NEW APPROACH TO DESCRIPTION OF
MAJORANA PROPERTIES OF NEUTRAL
PARTICLES

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Two mathematical models based on Pauli transformations including U(1) chiral group and Pauli SU(2) group, that mixes particle and antiparticle states, are developed for description of Majorana properties of neutral particles. The first one describes a system, incorporating left- and right-handed fermions of the same flavor, and it is a generalization of the Majorana model of his pioneer article of 1937 year. The second describes a two-flavor neutrino system with quantum numbers of Zel’dovich - Konopinsky - Mahmoud (ZKM) type. For massless fermions the Pauli symmetry is exact and leads to the conserved generalized lepton charge. It is a Pauli isospace vector, whose different directions are coordinated with Dirac or generalized Majorana properties. In nonzero - mass case the models describe the combined Dirac - Majorana properties of neutral particles, which are characterized either by the generalized lepton charges of ZKM - type or by the eigenvalues of the operator that is the product of the charge operator and chirality. The latter is connected with operator of the structure of Lagrangian mass term or with the generalized flavor number of the second model. The choice of the basic operator depends on the inversion classes (A-B or C-D - types) of the particles with respect to the space inversion. The modified second model can be used for description of neutrino oscillation in the simplest two - flavor case.

Investigation of Majorana properties of neutral particles [1] with neutrino as the most principal of them all is the chief goal of modern weak processes physics. An examination of the properties was essentially stimulated by the recent discovery of oscillations in atmospheric, solar and reactor neutrino fluxes [2] - [4]. Majorana models for neutrino description were developed in literature in two basic versions: either in schemes proceeded from Majorana and Pontecorvo [1,5] (in simplest case including one particle with left-handed

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and right-handed states) or in phenomenological models with two and more left-handed neutrinos of different species (flavors) \[6, 7\] (see also e.g. \[8 - 13\]) which are used today for analysis of the neutrino oscillations. However the absence of quantum characteristics suitable for the description of Majorana-type states is an essential disadvantage of the models.

Meanwhile a new approach to construction of Majorana schemes where the problem is partly solved is possible as it was demonstrated by the author in \[14 - 17\]. It is based on application of general Pauli (chiral - Pauli) transformations group \[18\] (which was Pauli - Pursey group in the terms of 1950’s years \[19 - 21\]). The use of the group allows to construct some examples of phenomenological models in which quantum numbers for description of Majorana states can be introduced. Below two simplest models of such type which are in line with above indicated versions will be presented: the model for one neutral particle with left-handed and right-handed states of the same flavor and a two-flavor neutrino model that includes states of different flavors, for example, electron and muon neutrino. The latter will be used then for description of neutrino oscillations.

The plan of the article is as follows: in §1 the Pauli group terms are introduced, in §2 and §3 the above indicated models are presented and §4 is consequently dedicated an application of the modified two-flavor model to describe the neutrino oscillations. Finally, in §5 we shall discuss peculiarities of Pauli models which make them different from Majorana neutrino phenomenological schemes presented in the literature.

§1. Introduction

As it was demonstrated for the first time by Pauli \[18\] the fermion fields of zero mass are symmetric relatively to the following transformations:

$$\psi'(x) = e^{i\pi \gamma^5/2}(a\psi(x) + b\gamma_5\gamma_3\psi^T(x)) = e^{i\pi \gamma^5/2}(a\psi(x) + b\gamma_5\psi^C(x)),$$

$$|a|^2 + |b|^2 = 1, \quad \psi^C(x) = C\psi^T(x) = \eta_C\gamma_2\gamma_4\psi^T(x), \quad (\eta_C = 1 \quad [22]).$$

These transformations involve pure Pauli SU(2) - group (type I of Pauli \[18\]) and chiral U(1) - transformations (type II) and conserve commutation relations of the fields. The former one for $a = e^{i\phi/2}, \ b = 0$ can be reduced to the phase transformation subgroup on whose base the conserved lepton charge can be put into the scheme. With introduction of $\hat{\kappa}_i \ (i = x, y, z)$ operators and the generalized two-component function $\Psi(x)$ one receives:

$$\hat{\kappa}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \hat{\kappa}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \hat{\kappa}_z = \begin{pmatrix} +1 & 0 \\ 0 & -1 \end{pmatrix},$$

$$\Psi(x) = \begin{pmatrix} \psi(x) \\ \gamma_5\psi^C(x) \end{pmatrix}, \quad (\hat{\kappa}_i \Psi(x) = \Psi(x)).$$
It is readily shown that for 
\[ a = e^{i\varphi/2}\cos\theta/2, \quad b = e^{-i\phi/2}e^{-i\phi/2} \]
transformation (1) can be reduced to the standard form:
\[ \Psi'(x) = e^{i\gamma}S(\chi)S(\phi, \theta)\Psi(x), \]
where 
\[ \kappa = \cos\theta\kappa_z + \sin\theta\cos\phi\kappa_x + \sin\theta\sin\phi\kappa_y, \quad S^+(\phi, \theta)\kappa_zS(\phi, \theta) = \kappa, \quad (3) \]
which can be interpreted in terms of rotations in chiral and Pauli subspaces. The latter includes 
\[ S(\phi) = e^{i\hat{\kappa}_z\phi/2} \]
rotations about \( \kappa_z \) axis and 
\[ S(\phi, \theta) = S^+(\phi, \theta)\kappa_zS(\phi, \theta) = \kappa, \]
transformation of \( \vec{\kappa}_z \) vector to the direction of \( \vec{\kappa} \), defined by the standard Euler angles \( \phi, \theta \). Some other representations of the generalized function are also possible. In this case the (3) form of Pauli transformations should be consequently modified.

The analysis of the conservation condition of Pauli transformations under CPT - operation \[22, 23\] shows (see details in \[16\]) that CPT - invariance leads to the following relation between phases \( \eta_P, \eta_T \) of discrete P- and T-transformations, introduced in coordination with the standard definition (see \[22\]), and \( b \) parameter of Pauli transformations:
\[ b(1 + \eta^2_P/\eta^2_T) = 0, \quad or \quad \frac{\eta^2_P}{\eta^2_T} = -1 \quad for \quad b \neq 0. \quad (4) \]
The latter relation is in accordance with the well known condition \( \eta^2_P = -1, \quad \eta^2_T = +1 \) adopted usually for physical particles \[22\]. Fermions of \( \eta_P = \pm i \) will be called particles of inversion of A - and B - classes \[24\]. However for Majorana particles one can not a priori exclude the other possible choice \( \eta_P = \pm 1 \), that Racah, for the first time, called attention to (see \[25\] and also \[26, 27\]). We shall describe such fermions as particles of inversion of C - and D - classes \[2\] and treat them hypothetical particles for which \( \eta^2_P = +1, \quad \eta^2_T = -1. \)

The inversion classes of neutral particles are connected with their Majorana properties. Indeed, let us consider the reasonably general conditions of Majorana type in the following forms:
\[ \psi^C(x, \zeta) = \lambda e^{i\phi}\psi(x, \zeta) \quad (A), \quad \psi^C(x, \zeta) = \lambda e^{i\phi}\gamma_5\psi(x, \zeta) \quad (B), \]
(\( \lambda - \) arbitrary real number, \( \zeta - \) quantum numbers of states), \quad (5)

It is easily shown that the former (5) (A) condition is fulfilled for particles of inversion A-B - classes only while the latter (5) (B) is fulfilled for C-D - ones.

\[2\] There is no prescribed designation of inversion classes of the particles in literature, here we are based on definitions of monograph \[24\], which are however opposite to designations of \[26\].
Note that the limitation of (5) \((A)\) condition is used in modern Majorana models [10] - [13]. It means that belonging of the particles to inversion A-B - classes is implicitly included in such models. However we shall not put this limitation into operation in this work and let the both capabilities be realized. In doing so the general Majorana conditions (5) \((B)\) should be used for particles of C-D - classes.

§2. Majorana properties of a neutral free fermion

The fundamental existence of Majorana properties of a neutral fermion was firstly demonstrated by E. Majorana. Basing on Dirac equation he showed that it has a special solution that meets condition \(\psi^C(x) = \psi(x)\) [1].

From the modern viewpoint he constructed a particular solution that is a superposition of particle and antiparticle states for the simplest case of one particle. We shall reveal below that basing on Pauli transformations one can generalize this result and construct a Majorana model for particles of one type that contains solutions obeying similar but more general conditions \((A), (B)\). Simultaneously a generalized concept of lepton charge will appear in the model. We shall initially develop the model for massless particles and then for the case of massive ones (this model was firstly proposed and investigated by the author in [14] - [16]).

Let us examine a fermion field of zero mass which is symmetric under Pauli transformations (1). The Lagrangian of the field presented in generalized functions (2) is as follows:

\[
L_0(x) = -\frac{1}{2} \left[ \overline{\Psi}(x) \gamma^\mu \partial_\mu \Psi(x) \right], \quad \Psi(x) = \left( \begin{array}{c} \psi(x) \\ \gamma^5 \psi^C(x) \end{array} \right),
\]

(here, wave functions are of secondary quantization). Note that the chosen form of generalized functions is universal and it is invariant with respect to Pauli transformations (1). The invariance of the Lagrangian under chiral \(S(\chi)\) and phase \(S(\varphi)\) transitions leads to the conserved chiral and lepton charges:

\[
Q^{CH} = \frac{1}{2} \int d^3x \left[ \overline{\psi}(x) \gamma_5 \psi(x) + \psi^C(x) \gamma_5 \psi^C(x) \right],
\]

\[
Q^L = Q^P = \frac{1}{2} \int d^3x \left[ \overline{\psi}(x) \gamma_5 \psi(x) + \psi^C(x) \right],
\]

\[
\hat{k}_z \left( \begin{array}{c} \psi(x) \\ \gamma^5 \psi^C(x) \end{array} \right) = \left( \begin{array}{c} \psi(x) \\ \gamma^5 \psi^C(x) \end{array} \right).
\]

They stipulate some characteristics of basic eigenfunctions \(\Psi_0(x)\), those are chirality \(\rho = \pm 1\) (\(L, R\)) and lepton charge \(q_z = \pm 1\):

\[
\gamma_5(\Psi_0)_{\rho,q_z}(x) = \rho(\Psi_0)_{\rho,q_z}(x), \quad \hat{k}_z(\Psi_0)_{\rho,q_z}(x) = q_z(\Psi_0)_{\rho,q_z}(x),
\]

\[
(\Psi_0)_{\rho,q_z=+1}(x) = \left( \begin{array}{c} \psi_0(x) \\ 0 \end{array} \right), \quad (\Psi_0)_{\rho,q_z=-1}(x) = \left( \begin{array}{c} 0 \\ \gamma_5 \psi^C_0(x) \end{array} \right) = \left( \begin{array}{c} 0 \\ \rho \psi^C_0(x) \end{array} \right).
\]
As it follows from (7) in the space of Pauli transformations the lepton charge operator \( \hat{\kappa}_z \) is connected with a vector oriented along \( z \)-axis in a way that particle solution \( q_z = +1 \) and antiparticle one \( q_z = -1 \) are matched with two possible projections of the vector to \( z \)-axis. The conservation of the lepton charge is evident from invariance of the Lagrangian under rotations about the vector. In Pauli scheme this vector describes the lepton charge. Note that in the case of massless particle there are no physical reasons for separation of \( z \)-axis in comparison with other directions.

