WILL IT BE POSSIBLE TO MEASURE INTRINSIC GRAVITOMAGNETISM WITH LUNAR LASER RANGING?

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Received 16 June 2008
Accepted 21 September 2008
Communicated by Jorge Pullin

In this note we mainly explore the possibility of measuring the action of the intrinsic gravitomagnetic field of the rotating Earth on the orbital motion of the Moon with the Lunar Laser Ranging (LLR) technique. Expected improvements in it should push the precision in measuring the Earth-Moon range to the mm level; the present-day Root-Mean-Square (RMS) accuracy in reconstructing the radial component of the lunar orbit is about 2 cm; its harmonic terms can be determined at the mm level. The current uncertainty in measuring the lunar precession rates is about $10^{-1}$ milliarcseconds per year. The Lense-Thirring secular, i.e. averaged over one orbital period, precessions of the node and the perigee of the Moon induced by the Earth’s spin angular momentum amount to $10^{-3} - 10^{-4}$ milliarcseconds per year yielding transverse and normal shifts of $10^{-1} - 10^{-2}$ cm yr$^{-1}$. In the radial direction there is only a short-period, i.e. non-averaged over one orbital revolution, oscillation with an amplitude of $10^{-5}$ m. Major limitations come also from some systematic errors induced by orbital perturbations of classical origin like, e.g., the secular precessions induced by the Sun and the oblateness of the Moon whose mismodelled parts are several times larger than the Lense-Thirring signal. The present analysis holds also for the Lue-Starkman perigee precession due to the multidimensional braneworld model by Dvali, Gabadadze and Porrati (DGP); indeed, it amounts to about $5 \times 10^{-3}$ milliarcseconds per year.

Keywords: Experimental studies of gravity; Moon

1. Introduction

In the framework of the linearized weak-field and slow-motion approximation of general relativity, the gravitomagnetic effects are induced by the off-diagonal components $g_{0i}$, $i = 1, 2, 3$ of the space-time metric tensor, which are proportional to the components of the matter current density of the source $j_i = \rho v_i$.

There are essentially two types of mass currents in gravity. The first type is induced by the rotation of the matter source around its center of mass and generates the intrinsic gravitomagnetic field which is closely related to the proper angular momentum $S$ (i.e. spin) of the rotating body. The other type is due to the translational motion of the source and is responsible for the extrinsic gravitomagnetic field.

A debate has recently arisen concerning the possibility of measuring some extrinsic gravitomagnetic orbital effects affecting the motion of the Earth-Moon sys-
tem in the Sun’s field with the Lunar Laser Ranging (LLR) technique. Another test of extrinsic gravitomagnetism concerning the deflection of electromagnetic waves by Jupiter in its orbital motion has been performed in a dedicated radio-interferometric experiment.

In this brief note we wish to consider in some details the possibility of measuring with LLR an effect induced by the intrinsic gravitomagnetic field of the spinning Earth

\[ B_g = \frac{G [3r (r \cdot S) - r^2 S]}{c^3 r^5} \]  

(1)

through the non-central, Lorentz-like acceleration

\[ a = -2 \left( \frac{v}{c} \right) \times B_g \]  

(2)

on the orbital motion of the Moon around the Earth. In eq. (1) and eq. (2), \( G \) is the Newtonian gravitational constant, \( c \) is the speed of light in vacuum, \( S \) is the Earth’s spin angular momentum and \( v \) is the velocity of the Moon with respect to the Earth. The orbital feature we are interested in consists of the Lense-Thirring precessions of the longitude of the ascending node \( \Omega \) and the argument of pericentre \( \omega \) of the orbit of a test particle. Twenty years ago, Bertotti in Ref. 14 wrote that the Lense-Thirring effect for the Moon was, at that time, too small to be detected; according to Ciufolini, intrinsic gravitomagnetism is still unmeasurable with the lunar orbit. Instead, the possibility of measuring it in view of the expected forthcoming improvements in LLR has recently been envisaged by Müller et al. in Ref. 15 and in Ref. 16; Kopeikin et al. in Ref. 17 retain highly plausible its measurement. An overview of other attempts to measure the Lense-Thirring effect in various Solar System scenarios with natural and artificial test particles can be found in Ref. 18. In particular, for the LAGEOS-LAGEOS II test in the gravitational field of the Earth see Ref. 19 and Ref. 20; for the test performed with the Mars Global Surveyor in the field of Mars see Ref. 22 and Ref. 23. Another effect induced by the intrinsic gravitomagnetic field of the Earth is the precession of orbiting gyroscopes currently under measurement by the GP-B mission; see on the WEB http://einstein.stanford.edu/Ref. 26 and Ref. 27.

