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A mathematical model of COVID-19 and the multi fears of the community during the epidemiological stage

Ali Yousef\textsuperscript{a}, Fatma Bozkurt\textsuperscript{a,b,\ast}, Thabet Abdeljawad\textsuperscript{c,d,**}, Emad Emreizeeq\textsuperscript{e}

\textsuperscript{a} Kuwait College of Science and Technology, Department of Mathematics, 27235 Kuwait City, Kuwait
\textsuperscript{b} Erciyes University, Department of Mathematics, 38039 Kayseri, Turkey
\textsuperscript{c} Prince Sultan University, Department of Mathematics and Sciences, 11586 Riyadh, Saudi Arabia
\textsuperscript{d} China Medical University, Department of Medical Research, 40402 Taichung, Taiwan
\textsuperscript{e} Abu Dhabi Polytechnic, Department of Mathematics, 111499 Abu Dhabi, United Arab Emirates

\textbf{A R T I C L E I N F O}

\begin{itemize}
  \item Article history:
    \begin{itemize}
      \item Received 12 April 2022
      \item Received in revised form 1 June 2022
    \end{itemize}
  \item Keywords:
    \begin{itemize}
      \item FDEs
      \item Local stability
      \item Global stability
      \item COVID-19
      \item Fear effect
      \item Vaccines
    \end{itemize}
\end{itemize}

\textbf{A B S T R A C T}

Within two years, the world has experienced a pandemic phenomenon that changed almost everything in the macro and micro-environment; the economy, the community’s social life, education, and many other fields. Governments started to collaborate with health institutions and the WHO to control the pandemic spread, followed by many regulations such as wearing masks, maintaining social distance, and home office work. While the virus has a high transmission rate and shows many mutated forms, another discussion appeared in the community: the fear of getting infected and the side effects of the produced vaccines. The community started to face uncertain information spread through some networks keeping the discussions of side effects on-trend. However, this pollution spread confused the community more and activated multi fears related to the virus and the vaccines.

This paper establishes a mathematical model of COVID-19, including the community’s fear of getting infected and the possible side effects of the vaccines. These fears appeared from uncertain information spread through some social sources. Our primary target is to show the psychological effect on the community during the pandemic stage. The theoretical study contains the existence and uniqueness of the IVP and, after that, the local stability analysis of both equilibrium points, the disease-free and the positive equilibrium point. Finally, we show the global asymptotic stability holds under specific conditions using a suitable Lyapunov function. In the end, we conclude our theoretical findings with some simulations.

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\section{Introduction}

\subsection{Epidemiological information of coronaviridae}

From ICTV (\textit{International Committee on Taxonomy of Viruses}), it is known that coronaviruses are members of a subfamily of Coronavirinae. This coronavirinae has four genera groups, while all these genera originated from animals and show mild symptoms in humans [1–3]. For example, HCoV-NL63 and HCoV-229E cause mild infections in the human body.

\textsuperscript{\ast} Corresponding author at: Erciyes University, Department of Mathematics, 38039 Kayseri, Turkey.

\textsuperscript{**} Corresponding author at: Prince Sultan University, Department of Mathematics and Sciences, 11586 Riyadh, Saudi Arabia.

\textit{E-mail addresses: f.bozkurt@kcst.edu.kw (F. Bozkurt), tabdeljawad@psu.edu.sa (T. Abdeljawad).}

https://doi.org/10.1016/j.cam.2022.114624
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and belong to $\alpha$-coronaviruses. For the SADS-CoV still, there is no evidence of human infections. On the other side, both HCoV-OC43 and HCoV-HKU1 are members of $\beta$-coronaviruses, mainly harmless to the human body. Some coronaviruses were isolated almost 50 years ago; for example, the HCoV-229E and HCoV-OC43.

In comparison, new viruses were identified in 2003 and named HCoV-NL63 and HCoV-HKU1. In general, coronaviruses were not seen as dangerous until the endemic spread of SARS-CoV in 2003 and MERS-CoV in 2012 [4–8]. Both viruses had endemic stages and showed respiratory and intestinal infections in the human body. Close to 2020, another virus of the Coronavirinae sub-family was seen in China (Wuhan). People assumed that it might be a similar outbreak of SARS-CoV seen in 2003, and it was expected that the virus would show an endemic form again. This virus, which the WHO named COVID-19, was characterized by two members of $\beta$-coronavirus; the human-origin SARS-CoV Tor2 and bat origin bat-SL-CoVZC45. However, intensive studies showed that this novel virus was mainly close to the bat-origin coronavirus [9]. In March 2020, the WHO upgraded the status of the novel coronavirus from epidemic to pandemic. Since then, the world has suffered from this virus, changing almost everything in ordinary life to stay alive.

1.2. The spread of COVID-19, fear of the community to the infection, and the vaccines

The world saw many forms of diseases that affected the environment of people, history, and their future. The plague, for example, showed up in the 14th century, and many people migrated to different places to protect themselves. Similar health protections were used, such as wearing masks and keeping social distance. The Spanish flu was a pandemic disease caused by the H1N1 influenza A virus and started to be seen in 1918. Some references estimate around 17–50 million deaths, reported as one of the deadliest pandemic events [10,11]. However, while the world reached the 4th Industrial Revolution, people did not expect a massive change in their lives caused by a virus. However, end of December 2019, the world experienced a virus that affected humans’ lives for almost more than two years. At the beginning of the spread, it was reported that the spread started from domestic animals. These were hunted for the fish market in Wuhan (China), which were infected by bats, mainly located in an area close to the Shatan River Valley. Later on, the virus spillover from the intermediate hosts to humans started to cause several severe diseases. Another scenario was spread through different networks that the virus was laboratory human-made, which spread in an institution located in Wuhan.

Besides all this information, we aim to discuss the psychological pressure on humans during the pandemic phenomenon. The world experienced many rules within the two years to prevent the spread of COVID-19. Those fundamental health care protections such as wearing masks and keeping social distance or heavy travel restrictions were, after a while, followed by long-term lockdowns and restricted to lives at home. Within this, the media and the social networks played essential roles, especially during the lock-down period, in informing the civilians about many things related to the spread and the medications. However, useless (unrealistic) information was spread from the same sources. As a result, people created scenarios related to the spread, the epidemiological stage, and the vaccines. Sequentially, some fear levels appeared in the confused community that wanted to know what was true in these stories. The community’s fear appeared not only to the virus but also to the vaccines. Many discussions appeared in the community that the vaccines might show side effects, which would affect our health in the future. This topic was spread through networks since the vaccines were produced shorter than expected. Mutated viruses, such as the delta form or omicron form and others, emphasized the long-existing talks related to the vaccines.

Considering all these pollutions that the community faced during the epidemiological stage, we emphasize the fear that appeared in our social life, which was spread through networks giving different and uncertain information about almost many things related to the virus. Studies show a correlation between the pandemic stage and mental health [12]. The well-known predator–prey model can be defined here as the virus in the form of the hunter and the community as prey. Thus, we adopt a medical (psychological) study to our work, which explains that the fear of the community of the virus (predation fears) can reduce the prey growth rate (psychological effect) by $\%40$ [13], which means that the immune system becomes weak. In the end, studies started to focus also on the psychosocial impact of COVID-19. Some of these studies can be seen in [14–17].

2. Mathematical model

Many phenomena in applied sciences can be characterized using mathematical models to visualize and prove predictions in the environment. Specifically, various mathematical models could explain biological and epidemiological phenomena such as infections, treatments, or diseases. While in the beginning, these studies were restricted to systems of ODEs, it was seen that many problems were able to be formulated by the FDEs that eventually emphasized the historical effect of the dynamical behavior [18–22]. For example, intensive studies related to the spread of the virus to specific regions with accurate data can be seen in [23–28].

In this study, we establish an $SEIQRV$ model that describes the spread of the coronavirus during the pandemic stage. We expand the work by incorporating a vaccinated compartment while also considering the fear of the susceptible compartment against the virus and the unknown vaccines. We assume that the transmission of the virus is only from human to human to avoid any discussions that were considered in 2019, where it was reported that the transmission started from the natural host to intermediate hosts and from there to humans. Studies related to the initial spread can be seen in [19,29]. The system is established in six classes. In this model, $S(t)$ shows the susceptible class, while $E(t)$ denotes...
the silent spreaders [30] and is the exposed compartment, which is infected but shows the symptoms late. The I(t) class is determined as infected by COVID-19. Another compartment is defined as Q(t), which denotes the individuals who are under quarantine. We denote the R(t) as the recovered compartment, and V(t) is the vaccinated class. As mentioned above, our main objective is to consider and analyze two types of fear in the susceptible compartment; the fear of being infected by COVID-19 and the fear of having side effects from the vaccines. One can notice that the fear of COVID-19 shows only a specific time interval of a few years. However, due to the rapid phases of the vaccines, people generally have worries about side effects in the further 5–10 years of their life related to vaccines. Therefore, there are two fear functions in the system, denoted by $\alpha_1$ and $\alpha_2$, which is the fear level in the memory of individuals denoted as “to get infected” and “to have side effects from the vaccine”, respectively. Thus, the fear functions are given as

$$f(\alpha_1, I) = \frac{1}{1 + \alpha_1} \text{ and } g(\alpha_2, V) = \frac{1}{1 + \alpha_2 V},$$

where

$$\Phi = \Theta_1 f(\alpha_1, I) + \Theta_2 g(\alpha_2, V),$$

and the process of fear related to the infection and the worries about the vaccines are denoted by $\Theta_1$ and $\Theta_2$, respectively.

The following statements are reasonably assumed to explain the process of the fear level;

**COVID-19 Fear**

- $f(0, I) = 1$: fear does not exist, and therefore, there is no kind of psychological pressure in the susceptible class.
- $f(\alpha_1, 0) = 1$: from the extinction of the virus, the fear and the psychological effects disappear.
- $\lim_{\alpha_1 \to \infty} f(\alpha_1, I) = 0$: the fear of the virus expands so that the “mental health” of the susceptible class decreases.
- $\lim_{\alpha_1 \to \infty} f(\alpha_1, I) = 0$: the existence of the pandemic stage continues to affect the susceptible class.
- $\frac{\partial f(\alpha_1, I)}{\partial I} < 0$: increase in the fear level also affects the offspring.
- $\frac{\partial f(\alpha_1, I)}{\partial \alpha_1} < 0$: increase in the infection population also affects the offspring.