Such a peculiarity of Pauli scheme lets one to introduce a concept of generalized lepton charge \( Q \) coordinated with an arbitrarily chosen direction of Pauli isospace. In this case its \( z \)-component conserves the connection with lepton charge \( Q_z \) and \( x \)-, \( y \)-components occur to coordinate with Majorana properties of the particle. The generalized lepton charge can be obtained from \( Q_z \) by using Pauli transformation \( S^+(\phi, \theta) \) of \( \Psi_0(x) \) wave function and \( \hat{\kappa}_z \) operator. It takes the form:

\[
Q^P = \frac{1}{2} \int d^3x \Psi^+(x) \hat{\kappa} \Psi(x) = \cos \theta Q^P_z + \sin \theta (\cos \phi Q^P_x + \sin \phi Q^P_y) = \frac{1}{2} \int d^3x \left[ \cos \theta (\psi^+(x) \psi(x) - \psi^C+(x) \psi^C(x)) + \sin \theta (\psi^+(x) e^{-i\phi} \gamma_5 \psi^C(x) + \psi^C+(x) e^{+i\phi} \gamma_5 \psi(x)) \right],
\]

and is coordinated with \( \kappa \) direction defined in Pauli isospace by standard Euler angles \( (\phi, \theta) \). The eigenfunctions of the generalized lepton charge operator with \( \rho, q = \pm 1 \) quantum numbers are constructed from the generalized functions \( \Psi_0(x) \) by using the same transformations:

\[
\hat{\kappa} \Psi_{\rho,q}(x) = q \Psi_{\rho,q}(x),
\]

\[
\Psi_{\rho,1}(x) = \begin{pmatrix} \psi_{\rho,1}(x) \\ \gamma_5 \psi^C_{\rho,1}(x) \end{pmatrix} = \begin{pmatrix} \cos \left( \frac{\theta}{2} \right) \psi_{\rho,0}(x) \\ e^{i\phi} \sin \left( \frac{\theta}{2} \right) \psi_{\rho,0}(x) \end{pmatrix},
\]

\[
\Psi_{\rho,-1}(x) = \begin{pmatrix} \psi_{\rho,-1}(x) \\ \gamma_5 \psi^C_{\rho,-1}(x) \end{pmatrix} = \begin{pmatrix} -e^{-i\phi} \sin \left( \frac{\theta}{2} \right) \psi_{\rho,0}(x) \\ \rho \cos \left( \frac{\theta}{2} \right) \psi_{\rho,0}(x) \end{pmatrix}.
\]

They describe consequently two independent solutions of different signs \( q = \pm 1 \). With fixed values of the generalized lepton charge \( q \) there are following relations between components \( \psi^C_{\rho,q}(x) \) and \( \psi_{\rho,q}(x) \) of new system of the eigenfunctions conserving their universal form:

\[
\psi^C_q(x) = q (\tan \left( \frac{\theta}{2} \right))^{q} e^{i\phi} \gamma_5 \psi_q(x), \quad (q = \pm 1), \quad \psi_q(x) = \sum_{\rho} \psi_{\rho,q}(x), \quad \Psi_q(x) = \sum_{\rho} \Psi_{\rho,q}(x) = \left( \begin{pmatrix} \psi_q(x) \\ \gamma_5 \psi^C_q(x) \end{pmatrix} \right), \quad \int d^3x \Psi^+_q(x) \Psi^T_q(x) = \delta_{q,q},
\]
They are projection conditions which set up the definite eigenvalues \( q \) and corresponding eigenfunctions of the generalized lepton charge. These conditions have a form of modified Majorana relations of (5) \((B)\) - type and are fulfilled only in a case when a particle under investigation belongs to inversion C-D - classes. It is seen that the choice of generalized lepton charge operator as the basic one is coordinated with definite inversion classes of the particle.

For description of particles of A-B - inversion classes it is necessary to choose the product of chirality and generalized lepton charge operator as basic operator. In doing so it is useful to represent the generalized wave function in a new universal form, that is also invariant under arbitrary chiral - Pauli transformations. In the case the general form of these changes of transformations (3) is following:

\[
\Phi(x) = \left( \psi_L(x) + \eta \psi_C^R(x) \right),
\]

\[
\Phi'(x) = e^{i\gamma_5/2} e^{i\hat{\kappa}_z \gamma_5/2} e^{i\eta (\cos \phi \hat{\kappa}_y - \sin \phi \hat{\kappa}_x \gamma_5)} \Phi(x) = S(\chi) S'_1(\varphi) S'_2(\phi, \theta) \Phi(x).
\]

In the new representation, the Pauli isospace is modified. Now it is based on \( \kappa_x \gamma_5, \kappa_y, \kappa_z \gamma_5 \) basic vectors coordinated with x, y, z - axes that corresponds to the form of Pauli rotations and lepton charge operators. In the basic case correspondent with (8) the lepton charge operator takes \( \hat{\kappa}_z \gamma_5 \) form and its product with the chirality operator is described by \( \hat{\kappa}_z \) form. In this representation the eigenfunctions of \( \kappa_z = \rho q_z \) quantum numbers and \( \eta = \pm 1 \) charge parity have the following form:

\[
\hat{\kappa}_z(\Phi_0)_{\kappa_z, \eta}(x) = \kappa_z(\Phi_0)_{\kappa_z, \eta}(x);
\]

\[
(\Phi_0)_{\kappa_z=+1, \eta}(x) = \left( \psi_{0L}(x) + \eta \psi_{0C}^R(x) \right), \quad (\psi_{0R}(x) = 0);
\]

\[
(\Phi_0)_{\kappa_z=-1, \eta}(x) = \left( \psi_{0R}(x) + \eta \psi_{0C}^L(x) \right), \quad (\psi_{0L}(x) = 0).
\]

The lepton charge in \( \Phi_0(x) \) state is defined by the relation:

\[
Q^L = Q^P_z = \int d^3x \Phi_0^*(x) \hat{\kappa}_z \gamma_5 \Phi_0(x).
\]

The functions (13) describe two Majorana solutions of \( \eta \) charge parity constructed either of the left particles and their right antiparticles \((\kappa_z = +1)\) or of the right particles and their left antiparticles \((\kappa_z = -1)\). Inserting them in (14) one can readily make sure that the mean value of the lepton charge
is equal to zero in these states. It is evident that \( \hat{\kappa} \) operator coordinates with \( z \)-axis of the modified Pauli space and two Majorana solutions coordinate with two projections onto \( z \)-axis of a vector, created as a product of chirality and the lepton charge vector. In the general case, that can be derived from (13) by using Pauli rotation \( S^+(\phi, \theta) \), the eigenfunctions of \( \hat{\kappa} \) operator, which correspond to the product of chirality and the generalized lepton charge, are as follows:

\[
\hat{\kappa}\Phi_{\kappa,\eta}(x) = \kappa\Phi_{\kappa,\eta}(x),
\]

\[
\Phi_{+1,\eta}(x) = \left(\frac{\cos\left(\frac{\eta}{2}\right)(\psi_{L}(x)+\eta\psi_{C}(x))}{\eta e^{i\gamma_5}\sin\left(\frac{\eta}{2}\right)(\psi_{L}(x)+\eta\psi_{C}(x))}\right);
\]

\[
\Phi_{-1,\eta}(x) = \left(\frac{-\eta e^{-i\gamma_5}\sin\left(\frac{\eta}{2}\right)(\psi_{L}(x)+\eta\psi_{C}(x))}{\cos\left(\frac{\eta}{2}\right)(\psi_{L}(x)+\eta\psi_{C}(x))}\right),
\]

The generalized lepton charge is consequently as follows:

\[
Q^P(\kappa) = \int d^3x \Phi_{\kappa,\eta}^+(x)\hat{\kappa}\gamma_5\Phi_{\kappa,\eta}(x),
\]

\[
\hat{\kappa}\gamma_5 = \cos \theta \hat{\kappa}_z\gamma_5 + \eta \sin \theta (\cos \phi \hat{\kappa}_x\gamma_5 + \sin \phi \hat{\kappa}_y).
\]

These expressions describe a pair of Majorana solutions \( \kappa = \pm 1 \) in terms of the previous system of eigenfunctions. Obviously that for the states with fixed \( \kappa \) quantum number the mean value of the generalized charge tends to zero in the general case as well. For fixed \( \kappa \) values the following connections arise between separate chiral components \( \psi_{\rho,\kappa}(x) \) and \( \psi_{\rho,\kappa}^C(x) \) entering into the universal form of the generalized function (12):

\[
\Phi_{\kappa,\eta}(x) = \left(\psi_{L}(x)+\eta\psi_{C}(x)\right),
\]

\[
\psi_{\rho,\kappa}^C(x) = \kappa(\tan \left(\frac{\rho}{2}\right))^{\rho \kappa} e^{i\phi} \psi_{\rho,\kappa}(x) \quad (\rho = \pm 1 \ (L, \ R)),
\]

They have the form of generalized Majorana conditions of (5) (A) type. They are projection conditions which set off the definite eigenvalues of \( \hat{\kappa} \) operator as it was in case (11), however, here they are consistent with the case when particles under investigation are of inversion A-B - classes. Thus, it occurs that in general case of Pauli scheme the choice of basic operators using for description of eigenfunctions of the states occurs to be directly correlated with inversion classes of the particles. In case of their being of inversion C-D - classes the generalized lepton charge operator should be chosen as a basic one. And if they are of inversion A-B - classes its product with chirality operator should be chosen. The projection conditions to set off fixed eigenfunctions of the basic operator, have the form of general ones of
either (3) (B) or (A) types. While the considerations have been made on the base of investigation of massless case the approach will also hold for particles of nonzero mass with some exception as we shall show below.

Let us turn now to the description of massive particles and state the general concept for this case. In accordance with views of Standard Model the masses of particles are determined by mean vacuum value of Higg’s field that has its origin in spontaneous breakdown of a basic symmetry proposed for them with their masses being zero. The introduction of the mass terms in Lagrangian (3) that is symmetric under Pauli transformations (1) is connected with the breakdown of the symmetry. It is possible to suppose that in the model under investigation the Pauli symmetry plays a role of the basic symmetry. Basing on example of a particular Dirac case one can see that the breakdown leads to production of separated directions in chiral and Pauli subspaces. If the mechanism of the broken symmetry is universal, the Higg’s vacuum values, because of different subspace directions, are connected after breakdown by the same transformations as the directions themselves. Basing on Dirac case with a well known mechanism of mass generation, one can construct in the same manner the Lagrangian mass terms of the general form including both Dirac and Majorana terms for the model under investigation. As a result the Pauli scheme for massive free particles, which in basic features is in coincidence with standard phenomenological scheme of the modern Majorana models [8] - [13], arises and it occurs to be a particular case of the models. However, it has some important peculiarities due to Pauli transformations which it is based on. The scheme is presented just below (for more details see [15] - [16]).

Let us begin from the Dirac case with describing it in representation (12) of generalized wave functions. In the representation the Lagrangian of Dirac type, Dirac equation and corresponding lepton charge have the following forms:

$$L(x) = L_0(x) - \frac{\hbar}{2} \overline{\Phi}_D(x) \hat{\kappa}_x \Phi_D(x),$$
$$Q^L = Q^P_z = \frac{1}{2} \int d^3 x \Phi_D^+(x) \hat{\kappa}_z \gamma_5 \Phi_D(x),$$
$$\left( \gamma_\mu \partial_\mu + M \hat{\kappa}_x \right) \Phi_D(x) = 0, \quad \Phi_D(x) = \left( \psi_{DL}(x) + \eta \psi_{DR}(x) \right) \left( \psi_{DL}(x) + \eta \psi_{DR}(x) \right).$$

As one can see from these expressions the invariance of Dirac Lagrangian under chiral and pure Pauli groups of general Pauli transformations is broken. The source of the breakdown is in the mass term of Lagrangian. However, it does not break the invariance under the phase subgroup of Pauli rotations about z-axis that is connected with the conserved lepton charge
\( Q^L = Q^P \). The expression of the lepton charge through generalized Dirac functions \( \Phi_D(x) \) is similar to (14) for massless fermions. Consequently it does not depend on mechanism of the breakdown introduced by mass terms. In the chosen representation the operator of lepton charge is coordinated with z - axis of the Pauli isospace while the operator of structure of the mass term is coordinated with x - axis. Thus in representation (12) for generalized wave functions in the Dirac case the spontaneous breakdown introduced by the mass term, leads to the production of two different isolated directions: z - axis of Pauli rotations responsible for the existence of conserved lepton charge and x - direction, whose operator determining the structure of the mass term is connected with.

It is easy to verify that the mass term of the Dirac Lagrangian is not sensitive to the inversion class of a particle. For this reason the alternative choice for the basic operator of representation of eigenfunctions is possible in the Dirac case: it is either the lepton charge operator or its product with chirality. The latter one occurs to be connected with the operator of the structure of the Lagrangian mass term.

