Fractionalized topological defects in optical lattices

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Abstract

Topological objects are interesting topics in various fields of physics ranging from condensed matter physics to the grand unified and superstring theories. Among those, ultracold atoms provide a playground to study the complex topological objects. In this paper we present a proposal to realize an optical lattice with stable fractionalized topological objects. In particular, we generate the fractionalized topological fluxes and fractionalized skyrmions on two-dimensional optical lattices and fractionalized monopoles on three-dimensional optical lattices. These results offer a new approach to study the quantum many-body systems on optical lattices of ultracold quantum gases with controllable topological defects, including dislocations, topological fluxes and monopoles.

1. Introduction

Topological excitations (i.e., domain walls, strings and monopoles) exist in a wide variety of systems in condensed matter physics, such as superfluids, superconductors and liquid crystals. Among them, BECs of dilute atomic gases [1, 2] are the ones of ideal testing grounds for investigating topological excitations. A variety of topologically interesting structures, such as vortices, knotted textures, skyrmions, and monopoles, have been fascinating for quite a few decades and recently have also been thoroughly studied in BECs of ultracold atoms [3–8].

In general, topological excitations can be characterized by an integer number. An example of typical 2D topological defects is quantized vortex in single component BECs, of which the circulation of supercurrent velocity $v$ is quantized by $N = \frac{M}{2\pi \hbar} \oint \mathbf{A} \cdot d\mathbf{l}$ (with $\mathbf{A}$ being the vector potential) owing to analyticity of single-valued order parameter (OP). A topological configuration with $|N| < 1$ (we call it fractionalized vortex) always traps a phase string that induces a (singular) branch-cut in the BECs and then is confined by a linear potential. In a special spinor BEC, deconfined topological excitations with half-quantized number may exist. In three-dimensions, an important topological defect is Dirac monopole, a point source of quantized flux. In [9], the monopole defect was created in a spin texture of spinor BEC by adiabatically modifying external magnetic fields. However, the monopole with fractional quantum number (we call it fractionalized monopole) has never been addressed.

On the other hand, researchers recognized that topological defects (such as the vortices, dislocations, monopoles) will have nontrivial quantum properties [10–12] due to the interplay between defect topology and the topology of the original states (the topological insulators, topological superconductors). Moreover, it is predicted that a Majorana-Fermion zero mode may be trapped around the core of a quantized vortex in topological superconductor and obeys non-Abelian statistics [13–16], which may be applied to realize topological quantum computation [17–20].

Here we propose a significantly simpler experimental protocol to embed topological configurations to realize optical lattices with manipulated lattice defects—dislocations. In particular, the dislocations may induce fractionalized flux/monopole on 2D/3D optical Peierls-lattice (see detailed discussions below). We demonstrate that an analogue of the fractionalized flux/monopole in BEC of an atomic gas can exist as a ground
Now we show how dislocations on Peierls lattice induce non-trivial states. As a result, these results offer a new method to study the quantum many-body system on an optical lattice of ultracold quantum gases with various fractionalized topological defects.

2. Model Hamiltonian

In this paper we study the one-component Bose–Hubbard (BH) model on 2D square Peierls-lattice and that on 3D cubic Peierls-lattice, for which the Hamiltonians are given by

\[
H = - \sum_{\langle ij \rangle} \left( t_{ij} b_i^\dagger b_j + \text{h.c.} \right) - U \sum_i n_i \left( n_i - 1 \right) + \mu \sum_i n_i,
\]

where \( b_i^\dagger \) and \( b_i \) are the bosonic creation and annihilation operators on site \( i \), respectively, \( n_i = b_i^\dagger b_i \) is the particle number operator on site \( i \). \( U \) is the strength of the repulsive interaction. \( \langle i, j \rangle \) represent all nearest neighboring links.

