MULTI-GAP SUPERFLUIDITY IN NUCLEAR MATTER
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Abstract

It is shown that under lowering density or temperature a nucleon Fermi superfluid can undergo a phase transition to a new superfluid state corresponding to superposition of states with singlet-triplet (ST) and triplet-singlet (TS) pairing of nucleons (in spin and isospin spaces). Such states arise as a result of branching from one-gap solution of the self-consistent equations, describing ST pairing of nucleons. The density and temperature dependence of the order parameters for new two-gap solutions is determined in the model with Skyrme effective forces.

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As it is known, at sufficiently low temperatures a Fermi-liquid (FL) becomes unstable with respect to formation of Cooper pairs and passes into a superconducting (superfluid) state. Corresponding examples are an electron liquid in metals and superfluid phases of $^3$He. There are some experimental facts in favour of superfluidity of atomic nucleus as well \[1\]. Since the study of superfluidity of atomic nuclei is difficult due to finiteness of their sizes, it is of interest to study superfluidity of infinite nuclear matter, especially in view of astrophysical applications. This question has been considered in many works. However, earlier the normal-to-superfluid phase transition in nuclear matter has been studied only. In principle, at further lowering of temperature a superfluid FL of nucleons can again become unstable passing into a new superfluid state. Such states are characterized by not by one but a few order parameters and have a lower symmetry than the initial superfluid state. Thus, we are speaking of the superfluid-to-superfluid phase transitions in nuclear matter. We shall clarify the mechanism of appearance of new superfluid states as well as find the temperature dependence of the order parameters for these states.

1. Basic equations. We study multi-gap superfluidity of nuclear matter using Landau’s semiphenomenological concept of a Fermi-liquid. The basic formalism is laid out in more detail in Ref. \[2\], where the phase transitions to one-gap superfluid states of symmetrical nuclear matter have been studied. The present work is an extension of Ref. \[2\] and its main aspect is analysis of the phase transitions to multi-gap superfluid states. For simplicity we assume that the energy functional is invariant under rotations in
the configurational, spin and isospin spaces (for infinite uniform nuclear matter the spin-orbit interaction is equal to zero). Hence, superfluid phases (Cooper pairs) are classified by specifying the total spin of the pair, \( S = 0, 1 \), the isospin \( T = 0, 1 \), their projections \( S_z \) and \( T_z \) on the \( z \) axis, and the orbital angular momentum \( L = 0, 1, 2, \ldots \) The possible values of the orbital angular momentum \( L \) for each value of \( S \) and \( T \) must be found according to the Pauli principle. Each superfluid phase is described by its own set of order parameters: at \( S = 0, T = 0 \) by the scalar order parameter \( \Delta_{00} \) (singlet-singlet pairing of nucleons), at \( S = 1, T = 0 \) or \( S = 0, T = 1 \) by the vector order parameter \( \Delta_{k0} \) or \( \Delta_{0k} \), respectively \((k = 1, 2, 3)\) (TS and ST pairing of nucleons) and at \( S = 1, T = 1 \) by the tensor order parameter \( \Delta_{ik} \) \((i, k = 1, 2, 3)\) (triplet-triplet pairing of nucleons). A nucleon superfluid FL is described by two distribution functions: the normal distribution function \( f_{\kappa_1\kappa_2} = \text{Tr} \, \hat{a}_{\kappa_2}^+ \, a_{\kappa_1} \) and the anomalous distribution function \( g_{\kappa_1\kappa_2} = \text{Tr} \, \hat{a}_{\kappa_2} a_{\kappa_1} \), where \( a_{\kappa}^+ \) and \( a_\kappa \) are the creation and annihilation operators of fermions with momentum \( \vec{p} \), spin (isotopic spin) projection \( \sigma(\tau) \), \( \kappa \equiv (\vec{p}, \sigma, \tau) \), \( \text{Tr} \ldots \) being the average of the operators with the density matrix of the system, \( g \). The energy of the system is specified as a functional of the distribution functions \( f \) and \( g \), \( E = E(f, g) \). It determines the fermion one-particle energy \( \varepsilon \) and the matrix order parameter \( \Delta \) of the system

\[
\varepsilon_{\kappa_1\kappa_2} = \frac{\partial E}{\partial f_{\kappa_2\kappa_1}}, \quad \Delta_{\kappa_1\kappa_2} = 2 \frac{\partial E}{\partial g_{\kappa_2\kappa_1}} \tag{1}
\]

