Supersymmetric Model Building
(and Sweet Spot Supersymmetry)

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Abstract

It has been more than twenty years since theorists started discussing supersymmetric model building/phenomenology. We review mechanisms of supersymmetry breaking/mediation and problems in each scenario. We propose a simple model to address those problems and discuss its phenomenology.

1 Introduction

There are three questions we should ask when we try to construct a model with weak scale supersymmetry; (1) How supersymmetry is broken? (2) How gauge/matter fields feel the supersymmetry breaking? (3) How the Higgs fields feel the supersymmetry breaking? There is a simple mathematical formulation of those questions. It is all about the interaction terms in the Lagrangian among the Goldstino multiplet $S$ and the fields in the minimal supersymmetric standard model (MSSM).

We reconsider problems in supersymmetric models by using the effective-field-theory approach. The low energy theory is described by the field $S$ and the matter/gauge/Higgs fields. By doing so, we can discuss different scenarios as different choices of functional form of $S$ which define an effective theory. In this formulation, we find that there is a sweet spot in between the gauge and gravity mediation ($m_{3/2} \sim O(1) \text{ GeV}$) where the theory is perfectly consistent with various requirements.

This note is based on [1, 2, 3, 4].

2 Formulation

2.1 Question (1): Supersymmetry Breaking Sector

The construction of the low-energy effective Lagrangian for supersymmetry breaking sector is completely analogous to the Higgs sector in the standard model. In the standard model, the
Figure 1: Any supersymmetry breaking model locally looks like this. We can parametrize this potential by two parameters $m^2$ and $\Lambda^2$. If it is a model of supersymmetry breaking by an $F$-term and if $m^4/\Lambda^4 \lesssim 4\pi$, the effective Lagrangian to describe physics around the minimum is given by Eq. (2).

Higgs potential is given by

$$V = \frac{\lambda_H}{4} \left(|H|^2 - v^2\right)^2 .$$

(1)

Here the Higgs field $H$ is a linear representation of the gauge group, and can be decomposed to the vacuum expectation value (VEV), three Goldstone bosons and a physical field. This theory makes sense as a low-energy effective theory as long as $\lambda_H \lesssim 4\pi$. Not only as the simplest model of the electroweak symmetry breaking, this model serves as the effective theory of a wide class of models which arises after integrating out heavy fields. The two parameters $\lambda_H$ and $v$ respectively set the strength of the self-interaction of the Higgs fields and the electroweak scale. The physical mode and gauge bosons get masses $\sqrt{\lambda_H}v$ and $gv$, respectively.

A wide class of supersymmetry breaking models also has a low-energy effective description in terms of the Goldstino multiplet $S$, that contains the $F$-component VEV, the Goldstino fermion and a physical scalar field. The Lagrangian is defined by a Kähler- and a superpotential as follows:

$$K = S^\dagger S - \frac{(S^\dagger S)^2}{\Lambda^2} , \quad W = m^2 S .$$

(2)

We eliminated the possible cubic terms $S^\dagger S^2 + \text{h.c.}$ by an appropriate field redefinition so that $S = 0$ is the minimum of the potential. This theory makes sense if $m^2 \lesssim \sqrt{4\pi\Lambda^2}$. The two parameters $1/\Lambda^2$ and $m^2$ respectively set the strength of the self-interaction of the $S$ field and the supersymmetry breaking scale $F_S$. The physical mode and the gravitino obtain masses: $m_S = 2m^2/\Lambda$ and $m_{3/2} = m^2/\sqrt{3}M_{\text{Pl}}$.

Since we have the standard model of supersymmetry breaking in Eq. (2), we do not need to consider detail of mechanisms or dynamics of the supersymmetry breaking for now. Each model of supersymmetry breaking simply maps to a parameter point or region in the two-dimensional parameter space $(m_{3/2}, \Lambda)$. We can leave those as parameters in the low-energy theory.