The representation (18) is in compliance with the choice of lepton charge operator. The eigenfunctions of \( q_z = \pm 1 \) charge take the following forms in this case:

\[
\begin{align*}
\hat{k}_z \gamma_5 \Phi_{D,q_z}(x) &= q_z \Phi_{D,q_z}(x), \\
\Phi_{D,q_z=+1}(x) &= (\psi_{DL}(x)) , \\
\Phi_{D,q_z=-1}(x) &= (\eta \psi_{CR}(x)), \\
\int d^3x \Phi^+_{D,q_z}(x) \Phi_{D,q'_z}(x) &= \delta_{q_z,q'_z}.
\end{align*}
\]

Such a representation of solutions is equivalent to a normal description of a particle as "Dirac" like ("Dirac neutrino"), when the particle and antiparticle states differ by the lepton charge values. An alternative representation can be deduced from (18) by Pauli rotation \( S^+ (\phi = 0, \theta = -\eta \pi/2) \) of wave function \( \Phi_D(x) \). Then the Dirac Lagrangian, the Dirac equation and the lepton charge are transformed to the form:

\[
L(x) = L_0(x) - \frac{M^2}{2} \overline{\Phi}_{MD}(x) \hat{k}_z \Phi_{MD}(x), \\
Q^P = -\frac{1}{2} \int d^3x \overline{\Phi}_{MD}^+ \hat{k}_z \gamma_5 \Phi_{MD}(x), \\
\Phi_{MD}(x) = e^{i\hat{k}_z \eta \pi/4} \Phi_D(x) = \frac{1}{\sqrt{2}} \left( \psi_{DL}(x) + \psi_{DR}(x) + \eta \psi_{CR}(x) \right), \\
\]

The basic operator \( \hat{k}_z \) defining the structure of the mass term in the representation is in compliance with product \( (\hat{k}_z \gamma_5) \) (which is the basic operator
of the charge representation (18), and $(\gamma_5)$ chirality. It is coordinated with $z$-axis and its eigenfunctions, specified by $\kappa_z = \pm 1$, $\eta = \pm 1$ eigenvalues, are as follows:

$$\hat{\kappa}_z(\Phi_{MD})_{\kappa_z,\eta}(x) = \kappa_z(\Phi_{MD})_{\kappa_z,\eta}(x);$$
$$\Phi_{MD}^{+1,\eta}(x) = \frac{1}{2}(\psi_D(x) + \eta\psi^*_D(x)) = \left(\psi_{DL}(x) + \eta\psi_{DR}(x)\right);$$
$$\Phi_{MD}^{-1,\eta}(x) = \frac{1}{2}(\psi_D(x) - \eta\psi^*_D(x)) = \left(\psi_{DL}(x) - \eta\psi_{DR}(x)\right);$$

(21)

and can be interpreted in terms of two independent Majorana particles $\psi_{MD1}(x), \psi_{MD2}(x)$. They generate a complete set of solutions similar to a particle - antiparticle pair, so that every solution can be represented as a superposition of two latter ones and vice versa. (Note that (21) fits the secondary quantized form and the normalization of the eigenfunctions, as compared with (20), changes when some additional Majorana conditions are taken into account.) The eigenvalue $\kappa_z = +1$ describes the Majorana solution of the $\eta$ charge parity, being set off $\psi^*_D(x) = \eta\psi_D(x)$ projection condition, and $\kappa_z = -1$ one describes the solution of the same charge parity, resulting from the condition $\psi^*_D(x) = -\eta\psi_D(x)$. The mean value of the lepton charge for each of them is equal to zero. These solutions are known as ”Majorana solutions” (”Majorana neutrino”). The first of them (for $\eta = +1$) was suggested for the first time by Majorana in [1].

Therefore, there are two alternative solutions in the Dirac case due to a different choice of the isolated directions in the Pauli isospace with a spontaneous symmetry breakdown being responsible for the appearance of the mass terms. In the charge (”Dirac”) representation the lepton charge is connected with a vector, directed along the $z$-axis, and the mass term alignes with a vector along the $x$-axis. In ”Majorana” (mass) representation the situation is contrary. Here, the $z$-axis is aligned with the vector connected with the operator of structure of the Lagrangian mass term and the lepton charge is aligned with the $x$-axis. The Pauli transformation connects them by the rotation about the $y$-axis to the $\frac{\pi}{2}$ angle. Owing to the independence of the Dirac Lagrangian from inversion classes, both alternative representations can be used for a description of physical particles of the inversion A-B - classes as well as hypothetical particles of C-D - classes.

In contrast with the Dirac case the choice of the basic operator of representation in the general case of the Pauli scheme is essentially connected
with inversion classes of the particle under investigation. In conventional
designations the most general form of the Lagrangian for the Pauli model
and its generalized charge are as follows:

\[
L(x) = L_0(x) - \frac{\mu}{2} \left\{ \cos \theta (e^{i\xi} \bar{\psi}_R(x) \psi_L(x) + e^{-i\xi} \bar{\psi}_L(x) \psi_R(x)) + e^{i\xi} \bar{\psi}_L(x) \psi_R(x) + e^{-i\xi} \bar{\psi}_R(x) \psi_L(x) + \sin \theta \bar{\psi}_R(x) \psi_L(x) + \bar{\psi}_L(x) \psi_R(x) \right\},
\]

\[Q^P = \frac{1}{2} \int d^4x \left[ \cos \theta (\psi^+_L(x) \bar{\psi}_L(x) - \psi^+_R(x) \bar{\psi}_R(x)) - \psi^+_R(x) \bar{\psi}_R(x) + \psi^+_L(x) \bar{\psi}_L(x) - \psi^+_L(x) \bar{\psi}_R(x) + \psi^+_R(x) \bar{\psi}_L(x) - \psi^+_L(x) \bar{\psi}_R(x) \right],
\]

(22)

\[
Q^P = \frac{1}{2} \int d^4x \left[ \cos \theta (\psi^+_L(x) \bar{\psi}_L(x) - \psi^+_R(x) \bar{\psi}_R(x)) + \sin \theta (\psi^+_L(x) e^{-i\xi} \bar{\psi}_R(x) + \psi^+_R(x) e^{-i\xi} \bar{\psi}_L(x) - \psi^+_L(x) e^{i\xi} \bar{\psi}_R(x) - \psi^+_R(x) e^{i\xi} \bar{\psi}_L(x)) \right],
\]

(23)

Here, the latter expressions define the designations due to a new discrete
operation of the generalized charge conjugation (GC - conjugation). In fact,
it is easy to verify that the Lagrangian (22) is invariant under the following
transformation:

\[
\psi_L(x) \leftrightarrow \psi_R^{GC}(x), \quad \psi_R(x) \leftrightarrow \psi_L^{GC}(x),
\]

which changes the sign of the generalized charge \(Q^P\). There are two phase
factors in the definition of GC - conjugation operation: the universal one
depending on \(\phi\) angle, whose introduction is equivalent to entering the enera!al
phase factor \(\eta_C = e^{-i\phi}\) into the definition of the standard charge conjugation
operation (1), and an extra one which depends on chiral characteristics of
a particle and is a peculiarity of the Pauli scheme. The latter takes into
account a fact that the phase factors \(\eta_C\) for left- and right - handed particles
can principally be different. By comparing the general Pauli Lagrangian (22)
with the existing Majorana models [10] - [12], [29] it is easy to verify that
the case in question is connected with the breakdown of CP - conservation.

Note a peculiarity of mass terms of the general Lagrangian and the
generalized charge (22) that is typical of the model under investigation. Their
Dirac parts (proportional to \(\cos \theta\)) are scalar under the space reflection inde-
dependently of inversion classes of particles. However the similar properties of
their Majorana terms (proportional to \(\sin \theta\)) are essentially dependent on the
inversion classes. For C-D - classes of particles the Majorana terms are scalar,
but for A-B - classes some of them are scalar and the others are pseudoscalar.
Consequently the choice of a basic operator of the representation using for
particles eigenfunctions description turns to be dependent in general on their inversion classes.

In move from the Dirac case \[^{(18)}\] to the general form of the charge representation using the generalized lepton charge as a basic operator, the Lagrangian and the generalized charge take the following forms:

\[
L(x) = \frac{M}{2} \Phi^{GC}(x) [\cos \theta (\cos \chi \kappa_x + \sin \chi \kappa_y) - \eta \sin \theta \kappa_z] \Phi^{GC}(x),
\]
\[
Q^P = \frac{1}{2} \int d^3 x \Phi^{GC^+}(x) [\cos \theta \kappa_z + \eta \sin \theta (\cos \chi \kappa_x + \sin \chi \kappa_y)] \gamma_5 \Phi^{GC}(x).
\]

GC - functions entered into expressions are consisted with the form \[^{(12)}\] with an operator of C - conjugation being exchanged by GC - one and with having their own Pauli transformations law of the following form:

\[
\Phi^{GC}(x) = \left(\psi_L(x) + \eta \psi_R^{GC}(x)\right) = \left(\psi_L(x) + \eta e^{-i (\chi + \phi)} \psi_R^{GC}(x)\right),
\]
\[
\psi^{GC}(x) = e^{i\eta (\cos \chi \kappa_y - \sin \chi \kappa_x) \theta/2} e^{i \kappa_x 5 \phi/2} e^{i \gamma 5 \chi/2} \Phi^{GC}(x) = S' (\chi, \theta) S'(\varphi) S(\chi) \Phi^{GC}(x).
\]

It is essentially to underline that with the use of GC - functions the values of \[^{(24)}\] expressions can be newly interpreted as vectors of the basic Pauli isospace constructed on \(\kappa_x, \kappa_y, \kappa_z\) vectors related to \(x, y, z\) - axes. In this case, the expressions \[^{(24)}\] contain \(\chi\) and \(\theta\) angles only while the \(\phi\) angle, after its having been introduced into the definition of the generalized GC - function, is not already explicitly included.

The eigenfunctions of the fixed generalized lepton charge, being originated from \[^{(19)}\] as a result of the general Pauli transformation, take the following forms:

\[
[\cos \theta \kappa_z + \eta \sin \theta (\cos \chi \kappa_x + \sin \chi \kappa_y)] \gamma_5 \Phi_q(x) = q \Phi_q(x),
\]
\[
\Phi_q(x) = S'^+ (\chi, \theta) e^{-i \kappa_x (\chi/2)} \Phi_{D^q}(x), (q = q_z),
\]
\[
\Phi^{GC}_q = S'^+ (\chi, \theta) e^{-i \kappa_x (\chi/2)} \Phi_{D^q}(x), (q = q_z) = \left(\psi_L(x) + \eta \psi_R^{GC}(x)\right) \left(\psi_L(x) + \eta \psi_R^{GC}(x)\right)_{q=+1} = \left(e^{-i \chi/2} \cos (\theta/2) \psi_{DL}(x) + \eta e^{-i \chi/2} \sin (\theta/2) \psi_{DR}(x)\right) \left(e^{-i \chi/2} \cos (\theta/2) \psi_{DL}(x) + \eta e^{-i \chi/2} \sin (\theta/2) \psi_{DR}(x)\right);
\]
\[
\Phi^{GC}_q = S'^+ (\chi, \theta) e^{-i \kappa_x (\chi/2)} \Phi_{D^q}(x), (q = q_z) = \left(\psi_L(x) + \eta \psi_R^{GC}(x)\right) \left(\psi_L(x) + \eta \psi_R^{GC}(x)\right)_{q=-1} = \left(-e^{-i \chi/2} \sin (\theta/2) \psi_{DL}(x) + \eta e^{-i \chi/2} \cos (\theta/2) \psi_{DR}(x)\right) \left(e^{i \chi/2} \sin (\theta/2) \psi_{DL}(x) + \eta e^{i \chi/2} \cos (\theta/2) \psi_{DR}(x)\right).
\]

As it follows from these relations, the modified projection Majorana conditions typical of the inversion C-D - classes particles are fulfilled for separate
components of these eigenfunctions with the quantum numbers \( q = \pm 1 \):

\[
\psi^C_q(x) = \sum_\rho \psi^C_{q,\rho}(x) = q(\tan(\theta/2))^q e^{i\phi} \gamma_5 \psi_q(x).
\]  

(27)

A different situation arises in a process of generalization of the Majorana representation (29) when the structure operator of the Lagrangian mass terms is chosen as basic. Then the general Lagrangian and the charge in terms of GC-functions take the forms:

\[
L(x) = L_0(x) - \frac{M}{2} \Phi^{GC}_M(x) \left[ \cos \theta \tilde{\kappa}_z + \eta \sin \theta (\cos \chi \tilde{\kappa}_x + \sin \chi \tilde{\kappa}_y) \right] \Phi^{GC}_M(x),
\]

\[
Q^2 = -\frac{1}{2} \int d^3x \Phi^{GC}_M(x) \left[ \cos \theta (\cos \chi \tilde{\kappa}_x + \sin \chi \tilde{\kappa}_y) - \eta \sin \theta \tilde{\kappa}_z \right] \gamma_5 \Phi^{GC}_M(x),
\]

\[
\Phi^{GC}_M(x) = e^{i(\cos \chi \tilde{\kappa}_y - \sin \chi \tilde{\kappa}_x)\pi/4} \Phi^{GC}(x) = \frac{1}{\sqrt{2}} \left( \psi_L(x) + e^{-i\chi} \psi_R(x) + \eta \psi^{GC}_L(x) + \eta e^{-i\chi} \psi^{GC}_R(x) \right).
\]

