Calculation of dynamic processes in relay systems of automatic control based on graph models

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Abstract. This article discusses the construction of a graph model of a relay system of automatic control. Based on the graph model, relations are derived for calculating transients in a relay system. To describe and analyse relay systems of automatic control, a specific feature of the relay element is used, consisting in the fact that its output value can take only certain constant values. The maximum possible set of structural states for relay elements is three. Accordingly, the relay element introduces a structure discretization effect into the system. This feature makes it possible to study relay systems with relatively simple means and makes it possible to develop methods for their calculation to a certain extent similar to methods for calculating linear systems. In developing the graph method for the description and analysis of relay automatic systems, this position was primarily taken as a basis. This method can be successfully used in any of the typical relay elements included in the system. A two-dimensional relay system is considered as an example. Based on the graph model, the output coordinates of the system are calculated and their graphs are constructed.

Keywords: graph, graph model, one-dimensional relay system, two-dimensional relay system, relay element switching, calculating transition processes

1. Introduction

In many sectors of the economy, including agriculture, automatic systems are used in which the control object is described by nonlinear equations.

The object of this study is the relay systems of automatic control. Relay systems of automatic control are one of the classes of nonlinear systems; they have been actively and widely used in various stationary and non-stationary, moving control objects, in measuring and regulatory complexes for a long time. They are distinguished by the simplicity of execution, settings, high reliability, resistance to the influence of non-stationary parameters, and better dynamic properties compared to continuous systems.

At present, when thanks to modern control theory it is possible to create digital control systems of any complexity, relay systems should seem to take a back seat. However, interest in them has not only not weakened, but in recent years has even increased. This can be confirmed by a review of works on this topic and the achievements of digital technology, in particular, the appearance of contactless keys, operating amplifiers on chips, microcontrollers, as well as theoretical developments in the field of relay systems in various fields of technology [1-9].
From a theoretical point of view, relay systems are essentially non-linear, which, on the one hand, was the limiting factor of their use due to the complexity of the calculation, and on the other hand, caused the development of theories created specifically for this class of control systems.

The list of issues related to the theory of relay systems, as well as to the theory of automatic control as a whole, is very wide. First of all, these are the features of their mathematical description, behavior in statics and dynamics, stability issues, special modes of relay systems of automatic control, etc. [10-15].

Thus, the urgency of the problem is determined by the widespread use of relay control systems and the need to study the dynamics of these systems in a wide range of parameters, the need to develop accurate methods that allow basic research and numerical calculations of specific nonlinear systems of automatic control. In this regard, the task of developing methods and algorithms for studying the dynamics of the functioning of relay systems of automatic control today is timely.

The purpose of this study is to develop, based on graph models, a topological method for modeling and researching relay systems of automatic control.

To achieve this goal, the following tasks were solved: a complex of graph models of one-dimensional and multidimensional relay automatic control systems with various types of relay elements was developed, algorithms for studying the dynamics of the functioning of this class of systems are constructed.

In this work, a specific feature of relay systems is used, consisting in the fact that the shape of the output quantity does not depend on the shape of the input signal. This allows a mathematical analysis of relay systems by relatively simple means [16-20]. Moreover, this feature makes it possible to use methods of calculation, which, in a sense, are similar to methods of calculating linear systems. Such an analogy allows us to preserve the familiar concepts and terminology of linear control theory. For example, the concept of the transfer function, frequency, and time characteristics.

2. Materials and methods
Consider the use of graph models to describe and analyze relay systems. Fig.1 is a structural diagram of a one-dimensional relay system. We pose the problem of finding the output signal of the system \( x_1(t) \) for all time instants \( t (t \geq t_0, t_0 \) is the initial moment of time). The linear part of the system is represented by a \( m \)-th order dynamic link.

When developing a topological method for the description and analysis of relay systems of automatic control, a feature of the relay element is used, which consists of the fact that its output value can take only certain constant values. If the relay element is without a dead band, then the maximum possible number of structural states for it is two. For relay elements with a dead zone, three structural states are possible.

Without loss of generality, let us assume that in the system under consideration the relay element has an ideal characteristic, its output signal can take only two values \(+b\) or \(-b\), where \(b\) is a certain constant number.

Let at the moment \( t_0 \) the value of the output signal of the relay element is \( Z(t_0) = b \) and the initial conditions are such that at some moment \( t_1 \) the relay element switches. On the time interval \( t \in [t_0, t_1] \) the system is characterized by a certain structural state \( S_j \).

We know the formation of the graph model [21-23] of the continuous part of the system, the relay element is taken into account by connecting an integrating link to the continuous part of the system.

As is known, in the study and design of control systems, the Laplace transform is used, therefore, the transfer function of the relay element is \( 1/p \), where \( p \) is a complex variable.