2.2 Minimal Supersymmetric Standard Model

Before going to the discussion of the communication between the MSSM sector and the supersymmetric sector, we review here important features of the MSSM.
The superpotential of the MSSM is

\[ W_{\text{MSSM}} = Q H_u U + Q H_d D + L H_d E + \mu H_u H_d , \tag{3} \]

where we suppressed the Yukawa coupling constants and flavor indices. The last term, the \( \mu \)-term, is needed to give a mass to the Higgsino, but it should not be too large. For supersymmetry to be a solution to the hierarchy problem, i.e., \( \langle H_u, d \rangle \ll M_{\text{Pl}} \), the \( \mu \)-term is necessary to be of the order of the electroweak scale (or scale of the soft supersymmetry breaking terms).

This is called the \( \mu \)-problem. The fact that \( \mu \) is much smaller than the Planck scale suggests that the combination of \( H_u H_d \) carries some approximately conserving charge.

There are many gauge invariant operators we can write down in addition to the above superpotential such as

\[ W_R = U D D + L L E + Q L D , \tag{4} \]

and

\[ W_{\text{dim.5}} = Q Q Q L + U D U E . \tag{5} \]

These are unwanted operators as they cause too rapid proton decays.

The \( \mu \)-problem and the proton decay problem above are actually related, and there is a simple solution to both problems. The Peccei-Quinn (PQ) symmetry with the following charge assignment avoids too large \( \mu \)-term and the proton decay operators.

\[ PQ(Q) = PQ(U) = PQ(D) = PQ(L) = PQ(E) = -\frac{1}{2} , \tag{6} \]

\[ PQ(H_u) = PQ(H_d) = 1 . \tag{7} \]

This symmetry is broken explicitly by the \( \mu \)-term, \( PQ(\mu) = -2 \). Since it is a small breaking of the PQ symmetry, the coefficients of the dimension five operators are sufficiently suppressed. The unbroken \( Z_4 \) symmetry, which includes the \( R \)-parity as a subgroup, still forbids the superpotential terms in Eq. (4) and ensures the stability of the lightest supersymmetric particle (LSP), leaving us to have a candidate for dark matter of the universe.

In fact, there is another symmetry which can play the same role as the PQ symmetry, called \( R \)-symmetry. The charge assignment is

\[ R(Q) = R(U) = R(D) = R(L) = R(E) = 1 , \tag{8} \]

\[ R(H_u) = R(H_d) = 0 . \tag{9} \]

Again, \( R(\mu) = 2 \) explicitly breaks the \( R \)-symmetry down to the \( R \)-parity.

In summary, there are approximate symmetries, \( U(1)_{\text{PQ}} \) and \( U(1)_R \), in the Lagrangian of the MSSM. If one of them is a good (approximate) symmetry of the whole system, it provides us with a solution to the \( \mu \) and the proton decay problems.
2.3 Question (2): Interactions between $S$ and gauge/matter fields

Now we discuss interaction terms between the $S$-sector and the MSSM sector. These interactions determine the pattern of supersymmetry breaking parameters which are relevant for low energy physics.

There are actually only three possibilities for gauginos to obtain Majorana masses. The Majorana mass of the gauginos is given by

$$m_{1/2} = \frac{[f]_F}{[f]_A},$$

where $f$ is the gauge kinetic function, $\mathcal{L} \ni f W^\alpha W_\alpha$. The gauge kinetic function $f$ is a one-loop exact quantity:

$$f = \frac{1}{g^2(\Lambda_0)} - \sum_i \frac{2b^i_H}{(4\pi)^2} \log \frac{M^i_H}{\Lambda_0} - \frac{2b^L_L}{(4\pi)^2} \log \frac{\mu_R}{\Lambda_0},$$

where $b^i_H$ and $M^i_H$ are the contributions to the beta-function and masses of heavy fields in the theory, respectively. The first term is the gauge coupling constant at tree level (defined by the running coupling constant at a cut-off scale $\Lambda_0$). The last term is the contribution from one-loop correction by light fields. In order for the function $f$ to be $S$ dependent so that r.h.s. of Eq. (10) is non-vanishing [5], we have three choices, i.e., which term(s) is $S$ dependent? Each of possibilities has been named as the gravity [6], gauge [7, 8, 9] and anomaly mediation [10], respectively.

