Electromagnetic and gravitational responses of photonic Landau levels

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Topology has recently become a focus in condensed matter physics, arising in the context of the quantum Hall effect and topological insulators. In both of these cases, the topology of the system is defined through bulk properties (‘topological invariants’) but detected through surface properties. Here we measure three topological invariants of a quantum Hall material—photonic Landau levels in curved space—through local electromagnetic and gravitational responses of the bulk material. Viewing the material as a many-port circulator, the Chern number (a topological invariant) manifests as spatial winding of the phase of the circulator. The accumulation of particles near points of high spatial curvature and the moment of inertia of the resultant particle density distribution quantify two additional topological invariants—the mean orbital spin and the chiral central charge. We find that these invariants converge to their global values when probed over increasing length scales (several magnetic lengths), consistent with the intuition that the bulk and edges of a system are distinguishable only for sufficiently large samples (larger than roughly one magnetic length). Our experiments are enabled by applying quantum optics tools to synthetic topological matter (here twisted optical resonators). Combined with advances in Rydberg-mediated photon collisions, our work will enable precision characterization of topological matter in photon fluids.

Topological phases of matter, which cannot be characterized by the spontaneous breaking of a local symmetry, have revolutionized modern condensed matter physics and materials science1. Such phases are so named because they are described by global invariants, which are insensitive to material imperfections. These invariants are widely applicable, including in the redefinition of the unit of electrical resistance, error-resilient spintronics2 and quantum computation3. Constructed as integrals of a type of curvature over a closed parameter space, these invariants are each defined as a global property that results from the integral of a local property, akin to the relationship between the (local) Gaussian curvature of a surface and the (global) Euler characteristic that determines the number of handles of the surface. In the integer quantum Hall effect, integration of the Berry curvature over the Brillouin zone (momentum space) defines an invariant called the first Chern number4,5. Two additional topological invariants, the mean orbital spin and the chiral central charge (hereafter central charge), are defined similarly to the Chern number, but over more abstract parameter spaces6–8.

Understanding the physical importance of the invariants that characterize topological matter remains a challenge. What is known is that each topological invariant is connected to a family of physical phenomena. In quantum Hall materials, the transverse (Hall) conductance is an experimentally quantized invariant, corresponding in the integer quantum Hall case to the Chern number4,5. Explorations of synthetic quantum matter composed of ultracold atoms has resulted in new experimental observables connected to the Berry curvature9–12, the anomalous velocity13, quantized charge transport in a Thouless pump14–16 and the linking number17. All of these observations relate directly back to the Chern number, all are global measurements, and each teaches us something different about its fundamental role in determining material properties. Meanwhile, understanding the physical importance of the mean orbital spin and the central charge has remained challenging because the transport coefficients that they affect are notoriously difficult to measure18, and no previous experimental work has investigated the locality of any of these three topological invariants.

Observing phenomena associated with new topological invariants provides insights into the importance of these quantum numbers and real-world tools for characterizing topological matter. Photonic topological materials offer especially promising routes to new experimental probes of topological invariants19–23 because of the time-, energy-, position- and momentum-resolved control of cold-atom experiments9–13,15,16,24,25 and the spectroscopic tools of electromagnetic systems19,26,27. Our platform for photonic Landau levels in curved space28 is compatible with spatially arbitrary excitation of the Landau level via holographic beam shaping and provides a conical singularity of spatial curvature that perturbs the Landau level—the conical geometry arises from a three-fold rotational symmetry imposed by the resonator configuration. Here we develop complex-valued tunnelling spectroscopy using holographic reconstruction of the response of the system to local excitations to access topological invariants through spatially localized observables. By harnessing a holographic reconstruction of the band projector, we measure the Chern number3,26 without relying on edge modes or non-local measurement. Using the conical defect in the photonic Landau level, we measure the mean orbital spin and the central charge through the gravitational response of the system, that is, its response to geometric deformation (via the amount of density build-up) and its structure at a singularity of spatial curvature.

