Super Black Hole from Cosmological Supergravity with a Massive Superparticle

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ABSTRACT

We describe in superspace a classical theory of two dimensional (1, 1) cosmological dilaton supergravity coupled to a massive superparticle. We give an exact non-trivial superspace solution for the compensator superfield that describes the supergravity, and then use this solution to construct a model of a two-dimensional supersymmetric black hole.

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1 Introduction

There are very few exact classical non-trivial solutions to supersymmetric field theories. As is elaborated on in [1], even more rare are exact superspace supergravity solutions. For classical supergravity theories, one looks for non-trivial solutions – those that cannot be reduced by infinitesimal supersymmetry transformations to purely bosonic solutions – using the method given in [2]. However, it is possible to sidestep this issue by examining classical supergravity problems in superspace [3]. A bona fide superspace supergravity solution – one which satisfies the constraints – has non-trivial torsion, a supercovariant quantity, and as such its value ultimately remains unchanged under a suitable gauge transformation. Hence an exact superspace supergravity solution must necessarily be non-trivial in this sense. Approaching classical supergravity problems from the superspace viewpoint obviates the triviality question. It is this approach that we take in this paper.

Following our previous motivation [1], we consider (1, 1) dilaton supergravity with a cosmological constant, coupled to a massive superparticle in (1 + 1) dimensions. The supergravity part of the theory is a supersymmetric generalization of the (1 + 1) dimensional “R=T” theory [4]. This theory has the unique feature that the dilaton superfield decouples from the classical equations of motion, so that supermatter induces superspace curvature, and the superspace curvature reacts back on the supermatter self-consistently. We obtain an exact solution for the supergravity compensator superfield that completely describes the supergravity in superconformal gauge, and use this compensator to construct a model of a supersymmetric black hole.

The outline of our paper is as follows. In section 2 we review bosonic cosmological dilaton gravity coupled to a massive particle. In section 3, we outline cosmological dilaton (1, 1) supergravity coupled to a massive superparticle. In section 4, we solve for the supergravity compensator in the presence of this superparticle, and in section 5, we discuss the construction of a super black hole model using this compensator.

2 Cosmological Dilaton Gravity

Before describing the supergravity action we use, we briefly review the bosonic Lagrangian of a massive point particle interacting with cosmological dilaton gravity [5], as it is simpler than the supersymmetric case, and illustrates the basic ideas.

The action for $R = T$ theory is

$$S = S_G + S_M = \frac{1}{2\kappa} \int d^2 x \left[ \sqrt{-g}(\psi R + \frac{1}{2}(\nabla \psi)^2) - \kappa L_M \right]$$

(2.1)
where the gravitational coupling $\kappa = 8\pi G$. The action (2.1) ensures that the dilaton field $\psi$ decouples from the classical equations of motion which, after some manipulation, are

$$R = \kappa T^\mu_{\mu} \quad (2.2)$$

$$\frac{1}{2} \left( \nabla_\mu \psi \nabla_\nu \psi - \frac{1}{2}(\nabla \psi)^2 \right) - \nabla_\mu \nabla_\nu \psi + g_{\mu\nu} \nabla^2 \psi = \kappa T_{\mu\nu} \quad (2.3)$$

where $T_{\mu\nu} = \frac{1}{\sqrt{-g}} \frac{\delta L}{\delta g^{\mu\nu}}$ is the stress-energy tensor and $R_{\mu\nu} = \partial_\lambda \Gamma^\lambda_{\mu\nu} - \partial_\nu \Gamma^\lambda_{\mu\lambda} - \Gamma^\lambda_{\mu\sigma} \Gamma^\sigma_{\lambda\nu} + \Gamma^\lambda_{\lambda\sigma} \Gamma^\sigma_{\mu\nu}$ is our convention for the Ricci tensor.

We take the matter Lagrangian to be that of a point particle in a spacetime with non-zero cosmological constant

$$\mathcal{L}_M = -\sqrt{-g} \Lambda + 2m \int d\tau \sqrt{-g} \frac{dz^\alpha}{d\tau} \frac{dz^\beta}{d\tau} \delta^{(2)}(x - z(\tau)) \quad (2.4)$$

so that

$$T_{\mu\nu} = \frac{1}{2} g_{\mu\nu} \Lambda + m \int d\tau \frac{1}{\sqrt{-g}} g_{\alpha\beta} \frac{dz^\alpha}{d\tau} \frac{dz^\beta}{d\tau} \delta^{(2)}(x - z(\tau)) \quad (2.5)$$

is the relevant stress energy, with $z^\mu(\tau)$ being the worldline of the particle.