Figure 1 (a) is an illustration of 2D square Peierls-lattice. For the BH model on a 2D square Peierls-lattice, the hopping parameters \( t_{ij} \) are not real numbers but have uniform phase factors along the \( x \)-direction \( t_{i+\hat{x},i} = e^{i\phi} \) (the blue links in figure 1 (a)) and the \( y \)-direction \( t_{i+\hat{y},i} = e^{i\phi} \) (the red links in figure 1 (a)) where \( \phi = (\phi_x, \phi_y) \) is the Peierls phase [23]. When a particle moves around a plaquette, there is no extra phase factor of its wavefunction due to cancelation effect as \( e^{i\phi_y} \cdot e^{-i\phi_y} \cdot e^{-i\phi_x} \equiv 1 \). We call this type of lattices with uniform hopping parameters but no extra flux as Peierls-lattice. Similar to 2D case, the hopping parameters \( t_{ij} \) for the 3D cubic Peiers-lattice (figure 1 (b)) defined by \( t_{i+\hat{x},i} = e^{i\phi_x} \), \( t_{i+\hat{y},i} = e^{i\phi_y} \), \( t_{i+\hat{z},i} = e^{i\phi_z} \) are characterized by a vector \( \phi = (\phi_x, \phi_y, \phi_z) \). Peierls lattice is a lattice with uniform magnetic vector potential, which is physically trivial and can be gauged away on flat space.

These models can be solved by the mean-field (MF) approximation [21, 22]. \( b_i^\dagger b_i \approx \langle b_i^\dagger \rangle \langle b_i \rangle + \langle b_i^\dagger \rangle \langle b_i^\dagger \rangle - \langle b_i^\dagger \rangle \langle b_i^\dagger \rangle \langle b_i^\dagger \rangle \langle b_i \rangle \), where \( \psi = \langle b_i \rangle \) is the superfluid (SF) OP. Using this MF approach, we derive the properties of the BH model on Peierls-lattices. For the case of strong coupling limit, \( U \gg t \), the Bose gas turns into the Mott insulator (MI) phase; for the case of weak coupling limit, \( U \ll t \), the ground state is the SF phase, \( \psi = \psi_0 e^{i\phi} \), where \( |\psi_0| = |\psi| = \sqrt{\mu} \) and \( \phi \) is an arbitrary real number from 0 to \( 2\pi \). Because the Peierls phase will never change the amplitude of the OPs, we get the same global phase diagram as that of the BHs without Peierls phase.

3. Dislocation and fractionalized fluxes on 2D Peierls-lattice

To describe the topological properties of the dislocations, we define the Burgers vector—a vector expressing the direction and magnitude of slip caused by a dislocation, \( \mathbf{B} = \oint_{\Gamma} \mathbf{u} \) [24, 25], with \( \mathbf{u} \) the displacement field vector and \( \Gamma \) the Burgers circuit defined on an ideal lattice. For the dislocation in figure 4(a), the corresponding Burgers vector around the dislocation \( O \) is \( -\mathbf{e}_y \). Now we show how dislocations on Peierls lattice induce fractional fluxes. We focus on the case \( \phi_x = 0 \) and \( \phi_y = \phi \) as \( \phi_z \) is trivial when \( \mathbf{B} \) is in the \( y \)-direction.

Similarly, we can consider a closed circuit \( \Gamma' \) around the dislocation on the dislocated lattice and find its image on an ideal lattice. We could get that \( \oint_{\Gamma'} \mathbf{u} = -\mathbf{B} \) [26]. As the magnetic vector potential is uniform on a Peierls lattice, we find that the net flux induced by the dislocation is \( \Phi = \oint_{\Gamma'} \mathbf{a} \cdot \mathbf{d}l = -\phi \cdot \mathbf{B} \).
On a 2D square lattice, the Peierls phases $\phi$ can also be eliminated by a gauge transformation $b_i \rightarrow b_i e^{i n_y \phi}$ and $b_i^\dagger \rightarrow b_i^\dagger e^{i n_y \phi}$ with $n_y$ the $y$-coordinate of the site. However, with the existence of topological defect—dislocation, the Peierls phases cannot be eliminated where the dislocation exists. We can perform a similar gauge transformation $b_i \rightarrow b_i e^{i n_y \phi}$ where $n_y$ is illustrated in figure 2.