The vertex characterizing the output signal of the relay element has a weight \( z = b \lor -b \). The graph model of the system, reflecting its behavior on the interval \( t \in [t_0, t_1] \), is presented in fig. 2.

From the consideration of the graph model [24-27], it is easy to write the equation of state of the system on the time interval \( t \in [t_0, t_1] \):
\[ \ddot{X}(p) = \hat{A}(p)X(p) + \hat{B}(p)Z(t_0) \]  
\[ (1) \]

Having performed the inverse Laplace transform, we have

\[ \ddot{X}(t) = \hat{A}(t-t_0)\ddot{X}(t_0) + \hat{B}(t-t_0)Z(t_0) \]
\[ (2) \]

\[ \begin{array}{c}
\text{f} \\
\hline
\text{relay} \\
\text{element} \\
\hline
\text{Controlled} \\
\text{process} \\
\hline
\text{x,}
\end{array} \]

**Figure 1.** Block diagram of a one-dimensional relay system.

\[ \begin{bmatrix}
f(t_0) \\
a(t_1) \\
-1 
\end{bmatrix}
\]

\[ \begin{bmatrix}
f(p) \\
1/(p) \\
e(p)
\end{bmatrix}
\]

\[ \begin{bmatrix}
Z(t_0) \\
x_m(t_0) \\
x_1(t_0) \\
x_2(t_0) \\
x_1(p)
\end{bmatrix}
\]

**Figure 2.** The graph model of the relay system.

It should be noted that the switching moment \( t_1 \) of the relay element is unknown in advance and is determined from the equation

\[ e(t_1) = a_t \]
\[ (3) \]

where \( e(t_1) = f(t_1) - x(t_1) \) and \( a_t \) are the sensitivity threshold of the relay element. In the general case, equation (3) is transcendental and is solved by one of the known numerical methods (Newton’s method, iterations, etc.) Having determined the switching moment of the relay element \( t_1 \), from relation (4) we can find the values of state variables at \( t = t_1 \):

\[ \ddot{X}(t_1) = \hat{A}(t_1-t_0)\ddot{X}(t_0) + \hat{B}(t_1-t_0)Z(t_0) \]
\[ (4) \]

These values are initial for determining the processes in the next step, i.e. on the time interval \( t \in [t_1, t_2] \), where \( t_2 \) is the second switching moment of the relay element. The structure of the graph model on the interval \( t \in [t_1, t_2] \), does not change, only the initial conditions change. Therefore, there is no need to rebuild the graph model; just use the formalized model shown in fig. 2.

On the time interval \( t \in [t_1, t_2] \) the equations for calculating the processes will be as follows:

\[ \dddot{X}(p) = \hat{A}(p)\ddot{X}(t_1) + \hat{B}(p)Z(t_1) \]
\[ (5) \]

where we will have
\[ X(t) = \tilde{A}(t-t_1)X(t_1) + \tilde{B}(t-t_1)Z(t_1) \]  

(6)

To determine the second moment of switching the relay element, it is necessary to solve the equation for the input signal of the relay element \( e(t_1) = a \). The values of the state variables at \( t = t_2 \), which are the initial conditions for determining the processes in the next time interval \( t \in [t_2, t_j] \), are found from the relation

\[ X(t_2) = \tilde{A}(t_2-t_1)X(t_1) + \tilde{B}(t_2-t_1)Z(t_1) \]  

(7)

In the general case, the ratios for calculating the processes on the segment \( t \in [t_{n-1}, t_n] \), where \( n = 1, 2 \), will have the form:

\[ X(t) = \tilde{A}(t-t_{n-1})X(t_{n-1}) + \tilde{B}(t-t_{n-1})Z(t_{n-1}) \]  

(8)

3. Results

As an example, consider a two-dimensional relay system of automatic control. Figure 3 shows the structural diagram of this system. Figure 4 shows its graph model, where the relay elements are taken into account by opening the system and adding to the continuous part of the system of links with the transfer function \( 1/p \). Depending on the input signals \( e_1 \) and \( e_2 \), the output signals of the relay \( z_1 \) and \( z_2 \) take the values: \( z_1 = -c_1 \lor c_1 \), \( z_2 = -c_2 \lor c_2 \). That is, with the known signals \( z_1 \) and \( z_2 \), the processes can be considered directly according to this graph model after replacing the transfers of arcs \( a_{ij}(p) \) by their originals \( a_{ij}(t) \). However, the switching moments of the relay elements are not known in advance and should be determined in this case from the solution of the equations

\[ e_1 = f_1 - y_1 \]  

(9)

\[ e_2 = f_2 - y_2 \]  

(10)

for \( \tau_1 \) and \( \tau_2 \) (\( \tau_1 \) and \( \tau_2 \) are the switching moments of the 1st and 2nd relay elements). This can be done using well-known methods for solving transcendental equations (iteration, Newton, chord method, combined method, etc.).

