Phantom Divide Crossing with General Non-minimal Kinetic Coupling

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Abstract

We propose a model of dark energy consists of a single scalar field with a general non-minimal kinetic couplings to itself and to the curvature. We study the cosmological dynamics of the equation of state in this setup. The coupling terms have the form $\xi_1 R f(\phi) \partial_\mu \phi \partial^\mu \phi$ and $\xi_2 R_{\mu\nu} f(\phi) \partial_\mu \phi \partial^\nu \phi$ where $\xi_1$ and $\xi_2$ are coupling parameters and their dimensions depend on the type of function $f(\phi)$. We obtain the conditions required for phantom divide crossing and show numerically that a cosmological model with general non-minimal derivative coupling to the scalar and Ricci curvatures can realize such a crossing.

PACS numbers: 95.36.+x, 98.80.-k, 04.50.kd

Keywords: Crossing of phantom divide; Non-minimal derivative coupling; Tachyon field.
1 Introduction

Recent observational data from CMB temperature fluctuations spectrum, Supernova type Ia redshift-distance surveys and other data sources, have shown that the universe is currently in a positively accelerated phase of expansion [1-4]. Nevertheless, there is not enough standard matter density in the universe to support this accelerated expansion. Therefore, we need additional cosmological component dubbed dark energy to explain this achievement. Dark energy (DE) has been one of most active field in modern cosmology [5]. The simplest candidate for DE is a tiny positive time-independent cosmological constant $\Lambda$. However, this scenario suffers from some difficulties such as lack of physical motivation, huge amount of fine-tuning to explain cosmological accelerated expansion and no dynamics for its equation of state [6].

As a possible solution to these problems, many dynamical scalar field models of DE have been proposed. Quintessence, phantom, k-essence and tachyon scalar fields belong to these sort of DE models (for review see [6]).

In the other hand, there are some datasets (such as the Gold dataset) that show a mild trend for crossing of the phantom divide line by the effective equation of state (EoS) parameter $\omega$ (the ratio of the effective pressure of the universe to the effective energy density of it) of dark component. The equation of state parameter crosses the phantom divide line ($\omega = -1$) at recent redshifts and current accelerated expansion requires $\omega < -1/3$.

The quintom scenario of dark energy is designed to understand the nature of dark energy with $\omega$ across $-1$. To realize a viable quintom scenario of dark energy it needs to introduce extra degree of freedom to the conventional theory with a single fluid or a single scalar field. The first model of quintom scenario of dark energy is given by Ref. [7] with two scalar fields (quintessence and phantom). Another attempts for constructing a quintom model are as follows: scalar field model with non-linear kinetic terms [8] or a non-linear higher-derivative one [9], braneworld models [10], phantom coupled to dark matter with an appropriate coupling [11], string inspired models [12], non-local gravity [13], modified gravity models [14] and also non-minimally coupled scalar field models in which scalar field couples with scalar curvature, Gauss-Bonnet invariant or modified $f(R)$ gravity [15-17]. Crossing of the phantom divide can also be realized with single imperfect fluid [18] or by a constrained single degree of freedom dust like fluids [19]. It has been shown in [45] that in the future the crossing of the phantom divide are the generic feature for all the existing viable $f(R)$ model such as Hu-Sawicki [46], Starobinsky [47], Tsujikawa [48] and the exponential gravity [49, 50] models. A phantom crossing DGP model has been constructed in [51] and the interacting chaplygin gas dark energy model which realizes phantom crossing investigated in [52] (for a detailed review on extended theories of gravity and their cosmological applications see [53]).

Furthermore, non-minimal couplings are generated by quantum corrections to the scalar field theory and they are essential for the renormalizability of the scalar field theory in curved space [20, 21]. One can extend the non-minimally coupled scalar tensor theories, allowing for non-minimal coupling between the derivatives of the scalar fields and the curvature [22]. Such a non-minimal coupling may appear in some Kaluza-Klein theories [23, 24] and also as quantum corrections to Brans-Dicke theory [25]. A model with non-minimal derivative
coupling was proposed in [22, 26, 27] and interesting cosmological behaviors of such a model in inflationary cosmology [28], quintessence and phantom cosmology [29, 30], asymptotic solutions and restrictions on the coupling parameter [31] have been widely studied in the literature. General non-minimal coupling of a scalar field and its kinetic term to the curvature as a source of late-time cosmic acceleration has been analyzed in [32]. Also, non-minimal coupling of modified $f(R)$ gravity and kinetic part of Lagrangian of a massless scalar field has been investigated in [33, 41, 42]. It has been shown that inflation and late-time cosmic acceleration of the universe can be realized in such a model. In this paper we consider the function $f(R)$ as linear in $R$ but we generalize the model allowing extra $R_{\mu\nu}$ coupling with kinetic term of the scalar field. We are interested in our analysis to the case of tachyon scalar field.

