The complete structure of the $WG_2$ algebra and its BRST quantization

Chuan-Jie Zhu

Abstract

The complete structure of the $WG_2$ algebra is obtained from an explicit realization by an abstract Virasoro algebra and a free boson field. We then construct its BRST operator and find a seven-parameter family of nilpotent BRST operators. These free parameters are related to the canonical transformations of the ghost, antighost fields which leave the total stress-energy tensor and the antighost field $b$ invariant.

1 Introduction

There exist only three generic nonlinear extension of the Virasoro algebra by a single spin-$s$ ($s > 2$) field. These algebras are related to the rank 2 Lie algebras. Two of them, $W_3$ and $WB_2$, related to the $A_2$ and $B_2$ ($= C_2$) algebras, are well-studied in the literature [1,2]. The other one, $WG_2$, related to the $G_2$ algebra, is a very complicated algebra. The structure of $WG_2$ is known by solving the bootstrap constraints or the Jacobi identities [3–5]. No explicit realization is known for $WG_2$. In application, it is important to know some realizations of the algebra. On the other hand, the authors of [6] starts directly from two free boson fields to formulate string theory with higher spin extension of the Virasoro algebra. As emphasized in [7,8], this may give rise to results which are not true for abstract $W$-algebras. So it is important to study things using only the abstract $W$-algebras. In this paper we will first derive the complete structure of the $WG_2$ algebra. We use only an abstract Virasoro algebra and a free boson field to construct an explicit realization of $WG_2$. Because the original Virasoro algebra and the boson field satisfy the Jacobi identities, the consistency of the $WG_2$ algebra is automatically guaranteed. Having the abstract $WG_2$ algebra in hand, we then forget its realization and study its BRST quantization. To simplify the construction of
the BRST operator, we require that \( \{ Q, b(w) \} \) gives the total stress-energy tensor. Even with this restriction, the BRST operator is not unique and has seven free parameters. We will show that these parameters are related to the canonical transformation of the ghost, antighost fields which leave the total stress-energy tensor and the antighost field \( b(z) \) invariant.

2 The \( W_{G_2} \) algebra

Let’s start from the basic fields. \( \phi \) is a free boson field. \( T_1 \) is the stress-energy tensor for an abstract Virasoro algebra. The basic OPEs are

\[
\partial \phi(z) \partial \phi(w) \sim -\frac{1}{(z-w)^2}, \\
T_1(z)T_1(w) \sim \frac{\tilde{c}/2}{(z-w)^4} + \left( \frac{2}{(z-w)^2} + \frac{1}{z-w} \partial_w \right) T_1(w).
\]

From these fields we can construct a new stress-energy tensor and a spin-6 field as follows:

\[
T = T_1 - \frac{1}{2} (\partial \phi)^2 + a_0 \partial^2 \phi, \\
W = x_1 (\partial \phi)^4 T_1 + x_2 (\partial \phi)^6 + (27 \text{ more terms}).
\]

By using the basic OPEs of \( T_1 \) and \( \partial \phi \), one then computes the OPEs for \( T \) and \( W \). The requirement that \( T \) and \( W \) form a (nonlinear) \( W_{G_2} \) algebra fixes all the coefficient \( x' \)'s. The OPEs for the \( W_{G_2} \) algebra can then be derived. The final result is\(^1\)

\[
T(z)T(w) \sim \frac{c/2}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \Lambda_1(w) + (z-w)^2 \Lambda_2(w) \\
+ (z-w)^4 \Lambda_4(w) + (z-w)^6 \Lambda_7(w), \\
T(z)W(w) \sim \frac{6}{(z-w)^2} W(w) + X_1(w) + (z-w) X_4(w) + (z-w)^2 X_2(w), \\
W(z)W(w) \sim \frac{c/6}{(z-ww)^{12}} + \frac{2T(w)}{(z-w)^{10}} + \frac{b_1 \Lambda_1(w)}{(z-w)^8}
\]

