Misleading signposts along the de Broglie-Bohm road to quantum mechanics

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Abstract
Eighty years after de Broglie’s, and a little more than half a century after Bohm’s seminal papers, the de Broglie–Bohm theory (a.k.a. Bohmian mechanics), which is presumably the simplest theory which explains the orthodox quantum mechanics formalism, has reached an exemplary state of conceptual clarity and mathematical integrity. No other theory of quantum mechanics comes even close. Yet anyone curious enough to walk this road to quantum mechanics is soon being confused by many misleading signposts that have been put up, and not just by its detractors, but unfortunately enough also by some of its proponents. This paper outlines a road map to help navigate one’s way.

1 De Broglie-Bohm theory in a nutshell

Already in its basic version without spin and magnetic interactions the theory yields a quite faithful Galilei covariant realist caricature of our world. Thus, there is a three-dimensional space, represented by \( \mathbb{R}^3 \), and there is time, represented by \( \mathbb{R} \); at any instant \( t \) in time this space is populated by a fixed number \( N \gg 1 \) of point particles at locations \( Q_k(t) \in \mathbb{R}^3 \), \( k = 1, \ldots, N \); as time goes on, these particles move according to the law of motion

\[
\frac{d}{dt} Q_k(t) = v^\psi_k(Q(t), t),
\]

where \( v^\psi(\cdot, t) : \mathbb{R}^{3N} \to \mathbb{R}^{3N} \) is a velocity field at time \( t \) on the space of generic configurations \( q \in \mathbb{R}^{3N} \), the \( k \)-th \( \mathbb{R}^3 \) component of which reads

\[
v^\psi_k(q, t) = \text{Re}\left((\psi^\dagger \psi)^{-1}( - i \frac{\hbar}{m_k} \nabla_k)\psi\right)(q, t);
\]

the complex-valued field \( \psi(\cdot, t) : \mathbb{R}^{3N} \to \mathbb{C} \) is Schrödinger’s wave function at time \( t \) for the \( N \)-particle system (a.k.a. its de Broglie wave), satisfying Schrödinger’s wave equation

\[
\ih \partial_t \psi(q, t) = H \psi(q, t).
\]

A quite faithful model of our world obtains when as Hamiltonian we take

\[
H = \sum_{1 \leq k \leq N} \frac{1}{2 m_k} (-\ih \nabla_k)^2 + \sum_{1 \leq k < \ell \leq N} \frac{e_k e_\ell - G m_k m_\ell}{|q_k - q_\ell|},
\]

with \( e_k = -e \) and \( m_k = m \) for \( k = 1, \ldots, N_e \), while \( e_k = Z_k e \) with \( Z_k \) a natural number and \( m_k \gg m \) for \( k = N_e + 1, \ldots, N \), and we also demand \( \sum_{k=1}^N e_k = 0 \); otherwise the symbols have their standard meaning.

* Version of Sept.19, 2007. Dedicated to Jeffrey Bub on occasion of his 65th birthday.
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The dynamical equations (1.1), (1.3) pose an initial value problem and as such need to be supplemented by initial data, say at $t = 0$, viz. by $\psi(\cdot, 0)$ and $Q(0)$. For $\psi(\cdot, 0)$ we demand it be antisymmetric w.r.t. permutations of particle indices in $\{1, ..., N_e\}$ (the electrons), while for other groups of particle indices (the various positive nucleus species) one imposes symmetry or antisymmetry w.r.t. permutations (we don’t need to bother to be more specific here). Also, the initial $\psi$ has to be sufficiently regular (formally a $C^\infty$ vector for $H$). For $Q(0)$ we demand that it be typical with respect to $|\psi(\cdot, 0)|^2 d^3N q$.

The model can easily be generalized to account for the effects of so-called “external” electromagnetic fields, as well as for electron spin, through the replacements $-i\hbar \nabla_k \rightarrow \sigma_k \cdot (-i\hbar \nabla_k - e_k A(q_k, t)/c)$ and $i\hbar \partial_t \rightarrow i\hbar \partial_t - e_k \phi(q_k, t)$, and with $\psi(\cdot, t) \in \mathbb{C}^{R^3N \times \{-1, 1\}^{N_e}}$ now an $N_e$-particle Pauli spinor w.r.t. the electron indices, such that $\psi^\dagger \psi$ now is the inner product over spinor space degrees of freedom, and $\sigma_k$ the 3-vector of Pauli matrices acting on the $k$-th electron index (similar modifications can be arranged for the nuclei).