The matrix self-consistent equation for determining the distribution functions \( f \) and \( g \) follows from the minimum condition for the thermodynamic potential \( \Omega = -S + Y_0E + Y_{4a}N_a + Y_{4b}N_b \) (\( S \) is the entropy, \( N_a, N_b \) are the numbers of particles of species \( a \) and \( b \), \( Y_0 = 1/T, Y_{4a} = -\mu_a/T \) and \( Y_{4b} = -\mu_b/T \) are the Lagrange multipliers, \( \mu_a \) and \( \mu_b \) are the chemical potentials of particles of species \( a \) and \( b \), \( T \) is the temperature):

\[
\hat{f} = \left\{ \exp(Y_0\varepsilon + \hat{Y}_4) + 1 \right\}^{-1} \equiv \left\{ \exp(Y_0\hat{\xi}) + 1 \right\}^{-1}, \quad \hat{g} = \begin{pmatrix} f g \\ g^+ 1 - \hat{f} \end{pmatrix}, \quad \hat{\varepsilon} = \begin{pmatrix} \varepsilon & \Delta \\ \Delta^+ & -\varepsilon \end{pmatrix}, \quad \hat{Y}_4 = \begin{pmatrix} Y_4 & 0 \\ 0 & -Y_4 \end{pmatrix} \tag{2}
\]

Here the quantities \( \varepsilon, \Delta, Y_4 \) are, in turn, matrices in the space of the \( \kappa \) variables, with \( Y_{4\kappa_1\kappa_2} = Y_{4\tau_1\delta_{\kappa_1\kappa_2}} \) \((\tau_1 = a, b)\) and the tilde stands for the transposition operation. Using the procedure of block diagonalization \cite{2,3} in an extended space \( 2 \times 2 \), where the operator \( \hat{f} \) acts, one can express evidently the distribution functions \( f \) and \( g \) in terms of the quantities \( \varepsilon \) and \( \Delta \). In what follows, we consider the case of symmetrical nuclear matter (equal numbers of protons and neutrons, or \( Y_{4a} = Y_{4b} \equiv Y_4, \mu_a = \mu_b \equiv \mu \)). In this case we obtain

\[
f = \frac{1}{2} \left( 1 - \frac{\xi}{E} \tanh \frac{Y_0E}{2} \right), \quad g = -\frac{1}{2E} \tanh \frac{Y_0E}{2} \cdot \Delta; \quad E = \sqrt{\xi^2 + \Delta \Delta^+}, \quad \xi = \varepsilon + \frac{Y_4}{Y_0} \tag{3}
\]
We emphasize that the quantities \(f, g, \xi\) and \(\Delta\) in Eq. (3) are matrices in the spin, isospin and momentum spaces. Further we shall study the unitary states of superfluid nuclear matter, for which the product \(\Delta \Delta^+\) is proportional to the product of the unit matrices in the spin and isospin spaces, \(\Delta \Delta^+ \propto \sigma_0 \tau_0\). We consider as a particular case two-gap unitary states, for which the order parameters possess by the same symmetry properties on momentum:

\[
\Delta(\vec{p}) = \Delta_{30}(\vec{p}) \sigma_3 \sigma_2 \tau_2 + \Delta_{03}(\vec{p}) \sigma_2 \tau_3 \tau_2, \quad \Delta_{30}(-\vec{p}) = \Delta_{30}(\vec{p}), \quad \Delta_{03}(-\vec{p}) = \Delta_{03}(\vec{p}) \quad (4)
\]

In this case the wave function of a Cooper pair describes the superposition of states with the triplet-singlet and singlet-triplet pairing of nucleons (TS-ST states):

\[
g(\vec{p}) = g_{30}(\vec{p}) \sigma_3 \sigma_2 \tau_2 + g_{03}(\vec{p}) \sigma_2 \tau_3 \tau_2, \quad g_{30}(-\vec{p}) = g_{30}(\vec{p}), \quad g_{03}(-\vec{p}) = g_{03}(\vec{p}), \quad (5)
\]

with the projections of total spin and isospin \(S_z = T_z = 0\).

