Constraints on non-universal soft terms from flavor changing neutral currents

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Abstract

The smallness of flavor changing neutral currents constrains the soft parameter space of supersymmetric extensions of the Standard Model. These low energy constraints are translated to the soft parameter space generated at some high energy scale $M_{\text{GUT}}$. For gaugino masses larger than the scalar masses and non-universal $A$-terms the constraints are significantly diluted at $M_{\text{GUT}}$ and do allow for the possibility of non-universal scalar masses. The strongest constraints arise in the slepton sector of the theory.

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The successful prediction of the smallness of flavor changing neutral currents (FCNC) is one of the cornerstones of the Standard Model (SM). Most extensions of the SM contain new sources for FCNC which lead to a delicate test of any new physics above the weak scale $M_Z$. $N = 1$ supersymmetric versions of the SM appear to be promising candidates for such new physics and most supersymmetric models do contain additional contributions to FCNC via gaugino exchange in box and/or penguin diagrams [1, 2]. The measurements of neutral meson mixing and radiative decays [3, 4]

$$
\frac{\Delta m_K}{m_K} = 7 \times 10^{-15}, \quad \frac{\Delta m_B}{m_B} = 7 \times 10^{-14}, \quad \frac{\Delta m_D}{m_D} \leq 7 \times 10^{-14}, \quad BR(B \to X_s\gamma) \leq 5.4 \times 10^{-4}, \quad BR(\mu \to e\gamma) \leq 5 \times 10^{-11}, \quad (1)
$$

impose severe constraints on the (off-diagonal elements of the) mass matrix of the squarks and sleptons [1, 2].

The masses of the squarks and sleptons arise as a consequence of supersymmetry and electroweak symmetry breaking. At present soft breaking of supersymmetry seems to be the most attractive mechanism for generating a phenomenologically acceptable scalar mass spectrum. Such softly broken supersymmetric theories appear rather naturally in the flat limit of spontaneously broken supergravity models where a hidden sector induces the breakdown and gravitational interactions communicate the breaking to the observable sector. This mechanism induces soft terms in the observable sector (which contains the quark and lepton supermultiplets) at some high energy scale $M_{GUT}$. However, the constraints (1) hold at the weak scale $M_Z$ and cannot be directly applied to the soft parameter space at high energies. Instead, renormalization corrections of the scalar masses have to be taken into account. As a consequence, the scalar masses at low energies can be (significantly) different from the soft input parameters generated at high energies ($M_{GUT}$) if large quantum corrections are present. The prime example of this phenomenon is the supersymmetric Coleman-Weinberg mechanism where renormalization effects turn a Higgs mass parameter negative and radiatively induce the electroweak symmetry breakdown [7].

In the simplest supergravity models [8] the soft terms are universal at $M_{GUT}$: all scalar masses $m^2_{ij}$ are determined by the gravitino mass $m_{3/2}$ ($m^2_{ij} = \delta_{ij} m^2_{3/2}$) and the
trilinear scalar couplings $A_{ij}$ are proportional to the corresponding Yukawa couplings $Y_{ij}$ ($A_{ij} = A m_{3/2} Y_{ij}$). Renormalization corrections to the scalar masses indeed induce some small non-universality at $M_Z$ but nevertheless the current bounds (1) are satisfied \[9,10,11\]. In more generic models (particularly in many string models \[10,11,12\]) non-universal soft terms do arise at $M_{\text{GUT}}$ \[13,14,15\] and from the point of view of (string-) model building it is of interest to analyze to what extent such non-universality can be tolerated without violating the low energy data (1). In other words one would like to translate the bounds (1) into constraints on the soft parameter space generated at $M_{\text{GUT}}$. We find two possible sources which can significantly dilute the bounds. On the one hand, large gaugino masses enhance the diagonal or ‘average’ squark masses $M_{\text{av}}$ \[14,12\] whereas non-universal $A$-terms can decrease the off-diagonal mass matrix elements. Both effects together lead to a dilution of the constraints at $M_{\text{GUT}}$ and allow for non-universal soft terms at the high energy scale.† It is the purpose of this letter to make these statements more quantitative.

2. In supersymmetric extensions of the SM all chiral fermions are promoted to chiral $N=1$ supermultiplets and one additional Higgs doublet is introduced. Let us assume that supersymmetry is broken by generic soft terms at some high energy scale $M_{\text{GUT}}$

$$-\mathcal{L}_{\text{soft}} = m_{3/2} \left( A_{ij}^u \bar{\tilde{u}}_R \tilde{q}_L^i h_u + A_{ij}^d \bar{\tilde{d}}_R \tilde{q}_L^j h_d + A_{ij}^l \bar{\tilde{e}}_R \tilde{l}_L^j h_d + \text{h.c.} \right) + (m_u^2)_{ij} \tilde{q}_L^i \tilde{q}_L^j + (m_d^2)_{ij} \tilde{u}_R^i \tilde{u}_R^j + (m_d^2)_{ij} \tilde{d}_R^i \tilde{d}_R^j + (m_l^2)_{ij} \tilde{l}_L^i \tilde{l}_L^j + (m_e^2)_{ij} \tilde{e}_R^i \tilde{e}_R^j$$

