Blocking Active-Sterile Neutrino Oscillations in the Early Universe with a Majoron Field

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Abstract

We propose a new mechanism to block the active-sterile neutrino oscillations in the Early Universe. We show that a typical consequence of theories where the lepton number is spontaneously broken is the existence of a coherent cosmological Majoron field with a strength proportional to the lepton and baryon numbers of the Universe. This field interacts with leptons and changes the potentials relevant for neutrino oscillations. If the scale of lepton number symmetry breaking is of the order of 1 GeV then a Majoron field and lepton number asymmetry of the order of the baryon asymmetry are strong enough to block the active-sterile neutrino oscillations with the atmospheric neutrino mass gap which otherwise would bring the sterile neutrino into equilibrium at the big bang nucleosynthesis epoch.

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I. INTRODUCTION

The explanation of the present neutrino puzzles [1] may require the existence of one or several species of extra light sterile neutrinos [2]. In particular, the sterile neutrino could be relevant for the explanation of the atmospheric neutrino problem (ANP) [3] in the presence of a significant $\nu_\mu - \nu_s$ mixing [1]. The typical required values are $\delta m^2_{\text{atm}} \sim 3 \times 10^{-3}$ eV$^2$ and large mixing angle, $\sin^2 2\theta_{\text{atm}} \simeq 1$.

On the other hand, for such a parameter range one can encounter a contradiction with the Big Bang nucleosynthesis (BBN) bounds [6] on the number of extra light particle species; namely, according to the analyses of ref. [7], the sterile neutrino comes into equilibrium with the particle thermal bath via $\nu_\mu - \nu_s$ oscillation, unless the condition $\delta m^2 \sin^4 2\theta \lesssim 3 \times 10^{-6}$ eV$^2$ is satisfied (updated constraints are given in ref. [8] for small mass differences, $\delta m^2 \lesssim 10^{-7}$ eV$^2$), which is certainly out of the range of parameters needed to explain the ANP.

However, it was found [9] that the $\nu_\mu - \nu_s$ oscillations are suppressed at temperatures $T \lesssim 3$ MeV (the decoupling temperature of $\nu_{\mu,\tau}$) if the lepton number asymmetry at these temperatures is very high, namely $L_a \gtrsim 10^{-5}$ (lepton number to photon number ratio). But this is 4–5 orders of magnitude larger than the observed baryon asymmetry of the universe ($B \lesssim 10^{-9}$) and in the most generic baryogenesis context one can expect that $L \sim B$ (e.g., in the context of grand unified theory (GUT) baryogenesis or leptogenesis [10,11] this is because the $B + L$ non-conserving sphaleron processes redistribute $B$ and $L$ among each other [12,13]). The same is true in the context of the electroweak baryogenesis ($B - L = 0$). In the Affleck-Dine mechanism $B$ and $L$ can in principle be independent of each other, but still of the same order.

It has been shown [13,8] that at much lower temperatures ($T < 100$ GeV) neutrino oscillations can actually produce a rapid increase of the lepton asymmetries from the initial very small values up to the order of $0.1$. This however only occurs for negative $\delta m^2 \cos 2\theta$ and very small active-sterile mixing angles, not directly relevant for the ANP.

In this paper we show that a lepton asymmetry as small as the present baryon asymmetry may be enough to block the sterile neutrino oscillations. The necessary new ingredient is the existence of a coherent Majoron field in the Early Universe and a low scale of spontaneous breaking of lepton number, $F_L \sim 1$ GeV (The Majoron [14] is the massless Nambu-Goldstone boson in models where the total or any partial lepton number is spontaneously broken.).

While it has been common wisdom that due to their derivative coupling nature [17] Nambu-Goldstone bosons cannot mediate long range interactions, it has recently been demonstrated [18] that a coherent source of a Majoron field, to be specific, is formed whenever the corresponding broken lepton number suffers a net increase or decrease in a certain region of space. The processes that violate this lepton number can be the very neutrino oscillations as exemplified in previous papers [18] or any other reactions. In the present work we show that a Majoron field can be produced due to lepto- and baryogenesis processes in

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1 The recent Super-Kamiokande data can be explained by $\nu_\mu - \nu_\tau$ oscillations while the situation where the ANP is exclusively due to $\nu_\mu - \nu_s$ oscillations is disfavored [4]. However, the more general case where $\nu_\mu$ oscillates into $\nu_\tau$ and $\nu_s$ with comparable rates is completely consistent with the data [3].
the Early Universe. The Majoron field interacts with neutrinos with a strength inversely proportional to the lepton breaking scale, $F_L$. If $F_L$ is around 1 GeV, a lepton asymmetry as small as $L \sim B \sim 10^{-9}$ can block the $\nu_\mu$ oscillation into sterile neutrinos with $\delta m^2 \sim 3 \times 10^{-3}$ eV$^2$ no matter how large their mixing angle is. That is our thesis.

The origin of the Majoron field is elaborated in section V and its role on neutrino oscillations into a sterile neutrino in section VI. But first we build in section II a specific model of neutrino masses with spontaneous breaking of lepton number and in section IV we derive the relations between the particle asymmetries in the Early Universe and the present baryon number. The model aims to fit the present known observations from solar, atmospheric and terrestrial neutrino experiments, including the Liquid Scintillation Neutrino Detector (LSND) result [19], with an extra sterile neutrino. However, it should be emphasized that the mechanism we propose of suppression of oscillations into a sterile neutrino at the BBN epoch, based on the existence of a Majoron field, does not depend on the particular model or set of neutrino mass parameters. The only fundamental assumptions are that the lepton number (or a partial lepton number) is spontaneously broken and the breaking scale is around the 1 GeV magnitude. In the next section we also make the point that in the absence of the LSND neutrino mass gap the oscillations of atmospheric and solar neutrinos into a sterile neutrino are no longer correlated with each other. In the last section we draw our conclusions.

