Supermassive gravitinos, dark matter, leptogenesis and flat direction baryogenesis

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Abstract: In general the gravitino mass and/or the soft supersymmetry breaking masses in the observable sector can be much larger than the TeV scale. Depending on the relation between the masses, new important channels for gravitino production in the early Universe can arise. Gravitinos with a mass above 50 TeV decay before big bang nucleosynthesis, which leads to relaxation of the well known bound on the reheating temperature \( T_R \leq 10^{10} \) GeV. However, if the heavy gravitinos are produced abundantly in the early Universe, their decay can alter the abundance of the lightest supersymmetric particle. Moreover, they may dominate the energy density of the Universe. Their decay will in this case increase entropy and dilute already created baryon asymmetry and dark matter. Such considerations put new constraints on gravitino and sfermion masses, and the reheating temperature. In this paper we examine various cosmological consequences of supermassive gravitinos. We discuss advantages and disadvantages of a large reheating temperature in connection with thermal leptogenesis, and find that large parts of the parameter space are opened up for the lightest right-handed (s)neutrino mass. We also discuss the viability of Affleck-Dine baryogenesis under the constraints from gravitino decay, and gravitino production from the decay of Q-balls.
1. Introduction

Primordial inflation is the most convincing paradigm for the early Universe [1]. The vacuum fluctuations created during inflation also explain the observed temperature anisotropy of the cosmic microwave background (CMB) radiation [2]. However inflation leaves the Universe cold and void of any thermal entropy. Entropy is believed to be created from the decay of the coherent oscillations of the inflaton which can happen perturbatively [3] and/or non-perturbatively into bosons [4, 5, 6] and
fermions \cite{8}. It is necessary that the standard model (SM) degrees of freedom are produced at this reheating stage, particularly baryons which are required for the synthesis of light elements during big bang nucleosynthesis (BBN) at a temperature $O(\text{MeV})$ \cite{3, 4, 10}.

On the other hand, we do not know the full particle content of the Universe beyond the electroweak scale, therefore we do not know what degrees of freedom were excited right after inflation. In this respect, supersymmetry (SUSY) acts as a building block which can explain the hierarchy between the Planck and electroweak scales, if it is softly broken in the observable sector at the TeV scale, see \cite{11}. Besides its phenomenological implications, this also has important cosmological consequences. The scalar potential of the minimal supersymmetric standard model (MSSM) has nearly 300 flat directions \cite{12}. These flat directions can address cosmological issues from reheating to density perturbations \cite{13}. If $R$-parity is conserved, the lightest supersymmetric particle (LSP) will be stable and can act as a cold dark matter (CDM) candidate. A neutralino LSP with a mass $m_\chi \sim 100 \text{ GeV}$ can match the current observational limit when produced thermally \cite{14}.

SUSY breaking at the TeV scale in the observable sector can be achieved via gravity \cite{11}, gauge \cite{15} and anomaly \cite{16} mediation, leading to different patterns of supersymmetric particle masses. However, there is a priori no fundamental reason why this scale should be favored by nature \cite{1}.

Inspired by the string landscape \cite{17, 18}, there has recently been an interesting proposal for SUSY breaking well above the electroweak (but below the Planck) scale \cite{20, 21}. In this new scheme, coined split SUSY, the masses of sfermions can be arbitrarily larger than those of fermions. Although such a scheme does not attempt to address the hierarchy problem, it removes fear from flavor changing and $CP$–violating effects induced by the light scalars at one-loop level. Successful gauge coupling unification requires that the gauginos be kept lighter than 100 TeV, while spontaneous breaking of the electroweak symmetry requires that the mass of the lightest Higgs be around $O(100) \text{ GeV}$.

A priori there is no fundamental theory which fixes the scale of SUSY breaking, but cosmological considerations can constrain it. For example, the theory permits a

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\footnote{String theory which is believed to be the most fundamental theory does not provide us with a concrete answer. Rather it provides us with a landscape with multiple vacua \cite{17}, where the SUSY breaking scale remains undetermined \cite{18}, this transcends into an uncertainty into the scale of inflation and the required number of minimal e-foldings \cite{19}.}

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very long-lived gluino. The annihilation of gluinos alone may not efficiently reduce their abundance below the experimental limits on the anomalous isotopes of ordinary matter [20]. The decay of gluinos within the lifetime of the Universe solves this potential problem, and requires sfermions masses to be less than $10^{13}$ GeV [20, 21]. Models which give rise to such a split pattern of SUSY breaking masses are typically more complicated than the conventional ones for TeV scale SUSY breaking [20, 21, 22]. Of course, one might also give upon supersymmetric gauge coupling unification and allow all supersymmetric particles to be much heavier than TeV. In this case SUSY will be irrelevant for physics at the electroweak scale. On the other hand, even in the context of MSSM, some of the sfermions can have a mass $\gg 1$ TeV, as happens in the so-called inverted hierarchy models [23]. Therefore, under general circumstances, it is possible that at least some of the sfermions are much heavier than TeV.

Local SUSY naturally embeds gravity, hence supergravity, and implies the existence of a new particle known as the gravitino which is the superpartner of the graviton which is fairly long-lived. Massive gravitinos consist of helicity $\pm 1/2$ (longitudinal) and helicity $\pm 3/2$ (transverse) components. In the early Universe gravitinos can be produced from thermal scatterings of gauge and gaugino quanta [24, 25], and from the decay of sfermions [25]. Gravitinos are also produced non-thermally from the direct decay of the inflaton [26, 27, 28], and from the vacuum fluctuations during the coherent oscillations of the inflaton field [29, 30, 31, 32]. In the minimal supergravity models, the gravitino mass $m_{3/2}$ is the same as the soft breaking mass of scalars [11]. However, gravitinos can be much heavier once one goes beyond the minimal supergravity, for example in no-scale models [33]. It is therefore possible that $m_{3/2} \gg 1$ TeV, even if SUSY is broken at TeV scale in the observable sector.

Gravitinos with a mass below 50 TeV decay during and/or after BBN. Depending on the nature of the decay, there exist tight bounds on the reheating temperature, see [34, 35]. Obviously these bounds do not apply to supermassive gravitinos, i.e. when $m_{3/2} \geq 50$ TeV. In such a case the reheating temperature could potentially be as large as the inflaton mass, leading to many interesting consequences. As an example, it opens up new regions of the parameter space for thermal leptogenesis [36], a scenario which is sensitive to the reheating temperature as it requires the excitation of the lightest right-handed (RH) neutrinos and their supersymmetric partners from the thermal bath.
However other considerations can constrain the abundance of supermassive gravitinos. Every gravitino produces one LSP upon its decay. If the gravitino decays after the thermal freeze-out of LSPs, then it can alter the LSP abundance. In addition, gravitinos can dominate the energy density of the Universe if they are produced abundantly. Entropy release from gravitino decay in this case dilutes any generated baryon asymmetry.

However, this may turn into a virtue, for example, in the case of Affleck–Dine (AD) baryogenesis [37]. Depending on the parameter space, the AD mechanism (which utilizes supersymmetric flat directions) can generate order one baryon asymmetry. This would then be diluted to the observed value if the gravitino decay generates enough entropy. Often the flat directions also fragment to form non-topological solitons such as supersymmetric Q-balls. The Q-balls decay slowly through their surface [38], and can themselves be a major source of late gravitino production.

In this article we consider various cosmological consequences of models with superheavy gravitinos and/or sfermions, without delving into model-building issues. Such particles will be inaccessible at future colliders, and hence cosmology will be essentially the only window to probe and/or constrain these models. The rest of this paper is organized as follows. In Section II we briefly review various sources of gravitino production and their individual contributions. We then discuss gravitino decay and constraints from the LSP dark matter on the abundance of supermassive gravitinos in Section III. We briefly review thermal leptogenesis and the effects of the gravitino in Section IV, and identify regions of the parameter space which allow successful leptogenesis. Section V is devoted to supersymmetric flat directions baryogenesis, including thermal effects and viability of the AD mechanism in the presence of supermassive gravitinos. We also show that long-lived Q-balls can be a source for copious production of gravitinos. We summarize our results and conclude the paper in the final Section VI.

2. Gravitino Production

The most important interaction terms of the gravitino field $\psi_\mu$ come from its coupling to the supercurrent. In the four-component notation, and in flat space-time, these terms are written as (see for instance [39])

$$
\mathcal{L}_{\text{int}} = -\frac{1}{\sqrt{2}M_p} \partial_\nu X^* \bar{\psi}_\mu \gamma^{\nu} \gamma_\mu \left( \frac{1 + \gamma_5}{2} \right) \chi - \frac{i}{8M_p} \bar{\psi}_\mu [\gamma^{\nu}, \gamma^\rho] \gamma^\mu \lambda^{(a)} F^{(a)}_{\mu \nu} + \text{h.c.} \quad (2.1)
$$
Here \( X \) and \( \chi \) denote the scalar and fermionic components of a general chiral superfield, respectively, while \( F^{(a)\mu\nu} \) and \( \lambda^{(a)} \) denote the gauge and gaugino field components of a given vector superfield respectively.

In the limit of unbroken SUSY gravitinos are massless and the physical degrees of freedom consist of the helicity \( \pm 3/2 \) (transverse) components. After spontaneous SUSY breaking gravitino eats the Goldstino and obtains a mass \( m_{3/2} \) through the super Higgs mechanism, and helicity \( \pm 1/2 \) (longitudinal) states appear as physical degrees of freedom. When the value of \( m_{3/2} \) is much smaller than the momentum of the gravitino, the wave-function of the helicity \( \pm 1/2 \) components of the gravitino can be written as

\[
\psi_\mu \sim i \sqrt{\frac{2}{3 m_{3/2}}} \partial_\mu \psi, \tag{2.2}
\]

with \( \psi \) being the Goldstino. The helicity \( \pm 1/2 \) states of the gravitino will in this case essentially interact like the Goldstino and the relevant couplings are given by an effective Lagrangian

\[
L_{\text{eff}} = \frac{i}{\sqrt{3 m_{3/2} M_P}} \left[ (m_X^2 - m_\chi^2) X^* \tilde{\psi} \left( \frac{1 + \gamma_5}{2} \right) \chi - m_\lambda \tilde{\psi} [\gamma^\mu, \gamma^\nu] \lambda^{(a)} F^{(a)\mu\nu} \right] + \text{h.c.} \tag{2.3}
\]

Here \( m_X \) and \( m_\chi \) denote the mass of \( X \) and \( \chi \) fields, respectively, while \( m_\lambda \) is the mass of gaugino field \( \lambda^{(a)} \). The interactions of helicity \( \pm 1/2 \) and helicity \( \pm 3/2 \) states of the gravitino have essentially the same strength when \( |m_X - m_\chi| \) and \( m_\lambda \) are smaller than \( m_{3/2} \). In the opposite limit, the rate for interactions of helicity \( \pm 1/2 \) states with \( X \) and \( \chi \), respectively gauge and gaugino fields, will be enhanced by a factor of \( (m_X^2 - m_\chi^2)/E^2 m_{3/2}^2 \) (\( E \) being the typical energy involved in the relevant process), respectively \( m_\lambda^2/m_{3/2}^2 \), compared to that of helicity \( \pm 3/2 \) states.

Gravitinos are produced through various processes in the early Universe, where the relevant couplings are given in (2.1) and (2.3).

- **Thermal scatterings:**

  Scatterings of gauge and gaugino quanta in the primordial thermal bath is an important source of gravitino production, leading to (up to logarithmic corrections) \([24, 25]\):

  Helicity \( \pm \frac{1}{2} \):

  \[
  \left( \frac{n_{3/2}}{s} \right)_{\text{sca}} \approx \left( 1 + \frac{M_g^2}{12 m_{3/2}^2} \right) \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left[ \frac{228.75}{g_*(T_R)} \right]^{3/2} 10^{-12},
  \]

  Helicity \( \pm \frac{3}{2} \):

  \[
  \left( \frac{n_{3/2}}{s} \right)_{\text{sca}} \approx \left( \frac{T_R}{10^{10} \text{ GeV}} \right) \left[ \frac{228.75}{g_*(T_R)} \right]^{3/2} 10^{-12}; \tag{2.4}
  \]
where $T_R$ denotes the reheating temperature of the Universe, $\tilde{M}_g$ is the gluino mass and $g_\ast(T_R)$ is the number of relativistic degrees of freedom in the thermal bath at temperature $T_R$. Note that for $M_\tilde{g} \leq m_{3/2}$ both states have essentially the same abundance, while for $M_\tilde{g} \gg m_{3/2}$ production of helicity $\pm 1/2$ states is enhanced due to their Goldstino nature. The linear dependence of the gravitino abundance on $T_R$ can be understood qualitatively. Due to the $M_P$-suppressed couplings of the gravitino, see (2.1) and (2.3), the cross-section for gravitino production is $\propto M^{-2}_P$. The production rate at temperature $T$ and the abundance of gravitinos produced within one Hubble time will then be $\propto T^3$ and $\propto T$ respectively. We remind that the Hubble expansion rate at temperature $T$ is given by $H = \sqrt{(g_\ast \pi^2/90)T^2/M_P}$, where $g_\ast$ is the number of relativistic degrees of freedom in the thermal bath at temperature $T$. This implies that gravitino production from scatterings is most efficient at the highest temperature of the radiation-dominated phase of the Universe, i.e. when $T = T_R$.

