Application of Taguchi Experimental Method in Numerical Simulation of Variable Viscosity Effect on Natural Convection in Porous Media

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Abstract

The study uses an optimization approach representation for the variable viscosity effect on the free convection over a vertical permeable plate in porous media. The surface of the vertical plate is uniform wall temperature (UWT). The surface blowing/suction velocity is also uniform. The viscosity of the fluid varies inversely as a linear function of the temperature. The partial differential equations are transformed into non-similar equations and solved by Keller box method (KBM). Compared with the previously published articles, the results are considered to be very consistent. Numerical results for the the local Nusselt number with the two parameters: 1) blowing/suction parameter $\xi$, 2) viscosity-variation parameter $\theta_r$ are expressed in tables. Through the Taguchi method to predict the best point of the maxima of the local Nusselt number is 1.1152.

Keywords: Taguchi method, Variable viscosity, Free convection, Vertical permeable plate, Porous media.

1. Introduction

The heat transfer of free convection in saturated porous media has many important applications in nature and engineering. Examples include geothermal flow, nuclear waste storage, and electronic heat transfer systems.

Minkowycz and Cheng\(^1\) solved the uniform lateral mass flux on natural convection over a vertical plate in porous media by using the local non-similarity (LNS) approximation. The uniform blowing/suction effect on free convection past a vertical plate and cone with uniform wall temperature (UWT) in porous media was solved by Yih\(^2\) utilizing Keller box method (KBM). He found that the local Nusselt number increase for the case of suction, but the blowing is the opposite.

The study of the variable viscosity in porous media has been widely discussed. Lai and Kulacki\(^3\) considered the effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium, and fluid viscosity has been assumed to be an inverse function of temperature. The effect of variable viscosity on flow and heat transfer characteristics for natural convection along an inclined plate in a saturated porous medium with an applied magnetic field has been explored by Tomer et al.\(^4\).

However, the Taguchi method to find the maximum values simulation of the research is still very few. Chou et al.\(^5\) studied application of Taguchi non-linear dynamic quality characteristics to fuzzy control system. Ho et al.\(^6\) reported adaptive network-based fuzzy inference system for prediction of surface roughness in end milling process using hybrid Taguchi-genetic learning algorithm. Taguchi optimization of bismuth-telluride based thermoelectric cooler was analyzed by Kishore et al.\(^7\).

In this exploring, the partial differential equations are transformed into non-similar equations and sloved in the Keller box method (KBM) proposed by Cebeci and Bradshaw\(^8\). The numerical simulation of two different parameters is carried out to obtain simulate the maximum value of the local Nusselt number and find the best parameter ratio. The study is divided into two phases: the first phase of the data is compared with the previously published article, the results are considered very consistent. The variable viscosity and blowing/suction effects on the local Nusselt number are presented in tabular and graphic forms. The second stage uses the Taguchi method to find the maximum point of the maxima of the local Nusselt number is 1.1152.
optimize the two parameters by Chou\cite{9}. We use the Taguchi experimental method to replace the single factor experiments to find the maximum local Nusselt number.

2. METHODS

2.1 Mathematical Equations

Phase 1: consider the influences of the variable viscosity and blowing/suction on the heat transfer by free convection flow over a vertical permeable flat plate embedded in a saturated porous medium. Figure 1 shows the concept map. The boundary condition is uniform wall temperature Tw (UWT).

Both a and $T_r$ are constants and their values depend on the reference state and the thermal property of the fluid, i.e., $\gamma$. In general, $a > 0$ ($\gamma > 0$) for liquid and $a < 0$ ($\gamma < 0$) for gas. The viscosity of the fluid usually reduces with increasing temperature while it enhances for gas.

To further show the appropriateness of equation (1), correlations between viscosity and temperature for air and water are given below because these two are the most common working fluids found in engineering applications.

