Cosmic Neutrinos from Unstable Relic Particles

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ABSTRACT

We derive constraints on the relic abundance of a generic particle of mass \( \sim 1 - 10^{14} \) TeV which decays into neutrinos at cosmological epochs, using data from the Fréjus and IMB nucleon decay detectors and the Fly’s Eye air shower array. The lifetime of such unstable particles which may constitute the dark matter today is bounded to be greater than \( \sim 10^{14} - 10^{18} \) yr, depending on the mass. For lifetimes shorter than the age of the universe, neutrino energy losses due to scattering and the expansion redshift become important and set limits to the ability of neutrino observatories to probe the early universe.
1. Introduction

Upper limits on the flux of high energy cosmic neutrinos obtained from nucleon decay experiments and cosmic ray observatories constrain the relic cosmological abundance of heavy unstable particles which decay into neutrinos. Given the energy spectrum of the decay neutrinos and the decay branching ratio, upper bounds can be obtained on the primordial abundance of the particle as a function of its lifetime. Earlier attempts to set such bounds [1,2] were made before any experimental data were available. We present here the bounds imposed by the non-observation of extraterrestrial high energy neutrinos in the Fréjus and IMB nucleon decay detectors and the Fly’s Eye air shower array. We improve on previous work by taking into account the experimental energy thresholds, the neutrino opacity of the early universe, neutrino absorption in the Earth and the appropriate neutrino interaction cross sections at high energies.

In section 2, we discuss the cosmological absorption of high energy neutrinos, and in section 3 calculate the spectrum of neutrinos generated by heavy particle decay. A discussion of the expected signals is given in section 4 and the constraints provided by present observations are presented in section 5. Our conclusions follow in section 6.

2. Cosmological neutrino absorption

High energy neutrinos can be absorbed in interactions with the relic thermal neutrino background and with nucleons in the early universe. The dominant processes are the annihilation of a high energy neutrino (or antineutrino) with a background antineutrino (or neutrino) and its inelastic scattering off a nucleon. We obtain below an analytical formula for the absorption redshift $z_a(E_e)$ at which the neutrino opacity of the universe $s_\nu$ is unity for a neutrino emitted with energy $E_e$; less than a fraction $1/e$ of the neutrinos emitted at redshifts larger than $z_a(E_e)$ propagate to the present epoch.
Consider a neutrino emitted with energy $E_e$ at time $t_e$ corresponding to redshift $z_e$. The cosmological neutrino opacity $s_\nu(t_e, E_e)$ is the mean number of scatterings undergone by the neutrino in which it could have been absorbed, given by

$$s_\nu(t_e, E_e) = \int_{t_e}^{t_0} \frac{dt}{\tau_\nu(t, E_\nu)}, \quad (2.1)$$

where $t_0 \sim 0.65 \times 10^{10}$ yr $(\Omega_0 h^2)^{-1/2}$ is the present age of the universe and $\tau_\nu(t, E_\nu)$ is the mean free time between collisions at time $t$ (and redshift $z$) for a neutrino of energy $E_\nu = E_e(1 + z)/(1 + z_e)$. Here $\Omega_0$ is the present mass density of the universe in units of the critical density $\rho_c \simeq 1.9 \times 10^{-29} h^2$ g cm$^{-3}$, where $h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$ ($0.4 \lesssim h \lesssim 1$). Taking into account the two absorption processes mentioned above:

$$\frac{1}{\tau_\nu(t, E_\nu)} = \frac{1}{\tau_{\nu\bar{\nu}}} + \frac{1}{\tau_{\nu N}}, \quad (2.2)$$

where the first term refers to neutrino-antineutrino annihilation and the second to neutrino-nucleon scattering.

To obtain $\tau_{\nu\bar{\nu}}$, we must average over the thermal energy distribution of the relic background (anti)neutrinos. Consider a decay neutrino and a background antineutrino; the same formulae apply to a decay antineutrino and a background neutrino. Indicating by $\theta_{\nu\bar{\nu}}$ the angle between the two colliding particles in the cosmic frame, we have

$$\frac{1}{\tau_{\nu\bar{\nu}}} = \langle (1 - \cos \theta_{\nu\bar{\nu}}) \sigma_{\nu\bar{\nu}} \rangle n_{\bar{\nu}}, \quad (2.3)$$

where $(1 - \cos \theta_{\nu\bar{\nu}})$ is the $\nu\bar{\nu}$ relative velocity (in units of $c$), $\sigma_{\nu\bar{\nu}}$ is the total $\nu\bar{\nu}$ annihilation cross section and $n_{\bar{\nu}}$ is the background antineutrino number density.
at temperature $T_\nu$, given by

$$n_\nu = \frac{3}{4} \frac{\zeta(3)}{\pi^2} T_\nu^3. \quad (2.4)$$

The angular brackets indicate an average over the antineutrino energy distribution,

$$f_\bar{\nu}(E_\bar{\nu}) = \frac{1}{2\pi^2} \frac{E_\bar{\nu}^2}{eE_\bar{\nu}/T_\nu + 1}. \quad (2.5)$$

We consider only annihilations into charged fermion pairs, $\nu\bar{\nu} \rightarrow f\bar{f}$. In this case, $\sigma_{\nu\bar{\nu}} = \sum_f \sigma_{\nu\bar{\nu} \rightarrow f\bar{f}}$, with the sum running over quarks and charged leptons. It is a good approximation for our purposes to consider massless fermions to estimate the annihilation cross section and simply add a new fermion channel whenever $E_\nu T_\nu > m_f^2$. In this case

$$\sigma_{\nu\bar{\nu}} = \frac{G_F^2 s}{4\pi} \left[ N_{\text{eff}}^{NC} P_Z(s) + N_{\text{eff}}^{CC} A_W(s) \right], \quad (2.6)$$

with $s = 2E_\nu E_\bar{\nu}(1 - \cos\theta_{\nu\bar{\nu}})$. The first term in the square brackets is due to neutral currents. The Z boson pole factor is defined as

$$P_Z(s) = \frac{M_Z^4}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}. \quad (2.7)$$

The second term takes into account the charge current contribution to the processes $\nu_e \bar{\nu}_e \rightarrow e^+ e^-$, $\nu_\mu \bar{\nu}_\mu \rightarrow \mu^+ \mu^-$, $\nu_\tau \bar{\nu}_\tau \rightarrow \tau^+ \tau^-$:

$$A_W(s) = \frac{M_W^6}{2s^3 \sin^2\theta_W} \left[ 1 - \frac{M_W^2}{s + M_W^2} + (1 - 2a) \frac{s}{M_W^2} + \frac{as^2}{M_W^4} + 2(a - 1) \ln \left( 1 + \frac{s}{M_W^2} \right) \right], \quad (2.8)$$

with

$$a = \left( \frac{1}{2} - \sin^2\theta_W \right) \frac{M_Z^2(s - M_Z^2)}{(s - M_Z^2)^2 + \Gamma_Z^2 M_Z^2}. \quad (2.9)$$

