Pressure Vessel Design by Design by Analysis Route

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Abstract: Pressure vessels are conventionally designed based on the Design by Rule (DBR) approach. Standard vessel geometries are designed using simple formulae and charts, formulated as standards and codes based on the rules proposed in the ASME Boiler and Pressure Vessel Code and the European Standard prEN13445-3. The stress categorization problem faced employing DBR methodology was overcome by employing Limit Load Analysis using the DBA approach. A simple unfired vertical cylindrical pressure vessel with torispherical pressure head and Y-forged skirt support was analyzed for its structural integrity using ANSYS software. The results obtained for two different materials, high strength steel P500-QT and the conventional low strength steel P355 employing DBA approach was compared with the calculated results using the DBR approach. The maximum allowable pressure (P MAX) for P500-QT material was found to be 2.5X the maximum pressure calculated using the DBR approach as per ASME codes and 3X times as per EN13445-3 standard. For P355 material the maximum allowable pressure (P MAX) was found to be 2.32X the maximum pressure calculated using the DBR approach as per ASME codes and 2.6X times as per EN13445-3. The DBR methodology proves to be highly conservative whereas the use of DBA leads to less conservative design parameters.

1. Introduction

Pressure vessels are employed as storage vessels, industrial processing chambers, reactor pressure vessels, space tanks, and integrated thermal units. Conventionally pressure vessels are designed using standard codes framed by international agencies like ASME BPV- Section VIII, Div 1 & 2 and European Standard prEN13445-3. DBR is a highly conservative and uneconomical approach, which considers safety as prime design criteria. Nevertheless, design by rule (DBR) approach is still dominant. Stress categorization route also referred to as the DBR approach is employed to calculate the primary, secondary and peak stresses reflecting the nature of their failure mechanisms such as gross plastic deformation (GDP), fatigue failure and ratcheting [1] [7]. The design of pressure vessel warrants for an explicit understanding of the material response, operating stresses and loads, external influence, failure modes, construction details, detection equipment, risk control mechanisms and feedback controls for the structural integrity of the vessel [2].

Elasto-plastic or non-linear analysis, considering geometric discontinuities using commercial FE software such as ANSYS, will be useful in determining the limit load capability of the pressure vessel [3-4]. The limit load is defined as the maximum load beyond which the pressure vessel collapses. In the textbooks, frequent reference is made to ASME Section VIII Division-1 and Division-2. In European countries prEN 13445-3, Annex-B, Direct Route for Design by Analysis (DBA) and EN 13445-3: Unfired pressure vessels - Part 3: Design, Annex C [5] for stress categorization (SCL) route are followed for designing pressure vessels [5]. DBR is based on a set of simple formulae used to determine either the minimum thickness or the maximum allowable working pressure for different pressure load conditions [3]. DBA allows approximation techniques like finite element methods, which can complement [7] DBR in the design of pressure vessels.
2. Design by Analysis (DBA) Methodology

The DBA approach is further classified into Stress Categorization (SCL) – indirect route and Design by Analysis (DBA) – direct route using FE codes.

2.1. DBA using Commercial FE Software ANSYS

There are two basic hardening rules used by ANSYS to prescribe the modification of the yield surface. For Kinematic hardening, the yield surface remains constant in size and translates in the direction of yielding as shown in figure.1.(a). For Isotropic hardening, the yield surface expands uniformly in all directions with the plastic flow as shown in figure.1.(b). The Proposed study is confined to rate-independent multilinear kinematic hardening (MKIN).

![Types of Hardening Rules used in ANSYS](image)

Figure.1. Types of Hardening Rules used in ANSYS [8]

2.2. Problem Formulation and Methodology

The study investigates the simple unfired vertical cylindrical pressure vessel (shell) with Y-forged skirt support and torispherical pressure head, analyzed for structural integrity using ANSYS elastic analysis for stress categorization and in-elastic analysis for limit load capability analysis. The high strength pressure vessel steel P500-QT and a low alloy steel P355 are taken for investigation [7]. The proposed methodology is presented in figure.2. Secondary loads like nozzle cylinder interactions, nozzle moments, discontinuity forces, accidental loads and temperature effects are not considered for the analysis. The design specification is listed in table.1, 2 respectively.

