Dynamical mixing between $2^3S_1$ and $1^3D_1$ charmed mesons

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In charmed $D$ and $D_s$ mesons sector, the matrix of a Hamiltonian in a quark potential model is computed in the $2^3S_1$ and $1^3D_1$ subspace. The masses of four mixed states of $2^3S_1$ and $1^3D_1$ are obtained. It is the off-diagonal part of spin-orbit tensor interaction that causes the mixing between the $2^3S_1$ and $1^3D_1$ states. The mixing angle between the $2^3S_1$ and $1^3D_1$ state is very small. Based on mass spectra analyses, $D_J^*(2600)$ is very possibly the $D_L^1(2600)$ which is predominantly the $2^3S_1$ $D$ meson. $D_{sL}^*(2700)$ is predominantly the $2^3S_1 D_s$ meson. Under the mixing, the $3P_0$ model is employed to compute the hadronic decay widths of all OZI-allowed decay channels of the four states. Based on the hadronic decay widths analyses, $D_J^*(2600)$ and $D_{sL}^*(2700)$ are possibly the mixed states of $2^3S_1$ and $1^3D_1$ with large mixing angles, which implies a large off-diagonal spin-orbit tensor interaction not existed in present Hamiltonian. For lack of experimental data, $D(2750)$ and $D_{sL}^*(2860)$ in PDG is difficult to be identified except that $D_J^*(2760)$ and $D_{sL}^*(2860)$ have been properly resolved from the experimental data.

PACS numbers:

I. INTRODUCTION

$D$ and $D_s$ mesons consist of a light quark ($u$, $d$ or $s$) and a heavy $c$ quark, they behave like a hydrogen atom. Both the heavy quark symmetry and the light quark chiral symmetry apply in these states. The study of the spectrum, decay and production of $D$ and $D_s$ mesons is helpful to detect the internal quark dynamics.

$S$-wave and $P$-wave charmed mesons ($D$ and $D_s$) without radial excitation have been well established. The higher located states are the $2S$ and $1D$ ones [1], which have not been definitely identified for some reasons. In experiment, the spin and parity are difficult to determine.

$D^*(2600)$ and $D^*(2760)$ were first observed in inclusive $e^+e^-$ collisions by the BaBar Collaboration [2] in the decay channels $D^+\pi^-$, $D^0\bar{\pi}^+$ and $D^{*+}\pi^-$, where they were suggested as the $2^3S_1$ and $3^3D_1$ charmed meson, respectively. In addition to their masses and widths, the branching ratios were measured

$$\frac{\Gamma(D^*(2600)^0\rightarrow D^+\pi^-)}{\Gamma(D^*(2600)^0\rightarrow D^0\pi^-)} = 0.32 \pm 0.02 \pm 0.09,$$

$$\frac{\Gamma(D(2750)^0\rightarrow D^{*+}\pi^-)}{\Gamma(D(2750)^0\rightarrow D^{*0}\pi^-)} = 0.42 \pm 0.05 \pm 0.11.$$

The helicity angle $\Theta_J$ distributions of $D^*(2600)$ were consistent with the expectations for a natural parity ($P = (-1)^J$) [2].

Three years later, two resonances named $D_J^*(2650)$ and $D_{L}^*(2760)$ with a natural parity were observed in the $D^{*+}\pi^-$ mass spectrum in inclusive $pp$ collision by the LHCb Collaboration [3]. In this experiment, $D_J^*(2650)$ was tentatively identified as a $J^P = 1^-$ radial excitation $2^3S_1$ charmed meson and $D_J^*(2760)$ was identified as a $J^P = 1^-$ orbital excitation $1^3D_1$ charmed meson. Subsequently, $D_J^*(2650)$ and $D_J^*(2760)$ are believed the previously observed $D^*(2600)$ and $D^*(2760)$, respectively.

In addition to inclusive production in $e^+e^-$ and $pp$ collisions, highly excited heavy flavor resonances were also produced in exclusive $B$ decays. In exclusive $B$ decays, $D_J^*(2760)$ was observed in the $B^- \rightarrow D_J^*(2760)^0K^-$ decay [4, 5] and $D_J^*(2760)$ was observed in $B^0 \rightarrow D^0\pi^+\pi^-$. The spin of $D_J^*(2760)$ was determined with 1 through a Dalitz plot analysis. In particular, the analysis indicates that $D_J^*(2760)$ observed in $e^+e^-$ and $pp$ collisions consists of $D_J^*(2760)$ and $D_{sL}^*(2760)$ [4, 5] observed in $B$ decays.

The observed $D^*(2760)$ in $D^{*+}\pi^-$, $D^0\pi^+$ and $D^{*+}\pi^-$ is denoted with $D(2750)$ in the charmed mesons list, but denoted with $D_J^*(2750)$ in a separate page in PDG2018. $D^*(2600)$ is denoted with $D_J^*(2600)$ in PDG2018, both $D_J^*(2600)$ and $D_{L}^*(2750)$ are omitted from summary table in PDG.

