Phonons dispersions in auxetic lattices

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Abstract. The modes of vibrations in auxetic structures are studied, with models where the two-dimensional lattice is represented by a planar mesh with rod-like particles connected by strings. An auxetic membrane can be obtained modifying a honeycomb one, according to a model proposed by Evans et al. in 1991 and used to explain a negative elastic Poisson's ratio. This property means that auxetic materials have a lateral extension, instead to shrink, when they are stretched. The models here proposed with rod-like particles inserted in the structure have interesting behaviour of acoustic and rotational branches of phonon dispersions. Complete bandgaps of vibrations can be obtained for a proper choice of lattice coupling parameters and distribution of masses in the unit cell of the lattice.

1. Introduction

Auxetic materials are characterised by an elastic Poisson's ratio that turns out to be negative [1]: this property means that these materials have a lateral extension, instead to shrink, when they are stretched. Although a negative value of the Poisson's ratio is not forbidden by thermodynamics, this property is not usually acknowledged, but in fact it is displayed by many elemental materials [2] and by several man-made compounds, from nano- till macroscales. Foams with auxetic structures built with polymeric materials are well known and used in the production of energy absorption and soundproofing materials [3-5]. Natural auxetic materials and structures are occurring in biological systems, in the skin and bone tissues and in the crystalline membranes [6].

A two-dimensional model for an auxetic mechanical system is that proposed in Ref.[1] and shown in Fig.1 on the left. The auxetic structure is a re-entrant honeycomb model and, if we assume the links \( L, L' \) as rigid rods, we can easily explain the behaviour of materials with negative Poisson's ratios. When the auxetic is stretched parallel to \( L' \), the structure expands instead to shrink in the transverse direction. We can then imagine an auxetic behaviour in structures where rigid parts are included. Elastic units such as fibers or strings can represent the interactions among the rigid extended masses, for instance rod-like particles. Here we study two auxetic two-dimensional membranes as networks where the junctions, connecting fibers and rod-like particles, are forming a two-dimensional lattice.

2. Auxetic models with rod-like particles.

Let us start to consider a re-entrant honeycomb structure composed of strings and rod-like particles as a model of auxetic membrane. In Fig.1 on the right we show the structure where thick lines represent the rigid units and thin lines the ropes. The membrane has an auxetic property when stretched in the \( Y \)-direction. The approach we follow in evaluating the vibrations of the structure is similar to that proposed by Martinsson and Movchan [7] for lattices with point-like particles connected with ropes.
Figure 1. The auxetic model of Evans et al. [1] on the left and the re-entrant honeycomb lattice with rod-like particles used as a model of auxetic membrane, on the right. Sites 0,B of lattice basis and lattice vectors are displayed too.

It is straightforward to investigate the harmonic vibrations of the two-dimensional mesh in Fig.1, if it is supposed to be infinite, with displacements of lines and sites in the direction perpendicular to lattice plane. $u_{i,b}$ is the displacement of node $b$ (0 or B) at lattice position $i$. $w_{j,b',b}$ is the displacement of a string connecting node $b$ of lattice cell at position $i$, with the nearest neighbour node $b'$ at position $j$. A linear co-ordinate $\zeta$ is ranging from zero to the length $L$ of the string. The equation for transverse vibrations of strings is the usual wave equation, with a phase velocity \( \rho = \frac{\omega}{v} \), where $\rho$ is the density and $T_o$ the axial tension. The time-harmonic oscillation frequency is $\omega$. When displaced, the string line unit is exerting a force on the rod-like particle:

$$ f = -T_o \frac{d^2 w_{j,b',b}}{d\zeta^2} \mid_{\zeta=0} = T_o \frac{\omega}{v} \frac{u_{i,b}}{\sin(\kappa L)} - u_{i,b} $$

where $\kappa = \omega/v$. $\zeta = 0$ corresponds to $b$ position. If we are looking for Bloch waves with wavevector $k$, it is possible to write for each lattice cell the displacements as $u_{i,0} = u_0 \exp(i\omega t - ik \cdot \mathbf{1}_i)$ and $u_{i,B} = u_B \exp(i\omega t - ik \cdot \mathbf{1}_i)$. Dispersion relations for frequency $\omega$ are then obtained substituting Bloch waves in the dynamics of rod-like particles, then solving the following equations of dynamics:

$$ -\omega^2 (u_{i,B} + u_{i,0}) = \frac{T_o}{M} \sum_{j,j'} \left( \frac{d^2 w_{j,B,0}}{d\zeta^2} + \frac{d^2 w_{j',B,0}}{d\zeta^2} \right) + \frac{\xi T_o}{M} \sum_{k,k'} \left( \frac{d^2 w_{k,B,0}}{d\zeta^2} + \frac{d^2 w_{k',B,0}}{d\zeta^2} \right) $$

