SAR Image Despeckling Based on Combination of Laplace Mixture Distribution with Local Parameters and Multiscale Edge Detection in Lapped Transform Domain

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Abstract

The speckle noise badly affects the tasks of automatic information extraction and scene analysis in Synthetic Aperture Radar (SAR) images. Therefore, the despeckling of SAR images while preserving edge and textures is highly important. In this paper, a lapped transform (LT) based SAR image despeckling algorithm is proposed. Since lapped orthogonal transform (LOT) is orthogonal and has good energy compaction, the statistical modeling of noise and signal can be done precisely in the same domain. The use of LOT in denoising applications is motivated by its low computational complexity and its feature of robustness to over-smoothing. Since the LOT is block transform, the dyadic remapping is carried out first and then the subband LOT coefficients are modeled similar to wavelet coefficients. The LOT coefficients of the logarithmically transformed reflectance and the speckle noise are modeled using 2-state laplace mixture pdf that uses local parameters and zero mean Gaussian pdf respectively. A MAP estimator within Bayesian framework based on proposed prior is developed. The mixture distribution parameters are estimated using Expectation-Maximization (EM) algorithm. Subband LOT coefficients at each scale are classified into edge and non-edge coefficients using LOT modulus maxima computation. The non-edge coefficients are filtered using the proposed Bayesian MAP estimator and edge coefficients are kept unmodified. The method is implemented in ‘cycle spinning’ mode to solve the problem of lack of shift-invariance property of the LOT. Experimental results carried out on real SAR images show that the proposed scheme very effectively preserve the edges of a SAR image with notable speckle suppression and outperform two undecimated wavelet transform based methods.

Keywords: Lapped orthogonal transform; Synthetic aperture radar (SAR) image; Despeckling; Laplace mixture pdf; Dyadic remapping; Cycle spinning; Maximum a posteriori (MAP) estimator.

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1. Introduction

SAR systems can work day and night regardless of weather conditions. SAR is very useful tool in remote sensing due to its high spatial resolution. Due to the coherent processing of the received scattered signals, the SAR images are contaminated with multiplicative speckle noise. The speckle noise badly affects the automatic interpretation of SAR images, therefore speckle reduction is an important preprocessing step.

The traditional spatial filters [1],[2] smooth the speckle very well but fail to preserve important details in SAR images. Discrete wavelet transform (DWT)[5],[7],[8] based despeckling methods perform better than spatial domain methods due to its multiresolution, sparsity and time frequency localization properties.

In the last few years, some attention has been paid to LT as a tool for image denoising due its important features like dyadic remapping, robustness to oversmoothing, sharper frequency attenuation and low computational complexity compared to orthogonal DWT [10-12]. In [11], Hazarika et. al. demonstrated that the subband lapped orthogonal transform (LOT)[15] coefficients of log transformed SAR images can be well modeled using generalized Gaussian distribution. Raghvendra et al.[4] modeled the LOT coefficients using Laplace mixture pdf (that uses no local parameters) for reducing additive white Gaussian noise in natural images. In [11], the authors observed that the histogram of subband LOT coefficients of log transformed SAR images are sharply peaked around zero and have heavy tails. The dyadic remapped LOT coefficients in a subband show clustering property i.e. if a given LOT coefficient is large in magnitude then the magnitude of adjacent coefficients are also likely to be large. This clustering property can be well modeled using local pdf's. Since the mixture models are highly efficient in modeling the heavy tailed processes, we model the subband LOT coefficients of log transformed SAR images using 2-state Laplace mixture distribution that uses local parameters. A MAP estimator using proposed prior is employed for estimating the clean LOT coefficients. For more effective edge preservation during despeckling, the above MAP estimate is combined with multi-scale edge detection in LOT domain.

This paper is organized as follows: Section 2 describes the statistical modeling of dyadic remapped LOT coefficients. In Section 3 we describe the proposed method. The experimental results are demonstrated in Section 4 and the conclusion is drawn from this paper in Section 5.

