Ergodic Sum-Rate Maximization for Fading Cognitive Multiple Access Channels without Successive Interference Cancellation

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Abstract

In this paper, the ergodic sum-rate of a fading cognitive multiple access channel (C-MAC) is studied, where a secondary network (SN) with multiple secondary users (SUs) transmitting to a secondary base station (SBS) shares the spectrum band with a primary user (PU). An interference power constraint (IPC) is imposed on the SN to protect the PU. Under such a constraint and the individual transmit power constraint (TPC) imposed on each SU, we investigate the power allocation strategies to maximize the ergodic sum-rate of a fading C-MAC without successive interference cancellation (SIC). In particular, this paper considers two types of constraints: (1) average TPC and average IPC, (2) peak TPC and peak IPC. For the first case, it is proved that the optimal power allocation is dynamic time-division multiple-access (D-TDMA), which is exactly the same as the optimal power allocation to maximize the ergodic sum-rate of the fading C-MAC with SIC under the same constraints. For the second case, it is proved that the optimal solution must be at the extreme points of the feasible region. It is shown that D-TDMA is optimal with high probability when the number of SUs is large. Besides, we show that, when the SUs can be sorted in a certain order, an algorithm with linear complexity can be used to find the optimal power allocation.

Index Terms

Cognitive Radio, Multiple Access Channel, Fading Channels, Spectrum Sharing, Ergodic Sum-Rate, Optimal Power Allocation, Non-convex Optimization.

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I. INTRODUCTION

The demand for frequency resources has dramatically increased due to the explosive growth of wireless applications and services in recent years. This poses a big challenge to the current fixed spectrum allocation policy. On the other hand, a report published by Federal Communications Commission (FCC) shows that the current scarcity of spectrum resource is mainly due to the inflexible spectrum regulation policy rather than the physical shortage of spectrum [1]. Most of the allocated frequency bands are under-utilized, and the utilization of the spectrum varies in time and space. Similar observations have also been made in other countries. In particular, the spectrum utilization efficiency is shown to be as low as 5% in Singapore [2]. The compelling need to improve the spectrum utilization and establish more flexible spectrum regulations motivates the advent of cognitive radio (CR). Compared to the traditional wireless devices, CR devices can greatly improve the spectrum utilization by dynamically adjusting their transmission parameters, such as transmit power, transmission rate and the operating frequency. Recently, FCC has agreed to open the licensed, unused television spectrum or the so-called white spaces to the new, unlicensed, and sophisticatedly designed CR devices. This milestone change of policy by the FCC indicates that CR is fast becoming one of the most promising technologies for the future radio spectrum utilization. This also motivates a wide range of research in the CR area, including the research work done in this paper.

A popular model widely adopted in CR research is the spectrum sharing model. In a spectrum-sharing CRN, a common way to protect primary users (PU) is to impose an interference power constraint (IPC) at the secondary network, which requires the interference received at PU receiver to be below a prescribed threshold [3]. Subject to such a IPC, the achievable rates of Additive White Gaussian Noise (AWGN) channels were investigated in [4]. In [5], the authors studied the ergodic capacity of a single-user CRN under IPC in different fading environment. In [6], the authors studied the outage performance of such a single-user spectrum-sharing CRN under a IPC. In [7], the authors studied the capacity and power allocation for a spectrum-sharing fading CRN under both peak and average IPC. In [8], the optimal power allocation strategies to achieve the ergodic and outage capacity for a spectrum-sharing fading CRN under different combinations of the transmit power constraint (TPC) and the IPC were investigated. However, the aforementioned works only focused on the point-to-point secondary networks. In [9], from an information
theoretic perspective, the authors investigated the achievable rate region of a Gaussian C-MAC. In [10] and [11], the authors investigated the optimal power allocation strategies for AWGN cognitive multiple access channels (C-MAC). In [12], the authors investigated the ergodic sum capacity for a fading C-MAC with multiple PUs. In [13], the authors studied the outage capacity region for a fading C-MAC. However, in these works, successive interference cancellation (SIC) decoders are assumed to be available, and thus no mutual interference among the secondary users (SU) is considered. Different from the aforementioned works, in this paper, we study the ergodic sum-rate and the corresponding optimal power allocation strategies of a fading C-MAC without SIC. Compared with the previous studies with SIC, the problem studied in this paper is much harder due to the existence of the mutual interference among SUs, which makes the problem a nonlinear, nonconvex constrained optimization problem.

Another line of related research [14]–[17] focused on the sum-rate maximization for MAC under non-CR setting (without IPC). In [14], the authors investigated the ergodic capacity region and its optimal power allocation for the fading MAC. In [15], the authors proposed the iterative water-filling algorithm to maximize the sum-rate of a multiple-input multiple-out (MIMO) MAC with SIC under individual power constraints. For sum-rate maximization of MAC without SIC, in [16], the authors were able to show the optimality of the binary power allocation for a two-user network. For arbitrary users, the authors only numerically illustrated the optimality of binary power allocation. While in [17], the authors analytically proved that binary power allocation is optimal for any number of users in terms of maximizing the sum-rate of the MAC without SIC. Compared with these works, the problem studied in this paper is more challenging due to the existence of the IPC, which changes the properties of the optimal power allocation. It is shown that binary power allocation is no longer optimal for our problem.

The main contribution and the key results of this paper are listed as follows:

- We investigate the optimal power allocation strategies to maximize the ergodic sum-rate of a fading C-MAC without SIC under both TPC and IPC. In particular, we consider two types of constraints: (1) average TPC and average IPC, (2) peak TPC and peak IPC.
- For the average TPC and average IPC case, we prove that the optimal power allocation is dynamic time-division multiple-access (D-TDMA), which is exactly the same as the optimal power allocation given in [12] to maximize the ergodic sum-rate of the fading C-MAC with SIC under the same constraints.
For the peak TPC and peak IPC case, we prove that the optimal solution must be at the extreme points of the feasible region. We show that D-TDMA is optimal when a certain condition is satisfied. It is also shown that D-TDMA is optimal with high probability when the number of SUs is large. Thus, we can solve the problem by searching the extreme points of the feasible region when the number of SUs is small, and by applying the D-TDMA scheme when the number of SUs is large.