**Vaccine Fear**

- $g(0, V) = 1$: if there is no fear of the vaccine, the individuals do not have concerns about the vaccine.
- $g(\alpha_2, 0) = 1$: if the percentage of vaccinations decreases, then the fear and uncertain information from networks increase, and the suspicion of vaccines increases.
- $\lim_{\alpha_2 \to \infty} g(\alpha_2, V) = 0$: if the fear and suspicions of the existed vaccines increase, which affects the percentage of the vaccinated compartments, then the infection of the susceptible class continues.
- $\lim_{\alpha_2 \to \infty} g(\alpha_2, V) = 0$: if the vaccination rate of the susceptible class increases because of having freedom in their regular life, however, still there are some suspicions related to side effects of the vaccines, then still the process of fear exists.
- $\frac{\partial g(\alpha_2, V)}{\partial \alpha_2} < 0$: if the fear effect increases, the “mentally healthy” susceptible class decreases.
- $\frac{\partial g(\alpha_2, V)}{\partial V} < 0$: if the vaccinated compartment increases, the non-vaccinated susceptible class decreases, while long-term suspicions still exist.

Thus, the system can be formulated as follows:

$$
\begin{align*}
D^\alpha S(t) &= \Lambda_1 + r (1 + \Phi) S(t) - \sigma S(t)^2 - \beta_1 E(t) S(t) - \beta_2 I(t) S(t) - \beta_3 V(t) S(t) - \delta S(t) - \eta S(t), \\
D^\alpha E(t) &= \Lambda_2 + \beta_1 (1 - \varepsilon_1) E(t) S(t) - \theta E(t) - \eta E(t), \\
D^\alpha I(t) &= \beta_1 \varepsilon_1 E(t) S(t) + \beta_2 I(t) S(t) + \theta E(t) - \gamma_1 I(t) - \eta I(t) - \mu I(t), \\
D^\alpha Q(t) &= \gamma_1 I(t) - \gamma_2 Q(t) - \eta Q(t) - \mu Q(t), \\
D^\alpha R(t) &= \gamma_2 Q(t) - \eta R(t), \\
D^\alpha V(t) &= \delta S(t) - \mu_1 V(t) - \eta V(t),
\end{align*}
$$

(2.1)

where $0 < \alpha \leq 1$ and the parameters belong to $R_+$. The initial values are given as

$$S(0) = S_0, E(0) = E_0, I(0) = I_0, Q(0) = Q_0, R(0) = R_0 \text{ and } V(0) = V_0,$$

(2.2)

while $(S, E, I, Q, R, V) \in R^6$.

The susceptible $S(t)$ denotes the people who can get infected by COVID-19. Thus, there are possibilities that $S(t)$ gets infected by the silent spreaders $E(t)$ with a rate of $\beta_1$ or it can also be infected by $I(t)$ with a rate of $\beta_2$, who do not follow the rules to isolate themselves. Here, $r$ is the growth rate, while $\sigma$ represents the density-dependent coefficient. $\Lambda_1$ denotes the recruitment rate of $S(t)$, $\delta$ shows the rate of vaccinations, who do not worry about taking the vaccines, while $\eta$ denotes the natural death rate for all compartments.

The exposed $E(t)$ compartment, also known as the silent spreaders, shows the symptoms late and therefore continues to spread the disease. This compartment decreases with a screening rate of $\theta$. Besides this, the recognition parameters $\varepsilon_1$ show that this compartment becomes aware of the disease and carefully follows the symptoms to avoid contact with individuals. Another vital piece of information about this compartment is that the individual can be the silent transmitter of COVID-19, as mentioned in the article of Harvard Medical School [30]. This is why all compartments need
to permanently follow the health protection rules since the community also shows individuals who have the virus but do not show the symptoms at all. Thus, we will consider a visible density of existence with $\Lambda_2$.

The density of $I(t)$, which is the infected compartment, increases with a rate of $\beta_1 \xi_1$ that is detected in the exposed compartment as COVID-19. Another scenario is that after the screening, with a rate of $\theta$, the individual notices that it is infected. In addition, we expect that a rate of $\gamma_1$ moves from this compartment to $Q(t)$ shown under quarantine. $\mu$ shows the death rate from COVID-19. The $Q(t)$ compartment shows the natural death rate and the death from corona with the parameters $\eta$ and $\mu$, respectively. The density of this compartment increases with a rate of $\eta$ and needs to be under quarantine. $R(t)$ is the recovered compartment. A density of $\gamma_2 Q(t)$ increases this class, which moves from class $Q(t)$ after sufficient successful treatments. $V(t)$ is the vaccinated compartment that increases with a rate of $\vartheta$, which is moved from the susceptible compartment. However, if this compartment does not follow the systematic vaccination procedure, it can get infected again, even if it shows mild influenza symptoms. Besides this information, according to a Forbes article published in January 2022, vaccinated people rarely die from COVID-19, but a minor death rate exists [31]. Therefore, in this compartment, we incorporate a death rate of $\mu_1$ that represents the death of corona.

In this study, we want to discuss the community’s fear caused by different networks. Our primary target is to recognize how the community lives during the epidemiological event. We want to clarify that precise information from central institutions would give the civilians sufficient awareness of how to live in this pandemic without being terrified by the virus. We want to focus on critical worldwide issues that can be explained to the community without terrifying them. Transparent information, successful treatments, and clearly explained information about the vaccines would help everybody return to “normal life”.

**Definition 2.1** ([32]). For a function $\varphi(t)$, the fractional integral with $\alpha > 0$ is given by Abdel’s formula as

$$I_\alpha \varphi (t) = \frac{1}{\Gamma (\alpha)} \int_0^t (t - x)^{\alpha - 1} \varphi (x) \, dx, \quad x > 0.$$

**Definition 2.2** ([32,33]). The Caputo fractional derivative of order $\alpha \in (n - 1, n)$, where $n$ is a positive integer and $\varphi : \mathbb{R}^+ \rightarrow \mathbb{R}$ is a continuous function is denoted as

$$D_0^\alpha \varphi (x) = \frac{1}{\Gamma (n - \alpha)} \int_0^x \varphi^{(n)}(t) \frac{t^{\alpha - 1}}{(x - t)^{\alpha + 1 - n}} \, dt.$$

When $\alpha = n$, the derivatives are the usual $n$th order derivatives.
3. The existence and uniqueness

In this section, we show that the domain is positive and that the system has a unique solution in \( \mathbb{R}^6_+ \). Let \( \mathbb{R}^6_+ = \{ M \in \mathbb{R}^6 : M \geq 0 \} \) and \( M(t) = (S(t), E(t), I(t), Q(t), R(t), D(t))^T \). Thus, we can show that the domain \( \mathbb{R}^6_+ \) is positively invariant by using the references in [34], [35], and [36].

**Lemma 3.1 ([34]).** Let \( f(x) \in C[a, b] \) and \( D^\alpha f(x) \in C[a, b] \) for \( 0 < \alpha \leq 1 \), then we have

\[
  f(x) = f(a) + \frac{1}{\Gamma(\alpha)} D^\alpha f(\xi) (x - a)^\alpha,
\]

where \( 0 \leq \xi \leq x \), \( \forall x \in (a, b) \).

**Lemma 3.2 ([35]).** Let \( f(x) \in C[0, b] \) and \( D^\alpha f(x) \in C[0, b] \) for \( 0 < \alpha \leq 1 \). Then, considering **Lemma 3.1**, we obtain the statements given as follows:

(i) \( f \) is non-decreasing if \( D^\alpha f(x) \geq 0 \), \( \forall x \in (0, b) \).

(ii) \( f \) is non-increasing if \( D^\alpha f(x) \leq 0 \), \( \forall x \in [0, b] \).

**Theorem 3.1.** The IVP in (2.1)–(2.2) has a unique solution, and it is bounded in \( \mathbb{R}^6_+ \).

**Proof.** From **Lemma 3.1, Lemma 3.2**, and the theory of [36], we want to show the existence and uniqueness of (2.1)–(2.2) in \((0, \infty)\). Notice that

\[
  D^\alpha S(t)|_{t=0} = \Lambda_1 \geq 0,
  D^\alpha E(t)|_{t=0} = \Lambda_2 \geq 0,
  D^\alpha I(t)|_{t=0} = \beta_1 \xi_1 E(t) S(t) + \theta E(t) \geq 0,
  D^\alpha Q(t)|_{t=0} = \gamma_1 I(t) \geq 0,
  D^\alpha R(t)|_{t=0} = \gamma_2 Q(t) \geq 0,
  D^\alpha V(t)|_{t=0} = \theta S(t) \geq 0,
\]

on each hyperplane, which is bounding the non-negative orthant. Thus, the domain \( \mathbb{R}^6_+ \) is invariant positively, which completes the proof.

4. Equilibria and local stability

We analyzed the local stability around the co-existing equilibrium point of (2.1) based on specific conditions using the Routh–Hurwitz Criteria.

