Comparison of high-accuracy numerical simulations of black-hole binaries with stationary-phase post-Newtonian template waveforms for initial and advanced LIGO

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Abstract
We study the effectiveness of stationary-phase approximated post-Newtonian waveforms currently used by ground-based gravitational-wave detectors to search for the coalescence of binary black holes by comparing them to an accurate waveform obtained from numerical simulation of an equal-mass non-spinning binary black hole inspiral, merger and ringdown. We perform this study for the initial- and advanced-LIGO detectors. We find that overlaps between the templates and signal can be improved by integrating the match filter to higher frequencies than used currently. We propose simple analytic frequency cutoffs for both initial and advanced LIGO, which achieve nearly optimal matches, and can easily be extended to unequal-mass, spinning systems. We also find that templates that include terms in the phase evolution up to 3.5 post-Newtonian (pN) order are nearly always better, and rarely significantly worse, than 2.0 pN templates currently in use. For initial LIGO we recommend a strategy using templates that include a recently introduced pseudo-4.0 pN term in the low-mass ($M \leq 35M_\odot$) region, and 3.5 pN templates allowing unphysical values of the symmetric reduced mass $\eta$ above this. This strategy always achieves overlaps within 0.3% of the optimum, for the data used here. For advanced LIGO we recommend a strategy using 3.5 pN templates up to $M = 12M_\odot$, 2.0 pN templates up to $M = 21M_\odot$, pseudo-4.0 pN templates up to 65 $M_\odot$, and 3.5 pN templates with unphysical $\eta$ for higher masses. This strategy always achieves overlaps within 0.7% of the optimum for advanced LIGO.

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1. Introduction
The coalescence of binary black holes is one of the most promising sources of gravitational waves for interferometric gravitational-wave detectors, such as LIGO, Virgo and GEO600 [1].
The first-generation LIGO detectors have achieved their design sensitivity and recorded over 1 year of coincident data [2]. This data, together with data from the Virgo detector, are currently being searched for gravitational waves from compact binary coalescence [3–8]. Upgrades to improve the sensitivity of these detectors by a factor of 2, and ultimately 10, are underway. Optimal searches using the enhanced detectors in 2009 will be sensitive to black-hole coalescence out to hundreds of megaparsecs [9]. The advanced detectors, operational next decade, could detect black-hole binaries at distances of over 1 Gpc [10].

Optimal searches for gravitational waves use matched filtering, which requires accurate knowledge of the waveform [1]. Previous searches in LIGO data have used post-Newtonian and phenomenological templates to search for the coalescence of black-hole binaries [5, 6, 8]. Over the last several years numerical relativity has made remarkable breakthroughs in simulating the late inspiral, merger and ringdown of black-hole binaries. The computational cost of these simulations is high, however, making it impractical to use them directly as template waveforms for use in a matched-filter search. It has been shown that there is good agreement between the waveforms generated by numerical relativity with analytic post-Newtonian waveforms to within just a few orbits of merger [11–21].

This paper uses the high-accuracy Caltech–Cornell numerical-relativity waveforms to suggest improvements to the analytic waveforms currently used in gravitational-wave searches by LIGO and Virgo. A similar study has been performed by Pan et al using numerical data from Pretorius and the Goddard groups [13]. Our main results are in agreement with their conclusion that a simple extension of the existing stationary-phase approximation to the adiabatic post-Newtonian waveforms (called TaylorF2 in [22]) yields high overlaps with numerical waveforms.

In section 2, we review the current techniques used for searching for gravitational waves in gravitational-wave detector data. We discuss the construction of the waveform—a pN–NR hybrid—in section 3. In section 4, we employ the hybrid waveform in a comparison of the detection efficiency of gravitational-wave templates that may be used in upcoming searches of LIGO and Virgo data. Finally, in section 5, we discuss improvements that may be made to the current data-analysis techniques to optimize overlaps.

Although template families such as EOB [42–45], EOBNR [46–51, 54] and phenomenological waveform families [52, 53] are being studied, in this paper we restrict attention to TaylorF2 as these are the waveforms currently used by in the search for binary black holes with total mass \( M \leq 35 M_\odot \). We provide overlaps for higher masses as a benchmark against which to compare the performance of other waveform families.

Throughout this paper, we use only the \((l, m) = (2, 2)\) component of the waveform \( \Psi_{4}^{2,2} \) (as defined, e.g., in [19]). For convenience, we drop the superscript. Whenever possible, we use dimensionless quantities, like \( r M |\Psi_4| \), where \( r \) is the areal radius of the observation sphere and \( M \) is the total apparent-horizon mass of the holes in the initial data. However, for any calculation involving the LIGO noise curve, we have a physical scale, and thus use standard mks units.

### 2. Searches for gravitational waves from black-hole binaries

#### 2.1. Matched filtering

Current searches for gravitational waves from binary black-hole coalescence use matched filtering to search for a waveform buried in noise. The matched filter is the optimal filter for detecting a signal in stationary Gaussian noise. Suppose that \( n(t) \) is a stationary Gaussian noise process with one-sided power spectral density \( S_n(f) \) given by

\[
\langle \hat{n}(f)\hat{n}^*(f') \rangle =
\]
\[ \frac{1}{2} S_n(|f|) \delta(f - f'). \]

For long integration times, the data stream \( s(t) \) output by the detector will always be dominated by the noise. Thus, we can simply approximate \( n \approx s \) to calculate \( S_n(f) \).

