Modeling and analysis of an open-drive Z-compressor

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Abstract. A rotary Z-compressor prototype for compressed air applications has been developed and tested. The Z-compressor working process resembles the one of a two-stage rolling piston compressor where the stages are phased by half rotation. In contrary to the traditional rolling piston design, the vane in a Z-compressor is positioned parallel to the main shaft and not perpendicular. In order to understand the impact of leakage and frictional losses and improve the design of such machine, a mechanic model has been developed to include governing equations within the working chambers (i.e., two suction chambers and two compression chambers), leakage flow models, detailed mechanical analysis, one-degree of freedom valve model, in-chamber heat transfer and an overall energy balance of the compressor shell. The model has been validated with preliminary experimental data and then exercised to identify the potential performance improvements over a range of clearances and working conditions.

1. Introduction

This paper analyzes an air-compressor prototype that belongs to the family of rotary compressors. A cut-away view of the compressor is shown in Figure 1. To be noted is that the design concept resembles a two-stage rolling compressor and presents two sets of compression and suction chambers with a half revolution shift between the upper and lower level. The volume of the chambers is the space between inner and outer cylinder walls and the height of the chambers changes periodically as function of the crank angle. The element that separates upper and lower chambers is named blade. Additionally, the chambers on the same side of the blade are separated by means of a point of zero height with the housing and by a sliding vane. Due to the shape of blade and inner cylinder from a front view (see right side of Figure 1), the compressor is denoted as Z-compressor.

The Z-compressor was first analyzed by Jovane [1] in his dissertation. Experimental work done on a prototype showed that the Z-compressor had low vibration and noise effects, but its performance was heavily affected by leakages and friction losses. A model of the dynamics of the Z-compressor was developed in order identify the important sources of losses, such as leakage, friction, and heat transfer and to optimize the geometry. The model was validated with internal and external experimental data generated using R410A as the working fluid. From the numerical results, it was concluded that volumetric efficiency and overall isentropic efficiency were lower compared to the efficiency of a rolling piston rotary compressor [2] with the exception of the indicated efficiency which was comparable.

This paper focuses on an improved Z-compressor design in which the blade is part of the outer cylinder. A mechanistic model has been developed to break down leakage, fric-
tional and heat transfer losses and further employed to investigate potential improvements.

**Nomenclature**

| Symbol | Description          | Unit          |
|--------|----------------------|---------------|
| $A$    | Area                 | $m^2$         |
| $h$    | Specific enthalpy    | $kJ/kg$       |
| $L$    | Length               | $m$           |
| $m$    | Mass                 | $kg$          |
| $\dot{m}$ | Mass flow rate     | $kg/s$        |
| $N$    | Rotational speed     | $-$           |
| $p$    | Pressure             | $kPa$         |
| $Q$    | Heat rate            | $kW$          |
| $Re$   | Reynolds number      | $-$           |
| $T$    | Temperature          | $K$           |
| $u$    | Specific internal energy | $kJ/kg$ (-) |
| $v$    | Specific volume      | $m^3/kg$      |
| $V$    | Volume               | $m^3$         |
| $\dot{V}$ | Volumetric flow rate | $m^3/s$      |
| $W$    | Power                | $W$           |

**η** Efficiency (-)

**Subscript**

| Subscript | Description |
|-----------|-------------|
| amb       | ambient     |
| dis       | discharge   |
| eff       | effective   |
| fr        | friction    |
| in        | inlet       |
| is        | isentropic  |
| JB        | journal bearing |
| m         | mechanical  |
| oa        | over-all    |
| out       | outlet      |
| sh        | shaft       |
| suc       | succion     |
| TB        | thrust bearing |
| th        | theoretical |
| v         | volume      |

**Figure 1.** Cutaway view (left) and transparent 3D view (right) of the Z-compressor.

### 2. Geometric Characteristics

The Z-compressor design considered in this work is an open-drive one. On each side of the compressor there are two ports allowing for suction and discharge processes. Reed valves are installed on the discharge ports. The main design difference with respect to the original Z-compressor analyzed by Jovane [1] is the blade integrated with the outer cylinder which avoids direct leakage paths from upper to lower chambers and vice-versa as shown in Figure 1.

