Mechanical Analysis and Measurement of Parameters of Wheel -Soil Interaction for a Lunar Rover

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Abstract. Mechanical models of the wheel-soil interaction are constructed for a rigid wheel of lunar rover when rolling and steering, based on the theories of terramechanics and passive earth pressure. The mechanical affect of different parameters of the soil to the rigid wheel is analysed, such as drawbar pull, rolling torque and steering torque. Experimental results of a rigid wheel in a kind of dry soil, whose mechanical parameters are some similar to lunar soil, validate the models correct. A novel test-bed and its data acquisition system are also designed for testing the performance of the wheel-soil interaction of lunar rover, and their key technologies concerned are discussed.

1. Introduction

Lunar rovers will meet challenge exploration missions on the deformable terrain on the Moon. The mobility of a lunar rover depends mostly on the mechanical performance of its wheels on the terrain, so many researchers pay attention to the wheel-soil interaction of the vehicles. Iagnemma K. developed a test-bed to estimate the parameters of the soil online [1, 2], Bauer R. simulated the wheel’s mechanical parameters with AS²TM, which is a commercially-available wheel-soil interface model in Matlab software, and the result fits the experiments well [3]. Andrade modeled global behavior of the soil using finite elements method to [4]. WANJII presented a visco-elastic soil model based on three-element Maxwell model to predict the performance of the wheel [5]. Reina improved the method to measure the slip and sinkage of the wheel [6].

This paper will focus on the wheel-soil interaction and construct mechanical models of a rigid wheel on the deformable terrain. The mechanical affect of different parameters of the soil to the rigid wheel will be analyzed and validated. A test-bed and its data acquisition system are designed for the further researches on wheel-soil interaction and test.

2. Mechanical models of wheel-soil interaction

2.1. Mechanical model when wheel rolling

The basic idea of constructing a wheel-soil interaction model is separating deformation in the wheel-soil interaction to vertical and horizontal ones, and described them with pressure-sinkage characteristics and shear-tension-displacement characteristics, respectively.

In figure 1, a vertical load \(W\) and drawbar pull \(DP\) are applied to the wheel by suspension. A torque \(T\) is applied to the wheel rotation axis by a motor. The angle velocity of the wheel is \(\omega\), \(\sigma(\theta)\) is the
normal stress of the point $P$ in the contact area, according to Bekker’s theory [7], $\sigma(\theta)$ can be written as:

$$\sigma = \left(\frac{k}{b} + k_{\phi}\right) \cdot z^{n}$$  \hspace{1cm} (1)

Where $\sigma$ is the pressure, $b$ is the width of the wheel, $z$ is the sinkage, and $n$, $k_{c}$, and $k_{\phi}$ are the pressure-sinkage parameters. The parameter $n$ is called the exponent of sinkage, while $k_{c}$ and $k_{\phi}$ are called the cohesive and frictional modulus of deformation, respectively.

The shear-stress to shear-deformation relationship, as proposed by Janosi [8], is described by:

$$\tau = \tau_{max} \left(1 - e^{-j/k}\right)$$  \hspace{1cm} (2)

Where $j$ is the shear deformation, and $k$ is the tangent modules of horizontal shear deformation, and $\tau_{max}$ is the maximum shear stress, which can be described by the Coulomb rule:

$$\tau_{max} = c + \sigma \tan \varphi$$  \hspace{1cm} (3)

Where $c$ is the cohesion of the soil, $\varphi$ represents the internal friction angle of the soil. From equation (2), $r$ is found to be a function of shear-deformation $j$, while here $j$ can be written as:

$$j = r[(\theta_{m} - \theta) - (1 - s)(\sin \theta_{m} - \sin \theta)]$$  \hspace{1cm} (4)

Where $\theta_{m}$ means the angle from which the wheel first makes contact with the terrain to the vertical, $s$ is the slip of the wheel, which represents the wheel’s slip, defined as:

$$s = \frac{(\omega - v)}{\rho \omega}$$  \hspace{1cm} (5)

According to Figure 1, the sinkage of the soil in point $P$ $z$ can be written as:

$$z = r(\cos \theta - \cos \theta_{m})$$  \hspace{1cm} (6)