II. NEUTRINO MASSES AND MIXING

The existence of a fourth, sterile, neutrino has been suggested as it is the only way of reconciling the atmospheric, solar, and LSND neutrino oscillation evidence and their very different $\delta m^2$ mass gap scales. The wide mass gap ($\mathcal{O}(1 \text{ eV})$) that is necessary to explain the LSND result in terms of $\nu_e - \nu_\mu$ mixing requires that the neutrino mass pattern should have a two-doublet structure [20]: one of the doublets consists of $\nu_e$ and $\nu_s$ or $\nu_\tau$, or a linear combination of both, and is responsible for the solar neutrino deficit, and the other one, responsible for the atmospheric neutrino anomaly, consists of $\nu_\mu$ and $\nu_\tau$ (or a linear combination of $\nu_s$ and $\nu_\tau$). These doublets are separated by the LSND mass gap.

It has been shown [21,22] that even after the recent SNO observations [23] both the sterile and active neutrino oscillations are viable solutions of the solar neutrino problem as well as a more general superposition of both. On the other hand, the atmospheric neutrino data seem [4] to favor the $\nu_\tau$ solution against the sterile neutrino case but the analysis [3] of the most recent data still allows a quite large relative probability, more than 50%, of oscillation into $\nu_s$ (the larger the probability, $\sin^2 \xi$, the smaller the allowed $\delta m^2$ range).

A consequence of the LSND large mass gap and the limits from reactor disappearance experiments such as Chooz [24] is that the solar neutrinos $\nu_e$ and the atmospheric neutrinos $\nu_\mu$ must oscillate into states that are essentially orthogonal to each other. In other words, if the solar electron neutrinos oscillate into the linear combination $\nu_e \equiv \cos \xi \nu_s - \sin \xi \nu_\tau$, then, the atmospheric muon neutrinos necessarily oscillate into the state $\nu_\mu \equiv \sin \xi \nu_s + \cos \xi \nu_\tau$. However, the situation would be totally different if the LSND evidence was not present.

To be more specific, let $\nu_1$, $\nu_2$ and $\theta_\odot$ be the mass eigenstates and mixing angle responsible for the solar neutrino deficit and $\nu_3$, $\nu_4$ and $\theta_{\text{atm}}$ the states and mixing angle relevant for atmospheric neutrinos. The two pairs are separated by the LSND mass gap and no other
specific mass hierarchy has to be assumed. The reactor experiments constrain the mixing matrix elements $U_{e3}$, $U_{e4}$, $U_{\mu 1}$, and $U_{\mu 2}$ to be small [20], but not $U_{\tau i}$ or $U_{si}$. If one neglects all the mixing angles that are necessarily small and irrelevant to explain the present bulk of data, the mixing matrix is given as

\begin{align}
\nu_1 &= \cos \theta_\odot \nu_e - \sin \theta_\odot (\cos \xi \nu_s - \sin \xi \nu_\tau), \\
\nu_2 &= \sin \theta_\odot \nu_e + \cos \theta_\odot (\cos \xi \nu_s - \sin \xi \nu_\tau), \\
\nu_3 &= \cos \theta_{\text{atm}} \nu_\mu - \sin \theta_{\text{atm}} (\sin \xi \nu_s + \cos \xi \nu_\tau), \\
\nu_4 &= \sin \theta_{\text{atm}} \nu_\mu + \cos \theta_{\text{atm}} (\sin \xi \nu_s + \cos \xi \nu_\tau).
\end{align}

The mass eigenstates $\nu_1$ and $\nu_2$ are separated by the gap $\delta m_\odot^2$ and $\nu_3, \nu_4$ by $\delta m_{\text{atm}}^2$.

Clearly, the less the atmospheric neutrinos oscillate into the sterile neutrino the more the solar neutrinos have to oscillate into $\nu_s$. There is a potential clash in the future if both solar and atmospheric neutrino experiments happen to constrain the respective sterile neutrino solutions to less than a 50% probability. In that case the conflict with the LSND data will be insoluble, which will call for new results from MiniBooNE [25], the next new independent accelerator experiment. Suppose for a moment that the LSND evidence does not exist or is going to be ruled out by the MiniBooNE experiment. We would like to stress that this does not rule out the sterile neutrino as a possible protagonist in the other solar and atmospheric neutrino problems, not even if both of them exclude dominant sterile neutrino solutions. On the contrary, the absence of the LSND mass gap increases the freedom in the neutrino mixing parameters. Then, the two-doublet mass pattern is no longer inevitable and the role of $\nu_s$ in the solar neutrino deficit is completely decoupled from its role in the atmospheric neutrino oscillations.

As a matter of proof we make explicit an extreme case, namely, where the atmospheric neutrinos oscillate into $\nu_\tau$ or $\nu_\mu$ with arbitrary relative probabilities, while the solar neutrinos oscillate exclusively to $\nu_\tau$ and $\nu_\mu$ but not to $\nu_s$. The mixing matrix can be described as follows:

\begin{align}
\nu_1 &= \cos \theta_\odot \nu_e - \sin \theta_\odot (-\sin \alpha \nu_\mu + \cos \alpha \nu_\tau), \\
\nu_2 &= \sin \theta_\odot \nu_e + \cos \theta_\odot (-\sin \alpha \nu_\mu + \cos \alpha \nu_\tau), \\
\nu_3 &= \sin \beta \nu_s + \cos \beta (\cos \alpha \nu_\mu + \sin \alpha \nu_\tau), \\
\nu_4 &= \cos \beta \nu_s - \sin \beta (\cos \alpha \nu_\mu + \sin \alpha \nu_\tau).
\end{align}