The tree-level gauge coupling $g$ can be $S$ dependent if

$$f(S)_{\text{tree}} = \frac{1}{g^2} + \frac{S}{M_{\text{Pl}}} + \cdots.$$  

(12)

We call this assumption as ‘gravity mediation.’ An important feature here is that the $S$ field must be singlet under any symmetry even including approximate ones at low energy. Symmetries are either absent or badly broken in order to obtain $O(1)$ valued gauge coupling constants for the standard model gauge interactions. Therefore, there is generically a cosmological moduli problem in this scenario [11]. Also, the absence of (approximate) symmetry makes the supersymmetric CP problem sharper as we see later. The gaugino masses are

$$M_{1/2} \sim F_S \sim m_{3/2}.$$  

(13)

The gravitino mass $m_{3/2}$ is $O(100 \text{ GeV})$ in this scenario.

The second possibility, making the second term $S$-dependent, is called ‘gauge mediation.’ The unique possibility of making this term $S$-dependent is

$$M^i_H \rightarrow M^i_H(S).$$  

(14)

The heavy fields whose masses depend on $S$ are called messenger fields. The simplest possibility is to take

$$M_H = kS,$$  

(15)

where $k$ is a parameter.
where $k$ is a dimensionless constant. In this case, in contrast to gravity mediation, $S$ can carry a charge since $S \rightarrow S e^{i\alpha}$ would only shift the $\theta$-term leaving the gauge coupling constant unchanged. We can, therefore, assign a (anomalous) U(1) charge to the $S$ field. (The $m^2 S$ term in Eq. (2) breaks this symmetry explicitly. This explains the smallness of the scale of supersymmetry breaking $m^2 \ll \Lambda^2$ which is consistent with the assumption made before ($m^2 \lesssim \sqrt{4\pi}\Lambda^2$). For small values of $m^2$, we can regard the U(1) symmetry as an approximate symmetry of the Lagrangian.) The gaugino masses are [8, 9]

$$M_{1/2} = \frac{g^2 b_H S}{(4\pi)^2 \langle S \rangle} = \frac{g^2 b_H}{(4\pi)^2} m_{3/2} \left( \frac{\langle S \rangle}{\sqrt{3}M_{Pl}} \right)^{-1}. \quad (16)$$

The gravitino mass is $m_{3/2} \ll O(100 \text{ GeV})$ in this scenario. Note that $S$ must have a VEV to be consistent if we take the assumption in Eq. (15).

The third possibility is called ‘anomaly mediation.’ In the supergravity Lagrangian with a requirement that the cosmological constant is vanishing, dimensionful parameters in the Lagrangian are accompanied with the compensator field $\phi = 1 + \frac{m_{3/2}^2}{2\theta}$, where $m_{3/2} \equiv F_S/\sqrt{3}M_{Pl}$. Therefore, the third term effectively becomes $S$-dependent:

$$-\frac{2b_L}{(4\pi)^2} \log \frac{\mu_R}{\Lambda_0 \phi}. \quad (17)$$

The gaugino masses are

$$M_{1/2} = \frac{g^2 b_L}{(4\pi)^2} m_{3/2}. \quad (18)$$

Therefore, $m_{3/2} \sim 10 - 100 \text{ TeV}$ in this scenario.

What we can do is to choose at least one of the above three for the functional form of the gauge kinetic function $f(S)$. For matter fields, we need to choose wave function factors $Z_{\Phi}(S, S^\dagger)$ where $K \ni Z_{\Phi}(S, S^\dagger)\Phi^\dagger\Phi$ and $\Phi = Q, U, D, L, E$.