For the peak TPC and peak IPC case, we show that when the SUs can be sorted in a certain order, an algorithm with linear complexity can be developed to find the optimal power allocation of our problem.

For the peak TPC and peak IPC case, we also show by simulations that the optimal power allocation to maximize the ergodic sum-rate of the fading C-MAC with SIC, which we refer to as SIC-OP, can be used as a good suboptimal power allocation for our problem. It is shown by simulations that SIC-OP is optimal or near-optimal for our problem when the D-TDMA is not optimal.

The rest of the paper is organized as follows. The system model and power constraints are described in Section II. The optimal power allocation strategies to maximize the ergodic sum-rate of the fading C-MAC without SIC are studied in Section III. Then, the simulation results are presented and analyzed in Section IV. Section V concludes the paper.

II. SYSTEM MODEL AND POWER CONSTRAINTS

A. System Model

In this paper, we consider a spectrum sharing CR network consists of one PU and a $K$-user secondary multiple access network. The communication links between each SU and the PU receiver (PU-Rx) are referred as the interference links. The links between the SUs and the secondary base station (SBS) are referred as the secondary links. For the convenience of exposition, all the channels involved are assumed to be block-fading (BF) [18], i.e., the channels remain constant during each transmission block, but possibly change from one block to another. As shown in Fig.1, the channel power gain of the interference link between SU-$i$ and the PU is denoted by $g_i$. The channel power gain of the secondary link between SU-$i$ and the SBS is denoted as $h_i$. All these channel power gains are assumed to be independent and identically distributed (i.i.d.) random variables (RVs) each having a continuous probability density function.
All the channel state information (CSI) is assumed to be perfectly known at both SUs. CSI of the secondary links can be obtained at SUs by the classic channel training, estimation, and feedback mechanisms. CSI of the interference links between SUs and primary receivers can be obtained at SUs via the cooperation of the primary receivers. The noise at SBS is assumed to be circular symmetric complex Gaussian variable with zero mean and variance $\sigma^2$ denoted by $\mathcal{CN}(0, \sigma^2)$.

### B. Power Constraints

In this paper, we denote the transmit power of SU-$i$ as $P_i$, then the instantaneous interference received at PU-Rx from SU-$i$ is $g_iP_i$. Then, the average and peak interference power constraint (IPC) can be described as

\[
\text{Average IPC: } \mathbb{E}\left[\sum_{i=1}^{K} g_i P_i\right] \leq I_{av}, \quad (1)
\]

\[
\text{Peak IPC: } \sum_{i=1}^{K} g_i P_i \leq I_{pk}, \quad (2)
\]

where $I_{av}$ denotes the limit of average received interference at the PU, and $I_{pk}$ denotes the maximum instantaneous interference that the PU can tolerate. $\mathbb{E}[\cdot]$ denotes the statistical expectation over all the involved fading channel power gains. Usually, the average IPC is used to guarantee the long-term QoS of the PU when it provides delay-insensitive services. When the service provided by the PU has an instantaneous QoS requirement, the peak IPC is usually adopted.

In this paper, we also consider the transmit power constraint (TPC) imposed at each SU. Same as the IPC, two types (both average and peak) of TPC are considered here. Let $P_{iav}$ and $P_{ipk}$ be the average and peak transmit power limit of SU-$i$, respectively. Then, the average and peak TPC can be described as

\[
\text{Average TPC: } \mathbb{E}[P_i] \leq P_{iav}, \quad \forall i, \quad (3)
\]

\[
\text{Peak TPC: } P_i \leq P_{ipk}, \quad \forall i, \quad (4)
\]

where $\mathbb{E}[\cdot]$ denotes the statistical expectation over all the involved fading channel power gains. The peak power limitation is usually due to the nonlinearity of power amplifiers in practice. The average TPC is usually imposed to meet a long-term transmit power budget.
III. ERGODIC SUM-RATE MAXIMIZATION FOR FADING C-MAC WITHOUT SIC

Without SIC decoders available at the SBS, the instantaneous transmission rate of each SU is given by

\[ r_i = \ln \left( 1 + \frac{h_i P_i}{\sum_{j=1, j\neq i}^{K} h_j P_j + \sigma^2} \right), \forall i. \]  

(5)

For BF channels, ergodic rate is defined as the maximum achievable rate averaged over all the fading blocks. Then, the ergodic sum-rate of the fading C-MAC considered in this paper can be written as

\[ E \left[ \sum_{i=1}^{K} \ln \left( 1 + \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j\neq i}^{K} h_j P_j} \right) \right]. \]  

(6)

In the following, we study power allocation strategy to maximize the ergodic sum-rate of the fading C-MAC subject to the power constraints given in Section II-B.

A. AVERAGE TPC AND AVERAGE IPC

Under average TPC and average IPC, the optimal power allocation to maximize the ergodic sum-rate of the fading C-MAC can be obtained by solving the following optimization problem:

**Problem 1:**

\[ \max_{P_i \geq 0, \forall i} \mathbb{E} \left[ \sum_{i=1}^{K} \ln \left( 1 + \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j\neq i}^{K} h_j P_j} \right) \right], \]  

(7)

s.t. (1), (3).

(8)

It is not difficult to observe that Problem 1 is a non-convex optimization problem. Thus, we cannot solve it by the standard convex optimization techniques. To solve Problem 1, we first look at the following problem.

**Problem 2:**

\[ \max_{P_i \geq 0, \forall i} \mathbb{E} \left[ \ln \left( 1 + \frac{\sum_{i=1}^{K} h_i P_i}{\sigma^2} \right) \right], \]  

(9)

s.t. (1), (3).

