New simulation setup for modelling the thermal-electric-magnetic behavior of superconductors.

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Abstract. This paper presents a new equation for the calculation of the thermal-electric behavior of superconductors. The equation is based on measurements of YBCO thin-films which were carried out in liquid nitrogen (LN₂), varying the temperature by application of different pressures. The principle of the experimental set-up is described in [1]. Thus, the temperature and current dependent resistivity characteristics can be obtained. The equation derived from those measurements describes the current-temperature dependency of the YBCO thin-film resistivity within the range of interest for resistive Superconducting Fault Current Limiters (FCL). In a first step the equation was used to program a superconducting thermal-electric element within the commercially available 3D finite element (FEM) multipurpose program ANSYS, which was also used to cross-check the measurements by simulations. In a second step the equation was incorporated into the FEM program COMSOL and expanded to consider the magnetic field too. The resulting simulation setup is shown along with some exemplary simulations.

1. Introduction
The electrical resistivity of high temperature superconductor materials depends on the parameters temperature, current density and external magnetic field. Superconducting Fault current limiters (FCL) are using these dependencies. One type of FCL – the Resistive Fault current limiter – utilizes the transition from the superconducting state to normal conducting state if the current exceeds the critical current of the superconductor during a short circuit. While the grid is operating normally, the current density in the superconducting material is well below the critical current density – the FCL is superconducting. During a short circuit the current exceeds the critical current, causing a very rapid increase of the electrical resistivity and therefore a very rapid rise of the temperature too. By this increased series resistance the current flow in the circuit is limited. The limitation process is a complex interaction between the current flow influenced by the current- and temperature-dependent resistivity, the temporal and local temperature evolution of the high temperature superconductor (HTS), its substrate and the external circuit [2,3,4]. The design and optimization of HTS current limiters is therefore a costly and time-consuming process. It can be optimized by software tools able to describe the HTS and predict the limitation behavior. But the coupled dependencies of the electrical resistivity on temperature and current density are not implemented in present commercially available software. To avoid long term software development the commercially available FEA programs ANSYS® and
later on COMSOL® have been customized for the simulation of HTS materials. Even complicated 3D structures are thus possible to simulate.

2. Model of the HTS resistivity without consideration of the magnetic field

2.1. Basic Equations

Without consideration of inductive couplings between the conductors and the influence of external magnetic fields on the resistivity of the HTS, the electric field is curl-free ($\vec{E} = -\text{grad}(\Phi)$). The current flow can be described by the Laplace Equation:

$$\text{div}(\sigma \cdot \text{grad}(\Phi)) = 0$$

(1)

where

$$\vec{J} = \sigma \cdot (-\text{grad}(\Phi))$$

$\Phi$ is the electric potential, $\sigma$ the electrical conductivity and $\vec{J}$ the current density. If the current flowing into the material is known it can be quantified as a boundary condition. Otherwise the current can be modelled through an external circuit using ANSYS® circuit elements.

The heat transfer is expressed by the following equation:

$$c_p \cdot \frac{\partial T}{\partial t} = Q_\lambda + \text{div}(\lambda \cdot \text{grad}(T))$$

$$Q_\lambda = \frac{J^2}{\sigma}$$

(2)

$T$ is the temperature, $c_p$ is the specific heat per volume, $\lambda$ is the thermal conductivity and $Q_\lambda$ the heat generated per volume by ohmic losses. Radiative and convective heat transfer can be incorporated as boundary conditions. The resistivity $\rho$ ($\rho = 1/\sigma$) of the superconductor couples the equations (1) and (2):

$$\rho = f(J, T)$$

(3)

Additional to the equations (1)-(3) the initial conditions for both $J$ and $T$ have to be defined. With the equation system (1)-(3) it is possible to calculate the distribution of temperature and electric potential. Based on these results current distribution can be calculated inside the superconducting material [5].