For air

$$\frac{1}{\mu} = -123.2(T - 742.6)$$

and for water

$$\frac{1}{\mu} = 29.83(T - 258.6)$$

The data used for these correlations are taken from Weast\cite{11}. While equation (3) is good to within 1.2% from 278$^\circ$K (5 $^\circ$C) to 373 K (100 $^\circ$C), equation (4) is good to within 5.8% from 283K (10 $^\circ$C) to 373 K (100 $^\circ$C). The reference temperatures thus selected for the correlations are very practical in most applications.

Introducing the boundary layer approximation and Boussinesq approximation, the governing equations and the boundary conditions based on the Darcy law can be written as follows:

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum (Darcy) equation:

$$u = -\frac{K}{\mu} \left( \frac{\partial p}{\partial x} + \rho g \right)$$
\[ v = -\frac{K}{\mu} \frac{\partial p}{\partial y} \quad (7) \]

Energy equation:
\[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \quad (8) \]

Boussinesq approximation:
\[ \rho = \rho_0 [1 - \beta_T (T - T_\infty)] \quad (9) \]

Boundary conditions:
\[ y = 0: v = V_w, T = T_w \quad (10.1-3) \]
\[ y \to \infty: u = 0, T = T_\infty \quad (11.1-3) \]

Here, \( u \) and \( v \) are the Darcian velocities in the \( x \)- and \( y \)-directions; \( \mu, p \), and \( \rho \) are the variable viscosity, the pressure and the density of the fluid, respectively; \( K \) is the permeability of the porous medium; \( g \) is the gravitational acceleration; \( T \) is the volume-averaged temperature; \( \alpha \) is the equivalent thermal diffusivity; \( \beta_T \) is the thermal expansion coefficients of the fluid; \( V_w \) is the uniform blowing/suction velocity.

The stream function \( \psi \) is defined by
\[ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x} \quad (12) \]

Thus, the continuity equation is automatically satisfied.

We note the governing equations (6) and (7). If we do the cross-differentiation \( \partial (\mu \cdot u)/\partial y - \partial (\mu \cdot v)/\partial x \), then the pressure terms in equations (6) and (7) can be removed. Next step, with the help of the equation (9) and the boundary layer approximation \( \partial / \partial x \ll \partial / \partial y \), then we can obtain
\[ \frac{\partial}{\partial y} (\mu \cdot u) = \rho_0 g K \left[ \beta_T \frac{\partial T}{\partial y} \right] \quad (13) \]

Integrating equation (13) once and via equations (1) and (11), then we get
\[ u = \left[ 1 + \gamma (T - T_\infty) \right] \frac{\rho_0 g K}{\mu_\infty} \left[ \beta_T (T - T_\infty) \right] \quad (14) \]

Using the following dimensionless non-similarity variables:
\[ \xi = \frac{2V_w x}{\alpha \cdot Ra^ {1/2}_x} \quad (15.1) \]
\[ \eta = \frac{y}{x \cdot Ra^ {1/2}_x} \quad (15.2) \]
\[ f(\xi, \eta) = \frac{\psi}{\alpha \cdot Ra^ {1/2}_x} \quad (15.3) \]
\[ \theta(\xi, \eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (15.4) \]
\[ Ra_x = \frac{\rho_0 g \beta_T K (T_w - T_\infty)}{\mu_\infty} \frac{x}{\alpha} \quad (15.5) \]

Substituting equation (15) into equations (14), (8) to (11), we obtain
\[ f' = (1 - \frac{\theta}{\theta_\ell}) \cdot \theta \quad (16) \]
\[ \theta' + \frac{1}{2} f \theta' = \frac{\xi}{2} (f' \cdot \frac{\partial \theta}{\partial \xi} - \theta' \cdot \frac{\partial f}{\partial \xi}) \quad (17) \]

The boundary conditions are defined as follows:
\[ \eta = 0: f = -\frac{\xi}{2} \cdot \theta = 1 \quad (18) \]
\[ \eta \to \infty: \quad \theta = 0 \quad (19) \]

