QCD VACUUM CHANGES IN NUCLEI *

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In this talk, I discuss how the changes in the QCD vacuum induced by increasing nuclear matter density affect nuclear properties under normal as well as extreme conditions. The quark condensate which is the order parameter for the mode in which chiral symmetry is manifested is expected to change as matter density or temperature is changed. The topics discussed are “BR scaling,” its connection to the structure of nuclei in Landau’s Fermi liquid theory and a variety of consequences on such nuclear properties as effective nucleon mass, nuclear gyromagnetic ratios, $g_A$, axial-charge transitions in nuclei and on the fluctuations of nuclear matter into the strange-flavor direction as observed in heavy-ion collisions. I will also present a simple explanation of the recent dilepton data in the CERN-CERES heavy-ion experiments involving densities greater than that of nuclear matter and of the Indiana results on the longitudinally polarized proton scattering from heavy nuclei, evidencing dropping vector-meson masses in medium which can be interpreted as signaling an aspect of chiral symmetry restoration in dense medium. The test of BR scaling provides a bridge between the physics of extreme conditions (e.g. relativistic heavy-ion collisions) and the physics of normal conditions which has conventional descriptions, thereby setting the stage for formulating many-body theory for nuclear matter starting from an effective chiral Lagrangian.

I. INTRODUCTION

In the presently accepted theory of strong interactions, the observed masses of low-energy particles such as the vector mesons $\rho$ and $\omega$, the baryons $N$, $\Delta$, · · · that make up the ingredient for nuclei and nuclear matter are understood to originate mainly from the spontaneous breaking of chiral $SU(N_f) \times SU(N_f)$ (where $N_f$ is the number of light-quark flavors) to its diagonal subgroup $SU(N_f)$ characterized by the condensate

$$\langle 0 | \bar{q} q | 0 \rangle$$

where $|0\rangle$ denotes the zero baryon density vacuum. Suppose one has a chunk of nuclear matter at a density $\rho_0$ and temperature $T$. In such an extended system of nuclear matter there is a good reason to believe as recent calculations (to be described below) indicate that the quark condensate will be modified such that its magnitude decreases as density and/or temperature is increased. We expect in general that

$$|\langle 0^* | \bar{q} q | 0^* \rangle| < |\langle 0 | \bar{q} q | 0 \rangle|$$

where $0^*$ stands for the “vacuum” in the presence of the medium. Now it is expected (although there is no rigorous proof) this condensate will vanish at certain critical density $\rho_c$ or temperature $T_c$.

An immediate and exciting question is then how the masses of the hadrons living in medium get affected by this change of the condensate and how the vacuum change affects – and manifests itself in – nuclear properties measured in the laboratory as density $\rho$ and/or temperature $T$ is increased. Can ordinary nuclear phenomena be expressed in terms of the vacuum change manifested in the condensate change? How do the properties of hadrons change as the quark condensate dissolves at the phase transition? Can many-body dynamics of nuclei be simplified if expressed in terms of the QCD vacuum variable? Indeed it has been argued recently that hadron masses of the light-quark constituent will decrease gradually – and not abruptly – to nearly zero at the critical point [1]. These and related questions will occupy much of the future experimental activities at CERN, RHIC and CEBAF.

In what follows, I would like to discuss a recent theoretical work done in answering the questions posed above. It will be based on “BR scaling” proposed by Gerry Brown and myself in [1].

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II. BR SCALING

In [1], it was argued that nuclear properties can be simply and concisely described in tree order with an effective chiral Lagrangian with the parameters of the Lagrangian given by the scaling

\[ \frac{m^*_M}{m_M} \approx \sqrt{\frac{g_A}{g_A^*}} \frac{m^*_B}{m_B} \approx \frac{f^*_\pi}{f_\pi} \equiv \Phi(\rho) \]  

(3)

where the subscripts \( M \) and \( B \) stand for chiral-quark mesons and baryons, respectively, and the star denotes quantities in medium. The BR scaling relation (3) that relates the dropping of light-quark non-Goldstone-boson masses to that of the nucleon mass which in turn is related to that of the pion decay constant was first derived by incorporating the trace anomaly of QCD into an effective chiral Lagrangian. The basic idea is that there are two components in the scalar field that interpolates the dimension-four operator \( \text{Tr} \, G_{\mu\nu}G^{\mu\nu} \), one the higher energy gluonium component \( G \) and the other lower-energy quarkonium component \( S \) and that the higher component is integrated out from the action. The resulting effective chiral action can then be written with the \( S \) field coupled in to the Goldstone boson fields and light-quark matter fields. We assume that at a given background with a density \( \rho \) (whether in equilibrium or not), we can rewrite the effective Lagrangian shifted around the background defined by the “vacuum” expectation value \( \langle S \rangle_\rho \) in the way both chiral invariance and scale invariance are preserved, with a suitable term that breaks the scale invariance as dictated by the trace anomaly of QCD. Symmetries demand that we have a low-energy effective chiral Lagrangian of the matter-free space with the parameters of the Lagrangian defined as in eq.(2). The argument used here is quite heuristic but one can obtain the same results starting with a chiral Lagrangian consisting of Goldstone bosons, baryons (as well as vector bosons) as matter fields and multi-Fermi terms appropriate for many-nucleon systems [3–5] with the four-Fermi interactions accounting for both the vector and scalar interactions needed in Walecka theory [7,6]. Multi-Fermi interactions in the scalar channel give rise to BR scaling.