As John Bell [1] put it, to the extent that (non-relativistic) orthodox quantum mechanics makes unambiguous predictions, the de Broglie-Bohm model makes the same predictions. What’s more, it provides a theory of our world: a three-dimensional world populated by point objects (particles) which move according to a (non-Newtonian) law of motion, their motion being such as to produce the overall material structures and processes we happen to recognize in the world we live in. Paraphrasing Dirac, the de Broglie-Bohm model accounts for all of chemistry and most of physics, from the existence of atoms and their main energy spectra (not the emission and absorption of (the relativistic) photons, though) to the existence and motion of things like planets and their moons — though Dirac of course said something like this about orthodox nonrelativistic quantum mechanics. Exactly how this theory accounts for all that, and without any magic, is now a fascinating matter of logical deduction, of which we will give no details here. You can read more about it in the exposition by Sheldon Goldstein in this volume [2], and in [3].

2 It’s the ontology, stupid!†

The purpose of the title of this section is to focus the attention on what is the truly important distinction of the de Broglie-Bohm theory, namely that this quantum theory is about something objectively real, in this case point particles which move according to a new law of motion, (1.1) with r.h.s. given by (1.2), which complements Schrödinger’s wave equation (1.3). It is this aspect, and this aspect alone, which restores intelligibility to the whole affair of nonrelativistic quantum physics, of which Feynman [4] famously said: “I think I can safely say that nobody today understands [orthodox] quantum mechanics.” While all of orthodox quantum physics suffers from a ‘reality-deficiency-disorder’ known as “the measurement problem,” whose symptoms are ameliorated by magical spells (‘whenever one performs a measurement, then (a miracle occurs and) reality becomes manifest, and only then’), the de Broglie-Bohm theory by contrast is a healthy realist theory of a

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† The title is obviously inspired by a famous Clinton campaign slogan from the nineties and should not be misconstrued as a personal attack by me on anyone.
nonrelativistic objective universe. This asset of the de Broglie-Bohm theory cannot be treasured too much.

Curiously enough, though, the theory is generally not well appreciated. The following quote, taken from the Encyclopedia Britannica (quoted in [2]), is typical in this regard:

“Attempts have been made by Broglie, David Bohm, and others to construct theories based on hidden variables, but the theories are very complicated and contrived. For example, the electron would definitely have to go through only one slit in the two-slit experiment. To explain that interference occurs only when the other slit is open, it is necessary to postulate a special force on the electron which exists only when that slit is open. Such artificial additions make hidden variable theories unattractive, and there is little support for them among physicists.”

This quotation from the Encyclopedia Britannica is quite illuminating in (at least) two ways.

First, the author of these Encyclopedia Britannica lines, and the editor who approved them, apparently recognize that in the de Broglie-Bohm theory an electron actually does have a position (and, therefore, definitely does “go through only one slit in the two-slit experiment”), but it is also quite apparent that it is either not recognized that this endows the theory with an ontology, something that is missing from the orthodox theory, or if it is recognized, then it is obviously not treasured at all! I can only hope that what’s expressed in the title of this section will eventually sink in.

Second, that author, and the editor, must have gotten their ‘insights’ about the de Broglie-Bohm theory from some really “very complicated and contrived” (looking) exposition, for he or she could not possibly be writing such things about equations (1.1)–(1.4). It sounds like there is a lesson to be learned here!

3 Newton’s ghost

Let’s be clear about this: the Encyclopedia Britannica entry about hidden variable theories totally misrepresents the de Broglie-Bohm theory — except for the claim that an electron has to “go through only one slit in the two-slit experiment,” and that “there is little support for them [hidden variable theories] among physicists.” It misrepresents the theory on three counts:

- The model (1.1)–(1.4) is *manifestly not* “very complicated,” certainly not by absolute standards, and especially not when compared with the orthodox formalism of nonrelativistic quantum mechanics, which shares with it the Schrödinger equation (1.3) with $H$ given by (1.4), but which lacks (1.1) and instead supplies a literally open-ended list of so-called measurement postulates — by that measure the de Broglie-Bohm model is even infinitely more simple.

