The Bi-moment Analysis for Thin-wall Component under Non-uniform Torsion

Runcheng Ye*

The University of Sydney, NSW, 2006 Australia

*Corresponding author email: joeyerun@outlook.com

Abstract. Thin-walled steel structural members are commonly used in order to reduce the weight of the building. It might be influenced by bi-moment, which was induced by torsion. In designs, engineers would normally neglect the bi-moment and let bending moment be the key parameter. However, for some situations, stress due to bi-moment would be greater. This study focus on the stress induced by bi-moment under eccentric load, applying on the flange of I section, which aims to find the critical eccentric to decide whether the bi-moment could be neglected. The model focuses on the plastic state with no buckling, using the finite element software Strand7. The result shows that for the I beam, the bi-moment has to be taken into account if the eccentricity is over 0.4.

Keyword: Bi-moment; Non-uniform torsion; Thin-wall structure; Finite element analysis.

1. Introduction

As people are increasing the sense of aesthetic, in addition to meeting the functional requirements, the design of buildings need to meet the requirement of architectural aesthetics. Therefore, many high-rises and large-scale buildings been designed, such as Georgia Dome in Atlanta, Fukuoka Yahoo Japan Dome, and Beijing National Stadium. For these buildings, the traditional reinforced concrete materials are obviously difficult to adapt, and thin-walled structures are widely used. The load on the components also different from traditional and regular buildings, and the calculation of internal forces could be complicated. The simplified calculation method is often used in the structural calculation. Although the work efficiency improved [1], the design always needs a big safe factor to ensure the safety of the building which would use more steel. In order to make a more accurate design, save materials and make the design work more efficient, to know in which situation, the stress needs to be calculated by the combination of bending and bi-moment is an essential project.

Bi-moment is a couple of opposite bending moments acting on parallel planes [2]. In most situations, the bi-moment would be avoided by the designs. To calculate the bi-moment, normally there were three methods: theoretical calculation, finite element methods, and physical model test. And finite element method was the most common way in recent researches as is universal, which could suit the most of structures and most of the mechanical phenomena.

The theoretical way of calculating the bi-moment and the stress induced by bi-moment has already been introduced [2], which is

\[
B = EI_\omega \phi''
\]

\[
\sigma = \frac{B \times \alpha}{I_\omega}
\]

\[
\alpha = \int \rho_0 ds
\]
Where $\phi''$ represents the angle derivative twice by $z$ direction. $\rho_0$ is vertical distance from center of shear to tangent to wall centerline, $B$ is bi-moment and $\alpha$ is warping function.

Also, recent research presented new ways to calculate the bi-moment, by using initial parameter solutions and transfer matrix method. [3] However, all the methods of calculating the value of bi-moment would be relatively complex. Thus, in designs, engineers would try to avoid the structure to induce bi-moment. For some situations, the bi-moment could not be avoided, engineers would always neglect the stress induced by it. This project uses finite element method, with the simulation of the stress due to bending and bi-moment, to find the situations that the bi-moment needs to be concerned.

To have an accurate result for the design of thin-wall component, many researchers contributed their results. Some researchers considered the elastic state of a beam under non-uniform torsion, by using finite element method to analyze the stress result. [4] The simulation considered the influence of twist, bi-moment, distortion, and bending, which would be accurate. However, the complexity of the calculation process not suitable for design work. Not only the result of stress, researchers also focusing on stability. Some scientists using finite-difference method to discretize the differential equations to calculate the buckling load of thin-wall members, and compared the result with numerical simulation to check the calculation. [5] Some researchers also discussed how the bi-moment induced by warping torsion contribute to the local buckling of thin-wall components by analyzing the experimental result and concluded the critical warping stress. [6] The buckling load normally would be smaller than the stress which leads to plastic failure, so there was more researches related to the stability of thin-wall structures than stress.

Both warping torsion and non-uniform torsion would induce bi-moment. For I beam, non-uniform torsion often caused by load with eccentricity while warping often induced by axial force with warping function [7]. This study would only consider situation under the non-uniform torsion.

