Calculation and Analysis of Truss Internal Force Based on Beam Element

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Abstract. For plane truss structure, starting from the analysis of ideal truss model, the influence of tangential deformation and angular deformation on the secondary internal force of the truss is fully considered through Python program. It is obtained through analysis that: in the ideal truss model, the $P-\delta$ second-order effect causes the member to produce tangential deformation and angular deformation, resulting in secondary internal forces. Numerical analysis shows that due to the influence of secondary internal force, the axial force error of ideal truss model can reach 19.731% and the secondary shear force is almost all the members of the truss, and the secondary moment only appears at the support. The research results have important reference value for the engineering design and high-precision internal force analysis of truss structures.

1. Introduction
Truss structure has the advantages of simple structure and low steel consumption. It is used for roof truss, bridge, TV tower and other structures [4–6, 8–11, 13]. In engineering design, it is usually regarded as an ideal truss model for analysis in order to simplify calculation. The ideal truss model has the following characteristics: 1) All members are smooth hinged. 2) All bars are straight bars. 3) All external loads imposed on nodes, and there is no non-nodal load. 4) The members intersect axially at common hinge points. 5) The internal force of the bar is only axial force without shear force and bending moment, and the truss only considers axial deformation.

Compared with the ideal truss model, the actual truss structure has some flexural deformation. Nair[1], Huixian Liu[2], Yongtao Bao[3], Yuehua Li[4], Hao Liu[5], Huijun Yan et all[6], Yue Cao et all[7], Hui Wang[8], Fanrong Liu[9], Rui Jiang[10], Xiaoqing Liu et all[11], Schmidt et all[12], Shundong Qiu[13] and so son blamed the difference to the secondary internal force or stress of the truss. And created anaytical methods including truss model, rigid frame model and non-bar element model, which is used to establishing finite element model.

Nair [1] saw the shear force and bending moment of the truss as secondary internal force, and the stress produced by secondary internal force is called secondary stress. When Huixian Liu [2] analyzed the secondary stress of the truss, he believed that the connecting plate at the node had an effect on the stiffening (stiffness strengthening) of the bar. And based on the stability principle of the pressure bar, the deflection caused by axial force is explained.
At present, it is generally believed that the main reasons for the secondary stress of the truss are: node rigidity, non-coincidence between the axis of the connected bar and the hinge node, and non-node load [4–6, 9, 12, 13]. Based on the finite element numerical analysis and experimental verification, it is considered that the main factors affecting the secondary internal force are: the line stiffness of the bar, the defect of the bar, the depth-span ratio of the bar, the connection mode of the node and the built-in function, the shape of the truss and the dead weight of the truss [1–12]. A large number of research results show that secondary internal forces are common in truss structures, and the analysis of secondary internal forces of truss structures is relatively mature. However, there are few literatures about the secondary internal force analysis of truss models. That is, does the ideal truss model itself have secondary internal forces?

Therefore, the secondary internal force analysis of truss finite element model will be carried out in this paper. Through Python finite element self-programmed program (Python program for short in this paper), the rigid frame model with discontinuous displacement is established, and the mechanism and influence factors of secondary internal force of truss model are explored, which provides a certain reference value for engineering design and finite element research.

2. Python Finite Element Program Introduction

Python program is based on the displacement method, can analyze bar structure, solid structure and composite structure, has the ability to analyze plane problems or space problems, and the loading process can be simulated once or multiple times through Python program. Figure 1 shows the block diagram of the program flow for the multi-condition mechanical model.

![Figure 1. The block diagram of the Python programmer for multi-condition.](image)

There are two types of iteration built into Python programs: general iteration and coupled iteration. General iteration means that there is no sequence between adjacent working conditions, and the initial data of the next working condition is the same as that of the previous working condition. Coupling iteration refers to the sequence of adjacent working conditions, and the solution result of the previous working condition will be loaded into the next working condition as the initial condition.

In Python programs, each degree of freedom has a corresponding serial number. When the total stiffness matrix is reduced (cut out row and column), the program will assign a new number to the degree of freedom to participate in the solution, which indicates the row and column order of the degree of freedom in the reduction stiffness matrix. The program is provided with a coupling element to simulate the functions of master-slave nodes. The coupling element can associate the corresponding
degrees of freedom of any two nodes. The associated degrees of freedom have the same number in the reduced stiffness matrix, and have the same displacement value in the process of solving linear equations.

3. Analysis of Secondary Internal Force of Truss

3.1. The Basic Assumptions
In order to facilitate the analysis of the secondary internal forces of the ideal truss model itself, the model should meet the following basic assumptions: 1) The members are hinged and have no built-in function. 2) The axes of adjacent bar are intersected at common hinge points, and all the bar are straight bar with uniform cross sections and the same material properties. 3) The load type is only node load, and all external loads imposed on the center of hinge point. 4) The model satisfies the assumption of plane section and small deformation.