III. BR CONJECTURE AND LANDAU FERMI-LIQUID THEORY

A contact of the present theory with many-body theory of nuclear matter can be made through the reinterpretation of the BR scaling in terms of a baryon chiral Lagrangian in the relativistic baryon formalism. Consider the Lagrangian containing the usual pionic piece \( \mathcal{L}_\pi \), the pion-baryon interaction \( \mathcal{L}_{N\pi} \) and the four-Fermi contact interactions

\[ \mathcal{L}_4 = \sum_\alpha \frac{C^2_\alpha}{2} (\bar{N} \Gamma_\alpha N)(\bar{N} \Gamma^\alpha N) \]  

(4)

where the \( \Gamma^\alpha \)'s are Lorentz covariant quantities – including derivatives – that have the correct chiral properties. The leading chiral order four-Fermi contact interactions relevant for the scaling masses are of the form

\[ \mathcal{L}_{4}^{(4)} = \frac{C^2_\sigma}{2} (\bar{N} N \bar{N} N) - \frac{C^2_\omega}{2} (\bar{N} \gamma_\mu N \bar{N} \gamma^\mu N). \]  

(5)

As indicated by our choice of notation, the first term can be thought of as arising when a massive isoscalar scalar meson (say, \( \sigma \)) is integrated out and similarly for the second term involving a massive isoscalar vector meson (say, \( \omega \)). Consequently, we can make the identification

\[ C^2_\sigma = \frac{g^2_\sigma}{m^2_\sigma}, \quad C^2_\omega = \frac{g^2_\omega}{m^2_\omega}. \]  

(6)

The four-Fermi interaction involving the \( \rho \) meson quantum number will be introduced below, when we consider the electromagnetic currents. As is well known [3], the first four-Fermi interaction in (5) shifts the nucleon mass in matter,

\[ m^*_N = m_N - C^2_\sigma (\bar{N} N). \]  

(7)

In [3] it was shown that this shifted nucleon mass mass scales the same way as the vector and scalar mesons

\[ \frac{m^*_V}{m_V} \approx \frac{m^*_\sigma}{m_\sigma} \approx \frac{m^*_N}{m_N} \approx \Phi(\rho). \]  

(8)
This relation was referred to in [1] as “universal scaling.” There are two points to note here: First as argued in [3], the vector-meson mass scaling applies also to the masses in (6). Thus, in medium the meson mass should be replaced by $m_{\star}^{\sigma,\omega}$. Consequently, the coupling strengths $C_{\sigma}$ and $C_{\omega}$ are density-dependent. Second, the scaling can be understood in terms of effects due to the four-Fermi interactions, which for nucleons on the Fermi surface correspond to the fixed-point interactions of Landau Fermi liquid theory according to Shankar and Polchinski [8]. We shall establish a direct connection to the Landau parameters of the quasiparticle interaction.

A. Landau’s Effective Mass of the Nucleon

The first quantity we will establish is the relation between the nucleon effective mass $m_{\star}^{\sigma}$ and the scaling parameter $\Phi$. In the Landau-Migdal Fermi liquid theory of nuclear matter [9,10], the interaction between two quasiparticles on the Fermi surface is of the form

$$F(p, p') = F(\cos \theta) + F'(\cos \theta)(\vec{\sigma} \cdot \vec{\sigma}') + G(\cos \theta)(\vec{\tau} \cdot \vec{\tau}') + G'(\cos \theta)(\vec{\sigma} \cdot \vec{\sigma}')$$

$$+ G(\cos \theta)(\vec{\tau} \cdot \vec{\tau}') + G(\cos \theta)(\vec{\sigma} \cdot \vec{\sigma}')$$

where $\theta$ is the angle between $\vec{p}$ and $\vec{p}'$. The function $F(\cos \theta)$ can be expanded in Legendre polynomials,

