Magnetic Phases in Three-Flavor Color Superconductivity

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Abstract

The best natural candidates for the realization of color superconductivity are quark stars -not yet confirmed by observation- and the extremely dense cores of compact stars, many of which have very large magnetic fields. To reliably predict astrophysical signatures of color superconductivity, a better understanding of the role of the star’s magnetic field in the color superconducting phase that realizes in the core is required. This paper is an initial step in that direction. The field scales at which the different magnetic phases of a color superconductor with three quark flavors can be realized are investigated. Coming from weak to strong fields, the system undergoes first a symmetry transmutation from a Color-Flavor-Locked (CFL) phase to a Magnetic-CFL (MCFL) phase, and then a phase transition from the MCFL phase to the Paramagnetic-CFL (PCFL) phase. The low-energy effective theory for the excitations of the diquark condensate in the presence of a magnetic field is derived using a covariant representation that takes into account all the Lorentz structures contributing at low energy. The field-induced masses of the charged mesons and the threshold field at which the CFL $\to$ MCFL symmetry transmutation occurs are obtained in the framework of this low-energy effective theory. The relevance of the different magnetic phases for the physics of compact stars is discussed.

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I. INTRODUCTION

At present the physics community is actively trying to find ways to differentiate a neutron star made up entirely of nuclear matter from one with a color superconducting quark matter core. Compact stars typically have very large magnetic fields. Hence any predicted signature of a color superconducting core should take into account the presence of the star’s magnetic field and its effects in the superconducting state. Given that magnetars can have surface fields as large as $10^{14} - 10^{16}$ G, it is reasonable to expect that the star’s interior fields can reach even higher values. Maximum strengths of $10^{18} - 10^{19}$ G are allowed by a simple application of the virial theorem.

Although a color superconductor (CS) is in principle an electric superconductor, because the diquark condensate carries nonzero electric charge, in the color-flavor-locked (CFL) phase (CFL is the color superconducting phase that realizes in a system of three-flavor massless quarks at high densities) there is no Meissner effect for a new in-medium electromagnetic field $\tilde{A}_\mu$. This in-medium electromagnetic field -called in the literature a ”rotated” electromagnetic field, where the ”rotation” takes place in an inner space- is a combination of the regular electromagnetic field and the $8^{th}$ gluon. As the quark pairs are all neutral with respect to the ”rotated” electromagnetic charge $\tilde{Q}$, the ”rotated” electromagnetic field $\tilde{A}_\mu$ remains long-range within the superconductor.

In this paper we are interested in the color-superconducting magnetic phases that realize in a very dense system of three-flavor massless quarks interacting in the background of a rotated magnetic field $\tilde{B}$. As shown in Ref, the color-superconducting properties of such a system are substantially affected by the penetrating $\tilde{B}$ field and as a consequence, a new phase, called Magnetic Color Flavor Locked (MCFL) phase, takes place. In the MCFL phase the pairing of (rotated) electrically charged quarks is reinforced by the field. Pairs of this kind have bounding energies which depend on the magnetic-field strength and are bigger than the ones existing at zero field. At field strengths of the order of the baryon chemical potential, the pairing reinforcement is sufficient to produce a distinguishable splitting of the gap in two pieces: one that only gets contributions from pairs of neutral quarks and one that gets contributions from both pairs of neutral and pairs of charged quarks.

Although the symmetry breaking patterns of the MCFL and CFL phases are different, the two phases are hardly distinguishable at weak magnetic fields. In the CFL phase the
symmetry breaking is given by

$$\mathcal{G} = SU(3)_C \times SU(3)_L \times SU(3)_R \times U(1)_B \times U(1)_{\text{e.m.}} \rightarrow SU(3)_{C+L+R} \times \tilde{U}(1)_{\text{e.m.}}.$$  (1)

This symmetry reduction leaves nine Goldstone bosons: a singlet associated to the breaking of the baryonic symmetry $U(1)_B$, and an octet associated to the axial $SU(3)_A$ group.

Once a magnetic field is switched on, the difference between the electric charge of the $u$ quark and that of the $d$ and $s$ quarks reduces the original flavor symmetry of the theory and consequently also the symmetry group remaining after the diquark condensate is formed. Then, the breaking pattern for the MCFL-phase [6] becomes

$$\mathcal{G}_B = SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{A} \times U(1)_B \times U(1)_{\text{e.m.}} \rightarrow SU(2)_{C+L+R} \times \tilde{U}(1)_{\text{e.m.}}.$$  (2)

The group $U(1)_A$ (not to be confused with the usual anomaly $U(1)_A$) is related to the current which is an anomaly-free linear combination of $s$, $d$, and $u$ axial currents [7]. In this case only five Goldstone bosons remain. Three of them correspond to the breaking of $SU(2)_A$, one to the breaking of $U(1)_A$, and one to the breaking of $U(1)_B$. Thus, an applied magnetic field reduces the number of Goldstone bosons in the superconducting phase, from nine to five.

The MCFL phase is not just characterized by a smaller number of Goldstone fields, but by the fact that all these bosons are neutral with respect to the rotated electric charge. Hence, no charged low-energy excitation can be produced in the MCFL phase. This effect can be relevant for the low energy physics of a color superconducting star’s core and hence for its transport properties. In particular, the cooling of a compact star is determined by the particles with the lowest energy; so a star with a core of quark matter and sufficiently large magnetic field can have a distinctive cooling process.

