I discuss vacuum alignment and CP violation in technicolor theories of electroweak and flavor symmetry breaking. I review the surprising appearance of rational phase solutions in the technifermion sector and propose a new solution of the strong CP problem of quarks. I then discuss the sources of weak CP violation, in both the CKM matrix and the suppressed extended technicolor and topcolor-assisted technicolor interactions. It is easy to reproduce the observed value of the neutral kaon CP-violating parameter $\epsilon$ from these interactions.

1 Outline

In this talk I discuss the dynamical approach to CP violation in technicolor theories and a few of its consequences. I will cover the following topics:

1. Vacuum alignment in technicolor theories and the rational–phase solutions.
2. A proposal to solve the strong CP problem without an axion or a massless up quark.
3. The structure of quark mass and mixing matrices in extended technicolor (ETC) theories with topcolor–assisted technicolor (TC2). In particular, realistic Cabbibo–Kobayashi–Maskawa (CKM) matrices are easily generated.
4. Flavor–changing neutral current interactions from extended technicolor and topcolor.
5. New results on the $K^0 - \bar{K}^0$ CP–violating parameter $\epsilon$.

2 Vacuum Alignment in the Technifermion Sector

In 1971, Dashen stressed the importance of matching the ground state $|\Omega\rangle$ of a theory containing spontaneously broken chiral symmetries with the perturbing Hamiltonian $H'$ that explicitly breaks those symmetries. He also showed that this process, known as vacuum alignment, can lead to a spontaneous breakdown of CP invariance: it may happen that the CP symmetry of $|\Omega\rangle$ is not the same as that of the the aligned $H'$. This idea found its natural home in dynamical theories of electroweak symmetry breaking—technicolor because they have large groups of flavor/chiral symmetries that are spontaneously broken by strong dynamics and explicitly broken by extended technicolor (ETC). Furthermore, the perturbation $H'$ generated by exchange of ETC gauge bosons is naively CP–conserving if CP is unbroken above the technicolor energy scale. Thus, in 1979, Eichten, Preskill and I proposed that CP violation occurs spontaneously in theories of dynamical electroweak

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"Invited talk at the Eighth International Symposium on Particles, Strings and Cosmology—PASCOS 2001, University of North Carolina, Chapel Hill, NC, April 10–15, 2001. Parts of this work were done in collaboration with Gustavo Burdman, Estia Eichten and Tonguç Rador."
symmetry breaking. Our goal, unrealized at the time, was to solve the strong–CP problem of QCD without invoking a Peccei–Quinn symmetry or a massless up quark.

This problem was taken up again a few years ago with Eichten and Rador. We studied the first important step in reaching this goal: vacuum alignment in the technifermion sector. We considered models in which a single kind of technifermion interacts with quarks via ETC interactions. There are \(N\) technifermion doublets \(T_{L,R I} = (u_{L,R I}, D_{L,R I})\), \(I = 1, 2, \ldots, N\), all transforming according to the fundamental representation of the technicolor gauge group \(SU(N_{TC})\). There are 3 generations of \(SU(3)_C\) triplet quarks \(q_{L,R i} = (u_{L,R i}, d_{L,R i})\), \(i = 1, 2, 3\). The left–handed fermions are electroweak \(SU(2)\) doublets and the right–handed ones are singlets. Here and below, we exhibit only flavor, not technicolor nor color, indices.

The technifermions are assumed for simplicity to be ordinary color–singlets, so the chiral flavor group of our model is \(G_f = [SU(2N)_L \otimes SU(2N)_R] \otimes [SU(6)_L \otimes SU(6)_R]\). When the TC and QCD couplings reach their required critical values, these symmetries are spontaneously broken to \(S_f = SU(2N) \otimes SU(6)\). We adopt as a “standard vacuum” on which to carry out chiral perturbation theory the ground state \(|\Omega\rangle\) whose symmetry group is the the vectorial \(SU(2N)_V \otimes SU(6)_V\), with fermion bilinear condensates given by

\[
\langle \Omega | \bar{u}_{LI} u_{RI} | \Omega \rangle = \langle \Omega | \bar{d}_{LI} d_{RI} | \Omega \rangle = -\delta_{IJ} \Delta_T,
\]

\[
\langle \Omega | \bar{q}_{LI} q_{RI} | \Omega \rangle = \langle \Omega | \bar{q}_{LI} d_{RI} | \Omega \rangle = -\delta_{IJ} \Delta_q.
\]

Here, \(\Delta_T \approx N_{TC} \Lambda^3_{ETC}\) and \(\Delta_q \approx N_C \Lambda^3_{QCD}\) when they are renormalized at their respective strong interaction scales. Of course, \(N_C = 3\).

All of the \(G_f\) symmetries except for the gauged electroweak \(SU(2) \otimes U(1)\) must be explicitly broken by extended technicolor interactions. In the absence of a concrete ETC model, we write the interactions broken at the scale \(M_{ETC}/g_{ETC} \sim 10^2–10^4\) TeV in the phenomenological four-fermion form (sum over repeated indices):

\[
\mathcal{H}' = \mathcal{H}'_{TT} + \mathcal{H}'_{Tq} + \mathcal{H}'_{qq}
= \Lambda_{TJKL}^{TT} \bar{T}_{LI} \gamma^\mu T_{LJ} \bar{T}_{RL} \gamma^\nu T_{RL} + \Lambda_{Tijj}^{Tq} \bar{T}_{LI} \gamma^\mu q_{Lj} \bar{q}_{Rj} \gamma^\nu T_{RL} + \text{h.c.}
+ \Lambda_{ijkl}^{qq} \bar{q}_{Li} \gamma^\mu q_{Lj} \bar{q}_{Rk} \gamma^\nu q_{Rl},
\]

where \(T_{L,R I}\) and \(q_{L,R i}\) stand for all \(2N\) technifermions and 6 quarks, respectively. Here, \(M_{ETC}\) is a typical ETC gauge boson mass and the \(\Lambda\) coefficients are \(O(g_{ETC}^2/M_{ETC}^2)\) times mixing factors for these bosons and group theoretical factors. The \(\Lambda\)’s may have either sign. In all calculations, we must choose the \(\Lambda\)’s to avoid unwanted Goldstone bosons. Hermiticity of \(\mathcal{H}'\) requires

\[
(\Lambda_{TJKL}^{TT})^* = \Lambda_{JILK}^{TT}, \quad (\Lambda_{Tijj}^{Tq})^* = \Lambda_{ijjj}^{Tq}, \quad (\Lambda_{ijkl}^{qq})^* = \Lambda_{jikk}^{qq}.
\]

The assumption of time–reversal invariance for this theory before any potential breaking via vacuum alignment means that the instanton angles \(\theta_{TC} = \theta_{QCD} = 0\) (at tree level) and that all \(\Lambda\)’s are real. Thus, e.g., \(\Lambda_{TJKL}^{TT} = \Lambda_{JILK}^{TT}\).

