How to Measure $CP$ Violation in Neutrino Oscillation Experiments?

Hisakazu Minakata$^1$ and Hiroshi Nunokawa$^2$

$^1$Department of Physics, Tokyo Metropolitan University
Minami-Osawa, Hachioji, Tokyo 192-03, Japan

$^2$Instituto de Física Corpuscular - C.S.I.C.
Departament de Física Teòrica, Universitat de València
46100 Burjassot, València, Spain

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Abstract

We propose a new method for measuring $CP$ violation in neutrino oscillation experiments. The idea is to isolate the term due to the $CP$-violating phase out of the oscillation probability by taking difference between yields of two (or three) detectors at path-lengths $L = 250 \left( \frac{E}{1.35 \text{GeV}} \right) \left( \frac{\Delta m^2}{10^{-2} \text{eV}^2} \right)^{-1}$ km and at $L/3$ (and also at $2L/3$ in the case of three detectors). We use possible hierarchies in neutrino masses suggested by the astrophysical and the cosmological observations to motivate the idea and to examine how the method works.

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CP violation in the lepton sector is an unexplored fascinating subject in particle physics. If observed, it should shed light on the deep relationship between quarks and leptons, the most fundamental structure of matter which we know to date. Moreover, it is suggested that CP violation in the lepton sector is the one of the key ingredients of the mechanism for generating baryon number asymmetry in the universe \[1\].

A viable way of observing CP violation in the lepton sector is to utilize the phenomenon of neutrino oscillation. It was pointed out in refs. \[2,3\] that the difference between oscillation probabilities of the neutrinos and its antineutrinos is proportional to the leptonic analogue of the Jarlskog factor \[4\], the unique (in three-flavor neutrinos) phase-convention independent measure for CP violation. Recently, measuring CP violation in long-baseline neutrino oscillation experiments is of considerable interest in the literature \[5–9\].

However, there exists a potential obstacle in measuring CP violation in long-baseline neutrino experiments. It is the problem of the contamination due to the matter effect. Since the earth matter is not CP symmetric its effect inevitably produces the fake CP violation \[10\] which acts as contamination to the genuine CP-violating effect due to the leptonic Kobayashi-Maskawa phase \[11\]. Even worse, the matter effect dominates over the CP phase effect in a certain region of the mixing parameters in the $\nu_\mu \rightarrow \nu_e$ experiment \[5–8\].

In this paper, we suggest a novel way of measuring CP violation in neutrino oscillation experiments by proposing the multiple detector difference method.\footnote{Preliminary descriptions of the two-detector difference method were given in ref. \[12\]. We should mention that the idea of placing multiple detectors in long-baseline experiments is not new. For example, it appeared in Brookhaven proposal \[13\]. But their motivation and the basic idea behind the use of the multiple detectors is entirely different from ours, and they do not discuss the possibility of measuring CP violation.} We will show that our method is relatively free from the problem of the matter effect contamination in long-baseline
neutrino oscillation experiments.

One way of avoiding the problem of “matter effect pollution” is to look for the oscillation channel in which genuine $CP$ violating effect dominates over the matter effect. This idea is examined in detail in ref. [8] within the restriction of the neutrino mass hierarchy to that motivated by hot dark matter and the atmospheric neutrino anomaly. It is found that under the constraints from the terrestrial experiments the unique case where the $CP$ phase effect dominates is the $\nu_\mu \rightarrow \nu_e$ channel in the region of large-$s_{13}$ and arbitrary-$s_{23}$ (the region (B) to be defined later). The $\nu_\mu \rightarrow \nu_\tau$ channel is relatively free from matter effect contaminations but they are not negligible. Unfortunately, the expected $CP$ violating effect is at most $\sim 1\%$ in this type of mass hierarchy due to the strong constraints on mixing angles from the terrestrial experiments.

