Cosmic decoherence: primordial power spectra and non-Gaussianities

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Abstract. We study the effect of quantum decoherence on the inflationary cosmological perturbations. This process might imprint specific observational signatures revealing the quantum nature of the inflationary mechanism being related to the longstanding issue of the quantum-to-classical transition of inflationary fluctuations. Several works have investigated the effect of quantum decoherence on the statistical properties of primordial fluctuations. In particular, it has been shown that cosmic decoherence leads to corrections to the curvature power spectrum predicted by standard slow-roll inflation. Equally interesting, a non-zero curvature trispectrum has been shown to be purely induced by cosmic decoherence but, surprisingly, decoherence seems not to generate any bispectrum. We further develop such an analysis by adopting a generalized form of the pointer observable, showing that decoherence does induce a non-vanishing curvature bispectrum and providing a specific underlying concrete physical process. Present constraints on primordial bispectra allow to put an upper bound on the strength of the environment-system interaction. In full generality, the decoherence-induced bispectrum can be scale dependent provided one imposes the corresponding correction to the power spectrum to be scale independent. Such scale dependence on the largest cosmological scales might represent a distinctive imprint of the quantum decoherence process taking place during inflation. We also provide a criterion that allows to understand when cosmic decoherence induces scale independent corrections, independently of the type of environment considered. As a final result, we study the effect of cosmic decoherence on tensor perturbations and we derive the decoherence corrected tensor-to-scalar perturbation ratio. In specific cases, decoherence induces a blue tilted correction to the standard tensor power spectrum.
1 Introduction

Inflation theory was conceived to solve the shortcomings of the standard Hot Big-Bang model, as the flatness and horizon “problems” [1–3]. Further, it was realized that it provides a mechanism which explains the origin of all the anisotropies and inhomogeneities we are observing nowadays, namely the Cosmic Microwave Background (CMB) anisotropies and the large-scale structure of the Universe (LSS), tracing them back to tiny primordial quantum fluctuations (of one or more scalar fields) during the early universe [4–10]. Similarly, inflation predicts through the same mechanism a relic background of primordial gravitational waves [11–16]. However, the adoption of the inflationary scenario of a quantum origin for the universe is still facing two puzzles that need to be cleaned away in order to have a self consistent formalism for the study of primordial perturbations [17–19].

It was shown in [20] that the initial, Bunch-Davies, vacuum state of the primordial fluctuations evolve into a highly squeezed quantum state. As consequence of the unitary dynamics that preserves the symmetries of initial state, the squeezed quantum state still represents a homogeneous and isotropic universe [21, 22]. Therefore, we may ask how the inhomogeneous and anisotropic universe we observe nowadays emerged from an initial homogeneous and isotropic one?

Second, we observe a classical universe rather than a quantum one, where we do not observe any of the distinctive properties of quantum mechanics, such as superposition of
states implied by the highly quantum squeezed state into which the primordial fluctuations state evolve. Hence, how did the classical universe emerged?

A quantitative answer to those questions, could shed light on new aspects of the primordial fluctuations providing us with new probes of the primordial era, which consequently, would lead to a better understanding of it. For example, they could bring new specific observational signatures able to reveal some aspects of the quantum processes characterizing the emergence of cosmological perturbations from inflation.

In the quest for an answer to the raised questions above, it was shown that in the large squeezing limit, the predictions inferred out of quantum processes are indistinguishable from those of classical stochastic processes [17, 23–25]. However, such equivalence does only justifies the analysis of cosmological data by relying purely on classical techniques, without answering the question of quantum to-classical transition in the early universe.

In the aim of studying the classicalization of the primordial universe, and find a signature of its quantum origin, many models have been suggested. Interestingly, cosmology offers a valuable arena to test and constrain those models on completely different physical scales compared to laboratories experiments [17–19, 25–57]. Among the many phenomenological models suggested so far, quantum decoherence emerges as a leading one, since, it is a well-established and physically tested concept [19, 58–63].

Quantum decoherence lays on the fact that a quantum system is not a closed but, rather, an open system interacting with its environment [64–69]. In cosmic decoherence, there are two main choices of the environment which have been investigated so far. The first is to consider that super-horizon, or light, modes of fluctuations to be decohered by sub-horizon, or heavy, modes representing the environment [32, 34, 35, 46, 53, 70]. The second choice is to consider an external field, such that decoherence is caused by field-field interactions, rather than by self interactions of the same field as in first case (i.e. by a long-short splitting of the inflationary fluctuations). Among the works which adopted such choice, we find [50, 51, 71, 72] (see also [73]), in addition to our current work. Such choice is motivated by the presence of more than one field in the early universe, at least, those populating radiation era. The system-environment interaction leads mainly to the suppression of off diagonal elements in the reduced density matrix of the system. However, decoherence occurs over a finite time scale, therefore, the environment does also affect the diagonal elements of the density matrix. Consequently, it alters the probabilities associated to the final outcomes of measurement [34, 35, 50, 71].

Indeed, several works showed that cosmic decoherence does affect the statistical properties of the primordial perturbations. Such corrections are shown to be essential to account for the quantum to classical transition in the early universe, in addition, they carry observational signatures of a quantum origin for the primordial fluctuations [32, 46, 50, 51, 71]. In particular, our work builds up, extends, and to some extent completes, works done in [50, 51, 71].

In this work, we present a generalized model to deal with cosmic decoherence of both curvature and tensor perturbations. In particular, we study the power spectra corrections

\footnote{Light modes represent our system, since they are the ones of observational interest for us. See however [35, 44] for a critical discussions about some short-comings of this approach.}
in addition to non-Gaussianities induced by decoherence. The results obtained for the different point correlation functions can be exploited to constrain the strength of the interaction between the system and the environment.\footnote{Some constraints were obtained, e.g., in \cite{50, 51} by using curvature power spectrum and tripsectrum.} Hence, this would allow to constrain the possible environments, and to reveal some properties of the inflation era.

The paper is organized as follows. In section 2 we present, briefly, the Lindblad equation which has been derived previously in several papers for cosmological applications. Then, we discuss in section 3 the cosmic decoherence of curvature fluctuations by providing a short summary of the approach followed by J.Martin and V.Vennin, in \cite{50, 51}, and of their main results on both curvature power spectrum and non-Gaussianities. In section 4 we present our approach that generalizes the previous one, and present a concrete physical process that motivate it. In order to shed light on some of the many interesting results implied by our approach, we derive the bispectrum expression and we discuss its specific scale-dependence, emphasizing the fact that such bispectrum is induced purely by the decoherence phenomenon, rather than being merely a correction to the leading bispectrum derived within standard inflation theory.\footnote{In standard single field slow-roll inflation, where primordial fluctuations are considered as a closed system, curvature non-Gaussianities are a result of (gravitational) self-interaction of curvature fluctuations manifested through cubic, and higher order, terms in the system action \cite{74, 75}.} In doing that, we also provide a general criterion to understand when one can achieve scale independence of the various decoherence induced corrections to the power spectrum, as well of the induced non-Gaussianities. We conclude this section by showing how our generalized approach can reproduce all the results derived with previous approaches, while adding at the same time some corrections which were missed due to the restricted form of pointer observable previously adopted. Besides the analysis dealing with the curvature perturbations, in section 5 we study the effect of cosmic decoherence on tensor perturbations, and derive the induced correction to their power spectrum and the corresponding modification to the tensor-to-scalar perturbation ratio $r$. Finally, we draw our conclusions in section 6. Appendix A contains various technical details of the bispectrum computation.

2 The Lindblad equation

Since we are about to discuss inflationary fluctuations we need to go beyond the homogeneous and isotropic Friedmann-Lemaitre-Robertson-Walker (FLRW) metric. In what follows we will focus for the moment on scalar perturbations which, up to first order, decouple from the tensor ones. The perturbed FLRW metric is given by \cite{76}

$$ds^2 = a^2(\eta) \left\{ -(1 - 2\phi) d\eta^2 + 2 \partial_i B dx^i d\eta + [(1 - 2\psi) \delta_{ij} + \partial_{ij} E] dx^i dx^j \right\}, \quad (2.1)$$

where $a$ is the FLRW scale factor, and $\phi$, $B$, $\psi$, $E$ are functions of space and time. Thanks to gauge freedom, the number of degrees of freedom can be reduced, such that, e.g., the scalar gravitational perturbations can be encoded into two gauge-invariant Bardeen
potentials \[\Psi = \psi - \frac{a'}{a} (B - E') .\] (2.2)

where a prime denotes a derivative with respect to the conformal time \(\eta\).

Besides the geometric part of perturbations, we turn now to matter sector. The inflaton is decomposed into a classical background \(\phi_0 (\eta)\) plus a fluctuation \(\delta \phi (\eta, x)\) to be quantized later. The latter, i.e \(\delta \phi (\eta, x)\), can be treated as a massless scalar field, which is an excellent approximation when the inflaton field satisfies the slow-roll conditions \[25\]

\[\phi (\eta, x) = \phi_0 (\eta) + \delta \phi (\eta, x) .\] (2.3)

Similarly, the scalar field perturbations can be conveniently described by a gauge invariant variable, namely,

\[\delta \phi'^{\mu} (\eta, x) = \delta \phi + \phi_0' (B - E') .\] (2.4)

The Bardeen potential and the gauge invariant field can be combined into a single gauge-invariant scalar variable called Mukhanov-Sasaki (MS) variable \[5, 78\]

\[v (\eta, x) = a \left[ \delta \phi'^{\mu} (\eta, x) + \frac{\phi_0'}{\mathcal{H}} \Psi \right] ,\] (2.5)

where \(\mathcal{H} = a' / a\) represents the conformal Hubble parameter; the MS variable \(v\) characterizes fully the scalar sector, being related to the comoving curvature perturbation \(\zeta\) through \[76, 79, 80\]

\[v (\eta, x) = - \frac{a \phi_0'}{\mathcal{H}} \zeta .\] (2.6)

In the simplest view of the standard inflation paradigm, the primordial quantum perturbations are considered as a closed system. The gravitational and matter degrees of freedom are encoded in field operators on the Hilbert space of physical states, and are governed by the standard quantum mechanics.

The curvature perturbations that describe jointly the inflaton fluctuations in addition to the scalar metric perturbations are governed, at the perturbative linear order, by the free Hamiltonian \[4\], written in Fourier space,

\[\hat{H}_v = \frac{1}{2} \int d^3 k \left[ \hat{p}_k \hat{v}_k^* + \omega^2 (\eta, k) \hat{v}_k \hat{v}_k^* \right] ,\] (2.7)

where \(\hat{v}_k\) is the Fourier transform of the quantized Mukhanov-Sasaki variable (2.6) and \(\hat{p}_k\) is its conjugate momentum, i.e \(\hat{p}_k = \hat{v}_k^*\).

\[4\] For the sake of studying the non-Gaussianities induced purely by cosmic decoherence, i.e by the interaction with an external environment, we are considering single-field slow roll inflation, and neglecting self-interactions of the curvature perturbation field.
\( \hat{H}_v \) represents a collection of parametric oscillators (one per each mode) with time dependent frequency given by

\[
\omega^2(\eta, k) = k^2 - \frac{(\sqrt{\epsilon a})''}{\sqrt{\epsilon a}},
\]  

(2.8)

with \( \epsilon \) being the first slow-roll parameter defined by \( \epsilon = 1 - \mathcal{H}'/\mathcal{H}^2 \), which we will assume to be constant throughout this work. In what follows, it will prove to be convenient to decompose the MS variable into its real and imaginary part as \( \hat{v}_k = (\hat{v}^R_k + \hat{v}^I_k) / \sqrt{2} \).

Obviously the Hamiltonian (2.7) describes a closed system, since it does not contain any interaction term. However, it is unlikely in reality for the primordial perturbations to be non-interacting, at least gravitationally, with the other degrees of freedom present in the early universe, possibly including the standard model fields, needed to give rise to radiation era [38, 50, 71]. Therefore, one should consider the inflationary perturbations rather as an open quantum system interacting with a given environment. On the other hand, by doing so, we will face the problem of being unable to describe the exact properties of all the possible degrees of freedom composing the environment. In addition, we are interested, mainly, on the effect of the environment on the system of interest, rather than the environment itself. It thus prove convenient to trace out the environmental degrees of freedom during the derivation of the master equation for the system, such that the effect of the environment will be encoded in a new term added to the Liouville equation, leading us to the quantum master equation governing the evolution of the system, namely the Lindblad equation.

The Lindblad equation is based on the Born and Markov approximations [61, 67, 81, 82] which could be summarized as follow [35, 50, 83]:

1. The environment evolves on a time scale that is much smaller than that of the system.
2. The backreaction of the system on the environment is negligible.
3. The influence of the environment on the system, that is here clearly crucial, can be treated perturbatively.

Generally, within a quantum decoherence context, the total Hamiltonian of the composite system, i.e system+environment, living in the Hilbert space \( \mathcal{E} = \mathcal{E}_{sys} \otimes \mathcal{E}_E \) is given by

\[
\hat{H} = \hat{H}_0 + \hat{H}_{int} = \hat{H}_{sys} \otimes \hat{I}_E + \hat{I}_{sys} \otimes \hat{H}_E + g \hat{H}_{int},
\]  

(2.9)

where \( \hat{H}_{sys} \) is the intrinsic Hamiltonian of the system acting on the Hilbert space \( \mathcal{E}_{sys} \), in our case given by \( \hat{H}_v \) in (2.7), \( \hat{H}_E \) is the free evolution Hamiltonian of the environment that acts on \( \mathcal{E}_E \) and it could be left unspecified. Finally, \( \hat{H}_{int} \) is the interaction Hamiltonian and \( g \) is a dimensionless coupling constant. Following [34, 35, 84] and, mainly, [50] we specialize to the case of interest for field theory, where interactions are local. Suppose, then, that system and environment interact through interactions of the local form such that \( \hat{H}_{int} \) could be written as

\[
\hat{H}_{int}(\eta) = \int d^3x \hat{A}(\eta, x) \otimes \hat{R}(\eta, x),
\]  

(2.10)
where $\hat{A}$ denotes a local functional of the fields describing the system sector, and similarly for $\hat{R}$ for the environment sector. According to the commutativity criterion, the decoherence superselected pointer basis (or preferred basis) is dictated by the system sector part of $\hat{H}_{\text{int}}(\eta)$, where it states that the preferred basis is the eigenstates of $\hat{A}(\eta, x)$; stated differently but equivalently, the pointer observables of the system are those observables commuting with the interaction Hamiltonian \cite{61, 64, 85}. However, in our case we will approach the problem in the reverse way, where based on the fact that CMB map is localized in the field amplitude of Mukhanov-Sasaki variable $\hat{v}$, we conclude that $|v_k\rangle$ constitutes the pointer basis \cite{86}. Therefore, using the commutativity criterion in the reverse way, we conclude that $\hat{A}(\eta, x)$ involves only $\hat{v}$ i.e

$$\hat{A} \equiv \hat{A}[\hat{v}^n],$$

with $n$ an integer. However, the computations that will be made all throughout this paper could also be applied for the case where $\hat{A}$ is a function of field momentum $\hat{p}$. Actually, there is another reason behind neglecting a dependence of $\hat{A}$ on the field momentum $\hat{p}$ and it consists of the fact that $\hat{p}$ is proportional to the decaying mode, therefore, any contribution from $\hat{p}$ would be subdominant compared to that from $\hat{v}$ (see \cite{50} for more details).

The reduced density matrix of the system $\hat{\rho}_{\text{sys}}(t)$ is defined by tracing out the environmental degrees of freedom as

$$\hat{\rho}_{\text{sys}}(t) = \text{Tr}_E[\hat{\rho}(t)].$$

Exploiting the aforementioned assumptions, one can show that $\hat{\rho}_{\text{sys}}(t)$ evolves according to the Lindblad equation\(^6\) \cite{35, 50, 81, 84, 87, 88}

$$\frac{d\hat{\rho}_{\text{sys}}}{d\eta} = i\left[\hat{\rho}_{\text{sys}}, \hat{H}_{\text{sys}}\right] - \frac{\gamma}{2} \int d^3x d^3y C_R(x, y) \left[\left[\hat{\rho}_{\text{sys}}, \hat{A}(x)\right], \hat{A}(y)\right],$$

with\(^7\) $\gamma = 2g^2t_c$. Here $t_c$ is the autocorrelation time of the environment, which, in order to assure the Markovianity of the process, should be small with respect to the time over which the system of interest evolves. On the other hand, $C_R(x, y)$ represents the equal-time environmental correlation function of $\hat{R}$, and is defined by

$$C_R(x, y) = \text{Tr}_E(\rho_E R(x) R(y)),$$

where $\rho_E$ is the density operator of the environment. The parameter $\gamma$ is generally time dependent so we adopt for it a power law dependence on the scale factor \cite{50}

$$\gamma = \gamma_s \left(\frac{a}{a_c}\right)^p,$$
where $p$ represents a free parameter, and $\ast$ refers to a reference time that is taken to be the time when the pivot scale $k_\ast = 0.051\text{Mpc}^{-1}$ crosses the Hubble radius, i.e $k_\ast = a_\ast H_\ast$.

We need also to adopt a convention for the correlator $C_R(x, y)$, so we assume the environment to be statistically homogeneous, i.e $C_R(x, y) \propto |x - y|$, and be in an isotropic configuration, i.e $C_R(x, y) \propto |x - y|$. In addition, if we assume that it is characterized by a physical correlation length scale $l_E$ then $C_R(x, y)$ must be a function of $a|x - y|/l_E$. For convenience, we assume it to be a top hat function [50]

$$C_R(x, y) = \bar{C}_R \Theta \left( \frac{a|x - y|}{l_E} \right), \quad (2.16)$$

with

$$\Theta(x) = \begin{cases} 
1 & \text{if } x < 1 \\
0 & \text{otherwise} 
\end{cases}, \quad (2.17)$$

and $\bar{C}_R$ is a constant. Such generic form of the environmental correlation function serves to capture the most relevant properties of the environment necessary to derive the master equation, in addition to the assumption about the environment short-range correlations. However, these requirements still allow some freedom on the choice of correlator generic form. For instance, a good alternative to the form suggested is

$$C_R(x, y) = \bar{C}_R \exp \left( -\frac{a|x - y|}{l_E} \right), \quad (2.18)$$

where we still have the property of decaying correlations within a characteristic length $l_E$. Needless to mention that such choice renders the computations more complicated, so we will stick to the top hat correlator form in our work. Another property that the correlator could reflect is whether the environmental correlations are local in time as in our case and in [50, 51, 89, 90], or non-local in time as in [71].

The precise form of the correlation function will be left unspecified in the following, since it depends on the type of field(s) constituting the environment, some examples can be found in [35, 50, 71].

Before proceeding with the application of the Lindblad equation it is important to mention, as a general property of it, that it is valid only at leading order in $\gamma$ [61]. This remark will be crucial for the results obtained, both, by us and by the previous works in the literature, as in [35, 50, 51, 91] and the papers cited therein.