We begin with a brief description of the local character of these topological invariants, connecting them to new observables. We then describe our measurement of the Chern number via a quantized bulk chiral phase response, and of the mean orbital spin and chiral central charge from precision measurements of density oscillations near singularities of spatial curvature and magnetic flux. We conclude with a brief discussion of extensions of this work to interacting quantum Hall materials.

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spin may be understood as the force (black arrows) on a rotating disk dissipationless transverse diffusion of momentum. The mean orbital carried by particles in a quantum Hall fluid; it gives rise to Hall viscosity—arrows). The mean orbital angular momentum carried by particles in a quantum Hall fluid; it gives rise to Hall viscosity—dissipationless transverse diffusion of momentum. The mean orbital spin may be understood as the force (black arrows) on a rotating disk (orange arrows). A. The mean orbital spin is the orbital angular momentum relative to ground.

New probes of topology

Although the Chern number $C$ is traditionally defined as an integral over the Brillouin zone\(^2\), the ‘bulk boundary correspondence’ connects a non-zero Chern number to robust chiral edge channels that extend around the boundary of the material\(^2\); indeed, the presence of these channels is often taken as proof that the bulk is topological\(^2,20,21,23,30\).

Accordingly, a conceptually simple local measure of the bulk Chern number results from cutting the system down to a patch a few magnetic lengths across and surrounding it with vacuum. The number and chirality of these edge modes directly reflects the Chern number.

In practice, it is difficult to cut the system. Kitaev proposed a way of extracting equivalent information from triple products of spatial projectors onto a spectrally isolated band\(^3\). We implement this approach using spatially resolved complex-valued tunnelling spectroscopy of a patch within the bulk of the lowest Landau level, thereby measuring a non-zero Chern number\(^2\).

Two additional topological invariants appear in quantum Hall physics. First, the mean orbital spin $\bar{\sigma}$ is a bulk invariant that quantifies the magnetic-like coupling of a particle to curvature and is related to the Hall viscosity and Wen–Zee shift (Supplementary Information section H). Second, the central charge\(^3\), which is related to the gravitational anomaly, is equal to the total number of edge modes (neutral and charged) in integer quantum Hall and Laughlin states and gives rise to the thermal Hall conductance\(^3,17,34\) (Fig. 1). These invariants have hitherto been understood in terms of thought experiments that require complicated ‘topological gymnastics’ (Fig. 1d) and measured through their connection to exotic transport coefficients (Fig. 1c, e). We are able to access them because they govern the coupling of local observables—namely the particle and angular-momentum densities—to spatial curvature.

The competition between phases in a topological fluid can be sensitive to its microscale properties—the lattice structure, the inter-particle interaction strength and form, the number of filled bands and disorder can each bias the system between competing ground states. However, a quantum Hall fluid near its many-body ground state may be described by a low-energy ‘effective theory’ that is fully characterized by only a few parameters that incorporate all of these properties. The effective action that describes this theory is\(^35,36\) $W(B, R, \nu, \xi, \bar{\sigma})$, where $B(x, y)$ is the magnetic field, $R(x, y)$ is the spatial (Ricci) curvature, $\nu c^2 h\sigma = \sigma H$ is the Hall conductance, $c$ is the electron charge and $h$ is the Planck constant. The Hall conductance specifies the current that is induced perpendicular to an applied electric field and is precisely quantized in the plateaus of the integer and fractional quantum Hall effects. For integer quantum Hall physics, $\nu = C$. The mean orbital spin $\bar{\sigma}$ and central charge $c$ complete the triplet of topological invariants that appear in quantum Hall systems. The effective action is fully specified by these five quantities, the derivatives of which correspond to several physical observables, including densities and transport coefficients.