The coefficients $N_{\text{eff}}$ are the effective numbers of annihilation channels. For neutral
currents, this is calculated as
\[ N_{\text{eff}}^{\text{NC}} = \sum_f \theta \left( E_{\nu} T_{\bar{\nu}} - m_f^2 \right)^2 \frac{2}{3} n_f \left( 1 - 8 t_{3f} q_f \sin^2 \theta_W + 8 q_f^2 \sin^4 \theta_W \right), \tag{2.10} \]

while for charged currents, the coefficient
\[ N_{\text{eff}}^{\text{CC}} = \theta \left( E_{\nu} T_{\bar{\nu}} - m_{l}^2 \right) \frac{16}{3} \sin^2 \theta_W \tag{2.11} \]
is non-zero only if the charged lepton \( l \) is in the same family as the annihilating neutrino. Above, \( n_f \) is the number of colours (1 for leptons, 3 for quarks), and \( t_{3f} \) and \( q_f \) are the third component of the weak isospin and the electric charge of the fermion in units of the positron charge respectively. We take the electroweak mixing angle to be given by \( \sin^2 \theta_W = 0.23 \). Inserting eq. (2.6) into eq. (2.3), we obtain
\[ \frac{1}{\tau_{\nu\bar{\nu}}} = \frac{G_F^2}{4\pi} \left( 1 - \cos \theta_{\nu\bar{\nu}} \right) \left[ N_{\text{eff}}^{\text{NC}} s P_Z(s) + N_{\text{eff}}^{\text{CC}} s A_W(s) \right] \right] n_{\bar{\nu}}, \tag{2.12} \]
where \( s = 2 E_{\nu} E_{\bar{\nu}} (1 - \cos \theta_{\nu\bar{\nu}}) \) is understood.

The thermal average has a peak at \( E_{\nu} T_{\bar{\nu}} \simeq M_Z^2/4 \), corresponding to the Z pole. All neutrinos emitted with energy \( E_{\nu} \simeq M_Z^2/4T_{\bar{\nu}} = 1.26 \times 10^{13} \text{TeV}/(1 + z_e) \) are absorbed. For \( E_{\nu} \lesssim M_W^2/4T_{\bar{\nu}} \), the factors \( P_Z(s) \) and \( A_W(s) \) in eq. (2.6) can be set equal to unity and \( \tau_{\nu\bar{\nu}} \) is easily evaluated as
\[ \frac{1}{\tau_{\nu\bar{\nu}}} = \rho_{\nu} \sigma_0 N_{\text{eff}} E_{\nu} \rho_{\bar{\nu}} = \rho_{\bar{\nu}} \sigma_0 N_{\text{eff}} (1 + z)^5 E_{\nu}. \tag{2.13} \]
Here, \( N_{\text{eff}} = N_{\text{eff}}^{\text{NC}} + N_{\text{eff}}^{\text{CC}} \) is the effective total number of annihilation channels (which varies between 0.33 and 10.1), \( \sigma_0 \) is defined as
\[ \sigma_0 \equiv \frac{2}{3\pi} G_F^2 = 1.12 \times 10^{-32} \text{cm}^2 \text{TeV}^{-2}, \tag{2.14} \]
and
\[ \rho_{\bar{\nu}} = 5.96 \times 10^{-14} \text{TeV cm}^{-3} \tag{2.15} \]
is the present antineutrino energy density (per species), taking the present photon
temperature to be 2.74 K [3]. Thus, the annihilation mean free time is

\[
\tau_{\nu\bar{\nu}} = 1.58 \times 10^{27} \text{ yr} \ N_{\text{eff}}^{-1} (1 + z)^{-5} (1 + z_e) \left( \frac{E_e}{\text{TeV}} \right)^{-1}.
\]  

(2.16)

Next we consider neutrino-nucleon scattering. The thermal motion of the non-relativistic nucleons can be neglected, and we have

\[
\frac{1}{\tau_{\nu N}} = n_N \sigma_{\nu N} = n_{N_0} \frac{\sigma_{\nu N} (1 + z)^4}{E_\nu} \frac{1}{1 + z_e} E_e,
\]

(2.17)

where \(n_N\) is the nucleon number density at redshift \(z\) and \(\sigma_{\nu N}\) is the neutrino nucleon scattering cross section at neutrino energy \(E_\nu\). The present nucleon mean density is in the range \(n_{N_0} \sim (0.25 - 1.5) \times 10^{-7} \text{ cm}^{-3}\) according to Big Bang nucleosynthesis calculations; this reflects the observational uncertainty in primordial \(^4\text{He}\) mass fraction, which is taken to be \(\sim 0.21 - 0.24\) [4].

For \(E_\nu \lesssim 1 \text{ TeV}\), the ratio \(\sigma_{\nu N}/E_\nu\) is constant and equal to \(0.67 \times 10^{-35} \text{ cm}^2 \text{ TeV}^{-1}\) for neutrinos and to \(0.34 \times 10^{-35} \text{ cm}^2 \text{ TeV}^{-1}\) for antineutrinos [5-6]. The neutrino scattering mean free time at these energies is

\[
\tau_{\nu N} \simeq 10^{24} \text{ yr} \ (1 + z)^{-4} (1 + z_e) \left( \frac{E_e}{\text{TeV}} \right)^{-1}.
\]

(2.18)

Comparing with the annihilation mean free time (2.16), we see that inelastic scattering upon nucleons dominates only at redshifts \(1 + z \approx 10^3 N_{\text{eff}}^{-1}\). However, now \(\tau_\nu \simeq \tau_{\nu N} \gtrsim 10^{15} \text{ yr} \gg t_0\), i.e. the universe has already become transparent to neutrinos. At higher neutrino energies, \(\sigma_{\nu N}/E_\nu\) decreases and neutrino-nucleon scattering is even less important, becoming negligible at all redshifts for \(E_\nu \gtrsim 10^6 \text{ TeV}\). Thus inclusion of \(\nu N\) scattering affects \(z_a(E_e)\) only marginally. For simplicity of presentation, we do not therefore write it explicitly in the formulae below, although we have included it in the numerical calculations.
The last ingredient necessary to compute the neutrino opacity is the relationship between the age of the universe and the redshift:

\[
 t = \begin{cases} 
 t_0(1 + z)^{-3/2}, & \text{for } z < z_{\text{eq}}, \\
 t_0(1 + z_{\text{eq}})^{1/2}(1 + z)^{-2}, & \text{for } z > z_{\text{eq}},
\end{cases}
\] (2.19)

where \(1 + z_{\text{eq}} = 2.25 \times 10^4 \Omega_0 h^2\) is the redshift at which the energy density of matter (with present density parameter \(\Omega_0\)) begins to dominate over that of radiation.