**Figure 3.** Block diagram of a two-dimensional relay automatic control system.
Figure 4. Graph model of a two-dimensional relay control system of automatic control.

Figure 5. Transient graph in a two-dimensional relay automatic control system.
After solving the equations for $\tau_1$ and $\tau_2$, the smallest of them is taken into account and the values of the variables $x_{11}^1, x_{21}^1, x_{12}^1, x_{11}^2$ for this switching moment are determined. They are the initial conditions for the next step in calculating the dynamics. In the next step, the calculations are again performed in parallel on both channels. Until the determination of the minimum time interval, after which one of the relay elements will switch. The described procedure is repeated for all steps of the process calculation, and is convenient from the point of view of computer simulation of processes.

For multidimensional relay systems, an algorithm for calculating transients based on a graph model can be formulated as follows.

Algorithm 1.
1. At the input of the relay elements, the macrostructure of the system is separated. Dynamic integrating links are introduced into all separate and cross channels. A dynamic graph model of the system is being built.
2. Based on automatic models of relay elements and solving transcendental equations, it is relatively determined $\tau_{r,i} = 1,2,\ldots,N$ determined

$$\tau_{\text{min}} = \min(\tau_i)$$

3. All state variables are calculated taking into account $\tau_{\text{min}}$ and the transition of the system to the next structural state is determined.
4. The state of finite state machines with memory is fixed (in the presence of relay elements with a hysteresis loop).
5. The return to paragraph 2 of the algorithm.

For the considered illustrative system, Fig. 5 shows the curves obtained on the basis of the above algorithm.

More than 1600 reclamation pumping stations work in the system of the Ministry of Water Resources. The largest share (up to 55%) among the installed pumps belongs to horizontal double-entry centrifugal pumps (type “D”) with a water supply of 320 to 2000 m$^3$/h and a lifting height of 21 to 125 meters. In practice, many pumping stations have problems of inefficient operation of equipment [2,3,4]. Many pumping stations have equipment that does not answerable to the technical requirements. The stations have pump’s technical characteristics increased in comparison with the necessary operational performance.

The operation of the power equipment of such pumping stations significantly affects the cost of electricity for water lifting. There is an over-expenditure of electricity for water lift and pumping of excess water. This is especially important in conditions of acute water shortage and energy resources of the Republic [2,3,4,5].

The operational efficiency of the pumping station depends primarily on the selected operating mode of the pumping units. Pump regulation is often used during the operation of pumping stations [7,8,9]. Pump regulation is the process of artificially changing its parameters (supply and pressure) to ensure the required values [10,11,12]. The operation of pumps outside the working area of the Q-H field leads to reduced energy efficiency, increased energy consumption. According to some data [1,2], as a result of the operation of pumping stations in uneconomical modes and the absence of effective methods for regulating the operating modes of pumping units, up to 5-25% of the consumed electricity is lost. Because of the above, it is extremely important for each pumping station to identify the optimal operating modes of the pumping units, i.e. to choose the most effective method of regulation.

The control methods currently used at pumping stations can be divided into two groups: quantitative and qualitative [8,9,10,11]. The method of controlling a valve on a pressure pipe (throttling) is the most common among quantitative methods. The regulation by changing the frequency of rotation of the impeller shaft and the cutting (or turning) of the impeller is the most used
method among the qualitative methods. Also, the method of controlling the operating time of the pumps (i.e., turning on/off the units) is widely used.

Irrigation pumping stations differ from water supply pumping stations both in terms of the requirements and operational conditions. Therefore, when choosing a regulatory method, these features must be taken into account.

4. Conclusions and discussions
From the considered example, we can conclude that modeling based on dynamic graphs is an effective method for solving problems of analysis of one-dimensional and multidimensional relay systems. We got a simple and convenient algorithm for calculating transients for multidimensional systems with various relay elements in the control channels.

The next stage of research is the use of graph modeling for the calculation of multidimensional relay systems with delay in individual channels. In such systems, there is a temporary delay of control signals. This phenomenon introduces difficulties in the calculation of such systems.

We also note that the proposed approach is convenient for solving the synthesis problem of relay automatic systems: the synthesis problem can be considered as determination among the set of admissible trajectories $St$, optimal in one sense or another, of the trajectory $St_i \in St$. The optimal trajectory can be realized by choosing the mode of operation of the relay part, or by changing the characteristics of individual structural states.

