Fundamental aspects of resolution and precision in vertical scanning white-light interferometry

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Keywords: height and lateral resolution in SWLI, batwing effect, scanning white-light interferometry (SWLI), phase-shifting interferometry (SPI), interferometric slope effect, envelope and phase evaluation, nonlinear envelope filtering

Abstract
We discuss the height and lateral resolution that can be achieved in vertical scanning white-light interferometry (SWLI). With respect to interferometric height resolution, phase-shifting interferometry (PSI) is assumed to provide the highest accuracy. However, if the noise dependence of SWLI phase evaluation and PSI algorithms is considered, SWLI measurements can be shown to be more precise. With respect to lateral resolution, the determination of the coherence peak position of SWLI signals seems to lead to better results compared to phase-based interferometric measurements. This can be attributed to the well-known batwing effect. Since batwing is a nonlinear effect applying nonlinear filters, e.g. a median filter, it reduces them significantly. If filtering is applied prior to the fringe order determination and phase evaluation, the number of artefacts known as ghost steps can be eliminated without changing the modulus of the phase. Finally, we discuss the dependence of measured height values on surface slope. We show that in interference microscopy there are additional limitations which are more rigid compared to the maximum surface slope angle resulting from the numerical aperture of the objective lens. As a consequence, the measurement precision breaks down at slope changes of steeper flanks even if the modulation depth of the interference signals is still good enough for signal analysis.

1. Introduction
Interference microscopes are mature instruments in context of optical 3D topography measurement. With respect to height resolution, phase-shifting interferometry (PSI) represents something like a gold standard. Nevertheless, today's optical profilers based on interference microscopy mostly use white or low-coherent light in order to overcome the phase ambiguity problem known from PSI measurement. It is generally assumed, that in the optimum case the height resolution capabilities of scanning white-light interferometry (SWLI) is of the same order as the height resolution of PSI instruments if phase evaluation techniques are used [1].

We will discuss the height resolution capabilities of interferometric profilers with respect to noisy signals in section 2.

The lateral resolution of optical profilers is typically obtained from the lateral resolution limit of the microscopic imaging system, e.g. using the well-known Rayleigh criterion [2]. However, in practice the situation is more complicated. As it is shown in section 3 the lateral resolution capabilities depend on the height of the surface structure and the signal processing strategy.

The maximum tolerable surface slope is generally obtained from the angle defining the numerical aperture of a microscope objective. In section 4 we show, that there is a further limitation of the maximum measurable surface slope in interference microscopy. This limitation is closely related to the lateral resolution capabilities of the microscope objective as it limits the maximum fringe density.

Finally, in section 5 we discuss an approach to reduce so-called ghost steps in the results of phase evaluation without filtering the phase values but only the topography data obtained from the coherence envelopes of SWLI signals.
2. Comparison of SWLI and PSI with respect to height measurement precision

One of the most significant advantages of interferometric measurement principles is the outstanding height resolution that can be reached.

The height resolution resulting from either PSI or SWLI is finally limited by the occurring noise and the algorithm used for phase estimation in the presence of noise. Since it is difficult to generate a well-defined noise level experimentally, Monte–Carlo simulations represent an appropriate method in this context as they allow to create ensembles of interference signals of well-defined signal-to-noise ratio (SNR). These signals can be analyzed with respect to their phase, and the standard deviation of the estimated phase value compared to the given phase value can be obtained in order to compare the performance of different algorithms and simulation results with theory.

The noise level of a signal is commonly described on the basis of the SNR, which is defined as the ratio of the mean square of the desired signal divided by the variance of the noise signal [7]:

\[
\text{SNR} = 10 \log_{10} \left( \frac{\text{mean square of signal}}{\text{variance of noise (bandwidth equal to Nyquist frequency)}} \right)
\]

(1)

If there is no bias, the uncertainty of the phase estimation decreases as the SNR increases.

In the following we compare several algorithms with respect to their performance in the presence of noise. Therefore, we first generate a series of random numbers equally distributed between 0 and 1. Then we multiply these random numbers by a given phase value interval \(\Delta \Phi\) and subtract \(\Delta \Phi/2\) resulting in random phase values equally distributed between \(-\Delta \Phi/2\) and \(+\Delta \Phi/2\).

