Degrees of Freedom of MIMO Two-Way X Relay Channel

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Abstract—In this paper, we study the degrees of freedom of a multiple-input multiple-output (MIMO) two-way X relay channel, i.e., a system with two groups of source nodes and one relay node, where each of the two source nodes in one group wants to exchange independent messages with both the two source nodes in the other group via the relay node. We only consider the symmetric case where each source node is equipped with M antennas while the relay is equipped with N antennas. We first show that the upper bound of the degrees of freedom is $2N$ when $N \leq 2M$. Then by applying physical layer network coding and joint interference cancellation, we propose a novel transmission scheme for the considered network. We show that this scheme can always achieve this upper bound when $N \leq \lfloor \frac{3M}{4} \rfloor$.

Index Terms—MIMO X channel, relay, degrees of freedom, network coding, signal alignment, joint interference cancellation.

I. INTRODUCTION

Wireless communication has been advancing at an exponential rate, due to the increasing expectations for multimedia services. This, in turn, necessitates the development of novel signaling techniques with high spectral efficiency and capacity. Among those factors limiting the capacity of wireless networks, interference has been seen as a key bottleneck. Recently, two novel signaling schemes have been proposed to deal with the interference problem: interference alignment and network coding.

Interference alignment was first proposed in [1] to achieve the maximum degrees of freedom (DoF) for the MIMO X channel. The key idea behind this technique is that we shall align these interference signals so that they occupy the smallest signal space, leaving more free space for the useful signals. With this powerful technique, authors in [2] have shown that the capacity of the K-user time varying interference channel is characterized as

$$C(SNR) = \frac{K}{2} \log(SNR) + o(\log(SNR)).$$

Thus, no matter the size of the interference network, it is theoretically possible that each user may be able to achieve half the DoF as if there were no interference at all. This surprising result reveals that interference is not a fundamental limitation for such networks. Based on the concept of interference alignment, an increasing variety of interference alignment schemes have emerged such as distributed interference alignment, ergodic alignment and blind interference alignment [3]-[5]. Also interference alignment can be implemented in time, frequency and spatial dimensions. A review of the current status of interference techniques is presented in [6].

Network coding is also a promising transmission technology to improve spectral efficiency and system throughput [7]. The key idea of network coding is to ask an intermediate node to mix the messages it received and forward the mixture to several destinations simultaneously. Compared with the conventional time sharing based schemes where destinations are served in different time slots, the implement of network coding can increase the overall throughput significantly. The first wireless communication scenario where the network coding was applied to is two-way relaying channel (TWRIC), where two source nodes exchange information with the help of a relay (sometimes referred as physical layer network coding) [8] and [9]. By applying physical layer network coding at the relay, the rate of information exchange between the two source nodes can be increased by two times compared with conventional schemes such as 802.11. Beyond this basic TWR-C, physical layer network coding has been applied to several generalized relay-aided wireless networks such as multiuser two-way relay networks, multipair two-way relay networks and multihop relay networks [10]-[14].

In this paper, we consider the network information flow problem for a multiple-input multiple-output (MIMO) two-way X relay channel and analyze its total DoF. In this network, all the four source nodes are equipped with M antennas and the relay is equipped with N antennas. Each of the two source nodes in one group exchanges two independent messages with both the two source nodes in the other group with the help of relay. We first derive an upper bound on DoF for such a network when $N \leq 2M$. By combining the physical layer network coding and joint interference cancellation techniques, we then propose a novel transmission scheme and show that our proposed scheme can always achieve this upper bound when $N \leq \lfloor \frac{3M}{4} \rfloor$. The relay-aided MIMO X channel has been considered in [15] to analyze the diversity and multiplexing tradeoff and in [16] [17] to study its DoF. However, all these works considered one-directional transmission, which we refer to as the MIMO one-way X relay channel. For the MIMO two-way X relay channel, it has been considered in [18] for a special case of $M = 3, N = 4$. In this paper, we consider the general case with arbitrary $M$ and $N$ and derive an upper bound.
bound on DoF which can be achieved by our proposed scheme when \( N \leq \frac{1}{2}M \).