Furthermore, in terms of the new variables, the Darcian velocities in \( x \)- and \( y \)-directions are, respectively, given by
\[ u = \frac{\alpha Ra^ {1/2}_x}{x} f' \quad (20) \]
\[ v = -\frac{\alpha Ra^ {1/2}_x}{x} \left[ \frac{1}{2} f + \frac{1}{2} (\xi \cdot \frac{\partial f}{\partial \xi} - \eta \cdot f') \right] \quad (21) \]

Where primes denote differentiation with respect to \( \eta, \xi \) defined in equation (15.1). \( \ell \) is the surface blowing/suction parameter; equation (18) can be obtained by integrating equation (21) versus \( \xi \) once and by setting \( \eta = 0 \) (at the surface, \( y = 0 \), then \( \eta = 0 \)), and with the help of equation

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(10). On the one hand, for the case of blowing, $V_w > 0$ and hence $\xi > 0$. On the other hand, for the case of suction, $V_w < 0$, and hence, $\xi < 0$.

Via the equations (15.4) and (2), the viscosity-variation parameter $\theta_r$ is a constant and is defined by

$$\theta_r = \frac{T_e - T_w}{T_w - T_e} = \frac{1}{\gamma(T_w - T_e)}$$

(22)

For equation (1), it is value referring that for $\gamma \to 0$, i.e. (constant viscosity) then $\theta_r \to \infty$. It is also important to note that $\theta_r$ is negative for liquid and positive for gas.

The result of practical interest in many applications is the surface heat transfer rate. The surface heat transfer rate is expressed in terms of the local Nusselt number

$$\frac{Nu_x}{Ra_x^{1/2}}$$

defined as following:

$$\frac{Nu_x}{Ra_x^{1/2}} = -\theta_r(\xi, 0)$$

(23)

For the case of $\theta_r \to \infty$, equations (16) to (19) are reduced to these of Yih(2) where a non-similar solution was obtained previously.

The analysis integrates the system of equations (16) to (19) by the implicit finite difference approximation together with the modified Keller box method of Cebeci and Bradshaw(8). First, the partial differential equation is converted to a system of three first-order equation. Then, these first-order equations are expressed in finite difference form and solved by the iterative scheme along with their boundary conditions. This method provides a better convergence rate and reduces the numerical calculation times.

The initial size of the calculated grid is $\Delta \eta_1 = 0.01$, the variable grid increment parameter is set to 1.01, the maximum value of $\eta_\infty$ is 15 and $\Delta \xi = 0.1$(uniform grid).

When the error value of $\theta_{\xi, 0}$ becomes less than $10^{-5}$, the iteration process is stopped and the final temperature distribution is given.

### 2.2 Taguchi method

Phase 2: Taguchi method is used to simulate three different values of two parameters, A: blowing/suction parameter $\xi(\xi = -2, 0, 2)$, B: viscosity-variation parameter $\theta_r(-2, \infty, 2)$.

If arranged in full factor method to set the parameter, it will take quite a large computation time. Therefore, this experiment selected Taguchi method for numerical optimization design by numerical simulation can reduce times. This study was chosen L9 ($3^3$) orthogonal table.

The two factors (A, B, 2 columns) have three levels (1, 2, 3), the number of simulations is 9 times. The levels of these two factors appear the same and this is a balance. This method will be used to establish the factor-level and preliminary data combinations as shown in Table 1. Select the Taguchi L9 ($3^3$) orthogonal table via Table 1 as shown in Table 2.

