Neutron charge form factor at large $q^2$

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Abstract. The neutron charge form factor $G_{En}(q)$ is determined from an analysis of the deuteron quadrupole form factor $F_{C2}(q)$ data. Recent calculations, based on a variety of different model interactions and currents, indicate that the contributions associated with the uncertain two-body operators of shorter range are relatively small for $F_{C2}(q)$, even at large momentum transfer $q$. Hence, $G_{En}(q)$ can be extracted from $F_{C2}(q)$ at large $q^2$ without undue systematic uncertainties from theory.

Introduction. Knowledge of the neutron charge form factor $G_{En}$ is of great importance for an understanding of its internal structure. It is also crucial for the calculation of nuclear charge form factors, since the latter depend on both $G_{En}$ and the proton charge form factor $G_{Ep}$.

Unfortunately, $G_{En}$ is still rather poorly known. The difficulties encountered in measuring $G_{En}$ are twofold: since there are no free neutron targets, $G_{En}$ has to be measured using composite systems, and this leads to complications due to the presence of other nucleons. In addition, the electron-neutron cross section is dominated by the contribution from the magnetic form factor $G_{Mn}$, thus making a determination of $G_{En}$ very difficult.

The traditional approach to determine $G_{En}$ uses the deuteron structure function $A(q)$, to which the deuteron magnetic form factor, and therefore $G_{Mn}$, contributes negligibly. After removing, via theoretical calculations, the effect of the deuteron structure and the contributions to the scattering process from two-body currents, subtraction of the $G_{Ep}$ contribution then allows one to extract $G_{En}$. This procedure is sensitive to systematic errors in the theory used, particularly those associated with the modeling of shorter-range two-body currents, which are still not very well controlled.

As a consequence, the resulting values for $G_{En}$ [1, 2] have fairly large uncertainties, and are limited to momentum transfers below $q^2=16$ fm$^{-2}$. This poses serious problems for the calculation of form factors of light nuclei, which one would want to calculate for the region covered by data, a region that extends to $q^2=30–100$ fm$^{-2}$. To the extent
that current parameterizations of nucleon form factors provide sensible extrapolations for $G_{En}$ at large $q^2$, one must conclude that the contribution of $G_{En}$, which seems to fall off with increasing $q^2$ much more slowly than $G_{Ep}$, becomes very important at these large momentum transfers.

More recently, the exploitation of a new technique to determine $G_{En}$ has become practical: when performing an $(e, e'n)$ coincidence experiment using polarized electrons and when measuring the polarization of the target nucleus or recoil neutron, it becomes possible to measure an interference term $G_{En}G_{Mn}$. This approach removes the difficulty associated with $G_{Mn}$-dominance, and is much less dependent on the nuclear structure of the target nucleus (deuteron or $^3$He). Several experiments of this type have been performed recently [3]–[11]. The resulting values for $G_{En}$ still have relatively large errors; they are, however, mainly statistical and thus can be reduced in the future using better technology. The limit in $q^2$ is presently 17 fm$^{-2}$. Two experiments are under way at JLAB to extend the $q^2$-range [12, 13]. Within the error bars the available results from the double-polarization experiments agree with the values determined from the deuteron structure function $A(q)$.

**Exploitation of the quadrupole form factor.** In this note, we again use elastic electron-deuteron data to determine $G_{En}(q)$ at high momentum transfer. The novel aspect of the present approach consists in exploiting the quadrupole form factor $F_{C2}(q)$ rather than the combination of monopole and quadrupole form factors represented by $A(q)$, as done in the past.

When using elastic e-d scattering, two sources of theoretical uncertainty must be considered, due to the model for the NN-interaction and the contribution of two-body currents. We address the two-body currents first.