$$ -\omega^2 (u_{i,B} - u_{i,0}) = \frac{L^2 T_o}{2I} \sum_{j,j'} \left( \frac{d^2 w_{j,B,0}}{d\zeta^2} - \frac{d^2 w_{j',B,0}}{d\zeta^2} \right) + \frac{L^2 T_o}{2I} \left( u_{i,B} - u_{i,0} \right) $$

$$ + \frac{L^2 \xi T_o}{2I} \sum_{k,k'} \left( \frac{d^2 w_{k,B,0}}{d\zeta^2} - \frac{d^2 w_{k',B,0}}{d\zeta^2} \right) - \frac{L^2 \xi T_o}{2I} \left( u_{i,B} - u_{i,0} \right) $$

We can define an axial tension $T_X$ of strings aligned in the X direction different from $T_o$, that is $T_X = \xi T_o$; indices $j,j'$ are used when sites are connected by ropes with tension $T_o$ and $k,k'$ when sites are connected with ropes with tension $\xi T_o$. Eq.2 rules the motion of centre of mass, Eq.3 the rotation around it. The phonon reduced frequency $\Omega = \omega/\omega_o$, where $\omega_o = T_o/M \nu$, is shown in Fig.2
as a function of wavevectors, for different values of the ratio $R = I/ML^2$ where $I$ is the inertia and $M$ the mass of rod-like units (all details on calculations for membranes with rigid units can be found in Ref.[8], where a conventional honeycomb mesh is discussed too). It is interesting to note in Fig.2, that an increase of parameter $R$ corresponds to a lower value of the reduced frequency of both acoustic and rotational modes - the optical modes in lattices with rigid unit. This result is in agreement with observations in Ref.[9]. Moreover, experiments show that auxetic structures absorb noise and vibrations more efficiently than non-auxetic equivalent ones [10]: a reason for this behaviour can be found in the low energy of rotational modes for high values of parameter $R$ [9]. The behaviour of a conventional honeycomb model is more or less the same but the rotational frequencies are higher of about 10% if compared with those of the auxetic model [8].

A gap in the phonon dispersions can be observed in Y-direction only if the axial tension $T_X$ of strings aligned in the X-direction is increased over a threshold value. The lowering of parameter $R$ does not create a gap in the Y-direction, and acoustic and rotational (optical) branches are always crossed. Acoustic waves propagating in Y-direction see the lattice as if it were composed of point-like particles and in fact dispersions are not depending on $R$.

Existence of complete bandgaps in the vibration spectra can be supposed too. Let us remember that band-gaps are intervals of frequencies where no propagating phonons exist. In crystalline lattices for instance, optical branches can be separated from acoustic branches for all the directions of phonon propagation [11]. Three-dimensional elastic media can show bandgaps and for this reason they are called "phononic crystals" [12], as the photonic crystals displaying bandgaps for light waves [13].

![Figure 2. Phonon dispersions for the auxetic lattice as a function of wavevectors. On the left, curves for different ratios $R$ show that the frequency of rotational modes decreases when $R$ increases. A gap in the Y-direction can be observed if the axial tension $T_X$ is increased (panel on the right). Arrows indicate the direction of parameter increases.](image)

The lattice proposed in Fig.1 is just one of the many possible "auxetic-like" two-dimensional structures. In Fig.3, it is shown an auxetic square lattice with rigid rod-like particles, which is auxetics in the two directions of the plane. The lattice basis has two rigid particles: if the two masses are different, a complete band gap is displayed (in the Figure 3, for a mass ratio equal to 2).
3. Conclusions

Auxetic-like membranes can be defined as structures, which do not collapse when stretched in one of the in-plane directions. For this reason, the mesh must be composed of strings and rigid parts: it is the presence of rigid particles that prevents the shrinking of membranes. Two-dimensional more complex structures can be proposed. In [9], the in-plane vibrations of rectangular rigid particles connected by harmonic elements are discussed.

Of course, different approaches to the problem of vibrations of auxetic structures are possible. For instance, solutions based on finite elements are proposed in Ref.[14] to solve a macroscopic mechanical system. These studies are in fact very important for applications, but specific. The aim of this paper is instead the proposal of lattices with rod-like particles, to investigate the vibrations of auxetic structures and how is possible to create a band-gap with proper mass differences or interaction anisotropy in the unit cell of the lattice.

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