Control and Measurement of Quantum Light Pulses for Quantum Information Science and Technology

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Manipulation of quantum optical pulses, such as single photons or entangled photon pairs, enables numerous applications, from quantum communications and networking to enhanced sensing. Common methods to shape laser pulses based upon filtering or amplification cannot be employed with quantum light pulses as these approaches introduce detrimental loss and noise to the system. Here, methods to control and measure quantum light pulses based upon deterministic application of targeted phases in time and frequency domains are reviewed, along with recent demonstrations of quantum applications.

1. Introduction

The spatial and polarization modes of light have been utilized for numerous applications in quantum information science and technology (QIST).\(^{[1–3]}\) Recently, the temporal and spectral properties of light have come to the fore as a promising platform for optical quantum technologies.\(^{[4,5]}\) This is driven by the compatibility of temporal modes with integrated optical platforms and their high information capacity. The ability to generate, manipulate, and measure ultrashort pulses of light has enabled a range of applications from probing the dynamics of molecular systems\(^{[6]}\) to precision metrology.\(^{[7]}\) Recently there has been significant interest in utilizing the temporal-spectral degree of freedom of light that underlies ultrashort optical pulses for quantum information science and technology.\(^{[4,8–16]}\) Quantum applications enabled by information encoded in the temporal mode of photons include linear optics quantum computing,\(^{[4,16–17]}\) boson sampling,\(^{[18]}\) quantum communications,\(^{[15,19–21]}\) and quantum sensing.\(^{[12,23]}\) Here we focus on manipulation of one- and two-photon states of light and applications enabled by this control. The methods to control the pulsed modes of light presented here can be applied to other states of the field, such as multi-mode squeezed states, which have also received significant attention in recent years.\(^{[24–26]}\)

There are three different approaches to encoding quantum information in the time-frequency (TF) domain put forth to date for QIST. Manipulation and measurement of frequency-bin\(^{[15,16,27]}\) and time-bin\(^{[17,18,28]}\) encoded single photons, equivalent to wavelength- and time–division multiplexing in classical communications, have been the focus of many studies. In these frequency- or time-bin encoding schemes, the intensities of pulses corresponding to a single, logical qudit overlap in one degree of freedom (time or frequency), but are fully distinguishable in the other. Pulsed or temporal modes, such as Hermite–Gaussian modes,\(^{[4]}\) overlap in both the temporal and spectral domains. These modes offer an alternative encoding and arise naturally in spontaneous parametric down conversion. This review focuses on such pulsed mode encodings that overlap in both time and frequency, but some of the techniques for manipulating temporal modes presented are applicable to time- and frequency-bin encodings.

Until recently, the approaches taken to manipulating quantum light pulses have involved time-stationary linear optics, such as spectral filtering or application of spectral phase by pulse shaping,\(^{[29]}\) time-varying amplitude modulation,\(^{[30–33]}\) and time-varying nonlinear optics in second- and third-order nonlinear optical media,\(^{[34]}\) as depicted in Figure 1. Targeted operations utilizing nonlinear optical interactions include frequency conversion using three-wave and four-wave mixing.\(^{[12,35–40]}\) Recently these techniques are being extended to perform mode-selective manipulation of individual spectral-temporal modes,\(^{[4,12,41,42]}\) an approach known as the quantum pulse gate (Figure 1c). Manipulation of the bandwidth of single photons has been demonstrated by use of a time lens in second-order nonlinear media.\(^{[38]}\) The former was applied to coherent control of time-bin-encoded quantum information\(^{[39,43]}\) and entangled photons...
were manipulated by temporal imaging. Quantum memories, which are typically based on third-order optical nonlinear interactions, have been used for frequency multiplexing of heralded single-photon sources.

The temporal-spectral degree of freedom enables multidimensional encoding and processing of quantum information by discretizing the continuous time–frequency state space into time or frequency “bins,” or spectral-temporal pulsed modes, the latter of which are the focus of this paper. Similarly, multidimensional quantum information encoding and processing can be realized by using the spatial degree of freedom of photons. Indeed many initial studies of multidimensional QIST were performed using the spatial degree of freedom including path- and orbital-angular-momentum encoding. Multidimensional QIST in the spectral-temporal domain offers natural compatibility with single-mode optical fibers and integrated optical circuits. Multiplexing quantum information in time or frequency is expected to allow increased rates of quantum information transfer. Crucially such multidimensional encoding can be often achieved using single-pixel photon detectors. The spectral-temporal degree of freedom is also naturally useful for quantum metrology of time and frequency. On the other hand multidimensional QIST in the spatial degree of freedom is a more mature topic, with better developed tools for manipulation of single photons. It is also indispensable for quantum metrology of spatial features, in particular in quantum imaging. Ultimately truly high dimensionality for QIST may be achieved by combining the spatial and spectral-temporal degree of freedom, as already suggested in the early work on hyperentanglement across all degrees of freedom of photons by Barreiro et al.

In this paper, we focus on approaches to perform transformations on temporal modes of the electromagnetic field in the optical regime that do not contribute to background noise and can, in principle, achieve the targeted transformation without added noise or loss. This is accomplished with the application of time- and frequency-dependent phases. We begin with the theoretical background of pulsed modes of light. This is followed by a brief overview of common approaches to manipulating light pulses in the classical regime, which employ filtering or amplification that do not preserve quantum states of light. We then turn to recent developments aimed at controlling pulsed modes occupied by quantum light by application of temporal and spectral phase across the pulse. Using the analogous evolution of spatial modes undergoing diffraction and pulse modes propagating in dispersive media, we describe how pulses are transformed by different temporal and spectral phases. Many of the methods described in this work are well-established for manipulation and characterization of bright classical pulses. Here we highlight the specific challenges that arise when applying those methods to quantum light pulses. In particular, we review methods that aim to avoid challenges arising from lossy filtering and inherently noisy amplification. Here we focus on electro-optic phase modulation as a deterministic linear optical means to apply temporal phase. We then provide an overview of applications of the control of ultrafast pulses, including mode-matching for the construction of a quantum network, time-to-frequency and...

Figure 1. Schematic representation of different approaches for manipulation of the spectro-temporal modes of quantum light pulses, which here takes a Gaussian input pulse to a first-order Hermite-Gaussian pulse shape. a) A pulse shaper in a free-space 4-f setup employing diffraction gratings (DG) and a spatial light modulator (SLM). By the space-frequency mapping one applies phase or/and loss by the SLM in the frequency domain. b) Application of amplitude modulation using an electro-optic amplitude modulator (EOAM) followed by an electro-optic phase modulation (EOPM) can be used to realize the transformation, albeit with intrinsic loss due to the amplitude filtering. Here one applies phase/loss directly in the time domain. c) A quantum pulse gate consisting of a strong optical pump field and appropriately chosen second-order nonlinear optical medium (χ(2)), selects an input pulse and maps it into a buffer (green) mode which can be reshaped into the target mode using an appropriately shaped pump pulse in an additional nonlinear optical process. d) Application of a sequence of n different spectral- and temporal-phase modulations can achieve the targeted temporal mode conversion. This sequence, depicted by brackets raised to the power n, consists of n applications of potentially different group delay dispersion (GDD) and temporal phase supplied by an EOPM.
frequency-to-time mapping, quantum state characterization, and quantum-enhanced sensing.

2. Background

2.1. Theory of Pulsed Modes

The quantum description of light begins by decomposing the electromagnetic field into an appropriately chosen set of orthonormal mode functions.\(^{[51]}\) The modes are selected to best describe the geometry and evolution of the source, channel, and detector under study. Here we envision light propagating in pulsed modes with a single polarization and transverse spatial mode, such as that found in paraxial beams or optical waveguides. In this way, we focus on the time-varying electric field amplitude

\[
E(t, z) = E_0 \sum_m \alpha_m \psi_m(t, z) e^{i(k_0 z - \omega_0 t)},
\]

where \(\tau\) is an absolute time, \(\omega_0\) is the central (carrier) frequency of the field, \(k_0\) the wave vector, and \(\{\psi_m(t, z)\}\) is a pulse mode envelope labeled by integer \(m\) traveling along the \(z\) axis with group velocity \(v_g\). The temporal modes \(\{\psi_m(t, z)\}\) are found to the wave equation describing propagation in a dispersive medium in the slowly-varying-envelope approximation\(^{[52-54]}\)

\[
\partial_t \psi_m(t, z) = -i \beta_m \partial^2_z \psi_m(t, z)/2,
\]

where \(\partial_z\) is the partial derivative with respect to the longitudinal coordinate \(z\), \(\beta_m\) is the group velocity dispersion (GVD), \(\partial_t\) is the partial derivative with respect to \(t\), and \(\tau = t - z/v_g\) is the retarded time in the reference frame traveling along the \(z\) axis at the group velocity of the optical pulse \(\psi_m\). We will use the notation \(\{\psi_m(t)\} = \{\psi_m(t - z/v_g, 0)\}\), and drop the explicit \(z\) dependence, working in the moving frame of the pulse to simplify the discussion. The temporal modes \(\{\psi_m(t)\}\) are chosen to form a complete set that can be used to expand an arbitrary pulsed signal centered about the pulse reference frame. An example of such modes are the Hermite–Gaussian (HG) pulse modes

\[
\psi_m(t) = \frac{1}{\sqrt{2^n n! \sqrt{\pi} \sigma_t}} H_n \left( \frac{t}{\sqrt{2} \sigma_t} \right) e^{-\left( \frac{t^2}{\sqrt{2} \sigma_t^2} \right)},
\]

where \(H_n(x)\) is the \(n\)-th order Hermite polynomial and \(\sigma_t\) parametrizes the temporal duration of the modes with the full-width at half maximum (FWHM) intensity duration of the zero-order HG mode given by \(\tau_m = 2\sqrt{2} \ln 2 \approx 2.35 \sigma_t\). Note that the HG modes considered here are temporal-spectral HG modes, not the more commonly encountered spatial HG modes. The parallel between the mathematical formalisms behind both the spatial and TF encoding is discussed in detail in the following Section 2.2.

