Investigation the success of semidefinite programming for the estimating of fuel cost curves in thermal power plants

Termik santrallerde yakıt maliyet eğrilerinin tahmini için yarı-kesin programlamanın başarısının araştırılması

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Investigation the Success of Semidefinite Programming for the Estimating of Fuel Cost Curves in Thermal Power Plants

**Highlights**
- SDP method has been proposed for fuel cost function parameter estimation problem.
- First, second and third order fuel cost curve functions.
- Power plants with different fuel types such as coal, oil, and gas.
- Comparison results are in favor of SDP.

**Graphical Abstract**
- Parameter estimation is an optimization problem in which the optimal values of the unknown parameters should be estimated.
- This paper presents a new and accurate method for estimating the parameters of thermal power plants fuel cost function.

**Aim**
The main goal of this paper is to optimize the fuel cost function coefficients of thermal generation units.

**Design & Methodology**
A semidefinite programming (SDP) method was proposed for the estimation of fuel cost functions' parameters in thermal power plants. The parameter estimation problem was designed as a minimization problem, where the objective function is the total absolute error (TAE). The coefficients of fuel cost curve functions are found for first, second, and third-order models. Different fuel types such as coal, oil and gas were taken into consideration in the simulation studies.

**Originality**
The first study applying the SDP method for estimating the parameters of the fuel cost function in thermal power plants.

**Findings**
The results achieved from the semidefinite programming method were compared with that of particle swarm optimization (PSO), artificial bee colony (ABC), crow search algorithm (CSA) and least error square (LES) methods, respectively. The results clearly demonstrate that SDP outperforms other techniques based on the total absolute error parameter. It has been observed that the higher the complexity (degree) of the fuel cost function, the higher the performance of the SDP.

**Conclusion**
The results showed that the SDP method is more robust and produces a lower error compared to the other methods. It is obvious that the SDP is a useful and powerful method for solving such a problem.

**Declaration of Ethical Standards**
The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.
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Research Article / Araştırma Makalesi

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ABSTRACT

Accurate estimation of fuel cost curve parameters in thermal power plants is of great importance because these parameters directly influence the economic dispatch calculations. In this paper, a semidefinite programming (SDP) approach was proposed for the estimation of fuel cost functions' parameters in thermal power plants. The parameter estimation problem was designed as a minimization problem, where the objective function was accepted as the total absolute error (TAE) in the study. Also, linear, quadratic, and cubic fuel cost functions were used to estimate the fuel cost parameters. Different fuel types such as coal, oil and gas were preferred for simulation studies. The results achieved from the semidefinite programming method were compared with that of particle swarm optimization (PSO), artificial bee colony (ABC), crow search algorithm (CSA) and least error square (LES) methods, respectively. The performance of the methods were compared according to the TAE parameter. Simulation results showed that SDP method is more successful than other methods considered in this paper. Clearly, the present paper showed that SDP has a higher potential to solve parameter estimation problems.

Keywords: Fuel cost function, parameter estimation problem, semidefinite programming.

Termik Santrallerde Yakıt Maliyet Eğrilerinin Tahmini İçin Yarı-Kesin Programlamanın Başarısının Araştırılması

ÖZ

Termik güç santrallerinde, yakıt maliyet eğrisi parametreleri ekonomik dağıtım hesaplamalarını doğrudan etkilediği için bu parametrelerin doğru tahmin edilmesi büyük önem taşımaktadır. Bu çalışmada, termik santrallerdeki yakıt maliyet fonksiyonu parametrelerinin tahmini için yarı kesin programlama (YKP) yaklaşımı önerildi. Parametre tahmin problemini, amaç fonksiyonunun toplam mutlak hata (TMH) olarak kabul edildiği bir minimizasyon problemini olarak tasarlandı. Ayrıca, yakıt maliyet eğrisi parametrelerini tahmin etmek için doğrusal, ikinci dereceden ve kübik yakıt maliyet fonksiyonları kullanıldı. Simülasyon çalışmalari için kömür, petrol ve gaz gibi farklı yakıt türleri tercih edildi. Yarı kesin programlama yönteminde elde edilen sonuçlar sırasında parçacık sürüsü optimizasyonu (PSO), yapay arı kolonisi (YAK), karga arama algoritması (KAA) ve en küçük hata karesi (EKHK) yöntemleriyle karşılaştırılırldı. Yöntemlerin performansı TMH parametrelerine göre karşılaştırılmıştır. Simülasyon sonuçları YKP yönteminin bu makalede dikkate alınan diğer yöntemlerden daha başarılı olduğu gösterdi. Bu makale YKP'nin parametre tahmin problemlerini çözme potansiyelinin yüksek olduğunu açıkça gösterdi.

