On the geometric phase and the scattering at the LHC

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Abstract

We discuss appearance at the LHC energies of the geometric phase, which in its turn can reflect a presence of the non-factorizable background geometry proposed in the RS-scenario with extra spatial dimensions.
An issue of the geometric phase in the high energy scattering is discussed. In quantum mechanics the geometric or Berry phase appears as a result of a time evolution resulting in the adiabatic loop (cyclic) variation of the Hamiltonian parameters [1, 2]. This phase was originally discovered for the states corresponding to the discrete spectrum and for the case of continuous spectrum in quantum mechanics the issue of the geometric phase has been studied in [3]. This adibatic loop evolution of the Hamiltonian parameters could lead to the phase difference of the initial and final states which is independent of the system dynamics and, in fact, depends only on the path the system evolves along of. Such phase difference is known as the Berry phase and its appearance demonstrates singular parameter dependence of the system dynamics. The interesting phenomenon of the geometric phase appearance can be found in many physical systems [4].

In the relativistic high energy scattering Hamiltonian is not known; instead, one should deal with the scattering matrix. Here, we consider the issue of the geometric phase in the case of relativistic high energy scattering on the base of the unitary representations for the scattering matrix and provide new details in addition to the ones mentioned in [5].

The elastic scattering $S$-matrix is connected with the elastic scattering amplitude by the relation $S(s, b) = 1 + 2i f(s, b)$, which is written here in the impact parameter representation. At high energies the respective function $S(s, b)$ can be rewritten in the form

$$S(s, b) = \kappa(s, b) \exp[2i\delta(s, b)],$$

(1)

where $\kappa(s, b)$ and $\delta(s, b)$ are the real functions, $b$ is an impact parameter of colliding hadrons. The function $\kappa$ ($0 \leq \kappa \leq 1$) is an absorption factor related to the contribution of the inelastic channels into unitarity relation, its zero value, $\kappa = 0$, corresponds to a complete absorption of the incoming channel. The factor $\kappa(s, b)$ is determined by the inelastic channels contribution to the unitarity equation for the elastic scattering amplitude $f(s, b)$, i.e.

$$\kappa^2(s, b) = 1 - 4h_{inel}(s, b),$$

(2)

with $h_{inel}(s, b)$ entering the equation

$$\text{Im}f(s, b) = h_{el}(s, b) + h_{inel}(s, b).$$

(3)

When elastic scattering amplitude is a pure imaginary function, the function $S(s, b)$ is a real one and it can vary in the range $-1 < S(s, b) < 1$. Evidently, at fixed value of $s$ the function $S(s, b) \to 1$ at $b \to \infty$, i.e. in this limit $\kappa \to 1$ and $\delta \to 0$.

Let consider another limiting behavior of $S(s, b)$, namely, when the impact parameter $b$ is fixed and $s \to \infty$. The standard assumption is that the function
$S(s, b) \to 0$ in this limit. It corresponds to approaching a black disk limit and the elastic scattering then is just a shadow of all inelastic processes. However, there is no reason to exclude existence of an another option, i.e. the function $S(s, b) \to -1$ when $b$ is fixed and $s \to \infty$, i.e. $\kappa \to 1$ and $\delta \to \pi/2$. This limiting behavior corresponds to a reflective scattering [5]. It implies appearance of the non-zero phase $\delta$ equal to $\pi/2$.

In the relativistic scattering one can consider the absorption factor $\kappa$ as a parameter which determine the hadron interaction dynamics at high energies. We can force $\kappa$ to perform a loop variation similar to a loop variation of the Hamiltonian parameters in quantum mechanics. Namely, we can vary variable $s$ (and/or $b$) in the way that $\kappa$ evolves cyclically and adiabatically from $\kappa_i > 0$ to $\kappa_{\text{min}} = 0$ and then to the value $\kappa_f$, where $\kappa_f = \kappa_i$ (i.e. performing a loop variation). As a result the non-zero phase appears ($\delta = \pi/2$) at small values of the impact parameter and this phase is independent of the details of the energy and/or impact parameter evolution. Therefore, we can treat it as a geometric phase, considering it as an analog of the Berry phase in quantum mechanics.

To illustrate the evolution which can lead to this loop variation one can use rational form of $S$–matrix unitarization which provide antishadowing in the elastic scattering [6]. This approach can be considered as a resulting one from the confinement condition implication for the colored states [7]. In the $U$–matrix approach the elastic scattering matrix element in the impact parameter representation is the following linear fractional transform:

$$S(s, b) = \frac{1 - U(s, b)}{1 + U(s, b)}.$$ (4)

$U(s, b)$ is the generalized reaction matrix, which is considered to be an input dynamical quantity. This form (4) is one-to-one transform and is easily invertible. We consider pure imaginary scattering amplitude $f$ and it leads to the pure imaginary function $U$. The absorption factor $\kappa(s, b)$ is connected to the function $U(s, b)$ by the relation

$$\kappa(s, b) = \frac{|1 - U(s, b)|}{|1 + U(s, b)|}. \tag{5}$$

The most of the models [5, 8] provide increasing dependence of the function $U(s, b)$ with energy (e.g. power-like one) and the exponential decrease with impact parameter at large values of $b$. The most recent quantitative analysis of the available experimental data with this form of unitarization has been performed in [9]. The value of energy corresponding to the black disk realization in central collisions $S(s, b)|_{b=0} = 0$ will be denoted as $s_0$ and it is determined by the equation $U(s, b)|_{b=0} = 1$; there is a model estimate for $\sqrt{s_0}$ at around $2 TeV$ [6]. In the energy region $s \leq s_0$ the scattering in the whole range of impact parameter variation has a pure shadow nature and the elastic $S$-matrix varies in the
range $0 \leq S(s, b) < 1$. But when the energy is higher than $s_0$ the scattering picture at small values of impact parameter $b \leq R(s)$ acquires geometric (of reflective) nature while shadow contribution is decreasing with energy, $R(s)$ (nodal point) is determined by $U(s, b = R(s)) = 1$. The $S$-matrix varies in the region $-1 < S(s, b) \leq 0$ at $s \geq s_0$ at $b \leq R(s)$. The impact parameter dependence of the absorption factor $\kappa(s, b)$ and phase $\delta(s, b)$ is depicted on Fig. 1 for the case of $s > s_0$. Thus, we can observe that despite the factor $\kappa$ performs loop variation

