Improving the economic and operational reliability of the designed structures by improving the acoustic methods for diagnosing invisible material microdefects

G A Arshinov
Kuban State Agrarian University, Russia, Kalinin str., 13, Krasnodar, 350044, Russia
E-mail: takhumova@rambler.ru

Annotation. The study aimed at increasing the economic and operational reliability of various designed building structures, improving non-destructive acoustic methods for detecting hidden microdefects in the materials used has been carried out. Construction, water, oil, gas pipelines must have the strength to ensure the trouble-free operation during the entire period of their operation, which determines the economic and operational reliability of structures. Strength can be significantly reduced if the latent microdefects are available in the material. In the field of such structural disturbances, material destruction, leading to loss of stability and bearing capacity of structural elements, accompanied by huge economic losses, environmental degradation and, in the worst case, human losses can develop. Therefore, it is necessary to improve the non-destructive acoustic methods for detecting latent microdefects. The mathematically refined models that describe the propagation of nonlinear deformation waves in thin-walled structural elements should be constructed for this. This is possible if the real physical and mechanical parameters, leading to more accurate characteristics of the deformation waves required for acoustic methods of the materials' defectoscopy are taken into account. Such models include nonlinear mathematical models describing the propagation of deformation waves in rods and cylindrical shells, when the real hereditary properties of the material, the possibility of developing large deformations in them, and the effects lost in the framework of models that consider only the linearly elastic properties of structural elements are taken into account. Such a simplification causes unacceptable errors in the calculation of wave parameters and, accordingly, gives far-from-truth results of an acoustic study of the presence of invisible microdefects in the material of thin-walled structural elements that weaken their strength, and the economic and operational reliability of the designed structures and pipelines depends on it.

1. Introduction
Thin-walled supporting elements - rods, plates, shells made of a material with viscoelastic and non-linear physical and mechanical properties are the basis of many building structures, and are also used in water, oil and gas pipelines.

The failure-free operation of such structures under the influence of the applied forces depends on the bearing elements’ strength and determines the economic efficiency of the structures. Therefore, increasing operational reliability, and therefore the economic efficiency of construction, water, oil and gas pipelines, is an urgent scientific problem.

One of the ways to solve it is to improve the non-destructive, and therefore cost-effective, acoustic methods for diagnosing microdefects. The basis for this can be the development of mathematical models for the propagation of nonlinear waves in rods and cylindrical shells, provided that the physical and mechanical properties of the material are more fully taken into account and the strict methods of mechanics of deformable solids are applied.
The proposed approach leads to a more accurate determination of the structural elements’ materials wave parameters’ theoretical values. The deformation wave speed experimental measurements in the rods using the nonlinear acoustic diagnostics and comparing the results with theoretically determined velocities by constructing and studying models that take into account the real inherited properties of the materials make it possible to predict more accurately the presence of material microdefects, in the area of which progressive accidental destruction of structures under the influence of power loads can develop. As a result, huge economic damage, disruptions in the work of entire industries and, in the worst case, the death of staff are inevitable.

Therefore, the task of determining the analytically accurate values of the rates of wave deformations in the rods by attraction of deformation models that take into account more precisely the physical and mechanical hereditary viscoelastic properties of the materials is relevant.

The theoretical justification for calculating the detailed values of the velocity of wave deformations is based on the nonlinear deformation waves occurrence and propagation mathematical modeling in rods with hereditary-rheological properties.

The growth of the Russian economy is determined not only by the creation of new industries and enterprises, but also by the stable and trouble-free operation of the already created ones. The economic and operational reliability of construction and water, oil and gas pipelines largely depends on the strength of thin-walled structural elements - rods, supports, beams, cylindrical shells, widely used in the construction of these structures.

Their strength is significantly reduced due to the presence of hidden microdefects in structural elements, the presence of which can lead to the loss of their bearing capacity, cause the destruction of structures, accompanied by the significant economic damage, environmental disruptions.

Therefore, the improvement of non-destructive acoustic methods for diagnosing the hidden microdefects based on the development of more accurate mathematical models for the propagation of nonlinear waves in thin-walled structural elements, taking into account their real physical and mechanical properties and finding the exact wave characteristics substantially used in non-destructive acoustic defectoscopy, determine the relevance of the study.

The growth of the Russian economy is determined not only by the creation of new industries and enterprises, but also by the stable and trouble-free operation of the already created ones. The economic and operational reliability of construction and water, oil and gas pipelines largely depends on the strength of thin-walled structural elements - rods, supports, beams, cylindrical shells, widely used in the construction of these structures. Their strength is significantly reduced due to the presence of hidden microdefects in structural elements, the presence of which can lead to the loss of their bearing capacity, cause the destruction of structures, accompanied by significant economic damage, environmental damage and, in the worst case, human casualties. Therefore, the improvement of non-destructive acoustic methods for the diagnosis of hidden microdefects based on the development of more accurate mathematical models for the propagation of nonlinear waves in thin-walled structural elements, taking into account their real physical and mechanical properties and finding the exact wave characteristics substantially used in acoustic defectoscopy, determine the relevance of the study.