3.2. Establishing Finite Element Model
When there is secondary internal force in the truss, not only the axial displacement, but also tangential displacement and angular displacement will occur. In the ideal truss model, only the axial deformation of the member is considered, and the tangential and angular deformation of the member are ignored. When the total stiffness is reduced, the rows and columns irrelevant to the axis deformation are removed, so that the tangential and rotational degrees of freedom of the member do not participate in the solution of the equations.

Therefore, based on Bernoulli-Euler elementary beam theory, a rigid frame model with discontinuous displacement as shown in figure 2 was established. In order to fully consider the influence of tangential deformation and angular deformation on the secondary internal forces of the truss, the model relates the corresponding degrees of freedom through coupling elements, and makes tangential and angular degrees of freedom participate in the solution of linear equations.

Figure 2. Mechanical model of truss.

Figure 3. Deformation decomposition diagram of truss model.
Figure 2 shows the mechanical model diagram of the truss in this paper. The node serial number and element serial number are marked in the figure. Nodes 2, 3, 4 and 5 are overlapped nodes, which are used to establish coupling elements and associate corresponding translational degrees of freedom. Similarly, the same method is used to establish coupling elements at other locations. Discontinuous displacement also exists at node 1 and node 10, but through analyzing maximum linearly independent group of Stiffness matrix, it is found that the rotation variables at node 1 and node 10 are not independent. Therefore, in order to make the stiffness matrix having been reduced non-singular, no coupling element is set at node 1 and node 10.

In order to facilitate data analysis, the units of model parameters are all SI units and set as follows: bar element length $L=1$, elastic modulus $E=1$, cross-sectional area $A=1$, moment of inertia $I_z=1$, shear coefficient $k_s=0$. In the boundary conditions, the degree of freedom of horizontal and vertical translational of node 1 is constrained, and the degree of freedom of vertical translational of node 10 is constrained. The following four analysis conditions are established: 1) Working condition 1: open axial deformation, close tangential deformation and angular deformation. 2) Working condition 2: open the axial deformation and tangential deformation, close the angular deformation. 3) Working condition 3: open axial deformation and angular deformation, close tangential deformation. 4) Working condition 4: open axial deformation, tangential deformation and angular deformation.

Working condition 1 is equivalent to the ideal truss model, working condition 4 is rigid frame model, and working condition 2 and 3 are only used for comparative analysis. The Python program stores the model data in Excel(.xlsx) file.

### 3.3. Model Validation and Result Analysis

The solved displacement result of working condition 1 is the same as the example in literature [14], and the calculation accuracy is much higher than the example, which verifies the feasibility of the truss model in figure 2 and the reliability of the Python calculation results.

The iteration type of the above four working conditions was set as general iteration, and the above four working conditions were analyzed simultaneously by Python program.

**Table 1. The summary table of bar end results.**

| Project               | Working condition 1 | Working condition 2 | Working condition 3 | Working condition 4 |
|-----------------------|---------------------|---------------------|---------------------|---------------------|
| Maximum axial force   | 1.666666667         | 0.041727838         | 1.666666667         | 1.337822671         |
| Minimum axial force   | -1.666666667        | -0.041727838        | -1.666666667        | -1.337822671        |
| Maximum tangential force | 1.80411e-16      | 0.890015913         | 5.2134e-15          | 1.026936027         |
| Minimum tangential force | -1.80411e-16    | -0.89005913         | -5.2134e-15         | -1.026936027        |
| Maximum moment        | 0                   | 0.445007957         | 9.53589e-16         | 1.026936027         |
| Minimum moment        | 0                   | -0.445007957        | -4.26091e-15        | -1.026936027        |
| Maximum displacement of X | 3.833333333     | 0                   | 3.833333333         | 4.0044893           |
| Minimum displacement of X | 0                   | -0.041679           | 0                   | 0                   |
| Maximum displacement of Y | 0                   | 0                   | 0                   | 0                   |
| Minimum displacement of Y | -7.684028          | -0.074168           | -7.684028           | -5.744108           |
| Maximum Angle         | 0                   | 0                   | 4.34837e-16         | 5.897586981         |
| Minimum Angle         | 0                   | 0                   | -1.26137e-15        | -5.897586981        |
The bar end displacement and bar end force of the four working conditions are compared, and the analysis results are as follows: 1) In working conditions 1 and 3, bar end displacement and bar end force of bar element are exactly the same. 2) In working conditions 1, 2 and 3, the bar element has no angular deformation. In working condition 4, except bar element 2 and bar element 10, all other bar elements have angular deformation. 3) In the working conditions 1 and 3, the bar end torque of the bar element is zero. In working condition 2, bar end torques exist except element 2 and element 10. In the 4 working condition, the bar end torque only appears at the support. 4) In the working conditions 1 and 3, the bar ends of the bar element have no tangential component force. In the working condition 2, except bar element 2 and bar element 10, there is tangential component force at the bar end of the bar element. In the working condition 4, the tangential force of bar end of bar element only appears in bar element 1, bar element 3, bar element 4 and bar element 9. 5) As shown in Table 1, compared with working condition 1, the concentrated force at the bar end of the bar element decreases, the torque at the bar end increases, and the rotation angle at the bar end increases. The relative errors of axial force, X displacement and Y displacement are 19.731%, 4.465% and 25.246% respectively.

