Generating an effective magnetic lattice for ultracold atoms

Xinyu Luo\textsuperscript{1,2}, Lingna Wu\textsuperscript{1}, Jiya Chen\textsuperscript{1}, Rong Lu\textsuperscript{1,3}, Ruquan Wang\textsuperscript{2,3} and I You\textsuperscript{1,3}

\textsuperscript{1} State Key Laboratory of Low Dimensional Quantum Physics, Department of Physics, Tsinghua University, Beijing 100084, People’s Republic of China
\textsuperscript{2} Institute of Physics, Chinese Academy of Sciences, Beijing 100080, People’s Republic of China
\textsuperscript{3} Collaborative Innovation Center of Quantum Matter, Beijing, People’s Republic of China

E-mail: hxuyin@mail.tsinghua.edu.cn and lyou@mail.tsinghua.edu.cn

Keywords: spin–orbit coupling, atomic Bose–Einstein condensate, magnetic lattice, optical lattice

Abstract

We present a general scheme for synthesizing a spatially periodic magnetic field, or a magnetic lattice (ML), for ultracold atoms using pulsed gradient magnetic field (GMF). Our scheme is immune to atomic spontaneous emission often encountered in optical lattices, and has the additional benefits of easy tunability for both the lattice period and depth. Technical requirements for the experimental protocol implementing our scheme is estimated and shown to be readily available in today’s cold atom laboratories. The effective Hamiltonian for atoms interacting with the synthesized two-dimensional ML has not been studied in quantum condensed matter physics previously. Its band structure shows interesting features reminiscent of lattice models in p-orbit physics. Realization of our proposal will significantly expand the repertoire for quantum simulation with ultracold atoms.

1. Introduction

Optical lattice (OL) is a highly controllable environment where many body physics can be studied with ultracold atoms [1, 2]. Additionally, atoms in OL promise exciting opportunities in quantum information science [3–5]. Many lattice geometries have been realized experimentally, from three-dimensional (3D) cubic lattices [6] to honeycomb lattices [7] and kagome lattices [8]. With spin-dependent OLs, atomic internal degrees of freedom such as its spin or pseudo-spin (group of internal states), are coupled to its spatial degrees of freedom. This can give rise to interesting phenomena [9–18] absent in spin-independent lattices. For example, attractive Fermi gases in 1D lattices support three-body bound states with only two-body interactions when the tunneling rates are spin-dependent [19]. Theoretical studies predict a exotic state with the coexistence of superfluid and normal components [20] in 3D systems. For bosons, a new phase supported by spin-dependent OLs is already revealed experimentally, where one spin component is Mott-insulating and the other superfluid, with the superfluid to Mott-insulating transition modulated by their mutual interactions [21].

A topical area of intense research interest in ultracold atoms concerns synthetic gauge fields [22]. Many theoretical studies have proposed ideas to synthesize artificial gauge potentials for atoms (with hyperfine spin $F$) in OL systems [23–27], often starting with simple forms of lattices $aF_2$ or spin-dependent lattices and periodically driven systems [28–30]. Some of these ideas are realized in recent experiments [31, 32]. They emulate atomic interactions with synthetic magnetic fields or spin–orbit coupling (SOC). More general spin-dependent lattices and artificial gauge fields, such as interaction forms that flip atomic spins can broaden the scope of quantum simulations and support exotic quantum phases [33–37]. For example, Cocks et al [33] find fermionic systems exhibit quantum phases such as topological and normal insulator, metal, or semi-metal, all with two or more Dirac cones even in the absence of atomic interactions when a staggered potential is added to an artificial Rashba-type SOC. In the presence of strong atomic interactions, semi-metal to antiferromagnetic insulator transition can occur.

Spin-dependent OL can be readily generated by light shifts with spin-dependent modulations [38], as in the familiar lin–$\theta$–lin laser cooling setup [12, 39]. 1D effective Zeeman lattice can be produced by combining a radiofrequency (RF) magnetic field with Raman laser fields [40]. However, due to the same scaling with laser intensity
and detuning, the ratio of the effective spontaneous emission rate to the spin-dependent lattice depth is determined by the ratio of atomic natural linewidth to its excite state fine structure splitting. The resulting spin-dependent lattice depth will be small in order to suppress spontaneous emission [41–43]. Furthermore, ideas based on light-atom interaction are limited by laser wavelengths, which set the typical scales of the resulting OL spatial periods, and are difficult to tune for given setups. Larger spacing spin-dependent lattice potentials can be realized with micro-fabricated wires or permanent magnet arrays on an atom chip [44–47]. Recently, sub-micron lattice spacing structures consisting of permanent magnet arrays are fabricated on an atom chip [48]. Magnetic lattice (ML) with spatial period as small as tens of nanometers is proposed using superconducting vortex arrays on an atom chip with reduced Johnson noise [49].

This work presents a new scheme for synthesizing a spatially periodic magnetic field or a ML using pulsed gradient magnetic field (GMF). It can be understood in terms of spatial dependent spin rotations, which couple atomic internal degrees of freedom to its spatial/orbital degrees of freedom. While sharing some features with an earlier proposal for synthesizing SOC with pulsed GMFs [50] as well as an analogous one using zero average modulated GMFs [51], the idea we present here is distinct. As we show in this work it allows for the generation of a new class of synthesized MLs not previously explored. The earlier SOC proposal [50, 51] is recently demonstrated experimentally [52], which makes the idea we present here an attractive follow up experiment. Our ML idea is implemented by introducing a bias magnetic field to the free evolution part of the earlier SOC protocol [50]. The lattice constant is found to be easily tunable and can be made smaller, even much smaller than the laser wavelength of an OL. Furthermore, the scheme we present can be generalized in a straightforward manner to more than one spatial dimension.