2. Statistical modeling of the dyadic remapped LOT coefficients

Let $Y_{SAR}(i,j)$, $Z_{SAR}(i,j)$ and $h_{SAR}(i,j)$ be the clean version of the image, observed image and multiplicative noise component respectively, then the SAR image model can be expressed as (speckle is assumed to be fully developed)

$$Z_{SAR}(i,j) = Y_{SAR}(i,j)h_{SAR}(i,j)$$

(1)

When logarithm transformation is applied on both sides of (1), the multiplicative model is converted into additive one and can be written as

$$Z(i,j) = Y(i,j) + N(i,j)$$

(2)

where $Z(i,j)$, $Y(i,j)$ and $N(i,j)$ denotes the logarithm of $Z_{SAR}(i,j)$, $Y_{SAR}(i,j)$ and $h_{SAR}(i,j)$ respectively.

In this paper, we tackle the despeckling problem as a maximum a posteriori (MAP) estimation problem in LOT domain. After applying LOT on both sides of (2), the block coefficients are rearranged in octave-like form which may be expressed as $z_{ab}^n(i,j) = y_{ab}^n(i,j) + n_{ab}^n(i,j)$ where $n_{ab}^n(i,j)$, $y_{ab}^n(i,j)$ and $z_{ab}^n(i,j)$ denotes the dyadic remapped LOT coefficient at $(i,j)^{th}$ position, at level $b$ with orientation $a$ of the log transformed speckle noise component, the corresponding log transformed clean version of SAR image and the corresponding log transformed observed SAR image respectively. The statistical distribution of log transformed speckle noise in LOT domain is assumed to be zero mean white Gaussian with standard deviation $s_n$.

Lapped orthogonal transform (LOT)[15] was introduced as an alternative to block DCT with significantly reduced blocking artifacts. The LOT has good energy compaction and is also orthogonal, hence signal and
noise modeling can be performed precisely in this domain. The LOT is block transform therefore first the block LOT coefficients are rearranged in octave-like form [10-13],[16] and then the subband coefficients are modeled for denoising applications [10-12].

In Fig. 1 we show the logarithmic histograms of one of the finest subband (for two SAR images) and the best Laplace, 2-state Gaussian mixture and 2-state Laplace mixture pdf fitted to this histogram. It is clearly observed that the 2-state Laplace mixture pdf provides the best fit to the empirical histogram. Therefore in this work, we propose to model the subband LOT coefficients of log transformed reflectance using 2-state Laplace mixture pdf that uses local parameters.

Fig.1 Log histograms of one of the subbands of dyadic remapped LOT coefficients for two approximately noise-free log transformed SAR images and the best Laplace, 2-state Gaussian mixture and 2-state Laplace mixture pdf fitted to this histogram in log domain

3. Proposed LOT based despeckling method

In this paper, the dyadic remapped LOT coefficients in a subband are modeled with 2-state Laplace mixture pdf that uses local parameters for the mixture model. This model exploits both, the intrascale dependencies between the dyadic remapped LOT coefficients and also simultaneously captures its heavy tailed behavior. A 2-state Laplace mixture pdf is given as [3-4]

$$P_{y(i)}(y(i)) = w(i)P_1(y(i)) + (1-w(i))P_2(y(i))$$ (3)

$$P_{y(i)}(y(i)) = w(i)\frac{1}{s_1(i)\sqrt{2}} e^{-\frac{y(i)}{s_1(i)}} + (1-w(i))\frac{1}{s_2(i)\sqrt{2}} e^{-\frac{y(i)}{s_2(i)}}$$ (4)

where $s_1(i)$ and $s_2(i)$ are the standard deviations of the individual Laplace pdfs. $w(i)$ and $(1-w(i))$ denotes their corresponding weights. Using Expectation-Maximization (EM) algorithm, these parameters are estimated from the noisy observation. The MAP estimator of the speckle free dyadic remapped LOT coefficients is expressed as [3-4]

$$\hat{y}(i) = \arg \max_{y(i)} P_{y(i)\mid z(i)}(y(i)\mid z(i))$$ (5)

