Drive force optimization of a pneumatically-driven Gifford-McMahon cryocooler by numerical modeling

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Abstract. Pneumatically-driven displacer mechanisms are widely used in Gifford-McMahon (G-M) cryocoolers. Particularly for large size G-M cryocoolers, this type of drive is preferable compared to the scotch yoke-type, as only a small motor is required for driving the rotary valve and, therefore, the entire cryocooler can be very compact. Though various numerical models of G-M cryocoolers have been presented in the past, modeling of the pneumatically-driven type and optimization of the driving force have rarely been done. This work presents a one-dimensional numerical model of a pneumatically-driven single stage G-M cryocooler running at 80 K and related studies. The transient model predicts the movement of the displacer and simulates the cryocooler performance simultaneously. An optimization of the drive force is implemented for the defined cryocooler operating at various speeds. It discovers that the driving force reshapes the P-V diagram effectively and has to be well designed according to the frequency to maximize the cooling capacity per mass flow rate.

1. Introduction

Gifford-McMahon cryocoolers have been broadly used in applications including recondensing the boil-off liquid helium in magnetic resonance imaging (MRI) systems, cooling scientific components, cryopumps and small-scale liquefiers [1]. As one of the key technologies, synchronizing the displacer movement with valve timing is critical to obtain the desired P-V (pressure-volume) diagram and cooling performance. Two types of drive mechanisms, scotch yoke and pneumatic drive, have been applied for decades to drive the displacer of G-M cryocoolers. Due to the direct mechanical coupling between the displacer movement and valve timing, the scotch yoke-type is preferable for timing control. In addition, the relative timing is independent of the operating frequency. However, this drive mechanism requires an additional torque to drive the displacer against the frictional force, gravity and other forces based on pressures. As a G-M cooler is scaled to a higher capacity, the size of the displacer and cylinder should increase simultaneously to keep the design near a thermodynamic optimum. To drive such a large displacer, an oversized and impractical motor sometimes is required. In contrast, the pneumatic drive system utilizes a small amount of high and low pressure gas from the compressor for driving purpose, and only a small motor with lower torque output is needed to drive a rotary valve, making it feasible for large-scale G-M cryocoolers especially.

In order to analyze pneumatically driven G-M cryocoolers, researchers have developed several analytical and one-dimensional (1D) numerical models. As the pressures in various chambers determine the total force applied on the displacer, predicting the displacer movement requires solving the
displacer’s momentum equation coupled with other governing equations describing the gas flow. This has only been done by a few existing models. Minas and Hualde [2] presented an analytical model to study the dynamic characteristics of a pneumatically-driven G-M cryocooler by solving a set of six simultaneous differential equations. Later, Minas [3] modified the previous model to study a two-stage cryocooler and non-impacting displacer motion was addressed. Kurihara and Fujimoto [4] developed a numerical model to predict the displacer movement, P-V diagram and performance of a two-stage pneumatically-driven 4 K G-M refrigerator. Kurihara et al. [5] improved the above-mentioned model and used it to analyze a two-stage 4 K modified Solvay cycle cryocooler. The experimentally-obtained optimum P-V diagram and valve timing were in good agreement with the numerical simulation results. However, the influence of the driving force has not been investigated thoroughly in these studies. Besides, the frequency plays an important role in controlling the relative timing in a pneumatically-driven system, which deserves a careful investigation at the same time. This paper uses an improved 1D numerical tool to investigate how the driving force and frequency interactively influence the P-V diagram, and then the cooling performance of a pneumatically-driven G-M cooler.

2. Numerical model
As shown in figure 1, a single-stage pneumatically-driven G-M cryocooler is defined for modeling. A displacer, which is packed with porous regenerative materials, is driven to reciprocate in a cylinder by pressurized gas in the drive, compression and expansion chambers. A rotary valve with four ports V1-V4 controls helium gas to flow into/from the drive space and cryocooler. In general, the gas intake of the cryocooler starts when the displacer is at the lowest position. Then the drive chamber is depressurized and the displacer is pushed up as the upward force becomes dominant. A similar process occurs when the displacer is at the highest position, that is, the drive chamber is pressurized after the exhaust process starts, which pushes the displacer down. The driving force can be adjusted by changing the diameter of a drive stem or the size of an orifice between the valves and the drive chamber. Here, a fixed orifice is used and this work mainly focuses on changing the drive stem to adjust the driving force.

