I. INTRODUCTION

The origin of the baryon asymmetry \( \eta_B = (n_B - n_{\bar{B}})/n_{\gamma} = 6.1 \pm 0.3 \times 10^{-10} \) is one of the most puzzling questions in Cosmology. A universe which was initially baryon-antibaryon symmetric will leave a baryon number of at least eight orders of magnitude smaller than the previous value. A set of criteria which must be satisfied by any model of baryogenesis was laid out by Sakharov [1] almost forty years ago for the purpose of calculating this asymmetry. Grand Unified Theories (GUT) contain the necessary ingredients for baryogenesis [2]: the out-of-equilibrium decay of a massive particle which violates baryon number as well as CP. However, there are several issues with this scenario. The most serious one is the presence of electroweak (EW) sphaleron processes at temperatures greater than the electroweak scale which conserve \( B - L \) but violate \( B + L \), where \( B \) and \( L \) are the baryon and lepton number respectively. It implies that any \( B + L \) asymmetry generated by GUT mechanisms would be “washed out” by the EW sphaleron processes [3]. It was then realized that one might need \( B - L \) to be violated itself in order to generate any baryon asymmetry.

What might be the possible sources of \( B - L \) violation?

A very promising mechanism under the name of leptogenesis was proposed in which an out-of-equilibrium decay of a heavy Majorana neutrino which violates \( B - L \) is responsible for a lepton asymmetry [4]. If this happens at high enough temperatures while the EW sphaleron processes are still in equilibrium, this lepton asymmetry can be converted into a baryon asymmetry. In these scenarios, the lepton number is associated with Standard Model (SM) leptons and the baryon number is associated with SM quarks. Let us recall that, in the SM, \( B \) and \( L \) are violated because the SM baryonic current, \( J_B^\mu \) and SM leptonic current, \( J_L^\mu \), have an anomaly given by

\[
\partial^\mu J_B^\mu = \partial^\mu J_L^\mu = \left( \frac{n_f}{32 \pi^2} \right) \left( -g_2 W_{\mu \nu} W^{\mu \nu} + g_2 B_{\mu \nu} B^{\mu \nu} \right),
\]

where \( W_{\mu \nu} \) and \( B_{\mu \nu} \) are the \( SU(2)_L \) and \( U(1)_Y \) gauge bosons respectively.

The aforementioned leptogenesis scenarios have spawned a considerable amount of very interesting works, especially in connection with constraints on neutrino masses (see e.g. the excellent review by Buchmüller, Peccei, and Yaganida in [4]). It goes without saying that much remains to be done along this path. From an experimental point of view, the question of whether neutrinos are Majorana or Dirac is far from being settled, with more experiments being planned to study this issue. The attractive and popular see-saw mechanism which gives rise to small neutrino masses, contains Majorana neutrinos, with the heavier ones being candidates for the leptogenesis scenario. (There are scenarios in which heavy Dirac neutrinos could be responsible for leptogenesis [5].) In view of these issues, it might be interesting to investigate alternative scenarios of leptogenesis. Could there be a mechanism of leptogenesis in which the \( B - L \) violation comes from the decay of some particle other than the heavy Majorana neutrino? After all, it is the SM lepton number violation which is at the heart of the matter, no matter what its source might be. Can one test this new scenario in terms of its particle physics implications?

There is indeed such a particle as described in [6]. It arises in the construction of a model of dark energy and dark matter [6, 7]. We summarize below the essence of...
that model in order to motivate the model of leptogenesis presented in this paper.

In recent papers [6, 7], a Quintessence model was proposed in which the quintessence field is an axion-like particle, a$_Z$, of a spontaneously broken global $U(1)_A$ symmetry whose potential is induced by the instantons of a new unbroken gauge group $SU(2)_Z$. The $SU(2)_Z$ coupling becomes large at a scale $\Lambda_Z \sim 10^{-3}$ eV starting from an initial value $M$ at high energy which is of the order of the Standard Model (SM) couplings at the same scale $M$. This last fact could come from the following Grand Unified path $E_6 \rightarrow SU(2)_Z \otimes SU(6)$ with $SU(6)$ ultimately breaking down to the Standard Model, the details of which is given in [8]. The scenario which was proposed in [6, 7] is one in which $a_Z$ gets trapped in a false vacuum of an instanton-induced potential with a vacuum energy density $\sim (10^{-3}$ eV)$^4$. This model of quintessence mimics a universe which is dominated by a cosmological constant and cold dark matter. In fact, the most recent analyses from the Supernova Legacy Survey (SNLS) and WMAP [9, 10] fits a flat $\Lambda$CDM with a constant equation of state $w = -0.97 + 0.07 - 0.09$. As noticed in [10], even without the prior that the universe is flat, the combination of WMAP, large scale structure and supernova data gives $w = -1.06 - 0.08 + 0.13$.

As discussed in [7], our model, beside providing a scenario for the dark energy, contains several other phenomenological and cosmological consequences, two of which involve a candidate for the cold dark matter and a candidate for a new scenario of leptogenesis. The purpose of the present paper is to present a detailed description of this new mechanism of leptogenesis.

These aforementioned candidates depend on each other in an interesting way. The $SU(2)_Z$ fermions (the shadow fermions), which transform as $(3, 1, 0)$ under $SU(2)_Z \otimes SU(2)_L \otimes U(1)_Y$, would not have any interaction with the SM particles (the visible sector) (other than the gravitational one) if it were not for the presence of a messenger scalar field $\phi(z) = (3, 2, -1/2)$ in our model. As discussed in [7], this presence manifests itself in a variety of ways: it helps maintain thermal equilibrium between the $SU(2)_Z$ and SM plasmas (so that the two sectors possess a common temperature) until it drops out of thermal equilibrium. Its decay into a SM lepton plus a $SU(2)_Z$ fermion, as we shall see below, generates an asymmetry of the SM lepton number which is subsequently reprocessed into a baryon number asymmetry through the electroweak sphaleron process. Furthermore, it will be seen below that the asymmetry depends on $m_{\psi_2}^2 - m_{\phi_2}^2$ besides other factors such as CP phase factors, etc..., where $m_{\psi_2}$ and $m_{\phi_2}$ are the masses of the $SU(2)_Z$ fermions and messenger field respectively. The non-vanishing asymmetry is seen to be linked to the breaking of a shadow “custodial” symmetry $SU(2)_{shadow}$ by the difference in mass among the two shadow fermions. By requiring the SM leptonic asymmetry to be of order $10^{-7}$, various upper bounds on the messenger mass are obtained. The messenger field can be as light as several hundreds of GeVs which makes it an interesting prospect for a search at future colliders such as the LHC.

This scenario of leptogenesis is drastically different from the “standard” one in that here it is a scalar field whose decays violate SM lepton numbers instead of the decays of the customary right-handed Majorana neutrinos. In some sense, it is reminiscent of the color-triplet Higgs scalar of $SU(5)$ with the difference being that, in our case, only SM lepton number is violated. Let us recall that in “standard” scenarios with two heavy particles, one being much heavier than the decaying particle, the computation of the asymmetry gives rise to a factor which is proportional to $1/x$ with $x \equiv (m_{heavy}/m_{light})^2$. In these models, SM particles which have masses much smaller than $m_{light}$, contribute a negligible amount to the asymmetry. In contrast, our model contains fermions, $\psi_i(z)$, whose masses are not too much smaller than that of the “light” messenger scalar field and whose contribution in the asymmetry turns out to be proportional to $(m_{\psi_1}/m_{\phi_1})^2$. This is a contribution which greatly dominates over that of $(m_{\tilde{\phi}_1}/m_{\tilde{\phi}_2}) < 10^{-26}$ in our model. These points will be made clear below.