The absorption redshift obtained from integration of eq. (2.1), using eqs. (2.13), (2.17) and (2.19), is shown as the diagonal full line in the \(E_e-t_e\) (or \(E_e-z_e\)) plane in figure 1 taking \(\Omega_0 h^2 = 1\). The dot-dashed line separates the two regions where annihilation and scattering absorption dominate. The region of interest extends from the present epoch \((z = 0)\), through the epoch of matter-radiation equality \((z \approx 2 \times 10^4)\), up to the epoch of light neutrino decoupling \((z \approx 10^{10})\), which are all indicated. The location of the Z boson pole is also shown as a diagonal dashed line.

Approximate expressions for the absorption redshift \(z_a(E_e)\) can be obtained for \(1 \ll z_e < z_{\text{eq}}\) and \(z_e \gg z_{\text{eq}}\). In these cases, the result of the integration simplifies to

\[
s_\nu = \begin{cases} 
 3.5 \times 10^{-17}(\Omega_0 h^2)^{-1/2}(1 + z_e)^{5/2} (E_e/\text{TeV}), & \text{for } 1 \ll z_e < z_{\text{eq}}; \\
 0.81 \times 10^{-14}(1 + z_e)^2 (E_e/\text{TeV}), & \text{for } z_e \gg z_{\text{eq}}.
\end{cases}
\] (2.20)

The absorption redshift \(z_a(E_e)\) is then obtained by setting \(s_\nu = 1\):

\[
 1 + z_a(E_e) = \begin{cases} 
 3.8 \times 10^6(\Omega_0 h^2)^{1/5}(E_e/\text{TeV})^{-2/5}, & E_e \gtrsim 5.2 \times 10^5 \text{TeV}(\Omega_0 h^2)^{-2}, \\
 1.1 \times 10^7(E_e/\text{TeV})^{-1/2}, & E_e \ll 5.2 \times 10^5 \text{TeV}(\Omega_0 h^2)^{-2}.
\end{cases}
\] (2.21)
3. Neutrino spectrum

We now determine the present energy spectrum of neutrinos originating from the decay of an unstable heavy particle $x$ with decay lifetime $\tau_x$. The number of neutrinos of type $\nu_i$ ($\nu_i = \nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu, \ldots$) produced at time $t$, per unit comoving volume and unit time, is

$$\gamma_e(t) = \frac{B_{\nu_i} Y_x(t)}{\tau_x} = \frac{B_{\nu_i} Y_{xp}}{\tau_x} \exp\left(-\frac{t}{\tau_x}\right) = \frac{B_{\nu_i} Y_{x0}}{\tau_x} \exp\left(-\frac{t - t_0}{\tau_x}\right), \quad (3.1)$$

where $B_{\nu_i}$ is the number of neutrinos of type $\nu_i$ produced per decaying $x$ particle, $Y_x(t) \equiv n_x(t)/n_\gamma(t)$ is the $x$ particle number density in ratio to the thermal photon density $n_\gamma(t) (= 412.7(1 + z)^3 \text{cm}^{-3})$, $Y_{xp}$ is its primordial value* and $Y_{x0}$ is its value today.

The number of neutrinos absorbed in the same volume, $\gamma_a(t)$, is proportional to the comoving density of decay-generated neutrinos $Y_{\nu_i}(t) = n_{\nu_i}(t)/n_\gamma(t)$ and is given by

$$\gamma_a(t) = \frac{Y_{\nu_i}(t)}{\tau_{\nu_i}(t, E_e)}, \quad (3.2)$$

where $\tau_{\nu_i}(t, E_e)$ is the neutrino absorption mean free time (see section 2).

The evolution of the comoving neutrino density $Y_{\nu_i}(t)$ is governed by

$$\frac{dY_{\nu_i}(t)}{dt} = \gamma_e(t) - \gamma_a(t) = \frac{B_{\nu_i} Y_{xp}}{\tau_x} \exp\left(-\frac{t}{\tau_x}\right) - \frac{Y_{\nu_i}(t)}{\tau_{\nu_i}(t, E_e)}, \quad (3.3)$$

with the following solution at the present epoch $t_0$:

$$Y_{\nu_i0} = B_{\nu_i} Y_{xp} \int_0^{t_0} \exp\left[-\frac{t_e}{\tau_x} - s_{\nu_i}(t_e, E_e)\right] \frac{dt_e}{\tau_x}. \quad (3.4)$$

Now differentiating with respect to the present neutrino energy $E_{\nu_i0} = E_e(1 + \ldots

* This is conveniently measured at the earliest epoch following which the comoving photon number is conserved, say at $T \sim 0.01 m_e$, corresponding to $t \sim 10^{-3} \text{yr}$; this is negligible compared to all other time-scales relevant here.
one obtains the present neutrino flux

\[
E_{\nu_0} \frac{d\phi_{\nu_0}}{dE_{\nu_0}} = \phi_{\gamma_0} B_{\nu_i} Y_{xp} \kappa \frac{t_e}{\tau_x} \exp \left[ -\frac{t_e}{\tau_x} - s_{\nu_i}(t_e, E_e) \right] \theta(E_e - E_{\nu_0}),
\]  

(3.5)

where

\[
t_e = t_0 \left( \frac{E_{\nu_0}}{E_e} \right)^{\kappa}, \quad \kappa = \begin{cases} 
2, & \text{for } E_{\nu_0} < E_e(1 + z_{eq})^{-1}, \\
\frac{3}{2}, & \text{for } E_{\nu_0} > E_e(1 + z_{eq})^{-1},
\end{cases}
\]

(3.6)

and \( \phi_{\gamma_0} = n_{\gamma_0}/4\pi = 0.98 \times 10^{12} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \) is the present background photon flux per unit solid angle. The decay neutrino flux is shown in figure 2 for \( \tau_x = 10^{-5} t_0 \) and \( 3E_e = 10^5 \text{ TeV}, 10^7 \text{ TeV}, 10^9 \text{ TeV} \). Notice that the present neutrino energy \( E_\nu \) is redshifted from \( E_e \). The dotted lines indicate what the flux would have been without cosmological neutrino absorption. These three curves are simple translations of each other, since the differential flux (3.5) with \( s_{\nu_i} = 0 \) depends only on the ratio \( E_{\nu_0}/E_e \).