$D_{sL}^*(2700)$ was first observed by BaBar [6] and then by Belle [7, 8] in $B^+ \rightarrow D^0_{sL}D_{sL} \rightarrow D^0D^0K^+K^-$ decay with $J^P = 1^-$. $D_{sL}^*(2760)$ was first reported by BaBar [9] in $D_{sL}^*(2760) \rightarrow D^0K^+K^-$ with a natural spin-parity. $D_{sL}^*(2700)$ and $D_{sL}^*(2760)$ were also observed in inclusive $e^+e^-$ collision by BaBar Collaboration [9]. Subsequently, it is found that $D_{sL}^*(2760)$ produced in $e^+e^-$ and $pp$ collisions by BaBar and LHCb consists of $D_{sL}^*(2680)$ and $D_{sL}^*(2860)$ [10, 11].

Both $D_{sL}^*(2700)$ and $D_{sL}^*(2860)$ have the decay channels $DK$ and $D^*K$. The ratios of branching fractions were given in the Review of Particle Physics (2018) [11]

$$\frac{\Gamma(D_{sL}^*(2700)^+\rightarrow D^{*+}K^+)}{\Gamma(D_{sL}^*(2700)^+\rightarrow DK^+)} = 0.91 \pm 0.13 \pm 0.12,$$

$$\frac{\Gamma(D_{sL}^*(2860)^+\rightarrow D^{*+}K^+)}{\Gamma(D_{sL}^*(2860)^+\rightarrow DK^+)} = 1.10 \pm 0.15 \pm 0.19.$$

The experimental results about their masses, decay widths and some branching fraction ratios are presented in Table. [11]

In theory, the spectroscopy of heavy-light mesons has been systematically studied in the relativized quark
Their predicted hadronic decay widths at this determined of the meson are charge conjugated into their anti-quarks. (2 states with the decay formula developed by Eichten, Hill, angle is studied through the hadronic decays of these angles such as the 2 eigenstates such as the 2

The mixing between different eigenstates may shift the predicted mass and change the decay widths. In Refs. 12, 37–39, it is noted that the mixing may arise from an internal quark dynamics or an interaction between the hadrons and their decay channels. In particular, the antisymmetric piece of the spin-orbit interaction can cause a 3Lj = 1 Lj mixing between the 2S+1 Lj eigenstates for unequal quark masses and the color hyperfine interaction can cause a 3Lj = 3 Lj mixing [12).

The mixing between the 3Lj = 1 Lj eigenstates such as the 1P1 = 1P1 mixing has been explored both through their mass spectra and through their strong decays 12, 13, 40, 41]. The mixing between the 3Lj = 3 Lj eigenstates such as the 2S1 = 1D1 mixing has also been explored 12]. However, the exploration is not sufficient.

In Ref. 12, the mixing angle is determined with θ = −0.5 radians from a simple mixing matrix of the masses of physical states (2.69 GeV and 2.81 GeV) and the predicted states of the 2S1 and 1D1 Ds mesons (2.71 GeV and 2.78 GeV, respectively). The mixing angle will change sign when the internal quark components of the meson are charge conjugated into their anti-quarks. Their predicted hadronic decay widths at this determined mixing angle in the 3P0 model is consistent with experimental data.

In Ref. 43, a similar mixing scheme of the 2S1 and 1D1 Ds as that in Ref. 12 is employed, and the mixing angle is explored through the hadronic decays of the Ds states. 1.12 ≤ θ ≤ 1.38 radians (opposite in sign with opposite internal quarks) is fixed for Ds*(2710) through a comparison of the predicted hadronic decay widths in the the 3P0 model with experimental data, while the mixing angle 1.26 ≤ θ ≤ 1.31 is fixed for D*3(2860).

In Refs. 46, 47, the similar mixing scheme of the 2S1 and 1D1 D and Ds is employed. The mixing angle is studied through the hadronic decays of these states with the decay formula developed by Eichten, Hill, and Quigg 15. The mixing angles are found small. θ = 4° → 17° and θ = −16° → −4° are obtained for D*(2600) and D*3(2700), respectively.

On one hand, the fixed mixing angles are different in different references. On the other hand, the mixing angles have not been consistently determined through the mass spectra and the decay properties. Of course, in order to identify the D*(2600), D(2750), D*(3)(2700) and D*3(2860), it is also important to systematically study the mixing between the 2S1 and 1D1 D and Ds mesons. For this purpose, we study the mixing between the 2S1 and 1D1 in the quark potential model firstly, and subsequently explore their strong decay in the 3P0 model.

The paper is organized as follows. In the second section, the mixing mechanism between the 2S1 and 1D1 D and Ds mesons is explored in the quark potential model, and the mixing angles are dynamically determined. The hadronic decays of the four mixed states are explored in the 3P0 model in Sec. III. In the final section, the conclusions and discussions are given.