We would like to mention that there exists models of baryogenesis where there is an asymmetry between SM particles and e.g. particles that are not affected by the electroweak sphalerons [11] or scalar condensates [12]. Our model is similar in spirit but is entirely different from the aforementioned interesting models.

In this paper, we will lay out the groundwork for the computation of the SM lepton asymmetry from $\phi(z)$ decays. First, we will give a brief summary of the salient points of the $SU(2)_Z$ model (nicknamed $Q2D$). In particular, we will focus on the particle content and the related interactions which are most relevant for this paper. We will discuss the reason for having two scalars: $\phi(z)$. We then proceed with the computation, at $T = 0$, of the SM lepton asymmetry, showing its dependence both on the masses of the particles involved and on the strengths of the couplings and the CP violation. A more complete treatment of the problem at $T \neq 0$ will be dealt with elsewhere. Here, the main aim will be to show that the SM lepton asymmetry, in our model, can be non-vanishing at zero temperature. We end with a brief discussion on a possible detection of the lightest scalar which is responsible for this SM lepton number asymmetry. An interesting feature of this model is the fact that this “messenger field” which is the “progenitor” of the lepton asymmetry could in fact be found and identified in future colliders such as the Large Hadron Collider (LHC).
II. A BRIEF REVIEW OF SU(2)_Z

In this section, we summarize the essential elements of the SU(2)_Z model used in [6], [7], restricting ourselves to the non-supersymmetric case. At some scale \( M \), the gauge group is described by

\[
G_{SM} \otimes SU(2)_Z \tag{2}
\]

where \( G_{SM} \) could be, for example, SU(6) as in the chain \( E_6 \rightarrow SU(6) \otimes SU(2)_Z \), with SU(6) breaking down to \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) via, e.g., the route \( SU(3)_C \otimes SU(3)_L \otimes U(1)_Y \). Fermion fields transform under the above gauge group as

\[
\psi_{L,R} = (R_{L,R},1) ; \psi_{L,R}^{(Z)} = (1,3) , \tag{3}
\]

where \( R_{L,R} \) denotes the representation of the left-handed and right-handed SM fermions under \( G_{SM} \). Notice that the fermions of each sector are singlets under the other’s gauge group. Notice also that \( SU(2)_Z \) is a vector-like gauge group, similar to ordinary QCD. Apart from the obvious gravitational interactions, the two sectors can communicate with each other through a messenger scalar field which carries quantum numbers of both sectors. We shall see below that one actually needs two of such scalars, one being much heavier than the other. In this paper, we will concentrate on the type of messenger fields which are crucial for our leptogenesis scenario. They are:

\[
\tilde{\phi}_i^Z = (\phi_i^Z,0) , \phi_i^Z = (1,2,Y_\tilde{\phi} = -1,3) \tag{4}
\]

under \( SU(3) \otimes SU(2)_L \otimes U(1)_Y \otimes SU(2)_Z \), where \( i = 1,2 \) and where \( Q = T_3L + Y/2 \). Since we wish \( SU(2)_Z \) to be unbroken and to grow strong at \( \Lambda_Z \sim 10^{-3} \text{eV} \), we will assume that the potential for \( \phi_i^Z \) is such that \( \langle \phi_i^Z \rangle = 0 \). As a consequence, it will not contribute to the breaking of the electroweak gauge group. The physical masses of the messenger fields are arbitrary. As explained in [7], one of the two messenger fields is assumed to have a mass less than 1 TeV so that the \( SU(2)_Z \) and SM plasmas maintain thermal equilibrium until \( \phi_1^Z \) drops out of equilibrium and the other, \( \phi_2^Z \), is assumed to be very massive, with a mass of the order of a typical GUT scale in order for the evolution of the \( SU(2)_Z \) coupling to yield the desired features of the model. This also turns out to be what we need for the leptogenesis scenario.

In addition to one of the above messenger fields (the heavy one with GUT-scale mass is not included in the evolution of the coupling), it is shown in [7] that the following fermions are needed in order for the \( SU(2)_Z \) coupling \( \alpha_Z = g_Z^2/4\pi \) to be of order unity at around \( \Lambda_Z = 3 \times 10^{-3} \text{eV} \): \( \psi_1^Z \) and \( \psi_2^Z \). As shown in [7], the masses of \( \psi_1^Z \) and \( \psi_2^Z \) come from a complex scalar which is a singlet under both the SM and \( SU(2)_Z \). (As shown in [7], the “axion”, which is the imaginary part of this complex scalar, is the quintessence field which gets trapped in a false vacuum and yields a scenario for the dark energy.) The vacuum expectation of the real part of that scalar is unconstrained by present particle physics data, although a recent model of “low scale” inflationary scenario did put a constraint on its VEV \( |\langle \phi_1^Z \rangle| \). As a result, the masses of \( \psi_1^Z \) and \( \psi_2^Z \) are arbitrary. However, as it is argued in [7], they (or at least one of them) can be a candidate for a WIMP cold dark matter if its mass is of \( O(100 - 200 \text{ GeV}) \). As discussed in [7], the most attractive WIMP scenario in our model is one in which \( \psi_1^Z \) and \( \psi_2^Z \) are close in mass to each other, with \( m_{\psi_1^Z} \sim m_{\psi_2^Z} \sim \mathcal{O}(100 \text{ GeV}) \). It is in this context that we will concentrate our discussion of leptogenesis.

To complete this review section, we show, for illustration, a couple of graphs of \( \alpha_Z \) and \( \alpha_{Z^{-1}} \) versus \( E \) taken from [7] for a given value of \( \phi_1^Z \) mass with two different values of \( \psi_1^Z \) masses. (The constraint is \( \alpha_Z = 1 \) at \( \Lambda_Z = 3 \times 10^{-3} \text{eV} \).) Although the calculations presented below are meant to be general, we will illustrate our results with masses which are in the range of values illustrated in Figures 1 and 2.

In [6], it was shown that the “minimum model” with \( \psi_1^Z, \psi_2^Z \) and \( \phi_1^Z \) was sufficient as a scenario for both the dark energy and dark matter. However, as discussed in [7], a simple extension of this “minimum model” by including one extra heavy messenger field \( \phi_2^Z \) has a far-reaching consequence: a non-vanishing SM lepton number asymmetry as we shall see below. We will assume that \( \langle \phi_2^Z \rangle = 0 \) and \( m_{\phi_2^Z} \gg m_{\phi_1^Z} > 0 \). Without this extra messenger field, the asymmetry will simply vanish. This is a well-known result of early models of baryogenesis [14]. It also turns out that the dominant contributions to this asymmetry is insensitive to the value of \( m_{\phi_2^Z} \) as long as \( m_{\phi_2^Z} \gg m_{\phi_1^Z} \). In this case, \( m_{\phi_2^Z} \) can be as large as a typical “GUT” scale.