The working process is similar to a twin rolling piston rotary compressor in which two compression processes occur in one rotation. Four working chambers can be identified: upper suction and compression chambers and lower suction and compression chambers as seen in the
cutaway view shown in Figure 1. Differently from a rolling piston rotary compressor, the Z-blade allows for volume variations. Two sliding vanes separate suction and compression chambers located on the same side of the Z-blade. The volume of each chamber is the space between inner and outer cylinders and the height of the chambers is dictated by the height of the portion of the Z-blade in contact with the vane, \( y_b \). The length of each portion of the vanes into the working chambers is given as:

\[
y_{up}, y_{low} = \frac{h_b}{2} [1 \mp \cos(\theta)] \tag{1}
\]

where \( h_b \) is the maximum height of the Z-blade, the minus sign is used for the upper chambers (up) and the plus sign for the lower chambers (low) and \( \theta \) is the rotation angle measured from the point of contact between the Z-blade and the vane to the point at with the Z-blade has zero height with respect to the housing.

The volume of the suction and compression chambers in both sides of the z-blade are obtained by defining a maximum chamber volume as

\[
V_{max} = \frac{\pi}{2} h_b (r_o^2 - r_i^2) \tag{2}
\]

where \( r_o \) and \( r_i \) are the inner and outer radius of the Z-blade, and the vane volume within the chamber which is a function of the height of the chambers and the shape of the vane, i.e., \( V_v = f(y(\theta), \text{geo}) \). Therefore, the upper suction and compression chambers are given by,

\[
V_{s,up} = \frac{V_{max}}{2\pi} (\theta - \sin \theta) - V_{v,up}(\theta) \tag{3}
\]

\[
V_{c,up} = \frac{V_{max}}{2\pi} (2\pi - \theta + \sin \theta) - V_{v,up}(\theta) \tag{4}
\]

The lower suction and compression chambers are obtained from Equation 3 and Equation 4 by including the half revolution shift:

\[
V_{s,up} = \frac{V_{max}}{2\pi} (\pm \pi + \theta - \sin(\theta \pm \pi)) - V_{v,up}(\theta \pm \pi) \tag{5}
\]

\[
V_{c,up} = \frac{V_{max}}{2\pi} (2\pi \mp \pi - \theta + \sin(\theta \pm \pi)) - V_{v,up}(\theta \pm \pi) \tag{6}
\]

where the upper sign on Equation 5 and Equation 6 is used for \( \theta \leq \pi \) and the lower sign for \( \theta > \pi \).

The main dimensions of the Z-compressor are reported in Table 1. The evolution of the working chamber volumes and their derivatives are shown in Figure 2 and they can be integrated into the compression process governing equations.

| Component                  | Dimension |
|----------------------------|-----------|
| Shaft radius (r_s)         | 1.6 mm    |
| Blade inner diameter radius (r_i) | 5 mm      |
| Blade outer diameter radius (r_o) | 13 mm    |
| Maximum chamber height (h_o) | 10 mm    |
| Vane thickness (t_v)       | 1.5 mm    |
3. Governing Equations

The Z-compressor is divided into five control volumes (CVs) of which four are associated with upper and lower suction and compression chambers and their volumes change according to the rotation angle (see Figure 2). A fifth static control volume is introduced to account for the space between the rotating cylinder and the external housing as well as the space where the vanes are installed. A system of conservation equations is applied simultaneously to each control volume to obtain the change in thermodynamic states. The properties within a control volume are assumed to be uniform, i.e. gradients within compressor chambers are neglected. As CoolProp [3] is used to retrieve the thermophysical properties of air, two independent thermodynamic properties need to be selected to derive the conservation equations of mass and energy. Temperature and density have been chosen. For each control volume, the general conservation of mass equation can be written as:

\[
\frac{d(\rho V)_CV}{d\theta} = \frac{1}{\omega} \sum_i \dot{n}_i 
\] (7)
The general conservation of energy in each chamber of the compressor can be expressed as:

$$\omega \frac{d}{d\theta} (\rho u V)_{CV} = \dot{Q} + \dot{W}_{CV} + \sum_i (\dot{m} h)_i$$

