Hadronic $B$ Decays to Charmless $VT$ Final States

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Abstract

Charmless hadronic decays of $B$ mesons to a vector meson ($V$) and a tensor meson ($T$) are analyzed in the frameworks of both flavor SU(3) symmetry and generalized factorization. We also make comments on $B$ decays to two tensor mesons in the final states. Certain ways to test validity of the generalized factorization are proposed, using $B \rightarrow VT$ decays. We calculate the branching ratios and CP asymmetries using the full effective Hamiltonian including all the penguin operators and the form factors obtained in the non-relativistic quark model of Isgur, Scora, Grinstein and Wise.

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I. INTRODUCTION

In the next few years $B$ factories operating at KEK and SLAC will provide plenty of new experimental data on $B$ decays. It is expected that improved new bound will be put on the branching ratios for various decay modes and many decay modes with small branching ratios will be observed for the first time. Thus more information on rare decays of $B$ mesons will be available soon. Experimentally several tensor mesons have been observed [1], such as the isovector $a_2(1320)$, the isoscalars $f_2(1270)$, $f_2'(1525)$, $f_2(2010)$, $f_2(2300)$, $f_2(2340)$, $\chi_{c2}(1P)$, $\chi_{b2}(1P)$ and $\chi_{c2}(2P)$, the isospinors $K^*_2(1430)$ and $D^*_2(2460)$. Experimental data on the branching ratios for $B$ decays involving a vector ($V$) and a tensor meson ($T$) in the final state provide only upper bounds, as follows [1]:

\[
\begin{align*}
B(B^+ \to \rho^+ D^*_2(2460)^0) &< 4.7 \times 10^{-3}, \\
B(B^0 \to \rho^+ D^*_2(2460)^-) &< 4.9 \times 10^{-3}, \\
B(B^+ \to \rho^0 K^*_2(1430)^+) &< 1.5 \times 10^{-3}, \\
B(B^0 \to \rho^0 K^*_2(1430)^0) &< 1.1 \times 10^{-3}, \\
B(B^+ \to \phi K^*_2(1430)^+) &< 3.4 \times 10^{-3}, \\
B(B^0 \to \phi K^*_2(1430)^0) &< 1.4 \times 10^{-3}, \\
B(B^+ \to \rho^0 a_2(1320)^+) &< 7.2 \times 10^{-4}.
\end{align*}
\]

(1)

In particular, the process $B \to K^*_2 \gamma$ has been observed for the first time by the CLEO Collaboration with a branching ratio of $(1.66^{+0.59}_{-0.53} \pm 0.13) \times 10^{-5}$ [3].

There have been a few works [3, 4] studying two-body hadronic $B$ decays involving a tensor meson $T$ ($J^P = 2^+$) in the final state using the non-relativistic quark model of Isgur, Scora, Grinstein and Wise (ISGW) [6] with the factorization ansatz. However, those works considered only the tree diagram contribution even in charmless $B$ decays to $PT$ ($P$ denotes a pseudoscalar meson) and $VT$, such as $B \to \eta^{(*)} a_2$ and $B \to \phi f_2^{(*)}$. In most cases of the charmless $\Delta S = 0$ processes, the dominant contribution arises from the tree diagram and the contributions from the penguin diagrams are very small. But in some cases such as $B \to \eta^{(*)} a_2$ and $\eta^{(*)} f_2^{(*)}$, the penguin diagrams could provide sizable contributions. Furthermore, in the charmless $|\Delta S| = 1$ decay processes, the penguin diagram contribution is enhanced by the CKM matrix elements.
$V_{tb}V_{ts}$ and becomes dominant.