The completeness relation for the set of modes \(\{\psi_m(t)\}\), which are solutions to Equation (2), implies that they are orthonormal

\[
\int_{-\infty}^{+\infty} \psi_m^*(t)\psi_n(t) \, dt = \delta_{mn},
\]

where \(n\) is the order. Note that a temporal mode \(\psi_m(t)\) can also be represented by its Fourier transform, \(\tilde{\psi}_m(\omega)\), in the frequency domain, where the orthonormality condition still holds

\[
\int_{-\infty}^{+\infty} \tilde{\psi}_m^*(\omega)\tilde{\psi}_n(\omega) \, d\omega = \delta_{mn}.
\]

The temporal modes have both amplitude and phase dependence, \(\psi_m(t) = |\psi_m(t)| e^{i\phi_m(t)}\), where \(\phi_m(t) = \arg[\psi_m(t)]\) is the temporal phase of the pulse. There is an equivalent frequency-dependent representation \(\tilde{\psi}_m(\omega) = |\tilde{\psi}_m(\omega)| e^{i\tilde{\phi}_m(\omega)}\), where \(\tilde{\phi}_m(\omega) = \arg[\tilde{\psi}_m(\omega)]\) is the spectral phase of the pulse. One should take care to note that the amplitude and phase in the frequency domain are not generally Fourier transforms of their time-domain analogs. Figure 2 shows the amplitude and phase of a Gaussian pulse with quadratic spectral phase, which introduces correlations between the frequency and arrival time known as “chirp.” The temporal duration \(\sigma_t\) of a pulse with flat phase is linked to the spectral bandwidth \(\sigma_\omega\) by the uncertainty relation \(\sigma_t \cdot \sigma_\omega \geq 1/2\), which is saturated only in the case of Gaussian pulses. Additionally, HG modes as depicted in Figure 3a–c have the property that their envelopes are self-similar under Fourier transformation.

2.2. Optical Space–Time Analogies

The spectral and temporal degrees of freedom of an optical mode are Fourier conjugates, just as transverse wave vector \(k = (k_x, k_y)\) and position \(x = (x, y)\) are to each other. Manipulation of the spectral–temporal degree of freedom of light can be likened to manipulation of the transverse spatial degree of freedom, performed with simple optical components, such as lenses or prisms. This analogy between temporal- and spatial-mode manipulation arises from the mathematical structure of the two-dimensional (2D), in-plane Helmholtz equation in the paraxial approximation

\[
\partial_z u(x, y, z) = -iV_z u(x, y, z)/(2k),
\]

being close to that of the wave equation describing propagation of an optical pulse in a dispersive medium in the slowly-varying-envelope approximation, Equation (2).\(^{[52-55,56]}\) The key difference is that the transverse spatial domain is 2D, whereas the temporal domain is one-dimensional. Here \(u(x, y, z)\) describes the transverse spatial mode profile of the beam at longitudinal position \(z\), \(k = 2\pi/\lambda\) is the wave-vector amplitude, and \(V_z = (\partial_z, \partial_t)\) is the transverse gradient operator. Upon examining Equations (2) and (6) it is clearly seen that transverse position can be matched with the retarded time \(t\), whereas the transverse wave vector is paired with frequency. Just as the temporal modes can be expanded in terms of HG pulsed modes, transverse spatial modes can also be expressed in terms of 2D HG modes, \(u_{mn}(x, y, z)\), as depicted in Figure 3d–f. We shall make use of this mathematical equivalency between spatial and temporal mode evolution when considering approaches to manipulating pulsed modes in Section 3.

We note that although Equations (2) and (6) are mathematically analogous, there are still several differences between the spatial-momentum and spectral-temporal degrees of freedom.
Figure 2. Spatial (top) and temporal (bottom) representations of free space propagation along the longitudinal $z$ axis. In both cases, the envelope of the modes spread, respectively in space ($x$) and in time ($t$) while the envelope in the conjugate space (momentum $k_x$ and spectrum $\omega$) is unaffected. Chronocyclic Wigner functions are plotted to further illustrate the similarities. (1) and (3) are taken at $z = 0$ while (2) and (4) are farther along the longitudinal axis.

Figure 3. a–c) HG pulse modes showing the envelope (blue) and carrier (red) of the field. Axes are in units multiple of the pulse second moment and the optical frequency is chosen arbitrarily to show only a few cycles. d–f) HG mode envelopes (solutions to the paraxial field propagation equation) plotted along one spatial direction. Insets: transverse field intensity. Axes are in waist units.
First, whereas spatial propagation always leads to acquisition of positive phase in the transverse momentum, in the time–frequency degree of freedom the phase acquired upon propagation in a dispersive medium can be both positive (corresponding to positive group-velocity dispersion) or negative (corresponding to negative group-velocity dispersion). This makes realization of temporal analogues of spatial relay lensing systems pointless: to compensate the effect of dispersion it is enough to use a dispersion compensating module. Moreover, the phase acquired due to diffractive propagation (in the spatial case) is always quadratic in the paraxial approximation, whereas it is possible to directly apply in-principle arbitrary spectral phases by sending the optical pulses through media with higher-order dispersion. In the spatial case application of higher-order phase in transverse momentum requires performing a Fourier transform (e.g., by a lens) and position phase modulation (e.g., by a spatial light modulator). There may be advantages to combining temporal and spatial mode shaping, but this goes beyond the scope of this review.

Note that the expansion of both spatial and temporal modes in the HG basis has a physical meaning. It can be shown that in the spatial domain the first-order HG mode follows naturally from an expansion of a beam of light that is displaced or tilted away from the beam axis, while the second-order HG mode is associated with defocusing, and the zero-order mode corresponds to an undisplaced reference beam. Due to the space–time analogy, a similar physical meaning holds for a pulse displaced in time and bandwidth, which is a superposition of HG1 and HG2 modes. Similarly, HG0 corresponds to an unperturbed reference pulse.

### 2.3. Quantum Treatment of the Field

Treating the field quantum mechanically in free space, each mode, \( \psi(\omega) \), is associated with a harmonic oscillator which is quantized and has the corresponding annihilation operator

\[
\hat{a}_\nu = \int \psi(\omega) \hat{a}(\omega) \, d\omega,
\]

where \( \hat{a}(\omega) \) is the annihilation operator for a monochromatic mode of frequency \( \omega \). In this section the limits of integration are \((-\infty, \infty)\). The corresponding creation operator is given by the conjugate transpose of Equation (7)

\[
\hat{a}_\nu^\dagger = \int \psi^*(\omega) \hat{a}^\dagger(\omega) \, d\omega.
\]

The monochromatic creation and annihilation operators obey the canonical bosonic commutation relation

\[
[\hat{a}(\omega), \hat{a}^\dagger(\omega')] = \delta(\omega - \omega').
\]

This imposes the following commutation relation between creation and annihilation operators of different temporal modes \( \nu \) and \( \phi \)

\[
[\hat{a}_\nu, \hat{a}_\phi^\dagger] = \int \psi^*(\omega) \hat{\phi}(\omega) \, d\omega,
\]

which is simply the overlap integral of the mode functions. The commutator in Equation (10) becomes a Kronecker-\( \delta \) when the modes are chosen from the same orthonormal set, such as the Hermite–Gaussian modes of Equation (3). The creation and annihilation operators for the temporal modes can also be represented in the time domain as

\[
\hat{a}_\nu = \int \psi(t) \hat{a}(t) \, dt
\]

and

\[
\hat{a}_\nu^\dagger = \int \psi^*(t) \hat{a}^\dagger(t) \, dt,
\]

where \( \hat{a}(t) \) (\( \hat{a}^\dagger(t) \)) is the annihilation (creation) operator for a photon at time \( t \). These obey the commutation relation

\[
[\hat{a}(t), \hat{a}^\dagger(t')] = \delta(t - t'),
\]

and are related to the frequency annihilation and creation operators by

\[
\hat{a}(t) = \frac{1}{\sqrt{2\pi}} \int \hat{a}(\omega) \exp(-i\omega t) \, d\omega
\]

and

\[
\hat{a}^\dagger(t) = \frac{1}{\sqrt{2\pi}} \int \hat{a}^\dagger(\omega) \exp(i\omega t) \, d\omega.
\]

A single-photon state of the electromagnetic field occupying a single mode \( \psi(t) \) is given by

\[
|1\rangle_\nu = \hat{a}_\nu^\dagger |\text{vac}\rangle,
\]

where \( |1\rangle_\nu \) represents a single photon occupying the mode \( \psi(t) \) with all other modes in the vacuum state and \( |\text{vac}\rangle \) is the vacuum state of the electromagnetic field for all modes. A single photon that occupies a single mode is said to be a pure state. Analogous to single-electron wave mechanics, one can treat the mode function that the photon occupies as its wave function.

