Breakdown of Scaling and Friction Weakening in the Critical Granular Flow

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The way granular materials respond to an applied shear stress is of the utmost relevance to both human activities and natural environment. One of the most intriguing and least understood behaviors is the stick-instability, whose most dramatic manifestation is earthquakes, ultimately governed by the dynamics of rocks and debris jammed within the fault gauge. Many of the features of earthquakes, i.e. intermittency, broad times and energy scale involved, are mimicked by a very simple experimental set-up, where small beads of glass under load are slowly sheared by an elastic medium. Analyzing data from long-lasting experiments, we identify a critical dynamical regime, that can be related to known theoretical models used for "crackling-noise" phenomena. In particular, we focus on the average shape of the slip velocity, observing a "breakdown of scaling": while small slips show a self-similar shape, large does not, in a way that suggests the presence of subtle inertial effects within the granular system. In order to characterize the crossover between the two regimes, we investigate the frictional response of the system, which we treat as a stochastic quantity. Computing different averages, we evidence a weakening effect, whose Stribeck threshold velocity can be related to the aforementioned breaking of scaling.

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Motivations

The way a granular bed responds to an applied shear stress reveals many of the peculiarities of this poorly comprehended "state" of matter. When a granular bed is sheared slowly enough by an elastic medium driven at constant velocity, nor the shear stress neither the shear rate can be directly controlled from outside. Rather, the system sets itself in a state at the edge between jamming and mobility, exhibiting intermittent flow also called stick-slip. This is an instance, among many others, of phenomena displaying intermittent and erratic activity, in the form of bursts, or avalanches, characterized by wild fluctuations of physical quantities, and for this reason named crackling noise. Examples include earthquakes, fractures, structural phase transitions and plastic deformation. These diverse phenomena share several common statistical features. In particular physical quantities display often long range correlations and self-similar distributions, i.e. power laws, over a wide range of values. Such properties are usually ascribed to the vicinity of some critical transition, which in granular media could be the jamming transition. Consistently, critical transitions bring about the existence of universality classes: systems that are microscopically very different, can display similar and universal statistical properties in their critical dynamics. Within this spirit we have designed an experimental setup suitable to observe such an irregular granular dynamics, characterized by critical fluctuations and reminiscent of that displayed by the aforementioned wide class of physical systems.

In order to compare different systems exhibiting critical dynamics, several quantities can be analyzed. Recent literature witnesses a surge of interest for the average avalanche (or burst) shape (or profile). Introduced in the context of Barkhausen noise in ferromagnetic materials, the average avalanche shape can provide a much sharper tool to test theory against experiments than the simple comparison of critical exponents characterizing probability distributions. As shown for simple stochastic processes, the geometrical and scaling properties of the average shape of a fluctuation depends on the temporal correlations of the dynamics. Such observation has allowed, for instance, to evidence a (negative) effective mass in magnetic domain walls.

Average avalanche shapes have been investigated for a variety of materials, well beyond magnetic systems. Among the others: dislocation avalanches in plastically deformed intermetallic compounds and in gold and niobium crystals; stress drop avalanches at the yielding transition in metallic glasses and, via numerical simulations, in amorphous systems; bursts of load redistribution in heterogeneous materials under a constant external load. Many biological studies have also measured average burst shape in cortical bursting activity, in transport processes in living cells, as well as in ants and human activity. Many other bursty dynamics have been investigated by means of this general tool, as stellar processes or Earth’s magneto-spheric dynamics, and earthquakes. The dependence of the avalanche shape from the interaction range has been studied in elastic depinning models.

In this paper we acquire and analyze for the first time the average shapes of slip velocity and of friction force in a sheared granular system, directly in the deep critical phase where it displays intermittent flow. Our findings also shed light on apparently contradictory recent obser-
vations [38, 39], and supply new essential elements to improve the formulation of new and more effective dynamical models, with important impact on the understanding of related natural and technological issues.