2 Results

The mathematical models’ development for the description of nonlinear viscoelastic deformation waves in the rods, which makes it possible to obtain the real wave parameters for the acoustic diagnostics of hidden microdefects in the structural material is the basis for increasing the economic and operational reliability of the designed building objects by improving the detection of invisible structural violations of the materials by the acoustic monitoring methods.

Mathematical modeling is performed using the strict methods of mechanics of a deformable solid body to specify the fields of displacement of medium points, the Green tensor for finite strains, variational principles, nonlinear heredity models, perturbation methods, and asymptotic methods of nonlinear wave dynamics.

Many media exhibit the linear elasticity of volumetric deformation, and the viscoelastic heredity is a characteristic of shear deformations. In order to study the deformation waves in thin-walled structures made of materials with similar properties, we consider an infinite rod having an invariable cross section in the absence of external volume and surface forces.

In the coordinate system with the x axis located at the center of gravity of the bar’s cross sections,
and the y and z axes in one of its cross sections, we define the displacements of the bar points by the formulas

\[ u_1 = u(x, t) ; \quad u_2 = -vy u_x ; \quad u_3 = -vz u_x , \]

(1)

and the rod deformations, by the Green tensor

\[ e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i} + u_{k,i}u_{k,j}) \]

(2)

Taking into account the linear elasticity of the volumetric deformations, the inherited viscoelastic properties of the rod material are described using the relations of the linear theory of viscoelastic elasticity:

\[ s_{ij}(t) = 2\mu e_{ij}(t) - \alpha \int_{-\infty}^{t} e_{ij}(\tau) \mathrm{d}\tau. \]

(3)

\[ \sigma(t) = K\theta(t) \]

In case of decomposing the function \( f(t) = e_{ij}(\tau) \) in a Taylor series in degrees \( (t - \tau) \), we pass from the integral operator in (3) to the differential:

\[ \sigma_{ij} = \tilde{\lambda}\theta\delta_{ij} + 2\tilde{\mu}e_{ij}, \]

(4)

where the notation is introduced \( \tilde{\lambda} = \lambda - \frac{2\mu}{3}, \quad 2\tilde{\mu} = 2\mu(1 + p) \).

Applying the operator \( p = \frac{\alpha}{\beta^2} \frac{\partial}{\partial t} - \frac{\alpha}{\beta} \) to the function \( f(t) \), we get: \( pf = \frac{\alpha}{\beta^2} f_t - \frac{\alpha}{\beta} f \).

The components of the strain deviator are given in the equations

\[ e_{11} = \frac{2(1+v)}{3} u_x + \frac{1}{3}(1-v^2)u_x^2 + \frac{v^2 r^2}{3} \]

\[ e_{22} = -\frac{1}{3}(1+v) u_x - \frac{1}{6}(1-v^2)u_x^2 - \frac{v^2 r^2}{6} \]

\[ e_{12} = \frac{-vy}{2} u_{xx} + \frac{v^2 y}{2} u_x u_{xx} ; \]

\[ e_{13} = \frac{-vz}{2} u_{xx} + \frac{v^2 z}{2} u_x u_{xx} . \]

(5)

where \( r^2 = z^2 + y^2 \).

We write the expressions for the strain variations

\[ \delta e_{11} = \delta u_x + u_x \delta u_x + v^2 r^2 u_{xx} \delta u_{xx} ; \quad \delta e_{22} = (1+v^2 u_x) \delta u_x ; \quad \delta e_{33} = (1+v^2 u_x) \delta u_x ; \]

\[ \delta e_{12} = \frac{-vy}{2} \delta u_{xx} + \frac{v^2 y}{2} (u_{xx} \delta u_x + u_x \delta u_{xx}) ; \quad \delta e_{13} = \frac{-vz}{2} \delta u_{xx} + \frac{v^2 z}{2} (u_{xx} \delta u_x + u_x \delta u_{xx}) \]

(6)

and move on to:

\[ \delta e_{11} = [\frac{-\partial}{\partial x} u_x + v^2 r^2 u_{xx} \delta u_{xx} \frac{\partial^2}{\partial x^2}] \delta u ; \]

\[ \delta e_{22} = [v \frac{\partial}{\partial x} u_x \frac{\partial^2}{\partial x^2}] \delta u ; \]

\[ \delta e_{12} = \frac{-vy}{2} \frac{\partial^2}{\partial x^2} + \frac{v^2 y}{2} u_{xx} \frac{\partial^2}{\partial x^2} ; \]

