Detection of BCS pairing in neutral Fermi fluids via Stokes scattering: the Hebel-Slichter effect

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Nearly a decade after the first experimental reports of atomic Bose-Einstein condensates (BEC) [1], the trapping and cooling of gases with Fermi statistics has become one of the central areas of research within the field of ultracold atomic gases. Such gases offer the exciting prospect of examining the properties of interacting Fermi gases, including BCS superfluid states, with unprecedented accuracy and flexibility.

Using Feshbach resonances, one can readily vary the interactions between atoms, allowing study of the crossover between a BEC of tightly bound molecules on the side of the resonance where \(0 < k_F a \ll 1\), with \(a\) the scattering length and \(k_F\) the Fermi momentum, to a BCS superfluid state on the other side of the resonance where \(0 < -k_F a \ll 1\), with an interesting crossover regime in between [2, 3]. Indeed, several groups have now reported clear experimental results for a BEC of diatomic molecules by looking at the momentum distribution of the gas [4]. Experiments have also probed the \(a < 0\) side of the resonance using a fast magnetic sweep to the BEC side [5]. Very recently, the \(a < 0\) side (\(-k_F a \gtrsim 3.3\)) was probed using a radio frequency (rf) transition to an unpaired hyperfine state [6]. The observed broadening of the rf spectral line was interpreted as arising from formation of Cooper pairs [7]. For a trapped system this effect is complicated by the fact that the mean (Hartree) field also yields a significant broadening of the spectrum; the unpaired hyperfine state in general sees a different spatially dependent Hartree field than the paired ones, giving non-trivial energy shifts in the hyperfine transitions. The broadening of the rf line due to this effect has been observed experimentally [8], and it has been demonstrated for a spherical gas in the BCS regime that it introduces broadening of the rf spectrum at least comparable to the effects coming from pairing [8]. An interpretation of the broadening of the rf spectrum in terms of pairing effects only is therefore not straightforward for an inhomogeneous system. The absence of unambiguous signatures of the presence of superfluidity in the BCS limit \((k_F a \ll 1)\), arises because the formation of large delocalized Cooper pairs does not affect the bulk properties. Clear observation of BCS superfluidity is one of the central problems for experimentalists. Suggestions to detect the onset of superfluidity include probing the collective mode spectrum [10], the quantization of angular momentum [11], off-resonance light scattering [12, 13], and by probing the dynamic structure factor using a scheme which avoids the complication due to the inhomogeneous Hartree field [12].

One of the hallmark experimental tests of BCS theory in conventional superconductors was the observed enhancement of the nuclear spin relaxation just below the transition temperature [14], an experiment which probed the detailed nature of the pair correlations. In this paper, we propose a related experiment to probe the pair correlations in trapped atomic gases, by looking at off-resonance inelastic (Stokes and anti-Stokes) light scattering of a laser beam on a two component atomic Fermi gas. The present scheme differs from the earlier suggestions using off-resonant light scattering [12, 13] crucially: we propose looking at Stokes and anti-Stokes inelastic light scattering involving a change of the hyperfine states of the atoms. The present proposal has several attractive features. First, there is no central coherent scattering peak, which is effectively insensitive to superfluid correlations. Also, as we show, the effects due to superfluidity on the scattered spectrum can be strikingly large close to the critical temperature, \(T_c\). Finally, since the present

![FIG. 1: Stokes (a) and anti-Stokes (b) scattering of an incident laser beam. In (a) the laser, detuned off resonance by a frequency \(\delta\), excites a particle in the lower level \(| \uparrow \rangle\) to an intermediate highly excited level \(| \uparrow \rangle\), which de-exites, emitting a lower frequency photon, \(\gamma\). The hyperfine splitting of the paired levels is \(\omega_{hf}\). In (b) the initial and final states are interchanged.](image-url)
scheme simply flips the atoms between the two paired hyperfine states, it is not beset by complications due to non-trivial energy shifts coming from non-superfluid effects, e.g., shifts in the Hartree energies $\tilde{\omega} \pm \frac{1}{2}|k|$.

We consider a trapped gas of atoms in two equally populated hyperfine states labelled by $|\sigma\rangle$, with $\sigma = \uparrow, \downarrow$, interacting via an effective attractive interaction. We take the state $| \downarrow \rangle$ to have energy $\omega_{hf}$ ($\hbar = 1$ in this paper) above the state $| \uparrow \rangle$, and assume that below a transition temperature, $T_c$, the gas is BCS paired and superfluid. Consider a laser beam of frequency $\omega_L$ and wave vector $\kappa$ illuminating the gas. As illustrated in Fig. 1, the light field can induce dipole radiation from the atoms by connecting the two hyperfine states $| \uparrow \rangle, | \downarrow \rangle$ to an electronically excited state $| e \rangle$, which we take to have energy $\omega_e$ above the state $| \uparrow \rangle$.

