Holographic conductivity in the massive gravity with power-law Maxwell field

A. Dehyadegari, M. Kord Zangeneh, A. Sheykhi

1Physics Department and Biruni Observatory, Shiraz University, Shiraz 71454, Iran
2Physics Department, Faculty of Science, Shahid Chamran University of Ahvaz, Ahvaz 61357-43135, Iran
3Center of Astronomy and Astrophysics, Department of Physics and Astronomy, Shanghai Jiao Tong University, Shanghai 200240, China
4Research Institute for Astrophysics and Astronomy of Maragha (RIAAM), P.O. Box 55134-441, Maragha, Iran

We obtain a new class of topological black hole solutions in (n + 1)-dimensional massive gravity in the presence of the power-Maxwell electrodynamics. We calculate the conserved and thermodynamic quantities of the system and show that the first law of thermodynamics is satisfied on the horizon. Then, we investigate the holographic conductivity for the four and five dimensional black brane solutions. For completeness, we study the holographic conductivity for both massless (m = 0) and massive (m ≠ 0) gravities with power-Maxwell field. The massless gravity enjoys translational symmetry whereas the massive gravity violates it. For massless gravity, we observe that the real part of conductivity, Re[σ], decreases as charge q increases when frequency ω tends to zero, while the imaginary part of conductivity, Im[σ], diverges as ω → 0. For the massive gravity, we find that Im[σ] is zero at ω = 0 and becomes larger as q increases (temperature decreases), which is in contrast to the massless gravity. It also has a maximum value for ω ≠ 0 which increases with increasing q (with fixed p) or increasing p (with fixed q) for (2 + 1)-dimensional dual system, where p is the power parameter of the power-law Maxwell field. Interestingly, we observe that in contrast to the massless case, Re[σ] has a maximum value at ω = 0 (known as the Drude peak) for p = (n + 1)/4 (conformally invariant electrodynamics) and this maximum increases with increasing q. In this case (m ≠ 0) and for different values of p, the real and imaginary parts of the conductivity has a relative extremum for ω ≠ 0. Finally, we show that for high frequencies, the real part of the holographic conductivity have the power law behavior in terms of frequency, ω^n where a ∝ (n + 1 − 4p). Some similar behaviors for high frequencies in possible dual CFT systems have been reported in experimental observations.

PACS numbers: 97.60.Lf, 04.70.-s, 71.10.-w, 04.70.Bw, 04.30.-w.

I. INTRODUCTION

A century after Einstein’s discovery namely general relativity, the domain of its applications has become as vast as it covers even condensed matter physics which seemed at the opposite end of physics building compared to gravity [1]. This strange topic which connects gravity to almost all fields of physics (see [2]) is called gauge/gravity duality (GGD); the extended version of AdS/CFT correspondence [3]. GGD has attracted increasing interests during recent years and become one of the most promising fields of physics which is hoped to be able to solve many of unsolved problems in different fields of physics including condensed matter physics.

Real materials in condensed matter physics do not respect the translational symmetry i.e. there is a dissipation in momentum. The momentum dissipation may come from the existence of a lattice or impurities. Although this dissipation has no important influence on the values of some observable, it affects the behavior of some others for instance conductivity. The DC conductivity in the presence of translational symmetry diverges, whereas in the absence of this symmetry (when momentum is dissipating) it has a finite value. In the context of GGD, it is important to study a gravity model which includes holographic momentum dissipation. There are some attempts to construct such gravity model [4]. One of these models proposed by D. Vegh [5], provides an effective bulk description of a theory in which momentum is no longer conserved. The conservation of momentum is due to the diffeomorphism invariance of stress-energy tensor in dual theory. In [5], the proposal is to break this symmetry holographically by giving a mass to graviton state. The resulting gravity is therefore massive gravity. One of the advantages of this theory is that the black hole solutions of it are solvable analytically and therefore it is an excellent toy model to study holographically the properties of materials without momentum conservation.
Thermal behaviors of black hole solutions in the context of massive gravity was explored extensively in recent years [5–8]. Thermodynamics of linearly charged massive black branes has been investigated in [5]. In [6], a class of higher-dimensional linearly charged solutions with positive, negative and zero constant curvature of horizon in the context of massive gravity accompanied by a negative cosmological constant has been presented and thermodynamics and phase structure of these black solutions have been studied in both canonical and grand canonical ensembles. In [7], van der Waals phase transitions of linearly charged black holes in massive gravity have been investigated and it has been shown that the massive gravity can present substantially different thermodynamic behavior in comparison with Einstein gravity. Also it has been shown that the graviton mass can cause a range of new phase transitions for topological black holes which are forbidden for other cases. The properties of massive solutions have been studied in different scenarios [9]. From holographic point of view, the behaviors of different holographic quantities have been studied [5, 10–22]. The behavior of holographic conductivity for systems dual to linearly charged massive black branes has been explored in [5]. In [11], a holographic superconductor has been constructed in the massive gravity background. [13] studies holographic superconductor-normal metal-superconductor Josephon junction in the massive gravity. Also the holographic thermalization process has been investigated in this context [14]. Analytic DC thermo-electric conductivities in the context of massive gravity have been calculated in [12]. In massive Einstein-Maxwell-dilaton gravity, DC and Hall conductivities have been computed in [15]. [16] presents a holographic model for insulator/metal phase transition and colossal magnetoresistance within massive gravity. Inspired by the recent action/complexity duality conjecture, it has been shown in [22] that the holographic complexity grows linearly with time in the context of massive gravity.

As we mentioned above, one of the quantities which is affected by momentum dissipation is conductivity. On the other hand, the choice of electrodynamics model has a direct influence on the behavior of conductivity. So, it is worthy to consider the effects of nonlinearity as well as massive gravity on the conductivity of the black hole solutions. It is well-known that the nonlinear electrodynamics brings reach physics compared to the linear Maxwell electrodynamics. For example, Maxwell theory is conformally invariant only in four dimensions and thus the corresponding energy-momentum tensor is only traceless in four dimensions. A natural question then arises: Is there an extension of Maxwell action in arbitrary dimensions that is traceless and hence possesses the conformal invariance? The answer is positive and the invariant Maxwell action under conformal transformation $g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$, $A_\mu \rightarrow A_\mu$ in $(n + 1)$-dimensions is given by [23],

$$S_m = \int d^{n+1}x \sqrt{-g}(-F)^p,$$

where $F = F_{\mu\nu}F^{\mu\nu}$ is the Maxwell invariant, provided $p = (n+1)/4$. The associated energy-momentum tensor of the above Maxwell action is given by

$$T_{\mu\nu} = 2 \left( p F_{\mu\eta} F_\nu^{\eta} F^{\rho\sigma} \frac{1}{4} g_{\mu\nu} F^p \right).$$

One can easily check that the above energy-momentum tensor is traceless for $p = (n+1)/4$. Also, quantum electrodynamics predicts that the electrodynamic field behaves nonlinearly through the presence of virtual charged particles that is reported by Heisenberg and Euler [24]. Hence, nonlinear electrodynamics has been subject of much researches [25–27]. This motivates us to extend the linearly charged black hole solutions of massive gravity [5, 6] to nonlinearly charged ones in the presence of power-law Maxwell electrodynamics and investigate the thermodynamics of them as well as the behavior of conductivity corresponding to the dual system. In addition to power-law Maxwell electrodynamics, other types of nonlinear electrodynamics have been introduced in [28–30]. In spite of the special property for $p = (n+1)/4$, different aspects of various solutions have been investigated for different $p$’s [31–33]. In the context of AdS/CFT correspondence, the power-law Maxwell field has been considered as electrodynamics source in [34–39].