Although the lepton asymmetry discussed in this paper comes primarily from the decay of \( \phi_1^Z \) when it drops out of thermal equilibrium, in principle the decay of the much heavier \( \phi_2^Z \) can also generate a SM lepton number asymmetry. However, this asymmetry will be washed out by the inverse-decay into the lighter \( \phi_1^Z \) at \( T > m_{\phi_1^Z} \). This is similar to the popular leptogenesis scenario with two Majorana neutrinos, one of which being much heavier than the other. Because of this fact, we will focus only on the decay of \( \phi_1^Z \) in this paper.

We now proceed to the discussion of our model of leptogenesis.

III. SM LEPTOGENESIS FROM \( \phi_1^Z \) DECAYS

In this section, we will show how the introduction of two messenger fields gives rise to the possibility of a new mechanism for SM leptogenesis, alternative to the popular scenario in which the SM lepton number asymmetry is generated by the decay of a heavy Majorana neutrino.
It will be shown that the value of the SM lepton asymmetry depends primarily on the ratio of the $SU(2)_Z$ fermion mass to that of the messenger scalar field $\tilde{\phi}_1$, namely $(m_{i}/m_{\tilde{\phi}_1})^2$, where $m_i$ with $i = 1,2$ denotes the mass of the $SU(2)_Z$ fermion. The (by-far) subdominant contributions are found to be proportional to $m_{\tilde{\phi}_2}^2/m_{\tilde{\phi}_1}^2$ and $m_{\tilde{\phi}_3}^2/m_{\tilde{\phi}_1}^2$ which are less than $10^{-26}$. Interestingly enough, as we shall see below, for $m_{\tilde{\phi}_3} \gg m_{\tilde{\phi}_1}$, the asymmetry depends (beside other factors such as the CP phases, etc.) mostly on the ratio $(m_{i}/m_{\tilde{\phi}_1})^2$ and is insensitive to the exact value of $m_{\tilde{\phi}_2}$ as long as $m_{\tilde{\phi}_2} \gg m_{\tilde{\phi}_1}$. Let us remind ourselves that $\psi_i^{(Z)}$ with $m_i = O(100\,\text{GeV})$ could be WIMP candidates as discussed in [6] and $\tilde{\phi}_1$ with a mass not-too-different from the electroweak scale can be searched for at colliders such as the LHC. One cannot fail but notice the interesting connection between the aforementioned ratio which appears in the SM lepton asymmetry and the “detectability” of $\psi_1^{(Z)}$ and $\tilde{\phi}_1^{(Z)}$. We will come back to this connection below.

Before writing down the interaction Lagrangian, let us notice a few facts. (a) $SU(2)$ representations are real: both $3 \times 3$ and $3 \times 3^*$ contain a singlet. (b) $\psi_{1,L}^{(Z)}$, transforms like a right-handed spinor. In order to construct the diagrams shown in Fig. 3, one can write the $SU(2)_L \otimes U(1)_Y \otimes SU(2)_Z$ invariant Lagrangian taking into account points (a) and (b) as follows:

$$\mathcal{L}_{\text{yuk}} = \sum_{i,m} \left( g_{\tilde{\phi}_1}^{(1)} \bar{m}_L \phi_{1}^{(Z)} \psi_{i}^{(Z)} + g_{\tilde{\phi}_2}^{(1)} \bar{m}_L \phi_{2}^{(Z)} \psi_{i}^{(Z)} + H.c. \right),$$

where

$$\psi_1^{(Z)} \equiv \psi_{1,R} \psi_{1,L}^{(Z)},$$

and where, in general, the couplings $g_{\tilde{\phi}_1,m}^{(1,2)}$ and $g_{\tilde{\phi}_2,m}^{(1,2)}$ are complex and where $m = 1,2,3$ and $i = 1,2$ refer to the lepton family number and the two $SU(2)_Z$ fermions respectively. In general, one has twelve complex Yukawa couplings in total. We write them as

$$g_{\tilde{\phi}_1}^{(1,2)} = |g_{\tilde{\phi}_1,m}^{(1,2)}| \exp(i\alpha_{(1,2),m}),$$

$$g_{\tilde{\phi}_2}^{(1,2)} = |g_{\tilde{\phi}_2,m}^{(1,2)}| \exp(i\beta_{(1,2),m}).$$

Notice that the interaction [5] violates “lepton” number in a general sense that it includes also the shadow fermions. It is then natural to define the following Majorana shadow fermions

$$N_1^{(Z)} = \psi_{1,L}^{(Z)} + \psi_{1,L}^{(Z)},$$

$$M_1^{(Z)} = \psi_{1,R}^{(Z)} + \psi_{1,R}^{(Z)}.$$
A remark is in order concerning (14). The asymmetry obtained when $K < 1$ is actually the largest value for a given scenario. However, when $K > 1$ but not too different from unity, the net lepton number asymmetry will be diluted by a factor which is approximately $1/K$ (modulo a factor which is less than 2 when $K$ is not too large). As we shall see below, the upper bound on the mass of the messenger “progenitor” field will be lowered if we allow for the possibility of $K > 1$. Roughly speaking, the dilution factor $1/K$ has to be compensated by an increase in the basic asymmetry $\epsilon_l^{\tilde{\psi}_1}$ discussed below which can come about when the mass of the messenger field is decreased. This will be discussed at the end of this section and in the section on phenomenology.

Once the out-of-equilibrium condition is fulfilled, the next thing to do is to estimate the SM lepton number asymmetry and relate it to the sought-after baryon asymmetry. Since the main aim of the present manuscript is to present a scenario for the computation of the SM lepton number asymmetry, we first present an estimate of that quantity in order to have some ideas on what to expect of the magnitude of the asymmetry parameter $\epsilon_l^{\tilde{\psi}_1}$ to be computed in our model. This asymmetry will be computed at $T = 0$. Although care should be taken to include finite temperature corrections (see e.g. (15)), one does not expect the final result to be too different from the zero temperature one. When $T < m_{\tilde{\psi}_1}$ and when $K < 1$, the number density of $\tilde{\psi}_1$ is approximately $n_{\tilde{\psi}_1} = T^3/\pi^2$ (overabundance) and the entropy is $s = (2/45) g_s \pi^2 T^3$, with $g_s \sim 114$ (including $SU(2)_L$ light degrees of freedom). The decay of $\tilde{\psi}_1$ and $\tilde{\psi}_1^*$ creates a SM lepton number asymmetry per unit entropy $n_{LSM}/s \sim 2 \times 10^{-3} \epsilon_l^{\tilde{\psi}_1}$. For the SM with three generations and one Higgs doublet, one has $n_B/s \sim -0.35 n_{LSM}/s \sim -10^{-3} \epsilon_l^{\tilde{\psi}_1}$, where $n_B$ is “processed” through the electroweak sphaleron. Since $m_B/s \sim 10^{-10}$, a rough constraint on $\epsilon_l^{\tilde{\psi}_1}$ is found to be

$$\epsilon_l^{\tilde{\psi}_1} \sim 10^{-7}.$$  \hspace{1cm} (15)

We will make use of the constraint (15) to gain some insights into the allowed ranges of masses in our model.