References
[1] Korobov V I and Bebiya M O 2017 Stabilization of one class of nonlinear systems Automation and Remote Control 78 pp 1–15
[2] Seifullaev R E and Fradkov A L 2017 Linear matrix inequality-based analysis of the discrete-continuous nonlinear multivariable systems. Automation and Remote Control 76, pp 989–1004
[3] Elkin V I 2010 Constructing subsystems for nonlinear controlled systems. Automation and Remote Control 71 pp 738–746
[4] Lampe B P and Rosenwasser E N 2007 Controllability and observability of discrete models of continuous plants with higher-order holds and delay Automation and Remote Control 68 pp 593–609
[5] Zhukov V P On the sufficient and necessary conditions for robustness of the nonlinear dynamic systems in terms of stability retention. Automation and Remote Control 69 pp 27–35
[6] Krutova I N and Sukhanov V M 2008 Dynamic features of flexible spacecraft control in process of its transformation into a large space structure. Automation and Remote Control 69 pp 774
[7] Gelig A.K 2002 Asymptotic Stability of Nonlinear Pulse Systems. Automation and Remote Control 63 pp 1217–1224
[8] Tkhai V N 2006 Oscillations and stability in quasiautonomous system I. Simple point of the one-parameter family of periodic motions Autom Remote Control 67 pp 1436–1444
[9] Serhiienko S and Serhiienko 2017 I Performance enhancement of the relay automatic control system with a fractional-order controller pp 76-79 10.1109/MEES 8248956.
[10] Guifang L Ch Y 2018 Controller design for stochastic nonlinear systems with matched conditions. Journal of Systems Engineering and Electronics vol. 29 no. 1 pp 160-165
[11] Huang J, Jiang Z and Zhao J 2016 "Component fault diagnosis for nonlinear systems," in Journal of Systems Engineering and Electronics, vol. 27, no. 6 pp 1283-1290
[12] Wei J and Liu Z 2019 "Asymptotic Tracking Control for a Class of Pure-Feedback Nonlinear Systems" in IEEE Access, vol. 7 pp 166-168
[13] Hassan M F and Hammuda M 2019 A New Observer for Nonlinear Systems With Application to Power Systems, in IEEE Access, vol. 7 pp 795-812
[14] Shiromoto H S Revay M and Manchester I R 2019 "Distributed Nonlinear Control Design Using Separable Control Contraction Metrics" in IEEE Transactions on Control of Network Systems vol 6 no 4 pp 1281-1290
[15] Pan W Yuan Y Gonçalves J and Stan G Jan 2016 "A Sparse Bayesian Approach to the Identification of Nonlinear State-Space Systems in IEEE Transactions on Automatic Control vol 61 no 1 pp 182-187
[16] Bozhenyuk A and Ginis L 2014 Modeling and analysis of complex systems on the basis of fuzzy graph models. Life Science Journal 11 pp 187-191

[17] Ausiello G Franciosa P and Frigioni D 2001 Directed Hypergraphs Problems Algorithmic Results and a Novel Decremental Approach. In: Theoretical Computer Science. ICTCS 2001 Lecture Notes in Computer Science vol 2202 Springer Berlin Heidelberg

[18] Andersen and Henrik Reif 1994 Model checking and Boolean graphs [MR1249946] Seventeenth Colloquium on Trees in Algebra and Programming (CAAP 92) and European Symposium on Programming (ESOP) (Rennes 1992) Theoret. Comput Sci 126 no 1 pp 3–30

[19] Molnar B A Beleczi and Benczür A 2018 Information systems modelling based on graph-theoretic background Journal of Information and Telecommunication 2:1 DOI: 10.1080/24751839.2017.1375223 pp 68–90

[20] Allamigeon X 2014 On the Complexity of Strongly Connected Components in Directed Hypergraphs Algorithmica 69 pp 335–369 https://doi.org/10.1007/s00453-012-9729-0

[21] Ausiello G Franciosa P G Frigioni D 2001 Directed Hypergraphs: Problems, Algorithmic Results, and a Novel Decremental Approach. In: Theoretical Computer Science ICTCS 2001 Lecture Notes in Computer Science vol 2202 Springer Berlin Heidelberg

[22] P Kennedy Laszlo 2018 Graph model-based analysis of technical systems IOP Conference Series: Materials Science and Engineering 393 DOI: 012007 10.1088/1757-899X/393/1/012007

[23] Tang Chung Man et al 2017 Accuracy Graphs of Spectrum-Based Fault Localization Formulas IEEE Transactions on Reliability 66 pp 403-424.

[24] Peter, Tamas and Szabo Krisztian 2012 A new network model for the analysis of air traffic networks. Periodica Polytechnica Transportation Engineering 40 pp 39-44

[25] Jones and Christopher V 1990 An introduction to graph-based modeling systems part I Overview ORSA Journal on Computing 2.2 136-151

[26] Latif S Afzaal H and Zafar N A 2018 Modelling of Graph-Based Smart Parking System Using Internet of Things 2018 International Conference on Frontiers of Information Technology (FIT), Islamabad, Pakistan pp 7-12

[27] Jalving J Cao Y and Zavala V 2019 Graph-Based Modeling and Simulation of Complex Systems. Computers and Chemical Engineering p 125