More recently, we have found that the magnetic field can also influence the gluon dynamics [8]. At field strengths comparable to the charged gluon Meissner mass a new phase can be realized giving rise to an inhomogeneous condensate of $\tilde{Q}$-charged gluons [8]. The gluon condensate anti-screens the magnetic field due to the anomalous magnetic moment of these spin-1 particles. Because of the anti-screening, this condensate does not give a mass to the $\tilde{Q}$ photon, but instead amplifies the applied rotated magnetic field. This means that at such applied fields the CS behaves as a paramagnet, thus we named this phase paramagnetic CFL (PCFL) [9]. This last effect is also of interest for astrophysics. Compact stars with
color superconducting cores could have larger magnetic fields than neutron stars made up entirely of nuclear matter, thanks to the gluon vortex antiscreening mechanism.

The above state of affairs underlines the need to discern the scales and field strengths at which one or another magnetic phase is physically relevant. If one ignores the quark masses, the main scales of the color superconductor are the baryon chemical potential $\mu$, the dynamically generated gluon mass $m_g \sim g\mu$ and the gap parameter $\Delta \sim \frac{\mu - \alpha}{g^5} e^{-\alpha/g}$, with $\alpha$ a constant that is dominated by magnetic gluon exchanges [10]. We can assume that at sufficiently high $\mu$, the running strong coupling $g$ becomes $g(\mu) \ll 1$, so the the hierarchy of the scales is $\Delta \ll m_g \ll \mu$. The main purpose of this paper is to elucidate how the different magnetic phases are related to the fundamental scales of the CS.

The plan of the paper is as follows. In Sec. II we develop the CFL low-energy effective theory in a magnetic background using a Lorentz covariant formalism. In this derivation the rotated magnetic field is introduced only through covariant derivatives, thus preserving the $\tilde{U}(1)_{em}$ gauge invariance. The threshold rotated magnetic field that decouples the charged mesons from the low-energy theory is found in Sec. III. In Sec. IV we discuss how the different magnetic phases are realized in a hierarchical order determined by the main energy scales of the CS. In the concluding remarks we state the major outcomes of the paper and discuss possible astrophysical implications of the realization of a PCFL-like phase at moderate densities.

II. LOW-ENERGY EFFECTIVE THEORY IN A MAGNETIC BACKGROUND

The physics at energies below the lower scale $\Delta$ can be explored by constructing the effective low-energy theory in the presence of a rotated magnetic field. Since in the CFL phase all the fermions are gapped and all the gluons have dynamically generated masses, the low energy theory of the CS is governed by the Goldstone modes arising from the breaking of the global symmetries in the presence of a magnetic field.

As discussed in the Introduction, once a magnetic field is present, the original symmetry group $G$ is reduced, due to the different electric charges of the quarks, to $G_B$. One would think that the low energy theory should correspond to the breaking pattern [2], hence be described by five neutral Goldstone bosons. However, it is clear that at very weak magnetic fields the symmetry of the CFL phase can be treated as a good approximated symmetry,
meaning that at weak fields the low-energy excitations are essentially governed by nine approximately massless scalars (those of the breaking pattern 1) instead of five.

A question of order here is: what do we exactly understand as a very weak magnetic field? In other words, what is the threshold-field strength that effectively separates the CFL low energy behavior from the MCFL one? A fundamental clue in this direction will come from the determination of the term in the low-energy CFL Lagrangian that can generate a field-induced mass for the charged Goldstone fields, disconnecting them from the low-energy dynamics at some field strength and thereby effectively reducing the number of Goldstone bosons from the nine of the CFL phase, to the five of the MCFL. A similar approach was previously followed in Ref. [11]. Nevertheless, as it will become clear below, our treatment and results will differ from those previously obtained.

Our strategy will consist of writing the effective low energy Lagrangian for the Goldstone bosons corresponding to the CFL breaking pattern $U(1)$, but in the presence of an external $\tilde{B}$ field, that is, ignoring the explicit breaking introduced by the electromagnetic interaction. To ensure that this Lagrangian is ”invariant” with respect to the original group of the CFL case, one treats the charge operator as a spurion field $[11]$, and assume it transforms conveniently under left/right flavor symmetries, as well as under the color symmetries, depending on whether the operator appears with flavor or color indexes, as it will be shown below.

There are two important points that separate our treatment from previous works. One is that we give the general form of the low-energy Lagrangian in an arbitrary reference frame, that is, we introduce all the possible covariant structures that can be formed at finite density and in the presence of an external magnetic field. When taken in the rest frame, the Lagrangian naturally reproduces the different Lorentz structures that characterize the problem with external magnetic field at finite density. The other is that when proposing the allowed terms of the low energy Lagrangian, we take into account that the coupling of the charged mesons with a rotated electromagnetic field can be traced back to the coupling of the fermions with the field. This coupling only occurs within a covariant derivative to preserve the gauge invariance of the theory under the $\tilde{U}(1)$ group. Then, the coupling between the Goldstone bosons and the rotated electromagnetic field always occurs within a covariant derivative too. We will show that in this $\tilde{U}(1)$ gauge invariant approach, the charged Goldstone bosons acquire field-induced masses that appear at the leading order of the low-energy theory.
To find the effective low-energy theory, we can follow a similar method to that used at zero field in Refs. [12, 13]. We start by introducing two scalar fields which describe the fluctuations of the diquark condensates, and are associated to left and right order parameters:

\[ X^{ai} \sim \epsilon^{abc} \epsilon^{ijk} \langle \psi^\dagger_L \psi^L_i \psi^L_j \psi^L_k \rangle, \quad Y^{ai} \sim \epsilon^{abc} \epsilon^{ijk} \langle \psi^\dagger_R \psi^R_i \psi^R_j \psi^R_k \rangle \]  

(3)

where \( i, j, k \) denote flavor indices, \( a, b, c \) denote color indices, and \( L/R \) denote left/right chirality, respectively. Under an \( SU(3)_C \times SU(3)_L \times SU(3)_R \) rotation, the above fields transform as

\[ X \rightarrow g_C X g_L^\dagger, \quad Y \rightarrow g_C Y g_R^\dagger, \]  

(4)

with \( g_C \in SU(3)_C \), and \( g_{R/L} \in SU(3)_{R/L} \). The expectation values of the \( X \) and \( Y \) fields define the ground state of the CFL phase which produces the symmetry breaking \( SU(3)_C \times SU(3)_L \times SU(3)_R \rightarrow SU(3)_{C+L+R} \).

