Two-level system with broken inversion symmetry coupled to a quantum harmonic oscillator

H.K. Avetissian and G.F. Mkrtchian
Centre of Strong Fields Physics, Yerevan State University, 1 A. Manukian, Yerevan 0025, Armenia

We study the generalized Jaynes-Cummings model of quantum optics at the inversion-symmetry-breaking and in the ultrastrong coupling regime. With the help of a generalized multiphoton rotating-wave approximation, we study the stationary solutions of the Schrödinger equation. It is shown that the problem is reduced to resonant interaction of two position-displaced harmonic oscillators. Explicit expressions for the eigenstates and eigenvalues of generalized Jaynes-Cummings Hamiltonian are presented. We exemplify our physical model with analytical and numerical considerations regarding collapse and revivals of the initial population of a two-level system and photon distribution function at the direct multiphoton resonant coupling.

PACS numbers: 42.50.Hz, 42.50.Pq, 85.25.Hv

I. INTRODUCTION

Two-level system coupled to a quantum harmonic oscillator (e.g., a single radiation mode) as a simple and tractable model has played a central role in many branches of contemporary physics ranging from quantum optics to condensed matter physics. In quantum optics it describes a two level atom resonantly coupled to a single mode electromagnetic radiation [1], so called Jaynes-Cummings (JC) model [2]. It accurately describes trapped ion experiments for quantum informatics [3]. In condensed matter physics we may include here Holstein model [4], graphene in the magnetic field [5] or in the quantized single mode radiation field [6], quantum dots coupled to photonic cavities [7], and circuit quantum electrodynamics (QED) setups where superconducting qubits are coupled to microwave cavities [8]. Even though the underlying setups of mentioned systems are different, the physics is similar to Cavity QED, where first experiments have been done toward the realization of JC model [9]. Cavity QED can be divided into three coupling regimes: weak, strong, and ultrastrong. For weak coupling atom-photon interaction rate is smaller than the atomic and cavity field decay rates. In this case one can manipulate by the spontaneous emission rate compared with its vacuum level by tuning discrete cavity modes [10]. In strong coupling regime, when the emitter–photon interaction becomes larger than the combined decay rate, instead of the irreversible spontaneous emission process coherent periodic energy exchange between the emitter and the photon field in the form of Rabi oscillations takes place [11]. Thanks to recent achievements in solid-state semiconductor [12] or superconductor systems [13] one can achieve ultrastrong coupling regime, where the coupling strength is comparable to appreciable fractions of the mode frequency. In this regime new nonlinear phenomena are visible [14]. Besides, in these setups one can enrich the conventional JC model including new interaction terms inaccessible in conventional Cavity QED setups. One of the new factor which can be incorporated into the JC model is an inversion-symmetry-breaking (ISB). Thus, in the conventional JC model, as well as in the Rabi model with classical radiation field one assumes that the diagonal matrix elements of the dipole moment operator are zero, that is the states possess a certain spatial parity, and the levels are not degenerated. Nevertheless, in various systems of interest, there is intrinsic or extrinsic reasons for ISB. The inversion symmetry of a system can be broken either by a system Hamiltonian or the stationary states may not have this symmetry. As has been shown in Refs. [15, 16], when the quantum system has permanent dipole moments in the stationary states, or the level is degenerated upon orbital momentum there are new multiphoton effects in the quantum dynamics of the system subjected to a strong laser field. Furthermore, these systems have an advantage, which allows to generate radiation with Rabi frequency [17] and moderately high harmonics by optical pulses [18]. For the semiconductor version of JC model one can achieve ISB by the asymmetric quantum dots [19]. In the circuit QED setups it appears naturally as a consequence of internal asymmetry. For flux qubit potential landscape is reduced to a double-well potential [12], for Cooper pair box ISB takes place at setup far from charge degeneracy point [20]. Thus, it is of interest to study the consequence of ISB on the quantized version of Rabi model, where multiphoton effects are expected in the ultrastrong coupling regime.