### 2.4. Representations of Partially Coherent Fields

Since the concept of a single-photon wave function was introduced, a number of ways of representing the time–frequency states of quantum optical fields have been proposed. In the case of pure single-photon states, the spectral (or temporal) mode function amplitude, \( \tilde{\psi}(\omega) \) (or \( \psi(t) \)), is sufficient to characterize the field. The mode function in the frequency domain is determined by its amplitude, \( \tilde{\psi}(\omega) \), and phase, \( \arg(\tilde{\psi}(\omega)) \), which also holds for the time domain arising from the unique relationship between Fourier-transform pairs. If the photon occupies a mixture of modes, then it is said to be mixed (or partially mixed) and cannot be represented by a mode function. For single photons occupying a probabilistic mixture of several modes, \( \psi(t) \), with probability \( P_n \), the spectral-temporal density matrix formalism can be used to describe the state of the photon. The single-photon density operator is given by

\[
\hat{\rho} = \sum_n P_n |1\rangle_n \langle 1|.
\]
where $|1\rangle_n$ is a single-photon state occupying the temporal mode $\psi_n(t)$. The matrix elements in the temporal and spectral bases for this single-photon state are

$$\rho(t_1, t_2) = \sum_n P_n \psi_n^*(t_1) \psi_n(t_2)$$

(18)

and

$$\hat{\rho}(\omega_1, \omega_2) = \sum_n P_n \hat{\psi}_n^*(\omega_1) \hat{\psi}_n(\omega_2).$$

(19)

An alternative description that has proven useful to describe a single-photon temporal-mode state is the chronocyclic Wigner function (CWF), introduced in ref. [68] for classical fields. This real-valued, 2D representation of the temporal modes is often helpful for visualizing effects of applied phases in the time or frequency domain and can describe both coherent and incoherent fields. For a single photon, it is defined by the Wigner transform of the density matrix $\rho(t_1, t_2)$

$$W(t, \omega) = \int_{-\infty}^{+\infty} \rho(t + t'/2, t - t'/2) \exp(-i\omega t') \, dt',$n

(20)

and can similarly be defined in terms of $\hat{\rho}(\omega_1, \omega_2)$. A generalized CWF has been also defined for multi-photon fields. [69] The Wigner function has also been applied to transverse spatial modes for classical light [70] and single-photon states. [71] This representation takes its inspiration from the Wigner function of the bosonic field developed for quantum optics, and the observation that the same properties can also be useful in a description of the time–frequency state. The marginal distributions of the CWF (obtained by integrating over either time or frequency) correspond to the intensity distributions in the other degree of freedom. In the case of a single photon, the CWF describes a classical field mode or an incoherent mixture of such modes. Figure 2 shows the CWF for a Gaussian pulse with flat phase and after acquiring quadratic spectral phase. Note that the quadratic spectral phase, typically arising from group-delay dispersion within a medium, lead to correlations between the time of arrival and instantaneous frequency within the pulse. In the same figure, the Wigner function for the transverse spatial distribution in the $x$-direction for a Gaussian beam depicts the minimum spot size at its focus. Analogous to quadratic spectral phase, diffraction arising from free-space propagation adds quadratic transverse-wave-vector phase. This leads to correlations between position $x$ and transverse wave vector $k_x$ after propagation arising from free-space diffraction, also depicted in Figure 2. This example highlights the utility of space–time analogies.

3. Pulse Manipulation with Spectral and Temporal Phase

Since the temporal modes $\psi(t)$ that carry quantum states of light introduced in Section 2.1 are solutions to the wave equation that arises from the classical Maxwell equations, they can be naturally transformed using many of the tools based on optical space–time analogies. The main challenge lies in realizing transformations in a manner that does not introduce significant photon noise (including spontaneous-emission noise from amplification) and does not impart an excessive amount of loss on the quantum light signals.

Classical pulse shaping techniques typically involve amplitude modulation that introduces gain or loss, and thus cannot be applied to quantum light. Note that although there are strategies to employ noiseless amplification using probabilistic heralding schemes, deterministic noiseless amplification of quantum states of light is ruled out by the “no-cloning” theorem. Both gain and loss inevitably introduce noise through background photons and vacuum fluctuations, respectively. Thus, to address temporal modes of light in the quantum regime requires phase-only manipulation of the pulse modes. The familiar behavior of light beam propagation due to diffraction can be used as an analogy to pulse propagation in dispersive media, as introduced above in Equations (2) and (6). We use this optical space–time analogy to describe the evolution of optical pulses when different spectral and temporal phases are imparted to the pulse.

### 3.1. Pulse Manipulation Viewed from the Optical Space–Time Analogy

Lossless, unitary manipulation of the spectral amplitude, $\hat{\psi}(\omega)$, requires phase-only operations to the conjugate temporal amplitude, and conversely for manipulation of the temporal amplitude, $\psi(t)$. To gain insight to how one can control the pulse mode structure of light using phase-only operations in the temporal and spectral domains, let us start with a simple example based on the optical space–time analogy - paraxial imaging by a thin lens.

To achieve spatial imaging, light emitted by an object travels toward a lens, undergoing diffractive, free-space propagation. According to the Fresnel–Kirchhoff formula, this corresponds to the plane wave components of the optical field acquiring a phase quadratic in the transverse component of their wave vector. Upon crossing the lens, the light acquires a quadratic phase in the transverse position. After further diffractive, free-space propagation behind the lens, corresponding again to acquiring a quadratic phase in the wave vector, a magnified or de-magnified image of the object can be formed when the two diffractive propagation distances satisfy the imaging formula.

An analogue of imaging by a thin lens in the spectral-temporal degree of freedom is realized by replacing the transverse position with the retarded time and the transverse wave vector component with angular frequency. An optical pulse with an initial temporal waveform $\psi(t)$ acquires a quadratic spectral phase by propagating in a dispersive medium, similar to free-space propagation. The analogue of a thin lens is a device that subjects the pulse to a phase that varies quadratically in time, within the reference frame traveling with the pulse, called a time lens. The temporal imaging scenario is completed by further propagation through a dispersive medium, subjecting the pulse to acquire additional quadratic spectral phase such that a temporal analogue of the spatial imaging formula is satisfied. Upon exiting from the dispersive medium, a pulse whose temporal waveform has been magnified (de-magnified) in time is obtained. By applying more complex temporal phase profiles one can realize more complicated transformations, such as analogues of aspheric optics, Fresnel lenses or arbitrary spatial light modulators.
Techniques based on the space–time analogy form a mature approach to manipulating bright, classical pulses of light. We refer the reader to excellent review papers summarizing the classical works within the field. The above techniques provide tools to perform coherent manipulation of the spectral-temporal state of optical pulses, just like lenses, free-form optical elements, spatial light modulators, and diffractive propagation perform coherent spatial transformations of beams of light. Thus the techniques based on the space–time analogy, implemented with pure phase modulations, are ideally suited for coherently manipulating the spectral temporal state of quantum light without introducing loss through filtering or amplification noise.

### 3.2. Phase Modulation Components

Spectral-temporal mode manipulation can be viewed as operations transforming the spectral-temporal modes of the electromagnetic field in the ideal lossless case, without directly altering the quantum excitations of the modes. This is analogous to a network of beam splitters and phase shifters that implement unitary mode manipulation of path-encoded quantum states used in many proof-of-concept quantum information protocols. Realistically, one needs to consider the unavoidable technical loss degrading the quantum states contained within the modes under consideration. Since the loss is independent of frequency and time, it will only result in reduced transmission of light, but will not affect the pulse mode transformation implemented. This is the regime in which we will work, where loss affects only the overall transmission of light.

We begin by considering the evolution of a pulse mode function when propagating in dispersive media and its basic applications. This is followed by a discussion of temporal phase manipulation approaches. We conclude this section with an overview of how a sequence of spectral and temporal phases can be used to implement general transformations on multiple pulsed modes.

#### 3.2.1. Dispersive Elements—Spectral Phase Modulation

One of the basic building blocks of temporal optical systems is a dispersive element. Upon propagation of an optical pulse in a dispersive medium its spectral amplitude \( \tilde{\psi}(\omega) \) acquires a spectral phase factor \( \Phi(\omega) \)

\[
\tilde{\psi}(\omega) \rightarrow \tilde{\psi}(\omega)e^{i\Phi(\omega)},
\]

in the slowly-varying-envelope approximation. For a pulse with frequency spectrum centered around \( \omega_0 \), the spectral phase factor per unit length of the medium \( \Phi(\omega) \) can be Taylor-expanded as

\[
\Phi(\omega) = \Phi_0 + \frac{\beta_2}{2!}(\omega - \omega_0)^2 + \frac{\beta_3}{3!}(\omega - \omega_0)^3 + \cdots,
\]

where \( \beta_n = \frac{d^n \Phi(\omega)}{d\omega^n} \bigg|_{\omega=\omega_0} \) is the n-th order dispersion coefficient of the medium, with \( n = 0, 1, 2, 3, \ldots \). \( \beta_2 \) is a constant phase factor, \( \beta_3 \) is linked to the group delay, and \( \beta_4 \) is the group-velocity dispersion. Typically, one can disregard the constant phase factor \( \beta_0 \), which is related to the carrier-envelope phase (CEP) offset and is of limited physical significance for many-cycle pulses. By solving Equation (2) in the slowly-varying-envelope approximation using Equation (22), one arrives at the following expression for the spectral phase acquired after propagation in dispersive medium of length \( z \)

\[
\Phi(\omega) = \sum_{n=0}^{\infty} \frac{\beta_n}{n!}(\omega - \omega_0)^n, \tag{23}
\]

where the first two terms, corresponding to carrier-envelope phase (CEP) and group delay offset, have been dropped. For basic temporal optical systems, such as a temporal imaging setup, a quadratic spectral phase factor is required: \( \Phi(\omega) = \Phi_2(\omega - \omega_0)^2/2 \), where \( \Phi_2 = \beta_2 z \) is the group-delay dispersion (GDD). Higher order terms in \( \omega \) may be desired for some applications, but they may also be a nuisance when a purely quadratic spectral phase is required.

The most common approach for introducing quadratic spectral phase involves the use of a length of glass in the form of an optical fiber. A length \( L \) of a fiber with group velocity dispersion coefficient \( \beta_2 \) will introduce a GDD of \( \Phi_2 = \beta_2 L \). Further higher-order spectral phase terms are always present in a realistic medium. Their significance depends mainly on the length of the medium and on the spectral bandwidth of the optical signal. See ref. [56] for a detailed discussion of these effects.

