High-Contrast Interference of Ultracold Fermions

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Many-body interference between indistinguishable particles can give rise to strong correlations
rooted in quantum statistics. We study such Hanbury Brown-Twiss-type correlations for number
states of ultracold massive fermions. Using deterministically prepared \(^6\)Li atoms in optical tweezers,
we measure momentum correlations using a single-atom sensitive time-of-flight imaging scheme.
The experiment combines on-demand state preparation of highly indistinguishable particles with
high-fidelity detection, giving access to two- and three-body correlations in fields of fixed fermionic
particle number. We find that pairs of atoms interfere with a contrast close to 80%. We show that
second-order density correlations arise from contributions from all two-particle pairs and detect
intrinsic third-order correlations.

Many-body interference describes processes by which non-interacting particles acquire strong correlations
solely due to their quantum statistics [1]. In such cases, interference occurs between many-particle paths and the
enhancement or suppression of particular outcomes is dictated by the exchange statistics of the particles.

The most famous example of many-body interference is the “bosonic bunching” of photons, as observed by
Hanbury Brown and Twiss in the development of stellar intensity interferometry [2, 3]. The experimental [4, 5]
and theoretical [6] study of correlations in photon fields have been driving the development of quantum optics [7].

In contrast to the statistical behavior of bosons, which can in certain cases be described in terms of classical waves [2], the interference of fermionic particles is a uniquely quantum mechanical phenomenon. Experimental access to correlations arising from fermionic interference allows the study of quantum systems through intensity interferometry, for example in heavy ion collisions [8] and ultracold atom systems [9]. Fermionic interference and associated anti-correlations have been observed for thermal sources of neutrons [10] and cold atoms [11], as well as electrons in free space [12] and in solid state [13, 14]. Interference between fermionic number states, however, has been far more elusive: Few particle species amenable to single-particle control possess fermionic statistics, and realizing high-quality single-fermion sources is an outstanding experimental challenge. Recent advances in this direction have enabled the observation of two-fermion interference in semiconductor architectures [15] and double ionization processes [16].

Here, we observe high-contrast interference of fermionic particles in pure quantum states of ultracold atoms. We use optical tweezers as a configurable, deterministic source of non-interacting fermions [17]. The particles interfere during a ballistic time-of-flight and are detected with high fidelity. We observe periodic antibunching of two fermions with close to full contrast. Adding a third source to the system, we measure the second- and third-order correlation properties of a three-fermion field and determine the contribution of intrinsic third-order correlations [18, 19].

FIG. 1. Many-body interference of fermionic particles. (a) The coincidence events for two detectors monitoring single-particle sources are given by the available sets of many-body paths. For indistinguishable particles, the paths add coherently, where the sign of their interference depends on quantum statistics. Two sources emitting indistinguishable fermions display suppressed coincidence counts. In a setting with more particles, correlations between detectors arise due to interference of all many-body paths. (b) We realize the thought experiment from (a) using ultracold fermions in optical tweezers. Time-of-flight expansion is performed in an optical dipole trap aligned with the \(x\)-axis connecting the tweezers and particles are detected in free-space fluorescence imaging. (c) The probability distribution \(|\Psi(x_1, x_2)|^2\) (plotted for the two-mode case) evolves from localized to delocalized states during time-of-flight, but retains a node at \(x_1 = x_2\) due to fermionic anti-symmetry.
are loaded independently from a degenerate Fermi gas and there is no tunnel coupling at any point during the experiment. This configuration corresponds to the scenario sketched in Fig. 1 (a) and (c): Fermionic correlations arise due to destructive interference of two sets of two-particle paths, or equivalently from the preservation of anti-symmetry of the two-particle wavefunction during time-of-flight. Our experiment directly realizes the fermionic analog of the Ghosh-Mandel experiment [4].

For each experimental run, we record the momenta of the two particles and construct the momentum density \( \langle n_k \rangle \) at wavevector \( k \) and its second-order correlator \( \langle n_k n_{k'} \rangle \) from a histogram over several thousand realizations. We only retain data from runs where two particles were successfully prepared and detected, which corresponds to about 80% of the data. In Fig. 2 (a) the experimentally measured momentum correlator is shown. The strong correlations in the relative momentum \( d = k_1 - k_2 \) immediately demonstrate the non-separability of the correlator into single-particle momenta. The modulations occur on a “lattice” momentum scale \( k_{lat} = \pi/a \) and carry an envelope set by the single-particle on-site momentum distribution.

