A Multi-Objective Stochastic Solid Transportation Problem with the Supply, Demand, and Conveyance Capacity Following the Weibull Distribution

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Abstract: This study addresses a multi-objective stochastic solid transportation problem (MOSSTP) with uncertainties in supply, demand, and conveyance capacity, following the Weibull distribution. This study aims to minimize multiple transportation costs in a solid transportation problem (STP) under probabilistic inequality constraints. The MOSSTP is expressed as a chance-constrained programming problem, and the probabilistic constraints are incorporated to ensure that the supply, demand, and conveyance capacity are satisfied with specified probabilities. The global criterion method and fuzzy goal programming approach have been used to solve multi-objective optimization problems. Computational results demonstrate the effectiveness of the proposed models and methodology for the MOSSTP under uncertainty. A sensitivity analysis is conducted to understand the sensitivity of parameters in the proposed model.

Keywords: multi-objective stochastic solid transportation problem; Weibull distribution; chance-constrained programming; global criterion method; fuzzy goal programming

1. Introduction

There have recently been an increasing number of natural or human-made disasters, including earthquakes, tsunamis, floods, typhoons, chemical explosions, and nuclear disasters. Those disasters bring massive human casualties and enormous economic losses. The disasters themselves are unpredictable, leading to high uncertainty as to the location and scale of damages. During or after the disaster impact, the relief goods pre-located at relief warehouses must be distributed quickly to the people in need to save their lives or to help them recover from damages [1]. This critical task in disaster management is called the relief distribution problem, where various uncertainties exist in demand points, demand, and road networks. The first disaster often causes the second disaster as aftershocks following the first earthquake impact. There are relief redistribution problems in these cases, which makes the amount of supplies uncertain [2]. In relief distribution and redistribution problems, multiple objectives [3] are required to be achieved because of timeliness, fairness, and costs. These relief distribution and redistribution problems offer the motivation to propose MOSSTPs with uncertainties in supply, demand, and conveyance capacity.

Another typical example can be found in temperature-controlled cold chain management. Fresh produce and fruits are highly perishable, and their shelf life is limited in time horizon. Unlike other products, the quality of the goods decreases continuously during the downstream activities in the supply chain. Dealing with food supply chain networks is complex, regardless of the type of products in such networks. One goal in distributing perishable products is to maintain their freshness while delivering them to demand destinations from supply sources. Because the value of such products decreases over time, the distribution network must focus on minimization of the transportation time and transportation cost.
A transportation problem (TP) is a well-known constrained linear programming problem, where the objective is to minimize the cost of distributing goods from a set of sources to a set of destinations. The solid transportation problem (STP) is an extension of the TP and includes a set of additional constraints for conveyance capacity [4]. Shell [5] initially considered the situations where the STP would arise and studied four cases of STP. The STP requires goods transported from supply sources to demand destinations through various transportation modes to minimize transportation costs. The STP has recently received significant attention, with various models and solution methods applied to deterministic and stochastic environments.

Supply at sources may be uncertain since any troubles or delays may occur for various reasons. Demand uncertainty cannot be avoided often due to inexact forecasting or demand volatility. The dynamic nature of the distribution network makes the conveyance capacities of transportation networks uncertain as well. In other words, the data about transportation systems, such as sources, demands, and conveyance capacities, are not always deterministic; indeed, they are inherently uncertain in nature because of insufficient information, lack of evidence, fluctuating financial markets, and so on. In such situations, the problem of scheduling shipments from a set of sources to a set of destinations is described as the stochastic STP with known distribution functions.

2. Literature Review

This section presents the literature review of TP and STP studied in stochastic environment as well as some other uncertain environments such as rough interval, uncertain measure, type-1 and type-2 fuzzy, etc. William [6] used a joint cumulative distribution function to define a stochastic variable for the demand and then studied a stochastic TP with a single source and a single destination. Ojha et al. [7] presented a stochastic discounted multi-objective STP with the demand as a stochastic variable and applied the expected value criterion to convert stochastic variables to deterministic ones. However, in practical applications, not only is the demand stochastic, but the supply and the conveyance capacity also are stochastic.

Mahapatra et al. [8] presented a multi-choice stochastic TP involving extreme value distributions using probabilistic constraints. The Weibull and extreme value distributions are closely related because the natural logarithm of the Weibull distribution yields the extreme value distribution. Holmberg and Tuy [9] proposed a branch-and-bound solution method to the TP with stochastic demand and concave production costs. Maity et al. [10] explained the importance of the multi-objective transportation problem and presented a multi-objective transportation problem to minimize the transportation cost and time.

In this study, we express the supply, demand, and conveyance capacity as stochastic variables. A TP becomes a chance-constrained programming problem if its linear constraints are probabilistic. Charnes and Copper [11,12] developed such a TP, and Kataoka [13] proposed a stochastic programming model for a single-objective TP. Chalam [14] developed a fuzzy goal programming technique for stochastic TPs under a budgetary constraint. Bhattacharya [15] presented a multi-objective TP with various costs, in which a random variable followed a normal distribution, and solved it using a fuzzy programming technique. Sreenivas et al. [16] performed a probabilistic study of TPs, and Mahapatra et al. [17] presented a multi-objective stochastic TP involving the log-normal distribution. Roy and Mahapatra [18] discussed a multi-objective interval-valued TP with probabilistic constraints involving the log-normal distribution. Sheng and Yao [19] presented a TP with uncertain costs and demands. Yang and Feng [20] studied a bi-criteria STP with a fixed-charge cost under a stochastic environment. Gessesse et al. [21] studied a multi-objective linear fractional transportation problem in a stochastic environment.

Most research has used a normal distribution to define the variables when modeling a stochastic TP or STP [22]. However, other distributions have also been used to define stochastic variables for TP or STP models (e.g., [23–25]). In addition, the fuzzy theory has been introduced to describe uncertainty. Recently, Das et al. [26] developed a TP
Das et al. ([27,28]) described type-2 fuzzy variables with two membership functions, such as the Gaussian and trapezoidal functions. Sinha et al. [29] presented a profit-maximizing STP with a trapezoidal interval type-2 fuzzy variable. The rough interval approach has been used to solve an uncertain profit-maximizing STP by Das et al. [30]. They described a multi-objective STP with probabilistic constraints and defined the stochastic variables of the probabilistic constraints using the Weibull distribution. Gao and Lee [2] studied a scenario-based multi-objective redistribution problem as a stochastic mixed-integer problem, where the supply and demand at relief centers are uncertain, and the transportation network availability is uncertain in disastrous situations. Agrawal and Ganesh [31] studied and presented a solution process based on Newton’s divided difference interpolation for a multi-choice transportation problem in a stochastic environment. Gupta et al. [32] considered a capacitated stochastic transportation problem in which the chance-constrained programming technique and maximum likelihood estimation are implemented to handle the uncertain parameters associated with supply and demand constraints.

Some works have studied multi-objective TP and STPs with the demand as probabilistic constraints, but multi-objective stochastic STPs with the supply, demand, and conveyance capacities as probabilistic constraints simultaneously, have not been investigated yet. In addition, the Weibull distribution function has not been used to define the stochastic variable in stochastic STP models. Since the Weibull distribution has flexibility in shape, scale, and location, it can model a wide range of failure rates. This study uses the Weibull distribution to define the stochastic variables in the multi-objective stochastic solid transportation problem (MOSSTP). A sensitivity analysis has been conducted in this study to understand the sensitivity of parameters in supply, demand, and conveyance capacity.

Table 1 presents a comparative analysis between the solution methods of some existing transportation problems in the stochastic environment and the method used to solve the STP in this paper. Although all of these existing works set their benchmark in the research society, some challenges exist in those solution methods. The solution method in this paper proposes a simple calculation-based conversion of probabilistic constraints to its equivalent deterministic form and then uses multi-objective compromise programming methods.