For a critical analysis about the validity of Lindblad equation (2.13) in the cosmological context, in addition to its link to non-perturbative methods, we refer the reader to [92].

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8Remember that $(x, y)$ are comoving coordinates.

9In [71] it has been shown that non-local time environmental correlation functions could cause a tiny decay in curvature power spectrum upon horizon crossing of perturbation modes.
3 Previous approaches

In dealing with cosmic decoherence, many previous works, as [34, 35, 50, 51, 91], assumed a monomial pointer observable of the form $\hat{A} = \hat{v}^n$, with $n$ being a given integer. In particular, J.Martin et al. considered in [50, 51] the cases of a linear and quadratic pointer observable, i.e $\hat{A} = \hat{v}$ or $\hat{A} = \hat{v}^2$, respectively; we will summarize their main results in the following section.\textsuperscript{10} However, as a key remark of our paper, we show that the form of the pointer observable is crucial in studying the cosmic decoherence effects. Indeed, our approach is based on adopting a pointer observable of the most general form $\hat{A} = \sum_n \alpha^{n-1} \hat{v}^n$, up to a certain order $n$ with $\alpha$ being an expansion parameter. Such generalized form of the pointer observable will not only reproduce the results obtained by [50, 51], but it leads also to corrections to those results in addition to new interesting outcomes regarding primordial non-Gaussianities.\textsuperscript{11}

To compute the correlation functions within cosmic decoherence, there are two methods:

1. We can solve the evolution equation (2.13) and obtain an explicit expression of the reduced system density matrix $\hat{\rho}_{sys}$, so that we use it to compute the correlation function $\hat{O} = f [\hat{v}, \hat{p}]$ through its expectation value

$$O \equiv \langle \hat{O} \rangle = \text{Tr}_{sys} \left( \hat{O} \hat{\rho}_{sys} \right).$$

However, unfortunately, in most of the cases it is highly challenging to solve (2.13) exactly, the only case doable with reasonable easiness being the linear case $\hat{A} \propto \hat{v}$. In that case, the Lindblad term is quadratic in $v$ just as the free Hamiltonian is, and we obtain a Gaussian density matrix with a width controlled by the environment [50].

2. Apart from the linear case it is better to pursue the second method in computing the correlation functions, which consists in solving directly the equation of motion governing $\langle \hat{O} \rangle$. Therefore, using (2.13) and (3.1) we obtain

$$\frac{d\langle \hat{O} \rangle}{d\eta} = \langle \frac{\partial \hat{O}}{\partial \eta} \rangle - i \left\langle \left[ \hat{O}, \hat{H}_v \right] \right\rangle - \gamma \int d^3x d^3y C_R (x, y) \left\langle \left[ \left[ \hat{O}, \hat{A} (x) \right], \hat{A} (y) \right] \right\rangle.$$

(3.2)

3.1 Corrected curvature power spectrum

As we just mentioned, for the case of a linear interaction with environment, $\hat{A} (x) = \hat{v} (x)$, there are two equivalent ways to determine the curvature power spectrum. The first consists

\textsuperscript{10} They also presented a diagrammatic approach to derive the cosmic decoherence induced correction to the curvature power spectrum for any order $n$ of $\hat{A} = \hat{v}^n$, see [50] for more details.

\textsuperscript{11} Within our approach, we will stop at the leading order that gives a non vanishing correction to the correlation function considered, except for the case of the power spectrum for which we will consider also the next to leading order to show that new contributions arise w.r.t. to the results obtained in the previous approach.
in solving equation (2.13) to get the explicit expression of the reduced density matrix to be used in computing the curvature power spectrum

\[ P_{vv} (k) = \langle |\hat{v}_k|^2 \rangle = \langle (\hat{v}_k^s)^2 \rangle = \text{Tr}_{\text{sys}} [(\hat{v}_k^s)^2 \hat{\rho}_{\text{sys}}] = \int d\hat{v}_k^s \langle \hat{v}_k^s | \hat{\rho}_{\text{sys}} | \hat{v}_k^s \rangle (\hat{v}_k^s)^2, \]  

(3.3)

with, \( s = R, I \), standing for the real and imaginary part of \( \hat{v}_k \), and summation upon repeated indices is adopted. However this method is applicable only at linear order in \( \hat{v} \), where (2.13) could be solved exactly as shown in [50], so for all higher order interactions we need the second method which consists in solving directly (3.2) to get the correlator expression, i.e. in this case \( \langle \hat{O} \rangle = \langle |\hat{v}_k|^2 \rangle \). Hence, in what follows, we will restrict ourselves to the second method. The interested reader is referred to [50] to see the equivalence of the two methods at the linear order \( \hat{A} \propto \hat{v} \). Using the second method yields [50]

\[ P''''_{vv} + 4 \omega^2 P'_{vv} + 4 \omega \omega' P_{vv} = S_1, \]  

(3.4)

where \( S_1 \) is a source function

\[ S_1 (k, \eta) = 2 (2\pi)^{3/2} \gamma \tilde{C}_R (k), \]  

(3.5)

\( \tilde{C}_R (k) \) being the Fourier transform of the environmental correlation function and \( \omega \) is given by Eq. (2.8) Needless to say that some approximations are needed to solve equation (3.4), among which we find the approximations used previously to derive the Lindblad equation. In particular, we solve (3.2) in the following two limits:

- The first limit uses the Markovian approximation that requires the environment autocorrelation time \( t_c \) to be very short compared to the typical time scale over which the system evolves \( \sim H^{-1} \). Assuming the environment correlation time \( t_c \) and length \( l_E \) to be of the same order \( t_c \sim l_E \) then

\[ Hl_E \ll 1, \]  

(3.6)

- The second limit is to evaluate the corrections at the end of inflation i.e \( -k\eta \to 0 \), when the modes of observational interest today are outside horizon during inflation.

Considering now the case of a quadratic interaction \( \hat{A} (x) = \hat{v}^2 (x) \), then the power spectrum is governed by\(^\text{12}\)

\[ P''''_{vv} + 4 \omega^2 P'_{vv} + 4 \omega \omega' P_{vv} = S_2, \]  

(3.7)

where the source function \( S_2 \) is now given by

\[ S_2 (k, \eta) = \alpha^2 \left( \frac{8\gamma}{(2\pi)^{3/2}} \right) \int d^3 k' \tilde{C}_R (k') P_{vv} (|k'| + k) . \]  

(3.8)

\(^\text{12}\)We will see later, in this paper, that our approach implies the existence of an additional source function \( S_3 \) in this equation, namely (3.7) will be given by \( P''''_{vv} + 4 \omega^2 P'_{vv} + 4 \omega \omega' P_{vv} = S_2 + S_3 \).
In order to have a better comparison between the predictions of the current model and those of standard inflation, i.e., inflation theory without cosmic decoherence, the curvature power spectrum is written as

\[ P_\zeta = \frac{k^3}{2\pi^2} \frac{1}{2a^2 M_{pl}\epsilon} P_{\text{vv}}(k) = P_\zeta|_{\text{standard}} (1 + \Delta P_k) \]  

(3.9)

\[ \epsilon = -\frac{\dot{H}}{H^2} \] being the usual first slow-roll parameter, \( H = \dot{a}/a \) the Hubble parameter and \( M_{pl} \) the Planck mass.

\( P_\zeta|_{\text{standard}} \) in (3.9) is the standard curvature power spectrum obtained using standard inflation that treats the primordial fluctuations as a closed system, while \( \Delta P_k \) is the cosmic decoherence induced correction which will be of interest for us. The explicit expressions of \( \Delta P_k \), as a function of the parameter \( p \) characterizing the system-environment coupling as in Eq. (2.15), were computed explicitly by J.Martin et al in [50], so we limit ourselves to mention the main results of interest for the goals of this paper. In particular, we will write the expressions of \( \Delta P_k \) for which there is a value of the free parameter \( p \) that makes it scale independent in accordance with the highly precise data collected so far [93].

It could be shown that the inhomogeneous part of the equations governing the power spectrum, (3.4) and (3.7), can be solved by the following ansatz \(^{13}[50]\)

\[ S_k(\eta) = -\frac{2}{W^2} \int_{\eta_0}^{\eta} d\eta' S_n(k, \eta') \Im \left[ v_k(\eta') v_k^*(\eta) \right] , \]  

(3.10)

where, \( n = 1, 2 \), depending on whether we are considering a linear or a quadratic interaction, and \( W \) is the Wronskian, \( v_k'(\eta) v_k^*(\eta) - v_k(\eta) v_k'^*(\eta) = i \). The initial comoving time \( \eta_0 \) is chosen such that \( S_k(\eta_0) = S_k'(\eta_0) = S_k''(\eta_0) = 0 \).

The full solution is then obtained by adding to \( S_k(\eta) \), the solution of the homogeneous equation in (3.4) and (3.7), which gives the standard power spectrum expression obtained in absence of quantum decoherence effects. Hence, the final expression of the power spectrum for scalar perturbations is given by

\[ P_{\text{vv}}(k) = |\hat{v}_k|^2 + S_k. \]  

(3.11)

Adopting the Bunch Davies vacuum as initial state, then we set in 3.10 \( \eta_0 \to -\infty \) in order for \( P_{\text{vv}}(k) \) to match Bunch Davies result in the infinite past [50].

Implementing the full power spectrum expression (3.11) into Eq. (3.9) yields the following corrections \(^{14}[50]\)

\[ |\Delta P_k| \simeq A(k) \left[ 1 + B e^t + C e^t + (D e^t + E e^t) \ln \left( \frac{k}{k_c} \right) \right] , \]  

(3.12)

\(^{13}\)It is worth to recall that the functions \( v_k(\eta) \) to be used in evaluating (3.10) are the free modes obtained with the free Hamiltonian (2.7).

\(^{14}\)These expressions are valid only for \( 3 + \frac{1}{1+p} < p < 3 + \frac{5}{1+p} \) in the linear case, and for \( 2 < p < 6 \) in the quadratic case. We refer the reader to [50] for the power spectrum expressions corresponding to other ranges of the free parameter \( p \).
with

\[ A(k) = \left( \frac{k\gamma}{k_*} \right)^2 \left( \frac{k}{k_*} \right)^{p-5} \frac{2^{6-p} (p-2) \sin(\pi p/2) \Gamma(p-3)}{2^6-p (p-2)} \]

\[ B = -2 \frac{(p-1)(p-3)}{(p-4)(p-2)} - 2 \frac{1}{2} \left( 4 - \frac{p}{2} \right) - \psi \left( -2 + \frac{p}{2} \right) \]

\[ C = \gamma_E + \ln(2) - 2 + \frac{6}{(p-2)(p-8)}, \quad D = p - 3, \quad E = 0 \]

and

\[ k_\gamma = \sqrt{\frac{2}{\pi} C_R \frac{\gamma_3}{a_3^3}}. \]  

(3.14)

where \( k_\gamma \) is homogeneous to the dimension of wavenumber, and \( \epsilon^* \) and \( \epsilon_2^* \), are first and second slow roll parameters, respectively, computed at Hubble exit time of pivot scale \( k^* \). The function \( \psi(x) \) refers to digamma function and \( \gamma_E \approx 0.577 \) is the Euler constant.

- For quadratic interaction

\[ \Delta P_k \simeq \frac{2^{p-1}(p-4)}{3\pi (p-1) \sin(\pi p/2)} \sigma_\gamma \left( \frac{k}{k_*} \right)^{p-3} \left[ \ln \left( \frac{\eta_R}{\eta_*} \right) + \frac{1}{p-4} - \frac{2(p-1)}{p(p-2)} \right] \]

\[ - \frac{p}{2} \cot \left( \frac{\pi p}{2} \right) + \ln(2) - \psi(p-2) + \ln \left( \frac{k}{k_*} \right) \],

(3.15)

with the dimensionless coefficient \( \sigma_\gamma \) being given by

\[ \sigma_\gamma = C_R \frac{\gamma_3}{a_3^3}. \]  

(3.16)

Notice that the difference of dimension between the scales \( k_\gamma \) and \( \sigma_\gamma \), in spite of the similar expressions, is due to the fact that they correspond to two different types of interactions, namely, linear and quadratic.

The scales \( k_\gamma \) and \( \sigma_\gamma \) are very important to constrain the interaction strength between the system and environment through the comparison of decoherence induced corrections with observations. In the quadratic case \( \eta_{IR} \) refers to an IR cutoff in the integral (3.10), with \( \ln \left( \frac{\eta_R}{\eta_*} \right) = N - N_{IR} \) giving the number of e-folds elapsed since the beginning of inflation.

We notice that the values \( p = 3 \) and \( p = 5 \) are very peculiar, since, as evident from (3.13) and (3.15), they correspond to (almost) scale independent corrections to the standard, well confirmed, quasi scale independent power spectrum. An interesting example
of the current model was studied in detail in [50], where a massive scalar field was considered as an environment for which the free parameter $p$ is given by

$$p \simeq 7 - 2n,$$  \hfill (3.17)

$n$ being the order of the interaction, i.e. the power of the Mukhanov-Sasaki operator $\hat{v}$ in the pointer observable $\hat{A}(\mathbf{x}) = \hat{v}^n(\mathbf{x})$. The interaction Hamiltonian considered in [50] for such an environment is given by

$$g\hat{H}_{int} = \lambda \mu^{4-n-m} a^4 \int d^3\mathbf{x} \hat{\varphi}^n \left( \hat{\psi}^m - \langle \hat{\psi}^m \rangle_{st} \right), \hfill (3.18)$$

where $\varphi$ is a scalar field representing our system, while $\psi$ is a heavy massive scalar field that represents the environment. So in this example the action describing the total system is given by

$$S = - \int d^4x \sqrt{-g} \left[ \frac{1}{2} g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi + V(\varphi) + \lambda \mu^{4-n-m} \langle \psi^m \rangle_{st} \varphi^n \right. \hfill (3.19)$$

$$\left. + \frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi + \frac{M^2}{2} \psi^2 + \lambda \mu^{4-n-m} \varphi^n (\psi^m - \langle \psi^m \rangle_{st}) \right].$$

Indeed, this model is interesting because it exactly corresponds, for a given interaction of order $n$, to the values of $p$ which give scale independent corrections to the power spectrum. On the other hand, if we keep the environment unspecified and impose the corrections to be scale independent, then we can obtain interesting constraints on the interaction strength with the environment by constraining $(k_\gamma/k_*)$. In particular we obtain upper bounds on $(k_\gamma/k_*)$, while a lower bound could be obtained by imposing decoherence to take place before the end of inflation as shown in [50].

Another interesting result found by J.Martin et al. in [50] was the computation of the decoherence corrected scalar spectral index $n_s$ and the tensor-to-scalar perturbation ratio $r$, which represent two important observables to remove the degeneracy among some inflation models. Considering a massive scalar field as environment and choosing $p = 5$ (corresponding to a linear interaction) they showed that the new scalar spectral index is given by

$$n_s = n_s \big|_{\text{standard}} - (6m - 2) \epsilon^*, \hfill (3.20)$$

where $\epsilon^*$ is the first slow roll parameter, and $m$ is the power of $\psi$ in (3.20). Similarly, they computed the new tensor-to-scalar perturbation ratio

$$r = \frac{r \big|_{\text{standard}}}{1 + \frac{5 \epsilon^*}{6 k_*^2}}, \hfill (3.21)$$

$^{15}$\langle \cdots \rangle_{st}$ refers to the expectation value with respect to the environmental stationary state, hence, the subscript "st". We remind that we used the Markov approximation in the derivation of the Lindblad equation.

$^{16}$Notice that it is not necessary for decoherence to take place before end of inflation. However, it must take place before recombination to give rise to the CMB classical fluctuations.
where by standard we refer as usual to the expressions obtained assuming standard inflation (i.e. when primordial perturbations are treated as a closed system).

However it is important to mention that the new tensor-to-scalar perturbation ratio (3.21) was obtained assuming the tensor perturbations to remain unaffected by the decoherence process. The latter assumption will turn out to be inaccurate as we will show in the last section.

Such modified $n_s$ and $r$ could indeed improve the fit to data for some inflationary models while worsening it for others [50].

3.2 The cosmic decoherence induced non-Gaussianities

No wonder that even if we consider the system to be intrinsically free, i.e. with no self-interactions, then its interaction with the environment will induce non-vanishing higher-order correlation functions. Indeed, J. Martin et al. extended their approach to the study of primordial non-Gaussianities following the same line of thoughts applied previously to the power spectrum [51]. In the following we briefly summarize their main results.

Considering the linear interaction, then, it is easy to see that all the higher-order correlation functions vanish. Indeed, the Lindblad term for a linear interaction is given by

$$-\frac{\gamma}{2} \int d^3k \bar{C}_R (k) \left\langle \left[ \hat{O}_k, \hat{V}_k \right], \hat{V}_{-k} \right\rangle,$$  (3.22)

so if we consider the three point correlator with less than two field momenta $\hat{p}_k$, as $\left\langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \right\rangle$ or $\left\langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{p}_{k_3} \right\rangle$, then the term (3.22) vanishes because of the vanishing commutator $[\hat{v}_k, \hat{v}_{k'}] = 0$. While if we consider correlators involving two field momentum operators or three, i.e $\left\langle \hat{v}_{k_1} \hat{p}_{k_2} \hat{p}_{k_3} \right\rangle$ and $\left\langle \hat{p}_{k_1} \hat{p}_{k_2} \hat{p}_{k_3} \right\rangle$, then using the commutation relation

$$[\hat{v}_p, \hat{p}_k] = i\delta^{(3)}(p + k),$$  (3.23)

we will end up with either $\left\langle \hat{v}_k \right\rangle$ or $\left\langle \hat{p}_k \right\rangle$, both of which gives zero due to $\left\langle \hat{a}_k \pm \hat{a}^\dagger_{-k} \right\rangle = 0$, where $(\hat{v}, \hat{p})$ are expressed as function of the creation and annihilation operators $(\hat{a}_k, \hat{a}^\dagger_{-k})$ defined, as usual, by

$$a_k (\eta) = \frac{1}{\sqrt{2}} \left( \sqrt{k} \hat{v}_k (\eta) + i \frac{1}{\sqrt{k}} \hat{p}_k (\eta) \right).$$  (3.24)

Regarding the trispectrum and when we consider solely the connected terms, the various contributions cancel out and decoherence does not induce a non-vanishing trispectrum for a linear interaction [51].