This paper is organized as follows. In section 2, our idea for synthesizing a ML with a bias magnetic field included during the free evolution period of the SOC protocol [50] is introduced. The dynamics governed by the synthesized Hamiltonian are analyzed, numerically simulated, and shown to support our claim that the protocol we present is valid and effective. The band structures for our ML are then computed and briefly discussed. In section 3, signatures for experimentally synthesizing and confirming our ML are discussed. Section 4 is devoted to discussions of experimental implementations and tunability issues of the ML idea we propose. Finally we summarize and discuss several potential experimental challenges for implementing our idea.

2. An effective ML from spatial dependent spin rotation

Our idea for generating a two-dimensional (2D) ML can be most easily appreciated in comparison to the earlier SOC protocol using GMF pulses [50]. As illustrated in figure 1(a) for one period, the first (second) half is composed of free evolution in a uniform magnetic field $B_0 \hat{z}$, sandwiched in between two short x- (y-) GMF pulses $B' \hat{x}$ ($B' \hat{y}$) with opposite amplitudes. $B'$ denotes the averaged first order spatial derivative, or the spatial gradient of the magnetic field. $\delta t'$ denotes the duration of each pulse, which is assumed small and the same for all pulses, while $\delta t$ is the duration between the two pulses. $T/2 = \delta t$ is half the period. Unlike the SOC protocol [50] with no bias along z-direction, a nonzero bias magnetic field along z-direction gives rise to a 2D ML in the x–y plane.

The dynamics from the first pulse are simple, with its evolution operator given approximately by $U_z' (\delta t') = \exp\left( -i g_\text{r} \mu_B B' x F_k \delta t' / \hbar \right)$, only due to the single atom Zeeman term $g_\text{r} \mu_B B' x F_k$, when it is assumed to overwhelm all other interactions. $\mu_B$ is the Bohr magneton, $g_\text{r}$ is the Lande g-factor for the ground state Zeeman manifold considered, e.g., with hyperfine spin $F = 1$ (as for $^{87}$Rb atom). $F_{x,y,z}$ denotes the x-, y-, and z-component of $\vec{F}$. As shown in the appendices A and B, with the use of Trotter expansion the pair of x-gradient pulses give (B.1)

$$U_z' (T/2, 0) = U_z' (\delta t') \exp\left( -i H_0 \delta t / \hbar \right) U_z'^\dagger (\delta t') = \exp\left( -i H_\text{eff}^{(2)} \delta t / \hbar \right),$$

which transforms the free evolution Hamiltonian $H_0 = \hbar^2 k^2/(2m) + \hbar \omega_0 F_z$ into an approximate one with a SOC term plus a ML term

$$H_\text{eff}^{(x)} = \frac{\hbar^2}{2m} \left( k_x - k_\text{so} F_z \right)^2 + k_y^2 + \hbar \omega_0 \left[ \cos (k_\text{so} x) F_z + \sin (k_\text{so} x) F_x \right],$$

where $\omega_0 = g_\text{r} \mu_B B_0 / \hbar$ is the Larmor frequency corresponds to the z-bias field $B_0$ and $k_x$ and $k_y$ are the x- and y-components of the atomic momentum $k$ respectively. The first term of equation (2) corresponds to the protocol for generating SOC [50], whose strength is given by the momentum impulse $\hbar k_\text{so} = g_\text{r} \mu_B B' \delta t'$ from the gradient pulse [50]. The second term effects the ML of our propose, which is $\cos (k_\text{so} x) F_z + \cos (k_\text{so} x) \hat{z}$ with wave vector $k_\text{so}$ as illustrated in figure 1(b). We can proceed with a pair of y-gradient pulses as shown in figure 1(a). If the effective action from each cycle is small such that we can use Trotter expansion (see equation (B.1)) to the first order and combine the non-commuting x- and y-dependent terms into the same exponent, we end up with the 2D version
\[
U_{2D}(T, 0) = U_y(T, T/2) U_x(T/2, 0) \approx \exp\left(-i H_{\text{eff}}^{(2D)} T / \hbar\right),
\]

with

\[
H_{\text{eff}}^{(2D)} = \frac{\hbar^2}{2m} \left( k_x - \frac{1}{2} k_{\omega x} F_x \right)^2 + \frac{\hbar^2}{2m} \left( k_y - \frac{1}{2} k_{\omega y} F_y \right)^2 + \frac{\hbar^2 k_{\omega z}^2}{8m} \left( F_x^2 + F_y^2 \right)
\]  
\[+ \frac{1}{2} \hbar \omega \left( F_x \cos \left( k_{\omega y} x \right) + F_y \sin \left( k_{\omega y} y \right) \right)\]
\[+ \frac{1}{2} \hbar \omega \left( F_x \cos \left( k_{\omega x} x \right) - F_y \sin \left( k_{\omega y} y \right) \right),\]  

