Development of methods for buildings calculation from sandwich panels for transformable low-rise buildings

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Abstract This paper focuses on calculating the main loads of prefabricated low-rise buildings from sandwich panels of arbitrary structural elements and connections in the form of a rigid spatial system. The main objective of this research are the theoretical and experimental substantiation of rational parameters of quickly erected transformable low-rise buildings from sandwich panels for the development of optimal design solutions to ensure spatial rigidity. The research used empirical research method - observation, experiment, measurement, comparison and analysis. The quickly erected transformable low-rise buildings of sandwich panels, like a prismatic shell consisting in the transverse direction of two closed rectangular contours, can bear an arbitrary power load. Determination of the cost of materials for additional structural elements in the form of ties (embedded parts), combining the individual elements in a holistic design that provides spatial rigidity of the structure.

Key words: prefabricated low-rise buildings of sandwich panels; power load; spatial stiffness; doubly connected shell; deformed state.

1. Introduction
Sandwich panels are in active use as enclosing constructions for different types of buildings [1]. Loads and temperature changes are essential in the analysis of sandwich panels’ behavior. The sandwich panel is a multi-layer structural element consisting of thermal insulation located between flat or profiled metal sheets and fixed to the buildings bearing structure by means of threaded screws. These screws often create “thermal bridges” cross the sandwich panel system [2-5].

The construction of low-rise buildings for various purposes in the mass modern construction is determined by their constructive-technological and space-planning decisions, as well as the requirements for ensuring the reliability of buildings [6].

Unsuccessful design solution (flat design scheme of the structure) causes the presence of an incorrect distribution of material and partly the work of the structure in the spatial version. This causes economic losses in the construction of structures and buildings.

Currently, the existing facilities can be divided into two main groups:

i. structures with a strongly pronounced nature of spatial work and;
ii. structures with a weakly pronounced nature of spatial work.

The first group of structures includes: domes, warehouses, arches, various shells (cylindrical, flat, double Gaussian curvature, negative Gaussian curvature, etc.), generous coverings, extended and high-rise frame-panel buildings. A clear line on the accessories in this group is difficult to hold [7].

In [8], the authors revealed the nature of the deformation of the fastening system on the basis of a series of preliminary numerical experiments performed with the application of a uniformly distributed
static load to the system, as well as loads arising from the difference in temperatures outside and inside the building. Calculated results obtained by authors and compared with theoretical calculations, however, during the experiment, the joint operation of the sandwich panel system with the building frame was not taken into account.

Investigations in this direction will make it possible to more accurately determine their stress – strain state, taking into account the boundary conditions, to design, construct and make them easier and more economical [9].

The second group includes the usual buildings: industrial workshops (multi-span in cross section), garages, hangars, pavilions, etc. Such structures very often can be represented as a set of trusses, arches, frames and other elements and structures interconnected by different types of connections into the spatial framework of structures. Basically, the supporting frame corresponds to a flat design scheme of a conventional building. The load from the coating is transferred to the trusses, from the trusses to the columns and from the columns to the foundations and the basis [10-12].

In such design schemes of structures, material costs are additionally required for additional structural elements in the form of various connections (embedded parts, welding materials, etc.). These links unite the individual elements into an integral structure and provide the spatial rigidity of the structure. Thus, it turns out that many constructions made up of flat structural elements work only partially as spatial systems and are too material-intensive [13-15].

This paper considers development of methods for calculating buildings from sandwich panels of three-layer, for quickly erected transformable low-rise buildings. Derivation of numerical methods for development of sandwich panels for structures will be considered.

2. Materials and methods

When calculating, the presented building is considered as a prismatic envelope of a multiply connected section. Bearing frame of the building creates longitudinal external walls and floors of the first and second floor. In the longitudinal direction along the three nodal points of the upper belt, a closed multiple-connected cross-section, external vertical loads are applied, and at the extreme points there are additional expansions (horizontal load) from the coating.

This prismatic shell with dimensions in the plan of 6 × 9 m. along the whole contour is supported by longitudinal and transverse supports (Figure 1). The cross section of the shell is divided into separate sections of different sizes, and then set of longitudinal strips is obtained. As a result, the horizontal and vertical stripes with a length of 9 m form a doubly connected shell (Figure 2).

![Figure 1](image1.png) **Figure 1.** Pre-fabricated transformable low-rise long building of sandwich panels.