The layout of this letter is as follows. In section II, we present the action of the massive gravity in the presence of power-Maxwell electrodynamics and then by varying the action we obtain the field equations. We also derive a class of topological black hole solutions of the field equations in higher dimensions. In section III, we study thermodynamics of the solutions and examine the first law of thermodynamics for massive black holes with power-law Maxwell field. In section IV, we investigate the holographic conductivity of black brane solutions in the presence of a power-law Maxwell gauge field. In particular, we shall disclose the effects of the power-law Maxwell electrodynamics as well as massive gravity on the holographic conductivity of dual systems. We finish with closing remarks in section V.
II. ACTION AND MASSIVE GRAVITY SOLUTIONS

The \((n+1)\)-dimensional \((n \geq 3)\) action describing Einstein-maxmassive gravity accompanied by a negative cosmological constant \(\Lambda\) in the presence of power-law Maxwell electrodynamics is

\[
S = \int d^{n+1}x \mathcal{L},
\]

\[
\mathcal{L} = \frac{\sqrt{-g}}{16\pi} \left[ R - 2\Lambda + (-F)^p + m^2 \sum_i c_i \mathcal{U}_i(g, \Gamma) \right],
\]

where \(g\) and \(R\) are respectively the determinant of the metric and the Ricci scalar and \(\Lambda = -n(n-1)/2l^2\) is the negative cosmological constant where \(l\) is the AdS radius. \(F = F_{\mu\nu}F^{\mu\nu}\) and \(F_{\mu\nu} = \partial_{[\mu}A_{\nu]}\) is electrodynamic tensor where \(A_\nu\) is vector potential. \(p\) determines the nonlinearity of the electrodynamic field. For \(p = 1\), the linear Maxwell gauge field will be recovered. In action (2), \(\Gamma\) is the reference metric, \(c_i\)’s are constants and \(\mathcal{U}_i\)’s are symmetric polynomials of eigenvalues of the \((n+1)\times(n+1)\) matrix \(K^\mu_\nu \equiv \sqrt{g^{\mu\alpha}\Gamma_{\alpha\nu}}\) so that

\[
\mathcal{U}_1 = [K],
\]
\[
\mathcal{U}_2 = [K]^2 - [K^2],
\]
\[
\mathcal{U}_3 = [K]^3 - 3[K][K^2] + 2[K^3],
\]
\[
\mathcal{U}_4 = [K]^4 - 6[K^2][K^2] + 8[K^3][K] + 3[K^2]^2 - 6[K^4],
\]

where the square root in \(K\) is related to mean matrix square root i.e. \((\sqrt{\mathcal{K}})^\mu_\nu \cdot \sqrt{\mathcal{K}}^\nu_\lambda = \mathcal{K}^\mu_\lambda\) and rectangular brackets mean trace \([K] \equiv \mathcal{K}^\mu_\mu\). Here \(m\) is the massive gravity parameter so that in limit \(m \to 0\), one recovers the diffeomorphism invariant Einstein-Hilbert action with a gauge field and a negative cosmological constant. The equations of motion for gravitation and gauge field are

\[
R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} - 2pF_{\mu\lambda}F^\lambda_{\nu} - \frac{1}{2}(-F)^p g_{\mu\nu} + m^2 \chi_{\mu\nu} = 0,
\]

\[
\nabla_\mu (\mathcal{F}^{\nu-1} F^\mu_{\nu}) = 0,
\]

which are obtained by varying the action (2) with respect to the metric tensor \(g_{\mu\nu}\) and gauge field \(A_\mu\) respectively. In Eq. (8), we have

\[
\chi_{\mu\nu} = -\frac{c_1}{2} (\mathcal{U}_1 g_{\mu\nu} - K_{\mu\nu}) - \frac{c_2}{2} (\mathcal{U}_2 g_{\mu\nu} - 2\mathcal{U}_1 K_{\mu\nu} + 2K^2_{\mu\nu}) - \frac{c_3}{2} (\mathcal{U}_3 g_{\mu\nu} - 3\mathcal{U}_2 K_{\mu\nu} + 6K^3_{\mu\nu})
\]
\[+ 6\mathcal{U}_4 K^2_{\mu\nu} - 6K^3_{\mu\nu}) - \frac{c_4}{2} (\mathcal{U}_4 g_{\mu\nu} - 4\mathcal{U}_3 K_{\mu\nu} + 12\mathcal{U}_4 K^2_{\mu\nu} - 24\mathcal{U}_4 K^3_{\mu\nu} + 24K^4_{\mu\nu}).
\]

The static spacetime line element takes the usual form

\[
ds^2 = -f(r)dt^2 + f^{-1}(r)dx^2 + r^2 h_{ij}dx^i dx^j,
\]

where \(f(r)\) is the metric function and \(h_{ij}\) is a function of coordinates \(x_j\) which spanned an \((n-1)\)-dimensional hypersurface with constant scalar curvature \((n-1)(n-2)k\) and volume \(\omega_{n-1}\). Without loss of generality, one can take \(k = 0, 1, -1\), such that the black hole horizon or cosmological horizon in (11) can be a zero (flat), positive (elliptic) or negative (hyperbolic) constant curvature hypersurface. The reference metric (fixed symmetric tensor) \(\Gamma_{\mu\nu}\) can be considered as [5, 6]

\[
\Gamma_{\mu\nu} = \text{diag}(0, 0, c_0^2 h_{ij}),
\]

where \(c_0\) is a positive constant. Using (11) and (12), one can easily calculates \(\mathcal{U}_i\)’s as

\[
\mathcal{U}_1 = \frac{(n-1)c_0}{r},
\]
\[
\mathcal{U}_2 = \frac{(n-1)(n-2)c_0^2}{r^2},
\]
\[
\mathcal{U}_3 = \frac{(n-1)(n-2)(n-3)c_0^3}{r^3},
\]
\[
\mathcal{U}_4 = \frac{(n-1)(n-2)(n-3)(n-4)c_0^4}{r^4}.
\]
Notice that $\mathcal{U}_3$ and $\mathcal{U}_4$ vanish for $(3+1)$-dimensional spacetime while $\mathcal{U}_4 = 0$ for $(4+1)$-dimensional spacetime. Using the metric (11), the electrodynamic field can be immediately found as

$$F_{tr} = -F_{rt} = \frac{q}{r^{(n-1)/(2p-1)}},$$

where $q$ is a constant parameter related to the total charge of black hole. Inserting Eqs. (12), (13) and (14) into field equations (8), one receives

$$f' + \frac{(n-2)f}{r^2} - \frac{(n-2)k}{r^2} + \frac{2\Lambda}{n-1} + \frac{2p-1}{n-1} \left(2q^2 r^{-2n-2}ight)^p - \frac{c_0 m^2}{r} \left(c_1 + \frac{(n-2)c_0 c_2}{r} + \frac{(n-2)(n-3)c_0^2 c_3}{r^2} + \frac{(n-2)(n-3)(n-4)c_0^3 c_4}{r^3}\right) = 0,$$

where $\prime$ denotes the derivative with respect to $r$. Solving above equations, $f(r)$ can be obtained as

$$f(r) = k - \frac{m_0}{r^{n-2}} - \frac{2\Lambda r^2}{n(n-1)} + \frac{2p q^{2p}(2p-1)^2}{(n-1)(n-2)p r^{2(np-3p+1)/(2p-1)}} + \frac{c_0 m^2 r}{n-1} \left(c_1 + \frac{(n-1)c_0 c_2}{r} + \frac{(n-1)(n-2)c_0^2 c_3}{r^2} + \frac{(n-1)(n-2)(n-3)c_0^3 c_4}{r^3}\right),$$

where $m_0$ is an integration constant which is related to total mass of black hole as we see later. One may note that the metric function (17) reduces to those of Refs. [5, 6] in the case $p = 1$. Also the solution (17), in the absent of massive parameter $(m = 0)$, leads to

$$f_0(r) = k - \frac{m_0}{r^{n-2}} - \frac{2\Lambda r^2}{n(n-1)} + \frac{2p q^{2p}(2p-1)^2}{(n-1)(n-2)p r^{2(np-3p+1)/(2p-1)}},$$