As far as the mass spectrum is concerned, the mass eigenstate $\nu_3$ is separated from the other three by mass gaps that are in the atmospheric neutrino range $\sim \delta m_{\text{atm}}^2 \sim 3 \times 10^{-3}$ eV$^2$. $\nu_1$ and $\nu_2$ are almost degenerate and separated by the solar neutrino mass gap and finally, $\nu_4$ is only subject to the condition $m_3^2 - m_4^2 \sim \delta m_{\text{atm}}^2$, as it, like $\nu_3$, does not participate in the solar neutrino oscillations. The mixing angles relate to the atmospheric mixing angle as $\cos \theta_{\text{atm}} = \cos \alpha \cos \beta$. The atmospheric neutrinos $\nu_\mu$ oscillate into $\nu_\tau$ with a probability proportional to $\sin^2 \alpha \cos^2 \beta$ whereas the probability of oscillation into $\nu_s$ is proportional to $\sin^2 \beta$. The ratio between them is given by $\tan^2 \xi = \tan^2 \beta/\sin^2 \alpha$. It is clear that the solar neutrinos do not oscillate into $\nu_s$.

This just shows that the potential problem raised by the possibility that the atmospheric neutrinos oscillate significantly into a sterile neutrino with its consequences for BBN is not
necessarily linked to the solar neutrino solutions and does not depend on the LSND evidence although it has been motivated by the coexistence of all three kinds of observation. In the present work we want to present a solution and a mechanism to block the oscillations of muon neutrinos into sterile neutrinos in the Early Universe at the time of BBN. The idea does not depend crucially on the particular neutrino mixing pattern but the actual numbers vary, of course, from model to model. We worked out in detail a particular model that is suitable to encompass all three types of neutrino oscillation evidence, including LSND.

III. NEUTRINO MASS MODEL

The seesaw mechanism [26] can be incorporated within a model where the lepton number is spontaneously broken at a relatively low energy scale by adding to the standard lepton doublets $\ell_i$ and charged singlets $e_i$ two heavy sterile neutrinos per lepton generation, $N_{Li}$ and $N^C_{Ri}$ (left-handed), with lepton numbers $+1$ and $-1$, respectively. The additional light sterile neutrino, $\nu_s$ (left-handed), has lepton number $L_s = -3$. The most general Yukawa interaction Lagrangian in the lepton sector is written in Majorana matrix form as

$$\mathcal{L}_Y = \frac{1}{2} \psi^T C \mathcal{M} \psi + \text{H.C.},$$

(3)

where $\psi \equiv (e^C_i, \ell_i, \nu_s, N_{Li}, N^C_{Ri})$ and $\mathcal{M}$ is the symmetric matrix

$$\mathcal{M} = \begin{pmatrix}
  e_j^C & \ell_j & \nu_s & N_{Li} & N^C_{Rj} \\
  e^C_i & 0 & h^T_e H_1 & 0 & 0 & 0 \\
  \ell_i & - & 0 & 0 & h_N H_2 \\
  \nu_s & - & - & h^T_s \sigma^* & 0 \\
  N_{Li} & - & - & h_L \sigma & M \\
  N^C_{Ri} & - & - & - & h_R \sigma^*
\end{pmatrix}.$$  

(4)

The omitted elements are obtained by symmetrization. $H_1$ and $H_2$ are two standard Higgs doublets under SU(2) and $\sigma$ is the singlet scalar field with lepton number $L_\sigma = -2$. $h_s$ is a $3 \times 1$ column and $h_e, h_N, h_L, h_R,$ and $M$ are $3 \times 3$ matrices.

Before lepton number spontaneous breaking the heavy sterile neutrinos form Dirac particles, namely, $N_i = N_{Li} + N^C_{Ri}$, with lepton number equal to 1 and masses $M_i$ in the basis where $M$ is diagonal: $M = \text{diag}(M_i)$. After lepton and gauge symmetry breaking the light neutrinos acquire masses and mix with the sterile neutrino in a $4 \times 4$ Majorana mass matrix. Denoting the $3 \times 3$ active, $3 \times 1$ active-sterile and $1 \times 1$ sterile neutrino blocks respectively, as $m_{\nu\nu}$, $m_{\nu s}$ and $m_{ss}$, we obtain in leading order in any basis where $M$ is a real matrix

$$m_{\nu\nu} = h_N M^{-1} h_L (h_N M^{-1})^T \langle \sigma \rangle v_2^2,$$

(5a)

$$m_{\nu s} = -h_N M^{-1} h_s \langle \sigma \rangle v_2,$$  

(5b)

$$m_{ss} = (M^{-1} h_s)^T h_R M^{-1} h_s \langle \sigma \rangle^3,$$  

(5c)

where $v_2 = \langle H^0_2 \rangle$. We take as reference scales $m_{\nu\nu} \sim 0.05$ eV to account for the atmospheric neutrino anomaly and $m_{\nu s} \sim 1$ eV for the LSND $\nu_\mu (\bar{\nu}_\mu) \rightarrow \nu_e (\bar{\nu}_e)$ evidence. Since we also assume $\langle \sigma \rangle \sim 1$ GeV the element $m_{ss} \sim 10^{-13}$ eV is completely negligible.
IV. ASYMMETRIES IN THE EARLY UNIVERSE