**Notation:** Boldface uppercase letters denote matrices and boldface lowercase letters are used for vectors. \( \mathbb{R} \) and \( \mathbb{C} \) denote the real and complex spaces. \( \mathbb{Z}^+ \) stands for the positive integer. \( (\cdot)^T, (\cdot)^H, (\cdot)^i \) and \( \text{Tr}(\cdot) \) stand for transpose, Hermitian transpose, Moore Penrose pseudoinverse and the trace, respectively. \( \mathbb{E}(\cdot) \) denotes the expectation operator. \( \text{Span}(\mathbf{H}) \) and \( \text{Null}(\mathbf{H}) \) stand for the column space and the null space for the matrix \( \mathbf{H} \), respectively. \( \dim(\mathbf{H}) \) denotes the dimension of column space of \( \mathbf{H} \). \( \mathbf{I}_N \) denotes the \( N \times N \) identity matrix and \( \ominus \) denotes the exclusive-OR operation.

II. MIMO TWO-WAY X RELAY CHANNEL

Consider an MIMO two-way X relay channel as shown in Fig. 1. The channel consists of four source nodes with \( M \) antennas each and a relay with \( N \) antennas. Each of the two source nodes on the left-hand-side wants to convey two independent messages to both the source nodes on the right-hand-side via the relay. So are the two source nodes on the right-hand-side.

In the multiple access (MAC) phase, all the four source nodes transmit their signals to the relay. The received signal at the relay is given by

\[
y_r = \sum_{i=1}^{4} \mathbf{H}_{i,r} x_i + n_r
\]

where \( y_r \) and \( n_r \) denote the \( N \times 1 \) received signal vector and additive white Gaussian noise (AWGN) vector at the \( r \)-th source node, respectively. \( x_i \) is the \( M \times 1 \) transmitting vector at source node \( i \) with the power constraint, i.e., \( \mathbb{E}(\text{Tr}(\mathbf{x}_i \mathbf{x}_i^H)) \leq P_i \). \( \mathbf{H}_{i,r} \) is the \( N \times M \) channel matrix from source node \( i \) to the relay. All the entries of the channel matrices \( \mathbf{H}_{i,j} \) for \( i = 1, 2, 3, 4 \) are independently and identically distributed (i.i.d) zero mean complex Gaussian random variables with unit variance, i.e. \( \mathcal{CN}(0, 1) \). Hence, all channel matrices will be of full rank with probability 1.

When receiving these signals successfully, the relay then generates new transmitting signals and broadcasts them to all the source nodes. This is called the broadcast (BC) phase. The received signal at the \( i \)-th source node is as below

\[
y_i = \mathbf{H}_{r,i} x_r + n_i
\]

where \( y_i \) and \( n_i \) denote the \( M \times 1 \) received signal vector and additive white Gaussian noise (AWGN) vector at the \( i \)-th source node, respectively. \( x_r \) is the \( N \times 1 \) transmitting vector at the relay with the power constraint, i.e., \( \mathbb{E}(\text{Tr}(\mathbf{x}_r \mathbf{x}_r^H)) \leq P_r \). \( \mathbf{H}_{r,i} \) is the \( M \times N \) channel matrix from the relay to source node \( i \). Throughout this paper, it is assumed that perfect channel state information (CSI) is available at all the source nodes and the relay. Additionally, we assume that the source nodes and the relay operate at full-duplex mode.

We then define the total DoF of the above network as

\[
d = d_{13} + d_{14} + d_{23} + d_{24} + d_{31} + d_{32} + d_{41} + d_{42}
\]

where \( d_{ij} \) is the DoF from the source node \( i \) to source node \( j \).

III. AN UPPER BOUND ON DOF

In this section, we derive an upper bound of the DoF for the MIMO two-way X relay channel when \( N \leq 2M \).