Anahtar Kelimeler: Yakıt maliyet fonksiyonu, parameter tahmin problemi, yarı-kesin programlama.

1. INTRODUCTION

Economic load dispatch (ELD) is crucial in power system planning and operation. The aim of ELD is to operate generators that produce energy in a power plant with minimum fuel costs, simultaneously, while satisfying the operational constraints and load demand [1]. The ELD problem can be devised as an optimization problem aimed at minimizing the fuel cost function. In ELD problem the fuel cost function is commonly represented by a linear, quadratic, or cubic function [2, 3]. Accurate estimation of fuel cost function parameters is critical in solving the ELD problem. Many algebraic models are being recommended for the fuel cost curve. Generally, fuel cost functions are represented by two essential classifications: smooth and non-smooth. [3-5].

In the literature, fuel cost curve parameters were estimated using heuristic optimization algorithms, artificial intelligence techniques, and traditional models. In [3], the particle swarm optimization (PSO) method
was presented for the fuel cost curve parameter estimation problem. In [4], a new differential evolution (DE) algorithm was proposed for the fuel cost parameter estimation problem. In [5], the authors proposed an improved differential evolution (IDE) algorithm for the parameter estimation problem. Ref. [6] presented an implementation of the artificial bee colony (ABC) algorithm to estimate the fuel cost curve parameters of thermal power plants (TPP). In [7], the crow search algorithm (CSA) was recommended for estimating the fuel cost curve parameters for TPP with and without the valve point effect. In [8], a new approach was proposed for parameter estimation based on the cuckoo search (CS) algorithm. The authors submitted the teaching-learning based optimization (TLBO) algorithm to estimate the fuel cost curve parameters in Ref. [9]. In [10], four algorithms were used to estimate the coefficients of the fuel cost curves. A new practice to solve the parameter estimation problem using a cuckoo search (CS) based algorithm was proposed in Ref. [11].

SDP has recently received much attention for various problems in power system analysis. Some applications are: In [12], it has been shown that SDP can effectively solve high dimension, non-smooth power system problems. In [13], the authors used an SDP method to solve the economic emission dispatch (EED) problem. Ref. [14] emphasized the success of SDP for optimal power flow problems. In [15], SDP was used to solve the renewable microgrid state estimation and its stabilization problem. In [16], the solution of the multi-objective optimal power flow problem was realized by SDP. It has been proved that SDP method is successful in solving complex problems with high number of variables.

The aim of this study is to present a new method based on semidefinite programming in order to estimate the fuel cost curve coefficients in TPP with high reliability. The parameter estimation problem was designed as a minimization problem, where the objective function is the total absolute error. In this paper, the smooth fuel cost functions are taken into consideration. Different study cases are presented to confirm the effectiveness of the proposed approach.

The rest of the paper is organized as follows: problem formulation is explained in Section 2. Following this, the main structure of the SDP is introduced in Section 3. In Section 4, the effectiveness of the SDP method was confirmed by comparisons with different optimization algorithms and traditional methods. Finally, we conclude in Section 5.

2. FORMULATION OF THE PROBLEM

The operating characteristic of TPP is represented by fuel cost functions. The fuel cost curve for the thermal generating unit (i) can be expressed by a polynomial function that relates its fuel cost to its real power output (MW) as [4, 5]:

\[ F_i(P_{ti}) = a_{0i} + \sum_{j=1}^{i} a_{ji} P_{ti}^j + r_i, \quad j = 1, 2, ..., N \]  

where \( F_i \) is the fuel cost of the \( i \)th generator, \( P_{ti} \) is the output power generated by the \( i \)th thermal unit, \( a_{0i} \) and \( a_{ji} \) are the cost coefficients for generator \( i \), \( r_i \) is the error associated with the \( i \)th equation, \( N \) is the polynomial order. \( N \) is the number of generation units.