Calculations of $F_{C2}(q)$ based on a variety of model interactions and currents indicate that contributions from two-body currents are relatively small, even at the high momentum transfers of interest here. This is consistent with the naive expectation that, since $F_{C2}(q)$ involves an integral of the product of deuteron S- and D-wave components with the spherical Bessel function $j_2(qr/2)$, it is presumably less sensitive to two-body currents, at least the short-range ones associated with vector-meson exchanges and/or transition mechanisms such as, for example, the $\rho\pi\gamma$ operator, whose contributions are quantitatively rather uncertain. This is illustrated in Fig. 1 where we show separately the contribution associated with the $\pi$-exchange two-body charge operator, as well as that including, in addition, the $\rho$-meson and $\rho\pi\gamma$ charge operators. The $\rho$-meson and $\rho\pi\gamma$ contributions have opposite sign, and tend to cancel each other. As a result, the total two-body contribution to $F_{C2}(q)$ is dominated, up to $q^2 \simeq 40$ fm$^{-2}$, by the long-range $\pi$-exchange operator.

In this context, it is worth noting that, while modern realistic interactions are essentially phase-equivalent — they all fit the Nijmegen data-base with a $\chi^2$ per datum close to one — they do differ in the treatment of non-localities. Some of them, like the Argonne $v_{18}$
model [14], are local (in LSJ channels), while some others, like the CD-Bonn model [15], have strong non-localities. In particular, the CD-Bonn interaction has a non-local one-pion-exchange (OPE) component. However, it has been known for some time [16], and recently re-emphasized by Forest [17], that the local and non-local OPE interactions are related to each other via a unitary transformation. Therefore, the differences between local and non-local OPE cannot be of any consequence for the prediction of observables, such as the deuteron electromagnetic form factors under consideration here, provided, of course, that two-body currents generated by the unitary transformation are also included. This fact has been demonstrated [18] in a calculation of the deuteron structure function \( A(q) \) and tensor observable \( T_{20}(q) \), based on the local Argonne \( v_{18} \) and non-local CD-Bonn models and associated (unitarily consistent) electromagnetic currents. The remaining small differences between the calculated \( A(q) \) and \( T_{20}(q) \) are due to the additional short-range non-localities present in the CD-Bonn. The upshot is that, provided that consistent calculations — in the sense above — are performed, present “realistic” interactions will lead to similar predictions for deuteron electromagnetic observables, at least to the extent that these are influenced predominantly by the OPE component. This is especially true for the \( F_{C2} \) form factor for which the \( \pi \)-exchange contributions dominate.

Because of these considerations, the theoretical uncertainties for \( F_{C2}(q) \) are small (smaller than for \( A(q) \)), which allows us to determine \( G_{En} \) with smaller systematic errors and extend our knowledge of it to larger \( q \). The use of \( F_{C2}(q) \) has now become possible with the measurements of the polarization observable \( T_{20}(q) \) in electron-deuteron scattering. With \( T_{20} \) known up to \( q^2=40 \) fm\(^{-2} \), the quadrupole form factor \( F_{C2}(q) \) can experimentally be determined up to that \( q \)-value.

![Figure 1: Effect of the \( \pi \)-exchange two-body charge operator (dashed) and that obtained by including the remaining, shorter range, two-body contributions (solid).](image)

Experimental \( F_{C2}(q) \). In order to determine \( F_{C2}(q) \), we have analyzed the world data on electron-deuteron elastic scattering [19] – [42]. Some 340 data points on e-d scattering are available for momentum transfers below 65 fm\(^{-2} \). The cross sections and
polarization observables are fitted with flexible parameterizations for \( F_{C0}(q) \), \( F_{M1}(q) \) and \( F_{C2}(q) \) [13]. The statistical errors of the data are calculated using the error matrix. The systematic errors, which in general are the largest ones by far, have been evaluated by changing each individual data set by the quoted error, and re-fitting the complete data set. The changes due to systematic errors of the different, independent, sets of data are evaluated separately, and added quadratically. The resulting \( F_{C2}(q) \) is used below.

**Determination of \( G_{E_n} \).** In order to extract \( G_{E_n} \) we compare to the predictions for \( F_{C2}(q) \) from a number of theoretical calculations. These calculations all use NN potentials that provide reasonably good fits to the modern scattering data base, and consistent two-body currents. We employ calculations using the Paris and Bonn-B potential by the Hannover group [14, 15], the calculation of Forest and Schiavilla [16] based on the Argonne V18 NN potential, and the results obtained recently by the Mainz group [17] using the Bonn OBEPQ-B potential. We also employ the results of the calculation by Van Orden, Gross and Devine [18] who use an OBE interaction directly fit to the NN scattering data.

