SUSY Violation in Effective Theories

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Abstract

We show that the effective theory of a supersymmetric model can violate SUSY at the level of dimension six operators and higher. This phenomenon occurs in gauge theories which involve heavy vector–superfields and different mass scales. It appears in SUSY GUT theories and is important in models of propagating Higgs boundstates.
The calculation of effective operators in supersymmetric theories plays an important role in the determination of the low energy implications of SUSY GUT theories and other supersymmetric high energy concepts [1]. In the following we show in a toy model that the expansion in local effective operators extracted from a supersymmetric underlying theory can violate SUSY. This phenomenon will turn out to be relevant in SUSY GUTs and supersymmetric models of Higgs boundstates.

An example of effective SUSY violation: Our toy model consists of the MSSM plus an extra heavy gauge vector field $V_S$ which acquires its mass by spontaneous symmetry breaking in an additional heavy Higgs sector. We calculate the effective operators at tree level the following way: We write down the component Lagrangian in the Wess–Zumino gauge. We extract the equations of motion from the full Lagrangian to get "constituent relations" for the heavy fields. These relations also include suppressed derivative terms of heavy fields coming from their kinetic terms. We insert these relations to eliminate the heavy fields in lowest order. Then we re-insert the same relations again to eliminate the suppressed derivative terms of heavy fields. This gives us the correct effective theory up to order $1/M^2$.

The part of the Lagrangian which involves heavy fields has the form

$$L_{\text{heavy}} = \int d^2\theta d^2\bar{\theta}(Q e^{(g_sV_S+g_2V_2+g_QV_1)}Q + R e^{(g_sV_S+g_RV_1)}R + H_S e^{(g_sV_S+g_HV_1)}H_S + \text{Pot}(H_S)) + \text{Pot}(H_S) \tag{1}$$

Throughout this paper superfields will always be denoted by capital letters and component fields by small letters except for the vectorfield which is identifiable by its Dirac index. $Q$ and $R$ are the left-handed and right-handed quark–superfields respectively, $H_S$ is the heavy Higgs, $V_S$ is the heavy vector–superfield, $V_2$ and $V_1$ are the SU(2) and U(1) MSSM gauge–superfields respectively. $\text{Pot}(H_S)$ denotes the operators which produce a scalar potential that breaks the $V_S$ gauge symmetry in a supersymmetric way. One could use a negative Fayet–Iliopoulos term to achieve this. However it is not necessary to specify $\text{Pot}(H_S)$ because the heavy Higgses do not couple to light fields which has the consequence that $\text{Pot}(H_S)$ does not contribute to the effective Lagrangian up to order $1/M^2$. Working in the Wess–Zumino(WZ) gauge the non-physical low-$\theta$ components of the gauge superfield are gauged away and eq. (1) produces, among others, the component terms

$$L_{\text{heavy}} = \tilde{q}^\dagger g_s V_S^\mu (g_2 V_2 + g_Q V_1) \tilde{q} + \tilde{r}^\dagger g_s V_S^\mu (g_R V_1) \tilde{r} + \bar{q} \gamma^\mu g_s V_S q + \bar{r} \gamma^\mu g_s V_S r + ..., \tag{2}$$

where the the small letters always denote the components of the superfield represented by...
the corresponding capital letter. They lead to the following effective terms involving one light vector field and 2 left and right–handed (s)quarks:

\[ L_{\text{eff}} = g_s^2 \bar{q}^\dagger (g_2 V_2 + g_Q V_1) \bar{q} \gamma^\mu r + g_s^2 \tilde{r} \bar{q} \gamma^\mu q + g_s^2 \tilde{r} \bar{q} \gamma^\mu q + g_s^2 \tilde{r} \bar{q} \gamma^\mu q \]

Now the requirement of a supersymmetric structure of the effective theory would imply that each contribution in (3) is part of a superfield D–term. (F–terms cannot include the gauge boson.) The corresponding D–term is

\[ \int d^2 \theta d^2 \bar{\theta} g_s^2 \bar{Q}(g_2 V_2 + (g_Q + g_R) V_1) Q \bar{R} R^c \]  

which, besides (3) and similar terms, includes the component terms

\[ g_s^2 \bar{q} (g_2 V_2 + g_Q V_1) \bar{q} \gamma^\mu r + g_s^2 \tilde{r} \bar{q} (g_2 V_2 + g_Q V_1) \bar{q} \gamma^\mu q + g_s^2 \tilde{r} \bar{q} (g_R V_1) \bar{q} \gamma^\mu q \]

\[ + (g_s^2 \tilde{q} (g_2 \lambda_2 + g_Q \lambda_1) \bar{q} \gamma^\mu r + (q \leftrightarrow r) \]

\[ + g_s^2 \tilde{q} (g_2 D V_2 + g_Q D V_1) \bar{q} \gamma^\mu r \].