Using this power spectral density (PSD), we can define the inner product between two real-valued signals—the data stream \( s \) and the filter template \( h \)—by

\[ (s| h) \equiv 2 \text{Re} \int_{-\infty}^{\infty} \frac{\tilde{s}(f) \tilde{h}^*(f)}{S_n(f)} \, df \]

(1)

\[ = 4 \text{Re} \int_{0}^{\infty} \frac{\tilde{s}(f) \tilde{h}^*(f)}{S_n(f)} \, df. \]

(2)

Then, given data \( s \) which may contain either noise \( n \) or noise and a gravitational wave signal \( h \),

\[ s = \begin{cases} n \\ n + h \end{cases} \]

(3)

the matched-filter signal-to-noise ratio (SNR) is defined as

\[ \rho = \frac{1}{\sqrt{(h|h)}} (s|h). \]

(4)

The SNR can then be used to construct a detection statistic (directly or in combination with other statistics). It is therefore important to ensure that the templates used in searches accurately model the expected waveforms to avoid reduction in the value of \( \rho \). The overlap between two templates \( h \) and \( h' \) is defined as

\[ \langle h|h' \rangle \equiv \frac{(h|h')}{\sqrt{(h|h)}(h'|h')} \]

(5)

The overlap encodes the fractional loss in SNR that results from using the template \( h' \) rather than the true waveform \( h \). In a search that uses \( \rho \) as a detection statistic this corresponds to the fractional loss in distance to which the search is sensitive.

The filter template includes arbitrary time and phase offsets, encoded by the arrival time and phase, \( t_a \) and \( \phi_a \). Under a change of these quantities, the Fourier transform behaves as

\[ \tilde{h}(f) \rightarrow \tilde{h}(f) e^{-2\pi if t_a - i\phi_a}. \]

(6)

Since the duration of the signal is short compared to the detector motion, we can maximize over the time of arrival \( t_a \)

\[ \max_{t_a} (s|h) = \max_{t_a} 4 \text{Re} \int_{0}^{\infty} \frac{\tilde{s}(f) \tilde{h}^*(f) e^{2\pi if t_a/i\phi_a}}{S_n(f)} \, df. \]

(7)

Note that this integral is just the (inverse) Fourier transform of the quantity \( \tilde{s}(f) \tilde{h}^*(f)/S_n(f) \) evaluated at \( t_a \). Thus finding the maximum over \( t_a \) involves taking the Fourier transform and selecting the largest element of the finite set that results from discretization. The unknown coalescence phase \( \phi_a \) can be maximized over by constructing the quantity [55]

\[ \max_{\phi_a} (s|h) = \sqrt{(s|h)^2 + (s|ih)^2}. \]

(8)

Combining equations (7) and (8) we obtain [56]

\[ \max_{t_a,\phi_a} (s|h) = 4 \max_{t_a} \left| \int_{0}^{\infty} \frac{\tilde{s}(f) \tilde{h}^*(f) e^{2\pi if t_a}}{S_n(f)} \, df \right|. \]

(9)
In this paper we are concerned with overlaps between pN waveforms and NR signals; to weight the inner product we use the following PSDs for initial and advanced LIGO: for initial LIGO we use an analytic approximation to the LIGO design PSD given by:

\[ S_n(f) = 3.136 \times 10^{-46} \left[ \frac{4.49 f}{150.0} \right]^{-56.0} + 0.16 \left( \frac{f}{150} \right)^{-4.52} + \left( \frac{f}{150.0} \right)^2 + 0.52 \]  \hspace{1cm} (10)

All integrals start from 40 Hz. As shown in figure 4, at this frequency the noise is an order of magnitude higher than its lowest value, and below this frequency it rises rapidly as \( \sim f^{-56} \). The region below 40 Hz therefore contributes very little signal power to the SNR [6].

The PSD for enhanced LIGO, which will begin operation in mid 2009, has a similar shape to that for initial LIGO although it has a factor of \( \sim 2 \) increase in strain sensitivity. Our results using the initial-LIGO PSD are therefore valid for enhanced LIGO, as the sensitivity factor cancels in equation (5); overlaps depend on the shape of the PSD.

For advanced LIGO we use the GWINC program [23] to generate the PSD. GWINC reports the PSD in increments of 0.0124 Hz. When calculating discrete integrals against signals sampled at other frequencies we obtain values for the PSD by linearly interpolating between the values provided by GWINC. We start integrals at 10 Hz as that is the point where the noise has increased by two orders of magnitude above its minimum, as also shown in figure 4.

### 2.2. Post-Newtonian template

Searches for gravitational waves in LIGO and Virgo use a post-Newtonian waveform known as TaylorF2 [8]. This is a frequency-domain waveform obtained via the stationary-phase approximation [24], which assumes that the frequency-domain amplitude is simply proportional to \( f^{-7/6} \) (the lowest-order behavior), while its phasing is given by the phase of the time-domain waveform, as a function of frequency. For a binary consisting of masses \( m_1 \) and \( m_2 \), located at an ‘effective’ distance \( D_{\text{eff}} \), we have

\[ \tilde{h}(f; M, \eta, f_c) = \Theta(f_c - f) \left( \frac{1}{D_{\text{eff}}} \right) A_{\text{1Mpc}}(M, \eta) f^{-7/6} e^{i\varphi(f; M, \eta)}, \]  \hspace{1cm} (11)

where

\[ A_{\text{1Mpc}}(M, \eta) = \left( \frac{5\pi}{24} \right)^{1/2} \left( \frac{G M_\odot}{1 \text{Mpc}} \right)^{1/6} \eta^{1/2} \left( \frac{M}{M_\odot} \right)^{1/3}, \]  \hspace{1cm} (12)

and the phasing \( \varphi \) of the frequency-domain waveform is given to 3.5 pN accuracy by the formula [25, 26]

\[ \varphi(f; M, \eta) = 2\pi f t_0 - 2\phi_0 - \pi/4 + \frac{3}{128\eta} \left[ v^{-5} + \left( \frac{3715}{756} + \frac{55}{9} \eta \right) v^{-3} - 16\pi v^{-2} \right. \]

\[ + \left. \left( \frac{15}{508032} + \frac{27}{504} \eta \right) \left( \frac{3085}{72} \eta^2 \right) v^{-1} \right] + \pi \left[ \frac{38645}{756} - \frac{65}{9} \eta \right] \left[ 1 + 3 \ln \left( \frac{v}{v_0} \right) \right] + \pi \left[ \frac{30456287}{3048192} - \frac{2255}{12} \pi^2 - \frac{47324}{63} - \frac{7948}{9} \right] \eta. \]
\[ v = \left( \frac{\Delta M}{\pi} f \right)^{1/3}, \quad M = m_1 + m_2 \text{ and } \eta = \frac{m_1 m_2}{m_1 + m_2}^2. \]  