As correlations do not depend on the centre of mass momentum, we define a normalized correlation function [9, 22]

\[
C^{(2)}(d) = \frac{\int \langle \hat{n}_k \hat{n}_{k+d} \rangle \, dk}{\int \langle \hat{n}_k \rangle \langle \hat{n}_{k+d} \rangle \, dk}. \tag{1}
\]

The correlation function in Fig. 2 (b) exhibits close to full-contrast sinusoidal oscillations consistent with a minimum at \( d = 0 \), corresponding to strong fermionic anti-bunching. The greyed out points near \( d = 0 \) mark the region of particle separation below 30 \( \mu \)m, where two particles cannot be distinguished reliably and coincidences cannot be detected in our experiment [20]. We exclude these points from all further analysis. To quantify the strength of periodic correlations, we fit a damped cosine function to the correlations away from \( d = 0 \), which gives a modulation contrast of 79(2)\%. The contrast certifies the high degree of indistinguishability between the particles during the preparation and the matterwave expansion. With a ground state preparation fidelity of 97\%, one would in principle expect correlations with a contrast of up to \( \sim 95\% \). The measured contrast is limited by alignment errors between the axis of the optical dipole trap and the tweezer axis, which provide some distinguishability between the particles during the expansion dynamics [20]. Nevertheless, the contrast significantly exceeds the minimum visibility of \( 1/\sqrt{2} \approx 71\% \) required to perform quantum optics experiments such as non-locality tests with massive particles [23, 24].

With this high-contrast, on-demand source of indistinguishable fermions, we can now study second- and third-order correlations of a triplet of sources, which we directly realize with the tweezer architecture (Fig. 1).
FIG. 3. Second-order correlations for three fermions. (a) (i): For an arbitrary configuration of single-fermion sources, each pair of particles contributes a sinusoidal oscillation to the correlation function $C^{(2)}(d)$. The full correlator can exhibit complex patterns due to beat notes between all spatial frequencies. (ii): Expected full correlation function for three regularly spaced sources. (b) Correlators for individual pairs of sources. (c) Three particles released from three equidistant tweezers exhibit correlations at the momentum scale $k_{lat}$ and its first harmonic, leading to a “superlattice” structure in $C^{(2)}(d)$. The bottom row displays the measured $C^{(2)}(d)$ for three particles (data points), together with the weighted sum of two-particle contributions (red line). (d) For three tweezers with irregular spacing, three distinct spatial frequencies contribute to the full second-order correlator. The sharp dip at $k_1 = k_2$ is partially due to our lack of sensitivity to coincidences at short distances.

For a system of $N$ localized sources each emitting a single fermionic particle, it can be shown that negative exchange symmetry between all pairs of particles gives rise to a second-order correlation function of the form

$$
C^{(2)}(d) = \frac{2}{N^2} \sum_{<i,j>} (1 - \cos (d(x_i - x_j))),
$$

where summation runs over all distinct pairs of emitters and $x_i$ refers to the center of the $i^{th}$ source [20].

This expression gives an intuitive picture for the expected second-order momentum correlations, which is illustrated in Fig. 3 (a): Every pair of particles generates a sinusoidal anti-bunching signal with a characteristic spacing given by the inverse of the source separation. The full second-order correlation function is then obtained as the sum over all pairwise correlation signals.

We probe the validity of this picture using three equidistant tweezers with $a_{12} = a_{23} = 2 \mu m$, where $a_{ij}$ refers to the spatial separation of tweezer $i$ and $j$. We expect two frequency contributions to the correlation function: Pairs from neighbouring tweezers contribute a ‘long wavelength’ modulation at frequency $k_{lat} = \pi/a_{12} = \pi/a_{23}$, whereas the outermost tweezers 1 and 3 give rise to a ‘short wavelength’ modulation with half the period $\pi/a_{13} = k_{lat}/2$. Here, the two contributions should result in a “superlattice” correlation structure, shown in Fig. 3 (a). Adding more sources, and hence Fourier components to the correlation function, would lead to narrower correlation minima and result in the delta-function correlations observed for fermionic band insulators in optical lattices [9].

Figure 3 (b) shows the experimentally measured second-order correlations. We first record the correlator for every tweezer spacing individually by loading only two of the three microtraps. The pairwise correlations (shown in the small panels (i) to (iv)) are identical to the two-tweezer case in Fig. 2, with a momentum correlation scale given by the inverse spacing of the active sources. Separately we measure the full second-order correlator for all three tweezers loaded simultaneously (Fig. 3 (c)). The three-source correlator shows flatter maxima and sharper minima than the two-source correlator, reflecting the presence of correlations at multiple wavelengths.

