Bending-induced local buckling during offshore installation of multi-layered FRP pipelines

D Pavlou and N D Adasooriya
Department of Mechanical and Structural Engineering and Material Science, Faculty of Science and Technology, University of Stavanger, Stavanger, Norway

* Contact author: dimitrios.g.pavlou@uis.no

Abstract. In the last two decades FRP pipelines have attracted the attention of the oil industry because of their high strength, excellent fatigue performance and low specific weight. On the other hand, the final cost of installation of FRP pipelines is comparable to the cost of carbon steel ones. Therefore, their implementation in offshore applications seems to be advantageous. During offshore installation, the curvatures of the pipes during the S-lay or J-lay installation processes cause high bending stresses and risk for bending-induced local buckling. Since the pipe wall is multi-layered and the laminae are anisotropic, the calculation of critical bending moments is difficult. In the present work, an analytical solution of critical bending moments for bending-induced local buckling is provided. The proposed method uses the classical lamination theory of multi-layered anisotropic materials and Flügge’s assumption for local buckling analysis of pipelines. Results for E-Glass fiber reinforced polymeric pipelines are provided and discussed.

1. Introduction
Composite FRP materials consist of a polymer matrix material reinforced by fibers. Since the strength of the fibers is much higher than the strength of the matrix, FRP materials have directionally-dependent properties. Glass fibers are low-cost fibers suitable for piping applications (conveying corrosive fluids), vessels, crafts, etc. Their microstructure is based on silica SiO₂ which in the form of a polymer (SiO₂)n does not melt but softens progressively till 2000°C. E-Glass is a borosilicate glass which exhibits very good corrosion resistance, and it is suitable for operating in water and mild chemical environment. The main physical and mechanical properties of E-Glass material used for fibers fabrication are presented in Table 1 [1].

Table 1. Mechanical properties of E-Glass fibers

| Property             | E-Glass |
|----------------------|---------|
| Tensile strength (MPa)| 3450    |
| Tensile modulus (GPa)| 72.4    |
| Elongation (%)        | 1.8-3.2 |
| Density (Kg/m³)       | 2541    |
| Diameter (μm)         | 8-13    |
Depending on the selected installation method, the installation loads are usually more critical for the pipeline dimensioning than the operation loads. For the case of offshore pipelines, the installation methods can be S-Lay or J-Lay (Figure 1a and b) [2, 3].

Figure 1. (a) S-Lay installation process, (b) J-Lay installation process
During above installation procedures the pipeline is subjected to pseudo-static loads. Bending is the predominant loading case, and the calculation of bending capacity is necessary. In the present work, the bending-induced local buckling during offshore installation [3, 4] of multi-layered FRP pipelines is investigated. To this end, the classical lamination theory of multi-layered anisotropic materials [5, 6] and Flügge’s assumption [7] for local buckling [8] analysis of pipelines is utilized in order to derive an analytical model. Details for this analysis can be found in [2].

2. Formulation of the model

When the bending moment $M_y$ is acting on a composite pipe (Figure 2a), the narrow strip AB is subjected to maximum compressive longitudinal strain $\varepsilon_0(0)$. In order to calculate the critical value of $M_y$ causing local buckling to the strip AB, we have initially to estimate.

The critical value of $\varepsilon_0(0)$, namely $\varepsilon_{c0}$, at the corresponding buckling state. To this scope, we’ll assume that above strip is a part of a same pipe which has reached the critical longitudinal compressive strain $\varepsilon_{c0}$ due to axial compression (Figure 2b).

We will approximate the critical value of $M_y$ for the case of Figure 2a considering the critical strain $\varepsilon_{c0}$ derived by the model shown in Figure 2b. As the maximum compressive stress causing buckling to the model of Figure 2b is acting on the whole perimeter of the pipe’s cross section, instead of acting on only to the point A (as in case of Figure 2a), above practical assumption is expected to be conservative, therefore, safe for designing purposes. This assumption has been checked in [7] for the case of isotropic thin-walled tubes. In that work it is seen that for pure bending the exact solution gives for the critical compressive stress a value which is about 30% higher than that obtained by the above assumption [8]. Concerning the Figure 2b, the critical longitudinal strain at the buckling state is:

$$\varepsilon_{c0} = \alpha_{11} N_c$$

where $N_c$ is the critical axial load per unit circumference given by

![Figure 2. (a) (α) Buckling of the narrow strip AB due bending (b) Buckling of narrow strip AB due to axial load](image-url)
\[ N_e = \lambda_{cr} N_o \] (2)

In this equation is \( N_o = 1 \). The parameter \( \lambda_{cr} \) is the minimum eigenvalue obtained by the solution of the following equation:

\[
\det \left( \begin{bmatrix} O & L \\ L & O \end{bmatrix} \begin{bmatrix} M_o & M_u \\ M_u & M_o \end{bmatrix} \begin{bmatrix} O & L \\ L & O \end{bmatrix} - \lambda \begin{bmatrix} \Phi_1 \cdot [J] \\ \Phi_2 \cdot [J] \end{bmatrix} \right) = 0
\] (3)

