Two-neutrino double electron capture on $^{124}\text{Xe}$ based on an effective theory and the nuclear shell model

E. A. Coello Pérez, J. Menéndez, and A. Schwenk

1 Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany
2 ExtreMe Matter Institute EMMI, Helmholtzzentrum für Schwerionenforschung GmbH, 64291 Darmstadt, Germany
3 Center for Nuclear Study, The University of Tokyo, Tokyo 113-0033, Japan
4 Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany

We study the two-neutrino double electron capture on $^{124}\text{Xe}$ based on an effective theory (ET) and large-scale shell model calculations, two modern nuclear structure approaches that have been tested against Gamow-Teller and double-beta decay data. In the ET, the low-energy constants are fit to electron capture and $\beta^-$ transitions around xenon. For the nuclear shell model, we use an interaction in a large configuration space that reproduces the spectroscopy of nuclei in this mass region. For the dominant transition to the $^{124}\text{Te}$ ground state, we find half-lives $T_{1/2}^{2\nu\text{ECEC}} = (1.3 - 18) \times 10^{22}$ y for the ET and $T_{1/2}^{2\nu\text{ECEC}} = (0.43 - 2.9) \times 10^{22}$ y for the shell model. The ET uncertainty leads to a half-life almost entirely consistent with present experimental limits and largely within the reach of ongoing experiments. The shell model half-life range overlaps with the ET, but extends less beyond current limits. Our findings thus suggest that the two-neutrino double electron capture on $^{124}\text{Xe}$ has a good chance to be discovered by ongoing or future experiments. In addition, we present results for the two-neutrino double electron capture to excited states of $^{124}\text{Te}$.

Introduction.—Second-order weak processes give rise to extremely rare decay modes of atomic nuclei. They have been observed in about a dozen nuclei with the longest half-lives in the nuclear chart of about $10^{19} - 10^{21}$ years [1]. All these are two-neutrino double beta ($\beta\beta$) decays, where the emission of two electrons is accompanied by two antineutrinos. An even rarer decay can occur if no neutrinos are emitted, the neutrinoless $\beta\beta$ decay. This process is particularly intriguing, because neutrinoless $\beta\beta$ decay is not allowed in the Standard Model, does not conserve lepton number, and can only happen if neutrinos are their own antiparticles (Majorana particles) [2].

Due to its unique potential for neutrino physics, beyond the Standard Model physics, and the understanding of the matter-antimatter asymmetry of the Universe, neutrinoless $\beta\beta$ decay searches are increasingly active [3-9]. The planning and interpretation of these experiments relies on a good understanding of the decay half-life, which depends on a nuclear matrix element. However, these are poorly known as neutrinoless $\beta\beta$ decay matrix-element calculations disagree by at least a factor two [10].

Second-order weak processes with neutrino emission are ideal tests of neutrinoless $\beta\beta$ decay matrix-element calculations. The initial and final states are common, and the transition operator is also very similar, dominated by the physics of spin and isospin. In addition to $\beta\beta$ decay, a related mode is the two-neutrino double electron capture (2$\nu\text{ECEC}$). Here, two K- or L-shell orbital electrons are simultaneously captured, rather than $\beta$ emitted. This mode is kinematically unfavored with respect to $\beta\beta$ decay, and at present only a geochemical measurement of $^{130}\text{Ba}$ [11,12], and a possible detection in $^{78}\text{Kr}$ [13,14] have been claimed. Moreover, a resonant neutrinoless ECEC could be fulfilled in selected nuclei [15,17]. For both ECEC modes limits of $10^{21} - 10^{22}$ years have been set in various isotopes [11,18-23].

