A new approach to the analysis of three body decays is presented. Model-independent results are obtained for the $S$-wave $K\pi$ amplitude as a function of $K\pi$ invariant mass. These are compared with results from $K^-\pi^+$ elastic scattering, and the prediction of the Watson theorem, that the phase behaviour be the same below $K\eta'$ threshold, is tested. Contributions from $I = \frac{1}{2}$ and $I = \frac{3}{2}$ are not resolved in this study. If $I = \frac{1}{2}$ dominates, however, the Watson theorem does not describe these data well.

1 Introduction

Decays of heavy-quark mesons are regarded as a potential source of information on the light-quark mesons they produce. For decays to three pseudo-scalar final states, kinematics and angular momentum conservation favor production of $S$-wave systems, so improving our knowledge of the particularly confusing scalar meson ($J^P = 0^+$) spectrum may be possible when the anticipated large, clean samples of such decays of $D$ mesons from the $B$ factories and the Tevatron collider become available. Extracting this information has, however, been done in model-dependent ways that make assumptions about the scalar states observed. Such assumptions can influence the results, so new approaches are required.

In this paper, we present a model-independent approach to the $S$-wave system in a study of the decays $D^+ \to K^-\pi^+\pi^+$ observed in data from Fermilab experiment E791. We compare the $S$-wave amplitudes so obtained with our earlier, isobar model analysis and also with $K^-\pi^+$ scattering.

Measurements of $K^-\pi^+$ scattering come principally from SLAC experiment E135 (LASS) and cover the invariant mass range only above 825 MeV/$c^2$. Data exist below this range, but

*Throughout the paper, charge conjugate states are implied unless explicitly stated otherwise.*
with less precision\textsuperscript{11}. More information in the low mass region is required if the possibility of the existence of a \( \kappa \) state is to be properly evaluated.

2 Data Sample

The selection process for events used in this analysis is described in Ref.\textsuperscript{1}. A signal consisting of 15,079 \( D^+ \to K^-\pi^+_a\pi^+_b \) decays, with a purity of \( \sim 94\% \), is obtained. Fig.\textsuperscript{4} shows the Dalitz plot with \( K^-\pi^+ \) squared invariant mass \( s \) plotted vs. \( s' \). Horizontal (and the symmetrized vertical) bands corresponding to the \( K^*(892) \) resonance are clearly seen. A striking and complex pattern of both constructive and destructive interference is seen near 2 \((\text{GeV}/c^2)^2\) due to either \( K^*_0(1430) \), \( K^*_1(1410) \) or \( K^*_2(1430) \). There is also evidence for \( K^*_1(1680) \), difficult to see due to smearing of the Dalitz plot boundary resulting from the finite resolution in the three-body \( D^+ \) mass.

![Figure 1: Dalitz plot for \( D^+ \to K^-\pi^+_a\pi^+_b \) decays. The squared invariant mass \( s \) is plotted against \( s' \). The plot is symmetrized, each event appearing twice. Lines in both directions indicate values equally spaced in squared effective mass at each of which the \( S \)-wave amplitude is determined by the model-independent partial wave analysis (MIPWA) described in section 3. Kinematic boundaries for the Dalitz plot are drawn for three-body mass values \( M = 1.810 \) and \( M = 1.890 \text{GeV}/c^2 \), between which data are selected for the fits.](image)

The most striking effect observed is the asymmetry in the \( K^*(892) \) bands, most easily described by interference with a significant \( S \)-wave contribution to the decay. In this paper we are able to extract information on the \( S \)-wave using the \( K^*(892) \), and also the other well established resonances in the Dalitz plot, as an interferometer.

3 Method

In Ref.\textsuperscript{1}, as in most earlier analyses of \( D \) decays to three pseudo-scalar particles \( ijk \), we use the “isobar model”. Details of this are given in Ref.\textsuperscript{4}. In this model, the decay amplitude \( A \) is described by a sum of quasi two-body terms \( D \to R+k \), \( R \to i+j \), in each of the three channels \( k = 1, 2, 3 \):

\[
A = d_0 e^{i\delta_0} + \sum_{n=1}^{\infty} d_n e^{i\delta_n} \frac{F_R(p, r_R, J)}{m_{Rn}^2 - s_{ij} - i m_{Rn} \Gamma_{Rn}(s_{ij})} \times F_D(q, r_D, J) \ M_J(p, q) \tag{1}
\]

In this, \( s_{ij} \) is the squared invariant mass of the \( ij \) system. \( J \) is the spin, \( m_{Rn} \) the mass and \( \Gamma_{Rn}(s_{ij}) \) the width of each of the \( N \) resonances \( R_n \) seen to be contributing to the decay. \( F_R \) and \( F_D \) are the amplitude factors for the \( R \) and \( D \) states, respectively.
and $F_D$ are form factors, with effective radius parameters $r_R$ and $r_D$, for all $R_n$ and for the parent $D$ meson, respectively. $p$ and $q$ are momenta of $i$ and $k$, respectively, in the $ij$ rest frame. $M_J(p,q)$ is a factor introduced to describe spin conservation in the decay. The complex coefficients $d_{ie}^j e^{ik \kappa}$ ($n = 0, N$) are determined by the $D$ decay dynamics and are parameters estimated by a fit to the data. The first, non-resonant ($NR$) term describes direct decay to $i + j + k$ with no intermediate resonance, and $d_0$ and $\delta_0$ are assumed to be independent of $s_{ij}$. For $D^+ \rightarrow K^- \pi^+_a \pi^+_b$ decays we Bose-symmetrize $\mathcal{A}$ with respect to interchange of $\pi^+_a$ and $\pi^+_b$.