After propagating through a dispersive medium, the width of the envelope of a transform-limited pulse of temporal duration \( \sigma \) will increase to a value \( \sigma' \) given by:

\[
\sigma' = \sigma\sqrt{1 + \left(\frac{\Phi_2}{2\sigma^2}\right)^2}. \tag{24}
\]

Either in the short-pulse or in the large GDD approximation, the temporal duration of the dispersed pulse then becomes:

\[
\sigma' = |D_s|L\sigma, \tag{25}
\]

where \( \sigma \) is the spectral bandwidth and \( D_s = -2\alpha c \beta_2 / \lambda_c^2 \) is called the dispersion parameter, usually given in units of [ps/nm/km]. Equation (24) may be used to compute the temporal spread of a short pulse after propagating through glass (\( \beta_2 \approx 50 \text{ fs}^2/\text{mm}^{-1} \)), while Equation (25) is generally applied to optical fibers (\( D_s \approx 100 \text{ (18 ps/nm/km at 800 (1550 nm central wavelength) } \))

Fiber-based dispersive elements are unavoidably associated with losses, which are of critical importance for quantum optical signals, since they destroy photon-number coherences of non-classical light and dramatically reduce multiphoton detection rates. The transmission of light through an optical fiber decreases exponentially with fiber length. The intensity transmission coefficient \( T \) of a length \( L \) of a fiber with an absorption coefficient \( \alpha \) is given by \( T = \exp(-\alpha L) \). The absorption coefficient \( \alpha \) is linked to loss \( \Lambda \), typically provided in the units of dB km\(^{-1} \), by \( \alpha \text{[km}^{-1}] = \Lambda [\text{dB/km}] \ln 10 / 10 \). With current telecommunication fibers, exhibiting down to 0.2 dB km\(^{-1} \) loss, \( \alpha L \) reaches the order of unity for propagation lengths on the order of 10 km, which presents a limitation for loss-sensitive quantum applications. For example a 100 km stretch of standard telecommunication fiber has a transmission of only 1%.
losses for other wavelengths are significantly higher, limiting the use of fibers as dispersive elements. Specialty fibers with engineered dispersion properties, such as dispersion compensating fibers, are also characterized by increased losses, albeit they provide significantly larger dispersion coefficients. Moreover, long fiber links may lead to increased latency of the optical setup.

In classical optical applications the problem of loss scaling has been addressed by using simultaneous amplification of optical signals, for example by Raman amplification. However this approach cannot be utilized for quantum optical signals due to the spontaneous emission noise introduced via amplification. An alternative approach to introduce large amounts of dispersion is through the use of chirped fiber Bragg gratings (CFBG). By inscribing an appropriately designed modulation of the refractive index along the length of an optical fiber a frequency-dependent delay is introduced, corresponding to a quadratic, or more complex, spectral phase. The periodic structure may provide a high reflection coefficient, approaching unity for highly dispersive structures. An optical circulator is required to couple light out of the structure, which typically introduces 2 to 3 dB of loss. Nevertheless for high dispersion values, corresponding to prohibitive lengths of optical fibers, the use of CFBG remains highly advantageous in terms of loss budget. The restrictions on CFBG fabrication include limited spectral bandwidth (which is directly linked to the length of the structure) and non-uniformity of the structure (so-called “ripple”, or oscillations of the introduced spectral phase).

There exist further techniques for spectral phase introduction, involving bulk optical or spatially multi-mode systems. They include diffraction grating or prism-based pulse stretchers and/or compressors and techniques involving spatial mode dependent dispersion in multi-mode fibers, which however are inherently lossy (since multiple spatial modes need to be projected onto a single output mode) and not well suited for all-fiber or integrated quantum photonic systems. For dispersive elements in integrated optical circuits, for example see refs. 80, 91.

Direct use of dispersive elements for quantum optical applications involves the realization of the dispersive Fourier transform for spectral-temporal mapping, which allows the spectrum of a single-photon source to be measured with a single-pixel detector. In this approach, a short pulsed signal is injected into the dispersive medium with sufficiently large GDD, $\Phi$, fulfilling the temporal analogue of the far-field Fraunhofer condition $\Phi \gg \sigma^2/4\pi$, where $\sigma$ is much less than the square root of the dispersion parameter, $\sigma \ll \sqrt{\Phi}$. The medium is followed by single-photon counting detector with high temporal resolution. Detection is typically performed in a coincidence with some reference signal (e.g., from heralding) The technique is especially beneficial for acquisition of joint spectra of correlated photon pairs, since no filtering is realized when using the technique, allowing for a high rate of coincident detection. This application will be further discussed in Sections 4.2 and 4.4.

Another interesting quantum application of dispersive propagation is to perform multiphoton interference analogous to the Boson sampling experiments utilizing spatial networks of beam splitters. Here a train of closely separated single-photon wavepackets is launched into a dispersive medium. In the dispersive Fourier transform limit, time-resolved single-photon detection at the output corresponds to frequency-resolved detection of the input pulses resulting in complex multi-photon coincidence detection patterns. Within the boson sampling paradigm, this may lead to computationally hard-to-predict photon-count probability distributions, especially if combined with temporal phase modulation. A proof-of-principle experiment with up to 3 photons has been performed using photons generated via SPDC and a CFBG as the dispersive medium.

### 3.2.2. Temporal Phase Modulation

Time-dependent phase modulation, which results in modification of the spectral amplitude of optical signals, is one of the key building blocks for temporal optical systems. This is similar to the role played by a spatial light modulator for wavefront shaping. Application of a time-varying phase, $\phi(t)$, to a wavepacket is represented mathematically by multiplication by a complex exponential phase term $\psi(t) \rightarrow \psi(t) \exp \{i\phi(t)\}$. This temporal phase factor does not modify the temporal envelope, $|\psi(t)|$, but does change the spectral envelope.

Deterministic approaches we consider here to experimentally introduce a time-varying phase rely on electro-optic or acousto-optic phase modulation. An alternative approach to deterministically imprint a time-varying phase is based on optical cross-phase modulation that utilizes the optical Kerr nonlinear interaction and group-velocity matched pulses. However, its application to single-photon-level signals is experimentally challenging, as will be discussed later in this section. The electro-optic approach enables larger phase modulation amplitude and bandwidth than is feasible with bulk acousto-optic modulators. Recent developments in integrated opto-mechanical modulation show that integrated acousto-optic modulation may provide phase modulation amplitudes and bandwidths comparable to integrated electro-optic phase modulators.

The most common realization of electro-optic phase modulation is based on the Pockels effect, where a time-varying voltage applied to an electro-optic crystal (typically lithium niobate) results in a time-varying change in refractive index, and thus a temporal phase imprint on an optical signal traversing the medium. The applied voltage signal must be synchronized with the optical pulses, as illustrated in Figure 4. The use of an electronic traveling-wave configuration combined with lithium niobate optical waveguide technology allows for long interaction lengths leading to high modulation amplitude and bandwidth. Further promising developments for application of time-varying phase to optical signals are based upon thin-film lithium-niobate technology, which has high potential for photonic integration. Realization of low-loss, phase modulators with high phase modulation amplitude and bandwidth capability within a silicon photonic platform remains a challenge, whereas it is possible in indium phosphide integrated optical circuits.

Note that the electro-optic phase modulation regime discussed here is distinct from the commonly encountered case of phase modulation of continuous-wave (CW) or quasi-continuous-wave
3.2.3. Linear Temporal Phase

The application of linear temporal phase across a pulse, as depicted in Figure 4, is analogous to the linear spatial phase applied to a transverse wavefront by a prism. Using the concept of space–time analogies, we refer to such a device as a time prism. Linear temporal phase, \( \phi(t) = \Omega(t - t_0) \), where \( t_0 \) is a constant temporal offset and \( \Omega \) denotes the slope of phase modulation, can be readily recognized as a spectral shift of the pulse by \( \Omega \) in angular frequency, that is \( \psi(\omega) \rightarrow \psi(\omega - \Omega) \). This is a direct consequence of the shift property of the Fourier transform. Achieving a large spectral shift requires a steep slope of temporal phase modulation combined with the linear phase variation extending over the entire duration of the pulse. Since maintaining a linearly increasing or decaying phase imprint over a long temporal extent is technically challenging, two approximate approaches have been used for electro-optic realization of the time prism.

The first approach involves using the approximately linear part of sinusoidally-varying phase modulation to achieve the spectral shift. In this technique it is sufficient to drive an electro-optic phase modulator with a single-tone radio-frequency (RF) signal, with angular frequency, \( \omega_{RF} \). It results in a temporal phase \( \phi(t) = \phi_0 \sin(\omega_{RF} t) \). Here the amplitude of modulation \( \phi_0 = \pi V_o / V_s \) is proportional to the ratio of the applied sinusoidal peak voltage, \( V_o \), and the characteristic half-wave voltage of the modulator, \( V_s \). Provided that the optical pulse duration, \( \tau_p \), is smaller than the extent of the approximately linear part of the sine, \( \tau_p \leq 1 / \omega_{RF} \), the temporal prism will be realized with slope \( \Omega = \phi_0 / \omega_{RF} \). The driving RF signal is synchronized with the optical pulses to be shifted. Note that for high-speed modulation (40 GHz), temporal jitter in the driving RF signal on the order of 2 ps can move the pulse out of the linear regime of the sinusoidal phase. One of the first experimental realizations of this approach with classical light was performed by Duguay et al.\(^{[105]}\). Recently the method has been applied to spectral shifting of heralded single-photon pulses using standard LiNbO\(_3\) waveguided electro-optic phase modulators (EOPMs) (40 GHz driving frequency, ±200 GHz spectral shift, see Figure 5)\(^{[106]}\) as well as using the integrated acousto-optic approach.\(^{[107]}\) In both works the preservation of the quantum character of the single photons was verified by means of high-visibility Hong-Ou-Mandel interference (HOMI) between the frequency-shifted photons. This electro-optic spectral shearing technique has enabled the recent demonstrations of heralded single photons in pure spectral-temporal modes from a spectrally entangled source using frequency multiplexing.\(^{[46,107]}\) Here, frequency-resolved detection of the idler photon provides information used to determine the spectral shift required to produce signal photons with the same central frequency. This information was used to generate an appropriate RF signal driving the electro-optic phase shifter.

A limitation of the sinusoidal method is that only appropriately short pulses will undergo an undistorted spectral shift. This places a limit on the spectral bandwidth of the pulses that can be used with this method. The second approach to applying linear temporal phase avoids this limitation by taking advantage of the fact that a linearly varying phase can be reset to 0 upon reaching a value that is a multiple of \( 2\pi \), known as phase wrapping. Thus an application of an ideal saw-tooth wave profile will result in a spectral shift equivalent to that induced by linear phase modulation with the same slope, a technique known as serrodyne optical frequency translation.\(^{[108]}\) The experimental challenge stems from the fact that an ideal saw-tooth waveform,
which would be necessary to drive the EOPM, contains infinite frequency components. In any real experiment it will be distorted due to the finite bandwidth of the components used (RF waveform generator, amplifier, EOPM). Therefore this approach is useful for realizing relatively small spectral shifts for pulses of long duration as well as for complex pulses with periods of low intensity, where the phase discontinuities can be strategically placed. As mentioned above, a major advantage of the serrodyne approach is that the temporal extent of the saw-tooth waveform is effectively infinite. This enables spectral shifting of narrowband signals by many multiples of their spectral widths. Serrodyne frequency shifting has been experimentally demonstrated in multiple settings with classical light, such as in refs. [108,109].

