On the toroidal nature of the low-energy E1 mode

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The nature of E1 low-energy strength (LES), often denoted as a “pygmy dipole resonance”, is analyzed within the random-phase-approximation (RPA) in $^{208}$Pb using Skyrme forces in fully self-consistent manner. A first overview is given by the strength functions for the dipole, compression- and toroidal operators. More detailed insight is gained by averaged transition densities and currents where the latter provide very illustrative flow pattern. The analysis reveals clear isoscalar toroidal flow in the low-energy bin 6.0-8.8 MeV of the LES and a mixed isoscalar/isodefect toroidal/compression flow in the higher bin 8.8-10.5 MeV. Thus the modes covered by LES embrace both vortical and irrotational motion. The simple collective picture of the LES as a “pygmy” mode (oscillations of the neutron excess against the nuclear core) is not confirmed.

During the last decade we observe an increasing interest in low-energy E1 strength (LES), for a recent review see \cite{1}. This interest is caused by a possible relation of LES to the neutron skin in nuclei and density dependence of the nuclear symmetry energy. This in turn may be important for building the isospin-dependent part of the nuclear equation of state and various astrophysical applications \cite{2}. Several different views of the LES origin come together. Most often the LES is interpreted as a "pygmy dipole resonance" (PDR) modeled as the oscillation of the neutron excess against the nuclear core \cite{1,3,4}. There are, however, serious objections against such a simplistic collective picture \cite{5,6}. In fact, the landscape of LES may be much richer. It can embrace also the toroidal resonance (TR) \cite{8,9} and anisotropic compression resonance (CR) \cite{10,11} which both are of actual interest \cite{12}. After exclusion of nuclear center-of-mass (c.m.) motion, the TR and CR dominate in the isoscalar (T=0) channel and constitute the low- and high-energy branches of the isoscalar giant dipole resonance (ISGDR). Following recent microscopic studies \cite{12,13}, the TR dominates in the LES region and the CR, being strongly coupled to TR, also significantly contributes there. The basic flow patterns of these three modes are shown schematically in Fig. 1. The panels illustrate the PDR oscillations of the neutron skin against the core (a), the typical vortices of the TR (b), and the dipole-compressional pattern of the CR (c). The latter can be viewed as oscillation of surface against core and thus shares some similarity with the PDR picture of panel (a). Unlike the irrotational PDR and CR, the TR is purely vortical in the hydrodynamical (HD) sense \cite{12,13}. Thus we see that the LES can involve quite different flows, vortical and irrotational.

Despite a great number of publications on PDR, TR and CR \cite{1}, their possible interplay in LES was only occasionally discussed. There is a study within the quasiparticle-phonon model which discusses PDR and TR side by side \cite{14}. It was shown that the vortical strength \cite{15} is peaked in the LES region and the isovector LES velocity field is mainly toroidal. Nevertheless, because of the dominant contribution of the neutron skin to the surface transition density and thus to $B(E1,T=1)$, the familiar PDR picture of LES was maintained \cite{14}. Similar arguments in favor of the PDR treatment were earlier presented in relativistic \cite{4} and Skyrme \cite{5} mean-field calculations, and taken up in most of subsequent publications. Recent explorations question the simple PDR-type collectivity of LES \cite{6,7,13}, though without analysis of LES velocity fields.

It is the aim of this paper to give a more thorough exploration of the interplay and structure of low lying dipole modes. Not only strength functions and transition densities but also the mode flow patterns will be considered. As we will see, the actual LES flow is predominantly of mixed TR/CR character. This conclusion may have far-reaching consequences for the information content of LES. If vorticity dominates, then only a minor irrotational fraction of LES is relevant for the nuclear symmetry energy and related problems (as was also worked out recently using the correlation analysis \cite{7}).