(7)
\[ \delta \varepsilon_3 = \left[-\frac{\nu z}{2} \frac{\partial^2}{\partial x^2} - \frac{\nu^2 z}{2} \frac{\partial}{\partial x} + \frac{\nu^2 z}{2} \frac{\partial^2}{\partial x^2}\right] \delta u. \]

Determining the variation of the internal energy of the rod by the formula:

\[ \delta W = \sigma_{11} \delta \varepsilon_1 + 2 \sigma_{22} \delta \varepsilon_2 + 2 \sigma_{12} \delta \varepsilon_2 + 2 \sigma_{13} \delta \varepsilon_3, \]

we get the expression

\[ \delta W = \left\{ \tilde{\lambda} (1 - 2\nu) + 2 \tilde{\mu} - 2\nu (1 - 2\nu) \tilde{\lambda} - 2 \tilde{\mu} \nu \right\} \delta u_x + \tilde{\mu} \nu v r^2 \delta u_{xxx} - \]

\[ -(B_1 + 2A_1 - 2\nu B_2 + 4 \nu^2 A_2) \delta u_x \delta u_{xx} - \tilde{\lambda} + 2 \tilde{\mu} - 2\nu \tilde{\lambda} - 2 \tilde{\mu} \nu \nu^2 r^2 \delta u_{xxxx} + \]

\[ + v^2 \tilde{\lambda} (A_1 - 2 \tilde{\mu} \nu) \delta u_x \delta u_{xx} - \frac{3B_1}{2} + 3 \nu^2 B_2 \delta u_x^2 \delta u_{xx} - \frac{1}{2} v^2 \tilde{\lambda} (\tilde{\lambda} + 2 \tilde{\mu}) + \]

\[ + 2v^2 \lambda + 2 \tilde{\mu} \nu^2 \left( \delta u_x^2 \right) + \frac{v^2 \tilde{\mu}^2}{2} (B_1 + 2 \tilde{\mu} \nu^2) \delta u_x^2 \delta u_{xx} + \frac{v^4}{2} (\tilde{\lambda} + 2 \tilde{\mu}) u_x^3 \delta u, \]

where \( A_1 = a(2\nu^3 - \nu + 1); \quad B_1 = \nu a(2\nu - 2\nu^2 + 1); \quad A_2 = a \nu^2 (1 - \nu); \quad B_2 = a \nu^3; \)

\[ a = \frac{1}{2(1 + \nu)(1 - 2\nu)}; \quad r^2 = z^2 + y^2. \]

Using the variational principle

\[ \delta J = \int_{t_1}^{t_2} \int \int \left\{ \rho \tilde{\mu} \delta \dot{u}_i - \sigma_{ij} \delta \varepsilon_{ij} \right\} dV = 0, \quad (8) \]

in which the dot denotes the time derivative \( t \); \( \rho \) - denotes the core material density; \( \delta \varepsilon_{ij} \) - denotes the strain variations; \( \delta u_i \) - denotes the variation of displacements, and the triple integral is calculated by the volume of the rod, we derive the equation of its motion:

\[ \rho (-u_{tt} + v^2 r^2 u_{tt xx}) + N_1 u_{xx} - \mu v^2 r^2 u_{xxxx} + N_2 u_x u_{xx} + \]

\[ + v^2 r^2 N_3 u_{xx} u_{xx} - v^2 r^2 N_4 (u_x u_{xx})_{xx} + N_5 u_x^2 u_{xx} + \]

\[ + \frac{1}{2} v^2 r^2 N_6 (u_x u_{xx})_x - \frac{1}{2} v^2 r^2 N_7 (u_x^2 u_{xx})_{xx} - \]

\[ - \frac{1}{2} v^4 r^4 N_8 ((u_{xx})^3)_{xx} = 0, \quad (9) \]

where

\[ N_1 = \tilde{\lambda} (1 - 2\nu)^2 + 2 \tilde{\mu} (1 + 2\nu^2); \quad N_2 = 3(1 - 2\nu)(1 + 2\nu) \tilde{\lambda} + 6 \tilde{\mu} (1 - 2\nu)^3; \]

\[ N_3 = (1 - 2\nu)^2 \tilde{\lambda} + 2 \tilde{\mu} (1 - \nu); \quad N_4 = \tilde{\lambda} (1 - 2\nu)^2 + 2 \tilde{\mu} (1 - 2\nu); \]

\[ N_5 = \frac{3}{2} \tilde{\lambda} (1 + 2\nu^2)^2 + 3 \tilde{\mu} (1 - 2\nu^4); \quad N_6 = (1 + 2\nu^2) \tilde{\lambda} + 2 \tilde{\mu} (1 + 2\nu^2); \]

\[ N_7 = (1 + 2\nu^2) \tilde{\lambda} + 2 \tilde{\mu} (1 + \nu^4); \quad N_8 = \tilde{\lambda} + 2 \tilde{\mu}. \]

