Rapid HeII→HeI recombination and radiation concerned with this process

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ABSTRACT
Recombination of the primordial helium plasma (HeII→HeI, z ≃ 1500–3000) is considered. This process has an effect on the CMBR anisotropy and CMBR spectrum distortion. In this work an influence of neutral hydrogen on kinetics of HeII→HeI recombination is investigated in the frame of the standard cosmological model. It is shown that small amount of neutral hydrogen (10⁻⁵–10⁻² of total number of hydrogen ions and atoms) leads to acceleration of HeII→HeI recombination at z ≲ 2000 and at z ≲ 1600 quasi-equilibrium HeII→HeI recombination (according to the Saha formula) becomes valid.

Key words: cosmology, primordial plasma, recombination, cmb, anisotropy

1 INTRODUCTION
The HeII→HeI primordial recombination has an effect on formation of the cosmic microwave background radiation (CMBR) anisotropy (e.g. Hu et al., 1995; Seager et al., 1999) and CMBR spectrum distortion (Lyubarsky and Sunyaev, 1983; Fähr and Loehr, 1991; Dubrovich and Stolyarov, 1997; Wong et al., 2006). The relative difference of CMBR anisotropy power spectrum corresponding to Saha HeII→HeI recombination and corresponding to HeII→HeI recombination by Seager et al. (1999) has a level of up to 5% for multipoles l ≃ 1500–3000 (Seager et al., 2000). Such changes can be measured in future experiments (e.g. Planck and others).

Kinetics of HeII→HeI recombination has been considered in a number of papers (Boschan and Bilizinger, 1998; Novosyadlyj, 2006; Wong and Scott, 2006; and references therein). The most used model of helium recombination is the model suggested by Matsuda et al. (1969) and developed by Seager et al. (1999). A significant modification of this model was suggested by Dubrovich and Grachev (2005) who took into account recombination through HeI ortho-states.

In the model by Seager et al. (1999) the effect of neutral hydrogen on HeII→HeI recombination was neglected although in a number of previous papers (e.g. Hu et al., 1995; Boschan and Bilizinger, 1998; and others) authors pointed out that this effect may be essential. This contradiction made us investigate the effect of neutral hydrogen on HeII→HeI recombination again and derive HeII→HeI recombination kinetic equation which takes this effect into account.

2 ADDITION TO THE RECOMBINATION MODEL
The main addition to the Seager’s et al. (1999) model is taking into account an interaction of resonant HeI quanta with neutral hydrogen. This interaction is described by the following elementary processes:

1. HeI resonant photon ionizes hydrogen atom with emission of electron which has a kinetic energy more than 6 eV: H + γ → H⁺ + e⁻.
2. The emitted electron loses the kinetic energy during electron-electron collisions with thermal (\(E_{e,th} = 3/2k_B T < 0.7\) eV) electrons of the primordial plasma. The typical time of this process can be estimated by the following formula (e.g. Spitzer, 1978):

\[ t_{ee} = \frac{m_e^2 v_e^3}{4\pi N_e e^4 \ln (\Lambda m_e v_e^2/3k_B T)} \cdot \Lambda = \frac{3}{2e^2} \sqrt{\frac{(k_B T)^3}{\pi N_e}} \]  \( (1) \)

where \(m_e\) is the electron mass, \(v_e\) is the electron velocity, \(N_e\) is the free electron concentration, \(T\) is the temperature of medium. Under considered conditions the value \(t_{ee}\) is less than 10⁷ s.

3. The hydrogen ion recombines with a thermal electron and emits a photon with energy \((13.6 + k_B T) \approx 14\) eV: \(H^+ + e^- \rightarrow H + \gamma\). The typical time of this process is larger than 10⁵ s.

Comparison of the typical times of electron-electron collisions and hydrogen recombination shows that between the ionization and the following recombination of hydrogen the energy of the non-thermal electrons is divided among the thermal electrons of the primordial plasma.
In the chain of elementary processes mentioned above HeI resonant photons disappear and do not keep the populations of HeI excited states.

The rate of hydrogen ionization by HeI resonant 2p → 1s photons \( J_H \) [cm\(^{-3}\)s\(^{-1}\)] determines the rate of disappearance of these photons. It was shown by Seager et al. (2000) that this rate is negligible in comparison with the total rate of 2p → 1s transitions (i.e. the difference of rates of forward 2p → 1s and backward 1s → 2p transitions) of HeI due to escape of resonant photons from line profile because of cosmological redshift \( J_{H\text{el},bg} \) (subscript \( b \) denotes 2p state of HeI, subscript \( g \) denotes 1s state of HeI, fig. 1). In this work we have found that the rate \( J_H \) was underestimated by Seager et al. (2000). To show this fact we considered the ratio of the rates \( J_H \) and \( J_{H\text{el},bg} \).

