Clearing an Orthogonal Polygon Using Sliding Robots

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Abstract
In a multi-robot system, a number of autonomous robots would sense, communicate, and decide to move within a given domain to achieve a common goal. In this paper, we consider a new variant of the pursuit-evasion problem in which the robots (pursuers) each move back and forth along an orthogonal line segment inside a simple orthogonal polygon $P$. A point $p$ can be covered by a sliding robot that moves along a line segment $s$, if there exists a point $q \in s$ such that $pq$ is a line segment perpendicular to $s$. In the pursuit-evasion problem, a polygonal region is given and a robot called a pursuer tries to find some mobile targets called evaders. The goal of this problem is to design a motion strategy for the pursuer such that it can detect all the evaders. We assume that $P$ includes unpredictable, moving evaders that have unbounded speed. We propose a motion-planning algorithm for a group of sliding robots, assuming that they move along the pre-located line segments with a constant speed to detect all the evaders with unbounded speed.

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1 Introduction

The mathematical study of the “pursuit-evasion” problem was first considered by Parson [11]. After that, the watchman route problem was introduced as a variation of the art gallery problem, which consists of finding static evaders in a polygon. The visibility-based motion-planning problem was introduced in 1997 by Lavalle et al. [6]. The aim was to coordinate the motions of one or more robots (pursuers) that have omnidirectional vision sensors to enable them to eventually “see” an evader that is unpredictable, has an unknown initial position, and is capable of moving arbitrarily fast. The process of detecting all evaders

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is also known as clearing the polygon. The pursuit-evasion problem has a broad range of applications such as air traffic control, military strategy, and trajectory tracking [6].

In 2011, Katz and Morgenstern introduced sliding camera guards for guarding orthogonal polygons [5]. We define the “sliding robots” to be the same as the sliding cameras, where the robot $r_i$ would travel back and forth along an axis-aligned segment $s$ inside an orthogonal polygon $P$. A point $p$ is seen by $s_i$ if there exists a point $q \in s_i$ such that $pq$ is a line segment perpendicular to $s_i$ and is completely inside $P$. The set of all points of $P$ that can be seen by $s_i$ is its sliding visibility polygon (see Fig. 1). The point $p$ is seen by $r_i$ if $r_i = q$.

![Figure 1](image_url) The shaded area shows the sliding visibility polygon of $s$.

According to the visibility-based motion-planning problem and the sliding robots, we study the new version of planning the motions for a group of robots for clearing an orthogonal polygon when robots are modeled as sliding cameras. The given orthogonal polygon $P$ has unpredictable, moving evaders with unbounded speed. Motion planning for a group of sliding robots to clear $P$ means presenting the sequence of motions for the sliding robots such that any evader is viewed by at least one robot. Moreover, a set of line segments, $S$, is given such that the union of their sliding visibility polygons is $P$.

Previous Works

Generally, in the pursuit-evasion problem, the pursuer is considered as an $l$-searcher with $l$ flashlights and rotates them continuously with a bounded angular rotation speed [13]. Thus, an $\infty$-searcher (also known as an omnidirectional searcher) is a mobile robot equipped with a $360^\circ$ view sensor for detecting evaders. Lavalle et al. proposed the first algorithm for solving the pursuit-evasion problem for an $l$-searcher [6]. They decomposed $P$ into cells based on visibility properties and converted the problem to a search on an exponential-sized information graph. Durham et al. [2] addressed the problem of coordinating a team of mobile robots with limited sensing and communication capabilities to detect any evaders in an unknown and multiply connected planar environment. They proposed an algorithm that guarantees the detection of evaders by maintaining a complete coverage of the frontier between cleared and contaminated regions while expanding the cleared region.

The art gallery problem is a classical and old problem in computational geometry. Over the years, many variants of this problem have been studied [10, 13, 4, 12]. Most of these have been proved to be NP-hard [7], containing the problem when the target region is a simple orthogonal polygon, and the goal is to find the minimum number of vertex guards to guard the entire polygon (e.g., [10, 12]). Some types of them, which consider the limited model of visibility, use polynomial time algorithms [9, 15].