\[\text{The fact that heavy quark chiral symmetries cannot be treated by chiral perturbative methods will be addressed below. We have excluded anomalous } U_A(1)’s \text{ strongly broken by TC and color instanton effects.}\]

\[\text{See the Appendix for estimates of } M_{ETC}/g_{ETC}.\]

\[\text{We assume that ETC interactions commute with electroweak } SU(2), \text{ though not with } U(1) \text{ nor color } SU(3). \text{ All fields in Eq. (2) are electroweak, not mass, eigenstates.}\]
Having chosen a standard chiral–perturbative ground state, $|\Omega\rangle$, vacuum alignment proceeds by minimizing the expectation value of the rotated Hamiltonian. This is obtained by making the $G_f$ transformation $T_{L,R} \rightarrow W_{L,R} T_{L,R}$ and $q_{L,R} \rightarrow Q_{L,R} q_{L,R}$, where $W_{L,R} \in SU(2N)_{L,R}$ and $Q_{L,R} \in SU(6)_{L,R}$:

$$\mathcal{H}(W, Q) = \mathcal{H}_T(W, W) + \mathcal{H}_q(Q, Q) + \mathcal{H}_q^\prime(Q, Q) + \mathcal{H}_W(W, Q)$$

$$= \Lambda_{ijkL}^T T_{LJ} W_{L,J}^I \gamma^\mu W_{L,J}^I T_{KL}^J W_{K,L}^I \gamma^\mu W_{R,L}^I T_{RL}^J + \cdots .$$

Since $T$ and $q$ transform according to complex representations of their respective color groups, the four–fermion condensates in the $S_f$–invariant $|\Omega\rangle$ have the form

$$\langle \Omega | \bar{T}_{LJ}^I \gamma^\mu T_{LJ}^I \bar{T}_{RK}^J \gamma^\mu T_{RL}^J | \Omega \rangle = - \Delta_T \delta_{IJ} \delta_{JK},$$

$$\langle \Omega | \bar{q}_{LJ}^I \gamma^\mu q_{LJ}^I \bar{q}_{RJ}^J \gamma^\mu q_{RJ}^J | \Omega \rangle = - \Delta_q \delta_{IJ} \delta_{JK}.$$ (5)

The condensates are positive, renormalized at $M_{ETC}$ and, in the large–$N_{TC}$ and $N_C$ limits, they are given by $\Delta_T \simeq (\Delta_T(M_{ETC}))^2$, $\Delta_q \simeq \Delta_T(M_{ETC}) \Delta_q(M_{ETC})$, and $\Delta_q \simeq (\Delta_q(M_{ETC}))^2$. In walking technicolor, $\Delta_T(M_{ETC}) \simeq (M_{ETC}/A_{TC}) \Delta_T(A_{TC}) = 10^2 \times A_{TC}$. In QCD, however, $\Delta_q(M_{ETC}) \simeq (\log(M_{ETC}/A_{QCD}))^\gamma \Delta_q(A_{QCD}) \simeq \Delta_q(\Lambda_{QCD})$, where $\gamma_m \simeq 2\alpha_C/\pi$ for $SU(3)_C$. Thus, the ratio

$$r = \frac{\Delta_T(M_{ETC})}{\Delta_T(M_{ETC})} \simeq \frac{\Delta_q(M_{ETC})}{\Delta_T(M_{ETC})}$$

is at most $10^{-10}$. This is 10 to $10^4$ times smaller than in a technicolor theory in which the coupling does not walk.

With these condensates, the vacuum energy is a function only of $W = W_L W_R^\dagger$ and $Q = Q_L Q_R^\dagger$, elements of the coset space $G_f/S_f$:

$$E(W, Q) = E_{TT}(W) + E_{Tq}(W, Q) + E_{qq}(Q)$$

$$= - \Lambda_{ijkL}^T t_{LJ}^I \Lambda_{ijL}^I \delta_{IJ} \delta_{JK} - (\Lambda_{ijL}^I \delta_{IJ} \delta_{JK} + c.c.) \Delta_T q_{LJ}^I \Lambda_{ijkL}^I \delta_{IJ} \delta_{JK}$$

$$= - \Lambda_{ijkL}^T t_{LJ}^I \Lambda_{ijL}^I \delta_{IJ} \delta_{JK} + O(10^{-10}).$$

Note that time–reversal invariance of the unrotated Hamiltonian $\mathcal{H}'$ implies that $E(W, Q) = E(W^*, Q^*)$. Hence, spontaneous CP violation occurs if the solutions $W_0$, $Q_0$ to the minimization problem are complex.

The last line of Eq. (4) makes clear that we should first minimize the technifermion sector energy $E_{TT}$. This determines $W$ up to corrections of $O(10^{-10})$. This result is then fed into $E_{Tq}$ to determine $Q_0$—and the nature of quark CP violation—up to corrections which are also $O(10^{-10})$.

So long as vacuum alignment preserves electric charge conservation, the alignment matrices will be block–diagonal:

$$W_{L,R} = \begin{pmatrix} W^U & 0 \\ 0 & W^D \end{pmatrix}_{L,R}, \quad Q_{L,R} = \begin{pmatrix} U & 0 \\ 0 & D \end{pmatrix}_{L,R}.$$ (6)

Two sorts of corrections to this statement are under study. The first are higher–order ETC and electroweak corrections to $E_{TT}$. The second are due to $TtT$ terms in $E_{Tq}$ which are important if the top condensate is large. I thank J. Donoghue and S. L. Glashow for emphasizing the importance of these corrections.

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In Ref. [8], it was shown that just three possibilities naturally occur for the phases in \( W \). (We drop the subscript “0” from now on.) Let us write \( W_{IJ} = |W_{IJ}| \exp(i\phi_{IJ}) \). Consider an individual term \(-\Lambda_{TT}^{IJKL} W_{JK} W_{LI}^\dagger \Delta_{TT}\) in the vacuum energy. If \( \Lambda_{TT}^{IJKL} > 0 \), this term is least if \( \phi_{IL} = \phi_{JK} \); if \( \Lambda_{TT}^{IJKL} < 0 \), it is least if \( \phi_{IL} = \phi_{JK} \pm \pi \). We say that \( \Lambda_{TT}^{IJKL} \neq 0 \) links \( \phi_{IL} \) and \( \phi_{JK} \), and tends to align (or antialign) them. Of course, the constraints of unitarity may partially or wholly frustrate this alignment. The three possibilities for the phases are:

1. The phases are all unequal, irrational multiples of \( \pi \) that are random except for the constraints of unitarity and unimodularity.
2. All of the phases may be equal to the same integer multiple of \( 2\pi/N \) (mod \( \pi \)). This occurs when all phases are linked and aligned, and the value \( 2\pi/N \) is a consequence of unimodularity. In this case we say that the phases are “rational”.
3. Several groups of phases may be linked among themselves and the phases only partially aligned. In this case, their values are various rational multiples of \( \pi/N' \) for one or more integers \( N' \) from 1 to \( N \).

We stress that, as far as we know, rational phases occur naturally only in ETC theories. They are a consequence of \( E_{TT} \) being quadratic, not linear, in \( W \). With these three outcomes in hand, we proceed to investigate the strong CP violation problem of quarks.

3 A Dynamical Solution to the Strong–CP Problem

There are two kinds of CP violation in the quark sector. Weak CP violation enters the standard weak interactions through the CKM phase \( \delta_{13} \) and, for us, in the ETC and TC2 interactions through phases in the quark alignment alignment matrices \( U_{L,R} \) and \( D_{L,R} \) discussed in Section 4. Strong CP violation, which can produce electric dipole moments \( 10^{16} \) times larger than in the standard model, is a consequence of instantons. No discussion of the origin of CP violation is complete which does not eliminate strong CP violation. Resolving the strong CP problem amounts to making \( \tilde{\theta}_q = \arg \det(M_q) \lesssim 10^{-10} \) (in a basis with instanton angle \( \theta_{QCD} = 0 \)), so that the neutron electric dipole moment is below its experimental bound of \( 0.63 \times 10^{-25} e\cdot \text{cm} \). Here, \( M_q \) is the hard or current algebra mass matrix of the quarks. It includes both the TC2 and ETC–generated parts of the top quark’s mass.

