Analysis of oscillations of framework at pulse influence taking into account physical non-linearity

N A Ziambaev
Department of Construction Industry and Theory of Structures, South Ural State University, 76, Lenin Avenue, Chelyabinsk 454080, Russia
E-mail: nikzyam@mail.ru

Abstract. The article illustrates the analysis of oscillations of a flat multi-storey framework as discrete dissipative systems (DDS) at pulse influence by the time analysis method (TAM). Brief theoretical prepositions of TAM, such as description of the equation of motion of the DDS and expressions of response parameters, are given. The design dynamic model of the structure represents the flat three-storey shear frame having three degrees of freedom. The calculation results are oscillograms of internal response characteristics and reaction parameters of the system on the time slice \( t \in [0;8] \) sec. The multi-cycle nature of deformation of racks and also the change of parameters of plastic zones of framework supporters in the process of non-linear oscillations are considered.

1. Introduction

The research of nonlinear work of building structure materials in the conditions of dynamic impacts is interested by specialists. Nowadays the analysis of such tasks is carried out by numerical methods generally [1,2]. These methods do not allow to write down the decision analytically and, therefore, to carry out full dynamic analysis on all interval of integration of an initial task. Besides, numerical methods do not permit to conduct a research of multi-cycle deformation of structure elements. Deficiency of researches in the field of oscillations of structures taking into account the nonlinear restoring force is noted in the works devoted to numerical methods. Besides, the numerical methods used in the nonlinear dynamic analysis of building and constructions, such as numerical integration of the equation of the movement [3], use the concept of linear acceleration that can give inaccurate results [4]. This paper considers application of one of the analytical approaches – the time analysis method (TAM). TAM gives the chance to determine all parameters of reaction analytically taking into account elasto-plastic work of material of a design. Besides, it allows to consider the multi-cycle nature of deformation of a framework.

2. Brief theoretical prepositions

All process of nonlinear dynamic response in time breaks into separate (consecutive) intervals of \( t \in [t_i, t_{i+1}] \) \( (i = 0, 1, 2, \ldots) \) and parameters of settlement model (elements of matrixes of masses, rigidity and damping) are constant for these intervals according to the principles of TAM. It provides consecutive creation of dynamic reaction according to the scheme of the elastic decision. Such
approach allows to use the device of the ordinary differential equations (ODE) with constant coefficients for the solution of a problem of oscillations of system.

The equation of the movement of system with \( n \) degrees of freedom (1) is written down in a matrix expression [5,6] for any quasilinear time interval with initial conditions (2) (\( t \in [t_i, t_{i+1}] \)):

\[
M \ddot{Y}(t) + C \dot{Y}(t) + R(t) = P(t) \quad (1)
\]

\[
Y_0 = Y(t_i), \quad \dot{Y}_0 = \dot{Y}(t_i) \quad (2)
\]

here \( M = \text{diag}(m_1, \ldots, m_n) \), \( C = C_i^T \in M_n(R) \) – matrices of masses and damping; \( Y(t) = \{y_j(t)\}, \ R(t) = \{R_j(t)\}, \ P(t) (j = 1, \ldots, n) \) – vectors of displacements, dynamic restoring forces (DRF) and external loading; \( j \) – number of a storey; \( t_i \) – the timepoint corresponding to transition of system through a critical point when there is a change of dynamic parameters of system owing to the beginning or the end of nonlinear work of racks of a framework. It should be noted that theory of disproportionate damping [7] is used for obtaining matrix \( C_i \).

The diagram of deformation of material of a framework is accepted bilinear with sites of elastic work, hardening and unloading [8]. In view of this fact, we will transform the equation (1) and we will transfer nonlinear members to the right part:

\[
M \ddot{Y}(t) + C \dot{Y}(t) + R_e(t) = f(t) \quad (3)
\]

here \( f(t) = P(t) - R_u(t_i) + R_p(t_i) \) – vector of the generalized dynamic loading; \( R_e(t) = K \dot{Y}(t) \) – elastic component of the restoring force; \( R_u(t_i) \) and \( R_p(t_i) \) – limit and residual components of the restoring force, respectively. Analytical expression for these members is given in [8].