(10)

Problem 2 gives the ergodic sum-rate for fading C-MAC with SIC, and it has been studied in [12]. It is shown in [12] (Lemma 3.1) that the optimal solution of Problem 2 is: at most one
user is allowed to transmit in each fading block. Based on this fact, we obtain the following theorem.

**Theorem 1:** The optimal solution of Problem 1 is the same as that of Problem 2

**Proof:** It is observed that the constraints of Problem 1 and Problem 2 are exactly the same. Thus, the feasible sets of Problem 1 and Problem 2 are the same. Now, suppose \( P = [P_1, P_2, \cdots, P_K]^T \) is a feasible solution of Problem 1. The rest of the proof consists of two steps.

**Step 1:** Since \( P \) is a feasible solution of Problem 1, it is also a feasible solution of Problem 2. Now, we show that the value of the objective function of Problem 2 under \( P \) is an upper-bound of that of Problem 1 under the same \( P \), i.e.,

\[
\mathbb{E} \left[ \ln \left( 1 + \sum_{j=1}^K \frac{h_j P_j}{\sigma^2} \right) \right] \leq \mathbb{E} \left[ \ln \left( 1 + \sum_{j=1}^K \frac{h_j P_j}{\sigma^2} \right) \right].
\]

Since the expectation operation is linear, it is equivalent to show that

\[
\ln \left( 1 + \sum_{i=1}^K h_i P_i \right) = \ln \left( \frac{\sigma^2 + \sum_{i=1}^K h_i P_i}{\sigma^2} \right)
\]

\[
= \ln \left[ \frac{\sigma^2 + \sum_{i=2}^K h_i P_i}{\sigma^2 + \sum_{i=2}^K h_i P_i} \right] \cdot \left( \frac{\sigma^2 + \sum_{i=3}^K h_i P_i}{\sigma^2 + \sum_{i=3}^K h_i P_i} \right) \cdots \left( \frac{\sigma^2 + \sum_{i=K}^K h_i P_i}{\sigma^2 + \sum_{i=K}^K h_i P_i} \right)
\]

\[
= \sum_{j=1}^K \ln \left( \frac{\sigma^2 + \sum_{i=j}^K h_i P_i}{\sigma^2 + \sum_{i=j+1}^K h_i P_i} \right)
\]

\[
= \sum_{j=1}^K \ln \left( 1 + \frac{h_j P_j}{\sigma^2 + \sum_{i=j+1}^K h_i P_i} \right)
\]

\[
\geq \sum_{j=1}^K \ln \left( 1 + \frac{h_j P_j}{\sigma^2 + \sum_{i=1, i \neq j}^K h_i P_i} \right), \quad (11)
\]

where we introduce a dumb item \( \sum_{i=K+1}^K h_i P = 0 \) in the equality “a” for notational convenience. The inequality “b” follows from the fact that \( \sum_{i=1, i \neq j}^K h_i P_i \geq \sum_{i=j+1}^K h_i P_i, \forall j \).

**Step 2:** Now, we show that the optimal solution of Problem 1 is the same as that of Problem 2. Since it is proved in [12] (Lemma 3.1) that the optimal solution of Problem 2 is: at most one user is allowed to transmit in each fading block. It is easy to observe that the optimal solution of Problem 2 is a feasible solution of Problem 1. Since we have shown in Step 1 that Problem 2 provides an upper-bound of Problem 1 for the same \( P \). Thus, it is easy to observe that the optimal solution of Problem 1 must be the same as that of Problem 2, which is: at most one user is allowed to transmit in each fading block.
Since in Theorem 1 we have shown that the optimal solution of Problem 1 is the same as that of Problem 2. Thus, the optimal power allocation strategies for Problem 1 can be obtained in the same way as [12]. Interested readers can refer to Lemma 3.1 and 3.2 in [12] for details.

B. Peak TPC and Peak IPC

Under peak TPC and peak IPC, the optimal power allocation to maximize the ergodic sum-rate of the fading C-MAC can be obtained by solving the following optimization problem:

**Problem 3:**

\[
\begin{align*}
\max_{P_i \geq 0, \forall i} & \quad \mathbb{E} \left[ \sum_{i=1}^{K} \ln \left( 1 + \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^{K} h_j P_j} \right) \right], \\
\text{s.t.} & \quad (2), (4).
\end{align*}
\]

(12)

Since all the constraints involved are instantaneous power constraints, Problem 3 can be decomposed into a series of identical subproblems each for one fading state, which is

**Problem 4:**

\[
\begin{align*}
\max_{P_i \geq 0, \forall i} & \quad \sum_{i=1}^{K} \ln \left( 1 + \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^{K} h_j P_j} \right), \\
\text{s.t.} & \quad (2), (4).
\end{align*}
\]

(14)

(15)

It can be verified that Problem 4 is non-convex. Thus, we cannot solve it directly by the standard convex optimization techniques. To solve Problem 4 we first investigate its properties.

**Lemma 1:** The optimal solution \( P^* \) of Problem 4 must be at the boundary of the feasible region of Problem 4.

**Proof:** This can be proved by contradiction. Suppose the optimal solution \( P^* \) of Problem 4 is in the interior of the feasible region, i.e., \( 0 < P_i^* < P_i^{pk}, \forall i \) and \( \sum_{i=1}^{K} g_i P_i^* < I \).