The rate \( J_H \) is given by the expression

\[
J_H \simeq \sigma_H (\nu_{bg}) N_{\gamma,bg} N_{HI}
\]

where \( c \) is the speed of light, \( \sigma_H (\nu_{bg}) \) is the hydrogen ionization cross section at frequency \( \nu_{bg} \) of the 2p → 1s transition of HeI, \( N_{HI} \) is the concentration of hydrogen atoms in ground state, \( N_{\gamma,bg} \) is HeI 2p → 1s resonant photon concentration given by the formula

\[
N_{\gamma,bg} \simeq \frac{8 \pi \nu_{bg}^3}{c^3} \eta_{bg} \Delta \nu D \nu_{bg} = \frac{8 \pi \nu_{bg}^3}{c^3} \eta_{bg} \sqrt{\frac{2 k_B T}{m_{He} c^2}}
\]

where \( \eta_{bg} \) is the number of photons per mode 2p → 1s, \( \Delta \nu D \) is the thermal width of line 2p → 1s, \( m_{He} \) is the mass of helium atom.

The rate \( J_{H\text{el},bg} \) [cm\(^{-3}\)s\(^{-1}\)] is given by the equation

\[
J_{H\text{el},bg} \simeq A_{bg}' N_{H\text{el},bg}
\]

where \( A_{bg}' \) is the effective coefficient [s\(^{-1}\)] of 2p → 1s transitions due to photon escape from the line profile because of cosmological redshift (superscript \( r \) denotes “redshifting”), \( N_{H\text{el},bg} \) is the concentration of HeI atoms in state 2p. The effective coefficient of transitions is given by the formula:

\[
A_{bg}' \simeq \frac{8 \pi \nu_{bg}^3 H}{c^3 \eta_{bg} N_{H\text{el},bg}}
\]

where \( H = H_0 \sqrt{\Omega_\Lambda + \Omega_m (1+z)^3 + \Omega_{\text{cld}(1+z)}^2} \) is the Hubble constant as function of redshift (parameters \( H_0 \), \( \Omega_\Lambda \), \( \Omega_m \) and \( \Omega_{\text{cld}} \) described in tab. 1), \( \eta_{bg} = 3 \) is the statistical weight of 2p state, \( N_{H\text{el},bg} \) is the concentration of HeI atoms in 1s state.

Optical depth of absorption in the line HeI 2p → 1s is much larger than unity, therefore \( \eta_{bg} \) is satisfied the following expression

\[
\eta_{bg} \simeq \frac{N_{H\text{el},bg}}{\eta_{bg} N_{H\text{el},bg}}
\]

Using equations (2) - (6) one can find the ratio of reaction rates

\[
\frac{J_H}{J_{H\text{el},bg}} \simeq \frac{c}{H} \sigma_H (\nu_{bg}) \frac{\Delta \nu D}{\nu_{bg}} N_{HI}
\]

In the case of \( J_H/J_{H\text{el},bg} \ll 1 \) the effect of neutral hydrogen on the HeI → HeI recombination is negligible. In the case of \( J_H/J_{H\text{el},bg} \gg 1 \) effect of neutral hydrogen dominates and recombination speeds up essentially.

Dependence of \( J_H/J_{H\text{el},bg} \) on \( z \) is shown in fig. 2. Exponential increase of \( J_H/J_{H\text{el},bg} \) (with decrease of \( z \)) is concerned with exponential increase of neutral hydrogen concentration \( N_{HI} \) which is satisfied the Saha ionization equation for HI in the epochs \( z = 1500 – 3000 \).

The ratio \( J_H/J_{H\text{el},bg} \) is equal to 1 at epoch \( z \simeq 1800 \) (fig. 2) in contradiction with results by Seager et al. (2000) where the ratio \( J_H/J_{H\text{el},bg} \) is much less than unity in the period \( z = 1500 – 3000 \).