For large laser detuning, $\delta = \omega_L - \omega_e$, the excited level $| e \rangle$ is not significantly populated and it can be adiabatically eliminated from the theory. The spectral intensity of the scattered light at position $r$ is then

$$S(r, \omega) = \sum_{\sigma_1 \sigma_2} I_{\sigma_2 \sigma_3}^{\sigma_1 \sigma_4}(r) \int_{-\infty}^{\infty} dt \int d^3r_1 d^3r_2 e^{i[\omega t + \Delta k (r_1 - r_2)]} \times \langle \psi_\sigma(r_1) \psi_\sigma(r_1) \psi_\sigma(r_2) \rangle$$

(1)

where the $\psi_\sigma$ are the field operators for the two low-lying hyperfine states $| \sigma \rangle$. Here $I_{\sigma_2 \sigma_3}^{\sigma_1 \sigma_4}(r)$ is the scattered light intensity from a single atom, including the dependence of the atomic levels involved and the various directions of the experiment, and $\Delta k$ is the change in momentum of the scattered light compared to the incident light. The frequency $\omega = \omega_S - \omega_L$ is the difference between the scattered light frequency $\omega_S$ and the incident light frequency $\omega_L$.

Equation (1) describes two types of off-resonant light scattering processes. The first, elastic or coherent scattering, is characterized by the initial and final atomic hyperfine states being identical, corresponding to $\sigma_1 = \sigma_3$ and $\sigma_2 = \sigma_4$. This process has been examined in detail by a number of authors and several effects of pairing have been identified. However, measurement of these effects is complicated by a large background coherent scattering process. The second type of scattering process is characterized by different initial and final atomic hyperfine states. If the initial state of the atom is $| \uparrow \rangle$ and the final is $| \downarrow \rangle$, the emitted light frequency is reduced by $\omega_{hf}$ from the incoming light frequency: this Stokes scattering process, Fig. 1a, corresponds to $\sigma_1 = \sigma_4 = \uparrow$ and $\sigma_2 = \sigma_3 = \downarrow$ in Eq. (1). If, on the other hand, the initial state of the atom is $| \downarrow \rangle$ and the final is $| \uparrow \rangle$, the emitted light frequency is increased by $\omega_{hf}$; this anti-Stokes scattering process is shown in Fig. 1b. Stokes and anti-Stokes scattering therefore probe correlation functions that involve a hyperfine state “spin” flip at positions $r_1$ and $r_2$ in Eq. (1). As we show, the rate of these Stokes and anti-Stokes transitions involving atomic hyperfine state flips are strongly affected close to $T_c$ by BCS pairing of the states, in contrast to elastic light scattering which does not involve a hyperfine flip. We note that for such inelastic scattering to occur, dipole transitions between the electronically excited (orbital p) state $| e \rangle$ and the two (orbital s) hyperfine states should be allowed by the selection rules. That is, both the dipole matrix elements relevant for Stokes and anti-Stokes scattering, $\langle e | d \cdot \mathcal{E} | \uparrow \rangle$ and $\langle e | d \cdot \mathcal{E} | \downarrow \rangle$, with $d$ the atomic dipole operator, and $\mathcal{E}$ the electric field of the incident laser, must be non-zero.

From Eq. (1), the problem of calculating the scattered light intensity is reduced to evaluating the Fourier transform of the correlation function $\langle \psi_\sigma^\dagger(r_1) \psi_\sigma(r_1) \psi_\sigma^\dagger(r_2) \rangle$. Experiments on the pairing transition for ultracold Fermi atomic gases use a Feshbach resonance to enhance the atom-atom interaction, thereby increasing $T_c$. The gas is then best regarded as a molecular BEC on one side of the resonance and a weakly coupled BCS superfluid on the other side of the resonance, with a crossover region in between. We are interested here in the problem of detecting the presence of large delocalized Cooper pairs on the BCS side of the resonance, where it is adequate to use mean-field BCS theory. We expand the field operators in Bogoliubov eigenmodes $|u_\eta(r), v_\eta(r)\rangle$ with energy $E_\eta$, which can be obtained from a solution of the Bogoliubov-de Gennes equations (18). With this expansion, Eq. (1) for Stokes and anti-Stokes scattering yields

$$S(\omega) \propto \sum_{\eta\eta'} \left\{ \frac{1}{2} |(u^*_{\eta u'_{\eta'}} - v^*_{\eta u'_{\eta'}})\Delta \omega|^2 \right. \times (1 - f_{\eta})(1 - f_{\eta'})\delta(\omega + E_{\eta'} + E_\eta)$$

$$+ \frac{1}{2} |(u_{\eta} v_{\eta'} - v_{\eta} u_{\eta'})\Delta \omega|^2 f_{\eta} f_{\eta'}\delta(\omega - E_{\eta'} - E_\eta)$$

$$+ \left. |(u_{\eta} u_{\eta'} + v_{\eta} v_{\eta'})\Delta \omega|^2 f_{\eta}(1 - f_{\eta'})\delta(\omega + E_{\eta'} - E_\eta) \right\},$$