$$F(\cos \theta) = \sum F_l P_l(\cos \theta),$$

with analogous expansions for the spin- and isospin-dependent interactions. The coefficients $F_l$ etc. are the Landau Fermi liquid parameters. Some of the parameters can be related to physical properties of the system. The relation between the effective mass and the Landau parameter $F_1$ (eq. (14)) is crucial for our discussion. In what follows, we will be concerned with the spin-independent Landau parameters, although the $g_{\star}^{\lambda}$ problem could be described in terms of the spin-isospin-dependent parameter $G'$. An important point of this paper is that one must distinguish between the effective mass $m_{\star}^{\sigma}$, which is of the same form as Walecka’s effective mass, and the Landau effective mass, which is more directly related to nuclear observables. To see what the precise relation is, we include the non-local four-Fermi interaction due to the one-pion exchange term, $L_4^{(\pi)}$. The total four-Fermi interaction that enters in the renormalization-group flow consideration à la Shankar-Polchinski is then the sum

$$\mathcal{L}_4 = L_4^{(\pi)} + L_4^{(\delta)}.$$ 

The point here is that the non-local one-pion-exchange term brings additional contributions to the effective nucleon mass on top of the universal scaling mass discussed above. We now compute the nucleon effective mass with the chiral Lagrangian and make contact with the results of Fermi liquid theory [11,12]. We start with the single-nucleon energy in the non-relativistic approximation

$$\epsilon(p) = \frac{p^2}{2m_{\star}^{\sigma}} + C_{\omega}^2 \langle N^\dagger N \rangle + \Sigma_{\pi}(p)$$

(12)

where $\Sigma_{\pi}(p)$ is the self-energy from the pion-exchange Fock term. The self-energy contribution from the vector meson (second term on the right hand side of (12)) comes from an $\omega$ tadpole (or Hartree) graph. The Landau effective mass $m_{\star}^{L}$ is related to the quasiparticle velocity at the Fermi surface

$$\frac{d}{dp} \epsilon(p)|_{p=p_F} = \frac{p_F}{m_{\star}^{L}} = \frac{p_F}{m_{\star}^{\sigma}} + \frac{d}{dp} \Sigma_{\pi}(p)|_{p=p_F}.$$ 

(13)

1 I should point out that for the purpose of the ensuing discussion, neither the detailed knowledge of the “heavy” degrees of freedom that give rise to the four-Fermi interactions nor the specific form of the density dependence will be needed. What really matters are the quantum numbers involved. The latter is invoked in reducing various density-dependent parameters to the universal one, $\Phi(\rho)$.

2 We treat the scalar and vector fields self-consistently and the self-energy from the pion exchange graph as a perturbation.
Using Galilean invariance, Landau [9] derived a relation between the effective mass of the quasi-particles and the velocity dependence of the effective interaction described by the Fermi-liquid parameter \( F_1 \):

\[
\frac{m^*_L}{m_N} = 1 + \frac{F_1}{3} = (1 - \frac{\tilde{F}_1}{3})^{-1},
\]

(14)

where \( \tilde{F}_1 = (m_N/m^*_N)F_1 \). The corresponding relation for relativistic systems follows from Lorentz invariance and has been derived by Baym and Chin [13].

With the four-Fermi interaction (11), there are two distinct velocity-dependent terms in the quasiparticle interaction, namely the spatial part of the current-current interaction and the exchange (or Fock) term of the one-pion-exchange. In the nonrelativistic approximation, their contributions to \( \tilde{F}_1 \) are (\( \tilde{F}_1 = \tilde{F}_\omega + \tilde{F}_\pi \))

\[
\tilde{F}_\omega = -C_\omega \frac{2p_F^3}{\pi^2m_N^2},
\]

(15)

\[
\tilde{F}_\pi = -3\frac{m_N}{p_F} \frac{d}{dp} \Sigma_\pi(p)|_{p=p_F},
\]

(16)

respectively.

Using eq. (13) we find

\[
\left( \frac{m^*_L}{m_N} \right)^{-1} = \frac{m_N}{m^*_N} + \frac{m_N}{p_F} \frac{d}{dp} \Sigma_\pi(p)|_{p=p_F} = 1 - \frac{1}{3} F_1,
\]

(17)

which implies that

\[
\frac{m_N}{m^*_N} = 1 - \frac{1}{3} F_\omega = \Phi^{-1}.
\]

(18)

This formula which relates a part of the effective nucleon mass to the universal scaling factor \( \Phi \) gives a relation between the \( \sigma \)-nucleon interaction (eq. (7)) and the \( \omega \)-nucleon coupling (eq. (15)). The \( \omega \)-exchange contribution to the Landau parameter \( F_1 \) is due to the velocity-dependent part of the potential, \( \sim \vec{p}_1 \cdot \vec{p}_2/m_N^2 \). This is an \( \mathcal{O}(p^2) \) term, and consequently suppressed in the naive chiral counting. Nonetheless it is this chirally non-leading term in the four-Fermi interaction (5) that appears on the same footing with the chirally leading terms in the \( \omega \) and \( \sigma \) tadpole graphs. This shows that there must be subtlety in the chiral counting in the presence of a Fermi sea.