- Neither is the model (1.1)–(1.4) “contrived,” for not only is the Schrödinger equation (1.3) with $H$ given by (1.4) the same as in the orthodox theory, but also the velocity field (1.2) is simply what in the orthodox theory is called the probability current density vector

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2 The Encyclopedia Britannica section on quantum physics is presided over by a Nobel laureate in physics as special editor, and its entries are written by professional physicists of some standing (just so you know I am not unfairly criticizing the production of some poorly trained and overtaxed professional staff writer).
\[ j(q,t) \] divided by the probability density \( \rho(q,t) = |\psi(q,t)|^2 \), so \( v\psi = j/\rho \). Given these established mathematical ingredients of the orthodox theory, if one now tries to make sense of the orthodox talk about particles, then it isn’t exactly “contrived” to contemplate that particles actually do have a position, so that \( N \) of them correspond to an actual point \( Q(t) \) in configuration space — and isn’t that, anyway, already suggested by the simple fact that Schrödinger’s wave function is a function on \( N \)-point configuration space? Once one has reached this insight, the next logical step is to look for a natural law of motion for these point particles, and having a velocity field on configuration space essentially ‘for free’ already, how “contrived” is it to contemplate that this velocity field evaluated at the actual point \( Q(t) \) actually is the velocity of \( Q(t) \)? Well, that’s exactly what (1.1) states.

- Neither does one have to “postulate a special force on the electron which exists only when that [other] slit is open [too]” in order “to explain that interference occurs.” All that is postulated beyond Schrödinger’s equation (1.3), is that the electron is guided by the velocity field (1.2) obtained from the solution \( \psi(q,t) \) of Schrödinger’s equation (1.3). Moreover, \( \psi \) develops an interference pattern only when both slits are open, a fact that is inevitably reflected in the manner how \( \psi \) guides the electron.

Needless to say, by misrepresenting the de Broglie-Bohm theory one also misleads uninformed readers: if I hadn’t known anything about that theory and read about it for the first time in the authoritative Encyclopedia Britannica, I would surely have gotten the impression that the theory is contrived and — presumably — very complicated. Incidentally, something like that actually did happen to me, though it was my professor in the course “Philosophical problems of physics” which I attended as a student, who told me basically the same things you read in the Encyclopedia Britannica article, and it did turn me against this theory. That was before I took a closer look myself. Bell’s book [1] was a revelation, and the writings by Dürr, Goldstein, Zanghì and their collaborators clarified matters for me considerably.

But who is responsible for such misleading misrepresentations of the theory? Of course, the ultimate responsibility always lies with their author, but I think it is fair to say that blame also should go to some of the main actors and their supporting cast for creating (unintentionally, I suppose) certain misperceptions of the theory. Thus, the original source of the particular misperception that the theory is postulating “special forces,” a Newtonian concept, hence a misguided attempt to “return to the womb of classical physics,” alas, is [5]. Indeed, while Bohm [5] does write down the theory which I’ve presented in section 1 — he does so in a small paragraph on p.6 of Part I, and again in the first paragraph of Part II, of his two-part article — much more prominence, by far, is given to a derived concept,
which is used as a point of departure for a speculative generalization of the theory. Namely, by taking the time derivative of (1.1), then using (1.2), (1.3), (1.4),5 a Newtonian-type second order equation of motion is obtained which exhibits, besides the familiar Coulomb and Newton forces, the gradient of a term that depends on $|\psi|$ and its second spatial derivatives. This term, which was already well-known (see, e.g., [6]) through rewritings of Schrödinger’s equation (1.3) into a system of equations, using the polar decomposition of $\psi = |\psi|e^{i\phi}$ (see next section), Bohm called the “quantum potential.” Of course, in order that everything else remains equal (i.e. the predictions derived from it remain unaltered), the first order equation (1.1) evaluated at the initial time still has to be imposed as an initial condition, and then it propagates for all times. So, with (1.1) understood as valid initially, Bohm’s reformulation of (1.1)–(1.4) into a Newtonian equation of motion coupled to Schrödinger’s equation is strictly equivalent to (1.1)–(1.4). But as such it is unnecessarily more complicated, for $\psi$ cannot be eliminated by taking the extra time derivative of (1.1).6 In any event, there is no doubt in my mind that Bohm’s pseudo-Newtonian reformulation has given more casual readers the impression that he tries to derive quantum mechanics from Newtonian point mechanics by postulating new special forces derived from a new potential, the quantum potential. Here is another victim (who gets it even more upside down, but who in his book actually seems quite sympathetic to Bohm) [7]: “Bohm was obliged to invent an agency – a guiding wave – to manipulate the particles. He called this guiding wave the “quantum potential”.”

