Hadroproduction of $t$ anti-$t$ pair with $b$ anti-$b$ pair with PowHel

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Abstract

We simulate the hadroproduction of a $t\bar{t}$ pair in association with a $b\bar{b}$ pair at 14 TeV LHC using the PowHel package. We use the generated events, stored according to the Les-Houches event format, to make predictions for differential distributions formally at the next-to-leading order (NLO) accuracy and we compare these to existing predictions accurate at NLO.
1 Introduction

According to the latest, most precise measurements, the t-quark mass is $m_t = 173.5 \pm 0.6 \pm 0.8$ GeV \cite{1}, indicating that the Yukawa coupling of the t-quark, $y_t = 0.997 \pm 0.008$ equals one with better than 1% accuracy. This remarkable result suggests that it is important to measure this coupling also directly as precisely as possible. After the recent discovery of a Higgs-like particle at the LHC \cite{2,3}, the focus of analyses is shifting towards determining whether the couplings of this new particle to other fundamental particles agree with the Standard Model (SM) expectations. The measurement of the $t\bar{t}H$ coupling is important also because deviation from the SM expectation provides signal of physics beyond the SM. If the Higgs-boson is indeed the particle discovered at the LHC recently then its mass is approximately 125 GeV. Such a Higgs-boson does not decay into t-quarks, therefore the best process to measure the $t\bar{t}H$ coupling in a model independent way is the study of hadroproduction of the Higgs-boson in association with a $t\bar{t}$ pair \cite{4,5}.

The produced Higgs-boson decays immediately and only its decay products can be observed. According to the SM predictions, such a Higgs-boson decays dominantly into a $b\bar{b}$ pair. Thus one can plan to measure the $t\bar{t}H$ coupling in the $pp \rightarrow t\bar{t}H \rightarrow t\bar{t}b\bar{b}$ process. Unfortunately, this process has a large background from the direct QCD process $pp \rightarrow t\bar{t}b\bar{b}$. In order to optimize the experimental selection of the $t\bar{t}H$ events, one needs simulation of the QCD background with high precision.

The most up to date method to simulate LHC processes with high precision is matching NLO QCD predictions to shower Monte Carlo programs (SMC). Presently, two formalisms are used frequently to match matrix element (ME) calculations at the NLO accuracy to parton showers (PS), the MC@NLO \cite{6,7} and POWHEG \cite{8,9} methods. Indeed, both methods were used to make predictions for $t\bar{t}H$ hadroproduction \cite{10,11}, and the two predictions were found in agreement in Ref. \cite{12}. Here we make one step further and describe our implementation of the $pp \rightarrow t\bar{t}b\bar{b}$ process within the PowHel framework.

The PowHel framework combines the POWHEG-Box \cite{13}, a flexible computer framework implementing the POWHEG method, and the HELAC-NLO package \cite{14}. The output of PowHel is simulated events stored according to the Les Houches accord \cite{15} (LHE). Those events can be fed into any shower Monte Carlo (SMC) program for generating events with hadrons. Using the same framework we already provided LHE’s for several processes at the LHC, namely to the production of a $t\bar{t}$-pair in association with a hard object, such as a jet \cite{16}, or vector bosons \cite{17}.

The scope of this letter is to present the simulation of the $t\bar{t}b\bar{b}$ LHE’s. The process under consideration represents an example of unprecedented complexity in PowHel (and also among all the processes considered for matched NLO and PS predictions), which raises some technical difficulties, worth of detailed description. Phenomenological analyses are left for a more detailed publication.
2 Method

We use the POWHEG formula [9] to estimate the cross section from events with either unresolved or resolved first radiation,

\[ \frac{d\sigma_{LHE}}{d\Phi_B} = \tilde{B}(\Phi_B) d\Phi_B \times \left[ \Delta(\Phi_B, p_{\perp}^{\min}) + d\Phi_{\text{rad}} \Delta(\Phi_B, k_{\perp}(\Phi_R)) \times \frac{R(\Phi_R)}{B(\Phi_B)} \Theta(k_{\perp}(\Phi_R) - p_{\perp}^{\min}) \right], \]  