Approximating the effect of the cosmological neutrino absorption with \( e^{-s_{\nu_i}} \simeq \theta(t_e - t_a) \), where \( t_a < t_0 \) corresponds to the absorption redshift \( z_a(E_e) \) at which the neutrino opacity is unity, eq. (3.5) can be easily integrated to obtain the total neutrino flux today,

\[
\phi_{\nu_0} \simeq \phi_{\gamma_0} B_{\nu_i} Y_{xp} (e^{-t_a/\tau_x} - e^{-t_0/\tau_x}).
\]

(3.7)

For \( \tau_x \ll t_0 - t_a \), this reduces to

\[
\phi_{\nu_0} \simeq \phi_{\gamma_0} B_{\nu_i} Y_{xp} e^{-t_a/\tau_x},
\]

(3.8)

while for \( \tau_x \gg t_0 - t_a \), it becomes

\[
\phi_{\nu_0} \simeq \phi_{\gamma_0} B_{\nu_i} Y_{xp} \frac{t_0 - t_a}{\tau_x}.
\]

(3.9)

The neutrino flux is exponentially suppressed for \( \tau_x \ll t_a \) and reaches a maximum of \( \phi_{\nu_0} \simeq B_{\nu_i} Y_{xp} \times 10^{12} \text{ cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \) for \( \tau_x \simeq t_0 - t_a \). This flux is potentially
enormous compared with present bounds on the diffuse extragalactic high energy neutrino flux (∼ $10^{-6}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ for $E_{\nu} \gtrsim 1$ TeV and ∼ $10^{-16}$ cm$^{-2}$ s$^{-1}$ sr$^{-1}$ for $E_{\nu} \gtrsim 10^7$ TeV) which may be inferred from data obtained with underground detectors and cosmic ray observatories (cf. section 4). Hence very restrictive bounds may be obtained on the abundance of the hypothetical decaying particle as demonstrated below.

4. Expected signals

At present, the best means to detect a diffuse background of high energy neutrinos is through the production of an energetic charged lepton in the collision of such a neutrino with a nucleon. We consider three possible types of signal according to where the interaction occurs. An event is called contained when the interaction occurs inside an underground detector, such as Fréjus, IMB and Kamiokande. A flux of through-going muons is registered when interactions occur in the material surrounding the detector (rock in the underground experiments and water in the forthcoming DUMAND, GRANDE and Lake Baikal experiments). Finally, if the interaction occurs in the atmosphere an extensive air shower (EAS) is generated, which can be detected by cosmic ray observatories such as Fly’s Eye and CASA.

We consider the following experimental constraints on the total neutrino flux:

(1) the rate of contained events in the Fréjus detector, with electron and/or muon energies greater than 3 GeV, does not exceed 17.7 kton$^{-1}$ yr$^{-1}$ [12];*

(2) the rate of contained events with energies between 100 MeV and 2.5 GeV in the IMB-3 detector is limited by 111.5 kton$^{-1}$ yr$^{-1}$ [13].†

* We consider the 11 electron and 14 muon charged current events over 3 GeV observed in 1.56 kton yr (see fig. 3 of ref. [12]), and apply the quoted identification efficiencies of 85% and 95% respectively.
† From the total number of 422 contained events in 3.4 kton yr, we exclude the 43 events below 100 MeV (see fig. 2 of ref. [13]) where the track reconstruction and identification efficiencies are low. For comparison the IMB-1 detector recorded 401 contained events in 3.77 kton yr [14].
In fact, the observed contained events are well accounted for by the expected neutrino flux from cosmic ray interactions in the atmosphere [15], within the uncertainty of $\sim 25\%$ in these computations. Hence the bound on contained events of non-atmospheric origin can, in principle, be improved by up to a factor of $\sim 10$ and the limits to be derived strengthened proportionally.

We also consider the following constraints on any extraterrestrial neutrino flux:

(3) the flux of upward-going muons (from directions with zenith angle larger than $98^\circ$) with energy greater than 2 GeV registered by the IMB-1 detector is less than $2.65 \times 10^{-13}$ cm$^{-2}$ s$^{-1}$ at the 90% confidence level, after subtraction of the expected atmospheric component [16];‡

(4) the Fly’s Eye array has set upper limits on the rate of neutrino-induced EAS’s of $10^{-45}$ sr$^{-1}$, $3.8 \times 10^{-46}$ sr$^{-1}$, $10^{-46}$ sr$^{-1}$ and $3.8 \times 10^{-47}$ sr$^{-1}$, all at the 90% confidence level, for neutrino energies higher than $10^5$ TeV, $10^6$ TeV, $10^7$ TeV and $10^8$ TeV respectively [18].

For the isotropic neutrino flux (3.5), the rate of contained events per unit detector mass, $R_c$, the upward-going muon flux, $\phi_\mu$, and the rate of EAS’s per unit solid angle, $J$, can all be written in the form

$$S = \sum_i \int dE_{\nu_i} \frac{d\phi_{\nu_i}}{dE_{\nu_i}} P_i(E_{\nu_i}) \Omega_i(E_{\nu_i}), \quad (4.1)$$

where the signal $S$ is $R_c$, $\phi_\mu$ or $J$, and the sum is over neutrino types ($\nu_i = \nu_e$, $\bar{\nu}_e$, $\nu_\mu$, $\bar{\nu}_\mu$, . . .). The effective aperture $\Omega_i(E_{\nu_i})$, which takes account of neutrino absorption by the Earth (if any), and the transfer functions $P_i(E_{\nu_i})$ depend on the experimental data set considered. (We have dropped the subscript 0 referring to the present neutrino energy.)

‡ The recent Kamiokande upper limit of $4 \times 10^{-14}$ muons cm$^{-2}$ s$^{-1}$ sr$^{-1}$ for zenith angles larger than $150^\circ$ [17] is slightly less stringent than the IMB limit we consider, which corresponds to $3.7 \times 10^{-14}$ muons cm$^{-2}$ s$^{-1}$ sr$^{-1}$. 

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For a simplified model of the Earth with uniform density \( \rho_\oplus = 5.5 \text{ g cm}^{-3} \) and radius \( R_\oplus = 6.37 \times 10^8 \text{ cm} \), the effective aperture is (neglecting the depth of the underground detector relative to \( R_\oplus \)),

\[
\Omega_i(E_{\nu_i}) = \int d\Omega \exp \left[ -2R_\oplus k_i(E_{\nu_i}) |\cos \vartheta| \right] \theta(-\cos \vartheta),
\]

where the integral extends over the geometrical aperture of the detector, \( \vartheta \) is the zenith angle and \( k_i(E_{\nu_i}) \) is the neutrino absorption coefficient in the Earth, given by

\[
k_i(E_{\nu_i}) = \frac{\rho_\oplus}{m_N} \sigma_{\nu_iN}(E_{\nu_i}),
\]

with \( m_N \) the nucleon mass and \( \sigma_{\nu_iN}(E_{\nu_i}) \) the total neutrino-nucleon cross section. For the neutrino energies under consideration, \( \sigma_{\nu_iN}(E_{\nu_i}) \) includes only the charged current cross section \( \sigma_{\nu_iN}^{\text{CC}}(E_{\nu_i}) \), because the energy and momentum fractions transferred to the nucleon in a neutral current process are negligible at these energies.