Table 1. A comparative analysis of the methodology with the existing methods.

| References          | Type of Problems | Stochastic Parameters | Solution Procedure                          | Remarks                                                                 |
|---------------------|------------------|-----------------------|---------------------------------------------|------------------------------------------------------------------------|
| William [6]         | TP               | Demand                | Linear programming approach                 | Provides some optimal solutions, but not unique, according to the author.|
| Ojha et al. [7]     | STP              | Demand                | Expected value criterion                    | The probabilities used are usually very subjective in this method.     |
| Mahapatra et al. [8]| TP               | Demand & supply       | Extreme value distribution conversion method| Extreme value distribution is a particular form of Weibull distribution  |
| Holmberg and Tuy [9]| TP               | Demand                | Branch and bound method                     | Extremely time-consuming: the number of nodes in a branching tree can be too large |
| Chalam [14]         | TP               | Demand                | Fuzzy goal programming & simplex method     | The related calculation is time-consuming.                              |
| Mahapatra et al. [17]| TP             | Demand & supply       | Chance constrained programming              | This method has its challenges like structural, convexity, and stability challenges. |
Table 1. Cont.

| References          | Type of Problems | Stochastic Parameters                        | Solution Procedure                              | Remarks                                                                 |
|---------------------|------------------|----------------------------------------------|-------------------------------------------------|------------------------------------------------------------------------|
| Roy and Mahapatra   | TP               | Demand & supply                              | Interval order relation & chance constrained    | The preference for order relation decision of intervals is usually very |
| [18]                |                  |                                              | programming                                     | subjective.                                                            |
| Yang and Feng       | STP              | Demand, supply, & conveyance capacity        | Expected value goal programming, chance-         | The expected value goal programming requires decision-makers preference, |
| [20]                |                  |                                              | constrained goal programming, dependent-chance  | same for the dependent chance goal programming                        |
|                     |                  |                                              | goal programming                                |                                                                        |
| This paper          | STP              | Demand, supply, & conveyance capacity        | Weibull distribution, global criterion method,  | The probabilistic constraints are defined with the Weibull distribution |
|                     |                  |                                              | fuzzy goal programming                          | and converted through simple mathematical calculation                |

The remainder of this article is organized as follows. The mathematical programming models for MOSSTP and its variants are studied and converted to the equivalent deterministic models in Section 3. The proposed solution methodology is presented in Section 4. The computational experiences are presented and analyzed in Section 5. Finally, Section 6 summarizes the conclusions of this study and identifies future directions.

3. Materials and Methods

The use of the Weibull distribution to describe real-life phenomena has a long history. A Swedish physicist, Waloddi Weibull [33], proposed it to model the stress distribution to break specimens. Since then, it has been adopted widely in many applications. For a comprehensive review of these applications, we refer readers to Johnson et al. [34] and Murthy et al. [35].

The probability density function (pdf) and cumulative distribution function (cdf) of random variable \( t \) following a three-parameter Weibull distribution are given, respectively, as

\[
f(t) = \frac{\beta}{\eta} \left( \frac{t - \gamma}{\eta} \right)^{\beta-1} e^{-\left(\frac{t - \gamma}{\eta}\right)^\beta},
\]

and

\[
F(t) = 1 - e^{-\left(\frac{t - \gamma}{\eta}\right)^\beta},
\]

where \( f(t) \geq 0, \ t \geq 0 \) or \( \gamma, \beta > 0, \eta > 0, \ -\infty < \gamma < \infty \). Note that \( \beta, \eta, \) and \( \gamma \) are the shape parameter (also known as the Weibull slope), the scale parameter, and the location parameter, respectively.

The Weibull distribution consists of the failure rate function, which defines the frequency at which an engineered system or component fails. The Weibull distribution is often suitable when the conditions for the strict randomness of the exponential distribution are not satisfied, where the shape parameter \( \beta \) depends on the fundamental nature of the problem. The location parameter \( \gamma \) shifts sample median and mode, and the value of scale parameter \( \eta \). However, it does not change \( \beta \) or the shape of the distribution. More importantly, it does not change the goodness of fit of data to the distribution function despite the commonly accepted idea that this additional parameter should improve the fit. This property is helpful to define the probabilistic constraints in the proposed mathematical model since the constraints are estimated using previously and partially known information on uncertain variables. For these reasons, the Weibull distribution has been adopted to describe the uncertainty in this study.
To convert the constraints in the proposed stochastic programming model to the deterministic ones, it is essential to obtain the quantiles of a probability distribution function in a closed form. The fact that the Weibull distribution has the closed form of the quantile is another reason to justify its use.

In this study, the multi-objective chance-constrained programming model for MOSSTP is defined as follows:

MOSSTP:

\[
\text{Min} Z_q = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk}, \quad q = 1, 2, \ldots, Q,
\]

subject to,

\[
P\left(\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq a_i\right) \geq p_{a_i}, \quad i = 1, 2, \ldots, m, \quad (4)
\]

\[
P\left(\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq b_j\right) \geq p_{b_j}, \quad j = 1, 2, \ldots, n, \quad (5)
\]

\[
P\left(\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq c_k\right) \geq p_{c_k}, \quad k = 1, 2, \ldots, l, \quad (6)
\]

\[x_{ijk} \geq 0, \quad \forall i, j \text{ and } k, \quad (7)\]

where probabilities \( p_{a_i}, p_{b_j}, \) and \( p_{c_k} \) are given. It is assumed that random variables \( a_i, b_j, \) and \( c_k, \) which correspond to the supply, demand, and conveyance capacity, follow the Weibull distribution, respectively. The Weibull distribution for \( a_i \) has three parameters, \( \beta_{a_i}, \eta_{a_i}, \) and \( \gamma_{a_i} \) as shape, scale, and location parameters. Similarly, the parameters for the Weibull distribution for \( b_j, \) and \( c_k, \) are also defined. Constraint (4) is the probabilistic constraint for the amount of supply at supply source \( i, \) which ensures that with a given probability \( p_{a_i}, \) the sum of amounts of shipments from supply source \( i \) cannot be more than \( a_i. \) In a similar manner, constraints (5) and (6) can be interpreted for the demand at demand destination \( j \) and the amount of conveyance capacity of transportation mode \( k. \)

Three cases, in which a single random variable among \( a_i, b_j, \) and \( c_k, \) is uncertain, are studied in Case I, II, and III. A general case where all random variables are uncertain is presented in Case IV.

3.1. Case (I) Only \( a_i (i = 1, 2, \ldots, m) \) Is Uncertain

It is assumed that \( a_i (i = 1, 2, \ldots, m) \) are independent random variables following the Weibull distribution with three known parameters \( \beta_{a_i}, \eta_{a_i}, \) and \( \gamma_{a_i}. \) Equation (4) can then be rearranged as

\[
P\left(a_i \geq \sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk}\right) \geq p_{a_i}, \quad i = 1, 2, \ldots, m. \quad (8)
\]

Assuming

\[\xi_{a_i} = \sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk}, \quad \forall j, k \text{ and } i = 1, 2, \ldots, m,\]

Equation (8) can be further expressed as \( P(a_i \geq \xi_{a_i}) \geq p_{a_i}, i = 1, 2, \ldots, m, \) which can then be written as

\[
\int_{\xi_{a_i}}^{\infty} \frac{\beta_{a_i}}{\eta_{a_i}} \left(\frac{a_i - \gamma_{a_i}}{\eta_{a_i}}\right)^{\beta_{a_i}-1} \exp\left(-\left(\frac{a_i - \gamma_{a_i}}{\eta_{a_i}}\right)^{\beta_{a_i}}\right)da_i \geq p_{a_i}, \quad i = 1, 2, \ldots, m. \quad (9)
\]
After integrating Equation (9), we obtain

\[
\exp \left\{ - \left( \frac{\xi_{a_i} - \gamma_{a_i}}{\eta_{a_i}} \right)^{\beta_{a_i}} \right\} \geq p_{a_i}, \quad i = 1, 2, \ldots, m. \tag{10}
\]

By applying the logarithm to Equation (10), we can obtain

\[
\xi_{a_i} \leq \gamma_{a_i} + \eta_{a_i} \left\{ - \ln(p_{a_i}) \right\}^{\frac{1}{\beta_{a_i}}}, \quad i = 1, 2, \ldots, m.
\]

Finally, this can be converted to a deterministic constraint in the form using the quantile of the Weibull distribution,

\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq \gamma_{a_i} + \eta_{a_i} \left\{ - \ln(p_{a_i}) \right\}^{\frac{1}{\beta_{a_i}}}, \quad i = 1, 2, \ldots, m. \tag{11}
\]

Hence, the multi-objective deterministic STP can be converted to the stochastic one when the supply is uncertain.