3. Design of Pressure Vessel by DBF Approach

3.1. Design of Cylindrical Shell

Allowable design stress (f or S) as per ASME code is denoted by equation-1. Allowable design stress (f or S) as per prEN 13445 -3 European standards is denoted by equation-2. The Thickness of the cylindrical shell is calculated using equation-3. The axial stress ($\sigma_{xy}$) was found using equation-4. Design specifications and results are presented in the table.3, 4.
Table.1 Pressure Vessel Design Specification [7]

| Component                        | Cylindrical Shell, Head, Y-Forged Skirt Support |
|----------------------------------|-------------------------------------------------|
| Material                         | P500-QT, P355 Pressure Vessel Steels             |
| Design Pressure, P               | P500-QT : 8.25 MPa; P355: 8.25 MPa               |
| Design Temperature               | 120°C                                           |
| Inner Diameter of Vessel / Shell, D | 2900 mm                                       |
| Cylindrical Shell Inner Radius, R | 1450 mm                                       |
| Maximum Allowable design Stress, S or f | P500-QT : 299 MPa; P355: 250 MPa |
| Load Case                        | Internal Pressure, P, MPa                      |
| Corrosion Thickness              | 2 mm to 4 mm maximum                           |
| Insulation thickness             | Not Required (Unfired Vessel Application)       |
| Pressure Head Type               | Torispherical Head                             |
| Skirt type                       | Y-Section Support (Forged Section)              |
| Skirt height                     | 1000mm                                         |
| Overall height of Vessel         | 5800mm                                         |
| Fluid /Application               | Hydrocarbon / Storage                          |
| Joint Efficiency, E              | 100% assumed                                    |

Table.2 Material Properties of Commercially Available Pressure Vessel Steels P500-QT and P355

| Material Properties          | P500-QT | P355 |
|------------------------------|---------|------|
| Young’s modulus, \(E\) (GPa) | 210     | 200  |
| Poisson Ratio (\(\gamma\))   | 0.29    | 0.28 |
| UTS, \(\sigma_u\) (R_{M20}), MPa | 640    | 600  |
| Yield value, \(\sigma_y\) (R_{P0.2}), MPa | 580    | 380  |
| Density, \(\rho\) (Kg/m^3)   | 7872    | 7850 |

Table.3 Cylindrical Shell Design Specification

| Design Pressure (P) | 8.25 MPa [7] |
| Corrosion Allowance (t) | 2mm to 4 mm |
| Cylindrical Shell Inner Radius (R) | 1450 mm |
| Joint efficiency (E or EJ) | 100% |

\[
f = \min \left( \frac{R_p(0.2/t)}{1.5}, \frac{R_m}{2.14} \right)
\]

\[
T = \frac{PR}{SE - 0.6P}
\]

\[
s_{xp} = \frac{PD}{4t}
\]

Table.4 Design Stress and Thickness of Cylindrical Shell Calculated using DBF Approach

| Material | Calculated Thickness, T mm | Corrosion Allowance | Final Shell thickness (Commercially available thickness), mm | Design Stress S or f, MPa |
|----------|-----------------------------|---------------------|------------------------------------------------------------|--------------------------|
| P500-QT  | 40                          | 4                   | 50                                                         | min (386.67 ; 299) = 299 |
| P355     | 48                          | 4                   | 50                                                         | min (253.33 ; 250) = 250 |

3.2. Design of Torispherical Head

For thickness and axial stress due to pressure load, equations 5 and 6 are used [3] [5]. Design specifications and results are presented in table.5 and 6.

