The scissors mode from a different perspective

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Abstract

The scissors mode, a magnetic dipole excitation—mainly orbital—is usually discussed in terms of a transition from a $I = 0^+$ ground state to a $I = 1^+$ excited state. This is understandable because it follows from the way the experiment is performed—e.g. inelastic electron scattering. Here however, we start with the excited $1^+$ state and consider all possible transitions to $I = 0^+, 1^+$ and $2^+$ states with final isospins. There is a larger transition to the $0^+_2$ state than to ground. This has a much richer structure. We note that the “sum of sums” is independent of the interaction.

1 Introduction

In a collective picture the scissors mode is an orbital magnetic dipole excitation, in which the deformed proton symmetry axis vibrates against the corresponding axis of the neutrons. Some early discussions of this mode are contained by Richter’s group, Bohle et al. [1, 2] as well as LoIudice and Palumbo [3], Suzuki and Rowe [4], Iachello [5], Dieperink [6] and Lipparini and Stringari [7]. A shell model approach was proposed by Zamick [8] and Poves, Retamosa and Moya de Guerra [9]. There has been continued interest in this topic as can be seen by the review article by K. Heyde et al. [10]. More recently, there has been work on M1 excitations by J. Beller et al. [11] in which the initial state has $I = 1^+$. This is of great relevance to the theme of the present work. There has been considerable work on $J^\text{max}$ pairing by Zhao and Arima [12], Cederwall [13], Xu et al. [14], Fu et al. [15] Zamick and Escuderos [16], Hertz-Kintish and Zamick [17].

In all the experiments which are mainly inelastic electron scattering, one starts with the $J = 0^+$ ground state and considers excitations to $I = 1^+$ states. The supporting calculations follow suit. However, since there are no practical constraints for theory, we will here start with the $I = 1^+$ scissors mode state and follow the various branches to which it can connect. Now we can go not only from $I = 1^+$ to $I = 0^+$ but also $I = 1^+$ to $I = 2^+$ which gives a much richer spectrum.

This work can be regarded as an extension of previous work by the authors [18]. In that work the main focus was on selection rules with a $J = 0 \ T = 1$ pairing interaction i.e. why certain B(M1)’s vanish. In this work we will make quantitative comparisons of the non-vanishing strengths with different interactions.

2 B(M1) Results for Various Interactions

We present results in Tables II through XXII, which are $^{44}\text{Ti} \ I = 1$ to 0, $^{44}\text{Ti} \ I = 1$ to 2, $^{46}\text{Ti} \ I = 1$ to 0, and $^{46}\text{Ti} \ I = 1$ to 2. As well as, $^{44}\text{Ti} \ I = 1$ to 1, $^{46}\text{Ti} \ I = 1$ to 1. There are four interactions used: $J = 0 \ T = 1$ Pairing, Q.Q, MBZE [1] and $J^\text{max} \ T = 0$ pairing. These are represented by 8 numbers (7 independent), corresponding to two nucleons coupled to $J = 0$ to $J = 7$. Here they are:
We also present the results in various figures. All B(M1)'s are in units of \((\mu_N)^2\).

### Table I. Matrix Elements for the Interactions

| State\((v, T, t)\) | \(I = 0\) Unshifted Energy | \(I = 2\) Unshifted Energy | \(I = 2\) Unshifted Energy | \(I = 2\) Unshifted Energy |
|----------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 000            | 0.000                     | 2.6996                    | 0                         | 0                         | 2.6996                    |
| 020            | 0.750                     | 8.0995                    | 0                         | 0                         | 8.0995                    |
| 400            | 2.250                     | 1.9300                    | 0.1117                   | 2.8922                   | 4.9339                    |
| 400            | 2.250                     | 0.8986                    | 7.7693                   | 1.9187                   | 10.4966                   |
| sum            |                           | 13.6277                   | 7.7910                   | 4.8109                   | 26.2296                   |

### Table II. Pairing B(M1) \(^{44}\)Ti \(I=1\) to \(I=0\)

| State\((v, T, t)\) | \(I = 1\) Unshifted Energy | \(I = 1\) Unshifted Energy | \(I = 1\) Unshifted Energy | \(I = 1\) Unshifted Energy |
|----------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 210            | 1.500                     | 2.250                     | 2.250                     | 2.250                     | sum                      |
| 411            |                           |                           |                           |                           | 4.3477                   |
| 411            |                           |                           |                           |                           | 52.1569                  |
| 411            |                           |                           |                           |                           | 9.8523                   |
| 222            |                           |                           |                           |                           | 19.9421                  |
| 221            |                           |                           |                           |                           | 22.1874                  |
| 411            |                           |                           |                           |                           | 22.5721                  |
| 250            |                           |                           |                           |                           | 131.1483                 |

### Table III. Pairing B(M1) \(^{46}\)Ti \(I=1\) to \(I=2\)

| State\((v, T, t)\) | \(I = 1\) Unshifted Energy | \(I = 1\) Unshifted Energy | \(I = 1\) Unshifted Energy | \(I = 1\) Unshifted Energy |
|----------------|---------------------------|---------------------------|---------------------------|---------------------------|
| 220            | 1.750                     | 2.0000                    | 2.0000                   | 2.5000                   | 2.5000                   | 2.5000                   | 2.5000                   | 2.5000                   | sum                      |
| 411            | 1.0799                    | 0                         | 0                         | 0                         | 0                         | 0                         | 0                         | 0                         | 1.7099                   |
| 411            | 9.7200                    | 0                         | 0                         | 0                         | 0                         | 0                         | 0                         | 0                         | 9.7200                   |
| 411            | 2.4344                    | 2.8794                   | 0.0491                   | 0.5611                   | 0.4150                   | 0                         | 0                         | 0                         | 6.3390                   |
| 411            | 2.7500                    | 0.3947                   | 0.7573                   | 5.7648                   | 0.1157                   | 2.0588                   | 0                         | 0                         | 6.3390                   |
| 611            | 2.7500                    | 0                         | 1.0423                   | 0.0987                   | 3.1539                   | 0.2640                   | 2.3989                   | 0.6317                   | 9.0913                   |
| 611            | 2.7500                    | 0                         | 0.0049                   | 0.1721                   | 0.0858                   | 0.4450                   | 0.0001                   | 1.7267                   | 7.5895                   |
| sum            |                           |                           | 13.6290                 | 4.6839                   | 6.0847                   | 3.9165                   | 3.1828                   | 2.3990                   | 2.3584                   | 36.2543                  |