4.1. Equilibria points

Let us rewrite the system

\[
  \left\{ \begin{array}{l}
  D^\alpha S(t) = h_1(\chi) = \Lambda_1 + r(1 + \Phi) S(t) - \omega S(t) S(t) - \beta_1 E(t) S(t) - \beta_2 I(t) S(t) - \theta S(t) - \eta S(t), \\
  D^\alpha E(t) = h_2(\chi) = \Lambda_2 + \beta_1 (1 - \epsilon_1) E(t) S(t) - \theta E(t) - \eta E(t), \\
  D^\alpha I(t) = h_3(\chi) = \beta_1 \epsilon_1 E(t) S(t) + \beta_2 I(t) S(t) + \theta E(t) - \gamma_1 I(t) - \eta I(t) - \mu I(t), \\
  D^\alpha Q(t) = h_4(\chi) = \gamma_1 I(t) - \gamma_2 Q(t) - \theta Q(t) - \eta Q(t), \\
  D^\alpha R(t) = h_5(\chi) = \gamma_2 Q(t) - \eta R(t), \\
  D^\alpha V(t) = h_6(\chi) = \theta S(t) - \mu_1 V(t) - \theta V(t),
  \end{array} \right. \tag{4.1}
\]

where \( h_i(\chi) = h_i(S(t), E(t), I(t), Q(t), R(t), V(t)) \) and \( i = 1, 2, 3, 4, 5, 6 \). We want to analyze the stability of (4.1) by using perturbation around the equilibrium points, such as \( \epsilon_i(t) > 0, i = 1, 2, 3, 4, 5, 6 \). Thus, we have;

\[
  S(t) - \bar{S} = \epsilon_1(t), \quad E(t) - \bar{E} = \epsilon_2(t), \quad I(t) - \bar{I} = \epsilon_3(t), \quad Q(t) - \bar{Q} = \epsilon_4(t), \quad R(t) - \bar{R} = \epsilon_5(t), \quad V(t) - \bar{V} = \epsilon_6(t), \tag{4.2}
\]
In this case, we have

\[
\begin{align*}
D^a (\psi_1 (t)) &\simeq h_1 (\chi) + \frac{\partial h_1}{\partial s} \psi_1 (t) + \frac{\partial h_1}{\partial t} \psi_2 (t) + \frac{\partial h_1}{\partial q} \psi_3 (t) + \frac{\partial h_1}{\partial \psi} \psi_4 (t) + \frac{\partial h_1}{\partial \chi} \psi_5 (t) + \frac{\partial h_1}{\partial S} \psi_6 (t), \\
D^a (\psi_2 (t)) &\simeq h_2 (\chi) + \frac{\partial h_2}{\partial s} \psi_1 (t) + \frac{\partial h_2}{\partial t} \psi_2 (t) + \frac{\partial h_2}{\partial q} \psi_3 (t) + \frac{\partial h_2}{\partial \psi} \psi_4 (t) + \frac{\partial h_2}{\partial \chi} \psi_5 (t) + \frac{\partial h_2}{\partial S} \psi_6 (t), \\
D^a (\psi_3 (t)) &\simeq h_3 (\chi) + \frac{\partial h_3}{\partial s} \psi_1 (t) + \frac{\partial h_3}{\partial t} \psi_2 (t) + \frac{\partial h_3}{\partial q} \psi_3 (t) + \frac{\partial h_3}{\partial \psi} \psi_4 (t) + \frac{\partial h_3}{\partial \chi} \psi_5 (t) + \frac{\partial h_3}{\partial S} \psi_6 (t), \\
D^a (\psi_4 (t)) &\simeq h_4 (\chi) + \frac{\partial h_4}{\partial s} \psi_1 (t) + \frac{\partial h_4}{\partial t} \psi_2 (t) + \frac{\partial h_4}{\partial q} \psi_3 (t) + \frac{\partial h_4}{\partial \psi} \psi_4 (t) + \frac{\partial h_4}{\partial \chi} \psi_5 (t) + \frac{\partial h_4}{\partial S} \psi_6 (t), \\
D^a (\psi_5 (t)) &\simeq h_5 (\chi) + \frac{\partial h_5}{\partial s} \psi_1 (t) + \frac{\partial h_5}{\partial t} \psi_2 (t) + \frac{\partial h_5}{\partial q} \psi_3 (t) + \frac{\partial h_5}{\partial \psi} \psi_4 (t) + \frac{\partial h_5}{\partial \chi} \psi_5 (t) + \frac{\partial h_5}{\partial S} \psi_6 (t), \\
D^a (\psi_6 (t)) &\simeq h_6 (\chi) + \frac{\partial h_6}{\partial s} \psi_1 (t) + \frac{\partial h_6}{\partial t} \psi_2 (t) + \frac{\partial h_6}{\partial q} \psi_3 (t) + \frac{\partial h_6}{\partial \psi} \psi_4 (t) + \frac{\partial h_6}{\partial \chi} \psi_5 (t) + \frac{\partial h_6}{\partial S} \psi_6 (t),
\end{align*}
\]

and

\[
\text{where we denote } h_i (\chi) = h_i (S, E, I, Q, R, \psi) \text{ and } i = 1, 2, 3, 4, 5, 6.
\]

We use the property that

\[
h_i (S, E, I, Q, R, \psi) = 0, \quad (4.3)
\]

for \( i = 1, 2, 3, 4, 5, 6 \). Thus, a linearized system around the fixed points is obtained, such as

\[
D^a V = J V,
\]

where \( V = (\psi_1 (t), \psi_2 (t), \psi_3 (t), \psi_4 (t), \psi_5 (t), \psi_6 (t)) \). Here, we denote \( J \) as the Jacobian matrix, and from \( W^{-1} J W = C \), the matrix \( C \) represents the diagonal matrix of \( \lambda_i (1 \leq i \leq 6) \). \( W \) denotes the eigenvectors of \( J \). In this case, we can write

\[
\begin{bmatrix}
D^a \psi_1 \\
D^a \psi_2 \\
D^a \psi_3 \\
D^a \psi_4 \\
D^a \psi_5 \\
D^a \psi_6 
\end{bmatrix} = \begin{bmatrix}
\lambda_1 \psi_1 \\
\lambda_2 \psi_2 \\
\lambda_3 \psi_3 \\
\lambda_4 \psi_4 \\
\lambda_5 \psi_5 \\
\lambda_6 \psi_6
\end{bmatrix}, \quad \text{where } \psi = \begin{bmatrix}
\psi_1 \\
\psi_2 \\
\psi_3 \\
\psi_4 \\
\psi_5 \\
\psi_6
\end{bmatrix}. \quad (4.5)
\]

The solutions are given, such as

\[
\begin{align*}
\psi_1 (t) &= \sum_{n=0}^{\infty} \frac{(\lambda_1)^n}{\Gamma (n+1)} \psi_1 (0) = E_a (\lambda_1 t^n) \psi_1 (0), \\
\psi_2 (t) &= \sum_{n=0}^{\infty} \frac{(\lambda_2)^n}{\Gamma (n+1)} \psi_2 (0) = E_a (\lambda_2 t^n) \psi_2 (0), \\
\psi_3 (t) &= \sum_{n=0}^{\infty} \frac{(\lambda_3)^n}{\Gamma (n+1)} \psi_3 (0) = E_a (\lambda_3 t^n) \psi_3 (0), \\
\psi_4 (t) &= \sum_{n=0}^{\infty} \frac{(\lambda_4)^n}{\Gamma (n+1)} \psi_4 (0) = E_a (\lambda_4 t^n) \psi_4 (0), \\
\psi_5 (t) &= \sum_{n=0}^{\infty} \frac{(\lambda_5)^n}{\Gamma (n+1)} \psi_5 (0) = E_a (\lambda_5 t^n) \psi_5 (0), \\
\psi_6 (t) &= \sum_{n=0}^{\infty} \frac{(\lambda_6)^n}{\Gamma (n+1)} \psi_6 (0) = E_a (\lambda_6 t^n) \psi_6.
\end{align*}
\]

From the study of [37,38], the stability criteria using the Mittag-Leffler functions are proven, and therefore, if \( |\arg (\lambda_i)| > \frac{\alpha}{\gamma} \) (1 \leq i \leq 6), then \( \psi_i (1 \leq i \leq 6) \) are decreasing. Thus, \( \psi_i (i = 1, 2, 3, 4, 5, 6) \) are decreasing.

Our study focuses on the stability analysis of the extinction of the disease \( \chi_1 = (S_1, 0, 0, 0, 0, \psi_1) \) and the co-existing (positive) equilibrium point, which is denoted as \( \chi_2 = (S_2, E_2, I_2, Q_2, R_2, \psi_2) \).

### 4.2 Disease-free and co-existing equilibrium points local stability analysis

In this section, we assume that the civilians have total awareness \( \varepsilon_1 = 1 \). Thus, we aim to focus mainly on the fear of the susceptible class and the other essential parameters during the pandemic stage, such as the screening effect, the treatment procedure, and the vaccine.

The Jacobian matrix around the disease-free equilibrium point \( \chi_1 = (S_1, 0, 0, 0, 0, \psi_1) \) is given as

\[
J (\chi_1) = \begin{bmatrix}
a_{11} & a_{12} & a_{13} & 0 & 0 & 0 \\
0 & a_{22} & 0 & 0 & 0 & 0 \\
0 & a_{32} & a_{33} & 0 & 0 & 0 \\
0 & 0 & a_{43} & a_{44} & 0 & 0 \\
0 & 0 & a_{54} & a_{55} & 0 & 0 \\
a_{61} & 0 & 0 & 0 & 0 & a_{66}
\end{bmatrix} \quad , \quad (4.6)
\]
where \( a_{11} = r - 2\sigma \bar{s}_1 - \theta - \eta, a_{12} = -\beta_{1}\bar{s}_1, a_{13} = -\beta_{2}\bar{s}_1, a_{22} = -\theta - \eta, a_{32} = \beta_{1}e_{1}\bar{s}_1 + \theta, a_{33} = \beta_{2}\bar{s}_1 - \gamma_1 - \eta - \mu, a_{43} = \gamma_1, a_{44} = -\gamma_2 - \eta - \mu, a_{54} = \gamma_2, a_{55} = -\eta, a_{61} = \theta, a_{66} = -\mu_1 - \eta \) and \( \Theta_1 = \Theta_2 = 0 \), since the fear process does not exist in the disease-free case.

The characteristic equation of (4.6) is obtained as

\[
(a_{11} - \lambda) (a_{22} - \lambda) (a_{33} - \lambda) (a_{44} - \lambda) (a_{55} - \lambda) (a_{66} - \lambda) = 0. \tag{4.7}
\]

The following theorem shows the stability conditions for the disease-free equilibrium point for a susceptible and vaccinated community.

