Simulations of tip vortex cavitation flows with nonlinear $k-\epsilon$ model

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Abstract. Effects of turbulence models on the simulations of tip vortex wetted flows and cavitating flows are studied. Two types RANS turbulence models, with and beyond the Boussinesq turbulent-viscosity hypothesis, are investigated by examining experimental results regarding strain rate and Reynolds shear stress distributions in the vortex region. The numerical results imply that the spatial phase shift between the mean strain rate and Reynolds stresses can be accurately modeled by the nonlinear $k-\epsilon$ turbulence model; the tip vortex cavitation region can only be predicted using the nonlinear turbulence model. The mechanism of the over-dissipation due to the turbulence model is analyzed in terms of the turbulence production, which is one of the dominant source terms in the transport equations of energy. The numerical results imply that the nonlinear $k-\epsilon$ model is a promising candidate for predicting tip vortex cavitation flows in practical applications at a reasonable computational cost.

1. Introduction
Tip vortex cavitation results in the vibration and cavitation erosion of propellers and rudders, particular for high-speed ships. Experimentation has been the primary tool for studying the formation and flow characteristics of tip vortex cavitation [1, 2, 3]. One of the most challenging task is to properly simulate the tip vortex cavitation. Using advanced computational fluid dynamics to accurately predict tip vortex flows is not simple but is possible. Great achievements have been accomplished using detached eddy simulations and large eddy simulation on cavitating flows. The Reynolds averaged Navier-Stokes equations remain the primary CFD solver used in practical applications due to their low computational cost. However, the widely used two-equation turbulence models based on the Boussinesq turbulent-viscosity hypothesis perform poorly for highly rotational flows. The intrinsic assumption of Boussinesq hypothesis that shear stress is locally determined by the mean strain rate, has no general validity [4], and fails when the flow stream is curved or strongly rotated. Chow et al. [5] and Giuni [6] measured the distribution of the strain rate and Reynolds shear stresses in the tip vortex region. It was found that the four-leaf clover pattern of these two contours was not aligned spatially, indicating that the flow is not isotropic. We denote this type of rotation as a spatial phase shift in the present work. These detailed experimental data imply that the turbulence models based on Boussinesq turbulent-viscosity hypothesis are not capable of describing the vortex evolution in the near...
wake of hydrofoils and propellers. The nonlinear relationship between the Reynolds shear stress and strain rate can fulfill the non-isotropic constitution relation by summation of strain rate tensor and rotation rate tensor. Although the theoretical foundation of the nonlinear $k$-$\epsilon$ model is significantly different from that of the standard $k$-$\epsilon$ model, the numerical implementation is straightforward in this paper by extension of the standard $k$-$\epsilon$ model, as the one proposed by Zhu & Shin [7].

In this paper we analyze the characteristics of turbulent tip vortex flow field, and verify the influence of turbulent dissipation on the formation of tip vortex cavitation flows.

2. Numerical method for flow solver
Turbulent viscous flows are solved using the Reynolds averaged Navier-Stokes equations. To simulate the cavitating flow around airfoils or propellers, the assumption of a homogeneous mixed fluid of water and vapor is introduced; the liquid and vapor phases are assumed to be fully mixed and share the same velocity and pressure in the flow field. A cavitation model is needed to simulate the phase transition between the liquid and vapor phases. The transport equation of Schnerr & Sauer is used in the present work [8].

The numerical models are solved using a finite volume method based on the OpenFOAM package. The PISO scheme is used to solve for velocity/pressure coupling between the momentum and continuity equations. A second-order upwind scheme is used for spatial discretization in the momentum and transport equations for $k$ and $\epsilon$, which are used to calculate the turbulent eddy viscosity according to different turbulence models. The QUICK scheme is used in the advection of the volume fraction function. The viscous terms are solved using a second-order central difference scheme. Details of the governing equations, numerical methods and mesh convergence tests will be reported elsewhere.

3. Numerical results and discussions for tip vortex flows
Considering wetted flow around a rounded wingtip NACA0012 hydrofoils following the setup as that in experiments [5], numerical simulation is conducted to study the spatial distribution of strain and Reynolds stresses. The number of total mesh cells is approximately 4.5M.