where \(\hbar^2 k_{\omega z}^2 (F_x^2 + F_y^2)/8m\) acts as a quadratic Zeeman shift (QZS). The leading order correction to the time evolution operator \(2\) for the effective Hamiltonian \(3\) including 2D ML is \(\delta U_{1D}^{(2)}(T, 0) \approx \max(\omega_{\omega z}^2, \omega_{\omega y}^2) \mathcal{O}(\delta t^2)\) assuming \(k_x, k_y \leq k_{\omega z}\) with \(\omega_{\omega z} = \hbar k_{\omega z}/2m\). When the Trotter expansion fails, one can simply reduce the free evolution time, which essentially constrains the strength of the synthesized ML \([50]\). More details on the relevant discussions can be found in the appendices. The synthesized \(\vec{B} (x, y) = B_0 [-\sin(k_{\omega y} y), \sin(k_{\omega x} x), \cos(k_{\omega x} x) + \cos(k_{\omega y} y) / 2] \) in our 2D ML \(H_{\text{eff}}^{(2D)}\) gives the following eigenvalues \((g_{\omega x} \mu_B |\vec{B}|, 0, -g_{\omega y} \mu_B |\vec{B}|)\) corresponding to spinor components \(|F = 1, M_F\rangle\) respectively for \(M_F = 0, 1, \text{and} -1\), for quantized along the position dependent direction of the local artificial ML field. The effective magnetic field \(|\vec{B}| = B_0 \sqrt{[1 + \cos(k_{\omega y} x) \cos(k_{\omega y} y)] / 2}\) is periodic in space with lattice constant \(a = 2\pi/k_{\omega z}\) as shown in Figure 1.
figure 2(a), which provides a periodic interaction potential in the \( x - y \) plane, where atoms can be trapped at local minima provided their kinetic energies are sufficiently small. The potential surface is shown by a 2D contour plot in figure 2(a). Along a line cut in one spatial direction as marked by the dashed line in figure 2(a), the potentials for the three spin states \( |M\rangle \) are shown in figure 2(b). Like in a real magnetic field, atoms moving across zeros of the synthetic magnetic field will undergo Majorana spin flips. To plug these zero field holes, an additional constant Zeeman term \( \alpha \) can be included in equation (3) between successive GMF pulse pairs for a duration \( \delta t \) to introduce extra free evolution. The net effective magnetic field then becomes \( \vec{B}_{\text{xy}}(x, y) = B_0 \left[ -\left( 1 - \alpha \right) \sin(k_{\text{so}}y), \left( 1 - \alpha \right) \sin(k_{\text{so}}x), \left( 1 - \alpha \right) \cos(k_{\text{so}}x) + \left( 1 - \alpha \right) \cos(k_{\text{so}}y) + 2\alpha \right] / 2 \) with \( \alpha = \delta t/T \) and \( T = 2\delta t + \delta \). A nonzero \( \alpha \) then plugs the zero field holes of the effective magnetic field.

The typical band structure of our 2D ML is shown in figure 3(a), which is found to be similar to the results for lattice models with p-orbital physics [53, 54], and distinctively different from usual OLs. The lowest two bands touch at \( \Gamma = (0, 0) \) and \( X = (2, 0) \) points along the \( \Gamma - X \) line. The third and fourth bands touch at \( M = (2, 2) \) and \( X \) points. The band touching points are found to be robust against tuning of lattice depth. This degeneracy can be broken leading to a gap if a constant Zeeman term \( \alpha \) is added to equation (3) (a nonzero \( \alpha \) as discussed previously) as shown in figure 3(b). The resulting band structure is found to be topologically trivial. Several variants of equation (3) generated by other pulse sequences, however, can indeed display nontrivial topological band structure. These new results will be further explored and published elsewhere [55]. To our knowledge, the type of 2D ML we generate here, is new to quantum condensed matter physics, and has not been discussed before.
Figure 4 compares a sampling of numerical solutions for the corresponding Gross–Pitaevskii equations \[56\]. For a fixed evolution time \( T = 100 \mu s \), the results propagated from the actual dynamics using 1, 5, and 20 pulses (corresponding to \( \delta t = 50, 10, 2.5 \mu s \) ) are shown in the 1st, 2nd, and 3rd columns, respectively. With increasing numbers of pulse cycles, atomic density distributions converge towards that from the effective dynamics by \( H_{\text{eff}}^{(2)} \), which are shown in the last column. We find that the required error bounds \( \delta \omega_\omega \leq \max (\omega_0, \omega_0') \delta t^2 \) can always be satisfied when \( \delta t \) is sufficiently short. Extensive simulations show that our idea is both effective and efficient. This conclusion is also supported by the analytic derivations of the effective interaction above as well as more detailed steps in the appendices, where all approximations used are found to be reasonable under most circumstances.

3. Experimental detection

In this section, we discuss how the ML we synthesize can be detected, by making use of lattice induced atomic diffractions for the simple 1D case.

The periodic ML potential induces atomic diffraction, which in turn can be used for its detection. Let us consider the simple 1D case, and focus on the parameter regime \( \omega_0 \gg \omega_R \), the Hamiltonian \( H_{\text{eff}}^{(x)} \) (equation (2)) then reduces to

\[
H_x = \hbar \omega_0 \left[ F_x \cos (k_{ax} x) + F_y \sin (k_{ax} x) \right],
\]

after neglecting the SOC term. It is invariant under spatial translations over multiple lattice spatial periods along \( x \)-direction. After a \( \pi/2 \) spin rotation transformation about the \( y \)-axis, it becomes

\[
H_y' = e^{-i\frac{\pi}{2} F_y} H_x e^{i\frac{\pi}{2} F_y} = \frac{1}{2} \hbar \omega_0 \left[ F_x e^{-i k_{ax} x} + F_y e^{i k_{ax} x} \right],
\]

where \( F_x = \sigma_x + i \sigma_y \) are the spin raising and lowering operators. Thus the ML represents a particular type of SOC, as an atom lowers (raises) its internal spin state, it gains (or loses) momentum \( \hbar k_{ax} \). If the ML is suddenly turned on, atoms will start to oscillate among different spin-momentum eigenstates \( | M_x, k_x \rangle \), with frequency \( \omega_0 \), where \( k_{ax}^{(0)} \) denotes the initial atomic kinetic momentum, and \( n \) denotes the diffraction order for an atom by the ML. The spin-momentum population oscillations can be detected by time of flight imaging after Stern–Gerlach magnetic field pulses, which reveal clear signatures of the synthesized 1D ML, as we illustrate in figure 5.

