Neuromorphic computing describes the use of very-large scale integrated logic (VLSI) systems to mimic neurobiological architectures [1] promising advances in computation density and energy efficiency in the post Moore’s law age of computing [2–6]. Oscillatory neural networks (ONNs) mimicking the human brain [7] are among the key challenges for implementation with nanotechnologies. Here we propose a new method of phase programming in ONs by manipulation of the saturation magnetization, and consequently the resonance frequency of a single oscillator via Joule heating by a simple DC voltage input. We experimentally demonstrate this method in a pair of stray field coupled magnetic vortex oscillators. Since this method only relies on the oscillatory behavior of coupled oscillators, and the temperature dependence of the saturation magnetization, it allows for variable phase programming in a wide range of geometries and applications that can help advance the efforts of high frequency neuromorphic spintronics up to the GHz regime.

Neurons in the brain behave as a network of coupled nonlinear oscillators processing information by rhythmic activity and interaction. Several technological approaches have been proposed that might enable mimicking the complex information processing of neuromorphic computing, some of them relying on nanoscale oscillators. For example, spin torque oscillators are promising building blocks for the realization of artificial high-density, low-power oscillatory networks (ON) for neuromorphic computing. The local external control and synchronization of the phase relation of oscillatory networks are among the key challenges for implementation with nanotechnologies. Here we propose a new method of phase programming in ONs by manipulation of the saturation magnetization, and consequently the resonance frequency of a single oscillator via Joule heating by a simple DC voltage input. We experimentally demonstrate this method in a pair of stray field coupled magnetic vortex oscillators. Since this method only relies on the oscillatory behavior of coupled oscillators, and the temperature dependence of the saturation magnetization, it allows for variable phase programming in a wide range of geometries and applications that can help advance the efforts of high frequency neuromorphic spintronics up to the GHz regime.
to be able to determine the phase relation between the applied rf current. In addition, STXM is used to obtain the vortex cores [29] in order to determine their radius. DC mode of LTEM is used to image the trajectories of the vortex core motion recorded in the LTEM image of the pair of magnetic vortex disks (radius \(d\)) remains at ambient temperature.

The dynamics of this system can be modeled using two coupled Thiele equations [23, 24], which describe the dynamics of the vortex core positions for the driven (index \(d\)) and the heated (index \(h\)) disk. For disk\(_d\) a spin polarized current density has been included [31, 32]:

\[
\mathbf{G}_d \times (\mathbf{V}_d + b_j \mathbf{j}) + \mathbf{D}_d(\alpha \mathbf{V}_d + \xi b_j \mathbf{j}) = \nabla_{\mathbf{R}_d} W(\mathbf{R}_d, \mathbf{R}_h)
\]

where \(\mathbf{G}_i = \mathbf{G}_d \times (\mathbf{V}_d + b_j \mathbf{j}) + \mathbf{D}_d(\alpha \mathbf{V}_d + \xi b_j \mathbf{j}) = \nabla_{\mathbf{R}_d} W(\mathbf{R}_d, \mathbf{R}_h)\)

\[
\mathbf{G}_h \times \mathbf{V}_h + \mathbf{D}_h \alpha \mathbf{V}_h = \nabla_{\mathbf{R}_h} W(\mathbf{R}_d, \mathbf{R}_h),
\]

Eigenmodes of the coupled oscillator system

The oscillators are excited by a cw excitation in the range of several hundred MHz applied to the right driven disk (disk\(_d\)) harnessing the Spin Transfer Torque effect (STT) [22, see Fig 1]. To manipulate the phase a DC voltage \(U_{\text{heat}}\) can be applied to the left "heated" disk (disk\(_h\)) to change its temperature via Joule heating and thereby reducing its saturation magnetization \(M_s\). The temperature of the heated disk can be estimated by \(T_h = T_0 + \frac{R_{\text{thermal}} U_{\text{heat}}^2}{R_d + R_c}\) with \(T_0 = 293\) K being ambient temperature, \(R_d = 90\) \(\Omega\) the electrical resistance of the disk, and \(R_c\) the resistance of the electrical contacts. \(R_{\text{thermal}} = 63300\) \(\frac{K}{W}\) is the thermal resistance calculated using 3D finite elements methods (FEM). The right disk\(_d\) stays close to ambient temperature as shown by the 3D FEM simulations (see Fig. 1), which can be explained by the low thermal conductivity through the thin SiN membrane.