The “primordial” quark mass matrix, the coefficient of the bilinear \( \bar{q}_R q_L \) of quark electroweak eigenstates, is generated by ETC interactions and is given by

\[
(M_q)_{ij} = \sum_{I,J} \Lambda_{Ij,j}^q W_{JI}^\dagger \Delta_T(M_{ETC}) \quad (q, T = u, U \text{ or } d, D). \tag{8}
\]

The \( \Lambda_{Ij,j}^q \) are real ETC couplings of order \((10^2–10^4 \text{ TeV})^{-2}\) (see the Appendix). Furthermore, the quark alignment matrices \( Q_{L,R} \) which diagonalize \( M_q \) to \( M_q \) are unimodular. Because \( W \) is block diagonal, \( E_{TT} \) factorizes into two pieces, \( E_{UU} + E_{DD} \), in which \( W_U \) and \( W_D \) may each be taken unimodular. Thus, totally aligned phases are multiples of \( 2\pi/N \), not \( \pi/N \).

The matrix element \( M_{tt} \) arises almost entirely from the TC2–induced condensation of top quarks. We assume that \( \langle \bar{t}t \rangle \) and \( M_{tt} \) are real in the basis with \( \theta_{QCD} = 0 \). Since technicolor, color, and topcolor groups are embedded in ETC, all CP–conserving condensates are real in this basis.

\[\text{pascos}'01: \text{submitted to Rinton on January 14, 2022}\]
Thus, \( \arg \det(M_q) = \arg \det(M_u) \equiv \arg \det(M_d) \), and the question of strong CP violation is determined entirely by the character of vacuum alignment in the technifermion sector, i.e., by the phases \( \phi_{IJ} \) of \( W \), and by how the ETC factors \( \Lambda^T_{ij} \) map these phases into the \((M_q)_{ij}\).

If the \( \phi_{IJ} \) are random irrational phases, \( \bar{\theta}_q \) could vanish only by the most contrived, unnatural adjustment of the \( \Lambda^T_{ij} \). If all \( \phi_{IJ} = 2m\pi/N \) (mod \( \pi \)), then all elements of \( M_u \) have the same phase, as do all elements of \( M_d \). Then, \( U_{L,R} \) and \( D_{L,R} \) will be real orthogonal matrices, up to an overall phase. There may be strong CP violation, but there will no weak CP violation in any interaction.

There remains the possibility, which we assume henceforth, that the \( \phi_{IJ} \) are different rational multiples of \( \pi \). Then, strong CP violation will be absent if the \( \Lambda^T_{ij} \) map these phases onto the primordial mass matrix so that each element \((M_q)_{ij}\) has a rational phase and these add to zero in \( \arg \det(M_q) \). In the absence of an explicit ETC model, we are not certain this can happen, but we see no reason that it cannot. For example, there may be just one nonzero \( \Lambda^T_{ij} \) for each pair \((ij)\) and \((IJ)\). An ETC model which achieves such a phase mapping will solve the strong CP problem, i.e., \( \bar{\theta}_q \lesssim 10^{-10} \), without an axion and without a massless up quark. This is, in effect, a “natural fine-tuning” of phases in the quark mass matrix. There is, of course, no reason weak CP violation will not occur in this model. We shall illustrate this with some examples in Sections 4 and 6.

Determining the quark alignment matrices \( Q_{L,R} \) begins with minimizing the vacuum energy

\[
E_{\bar{\theta}q}(Q) \approx -\frac{1}{2} \text{Tr} (M_q Q + \text{h.c.}) \Delta_q (M_{ETC}) \tag{9}
\]

to find \( Q = Q_R Q_L \). Whether or not \( \bar{\theta}_q = 0 \), the matrix \( Q^\dagger M_q \) is hermitian up to the identity matrix.\(^1\)

\[
M_q Q - Q^\dagger M_q^\dagger = i\nu_q 1, \tag{10}
\]

where \( \nu_q \) is the Lagrange multiplier associated with the unimodularity constraint on \( Q \), and \( \nu_q \) vanishes if \( \bar{\theta}_q \) does. Thus, \( Q^\dagger M_q \) may be diagonalized by the single unitary transformation \( Q_R \) and so \( \bar{\theta}_q \)

\[
M_q \equiv \left( \begin{array}{cc}
M_u & 0 \\
0 & M_d
\end{array} \right) = Q_R^\dagger M_q QQ_R = Q_R^\dagger M_q Q_L. \tag{11}
\]

### 4 Quark Mass and Mixing Matrices in ETC/TC2

#### 4.1 General Considerations

If \( \bar{\theta}_q = 0 \), the matrix \( M_q \) is brought to real, positive, diagonal form by the block–diagonal \( SU(6) \) matrices \( Q_{L,R} \). From these, one constructs the CKM matrix \( V = U_L^\dagger D_L \). Carrying

\(^1\)I thank C. Sommerfield for this description.

\(^2\)Since quark vacuum alignment is based on first order chiral perturbation theory, it is inapplicable to the heavy quarks \( c, b, t \). When \( \bar{\theta}_q = 0 \), Dashen’s procedure is equivalent to making the mass matrix diagonal, real, and positive. Thus, it correctly determines the quark unitary matrices \( U_{L,R} \) and \( D_{L,R} \) and the magnitude of strong and weak CP violation.
out the vectorial phase changes on the $q_{L,R}$ required to put $V$ in the standard Harari–Leurer form with the single CP–violating phase $\delta_{13}$, one obtains

$$V \equiv \begin{pmatrix}
V_{ud} & V_{us} & V_{ub} \\
V_{cd} & V_{cs} & V_{cb} \\
V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_{12} & \sin \theta_{12} e^{i \delta_{13}} & 0 \\
-s\sin \theta_{12} & \cos \theta_{12} e^{i \delta_{13}} & 0 \\
-s\cos \theta_{12} & s\sin \theta_{12} e^{i \delta_{13}} & e^{-2i \delta_{13}}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_{23} & 0 & \sin \theta_{23} \\
0 & 1 & 0 \\
-\sin \theta_{23} & 0 & \cos \theta_{23}
\end{pmatrix}
\begin{pmatrix}
\cos \theta_{13} & \sin \theta_{13} e^{i \delta_{13}} & 0 \\
-s\sin \theta_{13} & \cos \theta_{13} e^{i \delta_{13}} & 0 \\
-s\cos \theta_{13} & s\sin \theta_{13} e^{i \delta_{13}} & e^{-2i \delta_{13}}
\end{pmatrix}.

(12)

Here, $s_{ij} = \sin \theta_{ij}$, and the angles $\theta_{12}$, $\theta_{23}$, $\theta_{13}$ lie in the first quadrant. Additional CP–
violating phases appear in $U_{L,R}$ and $D_{L,R}$ and they are rendered observable by ETC and TC2
interactions. We will study their contribution to $\epsilon$ in Section 6. Before that, we need
to discuss the constraints on $M_{u,d}$ and $U_{L,R}$, $D_{L,R}$ imposed by ETC and TC2.

First, limits on flavor–changing neutral current (FCNC) interactions, especially those
mediating $|\Delta S| = 2$, require that ETC bosons coupling to the two light generations
have masses $M_{ETC} \gtrsim 1000$ TeV. These can produce quark masses less than about
$m_{u} (M_{ETC}) \approx 100$ MeV in a walking technicolor theory (see the Appendix). Extended
 technicolor bosons as light as 50–100 TeV are needed to generate $m_{d} (M_{ETC}) \sim 3.5$ GeV.
Flavor–changing neutral current interactions mediated by such light ETC bosons must be
suppressed by small mixing angles between the third and the first two generations.

