Dynamic Analysis of a Tension Leg Platforms (TLPs) Inspired by Parallel Robotic Manipulators

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ABSTRACT Dynamic performance of a tension leg platform (TLP) is analyzed using a robotic manipulator approach. The TLP is vertically anchored to the seabed by twelve mooring lines. A triangular mooring line configuration is studied. In order to provide necessary motion compensation to impinging water wave loads, necessary mooring line tensioning is accomplished by means of motors with reels mounted on the TLP which keep all mooring lines taut. Manipulator-based mass, damping stiffness matrices are calculated for the TLP. The forces due to the impinging water waves are estimated based on integral methods. The dynamic response is studied within the context of a rigid-body framework. The effect of increasing/decreasing the mooring line stiffness and pre-tensions on the dynamic behavior of the TLP are investigated.

INDEX TERMS Marine platforms, TLP, robotic manipulators, dynamic analysis, mooring line stiffness, water waves.

I. INTRODUCTION Floating marine platforms are key structures for offshore oil and gas exploration and production, particularly, in deep seas. These platforms’ operation depends on the harmonious management of the mooring system and the marine-riser. Mooring lines are used to anchor the platform with the seabed. They also restore the floating platform stability when it is exposed to loads caused by environmental conditions [1].

Understanding the dynamic behavior of floating marine platforms is critical for their design and operation. Predominantly, rigid body dynamics have been employed, considering the floating marine platform as a single rigid body having six degrees of freedom about a given inertial reference coordinate system. The floating platform motion can be divided into horizontal platform dynamics (surge, sway and yaw motions), which occur at the water wave frequency, and the vertical platform dynamics (heave, roll and pitch), which occur at high frequencies [2].

Tension leg platform has appeared in the literature nearly four decades ago. Paulling [3], [4] demonstrated how an uncoupled tension leg platform works, and this offers us one of the earliest tension leg platform models that is available in the literature. While three degrees of freedoms [3] were studied for the first tension leg platform model, six degrees of freedoms were studied by Paulling [4].

A number of studies focused on the dynamical analysis in tension leg platforms (TLPs). Lee and Wang [5] presented a detailed analytical solution for the dynamic behavior of a TLP system, in which the floating platform is moved through the sea wave-induced surge and drag motions. Chen et al. [6] examined the coupled dynamic interaction between the hull of a floating platform and its risers and tendons. Bae and Kim [7] studied the dynamic behavior of a mono-column-TLP under the effect of the second-order sum-frequency wave excitations. Chandraskaran and Jain [8] investigated the structural response of a triangular-shaped TLP under regular sea wave conditions, and they compared it with a four-legged TLP. Yang et al. [9] analyzed random dynamic responses of TLPs under different sea states and different wave approach angles. Reza and Sedighi [10] used the homotopy analysis method to study the dynamic behavior of a tension leg platform. Wang et al. [11] investigated the dynamic of a Tension Leg Platform by combining wind and wave loads within the typhoon area. Oyejobi et al. [12] used a linear wave superposition method to study the dynamic behavior of a tension leg platform for both unidirectional and directional seas. Han et al. [13] studied the stability and dynamic response of a submerged tension leg platform. Jameel et al. [14] investigated the platform motion and tether tension of tension leg platform subjected to different sea states. While many
researchers studied the dynamic behavior of TLP using different approaches, none of them used the concept of cable-driven parallel manipulators.

Cable-Driven Parallel Manipulators (CDPMs) are parallel manipulators that are driven by mooring lines instead of rigid-links. They are known for their high acceleration capability and their large workspace area, compared to those rigid link parallel manipulators. There have been different types of CDPM designs presented in the literature, such as NIST Robocrane [15], WARP [16], Falcon-7 [17], WiRo [18], the hybrid cable-actuated robot developed by Mroz and Notash [19] and DeltaBot [20].

