Abstract: Theoretical calculation and numerical simulation are used to investigate the lubricating oil demand of spur gears. In accordance with the function of lubricating oil during the meshing process, oil demand is regarded as the superposition of oil for lubrication and cooling. Oil for lubrication is calculated in accordance with meshing and elastohydrodynamic lubrication (EHL) theories. Oil for cooling is obtained from friction heat. The influence of different meshing positions on lubricating oil demand is analysed, and the effects of modulus, tooth number, transmission ratio, input speed and input torque on lubricating oil demand is investigated using a control variate method. Simulation results indicated that oil for lubrication and oil for cooling have two maxima each during a meshing circle. The influences of different gear parameters and working conditions on lubricating oil demand are compared. The results showed that the oil volume for lubrication increases and oil volume for cooling decreases as the modulus, tooth number and transmission ratio of the gear increase, the oil volume for lubrication and oil volume for cooling increases as the input speed and input torque increase.

Keywords: EHL; spur gear; lubrication oil; simulation; parameters
reasonably explains the formation mechanism of oil film; this mechanism is the theoretical basis of EHL [8]. The pressure–viscosity effect of lubricating oil and the elastic deformation of contact surfaces are considered in EHL. Thus, the EHL problem is described using a series of nonlinear coupled equations that are extremely difficult to solve. The earliest method used to solve EHL was a direct iteration method that produced considerable errors in the actual results. The inverse, multigrid [9] and finite element [10] methods were subsequently introduced, and their results can accurately describe the shape of the lubricating oil film and the pressure distribution in linear contact EHL.

Recent studies have used the Reynolds equation to solve the general lubrication problem in non-Newtonian fluids by regarding lubricating oil flow as the superposition of Poiseuille and Couette flows. The velocity of lubricating oil flow in this equation was solved using the characteristics of Poiseuille and Couette flows, and the other characteristics of lubricating oil film were solved on the basis of Newtonian fluids [11]. The traditional Integral Approach, the Single Domain Full-System Approach and the Double Domain Full-System Approach have been used to solve the EHL problem [12]. The comparisons of velocity and oil film thickness between this method and the conventional approach established the feasibility and validity of the method and illustrated its simplicity of calculation. The influence of the surface roughness of gear teeth on the lubricating oil film was significant [13]. Thus, the modified Reynolds equation in Patir and Cheng [14] and the surface deformation and statistical elasto-plastic asperity micro-contact model in Zhao et al. [15] were solved using a systematic approach under steady-state and isothermal operating conditions. The central oil film thickness formula, minimum oil film thickness and asperity-to-total load ratio were obtained via the regression of simulation data [13]. To examine the influence of roughness on EHL, the rough surface was regarded as a rough plane surface with uniformly distributed cylindrical asperities that were evenly truncated on top, and the inlet of the contact area was analysed. Several valuable conclusions, including the reduction of load capacity in the oil film and the increase in friction coefficient due to roughness, particularly at low speeds and under heavy loads, were drawn [16]. The load transmitted by a gear tooth during the meshing process may fluctuate significantly due to time-varying meshing stiffness and torque fluctuation. The influence of dynamic load on the minimum oil film thickness, pressure distribution and temperature distribution were analysed on the basis of the quasi-static tooth force solved via finite element contact analysis [17]. The results showed that a dynamic load will reduce the thickness of the oil film, increase its pressure and temperature and destroy the EHL of the contact surface. A tribo-dynamic model for spur gear pairs combining dynamic loading with EHL theory was proposed in 2016 [18]; this model can predict the contact force and velocity of dynamic teeth. A method that combined the Runge Kutta and multigrid methods was used to solve the aforementioned tribo-dynamic model, and the results were compared with those of the model under quasi-static conditions. Starved lubrication conditions will occur when an incomplete oil film is formed between contact surfaces. To investigate the influences of starved lubrication on minimum film thickness, maximum temperature and friction coefficient, a thermally starved EHL model was proposed [19]. This previous research found that the minimum oil film thickness was thinner, and the friction coefficient and maximum temperature were higher, in starved lubrication than those in EHL.

CFD is a new branch of fluid mechanics that has been widely used in gear lubrication. Liu et al. [20] built a CFD model based on the finite volume method to investigate the lubricating oil distribution and the churning power losses of a gearbox. The results were obtained by considering the 3D finite volume simulation model as a two-phase flow, and the results were compared with the measurements of churning losses and high-speed camera recordings. The comparisons exhibited good agreement in predicting lubricating oil distribution and churning power losses. Hu et al. [21] presented a CFD model combined with turbulence to investigate the lubricating oil distribution and churning power losses of a moving gearbox. The influences of gear rotation, immersion depth, gearbox motion frequency and amplitude on the results were determined.

On the basis of the preceding statement, current studies have explored effective methods for establishing a lubrication analysis model that can realistically and accurately describe the contact
state among tooth, the characteristics of oil film and the distribution of lubricating oil. However, these studies on gear lubrication are based on the condition that the lubricating oil supply and the lubrication state of the gear are known. A method that can analyse the lubricating oil demand of gears under specific operating conditions could help significantly in the design of gear lubrication systems.

In the current work, new calculation and analysis models of the lubricating oil demand of spur gears are proposed by considering lubricating oil demand as the superposition of oil for lubrication and cooling. This study aims to understand variations in oil for lubrication and cooling during a mesh cycle under different parameters and working conditions. The calculation models of lubricating oil demand are established in accordance with the meshing of spur gears and EHL theories. The influences of modulus, number of teeth, input speed and input torque on oil for lubrication and cooling are simulated. This study provides guidance for the appropriate design of gear lubrication systems.