We are interested in the fluctuations of the phases of the order parameters. Therefore, we can factor out the norm of the order parameters in (4) and work with unitary \( X \) and \( Y \) scalar fields. Although the axial group \( U(1)_A \) is anomalous at low densities, it becomes an approximate symmetry at high densities due to the suppression of the instanton interactions that produce the anomaly. Nevertheless, in our derivations neither the pseudo Goldstone mode associated to the breaking of this group, nor the Goldstone mode associated to the \( U(1) \)-baryon symmetry breaking will be considered, as these bosons are both neutral with respect to the rotated electromagnetic charge, and therefore irrelevant for the analysis of the present paper (we refer the interested reader to Ref. [13] to find out the contributions of these modes to the low energy theory; see also Ref. [14] for the diquark excitations of the CFL ground state at zero magnetic field in the framework of a Nambu-Jona-Lasinio model).

We can introduce the Goldstone canonical fields

\[ X = e^{i\Pi^a X_a}, \quad Y = e^{i\Pi^a Y_a}, \quad a = 1, \ldots, 8 \]  

(5)

with \( T_a \) being the \( SU(3) \) generators normalized to satisfy

\[ Tr[T_a, T_b] = \frac{1}{2} \delta_{ab} \]  

(6)

Thus, \( X \) and \( Y \) together contain a total of sixteen scalar fields. Strictly speaking only eight of them are genuine Goldstone bosons, that is, massless scalar fields associated to the breaking of global symmetries. The other eight are Higgs fields related to the spontaneous
breaking of the color gauge group SU(3) that gives mass to the gluons and therefore can be always eliminated from the theory by choosing a convenient (unitary) gauge \[13\]. Since these considerations remain valid in the presence of the external magnetic field, we will work in the unitary gauge and concentrate our analysis into the derivation of the low energy theory for the genuine Goldstone bosons.

At zero magnetic field, the low energy theory of the Goldstone bosons associated to the global symmetries can be written \[12\] as

$$\mathcal{L} = -\frac{f^2}{4} Tr[(J^\mu_X - J^\mu_Y)^2]$$  \(7\)

with currents \(J^\mu_X\) and \(J^\mu_Y\) defined by

$$J^\mu_X = X \partial^\mu X^\dagger, \quad J^\mu_Y = Y \partial^\mu Y^\dagger.$$  \(8\)

Notice that \(7\) does not take into account the breaking of the Lorentz invariance due to the finite density. The lack of Lorentz invariance was later incorporated in Ref. \[12\] by manually assigning different coefficients in front of the temporal and spatial derivative terms.

Using \(4\), we can verify that the currents \(J_X\) and \(J_Y\) transform as

$$J^\mu_X \rightarrow g_C J^\mu_X g_C^\dagger, \quad J^\mu_Y \rightarrow g_C J^\mu_Y g_C^\dagger.$$  \(9\)

The Lagrangian density \(7\) contains the leading (second) order in derivatives terms invariant under the \(SU(3)_C \times SU(3)_L \times SU(3)_R\) rotations.

Two important changes occur when a \(\tilde{B}\) field is switch on. First, the derivatives should be replaced by covariant derivatives containing the rotated electromagnetic potential \(\tilde{A}_\mu\) associated to the magnetic field \(\tilde{B}\). Second, the number of fundamental tensors available in the theory increases, because of the extra tensor \(\tilde{F}_{\mu\nu}\). As a consequence, we can construct an effective theory containing a larger number of independent terms which are quadratic in the (covariant) derivative.

The covariant derivative should be consistent with the fact that the VEV of the diquark fields, which only get contribution from the diagonal elements of the order parameter matrix, are all neutral in the rotated charge. We can generically denote the \(3 \times 3\) order parameter matrix by \(\Delta\). Taking into account that a rotated electromagnetic field is a combination of the \(8^{th}\)-gluon field and the conventional electromagnetic field and that each element of the
order parameter matrix carries an electric charge equal to the sum of the electric charges of
the quarks forming the corresponding pair, we can write the covariant derivative as

\[ [\partial_\mu + igG^8_\mu (T^8 \times 1) + ieA_\mu (1 \times Q_\Delta)]\Delta = [\partial_\mu - \frac{i\sqrt{3}}{2}gG^8_\mu (Q \times 1) - ieA_\mu (1 \times Q)]\Delta \]  

(10)

where the direct products denote (color × flavor). \(Q_\Delta\) is the conventional electromagnetic
charge operator of the quark pairs in the quark representation \((s, d, u)\). In this representation
the first column of \(\Delta\) will have \((d, u)\) pairs, the second \((s, u)\) pairs, and the third \((s, d)\) pairs.

Hence \(Q_\Delta = \text{diag}(1/3, 1/3, -2/3)\). Both the \(T^8\) generator of the \(SU(3)\) group and the usual
quark charge operator \(Q = \text{diag}(-1/3, -1/3, 2/3)\) are connected to \(Q_\Delta\) through the relations
\(T^8 = (\sqrt{3}/2)Q_\Delta = -(\sqrt{3}/2)Q\).