The central quantity to be computed in our model is the asymmetry

$$\epsilon_l^{\tilde{\psi}_2} = \frac{\Gamma(\tilde{\psi}_2 \rightarrow \tilde{\psi}_1 l) - \Gamma(\tilde{\psi}_2 \rightarrow \tilde{\psi}_1^* l)}{\Gamma(\tilde{\psi}_2 l) + \Gamma(\tilde{\psi}_2^* l)},$$  \hspace{1cm} (16)

where $\Gamma(\tilde{\psi}_2 l)$ and $\Gamma(\tilde{\psi}_2^* l)$ contain the sums over all three flavors of SM leptons. Also, in the numerator of (16), $\Gamma(\tilde{\psi}_2 l)$ and $\Gamma(\tilde{\psi}_2^* l)$ are computed up to one loop and therefore contain interferences between the tree-level and one-loop contributions. It is only this interference which contributes to $\epsilon_l^{\tilde{\psi}_1}$. As usual, the decay widths in the denominator of (16) are kept at tree level. To be more specific, we will define the following asymmetries corresponding to the decay of $\tilde{\psi}_1$ separately into $\psi_1^{(Z)}$ and $\psi_2^{(Z)}$.

$$\epsilon_1^{\tilde{\psi}_1} = \frac{\Gamma(\tilde{\psi}_1 \rightarrow \tilde{\psi}_1^{(Z)} + l) - \Gamma(\tilde{\psi}_1 \rightarrow \tilde{\psi}_1^{(Z)*} + l)}{\Gamma(\tilde{\psi}_1 \rightarrow \tilde{\psi}_1^{(Z)} + l) + \Gamma(\tilde{\psi}_1 \rightarrow \tilde{\psi}_1^{(Z)*} + l)}.$$  \hspace{1cm} (17)

$$\epsilon_2^{\tilde{\psi}_1} = \frac{\Gamma(\tilde{\psi}_1 \rightarrow \tilde{\psi}_2^{(Z)} + l) - \Gamma(\tilde{\psi}_1 \rightarrow \tilde{\psi}_2^{(Z)*} + l)}{\Gamma(\tilde{\psi}_1 \rightarrow \tilde{\psi}_2^{(Z)} + l) + \Gamma(\tilde{\psi}_1 \rightarrow \tilde{\psi}_2^{(Z)*} + l)}.$$  \hspace{1cm} (18)

In (17) and (18), a non-vanishing value for $\epsilon_1^{\tilde{\psi}_1}$ comes from the interference between the tree-level and one-loop contributions to the decay widths. In what follows, we will concentrate on these interference terms.

We will denote the tree-level-one-loop interference contribution to the decay width by $\Gamma_{\text{int,} \tilde{\psi}_1 l}^{(1)}$. The one-loop contribution includes both vertex and self energy corrections as shown in Fig. (3). In what follows, we will neglect the SM lepton masses in the one-loop calculations since their contributions to the asymmetry is tiny, of order $m_l m_{\psi_{1,2}}/m_{\tilde{\psi}_2}^2$. First, we concentrate on the vertex contribution. We obtain for $\tilde{\psi}_1^{(Z)} \rightarrow \tilde{\psi}_2^{(Z)} + l$,

$$\Gamma_{\text{int,} \tilde{\psi}_1 l}^{(1)} = \frac{\sum_l g_{\tilde{\psi}_1 l}^2 g_{\tilde{\psi}_2 l}^2 \sum_m g_{\psi_1 m}^2 g_{\psi_2 m}^2 I^{(1)}}{= \sum_l g_{\tilde{\psi}_1 l}^2 g_{\tilde{\psi}_2 l}^2 \sum_m g_{\psi_1 m}^2 g_{\psi_2 m}^2 I^{(2)}} + \text{c.c.},$$  \hspace{1cm} (19)

$$\Gamma_{\text{int,} \tilde{\psi}_1 l}^{(1)} = \frac{\sum_l g_{\tilde{\psi}_1 l} g_{\tilde{\psi}_2 l} \sum_m g_{\psi_1 m}^2 g_{\psi_2 m}^2 I^{(1)}}{=} + \text{c.c.}.$$  \hspace{1cm} (20)

where $g_{\tilde{\psi}_1 l}$ is defined in Eq. (5) and $I^{(1,2)}$ is an integral in which $\psi_1^{(Z)}$ propagates in the loop. (1,2) will be explicitly given below.) The sums are over all three flavors of leptons. Similarly, for the process $\tilde{\psi}_1^{(Z)} \rightarrow \tilde{\psi}_2^{(Z)*} + l$,

$$\Gamma_{\text{int,} \tilde{\psi}_1 l}^{(2)} = \frac{\sum_l g_{\tilde{\psi}_1 l} g_{\tilde{\psi}_2 l} \sum_m g_{\psi_1 m}^2 g_{\psi_2 m}^2 I^{(2)}}{=} + \text{c.c.},$$  \hspace{1cm} (21)

$$\Gamma_{\text{int,} \tilde{\psi}_1 l}^{(2)} = \frac{\sum_l g_{\tilde{\psi}_1 l} g_{\tilde{\psi}_2 l} \sum_m g_{\psi_1 m}^2 g_{\psi_2 m}^2 I^{(2)}}{=} + \text{c.c.}.$$  \hspace{1cm} (22)

It then follows that $\epsilon_1^{\tilde{\psi}_1}$ which are proportional to the difference between (19) and (20), and between (21) and
respectively, look as follows

\[
\epsilon_{1, V}^2 \propto \text{Im}\{ \sum_{l} g_{\phi_1 l}^{(1)*} g_{\phi_2 l}^{(1)} \sum_{m} g_{\phi_1 m}^{(1)*} g_{\phi_2 m}^{(1)} \} \text{Im}\{ I^{(1)} \}
\]

\[
+ \text{Im}\{ \sum_{l} g_{\phi_1 l}^{(2)*} g_{\phi_2 l}^{(1)*} \sum_{m} g_{\phi_1 m}^{(2)*} g_{\phi_2 m}^{(2)} \} \text{Im}\{ I^{(2)} \}.
\]  

(23)

where the subscript $V$ denotes the contribution coming from the interference of the tree-level and vertex-correction to the decay widths. Notice however that $\sum_{l} g_{\phi_1 l}^{(1)} g_{\phi_2 l}^{(1)*} \sum_{m} g_{\phi_1 m}^{(1)*} g_{\phi_2 m}^{(1)}$ is real and its imaginary part therefore vanishes. We are then left with

\[
\epsilon_{1, V}^2 \propto \text{Im}\{ \sum_{l} g_{\phi_1 l}^{(2)*} g_{\phi_2 l}^{(1)} \sum_{m} g_{\phi_1 m}^{(2)*} g_{\phi_2 m}^{(2)} \} \text{Im}\{ I^{(2)} \}.
\]  

(24)

Similarly, with $\sum_{l} g_{\phi_1 l}^{(1)*} g_{\phi_2 l}^{(2)} \sum_{m} g_{\phi_2 m}^{(2)*} g_{\phi_2 m}^{(2)}$ being real, we obtain

\[
\epsilon_{2, V}^2 \propto \text{Im}\{ \sum_{l} g_{\phi_1 l}^{(1)*} g_{\phi_2 l}^{(2)} \sum_{m} g_{\phi_1 m}^{(1)*} g_{\phi_2 m}^{(1)} \} \text{Im}\{ I^{(1)} \}.
\]  

(25)

Let us define

\[
C = \sum_{l} g_{\phi_1 l}^{(2)} g_{\phi_2 l}^{(1)*} \sum_{m} g_{\phi_1 m}^{(1)*} g_{\phi_2 m}^{(2)}.
\]  