### 2.4 Question (3): Interactions between $S$ and the Higgs fields

The interaction terms we can write down are

$$K_{Higgs} = Z_{H_u}(S, S^\dagger) H_u^\dagger H_u + Z_{H_d}(S, S^\dagger) H_d^\dagger H_d + Z_{H_uH_d}(S, S^\dagger) H_u H_d + \text{h.c.} \quad (19)$$

We expand $Z$-functions as

$$Z_{H_u} = 1 + (a_1 S + a_1^* S^\dagger) + a_2 S^\dagger S + \cdots, \quad (20)$$

$$Z_{H_d} = 1 + (b_1 S + b_1^* S^\dagger) + b_2 S^\dagger S + \cdots, \quad (21)$$

$$Z_{H_uH_d} = c_0 + (c_1 S + c_1^* S^\dagger) + c_2 S^\dagger S + \cdots, \quad (22)$$

where we have rescaled the fields $H_u$ and $H_d$ such that the kinetic terms are canonically normalized. By an appropriate shift we also take $\langle S \rangle = 0$. The coefficients $a_1, b_1, c_0, c_1$ and $c_2$ are complex parameter whereas $a_2$ and $b_2$ are real. Then soft terms are obtained to be

$$\mu = c_1 F_S^\dagger + c_0 m_{3/2}, \quad (23)$$
\[ B_\mu = c_2 |F_S|^2 - (a_1 + b_1) F_S \mu + m_{3/2} \mu , \tag{24} \]

\[ m_{H_u}^2 = -(a_2 - |a_1|^2) |F_S|^2 + O(m_{3/2}) , \tag{25} \]

\[ m_{H_d}^2 = -(b_2 - |b_1|^2) |F_S|^2 + O(m_{3/2}) . \tag{26} \]

\[ F_S \text{ and } m_{3/2} \text{ are related by } F_S = \sqrt{3} m_{3/2} M_{Pl}. \]

For natural electroweak symmetry breaking, we need
\[ \mu^2 \sim m_{H_u}^2 \sim m_{H_d}^2 \sim m_W^2 , \tag{27} \]
and
\[ B_\mu \lesssim m_W^2 , \tag{28} \]
where \( m_W \) is the electroweak scale \((O(100 \text{ GeV}))\). Also, to suppress CP violation at low energy we need
\[ \arg (M_{1/2} (B_\mu)^*) \ll 1 . \tag{29} \]

In gravity mediation, all the coefficients are expected to be non-zero and \( O(1) \) in the unit of \( M_{Pl} = 1 \) since we cannot assign any charge to \( S \). The relation in Eq. (27) can be naturally explained in this case. This is the Giudice-Masiero mechanism in gravity mediation \([12]\). However, at the same time, there is no reason to expect the CP phase above to be small.

The simplest possibility of realizing a small phase is to take
\[ B_\mu \ll m_W^2 \tag{30} \]
at an energy scale. This is possible if we can arrange
\[ a_1 = b_1 = c_2 = 0 , \quad m_{3/2} \ll c_1 F_S . \tag{31} \]
The relation in Eq. (27) implies
\[ c_1^2 \sim a_2 \sim b_2 , \quad c_1 F_S \sim m_W . \tag{32} \]

The above general discussion is suggesting that \( S \) field carries some approximately conserving charge. If we assign the PQ-charge
\[ PQ(S) = 2 , \tag{33} \]
we can naturally explain \( a_1 = b_1 = c_2 = 0 (= c_0) \). Gauge mediation is consistent with this hypothesis for generating gaugino masses because \( S \) can carry an anomalous U(1) charge and \( m_{3/2} \) is small. Note that anomaly mediation predicts \( B_\mu/\mu = m_{3/2} \sim 10 - 100 \text{ TeV} \) (see the last term in Eq. (24)) which is inconsistent with Eqs. (27) and (28). If \( S \) carries the PQ-charge, the forms of the \( Z \)-functions are
\[ Z_{H_u} = 1 + \frac{S^\dagger S}{\Lambda_H^2} , \quad Z_{H_d} = 1 + \frac{S^\dagger S}{\Lambda_H^2} , \quad Z_{H_u H_d} = \frac{S^\dagger}{\Lambda_H} . \tag{34} \]
Table 1: Problems in each scenario of mediation mechanisms. We have to choose at least one of the mechanisms to obtain Majorana masses for gauginos.

with $O(1)$ coefficients. The size of $\Lambda_H$ should be such that

$$\mu \sim \frac{F_S}{\Lambda_H} \sim O(100 \text{ GeV}) .$$

(35)

In gauge mediation, this means $\Lambda_H \ll M_{Pl}$. Therefore, the general picture we obtain is that supersymmetry breaking sector and the Higgs fields are directly coupled above some energy scale $\Lambda_H$ and the combined sector should respect the approximate PQ-symmetry.