Our study is performed for $^{208}$Pb using the Skyrme

\begin{figure}[h]
\centering
\includegraphics[width=0.9	extwidth]{fig1.png}
\caption{Schematic velocity fields for the E1 pygmy (a), toroidal (b), and high-energy compressional (c) flows. The driving field is directed along z-axis. The arrows indicate only directions of the flows but not their strength. In the plot (c), the compression (+) and decompression (-) regions, characterized by increased and decreased density, are marked.}
\end{figure}

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RPA approach with the techniques of [10]. The method is fully self-consistent as both the mean field and residual interaction are derived from the Skyrme functional [17, 13]. The RPA residual interaction takes into account all the terms of the Skyrme functional including the Coulomb (direct and exchange) energy. The center-of-mass correction (c.m.c.) is implemented for the ground state and T=0 dipole excitations. The parameterization SLy6 [20] is used which provides a satisfactory description of E1(T=1) strength in heavy nuclei [21]. The calculations are done in a 1D spherical coordinate-space grid with mesh size 0.3 fm and a calculation box of 21 fm. A large configuration space including 1ph states up to ~35 MeV and additional fluid dynamical basis modes is used. The later allows to include global polarization effects up to ~200 MeV [10], correctly extract the c.m. motion, and fully exhaust the Thomas-Reiche-Kuhn sum rule.

The excitation modes are first characterized by their strength function

\[ S_\alpha(E1;\omega) = \sum_\nu \omega_\nu (|\langle \Psi_\nu | \hat{M}_\alpha (E10) | \Psi_0 \rangle|^2 \xi_\alpha (\omega, \omega_\nu)) \]

where \( \xi_\alpha (\omega, \omega_\nu) = \Delta (\omega_\nu)/[2\pi((\omega - \omega_\nu)^2 + \Delta^2 (\omega_\nu)^2)] \) is a Lorentzian weight with energy-dependent smoothing width \( \Delta (\omega_\nu) = \max\{0.4 \text{ MeV}, (\omega_\nu - 8 \text{ MeV})/3\} \), for details see [22]. Further, \( \Psi_0 \) is the RPA ground state (g.s.) while \( \nu \) runs over the RPA spectrum with eigenvalues \( \omega_\nu \) and eigenstates \( |\Psi_\nu \rangle \). The \( \hat{M}_\alpha (E1\mu) \) is the transition operator of the type \( \alpha = \{E1, \text{tor}, \text{com}\} \).

For E1(T=1) transitions (\( \alpha = E1 \)), we consider the ordinary E1 operator \( (\propto r Y_{1\mu}) \) with effective charges \( e_t^0 = N/A \) and \( e_c^0 = -Z/A \) and the strength \( \xi_\alpha \) is weighted by the energy, i.e. \( l = 1 \). For \( \alpha = \text{tor}, \text{com} \), we implement \( e_t^0 = e_c^0 = 1 \) for \( T=0 \) (\( e_t^0 = -e_c^0 = 1 \) for \( T=1 \)) and no energy weight (\( l = 0 \)).

The TR and CR operators used in [11] read [12]

\[ \hat{M}_{\text{tor}} (E1\mu) = \frac{1}{10\sqrt{2}c} \int d^3r [r^3 - \frac{5}{3} r^2 \rho (\hat{r}) \cdot (\hat{\nabla} \times \hat{j}_c (\hat{r}))], \]

\[ \hat{M}_{\text{com}} (E1\mu) = -\frac{i}{10c} \int d^3r [r^3 - \frac{5}{3} r^2 \rho (\hat{r}) \cdot (\hat{\nabla} \times \hat{j}_c (\hat{r}))], \]

where \( \hat{j}_c (\hat{r}) \) is the operator of the convection nuclear current, \( \hat{Y}_{1\mu} (\hat{r}) \) and \( Y_{1\mu} (\hat{r}) \) are vector and ordinary spherical harmonics. The terms with the g.s. squared radius \( \langle r^2 \rangle_0 = \int d^3r r^2 \rho_0 (\hat{r})/A \) account for the c.m.c., \( \rho_0 (\hat{r}) \) is the g.s. density. Note that we describe CR and TR operators on the same footing using the current operator. There is the direct relation [12] \( \hat{M}_{\text{com}} (E1\mu) = -\hat{M}_{\text{com}} (E1\mu) \omega/c \) between the CR current-dependent operator [12] and its familiar density-dependent counterpart

\[ \hat{M}_{\text{com}} (E1\mu) = \frac{1}{10} \int d^3r \rho (\hat{r}) [r^3 - \frac{5}{3} r^2 \rho (\hat{r})] Y_{1\mu} (\hat{r}), \]

where \( \hat{\rho} (\hat{r}) \) is the density operator.