Therefore, one has to consider more complicated interactions, so choosing the quadratic interaction $\hat{A} = \hat{v}^2$ and implementing it again in (3.2) leads to eight equations that represent the various terms $\left\langle \hat{O}_{k_1} \hat{O}_{k_2} \hat{O}_{k_3} \right\rangle$ which could be built from $\hat{O}_k = \hat{v}_k$ or $\hat{p}_k$. However

\footnote{It is important to notice that here we are considering the primordial perturbations to be intrinsically (almost) Gaussian, i.e. all higher-order correlation functions are vanishing in the standard inflation scenario, and we will focus on the interactions with the environment as the main source of primordial non-Gaussianity.}
now the correlators which get modified by the Lindblad term, marked by \( \langle \rangle \), are proportional to the initial bispectrum coming from the system Hamiltonian, which is zero in our case since it corresponds to the free theory\(^{18}\), an example being

\[
\frac{d(\hat{v}_{k_1}\hat{p}_{k_2}\hat{p}_{k_3})}{dt} = \langle \hat{p}_{k_1}\hat{p}_{k_2}\hat{p}_{k_3} \rangle - \omega^2 (k_2) \langle \hat{v}_{k_1}\hat{v}_{k_2}\hat{p}_{k_3} \rangle - \omega^2 (k_3) \langle \hat{v}_{k_1}\hat{p}_{k_2}\hat{v}_{k_3} \rangle + \frac{4\gamma}{(2\pi)^{3/2}} \int d^3k \tilde{C}_R(|k|) \langle \hat{v}_{k_1}\hat{v}_{k_2-k}\hat{v}_{k_3} \rangle .
\]

(3.25)

On the other hand, the Lindblad equation is valid only at leading order in the coupling parameter \( \gamma \), therefore, we cannot use in the previous equation the bispectrum generated by decoherence since the latter would arise at \( \gamma^2 \) order, where we can no more rely on the Lindblad equation.

Having found a vanishing bispectrum, the authors of [51] considered the trispectrum, showing that in this case a non-vanishing trispectrum is indeed generated by decoherence effects. In particular, they found a remarkable expression in the range \( p < 4 \)

\[
g_{NL} \propto \sigma_c \left( \frac{k}{k_*} \right)^{p-3} ,
\]

(3.26)

where the non-linearity parameter \( g_{NL} \) is defined as

\[
\langle \hat{\zeta}_{k_1}\hat{\zeta}_{k_2}\hat{\zeta}_{k_3}\hat{\zeta}_{k_4} \rangle = \frac{25}{54} g_{NL} [P_{\zeta}(k_1) P_{\zeta}(k_2) P_{\zeta}(k_3) + 3 \text{ permutations}] \delta^{(3)}(k_1 + k_2 + k_3 + k_4) .
\]

(3.27)

We notice through (3.26) that the trispectrum is scale independent for \( p = 3 \) which, again, corresponds to a massive scalar field as an environment. Indeed, the fact that the bispectrum is vanishing lead the authors to state that decoherence is one of the rare examples where the bispectrum is perturbatively suppressed compared to the trispectrum.

However, we will show that such a conclusion is not completely accurate. Using a generalized form of the pointer observable \( \hat{\mathcal{A}} \), we will obtain a non vanishing bispectrum which is dominant with respect to the trispectrum. In addition, we will show that, at leading order in \( \gamma \), the bispectrum turns out to be scale independent for \( p = 4 \) (which again could correspond to a massive scalar field as environment, as will be explained in details below). Notice that, as an interesting consequence, one of our main results is that keeping the leading correction to the curvature power spectrum to be scale independent, in general does not guarantee the scale-independence of the bispectrum, rather it gives rise to a specific signature of the primordial bispectrum generated by decoherence effects, namely a linear-scale dependence of the primordial bispectrum.

Finally, we will give our own reasoning on how to understand when does decoherence induce scale independent corrections. We will show that scale independence is not related to a specific type of environment but, rather, a very general criterium holds that involves the power of \( \alpha \) in the Lindblad equation.

\(^{18}\)We remind the reader that we are considering an intrinsic system Hamiltonian of quadratic form, which does not contain cubic and higher-order self-interaction terms, see (2.7).
4 Our approach

As mentioned, the results obtained previously in the literature were based on restricted forms of the pointer observable \( \hat{A} \), namely monomial types, either \( \hat{v} \) or \( \hat{v}^2 \), or generally \( \hat{v}^n \). Therefore, even if such a choice leads to interesting results we extend this approach to get broader insights on the effect of cosmic decoherence on primordial perturbations. It is worth to mention that our generalization helps in shedding light on some of the general conclusions in \([50, 51, 71]\), which can then be scrutinized further under the light of the results obtained by our approach.

The approach we will adopt is to consider a pointer observable of the from

\[
\hat{A} = \sum_n \alpha^{n-1} \hat{v}^n ,
\]  

(4.1)

up to a certain order \( n \), with \( \alpha \) being an expansion constant.

One of the most interesting results we find is about the scale-dependence of the cosmic decoherence induced bispectrum. In general, guided by the observational constraints, and thus imposing the leading corrections to the curvature power spectrum to be (almost) scale independent does imply indeed a specific scale-dependence of the bispectrum. The latter represents a specific prediction of our scenario, starting from (4.1). On the other hand, we will show that there is a certain freedom in the free parameters in case the underlying physical model is that of a massive scalar field as environment, so that e.g. also a scale-independent bispectrum can be obtained (similarly to what previously seen for the inflationary power spectrum and trispectrum).

In order to discuss a concrete example, we provide some, possible, physical processes that could yield the form (4.1).

The first example consists in adopting an interaction Hamiltonian of the form

\[
g \hat{H}_{\text{int}} = \lambda \mu^{4-n-m} \alpha^4 \int d^3 x \left( \hat{\varphi} - \sigma \hat{I} \right)^n \left( \hat{\psi}^m - \left< \hat{\psi}^m \right>_{\text{st}} \right),
\]  

(4.2)

which leads us to a pointer observable of the form (4.1), where \( \sigma \) is a constant of the same dimension of \( \varphi \). The total action (3.20) now becomes

\[
S = -\int d^4 x \sqrt{-g} \left[ \frac{1}{2} g^{\mu \nu} \partial_\mu \hat{\varphi} \partial_\nu \hat{\varphi} + V (\hat{\varphi}) + \lambda \mu^{4-n-m} \left< \hat{\psi}^m \right>_{\text{st}} \hat{\varphi}^n + \frac{1}{2} g^{\mu \nu} \partial_\mu \hat{\psi} \partial_\nu \hat{\psi} + \frac{M^2}{2} \hat{\psi}^2 + \lambda \mu^{4-n-m} (\hat{\varphi} - \sigma)^n (\hat{\psi}^m - \left< \hat{\psi}^m \right>_{\text{st}}) \right] .
\]  

(4.4)

\(^{19}\)It could be intuitively understood how (4.2) produces (4.1) if we expand it as follows

\[
g \hat{H}_{\text{int}} \propto \lambda \mu^{4-n-m} \alpha^4 \int d^3 x \varphi^{n-t} \sigma^t (\hat{\psi}^m - \left< \hat{\psi}^m \right>_{\text{st}}),
\]  

(4.3)

and make the straightforward change of variables necessary to obtain (4.1), starting by the change of variable \( v (\eta, x) \equiv \alpha (\eta) \varphi (\eta, x) \) which mimics the MS variable definition, and ending up by defining \( k = n - t \). Then, it easy to get the expansion constant that is to be identified with \( \alpha \) in (4.1).
Notice the similarity between (4.2) and the one considered in [50] given by (3.18). Indeed, the physical process we suggest is a generalization of (3.18) where the environment is made of a massive scalar field $\psi$; the only difference being the displacement of the field $\varphi$ by $\sigma$.

Assuming small field values of the inflaton field, we can get more physical processes which could yield the pointer observable form (4.1). As case in point, we can have power law inflation potential [94] as the system contribution part to $\hat{H}_{\text{int}}$

$$g\hat{H}_{\text{int}} = \lambda \mu^{4-m} a^4 \int d^3 x \exp \left( -\frac{\alpha}{M_{\text{pl}}} \varphi \right) \left( \psi^m - \langle \psi^m \rangle_{\text{st}} \right),$$  \hspace{1cm} (4.5)$$

and expanding the potential in the limit $\alpha \varphi \ll M_{\text{pl}}$, we can easily recover the form (4.1).

Similarly, for a Starobinsky-like [95] inflaton contribution in $\hat{H}_{\text{int}}$

$$g\hat{H}_{\text{int}} = \lambda \mu^{4-m} a^4 \int d^3 x \left[ 1 - \exp \left( -\frac{\sqrt{2/3}}{M_{\text{pl}}} \varphi \right) \right]^2 \left( \psi^m - \langle \psi^m \rangle_{\text{st}} \right),$$  \hspace{1cm} (4.6)$$

after double\hspace{1cm}21\hspace{1cm}expanding the term \left[ 1 - \exp \left( -\frac{\sqrt{2/3}}{M_{\text{pl}}} \varphi \right) \right]^2 in the small field inflation limit, we obtain the form (4.1).

For the sake of simplicity, we will pick up the physical process given by (4.2) as reference in the following. However, as shown above, any physical mechanism yielding the pointer observable form (4.1) would lead to the same general conclusions regarding the effect of quantum decoherence on the inflationary observables.

The free parameter $p$ expression within our generalized physical process (4.2), instead of (3.18), could be found following the computations done by J.Martin et al. in appendix B of [50]. Doing so, we get the relation

$$p = 7 - k - l,$$  \hspace{1cm} (4.7)$$

that links the free parameter $p$ value to the parameters $(k, l)$ characterizing the physical process, where, $k$ and $l$ could be understood from the following equation

$$\frac{d \langle \dot{O} \rangle}{d \eta} = \left\langle \frac{\partial \dot{O}}{\partial \eta} \right\rangle - i \left\langle \left[ \dot{O}, \hat{H}_{\text{v}} \right] \right\rangle - \frac{1}{2} \sum_{k,l} \gamma (k, l) \alpha^{k+l-2} \int d^3 x d^3 y C_R (x, y) \left\langle \left[ \left[ \dot{O}, \dot{v}^k \right], \dot{v}^l \right] \right\rangle,$$  \hspace{1cm} (4.8)$$

where by writing $\gamma \equiv \gamma (k, l)$ and putting it inside the summation, we are taking into account the dependence of the parameter $\gamma$ on $k$ and $l$ as given by (4.7).

\hspace{1cm}21\hspace{1cm}We remind that the time dependence of $\gamma$ is given by 2.15.
The relation (4.7) generalizes the one obtained in [50] given by (3.17), where a monomial form for the pointer observable was considered, \( \hat{A}(x) = \hat{v}^n(x) \). Indeed for \( k = l = n \) one recovers \( p = 7 - 2n \).

It is worth to mention that with our generalized form of the pointer observable (4.1), given as an expansion in \( \hat{v} \), the coupling system-environment in the Lindblad equation, represented by \( \gamma \) might vary from one term to another in that expansion. Such variation of \( \gamma \) is captured by the fact that the free parameter \( p \) might not be the same, but it might depend on which term of (4.1) is contributing to the non-unitary term of the Lindblad equation. A case in point of such variation of \( \gamma \) could be seen through the suggested physical processes above, where \( p \) is a function of \((k,l)\) rather than being the same (i.e. constant) for all the terms, see (4.7) and (4.8).

We will soon see that the leading term giving a non zero bispectrum is

\[
\frac{\gamma}{2} \alpha \int d^3x d^3y C_R(x,y) \left\langle \left[ \hat{O}, \hat{v}^2 \right], \hat{v} \right\rangle,
\]

and it will turn out to be scale independent for \( p = 4 \). Therefore, from (4.7) we see that this value of \( p \) could correspond to a massive scalar field as environment, since for \( k = 2(1) \) and \( l = 1(2) \) equation (4.7) yields \( p = 4 \).

However, it is very important to mention that the results derived below, and those obtained in [50, 51], are independent from the type of environment adopted as long as it satisfies the Born and Markov approximations.

Finally, we remark that it is possible to have a pointer observable of the form (4.1) by considering a decoherence scenario where short wavelengths of primordial perturbations spectrum decohere the long wavelengths ones, which are those of observational interest. For more details on that, see [34, 35, 71].

For the bispectrum computation, we will stop at the order \( n = 2 \) in (4.1), which represents the leading order for which decoherence does induce a non-vanishing bispectrum.

After dealing with the computation of the bispectrum, we will go back to the power spectrum and show that, if one adopts as the pointer observable Eq. (4.1), then an additional contribution to the Lindblad equation arises w.r.t to the results obtained in [50] which used a quadratic interaction term. These new contributions are indeed of the same order in \( \alpha \) as the contribution coming from a pure quadratic interaction. This last observation is also applicable to the trispectrum computations in [51], however we will restrict ourselves to the computation details of the power spectrum case.

4.1 Computation of the cosmic decoherence induced bispectrum

First let us discuss briefly how our choice of \( \hat{A} \) as in Eq. (4.1) does reproduce and generalize the results obtained by J.Martin et al. in [50, 51], and most importantly the reason behind obtaining in this case a non vanishing bispectrum.

The starting point is the previously derived Lindblad equation

\[
\frac{d}{d\eta} \left\langle \hat{O} \right\rangle = \left\langle \frac{\partial \hat{O}}{\partial \eta} \right\rangle - i \left\langle \left[ \hat{O}, \hat{H}_v \right] \right\rangle - \frac{\gamma}{2} \int d^3x d^3y C_R(x,y) \left\langle \left[ \left[ \hat{O}, \hat{A}(x) \right], \hat{A}(y) \right] \right\rangle,
\]
where, now, we use the pointer observable $\hat{A}$

$$\hat{A}(x) = \hat{v} + \alpha \hat{v}^2,$$  \hspace{1cm} (4.11)

where $\alpha$ is a coupling constant, with dimension $[\text{momentum}]^{-1}$ which is introduced for the sake of dimensions homogeneity. Substituting (4.11) in (4.10) gives

$$\frac{d\langle \hat{O} \rangle}{d\eta} = \langle \frac{\partial \hat{O}}{\partial \eta} \rangle - i \langle \left[ \hat{O}, \hat{H}_v \right] \rangle - \frac{i}{2} \int d^3x d^3y C_R(x, y) \left\langle \left[ \left[ \hat{O}, \hat{v}(x) + \alpha \hat{v}^2(x) \right], \hat{v}(y) + \alpha \hat{v}^2(y) \right] \right\rangle$$

$$= \langle \frac{\partial \hat{O}}{\partial \eta} \rangle - \frac{i}{2} \int d^3x d^3y C_R(x, y) \left\langle \left[ \left[ \hat{O}, \hat{v}(x) \right], \hat{v}(y) \right] \right\rangle$$

$$+ \alpha^2 \left\langle \left[ \left[ \hat{O}, \hat{v}^2(x) \right], \hat{v}^2(y) \right] \right\rangle + \alpha \left\langle \left[ \left[ \hat{O}, \hat{v}^2(x) \right], \hat{v}(y) \right] \right\rangle \right\}.$$  \hspace{1cm} (4.12)

From this equation we can see how the interaction terms we are considering do contain all the results derived in [50, 51]. Indeed, in order to obtain their results, it is sufficient for their linear case to set $\alpha = 0$, where for this type of interaction only power spectrum receives correction while all the other correlation functions remain untouched. Then, to recover the pure quadratic contribution to the power spectrum correction we can add a dimensionless constant in front of $\hat{v}$ in $\hat{A}(x)$ and set it to zero for that purpose, though, at least within a perturbative approach, it is more legitimate to consider the quadratic order along with the linear one, since the latter usually gives the dominant contribution.25 In case of the trispectrum we do not need this constant since there is no correction to it from the linear order in $\hat{v}$. Similarly, there is no correction from the last term, marked with $\cdots$, of the real and non unitary part of (4.12), given by

$$\propto \alpha \left\langle \left[ \left[ \hat{O}, \hat{v}(x) \right], \hat{v}^2(y) \right] + \left[ \left[ \hat{O}, \hat{v}^2(x) \right], \hat{v}(y) \right] \right\rangle,$$  \hspace{1cm} (4.13)

since it gives for the power spectrum case, after contractions, terms proportional to $\langle v_k \rangle$ or $\langle p_k \rangle$ which are zero with respect to the Bunch-Davies vacuum, while, for the trispectrum case it gives terms proportional to the initial bispectrum coming from the system Hamiltonian, which is zero in our case since it corresponds to the free theory, see footnote 18 and

25We will see in the next section that the pointer observable $\hat{A}(x) = \hat{v} + \alpha \hat{v}^2 + \alpha^2 \hat{v}^3$, will induce a correction to both the power spectrum and the trispectrum, of the same order in $\alpha$ as that of a pure quadratic contribution $\alpha^2 \left\langle \left[ \left[ \hat{O}, \hat{v}^2(x) \right], \hat{v}^2(y) \right] \right\rangle$, namely of order $\alpha^2$. Those additional corrections come from $\alpha^2 \left\langle \left[ \hat{O}, \hat{v}^3(x) \right], \hat{v}(y) \right\rangle$ and $\alpha^2 \left\langle \left[ \hat{O}, \hat{v}(x) \right], \hat{v}^3(y) \right\rangle$ and were not considered in [50] because they show up only if we consider the most general form of the pointer observable mentioned above, thus we must consider also those extra corrections to get a complete and accurate result. We will come back to this point soon.
explanations below (3.25). However, it is this last term, given by (4.13), that generates a purely quantum decoherence induced bispectrum, at leading order in $\gamma$, and such a term is there thanks to the new form of the pointer observable $\hat{A}$ which enables cross terms of the type $\left\{ \left[ \hat{O}, \hat{v}^k (x) \right], \hat{v}^l (y) \right\}$ to show up with $k \neq l$. We remind that in [51] it was found that the first term of the third line in (4.12), which refers to a pure quadratic interaction, would generate at leading order in $\gamma$ a bispectrum, but this would turn out to be proportional to the initial bispectrum which is zero. On the other hand, our pointer observable choice leads to a bispectrum that is proportional to the power spectra, thus representing a direct consequence of the decoherence effects.