After this gauge transformation, the gauge phases in the hopping terms on the blue links are eliminated while the gauge phases on red links turn to $\phi$, as is illustrated in figure 3. The red links can be viewed to be a ‘phase’ string, whose ends are $\phi$ fluxes. For dislocations with higher Burgers vector $\mathbf{B} = -mi$, the gauge phases on red links are $m\phi$ and the ends of the string are $m\phi$ fluxes.

Near the dislocations the phase factors of the local SF OPs on different sites are not uniform due to the nonzero fractional flux on the dislocation. When a particle moves around a dislocation with the Burgers vector $\mathbf{B}$, an extra phase factor will be obtained: $e^{i\phi} = \exp[i\oint \mathbf{A} \cdot d\mathbf{l}]$. A universal relationship between the induced flux number (the circulation of supercurrent velocity, $\mathbf{N}$) and the Burgers vector of the dislocation is given by $\mathbf{N} = \frac{\partial}{\partial x} \oint \mathbf{A} \cdot d\mathbf{l} = \frac{1}{2\pi} \oint \mathbf{A} \cdot d\mathbf{l} = \frac{1}{2\pi} \oint \mathbf{B} \cdot d\mathbf{l} = \frac{\phi}{2\pi}$, where the effective vector potential is defined by $\mathbf{A} = \mathbf{L} \mathbf{B}$. $\mathbf{L}$ is the closed loop around the flux. Thus, the pair of dislocations have opposite topological fluxes: one is $-\phi \mathbf{B}$, and the other is $-\phi' \mathbf{B}'$. The total induced topological flux is zero due to the cancelation effect. For the case of $\phi \mathbf{B} = 0$, 2$\pi$, we have the induced topological fractionalized flux around the dislocations. In general, there is a twisted phase away from the dislocation as determined by $\Im \left( \frac{\phi}{2\pi} \right) = \varphi_1$. Here $\varphi_1$ is defined by $\varphi_1 = \sum_i (\frac{\phi \mathbf{B}}{2\pi}) \ln(z_i - z_0)$ and $z_0 = x_0 + iy_0$ represents the location of the flux. It is obvious that the
Based on the BH model in large-U limit, $U \to \infty$ (the hard-core boson limit), we discuss the fractionalized skyrmion on 2D Peierls-lattice. The Hamiltonian of hard-core bosons on Peierls-lattice can be written as

$$H = -\sum_{\langle ij \rangle} t_{ij} b_i^+ b_j - \sum_i V_i n_i,$$

where $b_i$ satisfies the non-double-occupancy constraint, $b_i^+ b_i \leq 1$. $V_i$ denotes the trap potential.

The above representation of the many-body quantum states is equivalent to the following operator identity:

$$\hat{\rho} = \sum_i \frac{1}{n_i + 1} \hat{n}_i$$

where $\hat{\rho}$ is the density operator. For a given state $\hat{\rho}$, the density operator $\hat{n}_i = \hat{\rho} \hat{\rho} \hat{n}_i \hat{\rho}$ is obtained as:

$$\hat{n}_i = \sum_j \hat{n}_j (\hat{\rho} \hat{n}_i \hat{\rho})$$

where $\hat{n}_i = b_i^+ b_i$.

Then, we study the properties of interacting bosons on the Peierls-lattice with dislocations by numerical calculations. We found that the local SF OPs show nontrivial topological properties. For example, for $\phi = 0, \pi$, there indeed exists a nontrivial topological phase variation:

$$\Delta n = n_i - n_o$$

where $n_i$ and $n_o$ denote the particle number at site $i$ and the end site, respectively.

