Can stellar wobble in triple systems mimic a planet?

J. Schneider* and J. Cabrera

Observatoire de Paris-Meudon, 92195, Meudon Cedex, France

Received; accepted

Abstract. The first extrasolar planets have been detected by the measurement of the wobble of the parent star. This wobble leads to the periodic modulation of three observables: the radial velocity, the position on the sky and the time of arrival of periodic signals. We show that the same wobble, and therefore the same modulation of the three observables, can be due to the presence of a more distant binary stellar companion. Thus, the observation of the wobble does not, by itself, constitute a proof of a planet detection. In particular, astrometric confirmation of a wobble does not necessarily provide a sufficient proof of the existence of a planet candidate detected by radial velocity. Additional conditions, which we discuss here, must be fulfilled. We investigate the observed wobble for the planet candidates already detected and we find that, for each case, a wobble due to a binary stellar companion can be excluded.

But for apparent Saturn-like planets in wide orbits, there may be an ambiguity in future detections, especially in spaceborne astrometric missions. We conclude that, in some cases, a definitive proof for the presence of a planet requires further observations such as direct imaging.

Key words. stars: planetary systems – astrometry – celestial mechanics

1. Introduction

The detection of the first extrasolar planets rests on an indirect method, namely the measurement of the reflex motion of the parent star. In cases where only the wobble is detected, one can ask whether the detection of radial velocity (RV) variations are indeed due to a stellar wobble and not to other effects (such as stellar rotation or variable stellar activity) are indeed due to a planet and not to other dynamical effects that the companion is indeed a planet (and not for instance a planetary mass black hole or strange matter object). In a few cases, the planet detection is confirmed by (or was preceded by) the detection of a transit of a planet, but the question of the planetary explanation of the wobble remains a priori open for the other candidates.

Here we consider the case where the wobble is real but due to the perturbation by a distant binary star first suggested by Schneider (1999).

2. An approximation: restricted 3 body problem for hierachical systems

Consider a triple hierarchical system consisting of a binary system (with masses $M_1 = M_2 = M$; separation between components $2a_B$) and a third companion orbiting the center of mass of the system in a bigger circular orbit (mass $M_3$; radius of the orbit $a_A$). The perturbation caused by the binary system on the orbit of the third companion can imitate the perturbation caused by a planet around the latter star. In the appendix we have derived the equations of motion of such an approximation. The result of the perturbation of the binary system is an elliptical periodic motion superimposed to the bigger orbit of the third star. Studying this perturbation in the plane of the orbit of the third star, let the X axis lay from the center of mass of the binary system to the target star, and the Y

Fig. 1. Orbital elements of the system (not to scale)
axis perpendicular to it (see fig.1), the magnitude of this perturbation (see appendix) in both axis is:

\[
\delta_x = 4.5 \frac{a_B}{a_A} \quad \delta_y = 3 \frac{a_B}{a_A}
\] (1)

Our approximation takes as starting points the sizes of the orbits of the binary system \((a_B)\) and the third star \((a_A)\) together with the mass of the stars in the binary system \((M)\) and gives the amplitude of the perturbation \((\delta_x; \delta_y)\) which the binary system causes in the motion of the third star.

Stellar wobble and planetary companion produce, in principle, the same perturbation: a periodical elliptical motion which can be measured either by radial velocity or astrometry.

We have made simulations of such triple systems with the code KAPPA. Taking as initial parameters \(a_B, a_A\) and \(M\) (see figure 1) we can calculate the initial position and velocities for the three bodies which are required by the code (using A.1, A.3 and A.4). Finally, we compare the results obtained in the simulations with those expected according to A.6.

As an example, we are going to choose \(a_B = 1\) AU; \(a_A = 90\) AU; \(M = 1\, M_\odot\). In fig.2 we see the radius of the orbit of the third star around the center of mass of the system. In a period of three years, it oscillates three times (half the period of the binary system, as expected). The radius fits very well to A.6.