Having said that and since we are focusing on the bispectrum, in this section we will keep only the new term in the Lindblad equation, so that we start the Fourier transform operation from

$$
\frac{d \langle \hat{O} \rangle}{d \eta} = \left\langle \frac{\partial \hat{O}}{\partial \eta} \right\rangle - i \left\langle \left[ \hat{O}, \hat{H}_v \right] \right\rangle - \frac{\gamma \alpha}{2} \int d^3 x d^3 y C_R (x, y) \left\{ \left\{ \left[ \hat{O}, \hat{v} (x) \right], \hat{v}^2 (y) \right\} + \left[ \left[ \hat{O}, \hat{v}^2 (x) \right], \hat{v} (y) \right]\right\}. \tag{4.14}
$$

Using

$$
\hat{v}^2 (x) = \frac{1}{(2\pi)^3} \int d^3 k' d^3 p \hat{v}_{k'} \hat{v}_{-k'} e^{ip \cdot x}, \tag{4.15}
$$

we get in Fourier space\(^{24}\)

$$
\frac{d \langle \hat{O} \rangle}{d \eta} = \left\langle \frac{\partial \hat{O}}{\partial \eta} \right\rangle - i \left\langle \left[ \hat{O}, \hat{H}_v \right] \right\rangle - \frac{\gamma \alpha}{2} \int d^3 k d^3 p \tilde{C}_R (|\mathbf{k}|) \left\{ \left\{ \left[ \hat{O}, \hat{v}_{-\mathbf{k}} \right], \hat{v}_p \hat{v}_{k-p} \right\} + \left[ \left[ \hat{O}, \hat{v}_p \hat{v}_{k-p} \right], \hat{v}_{-\mathbf{k}} \right]\right\}. \tag{4.16}
$$

Now, using the commutators property for $A$, $B$, $C$ being three operators

$$
[[A, B], C] = [A, [B, C]] \quad if \quad [B, C] = 0, \tag{4.17}
$$
equation (4.16) becomes

$$
\frac{d \langle \hat{O} \rangle}{d \eta} = \left\langle \frac{\partial \hat{O}}{\partial \eta} \right\rangle - i \left\langle \left[ \hat{O}, \hat{H}_v \right] \right\rangle - \gamma \alpha \int d^3 k d^3 p \tilde{C}_R (|\mathbf{k}|) \left\{ \left\{ \hat{O}, \hat{v}_p \hat{v}_{k-p} \right\}, \hat{v}_{-\mathbf{k}} \right\}. \tag{4.18}
$$

\(^{24}\)To arrive to this form, we use the Fourier transform $C_R (x, y) = \frac{1}{(2\pi)^3} \int d^3 k \tilde{C}_R (|\mathbf{k}|) e^{i\mathbf{k} \cdot \mathbf{x} - \mathbf{y}}$ in the first term $\left\{ \left[ \hat{O}, \hat{v} (x) \right], \hat{v}^2 (y) \right\}$ of the non unitary part of (4.14). Then, we make a variable change $\mathbf{k} \rightarrow -\mathbf{k}$ in the Fourier transform of $C_R (x, y)$ in the second term $\left\{ \left[ \hat{O}, \hat{v}^2 (x) \right], \hat{v} (y) \right\}$, and we take advantage of the invariance of the environment correlation function $\tilde{C}_R (|\mathbf{k}|)$ under such transformation, because of its dependence on the modulus $|\mathbf{k}|$. 

\[ - 19 - \]
The latter is the main equation that will enable us to get the eight differential equations satisfied by each bispectrum. Before listing them, notice that a particular comoving scale appears in the interaction term, indeed, in order for (4.12) (or equivalently (4.18)) to have the correct dimension, $\gamma \tilde{C}_R (|k|)$ must be homogeneous to the square of a comoving wavenumber which we define as in Eq. (3.14) [50]. Using the commutation relation (3.23) we get, after some straightforward computations, the following set of equations satisfied by the various bispectra

$$\frac{d\langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle}{d\eta} = \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle + \langle \hat{v}_{k_1} \hat{p}_{k_2} \hat{v}_{k_3} \rangle + \langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\frac{d\langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{p}_{k_3} \rangle}{d\eta} = \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{p}_{k_3} \rangle + \langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle - \omega^2 (k_3) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\frac{d\langle \hat{v}_{k_1} \hat{p}_{k_2} \hat{v}_{k_3} \rangle}{d\eta} = \langle \hat{v}_{k_1} \hat{p}_{k_2} \hat{v}_{k_3} \rangle + \langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle - \omega^2 (k_2) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\frac{d\langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle}{d\eta} = \langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle + \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle - \omega^2 (k_1) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\frac{d\langle \hat{v}_{k_1} \hat{p}_{k_2} \hat{p}_{k_3} \rangle}{d\eta} = \langle \hat{v}_{k_1} \hat{p}_{k_2} \hat{p}_{k_3} \rangle + \omega^2 (k_2) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{p}_{k_3} \rangle - \omega^2 (k_3) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\quad + 2\gamma \alpha \left( \tilde{C}_R (|k_2|) + \tilde{C}_R (|k_3|) \right) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\frac{d\langle \hat{p}_{k_1} \hat{p}_{k_2} \hat{p}_{k_3} \rangle}{d\eta} = \langle \hat{p}_{k_1} \hat{p}_{k_2} \hat{p}_{k_3} \rangle - \omega^2 (k_1) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{p}_{k_3} \rangle - \omega^2 (k_3) \langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\quad + 2\gamma \alpha \left( \tilde{C}_R (|k_1|) + \tilde{C}_R (|k_3|) \right) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\frac{d\langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle}{d\eta} = \langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle - \omega^2 (k_1) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{p}_{k_3} \rangle - \omega^2 (k_2) \langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\quad + 2\gamma \alpha \left( \tilde{C}_R (|k_1|) + \tilde{C}_R (|k_2|) \right) \langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$$

$$\frac{d\langle \hat{p}_{k_1} \hat{p}_{k_2} \hat{v}_{k_3} \rangle}{d\eta} = -\omega^2 (k_1) \langle \hat{v}_{k_1} \hat{p}_{k_2} \hat{p}_{k_3} \rangle - \omega^2 (k_2) \langle \hat{p}_{k_1} \hat{v}_{k_2} \hat{p}_{k_3} \rangle - \omega^2 (k_3) \langle \hat{p}_{k_1} \hat{p}_{k_2} \hat{v}_{k_3} \rangle$$

$$\quad + 2\gamma \alpha \tilde{C}_R (|k_1|) \left[ \langle \hat{p}_{k_2} \hat{v}_{k_1+k_3} \rangle + \langle \hat{p}_{k_3} \hat{v}_{k_1+k_2} \rangle \right] + 2\gamma \alpha \tilde{C}_R (|k_2|) \left[ \langle \hat{p}_{k_1} \hat{v}_{k_2+k_3} \rangle + \langle \hat{p}_{k_2} \hat{v}_{k_1+k_3} \rangle \right]$$

where $\omega^2$ is given by Eq. (2.8).

Combining those eight equations into a single one for $\langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle$ yields an equation of order eight which is not too illuminating. So to get a simpler equation of lower order we adopt the equilateral configuration limit in which all the momenta $k_1, k_2, k_3$ have
the same modulus $k = |k|$ so that $\omega^2 (k) \equiv \omega^2 (k)$ and $\tilde{C}_R (\{ k \}) = \tilde{C}_R (\{ k \}) = \tilde{C}_R (k)$. Doing so, the order of equation can be reduced into an equation of order four as we will see now. Moreover in this way we will be able to compute the overall amplitude $f_{\text{NL}}$ of the decoherence induced bispectrum. In the rest of this part, we will adopt the notation $\langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{v}_{k_3} \rangle \equiv B_{v^3}$, $\langle \hat{v}_{k_1} \hat{v}_{k_2} \hat{p}_{k_3} \rangle = B_{vpv}$ and similar expressions. Since we are interested in the bispectrum of curvature perturbations $B_{v^3}$, then, the strategy now is to differentiate the first equation of (4.19) and use the other equations in (4.19) in order to obtain a closed differential equation in $B_{v^3}$. Let us show how it works$^{25}$

\[
\frac{d^2 B_{v^3}}{d\eta^2} = 2 \left[ B_{vpv} + B_{ppp} + B_{pvp} \right] - 3\omega^2 B_{v^3} \\
\frac{d^3 B_{v^3}}{d\eta^3} = 6 B_{ppp} - 4\omega^2 \left[ B_{vvp} + B_{vpv} + B_{pvp} \right] - 3\omega^2 \frac{dB_{v^3}}{d\eta} - 6\omega \omega' B_{v^3} + 12\omega^2 B_{v^3} - 12\omega \omega' \frac{dB_{v^3}}{d\eta} - 3\omega^2 \frac{dB_{v^3}}{d\eta^2} - 6\omega^2 B_{v^3} - 6\omega' B_{v^3} \]
\]

\[d^4 B_{v^3} \]

\[\frac{d^4 B_{v^3}}{d\eta^4} = -14\omega^2 \left[ B_{vvp} + B_{vpv} + B_{pvp} \right] - 8\omega \omega' \left[ B_{vvp} + B_{vpv} + B_{pvp} \right] + 12\omega^2 B_{v^3} - 12\omega \omega' \frac{dB_{v^3}}{d\eta} - 3\omega^2 \frac{dB_{v^3}}{d\eta^2} - 6\omega^2 B_{v^3} - 6\omega' B_{v^3} \]

\[\frac{d^4 B_{v^3}}{d\eta^4} = -14\omega^2 \left[ B_{vvp} + B_{vpv} + B_{pvp} \right] - 8\omega \omega' \left[ B_{vvp} + B_{vpv} + B_{pvp} \right] + 12\omega^2 B_{v^3} - 12\omega \omega' \frac{dB_{v^3}}{d\eta} - 3\omega^2 \frac{dB_{v^3}}{d\eta^2} - 6\omega^2 B_{v^3} - 6\omega' B_{v^3} \]

and notice that from the the first equation in (4.20)

\[B_{vvp} + B_{ppp} + B_{pvp} = \frac{1}{2} \left( \frac{d^2 B_{v^3}}{d\eta^2} + 3\omega^2 B_{v^3} \right). \] (4.21)

Therefore, substituting it in the fourth derivative equation of $B_{v^3}$, in addition to using the first equation in (4.19), we get our final differential equation

\[\frac{d^4 B_{v^3}}{d\eta^4} + 10\omega^2 \frac{d^2 B_{v^3}}{d\eta^2} + 20\omega \omega' \frac{dB_{v^3}}{d\eta} + \left[ 9\omega^2 + 6\omega^2 + 6\omega \omega' \right] B_{v^3} = S (k, \eta), \] (4.22)

$^{25}$In these equations primes denote derivatives w.r.t. conformal time in order to simplify the notations.
where at leading order in $\gamma$, the source function $S(k, \eta)$ is given by

$$
S(k, \eta) = 24 \gamma^2 \alpha \tilde{C}_R(k) \left[ \Re \langle \hat{p}_{k_1} \hat{v}_{k_2+k_3} \rangle + \Re \langle \hat{p}_{k_1+k_2} \hat{v}_{k_3} \rangle + \Re \langle \hat{p}_{k_3} \hat{v}_{k_1+k_2} \rangle \right] \\
+ 4 \alpha \left( \gamma \tilde{C}_R(k) \right)' \left[ \langle \hat{v}_{k_1} \hat{v}_{k_2+k_3} \rangle + \langle \hat{v}_{k_2} \hat{v}_{k_1+k_3} \rangle + \langle \hat{v}_{k_3} \hat{v}_{k_1+k_2} \rangle \right] \\
+ 4 \gamma \alpha \tilde{C}_R(k) \frac{d}{d\eta} \left[ \langle \hat{v}_{k_1} \hat{v}_{k_2+k_3} \rangle + \langle \hat{v}_{k_2} \hat{v}_{k_1+k_3} \rangle + \langle \hat{v}_{k_3} \hat{v}_{k_1+k_2} \rangle \right].
$$

(4.23)

An important remark is that the terms involving $\gamma$ are not explicitly proportional to $\delta^{(3)}(k_1 + k_2 + k_3)$, where the presence of the Dirac delta function ensures the three Fourier modes of the bispectrum forming a triangle. However, remember that the system is solved through a perturbative expansion in $\gamma$ such that, during the first iteration, the Lindblad term containing the correlators $\langle \hat{p}_{k_1} \hat{v}_{k_2+k_3} \rangle, \langle \hat{v}_{k_1} \hat{v}_{k_2+k_3} \rangle, \ldots$ should be evaluated in the free theory. Therefore, we retrieve the expectation that the correlators turn out to be proportional indeed to $\delta^{(3)}(k_1 + k_2 + k_3)$, implying $k_2 + k_3 = -k_1 \equiv -k$ so that we can write $\langle \hat{p}_{k} \hat{v}_{-k} \rangle, \langle \hat{v}_{k} \hat{v}_{-k} \rangle$. Equation (4.23) becomes

$$
S(k, \eta) = 72 \gamma^2 \alpha \tilde{C}_R(k) \Re \langle \hat{p}_{k} \hat{v}_{-k} \rangle + 12 \alpha \left( \gamma \tilde{C}_R(k) \right)' \langle \hat{v}_{k} \hat{v}_{-k} \rangle + 12 \gamma \alpha \tilde{C}_R(k) \frac{d}{d\eta} \langle \hat{v}_{k} \hat{v}_{-k} \rangle.
$$

(4.24)

Our result (4.22) endorses the suggestion made in [51] according to which one can even conjecture that any correlator must obey a linear differential equation with a source term arising from the interaction with the environment.

The analogy between our equation (4.22) and the one for the power spectrum and trispectrum derived in [50, 51], makes us suggest that an exact solution is given by

$$
B_{wv} = \frac{4}{3W^3} \int_{-\infty}^{\eta} d\eta' S(k, \eta') 3m^3 \left[ v_{k} (\eta') v_{-k}^*(\eta) \right],
$$

(4.25)

$W$ being the Wronskian, as in Eq. (3.10). Indeed, using the Mukhanov-Sasaki equation $v_{k}'' + \omega^2 v_{k} = 0$ in addition to $v_{k} = (v_{k}^R + iv_{k}^I) / \sqrt{2}$, we can check with a straightforward but lengthy computation that (4.25) provides an exact solution to (4.22). It is worth to mention that the bispectrum we obtained is purely caused by decoherence, because we started with the quadratic Hamiltonian for the system which does not generate primordial non-Gaussianities.

Computing the integral (4.25) will enable us to discuss the non linearity-parameter $f_{NL}$ characterizing the amplitude of the curvature bispectrum, which is defined as the ratio between the bispectrum in the equilateral configuration to the square of power spectrum of curvature perturbation [74, 96]

$$
f_{NL} = \frac{5}{18} \frac{B_{\zeta\zeta\zeta}(k,k,k)}{P_{\zeta}^2(k)},
$$

(4.26)
where \( P_\zeta(k) = H^2/(4\epsilon M_\text{pl}^2 k^3) = P_{vv}/(2\epsilon M_\text{pl}^2 a^2) \) is the dimensionless power spectrum of the curvature perturbation related to Mukhanov-Sasaki variable by \( \zeta = -v/(\sqrt{2} M_\text{pl} a) \). Therefore, our \( f_{NL} \) could be expressed as

\[
(4.27)
\]

\[
f_{NL} = \frac{10}{27W^3} \frac{\sqrt{2} M_\text{pl} a}{P_{vv}^2(k)} \int_{-\infty}^{\eta} d\eta' S(k, \eta') \Im \left[ v_k(\eta') v_k^*(\eta) \right].
\]

Needless to say that computing the above integral is not straightforward, and some approximations are needed to render it tractable. The details of the computation are gathered in the appendix (see A), and here we will quote directly the final results we obtain for the \( f_{NL} \) expressions as a function of the free parameter \( p \).

- \( p = 1 \), in this case the contribution coming from \( B^{(1)}_{vvv} \) is negligible compared to the other two

\[
f_{NL} = \frac{5}{9\pi \sqrt{P_\zeta k_s}} \frac{\alpha k_s^2}{k_s} \left( \frac{k}{k_s} \right)^{-3} \left[ \frac{5}{3} (H\dot{E})^{-1} \sin (H\dot{E})^{-1} \left( \frac{k}{k_s} \right)^{-3} e^{3(N-N_*)} - \frac{32}{9} \cos (H\dot{E})^{-1} \left( \frac{k}{k_s} \right)^{-3} e^{(N-N_*)} + (H\dot{E})^{-1} \cos^3 (H\dot{E})^{-1} \right].
\]

(4.28)

- \( p = 2 \), in this case the contribution coming from \( B^{(1)}_{vvv} \) is negligible compared to the other two

\[
f_{NL} = \frac{5}{9\pi \sqrt{P_\zeta k_s}} \frac{\alpha k_s^2}{k_s} \left( \frac{k}{k_s} \right)^{-2} \left[ \cos^3 (H\dot{E})^{-1} + \frac{3}{4} Si \left( - (H\dot{E})^{-1} \right) - \frac{11}{16} Ci \left( e^{-\left( N-N_\ast \right)} \right) \right].
\]

(4.29)

- \( 3 \leq p \leq 5 \), in this case the contribution coming from both \( B^{(1)}_{vvv} \) and \( B^{(3)}_{vvv} \) is negligible compared to the \( B^{(2)}_{vvv} \), thus we get

\[
f_{NL} = \frac{5}{9\pi \sqrt{P_\zeta k_s}} \frac{\alpha k_s^2}{k_s} \left( \frac{k}{k_s} \right)^{p-4} (H\dot{E})^{p-2} \cos^3 (H\dot{E})^{-1}
\]

(4.30)

\[
3 \leq p \leq 5
\]

\[
\text{Notice that we are neglecting corrections at second-order in the perturbations connecting the Mukhanov-Sasaki variable with} \; \zeta, \; \text{since these are of gravitational strength and hence slow-roll suppressed w.r.t. to the contribution we are computing here.}
\]

\[
\text{We are using in the following expressions of} \; f_{NL} \; \text{the dimensionless curvature power spectrum} \; P_\zeta = H^2/(8\pi^2 M_\text{pl}^2 \epsilon) \simeq 2.2 \times 10^{-9}.
\]

\[
\text{The expressions corresponding to} \; B^{(1)}_{vvv}, \; B^{(2)}_{vvv}, \; \text{and} \; B^{(3)}_{vvv} \; \text{are given in the Appendix A.}
\]
\[ f_{\text{NL}} = \frac{5}{9\pi} \frac{\alpha k^2_{\text{NL}}}{k_*} \left( \frac{k}{k_*} \right)^2 (k_{E})^4 \cos^3 (H_{E})^{-1} + \frac{49}{30} \left( \frac{k}{k_*} \right)^3 e^{-(N-N_*)} \]

\[ - \frac{17}{60} \frac{k}{k_*} e^{-(N-N_*)} - 3 \left( \frac{k}{k_*} \right)^3 e^{-3(N-N_*)} \left( \ln \left( \frac{k}{k_*} \right) + N_* - N \right) - \frac{3}{2} \left( \frac{k}{k_*} \right)^3 e^{-3(N-N_*)} \]

\[ (4.31) \]

• for \( p > 6 \), taking into account all the terms, including the subleading, we get

\[ f_{\text{NL}} = \frac{5}{9\pi} \frac{\alpha k^2_{\text{NL}}}{k_*} \left( \frac{k}{k_*} \right)^3 \left[ (H_{E})^{p-2} \left( \frac{k}{k_*} \right)^{p-7} \cos^3 (H_{E})^{-1} - 7 \frac{(3p-4) e^{-(7-p)(N-N_*)}}{(p-4)(p-3)(p-2)(p-1)} \right. \]

\[ - 6 \left( \frac{k}{k_*} \right)^2 \frac{e^{-(9-p)(N-N_*)}}{(p-6)(p-5)(p-4)} - 6 \frac{e^{-(7-p)(N-N_*)}}{(p-6)(p-5)(p-4)} \]

\[ (4.32) \]

Some comments are in order here. First, at this level of the results, if we start from Eq. (4.10) where in full generality we adopt the same power of \( p \) as in (2.15) for the various contributions in the Lindblad term, then it is interesting to notice that, imposing (almost) scale-invariance of the power spectrum and negligible (scale-dependent) decoherence induced corrections to the power-spectrum on large (CMB) scales, guided by observational constraints, then the resulting bispectra would be strongly scale-dependent. This, e.g., could be the case of \( p = 5 \) (see Eq. (3.13) and Eq. (4.30), and the discussion there), leading to \( f_{\text{NL}} \propto k \). Notice that this remains true even when accounting for the additional corrections to the power-spectrum discussed in details in Sec. 4.2. \(^{29}\)

On the other hand it is quite remarkable that we can obtain a scale invariant bispectrum for \( p = 4 \). The latter result, as already mentioned, might correspond to a massive scalar field as environment as can be checked using the relation (4.7) and remembering

\(^{29}\)Indeed imposing (nearly) scale-invariance of the decoherence-induced corrections to the scalar power-spectrum might be a too restrictive condition, since the measured (almost) scale independent scalar power spectrum could be merely provided by the leading order, and if we increase the measurements sensitivity we might be able to detect a non-trivial weak scale-dependence which reflects the decoherence effect for \( p \neq 5 \).
that the bispectrum was induced by the Lindblad term
\[ \frac{\gamma}{2} \alpha \int d^3x d^3y \mathcal{C}_R (x, y) \left\langle \left[ \left[ \hat{O}, \hat{v}^2 \right], \hat{v} \right] \right\rangle, \tag{4.33} \]
thus corresponding to \( k = 2 \) and \( l = 1 \) in the massive scalar field scenario discussed previously.