Although the topological invariants could potentially be calculated from first principles, doing so has proven computationally challenging and depends on microscale properties which are often unknown. However, from the effective action it can be shown that introducing a conical geometry and threading the cone tip with a small tube of magnetic flux produces a localized density response that depends sensitively on both the mean orbital spin\(^3\) and the central charge (Supplementary Information section F). As such, we use a curve $R(x, y) = (8\pi/3)(x^2)$, $y$ and vary a magnetic field $B(x, y) = B_0 + \Phi_B/(2\pi)(x, y)$—where $B_0$ is the two-dimensional Dirac delta function and $\Phi_B/(2\pi) \in \{0, -1/3, -2/3\}$ is the additional magnetic flux threading—to induce a localized density response and thereby provide a window onto these topological invariants.

Electromagnetic response

To extract the Chern number we measure a quantized bulk chiral response\(^31,37\) (Fig. 1f). Particles that inhabit multiple Landau levels exhibit cyclotron orbits, which create a bulk circulating current. Although particles in a single Landau level do not display orbital dynamics, they still accrue a chiral (Aharanov–Bohm) phase when

**Fig. 1** | Topological invariants and their associated observables. a, In solids, the Chern number of a band is typically obtained by measuring the Hall conductance $\sigma_H$, which quantifies the current $I$ that is induced perpendicular to an electric field $E$, applied by an electrode at voltage $+V$ relative to ground. b, The precise quantization of the Hall conductance arises from the presence of disorder-protected chiral edge modes (orange arrows). c, The mean orbital spin is the orbital angular momentum carried by particles in a quantum Hall fluid; it gives rise to Hall viscosity—arrows). d, The central charge is the third topological invariant that characterizes quantum Hall fluids; it corresponds to the geometric phase $\Phi$ that is picked up by the wavefunction of the many-body ground state in response to a ‘Dehn twist’ of the torus on which the fluid lives. e, The central charge is typically measured through the thermal Hall conductance, which causes heat flow perpendicular to a temperature gradient, $f \rightarrow h$. In our system, we measure these three topological invariants using newly accessible observables. f, Working in flat space (away from the tip of the cone), we measure a bulk chiral phase response (black arrows; Fig. 3) to extract the Chern number (Fig. 4). g, h, We measure the accumulation of particles (yellow) at the tip of the cone (g) and the associated orbital angular momentum (curved black arrows) via the moment of inertia (width of the particle density distribution; horizontal black arrows) of the density at the tip (h) to extract the mean orbital spin and central charge (Fig. 5).
Holographic reconstruction of band projectors. Holographic beam shaping allows the injection of arbitrary light fields into our photonic quantum Hall system, while heterodyne imaging of the leakage field from the cavity enables full complex-valued electric-field reconstruction of the response of the system. a, A 780-nm laser field is directed onto a digital micromirror device (green) and diffraction off a hologram produced by this device is directed into the non-planar resonator (four plano-concave mirrors). The direct reflection is absorbed by a beam dump (black). Light that leaks from the resonator through one of its mirrors is split on a 50:50 beam splitter (grey) and directed to a photodiode (blue) and camera (purple), which images the transverse plane at the waist of the cavity (green grid). A few percent of the initial input light forms a reference beam that is also directed onto the camera but at a substantial angle relative to the resonator output to enable heterodyne imaging, akin to optical holography. b, The plane-wave reference beam interferes with the cavity output to produce an image (left) in which the fringe contrast provides information about the amplitude of the electric field and the fringe position provides information about the phase of the electric field. A dislocation in the fringe pattern identifies an optical vortex core, about which the phase of the electric field winds by an integer multiple of $2\pi$. This amplitude and phase information is extracted from the images via a filtering scheme in momentum space (Supplementary Information section A), providing the electric-field profile of the cavity mode (right). c, The projectors used to extract the Chern number are measured by injecting a magnetically translated Gaussian beam (Supplementary Information section D) and integrating (via long camera exposure) the heterodyned cavity response while sweeping the laser frequency across the Landau level. This procedure is robust to potential disorder that broadens the Landau level so long as the disorder is not strong enough to substantially admix other Landau levels. Although it is difficult to controllably introduce such disorder, we demonstrate this robustness by applying weak harmonic confinement: the Landau level broadens into spectrally resolved Laguerre–Gaussian rings (electric-field profiles shown in small colour-coded boxes and spectrum shown by the grey trace). A displaced Gaussian beam has substantial overlap with only a few of these modes (red trace), but integrating across the relevant frequency band (pink shading) yields a localized response (electric-field profile in the large pink box), from which we extract the projector. This is because the holographic reconstruction effectively integrates the complex electric field that leaks from the cavity, rather than its intensity, resulting in constructive interference of the various modes along the vertical dashed lines and destructive interference along the diagonal lines (Supplementary Information section A). This field integration is insensitive to potential disorder that broadens the band. In all the electric-field profiles, the brightness of each pixel corresponds to the amplitude of the electric field ($|E|$) and the hue corresponds to its phase $\text{Arg}(E)$.