In some cases, in order to remove degeneracies with schematic interactions we add -1.00 MeV to all the odd \(J, T=0\) matrix elements. If we did not do this, then states of different isospins would be degenerate and arbitrary mixtures of these states would appear in the computer output. This trick pushes up states of higher isospin to higher energies, but leaves the energies of lower isospin unchanged. We call these new energies shifted. These higher isospin states in \(^{44}\)Ti are indicated with a star (*) for \(T=1\) and two stars (**) for \(T=2\). Similarly, higher isospin states for \(^{46}\)Ti are given one star (*) for \(T=2\) and two stars (**) for \(T=3\). We give the seniority, isospin, and reduced isospin for the pairing interactions so we do not use the star notation for labeling the states.
Table V. Pairing B(M1) $^{46}$Ti $I=1$ to $I=2$

| State$(v,T,t)$ | $I=1$ | $I=2$ | 220 | 411 | 411 | 421 | 421 | 611 | 611 | sum |
|----------------|-------|-------|-----|-----|-----|-----|-----|-----|-----|-----|
|                |       | Unshifted Energy | 1.7500 | 2.0000 | 2.0000 | 2.500 | 2.500 | 2.7500 | 2.7500 |     |
| 211            | 1.0000 | 1.3712 | 0.9874 | 0.3326 | 0.0005 | 0.0019 | 0 | 0 | 2.6936 |
| 211            | 1.0000 | 0.1715 | 0.4367 | 0.1472 | 0.0813 | 0.3238 | 0 | 0 | 1.1605 |
| 221            | 1.5000 | 2.5716 | 2.2323 | 0.7524 | 0.0222 | 0.0883 | 0 | 0 | 5.6668 |
| 412            | 1.5000 | 0 | 0.0916 | 1.5360 | 0.0607 | 0.4819 | 0 | 0 | 2.1702 |
| 411            | 2.0000 | 0 | 0.0847 | 0.0914 | 0.5024 | 0.0261 | 0.4364 | 0.0065 | 1.1475 |
| 411            | 2.0000 | 0 | 0.0041 | 0.0186 | 0.0014 | 0.0668 | 1.5191 | 0.0152 | 1.6244 |
| 422            | 2.0000 | 0 | 0.2746 | 4.6069 | 0.1821 | 1.4454 | 0 | 0 | 6.5090 |
| 410            | 2.2500 | 12.1303 | 0.0646 | 1.6850 | 0.0832 | 0.5004 | 0 | 0 | 14.4635 |
| 410            | 2.2500 | 2.9785 | 3.5617 | 0.1189 | 0.6431 | 0.5838 | 0 | 0 | 7.8860 |
| 410            | 2.2500 | 5.6386 | 0.4668 | 2.4445 | 0.0273 | 0.9432 | 0 | 0 | 9.2804 |
| 231            | 2.2500 | 2.0572 | 0 | 0 | 1.1354 | 4.5230 | 0 | 0 | 7.7156 |
| 421            | 2.5000 | 0 | 0.1804 | 0.0338 | 0.6237 | 0.0188 | 0.6123 | 0.0630 | 1.5320 |
| 421            | 2.5000 | 0 | 0.0862 | 0.2962 | 0.8883 | 0.2597 | 5.2534 | 0.0019 | 6.7857 |
| 611            | 2.7500 | 0 | 2.1377 | 0.2523 | 6.7325 | 0.4370 | 2.3618 | 0.0555 | 11.9768 |
| 611            | 2.7500 | 0 | 0.2654 | 0.0135 | 0.4044 | 0.4321 | 0.5975 | 0.8390 | 2.1141 |
| 611            | 2.7500 | 0 | 0.0367 | 0.1344 | 0.0050 | 0.5082 | 7.1099 | 1.4178 | 9.2120 |
| 611            | 2.7500 | 0 | 0.0375 | 0.0024 | 0.1070 | 0.0127 | 0.0873 | 0.0461 | 0.2930 |
| 611            | 2.7500 | 0 | 0.1215 | 1.3291 | 0.3483 | 4.0036 | 0.0007 | 5.7321 | 11.5347 |
| sum            |       |       | 26.6789 | 11.0699 | 11.8488 | 14.6567 | 13.7952 | 17.5400 | 8.1771 | 103.7666 |

Table VI. Q.Q B(M1) $^{44}$Ti $I=1$ to $I=0$

| State $I=1$ | $I=0$ | Unshifted Energy | $I=2$ | $I=3$ | $I=4$ | sum |
|-------------|-------|------------------|------|------|------|-----|
|             |       | Unshifted Energy | $I_1$ | $I_2$ | $I_3$ |     |
|             |       | $I_1$ | $I_2$ | $I_3$ |      |     |
| $0_1$       | 0.0000 | 1.3174 | 0.0015 | 0.0007 | 1.3196 |
| $0_2$       | 3.6031 | 1.8021 | 6.1454 | 0.1535 | 8.1010 |
| $0_3$       | 7.5748 | 0.1833 | 9.0414 | 0.9530 | 10.1777 |
| $0_4$       | 10.9236 | 0.0414 | 0.0577 | 6.5323 | 6.6323 |
| sum         |       | 3.3442 | 15.2460 | 7.6404 |     | 26.2306 |

