Constraining primordial non-Gaussianity via multi-tracer technique with Euclid and SKA

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We forecast future constraints on local-type primordial non-Gaussianity parameter $f_{NL}$ with a photometric galaxy survey by Euclid, a continuum galaxy survey by Square Kilometre Array (SKA), and their combination. We derive a general expression for the covariance matrix of the power spectrum estimates of multiple tracers to show how the so-called multi-tracer technique improves constraints on $f_{NL}$. In particular we clarify the role of the overlap fraction of multiple tracers and the division method of the tracers. Our Fisher matrix analysis indicates that stringent constraints of $\sigma(f_{NL}) \lesssim 1$ can be obtained even with a single survey, assuming 5 mass bins. When Euclid and SKA phase 1 (2) are combined, constraints on $f_{NL}$ are improved to $\sigma(f_{NL}) = 0.34$ (0.31).

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Primordial non-Gaussianity of density fluctuations is a key to understand the physics of the early Universe. Among several types of primordial non-Gaussianity, the local-type one, $f_{NL}$, has been studied widely, partly because even the simplest inflationary models predict small but non-vanishing values of $f_{NL}$ of $O(0.01)$.

Primordial non-Gaussianity has primarily been constrained from the bispectrum in cosmic microwave background (CMB) temperature fluctuations. Recently, Planck 1 obtained a tight constraint of $f_{NL} = 2.7 \pm 5.8$ at 1$\sigma$ statistical significance. A complementary way to access non-Gaussianity is to measure its impact on large scale structure. Primordial non-Gaussianity induces the scale-dependent bias [2,3] such that the effect dominates at very large scales. Hence, based on a reasonable assumption that the galaxy bias is linear and deterministic on large scales, it has been shown that the galaxy survey can effectively constrain $f_{NL}$ to the level comparable to CMB temperature anisotropies [4,5].

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Future wide and deep surveys with Euclid 1 in optical and infrared bands and Square Kilometre Array (SKA) 2 in radio wavelengths will provide an unprecedented number of galaxies to measure the power spectra. The radio continuum survey conducted with SKA covers 30,000 deg$^2$ out to high redshifts, though the redshift information is not available. The authors in [7] found that even without the redshift information the multi-tracer technique improves constraints as $\sigma(f_{NL}) = O(1)$, while weaker constraints of $\sigma(f_{NL}) = O(10)$ without the multi-tracer technique. While the number of galaxies and covered area are smaller for the Euclid photometric survey (15,000 deg$^2$), it provides redshift information via photometric redshifts. Redshift information is expected to be highly advantageous for constraining $f_{NL}$ because the bias evolves strongly with redshift. As we show below, each of these two surveys provide constraints of $\sigma(f_{NL}) = O(1)$ and constraints improve to $\sigma(f_{NL}) = O(0.1)$ with their combination. To calculate expected constraints, in this paper, we employ the Fisher matrix formalism including the redshift binning as well as the mass binning, taking the overlap of the two survey regions into account.

First, we consider the non-Gaussian correction of the halo bias given by [3]:

$$\Delta b = \frac{2f_{NL}\delta_c}{MD_+} (b_L - 1) - \frac{1}{\delta_c} \frac{d}{d\ln \nu} \left( \frac{dn/dM}{dnG/dM} \right), \quad (1)$$

where $\nu = \delta_c/\sigma$, $\delta_c \approx 1.68$ is the critical linear density for spherical collapse and $\sigma(M,z) = \sigma_R(z)$ is the variance of the linear density field smoothed on the scale $R(M) = (3M/4\pi \rho_{b0})^{1/3}$ with $\rho_{b0}$ being the background density today. $D_+(z)$ is the growth factor, $M(k) = 2k^2T(k)/3\Omega_m H_0^2$, where $T(k)$ is the matter transfer function normalized to unity at large scales [12].

We employ a fit to simulation for the Gaussian mass function $dnG/dM$ and the linear bias factor $b_L$ given in [8]. We adopt a non-Gaussian correction of the mass function developed in [8], where we need the skewness of the density field that is proportion to $f_{NL}$ [3,11,11]. In this paper, for $\sigma S_3$, we adopt a fitting formula from [10].