A 1D numerical model is improved in order to simulate such a pneumatically-driven G-M cryocooler. Similar to the previous models presented by Wang [6], as well as Xu and Morie [7], the core of this model solves the governing equations of helium gas and the energy equation of the solid to obtain the solution of the flow field. In detail, the governing equation set consists of the continuity, simplified momentum and energy equations of helium gas:

\[
\frac{dm}{dt} = \dot{m}_{in} - \dot{m}_{out} \tag{1}
\]

\[
\frac{dP}{dx} = -f_{r} \rho u^2 \frac{u}{|u|} \tag{2}
\]

\[
\frac{\partial (\dot{m} h)}{\partial t} + (\dot{m} h)_{out} - (\dot{m} h)_{in} + \alpha_{ht} A_{ht} (T - T_{s}) - V \frac{dP}{dt} = 0 \tag{3}
\]

where \(m, \dot{m}, \rho, P, u, h, V\) and \(T\) are the mass, mass flow rate, pressure, temperature, density, velocity, enthalpy and volume of the helium gas; \(t\) and \(x\) are the time and spatial position; \(f_{r}\) is the friction factor; \(\alpha_{ht}\) and \(A_{ht}\) are the heat transfer coefficient and area; \(T_{s}\) is the temperature of the solid, respectively.

The regenerator is packed with wire screens. Correlations of the friction factor and heat transfer coefficient presented by Kays and London [8] are used in this model for this geometry.

![Figure 1. Schematic of a pneumatically-driven Gifford-McMahon cryocooler](image-url)
Further, the energy equation of the solid is:
\[
\frac{\partial (m_s c_s T_s)}{\partial t} = \alpha_h T_s (T - T_s)
\]
where \(m_s\) and \(c_s\) are the mass and specific heat of the solid. This equation is coupled with the energy equation of helium gas by the convection term \(\alpha_h (T - T_s)\). These four equations are discretized in both spatial and temporal domains based on the finite volume method and solved by iteration. More details about solving these equations numerically have been presented by Refs. [5], [6] and [7], which will not be repeated here. As this G-M cryocooler is assumed to work at 80 K, ideal gas properties are used.

The model is improved by adding the momentum equation of the displacer into the previous equation set, which is:
\[
m_{disp} \frac{d^2 x_{disp}}{dt^2} = F_{disp} = m_{disp} g + F_{drive} + F_{comp} - |F_{expa}| - |F_{seal}| \frac{v_{disp}}{|v_{disp}|}
\]
where \(m_{disp}, x_{disp}\) and \(v_{disp}\) are the mass, displacement and velocity of the displacer; \(F_{disp}, F_{drive}, F_{comp}, F_{expa}\) and \(F_{seal}\) are the total force applied on the displacer, force on the drive stem, force on the top of the displacer, force on the bottom of the displacer, and frictional force of the dynamic seals; \(g\) is the gravitational acceleration. \(F_{expa}\) always pushes upward and the absolute value of \(F_{seal}\) is assumed to be constant in this work. The lower and upper limits of \(x_{disp}\) are 0 and the stroke, respectively. The displacer stops immediately when it hits the top or bottom of the cylinder, and the deceleration process during the collision, which happens very fast in reality, is not considered.

This momentum equation is tightly coupled with the above-mentioned governing equations. The information about pressures in different chambers are needed to determine the forces \(F_{drive}, F_{comp}\) and \(F_{expa}\). Moreover, its solution, \(x_{disp}\), is required to calculate the chamber volume \(V\) for solving the flow field. Hence, an improvement over the original flow solver is implemented to solve the new equation set simultaneously. An initial guess of \(x_{disp}\) is given to the flow solver, and the solution is approached by iteration. Once the convergence is reached, the model continues to the next time step until the defined cryocooler reaches the steady state. At the end of modeling, the information of the last cycle, as well as other performance data, will be output.