(1)

In Eq. (1) \( \tilde{B}(\Phi_B) \) denotes the NLO-corrected fully differential cross section belonging to the underlying Born configuration \( \Phi_B \) (the integration over the momentum fractions is included implicitly),

\[ \tilde{B}(\Phi_B) = B(\Phi_B) + V(\Phi_B) + \int d\Phi_{\text{rad}} R(\Phi_R) + \int \frac{dx}{x} \left[ G_{\oplus}(\Phi_B) + G_{\ominus}(\Phi_B) \right], \]  

(2)

and \( \Delta(\Phi_B, p_{\perp}) \) is the POWHEG Sudakov form factor that exponentiates the integral of the ratio of the real radiation \( R(\Phi_R) \) and Born \( B(\Phi_B) \) contributions over the radiation phase space,

\[ \Delta(\Phi_B, p_{\perp}) = \exp \left\{ - \int d\Phi_{\text{rad}} \frac{R(\Phi_R)}{B(\Phi_B)} \Theta(k_{\perp}(\Phi_R) - p_{\perp}) \right\}. \]  

(3)

Using \( \tilde{B}(\Phi_B) \), we obtain the cross section at NLO accuracy as an integral over the Born phase space,

\[ \sigma_{NLO} = \int d\Phi_B \tilde{B}(\Phi_B). \]  

(4)

We used the HELAC-NLO package to generate the crossing symmetric matrix elements required in the POWHEG-Box as input. In particular, (i) the squared matrix elements for the flavour structures of the Born (\( gg \to t\bar{t} b\bar{b}, qq \to t\bar{t} b\bar{b}, \bar{q}g \to t\bar{t} b\bar{b} \)) and real radiation emission (\( qq \to t\bar{t} bb, gg \to t\bar{t} bb, \bar{q}g \to t\bar{t} bb, gq \to t\bar{t} bb, gq \to t\bar{t} bbq \)) subprocesses (\( q \in \{u,d,c,s\} \) – we neglect the contribution of b-quarks in the initial state), (ii) the colour-correlated and spin-correlated squared matrix elements for the Born flavour structures, and (iii) the finite part of the virtual correction contributions in dimensional regularization, on the basis of the OPP method [18] complemented by Feynman rules for the computation of the QCD \( R_2 \) rational terms [19]. With this input POWHEG-Box generates events with either unresolved or resolved first radiation. Then, one can use any shower Monte Carlo (SMC) program for generating events with hadrons. We leave the latter together with phenomenological analyses to a separate publication.
2.1 Checks

The consistency among real-emission, Born, colour-correlated and spin-correlated matrix elements was checked in randomly chosen phase space regions by taking the soft- and collinear limits of the real-emission squared matrix elements in all possible kinematically degenerate channels.

The process presented here was studied at the NLO accuracy in the literature \cite{20, 21}, which enabled us to make detailed checks of our calculation. We reproduced all figures of Ref. \cite{20} and found complete agreement.

2.2 Generation cuts and suppression

As we treat the b-quarks massless, the fully differential Born cross section becomes singular if any of the transverse momenta of the b-quarks $p_{\perp,b}$, or the invariant mass of the $b\bar{b}$-pair $m_{b\bar{b}}$ vanishes. To make the integral of $B$ finite over the whole Born phase space, we introduce generation cuts by requiring $p_{\perp,b} \geq p_{\perp}^{(g.c.)}$ and $m_{b\bar{b}} \geq m_{b\bar{b}}^{(g.c.)}$, where $p_{\perp}^{(g.c.)} = m_{b\bar{b}}^{(g.c.)} = 2$ GeV. We checked that these generation cuts are sufficiently low, so that the physical predictions are independent of those when selection cuts characteristic to physical analyses are superimposed. However, with such low generation cuts the event generation is very inefficient because most of the events are generated in regions of the phase space where $B$ is large, but those regions are usually not selected in the analyses. Therefore, we also introduce a suppression factor \cite{22} for $\tilde{B}$ of the form

$$F_{\text{supp}} = \left( \frac{m_{b\bar{b}}^2}{m_{b\bar{b}}^2 + (m_{b\bar{b}}^{\text{supp}})^2} \right)^3 \prod_{i=b,\bar{b}} \left( \frac{p_{\perp,i}^2}{p_{\perp,i}^2 + (p_{\perp}^{\text{supp}})^2} \right)^3,$$

where $p_{\perp}^{(\text{supp})} = m_{b\bar{b}}^{(\text{supp})} = 30$ GeV.