2. Theory

Lubricating oil has two major demands when gears mesh. A portion of lubricating oil enters the gaps between contact surfaces via entrainment, and an oil film is formed with sufficient thickness to separate the tooth surfaces. This film prevents wear and friction loss caused by the direct contact of the tooth surfaces. This type of lubricating oil, called oil for lubrication, meets the lubrication requirement of gears. Another type of lubricating oil, called oil for cooling, meets the cooling requirement of gears. Oil for cooling removes a large amount of the heat produced by the shear action of the oil film when the tooth surfaces slide. This prevents the rupture of the lubricating oil film caused by the high temperature of the lubricating oil. All the heat is assumed to be carried away by the lubricating oil. Given that the oil film in the contact area exerts a cooling effect, lubricating oil demand during the gear meshing process can be expressed as [22]

\[ Q = b \times \max(q_l, q_c), \]  

where \( Q \) is the total demand of lubricating oil, \( q_l \) is the oil for lubrication, \( q_c \) is the oil for cooling, and \( b \) is the width of the gear.

2.1. Oil for Lubrication

The role of oil for lubrication is to form an oil film with sufficient thickness between contact surfaces. Therefore, oil for lubrication is calculated using the sum of the instantaneous flow of lubricating oil between the surfaces of all gear teeth. Analysing the gear meshing process in detail and calculating the oil film thickness between the contact surfaces of all meshing teeth are necessary.

2.1.1. Analysis of Meshing Process

Figure 1 illustrates the meshing principle of an involute spur gear pair. The instantaneous meshing point \( K \) is located at the common tangent of the base circle of the two gears, \( N_1N_2 \) is the theoretical meshing line and \( P \) is the pitch point. The symbols \( r_{a_1}, r_{a_2}, \alpha, \alpha'_1 \) and \( \alpha'_2 \) represent the radius of the pinion base circle, radius of the gear base circle, pressure angle of the reference circle, pressure angle of the pinion addendum circle and pressure angle of the gear addendum circle, respectively. The intersection point \( B_1 \) between the addendum circle of a gear and the meshing line is the starting point of meshing, intersection point \( B_2 \) between the addendum circle of the pinion and the meshing line is the end point of meshing and \( B_1B_2 \) is the actual meshing line. When the instantaneous meshing point is located on line \( B_1C \), two pairs of teeth mesh simultaneously and the meshing point of another pair of teeth is located on line \( B_2D \). The distance between two meshing points on the meshing line is the base pitch \( P_b \). The line \( CD \) is the single tooth meshing area. A rectangular coordinate system is established with pitch point \( P \) as the origin and the direction from the start to the end of meshing as the positive direction. The coordinate of the instantaneous meshing point is \( s \). The range of \( s \) is \([-B_1P, B_2P]\) in a meshing period.
In accordance with the geometric character, the distance from the pitch point to the start of meshing and the that from the pitch point to the end of meshing can be described as [23]

\[ \begin{align*}
B_1P &= N_2B_1 - N_2P = r_{a2}(\tan \alpha'_2 - \tan \alpha) = \frac{mz_2}{2} \cos \alpha (\tan \alpha'_2 - \tan \alpha), \\
B_2P &= N_1B_2 - N_1P = r_{a1}(\tan \alpha'_1 - \tan \alpha) = \frac{mz_1}{2} \cos \alpha (\tan \alpha'_1 - \tan \alpha),
\end{align*} \]

The relative radius of curvature and entrainment velocity can be written as

\[ R = \frac{(r_{a1}\tan \alpha + s)(r_{a2}\tan \alpha - s)}{(r_{a1} + r_{a2})\tan \alpha}, \]

\[ u_E = \frac{u_1 + u_2}{2}, \]

where \( u_1 \) and \( u_2 \) denote the relative velocity between tooth surfaces and the meshing point, respectively, which can be written as

\[ u_1 = \frac{\pi n_1}{30} (r_{a1}\tan \alpha + s), \]

\[ u_2 = \frac{\pi n_1}{30} (r_{a1}\tan \alpha - s) \]

The load on the tooth surface is calculated using the load distribution coefficient, which can be described as follows:

\[ \zeta = \begin{cases} 
\frac{1}{3}(1 + \frac{P_{PB_1}}{P_{PB_1} - PD}) & (-PB_1 \leq s \leq -PC) \\
\frac{1}{1 - PC < s < PD) \\
\frac{1}{3}(1 + \frac{P_{PB_1}}{P_{PB_1} - PC}) & (-PB_1 \leq s \leq -PC)
\end{cases} \]

where

\[ \begin{align*}
PD &= B_1B_2 - B_1P - B_2D, \\
PC &= B_1B_2 - B_2P - B_1C, \\
B_1C &= DB_2 = (\epsilon - 1)p_b, \\
CD &= (2 - \epsilon)p_b, \\
p_b &= \pi m \cos \alpha.
\end{align*} \]
2.1.2. Analysis of EHL

Given that the width of the contact area is considerably smaller than the radius of the tooth surface curvature, lubrication in the spur gear can be simplified as linear contact EHL. In linear contact EHL, the steady-state Reynolds equation for Newtonian fluids and smooth surfaces can be written as

\[
\frac{d}{dx} \left( \frac{\rho h^3 \frac{dp}{dx}}{\eta} \right) = 12u_E \frac{d(\rho h)}{dx}, \tag{9}
\]

where \( \rho \) is the density of lubricating oil, \( p \) is the pressure of the oil film, and \( h \) is the thickness of the oil film.