For a quantitative analysis, the bottom panel of Fig. 3 (c) shows the correlation function $C^{(2)}(d)$ for the three particle system (blue data points). We compare the measured correlations to the weighted sum over all two-tweezer contributions (red line) [20]. The full second-order correlation function, including the decay of the contrast towards larger relative momenta $d$, is very well reproduced by the sum of the pair contributions.

For the above case of regularly spaced sources, all two-particle correlations occur at the lattice momentum scale and its first harmonic. In particular, both nearest neighbour pairs contribute the same signal to the full correlator ($a_{12} = a_{23}$). Source arrays with irregular spacing, on the other hand, can result in distinct correlation signals from all pairs of emitters. In such cases, each source pair...
FIG. 4. Third-order momentum correlations. Normalized correlation function \(C^{(3)}(d_1, d_2)\) for three fermions released from three wells with regular spacing \(a_{23} = a_{12}\), top row) and non-equal spacing \(a_{23} = 1.5a_{12}\), bottom row). The left column shows the full measured correlations, the central and right column show the disconnected and connected part of the correlation function, respectively. By construction, the function is symmetric under the transformation \((d_1, d_2) \leftrightarrow (-d_1, -d_2)\). The connected part of the correlation function \(C^{(3)}_{\text{con}}\) is scaled by a factor of two for better visibility. The measurement demonstrates the sensitivity to third-order correlations in the fermionic matterwave field.

generates sinusoidal correlations at a unique length scale, and their sum can give rise to complex structures in the full correlator.

We study the case of three tweezers with non-equal spacing, where three different frequency components contribute to the observed two-particle correlations. Figure 3 (d) shows experimental results for \(a_{12} = 1.6\, \mu m\), \(a_{23} = 1.5a_{12} = 2.4\, \mu m\). The incommensurate spacing leads to a doubling of the unit cell to four times the momentum \(k_{\text{lat}}\), which we define via the smallest tweezer spacing \(a_{12}\). Also here the correlations in the three-particle system are in perfect agreement with the weighted contributions from pairs of sources, elucidating the pairwise origin of second-order correlations in many-fermion systems.

To fully characterize the field produced by multiple sources, a measurement of correlation functions at higher orders is required. For bosonic particles, such measurements are routinely performed to assess the statistical properties of light sources \([7, 25, 26]\). Recently, measurements of higher-order correlations of massive bosonic particles have become possible with ultracold atoms \([27–29]\).

In our system, we measure the statistical properties of the matterwave field emanating from the three fermionic sources via the third-order correlator \(\langle n_{k_1} n_{k_2} n_{k_3} \rangle\) and the corresponding normalized correlation function

\[
C^{(3)}(d_1, d_2) = \frac{\int \langle \hat{n}_{k_1} \hat{n}_{k_1+d_1} \hat{n}_{k_1+d_2} \rangle \, dk}{\int \langle \hat{n}_{k_1} \rangle \langle \hat{n}_{k_1+d_1} \rangle \langle \hat{n}_{k_1+d_2} \rangle \, dk}.
\]

The correlation function is shown in Fig. S4 for the two data sets from Fig. 3. The equidistant and incommensurate tweezer configurations lead to clear and distinct correlation features at third order.

In order to interpret the third-order density correlations, it is useful to first remove contributions from lower order. This can be achieved by subtracting a suitable combination of first- and second-order correlators from the full third-order correlation function. Any remaining
correlations are intrinsic, that is they cannot be accessed from measurements at lower order. In interacting systems, such intrinsic correlations carry crucial information about the many-body state [28, 29], but they may be present even for free particle systems [18, 19]. For non-interacting bosons, for example, the intrinsic correlations contribute to a striking increase in zero-distance correlations at higher order [25, 27]. To assess the presence of intrinsic third-order correlations in our system, we combine our measurements at second order to construct the disconnected part of the third-order correlation function $C_{\text{dis}}^{(3)}(d_1, d_2)$. We define the disconnected correlator as [18, 19, 28]

$$\langle \hat{n}_{k_1} \hat{n}_{k_2} \hat{n}_{k_3} \rangle_{\text{dis}} =$$

$$s_1(N) \left( \langle \hat{n}_{k_1} \rangle \langle \hat{n}_{k_2} \hat{n}_{k_3} \rangle + \langle \hat{n}_{k_2} \rangle \langle \hat{n}_{k_1} \hat{n}_{k_3} \rangle + \langle \hat{n}_{k_3} \rangle \langle \hat{n}_{k_1} \hat{n}_{k_2} \rangle \right) - 2s_2(N) \langle \hat{n}_{k_1} \rangle \langle \hat{n}_{k_2} \rangle \langle \hat{n}_{k_3} \rangle.$$  