In this paper, three different cost fuel functions were used.

Model 1: Linear fuel cost function

\[ F_i(P_{ti}) = a_{0i} + a_{1i} P_{ti} + r_i \]  

Model 2: Quadratic fuel cost function

\[ F_i(P_{ti}) = a_{0i} + a_{1i} P_{ti} + a_{2i} P_{ti}^2 + r_i \]  

Model 3: Cubic fuel cost function

\[ F_i(P_{ti}) = a_{0i} + a_{1i} P_{ti} + a_{2i} P_{ti}^2 + a_{3i} P_{ti}^3 + r_i \]

where \( a_{0i}, a_{1i}, a_{2i}, a_{3i} \) are the fuel cost coefficients to be estimated and \( P_{ti} \) is the generated power of the \( i \)th unit [3].

Fig. 1 shows different smooth fuel cost curves. Entry to the unit is the total cost per hour \( F(\$/hr) \) and output is the net electrical power output of the unit \( P(MW) \) [7].

![Figure 1. Convex fuel cost curves](image-url)

A set of nonlinear equations of the parameter estimation problem can be formulated as follows [5]:

\[ Z_i = f_i(P_i, X_i) + r_i \]  

where \( Z_i \) is a vector of actual values of generation costs, \( X_i \) is the fuel cost parameters \((a_{0i}, a_{1i}, a_{2i}, a_{3i})\) to estimate for the \( i \)th generator and \( r_i \) is the error vector.

\[ r_i = F_i(\text{actual}) - F_i(\text{estimated}) \]  

The objective function is to minimize the total absolute error (TAE), subject to the equality and inequality constraints [7].

\[ \text{minimize} \quad \text{TAE} = \sum_{i=1}^{n} |r_i| \]  

subject to: \[ \sum_{i=1}^{n} P_i = P_D + P_L \]
\[ C^l_i \leq C_i \leq C^u_i \quad (7c) \]

where \( P_i \) is the total power generation, \( P_d \) is the load demand. Transmission losses are not considered \( (P_L = 0) \). \( C^l_i \) and \( C^u_i \) are the lower and upper bounds of fuel cost coefficients for the \( i \)th unit.

### 3. SEMIDEFINITE PROGRAMMING

Semidefinite programming is one of the most popular convex optimization methods. One of the main superiority of convex optimization is that whenever the problem is convex, the solution is globally optimal. Even when the problem is non-convex, SDP relaxation of the problem provides a good computable related to the optimal value [17, 18].

In heuristic methods, the parameter setting is required for optimum result, SDP does not have such an obligation [19, 20]. SDP is a generalization of the linear program (LP) in which the vector variables are modified by matrix variables and the element-wise non-negativity by positive semidefiniteness of the matrices [21, 22]. In this paper, an optimization problem is characterized by using primal SDP form [12]:

- **minimize** \( < A_0 X > \) \hspace{1cm} (8a)
- **subject to** \( < A_i X > = b_i \), \( i = 1, ..., m \) \hspace{1cm} (8b)
- \( X \geq 0 \) \hspace{1cm} (8c)

The related dual SDP problem is:

- **maximize** \( < b, y > \) \hspace{1cm} (9a)
- **subject to** \( \sum_{i=1}^{m} y_i A_i \leq A_0 \) \hspace{1cm} \( y \in R^m \) \hspace{1cm} (9b)

where \( X \in S^n \) is the decision variable \( b \in R^m \) and \( A_0, A_i \in S^n \). \( S^n \) is the set of all symmetric matrices in \( R^{nxn} \). The inner product between two matrices \( X, Y \in S^n \) is defined as \( < X, Y >= Trace(XY) = \sum_{i=1}^{n} \sum_{j=1}^{n} X_{ij} Y_{ij} \) [17].