Figure 1: Lorentz Transmission electron microscopy images of the excited double disk structure. LTEM image of the pair of magnetic vortex oscillators with a radius of \(r = 0.9\) \(\mu m\). The vortex gyration modes are driven by a cw excitation at 239 MHz applied to the right disk (disk\(_d\)). The trajectories of the vortex core motion recorded in the DC mode of the LTEM can be observed as bright ellipses at the centers of both disks. A DC voltage \(U_{\text{heat}}\) can be applied to the left “heated” disk (disk\(_h\)). \(U_{\text{heat}}\) is varied between 0 V to 0.4 V to manipulate the phase.

In addition, the contour lines of the temperature distribution at \(U_{\text{heat}} = 0.4\) V are plotted as can be seen, the driven disk (disk\(_d\)) remains at ambient temperature.
radius of gyration of both vortex cores was determined. When plotted against the applied frequency two resonance frequencies can be resolved as expected for the system of two coupled oscillators, see fig. 2. For the subsequent experiments described below we use $f_{in} = 238$ MHz and $f_{out} = 270$ MHz as the in-phase and out-of-phase resonances, respectively. As shown in the supplementary information the method of phase manipulation presented here is robust against small deviations of the driving frequency from the resonant condition (see fig S1).

**Determination of the phase relation between the two coupled oscillators**

To further investigate the phase relation between the two oscillators time-resolved STXM measurements at the MAXYMUS Beamline at Bessy II in Berlin were performed. First $disk_d$ was excited by a 0.15 V cw excitation at the two previously determined resonance frequencies ($f_{in} = 238$ MHz and $f_{out} = 270$ MHz). The gyration was resolved with a time resolution of 66 ps leading to a series of 62 images. The bright spot on the dark background is a direct image of the $z$-component of the magnetization $M_z$ and allows to determine the polarities $p_d = p_h = 1$. The sense of rotation for both disks is counter clockwise (ccw) and hence, the chirality can be determined to be $c_d = c_h = 1$ [35], which serves as an input for the simulations. The positions of the vortex cores were tracked by the Laplacian of Gaussian method [22] and are overlaid on the image as white spots for all 62 measurements, with the position of the shown image colored in red. This data can be plotted against time and fitted by a least squares sinusoidal fit with a fixed frequency equal to the excitation frequency. From the fit the phase between the two disks is retrieved together with the error from the covariance matrix. The error for the vortex core position is estimated by $\Delta x,y = \pm 50$ nm. Further the eccentricity $e = A_x/A_y$ of the elliptical trajectory is calculated.

| $U_{heat}$ [V] | $f$ [MHz] | $\varphi_{exp}$ | $\varphi_{th}$ | $\epsilon_{h,exp}$ | $\epsilon_{d,exp}$ | $\epsilon_{h,th}$ | $\epsilon_{d,th}$ |
|----------------|-----------|-----------------|----------------|-------------------|-----------------|----------------|----------------|
| 0              | 238       | 16.4 $\pm$ 7   | 15             | 0.80              | 0.88            | 0.95           | 0.96           |
| 0              | 270       | 176.3 $\pm$ 8  | 176            | 1.12              | 1.02            | 1.05           | 1.04           |
| 0.43           | 238       | 167.1 $\pm$ 12 | 169.5          | 1.11              | 0.95            | $> 1$          | $< 1$          |

**Table I**: Experimentally retrieved phase and eccentricity compared to calculated values determined by the extended Thiele equation model.