It is interesting to note that our analysis is equally valid also for the anomalous Lue-Starkman perigee precession predicted in the framework of the multi-dimensional braneworld model of modified gravity put forth by Dvali, Gabadadze and Porrati (DGP) to explain the observed acceleration of the Universe without resorting to dark energy; indeed, as we will see, the magnitude of such an effect is the same as the Lense-Thirring one for the Moon. Several researchers argued that it might be possible to measure the Lue-Starkman precession with LLR in view of the expected improvements in such a technique.
The physical and geocentric orbital parameters of the Moon are listed in Table 1.

| Parameter                        | Value                        | Units  |
|----------------------------------|------------------------------|--------|
| mass                             | $7.349 \times 10^{22}$ kg    | kg     |
| $S$ proper angular momentum      | $2.32 \times 10^{29}$ kg m$^2$s$^{-1}$ | kg m$^2$s$^{-1}$ |
| $Gm$                             | $4.902801076 \times 10^{12}$ m$^3$s$^{-2}$ | m$^3$s$^{-2}$ |
| $R$ radius                       | $1.738 \times 10^6$ m        | m      |
| $\alpha$ proper angular velocity | $2.66 \times 10^{-6}$ rad s$^{-1}$ | rad s$^{-1}$ |
| $C_{mR^2}$ normalized moment of inertia | 0.3932 | - |
| $\delta J_2$ mass quadrupole moment | $2.0326 \times 10^{-4}$ | - |
| $a$ semi-major axis              | $3.84400 \times 10^8$ m      | m      |
| $I$ mean inclination to the Earth’s equator | 23.5 | deg |
| $e$ eccentricity                 | 0.0549                       | -      |

2. The Lense-Thirring effect on the lunar orbit

By assuming a suitably constructed geocentric equatorial frame, it turns out that the node and the perigee of the Moon undergo the Lense-Thirring secular precessions

\[
\begin{align*}
\dot{\Omega}_{LT} &= \frac{2S_G}{c^2 a^3 (1-e^2)^{3/2}} = 0.001 \text{ mas yr}^{-1}, \\
\dot{\omega}_{LT} &= -\frac{6S_G \cos I}{c^2 a^3 (1-e^2)^{3/2}} = -0.003 \text{ mas yr}^{-1},
\end{align*}
\]

where $a$, $e$, $I$ are the semi-major axis, the eccentricity and the inclination to the Earth’s equator of the Moon’s orbit; mas yr$^{-1}$ stands for milliarcseconds per year. We used $S_G = 5.85 \times 10^{33}$ kg m$^2$s$^{-1}$ [34]. The Lue-Starkman [28] pericentre precession is just about $\pm 0.005$ mas yr$^{-1}$.

Since the ratio of the mass of the Moon to that of the Earth is $\mu = 0.0123000383$, one may argue that eq. (3), which has been derived for a test-particle like, e.g., an artificial satellite, does not apply to the Earth-Moon system. The intrinsic gravitomagnetic spin-orbit effects in the case of a two-body system with arbitrary masses $m_A$ and $m_B$ and spins $S_A$ and $S_B$ have been derived by Barker and O’Connell in Ref. [36] Damour in Ref. [37], Wex in Ref. [38]; for the sake of simplicity, we will reason in terms of the node. In this case, the total node precession

\[\text{The minus sign is related to the standard Friedmann-Lemaître-Robertson-Walker (FLRW) branch, while the plus sign is related to the self-accelerated branch which should be able to explain the observed acceleration of the Universe without resorting to dark energy.} \]
\( \dot{\Omega}_{\text{tot}} \) accounts for the spin-orbit contributions of both bodies and also for a spin-spin term. The expression of the node precession of a body A is

\[
\dot{\Omega}_A = \left( \frac{3 + x_A}{2c^2} \right) \frac{G(m_A + m_B)}{a^3(1 - e^2)^{3/2}} \frac{S_A}{m_A}, \quad x_A = \frac{m_A}{m_A + m_B},
\]

so that

\[
\dot{\Omega}_{\text{tot}} = \dot{\Omega}_A + \dot{\Omega}_B.
\]

