Non-Gaussian Features of Primordial Gravitational Waves

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We explore possible non-Gaussian features of primordial gravitational waves by constructing model-independent templates for nonlinearity parameters of tensor bispectrum. Our analysis is based on Effective Field Theory of inflation that relies on no particular model as such and thus the results are quite generic. The analysis further reveals that chances of detecting squeezed limit tensor bispectrum are fairly higher than equilateral limit. We also discuss prospects of detectability in upcoming CMB missions.

Introduction – With the recent detection of gravitational waves from binary mergers by LIGO-Virgo collaboration [1,2], this field of research has emerged as more exciting than ever. In it searches for primordial gravitational waves (GW henceforth) have found their relevance afresh. Primordial GW is generically sourced by inflation and it can serve as a missing link between early universe cosmology and its signatures on Cosmic Microwave Background (CMB) observations via tensor fluctuations. Apart from finding out primordial features of GW, which itself is quite appealing, the detection of tensor modes can also tell us about the energy scale of inflation. Nevertheless, it can put inflationary paradigm on a firm footing (or put it to tension the other way round). However, in spite of painstaking searches for the last few years, no signature of tensor modes via its two point correlation function has been observed. Latest observation from Planck satellite puts an upper bound on the tensor two point correlation, viz., tensor-to-scalar ratio as \( r \leq 0.07 \) at 95% CL [3]. Hence, searching for other characteristics of tensor perturbations, such as the three point function, which may have the potential to comment on primordial GW, is of extreme relevance these days.

Along with the above constraints, Planck [4,5] also set a constraint on tensor non-Gaussianities as \( f_{NL}^T = 400 \pm 1500 \) at 68% CL. Similar constraints can also be found in [6] based on WMAP. These early analyses show that the tensor perturbations can, in principle, significantly deviate from Gaussian nature. With future CMB missions like CORe [7], LiteBIRD [8], CMB-S4 [10], PRISM [11], PIXIE [12] etc. chipping in, it is high time the community have in their hand couple of useful, model-independent templates for tensor non-Gaussianities that can serve as a probe for primordial GW in these upcoming missions. This will help us analyze the prospects of detection and compare with the sensitivity of the upcoming missions. Nevertheless, the non-Gaussian signatures may serve as additional probes of tensor modes, along with the usual tensor to scalar ratio. It is worth mentioning that some of the future missions like LiteBIRD [8] already aim at detecting tensor non-Gaussianities at 3-\( \sigma \). So it is quite timely to explore the three point tensor correlation functions that can open up an altogether new direction towards investigation for primordial GW.

In the last couple of years, there has been some progress in this direction. To mention a few, in [13,14] tensor bispectrum is calculated for general single field slow roll inflationary model. Parity violating tensor non-Gaussianities have also been calculated in [15]. These works are extended for generalized G-inflation that takes into account the most general second-order equations of motion for single field models [16,17]. In [18] a large tensor non-Gaussianity and a nearly Gaussian scalar fluctuation is produced using a pseudo-scalar; and in [19] large non-Gaussianity is produced using the coupling between an axion field and a \( SU(2) \) gauge field. However, all of them are model-dependent approaches that rely either on particular (class of) models or on specific mechanisms.

In this Letter we propose a model-independent framework based on Effective Field Theory (EFT) of inflation developed in [20] and derive the most general model-independent third order action for tensor perturbation. We compute the generic tensor bispectrum therefrom. Finally, we search for possible templates for tensor bispectrum and comment on the prospects of detectability in upcoming CMB missions.

Graviton Lagrangian from EFT – In order to propose a generic template in this model-independent analysis, the graviton Lagrangian has to have the following properties: (i) it has to be more or less generic with no \textit{a priori} dependence of particular inflation models or particular mechanism to generate tensor non-Gaussianities, (ii) it has to be consistently derived from EFT and should finally have least number of free parameters. To construct such a generic Lagrangian for the graviton, we make use of the EFT approach developed in [20,21]. In this approach, the inflaton field \( \phi \) is a scalar under all diffeomorphisms but \( \delta \phi \) breaks the time diffeomorphism. Using this symmetry of the system and unitary gauge where \( \delta \phi = 0 \), the most general Lagrangian can be written as

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Since we are primarily interested in three point correlation function, in above Lagrangian terms upto third order in gravitation have been retained. This is purely gravitational Lagrangian where $\delta K^\mu_\nu$ is the fluctuation is the extrinsic curvature; the scalar perturbation is not explicit but can be reintroduced using Stückleberg trick. So the system has three degrees of freedom: one scalar and two tensor. Any time-dependence on the parameters is slow roll suppressed.