Many researchers studied different types of tension leg platforms (TLPs) and they used fixed values for the mooring line stiffness and tension which depend on the mooring lines properties. However, this paper analyzed a tension leg platform (TLP) that has adjustable mooring lines’ stiffness and tension which can be adjusted by the means of motors with reels mounted on the TLP. The TLP is analyzed based on the robot manipulator approach and it explores the effect of the mooring lines stiffness and tensions on the TLP’s dynamic behavior. In addition, it analyzes the dynamics of the TLP which is fixed by mooring lines (as depicted in Figure 1), whose motors and reels are mounted on the floating platform and their ends are anchored in the sea bed.

II. TLP CONFIGURATION

The general layout of the TLP used is shown in Figure 1 and Figure 2, and the layout has been arranged based on the TLP literature configurations [8], [21], [22]. Based on the comparative studies carried out on TLPs of different geometries, the triangular TLPs are cost-effective [8]. It is also shown that, in the surge and heave degrees of freedom of the triangular TLPs, the response exhibited is less than that of the square TLPs.

The mooring system for a TLP is made up of 12 mooring lines, which are grouped in 6 mooring bundles. Every two mooring lines are considered as one mooring bundle, and the centerlines of the mooring bundles are arranged as shown in Figure 2. The stiffness/tension of every mooring bundle is equal to the summation of the stiffness/tension of the two mooring lines.

The mooring bundles are connected with the anchors which are located, with respect to the positive X-axis, at angles $A_1, A_2, A_3, A_4, A_5$ and $A_6$. Those points are located around the perimeter of the base circle on the sea bed. Also, these mooring bundles are driven by motors/reels on the floating platform. Those motors/reels are located, with respect to the positive y axis, at angles $B_1, B_2, B_3, B_4, B_5$ and $B_6$. Table 1 shows the angles of $A_i$ and $B_i$ where $i = 1, 2, \ldots, 6$. Table 2 shows the TLP and sea wave parameters which are used in this paper analysis.

The platform and the mooring lines are considered as a single system and the analysis is carried out for the 6 DOF under different environmental loads. Different assumptions were considered in the analysis:

- The water is incompressible, inviscid and it has a constant density and temperature.
- The waves only move in one direction and the motion is irrotational.
- The waves have a small amplitude compared to their wavelength.
- The anchors’ positions of the motors are assumed to be parallel to the center of gravity for simplicity.
- The number of mooring lines/bundles should equal the number of degrees of freedom to have a fully constrained platform in the sea [23].

III. EQUATION OF MOTION

Modeling can be carried out when considering the platform as a rigid body supported by mooring lines with known stiffness [1], [2]. As with any dynamic system, a TLP can be described
by the equation of motion.

\[ M\ddot{x} + C\dot{x} + Kx = F \quad (1) \]

where \( M, C, \) and \( K \) are the mass, damping and stiffness matrices of the TLP, and \( F \) is the vector of the total forces and moments applied on the TLP.

### A. MASS MATRIX

The mass matrix is given as the sum of two parts. The first part is the TLP structural mass matrix, which consists of the inertial mass of TLP’s structure. The second part is the added mass matrix, known as the hydrodynamic mass matrix, which depends on the fluid domain around the TLP structure.

#### 1) STRUCTURAL MASS MATRIX

Structural mass is assumed to be constant, diagonal and is lumped at each platform degree of freedom. The structural mass matrix corresponding to the platform displacement at the center of gravity is given as

\[
M_s = \begin{bmatrix}
m_{11} & 0 & 0 & 0 & 0 & 0 \\
0 & m_{22} & 0 & 0 & 0 & 0 \\
0 & 0 & m_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & I_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & I_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & I_{66}
\end{bmatrix} \quad (2)
\]

where the notations 1-6 are referred to the surge, heave, sway, roll, pitch and yaw degree of freedoms respectively; \( m_{ii} \) is the total mass of the structural platform; \( I_{44}, I_{55}, \) and \( I_{66} \) are the total mass moments of inertia about the x-axis, y-axis, and z-axis, respectively.