which was presented in [32]. The mass parameter $(m_0)$ in Eq. (17) can be found as

$$m_0 = k r_+^{n-2} - \frac{2\Lambda r_+^2}{n(n-1)} + \frac{2p q^{2p} (2p-1)^2}{(n-1)(n-2)p r_+^{(n-2p)/(2p-1)}} + \frac{c_0 m^2 r_+^{n-1}}{n-1} \left(c_1 + \frac{(n-1)c_0 c_2}{r_+} + \frac{(n-1)(n-2)c_0^2 c_3}{r_+^2} + \frac{(n-1)(n-2)(n-3)c_0^3 c_4}{r_+^3}\right),$$

where $r_+$ is the radius of the event horizon given by the largest root of $f(r_+) = 0$. According to Eq. (14) and regarding $A_t(r) = \int F_{tr} dr$, the gauge potential $A_t$ can be calculated as

$$A_t(r) = \mu + \frac{q(2p-1)}{r^{(n-2p)/(2p-1)}}.$$

In (20), $\mu$ is the chemical potential of the quantum field theory locates on boundary which can be found by demanding the regularity condition on the horizon i.e. $A_t(r_+) = 0$ as

$$\mu = \frac{q(2p-1)}{(2p-n)r_+^{(2p-2)/(2p-1)}}.$$

One should note that the electric potential $A_t(r)$ has a finite value at infinity ($r \to \infty$) provided the parameter $p$ is restricted as

$$\frac{1}{2} < p < \frac{n}{2},$$

for the regularity condition to hold.
obtained from \((n - 2p)/(2p - 1) > 0\). One can also obtain the electric potential as

\[
U = A_\nu \chi^\nu \big|_{r=r_+ f} - A_\nu \chi^\nu \big|_{r=r_+},
\]

where \(\chi = C\partial_t\) is the null generator of the horizon and \(C\) is a constant. When one applies the power-law Maxwell electrodynamics, it is common to use a general Killing vector with a constant \(C\) [40, 41]. This is due to the fact that every linear combination of Killing vectors is also a Killing vector. Then, \(C\) is fixed so that the first law of thermodynamics is satisfied [40, 41]. For linear Maxwell case \((p = 1)\), the constant \(C\) reduces to 1. Choosing infinity as the reference point, one can calculate the electric potential energy

\[
U = C\mu.
\]

One can obtain the Hawking temperature of the black hole on the event horizon as

\[
T = \frac{f'(r_+)}{4\pi} = \frac{(n - 2)k}{4\pi r_+} - \frac{2\Lambda r_+}{4\pi(n - 1)} + \frac{2p q^2 p(1 - 2p)}{4\pi(n - 1)r_+^{2p(n - 2) + 1}/(2p - 1)} + \frac{c_0 m^2}{4\pi} \left( c_4 + \frac{(n - 2)c_0 c_2}{r_+} + \frac{(n - 2)(n - 3)c_0^2 c_3}{r_+^2} + \frac{(n - 2)(n - 3)(n - 4)c_0^2 c_4}{r_+^3} \right).
\]

The extremal black hole, whose temperature vanishes, can be also determined by an extremal charge,

\[
q_{ext}^{2p} = \frac{(n - 1)(n - 2)c_0^{2p(n - 3) + 1}/(2p - 1)}{2(2p - 1)2p} - \frac{\Lambda c_0^{2p(n - 1)}/(2p - 1)}{(2p - 1)2p^{-1}} + \frac{c_0 m^2 (n - 1)r_{ext}^{2p(n - 2) + 1}/(2p - 1)}{2(2p - 1)2p} \left( c_1 + \frac{(n - 2)c_0 c_2}{r_{ext}} + \frac{(n - 2)(n - 3)c_0^2 c_3}{r_{ext}^2} + \frac{(n - 2)(n - 3)(n - 4)c_0^2 c_4}{r_{ext}^3} \right),
\]

For \(q > q_{ext}\), there is a naked singularity in spacetime while \(q < q_{ext}\) describes solutions with two inner and outer horizons \((r_+ \text{ and } r_-)\). These two horizons degenerate for \(q = q_{ext}\). The behaviors of the metric function \(f(r)\) versus \(r\) for different topologies of horizon are depicted in Fig. 1.

Up to now, we have obtained the higher-dimensional black hole solutions in the context of massive gravity and in the presence of power-law Maxwell gauge field. In the next section, we will study the thermodynamics of the obtained solutions. To do that, we shall obtain the Smarr-type formula and check the satisfaction of the first law of black holes thermodynamics.

III. THERMODYNAMICS OF MASSIVE GRAVITY

The main purpose of this section is to examine the first law of thermodynamics for massive black holes with power-law Maxwell field. It was shown that the entropy of black holes in massive gravity still obeys the area law [6]. It
is easy to show that the entropy of black hole per unit volume $\omega_{n-1}$ as an extensive quantity of thermodynamics is given by [6]

$$S = \frac{r_{+}^{n-1}}{4},$$

(27)

which is a quarter of the event horizon area [6, 42]. The electric charge of black hole per unit volume $\omega_{n-1}$ can be calculated through the use of Gauss law

$$Q = \frac{1}{4\pi} \int r_{-1}^{n-1} (-F) \frac{1}{p-1} F_{\mu \nu} n^\mu u^\nu dr,$$

(28)

where $n^\mu$ and $u^\nu$ are respectively the unit spacelike and timelike normals to the hypersurface of radius $r$ defined by

$$n^\mu = \frac{1}{\sqrt{-g_{\mu \nu}}} dt = \frac{1}{\sqrt{f(r)}} dt, \quad u^\nu = \frac{1}{\sqrt{g_{rr}}} dr = \sqrt{f(r)} dr.$$

(29)

Thus, one can obtain

$$Q = \frac{2^{p-1} q^{2p-1}}{4\pi}.$$

(30)

In order to obtain the mass of black holes in massive gravity one can apply the Hamiltonian approach presented in Ref. [6]. The total mass ($M$) of massive black hole per unit volume $\omega_{n-1}$ can be calculated as [6]

$$M = \frac{(n-1)m_0}{16\pi},$$

(31)

where $m_0$ as a function of the horizon radius $r_+$ was given in Eq. (19). In order to check the first law of thermodynamic, we need to compute Smarr-type formula for mass $M$ as a function of extensive quantities entropy and electric charge. Using relations (27), (30) and (31), one can obtain the Smarr-type formula for mass as

$$M(S, Q) = \frac{k(n-1)(4S)^{(n-2)/(n-1)}}{16\pi} - \frac{\Lambda}{8\pi n} + \frac{Q^{2p/(2p-1)}(2p-1)^2}{2(n-2)p(4S)^{(n-2)/(n-1)}} \left( \frac{\pi}{2^{p-3}} \right)^{1/(2p-1)}$$

$$+ \frac{c_0 m^2 S}{4\pi} \left( c_1 + \frac{(n-1)c_0 c_2}{(4S)^{1/(n-1)}} + \frac{(n-1)(n-2)c_0^2 c_3}{(4S)^{2/(n-1)}} + \frac{(n-1)(n-2)(n-3)c_0^3 c_4}{(4S)^{3/(n-1)}} \right).$$

(32)

Now, one can show that the thermodynamic quantities satisfy the first law of thermodynamic as

$$dM = TdS + UdQ,$$

(33)

in which

$$T = \left( \frac{\partial M}{\partial S} \right)_Q \quad \text{and} \quad U = \left( \frac{\partial M}{\partial Q} \right)_S,$$

(34)

provided $C = p$ in (24). As it is clear, for linear Maxwell case ($p = 1$), the constant $C$ is reduced to 1. In the remainder of this work, we study the effect of power-law Maxwell electrodynamics on the holographic conductivity of dual systems with and without translational symmetry.