![Figure 2](image)

**Fig. 2.** Variation of the distance from the third star to the center of mass of the binary system.

3. Imitating a planet

The perturbation of the binary system can induce a motion in the target star imitating a planet. In other words, we see a star with a periodic elliptical wobble. We suppose that this wobble is caused by the orbit of the star around the center of mass of the system star-planet. However, this wobble can be in fact caused by a far binary system to which our target star is gravitationally linked. In our model, this motion will have the amplitude given by and the following period:

\[
P_* = P_{\text{pl}} = \frac{1}{2} P_{\text{binary}} = \frac{2\pi a_B^{3/2}}{\sqrt{GM}}
\] (2)

where \(M\) is the mass of each star of the binary system, \(a_B\) half the semi-major axis of the orbit of the binary system, as described previously. The subscripts \(*\) and \(\text{pl}\) refer to the target star and the supposed planet respectively. Other parameters are:

\[
a_{\text{pl}} = \left( \frac{GM_*}{4\pi^2 a_*^3} \right)^{1/3}
\] (3)

\[
e = \sqrt{1 - \left( \frac{\delta_y}{\delta_x} \right)^2}
\] (4)

\[
M_{\text{pl}} = M_* \frac{\delta_x}{a_{\text{pl}}}
\] (5)

From these expressions, one can derive the mass \(M\) required for each star of the hidden binary system to lead to the observed values for a pseudo-planet:

\[
M = (4.5)^{-3/5} M_{\text{pl}}^{3/5} M_*^{2/5} \left( \frac{a_A}{a_{\text{pl}}} \right)^{12/5}
\] (6)

4. Application to exoplanets detected by radial velocity

One may wonder if the low amplitude wobble detected more than 150 stars (for a permanent update, see [http://www.obspm.fr/planets](http://www.obspm.fr/planets)) is due to a planet or to a more distant binary system. From the point of view of radial velocity measurements, a star is considered as single if there is no long term drift in its velocity curve.

In our model, we take a star supposed to be single with a planet companion but in fact it is a star in a triple system. The absence of velocity drift imposes a minimum value for the distance of an hypothetical companion. \(\gamma = GM/a_A^2\) being the acceleration of the target star due to a companion at a distance \(a_A\), the velocity drift acquired over a time span \(\Delta T\) is \(\Delta V = \gamma \Delta T = GM/a_A^2 \Delta T\). The star is single if \(\Delta V\) is smaller than the observational limit. Taking from the last years of radial velocity surveys \(\Delta V < 10\) m/s and \(\Delta T = 5\) yrs, on gets, for \(M = 1\, M_\odot\), \(a_A > 300\) pc.

From equations A.7, 2, 3 and 4 we obtain a relation between the mass of the supposed planet and the period of the wobble (with \(M = M_* = 1\) solar mass):

\[
M_{\text{planet}} (M_\odot) = \left( \frac{35\, AU}{a_A} \right)^4 \left( \frac{P_*}{1\, \text{year}} \right)^{8/3}
\] (7)

In fig.3 we represent this relation for \(a_A = 300\) AU and \(a_A = 50\) AU together with the data for most of the known extrasolar planets known nowadays. Clearly, none of the planets found risk of being a triple system, because the effect that a binary system causes is small.
two spatial missions: GAIA (Sozzetti et al. 2003) and SIM detection.

a distant binary star can merely mimic low mass planets serious with future more accurate measurements of stellar wobble. One may wonder if in the future this problem can become discouraged the existence of the triple system and accept observational results do not point in that direction; we cannot be explained as due to the binary nature of the companion star. In γ Ceph the value of 2 solar masses is not aberrant, however, the mass found for the companion is 0.4 solar masses (Dvorak et al. 2003). Our model is consistent with a binary system of two solar masses each orbiting at 2.4 AU. That system has not been discovered, observational results do not point in that direction; we discourage the existence of the triple system and accept the presence of a planet.

4.1. Planets in binary star systems

There are presently around 15 planets detected in binary star systems (Eggenberger et al. 2004). One may wonder to what extent the companion to the target star hosting a planet is in fact a binary system inducing a stellar wobble imitating the effect of a planet.

Since the separation $a_A$, the orbital parameters of the planet and the mass of the star (given its spectral type) are then known; the mass of the hypothetical binary is given by equation 6. In other words, let’s suppose that there is no planet and that in fact there is a binary system perturbing the motion of the target star. We can calculate the mass of this system to see if this hypothesis is reliable. From the data of Eggenberger et al. 2004 table 1 shows in each case the value for $M$ derived from equation 6.

The values found for $M$ are aberrant (except in the case of γ Ceph); thus the corresponding stellar wobble cannot be explained as due to the binary nature of the companion star. In γ Ceph the value of 2 solar masses is not aberrant, however, the mass found for the companion is 0.4 solar masses (Dvorak et al. 2003). Our model is consistent with a binary system of two solar masses each orbiting at 2.4 AU. That system has not been discovered, observational results do not point in that direction; we discourage the existence of the triple system and accept the presence of a planet.