#### 2) ADDED MASS MATRIX

Strip theory, as described by Faltinsen and Newman [24], [25], has been used to obtain the added mass coefficients in the surge direction. Additionally, the flat disk approach, used by Mansouri and Hadidi [26], has been applied to find the added mass coefficient in the heave direction. Assume \( KG \) is the distance from the center of gravity (CG) of the moving platform to the still water level (SWL); \( c_a \) is the added mass coefficient, \( a \) is the radius of the moving platform, and \( \rho \) is the water density.

Using strip theory, the added mass force in the surge direction \( (F_{11}) \) is given as

\[ F_{11} = \int_{-b}^{0} \pi c_a \rho r^2 dz \ddot{X} \quad (3) \]

Therefore, the added mass is given as

\[ m_{11} = \pi c_a \rho r^2 b \quad (4) \]

In the heave direction, the added mass becomes, using the flat disk approach,

\[ m_{33} = \frac{4}{3} \rho r^3 \quad (5) \]

To obtain the added mass in the pitch direction, strip theory has been used.

\[ F_{51} = -\int_{-b}^{0} \pi c_a \rho r^2 (z - KG) d\dot{z} \ddot{X} \quad (6) \]

Therefore, the added mass is given as

\[ m_{51} = \pi c_a \rho r^2 (bKG + b/2) \quad (8) \]

On the other hand, the same criteria is used to find \( m_{15} \)

\[ F_{15} = \pi c_a \rho r^2 b(KG + b/2) \ddot{\theta} \quad (9) \]

Therefore, the added mass is given as

\[ m_{15} = \pi c_a \rho r^2 b(KG + b/2) \quad (11) \]

The added mass of the platform in the pitch direction is as follows:

\[ dF_{55} = -dF_{15}(z - KG) \quad (12) \]

Therefore,

\[ dF_{55} = \pi c_a \rho r^2 (z - KG)^2 d\dot{\theta} \quad (13) \]

Then, the added mass is given as

\[ m_{55} = \pi c_a \rho r^2 \left( \frac{b^3}{3} + KG^2 + KGb^2 \right) \quad (14) \]

Brunstad [27] stated that the mass force in the x-direction of a circular cylinder is equal to the one in the y-direction. Also, for a circular cylinder, the mass moment about the
x-axis is equal to the one about the y-axis; therefore, \( m_{22} = m_{11}, m_{44} = m_{55}, m_{23} = m_{13}, m_{24} = m_{31}, m_{24} = m_{15}, m_{42} = m_{51}, m_{34} = m_{35}, m_{43} = m_{53}. \) All other added masses are equal to zero because the TLP platform is vertical and symmetrical. The final added mass matrix of the TLP is given by

\[
M_a = \begin{bmatrix}
    m_{11} & 0 & 0 & 0 & m_{15} & 0 \\
    0 & m_{22} & 0 & m_{24} & 0 & 0 \\
    0 & 0 & m_{33} & 0 & 0 & 0 \\
    0 & m_{42} & 0 & m_{44} & 0 & 0 \\
    m_{51} & 0 & 0 & 0 & m_{55} & 0 \\
    0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

(15)

B. DAMPING MATRIX

There are many uncertainties in the damping matrix than the mass and stiffness matrices. In this study, the structural damping has been considered, while the damping that comes from the mooring lines is assumed to be negligible. Inceceik [28] used strip theory to calculate the damping matrix while in this paper the aforementioned has been conducted, based on [8], [9], [29]–[31], as shown in the following equation:

\[
C = 2 \begin{bmatrix}
    \xi_1\omega_1m_{ii} & 0 \\
    0 & \xi_2\omega_2m_{ii}
\end{bmatrix}, \quad i = 1, 2, 3
\]

(16)

where \( \xi_1\omega_1m_{ii} = \text{diag}[\xi_1\omega_1m_{11}, \xi_2\omega_2m_{22}, \xi_3\omega_3m_{33}] \), \( \xi_2\omega_2m_{ii} = \text{diag}[\xi_2\omega_1I_{11}, \xi_2\omega_2I_{22}, \xi_3\omega_3I_{33}] \), \( \xi_i \) is the damping ratio in the specified motion direction of the TLP, which assumed to be 0.0027, 0.0044, and 0.0026 for surge, sway, and heave respectively and 0.0005 for roll, pitch and yaw; and \( \omega_i \) is the natural frequency of the TLP in the specified degree of freedom.