### 3.2.4. Quadratic Temporal Phase

Let us now turn our attention to quadratic temporal phase modulation, that is, the time lens. An ideal time lens implements the following phase transformation of the temporal wavepacket

\[ \psi(t) \rightarrow \psi(t) \exp \left[ -iK(t-t_0)^2 / 2 \right], \]  

where \( t_0 \) corresponds to the temporal center of the lens, and \( K \) is the curvature or chirping rate, denoting the chirping power of the lens. In particular, when a monochromatic optical signal, \( \psi(t) = e^{i\omega_0 t} \), is subjected to the action of the time lens, it results in a time-dependent frequency shift, yielding a chirped output signal, with chirping rate \( K \)

\[ \psi_{out}(t) = e^{i\omega_0 t + K(t-t_0)^2 / 2}. \]  

By calculating the temporal derivative of the phase of the above expression one can easily find that the instantaneous frequency is linearly chirped at a rate \( K \), being given by \( \omega_0 + K(t - t_0) \). The experimentally important parameters of the time lens include the chirping rate \( K \), the duration over which the quadratic phase modulation can be realized, known as the temporal aperture and the deviations from perfect quadratic phase imprint, being a temporal analog of spatial aberrations introduced by lenses.\[110\]

Time lenses can be experimentally realized using either nonlinear optical approaches or electro-optic phase modulation. Most classical experiments use either the three-wave mixing\[111\] (in the form of sum- or difference-frequency generation) or four-wave mixing\[112–114\] processes with chirped pump beam(s) to realize the time lens. In this approach the signal is mixed with chirped pump pulses in a nonlinear medium. The chirp of the pump results in a time dependent spectral shift, leading to the time lens operation (combined with a significant spectral shift). These approaches achieve a large effective phase modulation amplitude over a large temporal aperture (especially in the case of a four-wave mixing time lens), enabling high-resolution temporal imaging\[115,116\] and complex temporal optical systems, such as a temporal telescope,\[117\] or a “spectral magnifier”.\[118\] However, the low conversion efficiency in the nonlinear mixing process limits the applicability of these approaches for quantum signals.

Temporal lensing of heralded single-photon signals has been demonstrated using three-wave mixing in a bulk crystal with a conversion efficiency on the order of 1\%\[38,39\], accompanied by a significant spectral shift. Other experiments were performed with single-photon level classical light.\[119,120\] The use of three-wave mixing as a time lens for quantum light, including both single-photon signals and squeezed light, has been theoretically analysed by Patera et al.\[121,122\]. Whereas coherent frequency shifting has been demonstrated using four-wave mixing for heralded single photons,\[123\] temporal lensing of quantum signals by means of four-wave mixing is yet to be demonstrated.

As we have mentioned before, a further possibility is to perform phase imprints by deterministic optical-optical, cross-phase modulation (by means of Kerr nonlinearity). Here a strong
of the maximum intensity of 8 enables possible gain in mode overlap, when including insertion loss of around 3 dB. Adapted with permission from Applied Physics Letters. Copyright 2020, AIP Publishing.

Figure 6. a) A clock signal is derived from the master laser. This is used to generate a reference electronic signal using a phase-locked loop (PLL), which can be delayed by time $\Delta t$ and sent to an RF generator that produces the appropriate RF signal. This can be subsequently amplified (AMP) prior to being sent to the EOPM to implement the time lens. b) An optical reference signal is used directly as an RF signal generator by a fast photodiode (PD) with a prior optical delay line for synchronization. The electronic output of the PD is subsequently amplified in an amplifier (AMP) and sent to the EOPM to implement the time lens. c) Experimental results for electro-optic quantum bandwidth converter showing compression factor of 18. The enhancement of the maximum intensity of 8 enables possible gain in mode overlap, when including insertion loss of around 3 dB. Adapted with permission from Applied Physics Letters. Copyright 2020, AIP Publishing.

parabolic pump pulse imprints a refractive index change, which results in a parabolic phase imprint on a co-propagating signal pulse. The process requires group-velocity matching between pump and signal, restricting the available operating wavelength ranges. Classical time lenses have been realized using this technique. Spectral shifts and spectral broadening have been shown with quantum light (in the form of heralded single photons), although no conclusive temporal lensing operation has been confirmed.

Direct electro-optic temporal phase inscription is a deterministic approach that is free from non-unit conversion efficiency and the need to reject a strong optical pump. As is the case for the time prism, the time lens is realized by applying a time-varying voltage signal, which is synchronized to the optical pulse, to a traveling-wave electro-optic phase modulator. The experimentally simplest approach is to approximate the required quadratic phase modulation by a maximum of a single-tone RF modulation signal. This approach was used in first realizations of deterministic temporal lensing of quantum light. The EOPM approach alleviates some of the challenges that arise with three- and four-wave mixing processes. In particular, the lack of strong optical pumps removes the risk of contaminating the few-photon level light with scattered photons. However, due to material limitations on the amplitude of electro-optic phase modulation, the effective resolution of spectral-temporal transformations utilizing the electro-optic time lens is limited. Recently it has been shown that a time lens can be realized by utilizing the amplified approximately quadratic signal around the maximum of an impulse response of a photodiode, see. The method enables temporal lensing of aperiodic signals and was shown to exhibit excellent stability of synchronization between the RF and optical signals.

3.3. Arbitrary TF Unitaries

More complex temporal phase profiles $\phi(t)$ can be viewed as a Fourier analogue to arbitrary spatial phase modulation using multipixel spatial phase modulators or deformable mirrors. It was shown in the spatial domain that targeted and, in principle, lossless spatial mode transformations can be realized by a sequence of appropriately chosen spatial phase modulation and diffractive propagation that realize the spatial Fourier transform of the wavefront. This result relies on the decomposition of unitary transformations on temporal modes in terms of spatial phase followed by diffractive Fourier transform, followed by spatial phase, followed by diffractive Fourier transform, etc. The number of phase pairs in the spatial and transverse wave-vector domain required for transformation scales approximately linear with the number of modes $N$. Following the optical space–time analogy, appropriately chosen temporal phase modulation combined with propagation in dispersive media, which acts as the analogue of the diffractive Fourier transform, can be used to implement specific lossless temporal mode transformations. This allows realization of unitary transformations of temporal modes, a key prerequisite from quantum information processing in the time–frequency degree of freedom of light and for mode matching for quantum networks. The use of cascaded quantum pulse gates has been proposed to achieve the same goal of temporal multimode transformations. Two-photon, Hong-Ou-Mandel interference, a key component to realizing a linear-optical transformations, has been recently demonstrated by implementation of a time–frequency domain beam splitter operation with a 50% conversion efficiency using similar nonlinear optical techniques.

Recently the feasibility of such electro-optic temporal mode transformations have been theoretically studied and confirmed. A proposed experimental scheme to achieve targeted temporal mode transformations by sequential application of alternating spectral and temporal phases, is depicted in Figure 7. Here a circulator (C) directs the input pulse to a chirped fiber Bragg grating (CFBG) that applies the dispersive Fourier transform mapping the spectral amplitude of the input pulse onto the temporal envelope. The circulator directs the reflected pulse from the CFBG to an EOPM that applies temporal phase $\phi(t)$. This implements the first spectral phase in the sequence since the first CFBG maps frequency to time. A second CFBG with opposite sign of GDD performs the inverse Fourier
transform mapping the pulse back to the time domain. A second EOPM applies a potentially different temporal phase, $\phi_j(t)$ to the pulse. This sequence of spectral and temporal phase applications is then applied several times (in the order of the number of modes to be addressed by the transformation). Numerical results have shown that targeted temporal mode transformations such as that depicted in Figure 7, which takes the fundamental to have shown that targeted temporal mode transformations such as that depicted in Figure 7, which takes the fundamental to

![Figure 7. Proposed scheme for targeted temporal mode transformation using a sequence of electro-optic phase modulators (EOPM), circulators (C) and chirped fiber Bragg gratings (CFBG). The dispersion of alternating sign, imprinted by the CFBGs, implements the Fourier transform (FT) and inverse Fourier transform (IFT). N given operations consisting of 2N consecutive temporal phases $\{\phi_j(t)\}$ ($j = 1 \cdots 2N$) is shown to realize a targeted unitary transformation, allowing for lossless conversion between a set of input modes to a set of output modes. Adapted with permission from ref. [49]. Copyright 2020, The Optical Society.](image)

3.4. Sources of Noise

To summarize, we have presented different methods whereby spectral and temporal phase manipulation may be utilized to achieve various goals. One can wonder what sources of noise may hinder these applications. In every previous case, two noise sources come into play: that of the optical field and that of the RF wave for temporal modulation. However, it is necessary for that RF wave to be coherent with the optical field (in the CW case) and it also has to be phase locked to the pulse train in the pulsed regime. This can be achieved with electrical schemes such as phase lock loop or by generating the RF wave from the pulse train itself, as demonstrated for instance in ref. [128]. Another source of RF noise may be the RF amplitude noise, imparted either by the amplifier or through limited amplitude resolution of arbitrary waveform generators. Initial numerical studies indicate that it is not a major source of performance reduction in the case of complex temporal phase modulation.[78]

In the case of utilizing a pulsed laser as a timing reference for RF signal generation, the main source of noise is that of the optical field. The noise analysis of a mode-locked laser is an active field of research, where a central goal is the stabilization of the frequency-comb structure over the longest possible timescale. A convenient description of the noise after the laser cavity can be given by performing a first-order Taylor expansion of the pulse train[22,61,130] which shows that the main parameters of a frequency comb are carried by orthogonal quadratures of the field. For instance, in the temporal domain, output power and repetition rate fluctuations (also called timing jitter) are carried by the amplitude quadrature and can be characterized by a single detector. While carrier-envelope phase (CEP) and center wavelength fluctuations are on the phase quadrature, which necessitates interferometric methods (such as an f-2f interferometer) to extract the frequency domain.