2.2. Analytical Description of the characteristic curve $\rho (J, T = \text{const.})$

To describe the dependency of the HTS-resistivity on the current density, the equation usually used for simulations of superconductors, can be described as a power law:

$$\frac{\rho}{\rho_C} = \left( \frac{|J|}{J_C} \right)^n$$

(4)

Where $\rho_C$ is the electrical resistivity at the critical current density, $J_C$ is the critical current density and $n$ the power law exponent. $n$ is a highly variable exponent and may go up to values above 30 in the vicinity of the critical current [5]. But Fig. 1 shows, that (4) describes only a small part of the characteristic curve well. Since the peak let-through current of a superconducting resistive fault current limiter is typically in the range of several times $J_C$, [5] a numerical calculation of the quench behavior of YBCO thin-films would have to incorporate the resistivity development up to these current densities. One approach to solve this problem could be a variation of the exponent $n$ and
therefore several different equations to stepwise approximate the characteristic curve. This approach is described in detail in [5] and exemplarily shown in Fig 2.

Fig. 1  Example of a characteristic curve $\rho(J, T = 77 \text{ K})$

Fig. 2  Example of the stepwise approximation as shown in [5].

But this approach has two serious disadvantages:

- The approximation of some parts of the characteristic is quite coarse. It is possible to divide the curve into finer areas, but this will lead, besides larger calculation times, to another disadvantage:
- Every transition from one approximation function to the next one incorporates a point of discontinuity. Calculating the HTS behavior in AC-grids with electrical energy storages (e.g. inductance), these discontinuities will lead to transient overvoltages which do not exist in reality.

Because of these disadvantages of the power law a new equation was developed. Through extensive measurements with YBCO thin-films on sapphire substrate and later with Coated Conductors over a broad range of currents (up to 3 times $J/J_C$) and temperatures (77K - 88K) the following equation was found:

$$\rho = \frac{|E|}{J_C(T = \text{const.}) \left( P \cdot \left( \frac{|E|}{E_C} \right)^m + (1 - P) \right)} \quad (5)$$

$E$ is the electric field, $J_C(T = \text{const.})$ is the critical current density at a constant temperature, $E_C$ the electric field criterion (e.g. 1µV/cm), $m$ and $P$ are constants which are material-specific. Many different YBCO thin-film samples were measured, for all samples the constants are in the range of $0.2 < m < 0.36$, $0.03 < P < 0.25$ and $0.8 \text{ MA/cm}^2 < J_C(T = \text{const.}) < 3 \text{ MA/cm}^2$. Details on how the measurements were conducted can be found in [1].

Using equation (5) a steady and exact interpolation of the resistivity curve of an HTS can be made. In Fig. 3 a comparison between measurements and the associated approximation for one sample is shown. The parameters used are $m = 1/3.1$ and $P = 0.036$ and $J_C,77K = 3 \text{ MA/cm}^2$. Through comparison of Fig 1 and Fig 3 it is evident that the fit made with (5) is better suited to approximate the current dependent electrical resistivity of a HTS at constant temperature over a broad range of currents which is necessary for FCL. If equation (5) is solved for $J$ and $\rho$ is set to 0, equation (6) follows:

$$J(\rho = 0) = (1 - P) \cdot J_C \quad (6)$$

At higher current densities $J$ the HTS is in the transition from superconduction to normalconduction. Below this current density the current-transport is without electrical loss, therefore the material is
superconducting. Assuming in accordance with the resistivity model from [6, 7] that the motion of depinned flux is the major loss mechanism of a YBCO superconductor, the current density calculated by equation (6) can be understood to be the depinning current density $J_{dp}$. If this current density is exceeded the current dependent, non-linear resistivity occurs for the first time. By extrapolation equation (5) connects the quite arbitrary parameter $J_C$ which depends on the chosen µV/cm-criterion of a measurement (see table 1), with $J_{dp}$.

![Fig. 3](image1.png)

**Fig. 3** Comparison between approximation and measurement. Approximation made with (5).

This is confirmed by measurements. In Fig. 4, the characteristic resistivity curve of a sample with some differently chosen $E_C$ is shown. Additionally to the sometimes used criteria of 1 µV/cm and 10 µV/cm an arbitrary and highly different definition of 165 µV/cm was chosen too, resulting in a critical current density $J_C$ of 3.32 MA/cm² in contrast to 3.12 MA/cm² (1 µV/cm) and 3.17 MA/cm² (10 µV/cm). Table 1 lists all critical current densities, the approximation parameters and via equation (6) calculated depinning current densities. The parameter $m$ is constant, $m = 1/3.1$ for all approximations.