To work with tensor perturbations we express the perturbed metric in unitary gauge

$$g_{ij} = a^2(t)[(1 + 2\zeta(t,x)\delta_{ij} + \gamma_{ij}]$$  

(2)

where $\zeta(t,x)$ is scalar perturbation and $\gamma_{ij}(t)$ is tensor perturbation which is transverse and traceless satisfying, $\gamma_{ii} = 0$ and $\partial_i\gamma_{ij} = 0$. In terms of $\gamma_{ij}$ the graviton Lagrangian [1] can be written as,

$$S = \int d^4x\sqrt{-\gamma}\left[\frac{1}{2}M^2_{pl}R - \Lambda(t) - c(t)g^{00} + \frac{1}{2}M_2(t)^4(g^{00} + 1)^2 - \frac{\tilde{M}_1(t)^3}{2}(g^{00} + 1)\delta K^\mu_\mu - \frac{\tilde{M}_2(t)^2}{2}\delta K^\mu_\nu\right.$$  

$$- \frac{\tilde{M}_3(t)^2}{2}\delta K^\nu_\nu\delta K^\mu_\mu + \frac{\tilde{M}_4(t)^3}{3!}(g^{00} + 1)^3 - \frac{\tilde{M}_5(t)^2}{3!}(g^{00} + 1)\delta K^\mu_\nu - \frac{\tilde{M}_6(t)^2}{3!}(g^{00} + 1)\delta K^\mu_\nu$$  

$$- \frac{\tilde{M}_7(t)^2}{3!}(g^{00} + 1)\delta K^\nu_\nu\delta K^\mu_\nu + \frac{\tilde{M}_8(t)}{3!}\delta K^\mu_\nu\delta K^\nu_\rho\delta K^\rho_\mu + ... \right]$$  

(1)

From [3] we get the solution for the mode function as

$$\gamma_k = \frac{H}{M_{pl}(c_\gamma k^3)^{1/2}}(1 + ic_\gamma k\tau)e^{-ic_\gamma k\tau}$$  

(6)

Thus we can derive the two point function for tensor modes in the super horizon limit as

$$\langle \gamma^*_k\gamma^*_k \rangle = \delta_{ss'}(2\pi)^3\delta^{(3)}(k - k')\frac{H^2}{M^2_{pl}c_\gamma k^3}$$  

(7)

To calculate the three point function for tensor perturbation we use IN-IN formalism. In this formalism, computation of the three point function will depend on the interaction Hamiltonian

$$H_{int} = \int d^3x a^3 \left[\frac{M^2_{pl}}{8}(2\gamma_{ik}\gamma_{jl} - \gamma_{ij}\gamma_{kl})\frac{\partial_\rho\delta\gamma_{ij}}{a^2}\right.$$

$$\left.+ \frac{\tilde{M}_9}{3!}\gamma_{ij}\gamma_{jk}\gamma_{ki}\right]$$  

(8)

The first term in (8) is the lowest order contribution of EFT (the so-called Einstein term) and the second term is the contribution of higher power EFT operator.

There can be at most eight combinations of three point functions: $\langle \gamma^*_k\gamma^*_k\gamma^*_k \rangle$, $\langle \gamma^*_k\gamma^*_k\gamma_0^* \rangle$, $\langle \gamma^*_k\gamma_0^*\gamma_0^* \rangle$ and its two cyclic permutations, $\langle \gamma^*_k\gamma_0^*\gamma^*_k \rangle$ and its two cyclic permutations. However, very few of them are independent as it will be revealed later on. Further, as there is no parity violating term we have $\langle \gamma^*_k\gamma^*_k\gamma^*_k \rangle = \langle \gamma^*_k\gamma_0^*\gamma_0^* \rangle$. Consequently, the correlation functions are given by

$$\langle \gamma^*_{k_1}\gamma^*_{k_2}\gamma^*_{k_3} \rangle = \frac{1}{(2\pi)^3}\delta^{(3)}(k_1 + k_2 + k_3)$$  

$$F(s_1k_1, s_2k_2, s_3k_3)\left(\frac{64H^4}{c^2M^4_{pl}}A(k_1, k_2, k_3)(s_1k_1 + s_2k_2 + s_3k_3)^2\right.$$  

$$\left.\frac{4M_9H^5}{M^6_{pl}}\frac{1}{k_1k_2k_3(k_1 + k_2 + k_3)^3}\right)$$  

(9)

where $A(k_1, k_2, k_3) = \frac{k^3_1}{32}(1 - \frac{1}{27}\sum_{i\neq j}k^2_i k_j - \frac{4k_1k_2k_3}{k^3_1})$ with $K = k_1 + k_2 + k_3$, and $F(x, y, z)$ is 

$$F(x, y, z) = -\frac{1}{64\pi^2y^{1/2}}(x + y + z)^3(x + y + z)(x - y - z)(y + z - x).$$

From equation (9) we find that the lowest order EFT contribution to bispectrum depends on the tensor sound
FIG. 1. The amplitude of bispectrum due to the lowest order EFT term as a function of $k_2/k_1$ and $k_3/k_1$.