(2)

where $i, j$ are summed over 1, 2, 3. $\tilde{q}_L$ ($\bar{\tilde{u}}_R, \bar{\tilde{d}}_R$) denotes the left- (right-) handed squarks, $\tilde{l}_L$ ($\bar{\tilde{e}}_R$) the left- (right-) handed sleptons and $h_u, h_d$ the two Higgs doublets. $\tilde{m}_a$ are the three gaugino masses of $SU(3), SU(2), U(1)$ respectively and for simplicity we take them to be universal (at $M_{\text{GUT}}$) $\tilde{m}_1 = \tilde{m}_2 = \tilde{m}_3 = \tilde{m}_{1/2}$.‡

† Other aspects of non-universal scalar mass terms have recently been discussed in refs. \[15,16,17\].

* An extended version of this letter can be found in refs. \[18,19\].

‡ In general, the soft terms (2) introduce new (and dangerous) CP-violating phases \[20\]. We do not address the related phenomenological problems but instead assume that all possible sources of CP-violation are small. (The constraints on the soft parameter space usually become stronger when arbitrary phases of $O(1)$ are present.)

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The soft terms displayed in eq. (2) determine the scalar masses at the weak scale after their renormalization corrections (between \( M_{\text{GUT}} \) and \( M_Z \)) have been taken into account. These corrections are determined by the solutions of the appropriate (one-loop) renormalization group (RG) equations [21,22]. For the purpose of this paper, it is instructive to first work in an approximation where only the gauge couplings and \( A \)-terms are kept while all Yukawa couplings are neglected in the RG-equations. This simplifies the discussion considerably and clearly shows the physical effects involved. The ‘zero Yukawa limit’ is a very good approximation except for the top-quark Yukawa coupling \( Y_t \) which might well be of order \( O(1) \). We postpone the discussion of its effects on our analysis to the end of this letter.

In the approximation where all Yukawa couplings are set to zero, the RG equations simplify as follows\(^5\)

\[
\begin{align*}
\partial_t \tilde{m}_a &= -\frac{1}{4\pi} b_a \alpha_a \tilde{m}_a, \quad b_a = (11, 1, -3), \\
\partial_t (m^2_{q,ij}) &= \frac{\delta_{ij}}{4\pi} \left( \frac{16}{9} \alpha_3 \tilde{m}_3^2 + 3 \alpha_2 \tilde{m}_2^2 + \frac{3}{2} Y_1 \tilde{m}_1^2 \right) - \frac{1}{16\pi^2} m^2_{3/2} ((A^u)^2_{ij} + (A^d)^2_{ij}), \\
\partial_t (m^2_{u,ij}) &= \frac{\delta_{ij}}{4\pi} \left( \frac{16}{9} \alpha_3 \tilde{m}_3^2 + \frac{10}{9} \alpha_1 \tilde{m}_1^2 \right) - \frac{1}{8\pi^2} m^2_{3/2} (A^u)^2_{ij}, \\
\partial_t (m^2_{d,ij}) &= \frac{\delta_{ij}}{4\pi} \left( \frac{16}{9} \alpha_3 \tilde{m}_3^2 + \frac{4}{9} \alpha_1 \tilde{m}_1^2 \right) - \frac{1}{8\pi^2} m^2_{3/2} (A^d)^2_{ij}, \\
\partial_t (m^2_{e,ij}) &= \frac{\delta_{ij}}{4\pi} \left( 4 \alpha_1 \tilde{m}_1^2 \right) - \frac{1}{8\pi^2} m^2_{3/2} (A^l)^2_{ij}, \\
\partial_t A_{ij}^u &= \frac{1}{4\pi} \left( \frac{s}{3} \alpha_3 + \frac{3}{2} \alpha_2 + \frac{13}{18} \alpha_1 \right) A_{ij}^u, \\
\partial_t A_{ij}^d &= \frac{1}{4\pi} \left( \frac{s}{3} \alpha_3 + \frac{3}{2} \alpha_2 + \frac{7}{18} \alpha_1 \right) A_{ij}^d, \\
\partial_t A_{ij}^l &= \frac{1}{4\pi} \left( \frac{3}{2} \alpha_2 + \frac{3}{2} \alpha_1 \right) A_{ij}^l,
\end{align*}
\]

where \( t = 2 \ln(M_{\text{GUT}}/Q) \). The solutions of eqs. (3) determine the mass parameters at low energies (\( Q = M_Z \)) in terms of their boundary values at the high energy scale \( M_{\text{GUT}} \). The important point in eqs. (3) is the different renormalization of the diagonal and off-diagonal mass terms. The diagonal matrix elements become larger at low energies if the gauginos are sufficiently heavy [14,12]. On the other hand the off-diagonal mass terms can renormalize down if off-diagonal \( A \)-terms are present. Thus their ratio can be significantly smaller at low energies than their boundary value at \( M_{\text{GUT}} \). This observation is the main physical effect we want to study.

\(^5\) For simplicity, we also assume \( A_{ij} \) to be symmetric in flavor space and neglect \( S = m_{h_u}^2 - m_{h_d}^2 + \text{Tr}(m_u^2 - m_t^2 - 2m_d^2 + m_s^2) \) whose coefficient in the RG-eq. is small.