With Bayes theorem, the above equation can be expressed as

$$\hat{y}(i) = \arg \max_{y(i)} \frac{P_{y(i)\mid z(i)}(y(i)\mid z(i))P_{y(i)}(y(i))}{P_{z(i)}(z(i))}$$ (6)
\[
\hat{y}(i) = \arg \max_{y(i)} P_{y(i)} \left( z(i) \mid y(i) \right) P_y(y(i))
\]  
(7)

\[
\hat{y}(i) = \arg \max_{y(i)} P_n(z(i) - y(i)) P_y(y(i))
\]  
(8)

When the noise is zero mean Gaussian with variance \( s_n^2 \), we have

\[
P_n = \frac{1}{\sqrt{2\pi s_n^2}} e^{-\frac{x^2}{2s_n^2}}
\]

\[
\hat{y}(i) = \arg \max_{y(i)} \frac{c}{s_n^2} (z(i) - y(i))^2 + f(y(i))
\]

(9)

where \( f(y(i)) = \log(P_y(y(i))) \). The MAP estimate is obtained by setting the derivative w.r.t \( y \) equals to zero

\[
\frac{(z(i) - \hat{y}(i))^2}{s_n^2} + f'(y(i)) = 0
\]

(10)

1. When \( P_{y(i)}(y(i)) \) assumes a single Laplace pdf with local parameters:

\[
P_{y(i)}(y(i)) = \frac{1}{s(i)\sqrt{2}} e^{-\frac{|y(i)|}{s(i)}}
\]

(11)

Here, we have \( f(y(i)) = -\log(s(i)\sqrt{2}) - \sqrt{2}|y(i)|/s(i) \). Therefore the estimate:

\[
\hat{y}(i) = \text{sign}(z(i)) \left( |z(i)| - \sqrt{2}s_n^2 / s(i) \right).
\]

(12)

where \( g = \max(g, 0) \). The approx. estimate of \( s(i) \) based on local neighborhood \( N(k) \) around noisy coefficient \( z(i) \) is given as

\[
s(i) = \max_{j \in N(k)} \frac{1}{M} \sum_{j=1}^{M} z(j)^2 - s_n^2 \frac{\delta}{\delta}
\]

(13)

The noise standard deviation is estimated from the first scale using

\[
\hat{s}_n(i) = K_s \text{Median}(\{|z(i)|\}) \cdot \frac{0.6745}{z(i, j)\hat{1}} HH_1
\]

(14)

where \( K_s \) is a smoothing factor. If the \( \text{SOFTLAMAP} \) operator is defined as

\[
\text{SOFTLAMAP}(r, g) = \text{sign}(r) \left( |r| - g \right)
\]

(15)

\[
\hat{y}(i) = \text{SOFTLAMAP} \left( \frac{\sqrt{2} s_n^2 \frac{\delta}{\delta}}{s(i) \frac{\delta}{\delta}} \right)
\]

(16)

2. When \( P_{y(i)}(y(i)) \) assumes mixture of two Laplace pdf's with local parameters:

When \( P_{y(i)}(y(i)) \) assumes the expression given in (3), the shrinkage function can be expressed as

\[
\hat{y}(i) = \frac{\text{SOFTLAMAP} \left( z(i), \sqrt{2} s_n^2 \frac{\delta}{\delta} \right) \text{SOFTLAMAP} \left( z(i), \sqrt{2} s_n^2 \frac{\delta}{\delta} \right) V}{1 + V}
\]

(17)

where \( V \) is

\[
V = \frac{1 - w(i)\frac{\delta}{\delta}}{s_2(i) \frac{\delta}{\delta}} \text{erfc} \frac{s_n - \frac{\delta}{\delta}}{s_2(i) \frac{\delta}{\delta}} + \frac{z(i) \frac{\delta}{\delta}}{\sqrt{2s_n^2 \delta}} + \frac{\text{erfc} \frac{s_n - \frac{\delta}{\delta}}{s_2(i) \frac{\delta}{\delta}}}{\sqrt{2s_n^2 \delta}} + \frac{z(i) \frac{\delta}{\delta}}{\sqrt{2s_n^2 \delta}}
\]

(18)
The estimation of mixture model parameters $s_1(i)$, $s_2(i)$ and $w(i)$ are carried out from the noisy observation using EM algorithm. For each data point $z(i)$, we consider a square window centered at $z(i)$. The parameter $s_1(i)$, $s_2(i)$ and $w(i)$ are initialized first and then expectation and maximization steps are iteratively performed until the parameters meet the convergence conditions.