Table VII. Q.Q B(M1) $^{44}$Ti $I=1$ to $I=2$

| State $I=2$ | $I=1$ | $I=2$ | $I=3$ | sum |
|-------------|-------|-------|-------|-----|
|             |       | $I_1$ | $I_2$ | $I_3$ |     |
|             |       | $I_1$ | $I_2$ | $I_3$ |      |     |
| $2_1$       | 0.9655 | 2.4898 | 0.0111 | 0.0016 | 2.5025 |
| $2_2$       | 3.6015 | 0 | 0 | 0 | 0 |
| $2_3$       | 4.7502 | 0.1735 | 20.6912 | 1.3251 | 22.1898 |
| $2_4$       | 6.4691 | 13.7051 | 8.2795 | 1.2061 | 23.1907 |
| $2_5$       | 7.5695 | 0 | 0 | 0 | 0 |
| $2_6$       | 7.6179 | 0.1271 | 0.8452 | 21.6001 | 22.5724 |
| $2_7$       | 7.7501 | 0.0545 | 46.3395 | 0.6632 | 47.0572 |
| $2_8$       | 9.7351 | 0 | 0 | 0 | 0 |
| $2_9$       | 10.4893 | 0.1723 | 0.0767 | 13.4996 | 13.6586 |
| sum         |       | 16.7223 | 76.2432 | 38.2057 | 131.1711 |
Table VIII. Q.Q B(M1) $^{46}$Ti $I=1$ to $I=0$

| $I = 0$ Unshifted Energy | $I = 1$ | $I = 2$ | $I = 3$ | $I = 4$ | $I = 5$ | $I = 6$ | $I = 7$ | sum |
|--------------------------|---------|---------|---------|---------|---------|---------|---------|-----|
| 0.0000                   | 4.3546  | 8.1095  | 8.7081* | 10.4611*| 10.5846 | 10.8481 | 11.6407*| 1.3947|
| 2.6505                   | 0.0003  | 0.3008  | 0.0015  | 0.00161 | 6.2831  |
| 7.9741                   | 0.1986  | 5.0379  | 0.3734  | 0.1137  | 0.0310  | 0.2988  | 0.0004  | 6.0534|
| 9.7327**                 | 0.0191  | 0.4441  | 0.2985  | 2.9265  | 0.1054  | 4.0417  | 0.8731  | 9.7262|
| 10.7392                  | 0.000099| 0.0097  | 0.0013  | 0.0020  | 0.8983  | 0.0079  | 4.1814  | 5.1007|
| sum                      | 4.2584  | 5.5820  | 9.7093  | 3.0425  | 1.3355  | 4.3499  | 7.9853  | 36.2929|

Table XI. Q.Q B(M1) $^{46}$Ti $I=1$ to $I=2$

| $I = 2$ Unshifted Energy | $I = 1$ | $I = 2$ | $I = 3$ | $I = 4$ | $I = 5$ | $I = 6$ | $I = 7$ | sum |
|--------------------------|---------|---------|---------|---------|---------|---------|---------|-----|
| 0.8630                   | 4.3546  | 8.1095  | 8.7081* | 10.4611*| 10.5846 | 10.8481 | 11.6407*| 0.6523|
| 1.4911                   | 0.0011  | 0.0030  | 0.0001  | 0.0017  | 1.9270  |
| 1.8998                   | 0.0151  | 0.0182  | 0.0005  | 0.0050  | 0.0017  | 2.3874  |
| 1.0660                   | 0.1476  | 0.0395  | 0.0276  | 0.1994  | 0.0026  | 0.6348  |
| 0.0486                   | 0.2012  | 0.3204  | 0.0569  | 0.0156  | 0.0226  | 1.2446  |
| 2.5182                   | 1.9267  | 0.0346  | 0.4418  | 0.0221  | 0.0027  | 4.7820  |
| 1.1676                   | 0.0683  | 0.2179  | 0.0103  | 0.0004  | 12.3131 |
| 8.5380                   | 0.0120  | 0.0168  | 2.4499  | 0.2874  | 0.0202  | 0.7989  |
| 9.6011*                  | 0.0372  | 0.2940  | 0.5506  | 0.5938  | 0.3825  | 3.3436  | 0.0549  | 8.2593|
| 9.6672                   | 0.0428  | 0.0018  | 5.5999  | 0.2733  | 2.7290  | 0.1841  | 2.1744  | 10.9653|
| 9.8751*                  | 0.1340  | 0.0460  | 0.0143  | 0.8130  | 4.0584  | 0.0469  | 0.0097  | 5.1723|
| 10.5511                  | 0.0154  | 0.0132  | 0.0092  | 4.2207  | 3.1012  | 0.1246  | 0.1755  | 7.6598|
| 10.8708**                | 0.0372  | 0.2940  | 0.5506  | 0.5938  | 0.3825  | 3.3436  | 0.0549  | 8.2593|
| 11.2619                  | 0.0079  | 0.0528  | 1.0658  | 0.0045  | 3.7622  | 0.0004  | 4.0527  | 8.9463|
| 11.3626                  | 0.0340  | 0.0075  | 0.1386  | 0.3283  | 10.7713 | 0.1488  | 11.4285 |
| 12.1399                  | 0.0005  | 0.0004  | 0.0345  | 0.5456  | 0.1533  | 0.0738  | 1.4694  | 2.27714|
| 12.4314                  | 0.0004  | 0.0099  | 0.0935  | 0.4677  | 1.4645  | 0.0974  | 6.3983  | 8.5227|
| 12.8660                  | 0.0084  | 0.0333  | 0.1017  | 0.0005  | 0.0112  | 0.7700  | 0.8951  |
| sum                      | 7.9429  | 10.6435 | 20.2928 | 14.1072 | 17.0165 | 14.9782 | 18.7921 | 103.7732|