In particular, from numerical calculations, we found that the local SF OPs show nontrivial topological properties. For example, for $\phi = 0, \pi$, there exists a $\pm \pi$ flux around the pair of dislocations, of which the induced flux number (the circulation of supercurrent velocity, $C$) is $\pm 1/2$. The phases patterns of topological vortexes near the dislocation $\phi$ are given in figure 4(c). The induced topological flux by the dislocations in the Peierls-lattice is protected by the topological properties of the dislocations. The fluctuations of the local hopping parameters will not change the topological flux. In experiments, people may detect the phase distribution in the SF order to observe the effect from the dislocations.

4. Fractionalized skyrmions on 2D Peierls-lattice

Based on the BH model in large-U limit, $U \to \infty$ (the hard-core boson limit), we discuss the fractionalized skyrmion on 2D Peierls-lattice. The Hamiltonian of hard-core bosons on Peierls-lattice can be written as

$$H = -\sum_{\langle ij \rangle} t_{ij} b_i^+ b_j - \sum_i V_i n_i,$$

where $b_i$ satisfies the non-double-occupancy constraint, $b_i^+ b_i \leq 1$. $V_i$ denotes the trap potential.

The above representation of the many-body quantum states is equivalent to the following operator identity:

$$\hat{\rho} = \sum_i \frac{1}{n_i + 1} \hat{n}_i$$

where $\hat{\rho}$ is the density operator. For a given state $\hat{\rho}$, the density operator $\hat{n}_i = \hat{\rho} \hat{\rho} \hat{n}_i \hat{\rho}$ is obtained as:

$$\hat{n}_i = \sum_j \hat{n}_j (\hat{\rho} \hat{n}_i \hat{\rho})$$

where $\hat{n}_i = b_i^+ b_i$. With this mapping, the hard-core bosons Hamiltonian, equation (3) becomes that of the XY model with a magnetic field applied along the $z$-direction: $H = -\sum_{\langle ij \rangle} t_{ij} (\hat{S}_i^x \hat{S}_j^y + \hat{S}_i^y \hat{S}_j^x) - \sum_i V_i (\hat{S}_i^z + \frac{1}{2})$. In the spin ordered state (that is just the
the function $i12_Bz = xV|zz\ 0)$ is not only analytic, but also meromorphic. However, for the $|U\ VU\ B4=q118x194$, the function $\mathcal{f}\ 1\ 0xx$ is set to be along the function $\mathcal{f}\ 0x\ xy\ d1\ c\ o\ s\ ni\ y\ plane$ around an edge dislocation, an extra phase factor is $\xi=\frac{r}{4\pi}$.

In this section, we study the properties of BH models on 3D cubic Peierls-lattice with dislocations. We take a BEC state for the hard-core bosons, core bosons, quarter-skyrmions in different potential traps.

In the spin ordered state, a dislocation may ‘nucleate’ a skyrmion with a fractionalized topological charge (we call it fractionalized skyrmion). We define the topological charge (Pontryagin index) of fractionalized skyrmions to be $\mathcal{Q} = \int d^4x (N \cdot \partial_x N \times \partial_y N) = \frac{2\pi}{4\epsilon} (1 - \cos \theta_{obs})$ where $\theta_{obs}$ denotes the z-direction of spin polarization far from the core of (fractionalized) skyrmion which is determined by the filling number of hard-core bosons, $\cos \theta_{obs} = n_0 - 1/2$. Let’s introduce a complex field $w = w_1(z) + iw_2(z)$ by a stereographic projection from north pole, $w_1(z) = \frac{n_z(z)}{1 - n_z(z)}$, $w_2(z) = \frac{n_y(z)}{1 - n_y(z)}$ [30, 31]. A typical (fractionalized) skyrmion’s solution with topological charge $\mathcal{Q}$ is given by $w(z) = (\xi^2 - z^2)^\frac{1}{2}$. Here $\xi$ is the radius of its core. For the skyrmions with integer $\mathcal{Q}$, the function $w$ is not only analytic, but also meromorphic. However, for the skyrmions with fractionalized topological charge $\mathcal{Q}$, the function $w$ is not analytic and there exists a dislocation-induced branch-cut in the function $w$.