Introduction

We study the stick-slip dynamics at the level of the single slip event, as illustrated in Fig. 1. The left panel reproduces the angular velocity, during a slip, of a slider that rotates while in contact with the granular bed. The middle panel shows the corresponding frictional torque experienced by the slider. The motion can be described as a function of the internal avalanche time, which starts at the beginning of the slip and ends when the system sticks. Each slip event has its own duration and its size (the grey area in Fig. 1, left panel). The average velocity shape is performed considering many slips with the same total duration as function of internal avalanche time. A similar averaging procedure is followed to obtain the average friction shape: i.e. the average friction torque exerted by the granular medium at the internal time during a slip event of total duration. The right panel of Fig. 1 shows the intricate, complex relation between friction and velocity during the intermittent, stick-slip dynamics.

In our study we observe the existence of a cross-over from small to large slips. We identify it as a breakdown of the critical scaling and show that such transition is in turn related to a change in the frictional properties of the system. Specifically, we find that the average velocity of the cross-over avalanches corresponds to a characteristic value marking a dynamical transition from weakening to thickening frictional behavior of the system. Average shape for avalanches of stress drop [39] and energy drop rate [38] have been recently investigated in slow but continuous flow, where velocity never drops to zero and the stress is the relevant fluctuating quantity. While in [39] average avalanches have been found to display symmetrical and self-similar shapes, in [38] these properties have been observed only in avalanches sufficiently small. Our investigations, conducted in the critical state, contribute to clarify the origin of these contradictory behavior observed in a different situation.

Experimental set up

The experimental set up (see Fig. 8) is similar to that employed in [5, 7, 40, 41] and described in more detail in the Appendix. The apparatus consists of an assembly of glass spheres laying in an annular channel and sheared by a horizontally rotating top plate driven by a motor. The instantaneous angular position of the plate and of the motor, respectively and are acquired by means of two optical encoders.

The plate is coupled to the motor through a soft torsion spring of elastic constant , so that the instantaneous frictional torque, , exerted by the granular medium can be derived from the equation of motion for the plate:

\[ \tau = -k(\theta - \theta_p) - I \ddot{\theta}, \]

where \( I \) is the inertia of the plate-axis system. The motor angular speed \( \omega_0 \) is kept constant, so that \( \theta(t) = \omega_0 t \), but the interaction between the top plate and the granular medium is crucial in determining the instantaneous plate velocity, leading to different possible regimes. When both the driving speed and spring constant are low enough the critical dynamics, in which the plate performs highly irregular and intermittent motion, is approached.

Scaling analysis

We have performed long experimental runs in the critical, stick-slip, regime measuring the angular coordinate of the plate \( \theta_p(t) \), from which we have derived the plate angular velocity \( \omega_p(t) \). We have collected statistics for a large number of avalanches: the distribution of corresponding durations and sizes \( S = \int_0^T \omega_p(t) dt \) are shown in Fig. 2. Both distributions exhibit a slow decay, roughly close to a power law, terminating by a bulging cutoff for large sizes. Similar broad distributions are shared by other quantities, e.g. the plate velocity, \( \omega_p \) (not shown). As recalled in the introduction, power law decay in distributions are generally considered the hallmark for criticality. If this is the correct scenario, one should
observe self-similar scaling relations in average quantities too. In particular, we consider the average shape of velocity during an avalanche of a fixed duration, defined as:

\[
< \omega_p(t)/T = \frac{1}{N_T} \sum_1 \omega_p^{(i)}(t),
\]

where \(\omega_p^{(i)}\) is the plate velocity during the \(i_{th}\) observed avalanche of duration \(T\), whose total number is \(N_T\), and \(t\) is the internal time within the slip: \(0 < t < T\). Although the average velocity shape \(< \omega_p(t) >_T\) depends on both \(t\) and \(T\), criticality should imply that an invariant function \(\Omega\) exists, such that it can be expressed as:

\[
< \omega_p(t) >_T = g(T)\Omega(t/T).
\]

The function \(g(T)\) determines how the average event size \(< S >\) scales with respect to the slip duration \(T\). In fact, integrating the above equation with respect to \(t\) one gets:

\[
<S> = Tg(T)
\]

(3)

