Coherence area measurements of bright entangled twin beams

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Quantum states of light with multiple spatial modes are fundamental for quantum imaging and parallel quantum information processing. Thus, their characterization, which can be achieved through measurements of the coherence area, is an important area of research. We present a comparative study between different measurement techniques for the coherence area of bright entangled twin beams of light. The first one is a direct measurement based on correlation measurements between spatial intensity fluctuations of the twin beams while the second one is based on a noise analysis of different spatial regions in the temporal domain. We generate the bright entangled twin beams with a four-wave mixing process in a hot rubidium vapor cell and capture images of the twin beams with an electron-multiplying charge-coupled-device camera. We show that the time domain measurements, which provides a significantly less complicated measurement scheme, provides an estimate of the size of the coherence area of the same order of magnitude as the direct measurement done in the spatial domain.

Spatial quantum correlations between entangled photons or bright twin beams have gaining attention due to their application in testing fundamental quantum physics [1], quantum imaging, quantum metrology, and quantum information processing [2]. In particular, spatial quantum correlations between photon pairs generated with spontaneous parametric down conversion (SPDC) have been extensively studied over the last two decades [2–13]. Owing to the conservation of momentum, or phase-matching, for the ideal case of a monochromatic planar pump beam and an infinitely thin crystal, there are point-to-point spatial quantum correlations between the down-converted photon pairs in the far field [3]. However, for realistic experimental conditions, such as a Gaussian pump beam and a finite crystal length, the spatial correlations are spread out over a region in the far-field. The minimum size of this region is called the coherence area [7]. The size of the coherence area plays an important role in quantum imaging as it limits the resolution of images [14, 15]. Moreover, the presence of many coherence areas in the generated photons is a signature of the multi-spatial mode nature of the generated fields [6]. Such a multi-spatial mode nature plays a fundamental role in quantum information, as it allows for parallel quantum information processing with each spatial mode playing the role of an independent quantum channel [1].

In this letter, we compare two different techniques that are typically used to characterize the size of the coherence area. The first technique provides a direct measurement and is based on spatial cross-correlation measurements between the entangled photons or beams of light captured with a high quantum efficiency CCD camera [5, 10]. The second one is based on noise measurements in the time domain of different spatial regions selected by apertures [16–18]. While the second technique provides an indirect measure, it is significantly easier to implement and more commonly used. To perform this comparison, we capture images of entangled twin beams, which we call probe and conjure with an electron-multiplying charge-coupled-device (EMCCD) camera. These images are then used to implement both techniques such that a direct comparison of the same data can be performed. We show that both techniques give an estimation of the size of the coherence area of the same order of magnitude, which validates the more commonly used temporal-domain technique as well as the model used to extract the size of the coherence area [17].

To perform the experiments, we use bright twin beams generated through a four-wave mixing (FWM) process in a double-Λ configuration in hot rubidium atomic vapor [10, 20]. This process is based on a third order non-linearity and produces narrowband macroscopic quantum correlated beams [21, 22] without the need of a cavity. This makes it possible to generate bright twin beams with a large number of spatial modes. The focus on bright twin beams instead of entangled photon pairs, is due to the control that is possible on the number of photons while maintaining the level of quantum correlations [20, 23]. This provides a significant advantages for quantum metrology and quantum information processing [24–28].

A schematic of the experimental setup is shown in Fig. 1. A strong pump beam (power of 2.4 W, 1/e² waist diameter of 4.5 mm) and an orthogonally polarized weak probe beam (power of 70 μW, 1/e² waist diameter of 0.2 mm), both derived from the same Ti-Sapphire laser, interact at an angle of 0.5 degree with ⁸⁵Rb atoms inside a glass cell heated to 110°C. The inset in Fig. 1 shows a schematic of the double-Λ energy level configuration on which the FWM process is based. In this process, the input probe beam is amplified and a new beam called the conjugate is generated, with the emission direction of these beams governed by the phase matching condition. The frequency of the laser is locked 1 GHz away.
from the atomic hyperfine transition \( F = 2 \) to \( F' = 3 \) of the \(^{85}\text{Rb} \) D1 line through a saturation absorption spectroscopy setup. The frequency of the probe beam is down shifted by 3.04 GHz with respect to the pump frequency with an acousto-optic modulator (AOM). After the cell, the pump beam is filtered with a polarization filter.

To study the spatial quantum correlations between the probe and conjugate beams in the far field, lenses are placed in the path of each of the beams after the cell to get the Fourier transform of the center of the cell onto the EMCCD camera. However, due to a cross-Kerr effect between the pump and the probe and conjugate, the Fourier planes do not lie at the expected planes. In order to determine the correct Fourier planes, we use an imaging system before the cell that places the Fourier transform of an object placed in the path of the input probe at the center of the cell. This makes it possible to determine the position of the lenses after the cell by optimizing the image of the object at the location of the EMCCD camera.