\(^1\) setting all derivatives of primary and quasi primary fields to 0, see the notation in [9].
Here Λ_{11} and Λ_{12} = -8Λ_{11}/5 are spin-9 quasi primary fields which don’t appear in the OPEs of the $WG_2$ algebra. The various coefficients appearing in (7) are given as follows:

\begin{align*}
T(z)\Lambda_1(w) &\sim (\text{singular terms}) + \Lambda_3(w) + (z-w)^2\Lambda_5(w) \\
&\quad + (z-w)^3\Lambda_{11}(w) + (z-w)^4\Lambda_8(w), \\
T(z)\Lambda_3(w) &\sim (\text{singular terms}) + \Lambda_6(w) + (z-w)\Lambda_{12}(w) + (z-w)^2\Lambda_9(w), \\
T(z)\Lambda_8(w) &\sim (\text{singular terms}) + \Lambda_{10}(w), \\
T(z)X_1(w) &\sim (\text{singular terms}) + X_3(w). \\
\end{align*}

where some quasi primary fields are defined in the above in terms of the nonsingular terms in OPEs and the rest are defined as follows:

\begin{align*}
&\frac{1}{(z-w)^6}(b_2\Lambda_2(w) + b_3\Lambda_3(w) + c_0W(w)) \\
&+ \frac{1}{(z-w)^4}(b_4\Lambda_4(w) + b_5\Lambda_5(w) + b_6\Lambda_6(w) + c_1X_1(w)) \\
&+ \frac{1}{(z-w)^2}\left(\sum_{i=7}^{10} b_i\Lambda_i(w) + c_2X_2(w) + c_3X_3(w)\right),
\end{align*}

(7)

where some quasi primary fields are defined in the above in terms of the nonsingular terms in OPEs and the rest are defined as follows:

\begin{align*}
b_1 &= \frac{62}{22 + 5c}, & b_2 &= \frac{-740 + 17c}{(-1 + 2c)(68 + 7c)}, \\
b_3 &= \frac{80 (35 + 139c)}{3 (-1 + 2c) (22 + 5c) (68 + 7c)}, \\
b_4 &= \frac{-6450504 - 5184658c + 17697c^2 + 1630c^3}{25 (-1 + 2c) (46 + 3c) (3 + 5c) (68 + 7c)}, \\
b_5 &= \frac{6310112 - 46423496c - 4678806c^2 + 121395c^3}{15 (-1 + 2c) (46 + 3c) (3 + 5c) (22 + 5c) (68 + 7c)}, \\
b_6 &= \frac{8 (-2179 + 57652c + 22992c^2)}{(-1 + 2c) (46 + 3c) (3 + 5c) (22 + 5c) (68 + 7c)}, \\
b_7 &= \frac{-3422146368 - 1675894344c - 7636990c^2 + 268425c^3 + 6625c^4}{25 (-1 + 2c) (46 + 3c) (3 + 5c) (68 + 7c) (232 + 11c)}, \\
b_8 &= \frac{2 (2591638496 - 45359110192c - 7010455500c^2 + 6482100c^3 + 2541375c^4)}{225 (-1 + 2c) (46 + 3c) (3 + 5c) (22 + 5c) (68 + 7c) (232 + 11c)}, \\
b_9 &= \frac{8 (-1089457712 - 16862376526c - 1015716375c^2 + 28458300c^3)}{585 (-1 + 2c) (46 + 3c) (3 + 5c) (22 + 5c) (68 + 7c) (232 + 11c)}, \\
b_{10} &= \frac{32 (15707 + 874936c + 172800c^2)}{(-1 + 2c) (46 + 3c) (3 + 5c) (22 + 5c) (68 + 7c) (232 + 11c)}, \\
c_0^2 &= \frac{400 (47 + 2c)^2 (516 + 13c)^2 (2 + c)(c^2 - 388c + 4)}{3(2c - 1)(3c + 46)(3c + 286)(5c + 3)(5c + 22)(7c + 68)(11c + 232)},
\end{align*}
\[ c_1 = \frac{186c_0}{13c + 516c}, \quad c_2 = \frac{9(13c - 694)c_0}{130(c + 2)(c + 47)}, \]
\[ c_3 = \frac{12(572c + 2089)c_0}{5(c + 2)(c + 47)(516 + 13c)}. \]  

(12)

We have checked that the above structure of the \( W_G^2 \) algebra is in agreement with that given in [3].