The interaction terms \( \alpha F_x, F_y \) in the ML we synthesize couple spatial diffractions with atomic spin flips, thus they can also be viewed as particular types of SOC. For the simple 1D case considered, the diffraction orders from our ML is finite, limited by the finite spin, rather than being infinite as for the usual spin–dependent lattice \( \alpha F_y \).

When spin flip pulses or additional free evolution periods are introduced between successive 1D ML pulse pairs,
The key component of our protocol is the pulsed 1D GMF $B' x x$ which enacts position dependent spin rotations. Such a magnetic field does not exist alone because the divergence and curl of a magnetic field vanishes in free space. However, a 1D GMF can be effectively implemented with the help of a strong bias field, which selects out the gradient from any inhomogeneous field configuration [52]. In actual experiments, it can also be achieved by the combination of a RF quadrupole magnetic field $\cos (\omega t + \phi) B'(x x - z z)$ with a strong bias field $B'_{y0} \hat{z}$, following an earlier suggestion from [51]. In this case, the depth of our ML depends explicitly on the detuning between the RF field and the bias field $B_{y0} \hat{z}$ Zeeman shift as detailed in appendix A. The validity conditions for our ML proposal require $\max (\omega_0^k, \omega_0^k \delta t] \ll 1$ and $\omega_0 \delta t \ll 1$, which are carefully derived in appendix B. Taking all considerations into account, we conclude that atom chip based setups are more suitable for generating the required fast GMF pulses. Figure 6 illustrates a possible setup for producing the required GMF in an atom chip, with a strong bias field $B'_{y0} \hat{z}$ applied along $z$-direction. When the four parallel wires (with 100 $\mu$m adjacent spacing) along x- (or y-) direction are respectively carrying RF modulated currents with alternating directions, a 2D quadrupole magnetic field $\cos (\omega t + \phi B') (x x - z z)$ [or $\cos (\omega t + \phi) B'(y y - z z)$] is generated [57]. A gradient field strength of 0.25 Gauss/$\mu$m results from a current of $\sim 1$ A, which is nowadays readily available in atom chip implementations. With realistic values of $B_0 = 10$ mili-Gauss, $\delta t = 10$ $\mu$s, $\delta t' = 2.5$ $\mu$s, and $B' = 0.25$ Gauss $\mu$m$^{-1}$ [58], we estimate typical atom chip based implementation can achieve $k_{y0} \sim 2.9$ $\mu$m$^{-1}$ $\sim 0.5 (k_y)$ (or lattice constant $\sim 2$ $\mu$m) and lattice depth $\omega_0 \sim (2\pi) 7$ kHz $\sim 14 \omega_0$, where $k_y$ is the one photon

![Figure 5](image1.png)

**Figure 5.** (a) The ML couples different spin-momentum states $|M_x, k_x = n k_{y0} + k^{(0)}_y\rangle$, where $k^{(0)}_y$ denotes the initial atomic kinetic momentum, and $nk_{y0}$ denotes the nth order diffracted momentum by ML. Atoms gain (or lose) momentum $\hbar k_{y0}$ when their internal spin states are lowered (raised). (b) The red dotted-dashed, blue dashed, and black solid lines respectively denote oscillating populations for states $|M_x = -1, n = 2\rangle$, $|M_x = 0, n = 1\rangle$, and $|M_x = 1, n = 0\rangle$.

![Figure 6](image2.png)

**Figure 6.** (a) A schematic of the envisaged wire configuration on an atom chip for producing the required gradient magnetic fields. A strong bias field $B'_{y0} \hat{z}$ is applied in the $z$-direction. RF quadrupole magnetic fields are generated by four parallel wires carrying RF currents with alternating current directions as indicated by the arrows. The strong bias field and the RF quadrupole field $\alpha B'(x x - z z)$ (or $\alpha \psi B'(y y - z z)$) together generate an effective 1D gradient magnetic field in $x$- (or $y$-) direction. Ultracold atoms (blue disks) are held by a quasi 2D dipole trap about 90 $\mu$m above the chip surface. (b) The quasi 2D dipole trap is formed by reflecting a focused far off-resonant laser beam (red arrows) from a substrate of high reflection coating layer on the atom chip. The drawing is not to scale.
recoil momentum for a typical laser wavelength $\lambda_2 = 1064$ nm. In the vicinity of the local minimum of the ML, the trapping frequency along x- (y-) direction is approximately given by $\sqrt{\hbar \omega_0 k_{\text{eff}}^2/2m}$, which is estimated to be about 2 kHz for the parameters mentioned above. Tighter trapping along the z-direction is realized with a 1D OL formed by reflecting a focused far off-resonant laser beam from the atom chip substrate surface as demonstrated in [59]. At red detuning, the focused laser beam also provides additional confinement along x- and y-directions, holding the atoms in the resulting quasi 2D trap, whose typical trap frequencies along x-, y-, z-directions are about $2\pi \times (30, 30, 1000)$ Hz respectively. With a condensate of $1 \times 10^5$ atoms, the transverse Thomas-Fermi in the quasi 2D trap is about 20 $\mu$m, which corresponds to about $20 \times 20$ lattice sites. In actual experimental implementation, uncertainties of controlling $k_{\text{in}}$ gives rise to overall shifts of the lattice sites. A 0.5% error in $k_{\text{in}}$ gives rise to a position shift of only 0.1 lattice constant, which seems rather small and acceptable. One can even go beyond $k_{\text{in}} \sim k_L$ with stronger and shorter gradient pulses [58, 60]. In real experiments, using higher GMF requires stronger bias field which is accompanied with increased QZS. Eventually this will compromise the spin level symmetry at weak magnetic fields and reduce a spin $F = 1$ atomic system to a two-level or pseudo-spin $1/2$ system, for instance, composed of the $|F = 1, M_F = -1\rangle$ and $|F = 1, M_F = 0\rangle$ states in $^{87}$Rb atom. This limit can alternatively be viewed as a ML for a two-component atomic condensates.