A framework with gauge mediation + approximate PQ symmetry + the Giudice-Masiero mechanism seems to be a good starting point according to the discussion of the $\mu$-term, proton decays and the CP problem. However, there is still an unexplained coincidence. For the gaugino masses in Eq. (16) and the $\mu$-term in Eq. (35) to be both $O(100 \text{ GeV})$, we need to explain a relation:

$$\langle S \rangle \sim \frac{\Lambda_H}{100} .$$

(36)

If there is a reasonable explanation for this coincidence in a stabilization mechanism of $S$, this hypothesis is going to be a perfect framework. We summarize three mediation mechanisms in Table 1.

3 Sweet Spot Supersymmetry

There have been many attempts to circumvent problems summarized in Table 1. For example, in Ref. [8, 13] it has been proposed to extend a model of gauge mediation to the NMSSM by introducing a new singlet field. However, for the successful electroweak symmetry breaking, further extension of the model were necessary such as introduction of vector-like matters. Similar attempts have been done in Ref. [14, 15] in anomaly mediation models. The gaugino mediation [16] is a variance of the gravity mediation and known to be a successful framework...
for solving the flavor problem. However, since the model relies on the $SW^\alpha W_\alpha$ term for the gaugino masses, the moduli problem and the CP problem remain unsolved. In Ref. [17], a mixture of anomaly and gauge mediation is proposed as a solution to the tachyonic slepton problem (see also [18]). The idea is to modify the structure of the anomaly mediation by introducing an additional light degree of freedom, $X$. It is claimed that the $\mu$-problem and the tachyonic slepton problem can be solved by assuming appropriate couplings of $X$ to the messenger and Higgs fields. However, it is unclear whether such a light degree of freedom is consistent with cosmological history.

We here propose a simple set-up to address all of the problems. The assumptions we make are the following: (1) There is an approximate PQ symmetry whose small explicit breaking triggers the supersymmetry breaking. (2) The Higgs fields and the supersymmetry breaking sector are directly coupled above an energy scale $\Lambda$. (3) There are messenger particles which obtain masses by a VEV of $S$. The effective Lagrangian written in terms of the Goldstino multiplet $S$ and the MSSM matter/gauge fields are then given by [4]:

$$K = S^\dagger S - \frac{c_S(S^\dagger S)^2}{\Lambda^2} + \left(\frac{c_\mu H_u H_d}{\Lambda} + \text{h.c.}\right) - \frac{c_H S^\dagger S(H_u^\dagger H_u + H_d^\dagger H_d)}{\Lambda^2} + \left(1 - \frac{4g^4 N_{\text{mess}}}{(4\pi)^4} C_2(R)(\log |S|^2)^2\right) \Phi^\dagger \Phi,$$

$$W = W_{\text{Yukawa}}(\Phi) + m^2 S + w_0,$$

$$f = \frac{1}{2} \left(\frac{1}{g^2} - \frac{2N_{\text{mess}}}{(4\pi)^2} \log S\right) W^\alpha W_\alpha.$$

The chiral superfield $\Phi$ represents the matter and the Higgs superfields in the MSSM, and $W_{\text{Yukawa}}$ is the Yukawa interaction terms among them. We defined $O(1)$ valued coefficients $c_S$, $c_\mu$, and $c_H$. We normalize the $\Lambda$ parameter so that $c_S = 1$ in the following discussion. The parameters $c_H$ and $\Lambda$ take real values whereas $c_\mu$ is a complex parameter. We consider the supergravity Lagrangian defined by the above Kähler potential $K$, superpotential $W$, and gauge kinetic function $f$. This is a closed well-defined system. The linear term of $S$ in the superpotential represents the source term for the $F$-component of $S$. The last term in the superpotential, $w_0$, is a constant, $|w_0| \simeq m^2 M_{\text{Pl}}/\sqrt{3}$, which is needed to cancel the cosmological constant. The scalar potential has a minimum at

$$\langle S \rangle = \frac{\sqrt{3} \Lambda^2}{6 M_{\text{Pl}}}$$

which avoids a singularity at $S = 0$ where messenger particles become massless [4].

