Tapered three-stage deployable tensegrity model

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Abstract. Tensegrity consisting pinned-jointed struts and cables is lightweight and flexible. Extensive research works on the shape change of tensegrity, especially the deployable tensegrity structures and tensegrity robots have been carried out. This paper presents the structural characteristics of a tapered three-stage tensegrity model during shape change analysis by using a shape change method. The method employed sequential quadratic programming in the optimization of forced elongation in cables, such that the model can advance to the targets. Structural characteristics of the tapered tensegrity model were examined under various displacement schemes. It was found that the tensegrity model demonstrated bending, axial and torsional deformations during the shape change analysis.

1. Introduction
A tensegrity system is defined by Motro as a system in a stable self-equilibrated state comprising a discontinuous set of compressed components inside a continuum of tensioned components [1]. Tensegrity has also been quoted as “islands of compression in an ocean of tension” by Fuller. Since the introduction of the system in 1950s, the system has been suggested in many applications such as roofs, towers, bridges, sculptures and even biotensegrity. The development of tensegrity has advanced to active deployable structures and robots in recent years. This development is credited to the characteristics of tensegrity for being lightweight and flexible. The abilities of tensegrity systems in altering their configurations and producing large displacement have been proven. Simple joints (pinned jointed), controllable cables and the ability of the cables to act as actuators or sensors make tensegrity structures fit for the design of deployable structures and tensegrity robots.

Furuya was the first who suggest the principle of tensegrity for deployable structures. An optimization method to determine the reference trajectory for deployment of an arbitrary tensegrity structure was developed by van de Wijdeven and de Jager [2]. The deployment of tensegrity structure at minimal energy and without external control was investigated numerically by Li et al. [3]. A control methodology for a deployment of a near full-scale deployable tensegrity footbridge was proposed by Veuve et al. [4]. The control method aimed to improve a number of deployment paths and control the shape of a structure after a perturbation or damage. Scolamiero patented a deployable tensegrity structure specifically for space applications [5]. Figure 1(a) shows a perspective view of a support structure of the patented deployable tensegrity structure. Tensegrity principle applied in the
construction of deployable structures can reduce the risk of failure during deployment.

On the other hand, tensegrity systems have been widely suggested as robotic tools. Paul et al. investigated the forward locomotion of triangular and quadrilateral tensegrity prisms [6]. Shibata examined the rolling ability of a six-strut tensegrity robot experimentally [7]. Moored et al. proposed an active tensegrity inspired by flapping manta ray as manipulator to drive an underwater vehicle [8]. Carreño presented design and prototype of a novel efficiency and simple control wheeled tensegrity robot [9], and this robot was designed with the goal of traversing air ducts. Zhang et al. evaluated deep reinforcement learning approach on locomotion of SUPERball tensegrity robot (figure 1(b)) with changes in system parameters such as sensor measurements and environmental conditions [10]. Studies on shape change of tensegrity structures mainly focused on the deployable procedures and gait controls of the tensegrity deployable structures and robots.

This paper presents the structural characteristics of a tapered three-stage tensegrity model using a shape change method. The shape change method ensures the tensegrity model to displace in a set of prescribed target coordinates. The remainder of the paper is organized as follows. Section 2 presents the equations for the proposed shape change method. Section 3 shows the numerical examples for tapered three-stage tensegrity model. Section 4 presents the structural characteristics of the tensegrity model during and after shape change analysis. Finally, section 4 shows the concluding remarks for the paper.

![Figure 1](image)

**Figure 1.** Application of tensegrity (a) tensegrity support [5], and (b) SUPERball tensegrity robot.

### 2. Method for shape change analysis

This section describes the method for shape change analysis of tensegrity models. Equations used in the shape change analysis are also presented in the section. The proposed shape change algorithm is summarized in figure 2.

The shape change analysis started with the preparation of several input parameters for tensegrity model. For instance, geometrical properties such as topology of tensegrity model could be obtained from current available form-finding method [11]. Selection of available materials for tensegrity model such as steel, aluminium or timber can be dependent on the strength, aesthetic or economic value of the design. Other information such as boundary conditions of the model (i.e. loadings and support conditions), coordinates of the monitored nodes and target displacements are also needed before the analysis. In this study, unconstrained nodes of tensegrity model are chosen as monitored nodes where these nodes are required to reach a set of prescribed target displacements in any magnitudes and directional modes. The target coordinates of monitored nodes are denoted as $\mathbf{x}$. 
The main objective function of the shape change analysis is to make sure the advancement of the analysis for the monitored nodes to approach to the target coordinates. The objective function is also set as one of the termination criteria in the analysis which can be expressed as:

$$\min f(x) = \left| x - \bar{x} \right|$$  \hspace{1cm} (1)

where $x$ is the prescribed target coordinates, and $x'$ is the current coordinate, for all the specified monitored nodes at current step during the shape change analysis.  