where the specific internal energy is obtained as $u = h - pv$, the boundary work rate of the control volume can be expressed as $\dot{W}_{CV} = -\omega p \left(\frac{dV}{d\theta}\right)$ and $\dot{Q}$ is the heat transfer rate between the components of the compressor chambers and the working fluid. By expanding the left hand side of Equation 8 according to [4], the derivative of temperature with respect to the crank angle can be obtained:

$$m_{CV} c_v \frac{dT}{d\theta} = -T \left(\frac{\partial p}{\partial T}\right)_v \left[\frac{dV}{d\theta} - v \frac{dm_{CV}}{d\theta}\right] - h \frac{dm_{CV}}{d\theta} + \frac{\dot{Q}}{\omega} + \frac{1}{\omega} \sum_i (\dot{m} h)_i$$

where the mass in the control volume is given by $m_{CV} = (\rho V)_{CV}$.

The mass flow rate through the discharge valves are estimated by applying a one-degree-of-freedom dynamic model to the reed valves [5] having two operating modes, i.e., pressure dominant when the net force on the valve is dominated by the pressure difference across the valve, and mass flux dominant when the gas velocity is the dominant force.

As shown in Figure 2, the lower chambers are phased by half rotation. As a consequence, there are two discontinuities in the volume curves when the rotation is at $\pi$. During the integration of the differential governing equations, such discontinuities would not allow the convergence of the code. This numerical issue has been handled by coupling an adaptive step-size solver with a step-callback function that helps the solver to step through the discontinuity by taking a sufficiently small step before and after the discontinuity. This numerical technique can be applied to other compressor types for example spool compressors [6], where a similar situation occurs. An example of step size variation with the crank angle can be seen in Figure 3. The Z-compressor involves continuous interactions between upper chambers and lower chambers via leakage paths. The stiffness of the governing equation is not constant and the adaptive solver decreases the step size to maintain the error within a certain threshold. At $\pi$, an even smaller step is enforced to step through the discontinuities of the control volumes and continue the integration.
Table 2. Leakage path description.

| Path | Description                                                                 | Area     |
|------|-----------------------------------------------------------------------------|----------|
| L1   | Leakage through the clearance of the vane and the bearing slot from the shell to the compression chamber | Constant |
| L2   | Leakage through the clearance of the vane and the bearing slot from the shell to the suction chamber | Constant |
| L3   | Leakage through the flat area of the blade                                   | Constant |
| L4a,b| Leakages through the clearance of the vane and cylinder                       | Variable |
| L5   | Leakage through the clearance of the vane and the blade                       | Variable |
| Lup  | Leakages from the upper compression chamber to the lower suction chamber and from the upper compression chamber to the lower compression chamber | Constant |
| Ldown| Leakages from the lower compression chamber to the upper suction chamber and from the lower compression chamber to the upper compression chamber | Constant |

The compressor model is closed by enforcing a steady-state overall energy balance with the shell of the compressor as shown in Figure 4:

\[
\dot{Q}_{amb} + \sum_i \dot{W}_{i,fr} + \sum_i \dot{Q}_i = \epsilon
\]

(10)

where \(\epsilon\) is the residual to be minimized, \(\dot{Q}_{amb}\) is the heat transfer between the shell and the ambient, \(\sum_i \dot{W}_{i,fr}\) is the sum of the mechanical and friction losses and the general \(\dot{Q}_i\) term represents the heat transfer interaction between the gas and the shell during suction and compression processes.

Figure 5. Leakage paths of the Z-compressor.
3.1. Leakage Flow Model

The Z-compressor presents several leakage paths. For the current design, the main leakage paths are identified in Figure 5. Furthermore, a more detailed description of each leakage path can be found in Table 2. By analyzing the leakage paths, it can be found that most of them present lengths that are relatively long compared to the cross-sectional area. In addition, the leakage path flow area is not always constant. Assuming isentropic flow of perfect gas will overestimate the leakage flows. To improve the leakage flow predictions, a detailed 1D-flow with friction analysis has been employed to obtain the mass flow rate estimations across each path. Coupling the three conservation equations, mass, momentum and energy, a general expression for the system of differential equations along the leakage path can be expressed as in [7],