Fig. 2 | Holographic reconstruction of band projectors. Holographic beam shaping allows the injection of arbitrary light fields into our photonic quantum Hall system, while heterodyne imaging of the leakage field from the cavity enables full complex-valued electric-field reconstruction of the response of the system. A 780-nm laser field is directed onto a digital micromirror device (green) and diffraction off a hologram produced by this device is directed into the non-planar resonator (four plano-concave mirrors). The direct reflection is absorbed by a beam dump (black). Light that leaks from the resonator through one of its mirrors is split on a 50:50 beam splitter (grey) and directed to a photodiode (blue) and camera (purple), which images the transverse plane at the waist of the cavity (green grid). A few percent of the initial input light forms a reference beam that is also directed onto the camera but at a substantial angle relative to the resonator output to enable heterodyne imaging, akin to optical holography. The plane-wave reference beam interferes with the cavity output to produce an image (left) in which the fringe contrast provides information about the amplitude of the electric field and the fringe position provides information about the phase of the electric field. A dislocation in the fringe pattern identifies an optical vortex core, about which the phase of the electric field winds by an integer multiple of $2\pi$. This amplitude and phase information is extracted from the images via a filtering scheme in momentum space (Supplementary Information section A), providing the electric-field profile of the cavity mode (right). The projectors used to extract the Chern number are measured by injecting a magnetically translated Gaussian beam (Supplementary Information section D) and integrating (via long camera exposure) the heterodyned cavity response while sweeping the laser frequency across the Landau level. This procedure is robust to potential disorder that broadens the Landau level so long as the disorder is not strong enough to substantially admix other Landau levels. Although it is difficult to controllably introduce such disorder, we demonstrate this robustness by applying weak harmonic confinement: the Landau level broadens into spectrally resolved Laguerre–Gaussian rings (electric-field profiles shown in small colour-coded boxes and spectrum shown by the grey trace). A displaced Gaussian beam has substantial overlap with only a few of these modes (red trace), but integrating across the relevant frequency band (pink shading) yields a localized response (electric-field profile in the large pink box), from which we extract the projector. This is because the holographic reconstruction effectively integrates the complex electric field that leaks from the cavity, rather than its intensity, resulting in constructive interference of the various modes along the vertical dashed lines and destructive interference along the diagonal lines (Supplementary Information section A). This field integration is insensitive to potential disorder that broadens the band. In all the electric-field profiles, the brightness of each pixel corresponds to the amplitude of the electric field ($|E|$) and the hue corresponds to its phase $\text{Arg}(E)$.

forced to travel in a closed path. This chiral phase is not apparent from the momentum-space definition of the Chern number, but in a given band $\mu$ it is highlighted by the following expression for the Chern number $^{31}$:

$$C^\mu = 12\pi i \sum_{\alpha \in A, \beta \in B} \gamma^\mu_{\alpha \beta \gamma} P^\mu_{\alpha \gamma} P^\mu_{\gamma \alpha} - P^\mu_{\alpha \gamma} P^\mu_{\gamma \alpha}$$