II. DYNAMICAL MIXING BETWEEN 2S1 AND 1D1

To describe the heavy-light meson states, two kinds of eigenstates are often employed. One is the |J, L, S⟩ (denoted with 2S Lj) with J = L + S and S = Sq + S̄q where L is the orbital angular momentum, and S, S̄q are the spins. Another one is the |J, j⟩ (denoted with jP), where P is parity, j = L + S̄q is the angular momentum of light quark freedom. Physical heavy-light mesons are usually not the eigenstates |J, L, S⟩ or |J, j⟩; they are the mixing states of these eigenstates. Eigenstates |J, L, S⟩ will be employed in the following.

In the quark potential model, the inter-quark interactions include the spin-spin interaction, the color-magnetic interaction, the spin-orbit interaction, and the tensor force 12, 55, 11. In our analysis, the relativised quark model 11 is employed for our analysis, where the Hamiltonian is

\[ H = T + V_{qq} \]

(1)

\[ V_{qq} = V_{conf} + V_{SD} \]

(2)

where \( V_{conf} \) is the standard Coulomb and linear scalar interaction, the spin-orbit and color tensor interaction

| State | Experiments | Mass (MeV) | Width (MeV) | Branching ratios |
|-------|-------------|------------|-------------|------------------|
| \( D^+_1(2600) \) | BaBar, LHCB | 2623 ± 12 | 130 ± 31 | \( \Gamma(D^+_1) / \Gamma(D^{*0}) = 0.32 ± 0.02 ± 0.09 \) |
| \( D(2750) \) | BaBar, LHCB | 2763.5 ± 3.4 | 66 ± 5 | \( \Gamma(D^*) / \Gamma(D) = 0.42 ± 0.05 ± 0.11 \) |
| \( D^*_1(2700) \) | BaBar, LHCb | 2708 ± 14.0 | 120 ± 11 | \( \Gamma(D^*_1) / \Gamma(D) = 0.91 ± 0.13 ± 0.12 \) |
| \( D^*_3(2860) \) | BaBar, LHCb | 2859 ± 12 ± 24 | 159 ± 23 ± 77 | \( \Gamma(D^*_3) / \Gamma(D) = 1.10 ± 0.15 ± 0.19 \) |

TABLE I: Experimental results of 2S and 1D candidates of D and Ds [4].
TABLE II: Masses of 1S and 1P D meson (MeV)

| State      | This Work | PDG  |
|------------|-----------|------|
| 1^1S_0     | 1867      | 1869 |
| 1^3S_1     | 2017      | 2010 |
| 1^1P_0     | 2257      | 2308 |
| 1^3P_2     | 2473      | 2460 |
| 1P         | 2399      | 2422 |
| 1P'        | 2429      | 2427 |

V_{SD} is rewritten as

\[
V_{SD} = \left( \frac{S_q}{2 m_q^2} + \frac{S_{\bar{q}}}{2 m_{\bar{q}}^2} \right) \cdot L \left( \frac{1}{r} \cdot \frac{dV_{conf}}{dr} + \frac{2}{r} \cdot \frac{dV_1}{dr} \right) + \left( \frac{S_q + S_{\bar{q}}}{m_q m_{\bar{q}}} \right) \cdot \left( \frac{1}{r} \cdot \frac{dV_2}{dr} \right) + 3 S_q \cdot \hat{r} S_q \cdot \hat{r} - S_q \cdot S_{\bar{q}} \cdot V_3 + \left( \frac{S_{\bar{q}} - S_q}{m_q^2} \right) + \frac{S_q - S_{\bar{q}}}{m_q m_{\bar{q}}} \cdot L V_4 + 32 \alpha_s \sigma^2 \bar{q} \cdot \hat{q} \cdot \hat{q} - \frac{9}{\pi} m_q m_{\bar{q}} \cdot S_q \cdot S_{\bar{q}} \cdot V_3 \right) (3)
\]

The explicit form of \( V_1, V_2, V_3 \), and \( V_4 \) are

\[
V_1(m_q, m_{\bar{q}}, r) = -br - C_F \frac{1}{2r} \left( \frac{\alpha_s}{\pi} (C_F - C_A (\ln[(m_q m_{\bar{q}})^{1/2} r] + \gamma_E)) \right)
\]

\[
V_2(m_q, m_{\bar{q}}, r) = \left( \frac{1}{r} \cdot \frac{dV_{conf}}{dr} + \frac{2}{r} \cdot \frac{dV_1}{dr} \right) + C_F \alpha_s \left( 1 + \frac{\alpha_s}{2\pi} b_0 \left( \ln[(m_q m_{\bar{q}})^{1/2} r] + \gamma_E \right) \right)
\]

\[
V_3(m_q, m_{\bar{q}}, r) = \frac{3}{\pi} C_F \alpha_s \left( 1 + \frac{\alpha_s}{2\pi} b_0 \left( \ln[(m_q m_{\bar{q}})^{1/2} r] + \gamma_E - \frac{4}{3} \right) \right)
\]

\[
V_4(m_q, m_{\bar{q}}, r) = \frac{1}{4r^3} C_F C_A \frac{\alpha_s^2}{\pi} \ln \frac{m_{\bar{q}}}{m_q} \left( \frac{m_q}{m_{\bar{q}}} \right) \left( \frac{m_{\bar{q}}}{m_q} \right) (4)
\]