We simplify the perturbation method (9). First, we convert it to the dimensionless variables:

\[ \xi = \frac{\bar{x}}{L} - \frac{c}{L} \tau; \quad \tau = \frac{v}{c} \tau; \quad u^* = \frac{u}{A}. \]

Let us assume that the characteristic wavelength \( L \) is significantly larger than the amplitude of the
deformation wave \( A \), the parameter is small, and the hereditary constants \( \alpha, \beta \) and the characteristic transverse size of the rod is such that the relations of orders get the following form:

\[
\frac{\alpha c}{\beta^2 L} = O(\varepsilon) ; \quad \frac{d}{L} = O(\sqrt{\varepsilon}) .
\]

Replacing the asymptotic expansion function \( u(\xi, \tau) \)

\[
u = u_0 + \varepsilon u_1 + \ldots ,
\]

we obtain the following equation for the rod motion:

\[
\frac{\rho c^2}{E} \left( -A \frac{u_{\xi\xi}}{\varepsilon^2} - 2 \varepsilon u_{\xi\tau} + \varepsilon^2 u_{\tau\tau} \right) + \nu^2 \varepsilon \frac{A}{\varepsilon^2} \left( u_{\xi\xi\xi\xi} - 2 \varepsilon u_{\xi\xi\tau} + \varepsilon^2 u_{\xi\tau\tau\tau} \right) + \]

\[
+ (1 - \frac{\alpha a_0}{\beta}) \frac{A}{\varepsilon^2} u_{\xi\xi} - (1 - \frac{\alpha}{\beta}) \frac{A}{\varepsilon^2} u_{\xi\xi\xi\xi} + (3 - \frac{\alpha a_3}{\beta}) \frac{A^2}{\varepsilon^2} u_{\xi\xi} + \]

\[
+ \frac{\alpha c}{\beta^2 \varepsilon} \left( \frac{\partial}{\partial \tau} - \frac{\partial}{\partial \xi} \right) \left[ a_0 \frac{A}{\varepsilon^2} u_{\xi\xi} - \frac{bA}{\varepsilon^2} u_{\xi\xi\xi\xi} + \frac{a_3 A^2}{\varepsilon^2} u_{\xi\xi} \right] = 0 .
\]

Having completed the transformations, we get the equation

\[
\frac{\rho c^2}{E} \left[ -u_{\xi\xi} + 2 \varepsilon u_{\xi\tau} - \varepsilon^2 u_{\tau\tau} + \varepsilon^2 \nu^2 \left( u_{\xi\xi\xi\xi} - 2 \varepsilon u_{\xi\xi\tau} + \varepsilon^2 u_{\xi\tau\tau\tau} \right) \right] + \]

\[
+ (1 - \frac{\alpha a_0}{\beta}) u_{\xi\xi} - \frac{\delta^2}{2(1 + \nu)} \left( 1 - \frac{\alpha}{\beta} \right) u_{\xi\xi\xi\xi} + \varepsilon (3 - \frac{\alpha a_3}{\beta}) u_{\xi\xi} + \]

\[
+ \frac{\alpha c}{\beta^2 \varepsilon} \left( \frac{\partial}{\partial \tau} - \frac{\partial}{\partial \xi} \right) \left[ a_0 u_{\xi\xi} - \frac{\varepsilon^2}{2(1 + \nu)} u_{\xi\xi\xi\xi} + \varepsilon a_3 u_{\xi\xi} \right] = 0 .
\]

In the zeroth approximation, taking into account relations of orders (11) and the expansion (12), we obtain the equality:

\[
\left[ -\frac{\rho c^2}{E} \left( 1 - \frac{\alpha a_1}{\beta} \right) \right] u_{0\xi\xi} = 0 ,
\]

where \( a_1 = \frac{2}{3}(1 + \nu) \).

As \( u_{0\xi\xi} \neq 0 \), then we obtain the formula for the wave propagation velocity deformations from (13):

\[
c = \sqrt{\frac{E}{\rho} \left[ 1 - \frac{2\alpha(1 + \nu)}{3\beta} \right]} .
\]

The solvability condition for the coefficient equation in the expansion (12), determined by the first approximation, leads to the Korteweg de Vries – Burgers equation:

\[
\psi_{\xi} + b_1 \psi \psi_{\xi} + b_2 \psi_{\xi\xi} + b_3 \psi_{\xi\xi\xi\xi} = 0 ,
\]

where

\[
\psi = u_{0\xi} ; \quad b_1 = \frac{E(3\beta - \alpha a_2)}{2pc^2\beta} ; \quad b_2 = \frac{E\alpha a_1}{2pc^2\beta} ;
\]
\[ a_2 = 2(1 - v^2) ; \quad b_3 = v^2 \left[ \frac{1}{2} - \frac{E(\beta - \alpha)}{4pc^2\beta(1 + v)} \right]. \]

We proceed to consider the physically and geometrically nonlinear rod, as in the linear case, using kinematic relations (1) to determine the final deformations of the rod using the formulas (2).