Table X. MBZB B(M1) $^{44}$Ti $I=1$ to $I=0$

| $I = 0$ Unshifted Energy | $I = 1$ | $I = 2$ | $I = 3$ | sum |
|--------------------------|---------|---------|---------|-----|
| 0.0000                   | 5.66864*| 7.58685*| 9.72619*| 1.35304|
| 5.58610                  | 0.13111 | 5.29543 | 0.05642 | 5.48296|
| 8.28402**                | 1.95508 | 6.07014 | 0.07579 | 8.10101|
| 8.7875                   | 0.17022 | 1.73958 | 9.38455 | 11.29435|
| sum                      | 3.43889 | 13.27571 | 9.51676 | 26.2314|
Table XI. MBZE B(M1) $^{44}$Ti $I=1$ to $I=2$

| $I=2$ | Unshifted Energy | $I=1$ | $I=2$ | $I=3$ | $I=4$ | $I=5$ | $I=6$ | $I=7$ |
|-------|------------------|-------|-------|-------|-------|-------|-------|-------|
| $2_1$ | 1.16313          |       | 1.34744 | 0.42560 | 0.04716 | 1.82020 |
| $2_2$ | 4.95650          |       | 12.97910 | 1.30523 | 0.27252 | 14.55685 |
| $2_3$ | 5.23665          |       | 0       | 0       | 0       | 0       |
| $2_4$ | 7.81197          |       | 0       | 0       | 0       | 0       |
| $2_5$ | 7.82336          |       | 1.09707 | 37.57634 | 12.68482 | 51.35823 |
| $2_6$ | 7.96963          |       | 1.53883 | 10.26440 | 6.83864 | 18.64187 |
| $2_7$ | 9.26771          |       | 0       | 0       | 0       | 0       |
| $2_8$ | 9.87032          |       | 0.09840 | 16.68741 | 5.40033 | 22.18614 |
| $2_9$ | 11.88190         |       | 0.11349 | 0.11945 | 22.34037 | 22.57331 |
| sum   | 17.19433         | 66.37843 | 47.58384 |       |       |       |

Table XII. MBZE B(M1) $^{46}$Ti $I=1$ to $I=0$

| $I=0$ | Unshifted Energy | $I=1$ | $I=2$ | $I=3$ | $I=4$ | $I=5$ | $I=6$ | $I=7$ |
|-------|------------------|-------|-------|-------|-------|-------|-------|-------|
| $0_1$ | 0.00000          | 3.65521 | 6.05887 | 7.78516 | 8.73868 | 9.46213* | 10.61597* | 11.36444* |
| $0_2$ | 4.62474          | 2.47374 | 0.18000 | 0.29729 | 0.51637 | 0.99056 | 0.14637 | 0.06077 |
| $0_3$ | 6.27338          | 0.67490 | 4.31054 | 0.11501 | 0.15137 | 0.78449 | 0.00119 | 0.07790 |
| $0_4$ | 7.93821          | 0.12817 | 0.46911 | 1.37366 | 0.36023 | 0.58752 | 2.16170 | 0.08942 |
| $0_5$ | 9.31823          | 0.00351 | 0.18194 | 3.21205 | 0.38400 | 0.04369 | 0.32570 | 5.58837 |
| $0_6$ | 13.20357**       | 0      | 0      | 0      | 0      | 4.6867 | 1.79917 | 3.25315 |
| sum   | 3.83994          | 5.15914 | 5.00393 | 1.52357 | 7.20572 | 4.44894 | 9.07959 | 36.26083 |

Table XIII. MBZE B(M1) $^{46}$Ti $I=1$ to $I=2$

| $I=1$ | Unshifted Energy | $I=2$ | $I=3$ | $I=4$ | $I=5$ | $I=6$ | $I=7$ |
|-------|------------------|-------|-------|-------|-------|-------|-------|
| $2_1$ | 1.14826          | 0.20333 | 0.03800 | 0.00273 | 0.00135 | 0.00364 | 0.00619 | 0.02923 |
| $2_2$ | 2.49693          | 1.20214 | 0.46560 | 0.06665 | 0.00391 | 0.08832 | 0.13831 | 0.00351 |
| $2_3$ | 3.42179          | 1.73449 | 0.01221 | 0.22106 | 0.00967 | 0.00787 | 0.00670 | 0.03729 |
| $2_4$ | 4.88264*         | 0.24575 | 1.20038 | 0.26325 | 0.00453 | 0.28514 | 0.00118 | 0.00090 |
| $2_5$ | 5.15177          | 0.50943 | 0.76123 | 0.03848 | 0.24204 | 0.48245 | 0.17814 | 0.01875 |
| $2_6$ | 6.15814          | 0.64804 | 0.07549 | 0.36327 | 0.03414 | 0.02360 | 1.45758 | 0.11907 |
| $2_7$ | 7.69411          | 0.09696 | 3.30106 | 0.23216 | 0.05538 | 0.54470 | 1.07923 | 1.26442 |
| $2_8$ | 7.25799          | 0.46379 | 1.32288 | 0.00002 | 0.23771 | 8.01263 | 0.25422 | 0.30383 |
| $2_9$ | 7.53733          | 0.00342 | 0.03632 | 4.01174 | 0.55212 | 0.14699 | 0.45188 | 0.21191 |
| $2_{10}$ | 8.22517        | 0.19893 | 0.00043 | 0.08763 | 5.40760 | 4.16620 | 0.02142 | 1.45222 |
| $2_{11}$ | 8.25484*        | 1.62241 | 0.47339 | 0.53906 | 0.06593 | 2.14453 | 0.18033 | 0.24099 |
| $2_{12}$ | 8.49974        | 0.04708 | 0.19257 | 6.07608 | 1.84693 | 0.62106 | 1.35405 | 0.32188 |
| $2_{13}$ | 9.50002*        | 0.39393 | 1.53351 | 3.33853 | 1.81402 | 0.00121 | 0.00053 | 0.46185 |
| $2_{14}$ | 9.91064*        | 0.01248 | 0.00940 | 0.07097 | 2.30873 | 0.02298 | 3.63939 | 0.03948 |
| $2_{15}$ | 10.18382        | 0.00954 | 0.06965 | 0.27111 | 0.94621 | 0.15988 | 1.72104 | 10.96380 |
| $2_{16}$ | 10.40254*        | 0.05283 | 0.56227 | 4.13858 | 2.90402 | 0.01365 | 2.05851 | 0.27184 |
| $2_{17}$ | 11.89813*        | 0.00532 | 0.00054 | 0.06234 | 0.61446 | 0.07426 | 0.44744 | 0.19571 |
| $2_{18}$ | 14.78987**       | 0      | 0      | 0      | 0      | 0.07617 | 0.00399 | 7.63472 |
| sum   | 7.42257          | 10.0549 | 16.05596 | 17.04875 | 16.87528 | 12.73913 | 23.57140 | 103.76801 |
Table XIV. $J_{\text{max}}$ B(M1) $^{44}\text{Ti}$ $I=1$ to $I=0$