- **Decaying sfermions:**
  If $m_{3/2} < \tilde{m}$, the decay channel $sfermion \rightarrow fermion + gravitino$ is kinematically open. When $\tilde{m} > few \times m_{3/2}$, the decay rate has a simple form

\[
\text{Helicity } \pm \frac{1}{2} : \quad \Gamma_{sferm \rightarrow ferm + \psi} \simeq \frac{1}{48\pi} \frac{\tilde{m}^5}{m_{3/2}^2 M_P^2},
\]

\[
\text{Helicity } \pm \frac{3}{2} : \quad \Gamma_{sferm \rightarrow ferm + \psi} \simeq \frac{1}{48\pi} \frac{\tilde{m}^3}{M_P^2}.
\]

(2.5)

Sfermions will reach thermal equilibrium abundances, provided that $T_R \geq \tilde{m}$. They promptly decay through their gauge interactions when the temperature drops below $\tilde{m}$. Gravitinos are produced from sfermion decays for the whole duration sfermions exist in the thermal bath $t \sim M_P/\tilde{m}^2$. The abundance of gravitinos thus produced will then be

\[
\text{Helicity } \pm \frac{1}{2} : \quad \left( \frac{n_{3/2}}{s} \right)_{\text{dec}} \simeq \left( \frac{\tilde{m}}{m_{3/2}} \right)^2 \left( \frac{\tilde{m}}{1 \text{ TeV}} \right) \left[ \frac{228.75}{g_\ast(\tilde{m})} \right]^{3/2} \left( \frac{N}{46} \right) 1.2 \times 10^{-19},
\]

\[
\text{Helicity } \pm \frac{3}{2} : \quad \left( \frac{n_{3/2}}{s} \right)_{\text{dec}} \simeq \left( \frac{\tilde{m}}{1 \text{ TeV}} \right) \left[ \frac{228.75}{g_\ast(\tilde{m})} \right]^{3/2} \left( \frac{N}{46} \right) 1.2 \times 10^{-19},
\]

(2.6)

where $g_\ast(\tilde{m})$ is the number of relativistic degrees of freedom at $T = \tilde{m}$, and $N$ is the number of all sfermions such that $m_{3/2} < \tilde{m} < T_R$. This result is
independent of \( T_R \), so long as \( T_R > \tilde{m} \). Note that for \( \tilde{m} \gtrsim m_{3/2} \) gravitinos of both helicities will be produced with approximately the same abundance.

If \( \tilde{m} \gg m_{3/2} \), helicity \( \pm 1/2 \) states interact with sfermions and fermions very efficiently and can actually reach thermal equilibrium, thus leading to \( (n_{3/2}/s)_{\text{dec}} \approx 10^{-2} \). This will happen when \( \tilde{m} \geq \left( 10^4 m_{3/2}^2 M_P \right)^{1/3} \), for example, \( \tilde{m} \geq 10^9 \) GeV if \( m_{3/2} \simeq 50 \) TeV.

- **Inflaton decay:**

  Reheating of the Universe also leads to gravitino production \([29, 30, 26, 27]\) (for related studies, see \([28, 40, 41]\)). Here we consider the case where inflaton decays perturbatively and a radiation-dominated Universe is established immediately after the completion of its decay.\(^2\) This in general provides a valid description of the last stage of inflaton decay, regardless of how fast and explosive the first stage of reheating might be due to various non-perturbative effects \([4, 3]\).

  Let us denote the SUSY-conserving mass of the inflaton multiplet by \( M_\phi \), and the mass difference between the inflaton \( \phi \) and inflatino \( \tilde{\phi} \) by \( \Delta m_\phi \).\(^3\) If \( \Delta m_\phi > m_{3/2} \), the decay \( \phi \to \tilde{\phi} + \text{gravitino} \) is kinematically possible. For \( \Delta m_\phi > \text{few} \times m_{3/2} \), the partial decay width will be \([26, 41]\)

\[
\begin{align*}
\text{Helicity } \pm \frac{1}{2} : & \quad \Gamma_{\phi \to \tilde{\phi} + \psi} \approx \frac{1}{48\pi} \frac{(m^2_\phi - m^2_\tilde{\phi})^4}{M_\phi^3 M_{3/2}^2 M_P^2}, \\
\text{Helicity } \pm \frac{3}{2} : & \quad \Gamma_{\phi \to \tilde{\phi} + \psi} \approx \frac{1}{48\pi} \frac{(m^2_\phi - m^2_\tilde{\phi})^4}{M_\phi^3 \Delta m^2_\phi M_P^2}. \quad (2.7)
\end{align*}
\]

We can estimate the abundance of produced gravitinos with the help of total inflaton decay rate \( \Gamma_\phi = \sqrt{(g_\ast(T_R)\pi^2/90)T_R^3/M_P} \), and the dilution factor due to final entropy release which is given by \( 3T_R^2/4M_\phi \).

\(^2\)Full thermal equilibrium is indeed achieved very rapidly, provided that inflaton decay products have interactions of moderate strength. For details on thermalization, see \([42]\).

\(^3\)The mass difference between the inflaton and inflatino \( \Delta m_\phi \) is in general different from that between the standard model fermions and sfermions \( \tilde{m} \). As an example, consider the case where the soft breaking (mass)\(^2\) of both the inflaton and sfermion fields is \( \tilde{m}^2 \). We will then have \( \Delta m_\phi \approx \tilde{m}^2/2M_\phi \ll \tilde{m} \) if \( \tilde{m} \ll M_\phi \), while \( \Delta m_\phi \simeq \tilde{m} \) when \( \tilde{m} \geq M_\phi \).
If $\Delta m_\phi \ll M_\phi$, inflaton decay gives rise to

\[
\text{Helicity } \pm \frac{1}{2} : \left( \frac{n_{3/2}}{s} \right)_{\text{reh}} \approx \left( \frac{\Delta m_\phi}{m_{3/2}} \right)^2 \left( \frac{\Delta m_\phi^2}{T_R M_P} \right) \left[ \frac{228.75}{g_*(T_R)} \right]^{1/2} \frac{1.6 \times 10^{-2}}{s},
\]

\[
\text{Helicity } \pm \frac{3}{2} : \left( \frac{n_{3/2}}{s} \right)_{\text{reh}} \approx \left( \frac{\Delta m_\phi^2}{T_R M_P} \right) \left[ \frac{228.75}{g_*(T_R)} \right]^{1/2} \frac{1.6 \times 10^{-2}}{s}.
\] (2.8)

An interesting point is that $M_\phi$ drops out of the calculation, and hence the final results in Eq. (2.8) have no explicit dependence on $M_\phi$. If $M_\phi \leq \Delta m_\phi$, the gravitino abundance will be smaller than that in (2.8) by a factor of 16. Note again that for $\Delta m_\phi \gtrsim m_{3/2}$ gravitinos of both helicities have approximately the same abundance.

Since from (2.8) $n_{3/2}/s$ is inversely proportional to $T_R$, gravitino production from inflaton decay becomes more efficient at lower reheating temperature. The reason is that a smaller $T_R$ means a smaller total decay rate $\Gamma_\phi$, while the partial decay width (2.7) is independent from $T_R$. Therefore decreasing $T_R$, while suppresses the production from thermal scatterings (2.4) and sfermion decays (2.6), can actually enhance the overall production of gravitinos. Obviously gravitino production from inflaton decay reaches saturation when the partial decay width equals the total decay rate $\Gamma_\phi$. In this case all inflatons decay to inflatino-gravitino pairs, and the subsequent decay of inflatins will reheat the Universe. In consequence, one gravitino will be produced per inflaton quanta, resulting in a gravitino abundance $(n_{3/2}/s)_{\text{reh}} = 3T_R/4M_\phi$.

Gravitino production in two-body decays of the inflaton will be kinematically forbidden if $m_{3/2} \geq \Delta m_\phi$. However, the inflaton decay inevitably results in gravitino production at higher orders of perturbation theory, provided that $M_\phi > m_{3/2}$ [13]. The leading order contributions come from the diagrams describing the dominant mode of inflaton decay with gravitino emission from the inflaton, its decay products and the decay vertex. The partial width for inflaton decay to gravitinos is in this case $\sim (M_\phi/M_P)^2 \Gamma_\phi$ which, after taking into account of the dilution factor, leads to $(n_{3/2}/s)_{\text{reh}} \sim (T_R M_\phi/M_P^2)$. If we impose the bound on the inflaton mass $M_\phi \leq 10^{13}$ GeV from the CMB for a simple chaotic type inflation model, and if $m_{3/2} \geq \Delta m_\phi$, gravitino production from inflaton decay is subdominant compared to that of thermal scatterings (2.4), and hence can be neglected.4

4A similar process, non-thermal production of gravitons from inflaton decay, can become impor-
• Non-perturbative production:

We note that besides various perturbative production mechanisms, both of the helicity states can be excited non-perturbatively during the coherent oscillations of the inflaton. This was first discussed in [29] and then elaborated in [30]. Right after inflation the helicity \( \pm 1/2 \) component, i.e. the Goldstino, is essentially the inflatino. For simple models with a single chiral superfield, it was shown that this component can be produced abundantly

\[
\left( \frac{n_{3/2}}{s} \right) \leq \left( \frac{T_R}{M_\phi} \right) [30].
\]

The reason is that its couplings, given in Eqs. (2.1) and (2.3), are not necessarily \( M_P \)-suppressed (contrary to the helicity \( \pm 3/2 \) states). However, as explicitly shown in [31], it also decays quickly along with the inflaton through derivative interactions, and hence poses no danger. Realistic models include at least two chiral superfields such that the inflation sector is different from the sector responsible for SUSY breaking in the vacuum. In these models also most of the spin-1/2 fermions produced during inflaton oscillations decay in form of inflatinos, provided that the scales of inflation and present day SUSY breaking are sufficiently separated [32]. The helicity \( \pm 3/2 \) components of the gravitino have \( M_P \) suppressed coupling all the time. In consequence, they are produced less abundantly

\[
\left( \frac{n_{3/2}}{s} \right) \leq \left( \frac{M_\phi}{M_P} \right) \left( \frac{T_R}{M_P} \right) [29],
\]

compared to the direct decay of the inflaton, thermal scatterings and sfermion decays. We will therefore ignore the contribution from non-perturbative production of gravitinos in the following.

To summarize, the total gravitino abundance is given by

\[
\left( \frac{n_{3/2}}{s} \right) = \left( \frac{n_{3/2}}{s} \right)_{\text{sca}} + \left( \frac{n_{3/2}}{s} \right)_{\text{dec}} + \left( \frac{n_{3/2}}{s} \right)_{\text{reh}}.
\]

(2.9)

As long as \( m_{3/2} \leq T_R \), gravitinos are always produced in thermal scatterings of gauge and gaugino quanta. In addition, sfermion and inflaton decays also contribute to gravitino production if \( m_{3/2} < \tilde{m} \leq T_R \) and \( m_{3/2} < \Delta m_\phi \) respectively. Then it turns out from (2.4), (2.6) and (2.8) that sfermion decays will be the dominant source of gravitino production unless \( T_R > 1.2 \times \left( \frac{\tilde{m}^2}{m_{3/2}^2} \right) \) and/or \( \Delta m_\phi > 0.37 \tilde{m} \). Hence, for \( m_{3/2} < \tilde{m} \leq T_R \), the most important contribution in (2.9) in general comes from sfermion decays (see also footnote 3 on page 6).