The pion contribution to \( F_1 \) can be evaluated explicitly [14]

\[
\frac{1}{3} \tilde{F}_\pi = \frac{3f_{\pi NN}}{8\pi^2p_F} \left( \frac{1}{2} m_\pi^2 + \ln \frac{m_\pi^2 + 2p_F^2}{m_\pi^2} - 2 \right) 
\approx -0.153.
\]

(19)

Here \( f_{\pi NN} \approx 1 \) is the non-relativistic \( \pi N \) coupling constant. The numerical value of \( \tilde{F}_\pi \) is obtained at nuclear matter density, where \( p_F \approx 2m_\pi \).

One of the important results of this paper is that eq. (18) relates the only unknown parameter \( \tilde{F}_\omega \) to the universal scaling factor \( \Phi \). Note that in the absence of the one-pion-exchange interaction – and in the nonrelativistic approximation – \( m^*_N \) can be identified with the Landau effective mass \( m^*_L \). In its presence, however, the two masses are different due to the pionic Fock term. We propose to identify the scaling nucleon mass defined in eq. (8) with the Landau effective mass:

\[
m^*_L = m^*_N.
\]

(20)

We note that the Landau mass is defined at the Fermi surface, while the scaling mass refers to a nucleon propagating in a “vacuum” modified by the nuclear medium. Although the two definitions are closely related, their precise connection is not understood at present. Nevertheless, eq. (20) is expected to be a good approximation.

**B. Axial-Charge Transitions in Nuclei**

One early evidence for the scaling factor \( \Phi \) was found in nuclear axial-charge transitions [15,16].
\[ A(J^+) \leftrightarrow B(J^-) \quad \Delta T = 1. \] 

(21)

This process has been extensively studied recently, experimentally by Warburton [16] and theoretically by several authors [17,18]. It has confirmed the prominent role of the pionic degree of freedom in nuclei as predicted sometime ago in [19] and recently given a strong theoretical support in terms of chiral perturbation theory [18].

The process (21) is dominated by the time component of the weak axial current \( A_0 \) and the axial charge operator relevant to nuclear processes is given to a very good accuracy by the single-particle current and a two-body soft-pion-exchange current with three- or higher-body contributions strongly suppressed [19,18], both of which goes as

\[ A_0 \sim f^{-1}_\pi. \] 

(22)

This shows that in medium the current will be modified by a factor \( \frac{f_\pi}{f_\pi} \equiv \Phi \) compared with the free-space operator.

Now if one denotes by \( R \) the ratio of the one-body matrix element to the two-body matrix element taken between the initial and final nuclear states

\[ R = \frac{\langle B|A_0^{(2\text{body})}|A \rangle}{\langle B|A_0^{(1\text{body})}|A \rangle} \] 

(23)

then the ratio studied by Warburton

\[ \epsilon_{MEC} \equiv \frac{\langle B|A_0|A \rangle}{\langle B|A_0^{\text{imp}}|A \rangle} \] 

(24)

where \( A_0^{\text{imp}} \) stands for the impulse-approximation operator with the free-space \( f_\pi \), is simply given by

\[ \epsilon_{MEC} = \Phi^{-1}(1 + R). \] 

(25)

Because of the soft-pion dominance in the two-body charge operator as found in [19], the ratio \( R \) is large, so one sees that the \( \epsilon_{MEC} \) can be rather large. The quantity \( \epsilon_{MEC} \) is then a direct measure of one part of the relation (3).

C. Effective Gamow-Teller Constant \( g^*_A \)

The renormalized axial-vector coupling constant \( g^*_A \) can be obtained by relating the Landau effective mass [15] and the effective chiral mass for the nucleon [3]:

\[ \frac{g^*_A}{g_A} = \left( 1 + \frac{1}{3} F_1 \right)^2 = \left( 1 - \frac{1}{3} \Phi \frac{F^\pi_1}{F_1} \right)^{-2}. \] 

(26)

Empirically the constant \( g^*_A \) is found to be very near 1 [20]. In the past the quenching of the axial constant was explained in terms of the Landau parameter \( g'_0 \) operative in the \( NN - \Delta N \) channel [21]. Here we obtained a relation that does not invoke an explicitly spin-isospin-dependent Landau parameter. At present it is not clear what the relation is between the two formulas, both of which predict about the same value for the constant.

D. Orbital Gyromagnetic Ratios in Nuclei

A low-energy observable that relates the “vacuum” factor \( \Phi \) to Landau parameters is the gyromagnetic ratios \( g_1^{(p,n)} \) of the proton and the neutron in heavy nuclei. We first recall the standard Fermi liquid theory result for the gyromagnetic ratio [3].

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3This behavior for the two-body soft-pion exchange current follows from the observation that the properties of the pion do not get modified in medium as one sees in nature.