Given that the volumetric deformations are linearly elastic, we apply the equations of the nonlinear heredity state in the form

\[ \sigma_{ij}(t) = \lambda \delta_{ij} + 2\mu e_{ij} - 2\mu \alpha \int_{-\infty}^{t} e^{-\beta(t-\tau)} [1 + \gamma e_{ij}^2(\tau)] e_{ij}(\tau) d\tau, \quad (16) \]

where \( \theta \) is the volume expansion; \( e_{ij} = e_{ij} - \frac{1}{3} \delta_{ij} \) are the strain deviator components; \( e_u^2 = \frac{2}{3} e_{ij} e_{ij} \) is the strain rate; \( \alpha, \beta, \gamma \) are the physical material constants.

Let us simplify the study by replacing the integral operator in the deformation law (16) with the differential one. For this, we decompose the function \( f(\tau) = [1 + \gamma e_u^2(\tau)] e_{ij}(\tau) \) in a Taylor series in degrees \( (t - \tau) \).

Preserving the two terms of the expansion, which corresponds to large values of the product \( \beta \cdot t \), we get the equality

\[ \sigma_{ij} = \tilde{\lambda} \delta_{ij} + 2\tilde{\mu} e_{ij} + 2\gamma \mu p (e_u^2 e_{ij}), \quad (17) \]

where

\[ \tilde{\lambda} = \lambda - \frac{2\mu}{3} p; \quad \tilde{\mu} = \mu(1 + p); \quad p = \frac{\alpha}{\beta^2} \frac{\partial}{\partial t} - \frac{\alpha}{\beta}; \]

We calculate the components of the strain deviator and their variations, respectively, according to the formulas (5), (6), the variation of the internal energy:

\[ \delta W = \sigma_{11} \delta e_{11} + 2\sigma_{22} \delta e_{22} + 2\sigma_{12} \delta e_{12} + 2\sigma_{13} \delta e_{13}. \]

The linear part of the variation \( \delta W \) can be defined by the formula:

\[ \delta W_a = [-\tilde{\lambda}(1 - 2v) + 2\tilde{\mu} - 2v((1 - 2v)\tilde{\lambda} - 2\tilde{\mu})]u_{xx} + 2\tilde{\mu} \frac{v^2 r^2}{2} u_{xxx} \]

and the nonlinear variation get the form:

\[ \delta W_a = [-B_1 u_x u_{xx} - v^2 r^2 (\tilde{\lambda} + 2\tilde{\mu}) u_{xx} + v^3 r^3 (\tilde{\lambda} + 2\tilde{\mu}) u_{xxx}] u_{xx} - 2\tilde{\mu} v^2 r^2 (e_u^2 e_{11})_x - 2\tilde{\lambda} u_x u_{xx} - \]

\[ -2u_x^2 u_{xx} - v^2 r^2 (\tilde{\lambda} + 2\tilde{\mu}) (u_x u_{xx})_x + 2\tilde{\mu} v^2 r^2 (e_u^2 e_{11})_x + \]

\[ + v^3 r^3 A_1 (u_x u_{xx})_x + v^3 r^3 B_1 (u_x u_{xx})_x + v^3 r^3 (\tilde{\lambda} + 2\tilde{\mu}) (u_{xx}^3)_x + \]

\[ + v^2 r^2 2\tilde{\mu} v^2 r^2 (e_u^2 e_{11})_x + 2v B_2 u_x u_{xx} + v^3 r^3 \tilde{\lambda} u_{xx} u_{xxx} + 2\tilde{\mu} v^2 r^2 (e_u^2 e_{22})_x - \]

\[ - 2v^2 A_2 u_x u_{xx} - \frac{3v^2}{2} B_2 u_x u_{xx} - \frac{v^3 r^3}{2} (\tilde{\lambda} (u_x u_{xx})_x - 2\tilde{\mu} v^2 r^2 (e_u^2 e_{22})_x)_x - \]

\[ - 2\tilde{\mu} v^3 r^2 (u_x u_{xx})_x - 2\tilde{\mu} v^2 r^2 (e_u^2 e_{12})_x + \tilde{\mu} v^3 r^2 u_{xx} u_{xxx} - \]

\[ - 2\tilde{\mu} v^3 r^2 (e_u^2 e_{12})_x - 2\tilde{\mu} v^2 r^2 (e_u^2 e_{22})_x - \]