Constraints on $f_{NL}$ come from the redshift- and mass-dependences of the bias. Thus, in order to take advantage
of the multi-tracer technique, we need a rough estimate of the halo mass of each galaxy. In the Euclid survey, assuming an accurate photometric redshift estimate of each galaxy, we can use various galaxy properties such as luminosity, color, and stellar mass to infer the halo mass. On the other hand, it is more challenging to estimate the halo mass of galaxies from radio surveys. In this paper, following [2], we assume that halo mass can be estimated from the galaxy type.

Estimates of the halo mass for individual galaxies involve large uncertainties. We take account of the uncertainties in halo mass estimation following [13]. Given the estimated mass $M_{\text{est}}$, the probability that the true mass is $M$ is assumed to be given by log-normal distribution with the variance $\sigma^2_{\ln M}$ and the bias $\ln M_{\text{bias}}$,

$$x(M_{\text{est}}; M) = \ln M_{\text{est}} - \ln M - \ln M_{\text{bias}} \over \sqrt{2\sigma^2_{\ln M}}.$$  \hspace{1cm} (2)

Furthermore, it is expected that these parameters depend on both halo mass and redshift. We assume the following functional form [14, 16]:

$$\ln M_{\text{bias}}(M, z) = \ln M_{b,0} + \sum_{i=1}^{3} q_{b,i} \ln \left( \frac{M}{M_{\text{piv}}} \right)^i + \sum_{i=1}^{3} s_{b,i} z^i,$$  \hspace{1cm} (3)

$$\sigma_{\ln M}(M, z) = \sigma_{\ln M, 0} + \sum_{i=1}^{3} q_{\ln M, i} \ln \left( \frac{M}{M_{\text{piv}}} \right)^i + \sum_{i=1}^{3} s_{\ln M, i} z^i,$$  \hspace{1cm} (4)

with $M_{\text{piv}} = 10^{12} h^{-1} M_\odot$. Here we included a large number of parameters to model the uncertainty of the halo mass estimate, which are fully marginalized over when deriving constraints on $f_{\text{NL}}$.

To apply the multi-tracer technique, we split galaxy samples into $N_M$ mass-divided subsamples for each redshift bin. The average density of galaxies in the $i$-th redshift bin $z_i < z < z_{i+1}$ and the $b$-th mass bin $M_{(b)} < M_{\text{est}} < M_{(b+1)}$ is given by

$$\bar{N}_{i(b)} = \int_0^\infty dz \frac{d^2V}{dzd\Omega} \int_0^\infty dM \frac{dn}{dM} S_i(b).$$  \hspace{1cm} (5)

Here $d^2V/dzd\Omega = \chi^2/H$ denotes the comoving volume element per unit redshift per unit steradian, and we have introduced $S_i(b)(M, z)$ to represent the selection function:

$$S_i(b)(M, z) = \Gamma_i(b) \Theta(z - z_i) \Theta(z_{i+1} - z) \times \frac{1}{2} \left[ \text{erfc}(x(M_{(b)}; M)) - \text{erfc}(x(M_{(b+1)}; M)) \right].$$  \hspace{1cm} (6)

where we have introduced the gray-body factor $\Gamma_i(b)$ to denote the fraction of observed halos for each mass bin, since we may not be able to observe all galaxies associated with the underlying dark matter halos. With these variables, the Limber-approximated angular power spectrum between $b$- and $b'$-th mass bins in the $i$-th redshift bin is expressed by [14]

$$C_{i(b)(b')}(\ell) = \int_0^\infty dz W_i(b) W_{i(b')} H \frac{\sigma_8}{\chi} P_\delta \left( \frac{\ell + 1/2}{\chi}, z \right),$$  \hspace{1cm} (7)

where $P_\delta(k, z)$ is the underlying dark matter power spectrum and $W_i(b)$ is the weight function defined as

$$W_i(b) = \frac{1}{N_i(b)} \frac{d^2V}{dzd\Omega} \int_0^\infty dM \frac{dn}{dM} S_i(b) b_i M, z, \frac{\ell + 1/2}{\chi}. $$  \hspace{1cm} (8)