![Figure 2](image2.png) **Figure 2.** Horizontal and vertical stripes 9 m form a doubly connected shell.
The width of the vertical stripes $d_1 = 2.7$ m. The width of the horizontal stripes $d_2 = 3$ m. In places of pairing, bands locate stringers, working only in tension and compression. The role of stringers is played by wooden bars with a rectangular section. Geometric and mechanical characteristics of the extreme and middle stringers are given in table 1.

### Table 1. Geometric and Mechanical Characteristics of Extreme and Medium Stringers.

| № | $a_{11}D^2-a_{11}'$ | $-a_{11}'$ | $-b_1D$ | $-b_1D$ | $\frac{1}{G}P_i$ |
|---|-------------------|------------|---------|---------|-----------------|
| 1 | $-a_{21}'$ | $a_{22}D^2-a_{22}'$ | $-b_{21}D$ | $-b_{21}D$ | $\frac{1}{G}P_i$ |
| 2 | $b_1D$ | $b_{21}D$ | $C_{11}D^2-\gamma_{11}$ | $C_{11}D^2-\gamma_{11}$ | $\frac{1}{G}P_i$ |
| 1 | $b_{21}D$ | $b_{22}D$ | $C_{21}D^2-\gamma_{21}$ | $C_{21}D^2-\gamma_{21}$ | $\frac{1}{G}P_i$ |
| 2 | $b_{22}D$ | $b_{22}D$ | $C_{22}D^2-\gamma_{22}$ | $C_{22}D^2-\gamma_{22}$ | $\frac{1}{G}P_i$ |

At the end, the multiple-connected shell has supporting diaphragms that are rigid in their plane and flexible from their own plane. The design of the end walls allows presentation of their work as a hinge. The end walls in the vertical plane (in their own plane) are not deformed, and in the horizontal direction (their plane) are considered absolutely flexible.

The design of the horizontal plates, i.e. the floors of the first and second floors are such that the longitudinal displacements of their cross sections cannot be obstructed, i.e. normal section forces will not be able to accept (or will accept them in a small volume).

Therefore, it is believed that these plates do not work in tension – compression, but work mainly in shear. Longitudinal normal forces are perceived only by stringers. Horizontal plates can perceive transverse normal forces and only after reinforcement with frames transverse bending moments.

In this case, the presented prismatic shell of a multiple-connected section without special reinforcement will have transverse frames in the transverse direction with a moment-less structure.

As a result, the shell reinforced in the longitudinal direction, by stringers operating exclusively in tension – compression is taken as the design scheme of the presented prismatic shell.

Thus, given the physical and structural properties of the building in question, when calculating it for a vertical load, it is sufficient to consider two states: symmetric and anti-symmetric.

Further, the calculation method is developed and used variation theory of V.Z. Vlasov [16] reduction of two-dimensional and three-dimensional problems to one-dimensional.

When calculating the required unknowns, the functions of longitudinal and tangential displacements are taken, which are represented as a product of two functions. One function is the desired one, and the second characterizes the pattern of its distribution in the transverse direction. It is believed that the prismatic shell has infinite in the longitudinal and finite in the transverse direction the degree of freedom. According to Vlasov, such systems are called discrete-continual.

The position of a point on the middle prismatic surface of the shell is determined by the coordinates $x$ and $s$. The first coordinate indicates the point's distance from the initial, extreme cross section $x = 0$, and the second coordinate $s$ determines the location of the point in the cross-sectional plane relative to the initial line (assumed as the initial) of the generator.

The longitudinal $u_i(x, s)$ and transverse $v_i(x, s)$ displacements of an arbitrary point with coordinates $(x, s)$ on a prismatic surface are represented as the following finite expansions:

$$ u(x, s) = \sum_{i=1}^{\infty} u_i(x)\xi_i(s) $$ (1)
Here, the first equation expresses the deformed state of an elementary transverse strip of width $dx = 1$, carried out by longitudinal displacements $u_i(x, s)$. The required functions $u_i(x), u_2(x), \ldots, u_m(x)$ are taken as longitudinal displacements of $m$ nodes of the shell. The functions $\xi_1(s), \xi_2(s), \ldots, \xi_m(s)$ corresponding to the longitudinal displacements must satisfy the conditions of continuity and continuity of the longitudinal displacements.