Taking into account that \(\tilde{\mathcal{A}}_\mu = A_\mu \cos \theta - G^8_\mu \sin \theta\), where \(\theta\) is the mixing angle \([4]\), Eq.(10)
can be rewritten as

\[ (\partial_\mu - i\tilde{e}\tilde{A}_\mu \tilde{Q})\Delta \]  

(11)

where \(\tilde{e} = e \cos \theta = (\sqrt{3}/2)g \sin \theta\), and \(\tilde{Q} = (1 \times Q - Q \times 1)\) is the “rotated” electric
charge operator. As expected, the charge operator \(\tilde{Q}\) assigns zero rotated charge to the
diagonal elements of \(\Delta\). Notice that we do not keep the orthogonal combination \(\tilde{G}^8_\mu = A_\mu \sin \theta + G^8_\mu \cos \theta\), as the field \(\tilde{G}^8_\mu\) acquires a mass larger than \(\Delta_{CFL}\), so it decouples from
the low-energy theory.

The straightforward generalization of the currents \([8]\) to the case with nonzero rotated
magnetic field should be done by substituting the derivative in \([8]\) by the covariant derivative
\([11]\). Taking into account that \(X X^\dagger = 1\) one can write the left current as

\[ (\tilde{J}^\mu_X)_{ab} = X_{ai}(\partial^\mu \delta_{ij} + i\tilde{e}\tilde{A}^\mu Q_{ij})X^\dagger_{jb} - i\tilde{e}\tilde{A}^\mu Q_{ab} = X(\partial^\mu X^\dagger + i\tilde{e}\tilde{A}^\mu Q^L X^\dagger) - i\tilde{e}\tilde{A}^\mu Q^C \]  

(12)

where we introduced the notations \(Q^C\) and \(Q^L\) to keep track of whether the operator \(Q\) is
an operator in color or (left) flavor space. A similar expression can be found for the right
current \(\tilde{J}^\mu_Y\), with the obvious substitution \(X \rightarrow Y\) and \(Q^L \rightarrow Q^R\).

As mentioned above, once a magnetic field is present one has an extra tensor in the
system that allows to create new structures in the Lorentz space. Working in a covariant
way, the leading order low-energy Lagrangian density can be written as

\[ \mathcal{L} = -\frac{f^2}{4} Tr[(\tilde{J}^\mu_X - \tilde{J}^\mu_Y)(\tilde{J}^\nu_X - \tilde{J}^\nu_Y)\Theta_{\mu\nu}] \]  

(13)
with

\[ \Theta_{\mu \nu} = C_1 g_{\mu \nu} + C_2 u_\mu u_\nu + C_3 \hat{F}_{\mu \rho} \hat{F}^{\rho \nu} \] (14)

being the most general Lorentz structure that can be formed with the vectors and tensors available at low energies. In (14) we considered the normalized electromagnetic tensor \( \hat{F}_{\mu \nu} = \frac{1}{\hat{B}} \tilde{F}_{\mu \nu} \), in addition to the usual metric tensor \( g_{\mu \nu} \) and the vector four-velocity of the center of mass of the dense medium \( u_\mu \). The \( C_i \)'s are just constant coefficients. In this covariant representation the magnetic field can be expressed as \( \tilde{B}_\mu = \frac{1}{2} \varepsilon_{\mu \nu \rho \lambda} u^\nu \tilde{F}^{\rho \lambda} \). In the rest frame, \( u^\nu = (1, 0, 0, 0) \), and the electromagnetic tensor becomes \( \tilde{F}_{\mu \nu} = \tilde{B}_\mu \delta_{\mu 1} \delta_{\nu 2} \) if we assume a magnetic field pointing along the third spatial direction. In the absence of a magnetic field, the coefficient \( C_3 \) is taken equal to zero and the Lagrangian density (13) reduces in the rest frame to that introduced in Refs. [12, 13], where the system was not Lorentz invariant due to the finite density (\( u_\mu \neq 0 \)), but it kept the rotational symmetry. In the presence of a magnetic field the structure associated to \( C_3 \) naturally separates between modes longitudinal and transverse to the field. A linear term in \( \hat{F}_{\mu \nu} \) is forbidden, since it would violate the theory CP invariance. Notice that in the leading (second) order in derivatives we do not need to introduce the momentum \( k_\mu \) in the structures contributing to (14).

Notice that the flavor symmetry \( SU(3)_L \times SU(3)_R \) is explicitly broken by the electromagnetic coupling in (13). So, strictly speaking, (13) is invariant under \( G_E \) but not under \( G \). However, one can make the theory invariant under the original group \( G \) if we treat the charge operators as spurion fields and assume that they transform as

\[ Q^L \rightarrow g^*_L Q^L g^*_L, \quad Q^R \rightarrow g^*_R Q^R g^*_R, \quad Q^C \rightarrow g_C Q^C g^*_C. \] (15)

Using these transformations, one can show that the new currents transform under \( G \) in the same way as the currents at zero field:

\[ \tilde{J}^\mu_X \rightarrow g_C \tilde{J}^\mu_X g^*_C, \quad \tilde{J}^\mu_Y \rightarrow g_C \tilde{J}^\mu_Y g^*_C \] (16)

yielding to the invariance of (13) under \( G \).