The charged current cross section \( \sigma_{\nu_iN}^{\text{CC}}(E_{\nu_i}) \) is well-known for \( E_{\nu_i} \lesssim 10 \text{ TeV} \):

\[
\sigma_{\nu_iN}^{\text{CC}} = 0.67 \times 10^{-35} \text{ cm}^2 \left( \frac{E_{\nu_i}}{\text{TeV}} \right),
\]

for a neutrino and

\[
\sigma_{\bar{\nu}_iN}^{\text{CC}} = 0.34 \times 10^{-35} \text{ cm}^2 \left( \frac{E_{\bar{\nu}_i}}{\text{TeV}} \right),
\]

for an antineutrino [5]. At higher energies the charged current cross section becomes more and more uncertain — by as much as a factor of 10 at \( E_{\nu_i} \simeq 10^9 \text{ TeV} \) — because of the poor knowledge of nucleon structure functions at small arguments [11]. For this reason, we have not attempted a precise calculation of \( \sigma_{\nu_iN}^{\text{CC}}(E_{\nu_i}) \) from a set of theoretical structure functions. We have used the differential charged current cross section up to \( E_{\nu_i} = 10^7 \text{ TeV} \) given in ref. [8]. At still higher energies, we have matched the asymptotic form of the cross section in ref. [7] to the results of ref. [8].
In figure 3 we show the effect of absorption in the Earth by plotting the effective aperture (4.2) integrated below the horizon:

\[
\Omega_{\text{below}}^{\nu_i}(E_{\nu_i}) = \frac{2\pi \sigma_{\oplus}}{\sigma_{\nu_i N}(E_{\nu_i})} \left[ 1 - \exp \left( -\frac{\sigma_{\nu_i N}(E_{\nu_i})}{\sigma_{\oplus}} \right) \right], \tag{4.6}
\]

with \( \sigma_{\oplus} = m_\odot/2R_\odot \rho_\odot = 2.4 \times 10^{-34} \text{ cm}^2 \). This effective aperture differs very little from the one obtained in ref. [19] using a more elaborate model of the Earth. As we see from the figure, the Earth severely attenuates the flux of neutrinos of energy exceeding \( \sim 10^5 \text{ TeV} \), becoming nearly opaque at \( \sim 10^{10} \text{ TeV} \).

Note that for \( \sigma \gtrsim 10^{-33} \text{ cm}^2 \), i.e. at \( E_{\nu_i} \gtrsim 10^3 \text{ TeV} \), the effective aperture from below the horizon is inversely proportional to the neutrino-nucleon scattering cross section,

\[
\Omega_{\text{below}}^{\nu_i}(E_{\nu_i}) \simeq 2\pi \frac{\sigma_{\oplus}}{\sigma_{\nu_i N}(E_{\nu_i})}. \tag{4.7}
\]

The resonant reaction \( \bar{\nu}_e e^- \rightarrow W^- \rightarrow \) “anything” severely depletes the \( \bar{\nu}_e \) flux from below the horizon at energies around \( E_{\bar{\nu}_e} \simeq 7 \times 10^3 \text{ TeV} \) [20]. However since we do not assume any predominant neutrino type in the decay neutrino flux, the \( \bar{\nu}_e \) flux from below accounts for only one eighth of the total rate of contained events. It is therefore a reasonable approximation, for our purposes, to neglect this resonance.

We present now the transfer functions \( P_i(E_{\nu_i}) \) for the experimental data sets under consideration. For contained events we have

\[
P_{i}^{\text{cont}}(E_{\nu_i}) = \theta(E_{\nu_i} - E_{\text{th}}) \frac{N_{\text{nucl}}}{M} \int_{E_{\text{th}}}^{\min(E_{\text{cut}},E_{\nu_i})} dE_{\nu_i} \frac{d\sigma_{\nu_i N}^{\text{CC}}}{dE_{\nu_i}}, \tag{4.8}
\]

where \( E_{\text{th}} \) is the experimental energy threshold for the lepton energy \( E_{\nu_i} \), \( E_{\text{cut}} \) is an experimental cutoff (2.5 GeV for IMB and infinite for Fréjus), \( M \) is the detector mass and \( N_{\text{nucl}} = 6.02 \times 10^{32} (M/\text{kton}) \) is the number of nucleons in
the detector. If the charged current cross section is written in units of $10^{-38}$ cm$^2$,
$$\sigma^{CC}_{\nu_iN}(E_{\nu_i}) = \sigma_{i,38} \times 10^{-38} \text{ cm}^2,$$
then the transfer function for contained events is
$$P^\text{cont}_i(E_{\nu_i}) \simeq 6.0 \times 10^{-6} \text{ cm}^2 \text{kton}^{-1} \theta(E_{\nu_i} - E_{\text{th}}) \int_{E_{\text{th}}}^{\text{min}(E_{\text{cut}},E_{\nu_i})} dE_{\nu_i} \frac{d\sigma_{i,38}}{dE_{\nu_i}}. \quad (4.9)$$

Both Fréjus and IMB data sets include neutrinos coming from all solid angles and their effective aperture computed from eq. (4.2) varies from $4\pi$ to $2\pi$ as the energy is increased; the neutrino flux is reduced at most by a factor of 2 at the highest energies.

In IMB, and other water-Čerenkov detectors, there is also the possibility that the hadronic fragments produced in the neutrino-nucleus collision give a detectable amount of Čerenkov light. Their contribution $P^\text{cont}_{i,\text{hadr}}(E_{\nu_i})$ should then be added to eq. (4.9). In the appendix, we present an estimate of the contribution from such ‘hadronic blasts’ and show that this is important only for very energetic neutrinos, $E_{\nu_i} \gtrsim 10^7$ TeV, where, however, the signal from EAS’s gives more stringent constraints.