Expressions for vectors of kinematic parameters of reaction of system (displacements, velocities and accelerations, respectively) register as follows:

\[
Y(t) = 2 \Re \{Z(t)\} + Y_e(t), \quad \dot{Y}(t) = 2 \Re \{S_i Z(t)\}, \quad \ddot{Y}(t) = 2 \Re \{S_i^2 Z(t)\} + M_i^{-1} P(t), \quad (4)
\]

here \( Z(t) = Z_{0R}(t) + Z_p(t) \). Here

\[
Z_{0R}(t_i) = \Phi(t - t_i) U_i^{-1} M_i \left[ -S_i \cdot \left( Y_0 - Y_e(t_i) \right) \right] + \dot{Y}_0 \quad (5)
\]

\[
Z_p(t) = U_i^{-1} \int_{t_i}^t \Phi^\tau(t - \tau) P(\tau) \, d\tau \quad (6)
\]

Expression (5) considers reaction of free oscillations of system from initial conditions and also from action of nonlinear components of the restoring force which contain in expression of a vector of quasistatic displacements \( Y_e(t_i) = K_i^{-1} \left[ -R_u(t_i) + R_p(t_i) \right] \). Expression (6) is reaction to the forced oscillations of the DDS. Formulas (5) and (6) contain \( \Phi(t - t_i) = e^{S_i(t-t_i)}, \ U_i = M_i S_i + S_i^T M_i + C_i \); the matrix \( S_i \) is a root of a characteristic matrix quadratic equation of the differential equation of the movement of the system (3) and also a matrix of internal dynamic characteristics of the DDS [3].

The force parameters of system responce (restoring, dissipative and inertial forces) are:

\[
R(t) = R_u(t_i) + R_u(t_i) - R_p(t_i); \quad F(t) = C_i \dot{Y}(t), \quad I(t) = -M \ddot{Y}(t) \quad (7)
\]

3. Creation of the function considering change of length of plastic zones in the racks of a multistorey framework in the course of its inelastic deformation

Figure 1 shows process of elasto-plastic deformation [9] of a rack of the multistorey framework subjected to the pulse impact. Variable \( x \) is the length of a plastic zone. Value of bending stiffness of material in this zone is lowered and equal to \( E_0 I \) and corresponds to stiffness in a stage of hardening
Stiffness of average piece of the column height is equal to \( EI \) (an elastic operational mode of a design, the initial elastic modulus). Length of this piece is \( h - 2x \) (figure 1). Right side of the figure contain a moment diagram. The moment diagram for ideally elastic material is designated by dash-dotted and dotted lines, and diagram for the nonlinear material is designated by solid line. Let's enter some designations. \( M_0 \) is value of the limit elastic moment, \( M \) is the value of the operational moment in the basic section of a column. Bending stiffness ratio of pieces of rack height mentioned above is equal to \( k = E_0/I/EI = E_0/E \), and relative length of a plastic zone \( \xi = 1 - 2x/h \) at \( x \leq h/2 \).

![Figure 1. Deformation of the framework rack taking into account change of length of plastic zones.](image)

We will use method of forces \([5,6,10]\) to receive value of the moment in the basic section of a rack in any timepoint at the shift of an upper node of a column on value \( \ddot{y}(t) \) (figure 1). After we get the solution of the canonical equations and get the value of unknown members we have expressions for the moments and shear forces in basic sections of columns of the storey of the framework \( M = (6EI \cdot (h^3)^{-1}) \cdot \ddot{y}(t)\phi(k,\xi), Q = (12EI \cdot (h^3)^{-1}) \cdot \ddot{y}(t)\phi(k,\xi) \). \( \phi(k,\xi) = k \cdot (1 - (1-k)\xi^2)^{-1} \) is a correction function considering change of length of plastic zones during inelastic work of the framework.