The overall frequency scale is set by the total mass \( M \), as can be seen by observing that each occurrence of \( f \) is accompanied by a factor of \( M^{5/3} \). Thus, going to a higher-mass system shifts the waveform to lower frequencies. On the other hand, to first order, the timescale for the rate of change of the frequency is given by the chirp mass:

\[ \mathcal{M} \equiv \left( \frac{m_1^3 m_2^3}{m_1 + m_2} \right)^{1/5} = M^{3/5} \eta^{3/5}. \]  

Clearly, the total mass and chirp mass give us two very different handles on the behavior of the waveform. These two handles will be important when we try to match the template to our waveform in regions where the post-Newtonian and stationary-phase approximations are poor. This is typically the case for high-mass systems, which only enter the detector band late in the inspiral. In this case, we can still obtain a high match, at the cost of using templates with the wrong values of \( M \) and \( \eta \).

We also note that physical binary systems are restricted to \( 0 < \eta \leq 0.25 \). However, for higher values of \( \eta \), the formulae shown above still give plausible waveforms; in fact, in some cases these templates match the true waveform better than any template with a physical value for \( \eta \). We will explore the implications of allowing unphysical values for \( \eta \) in searches over the templates in section 4.2.

Note the Heaviside function in equation (11). This contains a cutoff frequency \( f_c \) which is used to ensure that the template does not extend to frequencies much greater than the frequencies contained in the expected signal. This is essentially a third parameter for the template waveform and will be searched over. See section 4.1 for a discussion of strategies for optimizing detection by changing this cutoff.

Frequencies which are often used to characterize coalescing binary black holes are as follows: (i) the frequency at the innermost stable circular orbit (ISCO), \( r = 6M \), around a single Schwarzschild black hole with the total mass of the binary system; (ii) the frequency at the light ring, \( r = 3M \), around a single Schwarzschild black hole with the total mass of the binary system; (iii) the ringdown frequency of the 2,2 mode of the final black hole, which depends on both its spin and mass and (iv) an ‘effective ringdown’, \( f_{\text{ERD}} \equiv 1.07 f_{\text{Ringdown}} \) defined in terms of the 2,2 mode in [13].

Current searches use 2 pN stationary-phase approximation (SPA) TaylorF2 templates [8], where the phase evolution is given by equation (13) up to the \( v^{-1} \) term. It has previously been shown that such waveforms provide acceptable detection templates for binary neutron stars and sub-solar mass black holes [27], but not necessarily for higher-mass black holes. This is an issue we will investigate below by testing 2 pN and 3.5 pN templates.

3. PN–NR hybrid waveform

In order to perform our comparison we need to construct a ‘true’ black-hole binary waveform, which we might expect to observe with detectors. A numerical simulation will provide the data for the crucial nonlinear merger phase. We carefully extract the data and extrapolate it
to large radius, and investigate the effects of numerical error on the final result. Because this waveform is very computationally expensive to produce, it covers only about 32 cycles. This is only sufficient to investigate masses above about $33 \ M_{\odot}$ for initial LIGO and $110 \ M_{\odot}$ for advanced LIGO. Thus, we match the numerical waveform to a post-Newtonian waveform, producing a hybrid which extends for many thousands of cycles, covering the entire band of interest.

### 3.1. Numerical simulation, extraction and extrapolation

The numerical simulation is the same as that described in [16, 28]: an equal-mass, non-spinning, black-hole binary with reduced eccentricity [29], beginning roughly 16 orbits before merger, continuing through merger and ringdown [28]. It is performed with the Caltech–Cornell pseudospectral code, using boundary conditions designed to prevent constraint violations and gravitational radiation from entering the domain [30, 31].

Data are extracted from the simulation in the form of the Newman–Penrose scalar

$$ \Psi_4 = -C_{\alpha \beta \gamma \delta} \vec{m}^\beta l^\gamma \vec{m}^\delta, $$

(15)

where $l^\mu$ and the complex vector $\vec{m}^\beta$ are constructed with reference to the coordinate basis. Along the positive $z$-axis, we have

$$ l^\mu = \frac{1}{\sqrt{2}} (t^\alpha - z^\alpha) , $$

(16)

$$ \vec{m}^\beta = \frac{1}{\sqrt{2}} \left( \frac{\partial}{\partial x} - i \frac{\partial}{\partial y} \right)^\beta. $$

(17)

Here, $t^\alpha$ is the timelike unit normal to the spatial hypersurface and $z^\alpha$ is the unit vector in the positive $z$-direction. The vectors $\partial/\partial x$ and $\partial/\partial y$ are the standard coordinate vectors, which are not normalized. $\Psi_4$ is extracted as a function of time, at various radii along the positive $z$-axis. This is then extrapolated to large radii, as described in [16], and in greater detail in [32], using third-order polynomial extrapolation.

The measured (instantaneous) frequency at the beginning of the simulation is

$$ f_{\text{initial}} = (1.08 \pm 0.01) \times 10^3 \ Hz \frac{M_{\odot}}{M}. $$

(18)

The measured ringdown frequency is

$$ f_{\text{ringdown}} = (1.78 \pm 0.02) \times 10^4 \ Hz \frac{M_{\odot}}{M}. $$

(19)

The measured Christodoulou mass and spin of the final black hole are

$$ M_{\chi, \text{final}} = (0.95162 \pm 0.00002) M_{\chi, \text{initial}}, $$

(20)

$$ S_{\text{final}} = (0.68646 \pm 0.00004) M_{\chi, \text{final}}^2. $$

(21)

Using this value for the spin, a quasi-analytic formula due to Echeverria [33] predicts a value of $1.77 \times 10^4 \ Hz \frac{M_{\odot}}{M}$, for the ringdown frequency of the 2,2 mode, in close agreement with the measured frequency.

### 3.2. Accuracy of the numerical simulation

The numerical waveform will be the standard against which we will judge the TaylorF2 waveforms used in LIGO data analysis. To understand how precisely we should trust our
final results, we need to understand the accuracy of the waveform itself. The most accessible measure of the error in this fiducial waveform is its convergence with increasing numerical resolution, which dominates or is typical of all sources of error in the waveform [19]. Figure 1 shows the overlap (equation (5)) between waveforms computed at different resolutions. The data used here are the extrapolated $\Psi_4$ waveforms, integrated in time twice.