\footnote{tant in models with extra dimensions [44]. The large multiplicity of the Kaluza-Klein modes of the graviton can in this case easily overcome the suppression factor \( (M_\phi/M_P)^2 \).}
3. Gravitino Decay

- **Stable gravitino**:  
  First, we briefly consider the case for stable gravitinos. If the gravitino is the LSP, and $R$-parity is conserved, it will be absolutely stable. Its total abundance (including both helicity $\pm 1/2$ and $\pm 3/2$ states) will in this case be constrained by the dark matter limit $\Omega_{3/2} h^2 \leq 0.129$, leading to
  
  $$\left(\frac{n_{3/2}}{s}\right) \leq 4.6 \times 10^{-10} \left(\frac{1 \text{ GeV}}{m_{\chi}}\right). \quad (3.1)$$

  This implies that the individual contributions from Eqs. (2.4), (2.6) and (2.8) should respect this bound. As an example, consider the case with $m_{3/2} = 10$ GeV and $M_{\tilde{g}} \simeq 1$ TeV. This results in the constraints $T_R \leq 5.5 \times 10^8$ GeV, $\tilde{m} \leq 33$ TeV and $\Delta m_{\phi} \leq 140$ TeV.

- **Unstable gravitino**:  
  An unstable gravitino decays to particle-sparticle pairs through the couplings in (2.1), and the decay rate is given by [39]
  
  $$\Gamma_{3/2} \simeq \left(N_g + \frac{N_f}{12}\right) \frac{m_{3/2}^3}{32\pi M_P^2}, \quad (3.2)$$

  where $N_g$ and $N_f$ are the number of available decay channels into gauge-gaugino and fermion-sfermion pairs respectively. The gravitino decay is completed when $H \simeq \Gamma_{3/2}$, when the temperature of the Universe is given by
  
  $$T_{3/2} \simeq \left[\frac{10.75}{g_*(T_{3/2})}\right]^{1/4} \left(\frac{m_{3/2}}{10^5 \text{ GeV}}\right)^{3/2} 6.8 \text{ MeV}. \quad (3.3)$$

  Here $g_*(T_{3/2})$ is the number of relativistic degrees of freedom at $T_{3/2}$. If $m_{3/2} < 50$ TeV, gravitinos decay during or after BBN [8] and can ruin its successful predictions for the primordial abundance of light elements [4]. If the gravitinos decay radiatively, the most stringent bound $\left(n_{3/2}/s\right) \leq 10^{-14} - 10^{-12}$ arises for $m_{3/2} \simeq 100$ GeV – 1 TeV [34]. On the other hand, much stronger bounds are derived if the gravitinos mainly decay through hadronic modes. In particular, a branching ratio $\simeq 1$ requires that $\left(n_{3/2}/s\right) \leq 10^{-16} - 10^{-15}$ in the same gravitino mass range [35].

  To give a numerical example, consider the case when $m_{3/2} \simeq 1$ TeV. The abundance of a radiatively decaying gravitino is in this case constrained to be
Then Eqs. (2.4), (2.6) and (2.8) result in the bounds $T_R \leq 10^{10}$ GeV, $\tilde{m} \leq 203$ TeV and $\Delta m_\phi < 1.1 \times 10^6$ GeV, respectively. If a TeV gravitino mainly decays into gluon-gluino pairs, which will be the case if $m_{3/2} > M_\tilde{g}$, we must have $(n_{3/2}/s) \leq 10^{-16}$ \cite{[35]}. This leads to much tighter bounds $T_R \leq 10^{6}$ GeV, $\tilde{m} \leq 9.4$ TeV and $\Delta m_\phi \leq 11$ TeV.

### 3.1 Decay of Supermassive Gravitinos and Dark Matter Abundance

We now turn to supermassive gravitinos with a mass $m_{3/2} \geq 50$ TeV. If one insists on a successful supersymmetric gauge coupling unification, the gaugino masses should be below 100 TeV. This implies that the gravitino will not be the LSP. The decay of supermassive gravitinos happens sufficiently early in order not to affect the BBN. Nevertheless, their abundance can still be constrained due to different considerations. Gravitino decay produces one LSP per gravitino. This non-thermal component may exceed the dark matter limit if the decay happens below the LSP freeze-out temperature $T_f$. The freeze-out temperature is given by \cite{[14]}

$$T_f = \frac{m_\chi}{x_f}, \quad x_f = 28 + \ln \left\{ \frac{1 \text{ TeV}}{m_\chi} \frac{c}{10^{-2}} \left[ \frac{86.25}{g_*(T_{3/2})} \right]^{1/2} \right\}, \quad (3.4)$$

where $m_\chi$ is the LSP mass and we have parameterized the non-relativistic $\chi$ annihilation cross-section as

$$\langle \sigma_\chi v_{\text{rel}} \rangle = \frac{c}{m_\chi^2}. \quad (3.5)$$

Note that neutralinos reach kinetic equilibrium with the thermal bath, and hence become non-relativistic, very quickly at temperatures above MeV \cite{[15]}. The exact value of $c$ depends on the nature of $\chi$ and its interactions. For Bino-like LSP, $c$ can be much smaller than for a Wino- or Higgsino-like one. When sfermions are much heavier than the neutralinos, $c = 3 \times 10^{-3}$ for a Higgsino LSP and $c = 10^{-2}$ for a Wino LSP (including the effects of co-annihilation) \cite{[21]}. Gravitino decay occurs after the LSP freeze-out if $T_{3/2} < T_f$, which translates into an upper bound on the gravitino mass

$$m_{3/2} < \left( \frac{m_\chi}{1 \text{ TeV}} \right)^{2/3} \left[ \frac{g_*(T_{3/2})}{86.25} \right]^{1/6} 4.3 \times 10^7 \text{ GeV}. \quad (3.6)$$

This implies that for $m_\chi = 1$ TeV, gravitinos with a mass $m_{3/2} < 4 \times 10^7$ GeV decay when thermal annihilation of LSPs is already frozen. This decay produces one
LSP per gravitino. The dark matter limit $\Omega_\chi h^2 \leq 0.129$ constrains the total LSP abundance to obey

$$ \frac{n_\chi}{s} \leq 4.6 \times 10^{-10} \left( \frac{1 \text{ GeV}}{m_\chi} \right). \tag{3.7} $$

The final abundance of LSPs produced from gravitino decay depends on their annihilation rate. The rate of annihilation of non-relativistic LSPs is given by

$$ \Gamma_\chi = \langle \sigma v_{\text{rel}} \rangle n_\chi = c \frac{n_\chi}{m_\chi^2}. \tag{3.8} $$

If $\Gamma_\chi \geq \Gamma_{3/2}$, annihilation will be efficient and reduce the LSP abundance to

$$ n_\chi \approx 41.58 \left[ \frac{10^{-2}}{c} \right] \left[ \frac{86.25}{g_*(T_{3/2})} \right]^{1/4} \frac{m_\chi^2}{(m_{3/2} M_P)^{1/2}}. \tag{3.9} $$

Otherwise, gravitino decay contributes an amount $n_{3/2}/s$ to the LSP abundance.

Having a large abundance of gravitinos, i.e. $(n_{3/2}/s) > 4.6 \times 10^{-10} (1 \text{ GeV}/m_\chi)$, is therefore potentially dangerous and requires special attention.

The condition for efficient annihilation of LSPs whose abundance $(n_\chi/s) \geq 4.6 \times 10^{-10} (1 \text{ GeV}/m_\chi)$ at the time of gravitino decay translates into a lower bound on the gravitino mass

$$ m_{3/2} \geq \left[ \frac{10^{-2}}{c} \right]^{2/3} \left[ \frac{86.25}{g_*(T_{3/2})} \right]^{1/6} \left( \frac{m_\chi}{1 \text{ TeV}} \right)^2 2 \times 10^7 \text{ GeV}. \tag{3.10} $$

If $m_{3/2}$ is in the window given by (3.6) and (3.10), the final abundance of non-thermal LSPs will be given by (3.9). It satisfies the dark matter limit (3.7), and can account for the CDM for those values of $m_\chi$ and $m_{3/2}$ which saturate the inequality in (3.10).

Eqs. (3.6) and (3.10) can be simultaneously satisfied only if

$$ m_\chi \leq \left[ \frac{c}{10^{-2}} \right]^{1/2} \left[ \frac{g_*(T_{3/2})}{86.25} \right]^{1/4} 1.8 \text{ TeV}. \tag{3.11} $$

As a matter of fact, this is also the condition such that thermal abundance of LSPs at freeze-out respects the dark matter limit. It is not surprising as $T_{3/2} = T_f$ when the inequality in (3.11) is saturated. For the saturation value of $m_\chi$ thermal LSP abundance gives rise to $\Omega_\chi h^2 = 0.129$. For smaller $m_\chi$ the thermal component is not sufficient, while for larger $m_\chi$ thermal LSPs overclose the Universe.

For values of $m_\chi$ respecting the bound in (3.11), there always exists a window for $m_{3/2}$ such that gravitinos decay after the freeze-out while at the same time non-thermal LSPs efficiently annihilate and their abundance respects the dark matter
limit (3.7). In this mass window the abundance of thermal LSPs is too low to account for dark matter. On the other hand, non-thermal dark matter will be a viable scenario when the inequality in Eq. (3.10) is saturated (for non-thermal production of LSP dark matter from gravitino decay, see also [21, 46]). The gravitino mass window becomes narrower as \( m_{\chi} \) increases. It will eventually disappear when \( m_{\chi} \) reaches the upper bound in (3.11). For the canonical choice of \( c = 10^{-2} \), the gravitino mass window shrinks to a single point \( m_{3/2} = 6.3 \times 10^7 \text{ GeV} \) at the saturation value \( m_{\chi} = 1.8 \text{ TeV} \). For larger LSP masses it is necessary that gravitinos which decay after the freeze-out are not overproduced, i.e. that \( (n_{3/2}/s) \leq 4.6 \times 10^{-10} \left( \text{1 GeV}/m_{\chi} \right) \). Otherwise, gravitinos must decay above the freeze-out temperature, i.e. the opposite inequality as in (3.6) must be satisfied. Then gravitino decay does not affect the final LSP abundance as \( T_{3/2} > T_f \). However, for masses violating the bound in (3.11) thermal LSPs overclose the Universe. Therefore a viable scenario of LSP dark matter in this case requires late entropy generation.

3.2 Gravitino Non-domination

We now consider the constraints from dark matter abundance on gravitino decay in more detail. Assuming that there is no other stage of entropy generation, the Universe will remain in the radiation-dominated phase after reheating. During this period the scale factor the Universe increases as \( a \propto H^{-1/2} \). Gravitinos become non-relativistic at

\[
H_{\text{non}} \simeq \left( \frac{m_{3/2}}{E_p} \right)^2 H_p, \tag{3.12}
\]

where \( H_p \) denotes the expansion rate when (most of the) gravitinos are produced and \( E_p \) is the energy of gravitinos upon their production. If thermal scatterings are the main source of gravitino production, \( H_p \sim T_{R}^2/M_P \) and \( E_p \sim T_{R} \). On the other hand, if sfermion decays dominate gravitino production, \( H_p \sim \bar{m}^2/M_P \) and \( E_p = \bar{m}/2 \). Finally, if most of the gravitinos are produced in inflaton decay, \( H_p \sim T_{R}^2/M_P \) and \( E_p \simeq \Delta m_\phi \).

For \( H < H_{p} \), the energy density of the gravitinos is redshifted \( \propto a^{-3} \), compared to \( \propto a^{-4} \) for radiation. Initially the gravitino energy density is \( \rho_{3/2} = n_{3/2} E_p \), while the energy density in radiation is \( \rho_{\text{rad}} = (\pi^2/30) g_*(T_p) T_p^4 \). Gravitinos will dominate when \( \rho_{3/2}/\rho_{\text{rad}} \) is compensated by the slower redshift of \( \rho_{3/2} \). This happens at

\[
H_{\text{dom}} \simeq \frac{16}{9} \left( \frac{n_{3/2}}{s} \right)^2 \left( \frac{E_p}{T_p} \right)^2 H_{\text{non}} = 8.9 \left( \frac{g_*(T_p)}{228.75} \right)^{1/2} \left( \frac{n_{3/2}}{s} \right)^{2} \frac{m_{3/2}^2}{M_P}, \tag{3.13}
\]
where we have used $s = (2\pi^2/45) g_*(T_p) T_p^3$. Here $g_*(T_p)$ is the number of relativistic degrees of freedom in the thermal bath at the temperature $T_p$ when gravitinos are produced. Gravitino non-domination therefore requires that $H_{\text{dom}} < \Gamma^{3/2}$, i.e. that gravitinos decay while their energy density is subdominant. This translates into the bound

$$\frac{n_{3/2}}{s} < \left[ \frac{228.75}{g_*(T_p)} \right]^{1/4} \left( \frac{m_{3/2}}{10^5 \text{ GeV}} \right)^{1/2} 2.4 \times 10^{-8},$$

on the gravitino abundance.