From the analysis above, the press and shear pressure is different as the point $P$ changes, the force and torque balance equation in Figure 1 can be written as:

$$W = r b \int_{0}^{\theta_{m}} \sigma(\theta) \cos \theta \cdot d\theta + \int_{0}^{\theta_{m}} \tau(\theta) \sin \theta \cdot d\theta$$  \hspace{1cm} (7)

$$DP = r h \int_{0}^{\theta_{m}} \tau(\theta) \cos \theta \cdot d\theta - \int_{0}^{\theta_{m}} \sigma(\theta) \sin \theta \cdot d\theta$$  \hspace{1cm} (8)

$$T = r^{2} b \int_{0}^{\theta_{m}} \tau(\theta) \cdot d\theta$$  \hspace{1cm} (9)

where $\sigma(\theta)$, $\tau(\theta)$ can be computed from equation (1) and equation (2).

2.2. Mechanical model of the wheel-soil when steering

The torque of resistance applied on the wheel when steering consists of two parts: the first one is $RT1$, which is caused by the cohesion force between the bottom of the wheel and soil, the second one is $RT2$ caused by the wheel’s extruding the lateral soil.

Figure 2 shows the forces on the wheel when steering and $O'N$ is assumed to be the steering axis. $b$ is the width of wheel, $r$ is the radius, $\theta_{m}$ is the angle that the wheel contact with the soil, $\sigma$ is the normal stress, which can be computed through equation (1). $\tau_{s}$ is the friction force, which can be expressed as:

$$\tau_{s}(\theta) = \mu \cdot \sigma(\theta)$$  \hspace{1cm} (10)

Where, $\mu$ is the friction coefficient; the resistance torque $RT1$ in Figure 2 can be expressed as:

$$RT1 = r b \int_{0}^{\theta_{m}} \mu \cdot \sigma(\theta) \cdot r \sin \theta \cdot d\theta$$  \hspace{1cm} (11)

When wheel is steering, the wheel extrudes the lateral soil and causes the destruction of the soil. A coordinate system on the wheel is constructed with point $O$ as its origin, the horizontal orientation as $\zeta$.
axis and the vertical orientation as $\zeta$ axis. According to the theory of passive earth pressure, and assuming the stress in the lateral contact area between wheel and soil is a horizontal linear distribution,

\begin{equation}
\sigma_\theta(z,\zeta) = \frac{\zeta \gamma k_p (\zeta - r \cos \theta_m) + 2c \sqrt{|k_p|}}{\sqrt{r^2 - \zeta^2}}
\end{equation}

where $\gamma$ is the bulk density of soil; $k_p$ is the coefficient of Rankine passive earth pressure, which can be computed as:

\begin{equation}
k_p = t g^2 (45^\circ + \varphi / 2)
\end{equation}

then the torque of resistance on the lateral border can be written as:

\begin{equation}
RT_2 = 2 \int_{\cos \alpha_m}^{\cos \alpha_m} \int_{r^2 - \zeta^2} \sigma_\theta(z,\zeta) \zeta d\zeta d\xi
\end{equation}

The torque $RT$ which should be applied to the wheel when steering should get over the torque of the resistance $RT_1$ and $RT_2$, so $RT$ can be described as:

\begin{equation}
RT = RT_1 + RT_2
\end{equation}

3. Simulation and Experiment

3.1. Simulation and computation of wheel’s performance parameters

To analyze the affection of the parameters of the soil to the performance of the wheel, mechanical performance parameters are computed with different kind of soil. Table 1 shows the parameters of the three kind of soil.

\begin{table}[h]
\centering
\caption{Parameters of the soil.}
\begin{tabular}{|c|c|c|c|}
\hline
 & 1# & 2# & 3# \\
\hline
$c$ (kpa) & 0 & 0 & 0 \\
$\varphi$ & 28$^\circ$ & 32$^\circ$ & 35$^\circ$ \\
$n$ & 1 & 1 & 1 \\
k$_i$(kpa/m$^{n-1}$) & 0 & 0 & 0 \\
k$_o$(kpa/m$^2$) & 1.6 & 1.75 & 1.8 \\
k (mm) & 15 & 15 & 15 \\
\hline
\end{tabular}
\end{table}

Having known all the parameters of soil and wheel, mechanical parameters of wheel’s performance can be computed. According to equation (7), $W$ is a function of $\theta_m$ and slip $s$. $\theta_m$ and $z_m$ can be computed with iterative algorithms when the slip $s$ changes, then drawbar pull $DP$ and torque $T$ can be got, also the torque $RT$ when steering can also be computed.