MOSSTP-a:

\[
\text{Min} \ z^q = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk}, \quad q = 1, 2, \ldots, Q,
\]

subject to,

\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq \gamma_{a_i} + \eta_{a_i} \left\{ - \ln(p_{a_i}) \right\}^{\frac{1}{\beta_{a_i}}}, \quad i = 1, 2, \ldots, m, \quad j = 1, 2, \ldots, n,
\]

\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq b_j, \quad j = 1, 2, \ldots, n,
\]

\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \leq e_k, \quad k = 1, 2, \ldots, l,
\]

\[
x_{ijk} \geq 0, \quad \forall \ i, j \text{ and } k.
\]

3.2. Case (II) Only \(b_j (j = 1, 2, \ldots, n)\) Is Uncertain

It is assumed that \(b_j (j = 1, 2, \ldots, n)\) are independent random variables following the Weibull distribution with three known parameters \(\beta_{b_j}, \eta_{b_j}\) and \(\gamma_{b_j}\). Equation (5) can then be rearranged as

\[
P \left( b_j \leq \sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \right) \geq p_{b_j}, \quad j = 1, 2, \ldots, n. \tag{12}
\]

Let us suppose that

\[
\xi_{b_j} = \sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \quad \forall \ i, k \text{ and } j = 1, 2, \ldots, n.
\]

In this case, Equation (12) can be further expressed as \(P (b_j \leq \xi_{b_j}) \geq p_{b_j}, \quad j = 1, 2, \ldots, n\), which can be rewritten as

\[
\int_{-\infty}^{\xi_{b_j}} \frac{\beta_{b_j}}{\eta_{b_j}} \left( \frac{b_j - \gamma_{b_j}}{\eta_{b_j}} \right)^{\beta_{b_j}-1} \exp \left\{ - \left( \frac{b_j - \gamma_{b_j}}{\eta_{b_j}} \right)^{\beta_{b_j}} \right\} \, db_j \geq p_{b_j}, \quad j = 1, 2, \ldots, n \tag{13}
\]
Concurrently, \( b_j \geq \gamma_{b_j}, j = 1, 2, \ldots, n \), and so the integration of the above expression yields the form

\[
\int_{\gamma_{b_j}}^{\xi_{b_j}} \left( \frac{b_j - \gamma_{b_j}}{\eta_{b_j}} \right)^{\beta_{b_j} - 1} \exp \left\{ - \left( \frac{b_j - \gamma_{b_j}}{\eta_{b_j}} \right)^{\beta_{b_j}} \right\} db_j \geq p_{b_j}, \quad j = 1, 2, \ldots, n. \tag{14}
\]

Integrating Equation (14) gives

\[
1 - \exp \left\{ - \left( \frac{\xi_{b_j} - \gamma_{b_j}}{\eta_{b_j}} \right)^{\beta_{b_j}} \right\} \geq p_{b_j}, \quad j = 1, 2, \ldots, n. \tag{15}
\]

Equation (15) can be further simplified as

\[
\xi_{b_j} \geq \gamma_{b_j} + \eta_{b_j} \left\{ - \ln \left( 1 - p_{b_j} \right) \right\}^{\frac{1}{\beta_{b_j}}}, \text{ by applying the logarithm. }
\]

Finally, this can be expressed as a deterministic constraint in the form

\[
\sum_{i=1}^{m} \sum_{k=1}^{n} x_{ijk} \geq \gamma_{b_j} + \eta_{b_j} \left\{ - \ln \left( 1 - p_{b_j} \right) \right\}^{\frac{1}{\beta_{b_j}}}, \quad j = 1, 2, \ldots, n. \tag{16}
\]

Hence, the multi-objective stochastic STP is given when the demand is uncertain, as follows.

MOSSIP-b:

\[
\text{Min} z^Q = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk}, \quad q = 1, 2, \ldots, Q,
\]

subject to,

\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq a_i, \quad i = 1, 2, \ldots, m,
\]

\[
\sum_{i=1}^{m} \sum_{k=1}^{n} x_{ijk} \geq \gamma_{b_j} + \eta_{b_j} \left\{ - \ln \left( 1 - p_{b_j} \right) \right\}^{\frac{1}{\beta_{b_j}}}, \quad j = 1, 2, \ldots, n,
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq e_k, \quad k = 1, 2, \ldots, l,
\]

\[
x_{ijk} \geq 0, \quad \forall i, j \text{ and } k.
\]

3.3. Case (III) Only \( e_k (k = 1, 2, \ldots, l) \) Is Uncertain

It is assumed that \( e_k (k = 1, 2, \ldots, l) \) are independent random variables following the Weibull distribution with three known parameters \( \beta_{e_k}, \eta_{e_k} \text{ and } \gamma_{e_k} \). Equation (6) can then be rearranged as

\[
P \left( e_k \geq \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \right) \geq p_{e_k}, \quad k = 1, 2, \ldots, l. \tag{17}
\]

Assuming

\[
\xi_{e_k} = \sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk}, \quad i, j \text{ and } k = 1, 2, \ldots, l,
\]

Equation (17) can be further expressed as \( P(e_k \geq \xi_{e_k}) \geq p_{e_k}, k = 1, 2, \ldots, l \), which can be converted to

\[
\int_{\xi_{e_k}}^{\infty} \left( \frac{e_k - \gamma_{e_k}}{\eta_{e_k}} \right)^{\beta_{e_k} - 1} \exp \left\{ - \left( \frac{e_k - \gamma_{e_k}}{\eta_{e_k}} \right)^{\beta_{e_k}} \right\} de_k \geq p_{e_k}, \quad k = 1, 2, \ldots, l. \tag{18}
\]
Integration Equation (18) yields
\[
\exp \left\{ - \left( \frac{\xi_{e_k} - \gamma_{e_k}}{\eta_{e_k}} \right) \beta_{e_k} \right\} \geq p_{e_k}, \ k = 1, 2, \ldots, l. \tag{19}
\]

Applying the logarithm to Equation (19) gives \( \xi_{e_k} \leq \gamma_{e_k} + \eta_{e_k} \left\{ - \ln(p_{e_k}) \right\}^{\frac{1}{\beta_{e_k}}} \). Finally, this can be expressed as a deterministic constraint in the form
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \gamma_{e_k} + \eta_{e_k} \left\{ - \ln(p_{e_k}) \right\}^{\frac{1}{\beta_{e_k}}}, \ k = 1, 2, \ldots, l. \tag{20}
\]

Hence, the MOSSTP can be formulated when the conveyance capacity is uncertain, as follows.

MOSSTP-e:
\[
\text{Min} \ z_q = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk}, \ q = 1, 2, \ldots, Q,
\]
subject to,
\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq a_i, \ i = 1, 2, \ldots, m,
\]
\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq b_j, \ j = 1, 2, \ldots, n,
\]
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \gamma_{e_k} + \eta_{e_k} \left\{ - \ln(p_{e_k}) \right\}^{\frac{1}{\beta_{e_k}}}, \ k = 1, 2, \ldots, l,
\]
\[
x_{ijk} \geq 0, \ \forall \ i, \ j \text{ and } k
\]

3.4. Case (IV) \( a_i, b_j, \) and \( e_k \) Are Uncertain

It is assumed that \( a_i (i = 1, 2, \ldots, m) \), \( b_j (j = 1, 2, \ldots, n) \), and \( e_k (k = 1, 2, \ldots, l) \) are independent random variables following the Weibull distribution. Combining the derivations, Case I, II, and II, the following deterministic model for the MOSSTP is obtained in the following.