The most important feature of $M_{u}$ is that the TC2 component of $M_{u,1}$, $(m_{i})_{TC2} \simeq 160$ GeV,
is much larger than all its other elements, all of which are generated by ETC exchange. In particular, off-diagonal elements in the third row and column of $M_{u}$ are
expected to be no larger than the 0.01–1.0 GeV associated with $m_{u}$ and $m_{c}$. Thus, $M_{u}$ is
very nearly block–diagonal and, so, $|U_{L,R} u_{13}| \approx |U_{L,R} u_{14}| \approx \delta_{u_{13}}$.

The matrix $M_{d}$ has a triangular or nearly triangular structure. One reason for this is
the need to suppress $B_{d}–B_{d}$ mixing induced by the exchange of “bottom pions” of mass
$M_{\nu_{b}} \sim 300$ GeV. Furthermore, since $U_{L}$ is block–diagonal, the observed intergenera-
tional mixing in the CKM matrix must come from the down sector. These requirements are
met when the $d_{R}$, $s_{R} \leftrightarrow b_{L}$ elements of $M_{d}$ are much smaller than the $d_{L}$, $s_{L} \leftrightarrow b_{R}$ elements.

In Ref. [24] the strong topcolor $U(1)$ charges were chosen to exclude ETC interactions that
induce $M_{db}$ and $M_{bd}$. This makes $D_{R}$, like $U_{L,R}$, nearly $2 \times 2$ times $1 \times 1$ block–diagonal.

From these considerations and $V_{tb} \approx 1$, we have

$$V_{td} \approx V_{tb}^{*} V_{td} \approx U_{L,R}^{\ast} D_{L,R}^{\ast} U_{L,R}^{\ast} D_{L,R} \approx D_{L,R}^{\ast} D_{L,R}^{\ast} .

(13)

This relation, which is good to 10% (see Section 4.2 for examples), was used in Ref. [24] to
put strong limits on the TC2 $V_{8}$ and $Z'$ masses from $B_{d}–B_{d}$ mixing. We found that $M_{V_{8}}, M_{Z'} \gtrsim 5$ TeV $\gg (m_{1})_{TC2}$. This implies that the TC2 gauge couplings must be within 1%
or better of their critical values, a tuning we regard as unnaturally fine.

One more interesting property of the quark alignment matrices is this: The vacuum
energy $E_{Tq}$ is minimized when the elements of $U$ and $D$ have almost the same rational
phases as $M_{u}$ and $M_{d}$ do. In particular, all the large diagonal elements of $U,D$ have
rational phases (see Section 4.2). This is generally not true of $U_{L,R}$ and $D_{L,R}$ individually.
However, since $Q_{ii} = \sum_{j} Q_{Lij} Q_{Rij}^{\ast}$ ($Q = U,D$) has a rational phase, $E_{Tq}$ is likely to be
minimized when each term in the sum has the same rational phase. Thus, like DNA, in
which the patterns of the two strands are linked,

$$\arg Q_{Lij} - \arg Q_{Lik} = \arg Q_{Rij} - \arg Q_{Rik} \quad (\text{mod } \pi) \quad \text{for } i, j, k = u, c, t \text{ or } d, s, b. \quad (14)$$

In particular, \( \arg V_{id} \approx \arg D_{Lbd} \equiv \arg D_{Rbd} \equiv \arg D_{Rbb} \equiv \arg Q_{Rbb} \) (mod \( \pi \)) for \( d = d, s, b \).

4.2 Examples

Our proposal for solving the strong CP problem in technicolor theories rests on the fact that phases in the technifermion alignment matrices \( W = (W_U, W_D) \) can be difference rational multiples of \( \pi \), and on the conjecture that these phases may be mapped by ETC onto the primordial mass matrix \( (M_q)_{ij} = \Lambda_{ij}^T W_{ij}^T \Delta T \) so that \( \hat{\theta}_q = \arg \det(M_q) = 0 \). Corrections to \( \hat{\theta}_q \) are expected to be at most \( O(10^{-10}) \). In this section we present two examples of quark mass matrices for which we have engineered \( \hat{\theta}_q = 0 \). They lead to similar alignment and CKM matrices, except that one example has \( \delta_{13} = 0 \). Nevertheless, as we see in Section 6, both sections lead to successful calculations of CP-violating parameter \( \epsilon \).

**Model 1:**

In this model, \( \delta_{13} = 0 \), but CP violation will arise from phases in \( U_{L,R} \) and \( D_{L,R} \). The primordial quark mass matrices renormalized at \( M_{ETC} \) are taken to be of seesaw form with phases that are multiples of \( \pi/3 \):

\[
M_u = \begin{pmatrix}
(0, 0) & (200, 1/3) & (0, 0) \\
(15.6, -1/3) & (900, 1) & (0, 0) \\
(0, 0) & (0, 0) & (162620, 0)
\end{pmatrix}
\]

\[
M_d = \begin{pmatrix}
(0, 0) & (23.3, 0) & (0, 0) \\
(21.7, 0) & (102, 1/3) & (0, 0) \\
(17.0, 1/3) & (144, 2/3) & (3505, 0)
\end{pmatrix}
\]

The notation is \( |(M_q)_{ij}|, \arg[(M_q)_{ij}]/\pi \). Here, we have made \( \arg \det(M_u) = \arg \det(M_d) = \pi \). We imposed the same kind of structure on \( M_u \) as \( B_u - B_d \) mixing requires of \( M_d \). The quark mass eigenvalues may be extracted from \( M_q \). Their values at \( M_{ETC} \sim 10^4 \) TeV are (in MeV):

\[
m_u = 3.35, \quad m_c = 924, \quad m_t = 162620
\]

\[
m_d = 4.74, \quad m_s = 106, \quad m_b = 3508
\]

The alignment matrices \( U = U_L^T U_R \) and \( D = D_L^T D_R \) obtained by minimizing \( E_{Tq} \) are

\[
U = \begin{pmatrix}
(0.973, 0) & (0.232, 1/3) & (0, 0) \\
(0.232, -1/3) & (0.973, 1) & (0, 0) \\
(0, 0) & (0, 0) & (1, 0)
\end{pmatrix}
\]

\[
D = \begin{pmatrix}
(0.915, -2/3) & (0.404, 0) & (0.0046, -1/3) \\
(0.404, 0) & (0.914, -1/3) & (0.0400, -2/3) \\
(0.0119, -1/3) & (0.0384, -2/3) & (0.999, 0)
\end{pmatrix}
\]
The cloning of the $M_{u,d}$ phases onto $U,D$ is apparent. Diagonalizing the aligned quark mass matrices yields $Q_{L,R}$:

$$U_L = \begin{pmatrix}
(0.9999, -0.859) & (0.0164, 0.141) & (0, 0) \\
(0.164, -1.193) & (0.9999, -1.193) & (0, 0) \\
(0, 0) & (0, 0) & (1, -0.526)
\end{pmatrix}$$

$$D_L = \begin{pmatrix}
(0.980, 1.141) & (0.199, 1.141) & (0.00485, 1.141) \\
(0.199, -0.192) & (0.979, 0.808) & (0.0412, 0.808) \\
(0.00344, -0.526) & (0.0413, 0.474) & (0.999, -0.526)
\end{pmatrix}$$

$$U_R = \begin{pmatrix}
(0.976, -0.859) & (0.216, -0.859) & (0, 0) \\
(0.216, -1.193) & (0.976, -0.192) & (0, 0) \\
(0, 0) & (0, 0) & (1, -0.526)
\end{pmatrix}$$

$$D_R = \begin{pmatrix}
(0.977, -0.192) & (0.214, 0.808) & (0.000273, 0.808) \\
(0.214, 1.141) & (0.977, 1.141) & (0.00122, 1.141) \\
(5 \times 10^{-6}, -0.526) & (0.00125, 0.474) & (1, -0.526)
\end{pmatrix}.$$  \hfill (18)