The tachyon field in the world volume theory of the open string stretched between a D-brane and an anti-D-brane or a non-BPS D-brane plays the role of scalar field in the context of string theory [34]. What distinguishes the tachyon Lagrangian from the standard Klein-Gordan form for scalar field is that the tachyon action has a non-standard type namely, Dirac-Born-Infeld form [35]. Moreover, the tachyon potential is derived from string theory and should be satisfy some definite properties to describe tachyon condensation and other requirements in string theory. In summary, our motivation for investigating a model with non-minimal derivative coupling and tachyon scalar field is coming from a fundamental theory such as string/superstring theory and it may provide a possible approach to quantum gravity from a perturbative point of view [36-38].

An outline of the present work is as follows: In section 2 we introduce a model of DE in which the tachyon field plays the role of scalar field and the non-minimal couplings between scalar field, the derivative of scalar field, the Ricci scalar and the Ricci tensor are also present in the action. Then we derive field equations as well as energy density and pressure of the model in order to study the EoS parameter behavior in section 3. We obtain the conditions required for $\omega$ crossing $-1$ and using numerical method, we will show that the model can realize the $\omega = -1$ crossing. Section 4 is devoted to our conclusions.

## 2 Field Equations

We start with the following action for tachyon field with general non-minimal derivative couplings to the scalar and Ricci curvatures and also with itself,

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa^2} R - V(\phi) \sqrt{1 + g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} - \frac{1}{2} \xi_1 R f(\phi) \partial_\mu \phi \partial^\mu \phi \right. - \frac{1}{2} \xi_2 R_{\mu\nu} f(\phi) \partial^\mu \phi \partial^\nu \phi \left. - \frac{1}{2} \xi_3 R_{\mu\nu\rho\sigma} f(\phi) \partial^\mu \phi \partial_\rho \phi \partial^\nu \phi \partial^\sigma \phi \right],$$

where $\kappa^2 = 8\pi G = \frac{1}{M_{Pl}}$ while $G$ is a bare gravitational constant and $M_{Pl}$ is a reduced Planck mass, $V(\phi)$ is the tachyon potential which is bounded and reaching its minimum asymptotically. $f(\phi)$ is a general function of the tachyon field $\phi$ and $\xi_1$ and $\xi_2$ are coupling parameters and their dimensions depend on the type of function $f(\phi)$. Note that if we
consider the following restriction on parameters $\xi_1$ and $\xi_2$,

$$2\xi_1 + \xi_2 = 0,$$

then the last two terms in action (1) reduced to $\xi_1 f(\phi)G_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$ which corresponds to non-minimal derivative coupling of scalar field with the Einstein tensor. The cosmological implications of such a theory have been studied in Refs. [28-32].

The models of kind (1) with non-minimal coupling between derivatives of a scalar field and curvature are the extension of scalar-tensor theories and it is shown that these theories cannot be recasting into the Einstein gravity form by a conformal transformation of the metric [22]. A theory with the derivative coupling term $\xi_2 R_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$ has been considered in [31] and constraints on the coupling parameter $\xi_2$ have been obtained using precision tests of general relativity. A general model containing two derivative coupling terms $\xi_1 R\partial^\mu\phi\partial^\mu\phi$ and $\xi_2 R_{\mu\nu}\partial^\mu\phi\partial^\nu\phi$, has been discussed in [26, 27]. It was shown that the de Sitter spacetime is an attractor solution of the model if $4\xi_1 + \xi_2 > 0$. In here we study the model (1) without the restriction (2) on the coupling parameters.