| Factor | Level | 1 | 2 | 3 |
|--------|-------|---|---|---|
| blowing/suction parameter $\xi$ | A | $-2$ | 0 | 2 |
| viscosity-variation parameter $\theta_r$ | B | $-2$ | $\infty$ | 2 |

| No. | Row | 1 | 2 | 3 | 4 |
|-----|-----|---|---|---|---|
| 1   | 1   | 1 | 1 | 1 | 1 |
| 2   | 1   | 2 | 2 | 2 |   |
| 3   | 1   | 3 | 3 | 3 |   |
| 4   | 2   | 1 | 2 | 3 |   |
| 5   | 2   | 2 | 3 | 1 |   |
| 6   | 2   | 3 | 1 | 2 |   |
| 7   | 3   | 1 | 3 | 2 |   |
| 8   | 3   | 2 | 1 | 3 |   |
| 9   | 3   | 3 | 2 | 1 |   |

The measure used in science and engineering is the signal to noise ratios. For this purpose, the signal to noise ratio (S/N ratio) measured with dB is used. Minimizing quality characteristics is equivalent to maximizing the S/N ratio, defined by

$$S/N = -10 \log_{10} \frac{1}{n} \sum_{i=1}^{n} \left( \frac{1}{Y_i^2} \right)$$

(24)

Where Yi denote local Nusselt number in Table 6.
3. Results and Discussion

In order to verify the accuracy of our current method, we compared our results with those of Minkowycz and Cheng\(^{(1)}\) and Yih\(^{(2)}\). The results of comparing these articles are very consistent, as shown in Table 3.

Table 3. Comparison of the values of \(-\theta'(\xi,0)\) for various values of \(\xi\) with \(\theta_r = \infty\).

| \(\xi\) | Minkowycz and Cheng\(^{(1)}\) | Yih\(^{(2)}\) | Present results |
|--------|-----------------------------|-------------|-----------------|
| -2     | 1.0680                      | 1.0725      | 1.0726          |
| -1     | -                           | -           | 0.7117          |
| 0      | 0.4438                      | 0.4437      | 0.4437          |
| 1      | -                           | -           | 0.2595          |
| 2      | 0.1423                      | 0.1416      | 0.1411          |

The numerical results are presented for the blowing/suction parameter \(\xi\) ranging from \(-2\) to \(2\) and the viscosity-variation parameter \(\theta_r\) ranging from \(-2\) to \(2\).

Figure 2 plots the effects of blowing/suction parameter \(\xi\) and the viscosity-variation parameter \(\theta_r\) on the dimensionless temperature profile. For a fixed \(\xi\), it is observed that the dimensionless temperature profile increases with increasing the viscosity-variation parameter \(\theta_r\) from \(-2\) (liquid) to \(2\) (gas), thus decreasing the dimensionless wall temperature gradient. For \(\theta_r\) is given, as the blowing/suction parameter \(\xi\) is increased from \(-2\) (suction) to \(2\) (blowing), the dimensionless wall temperature gradient has the tendency to reduce.

![Graph showing the effects of \(\xi\) and \(\theta_r\) on the dimensionless temperature profile.](image)

The maximum value of the local Nusselt number is \(1.1152\) for the \(\xi(-2), \theta_r(-2)\). From the local Nusselt number table 4, it is confirmed that the value generated by the Taguchi method is the same as \(\xi(-2)\) and \(\theta_r(-2)\) in the orthogonal table 4. The maximum value of the local Nusselt number is \(1.1152\) for the \(\xi(-2), \theta_r(-2)\).

Table 4 lists the values of local Nusselt number for various values of \(\xi, \theta_r\). When \(\xi\) increases, the \(-\theta'(\xi, 0)\) decreases. As the \(\xi = -2\), the maximum value of the \(-\theta'(\xi, 0)\) is \(1.1152\). It has been found that both the local Nusselt number decreases with increasing the viscosity-variation parameter \(\theta_r\). This is because for the case of \(\theta_r > 0\) (gas) decreases the dimensionless surface temperature gradient, as shown in figure 2.