In the present work we study the effect of ISB on the quantum dynamics of a two-level system interacting with a quantized harmonic oscillator. Particularly, we consider the consequences of the ISB on the eigenstates and eigenenergies of generalized JC Hamiltonian, and on the dynamics of Rabi oscillations, collapse and revival. It is shown that ISB substantially alters the dynamics of the system compared with conventional JC one. Similar to quasiclassical case [16] it is possible direct multiphoton transitions, and as a consequence, there are Rabi oscillations with periodic exchange of several photons between the emitter and the radiation (bosonic) field. We consider ultrastrong coupling regime. Accordingly, the quantum dynamics of the considered system is investigated using
a resonant approximation.

The paper is organized as follows. In Sec. II the model Hamiltonian is presented and diagonalized in the scope of a resonant approximation. In Sec. III we consider temporal quantum dynamics of considered system and present corresponding numerical simulations. Finally, conclusions are given in Sec. IV.

II. BASIC MODEL HAMILTONIAN AND DRESSED STATES PICTURE

Assuming here two level system with ISB coupled to quantum harmonic oscillator, the model Hamiltonian can be written as

\[ \hat{H} = \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \frac{\hbar \omega_0}{2} \hat{\sigma}_z \]

\[ + \hbar (-\lambda_g \hat{\sigma}_+ + \lambda_e \hat{\sigma}_- + \lambda_{eg} \hat{\sigma}_x) (\hat{a}^\dagger + \hat{a}). \]  

The first two terms in Eq. (1) correspond to the free harmonic oscillator of frequency \( \omega \) and two level system with the transition frequency \( \omega_0 \), respectively. The final term gives the interaction between the oscillator and two level system. Creation and annihilation operators, \( \hat{a}^\dagger \) and \( \hat{a} \), satisfy the bosonic commutation rules, \( \hat{\sigma}_+ \), \( \hat{\sigma}_- \) are Pauli operators, \( \hat{\sigma}_+ = (\hat{I} + \hat{\sigma}_z) / 2 \) and \( \hat{\sigma}_- = (\hat{I} - \hat{\sigma}_z) / 2 \) are projection operators and are the result of ISB. These terms distinguish the systems being considered from conventional JC model. At \( \lambda_g = \lambda_e = 0 \), one will obtain usual Hamiltonian for JC model (including also counter-rotating terms) with coupling \( \hbar \lambda_{eg} \). In the case of atoms/molecules and quantum dots \( \lambda_g \) and \( \lambda_e \) correspond to mean dipole moments in states of indefinite parity, while \( \lambda_{eg} \) corresponds to transition dipole moment.

In the case of circuit QED see Refs. [13, 20]. Without loss of generality we have assumed that ground and excited states have mean dipole moments of opposite signs, and \( \lambda_e, \lambda_g \geq 0 \).

At first we will diagonalize the Hamiltonian (1), which is straightforward in the dressed states picture. As JC model our model does not admit exact analytical solution. One of the most powerful approximations for the solution of JC model is the resonant or so called rotating-wave approximation (RWA), which is valid at near-resonance \( |\omega_0 - \omega| << \omega \) and weak coupling between the two systems \( |\lambda_{eg}| << \omega_0 \). For our model generalized multiphoton RWA is needed. The first step is to rewrite Hamiltonian (1) in the form

\[ \hat{H} = \hat{H}_0 + \hat{V}, \]  

where \( \hat{H}_0 = \hat{H}_\uparrow \otimes \hat{\sigma}_+ + \hat{H}_\downarrow \otimes \hat{\sigma}_- \) represents two non-coupled position-displaced oscillators:

\[ \hat{H}_\uparrow = \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) + \frac{\hbar \omega_0}{2} \hat{\sigma}_z + \hbar \lambda_e (\hat{a}^\dagger + \hat{a}). \]

\[ \hat{H}_\downarrow = \hbar \omega \left( \hat{a}^\dagger \hat{a} + \frac{1}{2} \right) - \frac{\hbar \omega_0}{2} - \hbar \lambda_g (\hat{a}^\dagger + \hat{a}), \]  

