Explaining the $R_K$ and $R_{D^{(*)}}$ anomalies with vector leptoquarks

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Abstract

Recently the $B$ factories BaBar and Belle as well as the LHCb experiment have reported several anomalies in the semileptonic $B$ meson decays such as $R_K$ and $R_{D^{(*)}}$ etc. We investigate these deviations by considering the vector leptoquarks relevant for both $b \to s l^+ l^-$ and $b \to c l \bar{\nu}_l$ transitions. The leptoquark parameter space is constrained by using the experimentally measured branching ratios of $B_s \to l^+ l^-$, $\bar{B} \to X_s l^+ l^- (\nu \bar{\nu})$ and $B^+_u \to l^+ \nu_l$ processes. Using the constrained leptoquark couplings, we compute the branching ratios, forward-backward asymmetries, $\tau$ and $D^*$ polarization parameters in the $\bar{B} \to D^{(*)} l \bar{\nu}_l$ processes. We find that the vector leptoquarks can explain both $R_{D^{(*)}}$ and $R_K$ anomalies simultaneously. Furthermore, we study the rare leptonic $B^*_{u,c} \to l \bar{\nu}$ decay processes in this model.

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I. INTRODUCTION

The standard model (SM) of particle physics explains almost all the experimental data observed so far, to a very good level of accuracy. But it is unable to account for some of the fundamental problems of nature, such as the hierarchy in fermion masses, matter dominance of the universe and dark matter content etc. Therefore, we strongly believe that there exists some kind of new physics at high scale and the low-energy version of the same could be the SM. The study of nuclear beta decay has set the $V-A$ current structure of the weak interactions which describes various charged current interactions in all the generation of quarks and leptons to a high precision. However, the recently measured experimental data indicate that the processes involving third generation of fermions in both the initial and final states are comparably less precise than the first two generations. The couplings of third generation fermions to the electroweak gauge sector is comparatively stronger due to their larger masses and thus sensitive to new physics which could modify the $V-A$ structure of the SM. In this context, the study of $B_c^{(*)} \rightarrow \tau \bar{\nu}_l$ and $B \rightarrow D^{(*)} \tau \bar{\nu}_l$ charge current processes, involving the quark level transition $b \rightarrow c$ are captivating. Recently BaBar\cite{1, 2} and Belle \cite{3, 4} have measured the ratio of branching fractions of $\bar{B} \rightarrow D \tau \bar{\nu}_\tau$ over $\bar{B} \rightarrow D l \bar{\nu}_l$, where $l = e, \mu$ and the current experimental average \cite{5} is

$$R_D = \frac{\text{Br} (\bar{B} \rightarrow D \tau \bar{\nu}_\tau)}{\text{Br} (\bar{B} \rightarrow D l \bar{\nu}_l) = 0.397 \pm 0.040 \pm 0.028}, \quad (1)$$

which has 1.9$\sigma$ deviation from its SM result $R_{D}^{SM} = 0.300 \pm 0.008$ \cite{6}. In addition, both the $B$ factories and LHCb \cite{7} have reported 3.3$\sigma$ discrepancy \cite{5} in the measurement of $R_{D^*}$

$$R_{D^*} = \frac{\text{Br} (\bar{B} \rightarrow D^* \tau \bar{\nu}_\tau)}{\text{Br} (\bar{B} \rightarrow D^* l \bar{\nu}_l) = 0.316 \pm 0.016 \pm 0.010}, \quad (2)$$

from its SM prediction $R_{D^*}^{SM} = 0.252 \pm 0.003$ \cite{8}. These observations may be considered as the smoking gun signals for the violation of lepton flavour universality (LFU). The dominant theoretical uncertainties are reduced in these observables, as the hadronic uncertainties cancel out to a large extent in these ratios. The branching ratio of semileptonic $b \rightarrow c l \bar{\nu}_l$ process can be computed precisely due to the light mass of leptons in the final state, thus the deviation in $R_{D^{(*)}}$ could be from new physics affecting $\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}_\tau$ processes. Since these decays occur at tree level in the SM, new physics models with mass of the new particles near the TeV scale would be required to explain the $R_{D^{(*)}}$ anomalies. The branching ratios
of $\bar{B} \to \bar{D}^{(*)}\tau\bar{\nu}_\tau$ processes and the associated $R_{D^{(*)}}$ anomalies have been investigated in the literature both in the SM as well as in various new physics models [9–16].

Another interesting observable is the lepton non-universality parameter ($R_K$) in $B^+ \to K^+l^+l^−$ process, defined as [17]

$$R_K = \frac{\text{Br}(B^+ \to K^+\mu^+\mu^-)}{\text{Br}(B^+ \to K^+e^+e^-)}.$$  

(3)

This parameter has recently been measured at LHCb with the value $R_K = 0.745^{+0.090}_{-0.074}\pm0.036$ [18], which has $2.6\sigma$ deviation from its SM value $R_K = 1.0003 \pm 0.0001$ in the dilepton invariant mass squared bin $(1 \leq q^2 \leq 6)$ GeV$^2$. The deviation in the ratios of branching fractions of other exclusive and inclusive $b \to s$ semileptonic decays [19] into dimuon over the dielectron is a compelling reason to infer possible violation of lepton universality. Various new physics models have been considered in the literature [20] to explain the lepton non-universality ($R_K$) parameter. The decay rate [21] of $B \to K^*\mu^+\mu^-$ process and the famous $P_5'$ angular observable [22] also have $\sim 3\sigma$ deviation [23] from the corresponding SM predictions. Furthermore, the discrepancy of $3.3\sigma$ is found in the decay rate of $B_s \to \phi\mu^+\mu^-$ process in the low $q^2$ region [24].

In this paper, we pursue the analysis of semileptonic decays of $B$ meson mediated through charged-current $b \to c\ell\bar{\nu}$ and FCNC $b \to s\ell^+\ell^−$ transitions in the vector leptoquark (LQ) model. In most of the studies in the literature, the authors have discussed either $R_K$ or $R_{D^{(*)}}$ anomaly, but not both on the same footing. In the Ref. [25], both the $R_{D^{(*)}}$ and $R_K$ anomalies have been investigated in the $(3,2,1/6)$ scalar LQ model. According to the scenario presented in [9], the extension of SM with the $SU(2)_L$ singlet scalar LQ can accommodate $R_K$ through a loop correction and $R_{D^{(*)}}$ via the tree level LQ contribution. However, in Ref. [26], it has been argued that a simultaneous explanation of $R_K$ and $R_{D^{(*)}}$ is not realistic and would imply serious phenomenological problems elsewhere. In this work, we would like to focus on both the anomalies $R_{D^{(*)}}$ and $R_K$ as well as some other observables in the $b \to c\ell\bar{\nu}$ decay processes. We calculate the branching ratios, forward-backward asymmetries, the $\tau$ and $D^*$ polarizations of $B \to D^{(*)}\tau\bar{\nu}$ processes in the vector LQ model. We also estimate the branching ratios of the rare leptonic $B_{u,c}^{(*)} \to \tau\bar{\nu}$ decay processes. LQs can couple or decay to a quark and a lepton simultaneously and carry both baryon number ($B$) and lepton number ($L$). They can have spin 0 (scalar) or spin 1 (vector) and can be characterized by their fractional electric charge ($Q$) and fermion
number \((F = 3B + L)\). \(|F|\) can be either 0 or 2 depending on the coupling of LQ to the fermion-antifermion pair or fermion-fermion pair. Such LQs exist in some extended SM theories \([27]\) such as grand unified theories based on \(SU(5), SO(10)\) etc. \([27, 28]\), Pati-Salam model, technicolor model \([29]\) and composite model \([30]\). To avoid rapid proton decay, we consider the LQ which does not couple to diquarks and therefore conserve baryon and lepton numbers. The LQ model in the context of \(B\)-physics anomalies has been studied in the literature \([9, 10, 15, 16, 25, 31–34]\).

The outline of this paper is as follows. In section II, we describe the effective Hamiltonian involving \(b \rightarrow c\tau\bar{\nu}\) and \(b \rightarrow s l^+ l^-\) quark level transition in the SM. We also discuss the relevant vector LQ contributions to \(b \rightarrow c\bar{\nu}l\) and \(b \rightarrow s l^+ l^-\) processes. In section III, we compute the constraint on LQ parameter space by using the recently measured branching ratios of \(B_q \rightarrow l^+ l^-\), \(\bar{B} \rightarrow X_s l^+ l^-\) \((\bar{\nu}\bar{\nu})\) and \(B^+ \rightarrow l^+\nu\) processes, where \(l = e, \mu, \tau\). The branching ratios, forward-backward asymmetries, \(\tau\) and \(D(\star)\) polarization in \(B \rightarrow D(\star)\tau\bar{\nu}\) processes are presented in section IV. We also describe the deviation in lepton non-universality, \(R_{D(\star)}\) and \(R_{K(\star)}\) in this. We work out the branching ratios of the rare \(B^*_{u,c} \rightarrow l\nu\) decay processes in section V and section VI contains the summary and conclusion.