Loading of ultracold atoms into the synthetic ML is straightforward. With atoms initially prepared in their ground state, turning on the time modulating GMF adiabatically will shift atoms into the ground state of the effective Hamiltonian [51], as was demonstrated in the recent experiment which synthesizes SOC with a modulating gradient GMF [52]. Additionally, kinetics from terms neglected during the approximation leading to the effective Hamiltonian will likely slave the system to the ground state of the effective Hamiltonian, as already shown to occur for a variety of time dependent control protocols in atomic quantum gas systems [61]. Compared with most OL schemes, both the SOC strength $k_{\text{in}}$ and the ML depth $\hbar \omega_0$ can be independently tuned in our scheme. In the optical Raman scheme [40, 62], $k_{\text{in}}$ is usually fixed given single photon recoil momentum and the intersection angle of the two Raman laser beams. It is difficult to tune continuously in any experiment. A recent experiment achieved continuous tuning to weaker SOC strength with periodic modulations applied to Raman laser phases [63]. In our protocol, $k_{\text{in}}$ is determined by the momentum impulse from a single gradient pulse, which can be increased with higher gradient or longer pulses. Thus it is convenient to tune continuously in an experiment. We can also tune the effective Hamiltonian (3) from being Rashba type SOC dominated by increasing the gradient and decreasing the bias magnetic field, to being ML dominated by reducing the gradient and increasing the bias magnetic field.

Finally we discuss the possibility of tuning atomic interaction by Feshbach resonance in our synthetic ML. As discussed above, in the RF plus bias field implementation, the ML depth is given by the detuning between the RF field frequency and the bias Zeeman field $B_{\text{in}}^0 \sqrt{2}$ Larmor frequency. The bias Zeeman field strength can be independently changed to values around atomic Feshbach resonances. For our protocol to work, the two pseudo-spin states will need to possess different magnetic moments, a condition which is rightfully satisfied as tuning through a Feshbach resonance happens because the involved two atom states have different magnetic moments. This interesting prospect could even lead to spatial dependent atom–atom interactions using position dependent Feshbach resonances. In the presence of a strong GMF as required for our protocol, for example, at 0.25 Gauss $\mu m^{-1}$ as discussed earlier, a condensate with a size of 20 $\mu$m will correspond to the net bias magnetic field differing by up to 5 Gauss at both edges. For Feshbach resonances with widths much larger than this value, a uniform interacting situation arises; otherwise position dependent interaction strength happens. The SOC gauge field can change low energy atomic interaction properties, leading to modified thresholds [64, 65] and p-wave behaviors [66]. When combined with ML, the modified atomic collision interactions will likely bring in rich possibilities of interesting many body physics.

5. Conclusion

In conclusion, we propose an idea for dynamical generation of 2D ML. A GMF couples atomic internal states with its spatial motion, giving rise to spatial dependent spin rotations, which can be viewed as an effective ML field. Both the lattice constant and its depth are tunable, and we can tune the effective Hamiltonian (3) from being Rashba type SOC dominated to being ML dominated with corresponding choices for the gradient field strength and the bias field strength. An atom moving inside a ML experiences a spatially periodic potential, whose band structure displays desirable features reminiscent of lattice models with interesting p-orbit physics. The protocol we present in this work can be generalized to other geometries, some of which under certain circumstances can display nontrivial topological band structures. These will be further explored and published elsewhere [55]. Using the simple example of atomic diffractions from a 1D ML, we find population oscillations among spin-momentum states, which provides an easily observable signal for verifying the existence of synthesized ML. We also discuss protocols for implementing our proposal in today’s cold atom experiments.
Finally, it is perhaps worth emphasizing that the scheme we propose is quite general. It can be applied to any atoms with magnetic moments, and it is applicable to both bosonic and fermionic atomic species.

Acknowledgments

We thank Drs Ron Folman, Shuyu Zhou, and Zhifan Zhou for helpful discussions. This work is supported by MOST 2013CB922002 and 2013CB922004 of the National Key Basic Research Program of China, and by NSFC (No. 91121005, No. 11274195, No. 11404184, and No. 11374176).