It is seen that for $m_{3/2} \geq 50 \text{ TeV}$, gravitinos will dominate if $(n_{3/2}/s) > 1.7 \times 10^{-8}$. Thermal scatterings alone, see (2.4), can yield such large abundances for extremely large reheating temperatures $T_R > 10^{14} \text{ GeV}$ (we consider $M_{\tilde{g}} \leq 100 \text{ TeV}$ here). Therefore they do not lead to gravitino domination in general. The sfermion and inflaton decays, however, can produce a sufficiently large number of gravitinos for much lower $T_R$. As mentioned earlier, see the discussion after Eq. (2.9), sfermion decays are usually the dominant source of gravitino production when $T_R \geq \tilde{m}$. We therefore concentrate on the sfermions here.

Sfermion decays, see (2.4), will not lead to gravitino domination if

$$\tilde{m} < \left( \frac{m_{3/2}}{10^5 \text{ GeV}} \right)^{5/6} 1.3 \times 10^8 \text{ GeV}.$$ 

(3.15)

Here we have taken $g_*(\tilde{m}) = 228.75$ and $N = 46$ in (2.6).

A successful scenario with gravitino non-domination should take the constraints from LSP production into account. Fig. (1) depicts different regions in the $\tilde{m} - m_{3/2}$ plane for the choice $c = 10^{-2}$ and $m_\chi = 100 \text{ GeV}$. Above the solid line gravitinos dominate, i.e. the opposite inequality as in (3.15) is satisfied, and hence excluded. The region between the solid and dashed lines is defined by

$$\left( \frac{1 \text{ GeV}}{m_\chi} \right)^{1/3} \left( \frac{m_{3/2}}{10^5 \text{ GeV}} \right)^{2/3} 3.4 \times 10^7 \text{ GeV} < \tilde{m} < \left( \frac{m_{3/2}}{10^5 \text{ GeV}} \right)^{5/6} 1.3 \times 10^8 \text{ GeV}.$$ 

(3.16)

In this region $(n_{3/2}/s) > 4.6 \times 10^{-10} (1 \text{ GeV}/m_\chi)$, thus the density of LSPs produced in gravitino decay exceeds the dark matter limit. The dotted and dot-dashed vertical lines correspond to Eqs. (3.6) and (3.10) respectively. The regions in black color are excluded since either gravitinos dominate or gravitino decay produces too many LSPs which do not sufficiently annihilate. In region 1 gravitinos decay after the freeze-out but efficient annihilation reduces the abundance of produced LSPs below the dark matter limit. Gravitino decay occurs before the freeze-out in region 2 and
does not affect the final LSP abundance. Below the dashed line sfermion decays do not overproduce gravitinos. In fact, below the $\tilde{m} = m_{3/2}$ line such decays are kinematically forbidden altogether. In this part of the $\tilde{m} - m_{3/2}$ plane one has to worry about thermal scatterings though. If $T_R \geq 4.6 \times 10^9 \, (1 \, \text{GeV}/m_\chi) \, \text{GeV}$, scatterings will overproduce gravitinos. Therefore regions 3, 4 and 5 will not be acceptable in this case, due to inefficient LSP annihilation.

For the values of $c$ and $m_\chi$ chosen in this plot, thermal abundance of LSPs at freeze-out is too small to account for dark matter. Non-thermal LSP dark matter from gravitino decay will in this case be a viable scenario along the dot-dashed line.

**Figure 1:** Constraints from LSP production in gravitino non-domination case for $c = 10^{-2}$ and $m_\chi = 100 \, \text{GeV}$. The solid and dashed lines represent the upper and lower limits in Eq. (3.16) respectively. The dotted and dot-dashed lines correspond to Eqs. (3.6) and (3.10) respectively. The regions in black color are excluded. The solid red line is given by $\tilde{m} = m_{3/2}$.

### 3.3 Gravitino-dominated Universe

Gravitinos eventually dominate the energy density of the Universe if $\Gamma_{3/2} \leq H_{\text{dom}}$. 
Figure 2: Parameter constraints from LSP production in gravitino-dominated case for $c = 10^{-2}$ and $m_\chi = 100$ GeV. The dotted and dot-dashed lines correspond to Eqs. (3.6) and (3.10) respectively. The region in black color is excluded.

which happens for

$$\frac{n_{3/2}}{s} \geq \left[ \frac{228.75}{g_*(T_p)} \right]^{1/4} \left( \frac{m_{3/2}}{10^5 \text{ GeV}} \right)^{1/2} 2.4 \times 10^{-8}. \quad (3.17)$$

The scale factor of the Universe $a \propto H^{-2/3}$ in the interval $\Gamma_{3/2} \leq H < H_{\text{dom}}$. Gravitino decay will then increase the entropy density by the factor $d = (g_*(T_a)T_3^3/g_*(T_b)T_3^3)$. Here $T_a$, $T_b$ denote the temperature of the thermal bath before and after gravitino decay, respectively, while $g_*(T_a)$, $g_*(T_b)$ are the number of relativistic degrees of freedom at $T_a$ and $T_b$, respectively. Note that $d = \left( g_*(T_a)\rho_{3/2}^3/g_*(T_b)\rho_{R}^3 \right)^{1/4}$, with $\rho_{3/2}$ and $\rho_R$ being the energy density in the gravitinos and radiation, respectively, at the time of gravitino decay. The dilution factor $d$ is therefore given by

$$d = \left( \frac{H_{\text{dom}}}{\Gamma_{3/2}} \right)^{1/2} \left( \frac{g_*(T_a)}{g_*(T_b)} \right)^{1/4} \approx \left[ \frac{g_*(T_p)}{228.75} \right]^{1/4} \left( \frac{10^5 \text{ GeV}}{m_{3/2}} \right)^{1/2} \left[ \frac{(n_{3/2}/s)}{2.4 \times 10^{-8}} \right]. \quad (3.18)$$

\(^5\)To be more precise, the dilution factor is given by $1 + d$. The two definitions are essentially equivalent when $d \ll 1$. Obviously there is no gravitino domination, and hence no dilution, when $d < 1$. 

Here we have taken \((g_*(T_a)/g_*(T_b))^{1/4} \simeq 1\), which is a good approximation since in a wide range \(1 \text{ Mev} \leq T \leq \tilde{m}\) the number of relativistic degrees of freedom \(g_*\) changes between 10.75 and 228.75. To be more precise, the dilution factor is given by \(1 + d\).

Obviously for \(d < 1\) there is no gravitino domination, and hence no dilution. As mentioned before, sfermions are usually the main source of gravitino production for \(T_R \geq \tilde{m}\), and hence we concentrate on them here.

Gravitinos produced in sfermion decays dominate the Universe if

\[
\left( \frac{m_{3/2}}{10^5 \text{ GeV}} \right)^{5/6} \frac{1.3 \times 10^8 \text{ GeV} \leq \tilde{m} \leq T_R},
\]

in which case the dilution factor is given by

\[
d \simeq \left( \frac{10^5 \text{ GeV}}{m_{3/2}} \right)^{5/2} \left( \frac{\tilde{m}}{1.3 \times 10^8 \text{ GeV}} \right)^3.
\]

Gravitino decay dilutes the existing LSP abundance by a factor of \(d\), while at the same time producing one LSP per gravitino. Note that \((n_{3/2}/s) \leq 10^{-2}\) as gravitinos at most reach thermal equilibrium when \(\tilde{m} \gg m_{3/2}\). Eq. (3.18) then implies that \(d \leq 5.9 \times 10^5\). It is evident from (3.17) that the abundance of non-thermal LSPs upon their production from the decay of supermassive gravitinos exceeds the dark matter limit by several orders of magnitude. Therefore any acceptable scenario with gravitino domination requires that the LSP annihilation be efficient at the temperature \(T_{3/2}\), i.e. that (3.10) be satisfied.

Fig. (2) depicts different regions in the \(\tilde{m} - m_{3/2}\) plane for the choice \(c = 10^{-2}\) and \(m_\chi = 100\text{ GeV}\). Gravitinos dominate in the region above the solid line corresponding to Eq. (3.19). Therefore regions below this line are irrelevant. To the left of the dotted line, which represents Eq. (3.6), gravitino decay occurs after the freeze-out. To the right of the dot-dashed line, corresponding to Eq. (3.10), LSPs produced from gravitino decay efficiently annihilate. The region in black color is excluded due to inefficient LSP annihilation. Gravitinos decay after the freeze-out in region 1, but the final abundance of LSPs respects the dark matter limit. On the other hand, \(T_{3/2} \geq T_f\) in region 2, and hence gravitino decay does not affect the LSP abundance. For the values of \(c\) and \(m_\chi\) chosen here, thermal abundance of LSPs at freeze-out is too low to account for dark matter. However, non-thermal LSP dark matter from gravitino decay is successful along the dot-dashed line.

In passing we note that the same discussions also apply when \(T_R < \tilde{m}\). In this case, however, the inflaton decay will be the main source of gravitino production.
as sfermions are not excited by the thermal bath. Constraints from efficient LSP annihilation then lead to plots similar to those in Figs. (1), (2), with the $\tilde{m} - m_{3/2}$ plane replaced by the $\Delta m_{\phi} - m_{3/2}$ plane.

Entropy release from gravitino decay also dilutes any previously generated baryon asymmetry. This implies that baryogenesis should either take place after gravitino decay, or generates an asymmetry in excess of the observed value by the dilution factor given in (3.18). It is seen from (3.3) that $T_{3/2} \leq 100$ GeV unless $m_{3/2} > 10^8$ GeV. This implies that successful baryogenesis after gravitino decay will be possible only if gravitinos are extremely heavy. Therefore the more likely scenario in a gravitino-dominated Universe is generating a sufficiently large baryon asymmetry at early times which will be subsequently diluted by gravitino decay. We will discuss leptogenesis and baryogenesis, and the effect of gravitinos in detail in the next Sections.

One might invoke an intermediate stage of entropy release by the late decay of some scalar condensate (beside inflaton) to prevent gravitino domination. We shall notice, however, that any such decay will itself produce gravitinos with an abundance which is inversely proportional to the new (and lower) reheating temperature, see (2.8). This implies that any stage of reheating, while diluting gravitinos which are produced during the previous stage(s), can indeed produce more gravitinos. Therefore entropy generation via scalar field decay is in general not a helpful way to avoid a gravitino-dominated Universe.

One comment is in order before closing this subsection. In both of the gravitino non-domination and domination scenarios, having an LSP abundance in agreement with the dark matter limit constrains its mass through Eq. (3.11). Heavier LSPs overclose the Universe in one way or another. If $T_{3/2} \geq T_f$, gravitino decay will be irrelevant but thermal abundance of LSPs will be too high. If $T_{3/2} < T_f$, gravitino decay can in addition make an unacceptably large non-thermal contribution. In case of gravitino domination gravitino decay dilutes thermal LSPs. However, according to (3.17), the decay itself overproduces non-thermal LSPs which will not sufficiently annihilate. Therefore gravitino domination cannot rescue a scenario with thermally overproduced LSPs. Indeed, for $m_\chi \gg 1$ gravitinos should never dominate the Universe. The problem can be solved if $T_R < T_f$, or if another stage of entropy release below $T_f$ dilutes thermal LSPs. In both case, however, reheating can overproduce

If the scalar field does not dominate the Universe, the expression in (2.8) should be multiplied by the fraction $r$ of the total energy density $r$ which it carries.
gravitinos and the subsequent gravitino decay may lead to non-thermal overproduction of LSPs. If \( R \)-parity is broken, the LSP will be unstable and its abundance will not be subject to the dark matter bound. Obviously its thermal and/or non-thermal overproduction will not pose a danger in this case.

3.4 Solving the Boltzmann Equation

The entropy generated by gravitino decay can be estimated from Eq. (3.20). However, the evolution of gravitinos and relativistic particles can be followed directly by solving the Boltzmann equation. Assuming that gravitinos are non-relativistic at decay and that \( g_* \) is constant during the decay process, the Boltzmann equations for gravitinos take the form

\[
\dot{\rho}_{3/2} = -\Gamma_{3/2} \rho_{3/2} - 3H \rho_{3/2}
\]

and

\[
\dot{T} = -HT + \Gamma_{3/2} \frac{\rho_{3/2} T}{4\rho_R}.
\]

These should be solved together with the Friedmann equation

\[
H^2 = \frac{1}{3M_p^2} \left[ \rho_{3/2} + g_* (T) \frac{\pi^2}{30} T^4 \right].
\]

In Fig. (3) we show several examples of solving the Boltzmann equation for different initial conditions, always assuming the \( g_* = 10.75 \) during decay. The dilution factor \( d \) can then be found from

\[
d(t) = \frac{(a(t)T(t))^3}{(a_iT_i)^3},
\]

where \( a_i \) and \( T_i \) denote the initial values of the scale factor and the radiation temperature respectively. The bottom panel of Fig. (3) shows the dilution factor, and when gravitinos dominate it agrees quite well with Eq. (3.20).