with \( C_F = \frac{4}{3}, C_A = 3, b_0 = 9, \) and \( \gamma_E = 0.5772. \) The model parameters are: \( \alpha_s = 0.53, \mu = 1.0, \sigma = 1.13, b = 0.135, C_{\bar{c}u} = -0.305, \) and \( C_{\bar{c}s} = -0.254, \) were given in Ref. [11]. The quark masses are chosen as following: \( m_c = 1450 \text{ MeV}, m_u = m_d = 450 \text{ MeV}, \) and \( m_s = 550 \text{ MeV}. \) In term of these parameters, the predicted masses of the 1S and 1P D and \( D_s \) mesons agree well to the experimental data, which are presented in Table. [11] and Table. [11]

As well known, the \( H \) is not diagonal in the basis \( |J, L, S\rangle \) or \( |J, j\rangle. \) The relation between \( |J, L, S\rangle \) and \( |J, j\rangle \) can be found in Refs. [14] [40]. From Ref. [14], the off-diagonal interaction arises from the tensor interaction

\[
V_{tensor} = \left( \frac{3 S_q \cdot \hat{r} S_{\bar{q}} \cdot \hat{r} - S_q \cdot S_{\bar{q}} \cdot V_3}{3 m_q m_{\bar{q}}} \right) \cdot V_3(r) \quad (5)
\]

which can be written in an irreducible representation as

\[
V_{tensor} = 6 \sqrt{\frac{8 \pi}{15}} \frac{Y^{(2)}}{s^2} \cdot V_3(r)
\]

where \( Y^{(2)} \) is a rank 2 spherical harmonics and \( S^{(2)} = (S_q^{(1)} \times S_{\bar{q}}^{(1)})^{(2)} \) with spin operator \( S_q^{(1)} , S_{\bar{q}}^{(1)} \) in the spherical basis.

The matrix element of the tensor term is obtained through the Wigner-Eckhart theorem [45].

\[
\langle J, L, S|V_{tensor}|J', L', S\rangle = (-1)^{L+S+J} \left( \begin{array}{ccc} S & 2 & S \\ L & J & L' \end{array} \right) \langle L||Y^{(2)}||L' \rangle \langle S||S^{(2)}||S \rangle \times \langle J, L, S|V_3(r)|J', L', S\rangle
\]

where \( \langle L||Y^{(2)}||L' \rangle \) is a space reduced matrix element

\[
\langle L||Y^{(2)}||L' \rangle = (-1)^{L} \sqrt{\frac{5(2L+1)(2L'+1)}{4\pi}} \times \left( \begin{array}{ccc} L & 2 & L' \end{array} \right)
\]

and \( \langle S||S^{(2)}||S \rangle \) is the spin reduced matrix element which is \( \frac{\sqrt{3}}{2} \) for \( S = 1. \)

In the subspace of \( \langle 2^3 S_1 | \rangle \) and \( \langle 1^3 D_1 | \rangle \), the non-diagonal matrix of the Hamiltonian is

\[
\begin{pmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{pmatrix}
\]

The numerical matrix of \( H \) in the subspace of \( \langle 2^3 S_1 | \rangle \) and \( \langle 1^3 D_1 | \rangle \) for \( D \) and \( D_s \) mesons are

\[
\begin{pmatrix}
2635.16 & -0.21 \\
-0.21 & 2738.51
\end{pmatrix}
\]

and

\[
\begin{pmatrix}
2714.76 & -0.29 \\
-0.29 & 2805.49
\end{pmatrix}
\]

, respectively.

Without the off-diagonal tensor interaction, \( \langle 2^3 S_1 | \rangle \) and \( \langle 1^3 D_1 | \rangle \) are the eigenstates of the left \( H \). In this case, the eigenvalues of the \( \langle 2^3 S_1 | \rangle \) and \( \langle 1^3 D_1 | \rangle \) \( D \) mesons are 2635.16 MeV and 2738.51 MeV, respectively. The eigenvalues of the \( \langle 2^3 S_1 | \rangle \) and \( \langle 1^3 D_1 | \rangle \) \( D_s \) mesons are 2714.76 MeV and 2805.49 MeV, respectively. The masses of \( (2^3 S_1) \) charmed mesons are comparable to those in
Ref. [14], but the masses of \((1^3D_1)\) charmed states are lower than those in the same reference.