(1)

where the area probe is divided into thirds labelled A, B and C and the band projector

$$P^\mu_{\alpha \beta} = \left\langle x_\beta \left| \left( \sum_{j \neq \mu} |j\rangle \langle j| \right) \right| x_\alpha \right\rangle$$

maps eigenstates $|j\rangle$ residing in band $\mu$ to themselves and all other eigenstates to zero. Intuitively, this projector describes injecting a tiny probe (with a transverse size much smaller than the magnetic length $l_B$) at a desired location $x_\alpha = (x_\alpha, y_\alpha)$ and energy-integrating the resulting complex cavity response (leakage field) at another location $x_\beta = (x_\beta, y_\beta)$ across the band or Landau level $^{26,39}$ (Fig. 2c; Supplementary Information section B). For a Landau level (band), this response is exponentially localized with a characteristic scale of $l_B$ (within the magnetic unit cell).

We may then assemble triple products of these complex responses into a measurement of $C^\mu$ from which are chirality measurements:

for any triplet of points $(x_\alpha, x_\beta, x_\gamma)$, the first term of equation (1) $(P^\mu_{\alpha \gamma} P^\mu_{\gamma \alpha} - P^\mu_{\alpha \gamma} P^\mu_{\gamma \alpha})$ measures the particle current in a trajectory with one handedness $(x_\alpha \rightarrow x_\beta \rightarrow x_\gamma)$, while the second term $(P^\mu_{\alpha \gamma} P^\mu_{\gamma \alpha})$ measures the reverse trajectory $(x_\alpha \rightarrow x_\gamma \rightarrow x_\beta)$. The currents are equal in magnitude; however, owing to the vector potential, which provides an Aharonov–Bohm phase for particles traversing a closed loop, their phases differ. Each term in the sum is then the net non-reciprocity for that trajectory, and summing over all possible trajectories provides the Chern number (Fig. 3b–d).

We implement this protocol experimentally by using a digital micromirror device to excite each $|x_\alpha\rangle$ on a chosen grid (Fig. 2a). Holographic reconstruction of the transmitted resonator electric field (Fig. 2a, b) while sweeping the excitation laser frequency across the Landau level then provides the matrix elements of the band projector $P_{\alpha \beta}^\mu$, from $x_\alpha$ to all $x_\beta$ (Fig. 2c). We obtain all of the matrix elements of the projector by iterating the excitation location over all points on the chosen grid, and the Chern number is then computed using equation (1).

The terms in the sum in equation (1) fall off rapidly as $x_\alpha \rightarrow x_\beta$ and $x_\gamma$, become further apart because $P^\mu_{\alpha \beta} \propto \exp[-|x_\alpha - x_\beta|^2/(4l_B^2)]$. Consequently, the dominant contributions to the sum come from trios of points near the meeting point(s) of the three sectors; the contribution from any term associated with a point several magnetic lengths away from the centre is negligible. Accordingly, the sum in equation (1) can

\[ C^\mu = 12\pi i \sum_{\alpha \in A, \beta \in B} \gamma^\mu_{\alpha \beta \gamma} P^\mu_{\alpha \gamma} P^\mu_{\gamma \alpha} - P^\mu_{\alpha \gamma} P^\mu_{\gamma \alpha} \]

(1)
Gravitational response

The response of a quantum Hall fluid to manifold curvature is controlled by two topological invariants (the mean orbital spin and the central charge), which are specific to the particular quantum Hall state under consideration. Our platform, which consists of Landau levels on a cone with additional magnetic flux of $\Phi_B = -2\pi a/3$ ($a \in \{0, 1, 2\}$) threaded through the tip, provides an idealized source of manifold curvature localized precisely to the tip of the cone. In what follows, we connect variations in the local spatial density of states (LDOS) at this curvature singularity directly to the mean orbital spin and central charge (see also Supplementary Information section F).

