Energy loss of cluster ions in different concentration and temperature of plasma

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Abstract. The effects of the energy loss interference of homo and hetero nuclear di-cluster ions on the plasma in both Classic and Quantum Models are used to study of response dielectric function. The present work involves the interest ranges for Inertial Conference Fusion (ICF), Z-Pinch and Plasmas of Tokomak. The approximation of the Quantum Random Phase (RPA) is used, and the individual and collective Bohm-Pines model is calculated and the contribution of each mode will be calculated. For incident cluster ions, we present measurement and comparison of the stopping power for (homo nuclear di-cluster (H-H), (He-He)) and hetero-nuclear di-cluster ions like (He-H) in different plasma concentration and temperature of (ICF, Z-Pinch and TOKOMAK). The result appear the dependence of the interference term $I(r)$ on homo and hetero-nuclear di cluster distance $r_{12}$ and the di-cluster velocity. All equations have been programmed to present the work using FORTRAN-90 and the program has been written for numerical calculations.

1. Introduction:
The interactions of the energy cluster beams with plasma targets were an active area of research in recent years, both experimentally and theoretically since they have a lower current beam density. A weaker beam concentration and a smaller region of energy deposition, in fact, stopping the cluster is more complicated than stopping the atomic ions [1, 2]. The following basic processes include cluster ion interaction with a plasma target in the first step. In collisions with plasma, the molecule loses valence electrons and the molecular structure equilibrium is consequently disrupted because of the ionization of its constituent atoms. The ions will lose their energy because their kinetic energies are transferred to the target. Simultaneously, The repulsion induced by dynamically screened coulomb interaction between the like-charged ions drives a portion of them and the molecular ion will undergo the coulomb explosion path. After the initial break-up of the molecule, further movement of the molecule is followed by electronic target excitations, which display significant interference due to near spatial correlation inside the ions, known as the vicinage effect. This form of interference induced by the structure is designed to produce increasing stopping power for each ion compared to the energy loss of the monomer ion moving at the same speed as long as the inter-ionic distances inside the cluster are smaller than the characteristic length of the electronic excitation [3]. On the theoretical side, two key approaches were suggested to explain the interactions between ions and plasma beams: the dielectric formulation [4, 5] and the collisional approach [6-8]. On the basis of the latter, a further model was described using the Transport Cross-Section (TCS) method [9, 10]. Fermi has introduced a dielectric formalism to explain the energy
loss of charged particles in the medium [4], and the other authors have been further developed [5, 11-13]. This formulation includes the screening effects, in a self-consistent way, along with the individual and collective excitations generated in the medium. [14]. The aim of this research is to examine energy loss in detail of homo and hetero-nuclear di-cluster ions in the plasmas varying temperatures and densities. The analysis is expressed in the form of classical collision less plasma dielectric formalism and involves calculation using the approximation of the Quantum Random Phase (RPA) as well. We investigate the stopping of homo-and hetero-nuclear di-cluster ions, obtaining interference effects results and comparing them with previous work [15].

2. Theoretical procedure:
For a charged point particle, Ze and speed v, the expression of average energy loss or stopping power is given by.

\[ S = Z^2 s_0 = \frac{2(Ze)^2}{m^2} \int_0^\infty \frac{dk}{k} \int_0^{k/F}\omega d\omega \text{Im}\left( \frac{-1}{\epsilon(k,\omega)} \right). \]  

(1)

where \( \epsilon(k,\omega) \) is the Medium’s Dielectric Function. Previous authors have provided numerous calculations for plasma stopping power utilizing this technique [14]. The dielectric function can be conveniently parameterized in the case of a classical plasma as [15],

\[ \epsilon(k,\omega) = 1 + \left( \frac{k\omega}{k_D} \right)^2 W(\xi) \]

(2)

where the plasma dispersion function is \( W(\xi) \) [16],

\[ \xi = \frac{(\omega+i\gamma)/\omega_p}{k/k_D} \quad ; \quad k_D = \lambda_D^{-1} \]

(3)

where \( \gamma \) represents the damping. In the limit \( \gamma \to 0 \) for collision less plasma, Eq. (3) becomes:

\[ \xi = \omega/\omega_p \quad ; \quad k_D = \frac{\omega}{k_D} \]

(4)

And therefore Eq. (2) becomes,

\[ \epsilon(k,\omega) = 1 + \left( \frac{k\omega}{k_D} \right)^2 W\left( \frac{\omega}{\omega_p} \right) \]

(5)

The function \( W(z) \) can be expressed as a variable \( z = \omega/kv \) given in Appendix (A). The main parameters of the interaction between particles and plasmas problem are the velocity \( v \), its particle charge \( Z \), the density of plasma \( n_p \), and temperature (T). The inter nuclear distance \( r_{ij} \) should be used to describe interference effects as well as the correlated ions. Other interest quantities are the frequency of plasma \( \omega_p \), \( v_T \) velocity of thermal electrons, and Debye length \( \lambda_D \) defined by atomic units [16-17]

\[ \omega_p = \sqrt{4\pi n_p} \quad , \quad v_T = \sqrt{k_B T} \quad , \quad \lambda_D = \frac{v_T}{\omega_p} = \frac{\sqrt{k_B T}}{4\pi n_p} \]

where the plasma electron density is \( n_p \) and the temperature of the electron is \( T \).

2.1 Stopping power of di-cluster ions
Let us consider di-cluster ions with atomic numbers \( Z_1 \) and \( Z_2 \), depending on the dielectric formulation, the stopping power of di-cluster ions is given as follows [18]:

\[ S_{clus} = \left( -\frac{dE}{dx} \right) (Z_1) + \left( -\frac{dE}{dx} \right) (Z_1) + I(Z_1, Z_2, r_{12}) + I(Z_2, Z_1, r_{12}) \]

(6)

where \( -\frac{dE}{dx} \) for \( Z_1 \) and \( Z_2 \) are given in Eq.(1), \( I(Z_1, Z_2, r_{12}) \) and \( I(Z_2, Z_1, r_{12}) \) are the correlated functions of two charges in correlated motion with velocity \( \vec{v} \) and inter-nuclear separation \( \vec{r}_{12} = \vec{r}_1 - \vec{r}_2 \). The stopping power in Eq. (6) given either during (i) Parallel orientation:

\[ S_{clus} = \left[ -\frac{dE}{dx} \right] = \frac{e^2}{2\pi^2 v} \int dk^2 \vec{K} \frac{1}{k^2} \text{Im}\left( \frac{-1}{\epsilon(k,\vec{K})} \right) \left[ (Z_1^2 + Z_2^2) + 2Z_1Z_2 \cos(K,\vec{r}_{12}) \right] \]

(7)

Or in (ii) perpendicular (random) orientation:
\[ S_{\text{clus}} = \frac{e^2}{2\pi^2 v} \int dk^3 \, \frac{\bar{k}_z \cdot \bar{v}}{k^2} \, \text{Im} \left( \frac{-1}{\varepsilon(k, k, \omega)} \right) \left[ (Z_1^2 + Z_2^2) + 2Z_1Z_2 \frac{\sin(k \cdot \bar{r}_{12})}{k \cdot \bar{r}_{12}} \right] \]  

(8)

Using the variable \( \omega = k \cdot v \); \( dk^3 = 4\pi \frac{k}{\bar{v}} \, dk d\omega \)

(9)

Therefore, Eqns. (7, 8) become either

\[ \frac{-dE}{dx} = \frac{e^2}{2\pi^2 v} \int_0^k dk \, \frac{\bar{k}_z \cdot \bar{v}}{k^2} \, dw \, \text{Im} \left( \frac{-1}{\varepsilon(k, k, \omega)} \right) \left[ (Z_1^2 + Z_2^2) + 2Z_1Z_2 \frac{\sin(k \cdot \bar{r}_{12})}{k \cdot \bar{r}_{12}} \right] \]

Or,

\[ \frac{-dE}{dx} = \frac{2e^2}{\pi v^2} \int_0^{\infty} dk \, \frac{\bar{k}_z \cdot \bar{v}}{k} \, dw \, \text{Im} \left( \frac{-1}{\varepsilon(k, k, \omega)} \right) \left[ (Z_1^2 + Z_2^2) + 2Z_1Z_2 \frac{\sin(k \cdot \bar{r}_{12})}{k \cdot \bar{r}_{12}} \right] \]

(10)

The simplest case to be considered here is that of a pair of ions \( Z_1 = Z_2 = 1 \) in correlated motion, structures that may be obtained by the incidence of diatomic molecules. The study of this case is basic in order to understand more complicated cluster structures. The stopping power of cluster ion is given by [19].

\[ S_{\text{clus}} = \frac{2e^2}{\pi v^2} \int_0^{\infty} dk \, \frac{\bar{k}_z \cdot \bar{v}}{k} \, d\omega \, \omega \, \text{Im} \left[ \frac{-1}{\varepsilon(k, \omega)} \right] Z_{\text{clus}}^2 \]

(12)

where either,

\[ Z_{\text{clus}}^2 = (Z_{i1}^2 + Z_{i2}^2) + 2Z_{i1}Z_{i2} \cos(\bar{k} \cdot \bar{r}) \]

(13)

\[ Z_{\text{clus}}^2 = (Z_{i1}^2 + Z_{i2}^2) + 2Z_{i1}Z_{i2} \frac{\sin(\bar{k} \cdot \bar{r})}{\bar{k} \cdot \bar{r}} \]

(14)

Therefore, the stopping power of di-cluster divided into two parts. On the other hand, self-interaction stopping power and correlation interaction stopping power are shown below:

\[ S_{\text{self}} = \frac{2e^2}{\pi v^2} \left( Z_{i1}^2 + Z_{i2}^2 \right) \int_0^{\infty} dk \, \frac{\bar{k}_z \cdot \bar{v}}{k} \, d\omega \, \omega \, \text{Im} \left[ \frac{-1}{\varepsilon(k, \omega)} \right] \]

(15)

And correlated part either,

\[ S_{\text{corr}} = \frac{2e^2}{\pi v^2} \left( Z_{i1}^2 + Z_{i2}^2 \right) \int_0^{\infty} dk \, \frac{\bar{k}_z \cdot \bar{v}}{k} \, dw \, \text{Im} \left[ \frac{-1}{\varepsilon(k, \omega)} \right] \cos(\bar{k} \cdot \bar{r}) \]

Or,

\[ S_{\text{corr}} = \frac{2e^2}{\pi v^2} \left( Z_{i1}^2 + Z_{i2}^2 \right) \int_0^{\infty} dk \, \frac{\bar{k}_z \cdot \bar{v}}{k} \, dw \, \text{Im} \left[ \frac{-1}{\varepsilon(k, \omega)} \right] \sin(\bar{k} \cdot \bar{r}) \]

(17)

Then the stopping power of di-cluster ions is:

\[ S_{\text{clus}} = S_{\text{self}} + S_{\text{corr}} \]

(18)

There are some significant differences between correlated motion (di-cluster) and uncorrelated particles. In the simultaneous interaction between the two particles and the medium, these distinctions arise from interference effects. Let us now consider the effects of interference in the energy loss of the di-cluster produced by two particles in the motion at a velocity \( \bar{v} \) through the plasma and at inter-nuclear distance \( \bar{r} \). The di-cluster stopping power can be expressed as follows: [20, 22]:

\[ S = (Z_{i1}^2 + Z_{i2}^2)S_0 + 2Z_{i1}Z_{i2}I(k, r) \]

(19)

where \( I(k, r) \) is an interference function given by:

\[ I(k, r) = \frac{2e^2}{\pi v^2} \int_0^{\infty} \frac{dk}{k} \int_0^{k^2} dw \, \text{Im} \left[ \frac{-1}{\varepsilon(k, \omega)} \right] \cos(\bar{k} \cdot \bar{r}) \]

(20)

Or

\[ I(k, r) = \frac{2e^2}{\pi v^2} \int_0^{\infty} \frac{dk}{k} \int_0^{k^2} dw \, \text{Im} \left[ \frac{-1}{\varepsilon(k, \omega)} \right] \sin(\bar{k} \cdot \bar{r}) \]

(21)
$S_0$ is the stopping power of single ions defined in Eq. (1).

2.2 Stopping power of homo di-cluster ions:

Let's assume that the two particles have identical charges, $Z_1 = Z_2 = Z$, so that Eq. (19) becomes, 

$$S_{dcl} = 2Z^2 [S_0 + I(r)]$$

(22)

The predicted limitations of function $I(r)$ and the stopping power of the di-cluster are as follows:

i. for $r \to 0$: $I(r) \to S_0$ and $S_{dcl} \to (2Z)^2 S_0$ (one charge particle of 2Z)

ii. for $r \to \infty$: $I(r) \to 0$ and $S_{dcl} \to 2(Z^2 S_0)$ (two independent particles of charge).

Because of an ordinary experiment, the incident di-cluster is randomly oriented in this case. The average interference term can be taken with respect to the angle between $\vec{r}$ and $\vec{p}$, for H-H di-cluster ions, $Z=1$,

$$I(r) = \left( \frac{2}{\pi \nu^2} \right) \int_0^\infty dk \frac{\sin(kr)}{kr} f_{kv} w dz \Im \left( \frac{-1}{e^{(k,v)}} \right)$$

(23)

Using Eq (5) for the classical dielectric function collision less plasma, the integral of the interference term $I(r)$ can be written as follows:

$$I(r) = \frac{2\nu^2}{\pi} \int_0^\infty dk \frac{\sin(kr)}{kr} f_{kv} w dz \left( \frac{Y(z)}{(k_p^2 + X(z))^2 + [Y(z)]^2} \right)$$

(24)

The integration with the $k$ variability can be carried out analytically after certain algebra; the function $I(r)$ can be expressed as a single integral,

$$I(r) = \frac{2}{\pi} \left( \frac{\omega}{\nu} \right)^2 \left( \frac{1}{r} \right) \int_0^{\nu/\omega} z dz \Im \left[ F(z, k_{max}, r) \right]$$

(25)

where

$$F(z, k_{max}, r) = q \left\{ \sin qr [Ci((k_{max} - q)r) + Ci((k_{max} + q)r)] - Ci(-qr) - Ci(qr) \right\} +$$

$$\cos qr \left\{ \sin((k_{max} - q)r) - \sin((k_{max} + qr)r) - 2 \sin -qr \right\}$$

(26)

With

$$q = q(z) = K_p \sqrt{Z(z) + iY(z)}$$

(27)

$Si$ and $Ci$ are the integrals of sine and cosine. Analytical results for the interference term $I(r)$ in Eq. (25) can now be obtained for low and high velocities. [15]:

2.1 High-velocity approximations:

In the case of ion with velocity $v \gg v_T$, we can distinguish the contribution from the individual and collective types, also the integral $I(r)$ can be calculated by.

$$I(r) = \frac{2}{\pi} \left( \frac{\nu}{\omega} \right)^2 \left[ I_1 \left( \frac{2\nu}{\omega}, \frac{r}{K_p} \right) \Theta(v - v_T) + F_1 \left( \frac{v}{v_T}, \frac{r}{K_p}, k_{max} \right) \Theta(k_{max} - k_D) \right]$$

(28)

So,

$$I(r) = I^{ind}(r) + I^{coll}(r)$$

(29)

The above equations have been calculated numerically by programed the equations using Fortran-90 and writing a program $Diclustr-Classic.for$, and a copy of the program is available with authors.
And in general one can say that:

\[ H(u) = \left( \frac{\sin u}{u} \right) \mathcal{C}(u) \]  (33)

For \( \nu \gg \nu_T \) (\( x \gg 1 \)), \( F_1(x) \equiv 1 \) and get

\[ I(r) \equiv k_{\text{min}}^2 I_1(k_{\text{min}} r, k_{\text{max}} r) \]  (34)

With \( k_{\text{min}} = 1/b_{\text{max}} = \omega_p / \nu \). Functional dependency of the interference function at high speed. For large \( r (k_{\text{max}} r \gg 1) \) in Eq. (34) the term with \( k_{\text{max}} \) can be neglected. With regard to the lowest electron-ion approach distance in the nearest collisions, the inter-nuclear length is sufficiently large, so, the loss of energy is divided into individually and collectively excitation such that the tow ions behave as an individual scattering collision within the range of high velocities > \( \nu_T \). Usually, the participation of collective modes is a small portion of stopping term \( S_0 \). Additionally, the contribution of collective modes becomes very significant in the integration of the interference expression \( I(r) \) [23].

2.3 Low-velocity approximations:
Using the estimation \( z \ll 1 \), where, \( z = \omega / k \nu_T \), the function of energy loss \( Im[-1 / \varepsilon (k, w)] \) becomes helpful so,

\[ Im \left( \frac{-1}{\varepsilon(k,w)} \right) = \frac{\pi}{2} \left( \frac{k}{k_D} \right)^2 \frac{\exp \left( \frac{-z^2}{2} \right)}{\left[ 1 + (k/k_D)^2 \right]^{3/2}} \]  (35)

Integrating Eq. (24) one can get the di-cluster at low velocity approximation,

\[ I(r) = \left( \frac{2}{\pi} \right)^{1/2} \frac{1}{3 \lambda_D^2} I_2(r/\lambda_D, k_{\text{max}}/k_D) \left( \frac{\nu}{\nu_T} \right) F_2(\nu/\nu_T) \]  (36)

where \( F_2(x) \) is given by:

\[ F_2(x) = \frac{3}{x^3} \int_0^x z^2 \exp \left( \frac{-z^2}{2} \right) dz \]  (37)

And

\[ I_2(x, y) = \int_0^y dz \, z^3 \left( \frac{\sin(zx)}{zx} \right) (1 + z^2)^{-2} \]  (38)

3. Results and discussion
The calculations were made using the exact integral terms given by the dielectric forms. The classical dielectric function and quantum RPA procedure have been applied. Here, we examine the dependence of the interference term \( I(r) \) on the while homo nuclear di cluster distance \( r_{12} \) and velocity \( \nu \). For fast velocity, the behaviour of the interference expression \( I(r) \) could be seen in figure (3.5a). For corresponding (ICF) conditions of high velocities and big \( r \) quantities (with respect to \( \lambda_D \)) \( I(r) \) is rapidly tending to zero and it show low vibrational behaviour corresponding to distances. Though, \( I(r) \) tends rapidly to zero for low velocities and large \( r \)-quantities (with respect to \( \lambda_D \)). It displays elevated vibrations values. For very long distances, there is no oscillatory activity.

Here, with mutual distances greater than \( \lambda_D \), ions are scattered throughout a large area. Each ion is capable of generate a long range wake field that contributes to the electrical field at the location of all other ions. Constructive or destructive interferences may occur with respect to the characteristics phases of the wake field and result in increasing or decreasing stopping relative to uncorrelated stopping. Figure (1) shows the variation of correlation function \( \tilde{I}(r) \) and correlated stopping power of cluster ions \( S_{\text{clus}} \) like (a) H-H ICF, (b) He-He ICF and (c) He-H ICF di-cluster ions with atomic di-cluster ions distance \( r_{12} \). The similar patterns of the results in the all cases (a), (b) and (c) suggest the presence of a basic scale rule. The point to be examined below the figures, utilising either a classical value or a quantum value of \( k_{\text{max}} \), show a very strong agreement between these results. To discuss the effects of
the atomic distance $r_{12}$, (1) $r_{12} \leq 3$ the correlation function $I(r) \vert_{HH} < I(r) \vert_{HeH} < I(r) \vert_{HeHe}$ while, the stopping power of di-cluster ions $S_{clus}$: $S_{clus} (r) \vert_{HH} > S_{clus} (r) \vert_{HeH} > S_{clus} (r) \vert_{HeHe}$ as shown in figure (1). (2) at $r_{12} \rightarrow \infty$, i.e. $r_{12} \geq 20$, there is no contribution to the correlation function $I(r)$ and correlated stopping power $S_{clus}$. (3) Between these two regions, cosine or sine function appear in Eqns. (23 and 24) are strongly effect.

**Figure 1.** The correlated function term $I(r)$ and correlated stopping power $S_{clus}(r)$ for homo di-cluster ions (a) HH and (b) He-He and (c) hetero di-cluster ions He-H in ICF plasma as a function of inter atomic distances $r_{12}$ were calculated using the classical dielectric function, and the quantum values of $k_{max}$. 
**Stopping of Homo and Hetero di-cluster ions:**

Let us consider a di-cluster comprising two different forms of heavy atomic ions. For fast heavy ions, a number of electrons are typically carried with an increasing depth of penetration. The charge of the ion fluctuates around a certain value of the balance, which is generally calculated through the form of term:

$$q_i e \approx Z_i e \left[1 - \exp\left(-\nu / \nu_{TF}\right)\right]$$  \hspace{1cm} (39)

where, $\nu_{TF} = Z_i^{2/3} \nu_0$ is the velocity of Thomas-Fermi

The ionic effective charge is usually expressed via the ion charge $Z_i$ and the fractional effective charge of ion $\gamma_i$,

$$Z_{\text{eff}} = \gamma_i Z_i$$  \hspace{1cm} (40)

To connect the effective charge with the heavy $He$ – $H$ di-cluster ions let the effective charge of two ions are $Z^*_{He}$, where,

$$Z^*_{He} = Z_{He} \left(1 - e^{-\nu_i / Z_{He}^{2/3}}\right)$$  \hspace{1cm} (41)

Then, the stopping power of the di-cluster ions of charges $Z^*_{He}$ with atomic number $Z_{He} = 2$ and $Z = 1$ is given in Eqns. (13, 14 and 6).

Three kinds of di-cluster ions have been used in present work, Hydrogen-Hydrogen (H-H), Helium-Helium (He-He) and Helium-Hydrogen (He-H). Figure (2) shows the stopping power for Homo di-cluster ions like (a) H-H, (b) He-He, and (c) Hetero di-cluster ions, and He-H as a function of the velocity in plasma system ICF. Note that the difference in scale percentage between the three systems of di-clusters ions is why the interaction between incident homo nuclear di-cluster and electron target in ICF more Hetero di-cluster ions because the difference in atomic number of incident ions with the same density and thermal velocity of plasma matter.
Figure 2. Calculations the stopping power of homo nuclear di cluster for (a) H-H, (b) He-He and (c) Hetero di-cluster ions HeH) in plasma system ICF. As a function of ion velocities.

Figure (3) shows that the computations have been made using the exact integral terms given by the dielectric formula, both the classical dielectric function and the quantum (RPA) procedure have been used. We analyse here the dependent of the interference definition $I(r)$ on the hetero nuclear di cluster distance $r_{12}$ and on the di cluster velocity $\nu$. The behaviour of the interference concept $I(r)$ is displayed in figure (3a) for high velocities for the conditions corresponding to the (ICF) at large r values (with respect to $\lambda_D$). $I(r)$ tends to zero rapidly and exhibits low vibrational behaviour corresponding to distances, while for low velocities the behaviour of the interference definition $I(r)$ is displayed in figure (3b) for the conditions corresponding to the (Z-Pinch) tends to high oscillatory behaviour corresponding to distances.

Figure (4) shows the movement of hetero nuclear di-cluster (He-H) ion in different plasma systems (ICF, Z-Pinch) and in different distance between hetero nuclear di-cluster for ICF system the folds increase with velocity because of increasing the number of interaction between di-cluster and electron plasma. The effect of intensity and the stopping appear from the number of folds were increased with increasing the density, while for Z-Pinch the folds decrease with velocity because the density is low compare to ICF.
Figure 3. Show movement of di cluster (H-H) in the cases of (a) ICF, (b) Z-pinch and (c) Tokomak system characterized by the distance (r) between di cluster (H-H).
Figure 4. Shows the effect of the movement of hetero-nuclear di-cluster ions (He-H) inside (ICF and Z-pinch) systems plasma in different distance.

4. Conclusion
For different plasma conditions, the loss of energy of di-cluster ions in plasmas was analyzed numerically and analytically, detailed views the effect of the most important factors. The application of a classical dielectric function enables the stopping power and interference terms to be conveniently integrated.  
In the case of low velocity, the consequences of interference are only significant if the distances between the two ions are identical or less than the Debye length $\lambda_D$. In the case of molecular ions inserted into plasmas, this condition is very well satisfied. In the case of high speeds, the interference function generally scale with the parameter $r \omega_p / v$. As the value of $b_{max}$ increases with velocity, the effects of interference will become more relevant and large collective effects can be expected.

Appendix (A)

Using the parameter $\omega_p = \sqrt{4 \pi n_p}$, $v_T = \sqrt{k_B T}$, $\lambda_D = \sqrt{k_B T / 4 \pi n_p}$, 

And according to Eq.(2.15), with $\xi = \left( (\omega + i \gamma) / \omega_p \right)$, one can finds

$\epsilon(k, \omega) = 1 + \left( \frac{k_D}{k} \right)^2 W(\xi)$  \hspace{1cm} (A1)

where the plasma dispersion function is $W(\xi)$ [22]

For $\gamma \rightarrow 0$ (collisional plasma) the $W(\xi)$ function can be expressed in term of the variable

$W(z) = X(\xi) + i Y(\xi)$  \hspace{1cm} (A2)

With

$X(\xi) = 1 - \xi \exp \left( -\frac{\xi^2}{2} \right) \int_0^\infty \xi \exp \left( \frac{\xi^2}{2} \right) dx$  \hspace{1cm} (A3)

And

$Y(\xi) = \left( \frac{\pi}{2} \right)^{1/2} \xi \exp \left( -\frac{\xi^2}{2} \right)$  \hspace{1cm} (A4)

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