In a recent work \cite{7}, we have studied $B$ decays to a pseudoscalar meson and a tensor meson. In this work, the previous analysis is extended to charmless hadronic decays of $B$ mesons to a vector meson and a tensor meson in the frameworks of \textit{both} flavor SU(3) symmetry and the generalized factorization. We also comment on $B$ decays to \textit{two} tensor mesons in the final states. Purely based on the flavor SU(3) symmetry, we first present a model-independent analysis in $B \to VT$ decays. Then we use the \textit{full} effective Hamiltonian including all the penguin operators and the ISGW quark model to calculate the branching ratios for $B \to VT$ decays. Since we include both the tree and the penguin diagram contributions to decay processes, we are able to calculate the branching ratios for all the charmless $|\Delta S| = 1$ decays and the relevant CP asymmetries. In order to bridge the flavor SU(3) approach and the factorization approach, we present a set of relations between a flavor SU(3) amplitude and a corresponding amplitude in the factorization in $B \to VT$ decays. Certain ways to test validity of the generalized factorization are proposed by emphasizing interplay between both approaches.

This work is organized as follows. In Sec. II we discuss the notations for SU(3) decomposition and the full effective Hamiltonian for $B$ decays. We also make some comments on $B \to TT$ decays in Sec. II. In Sec. III we present a model-independent analysis of $B \to VT$ decays based on SU(3) symmetry. In Sec. IV the two-body decays $B \to VT$ are analyzed in the framework of generalized factorization. The branching ratios and CP asymmetries are calculated using the form factors obtained in the ISGW quark model. Finally, in Sec. V we conclude our analysis.

\section*{II. FRAMEWORK}

Since in $B \to VT$ decays there are three possible partial waves with $l = 1, 2, 3$ in the final state, $B \to VT$ processes are more complicated than $B \to PT$ processes. For the SU(3) analysis of $B \to VT$ decays, these partial waves in the final state need to be separated out. We will assume that this can be done by certain methods such as one using angular distributions in $B \to VV$ decays \cite{8}. In the flavor SU(3) approach, the decay amplitudes of two-body $B$ decays are decomposed into linear combinations of the SU(3) amplitudes, which are reduced matrix elements defined in Ref. \cite{9}. In SU(3) decomposition of decay amplitudes of the $B \to VT$
processes, we choose the notations given in Refs. [9–11] as follows: We represent the decay amplitudes in terms of the basis of quark diagram contributions, $T$ (tree), $C$ (color-suppressed tree), $P$ (QCD-penguin), $S$ (additional penguin effect involving SU(3)-singlet mesons), $E$ (exchange), $A$ (annihilation), and $PA$ (penguin annihilation). The amplitudes $E$, $A$ and $PA$ may be neglected to a good approximation because of a suppression factor of $f_B/m_B \approx 5\%$. For later convenience we also denote the electroweak (EW) penguin effects explicitly as $P_{EW}^C$ (color-favored EW penguin) and $P_{EW}^C$ (color-suppressed EW penguin), even though in terms of quark diagrams the inclusion of these EW penguin effects only leads to the following replacement without introducing new SU(3) amplitudes; $T \rightarrow T + P_{EW}^C$, $C \rightarrow C + P_{EW}$, $P \rightarrow P - \frac{1}{3} P_{EW}$, $S \rightarrow S - \frac{1}{3} P_{EW}$. We use the following phase convention for the vector and the tensor mesons:

$$\rho^+(a_2^+) = u\bar{d}, \quad \rho^0(a_2^0) = -\frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}), \quad \rho^-(a_2^-) = -\bar{u}d,$$

$$K^{*+}(K_2^{*+}) = u\bar{s}, \quad K^{*0}(K_2^{*0}) = d\bar{s}, \quad K^{*-}(K_2^{*-}) = -\bar{s}d,$$

$$\omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}), \quad \phi = s\bar{s},$$

$$f_2 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \cos \phi_T + (s\bar{s}) \sin \phi_T, \quad f_2' = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}) \sin \phi_T - (s\bar{s}) \cos \phi_T , \quad (2)$$

where the mixing angle $\phi_T$ is given by $\phi_T = \arctan(1/\sqrt{2}) - 28^\circ \approx 7^\circ$ [3,12].