Table 1. Experimental data for planets in binary systems. Last column is the mass (calculated with 6 for each star of an hypothetical binary system which would cause in the target star same wobble as the planet (see text). Data taken from Eggenberger et al. 2004 and from the Extrasolar Planets Encyclopaedia (http://www.obspm.fr/planets).

| Name     | $a_A$(AU) | $a_{pl}$(AU) | $M_{pl}$ ($M_J$) | $M$ ($M_\odot$) |
|----------|-----------|--------------|------------------|-----------------|
| HD 49979 | 6400      | 0.811        | 3.32             | $3.0 \times 10^4$ |
| GL 777 A | 3000      | 4.8          | 1.33             | $3.6 \times 10^4$ |
| HD 80606 | 1200      | 0.469        | 3.90             | $2.0 \times 10^6$ |
| 55 Cnc   | 1065      | 0.115        | 0.84             | $1.8 \times 10^7$ |
|          |           | 0.24         | 0.21             | $1.4 \times 10^6$ |
|          |           | 5.9          | 4.05             | $3.7 \times 10^4$ |
| 16 Cyg B | 850       | 1.6          | 1.5              | $2.8 \times 10^4$ |
| $v$ And  | 750       | 0.83         | 2.11             | $1.3 \times 10^5$ |
|          |           | 2.50         | 4.61             | $1.5 \times 10^4$ |
| HD 178911 B | 6400 | 0.32        | 6.292            | $1.6 \times 10^6$ |
| $\tau$ Boo | 240    | 0.05         | 4.08             | $1.1 \times 10^7$ |
| HD 195019 | 150      | 0.14         | 3.51             | $2.5 \times 10^5$ |
| HD 114762 | 130      | 0.35         | 11.03            | $3.9 \times 10^4$ |
| HD 19994 | 100       | 1.33         | 1.78             | $2.1 \times 10^2$ |
| $\gamma$ Ceph | 22 | 2.03         | 1.59             | $1.9 \times 10^2$ |
| Gl 86    | 20        | 0.11         | 4.0              | $3.5 \times 10^4$ |

$^a$ semimajor axis of the orbit of the binary stellar system in AU.

$^b$ semimajor axis of the orbit of the planet in AU.

$^c$ mass of the planet in Jupiter masses.

$^d$ mass calculated with 6 for each star of the hypothetical binary system which would imitate the wobble of a planet.

5. Application to future astrometric searches

One may wonder if in the future this problem can become serious with future more accurate measurements of stellar wobbles. Clearly, from fig. 8 the wobble induced by a distant binary star can merely mimic low mass planets on wide orbits. This configuration escapes planet detection by radial velocity but is well adapted to astrometric detection.

In fig.9 we represent the expected discovery space for two spatial missions: GAIA (Sozzetti et al. 2003) and SIM missions, together with the expected perturbation caused by binary systems at 50 and 300 AU.

In fig.10 we compare the expected results for SIM and PRIMA (Phase-Referenced Imaging and Microarcsecond
Astrometry at ESO VLTI). PRIMA has the same resolution as GAIA (10 micro arcseconds) but is not constrained by the three years lifetime, having access to a low-mass long-period region where this stellar wobble effect will be more important.

![Diagram](image)

**Fig. 5.** Discovery space for SIM, PRIMA and radial velocity missions, together with the expected perturbation caused by binary systems at 50 and 300 AU.

The possibility of stellar wobble simulating a planet is small, however, for long periods and if the distance from the target star to the binary system ($a_A$) is not that important (e.g. 50 AU), those effects will have to be taken into account.

### 6. Conclusion

By itself a stellar wobble is not a proof that a planet is detected. It is necessary to verify that no far binary star generates the wobble or to confirm that planet by transit or direct imaging observations. For the presently know planets, the explanation by a perturbing binary star can nevertheless be ruled out. But the sensitivity of GAIA, PRIMA and SIM is such that for some regions of the ($M_{pl}$, $P_{pl}$) plane there can be an ambiguity between a true planet detected by astrometry and a wobble induced by a binary star.

**Acknowledgements.** We are grateful to R. Dvorak for the permission of the use of his 3-body numerical simulation software KAPPA.