These noises may have intricate effects on the schemes presented in this section. It is however fair to point out that the timing jitter of the pulse train will dominate in most of the cases because of the need for a stable pulse train or a stable RF wave. For instance, timing jitter is the most undesirable source of noise for electro-optic shearing interferometry, where any fluctuation of the RF wave with respect to the pulse train will result in reduction of the spectral interference pattern.[111]

4. Applications

4.1. Quantum Interfaces—Mode Matching for Quantum Networks

Consider a network formed of nodes consisting of material quantum systems connected by photonic links.[112–116] The nodes may emit a photon that can be entangled with an internal degree of freedom of the node system. An entangling operation may then be performed between two nodes by performing a Bell-type measurement on two photons emitted from two separate nodes. A Bell measurement is necessarily based on interference between two incident photons. Its successful realization relies on high-visibility interference, which requires nearly perfect mode overlap between the photons. Spatial mode overlap is typically achieved by the use of single-mode fiber links. However, care must be taken to ensure spectral-temporal mode overlap in the case of photons coming from nodes of different types such as different trapped ion species.

In the case of spectral-temporal mode mismatch, mode converting devices are required to achieve high-visibility interference. A brute-force approach involves filtering out the unwanted
components of one of the modes. However, the efficiency of such a mode conversion process will at most be equal to the square-modulus of the overlap between the two modes, $|M_{1,2}|^2 = \frac{1}{2\pi} \int \psi_1^*(\omega) \psi_2(\omega) d\omega$, which may be very low or even zero. Mode conversion may improve the rate at which nodes may be entangled in a network provided the total transmission of the device is higher than the ratio of mode overlaps before and after mode conversion. Given a large number of links in a network the overall gain from increased mode matching can be substantial.

A significant body of work has been performed on matching the central wavelengths of single photons, both in the continuous-wave and pulsed regimes, for example refs. [37,137–140]. However vast differences may exist not only between central wavelengths, but also between spectral and temporal bandwidths of single-photon pulses, see Figure 8a.

A convenient tool to modify the bandwidth of approximately Gaussian pulses is the all-optical Fourier transform. It can be realized by using a dispersive element combined with a time lens, with GDD of the dispersive element equal to the inverse of the time lens chirping factor, $\phi_2 = 1/K$.[56] For an input transform-limited Gaussian pulse this operation produces an output Gaussian pulse of longer duration and proportionally narrower spectral bandwidth. This is in analogy to focusing a Gaussian beam using a thin lens placed in the beam waist. The all-optical Fourier transform can be also used to realize time-to-frequency conversion, which will be discussed in Section 4.2.

Single-photon spectral bandwidth compression has been initially demonstrated by a non-collinear three-wave mixing time lens.[38] A 40-fold compression factor was demonstrated, albeit with a system efficiency below 1%, which negated the possible gain in mode overlap. An electro-optic time lens was used to perform a sixfold spectral bandwidth compression experiment. With device transmission limited only by technical losses, stemming mainly from spatial mode mismatch, it was possible to demonstrate an improvement in single-photon flux rate through a reference narrowband spectral filter. With total system efficiency of 27 ± 1%, this enabled better than twofold increase in the transmission of single photons through the filter, demonstrating the potential of this approach for spectral mode matching in quantum networks.[126,141] Figure 8b,c. Recent progress[128] shows a compression factor of 18, as well as excellent stability at telecommunication wavelengths, see Figure 6c.

Bandwidth compression may also be realized based on principles other than a time-lens-based all-optical Fourier transform. In ref. [142] a Raman quantum memory in diamond was used to perform approximately twofold bandwidth modification through the use of a shaped read-out pulse. The system efficiency was 1%. In ref. [143] a 7.5-fold bandwidth modification was achieved by using the particular phase matching properties of a lithium niobate waveguide in a three-wave mixing process.[144] The system transmission was 16.9%, making it a converter that is useful for quantum network applications. However, the latter approach also introduces a significant spectral shift in the central frequency of light, which may be detrimental when trying to match emission from quantum systems with similar transition energies.

From the point of view of a future quantum network, of particular interest are spectral bandwidth transformations that would link the single- and sub-GHz bandwidths with multiple-GHz bandwidths.[145] The former are characteristic of quantum memories, trapped ions and solid-state single-photon sources,[146,147] whereas the latter of high-speed telecommunication systems and of pulsed quantum light sources based on parametric down-conversion. Here, bandwidth modification factors exceeding 100 are necessary. A possible way toward linear-optical spectral-bandwidth modification into the sub-GHz region has been proposed by applying the concept of serrodyne modulation to the time lens, realizing the temporal analogue of a spatial Fresnel lens. Here purely quadratic temporal phase modulation,
whose temporal extent is limited in practice, is replaced by a quadratic signal modulo-\(2\pi\). This deterministic technique may allow efficient linking of 100 GHz and sub-100 MHz spectral bandwidths.[78] Other temporal mode transformations have been studied theoretically. The important task of reversing an exponentially decaying pulse has been shown to be well approximated by using a single time lens.[148] More general transformations require application of more complex phase modulation analogous to those discussed in Section 3.3. These general transformations form the basis for code-division multiplexing schemes using temporal modes that are known to be optimal for achieving the ultimate limit to lossy, bosonic channel capacity,[149] which has application in deep space optical communications where the optical signal is extremely low.[150,151]

### 4.2. Time and Frequency Measurements

The ability to precisely measure a quantum optical signal in the conjugate frequency and time domains is an essential technique required for many applications in quantum optics, including characterization of quantum light[152–154] and quantum communications.[13,19–21] However, the time-bandwidth product of light pulses typically leads to challenges in measuring both time and frequency distributions. For example, single photons occupying an ultrashort (\(\approx 100 \text{ fs}\)) pulse have a broad spectrum (\(\approx 10 \text{ THz}\)) that can be easily measured with a spectrometer. However, current single-photon detection technology is not fast enough to directly measure the temporal structure of such short pulses with high precision. One approach to overcome this issue makes use of nonlinear optical gating, which has been used to probe ultrashort quantum optical biphoton pulses.[155,156] Time-resolved single-photon counting detectors typically have a temporal resolution in the range of 20 to 300 ps. Utilizing the dispersive Fourier transform (DFT) (see Section 3.2.1) one can map the frequency distribution of light onto its temporal envelope, and, using fast photon-counting detectors, measure the spectral distribution of quantum light sources with high resolution.[93,94,157] Conversely, time to frequency mapping by quadratic temporal phase modulation followed by frequency-resolved detection can enable measurement of arrival time with sub-picosecond precision.

In Section 3.2.1, we described spectral measurements utilizing the frequency-to-time mapping properties of the dispersive Fourier transform which enables high resolution spectral measurements.[84,158] This has been demonstrated at the single-photon level using optical fiber at telecommunications wavelengths[93] and CFBGs at visible wavelengths[94] to achieve \(\delta \omega \approx 2 \pi \cdot 15 \text{ GHz}\) spectral resolution. At telecommunication wavelengths 12 GHz resolution was experimentally demonstrated using a CFBG.[157] For a pulse of transform limited duration \(\tau_p\), with corresponding spectral bandwidth \(\Delta \omega \approx 2\pi / \tau_p\), one can resolve approximately \(N = \Delta \omega / \delta \omega\) independent spectral bins. To achieve a similar number of independent time-resolved measurements as present in the frequency domain, a task necessary to ensure equal information content in both domains, requires temporal resolution \(\delta t = \tau_p / N = 1 / (N^2 \delta \omega)\). This corresponds to the total chronocyclic time–frequency phase space occupied by the pulse with area \(\tau_p \Delta \omega\) tiled with \(N^2\) equally-sized squares of area \(\delta t \delta \omega\).

A concrete example of an application that requires an equal number of independent measurement outcomes in conjugate time–frequency bases arises in time–frequency quantum-key distribution (QKD).[13,20] Accessing conjugate arrival time and frequency bases requires measurements with compatible time and frequency resolution \(\delta t\) and \(\delta \omega\) as described above. Typically when performing direct measurements in one basis, the required resolution in the conjugate basis is not directly accessible for single-photon coincident measurements. For example, a measurement with temporal resolution of 100 ps, which corresponds to a frequency spread of 10 GHz, would require spectral measurements with sub-10-GHz resolution to provide frequency information within that time, as depicted in Figure 9. A similar experiment has been realized in ref. [20] with two bins in both time and frequency domains with state-of-the-art components at telecom wavelength.

The all-optical Fourier transform may be used to perform temporal measurements with resolution better than direct

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**Figure 9.** Experimental strategy to realize a prepare and measure time–frequency QKD protocol. It relies on frequency-to-time conversion utilizing dispersive elements and time-to-frequency conversion utilizing EOPM. a) Spectral bins are mapped to the temporal envelope of the pulse and subsequently detected with high temporal resolution. b) Time bins are mapped to the spectrum using dispersion and a time lens, and detected with a spectrometer. In both cases, the spectral and temporal states are either prepared by carving the spectrum and delaying time bins, or by directly mapping time/frequency to frequency/time with finite resolution resulting in well-defined bins.[13]
4.3. Single-Photon State Characterization

State characterization is indispensable for the development and operation of all elements of optical quantum technologies, including sources, detectors, memories, repeaters, interfaces and gates. While the large bandwidth of short optical pulses makes them an attractive high-dimensional platform for encoding quantum information, the fast timescales and low intensities present dual challenges to finding a reliable approach to fully reconstruct their spectral-temporal state.

A thorough review of established techniques to characterize ultrafast classical pulses is provided in ref. [159]. Among these are methods that utilize optical nonlinearities such as frequency-resolved optical gating (FROG) [160] and spectral phase interferometry for direct electric field reconstruction (SPIDER) [161], which both conventionally involve sum-frequency generation. However, at the single-photon level these techniques are challenging to realize due to the weakness of optical nonlinear effects. As such, a number of differing strategies have been developed to realize both partial and complete characterization of quantum optical pulses.