![Fig. 4](image2.png)

**Fig. 4** Characteristic curve for different $E_C$

| Criteria          | 1 µV/cm | 10 µV/cm | 165 µV/cm |
|-------------------|---------|----------|-----------|
| $J_c$ MA/cm²      | 3.12    | 3.17     | 3.32      |
| $P$               | 0.015   | 0.036    | 0.077     |
| $J_{dp}$ MA/cm²   | 3.07    | 3.06     | 3.06      |

The values of $J_{dp}$ derived from approximation (5) are identical in line with the measurement and rounding accuracies.

Since high temperature superconductivity is still not fully understood, the extrapolation for current densities much higher than $J_c$ is more complicated. In this work it is assumed that the approximation by equation (5) is valid for current densities $J > J_{dp}$ at least in the relevant range of current densities for resistive FCLs. Under the precondition that $m$ and $P$ are always smaller than 1 the limit of equation (5) is

$$ \lim_{J \to \infty} \rho = \lim_{J \to \infty} \sqrt[3]{E_c \cdot \frac{J^{-m} - (1 - P) \cdot J^{-m} \cdot J_c}{P \cdot J_c}} = \infty $$

That is a discrepancy to the model of [6, 7], because there a current density independent ohmic resistance in the free-flux-flow (FFF) range is postulated. But this model is one among many, other models are discussing e.g. an interaction between the flux-vortices among each other even with
complete released flux. Since there are many other models which are discussing the transition from the superconducting to the normal conducting state no definitive statement to the FFF can be made. Some exemplary theories can be found in [8-17].

Unaccounted for is the mechanism of the Cooper-Pair breaking. In principle there has to be a finite current density at which the breaking of Cooper-Pairs starts and therefore provides an additional part of the electrical resistance of the HTS. Likewise there has to be a finite current density at which the last Cooper-Pair is broken and therefore superconductivity is completely lost. In this work it is assumed that there are still free electrons, not bound into Cooper-Pairs, below this current density. Because of that normal-conduction coexists at all times to the FF. This theory of coexistence is used in other works too, e.g. [6]. For the electrical point of view this work assumes a parallel connection of the FF-resistance calculated with equation (5) and the extrapolated normal-conducting resistance. The maximal FF-resistance is therefore limited to the normal-conducting resistance.

2.3. Analytical Description of the characteristic curve \(\rho (J, T)\)

It is known from literature [18] that equation (8) describes the temperature dependency of the critical current density \(J_c(T)\), with \(r\) being a material parameter and \(T_c\) the critical temperature of the superconductor. For the measured samples \(r\) varies between 1.6 and 1.9 and \(T_c\) between 88 K and 90 K.

\[
J_c(T) = K \cdot \left(1 - \frac{T}{T_c}\right)^r
\]

\[
K = \frac{J_{c,77K}}{\left(1 - \frac{77K}{T_c}\right)^r}
\]

Fig. 5  Example of a set of resistivity characteristic curves, displayed by single characteristic curves \(\rho(J, T=\text{const.})\). The caption of the diagram shows the temperature at the measurements in K. Symbols represents measurements and lines represent simulations. For all simulations the parameters \(m = 1/3\) and \(P = 0.033\) were constant.
If the temperature dependent $J_c(T)$, as calculated with equation (8), is used in equation (5) instead of $J_c(T=\text{const.})$, the resulting equation is valid for all temperatures below $T_C$. The parameters $m$ and $P$ are the same for all temperatures and thus temperature independent material parameters. The resulting set of characteristic curves is shown in Fig. 5, comparing measurements and approximation.