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Additional contributions to the low energy masses arise from electroweak symmetry breaking and induce mass terms mixing left and right handed scalars. Altogether, the physical masses of the squarks and sleptons at $M_Z$ appear in three $3 \times 3$ mass matrices $M_{LL}^2(f), M_{RR}^2(f), M_{LR}^2(f)$ which together form the $6 \times 6$ mass matrix

$$ M^2(f) = \begin{pmatrix} M_{LL}^2(f) & M_{LR}^2(f) \\ M_{LR}^2(f) & M_{RR}^2(f) \end{pmatrix} . $$

There is one such matrix for each of the squarks (up and down) as well as the sleptons (i.e. $f = u, d, l$).

We have already observed that the RG-equations (3) are different for the diagonal and off-diagonal elements of the scalar mass matrix. Let us therefore denote the off-diagonal elements ($i \neq j$) at $M_Z$ by $\Delta M^2_{ij}$ while $\Delta m^2_{ij}$ indicates the off-diagonal terms at $M_{GUT}$. The solutions of eqs. (3) determine $\Delta M^2$ in terms of the high energy input parameters $\Delta m^2$ and $A$ as follows

\begin{align*}
(\Delta M^2_{LL}^{(u)})_{ij} &= (\Delta M^2_{LL}^{(d)})_{ij} = (\Delta M^2_{q})_{ij} - m^2_{3/2} (1.8 \ (A^u)^2_{ij} + 1.7 \ (A^d)^2_{ij}) , \\
(\Delta M^2_{RR}^{(u)})_{ij} &= (\Delta M^2_{RR}^{(d)})_{ij} = (\Delta M^2_{u})_{ij} - 3.6 \ m^2_{3/2} (A^u)^2_{ij} , \\
(\Delta M^2_{RR}^{(d)})_{ij} &= (\Delta M^2_{d})_{ij} - 3.4 \ m^2_{3/2} (A^d)^2_{ij} , \\
(\Delta M^2_{LL}^{(l)})_{ij} &= (\Delta M^2_{l})_{ij} - 0.7 \ m^2_{3/2} (A^l)^2_{ij} , \\
(\Delta M^2_{RR}^{(l)})_{ij} &= (\Delta M^2_{e})_{ij} - 1.4 \ m^2_{3/2} (A^l)^2_{ij} , \\
(\Delta M^2_{LR}^{(u)})_{ij} &= 3.7 \ m^2_{3/2} A^u_{ij} \langle h_u \rangle , \\
(\Delta M^2_{LR}^{(d)})_{ij} &= 3.6 \ m^2_{3/2} A^d_{ij} \langle h_d \rangle , \\
(\Delta M^2_{LR}^{(l)})_{ij} &= 1.5 \ m^2_{3/2} A^l_{ij} \langle h_d \rangle .
\end{align*}

(The numerical coefficients have been obtained for $\alpha_{GUT} = 1/24$, $M_{GUT} = 3.6 \times 10^{16} \ GeV, Q = 100 \ GeV$; $\langle h_d \rangle, \langle h_u \rangle$ denote the VEVs of the two Higgses and we use the typical values $\langle h_d \rangle = 60 \ GeV, \langle h_u \rangle = 150 \ GeV$ ($\tan \beta = 2.5$ throughout this paper.) We see that at low energies $\Delta M^2$ can be very small (even tuned to zero) if non-universal $A$-terms of order $O(1)$ are present at $M_{GUT}$.

The renormalization of the diagonal mass terms is conveniently discussed by defining (low energy) ‘average squark masses’ $M^2_{av}(f)$

$$ M^2_{av}(f) \equiv \frac{1}{6} (\text{Tr} \ M^2_{LL}(f) + \text{Tr} \ M^2_{RR}(f)) , $$

\begin{equation}
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\end{equation}
and the corresponding average masses at high energies

\[ m_{av}^2 (u) \equiv \frac{1}{6} (\text{Tr} m_q^2 + \text{Tr} m_u^2), \]
\[ m_{av}^2 (d) \equiv \frac{1}{6} (\text{Tr} m_q^2 + \text{Tr} m_d^2), \]
\[ m_{av}^2 (l) \equiv \frac{1}{6} (\text{Tr} m_l^2 + \text{Tr} m_e^2). \]

(7)

The solutions of eqs. (3) establish their connection

\[ M_{av}^2 (u) = m_{av}^2 (u) + 7 \tilde{m}_{1/2}^2 - m_{3/2}^2 (0.9 \text{Tr} (A^u)^2 + 0.3 \text{Tr} (A^d)^2) + O(M_Z^2), \]
\[ M_{av}^2 (d) = m_{av}^2 (d) + 7 \tilde{m}_{1/2}^2 - m_{3/2}^2 (0.3 \text{Tr} (A^u)^2 + 0.9 \text{Tr} (A^d)^2) + O(M_Z^2), \]
\[ M_{av}^2 (l) = m_{av}^2 (l) + 0.3 \tilde{m}_{1/2}^2 - 0.3 m_{3/2}^2 \text{Tr} (A^l)^2 + O(M_Z^2), \]

(8)

where the \( O(M_Z) \) contributions arise from electroweak symmetry breaking. Note that the \( A \)-terms as well as \( \tilde{m}_{1/2} \) appear in eqs. (5) and (8) due to renormalization effects. Eqs. (8) show the possible enhancement of \( M_{av}^{(u,d)} \) at low energies due to a large gaugino mass. However, this effect is much weaker in the slepton sector since here the renormalization is driven by \( \alpha_2 \) instead of \( \alpha_3 \) (see eqs. (3)).