When the low and high mixed states are denoted with \(|D_1^{L*}\rangle\) and \(|D_1^{H*}\rangle\) \([14,16]\), respectively, the matrix \(H\) can be diagonalized in the physical states (mixed states)

\[
\begin{bmatrix}
|D_1^{L*}\rangle \\
|D_1^{H*}\rangle
\end{bmatrix} =
\begin{bmatrix}
cos\theta & sin\theta \\
-sin\theta & cos\theta
\end{bmatrix}
\begin{bmatrix}
|2^3S_1\rangle \\
|1^3D_1\rangle
\end{bmatrix}
\]

with a mixing angle \(\theta\). After diagonalization, \(H\) is turned into \(H'=\)

\[
\begin{bmatrix}
H'_{11} & 0 \\
0 & H'_{22}
\end{bmatrix} =
\begin{bmatrix}
cos\theta & sin\theta \\
-sin\theta & cos\theta
\end{bmatrix}
\begin{bmatrix}
H_{11} & H_{12} \\
H_{21} & H_{22}
\end{bmatrix}
\begin{bmatrix}
cos\theta & sin\theta \\
-sin\theta & cos\theta
\end{bmatrix}^{-1}
\]

(7)

where \(H'_{11}\) and \(H'_{22}\) are the energy eigenvalues of the physical \(D_1^{L*}\) and \(D_1^{H*}\) states, respectively.

With previous formulas in hand, we obtain the physical masses of the mixed states and the mixing angles of \(2^3S_1\) and \(1^3D_1\) for \(D\) and \(D_s\) as follows

\[
M(D^{L*}) = 2635.16 \text{ MeV},
M(D^{H*}) = 2738.51 \text{ MeV},
\theta_{cij} \approx 0.12^\circ.
\]

\[
M(D_1^{L*}) = 2714.76 \text{ MeV},
M(D_1^{H*}) = 2805.49 \text{ MeV},
\theta_{c3} \approx 0.18^\circ.
\]

These four mixed states will be denoted with \(D_1^*(2635)\), \(D_1^*(2739)\), \(D_1^*(2715)\) and \(D_1^*(2805)\) throughout this paper. Obviously, the mixing angles between the \(2^3S_1\) and \(1^3D_1\) for \(D\) and \(D_s\) are very small, and the off-diagonal interactions resulting from the tensor interaction almost do not change the eigenvalues.

In comparison to the measured masses by experiments, \(D_1^*(2600)\) and \(D_1^*(2700)\) in PDG could be identified with the \(D_1^*(2635)\) and \(D_1^*(2715)\). That is to say, \(D_1^*(2600)\) and \(D_1^*(2700)\) are very possibly the predominant \(2^3S_1\) \(D\) and \(D_s\) mesons, respectively.

As analyzed in Refs. [4, 5, 10, 11, 14, 55], \(D^*(2670)\) \((D(2750))\) observed in \(e^+e^-\) and \(pp\) collisions in fact has been resolved into two \(D_1^*(2760)\) and \(D_1^*(2760)\) \(D\) states, \(D_{sJ}^*(2860)\) observed in \(e^+e^-\) and \(pp\) collisions has also been resolved into two \(D_{s1J}^*(2860)\) and \(D_{s2J}^*(2860)\) \(D_s\) states. Unfortunately, the analyses of the resolve are not sufficient, which may result in some uncertainties to the measured data of \(D_1^*(2760)\) and \(D_{s1J}^*(2860)\). For these reasons, the measured data of \(D^*(2670)\) \((D(2750))\) and \(D_{sJ}^*(2860)\) are not sufficient to give the right data of \(D_1^*(2760)\) and \(D_{s1J}^*(2860)\). In experiment, it is important to figure out proper ways to give the exact masses and some decay widths of the resolved states of \(D^*(2760)\) \((D(2750))\) and \(D_{sJ}^*(2860)\) in the future.

In Ref. [22], the \(D_{sJ}^*(2860)\) was regarded as the \(D_s^{H*}\), and a large mixing angle \(\theta = -0.5\) radians has been phenomenologically obtained. The mixing angles between the \(2^3S_1\) and \(1^3D_1\) for \(D\) and \(D_s\) could not be large if the off-diagonal tensor interaction is in its present form, but the mixing angles could be large if the off-diagonal interactions tensor interaction is in some other form which results in large. With large mixing angles, the masses of the mixed states \(D^{H*}\) and \(D_s^{H*}\) could be be large. In order to see how the masses depend on the mixing angles, the variation of the masses of the four mixed mesons with the mixing angles is plotted in Fig. 1. In a large range of the mixing angles, \(D^{H*}\) and \(D_s^{H*}\) have larger masses when the mixing angles turn largely, while \(D_1^{L*}\) and \(D_{s1J}^{L*}\) have smaller masses when the mixing angles turn largely.

![FIG. 1: Variation of the masses of the four mixed mesons with the mixing angles.](image-url)

It should be noted that an off-diagonal tensor interactions inversely proportional to the products of heavy quark and light quark mass in its present form cannot result in a large mixing. Which form of off-diagonal tensor interactions can bring in strong mixing deserves more exploration.

### III. HADRONIC DECAY OF \(D_1^*(2635)\), \(D_1^*(2739)\), \(D_1^*(2715)\) AND \(D_1^*(2805)\)

In order to learn the internal quark dynamics, another way is to study the strong decay of hadrons. In the case of \(2^3S_1\) and \(1^3D_1\) mixing, the hadronic decay of the four mixed states are explored in the \(3P_0\) model in this section.