Varying the action (1) with respect to metric tensor $g_{\mu\nu}$, leads to

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa^2(T_{\mu\nu} + \xi_1 T'_{\mu\nu} + \xi_2 T''_{\mu\nu}),$$

where

$$T_{\mu\nu} = V(\phi)\left(\nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu}u\right),$$

$$T'_{\mu\nu} = R(\nabla_\mu \phi \nabla_\nu \phi) + G_{\mu\nu}(\nabla_\phi)^2 - \nabla_\mu \nabla_\nu (\nabla_\phi)^2 + g_{\mu\nu}\Box(\nabla_\phi)^2,$$

and

$$T''_{\mu\nu} = -\frac{1}{2}g_{\mu\nu}\nabla_\gamma \phi \nabla_\lambda \phi R^{\gamma\lambda} + 2\nabla_\gamma \phi \nabla_\lambda \phi - \nabla_\gamma \nabla_\lambda (\nabla_\nu \phi \nabla_\nu \phi) + \frac{1}{2}g_{\mu\nu}\nabla_\gamma \nabla_\lambda (\nabla_\nu \phi \nabla_\nu \phi),$$

here $u = \sqrt{1 + \nabla_\mu \phi \nabla_\mu \phi}$.

One can obtain the scalar field equation of motion by variation of action (1) with respect to $\phi$,

$$\nabla_\mu \left(\frac{V(\phi)\nabla_\mu \phi}{u}\right) - \frac{dV(\phi)}{d\phi}u + \frac{1}{2}f(\phi)\nabla_\mu \left[\nabla_\nu \phi \left(\xi_1 g^{\mu\nu}R + \xi_2 R_{\mu\nu}\right)\right]$$

$$- \frac{1}{2} \left[\xi_1 R\partial_\mu \phi \partial^\mu \phi + \xi_2 R_{\mu\nu}\partial^\mu \phi \partial^\nu \phi\right] \frac{df(\phi)}{d\phi} = 0.$$  

(7)

In a spatially-flat Friedmann-Robertson-Walker (FRW) spacetime with the metric,

$$ds^2 = -dt^2 + a^2(t)(dr^2 + r^2d\Omega^2),$$

the components of the Ricci tensor $R_{\mu\nu}$ and the Ricci scalar $R$ are given by

$$R_{00} = -3(\dot{H} + H^2), \quad R_{ij} = a^2(t)(\dot{H} + 3H^2)\delta_{ij}, \quad R = 6(\dot{H} + 2H^2),$$

(9)
where $H = \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter and $a(t)$ is the scale factor. The equation (7) for a homogeneous time-depending $\phi$ in FRW background (8) reads

$$\frac{\ddot{\phi}}{1 - \dot{\phi}^2} + 3H\dot{\phi} + \frac{1}{V(\phi)} \frac{dV}{d\phi} + \sqrt{1 - \dot{\phi}^2} \left(\frac{3}{2} [(2\xi_1 + \xi_2)\dot{H} + (4\xi_1 + \xi_2)H^2] (2f(\phi)\ddot{\phi} + \frac{df}{d\phi} \dot{\phi}^2) \right. $$

$$\left. + 9(4\xi_1 + \xi_2)H^3 f(\phi)\dot{\phi} + 3(2\xi_1 + \xi_2)\ddot{H} f(\phi)\dot{\phi} + 3(14\xi_1 + 5\xi_2)H \dddot{H} f(\phi)\dot{\phi} \right) = 0. \quad (10)$$

Using equations (4)-(6), the $(0,0)$ component and $(i,i)$ components of equation (3) correspond to energy density and pressure respectively,

$$\rho = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}} + 9\xi_1 H^2 f(\phi)\dot{\phi}^2 + 3(2\xi_1 + \xi_2) \left(\dot{H} f(\phi)\dot{\phi}^2 - H f(\phi)\ddot{\phi} - \frac{1}{2} H \frac{df}{d\phi} \dot{\phi}^3 \right), \quad (11)$$

and

$$P = -V(\phi)\sqrt{1 - \dot{\phi}^2} + (\xi_1 + \xi_2) (3H^2 + 2\dot{H}) f(\phi)\dot{\phi}^2 + 2(\xi_1 + \xi_2) H \left(2f(\phi)\ddot{\phi} + \frac{df}{d\phi} \dot{\phi}^3 \right) $$

$$+ (2\xi_1 + \xi_2) \left(f(\phi)\ddot{\phi}^2 + f(\phi)\dot{\phi}^2 + \frac{5}{2} \frac{df}{d\phi} \dot{\phi}^2 \dot{\phi} + \frac{1}{2} \frac{d^2f}{d\phi^2} \dot{\phi}^4 \right). \quad (12)$$

The modified Friedmann equation for the $(0,0)$ component takes the form,

$$H^2 = \kappa^2 \left(\frac{V(\phi)}{1 - \dot{\phi}^2} \frac{1}{3} \left(9\xi_1 H^2 f(\phi)\dot{\phi}^2 + 3(2\xi_1 + \xi_2) \left(\dot{H} f(\phi)\dot{\phi}^2 - H f(\phi)\ddot{\phi} - \frac{1}{2} H \frac{df}{d\phi} \dot{\phi}^3 \right) \right) \right. $$

$$\left. + 9(4\xi_1 + \xi_2)H^3 f(\phi)\dot{\phi} + 3(2\xi_1 + \xi_2)\ddot{H} f(\phi)\dot{\phi} + 3(14\xi_1 + 5\xi_2)H \dddot{H} f(\phi)\dot{\phi} \right) = 0. \quad (13)$$

Next, we want to study the effects of general non-minimal derivative couplings on the cosmological evolution of the EoS and see how the present model can be used to realize a crossing of phantom divide $\omega = -1$.