The thickness of the oil film is the sum of the geometric clearance and elastic deformation of the tooth surface, which can be expressed as

\[
h(x) = h_0 + \frac{x^2}{2R} - \frac{2}{\pi} \int_{x_0}^{x} p(s) \ln (x - s)^2 ds, \tag{10}
\]

where \( h_0 \) is a constant. Notably, surface deformation is caused by total pressure. The comprehensive Young’s modulus \( E' \) is defined using

\[
\frac{2}{E'} = \frac{1 - \mu_1^2}{E_1} + \frac{1 - \mu_2^2}{E_2}, \tag{11}
\]

where \( E_1 \) and \( E_2 \) are the Young’s modulus of the pinion and gear materials, respectively, and \( \mu_1 \) and \( \mu_2 \) are the Poisson’s ratio of the pinion and gear materials, respectively.

The following viscosity and density of lubricating oil change with pressure can be written as

\[
\eta = \eta_0 \exp \left( (\ln \eta_0 + 9.67) \left[ 1 + 5.1 \times 10^{-9} p \right]^2 - 1 \right), \tag{12}
\]
\[
\rho = \rho_0 \left( 1 + \frac{0.6 \times 10^{-9} p}{1 + 1.7 \times 10^{-9} p} \right), \tag{13}
\]

where \( \eta_0 \) and \( \rho_0 \) are the viscosity and density of lubricating oil in atmospheric pressure, respectively.

With regard to EHL, the oil film pressure that must be balanced with the applied load can be described as

\[
w = \int p dx, \tag{14}
\]

where \( w \) represents the load per tooth width, and the equation for load balance is generally used as the termination criterion for iterative calculation.

2.1.3. Analysis of Lubricating Oil Flow

In the lubricating oil film between tooth surfaces, the force balance in the flow direction of Newtonian fluids can be expressed as

\[
\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial z} = \frac{\partial}{\partial z} \left( \eta \frac{\partial \bar{u}}{\partial z} \right), \tag{15}
\]

where \( \bar{u} \) is the flow velocity of lubricating oil between tooth surfaces.

Equation (15) can be integrated with the boundary conditions and expressed as follows:

\[
\begin{cases}
  z = 0, \bar{u} = u_1 \\
  z = h, \bar{u} = u_2 
\end{cases} \tag{16}
\]
The flow velocity of lubricating oil between tooth surfaces can be written as
\[
\bar{u} = \left(\frac{z^2 - zh}{2\eta}\right) \frac{\partial p}{\partial x} + \frac{z}{h} (u_2 - u_1) + u_1.
\] (17)

The mass flow in the flow direction per unit tooth width and time is expressed as
\[
m_x = \rho \int_0^h \bar{u} \, dz.
\] (18)

Equation (17) is substituted into Equation (18), and the mass flow rate can be written as
\[
m_x = -\rho h^3 \frac{\partial p}{12\eta \partial x} + \left(\frac{u_1 + u_2}{2}\right) \rho h.
\] (19)

2.1.4. Calculation of Oil for Lubrication

The lubricating oil entering the contact area must ensure the continuous formation of the oil film. Oil film pressure is approximately equal to the Hertz contact pressure at the centre of the contact area [24]. Given that the Hertz contact pressure reaches its maximum value at the centre of the contact area, the derivative of pressure at the centre of the contact area can be expressed as
\[
\left(\frac{\partial p}{\partial x}\right)_{x=0} = 0.
\] (20)

The mass flow of lubricating oil per unit tooth width and time at the centre of the contact area can be written as
\[
m_{x=0} = \left(\frac{u_1 + u_2}{2}\right) \rho_H h_c = u_E \rho_H h_c,
\] (21)

where \(h_c\) denotes the thickness of the oil film. The density of lubricating oil at the centre of the contact area can be written as
\[
\rho_H = \rho_0 \left(1 + \frac{0.6 \times 10^{-9} p_H}{1 + 1.7 \times 10^{-9} p_H}\right)
\] (22)

where \(p_H\) is the pressure at the centre of the contact area, and it is the same as the maximum Hertz contact pressure that can be written as
\[
p_H = \sqrt{\frac{w E'}{2\pi R}}.
\] (23)

The mass of lubricating oil is immutable during gear meshing. Thus, the volume flow of lubricating oil per unit tooth width and time can be described as
\[
q_{sl} = \frac{\rho_H}{\rho_0} u_E h_c,
\] (24)

where \(q_{sl}\) is the oil volume for the lubrication of a tooth that changes with load per unit tooth width and meshing point coordinates to ensure that the oil volume for the lubrication of a tooth can be described as
\[
q_{sl} = F_1(w, s),
\] (25)

where
\[
w = \frac{c T_1}{b r_1 \cos \alpha},
\] (26)

\(T_1\) denotes the input torque.
In a meshing circle, the oil volume for the lubrication of a gear pair can be written as

\[
q_l = \begin{cases} 
F_1(\varsigma w, s) + F_1((1 - \varsigma) w, s + p_b) & (PB_1 \leq s \leq -PC) \\
F_1(w, s) & (-PC < s < PD) \\
F_1(\varsigma w, s) + F_1((1 - \varsigma) w, s - p_b) & (PD \leq s \leq PB_2)
\end{cases}.
\]  

(27)

2.2. Oil for Cooling

The tooth surface velocity of a pinion is the same as that of a gear in the normal direction of the contact surface to ensure smooth transmission. However, the velocity of two gears in the tangential direction of a tooth surface is different. Relative sliding occurs between two gears, and the relative slide velocity and rate can be described using Equations (28) and (29), as follows:

\[
u_R = u_1 - u_2, \quad (28)
\]

\[
SR = \frac{\nu_R}{u_E} = \frac{2(u_1 - u_2)}{u_1 + u_2}. \quad (29)
\]