The functions $\xi_i(s)$ for the specified method of selecting longitudinal displacements $u_i(x)$ are non-zero only in straight sections of the contour of the cross section of the shell, at point $i$ have a single value and vary in these areas according to a linear law. At the remaining points of an elementary transverse strip of width $dx = 1$, these functions are equal to zero. Moreover, the chosen method of constructing a function $\xi_i(s)$ is not exclusively unique and depends on the choice of the desired functions $u_i(x)$.

Equation (2) determines the deformed state of the elementary strip $dx = 1$ in the cross section plane $x = \text{const}$. The transverse strip $dx = 1$ is considered as a core frame of a multiply connected section with inextensible, tensile or partially tensile elements. Then, for the required functions in equation (2), functions $v_j(x)$ are taken that characterize the shape of the deformation of the core system in its plane.

The number of degrees of freedom $n$ of the rod system under consideration is equal to the number of unknown functions $v_j(x)$ in its own plane and is determined by the formula:

$$n = 2m - c$$

where $m$ – number of nodes; $c$ – number of rods.

We compose the expressions of possible work of external and internal forces acting on an elementary transverse strip of a prismatic shell of a multiple-connected section on its possible displacements in and out of the plane. The resulting system with a total number of $(m + n)$ differential equations will represent the integral form of the equilibrium condition of an elementary transverse strip of width $dx = 1$.

Thus, the desired functions $u_i(x)$ and $v_j(x)$ will be determined from the specified systems of differential equations:

$$\int \frac{\partial \sigma}{\partial x} \xi_k(x, s) dF - \int \tau_{k}^{s} (x, s) dF + \int P_{x} \xi_k(x, s) ds = 0; \ (k = 1, \ldots, m)$$

$$\int \frac{\partial \tau}{\partial x} \eta_h(x, s) dF - \sum v_j \int \frac{M_{y}}{EY} dF + \int P_{y} \eta_h(x, s) ds = 0; \ (h = 1, \ldots, n)$$

These equations include normal and tangential stresses, bending $x = \text{const}$.

In these equations, the extreme members with a plus sign express the work of external forces $P_x$ and $P_y$, both the work of normal and shear internal forces.

The middle term with a minus sign represents the work of the shear forces $dF$ on the shear deformation represented by the derivative $\xi'_k(s)$, as well as the work of the bending moment $\sum v_j(x) M_y(s)$ on the mutual angle of rotation $\frac{M_y(s)}{EY} ds$.

Normal and tangential stresses $\sigma = \sigma(x, s) ; \ \tau = \tau(x, s)$ acting in the cross sections of the prismatic shell are considered uniformly distributed throughout the thickness of the section of the prismatic shell and are expressed by the known formulas:
\[
\sigma = E \frac{\partial u}{\partial x}, \quad \tau = G \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \right) \tag{6}
\]

Taking into account the decomposition (1) and (2) for these stresses, we obtain the following expressions:

\[
\sigma(x, s) = \sum_{j=1}^{n} u_j'(x) \xi_j(s) \tag{7}
\]

\[
\tau(x, s) = G \left[ \sum_{j=1}^{m} u_j'(x) \xi_j'(s) + \sum_{j=1}^{m} v_j'(x) \eta_j(s) \right] \tag{8}
\]

where \(E\) and \(G\) are the deformation and shear moduli; \(dS = d\alpha ds\) – differential cross-sectional area of the shell.

\(M_1(s)\) and \(M_2(s)\) represent the bending moments of the transverse strip \(dx = 1\) - the frames corresponding to the elementary states of deformation of this frame with \(v_k = 1\) and \(v_h = 1\).

\(J(s)\) - moment of inertia of the transverse strip width \(dx = 1\). If the shell in sections has only rectangular shapes, then the moment of inertia will be determined by the following formula:

\[
J = \frac{1}{12} \delta^3 \tag{9}
\]

In the case when the shell is reinforced with additional cross-frames, then the moment of inertia of the cross-section of the shell is calculated taking into account the average moment of inertia of these frames; in other words, the moment of inertia per unit length of the shell is taken.