(28)

The correspondent eigenfunctions of mass structure term operator have in general case the following form:

\[
\Phi^{GC}_{M=\pm 1}(x) = \frac{1}{\sqrt{2}} \left( \psi_L(x) + e^{-i\chi} \psi_R(x) + \eta \psi^{GC}_L(x) + \eta e^{-i\chi} \psi^{GC}_R(x) \right)_{\kappa=\pm 1} = \frac{1}{\sqrt{2}} \left( \psi_L(x) + e^{-i\chi} \psi_R(x) + \eta \psi^{GC}_L(x) + \eta e^{-i\chi} \psi^{GC}_R(x) \right)_{\kappa=\pm 1}
\]

(29)

In the case of \( \Phi^{GC}_{M}(x) \) expressions, the formulae meet projection conditions leading to the Majorana conditions given below. The latter are typical of the inversion A-B - classes particles and do not depend on the chiral angle:

\[
\psi_{\kappa,L}(x) = \kappa \eta(\tan(\eta_2^0 + \frac{\pi}{4}))^\kappa e^{i\phi} \psi_{\kappa,L}(x),
\]

\[
\psi_{\kappa,R}(x) = \kappa \eta(\cot(\eta_2^0 + \frac{\pi}{4}))^\kappa e^{i\phi} \psi_{\kappa,R}(x),
\]

\[
\psi_{\kappa,\rho}(x) = \kappa \eta(\tan(\eta_2^0 + \frac{\pi}{4}))^\kappa e^{i\phi} \psi_{\kappa,\rho}(x), \quad \rho = \pm 1(L, R).
\]

(30)

Thus, the representations (24) and (28) are two possible general forms of description of the particles in the model under investigation. The choice of the basic operator of the representation (24) or (28) forms is defined by the inversion class of the particles. For A-B - classes (physical) particles it is an operator that specifies the structure of the Lagrangian mass term. On the contrary for the C-D - classes (hypothetical ones) particles it is an
operator of the generalized lepton charge. It is easy to verify that the former (described in its basic representation) can be presented as a product of the latter one (in its own representation) and the operator of chirality. The application of GC - functions in general (24) and (28) forms lets to interpret these operators in terms of the Pauli isospace vectors constructed on the base of \( \kappa_x, \kappa_y, \kappa_z \) connected with x, y, z - axes consequently. Pauli transformations are interpreted in the case as rotations depended on \( \theta \) and \( \chi \) angles. The former ones introduce Majorana terms into the Lagrangian and into the generalized charge (22), while the latter, that incorporate \( \chi \) angle, describe the chiral transformations and introduce the violation of -parity. The correlation of the general form of Pauli Lagrangian with modern Majorana models [11, 12], [29] shows that it fits a special system including left - and right - handed particles of the same flavor with the complex Dirac mass \( M_D = M \cos \theta e^{i\chi} \) and left and right Majorana masses. In Pauli model under investigation the latter ones are equal in magnitude but are of opposite signs \( M_R = -M_L = M \sin \theta e^{i\chi} \). The value \( M^2 = |M_L(R)|^2 + |M_D|^2 \) is invariant of the group of general Pauli (chiral - Pauli) transformations.

Thus, the use of Pauli transformations as a theoretical base for a description of a neutral particle leads to a special Majorana model. In the frame of the model the general Majorana scheme is reduced to a special case that fits to one with an arbitrary Dirac mass and left and right Majorana masses being opposite in signs but equal in measure. Besides, in the frame of the model, in contrast with general Majorana phenomenological schemes, there is a possibility to generalize the concept of lepton charge and to link it with the form of the Lagrangian mass term. In this way the eigenfunctions of the generalized lepton charge operator and that of the mass structure term can be used for description of arbitrary Dirac and Majorana states of the Pauli model. In the case, the projection conditions for eigenfunctions of basic operators take the form of Majorana relations of general shape, determined by the inversion classes of investigated particles. The Dirac case is an exception from the general rule. In view of independence of its Lagrangian from the inversion classes of the particles there are two alternative representations of the solutions in the case. They are described by either eigenfunctions of the lepton charge operator or by those that define the structure of the Lagrangian mass term. This alternative meets well known "Dirac" or "Majorana" representations of the solutions. The model is described in the isospace of Pauli transformations so that the basic operators, determining the special representations, have properties of spatial vectors.
§3. Two-flavor Pauli model of Majorana neutrinos

Let us use the results obtained for the construction of a model describing two neutrinos of different flavor. For definiteness we shall assume that they are neutrino of electron (e) and muon (μ) flavors. Suppose that for the former case the left-handed state is stands for the particle (ν'_eL) and its right-handed state does for the antiparticle (ν^C_eR). For the latter case the opposite is true: the right-handed state is the particle (ν'_µR) and its left-handed state is the antiparticle (ν^C_µL). One can coordinate the model with the above developed scheme of §2 by using the substitutions:

\[
\psi_L(x) \rightarrow \nu_e(x), \quad \psi_R(x) \rightarrow \nu_\mu(x), \\
\psi^C_R(x) \rightarrow \nu^C_e(x), \quad \psi^C_L(x) \rightarrow \nu^C_\mu(x). \tag{31}
\]

Note that such a model is similar to Zel’dovich - Konopinsky - Mahmoud scheme (ZKM - scheme) [31, 32], that was proposed by them previously for the description of charged e^−, µ^+ leptons. Such schemes for neutrinos were described, for example in [10].

In accordance with general results of Pauli [18] such a model for massless neutrinos is invariant under chiral - Pauli transformations:

\[
\begin{align*}
\nu'_eL(x) &= e^{i\chi/2}e^{i\phi/2}[\cos(\theta/2)\nu_e(x) + \sin(\theta/2)e^{-i\phi}\nu^C_\mu(x)], \\
\nu'_\mu R(x) &= e^{-i\chi/2}e^{i\phi/2}[\cos(\theta/2)\nu_\mu(x) - \sin(\theta/2)e^{-i\phi}\nu^C_e(x)], \\
\nu^C_eR(x) &= e^{-i\chi/2}e^{-i\phi/2}[\cos(\theta/2)\nu^C_e(x) + \sin(\theta/2)e^{i\phi}\nu_\mu(x)], \\
\nu^C_\mu L(x) &= e^{i\chi/2}e^{-i\phi/2}[\cos(\theta/2)\nu^C_\mu(x) - \sin(\theta/2)e^{i\phi}\nu_e(x)].
\end{align*} \tag{32}
\]

These transformations are canonical ones. In this instance they mix electron and muon neutrino states of particle and antiparticle types of general chirality with conserving commutation relations of the massless fields.

The move to the two-flavor scheme by using (31) substitutions leads simultaneously to the changes of the standard definitions of discrete C -, P -, T - transformations [22] to the following forms:

\[
\begin{align*}
[\nu_eL(x)]^C &= \nu^C_eR(x), & [\nu_\mu R(x)]^C &= \nu^C_\mu L(x), \\
[\nu_eL(Px)]^{(P)} &= \eta_P \gamma_4 \nu_\mu R(x), & [\nu_\mu R(Px)]^{(P)} &= \eta_P \gamma_4 \nu_e L(x), \\
(P\vec{x} = -\vec{x}, \ P x_4 = x_4), \\
[\nu_eL(Tx)]^{(T)} &= -\eta_T \gamma_2 \nu^C_eL(x), & [\nu_\mu R(Tx)]^{(T)} &= \eta_T \gamma_2 \nu^C_\mu R(x), \\
(T\vec{x} = \vec{x}, \ T x_4 = -x_4). \tag{33}
\end{align*}
\]

In the frame of these new definitions all results of above investigated model due to properties of inversion classes of the particles hold true in the two flavor neutrino scheme.
The most general Lagrangian of the two-flavor model for massive neutrino, obtained from (22) by (31) substitutions, has the following form:

\[
L(x) = L_0(x) - \frac{M}{2} \left\{ \cos \theta (\mathcal{M}_{eR}(x)e^{-i\chi}\nu_{eL}(x) + \mathcal{M}_{\mu L}(x)e^{-i\chi}\nu_{\mu L}(x) + \mathcal{M}_{eR}(x)e^{+i\chi}\nu_{eR}(x) + \mathcal{M}_{\mu L}(x)e^{+i\chi}\nu_{\mu R}(x) + \sin \theta (\mathcal{M}_{eR}(x)e^{+i(x-\phi)}\nu_{eL}(x) + \mathcal{M}_{\mu L}(x)e^{-i(x-\phi)}\nu_{\mu R}(x) - \mathcal{M}_{eR}(x)e^{+i(x+\phi)}\nu_{eL}(x) - \mathcal{M}_{\mu L}(x)e^{-(x+\phi)}\nu_{\mu R}(x)) \right\}
\]

\[
= L_0(x) - \frac{M}{2} \left\{ \cos \theta (\mathcal{M}_{eR}(x)e^{+i\chi}\nu_{eL}(x) + \mathcal{M}_{\mu L}(x)e^{+i\chi}\nu_{\mu L}(x) + \mathcal{M}_{eR}(x)e^{+i\chi}\nu_{eR}(x) + \mathcal{M}_{\mu L}(x)e^{+i\chi}\nu_{\mu R}(x) + \sin \theta (\mathcal{M}_{eR}(x)e^{+i(x+\phi)}\nu_{eL}(x) - \mathcal{M}_{eR}(x)e^{-(x+\phi)}\nu_{eR}(x)) \right\}
\]

The peculiarity of the Pauli neutrino scheme is in existence of a conserved generalized lepton charge that is as follows for the Lagrangian (34):

\[
Q^P = \frac{1}{2} \int d^3x \cos \theta (\nu_{eL}(x)\nu_{eL}(x) - \nu_{\mu L}^{GC}(x)\nu_{\mu L}^{GC}(x) + \nu_{eR}(x)\nu_{eR}(x) - \nu_{\mu R}(x)\nu_{\mu R}(x) + \sin \theta (\nu_{eL}(x)e^{-i\chi}\nu_{eL}(x) + \nu_{\mu L}^{GC}(x)e^{+i\chi}\nu_{\mu L}^{GC}(x) - \nu_{eR}(x)e^{+i\chi}\nu_{eR}(x) - \nu_{\mu R}(x)e^{-i\chi}\nu_{\mu R}(x))
\]

Another important feature of the Pauli scheme is the invariance of the Lagrangian (34) under transformation:

\[
\nu_{eL}(x) \rightarrow (\nu_{eL}(x))^{GC}, \quad \nu_{eR}(x) \rightarrow \nu_{eL}(x), \quad \nu_{\mu L}(x) \rightarrow (\nu_{\mu R}(x))^{GC}, \quad \nu_{\mu R}(x) \rightarrow \nu_{\mu L}(x),
\]

that reverses the sign of the generalized lepton charge (35). This transformation is equivalent to the operation of the generalized charge (GC - ) conjugation (23) of the previous model. It transforms the electron (left) and muon (right) neutrinos into their own antiparticles with taking into account that the phase factors \( \eta_C \) of GC - operation can be different for electron and muon neutrino flavor. The latter assumption is considered by introduction of chiral \( \chi \neq 0 \) angle. As it was already underlined earlier in the context of introduction of GC - operation (see discussion after (23)) this difference is connected with CP - symmetry violation.