The terms of (3), (5) and (7) would be necessary to achieve a supersymmetric structure of our effective theory. But, because of the missing low-\(\theta\) components of the vector superfield in the WZ–gauge, they are not produced by integrating out the heavy sector. This can be most obviously seen in the case of (6)[see Fig. 1]. The light gauginos cannot couple to the heavy sector: Couplings to heavy gauge bosons are excluded because they belong to a different simple gauge group and couplings to heavy scalars would produce large gaugino masses. Thus a local effective coupling to the light gauginos could only come from covariant kinetic terms of squarks. But in the WZ–gauge the contributions of covariant kinetic terms that produce effective four–coupling terms have to include heavy gauge bosons, heavy gauginos or D–fields. As the \(\theta\)–structure forbids to couple a gaugino plus a second component of a vector superfield to the quark sector in one gauge coupling term, the couplings of type (3) cannot exist in the effective theory.
Fig. 1: SUSY violation due to missing light gaugino couplings. The effective diagrams on the right side belong to the same superfield coupling term. While the first one can be derived from a heavy gauge field exchange, the second one has no correspondence in the full theory. The outer lines denote (s)quark fields. L=1,2.

A similar phenomenon already occurs if one neglects the light gauge fields but takes into consideration the light Yukawa couplings. In this case the heavy Lagrangian has the form

$$L'_{\text{heavy}} = \int d^2\theta d^2\bar{\theta}(\mathcal{Q} e^{g_S V_S} \mathcal{Q} + R^c e^{g_S V_S} \mathcal{R}^c + H_S^e e^{g_S V_S} \overline{H_S^e}) + \text{Pot}(H_S) + \int d^2\theta g_8 HQR .$$

The effective theory now includes most but not all component contributions of the four–superfield operator

$$\frac{g_8^2}{M_S^2} \int d^2\theta d^2\bar{\theta}[(\mathcal{Q} \mathcal{R}^c)(\mathcal{Q} \mathcal{R}^c)] .$$

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The couplings to quark auxiliary fields $F_Q, F_R$ are missing in the effective theory. The missing terms would for example correspond to a term of the type $1/M_S^2(squark)^2(quark)^2(lightHiggs)$ after integrating out $D$– and $F$–fields. This term once again cannot be produced by integrating out heavy gauge bosons.

Only in a model which lacks both, light Yukawa terms and light gauge fields, SUSY is not violated in the effective theory. The lowest auxiliary field contributions now are of order $M_s^{-4}$ and correspond to two–$V_S$ exchange in the full theory. Thus the term (9) seems to be the correct effective operator in this case. The supersymmetric structure of the effective operators in this special case seems to be somewhat connected to the infinite increase of the number of fields which locally coupled in increasing order $1/M_S$ in the effective theory. However we did not find a satisfying structural explanation for the surviving of SUSY here.

The discussion above leads to the rather surprising conclusion that the low energy effective approximation of a supersymmetric theory can violate SUSY. In the described cases the supersymmetric structure of the dimension six four–superfield coupling cannot be achieved by integrating out heavy gauge bosons.