Then, we study the properties of hard-core bosons on the Peierls-lattice by numerical approach. From the numerical calculations, we found that for the hard-core boson on a uniform Peierls-lattice, the dislocation induces a vortex-like spin configuration on XY plane (a meron with a narrow core). When we add a smooth trap potential, the situation is different. We studied the hard-core bosons on three different types of trap potential, harmonic trap potential ($V(x) \sim x^2$), linear trap potential ($V(x) \sim x$) and square-root trap potential ($V(x) \sim \sqrt{x}$). All these trap potentials have minimum values at 0 site (the dislocation). The numerical results show that for the hard-core bosons on the Peierls-lattices with different trap potentials, the dislocation-induced fractionalized skyrmions have the same topological charge as $\mathcal{Q} = \frac{\phi_0}{4\pi}$. These topological configurations may be pictured as the similar picture: inside the core $|r - r_0| < \xi$, the spin is polarized along z-direction (that means $n_z \to 1$); outside it $|r - r_0| > \xi$, the spin is on XY plane (that means $n_z \to 1/2$ or $\theta_{obs} = \frac{\pi}{2}$). See the numerical results in figure 5(a), in which the spin vectors of a fractionalized skyrmion with $\mathcal{Q} = 1/4$ (we call it a quarter-skyrmion) induced by dislocation are shown for the interacting bosons, with $t/U = 0.01$, $V_x/U = 0.05 - 0.05 (\xi^2 - z^2)^\frac{1}{2}$. Figure 5(b) shows $\langle \hat{S}_z^i \rangle (n_i) \mbox{ via } |r| = r$. From these figures, one can see that the core’s radius differ for quarter-skyrmions in different potential traps.

5. Fractionalized flux-tubes and fractionalized monopoles on 3D Peierls-lattice

In this section, we study the properties of BH models on 3D cubic Peierls-lattice with dislocations. We take a dislocated 3D Peierls-lattice characterized by $\phi_0 = (0, \phi_x, 0)$ as an example.

For 3D cubic Peierls-lattice with an edge dislocation, of which the Burgers vector $\mathbf{B}$ is set to be along y-direction, $\mathbf{B} = (0, 1, 0)$. Now, the edge dislocation will trap a topological fractionalized flux-tube along it. See the illustration in figure 6(a). For simplify, we don’t plot the links along z-direction to emphasize the flux-tube on x-y plane. In the x-y plane, we get a 2D square Peierls-lattice with a 2D edge dislocation. When a particle moves on the x-y plane around an edge dislocation, an extra phase factor is $e^{i\phi} = \exp [i(\frac{\phi}{2} \cdot \mathbf{d} \cdot \mathbf{u})]$ where the induced
flux number and the Burgers vector of the dislocation is also given by \( \Phi = -\mathbf{\phi} \cdot \mathbf{B} \). Thus, as shown in figure 6(b), the edge dislocation may induce a fractionalized flux-tube. Then, we study the properties of interacting bosons on the Peierls-lattice with dislocations by the numerical approach. The phase pattern of a \( \pi \)-flux-tube on 3D Peierls-lattice is shown in figure 6(c). The result indicates that there indeed exists the dislocation-induced fractionalized flux-tube along \( z \)-direction.

Next, we study the fractionalized monopoles induced by dislocations on 3D cubic Peierls-lattice. There are two methods to generate fractionalized monopoles.