The study of the art gallery problem based on the sliding camera was started in 2011 by Katz and Morgenstern [5]. They studied the problem of guarding a simple orthogonal
polygon using minimum-cardinality sliding cameras (MCSC). They showed that, when the cameras are constrained to travel only vertically inside the polygon, the MCSC problem can be solved in polynomial time. They also presented a two-approximation algorithm for this problem when the trajectories that the cameras travel can be vertical or horizontal and the target region is an x-monotone orthogonal polygon. They left the computation of the complexity of the MCSC problem as an open problem. In 2013, Durocher and Mehrabi [1] studied these two problems: the MCSC problem and the minimum-length sliding camera (MLSC) problem, where the goal was to minimize the total length of the trajectories along which the cameras travel. They proved that the MCSC problem is NP-hard, where the orthogonal polygon can have holes. They also proved that the MLSC problem is solvable in polynomial time even for orthogonal polygons with holes. In 2014, Durocher et al. [8] presented an $O(n^{2.5})$-time (7/2)-approximation algorithm for solving the MCSC problem in simple orthogonal polygons. In 2014, De Berg et al. [11] presented a linear-time algorithm for solving the MCSC problem in an x-monotone orthogonal polygon. The complexity of this problem remains as an open problem.

### Our Result

Our aim is to plan the motions for a group of robots that move along the line segments of $S$ and find all unpredictable evaders such that the number of robots used is the cardinality of $S$. Owing to the difficulty of having multiple cooperating robots executing common tasks, we store some information (e.g., the status of some nearby regions that shows whether the regions have been cleared by some robots) on each reflex vertex.

We assume that the robots have the map of the environment and that they are capable of broadcasting a message (e.g., a region that is supposed to get cleared) to all the other robots by sending signals. This way, the robots can have some communications with each other to maintain the coordination process.

The best result of our algorithm is that, if $S$ is a set of MCSCs that guard the whole $P$, then our algorithm will detect all evaders with the minimum number of sliding robots.

### 2 Preliminaries and Notations

Let $P$ be an orthogonal polygon and $V(P) = \{v_1, v_2, ..., v_n\}$ be the set of all vertices of $P$ in counterclockwise order. We consider $V_{\text{ref}}(P)$ to be all of the reflex vertices of $P$ and assume a general position such that no four reflex vertices are collinear. Suppose that $P_1$ is a sub-polygon of $P$ whose boundary is from $a$ to $b$ ($a$ and $b$ are points on the boundary of $P$) in counterclockwise order. Then, we show $P_1$ by $(a, b)$.

Let $v_j$ be a reflex vertex of $P$. $v_j$ has two edges, $e_{j-1} = v_{j-1}, v_j$ and $e_j = v_j, v_{j+1}$, that can be extended inwardly until they reach the boundary of $P$. We call these extensions as the windows of $v_j$. Let $w_{i,j} = v_{i,j}$. We define the coordinates of $v_{i,j}$ as $(x_{i,j}, y_{i,j})$. We define the windows of $v_j$ as $w_{i,j} = v_{i,j}$. We define the coordinates of $v_{i,j}$ as $(x_{i,j}, y_{i,j})$. We define the windows of $v_j$ as $w_{i,j} = v_{i,j}$.

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windows of $P$. $L$ partitions $P$ into orthogonal rectangles.

![Figure 2](image.png) Shown are the windows and the sub-polygons of $v_j$.

## 3 The Proposed Algorithm

In this section, we present an algorithm for solving the pursuit-evasion problem using sliding robots. Assume that an orthogonal polygon $P$ and a set of orthogonal line segments $S = \{s_1, s_2, \ldots, s_k\}$ are given. We present a path-planning algorithm for finding the unpredictable evaders using a set of sliding robots $R = \{r_1, r_2, \ldots, r_k\}$ in which $r_i$ can move along the line segment $s_i$.