We adopt the Fisher analysis to estimate expected errors of model parameters for a given survey. The Fisher matrix is defined by

$$F_{\alpha\beta} = \sum_{\ell=\ell_{\text{min}}}^{\ell_{\text{max}}} \sum_{I,J} \frac{\partial C^I(\ell)}{\partial \theta^I} \left[ \text{Cov}(C(\ell), C(\ell)) \right]^{-1} \frac{\partial C^J(\ell)}{\partial \theta^J},$$  \hspace{1cm} (9)

where the indices $I$ and $J$ run over the redshift and mass bin, $(i, b, b')$, and $\theta^I$ are model parameters. Here, we consider 29 parameters in the Fisher matrix analysis: the primordial non-Gaussianity parameter $f_{\text{NL}}$, 14 parameters for systematic errors in the halo mass estimate for each of Euclid and SKA (see Eqs. [3] and [4]). We choose $\sigma_{\ln M, 0} = 0.3$ and zero for the other parameters as fiducial values. On the other hand, we fix standard cosmological parameters to those of the standard ΛCDM model: $\Omega_m, 0 = 0.266$, $\Omega_b, 0 = 0.04479$, $\Omega_\Lambda = 0.734$, $w = -1$, $h = 0.710$, $n_s = 0.963$, $k_0 = 0.05 \text{Mpc}^{-1}$ and $\sigma_8 = 0.801$. The marginalized error on each parameter is given by $\sigma(\alpha) = \sqrt{(F^{-1})_{\alpha\alpha}}$.

Now we derive the covariance matrix generalized to multiple tracers which are observed in different sky areas with some overlap. We introduce the observed density contrast as $\delta_w(b)(\theta) = w(b)(\theta) \delta^{(b)}(\theta)$, where $w(b)(\theta)$ is the survey window function on the sky for $b$-th tracer; $w(b) = 1$ if the direction $\theta$ on the sky is in the survey region, otherwise $w(b) = 0$. With the two-dimensional Fourier components of $\delta^{(b)}(\theta)$, $\delta_w^{(b)}(\ell) = \int d^2\ell (2\pi)^{-1/2} w(b)(\ell - \ell') \delta^{(b)}(\ell')$, where $\delta^{(b)}$ are Fourier transform of $w(b)$ and $\delta^{(b)}$ respectively, we can define an estimator of the angular power spectrum as [12]

$$\hat{C}_{i(b)(b')}(\ell) = \frac{1}{\Omega_{\text{w}}^{(b)}(\ell)} \int_{|\ell|} \frac{d^2\ell'}{\Omega_{\text{w}}^{(b')}} (\ell') \delta_w^{(b)}(\ell') (-\ell'),$$  \hspace{1cm} (10)

where we have considered the integral over a shell in the Fourier space of width $\Delta \ell$ and volume $\Omega_{\ell} = \int_{|\ell|} d^2\ell' \approx 2\pi^2 \Delta \ell$. Here the effective survey area was defined as $\Omega_{\text{w}}^{(b')} = \int d^2\theta w^{(b)}(\theta)$, which is the survey area of the $b$-th tracer for $b = b'$ and the overlapping area of the $b$- and $b'$-th tracers for $b \neq b'$. We have determined the functional form of the estimator so that it is unbiased in a sense that the ensemble average gives the true power spectrum, namely, $\langle \hat{C}_{i(b)(b')}(\ell) \rangle = C_{i(b)(b')}(\ell)$. Assuming


FIG. 1: The marginalized error on $f_{\text{NL}}$ as the function of the overlap fraction, for the single redshift bin of $0.7 < z < 1.2$. Different lines show results with different mass ratio $M_{(2)}/M_{(1)}$.