Choosing a frame at rest with respect to the particle, the trace of the stress energy is

$$T^\mu_{\mu} = -2me^{-\rho} \delta(x - x_0) + \Lambda \quad (2.6)$$

in conformal coordinates where $ds^2 = e^{2\rho} dx^+ dx^- = e^{2\rho}(-dt^2 + dx^2)/4$, with the location of the particle at $x = x_0$. The field equations (2.2) then become

$$\rho''(x) = -\frac{\kappa}{8} \Lambda e^{2\rho} + Me^\rho \delta(x - x_0) \quad (2.7)$$

with $M = 2\pi Gm$.

Setting $a^2 = \frac{\kappa}{8}|\Lambda|$, equation (2.7) has for $\Lambda > 0$ the solution

$$\rho = -\ln (\cosh(a|x - x_0| + b)) \quad (2.8)$$

where $\sinh(b) = \frac{M}{2a}$. If $\Lambda < 0$, the solution for $M < 2a$ is

$$\rho = -\ln (\cos(a|x - x_0| + b)) \quad (2.9)$$

where $\sin(b) = \frac{M}{2a}$, or

$$\rho = -\ln (\sinh(b - a|x - x_0|)) \quad (2.10)$$

if $M > 2a$, where $\cosh(b) = \frac{M}{2a}$. 

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In Schwarzschild-type coordinates the metric in the $\Lambda > 0$ case (2.8) becomes
\[ ds^2 = -\left(\frac{\kappa}{2} \Lambda Y^2 + 2M|Y| + 1\right) dT^2 + \frac{dY^2}{\frac{\kappa}{2} \Lambda Y^2 + 2M|Y| + 1} \] (2.11)
whereas for $\Lambda < 0$ the solutions (2.9) and (2.10) can respectively be transformed into
\[ ds^2 = -\left(\frac{\kappa}{2} \Lambda|Y|^2 + 2M|Y| + 1\right) dT^2 + \frac{dY^2}{\frac{\kappa}{2} \Lambda|Y|^2 + 2M|Y| + 1} \] (2.12)
and
\[ ds^2 = -\left(\frac{\kappa}{2} \Lambda|Y|^2 + 2M|Y| - 1\right) dT^2 + \frac{dY^2}{\frac{\kappa}{2} \Lambda|Y|^2 + 2M|Y| - 1} \] (2.13)
The metric (2.13) is that of an anti de Sitter black hole with mass parameter $M$. This solution, along with (2.11) and (2.12), has been discussed previously in ref. [5].

3 Cosmological (1, 1) Dilaton Supergravity

We now extend the previous theory to a superspace formulation of (1, 1) dilaton supergravity in two dimensions with a cosmological constant, $L$. We use light-cone coordinates $(x^+, x^-) = \frac{1}{2}(x^1 \pm x^0)$ and $(\theta^+, \theta^-)$. In superconformal gauge, the action is given by
\[ I_C = -\frac{2}{\kappa} \int d^2x d^2\theta(D_+ \Phi D_- \Phi + 4\Phi D_- D_+ S - 4e^{-2S}L) \] (3.1)
where $\Phi$ is the dilaton superfield, $S$ is the scalar compensator superfield that completely describes the supergravity in superconformal gauge, and the flat supersymmetry covariant derivatives are given by $D_\pm = (\partial_\pm + i\theta^+ \partial^\pm, \partial_\pm + i\theta^- \partial^\pm)$. We simply list the results here, but details can be found in [1]. The equations of motion are
\[ \Phi = -2S \] (3.2)
\[ D_- D_+ S = e^{-2S}L \] (3.3)
from which it is clear that the dilaton decouples from the theory, and that we recover the superLiouville equation for the compensator, $S$.

To obtain the component form of the superspace action (3.1), we identify the components of the superfields by theta expansion (dropping the fermionic fields),
\[ S = -\frac{1}{2} \rho + \sigma \theta^+ \theta^- \] (3.4)
\[ \Phi = -\frac{1}{2} \psi + \varphi \theta^+ \theta^- \] (3.5)
and eliminate the auxiliary fields \( \varphi, \sigma \) via their equations of motion. This yields

\[
I_C = \frac{1}{2\kappa} \int d^2x \left[ -4\psi \partial_x \partial_t \rho + \partial_t \psi \partial_x \psi - 16L^2 e^{2\rho} \right]
\]  (3.6)

for the component action. This is equivalent to (2.1) (using (2.4) with \( m = 0 \)) provided \( \Lambda = -\frac{32}{k}L^2 \). Since \( L \) must be real, this implies that from the superspace action, only the component action for anti de Sitter spacetimes is recovered. Alternatively, inserting the superfield expansions (3.4,3.5) into eqs. (3.2) and (3.3) in the static case yields after some manipulation

\[
\rho''(x) = 4L^2 e^{2\rho} = -\frac{k}{8} \Lambda e^{2\rho}
\]  (3.7)

which is equation (2.1) with \( M = 0 \).