**IV. HOLOGRAPHIC CONDUCTIVITY**

In this section, we will obtain the electrical transport behavior of the dual field theory in the presence of a power-law Maxwell gauge field. In order to do this, one should use the solution of the black brane ($k = 0$) found in the pervious section. First, we investigate the effects of the power-law Maxwell electrodynamics on the holographic conductivity of dual systems in which momentum is conserved ($m = 0$). Next, we consider the solutions dual to the systems which no longer possess momentum conservation ($m \neq 0$).
The planer \( (n+1) \)-dimensional metric can be rewritten as

\[
d ds^2 = -\mathcal{F}(u) dt^2 + l^2 \mathcal{F}(u)^{-1} u^{-4} du^2 + l^2 u^{-2} \sum_{i=1}^{n-1} dx_i^2, \tag{35}
\]

which is given by defining \( u = lr^{-1} \) in the metric (11). Accordingly, the event horizon of black brane is at \( u_+ = lr_+^{-1} \) and the \( n \)-dimensional thermal field theory lives at \( u = 0 \). The metric function of spacetime in absence of massive parameter is

\[
\mathcal{F}(u) = -m_0 l^{2-n} u^{n-2} + u^{-2} + 2^n q^{2p} (2p-1)^2 (n-1)^{-1} (n-2p)^{-1} \left[ l^{-1} u \right]^{2(np-3p+1)/(2p-1)}, \tag{36}
\]

obtained by substituting \( r = lu^{-1} \) and \( k = 0 \) in Eq. (18). Perturbing the vector potential component \( A_x \) and the metric component \( g_{tx} \) by turning on \( a_x(u) e^{-i\omega t} \) and \( g_{tx}(u) e^{-i\omega t} \) respectively, we can easily derive two linear equations of motion for electrodynamics

\[
a_x'' + \left[ (8p - n - 3) (2p - 1)^{-1} \right] a_x' + l^2 \omega^2 u^{-4} \mathcal{F}^{-2} a_x + h' \mathcal{F}^{-1} \left( g_{tx}' + 2u^{-1} g_{tx} \right) = 0, \tag{37}
\]

and for gravity

\[
g_{tx}' + 2u^{-1} g_{tx} + 2^{p+1} ph' (u^2 l^{-2} h'^2)^{p-1} a_x = 0, \tag{38}
\]

where now the prime means derivative with respect to \( u \) and \( h(u) \) is electric potential in the form

\[
h(u) = \mu + \frac{q(2p-1) u^{(n-2p)/(2p-1)}}{(n-2p) l^{(n-2p)/(2p-1)}}, \tag{39}
\]

which is obtained by transforming \( r \to lu^{-1} \) in Eq. (20). By eliminating \( g_{tx} \) between Eqs. (37) and (38), the differential equation for \( a_x \) is

\[
a_x'' + \left[ (8p - n - 3) (2p - 1)^{-1} \right] a_x' + a_x \mathcal{F}^{-1} \left( l^2 \omega^2 u^{-4} \mathcal{F}^{-2} - 2^{p+1} ph'^2 (u^2 l^{-2} h'^2)^{p-1} \right) = 0, \tag{40}
\]
The behavior of above relation near the boundary \((u \to 0)\) is
\[
a_x'' + (4p - n - 1)(2p - 1)^{-1}u^{-1}a_x' + \cdots = 0,
\]
which has the following solution
\[
a_x(u) = a_1 + a_2 u^{(n-2p)/(2p-1)} + \cdots,
\]
where \(a_1\) and \(a_2\) are two constant parameters. To calculate the expectation value of current for boundary theory, we can use the following formula [43, 44]
\[
\langle J_x \rangle = \frac{\partial \mathcal{L}}{\partial (\partial_t a_x)} \bigg|_{u=0},
\]
where \(\delta a_x = a_x(u)e^{-i\omega t}\) and \(\mathcal{L}\) was given in Eq. (3). So, it is obvious that the holographic conductivity can be obtained as
\[
\sigma = \frac{\langle J_x \rangle}{E_x} = -\frac{\langle J_x \rangle}{\delta a_x} = \frac{i\langle J_x \rangle}{\omega \delta a_x} = \frac{2p^{-3}p(n-2p)q^{2(p-1)}a_2}{(2p-1)\pi i \omega a_1}.
\]

It is easy to show that the holographic conductivity (44) reduces to \(\sigma = a_2/ (4\pi i \omega a_1)\) for \(n = 3\) and \(p = 1\) [5, 43]. In Figs. 2(a) and 3(a), the behaviors of real and imaginary parts of holographic conductivity for linear Maxwell case \((p = 1)\) are illustrated as a function of \(\omega/T\) and for various values of the charges of black brane \(q\) for \(n = 3\). This figure shows that the real part of conductivity \(\text{Re}[\sigma]\) decreases as \(q\) increases (temperature decreases) for \(\omega \to 0\) (Fig. 2(a)). Our numerical computations show that \(\text{Re}[\sigma]\) diverges at \(\omega = 0\) independent of the value of the charge parameter \(q\). Also, the maximum value of \(\text{Re}[\sigma]\) is greater for greater \(q\)'s. We observe that \(\text{Re}[\sigma]\) tends to a constant for high frequencies independent of the value of the charge parameter. Next, we turn to study imaginary part of the conductivity \(\text{Im}[\sigma]\) plotted in Fig. 3(a). Imaginary part of conductivity includes a minimum for different charges. This minimum is deeper for larger charges (lower temperatures). At \(\omega = 0\), imaginary part of conductivity \(\text{Im}[\sigma]\) diverges (Fig. 3(a)). This fact supports our numerical computation which shows that real part of conductivity blows up at zero frequency, according to Kramers-Kronig relation. For high frequencies, the imaginary part of conductivity vanishes independent of the value of charge. In Figs. 2(b) and 3(b), the behaviors of real and imaginary parts of holographic conductivity for linear Maxwell in terms of frequency for different values of black brane’s charge \(q\) for
n = 4 are depicted. For low frequencies the behavior of holographic conductivity is the same as the case n = 3. However, for high frequencies the behaviors are different. In n = 3 case, the real (imaginary) part of conductivity tend to a constant for high frequencies whereas for n = 4 case it increases (decreases) as \( \omega \) increases.

Now, we intend to study the effect of nonlinearity of the electrodynamics (power parameter \( p \) of the power-law Maxwell field) on holographic conductivity. Figs. 2(c), 2(d), 3(c) and 3(d) show the behavior of \( \text{Re}[\sigma] \) and \( \text{Im}[\sigma] \) as a function of \( \omega/T \) for different values of \( p \) (restricted by \( 1/2 < p < n/2 \)) for \( n = 3 \) and \( 4 \). In the \( \omega \to 0 \) limit, increasing \( p \) leads to the smaller \( \text{Re}[\sigma] \). For high frequencies, \( \text{Re}[\sigma] \) increases (decreases) as linear function of \( \omega/T \) and its slope increases (decreases) as \( p \) decreases (increases) for \( p < (n + 1)/4 \) \( (p > (n + 1)/4) \). For \( p = (n + 1)/4 \), \( \text{Re}[\sigma] \) and \( \text{Im}[\sigma] \) tend to a constant for high frequencies as one can see in Figs. 2(e) and 3(e). Above behaviors show that for high frequencies \( \text{Re}[\sigma] \propto \omega^n \) where \( \alpha \propto n + 1 - 4p \). This result is important from holographic point of view since similar results can be found in experimental observations [45, 46]. In [45], for a \((2+1)\)-dimensional graphene system, it was reported that the value of \( \text{Re}[\sigma] \) tends to a constant for large frequencies. We observed such a behavior in the conformally invariant case, \( p = (n + 1)/4 \). For conductivity of a \((2+1)\)-dimensional single-layer graphene induced by mild oxygen plasma exposure, a positive slope with respect to frequency for high frequencies has been reported in [46]. We observed similar behavior for conductivity in case of \( p < (n + 1)/4 \). For all values of \( p \), we see that \( \text{Im}[\sigma] \) blows up at zero frequency (Figs. 3(c) and 3(d)). For high frequencies, imaginary part of conductivity decreases for low values of \( p \), whereas it flattens for bigger \( p \)'s.