Because of the short extent of the numerical waveforms, we need to be careful when using their Fourier transforms. The signal can be corrupted easily by the non-periodicity of the integrated waveforms and the discontinuous jumps that result. For figure 1 we mitigate this problem by increasing the sampling frequency of the input data, which shifts the frequencies corrupted by the discontinuities to values above the physical values. The input data can easily be upsampled in the time domain by interpolating the phase and amplitude of the complex data to a finer time grid. We then perform the Fourier transform, and explicitly set the data to zero at frequencies below $f_{\text{initial}}$ and above $f_{\text{ringdown}}$, as given in equations (18) and (19). That is, we remove all data at frequencies which do not correspond to expected physical frequencies. While the results do depend on whether or not we impose these cutoffs, they do not depend sensitively on the actual cutoff frequencies.

The overlap between the lowest- and highest-resolution simulations (dashed lines) actually passes through zero, as shown in the upper panel, possibly because of loss of phase accuracy over the course of the simulation. All three simulations begin with the same initial data, so the waveforms are most similar at the beginning. Masses for which this is the most important segment (the lowest masses) will naturally have the highest overlap between resolutions. As the simulation progresses, numerical error accumulates—notably in the phase—so the overlap decreases with masses for which later segments dominate the overlap (higher masses). When the overlap is optimized over arrival time and phase, we can see that the overlap becomes much better, as shown in the lower panel, indicating sufficient accuracy within any frequency band.
for which phase coherence is required. In either case, the medium and highest resolutions are much more nearly the same. Without optimization, their overlap is within a few tenths of a percent of 1; after optimization, the overlap is within $10^{-6}$ of 1.

In the rest of our analysis we use the highest-resolution waveform. Because we always optimize over arrival time and phase, the lower panel of figure 1 is the most relevant and shows that the waveform has converged to very high accuracy. The overlaps we quote below will only be given to three decimal places at most, because this is roughly the accuracy of the single-precision numerical methods used in the rest of the paper. This accuracy is also sufficient for searches of gravitational-wave data. Thus, the truncation error of the simulated waveform is irrelevant for those purposes.

Other sources of error include residual eccentricity and spin, the influence of the outer boundary of the simulation, extrapolation errors and coordinate effects, as discussed in [16]. The eccentricity had a disproportionately large effect on the error quoted in that paper because of the matching technique, which is not used here. Restricting attention to the other effects of eccentricity, the uncertainty falls below that due to numerical error. Similarly, using the techniques of [34], the initial spins of the black holes have been measured more reliably, and found to be more than an order of magnitude smaller than previously determined, allowing us to reduce the estimate for that error to less than the numerical truncation error. The various coordinate effects were all estimated to be of roughly the same magnitude as the numerical error.

With the numerical error being many times more accurate than needed for this analysis and the other sources of uncertainty being of roughly the same size, these considerations indicate that the overall error in our fiducial waveform is substantially less than the precision needed for this analysis.

3.3. Hybrid waveform

Numerical simulations cannot simulate a very large portion of the inspiral of a black-hole binary system. Indeed, the longest such simulation currently in the literature is the one used here—which extends over just 32 gravitational wave cycles before merger. Fortunately, this is the only stage in which simulations are needed. It has been shown previously [16] that the TaylorT4 waveform with 3.5-pN phase and 3.0-pN amplitude matches the early part of this simulation to within $0.03\%$, ignoring junk radiation. We generate a TaylorT4 waveform of over 8000 gravitational wave cycles ($t \sim 1.2 \times 10^6 M$, starting at $M_f = 0.004$) and transition between the two to create a hybrid. This long waveform is sufficient to ensure that—even for the lowest-mass systems we will consider—the waveform begins well before it enters the frequency band of interest to LIGO.

We begin with $\Psi_4$ data, which will later be integrated to obtain $h$. Following [19], we stitch the numerical waveform to the pN waveform by adjusting the time and phase offsets of the pN waveform to minimize the quantity

$$\Xi(\Delta t, \Delta \phi) = \int_{t_1}^{t_2} [\phi_{\text{NR}}(t) - \phi_{\text{pN}}(t - \Delta t) - \Delta \phi]^2 dt.$$  \hspace{1cm} (22)

where the times are taken with respect to the start of the numeric waveform. Here, we choose $t_1 = 900 M$ and $t_2 = 1730 M$, which is closer to the beginning of the waveform than in the previous paper. This particular interval is chosen to begin and end at troughs of the small oscillations due to the residual eccentricity $e \sim 5 \times 10^{-5}$ in our numerical waveform. Taking a range from trough to trough or peak to peak—rather than node to node, for example—of the eccentricity effects minimizes their influence on the matching. The eccentricity oscillations
Figure 2. Amplitude and phase differences between the numerical and post-Newtonian waveforms, \( \Psi_4 \), that are blended to create the hybrid waveform. The vertical lines at 900\( M \) and 1730\( M \) denote the region over which matching and hybridization occur. Note that the agreement is well within the numerical accuracy of the simulation, represented by the horizontal bands, throughout the matching region. Also note that the phase difference is fairly flat for a significant period of time after the matching range, which indicates that the match is not sensitive to the particular range chosen for matching. The small oscillations in this flat region are due to eccentricity.

can be seen more easily after low-pass filtering the waveform, though we find filtering to be unnecessary for this paper. The junk radiation apparent in the waveform as shown here has no effect on the resulting match—as we have verified by filtering, and redoing the match. Because the final waveform will incorporate no numerical data before \( t_1 \) and very little immediately thereafter (as explained below), the junk radiation will have no effect on any of our results—as we have also explicitly verified. In particular, by integrating \( \Psi_4 \) to obtain \( h \), we will effectively smooth the junk radiation. In figure 2 we compare the phase of the numerical and pN waveforms. The quantities plotted are

\[
\delta \phi \equiv \phi_{pN} - \phi_{NR},
\]

\[
\delta A \equiv A_{pN} - A_{NR},
\]

shown over the interval on which both data sets exist. The vertical bars denote the matching region. Note that the phase difference is well within the accuracy of the simulation (about 0.01 radians, represented by the horizontal band) over a range extending later than the matching region. Also, the difference between the two is fairly flat, which implies that the match is not very sensitive to the region chosen for matching. Because of this, we expect that the phase coherence between the early pN data and the late NR data will be physically accurate to high precision.