To acquire the images of the bright twin beams, the pump and the input probe beams are pulsed with a timing sequence that is synchronized with the data acquisition for each frame of the camera. The details of the image acquisition with the EMCCD camera are described in Ref. \([29]\). We record 100 images with multiple frames in each image. Two consecutive frames in each image are then used for the analysis. Subtraction of these two frames gives an image that contains the spatial intensity fluctuations of each beam. These images are used to implement both techniques to estimate the size of the coherence area. We first use the direct method, based on the cross-correlation between the spatial intensity fluctuations of probe and conjugate beams. Due to phase-matching, the spatially correlated regions between the probe and conjugate beams are located diametrically opposite to each other with respect to the pump in the far field. Therefore, to calculate the cross-correlation, we rotate the image of the spatial intensity fluctuations of the conjugate beam by 180 degrees with respect to the one of the probe beam before the analysis. We then select a region of \(80 \times 80\) pixels of the image of the spatial intensity fluctuations of the conjugate beam by 180 degrees with respect to the one of the probe beam before the analysis. We then average over these cross-correlations for the 100 images to obtain the results shown in Figs. 2 (a, b). The width of the spatial cross-correlation gives a measure of coherence area of the beams. After a two-dimensional Gaussian fitting of the cross-correlations shown in Figs. 2 (a, b), we obtain a full width at half maximum (FWHM) of \(11.7 \times 11.6\) pixels and \(12.0 \times 11.7\) pixels, respectively.
The presence of correlated regions between the probe and the conjugate also plays a role in the spatial properties of the individual beams. To show that this is the case, we calculate the spatial auto-correlations of the spatial intensity fluctuations of the probe and conjugate beams. In order to do this, we implement a similar analysis as the one used to calculate the spatial cross-correlations, except that now a region of $80 \times 80$ pixels of the image of the spatial intensity correlations of the probe (conjugate) is scanned over the corresponding region of $120 \times 120$ pixels of the probe (conjugate) image. The auto-correlations that result from these calculations are shown in Figs. 2 (c, d). The FWHM of the auto-correlations for the probe and the conjugate beams are $10.5 \times 9.7$ pixels and $11.2 \times 11.0$ pixels, respectively. These values are consistent with the widths obtained from the cross-correlation measurements in Figs. 2 (a, b) and show that the coherence area information can also be obtained from measurements of a single beam.

In order to compare the direct measurement technique for estimating the size of the coherence area with the more commonly used time-domain technique, we perform a noise analysis of the intensity fluctuations of the stored images by selecting different spatial regions of the stored images. In particular, we effectively implement a slit of variable size for either the probe or the conjugate, along the lines of the measurements performed in [17]. While the analysis is done in the spatial domain, this approach allows us to use the same images to simulate the time-domain measurements and thus obtain a direct comparison of both techniques. The noise in each beam can be quantified through the Mandel Q-parameter, defined as

$$Q = \frac{\langle (\Delta \hat{N})^2 \rangle}{\langle \hat{N} \rangle} - 1,$$

where $\hat{N}$ is the photon number operator. For a coherent state $Q = 0$, which corresponds to the standard quantum limit.

The Q-parameter does not directly measure the coherence area; however, it tells if the given state is composed of a single spatial mode or of multiple spatial modes. For a single spatial mode the Q-parameter varies linearly with transmission independent on whether the intensity of the beam is attenuated uniformly with an ND filter or by spatially cutting different regions. On the other hand, for a multi-spatial mode beam, the behavior is not linear [15]. As the number of spatial modes in a given field depends on the size of the coherence area, it is possible to obtain an estimate of the size of coherence area from the non-linear behavior of the Q-parameter as the beam is clipped. Given that the information of the coherence area is contained in each of the twin beams, it is possible to estimate the size of the coherence area with this technique through a noise analysis of only one of the beam.

We use the theoretical model developed in Ref. [17] to obtain an estimate of the size of coherence area from the measurements of the Mandel Q-parameter. In this model the field operators are expanded in terms of a complete set of spatial basis modes. The functional dependence of the losses as a function of the spatial clipping is calculated for the basis modes and this information is then used to calculate the Q-parameter. We assume that each of the basis modes used for the expansion can be treated as a single spatial mode in terms of the behavior of the Q-parameter with losses. Finally, the Q-parameter is normalized to its value at a transmission of one to obtain the normalized Mandel Q-parameter, $Q_N$,

$$Q_N = \frac{\sum_i n_i^2 \langle \hat{n}_i \rangle}{\sum_i n_i \langle \hat{n}_i \rangle},$$

where $n_i$ is the transmission of basis mode $i$ when clipping the beam and $\hat{n}_i$ is the number operator for mode $i$. In deriving Eq. 2 we assume that all the basis modes have the same noise properties. This is a valid approximation as long as the beam does not occupy a significant portion of the spatial bandwidth. To extract the size of the coherence area from $Q_N$ as a function of transmission, we use a set of spatially localized modes that form a complete orthonormal basis set to perform the expansion. In particular, we use a set of 2-dimensional rect functions, which are equivalent to the CCD pixels. Each of these modes are taken to be of size $2a \times 2a$, where $a$ corresponds to the linear extent or radius of the coherence area.