We can see the non-trivial success of this framework in Fig. 2, where we see how $O(1)$ GeV gravitino mass is selected. The bands of $100$ GeV $< \bar{\mu} < 500$ GeV, $100$ GeV $< m_{\tilde{B}} < 500$ GeV, and $0.08 < \Omega_{3/2} h^2 < 0.12$ are shown, where we defined $\bar{\mu} \equiv m_{3/2} M_{\text{Pl}}/\Lambda$ (typical values of the $\mu$-term) and $\Omega_{3/2} h^2$ by

$$\Omega_{3/2} h^2 = 0.1 \times \left(\frac{m_{3/2}}{500 \text{ MeV}}\right)^{3/2} \left(\frac{\Lambda}{1 \times 10^{16} \text{ GeV}}\right)^{3/2}.$$
Figure 2: Phenomenologically required values of the Higgsino mass \( \bar{\mu} \) (with an \( O(1) \) ambiguity, see text), the Bino mass \( m_{\tilde{B}} \) and the gravitino energy density \( \Omega_{3/2}h^2 \). These three quantities have different dependencies on parameters \( m_{3/2} \) and \( \Lambda \). The three bands meet around \( m_{3/2} \approx 1 \) GeV and \( \Lambda \approx M_{\text{GUT}} \). The quantity \( \Omega_{3/2}h^2 \) is defined in Eq. (39). It represents the energy density of the non-thermally produced gravitinos through the decays of \( S \) if \( S \to hh \) is the dominant decay channel.

This is a contribution to the matter energy density from non-thermally produced gravitinos via decays of \( S \)-condensation in the early universe [3]. The Bino mass \( m_{\tilde{B}} \) is the mass of the \( U(1)_Y \) gaugino. Surprisingly, these three bands meet at \( m_{3/2} \approx 1 \) GeV and \( \Lambda \approx M_{\text{GUT}} \approx 10^{16} \) GeV. The factor of 100 in Eq. (36) is explained by the ratio \( M_{\text{Pl}}/M_{\text{GUT}} \approx 100 \).

The fact that \( \Lambda \) coincides with the unification scale, \( M_{\text{GUT}} \), is also quite interesting. It is reasonable that the Higgs fields are directly coupled to the GUT breaking sector above the GUT scale in order to achieve the doublet-triplet splitting. It is then also reasonable to have interaction terms suppressed by the GUT scale after integrating out heavy fields in the GUT breaking sector. The same “cut-off” scale \( \Lambda \) for \( S \) and Higgs fields suggests that the dynamics of GUT breaking is responsible for the supersymmetry breaking as well. The picture of unification of the Higgs sector, the supersymmetry breaking sector and the GUT breaking sector naturally comes out. Although it sounds like a very ambitious attempt to build a realistic model to realize this situation, it is quite possible and even very simple to build such a dream model. For an explicit example of such a GUT model, see Ref. [1].

The PQ symmetry plays an essential role in many aspects. It explains (1) smallness of the \( \mu \)-term (2) smallness of the supersymmetry breaking scale (3) absence of the proton-decay operators (4) stability of the dark matter (5) absence of the CP phase (6) smallness of \( \langle S \rangle \). Especially, (6) smallness of \( \langle S \rangle \) by \( \langle S \rangle \propto 1/M_{\text{Pl}} \) is important for solving the \( \mu \)-problem and also for gravitino cosmology. This simple framework is free from the \( \mu \)-problem, flavor problem, CP
problem, proton decay problem or cosmological moduli/gravitino problem.