(4)

The scale factors $s_1(N) = \frac{N(N-1)(N-2)}{N^2(N-1)}$ and $s_2(N) = \frac{N(10N-11)}{N^4(N-1)}$ account for correlations due to particle number conservation [20] and approach unity as $N \to \infty$. The corresponding correlation function $C_{\text{dis}}^{(3)}(d_1, d_2)$ represents the experimentally accessible knowledge of third-order correlations available from pairwise measurements and is shown in the central column of Fig. S4. Clearly, it does not include all features of the full correlation function and the connected part of the correlation function, $C_{\text{con}}^{(3)} = C^{(3)} - C_{\text{dis}}^{(3)}$, retains additional structure, shown in the right hand column of Fig. S4. The presence and functional form of the intrinsic correlations agrees very well with an analytic calculation and is in full agreement with a decomposition according to Wick’s theorem [20]. We conclude that our system displays strong correlations at third order consistent with ideal fermionic statistics. To our knowledge, this constitutes the first experimental characterization of a fermionic field beyond second order.

Our work opens the door for several interesting avenues of research: Our high-purity, on-demand source of indistinguishable fermions may enable quantum optics experiments with massive particles, such as fermionic ghost imaging or Bell tests [19, 24, 30]. Extending our methods to more particles and modes, the interplay of coherence, indistinguishability and quantum statistics can be studied in many-fermion interference [1].

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METHODS

Experimental Procedure

State Preparation and Imaging

We prepare $^6$Li atoms in optical tweezer traps following the procedure outlined in references [17, 31]: Using an accusto-optical deflector (AOD) and trapping light at wavelength $\lambda = 1064\,\text{nm}$ we create several tweezer traps with a waist of $w = 1.15\,\mu\text{m}$ and tunable spatial displacement [31]. We independently prepare particles in the ground state of each tweezer, ensuring that tunneling between the wells is fully suppressed at all times.

Time-of-flight (TOF) is performed in a weak elongated optical dipole trap (ODT) aligned along the axis connecting the tweezers [21] (see next section). For all experiments we extract the 1D momentum of the atoms along the $x$-axis of the ODT by measuring the position after TOF in the ODT with a single-atom state-resolved imaging technique [21]. The imaging system has an effective resolution of $4\,\mu\text{m}$ and the distance between two atoms of the same hyperfine state beyond which they can be distinguished with more than 90% fidelity is $\sim 30\,\mu\text{m}$. This minimal spacing sets a “dead volume” for the detectors, within which coincidences cannot be detected. We independently tune the trap depth of the tweezers and therefore the size of the on-site wave function, the separation of the tweezers and the trap frequency of the confining potential during TOF. We choose parameters to compromise between the single-particle envelope, correlation length scales, imaging resolution and imaging dead volume.

For our experiments, we prepare two particles per well, one each in two of the three lowest hyperfine states, commonly labelled $\left| 1 \right\rangle$ and $\left| 3 \right\rangle$, ascending in energy. Using a magnetic Feshbach resonance we tune the $s$-wave scattering length between the two hyperfine states to a zero crossing at $568\,\text{G}$, such that there are no interactions between them. We take separate images of the two hyperfine states during each experimental run [21] and consider the two spin manifolds as two independent realizations of the same setting, effectively increasing the statistics by a factor of two. We have verified that there are no correlations between the two different spin manifolds. Preparation fidelities are $\sim 95\%$ for two atoms in a single tweezer and correspondingly $\sim 85\%$ for a six-particle system.

Time-of-Flight

All experiments except the measurement for Fig. 2 of the main text are performed in a crossed-beam optical dipole trap (cODT) aligned along the axis connecting the tweezers [21] (see next section). For all experiments we extract the 1D momentum of the atoms along the $x$-axis of the ODT by measuring the position after TOF in the ODT with a single-atom state-resolved imaging technique [21]. The imaging system has an effective resolution of $4\,\mu\text{m}$ and the distance between two atoms of the same hyperfine state beyond which they can be distinguished with more than 90% fidelity is $\sim 30\,\mu\text{m}$. This minimal spacing sets a “dead volume” for the detectors, within which coincidences cannot be detected. We independently tune the trap depth of the tweezers and therefore the size of the on-site wave function, the separation of the tweezers and the trap frequency of the confining potential during TOF. We choose parameters to compromise between the single-particle envelope, correlation length scales, imaging resolution and imaging dead volume.