Now we are prepared to discuss the constraints imposed by (1) on the low energy scalar mass matrices. Apart from the SM contribution there exist additional contributions to FCNC processes induced by gaugino exchange in box and penguin diagrams. We do not repeat the explicit computation here but instead just summarize (and update) the relevant results obtained in refs. [5, 6]. The calculation can be performed in two different physically equivalent sfermion bases. For our purpose it is most convenient to use a basis where the fermion masses are diagonal, the gaugino-sfermion-fermion couplings are diagonal and the sfermion masses are arbitrary.† The experimental bounds (1) constrain \( \Delta M^2 \) and are conveniently expressed in terms of

\[ \langle \delta^{(f)} \rangle_{ij} \equiv \sqrt{\delta_{LL}^{(f)} \delta_{RR}^{(f)}}, \]

(9)

where \( M, N \) each take the values \( L, R \). In table 1 we display the constraints on \( \delta \) implied by (1) (cf. [5,15]). The numerical values are obtained from eqs. (1) and the formulas given in ref. [5].† The relevant Feynman diagrams depend on the gaugino

\[ \star \text{ For non-zero Yukawa couplings this implies a CKM rotation on the scalar mass matrices in order to keep the gaugino-sfermion-fermion couplings diagonal.} \]
\[ \dagger \text{ We thank F. Gabbiani for his assistance with some aspects of ref. [5].} \]
masses (at $M_Z$) via the ratios $x^{(u,d)} = \tilde{m}_3^2(M_Z)/\tilde{m}_{\tilde{u},\tilde{d}}^2$ in the squark sector and $x^{(l)} = \tilde{m}_1^2(M_Z)/\tilde{m}_{\tilde{l}}^2$ in the slepton sector. However, the dependence on $x^{(f)}$ is rather weak and in table 1 the typical values $x^{(u)} = x^{(d)} = 1$, $x^{(l)} = 0.5$ have been used. In the squark sector large hadronic uncertainties enter the estimates and the values of table 1 are only correct up to factors of $O(1)$. The slepton sector is not plagued by such uncertainties; however, here the relevant Feynman diagrams depend on the diagonal $A$-terms as well as the supersymmetric $\mu$-parameter (see ref. [5]). For a reasonable range of parameters the constraints in the slepton sector can vary by an order of magnitude. In table 1 we have chosen an appropriate average value but similar to the squark sector the numerical numbers should only be trusted up to factors of $O(1)$. Furthermore, only gaugino exchange diagrams are taken into account whereas all other contributions such as the SM diagrams and their supersymmetric analogues are neglected. In most cases this is justified since they are subleading and the leading gaugino contributions are only known within an accuracy of $O(1)$. However, in the up-squark sector for $\Delta M^2_{13}^{(u)}$ and $\Delta M^2_{23}^{(u)}$ the electroweak contributions become the leading constraint. Evaluation of the relevant electroweak diagrams [19] shows that they are suppressed by at least an order of magnitude compared to the constraints arising from $B - \bar{B}$ and $D - \bar{D}$ mixing and therefore are neglected in the analysis below. (A more detailed discussion of their possible effects can be found in ref. [18].)

3. It is instructive to split the analysis into two separate cases. First we concentrate on the situation where all $A$-terms are small and (together with the Yukawa couplings) can be neglected in the RG-analysis.‡ From eqs. (5) we learn that for vanishing $A$-terms all constraints on $\Delta M^2_{LR}^{(f)}$ are automatically satisfied. Furthermore, the $\Delta M^2$'s do not renormalize and are directly determined by their boundary values at $M_{GUT}$ (see eqs. (5)). Thus, each entry of table 1 can be translated into a constraint on a three-dimensional soft parameter space (at $M_{GUT}$) spanned by $(\tilde{m}_{\tilde{u},\tilde{d}}, \Delta m^2)$. Let us assume $\Delta m^2 \simeq m_{\tilde{u},\tilde{d}}^2$ (all matrix elements are of the same order of magnitude) and study the strongest constraints in the squark and slepton sector which arise from $K - \bar{K}$ mixing and $\mu \to e\gamma$. From eqs. (9), (5) and (8) we learn

\[
\begin{aligned}
(\delta^{(d)}_{RR})_{12} &= (\delta^{(d)}_{LL})_{12} = \frac{1}{1 + 7 x_0^{(d)}}, &
(\delta^{(l)}_{RR})_{12} &= (\delta^{(l)}_{LL})_{12} = \frac{1}{1 + 0.3 x_0^{(l)}},
\end{aligned}
\]

where $x_0^{(f)} \equiv \frac{\tilde{m}_{1/2}^2}{m_{\tilde{u},\tilde{d}}^2}$.