**Theorem 1:** Consider a MIMO two-way X relay channel with \( M \) antennas at every source node and \( N \) antennas at the relay. When \( N \leq 2M \), the total number of DoF is upper bounded by \( 2N \), i.e.,

\[
d \leq 2N.
\]

**Proof:** Without loss of generality, we assume that the power constraints at the source nodes and the relay are the same, i.e., \( P_i = P_r = P \), for \( i \in \{1, 2, 3, 4\} \). We first consider the network information flow of one direction, i.e., from source nodes 1, 2 to the source nodes 3, 4 via the relay. Then the signal model in (2) for the MAC phase can be written as follows

\[
y'_r = \mathbf{H}_{1,r} x_1 + \mathbf{H}_{2,r} x_2 + n'_r
\]

\[
= [\mathbf{H}_{1,r}, \mathbf{H}_{2,r}] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + n'_r
\]

\[
= \mathbf{H}_{12,r} x_2 + n'_r.
\]

Similarly, the BC phase in (3) can be written as follows

\[
y'_i = \mathbf{H}_{r,i} x'_r + n'_i
\]

\[
y'_i = \mathbf{H}_{r,i} x'_r + n'_i.
\]
Applying the cut-set theorem in [19] on each phase, as shown in Fig. 2, the cut-set bound of the rate is presented as below

\[ R_{13} + R_{14} + R_{23} + R_{24} \leq \min \{ I(\mathbf{y}_r'; \mathbf{x}_1; \mathbf{x}_2), I(\mathbf{y}_r'; \mathbf{y}_4; \mathbf{x}_1') \} \]  

(9)

where \( R_{13}, R_{14}, R_{23}, R_{24} \) stand for the information rates from the source nodes 1, 2 to the source nodes 3, 4, respectively; and \( I(\mathbf{A}; \mathbf{B}) \) stands for the mutual information between variables \( \mathbf{A} \) and \( \mathbf{B} \). For the first term on the right-hand-side of (9), we have

\[
I(\mathbf{y}_r'; \mathbf{x}_1; \mathbf{x}_2) = \log_2 \left( \det (\mathbf{I} + \frac{2P}{\min \{2M, N\}} \mathbf{H}_{12,r}) \right)
\]

\[ \leq \log_2 \left( \det (\mathbf{I} + \frac{2P}{\min \{2M, N\}} \mathbf{H}_{12,r}) \right) \]

(10)

and

\[
= \sum_{i=1}^{\min\{2M, N\}} \log_2 \left( 1 + \frac{2P \lambda_{12,r}^i}{\min \{2M, N\}} \right)
\]

(11)

where \( h(\mathbf{A}) \) stands for the differential entropy of the continuous variable \( \mathbf{A} \) and \( \lambda_{12,r}^i \) is the \( i \)th eigenvalue of the matrix \( \mathbf{H}_{12,r} \). For the second term in the right-hand-side of (9), we first rewrite (7) and (8) as follows

\[
\mathbf{y}_r' = \mathbf{H}_{r,34} \mathbf{x}_4 + \mathbf{n}_4
\]

(13)

where \( \mathbf{y}_r' \) and \( \mathbf{n}_4 \) denote the vectors \( \left[ \mathbf{y}_3^T, \mathbf{y}_4^T \right]^T \) and \( \left[ \mathbf{n}_3^T, \mathbf{n}_4^T \right]^T \), respectively; and \( \mathbf{H}_{r,34} \) denotes the matrix \( \left[ \mathbf{H}_{r,34}, \mathbf{H}_{r,34}^H \right]^T \). Then we can similarly have that

\[
I(\mathbf{y}_r'; \mathbf{y}_4' \mid \mathbf{x}_r') \leq \sum_{i=1}^{\min\{2M, N\}} \log_2 \left( 1 + \frac{P \lambda_{34,r}^i}{\min \{2M, N\}} \right)
\]

(14)

where \( \lambda_{34,r}^i \) is the \( i \)th eigenvalue of the matrix \( \mathbf{H}_{r,34} \mathbf{H}_{r,34}^H \). Based on the above results, we have that

\[
\frac{d_1 + d_4 + d_3 + d_4}{\log (5N)} \leq \min \{ I(\mathbf{y}_r'; \mathbf{x}_1; \mathbf{x}_2), I(\mathbf{y}_r'; \mathbf{y}_4'; \mathbf{x}_1') \}
\]

(15)

and

\[
\leq \min \left\{ \lim_{P \to \infty} \frac{I(\mathbf{y}_r'; \mathbf{x}_1; \mathbf{x}_2)}{\log(5N)}, \lim_{P \to \infty} \frac{I(\mathbf{y}_r'; \mathbf{y}_4'; \mathbf{x}_1')}{\log(P)} \right\}
\]

(16)

\[
= \min \{ \min \{2M, N\}, \min \{2M, N\} \}
\]

(17)

\[
= N.
\]

(18)

Considering the other direction of the network information flow, we can similarly obtain that

\[
d_3 + d_3 + d_4 \leq \min \{2M, N\}
\]

(19)

Combining (15)-(20), we conclude that

\[
d \leq 2N
\]

(21)

which completes the proof of Theorem 1.