Now, we look at the power allocation \( P_n \) of SU-n. For convenience, we denote (14) as \( f(P) \). Then, \( f(P) \) can be rewritten as

\[
\begin{align*}
f(P) = \ln \left( 1 + \frac{h_n P_n}{\sigma^2 + \sum_{j=1, j \neq n}^{K} h_j P_j} \right) + \sum_{i=1, i \neq n}^{K} \ln \left( 1 + \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^{K} h_j P_j} \right).
\end{align*}
\]

(16)
Taking the derivative of $f(P)$ with respect to $P_n$, we have
\[
\frac{\partial f(P)}{\partial P_n} = \frac{1}{1 + \frac{h_n P_n}{\sigma^2 + \sum_{j=1, j \neq n}^K h_j P_j}} \cdot \frac{h_n}{\sigma^2 + \sum_{j=1, j \neq n}^K h_j P_j}
+ \sum_{i=1, i \neq n}^K \frac{1}{1 + \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^K h_j P_j}} \cdot \left( \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^K h_j P_j} \right)^2 \cdot h_n
\]
\[
= \frac{h_n}{\sigma^2 + \sum_{j=1}^K h_j P_j} \left( 1 - \sum_{i=1, i \neq n}^K \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^K h_j P_j} \right).
\] (17)

It is observed that $q(P_n) = 1 - \sum_{i=1, i \neq n}^K \frac{h_i P_i h_n}{\sigma^2 + \sum_{j=1, j \neq i}^K h_j P_j}$ is a strictly increasing function with respect to $P_n$. Then, the solution to $q(P_n) = 0$ is unique. Consequently, the solution to $\frac{\partial f(P)}{\partial P_n} = 0$ is also unique since $\frac{h_n}{\sigma^2 + \sum_{j=1}^K h_j P_j}$ is strictly positive. Denote the solution of $\frac{\partial f(P)}{\partial P_n} = 0$ as $\hat{P}_n$, and we refer to $\hat{P}_n$ as the turning point. Then, on the left side of the turning point, $\frac{\partial f(P)}{\partial P_n}$ is always negative, thus $f(0) > f(P_n), \forall P_n \in [0, \hat{P}_n]$. On the right side of the turning point, $\frac{\partial f(P)}{\partial P_n}$ is always positive, thus $f(P_n) < f(P^*_n), \forall P_n \in [\hat{P}_n, P^*_n]$, where $P^*_n = \min \left\{ \frac{P^*_{nk}}{n}, \left( I - \sum_{i=1, i \neq n}^K g_i P^*_i / h_n \right) \right\}$. Thus, it is clear that the value of $f(P)$ can be increased by moving $P_n$ to the boundary. This contradicts with our assumption that $P^*$ is the optimal solution. Thus, Lemma 1 is proved.

Based on the result of Lemma 1, we are able to obtain the following theorem.

**Theorem 2**: The optimal solution $P^*$ of Problem 4 must be at the extreme point of the feasible region of Problem 4, i.e., at most one user’s power allocation is fractional.

**Proof**: Suppose the optimal solution is $P^*$. Thus, if $\sum_{i=1}^K g_i P^*_i < I$, based the results of Lemma 1, it is clear that $P^*_i, \forall i$ is either equal to 0 or $P^*_{nk}$. Thus, there is no fractional user.

Now, we consider the case that $\sum_{i=1}^K g_i P^*_i = I$. Suppose $P^*_1$ and $P^*_2$ are fractional, i.e., $0 < P^*_i < P^*_{nk}, \forall i \in \{1, 2\}$. The interference constraint can be rewritten as $g_1 P^*_1 + g_2 P^*_2 + \sum_{i=3}^K g_i P^*_i = I$. For convenience, we define $Q \triangleq I - \sum_{i=3}^K g_i P^*_i$. 
First, we consider the case that $\frac{h_1}{g_1} > \frac{h_2}{g_2}$. Under this assumption, we write $P_2^* = (Q - g_1 P_1^*)/g_2$. For convenience, we denote (14) as $f(P)$. Then, $f(P)$ can be rewritten as

$$f(P^*) = \ln \left( 1 + \frac{h_1 P_1^*}{\sigma + \sum_{j=3}^{K} h_j P_j^* + h_2(Q - g_1 P_1^*)/g_2} \right) + \ln \left( 1 + \frac{h_2(Q - g_1 P_1^*)/g_2}{\sigma + \sum_{j=3}^{K} h_j P_j^* + h_1 P_1^*} \right) + \sum_{i=3}^{K} \ln \left( 1 + \frac{h_i P_i^*}{\sigma + \sum_{j=3, j \neq i}^{K} h_j P_j^* + h_1 P_1^* + h_2(Q - g_1 P_1^*)/g_2} \right).$$

(18)

For notation convenience, define $C \triangleq \sigma^2 + \sum_{j=3}^{K} h_j P_j^*$ and $D_i \triangleq \sigma^2 + \sum_{j=3, j \neq i}^{K} h_j P_j^*$, then (18) can be rewritten as

$$f(P^*) = \ln \left( 1 + \frac{h_1 P_1^*}{C + h_2(Q - g_1 P_1^*)/g_2} \right) + \ln \left( 1 + \frac{h_2(Q - g_1 P_1^*)/g_2}{C + h_1 P_1^*} \right) + \sum_{i=3}^{K} \ln \left( 1 + \frac{h_i P_i^*}{D_i + h_1 P_1^* + h_2(Q - g_1 P_1^*)/g_2} \right).$$

(19)

Taking the derivative of $f(P^*)$ with respect to $P_1^*$, we have

$$\frac{\partial f(P^*)}{\partial P_1^*} = \frac{1}{C + h_2 Q + \left( h_1 - \frac{h_2 g_1}{g_2} \right) P_1^*} \left( \frac{h_1 C + h_2 g_1 Q}{C + \frac{h_2 g_1}{g_2} Q - \frac{h_2 g_1}{g_2} P_1^*} - \frac{h_2 g_1}{g_2} \frac{C + \frac{h_2 g_1}{g_2} Q}{C + h_1 P_1^*} \right) - \sum_{i=3}^{K} \frac{\left( h_1 - \frac{h_2 g_1}{g_2} \right) h_i P_i^*}{D_i + \frac{h_2 g_1}{g_2} Q + \left( h_1 - \frac{h_2 g_1}{g_2} \right) P_1^*}.$$  

(20)