(26)

It is then easy to see that the coefficient of the right-hand-side of (25) is just $C^*$ so that $\text{Im} C^* = - \text{Im} C$. One can then rewrite Eqs. (24) and (25) as

\[
\epsilon_{1, V}^2 = (\text{Im} C) \text{Im} \{ I^{(2)} \},
\]

(27)

\[
\epsilon_{2, V}^2 = -(\text{Im} C) \text{Im} \{ I^{(1)} \}.
\]  

(28)

In addition, the contribution to $\epsilon_{1, S}^2$ coming from the tree-level-self-energy interference is found to be

\[
\epsilon_{1, S}^2 \propto \text{Im}\{ \sum_{m} g_{\phi_1 m}^{(1)*} g_{\phi_2 m}^{(1)} \sum_{n} g_{\phi_1 n}^{(2)*} g_{\phi_2 n}^{(2)} \}
\]

\[
\times \left( \frac{m_{\phi_1}^2}{m_{\phi_1}^2 - m_{\phi_2}^2} \right) \frac{1}{16 \pi},
\]  

(29)

\[
\epsilon_{2, S}^2 \propto \text{Im}\{ \sum_{m} g_{\phi_1 m}^{(1)*} g_{\phi_2 m}^{(2)*} \sum_{n} g_{\phi_1 n}^{(2)*} g_{\phi_2 n}^{(2)} \}
\]

\[
\times \left( \frac{m_{\phi_1}^2}{m_{\phi_1}^2 - m_{\phi_2}^2} \right) \frac{1}{16 \pi}.
\]  

(30)

However, as discussed in [3], $m_{\phi_2}^2 \ll m_{\phi_1}^2$ (the ratio is approximately $\sim 10^{-26}$), the contribution of $\epsilon_{1, S}^2$ to $\epsilon_{1, V}^2$ is negligible and we will neglect it from hereon. In what follows, we will make the identification

\[
\epsilon_{1, 2} = \epsilon_{1, 2, V}^2,
\]  

(31)

and the total asymmetry is defined as

\[
\epsilon_{1, 2} = \epsilon_1 + \epsilon_2.
\]  

Therefore, we will focus below on constraints coming from the vertex corrections which will be given explicitly below.

Before presenting the results of the integrations, we write down explicitly the loop integrals $I$ in the case $m_{\psi(2)} = 0$ and $I^{(1, 2)}$ in the case $m_{\psi(2)} \neq 0$. All SM lepton masses are neglected in the propagators. They are:

\[
I = \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l + k)^2 (l + k')^2}
\]

\[
+ \int \frac{d^4l}{(2\pi)^4} \frac{m_{\psi(2)}^2}{[(l + k)^2][l^2 - m_{\psi(2)}^2]}.
\]  

(33)

\[
I^{(1, 2)} = \int \frac{d^4l}{(2\pi)^4} \frac{1}{(l + k)^2 (l + k')^2 - m_{\psi(2)}^2}
\]

\[
+ \int \frac{d^4l}{(2\pi)^4} \frac{m_{\psi(2)}^2}{[(l + k)^2][l^2 - m_{\psi(2)}^2][l^2 - m_{\psi(2)}^2]}.
\]  

(34)

where $k$ and $k'$ are the four-momenta of the external $\psi(2)$ and SM lepton $l$ respectively. In [34], the numerator of the second term actually should be $m_{\psi(2)}^2 + 4 m_l m_{\psi(2)} \sim m_{\psi(2)}^2$.

To make our discussion as concise as possible, we first present the result for the limit $m_{\psi(2)} \to 0$ to simply show that it coincides with the well-known results. For this purpose, let us define

\[
x \equiv \left( \frac{m_{\psi(2)}^2}{m_{\phi_1}^2} \right)^{2}.
\]  

(35)

In the limit $m_{\psi(2)} \to 0$, one has $\text{Im} I^{(1)} = \text{Im} I^{(2)} = I \text{Im}$ where

\[
\text{Im} I = \frac{1}{16 \pi} \left( 1 - x \ln(1 + \frac{1}{x}) \right).
\]  

(36)

A few remarks are in order here in order to clarify the contrast of the results of our model with those of “standard scenarios”. In the above result for $\text{Im} I$, the mass-independent part $1/16 \pi$ (first term on the right-hand side of (36) comes from the absorptive part of the first integral in (33), while the mass-dependent second term comes from the absorptive part of the second integral in (33). For $x \gg 1$, $(1 - x \ln(1 + \frac{1}{x})) = (1 - x(\frac{1}{x} - \frac{1}{2x^2} + ..)) \sim \frac{1}{2x}$ and one obtains the familiar result, namely $\text{Im} I \sim \frac{1}{32 \pi x}$. The mass-independent term cancels with the first term in the expansion of the logarithm. At this point, one might want to make contact with results that one obtains from an effective theory after integrating out the...
heavy degree of freedom, namely $\bar{\varphi}_2$. In an effective theory where the heavy degree of freedom is integrated out so that $\mathcal{L} = \mathcal{L}_{\text{Lagr}} + (1/m_{\varphi_1}^2) \mathcal{L}_{\text{vertex}} + \ldots$, one can construct an equivalent one-loop vertex correction and obtains the suppression factor $m_{\varphi_1}^2/m_{\varphi_2}^2$ directly without the aforementioned cancellation. In that sense, the effective theory "misses" that mass-independent term $1/16\pi$ which should be there in the full one loop calculation (used in most papers) but which gets cancelled for $x \gg 1$.

When $m_{\psi_{\varphi(z)}}$ is not too different from $m_{\varphi_1}$, there are some differences with the above "massless" results. The first term of Eq. (34) gives an identical absorptive part to that of the first term of Eq. (33), namely $(1/16\pi)$. The absorptive part of the second integral in (34) is now proportional to $\ln[(m_{\varphi_2}^2/m_{\varphi_1}^2 + 1 - m_{\varphi_{\varphi(z)}}^2/m_{\varphi_1}^2)/(m_{\varphi_2}^2/m_{\varphi_1}^2)]$. The first two terms inside the log are the well-known result for massless fermions while the last term cannot be neglected in this case because $m_{\psi_{\varphi(z)}} = O(m_{\varphi_2})$, in contrast with previous scenarios of baryogenesis or leptogenesis. This gives rise to an additional contribution in the vertex correction, namely

$$Im I^{(1,2)} = \frac{1}{16\pi} (1 - x \ln(1 + \frac{1}{x} \frac{m_{\varphi_2}}{m_{\varphi_1}})), \quad (37)$$

where

$$y_{1,2} \equiv \left(\frac{m_{\varphi_2}}{m_{\psi_{\varphi(z)}}}\right)^2. \quad (38)$$