In summary, the presented analytical description according to equation (5) associated with equation (8) as described within this chapter

- Can describe the complete characteristic curves, in the for FCL relevant current density range, precisely. This is in contrary to the usually used approximations (e.g. “power law”).
- Is continuous in the relevant current density range.
- Connects the depinning current density $J_{dp}$ and the critical current density $J_C$ by extrapolation.
- Actually only describes the FF-part of the electrical resistance of a superconductor. In parallel to the FF-resistance is always the normal-conducting resistance.

3. Extension of the HTS-resistivity model with the magnetic field

Starting from the thermal-electric dependence of equation (5) the next step in obtaining a simulation setup which incorporates all three critical values $T, J, B$ has to be the implementation of the influence of the magnetic field on the electrical resistance of an HTS. The starting point for the calculation of the distributions of magnetic fields and current densities in and around a conductor (not only a superconductor) is Amperes law (9) and the current conservation law in differential form (10).

$$\nabla \times \left( \frac{1}{\mu} \nabla \times \vec{A} \right) = \vec{J} = \vec{J}_W + \vec{J}_V = -\sigma \left( \frac{\partial \vec{A}}{\partial t} + \nabla V \right)$$ 

(9)

$$\nabla \cdot \vec{J} = -\nabla \cdot \left[ \sigma \left( \frac{\partial \vec{A}}{\partial t} + \nabla V \right) \right] = 0$$ 

(10)

The overall current density consists of the eddy current density $\vec{J}_W$ and the “ohmic” current density $\vec{J}_V$ due to the potential gradient. This system of equations allows the calculation of currents and magnetic fields in conductors with imposed voltages. If there are only induced currents, without predetermined electric potentials present, only the eddy current density exists, that means $\nabla V$ does not apply. This simplification is always possible, since the transport current can also be described as an induced current.

According to [19, 20] the so-called $A-\Phi$ formulation was chosen to incorporate the magnetic field into equations (5) and (8) which results in equation (11):

$$
\rho \left( \vec{J} , T, \vec{B} \right) = \frac{\frac{\partial \vec{A}}{\partial t}}{J_{C,77K} \left( 1 - \frac{T}{T_C} \right) \left( 1 - \frac{77K}{T_C} \right)^\nu \left[ \left( 1 - \frac{T}{T_C} \right)^\nu - P \left( \frac{\partial \vec{A}}{\partial t} \right)_c \right]} + P \left( \frac{\partial \vec{A}}{\partial t} \right)_c + (1 - P) \right)
$$

(11)

with

$$\vec{B} = \nabla \times \vec{A}$$ 

(12)
First tests with equation (11) were made with a simple 2D simulation model. Simulated was a round superconductor with a diameter of 0.4 mm. The other parameters used in the simulations are: $P = 0.033$, $m = 1/3$, $T_C = 88$ K, $J_{c,77K} = 1$ MA/cm², $r = 1.65$. The exiting current used in the simulation was a ramped DC current as shown in Fig. 6. The maximum current has a value of $I/I_C = 1.5$. The ramped current was used to make the simulation comparable with the measurements which used ramped DC currents as transport currents in the HTS. To prevent a temperature change in the model that would result in a change in the electrical conductivity the heat capacity for the YBCO was increased by a factor of 1000. Furthermore to prevent the propagation of the generated heat during the ramping of the current (0-1 ms) from areas with current flow into areas without current flow the heat conductivity for YBCO was decreased by a factor of 1000. These changes allow separating the effects of the current and temperature dependent electrical conductivities for easier testing of the simulation-setup and the underlying equations. The resulting thermal parameters used in the simulations are: $\lambda = 6.1$ mW/(m*K), $\rho = 6.3$ kg/dm³ and $c_P = 0.16$ MJ/(kg*K). In addition to simplify the simulation model further, no heat dissipation ("cooling") to the outside of the HTS was implemented. The initial temperature in the simulation is 77 K.

![Fig. 6](image.png)  
**Fig. 6**  Applied ramped DC current.