and

\[ \hat{V} = \hbar \lambda_{eg} \hat{\sigma}_x (\hat{a}^\dagger + \hat{a}). \]

is the interaction part. Hamiltonians (3) and (4) admit exact diagonalization. Schematic illustration of the two position-displaced harmonic oscillators with coupling \( \hat{V} \) is given in Fig. 1. It is easy to see that in each well the eigenstates are

\[ | \uparrow, N(\lambda_\uparrow) \rangle \equiv | \uparrow \rangle \otimes e^{-(\lambda_\uparrow \omega) (\hat{a}^\dagger - \hat{a})} | N \rangle, \]

\[ | \downarrow, N(\lambda_\downarrow) \rangle \equiv | \downarrow \rangle \otimes e^{(\lambda_\downarrow \omega) (\hat{a}^\dagger - \hat{a})} | N \rangle, \]

with energies

\[ E_{eN} = \frac{\hbar \omega_0}{2} + \hbar \omega (N + \frac{1}{2}) - \hbar \lambda_e^2 \omega, \]

\[ E_{gN} = -\frac{\hbar \omega_0}{2} + \hbar \omega (N + \frac{1}{2}) - \hbar \lambda_g^2 \omega. \]  

Hear \( D(\alpha) = e^{\alpha (\hat{a}^\dagger - \hat{a})} \) is the displacement operator and quantum number \( N = 0, 1, ..., \). The states \( | \uparrow \rangle, | \downarrow \rangle \) are eigenstates of \( \hat{\sigma}_z \) and the states \( | N(\lambda_\uparrow) \rangle, | N(\lambda_\downarrow) \rangle \) are position-displaced Fock states:

\[ | N(\lambda_\uparrow) \rangle = e^{-(\lambda_\uparrow \omega) (\hat{a}^\dagger - \hat{a})} | N \rangle = \sum_M I_{N,M} \left( \frac{\lambda_\uparrow^2}{\omega^2} \right) | M \rangle, \]

\[ | N(\lambda_\downarrow) \rangle = e^{(\lambda_\downarrow \omega) (\hat{a}^\dagger - \hat{a})} | N \rangle = \sum_M I_{M,N} \left( \frac{\lambda_\downarrow^2}{\omega^2} \right) | M \rangle, \]

where \( I_{N,M}(\alpha) \) is the Lagger function and defined via...
generalized Laguerre polynomials $L_n^l (\alpha)$ as follows:

$$I_{s,s'} (\alpha) = \sqrt{\frac{su^d}{s!}} e^{-\frac{\alpha}{2}} \alpha \frac{d^l}{d\alpha^l} I_{s'-s}^s (\alpha) = (-1)^{s-s'} I_{s',s} (\alpha),$$

$$L_n^l (\alpha) = \frac{1}{n!} e^{\alpha x_0} \frac{d^n}{dx_0^n} \left( e^{-\alpha x_0} x^n \right).$$  (9)

Particularly, $|0^{(\lambda_n)}\rangle$ and $|0^{(\lambda_p)}\rangle$ are the Glauber or coherent states with mean number of photons $\lambda_n / \omega$ and $\lambda_p / \omega$. Thus, we have two ladders shifted by the energy:

$$\hbar \omega_{eg} = \hbar (\omega_0 + \lambda_n^2 / \omega - \lambda_p^2 / \omega).$$  (10)

The coupling term $\hat{V}$ induces transitions between these two manifolds. At the resonance:

$$\omega_{eg} - \omega n = \delta_n; |\delta_n| << \omega$$  (11)

with $n = 1, 2, \ldots$ the equidistant ladders are crossed: $E_{eN} \simeq E_{gN+n}$, and the energy levels starting from the ground state of upper harmonic oscillators are nearly degenerated. The coupling $\hat{V}$ removes this degeneracy, leading to symmetric and asymmetric entangled states. The splitting of levels is defined by the vacuum multiphoton Rabi frequency. In this case we should apply secular perturbation theory resulting:

$$|\alpha, N\rangle = \left( C^\alpha_+ |\downarrow, N^{(\lambda_p)}\rangle + C^\alpha_+ |\uparrow, (N-n)^{(\lambda_n)}\rangle \right),$$

$$E_{\alpha, N} = \frac{1}{2} (E_{gN} + E_{eN-n}) + \alpha \sqrt{\frac{\delta_n^2}{4} + |V_N (n)|^2},$$  (12)

where $\alpha = \pm; C^\alpha_- \text{ and } C^\alpha_+$ are constant with ratio

$$C^\alpha_+ / C^\alpha_- = V_N (n) / (E_{\alpha, N} - E_{eN-n}),$$

and transition matrix element is:

$$V_N (n) = \langle \downarrow, N^{(\lambda_p)} | \hat{V} | \uparrow, (N-n)^{(\lambda_n)} \rangle = \hbar \lambda_n \left( \frac{\lambda_n - \lambda_p}{\omega} - \frac{n \omega}{\lambda_n + \lambda_p} \right) I_N^{N-n, N} \left( \frac{\lambda_n \lambda_p}{\omega^2} \right)^2.$$  (13)

For the exact resonance, starting from the $N = n$ we have symmetric and asymmetric entangled states

$$|\pm, N\rangle = \left( |\downarrow, N^{(\lambda_p)}\rangle \pm |\uparrow, (N-n)^{(\lambda_n)}\rangle \right) / \sqrt{2}$$  (14)

with energies $E_{\pm, N} = E_{gN} \pm |V_N (n)|$, while for $N = 0, 1, \ldots, n-1$, we have eigenstates $|\downarrow, N^{(\lambda_p)}\rangle$ and energy $E_{<N}$. For the conventional JC model there is a selection rule: $V_N (n) \neq 0$ only for $n = \pm 1$. This also follows from Eq. (15) in the limit $\lambda_n, \lambda_p \rightarrow 0$. That is why in that case only one photon Rabi oscillations take place. In our model due to ISB there are transition with arbitrary $n$ giving rise to multiphoton coherent transitions. Besides, at the $\lambda_p \neq 0$ in the ground state $|\downarrow, n\rangle \otimes |0^{(\lambda_p)}\rangle$ bosonic field is in the coherent state. The solutions (12) are valid at near multiphoton resonance $\omega_{eg} \simeq n \omega$ and weak coupling $|V_N (n)| << \omega$. The latter condition implies that for the multiphoton resonant transitions, systems with large dipole moments $(|\lambda_e + \lambda_g| >> |\lambda_{eg}|)$ are preferable.

III. MULTIPHOTON RABI OSCILLATIONS

Let us now consider the quantum dynamics of the two-level system and harmonic oscillator starting from an initial state, which is not an eigenstate of the Hamiltonian $H$. This is of particular interest for applications in quantum information processing. Assuming arbitrary initial state $|\Psi_0\rangle$ of a system, then the state vector for times $t > 0$ is just given by the expansion over dressed state basis obtained above:

$$|\Psi (t)\rangle = \sum_{N=0}^{n-1} |\downarrow, N^{(\lambda_p)}\rangle |\Psi_0\rangle e^{-i \omega_{eg} t} + \sum_{\alpha = \pm} \sum_{N=n}^{\infty} |\alpha, N\rangle |\Psi_0\rangle e^{-i \omega_{\alpha, N} t} |\alpha, N\rangle.$$  (15)

For concreteness we will consider two common initial conditions for harmonic oscillator: the Fock state and the coherent state. We will calculate the time dependence of the two level system population inversion $W_n (t) = \langle \Psi (t) | \hat{P} | \Psi (t) \rangle$ at the exact $n$-photon resonance (14) $\delta_n = 0$. For the field in the Fock state and two level system in the excited state $|\Psi_0\rangle = |\uparrow, 0\rangle$, we have

$$W_n (t) = \sum_{N=0}^{\infty} I_{N,0}^2 \left( \frac{\lambda_e^2}{\omega_0^2} \right) \cos \left( \Omega_{N+n} (n) t \right),$$  (16)

where $\Omega_{N} (n) = 2 |V_N (n)| / \hbar$ is the multiphoton vacuum Rabi frequency. For $\lambda_e^2 << \omega^2$ the main contribution in the sum (16) comes from the first term: $W_n (t) \simeq \cos (\Omega_{n} (n) t)$, which corresponds to Rabi oscillations with periodic exchange of $n$ photons between the two-level system and the radiation (bosonic) field.