Interference signals, each represented by a cosine function with a random phase value and additive Gaussian distributed white noise considering a certain SNR, are simulated using the above mentioned procedure. All algorithms studied below work on the same data set of noisy interference signals.

In order to study phase estimation in white-light interferometry we consider also a Gaussian envelope function multiplied by a cosine function as a simple mathematical signal model. Figures 1(a) and (b) show the two types of signal with and without Gaussian envelope. In both cases the upper signal has maximum SNR of 100 dB, and the lower signals are related to the minimum SNR value of 0 dB. According to our experience, typical interference signals obtained experimentally show SNR values between 20 and 60 dB.

The uncertainty of phase estimation is shown in figure 2 as the standard deviation of the estimated phase value compared to the given phase value for different phase detection algorithms. For each SNR value the phase values of 1000 signals are estimated leading to the desired standard deviation. The simulation algorithm assumes a wavelength of light of 640 nm and a sampling interval of 20 nm. Hence, the period of the interference signal is 320 nm, so that four times the sampling interval corresponds to a 90° phase shift. This allows us to study phase shifting algorithms as well as phase lock-in techniques which are not restricted to 90° phase shifts based on the same set of simulated interference data.

As representative examples of PSI algorithms the five point phase estimation algorithm introduced by Schwider and Hariharan [4, 5]:

\[
\varphi = \arctan \left( \frac{2(I_2 - I_4)}{2I_3 - I_1 - I_5} \right)
\]

(2)

and the well-known four step algorithm [3, 4]:

\[
\varphi = \arctan \left( \frac{I_2 - I_4}{I_1 - I_5} \right)
\]

(3)

are investigated with respect to their accuracy. It should be noted that these algorithms are unbiased, i.e. they are mathematically exact and yield perfect results in the absence of noise. Further PSI algorithms can be found in [4, 6], for example. However, since it is not our goal to give a complete overview of all algorithms we confine our studies to the two algorithms introduced in equations (2) and (3). The four step algorithm assumes exact 90° phase shifts in order
to avoid systematic errors, which are studied elsewhere \cite{6}. In contrast, the Schwider–Hariharan algorithm is more tolerant against constant phase shift calibration errors. The results obtained with these PSI algorithms are compared with the phase lock-in technique \cite{7}, assuming either 90° phase shifts (i.e. 80 nm sampling interval) or less (20 nm sampling interval).

The formula for the lock-in technique is given by:

\[
\varphi = \arctan \left( \frac{\text{Im} \{ S(f_0) \}}{\text{Re} \{ S(f_0) \}} \right),
\]

\[
S(f_0) = \sum_{n=0}^{N-1} w((n - n_0) \Delta z)e^{-2\pi i f_0 n \Delta z}
\]

with \( \Delta z \) being the sampling interval along the height axes during the depth scan, \( N \) the total number of sample points used for phase analysis, \( f_0 \) the spatial frequency of an interference signal, and \( w(n \Delta z) \) a window function. For the simulation results shown below we use a Blackman window function and calculate the arctangent function in equations (2)–(4) using a four-quadrant algorithm, which exhibits an unambiguity range of \( 2\pi \).

Figure 2 demonstrates that the performance of the phase lock-in technique is in most cases superior compared to phase-shifting techniques. This is due to the higher number of sample points which lead to a better accuracy of the lock-in algorithms in the presence of noise. It should be noted that our lock-in algorithm uses 101 sample points at 20 nm sampling interval in order to calculate a single phase value. These sample points as well as the sample points used for the PSI algorithms are centered with respect to sample point 512 according to figure 1.

The lock-in technique applied to all sample points related to the sampling interval of 20 nm shows lower uncertainty compared to the lock-in technique based on 90° phase shift, i.e. 80 nm sampling interval and 25 sample points used for phase estimation.