Let us pose

\[
m_A = m_{\oplus} \equiv M, \quad m_B = m_{\text{Moon}} \equiv m;
\]

thus, it is possible to obtain

\[
\dot{\Omega}_{\oplus} = \left( 1 + \frac{3}{4\mu} \right) \frac{2GS_{\oplus}}{c^2a^3(1 - e^2)^{3/2}},
\]

\[
\dot{\Omega}_{\text{Moon}} = \left( 1 + \frac{3}{4\mu} \right) \frac{2GS_{\text{Moon}}}{c^2a^3(1 - e^2)^{3/2}};
\]

recall that \( \mu \) is the Moon/Earth mass ratio. It results that the precession of eq. (7) is larger than the Lense-Thirring one of eq. (3) by the multiplicative factor \( \left( 1 + \frac{3}{4\mu} \right)^2 = 1.0092 \) yielding an error of \( 10^{-5} \) mas yr\(^{-1} \), which is completely negligible (see Section 3). Concerning the precession due to the lunar spin, we have

\[
\dot{\Omega}_{\text{Moon}} = \frac{3 + 4\mu}{4 + 3\mu} \frac{S_{\text{Moon}}}{\mu S_{\oplus}} = 2 \times 10^{-3},
\]

i.e. it is of the order of \( 2 \times 10^{-6} \) mas yr\(^{-1} \), which is negligible as well. The amplitude of the spin-spin term is proportional to

\[
\dot{\Omega}_{SS} \propto \frac{3}{2c^2} \sqrt{\frac{GM(1 + \mu)}{a^7}} \frac{1}{(1 - e^2)^2} \frac{S_{\oplus} S_{\text{Moon}}}{M m} = 6 \times 10^{-9} \text{ mas yr}^{-1}.
\]

Thus, we can conclude that the Lense-Thirring approximation is fully adequate for the Earth-Moon system.

3. Some sources of error

Let us now examine some sources os systematic errors. In regard to the potentially corrupting action of the mismodelling in the even (\( \ell = 2, 4, 6, \ldots \)) zonal (\( m = 0 \)) harmonic coefficients \( J_\ell \) of the multipolar expansion of the Newtonian part of the Earth’s gravitational potential, which is not the most important source of aliasing precessions in the case of the Moon\(^{32} \), only \( \delta J_2 \) would be of some concern. Indeed, the mismodelled secular precessions induced by it on the lunar node and perigee amount to \( -2.67 \times 10^{-4} \) mas yr\(^{-1} \) and \( 5.3 \times 10^{-4} \) mas yr\(^{-1} \), respectively; the impact of the other higher degree even zonals is negligible being \( \leq 10^{-8} \) mas yr\(^{-1} \).

\(^{b}\)The calibrated errors \( \delta J_\ell \) of the EIGEN-CCG01C Earth gravity field solution\(^ {33} \) were used.
As in the case of the spins, also the asphericity of the Moon has to be taken into account\(^{36,38}\), according to

\[
\dot{\Omega}_{J_{Moon}}^{2} = \frac{3}{2} \frac{n_{Moon} \cos F J_{Moon}^{2}}{(1 - e^2)^2} \left( \frac{R_{Moon}}{a} \right)^2,
\]

where \(n_{Moon} = \sqrt{GM(1 + \mu)/a^3}\) is the lunar mean motion and \(F\) is the angle between the orbital angular momentum and the Moon’s spin angular momentum \(S_{Moon}\); it is about 3.61 deg since the spin axis of the Moon is tilted by 1.54 deg to the ecliptic and the orbital plane has an inclination of 5.15 deg to the ecliptic\(^{33}\). Table 1 and eq. (11) yield a mismodeled node precession due to \(\delta J_{Moon}^{2}\) of about 0.006 mas yr\(^{-1}\), which is 6 times larger than the Lense-Thirring rate. For other sources of systematic errors induced by gravitational and even non-gravitational\(^{40}\) perturbations see Ref. 32 and references therein, especially Ref. 41. Among the N-body gravitational perturbations, the largest ones are due to the Sun’s attraction. In order to get an order-of-magnitude evaluation of their mismodelling, let us note that some of such effects are proportional to \(n_{\oplus}^2/n_{Moon}\); e.g. the node rate, referred to the equator, is

\[
\dot{\Omega}_{\oplus}^{\odot} = \frac{3 GM_{\odot} \cos I}{4 a_{\oplus}^3 n_{Moon}} \left( \frac{3}{2} \sin^2 \varepsilon - 1 \right) \approx -5 \times 10^7 \text{ mas yr}^{-1},
\]

where \(\varepsilon = 23.439\) deg is the obliquity of the ecliptic. Since \(\delta G M_{\odot} = 5 \times 10^{10} \text{ m}^3 \text{ s}^{-2}\) and \(\delta G M = 8 \times 10^9 \text{ m}^3 \text{ s}^{-2}\), we can assume a bias of \(\approx 0.07 \text{ mas yr}^{-1}\) which is 70 times larger than the Lense-Thirring precession.