In the shape change analysis, change in length in any cables of a tensegrity model other than elastic elongation is allowed. This change in length in cables is termed as forced elongation $\Delta l$. Forced elongation is not experienced in the cables that are constrained at both ends during the shape change analysis. Sequential Quadratic Programming method is used to optimize the forced elongation of cables as well as to solve the problem in equation (1) in the shape change analysis. The optimization programming can be written as

$$\min_{\Delta l} f(x) = g^T \Delta l + \frac{1}{2} \Delta l^T H \Delta l$$

$$g = P_2^T \left( x - \bar{x} \right)$$

$$H = P_2^T P_2$$

$$P_2 = P_1 \left( K^{-1} B'. C_L \right)$$

$$C_L = \frac{EA}{r^{-1}} \left( 1 + \frac{r^{-1}u}{r^{-1}l} \right)$$

where forced elongation, $\Delta l$, is the optimization variable and $H$ is a positive-definite approximation of Hessian matrix of the Lagrangian function, $K$ is tangent stiffness matrix and $B$ is force transformation matrix of tensegrity model. The element is assumed as linear elastic with axial stiffness $EA$, with the known length, $r^{-1}l$ and elastic elongation, $r^{-1}u$. Left superscripts $r$ and $t$ denotes the previous step and current step, respectively. The above sequential quadratic programming problem is subjected to inequality constraints: (i) the upper and lower limits of axial forces as well as (ii) the limit of the forced elongation.

After the search of optimal forced elongation, the nodal coordinates and axial forces of the tensegrity can be updated. The vector for the incremental nodal coordinates, $\Delta x$ and the incremental axial forces, $\Delta n$ at the shape change state are expressed as the following equations:

$$\Delta x = K^{-1} B'. C_L \cdot \Delta l$$

$$\Delta n = D_{\Delta l} \cdot \Delta l$$

$$D_{\Delta l} = \left( C_1 B^T K^{-1} B - I_m \right)^T C_L$$

$$C_1 = \frac{EA}{r^{-1}l}$$

The nodal coordinate and axial force of tensegrity model at current step are:

$$x' = \frac{r^{-1}l}{r^{-1}l} x + \Delta x$$

$$n' = \frac{r^{-1}l}{r^{-1}l} n + \Delta n$$

Objective function in equation (1) is calculated. The shape change analysis is checked against either one of the following two termination criteria: the objective function in equation (1) is less than
0.1 or the maximum iteration greater than 10000. If either of these criteria is satisfied, shape change analysis is terminated; otherwise step \( t=i+1 \) is set and the analysis is repeated.

**Figure 2.** Algorithm for shape change analysis.

### 3. Model characterization

The shape change method was applied to a tapered three-stage tensegrity model. The topology of the model searched from the form-finding method by Oh *et al.* was adopted in the study [11]. Figure 3 shows the topology of the tapered three-stage tensegrity models. The tapered tensegrity model was established with three independent simplex units (*i.e.* three-strut tensegrity model). Nominal height of the simplex is 450mm. Saddle depth of 180mm in between the upper and lower simplex resulted total height of 990mm. There are upper and lower equilateral triangular surfaces in a simplex. Twist angle of 25° was applied to upper relatively to lower equilateral triangular surfaces of each simplex unit. The sides of the lowest and topmost triangular surfaces in the model are 300mm and 100mm, respectively. The overall model of nine struts (*i.e.* members 1-9) and forty two cables (*i.e.* members 10-51) consists of three self-equilibrium stress modes and six rigid body displacement modes.

Table 1 shows the material properties that were assigned for the tapered tensegrity model [11]. Material properties required in the shape change analysis are elastic modulus, yield stress, density, cross sectional area of struts and cables. In the study, external load specifically selfweight of 3.2N was assigned to all nodes in the model. The model was constrained at base (*i.e.* node 1,2,3). It is noted that the length of the cables that are attached to fixed base (*i.e.* member 10,11,12) was fixed during the shape change analysis.
Figure 3. Inputs of tapered tensegrity model [11].