The addition of a semidefinite matrix variable in the direct formulation of some problems as SDP does not all the time produce a convex SDP problem. Reasons adduced for this include possible non-convexity in the objective function or in the constraints. Thus, the resulting SDP problem is rough to solve to global optimality. The relaxation of the SDP problem means that the non-convex constraints are embedded in a larger convex constraint set [12]. Contemplate the problem with non-convex constraint set \( K \).

\[ f^* = \min_{x \in K} f(x) \quad (10) \]

Given that there exists a convex set \( K' \) such that \( K \subset K' \). The relaxed problem becomes:

\[ f_* = \min_{x \in K'} f(x) \quad (11) \]

### 4. NUMERICAL RESULTS

The SDP approach for solving the fuel cost curve parameter estimation problem, have been tested using data published in [6]. The problem was solved in MATLAB environment using CVX with SDTP3 solver [23]. CVX is a modeling system used in MATLAB to solve convex optimization problems.

In this section, the proposed approach was applied to three different fuel cost functions, and the results obtained from the simulation studies were compared with other results in the literature, namely: Particle Swarm Optimization (PSO) [3], Artificial Bee Colony (ABC) [6], Crow Search Algorithm (CSA) [7], Least Error Square (LES) [10].

| Table 1. Estimated fuel cost coefficients for the linear model |
|---------------------------------|-----------------|-----------------|
| **System**                      | **Method**      | **Parameters**  |
| First-order model               |                 |                |
| **Unit 1 (Coal)**               | PSO             | 60.006          |
|                                 | ABC             | 45.212          |
|                                 | CSA             | 45.200          |
|                                 | LES             | 63.236          |
|                                 | SDP             | 45.200          |
|                                 |                 | 10.190          |
|                                 |                 | 10.560          |
|                                 |                 | 10.170          |
|                                 |                 | 10.560          |
| **Unit 2 (Oil)**                | PSO             | 66.051          |
|                                 | ABC             | 47.652          |
|                                 | CSA             | 47.600          |
|                                 | LES             | 66.160          |
|                                 | SDP             | 47.600          |
|                                 |                 | 10.570          |
|                                 |                 | 11.031          |
|                                 |                 | 11.030          |
|                                 |                 | 10.631          |
|                                 |                 | 11.030          |
| **Unit 3 (Gas)**                | PSO             | 66.002          |
|                                 | ABC             | 48.399          |
|                                 | CSA             | 48.400          |
|                                 | LES             | 66.700          |
|                                 | SDP             | 48.400          |
|                                 |                 | 10.780          |
|                                 |                 | 11.221          |
|                                 |                 | 11.220          |
|                                 |                 | 10.830          |
|                                 |                 | 11.220          |
Table 2. Simulation results for the linear model