MOSSTP-deterministic
\[
\text{Min} \ z_q = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} c_{ijk} x_{ijk}, \ q = 1, 2, \ldots, Q,
\]
subject to,
\[
\sum_{j=1}^{n} \sum_{k=1}^{l} x_{ijk} \leq a_i, \ i = 1, 2, \ldots, m, \tag{21}
\]
\[
\sum_{i=1}^{m} \sum_{k=1}^{l} x_{ijk} \geq b_j, \ j = 1, 2, \ldots, n, \tag{22}
\]
\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \leq \gamma_{e_k} + \eta_{e_k} \left\{ - \ln(p_{e_k}) \right\}^{\frac{1}{\beta_{e_k}}}, \ k = 1, 2, \ldots, l, \tag{23}
\]
\[
x_{ijk} \geq 0, \ \forall \ i, \ j \text{ and } k
\]

This section presented the deterministic mathematical programming models for MOSSTP, using the quantile of the Weibull distributions. In a real-life situation, it is possible that only some of the supply, demand, and conveyance capacity might be uncertain, and the others are certain. Therefore, MOSSTP-deterministic can be modified as
necessary, depending on the situation. This deterministic mathematical programming can be solved for optimality using the commercially available solvers.

4. Multi-Objective Optimization Solution Methods

The study in Section 3 was focused on the conversion of probabilistic constraints to deterministic ones and proposed the deterministic model for MOSSTP with other variants. In this section, two solution methods to handle the multiple objective optimizations are adopted to solve the deterministic model for MOSSTP.

There has been much research on multi-objective optimization problems. There may not be a single dominating solution for those problems that simultaneously optimizes all objective functions because the multi-objective functions may conflict with each other. For example, transportation times can be shortened by spending more on transportation costs. The transportation cost and times can be multiple objective functions in our MOSSTP. There have been several solution methods proposed by which one can obtain compromised solutions for multi-objective models. This study uses two solution methods, the global criterion method (GCM) and the fuzzy goal programming method (FGM), to produce solutions for the deterministic MOSSIP for further discussion.

4.1. Global Criterion Method

Most solution methods for multi-objective optimization problems use weighting factors [36]. To decide on the weighting factors for each objective, a clear indication is necessary about the favoritism done to a particular objective. For example, some decision-makers may be more inclined to minimize the total transportation cost than the total transportation time. Others may give minimization of total transportation time a higher priority than the total transportation cost. The task becomes difficult when no clear directive is given about the priority or all the components have equal weightage. The GCM is applied where no articulation of preference information is given [37]. It proves to be a straightforward yet fruitful choice, as it provides equal priority for all the objective functions.

The GCM measures the distance by using Minkowski’s $L_p$ metric, and it minimizes a global criterion function which is a measure of how close the decision-maker can approach the ideal solution.

The GCM generates the compromised optimal solutions to minimize some global criteria such as the square sum of the relative deviation of the criteria (objective functions) from individual objective function values or to minimize the relative distance between the individual optimal objective function values and the compromised optimal solutions. The aggregate objective function also helps to scale all the individual objective functions into a single criterion. The GCM for the multi-objective minimization model in this paper produces an aggregated objective function for global criteria as shown in Equation (24) [38].

The aggregated objective function is as follows:

$$
G = \left( \sum_{q=1}^{Q} \left[ \frac{Z_q(x) - Z_q(x^*)}{Z_q(x^*)} \right]^s \right)^{\frac{1}{s}},
$$

(24)

where $Z_q(x^*)$ is the value of objective function $q$ at its individual optimum $x^*$, $Z_q(x)$ is the objective function $q$, $s$ (1 $\leq$ $s$ $\leq$ $\infty$) is an integer-exponent that serves to reflect the importance of each objective. Setting $s = 1$ implies that equal importance is given to all deviations (Boychuk and Ovchinnikov [39]), while $s = 2$ implies that these deviations are weighted proportionately, with the largest deviations having the largest weight [40]. Setting $s > 2$ means that more weight is given to the largest of deviations.

The proposed MOSSTP model in this research is a multi-objective minimization model, and we solve the model using the above method; the MOSSTP involving the Weibull distribution can be transformed into its equivalent deterministic form as follows.
where \( q \) is the (unity) by minimizing under-deviations is taken into account in a solution search process. and constraint (21)–(23), mathematical models for the MOSSTP in this study using the FGP approach are as follows:

\[
\text{Minimize}\left\{ \sum_{q=1}^{Q} \left( \frac{Z_q(x) - Z_q(x^*)}{Z_q(x^*)} \right)^{\frac{1}{2}} \right\}, \quad q = 1, 2, \ldots, Q,
\]

subject to,

\[
z^q(x) = \sum_{i=0}^{m} \sum_{j=0}^{n} \sum_{k=0}^{l} c_{ijk} x_{ijk},
\]

and constraint (21)–(23),

\[x_{ijk} \geq 0, \quad \forall i, j \text{ and } k,
\]

where \( Q \) is the number of objective functions. The commercially available solvers can solve this problem.

4.2. Fuzzy Goal Programming Approach

Several fuzzy approaches have been proposed to solve the multi-objective transportation problem. Zangiabadi and Maleki [41] presented a fuzzy goal programming (FGP) approach to produce an optimal compromised solution for multi-objective transportation problems. Assuming that each objective function has a fuzzy goal, they assigned a nonlinear (hyperbolic) membership function to each objective function in order to solve decision problems with the multiplicity of objectives in an imprecise environment. In this approach, instead of measuring the achievement of fuzzy objective values directly, achieving membership values of objectives to the possible extent of the highest degree (unity) by minimizing under-deviations is taken into account in a solution search process.

The key idea behind the FGP is to minimize the distance between the individual objective values \( Z^q(x), \ (q = 1, 2, \ldots, Q) \) and the target (or aspiration) level of the objective function \( Z^q(x), \ (q = 1, 2, \ldots, Q), \) determined by the decision-maker.

The positive and negative deviational variables are introduced as follows.

\[
d_q^+ = \max\left(0, Z^q - \bar{Z}^q\right) = \frac{1}{2} \left\{ \left(Z^q - \bar{Z}^q\right) + \left|Z^q - \bar{Z}^q\right| \right\}, \quad q = 1, 2, \ldots, Q \text{ and}
\]

\[
d_q^- = \max\left(0, Z^q - \underline{Z}^q\right) = \frac{1}{2} \left\{ \left(Z^q - \underline{Z}^q\right) + \left|Z^q - \underline{Z}^q\right| \right\}, \quad q = 1, 2, \ldots, Q.
\]

The minimum distance between \( Z^q(x) \) and \( \bar{Z}^q(x) \) is achieved when \( d_q^+ \) is minimized for \( Z^q(x) \leq \bar{Z}^q(x) \) in the minimization problem. For details about the FGP approach adopted in this study, readers are referred to Zangiabadi and Maleki [40].

Mohamed in [42] used linear membership functions, but Zangiabadi and Maleki [40] introduced hyperbolic membership functions to minimize the deviational variables.