Bohm’s apparent motivation for presenting matters his way, as I understand [5], is his hope that by dropping ‘(1.1)-evaluated-at-the-initial-time’ as initial condition the so-obtained freedom to choose initial conditions also for the velocities will yield new physics in form of a generalization of quantum mechanics. This is certainly a legitimate hope to entertain, but what a considerable lapse of judgement it was, indeed, to pursue such a speculative idea which says that not everything else is equal in a two-part paper whose title declares that its main purpose is to suggest an “interpretation of the quantum theory in terms of “hidden” variables”7 (emphasis mine) — and which actually achieves the declared goal, for the first time ever! I wish Bohm had written his two-part paper [5] entirely in terms of equations (1.1)–(1.4) and relegated all the speculation about a possible generalization of quantum mechanics based on the second order Newtonian equation with its “quantum potential” to a separate, third paper entitled “A speculative generalization of quantum theory inspired by my [Bohm’s] previous two papers.” Of course, the reality is what it is, but next time you contemplate emphasizing “the importance of the quantum

5 To be more accurate, Bohm does not explicitly give the interaction term in (1.4) but just uses the more general $V(q_1,q_2,...)$ (in our notation), but this is irrelevant to our argument.

6 By comparison, Newtonian point mechanics in the Hamilton-Jacobi formalism operates with the same guiding equation (1.1) (when written in polar decomposition), but with the Hamilton-Jacobi partial differential equation for $\Phi_{\text{HJ}}$ in place of Schrödinger’s equation (1.3); while $\psi$ cannot be eliminated from the equations of motion for $Q$ by differentiation of (1.1), the Hamilton-Jacobi phase field $\Phi_{\text{HJ}}$ on configuration space can.

7 Incidentally, “interpretation of the quantum theory” is also not a lucky choice of words, for Bohm does not just interpret the theory in a new way, he presents a deeper theory which explains (nonrelativistic) quantum theory, namely the very theory de Broglie was pursuing in the 1920s before joining the Copenhagen bandwagon as a fallout from the 5th Solvay conference in 1927.
potential in order to understand quantum mechanics,’ think again!

I am not sure how much the original main actor, de Broglie, who in [8] states the de Broglie-Bohm N-particle model for the first time, has contributed to the misconception that the model is Newtonian through his later comments on Bohm’s papers. Yet he certainly contributed his own share to the general confusion about the model by not working things out properly, and by abandoning the pursuit of his model right after the 5th Solvay conference,\(^8\) resuming it only after Bohm’s 1952 papers, and then not in a consequential way but instead by chasing after his own favorite speculative ideas about the model’s origins in his “theory of the double solution” [9]. But that’s another story about which I say a few things in the last section.

4 Polar disorder

Both de Broglie [8] and Bohm [5] chose a polar representation of the Schrödinger wave function, viz. \( \psi = |\psi| e^{i\Phi} \) (though de Broglie frequently writes only the real part of it; why, I don’t know), through which the guiding equation (1.1) takes the particularly simple form

\[
\frac{d}{dt} Q_k(t) = \frac{\hbar}{m_k} \nabla_k \Phi(q,t) \bigg|_{q=Q(t)} \tag{4.1}
\]

while Schrödinger’s equation (1.3) with (1.4) becomes a coupled system for \( |\psi| \) and \( \Phi \): the continuity equation

\[
\partial_t |\psi|^2(q,t) = - \sum_{1 \leq k \leq N} \nabla_k \cdot \left( |\psi|^2(q,t) \frac{\hbar}{m_k} \nabla_k \Phi(q,t) \right) \tag{4.2}
\]

and a generalization of the classical Hamilton-Jacobi equation, viz.

\[
\hbar \partial_t \Phi(q,t) = - \sum_{1 \leq k \leq N} \frac{\hbar^2}{2m_k} \left( |\nabla_k \Phi(q,t)|^2 - \frac{\Delta_k |\psi|^2(q,t)}{|\psi|^2(q,t)} \right) - \sum_{1 \leq k < l \leq N} \frac{e_k e_l - Gm_k m_l}{|q_k - q_l|}. \tag{4.3}
\]