To distribute the movements of the robots, we define the “event points” as below:

- **Definition 1.** An event point happens when $r_i$ sees a reflex vertex, sees a waiting sliding robot, or reaches an endpoint of $s_i$.

### Overview of the Algorithm

Our algorithm has five steps. The “start step,” the “decision step,” the “sending a signal step,” the “move back-and-update step,” and the “termination step.”

To present our path-planning method, we start with an arbitrary sliding robot $r_i \in R$, which is on $s_i \in S$ (start step). $r_i$ starts moving from one endpoint of $s_i$. When $r_i$ reaches an event point, it updates the cleared sub-polygons. By the time that $D_i(2)$ becomes empty and $D_i(1) \neq \emptyset$, $r_i$ moves back along $s_i$ (move back-and-update step). Moreover, at each event point, $r_i$ stops and, according to the cleared sub-polygons of $P$, decides to continue its movement or send a signal to the other robots to clear a specific sub-polygon of $P$ (decision step). When $r_i$ sends a signal to the other robots to clear a sub-polygon $P_1$, a robot that can clear some parts of $P_1$ starts moving along its corresponding line segment (sending a signal step). When all parts of $P$ become cleared, the algorithm is finished (termination step).

### Details of the Algorithm

Now, we explain the steps of the algorithm in detail. We store the status of the regions in their corresponding reflex vertices, which are updated by the robots during the movements to keep track of the contaminated regions, which is helpful in the decision-making process.

For each $v_i \in V_{ref}(P)$, we store an array called $FF_i(i)$ ($1 \leq i \leq 4$) of size four in which the cells (of type Boolean) indicate whether the sub-polygons $P_j(j - 1, j)$, $P_j(j + 1, j)$,
For each \( r_i \in R \), we consider a triple storage, which is called \( D_i(j) \) for \( 1 \leq j \leq 3 \). Each storage includes an interval such as \((a, b)\), which indicates the boundary of \( P \) between \( a \) and \( b \). The first storage, \( D_i(1) \), indicates the cleared sub-polygon of \( P \) by \( r_i \) (partly or completely). The second storage, \( D_i(2) \), indicates the sub-polygon of \( P \) that should be cleared by \( r_i \) (partly or completely). The third storage, \( D_i(3) \), specifies the sub-polygon that should be cleared until \( r_i \) continues its movement. In the case where \( r_i \) is waiting, \( D_i(3) \) is not empty. Initially, for each \( r_i \in R \), \( D_i(1) = D_i(2) = D_i(3) = \emptyset \).

**Start Step**
As mentioned earlier, we start with one of the endpoints of an arbitrary \( s_i \) (\( r_i \) can move along \( s_i \)).

- If \( r_i \) starts from an endpoint that is on the boundary, \( r_i \) can see two consecutive vertices (suppose the endpoint is on the edge \( e_k = v_kv_{k+1} \)).
  1. If \( v_k \) and \( v_{k+1} \) are convex, then \( r_i \) starts clearing \( P \) by its movement and updates \( D_i(1) = (v_k, v_{k+1}) \). \( r_i \) continues its movement along \( s_i \) until an event point happens. At these times, \( r_i \) stops, updates \( D_i(1) \) and \( D_i(2) \), and makes a decision for its movement (decision step).
  2. If at least one of \( v_k \) or \( v_{k+1} \) is a reflex vertex, then \( r_i \) cannot start clearing \( P \); it therefore stops and waits on the endpoint to make a decision (decision step).

- If \( r_i \) wants to start from an endpoint that is not on the boundary, then \( r_i \) cannot start clearing \( P \); it therefore stops and waits on the endpoint (decision step). Suppose that the maximal normal line segment to \( s_i \) that passes through \( r_i \) is \( tr \). Let \( x \) and \( w \) be the first intersection of \( tr \) at the boundary of two sides. \( s_i \) can be inside the sub-polygon corresponding to \((x, w)\) or \((w, x)\). Assume that \( s_i \) is inside \((w, x)\). Therefore, \( r_i \) sends a signal to the other robots to clear \((x, w)\), and \( D_i(3) = (x, w) \) (sending a signal step). As shown in Fig.3 if \( r_i \) wants to start from \( z \), it stops and sends a signal to the other robots to clear the sub-polygon corresponding to \((x, w)\).