The hybrid waveform is then constructed by blending the two matched waveforms together according to

\[
A_{hyb}(t) = \tau(t) A_{NR} + [1 - \tau(t)] A_{pN}(t),
\]

(25)
\[ \phi_{\text{hyb}}(t) = \tau(t)\phi_{\text{NR}} + [1 - \tau(t)]\phi_{\text{pN}}(t). \]  

The blending function \( \tau \) is defined by

\[
\tau(t) = \begin{cases} 
0 & \text{if } t < t_1 \\
\frac{t - t_1}{t_2 - t_1} & \text{if } t_1 \leq t < t_2 \\
1 & \text{if } t_2 \leq t 
\end{cases}
\]  

The values of \( t_1 \) and \( t_2 \) are the same as those used for the matching. The amplitude discrepancy between the pN waveform and the NR waveform over this interval is within numerical uncertainty—roughly 0.4%. As with the matching technique (equation (22)), this method is similar to that of [35], but distinct, in that we blend the phase and amplitude, rather than the real and imaginary parts. This leads to a smoothly blended waveform, shown in Figure 3.

Up to this point, the waveform has been \( \Psi_4 \) data. With the long waveform in hand, we numerically integrate twice to obtain \( \dot{h} \), and set the four integration constants so that the final waveform has zero average and first moment [29]. Because of the very long duration of the waveforms, this gives a reasonable result, which is only incorrect at very low frequencies—lower than any frequency of interest to us. We have also checked that our results do not change when we effectively integrate in the frequency domain by taking

\[ \dot{h} = -\frac{\ddot{\Psi}_4}{4\pi f^2}, \]  

which is the frequency-domain analog of the equation \( \Psi_4 = \ddot{h} \).

4. Detection efficiency of gravitational-wave templates

We now compare the signal described in the previous section to restricted, stationary phase TaylorF2 post-Newtonian templates with terms up to order 2.0, order 3.5 and a ‘pseudo-4.0 pN-order’ term recommended in [13]. Overlaps are calculated using the techniques of
section 2.1, with the signal \( s \) being the hybrid waveform described in section 3 scaled to a range of masses. We consider both the initial- and advanced-LIGO noise curves.

Plots of the hybrid waveforms in comparison to the initial-LIGO noise curve are shown in figure 4. The masses are chosen so that various frequencies of interest (the final stitching frequency, the ISCO and the light-ring) occur at the ‘seismic wall’ for initial LIGO: 40 Hz. The waveforms \( \tilde{s} \) are scaled to depict the detectability of the signal, typically quantified by the SNR introduced in (4), which may be written as

\[
\rho^2 = \int_0^\infty \frac{4\tilde{s}^*(f)\tilde{s}(f)}{S_n(f)} \, df = \int_0^\infty \frac{2\tilde{s}(f)\sqrt{f}S_n(f)}{S_n(f)} \, d\ln f. \tag{29}
\]

In the final expression, the numerator and denominator have the same units, and are directly comparable. Because the square root of the denominator is familiar, we plot that along with the square root of the numerator. Plotting these two quantities together gives a graphical impression of the detectability of the waveform, and the relative importance of each part of the waveform, by its height above the noise curve. In [36], Brady and Creighton defined a slightly different quantity, the characteristic strain \( h_{\text{char}} \equiv \int |\tilde{s}(f)| \). The relative factor of \( \sqrt{f} \) they use is present so that they can plot \( h_{\text{char}} \) against \( \sqrt{f} S_n(f) \). Cutler and Thorne [37] defined still another quantity, the signal strength \( \tilde{h}(f) \), which is related to the Fourier transform by \( \tilde{h}(f) = \sqrt{T/N} \tilde{h}(s) \). The factor of \( \sqrt{5} \) comes from averaging over the orientation of the binary, which we do not do. \( T/N \) is the ratio of the threshold to the rms noise at the endpoint of signal processing.

For each template family we initially optimize over signal mass \( M \), symmetric mass ratio \( \eta = m_1m_2/(m_1 + m_2)^2 \), and upper cutoff frequency \( f_c \). The optimization is performed using a Nelder–Mead (‘amoeba’) algorithm [38]. The amoeba starts with a simplex in the parameter
space, and proceeds through a series of steps, each of which will improve the value of the function at at least one vertex. The algorithm terminates when all vertices have converged to the same point to within a specified tolerance. This process is deterministic and amounts to an enhanced steepest-ascent algorithm. It is therefore only guaranteed to find a local maximum, and indeed we find that an amoeba instance started at a random point in the parameter space is most likely to converge to a point that does not give the highest possible overlap. We interpret this as being due to a large region in parameter space containing a local maximum and a relatively smaller region containing the global maximum. For all optimizations performed in this paper we therefore supplement the basic amoeba by running 300 instances with random starting values and taking the best match obtained over all instances. In repeated runs the same optimal parameters were found by at least some of the amoebas, which supports the claim that this is the true maximum. At the least, the results obtained by this procedure provide a lower bound on the maximum overlap.