C. STIFFNESS MATRIX

The stiffness matrix is given as the summation of two parts. The first part is the mooring line stiffness matrix and it depends on the mooring line stiffness and tension, and the TLP position. The second part is the added stiffness matrix which is known as the hydrostatic stiffness matrix and it depends on the fluid domain around the TLP. Following researchers [32, 33], mooring lines considered linear springs.

1) JACOBEAN MATRIX

For the TLP, the Jacobian matrix \( J \) [34] can be represented as the relationship between the velocity of the moving platform’s centroid \( [Y_p \quad \omega_p]^T \) and the changes in mooring line lengths \( \dot{I} \). In FIGURE 1, \( P = [x \quad y \quad z]^T \) is the position vector of the moving platform with respect to the base frame; Roll (\( \psi \)), pitch (\( \theta \)) and yaw (\( \phi \)) are represented the Euler rotation angles between the moving \( F \) (u, v, and w) and base \( \Psi \) (X, Y, and Z) frames.

Equation (17) introduces the TLP Jacobian matrix as

\[
J = \begin{bmatrix}
    u^{(1)}T \\
    u^{(2)}T \\
    \vdots \\
    u^{(n)}T
\end{bmatrix}
\begin{bmatrix}
    \hat{b}^{(1)}_{xu}(1) \\
    \hat{b}^{(2)}_{xu}(2) \\
    \vdots \\
    \hat{b}^{(n)}_{xu}(n)
\end{bmatrix}
\]

(17)

where \( u^{(i)} \) is the unit vector along the \( i \)th mooring line direction, \( \hat{b}^{(i)} \) is the vector between the mobile platform center and the motors anchor locations and \( i = 1, \ldots, n \) (\( n \) is the mooring line number).

Using Figure 3 and Figure 4, vector \( u^{(i)} \) and \( \hat{b}^{(i)} \) are represented in Equations (18) and (19):

\[
u^{(i)} = \begin{bmatrix}
    \cos \gamma^{(i)} \sin \alpha^{(i)} \sin \phi^{(i)} \\
    \sin \gamma^{(i)} \sin \alpha^{(i)} \cos \phi^{(i)} \\
    \sin \alpha^{(i)} \cos \phi^{(i)}
\end{bmatrix}^T
\]

(18)

\[
\hat{b}^{(i)} = \Psi R_{T} \begin{bmatrix}
    \cos \beta^{(i)} \sin \phi^{(i)} \\
    \cos \beta^{(i)} \cos \phi^{(i)} \\
    \sin \beta^{(i)}
\end{bmatrix}^T
\]

(19)

where \( \alpha^{(i)} \) represents the angle between \( u^{(i)} \) and \( z' \) axis at point \( A^{(i)} \), \( \gamma^{(i)} \) represents the angle between the projection of \( u^{(i)} \) on \( x'y' \) plane and the \( x' \) axis at point \( A^{(i)} \), \( \hat{b}^{(i)} \) is represented with respect to the base frame. Based on the Euler angle representation, \( \Psi R_{T} \) represents the rotation matrix between the moving and the fixed frames.

2) MOORING LINE STIFFNESS MATRIX

The stiffness matrix of the platform is treated based on manipulator approach and it depends on the mooring line stiffness, mooring line tension and the TLP position. The TLP static force balance can be written as

\[
F = J^T \tau
\]

(20)

where \( F = [F_x \quad F_y \quad F_z \quad M_x \quad M_y \quad M_z]^T \) consists of external forces along with the X, Y and Z directions and external moments about the X, Y and Z directions [35]; \( \tau = [\tau^{(1)}, \tau^{(2)}, \ldots, \tau^{(n)}]^T \) is the vector of internal mooring line forces and \( n \) is the mooring lines’ number.