(where without loss of generality we have assumed \(\int_0^1 \Omega(x)dx = 1\)). The function \(\Omega\) represents the average invariant pulse shape, which is expected not to depend on the slip duration and can be computed via the above equations as

\[
\Omega(t/T) = T< \omega_p(t) >_T/< S >_T.
\]

(4)

The previous scaling scenario is produced by several theoretical models for critical dynamics. One paradigmatic model for crackling noise is the so called ABBM model [42], proposed to describe the intermittent statistics of electric noise recorded during hysteresis loops in ferromagnetic materials (Barkhausen noise). It is simple enough to allow exact analytical results [20, 22, 42–44], and it predicts power law distributions for avalanche sizes and durations, as well as parabolic average avalanche shape, in the scaling regime. In the conclusive section we will discuss the connections between this model and what we observe in our study.

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**FIG. 2.** Left: Probability distribution of slip extensions \((S)\). Right: Probability distribution of slip durations.

**FIG. 3.** Scatter plot of size \(S\) vs duration \(T\) of each single slip. Symbols (colors online) correspond to the different duration classes employed for the average shape analysis. Inset: Average slip velocity \((S <j>/T <j>\) for each class as a function of average duration \((T)\) of the class. Lines (both in main plots and in inset), are guide to the eyes for: power law behaviour \(S \approx T^{2.2}\), and linear behaviour \(S = \omega_c T\) (where \(\omega_c = 0.04\), see text and Fig. 5 for definition).

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**Average shape of slip velocity**

To investigate the properties of the average pulse shape, and to test its invariance and the scaling hypothesis, we have divided all the avalanches observed in the experiments into classes according to their duration (see Appendix). Figure 3 (main panel) shows the slip size as function of its duration for all the slips considered in the statistics, and the different colors correspond to the different classes of duration. For each class, \(j\), we have computed the average slip size \(< S >_j\) and duration \(< T >_j\), and the average velocity \(< \omega_p(t) >_j\) measured as function of the internal time \(t\). According to Eq. (4), in order to obtain \(\Omega(t/T)\), this average velocity has then been normalized to the ratio \(< S >_j / < T >_j\).

The resulting average shapes for each class of duration are shown in Fig. 4 (light, grey points in \(\square\) corresponding to very short slips at the limit of the system resolution, have been discarded). All classes exhibit comparable values of the rescaled maximum velocity implying that longer avalanches are also faster. However, rescaled average shapes unveil that there are two kinds of avalanches. Some of them, say short, have the shape described by a unique function \(\Omega(t/T)\), visible in Fig. 4 (left panel). That is, their size and duration are related by the well defined scaling law Eq. 2. On the contrary, the average velocity shapes of long avalanches (right panel) change with the duration and cannot be reduced to a universal form by a homogeneous rescaling of the variables. Moreover, they do not display the almost symmetric shape...
metries. A left asymmetry has been also observed in
and this could in principle be one origin of the asym-
between particles and supporting glass, and of nonlin-
the system enters the Bagnold’s regime \[47, 48\], a behav-
leftwards asymmetry \[18, 19\]).

In some magnetic materials, a leftwards asymmetry has been observed and related to memory effects acting as an effective negative mass of domain walls \[21\]. In our experiments we cannot exclude the existence of such an "effective" inertia of the system, due to some mem-
characterizing small avalanches.

As anticipated, Barés and coworkers \[38\] have recently measured the average shape of stress drop rate avalanches in a bidimensional granular system driven at constant shear rate. Similarly to the present findings, they observe that larger slips develop left asymmetries. They have hypothesized a possible role of the static friction between particles and supporting glass, and of nonlinear elasticity, given by the relatively soft nature of the grain material employed in their experiments. We can however exclude these factors in our experiments, where the interface grain-wall is small with respect to the bulk and the beads are made of glass. The leftwards asymmetry observed in experiments represents a very interesting phenomenon, which in general is expected from non trivial dynamical effects, and cannot be due to the simple inertia of the moving plate (which should produce opposite asymmetry \[18 \[19\]).