**Move Back-and-Update Step**
Assume that \( r_i \) moves along \( s_i \). When an event point happens, \( r_i \) updates \( D_i(1) \) (increases the cleared region) and \( D_i(2) \) (decreases the sub-polygon that should be cleared). At each
time that $D_i(2)$ becomes empty (and $D_i(1) \neq \emptyset$), $r_i$ moves back along $s_i$. It moves back until it sees a waiting robot or reaches an endpoint of $s_i$. When $r_i$ sees a reflex vertex $v_k$ during its movement, it updates $FF_k(j)$ for $1 \leq j \leq 4$ as detailed below:

- If the endpoints of $win_k(k - 1, k)$ and $v_{k+1}$ are inside the sub-polygon indicated by $D_i(1)$ (if $D_i(1) = (v_k, x_k)$, then $v_{k+1} \in D_i(1)$), then the sub-polygon $P_k(k-1, k)$ is cleared and $r_i$ updates $FF_k(1) = \text{true}$ (see Fig.3 when $r_1$ moves back from left to right and reaches $a$).
- If $D_i(1) = (x_k, v_k)$, then $r_i$ updates $FF_k(3) = \text{true}$ (see Fig.3 when $r_1$ moves back from right to left and reaches $a$).
- If $D_i(1) = (y_k, v_k)$, then $r_i$ updates $FF_k(2) = \text{true}$ (see Fig.3 when $r_1$ moves back from left to right and reaches $b$).
- If $D_i(1) = (v_k, y_k)$, then $r_i$ updates $FF_k(4) = \text{true}$ (see Fig.3 when $r_1$ moves back from right to left and reaches $b$).

As we explained earlier, $r_i$ moves back until it finishes clearing ($D_i(2) = \emptyset$). While it is moving back, if $r_i$ sees its corresponding waiting robot (supposedly $r_j$) and $D_j(1) = D_j(3)$, then $D_i(2) = \emptyset$. Therefore, $r_i$ updates $D_j(3) = \emptyset$, $D_j(1) = D_j(1) \cup D_i(1)$, and $D_j(2) = D_j(2) \cup D_i(1)$. Since $D_i(2)$ is empty, $r_i$ finishes its clearing and $r_j$ starts moving back (see Fig.3 when $r_1$ moves back from left to right and reaches $c$, it updates the information of $r_2$, and $r_2$ moves back), $r_j$ can be collinear with the endpoint of $s_j$. Moreover, if $r_i$ sees any reflex vertex $v_k$, $r_i$ updates $FF_k(j)$ for $1 \leq j \leq 4$ as explained above and continues moving back.

**Decision Step**

When $r_i$ stops and waits, it makes a decision and performs the following:

1. If $r_i$ is on the endpoint of $s_i$ (let ep be the endpoint), then
   a. If ep is on the boundary of $P$ (on the edge $(e_k = m, v_{k+1})$), then
      - If $v_k \in V_{ref}(P)$ and $P_k(k+1, k)$ is contaminated ($FF_k(2) = \text{false}$), then $P_k(k+1, k)$ should be cleared. Therefore, $r_i$ sends a signal to the other robots to clear $P_k(k+1, k)$ and updates $D_i(3) = (v_k, v_k)$ (As mentioned in Section2, $y_k$ and $v_k$ are two endpoints of $win_k(k +1, k)$, and since $P_k(k+1, k)$ includes $v_{k-1}$, $D_i(3)$ is from $v_k$ until $v_k$ in counterclockwise order). For an example, see Fig.4 assume that $r_3$ or $r_2$ is on the blue point of $s_3$ and $s_2$, respectively.
      - Else if $P_k(k+1, k)$ is cleared ($FF_k(2) = \text{true}$), then $D_i(1) = D_i(1) \cup (y_k, v_k)$ and $D_i(2) = D_i(2) \setminus (y_k, v_k)$.
   b. If $v_{k+1} \in V_{ref}(P)$ and $P_{k+1}(k, k+1)$ is contaminated ($FF_{k+1}(1) = \text{false}$), then $P_{k+1}(k, k+1)$ should be cleared. Therefore, $r_i$ sends a signal to the other robots to clear $P_{k+1}(k, k+1)$ and updates $D_i(3) = (v_{k+1}, x_{k+1})$. For an example, see Fig.4 assume that $r_1$ or $r_2$ is on the blue point of $s_1$ and $s_2$, respectively.
   c. Else if $P_{k+1}(k, k+1)$ is cleared ($FF_{k+1}(1) = \text{true}$), then $D_i(1) = D_i(1) \cup (v_{k+1}, x_{k+1})$ and $D_i(2) = D_i(2) \setminus (v_{k+1}, x_{k+1})$.
   d. If at least one of $v_k$ and $v_{k+1}$ is a reflex vertex, then ep is on $l(j) \in L$. If $l(j)$ includes two consecutive reflex vertices $v_m, v_{m+1}$, where $m \neq k$ (suppose that the nearest one to $r_i$ is $v_m$), then
      - For an example, see Fig.4 assume that $r_3$ is on the blue point of $s_3$.
         i. If $P_{m+1} = (m, m +1)$ is contaminated ($FF_{m+1}(1) = \text{false}$), then $r_i$ sends a signal to the other robots to clear $P_{m+1}(m, m +1)$ and updates $D_i(3) = (v_{m+1}, x_{m+1})$. 


b. If \( ep \) is not on the boundary of \( P \) and \( ep \) is collinear by at least one reflex vertex, then \( ep \) is on \( l(j) \) \( \in L \). Therefore,

- If \( l(j) \) consists of one reflex vertex \( v_k \) (assume that the consecutive vertex of \( v_k \) on \( l(j) \) is \( v_{k+1} \)) and \( s_i \) is inside \( P_h(k+1, k) \), then

For an example, see Fig. 5 assume that \( r_5 \) is on the blue point of \( s_5 \).

i. If \( P_h(k+1, k) \) is contaminated \( (FF_h(4) = false) \), then \( r_i \) sends a signal to the other robots to clear \( P_h(k+1, k) \) and updates \( D_i(3) = (v_k, y_k) \).

ii. Else \( (FF_h(4) = true) \), \( D_i(1) = D_i(1) \cup (v_k, y_k) \) and \( D_i(2) = D_i(2) \setminus (v_k, y_k) \).

- Else if \( s_i \) is inside \( P_h(k+1, k) \), then

i. If \( P_h(k+1, k) \) is contaminated \( (FF_h(2) = false) \), then \( r_i \) sends a signal to the other robots to clear \( P_h(k+1, k) \) and updates \( D_i(3) = (v_k, y_k) \).

ii. Else \( (FF_h(2) = true) \), \( D_i(1) = D_i(1) \cup (v_k, y_k) \) and \( D_i(2) = D_i(2) \setminus (y_k, v_k) \).

- If \( l(j) \) consists of two consecutive reflex vertices \( v_k \) and \( v_{k+1} \) (suppose that the nearest one to \( ep \) is \( v_k \), and \( s_i \) is inside \( P_h(k+1, k) \), then

For an example, see Fig. 5 assume that \( r_6 \) is on the blue point of \( s_6 \).

i. If \( P_{h+1}(k+1, k+1) \) is contaminated \( (FF_{h+1}(1) = false) \), then \( r_i \) sends a signal to the other robots to clear it \( (P_{h+1}(k, k+1) \) and updates \( D_i(3) = (v_{k+1}, x_{k+1}) \).

ii. Else \( (FF_{h+1}(1) = true) \) \( r_i \) sends a signal to the other robots to clear \( P_{h+1}(k+1, k+1) \cap P_h(k+1, k) \) and updates \( D_i(3) = (x_{k+1}, y_k) \).