Equation (20) can be differentiated to calculate the stiffness matrix as follows:

\[
\delta F = J^T \delta \tau + \delta J^T \tau
\]

(21)
The relation between the force and the stiffness matrix $K$ is given as

$$\delta F = K \delta r$$ (22)

where $\delta r$ is the TLP twist vector $\left( [\delta x, \delta y, \delta z, \delta \psi, \delta \theta, \delta \varphi]^T \right)$. For TLP, the stiffness of each mooring line has been modeled as a simple spring. Thus, the changes in forces of the mooring line can be written as follows:

$$\delta \tau = \Omega \delta l$$ (23)

where $\delta l = J \delta r$, $\Omega = \text{diag} \left( k^{(1)}, k^{(2)}, \ldots, k^{(n)} \right)$, and $k^{(i)}$ is the $i$th mooring line stiffness per one meter.

The transpose of the Jacobian matrix depends on the mooring line orientation and the mobile platform orientation [34]. Thus, $\delta J^{(iT)}$ can be written as

$$\delta J^{(iT)} = \frac{\partial J^{(i)}}{\partial \alpha^{(i)}} \delta \alpha^{(i)} + \frac{\partial J^{(i)}}{\partial \gamma^{(i)}} \delta \gamma^{(i)} + \frac{\partial J^{(i)}}{\partial \psi} \delta \psi + \frac{\partial J^{(i)}}{\partial \theta} \delta \theta + \frac{\partial J^{(i)}}{\partial \varphi} \delta \varphi$$ (24)

where $J^{(iT)}$ is the $i$th column of matrix $J^T$, $\delta \alpha = J_\alpha \left[ \delta x, \delta y, \delta z, \delta \psi, \delta \theta, \delta \varphi \right]^T$ and $\delta \gamma = J_\gamma \left[ \delta x, \delta y, \delta z, \delta \psi, \delta \theta, \delta \varphi \right]^T$. $J_\alpha$ and $J_\gamma$ can be found by differentiating the Jacobean matrix with respect to $\alpha$ and $\gamma$ respectively then dividing the result by the mooring line length [36].

Equation (21) can be formulated using Equations (23) and (24) as

$$\delta F = J^T \Omega \delta r + \sum_{i=1}^{n} \tau^{(i)} \left[ \frac{\partial J^{(i)}}{\partial \alpha^{(i)}} J_\alpha + \frac{\partial J^{(i)}}{\partial \gamma^{(i)}} J_\gamma + \frac{\partial J^{(i)}}{\partial \psi} \delta \psi + \frac{\partial J^{(i)}}{\partial \theta} \delta \theta + \frac{\partial J^{(i)}}{\partial \varphi} \delta \varphi \right]$$ (25)

where $\delta F = K \delta r + K_s \delta r$, $K_s$ and $K_r$ are represented the stiffness matrix due to the mooring line’s strength and the mooring line’s tension respectively.

The following equation represents the stiffness matrix.

$$\text{K} = \sum_{i=1}^{n} k^{(i)} + \sum_{i=1}^{n} \tau^{(i)} f^{(i)}$$ (26)

where $\tau^{(i)}$ is the $i$th internal mooring line force, $f^{(i)}$ is the ith mooring line length. The inside entries of the stiffness matrix are presented in appendix A.

3) ADDED STIFFNESS MATRIX

The added stiffness matrix is known as the hydrostatic stiffness matrix, and it depends on the fluid domain around the platform structure. In this paper, the aforementioned has been conducted, based on [26], [29], [31], as shown in the following equation.

$$K_o = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & k_{33} & 0 & 0 & 0 & 0 \\
0 & 0 & k_{44} & 0 & 0 & 0 \\
0 & 0 & 0 & k_{55} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}$$ (27)

where $k_{33} = \rho g A_s$, $k_{44} = k_{55} = \rho g A_s D (K_G - KB)$, $A_s = \frac{2}{3} B^2$, $KB$ is the distance between the Center of Buoyancy and the still water level, $D$ is the diameter of the moving platform, and $g$ is the gravitational acceleration.