**Theorem 4.1.** Let \( \chi_1 = (\bar{s}_1, 0, 0, 0, \bar{v}_1) \) be the disease-free equilibrium point of system (4.1). If

\[
\frac{r - \theta - \eta}{2\sigma} < \bar{s}_1 < \frac{\gamma_1 + \eta + \mu}{\beta_2},
\]

where \( r > \theta + \eta \), then \( \chi_1 \) is locally asymptotically stable.

**Proof.** It can be seen that

(i) \( \lambda_1 = r - 2\sigma \bar{s}_1 - \theta - \eta < 0 \), if \( \bar{s}_1 > \frac{r - \theta - \eta}{2\sigma} \) for \( r > \theta + \eta \).

(ii) \( \lambda_2 = -\theta - \eta < 0 \).

(iii) \( \lambda_3 = \beta_2 \bar{s}_1 - \gamma_1 - \eta - \mu < 0 \), if \( \bar{s}_1 < \frac{\gamma_1 + \eta + \mu}{\beta_2} \).

(iv) \( \lambda_4 = -\gamma_2 - \eta - \mu < 0 \).

(v) \( \lambda_5 = -\eta \).

(vi) \( \lambda_5 = -\mu_1 - \eta \).

The proof is completed.

**Remark 4.1.** The conditions in Theorem 4.1. show that the epidemiological spread stops if the infection rate of the \( S-I \) interaction reduces. In addition, with successful treatment medications, the individuals under quarantine can recover soon. Another essential stage is that individuals with zero worries about the virus and the vaccines return to a healthy susceptible compartment. The vaccination rate is high, and regular taking shows that a community without infection or minor cases tends to approach zero.

The Jacobian matrix of \( \chi_2 = (\bar{s}_2, \bar{v}_2, \bar{t}_2, \bar{r}_2, \bar{v}_2, \bar{v}_2) \) is given as

\[
J(\chi_2) = \begin{bmatrix}
    a_{11} & a_{12} & a_{13} & 0 & 0 & a_{16} \\
    0 & a_{22} & 0 & 0 & 0 & 0 \\
    a_{31} & a_{32} & a_{33} & 0 & 0 & 0 \\
    0 & 0 & a_{43} & a_{44} & 0 & 0 \\
    0 & 0 & 0 & a_{54} & a_{55} & 0 \\
    a_{61} & 0 & 0 & 0 & 0 & a_{66}
\end{bmatrix} \tag{4.9}
\]

where

\[
a_{11} = r \left( 1 + \bar{v}_1 \right) - 2\sigma \bar{s}_2 - \beta_{1}\bar{E}_2 - \beta_{2}\bar{T}_2 - \theta - \eta, a_{12} = -\beta_{1}\bar{s}_2, a_{13} = -\frac{\beta_{1}\bar{e}_1 \bar{s}_2}{(1+\alpha_{1})}, a_{22} = -\theta - \eta - \delta_1, a_{31} = \beta_{1}\bar{E}_2 + \beta_{2}\bar{T}_2, a_{32} = \beta_{1}\bar{s}_2 + \theta, a_{33} = \beta_{2}\bar{s}_2 - (\gamma_1 + \eta + \mu), a_{43} = \gamma_1, a_{44} = -(\gamma_2 + \eta + \mu), a_{54} = \gamma_2, a_{55} = -\eta, a_{61} = \theta \text{ and } a_{66} = -\mu_1 - \eta.
\]

From the Jacobian matrix in (4.9), we obtain

\[
\lambda_2 = -\theta + \eta + \delta_1 < 0, \lambda_4 = -\gamma_2 + \eta + \mu < 0 \text{ and } \lambda_5 = -\eta < 0, \tag{4.10}
\]

and the characteristic equation as

\[
\lambda^3 + (-a_{11} - a_{33} - a_{66}) \lambda^2 + (a_{11}a_{33} + a_{11}a_{66} + a_{33}a_{66} - a_{13}a_{31} + a_{16}a_{61} + a_{11}a_{33}a_{66} - a_{13}a_{31}a_{66} - a_{16}a_{61}a_{33}) \lambda + a_{11}a_{33}a_{66} - a_{13}a_{31}a_{66} - a_{16}a_{61}a_{33} = 0. \tag{4.11}
\]

Using the theory in [36,39], we apply the local stability conditions for the characteristic Eq. (4.11).

**Theorem A ([36,39]).** The positive equilibrium point \( \chi_2 \) of (4.1) is asymptotically local stable if one of the following holds. Let

\[
\lambda^3 + A_1 \lambda^2 + A_2 \lambda + A_3 = 0, \tag{4.12}
\]

where \( A_1, A_2 \) and \( A_3 \) are coefficients and \( \tilde{D} \) denotes the discriminant of (4.12) and is given by

\[
\tilde{D}(\lambda) = 18A_1A_2A_3 + (A_1A_2)^2 - 4A_3A_1^3 - 4A_2^3 - 27A_3^2 > 0; \tag{4.13}
\]
If $\bar{D}(\lambda) > 0$, $A_1 > 0$, $A_3 > 0$ and $A_1A_2 > A_3$.

(ii) If $\bar{D}(\lambda) < 0$, $A_1 \geq 0$, $A_2 \geq 0$, $A_3 > 0$ and $\alpha < \frac{2}{3}$.

(iii) If $\bar{D}(\lambda) < 0$, $A_1 < 0$, $A_2 < 0$ and $\alpha > \frac{2}{3}$.

**Theorem 4.2.** Let $\gamma_2$ be the co-existing equilibrium point of (4.1). Moreover, we assume that $\frac{\gamma_1+\eta+\mu}{\beta_2} < \bar{S}_2 < \frac{\gamma_1+2\eta+\mu+\mu_1}{\beta_2}$ and $r < \frac{2\sigma\bar{S}_2+\beta_2\bar{T}_2+\beta_2\bar{S}_2+\gamma_1+3\eta+\mu+\mu_1+\theta}{(1+\Phi)}$ for $2\sigma > \beta_2$. If

$$f(\alpha_1, \bar{T}_2) > \frac{(1 + \alpha_1\bar{T}_2)}{\alpha_1\Theta_1} \left( \beta_2\bar{S}_2 - (\gamma_1 + \eta + \mu) \right) \left( \frac{r_{\alpha\bar{f}(\alpha_1, \bar{T}_2)\Theta_1\bar{S}_2}}{1 + \alpha_1\bar{T}_2} \right) - 2 \left( r(1 + \Phi) - 2\sigma\bar{S}_2 - \beta_1\bar{E}_2 - \beta_2\bar{T}_2 - \theta - \eta \right) - \beta_2 \right) \left( \beta_1\bar{E}_2 + \beta_2\bar{T}_2 \right) \bar{S}_2$$

and

$$g(\alpha_2, \bar{V}_2) > \frac{2(\mu_1 + \eta)}{\alpha_2\Theta_2 + \bar{S}_2} \left( r(1 + \Phi) - 2\sigma\bar{S}_2 - \beta_1\bar{E}_2 - \beta_2\bar{T}_2 - \theta - \eta \right),$$

where

$$\Theta = \left( \frac{r_{\alpha\bar{f}(\alpha_1, \bar{T}_2)\Theta_1\bar{S}_2}}{1 + \alpha_1\bar{T}_2} \right) - 1 \left( \beta_2\bar{S}_2 - (\gamma_1 + \eta + \mu) \right).$$

Then from Theorem A/(i), the positive equilibrium point is local asymptotic stable.

**Proof.** Let the discriminant $\bar{D}(\lambda) > 0$, and the following conditions hold:

- We have $A_1 > 0 \implies a_{11} + a_{33} + a_{66} < 0$, if

$$[r(1 + \Phi) - 2\sigma\bar{S}_2 - \beta_1\bar{E}_2 - \beta_2\bar{T}_2 - \theta - \eta] < 0,$$

where we obtain

$$r < \frac{2\sigma\beta_2}{1 + \Phi}$$

for $2\sigma > \beta_2$.

Considering the condition for $A_3 > 0$, we get

$$\frac{r_{\alpha\bar{g}(\alpha_2, \bar{V}_2)\Theta_2\bar{S}_2}}{1 + \alpha_2\bar{V}_2} \Theta_1\bar{S}_2 \left( \gamma_1 + \eta + \mu \right) \left( \mu_1 + \eta \right) + \frac{(r_{\alpha\bar{f}(\alpha_1, \bar{T}_2)\Theta_1\bar{S}_2})}{1 + \alpha_1\bar{T}_2} \left( \beta_1\bar{E}_2 + \beta_2\bar{T}_2 \right) \left( \mu_1 + \eta \right),$$

for $\bar{S}_2 > \frac{\gamma_1+\eta+\mu}{\beta_2}$, where

$$r < \frac{2\sigma\beta_2 + \beta_1\bar{E}_2 + \beta_2\bar{T}_2 + (\eta + \theta)}{(1 + \Phi)},$$

and

$$\Theta = \left( \frac{r_{\alpha\bar{g}(\alpha_2, \bar{V}_2)\Theta_2\bar{S}_2}}{1 + \alpha_2\bar{V}_2} \right) - 1 \left( \beta_2\bar{S}_2 - (\gamma_1 + \eta + \mu) \right).$$

From both (4.17) and (4.19), we have

$$r < \frac{2\sigma\bar{S}_2 + \beta_1\bar{E}_2 + \beta_2\bar{T}_2 + (\eta + \theta)}{(1 + \Phi)} < \frac{2\sigma\bar{S}_2 + \beta_1\bar{E}_2 + \beta_2\bar{T}_2 + (\gamma_1 + 3\eta + \mu + \mu_1 + \theta)}{(1 + \Phi)},$$

since $\bar{S}_2 > \frac{\gamma_1+2\eta+\mu+\mu_1}{\beta_2}$.