2.3 Size of the remnant

Depending on the particular process being considered in the POWHEG method, the size of the $R/B$ ratio in the Sudakov exponent can become much larger than its collinear or soft approximation. In such cases the generation of the radiation becomes highly inefficient. This is especially true for the process under consideration. In order to speed-up calculations the POWHEG-Box contains the option of separating the real emission phase space into two disjoint regions, one containing the singular regions while in the other the squared matrix element remains finite. The latter is called the remnant contribution. Thus with this separation the real emission part can be cast into a sum $R = R_s + R_r$, where $R_s$ and $R_r$ are the singular and finite contributions, respectively. In the original POWHEG-Box implementation this separation is controlled by a step function in the form
\( R_s = (1 - \mathcal{F})R \) and \( R_t = \mathcal{F}R \), where

\[
\mathcal{F} = \begin{cases} 
1, & R > \chi \max(C_{\text{coll}}, C_{\text{soft}}) \\
0, & \text{otherwise}
\end{cases},
\]

(6)

with \( C_{\text{coll}} \) and \( C_{\text{soft}} \) being the sums of the collinear and soft counterterms and \( \chi \) is a free parameter that can be tuned to control the size of the phase space of the remnant. The larger \( \chi \), the smaller this region, and disappears completely in the limit \( \chi \to \infty \). With this separation the POWHEG formula in Eq. (1) becomes a sum of two terms,

\[
\sigma_{\text{LHE}} = \sigma_{\text{LHE,s}} + \sigma_{\text{LHE,r}}.
\]

(7)

The first one, \( \sigma_{\text{LHE,s}} \) is defined as in Eqs. (1)–(3) with the change that \( R \) is replaced with \( R_s \) everywhere. The second one contains the contribution of the remnant,

\[
\sigma_{\text{LHE,r}} = \int d\Phi R_{tt}(\Phi_R).
\]

(8)

Similarly, the cross section at NLO accuracy in Eq. (4) becomes a sum of two terms,

\[
\sigma_{\text{NLO}} = \sigma_{\text{NLO,s}} + \sigma_{\text{NLO,r}},
\]

(9)

where the first term is obtained as in Eq. (4) with the \( R(\Phi_R) \to R_s(\Phi_R) \) substitution and the second term is equal to \( \sigma_{\text{LHE,r}} \).

In principle the size of the remnant can be controlled freely by the parameter \( \chi \), but practical considerations limit this freedom. Unfortunately, when using the POWHEG method, the efficiency of event generation decreases with increasing \( \chi \) because over a large part of the phase space the value of the fully differential cross section is much smaller than its peak value. This decrease in efficiency depends on the process under consideration. For the complex final state of \( t\bar{t}b\bar{b} \)-production, the size of the phase space of the remnant is large if for instance, the value of \( \chi \) is chosen to its default value in the POWHEG-Box, \( \chi = 5 \).

The separation of the real emission contribution does not influence the NLO cross section. We show the result of an example computation in Table 1. These computations were done for proton-proton collisions at \( \sqrt{s} = 8 \) TeV, with CT10 parton distribution functions (PDF), equal renormalization and factorization scales \( \mu_0 = \mu_R = \mu_F = m_t = 173.2 \) GeV, and the virtual part was replaced with the half of the Born squared matrix element \( V \to B/2 \) (which we call ‘fake virtual’). We imposed the following set of cuts:

1. A track was considered as a possible jet constituent if \( |\eta'| < 5 \), t-quarks were excluded from the set of possible tracks.
2. Jets were reconstructed with the \( k_\perp \)-algorithm using \( p_{\text{min}}^{j} = 20 \) GeV and \( R = 0.4 \).
3. We required at least two jets, one \( b \) and one \( \bar{b} \)-jet, with \( |y_b(\bar{b})| < 2.5 \).
4. Events with invariant mass of the \( b\bar{b} \)-jet pair below \( m_{\text{min}}^{b\bar{b}} = 20 \) GeV were discarded.
The separation of the real emission into the singular and remnant contributions however, does influence the distributions computed from the generated LHE’s. We study this dependence in detail in the next section. Here we only point out that the ratio of the events with Born kinematics to those with first radiation depends on the value of $\chi$. In the last column of Table 1 we show this ratio, and find that for the default separation in the POWHEG-Box, $\chi = 5$, the number of generated events with Born kinematics is too large, making the transverse momentum distribution of the hardest non-$b$ jet ($p_{t,j_1}$) incorrect. Increasing $\chi$, this ratio decreases and becomes approximately 0% in the limit $\chi \to \infty$, and the $p_{t,j_1}$-distribution fits close to the NLO prediction for large transverse momenta. (At small $p_{t,j_1}$ we see the effect of the Sudakov-damping.) This suggests that one should use at least $\chi = 50$ to separate the remnant without introducing too large fraction of events with Born kinematics and, as a result, incorrect distribution of the hardest non-$b$ jet.

### 2.4 Precision and efficient evaluation of loop amplitudes

In the computations above we used a fake virtual part $V = B/2$. The reason for such a fake computation is the high degree of difficulty of evaluating the loop amplitudes. The high-rank of the tensor loop-integrals makes the numerical computation of the loop amplitude unstable, and thus, unreliable when double precision arithmetics is used in CutTools [18], as implemented in HELAC-1loop (part of HELAC-NLO). In order to control numerical instabilities we employ an $N = N$ test as implemented in CutTools. For a given numerator we determine the scalar-integral coefficients using double precision arithmetics. We check the accuracy of the integrand by reconstructing it using all the coefficients (and spurious terms) with a randomly chosen loop momentum. If the reached relative accuracy is worse than $10^{-4}$, we pass the same phase space point in double-double precision (computed simultaneously with the double-precision version) to HELAC-1loop@dd to recalculate all the coefficients (using the multiple precision part of CutTools). The HELAC-1loop@dd code is a straightforward extension of HELAC-1loop to double-double precision using the package QD [23]. This way we solved all numerical instabilities in the computation of the virtual correction. The above procedure however, makes the numerical computation of the loop

| $\chi$ | $\sigma_{NLO,s}[\text{fb}]$ | $\sigma_{NLO,r}[\text{fb}]$ | $\sigma_{NLO}[\text{fb}]$ | $E_B/E_R$ |
|-------|---------------------|---------------------|---------------------|----------|
| 5     | $446 \pm 1$         | $1109 \pm 9$       | $1555 \pm 9$       | 22%      |
| 10    | $514 \pm 2$         | $1040 \pm 9$       | $1554 \pm 10$      | 17%      |
| 50    | $693 \pm 2$         | $855 \pm 7$        | $1549 \pm 7$       | 1.9%     |
| $\infty$ | $1577 \pm 16$  | $0 \pm 0$          | $1577 \pm 16$      | $\simeq 0\%$ |

Table 1: The two contributions to $\sigma_{NLO}$ according to the decomposition of Eq. (9) computed with fake virtual. The last column shows the ratio of LHE’s without resolved radiation to those with resolved first radiation.
amplitudes rather cumbersome, requiring much CPU time. This problem is magnified by the hit-and-miss procedure, the way event generation is performed in the POWHEG-Box. During NLO integration the inclusive NLO cross section is computed simultaneously with the maximal value of $\tilde{B}$

$$\tilde{B}_{\text{max}} = \max_{\Phi_B} \tilde{B}(\Phi_B).$$

(10)