II. EFFECTIVE HAMILTONIAN FOR \(b \rightarrow c\tau\bar{\nu}\) AND \(b \rightarrow s l^+ l^-\) PROCESSES

In the SM, the effective Hamiltonian mediating the semileptonic decays \(b \rightarrow c\tau\bar{\nu}\), considering neutrinos only to be left handed, is given as \([10]\)

\[
\mathcal{H}_{eff} = \frac{4 G_F}{\sqrt{2}} V_{cb} \left[ (\delta_{l\tau} + C_{V_1}^l) \mathcal{O}_{V_1}^l + C_{V_2}^l \mathcal{O}_{V_2}^l + C_{S_1}^l \mathcal{O}_{S_1}^l + C_{S_2}^l \mathcal{O}_{S_2}^l + C_T^l \mathcal{O}_T^l \right],
\]

where \(G_F\) is the Fermi constant, \(V_{cb}\) is the Cabibbo-Kobayashi-Maskawa (CKM) matrix element and the index \(l\) stands for neutrino flavour, \(l = e, \mu, \tau\). The \(C_X^l\) coefficients, with \(X = V_{1,2}, S_{1,2}, T\) are the Wilson coefficients and the corresponding current-current operators are

\[
\begin{align*}
\mathcal{O}_{V_1}^l &= (\bar{c}_L \gamma^\mu b_L) (\bar{\tau}_L \gamma_\mu \nu_{LL}), \\
\mathcal{O}_{V_2}^l &= (\bar{c}_R \gamma^\mu b_R) (\bar{\tau}_L \gamma_\mu \nu_{LL}), \\
\mathcal{O}_{S_1}^l &= (\bar{c}_L b_R) (\bar{\tau}_R \nu_{LL}), \\
\mathcal{O}_{S_2}^l &= (\bar{c}_R b_L) (\bar{\tau}_R \nu_{LL}), \\
\mathcal{O}_T^l &= (\bar{c}_R \sigma^{\mu\nu} b_L) (\bar{\tau}_R \sigma_{\mu\nu} \nu_{LL}),
\end{align*}
\]
where \( q_{L(R)} = L(R)q \) are the chiral quark fields with \( L(R) = (1 \mp \gamma_5)/2 \) as the projection operators. Since the flavour of neutrino is not observed at \( B \)-factories all generations of neutrinos can be taken into account to reveal the signature of new physics (NP). In the standard model, the contribution to the \( b \to c \tau \bar{\nu}_\tau \) process is indicated as \( \delta_{l\tau} \) and the Wilson coefficients \( (C_X^l) \) are zero. These coefficients can only be generated in new physics models.

The effective Hamiltonian describing the processes induced by \( b \to s l^+ l^- \) transitions in the SM is given by

\[
H_{\text{eff}} = \frac{-4G_F}{\sqrt{2}} V_{tb} V^*_{ts} \left[ \sum_{i=1}^{6} C_i(\mu) O_i + \sum_{i=7,9,10,S,P} \left( C_i(\mu) O_i + C'_i(\mu) O'_i \right) \right],
\]

where \( V_{tb} V^*_{ts} \) is the product of CKM matrix elements and \( C_i \)'s are the Wilson coefficients evaluated at the renormalization scale \( \mu = m_b \). The corresponding effective operators are given as

\[
\begin{align*}
O_7^{(l)} &= \frac{e}{16\pi^2} \left( \bar{s}\sigma_{\mu\nu}(m_s L(R) + m_b R(L)) b \right) F^{\mu\nu}, \\
O_9^{(l)} &= \frac{\alpha}{4\pi} \left( \bar{s}\gamma^\mu L(R)b \right) \left( \bar{l}\gamma_\mu l \right), \\
O_S^{(l)} &= \frac{\alpha}{4\pi} \left( \bar{s}L(R)b \right) \left( \bar{l}l \right), \\
O_{10}^{(l)} &= \frac{\alpha}{4\pi} \left( \bar{s}\gamma^\mu L(R)b \right) \left( \bar{l}\gamma_\mu \gamma_5 l \right),
\end{align*}
\]

where \( \alpha \) is the fine structure constant. There is no contribution of primed Wilson coefficient as well as (pseudo)scalar coefficients in the SM and they arise only in the physics beyond SM. In the following subsections, we will discuss the possible LQ bosons relevant for the \( b \to c l \bar{\nu}_l \) and \( b \to s l^+ l^- \) quark level transitions.

**A. New physics contribution due to the exchange of vector leptoquark**

In the leptoquark model, the new particles, i.e., leptoquarks, interact with quarks and leptons simultaneously and carry both baryon and lepton numbers. Leptoquarks have ten different multiplets under the \( SU(3)_C \times SU(2)_L \times U(1)_Y \) SM gauge symmetries, with flavour non-diagonal couplings. Out of these, half are scalars and the rest have vectorial nature under the Lorentz transformation. The scalar (vector) LQs have spin 0 (1) and could potentially contribute to the FCNC processes involving the quark level transitions \( b \to s l^+ l^- \) and \( b \to c l^- \bar{\nu}_l \). Out of all possible LQ multiplets, six LQ bosons are relevant for the \( b \to c l \bar{\nu}_l \) processes whose quantum numbers are presented in Table I. Here \( S_{1,3} \) and...
$R_2$ are the scalar LQ bosons, $U_{1,3}^\mu$ and $V_2^\mu$ are the vector LQs. In this work, we investigate the $U_1^\mu = (3, 1, 2/3)$ and $U_3^\mu = (3, 3, 2/3)$ vector LQs, which have $Y = 2/3$, $F = 0$ and can mediate both $b \to s l^+ l^-$ and $b \to c l^- \bar{\nu}$ quark level transitions. The charge of LQ is related to hypercharge and weak isospin ($T_3$) through $Q = T_3 + Y$. In order to avoid rapid proton decay we do not consider diquark interactions, as the presence of both LQ and diquark interactions will violate baryon and lepton number. The interaction Lagrangian of $U_{1,3}^\mu$ LQs with the SM fermion bilinear is given as \cite{10, 14}

$$
\mathcal{L}^{LQ} = \left( h_{ij}^{1L} \bar{Q}_{iL} \gamma^\mu L_{jL} + h_{ij}^{1R} \bar{d}_{iR} \gamma^\mu l_{jR} \right) U_{1\mu} + h_{ij}^{3L} \bar{Q}_{iL} \sigma \sigma \gamma^\mu L_{jL} U_{3\mu} ,
$$

(8)

where $Q_L(L_L)$ is the left handed quark (lepton) doublet, $u_R(d_R)$ and $l_R$ are the right-handed up (down) quark and charged-lepton singlet respectively and $\sigma$ represents the Pauli matrices. Here the LQ couplings are represented by $h^{ij}$, where $i, j$ are the generation indices of quarks and leptons respectively.

The fermion fields in Eqn. (8) are represented in the gauge eigen basis in which Yukawa couplings of the up type quarks and the charged leptons are diagonal, whereas the down type quark fields are rotated into the mass eigenstate basis by the CKM matrix. Now performing the Fierz transformation, we obtain additional Wilson coefficients to the $b \to c \tau \bar{\nu}$ process as \cite{14},

$$
C_{V_1}^d = \frac{1}{2\sqrt{2} G_F V_{cb}} \sum_{k=1}^{3} V_{k3} \left[ \frac{h_{1L}^{2L} h_{k3}^{1L}}{M_{U_1}^{2/3}} - \frac{h_{1L}^{2L} h_{k3}^{3L}}{M_{U_3}^{2/3}} \right],
$$

(9a)

$$
C_{V_2}^d = 0,
$$

(9b)

$$
C_{S_1}^d = -\frac{1}{2\sqrt{2} G_F V_{cb}} \sum_{k=1}^{3} V_{k3} \frac{2h_{1L}^{2L} h_{k3}^{1R}}{M_{U_1}^{2/3}} ,
$$

(9c)

where $V_{k3}$ denotes the CKM matrix element, $M_{U_{1(3)}^{2/3}}$ is the mass of the leptoquark and the superscript denotes the charge of $U_{1(3)}$.

After expanding the $SU(2)$ indices of Eqn. (8), one can notice that $U_{1,3}$ vector LQs give
TABLE I: Possible relevant scalar and vector leptoquarks invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y$ SM gauge group.

| Leptoquarks | Spin | $F = 3B + L$ | $(SU(3)_C, SU(2)_L, U(1)_Y)$ |
|-------------|------|--------------|--------------------------------|
| $S_1$       | 0    | $-2$         | $(3^*, 1, 1/3)$                |
| $S_3$       | 0    | $-2$         | $(3^*, 3, 1/3)$                |
| $R_2$       | 0    | 0            | $(3, 2, 7/6)$                  |
| $U_1$       | 1    | 0            | $(1, 2/3)$                     |
| $U_3$       | 1    | 0            | $(3, 3, 2/3)$                  |
| $V_2$       | 1    | $-2$         | $(3^*, 2, 5/6)$                |

additional contributions to the Wilson coefficients of $b \to s l^+_i l^-_j$ processes as

\[
C^{NP}_9 = -C^{NP}_{10} = \frac{\pi}{\sqrt{2} G_F V_{tb} V_{ts}^*} \left[ \frac{h_{1L}^{2l} h_{1L}^{k3}}{M_{U_1}^{2/3}} + \frac{h_{3L}^{2l} h_{3L}^{k3}}{M_{U_3}^{2/3}} \right], \quad (10a)
\]

\[
C^{NP}_9 = C^{\prime NP}_{10} = \frac{\pi}{\sqrt{2} G_F V_{tb} V_{ts}^*} \left( \frac{h_{1R}^{2l} h_{1R}^{k3}}{M_{U_1}^{2/3}} \right), \quad (10b)
\]

\[
- C^{NP}_P = C^{NP}_S = \frac{\sqrt{2} \pi}{G_F V_{tb} V_{ts}^*} \left( \frac{h_{1L}^{2l} h_{1L}^{k3}}{M_{U_1}^{2/3}} \right), \quad (10c)
\]

\[
C^{\prime NP}_P = C^{\prime NP}_S = \frac{\sqrt{2} \pi}{G_F V_{tb} V_{ts}^*} \left( \frac{h_{1R}^{2l} h_{1R}^{k3}}{M_{U_1}^{2/3}} \right), \quad (10d)
\]

where $l, k$ are the generation indices and $C^{(\prime)NP}_{9,10,S,P}$ are the new Wilson coefficients which arise due to the exchange of vector LQs associated with their respective operators $O^{(\prime)}_{9,10,S,P}$.