**E-step:** This step computes the responsibility factors at each iteration

$$r_i(i) \leftarrow \frac{w(i)P_1(y(i))}{w(i)P_1(y(i)) + (1-w(i))P_2(y(i))}$$

(19)

and $r_2(i) \leftarrow (1 - r_1(i))$. The responsibility factors should satisfy $r_1(i) + r_2(i) = 1$.

**M Step:** The parameter $w(i)$ is computed using

$$w(i) \leftarrow \frac{1}{M} \sum_{j \in N(i)} r_i(i)$$

where $N(i)$ is a local square window with $M$ number of coefficients.

The parameters $s_1(i)$ and $s_2(i)$ are computed by

$$s_k(i) \leftarrow \frac{\sqrt{2\sum_{j \in N(i)} r_j(j)\{y(i) - \hat{a}_j\}^2}}{\sum_{j \in N(i)} r_j(j)}, k = 1, 2$$

(20)

### 3.1. Multiscale edge detection in dyadic remapped LOT domain

It has been observed in the literature that the despeckling techniques when combined with edge preserving strategies show highly effective despeckling results [14]. In this paper, we classify the LOT coefficients into edge and non edge coefficients using modulus maxima computation [6]. The main edge preserving strategy here is to filter only the non edge coefficients using non-linear shrinkage function (17) and edge coefficients are kept unchanged. The dyadic remapped LOT coefficients in horizontal and vertical subbands contain corresponding horizontal and vertical edge information of an image. Therefore at each scale $j$, the modulus maxima image is computed using horizontal and vertical subband coefficients[6]

$$M_j(u,v,2^j) = \sqrt{|L^H_j z(u,v,2^j)|^2 + |L^V_j z(u,v,2^j)|^2}$$

(21)

where $L^H_j z(u,v,2^j)$ and $L^V_j z(u,v,2^j)$ denotes horizontal and vertical subband coefficient at the same position.

![Fig. 2. Detection of edge coefficients in various scales for 'disk' image](image)

At each scale, the horizontal and vertical subband components build a gradient image. For the same pair of vertical and horizontal subbands the gradient angle matrix is given by[6]
The subband positions where  \( M_z(u,v,2^j) \) are local maxima in 1-D neighborhood of the gradient direction are probable edge positions. Fig. 2 shows detected edge coefficients at three different scales in dyadic remapped LOT domain for ‘disk’ image. The LOT modulus maxima computation using log transformed real SAR images show reasonably good detection of edges. The singularity information can be well separated out from the additive noise using thresholding of LOT modulus maxima.

The dyadic remapped LOT coefficients after filtering are rearranged back into block decomposition form and then inverse LOT is performed. Due to the use of log transform, the mean of log transformed speckle is biased and therefore require adjustment. Finally, exponential operation is carried out to obtain the despeckled image. Since the LOT lacks shift invariance, we implement the proposed method in cycle spinning mode [17] which significantly reduces the Gibbs effects and also ‘specks’ artifacts in smooth regions. Only nine random shifted versions are considered to avoid large number of computations.

4. Experimental Results

In this section, we use two real intensity SAR images (of size 256x256 pixels) taken from Sandia National Laboratories to validate the effectiveness of proposed despeckling method. The despeckling performance of proposed method is compared with two well known methods namely linear MMSE filtering in undecimated wavelet transform domain (UWT-LMMSE)[7] and MAP estimation in undecimated wavelet transform domain with coefficients of Laplacian reflectivity and Gaussian noise with segmentation (LGMAP-S)[8]. The results of these two methods were obtained from the authors through web based service. The implementation of UWT-LMMSE and LGMAP-S methods uses 9/7 biorthogonal wavelet with four number of multiresolution levels. The implementation of proposed method uses LOT with M=8 and window size of 9x9 in the EM algorithm.