\[
\begin{bmatrix}
0 & 0 & 1/V & 0 & 1/ho \\
0 & 1 & \rho V & 0 & 0 \\
0 & 0 & V & 1 & 0 \\
-\left(\frac{\partial h}{\partial T}\right) & 0 & 0 & 1 - \left(\frac{\partial h}{\partial \rho}\right)_T & 0 \\
-\left(\frac{\partial p}{\partial T}\right)_\rho & 1 & 0 & 0 - \left(\frac{\partial p}{\partial \rho}\right)_T & 0 \\
\end{bmatrix}
\begin{bmatrix}
dT/dx \\
dp/dx \\
dV/dx \\
dh/dx \\
dp/dx \\
\end{bmatrix}
= \begin{bmatrix}
-1 & dA \\
A_d & dx \\
2 & \rho V^2 A f_F \\
\end{bmatrix}
\begin{bmatrix}
-\frac{1}{D_h} \\
0 \\
0 \\
\end{bmatrix}
\]

where \(f_F\) is the Fanning friction factor for flow between infinite plates [7] and \(V\) is the mean fluid velocity. Solving the system of equations shown in (11) within the compressor model is computationally very expensive and also makes the convergence of the model more difficult to reach. Therefore, to predict the leakage flows in positive displacement machines, an isentropic flow model is generally adopted as it is simpler. However, it is known that such model can significantly overpredict the flow rate [7]. Therefore, the detailed 1D-flow model is used to determine a correction factor between the mass flow rate of leakage using the detailed flow with friction model and the mass flow rate predicted using a simpler isentropic flow model. For a given configuration, the ratio of the isentropic nozzle mass flow rate prediction to that of the detailed model is defined by

\[
M = \frac{\dot{m}_{\text{nozzle}}}{\dot{m}_{\text{fr}}} = f(Re^*, L^*, \delta^*)
\]

where the dimensionless characteristic length and dimensionless gap width are given by \(L^* = L/L_0\) and \(\delta^* = \delta/\delta_0\) with \(L_0\) and \(\delta_0\) being the reference length and gap width values. For brevity, only the case in which the detailed model is applied to the variable leakage path \(L_4\) is shown. In particular, Figure 6(a) shows the generated points of the mass flow rate correction factor \(M\) for different boundary conditions (pressure ratio across the path) and gap widths as function of the Reynolds number. The calibration results of Equation 12 is reported in Figure 6(b).

The correction factors are then used to compute the mass flow rate in and out each control volume during the integration of of Equation 7 and Equation 9.

3.2. Friction Losses

The average total shaft power is given by the sum of the compression power, \(\overline{W}_{PV}\), and the total friction power losses, \(\overline{W}_{\text{fr, tot}}\), due to contacts among the different parts,

\[
\overline{W}_{\text{sh}} = \overline{W}_{PV} + \overline{W}_{\text{fr, tot}}
\]

A detailed force and moments analyses have been included into estimate the friction losses. In particular, once the pressure distribution within the working chambers is known radial and axial loads on blade, shaft and bearings can be obtained as well as the associated frictional torques. Each frictional torque multiplied by the rotational speed gives the friction loss power.
been chosen to carry out the calculations: suction temperature 23 °C, have been defined by considering different clearance gap sizes. A nominal operating point has been identified and they are described in Table 3. As a first step, the Z-compressor shaft structure has been rethought to reduce friction losses, and its detailed compressor model developed is used as a tool to investigate the internal behavior of the compressor and to quantify the sources of losses. Figure 7(a) shows an example of pressure traces with respect to the rotation angle. The plot of the internal pressure of the compressor during the working process helped adapting the valve parameters to achieve the required operating points.