At temperatures below the heavy neutrino masses $M_i \gtrsim 10^6$ GeV, the Dirac masses $M_i$ still mediate scattering processes capable of producing the light singlet particles $\nu_s$ and $\sigma$ like $\ell H_2 \rightarrow \bar{\nu}_s \sigma$. They can be studied in terms of the effective operators

$$
\mathcal{L}_{\text{eff}} = \ell_i H_2 \frac{m_{ij}}{2\langle \sigma \rangle v_2} \ell_j H_2 \sigma + \ell_i H_2 \frac{m_{is}}{\langle \sigma \rangle v_2} \nu_s \sigma^* + \nu_s \frac{m_{ss}}{\langle \sigma \rangle^2 v_2} \nu_s \sigma^3 + \text{H.C.} ,
$$

which also give rise to the light neutrino masses after spontaneous breaking of lepton number. One obtains the c.m. cross sections of the scattering processes 1) $\bar{\ell}_i \ell_j \rightarrow H_2 H_2 \sigma$, 2) $\bar{\sigma} H_2 \rightarrow \ell_i \nu_s$, and 3) $\sigma \sigma \rightarrow \bar{\sigma} \nu_s \nu_s$ as

$$
\sigma_1 = \frac{6 s}{(8\pi)^3} \frac{|m_{ij}|^2}{\langle \sigma \rangle^2 v_2^4} \approx \frac{T^2}{\langle \sigma \rangle^2 v_2^4} \times 10^{-5} \text{ eV}^2 , 
$$

$$
\sigma_2 = \frac{1}{8\pi} \frac{|m_{is}|^2}{\langle \sigma \rangle^2 v_2^2} \approx \frac{4}{\langle \sigma \rangle^2 v_2^2} \times 10^{-2} \text{ eV}^2 ,
$$

$$
\sigma_3 = \frac{6 s}{(8\pi)^3} \frac{|m_{ss}|^2}{\langle \sigma \rangle^6} \approx \frac{T^2}{\langle \sigma \rangle^6} \times 10^{-28} \text{ eV}^2 ,
$$

respectively, where we have summed over initial and final weak isospin states ($\sqrt{s}$ is the c.m. energy).

In each case one compares the rate of collisions per particle, $\Gamma = \sigma n \approx 0.1 \sigma T^3$ (the boson number density is $n_b \approx 0.122 T^3$ and the fermion number density $n_f \approx 0.091 T^3$) with the Hubble rate $H \approx T^2/10^{18}$ GeV, assuming a total number of degrees of freedom around 100. The scalar singlet $\sigma$ is produced through the processes 1) $\bar{\ell}_i \ell_j \rightarrow H_2 H_2 \sigma$, $\bar{\ell}_i \bar{H}_2 \rightarrow \ell_j H_2 \sigma$, and $H_2 \bar{H}_2 \rightarrow \ell_i \ell_j \sigma$ with cross sections $\sigma_1$, $2\sigma_1/3$, and $\sigma_1/3$, respectively (for $i \neq j$), which gives a total rate per Hubble time $\Gamma_\sigma H^{-1} \approx 2(v/v_2)^4 T^3/10^{15}$ GeV$^3$ ($v \approx 174$ GeV is the electroweak breaking scale). This shows that $\sigma$ is in thermal equilibrium at temperatures larger than $T_\sigma \approx 10^5$ GeV, if one takes $v_2 = v$.

The sterile neutrino is produced in the processes 2) $\bar{\sigma} H_2 \rightarrow \ell_i \nu_s$, $\bar{\ell}_i \sigma \rightarrow H_2 \nu_s$, and $\ell_i \bar{H}_2 \rightarrow \bar{\sigma} \nu_s$, with cross sections $\sigma_2$, $2\sigma_2/3$, and $\sigma_2/3$, respectively, and a total rate $\Gamma_\sigma H^{-1} \approx (v/v_2)^2 T/T_s$, which makes the decoupling temperature of the light sterile neutrino $T_s \approx 4 \times 10^6$ GeV. If one or more heavy neutrinos $N_i$ have masses under that value, $\nu_s$ may decouple when some of the $N_i$ degrees of freedom are still present in the Universe (note that $M_i \gtrsim 10^6$ GeV). Finally, processes like $\sigma \sigma \rightarrow \bar{\sigma} \nu_s \nu_s$ are too weak to be relevant.

Above $T_s$ the sterile neutrino and scalar singlet $\sigma$ are in chemical equilibrium with the lepton and Higgs doublets and their number asymmetries are constrained by the equations of detailed balance. The precise relations between the particle asymmetries depend on which particles and processes are in thermodynamical equilibrium at a given time. To be definite we assume that by the time the sterile neutrino decouples, $B$ and $L$ are only violated by electroweak instanton processes while $B - L$ is conserved. On the other hand the right-handed electrons $e_R$ are not yet in chemical equilibrium and the quarks $u_R$, $d_R$ may or may not be in equilibrium depending on the exact values of their Yukawa couplings and temperature $T_s$. In either case the equations of detailed balance yield the particle asymmetries as functions of the $B - L$ asymmetry.
At temperatures above $T_s$ the operators of eq. (9) yield the chemical potential constraints

\[ \mu_\sigma + 2\mu_\ell + 2\mu_H = 0 , \]
\[ \mu_s - \mu_\sigma + \mu_\ell + \mu_H = 0 . \]

(10a) \hspace{1cm} (10b)

The other constraints come from standard model reactions [13,14]. To be definite we assume that $u_R$ and $d_R$ are in equilibrium at $T_s$ (the temperature at which they come into equilibrium increases with increasing Yukawa couplings and therefore with increasing number of Higgs doublets). The electroweak and QCD instantons and the Yukawa interactions imply that

\[ 3\mu_q + \mu_\ell = 0 , \]
\[ 2\mu_q - \mu_u - \mu_d = 0 , \]
\[ \mu_q - \mu_d - \mu_H = 0 , \]
\[ \mu_q - \mu_u + \mu_H = 0 , \]
\[ \mu_\ell - \mu_\tau - \mu_H = 0 , \]