2. Self-consistent equations. The case \(T=0\). To derive the self-consistent equations of a nucleon superfluid FL it is necessary to specify the energy functional, which we set in the form

\[
E(f, g) = E_0(f) + E_{int}(g), \quad E_0(f) = \frac{4}{V} \sum_{\vec{p}} \varepsilon_0(\vec{p}) f_{00}(\vec{p}), \quad \varepsilon_0(\vec{p}) = \frac{\vec{p}^2}{2m}, \quad (6)
\]

\[
E_{int}(g) = \frac{2}{V} \sum_{\vec{p}, \vec{q}} \left\{ g_{30}^*(\vec{p}) V_1(\vec{p}, \vec{q}) g_{30}(\vec{q}) + g_{03}^*(\vec{p}) V_2(\vec{p}, \vec{q}) g_{03}(\vec{q}) \right\}
\]

Here \(m\) is the effective nucleon mass, \(f_{00}\) is the coefficient of the product \(\tau_0 \sigma_0\) in expansion of the distribution function \(f\) in the Pauli matrices, \(V_1, V_2\) are the anomalous FL interaction amplitudes, which have the symmetry properties \(V_i(-\vec{p}, \vec{q}) = V_i(\vec{p}, -\vec{q}) = V_i(\vec{p}, \vec{q}), i = 1, 2\). The quantity \(m\) contains account of the normal FL effects and represents itself the mass of a free nucleon, renormalized by the normal FL interaction (in the Skyrme model the quantity \(m\) is given by Eq. (10)). With allowance for Eqs. (4), (5), we obtain expressions for the quantities \(\Delta_{30}, \Delta_{03}\) as functions of the quantities \(g_{30}\) and \(g_{03}\):

\[
\Delta_{30}(\vec{p}) = \frac{1}{V} \sum_{\vec{q}} V_1(\vec{p}, \vec{q}) g_{30}(\vec{q}), \quad \Delta_{03}(\vec{p}) = \frac{1}{V} \sum_{\vec{q}} V_2(\vec{p}, \vec{q}) g_{03}(\vec{q}) \quad (7)
\]

Conversely, expressions for the distribution functions \(g_{03}\) and \(g_{30}\) as functions of the quantities \(\xi, \Delta_{30}, \Delta_{03}\) can be found from Eqs. (3), (1). With account of them, we get the self-consistent equations for determining the order parameters of two-gap unitary states of superfluid nuclear matter:

\[
\Delta_{30}(\vec{p}) = -\frac{1}{V} \sum_{\vec{q}} V_1(\vec{p}, \vec{q}) \left\{ \frac{\Delta_+(\vec{q})}{4E_+(\vec{q})} \tanh \frac{Y_0 E_+(\vec{q})}{2} + \frac{\Delta_-(\vec{q})}{4E_-(\vec{q})} \tanh \frac{Y_0 E_-(\vec{q})}{2} \right\}, \quad (8)
\]
\[ \Delta_{03}(\vec{p}) = -\frac{1}{V} \sum_{\vec{q}} V_2(\vec{p}, \vec{q}) \left\{ \frac{\Delta_+ (\vec{q})}{4E_+ (\vec{q})} \tanh \frac{Y_0 E_+ (\vec{q})}{2} - \frac{\Delta_- (\vec{q})}{4E_- (\vec{q})} \tanh \frac{Y_0 E_- (\vec{q})}{2} \right\} \]

Here \( \Delta_\pm = \Delta_{30} \pm \Delta_{03} \), \( E_\pm = \sqrt{\xi^2 + |\Delta_\pm|^2} \). According to Eq. (8), considering a nucleon superfluid FL is characterized by two types of fermion excitations with the gaps \( \Delta_\pm \) in the spectrum. Eqs. (8) generalize equations of the BCS theory and contain the one-gap solutions with \( \Delta_{30} \neq 0 \), \( \Delta_{03} \equiv 0 \) (TS pairing) and \( \Delta_{30} \equiv 0 \), \( \Delta_{03} \neq 0 \) (ST pairing) as particular cases.