- Else if \( s_i \) is inside \( P_h(k+1, k) \), then

i. If \( P_{h+1}(k+1, k+1) \) is contaminated \( (FF_{h+1}(1) = false) \), then \( r_i \) sends a signal to the other robots to clear it \( (P_{h+1}(k, k+1) \) and updates \( D_i(3) = (v_{k+1}, x_{k+1}) \).

ii. If \( P_h(k+1, k) \) is contaminated \( (FF_h(2) = false) \), then \( r_i \) sends a signal to the other robots to clear it and updates \( D_i(3) = (y_k, v_k) \).

iii. If \( P_{h+1}(k+1, k) \) and \( P_h(k+1, k) \) are cleared \( (FF_{h+1}(1) = true \) and \( FF_h(2) = true) \), then \( D_i(1) = D_i(1) \cup (y_k, x_{k+1}) \) and \( D_i(2) = D_i(2) \setminus (y_k, x_{k+1}) \).
c. If ep is not on the boundary of P and ep is not collinear by any reflex vertex, then suppose that the maximal orthogonal line segment normal to $s_i$ at ep is l and let a and b be two endpoints of l. l partitions P into two sub-polygons. One of them consists of $s_i$. Therefore, $r_i$ sends a signal to the other robots to clear the sub-polygon that does not include $s_i$ and that is between a and b ($r_i$ updates $D_i(3)$ depending on its position to $D_i(3) = (a, b)$ or $D_i(3) = (b, a)$).

For an example, see Fig. 6 if $r_7$ is on the blue point of $s_7$, then the sub-polygon that is between (a, b) in counterclockwise order should be cleared.

2. Else if $r_i$ sees at least one reflex vertex ($r_i$ is on $l_j$), then
   a. If there are no two consecutive reflex vertices on $l_j$, then $r_i$ continues its movement along $s_i$.
   b. If there are two consecutive reflex vertices $v_k, v_{k+1}$ on $l_j$ (suppose that the nearest one to $r_i$ is $v_k$), then $r_i$ decides as below:
      For an example, see Fig. 5, assume that $r_i$ is on the point p of $s_1$.
      * If $P_{k+1}(k, k+1)$ is cleared ($FF_{k+1}(1) = true$), then $r_j$ updates $D_j(1) = D_j(1) \cup (v_{k+1}, x_{k+1})$ and $D_j(2) = D_j(2) \setminus (v_{k+1}, x_{k+1})$, and then continues its movement along $s_i$.
      * If $P_{k+1}(k, k+1)$ is contaminated ($FF_{k+1}(1) = false$), then $P_{k+1}(k, k+1)$ should be cleared. Therefore, $r_i$ waits and sends a signal to the other robots to clear $P_{k+1}(k, k+1)$ and updates $D_j(3) = (v_{k+1}, x_{k+1})$.

Waiting and Sending a Signal Step
Assume that $r_i$ waits and sends a signal to the other robots to clear sub-polygon $P_1$, which is between a and b in counterclockwise order ($D_i(3) = (a, b)$).

When $r_i$ sends a signal, a robot that can clear some portions of $P_1$ consisting of a starts clearing. At each time, one robot is clearing. Suppose that $r_j$ sees a and can start clearing $P_1$. Therefore, $r_j$ updates $D_j(2) = D_i(3)$.

If $r_j$ is outside of $P_1$, $r_j$ starts clearing from $a$ and $D_j(1)$ is the intersection of the boundary of $P_1$ and the orthogonal line segment that passes through $a$ and intersects $s_j$. Therefore, $r_j$ starts its movement (see Fig. 6). Otherwise ($r_j$ is inside $P_1$), $r_j$ starts clearing from one of its endpoints (for an example, see Fig. 5 if $r_5$ is on the blue point of $s_5$, $P_{k+1}(k, k+1)$ should be cleared).