III. CONSTRAINT ON LEPTOQUARK COUPLINGS FROM RARE DECAY PROCESSES OF $B$ MESON

After knowing all the possible vector LQs suitable for $B \to D^{(*)} l \bar{\nu}_l$ and $B \to K^{(*)} l^+ l^-$ processes and the contribution of additional new Wilson coefficients to the SM, we now proceed to constrain the new LQ parameter space. The relevant leptoquark couplings can be constrained using both $b \to s l^+ l^-$ and $b \to c l^- \bar{\nu}_l$ processes. In this analysis, we obtain the constraints on various LQ couplings by comparing the theoretical and experimental
branching ratio of $B_s \to l^+l^-$, $B \to X_s l^+l^-$ and $B \to X_s \nu\bar{\nu}$ processes, considering the LQ mass as $M_{LQ} = 1$ TeV. Using the constrained LQ couplings one can study the processes mediated by $b \to sl^+l^-$ and $b \to cl\bar{\nu}$ transitions. The new LQ parameter space contributing to $b \to ul\nu_l$ transition is constrained by $B_u \to l\nu_l$ processes.

A. $B_s \to l^+l^-$ processes

The rare leptonic $B_s \to l^+l^-$ processes, where $l = e, \mu, \tau$, mediated by $b \to sl^+l^-$ transitions are highly suppressed in the SM and occur via electroweak penguin and box diagrams. These processes are theoretically very clean and the only hadronic parameter involved is the decay constant of $B$ meson, hence well suited for constraining the LQ parameters. The branching ratio of $B_s \to l^+l^-$ process in the SM is given by

$$\text{Br}(B_s \to l^+l^-) = \frac{G_F^2}{16\pi^3} \tau_B \alpha e f_{B_s} |C_{10}^{\text{SM}}|^2 M_{B_s} m_l^2 |V_{tb} V_{ts}^*|^2 \sqrt{1 - 4m_l^2 M_{B_s}^2} \times \left(|P|^2 + |S|^2\right),$$

(11)

where $P$ and $S$ are defined as

$$P \equiv \frac{C_{10}^{\text{SM}} + C_{10}^{\text{NP}} - C_{10}^{\text{NP}}}{C_{10}^{\text{SM}}} + \frac{M_{B_s}^2}{2m_t m_b + m_s} \frac{m_b}{m_b + m_s} \left(\frac{C_{NP}^{\text{NP}} - C_{NP}^{\text{NP}}}{C_{10}^{\text{SM}}}\right),$$

$$S \equiv \sqrt{1 - \frac{4m_l^2 M_{B_s}^2}{2m_t m_b + m_s} \frac{m_b}{m_b + m_s} \left(\frac{C_{NP}^{\text{NP}} - C_{NP}^{\text{NP}}}{C_{10}^{\text{SM}}}\right)}.$$  

(12)

Here $C_{10,S,P}^{(\text{NP})}$ are the new Wilson coefficients arising due to the exchange of vector LQ, which are negligible in the SM. The theoretical predictions \cite{38} and the average experimental values of CMS and LHCb \cite{39,41} for the branching ratios of $B$ meson decaying to all charged leptonic modes are given as

$$\text{Br}(B_s \to ee)^{\text{SM}} = (8.54 \pm 0.55) \times 10^{-14} \ [38], \quad \text{Br}(B_s \to ee)^{\text{expt}} < 2.8 \times 10^{-7} \ [39],$$

$$\text{Br}(B_s \to \mu\mu)^{\text{SM}} = (3.65 \pm 0.23) \times 10^{-9} \ [38], \quad \text{Br}(B_s \to \mu\mu)^{\text{expt}} = (2.8^{+0.7}_{-0.6}) \times 10^{-9} \ [40],$$

$$\text{Br}(B_s \to \tau\tau)^{\text{SM}} = (7.73 \pm 0.49) \times 10^{-7} \ [38], \quad \text{Br}(B_s \to \tau\tau)^{\text{expt}} < 3.0 \times 10^{-3} \ [41].$$

(13)

If we consider the LQ couplings as chiral, then only $C_{10}^{\text{NP}}$ Wilson coefficient will give additional contributions. Now comparing the theoretical value of branching ratio of $B_s \to l^+l^-$ processes with the $1\sigma$ range of the experimental data, the allowed region of real and imaginary parts of the LQ couplings are shown in Fig. 1, for $B_s \to e^+e^-$ (top left panel),
$B_s \rightarrow \mu^+\mu^−$ (top right panel) and $B_s \rightarrow \tau^+\tau^−$ (bottom panel) processes. The constrained values of real and imaginary parts of LQ couplings are given in Table II.

As seen from Eqn. [12], the scalar and pseudoscalar Wilson coefficients are dominated by the $M_B^2/m_t$ multiplication factor, therefore the new physics contribution to the $C_{10}$ Wilson coefficient can be neglected. Now considering only the $C_{S,P}^{(NP)}$ new Wilson coefficients, the allowed region on real and imaginary parts of LQ couplings for $B_s \rightarrow e^+e^−$ (top left panel), $B_s \rightarrow \mu^+\mu^−$ (top right panel) and $B_s \rightarrow \tau^+\tau^−$ (bottom panel) processes are shown in Fig. 2 and the allowed range of LQ couplings are presented in Table II.

FIG. 1: Constraints on the real and imaginary parts of the leptoquark couplings from $B_s \rightarrow e^+e^−$ (top left panel), $B_s \rightarrow \mu^+\mu^−$ (top right panel) and $B_s \rightarrow \tau^+\tau^−$ (bottom panel) processes in $U(3,3,2/3)$ leptoquark model.
FIG. 2: Constraints on the real and imaginary parts of the leptoquark couplings from $B_s \rightarrow e^+ e^-$ (top left panel), $B_s \rightarrow \mu^+ \mu^-$ (top right panel) and $B_s \rightarrow \tau^+ \tau^-$ (bottom panel) processes in $U(3,1,2/3)$ leptoquark model.

**B. $\bar{B} \rightarrow X_s l^+ l^-$ processes**

In this subsection, we discuss the constraint on LQ couplings from the branching ratio of inclusive $\bar{B} \rightarrow X_s l^+ l^-$ decay process mediated via $b \rightarrow s l^+ l^-$ transitions. The branching ratio for this process in the SM is given by \[ 31 \] \[ 42 \]

\[
\left. \frac{d\text{Br}}{ds_1} \right|_{\text{SM}} = B_0 \frac{8}{3} (1 - s_1)^{2} \sqrt{1 - \frac{4t^2}{s_1}} \times \left[ (2s_1 + 1) \left( \frac{2t^2}{s_1} + 1 \right) |C_{9eff}|^2 + \left( \frac{2(1 - 4s_1)t^2}{s_1} + (2s_1 + 1) \right) |C_{10}|^2 + 4 \left( \frac{2}{s_1} + 1 \right) \left( \frac{2t^2}{s_1} + 1 \right) |C_7|^2 + 12 \left( \frac{2t^2}{s_1} + 1 \right) \text{Re}(C_7 C_{9eff}^*) \right],
\] (14)
TABLE II: Constraints on the real and imaginary parts of the leptoquark couplings from $B_s \to l^+l^-$ processes, where $l = e, \mu, \tau$.

| Leptoquark Couplings | Real part | Imaginary Part |
|----------------------|-----------|----------------|
| $h_{1(3)}^{21}Lh_{1(3)}^{31}$ | $-13.0 \to 13.0$ | $-13 \to 13$ |
| $h_{1(3)}^{22}Lh_{1(3)}^{32}$ | $-0.016 \to 0.0$ | $-0.008 \to 0.008$ |
| $h_{1(3)}^{23}Lh_{1(3)}^{33}$ | $-0.4 \to 0.4$ | $-0.4 \to 0.4$ |

where $t = m_t/m_b^{pole}$, $s_1 = q^2/(m_b^{pole})^2$ and $B_0$ is the normalization constant related to $\text{Br}(\bar{B} \to X_c e \bar{\nu}_e)$ process as

$$B_0 = \frac{3\alpha^2 \text{Br}(\bar{B} \to X_c e \bar{\nu}_e) |V_{tb}V_{ts}^*|^2}{32\pi^2 f(\hat{m}_c)\kappa(\hat{m}_c) |V_{cb}|^2}.$$  

(15)