As well known, the \(3P_0\) model is usually called as the quark-pair creation model. It has been employed extensively to study the Okubo-Zweig-Iizuka(OZI)-allowed hadronic decay processes. The model was first proposed by Micu [50] and developed by Yaouanc et al [51][55]. In
the model, the decay of a meson takes place through a $q\bar{q}$ pair creation with the vacuum quantum number $J^{PC} = 0^{++}$. The hadronic partial decay width $\Gamma$ of a decay process $A \to B + C$ in the center-of-mass frame, and the momentum of the final states $B$ and $C$ in the initial meson $J$ decay process $M$, is given by

$$\Gamma = \frac{\pi^2 |\vec{k}|}{m_A^2} \sum_{JL} |M^{JL}|^2$$

(8)

where $|\vec{k}| = \sqrt{[m_A^2 - (m_B - m_C)^2][m_A^2 - (m_B + m_C)^2]}$ is the momentum of the final states $B$ and $C$ in the initial meson $A$'s center-of-mass frame, and $M^{JL}$ is the partial wave amplitude of $A \to B + C$.

For mixed states $|D_1^{JL}|$ and $|D_1^{JH}|$ with mixing angle $\theta$,

$$\Gamma(|D_L>) = \frac{\pi^2 |\vec{K}|^2}{m_A^2} \sum_{JL} \cos \theta |M^{JL}(2^3 S_1) - \sin \theta |M^{JL}(1^3 D_1)|^2$$

(9)

In terms of the Jacob-Wick formula, $M^{JL}$ can be written as [54],

$$M^{JL}(A \to BC) = \frac{\sqrt{2L + 1}}{2J_A + 1} \times \sum_{M_{JB}, M_{JL}} \langle LOM_{JA}|J_A M_{JA}\rangle \times \langle J_B M_{JB} J_C M_{JL} |J, J M_{JA}\rangle \times M_{M_{JL}} |M_{JA} \chi_{MC}(\vec{K})$$

(10)

where $\vec{J} = \vec{J}_B + \vec{J}_C$, $\vec{J}_A = \vec{J}_B + \vec{J}_C + \vec{L}$ and $M_{JA} = M_{JB} + M_{JL}$. The $M_{M_{JL}} |M_{JA} \chi_{MC}(\vec{K})$ is the helicity amplitude

$$M^{M_{JL} M_{JA}}_{M_{JB} M_{JL}} = \frac{\sqrt{8 E_A E_B E_C \gamma}}{\sum_{M_{LA}, M_{SB}, M_{LC}, M_{SG}}} \langle L_A M_{LA} |S_A M_{SA} |J_A M_{JA}\rangle \times \langle R_B M_{LB} S_B M_{SB} |J_B M_{JB}\rangle \langle L_C M_{LC} S_C M_{SC} |J_C M_{JL}\rangle \times \langle 1m |1 - m|00\rangle \langle 13 \chi_{SA} M_{SA} |12 \chi_{SA} M_{SA} |1 - m\rangle \times \langle 13 \chi_{SB} M_{SB} |24 \chi_{SC} M_{SC} |12 \chi_{SA} M_{SA} |1 - m\rangle \times \langle \phi_B \phi_C |\phi_A \phi_0\rangle I^{M_{LA} M_{LB} M_{LC}}_{M_{JA} M_{JB} M_{JL}} (\vec{K})$$

(11)

where $\gamma$ is the pair-production strength constant. The detail of the flavor matrix element $\langle \phi_B \phi_C |\phi_A \phi_0\rangle$, the spin matrix element $\langle 13 \chi_{SA} M_{SA} |12 \chi_{SA} M_{SA} |1 - m\rangle$ and the momentum integral $I^{M_{LA} M_{LB} M_{LC}}_{M_{JA} M_{JB} M_{JL}} (\vec{K})$ can be found in Ref. [54].

In the $^3P_0$ model, numerical results depend on the parameters such as $\gamma$, the harmonic oscillator parameter $\beta$ and the constituent quark masses. In this paper, the $\gamma = 6.947$ ($\sqrt{96\pi}$ times as the $\gamma = 0.4$ in Ref. [14]) is employed as in Refs. [55, 57, 58]. For strange quark-pair $s\bar{s}$ creation, $\gamma_{s\bar{s}} = \gamma/\sqrt{3}$ [59]. The $\beta$ are taken from Ref. [54]. The constituent quark masses are chosen as $m_c = 1450$ MeV, $m_u = m_d = 450$ MeV, and $m_s = 550$ MeV [55].