Tooth surfaces are completely separated by the lubricating oil film in EHL, and the friction coefficient between two teeth can be written as [25]

\[
\mu = e^{f(SR, p_H, \eta_0, S)} - 8.91 - 0.35 |SR| p_H \log \eta_0 + 2.81 e^{-|SR| p_H \log \eta_0} + 0.62. \quad (30)
\]

where

\[
f(SR, p_H, \eta_0, S) = -8.91 - 0.35 |SR| p_H \log \eta_0 + 2.81 e^{-|SR| p_H \log \eta_0} + 0.62. \quad (31)
\]

The assumption is that the frictional losses of a gear are all converted into heat, and heat is removed by lubricating oil. The heat generated by friction and removed by oil per tooth width and unit time can be expressed as

\[
p_f = \mu w \nu_R, \quad (32)
\]

\[
p_f = q_{sc} \rho_0 c \Delta T, \quad (33)
\]

where \(q_{sc}\) is the oil volume for the cooling of a tooth, \(c\) is the specific heat capacity of lubricating oil and \(\Delta T\) is the maximum temperature increase in lubricating oil. The oil volume for the cooling of a tooth can be described using the function of load per unit tooth width and meshing point coordinates, which is expressed as

\[
q_{sc} = \frac{\mu w \nu_R}{\rho_0 c \Delta T}, \quad (34)
\]

\[
q_{sc} = F_2(w, s). \quad (35)
\]

In a meshing circle, the oil volume for the cooling of a gear can be expressed as

\[
q_c = \begin{cases} 
F_2(\varsigma w, s) + F_2((1 - \varsigma) w, s + p_b) & (PB_1 \leq s \leq -PC) \\
F_2(w, s) & (-PC < s < PD) \\
F_2(\varsigma w, s) + F_2((1 - \varsigma) w, s - p_b) & (PD \leq s \leq PB_2)
\end{cases}.
\]  

(36)

3. Lubricating Oil Demand during Meshing Process

The curvature radius and load of a tooth surface are continuously changing during the gear meshing process. Thus, the change in lubricating oil demand during the meshing process should be analysed. The parameters of gears, lubricating oil and working conditions are provided in Table 1.

The solution for lubricating oil demand can be attributed to solving complicated nonlinear equations. A compound iterative method is used to obtain the demand of lubricating oil.
Table 1. Parameters for simulation.

| Parameters                                      | Value                        |
|------------------------------------------------|------------------------------|
| Modulus, \( m \)                               | 5 mm                         |
| Number of teeth, \( z_1/z_2 \)                 | 23/68                        |
| Pressure angle on pitch circle, \( \alpha \)   | \( 20^\circ \)               |
| Tooth width, \( b \)                           | 30 mm                        |
| Poisson’s ratio of gear material, \( \mu_1/\mu_2 \) | 0.3                          |
| Elastic modulus of gear material, \( E_1/E_2 \) | 206 Gpa                      |
| Density of lubricating oil, \( \rho_0 \)       | 890 kg/m\(^3\)              |
| Viscosity of lubricating oil, \( \eta_0 \)     | 0.01 Pa.s                    |
| Viscosity-pressure index of lubricating oil, \( a \) | \( 2.2 \times 10^{-8} \) m\(^2\)/N |
| Specific heat capacity of lubricating oil, \( c \) | 2000 J·kg\(^{-1}\)·°C       |
| Maximum temperature rises of lubricating oil, \( \Delta T \) | \( 20^\circ \) C            |
| Input speed, \( n_1 \)                         | 8000 rpm                     |
| Input torque, \( T_1 \)                        | 80 N·m                       |

The central film thickness and entrainment velocity of each meshing tooth are compared during the meshing process, as shown in Figure 2. The two vertical coordinates shown in Figure 2 represent the central film thickness (\( \mu m \)) and entrainment velocity (m/s), respectively. The abscissa denotes the coordinates on the meshing line. Figure 3 shows the oil volume for the lubrication of each meshing tooth during the meshing process and the total oil volume for the lubrication of a gear.

Meshing teeth 1 and 2 mesh simultaneously in the first two tooth meshing area. The entrainment velocity of meshing teeth 1 and 2 in the first two tooth meshing area increases continuously with gear meshing. An oil film easily forms at high entrainment velocities. Thus, the oil film thickness of the two meshing teeth in the meshing area of the first two teeth increases with gear meshing. A positive correlation exists between oil film thickness and oil volume for lubrication. The oil volume for the lubrication of meshing teeth increases along the meshing line, reaching the maximum value at the end of the meshing area of the first two teeth. The total oil volume for the lubrication of a gear is the sum of the oil volume for the lubrication of meshing teeth 1 and 2.

As the gear rotates, meshing tooth 2 moves gradually away from the meshing. Thus, the gear is in the single tooth meshing area. The sudden increase in load of meshing tooth 1 reduces the oil film thickness. Therefore, the oil volume for the lubrication of meshing tooth 1 plummets at the intersection of the meshing areas of the first two teeth and the single tooth. The total oil volume for the lubrication of the gear is the same as the oil volume for the lubrication of meshing tooth 1. Oil film thickness and oil volume for the lubrication of meshing tooth 1 increase linearly in the single tooth meshing area.
Viscosity-pressure index of lubricating oil, $\nu = 2.2 \times 10^{-8} \text{m}^2/\text{N}$

Specific heat capacity of lubricating oil, $C_p = 2000 \text{J} \cdot \text{kg}^{-1} \cdot \degree \text{C}^{-1}$

Maximum temperature rises of lubricating oil, $\Delta T = 20 \degree \text{C}$

Input speed, $n_1 = 8000 \text{rpm}$

Input torque, $T_1 = 80 \text{N} \cdot \text{m}$

The solution for lubricating oil demand can be attributed to solving complicated nonlinear equations. A compound iterative method is used to obtain the demand of lubricating oil.