The product $P^\text{cont}_i(E_{\nu_i}) \Omega^\text{cont}_i(E_{\nu_i})$ thus obtained for the Fréjus and IMB contained events is shown in figure 4(a) for neutrinos (solid lines) and antineutrinos (dotted lines) as function of the neutrino (or antineutrino) energy $E_{\nu_i}$. The units are chosen such that the vertical axis directly gives the number of events per kiloton-year corresponding to a unit neutrino flux of $1 \text{ cm}^{-2} \text{s}^{-1} \text{ sr}^{-1}$. For $E_{\nu_i} \gtrsim 10^6$ TeV we calculate $P^\text{cont}_i(E_{\nu_i}) \Omega^\text{cont}_i(E_{\nu_i}) \simeq 3.8 \times 10^{-5} \text{ cm}^2 \text{ sr kton}^{-1}$ $\sigma_{i,38}$ for the Fréjus detector. The IMB curve (curve 1) above $10^7$ TeV is due to ‘hadronic blasts’ as discussed in the appendix.

The transfer function for the flux of up-going muons is (see ref. [21])
$$P^\mu_i(E_{\nu_i}) = \theta(E_{\nu_i} - E_{\text{th}}) \int_{E_{\text{th}}}^{\infty} \frac{dE_{\mu}'}{E_{\mu}'} \int_{0}^{E_{\mu}'} \frac{dE_{\mu}}{dX g(X, E_{\mu}', E_{\mu})} \frac{d\sigma_{\nu_iN}}{dE_{\mu}'}; \quad (4.10)$$
for $\nu_i = \nu_{\mu}, \bar{\nu}_\mu$, and $P^\mu_i(E_{\nu_i}) = 0$ for the other neutrino types. Here $E_{\mu}$ and
\( E'_\mu \) are the muon energies at production and at the detector respectively, and \( X = l \rho_{\text{rock}}/m_N \) is the column density of rock, i.e. the number of nucleons per unit area encountered by a muon travelling a length \( l \) in rock. Notice that \( P'_\mu \) is adimensional. The probability that a muon with initial energy \( E_\mu \) has an energy between \( E'_\mu \) and \( E'_\mu + dE'_\mu \) after traversing an amount \( x \) of rock is denoted by \( g(X, E'_\mu, E_\mu) \, dE'_\mu \). We assume a uniform rock density of \( \rho_{\text{rock}} = 2.6 \, \text{g cm}^{-3} \) in the region surrounding the detector. Following ref. [6], we make the approximation that the final muon energy \( E'_\mu \) coincides with its mean value (with no dispersion):

\[
\overline{E'_\mu} = (E_\mu + \epsilon)e^{-\gamma X} - \epsilon, \tag{4.11}
\]

with \( \epsilon \simeq 0.51 \, \text{TeV} \) and \( \gamma^{-1} = 1.54 \times 10^{29} \, \text{cm}^{-2} \). The integral over \( E'_\mu \) in eq. (4.10) can then be performed, and we obtain

\[
P^\mu_i(E_{\nu_i}) = \theta(E_{\nu_i} - E_{\text{th}}) \int_{E_{\text{th}}}^{E_{\nu_i}} \frac{dE'_\mu}{dE'_\mu} \frac{X_{\text{th}}(E_\mu)}{dE'_\mu} \frac{d\sigma_{\nu_i N}}{d\sigma_{\nu_i N}}, \tag{4.12}
\]

for \( \nu_i = \nu_\mu, \bar{\nu}_\mu \). Here

\[
X_{\text{th}}(E_\mu) = \gamma^{-1} \ln \left( \frac{1 + E_\mu/\epsilon}{1 + E_{\text{th}}/\epsilon} \right) \tag{4.13}
\]

is the column density traversed by muons produced with energy \( E_\mu \) which reach the detector with threshold energy \( E_{\text{th}} \). A plot of \( P^\mu_i(E_{\nu_i}) \) times \( \Omega^\mu_i(E_{\nu_i}) \), obtained by integrating eq. (4.2) over zenith angles larger than 98°, is shown in figure 4(b). The decrease of \( P^\mu \Omega^\mu \) for \( E_{\nu_i} \gtrsim 10^7 \, \text{TeV} \) is due to absorption by the Earth.

The final signal we consider is the rate of EAS’s per unit solid angle. Its transfer function is

\[
P^\text{EAS}_i(E_{\nu_i}) = \frac{1}{\Omega^\text{EAS}_i(E_{\nu_i})} \frac{d\sigma_{\nu_i N}(E_{\nu_i})}{d\sigma_{\nu_i N}} \theta(E_{\nu_i} - E_{\text{th}}), \tag{4.14}
\]

where the index \( i \) stands for \( \nu_e \) and \( \nu_\tau \), which can generate electromagnetic or hadronic showers in the atmosphere. Muons from charged current \( \nu_\mu \)-nucleon interactions do not trigger electromagnetic showers, since their radiation length for
bremstrahlung in air \((10^5 \text{ g/cm}^2)\) is much larger than the atmosphere thickness \((1030 \text{ g/cm}^2)\). The product \(P_{EAS}^i(E_{\nu_i}) \Omega_{EAS}^i(E_{\nu_i})\) is shown in figure 4(c) for the four experimental energy thresholds of the Fly’s Eye detector [18].

We are now in a position to compare the expected signals to the experimental limits.

5. Present constraints

We assume that the same numbers of neutrinos and antineutrinos of each type, \(B_{\nu_e} = B_{\bar{\nu}_e} = B_{\nu_\mu} = B_{\bar{\nu}_\mu} = \ldots \equiv B_{\nu}\), are produced in \(x\) decays and that their production energy is always \(E_e = \frac{1}{3} m_x\). Inserting the decay-generated neutrino flux, eq. (3.5), into eq. (4.1) and using the appropriate transfer functions and effective apertures described in section 3, we obtain the expected signals in terms of the decay lifetime \(\tau_x\) and the quantity \(B_{\nu} m_x Y_{x_p} = B_{\nu} m_x Y_{x_0} e^{t_0/\tau_x}\) which is proportional to the primordial energy density of the decaying particles. Notice that for \(\tau_x \gg t_0\), this quantity is \(\sim 25.5 \text{ eV} \left(B_{\nu} \Omega_{x_0} h^2\right)\), where \(\Omega_{x_0}\) is the present \(x\) mass density in units of the critical cosmological density.