Let \( L_q \) and \( U_q \) be the aspired level of achievement and the highest acceptable level of achievement for the \( q \)-th objective function, respectively. Steps to solve the proposed mathematical models for the MOSSTP in this study using the FGP approach are as follows:

Step 1. Solve the MOSSTP as a single objective STP, taking only one objective as the objective function at a time and ignoring all others. That is, \( X_1, X_2, \ldots, X_q \).
Step 2. Evaluate the value of each objective function at each solution derived in Step 1. We have at most \( q^2 \) values. That is, \( Z_1(X_1), Z_1(X_2), Z_1(X_3), \ldots, Z_1(X_q), Z_2(X_1), Z_2(X_2), Z_2(X_3), \ldots, Z_2(X_q), Z_3(X_1), Z_3(X_2), Z_3(X_3), \ldots, Z_3(X_q), Z_q(X_1), Z_q(X_2), Z_q(X_3), \ldots, Z_q(X_q) \).
Step 3. From Step 2, find the best \( (L_q, q = 1, 2, \ldots, Q) \) and the worst \( (U_q, q = 1, 2, \ldots, Q) \) values for each objective function, corresponding to the set of solutions. That is \( L_q = \min_{i} Z_{iq}(X_i) \) and \( U_q = \max_{i} Z_{iq}(X_i) \) in the minimization problem.
Step 4. Applying the min-max form of goal programming with the hyperbolic membership function, the following model is built. Solve this nonlinear optimization model.

\[ \text{Minimize } \Phi \]

subject to,

\[ \frac{1}{2} \left( e^{\left(\frac{U_q + L_q - z_q}{2}\right)\alpha_q} - e^{-\left(\frac{U_q + L_q - z_q}{2}\right)\alpha_q} \right) + d_q^+ - d_q^- = 1, \quad q = 1, 2, \ldots, Q, \quad (25) \]

\[ \Phi \geq d_q^-, \quad q = 1, 2, \ldots, Q, \quad (26) \]

\[ d_q^- d_q^+ = 0, \quad q = 1, 2, \ldots, Q, \quad (27) \]

\[ \Phi \leq 1 \]

and constraint (21)–(23),

\[ x \geq 0 \]

where \( \alpha_q = \frac{6}{(U_q - L_q)} \).

Solving the model obtained in step 4 using Lingo software, the optimal solution for the multi-objective model proposed in this paper can be obtained.

5. Numerical Experiments and Discussions

5.1. Implementation and Test Problem

This section presents a numerical example to show the effectiveness and applicability of our study on MOSSTP and its variants. We have presented the optimal compromised solutions for MOSSTP models proposed in Section 3, using GCM and FGP. All mathematical programming models were implemented using the Lingo solver to solve their deterministic counterparts.

A test problem is explained in this section. Consider a third-party logistics (TPL) company involved in a transportation network with four suppliers (source nodes) denoted by \( a_1, a_2, a_3, \) and \( a_4 \). Three wholesale market halls (demand nodes) are denoted by \( b_1, b_2, \) and \( b_3 \). Supplier \( a_1 \) provides apples, \( a_2 \) provides strawberries, \( a_3 \) provides grapes, and \( a_4 \) provides cherries. All produce is perishable. All suppliers have different amounts of their produce sent to the wholesale market halls. The wholesale market halls have their own demands for different products. The TPL is responsible for providing the appropriate conveyances. In this test problem, two conveyances with different loading capacities are considered and denoted by \( e_1 \) and \( e_2 \) respectively. Since the produce is perishable, it is crucial to consider the transportation time to be minimized with the least cost at the same time. Therefore, the goal of this transportation network is to minimize the total transportation cost and time.

The supply of fresh produce is uncertain due to unexpected events at the farms, including the weather condition and labor availability. We can easily consider situations where the supply is unstable or uncertain, making the availability of such produce probabilistic. Thus for the supplier \( a_1 \), the probability that the required amount of produce is available is \( p_{a_1} \). Similarly, probabilities \( p_{a_2}, p_{a_3}, \) and \( p_{a_4} \) are defined for suppliers \( a_2, a_3, \) and \( a_4 \), respectively.

The demand for fresh produce is uncertain by nature, too. It is caused by inexact forecasting, demand volatility, or unexpected delays in deliveries. Therefore, for market hall \( b_1 \), the probability that the expected demand is required is \( p_{b_1} \). Similarly, probabilities \( p_{b_2} \) and \( p_{b_3} \) are defined for market halls \( b_2 \) and \( b_3 \), respectively.

Similarly, the conveyance capacities are uncertain because of traffic congestions and road blockages. The probabilities that the capacities of two conveyances are available are defined as \( p_{e_1} \) and \( p_{e_2} \). These probabilities can be chosen by the decision-makers, according to the forecasting or insights.
Table 2a,b present the transportation costs $c_{ij}^1$ and $c_{ij}^2$ for two conveyances, respectively. Table 3a,b present the transportation costs $c_{ij}^2$ and $c_{ij}^2$ for two conveyances, respectively.

**Table 2. Transportation cost.**

|        | $b_1$ | $b_2$ | $b_3$ |
|--------|-------|-------|-------|
| $a_1$  | 12$   | 23$   | 20$   |
| $a_2$  | 10$   | 11$   | 17$   |
| $a_3$  | 20$   | 22$   | 13$   |
| $a_4$  | 14$   | 24$   | 21$   |

(a) Transportation cost for conveyance $e_1$ (i.e., $c_{ij}^1$)

|        | $b_1$ | $b_2$ | $b_3$ |
|--------|-------|-------|-------|
| $a_1$  | 14$   | 3$    | 23$   |
| $a_2$  | 12$   | 20$   | 14$   |
| $a_3$  | 12$   | 23$   | 12$   |
| $a_4$  | 14$   | 23$   | 21$   |

(b) Transportation cost for conveyance $e_2$ (i.e., $c_{ij}^1$)

**Table 3. Transportation time.**

|        | $b_1$ | $b_2$ | $b_3$ |
|--------|-------|-------|-------|
| $a_1$  | 2 h   | 4 h   | 5 h   |
| $a_2$  | 2 h   | 10 h  | 5 h   |
| $a_3$  | 5 h   | 8 h   | 9 h   |
| $a_4$  | 4 h   | 5 h   | 8 h   |

(a) Transportation cost for conveyance $e_1$ (i.e., $c_{ij}^2$)

|        | $b_1$ | $b_2$ | $b_3$ |
|--------|-------|-------|-------|
| $a_1$  | 5 h   | 3 h   | 3 h   |
| $a_2$  | 6 h   | 4 h   | 3 h   |
| $a_3$  | 2 h   | 3 h   | 2 h   |
| $a_4$  | 9 h   | 3 h   | 2 h   |

(b) Transportation time for conveyance $e_2$ (i.e., $c_{ij}^2$)

The proposed MOSSTP model for the test problem in this section can be formulated as follows:

\[
\begin{align*}
\text{Min } Z^1 &= \sum_{i=1}^{4} \sum_{j=1}^{3} \sum_{k=1}^{2} c_{ijk}^1 x_{ijk}, \\
\text{Min } Z^2 &= \sum_{i=1}^{4} \sum_{j=1}^{3} \sum_{k=1}^{2} c_{ijk}^2 x_{ijk}
\end{align*}
\]

subject to,

\[
\begin{align*}
P \left( \sum_{j=1}^{3} \sum_{k=1}^{2} x_{ijk} \leq a_i \right) &\geq p_{a_i}, \text{ for } i = 1, 2, 3, 4 \\
P \left( \sum_{i=1}^{4} \sum_{k=1}^{2} x_{ijk} \geq b_j \right) &\geq p_{b_j}, \text{ for } j = 1, 2, 3 \\
P \left( \sum_{i=1}^{4} \sum_{j=1}^{3} x_{ijk} \leq e_k \right) &\geq p_{e_k}, \text{ for } k = 1, 2
\end{align*}
\]
and constraint (21)–(23),
\[ x_{ijk} \geq 0, \forall i, j \text{ and } k. \]

For the numerical experiments in the following sections, the nominal values of uncertain constants are given as \( a_1 = 21, a_2 = 23, a_3 = 22, a_4 = 20, b_1 = 21, b_2 = 20, b_3 = 22, c_1 = 31 \) and \( c_2 = 34 \). In addition, arbitrary probabilities are given as \( p_{a_1} = 0.95, p_{a_2} = 0.93, p_{a_3} = 0.92, \) \( p_{b_1} = 0.36, p_{b_2} = 0.35, p_{c_1} = 0.08 \) and \( p_{c_2} = 0.07 \). The different values for the Weibull distribution parameters are considered as \( \beta_{a_i} = \beta_{b_j} = \beta_{c_k} = 2, \eta_{a_i} = \eta_{b_j} = \eta_{c_k} = 2, \) and \( \gamma_{a_i} = \gamma_{b_j} = \gamma_{c_k} = 20 \) since all constants \( a_i, b_j, \) and \( c_k \) are assumed to follow the Weibull distributions.