![Figure 1: Appearance of the geometric phase at the energies $s > s_0$ under the change of the impact parameter from $b_i$ to $b_f$ with $\kappa_i = \kappa_f$, where $\kappa_{i,f} \equiv \kappa(s, b_{i,f})$.](image)

when initial and final values are the same ($\kappa_i = \kappa_f$), the non-zero geometric or Berry phase appears.

One might relate it with the manifestation of the extra dimension with non-factorized metrics proposed in the RS-scenario [10]. There are two different realizations of the idea of the extra dimensions: one (ADD-model [11]) with a metrics of the compact extra dimensions factorized out of the metrics of the standard 3+1 manifold. In another theory (RS-model) called 5-dimensional warped geometry model [10] the space is a five-dimensional anti-de Sitter space with non-factorized metrics. The issues of extra dimensions are very well developed now and there are a lot of publications devoted to this subject (cf. review paper [12] and the references therein). Here we just comment on the the difference between the scenarios with factorizable and non-factorizable geometries. Namely, one should expect that the geometric phase will be presented in the latter case and absent in the former one.

We discuss now a possible instrumentation for the selection of the observable effects related to the presence of the geometric phase. Namely, for that purpose,
it is useful to consider the ratio of elastic to total impact parameter dependent cross-sections, i.e.

\[ \mathcal{R}(s, b) = \frac{\sigma_{el}(s, b)}{\sigma_{tot}(s, b)}. \]

The impact parameter dependent cross–sections \( \sigma_{el}(s, b) \), \( \sigma_{inel}(s, b) \) and \( \sigma_{tot}(s, b) \) can be extracted from the experimental data as it was performed in [13]. The function \( \mathcal{R}(s, b) \) at the energies \( s > s_0 \) has two different forms depending on the value of impact parameter, namely at \( b > R(s) \) this function has the form

\[ \mathcal{R}(s, b) = \frac{1 - \kappa(s, b)}{2}, \]

while in the region \( b < R(s) \) the presence of the geometric phase (i.e. \( \cos 2\delta = -1 \)) changes this dependence to

\[ \mathcal{R}(s, b) = \frac{1 + \kappa(s, b)}{2} \]

and it means that the hadron scattering is becoming predominantly geometric one at small impact parameters and high energies \( s > s_0 \). Thus, the scattering has a significant absorptive contribution only in the peripheral region of the impact parameters. The following asymptotic limits take place, namely, \( \mathcal{R}(s, b) \to 1 \) when impact parameter \( b \) is fixed and \( s \to \infty \) (i.e. geometric elastic scattering saturates the total cross-section in this limit), while \( \mathcal{R}(s, b) \to 0 \) at fixed energy \( s \) and \( b \to \infty \), i.e. at large impact parameters elastic scattering cross-section decreases faster than the cross-sections of the all inelastic processes, i.e.

\[ \mathcal{R}(s, b) = \mathcal{R}(s, b)/[1 + \mathcal{R}(s, b)], \]

where

\[ \mathcal{R}(s, b) \equiv \frac{\sigma_{el}(s, b)}{\sigma_{inel}(s, b)} \to 0 \]

in this limit. This is evident since

\[ \mathcal{R}(s, b) = f(s, b)/[1 - f(s, b)] \]

and \( f(s, b) \to 0 \) at fixed \( s \) when \( b \to \infty \). It is also clear that

\[ \mathcal{R}(s, b) \to \infty \]

at fixed \( b \) and \( s \to \infty \) in the presence of the above geometric phase.

It should be noted that the ratio \( \mathcal{R}(s, b) \) determines the function \( U(s, b) \), i.e.

\[ U(s, b) = \mathcal{R}(s, b) \]
and can be used as a tool to obtain a qualitative behavior of the function $U(s, b)$ from the experimental data.\(^1\)

The aim of this note was to discuss an issue of the geometric phase in the high energy scattering and to point out to the possible relation of the geometric phase with the presence of the extra dimensions with non-factorizable metrics along with discussion of the relation of this phase to some experimental facts observed or predicted at the LHC energies.

The appearance of the geometric phase at $s > s_0$ provides a change in the slope ("the knee") of the energy spectrum of the cosmic particles observed at the ground level as a result of the cosmic particles interactions with the air\(^5\).

It is the presence of geometric phase at the LHC energies means that geometric elastic scattering becomes dominating in the region of small impact parameters while inelastic processes are predominantly peripheral. It means that albedo is increasing with energy in the high energy hadron scattering at $s > s_0$\(^5\).

The perepherality of the inelastic amplitudes would lead to the ridge and double-ridge effects the the two-particle correlation functions in proton-proton collisions\(^7, 14\).

We also note that the presence of this phase should be interpreted in favor of the RS-scenario of the extra dimensions modeling.

References

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\(^1\)It should be reminded here that we consider for simplicity a case of pure imaginary scattering amplitude. It helps to separate the effects related to the geometric phase from the ones associated with the dynamical phase.
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