**A characterization of the phenomenon:** The phenomenon is connected to the special nature of the WZ–gauge. The WZ–gauge is not manifestly supersymmetric, it is not preserved by SUSY transformations. In a full gauge theory this is merely a formal problem. One can modify the SUSY transformation by adding a supersymmetric gauge transformation so that this modified SUSY transformation preserves the WZ–gauge. However the situation is different in an effective low energy approximation of this gauge theory: After the heavy vector superfield has been integrated out the effective theory is not gauge invariant anymore and the modified SUSY transformation cannot be applied. Therefore an effective theory gained from integrating out vector superfields in the WZ–gauge is not a supersymmetric theory. Now at first sight the conclusion could be tempting that it is illegal to integrate out in the WZ–gauge. Integrating out in a general gauge would give a supersymmetric result and the problem seems to be solved. We argue however that the opposite is true and the mistake would be made by not using the WZ–gauge: As a matter of fact gauge invariance of the full theory makes some of the vector superfield components non–physical. The non–physical character of these fields cannot be seen anymore in the non–gauge–invariant effective theory because the symmetry transformations to rotate them away are no more defined. Therefore, if one would integrate out in a general gauge, one would incorrectly believe in degrees of freedom which are not there. The only way to avoid this is to use up the gauge freedom already in the full theory to make all non–physical fields vanish. This means that it is necessary to use the WZ–gauge for integrating out heavy degrees of freedom. But that can
imply SUSY violation in the resulting effective theory.

It is instructive to have a short look at a situation where the low–θ components of the vector supermultiplet are physical. This is the case if we replace the heavy Higgs sector by an explicit gauge symmetry breaking mass term \( \int d^2\theta d^\theta m^2 V_S^2 \). This term corresponds to the component operators

\[
\int d^2\theta d^\theta m^2 V_S^2 = mCD + m(\bar{\chi}\lambda + \lambda\chi) + M^\dagger M + N^\dagger N + m^2 V^2 + \bar{\chi}\partial/\partial C + (\partial C)^2 ,
\]

with a scalar \( C \), a fermionic \( \chi \) and the auxiliary fields \( M \) and \( N \). In this case there is no gauge freedom, the low–θ components \( \chi \) and \( C \) get a kinetic term from the superfield mass term defined in eq. (10). Integrating out \( V_S \) in this model leads to a supersymmetric effective coupling. Argued in the framework of the discussion above the effective operators must be supersymmetric because the ‘trick’ with the WZ–gauge cannot be applied. From a different, maybe slightly more physical point of view one can argue the following way: If we have an explicit mass term, the propagation of the heavy fields could be set equal to zero except for the low–θ components which in this case come up to the derivative couplings of the effective coupling term. \( V_S \) can then be seen as something like a vector–like auxiliary field and the \( V_S \) exchange becomes a supersymmetric local operator. Now if we turn on again the kinetic term, this gives just contributions of higher order and cannot harm the supersymmetric effective dimension 6 operator any more. SUSY is protected because the propagation of the heavy fields is no essential feature of the model.

However a full gauge theory is a very different theory and it is exactly this difference that is overlooked by integrating out in a general gauge. In a gauge theory where the mass is provided by spontaneous symmetry breaking via the vev of a Higgs field, the propagation of this Higgs is necessary for the mass term which is of course necessary itself for the procedure of integrating out. Therefore the nonlocal character is essential in this case and SUSY which is no inner symmetry and therefore not blind against propagation is not protected in an effective theory.

It is important to notice the following subtlety in the nature of effective SUSY violation: The full \( 1/M_S^6 \)–expansion of the fundamental theory gives an infinite sum of non–supersymmetric local operators that produce a supersymmetric theory (= the fundamental theory). By neglecting operators of orders higher than e. g. \( 1/M_S^2 \) one violates SUSY. Therefore the actual SUSY violation is of order \( 1/M_S^3 \). Nevertheless the mistake one would make by misjudging the dimension six operators as supersymmetric is of the order \( 1/M_S^2 \) according
to the discussion of the previous section. A supersymmetric structure of the dimension six operators would require additional operators of that dimension which are simply missing. This means that there exist two similar but different supersymmetric theories, one being the theory of supersymmetric dimension six operators and the other being the full theory of heavy gauge superfield exchange. A low energy effective theory that wants to be correct up to order $1/M_S$ can differ from the second only at order $1/M_S^3$ but differs from the first at order $1/M_S^2$.

**Relevance of effective SUSY violation:** In our toy model we observed SUSY violating operators of dimension six. One can easily understand that dimension four and five operators cannot show the same phenomenon. The SUSY violating effect is intimately linked to supersymmetric gauge invariance and therefore only occurs in the supersymmetric sector. But in a scenario of vector superfield exchange effective operators of dimension four and five only stem from the soft breaking sector and thus are not endangered by the described SUSY violating mechanism.