4. Leptogenesis

The baryon asymmetry of the Universe (BAU) parameterized as \( \eta_B = (n_B - n_{\bar{B}})/s \), with \( s \) being the entropy density, is determined to be \( 0.9 \times 10^{-10} \) by recent analysis of WMAP data [2]. This number is also in good agreement with an independent determination from primordial abundances produced during BBN [17]. Three conditions are required for generating a baryon asymmetry: \( B- \) and/or \( L- \) violation, \( C- \) and \( CP- \) violation, and departure from thermal equilibrium [18]. Since \( B + L \)-violating
Figure 3: Plot of temperature, density and entropy increase as functions of time for $m_{3/2} = 100$ TeV. Curves are for $(n_{3/2}/s) = 10^{-10}, 10^{-9}, 10^{-8}, 10^{-7},$ and $10^{-6}$ in increasing order.

Sphalerons transitions are active at temperatures $100 \text{ GeV} \leq T \leq 10^{12} \text{ GeV}$ [49], any mechanism for creating a baryon asymmetry at $T > 100$ GeV must create a $B - L$ asymmetry. The final asymmetry is then given by $B = a(B - L)$, where $a = 28/79$ in case of SM and $a = 8/23$ for MSSM [50].

Leptogenesis postulates the existence of RH neutrinos, which are SM singlets, with a lepton number violating Majorana mass $M_N$. It can be naturally embedded in models which explain the light neutrino masses via the see-saw mechanism [51]. A lepton asymmetry can then be generated from the out-of-equilibrium decay of the
RH neutrinos into Higgs bosons and light leptons, provided $CP$-violating phases exist in the neutrino Yukawa couplings \cite{52,53,54}. The created lepton asymmetry will be converted into a baryonic asymmetry via sphalerons processes.

The on-shell RH neutrinos whose decay is responsible for the lepton asymmetry can be produced thermally via their Yukawa interactions with the standard model fields and their superpartners \cite{36}, for which $T_R \geq M_1 \sim 10^9$ GeV, \cite{55,56,57,59}, or non-thermally for which $T_R \leq M_N$, see \cite{1,60,61}. Non-thermal leptogenesis can also be achieved without exciting on-shell RH neutrinos \cite{62}. In supersymmetric models one in addition has the RH sneutrinos which serve as an additional source for leptogenesis \cite{63}. The sneutrinos are produced along with neutrinos in a thermal bath or during reheating, and with much higher abundances in preheating \cite{64}. There are also additional possibilities for leptogenesis from the RH sneutrinos some of which rely on soft SUSY breaking effects \cite{65,66,67,68,69,70}.

### 4.1 Thermal Leptogenesis

Let us concentrate on the supersymmetric standard model augmented with three RH neutrino multiplets in order to accommodate neutrino masses via the see-saw mechanism \cite{51}. The relevant part of the superpotential is

$$W \supset \frac{1}{2} M_i N_i N_i + h_{ij} H_u N_i L_j,$$

where $N$, $H_u$, and $L$ are multiplets containing the RH neutrinos $N$ and sneutrinos $\tilde{N}$, the Higgs field giving mass to, e.g., the top quark and its superpartner, and the left-handed (s)lepton doublets, respectively. Here $h_{ij}$ are the neutrino Yukawa couplings and we work in the basis in which the Majorana mass matrix is diagonal. The decay of a RH (s)neutrino with mass $M_i$ (we choose $M_1 < M_2 < M_3$) results in a lepton asymmetry per (s)neutrino quanta $\epsilon_i$, given by

$$\epsilon_i = -\frac{1}{8\pi} \frac{1}{[hh^\dagger]_{ii}} \sum_j \text{Im} ([hh^\dagger]_{ij})^2 f \left( \frac{M_j^2}{M_i^2} \right),$$

with \cite{54}

$$f(x) = \sqrt{x} \left( \frac{2}{x - 1} + \ln \left[ \frac{1 + x}{x} \right] \right).$$

The first and second terms on the right-hand side of Eq. (4.3) correspond to the one-loop self-energy and vertex corrections, respectively. Assuming strongly hierarchical
RH (s)neutrinos and an $\mathcal{O}(1)\ CP-$violating phase in the Yukawa couplings, it can be shown that [71]

$$|\epsilon_1| \lesssim \frac{3}{8\pi} \frac{M_1 (m_3 - m_1)}{\langle H_u \rangle^2},$$

(4.4)

where $m_1 < m_2 < m_3$ are the masses of light, mostly left-handed (LH) neutrinos. For a hierarchical spectrum of light neutrino masses ($m_1 \ll m_2 \ll m_3$), we then have

$$|\epsilon_1| \lesssim 2 \times 10^{-7} \left( \frac{m_3 - m_1}{0.05 \text{ eV}} \right) \left( \frac{M_1}{10^9 \text{ GeV}} \right).$$

(4.5)

To obtain this, we have used $m_3 - m_1 \approx m_3 \approx 0.05 \text{ eV}$ (as suggested by atmospheric neutrino oscillation data) and $\langle H_u \rangle \approx 170 \text{ GeV}.

If the asymmetry is mainly produced from the decay of the lightest RH states, after taking the conversion by sphalerons into account, we arrive at

$$\eta_B^{\text{MAX}} \approx 3 \times 10^{-10} \left( \frac{m_3 - m_1}{0.05 \text{ eV}} \right) \left( \frac{M_1}{10^9 \text{ GeV}} \right) \kappa,$$

(4.6)

where we have assumed maximal $CP$-violation. Here $\kappa$ is the efficiency factor accounting for the decay, inverse decay and scattering processes involving the RH states [55, 57].

The decay parameter $K$ is defined as

$$K \equiv \frac{\Gamma_1}{H(T = M_1)},$$

(4.7)

where

$$\Gamma_1 = \sum_i \frac{|h_{1i}|^2}{4\pi} M_1,$$

(4.8)

is the decay width of $N_1$ and $\tilde{N}_1$. One can also define the effective neutrino mass

$$\tilde{m}_1 \equiv \sum_i \frac{|h_{1i}|^2 \langle H_u \rangle^2}{M_1},$$

(4.9)

which determines the strength of $\tilde{N}_1$ and $N_1$ interactions, with the model-independent bound $m_1 < \tilde{m}_1$ [73].

- **case (1):**

  If $K < 1$, corresponding to $\tilde{m}_1 < 10^{-3} \text{ eV}$, the decay of RH states will be out-of-equilibrium at all times. In this case the abundance of RH states produced via Yukawa interactions does not reach the thermal equilibrium value. The lepton
number violating scatterings can be safely neglected. Hence this case is called
the weak washout regime. The efficiency factor is \( \kappa \simeq 0.1 \) when \( \tilde{m}_1 = 10^{-3} \) eV. Generating sufficient asymmetry then puts an absolute lower bound \( f_{ew} \times 10^9 \) GeV on \( M_1 \), and \( T_R \geq M_1 \) will be required in this case \[55\].

• case (2):

In the opposite limit \( K > 1 \), \( \tilde{N}_1 \) and \( N_1 \) will be in thermal equilibrium at
temperatures \( T > M_1 \). In particular, the efficiency of inverse decays erases any
pre-existing asymmetry (generated, for example, from the decay of heavier RH
states). This regime is called of strong washout. We note that this regime in-
cludes the entire favored neutrino mass range \( m_{sol} \lesssim \tilde{m}_1 \lesssim m_{atm} \). The inverse
decays keep the RH (s)neutrinos in equilibrium for sometime after \( T \) drops
below \( M_1 \). The number density of quanta which undergo out-of-equilibrium
decay is therefore suppressed and reduces the efficiency factor. Successful lep-
togenesis in the range \((m_{sol}, m_{atm})\) requires that \( 10^{10} \) GeV \( < M_1 \lesssim 10^{11} \) GeV
while, due to the efficiency of inverse decays, \( T_R \) can be smaller than \( M_1 \) by
almost one order of magnitude \[55\]. The efficiency factor \( \kappa \) in this window
varies between \( f_{ew} \times 10^{-3} \) and \( f_{ew} \times 10^{-2} \).

It is possible to obtain (approximate) analytical expressions for the efficiency
factor \( \kappa \). In the strong washout regime, the final efficiency factor is maximal when
\( \Delta L = 2 \) scatterings among the LH (s)leptons can be neglected. So long as the
scatterings can be neglected, the efficiency factor \( \kappa \) is independent from the lightest
RH (s)neutrino mass \( M_1 \) and \( \kappa(\tilde{m}_1) \) is given by \[55, 57, 58, 59\]
\[
\kappa \simeq 10^{-2} \left( \frac{0.01 \text{ eV}}{\tilde{m}_1} \right)^{1.1}.
\] (4.10)

Scatterings cannot be neglected for very large \( M_1 \) or large light neutrino masses. In
this case the efficiency factor can be approximated as \[55\]:
\[
\kappa(\tilde{m}_1, M_1, \tilde{m}^2) = \kappa(\tilde{m}_1) e^{-\frac{\omega}{z_B} \left( \frac{M_1}{m_{sol} \text{ GeV}} \right) \left( \frac{\tilde{m}_1}{m_{sol}} \right)^2},
\] (4.11)

where \( \omega \simeq 0.186 \), \( z_B = M_1/T_B \sim f_{ew} \), with \( T_B \) the temperature at which most of
the lepton asymmetry is produced, \( \tilde{m}^2 \) is the sum over the squares of the light neutrino
masses. The efficiency factor \( \kappa(\tilde{m}_1) \) is given in Eq. (1.10). For hierarchical light
neutrinos, \( \tilde{m}^2 = m_3^2 \simeq \Delta m_{atm}^2 \simeq 2.2 \times 10^{-3} \) eV\(^2\), where \( \Delta m_{atm}^2 \) is the mass squared
difference which controls the oscillations of atmospheric neutrinos. Washout effects
due to scatterings then become important for $M_1 \sim 10^{14}$ GeV. For quasi-degenerate neutrinos, $m_1 \simeq m_2 \simeq m_3$, larger values of $\bar{m}$ are possible and $\kappa$ will be exponentially suppressed even for much smaller values of $M_1$.

For negligible $\Delta L = 2$ scatterings, the baryon asymmetry is, in the case of maximal decay asymmetry, given by

$$\eta_B^{\text{MAX}} \simeq 0.9 \times 10^{-10} \left( \frac{m_3 - m_1}{0.05 \text{ eV}} \right) \left( \frac{0.01 \text{ eV}}{\bar{m}_1} \right)^{1.1} \left( \frac{M_1}{3.7 \times 10^{10} \text{ GeV}} \right). \quad (4.12)$$

The baryon asymmetry is proportional to $M_1$. The usual bound on the reheating temperature $T_R \leq 10^{10}$ GeV, given for $m_{3/2} < 50$ TeV, does not apply to supermassive gravitinos. Hence, RH (s)neutrinos with masses $M_1 \gg 10^{10}$ GeV can be thermally produced in the early Universe. This can lead to larger amounts of baryon asymmetry generated by thermal leptogenesis, with respect to the standard scenario. The lowest reheating temperatures required for thermally producing these RH (s)neutrinos is $T_R \sim M_1/few \ll 10^{14}$ GeV. Note that the initial assumption of gravitino non-domination is satisfied if gravitinos are dominantly produced by thermal scatterings.

4.2 Effects of the Gravitino on Thermal Leptogenesis

Thermal leptogenesis completes when $T \sim M_1/few \ [55, 57]$. Eq. (3.3) then implies that gravitino decay takes place after leptogenesis unless they are extremely heavy:

$$m_{3/2} > \left( \frac{M_1}{10^9 \text{ GeV}} \right)^{2/3} 10^{12} \text{ GeV}. \quad (4.13)$$

On the other hand, for $m_{3/2} \geq 50$ TeV, gravitino decay occurs at a temperature $T_{3/2} > 6.8$ MeV which is compatible with a successful BBN. Therefore both scenarios of gravitino domination and non-domination are in agreement with the BBN constraints. Nevertheless the effect of gravitino decay on the final baryon asymmetry need to be taken into account. We consider both scenarios of gravitino domination and non-domination.

- **Gravitino non-domination**: The condition for gravitino non-domination is given in Eq. (3.14). There will be practically no dilution by gravitino decay in this case, and thermal leptogenesis should generate $\eta_B \simeq 10^{-10}$ according to Eq. (4.6). Obtaining sufficient asymmetry in both of the weak and strong washout regimes requires
that $M_1 \gtrsim T_R > 10^9 \text{ GeV}$, and leptogenesis completes when $T \sim M_1 \geq 10^9 \text{ GeV}$ \cite{55, 57}. Sfermions with a mass $\tilde{m} \leq 10^9 \text{ GeV}$ certainly reach thermal equilibrium and, for $m_{3/2} < \tilde{m}$, their decay will produce gravitinos according to Eq. (2.6).