In flat space, the LDOS of a quantum Hall fluid is uniform, providing few signatures of the properties of the fluid; in curved space, however, the LDOS displays oscillations about its flat-space background that depend on the local curvature and threaded flux. The excess particle number localized to the tip of the cone, defined as the spatial integral of the excess density there, directly reflects the mean orbital spin (Fig. 1g); the width of this excess particle density reflects the orbital angular momentum attached to the curvature singularity and thus the central charge (Fig. 1h). We derive these connections in Supplementary Information section F. In the nth Landau level, the particle density $\rho$ near the cone tip and the excess angular momentum $l_B$ obey (Supplementary Information, equations (F5) and (F24), respectively):

$$\rho = n \frac{e^2}{\hbar c} \left(\frac{\Phi_B}{\Phi_0}\right)$$

$$l_B = 2\pi a/3 (a \in \{0, 1, 2\})$$
**Measuring the Chern number.** The sole length scale for physics in the lowest Landau level is the magnetic length $l_B$, so it is reasonable to expect that the Chern number will converge once the measurement area extends beyond this scale. Indeed, as the sum in equation (1) is taken out to larger radii on a grid (see Fig. 3c), the measured Chern number $c$ (red points) rapidly converges to one, in agreement with first-principles theory (grey curve; see Supplementary Information section C.1 for a derivation) with no adjustable parameters. The vertical dashed line at an enclosed flux $\Phi_0$ (grey curve; see Supplementary Information section C.1 for a derivation) to the finite number of states measured. The uniform-density region is uniform density at larger radii, and then a smooth decrease to zero owing to the finite number of states measured. The uniform-density region is averaged over all Landau levels to define a background density $\rho_{0t}$, on the basis of which we define the excess LDOS (red filling). For each Landau level $(n)$, the excess LDOS near the tip of the cone is integrated to measure the total excess particle number $nN$, from which we extract the mean orbital spin. We also compute the shifted second moment $\Delta M_2$, which provides a higher-precision measurement of the mean orbital spin and a measurement of the central charge. We benchmark the connection between these topological invariants and the LDOS oscillations by performing the same experiment in nine situations: for the lowest, first excited, second excited, and so on, Landau levels. In addition, these improvements provide an independent probe of the mean orbital spin and, most importantly, permit measurements to verify that the central charge is unity ($c_{tot} = 1$) in all Landau levels.

In Fig. 5 we present the LDOS of the lowest, first excited and second excited Landau levels on three cones differentiated by the effective magnetic flux threading the cone tip, following the previously described procedure\textsuperscript{28}. Near the tip of the cone, we observe oscillations in the

$$\rho^{(n)} = \frac{\nu e B}{h} + \frac{\nu \sigma_n R}{4\pi}$$

$$\bar{\rho}^{(n)} = \frac{c_n - 12 \sigma_n^2}{24} \left( \frac{1}{s} \right) + \frac{a}{2} \left( \frac{2\sigma_n}{a} - \frac{1}{s} \right)$$

Here $s$ is the number of unwrapped cones required to cover the plane (in our case $s = 3$), which provides an alternative parameterization of the curvature. This expression for the particle density indicates that whereas more curvature or a stronger magnetic field increases the particle density, its distribution depends non-monotonically on these quantities. The proportionality constants depend on both the mean orbital spin ($s_n$) and the central charge ($c_n$) in the $n$th Landau level; consequently, to determine these quantities we need to explore the dependence of the particle density on both the curvature and the magnetic field (parameterized by the number of excess flux quanta $a$ threading the cone tip).

Technical improvements to the original apparatus\textsuperscript{28} (Extended Data Fig. 1) enable undisturbed, high-precision access to the LDOS oscillations, thereby extending measurements of the mean orbital spin to excited Landau levels in which it takes on new values and is expected to obey $s_n = n + 1/2$, where $n = 0, 1, 2, \ldots$ specifies the lowest, first excited, second excited, and so on, Landau levels. In addition, these improvements provide an independent probe of the mean orbital spin and, most importantly, permit measurements to verify that the central charge is unity ($c_{tot} = 1$) in all Landau levels\textsuperscript{35,40–42}.