Suppose that $v_k$ is a reflex vertex and that $FF_k(x) = false$ (let $P_1$ be the corresponding sub-polygon of $FF_k(x)$); suppose also that $r_j$ is a robot that is waiting until $P_1$ becomes
cleared. At the time that \( r_i \) updates \( FF_k(x) \) to true, \( r_i \) finishes its clearance and updates \( D_j(1) = D_j(1) \cup D_i(1) \) and \( D_j(2) = D_j(2) \setminus D_i(1) \). Then, \( r_j \) continues its movement.

**Termination Step Algorithm**

We assume that, initially, all parts of \( P \) are contaminated and \( \forall r_i \in R D_i(1) = \emptyset \). Because of our algorithm, a robot can move and clear some parts of \( P \) at any time. When there is no waiting robot (\( \forall r_i \in R D_i(3) = \emptyset \)), all robots have cleared their corresponding sub-polygons (\( \forall r_i \in R D_i(2) = \emptyset \)), and all parts of \( P \) have been cleared (\( \forall r_i \in R D_i(1) = P \)), the motion-planning algorithm is finished.

**4 Analysis**

In this section, we shall prove that the proposed algorithm is deadlock free. Since \( S \) guards all parts of \( P \), then the algorithm will be terminated. Then, we will show that, starting with any arbitrary sliding robot, the algorithm can clear \( P \) completely.

▶ **Lemma 2.** The proposed algorithm is deadlock free.

**Proof.** Assume that \( r_i \) is waiting for sub-polygon \( P_i \) to be cleared by a sequence of robots. Inside \( P_i \), \( r_j \) may be waiting for sub-polygon \( P_j \) to be cleared. Therefore, there may exist a chain of waiting robots, say, \( r_{seq}(i) = \langle r_j, r_t, \ldots, r_m \rangle \), for clearing \( P_i \). If \( r_i \in r_{seq}(i) \), a deadlock occurs and the algorithm will not get terminated. Therefore, we shall show that the relation \( r_i \in r_{seq}(i) \) will never become valid.

Owing to the definition of the window and its corresponding sub-polygons, when \( r_i \) waits for the clearance of \( P_i \), it cannot see any points of \( P_i \), except its window. Since the sub-polygons corresponding to the other robots in \( r_{seq}(i) \) are inside \( P_i \), none of the waiting robots in \( r_{seq}(i) \) can wait for \( r_i \). Hence, the algorithm is deadlock free.

▶ **Lemma 3.** A simple orthogonal polygon can be completely cleared starting with an arbitrary sliding robot.

**Proof.** Assume that we start with an arbitrary robot \( r_i \). Because of Lemma 2, the proposed algorithm is deadlock free. Moreover, since \( S \) guards all parts of \( P \), the termination step
will happen. Based on the termination step, the relation $\bigcup_{i=1}^{[S]} D_i(1) = P$ becomes valid; therefore, there is no contaminated point in $P$ and the polygon gets cleared completely.

Theorem 4. Let $P$ be a simple orthogonal polygon consisting of unpredictable evaders, and let $S$ be a set of line segments such that the union of their sliding visibility polygons is $P$. We can propose a motion-planning algorithm for a group of sliding robots that move along the line segments of $S$ and find all evaders such that the number of sliding robots used is at most the cardinality of $S$.

Corollary 5. If $S$ is the set of minimum cardinality sliding cameras that guard the whole $P$, then our algorithm clears $P$ with the minimum number of sliding robots.

5 Conclusion

In this paper, we have proved that, in the case of having a known environment for sliding robots, there exists an algorithm for planning the motions of a group of sliding robots to detect all the unpredictable moving evaders that have unbounded speed. We assume that the speed of the sliding robots is unbounded ($\neq \infty$). We use a set of line segments $S$ where the sliding robots move along. In the case where $S$ is a set of minimum-cardinality sliding cameras that guard $P$, the proposed algorithm uses the minimum number of sliding robots to clear $P$.

Investigating the problem in which the environment is unknown to the robots, and in which the robots could only plan their motions based on the local visible area, would be challenging. Additionally, letting the robots send information only to those that are visible to them may make the problem more usable in real-life multi-robot systems.

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