To calculate the normalized Q-parameters for the probe and the conjugate, we select an analysis region of $100 \times 100$ pixels in the corresponding image of the spatial intensity fluctuations. Each of these regions is centered around the maximum intensity of the corresponding beam. In analogy to previous studies [16, 17], where the coherence area is estimated through a noise analysis on different spatial regions by clipping the beam with an aperture, here we select different spatial regions of the images. To emulate the symmetric clipping of a slit, we implement an algorithm that symmetrically sets rows (for horizontal clipping) or columns (for vertical clipping) on both sides of the analysis region to zero. As found from the cross-correlation measurements, the size of the coherence area is larger than the pixel size (16 µm) of the EMCCD camera. In order to obtain an accurate measure of the noise of the beam, the size of the detection area has to be comparable to the size of the coherence area. In order to make sure this is the case, we group or bin $10 \times 10$ pixels after setting the rows or columns to zero. Finally, we calculate the spatial noise variance $\langle (\Delta \hat{N})^2 \rangle$ as a function of the number of rows or columns that are set to zero, i.e. transmission. To obtain the mean number of photo-counts, i.e. $\langle \hat{N} \rangle$, in the probe and the conjugate beams, we fit the probe and the conjugate images with a two-dimensional Gaussian functions. We then estimate the total number of photo-counts by finding the area under the Gaussian fit. This allows us to avoid over-estimating the photo-counts due to dark noise or other artifacts that become relevant in the wings of the beams. We finally calculate the Q-parameters for the probe and
conjugate beams with Eq. (1). As described in Ref. [17], the discrete nature of the pixels leads to a Q-parameter as a function of transmission with discrete jumps. To avoid this artifact, we average the Q-parameter at each transmission for different starting points of the binning with respect to analysis region, in analogy to what is done in the theoretical model.

![Figure 3: Normalized Mandel Q-parameter for the probe (top row) and conjugate (bottom row) as a function of transmission. The open circles represent the experimental data while the solid lines give the optimum fit from our theoretical model. In the model a/ω is the only free parameter, where a gives a measure of the linear extent of coherence area while ω is the radius of the beam. The error bars represent one standard deviation over the 100 image sequences.](image)

Figure 3 shows the experimental and theoretical normalized Q-parameter as a function of transmissions for the probe (top row) and conjugate (bottom row). The experimental data is averaged over the 100 image sequences. For each beam, the normalized Q-parameters with clipping in the vertical (left column, Fig. 3) as well as in the horizontal (right column, Fig. 3) directions are shown. It can be seen that all the plots deviate from the linear behavior, which indicates the multi-spatial-mode nature of the beams. Each experimental plot is fitted with our theoretical model of Q-parameter that has as a free parameter a/ω, which corresponds to the ratio of coherence area radius (a) to the beam radius (ω). From the images of the twin beams, we measure the FWHM for the probe and conjugate to be 90 and 86 pixels in the vertical direction, and 70 and 66 pixels in the horizontal direction, respectively. From the optimum fit parameters (a/ω) given in Fig. 3 for the probe and conjugate beams, we can estimate the FWHM of the coherence area for both beams to be (4±1, 4±1) pixels in the horizontal and vertical directions, respectively. These values are of the same order of magnitude as the ones obtained with a direct measurement, which validates the indirect measurement technique as well as the theoretical model.

Given the importance of the size of the detection region, we study the dependence of the Q-parameter on binning. To do this, we calculate the normalized Q-parameter as described above for different binnings of the conjugate analysis region, as shown in Fig. 4. We observe that for binnings in the range of 8 to 16, the Q-parameter shows a consistent behavior. However, for a binning size smaller than the coherence area, the Q-parameter shows a slightly different behavior (as shown for a binning of 4 in Fig. 4). This is a result of the detection area defining the smallest scale for the spatial fluctuations instead of the coherence area, which leads to an apparent reduction in the size of the coherence area.

![Figure 4: Normalized Mandel Q-parameter for the conjugate for binnings of 4x4, 8x8, 10x10, and 16x16 pixels. The solid lines provide a guide for the eye.](image)

In conclusion, we have compared two different techniques to estimate the size of the coherence area. In particular, we consider the “direct” technique based on the cross-correlation of the spatial intensity fluctuations of the twin beams with the “indirect” technique based on a noise analysis of different spatial regions of a single beam. Results from both approaches are consistent and show the same order of magnitude estimation of the size of the coherence area of the twin beams. Thus, our analysis validates that the relatively less complicated “indirect” technique and corresponding theoretical model for estimating the coherence area provide an adequate characterization of the scale of the spatial correlations in the far field.

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