Since $\frac{h_1}{g_1} > \frac{h_2}{g_2}$, $\frac{\partial f(P^*)}{\partial P_1^*}$ is a strictly increasing function with respect to $P_1^*$. Thus, the solution to $\frac{\partial f(P^*)}{\partial P_1^*} = 0$ is unique. Denote the solution of $\frac{\partial f(P^*)}{\partial P_1^*} = 0$ as $\tilde{P}_1^*$, and we refer to $\tilde{P}_1^*$ as the turning point. Then, on the left side of the turning point, $\frac{\partial f(P^*)}{\partial P_1^*}$ is always negative, thus $f(0) > f(P_1), \forall P_1 \in [0, \tilde{P}_1^*]$. On the right side of the turning point, $\frac{\partial f(P^*)}{\partial P_1^*}$ is always positive, thus $f(P_1) < f(P_1^{pk}), \forall P_1 \in [\tilde{P}_1^*, P_1^{pk}]$. Thus, it is clear that the value of $f(P^*)$ can be increased by moving $P_1^*$ to 0 or $P_1^{pk}$. This contradicts with our assumption that $P^*$ is the optimal solution.

Now, we consider the case that $\frac{h_1}{g_1} < \frac{h_2}{g_2}$. For this case, we can write $P_1^* = (Q - g_2 P_2^*)/g_1$. Then, using the same approach, we can show that the value of $f(P^*)$ can be increased by moving $P_2^*$ to 0 or $P_2^{pk}$.

Combining the above results, it is observed that at most one user’s power allocation can be fractional. Theorem 2 is thus proved.

Based on Theorem 2, we can easily find the optimal solution $P^*$ of Problem 4 by searching the extreme points when the number of SUs is relatively small. However, when the number of SUs
is large, this scheme may not be practical due to the high computing complexity. Fortunately, we are able to show that with high probability, the optimal solution is D-TDMA when the number of SUs is large. This is given in Theorem 3.

To prove Theorem 3 we need the following lemma.

**Lemma 2:** The optimal solution $P^*$ of Problem 4 is $P^*_k = \min \left\{ P^{pk}_k, \frac{I^{pk}}{g_k} \right\}$ where $k = \arg\max_i \min \left\{ h_i P^{pk}_i, \frac{h_i I^{pk}}{g_i} \right\}$, and $P^*_i = 0, \forall i \neq k$, if the condition $\ln \left(1 + h_k P^*_k / \sigma^2\right) \geq 1$ holds.

**Proof:** It is shown that in [17] (Theorem 4), the optimal solution for Problem 4 without IPC is single-user transmission if at least one user satisfies $\ln \left(1 + h_i P_i / \sigma^2\right) \geq 1$, and the channel is assigned to the user with the largest $h_i P^{pk}_i$ at the current fading block. Our proof is mainly based on this result.

Define $T_i \triangleq \min \left\{ P^{pk}_i, \frac{I^{pk}}{g_i} \right\}$. Suppose there exists at least one user satisfying the condition $\ln \left(1 + h_i T_i / \sigma^2\right) \geq 1$. Since the condition $\ln \left(1 + h_i T_i / \sigma^2\right) \geq 1$ holds, it follows from [17] that the objective function of Problem 4 is maximized when only one user transmits in each fading block. When there is only one user transmitting, the objective function of Problem 4 reduces to $\ln \left(1 + h_i P_i / \sigma^2\right)$, and the constraints reduces to $P_i \leq P^{pk}_i$ and $g_i P_i \leq I^{pk}$. Clearly, the user with the largest $h_i T_i$ will maximize the objective function. Thus, the optimal allocation is $P^*_k = T_k$ where $k = \arg\max_i h_i T_i$, and $P^*_i = 0, \forall i \neq k$. Lemma 2 is thus proved.

**Theorem 3:** When the number of SUs is large, with high probability, the optimal solution of Problem 3 is D-TDMA, i.e., one user transmitting in each fading block.

**Proof:** From Lemma 2, it is known that if there exists at least one user satisfying the condition $\ln \left(1 + h_i T_i / \sigma^2\right) \geq 1$ where $T_i \triangleq \min \left\{ P^{pk}_i, \frac{I^{pk}}{g_i} \right\}$, the optimal solution of Problem 3 is dynamic TDMA. Since all the channel power gains are i.i.d., the probability of no user satisfying $\ln \left(1 + h_i T_i / \sigma^2\right) \geq 1$ is $1 - \left(\text{Prob} \left\{ \ln \left(1 + h_i T_i / \sigma^2\right) < 1\right\} \right)^K$. It is observed that this probability is a monotonic increasing function with respect to $K$. Thus, when the number of SUs is large, with high probability, the condition will hold. Theorem 3 is thus proved.

Based on these results, we can solve Problem 3 by searching the extreme points of the feasible region when the number of SUs is small, and by applying the D-TDMA scheme when the number of SUs is large. Readers may be interested in the number of SUs that is required to make the D-TDMA scheme optimal. We have investigated this issue in the simulation part given in Section IV. Please note that the condition given in Lemma 2 is only a sufficient condition. In practice,
the probability that D-TDMA is optimal is higher than 
\(1 - \left(\text{Prob} \left\{ \ln \left(1 + \frac{h_i T_i}{\sigma^2} \right) < 1 \right\} \right)^K\). For 
the commonly used parameters, D-TDMA can achieve a near-optimal performance when the 
number of SUs is moderate (such as \(K = 5\)).

In the above, we have presented the approach to solve Problem 4 in general. In the following, 
we show that if the SUs can be sorted in certain order according to their channel power gains, 
a simple algorithm with linear time complexity can be developed to solve Problem 4.

**Theorem 4:** If the SUs can be sorted in the following order: 
\(h_1 > h_2 > \cdots > h_K\) and 
\(\frac{g_1}{h_1} < \frac{g_2}{h_2} < \cdots < \frac{g_K}{h_K}\). Then, there exists an optimal solution, for any two users indexed by \(m\) 
and \(n\), if \(m < n\), their power allocation satisfies 
\(P^*_m \geq P^*_n\).