\[ - 2\tilde{\mu} v^3 r^2 (u_x u_{xx})_x - 2\tilde{\mu} v^2 r^2 (e_u^2 e_{12})_x + \tilde{\mu} v^3 r^2 u_{xx} u_{xxx} - \]
\[-2\mu \frac{v^4 r^2}{2} (u_x u_{xx})_x - 2\mu \gamma \nu^2\gamma (\epsilon_0^2 e_{12} u_{x x})_x - \mu \nu^3 r^2 (u_x u_{xx})_{xx} + \]
\[+ 2\mu \frac{v^4 r^2}{2} (u_x^2 u_{xx})_{xx} + 2\mu \gamma \nu^2\gamma (\epsilon_0^2 e_{12} u_{x})_{xx} - 2\mu \gamma \nu^2\gamma^2 (\epsilon_0^2 e_{13})_{xx} - \]
\[-2\mu \gamma \nu^2\gamma^2 (\epsilon_0^2 e_{13})_{xx})_x + 2\mu \gamma \nu^2\gamma (\epsilon_0^2 e_{13} u_{x x})_{xx} \vert \partial u ,\]

where:

\[A_1 = \tilde{\lambda}(1 - 2\nu) + 2\tilde{\mu} ; \quad B_1 = (1 + 2\nu)\tilde{\lambda} + 2\tilde{\mu} ; \]
\[A_2 = (1 - 2\nu)\tilde{\lambda} + 2\tilde{\mu} ; \quad B_2 = (1 + 2\nu^2)\tilde{\lambda} + 2\tilde{\mu} \nu^2 \]

are indicated.

We complete the internal energy variation:

\[\delta W = \{-N_1 u_{xx} + \tilde{\mu} \nu^2 r^2 u_{xxxx} - N_2 u_x u_{xx} - \nu^2 r^2 N_3 u_{xx} u_{xxx} + \]
\[+ \nu^2 r^4 N_4 (u_x u_{xx})_{xx} - N_5 u_x u_{xx} - \nu^2 r^2 N_6 (u_x u_{xx})_x + \nu^2 r^2 N_7 (u_x u_{xx})_{xx} + \]
\[+ \nu^4 r^2 N_8 (u_{xx})_{xx} - 2\mu \gamma \nu^2\gamma (\epsilon_0^2 e_{11} - 4\nu e_{22} + e_{11} u_x + 2\nu^2 e_{23} u_x + \]
\[+ \nu^2 y e_{12} u_{xx} + \nu^2 y e_{13} u_{xx}) + \nu^2 y e_{12} u_{xx} + \nu^2 y e_{13} u_{xx} + \nu^2 y e_{12} u_{xx} + \nu^2 y e_{13} u_{xx} - \]
\[- \nu^2 y e_{12} u_{xx} - \nu^2 y e_{13} u_{xx}) + \nu^2 y e_{12} u_{xx} + \nu^2 y e_{13} u_{xx} - \nu^2 y (e_{11} u_{xx})_x - \]
\[- \nu^2 y (e_{12} u_{xx})_x - \nu^2 y (e_{13} u_{xx})) \vert \partial u ,\]

where:

\[N_1 = \tilde{\lambda}(1 - 2\nu)^2 + 2\tilde{\mu}(1 + 2\nu) ; \quad N_2 = 3(1 - 2\nu)(1 + 2\nu^2)\tilde{\lambda} + 6\tilde{\mu}(1 - 2\nu^3) ; \]
\[N_3 = (1 - 2\nu)^2\tilde{\lambda} + 2\tilde{\mu}(1 - \nu) ; \quad N_4 = \tilde{\lambda}(1 - 2\nu) + 2\tilde{\mu}(1 - 2\nu) ; \]
\[N_5 = \frac{3}{2}(1 + 2\nu^2)^2 + 3\tilde{\mu}(1 - 2\nu^4) ; \quad N_6 = (1 + 2\nu^2)\tilde{\lambda} + 2\tilde{\mu}(1 + \nu^2) ; \]
\[N_7 = (1 + 2\nu^2)\tilde{\lambda} + 2\tilde{\mu}(1 + \nu^2) ; \quad N_8 = \tilde{\lambda} + 2\tilde{\mu} .\]

We substitute in the formula (8) and integrate over the cross-sectional region. Given that the variations are arbitrary, we obtain the rod motion equation:

\[
\rho(-u_{tt} + v^2 r^2 u_{txxx}) + \lambda_1 u_{xx} - \tilde{\mu} \nu^2 r^2 u_{xxxx} + \nu^2 r^2 N_3 u_{xx} u_{xxx} - \nu^2 r^2 N_4 (u_x u_{xx})_{xx} + N_5 u_x u_{xx} + \frac{1}{2} \nu^2 r^2 N_6 (u_x u_{xx}) - \frac{1}{2} \nu^2 r^2 N_7 (u_x^2 u_{xx})_{xx} - \frac{1}{2} \nu^4 r^2 N_8 (u_{xx}^3)_{xx} + 2\mu \gamma \nu^2\gamma \left(\frac{1}{3}(1 + u_x)(2(1 + v)u_x + (1 - u_x)^2 u_x^2 - \right)
\[+ \frac{1}{2}(1 - \nu^2)(u_x^2 u_{xx} - 4v)Q + \left(\frac{1}{3} v^2 u_{xx}^2 (1 + u_x) - \frac{1}{6} v^2 u_{xx}^2 (2v^2 u_x - \right)
\[- 4v) + \frac{1}{2} \nu^2(v^2 u_{xx}^2 - v u_{xx}^2) \vert \partial Q] + \left[(- \frac{1}{3} v^2 u_{xx} (2(1 + v)u_x + (1 - v^2)u_x^2) + \right]
\]
\[ + \frac{1}{2} (v - v^2 u_x) \cdot (v^2 u_x u_{xx} - v u_{xx}) \cdot Q - \frac{1}{2} v^4 A_1^3 (u_{xx})^3 + 2\mu \gamma \left[ \frac{\partial}{\partial x} \left( \frac{1}{3} (1 + u_x) + \frac{1}{3} v^3 u_{xx} Q \right) \right] = 0, \]