Appendix A. 1D gradient magnetic field

The protocol for synthesizing ML as discussed in the main text makes use of GMF $B' \hat{x}$ to enact position dependent spin rotations. Such a magnetic field, however, cannot simply exist because the divergence and curl of a magnetic field vanishes in free space. A convenient approach employs a 2D RF quadrupole magnetic field $\omega \phi + t B_x x z z \cos(\hat{z})$ together with a strong bias field $B_z \hat{z} (0)$ to provide an effective GMF $B' \hat{x}$ as proposed in [51]. With $\omega_0 = g_\mu_B B_z (0) / \hbar$ denoting the linear Zeeman shift, in the frame rotating at $\omega_L$ and under rotating-wave approximation, the Hamiltonian containing Zeeman interaction with the bias field and the 2D RF quadrupole magnetic field is

$$\hat{H}_B = -\hbar A \hat{x} + \hbar B_x \hat{x} \cos(\hat{z}) + g_\mu_B B_x \hat{x} \left( F_z \cos(\phi_z) + F_\phi \sin(\phi_z) \right), \quad (A.1)$$

provided $|B_z (0)| \gg |B_x|, |B'_x|$, where $\Delta = \omega - \omega_0$ is the detuning, and $|q| = \omega_0^2 / \Delta_{\text{HFS}}$ denotes the coefficient of the QZS for the bias field with $\Delta_{\text{HFS}}$ the ground state hyperfine splitting. When $\Delta = 0$ and neglecting QZS, choosing a particular phase of $\phi_z = 0$, this Hamiltonian reduces to $\hat{H}_B = g_\mu_B B_x \hat{x} F_z$, which describes Zeeman interaction of an atomic hyperfine spin $F$ with an effective 1D GMF $B' \hat{x}$. Similarly 1D GMF $B'_y \hat{y}$ can be generated by the combination of a 2D RF quadrupole magnetic field $\cos(\omega t + \phi_y) B_y \hat{y} (y' - z) z z$ with a strong bias field $B_z \hat{z}$, and choosing $\phi_y = 0$. The bias field $B_z \hat{z}$ in the rotating frame is equivalent to the detuning, or $\omega_0 = \Delta$.

Appendix B. The validity conditions for the effective Hamiltonian

In the main text, the effective 2D ML Hamiltonian is derived by employing two approximations. First, the Zeeman term $k_{\text{so}} x F_z$ is assumed to overwhelm all other interaction terms during GMF pulses; and second, higher order terms are neglected when non-commuting exponents $A$ and $B$ are combined into the same exponent according to $e^{A t} e^{B t} \approx e^{(A+B) t}$. This section provides the validity conditions for these two approximations by making use of the Trotter expansion to the first order

$$e^{A t} e^{B t} \approx e^{(A+B) t} e^{[A, B] t^2 / 2}. \quad (B.1)$$

B.1. The condition for neglecting atomic motion during the gradient pulse

Taking into account the atomic motion during the GMF, the lowest order Trotter expansion for the evolution operator of as single gradient pulse is found to be

$$\overline{U}_x (\delta t') = \exp \left\{ i k_{\text{so}} x F_z - i \frac{\hbar k_x^2}{2 m} \delta t' \right\} \approx \exp \left\{ i \frac{\hbar k_{\text{so}}}{2 m} k_x \delta t' \exp \left\{ -i \frac{\hbar k_x^2}{2 m} \delta t' \right\} \exp \left\{ i k_{\text{so}} x F_z \right\} \right\} = \exp \left\{ i \omega_0 \delta t' (k_x / k_{\text{so}}) F_z \right\} \exp \left\{ -i \omega_0 \delta t' \left( k_x / k_{\text{so}} \right)^2 F_z \right\} \exp \left\{ i k_{\text{so}} x F_z \right\}. \quad (B.2)$$

The last factor is the spatial dependent spin rotation discussed in the main text. The first two multiplying factors from accounts for the errors due to the neglect of atomic motion during the GMF pulse. The first factor denotes an 'extra' spin rotation caused by atomic motion, which can be safely neglected provided

$$\omega_0 \delta t' \ll 1, \quad (B.3)$$

New J. Phys. 17 (2015) 083048 X Luo et al
assuming \( k_x, k_y \lesssim k_{so} \). For ultracold atoms at tens or hundreds of nK considered in this study, \( k_x, k_y \lesssim k_{so} \) is always satisfied as it is simply equal to the statement of well below the recoil limit temperature, if we take a more typical momentum impulse of \( L = k_{so} \). The neglect of atomic motion during the gradient pulse can be well satisfied as we illustrate in figure B1, which compares density distributions from the effective Hamiltonian (in the first column) with that from the dynamics of the discussed pulse sequence \( \delta t = 2.5 \mu s, B_0 = 10 \) mGauss or \( \omega_0 = (2\pi) \times 0.024 \) kHz. The regions displayed correspond to \(-15a_0 < x, y < 15a_0\), with \( a_0 = \sqrt{\hbar / m \omega} \), the length scale for an isotropic harmonic oscillator \( \omega = \omega_r = \omega = (2\pi) \times 30 \) Hz.

**B.2. The condition for achieving the 2D ML**

We now proceed to investigate the condition for the second approximation. According to our protocol developed in the main text, we set

\[
A = \frac{\hbar}{2m} \left[ \left( k_x - k_{so} F_x \right)^2 + k_y^2 \right] + \omega_0 \left[ F_x \cos \left( k_{so} x \right) + F_y \sin \left( k_{so} x \right) \right],
\]

\[
B = \frac{\hbar}{2m} \left[ \left( k_y - k_{so} F_y \right)^2 + k_x^2 \right] + \omega_0 \left[ F_x \cos \left( k_{so} y \right) - F_y \sin \left( k_{so} y \right) \right],
\]