the Gaussian error covariance, we obtain the covariance matrix for multiple tracers as

$$\text{Cov} \left[ C_{i(b\bar{b})}(\ell), C_{j(\bar{b}b)}(\ell) \right] = \frac{\delta_{ij} \delta_{b\bar{b}'} (2\ell + 1) \Delta \ell}{2\ell + 1} \frac{4\pi \Omega_{w}(b\bar{b}')(\bar{b}b')}{\Omega_{w}(b\bar{b})} \left[ C_{i(b\bar{b})}(\ell) C_{i(\bar{b}b)}(\ell) + C_{i(\bar{b}b)}(\ell) C_{i(b\bar{b})}(\ell) \right] , \quad (11)$$

with $\Omega_{w}(b\bar{b}')(\bar{b}b') = \int d^2\vartheta w(b\bar{b}) w(\bar{b}b') w(\bar{b}b')$. Since the observed spectrum includes the shot noise contamination, we replace $C_{i(\bar{b}b)}(\ell)$ with $C_{i(b\bar{b})}(\ell) + \bar{N}_{i(b)} \delta_{b\bar{b}'}$.

As we stated above, we consider the Euclid photometric survey and the SKA continuum survey. For Euclid, a redshift range $0.2 < z < 4.2$ is considered and galaxy samples are split into 8 redshift-bins with the same interval ($\Delta z = 0.5$). We neglect the photometric redshift errors as they are expected to be much smaller than $\Delta z$. To include the effect of flux-cut for each redshift range, we adopt the following minimum mass for each bin, $M_{\text{cut}} > 0.7, 1, 2, 5, 10, 20, 50, 100$ in the unit of $10^{11} h^{-1} M_{\odot}$ and set $\Gamma_{(6)}^{\text{Euclid}} = 1$. Galaxy samples are further split according to the estimated halo mass.

We consider 5 mass-bins and take separating masses such that the 5 mass-bins of the same redshift bin have the same number of samples. Here it should be noted that the separating masses depend on the redshift. We will discuss other possibilities of the mass binning later. Summation of the power spectrum is taken for an $\ell$-range of $3 \leq \ell \leq 400$.

As for the SKA continuum survey, we have only one redshift-bin as no redshift information is available. Thus we simply drop the redshift dependent terms in Eqs. (3), (4). Following (7), we consider 5 types of galaxies as 5 tracers with the typical masses $M_{\text{SFG}} = 10^{11} h^{-1} M_{\odot}$ for star forming galaxies, $M_{\text{RQQ}} = 3 \times 10^{12} h^{-1} M_{\odot}$ for radio quiet quasars, $M_{\text{FRQ}} = 10^{13} h^{-1} M_{\odot}$ for FRI, $M_{\text{SB}} = 5 \times 10^{13} h^{-1} M_{\odot}$ for starburst galaxies and $M_{\text{FRQ}} = 10^{14} h^{-1} M_{\odot}$ for FRII. Accordingly, we consider 5 mass-bins, $M_{(i)} < M < M_{(i+1)}$ ($i = 1, \cdots, 4$) and $M > M_{(5)}$, with $M_{(1)} = 0.9 \times 10^{11} h^{-1} M_{\odot}$, $M_{(2)} = \sqrt{M_{\text{SFG}} M_{\text{RQQ}}}$, $M_{(3)} = \sqrt{M_{\text{RQQ}} M_{\text{FRQ}}}$, $M_{(4)} = \sqrt{M_{\text{FRQ}} M_{\text{SB}}}$, $M_{(5)} = \sqrt{M_{\text{SB}} M_{\text{FRQ}}}$. For the flux-cut, we adopt the gray body factor as $\Gamma_{(6)}^{\text{SKA1}} = \{0.013, 0.03, 0.1, 1, 1\}$ and $\Gamma_{(6)}^{\text{SKA2}} = \{0.5, 1, 1, 1, 1\}$, which are chosen to match the expected number density distribution of galaxies found in these surveys (see e.g., (11)). As for $\ell$-range, we consider $2 \leq \ell \leq 400$.

In computing the Fisher matrix for the combination of Euclid and SKA surveys, we adopt 9,000 deg$^2$ as the area of the overlap region and we neglect the contributions from the derivative of the cross correlations between Euclid and SKA for simplicity. We focus on constraints on $f_{\text{NL}}$ and marginalize over the other parameters.