We consider now extending the action (3.1) to include a massive superparticle. We use \( z = (x,t,\theta) \) as the coordinates of the superspace, and \( z_0(t) = (x_0(t),\theta_0(t)) \) as the coordinates of the superparticle. The technical details leading up to the action (3.8) below are the same as in [1], so we supply only the results here.

The action for the superparticle in superconformal gauge is

\[
I_P = 2m \int dt dx d\theta \left\{ g^{-1}e^{-4S} \left[ \frac{i}{4}(1 + \dot{x}_0) + i\dot{\theta}_0^+ \dot{\theta}_0^- \right] \left[ \frac{i}{4}(1 - \dot{x}_0) + i\dot{\theta}_0^- \dot{\theta}_0^+ \right] \right. \\
\left. + i \left[ \frac{i}{4}(1 + \dot{x}_0) + i\dot{\theta}_0^+ \dot{\theta}_0^- \right] D_+ G_+ + i \left[ \frac{i}{4}(1 - \dot{x}_0) + i\dot{\theta}_0^- \dot{\theta}_0^+ \right] D_- G_- \right. \\
\left. + \dot{\theta}_0^+ G_+ + \dot{\theta}_0^- G_- + \frac{g}{4} \right\} \delta(x - x_0(t)) \delta(\theta^+ - \theta_0^+(t)) \delta(\theta^- - \theta_0^-(t))
\]  (3.8)

where \( g \) is the einbein on the worldline of the superparticle, and \( G_\alpha \equiv e^S \Gamma_\alpha \). The general gauge superfield \( \Gamma_\alpha \) necessarily appears in the Wess-Zumino type term in the massive superparticle action in order for consistent coupling of the flat superparticle to supergravity. Requiring that the supergravity constraints be satisfied introduces a constraint on the gauge field \( G \), and we include this constraint in the supergravity action by means of a lagrange multiplier, \( \lambda \). Consequently, the dilaton supergravity part of the action is affected and becomes

\[
I_C = -\frac{2}{\kappa} \int d^2x d^2\theta \left[ D_+ \Phi D_- \Phi + 4\Phi D_- D_+ S - 4e^{-2S}L \right. \\
\left. + \kappa \lambda e^{-2S}(D_+ G_+ + D_- G_- - ie^{-2S}) \right]
\]  (3.9)

From the sum of (3.8) and (3.9), we obtain for the equation of motion for \( S \)

\[
D_- D_+ S(z) = e^{-2S}L \\
= \frac{\kappa m}{2} \int dt' \left\{ g^{-1}e^{-4S} \left[ \frac{i}{4}(1 + \dot{x}_0) + i\dot{\theta}_0^+ \dot{\theta}_0^- \right] \left[ \frac{i}{4}(1 - \dot{x}_0) + i\dot{\theta}_0^- \dot{\theta}_0^+ \right] \right\} \delta^4(z - z_0(t')) \\
= \frac{\kappa m}{4} \sqrt{\pi^2} e^{-2S} \delta(x - x_0(t)) \delta(\theta^+ - \theta_0^+(t)) \delta(\theta^- - \theta_0^-(t))
\]  (3.10)
where $\sqrt{\pi^2} = \frac{i}{2}\sqrt{1-x_0^2}$ for a free particle. Once we obtain the solution for $S$, it is possible to solve the constraint on $G$, but we shall not present this here.

### 4 Solution for Compensator

To solve for the compensator $S$ that describes the supergravity generated by a superparticle in the presence of a cosmological constant, we consider the superparticle to be stationary and fixed at $(x_0, \theta_0)$. In this case, (3.10) becomes

$$e^{2S}D_-D_+S(z) - L = \frac{M}{2} \delta(x - x_0)\delta(\theta^+ - \theta_0^+)\delta(\theta^- - \theta_0^-)$$

where $M = \frac{\kappa m}{4}$ as before, and $\sqrt{\pi^2} = \frac{i}{2}$ for $\dot{x}_0 = 0$. We rewrite the equation in terms of $T = e^{2S}$

$$TD_+D_- - D_+TD_- = -2T(L + \frac{M}{2}\delta(x - x_0)\delta^{(2)}(\theta - \theta_0))$$