These discussions of the absolute and the relative magnitudes of $CP$ violation is based on the $\nu - \bar{\nu}$ difference method. The major experimental problem with this method is the difficulty in determining relative normalization of the neutrino and the antineutrino beams. If the $CP$ violation to be measured is of the order of a few $\%$ it would require to calibrate the flux of neutrino beams to the accuracy better than it, which would be extremely difficult, if not impossible, experimentally.

The multiple detector difference method which we shall discuss in this paper aims to overcome this problem. Since the absolute flux of the neutrino beam is hard to determine to the accuracy better than $\sim 10\%$ [14] we may have to give up the comparison between the two beams, or two different experiments, if we want to do measurement with a few $\%$ level accuracy. Therefore, we stick to use a single neutrino beam, $\nu_\mu$ for example, in the experiment. Then, how can one measure $CP$ violation to such precision, or even to the accuracy of $1\%$ level?

In this paper we confine ourselves into the three-flavor mixing scheme of neutrinos. To

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2Related but different proposals in this context have been discussed in ref. [7].
develop the multiple detector difference method we work with the mass hierarchy

\[ \Delta M^2 \equiv \Delta m^2_{13} \approx \Delta m^2_{23} \gg \Delta m^2_{12} \equiv \Delta m^2, \] (1)

where \( \Delta m^2_{ij} \equiv m^2_j - m^2_i \) \( (i,j = (1,3), (2,3), (1,2)) \), motivated by the solar and the atmospheric neutrino observation \([15,16]\) and the neutrinos as hot dark matter in the mixed dark matter cosmology \([17]\).

We first focus on the case of mass hierarchy motivated by the dark-matter and the atmospheric neutrino observation, \( \Delta M^2 = 5 - 100 \text{ eV}^2 \) and \( \Delta m^2 = 10^{-3} - 10^{-2} \text{ eV}^2 \). To find a hint on how to isolate the \( CP \) violating term we first ignore the matter effect and analyze the structure of the neutrino oscillation probability in vacuum. With the mass hierarchy the oscillation probability in vacuum for the long-baseline experiments can be written as

\[ P(\nu_\beta \rightarrow \nu_\alpha) = A_{\beta\alpha} + B_{\beta\alpha} (1 - \cos \Delta) + C_{\beta\alpha} \sin \Delta \] (2)

with

\[ \Delta \equiv \frac{\Delta m^2 L}{2E}, \]

where \( L \) denotes the path-length of the baseline, and \( A_{\beta\alpha}, B_{\beta\alpha} \) and \( C_{\beta\alpha} \) are constants which depend on the mixing angles and the \( CP \) phase. We note that \( C_{\beta\alpha} = 2J \), up to the sign, where \( J \) indicates the Jarlskog factor whose explicit expression will be given later. (See eq.(10).) The rapid oscillation due to large \( \Delta M^2 \),

\[ \frac{\Delta M^2 L}{4E} = 127 \left( \frac{\Delta M^2}{1 \text{ eV}^2} \right) \left( \frac{L}{100 \text{ km}} \right) \left( \frac{E}{1 \text{ GeV}} \right)^{-1}, \] (3)

is averaged out which produces the first term in (2).

We want to isolate the last term, \( J \)-term, which is the measure for \( CP \) violation from the others. It is a simple matter to observe that the best way to carry this out is to do the measurements at \( \Delta = \frac{\pi}{2} \) and \( \frac{3}{2} \pi \):

\[ P(\nu_\beta \rightarrow \nu_\alpha; \; \Delta = \frac{3\pi}{2}) = A_{\beta\alpha} + B_{\beta\alpha} + 2J \]

\[ P(\nu_\beta \rightarrow \nu_\alpha; \; \Delta = \frac{\pi}{2}) = A_{\beta\alpha} + B_{\beta\alpha} - 2J \] (4)
where we took a particular sign for the $J$-term. Therefore, the difference $\Delta P$ between the oscillation probabilities at $\Delta = 3\frac{\pi}{2}$ and at $\Delta = \frac{\pi}{2}$ is nothing but the $CP$ violation $4J$.