As soon as the moment in basic sections of racks becomes \( M \geq M_0 / \xi \), length of the plastic zone increases by step size of \( x \). Then function \( \phi(k,\xi) \) is recalculated and, respectively, stiffness of the rack \( B_i = B*\phi(k,\xi) \), where \( B = EI \) — bending stiffness of racks during elastic oscillation mode.

### 4. Numerical implementation of the problem

As an example, consider the time analysis of the elasto-plastic oscillations of the three-storey framework. The design dynamic model (DDM) of the framework is adopted in the form of a flat shear frame with point masses located in the center of each storey (figure 2), thus the system has \( n = 3 \) dynamic degrees of freedom. Flooring joists and all nodes of the DDM are adopted absolutely rigid. Geometrical characteristics of left and right framework racks are equal. \( N_1 \ldots N_3 \) are the nodes of the frame and \( n_1 \ldots n_3 \) are the basic sections of framework racks.

Characteristics of DDM are:
- column heights: \( h_1 = 5.5 \text{ m}; h_2 = 4.8 \text{ m}; h_3 = 4.5 \text{ m} \). Column sections are double tees with inertia moments \( I = 21678 \text{ cm}^4 \) for 2nd and 3rd storeys, \( I = 38676 \text{ cm}^4 \) for 1st storey \([11]\).
- Material of racks — steel 10G2S1 (S 345) with the yield strength \( R_y = 33 \text{ kN/cm}^2 \) and ultimate tensile strength \( R_u = 49 \text{ kN/cm}^2 \) and initial elastic modulus \( E_0 = 206000 \text{ MPa} \). The material elastic modulus of the zone of hardening is less 40 times then initial one \([12-15]\).
- Elements of matrix of masses (taking into account the masses of floor slabs, flooring joists and framework racks): \( m_1 = 3,077 \text{ kN} \cdot \text{s}^2/\text{cm} \); \( m_2 = 3,074 \text{ kN} \cdot \text{s}^2/\text{cm} \); \( m_3 = 2,137 \text{ kN} \cdot \text{s}^2/\text{cm} \).
- Parameters of pulse impact and time analysis: start time of the analysis is \( t_0 = 0 \text{ s} \); pulse duration \( t_a = 0.7 \text{ s} \); \( t_m = 1.5 \text{ s} \); \( t_e = 2.2 \text{ s} \); duration of the whole analysis is 8 s and step time of the analysis is \( \Delta t = \)
amplitudes of pulse impact are \( P_0 = [240 240 150] \) kN; pulses are applied to the level of floor slab to each storey. Figure 3 shows a scheme of pulse impact.

Figures given below show oscillograms of response parameters of the system. Elastic mode of deformation of the structure is designated by black dotted line. Elasto-plastic mode with constant length of the plastic zone equal to whole height of the rack during nonlinear deformation \( (x = h/2) \) of the storey is designated by blue dot-dash line. Elasto-plastic mode with variable length of the plastic zone during nonlinear deformation (figure 1, \( x < h/2 \)) of the storey is designated by red solid line.

Figure 4 shows the diagram of deformation of 2nd storey of the framework. As we can see, elasto-plastic mode with \( x = h/2 \) gives bigger value of residual displacements then elasto-plastic one with \( x < h/2 \). It can be explained by smaller bending stiffness of storey during nonlinear deformation of the rack. The red line demonstrates bigger quantity of loops of plastic hysteresis because the bending stiffness of structure is higher than bending one for case of blue line. This fact influences spectrum of natural frequencies \( \omega \) of racks of 2nd storey (figure 5). We have smaller jumps of values of frequencies for red oscillogram. Frag. 1 of figure 5 shows steplike change of the spectrum because of steplike one of the length of plastic zones of column during nonlinear deformation of structure.

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**Figure 2.** The design dynamic model of framework.