(11a) \hspace{1cm} (11b) \hspace{1cm} (11c) \hspace{1cm} (11d) \hspace{1cm} (11e)

where $\mu_q$, $\mu_\ell$, and $\mu_H$ designate the flavor universal chemical potentials of the quark, lepton, and Higgs doublets, respectively, $\mu_u$, $\mu_d$ those of the right-handed quark isosinglets and $\mu_\tau$ the common chemical potential of the lepton isosinglets $\mu_R$ and $\tau_R$. Since the electron singlet $e_R$ is not in chemical equilibrium its chemical potential $\mu_e$ is an independent variable. We may assume that a baryon asymmetry originally produced in a GUT baryogenesis scenario is later communicated through electroweak instantons to the lepton sector ($T \lesssim 10^{12}$ GeV) but not to $e_R$. In that case $\mu_e$ remains zero until the $e_R$ Yukawa interactions come into equilibrium at temperatures lower than $T_s$.

A vanishing weak hypercharge implies

\[ 3(\mu_q + 2\mu_u - \mu_d - \mu_\ell) - 2\mu_\tau - \mu_e + 2n_H\mu_H = 0 . \]

(12)

Here $n_H$ is the total number of Higgs doublets; $n_H = 1$ if $H_1$ and $H_2$ are the same field and $n_H = 2$ otherwise. The above constraints and the condition $\mu_e = 0$ leave only one independent variable. It is convenient to choose this as $\mu_s$ because the $\nu_s$ abundance and number asymmetry are conserved after its decoupling. The other quantity that is conserved is $B - L$. Denoting the baryon number density as $dB/dV = \bar{B}T^3/6$ and likewise for $L$ and $B - L$, one has

\[ \bar{B} = 6\mu_q + 3\mu_u + \mu_d , \]
\[ \bar{L} = \bar{L}_\ell - 3\mu_s , \]
\[ \bar{L}_\ell = 6\mu_\ell + 2\mu_\tau + \mu_e - 4\mu_\sigma + 2n_N\mu_N , \]

(13a) \hspace{1cm} (13b) \hspace{1cm} (13c)

where $L_\ell$ stands for the lepton number of all particles except $\nu_s$ and $n_N$ is the number of relativistic Dirac heavy neutrinos at a given moment. The decay processes $N_i \to \ell_j H_2$ set the equation $\mu_N = \mu_\ell + \mu_H$. Putting everything together, one obtains

\[ B - L = \frac{1}{3} \left( 8 + 2n_N - \frac{15 + 4n_H}{9 + n_H} \right) (N_s - N_\bar{s}) , \]
\[ B - L_\ell = \frac{(9 + n_H)(17 + 2n_N) - 15 - 4n_H}{(9 + n_H)(8 + 2n_N) - 15 - 4n_H} . \]

(14) \hspace{1cm} (15)
After $\nu_s$ decoupling, $B - L$, $B - L_\delta$, and the $\nu_s$ number asymmetry $N_s - N_s$ are all conserved. Although the above relations are strictly valid only when $\nu_s$ is in thermal equilibrium, one expects that the decoupling process does not introduce very large perturbations and one may use these results as a first approximation. They give the $\nu_s$ number asymmetry and $B - L$ as functions of the primordial $B - L$ and number $n_N$ of heavy neutrinos that are relativistic when $\nu_s$ decouples.

The next transition is the decoupling of the scalar singlet $\sigma$ at a temperature $T_\sigma$ around $10^5$ GeV. This is close to the $e_R$ coupling epoch which starts at $T_{e R} \sim (v/v_1)^2 \times 10^4$ GeV. This temperature rises with increasing electron Yukawa coupling and if one assumes the existence of two Higgs doublets and $v_1 \lesssim v/3$ ($v \simeq 174$ GeV) then $e_R$ is already in equilibrium when $\sigma$ decouples. To be definite we assume so. One repeats the exercise with Eqs. (11) and (12), complemented with $\mu_\tau = \mu_e$ and Eq. (10a), to obtain all chemical potentials in terms of $\mu_\sigma$. Then Eqs. (13a) and (13c) with $n_N = 0$ yield the relation between the $\sigma$ number asymmetry and the baryon and lepton numbers. Denoting the total lepton number of the standard model particles as $L_\ell$ ($L_\ell = L_e + L_\mu + L_\tau$) one derives for $n_H = 2$,

$$ N_\sigma - N_\bar{\sigma} = \frac{12}{23} (B - L_\ell) , \quad (16) $$

$$ N_s - N_{\bar{s}} = \frac{47}{23} \frac{33}{65 + 22n_N} (B - L_\ell) , \quad (17) $$

$$ \frac{B - L}{B - L_\ell} = \frac{47}{23} \frac{164 + 22n_N}{65 + 22n_N} , \quad (18) $$

where in the last two equations we used Eqs. (14) and (15), keeping in mind that $n_N$ is the number of Dirac neutrinos $N_i$ that are relativistic when $\nu_s$ decouples. Again, one expects that the above results remain a reasonable approximation when $\sigma$ decouples. From then on, $B - L_\ell$, $B - L$, and the $\sigma$ and $\nu_s$ abundances are conserved by all effective interactions.