The Lagrangian (34) describes a particular case of a two-flavor Majorana neutrino system of ZKM - type, including a left-handed electron neutrino with \( M_L(\nu_e) \) Majorana mass and a right- handed muon neutrino with \( M_R(\nu_\mu) \) Majorana mass. As a consequence of Pauli-like scheme they are linked by the
expression: \( M_R(\nu_\mu) = -M_L(\nu_e) = M \sin \theta \). These neutrinos can be mixed in between to be described by the Lagrangian mass terms of "quasi-Dirac" type depended on \( M_D(\nu_\mu, \nu_e) = M \cos \theta e^{i\chi} \) parameter. The effective masses of the based electron and muon neutrino states, which arise in the process of diagonalization of the Lagrangian (34), are equal in magnitude and opposite in signs:

\[
M(\nu_1, \nu_2) = \mp \sqrt{(M_R(\nu_\mu) - M_L(\nu_e))^2 + |M_D(\nu_\mu, \nu_e)|^2} = \mp M, \quad (37)
\]

Thus, the length of the neutrino oscillation between the states is equal to infinity. The mixing angle \( \theta_{\text{mix}} \) introduced in a standard way [11] - [12], [28] is given by the ratio of "quasi-Dirac" to Majorana masses:

\[
\tan(2\theta_{\text{mix}}) = \cot \theta = \frac{2|\Im(M_D(\nu_\mu, \nu_e))|}{M_R(\nu_\mu) - M_L(\nu_e)} = \frac{|M_D(\nu_\mu, \nu_e)|}{M_R(\nu_\mu)}, \quad (38)
\]

It is evident that among Lagrangians (31) it is always possible to choose a basic one so that the others can be obtained by proper general Pauli (chiral and Pauli) transformations. In the model of one particle it was the Dirac Lagrangian, on whose basis the two main representations of eigenfunctions were introduced in §2. Those were the charge and Majorana (or mass) representations [15] [16]. Their basic operators were respectively the lepton charge operator \( \hat{k}_z \gamma_5 \) and \( \hat{k}_z \) operator, describing the Lagrangian mass term. In two-flavor neutrino model these basic operators are connected with different Lagrangians. Actually, in the standard phenomenological schemes the representation with absence of mixture of basic Majorana neutrino fields is usually chosen as the basic one. In the neutrino model under investigation such a representation is built on the basic \( \hat{k}_z \) operator that is consistent with the choice of parameter \( \theta = \eta \pi/2 \) (\( \theta_{\text{mix}} = 0 \)) in (34). In this case the Lagrangian and the generalized lepton charge obtain the following forms:

\[
L(x) = L_0(x) + \frac{M}{2} \{ \nu_0(x) \nu_0(x) - \nu_0(x) \nu_0(x) \} = L_0(x) + \frac{M}{2} \{ \nu_0(x) \nu_0(x) - \nu_0(x) \nu_0(x) \}, \quad (39)
\]

\[
Q^D = \frac{1}{2} \int d^3x \nu_0^+(x)(\cos \chi \hat{k}_x + \sin \chi \hat{k}_y)\gamma_5 \nu_0(x) = \\
\frac{n}{2} \int d^3x \nu_0(x) e^{-i\chi} \nu_0(x) + \nu_0(x) e^{i\chi} \nu_0(x) - \\
\nu_0(x) e^{i\chi} \nu_0(x) - \nu_0(x) e^{-i\chi} \nu_0(x),
\]

Here the two-component basic neutrino function \( \nu_0(x) \) is as follows:

\[
\nu_0(x) = \{ \nu_0(x), \nu_0(x) \},
\]

\[
\nu_0(x) = \nu_0(x) + \eta \nu_0^C(x) = \nu_0(x) + \eta e^{-i\chi} \nu_0^C(x),
\]

\[
\nu_0(x) = \nu_0(x) + \eta \nu_0^C(x) = \nu_0(x) + \eta e^{i\chi} \nu_0^C(x).
\]
It is similarly to \([12]\) modified with a transition to the GC - conjugation operation, and includes a pair of basic Majorana solutions which are the electron \(\nu_{eL}(x)\) and muon \(\nu_{0R}(x)\) neutrinos with no mixing. The solutions have the general charge parity \(\eta = \pm 1\) due to the GC - conjugation and differ from each other by the eigenvalues of \(\hat{\kappa}_z\) operator. The latter is an operator of neutrino flavor which simultaneously defines the structure of the Lagrangian \([39]\) mass term:

\[
\hat{\kappa}_z\nu_0 \kappa(x) = \kappa_0 \nu_0 \kappa(x),
\]

\[
\nu_{0+1}(x) = \left(\nu_0(x), \nu_0(x)\right), \quad (\kappa = +1), \quad \nu_{0-1}(x) = \left(\nu_0(x), 0\right), \quad (\kappa = -1).
\]

The corresponding equations in two-component and conventional forms are as follows:

\[
\gamma_\mu \partial_\mu \nu_0(x) - M_\kappa \nu_0(x) = 0,
\]

\[
\begin{align*}
\gamma_\mu \partial_\mu \nu_{0eL}(x) - M \eta \nu_{0eL}(x) &= 0, \\
\gamma_\mu \partial_\mu \nu_{0eR}(x) + M \eta \nu_{0eR}(x) &= 0, \\
\gamma_\mu \partial_\mu \nu_{0\mu L}(x) - M \eta \nu_{0\mu L}(x) &= 0, \\
\gamma_\mu \partial_\mu \nu_{0\mu R}(x) + M \eta \nu_{0\mu R}(x) &= 0,
\end{align*}
\]

It is necessary to underline that the lower pair of the equations is GC - conjugated to the previous one. Note that the equivalent equations which link the left - handed particle (electron neutrino) and right - handed antiparticle (electron antineutrino) components were obtained and investigated in \(\chi = \phi = 0, \eta = +1\) limit by Case \([30]\). In the case he considered the right - handed particle and left - handed antiparticle (muon) components ignored, so that he actually kept on investigating the \(\kappa = +1\) solution with additionally imposed conditions \(\nu_{0R}(x) = \nu_{0\mu L}(x) = 0\).

For the description of the basic representation of the charge type with an operator of the lepton charge as basic one it is useful to conserve the Dirac form of the Lagrangian. It is due to full mixing of electron and muon neutrino components of the generalized wave function \((\theta = 0, \theta_{mix} = \pi/4\) in \([44]\)) and it leads to the following forms of Lagrangian and lepton charge:

\[
\begin{align*}
L(x) &= L_0(x) - \frac{M}{2} \mathcal{P}_0(x) \left(\cos \chi \hat{k}_x + \sin \chi \hat{k}_y\right) \nu_0(x) = \\
L_0(x) - \frac{M}{2} (\mathcal{P}_{D\mu R}(x) e^{+ix_\mu} \nu_{D\mu L}(x) + \mathcal{P}_{D\mu L}(x) e^{-ix_\mu} \nu_{D\mu R}(x)) + \\
\mathcal{P}_{D\mu L}(x) e^{-ix_\mu} \nu_{D\mu R}(x) + \mathcal{P}_{D\mu R}(x) e^{+ix_\mu} \nu_{D\mu L}(x),
\end{align*}
\]

\[
Q^L = \frac{1}{2} \int d^3x \nu_0^2(x) \kappa_0 \gamma_5 \nu_0(x) = \frac{1}{2} \int d^3x (\nu_{D\mu L}(x) \nu_{D\mu L}(x) + \nu_{D\mu R}(x) \nu_{D\mu R}(x) - \nu_{D\mu L}(x) \nu_{D\mu L}(x) - \nu_{D\mu R}(x) \nu_{D\mu R}(x)),
\]

\[
\begin{align*}
\nu_D(x) &= \left(\nu_{D\mu L}(x), \nu_{D\mu R}(x)\right) = \left(\nu_{D\mu L}(x), \nu_{D\mu R}(x)\right),
\end{align*}
\]

18
The special case \( \chi = 0 \) is due to the real mass value and is similar to (18). One can construct it from the latter by using (31) substitutions in a way that it could be called as the "quasi-Dirac" case. The eigenfunctions of the solutions with fixed lepton charge \( q_\pm = \pm 1 \) are as follows:

\[
\begin{align*}
\hat{k}_z \gamma_5 \nu_{D,q_\pm}(x) &= q_\pm \nu_{D,q_\pm}(x), \\
\nu_{D,q_+}(x) &= \left(\frac{\nu_{D\mu L}(x)}{\nu_{D\mu R}(x)}\right), \quad \nu_{D,q_-}(x) = \left(\frac{\eta \nu_{D\mu R}^{GC}(x)}{\eta \nu_{D\mu L}^{GC}(x)}\right),
\end{align*}
\]

Note that in accordance with ZKM-scheme the lepton charge of the left-handed electron and right-handed muon antineutrinos is equal to +1 while that for the right-handed electron and left-handed muon antineutrinos is equal to -1.

It is interesting to note that one can obtain an alternative description of quasi-Dirac case in flavor variables when using Pauli transformation to \( \theta' = -\pi/2 \) angle of the wave functions and operators of (39) representation.

The reason is connected with the conservation of the flavor operator as the basic one although it takes a new form \( -\left(\cos \chi \hat{k}_x + \sin \chi \hat{k}_y\right) \) (see (43)) due to a nondiagonalized representation. The generalized function of quasi-Dirac representation \( \nu_D(x) \) takes the form:

\[
\begin{align*}
\nu_D(x) &= e^{-i\left(\cos \chi \hat{k}_y - \sin \chi \hat{k}_x\right)\theta'/2} \nu_0(x) = e^{i\left(\cos \chi \hat{k}_y - \sin \chi \hat{k}_x\right)\pi/4} \nu_0(x) = \\
&= \frac{1}{\sqrt{2}} \left(1 + e^{-i\chi} \right) \left(\frac{\nu_{0eL}(x)}{\nu_{0eR}(x)}\right) = \frac{1}{\sqrt{2}} \left(\nu_{0eL}(x) + e^{-i\chi} \nu_{0eR}(x)\right), \\
\nu_{D\mu L}(x) &= \frac{1}{\sqrt{2}} \left(\nu_{0eL}(x) + \eta e^{-i\chi} \nu_{0eR}(x)\right), \quad \nu_{D\mu R}(x) = \frac{1}{\sqrt{2}} \left(\nu_{0eL}(x) - \eta e^{-i\chi} \nu_{0eR}(x)\right), \\
\nu_{D\mu}^{GC}(x) &= \frac{1}{\sqrt{2}} \left(\nu_{0eL}^{GC}(x) + \eta e^{-i\chi} \nu_{0eR}^{GC}(x)\right), \\
\nu_{D\mu L}^{GC}(x) &= \frac{1}{\sqrt{2}} \left(\nu_{0eL}^{GC}(x) - \eta e^{-i\chi} \nu_{0eR}^{GC}(x)\right).
\end{align*}
\]

The generalized flavor \( \kappa = \pm 1 \) serves here as a quantum characteristic of the alternative representation. Its eigenfunctions can be obtained from (40) by using the same transformation:

\[
\begin{align*}
-(\cos \chi \hat{k}_x + \sin \chi \hat{k}_y) \nu_{D,\kappa}(x) &= \kappa \nu_{D,\kappa}(x), \\
\nu_{D,\kappa=+1}(x) &= \left(\nu_{D\mu L}(x) + \eta \nu_{D\mu R}^{GC}(x)\right) = \frac{1}{\sqrt{2}} \left(\nu_{0eL}(x) + \eta \nu_{0eR}^{GC}(x)\right), \\
\nu_{D,\kappa=-1}(x) &= \left(\nu_{D\mu L}(x) + \eta \nu_{D\mu R}^{GC}(x)\right) = \frac{1}{\sqrt{2}} \left(\nu_{0eL}(x) + \eta \nu_{0eR}^{GC}(x)\right).
\end{align*}
\]
forms:

\[\begin{align}
\nu_{D\mu L}^{C}(x) &= -\eta e^{i\phi}\nu_{D\mu L}(x), \\
\nu_{D\mu R}^{C}(x) &= -\eta e^{i\phi}\nu_{D\mu R}(x) \quad (\kappa = +1), \\
\nu_{D\mu L}^{C}(x) &= \eta e^{i\phi}\nu_{D\mu L}(x), \\
\nu_{D\mu R}^{C}(x) &= \eta e^{i\phi}\nu_{D\mu R}(x) \quad (\kappa = -1), \\
\nu_{D\mu}(x) &= -\kappa e^{i\phi}\nu_{D\epsilon}(x) \quad (\kappa = \pm 1).
\end{align}\] (47)

The latter expression means that the electron and muon Majorana components of the Dirac neutrino of ZKM - scheme in the states of fixed generalized flavor coincide in magnitude with accuracy to the phase. The solutions (43) and (16) are examples of an alternative description of the quasi - Dirac case in the two-flavor neutrino Pauli model in terms of either lepton charge of ZKM - type or generalized flavor.