Finally we turn to the case in which a two-level system begins in the ground state, while oscillator prepared in a coherent state with a mean excitation (photon) number $N$. From Eq. (3) follows that such state can be represented as $|\Psi_0\rangle = |\downarrow, 0\rangle \otimes |\lambda'_N\rangle$, where $\lambda'_N = N \omega$. Taking into account Eqs. (6) and (8), for population inversion we obtain

$$W_n (t) = 1 + 2 \sum_{N=n}^{\infty} I_{N,0}^2 (\rho) \sin^2 \frac{\Omega_N (n) t}{2},$$  (17)

where $\rho = (N + \lambda_g / \omega)^2$. In this case we have collapse and revival phenomenon of the multiphoton Rabi oscillations.

In this section, we also present numerical solutions of the time dependent Schrödinger equation with the full Hamiltonian (14) in the Fock basis considering up to $N_{\text{max}} = 200$ excitations. The set of equations for the probability amplitudes has been solved using a standard fourth-order Runge–Kutta algorithm [21].

In Figs. (2) and (3) photon number probability

$$P_N (t) = \langle \uparrow, N | \langle \Psi (t) \rangle | \Psi (t) \rangle |\uparrow, N\rangle$$
as a function of time is shown for two and three photon resonances. For an initial state we assume two level system in the excited state and the field in vacuum state - $|\uparrow\rangle \otimes |0\rangle$. As is seen only resonant multiphoton Fock states are excited. In Fig. 4 we show collapse and revival of the multiphoton Rabi oscillations. Two level system population inversion $W_n(t)$ is shown with the field initially in a coherent state. (a) Two-photon resonance with coupling parameters $\lambda_{eg}/\omega = 0.02$, $\lambda_g/\omega = 0$, and $\lambda_e/\omega = 0.1$ and mean photon number $N = 20$. (b) Three-photon resonance with parameters - $\lambda_{eg}/\omega = 0.02$, $\lambda_g/\omega = -0.1$, $\lambda_e/\omega = 0.1$ and mean photon number $N = 30$. (c) Same as (b) but for four-photon resonance and $N = 60$.

FIG. 5: Density plot of photon number probability distribution $P_N(t)$ (in arbitrary units) as a function of photon number and time (in units of oscillator period $T = 2\pi/\omega$) corresponding to setup of Fig. 4(a).

IV. CONCLUSION

We have presented a theoretical treatment of the quantum dynamics of a two-level system with ISB interacting with a quantized harmonic oscillator in the ultrastrong coupling regime. With the help of a resonant approach, we have solved the Schrödinger equation and obtained simple analytical expressions for the eigenstates and eigenenergies. The obtained results show that the effect of ISB on the quantum dynamics is considerable. For the $n$-photon resonance in addition to $n$ non-entangled states we have symmetric and asymmetric entangled states of a two level system and position-displaced Fock
states. The ground state is a not entangled, but the bosonic field may be in a coherent state. We have also investigated the temporal quantum dynamics of considered system and showed that similar to one-photon case due to ISB it is possible Rabi oscillations, collapse and revival of initial population with periodic multiphoton exchange between the two-level system and the radiation field. The proposed model may have diverse applications in Cavity QED experiments, especially in the variant of circuit QED, where the considered parameters are already accessible.

Acknowledgments

This work was supported by State Committee of Science (SCS) of Republic of Armenia (RA), Project No. 13RF-002.