Of course, $\Delta$ is a function of $L/E$. Therefore, the measurement of the difference $P(\Delta = 3\frac{\pi}{2}) - P(\Delta = \frac{\pi}{2}) \equiv \Delta P_2$ can be done either by varying $L$ or $E$, or both. However, we argue that a measurement of $CP$ violation with accuracy better than a few % would require measurements of $\Delta P_2$ by placing two detectors at $\Delta = \frac{\pi}{2}$ and $\Delta = 3\frac{\pi}{2}$. If we try to measure $\Delta P_2$ by varying the energy of the neutrino beam with the single detector the energy, of course, have to be varied by factor of 3. By such re-adjustment of the neutrino beam energy the relative normalization of the beam would become uncertain to the order of $\sim 10\%$. Therefore, the best thinkable way of avoiding the uncertainty of relative normalization of the neutrino flux is to do measurement at 2 detectors, using the same neutrino beam, one at $\Delta = 3\frac{\pi}{2}$, and the other at $\Delta = \frac{\pi}{2}$. For the KEK-PS→Superkamiokande experiment in which $L = 250$ km, the neutrino beam energy should be tuned to $E = 1.35(\frac{\Delta m^2}{10^{-2}eV^2})^{-1}$ GeV so that the location of Superkamiokande just corresponds to $\Delta = 3\frac{\pi}{2}$. For the MINOS experiment with $L = 730$ km the beam energy to be used is $E = 3.94(\frac{\Delta m^2}{10^{-2}eV^2})^{-1}$ GeV. Then, the second detector to be build should be located at $1/3$ of the baseline, $L_2 = 83.3$ km for the KEK-PS→Superkamiokande and $L_2 = 243$ km for the MINOS experiments.

Of course, one cannot make neutrino beam so monochromatic; it must have spread in energy. However, it seems possible to make the neutrino beam spread as small as $\sim 20\%$ of the beam energy [14]. Therefore, at least it is worth to think about the possibility.

Does the matter effect give rise to the serious contamination to the genuine $CP$ violating effect in the 2 detector difference method? It appears that the problem has a better shape

3The possibility of observing $CP$ violation in neutrino oscillation experiments by measurement at differing path-length, or by varying energy has been pointed out in ref. [18].

4Their geographical locations are, respectively, at around the city of Honjo in Saitama prefecture and a midpoint between Westfield and Packwaukee, about 55 Miles north of Madison, Wisconsin.
compared to that in the $\nu - \bar{\nu}$ difference method. To understand this point we write down the expression of neutrino oscillation probability with the correction of the earth matter effect. The expression is based on the adiabatic approximation and is valid to first order in matter perturbation theory. If we define the neutrino mixing matrix as $\nu_\alpha = U_{\alpha i} \nu_i$ the oscillation probability is given by

$$P(\nu_\beta \rightarrow \nu_\alpha) =$$

$$-2 \sum_{i=1,2} \left[ \text{Re}[U_{\alpha i} U_{\alpha i}^* U_{\beta i} U_{\beta i}] + \text{Re}(UUU\delta V)_{\alpha \beta i; 3} \right]$$

$$-4 \text{Re}[U_{\alpha 1} U_{\alpha 2}^* U_{\beta 1} U_{\beta 2}] \left[ \sin^2 \left( \frac{\Delta m^2}{4E_L} L \right) + \frac{1}{2} a_L \left( |U_{e 2}|^2 - |U_{e 1}|^2 \right) \sin \left( \frac{\Delta m^2}{2E_L} L \right) \right]$$

$$-2J \left[ \sin \left( \frac{\Delta m^2}{2E_L} L \right) + a_L \left( |U_{e 2}|^2 - |U_{e 1}|^2 \right) \cos \left( \frac{\Delta m^2}{2E_L} L \right) \right]$$

$$-4 \text{Re}(UUU\delta V)_{\alpha \beta ; 12} \sin^2 \left( \frac{\Delta m^2}{4E_L} L \right)$$

$$-2 \text{Im}(UUU\delta V)_{\alpha \beta ; 12} \sin \left( \frac{\Delta m^2}{2E_L} L \right)$$