### B. Nonvanishing \( m \)

Now, we intend to demonstrate the influence of power-law Maxwell parameter \( p \) on the holographic conductivity in massive gravity theory. Employing again \( r \to lu^{-1} \) and setting \( k = 0 \) in (17), we obtain

\[
F(u) = -m_0 l^{2-n} u^{n-2} + u^{-2} + 2p q^2 (2p - 1)^2 (n - 1)^{-1} (n - 2 p)^{-1} [u^{-1} u^{2 (np - 3 p + 1) / (2 p - 1)} + (n - 1)^{-1} c_0 m^2 l u^{-1} (c_1 + (n - 1) l^{-1} c_0 c_2 u + (n - 1) (n - 2) l^{-2} c_0 c_3 u) + (n - 1) (n - 2) (n - 3) l^{-3} c_0 c_4 u)] .
\]

(45)

Hereon, we should perturb the gauge field and the metric by turning on \( a_x(u) e^{-i\omega t} \), \( g_{tx}(u) e^{-i\omega t} \) and \( g_{ux}(u) e^{-i\omega t} \). At the linear regime, we have three independent differential equations for gauge field

\[
(Fa_x')' + (8p - n - 3) (2p - 1)^{-1} u^{-1} Fa_x' + l^2 \omega^2 u^{-1} F - a_x + h' (g_{tx}' + 2 u^{-1} g_{tx} + i \omega g_{ux}) = 0 .
\]

(46)
FIG. 5: The behaviors of imaginary parts of conductivity $\sigma$ versus $\omega/T$ for $m = 1$ with $t = r_+ = 1$, $c_0 = 1$, $c_1 = -1$ and $c_2 = 0$.

and for massive gravity

$$g''_{tx} + 2u^{-1}g'_{tx} + i\omega g_{ux} + 2^{p+1}p\hbar' (u^4l^{-2}h'^2)^{p-1} a_x + ic_0l^{-2}\omega^{-1}u^2\Upsilon g_{ux} = 0,$$  \hspace{1cm} (47)

$$g''_{tx} + (5-n)u^{-1}g'_{tx} - 2(n-2)u^{-2}g_{tx} + i\omega g'_{ux} + 2^{p+1}p\hbar' (u^4l^{-2}h'^2)^{p-1} a'_x - i(n-3)\omega u^{-1}g_{ux} + c_0\Xi u^{-2}\Upsilon^{-1} g_{tx} = 0,$$ \hspace{1cm} (48)

in which

$$\Xi = m^2 (c_1l u^{-1} + 2(n-2)c_0c_2 + 3(n-3)(n-2)c_0^2 c_3 l^{-1} u + 4(n-4)(n-3)(n-2)c_0^3 c_4 l^{-2} u^2).$$ \hspace{1cm} (49)

Eliminating $g_{tx}$ between Eqs. (46), (47) and (48), one arrives at the two following second-order differential equations

$$(\mathcal{F} a'_x)' + \left( (8p - n - 3)(2p - 1)^{-1} u^{-1} \mathcal{F} a'_x \right) + \left[ l^2 \omega^2 u^{-4} \mathcal{F}^{-1} - 2^{p+1}p\hbar'^2 (u^4l^{-2}h'^2)^{p-1} \right] a_x - ic_0l^{-2}\omega^{-1}\mathcal{F}^{1/2}u^{-2}g_{ux} = 0,$$ \hspace{1cm} (50)

$$l^{-2}u^2 \left( u^4\Xi^{-1}\mathcal{F} (u^2\Upsilon g_{ux})' \right)' - i2^{p+1} p\hbar c_0^{-1} u^{-2} \left[ -u^4\mathcal{F} a_x \left( h' (u^4l^{-2}h'^2)^{p-1} \right) \right]' + i(n-3)2^{p+1}p\hbar c_0^{-1}u^{-2} \left[ -u^3\mathcal{F} a_x h' (u^4l^{-2}h'^2)^{p-1} \right]' - (n-3)l^{-2}u^{-2}(u^5\mathcal{F}^{1/2} g_{ux})' + \omega^2 g_{ux} - i2^{p+1}p\hbar h' (u^4l^{-2}h'^2)^{p-1} a_x + c_0 u^2 l^{-2}\Upsilon\mathcal{F} g_{ux} = 0.$$ \hspace{1cm} (51)

One can show that the solution of differential equation (50) near boundary ($u \to 0$) is

$$a''_x + (4p - n - 1)(2p - 1)^{-1} u^{-1} a'_x + \cdots = 0,$$ \hspace{1cm} (52)

which is the same as (41) and also the holographic conductivity has the same form as (44). To solve above differential equations numerically, we impose incoming boundary conditions at the horizon

$$a_x(u), \ g_{ux}(u) \propto (u_+ - u)^{-i\omega/4\pi T},$$ \hspace{1cm} (53)
where $T$ is the Hawking temperature.

In Figs. 4 and 5, we depict the holographic conductivity for $(2+1)$- and $(3+1)$-dimensional dual systems including momentum dissipation in the presence of linear Maxwell and nonlinear electrodynamics. Fig. 5 shows that the imaginary part of conductivity near zero frequency does not have diverging behavior in the presence of momentum dissipation. Consequently, according to Kramers-Kronig relation, the real part of conductivity does not diverge at $\omega = 0$ and includes a Drude peak (in contrast with the case of previous subsection with no momentum dissipation where imaginary part of conductivity blows up at zero frequency and accordingly real part diverges there). Also, real part of DC conductivity becomes larger as $q$ ($p$) increases. For high frequencies, the behaviors of real and imaginary parts of conductivity for $n = 3$ and 4 in terms of black brane charge $q$ and nonlinear parameter $p$ are similar to the case of previous subsection with no momentum dissipation.

V. CLOSING REMARKS

A gravity theory called massive gravity [5] was proposed in order to describe a class of strongly interacting quantum field theories with broken translational symmetry via a holographic principle. In this letter, we consider the massive gravity theory when the gauge field is in the form of the power-Maxwell electrodynamics. First, we derive a class of higher dimensional topological black hole solutions of this theory. Then, we calculate the conserved and thermodynamic quantities of the system and check that these quantities satisfy the first law of black holes thermodynamics on the horizon.

The main purpose of this letter is to investigate the electrical transport behavior of the dual field theory in the presence of a power-law Maxwell gauge field for the obtained solutions. In order to clarify the effects of the massive gravity on the holographic conductivity, we have first considered the holographic conductivity of the dual systems in which momentum is conserved ($m = 0$). Then, we have extended our study to the case where translatinal symmetry is broken and consequently the system no longer possess momentum conservation ($m \neq 0$). For both cases, we have plotted the behaviour of the real and imaginary parts of the holographic conductivity in terms of the frequency per temperature ($\omega/T$) for $(2+1)$- and $(3+1)$-dimensional dual systems. In the former case ($m = 0$), we observed that the real part of conductivity $\text{Re}[\sigma]$ for $n = 3$ decreases as $q$ increases (temperature decreases) for $\omega \to 0$. Besides, $\text{Re}[\sigma]$ has a maximum which is greater for greater charges. Also, $\text{Re}[\sigma]$ tends to a constant for high frequencies independent of the value of charge. In addition, the imaginary part of conductivity $\text{Im}[\sigma]$ diverges as $\omega \to 0$. For high frequencies, the imaginary part of conductivity vanishes independent of the value of charge. The low frequencies behavior of holographic conductivity for $n = 4$ is the same as the case of $n = 3$. For high frequencies, in contrast with $n = 3$, the real (imaginary) part of conductivity increases (decreases) as $\text{Re}[\sigma]$ increases for $n = 4$. Next, we explored the effect of the power-law Maxwell field on holographic electrical transport. We observed that increasing $p$ leads to the smaller $\text{Re}[\sigma]$ for $\omega \to 0$ while for high frequencies $\text{Re}[\sigma] \propto \omega^a$ where $a \propto (n + 1 - 4p)$. Similar results for high frequencies can be found in experimental observations on $(2 + 1)$-dimensional graphene systems [45, 46]. This is important from holographic point of view.