3 The BRST quantization of the \( W_G^2 \) algebra

Now we study the BRST quantization of the \( W_G^2 \) algebra. Following the standard procedure we introduce ghost, antighost paris \((c(z), b(z))\) and \((\delta(z), \alpha(z))\) for \( T(z) \) and \( W(z) \) respectively. These ghost anti-ghost fields have spins \((-1, 2)\) and \((-5, 6)\) and their mode expansions are as follows

\[ c(z) = \sum_n c_n z^{-n+1}, \quad b(z) = \sum_n b_n z^{-n-2}; \quad (13) \]
\[ \delta(z) = \sum_n \delta_n z^{-n+5}, \quad \alpha(z) = \sum_n \alpha_n z^{-n-6}. \quad (14) \]

These modes satisfy the usual anti-commutation relations which can be derived from the following OPEs

\[ c(z)b(w) \sim \frac{1}{z - w}, \quad (15) \]
\[ \delta(z)\alpha(w) \sim \frac{1}{z - w}. \quad (16) \]

The other OPEs are all 0 (nonsingular).

Because of the complexity with normal ordering we will not use mode expansions. All our calculation are done with (the holomorphic) fields. The normal ordering for the ghost anti-ghost fields are such that the following equations are true

\[ c(z)b(w) = \frac{1}{z - w} + : c(z)b(w) :; \quad (17) \]
\[ \delta(z)\alpha(w) = \frac{1}{z - w} + : \delta(z)\alpha(w) :. \quad (18) \]

This is possible because all these fields are free fields.

With all the above knowledge, we now construct the quantum BRST operator. One way to start is to construct the corresponding classical BRST operator.
The quantum BRST operator is then assumed to be the same form as the classical one with possible renormalization of some coefficients and addition of some zero mode terms due to normal ordering. By imposing the nilpotent condition, one would determine all these coefficients. For linear algebras this route is quite successful. The same strategy has been applied to $W_3$ [10] and in [11] to a class of quadratic non-linear algebra. But the simplicity of this construction doesn’t apply to more complicated nonlinear algebras, such as $WB_2$ and $W_4$ [8,12].

We will follow the same strategy used in [8] for the BRST quantization of the $WB_2$ and $W_4$ algebras. The BRST operator is the contour integration of a spin-1 current $j(z)$ with ghost number 1. To simplify our calculations, we require that the (anti-)commutator of the BRST operator $Q$ with the stress-energy tensor antighost $b(z)$ gives the total stress-energy tensor:

$$\{Q, b(z)\} = T_{tot} \equiv T(z) + 2c'(z)b(z) + c(z)b'(z) + 6\delta'(z)\alpha(z) + 5\delta(z)\alpha'(z).$$ (19)

This fixes the dependence of the BRST current $j(z)$ on the ghost field $c(z)$ to the the following form

$$j(z) =: c(z)(T(z) + c'(z)b(z) + 6\delta'(z)\alpha(z) + 5\delta(z)\alpha'(z)) : + \cdots \quad (20)$$

We will group the rest terms by their $(\delta, \alpha)$-ghost number. By ghost number and spin counting, there are possible terms with $(\delta, \alpha)$-ghost number from 1 to 5. The ansatz for the $(\delta, \alpha)$-ghost number 1 terms is

$$j_1 = \delta(aw + m_1T^2\delta'\alpha + m_2T\delta'\alpha' + m_3T\delta'\alpha' + m_4T\delta'\alpha'' + m_5T\delta(3)\alpha + m_6\delta'\alpha(5) + m_7\delta(3)\alpha(3) + m_8\delta(5)\alpha + m_9\delta''\delta'\alpha\alpha).$$ (21)