The operators [21]. TR and CR are derived as second-order \( r^3 Y_{1\mu} \) terms in the low-momentum expansion of the ordinary E1 transition operator [13, 12]. Despite its second-order origin, TR and CR dominate in the E1(T=0) channel where the leading c.m. motion driven by the operator \( r Y_{1\mu} \) is removed as being the spurious mode. Following [21] [3-4], TR and CR deliver information on the curl \( \hat{\nabla} \times \hat{j}_c \) and divergence \( \hat{\nabla} \cdot \hat{j}_c \) of the nuclear current. As shown in [12], the corresponding velocity operators indicate that TR is purely vortical and CR is irrotational.

The strength functions [11] are shown in Fig. 2. In panel a), we see a good agreement of the computed giant dipole resonance (GDR) with the experiment [23], which confirms the accuracy of our description. For the LES region 6-10.5 MeV (marked as pygmy), we get two peaks at 7.5 and 10.3 MeV in accordance with previous RMF calculations [4]. Panels b) and c) show TR and CR strengths in T=0 and T=1 channels. For T=0, the TR and CR are believed to constitute the low- and high-energy branches of ISGDR observed in (\( \alpha, \alpha' \)) reaction [24] are denoted. c) The same as in the plot (b) but in T=1 channel.
dominant and strongly peaked part of TR(T=0), the left flank of TR(T=1), and a non-negligible low-energy fraction of CR. In other words, we expect here a complicated interplay of several modes.

To understand the LES structure, we need more detailed observables than the strength distribution. In the following, we will consider transition densities (TD) \( \delta \rho_{\nu}(r) = \langle \Psi_{\nu} | \rho_1 | \Psi_{\nu} \rangle \) and current transition densities (CTD) \( \delta j_{\nu}(r) = \langle \Psi_{\nu} | j_{1} | \Psi_{\nu} \rangle \) (analogous to velocity fields). As we have in \(^{208}\)Pb a high density of states, it is not worth to look at the pattern of individual states \( \nu \), which can vary from state to state and easily hide common features of the flow. Thus we will consider transition densities and velocity fields averaged over given energy intervals. Incoherent averaging requires expressions which are bi-linear in the excited states \( | \Psi_{\nu} \rangle \). This is achieved by summing TD and CTD weighted by the matrix elements \( D_{\nu \nu} \) of a probe operator \( D_{\nu}(E1) \):

\[
\delta \rho_{\nu}^{(D)}(r) = \sum_{\nu \in [\omega_1, \omega_2]} \sum_{q=n,p} e_{q}^{\nu} \delta \rho_{q}^{(D)}(r), \quad (5)
\]

\[
\delta j_{\nu}^{(D)}(r) = \sum_{\nu \in [\omega_1, \omega_2]} \sum_{q=n,p} e_{q}^{\nu} \delta j_{q}^{(D)}(r). \quad (6)
\]