Note that in paper by Boschan and Biltzinger (1998) other estimation of neutral hydrogen effect on helium recombination kinetics was proposed. In that work disappearance of HeI resonant photons was characterized by optical depth of hydrogen ionization by HeI 2p → 1s resonant photons

\[
\tau \simeq \frac{c}{H} \sigma_H (\nu_{bg}) N_{HI} \Delta t
\]

where \( \Delta t \) is the duration of period when considered photon is the HeI resonant photon. The value \( \Delta t \) is defined by deviation between photon frequency and central frequency of HeI resonant transition because only HeI resonant photons keep the populations of HeI excited states. Using \( \nu = -H \nu \) we find

\[
\tau \simeq \frac{c}{H} \sigma_H (\nu_{bg}) \frac{\Delta \nu D}{\nu_{bg}} N_{HI}
\]

Thus one can obtain that

\[
\tau \simeq \frac{J_H}{J_{H\text{el},bg}}
\]

and we can see that both of estimation (by reaction rate ratio and by optical depth) give the same results. The value \( \tau \) is equal to 1 at \( z \simeq 1800 \) according to (10) (see fig. 2). This result is similar to the result of paper by Boschan and Biltzinger (1998).

The simple estimation presented above shows that neutral hydrogen effect on helium recombination kinetics is essential. The fact made us consider this effect in detail and derive helium kinetic equation taking into account effect of neutral hydrogen.
3 MAIN EQUATION

The behaviour of HeII (i.e. He\textsuperscript{+}) fraction is described by the following differential equation

\[
\dot{x} = -C_{par} \left( \alpha_{par} N_e x - \frac{g_a}{g_b} \beta_{par} \exp \left( -\frac{E_a}{k_B T} \right) \right) (1 - x) - C_{or} \left( \alpha_{or} N_e x - \frac{g_a}{g_b} \beta_{or} \exp \left( -\frac{E_a}{k_B T} \right) \right) (1 - x)
\]

(11)

where \( x = N_{HeII}/N_{HeI} \) is the fraction of HeII ions relative to the total number of helium atoms and ions, \( C_{par} \) is the factor by which the ordinary recombination rate is inhibited by the presence of HeI \( 2^1p \rightarrow 1^1s \) resonance-line radiation, \( \alpha_{par} \) is the total HeI→HeII recombination coefficient to the excited para-states of HeI, subscript \( a \) denotes state \( 2^1s \) of HeI atom, \( N_e \) is the free electron concentration, \( g_b = 1 \) is the statistical weight of \( 2^1s \) state of HeI, \( g_a = 1 \) is the statistical weight of \( 1^1s \) state of HeI, \( \beta_{par} \) is the total HeI→HeII ionization coefficient from the excited para-states of HeI, \( E_{aa} \) is the \( 2^1s \rightarrow 1^1s \) transition energy, \( C_{or} \) is the factor by which the ordinary recombination rate is inhibited by the presence of HeI \( 2^3p \rightarrow 1^1s \) resonance-line radiation, \( \alpha_{or} \) is the total HeI→HeII ionization coefficient to the excited ortho-states of HeI, subscript \( a' \) denotes state \( 2^3s \) of HeI atom \( g_a' = 3 \) is the statistical weight of \( 2^3s \) state of HeI, \( \beta_{or} \) is the total HeI→HeII ionization coefficient from the excited ortho-states of HeI, \( E_{a'a} \) is the \( 2^3s \rightarrow 1^1s \) transition energy.

The para- and ortho- recombination and ionization coefficients are related by the following formulations

\[
\beta_{par} = \frac{g_c}{g_a} \beta_{par} \exp \left( -\frac{E_{ca}}{k_B T} \right), \quad \beta_{or} = \frac{g_c}{g_a} \alpha_{or} \exp \left( -\frac{E_{ca}}{k_B T} \right)
\]

(12)

where subscript \( c \) denotes continuum state of HeI atom, \( g_c = 4 \) is the statistical weight of continuum state of \((\text{He}^+ + e^-)\), \( g_c(T) \) is the transition function of free electrons, \( E_{ca} \) is the \( c \rightarrow 2^1s \) transition energy, \( E_{ca'} \) is the \( c \rightarrow 2^3s \) transition energy.

The inhibition factor \( C_{par} \) is given by the following formula

\[
C_{par} = \frac{(g_b/g_a) (A_{bg}^H + A_{bg}^L) \exp (-E_{ba}/k_B T) + A_{ag}}{\beta_{par} + (g_b/g_a) (A_{bg}^H + A_{bg}^L) \exp (-E_{ba}/k_B T) + A_{ag}}
\]

(13)

where \( A_{bg}^H \) is the effective coefficient [s\(^{-1}\)] of \( 2^1p \leftrightarrow 1^1s \) transitions due to effect of neutral hydrogen on HeI \( 2^1p \rightarrow 1^1s \) resonant radiation (superscript \( H \) denotes hydrogen), \( E_{ba} \) is the \( 2^1p \rightarrow 2^3p \) transition energy, \( A_{ag} \) is the coefficient of two-photon \( 2^1s \rightarrow 1^1s \) spontaneous decay.