In the latter case ($m \neq 0$), we find out that the imaginary part of the DC conductivity, $\text{Im}[\sigma]$, is zero at $\omega = 0$ and becomes larger as $q$ increases (temperature decreases). This is in contrast to the case without momentum dissipation. It also has a maximum value for $\omega \neq 0$ which increases with increasing $q$ (with fixed $p$) or increasing $p$ (with fixed $q$) for $n = 3$. For the real part of the conductivity, $\text{Re}[\sigma]$, we see that in case of $p = 1$ the maximum value (Drude peak) achieves at $\omega = 0$. Again this is in contrast to the former case ($m = 0$) in which the minimum value of $\text{Re}[\sigma]$ occurs for $\omega \to 0$. For different values of the power parameter, $p$, the real and imaginary part of the conductivity has relative minimum and maximum, respectively. Finally, we observed that both real and imaginary parts of the holographic conductivity are similar to the previous case for high frequencies.

In this work, we obtained the conductivity by applying the linear response theory where the electric field is treated as a probe. This may restrict the study from fully explaining the effects of nonlinearity of electrodynamics model. Therefore, it is an interesting issue for future researches to consider the case where the properties of the system are functions of electric field. In such case, nonlinear response happens. Some examples of such studies can be found in literature in Refs. [47–52].

Acknowledgments

AD and AS thank the Research Council of Shiraz University. MKZ would like to thank Shanghai Jiao Tong University for the warm hospitality during his visit. This work has been financially supported by the Research
Institute for Astronomy & Astrophysics of Maragha (RIAAM), Iran.

[1] J. Zaanen, Y. W. Sun, Y. Liu and K. Schalm, "Holographic Duality in Condensed Matter Physics" (Cambridge University Press, United Kingdom, Cambridge 2015).

[2] M. Natsuume, "AdS/CFT Duality User Guide" (Springer Press, Japan, Tokyo 2015).

[3] J. M. Maldacena, "The large-N limit of superconformal field theories and supergravity", Adv. Theor. Math. Phys. 2, 231 (1998) (Int. J. Theor. Phys. 38, 1113 (1999)) [hep-th/9711200];
E. Witten, "Anti de Sitter space and holonomy", Adv. Theor. Math. Phys. 2, 253 (1998) [hep-th/9802150];
S. S. Gubser, I. R. Klebanov and A. M. Polyakov, "Gauge theory correlators from non-critical string theory", Phys. Lett. B 428, 105 (1998) [hep-th/9802109].

[4] A. Karch and A. O'Bannon, "Metallic AdS/CFT", JHEP 0709, 024 (2007) [arXiv:0705.3870];
S. A. Hartnoll, J. Polchinski, E. Silverstein and D. Tong, "Towards strange metallic holography", JHEP 1004, 120 (2010) [arXiv:0912.1061];
T. Faulkner, N. Iqbal, H. Liu, J. McGreevy and D. Vegh, "Strange metal transport realized by gauge/gravity duality", Science 329, 1043 (2010);
T. Faulkner, N. Iqbal, H. Liu, J. McGreevy and D. Vegh, "From Black Holes to Strange Metals", arXiv:1003.1728;
R. Flauger, E. Pajer, and S. Papanikolaou, "A Striped Holographic Superconductor", Phys. Rev. D 83, 064009 (2011) [arXiv:1010.1775];
A. Aperis, P. Kotetes, E. Papantonopoulos, G. Siopsis, P. Skamagoulis, et al., "Holographic Charge Density Waves", Phys. Lett. B 702, 181 (2011) [arXiv:1009.6179];
T. Faulkner, N. Iqbal, H. Liu, J. McGreevy and D. Vegh, "Charge transport by holographic Fermi surfaces", Phys. Rev. D 88, 045016 (2013) [arXiv:1306.6396];
S. A. Hartnoll and D. M. Hofman, "Locally Critical Resistivities from Umklapp Scattering", Phys. Rev. Lett. 108, 241601 (2012) [arXiv:1201.3917];
G. T. Horowitz, J. E. Santos and D. Tong, "Optical Conductivity with Holographic Lattices", JHEP 1207, 168 (2012) [arXiv:1204.0519];
J. A. Hutasoit, G. Siopsis, and J. Therrien, "Conductivity of Strongly Coupled Striped Superconductor", JHEP 1401, 132 (2014) [arXiv:1208.2964];
H. Ooguri and C. S. Park, "Holographic End-Point of Spatially Modulated Phase Transition", Phys. Rev. D 82, 126001 (2010) [arXiv:1007.3737];
S. Kachru, A. Karch, and S. Yaida, "Holographic Lattices, Dimers, and Glasses", Phys. Rev. D 81, 026007 (2010) [arXiv:0909.2630];
S. Kachru, A. Karch, and S. Yaida, "Adventures in Holographic Dimer Models", New J. Phys. 13, 035004 (2011) [arXiv:1009.3288];
Y. Liu, K. Schalm, Y. W. Sun, and J. Zaanen, "Lattice Potentials and Fermions in Holographic non Fermi-Liquids: Hybridizing Local Quantum Criticality", JHEP 1210, 036 (2012) [arXiv:1205.5227];
G. T. Horowitz, J. E. Santos and D. Tong, "Further Evidence for Lattice-Induced Scaling", JHEP 1211, 102 (2012) [arXiv:1209.1098];
N. Iizuka and K. Maeda, "Towards the Lattice Effects on the Holographic Superconductor", JHEP 1211, 117 (2012) [arXiv:1207.2943];
G. T. Horowitz and J. E. Santos, "General Relativity and the Cuprates", JHEP 06, 087 (2013) [arXiv:1302.6586];
A. Donos and S. A. Hartnoll, "Interaction-driven localization in holography", Nature Phys. 9, 649 (2013) [arXiv:1212.2908];
S. A. Hartnoll, P. K. Kovtun, M. Muller and S. Sachdev, "Theory of the Nernst effect near quantum phase transitions in condensed matter and in dyonic black holes", Phys. Rev. B 76, 144502 (2007) [arXiv:0706.3215];
S. A. Hartnoll and C. P. Herzog, "Impure AdS/CFT correspondence", Phys. Rev. D 77, 106009 (2008) [arXiv:0801.1693];
R. J. Anantua, S. A. Hartnoll, V. L. Martin and D. M. Ramirez, "The Pauli exclusion principle at strong coupling: Holographic matter and momentum space", JHEP 1303, 104 (2013) [arXiv:1210.1590];
A. Donos and J. P. Gauntlett, "Holographic Q-lattices", JHEP 1404, 040 (2014) [arXiv:1311.3292];
A. Donos and J. P. Gauntlett, "Novel metals and insulators from holography", JHEP 1406, 007 (2014) [arXiv:1401.5077];
A. Donos and J. P. Gauntlett, "The thermoelectric properties of inhomogeneous holographic lattices", JHEP 1501, 035 (2015) [arXiv:1409.6875].

[5] D. Vegh, "Holography without translational symmetry", arXiv:1301.0537.

[6] R. G. Cai, Y. P. Hu, Q. Y. Pan and Y. L. Zhang, "Thermodynamics of Black Holes in Massive Gravity", Phys. Rev. D 91, 024032 (2015) [arXiv:1409.2369].

[7] S. H. Hendi, R. B. Mann, S. Panahiyan and B. Eslam Panah, "van der Waals like behaviour of topological adS black holes in massive gravity", Phys. Rev. D 95, 021501 (2017).