## 3 Crossing of the $\omega = -1$ with General Non-minimal Derivative Couplings

We now study the behavior of the equation of state for the present model. From the definition of EoS ($\omega = \frac{P}{\rho}$) one can obtain $P + \rho = (1 + \omega)\rho$. Using equations (11) and (12) we have the following expression,

$$\rho + P = \frac{V(\phi)\dot{\phi}^2}{\sqrt{1 - \dot{\phi}^2}} \frac{1}{3} \left(9\xi_1 H^2 f(\phi)\dot{\phi}^2 + 3(2\xi_1 + \xi_2) \left(\dot{H} f(\phi)\dot{\phi}^2 - H f(\phi)\ddot{\phi} - \frac{1}{2} H \frac{df}{d\phi} \dot{\phi}^3 \right) \right) $$

$$+ 9(4\xi_1 + \xi_2)H^3 f(\phi)\dot{\phi} + 3(2\xi_1 + \xi_2)\ddot{H} f(\phi)\dot{\phi} + 3(14\xi_1 + 5\xi_2)H \dddot{H} f(\phi)\dot{\phi} \frac{d^2f}{d\phi^2} \dot{\phi}^4 \right).$$
Since \( P + \rho = (1 + \omega) \rho \), one needs \( \rho + P = 0 \) when \( \omega \) goes to \(-1\). Then to check the possibility of the crossing of the phantom divide line \( \omega = -1 \), we must explore for conditions that \( \frac{d}{dt}(\rho + P) \neq 0 \) when \( \omega \) crosses over \(-1\).

Using equation (14) we derive the following equation,

\[
\frac{d}{dt}(\rho + P) = \sqrt{1 - \dot{\phi}^2} \cdot \left( \frac{V(\phi)\dot{\phi}^3}{1 - \dot{\phi}^2} + \frac{2V(\phi)\ddot{\phi}}{1 - \dot{\phi}^2} + \frac{V(\phi)\dddot{\phi}}{1 - \dot{\phi}^2} + 3(4\xi_1 + \xi_2)(2H^2f(\phi)\dddot{\phi} + \dot{H}\frac{df}{d\phi}\dddot{\phi} + \dddot{H}f(\phi)\dddot{\phi}) \right.
\]

\[
+ \frac{H^2}{\dddot{\phi}} \frac{df}{d\phi} \dddot{\phi}^3 + 2H\frac{df}{d\phi}f(\phi)\dddot{\phi}^3 + (8\xi_1 + 5\xi_2)(2Hf(\phi)\dddot{\phi} + \dddot{H}\frac{df}{d\phi} \dddot{\phi}^3 + \dddot{H}f(\phi)\dddot{\phi})
\]

\[
-(2\xi_1 - \xi_2)\left( \dot{H}f(\phi)\dddot{\phi} + Hf(\phi)\dddot{\phi} + \frac{1}{2}\frac{df}{d\phi}\dddot{\phi}^2(5H\dddot{\phi} + \dddot{H}) + \frac{1}{2}\frac{d^2f}{d\phi^2}\dddot{\phi}^4 \right)
\]

\[
+(2\xi_1 + \xi_2)\left( f(\phi)\dddot{\phi} + \frac{df}{d\phi}\dddot{\phi}(\dddot{\phi}^2 + \frac{7}{2}\dddot{\phi}) + \frac{9}{2}\frac{df}{d\phi}\dddot{\phi}^3\dddot{\phi} + \frac{1}{2}\frac{d^2f}{d\phi^2}\dddot{\phi}^4 \right). \tag{15}
\]