In the factorization scheme, we first consider the effective weak Hamiltonian. We then use the generalized factorization approximation to derive hadronic matrix elements by saturating the vacuum state in all possible ways. The method includes color octet non-factorizable contribution by treating $\xi \equiv 1/N_c$ ($N_c$ denotes the effective number of color) as an adjustable parameter. The generalized factorization approximation has been quite successfully used in two-body $D$ decays as well as $B \rightarrow D$ decays [13]. The effective weak Hamiltonian for hadronic $\Delta B = 1$ decays can be written as

$$H_{eff} = \frac{4G_F}{\sqrt{2}} \left[ V_{ub}V_{q1}^*(c_1O_1^u + c_2O_2^u) + V_{cb}V_{q2}^*(c_1O_1^c + c_2O_2^c) - V_{tb}V_{q3}^* \sum_{i=3}^{12} c_iO_i \right] + H.C. , \quad (3)$$

where $O_i$’s are defined as

$$O_1^I = (\bar{q}\gamma_\mu Lf)\tilde{f}\bar{\gamma}\gamma^\mu Lb, \quad O_2^I = (\bar{q}\gamma_\mu Lf_\beta)\tilde{f}\bar{\gamma}_\beta\gamma^\mu Lb, \quad O_3(5) = (\bar{q}\gamma_\mu L)(\Sigma\bar{q}'\gamma^\mu L(R)q'), \quad O_4(6) = (\bar{q}\gamma_\mu Lb_\beta)(\Sigma\bar{q}'_\beta\gamma^\mu L(R)q'_\alpha) ,$$
\[ O_{7(9)} = \frac{3}{2} (\bar{q} \gamma_\mu Lb)(\Sigma e_q \bar{q}' \gamma^\mu R(L)q') , \quad O_{8(10)} = \frac{3}{2} (\bar{q}_a \gamma_\mu Lb_\beta)(\Sigma e_q \bar{q}'_\beta \gamma^\mu R(L)q'_a) , \]
\[ O_{11} = \frac{g_s}{32\pi^2} m_b (\bar{q} \sigma^{\mu\nu} RT^a q) G_{\mu\nu}^a , \quad O_{12} = \frac{e}{32\pi^2} m_b (\bar{q} \sigma^{\mu\nu} Rb) F_{\mu\nu} . \]  

Here \( c_i \)'s are the Wilson coefficients (WC's) evaluated at the renormalization scale \( \mu \). And \( L(R) = (1 \mp \gamma_5)/2 \), \( f \) can be \( u \) or \( c \) quark, \( q \) can be \( d \) or \( s \) quark, and \( q' \) is summed over \( u, d, s, \) and \( c \) quarks. \( \alpha \) and \( \beta \) are the SU(3) color indices, and \( T^a \) (\( a = 1, \ldots, 8 \)) are the SU(3) generator with the normalization \( Tr(T^a T^b) = \delta^{ab}/2 \). \( g_s \) and \( e \) are the strong and electric couplings, respectively. \( G_{\mu\nu}^a \) and \( F_{\mu\nu} \) denote the gluonic and photonic field strength tensors, respectively. \( O_1 \) and \( O_2 \) are the tree-level and QCD-corrected operators. \( O_{3-6} \) are the gluon-induced strong penguin operators. \( O_{7-10} \) are the EW penguin operators due to \( \gamma \) and \( Z \) exchange, and box diagrams at loop level. We shall take into account the chromomagnetic operator \( O_{11} \) but neglect the extremely small contribution from \( O_{12} \). The dipole contribution is in general quite small, and is of the order of 10% for penguin dominated modes. For all the other modes it can be neglected \[ 34 \].

We use the ISGW quark model to analyze two-body charmless decay processes \( B \to VT \) in the framework of generalized factorization. We describe the parameterizations of the hadronic matrix elements in \( B \to VT \) decays \[ 33 \]:

\[ \langle 0|V^\mu|V \rangle = f_V m_V \epsilon^\mu , \]  
\[ \langle T|j^\mu|B \rangle = i\hbar (m_P^2) \epsilon^{\mu\nu\rho\sigma} \epsilon^\nu_{\alpha\beta} \bar{p}_B (p_B + p_T) \rho (p_B - p_T) \sigma + k (m_P^2) \epsilon^{\mu\nu} (p_B) \rho 
\[ + \epsilon^{\nu\sigma} \epsilon^{\nu\rho} \bar{p}_B [b_+ (m_P^2) (p_B + p_T)^\rho + b_- (m_P^2) (p_B - p_T)^\rho] , \]  