**Proof:** Assume that the users can be sorted in the following order: 
\(h_1 > h_2 > \cdots > h_K\) and 
\(\frac{g_1}{h_1} < \frac{g_2}{h_2} < \cdots < \frac{g_K}{h_K}\). Consider two users indexed by \(m\) and \(n\) with \(m < n\). Suppose at the 
optimal solution, 
\(P^*_m < P^*_n\). Now, we show this assumption does not hold by contradiction.

For convenience, we define 
\(P'_i \equiv h_i P_i\). Then, Problem 4 can be rewritten as

**Problem 5:**

\[
\max_{P'} \sum_{i=1}^{K} \ln \left( 1 + \frac{P'_i}{\sigma^2 + \sum_{j=1, j \neq i}^{K} P'_j} \right), \\
\text{s.t. } P'_i \geq 0, \forall i, \\
P'_i \leq h_i P^{pk}, \forall i, \\
\sum_{i=1}^{K} \frac{g_i}{h_i} P'_i \leq I. 
\]

In Theorem 2, we have proved that there is at most one fractional user. Thus, the value of 
\(P^*_m\) and \(P^*_n\) has the following two cases.

Case 1: \(0 < P^*_m < P^{pk}\) and \(P^*_n = P^{pk}\). It follows that 
\(P'_m = h_m P^*_m\) and \(P'_n = h_n P^{pk}\). Then, 
based on the relationship between \(P'_m\) and \(P'_n\), we have the following two subcases:

- **Subcase 1:** \(P'_m < P'_n\). Now, we swap the power allocation of these two users, i.e., 
  \(\tilde{P}'_m = h_n P^{pk}\) and \(\tilde{P}'_n = h_m P^*_m\). Since \(h_m > h_n\), it is clear that 
  \(\tilde{P}'_m = h_n P^{pk} < h_m P^{pk}\). Since 
  \(P'_m < P'_n\), it is clear that \(\tilde{P}'_n = h_m P^*_m < h_n P^{pk}\). At the same time, since 
  \(\frac{g_m}{h_m} < \frac{g_n}{h_n}\), we have that 
  \(\frac{g_m}{h_m} P'_m + \frac{g_n}{h_n} P'_n < \frac{g_m}{h_m} P'_m + \frac{g_n}{h_n} P'_n\). Thus, the power allocation \((\tilde{P}'_m, \tilde{P}'_n)\) is a feasible 
  solution of Problem 5. Besides, it is observed that the value of \(21\) under \((\tilde{P}'_m, \tilde{P}'_n)\) is the 
  same as that under \((P'_m, P'_n)\). Thus, \((\tilde{P}'_m, \tilde{P}'_n)\) is also an optimal solution of Problem 5.
Subcase 2: \( P'_m > P'_n \). Now, we consider the power allocation \( \tilde{P}'_m = P'_m + \Delta \) and \( \tilde{P}'_n = P'_n - \Delta \), where \( \Delta \) is a small constant such that \( \tilde{P}'_m \leq h_m P^k \) and \( \tilde{P}'_n \geq 0 \). Since \( \frac{\tilde{g}_m}{h_m} < \frac{\tilde{g}_n}{h_n} \), it is easy to verify that \( \frac{\tilde{g}_m}{h_m} \tilde{P}'_m + \frac{\tilde{g}_n}{h_n} \tilde{P}'_n < \frac{\tilde{g}_m}{h_m} P'_m + \frac{\tilde{g}_n}{h_n} P'_n \). Thus, the power allocation \( (\tilde{P}'_m, \tilde{P}'_n) \) is a feasible solution of Problem 5. Define (21) as \( f(P') \). It follows that

\[
f(P') = \ln \left( 1 + \frac{P'_m}{\sigma^2 + \sum_{j \neq i, m, n} P'_j + P'_n} \right) + \ln \left( 1 + \frac{P'_n}{\sigma^2 + \sum_{j \neq m, n} P'_j + P'_m} \right)
\]

\[
+ \sum_{i=1}^{K} \ln \left( 1 + \frac{P'_i}{\sigma^2 + \sum_{j \neq i, m, n} P'_j + P'_m + P'_n} \right).
\]

Case 2: \( P'_m = 0 \) and \( P'_n > 0 \). It follows that \( P'_m = 0 \) and \( P'_n = h_n P^*_n \). We swap the power allocation of these two users, i.e., \( \tilde{P}'_m = h_n P^*_n \) and \( \tilde{P}'_n = 0 \). Since \( h_m > h_n \), it is clear that \( \tilde{P}'_m = h_n P^*_n < h_m P^k \). At the same time, since \( \frac{\tilde{g}_m}{h_m} < \frac{\tilde{g}_n}{h_n} \), it can be verified that \( \frac{\tilde{g}_m}{h_m} \tilde{P}'_m + \frac{\tilde{g}_n}{h_n} \tilde{P}'_n < \frac{\tilde{g}_m}{h_m} P'_m + \frac{\tilde{g}_n}{h_n} P'_n \). Thus, the power allocation \( (\tilde{P}'_m, \tilde{P}'_n) \) is a feasible solution of Problem 5. Besides, it is observed that the value of (21) under \( (\tilde{P}'_m, \tilde{P}'_n) \) is the same as that under \( (P'_m, P'_n) \). Thus, \( (\tilde{P}'_m, \tilde{P}'_n) \) is also an optimal solution of Problem 5.

Thus, combining the results Case 1 and Case 2, it is clear that there exists an optimal solution: for any two users indexed by \( m \) and \( n \), if \( m < n \), their power allocation satisfies \( P^*_m \geq P^*_n \). Theorem 4 is thus proved.