Without the \( \Delta |\psi|/|\psi| \) term (4.3) would be the Hamilton-Jacobi equation for \( \Phi_{HJ} \); the \( \Delta |\psi|/|\psi| \) term is what Bohm called the “quantum potential.” Save some technical issues regarding the question whether globally differentiable functions \( |\psi| \) and \( \Phi \) can be found for a differentiable \( \psi \), at least locally Schrödinger’s equation (1.3) with (1.4) is entirely equivalent to (4.2) and (4.3). It should be noted that the gain in simplicity of the guiding equation is entirely offset by the gain in complexity and nonlinearity when rewriting (1.3) with (1.4) into (4.2) and (4.3). Rewriting (1.1)–(1.4) into the polar form (4.1)–(4.3) is very helpful when studying the classical limit of the de Broglie-Bohm model, but otherwise it makes things unnecessarily complicated. And, of course, if one wishes to compare the de Broglie-Bohm theory with orthodox quantum mechanics, the fact that the orthodox equation (1.3), with (1.4), is perfectly fine and doesn’t need to be transformed in any way, is quite important.

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\(^8\) In his acceptance speech for the Nobel prize in physics 2 years later, de Broglie not only omits mentioning his original ideas that particles are guided by a pilot wave, he actually presents the Copenhagen doctrine that particles do not have a position at all times.
Apropos the classical limit: in [10] Messiah uses the polar representation of Schrödinger’s equation and notes that whenever the \( \Delta |\psi|/|\psi| \) term is negligible in (4.3) then the classical Hamilton-Jacobi equation results, decoupled from the continuity equation. Strangely enough, Messiah then claims that thereby one has obtained classical Newtonian point mechanics — but this is of course a non sequitur without the guiding equation! The classical Hamilton-Jacobi PDE without the guiding equation (4.1) is not equivalent to Newtonian point mechanics, because this PDE, alone or with the continuity equation, does not supply the ontology of Newton’s theory: point particles whose velocities are determined by the configurational velocity field components \( m^{-1}_k \nabla_k \Phi \) evaluated at the actual configuration.

In the same vein, if in a paper on quantum mechanics the polar representation of the Schrödinger equation is used, either in disguise or (semi-)explicitly, and the ‘configurational velocity field’ \( v = j/\rho \) written as \( v_k = m^{-1}_k \nabla_k \Phi \), yet that paper does not contain the particle ontology together with the guiding equation (4.1) in some form (so that the ‘configurational velocity field’ \( v \) is not used to define the motion of an actual configuration point), then it is a non sequitur to assert that such a paper contains the essential equations of Bohm’s papers [5]. Such a non sequitur can actually be found in this volume in the paper by Hiley [11]. Hiley discusses the phase space representation of the Schrödinger equation, which goes back to the works of Wigner [12] and Moyal [13], though they pursued different goals. In this approach one works with the Wigner transform of \( \psi \),

\[
F(p, q, t) = \int \frac{1}{\hbar^{3N}} \psi^*(q - \frac{1}{2}q', t)\psi(q + \frac{1}{2}q', t)e^{ip\cdot q'/\hbar}d^{3N}q'.
\]

Since the time evolution for \( F(p, q, t) \) is equivalent to Schrödinger’s equation for \( \psi(q, t) \), it should come as no surprise that if one manipulates it long enough, as did Moyal and as does Hiley, then the polar (4.2), (4.3) show up again. Moreover, Moyal’s paper also contains the ‘configurational velocity field’ \( v = m^{-1} \nabla \Phi \), and he wrote his paper truly in the spirit of searching for a law of motion for actual particles, though his hope was that the quasi-Markovian equation on phase space for the Wigner-Moyal function (4.4) would reveal a stochastic process on phase space that describes the motion of the particles. In this sense Moyal had his heart in the right place. His hopes were dashed by the fact, unbeknownst to him at the time it seems, that the Wigner-Moyal function (4.4) is not always positive and as such cannot be interpreted as an ensemble probability density function, and the quasi-Markovian equation for it not as a Kolmogorov equation. However, Moyal did not have the guiding equation (4.1) in his paper! Having a particle ontology in mind\(^9\) doesn’t