(5)

where $(UUU\delta V)_{\alpha \beta ; ij}$ represent first-order corrections due to the matter effect and their expressions are given in ref. $[8]$. More precisely speaking, we made the following approximation to derive eq. (5): We took average over rapid oscillations with the period $[3]$, which produces the first two terms in (5). We ignored the terms of the order of $E a \Delta M^2$ because of the extreme hierarchy between the dark matter mass scale and the matter potential,

$$\frac{E a}{\Delta M^2} = 1.04 \times 10^{-4} \left( \frac{\rho}{2.72 \text{g/cm}^3} \right) \left( \frac{E}{1 \text{GeV}} \right) \left( \frac{\Delta M^2}{1 \text{eV}^2} \right)^{-1}. \quad (6)$$

We used the constant matter density approximation and ignored the terms of order $(aL)^2$ or higher, where $L$ is the path length of the baseline and $a = \sqrt{2} G_F N_e$ with $N_e$ being the electron number density. We note that

$$aL = 0.132 \left( \frac{\rho}{2.72 \text{g/cm}^3} \right) \left( \frac{L}{250 \text{km}} \right). \quad (7)$$

Therefore, ignoring $(aL)^2$ term should give a good approximation at least for the KEK→Superkamiokande experiment.

The difference between probabilities $\Delta P_2$ to be measured at 2 detectors up to the first order in matter potential $a$ is given by
\[ \Delta P_2(\nu_\beta \rightarrow \nu_\alpha) \equiv P(\nu_\beta \rightarrow \nu_\alpha; \Delta = \frac{3}{2} \pi) - P(\nu_\beta \rightarrow \nu_\alpha; \Delta = \pi) \]
\[ = 4J - 8\pi \frac{E_a}{\Delta m^2} \text{Re}[U_{\alpha_1}^* U_{\beta_1}^* U_{\alpha_2} U_{\beta_2}] \cos 2\theta_{12} c_{13}^2 \\
+ 8J \frac{E_a}{\Delta m^2} \cos 2\theta_{12} c_{13}^2 \]
\[ (8) \]

where \((\alpha, \beta) = (e, \mu), (\mu, \tau)\) and \((\tau, e)\). We have used the standard form of the CKM matrix
\[ U = \begin{bmatrix}
    c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{-i\delta} \\
    -s_{12} c_{23} - c_{12} s_{23} s_{13} e^{i\delta} & c_{12} c_{23} - s_{12} s_{23} s_{13} e^{i\delta} & s_{23} c_{13} \\
    s_{12} s_{23} - c_{12} c_{23} s_{13} e^{i\delta} & -s_{12} s_{23} - s_{12} c_{23} s_{13} e^{i\delta} & c_{23} c_{13}
\end{bmatrix} \]
\[ (9) \]
for the neutrino mixing matrix. With this parametrization, \(J\) is given as
\[ J \equiv \text{Im}[U_{\alpha_1} U_{\alpha_2}^* U_{\beta_1}^* U_{\beta_2}] = \pm c_{12} s_{12} c_{23} s_{23} s_{13}^2 s_{13} \sin \delta, \]
\[ (10) \]

where + sign is for cyclic permutations, i.e., \((\alpha, \beta) = (e, \mu), (\mu, \tau), (\tau, e)\) and – is for anti-cyclic ones.