In our computation, the masses of related mesons are input as follows: $m_\rho^0 = 134.977$ MeV, $m_\rho^+ = 139.570$ MeV, $m_K^0 = 497.611$ MeV, $m_{K^+} = 493.677$ MeV, $m_{K_L} = 775.26$ MeV, $m_{K_s} = 775.11$ MeV, $m_\eta = 547.862$ MeV, $m_{\eta'} = 782.65$ MeV, $m_{\eta'}(892) = 895.81$ MeV, $m_{K^{*+}(892)} = 891.66$ MeV, $m_{D^{*+}} = 1064.84$ MeV, $m_{D^{*0}} = 1086.91$ MeV, $m_{D_{s0}} = 2006.97$ MeV, $m_{D^{*0}} = 1010.27$ MeV, $m_{D_s(2550)^\circ} = 2539.4$ MeV, $m_{D_s(2420)^\circ} = 2421.4$ MeV, $m_{D_s(2420)^\circ} = 2423.2$ MeV, $m_{D_s(2430)^\circ} = 2427.0$ MeV, $m_{D_s(2460)^\circ} = 2462.6$ MeV, $m_{D_s(2460)^\circ} = 2464.3$ MeV, $m_{D_s(2460)^\circ} = 1968.3$ MeV, $m_{D_s(2460)^\circ} = 1968.3$ MeV. The masses of the four mixed states are chosen as: $m_{D_s(2635)^\circ} = 2635.16$ MeV, $m_{D_s(2739)^\circ} = 2738.51$ MeV, $m_{D_s(2715)} = 2714.76$ MeV, $m_{D_s(2705)} = 2805.49$ MeV [1].

### A. $D_1^*(2635)$ and $D_1^*(2739)$

$D_1^*(2635)$ and $D_1^*(2739)$ are mixed states of $2^3S_1$ and $1^3D_1$ $D$ mesons with mixing angle $\theta = 0.12^\circ$, possible hadronic decay channels and relevant partial decay widths are presented in Table IV. From this table, the total hadronic decay widths of $D_1^*(2635)$ and $D_1^*(2739)$

| Channels | $D_1^*(2635)$ | $D_1^*(2739)$ |
|----------|---------------|---------------|
| $D_1(2420)^\pi^+$ | 1.46 | 42.88 |
| $D_1(2420)^\pi^-$ | 2.79 | 85.51 |
| $D_1(2430)^\pi^+$ | 6.91 | 7.73 |
| $D_1(2430)^\pi^-$ | 13.62 | 15.78 |
| $D^0\pi^+$ | 0.09 | 18.06 |
| $D^0\pi^-$ | 0.13 | 36.52 |
| $D^*\pi^+$ | 0.25 | 12.51 |
| $D^*\pi^-$ | 0.34 | 12.11 |
| $D_1^*(2460)^\pi^+$ | 0.01 | 0.32 |
| $D_1^*(2460)^\pi^-$ | 0.02 | 0.58 |
| $D^*\rho^+$ | 2.36 | 9.95 |
| $D^*\rho^-$ | 4.90 | 20.02 |
| $D^*\rho^0$ | 1.62 | 5.01 |
| $D^*\rho^+$ | 0.34 | 3.74 |
| $D(2550)^\rho^+$ | × | 0.02 |
| $D(2550)^\rho^-$ | × | 0.03 |
| $D^0\rho^0$ | × | 7.29 |
| $D^*\rho^0$ | × | 13.91 |
| $D^*\rho^+$ | × | 6.80 |
| $\Gamma_{total}$ | 34.84 | 298.77 |

Table IV: Hadronic decay widths of $D_1^*(2635)^0$ and $D_1^*(2739)^0$ as mixed states of $2^3S_1$ and $1^3D_1$ with mixing angle $\theta = 0.12^\circ$ (in MeV).
are 34.84 MeV and 298.77 MeV, respectively. These total decay widths are largely different with those of the observed states.

The following ratios are also obtained

\[
\frac{\Gamma(D_s^*(2635)^0 \rightarrow D^+\pi^-)}{\Gamma(D_s^*(2635)^0 \rightarrow D^{*\pi^-})} = 0.03
\]

\[
\frac{\Gamma(D_s^*(2635)^0 \rightarrow D_s^+K^-)}{\Gamma(D_s^*(2635)^0 \rightarrow D_s^{*+}K^-)} = 0.74
\]

\[
\frac{\Gamma(D_s^*(2739)^0 \rightarrow D^+\pi^-)}{\Gamma(D_s^*(2739)^0 \rightarrow D^{*+}\pi^-)} = 1.82
\]

\[
\frac{\Gamma(D_s^*(2739)^0 \rightarrow D_s^+K^-)}{\Gamma(D_s^*(2739)^0 \rightarrow D_s^{*+}K^-)} = 3.34
\]

Obviously, the obtained branching ratios \(\Gamma(D^+\pi)/\Gamma(D^{*\pi^-})\) of \(D_s^*(2635)\) is smaller than the observed one of \(D_s^*(2635)^0\), while the branching ratios \(\Gamma(D^+\pi)/\Gamma(D^{*\pi^-})\) of \(D_s^*(2739)\) are larger than the observed one of \(D_s(2750)\). In other words, \(D_s^*(2635)\) and \(D_s(2750)\) are impossible to be identified with the combination of \(2^3S_1\) and \(1^3D_1\) mesons with a mixing angle \(\theta = 0.12^\circ\).

If the mixing angle is large, the predicted branching ratios are consistent with the observed ones as in Refs. [41, 46, 47]. At a large mixing angle, the masses of \(D_s^*(2635)\) turn smaller, and the masses of \(D_s^*(2739)\) turn larger as shown in Figure 1. However, the problem is which kind of off-diagonal spin-orbit tensor interaction can bring in a large mixing.