Let us now move to the general case of the two-flavor model. Starting from Lagrangian for the basic case of the flavor representation \(\theta = \eta\pi/2\) (39) and using Pauli transformation one can construct the general expression of the two-component wave function:

\[\nu(x) = \begin{pmatrix}
\nu_{e}(x) \\
\nu_{\mu}(x)
\end{pmatrix} = \begin{pmatrix}
\nu_{e L}(x) + \nu_{e R}(x) \\
\nu_{\mu L}(x) + \nu_{\mu R}(x)
\end{pmatrix} = e^{-i(\cos \chi \hat{\kappa}_{x} - \sin \chi \hat{\kappa}_{y})\theta'/2}\nu_{0}(x) =
\begin{pmatrix}
\cos (\theta'/2)\nu_{e}(x) - \sin (\theta'/2)e^{-i\phi}\nu_{\mu}(x) \\
\cos (\theta'/2)\nu_{\mu}(x) + \sin (\theta'/2)e^{i\phi}\nu_{e}(x)
\end{pmatrix}, \quad (\theta' = \eta\theta - \pi/2).\] (48)

The \(\theta'\) parameter of the Pauli transformation can be connected with the mixing angle introduced in a standard phenomenological description of two-flavor Majorana models [10] - [12]. It sets a degree of deviation of the investigated Lagrangian from the basic one, in which electron and muon components of the generalized function are not mixed. In this case the Lagrangian and the conserved generalized charge have the following forms:

\[L(x) = L_{0}(x) + \frac{M}{2}\bar{\nu}(x)[\cos \theta'\hat{\kappa}_{z} + \sin \theta'(\cos \chi \hat{\kappa}_{x} + \sin \chi \hat{\kappa}_{y})]\nu(x),
\]
\[Q^{F} = \frac{1}{2} \int d^{3}x i^{+}(x)[\cos \theta'(\cos \chi \hat{\nu}_{x} + \sin \chi \hat{\nu}_{y}) - \sin \theta'\hat{\kappa}_{z}]\gamma_{5}\nu(x),\] (49)

The corresponding system of equations for the separated components is as follows:

\[\begin{align}
\gamma_{\mu}\partial_{\mu}\nu_{e L}(x) - M \sin \theta' e^{-i\chi}\nu_{\mu R}(x) - M \eta \cos \theta' \nu_{e R}^{GC}(x) &= 0, \\
\gamma_{\mu}\partial_{\mu}\nu_{\mu R}(x) - M \sin \theta' e^{+i\chi}\nu_{e L}(x) + M \eta \cos \theta' \nu_{\mu L}^{GC}(x) &= 0, \\
\gamma_{\mu}\partial_{\mu}\nu_{e R}^{GC}(x) - M \sin \theta' e^{+i\chi}\nu_{\mu L}^{GC}(x) + M \eta \cos \theta' \nu_{\mu R}(x) &= 0, \\
\gamma_{\mu}\partial_{\mu}\nu_{\mu L}^{GC}(x) - M \sin \theta' e^{-i\chi}\nu_{e R}^{GC}(x) - M \eta \cos \theta' \nu_{e L}(x) &= 0.
\end{align}\] (50)

In the two-component form it is reduced to the equation:

\[\gamma_{\mu}\partial_{\mu}\nu(x) - M \sin \theta'(\cos \chi \hat{\kappa}_{x} + \sin \chi \hat{\kappa}_{y})\nu(x) - M \cos \theta'\hat{\kappa}_{z}\nu(x) = 0.\] (51)
It is useful to note that the conserved current \( J^\mu_\nu(x) \) has the following general form in the case:

\[
J^\mu_\nu(x) = \frac{\cos \theta'}{2} \left( \left[ \cos \theta' \chi \hat{\kappa}_x + \sin \theta' \hat{\kappa}_y \right] \nu(x) = \frac{1}{2} \left[ \cos \theta' (\mathcal{P}_L(x) \gamma_\mu \nu_{\nu L}(x) + \mathcal{P}_R(x) \gamma_\mu \nu_{\nu R}(x)) - \mathcal{P}_R(x) \gamma_\mu \nu_{\nu R}(x) \right] \right].
\]

It consists of two terms: the neutral current (proportional to \( \sin \theta' \)) that does not change flavor number, and the current which changes neutrino flavor from electron to muon type and vice versa (proportional to \( \cos \theta' \)). The latter includes \( \nu_\mu \rightarrow \nu_e (\Delta \kappa = +2) \) transitions as well as transitions of \( \nu_e \rightarrow \nu_\mu (\Delta \kappa = -2) \) type.

By using expression (48) one can construct the general form of the eigenfunctions with the generalized flavor operator describing the structure of the Lagrangian mass term as a basic one. The eigenfunctions with fixed quantum number \( \kappa = \pm 1 \) take the form:

\[
\nu_{\kappa = +1}(x) = \left( \frac{\nu_\mu(x)}{\nu_{\nu}(x)} \right)_{\kappa = +1} = \left( \frac{\cos \left( \theta'/2 \right) \nu_{\nu}(x)}{\sin \left( \theta'/2 \right) e^{i\kappa \theta_0(x)}} \right) (\kappa = +1),
\]

\[
\nu_{\kappa = -1}(x) = \left( \frac{\nu_\mu(x)}{\nu_{\nu}(x)} \right)_{\kappa = -1} = \left( \frac{\cos \left( \theta'/2 \right) e^{-i\kappa \theta_0(x)}}{\sin \left( \theta'/2 \right) \nu_{\nu}(x)} \right) (\kappa = -1).
\]

Their chiral components satisfy the relations:

\[
\nu^C_{\mu L}(x) = \kappa \eta (\tan \left( \theta'/2 \right))^{\kappa} e^{i\phi} \nu_{\nu L}(x),
\]

\[
\nu^C_{\mu R}(x) = \kappa \eta (\cot \left( \theta'/2 \right))^{\kappa} e^{i\phi} \nu_{\nu R}(x), \quad (\kappa = \pm 1),
\]

with the following combined form:

\[
\nu_{\mu,\kappa}(x) = \kappa (\tan \left( \theta'/2 \right))^{\kappa} e^{i\lambda} \nu_{\nu,\kappa}(x), \quad (\kappa = \pm 1).
\]

This relation generalizes expression (17) that was found above for the quasi-Dirac case. It follows that in states of fixed generalized flavor the contributions of Majorana electron and muon neutrino components are bound together. In the case of the basic Lagrangian (39) the contribution of muon constituent in the state of generalized electron flavor \( (\kappa = 1) \) is equal to zero. And in general case it is proportional to the tangent of half an angle \( \theta' \) which is responsible for flavor mixing. The similar value estimates the contribution of Majorana electronic constituent in the neutrino state of the generalized
muon flavor ($\kappa = -1$). In ”quasi-Dirac” states these contributions coincide in magnitude.

In the two-flavor ZKM model investigated these relations are analogous to Majorana relations of projection types of §2 model. The analysis of their behavior under space inversion shows that in premise of universality of phases of space inversion transformations they are realized for particles of inversion A-B - classes only ($\eta^\nu_P = -1$). However if electron and muon neutrinos belong to different inversion classes it is allowed that electron neutrinos are referred to C - class ($\eta_P(e) = +1$) and muon neutrinos to D - class ($\eta_P(\mu) = -1$) or vice versa ($\eta_P(e) = -1$, $\eta_P(\mu) = +1$).

In the alternative description with generalized lepton charge being used as a quantum characteristic its operator is chosen as basic. In this case the Lagrangian (43), in which Pauli rotation through angle $\theta' = -\pi/2$ relatively to (39) has been already executed, becomes the basic one, so that for coordination with the previous case it is necessary to make an additional transformation through the angle $\eta \theta$. Once the rotation is performed the following Lagrangian and the generalized lepton charge of (24) type will arise:

\[
L(x) = L_0(x) - \frac{M}{2} \bar{\nu}(x) \left[ \cos \theta (\cos \chi \delta x + \sin \chi \delta y) - \eta \sin \theta \delta z \right] \nu'(x),
Q^L = \frac{1}{2} \int d^4 x \nu'^+ \left[ \cos \theta \delta x + \eta \sin \theta \delta y \right] \gamma_5 \nu'(x).
\]

(56)

Note that expressions (43) and (56) are bound by Pauli transformation:

\[
\nu'(x) = e^{-i\eta (\cos \chi \delta y - \sin \chi \delta x) \frac{\pi}{2}} \nu_D(x) = \left( \begin{array}{c} \cos \theta/2 - i \sin \theta/2 e^{-i\chi} \\ \eta \sin \theta/2 e^{+i\chi} \cos \theta/2 \end{array} \right) \left( \begin{array}{c} \nu_D(x) \\ \nu_D(x) \end{array} \right)
\]

\[
\cos \theta/2 \nu_{DL}(x) - \eta \sin \theta/2 e^{-i\chi} \nu_{DR}(x) + \eta \cos \theta/2 \nu_{GC,DL}(x) - \sin \theta/2 e^{-i\chi} \nu_{GC,DR}(x),
\cos \theta/2 \nu_{DR}(x) + \eta \sin \theta/2 e^{+i\chi} \nu_{DL}(x) + \eta \cos \theta/2 \nu_{GC,DR}(x) + \sin \theta/2 e^{+i\chi} \nu_{GC,DL}(x)).
\]

(57)

The eigenfunctions of particles with fixed generalized lepton charge $q = \pm 1$ are obtained from (44) by using the same transformations. They take the form:

\[
[\cos \theta \delta x + \eta \sin \theta (\cos \chi \delta x + \sin \chi \delta y)] \gamma_5 \nu'_q(x) = q \nu'_q(x),
\nu'_q(x) = \left( \begin{array}{c} \nu'_q(x) \\ \nu'_q(x) \end{array} \right) \eta \sin \theta \delta x e^{-i\chi} \nu_{GC,DL}(x)
\]

\[
\eta \cos \theta/2 \nu_{GC,DL}(x) - \sin \theta/2 e^{+i\chi} \nu_{GC,DR}(x),
\eta \cos \theta/2 \nu_{GC,DR}(x) + \sin \theta/2 e^{+i\chi} \nu_{GC,DL}(x)).
\]

(58)

(59)

\[
(q = 1), \quad (q = -1), \quad (\eta \theta = \theta' + \pi/2).
\]

22
Their components satisfy the following projection Majorana conditions similar to (27):

\[
\nu_{\mu L}^c(x) = e^{i\phi} \nu_{\mu L}(x), \\
\nu_{e L}^c(x) = q(e^{i\phi} \nu_{e L}(x)), \\
\nu_{\mu R}^c(x) = -q(e^{i\phi} \nu_{\mu R}(x)) (q = \pm 1). 
\]

(59)

They connect electron neutrino components with charge-conjugated muon ones and muon neutrino components with charge-conjugated electron ones. It is evident that in the states of the fixed generalized lepton charge \(q\) and chirality, the contribution of the states with the opposite lepton charge for small angles \(\theta (\theta' \sim -\pi/2)\) is small. However it grows proportionally to the tangent of half an angle \(\theta\) value with receding from quasi-Dirac case and reaches equality with basic contribution for \(\theta = \eta \pi/2\), \((\theta' = 0)\), when Majorana solutions of fixed flavor due to Lagrangian \((39)\) exist. In this case the angle \(\theta\) plays a role of a parameter, describing the mixture of particle and antiparticle states of quasi-Dirac types for solutions of fixed generalized lepton charge.

From the analysis of behavior of expressions \((59)\) under space reflection it follows that in the case of universality of phase factors \(\eta_P\) they are fulfilled for particles of the inversion C-D-classes \((\eta_P^2 = +1)\) only. However if neutrinos are of different inversion classes they can be satisfied also for particles of inversion A-B-classes in the following cases: \(\eta_{P(e)} = i\) (A-class), \(\eta_{P(\mu)} = -i\) (B-class) or \(\eta_{P(\mu)} = +i\) (B-class). As a whole the expressions \((54)\) and \((59)\) show that in the general case of the two-flavor model the severe proportion of contributions of electron and muon neutrino components exist in eigenfunctions of basic operators. They play here the same role as projection Majorana relations do in the above one-particle model.