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* We will see that the values chosen for $x^{(f)}$ are also phenomenologically sensible.
‡ This essentially repeats and extends the analysis of ref. [14].
In fig. 1 we display the constraints on the ratio $\sqrt{x_0}$ as a function of the (physical) low energy average squark (slepton) mass $M_{\text{av}}$.\footnote{Both constraints are shown in the same plot for better comparison, strictly speaking there is a different $(M_{\text{av}}^{(d)}, x_{0}^{(d)})$ and $(M_{\text{av}}^{(l)}, x_{0}^{(l)})$ in each case.} We see that the constraints can be satisfied if the gaugino mass $\tilde{m}_{1/2}$ is significantly larger than $m_{\text{av}}$. By far the strongest constraint arises in the slepton sector from $\mu \to e\gamma$ and for small $M_{\text{av}}$ a rather large hierarchy is required ($\sqrt{x_0} \approx 30$). For the squarks this ratio can be lower ($\sqrt{x_0} \approx 10$) due to the large renormalization effect induced by $\tilde{m}_{1/2}$ (eq. (8)).\footnote{Note that with such a big hierarchy $M_{\text{av}}$ is almost entirely determined by $\tilde{m}_{1/2}$ and $m_{\text{av}} \approx 0$ for small $M_{\text{av}}$.}

For higher values of $M_{\text{av}}$ the slepton constraint falls off faster than the squark constraint due to the different scaling behavior of the penguin versus the box diagram (see table 1) and for large $M_{\text{av}}$ (1 TeV) one needs $\sqrt{x_0} \approx 6$ in both sectors. However, we should stress that fig. 1 should only be trusted up to factors of $O(1)$ due to the uncertainties implicit in table 1.

The ratio of the gaugino masses to the squark (slepton) masses at low energies does not require any hierarchy. The gaugino masses renormalize according to

\[
\begin{align*}
x^{(d)} &\equiv \frac{\tilde{m}_3^2(M_Z)}{M_{\text{av}}^{2(d)}} \approx \frac{9 x_0^{(d)}}{1 + 7 x_0^{(d)}} \rightarrow \frac{9}{7}, \\
x^{(l)} &\equiv \frac{\tilde{m}_1^2(M_Z)}{M_{\text{av}}^{2(l)}} \approx \frac{0.16 x_0^{(l)}}{1 + 0.3 x_0^{(l)}} \rightarrow 0.53. 
\end{align*}
\]

Thus $x^{(d)}$ only takes values in the interval $[0, \frac{9}{7}]$ and for large $\tilde{m}_{1/2}$ (large $x_0$) approaches $\frac{9}{7}$. Similarly, $x^{(l)}$ takes values in $[0, 0.53)$ and hence the photino is always lighter than the sleptons at $M_Z$. The fact that the ratios $x^{(f)}$ at low energies approach a fixed point for large $x_0$ is another reason for choosing $x^{(d)} = 1$, $x^{(l)} = \frac{1}{2}$ in table 1.

So far we have observed that in order to satisfy the constraints (1) a large hierarchy (at $M_{\text{GUT}}$) between $\tilde{m}_{1/2}$ and $m_{\text{av}}$ is required. However, this has been obtained under the assumption that all matrix elements of the scalar mass matrix are of the same order of magnitude. From eqs. (10) and table 1 it follows that an ‘inverse’ hierarchy $x_0 \ll 1$ requires

\[
\begin{align*}
\frac{\Delta m_{12}^2}{m_{\text{av}}^{2(d)}} &\approx 2 \cdot 10^{-2} \left( \frac{M_{\text{av}}^{(d)}}{1 \text{ TeV}} \right), \\
\frac{\Delta m_{12}^2}{m_{\text{av}}^{2(l)}} &\approx 0.3 \left( \frac{M_{\text{av}}^{(l)}}{1 \text{ TeV}} \right)^2.
\end{align*}
\]
i.e. some degree of universality of the scalar masses.

4. Let us turn to the case where also non-universal $A$-terms are kept in the RG-evolution. From eqs. (5) we learn that the constraints on $\Delta M_{LR}^2(f)$ directly translate into constraints on the off-diagonal $A$-terms. Table 1 implies that $K - \overline{K}$ mixing and $\mu \rightarrow e\gamma$ constrain $A_{12}^{(f)}$ and $A_{11}^{(f)}$ severely whereas the constraint on $A_{12}^{(f)}$ is significantly weaker and the diagonal terms $A_{11}^{(f)}, A_{22}^{(f)}$ are unconstrained by the bounds (1). To simplify the analysis we impose $A_{12}^{(f)} \simeq A_{11}^{(f)} \simeq A_{22}^{(f)} \simeq 0$ and choose the remaining $A$-terms in each sector to be identical, i.e. the $A$-matrices have the ‘texture’

$$A^{(f)} = \begin{pmatrix} 0 & 0 & A^f \\ 0 & 0 & A^f \\ A^f & A^f & A^f \end{pmatrix}. \tag{13}$$

As before, we take the off-diagonal elements of the scalar mass matrices to be of the same order of magnitude $\Delta m^2 = m_{\text{av}}^2 = m_{3/2}^2$ and hence the soft parameter space is spanned by $(m_{\text{av}}, x_0, A^u, A^d)$ in the squark sector and $(m_{\text{av}}, x_0, A^l)$ in the slepton sector.