The central film thickness and entrainment velocity of each meshing tooth are compared during the meshing process, as shown in Figure 2. The two vertical coordinates shown in Figure 2 represent the central film thickness ($\mu$ m) and entrainment velocity (m/s), respectively. The abscissa denotes the coordinates on the meshing line. Figure 3 shows the oil volume for the lubrication of each meshing tooth during the meshing process and the total oil volume for the lubrication of a gear.

![Figure 2. Central film thickness and entrainment velocity during the meshing process.](image)

Meshing tooth 3 begins to mesh at the end of the single tooth meshing area, and the gear is located in the second two tooth meshing area. Entrainment velocity, oil film thickness and oil volume for the lubrication of meshing tooth 1 are larger than those for meshing tooth 3, and they increase continuously with gear meshing. The oil volume for the lubrication of two meshing teeth reaches the maximum value at the end of the second two tooth meshing area. The total oil volume for the lubrication of the gear is the sum of the oil volume for the lubrication of meshing tooth 1 and 3.

The meshing process involves the continuous and repeated movement of a meshing point on a meshing line. In a meshing circle, two equal maxima for the oil volume for lubrication exist at the ends of the first two tooth meshing and the second two tooth meshing areas.

The analysis of the second section shows that the relative slide velocity and friction coefficient play key roles in the calculation of the oil volume for cooling. Therefore, the relative slide velocity and friction coefficient are compared during the meshing process, as shown in Figure 4. The two vertical coordinates that represent the relative slide velocity and friction coefficient are established in this figure. The abscissa denotes the coordinates along the meshing line. The area selected by the wireframe is magnified 10 times for clear visualization. Figure 5 illustrates the oil volume for the cooling of each tooth and the total oil volume for the cooling of a gear. The image is partially enlarged.

![Figure 4. Friction coefficient at the pitch point](image)

Figure 4 shows that the friction coefficient at the pitch point is equal to zero and that of the tooth surface increases as the distance between the meshing and pitch points increases, because the friction force between meshing tooth surfaces is the same as the shear force of the oil film in the EHL of a smooth surface. The friction coefficient increases as the relative slide velocity increases and oil film thickness decreases. The friction coefficient in the beginning of the meshing process is greater than that at the end of meshing process because the oil film thickness and relative slide speed at the end of the meshing process help reduce friction.

The friction coefficient of meshing tooth 1 is at its maximum value at the beginning of the first two tooth meshing area, resulting in the maximum value of oil volume for cooling, as shown in Figure 5. This finding is attributed to the relative slide velocity being at its maximum value and the oil film thickness being at its minimum value. In the first two tooth meshing area, the total oil volume for the cooling of the gear is the sum of the oil volume for cooling of meshing tooth 1 and 2. The relative slide velocity, oil film thickness and oil volume for the cooling of meshing tooth 1 decrease with gear rotation, and those of meshing tooth 2 increase with gear rotation, albeit with a small increment, and the total oil for cooling continuously decreases.
The total oil volume for the lubrication of a gear is the sum of the oil volume for the lubrication of meshing teeth 1 and 2. The end of the meshing area of the first two teeth. The total oil volume for the lubrication of a gear is the sum of the oil volume for the lubrication of meshing tooth 1 and 3. As the gear meshes, the relative slide velocity, oil film thickness and oil volume for the lubrication of meshing tooth 1 increase linearly in the single tooth meshing area. The sudden increase in load of meshing tooth 1 reduces the oil film thickness. Therefore, the oil volume for the lubrication of meshing tooth 1 plummets at the beginning of the single tooth meshing area due to the decrease in oil film thickness.

The relative slide velocity and friction coefficient are compared during the meshing process, as shown in Figure 4. The two maxima for the oil volume for lubrication exist at the intersection of the meshing areas of the first two teeth and the single tooth. The total oil volume for the lubrication of meshing tooth 1 are larger than those for meshing tooth 3, and they increase continuously with gear meshing. The oil volume for the lubrication of two meshing teeth reaches the maximum value at the end of the second two tooth meshing area. Entrainment velocity, oil film thickness and oil volume for the lubrication of meshing tooth 1 increase linearly in the single tooth meshing area. The sudden increase in load of meshing tooth 1 reduces the oil film thickness. Therefore, the oil volume for the lubrication of meshing tooth 1 plummets at the beginning of the single tooth meshing area.

The analysis of the second section shows that the relative slide velocity and friction coefficient play key roles in the calculation of the oil volume for cooling. Therefore, the relative slide velocity for the cooling of meshing tooth 3 occur at this point. The relative slide velocity for the cooling of each teeth and the total oil volume for the cooling of a gear increase suddenly at the beginning of the single tooth meshing area due to the decrease in oil film thickness.

Figure 4 shows that the friction coefficient at the pitch point is equal to zero and that of the tooth meshing line. In a meshing circle, two equal maxima for the oil volume for lubrication exist at the intersection of the meshing areas of the first two teeth and the second two tooth meshing areas. The total oil for cooling continuously decreases. The oil volume for the lubrication of the gear increases as the distance between the meshing and pitch points increases, because the friction coefficient increases as the relative slide velocity increases and oil film thickness decreases. The friction coefficient in the beginning of the meshing process is greater than that at the end of meshing process because the oil film thickness and relative slide speed at the end of meshing process help reduce friction.
When the gear is in the single tooth meshing area, meshing tooth 1 meshes separately. The friction coefficient and oil volume for the cooling of meshing tooth 1 increase suddenly at the beginning of the single tooth meshing area due to the decrease in oil film thickness. The oil volume for the cooling of the gear is equal to that of meshing tooth 1, but less than that of two tooth meshing area.

