Influence of geometric and physical Windbelt system parameters on instability pattern

A A Afanaseva\(^{1,2}\), A M Gouskov\(^{1,2}\), G Ya Panovko\(^{1,2}\)

\(^{1}\) Bauman Moscow State Technical University, 5 2nd Baumanskaya str., Moscow, 105005, Russia
\(^{2}\) Blagonravov Mechanical Engineering Research Institute of Russian Academy of Sciences, 4 Maly Kharitonyevsky Pereulok, Moscow, 101990, Russia

alexandra95_19@mail.ru

Abstract. Windbelt system non-linear aeroelastic vibrations are considered. In particular, how the various physical and geometric system parameters (overall dimensions, density of the air and ribbon material, internal damping) affects on the incoming air flow critical velocity value and the system's supercritical nature of the behaviour.

1. Introduction
Wind energy is one of the main directions of alternative energy based on the conversion of the kinetic energy of the air masses of the atmosphere into electrical, mechanical or thermal energy. Wind turbines typically accomplish the conversion of airflow energy into electrical energy. The possibility of generating electrical energy using a wind generator, which is called «Wind Belt» [1], is being actively explored.

2. Problem statement
The Windbelt device (figure 1) is a flexible belt stretched between supports transverse to the wind direction [1, 2].

![Figure 1. Windbelt device](image1)

![Figure 2. Location of magnets and coils.](image2)

Two magnets are attached to the tape (figure 2), and inductive coils are coaxially mounted on the frame. When the wind blows across it, the ribbon vibrates due to aeroelastic flutter. The vibrating
movement of the magnets induce current in nearby pickup coils by electromagnetic induction, which can then be straightened into a constant voltage if necessary.

The stationary equilibrium state of the tape is disrupted when a certain (critical) speed value is reached. Then its self-oscillations as flutter (joint bending-torsional vibrations) [3] arise.

3. Motion equations

We consider the oscillations of a long thin ribbon with hinged fastenings at the edges under the influence of entering an air flow to determine the critical velocity. The following dimensionless mathematical model [2] describes such a system:

\[
\begin{align*}
\frac{\partial^2 \xi}{\partial t^2} - \theta \alpha_1 \frac{\partial^2 \xi}{\partial z^2} + d_z \frac{\partial \xi}{\partial t} &= \Lambda^2 \beta_1 \varphi - \Lambda \gamma_1 \frac{\partial \varphi}{\partial t} - \Lambda \eta_1 \frac{\partial \xi}{\partial t} - (1/\Lambda) A_1 \left( \Lambda \varphi - \gamma_2 \frac{\partial \varphi}{\partial t} - \eta_2 \frac{\partial \xi}{\partial t} \right)^3, \\
\frac{\partial^2 \varphi}{\partial t^2} - \delta \frac{\partial \varphi}{\partial t} + d_\varphi \frac{\partial \varphi}{\partial t} &= \Lambda^2 \beta_1 \varphi - \Lambda \gamma_2 \frac{\partial \varphi}{\partial t} - \Lambda \eta_2 \frac{\partial \xi}{\partial t} - (1/\Lambda) A_1 \left( \Lambda \varphi - \gamma_2 \frac{\partial \varphi}{\partial t} - \eta_2 \frac{\partial \xi}{\partial t} \right)^3,
\end{align*}
\]

where \( \Lambda \) is the dimensionless wind velocity, \( \theta \) is the dimensionless tension force, and \( \alpha_1, \beta_1, \gamma_1, \eta_1, \gamma_2, \eta_2, d_z, d_\varphi, \varepsilon, \kappa \) are dimensionless complexes defined by the following expressions:

\[
\begin{align*}
\alpha_1 &= (1 + \varepsilon^2)/4\varepsilon; \\
\beta_1 &= (1 + \varepsilon^2)/3\varepsilon; \\
\gamma_1 &= \left(\sqrt{\frac{AE}{24}}\right) \frac{K}{\varepsilon (1 + \varepsilon^2)}; \\
\gamma_2 &= \left(\frac{\sqrt{AE}}{8}\right) \frac{1}{\varepsilon (1 + \varepsilon^2)}; \\
d_z &= 2\pi k \sqrt{\frac{\alpha_1}{\varepsilon}} \psi_z; \\
\varepsilon &= \frac{h}{b}; \\
d_\varphi &= 2\pi k \cdot \psi_\varphi; \\
\kappa &= \frac{\rho}{\rho_1}; \\
\eta_1 &= \left(\sqrt{\frac{AE}{6}}\right) \frac{K}{\varepsilon (1 + \varepsilon^2)}; \\
\eta_2 &= \left(\sqrt{\frac{AE}{2}}\right) \frac{K}{\varepsilon (1 + \varepsilon^2)},
\end{align*}
\]

where \( h \) is the thickness of the cross section, \( b \) is the width of the cross section, \( \rho \) is the density of the environment, \( \rho_1 \) is the density of the material, \( \psi_z, \psi_\varphi \) are the coefficients showing the ratio of real damping to critical [4], \( k \) is the harmonic number, \( A_1, A_4 \) are the coefficients of expansion of the lifting force in a Taylor series [2].