After substituting the values of normal and shear stresses into (4) and (5), \(\sigma_i, \tau_j\) from formulas (7) and (8) we obtain the system of ordinary differential equations of the second order:

\[
\sum_{j=1}^{m} a_{ij} \frac{d^2}{dx^2} u_j'(x) - \sum_{j=1}^{m} a_{ij} u_j' - \sum_{j=1}^{m} b_{ij} v_j' + \frac{1}{G} P_{ai} = 0 \tag{10}
\]

\[
\sum_{j=1}^{m} b_{ij} u_j'' + \sum_{k=1}^{n} C_{ik} v_k' - \gamma \sum_{j=1}^{m} \xi_j v_j + \frac{1}{G} P_{ai} = 0 \tag{10}
\]

The coefficients of this system are determined by the formulas:

\[
a_{ij} = \int \xi_i'(s) \xi_j(s) dF; \quad b_{ik} = \int \xi_i'(s) \eta_k(s) dF; \quad c = \int \eta_i(s) \eta_k(s) dF \tag{11}
\]

\[
a_{ij}' = \int \xi_i'(s) \xi_j'(s) dF; \quad b_{ik}' = \int \xi_i'(s) \eta_k'(s) dF; \quad c_{ik} = \frac{1}{E} \int M_i(s) M_k(s) ds \tag{11}
\]

In these formulas, the integrals apply to all elements of the cross section of a multiple-connected prismatic shell.

Based on the Betty theorem on the reciprocity of the elastic system, here is the following equality of coefficients:

\[
a_{ij} = a_{ji}; \quad b_{ik} = b_{ki}; \quad S_{ik} = S_{ki}; \quad a_{ij}' = a_{ji}' \tag{12}
\]

\[
a_{ij}' = a_{ji}' \quad \text{when} \quad h = k \tag{12}
\]

The free terms of the system of equations (10) in the presence of the surface forces of the shell \(P_s(x, s); \quad P_i(x, s)\) are according to the formulas:

\[
P_s(x) = \int P_s \xi_i(s) ds \tag{13}
\]
P_i(x) = \int P_s \eta_i(s) ds

Free members \( P_s, P_s' \) are generalized external forces. \( P_i(x) \) is calculated as the work of external surface forces \( P_s(x,s) \) on the longitudinal movement \( \xi_i(s) \) of the elementary transverse strip in the state \( u_j = 1 \). The transverse force \( P_j(x) \) is calculated as the work of external surface contour forces on the contour displacements of the elementary strip \( \eta_i(s) \) in its plane in the state \( v_n = 1 \).

The system of ordinary differential equations of the second order in expanded form is given in Table 1. In this table, each element is a linear ordinary differential operator with a constant coefficient.

Operators \( D^2 \) and \( D \) show that the corresponding required function is required to take the second and first derivative with respect to the \( x \) coordinate.

By the method of academician L.N. Krylov [17], symmetric system \((m + n)\) of ordinary linear differential equations with constant coefficients (10) can be reduced to an equivalent differential equation of order \( r \) \((m + n)\). However, if there is a large order of the system of equations, the reduction method becomes cumbersome and its implementation becomes more complicated. Therefore, the solution of equation (10) is easier to search in rows. Depending on the boundary conditions, the desired functions \( u_i(x) \) and \( v_j(x) \) longitudinal and tangential displacements are decomposed into trigonometric series.

\[
\begin{align*}
  u_i(x) &= \sum_{i=1}^{m} u_i \cos\left(\frac{(2n-1)n\pi}{l}\right), \\
  v_i(x) &= \sum_{i=1}^{m} v_i \sin\left(\frac{(2n-1)n\pi}{l}\right),
\end{align*}
\]

where \( u_i(x) \) and \( v_i(x) \) – unknown coefficients of the \( n \)-th row member; \( L \) – shell length (distance between the end diaphragms).

The longitudinal normal voltage of the \( i \)-th state is represented as the following series:

\[
\sigma_i'(x,s) = E\xi_i(s)u_i = -E\xi_i(s)\sum_{i=1}^{m} u_i \left(\frac{2n-1)n\pi}{l}\right) \sin\left(\frac{(2n-1)n\pi}{l}\right)
\]

With the hinge of the prismatic shell, the boundary conditions are as follows:

\[
\begin{align*}
  x &= 0; \quad \sigma_i(0) = 0; \quad v_i(0) = 0 \\
  x &= l; \quad \sigma_i(l) = 0; \quad v_i(l) = 0
\end{align*}
\]

In the presence of a uniformly distributed external vertical load with intensity \( q \) applied along the edges (nodes) of the prismatic shell, the decomposition form of the external load will have the following form:

\[
q(x) = \sum_{i=1}^{m} \frac{4q}{(2n-1)n\pi} \sin\left(\frac{(2n-1)n\pi}{l}\right)
\]

The system of algebraic equations for the \( n \)-th term of the expansion is obtained by substituting a differential matrix \( D^2 - \frac{(2n-1)n\pi^2}{l^2} \), for the first and fourth quadrants instead of the operator (see Table 1), and in the second and third quadrant \( D \) is replaced by \( \frac{(2n-1)n\pi}{l} \). Free terms are multiplied by a factor \( \frac{4}{(2n-1)n\pi} \). The final system of algebraic equations is given (see Table 1).