The scale invariance obtained in [50, 51] for the power spectrum and trispectrum was also corresponding to a massive scalar field as environment, and indeed within this model, one has enough freedom that the various correlators induced by decoherence effects turn out to be indeed scale-invariant.

In spite of these results, we think that the scale independence of various correlation functions is related to a more general factor that is independent of the type of environment considered. In the case of a linear interaction, \( \hat{A} (x) = \hat{v} \), for which the Lindblad term is given by
\[ \frac{\gamma}{2} \int d^3x d^3y \mathcal{C}_R (x, y) \left\langle \left[ \left[ \hat{O}, \hat{v} (x) \right], \hat{v} (y) \right] \right\rangle, \tag{4.34} \]
the scale independence of the curvature power spectrum\(^{30} \) was obtained for a value of \( p = 5 \).

In terms of the expansion parameter \( \alpha \), it correspond to the zeroth order, i.e to \( \alpha^0 \). Through our choice of the form of pointer observable (4.1), we considered \( \hat{A} (x) = \hat{v} + \alpha \hat{v}^2 \), and we noticed that the leading contribution to the curvature bispectrum was coming from a term in (4.34) that is \( \left( \hat{A} (x) \right)^2 \propto \alpha \hat{v}^3 \) as could be seen in (4.18), and scale independence was found for \( p = 4 \). While, for the case of a pure quadratic interaction where the Lindblad term (4.34) is \( \left( \hat{A} (x) \right)^2 \propto \alpha^2 \hat{v}^4 \), the scale invariance (of the power spectrum and trispectrum) was obtained in [50, 51] for \( p = 3 \). In terms of the expansion parameter \( \alpha \), it correspond to the second order, i.e to \( \alpha^2 \). Therefore we summarize our reasoning as follows

\[
\begin{align*}
\text{Value of } p \text{ corresponding} & \quad \left( \hat{A} (x) \right)^2 \propto \alpha \rightarrow p = 4 \\
\text{to scale independent} & \quad \left( \hat{A} (x) \right)^2 \propto \alpha^2 \rightarrow p = 3 \\
\text{corrections} & \quad \left( \hat{A} (x) \right)^2 \propto \alpha^0 \rightarrow p = 5
\end{align*}
\tag{4.35}
\]

Based on this analysis and reasoning we conjecture a relation that enables us to understand when decoherence does induce scale independent (contributions to the) correlation functions. Such a relation is simply linked to the order of \( \alpha \) in the Lindblad term
\[ \frac{\gamma}{2} \int d^3x d^3y \mathcal{C}_R (x, y) \left\langle \left[ \left[ \hat{O}, \hat{A} (x) \right], \hat{A} (y) \right] \right\rangle, \tag{4.36} \]

\(^{30}\)We remind that for the linear interaction only power spectrum receives corrections while all the other higher order correlation functions remain unaffected by the cosmic decoherence.
from which the decoherence induced contributions are coming, independently from the physical process and the type of environment considered. We can take further the analysis summarized in (4.35) and claim that the scale invariance of a contribution coming from the Lindblad term in $d \langle \hat{O} \rangle / d\eta$ of the order $\alpha^n$, with $n = k + l - 2$,

$$\frac{\gamma}{2} \alpha^{k+l-2} \int d^3 x d^3 y C_R(\mathbf{x}, \mathbf{y}) \left\langle \left[ \hat{O}, \hat{v}^k \right], \hat{v}^l \right\rangle ,$$

(4.37)
corresponds to the value of $p$ given by

$$p = 5 - n .$$

(4.38)

The latter claim is valid, at least, in all the computations made so far, namely power spectrum, bispectrum and trispectrum, as we explained in (4.35). Since the order of $\alpha$ in (4.37), i.e $n$, is related to $k$ and $l$, the relation (4.38) could be written, equivalently, as

$$p = 7 - k - l ,$$

(4.39)
surprisingly, this relation turns to be just identical to that of the physical process we suggested of a massive scalar field as environment, see equation (4.7). However, any physical process, or type of environment, which produces a pointer observable of the form (4.1), would produce corrections to the power spectrum or induce non-Gaussianities which are scale independent for a value of $p$ that is dictated by (4.38), according to the order of $\alpha$ in (4.37) from which the corrections were derived. For example, imposing the bispectrum to be scale independent, regardless of the type of environment, then will allow to get upper bounds on the interaction strength with the environment as a function of $p$.

Conversely, our general result (4.38) shows once more that in the case where it is possible to have a unique value of $p$ (see (2.15)) for the interaction strength $\gamma$ in the Lindblad term (3.2) such that, e.g., the power spectrum is scale-invariant, then necessarily this would imply that all the other correlators will unavoidably feature a specific and predictable scale-dependence.

Notice also that $f_{NL}$ is suppressed by the square root $^{31}$ of the slow-roll parameter $\epsilon$, which is to be compared with the standard bispectrum obtained in the standard single field slow-roll inflation that is slow-roll suppressed by the first and second slow roll parameters $\epsilon$ and $\epsilon_2$, respectively. Therefore the bispectrum induced by cosmic decoherence is parametrically greater w.r.t to the standard one. $^{32}$ Also, $M_{Pl}/H$ and specific scale dependencies contribute to increase the amplitude and to compensate the suppression by $Hl_E \ll 1$, present for some of the bispectra found, and by the coupling $\alpha$ which is supposed to be small.

We conclude this section by presenting some illustrating plots for the overall non-Gaussianity amplitude $f_{NL}$. In Fig. 1 we present $f_{NL}$ as a function of $k/k_s$ for various

\[ ^{31} \text{To see this, we remind that we expressed } f_{NL} \text{ in terms of the dimensionless curvature power spectrum } P_\zeta, \text{ see footnote 27.} \]

\[ ^{32} \text{It might well be that however the two bispectra are characterized by different shapes. A full analysis of this aspect is beyond the scope of this paper, where we have focused on the possibility of generating a non-vanishing bispectrum from decoherence effects, on its overall amplitude and scale dependence.} \]
values of the free parameter $p$. As mentioned before, two cases are of particular interest: $p=4$ corresponds to a scale-invariant bispectrum, in agreement with our general criterium given in (4.38); $p=5$ would correspond instead to a scale-invariant decoherence-induced contribution to the power spectrum (and negligible scale-dependent contributions on large CMB scales), resulting here in a non-linearity parameter scaling as $f_{NL} \propto k$. In Fig. 2 we present the plots of $f_{NL}$ for different values of $HI_E$, i.e. different values of environment correlation length compared to Hubble radius. We can see that this factor plays a major role for $2 < p \leq 7$, while has less effect on other values of $p$. If one looks at the analytical expressions for $f_{NL}$ this is clearly explained by the fact the dependence on $HI_E$ is strongly suppressed by the presence of $e^{N-N}N$ factors outside the range $2 < p \leq 7$. Indeed, we study the role of number of $e$-folds $N - N_s$ in Fig. 3, where, as expected from the analytical expressions of $f_{NL}$, this factor has less effect in the range $2 \leq p \leq 7$.

Since we are dealing with an open quantum system, it would be interesting to constrain the interaction system-environment. To this end, we take advantage of the observational constrains from Planck on equilateral bispectrum, from which we have $f_{NL} < 120$ at 95% CL [97]. We can proceed in two different ways to get such constraints. Either, we constrain the overall factor $(\alpha k^2 / k_s)$ appearing in front of the various expressions of $f_{NL}$, which sets the overall non-Gaussianity amplitude, as shown in Fig. 4, or we fix $\alpha kgamma$ and constrain the factor $k gamma / k_s$. This last choice serves to make a comparison with the previous constraints on $k gamma / k_s$ from decoherence effects on the curvature power-spectrum obtained in [50] possible. Obviously, the downside of second choice is the need to fix $\alpha kgamma$ with a given value. In Fig. 5 we present such constraints on $k gamma / k_s$ for different values of $\alpha kgamma$.

Finally, in Fig. 6, we shed light on the effect of environment correlation length on the obtained constraints. Here, we fix the value of $\alpha kgamma$ to $10^9$ and present the constraints for two different values of $HI_E$. Fig. 5 and Fig. 6 can be compared directly to, e.g., Fig. 6 of [50]. It is interesting to notice that there are regions in the parameter space according to different values of $\alpha kgamma$ and $HI_E$ for which observational constraints coming from Planck bispectrum measurements start to be competitive (if not stronger) than those from decoherence effects and constraints on the power spectrum. Of course this possibility is indeed model-dependent. One could still ask whether the values considered for $\alpha kgamma$ are physically relevant or constrained by other considerations. If one considers the parameter $\alpha$ as an expansion parameter in (4.11) then typically one has to impose that $|\alpha \delta \phi| \ll 1$. Therefore in this case one should check whether this might be consistent (and under which conditions) with values, e.g., of $\alpha kgamma = 10^9$. One has to account that the constraints from almost-scale invariance of the power spectrum and decoherence implies $k gamma / k_s < 10^{-1}$ [50] (with $k_s = 0.05$ Mpc $^{-1} \sim 3 \times 10^{39}$ GeV$^{-1}$) which means imposing a lower bound on $\alpha$. Thus one can check that the perturbativity condition $|\alpha \delta \phi| \ll 1$ implies an extremely

\[ \text{Here, for simplicity, we are smoothing the constraint from } -63 < f_{NL} < 120 \text{ into } |f_{NL}| < 120. \]

Also, a completely self-consistent data analysis would entail to compute the non-Gaussian shape of the bispectrum from cosmic-decoherence, and to apply to this shape a proper estimator for primordial non-Gaussianity, which is however beyond the scope of this paper. The constraints we obtain here are meant as a (conservative) estimate that one could obtain by applying an equilateral primordial non-Gaussianity estimator.
low values of the slow-roll parameter $\epsilon$ and hence of the standard tensor-to-scalar ratio $r$ (at least if one uses the (leading-order) dimensionless power-spectra such as the curvature power-spectrum $P_\zeta = P_{\delta\phi}/(2\epsilon M_{pl}^2) \sim 10^{-10}$). \(^{34}\)

On the other hand notice that not necessarily the expression in (4.11) should always be considered as an expansion, since the operator $\hat{A}$ enters in the interaction terms, thus defining the source to the power-spectra induced by the decoherence effects, and this source can be provided by fields that are different w.r.t. to the system field itself $\zeta$.

\[ P_{\zeta} = P_{\delta\phi}/(2\epsilon M_{pl}^2) \sim 10^{-10}. \]

\[^{34}\]In this case the decoherence induced corrections on the tensor sector discussed in the section 5 might become the dominant contribution.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Bispectrum parameter $f_{NL}$ absolute value rescaled by $\beta = \frac{\alpha k^2}{\kappa}$ as function of $\frac{k}{\kappa}$ for the various values of parameter $p$. We choose $H|E| = 10^{-3}$, and $N - N = 50$.}
\end{figure}

\subsection*{4.2 Additional corrections to the curvature power spectrum}

As we already mentioned in the previous section, \([50, 51]\) considered the effects induced by decoherence for two forms of the pointer observable, linear $\hat{A}(x) = \hat{v}$ and pure quadratic $\hat{A}(x) = \hat{v}^2$, and when it comes to the power spectrum, $P_{vv}$, they obtained non zero corrections for both cases. However, as we stated before, there are various motivations to consider a pointer observable which is expanded perturbatively in $\hat{v}$ with an expansion coupling $\alpha$. By doing so and in order to have a consistent and accurate result, we realized that actually there is an extra correction which must be considered along the pure quadratic one in the Lindblad term, because they are of the same order in $\alpha$, namely second order. In what follows, we show this last point and compute the additional correction that must be added to the ones already obtained in \([50]\).
Figure 2. Bispectrum parameter $f_{NL}$ absolute value rescaled by $\beta = \frac{\alpha k^2}{k^*}$ as function of $\rho$ for different values of $H_l E$. We choose $k/k^* = 1$, and $N - N_* = 50$.

Figure 3. Bispectrum parameter $f_{NL}$ absolute value rescaled by $\beta = \frac{\alpha k^2}{k^*}$ as function of $\rho$ for different values of $N - N_*$. We choose $H_l E = 10^{-3}$, and $k/k^* = 1$. 

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Figure 4. Constraints on $\frac{\alpha k^2}{\gamma^2}$ corresponding to $|f_{NL}| < 120$ for different values of $Hl_E$. We choose $k = 1$, and $N - N_* = 50$.

Figure 5. Constraints on $\frac{k}{H}$ corresponding to $|f_{NL}| < 120$ for different values of $\alpha k$. We choose $k = 1$, $N - N_* = 50$, and $Hl_E = 10^{-3}$. 
Figure 6. Constraints on $\frac{k}{\epsilon}$ corresponding to $|f_{NL}| < 120$ for different values of $H(t)$ and fixed value of $\alpha k = 10^9$. We choose $\frac{\epsilon}{k} = 1$, and $N - N_\ast = 50$.

Since we are interested in corrections up to order $\alpha^2$, we adopt the pointer observable

$$\hat{A}(x) = \hat{v} + \alpha \hat{v}^2 + \alpha^2 \hat{v}^3;$$

(4.40)

and (4.10) becomes up to second order in $\alpha$

$$\frac{d(\hat{O})}{\eta} = \left\langle \hat{O}, H_v \right\rangle - \frac{i}{2} \int d^3 x d^3 y C_R(x, y)$$

$$\times \left\langle \left[\hat{O}, \hat{v}(x) + \alpha \hat{v}^2(x) + \alpha^2 \hat{v}^3(x)\right], \hat{v}(y) + \alpha \hat{v}^2(y) + \alpha^2 \hat{v}^3(y) \right\rangle$$

$$= \left\langle \hat{O}, H_v \right\rangle - \frac{i}{2} \int d^3 x d^3 y C_R(x, y) \left\{ \left\langle \left[\hat{O}, \hat{v}(x)\right], \hat{v}(y) \right\rangle \right.$$}

$$+ \alpha \left\langle \left[\hat{O}, \hat{v}(x)\right], \hat{v}^2(y) \right\rangle + \left[\hat{O}, \hat{v}^2(x)\right], \hat{v}(y) \right\rangle \right.$$}

$$+ \alpha^2 \left( \left\langle \left[\hat{O}, \hat{v}^2(x)\right], \hat{v}^2(y) \right\rangle + \left[\hat{O}, \hat{v}^2(x)\right], \hat{v}(y) \right\rangle \right. \}.$$ 

(4.41)

From this last equation we can see in the last line that the pure quadratic interaction term \( \left\langle \left[\hat{O}, \hat{v}^2(x)\right], \hat{v}^2(y) \right\rangle \) is accompanied by a cross term coming from a combination of the linear and cubic interaction \( \left[\hat{O}, \hat{v}(x)\right], \hat{v}^3(y) \) + \left[\hat{O}, \hat{v}^3(x)\right], \hat{v}(y) \rangle \). It is this last term that could not be captured in the computations done in [50]. We now compute the
corresponding correction to the power spectrum $P_{vv}$. Fortunately, all what we will need to do at the end is to add this correction to the linear and quadratic ones, because, as we will see in a moment, the solution to the evolution equation of $P_{vv}$ is linear in the source functions which, on their turn, are proportional, apart from a numerical factor, to those real terms in (4.41).

