HARMONIC ANALYSIS OF THE COMBINED CIRCUIT OF SINGLE-PHASE SWITCHING ELECTRIC DRIVE WITH THYRISTOR CONTROL

Purpose. The purpose of this article is to provide a comparative analysis of influence of the phase controlled single-phase induction electric drive coordinates harmonic components on the operational characteristics of the electric drive under different variants of the power part. Methodology. A mathematical model of the combined circuit of a single-phase induction motor with a variable structure of the power part and a constant capacitance of the phase-shifting capacitor is generalized for various variants of connection of the power part of the electric drive with thyristor control. The comparative harmonic analysis of currents and torques of the motor is carried out and the factors influencing the energy efficiency of the electric drive at different circuit of connection of the power part are determined. Results. The analysis of the obtained data shows that the series-parallel circuit has the best harmonic composition of the consumed current in practically the whole range of changes of load torque and speed. The main effect on the loss increase is due to the asymmetry of the currents in the series-parallel circuit and the higher harmonics in the Steinmetz circuit. Originality. For the analysis of the processes in these circuits, a generalized thyristor control single-phase electric drive mathematical model, in which the structure of differential equations remains unchanged regardless of the circuit of connections of the power part, is developed. Practical value. Using of the combined circuit of single-phase switching on of a three-phase induction motor allows to use a constant capacity in the range of slips from nominal to critical without exceeding the nominal losses. References 4, tables 1, figures 5.

Key words: induction motor, single-phase power, thyristor control, harmonics.

Introduction. Circuits of single-phase switching of three-phase induction motors (IMs) with phase-shifting capacitor and permanent structure of the power section are widely used in unregulated electric drives due to the simplicity of design and sufficiently high energy characteristics [1-3]. In the case of a controlled electric drive (ED), the use of circuits with the connection of a phase-shifting capacitor with constant capacity can significantly impair the operational and energy characteristics of the electric drive in start-up and variable speed operation [2, 3]. The use of a capacitor with an adjustable capacity increases the overall dimensions of the ED. In [4], for the operation of controlled pumps at single-phase power, the use of a combined circuit controlled by a thyristor voltage converter (TVC) of the electric drive with variable structure of the power part, shown in Fig. 1 is proposed. The application of this circuit allows the use of a constant capacity in a wide range of speed regulation.

The goal of this work is a comparative analysis of the effect of the harmonic components of the coordinates of a single-phase electric drive arising from the phase control method on the operational characteristics of the electric drive in different variants of the power part connection.

Mathematical model. The circuit of combined connection (Fig. 1) has two variants of connection of a power part. The motor windings in the start mode and at low speed operation are switched on according to the series-parallel switching circuit (position 2 of contactor $K$), in the operating mode – according to the Steinmetz circuit (position 1 of contactor $K$). In order to analyze the processes in this circuit with thyristor control, a generalized mathematical model was developed in which the structure of the differential equations remains unchanged regardless of the circuit of the power unit connections.

In the stator coordinate system $\alpha, \beta, 0$, electromechanical processes in IM are described by the following system of equations in matrix form:

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\[
\begin{align*}
\overline{u}_s &= R_s \overline{i}_s + \sigma L_s \cdot \overline{\psi}_s + \frac{L_m}{L_r} \cdot \overline{p} \cdot \overline{\psi}_r; \\
0 &= -R_r \frac{L_m}{L_r} \overline{i}_s + \frac{R_r}{L_r} \overline{\psi}_r' + \overline{p} \cdot \overline{\psi}_r + \alpha \overline{\psi}_r; \\
M &= \frac{3}{2} \left( \overline{L_m} \left( \overline{\psi}'_{\alpha} - \overline{\psi}'_{\beta} \right) \overline{\psi}_{\alpha} \right), \\
J \frac{d\alpha}{dt} &= M - M_c,
\end{align*}
\]

where \( \overline{u}_s = [u_\alpha^s, u_\beta^s] \) is the vector of stator voltages; 
\( \overline{i}_s = [i_\alpha^s, i_\beta^s] \) is the vector of stator currents; 
\( \overline{\psi}_r' = [\psi'_\alpha, \psi'_\beta] \) is the vector of rotor flux linkages; 
\( L_s = L_m + L_\sigma s \), \( L_r \) are the full inductances of stator and rotor; 
\( L_m \) is the inductance of the magnetizing circuit; 
\( L_\sigma s \) is the rotor leakage inductance; \( R_s \), \( R_r \) are the resistances of stator and rotor; \( \sigma = 1 - \frac{L_m^2}{L_r \cdot L_s} \) is the leakage coefficient; \( J \) is the moment of inertia; 
\( \overline{\omega} = \begin{bmatrix} 0 & \alpha \\ -\alpha & 0 \end{bmatrix} \) is the matrix that determines the EMF of rotation; \( \overline{p} \) is the symbol of differentiation by time.