Above equation as well as the matrices \([L], [O], [J]\) and the parameters \( \Phi_1, \Phi_2 \) are given in ref. [6], i.e.:

\[
[O] = \begin{bmatrix}
-\alpha & 0 & \beta & 0 & 0 & 0 \\
0 & -\beta & \alpha & 0 & 0 & 0 \\
0 & 2 / D & 0 & a^2 + \beta^2 c_2^2 & \beta^2 + a^2 c_1^2 & -2a\beta(1+c_1c_2)
\end{bmatrix}
\] (4)

\[
[L] = \begin{bmatrix}
\beta c_2 & 0 & -ac_1 & 0 & 0 & 0 \\
ac_1 & -\beta c_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2a\beta c_2 & -2a\beta c_1 & 2(c_1a^2+c_2\beta^2)
\end{bmatrix}
\] (5)

\[
[M_o] = \begin{bmatrix}
A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\
A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\
0 & 0 & A_{66} & 0 & 0 & B_{66} \\
B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\
B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\
0 & 0 & B_{66} & 0 & 0 & D_{66}
\end{bmatrix}
\] (6)

\[
[M_u] = \begin{bmatrix}
A & B \\
B & D
\end{bmatrix} - [M_o]
\] (7)

\[
[J] = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (8)

\[
\Phi_1 = a^2 + \beta^2 c_2^2
\] (9)

\[
\Phi_2 = -2a\beta c_2
\] (10)

In the above matrices the four constants \( a, b, c_1, c_2 \) characterize the wave pattern of the buckling. For different combinations of the above constants, eq. (3) will result into different values of \( \lambda_{cr} \). The lowest value of \( \lambda_{cr} \) is the appropriate value to be used in eq. (2).

According to [2] the local strain \( \varepsilon_x \) at any point located on the mean diameter of a cross section of a pipe subjected to pure bending by moment is:
\[ \varepsilon_{\xi}^{\alpha} = \frac{1}{\rho_y} z \]  

(11)

where

\[ \frac{1}{\rho_y} = \frac{\hat{M}_y}{E I_{yy}} \]  

(12)

\[ \bar{E} I_{yy} = 2 \int_{0}^{\pi} \left( \frac{1}{\alpha_{11}} z^2 + \frac{1}{d_{11}} \cos^2 \theta \right) \frac{D}{2} d\theta \]  

(13)

\[ z = \frac{D}{2} \cos \theta \]  

(14)

The local strain \( \varepsilon_{\xi}^{\alpha} \) of the strip AB shown in Fig. 2a is taking place for \( \theta = 0 \).

Therefore:

\[ \varepsilon_{\xi}^{\alpha} = \frac{\hat{M}_y}{\bar{E} I_{yy}} \frac{D}{2} \]  

(15)

Combining equations (1), (2), (15), the critical bending moment \( \hat{M}_{yc} \) causing local buckling into the strip AB can be approximated by the following equation:

\[ \hat{M}_{yc} = \frac{2 \bar{E} I_{yy} a_{11} \lambda_{\xi}}{D} \]  

(16)

3. Implementation of the solution and results

The considering the described model, the critical bending moment \( \hat{M}_{yc} \) causing local buckling has been estimated for multilayered pipes made by E-Glass/Epoxy material [2]. The calculations have been performed with the aid of the standard software "Mathematica" [9]. In the derived diagrams are demonstrated the critical bending moments \( \hat{M}_{yc} \) causing local buckling for pipes of diameters \( \text{Dia} = 0.10, 1.2 \text{\,mm} \) consisting of plies of thickness \( 0.150 \text{\,mm} \) and fiber orientation \( \theta = \pm15^\circ, \pm30^\circ, \pm75^\circ \) for number of plies \( NP = 10, \ldots, 50 \). The results are shown in Figures. 3 (a)-(e).
Figure 3. Critical bending moment $M_{cr}$ causing local buckling in multi-layered E-Glass/Epoxy pipelines

4. Conclusions

1. In the present work, the bending-induced buckling of multi-layered FRP pipelines during S-lay and J-lay offshore installation is analysed.

2. With the aid of classical lamination theory (CLT) of anisotropic materials and taking into account the Flügge assumption for critical bending moment causing buckling in pipelines, a mathematical model is derived for bending-induced local buckling capacity of pipelines during offshore installation.
3. Unlike existing commercial software packages, the proposed analytical model is advantageous because it provides accurate results.
4. Implementation of the model on typical multilayered FRP pipelines made of E-Glass/Epoxy material has been carried out and useful nomographs for quick estimation of bending-induced local buckling moment capacity are provided.

References
[1] Hyer M 2009 Stress analysis of fiber reinforced composite materials, DEStech Publications.
[2] Pavlou D G 2013 Composite Materials in Piping Applications, Destech publications.
[3] Bai Y and Bai Q 2005 Subsea pipelines and risers, Elsevier.
[4] Reddy D V and Swamidas A S J 2014 Essentials of offshore structures, CRC press.
[5] Reddy J N 2004 Mechanics of laminated composite plates and shells, CRC press.
[6] Kollár L P and Springer G S 2003 Mechanics of composite structures, Cambridge University Press.
[7] Flügge W 1973 Stresses in shells, Springer-Verlag.
[8] Timoshenko S P and Gere J M 2009 Theory of elastic stability, Dover publications.
[9] Wolfram Mathematica, Mathamatica user manul, https://www.wolfram.com/mathematica/