Finally, from $A_1A_2 > A_3$, we have

$$(-a_{11} - a_{33} - a_{66}) (a_{11}a_{33} + a_{11}a_{66} + a_{33}a_{66} - a_{13}a_{31} - a_{16}a_{61}) > a_{11}a_{33}a_{66} - a_{13}a_{31}a_{66} - a_{16}a_{61}a_{13},$$

which is written in the form as follows:
\[ 2a_{16}a_{61}a_{33} + 2a_{13}a_{31}a_{66} - 4a_{11}a_{33}a_{66} > (a_{16}a_{61} - a_{33}^2)(-a_{11} - a_{66}) + (a_{13}a_{31} - a_{66}^2)(-a_{11} - a_{33}) + a_{11}^2(a_{33} + a_{66}). \]

(4.21)

This inequality holds the statements:

(I) \( a_{13}a_{31} < a_{66}^2 \implies - \left( \frac{r_\alpha \theta_2 \bar{S}_2}{(1 + \alpha \bar{I}_2)} + \beta_2 \bar{S}_2 \right) (\beta_1 \bar{E}_2 + \beta_2 \bar{I}_2) < (\mu_1 + \eta)^2. \)

(II) \( a_{16}a_{61} < a_{33}^2 \implies - \left( \frac{r_\alpha \theta_2 \bar{S}_2}{(1 + \alpha \bar{I}_2)} (\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu))^2. \)

(III) \( a_{11}^2(a_{33} + a_{66}) < 0, \)

(IV) \( a_{13}a_{31}a_{66} > a_{33}(-a_{16}a_{61} + 2a_{11}a_{66}) \)

\[ \implies \left( \frac{r_\alpha \theta_2 \bar{S}_2}{(1 + \alpha \bar{I}_2)} + \beta_2 \bar{S}_2 \right) (\beta_1 \bar{E}_2 + \beta_2 \bar{I}_2) > (\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu))^2, \]

\[ = -2 \left( r(1 + \bar{I}) - 2\alpha \bar{S}_2 - \beta_1 \bar{E}_2 - \beta_2 \bar{I}_2 - \vartheta - \eta \right). \]

if

\[ f(\alpha_1, \bar{I}_2) > \frac{(1 + \alpha \bar{I}_2)}{r_\alpha \theta_1} \left( \frac{r_\alpha \theta_2 \bar{S}_2}{(1 + \alpha \bar{I}_2)} - \beta_2 \right) \]

\[ \times \left( \frac{r_\alpha \theta_2 \bar{S}_2}{(1 + \alpha \bar{I}_2)} (\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu)) \right) \]

\[ \frac{(\beta_1 \bar{E}_2 + \beta_2 \bar{I}_2)}{\beta_2 \bar{S}_2} \]

and

\[ g(\alpha_2, \bar{V}_2) > \frac{2(\mu_1 + \eta)(1 + \alpha_2 \bar{V}_2)(r(1 + \bar{I}) - 2\alpha \bar{S}_2 - \beta_1 \bar{E}_2 - \beta_2 \bar{I}_2 - \vartheta - \eta)}{r_\alpha \theta_2 \bar{S}_2} \]  \( (4.23) \)

Remark 4.2. Theorem 4.2 shows that compartment \( S \) is dependent on the treated, less contact with the infected compartment, and taking the vaccines keeping a robust immune system to have a low percentage of mortality in \( \mu_1 \). The recognition of the susceptible class is very high, and the screening rate is considered permanently to avoid any increase in the silent spreaders compartment. The increase in vaccination reduces the worry of getting infected and seeing side effects from the vaccines. The decrease in both \( \theta_1 \) and \( \theta_2 \) can reduce the effect of fear functions, which shows that both fears have strong relationships in the psychological process of the human body.

Theorem 4.3. Let \( \bar{I}_2 \) be the co-existing equilibrium point of (4.1). Furthermore, we assume that \( \frac{\gamma_1 + \eta + \mu}{\beta_2} < \bar{S}_2 < \frac{\gamma_1 + 2\eta + \mu + \mu_1}{\beta_2} \) and \( r \leq \frac{2\alpha \bar{S}_2 + \beta_1 \bar{E}_2 + \beta_2 \bar{I}_2}{(1 + \bar{I})} \) for \( 2\alpha > \beta_2 \). If

\[ f(\alpha_1, \bar{I}_2) \geq \left( \frac{\beta_1 \bar{E}_2 + \beta_2 \bar{I}_2}{\beta_2 \bar{S}_2} \right) \left( \frac{\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu)}{(1 + \alpha \bar{I}_2)} (\mu_1 + \eta) - r_\alpha \theta_2 \bar{S}_2 \right) \]

\[ g(\alpha_2, \bar{V}_2) \leq \left( \frac{2(\mu_1 + \eta)(1 + \alpha_2 \bar{V}_2)(r(1 + \bar{I}) - 2\alpha \bar{S}_2 - \beta_1 \bar{E}_2 - \beta_2 \bar{I}_2 - \vartheta - \eta)}{r_\alpha \theta_2 \bar{S}_2} \right), \]

(4.24)

and

(4.25)

where

\[ \vartheta = \frac{(r_\alpha \theta_1 \theta_2 \bar{S}_2 (1 + \alpha_1 \bar{I}_2)^{-1} \beta_1 \bar{E}_2 + \beta_2 \bar{I}_2)(\mu_1 + \eta)}{r_\alpha \theta_2 (\alpha \bar{V}_2)(\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu))}. \]

Then from Theorem A(ii), the positive equilibrium point is local asymptotic stable and \( \alpha < \frac{\gamma_1}{2} \).

Proof. Let the discriminant \( \bar{D}(\lambda) < 0 \). Besides this, from \( A_1 \geq 0 \) we have \( a_{11} + a_{33} + a_{66} \leq 0, \) if

\[ r \left( 1 + \bar{I} \right) - 2\alpha \bar{S}_2 - \beta_1 \bar{E}_2 - \beta_2 \bar{I}_2 - 2\vartheta - (\gamma_1 + 3\eta + \mu + \mu_1 + \vartheta) \leq 0, \]

(4.26)

where we obtain

\[ r \leq \frac{(2\alpha - \beta_2) \bar{S}_2 + \beta_1 \bar{E}_2 + \beta_2 \bar{I}_2 + (\gamma_1 + 3\eta + \mu + \mu_1 + \vartheta)}{(1 + \bar{I})} \] for \( 2\alpha > \beta_2 \).
Considering the condition for $A_3 > 0$, we get
\[
\frac{r \alpha_2 g (\alpha_2, \bar{V}_2) \varTheta_2 \bar{S}_2}{(1 + \alpha_2 \bar{V}_2)} \theta (\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu)) > \left\{ r (1 + \bar{\Phi}) - 2 \sigma \bar{S}_2 - \beta_1 \bar{E}_2 - \beta_2 \bar{I}_2 - \bar{\theta} - \bar{\eta} \right\} \\
\times (\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu)) (\mu_1 + \eta) + \left( \frac{r \alpha_1 f (\alpha_1, \bar{T}_2) \varTheta_2 \bar{S}_2}{(1 + \alpha_1 \bar{T}_2)} + \beta_2 \bar{S}_2 \right) (\beta_1 \bar{E}_2 + \beta_2 \bar{I}_2) (\mu_1 + \eta),
\]
for $\bar{S}_2 > \frac{\gamma_1 + \eta + \mu}{\beta_2}$, where
\[
r \leq \frac{2 \sigma \bar{S}_2 + \beta_1 \bar{E}_2 + \beta_2 \bar{I}_2 + (\eta + \bar{\theta})}{(1 + \bar{\Phi})}
\]
and
\[
\theta > \frac{r \alpha_1 f (\alpha_1, \bar{T}_2) \varTheta_2 \bar{S}_2 (1 + \alpha_1 \bar{T}_2)^{-1} + \beta_2 \bar{S}_2}{r \alpha_2 g (\alpha_2, \bar{V}_2) \varTheta_2 \bar{S}_2 (1 + \alpha_2 \bar{V}_2)^{-1} (\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu))}.
\]
From both (4.27) and (4.29), we have
\[
r \leq \frac{2 \sigma \bar{S}_2 + \beta_1 \bar{E}_2 + \beta_2 \bar{I}_2 + (\eta + \bar{\theta})}{(1 + \bar{\Phi})} \leq \frac{2 \sigma - \beta_2 \bar{S}_2 + \beta_1 \bar{E}_2 + \beta_2 \bar{I}_2 (\gamma_1 + 3 \eta + \mu + \mu_1 + \bar{\theta})}{(1 + \bar{\Phi})},
\]
if $\bar{S}_2 < \frac{\gamma_1 + 2 \eta + \mu + \mu_1}{\beta_2}$.

In addition, from $A_2 \geq 0$, we get
\[
a_{11} a_{33} + a_{11} a_{66} + a_{33} a_{66} - a_{11} a_{31} - a_{16} a_{61} \geq 0
\]
\[
\implies (r (1 + \bar{\Phi}) - 2 \sigma \bar{S}_2 - \beta_1 \bar{E}_2 - \beta_2 \bar{I}_2 - \bar{\theta} - \bar{\eta}) (\beta_2 \bar{S}_2 - (\gamma_1 + 2 \eta + \mu + \mu_1)) - (\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu)) (\mu_1 + \eta)
\]
\[
+ \left( r \alpha_1 \theta_1 \bar{S}_2 (1 + \alpha_1 \bar{T}_2) + \beta_2 \bar{S}_2 \right) (\beta_1 \bar{E}_2 + \beta_2 \bar{I}_2) + \frac{r \alpha_2 \theta_2 \bar{S}_2 \bar{\theta}}{(1 + \alpha_2 \bar{V}_2)} \geq 0,
\]
where we have
\[
f (\alpha_1, \bar{T}_2) \geq \frac{1 + \alpha_1 \bar{T}_2}{r \alpha_1 \theta_1} \left\{ \frac{\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu)}{(\mu_1 + \eta)} - \frac{r \alpha_2 \theta_2 \bar{S}_2 \bar{\theta}}{(1 + \alpha_2 \bar{V}_2)} g (\alpha_2, \bar{V}_2) (1 + \alpha_2 \bar{V}_2)^{-1} - \beta_2 \right\}
\]
and
\[
g (\alpha_2, \bar{V}_2) \leq \frac{\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu)}{r \alpha_2 \theta_2 \bar{S}_2 \bar{\theta}} (1 + \alpha_2 \bar{V}_2) (\mu_1 + \eta).
\]

**Remark 4.3.** The proof in Theorem 4.3. showed that the fear of getting infected is higher than being vaccinated. Thus, the susceptible class is convinced that the vaccines do not show side effects now and in the future. The susceptible do this action considering mainly the information of successful treatments and low death rates in the vaccinated compartments. While the worry of getting infected still exists in the non-vaccinated group, any worry about vaccines shrinks because of professional and precise information about the vaccines.