In some magnetic materials, a leftwards asymmetry has been observed and related to memory effects acting as an effective negative mass of domain walls \[21\]. In our experiments we cannot exclude the existence of such an "effective" inertia of the system, due to some memory introduced by the underlying granular. For instance, in some experiments \[45\] researchers noted an increased inertia of the slider moving on a granular bed, due to the grains dragged by the slider itself and a similar augmented inertia has been observed also in our previous experiments \[9\]. Since the quantity of grains dragged by the disk during its motion could change during the irregular motion of the system one should consider the inertia as a dynamical quantity, rather than a constant, and this could in principle be one origin of the asym-
earthquakes \[36 \[46\].

### Breaking of scaling

More insight into the mechanisms leading to the scaling breakdown can be gained by looking again at the plot relating \(S\) and \(T\) shown in Fig. 3. The first information coming from this plot is that there exists a definite statistical scaling between slip size and duration, as shown by the scattering of data. The white squared symbols in the main plot represent the average slip size and duration of each class (statistical errors are negligible on these averages). It is seen that, at least for the four lower classes, they follow an algebraic relation: \(< S >_j \approx < T >^\alpha_j\) (red continuous line). The value of the exponent turns out to be \(\alpha \approx 2.2\). This is close to, but clearly different from, the value of \(\alpha = 2\) expected from extant models (for instance the ABBM model mentioned above \[22\]).

The other information supplied by the scatter plot of Fig. 3 is that this behavior changes at large slips, where a linear dependence, \(< S >_j \approx < T >\) looks more appropriate (yellow dashed line). Interestingly, the crossover between the two behaviors takes place around the fourth class, exactly where the scaling of the average pulse shape, shown in Fig. 1 breaks down. The inset of Fig. 3 puts into better evidence this cross-over. There, we have plotted the quantity \(< S >_j / < T >_j\) as function of \(< T >_j\). We observe a weakly superlinear relation for small slips, followed by a plateau at large slips. Note that the ratio between \(S\) and \(T\) is nothing but the plate average velocity during the slip. This observation allows to identify a critical velocity, as the ratio between the average slip size, \(< S_c > \approx 0.063\) rad, and the average duration, \(< T_c > \approx 1.57\) s, of the fourth class:

\[\omega_c = < S_c > / < T_c > \approx 0.04\ \text{rad/s} \]

We speculate that during large slips \((T > T_c)\), when the plate reaches high velocities \(\omega_p > \omega_c\), it could experience some sudden increase of friction. In the next section it will be seen that this increase indeed appears, as a dynamical effect.

### Stochastic friction

The forces ruling the slip dynamics are the spring torque and the granular friction. While the first one just depends linearly on the instantaneous angle, the second displays a complex behavior (see Fig. 7 central and right panel) from which interesting features emerge.

The classical Mohr–Coulomb criterion predicts constant friction at low shear, and increasing values when the system enters the Bagnold’s regime \[47 \[48\], a behavior well observed experimentally at constant shear (see e.g. \[49\]). However, it is doubtful whether this behavior could be relevant to the stick-slip dynamics observed in the critical regime. More generally, friction in granular

![Graph](image-url)
systems is usually measured under controlled shear strain or stress, but its properties can be dramatically different when observed in the self-organized state, as exemplified in Fig. 1 right panel. Some statistical features of friction in this state have been investigated in [4, 7], but despite this quantity plays a crucial role for the system dynamics, it has never been systematically measured to date during stick slip.

In the critical regime friction is a random quantity. It depends on the details of the network of contacts between particles in the granular bed. Fluctuations in the frictional response of the granular medium result from the stress propagation on the evolving network of grain contacts, and are at the very origin of the motion stochasticity. This fact has a number of consequences and some subtleties. A random friction force, as a stochastic quantity, can be described by statistical estimators like averages, moments, correlators, etc. Nevertheless, several averages can be defined, which depend on the driving protocol and can be very different from each other. More specifically, one can consider the time average of the friction over the full dynamics, but this is not always really meaningful, especially in the critical regime. As shown in [5] the statistical distribution of friction in this regime is characterized by fat tails, as opposite with the continuous sliding where it is normal. Another possible average, [5, 40] is the average friction conditioned to the instantaneous plate velocity:

\[
\tau_f(\omega) = \lim_{T \to \infty} \frac{\int_0^T \tau(t) \delta(\omega - \omega_p(t)) dt}{\int_0^T \delta(\omega(t) - \omega_p) dt}.
\]  