By using Equations (21)–(23), the probabilistic constraints can be easily converted to their deterministic forms. The complete formulation for this numerical example is omitted for brevity.

5.2. Computational Results by the GCM and FGP

To solve MOSSTP in Section 5.1, two approaches introduced in Section 4 are used and implemented in Lingo solver. This section presents the optimal solutions through GCM and FGP approaches for the test problem in Section 5.1. Table 4 shows that GCM obtained the optimal solutions for four different models proposed in Section 4. Lingo was able to produce optimal solutions for all four models using GCM.

| Table 4. Optimal solutions for 4 stochastic models using the GCM approach. |
|-----------------|---|---|---|
| Cost ($) | Time (h) | Shipment (x_{ijk}) |
| --- | --- | --- |
| Model I | 589.87 | 112 |
| \( x_{111} = 1.09, x_{121} = 0.22, x_{131} = 0.25, x_{112} = 0.25, x_{122} = 18.42, x_{132} = 0.24, x_{211} = 18.52, x_{221} = 0.24, x_{231} = 0.25, x_{212} = 0.25, x_{222} = 0.22, x_{322} = 1.07, x_{311} = 0.23, x_{321} = 0.21, x_{331} = 7.33, x_{312} = 0.24, x_{322} = 0.21, x_{332} = 12.35, x_{411} = 0.17, x_{421} = 0.25, x_{431} = 0.25, x_{412} = 0.26, x_{422} = 0.23, x_{432} = 0.26. |
| Model II | 587.85 | 112 |
| \( x_{111} = 0.25, x_{121} = 0.22, x_{131} = 0.24, x_{112} = 0.24, x_{122} = 19.71, x_{132} = 0.24, x_{211} = 19.64, x_{221} = 0.24, x_{231} = 0.24, x_{212} = 0.25, x_{222} = 0.23, x_{232} = 0.25, x_{311} = 0.24, x_{321} = 0.23, x_{331} = 8.03, x_{312} = 0.25, x_{322} = 0.22, x_{332} = 11.92, x_{411} = 0.23, x_{421} = 0.24, x_{431} = 0.24, x_{412} = 0.24, x_{422} = 0.22, x_{432} = 0.24. |
| Model III | 589.54 | 112 |
| \( x_{111} = 0.25, x_{121} = 0.22, x_{131} = 0.24, x_{112} = 0.24, x_{122} = 18.40, x_{132} = 0.23, x_{211} = 19.30, x_{221} = 0.24, x_{231} = 0.24, x_{212} = 0.25, x_{222} = 0.23, x_{232} = 0.25, x_{311} = 0.24, x_{321} = 0.22, x_{331} = 9.79, x_{312} = 0.25, x_{322} = 0.22, x_{332} = 10.77, x_{411} = 0.23, x_{421} = 0.24, x_{431} = 0.23, x_{412} = 0.24, x_{422} = 0.22, x_{432} = 0.24. |
| Model IV | 597.52 | 110 |
| \( x_{111} = 0.0, x_{121} = 0.23, x_{131} = 0.19, x_{112} = 0.25, x_{122} = 19.55, x_{132} = 0.24, x_{211} = 19.06, x_{221} = 0.24, x_{231} = 0.24, x_{212} = 0.24, x_{222} = 0.21, x_{232} = 0.55, x_{311} = 0.24, x_{321} = 0.22, x_{331} = 10.94, x_{312} = 0.24, x_{322} = 0.21, x_{332} = 8.72, x_{411} = 0.45, x_{421} = 0.42, x_{431} = 0.25, x_{412} = 0.86, x_{422} = 0.23, x_{432} = 0.25. |

The optimal solution for Model IV of the test problem in Section 5.1 is illustrated in Figure 1. The numbers in rectangles on sources are the nominal supplies. The numbers under rectangles are the right-hand-side values for the supplies in the deterministic model after the conversion. The numbers in circles on destinations are the nominal demands. The numbers under circles are the right-hand-side values for the demands in the deterministic model after the conversion. The right-hand-side values for the total capacities of conveyances 1 and 2 are 32.48 and 31.55, respectively. Note that the deterministic models converted from the MOSSTP are not balanced by nature. The numbers in the boxes on the connecting lines between suppliers and market halls indicate the optimal quantity of produce transported by vehicle \( k \).

Table 5 shows that FGP obtains the optimal solutions for four different stochastic models of the test problem. The FGP approach in Lingo solver was able to produce optimal solutions for all four models. These optimal solutions are compromised between transportation costs and times by the FGP approaches.
Table 5. Optimal solutions for four stochastic models using the FGP approach.

| Model   | Cost ($) | Time (h) | Shipment (\(x_{ijk}\)) |
|---------|----------|----------|-------------------------|
| Model I | 596.43   | 112      | \(x_{111} = 1.20, x_{121} = 0.25, x_{131} = 0.25, x_{112} = 0.25, x_{122} = 18.27, x_{132} = 0.24, x_{211} = 18.13, x_{221} = 0.25, x_{231} = 0.25, x_{212} = 0.25, x_{222} = 0.25, x_{232} = 1.42, x_{311} = 0.25, x_{321} = 0.25, x_{331} = 7.28, x_{312} = 0.25, x_{322} = 0.25, x_{332} = 12.08, x_{411} = 0.41, x_{421} = 0.25, x_{431} = 0.25, x_{412} = 0.25, x_{422} = 0.25, x_{432} = 0.25.\) |
| Model II| 589.41   | 108      | \(x_{111} = 0.16, x_{121} = 0.00, x_{131} = 0.16, x_{112} = 0.16, x_{122} = 19.68, x_{132} = 0.22, x_{211} = 19.96, x_{221} = 0.03, x_{231} = 0.23, x_{212} = 0.23, x_{222} = 0.15, x_{232} = 0.24, x_{311} = 0.19, x_{321} = 0.15, x_{331} = 9.42, x_{312} = 0.23, x_{322} = 0.93, x_{332} = 10.75, x_{411} = 0.18, x_{421} = 0.19, x_{431} = 0.18, x_{412} = 0.22, x_{422} = 0.18, x_{432} = 0.20.\) |
| Model III| 585.68   | 112      | \(x_{111} = 0.24, x_{121} = 0.14, x_{131} = 0.21, x_{112} = 0.22, x_{122} = 18.52, x_{132} = 0.19, x_{211} = 19.47, x_{221} = 0.21, x_{231} = 0.23, x_{212} = 0.23, x_{222} = 0.16, x_{232} = 0.24, x_{311} = 0.20, x_{321} = 0.15, x_{331} = 10.05, x_{312} = 0.23, x_{322} = 0.49, x_{332} = 10.69, x_{411} = 0.18, x_{421} = 0.19, x_{431} = 0.19, x_{412} = 0.22, x_{422} = 0.14, x_{432} = 0.20.\) |
| Model IV| 604.20   | 100      | \(x_{111} = 0.06, x_{121} = 0.00, x_{131} = 0.00, x_{112} = 0.24, x_{122} = 18.81, x_{132} = 1.34, x_{211} = 19.66, x_{221} = 0.22, x_{231} = 0.24, x_{212} = 0.24, x_{222} = 0.24, x_{232} = 0.18, x_{232} = 0.00, x_{311} = 0.21, x_{321} = 0.17, x_{331} = 9.89, x_{312} = 0.21, x_{322} = 0.16, x_{332} = 9.64, x_{411} = 0.23, x_{421} = 1.57, x_{431} = 0.24, x_{412} = 0.49, x_{422} = 0.20, x_{432} = 0.02.\) |

The comparison of optimal solutions by two solution methods for MOSSTP is given in Table 6 to understand the impact of the solution methods on the proposed MOSSTP.
models. In Table 6, one can observe that the GCM provides smaller transportation costs than the FGP, except Mode-III. The FGP provides shorter transportation times for Models II and IV than the GCM. For the other two models, transportation times produced by both solution methods remain the same. It is interesting to observe different objective values subject to the solution methods while the total shipments to satisfy the demands remain the same for each model.