Here $a$ is an arbitrary constant which set the normalization for the $\delta$ ghost. It is set to be $a = \sqrt{2560504830}$. $j_2$, $j_3$, $j_4$ and $j_5$ have 80, 124, 51 and 4 terms respectively. We will not give their explicit form here. After we have solved the nilpotent condition, we will give a simplified form of the BRST operator in an Appendix. Writing the BRST operator as the sum of various $(\delta, \alpha)$ ghost number terms:

$$Q = Q_0 + Q_1 + Q_2 + Q_3 + Q_4 + Q_5,$$

$$Q_i = \oint [dz]j_i(z),$$ (22)

the nilpotent condition $Q^2 = 0$ becomes
The first equation gives the critical central charge $c = 388$. The second equation (25) is satisfied only for $m_i = 0, i = 1, \ldots, 9$. We then solve equations (26) and (27) together to found a 7-parameter solution for the 204 coefficients in $Q_2$ and $Q_3$. The real time consuming part of the calculations is to check eqs. (28)–(30). The rest eqs. (31)–(34) are satisfied automatically by ghost, antighost counting. Because the calculations will take too long a time by simply using the OPEdefs.m Mathematica package [13], one must write some other programmes to do the calculation. Let us say a few words about how we actually did the calculations.

First it is a good idea to split the bosonic and fermionic part of every terms. Because we knew that only lower spin ($\leq 11$) bosonic fields could appear in $Q^2$, we can expand all the OPEs (including also nonsingular terms) to a certain degree approparitely. The OPEs for quasi primary fields are also needed. These can be obtained easily by using the OPEs and the definitions for quasi primary fields given in (5) to (11). The OPEs with the fermionic part are not so easy. One can just try a simple example by computing the OPEs of $B_i(z) = : c^{(i)}(z) \cdots c^{(i)}(z) b^{(i)}(z) \cdots b^{(i)}(z) b(z) :$ with itself. The time needed grows quite rapidly with $i$. Because all the fermionic ghost fields are free fields, we can use eqs. (17) and (18) to do all the possible contractions and then obtain the OPEs by expanding all fields around $w$. Actually, most of the computing time are used in this Laurent expansion. I have written a simple programme to do all these things but only for these free fermionic ghosts. For the problem in hand, the computations take much short time and it is possible to finish all the calculation in about two weeks. Here is the result: all the eqs. (28)–(30) are satisfied and the rest 55 coefficients in $Q_4$ and $Q_5$ are also found. The complete solution is quite long because there are seven free parameters. After explaining the meaning of the 7 free parameters in $Q$ and also putting some terms to 0, I will give an explicit solution in the Appendix.
4 The canonical transformations of the ghost, antighost fields

The canonical transformations of the ghost, antighost fields discussed in [12,8] are similar transformations. One easily verifies that the 3-parameter and 7-parameter canonical transformations for \( W_B^2 \) and \( W_4 \) ghost, antighost fields are the following transformations:

\[
\text{(field)} \rightarrow e^S \text{(field)} e^{-S} = \sum_{n=0}^{\infty} \frac{1}{n!} \left[ S, \cdots, [S, (\text{field})] \cdots \right],
\]

(36)

where \( S \) is the contour integration of a spin-1 current of ghost number 0:

\[
S = S_{W_B^2} = \oint [dz] \left( c_1 b' b \delta'' \delta + c_2 c b' b \delta + c_3 b' b b' \beta \delta \alpha \right),
\]

(37)

for \( W_B^2 \) and

\[
S = S_{W_B^2} + \oint [dz] \left( c_4 b b' \beta' \delta + c_5 b b' \beta' \delta' + c_6 b b' b b' \beta' \beta' \delta + c_7 b b' b b' \beta' \beta' \delta' \right),
\]

(38)

for \( W_4 \) ghost, antighost fields. From these results, we learnt that in order to obtain all the possible canonical transformations, one simply writes down all the possible ghost number 0 spin-1 currents modulo total derivatives. Of course, we omit some of the simplest canonical transformations generated by the currents \( c b, \delta \alpha \) and \( b \delta'' \). These are just simple rescaling and shifting of the ghost fields.