In addition to the dynamic wave forces, the water exerts a static upward force (buoyancy force $F_B$) which equals the weight of the water displaced by the submerged part of the platform. The platform is anchored to the seabed by pre-tensioned mooring lines. The static submerged depth can then be calculated as

$$b = \tilde{b} + b_w$$ (28)

where $\tilde{b} = \frac{\tilde{f}}{\rho g z_{\text{at}}}, \begin{bmatrix} 0 & 0 & \tilde{f} & 0 & 0 & 0 \end{bmatrix}^T = J \tilde{\tau}$, $\tilde{b}$ is the submerged depth due to the mooring line pre-tensions, $\tilde{\tau}$ is the vertical force applied downward on the platform by the mooring line pre-tensions, $\tilde{\tau}$ is a vector of the mooring line pre-tension, and $b_w = 0.5 m$ is the submerged depth due to weight.

IV. RIGID-BODY DYNAMICS

Modeling the platform as a rigid body supported by mooring lines, with known stiffness. The dynamic equation and the numerical approach that is explained in section III is validated experimentally by Horoub et al. [37]. The modal analysis
method is used to solve Eq. 1 by converting it to uncoupled differential equations as follows:

$$x_i(t) = Uq_i(t)$$ (29)

where the $x_i(t)$ the generalized coordinates, $q_i(t)$ are the natural coordinates, $U$ is the modal matrix (shape vectors), $i = 1, 2, 3 \ldots 6$.

The mooring line tensions can be calculated as

$$T_i = k\Delta L_i, \quad i = 1, \ldots, 6$$ (30)

where $\Delta L_i = L_{i,2} - L_{i,1}$, 1 and 2 denote the first and second TLP platform pose, $L_{i,j} = \sqrt{[P_j + b_{i,j} - a_i]^T[P_j + b_{i,j} - a_i]}$, $k$ is the mooring line stiffness, $a_i$ is the vector between the TLP base center and the bottom mooring lines anchor locations, and $\Delta L_i$ is the mooring line change length between two positions.

Using mooring line stiffness $k_1 = 30 \times 10^3$ N/m [1], [2] dynamic analysis is conducted on the TLP with the existence of the external forces which are due to the sea waves. The surge, sway, heave, roll, pitch and yaw responses of the TLP are shown in Figure 5. In the TLP heave DOF, the response exhibited is less than that in the surge DOF which is due to the high pre-tensions that TLP has, and this agrees with the literature. It can be seen that the results are different, not only in amplitude but also in the frequencies of their motion response. The pitch DOF has the highest amplitude compared to the other rotation DOFs. On the other hand, the sway DOF has the lowest amplitude which is due to the indirect effect of the sea wave loads on the TLP. The motion response that appeared in Figure 5 part b is due to the dynamic coupling.

V. STIFFNESS AND PRE-TENSION EFFECTS ANALYSIS

This section investigates the effect of increasing/decreasing the mooring line stiffness and pre-tensions on the dynamic behavior of the TLP.

The mooring line stiffness ($k_1 = 30 \times 10^3$ N/m) used in the previous study is increased to $k_2 = 60 \times 10^3$ N/m, then a dynamical analysis is conducted for the two cases at the static submerged depth of $b = 0.6$ m.

It is clearly shown in Figure 6 that increasing the mooring line stiffness decreases the amplitude displacement of the manipulator against the sea wave forces. This result is in agreement with the nature of the cable.

Table 3 represents the dynamical analysis for different mooring lines stiffness cases. It shows the effect of varying the mooring line stiffness on the dynamic behavior of the TLP.

The RMS values of the displacement, velocity, and acceleration are calculated against various values of mooring line stiffness for a static submerged depth of $b = 0.6$ m. It is clearly shown from Table 3 that increasing the mooring line stiffness decreases the displacement, velocity and acceleration of the TLP against the sea wave forces.