We present the results in figure 5. The shaded regions are excluded by the present experimental data. The solid lines refer to the limit on upward-going muons from IMB, the dotted and short-dashed lines to the Fréjus and IMB contained events, respectively, and the long-dashed lines to the Fly’s Eye EAS’s. These are essentially bounds on the relic energy density of the decaying particle, taking \(B_{\nu} = 1\); for \(B_{\nu} < 1\), these lines are to be proportionally shifted upward. As noted earlier, the experimental limits on contained events can, in principle, be improved by a factor of \(\sim 10\) if the signal due to atmospheric neutrinos is accounted for; the corresponding bounds should then be scaled downwards by the same factor. The short-dashed–dotted line corresponds to a present mass density \(\Omega_0 h^2 = 1\) either in \(x\) particles (for \(\tau_x \gg t_0\)) or in its decay products (for \(\tau_x \lesssim t_0\)); the region \(\Omega_0 h^2 > 1\) is excluded by the observational lower limits to the age and present expansion rate of the universe. For comparison we also show as a long-dashed–dotted line (in the
lower left quadrant) the upper bound on \( m_x Y_x \) for unstable particles decaying into electromagnetically interacting particles. This is obtained by requiring that the abundance of the primordially synthesised light elements \( D, \, ^3He, \, ^4He \) and \( ^7Li \) not be excessively altered from their observationally inferred values by the electromagnetic cascades initiated by the decay products [22]. In fact this bound also applies to unstable particles decaying into neutrinos, since the decay neutrinos can initiate similar electromagnetic cascades through the process \( \nu \bar{\nu} \rightarrow e^+ e^- \), where the target (anti)neutrinos belong to the thermal background. (This has also been considered in ref. [23]; however these authors do not calculate cascade generation correctly and obtain an overly restrictive bound.)

Figures 5(a-d) correspond to \( m_x = 1, \, 10^5, \, 10^6 \) and \( 10^{10} \) TeV respectively. As the \( x \) mass increases, the bounds at \( \tau_x \ll t_0 \) first shift to the left as the decay neutrinos become more energetic and the signals go further above the experimental thresholds. Then they proceed to move to the right because the neutrino absorption redshift decreases with increasing neutrino energy, hence the neutrinos produced by relatively short-lived particles do not survive until the present. The mass at the turning point is given approximately by solving \( m_x \simeq 3 E_{\text{th}}[1 + z_a(\frac{1}{3} m_x)] \) with the help of eqs. (2.21); its value is \( 1 \times 10^3 \) TeV and \( 1 \times 10^2 \) TeV for contained events in Fréjus and IMB respectively, \( 8 \times 10^2 \) TeV for the IMB upward-going muons, and \( 1 \times 10^8 \) TeV, \( 1 \times 10^9 \) TeV, \( 5 \times 10^9 \) TeV and \( 3 \times 10^{10} \) TeV for the four Fly’s Eye thresholds.

When \( \tau_x \gtrsim t_0 \), the best bounds on the relic abundance of the decaying particles come from the IMB limit on upward-going muons at \( m_x \lesssim 5 \times 10^5 \) TeV and the Fly’s Eye limits on EASs at \( m_x \gtrsim 5 \times 10^5 \) TeV. We can invert the argument and consider the interesting case \( \Omega_{x_0} h^2 \simeq 1 \), i.e. when the \( x \) particles are assumed to constitute the dark matter today.* A corresponding lower bound on its lifetime

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* In this case the actual spatial distribution of the relic particles should be taken into account, e.g. their likely concentration in the halo of our Galaxy. This would yield even stricter constraints. Preliminary work has been reported in ref. [24] and a more detailed study is in progress.
versus its mass can then be inferred and is plotted in figure 6. For $m_x \lesssim 30\,\text{TeV}$, this bound gets stronger with increasing $m_x$ as the mean energy of the decay generated neutrinos rises over the IMB detection threshold. For $m_x \gtrsim 30\,\text{TeV}$, the lower bound on $\tau_x$ is inversely proportional to $m_x$ (cf. eq. (3.9)), apart from the jumps at $\simeq 10^5\,\text{TeV}$, $\simeq 10^6\,\text{TeV}$, $\simeq 10^7\,\text{TeV}$ and $\simeq 10^8\,\text{TeV}$ corresponding to the Fly’s Eye energy thresholds. No bound exists for $m_x \gtrsim 5 \times 10^{14}\,\text{TeV}$, since the universe is opaque to such high energy neutrinos at the present epoch.

### 6. Conclusions

We have considered constraints on the lifetime, the abundance and the mass of unstable relic particles decaying into neutrinos at cosmological epochs, taking into account that both the early universe and the Earth are opaque to very high energy neutrinos. We have evaluated the signals expected from a diffuse background of decay-generated neutrinos in underground nucleon decay experiments and at cosmic ray observatories. Comparing these to the present limits on the flux of non-atmospheric neutrinos, we find severe bounds on the relic abundance of such heavy particles; in particular, such particles must be very long-lived indeed in order to constitute the dark matter today. These bounds are of relevance to massive metastable particles such as technicolour baryons and ‘cryptons’ (bound states in the hidden sector of superstring-inspired models) as discussed elsewhere [22].

### 7. Acknowledgements

We are grateful to the referee for suggesting that we consider the effects of hadronic ‘blasts’ in water-Čerenkov detectors.

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Note Added
Recently, we became aware of ref. [25], where the detection of neutrinos from cosmic relic particles is also studied, in particular the effects due to absorption in the early Universe. However this work assumes that the decaying particles were thermally produced (with a calculable abundance) in the early Universe, whereas we have presented results in a general form, applicable to any relic particle. This, in fact, is essential in order to consider particles with masses over a few hundred TeV. We also believe that we have addressed experimental issues in more detail.

APPENDIX

Here we present an estimate of the contribution of ‘hadronic blasts’ to the IMB transfer function for contained events. We calculate the visible energy equivalent to the Čerenkov light output from such a ‘blast’ and then compare it to the visible energy observable in the IMB detector.

Let $W$ be the energy transferred to the nucleus in the neutrino interaction. This is also presumably the energy available to the hadronic shower generated by the nuclear fragments. The visible energy $E_{\text{vis}}$ is defined to be the energy of a fictitious initial electron generating an electromagnetic shower with the same Čerenkov light output [13]. The physics of electromagnetic showers [26] then relates $E_{\text{vis}}$ to the ‘detectable’ track length $X_d$.

$$E_{\text{vis}} = E_c \frac{X_d(W)}{X_0 F(z)}, \quad (A.1)$$

where $E_c$ is the critical energy separating the domains where ionization and radiation energy losses dominate and is approximately given by

$$E_c \simeq \frac{800 \text{ MeV}}{Z + 1.2} = 71 \text{ MeV}. \quad (A.2)$$

Here $X_0(= 36.08 \text{ g/cm}^2 [27])$ is the electron radiation length in water and $F(z)$ is
approximately

\[ F(z) \simeq e^z \left( 1 + z \ln \frac{z}{1.526} \right), \quad (A.3) \]

with

\[ z \simeq 4.58 \frac{Z E_d}{A E_e} \simeq \frac{E_d}{28 \text{ MeV}} \quad (A.4) \]

(the numerical values refer to water). In the IMB detector, \( E_d = 1.52 m_e + 30 \text{ MeV} + 140 \text{ MeV} = 170 \text{ MeV} \) for an electron [14], hence eq. (A.1) reads

\[ E_{\text{vis}} = 4.8 \times 10^{-4} \text{ MeV} X_d(W) \quad (A.5) \]

with \( X_d(W) \) in g/cm\(^2\).