| First-order model | System | P (MW) | \( F_{\text{actual}} \) (GJ/h) | PSO \( F_{\text{estimated}} \) (GJ/h) | ABC \( F_{\text{estimated}} \) (GJ/h) | CSA \( F_{\text{estimated}} \) (GJ/h) | LES \( F_{\text{estimated}} \) (GJ/h) | SDP \( F_{\text{estimated}} \) (GJ/h) | Error |
|-------------------|--------|--------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------------------------------|--------|
| Unit 1 (Coal)     | 10     | 176.62 | 161.905 | 150.812 | 150.800 | 164.936 | 150.800 | 14.715 | 25.080 | 8.684 |
|                   | 20     | 256.40 | 263.803 | 256.412 | 256.400 | 266.636 | 256.400 | -7.403 | 0.012  | 10.236 |
|                   | 30     | 361.50 | 365.702 | 362.012 | 362.000 | 368.336 | 362.000 | -4.202 | -0.512 | 6.836  |
|                   | 40     | 467.60 | 467.600 | 467.612 | 470.036 | 467.600 | 0.000   | -0.012 | 0.000  | 2.436  |
|                   | 50     | 579.50 | 569.498 | 573.212 | 573.200 | 571.736 | 573.200 | 10.002 | 6.288  | 6.764  |
| SUM Error         |        |        |        |        |        |        |        |        |        | 36.332 |
|                   | 10     | 184.75 | 171.701 | 157.962 | 157.900 | 172.470 | 157.900 | 13.049 | 26.788 | 12.280 |
|                   | 20     | 268.20 | 277.400 | 268.272 | 268.200 | 278.780 | 268.200 | 9.200  | 0.072  | 10.580 |
|                   | 30     | 377.70 | 383.100 | 378.582 | 378.500 | 385.090 | 378.500 | 5.400  | -0.882 | 7.390  |
|                   | 40     | 488.80 | 488.800 | 488.892 | 491.400 | 488.800 | 0.000   | -0.092 | 0.000  | 2.600  |
|                   | 50     | 606.00 | 594.499 | 599.202 | 599.100 | 597.710 | 599.100 | 11.501 | 6.796  | 8.500  |
| SUM Error         |        |        |        |        |        |        |        |        |        | 39.151 |
|                   | 10     | 187.20 | 173.802 | 160.609 | 160.600 | 175.000 | 160.600 | 13.398 | 26.591 | 12.200 |
|                   | 20     | 272.80 | 281.601 | 272.819 | 272.800 | 283.300 | 272.800 | -8.801 | -0.019 | -0.502 |
|                   | 30     | 384.30 | 389.401 | 385.029 | 385.000 | 391.600 | 385.000 | 5.101  | -0.729 | -7.300 |
|                   | 40     | 497.20 | 497.200 | 497.239 | 497.200 | 499.900 | 497.200 | 0.000  | -0.039 | -2.700 |
|                   | 50     | 616.50 | 604.999 | 609.449 | 609.400 | 608.200 | 609.400 | 11.501 | 7.051  | 8.300  |
| SUM Error         |        |        |        |        |        |        |        |        |        | 38.801 |

4.1. Case 1: Linear Cost Function

In this case, the SDP method was applied to estimate the fuel cost function parameters of the quadratic model. The fuel cost coefficients computed by SDP and the results by techniques noted in this study are provided in Table 1. Actual fuel cost data for each unit; the estimated fuel cost values and total absolute error values obtained by PSO, ABC, CSA, LES and SDP methods are shown in Table 2. From Table 2, it can be noticed that SDP and CSA reach a minimum total absolute estimation error for each unit. SDP and CSA give the least objective function value (32.620 GJ/h) compared to PSO (36.332 GJ/h), ABC (32.632 GJ/h), and LES (38.956 GJ/h) for unit 1. In other words, SDP and CSA are 10.216%, 0.363%, and 16.264% less than the results of PSO, ABC, and LES respectively. For unit 2, SDP and CSA reach a minimum total absolute estimation error (34.550 GJ/h) compare to PSO (39.151 GJ/h), ABC (34.632 GJ/h), and LES (41.140 GJ/h). Briefly, SDP and CSA are 11.751%, 0.236%, and 16.018% less than the results of PSO, ABC, and LES respectively. The minimum absolute estimation error obtained with SDP and CSA methods for unit 3 is 34.400 GJ/h. SDP and CSA reduced the total error by 11.342%, 0.084%, and 16.097% respectively, compared to PSO, ABC, and LES.

4.2. Case 2: Quadratic Cost Function

The fuel cost coefficients obtained by SDP and the results by techniques considered in this study are reported in Table 3. The simulation results for the quadratic fuel cost function are presented in Table 4. From Table 4, it is clear that the SDP and CSA methods give the minimum objective function value 9.760 GJ/h, 9.975 GJ/h, 9.750 GJ/h for coal, oil and gas unit respectively. The minimum total absolute error of unit 1 obtained by SDP and CSA is 9.760 GJ/h. In other words, SDP and CSA are 0.357 (GJ/h), 0.050 (GJ/h), and 4.448 (GJ/h) less than the results of PSO, ABC, and LES respectively. For unit 2, SDP and CSA give the least objective function value 9.975 (GJ/h). Also, SDP and CSA reduced the total absolute error by 1.875 (GJ/h), 0.158 (GJ/h), and 4.489 (GJ/h) respectively, compared to PSO, ABC, and LES. For unit 3, SDP and CSA reach the least objective function value 9.750 (GJ/h) compare to other methods. The result obtained using the SDP method reduced the error value by 2.991 (GJ/h), 0.611 (GJ/h) and 4.466 (GJ/h) respectively, compared to PSO, ABC, and LES. It is seen that SDP can reduce the total absolute error significantly.
Table 3. Estimated fuel cost coefficients for the quadratic model