To solve the system (1), it is necessary to express the vector of stator voltages \( \overline{u}_s = [u_\alpha^s, u_\beta^s] \) through the parameters of the outer circuit of the motor and the state variables for which the stator currents \( \overline{i}_s = [i_\alpha^s, i_\beta^s] \) and rotor flux linkages \( \overline{\psi}_r' = [\psi'_\alpha, \psi'_\beta] \) are taken. Based on the Kirchhoff equation and the known relationships between electromagnetic variables in the orthogonal coordinate system \( \alpha, \beta, 0 \) and in the natural coordinate system A, B, C it is possible to determine that in the general case the stator voltage vector for the studied circuits (Steinmetz and series-parallel) is a function of the network voltage \( u \), the stator current vector \( \overline{i}_s \), the rotor flux linkage vector \( \overline{\psi}_r \) and the voltage on the capacitor \( u_c \) which in turn can be determined by the components of the vector \( \overline{i}_s \). Then the general expression for determining the vector of stator voltages will be:

\[
\overline{u}_s = \overline{k} \cdot u + \frac{1}{C_1} \overline{k}_2 \cdot i_s + \left( \overline{R}_s \cdot \overline{k}_3 + \sigma \cdot L_s \cdot \overline{k}_3 \cdot p \right) \cdot i_s + L_m \left( \overline{L_m} \overline{i}_s + L_m \overline{k}_4 \cdot p \right) \cdot \overline{p} \cdot \overline{\psi}_r'.
\]

In the right-hand side of expression (2), the first component is caused by the action of the network voltage, the second one by the voltage on the capacitor, the third one by the action of counter-EMF from zero sequence currents, and the fourth one by the counter-EMF of rotation.

The operation of the thyristor controller is described by a logical function

\[
F = X_1 \cdot \Pi + Y_1 + X_2 \cdot \Pi + Y_2,
\]

where the logic functions \( Y_1, Y_2 \) correspond to the nonzero value of currents through the thyristors, the logic functions \( X_1, X_2 \) to the reverse voltages on the thyristors, \( \Pi \) is the switching function corresponding to the control signal of the thyristors.

The switching function \( \Pi \) in phase control has the following form:

\[
\Pi = 1 \text{ at } \alpha + \pi \cdot k < \omega_0 \cdot t < (\alpha + \theta) + \pi \cdot k; \\
\Pi = 0 \text{ at } 0 + \pi \cdot k < \omega_0 \cdot t < \alpha + \pi \cdot k \cup \omega_0 \cdot t + \pi \cdot k,
\]

where \( \alpha \) is the control angle of the thyristors (in electrical degrees); \( \omega_0 \) is the frequency of supply voltage; \( \theta \) is the duration of control pulses (in electrical degrees), selected from the conditions of reliable opening of the thyristors; \( k = 0, 1, 2, 3, ... \) To reliably open thyristors, the TVC control system must generate pulses of duration of at least 70 electrical degrees.

Thyristor controller’s conductive state corresponds to \( F = 1 \), non-conductive one to \( F = 0 \).

The numerical values of the matrices of coefficients \( \overline{k}_1 - \overline{k}_4 \) that allow to associate the stator voltage vector of the two-phase IM model in the stator coordinates \( \alpha, \beta, 0 \) with the parameters of the external motor circuit (network voltage, capacitor capacity) and the state variables are given in Table 1 depending on the position of switch \( K \) and the state (conductive or non-conductive) of the thyristor controller.

Thus, changing the structure of the power circuit does not require a change in the system of differential equations, and is accompanied only by a change in the coefficients \( \overline{k}_1 - \overline{k}_4 \) in equation (2), without violating the laws of continuity of motor flux linkages and capacitor charges.

| Circuit (position of the key K) | Thyristors state (values of F) | \( \overline{k}_1 \) | \( \overline{k}_2 \) | \( \overline{k}_3 \) | \( \overline{k}_4 \) |
|---------------------------------|-------------------------------|-----------------|-----------------|-----------------|-----------------|
| 1 | 1 | 2/3 | 0 | -1/6 | -1/2 \( \sqrt{3} \) | 0 | 0 | 0 | 0 |
| | 0 | 0 | 0 | -1/2 \( \sqrt{3} \) | -1/2 | 0 | 0 | 0 | 0 |
| 2 | 1 | 2/3 | \( \sqrt{3} \) | -1 | 0 | -1/3 | 0 | 1/3 | 0 |
| | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
Results of investigations. Consider first the operation of the ED with a motor 4A71B2 of power of 1.1 kW when connected on the Steinmetz circuit. The curves of changes in time of linear current \( i \) and of the instantaneous electromagnetic torque \( M \) when connecting the motor windings according to the Steinmetz circuit are shown in Fig. 2.