Table 6. Comparison between the two results obtained using GCM and FGP.

| MOSSTP Models | Model-I | Model-II | Model-III | Model-IV |
|---------------|---------|----------|-----------|----------|
|               | GCM     | FGP      | GCM       | FGP      | GCM     | FGP      | GCM     | FGP      |
| Total shipment | 63.00   | 63.00   | 64.03     | 64.03    | 63.00   | 63.00    | 64.03   | 64.03    |
| Total cost ($) | 589.87  | 596.43  | 587.85    | 589.41   | 589.54  | 585.68   | 597.52  | 604.20   |
| Total time (h) | 112     | 112     | 112       | 108      | 112     | 112      | 110     | 100      |

5.3. Sensitivity Analysis and Discussions

A sensitivity analysis of optimal solutions concerning the variations of probabilities on uncertain parameters (source, demand, and conveyance capacity) in the MOSSTP is conducted in this section. We have used the same test problems for the sensitivity analysis while we only vary the probabilities on \( a_i \), \( b_j \), and \( e_k \). We solved Model IV of MOSSTP for the test problem by varying probabilities for one parameter while setting the other two probabilities to be 0.5. Both transportation cost and time in optimal solutions for each stochastic setting were obtained and are given in Tables 7–9. The results in bold indicate the minimum objective values. For this sensitivity analysis, the GCM has been used to handle multiple objective functions.

Table 7. Sensitivity analysis concerning the probability for \( a_i \).

| Sl. No. | Probability for \( a_i \) \( (p_{ai}) \) | Probability for \( b_j \) \( (p_{bj}) \) | Probability for \( e_k \) \( (p_{ek}) \) | Transportation Cost ($) | Transportation Time (h) |
|---------|-----------------------------------------|-----------------------------------------|-----------------------------------------|------------------------|------------------------|
| 1       | 0.99                                    |                                         |                                         | 619.6893               | 112                    |
| 2       | 0.95                                    |                                         |                                         | 614.0905               | 112                    |
| 3       | 0.90                                    |                                         |                                         | 606.2555               | 110                    |
| 4       | 0.85                                    |                                         |                                         | 602.7508               | 110                    |
| 5       | 0.80                                    |                                         |                                         | 602.4551               | 110                    |
| 6       | 0.75                                    |                                         |                                         | 599.6019               | **100**                |
| 7       | 0.70                                    |                                         |                                         | **595.2282**           | 112                    |
| 8       | 0.65                                    |                                         |                                         | 598.1236               | 112                    |
| 9       | 0.60                                    |                                         |                                         | 597.9150               | 112                    |
| 10      | 0.55                                    |                                         |                                         | 597.7031               | 112                    |
| 11      | 0.50                                    |                                         |                                         | **597.4853**           | 112                    |
| 12      | 0.45                                    |                                         |                                         | **597.4853**           | 112                    |
| 13      | 0.40                                    |                                         |                                         | **597.4853**           | 112                    |
| 14      | 0.35                                    |                                         |                                         | **597.4853**           | 112                    |
| 15      | 0.30                                    |                                         |                                         | **597.4853**           | 112                    |
| 16      | 0.25                                    |                                         |                                         | **597.4853**           | 112                    |
| 17      | 0.20                                    |                                         |                                         | **597.4853**           | 112                    |
| 18      | 0.15                                    |                                         |                                         | **597.4853**           | 112                    |
| 19      | 0.10                                    |                                         |                                         | **597.4853**           | 112                    |
| 20      | 0.05                                    |                                         |                                         | **597.4853**           | 112                    |
| 21      | 0.01                                    |                                         |                                         | **597.4853**           | 112                    |
Table 8. Sensitivity analysis concerning the probability for $b_j$.

| Sl. No. | Probability for $a_i$ ($p_{a_i}$) | Probability for $b_j$ ($p_{b_j}$) | Probability for $e_k$ ($p_{e_k}$) | Transportation Cost ($) | Transportation Time (h) |
|---------|---------------------------------|---------------------------------|---------------------------------|------------------------|------------------------|
| 1       | 0.50                            | 0.99                            |                                 | 635.9687               | 110                    |
| 2       | 0.95                            |                                 |                                 | 632.6581               | 110                    |
| 3       | 0.90                            |                                 |                                 | 629.8259               | 110                    |
| 4       | 0.85                            |                                 |                                 | 627.0953               | 110                    |
| 5       | 0.80                            |                                 |                                 | 623.8886               | 110                    |
| 6       | 0.75                            |                                 |                                 | 620.6356               | 110                    |
| 7       | 0.70                            |                                 |                                 | 612.6812               | 110                    |
| 8       | 0.65                            |                                 |                                 | 608.6513               | 112                    |
| 9       | 0.60                            |                                 |                                 | 604.7438               | 112                    |
| 10      | 0.55                            |                                 |                                 | 601.0435               | 112                    |
| 11      | 0.50                            |                                 |                                 | 597.4853               | 112                    |
| 12      | 0.45                            |                                 |                                 | 594.2397               | 112                    |
| 13      | 0.40                            |                                 |                                 | 591.0414               | 112                    |
| 14      | 0.35                            |                                 |                                 | 587.8478               | 112                    |
| 15      | 0.30                            |                                 |                                 | 584.6131               | 112                    |
| 16      | 0.25                            |                                 |                                 | 581.2823               | 112                    |
| 17      | 0.20                            |                                 |                                 | 577.7800               | 112                    |
| 18      | 0.15                            |                                 |                                 | 573.9885               | 112                    |
| 19      | 0.10                            |                                 |                                 | 569.6865               | 112                    |
| 20      | 0.05                            |                                 |                                 | 564.3107               | 112                    |
| 21      | 0.01                            |                                 |                                 | 557.3909               | 112                    |

Table 9. Sensitivity analysis with respect to the probability for $e_k$.

| Sl. No. | Probability for $a_i$ ($p_{a_i}$) | Probability for $b_j$ ($p_{b_j}$) | Probability for $e_k$ ($p_{e_k}$) | Transportation Cost ($) | Transportation Time (h) |
|---------|---------------------------------|---------------------------------|---------------------------------|------------------------|------------------------|
| 1       | 0.50                            | 0.50                            | 0.99                            | 594.8254               | 112                    |
| 2       | 0.95                            |                                 | 0.95                            | 595.6662               | 112                    |
| 3       | 0.90                            |                                 | 0.90                            | 596.0983               | 112                    |
| 4       | 0.85                            |                                 | 0.85                            | 596.3820               | 112                    |
| 5       | 0.80                            |                                 | 0.80                            | 596.6022               | 112                    |
| 6       | 0.75                            |                                 | 0.75                            | 596.7869               | 112                    |
| 7       | 0.70                            |                                 | 0.70                            | 596.9492               | 112                    |
| 8       | 0.65                            |                                 | 0.65                            | 597.0963               | 112                    |
| 9       | 0.60                            |                                 | 0.60                            | 597.2328               | 112                    |
| 10      | 0.55                            |                                 | 0.55                            | 597.3617               | 112                    |
| 11      | 0.50                            |                                 | 0.50                            | 597.4853               | 112                    |
| 12      | 0.45                            |                                 | 0.45                            | 597.6055               | 112                    |
| 13      | 0.40                            |                                 | 0.40                            | 597.7239               | 112                    |
| 14      | 0.35                            |                                 | 0.35                            | 597.8421               | 112                    |
| 15      | 0.30                            |                                 | 0.30                            | 597.9618               | 112                    |
Table 9. Cont.