Based on this theorem, we can develop the Algorithm 1 with linear complexity to solve Problem 4 when the users can be sorted in the order stated in Theorem 4.
Algorithm 1 Optimal power allocation for Problem 4 with channel ordering

1: if $g_1 P^{pk} > Q$ then
2: \[ R^* = \log \left( 1 + \frac{h_1 Q^{pk}}{g_1 \sigma^2} \right), \quad k^* = 1. \]
3: else
4: Initialize $k = 1$.
5: Find the largest $k$ that satisfies $\sum_{i=1}^{k} g_i P^{pk} \leq I^{pk}$ and $k \leq K$. Denote this $k$ as $k^L$.
6: Initialize $R(1) = \log \left( 1 + \frac{h_1 P^{pk}}{\sigma^2} \right)$, $k^* = 1$, $R^* = R(1)$.
7: for $k = 2$ to $k^L$ do
8: \[ R(k) = \sum_{i=1}^{k} \ln \left( 1 + \frac{h_i P^{pk}}{\sigma^2 + \sum_{j=1,j\neq i}^{k-1} h_j P^{pk}} \right). \]
9: \[ \text{if } R(k) > R^* \text{ then} \]
10: \[ R^* = R(k), \quad k^* = k. \]
11: \[ \text{end if} \]
12: \[ \text{end for} \]
13: \[ R(k^L + 1) = \sum_{i=1}^{k^L+1} \ln \left( 1 + \frac{h_i P_i}{\sigma^2 + \sum_{j=1,j\neq i}^{k^L+1} h_j P_j} \right), \quad \text{where } P_i = P^{pk}, \forall i \leq k^L, \text{ and } P_{k^L+1} = \frac{I^{pk} - \sum_{i=1}^{k^L} g_i P^{pk}}{g_{k^L+1}}. \]
14: \[ \text{if } R(k^L + 1) > R^* \text{ then} \]
15: \[ R^* = R(k^L + 1), \quad k^* = k^L + 1. \]
16: \[ \text{end if} \]
17: \[ \text{end if} \]

IV. Numerical results

In this section, several numerical results are given to evaluate the performances of the proposed studies. All the channels involved are assumed to be Rayleigh fading, and thus the channel power gains are exponentially distributed. Unless specifically stated, we assume the mean of the channel power gains is one. The noise power $\sigma^2$ at SBS is also assumed to be 1. For convenience, the transmit power constraint at each SU is assumed to be the same. The numerical results presented here are obtained by taking average over 10000 rounds simulations. In this section, we only provide the simulation results for the ergodic sum-rate under peak TPC and peak IPC. No simulation results for the ergodic sum-rate under average TPC and average IPC are provided. This is due to the fact that we have shown that the optimal power allocation for the ergodic
sum-rate with/without SIC under average TPC and average IPC is the same. As a result, the simulation results for this case are exactly the same as those shown in [12].

A. Ergodic Sum-Rate with/without SIC

First, we compare the ergodic sum-rate for the fading C-MAC with/without SIC under different combinations of TPC and IPC. In Fig. 2, Fig. 3 and Fig. 4, we show the results for the fading C-MAC with $K = 2, 5, 10$, respectively. It is observed from all the figures that the ergodic sum-rate with SIC is always larger than that without SIC under the same TPC and IPC. This verifies our result that the ergodic sum-rate with SIC is a upper-bound of that without SIC. It is also observed that the gap between the ergodic sum-rate with SIC and that without SIC in general increases with the increasing of the number of SUs ($K$). The engineering insight behind this is that when the number of SUs is small in the C-MAC, it is not necessary to implement SIC at the SBS due to the cost and complexity. While when the number of SUs is large, it is worthwhile implementing SIC at the SBS to achieve a larger sum-rate. It is observed from all the curves that when the TPC of SU is large, the ergodic sum-rate gap with/without SIC is negligible. This is due to the following fact. When TPC is very large, TPC will not be the bottleneck, and the performance of the C-MAC will only depend on the IPC. It is proved in [19] that the ergodic sum-rate with/without SIC under only the IPC is the same. The optimal resource allocation for both cases are D-TDMA, and let the SU with the best $h_i/g_i$ to transmit in each fading block. Thus, from engineering design perspective, it is not necessary to implement SIC at the SBS when the TPC is relatively large as compared to the IPC.

B. Optimality of the D-TDMA

In Fig. 5 we numerically compute the probability of D-TDMA being optimal for different number of SUs based on the condition given in Lemma 2. First, it is observed that the probability increases with the increasing of the number of SUs. It is also observed that the probability increases with the increasing of $P_{pk}$ for the same number of SUs. When $P_{pk} = 5dB$ or $10dB$, with only 10 SUs, the probability of D-TDMA being optimal is close to 1. When $P_{pk} = 0dB$, with 20 SUs, the probability of D-TDMA being optimal is more than 95%. These indicates that when the number of SUs is sufficiently large, the D-TDMA is optimal with a high probability.
As pointed out previously, the condition given in Lemma 2 is a sufficient condition. In practice, the probability that D-TDMA is optimal is higher than the probability shown in Fig. 5.

In Fig. 6 we compare the ergodic sum-rate under the optimal power allocation and that under the D-TDMA when the number of SUs is 5. The results are obtained by averaging over 10000 rounds simulations. It is observed from the figure that when the TPC is larger than 0dB, D-TDMA can achieve the same ergodic sum-rate as the optimal power allocation. Even when the TPC is less than 0dB, the gap between the ergodic sum-rate under the optimal power allocation and that under the D-TDMA is not large. Thus, in general, we can use the D-TDMA scheme as a good suboptimal scheme when the number of SUs is larger than 5.

C. Ergodic Sum-Rate under the SIC-OP

In this subsection, we compute the optimal power allocation for C-MAC with SIC first, and then apply the obtained power allocation to C-MAC without SIC (Problem 3) as a suboptimal power allocation. For convenience, we denote the optimal power allocation for C-MAC with SIC as SIC-OP. We then compare the ergodic sum-rate (without SIC) under the SIC-OP with that under the D-TDMA. The ergodic sum-rate (without SIC) under the optimal power allocation are also included as a reference.

