LOW-SURFACE-DENSITY GALAXIES AND THE MODIFIED DYNAMICS

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ABSTRACT

Very-low-surface-density galactic systems have very low mean accelerations. They thus provide quintessential tests of the Modified Dynamics (MOND), which predicts an increasing mass discrepancy with decreasing acceleration. We describe succinctly the results pertinent to several classes of such objects: Low-luminosity (dwarf) spirals, irregular dwarf spirals, normal-luminosity-but-low-surface-density spirals, and dwarf-spheroidal satellites of the Milky Way.

1. The modified dynamics

As some of you may know, I have been advocating, with others, that there is not much dark matter in galactic systems. The mass discrepancy observed in galaxies is then due to a breakdown of Newtonian dynamics, which is used to determine the gravitational masses. The specific alternative proposed\cite{8,9}, called MOND, assumes that Newtonian dynamics (law of inertia and/or gravity) break down when the acceleration of a test particle in a system is much smaller than some borderline acceleration $a_0$. The Newtonian acceleration $g_N = GM/r^2$ that an attracting mass $M$ produces on a test particle, a distance $r$ away from it, is assumed to be valid only in the limit $g_N \gg a_0$. In the opposite limit, $g_N \ll a_0$, the test-particle acceleration, $g$, is given by $g^2/a_0 \approx g_N = MG/r^2$. This basic idea may be interpreted as either a modification of Newtonian gravity, or a modification of the law of inertia\cite{8}, and can be incorporated into Lagrangian theories in both the former\cite{3} and the latter\cite{10} interpretation. There have been several attempts to develop relativistic extensions for the modified-gravity approach (see e.g. refs.\cite{2,3,14}), but none of these is without problems.

The salient ramifications of MOND are captured by the simplistic formulation that relate the
acceleration $\vec{g}$ to the acceleration, $\vec{g}_N$, calculated with Newtonian gravity, by

$$\mu(g/a_0)\vec{g} = \vec{g}_N,$$

(1)

where $g \equiv |\vec{g}|$, and $\mu(x)$ is some extrapolating function whose limiting behaviour at the two extreme values of its argument are given by: $\mu(x \gg 1) \approx 1$, to recover Newtonian dynamics in this limit, and $\mu(x \ll 1) \approx x$. Otherwise, $\mu$ remains unspecified. The implications for galaxy dynamics do not depend critically on the exact form of $\mu$, as long as it is assumed to be increasing. Accelerations in galactic systems are never much larger than $a_0$ (see below) so $\mu(x)$ has to be known only up to $x$ of a few, in this context. In contrast, aspects such as solar-system tests of the theory, which probe the region $g \gg a_0$, depend critically on just how fast $\mu(x)$ approaches 1 at large $x$. For instance, the two choices $\mu(x) = 1 - e^{-x}$, and $\mu(x) = x/(1 + x)$ make very similar predictions for galactic dynamics, but very different ones for, say, the perihelion shift of planetary orbits: The former predicts a totally negligible effect, while the latter produces an effect that is already in conflict with the measurements [8].

The main predictions of MOND regarding galaxies are [9]:

1. The orbital velocity on a circular orbit far from a finite mass is independent of the orbital radius. This leads to asymptotically flat rotation curves of disk galaxies.
2. The asymptotic velocity $V_\infty$, depends only on the total mass, $M$, of the system (galaxy): $V_\infty^4 = MGa_0$. This gives the Tully-Fisher relation.
3. The mean velocity dispersion, $\sigma$, of a self-gravitating system supported by random motions is strongly correlated with the total mass: $\sigma^4 \sim MGa_0$. This leads to the Faber-Jackson relation for elliptical galaxies.
4. Thin galactic disks are more stable when their mean surface density, $\Sigma$, satisfies $\Sigma < \Sigma_0 \equiv a_0 G^{-1}$ (i.e. their mean acceleration is much smaller than $a_0$). This explains the marked paucity of galaxies with surface density above some cutoff value, known as the Freeman law.
5. Isothermal spheres do not exist that have a mean surface density much exceeding $\Sigma_0$. This accounts for the observed analogue of the Freeman law for elliptical galaxies, known as the Fish law.
6. The rotation curve calculated for a galaxy using MOND, and assuming the presence of only the visible matter, should agree with the observed rotation curves. This most detailed prediction was tested repeatedly (see e.g. [1]).

The constant $a_0$ appears in predictions 2-6 above, and can thus be determined (in several independent ways) by comparing the predictions with the data. All these methods yield $a_0 \sim (1 - 2) \times 10^{-8} cm sec^{-2}$.

It may be most significant that $a_0$ turns out to be of the same order as $cH_0$ ($H_0$ being the Hubble constant). This may betoken some connection of MOND with cosmology, in the spirit of Mach’s principle (see more details in [10]).

The mass discrepancy in clusters of galaxies (at radii of a few Mpc) is well accounted for by MOND (e.g. ref. [13]), and so is the discrepancy in small galaxy groups.

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The only place where MOND fails systematically to explain away dynamical dark matter is in the cores (up to a few hundred kpc) of rich, x-ray galaxy clusters (e.g. ref.[4]). Even with MOND, most of the mass required in these cores by x-ray gas hydrostatics, and by lensing, must be genuine, yet-undetected dark matter. Cooling flows are known to carry cool gas into these cores whose fate is not clear, and which have so far escaped detection. While the estimated, present-day mass deposition rates are too small to account for the total dark mass required in the cores, they could have been much larger in the past.