Meshing tooth 3 starts to mesh at beginning of the second two tooth meshing area. The maximum friction coefficient and oil volume for the cooling of meshing tooth 3 occur at this point. The total oil volume for the cooling of the gear is the sum of the oil volume for the cooling of meshing teeth 1 and 3. As the gear meshes, the relative slide velocity, oil film thickness and oil volume for the cooling of meshing tooth 3 decrease, but those of meshing tooth 1 increase. The total oil for the cooling of the gear still decreases because of the small increment of oil for the cooling of meshing tooth 1. Oil volume for the cooling of the gear has two maxima, which are located at the beginning of the first two tooth meshing and the second two tooth meshing areas.

4. Effect of Gear Parameters

Under the same working conditions, various parameters of the gear pair, such as number of teeth, modulus and transmission ratio, will lead to different contact states between tooth surfaces that will further affect the oil volumes for the lubrication and cooling of the gear.

4.1. Effect of Modulus

Modulus is a basic parameter of the gear that determines its size and exerts a significant impact on transmission performance. Moduli are set to 3, 4, 5, 6 and 8, and the other parameters listed in Table 1 remain the same. Several lubrication simulations are conducted to analyse the influence of modulus on lubricating oil demand.

The variation in oil volume for the lubrication of the gear under different moduli during the meshing process is obtained, as shown in Figure 6a. This figure indicates that the length of the actual meshing line, the width of the first two teeth area, the width of the single tooth area and the width of the second two tooth area increase as the modulus of the gear increases. In addition, the oil volume for the lubrication of the gear increases with increasing modulus of the gear because the diameter of the gear increases with increasing modulus. The entrainment velocity and oil film thickness at the same rotational speed increase as the modulus of the gear increases. Notably, the gaps of the oil volume for the lubrication between the two tooth meshing and the single tooth meshing areas increase as the modulus of the gear increases.

The preceding analysis demonstrates that the friction coefficient is the primary influencing factor of the oil volume for cooling. The variations in the friction coefficient at different moduli with gear rotation are obtained, as shown in Figure 6b, including a partially enlarged image. The friction coefficient decreases as the modulus increases, because the radius of the tooth surface curvature increases as the modulus of the gear increases and the friction coefficient is inversely proportional to the radius of the curvature.

A series of simulations is performed to examine the influence of different moduli of the gear on the oil volume for cooling, and an enlarged image of the single meshing area is shown in Figure 6c. The oil volume for cooling decreases as the modulus of the gear increases due to the friction coefficient.
4. Effect of Gear Parameters

Under the same working conditions, various parameters of the gear pair, such as number of teeth, modulus and transmission ratio, will lead to different contact states between tooth surfaces that will further affect the oil volumes for the lubrication and cooling of the gear.

4.1. Effect of Modulus

Modulus is a basic parameter of the gear that determines its size and exerts a significant impact on transmission performance. Moduli are set to 3, 4, 5, 6 and 8, and the other parameters listed in Table 1 remain the same. Several lubrication simulations are conducted to analyse the influence of modulus on lubricating oil demand.

The variation in oil volume for the lubrication of the gear under different moduli during the meshing process is obtained, as shown in Figure 6a.

Figure 6. Oil volume for lubrication (a), friction coefficient (b), and the oil volume for cooling (c) under different moduli during the meshing process.
4.2. Effect of Number of Teeth

Number of teeth is an important parameter of gears that significantly affects the lubrication state between tooth surfaces during the meshing process and the performance of a gear transmission device. This section focuses on the influence of different numbers of teeth on the oil volumes for lubrication and cooling. The numbers of teeth in the pinion are set to 20, 23, 25 and 30. The other gear parameters listed in Table 1 remain the same. A series of calculations is used to analyse the influence of different numbers of teeth on the oil volume for lubrication during the meshing process, as shown in Figure 7a.

This figure indicates that the width of the single tooth meshing area decreases as the number of teeth increases, and the oil volume for the lubrication of the gear increases as the number of teeth increases because the size of the gear increases as the number of teeth increases. Consequently, the entrainment velocity and oil film thickness of all the meshing teeth increase at the same rotational speed. Notably, the difference in the oil volume for cooling increases as the number of teeth increases.

To compare the oil volume for cooling at different numbers of teeth, the variations in the friction coefficient that play a crucial role in the oil volume for cooling are obtained first, as shown in Figure 7b, which includes a partially enlarged image.

Figure 7b shows that the friction coefficient decreases as the number of teeth increases because oil film thickness increases as the number of teeth increases. A series of simulations is performed to examine the influence of different numbers of teeth, as shown in Figure 7c, and the enlarged image of the single meshing area is presented.

Figure 7c shows the decrease in the oil volume for cooling as the number of teeth increases. This behaviour is affected by the friction coefficient.

4.3. Effect of Transmission Ratio

In this section, the transmission ratio of a gear is used as the variable to examine the influence of transmission ratio on the oil volume for lubrication and cooling. The transmission ratios are set to 1, 2.95, 4.7 and 9.3. The other gear parameters are identical to those described in Section 3. The variations in the oil volume for lubrication during the meshing process at different transmission ratios are obtained using a series of simulations, as shown in Figure 8a.

This figure indicates that the oil volumes for lubrication in the first two tooth meshing and the second two tooth meshing areas are invariable, and their values are identical when the transmission ratio of the gear is 1. The oil volume for lubrication in the single tooth meshing area is half that in the two tooth meshing area, because the entrainment velocity and oil film thickness of each meshing tooth during the meshing process are invariable. Moreover, the distance between the start of the meshing and pitch point increases as the transmission ratio increases and the width of the single meshing area decreases. The rangeability of the oil volume for lubrication along the mesh line (i.e., the slope of the curve of the oil volume for lubrication) increases as the transmission ratio increases. The oil volume for lubrication at the beginning of meshing decreases and the oil volume for lubrication at the end of meshing increases as the transmission ratio increases, because a similar variation trend occurs in entrainment velocity and oil film thickness at different transmission ratios.