In Fig. 5 we plot such conditioned average friction during the stick slip critical regime. As noted in [4], an interesting Stribeck-shaped (that is, a shear weakening followed by a thickening) friction curve appears, featuring weakening for small velocities and recovering the Bagnold behavior at high velocities. However, this velocity weakening arises as a dynamical effect. In fact, a different driving protocol can give different results: for instance, at constant shear [40] the average friction is constant at low and intermediate speeds.

The analysis of Fig. 5 allows to identify a velocity corresponding to the position of the minimum of the average friction \(\tau_f\). Our experiments clearly indicate that the position of this minimum does not depend on the drive velocity (Fig. 13 in Appendix) and it is always attained near the velocity \(\omega \approx 0.04\) rad/s. This value is very close to the value \(\omega_c\) marking the crossover in the scaling of \(S\) vs \(T\) (see Fig. 5), which in turn is related to the breakdown of scaling of the average avalanche shape shown in Fig. 4. This corroborates the previous interpretation of the crossover phenomena and the breaking of the critical scaling of the dynamics as due to the weakening followed by the increase of friction experienced by the plate during larger, faster avalanches Fig. 4.

In order to better investigate whether and how friction dynamical behavior can influence the average velocity shape we have also analyzed the average shape of friction along the slip. In an analogous fashion to what done for computing the velocity shapes, one can define \(\langle \tau(t) \rangle_T\) as the average frictional force for slips of the same duration \(T\). In practice, we have computed the average value of the friction torque over slips of similar duration \(T\), according to the same classes of duration adopted for velocities (see Fig. 3 and Appendix). The results, presented in Fig. 6, show that the breaking of scaling of the velocity shapes corresponds to a change in the frictional properties of \(\langle \tau(t) \rangle_T\). For small avalanches, i.e. those corresponding to the cases in which average velocity shape obeys scaling (curves plotted in the left graph of Fig. 6), the average friction maintains an almost constant value along the whole slip, whose value is independent from the slip duration. On the contrary, the curves corresponding to longer slips (shown in the right panel of Fig. 6) display different shapes that, as in the case of velocity (Fig. 4), strongly depend on \(T\), and cannot be collapsed. Note also that higher frictions are experienced during longer slips.

Let us stress here the difference between the two average procedures considered in this work. The average \(\langle \tau \rangle_T\) shown in Fig. 6 are performed over slips of similar duration, at the same internal avalanche time \(t\). Instead, the (conditional) average \(\tau_f\), defined in Eq. 5 and shown in Fig. 5, mixes events of any duration, and it depends on the instantaneous plate velocity \(\omega_p\). The two quantities give different aspects of the same (stochastic) physical phenomenon. Nevertheless, the combination of the two analysis suggests that the quite complex friction weakening behavior of \(\tau_f\) is mainly due to large slips, which show a non constant average friction \(\langle \tau \rangle_T\) in time (see Fig. 4 right panel), in contrast with small avalanches, where the average keeps mainly constant.
Our experiments show a good scaling of the average velocity shape for small avalanches, with an almost symmetric average shape. For larger avalanches however, scaling (Eq. 2) is broken: for large slips the shape takes a clear leftwards shape in agreement with what observed in seismic data \cite{38, 46} (and recently in \cite{51}).