For comparison we added a constant line in the diagrams displayed in figures 2(a)–(c), which corresponds to the standard deviation \( \sigma_{\text{eq}} \) of equally distributed phase values:
\[
\sigma_{\text{eq}} = \frac{\Delta \Phi}{\sqrt{12}}. \tag{5}
\]

Another theoretical curve shown in the diagrams of figures 2(a)–(c) indicated by ‘Brophy (theoret.)’ represents the theoretical standard deviation of the phase estimation calculated by the formula [8]:
\[
\sigma_p^2 = \frac{k \sigma_\text{I}^2}{b_0^2} \tag{6}
\]

with the standard deviation \(\sigma_\text{I}\) related to the noisy sinusoidal interference signal of amplitude \(b_0 = 1\) and a constant \(k\) which equals \(7/16\) for the Schwider–Har-iharan algorithm. This formula assumes uncorrelated intensity noise with equal variance from frame to frame. If the same calculation is done for the four step algorithm the resulting standard deviation \(\sigma_p\) will increase by a factor of \(\sqrt{8}/7\) i.e. by 6.9%. A theoretical calculation of the minimum variance known as the unbiased Cramér–Rao bound resulting from phase estimation based on the lock-in technique according to equation (4) under the assumption of sinusoidal input signals of known frequency and a rectangular window comprising \(N\) sample points has been derived by Rife and Boorstyn [9]:
\[
\sigma_p^2 = \frac{\sigma_\text{I}^2}{(b_0^{-2}N)}. \tag{7}
\]

This formula agrees with equation (6) if the constant \(k\) equals \(1/N\). However, since we use a Blackman window function \(w(n)\) in order to get rid of leakage effects equation (7) must be modified as follows:
\[
\sigma_p^2 = \frac{\sigma_\text{I}^2}{(b_0^{-2} \sum_{n=0}^{N-1} w(n))}. \tag{8}
\]

Assuming unity amplitude again, the theoretical results according to equation (8) are plotted in figures 2(a)–(c) indicated as ‘CR-bound (Lock-in)’ for \(N = 1 = 100\). This corresponds to the sampling interval of 20 nm. For the 90° sampling interval of 80 nm the standard deviation \(\sigma_p\) according to equation (7) will increase by a factor of 2. This agrees with the reduction of the standard deviation by averaging which is known from statistics.

Figure 2(b) indicates that for a Gaussian envelope or, more general, for any amplitude modulation the PSI algorithms are no longer unbiased estimators as the uncertainty of the algorithms tends to a constant value for SNR > 20 dB. In contrast, the lock-in algorithms are less affected by the envelope of an interference signal and their uncertainty still decreases as the SNR increases. From this result one may conclude that in coherence scanning interferometry where the interference signals show a characteristic envelope PSI algorithms are not the best choice for phase estimation.

In figures 2(a) and (b) the phase difference values to be estimated are limited to the interval \(-\pi/2 < \Delta \Phi \leq \pi/2\). As a consequence, no \(2\pi\) phase jumps occur.

Figure 2(c) shows that the total range \(\Delta \Phi\) of phase values which are to be estimated is an important point with respect to the performance of phase estimation algorithms.

In general, the phase shifting algorithms need a higher SNR in order to work correctly if the phase values are distributed over a complete \(2\pi\) interval. If a phase estimation error occurs close to the boundary of such an interval, \(2\pi\) phase errors result which enhance the phase estimation uncertainty significantly. This can be seen by comparison of the results obtained for a range of phase values of \(2\pi\) (figure 2(a)) compared to figure 2(a) where an interval of \(\pi\) was chosen. In figure 2(d) each point represents the deviation of a phase value estimated by the four step algorithm with respect to the given phase value. If, for example, the given phase value is \(179°\) and the phase estimation error due to noise is \(2°\) an estimated phase value of \(−179°\) will result and the deviation plotted in figure 2(d) will be 358°. In figure 2(d) the highest SNR value where this situation occurs is 35 dB. Consequently, in figure 2(c) beginning with SNR = 40 dB the standard deviation of the phase values estimated by the four point algorithm drops down and the results are close to the theoretical ‘Brophy’ curve. It should be mentioned that the SNR value where this drop occurs depends strongly on the concrete set of random numbers used for the simulation. Practical consequences of such \(2\pi\) phase jumps can be observed in example 5.1 of [10], which shows noisy interference patterns that lead to ragged edges where the phase value jumps from one \(2\pi\) interval to another.