As required, all the mixing in $U_{L,R}$ and $D_R$ is between the first two generations; mixing of these two with the third generation comes entirely from $D_L$. A perusal of the phases will reveal differences which are multiples of $\pi/3$. Finally, the CKM matrix is

$$V = \begin{pmatrix}
(0.977, 0) & (0.215, 0) & (0.00552, 0) \\
(0.215, 1) & (0.976, 0) & (0.0411, 0) \\
(0.00344, 0) & (0.0413, 1) & (0.999, 0)
\end{pmatrix}.$$  \hfill (19)

Note its similarity to $D_L$ (including phase differences). This corresponds to the angles

$$\theta_{12} = 0.217, \; \theta_{23} = 0.0411, \; \theta_{13} = 0.00552, \; \delta_{13} = 0.$$  \hfill (20)

The angles $\theta_{ij}$ are in good agreement with those in the Particle Data Group’s book.\footnote{We will see in Section 6 that, even though $\delta_{13} = 0$, the CP-violating angles in $D_{L,R}$ can easily account for the measured value of $\epsilon$.}

**Model 2:**

The second model is based on a $W$–matrix whose phases are multiples of $\pi/5$. The primordial quark mass matrices renormalized at $M_{ETC}$ are again taken to be of seesaw form, but we allow off–diagonal terms $|M_{ij}| \sim \sqrt{|M_{ii}M_{jj}|}$ (all masses refer to the ETC contribution only):

$$M_u = \begin{pmatrix}
(7, 0.2) & (2, -0.4) & (0, 0) \\
(100, 0.4) & (890, -0.2) & (0, 0) \\
(50, -0.4) & (500, 0.2) & (160000, 0)
\end{pmatrix}$$

$$M_d = \begin{pmatrix}
(8, 0) & (1, -0.2) & (0, 0) \\
(25, -0.2) & (100, -0.4) & (0, 0) \\
(10, 0) & (140, -0.4) & (3500, 0.4)
\end{pmatrix}.$$  \hfill (21)
Again, the cloning of the M_q book. In this model, the CKM matrix is (compare it to D

\begin{equation}
M = 6.84, \ m_c = 896, \ m_t = 160000 \\
m_d = 7.52, \ m_s = 103, \ m_b = 3503 
\end{equation}

Here, we have made \( \arg \det(\mathcal{M}_u) = \arg \det(\mathcal{M}_d) = 0 \). We again imposed the same kind of structure on \( \mathcal{M}_u \) as \( \bar{B}_d - B_d \) mixing requires of \( \mathcal{M}_d \). The quark mass eigenvalues are (in MeV):

\begin{align*}
U &= \begin{pmatrix}
(0.994, -0.2) & (0.110, -0.4) & (0.00031, 0.4) \\
(0.110, -0.6) & (0.994, 0.2) & (0.0031, -0.2) \\
(0.00062, 0.505) & (0.00306, -0.6) & (1, 0)
\end{pmatrix} \\
D &= \begin{pmatrix}
(0.976, 0) & (0.217, 0.2) & (0.00265, -0.0178) \\
(0.217, -0.8) & (0.975, 0.4) & (0.0389, 0.4) \\
(0.00664, -0.679) & (0.0384, 0.603) & (0.999, -0.4)
\end{pmatrix} 
\end{align*}

The alignment matrices \( U = U_L^\dagger U_R \) and \( D = D_L^\dagger D_R \) obtained by minimizing \( E_{Tq} \) are

\begin{align*}
U_L &= \begin{pmatrix}
(0.994, 0.873) & (0.112, 0.336) & (0.00031, 0.535) \\
(0.112, 0.472) & (0.994, 0.936) & (0.00313, -0.0652) \\
(0.00063, -0.422) & (0.00308, 0.138) & (1, 0.135)
\end{pmatrix} \\
D_L &= \begin{pmatrix}
(0.970, 0.881) & (0.245, 0.727) & (0.00286, 0.535) \\
(0.245, 0.0810) & (0.969, 0.927) & (0.0400, 0.936) \\
(0.00771, 0.213) & (0.0394, 1.131) & (0.999, 0.135)
\end{pmatrix} \\
U_R &= \begin{pmatrix}
(1, 1.073) & (0.00198, 0.534) & (0, 0) \\
(0.00198, 0.274) & (1, 0.736) & (1.8 \times 10^{-5}, -0.322) \\
(0, 0) & (1.8 \times 10^{-5}, 0.192) & (1, 0.135)
\end{pmatrix} \\
D_R &= \begin{pmatrix}
(1, 0.881) & (0.0284, 0.727) & (1.7 \times 10^{-5}, 0.661) \\
(0.0284, -0.319) & (1, 0.527) & (0.00116, 0.531) \\
(1.7 \times 10^{-5}, 0.611) & (0.00116, -0.470) & (1, 0.535)
\end{pmatrix} 
\end{align*}

Again, the cloning of the \( M_{u,d} \) phases onto the large elements of \( U, D \) is apparent. The \( Q_{L,R} \) are:

\begin{align*}
\theta_{12} = 0.236, \ &\theta_{23} = 0.0431, \ &\theta_{13} = 0.00315, \ &\delta_{13} = -0.957.
\end{align*}

The CKM matrix is (compare it to \( D_L \))

\begin{equation}
V = \begin{pmatrix}
(0.972, 0) & (0.234, 0) & (0.00315, 0.305) \\
(0.233, 0.9999) & (0.971, 8.6 \times 10^{-6}) & (0.0431, 0) \\
(0.00867, 0.0930) & (0.0423, 0.995) & (0.999, 0)
\end{pmatrix} 
\end{equation}

Again, the angles \( \theta_{ij} \) are in reasonable agreement with those in the Particle Data Group’s book. In this model, \( \delta_{13} \) is large.
5 ETC and TC2 Four–Fermion Interactions

The FCNC effects that concern us arise from four–quark interactions induced by the exchange of heavy ETC gauge bosons and of TC2 color–octet “colorons” $V_8$ and color–singlet $Z'$. Lepton interactions are not dealt with here.