On the level of dimension six effective operators however SUSY violation is a general feature of every model with heavy gauge vector superfield exchange plus couplings that involve just light fields. (The second condition is necessary to exclude the case of a pure heavy gauge field exchange which, as we argued above, is supersymmetric.)

One model where this SUSY violating phenomenon is of special importance is the model of propagating Higgs boundstates like it is realized in supersymmetric top condensation [2]. In a recent paper [3] we argued that the non-renormalizable dimension six four-superfield operator which is introduced in this model to achieve dynamical electroweak symmetry breaking cannot be interpreted as an effective operator for heavy gauge superfield exchange. Now we find one more argument against this interpretation: By neglecting the light Higgs sector of our toy model and adding some soft breaking terms we get exactly the type of heavy gauge theory which had to be considered as the underlying gauge theory of SUSY top condensation. However we saw that this theory does not produce supersymmetric dimension six operators and therefore is fundamentally incapable of leading to SUSY top condensation.

An important field where effective operators of a high scale SUSY theory play a role are SUSY GUT theories. Here the SUSY violating effects also occur albeit not on a level relevant for the low energy effects currently discussed. The only difference between a SUSY GUT and our toy model concerns the gauge structure: While in the toy model we had separate gauge groups for light and heavy vector bosons now both sit in the adjoint representation of one simple group. This different gauge structure implies some differences, if we reconsider our procedure
of integrating out heavy component fields in the WZ–gauge. Concerning for example the light gaugino case now there exist effective operators involving the light gaugino $\lambda_L$. These stem from coupling terms of the form

$$g_{gauge} V_S \lambda_L \lambda_S$$

in the full theory. However these effective operators are of dimension 7 and fail to produce a supersymmetric structure of dimension six operators. The different gauge structure does not change the principle of the discussed SUSY violating effect.

Fig. 2: Diagrams leading to effective dimension 7 operators with a light gaugino in SUSY GUTs. The outer lines again denote (s)quark fields.

In a recent work [3] soft breaking GUT contributions have been calculated by integrating out the superfields in superspace which per definition must always lead to a supersymmetric result. This comes up to integrating out in components in a general gauge, as there is no gauge defined before expanding in components. The method is perfectly valid if one knows in advance that no SUSY violation can occur in the effective terms one wants to calculate. This is the case for dimension 4 and dimension 5 operators like those calculated in [3]: In general one would have to be careful.

A generally correct method would be the following:
- Calculate the equations of motion in superfields.
- If just chiral fields are integrated out, SUSY is safe and the whole discussion can be done in superfields.
- If vector superfields are to be integrated out, expand the equations of motion into component fields, write the vector superfields in the WZ–gauge and carry out a comparison of coefficients.
- The result are constituent relations for the high $\theta$–valued components of the vector superfield plus conditions which force the low $\theta$–valued components of the supercurrent that couples
to the vector superfield to be zero.
- Inserting the constituent conditions and applying the zero conditions one gets the correct low energy effective theory.

**Conclusions:** We described the possibility of SUSY violating effects in non–renormalizable low energy effective theories of supersymmetric models. These effects occur in any fundamental theory with heavy gauge superfields plus any type of Yukawa or gauge coupling between light fields. Examples for this scenario are SUSY GUT theories or attempts to produce SUSY top condensation by heavy vector exchange. SUSY violation happens first at the level of dimension six operators where some of the component operators which would be necessary to provide a supersymmetric structure at this level are not produced in the effective theory. The foundation of the phenomenon lies in the specific character of supersymmetric gauge invariance which is lost in the effective theory. While the missing operators are of order $1/M_S^2$ the SUSY violating effect is nevertheless of higher order because the inclusion of the neglected higher order operators would completely define the full theory and therefore also reinstall SUSY. Still the mistake one would make by constructing the effective theory supersymmetrically would be of order $1/M_S^2$. By integrating out either in components in a general gauge or in superfields one is insensitive to effective SUSY violation and gets wrong results if this phenomenon occurs. It is therefore necessary to integrate out in the WZ–gauge. The most convenient method is to extract the equations of motion in superfield formalism and fix the WZ–gauge by doing a comparison of coefficients in the equations of motion before reinserting them into the Lagrangian.

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