2. Methodology

2.1. Model

I section is symmetry, for which in most situations, there would be no bi-moment. However, in some special cases, the bi-moment must be considered. This study would focus on one load case shows in figure 1. With simply supported on both edges.

![Figure 1. Load on the beam.](image)

This load case would cause non-uniform torsion which would induce bi-moment. According to Melcher [8], both pure torsion and warping could cause bi-moment. Some researchers have already analyzed the formulation of warping, which would normally occur to the structure with axial force applied [9]. This study would only focus on the bi-moment induced by non-uniform torsion.
Table 1. Size of the model.

| part size (mm) | height | width | flange | web | length | mesh |
|---------------|--------|-------|--------|-----|--------|------|
| 600           | 300    | 20    | 10     | 6000| 30*30  |

2.2. Boundary Conditions
The cross section sticks rigid links to link all nodes on this cross section to the center as a rigid body. Because the rigid links could avoid the distortion, which would affect the stress simulated.

![Figure 2. Rigid links.](image)

If there is no rigid links on the cross section, the distortion would occur to the structure while the stress would be induced in elastic way. However, for design in real situation, the stress in the steel need to under the yield stress, thus, the elastic situation would not be considered in this study. The shape of the I section under the load case with bi-moment would be shown in figure 3.

![Figure 3. Distortion of the structure.](image)

By changing the distance between the load point to center, and to generate the ratio of the distance (a) over the width of half flange (b). This ratio could help to analysis how the stress by bi-moment changing due to the eccentricity.

3. Results and discussion

3.1. Results
Set the load shown on figure 1 as 100KN, for the case that the parameter (a) equal to 0, the value of pure bending could be obtained, and the results from the software is 38.8MPa. The ratio of (a) over (b) versus the stress ratio which caused from bi-moment and bending moment would be an important reference to show the changes of bi-moment. So, this project chose the ratio of distance as 0, 0.2, 0.4, 0.6, 0.8 and 1, to plot the curve in figure 4.
According to the figure, when the ratio is about 0.42, the stress induced by bi-moment would be the same as the stress due to bending moment.

When the load point at the edge of the flange, the stress would be biggest and the result shows in figure 5.

3.2. Discussion

The buckling of the I beam under the influence of bi-moment also needs to take into account, for most situations, the buckling would happen before the stress come to the capacity of the steel. [10] Some researchers already simulated the theoretical calculation of the global torsional buckling. Further research could focus on the failure mode of the steel beam under non-uniform torsion, and the stress of the beam caused by the axial force which caused warping torsion. And for some steel structures with no symmetry such as Z section or channel, for most load cases, the non-uniform torsion would be induced, and whether the stress due to bi-moment need to be taken into account would be another further research direction.

Compared with the existing models and method mentioned in the introduction part, the advantage of this model is that the model concluded a critical eccentric focusing on the I beam, which could simplify the design work. Following the former researches, the accurate results of critical stress could be calculated, but the complexity of the process of calculation made it unsuitable for real design works. Therefore, the result of this study could be a supplement to those former researches. Engineers could use the critical eccentric from this study to decide if those calculations needed.

The limitation of this model is the model not include all possible failures for the I beam. The design needs to consider both elastic and plastic state, also the stability should be discussed, and the distortion could influence the results. The warping torsion also needs to be considered which the software strand7 could not be calculated as the axial point load would make the stress distribution inaccurate.
4. Conclusion
This study analyzed the eccentric point load on the I section. According to the result, if the ratio was over 0.4, the stress induced by bi-moment would be too large to be neglected compared with the stress caused by bending moment. Moreover, the stress ratio could be a method to estimate the stress by bi-moment by using the stress induced by bending times the ratio in the result under the situation of point load acting on the flange. It could be a supplement to the existing calculation method mentioned in introduction part to simplify the design work. For future research, the other critical value other than plastic stress failure should be came forward. Also, the method of adjusting the thin-wall members to make the structure safe while the eccentric is over 0.4 is another future research scope, which might simplify the process of design.

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