We solve the equation now for $T$ by analogy with the previous bosonic solution, and also by experience with the form of the compensator in the $L = 0$ case [1]. We find that just as in the bosonic case, the solution can be chosen either as

$$T(x, \theta) = 2L(\theta^+ - \theta_0^+)(\theta^- - \theta_0^-) + \cos[2L|X| + c]$$

with $c = -\sin^{-1}(\frac{M}{4L})$, or as

$$T(x, \theta) = 2L(\theta^+ - \theta_0^+)(\theta^- - \theta_0^-) + \sinh[c - 2L|X|]$$

with $c = \cosh^{-1}(\frac{M}{4L})$. Note that in the former case $M < 4L$, whereas in the latter case $M > 4L$. In these expressions,

$$|X| \equiv |x - x_0 - i(\theta^+\theta_0^+ + \theta^-\theta_0^-)|$$

$$\equiv |x - x_0| - i(\theta^+\theta_0^+ + \theta^-\theta_0^-)[\Theta(x - x_0) - \Theta(x_0 - x)] + 2\theta^+\theta^-\theta_0^+\theta_0^-\delta(x - x_0)$$

is to be understood as a Taylor series expansion, and $\Theta(x - x_0)$ is the Heaviside function. We note that these are specific non-trivial solutions. The most general non-trivial solution to this problem will be presented elsewhere [6].

Although we obtained the solution for $S$ assuming the particle was held fixed at $(x_0, \theta_0)$, we note that the derivatives of $S$ with respect to the particle coordinates all vanish when evaluated at the particle position. This is sufficient to show that the “force” on the particle due to the supergravity fields is zero, and hence this solution is in fact a solution to the full coupled equations of motion.
5 Discussion

The solution (4.4) is the supersymmetric analogue of the solution (2.13), and can be regarded as an anti de Sitter super black hole in two-dimensional (1,1) superspace. This solution is written in the superconformal coordinates \( z = (x, \theta) \); to facilitate comparison with the results of \([5]\), we transform now to superspace coordinates \( w = (u, \lambda) \) that correspond to Schwarzschild gauge, in which the dyad of the bosonic subspace takes the form

\[
e_m^a = \begin{bmatrix} \sqrt{\alpha} & 0 \\ 0 & \sqrt{\alpha^{-1}} \end{bmatrix}
\]

(5.1)

We find that the transformation from superconformal to Schwarzschild coordinates for \( T \) of (4.4) is

\[
(c - 2L|X|) = -\coth^{-1}\left[\frac{4L}{\sqrt{1 - \frac{M^2}{16L^2}}}(|U| + U_0)\right]
\]

(5.2)

where

\[
|U| \equiv |u - u_0 - i(\lambda^+ \lambda_0^+ + \lambda^- \lambda_0^-)|
\]

(5.3)

\[
= |u - u_0| - i(\lambda^+ \lambda_0^+ + \lambda^- \lambda_0^-)[\Theta(u - u_0) - \Theta(u_0 - u)] + 2\lambda^+ \lambda^- \lambda_0^+ \lambda_0^- \delta(u - u_0)
\]

and where \( U_0 \) is a constant. One can choose \( u_0 = 0 \) without loss of generality. It is straightforward to work out the explicit relationship between \((x, \theta^\pm)\) and \((u, \lambda^\pm)\), and also to compute explicit expressions for the gravitini, but we shall not do that here. The transformation (5.2) is actually valid only outside the event horizon for sufficiently large \(|u - u_0|\). However, once one has the expression in Schwarzschild coordinates it is easy to continue across the event horizon in a manner analogous to that for the solution (2.13). The compensator associated with the solution (4.4) transformed via (5.2) can then be used to compute the full vielbein associated with the super black hole.

We can perform a similar transformation to facilitate comparison between (2.12) and its supersymmetric analogue (1.3). The former corresponds to the spacetime of a bosonic particle in anti de Sitter space, and the latter is its superspace counterpart. Here the supercoordinate transformation that takes us from \( z = (x, \theta) \) to \( w = (u, \lambda) \) is given by

\[
(2L|X| + c) = \tan^{-1}\left[\frac{4L}{\sqrt{1 - \frac{M^2}{16L^2}}}(|U| + U_0)\right]
\]

(5.4)

We have found expressions for the supergravity compensator that completely determine the vielbeins of a super anti de Sitter black hole and of a point particle in
super anti de Sitter space. As with the bosonic case, the super black hole solution can only be obtained provided $M > 4L$. A further exploration of these solutions will be given in ref. [6].

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