Here $\hat{m}_c = m_c^{pole}/m_b^{pole}$ and the functions $f(\hat{m}_c)$ and $\kappa(\hat{m}_c)$ are defined in Ref. [31, 42]. For the numerical estimation, we use the numerical parameters as $\hat{m}_c = 0.29 \pm 0.02$ [33] and $\text{Br}(\bar{B} \to X_c e \bar{\nu}_e) = (10.1 \pm 0.4)\%$ [19]. For the CKM matrix elements we use the Wolfenstein parameters with values $A = 0.814^{+0.023}_{-0.024}$, $\lambda = 0.22537 \pm 0.00061$, $\bar{\rho} = 0.117 \pm 0.021$ and $\bar{\eta} = 0.353 \pm 0.013$ [19]. Now using these parameters, the branching ratios of $\bar{B} \to X_s l^+l^-$ processes in the SM for the low $q^2 \in [1, 6]$ GeV$^2$ region are found as

$$\begin{align*}
\text{Br}(\bar{B} \to X_s e^+e^-)|_{q^2 \in [1,6] \text{ GeV}^2} &= (1.67 \pm 0.06) \times 10^{-6}, \\
\text{Br}(\bar{B} \to X_s \mu^+\mu^-)|_{q^2 \in [1,6] \text{ GeV}^2} &= (1.6 \pm 0.61) \times 10^{-6},
\end{align*}$$

(16) (17)

and the predicted branching ratios in the high $q^2 \geq 14.2$ GeV$^2$ region are given as

$$\begin{align*}
\text{Br}(\bar{B} \to X_s e^+e^-)|_{q^2 \geq 14.2 \text{ GeV}^2} &= (3.9 \pm 0.15) \times 10^{-7}, \\
\text{Br}(\bar{B} \to X_s \mu^+\mu^-)|_{q^2 \geq 14.2 \text{ GeV}^2} &= (3.8 \pm 0.25) \times 10^{-7}, \\
\text{Br}(\bar{B} \to X_s \tau^+\tau^-)|_{q^2 \geq 14.2 \text{ GeV}^2} &= (1.78 \pm 0.29) \times 10^{-7}.
\end{align*}$$

(18) (19) (20)
The corresponding experimental results \[44\] for both low and high \(q^2\) regions are given by

\[
\text{Br}(\bar{B} \rightarrow X_s e^+ e^-) = (1.93^{+0.47}_{-0.45} +0.21_{-0.16} \pm 0.18) \times 10^{-6} \quad \text{for low } q^2, \quad (21)
\]
\[
= (0.56^{+0.30}_{-0.18} +0.03_{-0.03} \pm 0.00) \times 10^{-6} \quad \text{for high } q^2, \quad (22)
\]
\[
\text{Br}(\bar{B} \rightarrow X_s \mu^+ \mu^-) = (0.66^{+0.82}_{-0.76} +0.30_{-0.24} \pm 0.07) \times 10^{-6} \quad \text{for low } q^2, \quad (23)
\]
\[
= (0.60^{+0.31}_{-0.29} +0.05_{-0.04} \pm 0.00) \times 10^{-6} \quad \text{for high } q^2, \quad (24)
\]

where the first uncertainties are statistical, the second experimental systematics and the third model-dependent systematics. Since there is no experimental measurement for the branching ratio of \(\bar{B} \rightarrow X_s \tau^+ \tau^-\) process, we consider the limit as \(\sim 1\%\) in our analysis. Including the new physics contribution, the total branching ratio of \(\bar{B} \rightarrow X_s l^+ l^-\) process is given by \[31, 42\]

\[
\left(\frac{d\text{Br}}{ds_1}\right)_{\text{Total}} = \left(\frac{d\text{Br}}{ds_1}\right)_{\text{SM}} + B_0 \left[ \frac{16}{3} (1 - s_1)^2 (1 + 2s_1) \left[ |C_{9}^{NP}|^2 + |C_{10}^{NP}|^2 + |C_{9}^{NP}|^2 \right] + 32 (1 - s_1)^2 \text{Re}(C_7 C_{10}^{NP*}) \right], \quad (25)
\]

where \(C_{9,10}^{(c)NP}\) are the new Wilson coefficients. The particle masses and the lifetime of \(B\) meson are taken from \[19\]. Now comparing the theoretical and experimental branching ratios, we show the constraints on \(U(3,3,2/3)\) LQ couplings from \(\bar{B} \rightarrow X_s e^+ e^-(\mu^+ \mu^-)\) process for low \(q^2\) (left panel) and high \(q^2\) (right panel) in Fig. 3 (Fig. 4) respectively. Similarly in Fig. 5, we show the allowed region from \(\bar{B} \rightarrow X_s \tau^+ \tau^-\) process in high \(q^2\) region. From these figures, the allowed range of real and imaginary parts of LQ parameter space in the low and high \(q^2\) regime are presented in Table III.

\textbf{C. } \(\bar{B} \rightarrow X_s \nu \bar{\nu}\) process

The study of the processes involving \(b \rightarrow s \nu \bar{\nu}\) transitions are quite important, as they are related to \(b \rightarrow s l^+ l^-\) processes by \(SU(2)_L\) and are also very sensitive to the search for new physics beyond the SM. The inclusive decay \(\bar{B} \rightarrow X_s \nu \bar{\nu}\) is theoretically very clean since both the perturbative and the non-perturbative corrections are small. Thus, these decays do not suffer from the form factor uncertainties.
FIG. 3: Constraints on the real and imaginary parts of the leptoquark couplings from $\bar{B} \rightarrow X_s e^+ e^-$ process in low $q^2$ (left panel) and high $q^2$ region (right panel) in the $U(3, 3, 2/3)$ leptoquark model.

FIG. 4: Constraints on the real and imaginary parts of the leptoquark couplings from $\bar{B} \rightarrow X_s \mu^+ \mu^-$ process in low $q^2$ (left panel) and high $q^2$ region (right panel) in the $U(3, 3, 2/3)$ leptoquark model.

The effective Hamiltonian for $b \rightarrow s \nu \bar{\nu}$ process is given by [15]

$$H_{\text{eff}} = \frac{-4 G_F}{\sqrt{2}} V_{tb} V_{ts}^* (C_L^\nu \mathcal{O}_L^\nu + C_R^\nu \mathcal{O}_R^\nu) + h.c., \quad (26)$$

where the six-dimensional operators are

$$\mathcal{O}_L^\nu = \frac{\alpha}{4\pi} (\bar{s} \gamma_\mu L b) \left( \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \right), \quad \mathcal{O}_R^\nu = \frac{\alpha}{4\pi} (\bar{s} \gamma_\mu R b) \left( \bar{\nu} \gamma^\mu (1 - \gamma_5) \nu \right). \quad (27)$$

In the SM, the $C_L^\nu$ coefficient is computed using the loop functions [16] and is given by

$$C_L^\nu = -X(x_t)/\sin^2 \theta_w, \quad (28)$$
FIG. 5: Constraints on the real and imaginary parts of the leptoquark couplings from $\bar{B} \rightarrow X_s \tau^+ \tau^-$ process in the $U(3, 3, 2/3)$ leptoquark model

TABLE III: Constraints on the real and imaginary parts of the leptoquark coupling for low and high $q^2$ region from $\bar{B} \rightarrow X_s l^+ l^-$ process, where $l = e, \mu, \tau$

| $q^2$ bin | Leptoquark Couplings | Real part | Imaginary Part |
|-----------|-----------------------|-----------|---------------|
| low $q^2$ | $h_{1(3) L}^{21} h_{1(3) L}^{31*}$ | $-0.01 \rightarrow 0.01$ | $-0.01 \rightarrow 0.01$ |
|           | $h_{1(3) L}^{22} h_{1(3) L}^{32*}$ | $-0.008 \rightarrow 0.008$ | $-0.008 \rightarrow 0.008$ |
| high $q^2$ | $h_{1(3) L}^{21} h_{1(3) L}^{31*}$ | $-0.022 \rightarrow 0.022$ | $-0.022 \rightarrow 0.022$ |
|           | $h_{1(3) L}^{22} h_{1(3) L}^{32*}$ | $-0.018 \rightarrow 0.018$ | $-0.018 \rightarrow 0.018$ |
|           | $h_{1(3) L}^{23} h_{1(3) L}^{33*}$ | $-3.8 \rightarrow 3.8$ | $-3.8 \rightarrow 3.8$ |

whereas the $C_R^\nu$ coefficient is negligible. The branching ratio of $\bar{B} \rightarrow X_s \nu \bar{\nu}$ process is

$$
\frac{d\Gamma}{ds_b} = m_b^5 \frac{\alpha^2 G_F^2}{128\pi^5} |V_{ts}^* V_{tb}|^2 \kappa(0) \left( |C_L^\nu|^2 + |C_R^\nu|^2 \right) \lambda^{1/2}(1, \tilde{m}_s^2, s_b) \\
\times \left[ 3s_b \left( 1 + \tilde{m}_s^2 - s_b - 4\tilde{m}_s \frac{Re(C_L^\nu C_R^{\nu*})}{|C_L^\nu|^2 + |C_R^\nu|^2} \right) + \lambda(1, \tilde{m}_s^2, s_b) \right],
\tag{29}
$$

where $\tilde{m}_s = m_s/m_b$, $s_b = s/m_b^2$ and $\kappa(0) = 0.83$ is the QCD correction to the $b \rightarrow s \nu \bar{\nu}$ matrix element [47]. For numerical analysis, we have used the quark masses as $m_s = 0.1$ GeV and $m_b = 4.8$ GeV. It should be noted from [8] that $U_3$ leptoquark has additional Wilson coefficient contribution to $b \rightarrow s \nu \bar{\nu}_1$ process, which is given by

$$
C_L^{LQ} = \frac{2\pi}{\sqrt{2} G_F \alpha V_b V_{ts}^*} \sum_{m, n = 1}^{3} V_{m3} V_{n2}^* \frac{h_{3L}^{ni} h_{3L}^{mi*}}{M^2_{U_3}^{1/3}},
\tag{30}
$$