The inhibition factor \( C_{or} \) is given by the following formula

\[
C_{or} = \frac{(g_b/g_a') (A_{bg}^H + A_{bg}^L) \exp (-E_{ba}/k_B T) \exp \left( -\frac{E_{ba'/k_B T}}{k_B T} \right)}{\beta_{or} + (g_b/g_a') (A_{bg}^H + A_{bg}^L) \exp \left( -\frac{E_{ba'/k_B T}}{k_B T} \right)}
\]

(14)

where subscript \( b' \) denotes state \( 2^3p \) of HeI, \( g_{b'} = 9 \) is the statistical weight of \( 2^3p \) state of HeI, \( A_{b'g}^H \) is the effective coefficient of \( 2^3p \leftrightarrow 1^1s \) transitions due to effect of neutral hydrogen on HeI \( 2^3p \rightarrow 1^1s \) resonant radiation, \( A_{b'g}^L \) is the effective coefficient of \( 2^3p \leftrightarrow 1^1s \) transitions due to escape of photon from the \( 2^3p \rightarrow 1^1s \) line profile because of cosmological redshift, \( E_{b'a'} \) is the \( 2^3p \rightarrow 2^3p \) transition energy.

The coefficient \( A_{b'g}^L \) is given by the formula (15). The coefficient \( A_{b'g}^H \) is given by the following formulae

\[
A_{b'g}^L = A_{b'g} \tau_{b'g}^{-1} \left( 1 - \exp(-\tau_{b'g}) \right), \quad \tau_{b'g} \approx \frac{g_{b'} A_{b'g}^0 c^2 N_{HeII} \gamma_{f_{b'}}}{2 \pi H \nu_{b'g}^0}
\]

(15)

where \( A_{b'g} = g_{2^3P_1} A_{2^3P_1 \rightarrow 1^1S_0}/g_{b'} \) is the coefficient of \( 2^3p \rightarrow 1^1s \) spontaneous transition, \( g_{2^3P_1} = 3 \) is the statistical weight of \( 2^3P_1 \) state.

The coefficient \( A_{b'g}^H \) (subscript \( f \) denotes \( 2^3p \) \((b')\) state) can be obtained by consideration of kinetic equation for HeI \( f \rightarrow g \) transitions and transfer equation for HeI \( f \rightarrow g \) resonance radiation in the homogeneous expanding Universe in the presence of neutral hydrogen. Detailed calculation (paper in preparation) shows that exact expressions for the coefficients \( A_{b'g}^H \) and \( A_{b'g}^L \) can be approximated by the following formulae:

\[
A_{b'g}^H = \frac{A_{b'g}}{1 + \gamma_{f_{b'}} - 1}, \quad \gamma_{f_{b'}} = \frac{(g_{f'/g_{b'}}) A_{b'g} N_{HeII} \gamma_{f_{b'}}^2}{\sqrt{\pi} \sigma_{b'} (\nu_{b'g})} \frac{\Delta \nu_{b'g} \Delta \nu_{f_{b'}}}{\nu_{b'g}^3} N_{b'}
\]

(16)

Parameters \( \gamma_{f} \) and \( \gamma_{b'} \) are the following: \( p_{b} = 0.36, q_{b'} = 0.97, p_{f} = 0.66, q_{f} = 0.9 \).

The equation (11) has been solved numerically. Results of calculation are presented in fig. 3.

The equation (11) combined with equations for description of fractions of HII (Zel’dovich et al., 1968; Peebles, 1968) and HeII (Matsuda et al., 1969; Seager et al., 1999) allows us to calculate ionization history of the Universe in the epochs \( z = 10^2 - 10^2 \).
we can find the spectrum of radiation concerned with redshift (fig. 4). The calculation shows that during epochs fraction on redshift (fig. 3) and free electron fraction on
The main results of this paper are the dependencies of HeII on redshift z, which is shown. The dotted curve shows the unity level.

4 RADIATION CONCERNED WITH HEII→HEI RECOMBINATION

The most of recombination photons corresponding to transitions to ground state of helium atoms is absorbed by the neutral hydrogen atoms and converts to the Lyα-photons of HI.