At low energies and to lowest order in $\alpha_{ETC}$, the ETC interaction involves products of chiral currents. Still assuming that the ETC gauge group commutes with electroweak $SU(2)$, it has the form

$$\mathcal{H}_{ETC} = \Lambda^{L}_{ijkl} (u'_L i\gamma^\mu u'_L + \bar{d}'_L i\gamma^\mu d'_L) + (u'_L i\gamma^\mu u'_L + \bar{d}'_L i\gamma^\mu d'_L) (\Lambda^{L}_{ijkl} u'_L i\gamma^\mu u'_L + \Lambda^{d,LR}_{ijkl} d'_R \gamma^\mu d'_L)$$

where primed fields are electroweak eigenstates. The ETC gauge group contains technicolor, color and topcolor, and flavor as commuting subgroups. It follows that the flavor currents in $\mathcal{H}_{ETC}$ are color and topcolor singlets. The $\Lambda$’s in $\mathcal{H}_{ETC}$ are of order $g_{ETC}/M_{ETC}^2$, whose magnitude is discussed below, and the operators are renormalized at $M_{ETC}$. Hermiticity of $\mathcal{H}_{ETC}$ implies that $\Lambda_{ijkl} = \Lambda_{jikl}$. We assume that this primordial ETC interaction conserves CP, i.e., that all the $\Lambda$’s are real. When written in terms of mass eigenstate fields $q_{L,R} = \sum_j (Q_{L,R}^i)_{ij} q'_{L,R}$ with $Q = U, D$, an individual four–quark term in $\mathcal{H}_{ETC}$ has the form

$$\left( \sum_{i,j',k',l'} \Lambda_{ij'k'l}^{q_1q_2\lambda_1\lambda_2} Q_{ij'}^l Q_{ij}^{\lambda_1} Q_{ij}^{\lambda_2} Q_{ij}^{\lambda_2} \right) \tilde{q}_{\lambda_1} \gamma^\mu q_{\lambda_2} \bar{q}_{\lambda_2} \gamma_\mu q_{\lambda_2}$$.

A reasonable and time–honored guess for the magnitude of the $\Lambda_{ijkl}$ is that they are comparable to the ETC masses that generate the quark mass matrix $M_q$. We elevate this to a rule: The ETC scale $M_{ETC}/g_{ETC}$ in a term involving weak eigenstates of the form $q'_i \bar{q}'_i \bar{q}'_j \bar{q}'_j$ (for $q'_i = u'_i$ or $d'_i$) is approximately the same as the scale that generates the $\bar{q}_{Ri} q_{Li}$ mass term, $M_{ij}$. A plausible, but approximate, scheme for correlating a quark mass $m_q(M_{ETC})$ with $M_{ETC}/g_{ETC}$ is presented in the Appendix. The results are shown in Fig. 1. There, $\kappa > 1$ parameterizes the departure from the strict walking technicolor limit; i.e., $\alpha_{TC} = \text{constant}$ and the anomalous dimension $\gamma_m$ of $TT$ equals one up to the highest ETC mass scale divided by $k$, and $\gamma_m = 0$ beyond that. The ETC masses run from $M_{ETC}/g_{ETC} = 46$ TeV for $m_q = 5$ GeV to $2.34/\kappa \times 10^4$ TeV for $m_q = 10$ MeV. We rely on Fig. 1 for estimating the $\Lambda$’s in $\mathcal{H}_{ETC}$.

Extended technicolor masses, $M_{ETC}/g_{ETC} \gtrsim 1000$ TeV, are necessary, but not sufficient, to suppress FCNC interactions of light quarks to an acceptable level. This is especially true for $\Delta M^0_R$ and $\epsilon_{111}$ Thus, we assume that $\mathcal{H}_{ETC}$ is electroweak generation conserving, i.e.,

$$\Lambda_{ijkl}^{q_1q_2\lambda_1\lambda_2} = \delta_{i1} \delta_{j1} \Lambda_{ijkl}^{q_1q_2\lambda_1\lambda_2} + \delta_{i2} \delta_{j2} \Lambda_{ijkl}^{q_1q_2\lambda_1\lambda_2}$$.

Considerable FCNC suppression then comes from off–diagonal elements in the alignment matrices $Q_{L,R}$.

In all TC2 models, color $SU(3)_c$ and weak hypercharge $U(1)_Y$ arise from the breakdown of the topcolor groups $SU(3)_1 \otimes SU(3)_2$ and $U(1)_1 \otimes U(1)_2$ to their diagonal subgroups.
Here, $SU(3)_1$ and $U(1)_1$ are strongly coupled, $SU(3)_2$ and $U(1)_2$ are weakly coupled, with the color and weak hypercharge couplings given by

$$
g_C = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \equiv \frac{g_1 g_2}{g_V} = g_2 \cos \theta_C \simeq g_2;
$$

$$
g_Y = \frac{g_1' g_2'}{\sqrt{g_1'^2 + g_2'^2}} \equiv \frac{g_1' g_2'}{g_Y} = g_2' \cos \theta_Y \simeq g_2'. \quad (30)
$$

Top and bottom quarks are $SU(3)_1$ triplets. The broken topcolor interactions are mediated by a color octet of colorons, $V_8$, and a color singlet $Z'$ boson, respectively. By virtue of the different $U(1)_1$ couplings of $t_R$ and $b_R$, exchange of $V_8$ and $Z'$ between third generation quarks generates a large contribution, $(m_t)_{TC2} \simeq 160 \text{ GeV}$, to the top mass, but none to the bottom mass.

If topcolor is to provide a natural explanation of $(m_t)_{TC2}$, the $V_8$ and $Z'$ masses ought to be $\mathcal{O}(1 \text{ TeV})$. In the Nambu–Jona-Lasinio (NJL) approximation, the degree to which this naturalness criterion is met is quantified by the ratio

$$
\frac{\alpha(V_8) + \alpha(Z') - (\alpha^*(V_8) + \alpha^*(Z'))}{\alpha^*(V_8) + \alpha^*(Z')} = \frac{\alpha(V_8) r_{V_8} + \alpha(Z') r_{Z'}}{\alpha(V_8)(1 - r_{V_8}) + \alpha(Z')(1 - r_{Z'})}. \quad (31)
$$
Here,

$$\alpha(V_8) = \frac{4\alpha V_8 \cos^4 \theta_C}{3\pi} = \frac{4\alpha_C \cot^2 \theta_C}{3\pi},$$

$$\alpha(Z') = \frac{\alpha_Z Y_{t_L} Y_{t_R} \cos^4 \theta_Y}{\pi} = \frac{\alpha_Y Y_{t_L} Y_{t_R} \cot^2 \theta_Y}{\pi};$$

$$\tan \theta_C = \frac{g_2}{g_1}, \quad \tan \theta_Y = \frac{g_2'}{g_1'}, \quad r_i = \frac{(m_i^2)_{TC2}}{M^2_{i^\prime}} \ln \left(\frac{M^2_{i^\prime}}{(m_i^2)_{TC2}}\right), \quad (i = V_8, Z');$$

and $Y_{t_{L,R}}$ are the $U(1)_t$ charges of $t_{L,R}$. The NJL condition on the critical couplings for top condensation is $\alpha^*(V_8) + \alpha^*(Z') = 1$. In Ref.\textsuperscript{[1]} we showed that, for such large couplings, TC2 is tightly constrained by the magnitude of $B_d - B_s$ mixing, requiring $M_{V_8} \simeq M_{Z'} > 5$ TeV. This implies that the topcolor coupling $\alpha(V_8) + \alpha(Z')$ must be within less than 1% of its critical value, a tuning we regard as unnaturally fine. Other limits on $M_{V_8}$ were obtained in Refs.\textsuperscript{[2]} One way to eliminate this fine–tuning problem is to invoke the “top seesaw” mechanism in which the topcolor interactions operate on a quark whose mass is several TeV, and the top’s mass comes to it by a seesaw mechanism\textsuperscript{[3]}. There are two variants of TC2: The “standard” version\textsuperscript{[4]} in which only the third generation quarks are $SU(3)$ triplets, and the “flavor–universal” version\textsuperscript{[5]} in which all quarks are $SU(3)$ triplets. In standard TC2, $V_8$ and $Z'$ exchange gives rise to FCNC that mediate $|\Delta S| = 2$ and $|\Delta B| = 2$. In flavor–universal TC2, only $Z'$ exchange generates such FCNC. We shall write the four–quark interaction for standard TC2, but our results apply to $Z'$ exchange interactions in flavor–universal TC2 as well.