The results of optimizing over all of \( M, \eta \) and \( f_c \) for selected masses for initial LIGO are given in table 1 and summarized in figure 5. In all cases \( \eta \) is allowed to range over unphysical values up to \( \eta = 1.00 \), see section 4.2. Although it is possible that any particular optimization could have obtained only a local maximum, the smoothness of the curves suggests that in all cases the global maximum is found. For initial LIGO, in the range covered by the current compact binary coalescence (CBC) low-mass search (\( M < 35 M_\odot \)) [8], the pseudo-4.0 pN TaylorF2 waveforms achieve the highest overlaps, exceeding those obtained with 3.5 pN waveforms by \( \sim 1\% \). Above 35 \( M_\odot \) the 3.5 pN waveforms produce overlaps as much as 4% greater than those obtained with pseudo-4.0 pN waveforms over a range from 40 to 80 \( M_\odot \). With the advanced-LIGO noise curve, in the CBC low-mass range, the 3.5 pN and pseudo-4.0 pN waveforms produce overlaps within 2% of each other, with 3.5 pN producing higher overlaps below 20 \( M_\odot \) and pseudo-4.0 pN producing higher overlaps in the range 20–35 \( M_\odot \). Pseudo-4.0 pN continues to give the highest overlaps up to 60 \( M_\odot \), producing overlaps as much as 4% greater than those obtained with 3.5 pN waveforms. Above 60 \( M_\odot \) 3.5 pN waveforms again yield the best overlaps, by as much as 6% around 90 \( M_\odot \).

A significant feature of Tables 1 and 2 is the size of the error bars on the cutoff frequencies. For \( M = 20 M_\odot \) the cutoff frequency can vary as much as 128% above and 28% below the optimal value while losing no more than 1% of overlap. This leads us to consider the range of possible template parameters which may give high overlaps. In the following section we consider the reduction in overlap as the parameters \( f_c \) and \( \eta \) are independently varied from the optimal value.

### 4.1. Effect of upper frequency cutoff

As shown in figure 4 the amplitude of the NR waveforms drops sharply at around the ringdown frequency, which depends on the total mass of the binary. The TaylorF2 waveforms do not model the late inspiral, merger or ringdown and hence will continue to evolve as \( f^{-1/6} \) at all frequencies, increasingly deviating from the NR waveform. This suggests that the upper frequency cutoff of the TaylorF2 waveform should be chosen to be below the frequency at which the two diverge. However, the effect of the divergence is mitigated by the PSD. The denominator of the overlap, equation (5), depends on \( \langle s | s \rangle \) which is a constant and \( \langle h | h \rangle \) which would increase without limit if not for the PSD. Figure 6 shows \( |\tilde{h}(f)|^2 / S_n(f) \)—the integrand of \( \langle h | h \rangle \)—for the initial-LIGO noise curve for an example TaylorF2 waveform for an equal-mass 10 \( M_\odot \) binary. We see that above about 450 Hz there is very little contribution to the integrand, and so extending the cutoff frequency above this will not impact the overlap.
ensures that the signals will be correlated over most of their length. In our studies we have
For anti-correlated signals the integrand can be negative, however the maximization over
limitations. For example, a 10% decrease in the noise level can lead to a 10% increase in the
We restrict the search to 0 \( \leq \eta \leq 1,000 \), so the upper error bounds when \( \eta \approx 1,000 \) may be artificially small. Section 4.2 discusses unphysical values of \( \eta \).

Table 1. Maximum overlaps between Caltech–Cornell hybrid waveforms and restricted stationary-phase pN templates using the initial-LIGO noise curve. The first number in each block is the overlap; subsequent numbers are the template parameters that achieve this overlap. Parameter values within the specified ranges keep the overlap within 1% of the maximum by varying that parameter, while leaving others fixed. We restrict the search to 0 \( \leq \eta \leq 1,000 \), so the upper error bounds when \( \eta \approx 1,000 \) may be artificially small. Section 4.2 discusses unphysical values of \( \eta \).

| \( M/M_\odot \) | \( \langle \eta \rangle \) | \( f_{\text{cut}} \) (Hz) |
|-----------------|-----------------|------------------|
| \( (10 + 10) M_\odot \) | 0.99 | 501.18+184.52+233.00 −350.00 |
| \( (20 + 20) M_\odot \) | 0.98 | 431.34+338.00 −77.00 |
| \( (30 + 30) M_\odot \) | 0.97 | 296.05+53.00 −31.00 |
| \( (50 + 50) M_\odot \) | 0.96 | 190.56+20.00 −14.00 |

Table 2. Maximum overlaps between Caltech–Cornell hybrid waveforms and restricted stationary-phase pN templates using the advanced-LIGO noise curve. The first number in each block is the overlap; subsequent numbers are the template parameters that achieve this overlap. Parameter values within the specified ranges keep the overlap within 1% of the maximum by varying that parameter, while leaving others fixed. We restrict the search to 0 \( \leq \eta \leq 1,000 \), so the upper error bounds when \( \eta \approx 1,000 \) may be artificially small. Section 4.2 discusses unphysical values of \( \eta \).

| \( M/M_\odot \) | \( \langle \eta \rangle \) | \( f_{\text{cut}} \) (Hz) |
|-----------------|-----------------|------------------|
| \( (10 + 10) M_\odot \) | 0.98 | 509.47−140.00 352.44+64.00 |
| \( (20 + 20) M_\odot \) | 0.96 | 309.53+20.00 −7.00 |
| \( (30 + 30) M_\odot \) | 0.95 | 195.63+21.00 −15.00 |
| \( (50 + 50) M_\odot \) | 0.96 | 112.00−18.00 177.00+64.00 |

For anti-correlated signals the integrand can be negative, however the maximization over \( t_c \) ensures that the signals will be correlated over most of their length. In our studies we have
found that the loss in SNR due to regions where the signal and template are anti-correlated
Figure 5. Left: overlaps between Caltech–Cornell hybrid waveforms, scaled to various masses, and restricted stationary-phase pN waveforms for initial-LIGO PSD. Optimization is over $M$ and $\eta$, while the cutoff frequency $f_c$ is prescribed by the weighted average described below. The mass ratio $\eta$ is allowed to range over unphysical values. The best-fit values found for the pseudo-4.0 pN templates are always physical in this case. See section 4.2. Right: the same, for the advanced-LIGO PSD.