If $m_{3/2} \geq \tilde{m}$, gravitino production from sfermion decays is kinematically forbidden. Scatterings of gauge and gaugino quanta in the thermal bath will nevertheless produce gravitinos so long as $m_{3/2} \leq T_R$. Late time domination of gravitinos thus produced requires that the reheating temperature $T_R \geq 10^{14} \text{ GeV}$, see Eq. (2.4). However, gravitinos can be overproduced for much smaller $T_R$. For $m_\chi = 100 \text{ GeV (1 TeV)}$, gravitino decay results in LSP overproduction when $T_R \geq 3 \times 10^{10} (3 \times 10^9) \text{ GeV}$. This indeed occurs for the bulk of the parameter space compatible with thermal leptogenesis, particularly in the favored neutrino mass window $m_{\text{solar}} \leq \tilde{m}_1 \leq m_{\text{atm}}$ \cite{55}. The condition for sufficient annihilation of non-thermal LSPs in this case sets a lower bound on the gravitino mass through Eq. (3.10), independently of whether $m_{3/2} < \tilde{m}$ or $m_{3/2} \geq \tilde{m}$. Constraints from LSP annihilation, which determine acceptable regions of the $\tilde{m} - m_{3/2}$ plane, are summarized in Fig. (1) and the related discussion.

In the case of gravitino non-domination, the produced lepton asymmetry is not subsequently diluted by gravitino decays, even if taking place after leptogenesis. As the bound on the reheating temperature $T_R \leq 10^{10} \text{ GeV}$ does not apply for supermassive gravitinos, RH neutrinos with masses $M_1 \gg 10^{10} \text{ GeV}$ can be fully thermalized for a sufficiently large reheating temperature. From Eqs. (4.6), (4.10) and (4.11), it follows that the baryon asymmetry is proportional to the lightest RH neutrino mass, $M_1$, up to $M_1 \sim 10^{14} \text{ GeV}$, for $\tilde{m}^2 \sim \Delta m^2_{\text{atm}}$. For larger values of $M_1$, the lepton asymmetry is washed out by $\Delta L = 2$ scatterings. Then, the maximum baryon asymmetry is produced for $M_1 \sim 5 \times 10^{12} \frac{z_B}{\omega} \text{ GeV}$. Here we have again taken $\tilde{m}^2 \sim \Delta m^2_{\text{atm}}$. From Eq. (4.12), we notice that for large values of $M_1$ the generated $\eta_B$ can be much larger than the one required to explain the observations, if the decay asymmetry is maximal. Models which implement the see-saw mechanism of neutrino mass generation typically assume the conservation of flavor symmetries and/or special forms of the Yukawa couplings, in order to explain the low energy neutrino masses and mixing. In many of these models, the decay asymmetry is
constrained to be non maximal and $M_1$ larger than the typical values $10^9$–$10^{10}$ GeV are needed to generate a sufficient baryon asymmetry. Models with supermassive gravitinos allow to have reheating temperatures high enough to thermally produce such heavy RH neutrinos. In each specific model, a detailed analysis is required for establishing the feasibility of successful leptogenesis and, at the same time, the possibility to explain the low energy neutrino mass matrix (for a discussion of CP-violation in specific see-saw models and leptogenesis, see, e.g., Ref. [72]).

- **Gravitino-dominated Universe:**

  If gravitinos are produced very abundantly, see Eq. (3.17), the Universe will become gravitino-dominated. Sfermion decays (which, as mentioned earlier, usually dominate over thermal scatterings and inflaton decay) produce such abundances of gravitinos when Eq. (3.19) is satisfied.

  Gravitino decay reheats the Universe to a temperature $T_{3/2}$, see Eq. (3.3), and increases the entropy density by a factor $d$ given in Eq. (3.18). Successful thermal leptogenesis *after* gravitino decay will be only possible if gravitinos are extremely heavy $m_{3/2} > 10^{12}$ GeV, see Eq. (4.13). According to Eq. (3.17), gravitino domination in this case requires that $(n_{3/2}/s) > 10^{-5}$. It follows from Eq. (3.19) that sfermion decays yield this only if $\tilde{m}$ (and $T_R$) is $> 10^{14}$ GeV. In addition, non-perturbative production of gravitinos with the necessary abundance is also questionable.

  Therefore, it is more realistic to consider the opposite situation where leptogenesis occurs *before* gravitino decay. In this case, due to the entropy release by gravitino decay, the generated asymmetry must exceed the observed value of $\eta_B \simeq 10^{-10}$ by a factor of $d$. Eqs. (3.20) and (4.3) then imply that the final asymmetry is

  $$\eta_B \simeq 3 \times 10^{-10} \left( \frac{m_{3/2}}{10^5 \, \text{GeV}} \right)^{5/2} \left( \frac{1.3 \times 10^8 \, \text{GeV}}{\tilde{m}} \right)^3 \left( \frac{M_1}{10^9 \, \text{GeV}} \right)^\kappa. \quad (4.14)$$

  As a specific example, consider leptogenesis in the favored neutrino mass window $m_{\text{sol}} \leq \tilde{m}_1 \leq m_{\text{atm}}$. In this interval, which entirely lies within the strong washout regime, the efficiency factor $\kappa \sim 10^{-2}$, and the reheating temperature follows $T_R \geq 0.1 M_1$ [55]. Therefore successful leptogenesis requires that

  $$M_1 \simeq \left( \frac{10^5 \, \text{GeV}}{m_{3/2}} \right)^{5/2} \left( \frac{\tilde{m}}{1.3 \times 10^8 \, \text{GeV}} \right)^3 3 \times 10^{10} \, \text{GeV}. \quad (4.15)$$
In Fig. 4 we show the value of $M_1$ needed to produce the correct baryon asymmetry of $\eta_B = 0.9 \times 10^{-10}$ as a function of the lightest RH (s)neutrino mass $m_{3/2}$ and $\tilde{m}$. The plot is produced assuming $\kappa = 10^{-2}$, but since $\eta_B \propto \kappa$ it can easily be rescaled for other values. Without dilution, successful leptogenesis for $\kappa = 10^{-2}$ requires that $M_1 \simeq 3 \times 10^{10}$ GeV, see Eq. (4.6). Therefore having contours with $M_1 > 3 \times 10^{10}$ GeV indicates gravitino domination. As mentioned earlier, thermal leptogenesis fails for $M_1 \geq 10^{14}$ GeV due to the erasure of generated asymmetry by $\Delta L = 2$ scattering processes. This happens in the light colored region, and hence excludes it. The overlap between Figs. (1) and (3) combines the constraints from leptogenesis and dark matter considerations.

Thermal leptogenesis cannot generate a baryon asymmetry which exceeds $10^{-2}$. The maximal value is obtained in the (hypothetical) case when RH (s)neutrinos have thermal equilibrium abundance, and the efficiency factor $\kappa$ and asymmetry parameter $|\epsilon_1|$ are both 1, see Eq. (4.6). This implies that successful leptogenesis in a gravitino-dominated Universe would be impossible if the dilution factor $d$ was larger than $10^8$. However, since gravitinos can at most reach thermal equilibrium, we always have $d \leq 5.9 \times 10^8$, see the discussion after Eq. (3.20). This ensures that there will be no such case where thermal leptogenesis is absolutely impossible in a gravitino-dominated Universe.

5. Baryogenesis from Supersymmetric Flat Directions

There are many gauge invariant combinations of the Higgs, squark and slepton fields along which the scalar potential identically vanishes in the limit of exact SUSY. Within the MSSM there are nearly 300 flat directions which are both $F$- and $D$-flat and conserve R-parity [12]. Soft terms, as well as non-renormalizable superpotential terms, lift the flat directions when SUSY is broken.

A homogeneous condensate along can be formed along a flat direction in the inflationary epoch, provided that the flat direction mass $\tilde{m}$ is smaller than the Hubble expansion rate during inflation. The condensate starts oscillating coherently when the expansion rate $H \simeq \tilde{m}$. During this epoch the inflationary fluctuations of the condensate can be converted to density perturbations [74]. The condensate can also help an efficient reheating to the SM degrees of freedom [76]. In addition, it can
Figure 4: The value of $\log(M_1)$ (in GeV) needed to produce $\eta_B = 0.9 \times 10^{-10}$ as a function of $m_{3/2}$ and $\tilde{m}$ in the gravitino-dominated scenario. We have chosen the efficiency factor $\kappa = 10^{-2}$, which is the typical value in the favored neutrino mass window. The dark color region does not have gravitino domination, while in the light color region thermal leptogenesis fails due to washout by lepton number violating scatterings.

Excite vector perturbations to explain the observed large scale magnetic field \cite{75}. If the condensate carries a non-zero baryon and/or lepton number, then the flat direction dynamics can be responsible for baryogenesis via the Affleck-Dine (AD) mechanism \cite{77} (for a review, see Ref. \cite{13}). In the following section we make a general discussion on the viability of AD baryogenesis for arbitrarily heavy gravitinos and/or sfermions.

5.1 Late Baryogenesis via the Affleck–Dine Mechanism

The scalar potential for a flat direction $\phi$ (not to be confused with the inflaton field in Section II) is given by \cite{77}

$$V = (\tilde{m}^2 + c_H H^2) |\phi|^2 + \left( \frac{A + a H}{n M_{n-3}} \lambda \phi^n + h.c. \right) + \frac{\lambda^2}{M(2n-3)} |\phi|^{2(n-1)}. \quad (5.1)$$

Here $\tilde{m}^2$ and $c_H H^2$ are the soft mass$^2$ from SUSY breaking in the vacuum and SUSY breaking by the non-zero energy density of the inflaton respectively. Here $c_H$ can
have either sign, and its sign is also affected by radiative corrections \cite{78, 79}. The last term on the right-hand side of (5.1), which lifts the flat direction, arises from a non-renormalizable superpotential term (i.e. \(n \geq 4\)) induced by new physics at a high scale \(M\). In general \(M\) could be a string scale, below which we can trust the effective field theory, or \(M = M_p\). SUSY breaking in the vacuum and by the inflaton energy density generate \(A\)-terms \(A\) and \(aH\), respectively, corresponding to this non-renormalizable superpotential term. For minimal Kähler terms and in case of gravity mediation \(\tilde{m} \sim A \sim m_{3/2}\) and \(0 < c_H \sim 1\). Depending on the symmetries of the inflaton sector, both \(a \sim \mathcal{O}(1)\) and \(a \ll 1\) are possible.

The equation of motion for the flat direction is given by

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi^*} = 0. \tag{5.2}
\]

The evolution is easiest to analyze by the field parameterization

\[
\phi = \frac{1}{\sqrt{2}} \varphi e^{i\theta}, \tag{5.3}
\]

where \(\varphi, \theta\) are real fields. Then the scalar potential can be written in the form

\[
V(\varphi, \theta) = \frac{1}{2} (\tilde{m}^2 + c_H H^2) \varphi^2 + \frac{|\lambda| f(\theta)}{2^{(n-2)/2} n M^{n-3}} \varphi^n + \frac{|\lambda|^2}{2^{n-1} M^{2(n-3)}} \varphi^{2(n-1)}. \tag{5.4}
\]

Here

\[
f(\theta) = |A| \cos(n\theta + \theta_A + \theta_\lambda) + |a| H \cos(n\theta + \theta_a + \theta_\lambda), \tag{5.5}
\]

with \(\theta_A, \theta_a, \theta_\lambda\) being the angular directions for \(A, a, \lambda\) respectively. The baryon/lepton number density is given by

\[
n_{BL} = \frac{\beta}{i} \left( \phi^* \dot{\phi} - \dot{\phi}^* \phi \right) = \beta \dot{\theta} \varphi^2, \tag{5.6}
\]

where \(\beta\) is the baryon/lepton number carried by the flat direction. At the minima of the potential

\[
\varphi_{\text{min}}^{n-2} = \frac{2^{n/n-2} M^{n-3}}{(n-1) |\lambda|} \left\{ -f(\theta) \pm \left[ f(\theta)^2 - 4(n-1)(\tilde{m}^2 + c_H H^2) \right]^{1/2} \right\}, \tag{5.7}
\]

and \(n_{\theta_{\text{min}}} = (2p + 1) \pi - \theta_a - \theta_\lambda\) if \(|a| H \gg |A|\), while \(n_{\theta_{\text{min}}} = (2p + 1) \pi - \theta_A - \theta_\lambda\) if \(|a| H \ll |A|\) (with \(p = 0, 1, \ldots, n-1\)).