A series of simulations used to examine the influence of different transmission ratios on the friction coefficient during the meshing process and the variations in the oil volume for cooling at different transmission ratios during the meshing process are shown in Figure 8b,c, respectively. These figures contain partially enlarged images.

Figure 8b indicates that the friction coefficient curve is symmetric around the pitch point when the transmission ratio of the gear is 1 and the friction coefficients at the start and end of meshing are consistent. The friction coefficient during the meshing process decreases as the transmission ratio increases. However, the friction coefficient at the start of meshing increases as the transmission ratio increases.
Figure 7. Oil volume for lubrication (a), friction coefficient (b), and oil volume for cooling (c) under different numbers of teeth during the meshing process.
Different transmission ratios during the meshing process are shown in Figure 8b,c, respectively. These figures contain partially enlarged images. The friction coefficient during the meshing process and the variations in the oil volume for cooling at the end of meshing increases as the transmission ratio increases, because a similar consistent. The friction coefficient during the meshing process decreases as the transmission ratio increases because two teeth are meshed simultaneously at these meshing points. However, the friction coefficient at the start of meshing increases as the transmission ratio increases because the entrainment velocity and oil film thickness of each meshing tooth during the meshing process are invariable. Moreover, the distance between the start of the two tooth meshing area decreases. The rangeability of the oil volume for lubrication along the mesh line (i.e., meshing and pitch point increases as the transmission ratio increases and the width of the single meshing area decreases. The rangeability of the oil volume for lubrication in the single tooth meshing area is half that in the two tooth meshing area, because the entrainment velocity and oil film thickness of each meshing tooth are different. The friction coefficient that play a crucial role in the oil volume for cooling are obtained first, as shown in Figure 7b, which includes a partially enlarged image.

Figure 8. Oil volume for lubrication (a), friction coefficient (b), and oil volume for cooling (c) under different transmission ratios during the meshing process.
Figure 8c illustrates that the oil volume for cooling decreases as the transmission ratio increases. However, the oil volumes for cooling at the start of meshing and at the intersection of the single meshing teeth and the second meshing teeth areas initially decrease and then increase as the transmission ratio increases because two teeth are meshed simultaneously at these meshing points. One of the meshing teeth is found at start of meshing, and the friction coefficient and oil volume for cooling of this meshing tooth increase as the transmission ratio increases. However, the friction coefficient and oil volume for cooling of the other meshing tooth decrease as the transmission ratio increases. When the increase in the transmission ratio is small, the increase in the oil volume for cooling of the meshing tooth at the start of meshing is smaller than the decrease in the oil volume for cooling of the other meshing tooth. Thus, the total oil volume for cooling of the gear decreases. Moreover, when the transmission ratio increases considerably, the increase in the oil volume for cooling of the meshing tooth at the start of meshing is greater than the decrease in the oil volume for cooling of the other meshing tooth. Hence, the total oil volume for cooling of the gear increases.

5. Effect of Working Conditions

The contact and oil film states between the tooth surfaces under different working conditions, including the input speed and input torque, exhibit significant differences. This situation results in different oil volumes for lubrication and cooling. Therefore, the influence of working conditions on the oil volumes for lubrication and cooling during the meshing process should be examined.

An analysis method similar to that described in Section 4 is used to investigate the influence of input speed on lubricating oil demand. The input speeds are set to 4000, 8000, 10,000 and 12,000 rpm. The parameters of the gear and input torque are identical to those described in Section 3. The variations in the oil volume for lubrication during the meshing process at different input speeds are obtained through a series of simulations, as shown in Figure 9a.

This figure indicates that the input speed of the gear has no effect on the meshing characteristics of the gear. The oil volume for lubrication increases as the input speed increases because the entrainment velocity and oil film thickness of each meshing tooth during the meshing process increases as the input speed increases. A series of simulations is performed to examine the influence of different input speeds on the friction coefficient during the meshing process, as shown in Figure 9b, and partially enlarged images are obtained concurrently.

Oil film thickness and relative slide velocity increase as the input speed increases. The friction coefficient decreases as oil film thickness increases, but it increases as relative slide velocity increases. Figure 9b illustrates that the friction coefficient decreases slightly as the input speed increases because of the considerable influence of oil film thickness on the friction coefficient. The oil volume for cooling at different input speeds during the meshing process is compared, as shown in Figure 9c with a partially enlarged image.

This figure indicates that the oil volume for cooling increases as the input speed increases because the relative slide velocity exerts a significant influence on the oil volume for cooling although the friction coefficient decreases as input speed increases. The oil for cooling increases as the relative slide velocity increases.

To compare lubricating oil demand at different input torques during the meshing process, the input torques are set to 40, 60, 80 and 100 Nm. The parameters of the gear and input speed are the same as those described in Section 3. The variations in the oil volume for lubrication during the meshing process at different input torques are obtained using a series of simulations, as shown in Figure 10a.

This figure illustrates that the oil volume for lubrication decreases as the input torque increases because oil film thickness decreases as the input torque increases. The thin oil film leads to an increase in the friction coefficient as the input torque increases, as shown in Figure 10b. A series of simulation models is used to examine the influence of different input torques on the oil volume for cooling during the meshing process, as shown in Figure 10c.
5. Effect of Working Conditions

The contact and oil film states between the tooth surfaces under different working conditions, including the input speed and input torque, exhibit significant differences. This situation results in different oil volumes for lubrication and cooling. Therefore, the influence of working conditions on the oil volumes for lubrication and cooling during the meshing process should be examined.