In Fig. 7 and Fig. 8, we assume that there are 5 SUs in the network, and the IPC is assumed to be 0dB. In Fig. 7 we assume that the mean of the channel power gain is 1, i.e., $E\{h_i\} = E\{g_i\} = 1, \forall i$. It is observed from Fig. 7 that there exists one crossing-point, before which SIC-OP performs better than the D-TDMA. Actually, SIC-OP can achieve the same performance as the optimal power allocation when the TPC is sufficiently small. After the crossing point, D-TDMA performs better than the SIC-OP. When the TPC is sufficiently large, D-TDMA can achieve the same performance as the optimal power allocation. Similar results can be observed in Fig. 8, in which we assume that the mean of the channel power gain is 0.1, i.e., $E\{h_i\} = E\{g_i\} = 0.1, \forall i$. The difference between Fig. 8 and Fig. 7 is that the crossing point of Fig. 8 has a larger value of $P^\text{pk}$ as compared to the the crossing point of Fig. 7. This can be explained as follows. According to Lemma 2, the condition for D-TDMA being optimal is $\ln (1 + h_k P^*_k / \sigma^2) \geq 1$. Thus, when the mean of $h_k$ is small, a larger $P^*_k$ is needed to make D-TDMA optimal.

In the following, we explain why SIC-OP can achieve the same performance as the optimal power allocation when the TPC is small. Now, we look at the sum-rate of MAC without
SIC, which is $\sum_{i=1}^{K} \ln \left(1 + \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^{K} h_j P_j}\right)$. When $\frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^{K} h_j P_j}$ is small, it is equivalent to $\sum_{i=1}^{K} \frac{h_i P_i}{\sum_{j=1, j \neq i}^{K} h_j P_j}$, since $\ln(1 + x) \approx x$ when $x$ is small. Further, since the TPC is small, $\sum_{j=1, j \neq i}^{K} h_j P_j \approx \sum_{j=1}^{K} h_j P_j$. Thus, $\sum_{i=1}^{K} \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^{K} h_j P_j} \approx \frac{\sum_{i=1}^{K} h_i P_i}{\sum_{i=1}^{K} h_i P_i} = \frac{1}{1 + \frac{\sigma^2}{\sum_{i=1}^{K} h_i P_i}}$. Thus, maximizing $\sum_{i=1}^{K} \ln \left(1 + \frac{h_i P_i}{\sigma^2 + \sum_{j=1, j \neq i}^{K} h_j P_j}\right)$ is equivalent to maximizing $\sum_{i=1}^{K} h_i P_i$. The sum-rate of MAC with SIC is obtained by maximizing $\ln \left(1 + \sum_{i=1}^{K} h_i P_i\right)$, which is also equivalent to maximizing $\sum_{i=1}^{K} h_i P_i$, since the log function is a monotonic increasing function.

Now, we explain why this observation is important. With this observation, we can solve Problem 3 by $\max \{\text{SIC-OP}, \text{D-TDMA}\}$, which achieve the same performance as the optimal power allocation for most cases. Besides, the complexity is much lower than searching the extreme points, especially when the number of SUs is large.

V. CONCLUSIONS

In this paper, we studied the ergodic sum-rate of a spectrum-sharing cognitive multiple access channel (C-MAC), where a secondary network (SN) with multiple secondary users (SUs) shares the spectrum band with a primary user (PU). We assumed an interference power constraint at the PU, individual transmit power constraints at the SUs, and to reduce decoding complexity, no successive interference cancellation (SIC) at the C-MAC. We investigated the optimal power allocation strategies for two types of power constraints: (1) average TPC and average IPC, and (2) peak TPC and peak IPC. For the average TPC and average IPC case, we proved that the optimal power allocation is dynamic time-division multiple-access (D-TDMA). For the peak TPC and peak IPC case, we proved that the optimal solution must be at the extreme points of the feasible region. We showed that D-TDMA is optimal with high probability when the number of SUs is large. We also showed through simulations that the optimal power allocation to maximize the ergodic sum-rate of the fading C-MAC with SIC is optimal or near-optimal for our setting when D-TDMA is not optimal. In addition, when some channel conditions are met, we gave a linear time complexity algorithm for finding the optimal power allocation.

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Fig. 1. System model for a two-user fading C-MAC

Fig. 2. Ergodic Sum-Rate vs. the transmit power of SUs ($K = 2$, $\sigma^2 = 1$, $\mathbb{E}\{h_i\} = \mathbb{E}\{g_i\} = 1$)
Fig. 3. Ergodic Sum-Rate vs. the transmit power of SUs ($K = 5$, $\sigma^2 = 1$, $\mathbb{E}\{h_i\} = \mathbb{E}\{g_i\} = 1$)

Fig. 4. Ergodic Sum-Rate vs. the transmit power of SUs ($K = 10$, $\sigma^2 = 1$, $\mathbb{E}\{h_i\} = \mathbb{E}\{g_i\} = 1$)
Fig. 5. Probability of D-TDMA being optimal vs. the number of SUs ($I_{pk} = 0$ dB, $\sigma^2 = 1$, $E\{h_i\} = E\{g_i\} = 1$)

Fig. 6. Comparison of the ergodic sum-rate: Optimal vs. D-TDMA ($K = 5$, $\sigma^2 = 1$, $I_{pk} = 0$ dB, $E\{h_i\} = E\{g_i\} = 1$)
Fig. 7. Comparison of the ergodic sum-rate: Optimal vs. D-TDMA vs. SIC-OP ($K = 5$, $\sigma^2 = 1$, $I_{pk} = 0$ dB, $E\{h_i\} = E\{g_i\} = 1$)

Fig. 8. Comparison of the ergodic sum-rate: Optimal vs. D-TDMA vs. SIC-OP ($K = 5$, $\sigma^2 = 1$, $I_{pk} = 0$ dB, $E\{h_i\} = E\{g_i\} = 0.1$)