Let us give an analysis of Eqs. (8), using the model representations of the BCS theory. Precisely, we assume that the interaction amplitudes \( V_1 \) and \( V_2 \) are not equal to zero only in a narrow layer near the Fermi surface: \( |\xi| \leq \theta, \theta \ll \varepsilon_F \) (in numerical calculations we set \( \theta = 0.1 \varepsilon_F \)). Besides, we choose the effective Skyrme forces \( \mathbb{F} \) as the amplitudes of NN interaction. Relation between the anomalous FL amplitudes \( V_1, V_2 \) and the amplitudes of NN interaction has been set in Ref. [2]. Taking into account it, for the quantities \( V_1, V_2 \) we have:

\[ V_{1,2}(\vec{p}, \vec{q}) = t_0 (1 \pm x_0) + \frac{1}{6} t_3 n^\alpha (1 \pm x_3) + \frac{1}{2\hbar^2} t_1 (1 \pm x_1) (\vec{p}^2 + \vec{q}^2), \]  \hspace{1cm} (9)

where \( n \) is the density of symmetrical nuclear matter, \( t_i, x_i, \alpha \) are some phenomenological parameters. Note that the amplitudes \( V_1, V_2 \) contain no dependence from the parameters \( t_2, x_2 \), because the amplitudes \( V_1, V_2 \) are the even functions of the arguments \( \vec{p}, \vec{q} \) (see details in Ref. [2]). There are sets of parameters \( t_i, x_i, \alpha \), which differ for various versions of the Skyrme forces (we shall use the Ska, SkM, SkM* and RATP potentials [4] as well as the SkP potential [5]). In the Skyrme model the effective nucleon mass is given by the expression

\[ \frac{\hbar^2}{2m} = \frac{\hbar^2}{2m_0} + \frac{1}{16} (3t_1 + t_2 (5 + 4x_2)) n, \]  \hspace{1cm} (10)

\( m_0 \) being the mass of a free nucleon.

Let us consider the case \( T = 0 \). As a result of the above assumptions, we arrive at equations for determining the quantities \( \Delta_{30} \equiv \Delta_{30}(p = p_F), \Delta_{03} \equiv \Delta_{03}(p = p_F) \):

\[ \Delta_{30} = g_1 \int_{-\theta}^\theta d\xi \{ (\frac{1}{E_+} + \frac{1}{E_-}) \Delta_{30} + (\frac{1}{E_+} - \frac{1}{E_-}) \Delta_{03} \}, \]  \hspace{1cm} (11)

\[ \Delta_{03} = g_2 \int_{-\theta}^\theta d\xi \{ (\frac{1}{E_+} + \frac{1}{E_-}) \Delta_{03} + (\frac{1}{E_+} - \frac{1}{E_-}) \Delta_{30} \} \]

Here \( g_{1,2} = -\nu_F V_{1,2}(p = p_F, q = p_F) \) and \( \nu_F = \frac{mp_F}{2\pi^2 \hbar^3} \) being the density of states at the Fermi surface of a proton (neutron) with the given spin projection. Eqs. (11) allow to determine the order parameters \( \Delta_{30}, \Delta_{03} \) in the TS–ST states of paired nucleons. In general case this can be done only numerically. Analytical consideration is possible if the
conditions $\Delta_{30}, \Delta_{03} \ll \theta$ are fulfilled (logarithmic approximation). Introducing the ratio of the order parameters $\alpha = \Delta_{30}/\Delta_{03}$ and performing integration in Eqs. (11) in this approximation, we arrive at the following equations for the quantities $\alpha$ and $\Delta_{03}$:

\[
\frac{1}{g_1} = 4 \ln \frac{2\theta}{\Delta_{03}} + 2 \left( \frac{1}{\alpha} \ln \left| \frac{1 - \alpha}{1 + \alpha} \right| - \ln |1 - \alpha^2| \right),
\]

\[
\frac{1}{g_2} = 4 \ln \frac{2\theta}{\Delta_{03}} + 2 (\alpha \ln \left| \frac{1 - \alpha}{1 + \alpha} \right| - \ln |1 - \alpha^2|).
\]