An analysis method similar to that described in Section 4 is used to investigate the influence of input speed on lubricating oil demand. The input speeds are set to 4000, 8000, 10,000 and 12,000 rpm. The parameters of the gear and input torque are identical to those described in Section 3. The variations in the oil volume for lubrication during the meshing process at different input speeds are obtained through a series of simulations, as shown in Figure 9a.

This figure indicates that the input speed of the gear has no effect on the meshing characteristics of the gear. The oil volume for lubrication increases as the input speed increases because the entrainment velocity and oil film thickness of each meshing tooth during the meshing process increases as the input speed increases. A series of simulations is performed to examine the influence of different input speeds on the friction coefficient during the meshing process, as shown in Figure 9b, and partially enlarged images are obtained concurrently.

Oil film thickness and relative slide velocity increase as the input speed increases. The friction coefficient decreases as oil film thickness increases, but it increases as relative slide velocity increases. Figure 9b illustrates that the friction coefficient decreases slightly as the input speed increases because of the considerable influence of oil film thickness on the friction coefficient. The oil volume for cooling at different input speeds during the meshing process is compared, as shown in Figure 9c with a partially enlarged image.

This figure indicates that the oil volume for cooling increases as the input speed increases because the relative slide velocity exerts a significant influence on the oil volume for cooling although the friction coefficient decreases as input speed increases. The oil for cooling increases as the relative slide velocity increases.

To compare lubricating oil demand at different input torques during the meshing process, the input torques are set to 40, 60, 80 and 100 Nm. The parameters of the gear and input speed are the same as those described in Section 3. The variations in the oil volume for lubrication during the meshing process at different input torques are obtained using a series of simulations, as shown in Figure 10a.

![Figure 9. Oil volume for lubrication (a), friction coefficient (b) and oil volume for cooling (c) at different input speeds during the meshing process.](image-url)
Figure 9. Oil volume for lubrication (a), friction coefficient (b) and oil volume for cooling (c) at different input speeds during the meshing process.

This figure indicates that the input speed of the gear has no effect on the meshing characteristics of the gear. The oil volume for lubrication increases as the input speed increases because the entrainment velocity and oil film thickness of each meshing tooth during the meshing process increases as the input speed increases. A series of simulations is performed to examine the influence of different input speeds on the friction coefficient during the meshing process, as shown in Figure 9b, and partially enlarged images are obtained concurrently.

Oil film thickness and relative slide velocity increase as the input speed increases. The friction coefficient decreases as oil film thickness increases, but it increases as relative slide velocity increases. Figure 9b illustrates that the friction coefficient decreases slightly as the input speed increases because of the considerable influence of oil film thickness on the friction coefficient. The oil volume for cooling at different input speeds during the meshing process is compared, as shown in Figure 9c with a partially enlarged image.

This figure indicates that the oil volume for cooling increases as the input speed increases because the relative slide velocity exerts a significant influence on the oil volume for cooling although the friction coefficient decreases as input speed increases. The oil for cooling increases as the relative slide velocity increases.

To compare lubricating oil demand at different input torques during the meshing process, the input torques are set to 40, 60, 80 and 100 Nm. The parameters of the gear and input speed are the same as those described in Section 3. The variations in the oil volume for lubrication during the meshing process at different input torques are obtained using a series of simulations, as shown in Figure 10a.

Figure 10. Oil volume for lubrication (a), friction coefficient (b), and oil volume for cooling (c) at different input torques during the meshing process.

This figure illustrates that the oil volume for lubrication decreases as the input torque increases because oil film thickness decreases as the input torque increases. The thin oil film leads to an increase in the friction coefficient as the input torque increases, as shown in Figure 10b. A series of simulation models is used to examine the influence of different input torques on the oil volume for cooling during the meshing process, as shown in Figure 10c.

The oil volume for cooling increases as the input torque increases due to the friction coefficient, as shown in Figure 10c.

6. Conclusions

The theoretical calculation of the oil volumes for lubrication and cooling is established in this study. Lubricating oil demand during the meshing process is obtained by using different gear parameters under different working conditions. The following conclusions can be drawn from this study.

1) During the meshing of gears, two equal maxima of the oil volume for lubrication at the end of first two tooth meshing and second two tooth meshing areas exist in a meshing circle. Meanwhile, the two maxima of the oil volume for cooling of the gear are located at the start of the first two tooth meshing and the second two tooth meshing areas.

2) At the beginning and end of meshing, gears are prone to failure.

3) The maximum lubricating oil demand during the meshing process should be used as the oil supply for lubrication system design.
During the meshing of gears, two equal maxima of the oil volume for lubrication at the end of first two tooth meshing and second two tooth meshing areas exist in a meshing circle. Meanwhile, the two maxima of the oil volume for cooling of the gear are located at the start of the first two tooth meshing and the second two tooth meshing areas.

At the beginning and end of meshing, gears are prone to failure.

The maximum lubricating oil demand during the meshing process should be used as the oil supply for lubrication system design.

Oil for lubrication increases, but oil for cooling decreases as the modulus increases.

As the number of teeth increases, the oil volume for lubrication increases, but the oil volume for cooling decreases.

The rangeability of the oil volume for lubrication along the mesh line (i.e., the slope of the curve of the oil volume for lubrication) increases as the transmission ratio increases. The oil volume for cooling decreases as the transmission ratio increases. However, the oil volume for cooling at the start of meshing and at the intersection of the single tooth meshing and the second two tooth meshing areas initially decreases and then increases as the transmission ratio increases.

The oil volume for lubrication increases as the input speed increases but decreases as the input torque increases. The oil volume for cooling increases as the input speed and torque increase.

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