Of course, this kind of phase jumps can be often eliminated by phase unwrapping algorithms [3]. However, for higher densities of phase jumps phase unwrapping algorithms will run into problems.

Only for very high SNR values of more than 80 dB and cosine signals the Schwider–Har-iharan algorithm shows a better performance compared to the phase lock-in technique. This is due to the bias, which is inherent in the phase lock-in detection algorithms and limits the uncertainty to approximately 0.003°. These systematic errors of the phase estimation are related to the window function which leads to a different weighting of sine and cosine components. In figures 2(a) and (c) the phase uncertainty due to bias corresponds to a theoretical height uncertainty of approximately 2.5 pm. Therefore, it can be neglected in practice since there are other more dominating effects influencing the uncertainty of height measurements [11]. In addition, the bias can be further reduced if a broader window function comprising e.g. 200 sample points is used.

In general, the performance of the Schwider–Har-iharan algorithm compared to the four step algorithm is slightly better as it uses five instead of only four sample points and thus shows a better noise compensation. This agrees with the well-known fact that PSI algorithms using more steps provide better accuracy [6, 12].
related to surface height differences. If the contrast of the interference pattern is directly proportional to the phase object, then the amplitude object, the phase object is converted into an amplitude object, so the wavelength of the illuminating light, interferometry

tudes are very small compared to a quarter of the wavelength. Under the assumption that the surface amplitudes are small, the lateral resolution is improved.

3. Lateral resolution—comparison of phase and envelope evaluation

Besides their outstanding height resolution capabilities, the lateral resolution is another important feature of 3D interference microscopes. As the lateral dimensions of micro- and nano-components steadily increase, the lateral resolution of measuring instruments is gaining more and more interest. Unfortunately, the lateral resolution in microscopic 3D measurement is not uniquely defined. Sometimes the lateral resolution known from conventional imaging microscopy is simply used even in 3D microscopy. The problem is, that these definitions, which are based on the Rayleigh or the Sparrow criterion for incoherent illumination, are not directly related to the measured surface topography [13]. This is a consequence of the fact that the surfaces under investigation typically behave as phase objects, i.e. the surface height function modulates the phase of an electromagnetic wave. Under the assumption that the surface amplitudes are very small compared to a quarter of the wavelength of the illuminating light, interferometry converts the phase object into an amplitude object, so that the contrast of the interference pattern is directly related to surface height differences [14]. Under this condition the 3D-interference microscope shows linear transfer characteristics and the lateral resolution limit of the 3D microscope corresponds to the 2D resolution of the microscopic imaging system. In this context the lateral period limit of an instrument is defined as the period of a sinusoidal surface which results in a measured amplitude that equals 50% of the real amplitude of the surface [13, 15]. Under the above mentioned approximation for weak amplitudes the lateral period limit corresponds to twice the Rayleigh resolution, i.e. to the diameter of the Airy disk [15].