Before doing the computations, we need to Fourier transform (4.41) using for the cubic terms

$$
\hat{v}^3(x) = \frac{1}{(2\pi)^{9/2}} \int d^3p d^3q d^3r \hat{v}_{p} \hat{v}_{q} \hat{v}_{r} e^{i(p-q-r)x}.
$$

(4.42)

Noticing that the term linear in $\alpha$ in (4.41) gives a vanishing contribution to the correction of $P_{vv}$, and exploiting (4.17), the Fourier transform is given by

$$
\frac{d\langle \hat{O} \rangle}{d\eta} = \left\langle \frac{\partial \hat{O}}{\partial \eta} \right\rangle - i \left\langle \left[ \hat{O}, \hat{H}_v \right] \right\rangle - \frac{\gamma}{2} (2\pi)^{3/2} \int d^3k C_k (|k|) \left\langle \left[ \left[ \hat{O}, \hat{v}_k \right], \hat{v}_{-k} \right] \right\rangle
$$

$$
- \frac{\gamma^*}{2(2\pi)^{3/2}} \int d^3k d^3p d^3q C_p (|k|) \times \left\langle \left[ \left[ \hat{O}, \hat{v}_p \hat{v}_{k-p_1} \right], \hat{v}_{p_2} \hat{v}_{k-p_2} \right] \right\rangle
$$

$$
+ 2 \left\langle \left[ \left[ \hat{O}, \hat{v}_k \right], \hat{v}_p \hat{v}_{k-p_1} \hat{v}_{k-p_2} \right] \right\rangle. \tag{4.43}
$$

The next step is to derive the equations governing the various two-point correlators

$$
\frac{d\langle \hat{v}_{k_1} \hat{v}_{k_2} \rangle}{d\eta} = \langle \hat{v}_{k_1} \hat{p}_{k_2} \rangle + \langle \hat{p}_{k_1} \hat{v}_{k_2} \rangle
$$

$$
\frac{d\langle \hat{p}_{k_1} \hat{v}_{k_2} \rangle}{d\eta} = \langle \hat{p}_{k_1} \hat{p}_{k_2} \rangle - \omega^2 (k_2) \langle \hat{v}_{k_1} \hat{v}_{k_2} \rangle
$$

$$
\frac{d\langle \hat{p}_{k_1} \hat{v}_{k_2} \rangle}{d\eta} = \langle \hat{p}_{k_1} \hat{p}_{k_2} \rangle - \omega^2 (k_1) \langle \hat{v}_{k_1} \hat{v}_{k_2} \rangle
$$

$$
\frac{d\langle \hat{p}_{k_1} \hat{p}_{k_2} \rangle}{d\eta} = -\omega^2 (k_2) \langle \hat{p}_{k_1} \hat{v}_{k_2} \rangle - \omega^2 (k_1) \langle \hat{v}_{k_1} \hat{p}_{k_2} \rangle + \gamma (2\pi)^{3/2} C_k (|k|) \delta (k_1 + k_2) + \alpha^2 \frac{\gamma}{2(2\pi)^{3/2}}
$$

$$
\times \left[ 4 \int d^3k C_k (|k|) \langle \hat{v}_{k_1+k_2} \hat{v}_{-k-k_2} \rangle + 3 \left( C_k (|k_1|) + C_k (|k_2|) \right) \int d^3k \langle \hat{v}_{k_1+k_2} \hat{v}_{-k-k_2} \rangle \right]. \tag{4.44}
$$

The term marked with $\gamma$ is the new contribution to the power spectrum correction. Similarly to what happens for the system of equations governing the various bispectra in Eq. (4.19), it is worth mentioning that, though decoherence effects appear explicitly only in the correlator $\langle \hat{p}_{k_3} \hat{p}_{k_2} \rangle$, they do affect all the correlators since the latter are governed by a coupled system of equations, as it is clear from (4.44). Another point to mention, which we
already discussed in the case of the bispectrum, is the appearance of \( \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \) in the linear contribution to \( \langle \hat{p}_{\mathbf{k}_1} \hat{p}_{\mathbf{k}_2} \rangle \), which ensures that the interaction with the environment preserves statistical homogeneity. We do not see the Dirac delta function in the last two terms \( \propto \alpha^2 \), however, remember that this term is also \( \propto \gamma \) and the system is solved through a perturbative expansion in \( \gamma \). During the first iteration, the last two Lindblad terms contain the correlators \( \langle \hat{v}_{\mathbf{k} + \mathbf{k}_1} \hat{v}_{-\mathbf{k} + \mathbf{k}_2} \rangle \) and \( \langle \hat{v}_{\mathbf{k}} \hat{v}_{\mathbf{k}_1 + \mathbf{k}_2} \rangle \) to be computed in the free case, so they turn out to be proportional to \( \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2) \) thus guaranteeing indeed that the solution is statistically homogeneous. In addition we adopted the case \( \tilde{C}_R(k) \equiv \tilde{C}_R(k) \), i.e an isotropic environmental correlation function. Thus the system for isotropic solutions reduces to

\[
\frac{dP_{vv}(k)}{d\eta} = P_{vp}(k) + P_{pv}(k) \tag{4.45}
\]

As it was done for the bispectrum case, one can combine the above equations in order to get a single differential equation for \( P_{vv} \), this time of third order, given by

\[
P''_{vv} + 4\omega^2 P'_{vv} + 4\omega \omega' P_{vv} = S_1 + S_2 + S_3, \tag{4.46}
\]

where \( S_1 \) is a source function coming from the linear contribution to the correction

\[
S_1 = 2(2\pi)^{3/2} \gamma \tilde{C}_R(k), \tag{4.47}
\]

while \( S_2 \) and \( S_3 \) are source functions arising from the pure quadratic interaction correction and the crossed term between linear and cubic interaction, respectively. They are functions of time and involve the power spectrum \( P_{vv} \) itself evaluated at all scales, namely

\[
S_2(k, \eta) = \alpha^2 \frac{\gamma}{(2\pi)^{1/2}} \int d^3k' \tilde{C}_R(k') P_{vv}(|k' + k|), \tag{4.48}
\]

\[
S_3(k, \eta) = \alpha^2 \frac{12\gamma}{(2\pi)^{1/2}} \tilde{C}_R(k) \int d^3k' P_{vv}(|k'|). \tag{4.48}
\]

Guided by the intuition based on previous computations, both those in [50, 51] and that of the bispectrum, we introduce the following functions which will constitute a solution to (4.46)

\[
P_{vv}^{(i)} = -\frac{2}{W_2} \int_{-\infty}^{\eta} d\eta' S_i(k, \eta') \text{Im} \left[ v_k(\eta') v_k^*(\eta) \right], \tag{4.49}
\]

\[\text{– 33 –}\]
with $i = 1, 2, 3$. So the general solution to (4.46) is given by

$$P_{vv} = v_k(\eta) v_k^*(\eta) + P_{vv}^{(1)}(\eta) + P_{vv}^{(2)}(\eta) + P_{vv}^{(3)}(\eta),$$

(4.50)

where the first term is the standard result, i.e. without decoherence; $P_{vv}^{(1)}$ and $P_{vv}^{(2)}$ have already been computed in [50], and the aim now is to compute explicitly $P_{vv}^{(3)}$ which represents the new correction we obtained. However, before starting, it is useful to make a comparison between $P_{vv}^{(2)}$ and $P_{vv}^{(3)}$ expressions, since they are of the same perturbative order in $\alpha$. It is easy to notice that $P_{vv}^{(2)}$ differs from $P_{vv}^{(3)}$ by the fact that in the former the environment correlation function is inside the integral while in the latter it is outside of it. Combining this last observation with the computations carried out for $P_{vv}^{(2)}$ in [50] in the limit $k\eta \to 0$ and $H_E \ll 1$, we find a relation between the source functions $S_2(k, \eta)$ and $S_3(k, \eta)$. Indeed, it has been shown in [50] that the environment correlation function $\tilde{C}_R(k')$ which is inside the integral in $S_2(k, \eta)$ could be replaced by $\tilde{C}_R(k)$ and be factorized outside of the integral, in such case we simply get

$$S_2(k, \eta) = \alpha^2 \frac{8\pi^3}{(2\pi)^3/2} \int d^3k' \tilde{C}_R(k') P_{vv}(|k' + k|)$$

$$\approx \alpha^2 \frac{8\pi^3}{(2\pi)^3/2} \tilde{C}_R(k) \int d^3k' P_{vv}(|k' + k|)$$

(4.51)

where $\eta_{IR}$ is an IR cutoff to make the integral finite. Thus from (4.51) we deduce that there is no need to compute $P_{vv}^{(3)}$ from scratch, since it is related to $P_{vv}^{(2)}$, therefore, the overall correction at order $\alpha^2$ is given by $\frac{5}{3} P_{vv}^{(2)}$ and this factor of $5/3$ represents the correction brought by considering the pointer observable (4.40) instead of a pure quadratic one.

The second important fact to be deduced from (4.51), is that $P_{vv}^{(3)}$ will represent a scale independent correction to the standard power spectrum for $p = 3$ since it was shown in [50] that $P_{vv}^{(2)}$ is scale independent for this value of $p$, and this endorses once again the analysis we made below equation (4.34). In this case, despite that $P_{vv}^{(2)}$ and $P_{vv}^{(3)}$ are originated from different terms in (4.43), namely $\alpha^2 \left\langle \left\{ \hat{O}, \hat{v}_p \hat{v}_{k-p_1} \right\}, \hat{v}_{p_2} \hat{v}_{-k-p_2} \right\rangle$ and $\alpha^2 \left\langle \left\{ \hat{O}, \hat{v}_k \right\}, \hat{v}_{p_1} \hat{v}_{p_2} \hat{v}_{-k-p_1-p_2} \right\rangle$, respectively, they are both $\propto \alpha^2$. Thus, according to the conjecture we made, they should share the same value of the parameter $p$, see (4.38), for which their expressions are scale independent, and this is exactly what happens.

Notice that the same line of thoughts should be applied to the computation of the trispectrum which receives its first non vanishing correction at order $\alpha^2$. So in addition to the pure quadratic interaction contribution, computed in [51], we must add the new term marked with $\tilde{3}$ in (4.41). We expect that the difference between the pure quadratic pointer observable and ours will be more relevant in the trispectrum than it is in the case of the power spectrum.
5 Decoherence of tensor perturbations

The aim of this section is to compute the decoherence induced corrections to the tensor (i.e. inflationary gravitational waves) power spectrum. Since the ultimate goal is to investigate the effect of the interaction with the environment on the scalar and tensor perturbations we will apply the same Lindblad equation (4.10) to the tensor perturbations that represent, now, our system. In fact the Lindblad equation (4.10) was derived in full generality regardless of the nature of the system, provided that the environment should satisfy some properties and conditions encoded in the correlation function

\[ C_{R} (x, y) = \text{Tr}_{E} (\rho_{E} R(x) R(y)) \]  

(5.1)

We introduce the following Fourier expansion of the tensor perturbations \( h_{ij}(\eta, x) \), which will be useful in the following computations,

\[ h_{ij}(x) = \frac{1}{(2\pi)^{3/2}} \int d^{3}k h_{ij}(k) e^{ik \cdot x} \]  

(5.2)

with

\[ h_{ij}(k) = \sum_{\lambda=+,x} h_{\lambda}(k) e_{ij}^{\lambda}, \quad e_{ij}^{\lambda} e_{ij}^{\lambda'} = 2\delta^{\lambda\lambda'}, \]  

(5.3)

where we adopted the well-known case with \( k \) aligned along z-direction i.e \( k \equiv k(0,0,1) \) and the polarization tensors given by

\[ e^{+} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad e^{x} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]  

(5.4)

and for canonical normalization, we introduce

\[ v_{\lambda} = \frac{aM_{pl}}{\sqrt{2}} h_{\lambda}. \]  

(5.5)

Now, we have to choose \( \hat{H}_{\text{int}} \) between the tensor perturbations and the environment of the following local form

\[ \hat{H}_{\text{int}} = \int d^{3}x A(\eta, x) \otimes E(\eta, x). \]  

(5.6)

The first obvious choice is to have linear interaction in tensor perturbations since it is expected to give the dominant contribution

\[ A(\eta, x) = h_{ij}(\eta, x), \]  

(5.7)

and in order to contract the spatial indices in \( h_{ij} \), the environment interaction operator should be of the type
\[ E (\eta, \mathbf{x}) \equiv E_{ij} (\eta, \mathbf{x}) . \]  
(5.8)

To make contact with our previous computations of the curvature perturbation case and with those in [50, 51], we take the case where indices refer to spatial derivatives so that one could write

\[ E_{ij} (\eta, \mathbf{x}) = \partial_i R_1 (\eta, \mathbf{x}) \partial_j R_2 (\eta, \mathbf{x}) . \]  
(5.9)

For example, if we consider a scalar field \( \psi \) as environment\(^{35} \) then among the possible interactions we have

\[ \mathcal{H}_{int} \propto h_{ij} \partial_i \partial_j \psi, \ h_{ij} \partial_i \psi \partial_j \psi, \ldots. \]  
(5.10)

We adopt this convention remembering that the environment states with respect to which we are taking the trace are usually considered to be the Bunch Davies vacuum [53, 71] which is homogeneous and isotropic. Using the linearity of the trace, then we can exchange between trace and derivatives in

\[ C_E (\mathbf{x}, \mathbf{y}) = \text{Tr}_E (\rho_E E (\mathbf{x}) E (\mathbf{y})) , \]  
(5.11)

so that\(^{36} \)

\[ C_{Eijmn} (\mathbf{x}, \mathbf{y}) = \partial^x_i \partial^y_j \partial^y_m \partial^y_n \text{Tr}_E (\rho_E R (\mathbf{x}) R (\mathbf{y})) = \partial^x_i \partial^y_j \partial^y_m \partial^y_n C_R (\mathbf{x}, \mathbf{y}) , \]  
(5.12)

where we will preserve the convention adopted in [50] regarding the environmental correlation function \( C_R (\mathbf{x}, \mathbf{y}) \), that is given by (2.16).

We will show in a moment that at linear order in \( h_{ij} \) the tensor power spectrum does not receive a correction due to fact that \( h_{ij} \) is transverse, i.e., \( \partial^i h_{ij} = 0 \).

Since the linear order gives vanishing correction to the tensor power spectrum, we consider a second possible choice of \( A (\eta, \mathbf{x}) \) consisting in a quadratic interaction. As the simplest example we consider\(^{37} \)

\[ A (\eta, \mathbf{x}) = h_{ij} (\eta, \mathbf{x}) h_{ij} (\eta, \mathbf{x}) . \]  
(5.13)

Therefore in this case by relabeling (5.11) we have

\[ C_E (\mathbf{x}, \mathbf{y}) \equiv C_R (\mathbf{x}, \mathbf{y}) , \]  
(5.14)

\(^{35} \)This is the case we might have mostly in mind but, still, we are not excluding an environment made of tensor fields themselves, since we can have for example \( E_{ij} (\eta, \mathbf{x}) = \partial_i \partial_j (\chi_k \chi^k) \). Also note that the scalar field environment could be provided by small-scale curvature perturbations \( \zeta \) acting on a system of (long-wavelength) tensor perturbation modes.

\(^{36} \)By \( \partial^x_i \) we mean \( \frac{\partial}{\partial x^i} \), and same for others.

\(^{37} \)Such an example, which here serves as a toy-model, can be extended to the case where (spatial) derivatives act on the tensor fields appearing in Eq. (5.13). Indeed such an interaction term (5.13) can arise, e.g., from cubic interactions between curvature perturbations \( \zeta \) and tensor modes \( h_{ij} \) after integration by parts and considering small scale curvature perturbations as the environment.
and we will see that the results will differ slightly from the curvature power spectrum case. However, it is still important to emphasize the fact that the curvature power spectrum receives corrections already for an $A(\eta, x)$ linear in the Mukhanov-Sasaki variable, while the tensor power spectrum starts receiving decoherence induced corrections only at quadratic order. This last remark implies that we expect the tensor corrections to be suppressed with respect to those of the curvature perturbation case.

5.1 Linear interaction

Considering a pure linear pointer observable, $A(\eta, x) = h_{ij}(\eta, x)$, and using our convention in (5.12), then (4.10) is given by

$$\frac{d\langle \hat{O} \rangle}{d\eta} = \left< \frac{\partial \hat{O}}{\partial \eta} \right> - i \left< \left[ \hat{O}, \hat{H}_h \right] \right> - \frac{\gamma}{2} \int d^3x d^3y \partial_i^k \partial_j^l \partial_k^p C_R(x, y) \left< \left[ \left[ \hat{O}, h_{ij}(x) \right], h_{kl}(y) \right] \right> ,$$

(5.15)

where for a tensor perturbations system $\hat{H}_h$ is given by

$$\hat{H}_h = \frac{M^2 a^2}{8} \int d^3x \left[ h_{ij}^2 + (\nabla h_{ij})^2 \right] .$$

(5.16)

As in the scalar case, we prefer to work in Fourier space so transforming this last equation using (5.3) leads to

$$\frac{d\langle \hat{O} \rangle}{d\eta} = \left< \frac{\partial \hat{O}}{\partial \eta} \right> - i \left< \left[ \hat{O}, \hat{H}_v \right] \right> - \frac{\beta^2 \gamma}{2} \frac{1}{(2\pi)^{3/2}} \sum_{\lambda\lambda'} \int d^3k d^3p_1 d^3p_2 \tilde{C}_R(|k|) \left< \left[ \left[ \hat{O}, \hat{v}_\lambda^i \hat{v}_{\lambda'}^j \right], \hat{v}_{\lambda'}^{i'} \hat{v}_{\lambda}^{j'} \right] \right> ,$$

(5.17)

where $\xi^2$ is an expansion constant that will serve later to set up the right dimensions.

5.2 Quadratic interaction

The result obtained in the previous section forces us to consider the next order in $h_{ij}(\eta, x)$, so having (5.13) and (5.14), the Lindblad equation now reads

$$\frac{d\langle \hat{O} \rangle}{d\eta} = \left< \frac{\partial \hat{O}}{\partial \eta} \right> - i \left< \left[ \hat{O}, \hat{H}_h \right] \right> - \frac{\gamma \xi^2}{2} \int d^3x d^3y C_R(x, y) \left< \left[ \left[ \hat{O}, h_{ij}(x) \right], h_{mn}(y) \right] \right> ,$$

(5.18)

where $\xi^2$ is an expansion constant that will serve later to set up the right dimensions. Fourier transforming equation (5.18) using, again, (5.3) and (5.5), yields

$$\frac{d\langle \hat{O} \rangle}{d\eta} = \left< \frac{\partial \hat{O}}{\partial \eta} \right> - i \left< \left[ \hat{O}, \hat{H}_v \right] \right> - \frac{\beta^2 \gamma}{2} \frac{1}{(2\pi)^{3/2}} \sum_{\lambda\lambda'} \int d^3k d^3p_1 d^3p_2 \tilde{C}_R(|k|) \left< \left[ \left[ \hat{O}, \hat{v}_\lambda^i \hat{v}_{\lambda'}^j \right], \hat{v}_{\lambda'}^{i'} \hat{v}_{\lambda}^{j'} \right] \right> ,$$

(5.19)

38 The indices of $h_{ij}$ are raised and lowered by $\delta_{ij}$ so we do not sharply distinguish the upper and lower indices of $h_{ij}$.
where now

\[ \hat{H}_v = \frac{1}{2} \sum_{\lambda} \int d^3k \left[ \hat{p}_{\lambda k}^2 \hat{p}_{-\lambda k}^2 + \omega^2 \hat{v}_{\lambda k}^2 \hat{v}_{-\lambda k}^2 \right] \quad \text{with} \quad \omega^2 = k^2 - \frac{a''}{a}. \quad (5.20) \]

Here we have also defined

\[ \beta = \frac{2\xi}{M_{pl}^2}, \quad (5.21) \]

being a dimensionless coupling constant so that we can preserve the definition made in (3.16) for, the dimensionless coupling, \( \sigma_\gamma \). It may seem that there is a missing factor of \( a^{-4} \) in the real part of (5.19) but actually it has been absorbed in the definition of \( \gamma \) given in (2.15). This last step was made because, on the one hand, we want to compute the decoherence corrected tensor-to-scalar perturbation ratio, \( r \), and for this purpose, as the simplest possibility, we adopt the same form of the scalar case for the various parametrizations, and on the other hand for the scalar case we considered an interaction system operator \( A(\eta, x) \) directly proportional to the Mukhanov-Sasaki variable rather than the inflaton or the metric scalar fluctuation so the \( a \) relating the two was already absorbed in the definition (3.18), see appendix B in [50] for more details.