FIG. 2. The amplitude of bispectrum due to higher power EFT term as a function of $k_2/k_1$ and $k_3/k_1$.

speed $c_\gamma$ (or, equivalently, on the EFT parameter $\bar{M}_9$) whereas the size of the higher power EFT term is solely measured by the parameter $\bar{M}_9$.

In figures 1 and 2 the amplitude of bispectrum (for lowest order and higher power EFT terms respectively) have been plotted as a function of $k_2/k_1$ and $k_3/k_1$. Figure 1 shows that the bispectrum has a peak in the squeezed limit and figure 2 has a peak in equilateral limit. As we will justify later on, the lowest order term has the dominant contribution to the bispectrum. Therefore, chances of detecting squeezed limit tensor bispectrum are fairly higher than equilateral limit. The figures also reveal that only these two limits should be our point of interest while searching for tensor non-Gaussianities in future CMB missions.

Templates and Detectability – Let us first obtain the correlators for tensor modes. The equilateral limit of the correlators look

$$
\langle \gamma_{k_1}^+ \gamma_{k_2}^+ \gamma_{k_3}^+ \rangle |_{eq} = \langle \gamma_{k_1}^- \gamma_{k_2}^- \gamma_{k_3}^- \rangle |_{eq} = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) \frac{H^4}{16c_s^2 M_{pl}} \left(\frac{459}{2} - \frac{\bar{M}_9 H^2 c_s^4}{M_{pl}}\right) \frac{1}{k_1^3},
$$

$$
\langle \gamma_{k_1}^+ \gamma_{k_2}^- \gamma_{k_3}^- \rangle |_{eq} = \langle \gamma_{k_1}^- \gamma_{k_2}^+ \gamma_{k_3}^- \rangle |_{eq} = \langle \gamma_{k_1}^- \gamma_{k_2}^- \gamma_{k_3}^+ \rangle |_{eq} = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) \frac{H^4}{c_s^2 M_{pl}} \left(\frac{17}{96} - \frac{\bar{M}_9 H}{144M_{pl}^2 c_s^2}\right) \frac{1}{k_1^3},
$$

whereas in the squeezed limit, they take the form

$$
\langle \gamma_{k_1}^+ \gamma_{k_2}^+ \gamma_{k_3}^+ \rangle |_{sq} = \langle \gamma_{k_1}^- \gamma_{k_2}^- \gamma_{k_3}^- \rangle |_{sq} = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) \frac{H^4}{16c_s^2 M_{pl}} \left(\frac{3}{k_1^3} - \frac{\bar{M}_9 H c_s^2}{2M_{pl}^2}\right) \frac{1}{k_2^3},
$$

$$
\langle \gamma_{k_1}^+ \gamma_{k_2}^- \gamma_{k_3}^- \rangle |_{sq} = \langle \gamma_{k_1}^- \gamma_{k_2}^+ \gamma_{k_3}^- \rangle |_{sq} = \langle \gamma_{k_1}^- \gamma_{k_2}^- \gamma_{k_3}^+ \rangle |_{sq} = (2\pi)^3 \delta^{(3)}(k_1 + k_2 + k_3) \frac{H^4}{c_s^2 M_{pl}} \left(\frac{3}{k_1^3} - \frac{\bar{M}_9 H^2 c_s^2}{2M_{pl}^2}\right) \frac{1}{k_2^3}.
$$

We can now define the dimensionless “nonlinearity parameter” in the equilateral limit as $f_{NL}^T \sim \frac{\langle \gamma^2 \gamma \rangle}{\langle \gamma^3 \rangle^2}$, where $P_\gamma$ is the dimensionless scalar power spectrum. Using $P_\gamma(k) = \frac{H^2}{8\pi^2 M_{pl}^2 c_s^2} \left(\frac{k}{k_*}\right)^{n_s-1}$ we end up at two independent nonlinearity parameters in equilateral limit:

$$
f_{NL}^{++} |_{eq} = f_{NL}^{--} |_{eq} = \frac{5}{18} \left(\frac{c_s}{c_\gamma}\right)^2 \left(\frac{459}{2} - \frac{\bar{M}_9 H^2 c_s^4}{M_{pl}}\right) \left(\frac{k_1}{k_*}\right)^{-2(n_s-1)}
$$

$$
f_{NL}^{++} |_{eq} = f_{NL}^{--} |_{eq} = \text{cyclic perms} = \frac{40}{9} \left(\frac{c_s}{c_\gamma}\right)^2 \left(\frac{17}{96} - \frac{\bar{M}_9 H}{144M_{pl}^2 c_s^2}\right) \left(\frac{k_1}{k_*}\right)^{-2(n_s-1)}
$$

where the scale $k_1$ is the pivot scale used in the estimation of all cosmological parameters, and hence in estimating tensor $f_{NL}$.