The measured phases (see table I) are in excellent agreement with the results from the calculations. The change of the eccentricity from $e < 1$ to $e > 1$ from the in-phase resonance to the out-of-phase resonance is typical [35].
Phase manipulation

Now, to manipulate the phase relation between the vortex core trajectories, a voltage $U_{\text{heat}}$ is applied to disk $h$. The increase of temperature causes a decrease of the saturation magnetization $M_{S,h}$ of disk $h$ while $M_{S,d}$ remains constant. $U_{\text{heat}}$ is increased stepwise from 0 V to 0.43 V while the frequency of the excitation is kept constant at 239 MHz. For each heating voltage a series of images with the same time resolution is captured (see fig. 3 a)) and the phase is determined from the image series. The results are summarized in fig. 3 b) with $U_{\text{heat}}$ being the upper axis. The series starts again at a phase of 16.4° for $U_{\text{heat}} = 0$ V and as can be seen the phase shifts up to a maximum of 167° for $U_{\text{heat}} = 0.4$ V caused by a temperature increase of disk $d$ of up to 85 K. At the highest applied voltage the vortices are almost on the opposite side of the elliptical trajectory in contrast to the small phase shift observed for $U_{\text{heat}} = 0$ V (see fig. 3 a) and b)). In the simulation the largest observed phase shift corresponds to a ratio of $M_{S,h}/M_{S,d} = 0.85$, see lower axis of fig. 3 b). To match the experimentally retrieved phase data for different temperatures to the analytically calculated phase values depending on the ratio $M_{S,h}/M_{S,d}$, the temperature dependence of $M_{S,h}(T)/M_{S,d}$ can be estimated via Bloch’s law [36]:

$$M_{S,h}(T) = M_0(1 - \left(\frac{T}{T_c}\right)^\frac{1}{2}) = M_0(1 - \left(\frac{T_0 + \beta U_{\text{heat}}^2}{T_c}\right)^\frac{1}{2})$$

By doing so, the Curie temperature $T_c$ can be used as a parameter in a least square fit of the calculated phase to the experimentally retrieved values. Obviously, this is a very indirect method of determining $T_c$, however, the obtained value of $T_c = 885$ K ± 200 K is in good agreement with literature values [37] and serves merely as a sanity check for the analytic modeling. The resulting dependence of the phase as a function of $M_{S,h}/M_{S,d}$ (fig. 3 b)) is in good agreement with theory. When going from the "in-phase" to the "out-of-phase" mode via manipulation of $M_{S,h}$ the eccentricity of disk $d$ crosses from $e < 1$ to $e > 1$; the values change from 0.8 to 1.11. For disk $d$ the eccentricity stays below 1 and changes from 0.88 to 0.95. Both observations are in good agreement with the simulations (see table 1). The complete transition is shown in fig. S2.

Conclusions

In summary, we have successfully shown fine-grain phase manipulation of a pair of magnetic vortex oscillators in a controlled manner with high resolution (basically only limited by the measurement time) by a simple DC voltage input. The measurements were carried out at 239 MHz but vortex dynamics are easily scalable from the kHz to the GHz regime [38]. The power needed to control the phase is significant at 1.7 mW but scales down orders of magnitudes when going to nano-oscillators. Moreover, we have developed a method of analog phase programming over a wide range from 16.4° of up to 167°. The used method for phase programming relies solely on the oscillatory behavior of coupled oscillators, and the temperature dependence of the saturation magnetization. Hence, it allows for variable phase programming by a simple DC voltage input in a wide range of geometries and applications that can help advance the efforts of high frequency neuromorphic spintronics up to the GHz regime.

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Figure 3: Phase control of the coupled oscillators. a) STXM image series for $f_{ex} = 239$ MHz and $U_{\text{heat}} = 0$ V (top) as well as $U_{\text{heat}} = 0.43$ V (bottom). For $U_{\text{heat}} = 0$ V the phase difference between both oscillators is $16.4^\circ \pm 7^\circ$. For $U_{\text{heat}} = 0.43$ V the phase is increased to $167.1^\circ \pm 12^\circ$, and the cores are almost on the opposite side of the trajectory. Only 5 out of the 62 frames are plotted. All vortex core positions are overlaid in white. b) The phase $\phi_{ex}$ retrieved from STXM data is plotted against $U_{\text{heat}}$ and the corresponding ratio of $\frac{M_{Sh}}{M_{Sd}}$. It agrees with the calculated phase change $\phi_{th}$. 
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Supplemental Materials: Phase programming in coupled spintronic oscillators