**Theorem 4.4.** Let $\chi_2$ be the co-existing equilibrium point of (4.1). Moreover, assume that $\bar{S}_2 < \frac{\gamma_1 + 2 \eta + \mu + \mu_1}{\beta_2}$ and $r > \frac{2 \sigma - \beta_2 \bar{S}_2 + \beta_1 \bar{E}_2 + \beta_2 \bar{I}_2 (\gamma_1 + 3 \eta + \mu + \mu_1 + \bar{\theta})}{(1 + \bar{\Phi})}$ for $2 \sigma > \beta_2$. If
\[
f (\alpha_1, \bar{T}_2) < \frac{1 + \alpha_1 \bar{T}_2}{r \alpha_1 \theta_1} \left\{ \frac{\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu)}{(\mu_1 + \eta)} - \frac{r \alpha_2 \theta_2 \bar{S}_2 \bar{\theta}}{(1 + \alpha_2 \bar{V}_2)} g (\alpha_2, \bar{V}_2) (1 + \alpha_2 \bar{V}_2)^{-1} - \beta_2 \right\}
\]
and
\[
g (\alpha_2, \bar{V}_2) \leq \frac{\beta_2 \bar{S}_2 - (\gamma_1 + \eta + \mu)}{r \alpha_2 \theta_2 \bar{S}_2 \bar{\theta}} (1 + \alpha_2 \bar{V}_2) (\mu_1 + \eta),
\]
then from Theorem A/(iii), the positive equilibrium point is local asymptotic stable and $\alpha > \frac{\bar{\theta}}{\bar{\eta}}$.

**Proof.** Let the discriminant $\bar{D} (\lambda) < 0$. Moreover, from $A_1 < 0$ we have $a_{11} + a_{33} + a_{66} > 0$, if
\[
r (1 + \bar{\Phi}) - 2 \sigma \bar{S}_2 - \beta_1 \bar{E}_2 - \beta_2 \bar{I}_2 - \beta_2 \bar{S}_2 - (\gamma_1 + 3 \eta + \mu + \mu_1 + \bar{\theta}) > 0,
\]
where we obtain
\[
r > \frac{(2\sigma - \beta_2)S_2 + \beta_1E_2 + \beta_2I_2 + (\gamma_1 + 3\eta + \mu + \mu_1 + \vartheta)}{(1 + \Phi)} \text{ for } 2\sigma > \beta_2.
\] (4.37)

Considering now \( A_2 < 0 \), we have
\[
a_{11}a_{33} + a_{11}a_{66} + a_{33}a_{66} - a_{13}a_{31} - a_{16}a_{61} < 0
\]
\[
\implies r \left( \frac{1 + \Phi}{1 + \Phi} - 2\sigma \right) - \beta_1E_2 - \beta_2I_2 - \vartheta - \eta \left( \beta_2S_2 - (\gamma_1 + 2\eta + \mu + \mu_1) \right) - \left( \beta_2S_2 - (\gamma_1 + \eta + \mu) \right) (\mu_1 + \eta)
\]
\[
+ \left( \frac{r}{1 + \frac{1}{(1 + \alpha_1)I_2^2}} + \beta_2S_2 \right) \left( \beta_1E_2 + \beta_2I_2 \right) > 0,
\] (4.38)

where we obtain
\[
r \left( \frac{1 + \Phi}{1 + \Phi} - 2\sigma \right) - \beta_1E_2 - \beta_2I_2 - \vartheta - \eta > 0 \implies r > \frac{2\sigma S_2 + \beta_1E_2 + \beta_2I_2 + \vartheta + \eta}{(1 + \Phi)},
\] (4.39)

\[
\beta_2S_2 - (\gamma_1 + 2\eta + \mu + \mu_1) < 0 \implies S_2 < \gamma_1 + 2\eta + \mu + \mu_1, \quad (4.40)
\]

\[
f(\alpha_1, T_2) < \frac{1 + \alpha_1T_2}{r\alpha_1} \left( \frac{\beta_2S_2 - (\gamma_1 + \eta + \mu_1)}{\beta_2} \right) \left( 1 + \alpha_2, V_2 \right) + \frac{r\alpha_2\beta_2S_2}{r\alpha_2\beta_2S_2} \left( \beta_1E_2 + \beta_2I_2 \right) S_2 \left( \mu_1 + \eta \right)
\] (4.41)

and
\[
g(\alpha_2, V_2) < \frac{\beta_2S_2 - (\gamma_1 + \eta + \mu_1)}{r\alpha_2\beta_2S_2} \left( 1 + \alpha_2, V_2 \right) \left( \mu_1 + \eta \right),
\] (4.42)

which completes the proof.

**Remark 4.4.** In Theorem 4.4., where \( \alpha > \frac{1}{r} \), we notice that the worry and the fear of the vaccines and the virus’s infection decreases. The individuals show this psychological stage in the last variant of the corona, the Omicron form. Most people were vaccinated, and infected individuals see mild symptoms similar to influenza. Vaccination of a wide age interval is considered, and the minor death rates with corona, including successful treatments, support the return to everyday life.

### 5. Global stability of the equilibrium points

We apply the discretization process to analyze the global stability of both equilibrium points, the disease-free and the co-existing. Since many decisions and actions were made in discrete time intervals during the two years of the epidemiological spread, we prefer to consider the global stability of system (4.1) as a system of difference equations. The discretization is an approximation for the right-hand side of the FDE \( D^\alpha X(t) = f(X(t)), t > 0 \) where \( \alpha \in (0, 1) \). We consider the discrete-time effect and rewrite system (4.1) according to this, while the recognition parameter is chosen as \( \varepsilon = 1 \). The discretization of the system (4.1) is as follows:

\[
\left\{
\begin{array}{l}
D^\alpha S(t) = A_1 + \tau \left( 1 + \Phi \right) S \left( \frac{1}{2} \right) x - \sigma S \left( \frac{1}{2} \right) x^2 - \beta_1E \left( \frac{1}{2} \right) x - \beta_2I \left( \frac{1}{2} \right) x - S \left( \frac{1}{2} \right) x \\
- \vartheta S \left( \frac{1}{2} \right) x - \eta S \left( \frac{1}{2} \right) x \\
D^\alpha E(t) = A_2 - \theta E \left( \frac{1}{2} \right) x - \eta E \left( \frac{1}{2} \right) x \\
D^\alpha I(t) = \beta_1E \left( \frac{1}{2} \right) x + \beta_2I \left( \frac{1}{2} \right) x + \theta E \left( \frac{1}{2} \right) x - \gamma I \left( \frac{1}{2} \right) x - \eta I \left( \frac{1}{2} \right) x - \vartheta I \left( \frac{1}{2} \right) x - \mu I \left( \frac{1}{2} \right) x \\
D^\alpha Q(t) = \gamma I \left( \frac{1}{2} \right) x - \gamma Q \left( \frac{1}{2} \right) x - \eta Q \left( \frac{1}{2} \right) x - \mu Q \left( \frac{1}{2} \right) x \\
D^\alpha R(t) = \gamma Q \left( \frac{1}{2} \right) x - \eta R \left( \frac{1}{2} \right) x \\
D^\alpha V(t) = \vartheta S \left( \frac{1}{2} \right) x - \mu V \left( \frac{1}{2} \right) x - \eta V \left( \frac{1}{2} \right) x.
\end{array}
\right.
\] (5.1)

For \( t \in [0, h], \frac{1}{h} \in (0, 1) \), we have

\[
\left\{
\begin{array}{l}
D^\alpha S(t) = A_1 + \tau \left( 1 + \Phi \right) S_0 - \sigma S_0^2 - \beta_1E_0S_0 - \beta_2I_0S_0 - \vartheta S_0 - \eta S_0 \\
D^\alpha E(t) = A_2 - \theta E_0 - \eta E_0 \\
D^\alpha I(t) = \beta_1E_0S_0 + \beta_2I_0S_0 + \theta E_0 - \gamma_1I_0 - \eta I_0 - \mu I_0 \\
D^\alpha Q(t) = \gamma_1I_0 - \gamma Q_0 - \eta Q_0 - \mu Q_0 \\
D^\alpha R(t) = \gamma Q_0 - \eta R_0 \\
D^\alpha V(t) = \vartheta S_0 - \mu V_0 - \eta V_0.
\end{array}
\right.
\] (5.2)
The solution of (5.2) reduces to

\[
\begin{align*}
S_1(t) &= S_0 + \frac{r^\mu}{\rho(\alpha+1)} \{ A_1 + r (1 + \Phi) S_0 - \sigma S_0^2 - \beta_1 E_0 S_0 - \beta_2 l_0 S_0 - \vartheta S_0 - \eta S_0 \} \\
E_1(t) &= E_0 + \frac{r^\mu}{\rho(\alpha+1)} \{ A_2 - \vartheta E_0 - \eta E_0 \} \\
l_1(t) &= l_0 + \frac{r^\mu}{\rho(\alpha+1)} \{ \beta_1 E_0 S_0 + \beta_2 l_0 S_0 + \vartheta E_0 - \gamma l_0 - \eta l_0 - \mu l_0 \} \\
Q_1(t) &= Q_0 + \frac{r^\mu}{\rho(\alpha+1)} \{ \vartheta l_1 - \gamma_2 Q_0 - \eta Q_0 - \mu Q_0 \} \\
R_1(t) &= R_0 + \frac{r^\mu}{\rho(\alpha+1)} \{ \gamma_2 Q_0 - \eta R_0 \} \\
V_1(t) &= V_0 + \frac{r^\mu}{\rho(\alpha+1)} \{ \vartheta S_1 - \mu_1 V_0 - \eta V_0 \}.
\end{align*}
\]