When the bar has tangential deformation (lateral deformation) or angular deformation, due to the existence of certain lateral stiffness or bending stiffness of the bar, the corresponding secondary shear force or secondary bending moment will be generated after the bar moves sideways or turns. Based on Bernoulli-Euler elementary beam theory, the relation between shear force and tangential equation (1) and the relation between bending moment and angular equation (2) were obtained:

\[ F_s = K_s \delta_s = \zeta \frac{EI}{L} \delta_s \]  
\[ M = K_\theta \theta = \eta \frac{EI}{L} \theta \]  

In equations 1 and 2:
- \( F_s \): Shear force.
- \( M \): Bending moment.
- \( K_s \): Lateral stiffness.
- \( K_\theta \): Rotational stiffness.
- \( \delta_s \): Lateral deformation
- \( \theta \): Angular deformation.
- \( \zeta \): The lateral stiffness coefficient, related to the restraint form at both ends of the bar.
- \( \eta \): The rotational stiffness coefficient, related to the restraint form at both ends of the bar.

Based on equations (1) and (2), when the lateral stiffness \( K_s \) and rotational stiffness \( K_\theta \) remain unchanged, the secondary internal force is proportional to the corresponding member displacement. Therefore, in the ideal truss model, the tangential (lateral) and angular displacements are the main causes of secondary internal forces.

3.4. Mechanism of Tangential Deformation and Angular Deformation

It can be concluded that there are differences between the ideal truss model and the discontinuous rigid frame model. The main reason for this difference is that the \( P-\delta \) second-order effect of ideal truss model is ignored in internal force calculation, that is, the calculation of internal force precedes the occurrence of deformation. As shown in Figure 3, the force and deformation process of beam element ① in working condition 4 is decomposed into the following three stages: 1) Loading stage (a): at this time, the beam element is in a balanced state of force, and the resultant force \( F_1 \) of the bar end is parallel to the beam axis, but it is considered that the beam element is not deformed at this time. 2) Deformation stage (b): only axial deformation occurs in the beam element, and the final state, which is the same as working condition 1, is reached. 3) Adjustment stage (c): The beam element is in equilibrium under the joint action of bar end forces \( F_1 \) and \( F_2 \). Loading stage (a) and deformation stage (b) are the deformation processes of the ideal truss model. However, the second order effect of \( P-\delta \) is ignored, so the truss can not keep the equilibrium state after the deformation stage (b). Under the action of unbalanced external forces, the bar produces tangential deformation and axial deformation. According to the relation between equations (1) and (2), secondary
shear force and secondary bending moment are generated inside the bar, and the resultant force of secondary shear and secondary bending moment forms the “additional” bar end force $F_2$.

4. Conclusion and Prospect
Based on Python program, this paper analyzes the secondary internal force of truss model and draws the following conclusions: 1) The secondary internal forces of the truss model are generated by the tangential and angular displacements of the members. The secondary internal force is proportional to the corresponding displacement. 2) The main reason why the ideal truss model cannot calculate the secondary internal forces is that the $P$-$\delta$ second-order effect is neglected in the calculation of internal forces. 3) The angular displacement of the truss model only appears at the support, and the angular displacement at the non-support does not affect the calculation of other degrees of freedom. Tangential displacement is the main reason affecting the secondary internal force of truss non-support member. The secondary moment occurs only at the support, and the secondary shear force is almost all the members. 4) When the ideal truss model is used for analysis, the axial force error, vertical settlement error and lateral displacement error of the model are at least 19.731%, 25.246% and 4.465% respectively.

Based on the research results of this paper, the following suggestions are given for the analysis and design of truss structures: 1) The secondary internal force should be decomposed into the secondary internal force of truss model and the secondary internal force of truss structure. The secondary internal force of truss model is caused by neglecting $P$-$\delta$ second-order effect in internal force calculation. The secondary internal force of truss structure is caused by node rigidity, non-coincidence between the axis of connected bar and hinge node, non-node load and so on. 2) Strengthen the bending stiffness of the member at the support, strengthen the shear stiffness of all the members.

Although the analysis conclusions of this paper confirm the research results of relevant literature, the analysis model of this paper cannot contain all possibilities. The influence of load forms and truss structure forms on the secondary internal forces of the truss model will be further advanced in subsequent studies.

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