Solving a system of algebraic equations table 1, we obtain for each term the decomposition of the longitudinal \( u_i \) and transverse (vertical) \( v_k \) displacements.
Thus, the type of functions $U_i$ and $V_k$ is set. After their substitution in (1) and (2) for the $n$-th term of the expansion, we find the values of the sought functions:

$$
U_n(x,s) = \sum_{j=1}^{n} u_j(x) \xi_j(s) = \sum_{j=1}^{n} u_j \cos \left( \frac{(2n-1)\pi}{l} \xi_j(s) \right)
$$

$$
V_n(x,s) = \sum_{j=1}^{n} v_j(x) \eta_j(s) = \sum_{j=1}^{n} v_j \sin \left( \frac{(2n-1)\pi}{l} \eta_j(s) \right)
$$

On the basis of the data obtained (meaning the displacement of characteristic points), the stress–strain field of the prismatic shell is established by the usual methods of structural mechanics.

3. Results

The prismatic shell of a quickly erected transformable low-rise buildings from sandwich panels, consisting in the transverse direction of two closed rectangular contours, is considered. The geometric dimensions of the shell elements are given. The cross section of the shell has two axes of symmetry: vertical and horizontal. Such a shell can carry an arbitrary power load. In the particular case, it is possible to consider the option when an external vertical load applied on the ribs (along the nodes) according to an arbitrary law acts on the shell. Due to symmetry, the external load can be expanded and its two states under load can be considered: the shell is operating under symmetric and anti-symmetric loads.

Each horizontal plate of the shell has a width $d_0$ (cm), vertical plates have a width $d_1$ (cm), let us assume that in the first approximation all plates have the same thickness $t$. For each plate, the law of flat sections is assumed to be fair. Also, in the first approximation, the plates on their width are considered inextensible. Note also that formulas (11; 13) allow us to take into account the compressibility of the plates. For the generalized coordinates $\xi_1(s), \xi_2(s)$ the functions that are symmetric with respect to the vertical and back symmetric with respect to the horizontal plane are selected (see Fig. 1). With this choice of functions $\xi_1(s), \xi_2(s)$ the sought generalized displacements $U_1(x), U_2(x)$ will be numerically equal to the longitudinal displacements at the nodal points of the cross-section contour.

The functions $\xi_1(s), \xi_2(s)$ determine two independent components of the cross section deplanation related to the equilibrium state of the shell, symmetrical about the vertical plane of symmetry (figure 3).

In principle, horizontal movements or movements of vertical plates are taken as generalized movements. Type of functions $\eta_1(s)$ when $v_1 = v_2 = 1$ are given (see figure 4).

In the case of a symmetrical load, the deflections of horizontal plates in the horizontal direction are assumed to be zero. In figure 4 the arrows indicate the positive direction of deflection of the plate. Based on the above, the generalized equilibrium conditions of a strip of width $dx = 1$ in the form of ordinary differential equations with the unknown functions $U_1(x), U_2(x), V_1(x), V_2(x)$.

Figure 3. State of equilibrium of the shell, symmetrical with respect to the vertical plane of symmetry.
Figure 4. Positive direction of plate deflection.
4. Conclusion

Derivatives \( \xi_1'(s), \xi_2'(s) \) of the given functions \( \xi_1(s), \xi_2(s) \) in certain parts of the cross-section contour are constant values (not dependent on the \( s \) coordinate). The arrows indicate the directions in which the corresponding differentiable function increases.

Similarly, the functions of the transverse distribution of displacements are considered \( \eta_j(s) \). They characterize the deformation of an elementary transverse strip in the section plane \( x = \text{const} \). The degree of freedom of an elementary transverse strip, as a flat hinge-rod system, under the condition of the inextensibility of its elements in a symmetric state is equal to two. This means that the deformed symmetric state of the contour of the shell in the plane of its cross section is determined by two parameters. These parameters are generalized transverse displacements \( v_1(x), v_2(x) \) depending on the \( x \) coordinate.

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