14
In the presence of LQ the total decay rate of $B \to X_s \nu \bar{\nu}$ process can be obtained from (29) by replacing the Wilson coefficient $C'_L \to C'_L + C'^{LQ}_L$. Using all the particle masses and the lifetime of $B$ meson from [19], the branching ratio in the SM is found to be

$$\text{Br}(\bar{B} \to X_s \nu \bar{\nu}) = (2.74 \pm 0.16) \times 10^{-5},$$

and the corresponding experimental upper limit measured by the ALEPH collaboration is given by [48]

$$\text{Br}(\bar{B} \to X_s \nu \bar{\nu}) < 6.4 \times 10^{-4}.$$  

Since $U^{2/3}_3$ and $U^{-1/3}_3$ LQs are coming from the same $SU(2)$ triplet, one can constrain $h^2_{3L}h^{3\ast}_{3L}$ couplings by assuming that both the LQs have the same mass. Now comparing the theoretical and experimental branching ratio, we show the constraints on $U(3,3,2/3)$ leptoquark couplings in Fig. 6. From the figure, the allowed ranges of real and imaginary part of the couplings are found as

$$-0.02 \leq \text{Re}[h^2_{3L}h^{3\ast}_{3L}] \leq 0.02, \quad -0.02 \leq \text{Im}[h^2_{3L}h^{3\ast}_{3L}] \leq 0.02.$$  

(Fig. 6: Constraints on the real and imaginary parts of the leptoquark couplings from $\bar{B} \to X_s \nu \bar{\nu}$ process in the $U(3,3,2/3)$ leptoquark model.)

D. $B^+_u \to l^+\nu_l$ processes

The rare leptonic $B^+_u \to l^+\nu_l$ decay modes, where $l = e, \mu, \tau$ mediated by $b \to ul\nu$ transitions can provide significant constraints on models of new physics. Neglecting the
electromagnetic radiative corrections, the branching ratios of the $B^+_u \to l^+\nu_l$ processes in the $U_{1,3}$ leptoquark model are given by [11],

$$\text{Br}(B^+_u \to l^+\nu_l) = \frac{G_F^2 M_{B_u} m_l^2}{8\pi^2} \left(1 - \frac{m_l^2}{M_{B_u}^2}\right)^2 f_{B_u}^2 |V_{ub}|^2 \tau_{B^+} \times \left|(1 + C_{V_1} - C_{V_2}) + \frac{M_{B_u}^2}{m_b (m_b + m_u)} C_{S_1}\right|^2,$$

(34)

where $C_{V_1,2}$ and $C_{S_1}$ Wilson coefficients arise due to $U_{1,3}$ leptoquark exchange and are negligible in the SM. Using the particle masses and life time of $B^+_u$ meson from [19], the decay constants $f_{B_{u,d}} = 190.5(4.2)$ MeV [49] and $|V_{ub}| = 4.13(49) \times 10^{-3}$ [19], the branching ratios in the SM are found to be

$$\text{Br}(B^+_u \to e^+\nu_e) = (8.9 \pm 0.23) \times 10^{-12},$$
$$\text{Br}(B^+_u \to \mu^+\nu_\mu) = (3.83 \pm 0.1) \times 10^{-7},$$
$$\text{Br}(B^+_u \to \tau^+\nu_\tau) = (8.48 \pm 0.28) \times 10^{-5},$$

(35)

and the corresponding averaged experimental values are [19]

$$\text{Br}(B^+_u \to e^+\nu_e) < 9.8 \times 10^{-7},$$
$$\text{Br}(B^+_u \to \mu^+\nu_\mu) < 1.0 \times 10^{-6},$$
$$\text{Br}(B^+_u \to \tau^+\nu_\tau) = (1.14 \pm 0.27) \times 10^{-4}.$$

(36)

If we apply chirality on LQ, then only $C_{V_1}$ Wilson coefficient will contribute to the branching ratios. Now comparing the theoretical (35) and experimental (36) values, the allowed region of real and imaginary part of LQ couplings from $B^+_u \to e^+\nu_e$ (left panel), $B^+_u \to \mu^+\nu_\mu$ (right panel) and $B^+_u \to \tau^+\nu_\tau$ (bottom panel) processes are shown in Fig. 7 and the constrained values are given in Table IV. From (34), it should be noted that the contribution of $C_{S_1}$ Wilson coefficient is enhanced by the factor $M_{B_u}^2/m_l$, so we will neglect the NP in $C_{V_1}$ for simplicity. Then the branching ratio is only sensitive to the $C_{S_1}$ Wilson coefficient. In Fig. 8, we show the constraint on $U(3, 1, 2/3)$ LQ couplings from $B^+_u \to e^+\nu_e$ (left panel), $B^+_u \to \mu^+\nu_\mu$ (right panel) and $B^+_u \to \tau^+\nu_\tau$ (bottom panel) processes and the allowed ranges are given in Table IV.
FIG. 7: Constraints on the real and imaginary parts of the leptoquark couplings from $B_u^+ \to e^+ \nu_e$ (left panel), $B_u^+ \to \mu^+ \nu_\mu$ (right panel) and $B_u^+ \to \tau^+ \nu_\tau$ (bottom panel) processes in $U(3, 3, 2/3)$ leptoquark model.

IV. $B \to D^{(*)} \ell \bar{\nu}$ PROCESS

In this section, we discuss the theoretical framework to compute the branching ratios and other physical observables in $B \to D^{(*)} \ell \bar{\nu}$ processes. The hadronic matrix elements between the initial $B$ meson and final $D$ meson can be parameterized in terms of the form factors $F_0(q^2), F_1(q^2)$ and $F_T(q^2)$ as [10]

$$
\langle D(k)|\bar{c}\gamma_\mu b|\bar{B}(p)\rangle = \left[(p + k)_\mu - \frac{M_B^2 - M_D^2}{q^2} q_\mu\right] F_1(q^2) + q_\mu \frac{M_B^2 - M_D^2}{q^2} F_0(q^2),
$$

$$
\langle D(k)|\bar{c}\sigma_{\mu\nu} b|\bar{B}(p)\rangle = -i (p_\mu k_\nu - k_\mu p_\nu) \frac{2F_T(q^2)}{M_B + M_D},
$$

where $p, k$ are the 4-momenta of the $B$ and $D$ mesons respectively and $q^2 = (p - k)^2$ is the momentum transfer to the dilepton system. The expression for $F_{1,0,T}(q^2)$ form factors in
FIG. 8: Constraints on the real and imaginary parts of the leptoquark couplings from $B_u^+ \to e^+\nu_e$ (left panel), $B_u^+ \to \mu^+\nu_\mu$ (right panel) and $B_u^+ \to \tau^+\nu_\tau$ (bottom panel) processes in $U(3, 1, 2/3)$ leptoquark model.

terms of heavy quark effective theory (HQET) form factors $(h_{\pm, T}(q^2))$ are given in Appendix A [10, 13]. Using Eqn. (37) the differential decay rate of $B \to D\tau\bar{\nu}_\ell$ process with respect to $q^2$ is given by [10, 13]

$$
\frac{d\Gamma(B \to D\tau\bar{\nu}_\ell)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 m_B^3} q^2 \sqrt{\lambda_D(q^2)} \left( 1 - \frac{m_\tau^2}{q^2} \right)^2 \times \left[ \left| \delta_{l\tau} + C_{V_1}^l \right|^2 \left( \left(1 + \frac{m_\tau^2}{2q^2}\right) H_{V,0}^s + \frac{3}{2} \frac{m_\tau^2}{q^2} H_{V,t}^s \right) \right.
$$

$$
+ \left. \frac{3}{2} \left| C_{S_1}^l \right|^2 H_S^2 + 3 \text{Re} \left[ \left( \delta_{l\tau} + C_{V_1}^l \right) C_{S_1}^l \right] \frac{m_\tau}{\sqrt{q^2}} H_S^s H_{V,t}^s \right],
$$

where $\lambda_D(q^2) = [(M_B - M_D)^2 - q^2] [(M_B + M_D)^2 - q^2]$ and the hadronic amplitudes $(H_{V,(0,t)}^s)$ and $H_S^s$) are given in Appendix A.
Another interesting observable, i.e., the lepton non-universality parameter, is the ratio of lepton processes, where $l = e, \mu, \tau$

| Leptoquark Couplings | Real part | Imaginary Part |
|----------------------|-----------|----------------|
| $h_{1/3}^{11} h_{1/3}^{31*}$ | $-40.0 \rightarrow 40.0$ | $-40.0 \rightarrow 40.0$ |
| $h_{1/3}^{12} h_{1/3}^{32*}$ | $-0.08 \rightarrow 0.32$ | $-0.2 \rightarrow 0.2$ |
| $h_{1/3}^{13} h_{1/3}^{33*}$ | $0.24 \rightarrow 0.32$ | $-0.2 \rightarrow 0.2$ |
| $h_{1/3}^{11} h_{1/3}^{31*}$ | $-0.002 \rightarrow 0.002$ | $-0.002 \rightarrow 0.002$ |
| $h_{1/3}^{12} h_{1/3}^{32*}$ | $-0.0008 \rightarrow 0.0032$ | $-0.002 \rightarrow 0.002$ |
| $h_{1/3}^{13} h_{1/3}^{33*}$ | $-0.034 \rightarrow 0.046$ | $-0.028 \rightarrow 0.028$ |

The matrix element in the $B \rightarrow D^* \tau \bar{\nu}_l$ process can be parametrized as [10]

$$
\langle D^*(k, \varepsilon) | \bar{c} \gamma_{\mu} b | B(p) \rangle = -i \epsilon_{\mu \rho \sigma} \varepsilon^{* \rho} p^\sigma \frac{2V(q^2)}{M_B + M_{D*}},
$$

$$
\langle D^*(k, \varepsilon) | \bar{c} \gamma_{\mu} \gamma_5 b | B(p) \rangle = \varepsilon^{* \mu} (M_B + M_{D*}) A_1(q^2) - (p + k)_\mu (\varepsilon^* \cdot q) \frac{A_2(q^2)}{M_B + M_{D*}} - q_\mu (\varepsilon^* \cdot q) \frac{2M_{D*}}{q^2} [A_3(q^2) - A_0(q^2)],
$$

where

$$
A_3(q^2) = \frac{M_B + M_{D*}}{2M_{D*}} A_1(q^2) - \frac{M_B - M_{D*}}{2M_{D*}} A_2(q^2),
$$

and the $V(q^2)$ and $A_{0,1,2}(q^2)$ in terms of HQET form factors are presented in Appendix B.