[8] S. G. Ghosh, L. Tamukjik and P. Wongjun, "A class of black holes in dRGT massive gravity and their thermodynamical properties", Eur. Phys. J. C 76, 119 (2016) [arXiv:1506.07119];
S. H. Hendi, S. Panahiyan, B. Eslam Panah and M. Momennia, "Geometrical thermodynamics of phase transition: charged..."
black holes in massive gravity, arXiv:1506.07262.
S. H. Hendi, S. Panahiyan and B. Eslam Panah, Charged Black Hole Solutions in Gauss-Bonnet-Massive Gravity, JHEP 1601, 120 (2016) [arXiv:1507.06563];
S. H. Hendi, B. Eslam Panah and S. Panahiyan, Einstein-Born-Infeld-Massive Gravity: AdS-Black Hole Solutions and their Thermodynamical properties, JHEP 1511, 157 (2015) [arXiv:1508.01311];
S. H. Hendi, B. Eslam Panah and S. Panahiyan, Thermodynamical Structure of AdS Black Holes in Massive Gravity with Stringy Gauge-Gravity Corrections, Class. Quant. Grav. 33, 235007 (2016) [arXiv:1510.00108];
S. H. Hendi, B. Eslam Panah and S. Panahiyan, Charged Black Holes in Massive Gravity's Rainbow, arXiv:1602.01832;
S. H. Hendi, B. Eslam Panah and S. Panahiyan, Massive charged BTZ black holes in asymptotically (A)dS spacetimes, JHEP 1605, 029 (2016) [arXiv:1604.00370];
S. H. Hendi, G. Q. Li, J. X. Mo, S. Panahiyan and B. Eslam Panah, New perspective for black hole thermodynamics in Gauss-Bonnet-Born-Infeld massive gravity, Eur. Phys. J. C 76 (2016) 571 [arXiv:1608.03148];
K. Meng and J. Li, Black hole solution of Gauss-Bonnet massive gravity coupled to Maxwell and Yang-Mills fields in five dimensions, Europhysics Letters 116, 10005 (2016);
J. Xu, L.-M. Cao, and Y.-P. Hu, P-V criticality in the extended phase space of black holes in massive gravity, Phys. Rev. D 91 (2015) 124033, [arXiv:1506.03578];
D. C. Zou, R. Yue and M. Zhang, Reentrant phase transitions of higher-dimensional AdS black holes in dRGT massive gravity, arXiv:1612.08056;
D. C. Zou, Y. Liu and R. H. Yue, Behavior of Quasinormal Modes and Van der Waals like phase transition of charged AdS black holes in massive gravity, arXiv:1702.08118.

[9] Y. P. Hu, X. M. Wu and H. Zhang, Generalized Vaidya Solutions and Misner-Sharp mass for n-dimensional massive gravity, Phys. Rev. D 95 (2017) 084002, [arXiv:1611.09042];
Y. P. Hu and H. Zhang, Misner-Sharp Mass and the Unified First Law in Massive Gravity, Phys. Rev. D 92, 024006 (2015) [arXiv:1502.00069];
T. Q. Do, Higher dimensional massive bigravity, Phys. Rev. D 94, 044022 (2016) [arXiv:1604.07568];
T. Q. Do, On five-dimensional massive (bi-)gravity, arXiv:1604.07568;
T. Q. Do, Higher dimensional nonlinear massive gravity, Phys. Rev. D 93 (2016) 104003 [arXiv:1602.05672];

[10] R. A. Davison, Momentum relaxation in holographic massive gravity, Phys. Rev. D 88, 086003 (2013) [arXiv:1306.5792];
M. Blake and D. Tong, Universal Resistivity from Holographic Massive Gravity, Phys. Rev. D 88, 106004 (2013) [arXiv:1308.4970];
L. Alberte and A. Khmelitsky, Stability of Massive Gravity Solutions for Holographic Conductivity, Phys. Rev. D 91, 046006 (2015) [arXiv:1411.3027];
L. Alberte, M. Baggioli, A. Khmelitsky and O. Pujolas, Solid Holography and Massive Gravity, JHEP 1602, 114 (2016) [arXiv:1510.09089];
X. X. Zeng, H. Zhang and L. F. Li, Phase transition of holographic entanglement entropy in massive gravity, Phys. Lett. B 756, 170 (2016) [arXiv:1511.00383];
W. J. Pan and Y. C. Huang, Fluid/gravity correspondence for massive gravity, Phys. Rev. D 94, 104029 (2016) [arXiv:1605.02481];
S. H. Hendi, N. Riazi and S. Panahiyan, Holographical aspects of dyonic black holes: Massive gravity generalization, arXiv:1610.01505.

[11] H. B. Zeng and J. P. Wu, Holographic superconductors from the massive gravity, Phys. Rev. D 90, 046001 (2014) [arXiv:1404.5321];
A. Amoretti, A. Braggio, N. Maggiore, N. Magnoli and D. Musso, Analytic DC thermo-electric conductivities in holography with massive gravitons, Phys. Rev. D 91, 025002 (2015) [arXiv:1407.0306];
Y. P. Hu, H. F. Li, H. B. Zeng and H. Q. Zhang, Holographic Josephson Junction from Massive Gravity, Phys. Rev. D 93, 104009 (2016) [arXiv:1512.07035];
Y. P. Hu, X. X. Zeng and H. Q. Zhang, Holographic Thermalization and Generalized Vaidya-AdS Solutions in Massive Gravity, Phys. Lett. B 765, 120 (2017) [arXiv:1611.00677];
Z. Zhou, J. P. Wu and Y. Ling, DC and Hall conductivity in holographic massive Einstein-Maxwell-Dilaton gravity, JHEP 1508, 067 (2015) [arXiv:1504.00535];
R. G. Cai and R. Q. Yang, Insulator/metal phase transition and colossal magnetoresistance in holographic model, Phys. Rev. D 92, 106002 (2015) [arXiv:1507.03105];
R. G. Cai, X. X. Zeng and H. Q. Zhang, Influence of inhomogeneities on holographic mutual information and butterfly effect, arXiv:1704.03899.

[12] M. Baggioli and O. Pujolas, Holographic Polarons, the Metal-Insulator Transition and Massive Gravity, Phys. Rev. Lett. 114, 251602 (2015) [arXiv:1411.1003];
L. Alberte, M. Baggioli, A. Khmelitsky, and O. Pujolas, Solid Holography and Massive Gravity, JHEP 02, 114 (2016) [arXiv:1510.09089];
M. Baggioli and M. Goykhman, Under The Dome: Doped holographic superconductors with broken translational symmetry, JHEP 01, 011 (2016) [arXiv:1510.06363];
M. Baggioli and M. Goykhman, Phases of holographic superconductors with broken translational symmetry, JHEP 07, 035(2015) [arXiv:1504.05561];
W. J. Pan and Y. C. Huang, Holographic complexity and action growth in massive gravities, arXiv:1612.03627.
M. Hassaine and C. Martinez, Higher-dimensional black holes with a conformally invariant Maxwell source, Phys. Rev. D
W. Heisenberg and H. Euler, *Folgerungen aus der Diracschen Theorie des Positrons*, Z. Phys. 98, 714 (1936).

G. W. Gibbons and D. A. Rasheed, *Electric-magnetic duality rotations in nonlinear electrodynamics*, Nucl. Phys. B 454, 185 (1995).

Olivera Miskovic and Rodrigo Olea, *Conserved charges for black holes in Einstein-Gauss-Bonnet gravity coupled to nonlinear electrodynamics in AdS space*, Phys. Rev. D 83, 024011 (2011) [arXiv:1009.5763];

H. P. de Oliveira, *Nonlinear charged black holes*, Class. Quant. Grav. 11, 1469 (1994).

M. Born and L. Infeld, *Foundations of the new field theory*, Proc. R. Soc. A 144, 425 (1934).

H. H. Soleng, *Charged black points in general relativity coupled to the logarithmic U(1) gauge theory*, Phys. Rev. D 52, 6178 (1995) [hep-th/9509035].