The TC2 interaction at energies well below $M_{V_8}$ and $M_{Z'}$ is

$$\mathcal{H}_{TC2} = \frac{g_{V_8}^2}{2M_{V_8}^2} \sum_{A=1}^{8} J^A_{\mu} J^A_{\mu} + \frac{g_{Z'}^2}{2M_{Z'}^2} J^a_{Z\prime \mu} J^a_{Z\prime \mu}.$$ \hspace{1cm} (33)

The coloron and $Z'$ currents written in terms of electroweak eigenstate fields are given by (color indices are suppressed)

$$J^A_\mu = \cos^2 \theta_C \sum_{i_t,b} \bar{q}^i_{\gamma \mu} \frac{\lambda_A}{2} q^i - \sin^2 \theta_C \sum_{i=d,c,s} \bar{q}^i_{\gamma \mu} \frac{\lambda_A}{2} q^i;$$

$$J^a_{Z' \mu} = \cos^2 \theta_Y J^a_\mu - \sin^2 \theta_Y J^a_\mu \equiv \sum_{i=L,R} \sum_{i} \left(\cos^2 \theta_Y Y_{i\lambda i} - \sin^2 \theta_Y Y_{2\lambda i}\right) \bar{q}^i_{\lambda \gamma \mu} q^i_{\lambda i}.$$ \hspace{1.5cm} (34)

The $U(1)_1$ and $U(1)_2$ hypercharges satisfy $Y_{i\lambda i} + Y_{2\lambda i} = Y_{i\lambda i} = 1/6, Q_{EM}$ for $\lambda = L, R$. Consistency with $SU(2)$ symmetry requires $Y_{Lt} = Y_{Lb},$ etc. The suppression of light quark FCNC requires $Y_{1Li} = Y_{1Li}$ for $i = u, d, c, s$ and $Y_{1Li} = Y_{1Li}$. Remaining FCNC are suppressed by small mixing angles.

6 ETC and TC2 Contributions to the CP–Violating Parameter $\epsilon$

The CP–violating parameter $\epsilon$ is defined by

$$\epsilon = \frac{A(K_L \to (\pi\pi)_{I=0})}{A(K_S \to (\pi\pi)_{I=0})} = \frac{e^{i\pi/4} \text{Im} M_{ KL}}{\sqrt{2} \Delta M_K},$$ \hspace{1cm} (35)
where $2M_KM_{12} = \langle K^0|H_{\Delta S}=2|K^0\rangle$ and we use the phase convention that $A_0$, $(\pi\pi)^{-\frac{1}{4}}H_{\Delta S}=1|K^0\rangle$ is real. Experimentally, $\epsilon = (2.271 \pm 0.017) \times 10^{-3}\exp(i\pi/4)$

The standard model contribution to $\epsilon$ is

$$\epsilon_{SM} = \frac{\epsilon_{\pi}^4 f_K^2 M_K^2 \hat{B}_K |M_K|}{3\sqrt{2}\Delta M_K} \text{Im} \left[ \lambda_2^2 \eta_1 S_0(x_c) + \lambda_2^4 \eta_2 S_0(x_t) + 2\lambda_2^4 \lambda_3 \eta_3 S_0(x_c, x_t) \right],$$

(36)

where $f_K = 112\text{MeV}$ is the kaon decay constant, $\hat{B}_K = 0.80 \pm 0.15$ is the kaon bag parameter, $\lambda_i = \lambda(t)$, and the other quantities are defined in Ref.13.

Despite the large ETC gauge boson masses of several 1000 TeV and the stringent $\bar{B}_d - B_d$ mixing constraint leading to TC2 gauge masses of at least 5 TeV, both interactions can contribute significantly to $\epsilon$. The main ETC contribution comes from $s's's's'$ interactions and is given by

$$\epsilon_{ETC} \simeq \frac{\epsilon_{\pi}^4 f_K^2 M_K^2 \hat{B}_K}{3\sqrt{2}\Delta M_K} \left\{ -\left( \frac{M_K}{m_s + m_d} \right)^2 + \frac{3}{2} \right\} \Lambda_{ss}^{LR} \text{Im} \left( D_{Lss} D_{Rss}^* D_{Lsd} D_{Rsd}^* \right)$$

$$+ 2 \Lambda_{ss}^{LL} \text{Im} \left( D_{Lss}^* D_{Rss}^* D_{Lsd}^* \right) + \Lambda_{ss}^{RR} \text{Im} \left( D_{Rss}^* D_{Rsd}^* \right) \right\}.$$  

(37)

Note the suppression of $O((\theta_1^2)^2)$ from mixing angle factors. This $s's's's'$ contribution as well as the those from the standard model and TC2 vanish for Model 1. For that model, $\text{Im}(M_{12})_{ETC}$ comes from $s'd'd's'$ terms and has a form similar to Eq. (37).

The dominant TC2 contribution comes from $b'^*_L b'^*_L b'^*_L b'^*_L$ interactions; terms involving $b'^*_R$ are suppressed by the very small $D_{Rbs}$ and $D_{Rbd}$:

$$\epsilon_{TC2} \simeq \frac{\epsilon_{\pi}^4 f_K^2 M_K^2 \hat{B}_K}{3\sqrt{2}\Delta M_K} \left[ \frac{\alpha_C \cot^2 \theta_C}{M_{s_b}^2} + \frac{\alpha_H (\Delta Y_L)^2 \cot^2 \theta_Y}{M_{Z'}^2} \right] \text{Im} \left( D_{Lbs}^2 D_{Lbd}^* \right).$$

(38)

The couplings and mixing angles were defined in Eq. (22) and $\Delta Y_L = Y_{bs} - Y_{bl} - Y_{bs} - Y_{sl}$ is a difference of strong $U(1)$ hypercharges; we take $\alpha_C \cot^2 \theta_C = \alpha_H (\Delta Y_L)^2 \cot^2 \theta_Y = \frac{3\pi}{8}$.13

The various contributions to $\epsilon$ for several different “models” of the primordial quark mass matrix $M_q$ are given in Table 1. Models 1 and 2 are present as are some related ones (e.g., model 2' is similar to model 2, but the complex conjugate input $M_{q'}$ is used; differences apart from signs are due to computer round-off error). As indicated, typical ETC masses from Fig. 1 are used for $\Lambda_{ss}$ and $\Lambda_{sd}$ (the latter for model 1 only). Note that model 1 accounts very well for $\epsilon$ from ETC interactions alone. ETC interactions that lead to models 1' and 2' are ruled out. In the other models, $\epsilon$ is easily accounted for because large cancellations occur among the ETC contributions or between ETC and TC2 contributions. These would be disturbing if we had not already seen them in a standard context: the large cancellations between QCD and electroweak penguin terms in the calculation of $\epsilon'/\epsilon$.14

7 Summary and Conclusions

We have presented a dynamical picture of CP nonconservation arising from vacuum alignment in extended technicolor theories. This picture leads naturally to a mechanism for evading strong CP violation without an axion or a massless up quark. We derived complex...
Table 1. Contributions to $\epsilon e^{-i\pi/4} \times 10^3$ for various “models” of $\mathcal{M}_q$ with $\theta_q = 0$. Unless otherwise indicated, all $\Lambda_{ss} = (2000 \text{ TeV})^{-2}$ and $M_{Z'} = 10 \text{ TeV}$.