This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France.
Appendix A:

The perturbation of the trajectory of a body in a triple system by a distant binary system is a classical issue in the 3-body problem (Roy 1979). Here we remind the analysis only for self-consistency of our paper.

Consider a simplified triple hierarchical system consisting of a binary system of two equal mass bodies \((M_1 = M_2 = M)\) in a circular orbit with radius \(a_B\) plus a third companion orbiting the center of mass of the binary system in a bigger circular orbit with radius \(a_A \gg a_B\).

Suppose that the motion of the binary system is not perturbed by the third body (let \(M_3 = 0\)) and let them move in the plane \(z = 0\). As we want a circular orbit with a given angular velocity \(\omega\), we find that the equation of motion of the bodies \(M_1\) and \(M_2\) is:

\[
\begin{align*}
\vec{r}_{M_1} &= a_B \left( \cos \omega t \hat{i} + \sin \omega t \hat{j} \right) \\
\vec{r}_{M_2} &= -a_B \left( \cos \omega t \hat{i} + \sin \omega t \hat{j} \right)
\end{align*}
\]

\[\omega = \sqrt{\frac{2GM}{(2a_B)^3}} = \sqrt{\frac{GM}{4a_B^3}}\]  

The gravitational force per unit of mass in any point \(\vec{r} = (x, y, z)\) caused by these two bodies of mass \(M\) is:

\[
\vec{F} = -GM \left( \frac{\vec{r}_1}{|\vec{r}_1|^3} + \frac{\vec{r}_2}{|\vec{r}_2|^3} \right) \quad \text{where} \quad \vec{r}_i = \vec{r} - \vec{r}_{M_i}, \quad i = 1, 2
\]

(A.1)

To make the problem even simpler, we are going to suppose that the motion of the third star is in the plane defined by the orbit of the binary system. In the plane we keep only the cartesian coordinates \((x, y)\) or their polar equivalent \((r, \theta)\) with the identities for the unitary vectors:

\[
\begin{align*}
\dot{r} &= \cos \theta \dot{i} + \sin \theta \dot{j} \\
\dot{\theta} &= -\sin \theta \dot{i} + \cos \theta \dot{j}
\end{align*}
\]

then we can separate (A.2) into the central force (in the direction of the center of mass of the binary system, parallel to \(\dot{r}\)) and the angular force (perpendicular to the former, parallel to \(\dot{\theta}\)):

\[
\vec{r}_{1/2} = \vec{r} - \vec{r}_{M_{1/2}} = \left[ r \cos \theta \dot{i} + r \sin \theta \dot{j} \right] - \left[ \pm a_B \left( \cos \omega t \dot{i} + \sin \omega t \dot{j} \right) \right] = \\
= \left[ r \mp a_B \cos (\omega t - \theta) \right] \dot{r} \mp a_B \sin (\omega t - \theta) \dot{\theta}
\]

\[
|\vec{r}_{1/2}| = r^2 + a_B^2 \mp 2ra_B \cos (\omega t - \theta)
\]

\[
\frac{1}{r_{1/2}^3} \approx \frac{1}{r^3} \left[ 1 \mp 3 \epsilon \cos (\omega t - \theta) + \frac{15}{2} \epsilon^2 \cos^2 (\omega t - \theta) - \frac{3}{2} \epsilon^2 \right]
\]

where \(\epsilon \equiv \frac{a_B}{r} \ll 1\)

(A.2)

\[
\vec{F} = \vec{\Phi} = -GM \left( \frac{\vec{r}_1}{|\vec{r}_1|^3} + \frac{\vec{r}_2}{|\vec{r}_2|^3} \right) \approx \\
\approx -2GM \left( \left[ 1 + \frac{3}{4} \epsilon^2 + \frac{9}{4} \epsilon^2 \cos^2 (\omega t - \theta) \right] \dot{r} - \frac{3}{2} \epsilon^2 \sin 2(\omega t - \theta) \dot{\theta} \right)
\]

(A.3)

correct to order 2 in \(\epsilon\) and which is the same expression we can find in equation 14.25 of (Roy 1979) (under our assumption that the masses of the binary system are equal and the third mass is zero). As \(\vec{\Phi} = \left( \dot{r} - r \dot{\theta}^2 \right) \dot{r} + \left( 2 \dot{\theta} \dot{r} + r \dot{\theta} \right) \dot{\theta} \); we finally arrive to:

\[
\dot{r} - r \dot{\theta}^2 \approx -2GM \frac{r^2}{r^2} \left[ 1 + \frac{3}{4} \epsilon^2 + \frac{9}{4} \epsilon^2 \cos^2 (\omega t - \theta) \right]
\]

\[
2\dot{\theta} + r \dot{\theta} \approx \frac{3GM}{r^2} \epsilon^2 \sin 2(\omega t - \theta)
\]