In Fig. 5 we present the LDOS of the lowest, first excited and second excited Landau levels on three cones differentiated by the effective magnetic flux threading the cone tip, following the previously described procedure\textsuperscript{28}. Near the tip of the cone, we observe oscillations in the
radial LDOS profile $\rho(r)$, which settle to a uniform background level by $r \approx 4L_0$. At large radii, the LDOS drops to zero only because a finite number of single-particle states were included, the number being limited by the size of the digital micromirror device used for mode injection. The background level is equal for all nine LDOS measurements (Supplementary Information section E.1) and their average is used to define the background LDOS $\rho_0$ for all measurements. We then compute the total excess particle number

$$\delta N = \int [\rho(r) - \rho_0] \, d^3r$$

and a measure of the width of the excess density, the shifted second moment

$$\Delta M_2 = \int [\rho(r) - \rho_0] \left( \frac{r^2}{2} - (2n + 1) \right) \, d^3r$$

These quantities provide the mean orbital spin and the central charge (our primary theoretical result; see Supplementary Information sections F and G).

From the excess particle number, we measure the mean orbital spin and average the result over flux threading in the lowest three Landau levels, finding $\delta \nu = 0.44(4), 1.46(2)$ and $2.44(2)$ for $n = 0, 1$ and 2, respectively. We can also use measurements of the shifted second moment to extract the mean orbital spin, because the linear component of the dependence of the shifted second moment on the flux is exactly $(\delta \nu - n) a$ (Supplementary Information section F). This provides a much more precise determination of $\delta \nu = 0.499(15), 1.500(1)$ and $2.505(68)$ for $n = 0, 1$ and 2, respectively. We use these measurements of the mean orbital spin along with measurements of the shifted second moment to calculate the central charge in each Landau level, finding $\delta \nu = 0.98(8), 1.29(1)$ and $1.84(24)$ for $n = 0, 1$ and 2, respectively. The precision of the measurement of the central charge decreases in higher Landau levels, owing to the finite field of view and the increased sensitivity to error in the mean orbital spin. Nonetheless, all measurements of the mean orbital spin and the measurement of the central charge in the lowest Landau level are in agreement with theoretical expectations for an integer quantum Hall fluid.

Outlook
We have developed and measured local observables that characterize bulk invariants of topological materials. We elucidate the physical importance of these invariants and our approach reduces the (non-physical) sensitivity of the standard definitions of the invariants to experimental imperfections such as disorder. For example, the standard definition of the Chern number assumes discrete translational symmetry, whereas our approach is applicable to systems composed of spatial domains separated by topological phase boundaries, for which spatial resolution would provide a Chern map.

Although the Hall conductance, mean orbital spin and central charge do not fully characterize a generic quantum Hall state, they often provide sufficient information to distinguish between candidate phases in the laboratory. In the case of the electronic $\nu = 5/2$ fractional quantum Hall plateau, a measurement of either the mean orbital spin or the central charge would suffice to choose among the more than nine candidate states. The photonic fractional quantum Hall system is bosonic, so similar physics is expected at $\nu = 1$, permitting the exploration of and state-reconstruction tools that are inaccessible to other platforms.
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Additional information