![Fig. 2. Current and torque change curves when connected according to the Steinmetz circuit](image)

During the non-conducting state of the thyristors, the phase A current is zero, and the capacitor is periodically discharged through phases B and C. The motor goes into braking at these intervals resulting in the maximum (in modulus) value of the oscillating torques \( \Delta M \) much greater than the average torque \( M_{\text{avg}} \) (\( \Delta M_{\text{max}}=3...4 M_{\text{avg}} \)).

The harmonic analysis of the linear current of the motor showed that all odd harmonics are present in the current curve at phase control. The most significant higher harmonic in the curve of the linear current of the motor is the third one whose amplitude reaches at angles \( \alpha \) greater than 90 electrical degrees, values of 50...60 % of the amplitude of the fundamental harmonic. The amplitudes of the fifth and seventh harmonics in this case are respectively 30...35 % and 15...20 %.

The dependences of the ratio of power consumed by the motor when powered from TVC with a given opening angle \( P_{1} \) and when powered by sinusoidal voltage equal to the value of the first harmonic at thyristor control, \( P_{1} \) per units (p.u.) on the sliding (Fig. 3) show that at when powered from TVC the power consumption is higher than at sinusoidal power supply. The efficiency of the motor when powered from TVC is lower than when at sinusoidal power supply (Fig. 4) while reducing the efficiency is greater, the greater the opening angle of the thyristors, reaching at \( \alpha = 90...110 \) electrical degrees 20 %.

![Fig. 3. Dependencies of the ratio of powers consumed by the motor](image)

![Fig. 4. Dependencies of the motor’s efficiency ratio](image)

In order to improve energy performance, it is necessary to use means of reducing the higher harmonic components of the motor current.

One possible way to improve the harmonic composition of the linear current of the motor at thyristor control is to connect the motor in a series-parallel circuit with voltage regulation of only one phase (position 2 of the contactor \( K \) in Fig. 5). In this case, the unregulated phase winding acts as a filter. The curves of changes in time of linear current \( i \) and the instantaneous electromagnetic torque \( M \) of the series-parallel circuit are presented in Fig. 5.

![Fig. 5. Current and torque change curves when connected according to the series-parallel circuit](image)
This circuit with the connection of TVC in series with the main phase is characterized by the best harmonic composition of the linear current (the amplitude of the third harmonic is 20-30 \% of the amplitude of the first harmonic). The analysis of the simulation results of this circuit showed a significant decrease in the harmonic coefficient (at thyristor opening angles of 90…120 electrical degrees $\alpha$, does not exceed 40 \%) and a decrease in the amplitude of the third harmonic of the linear current by 30…40 \% in the whole range of change of torque and speed.

However, it should be noted that the improvement of the harmonic composition in this circuit is achieved by the price of increasing the asymmetry coefficient and, consequently, the loss from reverse sequence currents. Increasing the coefficient of asymmetry at the phase control of the capacitor IM according to the series-parallel scheme is caused by the delayed nature of the phase shift angle of the first harmonic of the main phase voltage relative to the supply voltage due to the use of thyristors with natural switching.

When comparing Fig. 2, 5 it can be seen that the series-parallel circuit has smaller amplitude of oscillatory torques. The maximum value (in modulus) of the oscillating torques $\Delta M$ does not exceed the values 1…1.5 of the average torque $M_{avg}$. This is due to the fact that in the intervals of the non-conducting state of the thyristors, the motor continues to operate in single-phase connection motor mode and does not go into braking mode, as in the Steinmetz circuit. Figure 5 shows that the non-sinusoidal oscillating component caused by the switching of the thyristors is weakly expressed in the instantaneous electromagnetic torque curve in a series-parallel circuit.

**Conclusions.**

The use of the combined circuit of single-phase connection on of a three-phase induction motor allows to use a constant capacity in the range of slips from nominal to critical without exceeding the nominal losses.

The analysis of the obtained data shows that the series-parallel circuit has a better harmonic composition of the current consumed and smaller amplitudes of the torque oscillations than the Steinmetz circuit, in practically the whole range of changes of the load torque and speed.

The main influence on the increase of losses in comparison with symmetric sinusoidal amplitude control is due to the asymmetry of the currents in the series-parallel circuit and the higher harmonics of the currents in the Steinmetz circuit.

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