4This quantity has been extensively analyzed in terms of standard exchange currents and their relations, via vector-current Ward identities, to nuclear forces [22].
1. Migdal’s formula

The response to a slowly-varying electromagnetic field of an odd nucleon with momentum \( \vec{p} \) added to a closed Fermi sea can, in Landau theory, be represented by the current [10,23]

\[
\vec{J} = \frac{\vec{p}}{m_N} \left( \frac{1 + \tau_3}{2} + \frac{F'_1 - F_1}{6 + F_1/3} \tau_3 \right)
\]

(27)

where \( m_N \) is the nucleon mass in medium-free space. The long-wavelength limit of the current is not unique. The physically relevant one corresponds to the limit \( q \to 0, \omega \to 0 \) with \( q/\omega \to 0 \), where \( (\omega, q) \) is the four-momentum transfer. The current (27) defines the gyromagnetic ratio

\[
g_l = \frac{1 + \tau_3}{2} + \delta g_l
\]

(28)

where

\[
\delta g_l = \frac{1}{6} \frac{F'_1 - F_1}{1 + F_1/3} \tau_3 = \frac{1}{6} (\vec{F}' - \vec{F}) \tau_3.
\]

(29)

2. From Chiral Lagrangian

Let us first compute the gyromagnetic ratio using the chiral Lagrangian and demonstrate that Migdal’s result (29) is reproduced. The derivation will be made in terms of Feynman diagrams. The single-particle current \( \vec{J}_1 = \vec{p}/m_N \) is given by a diagram with the external nucleon lines dressed by the scalar and vector fields. Note that it is the universally scaled mass \( m_N' \) that enters, not the Landau mass. This leads to a gyromagnetic ratio

\[
(g_l)_{sp} = \frac{m_N}{m_N'} \frac{1 + \tau_3}{2}.
\]

(30)

At first glance this result seems to imply the enhancement of the single quasiparticle gyromagnetic ratio by the factor \( 1/\Phi \) (for \( \Phi < 1 \)) over the free space value. However this interpretation, often made in the literature, is not correct. We have to take into account the corrections carefully.

The first correction to (30) is the contribution from short-ranged high-energy isoscalar vibrations corresponding to an \( \omega \) meson. This contribution has been computed by several authors [24,25]. In the nonrelativistic approximation one finds

\[
g'_l = -\frac{1}{6} C^2 \frac{2p_F}{\pi^2} \frac{1}{m_N^2} \frac{1}{6} \vec{F}' = \frac{1}{6} \vec{F}'_1.
\]

(31)

Now using (18), we obtain the second principal result of this paper,

\[
g''_l = \frac{1}{6} \vec{F}'_1 = \frac{1}{2} (1 - \Phi(\rho)^{-1}).
\]

(32)

The corresponding contribution with a \( \rho \) exchange in the graph yields an isovector term

\[
g'_l = -\frac{1}{6} C^2 \frac{2p_F}{\pi^2} \frac{1}{m_N^2} \tau_3 = \frac{1}{6} (\vec{F}'_1)' \tau_3
\]

(33)

where the constant \( C_\rho \) is the coupling strength of the four-Fermi interaction

\[
\delta L = -\frac{C_\rho^2}{2} (\bar{N}\gamma_\mu r^a N \bar{N}\gamma^\mu r^a N).
\]

(34)

In analogy with the isoscalar channel, we may consider this as arising when the \( \rho \) is integrated out from the Lagrangian, and consequently identify

\[
C_\rho^2 = g_\rho^2/m_\rho^2.
\]

(35)
Again in medium, $m_\rho$ should be replaced by $m_\rho^*$. The results (31) and (33) can be interpreted in the language of chiral perturbation theory as arising from four-Fermi interaction counterterms in the presence of electromagnetic field, with the counterterms saturated by the $\omega$ and $\rho$ mesons respectively (see eq. (92) of [3]).

The next correction is the pionic exchange current (known as Miyazawa term) which yields [14]

$$g_l = \frac{m_N}{m_\pi^*} \frac{1 + \tau_3}{2} + \frac{1}{6} \left( \tilde{F}_1^\omega + (\tilde{F}_1^\rho)' \tau_3 \right) = \frac{1 + \tau_3}{2} + \frac{1}{6} \left( \tilde{F}_1' - \tilde{F}_1 \right) \tau_3$$

(37)

where eq. (18) was used with

$$\tilde{F}_1 = \tilde{F}_1^\omega + \tilde{F}_1^\pi,$$

$$\tilde{F}_1' = (\tilde{F}_1^\pi)' + (\tilde{F}_1^\rho)'.$$  (38)  

$$\tilde{F}_1^\pi = \tilde{F}_1^\pi,'$$  (39)