One method is to consider the nonuniform Peierls phases: on upper half system (for \( y > 0 \)), \( \phi = (0, 0, 0) \), on lower half system (for \( y < 0 \)), \( \phi = (0, 0, 0) \). Thus the dislocation will induce flux-tube on upper half system, but not on lower half system. The flux-tube is terminated at \( y = 0 \), or we get a fractionalized monopole at \( y = 0 \). To characterize the fractionalized monopole, we define the ‘monopole’ number, \( \mathcal{M} = \frac{1}{2\pi} \int S \mathbf{B} \cdot d\mathbf{S} \int \mathbf{a} \cdot dl = \frac{\alpha B}{2\pi} \) where the effective magnetic field is defined by \( \mathbf{B} = \nabla \times \mathbf{a} \). \( S \) is the closed surface around the monopole. Thus, we have a fractionalized monopole attaching a fractionalized flux-tube (a true Dirac string). Here, the singularity is physical, as it can not be gauged away.

The other method is to consider a half-dislocation on a 3D cubic uniform Peierls-lattice: the dislocations exist in lower half system. See the illustration in figure 7(a). As a result, the dislocation-induced flux-tube terminates in the middle of the system. As shown in figure 7(b), the end of the flux-tube will play the role of a fractionalized monopole.

We use the second method to generate a fractionalized monopole and the corresponding BH model is calculated by numerical approach. From numerical calculations, we found there indeed exists a fractionalized monopole.
monopole around the dislocation, of which the monopole number is $\mathcal{M} = 1/2$. We call this topological object half-monopole. The phase pattern near the half-monopole is given in figure 7(c). Now, the half-monopole can be viewed as the end of a $\pi$-flux-tube while the Dirac monopole is the end of an unphysical flux tube—the Dirac string.

6. The physical realization

6.1. Optical Peierls-lattice

Firstly, we show how to realize the optical Peierls-lattice. In recent experiments, tunable Peierls phases in a 1D ‘Zeeman lattice’ have been realized using a combination of radio-frequency and Raman coupling [32]. Now, a hot topic is to realize the spatially dependent Peierls phases that leads to an effective (staggered) magnetic flux through one unit cell of the optical lattice. Thus, to engineer uniform complex hopping amplitudes with Peierls phases along the $y$-direction [23], $t_{i,j+e_y} = te^{i\phi}$, the Raman-assisted hoppings are used by a pair of far-detuned running-wave laser beams with different frequencies, $\omega_0$ and $\omega_0 + \Delta$ [33–36]. A linear potential is added along the $y$-direction, for which the energy difference between two nearest neighbor sites is $\Delta$. Thus the Raman-assisted hoppings are restored along the $y$-direction with the resonance energy $\hbar \omega = \Delta$. However, along the $x$-direction, there is no such Raman-assisted hoppings. As a result, we can obtain uniform complex hopping amplitudes with Peierls phases along the $y$-direction, $t_{i,j+e_y} = t$, $t_{i,j+e_x} = te^{i\phi}$. By similar approach, we can realize a 3D optical Peierls-lattice, for example $t_{i,j+e_x} = t$, $t_{i,j+e_z} = te^{i\phi}$, $t_{i,j+e_y} = t$.

6.2. 2D optical lattices with edge dislocations

Secondly, we discuss the physical realization of the 2D optical lattice with dislocations.  

An optical vortex is a beam of light whose phase varies in a screw thread like manner along its axis of propagation. The optical vortex waves possess a phase singularity which occurs at a point or a line where the physical property of the wave becomes infinite or changes abruptly. We use the properties of vortices to realize the dislocations [37–42].