(B.4)

the commutator between \( A \) and \( B \) is found to be

\[
[A, B] = i\omega_0^2 \left\{ \frac{k_x}{k_{so}} - F_x, \frac{k_y}{k_{so}} - F_y, F_z \right\}
\]

\[
+ i\omega_0 \omega_0 \left[ \left\{ \frac{k_x}{k_{so}} - F_x, F_y \cos \left( k_{so} y \right) \right\} + \left\{ \frac{k_y}{k_{so}} - F_y, F_x \cos \left( k_{so} x \right) \right\} \right]
\]

\[
+ i\omega_0 \omega_0 \left[ \left\{ \frac{k_y}{k_{so}} - F_y, F_x \sin \left( k_{so} y \right) + F_x \cos \left( k_{so} y \right) \right\} \right]
\]

\[
- \left\{ \frac{k_x}{k_{so}} - F_x \right\} \left\{ F_x \sin \left( k_{so} x \right) - F_y \cos \left( k_{so} x \right) \right\}
\]

\[
+ i\omega_0 \omega_0 \left[ F_x \sin \left( k_{so} x \right) \cos \left( k_{so} y \right) - F_y \cos \left( k_{so} x \right) \sin \left( k_{so} y \right) \right]
\]

\[
+ F_x \sin \left( k_{so} x \right) \sin \left( k_{so} y \right) \right\},
\]

(B.5)

where \( \omega_R = \hbar k_{so}^2 / 2m \), and \( \{ F_i, F_j \} = F_i F_j + F_j F_i \) denotes anti-commutator. For spin-1/2 particles, \( F_j = \sigma_j / 2 \), which gives \( \{ F_i, F_j \} = 0 \) if \( i \neq j \). Equation (B.5) in the above reduces to
\[ [A, B] = i\omega_0^2 \frac{2k_y}{k_{\text{so}}} \sigma_x + i \frac{\omega_0 \omega_0}{2k_{\text{so}}} \left[ k_y \cos(k_{\text{so}}x) + \cos(k_{\text{so}}y) \right] \sigma_x \]

\[ + i \frac{\omega_0 \omega_0}{2k_{\text{so}}} \left[ k_x \cos(k_{\text{so}}x) + \cos(k_{\text{so}}y) \right] \sigma_y \]

\[ + \left( \{ k_y, \sin(k_{\text{so}}y) \} - \{ k_x, \sin(k_{\text{so}}x) \} \right) \sigma_z \]

\[ + \frac{i \omega_0^2}{2} \left[ \sigma_x \sin(k_{\text{so}}x) \cos(k_{\text{so}}y) - \sigma_y \cos(k_{\text{so}}x) \sin(k_{\text{so}}y) \right] \sigma_z \sin(k_{\text{so}}y) \sin(k_{\text{so}}y) \cdot (B.6) \]

The leading order correction to the time evolution operator for the 2D ML is therefore

\[ \delta U_{(2D)}(T, 0) \approx \max\left( \omega_0^2, \omega_0^2 \right) \mathcal{O}(\delta t^2). \]

The condition for combining the otherwise non-commuting \( x \) and \( y \)-dependent exponents to forming the 2D ML in the same exponent is then simply given by

\[ \max\left( \omega_0^2, \omega_0^2 \right) \delta t^2 \ll 1, \]

which is easily seen to be satisfied as shown in figure 4.