where

\[ Q = R_1 + R_2 + R_3; \]
\[ R_1 = \frac{1}{3} [2(1 + v) u_x + (1 - v^2) u_x^2]^2; \quad R_3 = \frac{1}{3} v^4 u_{xx}; \]
\[ R_2 = \frac{2}{3} v^2 u_{xx}^2 [2(1 + v) u_x + (1 - v^2) u_x^2]^2 + (v^2 u_x u_{xx} - v u_{xx})^2. \]

3. Discussion

We study the motion equation (18) by the perturbation method. For this, as in the linear case, we introduce the dimensionless variables (10) and order relations (11).

We use the function using the asymptotic expansion (12), as a result we obtain the motion equation of a physically and geometrically nonlinear rod:

\[ \frac{\rho c^2}{E} \left[ - u_{xx} - 2 v u_{x} + 2 u_{tt} + v^2 I (u_{xx}^2 - 2 u_{x} u_{xx}^2 + u_{x}^2) \right] + \]
\[ + (1 - \frac{\alpha a_0}{\beta}) A \frac{u_{xx}}{e^2} u_{xx} - b(1 - A) A \frac{u_{xx}^2}{e^2} + (3 - \frac{\alpha a_3}{\beta}) A^2 \frac{u_{x} u_{xx}}{e^3} - \]
\[ - \frac{\alpha}{\beta} \gamma a_1 A^3 \frac{u_{x}^2 u_{xx}}{e^4} + \frac{a c}{\beta e^2} (e \frac{\partial}{\partial x} - \frac{\partial}{\partial x}) [a_0 A \frac{u_{xx}}{e^2} - b A A \frac{u_{xx}^2}{e^4}] + \]
\[ + \frac{a_3 A^2}{e^3} u_{x} u_{xx} + \frac{\gamma a_1 A^3}{e^3} u_{x}^2 u_{xx} = 0 \]

or

\[ \frac{\rho c^2}{E} \left[ - u_{xx} + 2 v u_{x} - v^2 u_{tt} + v^2 I (u_{xx}^2 - 2 u_{x} u_{xx}^2 + u_{x}^2) \right] + \]
\[ + (1 - \frac{\alpha a_0}{\beta}) A \frac{u_{xx}}{e^2} u_{xx} - \frac{e v^2 I}{2(1 + v)} (1 - A) A \frac{u_{xx}^2}{e^2} + e^2 A_3 A \frac{u_{xx}^2}{e^2} + \]
\[ + \frac{e c}{\beta e^2} (e \frac{\partial}{\partial x} - \frac{\partial}{\partial x}) [a_0 u_{xx} - \frac{e v^2 I}{2(1 + v)} u_{xx}^2 + e A_3 u_{xx} u_{xx}] + \]
\[ + e^2 \gamma a_1 u_{x}^2 u_{xx}] = 0, \]

where

\[ a_0 = \frac{2(1 + v)}{3}; \quad a_3 = \frac{-4v^3 - 2v^2 + 2v + 8}{3(1 + v)}; \quad b = \frac{v^2 I}{2(1 + v)}. \]

After transformations in the zero\(^{th}\) approximation, we receive the formula:

\[ [- \frac{\rho c^2}{E} + (1 - \frac{\alpha a_1}{\beta})] u_{xx} = 0, \]
where $E$ is the elastic modulus; $a_1 = \frac{2}{3}(1 + \nu)$.

