STABILITY OF SATELLITES IN CLOSELY PACKED PLANETARY SYSTEMS

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1. INTRODUCTION

Many multi-planet systems discovered by the Kepler mission are systems with tightly packed inner planets (STIPs). We find that the majority of closely spaced stable two-planet systems can stably support satellites across a range of parameter-space which is only slightly decreased compared to that seen for the single-planet case. In particular, circular prograde satellites remain stable out to \( \sim 0.4 R_H \) (where \( R_H \) is the Hill radius) as opposed to \( 0.5 R_H \) in the single-planet case. A similarly small restriction in the stable parameter-space for retrograde satellites is observed, where planetary close approaches in the range \( 2.5 \sim 4.5 \) mutual Hill radii destabilize most satellites orbits only if \( a \sim 0.65 R_H \). In very close planetary pairs (e.g., the 12:11 resonance) the addition of a satellite frequently destabilizes the entire system, causing extreme close approaches and the loss of satellites over a range of circumplanetary semi-major axes. The majority of systems investigated stably harbored satellites over a wide parameter-space, suggesting that STIPs can generally offer a dynamically stable home for satellites, albeit with a slightly smaller stable parameter-space than the single-planet case. As we demonstrate that multi-planet systems are not a priori poor candidates for hosting satellites, future measurements of satellite occurrence rates in multi-planet systems versus single-planet systems could be used to constrain either satellite formation or past periods of strong dynamical interaction between planets.

**Key words:** celestial mechanics – methods: numerical – planetary systems – planets and satellites: dynamical evolution and stability

**Online-only material:** color figures

To understand the stability of satellites around planets in STIPs, we select two-planet systems which are stable, while being as closely spaced as it is possible for two planets to be (without being co-orbital). We add satellites to these systems and examine whether the stability boundary at \( a_{max} \sim 0.5 R_H \) is changed by the close planetary approaches.

We neglect planetary obliquity and tides, as (1) their effects are strongest at small satellite semi-major axes, \( a_{Sat} \) (Murray & Dermott 1999), while we anticipate corrections due to planetary perturbations will occur at large \( a_{Sat} \), and (2) such effects occur in the single-planet case as well as the two-planet case, and we wish to focus on the differences caused by the additional planet. For small planetary semi-major axes and large satellite:planet mass ratios, tides can cause otherwise stable satellites to be lost from their parent planets within the age of the stellar system via inward and/or outward migration of satellites (Barnes & O’Brien 2002). Any reduction in \( a_{max} \) seen in our simulations will accelerate this tidal-driven loss of satellites.

2. METHODOLOGY

Our numerical integrations (of length \( t_{final} \)) use the same methodology as Deck et al. (2012, 2013), i.e., using a symplectic integrator (Wisdom & Holman 1991) with corrector (Wisdom et al. 1996; Wisdom 2006) to evolve suites of initial conditions, while concurrently integrating the tangent equations (Lichtenberg & Lieberman 1992).

At \( t_{final} \), the length of the tangent vector, \( d \), is reported. In chaotic orbits, \( \log d \) grows exponentially in time, while for regular orbits, \( \log d \) grows polynomially. The Lyapunov time, \( T_{Ly} = t_{final} / \log d \), hence regular orbits have longer Lyapunov times.
For orbits that do not suffer close encounters, our simulations conserve energy to $dE/E \sim 10^{-9}$ (note that $m_{\text{Sat}}/m_\star = 0.3 \times 10^{-7}$). Energy conservation when close-encounters occur is significantly worse, but we do not follow the outcomes of such systems, instead we simply flag them as chaotic.

Our timesteps were chosen to be $<0.05 \times$ the shortest physical timescale in the problem (Rauch & Holman 1999): a timestep of 0.01 days for planet-plus-satellite integrations having satellite periods down to $\sim 1$ day.

We measure distances in terms of (1) the parent planet’s Hill radius, $R_H,\text{parent} = a,\text{Parent} (m,\text{Parent}/3)^{1/3}$ and (2) the mutual Hill radius of parent planet and perturbing planet, $R_H,\text{Mutual} = 0.5(a,\text{Parent} + a,\text{Perturber}) \times (m,\text{Parent} + m,\text{Perturber}/3)^{1/3}$, where $a$ and $m$ are the semi-major axis and the mass ratio with the central star.

2.1. Selecting Closely Spaced Two-planet Systems

Two-planet systems which are both stable and closely spaced experience repeated relatively close approaches between the two planets, which may destabilize some satellite orbits, while the planet themselves remain stable.

The results of Deck et al. (2013) provide a map of the closest (minimum $a_2/a_1$) stable orbits for systems with planets of various masses. Figure 9 in Deck et al. (2013) is reproduced in our Figure 1, along with a version of their Figure 11 in which the mean anomaly of the planet is held fixed.

Figure 1 shows the chaotic structure of phase space as a function of mean anomaly, $M_1$, and period ratio (left-hand side), and as a function of the period ratio of the planets and the eccentricity (right-hand side). In the simulations in Figure 1, the following were held constant: for the inner planet, $(i_1, \omega_1, \Omega_1) = (0, 0, 0)$. For the outer planet $(a_2, e_2, i_2, \omega_2, \Omega_2, M_2) = (1 \text{ AU}, 0, 0, 0, 0, 0)$, where $a, e, i, \omega, \Omega$ and $M$ are the semi-major axis, eccentricity, inclination, argument of pericenter, ascending node, and mean anomaly respectively.

For the plots on the left, while $M_1$ and $a_2$ are varied, $e_1$ is fixed at $e_1 = 0.01$, while for the plots on the right, $M_1 = 0^\circ$ while $e_1$ and $\Omega_1$ are varied.

The panels show the results for different masses ($m_1 = m_2 = m_1$), with: top: $m_1 = 10^{-3}$; middle: $m_1 = 10^{-4}$; bottom: $m_1 = 10^{-5}$; darker colors indicate shorter Lyapunov times. The yellow regions indicate very long Lyapunov times, i.e., stable planets. Labeled boxes indicate the regions we use to investigate the stability of planetary satellites. The boxes above are approximate: the precise definitions for the sets are provided in the text. Because the plots are slices at constant $e_1$ (left) and constant $M_1$ (right), regions which appear unstable at one eccentricity or mean anomaly can be stable at a different eccentricity or mean anomaly: e.g., set $F$ at the top-left appears unstable, but this is because the left-hand plots have $e_1 = 0.01$, where-as set $F$ has $e_1 \sim 0.1$, placing it in a stable region of parameter-space (top-right).

(A color version of this figure is available in the online journal.)

![Figure 1. Lyapunov time plots for two-planet systems following Deck et al. (2013). The chaotic structure of phase space as a function of: left: the mean anomaly, $M_1$, and period ratio (at fixed $e_1 = 0.01$); right: the period ratio and the eccentricity (at fixed $M_1 = 0^\circ$). The top-to-bottom panels show the results for different values of the planetary masses: top: $m_1 = m_2 = 10^{-3}$; middle: $m_1 = m_2 = 10^{-4}$; bottom: $m_1 = m_2 = 10^{-5}$; darker colors indicate shorter Lyapunov times. Yellow regions indicate very long Lyapunov times, i.e., stable planets. Labeled boxes indicate the regions we use to investigation the stability of planetary satellites. The boxes above are approximate: the precise definitions for the sets are provided in the text. Because the plots are slices at constant $e_1$ (left) and constant $M_1$ (right), regions which appear unstable at one eccentricity or mean anomaly can be stable at a different eccentricity or mean anomaly: e.g., set $F$ at the top-left appears unstable, but this is because the left-hand plots have $e_1 = 0.01$, where-as set $F$ has $e_1 \sim 0.1$, placing it in a stable region of parameter-space (top-right).](image)
We take sets $(A, G, \text{and } U)$ and add a $m_L$-mass satellite to the circular outer planet at 1 AU, with $a_{Sat}$ and $e_{Sat}$ drawn randomly from uniform distributions $0.01 \lesssim R_H < a_{Sat} < 0.6 R_{H,Parent}$ and $0 < e_{Sat} < 1.0$. We integrate $N = 10^4$ versions of each set for $3 \times 10^3$ planetary orbits.

We provide Lyapunov-exponent maps for all of the sets in Figure 2. Since the planetary orbits are regular a small Lyapunov time indicates a chaotic satellite orbit. Note that, in Figure 2,
set $U$ has chaotic planets, so the Lyapunov indicator provides a confusing mixture of information regarding chaos in both the planetary and satellite orbits. Hence for set $U$ we simply plot (in “olive”) the initial orbital elements for the satellites that survive at the end of the simulation.

In sets $A$ and $B$, unstable orbits occur across much of parameter-space due to destabilizing close planet–planet approaches (see Section 3.4) as the entire planetary system destabilizes.

Importantly, in sets $C - G$, the broad range of satellite orbits in Figure 2 remain stable. Only satellites close to the standard stability boundary are destabilized. While different sets are each affected differently, we observe that the stability boundary changes from the single planet result ($a_{\text{max}}/R_{\text{H,Parent}} = 0.49 \times (1.0 - 0.27e_{\text{Sat}})$) to something approximating

$$
a_{\text{max}}/R_{\text{H,Parent}} \approx 0.4 \times \left(1 - \frac{e_{\text{Sat}}}{4}\right). \tag{1}
$$

Equation (1) is intended to be extremely approximate and is not a “fit” to the data: both the slope and intercept of the equation for $a_{\text{max}}$ as a function of $e_{\text{sat}}$ varies across the sets. A general version of Equation (1) which explicitly accounts for the perturbing planet’s mass and orbital elements requires a much broader series of investigations than we can undertake in this Letter.

Close to $a_{\text{max}}$, many satellites remain bound at $t = 10^6$ days, but have short Lyapunov times indicative of chaos (e.g., region $X$ in Figure 2) (this is similar to the behavior of many solar system irregular satellites (Nesvorny et al. 2003; Frouard et al. 2010, 2011, and references therein).

In Figure 2, there exists small stable “islands” at $a_{\text{Sat}} \approx 0.43$ (see region $Y$), hinting at possible resonances between satellite and planetary orbital periods.

An analytic approximation (and extension to non-coplanar systems) to the results of Domingos et al. (2006, D06) in Donnison (2010, D10), whose derivation required that both the mass ratio between star and planet–satellite, and semi-major axis ratio between planetary orbit and satellite orbit be large. The results of D10 agree with the prograde results of D06, but fail significantly for the retrograde case. While the mass and semi-major axes of our single-planet simulations satisfy the requirements of D10, the planet–planet masses and separations in the two-planet simulations do not, hence D10’s results do not apply and cannot guide our understanding of the destabilization.

### 3.3. Retrograde, two-planet, one-satellite

We plot in Figure 3 the stability of retrograde orbits. The satellites in these retrograde simulations are generated as for the prograde simulations with the exception that $a_{\text{Sat}}$ is now drawn randomly from a uniform distribution $0 < a_{\text{Sat}} < 0.99 R_{\text{H,Parent}}$.

The one-planet plus one-(retrograde)-moon case is plotted at the top-right, with the stability boundary $a_{\text{max}}/R_{\text{H,Parent}} = 0.9309 \times (1.0 - 1.0764 e_{\text{planet}} - 0.9812 e_{\text{Sat}})$, of D06 over-plotted in red. This is in reasonable agreement with our results, although in the high-eccentricity regime that D06 did not explore, their fitted boundary slightly underestimates the stability region.

The systems ($C$–$G$ and $U$), have slightly reduced satellite stability under the influence of a perturbing planet, compared to the single-planet case (top-right). Systems which experience
close approaches $>4R_{H,\text{Mutual}}$ ($D$ and $E$—for details of close approaches see Figure 5) are able to maintain retrograde moons out to beyond $0.9R_{H,\text{Parent}}$, albeit with some fraction of the cases becoming unbound. Systems which experience close approaches $<4R_{H,\text{Mutual}}$ ($C$, $F$, $G$ and $U$) are only able to stably support moons out to $\sim0.6R_{H,\text{Parent}}$. All of these stated boundary values are maximal, i.e., they refer to the $e_{\text{Sat}} = 0$ case. Higher eccentricities significantly reduce $a_{\text{max}}$.

### 3.4. Causes of Instability

We plot in Figure 4 comparisons of (1) The Lyapunov-exponent maps, and (2) Maps of the close-approach distance (between satellite and perturber) in units of $R_{H,\text{Mutual}}$.

We observe (1) a close correspondence between the Lyapunov-exponent maps and the close-approach maps, demonstrating that the close approaches between satellite and perturbing planet drive the chaos at the outer edges of the plots. (2) The “islands of stability” noted in Figure 2 are associated with more distant approaches, possibly coincident with commensurabilities between satellite and perturber’s orbital period. (3) The close-approach distance maps correspond to the Lyapunov-exponent maps, and the close-approach maps, demonstrating that the close approaches between satellite and perturbing planet drive the chaos.

In Figure 5 we plot the minimum separation observed between the two planets without a satellite in the simulation (top row), the two planets with a satellite present (second row), and the satellite and the perturbing planet (third row).

We see that: (1) in sets $A$ and $B$ the addition of the satellite has caused close approaches between the planets (second row) that were absent during the no-satellite integrations (top row), and (2) in sets ($C$ — $G$ and $U$), the planet—planet separations remain (relatively) large, while satellite—planet separations can become small.

We plot (fourth row of Figure 5) the Lyapunov exponent as a function of the minimum satellite—planet separation (in Hill radii), demonstrating that a critical close-approach distance is apparent in all of the stable systems ($C$ — $G$), but that the value of this critical close-approach distance differs across the different sets in a non-trivial fashion.

At the bottom of Figure 5 we plot the Lyapunov exponent as a function of the ratio $(r_{\text{Satellite}}−r_{\text{Parent}})/(r_{\text{Satellite}}−r_{\text{Perturber}})$ at the time of minimum recorded separation between satellite and perturber. When the perturbing planet comes closer to the satellite than $\sim5−10\times$ the satellite’s distance from its parent planet, the satellite becomes unstable. Closer approaches than this lead to satellites being ejected on to planet-crossing orbits. They subsequently experience strong scattering, leading to extremely small minimum recorded close approaches with $r_{\text{Satellite}}−r_{\text{Perturber}} \lesssim 1R_{H,\text{Mutual}}$.

We speculate that the velocity perturbations induced by the perturbing planet are sufficient to excite eccentricities that bring the satellite’s apocenter beyond the stellar-induced stability boundary, leading to the destabilization of the planet.

While the ratio of separations is important in determining the stability boundary, the transition from stability to instability happens gradually and at different values in the different systems.

This study compliments that of Gong et al. (2013) whose work concerned the satellite stability in systems of three giant planets undergoing very close encounters leading to gross instability (planetary ejections and mergers), while our study concerns the satellite stability in primarily stable two planet systems, undergoing frequent but weak perturbations.

A more comprehensive investigation is required to understand the dependence of $a_{\text{max}}$ on the close-approach distance of the perturbing planet, but our results indicate that repeated weaker interactions do not destabilize moon orbits on short timescales. Similarly, we defer for future work the study of the effects of non-coplanarity and possible resonances between the satellite and planetary periods.
4. DISCUSSION AND CONCLUSION

We numerically integrated four-body (star, planet, planet, satellite) systems to investigate the stability of satellites in planetary systems with tightly packed inner planets, finding that:

1. In the most closely spaced systems investigated in sets A and B, the addition of a lunar-mass satellite is sufficient to catastrophically destabilize many of the planetary systems.
2. In the majority of systems investigated (sets C − G), with period ratios \( \geq 1.1 \), close approaches between planets occur with minimum separations \( \geq 2.5 \) mutual Hill radii. Such close encounters only slightly reduce the region of parameter-space over which prograde satellites are stable, reducing the zero-eccentricity outer stability boundary from \( a_{\text{max}} \sim 0.5 R_H \) to \( a_{\text{max}} \sim 0.4 R_H \).
3. Retrograde satellites are similarly mildly affected, with planetary close approaches \( < 4 R_{H,Mutual} \) causing the stable parameter-space for satellites to be restricted to \( \lesssim 0.6 R_{H,Parent} \), while more distant planetary approaches leave the satellite stability zone broadly the same as in the single-planet case.
4. There is a critical close approach for destabilization, \( (r_{Satellite} - r_{Perturber})/(r_{Satellite} - r_{Parent}) \sim 5-10 \).

While further investigations may be valuable, the approximately coplanarity (Fabrycky et al. 2012) and equal size planets (Ciardi et al. 2013) of the majority of STIPs suggests that the investigation presented here provides strong evidence that STIPs can generally offer a dynamically stable home for satellites and hence provide suitable targets for exo-moon detection campaigns. Moreover, the STIPs in the Kepler catalog have wider separations (in Hill radii) than the systems examined here, thus any disturbance to their satellite stability zone will be further reduced.

We have demonstrated that multi-planet systems are not a priori poor candidates for hosting satellites. Hence we suggest that future measurement of satellite occurrence rates in multi-planet systems versus single-planet systems could be used to constraint satellite formation and/or previous periods of strong dynamical interaction between planets.

This material is based upon work supported by the NASA Origins of Solar Systems Program grant NNX13A124G, and by the BSF grant number 2012384. K.M.D. acknowledges support from an NSFGRF. We thank the anonymous referee for their valuable input.

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