From the above theorem, we can see that when \( N \leq 2M \), the total DoF for the above network is exactly bounded by the number of the relay’s antennas.

**IV. NOVEL TRANSMISSION SCHEM**

In this section, by applying the physical layer network coding and joint interference cancellation, we propose a novel transmission scheme for the considered network to maximize its total DoF.

To explain our scheme and its benefits clearly, a system where each source node has \( M = 3 \) antennas and a relay has \( N = 4 \) antennas is assumed in this subsection. Each of the two source nodes in one group will transmit two independent data streams to both the source nodes in the other group. Source node 1 transmits codewords \( s_{1,3}, s_{1,4} \) for messages \( W_{1,3}, W_{1,4} \) by using beamforming vectors \( \mathbf{v}_{3,1}, \mathbf{v}_{4,1} \) to source nodes 3, 4 via the relay, respectively. The other three source nodes are in a same manner.

**Step 1: Signal alignment during the MAC phase**

During the MAC phase, there are totally 8 data streams arriving at the relay. Since the relay has only 4 antennas, it is impossible for it to decode all the 8 data streams. However, based on the idea of physical layer network coding, the relay node only needs to decode the mixture of the symbols. Thus the key point of the proposed scheme is to obtain the network coded messages \( W_{1,3} \oplus W_{3,1}, W_{1,4} \oplus W_{4,1}, W_{2,3} \oplus W_{3,2} \) and \( W_{2,4} \oplus W_{4,2} \) at the relay. Inspired by the signal alignment for network coding [14], we should carefully design the beamformers so that two desired signals for network coding are aligned within the same spatial dimension. Taking source node 1 as an example, we should align its transmitted data streams with these streams from source node 3, 4 as follows

\[
\text{span}(\mathbf{H}_{1,3}, \mathbf{v}_{1,3}) = \text{span}(\mathbf{H}_{3,1}, \mathbf{v}_{3,1}) \triangleq \mathbf{g}_{1}^1
\]

(22)

\[
\text{span}(\mathbf{H}_{1,4}, \mathbf{v}_{1,4}) = \text{span}(\mathbf{H}_{4,1}, \mathbf{v}_{4,1}) \triangleq \mathbf{g}_{1}^2
\]

(23)

where \( \mathbf{g}_{1}^1, \mathbf{g}_{1}^2 \) are the signal vectors seen by the relay. Fig. 3 illustrates the concept of the signal alignment in the MAC phase. Then the relay can obtain the above four network coded messages.

**Step 2: Joint interference cancellation during the BC phase**

For the BC phase, the relay broadcasts these four network coded messages using beamformers \( \mathbf{u}_{i,3}^1, \ldots, \mathbf{u}_{i,4}^4 \). Then we can
interference by itself. For source node relay and the source nodes jointly cancel these interference. In order to handle these interference properly, we let the nodes. Fig. 4 illustrates the process of the joint interference streams along beamforming vectors network can be achieved when $u \subseteq \mathbb{R}^2$.

We first consider the nontrivial case $4, N, 4, \{s, d\}$, and $3, 2$, and $3, 2$, respectively. The transmitted signals for the other source nodes are in a similar form. In order for the relay to obtain the network coded messages $W_{1,3} \oplus W_{3,1}, W_{1,4} \oplus W_{4,1}, W_{2,3} \oplus W_{3,2}$ and $W_{2,4} \oplus W_{4,2}$, we should carefully choose the beamforming vectors to satisfy the signal alignment conditions as below

$$H_{1,r}v_{1,3} = H_{3,r}v_{3,1} \triangleq \mathbf{g}_r^i, \quad 1 \leq i \leq \frac{N}{4}$$

$$H_{1,r}v_{1,4} = H_{4,r}v_{4,1} \triangleq \mathbf{g}_r^{i+1}, \quad 1 \leq i \leq \frac{N}{4}$$