| Sl. No. | Probability for $a_i$ ($p_{ai}$) | Probability for $b_j$ ($p_{bj}$) | Probability for $c_k$ ($p_{ck}$) | Transportation Cost ($) | Transportation Time (h) |
|---------|----------------------------------|----------------------------------|----------------------------------|------------------------|------------------------|
| 16      | 0.25                             | 0.1                               | 0.25                             | 598.0850               | 112                    |
| 17      | 0.20                             | 0.1                               | 0.20                             | 598.2090               | 112                    |
| 18      | 0.15                             | 0.1                               | 0.15                             | 598.3492               | 112                    |
| 19      | 0.10                             | 0.1                               | 0.10                             | 598.5082               | 112                    |
| 20      | 0.05                             | 0.1                               | 0.05                             | 598.6488               | 112                    |
| 21      | 0.01                             | 0.1                               | 0.01                             | 598.7756               | 112                    |

The sensitivity analysis concerning the probability for $a_i$ shows interesting patterns. Figure 2a,b show the illustrations of the sensitivity analysis of the transportation cost and time concerning the probability for $a_i$, respectively.

![Figure 2a](image1.png)  ![Figure 2b](image2.png)

**Figure 2.** Sensitivity analysis of the optimal transportation cost (a) and time (b) concerning the probability of supply availability.

In Figure 2a, the least transportation cost was observed when $p_{ai} = 0.7$. When $0 \leq p_{ai} < 0.7$, the transportation cost remains the same or increases gradually. When $p_{ai} > 0.7$, the transportation cost increases sharply. In Figure 2b, the shortest transportation time was achieved when $p_{ai} = 0.75$. When $0 \leq p_{ai} < 0.75$, the transportation time remains unchanged. When $p_{ai} > 0.75$, the transportation time increases. This sensitivity analysis indicates that both objective functions of the test problem are quite sensitive to the variation of the probability for $a_i$. This analysis helps a decision-maker to choose appropriate probabilities for the supply availability.

Table 8 shows the results of sensitivity analysis concerning the probability for $b_j$. Figure 3a,b represent the graphical views of this analysis for the transportation cost and time concerning the probability for $b_j$, respectively. In Figure 3a, one can observe that the transportation cost increases gradually with the probability for $b_j$. Note that the transportation cost is sensitive to the variation of the probability for demand requirements. For the transportation time in Figure 3b, it is shown that the transportation time remains unchanged for $0 \leq p_{ai} < 0.7$. When $p_{ai} = 0.7$, the transportation time decreases to the shortest, and then it remains at the shortest transportation time for $7 \leq p_{ai} \leq 1$. Understanding the sensitivity patterns of probabilities for the uncertain parameters can provide decision-makers with insight and ability to design the transportation network.
Figure 3. Sensitivity analysis of the optimal transportation cost (a) and time (b) concerning the probability for demand requirements.

Sensitivity analysis concerning the conveyance capacity shows a significant impact on transportation cost while there was no influence on the transportation time. This is because the conveyance capacity has nothing to do with the speed of transportation conveyance. Table 9 presents that the transportation cost decreases gradually as the probability for conveyance capacity increases, and the transportation time remains the same for all experiments. The sensitivity against the probability for conveyance capacity is shown in Figure 4. Given the test problem in Section 5.1, the transportation cost is very sensitive to the conveyance constraints.

Figure 4. Sensitivity analysis of the optimal transportation cost concerning the probability for conveyance capacities.

It is concluded that the proposed MOSSTP models in this study produce the optimal solutions in an uncertain environment. It is trivial to consider the more conservative decisions in highly uncertain situations. Therefore, it is observed that as the probabilities
for $a_i$, $b_j$, and $e_k$ decrease, more uncertainty prevails in the STP. If more uncertainty exists in the optimization problem, it is natural to choose more conservative solutions. By modeling MOSSTP with the uncertainty introduced through the probabilities for $a_i$, $b_j$, and $e_k$, it was observed that more conservative solutions are chosen as the optimum. However, the sensitivity analysis shows that it is crucial to understand the sensitivity of multiple objectives against increasing uncertainty. It provides insight to a decision-maker for choosing appropriate uncertainty levels probability for uncertain parameters.

6. Conclusions and Future Research

This research aimed to present a solution methodology for the MOSSTP under probabilistic constraints in which the stochastic variables are defined using the Weibull distribution function. Primarily, the proposed model was converted to a deterministic model using chance-constrained programming. The GCM and FGP were used to derive the objective functions for equivalent models and provide optimal solutions.

Most previous research has considered one deterministic and one probabilistic constraint. However, in our methodology, we tested three different probabilistic constraints following the Weibull distribution. Therefore, our technique is advantageous in terms of real-life applications. A numerical example was provided to explain the proposed solution method, and various experimental results were presented for analysis. Sensitivity analysis-based results were plotted using the solutions given by Model-IV.

The limitations of the current study are presented. This research considered an STP where the transportation cost or time are not considered uncertain. The market situations or traffic congestions may often cause uncertainty in the transportation cost and time. We plan to investigate STP scenarios by considering transportation cost and time uncertainty in the future study.

The STP in this study considered three parameters (supply, demand, and conveyance) to be uncertain. However, it is observed that various factors play essential roles in controlling the total transportation time and cost in distributing perishable products, including produce, vaccine, breakable items. Therefore, it is important to model these factors as the parameters for STPs and introduce the corresponding constraints.

In future work, our proposed solution method could be used for decision-making in vehicle routing problems, traveling salesman problems, supply chain management, disaster management, and humanitarian logistics, where the occurrence of uncertain situations is widespread. Our proposed methodology can also be used in data envelopment analysis. The methodology presented in this paper can be applied to solve the economic order quantity model as uncertainty often occurs in such models due to demand fluctuations. In addition, we plan to collect the historical data from the responsible authority and apply statistical regularity criteria to obtain its probability distribution for future research. In such a case, the Weibull distribution has a wide range of applications.

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Abbreviations

To define the mathematical models, the following parameters and variables are used.

Parameters

- \( Q \): number of objective functions
- \( m \): number of supply sources
- \( n \): number of demand destinations
- \( l \): number of conveyances
- \( z^q(x) \): \( q \)-th objective function
- \( c_{ijk} \): Unit cost in the \( q \)-th objective function
- \( a_i \): Amount of supply at the \( i \)-th supply source
- \( b_j \): Amount of demand at the \( j \)-th demand destination
- \( e_k \): Amount of conveyance capacity of the \( k \)-th transportation mode
- \( p_{ai} \): Probability for \( a_i \)
- \( p_{bj} \): Probability for \( b_j \)
- \( p_{ek} \): Probability for \( e_k \)
- \( \beta_{ai} \): Shape parameter for \( a_i \) following the Weibull distribution
- \( \beta_{bj} \): Shape parameter for \( b_j \) following the Weibull distribution
- \( \beta_{ek} \): Shape parameter for \( e_k \) following the Weibull distribution
- \( \eta_{ai} \): Scale parameter for \( a_i \) following the Weibull distribution
- \( \eta_{bj} \): Scale parameter for \( b_j \) following the Weibull distribution
- \( \eta_{ek} \): Scale parameter for \( e_k \) following the Weibull distribution
- \( \gamma_{ai} \): Location parameter for \( a_i \) following the Weibull distribution
- \( \gamma_{bj} \): Location parameter for \( b_j \) following the Weibull distribution
- \( \gamma_{ek} \): Location parameter for \( e_k \) following the Weibull distribution

Decision variables

- \( x_{ijk} \): Amount of shipment from \( i \)-th supply source to \( j \)-th demand destination using \( k \)-th transportation mode

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