Excluding $\Delta_{03}$ from Eqs. (12), we obtain the equation

\[
\varphi(\alpha) = \frac{1}{g_1} - \frac{1}{g_2}, \quad \varphi(\alpha) \equiv 2 (\alpha - \frac{1}{\alpha}) \ln \left| \frac{1 + \alpha}{1 - \alpha} \right|.
\]

Since $\varphi_{\min} = \varphi(0) = -4$ (the point $\alpha = 0$ is the point of removable discontinuity) and $\varphi_{\max} = \varphi(\pm \infty) = 4$, then Eq. (13) has the solution for $\alpha$, if the coupling constants $g_1$ and $g_2$ satisfy the inequalities $-4 < 1/g_1 - 1/g_2 < 4$. These inequalities have to be fulfilled in the logarithmic approximation for existence of the TS–ST mixed state. In this case, they impose certain restrictions on the coupling constants $g_1$ and $g_2$. The sense of these restrictions is that the coupling constants in TS and ST pairing channels must be of the same order. Clearly that similar restrictions exist in a general case, when the conditions of the logarithmic approximation are not fulfilled.

The results of numerical determination of the order parameters $\Delta_{30}(n), \Delta_{03}(n)$ as functions of density on the base of Eqs. (11) are displayed in Fig. 1. It is seen that at some critical density there appear two pair $(\pm \Delta_{30}(n), \Delta_{03}(n))$ of TS-ST solutions which thermodynamically are indistinguishable. We conclude that two–gap TS–ST superfluid states can arise in nuclear matter as a result of the density phase transition (at fixed temperature) from one-gap ST superfluid state (in the model with the Skyrme effective forces). Note that Eqs. (11) have two–gap solutions with $\Delta_{30} \neq 0, \Delta_{03} \neq 0$ in the case of the SkM, SkM* and Ska potentials but have no such solutions for the RATP and SkP potentials. For the latter potentials the TS coupling constant is considerably larger than the ST coupling constant and, hence, the conditions for existence of two-gap solutions are broken. Concerning the potential SkP we remark that it has been used for the description of the pairing correlations in highly asymmetric nuclear matter [5] unlike to our case where we study superfluidity of symmetric nuclear matter.

3. Temperature behaviour of the order parameters. Given in the section 2 analysis relates to the case $T = 0$. It is clear, that if TS–ST states exist at $T = 0$ then such states arise first at some critical temperature $T_{\text{cst}}$. To describe the temperature behaviour of the order parameters and the mechanism of appearance of the new solutions it is convenient to pass to the new independent variables $x = \Delta_{30} + \Delta_{03}, y = \Delta_{30} - \Delta_{03}$.
With allowance for Eq. (8), where we have replaced summation by integration, the self-consistent equations in terms of the variables $x$ and $y$ are written in the form

$$\frac{x + y}{2g_1} = x\lambda(x, T) + y\lambda(y, T), \quad \frac{x - y}{2g_2} = x\lambda(x, T) - y\lambda(y, T),$$

(14)

$$\lambda(x, T) = \int_{-\theta}^{\theta} \frac{d\xi}{E} \tanh \frac{E}{2T}, \quad E = \sqrt{\xi^2 + x^2}$$

Excluding the variable $y$ from Eq. (14), for determining the function $x = x(T)$ we obtain the equation

$$d(x, T) \cdot d(x \cdot d(x, T), T) \equiv D(x, T) = 1, \quad d(x, T) = \frac{4g_1g_2\lambda(x, T) - g_1 - g_2}{g_2 - g_1}$$

(15)

It is not difficult to see that the variables $x$ and $y$ are related by the formula $y = xd(x, T)$.