Thus, when the corrections are suitably calculated, we do recover the familiar single-particle gyromagnetic ratio $(1 + \tau_3)/2$ and reproduce the Fermi-liquid theory result for $\delta g_l$ (29)

$$\delta g_l = \frac{1}{6} (\tilde{F}_1' - \tilde{F}_1) \tau_3$$

(40)

with $\tilde{F}$ and $\tilde{F}'$ in the theory given entirely by (38) and (39), respectively. Equation (37) shows that the isoscalar gyromagnetic ratio is not renormalized by the medium (other than binding effect implicit in the matrix elements) while the isovector one is. It should be emphasized that contrary to naive expectations, BR scaling is not in conflict with the observed nuclear magnetic moments. We will show below that the theory agrees quantitatively with experimental data.

### IV. TESTING BR SCALING

#### A. $\Phi$ from QCD Sum Rules

It is possible to extract the scaling factor $\Phi(\rho)$ from QCD sum rules – as well as from an in-medium Gell-Mann-Oakes-Renner relation – and compare with our theory. In particular, the key information is available from the calculations of the masses of the $\rho$ meson [26,27] and the nucleon [28,29] in medium. In their recent work, Jin and collaborators find (for $\rho = \rho_0$) [27]

$$m_\rho^* = 0.78 \pm 0.08.$$  (41)

We identify the $\rho$-meson scaling with the universal scaling factor,

$$\Phi(\rho_0) = 0.78.$$  (42)

This is remarkably close to the result that follows from the GMOR relation in medium [31,4,30]

$$\left( \frac{f_\pi}{f_\pi} \right)^2 (\rho_0) \approx \frac{m_\pi^*}{m_\pi} (1 - \Sigma_{\pi N} \rho_0 \frac{\rho_0}{f_\pi^2 m_\pi^2} + \cdots) \approx 0.63,$$

(43)

where the pion-nucleon sigma term $\Sigma_{\pi N} \approx 45$ MeV is used. In fact, in previous papers by Brown and Rho, the scaling factor $\Phi$ was inferred from the in-medium GMOR relation. How this scaling factor behaves as a function of density will have to be determined by heavy-ion experiments. First-principle calculations anchored on QCD – QCD sum rules or lattice – are not likely to be forthcoming in the near future.
B. Predictions by Chiral Lagrangian

Our theory has only one quantity that is not fixed by the theory, namely the scaling factor $\Phi(\rho)$ ($\tilde{F}_{1}^{\pi}$ is of course fixed for any density by the chiral Lagrangian). Since this is given by the QCD sum rule for $\rho = \rho_0$, we use this information to make quantitative predictions.

1. Effective nucleon mass

The Landau effective mass of the nucleon (17) is

$$\frac{m_N^*}{m_N} = \Phi \left( 1 + \frac{1}{3} \tilde{F}_{1}^{\pi} \right)$$

$$= \left( \Phi^{-1} - \frac{1}{3} \tilde{F}_{1}^{\pi} \right)^{-1}$$

$$= (1/0.78 + 0.153)^{-1} = 0.69(7)$$ (44)

where we used (19) and (42). This is in agreement with the QCD sum-rule result of [29]:

$$\frac{m_N^*}{m_N} = 0.67 \pm 0.05.$$ (45)

The agreement is both surprising and intriguing since as mentioned above, the Landau mass is “measured” at the Fermi momentum $p = p_f$ while the QCD sum-rule mass is defined in the rest frame, so the direct connection remains to be established. The prediction (44) is also in excellent agreement with observations in the spectroscopy of heavy nuclei and in the electron scattering from nuclei.

2. Scaling of the vector-meson masses

The scaling of the vector meson mass as given by the QCD sum-rule prediction has recently been discussed in two different contexts.

The first case concerns the recent CERN-CERES experiments on dileptons produced in the central collision of Si on Au at 200 GeV where an excess dilepton production was observed around the invariant masses of 300 – 500 MeV [32,33]. This enhanced low-mass dilepton production was simply explained by Li, Ko and Brown [34] in terms of the scaled mass for the $\rho$ meson

$$m_{\rho}^* \approx \Phi(\rho)m_{\rho}.$$ (46)

Here the scaling factor $\Phi$ is somewhat arbitrarily chosen with its normalization fixed at $\rho = 0$ and $\rho = \rho_0$. Many other mechanisms for the dilepton excess have been proposed but up to now, it appears that the explanation in terms of BR scaling is the only viable explanation. It may be possible to arrive at a similar result starting with a Lagrangian defined at zero density and doing dynamical calculations [35]. Whether or not the dynamical description will justify the inherently quasiparticle picture that is implicit in BR scaling is not clear at the moment and will eventually be clarified by the calculations in progress.