From eqs. (5) we observe that $A$-terms do renormalize the off-diagonal scalar mass terms such that they become smaller at low energies. Thus we expect the the bounds (1) to be more easily satisfied when non-universal $A$-terms are present. Using eqs. (5), (8), (13) (and neglecting terms of $O(M_Z^2)$ as before) we find in the slepton sector

$$\begin{align*}
(\delta_{LL}^{(l)})_{12} &= (\delta_{LL}^{(l)})_{13} = (\delta_{LL}^{(l)})_{23} = \frac{1 - 0.7 (A^l)^2}{1 - 1.5 (A^l)^2 + 0.3 x_0^{(l)}}, \\
(\delta_{RR}^{(l)})_{12} &= (\delta_{RR}^{(l)})_{13} = (\delta_{RR}^{(l)})_{23} = \frac{1 - 1.4 (A^l)^2}{1 - 1.5 (A^l)^2 + 0.3 x_0^{(l)}}, \\
(\delta_{LR}^{(l)})_{12} &= 0, \quad (\delta_{LR}^{(l)})_{13} = (\delta_{LR}^{(l)})_{23} = \frac{1.5 A^l \langle h_d \rangle}{m_{\text{av}} (1 - 1.5 (A^l)^2 + 0.3 x_0^{(l)})}.
\end{align*} \tag{14}$$

We see that for $A^l$ of $O(1)$ one of the numerators in $\delta_{LL}$ or $\delta_{RR}$ can be made small and even tuned to zero but not both simultaneously. In addition, one has to make sure

- This texture is also suggested in some superstring models where typically $A^f \simeq Y_{33}^f$ holds. This implies large quantum corrections to the fermion masses and opens up the possibility of generating the entire fermion mass hierarchy from $A$-terms as a quantum effect. Work along these lines is in progress.
- A similar situation occurs in the supersymmetric Coleman-Weinberg mechanism where a Higgs parameter is tuned negative.

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that $M_{\text{av}}^{(l)} > 50 \, \text{GeV}$ or in other words the denominators in eqs. (14) stay positive. This again requires a hierarchy between gaugino and slepton masses $x_0^{(l)} > 1$. The absolute value of the numerators in $\delta_{LL}$ and $\delta_{RR}$ should both be small and for fixed $x_0^{(l)}$ this leads to a lower and upper bound for $A^l$. Furthermore, the constraints for $\delta_{LR}$ and $M_{\text{av}}^{(l)} > 50 \, \text{GeV}$ also imply upper bounds on $A^l$. Together, the upper and lower bounds lead to a ‘wedge’ shaped curve which we show in fig. 2 as a function of the low energy slepton mass $M_{\text{av}}^{(l)}$ for fixed $\sqrt{x_0^{(l)}} = 8, 16$. Leaving $x_0^{(l)}$ arbitrary one can scan over the parameter space $(m_{\text{av}}^{(l)}, x_0^{(l)}, A^l)$ and demand all constraints in table 1 to be satisfied. In fig. 3 we display the lowest allowed $x_0^{(l)}$ as a function of $M_{\text{av}}^{(l)}$ for an arbitrary $A^l$ and for fixed $A^l = Y_\tau$ (which essentially coincides with $A^l = 0$). We indeed see that it is easier to obey the constraints for arbitrary $A^l$ and as a consequence a smaller value for $\sqrt{x_0}$ is tolerable.

In the squark sector we perform a similar analysis which becomes somewhat more involved since we have two independent $A$-terms ($A^u, A^d$) to play with. Using eqs. (5), (8), (13) we have

\[
\begin{align*}
(\delta_{LL}^{(d)})_{12} &= (\delta_{LL}^{(d)})_{13} = (\delta_{LL}^{(d)})_{23} = \frac{1 - (1.8 (A^u)^2 + 1.7 (A^d)^2)}{1 - 1.5 (A^u)^2 - 4.5 (A^d)^2 + 7 x_0}, \\
(\delta_{RR}^{(d)})_{12} &= (\delta_{RR}^{(d)})_{13} = (\delta_{RR}^{(d)})_{23} = \frac{1 - 3.4 (A^d)^2}{1 - 1.5 (A^u)^2 - 4.5 (A^d)^2 + 7 x_0}, \\
(\delta_{LR}^{(d)})_{12} &= 0, \quad (\delta_{LR}^{(d)})_{13} = (\delta_{LR}^{(d)})_{23} = \frac{3.6 A^d \langle h_d \rangle}{m_{\text{av}} (1 - 1.5 (A^u)^2 - 4.5 (A^d)^2 + 7 x_0)},
\end{align*}
\]

(15)

and a similar set in the up-squark sector. We numerically scan over the parameter space $(A^u, A^d, x_0, m_{3/2})$ and check that all constraints in table 1 are satisfied simultaneously.† In fig. 4 we display the lowest possible $\sqrt{x_0}$ as a function of the physical (low energy) squark mass $M_{\text{av}}^{d}$ for arbitrary (allowed) $A^u, A^d$, for fixed $A^u = Y_t$, $A^d = Y_b$ and also for $A^u = A^d = 0$. As expected for non-zero $A$ the allowed values for $\sqrt{x_0}$ are significantly lower than for $A = 0$ but we again need $\sqrt{x_0} > 1$. For fixed $x_0$ the $A^u$ and $A^d$ satisfy again upper and lower bounds in analogy with the slepton sector.