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METHODS

The experimental resonator. The four-mirror, running-wave, non-planar resonator that we used consists of four high-reflectivity mirrors, each with a radius of curvature of 100 mm and coated for both 780-nm and 1,560-nm light, mounted in two steel structures that define a stretched-tetrahedral resonator geometry characterized by an axial length of 3.1 cm and an opening half-angle of 10°. The two steel mounts are aligned via rods and a micrometre stage controls the relative separation. This enables smooth length adjustments to tune the resonator to degeneracy. One mirror is mounted on a piezoelectric transducer, which allows the cavity length to be stabilized via proportional–integral feedback based on a Pound–Drever–Hall error signal that is generated by the 1,560-nm light reflected off a resonator mirror. The free spectral range of the resonator is 1,400 MHz and the finesse for 780 nm is 8,000. The cyclotron frequency is \( \omega_C = 2\pi \times 900 \text{ MHz} \). The 1/e^2 intensity radius of the waist of the resonator is \( w_0 = 105 \mu m \), and this sets the magnetic length, \( l_B = w_0/2 \).

Digital micromirror device. To excite the resonator with light with arbitrary amplitude and phase profiles, we shine 780-nm narrowband laser light onto computer-generated holograms produced by a phase-corrected digital micromirror device, and direct the resulting diffracted light into the resonator. We then extract the full amplitude and phase information of the transmitted resonator field by interference with a reference beam (Supplementary Information section A).

Projector measurement. We use the digital micromirror device as a generalized scanning tunnelling microscope, enabling us to inject light with arbitrary position, momentum or angular momentum. The holographic reconstruction technique allows us to measure the (spatially localized) band projectors even when disorder or harmonic confinement breaks the degeneracy between the modes in the Landau level; it is only necessary that the probe sweeps across the band of states in the Landau level and that the resulting resonator response interferes with the reference beam before ‘integration’ of the intensity on the camera. At each laser frequency in the sweep, the resonator response is a ring carrying orbital angular momentum; these rings can interfere with one another, even if they arrive on the camera at different times, because the interference with the heterodyne beam converts phase to intensity, resulting in the desired localized mode (Fig. 2c). The sweep must be sufficiently slow compared to the lifetime of the photons in the resonator. For our experiments, an 8-MHz sweep is performed in 10 ms whereas the line width is \( 2\pi \times 180 \text{ kHz} \), resulting in a very slow sweep of the laser compared to the cavity lifetime: 0.004 cavity line widths per cavity lifetime.

Apparatus improvements. Our apparatus is based on that used previously28. We rebuilt the experimental resonator with new mirrors and mirror mounts and in a new configuration. The resonator housing is steel (rather than plastic) and the resonator length is stabilized with a Pound–Drever–Hall error signal that controls the proportional–integral feedback circuitry, which actuates a piezo-stack glued to a mirror. This enables the precise control of the probe laser detuning from the resonator resonance is necessary for the measurement of local projectors. We image the transverse plane of the resonator by collecting light transmitted through one of the mirrors. In passing through the glass substrate of a curved mirror at substantial non-normal incidence, the light is defocused by an effective cylindrical lens. This appears as an artificial breaking of rotational symmetry in the resonator modes and limited previous LDOS measurements. In particular, it made measurements of the second moment impossible because, unlike the integrated excess density, the second moment is not invariant to astigmatic distortion. Increasing the radii of curvature of the mirrors from 25 mm for two of them and 50 mm for the other two to 100 mm for all four of them and reducing the non-planar opening half-angle from 16° to 10° reduce the effect of this aberration (Extended Data Fig. 1).

Data availability
The raw data the support the findings of this study are available from the corresponding author on reasonable request.
Extended Data Fig. 1 | Resonator imaging comparison. The LDOS in the second excited Landau level with an effective magnetic flux of $\Phi_B/(2\pi) = -2/3$ threading the cone tip highlights the improvements in the resonator design and the imaging system. The previous resonator28 (top left) exhibits substantial diagonal astigmatism, which has been removed in the resonator used here (top right). Images of modes in the lowest Landau level provide estimates of the expectation value of $r^2$ (bottom), errors in which directly cause systematic errors in measurements of the shifted second moment. The substantial reduction in deviations from the ideal system enables measurements of the central charge and extensions to higher Landau levels. Error bars are calculated from the uncertainty in the centre location and waist size of the modes and are all smaller than the symbol size.