The second case concerns polarized proton scattering from a heavy nucleus. By separating the longitudinal component of the isovector spin response function from the transverse, it has been possible to extract information of the propagation of the $\rho$ meson in medium. In the 198.5 MeV $^{28}\text{Si} (\vec{p}, \vec{p}')^{28}\text{Si}$ reaction [36], the Indiana group demonstrated that the suppression of the partial cross section for spin-longitudinal response can be understood simply if one assumes that the propagating vector meson has a scaling behavior consistent with the BR scaling.
3. Warburton’s $\epsilon_{MEC}$

The ratio $R$ turns out to be quite insensitive to nuclear models used and does not depend sensitively on the mass number \[^{19}\]. It comes out to the next-to-next-to-leading order in chiral perturbation theory \[^{18}\] to be

$$R = 0.5 \pm 0.1$$ \hspace{1cm} (47)

for medium to heavy nuclei. For the heavy nuclei considered by Warburton (lead region), we may take $\rho \approx \rho_0$ and hence using $(f^*/f_\pi)^2 = \Phi^2 \approx 0.63$ in (25), we have

$$\epsilon_{MEC} = \frac{1}{0.78}(1 + 0.5 \pm 0.1) = 1.9 \pm 0.1$$ \hspace{1cm} (48)

which should be compared with Warburton’s “empirical” value

$$\epsilon_{MEC}^{exp} = 1.8 \sim 2.0.$$ \hspace{1cm} (49)

Here Warburton’s range indicates the theoretical uncertainty in the estimate of the single-particle matrix element that depends on the strength of the tensor force in nuclei. In fact, BR scaling implies the suppression of the tensor force, so the uncertainty will remain unless the single-particle matrix element is reevaluated with the BR scaling taken into account.

4. Effective axial-vector coupling constant

The next quantity of interest is the axial-vector coupling constant in medium, $g^*_A$

$$\frac{g^*_A}{g_A} = \left(1 + \frac{1}{3} F_1 \right)^2 = \left(1 - \frac{1}{3} \Phi F_1 \right)^{-2},$$ \hspace{1cm} (50)

which at $\rho = \rho_0$ gives

$$g^*_A = 1.0(0).$$ \hspace{1cm} (51)

This agrees well with the observations in heavy nuclei \[^{20}\]. Again this is an intriguing result. While it is not understood how this relation is related to the old one in terms of the Landau-Migdal parameter $g'_0$ in $NN \leftrightarrow N\Delta$ channel \[^{21}\], it is clearly a short-distance effect in the “pionic channel” involving the factor $\Phi$. This is consistent with the argument \[^{18}\] that the renormalization of the axial-vector coupling constant in medium cannot be described in low-order chiral perturbation theory.

5. Orbital gyromagnetic ratio

The correction to the single-particle gyromagnetic ratio can be rewritten as

$$\delta g_1 = \frac{4}{9} \left[ \Phi^{-1} - 1 - \frac{1}{2} \tilde{F}_1 \right] \tau_3$$ \hspace{1cm} (52)

\footnote{This ratio is dominated as mentioned by the soft-pion exchange term with the loop corrections amounting to $\sim 10\%$ of this value, so included in the error estimate.}

\footnote{The $g^*_A$ calculated here is for a quasiparticle sitting on top of the Fermi sea and is presumably a fixed-point quantity as one scales down in the sense of renormalization group flow. As such, it should be applicable within a configuration space restricted to near the Fermi surface. I think this is the reason why $g^*_A = 1$ was required in the $0\hbar\omega$ Monte Carlo shell-model calculation of Langanke et al \[^{20}\]. The consequence of this result is that if one were to calculate core-polarization contributions involving multiparticle-multihole configurations mediated by tensor forces, one should obtain only a minor correction. As Gerry Brown has been arguing for some time, this can happen because of the suppression of tensor forces in the presence of BR scaling. Note also that the effective $g^*_A$ obtained here has nothing to do with the so-called “missing Gamow-Teller strength” often discussed in the literature.}
where we have used (36) and the assumption that the nonet relation $C^2_\rho = C^2_\omega/9$ holds. The nonet assumption would be justified if the constants $C_\omega$ and $C_\rho$ were saturated by the $\omega$ and $\rho$ mesons, respectively. At $\rho = \rho_0$, we find

$$\delta g_l = 0.22(7) \tau_3. \quad (53)$$

This is in agreement with the result $^{34}$ for protons extracted from the dipole sum rule in $^{209}$Bi using the Fujita-Hirata relation $^{38}$:

$$\delta g_l^{proton} = \kappa/2 = 0.23 \pm 0.03. \quad (54)$$

Here $\kappa$ is the enhancement factor in the giant dipole sum rule. Given that this is extracted from the sum rule in the giant dipole resonance region, this is a bulk property, so our theory is directly relevant.