An ideal optical vortex propagating in the $z$-direction may be written in the cylindrical coordinate as $E(r, \varphi, z) = A(r, z)\exp(i m \varphi)\exp(-ikz)$, where $A(r, z)$ is amplitude function, $k = 2\pi/\lambda$ is the wave number, and $m$ is known as the topological charge. The optical system is shown in figure 8: a plane wave and an optical vortex wave interfere in the $x$–$z$ plane with wavelength $\lambda_1$, and two plane waves form an optical standing wave along the $y$-axis with wavelength $\lambda_2$. We use different frequencies for the laser fields in order to eliminate any residual interferences between the standing waves and the optical vortex. The optical vortex wave propagates along the $z$-axis. The plane wave in $x$–$z$ plane is traveling at an angle $\theta$ to the $z$-axis. The intensity of light in receiving plane at $z_0$ can be written as $I \propto \cos(k_1(x \sin \theta + z_0 \cos \theta - z_0) + m \varphi + \phi_0) + \cos^2(k_2 y)$, where $k_1 = 2\pi/\lambda_1$, $k_2 = 2\pi/\lambda_2$, $m$ is the topological charge, $\phi_0$ is the phase shift from the optical path difference between the optical vortex and the plane wave in the $x$–$z$ plane, $\varphi = \tan^{-1}(x/y)$. We will set the interfering plane at $z_0 = 0$ in all our simulation. The results are shown in figure 8. The simulation region is $5 \times 5$ um. $\lambda_1 = 500$ nm, $\lambda_2 = 600$ nm are used in our simulations.

From figure 8, we can see that the dislocation emerges at the phase singularity point. The simulation results show that the dislocation is sensitive to the phase shift between the optical vortex and the plane wave (not shown). The intensity distribution will be reversed when the phase difference is $\pi$, as shown in figures 8(a), (b). So we need to adjust the phase shift properly to obtain the designed dislocation optical lattice, which can be realized by tunable phase plate. The fringe spacing would increase due to the decrease of the angle $\theta$. We can also
generate more dislocation arms if the topological charge $|m| > 1$ (not shown). The advantage of this system is we can obtain the dislocations relative conveniently, but it is difficult to achieve the multiple or periodic dislocations.

6.3. Three-dimensional optical lattices with edge dislocations

It is impossible to achieve 3D optical lattice with perfect dislocation simply using the method described in figure 8. The dislocation line will drift when penetrating through layers. We improved the optical system, and realized a periodic 3D optical lattice with stable dislocations. The optical system is shown in figure 9. The main structure is the same as 2D optical lattice except two differences. First, we obtain the periodic array of the dislocation according to the optical vortex reflected from the new added mirror in $z$-direction. The second and more important is that the plane wave in $x$–$z$ plane can only propagate along the $x$-direction or be perpendicularly to the optical vortex which is different to realize in the 2D situation. If not, one needs to add another proper plane wave, and can also obtain the 3D dislocation optical lattice.

The intensity of the 3D dislocation optical lattice can be written as:

$$I \propto \cos(2kz) + \cos(kz)\cos(kx - m\varphi + \phi_0) + \cos^2(ky).$$

All the meanings and value of the parameters are the same as the 2D case. Using this method, we get the periodic dislocation optical lattice. The style of the dislocations is associated with the topological charge and the phase shift. When given a certain topological charge and phase shift, we can get two styles of dislocation emerged in $z$-direction periodically. The period depends on the wavelength of the optical vortex wave.

6.4. Three-dimensional optical lattices with screw dislocations

The 3D simple cubic optical lattice has been realized and studied before. In our proposed experiment setup to realize 3D optical lattice with screw dislocations, most of the experimental conditions and designs are the same as 3D simple cubic optical lattice except for two differences. In order to realize the topological defects with the screw dislocations, we utilize an optical vortex wave and a phase conjugate mirror. As shown in figure 10, the blue cylinders are optical vortex which travels in $z$-direction and reflected by a phase conjugate mirror. The red cylinders are standing waves orthogonal to each other in $x$–$y$ plane. When the topological charge of the optical vortex wave $m = 0.5$, the 3D optical lattice with screw dislocations is obtained, shown in figure 10 (right).
7. Discussion

In this paper, we studied the physics of ultracold quantum gases on a 2D and 3D optical lattice with dislocations. We found that the dislocations may induce fractionalized fluxes or fractionalized monopoles on a Peierls optical lattice due to the interplay between the hopping amplitudes of the Peierls phases and the nonlocal properties of the dislocations. In this case, a topological optical vortex eventually produces various fractionalized topological defects of bosons on the optical lattices. In the future, this realization may be applied to realize nontrivial topology in topological states.

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