Figure 6. Left: integrand of equation (1) for a TaylorF2, 3.5 pN waveform with $M = 10$ and $\eta = 0.25$, at a distance of 100 Mpc, using the initial-LIGO noise curve. Note that the shape of this curve does not change as we change $M$ and $\eta$; only the vertical scale changes. Right: overlap between Caltech–Cornell waveform scaled to $M = 40 M_\odot$ and restricted TaylorF2, 3.5 pN waveform using the best-match values for $M$ and $\eta$, as a function of the cutoff frequency $f_c$, with the initial-LIGO noise curve. The vertical bars are meant to delineate 1% loss. Note that the upper bound extends to higher frequencies indefinitely.

is typically small, and in particular this loss is completely negligible for optimal templates. However frequencies above the lightring where the waveforms have diverged will contribute very little. The effect of including higher frequencies on the overlap is therefore determined by the $\langle h|/h \rangle$ term in the denominator. For systems with ringdown frequencies well above the peak
of the integrand in figure 6, this term will not significantly reduce the overlap. For example, binaries of total mass roughly \(40 M_\odot\) have ringdown frequencies at roughly 450 Hz. Only a small fraction of the SNR comes from higher frequencies. Thus, we expect that systems with lower masses should not suffer great loss in overlap if the cutoff frequency is higher than ringdown. This is indeed what we find, as shown by a representative example on the right side in figure 6. For this 40 \(M_\odot\) system, using the initial-LIGO noise curve, the optimal cutoff frequency is around 450 Hz—roughly the ringdown frequency. Decreasing the cutoff quickly decreases the overlap. The cutoff may be increased almost indefinitely, however, with only 0.5% loss in overlap. This, of course, changes when using the advanced-LIGO noise curve. For higher-mass systems, where the lightring frequency is lower, the PN and NR waveforms diverge earlier and the overlap can be significantly reduced if the upper frequency cutoff is too large. We revisit this issue in section 5.

4.2. Unrestricted \(\eta\)

The physical symmetric mass ratio is restricted to the range \(0 < \eta \leq 0.25\), values above this imply complex-valued masses. However the pN waveforms are well behaved for \(0 < \eta < 1.0\), and as seen from Tables 1 and 2, the highest overlaps are often obtained at unphysical values of \(\eta\). Allowing such unphysical values is clearly a problem for parameter estimation, however in this paper we are focusing on the issue of detection, where unphysical values are not a concern. In figure 7 we show the effect of limiting the optimization to physical \(\eta\). At high masses, the limitation reduces the optimal overlap by up to 11%. TaylorF2 waveforms with \(\eta \leq 0.25\) would not be expected to accurately model the late-inspiral and merger part of the waveform, as non-Newtonian effects are increasingly significant in this region. We find that allowing unphysical \(\eta\) broadens the space of waveforms covered by the TaylorF2 approximation sufficiently to capture more of the late-inspiral and merger.

5. Recommendations for improvements

Based on the analysis of the previous sections we propose a series of adjustments to searches using TaylorF2 template waveforms to enhance the efficiency of those searches. First, as seen in figure 5 for initial LIGO, adding terms up to 3.5 pN order produces overlaps as large or larger than the current 2.0 pN templates over most of the mass range, while the pseudo-4.0 pN templates recommended in [13] produce slightly larger overlaps at masses near 20 \(M_\odot\). Thus, we recommend pseudo-4.0 pN templates for the low mass range, \(M < 35 M_\odot\), and 3.5 pN templates for higher masses. The improvement due to 3.5 pN templates over 2.0 pN generally holds for advanced LIGO as well. The 3.5 pN templates produce larger overlaps than 2.0 pN templates above 50 \(M_\odot\) without a significant loss (within 2%) at lower masses. However, there is a large region for which the pseudo-4.0 pN term does significantly better. When using an advanced-LIGO noise curve, we recommend 3.5 pN templates generally, 2.0 pN templates in the range 12–21 \(M_\odot\) and pseudo-4.0 pN templates for masses in the range 21–65 \(M_\odot\).

As a second improvement, we note from figure 7 that allowing \(\eta\) to range over unphysical values significantly improves matches with 3.5 pN templates above 30 \(M_\odot\). In preliminary studies we have found that extending to \(\eta \leq 1\) roughly doubles the size of the template bank, and the advantages must therefore be weighed against the increase in false alarm rate.

Our third recommendation involves the cutoff frequency used for the template waveform. Optimization over the cutoff frequency is too computationally intensive to be done in searches. Currently, the cutoff frequency is typically taken to be the Schwarzschild ISCO frequency. To
examine the effect of this choice we vary $f_c$ while keeping the mass and $\eta$ at their optimal values, for each of the signal masses in our range. In general the parameters are correlated; the loss of overlap by moving along one parameter direction can be compensated for by changing the values of the other parameters. However, as we show below, the overlap depends only weakly on $f_c$ over much of the mass range, and so the compensatory changes in $M$ and $\eta$ are expected to be small. The result of one such variation is shown in figure 6 (right). Figures 8 shows the variations for all masses, highlighting the regions within which the overlap drops by less than 1% (dark gray) and 3% (light gray) of the optimal value. This figure also shows the ISCO and ERD frequencies, neither of which stays within the 1% band for both initial and advanced LIGO. In particular, the ISCO is a poor choice for both initial and advanced LIGO except at very low masses, where the precise value of the cutoff is almost irrelevant.

The ISCO is often pointed to—somewhat arbitrarily—as a good estimate of the breakdown of post-Newtonian approximations [39]. So, for instance, if we were to match a pN template to a physical waveform, beginning at some point in the distant past, we might expect them to separate quite badly near the ISCO. Of course, for realistic black-hole binaries, the gravitational waves will only enter the LIGO band late in the inspiral—just before the ISCO for low-mass systems, or after the ISCO for high-mass systems. We can see from figure 4 that, for masses below about 30 $M_\odot$, the ISCO is high enough that lower-frequency parts of the waveform contribute the most to the SNR. For very high masses, however, this basically cuts the waveform down to nothing. In initial LIGO, the ISCO is completely buried in seismic noise for masses above about 100 $M_\odot$. Thus, we must move the cutoff frequency up. We cannot push the cutoff far above ringdown, because the physical waveform simply ceases to exist (see figure 4). It has been suggested that an ‘effective ringdown’ (ERD) frequency $f_{\text{ERD}} \equiv 1.07 f_{\text{ringdown}}$ is a useful upper limit [13]. For intermediate masses, we would like to interpolate somehow between these two extremes of ISCO and ERD. We suggest setting the cutoff frequency to
Figure 8. Left: candidate $f_c$ values for 3.5 pN templates with initial LIGO. The dark gray band contains cutoff frequencies with matches within 1% of the value at which the best overlap was obtained. The light gray band contains frequencies with matches within 3%. Right: candidate $f_c$ values for 3.5 pN templates with advanced LIGO. The dark gray band contains cutoff frequencies with matches within 1% of the value at which the best overlap was obtained. The light gray band contains frequencies with matches within 3%. Note that the weighted-average cutoff extends past the 1% error bars for $12 < M/M_\odot < 40$. However, in that same region, the 3.5 pN templates do poorly overall and we recommend pseudo-4.0 pN templates. The optimal cutoff frequency for pseudo-4.0 pN templates is much closer to the weighted-average cutoff in this mass range.