3. Results and discussion

Simulations assume the supply and return pressures to be 2.1 and 0.7 MPa, respectively, and they are kept constant. This also means that the compressor has an infinite capacity to supply helium gas. The cryocooler operates at 80 K. Both compression and expansion chambers have a diameter of 50 mm and the stroke of the displacer is 25 mm. The frictional force applied on the displacer is 20 N, and its direction is always opposite to the displacer’s velocity. The displacer weighs 1.5 kg. For both the gas intake and exhaust, each process takes 1/3 of the cycle time and the timing leads the depressurization or pressurization process in the drive chamber, respectively. Here, no attempt is made to optimize the valve timing or cryocooler geometry and this paper emphasizes how the drive mechanism affects the P-V diagram and the cryocooler’s performance. For a pneumatically-driven G-M cooler, the opening or closing of the valve to the drive chamber determines when the displacer dispatches from the highest or lowest position. However, the displacer speed depends on the driving force and may strongly influence the P-V diagram. The driving force is mainly determined by the pressures and size of the drive stem. Moreover, unlike the scotch yoke, the relative timing between the gas flow into/from the cryocooler and the displacer movement shifts with the frequency, which also reshapes the P-V diagram consequently. The main parameters used in modeling are summarized in Table 1.

**Figure 2** shows the interactive effect of both the diameter of the drive stem and frequency on P-V diagrams. In general, with the same frequency, the P-V diagram is close to a rectangle when the drive stem is the largest, e.g., the case with a frequency of 2.5 Hz and a stem diameter of 8.25 mm in figure 2 (a). Due to the effects of the large driving force and the relatively long period of gas intake, the displacer moves too fast and reaches the highest position before the intake valve closes. Afterwards, when the
exhaust valve opens, the expansion occurs from the highest pressure with the displacer stationary, which forms the rectangular P-V diagram.

| Parameter                              | Value       |
|----------------------------------------|-------------|
| Supply pressure                        | 2.1 MPa     |
| Return pressure                        | 0.7 MPa     |
| Working fluid                          | Helium      |
| Cooling temperature                    | 80 K        |
| Diameter of compression chamber        | 50 mm       |
| Diameter of expansion chamber          | 50 mm       |
| Regenerator material                   | Wire screens|
| Stroke of the displacer                | 25 mm       |
| Frictional force applied on the displacer | 20 N       |
| Displacer mass                         | 1.5 kg      |
| Gas intake time                        | 1/3 cycle   |
| Gas exhaust time                       | 1/3 cycle   |

When a smaller drive stem is used, both the top-right and bottom-left corners of the P-V diagrams are cut off, and the trimmed areas become larger in all four plots of figure 2. In those cases, the displacer is still in the middle of the stroke and keeps moving up, when the gas intake stops. At this moment, the cryocooler is isolated from the external gas source, and the moving displacer forces the warm gas in the compression chamber to flow into the expansion chamber through the regenerator. As the helium gas is pre-cooled by the regenerator, the pressure in the expansion chamber, as well as the whole cryocooler, decreases, and this forms a “pre-expansion” process. This is reflected by the cut-off corner at the top-right side of the P-V diagram. Next follows the expansion process from a middle pressure when the exhaust valve opens. However, once the stem size is smaller than a certain value, e.g., 7 mm stem for figure 2 (a) with a frequency of 2.5 Hz, the exhaust period starts before the displacer hits the top of the cylinder, which leads to insufficient expansion and significantly cuts the P-V diagram. Another cut-off corner in the bottom-left side is also observed when the stem becomes smaller, which is formed during the downward movement of the displacer. With such a smaller stem, the displacer moves slower and the exhaust period stops before the displacer hits the cylinder bottom, which results in a “pre-compression” process due to the re-heating in the regenerator. With a closer look, it is found that the cut-off corner for pre-compression is always smaller than that of the pre-expansion. That’s because the displacer moves faster during the downward movement, as a result of the large force based on pressures and gravity.

The P-V diagrams at 3.0, 3.5 and 4.0 Hz are illustrated in figures 2 (b)-(d). With higher frequency, the periods of the gas intake, gas exhaust and dwell become shorter inherently. If a scotch yoke system is used, the displacer’s speed increases proportionally to the frequency. However, in a pneumatic drive, the displacer speed relies on the driving force based on pressures. This leads to a relatively earlier start of the pre-expansion or pre-compression process for high frequency operation. Therefore, the cut-off corners become larger with a higher frequency as displayed in figure 2 (b)-(d). It also indicates that the driving force has to be optimized to match the frequency for a desired P-V diagram.