where \( j^\mu = V^\mu - A^\mu \). \( V^\mu \) and \( A^\mu \) denote a vector and an axial-vector current, respectively. \( f_P \) denotes the decay constant of the relevant pseudoscalar meson. \( h(m_P^2), k(m_P^2), b_+ (m_P^2), \) and \( b_- (m_P^2) \) express the form factors for the \( B \to T \) transition, \( F^{B \to T}(m_P^2) \), which have been calculated at \( q^2 = m_P^2 \) (\( q^\mu \equiv p_B^\mu - p_T^\mu \)) in the ISGW quark model \[ 33 \]. \( p_B \) and \( p_T \) denote the momentum of the \( B \) meson and the tensor meson, respectively.

The polarization tensor \( \epsilon^{\mu\nu} \) of the tensor meson \( T \) satisfies the following properties \[ 33 \]:

\[ \epsilon^{\mu\nu} (p_T, \lambda) = \epsilon^{\nu\mu} (p_T, \lambda) , \]  
\[ p_\mu \epsilon^{\mu\nu} (p_T, \lambda) = p_\nu \epsilon^{\mu\nu} (p_T, \lambda) = 0 , \]  
\[ \epsilon^\mu (p_T, \lambda) = 0 \]  

...
where $\lambda$ is the helicity index of the tensor meson. We note that due to the above properties of the polarization tensor, the matrix element $\langle 0|j^\mu|T \rangle$ vanishes:

$$\langle 0|j^\mu|T \rangle = p_\nu \epsilon^{\mu\nu}(g_T, \lambda) + p_T^\mu \epsilon^{\nu}(g_T, \lambda) = 0.$$ (10)

Thus, in the generalized factorization scheme, just as in the case of $B \to PT$ decays, the decay amplitudes for $B \to VT$ can be considerably simplified, compared to those for other two-body charmless decays of $B$ mesons such as $B \to PP$, $PV$, and $VV$: Any decay amplitude for $B \to VT$ is simply proportional to the decay constant $f_V$ and a certain linear combination of the form factors $F^{B \to T}$, i.e., there is no such amplitude proportional to $f_T \times \text{(form factor for } B \to V)$.

We would like to make comments on decays of $B$ mesons to two tensor mesons in the final state. Since $\langle 0|j_\mu|T \rangle = 0$, in the factorization scheme the decay amplitude for $B \to TT$ decays always vanishes:

$$\langle TT|H_{eff}|B \rangle \sim \langle T|j_\mu|B \rangle \langle 0|j_\mu|T \rangle = 0$$ (11)

Non-zero of a rate for any $B \to TT$ decay would arise from non-factorizable effects or final state interactions. Therefore, search for any $B \to TT$ modes in future experiment can provide a critical test of the factorization ansatz.

**III. FLAVOR SU(3) ANALYSIS OF $B \to VT$ DECAYS**

We list the $B \to VT$ decay modes in terms of the SU(3) amplitudes. The coefficients of the SU(3) amplitudes in $B \to VT$ are listed in Tables I and II for strangeness-conserving ($\Delta S = 0$) and strangeness-changing ($|\Delta S| = 1$) processes, respectively. In the tables, the unprimed and the primed letters denote $\Delta S = 0$ and $|\Delta S| = 1$ processes, respectively. The subscript, $V$ in $T_V$, $C_V$, ... or $T$ in $T_T$, $C_T$, ..., on each SU(3) amplitude is used to describe such a case that the meson, which includes the spectator quark in the corresponding quark diagram, is the vector $V$ or the tensor $T$. Note that the coefficients of the SU(3) amplitudes with the subscript $V$, which would be proportional to $f_T \times F^{B \to V}$, are expressed in square brackets. As explained in Sec. II, the contributions of the SU(3) amplitudes with the subscript $V$ vanish in the framework of factorization, because those contributions contain the matrix element $\langle T|j_\mu^{\text{weak}}|0 \rangle$ which is zero, see Eq. (10). Thus, it will be interesting to compare the results obtained in the SU(3) analysis.
with those obtained in the factorization scheme, as we shall see. We will present some ways to
test validity of both schemes in future experiment.