B. \(D_s^*(2715)\) and \(D_s^*(2805)\)

\(D_s(2715)\) and \(D_s^*(2805)\) are also mixed states of \(2^3S_1\) and \(1^3D_1\) mesons with mixing angle \(\theta = 0.18^\circ\). Possible hadronic decay channels and relevant partial decay widths are presented in Table 1.

From this table, the total hadronic decay width (39.27 MeV) of \(D_s^*(2715)\) is much smaller than the observed one of \(D_s^*(2700)\), while the total hadronic decay width (184.63 MeV) of \(D_s^*(2805)\) is comparable to that of \(D_s^*(2800)\).

The obtained ratios

\[
\frac{\Gamma(D_s^*(2715)^+ \rightarrow D^0K^+)}{\Gamma(D_s^*(2715)^+ \rightarrow D^{*0}K^+)} = 0.09
\]

\[
\frac{\Gamma(D_s^*(2805)^+ \rightarrow D^0K^+)}{\Gamma(D_s^*(2805)^+ \rightarrow D^{*0}K^+)} = 1.94
\]

are largely different with the observed ones of \(D_s^*(2700)\) and \(D_s^*(2800)\).

Obviously, \(D_s^*(2700)\) and \(D_s^*(2805)\) are impossible to be identified with the combination of \(2^3S_1\) and \(1^3D_1\) mesons with a mixing angle \(\theta = 0.18^\circ\). Similarly, the theoretical results of these ratios could be consistent with the observed data at a large mixing angle.

IV. CONCLUSIONS AND DISCUSSIONS

In this paper, the masses of \(1S, 1P, 1D\) and \(2S\) states of \(D_s\) have been calculated in the quark potential model. The off-diagonal tensor interactions result in small mixing angles between the \(2^3S_1\) and \(1^3D_1\) mesons. The mass difference of the light q quark and s quark changes the mixing angle little. The masses of the four mixed \(D_s^*(2635), D_s^*(2739), D_s^*(2715)\) and \(D_s^*(2805)\) mesons are obtained as 2635 MeV, 2739 MeV, 2715 MeV and 2805 MeV, respectively.

The hadronic partial decay widths of the four mixed states are computed, and some branching fraction ratios are given. Masses of \(D^{+\pi}\) and \(D_s^{*+}\) turn a little larger when the mixing angles turn largely, while masses of \(D^{*+}\) and \(D_s^{*+}\) turn a little smaller when the mixing angles turn largely. The hadronic partial decay widths and the branching fraction ratios depend heavily on the mixing angles.

Based on mass spectra and hadronic decay analyses, the \(D_s^*(2600)\) is very possibly the \(D_s^*(2700)\) of charm mesons. The \(D_s^*(2600)\) and \(D_s^*(2700)\) are predominantly the \(2^3S_1\) and \(1^3D_1\) mesons, respectively. In fact, the resolve of \(D_s(2750)\) and \(D_s^{*}(2860)\) is not sufficient for the identification of the observed states. Until the \(D_s^*(2700)\) and \(D_s^{*}(2860)\) have been separately resolved, it is difficult to identify the \(D_s(2750)\) and \(D_s^{*}(2800)\) from a single measurement.

As pointed out in Ref. [48], the leptonic or electronic decay width is more sensitive to the \(3^1S_0\) and \(3^3D_1\) mixing detail. The measure of the leptonic or electronic decay widths will be helpful to the understanding of these mixed states.

Of course, if the mixing of the \(2^3S_1\) and \(1^3D_1\) is large, there implies higher \(D_s^{*+}\) and \(D_s^{*+}\) charm mesons. In this case, the present form of the off-diagonal tensor interactions does not provide such a large mixing, and the exact form of the off-diagonal tensor interactions deserves deep exploration.

Acknowledgments

This work is supported by National Natural Science Foundation of China under the grants: 11975146 and 11475111.
TABLE V: Hadronic decay widths of $D_s^*(2715)^+$ and $D_{s1}^*(2805)^+$ as mixed states of $2S_1$ and $1D_1$ with mixing angle $\theta = 0.18^\circ$ (in MeV)

| Channels               | $D_s^*(2713)^+$ | $D_{s1}^*(2773)^+$ |
|------------------------|-----------------|--------------------|
| $D^+ K^0$              | 1.79            | 51.79              |
| $D^0 K^+$              | 1.63            | 51.30              |
| $D^{*+} K^0$           | 17.23           | 26.43              |
| $D^{*0} K^+$           | 17.18           | 26.47              |
| $D_s^{+} \eta^0$       | 0.50            | 10.40              |
| $D_s^{*+} \eta^0$      | 0.94            | 3.36               |
| $D^0 K^{*+}$           | ×               | 8.36               |
| $D^{*+} K^{*0}$        | ×               | 6.52               |
| $\Gamma_{total}$       | 39.27           | 184.63             |

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