PHASE PROGRAMMING IN SLIGHT OFF RESONANT EXCITATION

To study the influence of slight deviations $\Delta f$ of the driving frequency from the first resonant frequency $f_{in}$ to the first peak in the frequency spectra further analytically simulations using the coupled Thiele equation model have been carried out. Here $disk_d$ is driven by an AC current which has an amplitude of $5 \times 10^9$ A/m² at a fixed frequency $f_{in} + \Delta f$. The heating of $disk_h$ is included by changing the saturation magnetization $M_{sh}$ of $disk_h$, while the saturation magnetization $M_{sd}$ of the driven disk is kept constant. As a result one can analyze the resulting phase shift between the gyrotropic motion of the coupled vortices as a function of the ratio of the saturation magnetizations $M_{sh}/M_{sd}$ in combination with different values for $\Delta f$ (see fig. S1). The overall behavior of the phase shift is independent on the size of $\Delta f$, as is the final phase shift at small ratios of $M_{sh}/M_{sd}$ (high heating powers). However, within the transition zone the phase shift dependends sensitively on the size of $\Delta f$.

Figure S1: Phase manipulation for slightly off-resonant excitation. Calculated phase difference for the two vortex gyrations as a function of the saturation magnetization ratio $M_{sh}/M_{sd}$ of the heated disk and the driven disk for excitation frequencies shifted by $\Delta f$ from the first resonance of the coupled system shown for $(\Delta f = -5$ MHz, $-3$ MHz, $0$ MHz, $3$ MHz and $5$ MHz). Data derived from a model of coupled Thiele equations.
DEPENDENCE OF THE ECCENTRICITY ON PHASE DIFFERENCE

As shown in table I in the main text the transition from the in-phase to the out-of-phase-state by heating disk\(_h\) is combined by a change of the eccentricity from \(e_h < 1\) to \(e_h > 1\) for disk\(_h\) while the eccentricity of the excitation in disk\(_d\) does not. The change of the eccentricity from \(e > 1\) to \(e < 1\) indicates a change in the overall elongation of the ellipse. If \(e > 1\) the ellipse is elongated along the \(x\)-axis, and if \(e < 1\) the ellipse is elongated along the \(y\)-axis. This behavior is also reproduced by the simulation derived from the coupled Thiele equation model (see solid lines in fig. S2). During the shift from the in-phase excitation to the out-of-phase state the eccentricity of the driven disk only exhibits a value greater than 1 over a limited phase range. For larger phase shifts the excitation again becomes elongated along the \(y\)-axis. The eccentricity of the vortex core gyration in the heated changes from values below 1 to values above 1. The change from values below 1 to values above 1 takes place around a phase difference of 90°. The eccentricity as well as the phase difference are two experimentally accessible values and can be directly compared with the experimental results (see crosses in fig. S2). The error for the phase is the same as in fig. 3 b). The error of the eccentricity follows from the error of the recorded values for the two axes \(A_x\) and \(A_y\) of the elliptical excitation by error-propagation. As can be seen \(e_{d,exp}\) stays below a value of 1 (green line) for the described transition between the two states. The eccentricity \(e_{h,exp}\) for the heated disk disk\(_h\) on the other hand changes from \(e_{h,exp} < 1\) for phase shifts below 90° to \(e_{h,exp} > 1\) for phase shifts above 90°. This agrees with the prediction from the simulations. Due to the nature of the sharp phase transition depending on the applied heating power (see fig. 3 b). The exact transition is experimentally hard to resolve.

Figure S2: Eccentricity of the elliptical vortex gyration. Eccentricity \(e\) for the elliptical trajectory of the two vortex gyrations as a function of phase difference. The solid lines are the results derived from the simulations (cyan for disk\(_d\) and magenta for disk\(_h\)). The eccentricity for the experimentally resolved elliptical trajectory of the two vortex gyrations together with the derived errors is shown as crosses. The eccentricity \(e_{d,exp}\) for the driven disk disk\(_d\) is shown in blue. For the heated disk disk\(_h\) the eccentricity \(e_{h,exp}\) is shown in red. To exemplify the change of \(e_{h,exp} < 1\) to \(e_{h,exp} > 1\) the green line at \(e = 1\) is shown.