Now, we mention the following point: if we consider the restriction (2) then one of the possibilities to have \( \rho + P = 0 \) and \( \frac{d}{dt}(\rho + P) \neq 0 \) is \( \dddot{\phi} = 0 \) [39]. But in the case of our interest \((2\xi_1 + \xi_2 \neq 0)\), the condition \( \dddot{\phi} = 0 \) leads to \( \rho + P = 0 \) and \( \frac{d}{dt}(\rho + P) = 0 \) i.e. the impossibility for having \( \omega = -1 \) crossing. So, in order to have crossing of the phantom divide the only possibility is as follows,

\[
\dddot{\phi}^2\left( \frac{V(\phi)}{1 - \phi^2} + 3(4\xi_1 + \xi_2)H^2f(\phi) + (8\xi_1 + 5\xi_2)\dot{H}f(\phi) - \frac{1}{2}(2\xi_1 - \xi_2)H\frac{df}{d\phi}\dddot{\phi}
\]

\[
+ \frac{1}{2}(2\xi_1 + \xi_2)\left( \frac{5}{2}\frac{df}{d\phi}\dddot{\phi} + \frac{d^2f}{d\phi^2}\dddot{\phi} \right) \right) = (2\xi_1 - \xi_2)Hf(\phi)\dddot{\phi} - (2\xi_1 + \xi_2)f(\phi)(\dddot{\phi}^2 + \dddot{\phi}). \tag{16}
\]

The above condition simplifies the equation (15) as,

\[
\frac{d}{dt}(\rho + P) = \sqrt{1 - \dot{\phi}^2} \cdot \left( \frac{V(\phi)\dot{\phi}^3}{1 - \dot{\phi}^2} + \frac{\dddot{\phi}}{1 - \dot{\phi}^2} + 3(4\xi_1 + \xi_2)H^2\frac{df}{d\phi}\dddot{\phi}^3 + (8\xi_1 + 5\xi_2)\dot{H}f(\phi)\dddot{\phi}^2
\]

\[
-(2\xi_1 - \xi_2)\left( \dot{H}f(\phi)\dddot{\phi} + \frac{3}{2}H\frac{df}{d\phi}\dddot{\phi}^2\dddot{\phi} + \frac{1}{2}\frac{d^2f}{d\phi^2}\dddot{\phi}^4 \right) + (2\xi_1 + \xi_2)\left( f(\phi)\dddot{\phi} \right.
\]

\[
+ \dddot{\phi} - \frac{2\dddot{\phi}^3}{\dddot{\phi}} + \frac{df}{d\phi}\dddot{\phi}(\dddot{\phi}^2 + \frac{7}{2}\dddot{\phi}) + \frac{7}{2}\frac{df}{d\phi}\dddot{\phi}^3\dddot{\phi} + \frac{1}{2}\frac{d^2f}{d\phi^2}\dddot{\phi}^5 \right). \tag{17}
\]

One can see from (17) that, even if \( \dddot{\phi} = 0 \) and \( \dddot{\phi} = 0 \), crossing \(-1\) can happen. So, crossing of the phantom divide in our model can occur in the minimum of the tachyon potential where one expects \( \phi \neq 0 \) and \( \phi = \phi = 0 \).
This outcome is the same as the result of Ref. [17] where tachyon field non-minimally coupled to Gauss-Bonnet invariant but in contrast with the result of Ref. [40], where the authors have added a term $\phi \Box \phi$, in the square root part of action (1) without non-minimal derivative coupling terms and concluded that crossing over $-1$ must takes place before reaching potential to its minimum. Note that in our model there is no extra term but we have included non-minimal coupling of tachyon field with its derivative and curvatures.

In summary, it seems that in studying phantom divide crossing cosmology the non-minimal coupling of tachyon field with its derivative and the Ricci curvatures has the same effects as coupling of tachyon to Gauss-Bonnet invariant where crossing over $-1$ can be happen when tachyon potential reaches its minimum asymptotically.

In order to show that our model can realize crossing of $\omega = -1$ more clearly, we choose two specific tachyon potentials and study evolution of EoS numerically. Figure 1 shows such a numerical calculations for $V(\phi) = V_0 e^{-\alpha \phi^2}$ with constant $\alpha$ and for another tachyon potential $V(\phi) = \frac{V_0}{\phi^2}$. It has been shown that crossing of $\omega = -1$ can be realized in our model. Also we have used the function $f(\phi) = b \phi^n$ with constants $b$ and $n$.

![Figure 1](image.png)

Figure 1: The plots of EoS versus redshift $z$, (left for the potential $V(\phi) = V_0 e^{-\alpha \phi^2}$ and right for the potential $V(\phi) = \frac{V_0}{\phi^2}$), $\phi = \phi_0 t$, $f(\phi) = b \phi^n$ and $H = \frac{H_0}{t}$, (with $\xi_1 = \xi_2 = 10$, $b = 1$, $n = 5$, $V_0 = 4$, $h_0 = 100$, $\phi_0 = 0.5$ and $\alpha = 5$).