Finally, the ‘inverse’ hierarchy $x_0 < 1$ does not allow large $A$-terms since $M_{\text{av}}$ would become too small. Thus in this case one is forced to the case of vanishing $A$-terms and the situation discussed in eq. (12).

† We also check that the lowest eigenvalue of the squark mass matrix is above 100 GeV.
5. So far the discussion has been in the limit of vanishing Yukawa couplings. Turning on the Yukawa couplings we first need to transform to the fermion mass basis with an appropriate CKM rotation. Since the constraints in table 1 are given in a basis where also the fermion-sfermion-gaugino couplings are diagonal one needs to perform a ‘compensating’ CKM rotation on the scalar degrees of freedom. It is possible that after such a rotation the scalar mass matrices are automatically diagonal. This ‘squark-quark alignment mechanism’ has been recently discussed in ref. [15] and shown to arise naturally in theories with horizontal symmetries. Here we assume that no such alignment occurs and the CKM-rotated mass matrix is still arbitrary with \( \Delta m^2 = O(m^2_{3/2}) \). In the RG-analysis we now keep the top-quark Yukawa coupling \( Y_t \) which is the leading effect.† It is important to note that in this approximation the fermion mass basis does not rotate between \( M_Z \) and \( M_{GUT} \) and one stays in the fermion mass basis chosen at \( M_{GUT} \) along the RG-trajectory.

For universal \( A \)-terms a non-zero \( Y_t \) only affects the renormalization of \( M^{(u,d)}_{atv} \) and only via the renormalization of \( (m^2_q)_{33} \) and \( (m^2_u)_{33} \). This changes the coefficient of \( (m^2_q)_{33} \) and \( (m^2_u)_{33} \) in eqs. (7), (8) by at most a factor of 2 which gets weakened by taking the average mass. (Also the coefficient of \( \tilde{m}_{1/2} \) changes slightly.) Hence, for universal \( A \)-terms the effect of a large \( Y_t \) is well within the (hadronic) uncertainties of the constraints given in table 1 and hence fig. 1 still holds in our approximation.

For non-universal \( A \)-terms the effect of a large \( Y_t \) is more significant (the RG-analysis can be found in ref. [18]). Again only the squark sector is affected whereas the sleptons (and hence figs. 2,3) are unchanged. The most important effect is that all coefficients in front of the \( A^u \)-terms in eqs. (15) become \( Y_t \) dependent and for a large \( Y_t \) decrease significantly (they approach 0 for \( Y_t \) at its IR (quasi-) fixed point.) Hence the ‘help’ from the \( A \)-terms in satisfying the FCNC constraints becomes weaker for large \( Y_t \). In fig. 5 we display this effect for three different values of \( Y_t \) (for \( \tan \beta = 2.5 \) they correspond to \( m_{top} = 0, 160, 175, 180 \text{ GeV} \)). When \( Y_t \) approaches the fixed point the constraint on \( \sqrt{x_0} \) moves towards the \( A = 0 \) case. However, as before the most stringent constraint arises in the slepton sector which is independent of \( Y_t \).

6. Let us summarize the main points of our analysis. The smallness of FCNC imposes severe constraints on the off-diagonal elements of the scalar mass matrices

† For large \( \tan \beta \) one should keep all Yukawa couplings of the third generation but we do not discuss this case here.
at low energies. In translating these constraints to some high energy scale \( M_{\text{GUT}} \) (which is of interest for model building) quantum corrections have to be included and can (significantly) change the conditions on the supersymmetric parameter space at \( M_{\text{GUT}} \). In particular, a (large) hierarchy between the (high energy) gaugino mass and the (high energy) scalar masses ensures that at low energies the FCNC constraints are satisfied even for non-universal scalar masses at \( M_{\text{GUT}} \). Non-universal \( A \)-terms also dilute the constraints at \( M_{\text{GUT}} \) but one still needs the gaugino mass to be larger than the scalar masses. An inverse hierarchy with scalar masses bigger than the gaugino mass (again at \( M_{\text{GUT}} \)) is only possible if there is a (significant) degree of universality within the scalar mass matrices. By far the strongest constraints on the high energy parameter space arise in the slepton sector of the theory.