Data Processing

For each experimental setting we record several thousand realizations and postselect all recorded images for the correct atom number. We achieve a postselection rate of $\sim 60\%$ for the three-atom setting and $\sim 80\%$ for the two-atom experiments. The difference between the preparation fidelity and the postselection rate is caused primarily by atoms in the high-$k$ wings of the distributions, which we neglect, and by the fact that atoms in close proximity to each other cannot be detected.

Prior to analysis, we group the measured positions after TOF into bins defined by two to four camera pixels.

Fitting the Correlator

In order to extract the contrast of the measured correlations we fit a phenomenological model to the correlation functions. For experiments with two fermions we fit...
Reconstruction of the Second-Order Correlation Function

The second-order correlation function for three atoms released from three wells is a sum over all mutual two-particle correlation functions (see Eq. (2) of the main text). The two-particle correlator for every tweezer spacing was measured in the experiment and is shown in Fig. 3 of the main text. Figure S3(a) shows the reconstructed correlator, i.e. the sum of the three two-atom correlators for the two different well configurations. The measured three-atom correlator is shown in (b). Note that the dataset for the latter is much larger than those for the two-atom correlator and therefore the noise is lower in (b). Figure S3(c) shows the difference of (a) and (b). There is almost no structure left, verifying that the full second-order correlator can be reconstructed from the individual two-atom correlators,

\[
\langle n_{k_1} n_{k_2} \rangle_{123} = \langle n_{k_1} n_{k_2} \rangle_{12} + \langle n_{k_1} n_{k_2} \rangle_{23} + \langle n_{k_1} n_{k_2} \rangle_{13}. \tag{6}
\]

Here, \( \langle n_{k_1} n_{k_2} \rangle_{ijk} \) denotes the correlator for the initial state with atoms loaded into tweezer \( i, j, k \).

Theoretical Analysis

We calculate the theoretically expected correlation functions for pure states of noninteracting fermions. In the analysis we neglect terms on the diagonal planes \( k_1 = k_2, k_1 = k_3, k_2 = k_3 \), where additional autocorrelation peaks arise. This restriction significantly simplifies the analysis and is reasonable as we are unable to access the \( \Delta k=0 \) correlations experimentally due to the imaging dead volume. In particular, the momentum correlators are then equal to their normal ordered form, \( \langle n_{k_1} n_{k_2} n_{k_3} \rangle = \langle n_{k_1} n_{k_2} n_{k_3} \rangle \) and standard results such as Wick’s theorem [33] can readily be applied. In the following calculations, normal order of the theoretically expected correlators is implied everywhere.

Second-Order Density Correlation Function

Let us consider the second-order momentum correlator expected for an array of \( N \) single-fermion sources.

We define the fermionic field operators \( \hat{\Psi}(x) \), in terms of the fermionic annihilation operators \( \hat{a}_i \) via

\[
\hat{\Psi}(x) = \sum_{i=1}^{N} \Phi(x-x_i) \hat{a}_i. \tag{7}
\]

\( \Phi(x-x_i) \) is the on-site wave function at lattice position \( x_i \), with \( \Phi(k) \) its Fourier transform.

Quadratic Corrections of Single Particle Momenta

For an expansion for a quarter of a trap period in a harmonic trap, there is an exact linear relation \( k(x) = kx \) between initial momentum \( k \) and position \( x \) after TOF [32]. However, the geometry of the ODT is not strictly harmonic but rather Gaussian and as we expand the atomic wave function to roughly the waist of the trap, anharmonic corrections start to play a role. As a consequence the correlator is distorted, influencing the contrast \( c \) and decay length \( \xi \) in Eq. (5), see left panel in Fig. S2. We correct this effect by taking a quadratic correction to the mapping \( k(x) \) into account. We rescale the momentum of each particle as \( k_\pm \rightarrow k_\pm + \alpha_\pm k_\pm^2 \) and choose \( \alpha_\pm \) to maximize \( c \) and \( \xi \) of the fit to the correlation function. To account for the asymmetric distortion of the correlator, we use two rescaling parameters, \( \alpha_+ \) for positive and \( \alpha_- \) for negative momenta. This rescaling removes the distortions from the correlator, see right panel in Fig. S2. For momenta measured in units of \( k_{\text{lat}} \), \( \alpha_\pm \) takes typical values of \(~0.025\).