However, in practical situations the measured amplitude may be 50% of the real amplitude even if the period differs from the Airy disk diameter. In particular, the lateral resolution in 3D microscopy depends on the shape and the amplitude of the measuring object or calibration specimen (rectangular or sinusoidal grating structure), the center wavelength of the used illumination source and the kind of signal processing, e.g. envelope of phase based evaluation. This is established in figures 3 and 4 by measurement results obtained from a one-dimensional rectangular silicon grating and a digital versatile disc (DVD). For these measurements a custom-made vertical scanning Linnik interferometer equipped with two 100X, NA = 0.9 objectives was developed in our lab. In figure 3(a) a grating of 300 nm period and 140 nm peak-to-valley (PV) amplitude is measured using a blue LED (λ = 460 nm) for illumination. Hence, a Rayleigh resolution of 312 nm results. The measured profile demonstrates that, although the period is slightly below the Rayleigh resolution limit, the measured grating amplitude is 50% of the real amplitude. Therefore, if the envelope position of the white-light interference signals is evaluated, the structure is laterally resolved according to the 50% criterion, although this criterion is not really applicable here, since we are out of the linear range of the measuring instrument. The profile obtained from phase evaluation shows a significantly lower amplitude, so that for this kind of signal evaluation the 50% resolution criterion is no longer fulfilled. However, the period of the grating structure can still be recognized. The result shows that the envelope evaluation seems to provide a better lateral resolution in this situation. This is a consequence of the batwing effect which typically occurs at edges [16–18]. The batwing height reaches its maximum value if the step height equals a quarter of the effective
wavelength [18]. Due to the rather high numerical aperture of 0.9 the effective wavelength is close to 560 nm even for blue LED illumination, so that a quarter of this effective wavelength is close to the step height of the profile. In figure 3 the grating period is so small, that batwings occurring at different flanks overlap and the surface amplitude resulting from envelope evaluation is finally overestimated in comparison with the amplitude obtained from phase evaluation. Consequently, the surface profile obtained from envelope evaluation seems to be closer to the real profile and this leads to the impression of better lateral resolution. In addition, the profile obtained from phase evaluation in figure 3 is 180° phase shifted with respect to the profile resulting from envelope evaluation. This occurs if the grating amplitude is slightly higher than a quarter of the evaluation wavelength [18, 19].

If, on the other hand, a rectangular profile with a step height that equals half the effective wavelength is measured, the phase evaluation will result in a constant phase value and the modulation depth of the profile obtained from the envelopes will be dramatically reduced due to inherent low pass filtering. In this case the grating period is hardly resolved although it may be the same as discussed in context with figure 3.

Figure 4 shows that these effects have to be considered in practice as soon as the lateral dimensions of the measured surface structure is of the same order as the lateral resolution of the measuring instrument. The color images in figures 4(a) and (b) represent the results of envelope and phase evaluation, respectively. The pits of the DVD structure can be clearly seen. Figures 4(c) and (d) display the profiles taken from the straight lines plotted in figures 4(a) and (b). Although the pits are laterally resolved in both cases, the depth of the pits (approximately 120 nm) is overestimated by the envelope evaluation whereas it is significantly underestimated by the phase evaluation [20]. This agrees with the above explanation related to figure 3. In conclusion, due to the batwing effect the lateral resolution may lead to the impression of being smaller if the envelope position is evaluated, whereas the topographies obtained from phase evaluation show a lower vertical resolution but a stronger low-pass filtering effect due to the diffraction limit.

The wavelength dependence of batwings can be obtained from figure 5, which shows results of measurements conducted with the above mentioned Linnik interferometer on a rectangular grating structure of 6 μm period and 191 nm PV-amplitude. For diagrams (a) and (c) a red LED was used to illuminate the specimen, whereas for (b) and (d) the interferometer was equipped with a blue LED. Obviously, the batwings are significantly stronger in the profiles according to figure 5(a) where the effective wavelength is close to four times the step height as it is shown in figure 5(c). The effective wavelength is obtained experimentally from the center frequency of the Fourier transformed SWLI interference signals. Using the blue LED for illumination and keeping everything else unchanged the batwings could be significantly reduced, since now the PV-amplitude differs from a quarter of the effective wavelength according to figure 5(d). Another effect that can be observed in figure 5 is that the effective wavelength changes at the edges of the profiles compared to the plateaus. In figure 5(c) the effective
wavelength increases whereas in figure 5(d) it decreases at the edges. This is a consequence of the destructive interference which occurs at the edges due to diffraction. If red light is used the higher frequency components of the signals occurring at the edges are missing because of destructive interference of electromagnetic waves coming from the different height levels. The lower frequency components corresponding to a higher effective wavelength are still present. If blue light is used the situation changes. Now, destructive interference occurs for the signal contributions of lower frequency, i.e. the higher frequency components still contribute to the interference signals and this corresponds to a shorter effective wavelength at the edges.

### 4. Interferometric slope effect

Besides vertical and lateral resolution the tolerance against surface tilt and slope is another important criterion to characterize a measuring instrument.