The differential decay distribution with respect to $q^2$ is given as [10, 13]

$$
\frac{d\Gamma (B \rightarrow D^* \tau \bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 M_B^3} q^2 \sqrt{\lambda_{D*}} (q^2) \left( 1 - \frac{m_\tau^2}{q^2} \right)^2 
\times \left[ \left| \delta_\tau + C_{V_i}^l \right|^2 \left( 1 + \frac{m_\tau^2}{2q^2} \right) (H_{V^+}^2 + H_{V^-}^2 + H_{V^0}^2) + \frac{3m_\tau^2}{2q^2} H_{V^*}^2 \right]
\times \frac{3}{2} \left| C_{S_1}^{l*} \right|^2 H_S^2 + 3\text{Re} \left[ (\delta_\tau + C_{V_i}^l) C_{S_1}^{l*} \frac{m_\tau}{\sqrt{q^2}} H_S H_{V^*} \right],
$$

where $H_{V^\pm}, H_{V^0}, H_{V^*}$ and $H_S$ are the hadronic amplitudes described in Appendix B. Another interesting observable, i.e., the lepton non-universality parameter, is the ratio of branching fractions of $B \rightarrow D^{(*)} \tau \bar{\nu}_l$ to $B \rightarrow D^{(*)} l \bar{\nu}_l$ processes, defined as [9, 11, 13, 15]

$$
R_{D^{(*)}} = \frac{\text{Br} (B \rightarrow D^{(*)} \tau \bar{\nu}_l)}{\text{Br} (B \rightarrow D^{(*)} l \bar{\nu}_l)},
$$

19
which probes lepton flavour dependent term in and beyond SM. Similarly in the transition, the lepton non-universality is given by \[ R_{K^{(*)}} = \frac{Br (\bar{B} \rightarrow K^{(*)} \mu^+ \mu^-)}{Br (\bar{B} \rightarrow K^{(*)} e^+ e^-)}. \] (43)

The decay rate expressions for \( \bar{B} \rightarrow K^{(*)} \mu^+ \mu^- \) are taken from \[ [17, 33] \]. One can also see the \( q^2 \) variation of these parameters using the relations

\[
R_{D^{(*)}}(q^2) = \frac{d\Gamma (\bar{B} \rightarrow D^{(*)} \tau \bar{\nu}_\tau)}{d\cos \theta /dq^2} \quad \quad \quad R_{K^{(*)}}(q^2) = \frac{d\Gamma (\bar{B} \rightarrow K^{(*)} \mu^+ \mu^-)}{d\Gamma (\bar{B} \rightarrow K^{(*)} e^+ e^-) /dq^2}. \] (44)

Besides the branching ratios and lepton non-universality parameters, the following interesting observables could be sensitive to new physics.

- The \( \tau \) forward-backward asymmetry in the \( B \rightarrow D^{(*)} \tau \bar{\nu}_\tau \) processes is defined as \[ [10, 11] \]

\[
A_{FB}(q^2) = \frac{\int_1^0 d\cos \theta d\cos \theta - \int_0^- d\cos \theta d\cos \theta}{\int_1^- d\cos \theta d\cos \theta} = \frac{b_\theta(q^2)}{d\Gamma /dq^2}, \] (45)

where \( \theta \) is the angle between the direction of the charged lepton and the \( D^{(*)} \) meson in the \( \tau \bar{\nu}_\tau \) rest frame. The expression for \( b_\theta(q^2) \) can be found in \[ [10] \].

- \( \tau \) polarization parameter is defined as \[ [10] \]

\[
P_\tau(q^2) = \frac{d\Gamma (\lambda_\tau = 1/2)/dq^2 - d\Gamma (\lambda_\tau = -1/2)/dq^2}{d\Gamma (\lambda_\tau = 1/2)/dq^2 + d\Gamma (\lambda_\tau = -1/2)/dq^2}, \] (46)

where the decay distribution \( d\Gamma (\lambda = \pm 1/2)/dq^2 \) is given in Appendix C .

- The longitudinal and transverse polarization of \( D^{*} \) can be defined as \[ [11] \]

\[
F_{L,T}^{D^*}(q^2) = \frac{d\Gamma_{L,T} (B \rightarrow D^* \tau \bar{\nu}_\tau)}{d\Gamma (B \rightarrow D^* \tau \bar{\nu}_\tau) /dq^2}, \] (47)

where the subscripts \( L, T \) denote the longitudinal and transverse components respectively, and \( d\Gamma_{\tau}/dq^2 = d\Gamma_+/dq^2 + d\Gamma_-/dq^2 \). The complete expression for \( d\Gamma_{\pm}/dq^2 \) is presented in Appendix C.

- Analogous to \( R_{D^*} \), one can also define the ratio of longitudinal and transverse \( D^{*} \) polarization distribution of \( B \rightarrow D^* \tau \bar{\nu}_\tau \) to the corresponding \( B \rightarrow D^* l \bar{\nu}_l \) process as \[ [11] \]

\[
R_{L,T}^{D^*}(q^2) = \frac{d\Gamma_{L,T} (B \rightarrow D^* \tau \bar{\nu}_\tau)}{d\Gamma_{L,T} (B \rightarrow D^* l \bar{\nu}_l) /dq^2}. \] (48)
After getting familiar with the expressions for branching ratios and different physical observables of $B \to D^{(*)} l \bar{\nu}_l$ processes, we now proceed for numerical estimation. All the particle masses and the life time of $B$ meson are taken from [19] and the CKM matrix element $|V_{cb}| = 0.0424(9)$ [51]. Now using the constrained leptoquark parameter space as discussed in section III and the Eqns. (9a, 9b, 9c), we calculate bound on the new Wilson coefficients $C_{V1}(C_{S1})$. If we apply chirality on vector LQs, then $C_{V1}$ is the only additional Wilson coefficient to the SM. As the constraint on $C_{V1}$ is found to be same for both $U_{1,3}$ leptoquark (with only a sign difference), we present the effects of only $U_3$ leptoquark in our analysis. We show in Fig. 9, the branching ratio of $B \to D e \bar{\nu}$ (top-left panel), $B \to D \mu \bar{\nu}$ (top-right panel) and $B \to D \tau \bar{\nu}$ processes (bottom panel) with respect to $q^2$ in $U_3$ vector LQ model. Here darker blue dashed lines represent the SM contribution and the orange bands are due to new physics contribution from LQ model. The lighter blue bands correspond to the uncertainties arising in the SM due to the uncertainties associated with the CKM matrix elements and the hadronic form factors. Similarly the $q^2$ variation of branching ratio of $B \to D^* e \bar{\nu}$ (top-left panel), $B \to D^* \mu \bar{\nu}$ (top-right panel) and $B \to D^* \tau \bar{\nu}$ (right panel) processes in the LQ model are presented in Fig. 10. The branching ratios of $B \to D^{(*)}\tau \bar{\nu}$ process has significant deviation from its SM value whereas the deviation in $B \to D^{(*)}l \bar{\nu}_l$ process is negligible. The integrated values of branching ratios of these processes in SM and LQ model are given in Table V. In Fig. 11, we present the plot for the $D^*$ polarization distributions in $B \to D^* l \nu_l$. The left panel of the figure is for $R_{L}^{D^*}$ and right panel for $R_{T}^{D^*}$. The predicted numerical values are given in Table V. Since the LQ contribution does not affect some observables like forward-backward asymmetry, $\tau$ polarization and $F_{L,T}^{D^*}(q^2)$, we don’t provide the corresponding results.

In Fig. 12, we show the variation of lepton non-universality parameters, $R_D(q^2)$ (left panel) and $R_{D^*}(q^2)$ (right panel) with respect to $q^2$ and the corresponding numerical values are presented in Table VI. Now using the constraints on real and imaginary part of the LQ couplings as given in Table II and III, and the Eqns. (10a, 10b, 10c, 10d), we compute the constraint on the new $C^{(i)NP}_{9,10,S,P}$ Wilson coefficients. Using the constrained parameters, the plot for $R_{K}^{\mu e}(q^2)$ in low $q^2$ (left panel) and in high $q^2$ (right panel) are presented in Fig. 13. Fig. 14 shows the $R_{K^*}^{\mu e}(q^2)$ anomaly plots in low $q^2$ (left panel) and high $q^2$ (right panel) in the LQ model. The predicted numerical values of lepton non-universality ($R_{K^{(*)}}$) are given in Table VI. From Table VI, one can see that the predicted values of lepton non-universality
parameters in the LQ model have significant deviation from the SM and are within the $1\sigma$ range of experimental limit. We observe that the addition of new vector LQ can explain both the $R_{D(*)}$ and $R_{K(*)}$ anomalies very well.