Our analyses show that the breakdown of velocity scaling goes along with changes in the friction behavior, pointing out a strict relation between the two phenomena. On the opposite, spring-block models with only Coulomb friction generate symmetric slips \cite{52, 53}. Effective friction laws accounting for elapsed time and/or space have been incorporated in solid-on-solid interface models, through the dependence on so called state variables \cite{54–56}. These rate-and-state laws are often adopted for studying and modeling co-seismic fault shearing, together with their other simpler forms \cite{53, 57–59}. They are essentially phenomenological and can describe both velocity weakening and hardening, depending on the adopted parameters (which are not derivable from microscopic principles). These laws have shown to work to some extent also for interstitial granular matter \cite{60}, but with some inconsistencies \cite{61}. Moreover they have been drawn from experiments where velocity is forced to change in sudden steps and they don’t seem to have been never investigated in the critical stick-slip. Attempts to do this with smoothly varying velocity have been done in \cite{40}. In a very recent work \cite{62} both friction weakening and hysteresis have been numerically investigated during granular shear cycles, showing that these are due to contact instabilities induced by the acoustic waves generated during granular fluidization. It is thus clear that granular flow cannot be effectively modeled without the inclusion of more refined and realistic friction laws.

An effective modeling approach to the critical granular dynamics cannot as well exclude a stochastic description of friction, which generates the slip unpredictability and their range of variability, with the following change in the slip shapes. To our knowledge, the only few attempts in this direction \cite{6, 40, 63} are inspired to the aforementioned ABBM model \cite{42}, which represents the mean field approximation for the motion of a driven elastic interface in a random environment \cite{64, 65}. From the dynamical point of view, it describes a spring–slider model subjected to a friction where both viscous and a random pinning components are present, in the overdamped (i.e. negligible inertia) approximation. At small, but finite driving rate, the ABBM model predicts an intermittent, critical dynamics for the block motion. Similarly to our observations, avalanche statistics show a scaling regime for short slips, whose average velocity has parabolic shapes. However, an exponent $\alpha = 2$ relates $\langle S \rangle$ to $\langle T \rangle$, which is different from what observed in our experiment. More-
over, for longer slips, ABBM predicts flatter symmetrical shapes, witnessing a cut-off in the velocity correlation. No inertial effects are present, due to the overdamped approximation.

A variant of the ABBM model for critical granular dynamics has been introduced in [6], where, based on empirical observations, a simple Strubeck-like rate dependence, showing a minimum, of the average granular friction was adopted. Moreover, more physically, a cut off in the spatial correlation of the random force was considered and, at odds with the original model, inertia was taken in account. Later on, attempts to introduce in the model a state dependent weakening friction have been done [40], and further investigations are in progress.

We think that the insights provided by the present study can explain the contradictory recent observations in [38, 39] and can be useful to advance such efforts to improve models. In particular, they show that inertia can play an important role in both weakening-hysteresis [62] and in the determining scaling exponent $\alpha$ (an inertial ABBM model has been studied in [66]). Even at the microscopic level, grain inertia can influence the avalanche statistics. For instance, in sandpile models, largely studied in the context of SOC (Self Organized Criticality), the tendency of real sand grains to keep moving once they start facilitate the emergence of huge avalanches. Recent theoretical developments propose, in the presence of such facilitation effects, a scenario called Self-Organised Bistability [67], where again a breaking of scaling is associated to the appearance of large avalanches ("kings").

We conclude that weakening is a genuine property exhibited by granular dynamics at variable shear rate, and that randomness and memory are a general features of friction that cannot be overlooked in the formulation of effectual models. Such models can have impact on the understanding of many phenomena occurring in the large realm of granular systems, and in particular of self organized natural phenomena like landslides, and earthquake, where it is not yet clear the way different mechanisms can contribute to the shear weakening observed in coseismic fault shearing [68]. Further investigation on theoretical models incorporating more realistic, specifically memory dependent friction laws, and new experiments will allow to better understand the mechanism for which criticality breaks down.

ACNOKWLEGMENTS

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APPENDIX

The experimental set up

FIG. 8. Photo (up) and schema (down) of the experimental set up.