In general, the P-V power is proportional to the cycle frequency and swept area of a P-V diagram. As shown in figure 3, the calculated P-V power increases with the drive stem size, due to the larger P-V diagram as depicted in figure 2. When the drive stem is as small as 6.50 and 6.75 mm, lower frequency leads to slightly higher P-V power, as the swept area in one cycle is larger. The P-V power reaches its maximum when the P-V diagram becomes rectangular. When the stem size is larger than 7.25 mm,
higher frequency always provides larger P-V power and it contributes more to the increase than the stem size. Correspondingly, the cooling capacity in figure 4, which is the heat absorbed from the heat station, has a very similar pattern as the P-V power. It increases with the stem size and reaches its maximum when the P-V diagram becomes rectangular. The maximal cooling capacity is about 142 W when the frequency is 2.5 Hz and it increases to about 195 W with a 4 Hz operation.

Although a higher cooling capacity is obtained with such a rectangular P-V diagram, it requires a larger mass flow rate supplied from the compressor as well. The average mass flow rates in half a cycle (intake or exhaust) are calculated and presented in figure 5. As the drive stem size increases, the cut-off corners of the P-V diagram become smaller, which also increase the average mass flow rate. Similar to the P-V power, the average mass flow rate reaches its maximum when the driving force is large enough.

Note that there is another small loss associated with the pneumatic drive, which is the gas utilized for driving the displacer. It should be counted from the point of view of the total power consumption. This loss is proportional to the ratio of the mass flow rate used for driving to total mass flow rate, which is about 1.9 - 3.5% in this study.

The difference between the P-V power and cooling capacity is actually the regenerator loss due to imperfect heat transfer in the regenerator with a finite volume. The regenerator ineffectiveness increases with the mass flow rate and is normally required to be less than 1%. As shown in figure 6, a large stem size leads to higher regenerator loss, which is mainly attributed to the increased mass flow rate. It takes about 12.4-14.6% of the P-V power.
As mentioned above, the mass flow rate varies in different work conditions. Therefore, a direct comparison of the cooling capacity is not so reasonable. Specific cooling capacity, which is the ratio of the cooling capacity to mass flow rate, measures the utilization of helium gas flow from the compressor and is a more meaningful parameter for comparison purposes. The results of the specific cooling capacity in Figure 7 show that an optimal drive stem size exists for maximizing the specific cooling capacity. The optimal stem size increases from about 6.8 to 7.6 mm with a frequency ranging from 2.5 to 4.0 Hz, which leads to a growth in the specific cooling capacity from about 84 to 105 J/g. With a too small drive stem, i.e., little driving force, the exhaust process starts before the displacer reaches the highest position and no more gas is entering the expansion chamber and the gas in the whole cryocooler begins to exhaust. As a result, the P-V power becomes much smaller and an insufficient expansion occurs, which results in a low specific cooling capacity naturally. However, too large of a driving force also reduces efficiency, because the large mass flow rate associated with a rectangle-like P-V diagram lowers the regenerator effectiveness, and then increases the regenerator loss. Consequently, the specific cooling capacity becomes smaller as well.

4. Conclusions
A pneumatically-driven G-M cryocooler is simulated and analyzed using an improved 1D numerical model. The modified model solves the momentum equation of the displacer by iteration together with a
flow solver, which directly outputs the displacer movement and the information of the flow field. It is clearly shown that, by adjusting the size of the drive stem, i.e., the driving force, the P-V diagram can be largely tuned. A small driving force (drive stem) or a higher frequency leads to an earlier start of the “pre-compression” and “pre-expansion” processes and enlarges the cut-off corners of the P-V diagram. The calculated P-V power and cooling capacity, as well as the regenerator loss and mass flow rate, increase with the driving force in general. The simulations discover that an optimal drive stem size exists for maximizing the cooling capacity per mass flow rate and it has to be well designed to match the frequency. This is a result of the balance between the mass flow rate, P-V power and regenerator loss. Proper “pre-compression” and “pre-expansion” processes are necessary for fully utilizing the supplied helium flow from the compressor. It’s also found that the amount of required helium flow for driving the displacer is relatively small compared to the total mass flow rate.

5. References
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