We first consider the important case of a single photon in an unknown pulsed mode. For a photon occupying a given transverse spatial and polarization modes, characterization of the temporal-spectral mode completely determines the single-photon state, which may be thought of as the spectral-temporal wave function introduced in Equation (16) in Section 2.3. Since a complete description of this state requires more information than can be extracted from a single measurement, all methods to achieve a full reconstruction require many identical copies of the state to be prepared.

Several demonstrations have successfully reconstructed the state by linear optical interference with a known reference field. The main two approaches are based on either Hong-Ou-Mandel interference (HOMI) [66,67] or homodyne tomography [162,163]. All such “externally referenced” approaches suffer the drawback of requiring stable, mode-matched, well-characterized reference pulses for reliable measurements, since some mode overlap with the reference field is needed to obtain the necessary interference. For this reason, some a priori knowledge of the test pulse mode is also necessary. On the other hand, externally referenced methods have the advantages of being able to detect discontinuities in the spectral phase and to characterize photons with discontinuous spectra, which is not straightforward for self-referencing approaches such as SPIDER [67].

In the first full reconstruction of a single-photon spectral density matrix [166], the test pulse is interfered at a beam splitter with a highly attenuated coherent state reference field in a double-pulse mode, where both the spacing of the double pulse and its absolute delay relative to the test field are controllable parameters. Bucket detectors at the output of this Hong-Ou-Mandel interferometer operate in coincidence, with the probability of bunching related to the mode overlap between the test field and the reference double pulse. By analyzing the coincidence rate as a function of the two mode-control parameters, the authors reconstruct the full spectral–temporal density matrix of the unknown photon. To improve acquisition times and eliminate the need to scan the reference, a mode-resolved approach to single-photon characterization by HOMI was demonstrated in ref. [67]. In this demonstration, the test photon was interfered with a single photon in a fixed pulsed mode. Both outputs of the interferometer are spectrally resolved in coincidence. The resulting two-photon interference pattern is inverted using Fourier transform techniques, recovering the full density matrix. HOMI-based methods have the drawback that being based on second-order correlation function measurements, they are sensitive to the photon-number statistics of the test pulse and cannot easily be generalized to pulsed modes containing higher photon-number contributions.

The spectral density matrix of an arbitrary ultrafast single-photon pulse can also be reconstructed by homodyne detection. An adaptive scheme is used in ref. [162] with the spectral amplitude and phase of the local oscillator (LO) varied using classical ultrafast pulse shaping methods. A genetic algorithm is then used to optimize the mode overlap. The complex spectral amplitude of the quantum state is then fully determined from the mode-optimized local oscillator. Later, in ref. [163] the density matrix was recovered using polychromatic optical heterodyne tomography, which extracts the spectral coherence properties from the autocorrelation function of the homodyne photocurrent at multiple LO frequencies.

In addition to linear optical approaches to pulse reconstruction, methods using nonlinear interaction with a known optical reference field have also been demonstrated. In ref. [164] a quantum pulse gate allows full state tomography by projection onto arbitrarily chosen temporal modes, controlled by shaping a bright “gate” pulse prior to a sum frequency generation interaction with the test pulse. It should be noted that techniques that utilize nonlinear interaction between a reference field and the single-photon state under examination require group velocity and phase-matching conditions to be satisfied.

A self-referencing approach to single-photon pulse characterization, which completely eliminates the need for a second
This approach, based on spectral shearing interferometry, enables characterization of the single-photon spectral–temporal wave function using electro-optic spectral shearing and spectrally-resolved measurement of single photons by dispersive Fourier transform spectrometers (Section 3.2.1). Spectral shearing interferometry is similar to SPIDER in that the pulse is sent into an interferometer with one arm experiencing a spectral shift relative to the other, but differs in that the shift is achieved deterministically by a time prism (Section 3.2.2). In this demonstration, the time prism was realized using electro-optic phase modulation, although in principle the spectral shift could also be achieved by cross-phase modulation if group velocity matching conditions can be satisfied. In contrast with externally referenced approaches, this technique is self-referencing: it requires no additional optical fields besides the test pulse, making it applicable across a broad range of wavelengths. Furthermore, this method does not require scanning of a reference field to achieve complete state reconstruction, which enables real-time feedback for source optimization. Since this measurement is based only on first-order intensity correlations, it requires no assumptions about the photon number statistics of the test field. However, it does assume that the pulse is in a pure state.

4.4. Two-Photon Time–Frequency States

Going beyond single-photon fields, many quantum technological protocols require well-characterized systems of several spatially remote pulses exhibiting quantum correlations in the time–frequency basis. Examples include entanglement distribution for certain secure communication protocols, distributed quantum computing, and certain measurement and sensing applications (see Section 4.5). The most commonly encountered example of such a system is the spectrally entangled photon pair. The wave function of a pure photon pair may be written

$$|\Psi_{\gamma,i}\rangle = \int f(\omega_1, \omega_2) \hat{a}^\dagger_1(\omega_1) \hat{a}^\dagger_2(\omega_2) d\omega_1 d\omega_2 |\text{vac}\rangle,$$

where \(\hat{a}^\dagger_1\) and \(\hat{a}^\dagger_2\) are the creation operators for monochromatic modes in the signal and idler paths, respectively. The complex quantity \(f(\omega_1, \omega_2)\) is known as the joint spectral amplitude and contains complete information about the state. Such states are naturally generated as the lowest-order non-vacuum component of the output of type-II spontaneous parametric down-conversion sources and some spontaneous four-wave mixing sources.

There has been significant interest in harnessing and verifying time–frequency entanglement in a photon pair. One approach to verify TF entanglement is Franson interferometry, which is useful for time-bin entanglement. Frequency-bin entanglement has seen a surge in research, in part due to the development of quantum optical microcombs. Here, a single-tone electro-optic modulator can be used to interfere adjacent frequency bins, similar to a frequency-domain Franson interferometer. Using TF entangled photon pairs, Yang et al. demonstrated nonlocal wavelength-to-time mapping. Here spectrally filtered detection of one photon heralds its twin in a particular frequency state, which is converted into the temporal domain using the dispersive Fourier transform. Recent work has shown that appropriate design of correlations between photon pairs can achieve tight temporal correlations after propagation through a dispersive medium. By using temporal imaging, TF correlations of spectrally entangled biphotons may be reversed, for example, spectrally-correlated...
(temporally-anticorrelated) photons can be transformed into spectrally-anticorrelated (temporally-correlated) photons.\textsuperscript{[174]} This requires applying a temporal imaging circuit with negative magnification to one photon of the entangled photon pair. The first experimental demonstrations used a three-wave mixing time lens.\textsuperscript{[44]} However, entanglement verification is not generally enough to determine many properties of the joint two-photon state and for practical purposes a more complete characterization must be obtained.

As mentioned above, measurement of the joint spectral intensity (JSI), $S(o_1, o_2) = |f(o_1, o_2)|^2$, is now a standard characterization method.\textsuperscript{[165]} A related quantity is the joint temporal intensity (JTI), $S(t_1, t_2) = |f(t_1, t_2)|^2$ (where $f$ is the Fourier transform of $I$), which gives the probability of detecting the photons at given arrival times. The JTI is more challenging to measure directly for broadband light as the timescales involved can be on the order of tens to hundreds of femtoseconds, which is generally below the timing resolution of available single-photon detectors. The JTI was measured in ref. [156] by using optical gating with classical femtosecond pulses. Since JSI and JTI measurements are insensitive to spectral and temporal phase respectively, they cannot reveal the full extent of possible nonclassical correlations.\textsuperscript{[165]} Taken together, joint measurements in both frequency and time have enabled partial characterization of entangled ultrafast photon pairs,\textsuperscript{[155]} but even so still cannot provide the full two-photon state for an arbitrary photon pair. Straightforward generalizations of the interferometric methods used to characterize single photons, such as mode-selective homodyne tomography, are hampered by the larger state space that must be spanned in the multi-photon case. Nonetheless, there have recently been a number of significant results in the field of full two-photon state characterization, employing both linear-optical and nonlinear methods.

One approach that has had considerable success in characterizing parametric photon-pair sources is stimulated emission tomography. Here one or both of the output modes of the source are first seeded by classical light, allowing the spectral-temporal properties of the spontaneous pair production to be inferred from observations of the corresponding classical process.\textsuperscript{[175]} One recent experiment by Huo et al.\textsuperscript{[176]} was built on the principle of adaptively seeding the signal mode of the photon pair source whilst feeding back with the spectrum of the amplified seed, ultimately converging on the eigenmode of the amplifier with largest eigenvalue. Subsequent eigenmodes are then recovered by repeating the process under the constraint of orthogonality with the known modes. With the Schmidt modes of the photon source known, full characterization can proceed by joint homodyne tomography. A modified version of this scheme in which both input modes are seeded allows for source characterization in the low-gain or near-degenerate case.\textsuperscript{[177]} Despite these successes, stimulated emission tomography has the significant drawback that it is only applicable to photon pairs emerging directly from a parametric source. This excludes many potential applications, such as determining the coherence-preserving characteristics of a node or an interface in a quantum network or the properties of a photon pair from a non-parametric source. Moreover, since the output of the spontaneous emission process is only inferred from the corresponding stimulated process, stimulated emission tomography may understate the role of noise generated by other mechanisms.

A recent demonstration by MacLean et al.\textsuperscript{[178]} based on sum-frequency generation with well-characterized classical pulses achieved a full reconstruction of the state of a pure entangled photon pair. This approach is agnostic to the nature of the pair source, and indeed the photons may have very different spectral-temporal characteristics, but they must be compliant with the group velocity and phase matching conditions of the nonlinear gates. In the linear optical domain, complete characterization of restricted classes of two-photon states with frequency correlations has been achieved by either time-resolved\textsuperscript{[179]} or frequency-resolved\textsuperscript{[180]} photon-pair self-interference measurements. This approach is limited to nearly degenerate, tightly correlated photon-pair sources with joint temporal amplitudes that depend only on the two-photon time difference and cannot address each photon independently.

