Method to produce an uniform magnetic field in a dilution refrigerator

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Abstract. Development of superconducting chips often requires an extended region with a highly uniform magnetic field. Frequently used for such a purpose Helmholtz, solenoid and Maxwell coils produce a limited volume where magnetic field is uniform. The report suggests a method for calculating a multi-coil device with the maximum value of equilibrium field. An influence of manufacturing tolerance is discussed.

1. Introduction
The operation of D-Wave’s quantum processor [1] greatly depends on the ambient magnetic field within the environment of the chip. A special magnetic vacuum system (MVS) was developed to protect the processor from influences of magnetic field of Earth and sources of interference. The last stage of the MVS is an active compensation system based on magnetic sensors and compensation coils creating a uniform field to reduce a residual field of mu-metal shields. The uniform magnetic field created by the compensation coil defines the quality of the MVS and the volume where the residual field is acceptable for the processor operation.

An installation of 2 or 3 of Helmholtz or Maxwell coils [2,3] is usually used to create an equilibrium magnetic field. The coils create an equilibrium magnetic field in a small volume compared to the installation size. D-Wave’s quantum processor operates in a limited space of a dilution fridge and we developed a way to calculate a coil system to use fridge space more efficiently.

2. Implementation of compensation coils into dilution fridge
In general the design of a dilution fridge is a system of concentric cylinders of radiation shields inside a vacuum chamber. It is convenient to use one of these cylinders as a base for the coils. Another advantage of such design is the minimization of coil connections.

In the MVS of the D-Wave One™ computer the last radiation shield is made from an aluminium alloy and used as a base for compensation coils and a superconducting shield. Separate coils arrangements are used for all of three field component. In this report we describe method of calculation and design of axial coils.

3. Coils design
In order to produce uniform compensation fields in the z-direction (axial field) in a cylindrical shield, there would need to be at least one pair of Helmholtz coils wound around the shield. With a single Helmholtz pair, the coils will only span the length of the radius. In theory, if this pair could be placed

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at exactly the height of the sample holder, the single pair might be sufficient to cancel any field around the chip. However, changes in hardware design are likely to result in this height changing over time, requiring rewiring of the coils, making this an undesirable option. Also, it would be preferable to be able to compensate a larger portion of the volume of the superconducting shield, in order to also include other parts inside.

For the Helmholtz coils an expression for the field value at the point $z$ is:

$$B(z) = \frac{1}{2} \mu_0 N R^2 \left[ \left( R^2 + \left( z + \frac{R}{2} \right)^2 \right)^\frac{3}{2} + \left( R^2 + \left( z - \frac{R}{2} \right)^2 \right)^\frac{3}{2} \right].$$  \hspace{1cm} (1)

Now consider an arbitrary axial symmetric magnetic field aligned with the $z$-axis, with the centre placed at the origin. We can write the Taylor Series expansion of the field as:

$$B(z) = B(0) + \frac{1}{2} B^{(2)}(0) z^2 + \frac{1}{24} B^{(4)}(0) z^4 + \frac{1}{720} B^{(6)}(0) z^6 + \ldots .$$  \hspace{1cm} (2)

The choice of separation of the identical coils by one radius gives the condition $B^{(2)}(0) = 0$. This means that the constant $B(0)$ is the only surviving term from equation (2) up to order $z^4$.

To use fridge space efficiently, it would be preferable to be able to create a uniform field in a large portion of the operating volume. Multi-coil configuration is possible and results in a more homogenous field. For instance, Maxwell’s Tricoil is an adaptation due to Maxwell in which 3 coils are used with a ‘great circle’ coil of radius $R$, and $N = 64$ turns placed at the centre and two ‘small circles’ of radius $(3/7)^{1/2} R$ and $N = 49$ turns, each placed at a distance of $(3/7)^{1/2} R$ from the central coil. This configuration results in both $B^{(2)}(0) = 0$ and $B^{(4)}(0) = 0$, so the homogeneity of the field is improved from fourth to sixth order in $z$. In fact, the volume of the region in which the field varies less that 1% is improved by 1.27 times that of the Helmholtz coil using Maxwell’s Tricoil. However, configurations such as Maxwell’s Tricoil are not possible in a fridge because the radii of the coils are not identical so they cannot be wound simply onto the shield.

To wind the Helmholtz coil directly onto our shield, we need to use a modification of the Helmholtz coil using coils of the same radius. Wang et al investigated this idea [4] using a three coil array with the third coil having the same radius as the other two. In order to optimize the turns ratio of the outer two coils relative to the inner coil and the spacing of the two coils, Wang et al aimed to satisfy the condition that both $B^{(2)}(0) = 0$ and $B^{(4)}(0) = 0$ (field with sixth order uniformity in $z$). The authors determined that the volume of the region in which the field varies less that 1% is improved by 2.20 times when their improved coil is used instead of the traditional Helmholtz coil.