4 Low energy phenomenology

4.1 Spectrum

The set-up in Eq. (37) provides a characteristic spectrum of the supersymmetric particles. It is different from conventional gauge or gravity mediation models. Since the Higgs sector directly couples to the supersymmetry breaking sector at the GUT scale, the soft mass terms for the Higgs fields are generated at the GUT scale. The gaugino masses and sfermion masses are, on the other hand, generated at the messenger scale. This hybrid feature provides interesting predictions on the low energy spectrum.

Unfixed parameters in this model are

\[
\begin{align*}
m_H^2 &\equiv \frac{c_H|m_0|^2}{\Lambda^2}, \\
\mu &\equiv \frac{c_\mu m_0^2}{\Lambda}, \\
\tilde{M} &\equiv \frac{1}{(4\pi)^2} \frac{m_{\tilde{X}}^2}{\langle S \rangle}, \\
M_{\text{mess}} &\equiv k\langle S \rangle, \\
N_{\text{mess}} 
\end{align*}
\]

We take the scale \(\Lambda\) to be the unification scale \(M_{\text{GUT}}\).

All the soft supersymmetry breaking parameters at the electroweak scale can be expressed in terms of these five parameters. One combination of the parameters should be fixed by the condition for the electroweak symmetry breaking, i.e., \(M_Z = 91.2\) GeV. We take the \(m_H^2\) parameter as an output of the calculation. The model parameters are now defined by \((\mu, M_{\text{mess}}, \tilde{M}, N_{\text{mess}})\).

We show in Fig. 3 an example of the RG evolution of soft supersymmetry breaking parameters for \((\mu, M_{\text{mess}}, \tilde{M}, N_{\text{mess}}) = (300, 10^{10}, 900, 1)\). The horizontal axis \(\mu_R\) is the RG scale. In the left panel of Fig. 3, scalar masses and the \(\mu\)-parameter are plotted. We have defined mass parameters \(\tilde{m}_X \equiv \text{sgn}(m_X^2)|m_X^2|^{1/2}\) for each scalar mass parameter \(m_X^2\). The gaugino masses are generated at the threshold of the messenger particles.

Figure 3: The RG evolution of the supersymmetry breaking parameters. We take \(N_{\text{mess}} = 1\).
Figure 4: The distribution of the invariant mass $M_{\tilde{\tau}\tau}$. The shaded histogram shows the events with a mis-identified $\tau$-jet which is simulated by assuming a mis-tagging probability of a non-$\tau$-labelled jet to be 1%. The small allows and dashed lines denote the input values of three neutralino masses. Three curves are fitting functions of three endpoints which correspond to the endpoints of $\chi^0_{1,2,3}$ from left to right, respectively. The third endpoint is statistically not very significant. We have used HERWIG 6.50 [22] for event generation, TAUOLA 2.7 [23] for simulation of tau decays and AcerDET 1.0 [24] for the detector simulation.

4.2 LHC signatures

An interesting possibility in this scenario is that the scalar tau lepton can be the next to lightest supersymmetric particle (NLSP) due to the negative contribution from the one-loop running above the messenger scale. This situation is actually well-motivated from the discussion of the ultra-violet completion of the framework [4]. If the stau is the NLSP, the lifetime is of $O(1000)$ seconds with our assumption of the $O(1)$ GeV gravitinos. The LHC signals with such a long-lived stau will be quite different from ones with the usual assumption of the neutralino LSP.

We study the LHC signals with the parameter set we have used in Fig. 3 where the stau is the NLSP. The stau will be produced at the LHC mainly from the neutralino decays $\chi^0_i \rightarrow \tilde{\tau}\tau$, and most of them reach to the muon system. Therefore, we can reconstruct the four-momenta of staus event by event basis with a good accuracy [19]. The reconstruction of neutralino masses is also possible by looking at the edge of the invariant mass of the stau and a tau-jet, $M_{\tilde{\tau}\tau}$ [21] (see [22] for a recent study in minimal supergravity model).

Fig. 4 shows the distribution of the invariant mass. We can clearly see the edges at the input neutralino masses. We can confirm/exclude the model by testing if relations among stau and neutralino masses are consistent with the model predictions.
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