Thus we have demonstrated that in the frame of the Pauli concept it is possible to construct a two-flavor neutrino model, which incorporates two sorts of massive neutrinos of different flavor and has quantum characteristics of Zel’dovich-Konopinsky-Mahmoud type. The general Lagrangian of such a model includes the mass terms connected with Majorana masses of the neutrinos as well as the terms responsible for their mixing (quasi-Dirac ones). In this case Majorana masses of different neutrinos are equal in magnitude but differ in signs, so that the length of neutrino oscillation between them tends to infinity. Such Pauli model is a special case of general models of ZKM-type but has a set of important properties which are absent...
in the general schemes. The existence of special quantum numbers, universally describing either Majorana or quasi-Dirac states of the model, is the main feature. In accordance with the choice of basic operators two following possible alternative representations can be realized. They are either flavor basic operator, which simultaneously defines the mass structure of the specific Lagrangian, or the generalized lepton charge, describing all the states of the system by using these characteristics. With all this going on the choice of the basic operator is dependent on the inversion class of a particle under investigation. There is an exclusion from the general law which is the quasi-Dirac case, with the alternative use of both representations being possible. The chiral-Pauli transformation, transferring the basic form of definite representation into the general one, includes the angle parameter, that sets the proper rotation in the isospace of Pauli transformations as well as degree of mixture of basic solutions of the basic form in a solution with the fixed quantum number of the generalized type.

However, the application of this scheme to the analysis of the experimental data on neutrino oscillation needs taking into proper account an inequality of Majorana electron and muon neutrino masses. Below we shall show that a necessary extension of the scheme presented can be given with conservation of the peculiarities of the Pauli scheme, provided that the proper general suppositions of the system neutrino Lagrangian are adopted.

§4. Neutrino oscillations in two-flavor Pauli model

Let us consider the most general Lagrangian of phenomenological Majorana models presented for example in [11] and set off the term of (34) type, which can be described by the two-flavor Pauli scheme. Transforming it to notations of the model by (31) substitutions we can get the model Lagrangian:

\[
L_{\text{md}}(x) = L_0(x) - \frac{1}{2} \{ \overline{\nu}_{\mu R}(x) m_D \nu_{e L}(x) + \overline{\nu}_{e R}(x) m_D^* \nu_{\mu L}(x) + \overline{\nu}_{\mu L}(x) m_D^* \overline{\nu}_{e R}(x) + \overline{\nu}_{\mu R}(x) (m_1 - im_2) \nu_{\mu L}(x) + \overline{\nu}_{e L}(x) (m_1^* + im_2^*) \nu_{\mu R}(x) + \overline{\nu}_{e R}(x) (m_1 + im_2) \nu_{e L}(x) + \overline{\nu}_{e L}(x) (m_1^* - im_2^*) \nu_{e R}(x) \}\].

(60)

It depends on three phenomenological mass parameters \(m_1, m_2, m_D\), which are arbitrary complex values usually connected with Majorana - \(m_1, m_2\), and Dirac (quasi-Dirac) - \(m_D\) masses. As it was demonstrated above the Pauli scheme is a special case of a general Majorana one in which Majorana mass terms of L and R types are equal in value and of different signs. For comparison with Pauli (34) form let us connect the mass parameters of
with ones of the two-flavor neutrino model of §3 by using the following relations:

\[ m_1 - im_2 = M_R(\nu_\mu)e^{i(\chi-\phi)} = M'_R(\nu_\mu)e^{-i\phi}, \quad m_D = M_D(\nu_\mu\nu_e)e^{+i\chi}, \]
\[ m_1 + im_2 = M_L(\nu_e)e^{i(\chi+\phi)} = M'_L(\nu_e)e^{+i\phi}, \quad m^*_D = M_D(\nu_\mu\nu_e)e^{-i\chi}, \]  
\[ ReM_D(\nu_\mu\nu_e) = M \cos \theta, \quad \frac{1}{2}(M_R(\nu_\mu) - M_L(\nu_e)) = M \sin \theta, \]  

(61)

If in the further course one uses in these expressions the neutrino GC - functions (34) then the model Lagrangian (60) can be rewritten in the form:

\[ L_{md}(x) = L_0(x) - \frac{M}{2}[\cos \theta(\nabla_{\mu R}(x)e^{i\chi}\nu_{eL}(x) + \nabla_{\mu L}^G(x)e^{i\chi}\nu_{eR}(x) + \nabla_{\mu R}^G(x)\nu_{R}(x) + \nabla_{\mu L}^G(x)\nu_{L}(x)) - \sin \theta(\nabla_{\mu R}(x)\nu_{eL}(x) + \nabla_{\mu L}^G(x)\nu_{R}(x) - \nabla_{\mu R}^G(x)\nu_{L}(x)\nu_{eR}(x) - \nabla_{\mu L}^G(x)\nu_{eL}(x)) - \bar{\nu}_{\mu R}^G(x)\nu_{eL}(x)] - \frac{1}{2}\{M_R(\nu_\mu) + M_L(\nu_e)\} (\nabla_{\mu R}(x)\nu_{eL}(x) + \nabla_{\mu L}^G(x)\nu_{R}(x) + \nabla_{\mu R}^G(x)\nu_{L}(x)) + \frac{M_D(\nu_\mu\nu_e) - M^*_D(\nu_\mu\nu_e)}{2} (\nabla_{\mu R}(x)e^{i\chi}\nu_{eL}(x) + \nabla_{\mu R}^G(x)e^{-i\chi}\nu_{eL}(x)) \]  

(62)

The second term of the expression is in coincidence with the mass term of the Lagrangian (34) and two latter ones comprise the additive mass term which takes into account the inequality of Majorana masses of electron and muon neutrinos and possible complexity of the quasi - Dirac \( M_D(\nu_\mu\nu_e) \) parameter. Note, that the term, depending on half a sum of Majorana masses of the electron and muon neutrinos, is symmetric under operation of the generalized charge GC - conjugation as the Pauli Lagrangian (34) does. It allows in some way to simplify the form (62) by implying the similar symmetry condition to the whole Lagrangian of the model as a general requirement. For \( ImM_D(\nu_\mu\nu_e) = 0 \) the Lagrangian does not incorporate the latter term but however it is in consistency with a rather general Majorana model for two-flavor neutrinos with unequal Majorana masses as before. Since GC - conjugation takes into account different phase factors \( \eta_{\nu_\mu\nu_e} \) for neutrinos of different flavors, the introduction of a similar general symmetry condition under that operation can be considered as natural and physically non contradictory for the general Majorana scheme.

The two-flavor neutrino oscillations have a finite oscillation length in the modified general model. Indeed, let us introduce a \( \nu_{md}(x) \) neutrino function that is an analogue of \( \nu(x) \) (45) in the Lagrangian of the model, and conduct a transformation, leading to reduction of the Pauli mass term of the Lagrangian to the form (39). To realize it one has to transform the shortened
Pauli transformation with using rotation inversed to (48):

Lagrangian (62) to its general form (49) and then perform the correspondent Pauli transformation with using rotation inversed to (48):

\[ L_{md}(x) = L_0(x) - \frac{M_0}{2} v_{md}(x) \nu_{md}(x) + \frac{M}{2} v_{md}(x) \hat{k} \nu_{md}(x), \]

\[ M_0 = \frac{M_R(\nu_e) + M_L(\nu_e)}{2}, \quad \hat{k} = \cos \theta' \hat{k}_z + \sin \theta' (\cos \chi \hat{k}_x + \sin \chi \hat{k}_y), \]

It reduces the Lagrangian to the following diagonal form:

\[ L_{md}(x) = L_0(x) - \frac{M_0}{2} v_{md0}(x) \nu_{md0}(x) + \frac{M}{2} v_{md0}(x) \hat{k} \nu_{md0}(x), \]

\[ \nu_{md0}(x) = (v_{md0}(x), \nu_{md0}(x) = e^{i(\cos \chi \hat{k}_y - \sin \chi \hat{k}_z)} \nu_{md}(x), \]

\[ \gamma_\mu \bar{\nu}_{md0}(x) + M_0 \nu_{md0}(x) = M \hat{k} \nu_{md0}(x) = 0, \quad \theta' = \eta \theta - \pi / 2. \]

In accordance with the standard procedure of Majorana phenomenology (see, e.g. [11, 12]) physical neutrinos entered into $\nu_{md}(x)$ are superpositions of the basic neutrino states of definite masses $(\nu_{md0}(x))$. The latter ones obey equations (64) and are described by the universal flavor number $\kappa$ of the two-flavor model of the previous section. In passing from Lagrangian (63) to the basic form the effective Majorana masses of electron and muon neutrino take the values:

\[ \eta \frac{M_R(\nu_e) + M_L(\nu_e)}{2} - \sqrt{\frac{1}{4} (M_R(\nu_e) - M_L(\nu_e))^2 + (Re M_D(\nu_e))^2}, \]

\[ M_R(\nu_e) = M_0 - M = \]

\[ \eta \frac{M_R(\nu_e) + M_L(\nu_e)}{2} + \sqrt{\frac{1}{4} (M_R(\nu_e) - M_L(\nu_e))^2 + (Re M_D(\nu_e))^2}. \]

As a result, the known formulae arise for effective Majorana masses of neutrino of two types, that define the length of their mutual oscillations through the difference of their squared masses:

\[ |M^2(\nu_e) - M^2(\nu_e)| = 4|M_0|M, \]

\[ L_{osc} = 4\pi E/|M^2(\nu_e) - M^2(\nu_e)| = \pi E/|M_0|M. \]

The new principal feature of the Pauli model, following from the structure of Lagrangian (63) and mass formula (65), is a fact that the effective masses of free Majorana particles consist of two terms different in their nature. They have properties of Pauli isoscalar and Pauli isovector. The former is universal and depends on mass characteristics of neutrinos only, the latter is described by the flavor quantum number $\kappa$ and depends on the mixing angle $\theta'$. The universality of the former means that separate components of wave function
\( \nu_{md}(x) \) can be determined by the same quantum numbers of flavor type as components of \( \nu(x) \) function of Pauli model. Besides, such an interpretation shows that in the modified two-flavor Pauli model the effective neutrino Majorana masses are formed with participation of not one but two different Higgs scalar fields, which also have properties of a scalar and a vector of the Pauli isospace.

In parallel with the length of neutrino oscillations the other important experimental parameter of physical neutrino is the mixing angle. In phenomenological neutrino schemes it is included on the base of the connection between wave functions of physical left-handed neutrino states and eigenfunctions of states \( \nu_{1L}(x), \nu_{2L}(x) \) of fixed masses. It is described by the standard relations:

\begin{align}
\nu_{eL}(x) &= \nu_{1L}(x) \cos \theta_{mix} + \nu_{2L}(x) \sin \theta_{mix}, \\
\nu_{\mu L}(x) &= \nu_{2L}(x) \cos \theta_{mix} - \nu_{1L}(x) \sin \theta_{mix}.
\end{align}

(67)

In the two-flavor Pauli model they appear as a consequence of Pauli transformations (64) reducing Lagrangian (63) to a diagonal form with eigenfunctions constructed of the wave functions of fixed masses and flavors \( \nu_{md}(x) e^{i\phi} \) (\( \rho = L, R \)). The \( \theta' \) angle that specifies these transitions, can be coordinated with the mixing angle \( \theta_{mix} \) defined from (67) and can be deduced from the neutrino oscillation experiments. It is obvious that the connections equivalent to (67) are described in the model under investigation by using a transformation of Pauli type, inverted to (64). They are as follows:

\begin{align}
(\nu_{md})_{eL}(x) &= \cos (\theta'/2)(\nu_{md0})_{eL}(x) - \eta \sin (\theta'/2)(\nu_{GC_{md0}})_{\mu L}(x), \\
(\nu_{md})_{\mu L}(x) &= \cos (\theta'/2)(\nu_{GC_{md0}})_{\mu L}(x) + \eta \sin (\theta'/2)(\nu_{md0})_{eL}(x), \\
(\nu_{GC_{md}})_{\mu L}(x) &= \cos (\theta'/2)(\nu_{md0})_{eL}(x) - \eta \sin (\theta'/2)(\nu_{GC_{md0}})_{\mu L}(x), \\
(\nu_{GC_{md}})_{eL}(x) &= \eta \theta' - \pi/2.
\end{align}

(68)

(The phase factor \( e^{-i\phi} = \eta_{GC} \) is included in the definition of charge GC-conjugation operation). The relations (68) are analogues of formulae (67) to present the eigenfunctions of the physical neutrino through the states of a fixed masses. The latter are basic flavor states of Pauli model and coincide
with (67) under the following conditions:

\[
\begin{align*}
\nu_{eL}(x) &= (\nu_{md})_{eL}(x), \quad \nu_{\mu L}(x) = (\nu_{md})_{\mu L}(x), \quad \nu_{1L}(x) = (\nu_{md0})_{eL}(x), \\
\nu_{2L}(x) &= \nu_{GC} \nu_{md0}, \\
\tan(2\theta_{mix}) &= \eta \cot \theta, \\
2\theta_{mix} &= -\theta' = \pi/2 - \eta \theta, \quad \eta = +1.
\end{align*}
\]

(69)

The first ones describe transition from the standard scheme to Majorana one of Zel’dovich - Konopinsky - Mahmoud type, the second ones state the connection between the experimental mixing angle $\theta_{mix}$ and the $\theta$ angle of the Pauli scheme. The former sets the direction of the vector of the generalized lepton charge relatively to z - axis of the Pauli isospace for two neutrinos of different flavors.