(A.4)

As a solution to the system of differential equations (A.4), we propose that the third body is making a circular orbit of radius \(a_A\) around the center of mass of the binary system with angular velocity \(\Omega\) perturbed by another elliptical
motion of semiaxes $\delta_x$ and $\delta_y$ both much smaller than $a_A$. So the equation of motion of this body is, in rectangular coordinates:

\[
x(t) = a_A \cos \Omega t + \delta_x \cos 2\omega t \\
y(t) = a_A \sin \Omega t - \delta_y \sin 2\omega t
\]

(A.5)

Notice:

1. this is not the most general solution. In fact, it is closer to a perturbation solution: these equations are solution of the differential equation system (A.4).

2. the angular velocity of the perturbation is $-2\omega$. As the two bodies of the binary system have the same mass, every half a revolution of the system the third body sees the same configuration of the binary system, in other words, the same configuration of the perturbation.

3. the Keplerian angular velocity goes as $a^{-1.5}$. As $a_A \gg a_B \Rightarrow \omega \gg \Omega$.

Taking this into account we arrive to the following equations to order one in the perturbation (that is, taking into account that $\delta_x/a_A \ll 1$; $\delta_y/a_A \ll 1$; $\Omega/\omega \ll 1$; $\omega^2/\Omega^2$);

\[
r \equiv \sqrt{x^2 + y^2} \approx a_A + \delta_x \cos 2\omega t \\
\dot{r} \equiv \frac{d}{dt} \sqrt{x^2 + y^2} \approx -4\omega^2 \delta_x \cos 2\omega t \\
\theta \equiv \arctan \frac{y}{x} \approx \Omega t \\
\dot{\theta} \equiv \frac{d}{dt} \arctan \frac{y}{x} \approx 4\omega^2 \frac{\delta_x}{a_A} \sin 2\omega t
\]

(A.6)

Introducing this solution (A.6) in the system of equations (A.4) we obtain the following identities:

\[
a_A \Omega^2 + 4\omega^2 \delta_x \cos 2\omega t = 2GM \frac{a^2}{a_A^2} \left(1 + \frac{3}{4} \epsilon \epsilon^2 + \frac{9}{4} \epsilon^2 \cos 2\omega t \right) \\
4\omega^2 \delta_y \sin 2\omega t = 3GM \frac{a^2}{a_A^2} \epsilon^2 \sin 2\omega t
\]

from which we arrive to:

\[
\delta_x = 4.5 \frac{a_B^5}{a_A^4} \\
\delta_y = 3 \frac{a_B^5}{a_A^4} \\
\Omega^2 = 2GM \frac{a^2}{a_A^2} \left(1 + 0.75 \frac{a_B^2}{a_A^2} \right)
\]

(A.7)

The mass of the binary system does not affect the amplitude of the perturbation but does affect the period ($T_{\text{wobble}} \sim (2\omega)^{-1} \sim M^{-0.5}$). The angular velocity $\Omega$ is bigger than the Keplerian one, that means that for the same mass, the system is less bounded (the total energy $E = T - U$ is less negative).

References

Dvorak, R., Pilat-Lohinger, E., Funk, B., Freistetter, F., 2003. Planets in habitable zones: a study of the binary Gamma Cephei. A&A 398, L1-L4.

Eggenberger A., Udry S. & Mayor M., 2004. Statistical properties of exoplanets. III. Planet properties and stellar multiplicity. A&A 417, 353-360.

Roy, A. E., 1979, Orbital Motion. Adam Hilger Ltd, Bristol.

Schneider, J., 1999. The wobble method of extrasolar planets detection revisited. American Astronomical Society, DPS meeting 31, No. 4, # 5.02

Sozzetti A., Casertano S., Lattanzi M. & Spagna A., 2003. The GAIA astrometric survey of the solar neighborhood and its contribution to the target database for DARWIN/TPF. In Proceedings of the conference on Toward Other Earths: DARWIN/TPF and the Search for Extrasolar Terrestrial Planets. ESA SP-539, Noordwijk, Netherlands, p. 605-610.