The radial field \(\varphi\) quickly settles at one of the minima given in (5.7) during inflation. If \(|a| H \gg |A|\), the phase field \(\varphi \theta\) (since \(\theta\) is dimensionless) has a mass of
order $|a|H$ and it ends up in one of the discrete minima $n \theta_{\text{min}} = \pi - \theta_a - \theta_\lambda$. The phase field has a mass $\ll H$ if $|a|H \ll |A|$, and hence it freezes at a random value.

After inflation the Hubble rate decreases as the Universe expands, and so does $\varphi_{\text{min}}$. The $\varphi$ field tracks the instantaneous minimum of the potential, so its evolution can be qualitatively understood by looking at the evolution of the minimum. Once $\tilde{m}^2 \simeq |c_H|H^2$, the minimum of the potential changes from $\varphi_{\text{min}}$ to $\varphi = 0$ in a non-adiabatic manner. At this time $\phi$ starts oscillating in the radial direction with frequency $\tilde{m}$. The motion of the phase field, which is necessary for generating a baryon/lepton asymmetry, requires the exertion of a torque. If $|a|H \sim |A|$, a non-adiabatic change in the position of the minimum from $n \theta_{\text{min}} = \pi - \theta_a - \theta_\lambda$ to $n \theta_{\text{min}} = \pi - \theta_A - \theta_\lambda$ generates a torque. If $|a|H \ll |A|$, the freezing of the phase field at a random value generates the torque and leads to its motion towards the minimum $n \theta_{\text{min}} = \pi - \theta_A - \theta_\lambda$ if $|a|H \ll |A|$. The potential along the angular direction will quickly decrease due to the redshift of $\varphi$, see Eq. (5.4), once $\varphi$ starts its oscillations.

In consequence, $\phi$ starts freely rotating in the angular direction at which time a net baryon/lepton asymmetry is generated.

Based on Eqs. (5.2) and (5.6), the baryon/lepton asymmetry obeys the equation

$$\dot{n}_{B,L} + 3Hn_{B,L} = -\beta \frac{\partial V}{\partial \theta}. \quad (5.8)$$

This can be integrated to give at late times $t \gg H_{\text{osc}}^{-1}$ (see [13])

$$n_{B,L} \simeq \beta \frac{2(n-2)}{3(n-3)} \frac{\sin \delta}{(H_{\text{osc}}t)^2} |A| \varphi_{\text{osc}}^2,$$

where $\varphi_{\text{osc}}$, $H_{\text{osc}}$ denote the value of $\varphi$, $H$ when the condensate starts oscillating. Here $\delta$ is a measure of spontaneous $CP$-violation in the $\phi$ potential and $\sin \delta \sim 1$.

The baryon to entropy ratio will then be

$$\frac{n_{B,L}}{s} = \frac{3T_R n_{B,L}}{4\rho_R} = \frac{T_R n_{B,L}}{4M_P^2H_{\text{osc}}^2}. \quad (5.10)$$

We parameterize the $A$-term as $|A| = \gamma \tilde{m}$. For gravity-mediated SUSY breaking typically $\tilde{m} \sim |A| \sim m_{3/2}$, while in gauge-mediated models $|A| \sim m_{3/2} \ll \tilde{m}$. In split SUSY $A \ll \tilde{m}$, see the discussion in Ref. [21]. Here we consider $\gamma$ to be arbitrary, and hence $\tilde{m}$ and $|A|$ be unrelated.

Since $H_{\text{osc}} \simeq \tilde{m}$ and $\varphi_{\text{osc}}^2 \sim 2^{(n-2)/2}M^{n-3}\tilde{m}/|\lambda|$, we find

$$\frac{n_{B,L}}{s} \simeq \frac{n-2}{6(n-3)} \frac{|A| T_R}{\tilde{m}^2} \left( \frac{\tilde{m}}{|\lambda|M_P} \right)^{2/(n-2)}, \quad (5.11)$$
where $M = M_P$ is taken.

For the lowest dimensional non-renormalizable term $n = 4$, and $\lambda \sim \mathcal{O}(1)$, the generated baryon asymmetry is $n_{BL}/s \sim 0.1(|A|T_R/\tilde{m}M_P)$. If $|A| \sim \tilde{m}$ then $T_R \sim 10^9$ GeV is adequate to generate baryon asymmetry of order $10^{-10}$. If $|A| \ll \tilde{m}$, one would require even larger $T_R$. However the most desirable feature of AD baryogenesis lies in its flexibility to generate a desirable baryon asymmetry even for very low reheating temperatures. For instance, if $\tilde{m} \sim 10^7$ GeV and $n = 6$, the required asymmetry can be generated for $T_R \sim 10^3$ GeV when $|A| = \tilde{m}$.

At late times, i.e. $H \ll \tilde{m}$, contributions from SUSY breaking by the inflaton energy density are subdominant to soft terms from SUSY breaking in the vacuum. If $|A|^2 < 4(n - 1)\tilde{m}^2$, the $\phi$ potential has only one minimum at $\varphi = 0$. However, another minimum appears away from the origin when $4(n - 1)\tilde{m}^2 \leq |A|^2$, see (5.7). In the AD scenario the $\phi$ field starts at large values of $\varphi$, and hence it gets trapped in this secondary minimum in the course of its evolution in the early Universe. If $4(n - 1)\tilde{m}^2 \leq |A|^2 < n^2\tilde{m}^2$, the true minimum is still located at the origin. Tunneling from the false vacuum could still save the situation in this case. However, for $|A| \geq n\tilde{m}$, the true minimum will be at $\varphi \neq 0$. Since flat directions have non-zero charge and color quantum numbers, this will lead to an unacceptable situation with charge and color breaking in the vacuum. It is therefore necessary to have $|A|^2 < 4(n - 1)\tilde{m}^2$ in order to avoid entrapment in such vacuum states. This is the case in gravity-mediated and gauge-mediated models, as well as split SUSY. Note that the same discussion applies to the soft terms induced by non-zero energy density of the inflaton. However, these terms disappear at late times and will be irrelevant in the present vacuum.

- Thermal effects:

According to the potential given in Eqs. (5.1) and (5.4), the flat direction condensate starts oscillating when $H \simeq \tilde{m}$. However thermal effects from reheating may trigger an earlier oscillation and lead to a larger value of $H_{osc}$ [80, 81]. The inflaton decay (in the perturbative regime) is a gradual process which starts after the end of inflation. Hence, even before the inflaton decay is completed, the decay products constitute a thermal bath with instantaneous temperature $T \simeq (HT_R^2M_P)^{1/4} > T_R$ [82]. The flat direction has gauge and Yukawa couplings, collectively denoted by $y$, to other fields. Its VEV gives a mass $\sim y\varphi$ to these fields. If $y\varphi \leq T$, these fields are excited in the thermal bath and reach
Figure 5: The reheating temperature as a function of the scalar mass, given by Eq. (5.11), for successful AD baryogenesis. The cases \( n = 4 \) and \( n = 6 \) are represented by solid and dashed lines respectively. The black and red lines are plotted for \( |A| = \tilde{m} \) and \( |A| = 1 \) TeV respectively.

thermal equilibrium. This, in turn, results in a thermal correction \( \sim yT \) to the \( \phi \) mass. If \( yT \) exceeds the Hubble parameter at early times, i.e. for \( H \gg \tilde{m} \), the condensate starts early oscillations [80]. On the other hand, if \( y\phi > T \), the fields coupled to \( \phi \) will be too heavy to be excited. They will decouple from the running of gauge coupling(s) at temperature \( T \) instead, which induces a logarithmic correction to the free energy \( \sim T^4 \log(T/\phi) \). This triggers early oscillations of the condensate if \( T^2/\phi > H \) when \( H \gg \tilde{m} \) [81]. Note that according to Eq. (5.10) a larger value of \( H_{osc} \) results in a smaller baryon/lepton asymmetry.

Refs. [80, 81] have studied thermal effects for the conventional case with \( \tilde{m} \sim 1 \) TeV. Thermal corrections of the former type will become less important as \( \tilde{m} \) increases. The reason is that \( \phi^{n-2} \propto H \), see (5.7), and hence \( y\phi > H \) will be more difficult to satisfy for larger \( \tilde{m} \). Also, since \( H \propto T^4 \) at early times, \( (T^2/\phi) \) increases more slowly that \( H \). Therefore thermal corrections of the latter type will also become less important when \( \tilde{m} \gg 1 \) TeV.
5.2 Effects of Gravitino on Affleck-Dine Baryogenesis

In Eq. (5.11) no specific assumption is made about the source which reheats the Universe. It can be either the inflaton decay, as usually considered, or the decay of the flat direction condensate. In case the inflaton decay reheats the Universe $T_R < \tilde{m}$ and $\tilde{m} \leq T_R$ are both possible. However, $T_R \geq \tilde{m}$ if the flat direction is responsible for reheating the Universe. The energy density in the condensate oscillations is $\tilde{m}^2 \varphi^2$. The flat direction has gauge and Yukawa couplings to other fields through which it induces a mass $\varphi$ for the decay products. The condensate will decay no later than the time when $\varphi \lesssim \tilde{m}$. Hence, since energy density in the radiation is $\sim T^4$, we will have $T_R \geq \tilde{m}$ in this case. According to Eq. (5.11), the yielded baryon asymmetry is $\propto T_R$. This implies a larger asymmetry for larger values of $T_R$. Having an unacceptably large baryon asymmetry is indeed typical when the flat direction condensate has a very large VEV such that it dominates the energy density and, subsequently, reheats the Universe [37]. Gravitino domination can in this case help to dilute the excess of baryon asymmetry. It is interesting that such a solution, invoked from the early days of AD baryogenesis [37], can be naturally realized with supermassive gravitinos.

When $T_R \geq \tilde{m}$ sfermions reach thermal equilibrium after reheating and their decay will be the dominant source of gravitino production. The condition for gravitino domination and the dilution factor from gravitino decay are then given by Eqs. (3.19) and (3.20) respectively. The final asymmetry generated via the AD mechanism in case of gravitino domination will then be, see (5.11)

$$\frac{n_B}{s} \approx \frac{n - 2}{6(n - 3)} \left( \frac{\tilde{m}}{|A| M_P} \right)^{2/n - 2} \left( \frac{m_3/2}{|A| 10^5 \text{ GeV}} \right)^{5/2} \left( \frac{1.3 \times 10^8 \text{ GeV}}{\tilde{m}} \right)^{3/2} \left( \frac{\Delta m_{\phi}}{m_{3/2}} \right)^2.$$ (5.12)

The results for the two extreme cases $|A| = \tilde{m}$ and $|A| = 1$ TeV are summarized in Fig. (6) when $n = 4, 6$. The combined constraints from baryogenesis and dark matter considerations will be included in the overlap of Figs. (1) and (6).

If $T_R \ll \tilde{m}$, sfermion quanta will not be excited in the thermal bath. However, inflaton decay can in this case result in efficient production of gravitinos according to Eq. (2.8). One can then repeat the same steps to find an expression for the initial asymmetry similar to that in (5.12). Such an expression, and plots similar to those in Fig. (6), will however depend on $\Delta m_{\phi}$ as well as $\tilde{m}$ and $m_{3/2}$. This leads to a more complicated and model-dependent situation. Moreover, the scenario with gravitino domination will be more constrained when $T_R \ll \tilde{m}$ (specially if $|A| \ll \tilde{m}$). The
initial baryon asymmetry is already suppressed in this case, see (5.11), and gravitino decay may dilute it to unacceptably small values.

![Graph](image)

**Figure 6:** The scalar mass as a function of gravitino mass, given by Eq. (5.12), for successful AD baryogenesis in a gravitino-dominated Universe. The conventions are the same as in Fig. (5).

### 5.3 Late Gravitino Production from Q-ball Decay

So far we have assumed that gravitinos are mainly produced in sfermion decays (if $T_R \geq \tilde{m}$), or in inflaton decay (if $T_R \ll \tilde{m}$). The flat direction condensate consists of zero-mode quanta of the sfermions, and hence its decay too can lead to gravitino production. If $T_R \geq \tilde{m}$, this will be subdominant to the contribution from the decay of thermal sfermions. The reason is that the zero-mode quanta have at most an abundance $(n/s)$ which is comparable to that of thermal sfermions. If $T_R \ll \tilde{m}$, sfermions will not be excited in the thermal bath. As mentioned before, the condensate certainly decays no later than the time when $\varphi \lesssim \tilde{m}$. Even if $\varphi \sim M_P$ initially, the condition $\varphi \lesssim \tilde{m}$ is satisfied at $H \gtrsim \tilde{m}^2/M_P$. Not that the scale factor of the Universe $a \propto H^{-2/3}$ during reheating, in which phase the Universe is dominated by inflaton oscillations. Also, the abundance of zero-mode quanta $(n/s) < (3T_R/4\tilde{m}) \ll 1$. This implies that the condensate contains a much smaller
number of quanta which survive (much) shorter than thermal sfermions. Therefore gravitino production from the decay of the flat direction condensate will not be as constraining as that in the decay of thermal sfermion. In most case, it can be simply neglected.