$$H_{2,r}v_{2,3} = H_{3,r}v_{3,2} \triangleq \mathbf{g}_r^{i+2}, \quad 1 \leq i \leq \frac{N}{4}$$

$$H_{2,r}v_{2,4} = H_{4,r}v_{4,2} \triangleq \mathbf{g}_r^{i+3}, \quad 1 \leq i \leq \frac{N}{4}$$

where $\mathbf{g}_r^1, \ldots, \mathbf{g}_r^N$ are $N$ transmitting vectors seen by the relay. The above conditions imply that

$$\text{span} \left( \left[ \mathbf{g}_r^1, \ldots, \mathbf{g}_r^N \right] \right) \subseteq \text{span} \left( H_{1,r} \right) \cap \text{span} \left( H_{3,r} \right)$$

$$\text{span} \left( \left[ \mathbf{g}_r^{i+1}, \ldots, \mathbf{g}_r^{N} \right] \right) \subseteq \text{span} \left( H_{1,r} \right) \cap \text{span} \left( H_{4,r} \right)$$

$$\text{span} \left( \left[ \mathbf{g}_r^{i+2}, \ldots, \mathbf{g}_r^{N} \right] \right) \subseteq \text{span} \left( H_{2,r} \right) \cap \text{span} \left( H_{3,r} \right)$$

$$\text{span} \left( \left[ \mathbf{g}_r^{i+3}, \ldots, \mathbf{g}_r^{N} \right] \right) \subseteq \text{span} \left( H_{2,r} \right) \cap \text{span} \left( H_{4,r} \right)$$

For each pair of source nodes $i$, according to dimension theorem [20], we obtain that

$$\dim \left( \text{span} \left( H_{1,r} \right) \cap \text{span} \left( H_{3,r} \right) \right) = \dim \left( \text{span} \left( H_{1,r} \right) \right) + \dim \left( \text{span} \left( H_{3,r} \right) \right) - \dim \left( \text{span} \left( H_{1,r} \right) \cap \text{span} \left( H_{3,r} \right) \right)$$

Since all the entries of the channel matrices are i.i.d. zero mean complex Gaussian random variables, there exists a $(2M - N = \frac{N}{2})$-dimensional intersection subspace constituted by the column space of channel matrices for each user pair with probability 1. Then we can always choose $\frac{N}{2}$ linearly independent transmitting vectors $\{ \mathbf{g}_r^i \}$ for each source node pair. As a result, the received signal in (2) is rewritten as

$$y_r = \mathbf{G}_r s_r + n_r$$

V. ACHIEVABILITY OF THE UPPER BOUND

In this section, we show that our proposed scheme can always achieve this upper bound when $N \leq \left\lfloor \frac{4M}{3} \right\rfloor$.

**Theorem 2**: The upper bound of the DoF for the considered network can be achieved when $N \leq \left\lfloor \frac{4M}{3} \right\rfloor$, i.e.,

$$d_{max} = 2N, \forall N \leq \left\lfloor \frac{4M}{3} \right\rfloor.$$  \hspace{1cm} (24)

**Proof**: We first consider the nontrivial case that $N = \frac{4M}{3}, \forall M = 3k, k \in \mathbb{Z}^+$ and show that $(d_{13}, d_{14}, d_{23}, d_{24}, d_{31}, d_{32}, d_{41}, d_{42}) = (N, N, N, N, N, N, N, N)$ is achieved by using the proposed scheme.

During the MAC phase, the $i$th source node sends message $W_{i,j}$ to the $j$th source node using $\frac{N}{4}$ independently encoded streams along beamforming vectors $\mathbf{V}_{i,j} = [v_{i,j}^1, \ldots, v_{i,j}^N]$.