$^{124}\text{Xe}$ is one of the most promising isotopes to observe 2$\nu\text{ECEC}$ due to its largest Q-value of 2857 keV [24]. Large-volume liquid-xenon experiments primarily designed for the direct detection of dark matter such as XMASS [25], XENON100 [26], or LUX [27] are sensitive to ECEC and $\beta\beta$ decays in $^{124}\text{Xe}$, $^{126}\text{Xe}$ and $^{134}\text{Xe}$ [28,29]. Enriched xenon gas detectors are also very competitive [30,31]. Recent searches have reached a sensitivity comparable to the half-lives expected by most theoretical calculations [32,33]. Moreover the latest limits set by the XMASS collaboration [34] exclude most theoretical predictions.

Calculations of 2$\nu\text{ECEC}$ and two-neutrino $\beta\beta$ decay are challenging because they involve the quantum many-body problem of heavy nuclei with even-even and odd-odd numbers of neutrons and protons. The methods of choice are the quasiparticle random-phase approximation (QRPA) [35,36], extensively used for $^{124}\text{Xe}$ 2$\nu\text{ECEC}$ [37,38], and the large-scale nuclear shell model [40], which predicted successfully the $^{48}\text{Ca}$ $\beta\beta$ half-live before its measurement [41,42]. In this mass region, shell model studies have covered $^{124}\text{Sn}$, $^{128,130}\text{Te}$ and $^{136}\text{Xe}$ [43,44], but no shell-model calculation exists for $^{124}\text{Xe}$ 2$\nu\text{ECEC}$. In addition to QRPA, other more schematic approaches have also been applied to $^{124}\text{Xe}$ [48,51].

Further state-of-the-art $^{124}\text{Xe}$ 2$\nu\text{ECEC}$ calculations are thus required given the tension between theoretical predictions and experimental limits. In this Letter, we calculate the corresponding nuclear matrix elements using an effective theory (ET), introduced in Ref. [52], which describes well Gamow-Teller and two-neutrino $\beta\beta$ decays of heavy nuclei, including $^{128,130}\text{Te}$. One of the advantages of the ET is to provide consistent theoretical uncertainties. Similar ETs have been used
to study electromagnetic transitions in spherical \[123, 124\] nuclei. In addition, we present the first large-scale nuclear shell model calculation for \[124\] Xe \(2\nu\)ECEC. We focus on transitions to the \(124\) Te \(0^+\) ground state, but also consider \(2\nu\)ECEC to the lowest excited \(0^+_2\) and \(2^+_1\) states. The relation between the calculated nuclear matrix element \(M^{2\nu\text{ECEC}}\) and the \(2\nu\)ECEC half-life is given by

\[
(T_{1/2}^{2\nu\text{ECEC}})^{-1} = G^{2\nu\text{ECEC}} g_A^4 |M^{2\nu\text{ECEC}}|^2 ,
\]

where \(G^{2\nu\text{ECEC}}\) is a known phase-space factor \[62\] and \(g_A = 1.27\) the axial-vector coupling constant.

Effective theory.—We use an ET that describes the initial \(124\) Xe and final \(124\) Te nuclei, both with even number of protons and neutrons, as spherical collective cores. The intermediate nucleus, \(124\) I, has odd number of protons and neutrons. The ET describes its lowest \(1^+_1\) state as a double-fermion excitation of a \(0^+\) reference state that represents the ground state of either \(124\) Xe or \(124\) Te, \(|1^+_1; f_p; j_n⟩ = (n^l \otimes p^l) |0^+⟩\). Depending on the reference state, \(n^l (p^l)\) creates a neutron (proton) particle or hole in the single-particle orbital \(j_n (f_p)\). At leading order, higher \(1^+\) states are described as multiphonon excitations, with energies with respect to the reference state \(E(1^+_n+1) = E(1^+_1) + n\omega\), where \(\omega\) is the excitation energy and \(n\) is the number of phonon excitations.

The effective spin-isospin \(\sigma (\tau)\) Gamow-Teller operator is systematically constructed as the most general rank-one operator. At leading order it takes the form \[52\]

\[
O_{\text{GT}} = C_\beta \left( \hat{p} \otimes \hat{n} \right)^{(1)} + \sum_L C_{\beta L} \left[ \left( \hat{d}_L^\dagger \otimes \hat{n} \right)^{(L)} \right]^{(1)} + \sum_{L_L} C_{\beta L_L} \left[ \left( \hat{d}_L^\dagger \otimes \hat{d}_L \otimes \hat{d}_L \otimes \hat{n} \right)^{(L_L)} \right]^{(1)} + \ldots ,
\]

where the tilde denotes well defined annihilation operators, and the phonon \((\hat{d}, \hat{d}_L)\) and nucleon operators are tensor coupled. The low-energy constants \(C\) must be fitted to data, and the expansion above is truncated after terms involving more than two phonon creation or annihilation operators. The first term in Eq. \[2\] couples the reference state to the lowest \(1^+_1\) state of the odd-odd nucleus, so that \(C_\beta\) can be extracted from the known \(\log(ft)\) value of the corresponding \(\beta\) decay or EC:

\[
|0^+⟩\langle O_{\text{GT}}|1^+_1⟩ = \frac{3\kappa}{g_A^2 a_{0\text{EC}}^{|O_{\text{GT}}|1^+_1} |f|^2} ,
\]

where \(\kappa = 6147\) is a constant. The power counting of the ET \[52, 54\] relates the Gamow-Teller matrix elements from the lowest and higher \(1^+\) initial states to the common final reference state by \(⟨0^+|O_{\text{GT}}|1^+_1⟩ \sim (\omega/\Lambda)^{\nu/2}⟨0^+|O_{\text{GT}}|1^+_1⟩\), where \(\Lambda \sim 3\omega\) is the breakdown scale of the ET. This allows us to estimate the values of \(C_{\beta L}\) and \(C_{\beta LL}\) with consistent theoretical uncertainties.

The \(2\nu\)ECEC matrix element from the ground state of the initial nucleus to a \(0^+\) state of the final one is

\[
M^{2\nu\text{ECEC}} = \sum_j \frac{⟨0^+_1|O_{\text{GT}}|1^+_j⟩⟨1^+_j|O_{\text{GT}}|0^+_{\text{gs,i}}⟩}{D(1^+_j)/m_e} ,
\]

where \(j\) sums over all \(1^+_j\) states of the intermediate nucleus. The electron mass \(m_e\) keeps the matrix element dimensionless, and the energy denominator is \(D(1^+_j) = E(1^+_j) - E(0^+_{\text{gs,i}}) + [E(0^+_1) - E(0^+_{\text{gs,i}})]/2\), neglecting the difference in electron binding energies. The expression for the \(2\nu\)ECEC to a final \(2^+\) state is similar \[39\], but the energy denominator appears to the third power.

Because the ET is designed to reproduce low-energy states, we calculate the \(2\nu\)ECEC matrix elements within the single-state dominance (SSD) approximation:

\[
M^{2\nu\text{ECEC}} \approx \frac{⟨0^+_1|O_{\text{GT}}|1^+_1⟩⟨1^+_1|O_{\text{GT}}|0^+_{\text{gs,i}}⟩}{D(1^+_1)/m_e} ,
\]

which implies that only the matrix elements involving the lowest \(1^+_1\) state contribute. The advantage is that the ET can fit these using Eq. \[3\]. The contribution due to omitted higher intermediate \(1^+\) states is estimated within the ET and treated as a theoretical uncertainty \[52\]:

\[
\frac{\Delta M^{2\nu\text{ECEC}}}{M^{2\nu\text{ECEC}}} = \frac{D(1^+_1)/m_e}{\Lambda} \Phi \left( \frac{\omega}{\Lambda}, 1, \frac{D(1^+_1)}{\Lambda} + \frac{\omega}{\Lambda} \right) ,
\]

where \(\Phi(z, s, a) = \sum_{n=0}^{\infty} z^n/(s + n)^a\) is the Lerch transcendent. The ET describes very well the experimentally known two-neutrino \(\beta\beta\) decay half-lives once the ET uncertainties, including from Eq. \[6\], are taken into account \[52\]. This agreement includes \(128, 130\) Te among other heavy nuclei.

The ET \(2\nu\)ECEC matrix element calculation thus requires the known ground-state energies and the lowest \(1^+_1\) excitation energy to calculate the energy denominator, as well as the Gamow-Teller \(\beta\) decay and EC matrix elements from the \(1^+_1\) to the initial and final states of the \(2\nu\)ECEC to fit the low-energy constants. In addition, the collective mode \(\omega\) sets the ET uncertainty. Unfortunately, for \(124\) Xe there are no direct measurements for Gamow-Teller \(\beta\) decay or EC from the lowest \(1^+_1\) state in \(124\) I (the ground state is \(2^+\)), or alternatively zero-angle charge-exchange reaction cross sections involving the nuclei of interest. The \(1^+_1\) excitation energy in \(124\) I is also unknown.

Therefore, we adopt the following strategy. First, we set \(\log(ft)^{\text{EC}} = 5.00(10)\) for the EC on the lowest \(1^+_1\) state in \(124\) I, based on the experimental range of known EC on iodine isotopes with nucleon number
A = 122 – 128. This quantity varies smoothly for nuclei within an isotopic chain. For the $\beta^-$ decay, we set
\[ \log(f_t)^{\beta^-} = 1.06(1) \log(f_t)^{\text{EC}} \]
based on the systematics of odd-odd nuclei in this region of the nuclear chart. Guided by the known spin-unassigned excited states of $^{124}$I and the systematics in neighboring odd-odd nuclei, we set the excitation energy of the first $1^+_1$ state in $^{124}$I as 105 – 170 keV. Because this range is much smaller than the energy differences in $D(1^+_1)$, the associated uncertainty in the matrix elements is only a few percent. From the above considerations we obtain a range for the $^{124}$Xe $2\nu$ECEC matrix element based on our choice of parameters entering the ET. Finally, we set the excitation energy to $\omega = 478.3$ keV, the average of the excitation energies of the lowest $2^+_1$ states in the corresponding even-even nuclei $^{136}$Xe. This allows us to estimate the ET uncertainty associated to the SSD approximation, Eq. (6).

Nuclear shell model.—Next we perform large-scale shell model calculations to obtain the nuclear matrix element using the full expression Eq. (1). We solve the many-body Schrödinger equation $H |\psi\rangle = E |\psi\rangle$ for $^{124}$Xe, $^{124}$Te, $^{124}$I, using a shell model Hamiltonian $H$ in the configuration space comprising the $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ single-particle orbitals for neutrons and protons. To keep the dimensions of the shell model diagonalization tractable, especially for the largest calculations $^{124}$Xe, we need a truncated configuration space. In a first truncation scheme, similar to the one used in Ref. 62, we limit to two the number of nucleon excitations from the lower energy $0g_{7/2}$, $1d_{5/2}$ orbitals to the higher lying $1d_{3/2}$, $2s_{1/2}$, $0h_{11/2}$ orbitals. Second, we adopt a complementary truncation scheme that keeps a maximum of two neutron excitations but does not limit the proton excitations from lower to higher lying orbitals (a maximum of six nucleon excitations are permitted in $^{124}$Xe). This keeps the $0g_{7/2}$ orbital fully occupied. A third scheme with the $1d_{5/2}$ orbital fully occupied gives results within those of the other two truncations.

We use the shell model interaction GCN5082 63, 65, fitted to spectroscopic properties of nuclei in the mass region of $^{124}$Xe. The shell model interaction has been tested against experimental data on Gamow-Teller decays and charge-exchange transitions in this region, showing a good description of data with a renormalization, or “quenching”, of the $\sigma \tau$ operator $\alpha = 0.57$ 43. For the two-neutrino $\beta\beta$ decay of $^{128}$Te, $^{130}$Te, and $^{136}$Xe, however, this interaction fits data best after a larger renormalization $\alpha = 0.48$ 43. An extreme case is the very small $\beta\beta$ $^{136}$Xe matrix element, which is only reproduced with $\alpha = 0.42$ 43. The renormalization of the spin-isospin operator is needed to correct for the approximations made in the many-body calculation, such as unaccounted correlations beyond the configuration space or neglected two-body currents 10, 66. A full understanding of its origin would require an ab initio study that is currently possible only for nuclei lighter than xenon 67, 70. Here we follow the strategy of previous shell model $\beta\beta$ decay predictions 111, 112 and include the above “quenching” factors phenomenologically to predict the half-life of $^{124}$Xe.

The low-energy excitation spectra of the three isotopes are well reproduced. Figure 1 compares the experimental and calculated spectra for $^{124}$Xe and $^{124}$Te, obtained with the first truncation scheme described above. The spectra corresponding to the second truncation scheme is of similar quality. When additional excitations to the higher lying orbitals are permitted, the first excited $0^+_2$ in $^{124}$Te is raised to 1.6 MeV, in much better agreement with experiment. However, such extended truncation yields too large dimensions for $^{124}$Xe, and cannot be used in our $2\nu$ECEC calculations. The spectra of the intermediate $^{124}$I is not well known besides the ground state and few tentative spin assignments. The GCN5082 interaction reproduces correctly the spin and parity of the $2^+$ ground state, although with a lowest $1^+_1$ state at only about 10 keV, below any measured level. For the $2\nu$ECEC, as in the ET calculation, we consider the lowest $1^+_1$ state at 105 – 170 keV excitation energy. All shell model calculations have been performed with the codes ANTOINE and NATHAN 10, 72.

Results and discussion.—The calculated nuclear matrix elements are common for the capture of K- or L-shell electrons. However, the presented half-lives correspond to the $^{124}$Xe $2\nu$ECEC of two K-shell electrons, as this is the mode explored in recent experiments 30, 31.

Table I summarizes our main results. The ET predicts a smaller central value for the $^{124}$Xe $2\nu$ECEC matrix element than the NSM, even though both results are consistent when taking uncertainties into account. The ET uncertainty results from combining the uncertainty associated to the SSD approximation, Eq. (6), with the range of the parameters used as input for the ET. Both contributions are of similar size. For the NSM, one part of the
NSM half-lives are in general shorter than the QRPA ones,
transitions to excited states are extremely suppressed
because of the small nuclear matrix element.
requires the capture of K- and L-shell electrons, is further
suppressed because of the small nuclear matrix element.
In addition, it has to be noted that the theoretical
uncertainty is given by the range of results obtained
with different truncation schemes. The dominant part,
however, is given by the three “quenching” values
considered: the average $q = 0.57$ and $q = 0.48$,
corresponding to the best description of Gamow-Teller
transitions and $\beta\beta$ decays, respectively, plus the addi-
tional conservative $q = 0.42$ needed in the $^{136}$Xe $\beta\beta$ decay. The
NSM ranges in Table 1 cover the results obtained with
the two truncations and three “quenching” values.

Table 1 also shows our predictions for the $2\nu$ECEC into
excited states of $^{124}$Te. For both final $0^+_1$ and $2^+_1$ states,
the ET and NSM matrix elements are consistent, even
though the central values predicted by the shell model
are about one third of the ET ones. The suppressed
NSM matrix element to the final $0^+_1$ state with respect
to the transition to the ground state is consistent with the
results on neutrinoless $\beta\beta$ decay in $^{128,130}$Te and $^{136}$Xe,
using the same interaction \[33\]. While the shell model
uncertainties are somewhat smaller than in the $2\nu$ECEC
to the ground state, the ET ones are much larger, because
of the limitations of the SSD approximation when the
energy denominator $D(1^+_1)$ is small \[52\]. The ET and
NSM half-lives are in general shorter than the QRPA ones
for the $0^+_2$ $2\nu$ECEC \[38,39\], while for the $2^+_1$ $2\nu$ECEC
the NSM and QRPA \[39\] predictions are very similar.
Transitions to excited states are extremely suppressed
because of the reduced $Q$-value and corresponding
phase-space factor. The $2\nu$ECEC to the final $2^+_1$ state,
which requires the capture of K- and L-shell electrons, is further
suppressed because of the small nuclear matrix element.

Figure 2 compares our theoretical predictions for the
$2\nu$ECEC on $^{124}$Xe to the $^{124}$Te ground state with
the most advanced QRPA results from Refs. \[38,39\] and the
most recent experimental $2\nu$ECEC limits \[31,34\]. Theoretical
half-lives are shown as black bars. The predictions
from the ET, NSM, and QRPA are consistent. However,
the ET shows a clear preference for longer half-lives than
the NSM. On the other hand, the QRPA spans much
shorter half-lives than those predicted by the ET or NSM.

Figure 2 shows that the theoretical predictions are
consistent with the lower half-life limits established by
the first results of the XMASS \[32\] and XENON100 \[33\]
collaborations and with Ref. \[31\], shown as red, blue
and purple horizontal lines in Fig. 2 respectively. However,
the most recent limit established very recently by
XMASS \[34\] (green line) excludes most of our NSM results,
but a part of the predicted range remains permitted.
Note that since the shell model configuration space
had to be truncated, we could not obtain the exact nu-
clear matrix element without “quenching”. On the other
hand, the ET half-life is almost fully consistent with the
current XMASS limit. The ET central half-life is only
about five times longer, and the range predicted by the
ET lies largely within the sensitivity of ongoing exper-
iments \[33\]. The QRPA predictions are mostly excluded
including error bars, except the very recent results from
Ref. \[39\], just at the border of the permitted region. Most
other older theoretical calculations are also in tension
with the XMASS limit \[34\]. Overall, our results suggest
that the $^{124}$Xe $2\nu$ECEC could very well be discovered in
ongoing or upcoming experiments in the near future.

Summary.—We have calculated the nuclear matrix el-
ements for the $2\nu$ECEC on $^{124}$Xe using an ET and the
large-scale nuclear shell model, two of the nuclear many-
body approaches best suited to describe $\beta$ and EC trans-
itions in heavy nuclei. The ET results are based on
$\beta$ decay and EC on neighboring nuclei, while the shell
model uses an interaction that describes well $\beta\beta$ decays
describing neighboring nuclei. The ET provides consistent the-

| $2\nu$ECEC | $G^{2\nu\text{ECEC}}$ | $T^{2\nu\text{ECEC}}$ |
|----------|-----------------|-----------------|
| ET       | $0.011 - 0.041$ | $0.002 - 0.050$ | $0.8 - 9.0 \times 10^{-4}$ |
| NSM      | $0.028 - 0.072$ | $0.005 - 0.010$ | $1.1 - 2.3 \times 10^{-4}$ |

![Figure 2](image-url)
theoretical uncertainties set by the order of the ET calculation, while the shell model uncertainty is dominated by the range of “quenching” considered for the 2νECEC operator. The ET predicts a half-life consistent and up to several times longer than current experimental limits, while the shell model prediction extends less beyond current limits. When all uncertainties are taken into account, the ET and NSM results are consistent, as well as with the most advanced QRPA results.

Future directions include higher-order calculations in the ET to reduce the uncertainties, and improved NSM studies with a better understanding of the “quenching” of the operator, and limiting truncations in the configuration space. Our findings suggest that the $^{124}$Xe 2νECEC has a good chance to be discovered by ongoing or future experiments, so that these predictions can be tested by upcoming analyses of ongoing experiments and can further stimulate future searches.

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