![Fig. 3. Despeckled images for ‘horsetrack’](a) Original (b) Using UWT-LMMSE [7] (c) Using LGMAP-S [8] (d) Using proposed method)

In this paper, the despeckling performance is evaluated using equivalent number of looks (ENL), edge saving index (ESI) and visual analysis of ratio images. The ENL and ESI indexes respectively indicate the degree of speckle suppression (in homogeneous regions) and the edge preservation. ENL and ESI (in horizontal direction \( ESI^H \) and vertical direction \( ESI^V \)) are computed using following expressions:

\[
ENL = \frac{E(\hat{Y}_{SAR})^2}{\text{var}(\hat{Y}_{SAR})}
\]

where \( E(\hat{Y}_{SAR}) \) and \( \text{var}(\hat{Y}_{SAR}) \) are mean and variance in a given homogeneous region.
\[ ESI^H = \frac{\sum_{i=1}^{r-1} \sum_{j=1}^{c-1} \left| Y_{SAR}(i,j+1) - \tilde{Y}_{SAR}(i,j) \right|^2}{\sum_{i=1}^{r-1} \sum_{j=1}^{c-1} \left| Z_{SAR}(i,j+1) - Z_{SAR}(i,j) \right|^2} \]

\[ ESI^V = \frac{\sum_{i=1}^{r-1} \sum_{j=1}^{c-1} \left| Y_{SAR}(i+1,j) - \tilde{Y}_{SAR}(i,j) \right|^2}{\sum_{i=1}^{r-1} \sum_{j=1}^{c-1} \left| Z_{SAR}(i+1,j) - Z_{SAR}(i,j) \right|^2} \]

where \( \tilde{Y}_{SAR} \) is filtered image and \( Z_{SAR} \) is the observed SAR image. \( r \) and \( c \) denotes the number of rows and columns in the SAR image.

Two homogeneous regions namely Region 1 and Region 2 in each intensity SAR image, is used for ENL calculations. Region 1 and Region 2 comprise of 30x43 and 40x35 in ‘horsetrack’ and 25x43 and 37x21 pixels in ‘stadium’ respectively. Since the ratio of real image to the despeckled one is a speckle, the visual inspection of ratio images provide vital information on speckle suppression and edge preservation. In Fig. 3, it is observed that all methods suppress speckle to some extent, however the proposed method (Fig.3(d)) shows best speckle smoothing among all. In terms of edge preservation, UWT-LMMSE shows edge blurring in many regions whereas the LGMAP-S and proposed method show very close performance. The ratio image obtained using our method when compared to other two exhibits least structural details (Fig. 4(d)). The ratio image obtained using UWT-LMMSE shows clear and sharp traces of edge patterns from original image indicating that some important edge structures are oversmoothed during despeckling. The results in Table 1 show that the proposed method performs best in terms of ENL and outperform others by a good margin. UWT-LMMSE exhibits low
value of ESI in most of the cases. The LGMAP-S method shows best values of ESI in most of the cases, however it is obtained at the cost of low values of ENL. The results of Table I seems to be consistent with visual results provided in Fig.3 and ratio image inspection (Fig.4). It is clear from the results that the proposed method outperforms UWT-LMMSE and LGMAP-S in terms of simultaneous speckle suppression and edge preservation.

5. Conclusion

In this paper, we have proposed a new LOT based SAR image despeckling technique which is based on a 2-state Laplace mixture distribution that uses local parameters. This model is able to exploit both the ‘clustering’ property and the heavy tailed nature of dyadic remapped LOT coefficients. We classify the dyadic remapped LOT coefficients into edge and non-edge coefficients using modulus maxima computation strategy. For more effective edge preservation, only the non-edge coefficients are modified using Bayesian MAP estimate which uses proposed prior and edge coefficients are kept unchanged. The simulation results show that the proposed method achieves better performance than a few well known undecimated wavelet domain methods in terms of simultaneous edge preservation and speckle reduction.

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