Direct comparison with magnetic moment measurements is difficult since BR scaling is expected to quench the tensor force which is crucial for the calculation of contributions from high-excitation states needed to extract the $\delta g_l$. Calculations with this effect taken into account are not available at present. Modulo this caveat, our prediction (53) compares well with Yamazaki’s analysis $^{39}$ of magnetic moments in the $^{208}$Pb region

$$\delta g_l^{proton} \approx 0.33, \quad \delta g_l^{neutron} \approx -0.22 \quad (55)$$

and also with the result of Arima et al. $^{40}$ $^{39}$

$$\delta g_l \approx 0.25 \tau_3. \quad (56)$$

6. Kaons in dense matter

Given the identification of the background field of the ground-state nuclear matter with the mean fields of the chiral Lagrangian with the BR-scaled parameters as argued in $^{3}$, one can then describe fluctuations around that background in various flavor directions. Of particular current interest is the kaonic excitation in nuclear matter as well as in dense matter. Specifically one can ask what happens to a $K^\pm$ propagating in dense medium as in relativistic heavy-ion collisions. One can address this issue in terms of an optical potential felt by the kaon in medium or equivalently in terms of an effective density-dependent mass of the kaon.

It is shown in $^{3}$ that a $K^-$ in nuclear matter feels an effective potential given by

$$S_{K^-} + V_{K^-} \approx \frac{1}{3}(S_N - V_N) \quad (57)$$

where $S$ and $V$ stand respectively for scalar and vector potentials and the subscripts stand for the particles in the potential. The appearance of the factor 1/3 indicates that a constituent quark (or quasiquark) picture has emerged here. From the phenomenology in Walecka mean-field theory, we have

$$(S_N - V_N) \lesssim -600 \text{ MeV.} \quad (58)$$

This gives the prediction

$$S_{K^-} + V_{K^-} \lesssim -200 \text{ MeV.} \quad (59)$$

This can be compared with the result of an analysis in $K$-mesic atoms made by Friedman, Gal and Batty $^{41}$ who find attraction at $\rho \approx 0.97\rho_0$ of

$$S_{K^-} + V_{K^-} = -200 \pm 20 \text{ MeV.} \quad (60)$$

Extended to neutron-rich matter and to higher density, one expects that negatively charged kaons will condense at a matter density between 2 and 3 times the normal matter density $^{3}$. This means that the effective mass of the kaon at that density is comparable to the electron chemical potential in compact-star matter.

This is an extremely simple prediction which would be very exciting if confirmed. One cannot take this prediction seriously, however, unless one can show that corrections to the mean-field result are small. Indeed, constrained by chiral perturbation theory and the ensemble of available data on kaon-nucleon interactions, it does not appear possible to obtain condensation at this low density $^{42}$ unless one adds additional four-Fermi interactions that provide attraction $^{13}$ as seems required for the kaonic atom data. But these are not unique as (many) such four-Fermi terms cannot yet be constrained by experiments. What is significant, however, is that the presently available data on kaon properties in medium obtained by the KaoS and FOPI collaborations at GSI are seen to provide a non-trivial support to this additional attraction $^{16}$. More accurate data expected to be available soon will confirm or refute this theory.
V. CONCLUSION

The assumptions that underlie the preceding arguments are:

• the BR-scaled chiral Lagrangian in mean field represents, at the nuclear matter saturation density $\rho_0$, the Landau Fermi-liquid fixed point theory;

• it is sensible to extrapolate the BR-scaled Lagrangian to describe, by a simple scaling of the parameters, processes occurring at any density $\rho \neq \rho_0$;

• the masses and constants of the effective chiral Lagrangian scale smoothly all the way to the chiral phase transition point.

I have shown that it is possible with the above assumptions to link what one observes in relativistic heavy-ion collisions that probe densities higher than normal to what one measures in low-energy experiments. This is suggested to be an indication that nuclear physics can be described in a continuous way from normal to the phase transition point with the vacuum suitably modified through the scaling parameters. The crucial underlying assumption is that by shifting the “vacuum” to that characterized by a given density, one converts an intrinsically strong-coupling theory to a weak-coupling one, thus justifying the mean-field approximation.

The scheme described here seems to work remarkably well. This makes the following unresolved basic questions more poignant:

• How is nuclear matter obtained starting from an effective chiral Lagrangian?

• How good is the notion of quasiparticles – for baryons for which Pauli principle could play a role and for mesons for which no such mechanism is apparent – in a density regime where coupling is strong from the viewpoint of a theory defined at zero density?

• How can one justify the notion of Landau Fermi liquid for nuclear matter when hyperons can enter the process as in kaon condensation?

• How can one systematically calculate corrections to the mean-field approximation with a chiral Lagrangian, the parameters of which are BR-scaled?

Some of these questions are being addressed in a variety of ways [35,44,45] and will be answered when the experimental data being measured become available.

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