(2)

where $f_{\eta} = [\exp(\beta E_{\eta}) + 1]^{-1}$ is the Fermi function and $h_k = \int d^3r \exp(-i kr) h(r)$ denotes the Fourier transform. Equation (2) describes the creation and annihilation, respectively, of two quasiparticles. The third term describes the scattering and hyperfine “spin” flip of a quasiparticle. For Stokes scattering, the quasiparticle is in the initial state $(\eta \uparrow)$ with energy $E_\eta$ and it scatters into the state $(\eta' \downarrow)$ with energy $E_{\eta'} + \omega_{hf}$. Energy conservation for this process reads $\omega_L + E_{\eta'} = \omega_S + E_\eta + \omega_{hf}$. For anti-Stokes scattering the initial and final quasiparticle states are $(\eta \downarrow)$ and $(\eta' \uparrow)$ with energies $E_{\eta'} + \omega_{hf}$ and $E_\eta$ respectively.

We first consider light scattering on a homogeneous system, which can be approximately realized experiment-
tally for a trapped gas by focusing the laser beam on an area much smaller than the size of the atomic cloud. The quasiparticle eigenfunctions are then plane waves; for small frequency shifts, $\tilde{\omega} \ll \Delta$, where $\Delta$ is the superfluid gap, we can neglect the terms in Eq. (2) describing the creation and annihilation of two quasiparticles, and obtain

$$S(\omega) \propto \sum_q |u_q u_{q+\Delta k} + v_q v_{q+\Delta k}|^2 f_q (1 - f_{q+\Delta k}) \times \delta (\omega + \omega_{hf} + E_{q+\Delta k} - E_q),$$

with $u_q^2 = (1 + \xi_q/E_q)/2$ and $v_q^2 = (1 - \xi_q/E_q)/2$. Here $\xi_q = q^2/2m - \mu$, and $E_q = \sqrt{\xi_q^2 + \Delta^2}$. In the limit, $\Delta k \ll k_F$, and $\tilde{\omega} \ll \Delta k k_F/m$ this expression reduces to,

$$S(\omega) \propto \frac{1}{\Delta k} \int_{\min(\Delta, \Delta+\omega)}^{\infty} dE \frac{E}{\sqrt{E^2 - \Delta^2}} \frac{E'}{\sqrt{E'^2 - \Delta^2}} \left(1 + \frac{\Delta^2}{EE'}\right) f(1 - f'),$$

(4)

where $E' = E - \tilde{\omega}$. Equation (4) is identical to the expression for the nuclear relaxation rate for polarized nuclei in a conventional superconductor [16]. In particular, we note that the scattered intensity is logarithmically divergent for $\tilde{\omega} = 0$ in an infinite size superfluid system. This apparent divergence, which arises from the divergent density of states at the Fermi surface for a superconductor, only appears in “spin”-flip processes such as Stokes and anti-Stokes scattering. In reality the divergence is mediated by finite size and finite lifetime effects.

Had we instead considered elastic scattering processes where the initial and final hyperfine states of the atom are identical, the coherence factor in the last term in Eq. (2) describing quasiparticle scattering would read $u u^* - v v^*$. This factor vanishes at the Fermi surface for $\tilde{\omega} = 0$, and the coherence term for elastic scattering exactly cancels the divergent density of states at the Fermi surface. For Stokes and anti-Stokes scattering on the other hand, the coherence factor in Eq. (2) reads $u u^* + v v^* \sim 1$ and the divergence in the density of states at the Fermi surface due to pairing shows up in the response of the gas. The coherence factor reflects the fact that the coupling of the quasiparticles to a “spin flip” perturbation is essentially the same in the superfluid and normal phases. It is important that the measurement be made on a finite $\Delta k$ such that $\tilde{\omega} \ll \Delta k k_F/m$. For $\Delta k = 0$, the overlap integrals in Eq. (2) simply yield a $\delta n_n$ selection rule, and the scattered signal becomes $\propto \sum_q f_q (1 - f_q)$, which does not exhibit a peak below $T_c$ due to pairing. Measuring the intensity of the Stokes and anti-Stokes lines allows one to test the detailed nature of the pairing correlations reflected in the coherence factors, as was done for the BCS theory [16].