The boundary conditions require some careful consideration. From a mathematical point of view, it is necessary to specify either the magnetic potential $A_z$ (corresponding to a Dirichlet type condition) or the normal derivative of the same field (a Neumann condition) on the outer surface. These quantities, however, have little or no significance in applied engineering. In COMSOL Multiphysics the Neumann type condition on the $A_z$ field is implemented by specifying a surface current equal to the negative of the tangential magnetic field at the boundary. The current might not be physically real. On exterior boundaries it can be interpreted as the surface current necessary to make the magnetic field $H$ vanish outside the domain.

Even better from the pure engineering point of view is to specify the overall current throughput, which is rather straightforward in this case. Making the magnetic field disappear everywhere outside the circular domain requires that the overall current through the domain equals zero. One way of achieving this is by adding a “virtual” surface current of opposite sign to the “real” current inside the conductor. Because the problem is rotationally symmetric, it can be written as the necessary virtual surface current density:

$$\vec{J}_s = -\frac{I_{tot}}{2\pi R}$$

(12)
where $I_{\text{tot}}$ is the real overall current throughput, and R is the radius of the wire [21]. Because equation (5), and therefore equation (11) too, is only valid for the flux-flow range of the HTS the resistivity development stops if the superconductor is fully normal conducting. The resistivity development for a normalconductor can be described by:

$$\rho(T) = \rho_0 \cdot (1 + \alpha(T - T_0)) \quad (13)$$

Equation (13) is not implemented into the simulation setup yet, but because of the chosen boundary conditions and material parameters, only a very small temperature rise occured during the simulation. The temperature at the end of the simulation was $T_{\text{End}} = 77.1$ K. Therefore the temperature did not exceed the critical temperature and the missing implantation of (13) is not a problem in the simulation shown later in this work.

![Graph showing current density across the cross-section at different times with a current density of $J/J_C = 1.5$.](image1)

**Fig. 7** Current density across the cross-section at different times with a current density of $J/J_C = 1.5$.

![Graph showing current density and electrical conductivity over time ($J/J_C = 1.5$).](image2)

**Fig. 8** Current Density, and electrical conductivity over time ($J/J_C = 1.5$).
To demonstrate the functionality of the model and the equations the results of an exemplary simulation with a current of $I/I_c = 1.5$ is shown in Fig. 7 and 8. Fig. 7 shows the current density with respect to the cross-section of the superconductor at different times. The current density penetrates the superconductor with the rising transport current from the surface to the inner regions until $J_c$ is reached. That the transition from the area with current flow to the area without current flow is not sharp is an artefact of the numerical approach (interpolation in finite elements). After the current exceeds $I_c$ the current flow has spread over the entire cross-section of the superconductor. A further increase of the current means the conductor is no longer superconducting and works now like a normalconducting material. Thus the current density now rises equally over the entire cross-section. This behavior is in good agreement to the known critical state models. In Fig. 8 the development of the current density and the electrical conductivity over time are shown at a point near the surface of the superconductor. The model was calculated up to 100 ms but for a better visibility of the transition-region the curves in Fig. 8 are shown only up to 1.5 ms. Up to 1 ms the strong decrease of the conductivity in relation with the rising current density can be observed, which is typical for superconductors. There is only a very small temperature change ($T_{End} \approx 77.1$ K) in the model during that time, because the boundary conditions and material parameters of the simulation-model were made specially to separate the effects of the current and temperature dependent conductivities for easier testing of the simulation-setup and the underlying equations. In summary the model shows a good conformity to the known behavior of superconductors. Especially the penetration of the current into the superconductor is reproduced very well as well as the current dependent conductivity.

4. Conclusion
A new approximation for $\rho(J, T = \text{const.})$ is found and shows good agreement to measurements with YBCO thin-films. Through the integration of equation (8), which is known from the literature [8], it is possible to include the temperature dependency of HTS as well. The fit parameters used in the new equation (5) are temperature independent. Because of this and since the approximation introduced in this work describes the resistivity characteristic curves exactly and continuously in the for FCL relevant current density range, it is well-suited especially for numerical calculations. Therefore the approximation presented in this work is better suited for the prediction of the behavior of a FCL than the usually used approximations (e.g. “power law”). In the second part of this paper a way is described to additionally incorporate the magnetic field dependency of HTS into the numerical calculations.

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