Table.5 Cylindrical Shell Design Specification

| Design Pressure(P) | 8.25 MPa [7] |
| Head Inner Radius (R_{h}) | 1450 mm |
| Allowable Stress (f or S) | P500-QT : 299 MPa; P355 : 250 MPa |
| Joint Efficiency (E or EJ) | 100t% |
| Thickness of cylindrical shell (T=t=e_a) | 50 mm |
Table 6. Design Stress and Thickness of Torispherical Head Calculated using DBF Approach

| Material    | Calculated Thickness, \( T_2 \) mm | Corrosion Allowance 2 mm | Final shell thickness (Commercially available thickness), mm | Design Stress \( S_{or f} \), MPa |
|-------------|-----------------------------------|--------------------------|-------------------------------------------------------------|----------------------------------|
| P500-QT     | 20                                | 2                        | 25                                                          | 299                              |
| P355        | 23                                | 2                        | 25                                                          | 250                              |

3.3. Stresses in Cylindrical Shell and Torispherical Head

The stresses at or near the juncture of pressure vessels are called as the discontinuity or secondary stresses. They are secondary in extent but can be of very high magnitude. The secondary stresses acting at the cylindrical shell are calculated using equations 7 and 8 to calculate the longitudinal \( \sigma_y \) and hoop stresses \( \sigma_x \) respectively [3] [5]. But in the spherical portion (refer equation 9) both the stresses are equal i.e. \( \sigma_y = \sigma_x \) [3]. The value of ‘t’ for the shell portion is 50mm. and for the spherical region is 25mm. Where \( P=8.25 \)MPa [7] and \( R=1450 \)mm. Thus from the above design calculations, the thickness of the vessel components like shell, head, and the skirt was found.

\[
\sigma_y = \frac{PR}{2t} \quad (7) \quad \sigma_y = \sigma_x = \frac{PR}{2t} \quad (9) \quad \sigma_x = \frac{PR}{t} \quad (8)
\]

Design Check: P500-QT and P355 Material

| Stress Type                          | P500-QT (MPa) | P355 (MPa) |
|--------------------------------------|--------------|------------|
| Longitudinal stress in cylindrical shell | 119.625 < S | 239.25 < S |
| Hoop stress in cylindrical shell      |              |            |
| Stresses in spherical head, \( \sigma_y = \sigma_x \) | 239.25 < S |            |

3.4. Maximum Allowable Stresses in Cylindrical Shell and Torispherical Head

The maximum permissible pressure according to the design by rules approach is calculated as laid out in Section 7.6.6.3 in prEN13445. The general formula used for the calculation of the maximum permissible pressure in EN13445 for cylindrical pressure vessels is represented by Equation 10 [3] [5]. As per the DBR the allowable pressure \( (P_{max}) \) for the material under study is tabulated in table 7.

\[
P_{max} = \frac{2fz e_a}{D_M} \quad (10)
\]

Table 7. Design Stress and Thickness of Torispherical Head Calculated using DBR Approach

| Material       | Mean diameter of a cylindrical shell \( (D_M) \) | Weld efficiency \( (z) \) | Thickness of pressure vessel under study \( (e_a) \) | Allowable design stress \( (f) \) | Maximum allowable pressure \( (P_{max, DBR}) \) MPa |
|----------------|-----------------------------------------------|--------------------------|--------------------------------------------------|---------------------------------|-----------------------------------------------|
| P500-QT        | 3000 mm                                       | 100T%                    | 50 mm                                            | P500-QT: 299 MPa; P355: 250 MPa |                                |
| P355           |                                               |                          |                                                  |                                 |                                |

4. Design of Pressure Vessel by DBA Approach

Initially, the elastic analysis was carried out to find the stresses at the junctions of shell and head, shell and skirt and the yield pressure is observed from the elastic analysis. Secondly, nonlinear analysis considering the material nonlinearity was conducted. A 2D axisymmetric analysis was carried out using ANSYS software. The material modeling for nonlinearity is done by
selecting the inelastic, rate-independent Mises plasticity [8] and incorporating the multilinear kinematic hardening option by assigning the true stress strain data for P500-QT and P355 steel material as shown in figure.3.