\(^9\) Paraphrasing John Stewart Bell [1]: ‘It has to be in the mathematics, not just in the talk!’ Indeed, if having a particle ontology in mind were enough, then we would have to call it the ‘de Broglie-Bohm-Born-Moyal-Wigner-...’ model, instead of de Broglie-Bohm, for also Born in [14] and Wigner in [15] talk very explicitly about \( \psi \) being a “Führungsfeld” which is guiding the particles. All the same, both Born and Wigner are critical of attempts to supply an actual guiding equation, though for different reasons: Born gives logical-positivistic philosophical reasons, while Wigner argues conceptually: since \( |\psi|^2 \), which is an ensemble probability density (viz. standing for our ignorance), enters the “quantum” Hamilton-Jacobi equation (4.3) for \( \Phi \), it follows that \( \nabla \Phi \) cannot possibly guide an individual member of the ensemble. But of course \( |\psi|^2 \) is not fundamentally a probability density. Now that sounds paradoxical, for Born’s law says that \( |\psi|^2 \) is a probability density. How the paradox is resolved by the de Broglie-Bohm theory you can read in [2] and especially [3].
mean it’s in the theory. Wigner in [12], on the other hand, was not at all looking for any ontology but instead was concerned with purely practical matters of computing quantum corrections to the classical Boltzmann-Gibbs statistical ensemble predictions.

While it is thus misleading to claim the basic equations of the de Broglie-Bohm model were contained in a paper featuring (4.2), (4.3) and \( v_k = m_k^{-1} \nabla_k \Phi \) without an ontology, even if you see (4.2) and (4.3) together with \( v_k = m_k^{-1} \nabla_k \Phi \) in a paper with an ontology, that doesn’t mean you have the de Broglie-Bohm model in front of you. As noted earlier, the rewriting of (1.3) with (1.4) into polar form (4.2) and (4.3) was there almost from the beginning. Beside de Broglie, also Madelung [6] made use of it. Madelung also had an ontology! However, Madelung pointed out that the polar representation gives a system of PDEs which, when \( N = 1 \), can be given a fluid-dynamical interpretation. This is very much in the spirit of Schrödinger’s concept of \( \psi \) as matter waves. There are no fundamental particles in either Madelung’s or Schrödinger’s interpretation. But Madelung’s, like Schrödinger’s, ideas of a physical material continuum ontology work only on three-dimensional space (conceptually, that is; they do not produce the correct physics), while they are absurd for an equation on \( N \)-particle configuration space. Oddly enough, while Madelung noted that no such fluid ontology interpretation seems feasible when \( N > 1 \), his model enjoys a large following these days in the semi-conductor modeling community.

5 Quo vadis?

Richtiges Auffassen einer Sache und Mißverstehen der gleichen Sache schließen einander nicht vollständig aus. — Franz Kafka

In the previous four sections, I have first presented the de Broglie-Bohm model (a.k.a. Bohmian mechanics), which is found both in [8] and in [5] except for the notation and the different representation of Schrödinger’s equation (cf. my section 4). Next I’ve emphasized that the model is presumably the simplest and conceptually most natural realist completion of non-relativistic orthodox QM, which removes all the paradoxes of the orthodox theory, and that as such it is a milestone. I have then addressed some misconceptions of the theory, explained what’s wrong with them, and argued that these misconceptions were (and are) at least facilitated by confusing presentations of the theory, and not just by its detractors but, alas, by some of its very proponents. The lesson one learns from this is how immensely important it is to clearly focus on what the theory is about — its ontology, in this case point particles which move according to a simple and obvious (given Schrödinger’s equation) non-Newtonian law of motion — and to present the theory in a technically clean and simplest possible manner. Ignore this and you confuse others and, perhaps, yourself.

In this last section I want to ask: “Where does one go from here?” After all, the model (1.1)–(1.4) presented in section 1 is non-relativistic, so it can only be a stepping stone en route to a deeper, more general theory. De Broglie and Bohm were of course aware of this fact. Both strived, each one in his own way, to find such a deeper theory. Neither of them held monolithic views about it.