It remains now to estimate \( X_d(W) \), the path length of charged particles in the hadronic shower with energy above the detection threshold. The mean number of charged particles \( n_{\text{ch}}(W) \) as a function of the available energy \( W \) has been studied in deep inelastic scattering and incorporated into the Lund Monte Carlo program [28]. By fitting fig. 8 of this reference we obtain:

\[ n_{\text{ch}}(W) = 1.67 + 0.211 \exp \left[ 3.06 \ln^{1/2} \left( \frac{W}{\text{GeV}} \right) \right]. \quad (A.6) \]

We assume now that all charged particles in the first generation of the shower are above the Čerenkov threshold. This is a good approximation at the high energies that will turn out to be relevant for hadronic blasts. With this assumption, \( X_d(W) \) is the product of \( n_{\text{ch}}(W) \) and the mean path length of a charged particle. A lower bound to the latter is one nuclear interaction length, \( X_{\text{nucl}} = 84.9 \text{ g/cm}^2 \) in water. This leads to a lower bound for \( X_d(W) \):

\[ X_d(W) > X_{\text{min}}(W) \simeq n_{\text{ch}}(W) X_{\text{nucl}}. \quad (A.7) \]

An upper bound is obtained by multiplying \( X_{\text{min}}(W) \) by the typical length of a
hadronic shower in units of interaction lengths [29]:

\[ X_d(W) < X_{\text{max}}(W) \simeq n_{\text{ch}}(W)X_{\text{nuc}} \left[ 5.45 + 0.89 \ln \left( \frac{W}{\text{GeV}} \right) \right]. \tag{A.8} \]

A lower and an upper bound to the visible energy \( E_{\text{vis}}(W) \) can then be obtained from eq. (A.5).

This range then has to be compared with the IMB energy threshold for \( \pi^\pm \) detection, \( E_\pi = 1.52m_\pi + 30\text{ MeV} + 140\text{ MeV} = 382\text{ MeV} \), and with the highest energy analyzed, \( E_{\text{vis}} = 2500\text{ MeV} \) [13]. The result of such a comparison is that for \( W \lesssim 10^2\text{ TeV} \) there is probably not enough Čerenkov light for the ‘blast’ to be seen, while for \( W \gtrsim 10^6\text{ TeV} \) the detector is probably overloaded (more than 900 PMTs fired [13]) or otherwise not sufficiently efficient to detect the blast. However for \( 10^2\text{ TeV} \lesssim W \lesssim 10^6\text{ TeV} \), such blasts, if they do occur, should already be present in the sample of ref. [13], probably as multiple-ring events.

However, the neutrino energy \( E_\nu \) required to have \( W \gtrsim 10^2\text{ TeV} \) is \( E_\nu \gtrsim 10^7\text{ TeV} \). This comes from the kinematic relation \( W^2 \simeq 2m_\nu E_\nu(1-x)y \), with \( m_\nu \) the nucleon mass and \( x \) and \( y \) the usual deep inelastic scattering variables, together with the consideration that at high energies the W or Z propagator restricts the important values of \( x \) and \( y \) to \( x \simeq 0 \) and \( y \simeq 1 \). So ‘hadronic blasts’ in Čerenkov detectors turn out to be important only for very high energy neutrinos, where (at least for the purposes of this paper) there already are much better bounds on cosmic neutrino fluxes from EAS arrays.

For the sake of completeness, we show in fig. 4a (curve 1 at high energies) the contribution of hadronic blasts in IMB, obtained by numerical integration of:

\[
P_{\text{cont hadr}}(E_\nu) = \frac{N_{\text{nuc}}}{M} \int_0^{2m_\nu E_\nu} \frac{dW_{\text{max}}^2}{(2m_\nu E_\nu)^2y} \left[ \frac{d\sigma^{\text{CC}}}{dx dy} + \frac{d\sigma^{\text{NC}}}{dx dy} \right]. \tag{A.9} \]

Here \( Q^2 \simeq 2m_\nu E_\nu xy \), \( W_{\text{min}} = 10^2\text{ TeV} \), \( W_{\text{max}} = 10^6\text{ TeV} \) and the contributions from charged and neutral currents have been summed. The EHLQ structure functions
have been used together with a McKay-Ralston asymptotic form at low $x$ (as in ref. [10]).
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FIGURE CAPTIONS

Fig. 1. The absorption redshift $z_a$ (line 1) for cosmic neutrinos as a function of the neutrino energy at emission $E_e$ taking $\Omega_0 h^2 = 1$. The other lines indicate: (2) the boundary between the regions where absorption due to annihilation and scattering dominate; (3) the present epoch; (4) the Z boson pole; (5) the epoch of matter-radiation equality; (6) the epoch of light neutrino decoupling.

Fig. 2. The present energy spectrum of decay generated neutrinos for $\tau_x = 10^{-5} t_0$ and $3E_e = 10^5$ TeV (line 1), $10^7$ TeV (line 2) and $10^9$ TeV (line 3). The full lines show the effects of cosmological neutrino absorption.

Fig. 3. The effective detector aperture, integrated below the horizon, for neutrinos (solid line) and antineutrinos (dotted line), demonstrating the opacity of the Earth at high energies.

Fig. 4. The product of the transfer function $P_i(E_{\nu_i})$ and of the effective aperture $\Omega_i(E_{\nu_i})$ for the three experimental data sets we consider: (a) contained events in IMB (curve 1) and Fréjus (curve 2) (b) IMB upward-going muons and (c) Fly’s Eye EAS’s (four thresholds). The dotted lines correspond to antineutrinos.

Fig. 5. The bound on the primordial energy density of the decaying particle multiplied by the branching ratio into neutrinos, $B_{\nu} m_x Y_{x\nu}$, as a function of its lifetime $\tau_x$, for various choices of its mass $m_x$. The shaded regions are excluded by the present experimental data. The various lines refer to: IMB upward-going muons (solid lines), Fréjus and IMB contained events (dotted and short-dashed lines respectively), Fly’s Eye EAS’s (long-dashed lines). Also indicated are the upper bound on the total energy density $\Omega_0 h^2 = 1$ (short-dashed–dotted line) and the upper bound inferred from considerations of primordial light element abundances (long-dashed–dotted line).

Fig. 6. The lower bound on the $x$ particle lifetime versus its mass for a present density $\Omega_{x_0} h^2 = 1$ in the relic particles, assuming unit branching ratio into
neutrinos