From formula (19), when the component $u_{0_{\xi\xi}} \neq 0$, the expression for the deformation wave velocity is propagated in the rod with the indicated properties

$$c = \sqrt{\frac{E}{\rho}[1 - \frac{2\alpha}{3\beta}(1 + \nu)]}.$$

From the first approximation, the modified Korteweg de Vries – Burgers equation it follows:

$$\psi_t + b_1\psi\psi_x - b_2\psi^2\psi_x + b_3\psi_{\xi\xi} + b_4\psi_{\xi\xi\xi} = 0,$$

where

$$\psi = u_{0\xi}; \quad b_1 = m(3 - \frac{\alpha a_2}{\beta}); \quad m = \frac{E}{2\rho c^2};$$

$$b_2 = \frac{ma_2\gamma a_3}{\beta}; \quad b_3 = -\frac{ma_2 a_1}{\beta^2 Lc}; \quad b_4 = mv^2[\frac{1}{2m} - \frac{\beta - \alpha}{2\beta(1 + \nu)}];$$

$$a_2 = 2(1 - \nu^2); \quad a_3 = \frac{8}{3}(1 + \nu^2)(1 + 2\nu).$$

4. **Summary**

To increase the economic and operational reliability of the designed building structures by improving the acoustic diagnostics of the thin-walled structural elements materials’ invisible microdefects, the new mathematical models of nonlinear wave dynamics of wave processes in rods made of non-linear viscoelastic material have been developed.

Stricter relationships between the geometric, physical and wave characteristics of the deformation process have been established. They allow to calculate more accurate values of the deformation wave speed in the rod, and to significantly increase the accuracy of the material hidden microdefects’ registration. As a result, the use of unreliable structural elements in construction is excluded, thereby increasing the economic and operational reliability of the designed building structures.

It is shown that the compensation of the effects of nonlinearity, dispersion and dissipation leads to the longitudinal solitary waves’ formation in the rods, the speed of which increases with increasing the wave amplitude, i.e. depends on the non-linearity degree of the process.

The linear models used earlier do not even allow the qualitative detection of this effect. The established relationships between geometric, physical, mechanical and the wave characteristics make it possible to correctly apply the acoustic methods for recording the invisible microdefects of the material.

**Acknowledgements**

Funding: The reported study was funded by RFBR, project number 19-010-00385 A

**References**

[1] Nigul U K 1981 Nonlinear Acoustodynamics (Shipbuilding, L.).
[2] Moskvitin V V 1972 Resistance of viscoelastic materials (Nauka, Moscow).
[3] Ilyushin A A, Pobedrya B E 1970 Fundamentals of the mathematical theory of thermo-viscoelasticity (Nauka, Moscow).
[4] Loyko V I Arshinov G A, Arshinov V G 2015 Mathematical modeling of mutually beneficial relations between producers of raw materials and its processors based on a non-linear demand function *Political network electronic scientific journal of the Kuban State Agrarian University* **110** 1691–1706.
[5] Bogomolova Irina P, Krivenko Elena I, Larionova Anna A, Eroshenko Vasily I, Zaitseva Natalia A 2019 The role and features of resource-saving processes in modern conditions of
managing the national economy and the implementation of state strategic initiatives Journal of Environmental Treatment Techniques 7 (3) 426-431.

[6] Bondarenko S V 2019 Economic and organizational mechanisms of forming business networks in the construction industry IOP Conf. Ser.: Mater. Sci. Eng. 698 077040

[7] Lazareva N, Fedorkova A, Sverchkova O, Gornostaeva Z 2019 An attractive assessment on building enterprises into conditions for regional sustainable development International Management, & Applied Sciences & Technologies 10 (16). ISSN 2228-9860 eISSN 1906-9642 CODEN: ITJEAS Paper ID:10A16N http://TUENGR.COM/V10A/10A16N.pdf DOI: 10.14456/ITJEMAST.2019.224

[8] Rodina E E, Filatov V V, Larionova A A, Makarova L M, Bereznjakovskii V S 2018 Revitalization of Depressed Industrial Areas Based on Ecological Industrial Parks Eurasian Journal of Analytical Chemistry 13 (1b).

[9] Takhumova O V, Kadyrov Marsel A, Titova Evgenia V, Ushakov Denis S and Ermilova Mariia I 2018 Capital Structure Optimization in Russian Companies: Problems and Solutions Journal of Applied Economic Sciences XIII 7(61) 1939-1945.

[10] Takhumova O V, Degtyareva O G 2019 Cost-benefit analysis of the cast-in-place building framework vertical bearing structural members IOP Conf. Ser.: Mater. Sci. Eng. 698 077041.

[11] Takhumova O V, Vershitskiy A V, Kobylatova M F, Bludova S N, Asanova N 2018 Development of Entrepreneurial Structures Of Production And Trade Sphere On The Basis Of Integration Research Journal of Pharmaceutical, Biological and Chemical Sciences 9(6) 1624-1629

[12] Popov R A, Sekisov A N, Shipilova N A 2016 The Economics of Innovation in Modern Russia: Practice, Problems and Prospects Prospects) International Journal of Economics and Financial 6.(8) 184-188. Information on http://www.econjournals.com/index.php/ijefi/article/view/3727/pdf.

[13] Popov R A, Shipilova N A, Sekisov A N, Soloveva E V, Gura D A 2019 Innovative development of construction in russia: economics, technologies, management Amazonia Investiga 8(19) 653-663 Information on http://amazoniainvestiga.info/index.php/amazonia/article/view/281/258.