The error function parameterized by \( s \) and \( w \) accounts for the dead volume of the imaging system. The exponential length scale \( \xi \) accounts for the decay of the contrast \( c \) for large separations \( d \). \( k_{\text{lat}} \) is the lattice momentum and \( y_0 \) the offset. The free parameters of the fit are \( s, w, y_0, \xi, c, \) and \( k_{\text{lat}} \).

\[
C^{(2)}(d) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{|d| - s}{w} \right) \right) \left( y_0 - e^{-\frac{d^2}{2c^2}} \cos \left( \pi d/k_{\text{lat}} \right) \right). \tag{5}
\]

Reconstruction of the Second-Order Correlation Function

The error function parameterized by \( s \) and \( w \) accounts for the contrast and the decay length \( \xi \) of the momentum correlator \( C^{(2)}(d) \). The procedure is shown here for the data presented in Fig. 3 (c) of the main text.

The measurement of second-order correlators was performed by measuring \( \Delta \Phi(k_1) \Phi(k_2) \Phi(k_3) \). The second-order correlator can be reconstructed from the individual two-atom correlators,

\[
\langle n_{k_1} n_{k_2} \rangle_{123} = \langle n_{k_1} n_{k_2} \rangle_{12} + \langle n_{k_1} n_{k_2} \rangle_{23} + \langle n_{k_1} n_{k_2} \rangle_{13}. \tag{6}
\]
FIG. S3. Reconstruction of the second-order three-atom correlator. (a) shows the mean of all mutual measured two-atom correlators. (b) shows the measured second-order correlator for three fermions released from three tweezers. (c) shows the difference (b)-(a). There is almost no structure in (c), verifying that the full second-order correlator can be reconstructed from the measured two-atom correlators.

The momentum representation of the field operator is then

$$\hat{\Psi}(k) = \sum_i \tilde{\Phi}(k) e^{-ikx_i} \hat{a}_i.$$  \hspace{1cm} (8)

The momentum density is given by

$$\langle \hat{n}_{k_1} \rangle = \langle \hat{\Psi}^\dagger(k_1) \hat{\Psi}(k_1) \rangle$$

$$= |\tilde{\Phi}(k_1)|^2 \sum_{j,k} e^{i k_1 (x_j - x_k)} \langle \hat{a}_j^\dagger \hat{a}_k \rangle$$

$$= N |\tilde{\Phi}(k_1)|^2.$$  \hspace{1cm} (9)

The expectation value $\langle \hat{a}_j^\dagger \hat{a}_k \rangle$ has to be evaluated with respect to the product state containing one particle per mode, which gives $\langle \hat{a}_j^\dagger \hat{a}_k \rangle = \delta_{jk}$. The two-point correlation function is given by

$$\langle n_{k_1} n_{k_2} \rangle = \langle \hat{\Psi}^\dagger(k_1) \hat{\Psi}^\dagger(k_2) \hat{\Psi}(k_2) \hat{\Psi}(k_1) \rangle$$

$$= |\tilde{\Phi}(k_1)|^2 |\tilde{\Phi}(k_2)|^2 \sum_{j,k,l,m} e^{i [(k_1 - k_2)(x_j - x_k) + k_2 (x_l - x_m)]} \langle \hat{a}_j^\dagger \hat{a}_k^\dagger \hat{a}_l \hat{a}_m \rangle.$$  \hspace{1cm} (10)

Due to fermionic statistics, the expectation value takes the form $\langle \hat{a}_j^\dagger \hat{a}_k^\dagger \hat{a}_l \hat{a}_m \rangle = \delta_{jm} \delta_{kl} - \delta_{jl} \delta_{km}$. The correlator hence is

$$\langle n_{k_1} n_{k_2} \rangle = \sum_{j,k} (1 - e^{i (k_1 - k_2)(x_j - x_k)})$$

$$= \sum_{<j,k>} (2 - 2 \cos (d(x_j - x_k))).$$  \hspace{1cm} (11)

Summation over distinct pairs of sites is denoted by $\langle j,k \rangle$ and $d = k_1 - k_2$. Together with Eq. (9), this result gives for $C^{(2)}(d)$ in Eq. (2) of the main text.
Wick’s Theorem

The correlations of non-interacting fermionic states can be factorized into lower-order correlators using Wick’s theorem [33].

\[ \langle \hat{n}_{k_1} \hat{n}_{k_2} \rangle = n_{k_1} n_{k_2} - G_{k_1,k_2}^{(1)} G_{k_2,k_1}^{(1)} \]  