| System     | Method | Parameters | \(a_0\)  | \(a_1\)  | \(a_2\)  |
|------------|--------|------------|----------|----------|----------|
|            | PSO    | 96.279     | 7.592    | 0.042    |          |
|            | ABC    | 96.604     | 7.587    | 0.041    |          |
| Unit 1 (Coal) | CSA    | 96.600     | 7.588    | 0.041    |          |
|            | LES    | 95.856     | 7.374    | 0.047    |          |
|            | SDP    | 96.600     | 7.588    | 0.041    |          |
|            | PSO    | 101.000    | 7.800    | 0.046    |          |
|            | ABC    | 101.536    | 7.877    | 0.044    |          |
| Unit 2 (Oil) | CSA    | 101.531    | 7.880    | 0.044    |          |
|            | LES    | 100.710    | 7.670    | 0.049    |          |
|            | SDP    | 101.531    | 7.880    | 0.044    |          |
|            | PSO    | 102.000    | 7.900    | 0.048    |          |
|            | ABC    | 101.817    | 8.099    | 0.043    |          |
| Unit 3 (Gas) | CSA    | 101.812    | 8.100    | 0.043    |          |
|            | LES    | 101.100    | 7.881    | 0.049    |          |
|            | SDP    | 101.102    | 8.100    | 0.043    |          |

Table 4. Simulation results for the quadratic model

| System     | Method | P (MW)  | \(F_{\text{actual}}\) (GJ/h) | \(F_{\text{estimated}}\) (GJ/h) | Error  |
|------------|--------|---------|-------------------------------|---------------------------------|--------|
|            | PSO    | 10      | 176.23                         | 176.38                          | -0.97  |
|            | ABC    | 20      | 256.40                         | 264.75                          | 8.35   |
|            | CSA    | 30      | 361.50                         | 361.50                          | 0.00   |
|            | LES    | 40      | 467.60                         | 466.56                          | 1.04   |
|            | SDP    | 50      | 579.50                         | 579.50                          | 0.00   |
| Unit 1 (Coal) | PSO    | 10      | 176.23                         | 176.38                          | -0.97  |
|            | ABC    | 20      | 256.40                         | 264.75                          | 8.35   |
|            | CSA    | 30      | 361.50                         | 361.50                          | 0.00   |
|            | LES    | 40      | 467.60                         | 466.56                          | 1.04   |
|            | SDP    | 50      | 579.50                         | 579.50                          | 0.00   |
| System     | Method | P (MW)  | \(F_{\text{actual}}\) (GJ/h) | \(F_{\text{estimated}}\) (GJ/h) | Error  |
|            | PSO    | 10      | 184.75                         | 183.60                          | -0.15  |
|            | ABC    | 20      | 268.20                         | 275.40                          | 7.20   |
|            | CSA    | 30      | 377.70                         | 376.40                          | 1.30   |
|            | LES    | 40      | 488.80                         | 486.60                          | 2.20   |
|            | SDP    | 50      | 606.00                         | 606.00                          | 0.00   |
| Unit 2 (Oil) | PSO    | 10      | 184.75                         | 183.60                          | -0.15  |
|            | ABC    | 20      | 268.20                         | 275.40                          | 7.20   |
|            | CSA    | 30      | 377.70                         | 376.40                          | 1.30   |
|            | LES    | 40      | 488.80                         | 486.60                          | 2.20   |
|            | SDP    | 50      | 606.00                         | 606.00                          | 0.00   |
| System     | Method | P (MW)  | \(F_{\text{actual}}\) (GJ/h) | \(F_{\text{estimated}}\) (GJ/h) | Error  |
|            | PSO    | 10      | 187.20                         | 185.78                          | -1.42  |
|            | ABC    | 20      | 272.80                         | 279.12                          | -6.25  |
|            | CSA    | 30      | 384.30                         | 382.02                          | -2.28  |
|            | LES    | 40      | 497.20                         | 494.48                          | 2.71   |
|            | SDP    | 50      | 616.50                         | 616.50                          | -0.00  |
| Unit 3 (Gas) | PSO    | 10      | 187.20                         | 185.78                          | -1.42  |
|            | ABC    | 20      | 272.80                         | 279.12                          | -6.25  |
|            | CSA    | 30      | 384.30                         | 382.02                          | -2.28  |
|            | LES    | 40      | 497.20                         | 494.48                          | 2.71   |
|            | SDP    | 50      | 616.50                         | 616.50                          | -0.00  |