For \( t \in [h, 2h) \), \( \frac{t}{h} \in [1, 2) \) we obtain

\[
\begin{align*}
S_2(t) &= S_1 + \frac{(t-h)^\mu}{\rho(\alpha+1)} \{ A_1 + r (1 + \Phi) S_1 - \sigma S_1^2 - \beta_1 E_1 S_1 - \beta_2 l_1 S_1 - \vartheta S_1 - \eta S_1 \} \\
E_2(t) &= E_1 + \frac{(t-h)^\mu}{\rho(\alpha+1)} \{ A_2 - \vartheta E_1 - \eta E_1 \} \\
l_2(t) &= l_1 + \frac{(t-h)^\mu}{\rho(\alpha+1)} \{ \beta_1 E_1 S_1 + \beta_2 l_1 S_1 + \vartheta E_1 - \gamma l_1 - \eta l_1 - \mu l_1 \} \\
Q_2(t) &= Q_1 + \frac{(t-h)^\mu}{\rho(\alpha+1)} \{ \vartheta l_1 - \gamma_2 Q_1 - \eta Q_1 - \mu Q_1 \} \\
R_2(t) &= R_1 + \frac{(t-h)^\mu}{\rho(\alpha+1)} \{ \gamma_2 Q_1 - \eta R_1 \} \\
V_2(t) &= V_1 + \frac{(t-h)^\mu}{\rho(\alpha+1)} \{ \vartheta S_1 - \mu_1 V_1 - \eta V_1 \}.
\end{align*}
\]

Repeating the process \( n \) times, we have

\[
\begin{align*}
S_{n+1}(t) &= S_n + \frac{(t-nh)^\mu}{\rho(\alpha+1)} \{ A_1 + r (1 + \Phi) S_n - \sigma S_n^2 - \beta_1 E_n S_n - \beta_2 l_n S_n - \vartheta S_n - \eta S_n \} \\
E_{n+1}(t) &= E_n + \frac{(t-nh)^\mu}{\rho(\alpha+1)} \{ A_2 - \vartheta E_n - \eta E_n \} \\
l_{n+1}(t) &= l_n + \frac{(t-nh)^\mu}{\rho(\alpha+1)} \{ \beta_1 E_n S_n + \beta_2 l_n S_n + \vartheta E_n - \gamma l_n - \eta l_n - \mu l_n \} \\
Q_{n+1}(t) &= Q_n + \frac{(t-nh)^\mu}{\rho(\alpha+1)} \{ \vartheta l_n - \gamma_2 Q_n - \eta Q_n - \mu Q_n \} \\
R_{n+1}(t) &= R_n + \frac{(t-nh)^\mu}{\rho(\alpha+1)} \{ \gamma_2 Q_n - \eta R_n \} \\
V_{n+1}(t) &= V_n + \frac{(t-nh)^\mu}{\rho(\alpha+1)} \{ \vartheta S_n - \mu_1 V_n - \eta V_n \}.
\end{align*}
\]

Let \( t \in [nh, (n+1) h) \) and \( t \to (n+1) h \) for \( \alpha \to 1 \). Then we have

\[
\begin{align*}
S_{n+1}(t) &= S_n + \frac{r^\mu}{\rho(\alpha+1)} \{ A_1 + r (1 + \Phi) S_n - \sigma S_n^2 - \beta_1 E_n S_n - \beta_2 l_n S_n - \vartheta S_n - \eta S_n \} \\
E_{n+1}(t) &= E_n + \frac{r^\mu}{\rho(\alpha+1)} \{ A_2 - \vartheta E_n - \eta E_n \} \\
l_{n+1}(t) &= l_n + \frac{r^\mu}{\rho(\alpha+1)} \{ \beta_1 E_n S_n + \beta_2 l_n S_n + \vartheta E_n - \gamma l_n - \eta l_n - \mu l_n \} \\
Q_{n+1}(t) &= Q_n + \frac{r^\mu}{\rho(\alpha+1)} \{ \vartheta l_n - \gamma_2 Q_n - \eta Q_n - \mu Q_n \} \\
R_{n+1}(t) &= R_n + \frac{r^\mu}{\rho(\alpha+1)} \{ \gamma_2 Q_n - \eta R_n \} \\
V_{n+1}(t) &= V_n + \frac{r^\mu}{\rho(\alpha+1)} \{ \vartheta S_n - \mu_1 V_n - \eta V_n \}.
\end{align*}
\]

**Lemma 5.1.** Assume that \( \{X(n)\}_{n=0}^\infty \) is a positive solution of (5.6). Thus, the following conditions hold.

(i) If

\[
\begin{align*}
A_1 &> (\sigma S_n + \beta_1 E_n + \beta_2 l_n + \vartheta + \eta - r (1 + \Phi)) S_n, \\
A_2 &> (\theta + \eta) E_n, \\
E_n &> \frac{(\gamma_2 + \eta + \mu - 2 \beta_2 S_n) l_n}{(\beta_2 S_n + \beta_1 E_n + \beta_2 l_n + \vartheta + \eta - r (1 + \Phi))}, \\
l_n &> \frac{\gamma_1}{(\gamma_2 + \eta + \mu - 2 \beta_2 S_n) l_n}, \\
Q_n &> \frac{\eta R_n}{\gamma_1}, \\
S_n &> \frac{\vartheta S_n}{\mu_1}.
\end{align*}
\]

Then the positive solution \( \{X(n)\}_{n=0}^\infty \) of system (5.6) is monotonic increasing.

(ii) If

\[
\begin{align*}
A_1 &< (\sigma S_n + \beta_1 E_n + \beta_2 l_n + \vartheta + \eta - r (1 + \Phi)) S_n, \\
A_2 &< (\theta + \eta) E_n, \\
E_n &< \frac{(\gamma_2 + \eta + \mu - 2 \beta_2 S_n) l_n}{(\beta_2 S_n + \beta_1 E_n + \beta_2 l_n + \vartheta + \eta - r (1 + \Phi))}, \\
l_n &< \frac{\gamma_1}{(\gamma_2 + \eta + \mu - 2 \beta_2 S_n) l_n}, \\
Q_n &< \frac{\eta R_n}{\gamma_1}, \\
S_n &< \frac{\vartheta S_n}{\mu_1}.
\end{align*}
\]
Then the positive solution $[X(n)]_{n=0}^\infty$ of system (5.6) is monotonic decreasing.

**Proof.** The following computation can be obtained to analyze the monotonic behavior of the solution in system (5.6), such as
\[
\begin{align*}
S_{n+1} - S_n &= \frac{\nu}{\beta'_{S11}} \left( A_1 + r (1 + \Phi) S_n - \sigma S_n^2 - \beta_1 E_n S_n - \beta_2 I_n S_n - \theta S_n - \eta S_n \right) \\
E_{n+1} - E_n &= \frac{\nu}{\beta'_{E11}} \left( A_2 - \theta E_n - \eta E_n \right) \\
I_{n+1} - I_n &= \frac{\nu}{\beta'_{I11}} \left( \beta_1 E_n S_n + \beta_2 I_n S_n + \theta E_n - \gamma I_n - \eta I_n - \mu I_n \right) \\
Q_{n+1} - Q_n &= \frac{\nu}{\beta'_{Q11}} \left( \gamma I_n - \gamma_2 Q_n - \eta Q_n - \mu Q_n \right) \\
R_{n+1} - R_n &= \frac{\nu}{\beta'_{R11}} \left( \gamma_2 Q_n - \eta R_n \right) \\
V_{n+1} - V_n &= \frac{\nu}{\beta'_{V11}} \{ \theta S_n - \mu V_n - \eta V_n \}.
\end{align*}
\]
(5.9)
Thus it can be seen that for the conditions in (i), system (5.9) shows
\[
S_{n+1} > S_n, \ E_{n+1} > E_n, \ I_{n+1} > I_n, \ Q_{n+1} > Q_n, \ R_{n+1} > R_n, \text{ and } V_{n+1} > V_n
\]
and based on the conditions in (ii), we have
\[
S_{n+1} < S_n, \ E_{n+1} < E_n, \ I_{n+1} < I_n, \ Q_{n+1} < Q_n, \ R_{n+1} < R_n, \text{ and } V_{n+1} < V_n.
\]
(5.10, 5.11)

**Theorem 5.1.** Let $\chi_1$ be the disease-free equilibrium of (5.6). Assume that the local stability conditions and Lemma 5.1/(ii) hold. If
\[
h_1 < \left( \frac{2 (S_n - \bar{S}_1) \Gamma (\alpha + 1)}{(\sigma S_n + \beta_1 E_n + \beta_2 I_n + \theta + \eta - r (1 + \Phi)) S_n - \Lambda_1} \right)^{\frac{1}{2}} \text{ and } h_6 < \left( \frac{2 (V_n - \bar{V}_1) \Gamma (\alpha + 1)}{(\mu_1 + \eta) V_n - \theta S_n} \right)^{\frac{1}{2}},
\]
where $S_n > \bar{S}_1$ and $V_n > \bar{V}_1$, then $\chi_1$ is global asymptotically stable.

**Proof.** Let us establish a suitable Lyapunov function $L(n)$ defined by
\[
L(n) = \left( X(n) - \chi_1 \right)^2, \ n = 0, 1, 2, \ldots
\]
(5.13)
where $X(n) = (S(n), E(n), I(n), Q(n), R(n), V(n))$ and $\chi_1 = (\bar{S}_1, 0, 0, 0, 0, \bar{V}_1)$.