![Figure 3](image1)

**Figure 3. True Stress-Strain Curve for P500-QT and P355 Material**

5. Elastic Analysis of Pressure Vessel

The stress distribution in the discontinuity regions is marked as SCL (Stress Categorization Lines) as shown in figure.4. Five nodal points equally spaced along with the thickness, designated as O-outer, NO- near outer, M-middle, NI- near inner and I-inner are considered for elastic analysis. The following junctions are considered for stress categorization analysis:

a) SCL-1: Shell to pressure head taper start junction; b) SCL-2: Shell to pressure head taper end junction; c) SCL-3: Y-forged section to the head junction; d) SCL-4: Y-forged section to the skirt junction; e) SCL-5: Y-forged section near the curvature and the head junction; f) SCL-6: Y-forged section near the curvature and the skirt junction.

![Figure 4](image2)

**Figure 4. a) Pressure Vessel Schematic b) 2D Finite Element Model with Axisymmetric Boundary Conditions c) Stress Categorization Lines (SCL – Inner, Near Inner, Middle, Near Outer, Outer)**

The discontinuity stresses developed in the shell to skirt junctions and the shell to head junctions can be determined using elastic FE analysis, which by analytical method becomes tedious and time-consuming. The stress distributions in the discontinuity region from the inner area to the outer area of the vessel are captured (refer figure 5-8). Also, in-elastic or plastic analysis of the vessel was conducted considering the material nonlinearities, modeled using true stress-strain material data. An increment of 1MPa is given to the yield pressure until we obtain the limit load of the vessel. Thus using ANSYS finite element package we conduct both elastic and elastic-plastic analysis for the pressure vessel taken for study.
Figure 5. Stress Distribution Curves at Stress Categorization Lines 1 to 6 for P500-QT Material Subjected to Yield Pressure - Elastic Analysis

Figure 6. Deformation and Von Mises Plot at Yield Pressure for P500-QT Material - Elastic Analysis
6. In-Elastic Analysis of Pressure Vessel

The in-elastic analysis was carried out to find the limit or collapse load of the pressure vessel. The deformation along axial (UY) and radial (UX) are captured for different pressure loading. The pressure loading cycle with an increment of 1MPa starting from yield pressure \[7\] was followed: \(P_y\) (pressure at yield) to \(P_L\) (limit pressure- to be determined from analysis). The criteria for finding the collapse load in the elastoplastic analysis are presented below:

(a) With a sudden increase in the pressure, the vessel deforms drastically, a condition at which the finite element analysis gives an error stating that the element is highly deformed. This happens in any large strain analysis problems.
(b) Also, the principal plastic strain increment for large strain analysis should be less than or equal to 5% \[7\]. Thus the limit pressure value should have a limiting strain increment (refer figure 9).
In our case, this criterion was satisfied to a major extent but produced an error of +1.4% (i.e. 6.4% maximum) increase in plastic strain (refer figure.9) for the pressure vessel steel P500-QT. But P355 material falls within the limit. Thus from the investigation, the limit or collapse load (P_L) for the pressure vessel and the plastic strain increment (successive strain increment) values obtained from the analysis using ANSYS package is tabulated in table 8.

![Figure 9: Principal Plastic Strain versus Load for P500-QT and P355 Material](image)

Table 8. Limit Load (P_L) for P500-QT and P355 Pressure Vessel Steel: ANSYS Results

| Material | Limit load, P_L, MPa | Principal plastic strain Increment (%) | Principal plastic strain Error (%) |
|----------|----------------------|----------------------------------------|-----------------------------------|
| P500-QT  | 37.25                | 6.4                                    | 1.4                               |
| P355     | 29.05                | -0.17 (negligible)                     | NIL                               |