Then the transmitted signals for source node 1 are

$$x_1 = \mathbf{V}_{1,3}s_{1,3} + \mathbf{V}_{1,4}s_{1,4}$$

$$= \sum_{i=1}^{N} v_{1,3}^i s_{1,3}^i + v_{1,4}^i s_{1,4}^i$$

where $s_{1,3}$ and $s_{1,4}$ are the $\frac{N}{4} \times 1$ encoded symbol vectors for $W_{1,3}$ and $W_{1,4}$, respectively. The transmitted signals for the other source nodes are in a similar form. In order for the relay to obtain the network coded messages $W_{1,3} \oplus W_{3,1}, W_{1,4} \oplus W_{4,1}, W_{2,3} \oplus W_{3,2}$ and $W_{2,4} \oplus W_{4,2}$, we should carefully choose the beamforming vectors to satisfy the signal alignment conditions as below
matrices are independently drawn from the Gaussian distribution, the probability that a basis vector in the intersection space of one pair of source nodes’ channel matrices lies in another intersection space of another pair is zero. Thus $G_r$ is full-rank with probability 1, which guarantees the decidability of $s_r$ at the relay. The four network coded messages $W_{13} = W_{1,3} \oplus W_{3,1}, W_{14} = W_{1,4} \oplus W_{4,1}, W_{23} = W_{2,3} \oplus W_{3,2}$ and $W_{24} = W_{2,4} \oplus W_{4,2}$ are then obtained by applying the physical layer network coding modulation-demodulation mapping principle [8] into each entry of $s_r$.

For the BC phase, the relay broadcasts the network coded messages $W_{13}, W_{14}, W_{23}$ and $W_{24}$ to all the source nodes using encoded symbols $q_r = [q_1^r, ..., q_N^r]^T$ along the beamforming vectors $U_r = [u_1^r, ..., u_N^r]$. More specifically, $[q_1^r, ..., q_{\frac{N}{2}}^r]^T$, $[q_{\frac{N}{2}}^r + 1, ..., q_N^r]^T$, $[q_{\frac{N}{2}}^r + 1, ..., q_{\frac{3N}{4}}^r]^T$ and $[q_{\frac{3N}{4}}^r + 1, ..., q_N^r]^T$ are the $\frac{N}{2} \times 1$ encoded symbol vectors for $W_{13}, W_{14}, W_{23}$ and $W_{24}$, respectively. Then the transmitted signal at the relay in (3) is rewritten as

$$x_r = \sum_{i=1}^{N} u_i^r q_i^r.$$  
(38)

The received signal for source node 1 can be expressed as

$$y_1 = H_{r,1} \left( \sum_{i=1}^{\frac{N}{2}} u_i^r q_i^r + \sum_{i=\frac{N}{2}+1}^{\frac{3N}{4}} u_i^r q_i^r + \sum_{i=\frac{3N}{4}+1}^{N} u_i^r q_i^r \right) + n_1$$  
(39)

where the first term in the bracket represents the combination of the desired network-coded messages $W_{13}$ and $W_{14}$, while the remaining two terms are the unwanted interference $W_{23}$ and $W_{24}$. The received signals for the other source nodes are in a similar way. We can see that each source node suffers from two parts of interference, i.e., the signals intended for the other two pairs. By applying the joint interferences cancellation, the relay chooses beamformers which shall satisfy the following conditions:

- $\text{span}\left( [u_1^r, ..., u_{\frac{N}{2}}^r] \right) \subseteq \text{Null}(H_{r,4})$  
- $\text{span}\left( [u_{\frac{N}{2}+1}^r, ..., u_{\frac{3N}{4}}^r] \right) \subseteq \text{Null}(H_{r,2})$  
- $\text{span}\left( [u_{\frac{3N}{4}+1}^r, ..., u_{\frac{N}{2}}^r] \right) \subseteq \text{Null}(H_{r,1})$  
- $\text{span}\left( [u_{\frac{N}{2}+1}^r, ..., u_N^r] \right) \subseteq \text{Null}(H_{r,3})$.