We note that in the mass hierarchy with dark matter mass scale the reactor and the accelerator experiments constrain the mixing angles into three regions on the plane spanned by \(s_{13}^2\) and \(s_{23}^2\) [19]. Since our present analysis is motivated by the atmospheric neutrino anomaly we restrict ourselves into the two regions (A) or (B):

(A) small-\(s_{13}\) and small-\(s_{23}\)

(B) large-\(s_{13}\) and arbitrary \(s_{23}\)

In long-baseline experiments, we expect large (\(\sim\) order 1) oscillation probabilities for \(\nu_\mu \rightarrow \nu_e\) and \(\nu_\mu \rightarrow \nu_\tau\) channels in the regions (A) and (B), respectively. For brevity we shall call these channels the dominant channels in the respective parameter regions, and the alternative ones, i.e., \(\nu_\mu \rightarrow \nu_\tau\) in the regions (A) etc. the minor channels. Note that
\[- 4\text{Re}[U_{\alpha_1} U_{\alpha_2}^* U_{\beta_1}^* U_{\beta_2}]\]
is nothing but the coefficient of \(\sin^2(\frac{\Delta m^2}{4E}L)\) term in the oscillation probabilities, and therefore is of order unity for the dominant channels. For \(\Delta M^2 = 5 \text{ eV}^2\), \(4J\) is at most \(\sim 10^{-2}\) in both regions (A) and (B). See Fig. 1 of ref. [8]. We note that
\[ \frac{E_a}{\Delta m^2} \sim 10^{-2}\]
in the mass hierarchy with which we are dealing.
Now we can roughly estimate the relative magnitude of the matter and the \( CP \) violating effects in \( \Delta P_2 \). It depends upon the regions (A) and (B); In the region (A), the \( CP \) phase and the matter effects are, roughly speaking, comparable unless \( \cos 2\theta_{12} \) is tuned to be small. We have to do the experiments in the minor channel \( \nu_\mu \to \nu_\tau \) to avoid matter effect contamination. In the region (B) on the other hand, \( c_{13} \) is small, \( c_{13}^2 \sim 10^{-2} \), and therefore the \( CP \) phase effect is always dominant over matter effect. One can do experiments in the dominant channel in the region (B). However, to do subtraction between the yields of the end and the intermediate detectors it may be better to do experiments always in the minor channel; One has to subtract a large number from a comparable large number to have a small number in the case of dominant channels.

A special attention has been paid to the long-baseline neutrino experiment because with the dark matter motivated mass hierarchy the effect of \( CP \) violation cannot be observed in short-baseline experiments. It can be shown that \( CP \) violating effect is suppressed by a factor of \( \frac{\Delta m^2}{\Delta M^2} \).

Let us confirm by numerical computation that the qualitative results we have obtained so far are correct. First we pick up the following two sets of parameters (a) and (b) from the allowed regions (A) and (B), respectively,

\[
\begin{align*}
\text{(a)} & \quad s_{23}^2 = 3.0 \times 10^{-3}, \quad s_{13}^2 = 2.0 \times 10^{-2} \\
\text{(b)} & \quad s_{23}^2 = 2.0 \times 10^{-2}, \quad s_{13}^2 = 0.98.
\end{align*}
\]

They are chosen so that the largest \( CP \) violating effect is expected in each region (A) or (B), and the same set of parameters are used in our analysis done in ref. [8].

In Fig. 1 we present \( \Delta P_2(\nu_\mu \to \nu_e) \) and \( \Delta P_2(\nu_\mu \to \nu_\tau) \) for these two parameter sets for the KEK–Superkamiokande distance \( L = 250 \) km. We have carried out the calculation using the exact solutions obtained in ref. [20] for a constant matter density and we used \( \rho = 2.72 \) gcm\(^{-3}\) with the electron fraction \( Y_e = 0.5 \). We took average over the rapid oscillations due to the dark matter scale \( \Delta M^2 \). In the same figure we also plot the values obtained by our analytic formula [8] to indicate that it gives a reasonably good approximation.
In Fig. 2 we present the same quantities but for $L = 730$ km, i.e., Fermilab-Soudan2 detector distance (MINOS experiment). This is important for $\nu_\mu \to \nu_\tau$ channel because $\tau$ cannot be produced with such low neutrino energy as $E = 1.35$ GeV. The nice feature of $\Delta P_2$ in Figs. 1 and 2 is that the matter effect contamination is relatively smaller than that in the $\nu - \bar{\nu}$ difference method discussed in ref. [8]. This is particularly true for $L = 250$ km. Therefore, if we can measure $\Delta P_2$ to the accuracy of $\sim 1\%$ level it is, in principle, possible to observe CP violation in neutrino oscillation experiments.