Having derived the Lindblad equation governing the expectation values of our system observables, all what remains to do is to derive explicitly the equations obeyed by the different correlators, as was done in (4.44). However, this time we should add the polarization indices to (3.23). To this end, we use the relation \( \left[ \hat{v}_{\lambda p}, \hat{p}_{\lambda k} \right] = i\delta^{s\lambda}(p + k) \). Thus following the same steps as before we obtain the following system of equations

\[
\begin{align*}
\frac{d\langle \hat{v}_{\lambda k_1}^s \hat{p}_{\lambda k_2} \rangle}{d\eta} &= \langle \hat{v}_{\lambda k_1}^s \hat{p}_{\lambda k_2} \rangle + \langle \hat{p}_{\lambda k_1}^s \hat{v}_{\lambda k_2} \rangle \\
\frac{d\langle \hat{p}_{\lambda k_1}^s \hat{v}_{\lambda k_2} \rangle}{d\eta} &= \langle \hat{p}_{\lambda k_1}^s \hat{v}_{\lambda k_2} \rangle - \omega^2 \langle k_2 \rangle \langle \hat{v}_{\lambda k_1}^s \hat{v}_{\lambda k_2} \rangle \\
\frac{d\langle \hat{v}_{\lambda k_1}^s \hat{v}_{\lambda k_2} \rangle}{d\eta} &= \langle \hat{v}_{\lambda k_1}^s \hat{v}_{\lambda k_2} \rangle - \omega^2 \langle k_1 \rangle \langle \hat{v}_{\lambda k_1}^s \hat{v}_{\lambda k_2} \rangle \\
\frac{d\langle \hat{p}_{\lambda k_1}^s \hat{p}_{\lambda k_2} \rangle}{d\eta} &= -\omega^2 \langle k_2 \rangle \langle \hat{p}_{\lambda k_1}^s \hat{v}_{\lambda k_2} \rangle - \omega^2 \langle k_1 \rangle \langle \hat{v}_{\lambda k_1}^s \hat{p}_{\lambda k_2} \rangle + \beta^2 \frac{4\gamma}{(2\pi)^{7/2}} \int d^3k \bar{C}_R \langle |k| \rangle \langle \hat{v}_{\lambda k + k_1}^s \hat{v}_{\lambda -k + k_2}^s \rangle.
\end{align*}
\]

Apart from the polarization indices and the expansion constant (\( \beta \) vs \( \alpha \) compared to the scalar case), the previous system of equations does not differ from the scalar case.

Therefore, all what we need to do is to exchange \( \alpha \) by \( \beta \) in the solutions of the scalar case and multiply the result by 2 to account for the two possible polarization modes (+, ×). The tensor power spectrum \( P_{vv} = \sum_s \langle \hat{v}_{\lambda k_1}^s \hat{v}_{\lambda k_2}^s \rangle \) is governed by

\[ P_{vv}'' + 4\omega^2 P_{vv} + 4\omega P_{vv}' = S(k, \eta), \quad (5.23) \]
with
\[ S(k, \eta) = \beta^2 \frac{16 \gamma}{(2\pi)^{3/2}} \int d^3k' \tilde{C}_R(k') P_{vv}(|k' + k|), \] (5.24)
which, as we know, admits as solution
\[ P_{vv} = \sum_s v^s_k(\eta) v^{s*}_k(\eta) + 2 \sum_s \int d\eta' S(k, \eta') \Im \left[ v^s_k(\eta') v^{s*}_k(\eta) \right]. \] (5.25)
The first term is the standard result while the second term is the correction induced by the interaction with the environment. Since we are interested in the power spectrum of tensor perturbations $h$ and not the variable $v$, and in order to facilitate the task of computing the corrected tensor-to-scalar ratio $r$, then, as was done for the scalar perturbations in [50], we adopt the following notation for the total dimensionless tensor power spectrum $P_T$:
\[ P_T = \frac{k^3}{2\pi^2} \frac{2P_{vv}}{M_{pl}^2 a^2} = P_T^{\text{standard}} [1 + \Delta P_T], \] (5.26)
with
\[ P_T^{\text{standard}} = \frac{8}{M_{pl}^2} \left( \frac{H}{2\pi} \right)^2 \left( \frac{k}{aH} \right)^{-2\epsilon} \approx \frac{8}{M_{pl}^2} \left( \frac{H}{2\pi} \right)^2, \] (5.27)
and
\[ \Delta P_T = \frac{2}{\sum_s} \int_{-\infty}^{\eta} d\eta' S(k, \eta') \Im \left[ v^s_k(\eta') v^{s*}_k(\eta) \right]. \] (5.28)
The computation of the integral in (5.25) was already done in [50] for curvature perturbations, so we need just to make the modifications already mentioned above to get the results corresponding to the tensor case. We will report the expressions for $\Delta P_T$ as a function of the parameter $p$ defined in (2.15). We have three regimes in addition to two singular cases,

- for $p > 6$

\[ \Delta P_T \approx \frac{8\beta^2 \sigma_\gamma}{27\pi} \left( \frac{k_{fs}}{k_\ast} \right)^3 \left( \frac{\eta}{\eta_\ast} \right)^{6-p} \left[ \frac{1}{p^2} - \frac{2}{(p-3)^2} + \frac{1}{(p-6)^2} + \frac{18}{p(p-3)(p-6)} \ln \left( \frac{\eta_{IR}}{\eta} \right) \right], \] (5.29)

where the expression of $\sigma_\gamma$ is given by (3.16), $(-1/\eta_{IR})$ is an IR cutoff in the integral (5.24) and $k_\ast$ refers to a pivot scale; $\ln (\eta_{IR}/\eta) = N - N_{IR}$ gives the number of $\epsilon$-folds elapsed since the beginning of inflation. We notice that in this regime the power spectrum correction scales as $k^3$, in addition to be not frozen on large scales and continues to increase, leading to a very large enhancement of the correction compared to the standard power spectrum at late time. This behavior of the tensor power spectrum correction corresponds to the analogous case of the curvature case in [50].

---

39By total we mean the standard result plus the correction induced by decoherence.
For $2 < p < 6$

$$
\Delta P_T \simeq \frac{2^{p-1}(p-4)}{3p^2 (p-1) \sin (\pi p/2)} \beta^2 \sigma_\gamma \left( \frac{k}{k_*} \right)^{p-3} \left[ \ln \left( \frac{\eta_{IR}}{\eta_*} \right) + \frac{1}{p-3} - \frac{2(p-1)}{p(p-2)} \right]
$$

$$
- \frac{\pi}{2} \cot \left( \frac{\pi p}{2} \right) + \ln (2) - \psi (p-2) + \ln \left( \frac{k}{k_*} \right) \right],
$$

(5.30)

where here $\psi$ is the digamma function. In this case we obtain a scale invariant correction for $p = 3$ which was also the case for the correction to the curvature power spectrum coming from the quadratic interaction, i.e $P_{vv}^{(2)} + P_{vv}^{(3)}$ in (4.50). However let us remember that, in contrast to the tensor case, the dominant correction to the curvature power spectrum comes from the linear interaction, providing a scale invariant result for $p = 5$. Therefore at leading order, the scale invariant corrections to the scalar and tensor power spectra correspond to different values of $p$. In particular notice that, in the cases where the parameter $p$ would be the same for the scalar and tensor cases, then by imposing a scale invariant scalar power spectrum, implies a strong blue-tilted tensor power spectrum.

For $p < 2$

$$
\Delta P_T \simeq \frac{4 (H_s l_E)^{p-2}}{3 \pi (2-p)} \beta^2 \sigma_\gamma \left( \frac{k}{k_*} \right)^{p-3} \left[ \frac{1}{2-p} + N_* - N_{IR} + \ln (H_s l_E) + \ln \left( \frac{k}{k_*} \right) \right].
$$

Notice that in this case the decoherence induced corrections to the power spectrum become negligible on very small scales.

For $p = 2$ and $p = 6$ which are singular we find

$$
\Delta P_T \mid_{p=2} \simeq \frac{1}{18 \pi} \beta^2 \sigma_\gamma \left( \frac{k}{k_*} \right)^{-1} \left[ 12 - \pi^2 + 12C (2+C) - 12 \ln^2 (H_* l_E) + 24 \left[ C + 1 - \ln (H_* l_E) \right] \right]
$$

$$
\times \left[ N_* - N_{IR} + \ln \left( \frac{k}{k_*} \right) \right]
$$

$$
\Delta P_T \mid_{p=6} \simeq \frac{1}{16 \pi} \beta^2 \sigma_\gamma \left( \frac{k}{k_*} \right)^3 \left[ 2 \pi^2 - 21 - 12C (1 + 2C) - 12 \left( 3 + 4C \right) (N - N_{IR}) \right]
$$

$$
+ 12 (1 + 4C) (N - N_* + 24 (N - N_*) \left[ 2 (N - N_{IR}) - (N - N_*) \right]
$$

$$
- 12 \ln \left( \frac{k}{k_*} \right) \left[ 1 + 4 \left( C + N_* - N_{IR} \right) 2 \ln \left( \frac{k}{k_*} \right) \right]
$$

(5.32)

where $C$ is a constant.
5.3 Decoherence induced Corrections to the tensor-to-scalar perturbation ratio $r$

In Section 3, we presented the computations made in [50] to get the decoherence induced corrections to $n_s$ and $r$. However, in their computations of $r$ they assumed the tensor power spectrum $P_T$ to remain unaffected by the environment. However, as we showed with the computations we just carried out in the previous section, the tensor sector can be affected by decoherence induced contributions, which therefore should be accounted for. Therefore, our aim now is to give the decoherence corrected expression of $r$ taking into account our results regarding the effect of environment on tensor modes.

Once we come to compute the corrected $r$, the scale independence of the leading decoherence induced corrections to the curvature and tensor power spectra correspond to different values of $p$, namely $p = 5$ and $p = 3$, respectively. However, it is important to remember that curvature corrections are dominant with respect to tensor ones, since the latter receive their first non vanishing correction only at quadratic level. Therefore, if we restrict ourselves to linear interactions, then, the decoherence corrected $r$ is still given by the expression computed by J.Martin et al in [50]

$$r = r|_{standard} \frac{1}{1 + \frac{\pi k^2}{6 k_f^2}}. \quad (5.33)$$

However if we consider the leading corrections of both power spectra, curvature and tensor, regardless of their order, then, the previous equation will be modified. Since curvature power spectrum has been well confirmed to be quasi scale independent, at least up to certain sensitivity, then we choose the value for which its correction is also scale independent, namely $p = 5$ (see in any case the comment in footnote 29). This will lead to a tensor-to-scalar perturbation ratio $r$ which in full generality will be given by

$$r = r|_{standard} \frac{1}{1 + \frac{\pi k^2}{6 k_f^2}} \left[1 + \Delta P_T\right]_{k=k_\ast}, \quad (5.34)$$

where one should consider the various cases for $\Delta P_T$ obtained above for the tensor perturbations in Eqs. (5.29), (5.30), (5.31) and (5.32). For example, at least in the cases where the parameter $p$ for the tensor sector is taken to be the same as for the scalar power-spectra, i.e. $p = 5$, then the tensor power spectrum correction is blue titled with a spectral index $n_T \approx 2$, as seen from (5.30), and is given up to leading order by$^{40}$

$$\Delta P_T \approx \frac{2}{45} \beta^2 \sigma_\gamma \left(\frac{k}{k_\ast}\right)^2 \left[\ln \left(\frac{\eta_{IR}}{\eta_\ast}\right) + \ln \left(\frac{k}{k_\ast}\right)\right]. \quad (5.35)$$

Substituting (5.35) in (5.26) yields

$$P_T \approx P_T|_{standard} \left[1 + \frac{2}{45} \beta^2 \sigma_\gamma \left(\frac{k}{k_\ast}\right)^2 \left[\ln \left(\frac{\eta_{IR}}{\eta_\ast}\right) + \ln \left(\frac{k}{k_\ast}\right)\right]\right]. \quad (5.36)$$

$^{40}$By leading order we mean to neglect the constant numbers that are much smaller than $\ln \left(\frac{\eta_{IR}}{\eta_\ast}\right)$. 

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So combining (5.34) with (5.36) in this specific case leads to a decoherence corrected 
\( r \), evaluated at the pivot scale i.e \( k = k_* \), given by

\[
r \simeq r \left|_{\text{standard}} \right. \frac{1 + \frac{2}{45} \beta^2 \sigma_\gamma (N_* - N_{IR})}{1 + \frac{6 k_*^2}{5 k^2}}.
\] (5.37)

Notice that in principle \( (N_* - N_{IR}) = \ln(a_* / a_i) \) (where \( a_* \) is the scale-factor when the reference mode \( k_* = 0.05 \) Mpc\(^{-1}\) crosses the horizon during inflation and \( a_i \) is the scale factor at the beginning of inflation) can be of the order of \( 10^4 \), even if this is model dependent. Therefore, even very low values of the coupling-parameter \( \beta^2 \sigma_\gamma \) between the tensor fluctuation system and the environment, can indeed lead to a non-negligible correction to the tensor-to-scalar perturbation ratio. For example, given that the interaction between tensor and environment considered here resembles the quadratic interaction case of [50], and indeed we get similar results, as a first estimate one could use their constraints. So, choosing \( p = 5 \) to be consistent with (almost) scale-invariance of the scalar perturbations, such value of \( p \) then leads to an upper bound \( \beta^2 \sigma_\gamma < 10^{-5} \) from their results in the quadratic interactions (see, e.g. their Fig. 8) which still, as discussed above, leads in any case to observable consequences in the tensor-to-scalar perturbation ratio.

A high precise measurement of the tensor power spectrum \( P_T \) and the tensor-to-scalar perturbation ratio \( r \) will provide tight constraints on the interaction strength, encoded in \( \beta^2 \sigma_\gamma \), between the primordial tensor fluctuations and their environment. Those constraints can be compared or used in combination with those obtained from the scalar correlation functions, namely, power spectrum, bispectrum and trispectrum.

6 Conclusions

In this paper we addressed the question of the quantum to classical transition in the early universe. In particular, we aimed at the identification of observational connections between the quantum initial state and the classical universe we observe today through the completion of the inflation formalism with a model that accounts for the quantum to classical transition of the primordial quantum fluctuations. A well motivated model for such a goal is quantum decoherence, where many attempts have been made to apply it on large cosmological scales yielding the so called cosmic decoherence. Several works in this direction were carried out [17, 25, 34, 35, 46, 50, 51, 86], which inspired the work done in this paper. In particular, our work builds up, extends, and to some extent completes, works done in [50, 51, 71].

We started by presenting briefly the derivation of the Lindblad equation which is the corner stone in studying the quantum to classical transition of an open system. Then, for the sake of smoothing the flow of ideas, we summarized in Section 3 the main results of J.Martin et al. in [50, 51], based on which we built our model. In particular we discussed the approach adopted by them in studying the corrections induced by cosmic decoherence to the various curvature correlation functions. Confronting the corrections obtained with cosmological data, they were able to constrain the set of the possible environments which could decohere the primordial fluctuations. Since cosmological data has well confirmed
the quasi scale independence of curvature power spectrum, a particular and quite remark-
able environment was a massive scalar field which induces scale independent corrections
to the curvature power spectrum and trispectrum. Another remarkable result obtained
by J.Martin et al. was computing the correction induced by decoherence into the scalar
spectral index $n_s$ and the tensor-to-scalar perturbation ratio $r$, assuming the tensor per-
turbations to remain unaffected. They showed how the corrected $(n_s, r)$ affect the status
of various inflation models when compared to cosmological data. An intriguing result was
also obtained by J.Martin et al., where they showed that decoherence induces vanishing
bispectrum along with non vanishing trispectrum. Therefore, decoherence was stated as one
of the rare examples where the bispectrum is perturbatively suppressed compared to the
trispectrum, which therefore would contain the relevant non-Gaussian signal.

This last result was one of the reasons that sparked the work done in this paper,
where we realized that its validity is not as general as it may seems. Indeed, adopting a
different approach than that of [50, 51] we were able to obtain interesting results which
we summarize as follow:

1. First of all, through (4.1) we were able not only to reproduce all the results obtained
   in [50, 51], as can be seen using (4.12), but we also showed in 4.2 that there were
   missing terms in the power spectrum and trispectrum due to the restricted choice of
   the form of the pointer observable. Therefore, through the pointer observable (4.1)
   we have chosen, we generalized the model initiated by J.Martin et al in [50, 51]. In
   addition, we motivated the form of such a pointer observable by a physical process
   that could produce it. It turned out to be, again, a generalization of the physical
   process considered by J.Martin et al with many new implications.

2. We showed in 4.1 that decoherence does induce a non vanishing bispectrum. More-
   over, it is dominant with respect to the trispectrum. Indeed the highest the order of
correlation function is, then the highest is the order of $\alpha$ at which it receives its first
non vanishing contributions. In particular, we saw in the first part of our work that
the first non vanishing correction to power spectrum, bispectrum, and trispectrum
was found at the order $\alpha^0$, $\alpha^1$, and $\alpha^2$, respectively. Another interesting remark
consists on the possibility of the bispectrum induced by decoherence to be dominant
with respect to the one arising in standard single field inflation.

3. Generally the decoherence induced bispectrum can have a specific scale dependence.
On the other hand, similarly to the power spectrum and trispectrum, we showed
that the bispectrum could be, at leading order, scale independent in the case of a
massive scalar field as environment. In this respect we conjectured that the scale
independence of the corrections induced by cosmic decoherence is governed by a cri-
terion which surpasses the type of environment. As an attempt to find such criterion,
we conjectured that obtaining a scale independent corrections of a given correlation
function is related to the order in the expansion parameter $\alpha$ at which the correction
arises, independently from any choice of the environment including the massive scalar
field.
4. Another quite interesting result, that we derived in this paper, consisted in computing cosmic decoherence effects on primordial tensor perturbations. We have shown in 5.1 that by considering a linear interaction, decoherence does not affect tensor fluctuations due to the transversality property of tensor modes. Therefore, we considered the next order and studied a quadratic interaction which indeed modifies the tensor power spectrum. In its turn this implies that the tensor-to-scalar perturbation ratio $r$ computed in [50] is modified, see (5.33) and, e.g., (5.37). By doing so, we saw that scalar and tensor power spectra corrections are scale independent for different values of the free parameter(s) $p$. So, e.g., in the case where the parameter $p$ is the same for scalar and tensor perturbations, by adopting the value $p = 5$ that makes (the dominant) scalar correction scale independent, leads to a strongly blue tilted tensor correction. At the same time, interestingly enough, for the same value of $p = 5$, the curvature bispectrum turns out to be also linearly scale dependent.

The work done so far about cosmic decoherence shows that the question of quantum to classical transition in the early universe is not a mere foundational question, but its answer could contain important observational signatures which could, in their turn, bring new insights into our current standard predictions.