When the temperature drops down to the electroweak phase transition the weak isospin and hypercharge are no longer conserved, contrary to the electric charge. The quarks and charged leptons form Dirac mass eigenstates whose well defined chemical potentials are subject to a new set of constraints [14], together with the neutrinos and charged Higgs and $W$ bosons as follows:

$$ \mu_{W^+} = \mu_u - \mu_d = \mu_\nu - \mu_e = \mu_{H^+} , \quad (19) $$

$$ 3\mu_u + 3\mu_d + \mu_\nu + \mu_e = 0 . \quad (20) $$

The latter constraint is due to sphaleron processes. On the other hand, the net electric charge is zero:

$$ 3(4\mu_u - 2\mu_d - 2\mu_e) + 2(2 + n_H)\mu_{W^+} = 0 . \quad (21) $$

Definite $B$ and $L$ numbers are predicted in terms of the preexisting $B - L_\ell$ number, namely [14],

$$ B = \frac{32 + 4n_H}{98 + 13n_H} (B - L_\ell) , \quad (22) $$

$$ L_\ell = \frac{66 + 9n_H}{98 + 13n_H} (B - L_\ell) . \quad (23) $$
These relations are preserved during the phase transition if there is no intrinsic electroweak baryogenesis. After that the sphaleron processes stop being effective and the baryon number is separately conserved. This allows us to predict the scalar $\sigma$ particle and sterile neutrino asymmetries in terms of the present baryon number. From Eqs. (13) and (17), valid for two Higgs doublets ($n_H = 2$), one derives the asymmetries and lepton numbers $L_{(\sigma)} = -2(N_\sigma - \overline{N_\sigma})$ and $L_{(s)} = -3(N_s - \overline{N_s})$ carried by $\sigma$ and $\nu_s$ as

\begin{align}
L_{(\sigma)} &= -A_\sigma B = -\frac{2431}{2310} B, \\
L_{(s)} &= -\frac{4731}{2310} \frac{65 + 22n_N}{65 + 22n_N} B.
\end{align}

It is important to notice that, after spontaneous breaking of lepton number, the $B - L$ violating processes are too weak to be in equilibrium, in particular during the electroweak phase transition if it occurs after $L$ breaking. If that was not the case the baryon and lepton numbers would be washed out. As soon as the sphalerons decouple the baryon and lepton numbers start to be separately conserved. This is also true after $L$ spontaneous breaking because the $L$ violating reactions are weak. The lepton number may only be significantly violated at much lower temperatures of the order of 1 to 10 MeV when neutrino oscillations from active to sterile neutrinos become possible. Another point is that, after $L$ breaking, the lepton number $L_{(\sigma)}$ carried by the scalar singlet $\sigma$ still exists but is then associated with a coherent Majoron field. This is the subject of the next section.

V. MAJORON FIELD

The Majoron equation of motion is determined by the equation of conservation of the lepton number Nöther current. The lepton current of the scalar field $\sigma$ with lepton number $L_\sigma = -2$ is

$$J_\sigma^\mu = L_\sigma i \langle \sigma^* \nabla^\mu \sigma - \sigma \nabla^\mu \sigma^* \rangle.$$  \hspace{1cm} (26)

At the classical level the total lepton number of charged leptons, neutrinos and scalar $\sigma$ is conserved but electroweak instanton effects break $L$ explicitly. $B - L$ remains conserved and its equation of conservation reads as

$$\nabla_\mu J_\sigma^\mu + \nabla_\mu J_f^\mu = 0,$$  \hspace{1cm} (27)

where $J_f^\mu$ is the $L - B$ current of all the other particles, in our case leptons and quarks:

$$J_f^\mu = -\sum (B_f - L_f) (n_f - \overline{n_f}) v^\mu,$$  \hspace{1cm} (28)

where $n_f$ and $\overline{n_f}$ are the particle and antiparticle densities and $v^\mu = (1, \mathbf{v})$ the macroscopic velocity vector.

Before spontaneous breaking of the lepton number the $\sigma$ current is related to the $\sigma$ particle asymmetry, $J_\sigma^\mu = L_\sigma (n_\sigma - \overline{n_\sigma}) v^\mu$, but after lepton symmetry breaking, the mass
eigenstates are no longer the complex field $\sigma$ but rather the massive Higgs particle $\rho$ and massless Majoron boson $\varphi$. They relate to each other as

$$\sigma = \frac{1}{\sqrt{2}}(v_\sigma + \rho) \exp(-i \varphi/v_\sigma)$$

(29)

and the lepton current is expressed as

$$J^\mu_\sigma = L_\sigma v_\sigma \left(1 + \frac{\rho}{v_\sigma} \right)^2 \nabla^\mu \varphi.$$ 

(30)

As emphasized in references [13,17], after symmetry breaking the global symmetry is realized as an invariance under translations of the Majoron field whose equation of motion is determined by the still valid Eq. (27) of lepton number conservation.

After lepton breaking the current $J^\mu_\sigma$ can be realized only through a coherent Majoron field $\varphi$; in other words, the expectation value of $\sigma$ has a variable phase:

$$\langle \sigma \rangle = \frac{v_\sigma}{\sqrt{2}} \exp(-i \varphi/v_\sigma).$$

(31)

We obtain for the current

$$J^\mu_\sigma = L_\sigma v_\sigma \left(1 + \frac{\langle \rho^2 \rangle}{v_\sigma^2}\right) \nabla^\mu \varphi,$$

(32)

where the term $\langle \rho^2 \rangle$ is an average over quantum fluctuations. i.e., the thermal bath of massive Higgs particles $\rho$. When the temperature of the $\rho$ bosons, lower than the photon temperature when the number of relativistic degrees of freedom drops down one order of magnitude, is much smaller than the breaking scale $v_\sigma$, the $\langle \rho^2 \rangle$ term can be neglected. Then, $J^\mu_\sigma = L_\sigma v_\sigma \nabla^\mu \varphi$. In a homogeneous and isotropic Universe $\varphi$ depends only on time and

$$J^0_\sigma = L_\sigma v_\sigma \dot{\varphi} = F_\varphi \dot{\varphi}.$$ 

(33)