Following Ref. [12], we introduce now the color singlet,

\[ \Sigma = Y^\dagger X, \] (17)
which transforms under $SU(3)_C \times SU(3)_L \times SU(3)_R$ as $\Sigma \rightarrow g^*_R g^*_L$. In terms of $\Sigma$ the Lagrangian density \[13\] can be written as

$$\mathcal{L} = \frac{f^2_\pi}{4} \text{Tr}[(D_\mu \Sigma)(D_\nu \Sigma)^\dagger \Theta_{\mu\nu}]$$

(18)

where the covariant derivative acting on $\Sigma$ is

$$D_\mu \Sigma = \partial_\mu \Sigma + i \tilde{e} \tilde{A}_\mu (Q^R \Sigma - \Sigma Q^L)$$

(19)

In the rest-frame, \[18\] becomes

$$\mathcal{L} = \frac{f^2_\pi}{4} \left[ \text{Tr} (D_0 \Sigma)(D_0 \Sigma)^\dagger \right] + v^2_\perp \text{Tr} (D_\perp \Sigma)(D_\perp \Sigma)^\dagger + v^2_\parallel \text{Tr} (D_\parallel \Sigma)(D_\parallel \Sigma)^\dagger .$$

(20)

showing the expected separation between longitudinal $D_\parallel$ and transverse $D_\perp$ (to the field) components of the covariant derivatives, a direct consequence of the partial breaking of the rotational symmetry in the presence of the magnetic field. The decay constant $f_\pi$, and the meson maximum velocities $v_\perp$, and $v_\parallel$, are parameters to be computed from the microscopic theory. They can in general depend on the baryonic chemical potential and the applied magnetic field. In the weak-field limit ($eB \ll \mu^2$), at asymptotically large values of the chemical potential $\mu$, they can be approximated by their zero-field values \[13\]

$$f^2_\pi \approx \frac{21}{18} - \frac{8 \ln 2}{18} \frac{\mu^2}{2\pi^2}, \quad v_\perp \approx v_\parallel \approx \frac{1}{\sqrt{3}}$$

(21)

As we shall see in the next Section, the low-energy theory \[20\] supports the generation of field-dependent masses for the charged mesons at fields larger than a threshold value, even though no quadratic-in-$\tilde{B}$ term, like the one proposed in Ref. \[11\] (Eq. (2.16)), is present at the leading order. To understand this, one should keep in mind that the interaction between $\Sigma$ and the electromagnetic field always occur through the covariant derivative. To generate a quadratic-in-$\tilde{B}$ (and quadratic-in-$Q$) term, one needs to consider a higher order contribution involving four covariant derivatives

$$\int d^4x Tr[(D_\mu \Sigma)(D_\nu \Sigma)^\dagger(D_\rho \Sigma)(D_\lambda \Sigma)^\dagger \hat{F}_{\mu\nu} \hat{F}_{\rho\lambda}]$$

(22)

Using (we dropped total derivatives)

$$D_\mu \Sigma(D_\nu \Sigma)^\dagger \hat{F}_{\mu\nu} = \frac{1}{2} \hat{F}_{\mu\nu} [i \tilde{e} \tilde{F}_{\mu\nu}(Q^R - \Sigma Q^L)^\dagger] + 2i \tilde{e} \tilde{A}_\mu (Q^R \Sigma \partial_\nu \Sigma^\dagger - \Sigma \partial_\nu \Sigma^\dagger Q^R - 2 \Sigma (\partial_\nu \Sigma^\dagger) \partial_\mu] ,$$

(23)
it can be straightforwardly shown that the term proposed in [11]
\[
\int d^4x \bar{e}^2 \tilde{B}^2 Tr [Q^R \Sigma Q^L \Sigma^\dagger]
\] (24)
is just one of the several terms coming out of (22). However, as proved below, the threshold field for the decoupling of the charged mesons will be determined by a contribution more relevant than (24).

III. CFL-MCFL THRESHOLD FIELD

The unitary matrix $\Sigma$ can be parametrized in term of the elementary Goldstone bosons $\phi^A$ as
\[
\Sigma = \exp \left(i \frac{\phi^A T^A}{f_\pi}\right), \quad A = 1, ..., 8,
\] (25)
where $T^A$ are the $SU(3)$ generators. Expanding (25) up to linear terms in the fields, we can write (19) as a sum of covariant derivatives for the $\phi^A$ fields
\[
D_\mu \Sigma = \frac{i}{f_\pi} \left( \sum_{A=1}^{3} T^A \partial_\mu \phi^A + T^8 \partial_\mu \phi^8 + \sum_{\pm} \tau^\pm (\partial_\mu \pm i e \tilde{A}_\mu) \Pi^\pm + \sum_{\pm} \Lambda^\pm (\partial_\mu \pm i e \tilde{A}_\mu) \kappa^\pm \right)
\] (26)
where
\[
\Pi^\pm = \frac{1}{\sqrt{2}} [\phi^4 \mp i \phi^5], \quad \kappa^\pm = \frac{1}{\sqrt{2}} [\phi^6 \mp i \phi^7],
\] (27)
\[
\tau^\pm = \frac{1}{\sqrt{2}} [T^4 \pm i T^5], \quad \Lambda^\pm = \frac{1}{\sqrt{2}} [T^6 \pm i T^7]
\] (28)
and the rotated electromagnetic potential is taken in the Landau gauge
\[
\tilde{A}_\mu = (0, 0, \tilde{B}x, 0)
\] (29)

From (20), the low-energy Lagrangian for the Goldstone bosons can be written as
\[
L = \int d^4x \left\{ \frac{1}{4} \sum_{A=1}^{3, 8} |\partial_\mu \phi^A|^2 + |\partial_\mu \Upsilon|^2 \right\} + \frac{v^2}{4} \left( \sum_{A=1}^{3, 8} |\partial_{\parallel} \phi^A|^2 + |\partial_{\parallel} \Upsilon|^2 \right)
\]
\[
+ \frac{v^2}{4} \left( \sum_{A=1}^{3, 8} |\partial_{\perp} \phi^A|^2 + |(\partial_{\perp} + i e \tilde{A}_{\perp}) \Upsilon|^2 \right)
\] (30)
where we introduced the charged meson doublet
The Lagrangian (30) represents the CFL low-energy theory in the presence of a weak $(k^2 \sim \tilde{e} \tilde{B} \ll \mu^2)$ constant magnetic field. Now we are ready to determine the strength of the threshold field for which the effective symmetry transmutation from CFL to MCFL occurs.