Let us, now, consider the precision of LLR in reconstructing the lunar orbit with respect to the Lense-Thirring effect. Concerning the precision in measuring the lunar precession rates, it amounts to about 0.1 mas yr\(^{-1}\), i.e. it is two orders of magnitude larger than the Lense-Thirring precessions of eq. (3). The orbital perturbations experienced by a test particle are usually decomposed along three orthogonal directions of a frame co-moving with it; they are named radial \(R\) (along the radius vector), transverse \(T\) (orthogonal to the radius vector, in the osculating orbital plane) and normal \(N\) (along the orbital angular momentum, out of the osculating orbital plane). According to, e.g., Ref. 46, the \(R-T-N\) perturbations can be expressed in terms of the shifts in the Keplerian orbital elements as

\[
\begin{align*}
\Delta R &= \sqrt{(\Delta a)^2 + \frac{(\epsilon \Delta a + a \Delta e)^2 + (ae \Delta M)^2}{2}}, \\
\Delta T &= a \sqrt{1 + \frac{e^2}{2}} \left[ \Delta M + \Delta \omega + \cos I \Delta \Omega + \sqrt{(\Delta e)^2 + (e \Delta M)^2} \right], \\
\Delta N &= a \sqrt{\left(1 + \frac{e^2}{2}\right)} \left[ \frac{(\Delta I)^2}{2} + (\sin I \Delta \Omega)^2 \right].
\end{align*}
\]

\[(13)\]
where \( \mathcal{M} \) is the mean anomaly. The lunar Lense-Thirring shifts after one year are, thus

\[
\begin{align*}
\Delta R_{LT} &= 0, \\
\Delta T_{LT} &= a\sqrt{1 + \frac{2}{e^2}} (\Delta \omega_{LT} + \cos I \Delta \Omega_{LT}) = -0.38 \text{ cm}, \\
\Delta N_{LT} &= a\sqrt{1 + \frac{2}{e^2}} \sin I \Delta \Omega_{LT} = 0.07 \text{ cm}.
\end{align*}
\]

(14)

It is important to note that there is no Lense-Thirring secular signature in the Earth-Moon radial motion on which all of the efforts of LLR community have been concentrated so far. It can be shown that a short-period, i.e. not averaged over one orbital revolution, radial signal exists; it is proportional to

\[
\Delta r \propto \frac{2 \mathcal{G} S_0}{c^2 na^2} = 2 \times 10^{-5} \text{ m},
\]

(15)

which is too small to be detected since the present-day accuracy in estimating the amplitudes of radial harmonic signals is of the order of mm. Major limitations come from the post-fit Root-Mean-Square (RMS) accuracy with which the lunar orbit can be reconstructed; the present-day accuracy is about 2 cm in the radial direction \( R \) along the centers-of-mass of the Earth and the Moon. Improvements in the precision of the Earth-Moon ranging of the order of 1 mm are expected in the near future with the APOLLO program. Recently, sub-centimeter precision in determining range distances between a laser on the Earth’s surface and a retro-reflector on the Moon has been achieved. However, it must be considered that the RMS accuracy in the \( T \) and \( N \) directions is likely worse than in \( R \).

4. Conclusions

In this note we have examined the possibility of measuring the action of the intrinsic gravitomagnetic field of the spinning Earth on the lunar orbital motion with the LLR technique. After showing that the Lense-Thirring approximation is adequate for the Earth-Moon system, we found that the Lense-Thirring secular precessions of the Moon’s node and the perigee induced by the Earth’s spin angular momentum are of the order of \( 10^{-3} \) mas yr\(^{-1} \) corresponding to transverse and normal secular shifts of \( 10^{-1} - 10^{-2} \) cm yr\(^{-1} \). The intrinsic gravitomagnetic field of the Earth does not secularly affect the radial component of the Moon’s orbit; a short-period, i.e. not averaged over one orbital revolution, radial oscillation is present, but its amplitude is of the order of \( 10^{-5} \) m. The current RMS accuracy in reconstructing the lunar orbit is of the order of cm in the radial direction; the harmonic components can be determined at the mm level. Forthcoming expected improvements in LLR should allow to reach the mm precision in the Earth-Moon ranging. The present-day accuracy in measuring the lunar precessional rate is of the order of \( 10^{-1} \) mas yr\(^{-1} \). Major limitations come also from some orbital perturbations of classical origin like,
e.g., the secular node precessions induced by the Sun and the oblateness of the Moon which act as systematic errors and whose mismodeled parts are up to 70 times larger than the Lense-Thirring effects. As a consequence of our analysis, we are skeptical concerning the possibility of measuring intrinsic gravitomagnetism with LLR in a foreseeable future. The same conclusion holds also for the Lue-Starkman perigee precession predicted in the framework of the multidimensional braneworld DGP model of modified gravity; indeed, it is as large as the Lense-Thirring one for the Moon.

Acknowledgments

I thank T. Murphy and J.G. Williams (NASA-JPL) for useful information and material supplied.

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