The value of $\dot{\varphi}$ is subject to the equation of conservation (27). Integrating over space, the $\sigma$ lepton charge $L_\sigma(t) = \int dV J^0_\sigma$ is determined at a given time by its initial value and the variation of the $B - L$ number carried by leptons and quarks:

$$L_\sigma(t) = L_\sigma(t_i) + (B - L)_f(t) - (B - L)_f(t_i).$$

(34)

The initial value of $L_\sigma(t)$ is the lepton charge carried by the complex bosons $\sigma$ before spontaneous lepton breaking. This value is proportional to the initial $B - L$ or to the present baryon number, as Eq. (24) shows for the particular model we worked out. On the other hand, $B - L$ is possibly violated by neutrino oscillations only at very low temperatures when $B$ is conserved. As a result, $L_\sigma(t) = -A_\sigma B - \Delta L_f$ and the Majoron time derivative is obtained from Eq. (33) as

$$\dot{\varphi} = \frac{n_\gamma}{F_\varphi} (A_\sigma \dot{B} + \Delta \dot{L})$$

(35)

in terms of the baryon number and lepton number variation per photon

$$\dot{B} = \frac{B}{N_\gamma}, \quad \Delta \dot{L} = \frac{\Delta L_f}{N_\gamma}.$$ 

(36)

Notice that $L_f$ and $\Delta \dot{L}$ count only the fermion particles, charged leptons and neutrinos, but not the $\sigma$ field.
VI. NEUTRINO OSCILLATIONS

Neutrino oscillations \cite{27} are governed by the neutrino masses and mixing angles and interactions with the background medium \cite{28}, which in turn depend on the temperature and particle number asymmetries. As far as standard model weak interactions are concerned the electron, proton, and neutron asymmetries, closely related to the baryon asymmetry, are too small to play a role in the oscillation of active neutrinos into sterile neutrinos. However, the neutrino asymmetries can in principle be much larger than the baryon asymmetry. Normalizing them to the photon density as

\begin{equation}
\hat{L}_a = \frac{N_{\nu_a} - N_{\bar{\nu}_a}}{N_\gamma},
\end{equation}

the potential of the flavor $\nu_a = \nu_e, \nu_\mu, \nu_\tau$ induced by electroweak interactions is given at low temperatures ($T < m_\mu \ll M_W$) by \cite{29}

\begin{equation}
V_{\text{EW}} = \pm \sqrt{2} G_F n_\gamma (\hat{L}_a + \hat{L}_e + \hat{L}_\mu + \hat{L}_\tau \mp A_a T^2 M_W^{-2}),
\end{equation}

where $A_e = 55$ and $A_{\mu,\tau} = 15.3$ (the electron and nucleon asymmetries are neglected). The upper sign holds for neutrinos and the lower sign for antineutrinos. The sterile neutrino has no standard model potential by definition.

In the case of interest, $\delta m^2 \sim 3 \times 10^{-3}$ eV$^2$, the thermal contribution proportional to $n_\gamma T^2$ prevents the oscillation into a sterile neutrino at temperatures above $\sim 10$ MeV. At smaller temperatures that term becomes ineffective and the active neutrino oscillates into the sterile flavor as in vacuum, violating the bounds on the number of light degrees of freedom at BBN \cite{3}. Foot and Volkas \cite{9} pointed out that if there is an initial asymmetry $\hat{L}_a = \hat{L}_a + \hat{L}_e + \hat{L}_\mu + \hat{L}_\tau$ larger than $7 \times 10^{-5}$ and $\delta m^2 \lesssim 10^{-2}$ eV$^2$, the active neutrino $\nu_a$ cannot significantly oscillate into the sterile $\nu_s$ and the initial neutrino asymmetries are preserved until the active neutrino $\nu_a$ decouples or, in the case of $\nu_e$, until the protons and neutrons stop being in equilibrium. In these conditions the BBN bounds on the extra light degrees of freedom are satisfied. These straightforward considerations have one price, which is the assumption of an initial neutrino asymmetry five orders of magnitude larger than the baryon asymmetry. \footnote{Foot and Volkas explored the case where $\nu_\tau \rightarrow \nu_s$ oscillations with $-\delta m^2 \gtrsim 10$ eV$^2$ and $\sin^2 2\theta \lesssim 10^{-5}$ create a lepton asymmetry large enough to block $\nu_\mu \rightarrow \nu_s$ oscillations with ANP parameters. In any case, this cannot be a generic situation and requires some agreement in the parameter space – the masses and mixing of all neutrino species.}

The situation changes if there is a Majoron field. Majorons, like any Nambu-Goldstone boson have only derivative couplings. As a result, a coherent Majoron field produces neutrino potentials proportional to its gradient \cite{18}. If $\Lambda$ is the spontaneously broken lepton number, in general, any combination of partial lepton numbers, and $\Lambda_a$ the quantum number of the flavor $\nu_a$, a Majoron field $\varphi$ produces the potential

\begin{equation}
V_\Lambda = -\frac{1}{F_\Lambda} \Lambda_a \nu^\mu \partial_\mu \varphi
\end{equation}

\[\text{In ref.} \text{\cite{30}} \text{Foot and Volkas explored the case where } \nu_\tau \rightarrow \nu_s \text{ oscillations with } -\delta m^2 \gtrsim 10 \text{ eV}^2 \text{ and } \sin^2 2\theta \lesssim 10^{-5} \text{ create a lepton asymmetry large enough to block } \nu_\mu \rightarrow \nu_s \text{ oscillations with ANP parameters. In any case, this cannot be a generic situation and requires some agreement in the parameter space – the masses and mixing of all neutrino species.} \]
for the neutrino $\nu_a$ and the symmetric one for the antineutrino $\bar{\nu}_a$, where $v^\mu = (1, v)$ is the neutrino four-velocity ($|v| = 1$ in leading order). In the present case the Majoron is associated with the total lepton number $L$ and the Majoron field is a uniform field in the Early Universe given by Eq. (35). Hence, it induces the potentials

$$V_L = F_L^{-2} n_\gamma L_a (A_\sigma \hat{B} + \Delta \hat{L}) ,$$

where $A_\sigma$ is a model dependent coefficient of order 1; $A_\sigma \approx 3.2$ in the case we are considering. The quantum numbers are $L_a = +1 \ (-1)$ for an active neutrino (antineutrino) and $L_a = -3 \ (+3)$ for the sterile neutrino $\nu_s \ (\bar{\nu}_s)$.