For each channel matrix $H_{r,i}, i = 1, ..., 4$, there exists a $(N - M = \frac{N}{2})$-dimensional null space with probability 1; and $\text{Null}(H_{r,i}) \cap \text{Null}(H_{r,j}) = \text{span}(0), \forall i \neq j$. So we can always choose $N$ linearly independent beamformers which satisfy the above conditions. Thus received signals for source node 1 can be rewritten as below

$$y_1 = H_{r,1} \left( \sum_{i=1}^{\frac{N}{2}} u_i^r q_i^r + \sum_{i=\frac{N}{2}+1}^{\frac{3N}{4}} u_i^r q_i^r \right) + n_1.$$  
(44)

Source node 1 has total $M = \frac{2N}{3}$-dimensional space and the space of useful signal is $\frac{2N}{3}$-dimensional. So it has exactly $(\frac{2N}{3} - \frac{N}{2} = \frac{N}{6})$-dimensional free space for the interference signal whose dimension is also $\frac{N}{6}$. Thus source node 1 can cancel the other part of the interference and decodes the useful signals which contains the messages $W_{13}$ and $W_{14}$. Using its side information, source node 1 can obtain the messages from source node 3 and source node 4 as follows

$$W_{3,1} = W_{1,3} \oplus W_{13}, W_{4,1} = W_{1,4} \oplus W_{14}.$$  
(45)

In the same manner, the other source nodes can also obtain the messages intended for themselves. Therefore, $2N$ DoF is achieved by using the proposed scheme on the MIMO two-way X relay channel.

For the other cases that $M$ is not multiple of 3 or $N$ is smaller than $\lfloor \frac{2N}{3} \rfloor$, we can choose the DoF for each pair as below\footnote{Note that the assignment of the DoF is not unique, and $d_{ij} = d_{ji} = 0$ means that there is no information exchange for this pair of users.}

$$d_{13} = d_{31} = \left\lfloor \frac{N}{4} \right\rfloor, d_{14} = d_{41} = \left\lfloor \frac{N}{4} \right\rfloor$$  
(46)

$$d_{23} = d_{32} = \left\lfloor \frac{N}{4} \right\rfloor, d_{24} = d_{42} = N - 3 - \left\lfloor \frac{N}{4} \right\rfloor.$$  
(47)

Then we can similarly apply the above scheme to achieve the upper bound $2N$ and the details are omitted here.

The total DoF of the considered network for typical values $M$ and $N$ are summarized in Table 1. From the table, we can see that in some cases, the DoF for each source node may not be the same. However, we can apply time slot extension to make the average DoF for each source node equal to $\frac{N}{2}$ [2].

For the conventional time division multiple access (TDMA)-based scheme, each source node transmits signals in orthogonal time slots via the relay. We can easily see that it can achieve at most $\min\{M, N\}$ DoF. Thus, our proposed transmission scheme outperforms the conventional TDMA-based scheme significantly.

Remark 1: The proposed transmission scheme only achieves the upper bound when $N \leq \lfloor \frac{4M}{3} \rfloor$. As an extension of this work, we show in [21] that by considering the joint transceiver design for interference cancellation in the BC phase, the upper bound when $N \leq \lfloor \frac{4M}{5} \rfloor$ can be achieved.

VI. Simulation Results

In this section, we provide numerical results to show the ergodic sum rate performance of the proposed scheme for the proposed transmission scheme. Each entry of the channel matrices is drawn from $CN(0, 1)$. For simplicity, we assume that the transmitted power for the source nodes and the relay are the same, i.e., $P_1 = P_2 = P_3 = P_4 = P_r = P$, and equal power allocation is employed for each data stream. A common noise variance is set to be $\sigma_1^2 = $ $\sigma_2^2 = $ $\sigma_3^2 = $ $\sigma_4^2 = $ $\sigma_r^2 = \sigma^2$. The numerical results are illustrated with respect to the ratio of the total transmitted signal power to the noise variance
at each receive antenna in decibels (SNR = $\frac{P_r}{P_n}$). In total 10000 channel realizations are simulated for each network architecture.

From Fig. 5, we can see that the proposed scheme indeed achieves the DoF upper bound. In specific, we can always observe a sum-rate increase of $2N$ bps for every $3\,\text{dB}$ increase in SNR. For instance, when $M = 6$, $N = 8$, the curve has a slope of $2N = 16$. Also we can see that our proposed scheme outperforms the conventional TDMA-based scheme significantly.

VII. CONCLUSION

This paper considered the total DoF for the MIMO two-way X relay channel. We analyzed the upper bound of the DoF for such a network when $N \leq 2M$. Then by exploiting physical layer network coding and joint interference cancellation, we proposed a novel transmission scheme to achieve the upper bound when $N \leq \left\lceil \frac{4M}{3} \right\rceil$.