In a recent publication, full time–frequency characterization of an entangled pair of broadband photons in a linear-optical setup was achieved by performing electro-optic shearing interferometry on one photon in conjunction with spectrally-resolved measurements on the other.\textsuperscript{[184]} This method is fully self-referenced, introducing no requirements on mode overlap between either photon and an external reference field or between the two photons themselves. As such, the device can accept input states across a large bandwidth and temporal aperture, with each input photon pair contributing a measurement outcome to the final reconstruction with a probability limited only by technical challenges such as optical loss. This demonstration realized the first direct measurement of the nonlocal spectral phase (NSP) of a photon pair, $\text{Arg}[\phi(o_1, o_2)] = \phi(o_2) - \phi(o_1)$, where $\phi_1$, $\phi_2$ are local phase terms on the signal and idler. The NSP, which is fixed at the generation of the pair and is invariant under local phase operations on either photon, gives rise to spectral entanglement in spite of the absence of correlations in the JSI. The introduction of nonzero NSP by chirping the pump was shown experimentally, demonstrating the ability to quickly toggle between generating entangled and separable photon pairs. The measured joint spectral amplitudes are shown in Figure 10 (right), showing the reconstruction of $f(o_1, o_2)$ for a separable pair (top) and a pair exhibiting non-local phase correlations (bottom) that increase the state entanglement.

### 4.5. Measurement and Sensing

The physical properties of ultrafast optical pulses, such as high bandwidth and rapid intensity variation, make them suitable probes for various measurement applications including range-finding,\textsuperscript{[181]} dispersion measurements, all-optical clocks,\textsuperscript{[182]} and broadband spectroscopy.\textsuperscript{[183]} Consequently, in addition to their uses in quantum information processing and communication, these pulses have been put forward as a platform for a number of applications in quantum-enhanced metrology.\textsuperscript{[184]}

For several of these applications, a quantum advantage can be obtained within the general framework of the phase estimation problem. Many optical metrology techniques reduce to imprinting the parameter of interest onto the phase of an optical signal in some given mode, which is then itself measured with the highest achievable precision. This precision is ultimately limited by the quantum Cramer–Rao bound (QCRB).\textsuperscript{[185]} The QCRB relates $\sigma_\phi^2$.
the minimum achievable variance in the estimated value of the target parameter $\theta$, to the number of experimental trials $v$ and the quantum Fisher information $I_Q$:

$$\sigma_{\theta}^2 = \frac{1}{vI_Q}.$$ (30)

The QCRB is saturated for a unique, optimal choice of measurement operator, but varies depending on the choice of probe state. In this context, a quantum advantage can be obtained by using probe states that are chosen to optimize the value of $I_Q$. For ideal phase estimation, precision is limited by shot noise in the interferometer. With semiclassical coherent state probes, this imposes a scaling of $I_Q \approx N$ (where $N$ is the number of photons in the probe mode), known as the standard quantum limit (SQL). It is well known that quadrature squeezing can provide a significant improvement over the SQL – a fact which has already found mature application in experiments with narrowband lasers, most notably in the LIGO observatory.[186]

To date, there has been comparatively less work on using engineered quantum ultrafast pulses to realize an analogous quantum advantage in broadband applications, although some early results exist. In ref. [130] the authors use an ultrafast squeezed light pulse to obtain measurements of the central frequency and mean energy of a mode with precision beyond the SQL. Frequency comb modes are of particular interest for certain multi-parameter measurements as they are simultaneously sensitive to small perturbations in both the temporal and the conjugate spectral domains (being composed of short, rapidly varying pulses in the time domain, and narrow spectral lines in the frequency domain). This apparent suppression of both temporal and spectral uncertainty is discussed in ref. [187], which presents frequency combs as an alternative approach to quantum operations beyond the uncertainty limit. A number of proposed experiments employing ultrafast pulses aim to use information from both time–of-flight measurements, which can be sensitive at the femtosecond level, and interferometric measurements, which are sensitive below the timescale of a single optical cycle (about one femtosecond at optical wavelengths), but ambiguous modulo the period.[22,61,188] Using squeezed light, such schemes thus permit absolute, sub-wavelength range-finding measurements beyond the SQL.

Another potential application of entangled broadband pulses to quantum metrology is provided by the dispersion cancellation paradigm. In 1992 Steinberg et al. showed that the even-order terms in the group-velocity dispersion (GVD) of highly correlated photon pairs do not influence the position or visibility of the Hong-Ou-Mandel dip.[189] The same year, Franson observed that the effect of local dispersion on the state of an entangled photon pair can be completely canceled by local operations on the second photon.[190] It has since been suggested that this dispersion cancellation may provide an advantage by canceling the distorting effect of dispersion on some measurements. Several protocols have made use of these phenomena, such as high-precision clock synchronization[191] and quantum optical coherence tomography.[192] However, the scope with which truly quantum effects such as entanglement play a role in dispersion cancellation for precision metrology has been contested.[193] In any case, a number of proposals claiming to be “quantum-inspired” make use of dispersion cancellation in optical pulses for practical measurement applications (e.g., ref. [194]), and the topic remains an area of active research.

In a separate work also claiming to be “quantum inspired,” Donohue et al. show how temporal-mode-resolved measurements enable resolution of spectral-temporal features below the theoretical limit set for time-resolved intensity measurements.[23] This work shows how techniques developed in the framework of quantum metrology can be used to optimize over all possible quantum measurements to achieve a practical enhancement.

We also briefly mention an increasing recognition of the potential of quantum probe states in the fields of nonlinear microscopy and spectroscopy. Illumination with entangled photon pairs can provide better scaling with intensity when probing two-photon absorption processes, for example in biological samples. Entangled light also provides new methods to shape and manipulate excitation pathways in matter in a manner that is classically forbidden, with applications in microscopy, imaging, interferometry and coherent control. For more information, a recent and wide-ranging roadmap detailing these concepts is presented in ref. [183].

At present, ultrafast quantum optical metrology is less developed than the continuous-wave approach. By way of outlook, however, this approach shows great potential by analogy with existing results in other highly multimode bases. Early results have been achieved for several quantum measurement and sensing protocols with existing demonstrations in the spatial domain, including ghost imaging[195] and quantum radars.[196] A number of research groups are working toward replicating and building on these results in the temporal-spectral domain.

5. Outlook

Over the past decade there has been a surge in research aimed at the preparation, manipulation, and measurement of pulsed modes of quantum light. These efforts are driven, in part, by the potential for significantly increased information content derived from temporal-mode encoding. Although the manipulation of temporal modes is described by the classical Maxwell equations, techniques to shape quantum light pulses require low-loss, low-noise implementation that are often not employed when addressing classical pulses. Here, we have presented methods to manipulate optical pulses based on the deterministic application of temporal and spectral phase. This approach is, in principle, compatible with quantum light and can enable implementation of unitary transformations of temporal modes. The ability to perform targeted temporal-mode transformations on multiple modes, which can be viewed as a multiplexer/demultiplexer for temporal modes, underpins a number of quantum applications including linear optics quantum computing, high-dimensional quantum communications, quantum-enhanced sensing, and more specific applications such as temporal mode matching, time–frequency measurements, and few-photon state characterization. There are several technical challenges to be overcome to realize the multimode transformation presented in Section 3.3. We have also reviewed the state of the art in the measurement and characterization strategies suitable for quantum pulses, focusing on those based on deterministic processes such as electro-optic spectral shearing interferometry. Together, these
techniques complement the nonlinear optical methods to manipulate and measure temporal modes, such as the quantum pulse gate and quantum memories. Last, we have provided an introduction to proposed applications of these emerging technologies, such as quantum network interfacing, enhanced metrology and sensing, and communications.

As we look to the future of temporal mode encoding for quantum applications, key challenges include the engineering of low-loss components, for example, electro-optic phase modulators, switches, circulators, and chirped fiber Bragg gratings. These systems should also have matching spatial modes to reduce coupling losses. Moving to a monolithic integrated optical platform is one approach to be explored for reduced loss. The framework for the temporal multimode transformation presented in Section 3.3 requires a sequence of temporal phases implemented using electro-optic phase modulators. The modulators must be driven by RF voltage that is synchronized with the input pulses. There is significant room for improving the RF signals used to drive the phase modulation—including length, amplitude, and bandwidth of the driving voltage. Current work in this direction, building on approaches used to manipulate classical light, produces RF signals from shaped optical pulses derived from the pulsed pump laser that generates photon pairs. This ensures that the RF signal is synchronized with the optical pulses. Although promising, this approach will require increased bandwidth and signal duration to compete with electronic arbitrary waveform generators.

The techniques presented here can, in principle, address pulses of any duration. However, in practice going from the femtosecond to nanosecond or longer regime, as required for quantum networks based upon photons from atomic or color-center systems, present new challenges. Although the RF signals become relatively easy to generate for nanosecond pulses, achieving significant spectral phase across the small bandwidth will be difficult to implement with low loss. Tunable optical cavities may provide a path for on-chip dispersion management in this regime. In addition, these techniques can operate on any state of the field, not only photon number states as previously presented here. Utilizing the pulse mode manipulation and measurements techniques presented here for continuous-variable encoding has yet to be implemented, offering a new direction in which to move.

It is clear that quantum networking, modular quantum computation, and sensing applications where the spectral and temporal properties of light are essential, will require the capability to manipulate and measure the temporal mode structure of quantum light. Additional applications of the tools to manipulate and measure pulses of light presented here are sure to arise. Indeed, in the writing of this manuscript, we found connections to this work with photonic approaches to machine learning hardware as a potential application and deep-space communications that can achieve the channel capacity. There is much to be explored in this area, which can be guided in some ways by applications in the spatial domain through the space–time analogy. With further advances in the opto-electronic components of these systems there will certainly be major improvements in the dimensionality and complexity of temporal modes that can be addressed that will bring new applications into light.

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Conflict of Interest
The authors declare no conflict of interest.

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