Update on the hadron spectrum with two flavors of staggered quarks

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We present an update on the MILC Collaboration’s light hadron spectrum calculation with two flavors of dynamical, staggered quarks. Results are presented for gauge couplings 5.30, 5.415, 5.50 and 5.60, with a range of quark masses for each value of the coupling. We present extrapolations of \( m_N/m_\rho \) to the continuum limit for fixed values of \( m_\pi/m_\rho \) including the physical one.

Over the past few years the MILC Collaboration has been engaged in a series of spectrum calculations with dynamical quarks and in the quenched approximation, using both standard and improved actions[1]. One objective of this work is to obtain sufficient control over systematic errors so that one can make reliable extrapolations to the limits of zero lattice spacing and physical quark masses. Another objective is to make detailed comparisons between the spectra with quenched and dynamical quarks, and with standard and improved actions. Clearly, to reach these objectives requires high statistics calculations for a sufficiently wide range of lattice spacings and quark masses to enable us to make extrapolations to the continuum and chiral limits.

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It also requires that these calculations be carried out on large enough lattices to avoid finite size effects.

In this note we provide an update on our results for two flavors of dynamical, staggered quarks using the standard gauge and quark actions. Results for the quenched approximation and with improved actions are presented elsewhere in these proceedings[2]. We have carried out simulations at four values of the gauge coupling, \( 6/g^2 = 5.30, 5.415, 5.50 \) and 5.60 using at least four values of the quark mass at each coupling. Lattices were generated with the refreshed hybrid molecular dynamics algorithm. Once the lattices were equilibrated the light hadron spectrum was measured every fifth molecular dynamics time unit. Lattices were saved after every tenth time unit for use in other projects. With the exception of a few runs at large quark mass, measurements were made on
at least four hundred lattices at each values of the

gauge coupling and quark mass.

As one would expect, the hadron masses evaluated
in lattice units are strongly dependent on the
gauge coupling and bare lattice quark mass, \( am_q \).
Here \( a \) is the lattice spacing. The one exception
is the mass of the Goldstone pion which is weakly
dependent on the gauge coupling. In Fig. 1, we
plot the \( \rho \) mass as a function of the bare quark
mass for the two weakest values of the gauge
coupling that we have studied. For comparison we
also include in this figure results from quenched
calculations with staggered quarks at gauge cou-
plings 5.70, 5.85 and 6.15. Notice that for fixed
values of the gauge coupling, \( am_\rho \) decreases more
rapidly for decreasing \( am_q \) in full QCD than in
the quenched approximation. This is due to the
dependence of the lattice spacing on the quark
mass in full QCD.

An important question regarding any simula-
tion with staggered quarks is the extent to which
flavor symmetry is restored as the lattice spac-
ing and quark mass are decreased. One mea-
sure of flavor symmetry violation is the quantity
\( \delta_\pi = (m_\pi^2 - m_\rho^2)/m_\rho^2 \). In Fig. 2, we plot \( \delta_\pi \)
as a function of \( am_q \) for each of the couplings we
have studied. The trends are as expected. The
value of \( \delta_\pi \) for \( 6/g^2 = 5.60 \) is close to, but slightly
below that found in the quenched approximation
for gauge coupling 5.85.

In Fig. 3, we show the Edinburgh plot for our
two weakest gauge couplings using the Goldstone
pion in the ratio \( m_\pi/m_\rho \). Once again we include
quenched results for gauge couplings 5.70, 5.85
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Figure 4. $m_N/m_ρ$ as a function of the square of the lattice spacing for $m_π/m_ρ = 0.5$

and 6.15 in this graph. Here too, the full QCD results at 5.60 are quite close to the quenched ones at 5.85.

Because we have carried out calculations with a number of quark masses at each value of the coupling we have studied, we can perform fits to the hadron masses as a function of $am_ρ$ in order to extrapolate to the chiral limit and to interpolate for comparisons with quenched and improved action calculations. We are analyzing a variety of fitting functions. Here we present results from fits of the nucleon and rho masses to the form

$$m = m_0 + αm_ρ + βm_ρ^3/2 + γm_ρ^2.$$  

In Fig. 4 we show $m_N/m_ρ$ for the four values of the gauge coupling we have studied, in each case interpolating to the value of $am_ρ$ for which $m_π/m_ρ = 0.5$. The $x$-axis in this figure is $(am_ρ)^2$ interpolated to the same value of the quark mass. This quantity gives a measure of the square of the lattice spacing.

Fig. 5 is a repeat of Fig. 4, this time with extrapolations of $m_N/m_ρ$ to the continuum limit for fixed values of $m_π/m_ρ$, and they provide support of the expectation that for staggered quarks the leading corrections are of order $a^2$. These results are quite encouraging; however, a better understanding of the chiral extrapolation, as well as additional calculations at weaker coupling and smaller quark masses are needed in order to obtain definitive results.

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