7. Maximum Allowable Pressure as per DBA

From DBA-inelastic analysis, P_L = 29.05 MPa (P355 material); 37.25 MPa (P500-QT material) was observed. As per ASME standard and pr EN13445 standard, the maximum allowable pressure (P_max,DBA) is calculated using equations 11 and 12 respectively. The maximum allowable pressure using FE analysis following the DBA approach is depicted in table 9.

\[
P_{\text{max, DBA}} = \frac{2}{3}P_L \quad (11) \quad P_{\text{max, DBA}} = \frac{P_L}{\gamma_p} \quad (12)
\]

Where, P_L = Limit pressure obtained through analysis, MPa
\(\gamma_p\) = Partial factor of safety for ferritic steel material [7]. (\(\gamma_p\) = 1.25 for P500-QT and \(\gamma_p\) = 1.3383 for P355)

Table 9. Maximum Allowable Pressure from FE analysis following DBA approach

| Material | Maximum Allowable Pressure (P_max,DBA), MPa, as per ASME standard | Maximum Allowable Pressure (P_max,DBA), MPa, as per EN13445-3 European standard |
|----------|---------------------------------------------------------------------|--------------------------------------------------------------------------------|
| P500-QT  | 24.83                                                               | 29.8                                                                         |
| P355     | 19.37                                                               | 21.70                                                                        |

(c) Principal plastic strain at 19.25 MPa
(e) Principal plastic strain at 37.25 MPa
Comparison of DBR with DBA

The maximum allowable pressure calculated using the design by rule (DBR) and design by analysis (DBA) approach following both ASME and European standards is presented in Table. 10. The results reveal that steels with high yield to tensile ratio are penalized using the DBR design approach [7].

| Material | \( P_{\text{max, DBA}} \) MPa | \( P_{\text{max, DBA}} \) MPa (ASME) | \( P_{\text{max, DBA}} \) MPa (EN13445-3) |
|----------|-------------------------------|--------------------------------|-----------------------------------|
| P500-QT  | 9.96                          | 24.83                          | 29.8                              |
| P355     | 8.33                          | 19.37                          | 21.70                             |

Figure 10. Principal Plastic Strain at different Pressure Loads for P500-QT Material: In-Elastic Analysis.

Figure 11. Principal Plastic Strain at different Pressure Loads for P355 Material: In-Elastic Analysis.
9. Results And Discussion

The maximum allowable pressure for the pressure vessel taken for analysis as per the DBR method is found to be highly conservative. Both the high strength and low strength steel materials are penalized by the DBF methodology [7] in terms of the load carrying capacity of the material (refer table.11). This means the designed pressure vessel can withstand more pressure than the maximum allowable pressure as designated by DBF methodology.

| Material  | Comparison of DBA and DBF as per ASME standard | Comparison of DBA and DBF as per EN13445-3 standard |
|-----------|-----------------------------------------------|--------------------------------------------------|
| P500-QT   | \( P_{DBA}=2.5 \ P_{DBF} \)                   | \( P_{DBA}=3.0 \ P_{DBF} \)                     |
| P355      | \( P_{DBA}=2.32 \ P_{DBF} \)                  | \( P_{DBA}=2.6 \ P_{DBF} \)                     |

The conclusions drawn from this study can be summarized as follows:

a) The designed pressure vessel can withstand more pressure than the maximum allowable pressure as designated by DBR methodology. The application of DBA leads to much less conservative design parameters [7] for less critical pressure vessel applications.

b) High safety performance is mandatory for pressure vessel applications. Pressure vessels used in nuclear reactors and operating in adverse conditions should necessarily operate under high safety. But this may not be required for vessels used for storage applications (other than harmful liquids and gases).

c) Plastic strain increment error of 1.4% from its limit of maximum 5% for P500-QT material, because of modeling and meshing errors occurred in ANSYS package during the analysis phase. Mesh convergence study could reduce such errors. DBA can be integrated with DBR in designing pressure vessels for normal and less critical storage applications.

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