Number of HI Lyα-photons per mode concerned with HeII→HeI recombination is given by the formula

\[ \Delta \eta(\nu, z_b) = \frac{e^3 N_{He} (z_b) |\hat{x}(z_b)|}{8 \pi \nu_0^3} \frac{H(z_b)}{\eta} \]  

where \( \Delta \eta \) is the number of photons per mode depending on frequency \( \nu \) and epoch \( z_b \), \( \nu_0 \) is the HI Lyα frequency, \( z_b \) is the epochs of photon birth. Using relation

\[ \Delta \eta(\nu, z) = \Delta \eta \left( \frac{1 + z'}{1 + z} \right)^3 \]  

we can find the spectrum of radiation concerned with HeII→HeI recombination at the present epoch.

5 RESULTS

The main results of this paper are the dependencies of HeII fraction on redshift (fig. 3) and free electron fraction on redshift (fig. 4). The calculation shows that during epochs \( z = 1600 \sim 2000 \) kinetics of HeII→HeI recombination (this work, black curve) changes from strongly non-equilibrium (Seager et al., 1999, blue curve) to quasi-equilibrium (according to the Saha formula for HeI, red curve). This change is due to absorption of HeI resonant photons by neutral hydrogen whose concentration increases exponentially with decrease of temperature. At the epoch \( z = 1600 \) the fraction of HeII relative to the total number of helium ions and atoms is less than \( 10^{-9} \). To compare our results with those of previous papers we have calculated HeII→HeI recombination kinetics at the following parameters: \( A_{fg}^H = 0, A_{23}, P_1 \rightarrow 1^1S_0 = 233 \, s^{-1} \) (Lin et al., 1977) corresponding to paper by Dubrovich and Grachev (2005) - violet curve, and \( A_{23}, P_1 \rightarrow 1^3S_0 = 177.58 \, s^{-1} \) (Lach and Pachucki, 2001) [violet curve practically overlaps magenta curve], green curve corresponds to taking into account effect of neutral hydrogen on HeII→HeI recombination, black curve corresponds to taking into account both of the effects, red curve corresponds to recombination according to the Saha formula for HeI. Top panel corresponds to linear scale, bottom one does logarithmic scale.

In the fig. 4 one can see that recombination of helium ends \( (x < 10^{-6}) \) before recombination of hydrogen begins \( (x_{HI} \geq 0.99) \) in contradiction of results by Seager et al. (1999). This change of primordial plasma recombination kinetics should led to changes of calculated CMBR anisotropy at the level of up to 5% for multipoles \( l \simeq 1500 \sim 3000 \) (Seager et al., 2000).

Results of calculation of radiation intensity are presented in the fig. 5: 1) Planck spectrum radiation at the temperature \( T_0 = 2.726 \, K \) (red curve); 2) HI Lyα radiation concerned with hydrogen recombination (blue curve, Grachev and Dubrovich, 1991; Rubino-Martin et al., 2006;
Kholupenko and Ivanchik, 2006; and references therein); 3) HI Lyα radiation concerned with HeII→HeI recombination (magenta curve). Wien-tail CMBR intensity distortion concerned with HeII→HeI recombination has the maximum value about 9.3·10^{-25} \text{erg cm}^{-2}\text{ster}^{-1}\text{s}^{-1}\text{Hz}^{-1} being at frequency 1350 GHz. This frequency is about 2 times smaller than frequency calculated by Wong et al. (2006). This difference is concerned with re-emission of helium recombination photons by neutral hydrogen atoms in Lyα-line of HI while in the paper by Wong et al. (2006) helium recombination photon emission was considered at transitions 2^1p \rightarrow 1^1s and 2^1s \rightarrow 1^1s of HeI.

Table 1. Parameters of the standard cosmological model

| Value matter description | Symbol | Value |
|--------------------------|--------|-------|
| total matter             | \Omega_{\text{tot}} | 1     |
| non-relativistic matter  | \Omega_{m}   | 0.27  |
| baryonic matter          | \Omega_{b}   | 0.04  |
| relativistic matter      | \Omega_{\text{rel}} | \sim 10^{-4} |
| vacuum-like energy        | \Omega_{\Lambda} | 0.73  |
| Hubble constant           | \mathcal{H}_0 | 70 \text{ km/s/Mpc} |
| radiation temperature    | \mathcal{T}_0 | 2.726 \text{ K} |
| helium mass fraction      | Y          | 0.24  |

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