Table 4. The orthogonal table numerical of the local Nusselt number, the local Nusselt number of signal/noise ratio.

| \(\xi\) | \(\theta_r\) | local Nusselt number | Nusselt number signal/noise ratio |
|--------|-------------|----------------------|----------------------------------|
| \((-2)\) | \((-2)\)    | 1.1152               | 0.9471                           |
| \((-2)\) | \((\infty)\) | 1.0726               | 0.6088                           |
| \((-2)\) | \((2)\)     | 1.0342               | 0.2921                           |
| \((0)\) | \((-2)\)    | 0.5180               | -5.7134                          |
| \((0)\) | \((\infty)\) | 0.4437               | -7.0582                          |
| \((0)\) | \((2)\)     | 0.3536               | -9.0298                          |
| \((2)\) | \((-2)\)    | 0.2033               | -13.8373                         |
| \((2)\) | \((\infty)\) | 0.1411               | -17.0095                         |
| \((2)\) | \((2)\)     | 0.0706               | -23.0239                         |
| average |             | 0.5503               | -8.2027                          |

Table 5. The results of the local Nusselt number and signal noise ratio orthogonal table simulation.

| Factor | Nusselt number signal noise ratio |
|--------|----------------------------------|
| Level  | А                            | Б                |
| 1      | 0.6160                        | -6.2012          |
| 2      | -7.2671                       | -7.8196          |
| 3      | -17.9569                      | -10.5872         |
| Delta  | 18.5729                       | 4.3860           |
| Ranking| 1                             | 1                |
| Level  | Mean Value                    |
| 1      | 1.0740                        | 0.6122           |

Fig 2. The dimensionless temperature profile for two values of \(\xi\) and \(\theta_r\).
for “Local non-similar solutions for free convective flow with uniform lateral mass flux in porous medium”, Lett Heat Mass Trans, Vol. 9, pp. 159-168, 1982
2) K. A. Yih : “The effect of uniform lateral mass flux on free convection about a vertical cone embedded in a saturated porous medium”, Int Comm Heat Mass Tran, Vol. 24, pp. 1195-1205, 1997
3) F. C. Lai, and F. A. Kulacki : “The effect of variable viscosity on convective heat transfer along a vertical surface in a saturated porous medium”, Int J Heat Mass Tran, Vol. 33, pp. 1028-1031, 1990
4) N. S. Tomer, P. Singh, and M. Kumar : “Effect of variable viscosity on convective heat transfer along an inclined plate embedded in porous medium with an applied magnetic field”, WASET, Vol. 51, pp. 992-996, 2011
5) J. H. Chou, and X. X. Wu : “Application of Taguchi nonlinear dynamic quality characteristics to fuzzy control system”, J Sci Educ Technol, Vol. 8, pp. 1-7, 1999
6) W. H. Ho, J. T. Tsai, B. T. Lin, and J. H. Chou : “Adaptive network-based fuzzy inference system for prediction of surface roughness in end milling process using hybrid Taguchi-genetic learning algorithm”, Expert Syst Appl, Vol. 36, pp. 3216-3222, 2009
7) R. A. Kishore, P. Kumar, M. Sanghadasa, and S. Priya : “Taguchi optimization of bismuth-telluride based thermoelectric cooler”, J Appl Phys, Vol. 122, pp. 025109-1-025109-12, 2017
8) T. Cebeci, and P. Bradshaw : “Physical and Computational Aspects of Convective Heat Transfer”, New York, Springer-Verlag, 1984
9) J. H. Chou : “Optimization Methodology Course Handout”, Department of Electrical Engineering, National Kaohsiung University of Applied Sciences, 2012
10) K. Vajravelu, K. V. Prasad, R. A. V. Gorder, and J. Lee : “Free convection boundary layer flow past a vertical surface in a porous medium with temperature-dependent properties”, Transport Porous Med, Vol. 90, pp. 977-992, 2011
11) R. C. Weast : “CRC Handbook of Chemistry and Physics”, CRC Press, Boca Raton, Florida, 1990