The experimental apparatus utilized for this research consists of a circular PPMI channel of outer and inner radii $R = 19.2$ cm and $r = 12.5$ cm respectively. The channel is 12 cm height and is almost filled with a bidisperse mixture 50%-50% of glass beads, with radii $r_1=1.5$ mm $\pm$ 10% and $r_2=2$ mm $\pm$ 10%.
TABLE I. Features of the analyzed series of experiments with different drives

| series | duration (minutes) | # of points driving $\omega_d$ | # of slips used in analysis |
|--------|-------------------|-------------------------------|-----------------------------|
| (EA)   | 3900              | 5849962                       | 0.0015                      |
| (EB)   | 673               | 202079                        | 0.0222                      |
| (EC)   | 1200              | 36000000                      | 0.0044                      |
| (ED)   | 4080              | 12240020                      | 0.0055                      |
| (EE)   | 360               | 1079977                       | 0.011                       |
| (EF)   | 240               | 720007                        | 0.021                       |
| (EG)   | 210               | 630014                        | 0.033                       |

TABLE II. Classes of avalanche duration adopted for the analysis, and the resulting number of avalanches for the data set (EA) discussed in the main text.

| duration | # of avalanches |
|----------|-----------------|
| $0.309 \leq T < 0.489$ | 929 |
| $0.489 \leq T < 0.722$ | 866 |
| $0.772 \leq T < 1.219$ | 987 |
| $1.219 \leq T < 1.925$ | 1694 |
| $1.925 \leq T < 3.04$ | 1380 |
| $3.04 \leq T < 4.8$ | 158 |

A top plate, fitting the channel, can be rotated and has a few layer of grains glued to its lower face in order to better drag the underlying granular medium. The plate has mass $M = 1200$ g and moment of inertia $I = 0.026$ kg m$^2$, and it is free to move vertically, implying that in our experiments the medium can change volume under a nominal pressure of $p = Mg/[\pi(R^2 - r^2)] \approx 176$ Pa. The plate is connected to an end of torsion spring, of elastic constant $\kappa = 0.36$ Nm/rad, while the other end of the spring is rotated by a motor at constant angular velocity $\omega_d$. The angular positions of motor and plate are supplied by two optical encoders positioned on either side of the torsion spring, each one having a spatial resolution of $3 \cdot 10^{-5}$ rad and being sampled at 50 Hz. These measures provides the plate instantaneous position and velocity, $\theta_p$ and $\omega_p$, as well as the friction torque, which is proportional to the angular difference between motor and plate (see Eq. (1)).

Experimental analysis

In principle each single slip event, or avalanche, begins when $\omega_p$ starts to differ from zero and ends when $\omega_p$ goes back to zero. However, in practice it is necessary to choose a threshold value $\omega_{th}$ to cross, in order to get rid of the instrumental noise. This choice is to some extent arbitrary, however all the results have been observed to be independent from the chosen threshold, as long as it is small and different from 0. For our analysis we have set $\omega_{th} = 0.00175$ rad/s, and considered the seven time series reported in table I.

The avalanches of each series have been grouped into classes on the base of their duration, according to the first column of Table II. For each class $j$ the average avalanche duration $< T >_j$ and size $< S >_j$ have been evaluated, and instantaneous average velocity has been computed at a set of discrete times, $0 \leq t_i \leq < T >_j$ (see main text).

Avalanches at the extremes of distributions have been dropped out. For example duration and size for avalanches from the series (EA) are plotted in Fig. 2 of the main text, with different colors for each interval. Avalanches in gray, shorter than 0.31 s, are too small to perform meaningful analysis (less than 15 points at 50Hz of sampling rate). The total number of avalanches employed in this statistics has then been 6014, distributed according to the second column of table II.

The main text presents results from the series (EA). The results from the other datasets, with the different drives reported in Table I, display similar behaviors and are shown in Figs. 2-7 of this Appendix, to be compared with the corresponding Figs. 2-7 in the main text. Analogous results were obtained adopting different sampling frequencies and threshold values.

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FIG. 10. Avalanche sizes vs durations, and class definitions, for different drive velocities (see Fig. 2 in the main text)

FIG. 11. Average velocity shapes for different drive velocities (see Fig. 4 in the main text)

FIG. 12. Average friction shapes for different drive velocities (see Fig. 5 in the main text)

FIG. 13. Average friction vs average velocity for different drive velocities (see Fig. 6 in the main text)

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FIG. 14. Conditional average friction vs instantaneous plate velocity for different drive velocities (see Fig. 7 in the main text)
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