Table 1. Material properties [11].

| Elastic Modulus, E (MPa) | Yield Stress, \(\sigma_s\) (MPa) | Density, \(\gamma\) (10^6 kg/mm^3) | Strut Cross sectional Area, \(A_s\) (mm^2) | Cable Cross sectional Area, \(A_c\) (mm^2) |
|-------------------------|-------------------------------|---------------------------------|--------------------------------|------------------------|
| 200 000                 | 250                           | 7.85                            | 113.10                         | 19.63                  |

4. Structural characteristics during shape change

This section presents the results on structural characteristics of the tapered three-stage tensegrity model during shape change analysis. The characteristics in term of the configurations during the shape change analysis and the axial forces at the final step of shape change analysis are presented. The structural characteristics of tapered three-stage tensegrity model were investigated through three shape change analysis cases. Various target displacements prescribed in the analysis cases are shown in table 2.

Table 2. Target displacements.

| Analysis cases | Target displacements (mm) |
|----------------|----------------------------|
|                | X - axis | Y - axis | Z - axis |
| Xp200Z         | 200      | 0        | 0        |
| Xp200Zp100     | 200      | 0        | 100      |
| Xp200Zn100     | 200      | 0        | -100     |

4.1 Incremental shape change configurations

Figure 4 shows the normalized objective function (NOF) over computational steps for all the shape change analysis cases. It could be seen that the tapered tensegrity model advanced to the prescribed target coordinates with lesser computational steps in case Xp200Z, followed by case Xp200Zn100 and then case Xp200Zp100.
Figure 4. Normalized objective function over computational steps.

Figure 5 to figure 7 show the configurations at first step, intermediate step and final step of shape change analysis for cases Xp200Z, Xp200Zp100 and Xp200Zn100, respectively. Target coordinates for all the analysis cases were also illustrated in the figures.

In case Xp200Z (figure 5), since the target displacements were prescribed only in x direction, the final coordinates of monitored nodes in y and z directions can be in any magnitudes. It is found that the monitored nodes of the model displaced 200mm in positive x-direction (as prescribed), approximately 90mm in negative y-direction and about 50mm in negative z-direction. The model generally demonstrated bending deformation.

In case Xp200Zp100 (figure 6), the monitored nodes of the model displaced 200mm in positive x-direction and 100mm in positive z-direction (as prescribed). The monitored nodes displaced in range of 15mm to 71mm in negative y-direction. In addition to bending, the model showed axial deformation. Rotational deformation was also demonstrated during the shape change analysis.

(a)  (b)  (c)

Figure 5. Configurations during shape change analysis for case Xp200Z at (a) step 1, (b) step 7 and (c) final step (step 14).
In case Xp200Zn100 (figure 7), the monitored nodes of the model displaced 200mm in positive x-direction and 100mm in negative z-direction (as prescribed). The monitored nodes displaced in range of 267mm to 332mm in negative y-direction. This model mainly showed bending deformation during the shape change analysis.

4.2 Changes in Axial forces during shape change
Figure 8 to figure 10 show axial forces for the tapered tensegrity model at first step and final step of shape change analysis for cases Xp200Z, Xp200Zp100 and Xp200Zn100, respectively.

It could be seen from all cases that almost the axial forces of the model showed decrement at final step compared to step 1 of shape change analysis. Most of the cables (about 90%) showed axial forces less than 1000N and only two to three struts showed compressive forces more than 1000N at final step. There is no specific trend in the axial forces distribution for the analysis cases. The axial forces...
of the model during the shape change analysis were examined and found to be within the upper and lower limit axial forces.

Figure 8. Axial forces for case Xp200Z at step 1 and final step.

Figure 9. Axial forces for case Xp200Zp200 at step 1 and final step.

Figure 10. Axial forces for case Xp200Zn200 at step 1 and final step.
5. Conclusions
The deployability of the tapered three-stage tensegrity model was investigated through a proposed shape change algorithm. The Sequential Quadratic Programming method in the proposed algorithm is able to determine the optimal forced elongation in cables and effectively ensure the advancement of monitored nodes to the target coordinates. The tapered tensegrity model demonstrated bending, torsional and axial deformation in the study of three shape change analysis cases (i.e. Xp200Z, Xp200Zn100 and Xp200Zp100). The model showed potential to displace in y direction for all the cases. Simulation results from analysis cases under the studied cases also reveal the capability of the tapered tensegrity model in force distribution. The force distribution in the tensegrity model is spontaneously at every incremental step while the forces are kept within the upper and lower limit during the shape change analysis. The tapered three-stage tensegrity model could be suggested as deployable structures and tensegrity robots. Algorithms considering real environmental loadings and operational challenges may be carried out in future works.

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