For \( W_{G_2} \) ghost, antighost field system, there are altogether 56 independent ghost number 0 spin-1 currents (including simple rescaling and shifting). One thing to be noticed here is that these currents can also involve the Virasoro field because the spin of the \( \delta \) ghost is so low that there exists many ghost, antighost currents with very low spin. The canonical transformation for \( W_{G_2} \) is a 56-parameter family transformation. It is not difficult to obtain the full transformation group. The more interesting thing is the subgroup of canonical transformations which leave the total stress-energy tensor and the antighost \( b(z) \) invariant. This subgroup is a 10-parameter group. Nevertheless, 3 parameters are fixed by our ansatz for the BRST operator:

\[
j(z) = c(z) T(z) + a \delta(z) W(z) + (3 \text{ or more ghost terms}).
\]

(39)

Simple rescaling of \( c(z) \) and \( \delta(z) \) are not allowed. Also simple shifting of \( c(z) \) by \( \delta^{(4)}(z) \) is not allowed. This then leaves a 7-parameter subgroup of canonical transformations which leaves the total stress-energy tensor and the antighost
Now we give the explicit form of the seven parameter generator for the canonical transformations. We split it into 2 groups according the number of ghost antighost fields. We have

\[
J_1 = c_1 T b'' b' b \delta^{(3)} \delta'' \delta + c_2 T b'' b' b \delta^{(4)} \delta' \delta + c_3 T b'' b' b \delta^{(3)} \delta' \delta
+ \left( \frac{2683 c_1}{89} + \frac{4508 c_2}{89} - \frac{2877 c_3}{89} \right) T^2 b'' b' b \delta'' \delta' \\
- \left( \frac{901 c_1}{89} + \frac{1521 c_2}{89} - \frac{894 c_3}{89} \right) T b^{(4)} b' b \delta'' \delta' \\
\left( \frac{421687906 c_1}{666165} + \frac{745270691 c_2}{666165} - \frac{159173993 c_3}{666165} - \frac{193925 c_4}{666165} \right) b'' b' b \delta^{(4)} \delta'' \delta'
\]

and

\[
J_2 = c_5 T b' b \delta^{(3)} \delta' + c_6 T b' b \delta^{(4)} \delta + c_7 T b'' b \delta'' \delta'
+ \left( \frac{356406389669 c_5}{2979256008} - \frac{182203755439 c_6}{14896280340} + \frac{1385455753 c_7}{9930853536} \right) b' b \delta^{(4)} \delta''
+ \left( \frac{111038273687 c_5}{37240700760} + \frac{74234639611 c_6}{18620350380} - \frac{981306029 c_7}{12413566920} \right) b' b \delta^{(5)} \delta'
+ \left( \frac{1261098246353 c_5}{446888409120} - \frac{3548929059731 c_6}{14896280340} + \frac{19561388891 c_7}{1099356480} \right) b' b \delta^{(6)} \delta
\]
By using these expressions, I have checked explicitly that the 4 transformations associated with \( J_1 \) which change the \( \delta \) ghost field give exactly the form of the BRST operator obtained by explicit computations. In particular one can use these canonical transformations to put all \( W \)-dependent terms in \( Q_3 \) and \( Q_4 \) to 0. After doing that, the BRST operator can be written as the sum of four anticommuting nilpotent operators:

\[
Q = \tilde{Q}_0 + c_1 \tilde{Q}_1 + c_2 \tilde{Q}_2 + c_3 \tilde{Q}_3, \quad \{\tilde{Q}_i, \tilde{Q}_j\} = 0.
\]  

(42)

The dependence of the BRST operator on the rest three free parameters is linear. Also the rest canonical transformations associated with \( J_2 \) are just linear transformations. The explicit expression given in the Appendix is \( \tilde{Q}_0 \), arranged by their \((\delta, \alpha)\) ghost number. The other three \( \tilde{Q}_i \)'s are \( \tilde{Q}_0 \) exact.
Appendix: The explicit expression of the BRST current

The BRST current of $\hat{Q}_0$ contains various $(\delta, \alpha)$-ghost number terms. The 80 ghost number 2 terms are

| Term | Expression |
|------|------------|
| $b\delta(5)\delta(4)$ | $31195928179 \over 207909000$ |
| $b\delta(6)\delta(3)$ | $-3256883498089 \over 8731800000$ |
| $b\delta(7)\delta''$ | $5574877217 \over 81675000$ |
| $b\delta''\delta\delta(5)$ | $-5574877217 \over 81675000$ |
| $b\delta''\delta\delta(4)$ | $2650215533 \over 32670000$ |
| $b\delta''\delta\delta''$ | $328840539 \over 1089000$ |
| $b\delta''\delta\delta''\delta''$ | $328840539 \over 1089000$ |