For $x, y_{1,2} \gg 1$, one can again expand the logarithm term as $1 - x \ln(1 + 1/x - 1/y_{1,2}) = 1 - x(1 - 1/y_{1,2} - (1/2)(1 - 1/y_{1,2})^2 + \ldots) \approx x/y_{1,2} + (1/2) x (1 - 1/y_{1,2})^2 + \ldots$. Again, the mass-independent term cancels again $x(1/x)$ but now there remains an extra term $x/y_{1,2}$ which would vanish if $m_{\psi_{\varphi(z)}} = 0$. The second term in the expansion is now a subdominant term equal to $(1/2)(m_{\psi_{\varphi(z)}}^2 - m_{\varphi_1}^2)^2/(m_{\psi_{\varphi(z)}}^2 m_{\varphi_1}^2)$. Taking into account the explicit definitions of $x$ and $y_{1,2}$, one can expand Eq. (37) to find

$$Im I^{(1,2)} = \frac{1}{16\pi} \left(\frac{m_{\psi_{\varphi(z)}}}{m_{\varphi_1}}^2 + \ldots\right), \quad (39)$$

where $\ldots$ in Eq. (39) denotes terms of order $(m_{\varphi_1}/m_{\varphi_2})^2 < 10^{-26}$, $(m_{\psi_{\varphi(z)}}/m_{\varphi_2})^2 < 10^{-26}$ and higher which are negligible compared with $(m_{\psi_{\varphi(z)}}/m_{\varphi_1})^2 = x/y_{1,2}$. The dominant term in (39) which comes from the full one-loop calculation and which cannot be neglected because $m_{\psi_{\varphi(z)}} = O(m_{\varphi_2})$, cannot be seen from an effective theory. This might appear to be "unexpected" because in previous calculations, the decaying particle is much more massive than any SM particles (either in SU(5) baryogenesis or in standard scenarios of leptogenesis). It is this quantity which will determine the size of $Im I^{(1,2)}$ and hence that of the asymmetry $\epsilon_{1,2}$. From (39), one notices that $Im I^{(1,2)}$ is not sensitive to the value of the mass of $\bar{\varphi}_2$ as long as $m_{\bar{\varphi}_2} \gg m_{\varphi_1}, m_{\psi_{\varphi(z)}}$

From the above discussions, one notices the clear distinction between our model for leptogenesis and the "standard scenario" involving heavy Majorana neutrinos. Let us enumerate the differences.

- For the "standard" scenario, SM particle masses are neglected compared with the heavy Majorana neutrino masses in the one-loop computations of the vertex and wave function corrections. As a result, the lepton number asymmetry in these scenarios depend only on the ratios of heavy Majorana neutrino masses, apart from the couplings and CP phase(s).

- In our scenario, the decaying particle which gives rise to the net lepton asymmetry is the messenger scalar field $\bar{\varphi}_1$ whose mass is within the range of the electroweak scale while the other messenger field $\bar{\varphi}_2$ has a mass of the order of a typical GUT scale. If these were the only particles that one takes into account in the computation of the asymmetry, the ratio of the mass of $\bar{\varphi}_1$ to that of $\bar{\varphi}_2$ would negligibly small to play any role in the asymmetry. The difference with the "standard" scenario lies in the existence of the $SU(2)_L$ fermions whose masses are not too different from that of the decaying $\bar{\varphi}_1$. This gives, as a result, an asymmetry which mainly depends on the ratio of the masses of the $SU(2)_L$ fermions to that of $\bar{\varphi}_1$ and which is no longer suppressed by large mass ratios such as $(m_{\bar{\varphi}_2}/m_{\bar{\varphi}_1})^2$ and $(m_{\bar{\varphi}_2}/m_{\psi_{\varphi(z)}})^2$. In fact, the asymmetry is not sensitive to mass of the very heavy messenger field but depends instead on the ratio of the masses of the two "lighter" particles.

Let us now turn to the question of how big or how small $\epsilon_{total}$ might be. In particular, it would be illuminating to see under what conditions $\epsilon_{total}$ vanishes so that we might learn about the reasons why it does not vanish. From the definitions (17, 18) and the results (27, 28), we obtain

$$\epsilon_{total} = \epsilon_1 + \epsilon_2 = \frac{ImC}{\sum_i |g^{(1)}_{\bar{\varphi}_i}|^2} Im\{I^{(2)}\} - \frac{ImC}{\sum_i |g^{(2)}_{\bar{\varphi}_i}|^2} Im\{I^{(1)}\}. \quad (40)$$

Using the results for $Im\{I^{(1,2)}\}$ (Eq. (39)), we can rewrite (40) as

$$\epsilon_{total} = \frac{1}{16\pi} \left(\frac{m_{\psi_{\varphi(z)}}^2 - m_{\varphi_1}^2}{m_{\varphi_1}^2}\right) \left(\frac{ImC}{\sum_i |g^{(1)}_{\bar{\varphi}_i}|^2}\right) + \frac{1}{16\pi} \left(\frac{m_{\psi_{\varphi(z)}}^2}{m_{\varphi_1}^2}\right) \left(ImC\right) \left(\frac{1}{\sum_i |g^{(1)}_{\bar{\varphi}_i}|^2} - \frac{1}{\sum_i |g^{(2)}_{\bar{\varphi}_i}|^2}\right). \quad (41)$$
A close examination of (41) reveals some interesting features of our model.

- $\epsilon_{\text{tot}}$ vanishes if $\text{Im}C = 0$. This fact is self-evident since, in order to obtain a non-vanishing asymmetry, one has to have CP-violating complex Yukawa couplings and hence $\text{Im}C \neq 0$ in our model.

- $\epsilon_{\text{tot}}$ vanishes, even when $\text{Im}C \neq 0$, if we have both

$$\frac{1}{\sum_i |g^{(1)}_{\tilde{\phi}_1 i}|^2} = \frac{1}{\sum_i |g^{(2)}_{\tilde{\phi}_2 i}|^2}, \quad (42)$$

and

$$m_{\psi_1^{(2)}} = m_{\psi_2^{(2)}}. \quad (43)$$

Point # 2 is quite interesting in its own right. Since, in the limit (42, 43), $\epsilon_{\text{tot}} = 0$, this seems to suggest some kind of symmetry in the shadow sector involving $\psi_1^{(2)}$. To see what this might be, let us again look at Eq. (41). If we put

$$|g^{(1)}_{\tilde{\phi}_1 i}| = |g^{(2)}_{\tilde{\phi}_2 i}|, \quad (44)$$

in (41), we obtain

$$\epsilon_{\text{tot}} = \frac{1}{16\pi} \left( \frac{m_{\psi_2^{(2)}}^2 - m_{\psi_1^{(2)}}^2}{m_{\tilde{\phi}_1^1}^2} \right) \sum_i \frac{\text{Im}C}{|g^{(1)}_{\tilde{\phi}_1 i}|^2}. \quad (45)$$

The equality (44) suggests the existence of a custodial symmetry, at tree-level (where only the magnitudes of the couplings are involved), in the shadow sector which is explicitly broken by the mass difference between $\psi_1^{(2)}$ and $\psi_2^{(2)}$. Let us assume this custodial symmetry to be a global $SU(2)$ and let us denote it by $SU(2)_{\text{shadow}}$. The shadow fermions then belong to a doublet of $SU(2)_{\text{shadow}}$: $(\psi_1^{(2)}, \psi_2^{(2)})$. The breaking of this custodial symmetry by the shadow mass difference such that $\epsilon_{\text{tot}} \neq 0$ is reminiscent of the breaking of the SM custodial symmetry by mass differences among the up and down members of the $SU(2)_L$ doublet such that $\rho \neq 1$. The factor $\left( \frac{m_{\psi_2^{(2)}}^2 - m_{\psi_1^{(2)}}^2}{m_{\tilde{\phi}_1^1}^2} \right)$ plays a similar role to the radiative correction to the $\rho$ parameter coming from, e.g. a quark doublet, which is proportional to $\left( \frac{m_q^2 - m_{\tilde{\chi}_1}^2}{m_{\tilde{\chi}_1}^2} \right)$.