A comprehensive description of such approach can be found in [1]. The average total friction loss power is given as:

$$\bar{W}_{fr,\text{tot}} = \sum_i \bar{W}_{fr,i} = \bar{W}_{fr,\text{pt}} + \bar{W}_{fr,v} + \bar{W}_{fr,\text{flat}} + \bar{W}_{fr,\text{cyl-cyl}} + \bar{W}_{fr,\text{cyl-flat}} + \bar{W}_{fr,\text{TB}} + \bar{W}_{fr,\text{JB-low}} + \bar{W}_{fr,\text{JB-up}} \tag{14}$$

where $\bar{W}_{fr,\text{pt}}$ is the friction power associated with the tangential component of the pressure forces acting on the Z-blade, $\bar{W}_{fr,v}$ is the total friction power due to the contact between vane and blade on both sides of the blade, $\bar{W}_{fr,\text{flat}}$ is the frictional loss due to the flat portions of the Z-blade, $\bar{W}_{fr,\text{cyl-cyl}}$ is the friction loss between the outer cylinder wall and the compressor inner shell, $\bar{W}_{fr,\text{cyl-flat}}$ is the friction losses induced by the upper and lower flat circular crowns of the outer cylinder and $\bar{W}_{fr,\text{TB}}, \bar{W}_{fr,\text{JB-low}}, \bar{W}_{fr,\text{JB-up}}$ are the losses related to thrust bearing, lower and upper journal bearings, respectively.

4. Results and Discussion
The detailed compressor model developed is used as a tool to investigate the internal behavior of the compressor and to quantify the sources of losses. Figure 7(a) shows an example of pressure traces with respect to the rotation angle. The plot of the internal pressure of the compressor during the working process helped adapting the valve parameters to achieve the required operating points.

Starting from the baseline design, i.e. #1, the model has been exercised to improve such design. A total of five possible prototypes have been identified and they are described in Table 3. As a first step, the Z-compressor shaft structure has been rethought to reduce friction losses, i.e. design #2. A more compact design led to significantly reduce the friction losses between the vane and the blade as well as eliminating the need for two journal bearings, as shown in Figure 7(b). Next, by keeping the same design configuration as design #2, three additional designs have been defined by considering different clearance gap sizes. A nominal operating point has been chosen to carry out the calculations: suction temperature 23 °C, suction temperature 100
Figure 7. (a) Evolution of the pressure within the working chambers throughout one rotation. The colors of the solid lines are identical to the ones in Figure 2; (b) Comparison of the friction loss contributions between design #1 and the improved design #2. The other designs are equivalent to design #2 in terms of shaft configuration.

Friction Power [W]

Friction Loss

Design #1

Design #2

(a)

(b)

Figure 8. Leakage mass flow rate distribution among all the paths for different Z-compressor designs.

The improvements or degradations of the compressor performance are determined by calculating the volumetric efficiency, as shown in Figure 9. In the baseline design, all the gaps are kept the same leading to a volumetric efficiency close to 40%. A new vane design and reduced gaps of $L_1$ and $L_2$ of design #2 allowed to improve the volumetric efficiency by almost 50%. The best performance has been obtained with design #3 in which the gaps have been reduced to minimum and with a thicker vane. In the last two designs, a piston-ring has been added to eliminate the leakage flows between upper and lower chambers. However the gap sizes have been relaxed. As a result, design #4 reached a 51% volumetric efficiency while design #5 showed a 15% improvement over the baseline design. The parametric analyses have been further used to define a design #6 with tip seals for prototyping.
Table 3. Description of the different designs analyzed.

| Design | Description |
|--------|-------------|
| #1     | Baseline (as Figure 1) |
| #2     | Shorter vane geometry and modified shaft configuration (one journal bearing and one thrust bearing); $L_1$ and $L_2$ gaps reduced by a factor of 5 |
| #3     | Same as #2 with thicker vane and minimum gap size everywhere |
| #4     | Same as #3 with thicker vane and minimum gap size everywhere |
| #5     | Same as #2; gaps as baseline and added piston-ring to reduce $L_{up}$ and $L_{down}$ |
| #6     | Same as #4 with $L_1$ and $L_2$ gaps increased by a factor of 2 compared to baseline |

5. Conclusions

A comprehensive mechanistic model of a Z-compressor has been developed to identify the losses of a baseline prototype. From the experimental work, friction losses and leakage flows were detrimental to the performance of such compressor. The model has been used to evaluate five potential designs to improve the baseline performance. A new shaft configuration and vane design led to decrease the friction losses by approximately 70%. By reducing the clearance gaps, the volumetric performance of the compressor improved by up to 90%. However, due to manufacturing limitations, an optimum solution has been found by compromising between design and gap sizes, which led to developing design #6.

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