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| Process | Constraints |
|---------|-------------|
| $K\bar{K}$ | | |
| $(\delta_{LL/RR}^{(d)})_{12}(x^{(d)} = 1)$ | $(\delta_{LR}^{(d)})_{12}(x^{(d)} = 1)$ | $\langle \delta_{12}^{(d)} \rangle (x^{(d)} = 1)$ |
| $1 \cdot 10^{-1} M_{av}/1TeV$ | $1 \cdot 10^{-2} M_{av}/1TeV$ | $8 \cdot 10^{-3} M_{av}/1TeV$ |
| $B\bar{B}$ | | |
| $(\delta_{LL/RR}^{(d)})_{13}(x^{(d)} = 1)$ | $(\delta_{LR}^{(d)})_{13}(x^{(d)} = 1)$ | $\langle \delta_{13}^{(d)} \rangle (x^{(d)} = 1)$ |
| $2 \cdot 10^{-1} M_{av}/1TeV$ | $5 \cdot 10^{-2} M_{av}/1TeV$ | $3 \cdot 10^{-2} M_{av}/1TeV$ |
| $D\bar{D}$ | | |
| $(\delta_{LL/RR}^{(u)})_{12}(x^{(u)} = 1)$ | $(\delta_{LR}^{(u)})_{12}(x^{(u)} = 1)$ | $\langle \delta_{12}^{(u)} \rangle (x^{(u)} = 1)$ |
| $2 \cdot 10^{-1} M_{av}/1TeV$ | $5 \cdot 10^{-2} M_{av}/1TeV$ | $3 \cdot 10^{-2} M_{av}/1TeV$ |
| $b \to s\gamma$ | | |
| $(\delta_{LL/RR}^{(d)})_{23}(x^{(d)} = 1)$ | $(\delta_{LR}^{(d)})_{23}(x^{(d)} = 1)$ | $\langle \delta_{12}^{(d)} \rangle (x^{(d)} = 1)$ |
| $3 \cdot 10^2 M_{av}^2/(1TeV)^2$ | $4 \cdot 10^{-1} M_{av}/1TeV$ | $\langle \delta_{12}^{(d)} \rangle (x^{(d)} = 1)$ |
| $\mu \to e\gamma$ | | |
| $(\delta_{LL/RR}^{(l)})_{12}(x^{(l)} = 0.5)$ | $(\delta_{LR}^{(l)})_{12}(x^{(l)} = 0.5)$ | $\langle \delta_{12}^{(l)} \rangle (x^{(l)} = 0.5)$ |
| $1 \cdot 10^{-1} M_{av}^2/(1TeV)^2$ | $2 \cdot 10^{-5} M_{av}/1TeV$ | $\langle \delta_{12}^{(l)} \rangle (x^{(l)} = 0.5)$ |
| $\tau \to e\gamma$ | | |
| $(\delta_{LL/RR}^{(l)})_{13}(x^{(l)} = 0.5)$ | $(\delta_{LR}^{(l)})_{13}(x^{(l)} = 0.5)$ | $\langle \delta_{13}^{(l)} \rangle (x^{(l)} = 0.5)$ |
| $4 \cdot 10^3 M_{av}^2/(1TeV)^2$ | $2 \cdot M_{av}/1TeV$ | $\langle \delta_{13}^{(l)} \rangle (x^{(l)} = 0.5)$ |
| $\tau \to \mu\gamma$ | | |
| $(\delta_{LL/RR}^{(l)})_{23}(x^{(l)} = 0.5)$ | $(\delta_{LR}^{(l)})_{23}(x^{(l)} = 0.5)$ | $\langle \delta_{23}^{(l)} \rangle (x^{(l)} = 0.5)$ |
| $7 \cdot 10^2 M_{av}^2/(1TeV)^2$ | $2 \cdot 10^{-1} M_{av}/1TeV$ | $\langle \delta_{23}^{(l)} \rangle (x^{(l)} = 0.5)$ |

Table 1: constraints from FCNC processes
**Figure Captions**

Figure 1: Constraints on the ratio $\sqrt{x}_0 = \frac{\tilde{m}_{1/2}}{m_{\text{av}}}$ as a function of the (low energy) average mass $M_{\text{av}}^{(l)}$.

Figure 2: Upper and lower bounds on $A^l$ as a function of the average mass $M_{\text{av}}^{(l)}$ for a fixed $\sqrt{x}_0 = 8, 16$.

Figure 3: The lowest allowed $\sqrt{x}_0$ as a function of $M_{\text{av}}^{(l)}$ for a) arbitrary $A^l$; b) $A^l = Y_\tau$.

Figure 4: The minimal allowed $\sqrt{x}_0$ as a function of the average squark mass $M_{\text{av}}^{(d)}$ for a) arbitrary $A^u, A^d$; b) $A^u = Y_t, A^d = Y_b$; c) $A^u = A^d = 0$.

Figure 5: The minimal allowed $\sqrt{x}_0$ as a function of $M_{\text{av}}^{(d)}$ for $\tan\beta = 2.5$ and a) $m_{\text{top}} = 0$; b) $m_{\text{top}} = 160\,\text{GeV}$; c) $m_{\text{top}} = 175\,\text{GeV}$; d) $m_{\text{top}} = 180\,\text{GeV}$. Case d) coincides with $A = 0$. 
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9408275v2
Fig. 1

$\mu \rightarrow e\gamma$

Fig. 2

$A^1$

Fig. 3

$M_{av} (GeV)$

Fig. 4

$M_{av}^l (GeV)$

Fig. 5

$M_{av}^{d} (GeV)$