| $I = 0$ | Unshifted Energy | $1_1$ | $1_2$ | $1_3$ | sum |
|---------|------------------|-------|-------|-------|------|
| 01      | 0.0758           | 1.3441| 0     | 0     | 1.3441|
| 02      | 5.0769           | 0.2309| 4.6967| 1.3398| 6.2674|
| 03      | 5.0769           | 0.2627| 5.8869| 4.3617| 10.5173|
| 04      | 5.0769**         | 1.1300| 5.6054| 4.3646| 11.1000|
| sum     |                  | 2.9737| 16.189| 7.0661| 26.2288|

Table XV. $J_{\text{max}}$ B(M1) $^{44}\text{Ti}$ $I=1$ to $I=2$

| $I = 2$ | Unshifted Energy | $1_1$ | $1_2$ | $1_3$ | sum |
|---------|------------------|-------|-------|-------|------|
| 01      | 1.0776           | 2.8055| 0     | 6.0010| 14.8682|
| 02      | 3.0518           | 10.7765| 0.0698| 4.6408| 15.4871|
| 03      | 3.0676*          | 0     | 0     | 0     | 0    |
| 04      | 5.0769           | 0.4151| 54.3381| 2.0636| 56.8168|
| 05      | 5.0769           | 0.0086| 0.2783| 10.9842| 11.2711|
| 06      | 5.0769*          | 0     | 0     | 0     | 0    |
| 07      | 5.0769*          | 0     | 0     | 0     | 0    |
| 08      | 5.0769**         | 0.6861| 23.0100| 8.3647| 32.0593|
| 09      | 5.0769**         | 0.1578| 3.2658| 9.2783| 12.7019|
| sum     |                  | 14.8682| 80.9620| 35.3326| 131.1628|

Table XVI. $J_{\text{max}}$ B(M1) $^{46}\text{Ti}$ $I=1$ to $I=0$

| $I = 0$ | Unshifted Energy | $1_1$ | $1_2$ | $1_3$ | $1_4$ | $1_5$ | $1_6$ | $1_7$ | sum |
|---------|------------------|-------|-------|-------|-------|-------|-------|-------|------|
| 01      | 1.0143           | 1.6533| 0.0134| 0.0005| 0     | 0     | 0     | 0     | 1.6670|
| 02      | 2.4037           | 0.0905| 2.8076| 0.3393| 0.0991| 0.0183| 0.0161| 0.0002| 2.2811|
| 03      | 4.0284           | 1.8661| 1.0119| 0.0091| 0.4326| 0.0680| 2.0710| 0.0018| 5.4605|
| 04      | 4.9091           | 0.0136| 0.5472| 4.7113| 0.2183| 2.8754| 0.0207| 0.3620| 8.7485|
| 05      | 7.0280           | 0     | 0.0002| 0.3270| 1.4752| 0.0037| 0.1439| 5.4312| 7.3812|
| 06      | 7.0280**         | 0     | 0     | 0     | 0     | 0.0956| 3.7398| 5.8845| 9.7199|
| sum     |                  | 3.6236| 4.3803| 5.3863| 2.1352| 3.0610| 5.9916| 11.6797| 36.2577|
Table XVII. $J_{\text{max}}$ B(M1) $^{46}\text{Ti}$ $I=1$ to $I=2$

| $I = 2$ | Unshifted Energy | $I = 1$ | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | sum |
|---------|------------------|---------|-------|-------|-------|-------|-------|-------|-------|-----|
| 21      | 1.0281           | 2.4966  | 3.0068| 4.8057| 5.1080*| 5.4724| 5.6332*| 7.0280*| 1.0079|
| 22      | 1.7145           | 1.4089  | 1.8389| 0     | 0.0055| 0.0002| 0.0036| 0.0008| 3.2572|
| 23      | 2.4212           | 0.0064  | 0.2088| 0.0189| 0.0048| 0.0114| 0.0079| 0.0011| 0.2593|
| 24      | 2.7178           | 0.0639  | 1.4115| 0.0500| 0.0011| 0.0679| 0.0153| 0.0019| 1.6116|
| 25      | 3.1507           | 0.0964  | 1.5097| 0.8884| 0.0500| 0.0149| 0.1374| 0.0035| 2.7003|
| 26      | 3.7368           | 0.0392  | 2.2568| 0.0312| 4.6084| 0.1493| 0.0380| 0.0006| 7.1235|
| 27      | 3.9423           | 0.1232  | 0.0827| 0.0085| 0.7506| 0.6391| 0.1956| 0.0019| 1.8016|
| 212     | 4.0692*          | 2.6849  | 0.0001| 0.0410| 0.0132| 2.1248| 0.7631| 0.0010| 5.6281|
| 28      | 4.1408           | 1.2421  | 0.0179| 0.1064| 0.0300| 0.5087| 4.4675| 0.0108| 6.3834|
| 29      | 4.6429           | 0.00005| 0.1848| 8.8388| 0.8913| 0.0559| 0.0605| 0.0219| 10.0533|
| 213     | 4.8658*          | 0.0005  | 0.9376| 4.6612| 2.0247| 0.0229| 0.0774| 0.1122| 7.8365|
| 210     | 5.2300           | 0.0003  | 0.4780| 0.1015| 0.4972| 7.5926| 0.4851| 0.0425| 9.1972|
| 216     | 5.4124*          | 0.0017  | 0.2989| 0.1136| 0.1323| 3.1652| 1.1913| 0.5122| 5.4152|
| 211     | 5.5500           | 0.0010  | 0.1784| 0.8460| 0.4731| 3.0674| 7.7698| 3.0925| 15.3652|
| 214     | 7.0280           | 0       | 0.00005| 0.0058| 0.4008| 0.1698| 0.0383| 1.3172| 1.9320|
| 215     | 7.0280           | 0.00001| 0.0010| 0.3228| 0.1072| 0.5371| 0.0262| 13.8722| 14.8665|
| 217     | 7.0280*          | 0       | 0.0007| 0.3348| 0.3573| 0.0048| 0.0049| 0.9106| 1.6131|
| 218     | 7.0280**         | 0       | 0      | 0     | 1.5404| 0.6822| 5.4929| 7.7155| 25.3321|