Here, \( n_{k_1} = \langle \hat{n}_{k_1} \rangle = \langle \hat{\Psi}^\dagger(k_1) \hat{\Psi}(k_1) \rangle \) and \( G_{k_1,k_2} = \langle \hat{\Psi}^\dagger(k_1) \hat{\Psi}(k_2) \rangle \) denote the diagonal and off-diagonal part of the one-body density matrix, respectively. Also \( G_{k_1,k_2}^{(1)} = G_{k_2,k_1}^{(1)*} \) such that \( G_{k_1,k_2}^{(1)} G_{k_2,k_1}^{(1)} = |G_{k_1,k_2}^{(1)}|^2 \). For the third order correlations

\[ \langle \hat{n}_{k_1} \hat{n}_{k_2} \hat{n}_{k_3} \rangle = n_{k_1} n_{k_2} n_{k_3} - n_{k_1} G_{k_2,k_3}^{(1)} G_{k_3,k_2}^{(1)} - n_{k_2} G_{k_1,k_3}^{(1)} G_{k_3,k_1}^{(1)} - n_{k_3} G_{k_1,k_2}^{(1)} G_{k_2,k_1}^{(1)} + G_{k_1,k_2}^{(1)} G_{k_2,k_3}^{(1)} G_{k_3,k_1}^{(1)} + G_{k_1,k_3}^{(1)} G_{k_2,k_3}^{(1)} G_{k_3,k_2}^{(1)} + G_{k_1,k_2}^{(1)} G_{k_2,k_1}^{(1)} G_{k_3,k_3}^{(1)} 
\]

\[ = n_{k_1} n_{k_2} n_{k_3} - n_{k_1} |G_{k_2,k_3}^{(1)}|^2 - n_{k_2} |G_{k_1,k_3}^{(1)}|^2 - n_{k_3} |G_{k_1,k_2}^{(1)}|^2 + G_{k_1,k_2}^{(1)} G_{k_2,k_3}^{(1)} G_{k_3,k_1}^{(1)} + G_{k_1,k_3}^{(1)} G_{k_2,k_3}^{(1)} G_{k_3,k_2}^{(1)} + G_{k_1,k_2}^{(1)} G_{k_2,k_1}^{(1)} G_{k_3,k_3}^{(1)} 
\]

\[ = -2n_{k_1} n_{k_2} n_{k_3} + n_{k_1} \langle \hat{n}_{k_2} \hat{n}_{k_3} \rangle + n_{k_2} \langle \hat{n}_{k_1} \hat{n}_{k_3} \rangle + n_{k_3} \langle \hat{n}_{k_1} \hat{n}_{k_2} \rangle + 2 R \{G_{1,2} G_{1,3}^{(1)}\} \]  

The most common form (denoted by the horizontal bar) for the disconnected correlation function at third order is [18, 28]

\[ \langle n_{k_1} n_{k_2} n_{k_3} \rangle_{\text{dis}} = -2n_{k_1} n_{k_2} n_{k_3} + n_{k_1} \langle \hat{n}_{k_2} \hat{n}_{k_3} \rangle + n_{k_2} \langle \hat{n}_{k_1} \hat{n}_{k_3} \rangle + n_{k_3} \langle \hat{n}_{k_1} \hat{n}_{k_2} \rangle \]  

With this definition, one finds

\[ \langle n_{k_1} n_{k_2} n_{k_3} \rangle_{\text{con}} = 2 R \{G_{1,2} G_{1,3}^{(1)} G_{2,3}^{(1)} G_{3,1}^{(1)}\}. \]  

It is precisely the triple propagator that gives rise to intrinsic correlations at third order [19]. From measurements at first and second order, we can extract the magnitudes \( |G_{k_1,k_2}^{(1)}| \) of the propagators, but not their phase. The full information about the propagator \( G_{k_1,k_2}^{(1)} \) could be obtained by performing an additional first order interference experiment. With this information, a full factorization of all correlation functions could be achieved.

Particle Number Conservation

We note that in states with fixed particle number, significant correlations can arise purely due to the conservation of total particle number. Intuitively speaking, detecting one of \( N \) completely uncorrelated particles at a particular location reduces the probability of finding a particle at a different location by \( 1 - \frac{1}{N} \). This correlation term is absent in the case of pure fermionic statistics on account of the Pauli principle. However, any degree of distinguishability introduced, most notably by integrating out transversal excitation in the tweezer in our effective one-dimensional description, will reintroduce correlation terms due to particle number conservation. In the thermodynamic limit \( N \to \infty \), such correlations can be neglected, but they are highly significant for few-body states.