For that aim, it is convenient to work in momentum space. Transforming to momentum space in the presence of the magnetic field can be done by applying to the scalar fields the same method originally developed for fermions in [15] and later extended to vector fields in [16]. In this approach the transformation to momentum space can be carried out using the wave functions $S_k(x)$ of the asymptotic states of the charged mesons in a uniform magnetic field. These functions play the role in the magnetized medium of the usual plane-wave (Fourier) functions $e^{ipx}$ at zero field. Then, for the charged field $\Upsilon$ we have

$$\Upsilon(k) = \int d^4x S_k(x) \Upsilon(x)$$

where

$$S_k(x) = N \exp(ik_0x^0 + ik_2x^2 + ik_3x^3)D_n(\rho)$$

with $D_n(\rho)$ being the parabolic cylinder functions with argument $\rho = \sqrt{2|\tilde{e} \tilde{B}|(x_1 - k_2/\tilde{e} \tilde{B})}$, $N$ the normalization constant ($N = (4\pi|\tilde{e} \tilde{B}|)^{1/2}/\sqrt{n!}$), and $n = 0, 1, ...$ denoting the Landau levels. It is easy to check that the transformation functions (33) satisfy the orthonormality condition

$$\int_{-\infty}^{\infty} d^4x S_k(x) S_{k'}(x) = (2\pi)^4 \tilde{\delta}(k' - k)$$

where $\tilde{\delta}(k' - k) = \delta(k'_0 - k_0)\delta(k'_2 - k_2)\delta(k'_3 - k_3)\delta_{n'n}.$

Using this transformation in (30) we can derive the Klein-Gordon equation for the charged mesons in momentum space

$$[k_0^2 - \tilde{e} \tilde{B}(2n + 1)v_\perp^2 - k_3^2 v_\parallel^2] \Upsilon(k) = 0.$$  

and from it the corresponding dispersion relation

$$E^2 = \tilde{e} \tilde{B}(2n + 1)v_\perp^2 + k_3^2 v_\parallel^2,$$
We see that at zero momentum ($k_3 = 0, n = 0$) the rest energy of the charged mesons is $M_B^2 = \tilde{e}\tilde{B}v^2$, meaning they acquire a mass induced by the magnetic field. For a meson to be stable, its mass should be less than twice the gap, otherwise it will decay into a quasiparticle-quasihole pair. Here we are assuming that the interactions between the Goldstone bosons and the quasiparticles are described by a Yukawa term that originates within a NJL-type model. Some authors (see [17] for details) have argued that the microscopic structure of the NG bosons in the CFL phase should be that of a quartic-quark state. In the quartic-quark picture the threshold would presumably be four times the gap energy. Within the NJL-approach, the threshold field where the number of stable Goldstone bosons effectively changes, thus the symmetry of the CFL phase can no longer be treated as a good approximated symmetry is given by

$$\tilde{e}\tilde{B}_{MCFL} = 4\frac{\Delta_{CFL}^2}{v^2} \approx 12\Delta_{CFL}^2,$$

where we used the weak-field approximation $v_\perp \approx 1/\sqrt{3}$ [13]. Contrary to the result found in [11], our threshold field does not depend on the decay constant $f_\pi$, therefore it depends on $\mu$ only through $\Delta_{CFL}$. The $f_\pi$ dependence found in [11] is a direct consequence of considering a subleading contribution in the derivation of the threshold field value, as previously shown.

For $\Delta_{CFL} \sim 15MeV$ we get $\tilde{e}\tilde{B}_{MCFL} \sim 10^{16}G$. At these field strengths, the charged mesons decouple from the low-energy theory. When this decoupling occurs, the five neutral Goldstone bosons (including the one associated to the baryon symmetry breaking) that characterize the MCFL phase will drive the low-energy physics of the system. Therefore, coming from low to higher fields, the first magnetic phase that effectively shows up in the magnetized system will be the MCFL, even though at fields near the threshold field the splitting of the gaps found at much stronger fields [6] may still be negligible.

We underline that the phenomenon occurring at the threshold field (37) is not a phase transition, as no symmetry is broken there. At any nonzero magnetic field strength, below or above the threshold field (37), the symmetry of the system is, strictly speaking, that of the MCFL. However, at $\tilde{B} < \tilde{B}_{MCFL}$ the charged Goldstone bosons are so light that the observable impact of the smaller symmetry of the MCFL phase, compared to that of CFL, is irrelevant and the nine Goldstone bosons of the CFL phase provide a good approximated description of the low-energy physics. Based on these considerations we choose to call the CFL-MCFL transition a symmetry transmutation.
On the other hand, it is worth to call attention to the analogy between the CFL-MCFL transmutation and what could be called a "field-induced" Mott transition. Mott transitions were originally considered in condensed matter in the context of metal-insulator transitions in strongly-correlated systems [18]. Later on, Mott transitions have been also discussed in QCD to describe delocalization of bound states into their constituents at a temperature defined as the Mott temperature [19]. By definition, the Mott temperature $T_M$ is the temperature at which the mass of the bound state equals the mass of its constituents, so the bound state becomes a resonance at $T > T_M$. In the present work, the role of the Mott temperature is played by the threshold field $\tilde{B}_{MCFL}$. Mott transitions typically lead to the appearance of singularities at $T = T_M$ in a number of physically relevant observables. It is an open question worth to be investigated whether similar singularities are or not present in the CFL-MCFL transmutation at $\tilde{B}_{MCFL}$.