The sums in (5)-(6) involve all the RPA states \( | \nu \rangle \) in the energy interval \([\omega_1, \omega_2]\). Since states \( | \nu \rangle \) contribute twice (to \( D_{\nu \nu} \) and transition densities \( \delta \rho_{q}^{(D)} / \delta j_{q}^{(D)} \)), the expression is independent of the phase of each state \( | \Psi_{\nu} \rangle \) as it should be. In (5)-(6), the index \( \beta = p,n,0,1 \) defines the type of TD or CTD (neutron, proton, T=0, T=1) by the proper choice of the effective charges: \( e_{p}^{p} = 1, e_{n}^{p} = 0; \ e_{p}^{n} = 0, e_{n}^{n} = 1; \ e_{p}^{0} = e_{n}^{0} = 1; \ e_{p}^{1} = N/A, e_{n}^{1} = -Z/A \). We use two different dipole probe operators: the isovector \( D_{1} = (N/A) \sum_{i}^{A} (r_{Y_{1}}_{i})_{z} (Z/A) \sum_{i}^{A} (r_{Y_{1}}_{i})_{x} \) relevant for reactions with photons and electrons, and isoscalar compressional \( D_{0} = \sum_{i}^{A} (r_{Y_{0}}_{i})_{z} \) relevant for \( (\alpha, \alpha') \) reaction. Due to \( D_{\nu \nu}^{2} \) weights, the contribution of RPA states with a large \( D_{\nu \nu} \) strength is enhanced.

In Fig. 3, the TD summed over two bins of the LES, 6.0-8.8 MeV and 8.8-10.5 MeV, are shown. One sees that, up to a scale factor, the TD for the probes \( D_{1} \) and \( D_{0} \) are rather similar, especially at 6.0-8.8 MeV. Panels a) and c) show that at 6.0-8.8 MeV the protons and neutrons oscillate in phase in the interior area \( 4 \text{ - } 7 \text{ fm} \) (isoscalar flow of the core) but neutrons dominate at larger distances \( r > 7 \text{ fm} \) (contribution of the neutron excess). Due to \( r^{2} \)-factor, mainly the surface area \( r > 7 \text{ fm} \) contributes to the B(E1) \( \propto \int dr r^{2} \rho(r) \). This would favor the simple PDR picture of neutron-core oscillations. At the higher energy, 8.8-10.5 MeV, we see mainly isovector motion at 6-8 fm and again dominance of neutrons at \( r > 8 \text{ fm} \). So, LES here is more isovector and does not support the PDR picture. Furthermore, the isoscalar \( D_{0} \) leads to much weaker TD at 8.8-10.5 MeV relative to the region 6.0-8.8 MeV. This is probably caused by the fact that LES at 6.0-8.8 MeV is mainly isoscalar thus gaining more from the isoscalar weight. Note that the \( r^{2} \)-factor amplifies the pattern in the nuclear surface and damps it in the interior area at \( r < 4 \text{ fm} \). The TD in Fig. 3 indicate that there are sizable effects in the interior. This will be corroborated by the flow pictures below.

A thorough analysis of excitations should also look at CTD which reveal even more details than mere TD. The CTD for dipole states \( \lambda \mu = 10 \) are presented in Figs. 4-7. In Fig. 4, the fields for the isovector GDR (10.5-15 MeV) and high-energy isoscalar CR (22-30 MeV) are given as reference examples. They show typical GDR and CR flows, see \([14, 25, 26]\) for a comparison. In the CR case, the compression and decompression zones along the \( z \)-axis are visible, as in Fig. 1c). These plots for well known modes serve as benchmark and assert the validity of our prescription.

In Figs. 5-7 the fields for the two LES bins, 6.0-8.8 and 8.8-10.5 MeV, are depicted. Since \( D_{1} \) and \( D_{0} \) fields look, up to a scale factor, rather similar (especially for the bin 6.0-8.8 MeV), we will show further on only \( D_{1} \) weighted CTD. In every figure, the actual CTD scale (common for all the plots) is adjusted for better view.
for CTD are used.

Fig. 5 shows the fields in the bin 6.0-8.8 MeV. The neutron flow dominates in both interior and surface. Since protons and neutrons move in phase, the total flow is essentially isoscalar, see panels c) and d) in comparison. The T=0 character of the lower bin of LES is in accordance with previous theoretical results \[4,27\] and experimental findings, e.g. for \(^{124}\text{Sn} \[28\]. What is important for our aims, Fig. 5 clearly demonstrates the overwhelming toroidal flow in neutron and T=0 cases (less in the proton case). This is in accordance with the TR(T=0) strength from Fig. 2b, which is strictly peaked just at 7-8 MeV and dominates over the CR(T=0). Therefore, LES at 6.0-8.8 MeV is of almost pure toroidal (vortical) nature! The irrotational PDR flow is not seen at all.

