Impact of a deep foundation on enclosing wall structure of excavation

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Abstract. The article provides a solution to the problem of the stress-strain statement of a soil mass with excavation, under the impact of a deep foundation. The solution to the problem is obtained by integrating Melan's solution. The final expressions for the stress-strain statement components are presented. Isolines of stresses in the soil massif and reactive forces of enclosing wall structure are also given.

Introduction

Probably, for constructing high-rise buildings with a developed underground part in deep pits it is important to define the stress-strain state of soils behind the pit fencing, taking into account the interaction with the surrounding buildings and the conditions of contact with the enclosing structure. Further development of theoretical and soil mechanics, solutions of engineering problems were appeared based on the theory of elasticity. Many of them remain relevant today, since on their basis it is possible to develop new analytical solutions for nonlinear soil mechanics, which make it possible to take into account a wide range of factors that form the stress-strain state of soil massifs. In addition, with an accurate analytical solution, it is possible to verify the numerical modeling of an engineering problem, which are widely used in the practice of designing the bases and foundations of buildings and structures.

For excavations of soils with enclosing structures, a complex non-uniform stress-strain state is created in soil massif. In the case of the impact of a deep foundation, additional normal and shear stresses are created, which have additional effects on the enclosing structures. This article provides an example of an analytical solution to the problem of the stress-strain state of a foundation near an excavation site with the influence of a uniformly distributed load applied at depth.

One of the first solutions of definition of the stress state with a linear load on the surface is the Flaman solution (1892)\textsuperscript{1} Also known are the Mitchell problem (1902) on the action of a line uniformly distributed load of intensity $q$, the Boussinesq problem (1885) for a point load $P$ acting on a linearly deformed half-space, the Kelvin problem on a point load applied to an infinitely long body, and others. It should be noted that many scientists were engaged in the issue of stress distribution in a soil layer resting on a rigid foundation: Melan (1919)\textsuperscript{2}, Marger (1931), Burmister (1956), Shekhter O Ya (1937), Gersevanov N M [3], Gorbunov-Posadov M I (1946-1953)\textsuperscript{4-5}, Egorov K.E. (1961)\textsuperscript{6}, Tsytovich N A (1943)\textsuperscript{7}, Ter-Martirosyan Z G [8-9], Malyshev M V (1980)\textsuperscript{10}, Zaretsky Yu K (1989)\textsuperscript{11}, and a number of foreign scientists [12-15].

On the basis of the solution of the Melan problem, the formulation and solution of problem for definition of the stress-strain state of a soil massif with distributed load $q = \text{const}$ along a line of width $b = 2a + c$ at depth, the lower boundary of which lies on rigid foundation base, is presented.
Materials and methods

For definition of the stress-strain state of soil massif with the influence of a distributed load \( q = \text{const} \) along a line of width \( b = 2a \) at depth \( d \), we used the solution of E. Melan [2], given in the work of V.A. Florin [16], about the force applied inside a linearly deformable half-plane (figure 1) at the point \( z = d \) with conditions of plane deformation.

For presenting a uniformly distributed load on distance \( c \) from the vertical excavation site, we provide the scheme as a result of the sum of line-loads of intensity \( q \) over a line \( b=2a \) and intensity \(-q\) over a line \( 2c \) as shown on figures 2-4.

**Figure 1.** Scheme of determine stress-strain statement of half-plane with line-load \( q=\text{const} \) over a line \( b=2a \) at depth \( d \) (Melan solution).

**Figure 2.** Design scheme of determine stress-strain statement for quarter-plane with line-load \( q=\text{const} \) over a line \( b=2a \) at depth \( d \). The enclosing wall has no horizontal displacements.

**Figure 3.** Design scheme of determine stress-strain statement for half-plane with symmetrical line-loads of intensity \( q=\text{const} \) over a line \( b=2a \) on distance \( c \) at depth \( d \) relating to line 0-0. The left part of the scheme is equivalent to scheme on figure 2.
The stress distribution in homogeneous soil massif with a distributed line-load over a line of width $b = 2a$ at a depth $d$ can be obtained by integrating the equations presented by E. Melan in the range from $-a$ to $a$ in accordance with the scheme, replacing $x$ on $(x-\xi)$ and $q$ replacing on $q d\xi$, as shown in Figure 1. The results of integration are shown below:

$$
\sigma_x = \int_{-a}^{a} q \frac{m+1}{2m} \left[ \frac{z-d}{r_1^2} + \frac{(z+d)(z+d)^2 + 2dz}{r_2^4} - \frac{8dz(z+d)(x+\xi)}{r_2^6} \right] \ d(x-\xi) 
$$

$$
\sigma_z = \int_{-a}^{a} q \frac{m-1}{2m} \left[ \frac{z-d}{r_3^2} + \frac{3z+d}{r_2^4} - \frac{4z(x-\xi)^2}{r_3^6} \right] \ d(x-\xi) 
$$

$$
\tau = \int_{-a}^{a} q(x-\xi) \left[ \frac{m+1}{2m} \left( \frac{z-d}{r_3^2} + \frac{z^2 - 2dz - d^2}{r_2^4} + \frac{8dz(z+d)^2}{r_3^6} \right) \right] \ d(x-\xi) 
$$

The variables $r1$ and $r2$ can be described by the formulas:

$$
r_1^2 = (x-\xi)^2 + (z-d)^2 
$$

$$
r_2^2 = (x-\xi)^2 + (z+d)^2 
$$
Results
Analytical calculation using the MathCAD software made it possible to determine the stress components. Line-load is $q = 250 \, \text{kPa}$, depth of foundation is $d = 20 \, \text{m}$, width of foundation $b = 2a + c = 8 \, \text{m}$. The isolines of the stress components are shown on figures 5 to 7.

![Diagram of isobars](image-url)  
*Figure 5. Isolines of horizontal stresses with a deep foundation at depth $d$.**
Figure 6. Isolines of vertical stresses with a deep foundation at depth \(d\).

Figure 7. Isolines of shear stresses with a deep foundation at depth \(d\).
Also, diagrams of additional normal and shear stresses were obtained at the contact between the enclosing structure and the soil massif. The results are shown in figures 8 to 10.

**Figure 8.** The diagram of additional vertical stresses at the contact of the enclosing structure with the soil massive with a deep foundation at depth $d$.

**Figure 9.** The diagram of additional horizontal stresses at the contact of the enclosing structure with the soil massive with a deep foundation at depth $d$. 
Discussions
Figures 8-10 show that impact of deep foundation at a distance from the vertical excavation creates significant additional vertical, horizontal and shear stresses at the contact of the enclosing structure with the soil massive, which requires taking into account in constructing the enclosing structures of the excavation sites. Thus, an analytical solution to the problem of the stress-strain state of a soil mass near a vertical excavation with a deep foundation has been obtained.

Summary
As a result of the analytical solution of the problem of the impact of a deep foundation near a vertical excavation, based on Melan's solution, formulas were obtained to determine the components of normal and shear stresses of distributed line-load at a depth near a vertical excavation. Analysis of the solution results shows that the impact of deep foundation at a distance from the vertical excavation creates significant additional vertical, horizontal and shear stresses at the contact of the enclosing structure with the soil massive, which requires taking into account in constructing the enclosing structures of the excavation sites.

The results of the solution can be used to determine the active pressure on the enclosing structures for taking into account additional stresses with deep foundations near excavation sites.

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