Therefore, the order parameters $\Delta_{30}(T), \Delta_{03}(T)$ can be found by the formulae:

$$\Delta_{30} = \frac{1}{2}x(1 + d(x, T)), \quad \Delta_{03} = \frac{1}{2}x(1 - d(x, T))$$

(16)

One–gap solutions are obtained from Eqs. (14) as solutions of the equations $d(x, T) = 1$ (triplet–singlet) and $d(x, T) = -1$ (singlet–triplet) while corresponding critical temperatures $T_{ts}, T_{st}$ are determined from the equations $d(0, T_{ts}) = 1, d(0, T_{st}) = -1$. As follows from evaluations with the Skyrme forces, for all densities, where the superfluid states exist, it holds $g_1 > g_2$. Analyzing the behaviour of the function $D(x, T)$, one can conclude that at temperatures $T > T_{ts}$ there is no one–gap solutions; at temperatures $T_{st} < T < T_{ts}$ there is only one TS solution; at temperatures $T_{stst} < T < T_{st}$ the system is characterized by two one–gap (TS and ST) solutions. Finally, at temperatures $T < T_{tsst}$ we have, besides the previous solutions, two new TS–ST solutions as well.

To determine the critical temperature $T_{tsst}$, at which TS–ST solutions arise first we use the following considerations. Clearly, TS–ST solutions arise as a result of branching from TS or ST branch of solutions Eq. (15). In moment, when TS–ST solutions arise, the derivative $D_x'(x, T)$ vanishes in the critical point $(x_{tsst}, T_{tsst})$. If branching occurs from TS solution, i.e., $d(x, T) = 1$, then $D_x' = d_x'(2 + xd_x') > 0$ for $x > 0$ (since the function $D(x, T)$ is even with respect to $x$; in this case $d_x' > 0$) and the equation $D_x' = 0$ has no solution. If branching occurs from ST solution, i.e., $d(x, T) = -1$, then $D_x' = d_x'(-2 + xd_x')$ and the critical point $(x_{tsst}, T_{tsst})$ is determined from the equations

$$d(x, T) = -1, \quad xd_x'(x, T) = 2$$

(17)

Calculation of the second derivative $D_{xx}''(x, T)$ in the critical point gives $D_{xx}''(x_{tsst}, T_{tsst}) = 0$, i.e., the mechanism of branching of TS–ST solutions is formation of inflection on the
curve $z = D(x, T_{tss})$ for $x = x_{tss}$. All said above allows to approximate the function $D(x, T)$ for $T < T_{st}$ in vicinity of the inflection point in the form

$$D(x, T) = A(x - x_{st}(T))^3 + C(T)(x - x_{st}(T)) + 1 \quad (18)$$

($x_{st}(T)$ is the singlet–triplet branch of the equation $D(x, T) = 1$), where for the coefficients $A \equiv \frac{1}{6}D'''_{xxx}(x_{tss}, T_{tss})$, $C(T) \equiv D'_x(x, T)|_{x=x_{st}(T)}$ it is not difficult to obtain the expressions

$$A = \frac{1}{3}d'''_{xxx} - (d'_x)^3 - \frac{3}{2}d'_x d''_{xx} \frac{1}{2}x(d''_{xx})^2, \quad C(T) = -d'_x(x_{st}(T), T)(2 - x_{st}(T)d'_x(x_{st}(T), T)).$$

Retaining in expansion of the function $C(T)$ the linear with respect to $T_{tss} - T$ term, we obtain the formulae, describing the behaviour of TS–ST solutions near the critical point

$$x^{(\pm)}_{tss}(T) = x_{tss} \pm \sqrt{\beta(T_{tss} - T)/A}, \quad \beta = 2d''_{xt} - d''_T(d'_x + x_{tss}d''_{xx}); \quad (19)$$

In this case, it is necessary that $\beta/A > 0$.