There is a spatial phase shift of the four-lobe pattern between the strain rate and the Reynolds stress [5, 6]: a phase shift of $\pi/4$ can be observed between the strain rate $S_{xy}$, $S_{yz}$ and the corresponding components of Reynolds stress; and a phase shift of $\pi/2$ can also be observed between the strain rate $S_{zz}$ and the Reynolds stress $-u'u''$. Numerical results of the distribution of strain rate and Reynolds stress imply the advantages of the nonlinear $k$-$\epsilon$ model in the numerical simulations of tip vortex flows in Fig.1. The advantages of this model can be explained from the perspective of an energy budget. Defining the kinetic energy of the mean flow to be $E = u_i u_j / 2$ and the turbulent kinetic energy to be $k = u'_i u'_j / 2$, the transport equations of these two types of energy are:

$$\frac{D \rho_m E}{Dt} + \nabla \cdot \vec{T} = -P - \rho_m \bar{e}, \quad \frac{D \rho_m k}{Dt} + \nabla \cdot \vec{T}' = P - \rho_m \epsilon$$  \hspace{1cm} (1)

where $D/Dt = \partial/\partial t + u_i \partial/\partial x_i$ is total derivative, $\rho_m \bar{e} = \rho_m S_{ij} S_{ij}$ is the viscous dissipation rate of mean flow strain rate $S_{ij} = (u_{i,j} + u_{j,i})/2$ due to molecule viscosity, $\rho_m \epsilon = \mu_m \delta_{ij} S_{ij}$ is turbulent kinetic dissipation rate, relating the strain rate of fluctuate velocity field $s_{ij} = (u'_{i,j} + u'_{j,i})/2$. $\vec{T}$ and $\vec{T}'$ are convection of Reynolds stress, pressure stress and molecule viscosity by mean and fluctuated velocities.

Production is defined as $P = \tau_{ij}^D S_{ij}$ in which $\tau_{ij}^D$ is the deviatoric part of the Reynolds stress. We have $P \ll \tau$ due to the fact that the turbulent eddy viscosity is much larger than the molecular viscosity $\mu_t \gg \mu_m$. Therefore, the primary source in the right hand side of Eq.1(a) is
(a) isosurface of $Q = 10^3$ with the same grid

(b) $-S_{12} = -(u_y + v_z)$ and $\overline{u'v'}$

(c) $-S_{13} = -(u_z + w_x)$ and $\overline{u'w'}$

(d) $-S_{23} = -(v_z + w_y)$ and $\overline{v'w'}$

Figure 1. Numerical results of wetted flows around NACA0012 airfoil, attach angle 10$^\circ$, Reynolds number $Re = 4.6 \times 10^6$. (a) isosurface of $Q = 10^3$. (b)-(d) Comparison of spatial distribution of negative strain rate $-S_{ij}$ and deviatoric Reynolds stress $\overline{u'v'}$ among experimental results [5], numerical results with standard $k$-$\epsilon$ and nonlinear $k$-$\epsilon$ models.

contributed by the production term $P$. The production $P$ extracts the total kinetic energy from the mean flow and transfers it to the turbulent kinetic energy. The production $P$ thus serves as a sink term in Eq.1(a) and a source term in Eq.1(b). Correct prediction of the magnitude of production $P$ influences the energy transfer between the mean and fluctuating flow. Finally, the amount of transfer energy is dissipated at the turbulent kinetic dissipation rate $\rho m \epsilon$. For homogeneous and steady turbulent flows, we have $P \approx \epsilon$. Therefore, the turbulence dissipation may be estimated by the magnitude of production $P$. If $P$ is over estimated, the strength of the vortex will be under estimated.

To illustrate the effects of phase shift on the production term, a typical four-lobe distribution of strain rate and Reynolds stress components is plotted at a prescribed circle around the vortex center in Fig.2. For turbulence models with the Boussinesq hypothesis, the strain rate and Reynolds stress takes the same sign along the circle and thus obtains a maximum production $P = \tau_{ij}S_{ij}$. In the nonlinear $k$-$\epsilon$ model, there is a phase shift between the strain rate and Reynolds stress, as observed experimentally. Thus the integration of $P$ over the cross-section will be partially canceled. Therefore, the nonlinear model must predict less production and turbulence dissipation than the standard $k$-$\epsilon$ model. Numerical results shows that the integration $\int \int P \ dydz = 4.08$ and $2.83$ at section of $x = 1.45$ times chord by these two turbulence models.

4. Applications for cavitating flows around E779A propeller
The numerical simulations of tip vortex flows around INSEAN E779A four-blade propeller with a diameter $D = 227$ mm, nominal pitch ratio 1.1, hub diameter 45.3 mm are examined. The incoming velocity is $U_\infty = 2.808$ m/s, and the advance ratio $J = U/nD = 0.71$. The cavitation number is $\sigma_n = (p - p_v)/2(nD)^2 = 1.763$. Fully structural mesh are used in these simulation. After convergence tests, the total number of computational cells number is approximately 4.5M
The numerical results shown in this paper imply that the nonlinear $k-\epsilon$ model is a promising candidate for predicting tip vortex cavitation flows in practical applications at a much lower computational cost compared to other advanced turbulence models, including detached eddy simulations and large eddy simulations. Finally, we need to mention that the mesh resolution was also recognized to affect the vortex wake. Using a fine mesh can resolve tip vortices further downstream. Therefore, adaptive mesh refinement in the vortex wake region is important to capture the detail flow field away from the propeller.

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