S. H. Hendi, *Asymptotic charged BTZ black hole solutions*, JHEP 1203 (2012) 065 [arXiv:1405.4941];

H. A. Gonzalez, M. Hassaine, and C. Martinez, *Thermodynamics of charged black holes with a nonlinear electrodynamics source*, Phys. Rev. D 80, 104008 (2009) [arXiv:0909.1365];

S. H. Hendi and B. Eslam Panah, *Thermodynamics of rotating black branes in Gauss-Bonnet-nonlinear Maxwell gravity*, Phys. Lett. B 684, 77 (2010) [arXiv:1008.0102];

S. H. Hendi, *Rotating black branes in the presence of nonlinear electromagnetic field*, Eur. Phys. J. C 69, 281 (2010) [arXiv:1008.0168];

A. Bazzanash, M. H. Dehghani and M. Ghanaatian, *Surface terms of quartic quasitopological gravity and thermodynamics of nonlinear charged rotating black branes*, Phys. Rev. D 86, 104043 (2012) [arXiv:1209.0246];

M. H. Dehghani, A. Sheykhi, and S. E. Sadati, *Thermodynamics of nonlinear charged Lifshitz black branes with hyperscaling violation*, Phys. Rev. D 91, 124073 (2015) [arXiv:1505.01134];

M. Kord Zangeneh, A. Sheykhi, and M. H. Dehghani, *Thermodynamics of topological nonlinear charged Lifshitz black holes*, Phys. Rev. D 92, 024050 (2015) [arXiv:1506.01784];

M. Kord Zangeneh, A. Sheykhi, and M. H. Dehghani, *Thermodynamics of charged rotating dilaton black branes with power-law Maxwell field*, Eur. Phys. J. C 75, 497 (2015) [arXiv:1506.04077];

M. Kord Zangeneh, A. Sheykhi, and M. H. Dehghani, *Thermodynamics of higher dimensional topological dilaton black holes with a power-law Maxwell field*, Phys. Rev. D 91, 044035 (2015) [arXiv:1505.01103];

M. Kord Zangeneh, M. H. Dehghani and A. Sheykhi, *Thermodynamics of topological black holes in Brans-Dicke gravity with a power-law Maxwell field*, Phys. Rev. D 92, 104035 (2015) [arXiv:1509.05990].

S. H. Hendi and M. H. Vahidinia, *Extended phase space thermodynamics and P-V criticality of black holes with nonlinear source*, Phys. Rev. D 88, 084045 (2013) [arXiv:1212.6128];

M. Hassaine and C. Martinez, *Higher-dimensional charged black hole solutions with a nonlinear electrodynamics source*, Class. Quantum Gravit. 25, 195023 (2008) [arXiv:0803.2946];

H. Maeda, M. Hassaine and C. Martinez, *Lovelock black holes with a nonlinear Maxwell field*, Phys. Rev. D 79, 044012 (2009) [arXiv:0812.2038];

A. Sheykhi, *Higher dimensional charged f(R) black holes*, Phys. Rev. D 86, 024013 (2012) [arXiv:1209.2960];

A. Sheykhi and S. H. Hendi, *Rotating black branes in f(R) gravity coupled to nonlinear Maxwell field*, Phys. Rev. D 87, 084015 (2013);

Z. Dayyani, A. Sheykhi, M. H. Dehghani, *Counterterm method in dilaton gravity and the critical behavior of dilaton black holes with power-Maxwell field*, [arXiv:1611.00590].

J. Jing, Q Pan, S. Chen, *Holographic Superconductors with Power-Maxwell field*, JHEP 1111, 045 (2011) [arXiv:1106.5181];

J. Jing, L. Jiang and Q. Pan, *Holographic superconductors for the Power-Maxwell field with backreactions*, Class. Quantum Grav. 33, 025001 (2016).

D. Roychowdhury, *AdS/CFT superconductors with Power Maxwell electrodynamics: reminiscent of the Meissner effect*, Phys. Lett. B 718, 1089 (2013) [arXiv:1211.1612];

A. Sheykhi, H. R. Salahi, A. Montakhab, *Analytical and Numerical Study of Gauss-Bonnet Holographic Superconductors with Power-Maxwell Field*, JHEP 04, 058 (2016) [arXiv:1603.00075];

H. R. Salahi, A. Sheykhi and A. Montakhab, *Effects of backreaction on power-Maxwell holographic superconductors in Gauss-Bonnet gravity*, Eur. Phys. J. C 76, 575 (2016) [arXiv:1608.05025];

A. Sheykhi, F. Shamsi, S. Davatolhigg, *The upper critical magnetic field of holographic superconductor with conformally invariant power-Maxwell electrodynamics*, Canadian Journal of Physics 95, 450 (2017) [arXiv:1609.05040];

M. Kord Zangeneh, A. Sheykhi, and M. H. Dehghani, *Thermodynamics of higher dimensional topological dilaton black holes with a power-law Maxwell field*, Phys. Rev. D 91, 044035 (2015).

M. Kord Zangeneh, A. Sheykhi, and M. H. Dehghani, *Thermodynamics of topological nonlinear charged Lifshitz black holes*, Phys. Rev. D 92, 024050 (2015).

J. D. Beckenstein, *Black holes and entropy*, Phys. Rev. D 7, 2333 (1973);

S. W. Hawking, *Black hole explosions*, Nature (London) 248, 30 (1974);

G. W. Gibbons and S. W. Hawking, *Action integrals and partition functions in quantum gravity*, Phys. Rev. D 15, 2738 (1977).

S. A. Hartnoll, *Lectures on holographic method for condensed matter physics*, Class. Quant. Grav. 26, 224002 (2009) [arXiv:0903.3246];

D. Tong, *Lectures on holographic conductivity*, talk presented at Cracow school of theoretical Physics, 2013, http://www.damtp.cam.ac.uk/user/tong/talks/zakopane.pdf.

Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer and D. N. Basov, *Dirac charge dynamics*
in graphene by infrared spectroscopy, Nature Physics 4, 532 (2008) [arXiv:0807.3780 [cond-mat.mes-hall]].

[46] I. Santoso, R. S. Singh, P. K. Gogoi, T. C. Asmara, D. Wei, W. Chen, A. T. S. Wee, V. M. Pereira and A. Rusydi, Tunable optical absorption and interactions in graphene via oxygen plasma, Phys. Rev. B 89, 075134 (2014) [arXiv:1370.1358 [cond-mat.mes-hall]].

[47] A. Karch and S. L. Sondhi, Non-linear, Finite Frequency Quantum Critical Transport from AdS/CFT, JHEP 1101, 149 (2011) [arXiv:1008.4134].

[48] J. Sonner and A. G. Green, Hawking Radiation and Non-equilibrium Quantum Critical Current Noise, Phys. Rev. Lett. 109, 091601 (2012) [arXiv:1203.4908].

[49] A. Kundu and S. Kundu, Steady-state Physics, Effective Temperature Dynamics in Holography, Phys. Rev. D 91, 046004 (2015) [arXiv:1307.6607].

[50] G. T. Horowitz, N. Iqbal and J. E. Santos, A Simple Holographic Model of Nonlinear Conductivity, Phys. Rev. D 88, 126002 (2013) [arXiv:1309.5088].

[51] H.-B. Zeng, Y. Tian, Z. Y. Fan and C.-M. Chen, Nonlinear transport in a two dimensional holographic superconductor, Phys. Rev. D 93, 121901(R) (2016) [arXiv:1604.08422].

[52] H.-B. Zeng, Y. Tian, Z. Y. Fan and C.-M. Chen, Nonlinear Conductivity of a Holographic Superconductor Under Constant Electric Field, Phys. Rev. D 95, 046014 (2017) [arXiv:1611.06798].