![Graphs showing branching ratios](image)

FIG. 9: The variation of branching ratios of $B \to D e \bar{v}$ (left panel), $B \to D \mu \bar{v}$ (right panel) and $B \to D \tau \bar{v}$ (bottom panel) processes with respect to $q^2$ in the leptoquark model. Here darker blue dashed lines are for SM and orange bands represent leptoquark model. The lighter blue bands stand for the theoretical uncertainties arise due to the input parameters in the SM.

V. $B^{*+}_{u, c} \to l^+ \nu$ PROCESS

The rare leptonic $B^{*+}_{u, c} \to l^+ \nu_l$ processes of unstable $B^{*+}_{u, c}$ mesons mediated by $b \to ul\nu$ and $b \to cl\nu$ transitions are studied in this section. Unlike their pseudoscalar partners these decays are not helicity suppressed, but their shorter lifetimes make the branching ratios to be small. The interaction Lagrangian of charged-current leptonic decays of $B^{(*)}_{u, c}$ mesons are given by $[52]$

$$\mathcal{L} = -\frac{4G_F}{\sqrt{2}} V_{q'b} \left[ (1 + C_{V_3}) \left( \bar{q}' \gamma^\mu Lb \right) (\bar{l}\gamma_\mu L\nu) + C_{V_2} \left( \bar{q}' \gamma^\mu Rb \right) (\bar{l}\gamma_\mu L\nu) \right], \quad (49)$$
FIG. 10: The variation of branching ratios of $B \rightarrow D^* e \bar{\nu}$ (left panel), $B \rightarrow D^* \mu \bar{\nu}$ (right panel) and $B \rightarrow D^* \tau \bar{\nu}$ (bottom panel) processes with respect to $q^2$ in the leptoquark model.

FIG. 11: The plot for $R_L^{D^*}$ (left panel) and $R_T^{D^*}$ (right panel) in the leptoquark model

where $q' = u, c$ and $C_{V_{1,2}}$ are the new Wilson coefficients arising due to the exchange of vector LQ. The transition amplitudes can be expressed in terms of the decay constants, defined as

$$\langle 0|\bar{q}'\gamma^\mu\gamma_5b|B_{q'}(p_{B_{q'}})\rangle = -i f_{B_{q'}} p^\mu_{B_{q'}},$$

$$\langle 0|\bar{q}'\gamma^\mu b|B_{q'}^*(p_{B_{q'}^*}, \epsilon)\rangle = f_{B_{q'}^*} M_{B_{q'}^*} \epsilon^\mu,$$

(50)
FIG. 12: The \( q^2 \) variation of lepton non-universality \( R_D(q^2) \) (left panel) and \( R_{D^*}(q^2) \) (right panel) in leptoquark model.

FIG. 13: The plot for \( R_K(q^2) \) in low \( q^2 \) (left panel) and high \( q^2 \) (right panel) in the leptoquark model.

where \( f_{B_{q'}} \) are the decay constant of \( B_{q'}^{(*)} \) mesons and \( \epsilon \) is the polarization vector of \( B_{q'}^* \).

Using Eqn. (50), the differential decay distribution of \( B_{u,c}^{(s)*} \to l^+\bar{\nu}_l \) processes in the LQ model are

\[
\Gamma(B_{q'}^+ \to l^+\nu) = \frac{G_F^2}{8\pi} |V_{q'b}|^2 (1 + C_{V_1} - C_{V_2})^2 M_{B_{q'}} f_{B_{q'}}^2 m_l^2,
\]

(51)

and

\[
\Gamma(B_{q'}^{+*} \to l^+\nu) = \frac{G_F^2}{12\pi} |V_{q'b}|^2 (1 + C_{V_1} + C_{V_2})^2 M_{B_{q'}^*} f_{B_{q'}^*}^2,
\]

(52)
FIG. 14: The plot for $R_{K^*}(q^2)$ in low $q^2$ (left panel) and high $q^2$ (right panel) in the leptoquark model.

TABLE V: The predicted values of branching ratios and $D^*$ polarizations of $\bar{B} \to D^{(*)}\tau\bar{\nu}$ processes in the vector leptoquark model.

| Observables       | SM Predictions       | Values in LQ Model |
|-------------------|----------------------|---------------------|
| $\text{Br}(\bar{B} \to Dl\bar{\nu})$ | $(2.18 \pm 0.13) \times 10^{-2}$ | $(2.13 - 2.25) \times 10^{-2}$ |
| $\text{Br}(\bar{B} \to D\tau\bar{\nu})$ | $(6.75 \pm 0.08) \times 10^{-3}$ | $(2.48 - 8.2) \times 10^{-3}$ |
| $\text{Br}(\bar{B} \to D^*l\bar{\nu})$ | $(5.18 \pm 0.31) \times 10^{-2}$ | $(5.04 - 5.32) \times 10^{-2}$ |
| $\text{Br}(\bar{B} \to D^*\tau\bar{\nu})$ | $(1.33 \pm 0.14) \times 10^{-2}$ | $(1.3 - 1.6) \times 10^{-2}$ |
| $R^D_L$           | 0.227                | 0.215 - 0.283       |
| $R^D_T$           | 0.29                 | 0.274 - 0.36        |

respectively. The input values of masses of $B_{u,c}^{(*)}$ mesons are taken from [19] and the decay constants of $B_{u,c}^{(*)}$ mesons are $f_{B^*}/f_B = 0.941(26)$ [53], $f_{B_c} = 489$ MeV [54] and $f_{B_c^*}/f_{B_c} = 1$ [52]. The branching ratios of $B_c \to l\nu_l$ processes in the SM are

\begin{align}
\text{Br}(B_c^+ \to e^+\nu_e)|_{\text{SM}} &= (2.94 \pm 0.12) \times 10^{-9}, \\
\text{Br}(B_c^+ \to \mu^+\nu_\mu)|_{\text{SM}} &= (1.26 \pm 0.05) \times 10^{-4}, \\
\text{Br}(B_c^+ \to \tau^+\nu_\tau)|_{\text{SM}} &= (3.6 \pm 0.14) \times 10^{-2}.
\end{align}
TABLE VI: The predicted values of $R_{D(\ast)}$ and $R_{K(\ast)}$ in the vector leptoquark model.

| Observables | SM Predictions | Values in LQ Model | Experimental Limit |
|-------------|----------------|-------------------|--------------------|
| $R_D$       | 0.31           | 0.11 - 0.386      | 0.397 ± 0.040 ± 0.028 |
| $R_{D*}$    | 0.26           | 0.243 - 0.32      | 0.316 ± 0.016 ± 0.010 |
| $R_{K^{\mu\nu}_{q^2\in[1,6]}}$ | 1.006       | 0.75 - 1.006      | 0.745$^{+0.090}_{-0.074}$ ± 0.036 |
| $R_{K^{\mu\nu}_{q^2\geq14.18}}$ | 1.004       | 0.74 - 1.004      | ... |
| $R_{K^{\ast}_{q^2\in[1,6]}}$ | 0.996       | 0.725 - 0.996     | ... |
| $R_{K^{\ast}_{q^2\geq14.18}}$ | 0.999       | 0.816 - 0.999     | ... |

The decay width of $B_{u,c}^* \rightarrow l^+\nu_l$ processes in the SM are

$$\Gamma(B_u^* \rightarrow l\nu_l) = (2.98 \pm 0.12) \times 10^{-16} \text{ GeV}, \quad (56)$$

$$\Gamma(B_c^* \rightarrow l\nu_l) = (3.9 \pm 0.16) \times 10^{-13} \text{ GeV}. \quad (57)$$

The decay width of $B_{u,c}^* \rightarrow l\nu_l$ processes are independent of the mass of the final leptons, hence same for all generation in the SM. In order to calculate the branching ratios we need the values of lifetime or the total decay width of $B_{u,c}^*$ mesons. We have taken the decay width of $B_{u,c}^*$ meson as $\Gamma_{B_u^*} = 0.50(25)$ KeV and $\Gamma_{B_c^*} = 0.03(7)$ KeV respectively, which are computed in Ref. [52]. The predicted branching ratios in the vector LQ model are presented in Table VII. The branching ratios of $B_u^{\ast+} \rightarrow l^+\nu_l$ processes are of the order of $\sim 10^{-5}$, which are not very suppressed, and they could be observed in the LHCb experiment. However, the branching ratios of $B_{u,c}^* \rightarrow l^+\nu_l$ are found to be rather small. We do not find much deviation from the SM in the $B_{u,c}^* \rightarrow l\nu$ processes in the LQ model.