Let us come back to the statement that $\epsilon_{\text{tot}} = 0$ when $\text{Im}C = 0$ (Point # 1). This can be achieved when

$$g^{(1)}_{\tilde{\phi}_1 m} = g^{(2)}_{\tilde{\phi}_1 m} = |g_{\tilde{\phi}_1 m}| \exp(i\alpha_m), \quad (46)$$

and

$$g^{(1)}_{\tilde{\phi}_2 m} = g^{(2)}_{\tilde{\phi}_2 m} = |g_{\tilde{\phi}_2 m}| \exp(i\beta_m). \quad (47)$$

With (46, 47), one can easily see that $C$ is real and hence $\text{Im}C = 0$. What (46, 47) also imply is that the shadow custodial symmetry $SU(2)_{\text{shadow}}$, when it is applied beyond the tree-level to the Yukawa couplings, gives rise to $\epsilon_{\text{tot}} = 0$ even if it is broken in the mass sector by having $m_{\psi_1^{(2)}} \neq m_{\psi_2^{(2)}}$.

Last but not least, one might want to know whether or not we can put $\tilde{\phi}_1$ and $\tilde{\phi}_2$ together into a doublet of the shadow custodial symmetry. In what follows, we will see that the constraint from the $K$ factor appears to rule out such a possibility. In the last section on phenomenology, we will be pointing out that from the constraint $0.04 < K < 1$ (44), one can deduce that $8 \times 10^{-17} < \alpha_{\tilde{\phi}_1} < 2 \times 10^{-15}$. The shadow custodial symmetry that includes $\tilde{\phi}_1$ and $\tilde{\phi}_2$ would then imply $\alpha_{\tilde{\phi}_1} = \alpha_{\tilde{\phi}_2}$. As seen below, the asymmetry is proportional to $\alpha_{\tilde{\phi}_2}$ and the K factor constraint would make it many orders of magnitude below the required value if the shadow custodial symmetry also includes the messenger fields.

The above discussions suggest a deep connection between the possible existence of a custodial symmetry in the shadow sector and the size of the lepton number asymmetry $\epsilon_{\text{tot}}$: the breaking of that custodial symmetry at loop levels gives rise to a non-vanishing asymmetry. Our next step is to make an estimate for the asymmetry as a function of various quantities which might have direct phenomenological implications such as the messenger mass and the shadow fermion masses.

As mentioned above, the constraint we will use is $|\epsilon_{\text{tot}}| \sim 10^{-7}$. Also for simplicity, we will use the formula (44) for $\epsilon_{\text{tot}}$. Let us first define

$$|\Delta m_{\psi_1^{(2)}}^2| \equiv |m_{\psi_2^{(2)}}^2 - m_{\psi_1^{(2)}}^2|. \quad (48)$$

As an example, let us take some numbers presented in Fig. (41), namely $m_{\tilde{\phi}_1^{(2)}} = 300 \text{GeV}$, $m_{\psi_1^{(2)}} = 100 \text{GeV}$ and $m_{\psi_2^{(2)}} = 50 \text{GeV}$. We then obtain

$$\frac{1}{16\pi} \left( \frac{|\Delta m_{\psi_1^{(2)}}^2|}{m_{\tilde{\phi}_1^1}^2} \right) \sim 1.7 \times 10^{-3}. \quad (49)$$

One can then obtain the following estimate

$$\frac{|\text{Im}C|}{\sum_i |g^{(1)}_{\tilde{\phi}_1 i}|^2} \sim 10^{-4}. \quad (50)$$

For the sake of estimation, let us assume that $|g_{\tilde{\phi}_1 e}| \sim |g_{\tilde{\phi}_1 \mu}| \sim |g_{\tilde{\phi}_1 \tau}| = |g_{\tilde{\phi}_2 e}|$ and $|g_{\tilde{\phi}_2 \mu}| \sim |g_{\tilde{\phi}_2 \tau}| = |g_{\tilde{\phi}_2 z}|$. One then obtains

$$\frac{|\text{Im}C|}{\sum_i |g^{(1)}_{\tilde{\phi}_1 i}|^2} \sim |g_{\tilde{\phi}_2 z}|^2 \sin \chi \sim 10^{-4}, \quad (51)$$

where $\sin \chi$ is a function of the various phases. Notice that (51) is practically independent of the size of the Yukawa couplings of the lighter decaying messenger field.

- To obtain a rough estimate on the lower bound on the light messenger field mass, we set $|g_{\tilde{\phi}_2 z}|^2 \sin \chi <$
1, giving the following bound, for $m_{\psi_2(z)} = 100 \text{ GeV}$
and $m_{\psi_1(z)} = 50 \text{ GeV}$,
\[ m_{\tilde{\varphi}_1(z)} < 38 \text{ TeV}. \] (52)

- A repeat of the above estimate with $m_{\psi_2(z)} = 100 \text{ GeV}$
and $m_{\psi_1(z)} = 98 \text{ GeV}$, for example, yields
\[ m_{\tilde{\varphi}_1(z)} < 9 \text{ TeV}. \] (53)

- Our last example is with $m_{\psi_2(z)} = 100 \text{ GeV}$,
$m_{\psi_1(z)} = 50 \text{ GeV}$ and $|g_{\varphi_2}|^2 \sin \chi \sim 10^{-3}$. This

\[ m_{\tilde{\varphi}_1(z)} < 1.2 \text{ TeV}. \] (54)

The next interesting question to ask is how small can $|\Delta m_{\psi_1(z)}^2|$ be. First, the messenger field cannot be too light since it has to decay while the sphaleron process is
still in thermal equilibrium as we have discussed above.

Using the constraint (14), let us for definiteness set the minimum value for the mass of the messenger field to be approximately 100 GeV. One now obtains a lower bound
on $|\Delta m_{\psi_1(z)}^2|$, namely
\[ |\Delta m_{\psi_1(z)}^2| > 0.05 \text{ GeV}^2. \] (55)

Using more “reasonable” values for $\frac{|MC|}{\sum_i |g_{\varphi_1}|^2}$, say $10^{-3}$,
and $m_{\tilde{\varphi}_1(z)} \sim 300 \text{ GeV}$, one obtains
\[ |\Delta m_{\psi_1(z)}^2| > 452 \text{ GeV}^2. \] (56)

From the above estimates, one can infer that, unless there is a high degree of degeneracy in the shadow fermion sector, the masses $m_{\psi_1(z)}$ can be naturally in the range
which is suitable for them to be candidates for Cold Dark Matter (CDM), namely $O(100 \text{ GeV})$.

What the previous estimates show is that, for a given value of the factor $|g_{\varphi_2}|^2 \sin \chi$, the more degenerate $\psi_1(z)$
and $\psi_2(z)$ are, the lower the $\phi_1(z)$ mass should be in order
to get a reasonable value for the asymmetry. Turning the argument around, one infers that the shadow fermions
cannot be too degenerate and that their masses can naturally
be in the favored range to be CDM candidates.