IV. $\tilde{B}$ VS $\mu$ PHASES IN A COLOR SUPERCONDUCTOR WITH THREE QUARK FLAVORS

What will happen if we keep increasing the magnetic field until it reaches the next energy scale $g\mu$? As known [8], due to the interaction of the applied magnetic field with the charged gluon anomalous magnetic moment $(\epsilon^\mu e f_{\mu\nu} G^+ G^-)$, once $\tilde{B} \geq \tilde{B}_{PCFL} = m^2_M$, with $m_M \sim g\mu$ being the magnetic mass of the charged gluons, one of the modes of the charged gauge field becomes tachyonic (this is the well known "zero-mode problem" for spin-1 charged fields in the presence of a magnetic field found for Yang-Mills fields [20], for the $W^{\pm}_\mu$ bosons in the electroweak theory [21, 22], and even for higher-spin fields in the context of string theory [23]). Similarly to other spin-1 theories with magnetic instabilities [20]-[22], the solution of the zero-mode problem leads to the restructuring of the ground state through the formation of an inhomogeneous gauge-field condensate $G$, as well as an induced magnetic field due to the back reaction of the $G$ condensate on the rotated electromagnetic field. The magnitude of the $G$-condensate plays the role of the order parameter for the phase transition occurring at $\tilde{B} = \tilde{B}_{PCFL}$. Near the transition point, the amplitude of the condensate $G$ is very small [8]. Then, the condensate solution can be found using a Ginzburg-Landau (GL) approach similar to Abrikosov’s treatment of type II metal superconductivity near the critical field $H_c^2$ [24]. As in Abrikosov’s case, the order parameter $|G|$ continuously increases from zero...
with the applied magnetic field, signalizing a second-order phase transition towards a gluon crystalline vortex state characterized by the formation of flux tubes. Both spatial symmetries -the rotational symmetry in the plane perpendicular to the applied magnetic field and the translational symmetry- are broken by the vortex state.

It turns out that, contrary to what occurs in conventional type-II superconductors, where the applied magnetic field only penetrates through flux tubes and with a smaller strength than that of the applied field, the gluon vortex state exhibits a paramagnetic behavior. That is, outside the flux tube the applied field $\tilde{B}$ totally penetrates the sample, while inside the tubes the magnetic field becomes larger than $\tilde{B}$. This antiscreening behavior is similar to that found in the electroweak system at high magnetic field [22]. Hence, since the $\tilde{Q}$ photons remain long-range in the presence of the condensate $G$, the $\tilde{U}(1)_{em}$ symmetry remains unbroken. At asymptotically large densities, because $\Delta_{CFL} \ll m_M$, we have $\tilde{B}_{MCFL} \ll \tilde{B}_{PCFL}$ for each $\mu$ value.

At fields $\tilde{B} \gtrsim \mu^2$ the density of states on the Fermi surface of the charged quarks will be larger than that of neutral quarks [6]. Because of the different density of states, the magnitude of the gap receiving contributions from pairs of rotated charged quarks will split from the magnitude of the gap receiving contributions only from pairs of rotated neutral quarks, as shown to happen, without taking into account the vortex state, in Ref. [6]. The splitting of the gaps at this scale, however, would not break any new symmetry that has not been already broken at much lower scales. Hence, no new phase transition occurs at fields of order $\mu^2$.

For fields much larger than $\mu^2$, i.e. sufficiently strong as to surpass all the energy scales of the system, the quark infrared dynamics will become predominant, and the phenomenon of magnetic catalysis of chiral symmetry breaking [25] will be activated, producing a phase that favors quark-antiquark condensates over quark-quark condensates. However, the exploration of this region goes beyond the scope of this paper.

Fig.1 provides a qualitative sketch of the magnetic phases that exist at asymptotically high densities in the framework of a three-flavor color superconductor. In that region, at very weak magnetic fields, the color superconducting state is practically described by the CFL phase, because the charged mesons, although massive, are so light that they cannot decay in pairs of quasiparticle-quasihole. When the field strength is of the order of the quarks’ energy gap, the charged mesons become heavy enough to decouple and the low-
energy physics is indeed that of the MCFL phase, where five neutral massless bosons drive the low-energy behavior. At fields comparable to the magnetic masses of the charged gluons, a chromomagnetic instability is developed for these gluons leading to the formation of a vortex state and the antiscreening of the magnetic field [8, 9]. The vortex state breaks the translational symmetry, as well as the remaining rotational symmetry in the plane...
perpendicular to the applied magnetic field, hence the vortex formation corresponds to a phase transition from the MCFL to a PCFL phase.