4. Design of a new multi-coil array

Because our shields most often have a large length relative to the diameter size, we will consider modifications of the Helmholtz coil with multiple coils. In order to calculate the parameters for multi-coil systems, we first consider the field inside an array of coils aligned along the $z$-axis and symmetric about the origin. Similarly to how we calculated this for the single Helmholtz pair, the field along the axis is a superposition of the field due to each of the loops. In general, this axial field will be of the form:

$$B(z) = \frac{1}{2} \mu_0 N R^2 \left[ N_0 \left[ R^2 + \left( z + a_i \right)^2 \right]^\frac{3}{2} + \sum_{i=1}^{\infty} N_i \left[ R^2 + \left( z + a_i \right)^2 \right]^\frac{3}{2} + \left[ R^2 + \left( z - a_i \right)^2 \right]^\frac{3}{2} \right] \right].$$  \hspace{1cm} (3)
The first term in the brackets corresponds to an unpaired loop with a number of turns \( N_0 \) placed at the origin, which may or may not be present. The remaining terms correspond to pairs of loops placed at \( z = a_i \) and \( z = -a_i \) with a turns ratio of \( k \) turns relative to the central loop. If a central loop is not present, the first term in the brackets vanishes, \( k = 1 \), and the turns ratios of the successive pairs will be referenced to this (usually the inner-most) pair.

For a modified Helmholtz coil with \( n \) coils including a central coil, there will be \( (n-1)/2 \) spacings and \( (n-1)/2 \) turns ratios to solve for, resulting in a total of \( n-1 \) unknown configuration parameters. For a modified Helmholtz coil with \( n \) coils not including a central coil, there will be \( n/2 \) spacings and \( (n-2)/2 \) turns ratios to solve for, again resulting in a total of \( n-1 \) unknown configuration parameters. These configuration parameters can be determined by requiring that they satisfy the condition that the derivatives \( B^{(2)}(0), B^{(4)}(0), \ldots, B^{(2(n-1))}(0) \) vanish. Noting that the odd numbered derivatives vanish at the origin automatically by symmetry, we see that this condition results in a field with uniformity of \((2n)^{th}\) order in \( z \). This means that increasing the number of coils increases the uniformity. Solving for these parameters for an \( n \)-coil configuration necessarily involves solving a non-linear system of \((n-1)\) equations.

5. Solving the equation

For an \( n \)-coil configuration, solving the system of \((n-1)\) non-linear equations is not entirely trivial. It should be done using a computational iterative simultaneous equation solver. This was done in two ways: using the Newton-Raphson method and using the Secant method. When doing numerical calculations with the Newton-Raphson method, only a set of starting values (or “guessed solution”) need be input to the solver, whereas when the Secant method is used, an input of two values in between which the solution is expected to lie and possibly (depending on the solver) a maximum value should be given for each parameter.

6. Correction of manufacturing tolerance

The roots of system of equations are real numbers but numbers of turns are integer. Distances between coils should be also set in mm or in decimal parts of mm because of manufacturing reasons. To reduce a worsening of field uniformity coming from rounding, an iteration process was introduced in the calculation. After the first step, when real numbers are calculated, we did rounding of number of turns and distance \( a \) for the coil pair the nearest to the centrum of the coil system. The calculations are repeated for the rest of coils with reduced number of unknown parameters. The iterations are repeated for all of the coil pair consequently.

7. Modelling field uniformity

A measure of uniformity must be defined:

\[
\varepsilon_z(r,z) = \frac{[B(r,z) - B(0,0)]}{B(0,0)} \times 100\%.
\] (4)

This quality factor \( \varepsilon_z(r,z) \) tells us by which percentage the axial field at a given point \((r,z)\) varies from the axial field at the centre of the coil array (the origin). The volume of the regions in which \( \varepsilon_z(r,z) < 0.1\% \) and \( \varepsilon_z(r,z) < 1\% \) are a useful measure of the uniformity of the field within the coils. The regions for different coil arrays are shown on figure 1.
Figure 1. Two-dimensional plots of a cylinder of radius $R$ and height $6R$ with the regions of $\varepsilon_z(r, z) < 0.1\%$ and $\varepsilon_z(r, z) < 1\%$ highlighted for arrays of 2 to 8 coils. For each coil array, the plot on the left-hand side shows the region where $\varepsilon_z(r, z) < 0.1\%$ and the plot on the right-hand side shows the region where $\varepsilon_z(r, z) < 1\%$.

8. Conclusions

In summary, a method of improving the uniformity of the axial fields used for field compensation has been presented. It is shown that the field uniformity volume can be enlarged increasing the number of coils.

[1] Johnson M W et al. 2011 Nature 473 194-198
[2] See, for example, http://demonstrations.wolfram.com/HelmholtzCoilFields/
[3] Clerk-Maxwell J 1873 Treatise on electricity and magnetism (Oxford: The Clarendon Press)
[4] Wang J, She S and Zhang S 2002 Rev. Sci. Instrum. 73 5