If, in fact, the mass discrepancy bespeaks the presence of DM and not of new physics, then MOND is, in the least, a very economical description of the mass distribution, and tells us that the amount and distribution of DM in galaxies uncannily follows a very strict rule involving only one parameter ($a_0$), and is fully determined by the distribution of visible matter. This is quite hard to believe.

2. Low-surface-density galaxies

MOND was introduced to account for the behaviour of the rotation curves of "normal" disc galaxies at large radii, where the accelerations become very small. The asymptotics of rotation curves actually determine the essential phenomenology of MOND: The linear form of $\mu(x)$ for small $x$ is dictated by the asymptotic flatness of rotation curves. The value of $a_0$ is fixed by the intercept of the Tully-Fisher relation. However, small accelerations are found in the realm of the galaxies not only in the outskirts of galaxies. There are systems in which the accelerations are very small everywhere from the centre out. These are the low-surface-density (LSD) galaxies, which are particularly crucial in testing the modified dynamics. This has to do with the fact that the mean surface density of a galaxy, $\Sigma = M/\pi R^2$, is a direct measure of its mean (Newtonian) acceleration $GM/R^2$. Defining thus $\Sigma_0 \equiv G^{-1}a_0$, we see that systems with a mean surface density $\Sigma \ll \Sigma_0$ are deep in the MOND regime. In predicting their behaviour no leeway is left in adjusting the theory. Such systems afford particularly sharp tests of MOND because a. The expected mass discrepancy is large. b. The dynamics is practically independent of the assumed form of $\mu(x)$ because in the relevant, $x \ll 1$, regime we have $\mu \approx x$. c. The shape of the rotation curves predicted by MOND for an LSD galaxy is independent of various galaxy parameters that are not always known with good accuracy: The distance to the galaxy, its stellar $M/L$, its inclination, as well as the value of $a_0$ all enter together only in the normalization of the predicted curve. d. Many LSD galaxies are dominated by gas mass (relative to stellar mass) and hence their analysis depends rather weakly on the assumed stellar $M/L$ values, yielding almost parameter-free MOND predictions.

Various galaxy types fall in the class of LSD galaxies: 1. dwarf spirals 2. dwarf irregulars (described e.g. in [3]). One obtains 3. normal-, or high-luminosity spirals with low surface brightness (see e.g. ref.[7]). 4. The dwarf spheroidal satellites of the Milky Way with stellar $\Sigma$ down to a few
percent of $\Sigma_0$ (see for a review).

3. MOND predictions and observations of LSD galaxies

3.1 LSD, Dwarf Spirals

There are now quite a few dwarf spirals for which the rotation curves, as well as the (stellar and gaseous) mass distributions have been measured. They afford very acute tests of MOND for the reasons explained above. They were predicted to show a large mass discrepancy right from the center of the galaxy, long before any of the above data was available. Rotation-curve analysis of these show very good agreement with the RCs predicted by MOND. Some examples of these are given e.g. in ref. [1].

3.2 Dwarf-Irregular Spirals

Milgrom [11] has analyzed the data of ref. [5] using a generalized MOND virial relation that relates the total mass to the rms velocity dispersion in LSD systems. He found that the masses predicted by MOND from the observed dispersions agree with the observed masses (gas plus stars with reasonable $M/L$ values of order one solar unit). In contrast, the Newtonian $M/L$ values found in ref. [5] range between 7 and 26.

3.3 LSD, Normal-Luminosity Spirals

McGaugh et al. (ref. [7]) have analyzed some twenty-five galaxies spanning a large range of surface densities, most of which having low surface densities in the sense we discuss here. They find a strong correlation between the Newtonian $M/L$ value and their mean surface density, in just the way predicted by MOND: The-lower-surface-density galaxies have higher Newtonian $M/L$ values (up to a few tens solar units). Their MOND analysis of the same sample gives a mean MOND $M/L$ value of order unity across the full surface-density range.

3.4 Dwarf-Spheroidal Satellites of the Milky Way

As was predicted by MOND [9] these are now known to evince large mass discrepancies when analyzed by Newtonian dynamics (see e.g. ref. [1], and references therein). In a recent MOND analysis, using updated velocity dispersions for some of the dwarfs, it was found [12] that the dynamics is explained with $M/L$ values typical of globular-cluster stellar populations, i.e. with no need for dark matter. The results are summarized in Table 1, together with the estimated Newtonain $M/L$ values.

Acknowledgement I thank Stacy McGaugh for permission to quote from the results of ref. [7] before their publication.
Dwarf galaxy                  | Sculptor | Sextans | Carina | Draco | LeoII | U Minor | Fornax |
--------------------------------|----------|---------|--------|-------|-------|---------|--------|
Newtonian $M/L$                | $\sim 12$ | $\sim 18$ | 16–62 | 50–120 | 7–15 | 35–100 | 5–26 |
MOND $M/L$                     | 0.7–2     | 0.7–3   | 1.5–7  | 3–6   | 0.7–4 | 2.5–6.5 | 0.1–1.4 |

Table 1: The estimated Newtonian and MOND $M/L$ ranges for the seven dwarf spheroidals with measured velocity dispersions

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