The above upper bounds on the light messenger mass
can be lowered if we allow for the factor $K$ (see Eq. 13)
to be greater than unity. Because of the dilution factor $1/K$,
$\epsilon_{\text{tot}}$ changes to $\sim -10^{-7} K$. For example, the bound
is lowered to 2.8 TeV and 890 GeV for $K = 10, 100$
respectively.

The examples given above are far from being exhaustive
and are simply meant to be illustrative of the deep
connection, in our model, between the SM lepton asymmetry,
which is eventually transmogrified into a baryon asymmetry,
and the mass of the messenger scalar field
\[ \tilde{\varphi}_1 \] responsible for this asymmetry. They point to the
fact that $\tilde{\varphi}_1$ could be relatively “light” and has thus a
“chance” to be found if it exists. In addition, it was also shown that a deep connection exists between the asymmetry
and the breaking of the shadow custodial symmetry
discussed above.

As we have mentioned above, a complete treatment of the
asymmetry linking $\epsilon_{\text{tot}}$ to the actual SM lepton number asymmetry,
and eventually to the baryon asymmetry, requires one to solve the Boltzman equation taking into account various factors such as decays and inverse decays, etc...
This however will not significantly change the various
bounds derived above. A more detailed study will
be presented elsewhere.

Last but not least, we would like to remark that the
lepton flavour effects encountered in see-saw leptogenesis
scenarios (10) do not affect our model since the decays
here are very out-of-equilibrium and the single-flavour
analysis is justified.

IV. PHENOMENOLOGICAL CONSEQUENCES
OF THE “LEPTON NUMBER PROGENITOR”
\[ \tilde{\varphi}_1(z) \] WITH MASS $\lesssim 1 \text{ TeV}$

Since this section is slightly out of the main topic of
the paper, it will be very short and the details will be
presented elsewhere. The main purpose for including
it here is to show that there are consequences of our
proposed SM leptogenesis scenario that can be tested
experimentally in a not-too-distant future at the LHC.

The progenitor for the aforementioned lepton asymmetry
can possibly be found and identified experimentally!
As we had mentioned in Section (II), we require
$SU(2)_Z$ to be confining and, as the result, the messenger
fields which carry both $SU(2)_Z$ and electroweak
quantum numbers cannot have a vacuum expectation value.
In the kinetic terms for the messenger fields, and in particular
for $\tilde{\varphi}_1(z)$, one is interested in the following
interactions: $W^+ W^- (\tilde{\varphi}_1(z),0 \pm \tilde{\varphi}_1(z),+) + \tilde{\varphi}_1(z)^{-} \pm \tilde{\varphi}_1(z)^{+}$ and
$Z Z (\tilde{\varphi}_1(z),0 \pm \tilde{\varphi}_1(z),0 + \tilde{\varphi}_1(z)^{-} \pm \tilde{\varphi}_1(z)^{+})$. These interactions
will provide the dominant weak boson fusion (WBF)
production mechanism for a pair of $\tilde{\varphi}_1(z)$. A rough
expectation for the production cross section for $\tilde{\varphi}_1(z)$ with
a mass around 300 GeV is around 1 pb and 0.1 pb for a
mass around 500 GeV. The decay $\tilde{\varphi}_1(z)^{+} \rightarrow \psi_{1,2}^+ + l^+_i$ is
practically unobservable while $\tilde{\varphi}_1(z)^{-} \rightarrow \psi_{1,2}^- + l^-_i$
and $\tilde{\varphi}_1(z)^{\pm} \rightarrow \psi_{1,2}^{\pm} + l^+_i$ will have charged SM leptons with
unconventional geometry, perfectly distinguishable from
the decay of a 600 GeV SM Higgs boson.

For the decays of $\tilde{\varphi}_1(z)^{\pm}$, one might want to have
a rough idea on the length of the charged tracks
before the decays occur. For definiteness, let us take
$m_{\tilde{\varphi}_1} = 500 \text{ GeV}$ as an example. From the definition of
$K$ (13) and from the requirement $0.04 < K < 1$ (14),
one can deduce that $8 \times 10^{-17} < \alpha_{\tilde{\varphi}_1} < 2 \times 10^{-15}$.
It can occur at one loop with \( \Gamma_{\tilde{\varphi}_1} \sim \alpha_{\tilde{\varphi}_1} m_{\tilde{\varphi}_1} \), the decay length is roughly \( 0.02 \text{ cm} < \ell_{\tilde{\varphi}_1} < 0.5 \text{ cm} \). This decay length falls within the range of the radial region of a typical silicon detector at CMS and ATLAS (40 cm and 60 cm respectively). Notice that when \( K > 1 \) implying a larger \( \alpha_{\tilde{\varphi}_1} \), the decay length is even smaller than the previous upper bound, again well within reach of the aforementioned silicon detectors. It is also conceivable that, if these decays were to be observed, one might be able to measure the CP violating phases in Eq. (5) and, as a consequence, the size of the SM lepton number asymmetry needed in this leptogenesis scenario. It is interesting to note that our scenario allows for a direct search of the progenitor of the lepton, and hence baryon, asymmetry at future colliders.

The detection of \( \tilde{\varphi}_{1,2} \) would fall into the domain of Dark Matter search since it is electrically neutral and interacts very weakly with normal matter [7]. However, it can also be indirectly “observed” as missing energy in the decay of the messenger field.

One might also ask whether or not the process \( \mu \to e\gamma \) can be affected by the couplings discussed in this paper. It can occur at one loop with \( \psi_{1,2} \) and \( \tilde{\varphi}_1 \) or \( \tilde{\varphi}_2 \) propagating in the loop. First there is no enhancement factor encountered in \( Im I^{(1,2)} \). Second the rate is negligibly small because of constraints such as \( 8 \times 10^{-17} < \alpha_{\tilde{\varphi}_1} < 2 \times 10^{-15} \) or in the second case because of the suppression factor \( (\frac{m_{\tilde{\psi}_1}}{m_{\tilde{\psi}_2}})^2 \).

Finally, we wish to point out that our shadow fermions \( \psi_{1,2} \) do not acquire a millicharge which would have happen if \( SU(2)_Z \) is broken down to \( U(1) \) which could mix through vacuum polarization with \( U(1)_{em} \) [17]. In our model \( SU(2)_Z \) is unbroken and this is the reason why it grows strong at a very low scale \( \sim 10^{-3} \text{ eV} \).

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FIG. 1: $\alpha_Z(E)$ and $\alpha_Z^{-1}(E)$ versus $t = \ln(E/\Lambda_Z)$ for $m_{\psi'(z)} = 300 \text{ GeV}$, $m_{\psi(z)} = 100 \text{ GeV}$ and $m_{\psi_1(z)} = 50 \text{ GeV}$. Here $\Lambda_Z = 3 \times 10^{-3} \text{ eV}$. 
FIG. 2: $\alpha_Z(E)$ and $\alpha_Z^{-1}(E)$ versus $t = \ln(E/\Lambda_Z)$ for $m_{\psi_1^{(2)}} = 300 \text{ GeV}$, $m_{\psi_{13}^{(2)}} = 200 \text{ GeV}$ and $m_{\psi_1^{(2)}} = 100 \text{ GeV}$. Here $\Lambda_Z = 3 \times 10^{-3} \text{ eV}$. 
FIG. 3. The decay $\tilde{\phi}_1^{(2)} \to \tilde{\psi}_{1,2}^{(2)} + l$ at tree-level and at one loop.