A weighted average of the two, where the weights are the contributions to the SNR below the given frequency. If we assume coherent phasing between the template and the physical waveform, we can simply take the amplitudes of the two waveforms. Also, note that the restricted SPA approximation for the amplitude is reasonable. Thus, define

$$\rho^2_{\text{ISCO}} \equiv \int_0^{f_{\text{ISCO}}} \frac{f^{-7/3}}{S_n(f)} \, df,$$  \hspace{1cm} (30)

$$\rho^2_{\text{ERD}} \equiv \int_{f_{\text{ISCO}}}^{f_{\text{ERD}}} \frac{f^{-7/3}}{S_n(f)} \, df,$$  \hspace{1cm} (31)

$$\rho^2_{\text{tot}} \equiv \int_0^{f_{\text{ERD}}} \frac{f^{-7/3}}{S_n(f)} \, df,$$  \hspace{1cm} (32)

$$f_{\text{cut}} \equiv \frac{f_{\text{ISCO}} \rho_{\text{ISCO}} + f_{\text{ERD}} \rho_{\text{ERD}}}{\rho_{\text{tot}}}.$$  \hspace{1cm} (33)

We have already dropped constant factors in the expressions for $\rho$ that will cancel out.

Note that these expressions only depend on the total mass by way of the limits of integrations—which are very simple, known functions of the mass—so these integrals could be done just once for a given noise curve, storing the intermediate values. When the cutoff needs to be calculated, the cumulative integral could be evaluated at the given ISCO and ringdown frequencies. Hence, this would be a fast way of calculating the cutoff, with no need to do the integrals each time the cutoff is needed.
We can test this recommended frequency by comparing it to the optimal cutoff frequency found by the amoeba search described in section 4. For 3.5 pN templates in initial LIGO, we find that it is an excellent match to the optimal frequency. Figure 8 shows these two values, along with dark and light bands showing the regions in which changing $f_c$ results in a loss of overlap of 1% and 3%, respectively. Of course, the same figure shows that using the ERD recommendation would stay within the 1% error bounds. Nonetheless, the close match between this recommendation and the true optimum suggests that it is sound. The smoothness of these curves again suggests that the optimization is finding the global maximum. The jagged features of the ‘optimal’ line are all well within 1% of the maximum; as we use single-precision floating point numbers in overlap calculations the third digit, where these variations appear, is subject to numeric noise. Thus, our final recommendation is to use the weighted-average frequency cutoff throughout the entire mass range. While our analysis has been restricted to equal-mass systems, the cutoff frequency we have defined here could be applied to unequal-mass systems as well. It will be interesting to see how this cutoff fares in those situations.

Similar results hold for advanced LIGO, when using our recommended template for each mass. That is, in regions where 3.5 pN templates do poorly (see figure 5), the weighted average is a poor predictor of the optimal cutoff frequency using those templates, as shown in figure 8. However, in those same regions—where pseudo-4.0 pN templates do well—the weighted average is a good predictor of the optimal cutoff frequency for 4.0 pN templates. Thus, again, we recommend using the weighted-average frequency cutoff throughout the entire mass range with advanced LIGO.

By prescribing a cutoff frequency, the search does not need to extend over that parameter. Similarly, by prescribing a post-Newtonian order, we need use only one template for a given total mass. On the other hand, if these recommendations decrease the overlap found by too much when using them compared to the overlap found by an unconstrained search, it may be better to search the larger parameter space. We can evaluate the loss in overlap by comparing the results found using our recommendations to the results found when searching over the set of all three template families, and all masses, mass ratios and cutoff frequencies. We have determined that this loss in overlap when using our recommendations is always less than 0.0025 for initial LIGO and less than 0.007 for advanced LIGO.

6. Conclusions

We have compared high-accuracy NR waveforms for equal-mass binary black holes from the Caltech–Cornell group to stationary phase post-Newtonian waveforms. We examined a number of factors that influence the matches between the two, with the goal of optimizing the matches and hence improving the efficiency of templated searches in initial and advanced LIGO. We first considered the effect of the post-Newtonian order to which the phase evolution is taken, and found that adding terms up to 3.5 pN or pseudo-4.0 pN to the currently used 2.0 pN templates significantly improves the matches over a large range of masses, as shown in figure 5. We then studied the effect of varying the upper cutoff frequency of the templates. The frequency that achieves the optimal match is a function of mass, and we find this function is well approximated by an average between ISCO and ERD, weighted by contribution to the SNR, as shown in figure 8. Finally, we allow the symmetric mass ratio $\eta$ to range over unphysical values up to 1.0 and find that this dramatically improves matches, as shown in figure 7. Based on the results we recommend that, if TaylorF2 template waveforms are to be used in searches, they should go up to 3.5 pN or pseudo-4.0 pN over most of the mass range, integrations should extend up to our recommended cutoff, and $\eta$ should be allowed...
to extend up to 1. For initial LIGO, the overlaps obtained using these parameters is always within 0.0025 of overlaps achievable by optimizing over all three parameters.

In future work we plan to extend this analysis to unequal-mass and spinning black-hole systems. We have found that allowing unphysical values of $\eta$ roughly doubles the size of the template bank and we also plan to study the impact of this on the false alarm rate.

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