As an example of application of the model under investigation we propose a qualitative interpretation of the experimental neutrino oscillation results. The modern data on the mixing angles and the squared neutrino mass differences [33] are as follows:

\[
\begin{align*}
(\theta_{mix})_{12} &= (34 \pm 2.3)\degree, \quad (\theta_{mix})_{23} = (45 \pm 8.2)\degree, \quad (\theta_{mix})_{13} \leq 13\degree, \\
\Delta M^2_{12} &= M_1^2 - M_2^2 \sim 8 \cdot 10^{-5} \text{ eV}^2, \quad \Delta M^2_{23} = M_2^2 - M_3^2 \sim 2.5 \cdot 10^{-3} \text{ eV}^2.
\end{align*}
\]

(70)

It is evident that as a consequence of the small mixing angle $(\theta_{mix})_{13}$ the physical neutrinos of electron and $\tau$ types can be approximately described in the frame of two-component mixing. However the muon neutrino is a mixture of three components and can not be described by such a simple model. For Pauli parameters $\theta$, which set $\nu_e - \nu_\mu$ and $\nu_\tau - \nu_\mu$ mixtures, one can extract the following experimental values by using (69):

\[
(\theta_{exp})_{12} = (22 \pm 4.6)\degree, \quad (\theta_{exp})_{23} = (0 \pm 16)\degree.
\]

(71)

These values show that $\tau$ - neutrino is a "quasi-Dirac" mixture of $\nu_{\tau L}$ and $\nu_{\mu R}$, which can be described as the state of lepton charge of ZKM -type with $q = +1$. In accordance with (63) the electron neutrino is described as a mixture of Majorana states $\nu_{e,k}$ and $\nu_{\mu,k}$, specifying by the generalized flavor number $\kappa = +1$. Using (65) one can estimate that the fraction of the muon neutrino Majorana component is about half of the electron neutrino fraction value. The muon neutrino is a complex mixture of all three neutrino states and is not described in our scheme. The three-flavor neutrino Majorana model on the base of Pauli transformations will be presented by the author in a special separate article.
§5. Conclusion

Thus, the two models for the description of Majorana properties of neutral free fermions have been constructed on the base of general Pauli (chiral-Pauli) transformations. The first one is a model for one particle with states of left and right chirality that develops the simplest initial scheme proposed by Majorana [1] (see also [30]). It extends the latter to the case when the mass part of its Lagrangian includes both Dirac and Majorana mass terms. The second one describes a system of neutral particles of two different flavors so that the particle state of the first one (conditionally called as electron neutrino) has the left chirality and that of the other (called as muon neutrino) has the right chirality, i.e., it goes in consistence with the Zel’dovich-Konopinsky-Mahmoud scheme. The models under investigation are special cases of general Majorana schemes presented in literature. The latter one can be modified and used for description of the simplest, two-flavor version of the neutrino oscillation in ZKM-scheme.

As a consequence of connection with Pauli transformations these Pauli schemes possess the following peculiarities:

1. These models describe a special class of Majorana Lagrangians of neutral particles connected by general Pauli (chiral-Pauli) transformations, that lets one to get an arbitrary Lagrangian by starting from the basic Lagrangian of definite representation of the model. The chiral-Pauli transformations consist of pure Pauli SU(2)-group and chiral group of U(1)-type. For massless particles the general Pauli symmetry is exact, however it is destroyed with introduction of mass terms in the Lagrangian.

2. In coordination with general suggestions of the Standard model the mass terms of Pauli Lagrangians are interpreted as a result of the spontaneous symmetry breakdown that pick out a special direction in the space of Pauli transformations with nonzero mean vacuum value of Higgs field. In assumption of universality of the breakdown mechanism the mass terms of different Pauli Lagrangians are connected with the same transformations as the correspondent specified directions outlined above. The hypothesis of universality is equivalent to the assumption that the Higgs field has the properties of a vector in Pauli isospace. In this case the mean vacuum value set fixed its modulus and specifies the effective mass of basic particles, and the angle coordinates of the vector, defining the relative values of Dirac and Majorana mass terms, are not changed with the spontaneous breakdown. The spontaneously broken symmetry due to chiral transformations is connected with the breakdown of CP-invariance.
The modified version of the two-flavor Pauli scheme introduces an additional mass term of the Lagrangian with properties of isoscalar of Pauli space, so that quantum characteristics of the states due to isovector part are conserved in the modified scheme as well. The Higgs field of the modified model consists of two components related to Pauli isovector and isoscalar types of mass terms of the Lagrangian.

3. The conception of the lepton charge of neutral particles is extended to the generalized lepton charge, that is coordinated with an arbitrary direction of the Pauli isospace. For an arbitrary Lagrangian the corresponding operator of the generalized lepton charge includes parameters of Pauli rotations. On the base of this operator the product of the generalized lepton charge and chirality that can be used as an alternative basic operator for description of neutral particles is constructed. The quantum numbers of these operators allow to describe universally the states of Dirac (quasi-Dirac) as well as those of pure Majorana or mixed Dirac-Majorana types. The generalization of the lepton charge leads to a modification of the operation of charge conjugation. In the generalized charge GC - conjugation $\eta_C$ phase factors for the left-handed and right-handed particles can be different; that is connected with a violation of CP-symmetry.

4. The basic representations of the models are given by their Lagrangians and basic operators, the other representations are connected with them by Pauli transformations. In the first model it is Dirac Lagrangian with two basic representations: the charge ("Dirac") one, where the solutions are described by lepton charges, and the Majorana ("mass") one with two Majorana solutions of fixed mass which differ in signs of $\psi_C(x) = \pm \psi(x)$ condition. Their basic operators are correspondingly either the operator of the lepton charge or the operator of the structure of the Lagrangian mass term. The latter is connected with the product of lepton charge and chirality. In the basic "mass" ("flavor") representation of the second model the basic neutrino Majorana states of electron and muon flavors are not mixed so that the mean value of the lepton charge is equal to zero. The basic charge representation of the second model is analogous of "Dirac" one ("quasi-Dirac") of the first model. It has the basic states of fixed lepton charge of ZKM-type, which are the mixture of electron and muon neutrino Majorana components with their contributions being coincident in values.

5. The interpretation of Pauli models depends on the inversion classes of particles under investigation, which are defined by the phase factors of the space inversion operation. They are $\eta_P = \pm i$ for particles of inversion
A and B - classes and $\eta_P = \pm 1$ for inversion C and D - classes respectively. In modern models it is supposed that neutrinos as well as the other physical particles belong to A-B - inversion classes; however in Majorana neutrino schemes this hypothesis should be controlled experimentally. In Pauli models the form of Majorana conditions is connected with the inversion classes of particles. For example, $\psi^C(x) = \pm \psi(x)$ condition is realized for inversion A-B - classes only and its analogue for C-D - classes is $\psi^C(x) = \pm \gamma_5 \psi(x)$ condition. In general case the form of Majorana conditions is generalized but defined as above by inversion classes of particles under investigation. This rigid connection between the form of Majorana conditions and inversion class of particles can be destroyed if one lets the phase factors $\eta_P$ be not universal, so that different physical neutrino states could belong to different inversion classes.

6. The choice of the basic operator, its eigenfunctions and projection conditions of fixed eigenvalue is connected with the inversion class of particles under consideration. In general case there are two alternative representations for wave functions of the states: the mass (flavor) representation with basic operator of the structure of Lagrangian mass term (generalized flavor) and the charge representation, when the operator of the generalized lepton charge is basic. These operators do not commute and are complementary. For universal definition of the phases of space inversion operation, the first representation is realized for particles of inversion A-B - classes and the second one is realized for particles of inversion C-D - classes. If Lagrangian is not sensitive to the inversion classes of particles, as for example in Dirac ("quasi - Dirac") case, the alternative description is possible in every representation. The projection conditions for eigenfunctions take either form of Majorana conditions (first model) or the form of connection between electron and muon neutrino components of particle and antiparticle type (second model). For particles of inversion A-B - classes their forms generalize the known Majorana conditions but for C-D - classes those include, in addition, the $\gamma_5$ operator.

7. The basic operator of the basic representation is a vector. It is coordinated with $z$ - axis of the Pauli isospace and its eigenvalues are coordinated with projections of the vector to the axis. For arbitrary directions the basic operators are coordinated with vectors of the correspondent directions. For the representations using the operator of the generalized lepton charge, the basic case corresponds to Dirac (quasi - Dirac) description on the ground of independent particle and antiparticle states and correlates with $z$ - axis. The
general Pauli Lagrangian is characterized by \( \theta \) angle between the vector of the generalized lepton charge and \( z \) - axis of the Pauli isospace. The intermediate cases are connected with the Dirac - Majorana mixture defined by the \( \theta \) angle, and the Majorana case corresponds to \( \theta = \eta \pi/2 \), when particle and antiparticle contributions in eigenfunctions of the generalized lepton charge are equal. For the representations using operator of the generalized flavor ("mass") type, the mixture of Majorana states of different flavor is absent in the basic representaion. The intermediate cases are connected with incorporation of the mixture given by \( \theta' \) angle. For pure Dirac (quasi - Dirac) case \( \theta' = -\pi/2 \), that corresponds to maximal mixing of basic flavor ("mass") states. In neutrino oscillation experiments the mixing angle is usually measured starting from the pure Majorana case and it characterizes the extend of mixture of basic, flavor Majorana states in the state of physical neutrino.

8. The mass part of Majorana Lagrangians contains terms of two types: those that specify Majorana masses of the particles \((M_{M1}, M_{M2})\) and those that specify their mixture \((M_{12})\). The latter are interpreted in terms of mixture of Majorana particles and associated with Dirac (quasi - Dirac) terms of the model. The Pauli models are specified by the fact that Majorana mass terms of the described particles (they are left and right ones - in the first model or electron and muon neutrinos - in the second one respectively) are equal in magnitude and opposite in signs \((M_{M2} = -M_{M1})\). In the conventional method of diagonalization of Lagrangians with transition to Majorana representation the former ones are set to the terms which depend on effective masses of particles \((M_1, M_2)\). The latter ones tend to zero so that the states of fixed effective masses are not mixed. In Pauli case the application of diagonalization method is realized by chiral - Pauli transformations and leads to the following effective masses of the particles:

\[
M_{1,2} = \pm \sqrt{\frac{1}{2} |M_{M1} - M_{M2}|^2 + |M_{12}|^2}, \quad M_{M1} + M_{M2} = 0,
\]

(72)

In this case the lengths of oscillations between electron and muon neutrino states of the second model occur to tend to infinity:

\[
L_{osc} = 4\pi E/|M_1^2 - M_2^2| = \infty.
\]

(73)

In the modified neutrino Pauli scheme the additional universal term is included into the Lagrangian of the system, so that the effective masses of neutrinos differ in measure and the length of their mutual oscillations becomes finite:

\[
M_{1,2} = M_0 \pm \sqrt{|M_{M2}|^2 + |M_{12}|^2}, \quad L_{osc} = \frac{\pi E}{M_0 \sqrt{|M_{M2}|^2 + |M_{12}|^2}}.
\]

(74)
In this case the oscillation parameter $\Delta M^2$ depends on mass characteristics of the Pauli model only, which are invariants of isovector and isoscalar terms of Pauli Lagrangian and do not depend on the mixing angle.

The investigation of the models arises the following important general question: "Do Majorana particles of inversion C-D - classes exist in the nature?" The similar particles should have nonstandard properties under an operation of the space inversion and special Majorana conditions which include additional $\gamma_5$ operator, different from the standard ones in existing Majorana schemes.

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34
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