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**A Computation of the non linearity parameter $f_{NL}$**

In this appendix we give more details on the computation of the non linear term $f_{NL}$. Before doing so, let us remind some definitions which will be needed to evaluate (4.25). First, we will put ourselves in the de-Sitter limit, thus, the scale factor is given by $a = \frac{1}{(H\eta)}$ and the MS variable , $v_k(\eta)$, by

$$v_k(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}} \left(1 - \frac{i}{k\eta}\right), \quad (A.1)$$
which by turn yields
\[
\Im^{3} \left[ v_{k} (\eta') v_{k}^* (\eta) \right] = \frac{\cos(k(\eta-\eta'))}{2k} \left( \frac{1}{k\eta} - \frac{1}{k\eta'} \right) + \frac{\sin(k(\eta-\eta'))}{2k} \left( 1 + \frac{1}{k^2 \eta \eta'} \right)
\]
\[
\langle \hat{v}_{k} \hat{v}^{*} \rangle_{k} = \frac{1}{2k} \left[ 1 + \frac{1}{(-k\eta)^2} \right]. \quad (A.2)
\]
\[
\Re \langle \hat{p}_{k} \hat{v}^{*} \rangle_{k} = \frac{1}{2k(-k\eta)^2}
\]

For convenience, we remind also
\[
\tilde{C}_{R} (k) = \sqrt{\frac{2}{\pi}} \bar{C}_{R} \frac{l_{E}}{3a^3} \Theta \left( \frac{k l_{E}}{a} \right), \quad (A.3)
\]
so using \( k_* = a_* H_* \) in addition to the fact that \( H \simeq H_* \simeq constant \), then, (4.25) could be written as
\[
B_{vvv} = B_{vvv}^{(1)} + B_{vvv}^{(2)} + B_{vvv}^{(3)}, \quad (A.4)
\]
which \(^{41}\) by adopting the new variable \( z \) defined as \( z = k\eta' \) leads to
\[
B_{vvv}^{(1)} = 7\alpha \frac{k^2}{k_*^2} \left( \frac{k}{k_*} \right)^{p-7} \int_{-\infty}^{k\eta} dz \left\{ \Theta (-z H l_{E}) (-z)^{-p} \right. \\
\times \left[ \cos (k\eta - z) \left( \frac{1}{k\eta} - \frac{1}{z} \right) + \sin (k\eta - z) \left( 1 + \frac{1}{k\eta z} \right) \right] \right\}^3.
\]
\[
B_{vvv}^{(2)} = \alpha \frac{k^2}{k_*^2} \left( \frac{k}{k_*} \right)^{p-7} H l_{E} \int_{-\infty}^{k\eta} dz \left\{ \delta (1 + z H l_{E}) (-z)^{-3-p} \left( 1 + \frac{1}{(-z)^2} \right) \right. \\
\times \left[ \cos (k\eta - z) \left( \frac{1}{k\eta} - \frac{1}{z} \right) + \sin (k\eta - z) \left( 1 + \frac{1}{k\eta z} \right) \right] \right\}^3, \quad (A.5)
\]
\[
B_{vvv}^{(3)} = \alpha \frac{k^2}{k_*^2} \left( \frac{k}{k_*} \right)^{p-7} (p-3) \int_{-\infty}^{k\eta} dz \left\{ \Theta (-z H l_{E}) (-z)^{2-p} \left( 1 + \frac{1}{(-z)^2} \right) \right. \\
\times \left[ \cos (k\eta - z) \left( \frac{1}{k\eta} - \frac{1}{z} \right) + \sin (k\eta - z) \left( 1 + \frac{1}{k\eta z} \right) \right] \right\}^3.
\]

Notice that the presence of \( \Theta (-z H l_{E}) \) in the first and third integral sets a lower bound on our variable \( z \), where \( z_{\text{min}} = - (H l_{E})^{-1} \), and this ensures that those integrals are finite, it is also obvious that the presences of delta function in second integral is due to the derivative

\(^{41}\) Notice that the in \( B_{vvv}^{(2)} \), we have \( \delta (1 + z H l_{E}) \) instead of \( \delta (z H l_{E}) \) and this is due to our definition of top hat function as being non zero if \( - z H l_{E} < 1 \), therefore we shifted the argument and used the definition of the derivative of Heaviside function.
of Heaviside function. The above integrals are not exactly analytically computable, but the fact that we are interested on super Horizon modes $-k\eta \to 0$ will help to simplify them, the behavior of the correlators can be obtained by identifying the region in the integration domain from where the integral receives its main contribution.

1. For $B_{vvv}^{(1)}$ we see that for $42 > p > 0$ the main contribution is always coming from the upper bound where $-k\eta \ll 1$ and $-z \ll 1$ (with $H_E \ll 1$) thus we can expand the integrand in this limit and $B_{vvv}^{(1)}$ becomes

$$B_{vvv}^{(1)} = -7 \alpha k^2 \left( \frac{k}{k_*} \right)^{p-7} \int_{-(H_E)^{-1}}^{k\eta} dz \left( \frac{z-k\eta}{z} \right)^3,$$  

(A.6)

For $p \neq 1, 2, 3, 4$, which are singular cases to be computed apart, the above integral gives for $4 > p$

$$B_{vvv}^{(1)} = -7 \alpha k^2 \left( \frac{k}{k_*} \right)^{p-7} (-k\eta)^{4-p} \left[ \frac{1}{p-1} - \frac{3}{p-3} + \frac{3}{p-2} - \frac{1}{p-1} \right],$$  

(A.7)

- For $p = 1$, (A.6) gives

$$B_{vvv}^{(1)} = 7 \alpha k^2 \left( \frac{k}{k_*} \right)^{-6} (k\eta)^3 \left( \ln (|k\eta|) - \frac{11}{6} \right),$$  

(A.8)

- For $p = 2$, (A.6) gives

$$B_{vvv}^{(1)} = 21 \alpha k^2 \left( \frac{k}{k_*} \right)^{-5} (k\eta)^2 \left( \ln (|k\eta|) - \frac{1}{2} \right),$$  

(A.9)

- For $p = 3$, (A.6) gives

$$B_{vvv}^{(1)} = 21 \alpha k^2 \left( \frac{k}{k_*} \right)^{-4} (k\eta) \left( \ln (|k\eta|) + \frac{1}{2} \right),$$  

(A.10)

- For $p = 4$, (A.6) gives

$$B_{vvv}^{(1)} = 7 \alpha k^2 \left( \frac{k}{k_*} \right)^{-3} \left( \ln (|k\eta|) + \frac{11}{6} \right),$$  

(A.11)

Supposing $p$ positive is due the fact that classicalization of our perturbations becomes more efficient as our modes exit horizon, which is equivalent to saying that decoherence becomes more efficient as the coupling system-environment increases.
We see that in the singular cases and in super horizon limit the first term $\propto \ln (|k\eta|)$ gives the largest contribution, we notice also that in all cases, singular and non-singular, $B_{vvv}^{(1)} \propto k^{-3}$.

2- For $B_{vvv}^{(2)}$ the delta function renders the integration task simple where considering only the dominant terms for in the limit $-k\eta \ll 1$ and $(H_lE)^{-1} \gg 1$ we get

$$B_{vvv}^{(2)} = \alpha \frac{k^2}{k_s^2} \left( \frac{k}{k_s} \right)^{p-7} (H_lE)^{-p-2} \cos^3 (H_lE)^{-1}, \quad (A.12)$$

3- For $B_{vvv}^{(3)}$ in the case $p > 2$ and we find ourselves again in the limit where the main contribution comes from the upper bound thus for $-k\eta \ll 1, -z \ll 1$ $B_{vvv}^{(3)}$ could be written as

$$B_{vvv}^{(3)} = -\alpha \frac{k^2}{k_s^2} \left( \frac{k}{k_s} \right)^{p-7} (p-3) \int_{-(H_lE)^{-1}}^{k\eta} dz \frac{(z - k\eta)^3 (z^2 + 1)}{(-z)^p}, \quad (A.13)$$

which gives

$$B_{vvv}^{(3)} = -\alpha \frac{k^2}{k_s^2} \left( \frac{k}{k_s} \right)^{p-7} (p-3) \left\{ (k\eta)^6 - \frac{1}{p-6} \left[ \frac{3}{p-5} - \frac{3}{p-4} - \frac{1}{p-3} \right] \right\}, \quad (A.14)$$

simplifying this last we get

$$B_{vvv}^{(3)} = -6\alpha \frac{k^2}{k_s^2} \left( \frac{k}{k_s} \right)^{p-7} \left\{ (k\eta)^{6-p} \frac{1}{(p-6)(p-5)(p-4)} + (k\eta)^{4-p} \frac{1}{(p-4)(p-2)(p-1)} \right\}, \quad (A.15)$$

as in the previous case we got some singular cases which need to be computed apart.

• For $p = 4$ (A.13) gives

$$B_{vvv}^{(3)} = -\alpha \frac{k^2}{k_s^2} \left( \frac{k}{k_s} \right)^{p-7} \int_{-(H_lE)^{-1}}^{k\eta} dz \frac{(z - k\eta)^3 (z^2 + 1)}{z^4} = -\alpha \frac{k^2}{k_s^2} \left( \frac{k}{k_s} \right)^{p-7} \left\{ (k\eta)^2 \left( 3 \ln (|k\eta|) - \frac{5}{2} \right) + \ln (|k\eta|) - \frac{17}{6} \right\}, \quad (A.16)$$
• For $p = 5$ (A.13) gives
\[ B_{vuv}^{(3)} = 2\alpha k^2 \left( \frac{k}{k^*} \right)^{-3} \int_{-(H_{1E})^{-1}}^{k\eta} \frac{(z - k\eta)^3 (z^2 + 1)}{z^5} \, dz, \]  \hspace{1cm} \text{(A.17)}
\[ = -2\alpha k^2 \left( \frac{k}{k^*} \right)^{-3} \left[ 3 (k\eta) \left( \frac{7}{2} + \ln(|k\eta|) \right) + \frac{1}{4k^*} \right] \]

• For $p = 6$ (A.13) gives
\[ B_{vuv}^{(3)} = -3\alpha k^2 \left( \frac{k}{k^*} \right)^{-3} \int_{-(H_{1E})^{-1}}^{k\eta} \frac{(z - k\eta)^3 (z^2 + 1)}{z^6} \, dz, \]  \hspace{1cm} \text{(A.18)}
\[ = 3\alpha k^2 \left( \frac{k}{k^*} \right)^{-3} \left[ \ln(|k\eta|) + \frac{3}{2} + \frac{17}{60} \frac{1}{(k\eta)^2} \right] \]

Now we turn to the cases $p = 2, 1$

$p = 2$ given by
\[ B_{vuv}^{(3)} = -\alpha k^2 \left( \frac{k}{k^*} \right)^{-5} \int_{-(H_{1E})^{-1}}^{k\eta} \frac{(z - k\eta)^3}{z^2} \, dz \left\{ \left( 1 + \frac{1}{z^2} \right) \right\} \times \left[ \cos(k\eta - z) \left( \frac{1}{k\eta} - \frac{1}{z} \right) + \sin(k\eta - z) \left( 1 + \frac{1}{zk\eta} \right) \right]^3, \]  \hspace{1cm} \text{(A.19)}
we see two contributions to the above integral $B_{vuv}^{(3)} = B_{vuv}^{(3)} \big|_1 + B_{vuv}^{(3)} \big|_2$, the first is dominated by the upper bound therefore expanding the integrand again around $-k\eta \ll 1, -z \ll 1$ leads to
\[ B_{vuv}^{(3)} \big|_1 = \alpha k^2 \left( \frac{k}{k^*} \right)^{-5} \int_{-(H_{1E})^{-1}}^{k\eta} \frac{(z - k\eta)^3}{z^2} \, dz \left( 1 + \frac{1}{z^2} \right) \]  \hspace{1cm} \text{(A.20)}
\[ = -3\alpha k^2 \left( \frac{k}{k^*} \right)^{-5} (k\eta)^2 \left[ \ln(|k\eta|) - \frac{1}{2} \right]. \]
second contribution is given by
\[ B_{vuv}^{(3)} \big|_2 = \alpha k^2 \left( \frac{k}{k^*} \right)^{-5} \int_{-(H_{1E})^{-1}}^{k\eta} \frac{(z - k\eta)^3}{z^2} \, dz \left( \cos(z - k\eta) \left( 1 + \frac{1}{z^2} \right) - \sin(z - k\eta) \left( 1 + \frac{1}{zk\eta} \right) \right)^3, \]  \hspace{1cm} \text{(A.21)}
this last could be computed exactly, and since the antiderivative is too involved we will pick only the leading terms
\[ B_{vuv}^{(3)} \big|_2 = \alpha k^2 \left( \frac{k}{k^*} \right)^{-5} \left[ \frac{3}{2} \frac{Si(-(H_{1E})^{-1})^{(1)}}{(k\eta)^4} - \frac{11}{16} C_i(-k\eta) \right], \]  \hspace{1cm} \text{(A.22)}
where \(Ci (x)\) and \(Si (x)\) are the CosIntegral and SinIntegral, respectively. Thus we see that in the limit \(k\eta \to 0\) the \(B^{(3)}_{vvv} |_1\) is subdominant with respect to \(B^{(3)}_{vvv} |_2\), therefore we may safely conclude that

\[
B^{(3)}_{vvv} = \alpha \frac{k^2}{k^2} \left( \frac{k}{\kappa} \right)^{-5} \left[ \frac{3}{4} \frac{Si(-H_E^{-1})}{(k\eta)^3} - \frac{11}{16} \frac{Ci(-k\eta)}{(k\eta)^3} \right].
\]

Finally we compute \(B^{(3)}_{vvv}\) for \(p = 1\) which is given by

\[
B^{(3)}_{vvv} = 2\alpha \frac{k^2}{k^2} \left( \frac{k}{\kappa} \right)^{-6} \int_{-H_E^{-1}}^{k\eta} dz \left( \frac{z - \frac{1}{z}}{z^3} \right) \\
\times \left[ \cos (k\eta - z) \left( \frac{1}{k\eta} - \frac{1}{z} \right) + \sin (k\eta - z) \left( 1 + \frac{1}{k\eta z} \right) \right]^3,
\]

the first integral could be computed exactly, but again for simplicity we write down the leading terms

\[
\int_{-H_E^{-1}}^{k\eta} dz \left( \frac{z - \frac{1}{z}}{z^3} \right) \left[ \cos (k\eta - z) \left( \frac{1}{k\eta} - \frac{1}{z} \right) + \sin (k\eta - z) \left( 1 + \frac{1}{k\eta z} \right) \right]^3 \\
\approx \frac{16}{9} \frac{\cos (H_E^{-1})}{(k\eta)^3} - \frac{5}{6} \frac{(H_E^{-1})^2 \sin (H_E^{-1})^{-1}}{(k\eta)^3},
\]

the second integral receives its main contribution from the upper bound thus as usual expanding the integrand in the limit \(-k\eta \ll 1, -z \ll 1\) we end up with

\[
\int_{-H_E^{-1}}^{k\eta} dz \left( \frac{z - a}{z^3} \right) \\
\approx (k\eta)^3 \left[ \frac{5}{3} \ln \left( \frac{2}{|k\eta|} \right) \right],
\]

so in total, and considering only the leading term in (A.26), namely the one \(\propto \ln (|k\eta|)\), which becomes now subdominant it the total \(B^{(3)}_{vvv}\)

\[
B^{(3)}_{vvv} = 2\alpha \frac{k^2}{k^2} \left( \frac{k}{\kappa} \right)^{-6} \left[ \frac{16}{9} \frac{\cos (H_E^{-1})}{(k\eta)^3} - \frac{5}{6} \frac{(H_E^{-1})^2 \sin (H_E^{-1})^{-1}}{(k\eta)^3} - (k\eta)^3 \ln (|k\eta|) \right],
\]

Having finished the computations of of the different \(B^{(i)}_{vvv}\), we substitute the results found for the different values of \(p\) in (4.27) in order to obtain the decoherence induced\(^{43}\) \(f_{NL}\).

\(^{43}\)We are using in the following expressions of \(f_{NL}\) the dimensionless curvature power spectrum \(P_\zeta = H^2 / (8\pi^2 M_{pl}^2) \simeq 2.2 \times 10^{-9}\)
• $p = 1$, in this case the contribution coming from $B_{vvv}^{(1)}$ is negligible compared to the other two

\[
f_{NL} = \frac{5}{9\pi\sqrt{P_\zeta}} \frac{\alpha k^2}{k_*} \left( \frac{k}{k_*} \right)^2 \left[ \frac{5}{3} (Hl_E)^{-1} \sin (Hl_E)^{-1} \left( \frac{k}{k_*} \right)^{-3} e^{3(N-N_*)} - \frac{32}{9} \cos (Hl_E)^{-1} \left( \frac{k}{k_*} \right)^{-3} e^{(N-N_*)} \right]
\]  

(A.28)

• $p = 2$, in this case the contribution coming from $B_{vvv}^{(1)}$ is negligible compared to the other two

\[
f_{NL} = \frac{5}{9\pi\sqrt{P_\zeta}} \frac{\alpha k^2}{k_*} \left( \frac{k}{k_*} \right)^-2 \left[ \cos^3 (Hl_E)^{-1} + \frac{3}{4} Si \left( - (Hl_E)^{-1} \right) - \frac{11}{16} Ci \left( \frac{k}{k_*} e^{-(N-N_*)} \right) \right]
\]  

(A.29)

• $3 \leq p \leq 5$, in this case the contribution coming from both $B_{vvv}^{(1)}$ and $B_{vvv}^{(3)}$ is negligible compared to the $B_{vvv}^{(2)}$, thus we get

\[
f_{NL} = \frac{5}{9\pi\sqrt{P_\zeta}} \frac{\alpha k^2}{k_*} \left( \frac{k}{k_*} \right)^{p-4} (Hl_E)^{p-2} \cos^3 (Hl_E)^{-1}
\]  

(A.30)

• $p = 6$

\[
f_{NL} = \frac{5}{9\pi\sqrt{P_\zeta}} \frac{\alpha k^2}{k_*} \left[ \left( \frac{k}{k_*} \right)^2 (Hl_E)^4 \cos^3 (Hl_E)^{-1} + \frac{49}{30} \left( \frac{k}{k_*} \right)^3 e^{-(N-N_*)} - \frac{17}{60} \left( \frac{k}{k_*} \right)^3 e^{-(N-N_*)} - 3 \left( \frac{k}{k_*} \right)^3 e^{-3(N-N_*)} \right]
\]  

(A.31)

• for $p > 6$, taking into account all the terms, including the subleading, we get

\[\text{We are using in the following expressions } k\eta = \frac{k}{k_*} \eta = - \frac{k}{k_*} e^{-(N-N_*)}\]
\[ f_{NL} = \frac{5}{9\pi \sqrt{p}} \frac{\alpha k^2}{k_s} \left( \frac{k}{k_s} \right)^3 \left( H(E) \right)^{p-2} \frac{\left( k \right)^{p-7}}{\left( k_s \right)^{p-7}} \cos^2 \left( H(E) \right)^{-1} - 7 \frac{(3p - 4)}{(p - 4)(p - 3)(p - 2)(p - 1)} \]

\[ -6 \left( \frac{k}{k_s} \right)^2 \frac{e^{-(9-p)(N-N_*)}}{(p - 6)(p - 5)(p - 4)} \cdot 6 \frac{e^{-(7-p)(N-N_*)}}{(p - 6)(p - 5)(p - 4)} \]

(A.32)

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