A well-known fundamental limit of maximum measurable surface slope is usually obtained from the numerical aperture of the used objectives. According to figure 6(a) the maximum slope angle directly corresponds to the angle $\theta_{\text{NA}}$ defining the NA. If the tilt angle is steeper than $\theta_{\text{NA}}$ no reflected light is collected by the objective lens anymore and thus no interference signal appears. However, in interferometry there is another consideration limiting the maximum surface slope as a consequence of the interference fringe density. A tilted surface can be measured without difficulty as long as the fringe spacing in an interference image is well above the lateral resolution limit. If the lateral resolution limit is reached, low-pass filtering occurs and the fringe contrast will be reduced until it disappears when the interference fringes are no longer laterally resolved.

A significant low-pass filtering effect can be expected if the lateral distance between two fringes corresponds to the Airy disk diameter as it is sketched in figure 6(b). The height difference corresponding to the fringe spacing is half the effective wavelength $\lambda_{\text{eff}}$. This leads to a critical angle

$$\tan (\theta_{50\%}) = \frac{\Delta z}{\Delta x} \rightarrow \theta_{50\%}$$

$$= \arctan \left( \frac{\lambda_{\text{eff}} \cdot \text{NA}}{2.44 \cdot \lambda} \right).$$

$$\lambda_{\text{eff}} = \lambda \left( 1 - \frac{1}{2} \sin^2 \left( \frac{\theta_{\text{NA}}}{2} \right) \right),$$

where the effective wavelength definition according to Abdulhalim [21] is used. The angle $\theta_{50\%}$ corresponds

![Figure 5. Measured profiles of a rectangular grating structure (SiMetrics RS-N) of 6 μm period and 191 nm PV-amplitude, (a) result of envelope and phase evaluation obtained with red LED illumination, (b) result of envelope and phase evaluation obtained with blue LED illumination (the profiles obtained from phase evaluation are offset by a constant height value for clarity), (c) effective wavelength for the profiles shown in (a) and (d) effective wavelength for the profiles shown in (b).](image)

![Figure 6. Maximum tilt angle due to NA (a), tilt angle related to significant low-pass filtering of interference fringes due to limited lateral resolution (b).](image)
to a reduction of 50% of the modulation depth of a measured SWLI signal compared to a signal obtained from a perfectly aligned plane without any slope. The effective wavelength is a consequence of the high NA values leading to a significant influence of obliquely incident rays on the fringe spacing [22]. It should be noted that different formulae for the calculation of the effective wavelength can be found in literature [21, 22].

The fringe visibility tends to zero as the fringe spacing reaches the lateral resolution limit according to the Rayleigh criterion. In addition, the above definition of the effective wavelength is valid only for a perfectly aligned smooth surface. If the surface is tilted the effective wavelength shifts to higher values. At the maximum tilt according to equation (6) the effective wavelength results in $\lambda_{\text{eff}} = \lambda / \cos \Theta_{\text{NA}}$. Hence, the height difference $\Delta z$ corresponding to the interference fringe spacing is $\Delta z = \lambda / (2 \cos \Theta_{\text{NA}})$ and therefore:

$$\tan (\Theta_{\text{NA}}) = \frac{\Delta x}{\Delta z} \rightarrow \Delta z = \frac{\lambda}{(2 \cos \Theta_{\text{NA}})} = \frac{\lambda}{2 \sin \Theta_{\text{NA}}} = \frac{\lambda}{2 \text{NA}}. \quad (10)$$

Obviously, here $\Delta x$ corresponds to the Abbe resolution limit for oblique incidence instead of the lateral resolution according to the Rayleigh criterion.

Due to the dependence of the effective wavelength on the surface tilt, slope changes, i.e. surface curvature, may be critical because of the changing spectral composition of the interference signals.

