central peak behavior is replaced by a smoother-one in the frequency domain. This result is a direct consequence of the dissipative regime of the 2\( \pi \)-soliton and, as the damping constant \( \gamma \) can be controlled by changing the temperature and the magnetic field, a possible indication of a non-ballistic regime has been found. Another quantity that can be computed in order to get a better idea of the influence of the dissipative motion of magnetic solitons in TMMC is the \( T_1 \) time of NMR. This problem is currently being investigated by one of us.

V. CONCLUSIONS

In this paper we have analyzed the possibility of identifying two different regimes of motion for the magnetic solitons in the TMMC antiferromagnet. We were able to show that above the transition temperature \( T_N \), the \( \pi \)-soliton moves without dissipation, even from the field theoretical point of view. This result is in complete agreement with the ballistic regime adopted to understand the experimental data reported in Ref. 11.

On the other hand, for \( T \) below the transition temperature the 2\( \pi \)-solitons in the system move with finite mobility. Therefore, a dissipative equation of motion has to be used in the description of the soliton’s center of mass motion. This difference in the regime of motion is directly reflected in the longitudinal structure factor and, therefore, can be used as an indication of the finite mobility of the solitonic solutions below the transition temperature. The results presented here could be directly compared to the experimental data one may obtain when testing the TMMC antiferromagnet in this temperature regime.

Although the formulation used to compute the damping parameter is valid for all values of the external magnetic field, we have restricted ourselves to the study of very weak fields. However, our formulation can be used to study situations with arbitrarily stronger magnetic fields and to other magnetic materials that support solitonic solutions without any major qualitative difference. For instance, we could treat systems modelled by the 1-D Dzyaloshinski-Moriya antiferromagnet which can naturally be described by a 2DSG Hamiltonian independently of the temperature.

VI. ACKNOWLEDGMENT

AVF wishes to thank Fundação de Amparo à Pesquisa do Estado de São Paulo (FAPESP) for financial support, whereas AOC kindly acknowledges partial support from Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq).