Out-of-equilibrium dynamics of the vortex glass

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We study the relaxational dynamics of flux lines in high-temperature superconductors with random pinning using Langevin dynamics. At high temperatures the dynamics is stationary and the fluctuation dissipation theorem (FDT) holds. At low temperatures the system does not equilibrate with its thermal bath: a simple multiplicative aging is found, the FDT is violated and we found that an effective temperature characterizes the slow modes of the system. The generic features of the evolution – scaling laws – are dictated by the ones of the single elastic line in a random environment.

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The effect of quenched disorder on the vortex phase diagram of superconductors has attracted much interest since the discovery of high $T_c$ cuprates. Since the long-range order of the vortex lattice is destroyed by any small amount of disorder [1], an amorphous “vortex glass” (VG) phase was predicted [2]. While it is currently understood that there is a “Bragg glass” phase for weak disorder [3], there is still some controversy about the nature of the amorphous phase at strong disorder. Experimentally, one finds an irreversibility line (IRL) below which the magnetization is irreversible and the linear resistivity drops to nearly zero [4]. The original proposal [5] as in structural glasses [6] includes screening) showed that below a crossover temperature, where the VG-type of criticality is arrested, the temperature, the $\Gamma$-type of criticality is arrested, the

\[ \eta \frac{\partial r_{iz}(t)}{\partial t} = -\frac{\delta H[r_i(z)]}{\delta r_{iz}} + f_{iz}^T(t), \]

where $\eta$ is the Bardeen-Stephen friction coefficient. The thermal force $f_{iz}^T(t)$ satisfies (\$f_{iz,\mu}(t) = 0\$, \$f_{iz,\mu}(t)f_{iz',\mu}(t'') = 2\hbar k_B T\delta(t - t')\delta_{zz'}\delta_{\mu\mu'}\delta_{\mu\mu'}\$), and $\mu, \mu' = x, y$ and $T$ is the thermal bath temperature.

We therefore choose parameter values corresponding to $\lambda_{ab}/\xi_{ab} = 100$, and $\lambda_f/\xi_{ab} = 16$; and we use $c = \epsilon'\lambda_0^2(1 + \ln(\lambda_{ab}/d_0))/\eta$ [13]. The strength of disorder is set to $\gamma = 10^{-5}$, for which case we find that above $B_{cr} \sim 0.002H_{c2}$ the Bragg peaks disappear and the flux lines are frozen in a highly amorphous structure at low temperatures. We therefore choose to study the case with $B = 0.01H_{c2} > B_{cr}$, which is deep
within the VG regime at low \( T \). Time is normalized by \( t_0 = \zeta_0^2/\eta/\epsilon_0 \), length by the vortex lattice parameter \( a_0 = |2\Phi_0/(\sqrt{3}B)|^{1/2} \), energy by \( \epsilon_0 d_x \), and temperature by \( \epsilon_0 d_x / k_B \). We simulate \( N = 56 \) vortices in a box of size \( 7a_0 \times 8a_0 \sqrt{3}/2 \) with periodic boundary conditions for the in-plane coordinates. The \( z \) direction is discretized in \( L = 50 \) planes with free boundary conditions. Averages are performed over 10 realizations of the disorder.

In order to study the out-of-equilibrium dynamics we use the following protocol. First we equilibrate the system at an initial high temperature \( T \) where the full system size separation of the vortex lines is achieved. The system is let evolve during a waiting time \( t_w \) after which the quantities of interest are measured. In general, one defines two-time correlation or response functions \( C(t, t_w) \). When the system reaches equilibrium, these quantities become independent of \( t_w \) and depend only on the difference \( \tau = t - t_w \). If the system is not able to reach equilibrium within the observation time window, \( C(t, t_w) \) depends on the two times.

In particular, if the decay gets slower for longer \( t_w \)’s we say that the system “ages”. We study: (a) The dynamic wandering \( W(z, t, t_w) \), defined by

\[
NW(z, t, t_w) = \sum_i \langle [r_{iz}(t) - r_{iz}(t_w)] - [r_{iz}(t) - r_{iz}(t_w)] \rangle^2
\]

which measures how the displacement of the vortex segment in the \( z \)-th plane with respect to the bottom plane correlates between \( t \) and \( t_w \) \((\ldots)\) means average over thermal noise and disorder). (b) The mean square displacement (MSD) in the planes,

\[
B(t, t_w) = \frac{1}{LN} \sum_{iz} \langle [x_{iz}(t) - x_{iz}(t_w)]^2 \rangle.
\]

Firstly, we analyze the single, non-interacting, flux line without disorder. In Fig. (a) and (b) we show the dynamic wandering and the MSD, respectively, when the line is quenched to \( T = 0.06 \). In this case, the flux line reaches equilibrium in a short time: both correlation functions are independent of \( t_w \) and depend only on \( \tau = t - t_w \) (data for several \( t_w \) overlap in the plots). From the behavior of \( W \), we define a set of characteristic times \( t_\tau \), such that when \( \tau = t - t_w > t_\tau \), \( W(z, \tau \gg t_\tau) \sim z^{2\zeta} \) saturates to a constant value. Here \( \zeta \) is the roughness exponent given by thermal fluctuations, i.e. \( \zeta = \zeta_T = 1/2 \). \( t_\tau \) is the time needed for the line element in the \( z \)-th plane to feel the elastic interaction with the line element in the plane \( z = 0 \). In Fig. (a) we show the times scales \( t_1 \) for neighboring planes separation and \( t_L \) for full system size separation. We found that for \( \tau < t_1 \), \( W(z, \tau) \sim \tau \) for all \( z \). For times \( \tau > t_L \), \( W(L, \tau) \) saturates. For times \( t_1 < \tau < t_2 \), the dynamic wandering shows an intermediate regime between these two extreme behaviors. In Fig. (b) the MSD is shown and the same regimes are identified. First, for \( \tau < t_1 \) the MSD follows the 2D diffusion of individual line elements (“pancakes”), \( B(\tau) \sim \tau \), before the elastic interplane interaction becomes relevant. Second, for \( t_1 < \tau < t_L \), flux line thermal relaxation is observed, characterized by sublinear diffusion \( B(\tau) \sim \tau^\alpha \), with \( \alpha = 1/2 \). Third, for \( \tau > t_L \), we observe diffusion of the center of mass of the flux line \( B(\tau) \sim \tau \), \( t_L \) is the time scale above which finite size effects dominate. The time regime we wish to study is the one without finite size effects, \( \tau < t_L \) \((\tau \lesssim 10^4 \) for \( L = 50 \)).

Secondly, we analyze the evolution of interacting lines in the presence of disorder. When we quench the system to relatively high temperatures \( T > 0.2 \) it reaches equilibrium, \( W(t, t_w) \) and \( B(t, t_w) \) are independent of \( t_w \) and they behave as in Fig. (a) for non-interacting flux lines meaning that we are clearly within the VL. Below a crossover temperature \( T_g \approx 0.18 \) the system is no longer able to equilibrate, and the correlation functions depend on \( t_w \). The MSD at \( T = 0.02 \) is shown in Fig. (b). For \( \tau < t_1 \), the dynamics is governed by single pancake fluctuations and the MSD is independent of \( t_w \). For \( \tau > t_1 \), there is “aging”, the longer the waiting time \( t_w \), the slower the relaxation of the flux lines.

In order to study the modifications of the fluctuation-dissipation theorem (FDT) in the out-of-equilibrium regime, a response function should be measured. To this end, a random force of the form \( f_{iz} = \delta s_{iz} \hat{x} \) is switched on at a time \( t_w \) on a replica of the system, where \( \delta \) is the intensity of the perturbation, and \( s_{iz} = \pm 1 \) with equal
probability $[18]$. The integrated response is

$$\chi(t, t_w) = \frac{1}{L N \delta} \sum_i \langle s_i \delta [x_i(t) - x_i(t)] \rangle,$$

where $x_i$ and $x_i(t)$ correspond to the position evaluated in two replicas of the system, with and without the perturbation. In equilibrium, FDT implies $2T\chi(\tau) = B(\tau)$. In Fig. 2(b), we show $2T\chi(t, t_w)$ at $T = 0.02$. $\chi$ depends on $t_w$, showing aging. It is also clear that for long $\tau$ and long $t_w$, $2T\chi$ is not proportional to $B$, violating FDT. This type of behavior: aging in $\tilde{B}$ and $\chi$ and violation of FDT for long $t_w$ and $\tau$ is observed at all $T < T_g$.

Typically, a correlation function $C(t, t_w)$ in structural glasses has an additive scaling form $C(t, t_w) = C_q(t - t_w) + C_{ag}[h(t)/h(t_w)]$, with, very often, $h(t) = t$ (known as simple aging) $[11, 20]$. This does not hold in the vortex problem. Instead, we found a “multiplicative” scaling similar to the one proposed for the low-temperature behavior of a directed polymer in random media $[21]$ and the out-of-equilibrium critical dynamics of the 2d XY model $[22]$. Following Yoshino $[21]$ we tried the scaling form $B(t, t_w) = \tilde{B}(\tilde{t})^{\alpha_{t_w}}$, and $2T\chi(t, t_w) = \tilde{\chi}(\tilde{t})^{\alpha_{t_w}}$, with $\tilde{t} = t/t_w$ and $\tilde{B}$ and $\tilde{\chi}$ given by

$$\tilde{B}(\tilde{t}) = \begin{cases} c_1(T)(\tilde{t} - 1)^{\alpha(T)} & \tilde{t} \ll 1, \\ c_2(T)(\tilde{t} - 1)^{\alpha(T)} & \tilde{t} \gg 1, \end{cases}$$

$$\tilde{\chi}(\tilde{t}) = \begin{cases} c_1(T)(\tilde{t} - 1)^{\alpha(T)} & \tilde{t} \ll 1, \\ \chi(T)c_2(T)(\tilde{t} - 1)^{\alpha(T)} & \tilde{t} \gg 1, \end{cases}$$

where $c_1$, $c_2$, and $y$ are temperature dependent coefficients $[23]$. $y(T)$ modifies the characteristic relaxation time by evaluating $C(t, t_w) = \exp[-B(t, t_w)]$. In the VL we ex-
the inset of Fig. 4(a). In the VL we obtained 

t but we can extract a "multiplicative scaling, and the same definitions of \( \tilde{\chi} \) size effects [cfr. Fig. 1]. Below a crossover temperature 

\( T \alpha \) the largest 

\( T \alpha \) scaling as for single lines, but with a smaller exponent 

\( \alpha(T) \). Do vortices freeze like a structural (fragile) glass? Yes and No. Yes, because we found aging and a simple violation of FDT with an effective temperature \( T_{\text{eff}} \) which is independent of \( T \) within our numerical accuracy. No, because aging follows a multiplicative scaling, similar to polymers in random media and, more precisely, critical systems like the 2d XY model. Longer times are needed to decide whether this "critical" scaling holds in the limit \( t_w \to \infty \) and to grasp the nature of the low- 

\( T \) phase. Our results suggest to do experiments with a fast quench to low \( T \): with relaxational measurements of transport or magnetic properties one could study the aging regime while with voltage or flux noise measurements one could test how the FDT is violated.

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\[ C(r) \sim \exp[-(r/t_r)^\alpha] \] For single flux lines the \( T \) dependence of \( t_r \) should be \( t_r \propto 1/T^2 \) \[3, 14\]. We analyzed this correlation function for different \( T \), fitting the corresponding stretched exponential with \( \alpha = 1/2 \) and estimating the relaxation time \( t_r(T) \), that is plotted in the inset of Fig. 4(a). In the VL we obtained \( t_r \propto 1/T^2 \), as expected. Below \( T_g \approx 0.18 \) the system does not equilibrate but we can extract a "\( t_w \)" by fitting \( C(t, t_w) \) for the largest \( t_w \) with the same exponential form. The obtained \( t_r \) increases very rapidly below \( T_g \), reflecting that the system falls out of equilibrium in this aging regime.

Finally, we performed the same analysis for single flux lines in the presence of disorder. For high temperatures we found no aging and \( \alpha \approx 1/2 \), with important finite size effects [cfr. Fig. 4]. Below a crossover temperature we observe similar aging to the one shown in Fig. 4. The multiplicative scaling, and the same definitions of \( B \) and \( \tilde{\chi} \), describe the data very accurately, the only differences being that \( \alpha(T) \) takes larger values that are closer to \( \alpha = 1/2 \) than in the VG [see Fig. 4(a)], and that \( T_{\text{eff}} \) takes a smaller value, \( T_{\text{eff}} = 0.156 \), than in the VG.

The results described in the last paragraph imply that, rather surprisingly, the out-of-equilibrium slow dynamics of the VG is dominated by the relaxation of the elastic lines along the \( z \)-direction. Aging follows a multiplicative scaling as for single lines, but with a smaller exponent \( \alpha(T) \). Do vortices freeze like a structural (fragile) glass? Yes and No. Yes, because we found aging and a simple violation of FDT with an effective temperature \( T_{\text{eff}} \) which is independent of \( T \) within our numerical accuracy. No, because aging follows a multiplicative scaling, similar to polymers in random media and, more precisely, critical systems like the 2d XY model. Longer times are needed to decide whether this "critical" scaling holds in the limit \( t_w \to \infty \) and to grasp the nature of the low- \( T \) phase. Our results suggest to do experiments with a fast quench to low \( T \): with relaxational measurements of transport or magnetic properties one could study the aging regime while with voltage or flux noise measurements one could test how the FDT is violated.

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![FIG. 4: (a) Temperature dependence of the dynamic exponent. The equilibrium value for single elastic lines, \( \alpha = 1/2 \), is highlighted. (b) The slope \( y(T) \). The data is fitted linearly with \( y = T/T_{\text{eff}} \) and the values of \( T_{\text{eff}} \) are indicated. Inset shows the relaxation time at the VL-VG transition.](image-url)