The results of numerical determination of the order parameters $\Delta_{30}(T)$, $\Delta_{03}(T)$ on the base of Eqs. (15), (16) are displayed in Fig. 2. It is seen that two pairs of TS–ST solutions $(\pm \Delta_{30}(T), \Delta_{03}(T))$ exist for the temperatures $T < T_{tss} = T_{tss}$ and besides, in the critical point, in accordance with Eq. (16), $\Delta_{30} = 0$, $\Delta_{03} = \Delta_{03}^{st}$ ($\Delta_{03}^{st}$ being the one–gap ST solution). We recall, that we consider the case $x > 0$. Existence of two pairs of the solutions $(\pm \Delta_{30}(T), \Delta_{03}(T))$ follows from the fact that, if Eqs. (14) have as a solution the pair of functions $(x(T), y(T))$, then after substitution $x \rightarrow -y, \quad y \rightarrow -x$ we obtain the new pair that is a solution too. In the case $x < 0$ we would be able to obtain another two pairs of TS–ST branches $(\pm \Delta_{30}(T), -\Delta_{03}(T))$.

**Conclusion.** Thus, we pointed out the possibility of arising of two-gap unitary states in superfluid nuclear matter corresponding to the superposition of ST and TS pairing of nucleons. The self-consistent equations for these states differ essentially from the equations of the BCS theory and contain the one-gap solutions (ST and TS) as some particular cases. Analysis of the self-consistent equations at the temperature $T = 0$ in the logarithmic approximation shows that new states can arise only under quite specific restrictions on the coupling constants, which describe interaction of nucleons in ST and TS pairing channels. Since the constants of the effective interaction depend from density, there exist such density intervals for which either one-gap or two-gap superfluidity of nuclear matter is realized. Calculations with the Skyrme interaction being chosen as a potential of NN interaction indicate that two-gap unitary states arise in nuclear matter as a result of the phase transition in density from one-gap ST superfluid state (results have been obtained for the Ska, SkM and SkM* potentials). Analysis of the temperature behaviour of the
order parameters shows that new two-gap states can arise under lowering temperature from the superfluid state with ST pairing of nucleons as well. Thus, we conclude that a superfluid nucleon FL under lowering density or temperature can undergo the new phase transition leading to emergence of two-gap superfluid states. Among the other problems we note here the study of influence of asymmetry [6] and real bound states (deuterons) [7, 8] on the thermodynamic properties of multi-gap superfluid states in nuclear matter.

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Figure captions:

Fig. 1. Order parameters $\Delta_{30}, \Delta_{03}$ vs density of symmetric nuclear matter at $T = 0$ in the case of TS ($\Delta_{30} \neq 0, \Delta_{03} = 0$), ST ($\Delta_{30} = 0, \Delta_{03} \neq 0$) and mixed TS-ST ($\Delta_{30} \neq 0, \Delta_{03} \neq 0$) states of paired nucleons for the SkM* (a), SkM (b), Ska (c) Skyrme forces; $tsst(ts_+), tsst(ts_-)$ and $tsst(st_{\pm})$ are notations for the dependences of the TS and ST order parameters $\Delta_{30}$ and $\Delta_{03}$ in two pair $(\pm \Delta_{30}(n), \Delta_{03}(n))$ of TS-ST solutions of Eq. (15). Branching of TS-ST solutions occurs at $n = 0.049 \text{ fm}^{-3}$ for the SkM* potential, at $n = 0.052 \text{ fm}^{-3}$ for the SkM potential and at $n = 0.064 \text{ fm}^{-3}$ for the Ska potential.

Fig. 2. Order parameters $\Delta_{30}, \Delta_{03}$ vs temperature. Notations are the same as in Fig. 1. Calculations have been performed for the SkM* (a), SkM (b) potentials at $n = 0.04 \text{ fm}^{-3}$, for the Ska (c) potential at $n = 0.05 \text{ fm}^{-3}$. Branching of TS-ST solutions occurs at $T = 0.162 \text{ MeV}$ for the SkM* potential, at $T = 0.179 \text{ MeV}$ for the SkM potential and at $T = 0.203 \text{ MeV}$ for the Ska potential.