Table XVIII. ALL INTERACTIONS B(M1) $^{44}\text{Ti}$ $I=1$ to $I=1$

| $I = 1$ | Unshifted Energy | $I = 1$ | $I_1$ | $I_2$ | $I_3$ | sum |
|---------|------------------|---------|-------|-------|-------|-----|
| $I_1$   | –                | 0.1466  | 0     | 0     | 0.1466|
| $I_2$   | –                | 0       | 0.1466| 0     | 0.1466|
| $I_3$   | –                | 0       | 0     | 0.1466| 0.1466|

Table XIX. Pairing B(M1) $^{46}\text{Ti}$ $I=1$ to $I=1$

| $I = 1$ | Unshifted Energy | $I = 1$ | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | sum |
|---------|------------------|---------|-------|-------|-------|-------|-------|-------|-------|-----|
| $I_5$   | 1.7500           | 0.1466  | 0     | 0     | 0     | 0     | 0     | 0     | 0.1466|
| $I_1$   | 2.0000           | 0.6726  | 0.0666| 0.5047| 0.0883| 1.4888| 0.00001| 0.1466|
| $I_2$   | 2.0000           | 0.0066  | 0.1744| 0.7997| 0.3788| 0.5271| 0.1994|
| $I_6$   | 2.5000           | 0.5047  | 0.1744| 1.6374| 1.3985| 0.4689| 5.2148|
| $I_7$   | 2.5000           | 0.0883  | 0.7997| 1.6374| 0.3968| 4.2042| 1.1124| 8.2388|
| $I_3$   | 2.7500           | 0.1488  | 0.3788| 1.3985| 4.2042| 6.5634| 1.0992| 15.1338|
| $I_4$   | 2.7500           | 0.00001| 0.5271| 0.4689| 1.1124| 1.0992| 0.2795| 3.48714| 36.9016|

sum 0.1466 2.76104 1.9194 5.2148 8.2388 15.1338 3.48714 36.9016
### Table XX. Q.Q B(M1) $^{46}$Ti $I=1$ to $I=1$

| $I$ = 1 | $I$ = 1 Unshifted Energy | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | sum |
|---------|--------------------------|-------|-------|-------|-------|-------|-------|-------|-----|
| $1$     | 4.3546                   | 0.1475| 0.0001| 0.0003| 0.0016| 0.0023| 0.0012| 0.0006| 0.15351|
| $2$     | 8.1095                   | 0.0001| 0.1417| 0.0026| 0.0084| 0.0320| 0.0075| 0.0060| 0.1982 |
| $3$     | 8.7081*                  | 0.0003| 0.0026| 0.4191| 0.3493| 0.6084| 0.2609| 0.1632| 1.8038 |
| $4$     | 10.4611*                 | 0.0016| 0.0084| 0.3493| 0.7721| 4.4337| 1.3977| 0.8162| 7.7790 |
| $5$     | 10.5846                  | 0.0023| 0.0320| 0.6084| 4.4337| 6.9384| 3.3583| 1.4211| 16.7942|
| $6$     | 10.8481                  | 0.0012| 0.0075| 0.2609| 1.3977| 3.3583| 0.5204| 0.6096| 6.1556 |
| $7$     | 11.6407*                 | 0.0006| 0.0060| 0.1632| 0.8162| 1.4211| 0.6096| 1.0009| 4.0176 |
| sum     |                          | 0.15351| 0.1982 | 1.8038| 7.7790| 16.7942| 6.1556 | 4.0176| 36.9019|

### Table XXI. MBZE B(M1) $^{46}$Ti $I=1$ to $I=1$

| $I$ = 1 | $I$ = 1 Unshifted Energy | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | sum |
|---------|--------------------------|-------|-------|-------|-------|-------|-------|-------|-----|
| $1$     | 3.65521                  | 0.20475| 0.00235| 0.03278| 0.17333| 0.04688| 0.00029| 0.06086| 0.52124|
| $2$     | 6.05887                  | 0.00235| 0.12924| 0.02634| 0.19778| 0.01634| 0.00696| 0.00190| 0.44391|
| $3$     | 7.78516                  | 0.03278| 0.02634| 0.35892| 3.47085| 0.20043| 1.73496| 0.19646| 6.02074|
| $4$     | 8.73868                  | 0.17333| 0.19778| 3.47085| 6.05178| 1.84761| 0.33556| 4.24016| 16.31977|
| $5$     | 9.46213*                 | 0.04688| 0.01634| 0.20043| 1.84761| 1.09868| 0.02403| 0.46457| 3.69854|
| $6$     | 10.61597*                | 0.00029| 0.06996| 1.73496| 0.33556| 0.02403| 0.52859| 1.02258| 3.71957|
| $7$     | 11.36444*                | 0.06086| 0.00190| 0.1632| 4.24016| 0.46457| 1.02258| 0.20025| 6.18678|
| sum     |                          | 0.52124| 0.44391| 6.02074| 16.31977| 3.69854| 3.71957| 6.18678| 36.90425|