While the first three calculations are based on an essentially non-relativistic framework (with relativistic corrections), the calculation [17] starts from a system of coupled nucleon and meson fields and, by means of the Foldy-Wouthuysen transformation, derives the non-relativistic limit including all the leading order relativistic contributions. The calculation of Van Orden \textit{et al.} starts from the Bethe-Salpeter equation, which has been reduced to a quasi-potential equation by assuming that one of the nucleons is on mass shell. This calculation is Lorentz covariant and gauge invariant. All calculations include the relevant two-body terms.

In general, these calculations have used proton form factors as given by the Hoehler parameterization [49], which explains well the e-p scattering data up to the \( q^2 \) of interest here, including the recent \( G_{Ep}/G_{Mp} \)-data [50, 51, 52, 53]. The calculations of refs. [17, 18] have been carried out using the dipole form factor for the proton, which only roughly reproduces the proton data; here we use the calculation of Arenhoevel \textit{et al.} performed with the Hoehler form factors, while the calculation of van Orden \textit{et al.} has been renormalized to the Hoehler proton form factor. All calculations use the Galster [2] neutron charge form factor, or the one by Hoehler, which is very close in the range of \( q^2 \) of interest.

In fig. 2 we show the ratio of these theoretical \( F_{C2}(q) \) form factors to the experimental ones. This figure shows that for the C2 form factor the different theoretical predictions are quite close. The effect of \( G_{E_n} \) is appreciable at the higher momentum transfers, large enough to be extracted despite the differences between the theoretical predictions.

In order to determine \( G_{E_n} \), we use the following approach: As the “theoretical prediction” we use the \textit{average} of the five calculations discussed above. For the “theoretical error bar” we take the quadratically added deviation of the individual calculations from the average. The deviation of this average from experiment we then take as an indication that the Galster (or Hoehler) \( G_{E_n} \) used in the calculation is not quite the correct one, and
we determine $G_{En}$ to get perfect agreement between experiment and the theory average. The resulting values of $G_{En}$, together with the error bars that include both the spread of the theoretical predictions and the experimental uncertainty on $F_{C2}$, is shown in fig. 3.

Figure 3 shows that the form factors extracted from the C2 deuteron structure function are reasonably accurate in comparison with the results obtained from double-polarization measurements, and they agree with them in the $q^2$-region of overlap. In comparison to the mean values of $G_{En}$ determined by Platchkov et al. from the deuteron $A(q)$ structure function, the present results are somewhat higher in the region above $q^2=8$ fm$^{-2}$, but compatible with them given the spread of the theoretical predictions available to Platchkov et al. at the time. The $G_{En}$ extracted from the $C2$-data have larger uncertainties at low $q^2$, where the $C0$ multipolarity dominates the cross section and where the available $T_{20}$ data are not very accurate. There, the usage of $A(q)$ leads to superior results.

The determination of $G_{En}$ from $F_{C2}$ extends to larger momentum transfer than all previous determinations, which were limited to $q^2 \approx 16$ fm$^{-2}$. Somewhat surprisingly, the extrapolation of the Galster parameterization beyond its limit of validity ($q^2=16$ fm$^{-2}$) does quite well in reproducing the data. As pointed out above, double-polarization experiments presently under way at JLAB are expected to provide data in this higher-$q^2$ region.

Conclusions. In this note, we have determined the neutron charge form factor $G_{En}$ starting from the data on electron-deuteron elastic scattering. Contrary to previous analyses, we use the deuteron quadrupole form factor, which is less sensitive to the short-range two-body currents that are not well under control. We employ a representative selection
Figure 3: The $G_{En}$ extracted from the C2 data (⋄). Also shown are the values obtained from double-polarization experiments, and the Galster parameterization with its extrapolation into the region not covered by previous experiments (dotted).

of both non-relativistic and relativistic theoretical calculations to predict deuteron structure functions and contributions of two-body currents, thus allowing use to produce a fair estimate of the theoretical uncertainties involved in our procedure. Using this approach, we for the first time provide data (other than upper limits [55]) for $G_{En}$ at large $q^2$.

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