References

[1] Bloch I, Dalibard J and Zwerger W 2008 Rev. Mod. Phys. 80 885
[2] Jaksch D and Zoller P 2005 Ann. Phys. 315 52
[3] Jaksch D, Bierig H J, Cirac J I, Gardiner C W and Zoller P 1999 Phys. Rev. Lett. 82 1975
[4] Brennen G K, Caves C M, Jezek P S and Deutsch I H 1999 Phys. Rev. Lett. 82 1060
[5] Karski M, Förster L, Choi J M, Steffen A, Alt W, Meschede D and Widera A 2009 Science 325 174
[6] Greiner M, Mandel O, Esslinger T, Hänsch T W and Bloch I 2002 Nature 415 39
[7] Bockelmann U, Greif D, Uehlinger T, Jotzu G and Esslinger T 2012 Nature 483 302
[8] Jo G B, Guzman J, Thomas C K, Hosur P, Vishwanath A and Stamper-Kurn D M 2012 Phys. Rev. Lett. 108 045305
[9] Han W, Zhang S, Jin and Liu W M 2012 Phys. Rev. A 85 043626
[10] McKay D C, Meldgin G, Chen D and DeMarco B 2013 Phys. Rev. Lett. 111 060302
[11] Berman J, Sengupta K and Kim Y B 2006 Phys. Rev. B 74 135124
[12] Mandel O, Greiner M, Widera A, Rom T, Hänsch T W and Bloch I 2003 Phys. Rev. Lett. 91 010407
[13] Lee P J, Anderlini M, Brown B L, Sebby-Strabley J, Phillips W D and Porto V 2007 Phys. Rev. Lett. 99 020402
[14] Soltan-Panahi P, Lührmann D S, Struck J, Windpassinger P and Sengstock K 2012 Nat. Phys. 871
[15] Hubener A, Snoek M and Hofstetter W 2009 Phys. Rev. B 80 245109
[16] Zapata I, Wunsch B, Zinner N T and Demler E 2010 Phys. Rev. Lett. 105 095301
[17] Hassan S R, Sriluckshmy P V, Vovk S K, Shankar R and Sénéchal D 2013 Phys. Rev. Lett. 110 037201
[18] Ostrovskaya E A and Kivshar Y S 2004 Phys. Rev. Lett. 92 180405
[19] Orso G, Burovski E and Jolicoeur T 2010 Phys. Rev. Lett. 104 065301
[20] Liu W V, Wilczek F and Zoller P 2004 Phys. Rev. A 70 033603
[21] Soltan-Panahi P, Struck J, Hauke P, Bick A, Plenkers W, Meineke G, Becker C, Windpassinger P, Lewenstein M and Sengstock K 2011 Nat. Phys. 7 434
[22] Dalibard J, Gerbier F, Juzeliūnas G and Öhberg P 2011 Rev. Mod. Phys. 83 1523
[23] Jaksch D and Zoller P 2003 New J. Phys. 5 56
[24] Gerbier F and Dalibard J 2010 New J. Phys. 12 033007
[25] Cooper N R 2011 Phys. Rev. Lett. 106 175301
[26] Cooper N R and Dalibard J 2011 Europhys. Lett. 95 66004
[27] Beri B and Cooper N R 2011 Phys. Rev. Lett. 107 145301
[28] Sørensen A S, Demler E and Lukin M D 2005 Phys. Rev. Lett. 94 086803
[29] Kitagawa T, Berg E, Rudner M and Demler E 2010 Phys. Rev. B 82 235114
[30] Osterloh K, Baig M, Santos L, Zoller Pand Lewenstein M 2005 Phys. Rev. Lett. 95 010403
[31] Aidelsburger M, Atala M, Lohse M, Barreiro J T, Paredes B and Bloch I 2013 Phys. Rev. Lett. 111 185301
[32] Miyake H, Siviglia G A, Kennedy C J, Burton W C and Ketterle W 2013 Phys. Rev. Lett. 111 185302
[33] Cocks D, orch P P, Rachel S, Buchhold M, Le Hur K and Hofstetter W 2012 Phys. Rev. Lett. 109 205303
[34] Gong M, Qian Y, Yan M, Scoracle V W and Zhang C 2015 Sci. Rep. 5 10050
[35] Cole W S, Zhang S, Paramekanti A and Trivedi N 2012 Phys. Rev. Lett. 109 085302
[36] Radić J, di Ciolo A, Sun K and Galitski V 2012 Phys. Rev. Lett. 109 085303
[37] Cai Z, Zhou X and Wu C 2012 Phys. Rev. A 85 061605
[38] Grynyberg G and Robilliard C 2001 Phys. Rep. 355 335
[39] Dalibard J and Cohen-Tannoudj C 1989 J. Opt. Soc. Am. B 6 2023
[40] Jimenez-García K, LeBlanc L J, Williams R A, Becker M C, Perry A R and Spielman I B 2012 Phys. Rev. Lett. 108 225303
[41] Windpassinger P and Sengstock K 2013 Rep. Prog. Phys. 76 086401
[42] Kennedy C J, Siviglia G A, Miyake H, Burton W C and Ketterle W 2013 Phys. Rev. Lett. 111 225301
[43] Cheuk I W, Sommer A T, Hadzibabic Z, Yefsah T, Bakr W S and Zwierlein W M 2012 Phys. Rev. Lett. 109 095302
[44] Fortagh J and Zimmermann C 2007 Rev. Mod. Phys. 79 235
[45] Singh M, Volk M, Akulshin A, Sidorov A, McLean R and Hannaford P 2008 J. Phys. B: At. Mol. Opt. Phys. 41 065301
[46] Whitlock S, Gerritsma R, Fernholz T and Sreenuew R 2009 New J. Phys. 11 025021
[47] Jose S, Surendran P, Wang Y, Herrera I, Krzemien L, Whitlock S, McLean R, Sidorov A and Hannaford P 2014 Phys. Rev. A 89 051602
[48] Herrera I, Wang Y, Michaux P, Nissen D, Surendran P, Juodkazis S, Whitlock S, McLean R J, Sidorov A, Albrecht M and Hannaford P 2015 J. Phys. D: Appl. Phys. 48 115002
[49] Romero-Isart O, Navau C, Sanchez A, Zoller P and Cirac J I 2013 Phys. Rev. Lett. 111 115304
[50] Xu Z F, You L and Ueda M 2013 Phys. Rev. A 87 063634
[51] Anderson B M, Spielman I B and Juzeliūnas G 2013 Phys. Rev. Lett. 111 125301
[52] Luo X, Wu L, Chen J, Guan Q, Gao K, Xu Z F, You L and Wang R 2015 arXiv:1502.07091
[53] Liu W V and Wu C 2006 Phys. Rev. A 74 013607
[54] Wirth G, Ölschläger M and Hemmerich A 2011 Nat. Phys. 7 147
[55] Yu J, Xu Z F, Lu R and You L 2015 arXiv:1506.02418
[56] Xu Z F, Zhang P, Lu R and You L 2010 Phys. Rev. A 81 053619
[57] Dekker N H, Lee C S, Lorent V, Thywissen J H, Smith S P, Drndić M, Westervelt R M and Prentiss M 2000 Phys. Rev. Lett. 84 1124
[58] Machluf S, Japha Y and Folman R 2013 Nat. Commun. 4 2424
[59] Gallego D, Hofferberth S, Schumm T, Krüger P and Schmiedmayer J 2009 Opt. Lett. 34 3463
[60] Shtrirberg L, Twig Y, Dikarov E, Halevy R, Levit M and Blank A 2011 Rev. Sci. Instrum. 82 043708
[61] Struck J, Ölschläger C, Weinberg M, Hauke P, Simonet J, Eckardt A, Lewenstein M, Sengstock K and Windpassinger P 2012 Phys. Rev. Lett. 108 225304
[62] Lin Y J, Jiménez-Garcia K and Spielman I 2011 Nature 471 83
[63] Jiménez-Garcia K, LeBlanc L J, Williams R A, Beeler M C, Qu C, Gong M, Zhang C and Spielman I B 2015 Phys. Rev. Lett. 114 125301
[64] Duan H, You L and Gao B 2013 Phys. Rev. A 87 052708
[65] Wang S J and Greene C H 2015 Phys. Rev. A 91 022706
[66] Williams R A, LeBlanc L J, Jiménez-Garca K, Beeler M C, Perry A R, Phillips W D and Spielman I B 2012 Science 335 314