Figure 3~Figure 4 shows the relationships between $DP$, $T$ and $s$ in three different kind of soil. Figure 5 shows the relationships between $RT$ and $z_m$. In these figures, we can see when the slip of wheel is small, $DP$ and $T$ grows quickly as $s$ grows, but when slip $s$ turns bigger, $DP$ and $T$ grows slowly. The shear force will grows as a result of the growing of the internal friction angle, which makes the growing of $DP$, $T$ and $RT$. 

\begin{figure}[h]
\centering
\caption{Force diagram of the wheel when rolling.}
\end{figure}

\begin{figure}[h]
\centering
\caption{Force diagram of the wheel when steering.}
\end{figure}
3.2. Experiments and Results

To justify the model constructed above, experiments were carried out in an easily test-bed. The cohesion of the soil in moon is about 0.1–1kpa, internal friction angle is about 35°–50°, and bulk density is 0.9–1.9g·cm⁻³ [9]. To simulate the soil of moon in some degree, experiments were done in soil 3#. Some other parameters of soil 3# are as such: $c$ is 0.37, $\varphi$ is 35.8°, $k$ is 12.1mm, $n$ is 1, $k_c$ is 0, $k_\varphi$ is 1850kpa/mm, the bulk density is 15.65 kN/m³, $k_p$ is 3.82. The results of the experiments are showed in Figure 7 and Figure 8.

Compared $DP$ and $RT$ in experiments with them in theory, the draw bar pull $DP$ is within 75.9% confidence intervals, and $RT$ are within 81.7% confidence intervals.

As the simple test-bed can’t measure the $DP$ and $RT$ in high precision and can’t measure more parameters of the wheel’s performance, another test-bed is designed.

4. Design of the test-bed and its data acquisition

4.1. Structure of test-bed

All the parameters of wheel’s performance must be measured in the test-bed such as drawbar pull $DP$, torque of the wheel when rolling $T$, torque applied to wheel when steering $RT$ and the sinkage of the wheel $zm$. All the parameters are a function of slip, so the slip $s$ should be measured. According to these, a test-bed is designed; figure 9 shows its structure.

The wheel is mounted on a bracket, which is connected with torque sensor and driving motor. Different wheels with different widths should be measured in the test-bed, so the distance of the bracket should be designed adjustable, and some allowance should be given in the lateral direction. As different wheels should steer with different radius, so the radius when steering should be designed adjustable too. The drag force of the system is applied by synchronous belt driven by the torque motor, which provided resistance for the wheel. As the alignment error of the synchronous belt, the extra friction will exist in the system. To delimit the friction force, a mechanical structure is designed.

4.2. Design of data acquisition system

There are fourteen parameters needed to be measured such as the linear velocity of the wheel $v$, angular velocity $\omega$, drawbar pull $DP$, driven torque $T$, steering torque $RT$, sinkage $zm$ and so on. To achieve different slip, the speed of the motors in the system should be designed controllable. Figure 10 shows the block diagram of the data acquisition system. The data acquisition system consists of PC,
DSP, A/D Converter, D/A Converter and some sensors related. The principle of the system is as follows: PC communicates with DSP through RS232, control signal and data of the expected motor velocity are sent to the DSP, PC also received the data sent from DSP. DSP dealt with the data that transported from the feedback of motor’s encoder and received from PC first, then the data is transported to DA, and the desired speed and slip can be controlled.

Figure 9. Structure of the test-bed.  
Figure 10. Block diagram of the acquisition system.

5. Conclusion
Mechanical Model of wheel-soil interaction for lunar rover’s rigid wheel is constructed when wheel is rolling and steering based on terramechanics and the theory of passive earth pressure. The parameters of wheel’s performance are computed with three different kind of soil. The result shows the draw bar pull DP, driven torque T, and steering torque RT will grow as the internal friction angle grows, and the increase of slip s also cause them turns bigger. The module constructed is nearly correct according to the simple experiments. To justify the module further, another test-bed which can measure all the parameters of wheel’s performance is designed, and the test-bed can test a wide range of wheel with different width and radius.

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