The above discussion strictly applies to the oscillations of a homogeneous condensate. However, it usually happens that the flat direction oscillations fragment and forms non-topological solitons known as Q-balls [83]. These Q-balls can decay much later than a homogeneous condensate. Their late decay could then efficiently produce gravitinos, even if Q-balls do not dominate the energy density of the Universe.

To elucidate, let us consider the case when tree-level sfermion masses at a high scale $M$ are given by $\tilde{m}$. The potential for the sfermions, after taking into account of one-loop corrections, reads

$$V = \tilde{m}^2 |\phi|^2 \left[ 1 + K \ln \left( \frac{|\phi|^2}{M^2} \right) \right], \quad (5.13)$$

where $K$ is a coefficient determined by the renormalization group equations, see [11, 84]. In order to form Q-balls it is necessary that the potential be flatter than $|\phi|^2$ at large field values, i.e. that $K < 0$. Loops which contain gauginos make a negative contribution $\propto -m_{1/2}^2$ to $K$, with $m_{1/2}$ being the gaugino mass. On the other hand, loops which contain sfermions contribute $\propto +\tilde{m}^2$. Then $K < 0$ can be obtained, provided that $2m_{1/2} \gtrsim \tilde{m}$ [84]. In models of gravity-mediated SUSY breaking $\tilde{m} \sim m_{1/2}$, and hence $K < 0$ is obtained for many flat directions. When the spectrum is such that $\tilde{m} \gg m_{1/2}$, like in the case of split SUSY, there are no Q-balls as $K > 0$.

The potential can also be much flatter $\propto \ln|\phi|$ at large field values. This happens in models of gauge-mediated SUSY breaking [25], and can arise from thermal corrections [81]. The important point in any case is that the scalar field profile within a Q-ball is such that the field value is maximum at the center $\varphi_0$ and decreases towards the surface. This implies that for $g\varphi_0 \geq \tilde{m}$, with $g$ being a typical coupling of the $\phi$ field to other fields, the Q-ball decays through its surface as decay inside the Q-ball is not energetically allowed [80]. Note that for a typical gauge or Yukawa coupling $g\varphi_0 \geq \tilde{m}$ if $\varphi_0 \gg \tilde{m}$. The decay rate of Q-ball which contains a (baryonic/leptonic) charge $Q$ is in this case given by [80]

$$\frac{dQ}{dt} \leq \frac{\omega^3 A}{192\pi^2},$$

where $\omega \simeq \tilde{m}$ and $A = 4\pi R_Q^2$ is the surface area of the Q-ball. For example, consider Q-ball formation for the potential given in (5.13). A Q-ball with total charge $Q$ then
has a decay lifetime \[ \tau_Q \gtrsim \left( \frac{|K|}{0.03} \right) \left( \frac{1 \text{ TeV}}{\tilde{m}} \right) \left( \frac{Q}{10^{20}} \right) \times 10^{-7} \text{ sec}, \] (5.15)

which corresponds to the decay temperature

\[ T_d \lesssim \left( \frac{0.03}{|K|} \right)^{1/2} \left( \frac{\tilde{m}}{1 \text{ TeV}} \right)^{1/2} \left( \frac{10^{20}}{Q} \right)^{1/2} \times 2 \text{ GeV}. \] (5.16)

Here we have used \( \tau_d^{-1} \sim (T_d^2 / M_P) \). The total baryonic/leptonic charge \( Q \) of a Q-ball is given by the multiplication of baryon/lepton number carried by the flat direction and the total number of zero-mode quanta inside the Q-ball. This, after using (2.5), leads to

\[
\text{Helicity } \pm \frac{1}{2}: \left( \frac{n_{3/2}}{s} \right)_{\text{Q-ball}} \sim \left( \frac{\tilde{m}}{m_{3/2}} \right)^2 \left( \frac{\tilde{m}}{1 \text{ TeV}} \right)^2 \left( \frac{Q}{10^{20}} \right) \left( \frac{n_B}{s} \right) 2.5 \times 10^{-13},
\]

\[
\text{Helicity } \pm \frac{3}{2}: \left( \frac{n_{3/2}}{s} \right)_{\text{Q-ball}} \sim \left( \frac{\tilde{m}}{1 \text{ TeV}} \right)^2 \left( \frac{Q}{10^{20}} \right) \left( \frac{n_B}{s} \right) 2.5 \times 10^{-13},
\] (5.17)

Obviously \( \left( \frac{n_{3/2}}{s} \right)_{\text{Q-ball}} \leq \left( \frac{n_B}{s} \right) \), since the decay of each quanta inside the Q-ball can at most produce one gravitino. This implies that gravitinos produced from the decay of Q-balls will not dominate the Universe if the decay generates a baryon asymmetry \( \left( \frac{n_B}{s} \right) \simeq 10^{-10} \), see Eq. (3.17). However, the situation will be different for larger Q-balls which yield an asymmetry \( \gg 10^{-10} \). If gravitinos from Q-ball decay dominate the Universe, they will dilute the baryon asymmetry. The final asymmetry will then have the correct size, see Eqs. (3.18) and (5.17), if

\[ Q \sim \left( \frac{m_{3/2}}{10^5 \text{ GeV}} \right)^{1/2} \left( \frac{1 \text{ TeV}}{\tilde{m}} \right)^2 \left( \frac{m_{3/2}}{\tilde{m}} \right)^2 \times 10^{35}. \] (5.18)

Assuming that the Q-balls do not dominate the energy density of the Universe, we must have \( \left( \frac{n_B}{s} \right) < (3T_d/4\tilde{m}) \). The necessary condition for gravitino domination, after using Eqs. (3.17), (5.16) and Eq. (5.17), is then obtained to be

\[ Q > \left( \frac{m_{3/2}}{10^5 \text{ GeV}} \right) \left( \frac{m_{3/2}}{\tilde{m}} \right)^4 \left( \frac{1 \text{ TeV}}{\tilde{m}} \right)^3 2.5 \times 10^{35}. \] (5.19)

For \( \tilde{m} \gg 1 \text{ TeV} \) and \( m_{3/2} \ll \tilde{m} \), this lower bound on the Q-ball charge is compatible with the value in Eq. (5.18) required for successful baryogenesis. Note that the Q-ball decay temperature \( T_d \) may be smaller than the LSP freeze-out temperature \( T_f \). In this case Q-ball decay can be dangerous as three LSP per baryon number will
be produced. However, this will not lead to problem so long as the gravitino decay temperature \(T_{3/2} < T_d\) (which is typically the case) and the condition for efficient LSP annihilation in Eq. (3.10) is satisfied. The Q-balls will dominate the Universe if they carry a very large charge. The initial baryon asymmetry released by the Q-balls then has a simple expression \((n_B/s) \sim (T_d/\bar{m})\).

The results in Eqs. (5.15), (5.16), (5.17), (5.18) and (5.19) are valid for the potential given in Eq. (5.13). The same steps (though more involved) can be followed to obtain similar results for logarithmic potentials as in the case of gauge-mediated models. The remarkable point in all cases is that the Q-ball decay lifetime increases with its charge, implying a more efficient production of gravitinos from the Q-ball decay. This leads to an attractive solution that the large baryon asymmetry released by the Q-ball decay can be naturally diluted by gravitinos produced in the same process.

Finally, we shall notice that in models of running mass inflation even the inflaton condensate could fragment into non-topological solitons [87]. In that case Q-balls would naturally dominate the Universe. We do not discuss such possibility here.

6. Summary and Conclusions

In this paper we have investigated cosmological consequences of models with super-heavy gravitinos and/or sfermions. A priori there is no fundamental reason which fixes the scale of SUSY breaking. Models with weak-scale SUSY breaking in the observable sector have the promise to solve the hierarchy problem. However this may not necessarily be the case and the SUSY breaking scale can turn out to be very high. Under general circumstances, arbitrarily heavy gravitino mass \(m_{3/2}\) and/or sfermion masses \(\bar{m}\) are quite plausible. Therefore, inspired from the recent models of large scale SUSY breaking, it becomes pertinent to re-examine the cosmological and phenomenological consequences.

Gravitino are produced through various processes in the early Universe. Scatterings of gauge and gaugino quanta in thermal bath, sfermion decays and the inflaton decay are the main sources for gravitino production. The main results are presented in Eqs. (2.4), (2.6) and (2.8). Sfermion decays usually dominate when the reheating temperature \(T_R \geq \bar{m} > m_{3/2}\). On the other hand, the contribution from the inflaton decay dominates when \(T_R \ll \bar{m}\).
Gravitinos which are heavier than 50 TeV decay before primordial nucleosynthesis, and hence are not subject to BBN bounds. However, each gravitino produces one LSP upon its decay. Hence, in models with conserved $R$-parity, the abundance of supermassive gravitinos is constrained by the dark matter limit. Indeed, efficient annihilation of LSPs produced in gravitino decay sets a lower bound on $m_{3/2}$. When this lower bound is saturated, gravitino decay can successfully produce non-thermal LSP dark matter.

This is also valid in a gravitino-dominated Universe, which happens when gravitinos are produced very abundantly. However, for $m_{3/2} \geq 50$ TeV, gravitino domination cannot rescue a scenario where thermal LSP abundance at the freeze-out exceeds the dark matter bound. For a Wino- or Higgsino-like LSP this is the case when the LSP mass $m_\chi > 2$ TeV. The reason is that in this case gravitino decay, while diluting the thermal abundance, leads to non-thermal overproduction of LSPs. Therefore, if $R$-parity is conserved, gravitinos should never dominate in models with such heavy LSPs. The results for gravitino production in conjunction with the constraints from the dark matter bound and LSP annihilation are summarized in Eqs. (3.6), (3.10), (3.15) and (3.19). Figs. (1) and (2) depict the acceptable parts of the $\tilde{m} - m_{3/2}$ plane for successful scenarios of gravitino non-domination and domination respectively.

We discussed some specific scenarios of baryogenesis in the presence of supermassive gravitinos. The parameter space for thermal leptogenesis is substantially relaxed when $m_{3/2} \geq 50$ TeV, as a considerably larger reheating temperature $T_R$ and/or right-handed (s)neutrino mass $M_1$ will be allowed. This, however, implies that gravitinos can also be efficiently produced for a wide range of sfermion masses. Since gravitino decay takes place after the completion of leptogenesis, unless they are extremely heavy, the generated baryon asymmetry will be diluted in a gravitino-dominated Universe. Successful leptogenesis then requires (much) larger right-handed (s)neutrino masses than usual. However, it is known that thermal leptogenesis fails for $M_1 \geq 10^{14}$ GeV since lepton number violating scatterings in this case erase the generated asymmetry. This leads to additional constraints on the $\tilde{m} - m_{3/2}$ parameter space in case of gravitino domination. Our results are summarized in Eqs. (4.14), (4.15) and Fig. (4).

We also considered late time baryogenesis from supersymmetric flat directions via the Affleck-Dine mechanism. Thermal effects which can trigger early oscillations
of the flat direction condensate, thus suppressing the generate asymmetry, tend to be less important for $\tilde{m} \gg 1$ TeV. A large expectation value for the condensate at the onset of its oscillations usually leads to a baryon asymmetry $(n_B/s) \gg 10^{-10}$, as well as a large reheating temperature $T_R \geq \tilde{m}$. Gravitinos produced from sfermion decays can then dominate the Universe and dilute the initially large asymmetry down to acceptable values. The main results in Eqs.(5.11), (5.12) are depicted in Figs (5), (6).

There is even a closer connection between large baryon asymmetry and efficient gravitino production when oscillations of the flat direction condensate fragment into Q-balls (as happen in many cases). Q-balls decay (much) later than the homogeneous condensate, and the larger the baryonic/leptonic charge they carry the longer their decay lifetime. Hence the decay of large Q-balls is a natural source for copious production of gravitinos which can even dominate the Universe and dilute the large baryon asymmetry released by Q-balls. We explicitly demonstrated this for a potential with logarithmic corrections, Eqs. (5.18) and (5.19), but the same conclusions hold for other types of flat potentials.

To conclude, models with superheavy gravitinos and/or sfermions have very interesting cosmological consequences. These models can naturally give rise to a large gravitino abundance in the early Universe. This, contrary to models with a weak scale gravitino mass, can turn to a virtue and lead to successful production of dark matter and baryon asymmetry generation.

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