To illustrate the effect, we evaluate Eq. (4) numerically, with a pairing gap obtained by solving the BCS gap equation as a function of $T$. We have chosen $k_F|a| = 0.3$ and a frequency shift $\tilde{\omega}/\epsilon_F = 0.0001$, where $\epsilon_F$ is the Fermi energy. Figure 2 shows the temperature dependence of the scattered light intensity of the Stokes and anti-Stokes lines. We see that close to $T_c$, the intensity of light scattered from the superfluid state is significantly larger than from the normal state. The intensity for scattering from the normal state is $\propto T$. The scattered light from the superfluid is very large close to $T_c$ due to the density of states effect described above, while it becomes exponentially suppressed for $T \to 0$, since the density of quasiparticles available for scattering scales as $\exp(-\beta \Delta)$. Due to the large peak in the scattering intensity below $T_c$, a Stokes–anti-Stokes experiment could clearly reveal the presence of pairing. Note that as $\tilde{\omega}$ decreases, the peak below $T_c$ grows.

To study the effect of the trapping potential, we now examine the scattered light from a gas in a spherical trap $V_{\text{pot}}(r) = mw_T r^2/2$. The Cooper pairing is between atoms in time-reversed angular momentum states, $(l, m)$ and $(l, -m)$, where $l$ is the single particle orbital angular momentum, and $m$ its component along the incident laser beam. We solve the Bogoliubov–de Gennes equations numerically using the method described in Ref. [16]. With the quasiparticle energies and wave functions obtained from this calculation, we then compute the scattered intensity from Eq. (2), where the quantum number $n$ now stands for $(n, l, m)$, with $n$ the radial quantum number.

In Fig. 2, we show the calculated intensities, for $1.6 \times 10^4$ particles trapped with a critical temperature $k_B T_c \approx 0.09 \epsilon_F$, $\tilde{\omega} = 0.003 \epsilon_F$, and $\Delta k = 2/l_0$ with $l_0 = (m \omega_T)^{-1/2}$ the trap length. As in the homogeneous case, the scattered intensity from the superfluid gas has a large maximum below $T_c$. Again, this peak is due to the increased density of states close to the Fermi level in the superfluid phase for $T$ close to $T_c$.

FIG. 2: The scattered Stokes and anti-Stokes light intensity in a homogeneous system as a function of the reduced temperature $T/T_c$, normalized to the scattered intensity at $T_c$. The inset shows the BCS gap $\Delta(T)$.
The shell structure of normal phase quasiparticle levels is less pronounced than in the non-interacting case since the Hartree field breaks the degeneracy of each harmonic oscillator level in the normal phase [20]. This introduces a dispersion in the quasiparticle energies as a function of angular momentum $l$. The pairing suppresses this effect bringing the quasiparticles closer in energy, and yielding a maximum in the density of states close to the Fermi level. This effect is shown in the inset in Fig. 3 where the lowest quasiparticle energies are plotted as a function of $l$ for the superfluid state (×) and the normal state (+) for $T/T_c \sim 0.7$. We see that the dispersion of the energy levels as a function of $l$ is larger in the normal phase than in the superfluid phase. Again, it is crucial to choose a finite momentum shift $\Delta k = 2/l_c$ such that the total angular momentum $l$ of a quasiparticle is not conserved in the light scattering process. However for $m$ the angular momentum along the beam axis, a given quasiparticle with angular quantum numbers $(l, m)$ can scatter to any quasiparticle state with quantum numbers $(l', m)$ with $l' \neq l$, and the peak in the density of states close to the Fermi energy in the superfluid state shows up in the intensity of the scattered light. For a very small momentum shift $\Delta k \ll l_c^{-1}$, the scattered signal would, as for the homogeneous case, from Eq. (2), be $\propto \sum \delta n_l (1 - \delta n_l)$, which makes it not exhibit any peak below $T_c$ due to pairing. In contrast to the homogeneous case, the increase in the scattering intensity due to pairing is smooth at $T_c$ due to the finite size of the spherical system. Again, the size of the peak increases with decreasing $\omega$.

In summary, we propose the detection of pairing in a two component atomic Fermi gas by looking at off-resonant light scattering. The onset of Cooper pairing yields a large peak in the intensity of the Stokes and anti-Stokes lines below $T_c$ for both a homogeneous and a trapped gas. The proposed effect, which is the light scattering analog of the famous Hebel-Slichter effect on the nuclear relaxation rate in conventional superconductors, is unique in that the superfluidity gives an enhanced signal close to $T_c$. The interpretation of the results it is not beset by non-trivial mean field energy shifts.

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