### Table XXII. $J_{\text{max}}$ B(M1) $^{46}$Ti $I=1$ to $I=1$

| $I$ = 1 | $I$ = 1 Unshifted Energy | $I_1$ | $I_2$ | $I_3$ | $I_4$ | $I_5$ | $I_6$ | $I_7$ | sum |
|---------|--------------------------|-------|-------|-------|-------|-------|-------|-------|-----|
| $1$     | 2.4966                   | 0.1456| 0.0004| 0.0022| 0.0022| 0.0003| 0.0001| 0| 0.1508 |
| $2$     | 3.0668                   | 0.0004| 0.0098| 0.5060| 0.4823| 0.1481| 0.0067| 0.0005| 1.2150|
| $3$     | 4.8057                   | 0.0022| 0.5060| 1.4629| 2.6995| 0.0218| 0.3418| 0.0659| 5.1001|
| $4$     | 5.1080*                  | 0.0022| 0.4823| 2.6995| 1.6023| 0.2780| 0.1540| 0.0105| 5.2288|
| $5$     | 5.4724                   | 0.0003| 0.1481| 0.0218| 0.2780| 11.5766| 3.0925| 1.7817| 16.8990|
| $6$     | 5.6332*                  | 0.0001| 0.0067| 0.3418| 0.1540| 3.0925| 1.2682| 0.4129| 5.2762|
| $7$     | 7.0280*                  | 0| 0.0005| 0.0659| 0.0105| 1.7817| 0.4129| 0.8250| 3.0965|
| sum     |                          | 0.1508| 1.2150| 5.1001| 5.2288| 16.8990| 5.2762| 3.0965| 36.9051|
Figure 1: Strong B(M1) Diagrams

We here repeat the expressions for the B(M1)’s given by Harper and Zamick [12]

\[ B(M1) = \frac{3}{4\pi} \frac{2I_f + 1}{2I_i + 1} \left[ g_{jp} X_1 + (-1)^{I_f-I_i} g_{jn} X_2 \right]^2 \]  

(1)

Here \( g_j = g_l \pm \left( \frac{g_s - g_l}{2l + 1} \right) \)  

(2)

\( g_{sp} = 5.586 \)
\( g_{lp} = 1 \)
\( g_{sn} = -3.826 \)
\( g_{ln} = 0 \)  

(3)

(4)

For the case \( I_f \) is not equal to \( I_i \) we find:

\[ X_1 = (-1)^{I_f-I_i+1} X_2 \]  

(5)

\[ B(M1) = \frac{3}{4\pi} \frac{2I_f + 1}{2I_i + 1} (g_{jp} - g_{jn})^2 X_1^2 \]  

(6)

3 Selection Rules for the Pairing Interaction

In a previous work we commented on selection rules for vanishing B(M1)’s with a \( J=0 \) T=1 pairing interaction. The basis states were written \((v,T,t)\)-seniority, isospin and reduced isospin. We briefly repeat the selection rules here and refer to tables II,III,IV, and V. For the \( J = 0 \ T = 1 \) pairing interaction we previously found the following:

a. Transitions with \( \Delta T=2 \) or more are forbidden.
b. For \( N=Z \) nuclei \( T=1 \) to \( T=1 \) M1 transitions are zero.
c. \( \Delta v=4 \) M1 transitions are forbidden.
d. Transitions in which both \( v \) and \( t \) change are forbidden.

Here we discuss case d in more detail than we did in ref 9. In say, \( ^{46}\text{Ti} \) we break the six nucleons into three pairs. We cannot have an M1 transition involving only a pair of identical particles–we must consider a neutron-proton pair. The only way to change seniority is to create or destroy a \( J=0^+ T=1 \) pair. The reduced isospin excludes \( J=0^+ T=1 \) pairs. If the M1 operator acts on a \( J=0^+ T=1 \) pair it creates a \( J=1^+ T=0 \) pair. Since this new pair has \( T=0 \) it will not change the reduced isospin. Alternately, if we act on a \( J=1^+ T=0 \) pair we note that because it has \( T=0 \), it does not contribute to the reduced isospin. The M1 operator will change this to a \( J=0^+ T=1 \) pair and such pairs are excluded from the reduced isospin set. Hence, if we change seniority we cannot change the reduced isospin. These arguments of course also explain c, why \( v \) cannot change by more than two units.

In tables IV and V, we show using the \( J=0^+ \) pairing interaction, \( ^{46}\text{Ti} \) transitions from \( 1^+ \) to \( 0^+ \) states and \( 1^+ \) to \( 2^+ \) states respectively. We find an abundance of confirmations of rule d. We note in table IV that all transitions from the \( I=0^+ (220) \) configuration to \( 1^+ \) states except for \( (220) \) vanish. These latter \( 1^+ \) states have configurations \( (411) \), \( (611) \) and \( (421) \). In table V we see that \( B(M1)'s \) from \( J=2^+ (410) \) to \( 1^+ (611) \) vanish.

In table V we also see that \( \Delta v=4 \) \( B(M1)'s \) are zero e.g. \( (211) \) to \( (611) \). Note that the \( \Delta T=2 \) transitions from the last \( 2^+ \) state \( (231) \) to \( I=1^+ T=1 \) states all vanish. This selection rule is the easiest to understand i.e. in terms of the Wigner-Eckart theorem.

4 Selection Rules for the Q.Q and \( J_{\text{max}} \ T=0 \) Interactions.

We find also some vanishing \( B(M1)'s \) when the quadrupole-quadrupole interaction Q.Q is employed. Some are not surprising like the \( T=1 \) to \( T=1 \) transitions in \( N=Z \) \( ^{44}\text{Ti} \) shown in Table VII. Likewise, the \( \Delta T=2 \) transitions in Table IX from the \( 2_{18}^+ \ T=3 \) state in \( ^{46}\text{Ti} \) to all \( T=1 \) \( I=1^+ \) states. However, the vanishing \( B(M1)'s \) in the top line of Table VIII involving \( I=0^+ \) and \( I=1^+ \) states in \( ^{46}\text{Ti} \) are hard to explain and we will not attempt to do so here. The vanishings are from the lowest \( 0^+ \) state to two \( T=1 \) states and one \( T=2 \) state. But we have non-vanishings to other \( T=1 \) and \( T=2 \) states, so there is no simple connection with isospin.