In Ref.\[4] we reported that the $NR$ term was smaller than previously thought, and that a further term, parametrized as a new $J = 0$ resonance $\kappa (800)$ with $m_\kappa = (797 \pm 19 \pm 43) \text{MeV}/c^2$ and $\Gamma_R = (410 \pm 43 \pm 87) \text{MeV}/c^2$, gave a much better description of the data. Here, we examine the $K^-\pi^+$ $S$-wave in a model-independent way. The $S$-wave part of Eq.\[4] (all terms with $J = 0$, including the $NR$ term) is factored

$$S = S(s_{K\pi}) \times M_0^\kappa(p,q)F_D(q,r_D) = \text{Interp} \left(c_k e^{i\gamma_k}\right) \times M_0^\kappa(p,q)F_D(q,r_D) \quad (2)$$

into a partial wave $S(s_{K\pi})$, describing $K^-\pi^+$ scattering, and the product $M_0^\kappa(p,q)F_D(q,r_D)$ describing the $D$ decay. $S(s_{K\pi})$ is interpolated between a set of points $c_k e^{i\gamma_k}$ defined at 40 $K^-\pi^+$ invariant mass squared values $s_{K\pi}^k$ indicated by the lines in Fig.\[4]. Each $c_k$ and $\gamma_k$ is regarded as an independent parameter determined by the data.

We factor the $P$- and $D$- reference waves in the same way:

$$P = P(s_{K\pi}) \times M_1^\kappa(p,q)F_D(q,r_D); \quad D = D(s_{K\pi}) \times M_2^\kappa(p,q)F_D(q,r_D), \quad (3)$$

however, we parametrize the partial waves $P(s_{K\pi})$ and $D(s_{K\pi})$ exactly as in Eq.\[4].

We make an unbinned likelihood fit to the data. Using the method described in Ref.\[4]. This incorporates an incoherent background function describing the 6% of our sample not corresponding to true $D$ decays. We measure 86 parameters - all $(c_k, \gamma_k)$ and the coefficients $d_{k} e^{i \delta_k}$ for $K^+(892), K^+_\ast(1680)$ in the $P$-wave and $K^+_2(1430)$ in the $D$-wave. For the $K^+(892)$, we define $d_k e^{i \delta_k} = 1$ to provide the reference phase.

The fit results in an excellent description of the data. Comparison of the observed and predicted population of the Dalitz plot gives a $\chi^2$ probability of 50% for 363 bins.

### 4 Results

The $S$-, $P$- and $D$-waves resulting from the fit are shown in Fig.\[2]. They are compared with the model-dependent fit from Ref.\[4]. The main $S$-wave features of both fits agree well. Resonant fractions and the total $S$-wave fraction (about 75%) also agree within statistical limits.

We turn now to a comparison of the $S$-wave amplitudes $S(s_{K\pi})$ measured here with the amplitudes $T(s_{K\pi})$ measured in $K^-\pi^+$ elastic scattering. We expect, for each partial wave $J$ (for each isospin $I$) that $S(s_{K\pi}) = \sqrt{s_{K\pi}/p}^{(J+1)} Q(s_{K\pi}) T(s_{K\pi})$ where $Q(s_{K\pi})$ describes the dependence of $K^-\pi^+$ production in $D$ decays on $s_{K\pi}$. The Watson theorem\[5] requires that, provided there is no re-scattering of the $K^-\pi^+_a$ from $\pi^+_b$, that $Q$ is a real function, so that phases found in $D$ decay should match those in $K^-\pi^+$ elastic scattering data.

$I = 1/2$ phases measured by LASS are plotted in Fig.\[2]. There is a large offset in the $S$-wave, about 75°, not seen in $P$- or $D$-waves. The shapes of $S$- and $P$-waves are also not the same. Unless significant admixture of $I = 3/2$ $K^-\pi^+$ production occurs, these results suggest that the conditions for the Watson theorem are not met in these data.

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Figure 2: (a) Phases (left) and magnitudes (right) of $S$-wave amplitudes for the $K^-\pi^+$ systems from $D^+ \rightarrow K^-\pi^+\pi^+$ decays obtained from the MIPWA fit described in the text. The effect of adding systematic uncertainties in quadrature with statistical errors is indicated by extensions on the error bars. The $P$-wave and $D$-wave amplitudes are plotted in (b) and (c), respectively, as curves derived from the isobar parameters used in these waves, and from the full error matrix resulting from the fit. Shaded areas represent one standard deviation limits on the amplitudes. In all plots, the dashed curves show one standard deviation limits for the prediction of the isobar model fit. $I = 1/2$ phase measurements from the LASS experiment, with vertical and horizontal error bars, are included in the phase plots. The vertical lines mark the $K^-\eta'$ threshold, the upper limit of elastic scattering.

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