For squeezed limit $k_1 << k_2 = k_3$, $k_1 \sim H$ and we have to measure $f_{NL}^{++} |_{sq}$ at $k_2 = k_3 = aH$. So, we may safely assume that the same definition as equilateral limit i.e., $f_{NL}^{++} |_{sq} \sim \frac{\langle \gamma_{k_1}^+ \gamma_{k_2}^+ \gamma_{k_3}^+ \rangle |_{eq}}{\langle \gamma_{k_1}^+ \gamma_{k_2}^+ \gamma_{k_3}^+ \rangle |_{eq}}$ holds good, at least at the first go. Consequently, in squeezed limit we have three independent nonlinearity parameters:

$$
f_{NL}^{++} |_{sq} = f_{NL}^{++} |_{eq} = \frac{40}{9} \left(\frac{c_s}{c_\gamma}\right)^2 \left(\frac{3}{k_1^3} - \frac{\bar{M}_9 H^2 c_s^2}{8M_{pl}^2}\right) \left(\frac{k_2}{k_*}\right)^{-2(n_s-1)}
$$
are going to detect tensor non-Gaussianity some day, it by future CMB missions. If they are sensitive enough and puts additional constraints. One needs to keep this to-scalar ratio squared for single field inflation models, factor (\(c_{s}c_{\gamma}\))

\[
\frac{5}{18} \left( \frac{c_{s}c_{\gamma}}{c_{s}} \right) ^{2} \left( 3 - \frac{M_{0}H}{2M_{pl}^{2}c_{\gamma}^{2}} \right) k_{1} k_{2}^{-2(n_{s}-1)}
\]

(15)

\[
\frac{40}{9} \left( \frac{c_{s}c_{\gamma}}{c_{s}} \right) ^{2} k_{3} k_{1} k_{2}^{3} \left( \frac{3}{8M_{pl}^{2}c_{\gamma}^{2}} \right) k_{1} ^{2} k_{2}^{-2(n_{s}-1)}
\]

(16)

Equations (12) - (16) are generic, model-independent expressions for all tensor nonlinearity parameters \(f_{NL}^{T}\) of our interest. In this model-independent analysis, along with the usual inflationary parameters (scalar sound speed, scalar spectral index and first slow roll parameter), \(c_{\gamma}\) (or, equivalently, \(M_{3}\)) and \(M_{0}\) are the only two parameters that control the numerical value of \(f_{NL}^{T}\). But even if \(M_{0}\) is as large as \(M_{pl}\) the size of the non-Gaussianity from this operator will be at most of the order of \(\frac{M_{0}}{M_{pl}}\). The well-known bounds on H on pivot scale being \(\left( \frac{H}{M_{pl}} \right)^{5} < 3.6 \times 10^{-5}\) [4]. \(f_{NL}^{T}\) is practically insensitive to \(M_{0}\). Also, \(c_{s}^{2} \leq 1\) anyway. One can also set it to unity by a disformal transformation [22] that results in a modified Hubble parameter. Thus, any effect of the higher derivative EFT term, that mostly reflects the equilateral limit, is sub-dominant. Therefore, the lowest order EFT term, that has a peak in the squeezed limit, has the dominant contribution to the tensor non-linearity parameters \(f_{NL}^{T}\). However, there is an overall factor \((c_{s}c_{\gamma}/c_{s})^{2}\) outside, that is nothing but the tensor-to-scalar ratio squared for single field inflation models, and puts additional constraints. One needs to keep this in mind while investigating for tensor non-Gaussianities by future CMB missions. If they are sensitive enough and are going to detect tensor non-Gaussianity some day, it is expected that they will detect squeezed limit tensor bispectrum first.

The bottomline is that, only equilateral and squeezed limits should be our point of interest while searching for tensor non-Gaussianities in future CMB missions and should be investigated further for analysis and comparison with the sensitivity of upcoming CMB missions. Nevertheless, chances of detecting squeezed limit tensor bispectrum are fairly higher than equilateral limit. We believe these two aspects can indeed serve as motivation to investigate for the non-Gaussian effects of primordial GW in future CMB missions using the generic templates proposed in this article.

Conclusions – In summary, let us highlight the major developments made in this Letter:

- Developed a model-independent framework for calculating tensor non-Gaussianities based on EFT of inflation.
- Proposed generic templates for tensor non-Gaussianities that may be useful for future CMB missions.
- Discussed why only equilateral and squeezed limits should be our point of interest while searching for tensor non-Gaussianities in future CMB missions.
- Found that chances of detecting squeezed limit tensor bispectrum are fairly higher than equilateral limit.

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