[1] M. O. Scully, M. S. Zubairy, Quantum Optics (Cambridge University Press, Cambridge, U.K., 1997).
[2] E. T. Jaynes, F. W. Cummings, Proc. IEEE 51, 89 (1963).
[3] D. Leibfried, R. Blatt, C. Monroe, D. Wineland, Rev. Mod. Phys. 75, 281 (2003).
[4] T. Holstein, Ann. Phys. (N.Y.) 8, 325 (1959).
[5] B. Dóra, K. Ziegler, P. Thalmeier, M. Nakamura, Phys. Rev. Lett. 102, 036803 (2009).
[6] O. V. Kibis, Phys. Rev. B 81, 165433 (2010).
[7] E. Peter et al., Phys. Rev. Lett. 95, 067401 (2005); K. Hennessy et al., Nature 445, 896 (2007).
[8] M. O. Scully, M. S. Zubairy, Quantum Optics (Cambridge University Press, Cambridge, U.K., 1997).
[9] E. T. Jaynes, F. W. Cummings, Proc. IEEE 51, 89 (1963).
[3] D. Leibfried, R. Blatt, C. Monroe, D. Wineland, Rev. Mod. Phys. 75, 281 (2003).
[4] T. Holstein, Ann. Phys. (N.Y.) 8, 325 (1959).
[5] B. Dóra, K. Ziegler, P. Thalmeier, M. Nakamura, Phys. Rev. Lett. 102, 036803 (2009).
[6] O. V. Kibis, Phys. Rev. B 81, 165433 (2010).
[7] E. Peter et al., Phys. Rev. Lett. 95, 067401 (2005); K. Hennessy et al., Nature 445, 896 (2007).
[8] A. Wallraff et al., Nature 431, 162 (2004).
[9] J. M. Raimond, M. Brune, S. Haroche, Rev. Mod. Phys. 73, 565 (2001).
[10] F. Goy, J. M. Raimond, M. Gross, S. Haroche, Phys. Rev. Lett. 50, 1903 (1983).
[11] H. Walther, B. T. H. Varcoe, B. G. Englert, T. Becker, Rep. Prog. Phys. 69, 1325 (2006).
[12] G. Günter et al., Nature 458, 178 (2009).
[13] T. Niemczyk et al., Nature Physics 6, 772 (2010).
[14] J. Casanova, G. Romero, I. Lizuain, J.J. Garcia-Ripoll, E. Solano, Phys. Rev. Lett. 105, 263603 (2010); D. Ballester et al., Physical Review X 2, 021007 (2012); G. Romero, D. Ballester, Y. M. Wang, V. Scarani, E. Solano, Phys. Rev. Lett. 108, 120501 (2012); Shu He et al., Phys. Rev A 86, 033837 (2012).
[15] A. Brown, W. J. Meath, P. Tran, Phys. Rev. A 63, 013403 (2000); ibid. 65, 063401 (2002).
[16] H. K. Avetissian, G. F. Mkrtchian, Phys. Rev. A 66, 033403 (2002); H. K. Avetissian, G. F. Mkrtchian, M. G. Poghosyan, ibid. 73, 063413 (2006); H. K. Avetissian, B. R. Avchyan, G. F. Mkrtchian, ibid. 74, 063413 (2006).
[17] O. V. Kibis, G. Y. Slepyan, S. A. Maksimenko, and A. Hoffmann, Phys. Rev. Lett. 102, 023601 (2009); H. K. Avetissian, B. R. Avchyan, G. F. Mkrtchian, Phys. Rev. A 82, 063412 (2010).
[18] H. K. Avetissian, B. R. Avchyan, G. F. Mkrtchian, Phys. Rev. A 77, 023409 (2008); J. Phys. B 45, 025402 (2012).
[19] A. Balandin, K. L. Wang, Superlattices Microstruct 25, 509 (1999).
[20] A. Blais, R.S. Huang, A. Wallraff, S.M. Girvin, R. J. Schoelkopf, Phys. Rev. A 69, 062320 (2004).
[21] W. H. Press, S. A. Teukolsky, W. T. Vetterling, B. P. Flannery, Numerical Recipes in C (Cambridge University Press, Cambridge, U.K., 1992).