The explicit expression of the BRST current

$$Tb\delta(7)\delta'' = -6911369737 \over 52270000$$

$$Tb\delta(7)\delta'' = -6911369737 \over 52270000$$
There are 124 possible ghost number 3 terms. Some of them are fixed by using the seven-parameter canonical transformations. The rest 115 terms are as follows:

| Term                                      | Expression                  |
|-------------------------------------------|----------------------------|
| $b'\delta^{(5)}(3)\delta'$              | $-16673985840229352563972999$ |
| $b'\delta^{(6)}(3)\delta'$              | $253075626570900000000$      |
| $b'\delta^{(7)}(3)\delta'$              | $-5868998946803815515773777$ |
| $b'\delta^{(8)}(3)\delta'$              | $37961343985605350000000$    |
| $b'\delta^{(9)}(3)\delta'$              | $-239503785557943931129563$  |
| $b'\delta^{(10)}(3)\delta'$             | $84358542199230000000000$    |
| $b'\delta^{(11)}(3)\delta'$             | $-17742607936819073181144427$ |
| $b'\delta^{(12)}(3)\delta'$             | $96628875599718000000000$    |
| $b'\delta^{(13)}(3)\delta'$             | $-2333725362235177565915033$ |
| $b'\delta^{(14)}(3)\delta$              | $35430587719896600000000$    |
| $b'\delta^{(15)}(3)\delta$              | $162455566957420711023713$   |
| $b'\delta^{(16)}(3)\delta$              | $506152531341380000000000$   |
| $b'\delta^{(17)}(3)\delta$              | $6621709637439158191997261$  |
| $b'\delta^{(18)}(3)\delta$              | $379613439856053500000000$   |
| $b'\delta^{(19)}(3)\delta$              | $6168105439889800353741563$  |
| $b'\delta^{(20)}(3)\delta$              | $885764692997415000000000$   |
| $b'\delta^{(21)}(3)\delta$              | $-840267973121435332827993$  |
| $b'\delta^{(22)}(3)\delta$              | $253075626570900000000000$   |
| $b'\delta^{(23)}(3)\delta$              | $209611988729928007956861$   |
| $b'\delta^{(24)}(3)\delta$              | $664323519748061250000000$   |
| $b'\delta^{(25)}(3)\delta$              | $123609963760510471593171$   |
| $b'\delta^{(26)}(3)\delta$              | $579772353598389000000000$   |
| $b'\delta^{(27)}(3)\delta'\delta'\delta'\alpha$ | $31066340668606580723211$ |
| $b'\delta^{(28)}(3)\delta'\delta'\delta\alpha$ | $253075625657900000000000$ |
| $b'\delta^{(29)}(3)\delta'\delta'\delta\alpha$ | $613439072671041414331$   |
| $b'\delta^{(30)}(3)\delta'\delta'\delta\alpha$ | $158172266666825000000000$ |
| $b'\delta^{(31)}(3)\delta'\delta'\delta\alpha$ | $6187669507681853728954331$ |
| $b'\delta^{(32)}(3)\delta'\delta'\delta\alpha$ | $126537813285345000000000$ |
| $b'\delta^{(33)}(3)\delta'\delta'\delta\alpha$ | $-16895103139354594512819$ |
| $b'\delta^{(34)}(3)\delta'\delta'\delta\alpha$ | $329924254877500000000000$ |
| $b'\delta^{(35)}(3)\delta'\delta'\delta\alpha$ | $572365691482740988789$    |
| $b'\delta^{(36)}(3)\delta'\delta'\delta\alpha$ | $200836371881590000000000$ |
| $b'\delta^{(37)}(3)\delta'\delta'\delta\alpha$ | $-1225805164496011395541$  |
| $b'\delta^{(38)}(3)\delta'\delta'\delta\alpha$ | $115034375731950000000000$ |
| $b'\delta^{(39)}(3)\delta'\delta'\delta\alpha$ | $602360975862927956467$     |
| $b'\delta^{(40)}(3)\delta'\delta'\delta\alpha$ | $669512338605000000000000$ |
| $b'\delta^{(41)}(3)\delta'\delta'\delta\alpha$ | $242505425701622741129841$ |
| $b'\delta^{(42)}(3)\delta'\delta'\delta\alpha$ | $253075625657900000000000$ |
| $b'\delta^{(43)}(3)\delta'\delta'\delta\alpha$ | $138007166586462117063$     |
| $b'\delta^{(44)}(3)\delta'\delta'\delta\alpha$ | $421792710901515000000000$ |
| $b'\delta^{(45)}(3)\delta'\delta'\delta\alpha$ | $-1020068155852307934892$  |
| $b'\delta^{(46)}(3)\delta'\delta'\delta\alpha$ | $316344353231362500000000$ |
| $b'\delta^{(47)}(3)\delta'\delta'\delta\alpha$ | $-39588958127547857672293$ |
| $b'\delta^{(48)}(3)\delta'\delta'\delta\alpha$ | $253075625657900000000000$ |
| $b'\delta^{(49)}(3)\delta'\delta'\delta\alpha$ | $885764692997415000000000$ |
| $b'\delta^{(50)}(3)\delta'\delta'\delta\alpha$ | $-17835687962729623221467$ |
| $b'\delta^{(51)}(3)\delta'\delta'\delta\alpha$ | $337434168766920000000000$ |
| $b'\delta^{(52)}(3)\delta'\delta'\delta\alpha$ | $6022597968898920098814047$ |
| $b'\delta^{(53)}(3)\delta'\delta'\delta\alpha$ | $506151253311580000000000$ |
| $b'\delta^{(54)}(3)\delta'\delta'\delta\alpha$ | $-1686234259781223760353$  |
| $b'\delta^{(55)}(3)\delta'\delta'\delta\alpha$ | $613616670474480000000000$ |
There are 51 ghost number 4 terms. Only one term depending on $W$ is set to zero. The rest 50 terms are as follows:

| Term 1 | Term 2 | Term 3 | Term 4 | Term 5 | Term 6 | Term 7 | Term 8 | Term 9 | Term 10 | Term 11 | Term 12 | Term 13 | Term 14 | Term 15 | Term 16 | Term 17 | Term 18 | Term 19 | Term 20 | Term 21 | Term 22 | Term 23 | Term 24 | Term 25 | Term 26 | Term 27 | Term 28 | Term 29 | Term 30 | Term 31 | Term 32 | Term 33 | Term 34 | Term 35 | Term 36 | Term 37 | Term 38 | Term 39 | Term 40 | Term 41 | Term 42 | Term 43 | Term 44 | Term 45 | Term 46 | Term 47 | Term 48 | Term 49 | Term 50 | Term 51 |
|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 89401293254112689160479 | 26896688280000000 | 40752983787784718777541 | 141326713478500000000 | 3976133177714728853439929 | 313794969680000000000 | 1074277510601482925641111 | 411856539287568900000 | 8856802844914614118822683 | 19769065885886000000000 | 509201803690000030000 | 18287681796000000000000 | 61022858910007664872830 | 26896688280000000000000 | 42148025841537990240191 | 20919646440000000000000 | 126031016562000000000000 | 104598223200000000000000 | 104598223200000000000000 | 781448214500000000000000 | 709657121899622308570623 | 219580268762000000000000 | 450115831950521999637983 | 470692044900000000000000 | 2855104267497065579089 | 725638939200000000000000 | 129301085920417149999195 | 781448214500000000000000 | 8967245370604332939933 | 781448214500000000000000 | 8857370932894131972631 | 781448214500000000000000 | 64683767042985969831477 | 158893483000000000000000 | 4823743794405889834211 | 285267906000000000000000 | 11029849252615103882357 | 781448214500000000000000 | 139736131503162961517 | 348650774000000000000000 |

Finally the 4 ghost number 5 terms are:
Acknowledgements

I would like to thank Prof. C. Pope for reading this paper and for discussions. Thanks also go to Prof. S. Randjbar-Daemi for inviting me to visit ICTP where part of this work is done. This work is also supported in part by Pan-Deng-Ji-Hua special project and Chinese Center for Advanced Science and Technology (CCAST).

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