**Figure 3.** The scheme of pulse loading applied to the framework.

**Figure 4.** Diagram of deformation ‘resistance force – relative displacements’ for the racks of 2nd storey.

**Figure 5.** Spectrum of natural frequencies of racks of 2nd storey.
Figure 6 shows oscillograms of relative displacements of the nodes of 2nd storey. Comparison of different modes of structure deformation is given. The timepoint of start of nonlinear deformation of the rack is shown by circle and end of nonlinear one of the column is shown by asterisk. Horisontal lines are asymptotes. Distance between the line of the beginning of coordinates and asymptotes is value of residual plastic displacements of the storey. Figure 7 compares oscillograms of restoring forces for the nodes of 2nd storey for different oscillation modes of structure.

Figure 8 shows oscillograms of efficiency of normal stresses $N_\sigma(t)$ in basic sections of racks of 2nd storey of the framework. This parameter shows ratio of actual stress in the section to ultimate tensile strength of the structure material. As we can see, elastic line shows values $N_\sigma(t)>1$ that indicates structural failure. That’s why we have to optimize the sections of racks and spend more material for design. On the other side, the oscillation mode with $x = h/2$ (blue line) gives smaller values of $N_\sigma(t)$ which does not correspond to real process of deformation of structure. The oscillation mode with $x < h/2$ (red line) considers the step increase of length of plastic zones of racks that is much closer to real process of deformation of structure with continuous change of stiffness parameters during nonlinear oscillation mode [16-19].
Accuracy of obtained results is proved by spectrum of discrepancies $\Delta f$ of the equation of motion (1). Solution is accurate if $\Delta f = R(t) + F(t) - I(t) - P(t) \to 0$. Figure 9 shows that TAM solution of the problem of nonlinear oscillations is accurate because maximum value of $\Delta f$ is about $2 \times 10^{-11}$ kN.

![Figure 9. Oscillograms of discrepancies of the equation of motion of the DDS.](image)

The implementation of the mathematical model and the solution of the problem of nonlinear oscillations are carried out using the language of technical and science calculations MATLAB [20].

5. Conclusion
1. The expressions of kinematic and force response parameters of the discrete dissipative system are given in closed form in relation to the problem of nonlinear oscillations of structures.
2. The time analysis of the elasto-plastic response of a three-storey framework is simulated for different modes of oscillations of the system.
3. The multi-cycle nature of deformation of racks and step change of parameters of plastic zones of framework supporters in the process of nonlinear oscillations are considered.

References
[1] Bate K 1982 Numerical methods of the analysis and finite element method (Moscow: Stroyizdat Publ) p 447
[2] Il’in V P Numerical methods of the solution of building mechanics problems: manual (Moscow: ASV Publ) p 426
[3] Shaposhnikov N N, Kashaev S K and Belozerskaya O V 1997 Development of methods of numerical integration of the equations of the movement of dynamic systems News of universities. Construction (Novosibirsk) 7 pp 89–93
[4] Clough R W and Penzien J 1995 Dynamics of Structures (USA: Computers & Structures, Inc.) p 752
[5] Kiselev 1980 Building mechanics. Special course. Dynamics and stability of structures. Textbook for universities (Moscow: Stroyizdat Publ) p 616
[6] Trushin C I 2016 Building mechanics. Method of finite elements: manual (Moscow: NITs INFRA-M Publ) p 305
[7] Potapov A N 2003 The dynamic analysis of discrete dissipative systems at non-stationary influences (Chelyabinsk: SUSU Publ.) p 167
[8] Potapov A N and Ziambaev N A 2017 The creation of mathematical model of physically nonlinear vibrations of a multistorey frame Bulletin of the South Ural State University. Ser. Construction Engineering and Architecture 17 (3) pp 12–17
[9] Birger I A 1993 Durability calculation of details of mechanisms: reference book (Moscow: Mechanical engineering Publ) p 640
[10] Darkov A V 2004 Building mechanics: textbook for universities (St. Petersburg: Lan’ Publ) p 656
[11] STO ASChM 20-93 Rolled steel sections. I-beers with parallel adgus of flanges. Specifications (Moscow) p 9
Acknowledgments
The work was supported by Act 211 Government of the Russian Federation, contract no. 02.A03.21.0011.