When the underlying Born kinematics $\Phi_B$ is generated for an event, it is accepted if $\tilde{B}(\Phi_B) \geq \xi \tilde{B}_{\text{max}}$, where $\xi$ is a random number picked between zero and one. Using this hit-and-miss method, the $\tilde{B}$ function has to be evaluated several times to find a suitable phase space point which is selected. If $\tilde{B}$ is calculated multiple times per event, event generation becomes highly inefficient because $\tilde{B}$ contains the virtual part, whose evaluation is very time consuming. To improve the efficiency, we first generate events with the fake virtual. We select those events that pass the selection cuts, whose weights in a final step are reweighted by the true virtual contribution. This way we can reduce
Figure 2: Distribution of weights before and after reweighting.

The computation of the virtual contributions to few hundred thousand phase space points. The plots shown in the next section contain 200k points.

The POWHEG method produces unweighted events. When we generate events with the fake virtual part, unweighting must be done using the NLO cross section obtained also with the fake virtual. After the events that pass the selection cuts are collected, we reweight those

\[ \mathcal{W}\big|_{V=B/2} \rightarrow \mathcal{W} = \mathcal{W}\big|_{V=B/2} \frac{\tilde{B}(\Phi_B)}{B\big|_{V=B/2}(\Phi_B)}, \]

where \( \mathcal{W}\big|_{V=B/2} \) is the weight with fake virtual, and \( \mathcal{W} \) is the weight with the true virtual part. Clearly, the new weight becomes dependent upon the underlying Born kinematics.

The majority of events generated by the POWHEG-Box have either positive or negative, but in magnitude equal weights. As seen on Fig.2 the relative fraction of the negative weights is at the percent level. (Due to the suppression in Eq. (5) there are also several large weight events starting at about 50 in Fig.2) The drawback of the reweighting procedure is the loss of equal weights: although about 90% of the events have equal positive weights, LHE’s with many different weights also appear.
3 Predictions from the LHE’s

Using PowHel one can make predictions at five different stages in the evolution of the final state: (i) at the parton level using NLO accuracy, (ii) from the LHE’s, formally at the NLO accuracy, (iii) after decay of the heavy particles, (iv) after parton shower, (v) at the hadron level after full SMC. While in an experimental analysis the last option is the most useful, for examining the effect of the different stages in the evolution, and also for checking purposes, it is useful to study the predictions at intermediate stages. In the previous section we mentioned that we made comparison with existing predictions at stage (i) and found agreement. We now show predictions at stage (ii), and discuss how these depend on the separation of the remnant (on $\chi$) and on the choice of the scale $\mu_0$, and leave the phenomenological analyses at the remaining stages to a more detailed publication.

Our selection cuts are those used in setup 1 of Ref. [21], which amounts to the cuts (1–3) above supplemented with an invariant mass cut of the $b\bar{b}$-jet pair, $m_{b\bar{b}} > 100$ GeV. In Figs. 3–5.a we show our predictions from the LHE’s as bands obtained by varying the scale in the range $[\mu_0/2, 2\mu_0]$ around the fixed default scale $\mu_0 = m_t$ and the predictions at NLO accuracy with scale varied in the same range. The LHE’s were obtained with remnant separation parameter $\chi = 5$, but for comparison we also show the predictions with the default scale from LHE’s generated with $\chi = 50$, which describes the distribution of the leading non-b jet more accurately. In Figs. 3–5.b we show our predictions from the LHE’s as bands obtained by varying the scale in the range $[\mu_0/2, 2\mu_0]$ around a dynamical scale $\mu_0^2 = m_t \sqrt{p_{\perp,1} p_{\perp,\bar{b}}}$. The LHE’s were obtained with $\chi = 50$. For comparison on each plot we also show the NLO predictions of Ref. [21] obtained with their default dynamical scale, and the band of our NLO predictions obtained by varying the fixed scale in the range $[\mu_0/2, 2\mu_0]$.

We present kinematical distributions of the $b$-jet pair: transverse momentum in Fig. 3, rapidity in Fig. 4, and invariant mass in Fig. 5. Our first observation is that the predictions from the LHE’s show much smaller scale dependence than the corresponding predictions at NLO accuracy. Nevertheless, the latter bands overlap with the former ones.