Now we turn to the question of how the multiple detector difference method works for the neutrino mass hierarchy motivated by the atmospheric [16] and the solar [15] neutrino observations, $\Delta M^2 = 10^{-3} - 10^{-2}$eV$^2$ and $\Delta m^2 = 10^{-6} - 10^{-4}$eV$^2$. In this case we cannot use the matter perturbation theory developed in ref. [8] because $\frac{E_\alpha \Delta m^2}{\Delta M^2} \simeq 1 - 10^2$ cannot be used as an expansion parameter. In this paper we rely on the approximate formula derived by Arafune, Koike and Sato [7] who use as expansion parameters $aL$ in (7) and $\Delta m^2 L^2 E = 6.4 \times 10^{-2} \left( \frac{\Delta m^2}{10^{-4} \text{eV}^2} \right) \left( \frac{L}{250 \text{km}} \right) \left( \frac{E}{1 \text{GeV}} \right)^{-1}$. (13)

The general structure of the oscillation probability [7] can be expressed, to leading order of these expansion parameters, as

$$P(\nu_\beta \to \mu_\alpha) = \tilde{A}_{\beta\alpha}(1 - \cos \Delta) + \tilde{B}_{\beta\alpha} \Delta \sin \Delta + \tilde{C}_{\beta\alpha} \Delta (1 - \cos \Delta)$$ (14)

where $\Delta \equiv \frac{\Delta M^2 L}{2E}$. The coefficient of the last term is given by $\tilde{C}_{\beta\alpha} = 2J \frac{\Delta m^2}{\Delta M^2}$. The coefficient $\tilde{B}_{\beta\alpha}$ contains the terms proportional to either $\frac{\Delta m^2}{\Delta M^2}$ or $\frac{E_\alpha \Delta m^2}{\Delta M^2}$ and they have similar order of magnitude as the last term. Enriched by three terms with different $\Delta$ dependence and with similar magnitudes it is unlikely that one can separate the last term by using only 2 detectors. Then, we have to generalize our method to that of 3 detectors, the ones at $\Delta = \pi/2, \pi, \text{and } 3\pi/2$. Then, it is simple to show that

$$\Delta P_3(\nu_\beta \to \nu_\alpha) \equiv P(\Delta = \frac{3}{2} \pi) + 3P(\Delta = \frac{\pi}{2}) - 2P(\Delta = \pi) = 2\pi J \Delta m^2 \Delta M^2. \quad (15)$$

To have a feeling of the magnitude of CP violation we choose $\Delta m^2 = 10^{-4}$eV$^2$ and $\Delta M^2 = 10^{-2}$eV$^2$, and take $s_{12} = 1/2, s_{23} = 1/\sqrt{2}$, and $s_{13} = \sqrt{0.1}$, as done by Arafune et al.
Then, the RHS of (13) is estimated to be $0.39 \times 10^{-2}$. But, we should note that the parameters we picked up are not those which maximize $\Delta P_3$.