Table 4 studied the effect of varying the submerged depth due to mooring line pre-tension on the dynamic behavior of the TLP. The RMS values of the displacement, velocity, and acceleration are calculated against various values of submerged $b$ for $k_1 = 30 \times 10^3$ N/m. In VI, the displacement, velocity and acceleration of the TLP increased because the pre-tension is increased. This is due to the fact that increasing
the mooring line tensions increases the stiffness of the manipulator as shown in Equation (26).

VI. CONCLUSION

In conclusion, the stiffness matrix which is used in the dynamical analysis has been calculated based on the manipulator approach. Moreover, the dynamical model that was mentioned, is a universal model that can be used for any marine platform structure. In this paper, the dynamic analysis was conducted for a TLP in a marine environment. The TLP is driven by twelve mooring lines, assumed as six mooring bundles, anchored in the seabed and driven by reels/motors mounted on the TLP. Dynamic analysis was conducted showing the mooring bundles tensions and TLP displacements, velocities and accelerations against sea wave forces.

The TLP positive tensions were maintained by varying the static submerged depth to balance the variable sea wave forces. It was also shown that increasing the mooring line stiffness or decreasing the pre-tension, reduces the amplitudes of the platform displacement, velocity and acceleration responses.

### TABLE 3. Effect of mooring line stiffness on TLP dynamic displacement, velocity, and acceleration.

| Stiffness ($k_s$) (N/m) | RMS $\psi$ (rad) | RMS $\theta$ (rad) | RMS $\dot{\psi}$ (rad/s) | RMS $\dot{\theta}$ (rad/s) | RMS $\ddot{\psi}$ (rad/s²) | RMS $\ddot{\theta}$ (rad/s²) |
|-------------------------|------------------|------------------|-------------------|-------------------|-----------------|-------------------|
| $10^3$                   | $10^{-2}$        | $10^{-2}$        | $10^{-2}$         | $10^{-2}$         | $10^{-2}$       | $10^{-2}$       |
| 30                      | 0.000019        | 0.0059          | 0.00016           | 0.0042            | 0.0017          | 0.2173           | 0.00023 | 0.0985 |
| 60                      | 0.000009        | 0.00298         | 0.00004           | 0.0023            | 0.0002          | 0.2146           | 0.00006 | 0.0601 |
| 90                      | 0.000008        | 0.00206         | 0.00002           | 0.0016            | 0.0002          | 0.2136           | 0.00002 | 0.0448 |
| 120                     | 0.000008        | 0.0161          | 0.0001            | 0.0012            | 0.0002          | 0.2130           | 0.00001 | 0.0363 |

### TABLE 4. Effect of submerged depth due to mooring line pre-tension on TLP dynamic displacement, velocity, and acceleration.

| $\bar{d}$ (m) | RMS $\psi$ (rad) | RMS $\theta$ (rad) | RMS $\dot{\psi}$ (rad/s) | RMS $\dot{\theta}$ (rad/s) | RMS $\ddot{\psi}$ (rad/s²) | RMS $\ddot{\theta}$ (rad/s²) |
|----------------|------------------|------------------|-------------------|-------------------|-----------------|-------------------|
| $10^{-2}$      | $10^{-2}$        | $10^{-2}$        | $10^{-2}$         | $10^{-2}$         | $10^{-2}$       | $10^{-2}$       |
| 0.6            | 1.87             | 0.000019        | 0.0559            | 0.00016           | 0.0042          | 0.0017           | 0.2173 | 0.00023 | 0.0985 |
| 0.8            | 2.0644           | 0.000027        | 0.1011            | 0.00021           | 0.0042          | 0.0017           | 0.2145 | 0.00033 | 0.1618 |
| 1.0            | 2.1387           | 0.000037        | 0.1539            | 0.00027           | 0.0042          | 0.0016           | 0.2572 | 0.00045 | 0.2309 |
| 1.2            | 2.1968           | 0.000046        | 0.2089            | 0.00034           | 0.0042          | 0.0014           | 0.2679 | 0.00057 | 0.3021 |