Looking at the distributions obtained with the fixed scale, it is remarkable that the predictions from the LHE’s obtained with $\chi = 5$ coincide almost exactly with the NLO predictions of Ref. [21] obtained with a dynamical scale, which is true for all kinematical distributions presented in Ref. [21]. This suggests that these LHE’s can be used if one is interested in distributions related to observables of particles emerging from the primary $b$-quarks. Unfortunately, as seen in Fig. 1 the events obtained with $\chi = 5$ do not describe the non-$b$ radiation. Instead, $p_{\perp,1}$ is described correctly if $\chi = 50$. However, the predictions from the LHE’s obtained with $\chi = 50$ and fixed scale fall below the distributions obtained with $\chi = 5$, though in agreement with it within scale uncertainty. There is one exception, the invariant mass distribution of the $b\bar{b}$-jet pair in the range of $[100,150]$ GeV, relevant to Higgs studies. It is encouraging that in this range the various predictions are robust against variations of the parameters.
Figure 3: Transverse momentum distribution of the $b\bar{b}$-jet pair at the LHC at $\sqrt{s} = 14 \text{ TeV}$ using PowHel. Distributions from LHE’s are denoted PowHel LHE, while those at NLO accuracy by PowHel NLO using a fixed scale $\mu_0 = m_t$. The shaded band corresponds to cross section obtained with varying the scale in the range $[\mu_0/2, 2\mu_0]$. For comparison we also show the NLO predictions from Ref. [21] (BDDP NLO). (a) PowHel LHE’s generated using a fixed scale $\mu_0 = m_t$ and with remnant separation $\chi = 5$ (solid line with solid band as scale dependence), and with $\chi = 50$ (dash-dotted). (b) PowHel LHE’s generated using a dynamical scale $\mu_0^2 = m_t \sqrt{p_{t,bb} p_{t,bb}}$, and with $\chi = 50$ (solid line with solid band as scale dependence). The left scale on the lower panels show the ratio of the bands of scale dependences to the distribution of the PowHel LHE’s while the right scale on the lower panels shows the ratio of the NLO prediction to that obtained from the same PowHel LHE’s generated with (a) fixed scale and $\chi = 5$, (b) dynamical scale and $\chi = 50$.

Figure 4: Same as Fig. 3 for the rapidity distribution of the $b\bar{b}$-jet pair.

It is also convincing that looking at the distributions obtained from the LHE’s generated with the default dynamical scale and remnant separation parameter $\chi = 50$, shown in Figs. 3b, the predictions are close (within 10%) to the NLO ones of Ref. [21], obtained also with dynamical scale. This observation is valid for the comparison of all kinematical distributions from our LHE’s and the NLO predictions of Ref. [21]. Thus we conclude that these events can be used to produce distributions that agree with the NLO predictions up to corrections beyond NLO accuracy, and can be used as input in the SMC’s for generating events at stages (iii–v).
4 Conclusions

We have implemented the hadroproduction of a $t\bar{t}$-pair in association with a $b\bar{b}$-pair, $pp \to t\bar{t} b\bar{b}$, in the PowHel framework, which can be used to generate LHE files. We discussed the technical subtleties of such event simulations, in particular, issues related to the efficient generation of the events for such a complex process: (i) the advantage of separating the phase space into singular and remnant regions and the dependence of the predictions on this separation, (ii) the implementation of higher precision arithmetics in computing the loop amplitudes with sufficient precision, (iii) the advantage of performing the event generation with fake virtual part and the method of reweighting.

The event files produced by PowHel, together with further detail and results of our project, the corresponding version of the program are available at [http://grid.kfki.hu/twiki/bin/view/DbTheory](http://grid.kfki.hu/twiki/bin/view/DbTheory). We are confident that these LHE’s are suitable input into SMC’s to produce distributions at the hadron level needed for experimental analyses. Thus using those events one can make a detailed analysis of $t\bar{t}H$ signal and $t\bar{t}b\bar{b}$ background predictions at the hadron level.

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