4.3. Case 3: Cubic Cost Function

In this case, the cubic fuel cost function, which has a more complex structure than other fuel cost functions, was preferred to demonstrate the effectiveness of the proposed approach. Table 5 presents the estimated coefficients of the cost function obtained using considered methods for case 3. In Table 6, the actual fuel cost data for each unit; estimated fuel cost data obtained from the noted methods in this paper; error values calculated from the difference between actual and estimated values; and total absolute error values for each algorithm are presented. Total absolute error values from the PSO, ABC, CSA, LES, and SDP methods are 8.641, 5.422, 4.862, 10.329 and 4.853 GJ/h for unit 1. As the simulation results show, the minimum objective value obtained by the SDP method is better than others. That is, the SDP is 43.837%, 10.494%, 0.185%, and 53.015% lower than compared with the results of the PSO, ABC, CSA, and LES algorithms. For unit 2, the obtained simulation results from the PSO, ABC, CSA, and LES methods are 5.547, 5.240, 4.841, 11.059 and 4.825 GJ/h, respectively. The SDP method is 0.722, 0.415, 0.023, 8.100, 0.023, and 6.321 GJ/h, respectively.
The SDP method is 0.883, 0.863, 0.019, and 5.232 GJ/h less than the results of PSO, ABC, CSA and LES, respectively. Also, the proposed method is 15.226%, 14.889%, 0.385%, and 51.556% lower than those of the other algorithms, respectively. Simulation results proved that SDP method is more successful than PSO, ABC, CSA and LES methods in estimating the parameters of cubic cost function.

Table 5. Estimated fuel cost coefficients for the cubic model.

| System       | Method | Parameters |   |   |   |   |
|--------------|--------|------------|---|---|---|---|
|              |        | $a_0$      | $a_1$ | $a_2$ | $a_3$ |
| Unit 1 (Coal)| PSO    | 120.241    | 3.979  | 0.184 | -0.002 |
|              | ABC    | 124.536    | 3.485  | 0.187 | -0.001 |
|              | CSA    | 127.036    | 3.122  | 0.199 | -0.001 |
|              | LES    | 123.180    | 3.535  | 0.193 | -0.002 |
|              | SDP    | 127.066    | 3.118  | 0.199 | -0.001 |
| Unit 2 (Oil) | PSO    | 130.278    | 3.542  | 0.200 | -0.002 |
|              | ABC    | 129.235    | 3.485  | 0.187 | -0.001 |
|              | CSA    | 132.463    | 3.336  | 0.205 | -0.001 |
|              | LES    | 128.640    | 3.746  | 0.199 | -0.002 |
|              | SDP    | 132.500    | 3.332  | 0.205 | -0.001 |
| Unit 3 (Gas) | PSO    | 128.376    | 4.146  | 0.188 | -0.002 |
|              | ABC    | 126.014    | 3.804  | 0.189 | -0.001 |
|              | CSA    | 132.428    | 3.608  | 0.203 | -0.001 |
|              | LES    | 128.400    | 4.046  | 0.195 | -0.002 |
|              | SDP    | 132.333    | 3.625  | 0.202 | -0.001 |