Figure 7 shows a simulation of a measurement result assuming a sinusoidal surface. The simulation result exhibits something like a batwing at the flank of the sinusoid. This systematic error appears only in the result of envelope evaluation at the lower numerical aperture of 0.55. It leads to an error in fringe order determination resulting in a ghost step in the results of phase evaluation. The numerical aperture of 0.9 is high enough to avoid this effect due to the better lateral resolution of the optical system. Figure 8 shows a similar result obtained experimentally [23]. The measuring object was a chirp standard manufactured by the PTB [24]. The minimum period of this standard is 3.6 $\mu$m and the PV-amplitude is 400 nm corresponding to a maximum surface slope of 20°. Even in the experimental case the Mirau interferometer of NA = 0.55 ($\Theta_{\text{NA}} = 33.4^\circ$) reveals systematic measurement errors at the flanks of the sinusoidal structures of lower period. Similar effects have been observed by Gao et al [25]. According to figure 8(a) these errors can be avoided again if the Linnik interferometer equipped with objective lenses of 0.9 NA is utilized.

5. Filtering approach

In the previous sections we have shown that the position of the envelope of a white-light interference signal is much more sensitive with respect to disturbing effects such as batwings (see section 3) or the interferometric slope effect described in section 4. These phenomena may lead to $2\pi$ phase jumps in the result of phase evaluation if the fringe order estimation is based on the envelope position. As a consequence, fringe order errors may be avoided if the profiles obtained from the position of the signal envelopes are filtered in order to reduce these effects prior to the fringe order estimation.

This idea is known as phase gap reduction and a detailed procedure is described in [17].

A simple approach to reduce the phase gap is demonstrated by figure 9. Figure 9(a) shows profiles obtained from envelope and phase evaluation. These profiles suffer from batwings. The batwings are significantly reduced by a 7 points median filter as it can be seen in figure 9(b). In addition, figure 9(b) shows the profile resulting from phase evaluation. Here, the fringe order is determined from the median filtered profile obtained from the envelope evaluation, which is also displayed in figure 9(b). Obviously, without filtering phase jumps occur leading to peaks of half the evaluation wavelength in the result of phase evaluation according to figure 9(a). If the phase evaluation is based on the median filtered envelope profile plotted in figure 9(b) the artefacts coming from an erroneous fringe order determination can be avoided. Nevertheless, the profile obtained from phase evaluation

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**Figure 7.** Simulated sinusoidal profiles of 3.6 $\mu$m period and 250 nm amplitude resulting from SWLI measurement, (a) assuming an objective lens of 0.55 numerical aperture, (b) assuming 0.9 numerical aperture.
displayed in figure 9(b) uses the same phase data as the phase based profile shown in figure 9(a). Finally, figure 9(c) shows that the procedure described here is different from phase unwrapping procedures, since in this situation unwrapping errors lead to the stair-like profile of the unwrapped height values depicted in figure 9(c).

This example demonstrates that median filtering of the profiles obtained from envelope evaluation can be used to avoid phase jumps in the results of phase evaluation. The batwing effect is strongly nonlinear. Consequently, nonlinear filtering such as median filtering seems to be appropriate in context with batwings.

Another option to correct for phase jumps is to apply the dual-wavelength evaluation approach we introduced in an earlier paper [26].

6. Conclusion

In this paper influences restricting the resolution and precision in vertical SWLI are addressed and discussed. We first demonstrate that in many relevant cases the axial measurement uncertainty in SWLI measurements based on phase evaluation is superior compared to PSI. The lateral resolution in phase measuring interferometry is affected by the diffraction limit which leads to low-pass filtering effects in the profiles resulting from the phase modulation of the wave front reflected by the measuring object. On the other hand, the results of analyzing the amplitude modulation of SWLI signals, i.e. applying the envelope evaluation technique, seem to show a better lateral resolution compared to phase evaluation. We suppose that this is a consequence of overlapping batwings which result in higher height differences but depend on the ratio of the height step to the effective wavelength of light. In addition, the envelope positions may be affected by slope changes of tilted surface areas. Finally, a practical method to get rid of phase jumps without manipulating the measured phase values is introduced and verified by experimental data.

This paper is intended as a basis for further discussion of fundamental effects in white-light interferometry also in comparison with other mature techniques such as confocal microscopy.
Acknowledgments

The authors would like to thank the Deutsche Forschungsgemeinschaft (DFG, LE 992/6-2) for the support for a part of this research work.

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