The chemical potential $\mu_{CFL}$ in Fig. 1 is used to schematically separate the regions where the effects of an s-quark mass $M_s$ can (right of $\mu_{CFL}$) or cannot (left of $\mu_{CFL}$) be neglected. The region of moderately high densities to the left of $\mu_{CFL}$ along the zero-magnetic-field line has been subjected to intense scrutiny in the literature [26]-[32], as this is the most important region for applications of CS in realistic systems as compact stars. A main problem has been to find the stable phase at these moderately high densities. Despite many clever propositions that include a modified CFL-phase with a condensate of kaons [29]; a LOFF phase on which the quarks pair with nonzero total momentum [30]; as well as homogeneous [31] and inhomogeneous [32] gluon condensate phases, it is still unclear which of these phases produces the lowest free energy. Given that nobody has yet studied this question at finite magnetic field, we choose to indicate it in the figure with question marks.

V. CONCLUDING REMARKS

Summarizing, in a color superconductor with three-flavor quarks at very high densities an increasing magnetic field produces a phase transmutation from CFL to MCFL first, and then a phase transition from MCFL to PCFL. During the phase transmutation no symmetry breaking occurs, since in principle once a magnetic field is present the symmetry is theoretically that of the MCFL, as discussed above. However, in practice for $\tilde{B} < \tilde{B}_{MCFL} \sim \Delta_{CFL}^2$ the MCFL phase is almost indistinguishable from the CFL. Only at fields comparable to $\Delta_{CFL}^2$ the main features of MCFL emerge through the low-energy behavior of the system. At the threshold field $\tilde{B}_{MCFL}$, only five neutral Goldstone bosons remain out of the original nine characterizing the low-energy behavior of the CFL phase. These are precisely the five Goldstone bosons determining the new low-energy behavior of the genuinely realized MCFL phase. Going from MCFL to PCFL is, on the other hand, a real phase transition [8], as the translational symmetry, as well as the remaining rotational symmetry in the plane perpendicular to the applied magnetic field are broken by the vortex state.

Throughout this paper we have ignored the effects due to quark masses because we assumed very large baryon density. However, the densities of interest for most astrophysical applications are just moderately high. At moderate densities the s-quark mass can and do
play an important role \[26\]. In this case, the color superconductor develops chromomagnetic instabilities even in the absence of an external magnetic field \[27\]. At even lower densities, where the s-quark decouples due to its larger mass, a two-flavor color superconductivity -the so-called 2CS phase- realizes. In this phase, when color neutrality and \(\beta\) equilibrium conditions are imposed, some chromomagnetic instabilities can also develop at certain density values \[28\]. Finding the stable superconducting ground state at moderate densities is one of the main questions in the field at the present moment \[29\]-\[32\]. The understanding of this problem in the presence of a magnetic field, which no doubt is another crucial player in the physical scenario of a compact star, is also an important open question. In this regard, as discussed in Ref. \[32\], the removal of the chromomagnetic instabilities found at moderate densities in the two-flavor system may be related to the \textit{spontaneous generation} of an inhomogeneous gluon condensate with a corresponding induced magnetic field. If this proposition is proved to be correct also for the three-flavor case, the star’s core could be in a PCFL-like magnetic phase, even if the core’s original magnetic field is zero or relatively low. That is, the PCFL-like phase will not be triggered by instabilities produced at a critical value of some pre-existing inner magnetic field, but by instabilities connected to the interplay of the neutrality conditions and the s-quark mass at some baryon density. An interesting consequence of a PCFL core is that a star’s core in this phase can generate and/or boost its inner magnetic field. Exploring the magnetic phases that realize at realistic densities is an important pending task.

It is plausible that if compact stars are the natural playground for color superconductivity, the magnetic phases described in this paper, or more precisely, the version of these phases at more realistic densities, may be relevant for the physics of the core of highly magnetized compact objects like magnetars \[2,33\], and the so-called Central Compact Objects (CCO) \[34\]. CCO are point-like sources located near the center of supernova remnants that cannot be identified as active radio pulsars or magnetars \[34\]. Some of them may have magnetar’s strength \(B\) fields and much smaller radii. For typical neutron star masses \(\sim 1.4M_\odot\), a smaller radii means a denser star’s core; the denser the core, the greater the chance it can be in a color superconducting phase. Magnetars’ surface magnetic fields are typically in the range of \(10^{14}\)\(G\) – \(10^{16}\)\(G\), and the fields at their much denser cores can probably reach even larger values. Any of these compact objects could be a candidate for the realization of a color superconducting phase in a highly magnetized background.
Moreover, the standard explanation of the origin of the magnetars’ large magnetic fields cannot explain all the features of the supernova remnants surrounding these objects. Magnetars are supposed to be created by a magnetohydrodynamic-dynamo mechanism that amplifies a seed magnetic field due to a rapid rotating (spin period < 3ms) protoneutron star. Part of this rotating energy is supposed to power the supernova through rapid magnetic braking, implying that the supernova remnants associated with magnetars should be an order of magnitude more energetic than typical supernova remnants. However recent calculations indicate that their energies are similar. In addition, one would expect that when a magnetar spins down, the rotational energy output should go into a magnetized particle wind of ultrarelativistic electrons and positrons that radiate via synchrotron emission across the electromagnetic spectrum. Nevertheless, so far nobody has detected the expected luminous pulsar wind nebulae around magnetars.

Although more observations are needed to confirm the above, current observations indicate that alternative models to the standard magnetar model need to be considered. For example, some authors have suggested that magnetars could be the outcome of a stellar progenitor with highly magnetized cores. A progenitor star with a PCFL-like core would be capable to induce and/or enhance the star’s magnetic field due to the antiscreening mechanism inherent to this color superconducting phase, and as such provides an alternative to the observational conundrum of the standard magnetar’s paradigm.

Only after the stable color-superconducting phase at moderate densities is well established with and without an external magnetic field, we will be in a well-grounded position to investigate and reliably predict observable signatures of color superconductivity in compact stars.

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