The above discussions on relative and absolute magnitudes of $CP$ violations can be confirmed by the precise computation of $\Delta P_3$ as we did for $\Delta P_2$. We plot in Fig. 3 $\Delta P_3(\nu_\mu \to \nu_e)$ and $\Delta P_3(\nu_\mu \to \nu_\tau)$ as a function of $s_{13}$. We see that the matter effect contamination is small in both channels and $\Delta P_3 \sim 0.5$ for $L = 250$ km. In Fig. 4 we plot the same as in Fig. 3 but for $L = 730$ km. In this case, however, the magnitude of matter effect can be comparable to the genuine $CP$ effect, which also implies that our analytic formula (15) is not accurate.

What are the required statistics for the experiments designed for measurements of $CP$ violation at 1 % level? Suppose that we design a long-baseline experiment which will produce $N$ muon events at each detector in the absence of oscillation. We assume that number of appearance events in the dominant channel is $\sim N/2$. Then, the number of events in the minor channel may be of the order of $\sim 10^{-3}N$. We have to make subtraction between 2 or 3 detectors. If we require the accuracy of less than 10 % uncertainty in the number of events in the minor channel it means that $N \sim 10^5$. Having such statistics with very narrow-band beam is certainly not easy to achieve, but may be possible in future Japan Hadron Project and the MINOS experiments.

The potential problem, though technical, with the multiple detector difference method is that we have to dig a tunnel down from the earth surface to build an intermediate detector. We have to dig down to 1.1 km and 9.3 km for the KEK-PS→Superkamiokande and the MINOS experiments, respectively. We hope that the technical problem can be overcome at the stage when the neutrino experiments for measuring $CP$ violation is on the time table.

To establish the multiple detector difference method much more careful studies are required on various aspects including beam design. Among them one of the most important is the effects of averaging over finite energy width of the beam; our preliminary investigation indicates the sensitivity of $\Delta P$ against the variation of neutrino energy. Keeping these problems in mind, we want to emphasize that this method may be the only practical way
of measuring $CP$ violation to the accuracy of a few $\%$ level.

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Fig. 1: We plot in (i) and (ii) $\Delta P_2(\nu_\mu \to \nu_e)$ and in (iii) and (iv) $\Delta P_2(\nu_\mu \to \nu_\tau)$ as a function of $s_{12}^2$, for the cases where only the genuine $CP$ effect (open circles), only the matter effect (dotted lines), and both effects (solid lines) exist. We fixed the other mixing parameters as $s_{23}^2 = 3.0 \times 10^{-3}$, $s_{13}^2 = 2.0 \times 10^{-2}$ for the left two panels (i) and (iii) and $s_{23}^2 = 2.0 \times 10^{-2}$, $s_{13}^2 = 0.98$ for the right two panels (ii) and (iv). The other parameters are fixed to be the same for all the case (i-iv), i.e., $E=1.35$ GeV, $\Delta M^2 = 5$ eV$^2$, $\Delta m^2 = 10^{-2}$ eV$^2$ and $\delta = \pi/2$ and $L = 250$ km. We also plot the approximated values for the case where only the matter (open squares) and the matter + $CP$ (asterisks) effects exist except for (ii) where no appreciable difference between the exact and the approximated values can be seen.
Fig. 2: The same as in Fig. 1 but for $L = 730$ km and $E = 3.94$ GeV.
Fig. 3: We plot in (i) $\Delta P_3(\nu_\mu \rightarrow \nu_e)$ and in (ii) $\Delta P_3(\nu_\mu \rightarrow \nu_\tau)$ as a function of $s_{13}^2$, for the cases where only the genuine CP effect (open circles), only the matter effect (dotted lines), and both effects (solid lines) exist. We fixed the other parameters as $s_{12}^2 = 0.25$, $s_{23}^2 = 0.5$, $E = 1.35$ GeV, $\Delta M^2 = 10^{-2}$ eV$^2$, $\Delta m^2 = 10^{-4}$ eV$^2$, $\delta = \pi/2$ and $L = 250$ km.
Fig. 4 : The same as in Fig. 3 but for $L = 730$ km and $E = 3.94$ GeV.