Before showing expected constraints from Euclid and SKA surveys, let us check the dependence of the efficiency of the multi-tracer technique on the overlapping survey area and different mass-binning, considering a simple case of 2 tracers observed by a Euclid-like survey. In Figure 1, we plot the marginalized error on $f_{\text{NL}}$ as a function of the overlap fraction $\Omega_{w}(12)/\Omega_{w}$ for a single redshift-bin $0.7 < z < 1.2$. Different curves represent different mass-binning varying the mass ratio $M_{(2)}/M_{(1)}$. Here we assume that the sky coverages for both tracers are the same, $\Omega_{w}(11) = \Omega_{w}(22) = \Omega_{w}$. We find that the non-vanishing overlap region leads to improved constraints on $f_{\text{NL}}$, which becomes smallest in the case of the maximal overlap. One can also see that in the case of the maximal overlap there is a critical value of the mass ratio $M_{(2)}/M_{(1)}$ which results in the tightest constraint. This behavior can be understood as follows: once we fix the mass ratio, the number density for each mass bin, $N_{i(b)}$, is determined through Eq. (5). Changing the value of the mass ratio leads to the larger shot noise for one of the mass-bins and smaller shot noise for the other. We find that the tightest constraint is obtained when the shot noise for the two mass-bins becomes comparable. This is the reason for our choice of separating masses by the same number density, as explained above.

Next, we focus on the Euclid survey. Figure 2 shows the marginalized constraints on $f_{\text{NL}}$ as a function of the number of tracers for a single redshift-bin $0.7 < z < 1.2$ with the maximal overlap among tracers. We find that the constraining power increases with $N_{M}$. Even 2 tracers drastically improve the constraint, simply because the multi-tracer technique does not take effect for the 1 tracer case. Furthermore, combining multiple redshift-bins improves substantially the constraint, as is shown in Figure 3. We find that galaxy samples as far as $z = 3.2$ (6th bin) contribute significantly to the constraint. When all 5 mass-bins and 8 redshift-bins are taken into account, the Euclid photometric survey can reach $\sigma(f_{\text{NL}}) = 0.46$.

Finally, Figure 4 shows the expected marginalized constraints on $f_{\text{NL}}$ for each survey and their combinations. The constraints on $f_{\text{NL}}$ from SKA1 and SKA2...
are $\sigma(f_{\text{NL}}) = 1.64, 0.65$, respectively, which are consistent with Ref. [2]. These are relatively weaker than that from Euclid, presumably because the redshift information obtained from the photometric survey is more advantageous than the larger sky coverage and the larger number of galaxy samples from SKA survey. Combining Euclid and SKA, the constraint can improve further to $\sigma(f_{\text{NL}}) = 0.34$ (Euclid+SKA1), 0.31 (Euclid+SKA2), suggesting that the joint analysis between Euclid and surveys are quite effective to constrain primordial non-Gaussianity.

To summarize, we have discussed the potential power of multi-tracer technique for the combination of the Euclid photometric survey and the SKA continuum survey. Splitting the galaxy samples into the subsamples by the inferred halo mass and redshift, constraints on $f_{\text{NL}}$ drastically improve. We have shown that constraints of $\sigma(f_{\text{NL}}) = \mathcal{O}(1)$ can be obtained even with a single survey. Combining Euclid and SKA, even stronger constraints of $\sigma(f_{\text{NL}}) = \mathcal{O}(0.1)$ can be obtained.

In this paper, we have made several simplified assumptions. In future galaxy surveys, the systematic uncertainties likely play a more important role than statistical errors. Here we considered only the uncertainty in the halo mass estimation. For instance, the uncertainty in photometric redshifts and the effect of the stochastic bias may become important. We should also address the identification of the optical and infrared counterparts in the overlap region of SKA and Euclid surveys. While we conservatively assumed no redshift information for the SKA survey, checking the counterparts in Euclid or other surveys would provide valuable information on redshifts of individual SKA sources, which may allow the tomographic analysis in the SKA survey to lead further improvements of the constraints (see [13]). We hope to come back these issues in the near future.

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