The LES fields in the bin 8.8-10.5 MeV in Fig. 6 are more complicated. The flow is mainly isovector in the nuclear interior and isoscalar at the surface, thus demonstrating an isospin-mixed character (again in accordance to RMF findings \[4,27\]). Furthermore, the TR flow is faint. Actually there are hints of several flows: TR (b-c), CR (b), and familiar linear dipole (a,d). This complex picture reflects the fact that, following Figs. 2b and c, the region 8.8-10.5 MeV hosts various modes and feels already the vicinity to the GDR.

Finally, Fig. 7 exhibits the \(r^2\)-weighted CTD to highlight the role of surface nucleons (e.g. the neutron excess) in the peripheral reactions like \((\alpha, \alpha')\) and, to a lesser extent, photo-absorption. Flows in Fig. 7 correspond to the TD in Fig. 3 (a-b). The \(r^2\)-weighted presentation weakens the interior flow and emphasizes the role of the neutron excess, see Fig.7 b). Nevertheless, the LES still keeps their TR and mixed (TR/CR) nature in both bins, 6.0-8.8 and 8.8-10.5 MeV. Again we cannot find any sizable evidence for PDR flow.

The question remains how to observe the velocity fields experimentally and thus disclose the true nature of LES. The typical reactions mentioned above are mainly sensitive to the nuclear surface and lose the important information on the nuclear interior. This is especially the case for the isoscalar \((\alpha, \alpha')\) whose response is driven by the operator \(r^3Y_1\) with a huge surface enhanced fac-
tor. (Note also that the most relevant \((\alpha, \alpha')\) measurements of ISGDR in \(^{208}\text{Pb}\) [24] consider the energy interval \(\omega > 8\) MeV and so, following our calculations, lose the TR\((T=0)\) peaked at 7-8 MeV). Perhaps, the \((e, e')\) reaction which can cover both nuclear surface and interior is the most promising tool to examine LES flows.

In this study we do not take into account the coupling with complex configurations which may be essential for LES [28, 29]. However the TR/CR signatures in LES look strong enough to be appreciably smeared out by this effect.

In conclusion, Skyrme-RPA calculations have been performed to inspect the nature of the E1 low-energy strength (LES), often denoted as the pygmy dipole resonance (PDR) and associated with the picture that the neutron skin oscillates against the nuclear core. Strength functions, averaged transition densities and averaged current fields (collecting contributions of all RPA states in a given energy interval) were used for the analysis. The current fields turned out to be most important to illustrate the LES flows. The results show that, in agreement with previous studies [4, 27], LES may be divided into two energy regions, 6.0-8.8 MeV and 8.8-10.5 MeV in our case, where the lower one is basically isoscalar and higher one is isospin-mixed.

What is most interesting, LES at 6.0-8.8 MeV shows a clear toroidal (vortical) nature while the interval 8.8-10.5 MeV gives a mixed toroidal/compression/linear flow. No convincing indicator of PDR-like flow is found. This means that the familiar treatment of LES as the out-of-phase motion of the neutron excess against the nuclear core (arising from the analogy with light halo nuclei and suggested from \(r^2\)-weighted transition densities) is misleading. Our study does not deny the important contribution of the neutron excess to various (basically peripheral) reactions. At the same time, we find that LES flow pattern is far from a simple PDR picture and actually involve various types of motion, irrotational (compression) and vortical (toroidal). In particular, LES at 6.0-8.8 MeV constitutes an almost pure toroidal \(T=0\) resonance. This conclusion may have far-reaching consequences for further exploration of LES and related observables.

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