The variation of the lepton number in the neutrino sector can be caused only by oscillations into the sterile neutrino because this is the only one with lepton number different from $\pm 1$ and the types of oscillations we are considering conserve chirality. The oscillations $\nu_a \leftrightarrow \nu_s \ (\bar{\nu}_a \leftrightarrow \bar{\nu}_s)$ produce a lepton number variation $\Delta L = \Delta N_a - 3 \Delta N_s = -4 \Delta N_s \ (\Delta L = 4 \Delta N_s)$; hence,

$$\Delta \hat{L} = -4 \frac{\Delta N_s - \Delta N_s}{N_\gamma} .$$

Combining Eqs. (38) and (40), the difference between active and sterile neutrino potentials is

$$V_a - V_s = \sqrt{2} G_F n_\gamma (\pm \hat{L}^a - A_\sigma T^2 M_W^{-2}) \pm 4 F_L^{-2} n_\gamma (A_\sigma \hat{B} + \Delta \hat{L}) ,$$

where $\hat{L}^a = \hat{L}_a + \hat{L}_e + \hat{L}_\mu + \hat{L}_\tau$ and the lower signs apply to antineutrinos. It is now clear that if, in the case of standard weak interactions, an asymmetry $\hat{L}^a > 7 \times 10^{-5}$ is enough to block the $\nu_a \leftrightarrow \nu_s$, $\bar{\nu}_a \leftrightarrow \bar{\nu}_s$ oscillations then, in the presence of a Majoron field, the known baryon asymmetry $\hat{B} = (4 - 7) \times 10^{-10}$ can do the same if the scale of lepton number breaking $F_L$ obeys the condition

$$F_L^2 < \frac{4 A_\sigma \hat{B}}{7 \sqrt{2} G_F} \times 10^5 = 5 - 9 \text{ GeV}^2 .$$

Such a low scale of lepton number breaking is perfectly consistent with the existing bounds for this kind of singlet Majoron model [16,32,18] including the astrophysical bounds [33]. The reason is that in scattering processes the Majorons couple primarily to neutrinos with strengths proportional to the neutrino masses $g \sim m_\nu / F_L$, which are therefore negligibly small for the assumed neutrino mass spectrum even if $F_L \sim 1 \text{ GeV}$.

**VII. CONCLUSIONS**

It is well known that an explanation of the atmospheric neutrino problem [3] in terms of oscillations of the muon neutrino into a, at least in part, sterile neutrino is in contradiction

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3 They should not be confused with the potentials induced by a thermal bath of Majoron particles, proportional to the neutrino masses, which could be significant for rather large neutrino masses like 17 keV as considered in ref. [31].
with the BBN limits on the number of extra light degrees of freedom [6]. Indeed, for such a mass gap, \( \delta m^2 \sim 3 \times 10^{-3} \text{ eV}^2 \), in view of the large mixing angle, the sterile neutrino would come into equilibrium with the particle thermal bath via \( \nu_\mu - \nu_s \) oscillations [7].

In this paper we formulate a mechanism capable of blocking the active-sterile neutrino oscillations that operates in the framework of theories where the lepton number is spontaneously broken. It has been shown [18] that a generic feature of these theories is the production of coherent, long-range Majoron fields. While they can appear in stars as a result of neutrino oscillations or any other lepton number violating large scale process, we found that in the Early Universe a cosmological Majoron field emerges also as a result of a primordial lepton asymmetry carried by the scalar particles and complex scalar field (\( \sigma \)) whose expectation value spontaneously breaks the lepton number. The Majoron field amplitude is thus naturally proportional to the lepton number of the Universe, and baryon number as well, due to the \( L \) and \( B \) violating sphaleron processes.

The leptons, and neutrinos in particular, interact with the derivatives of the Majoron field, which gives rise to new neutrino potentials that are relevant for the oscillation phenomena. The potentials are inversely proportional to the second power of the scale of lepton number symmetry breaking (\( \langle \sigma \rangle \)) but this scale can be much smaller than the electroweak breaking scale. For a lepton number breaking scale of the order of 1 GeV, a Majoron field associated with a lepton number asymmetry of the same order of magnitude as the baryon asymmetry can block the active-sterile neutrino oscillations at temperatures in the MeV range for a neutrino mass gap \( \delta m^2 \sim 3 \times 10^{-3} \text{ eV}^2 \).

This provides an interesting way to block the sterile neutrino oscillations as it does not require the extraordinarily high lepton asymmetries, \( \sim 10^{-5} \), that are necessary [9] in the framework of the standard model weak interactions. In fact, the standard model neutrino potentials that are proportional to the background lepton asymmetries are suppressed by the Fermi constant. In contrast, the potentials due to a Majoron field vary as the inverse square lepton number symmetry breaking scale. For a 1 GeV energy scale one immediately obtains the five orders of magnitude increase factor that brings \( 10^{-5} \) down to the baryon number asymmetry \( B \sim 10^{-10} \).

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