A change along (5.13) would show the following:
\[
\Delta L(n) = L(n+1) - L(n) = \left( X(n+1) - \chi_1 \right)^2 - \left( X(n) - \chi_1 \right)^2 = \left( X(n+1) - X(n) \right) (X(n+1) + X(n) - 2 \chi_1).
\]
(5.14)

Considering the susceptible compartment of system (5.6), we have
\[
\Delta L_1(n) = (S(n+1) - S(n)) (S(n+1) + S(n) - 2 \bar{S}_1).
\]
(5.15)
Using Lemma 5.1/(ii), we can see that $S(n+1) < S(n)$. Thus, we need to show only that
\[
S(n+1) + S(n) - 2 \bar{S}_1 > 0,
\]
(5.16)
which holds for
\[
h < \left( \frac{2 (S_n - \bar{S}_1) \Gamma (\alpha + 1)}{(\sigma S_n + \beta_1 E_n + \beta_2 I_n + \theta + \eta - r (1 + \Phi)) S_n - \Lambda_1} \right)^{\frac{1}{2}} \text{ and } S_n > \bar{S}_1.
\]
(5.17)
Thus, we obtain $\Delta L_1(n) < 0$. From Lemma 5.1/(ii), we can find that also $\Delta L_i(n) < 0$ for $i = 2, 3, 4, 5$. Finally, similar to the computations of the susceptible compartment, we obtain that $\Delta L_6(n) < 0$, if
\[
V(n+1) + V(n) - 2 \bar{V}_1 > 0,
\]
(5.18)
which holds if
\[
h < \left( \frac{2 (V_n - \bar{V}_1) \Gamma (\alpha + 1)}{(\mu_1 + \eta) V_n - \theta S_n} \right)^{\frac{1}{2}} \text{ for } V_n > \bar{V}_1.
\]
(5.19)
The proof is completed.

The following theorem is similar to Theorem 5.1. and is omitted.
Theorem 5.2. Let \( \chi_2 \) be the disease-free equilibrium point of system (5.6). Moreover, assume that the local stability conditions and Lemma 5.1(ii) hold. If
\[
\begin{align*}
    h_1 &< \left( \frac{2(Q_n - \overline{Q}_n) \Gamma (\alpha + 1)}{(\Theta_n + \beta_1 I_n)^2 + \beta_2 I_n + \gamma_1 I_n} \right)^{\frac{1}{3}}, \\
    h_2 &< \left( \frac{2(\Theta_n - \overline{\Theta}_n) \Gamma (\alpha + 1)}{(\Theta_n + \eta + \mu) \Theta_n - \gamma_1 I_n} \right)^{\frac{1}{3}}, \\
    h_3 &< \left( \frac{2(\Theta_n - \overline{\Theta}_n) \Gamma (\alpha + 1)}{(\Theta_n + \eta + \mu) \Theta_n - \gamma_1 I_n} \right)^{\frac{1}{3}}, \\
    h_4 &< \left( \frac{2(Q_n - \overline{Q}_n) \Gamma (\alpha + 1)}{(\gamma_2 + \eta + \mu) \Theta_n - \gamma_1 I_n} \right)^{\frac{1}{3}}, \\
    h_5 &< \left( \frac{2(\Theta_n - \overline{\Theta}_n) \Gamma (\alpha + 1)}{\eta \Theta_n - \gamma_1 Q_n} \right)^{\frac{1}{3}}, \\
    h_6 &< \left( \frac{2(Q_n - \overline{Q}_n) \Gamma (\alpha + 1)}{\eta \Theta_n - \gamma_1 Q_n} \right)^{\frac{1}{3}},
\end{align*}
\]
where \( S_n > \overline{S}_n, E_n > \overline{E}_n, I_n > \overline{I}_n, Q_n > \overline{Q}_n, R_n > \overline{R}_n \) and \( V_n > \overline{V}_n \), then the equilibrium point \( \chi_2 \) is globally asymptotically stable.

6. Some simulations considering the fear level and the process of fear

We illustrate the fear effect of the susceptible compartment denoted as \( \alpha_1 \) and \( \alpha_2 \) and the process of worrying with \( \Theta_1 \) and \( \Theta_2 \) on using MATLAB 2019. Here we want to show the split fear of the susceptible class getting infected and having side effects from taking the vaccines. Many discussions about the virus were made in the media and on social networks for a long time. This was the first time in the millennium that the world faced a pandemic event. The unspecific and polluted information was made to terrify the civilians during the lull-down periods that expanded to long-term locks. Suddenly, there was hope for the produced vaccines. However, again, polluted information was spread from different media sources and social networks that these vaccines have strong side effects. Thus, not only was the virus a challenge for the people, to find the precise and accurate facts was also tricky.

We determine the initial values of (4.1) as \( S(0) = 2000, E(0) = 80, I(0) = 40, Q(0) = 30, R(0) = 10, V(0) = 2 \) (see Table 2).

In Fig. 1, we see a dynamical behavior when the awareness is assumed to be \( \varepsilon = 1 \). We consider a full recognition of the civilians to the information related to the virus. However, still, the screening rates are around \( s \% 30 \). We notice that the vaccines’ recognition and the infection’s knowledge keep the susceptible class strong without fear. The process of worry is \( \Theta_1 = \Theta_2 = 0 \). The vaccination rate is still low but started to be taken by individuals. Even this tiny start of being vaccinated reduces the exposed compartment (including the infected compartment). Besides this, since the treatments are successful and, in addition, they are explained well to the civilians through health organizations, we can see rapid movements from the quarantine to the recovery class.

In Fig. 2, we assume that the susceptible class shows a process of fear for both cases of being infected and taking the vaccines, such as is \( \Theta_1 = \Theta_2 = 0.2 \). The process of worry is still significantly low; however, considering the permanent spread of unrealistic and polluted information activates the fear to \( \alpha_1 = \alpha_2 = 0.5 \). While vaccinations started to be applied, people preferred to wait and see side effects. This waiting time allows for the existence of the virus in the environment.

| Table 2 | Parametric values of Table 1. |
|-----------------|-----------------|-----------------|
| Parameter       | Parameter description | Rates          |
| \( \alpha_1 \)  | The fear level stored in the memory of compartment | \([0, 1]\) |
| \( \alpha_2 \)  | Fear level in the memory of the susceptible class | \([0, 1]\) |
| \( \Theta_1 \)  | The process of fear occurs when the individual notices from different sources uncertain information about the virus | \([0, 1]\) |
| \( \Theta_2 \)  | The process of fear occurs when the individual notices from different sources uncertain information about the vaccines | \([0, 1]\) |
| \( \lambda_1 \)  | The recruitment rate of the susceptible class | 12000 |
| \( \lambda_2 \)  | The visible density of existence of the exposed compartment | 100 |
| \( \sigma \)  | Density dependent coefficient of the susceptible class | 0.0001 |
| \( \beta_1 \)  | Rate of infection from S-E interaction | 0.0002 |
| \( \beta_2 \)  | Rate of infection from S-I interaction | 0.0003 |
| \( \nu \)  | Rate of vaccination | \([0, 1]\) |
| \( \varepsilon_1 \)  | Recognition of infection | \([0.5, 1]\) |
| \( \mu \)  | Rate of isolation of infected people | 0.4 |
| \( \gamma_2 \)  | Rate of recovery due to treatment | \([0, 0.8]\) |
| \( \mu_1 \)  | Death rate from COVID-19 | 0.00019 |
| \( \eta \)  | The natural death rate | 0.0012 |

The process of worry occurs when the individual is \( \alpha \) times infected by COVID-19. let \( \lambda = \( \alpha \) \lambda \). The process of worry occurs when the individual notices from different sources uncertain information about the virus. The process of worry occurs when the individual notices from different sources uncertain information about the vaccines. The recruitment rate of the susceptible class is 12000. The visible density of existence of the exposed compartment is 100. The density dependent coefficient of the susceptible class is 0.0001. The rate of infection from S-E interaction is 0.0002. The rate of infection from S-I interaction is 0.0003. The rate of vaccination is \([0, 1]\).
We can see that the belief in the medications is deficient. The pressure on the civilians caused by “trend discussions”, whether it is realistic or not, decreases their recognition rate up to %60. While the screening rate is fixed at %30, Fig. 2 shows that we have an uncontrollable spread of the infected compartments in this scenario.

7. Conclusion

We proposed a SEIQRV model as an FDE system to describe the fear process and the fear level of the susceptible class. The pandemic spread during the digital world had its benefits to have updated information on time and a negative effect, which leads to uncertain and polluted information spread from networks, who were incredibly unprofessional people who aimed to activate civilians’ fear. This work studied a split form of fear in the susceptible class. This compartment followed and focused on two crucial news; the infection and the vaccines. Any polluted information in social networks related to these two topics created worries in the susceptible class, which prefers to avoid any vaccine for a specific time. In this study, the recognition parameter showed the key point distinguishing between “keeping the community aware of the infection and the vaccines” and “to let the people be scared during the two years of the pandemic stage”. Fig. 1 showed that the recognition parameters kept the people aware of the symptoms of the infection so that they act quickly with PCR test to move from the exposed compartment to the infected one if they feel any symptoms of COVID-19. Besides this, worry and fear do not exist; therefore, we expect high trust in the medications and vaccinations. While in Fig. 2, the worry increased because of trend discussions to scarify the community, the pandemic stage continues for a long time.

Moreover, as necessary as the awareness, we want to conclude that clear information about treatment processes and vaccines is also essential to return to the desired everyday life. Therefore, the pandemic phenomena should not create unrealistic stories that might affect the psychology of the human body.
Fig. 2. Dynamical behavior of the compartments in the system (4.1), when $\theta_1 = 0.3, \theta_2 = 0.2, \alpha_1 = \alpha_2 = 0.5, \varepsilon = 0.6$ and $v = 0.01$.

Our future works will focus mainly on the environmental effects that appeared after corona, based on fractional and hybrid constructed models.

CRediT authorship contribution statement

Ali Yousef: Conceptualization, Methodology, Investigation, Visualization, Writing – original draft, Supervision. Fatma Bozkurt: Conceptualization, Methodology, Investigation, Visualization, Writing – original draft, Supervision. Thabet Abdeljawad: Conceptualization, Methodology, Investigation, Visualization, Writing – original draft, Supervision. Emad Emreizeeq: Visualization, Writing – original draft, Supervision.

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