4. Results

The paper [2] shows the influence of the main external parameters \( \Lambda \) and \( \theta \) on the stability of the solution to system (1). In this paper, the influence of geometric \( \varepsilon, l \) and physical parameters \( \kappa, d_z, d_\varphi \) is considered.

In order to analyze the influence of the geometric parameter, which is included in dimensionless complexes (2), on the critical velocity \( \Lambda_0 \), the parameter values are fixed:

\[ \kappa = 0.03, \theta = 0.002, \psi_z = \psi_\varphi = 0.05 \]

The plate is considered thin when the ratio of the width of the section to its thickness equal to 1/20. Therefore, \( \varepsilon \) is considered in the interval \((0, 0.05]\). The results are presented in figure 3. As it shown, with a small \( \varepsilon \) value from 0 to \( \varepsilon_* \), the critical parameter of the wind speed is \( \pi \), which corresponds to a static stability loss [2].
Subsequently, we consider the effect of the dimensional geometric parameter (ribbon length \( l \)) on the dimensional critical velocity \( U_* = \Lambda_* U^* \). In contrast to \( \varepsilon \), length \( l \) does not explicitly enter into system of equations (1), since it is not part of dimensionless complexes (2). However, \( l \) is one of the parameters for scale factors [2], in particular, in \( U^* \). The scale for speed is as follows:

\[
U^* = \sqrt{\frac{8Gh^3}{(3\rho_0 a_0 b)^2}} = c/l, \quad c = \sqrt{\frac{8Gh^3}{(3\rho_0 a_0 b)}} \Rightarrow U_* = \Lambda_* c/l
\]  

(3)

Dependence (3) has a hyperbolic character. It is presented in figure 4 with the following parameters:

\[
\kappa = 0.03, \varepsilon = 0.002, \theta = 0.002, \psi_x = \psi_y = 0.05
\]

The effect of \( \kappa \) on the critical velocity \( \Lambda_* \) is studied in a similar way. The following parameters are fixed:

\[
\varepsilon = 0.002, \theta = 0.002, \psi_x = \psi_y = 0.05
\]
And then a graph is plotted (figure 5). At a certain value of $\kappa = \kappa_*$, the critical velocity reaches a value equal to $\pi$, and regardless of the increase in the parameter remains constant, which corresponds to divergence.

![Figure 5. The dependence of the critical velocity on $\kappa$.](image)

In addition, the dependence of the critical velocity on the value of the internal damping $d_\phi$ and $d_\zeta$ is of interest. From expression (2), the damping coefficients vary with respect to their critical values using $\psi_\zeta$ and $\psi_\phi$. For function $\Lambda_\kappa(\psi_\zeta)$, the parameters are accepted as follows:

$$\kappa = 0.03, \varepsilon = 0.002, \theta = 0.002, \psi_\phi = 0.05$$

Similarly for $\Lambda_\kappa(\psi_\phi)$:

$$\kappa = 0.03, \varepsilon = 0.002, \theta = 0.002, \psi_\zeta = 0.05$$

The result is shown in figure 6.

![Figure 6. Dependence of the critical velocity on $\psi_\zeta$ and $\psi_\phi$.](image)
5. Conclusion
As a conclusion, we note that such an analysis allows one to select the system parameters so that the dynamic loss of stability (flutter) occurs earlier than the static (divergence).

Acknowledgments
The Russian Science Foundation supported the work (project №18-19-00708)

References
[1] Frayne, S (2009) Generator utilizing fluid induced oscillations. US7573143 B2. – https://patents.google.com/patent/US20090309362
[2] A A Afanaseva, A M Gouskov, G Ya Panovko, « Nonlinear dynamics of a thin narrow tape in a subsonic air stream. ». // Journal of Machinery Manufacture and Reliability. – 2019. №7 (in Russian)
[3] Y C Fung, An Introduction to the Theory of Aeroelasticity / Translation from English A I Smirnov, edited by E I Grigolyuk - M.: State publishing house of physical and mathematical literature, 1959. - 523 p. (in Russian)
[4] Vibratsii v teckhnike, T 1 Kolebaniya v lineynih sistemah [Vibrations in tehnique, vol. 1: Linear systems oscillations]. Moscow, Mashinostroenie Publ, 1978, 352 p. (in Russian)