VI. CONCLUSION

In this work we considered the vector leptoquark model to explain the anomalies observed in semileptonic $\bar{B} \rightarrow D^{(*)}\tau\bar{\nu}$ decay process in light of recent $B$-factories result, especially the deviation of $R_{D(\ast)}$ observables from the SM predictions. There are two relevant vector leptoquark ($U_1, U_3$) states which conserve baryon and lepton numbers and can simultaneously explain the processes mediated by quark level transitions $b \rightarrow c l\nu_l$ and $b \rightarrow s l^+ l^-$. We constrained the leptoquark couplings by using the branching ratios of $B_s \rightarrow l^+ l^-$, $\bar{B} \rightarrow X_s l^+ l^-$,
TABLE VII: The predicted values of branching ratios of $B^{*+} \rightarrow l^+\nu$ processes in the leptoquark model.

| Observables | SM predictions | Values in LQ Model |
|-------------|----------------|--------------------|
| $\text{Br}(B^*_u \rightarrow e\nu)$ | $(5.97 \pm 0.24) \times 10^{-10}$ | $(0.94 - 1.01) \times 10^{-6}$ |
| $\text{Br}(B^*_u \rightarrow \mu\nu)$ | $(5.97 \pm 0.24) \times 10^{-10}$ | $(3.67 - 6.793) \times 10^{-10}$ |
| $\text{Br}(B^*_u \rightarrow \tau\nu)$ | $(5.97 \pm 0.24) \times 10^{-10}$ | $(4.2 - 3.67) \times 10^{-10}$ |
| $\text{Br}(B^*_c \rightarrow e\nu)$ | $(1.3 \pm 0.052) \times 10^{-5}$ | $(1.27 - 1.34) \times 10^{-5}$ |
| $\text{Br}(B^*_c \rightarrow \mu\nu)$ | $(1.3 \pm 0.052) \times 10^{-5}$ | $(1.26 - 1.34) \times 10^{-5}$ |
| $\text{Br}(B^*_c \rightarrow \tau\nu)$ | $(1.3 \pm 0.052) \times 10^{-5}$ | $(1.26 - 1.58) \times 10^{-5}$ |

$\bar{B} \rightarrow X_s\nu\bar{\nu}$ and $B^+_u \rightarrow l^+\nu_l$ processes, where $l$ is any charged lepton. We estimated the branching ratios, forward backward asymmetries, lepton non-universality, $\tau$ and $D^*$ polarization parameters in the $\bar{B} \rightarrow D^{(*)}l\bar{\nu}_l$ processes. We looked into the lepton non-universality parameters in both $\bar{B} \rightarrow D^{(*)}l\bar{\nu}_l$ and $\bar{B} \rightarrow K^{(*)}l^+l^-$ processes and found that both the $R_{K^{(*)}}$ and $R_{D^{(*)}}$ anomalies could be explained by $U_{1,3}$ vector leptoquarks. We also studied the rare $B^{*+}_{u,c} \rightarrow l\nu$ decay processes of $B^{*+}_{u,c}$ vector mesons. The branching ratios of the decay modes $B^*_c \rightarrow l\nu$ are not very suppressed, i.e., $\mathcal{O}(10^{-5})$, which could be observed in the LHCb experiment.

**Appendix A: $B \rightarrow D\tau\bar{\nu}_l$ form factors**

The nonzero hadronic amplitudes for $B \rightarrow D\tau\bar{\nu}_l$ process are

$$H_{s,0}^s(q^2) \equiv H_{V_{1,0}}^s(q^2) \equiv H_{V_{2,0}}^s(q^2) = \sqrt{\frac{\lambda_D(q^2)}{q^2}} F_1(q^2),$$

$$H_{s,t}^s(q^2) \equiv H_{V_{1,t}}^s(q^2) \equiv H_{V_{2,t}}^s(q^2) = \frac{M_B^2 - M_D^2}{\sqrt{q^2}} F_0(q^2),$$

$$H_{S}^s(q^2) \equiv H_{S_{1}}^s(q^2) = H_{S_{2}}^s(q^2) \simeq \frac{M_B^2 - M_D^2}{m_b - m_c} F_0(q^2),$$

(A1)
where the form factors $F_{0,1}$ are defined as

$$F_1(q^2) = \frac{1}{2\sqrt{M_B M_D}} \left[ (M_B + M_D) h_+ (\omega(q^2)) - (M_B - M_D) h_- (\omega(q^2)) \right],$$

$$F_0(q^2) = \frac{1}{2\sqrt{M_B M_D}} \left[ \frac{(M_B + M_D)^2 - q^2}{M_B + M_D} h_+ (\omega(q^2)) - \frac{(M_B - M_D)^2 - q^2}{M_B - M_D} h_- (\omega(q^2)) \right].$$

Here $h_{\pm} (\omega(q^2))$ are the HQET form factors taken from the Ref. [10, 55].

**Appendix B: $B \to D^* \bar{l} \nu$ form factors**

The hadronic amplitude for $B \to D^* \bar{l} \nu$ process are

$$H_{V,\pm}(q^2) \equiv H_{V_1,\pm}(q^2) = -H_{V_2,\pm}(q^2) = (M_B + M_{D^*}) A_1(q^2) \pm \frac{\sqrt{\lambda_{D^*}(q^2)}}{M_B + M_{D^*}} V(q^2),$$

$$H_{V,0}(q^2) \equiv H_{V_1,0}(q^2) = -H_{V_2,0}(q^2) = \frac{M_B + M_{D^*}}{2M_{D^*}\sqrt{q^2}} \left[ - (M_B^2 - M_{D^*}^2 - q^2) A_1(q^2) + \frac{\lambda_{D^*}(q^2)}{(M_B + M_{D^*})^2} A_2(q^2) \right],$$

$$H_{S}(q^2) \equiv H_{S_1}(q^2) = -H_{S_2}(q^2) \simeq -\frac{\sqrt{\lambda_{D^*}(q^2)}}{m_b + m_c} A_0(q^2),$$

where the form factors are defined as

$$V(q^2) = \frac{M_B + M_{D^*}}{2\sqrt{M_B M_{D^*}}} h_V (\omega(q^2)),$$

$$A_1(q^2) = \frac{(M_B + M_{D^*})^2 - q^2}{2\sqrt{M_B M_{D^*}}(M_B + M_{D^*})} h_{A_1} (\omega(q^2)),$$

$$A_2(q^2) = \frac{M_B + M_{D^*}}{2\sqrt{M_B M_{D^*}}} \left[ h_{A_3} (\omega(q^2)) + \frac{M_{D^*}}{M_B} h_{A_2} (\omega(q^2)) \right],$$

$$A_0(q^2) = \frac{1}{2\sqrt{M_B M_{D^*}}} \left[ \frac{(M_B + M_{D^*})^2 - q^2}{2M_{D^*}} h_{A_1} (\omega(q^2)) - \frac{M_B^2 - M_{D^*}^2 + q^2}{2M_B} h_{A_2} (\omega(q^2)) - \frac{M_B^2 - M_{D^*}^2 - q^2}{2M_{D^*}} h_{A_3} (\omega(q^2)) \right].$$

The complete expression for HQET form factors $h_i, i = V, A_{1,2,3}$ are given in [10, 55].
Appendix C: $\tau$ and $D^*$ polarizations

For a fixed polarization of $\tau$, the decay distribution of $B \to D\tau\bar{\nu}_l$ process with respect to $q^2$ are given as

$$
\frac{d\Gamma^{\lambda_{\tau}=1/2}_{\tau} (B \to D\tau\bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 M_B^3} q^2 \sqrt{\lambda_D(q^2)} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \times
\left[ \frac{1}{2} \delta_{l\tau} + C_{V_1}^l \frac{m_\tau^2}{q^2} (H_{V,0}^2 + 3H_{V,t}^2) + \frac{3}{2} C_{S_1}^l H_S^2 \right]
+ 3\text{Re} \left[ (\delta_{l\tau} + C_{V_1}^l) C_{S_1}^{l*} \frac{m_\tau}{\sqrt{q^2}} H_S H_{V,t} \right],
$$

(C1)

and for $B \to D^*\tau\bar{\nu}_l$ process

$$
\frac{d\Gamma^{\lambda_{\tau}=-1/2}_{\tau} (B \to D\tau\bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 M_B^3} q^2 \sqrt{\lambda_{D^*}(q^2)} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \times
\left[ \frac{1}{2} \delta_{l\tau} + C_{V_1}^l \frac{m_\tau^2}{q^2} (H_{V,+}^2 + H_{V,-}^2 + H_{V,0}^2 + 3H_{V,t}^2) \right]
+ 3\text{Re} \left[ (\delta_{l\tau} + C_{V_1}^l) C_{S_1}^{l*} \frac{m_\tau}{\sqrt{q^2}} H_S H_{V,t} \right],
$$

(C2)

The $q^2$ distributions of $B \to D^*\tau\bar{\nu}_l$ process for a given polarization of $D^*$ are given as

$$
\frac{d\Gamma^{\lambda_{D^*}=\pm 1}_{\tau} (B \to D\tau\bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 M_B^3} q^2 \sqrt{\lambda_{D^*}(q^2)} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \times
\left[ \delta_{l\tau} + C_{V_1}^l \frac{m_\tau^2}{2q^2} \right] H_{V,\pm}^2,
$$

(C3)

and

$$
\frac{d\Gamma^{\lambda_{D^*}=0}_{\tau} (B \to D\tau\bar{\nu}_l)}{dq^2} = \frac{G_F^2 |V_{cb}|^2}{192\pi^3 M_B^3} q^2 \sqrt{\lambda_{D^*}(q^2)} \left(1 - \frac{m_\tau^2}{q^2}\right)^2 \times
\left[ \frac{1}{2} \delta_{l\tau} + C_{V_1}^l \frac{m_\tau^2}{2q^2} (H_{V,0}^2 + \frac{3}{2} m_\tau^2 H_{V,t}^2) \right]
+ 3\text{Re} \left[ (\delta_{l\tau} + C_{V_1}^l) C_{S_1}^{l*} \frac{m_\tau}{\sqrt{q^2}} H_S H_{V,t} \right].
$$

(C4)
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