Also, what is special about the lowest \( 0^+ \) state with \( T=1 \) from the other \( 0^+ T=1 \) states in the lower rows?? There are no vanishings for the latter states except of course in the bottom row where we have the \( \Delta T=2 \) selection rule . That is to say the \( 0_0 \) state has \( T=3 \) and will not connect to \( I=1^+ T=1 \) states.

There are other peculiarities with the Q.Q interaction. As noted in [18], in \( ^{44}\text{Ti} \) there is a degenerate pair of \( I=2^+ \) states at \( 7.75 \text{ MeV} \)–one has isospin \( T=0 \) and the other \( T=2 \).

Likewise we find some hard to understand selection rules for the \( J_{\text{max}} \ T=0 \) interaction. In Table XVI we find for \( ^{46}\text{Ti} \), from the lowest \( 0^+ \) state there are vanishing \( B(M1)'s \) to one \( T=1 \) state and two \( T=2 \) states. As in the case with Q.Q this is hard to understand.

5 Sums of Sums

Note that the sum of sums, i.e. sum of all \( B(M1)'s \) from all \( 1^+ \) states to all \( 0^+ \) states, is independent of the interaction–same for pairing as for Q.Q.

This is easy to show, utilizing the fact that the \( D \)'s form a complete set and the wave functions are normalized to unity.

\[
\sum_{\alpha} D^\alpha(J_pJ_n)D^\alpha(J'_pJ'_n) = \delta_{J_pJ'_p}\delta_{J_nJ'_n} \tag{7}
\]

\[
\sum_{J_pJ_n} D^\alpha(J_pJ_n)^2 = 1 \tag{8}
\]
This leads to the following expression for the sum of sums.

\[
SS = \frac{3}{4\pi} \frac{2I_f + 1}{2I + 1} (g_p - g_n)^2 \sum_{J_p J_n} U(1, J_p J_f J_n; J_p J_i) \times J_p (J_p + 1)
\]  

(9)

6 Non-Monotonic Behavior of the B(M1) $1^+_1$ to $0^+_1$ as One Switches from $J = 0$ pairing to $J_{max}$ Pairing

Let us focus on the $1^+_1$ transitions. The conventional scissors mode excitation is from $0^+_1$ to $1^+_1$ which will of course be a factor of three larger than the reverse transition $1^+_1$ to $0^+_1$. With the Q.Q interaction we note however that there are even larger B(M1)’s to other states. In $^{46}$Ti whereas the B(M1) for $1^+_1$ to $0^+_1$ is 1.3901, from $1^+_1$ to $0^+_2$ it is 2.6505, almost twice as large. One possible explanation of this is that the $0^+_2$ state is a double scissors mode excitation.

Let us however now focus on the $1^+_1$ to $0^+_1$ in $^{46}$Ti, i.e. the conventional spin-scissors mode. Here are some values from the above tables:

Table XXIII. Comparison of $1^+_1$ to $0^+_1$ in $^{46}$Ti

| Interaction | Table | B(M1) |
|-------------|-------|-------|
| $J = 0$ Pairing | VI | 1.0799 |
| Q.Q | VII | 1.3901 |
| MBZE | XII | 0.55962 |

It is puzzling that Q.Q and MBZE are so different because there is a big overlap between their respective wave functions.

To better understand this we now consider simple interactions which are mixtures of $J=0$ pairing and $J_{max}$ pairing:

\[
V = a\delta_{J = 0} + b\delta_{J = 7}
\]  

(10)

We present the B(M1) for selected values of $(a, b)$,

Table XXIV. B(M1) for a Mixture of Pairing and $J_{max}$

| a  | b  | B(M1) |
|----|----|-------|
| -1 | 0  | 1.082 | $J=0$ pairing |
| -1.15 | -1 | 0.210 | close to lowest B(M1) |
| -1 | -1 | 0.260 | equal $J=0$ and $J = J_{max}$ pairing |
| 0 | -1 | 1.641 | $J_{max}$ pairing |

We see a fairly complicated behavior—relatively large B(M1)’s at the two limits, $J=0$ pairing and $J_{max}$ pairing. However, for equal $J=0$ and $J = J_{max}=7$ pairing the value is much smaller, 0.210 as compared with 1.080 and 1.641. We get a non-monotonic behavior for this spin scissors mode. We get the lowest possible B(M1) for $(a, b)$ close to (-1.15,-1) i.e. B(M1)=0.210.

Going back to Q.Q and MBZE, evidently there is more $J = 0$ and $J = J_{max}$ interference in MBZE than there is in Q.Q.

7 Additional Comments

We note that the B(M1) from the lowest $1^+$ to the lowest $0^+$ (generally considered the scissors mode transition) is considerably smaller than the transition from this $1^+$ to all $0^+$ states. For example with MBZE (the most realistic interaction here) the respective numbers are 1.6533 and 3.6136. The respective numbers from $1^+$ to $2^+$ are 1.0252 and 6.6738.
Note in Table XIV that along the diagonal of the one to one “transitions” in $^{44}\text{Ti}$ the values of “B(M1)” are all the same. Of course they are not real transitions, but they can be related to the magnetic moments. Note that for N=Z nuclei in the single $j$ approximation the magnetic $g$ factor is independent of the details of the wave function. As seen in the appendix of [18] the value is

$$g = \frac{g_p + g_n}{2} = 0.55 \tag{11}$$

This explains why all the diagonal “B(M1)’s” are the same in $^{44}\text{Ti}$. This is not the case in $^{46}\text{Ti}$. The off diagonal zeros in Table XV are due to the fact, as mentioned in [18] that in N=Z nuclei transitions from $T$ to the same $T$ (in this case $T=1$) are forbidden.

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