Now we discuss on the stability of the model. The sound speed expresses the phase velocity of the inhomogeneous perturbations of the tachyon field. To achieve the classical
stability, we must have $C_s^2 \geq 0$, where

$$C_s^2 = \frac{P'}{\rho'}$$

$$= \frac{1}{2} \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}} + (\xi_1 + \xi_2) \left( (3H^2 + 2\dot{H})f + 2H \left( f\ddot{\phi} + \frac{3}{2} \frac{df}{d\phi} \dot{\phi} \right) \right) + (2\xi_1 + \xi_2) \left( \frac{1}{2} f \frac{d^2 f}{d\phi^2} + \frac{5}{2} \frac{df}{d\phi} \dot{\phi} + \frac{d^2 f}{d\phi^2} \dot{\phi} \right)$$

$$= \frac{1}{2} \frac{V(\phi)}{(1-\dot{\phi}^2)^{\frac{3}{2}}} + 9\xi_1 H^2 f + 3(2\xi_1 + \xi_2) \left( \dot{H} f - \frac{1}{2} H f \frac{d^2 f}{d\phi^2} - \frac{3}{4} H \frac{df}{d\phi} \dot{\phi} \right),$$

(18)

where a prim denotes derivative with respect to $\dot{\phi}^2$.

In figure 2, we have plotted the $C_s^2$ for the models considered in this paper for the numerical calculations. From this figure, we can see that the sound speed parameter is positive throughout the phantom divide crossing phase.

Figure 2: The plots of the sound speeds versus redshift $z$, (left for the potential $V(\phi) = V_0 e^{-a\phi^2}$ and right for the potential $V(\phi) = \frac{V_0}{\phi}$) , $\phi = \phi_0 t$, $f(\phi) = b\phi^n$ and $H = \frac{h_0}{t}$, (with $\xi_1 = \xi_2 = 10$, $b = 1$, $n = 5$, $V_0 = 4$, $h_0 = 100$, $\phi_0 = 0.5$ and $a = 5$).

4 Conclusion

In order to solve cosmological problems and because the lack of our knowledge, for instance to determine what could be the best candidate for DE to explain the accelerated expansion of the universe, the cosmologists try to approach to best results as precise as they can by considering all the possibilities they have.

The two most reliable and robust SnIa datasets existing at present are the Gold dataset [1] and the Supernova Legacy Survey (SNLS) [43] dataset. The Gold dataset compiled by Riess
et. al. is a set of supernova data from various sources analyzed in a consistent and robust manner with reduced calibration errors arising from systematics. It contains 143 points from previously published data plus 14 points with \( z > 1 \) discovered recently with the HST. The SNLS is a 5-year survey of SnIa with \( z < 1 \). The published first year SNLS dataset consists of 44 previously published nearby SnIa with \( 0.015 < z < 0.125 \) plus 73 distant SnIa \( 0.15 < z < 1 \). The following comments can be made for phantom divide crossing based on the cosmological data [44]: The Gold dataset mildly favors dynamically evolving dark energy crossing the phantom divide at \( z \approx 0.2 \) over the cosmological constant while the SNLS does not. Dark energy probes other than SnIa that include the CMB, BAO, Clusters Baryon Fraction and growth rate of perturbations mildly favor crossing of the phantom divide for low values of \( \Omega_{0m} \) (\( \Omega_{0m} \leq 0.25 \)).

Within the different candidates to play the role of the DE, the quintom model, has emerged as a possible model with EoS across \(-1\). In this paper, we have proposed a model of dark energy with non-minimally kinetic coupled scalar field, where the kinetic term is not only coupled to itself through the function \( f(\phi) \), but to the Ricci scalar and the Ricci tensor. We have studied cosmological evolution of EoS in this setup where tachyon field played the role of scalar field. We considered the non-minimal kinetic couplings, without the restriction on the coupling constants \( \xi_1 \) and \( \xi_2 \) namely equation (2) and obtained the condition required for phantom divide crossing as equation (16). It has been shown that the \( \omega = -1 \) crossing can be realized even if the potential goes to its minimum asymptotically and this result is the same as that in [17].

Using the numerical methods we showed that the crossing of phantom divide occur for special potentials and coupling function. It may be interesting to consider different potentials and coupling functions in this setup.

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