Bohm, who started out a ‘Copenhagenist’ before his interaction with Einstein eventually turned him against the Copenhagen doctrine, looked for a deeper theory already in his 1952 papers [5], where the de Broglie-Bohm theory of quantum mechanics is presented only as a special case of his unhappy speculative generalization of it, as I’ve explained in section 3; yet this work also contains a generalization of de Broglie-Bohm theory to include
photons, in an appendix. Shortly after he also knew how to extend the de Broglie-Bohm model to the Dirac equation [16]. After those initial steps toward a relativistic generalization of the de Broglie-Bohm model came a longer intermission during which Bohm explored other ontologies. For instance, together with his Brazilian collaborators he worked on the extension of the Madelung fluid ontology to include spin [17]; this paper gives you the impression that Bohm had already abandoned the point particle ontology around 1955. Then there is his joint work with de Broglie’s student Vigier, and his work with Bub [18]. But eventually Bohm picked up the de Broglie-Bohm model again in his collaboration with Hiley [19]. Bohm’s pet idea became the “implicate order,” something I haven’t digested.

De Broglie’s path was not straight either, but of course different in many ways. He began his career a realist, developing his pet idea of the “double solution theory” early on, dropping it, then picking it up again after a 25 year intermission [9]. In those intermediate 25 years he was a faithful follower of the Copenhagen doctrine; cf. footnote 8. De Broglie’s double solution theory itself evolved over time: early on it involved looking for singular solutions of a system of $N$ one-body Schrödinger (or Klein-Gordon) equations, suitably constrained, the singularities of which he hoped to show would move according to the guiding field obtained from the gradient of the phase of a singularity-free Schrödinger wave function which solves an associated Schrödinger $N$-body equation; later it involved also the Dirac equation and finally nonlinear Schrödinger-type equations [9]. As is well-known, the study of nonlinear Schrödinger (and Klein-Gordon, and Dirac) equations on physical spacetime has become quite popular in recent decades, though for different reasons.

De Broglie’s pursuit of the double solution is, in my opinion, another example of a serious lapse of judgement, though a quite different one from Bohm’s (cf. section 3). Namely, it is a great mystery to me why de Broglie — who was thinking about point electrons as singularities in a field on physical space — was thinking of singularities in the field solutions to some Schrödinger-type equation. After all, at least in classical electrodynamics point electrons are already given to us as point singularities — in the classical electromagnetic Maxwell fields; furthermore, the energy of an electrostatic field with $N$ of these point singularities (with the infinite self-energies subtracted) is precisely the Coulombic interaction term in the Hamiltonian (1.4) which enters Schrödinger’s equation (1.3).

Incidentally, I have reached a point where it is not inappropriate to say a few words about where I myself have been going. The so-called UV problems caused by such point singularities in the electromagnetic fields of Maxwell’s linear field theory are precisely the point of departure for my own pursuit of an intelligible realistic quantum field theory [20]. Nonlinear field equations are an important ingredient in the model I am working with, though not a nonlinear modification of some Schrödinger equation but the nonlinear Maxwell-Born-Infeld field equations of classical electromagnetic theory [21]. These nonlinear field equations remove the infinite self-energies of point electrons without any artificial cutoffs; thus, point electrons now appear as much milder point defects of the electromagnetic field. But the Maxwell-Born-Infeld field equations do not provide the law of motion of those point defects — it is here where Hamilton-Jacobi and de Broglie-Bohm formalisms appear to be ‘just what the doctor ordered.’ The works [20] are about the classical and quantum electromagnetic theory of point electrons without spin. Spin, handled by Dirac’s equation, and curved spacetimes have by now been included; this is being written up for
publication. Photons are still somewhat elusive, but I will have something to say about those also. This theory is as relativistic as possible, but there is a price to be paid.

Namely, I haven’t yet mentioned the most striking problem. De Broglie-Bohm theory is manifestly non-local, in a manner that ‘infinitely’ trumps the extent of non-locality in Newton’s point mechanics, where the electrical and gravitational forces between different constituents decay reciprocally with the square of their separation. Bell’s work [1] brought to the fore that the measurement formalism of orthodox quantum theory makes the very quantum theory non-local. So the real question, as Bell [1] has emphatically stressed, is how to replace the ontology-free non-local measurement formalism with an intelligible, realistic, physically objective non-local mathematical theory which obeys special, and eventually general relativity — the paradigms of locality!

One intriguing insight which the pursuit of a relativistic generalization of de Broglie-Bohm theory has produced so far is that it seems necessary to introduce a new element of reality, namely either a spacelike foliation of spacetime, or its dual concept, a timelike vector field on spacetime; see [22], [23]. This is also the case in [20], and in the model contemplated in [24]. The foliation would have to satisfy its own set of dynamical equations, which one can borrow from general relativity. Where all this will lead us nobody knows, but hopefully the end-product will teach us something new about nature.

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