Table 6. Simulation results for the cubic model

| System       | P (MW) | $F_{\text{actual}}$ (GJ/h) | $F_{\text{estimated}}$ (GJ/h) | PSO $F_{\text{estimated}}$ (GJ/h) | ABC $F_{\text{estimated}}$ (GJ/h) | CSA $F_{\text{estimated}}$ (GJ/h) | LES $F_{\text{estimated}}$ (GJ/h) | SDP $F_{\text{estimated}}$ (GJ/h) | Error |
|--------------|--------|-----------------------------|-------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|-------|
|              | 10     | 176.62                      | 176.806                       | 176.615                          | 176.617                          | 176.227                          | 176.620                          | -0.186                          | 0.003 | 0.393 | 0.000 |
| Unit 1 (Coal)| 20     | 256.40                      | 260.557                       | 257.134                          | 256.405                          | 258.274                          | 256.400                          | -0.157                          | -0.734 | -0.005 | 1.874 |
|              | 30     | 361.50                      | 361.951                       | 357.093                          | 356.649                          | 359.721                          | 356.664                          | -0.451                          | 4.406 | 4.851 | 1.779 |
|              | 40     | 467.60                      | 471.446                       | 467.492                          | 467.597                          | 470.968                          | 467.600                          | 3.846                           | 0.107 | 0.003 | -3.368 |
|              | 50     | 579.50                      | 579.500                       | 579.331                          | 579.500                          | 582.415                          | 579.500                          | 0.000                           | 0.168 | 0.000 | -2.915 |
|              |        |                             |                               |                                  |                                  |                                  |                                  | 8.641                           | 5.422 | 4.862 | 10.329 |
| $\Sigma$Error|       |                             |                               |                                  |                                  |                                  |                                  |                                  | 5.747 | 5.240 | 4.841 |
|              | 10     | 184.75                      | 184.076                       | 184.739                          | 184.744                          | 184.301                          | 184.750                          | 0.674                           | 0.010 | 0.006 | 0.449 |
| Unit 2 (Oil) | 20     | 268.20                      | 268.200                       | 269.163                          | 268.213                          | 269.562                          | 268.200                          | 0.000                           | -0.963 | -0.013 | -1.362 |
|              | 30     | 377.70                      | 373.010                       | 373.507                          | 372.896                          | 374.232                          | 372.875                          | 4.690                           | 4.192 | 4.804 | 3.477 |
|              | 40     | 488.80                      | 488.863                       | 488.771                          | 488.816                          | 488.884                          | 488.800                          | -0.063                          | 0.028 | -0.016 | 0.716 |
|              | 50     | 606.00                      | 606.119                       | 605.955                          | 605.998                          | 606.945                          | 606.000                          | -0.119                          | 0.044 | 0.002 | 5.055 |
| $\Sigma$Error|       |                             |                               |                                  |                                  |                                  |                                  | 5.547                           | 5.240 | 4.841 | 11.059 |
|              | 10     | 187.20                      | 187.101                       | 187.188                          | 187.200                          | 186.804                          | 187.200                          | 0.099                           | 0.016 | 0.000 | 0.369 |
| Unit 3 (Gas) | 20     | 272.80                      | 274.326                       | 274.632                          | 272.800                          | 274.688                          | 272.800                          | -1.526                          | -1.832 | 0.000 | -1.888 |
|              | 30     | 384.30                      | 381.000                       | 380.561                          | 379.421                          | 382.452                          | 379.383                          | 3.300                           | 3.738 | 4.879 | 1.848 |
|              | 40     | 497.20                      | 498.074                       | 497.170                          | 497.256                          | 500.496                          | 497.200                          | -0.874                          | 0.029 | -0.056 | -3.296 |
|              | 50     | 616.50                      | 616.500                       | 616.659                          | 616.500                          | 619.220                          | 616.500                          | 0.000                           | -0.159 | 0.000 | -2.720 |
| $\Sigma$Error|       |                             |                               |                                  |                                  |                                  |                                  | 5.799                           | 5.776 | 4.935 | 10.148 |

5. CONCLUSION

In this paper, the SDP algorithm was applied to find the optimal fuel cost curve parameters of thermal power plants. The parameter estimation problem has been expressed as an optimization problem where the objective is to minimize the total absolute error. To evaluate the success of the proposed method, three different test cases were evaluated for three different power plants with different fuel types such as coal, oil, and gas. The performance of the SDP method was compared with the PSO, ABC, CSA and LES methods. The simulation results showed that the SDP algorithm is more robust and produces a lower error between the actual and estimated parameters compared to the others for all test cases.
DECLARATION OF ETHICAL STANDARDS
The author(s) of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

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