| Model | $\Lambda_{sd}$ | (ETC)LR | (ETC)LL | (ETC)RR | (TC2)LL | Comments |
|-------|----------------|---------|---------|---------|---------|----------|
| 1     | 0              | 2.38    | 0       | 0       | 0       | Fit for $\Lambda_{sd} = (4000 \text{ TeV})^{-2}$ |
| 1'    | 2.28           | 9.61    | 0.88    | 1.02    | 8.34    | Fit for $M_{ETC} \to \infty$, $M_{Z'} \to \infty$ |
| 2     | -1.98          | 9.44    | -7.68   | -0.11   | -4.57   | $\epsilon_{ETC+TC2} = 4.22$ for $\Lambda_{ss} = (1250 \text{ TeV})^{-2}$, $M_{Z'} \to \infty$ |
| 2'    | 1.97           | -6.30   | 7.76    | 0.05    | 4.52    | Approximate fit for $M_{ETC} \to \infty$, $M_{Z'} \to \infty$ |
| 2''   | -2.02          | 31.25   | -7.92   | -1.21   | -4.68   | $\epsilon_{ETC+TC2} = 4.33$ for $\Lambda_{ss} = (2500 \text{ TeV})^{-2}$, $M_{Z'} = 6.9 \text{ TeV}$ |
| 3     | 2.18           | -8.94   | -0.97   | -0.80   | 8.20    | $\epsilon_{ETC+TC2} = 0.10$ for $M_{Z'} = 8.7 \text{ TeV}$ |


quark mixing matrices from ETC/TC2–based constraints on the primordial mass matrices $\mathcal{M}_u$ and $\mathcal{M}_d$. These led to very realistic–looking CKM matrices. We categorized 4–quark contact interactions arising from ETC and TC2 and proposed a scheme for estimating the strengths of these interactions. Putting this together with the quark mixing matrices, we calculated the contributions to the CP–violating parameter $\epsilon$, obtaining quite good (or powerfully constraining) results for a variety of “models” of $\mathcal{M}_q$. Future work will include calculating $\epsilon'/\epsilon$ and $\sin(2\beta)$ in these models.

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Appendix. ETC Gauge Boson Mass Scales

To set the ETC mass scales that enter $\mathcal{H}_{ETC}$ in Eq. (27), we assume a model containing $N$ identical electroweak doublets of technifermions. The technipion decay constant (which helps set the technicolor energy scale) is then $F_T = F_\pi/\sqrt{N}$, where $F_\pi = 246 \text{ GeV}$ is the fundamental weak scale. We estimate the ETC masses in $\mathcal{H}_{ETC}$ by the rule stated in Section 5: The ETC scale $M_{ETC}/g_{ETC}$ in a term involving weak eigenstates of the form $\bar{q}_i'q_j'\bar{q}_i'q_j'$ or $\bar{q}_i'q_j'q_j'$ (for $q'_i = u_i'$ or $d_i'$) is approximately the same as the scale that generates the $\bar{q}_Rq_L$ mass term, $(\mathcal{M}_q)_{ij}$.

The ETC gauge boson mass $M_{ETC}(q)$ giving rise to a quark mass $m_q(M_{ETC})$—an
element or eigenvalue of $M_q$—is defined by

$$m_q(M_{ETC}) \simeq \frac{g_{ETC}^2}{M_{ETC}^2(q)} \langle \bar{T}T \rangle_{ETC}. \quad (39)$$

Here, the quark mass and the technifermion bilinear condensate, $\langle \bar{T}T \rangle_{ETC}$, are renormalized at the scale $M_{ETC}(q)$. The condensate is related to the one renormalized at the technicolor scale $\Lambda_{TC} \simeq F_T$ by the equation

$$\langle \bar{T}T \rangle_{ETC} = \langle \bar{T}T \rangle_{TC} \exp \left( \int_{\Lambda_{TC}}^{M_{ETC}(q)} \frac{d\mu}{\mu} \gamma_m(\mu) \right). \quad (40)$$

Scaling from QCD, we expect

$$\langle \bar{T}T \rangle_{TC} \equiv \Delta_T \simeq 4\pi F_T^3 = 4\pi F_\pi^3 / N_{TC}^{3/2}. \quad (41)$$

The anomalous dimension $\gamma_m$ of the operator $\bar{T}T$ is given in perturbation theory by

$$\gamma_m(\mu) = \frac{3C_2(R)}{2\pi} \alpha_{TC}(\mu) + O(\alpha_{TC}^2). \quad (42)$$

where $C_2(R)$ is the quadratic Casimir of the technifermion $SU(N_{TC})$ representation $R$. For the fundamental representation of $SU(N_{TC})$ to which we assume our technifermions $T$ belong, it is $C_2(N_{TC}) = (N_{TC}^2 - 1) / 2N_{TC}$. In a walking technicolor theory, however, the coupling $\alpha_{TC}(\mu)$ decreases very slowly from its critical chiral symmetry breaking value at $\Lambda_{TC}$, and $\gamma_m(\mu) \simeq 1$ for $\Lambda_{TC} < \mu < M_{ETC}$. An accurate evaluation of the condensate enhancement integral in Eq. (40) requires detailed specification of the technicolor model and knowledge of the $\beta(\alpha_{TC})$–function for large coupling. Lacking this, we estimate the enhancement by assuming that

$$\gamma_m(\mu) = \begin{cases} 1 & \text{for } \Lambda_{TC} < \mu < M_{ETC}/\kappa^2 \\ 0 & \text{for } \mu > M_{ETC}/\kappa^2 \end{cases} \quad (43)$$

Here, $M_{ETC}$ is the largest ETC scale, i.e., the one generating the smallest term in the quark mass matrix for $\kappa = 1$. The number $\kappa > 1$ parameterizes the departure from the strict walking limit (i.e., $\gamma_m = 1$ constant all the way up to $M_{ETC}/\kappa^2$). Then, using Eqs. (39,40), we obtain

$$\frac{M_{ETC}(q)}{g_{ETC}} = \begin{cases} \sqrt{\frac{64\pi^3\alpha_{ETC}}{\kappa^2 N_{mq}}} \frac{F_T^2}{N_{mq}} & \text{if } M_{ETC}(q) < M_{ETC}/\kappa^2 \\ \sqrt{\frac{4\pi M_{ETC} F_T^2}{\kappa^2 N_{mq}}} & \text{if } M_{ETC}(q) > M_{ETC}/\kappa^2 \end{cases} \quad (44)$$

To evaluate this, we take $\alpha_{ETC} = 3/4$, a moderately strong value as would be expected in walking technicolor, and $N = 10$, a typical number of doublets in TC2 models with topcolor breaking. Then, taking the smallest quark mass at the ETC scale to be 10 MeV, we find $M_{ETC} = 7.17 \times 10^3$ TeV. The resulting estimates of $M_{ETC}/g_{ETC}$ were plotted in Fig. 1 for $\kappa = 1$, $\sqrt{10}$, and 10. They run from $M_{ETC}/g_{ETC} = 46$ TeV for $m_q = 10$ GeV to $2.34 / \kappa \times 10^3$ TeV/ for $m_q = 10$ MeV. Very similar results are obtained for $\alpha_{ETC} = 1/2$ and $N = 8$.

\footnote{See Ref. [4] for an attempt to calculate this integral in a walking technicolor model.}
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