### APPENDIX

The following equation represents the elements of the stiffness matrix due to mooring line strength ($K_s$) and due to mooring line tension ($K_t$).

\[
K_s = \sum_{i=1}^{n} k^{(i)}
\]

\[
K_t = \sum_{i=1}^{n} t^{(i)}
\]

where the inside entries of the stiffness matrix as follows

\[
k_{11} = C(y^{(i)}S(a^{(i)})^2)
\]

\[
k_{41} = C(y^{(i)}S(a^{(i)})S(y^{(i)})
\]

\[
k_{12} = C(y^{(i)}S(a^{(i)})^2)
\]

\[
k_{42} = S(a^{(i)})S(y^{(i)})
\]



RMS: ROOT MEAN SQUARE.
\[ k_{13} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ k_{14} = C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ k_{15} = C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ k_{16} = -C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ k_{17} = C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ k_{18} = C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ k_{19} = C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ k_{20} = C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ k_{21} = C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ k_{22} = S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{23} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{24} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{25} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{26} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{27} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{28} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{29} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{30} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{31} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{32} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{33} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{34} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{35} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
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\[ k_{37} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{38} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{39} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{40} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{41} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{42} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{43} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{44} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{45} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]
\[ k_{46} = C(\gamma^{(i)})S(\alpha^{(i)})S(\gamma^{(i)}) \]

where

\[ \xi_1 = b \left[ \cos(\beta^{(i)}) \sin(\theta) - \sin(\beta^{(i)}) \cos(\theta) \sin(\psi) \right] \]
\[ \xi_2 = b \left[ \sin(\beta^{(i)}) \left[ \cos(\psi) \cos(\theta) + \sin(\theta) \sin(\psi) \right] \right. \]
\[ k_{50} = \left. \left. + \cos(\beta^{(i)}) \cos(\theta) \sin(\varphi) \right) \right) \]
\[ \xi_3 = b \left[ \sin(\beta^{(i)}) \left[ \cos(\theta) \sin(\varphi) - \cos(\varphi) \sin(\theta) \sin(\psi) \right] \right. \]
\[ k_{51} = \left. \left. - \cos(\beta^{(i)}) \cos(\theta) \cos(\psi) \right) \right) \]
\[ \xi_4 = b \left[ \sin(\beta^{(i)}) \cos(\theta) \cos(\psi) \right] \]
\[ \xi_5 = b \left[ \sin(\beta^{(i)}) \left[ \cos(\theta) \sin(\varphi) - \sin(\varphi) \sin(\theta) \cos(\psi) \right] \right. \]
\[ k_{52} = \left. \left. + \cos(\beta^{(i)}) \cos(\theta) \cos(\psi) \right) \right) \]
\[ \xi_6 = b \left[ \sin(\beta^{(i)}) \left[ \cos(\theta) \sin(\varphi) + \sin(\varphi) \sin(\psi) \right] \right. \]
\[ k_{53} = \left. \left. - \cos(\beta^{(i)}) \cos(\theta) \cos(\psi) \right) \right) \]
\[ \xi_7 = b \left[ \sin(\beta^{(i)}) \cos(\theta) \sin(\theta) \right. \]
\[ \xi_8 = b \left[ \cos(\beta^{(i)}) \sin(\theta) \right. \]
\[ k_{54} = \left. \left. \left[ \cos(\beta^{(i)}) \sin(\theta) \right] \right) \right) \]
\[ \xi_9 = b \left[ \cos(\beta^{(i)}) \cos(\theta) \sin(\theta) \right. \]
\[ k_{55} = \left. \left. \sin(\beta^{(i)}) \sin(\theta) \right) \right) \]
\[ \xi_{10} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{11} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{12} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{13} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{14} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{15} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{16} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{17} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{18} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{19} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{20} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
\[ \xi_{21} = C(\alpha^{(i)})C(\gamma^{(i)})S(\alpha^{(i)}) \]
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