We therefore modify the standard definition of disconnected and intrinsic correlation functions to include the scaling factors \( s_1(N) \) and \( s_2(N) \) given in Eq. (4) of the main text. This renormalization of terms in the correlation function ensures that: (a) Correlations from number conservation are removed, i.e. uncorrelated particles give \( \langle \hat{C}^{(3)}(d_1,d_2)\rangle = 0 \) for any \( N \) and (b) in the limit \( N \to \infty \) our definition of the disconnected correlation functions agrees with the standard form from Wick’s theorem (Eq. (14)). The baseline of intrinsic correlations, \( \langle \hat{C}^{(3)}(d_1,d_2)\rangle = 0 \), then refers to the expectation for exactly \( N \) uncorrelated particles.

Explicit Correlation Functions

Using the framework outlined above, we explicitly calculate the expected correlator for three fermions emitted from three sources spaced by \( a_{12} \) and \( a_{23} \). The correlators for \( N \) particles are normalized such that

\[ \int \langle \hat{n}_{k_1} \rangle \, dk_1 = N \]

\[ \int \int \langle \hat{n}_{k_1} \hat{n}_{k_2} \rangle \, dk_1 \, dk_2 = N(N - 1) \]

\[ \int \int \int \langle \hat{n}_{k_1} \hat{n}_{k_2} \hat{n}_{k_3} \rangle \, dk_1 \, dk_2 \, dk_3 = N(N - 1)(N - 2). \]  

Measuring momentum in units of the lattice momentum \( k_{\text{lat}} = \pi / a_{12} \), we obtain for \( a_{23} = a_{12} \):

\[ \langle \hat{n}_{k_1} \hat{n}_{k_2} \rangle = \frac{n_{k_1} n_{k_2}}{2} (3 - 2 \cos(2\pi(k_1 - k_2))) - \cos(4\pi(k_1 - k_2))) \]

and for \( a_{23} = 1.5 a_{12} \)

\[ \langle \hat{n}_{k_1} \hat{n}_{k_2} \rangle = \frac{n_{k_1} n_{k_2}}{2} (3 - \cos(2\pi(k_1 - k_2))) - \cos(3\pi(k_1 - k_2)) - \cos(5\pi(k_1 - k_2))). \]
The theoretically expected correlation functions $C^{(2)}(d)$
are plotted with full contrast in the lower panel of
Fig. 3(a) of the main text. For the third-order correla-
tor we obtain for $a_{23} = a_{12}$

\[ \langle \hat{n}_{k_1} \hat{n}_{k_2} \hat{n}_{k_3} \rangle = \frac{2n_{k_1}n_{k_2}n_{k_3}}{27} (3 \]
\[ - 2 \cos(2\pi(k_1 - k_2)) - \cos(4\pi(k_1 - k_2)) \]
\[ - 2 \cos(2\pi(k_1 - k_3)) - \cos(4\pi(k_1 - k_3)) \]
\[ - 2 \cos(2\pi(k_2 - k_3)) - \cos(4\pi(k_2 - k_3)) \]
\[ + 2 \cos(2\pi(2k_1 - k_2 - k_3)) \]
\[ + 2 \cos(2\pi(2k_2 - k_1 - k_3)) \]
\[ + 2 \cos(2\pi(2k_3 - k_1 - k_2)) \]  \hspace{1cm} (19)

and for $a_{23} = 1.5a_{12}$

\[ \langle \hat{n}_{k_1} \hat{n}_{k_2} \hat{n}_{k_3} \rangle = \frac{2n_{k_1}n_{k_2}n_{k_3}}{27} (3 \]
\[ - \cos(2\pi(k_1 - k_2)) - \cos(3\pi(k_1 - k_2)) \]
\[ - \cos(5\pi(k_1 - k_2)) \]
\[ - \cos(2\pi(k_1 - k_3)) - \cos(3\pi(k_1 - k_3)) \]
\[ - \cos(5\pi(k_1 - k_3)) \]
\[ - \cos(2\pi(k_2 - k_3)) - \cos(3\pi(k_2 - k_3)) \]
\[ - \cos(5\pi(k_2 - k_3)) \]
\[ + \cos(2\pi(3k_1 + 2k_2 - 5k_3)) \]
\[ + \cos(2\pi(2k_1 + 3k_2 - 5k_3)) \]
\[ + \cos(2\pi(-5k_1 + 2k_2 + 3k_3)) \]
\[ + \cos(2\pi(-5k_1 + 3k_2 + 2k_3)) \]
\[ + \cos(2\pi(3k_1 - 5k_2 + 2k_3)) \]
\[ + \cos(2\pi(2k_1 - 5k_2 + 3k_3)) \]  \hspace{1cm} (20)

Figure S4 shows the theoretical expectation for the cor-
relator shown in Fig. 4 of the main text.
FIG. S4. Theoretical expectation for the correlator shown in Fig. 4 of the main text according to Eqs. (19) and (20).