Among the $\Delta S = 0$ amplitudes, the tree diagram contribution is expected to be largest
so that from Table I the decays $B^+ \to \rho^+ a^+_2$, $\rho^+ f_2$, and $B^0 \to \rho^+ a^-_2$ are expected to have
the largest rates. Here we have noticed that in $B^+ \to \rho^+ f^{(t)}_2$ decays, $\cos \phi_f \approx 0.99$ and
$\sin \phi_f \approx 0.13$, since the mixing angle $\phi_f \approx 7^\circ$. The amplitudes for the processes $B \to \phi f^{(t)}_2$,
$\phi a_2$, and $K^* K^*_2$ have only penguin diagram contributions, and so they are expected to be
small. In principle, the penguin contribution (combined with the smaller color-suppressed EW
penguin) $p_T \equiv P_T - (1/3)P_{EW,T}$ can be measured in $B^+(0) \to \bar{K}^{*0} K^+_2$. The tree contribution
(combined with much smaller color-suppressed EW penguin) $t_T \equiv T_T + P_{EW,T}$ are measured by
the combination $A(B^+(0) \to \bar{K}^{*0} K^+_2) - A(B^0 \to \rho^+ a^-_2)$. The amplitudes for $B^0 \to \rho^0 f'_2$ and
$\omega f'_2$ have the color-suppressed tree contributions, $C_T(C_V)$, but are suppressed by $\sin \phi_f$ so that
they are expected to be small. We shall see that these expectations based on the SU(3) approach
are consistent with those calculated in the factorization approximation. However, there exist
some cases in which the predictions based on both approaches are inconsistent. Note that in
Table I the amplitudes for $B^0 \to \rho^- a^+_2$ and $B^{+(0)} \to K^{*+(0)} K_2^{*0}$ can be decomposed into linear
combinations of the SU(3) amplitudes as follows:

$$A(B^0 \to \rho^- a^+_2) = -T_V - P_V - (2/3)P_{EW,V}^C,$$
$$A(B^+ \to K^{*+} K^*_2) = A(B^0 \to K^{*0} K_2^0) = P_V - (1/3)P_{EW,V}^C.$$\hspace{1cm}(12)\hspace{1cm}(13)

As previously explained, in factorization the rates for these processes vanish because all the
SU(3) amplitudes are with the subscript $V$. Non-zero of decay rates for these processes would
arise from non-factorizable effects or final state interactions. Thus, in principle one can test
validity of the factorization ansatz by measuring the rates for these decays in future experiment.
Furthermore, the non-factorizable penguin contribution, if exists, (combined with the smaller
color-suppressed EW penguin) $p_V \equiv P_V - (1/3)P_{EW,V}$ can be measured in $B^{+(0)} \to \bar{K}^{*+(0)} K_2^{*+(0)}$.
Also, supposing that $P_V$ is very small compared to $T_V$ as usual, one can determine the magnitude
of $T_V$ by measuring the rate for $B^0 \to \rho^- a^+_2$.

In the $|\Delta S| = 1$ decays, the (strong) penguin contribution $P'$ is expected to dominate
because of enhancement by the ratio of the CKM elements $|V_{t\bar{d}}V_{ts}|/|V_{u\bar{s}}V_{us}| \approx 50$. We note
that the amplitudes for $B^+ \to K^{*0} a^+_2$ and $B^+ \to \rho^+ K_2^{*0}$ have only penguin contributions,
respectively, as follows:

\[ A(B^+ \rightarrow K^{*0} a^+_2) = P'_T - \frac{1}{3} P'^{Cl}_{EW,T} , \tag{14} \]
\[ A(B^+ \rightarrow \rho^+ K^{*0}_2) = P'_V - \frac{1}{3} P'^{Cl}_{EW,V} . \tag{15} \]

Thus the penguin contribution (combined with the smaller color-suppressed EW penguin) \( p'_T \equiv P'_T - \frac{1}{3} P'^{Cl}_{EW,T} \) is measured in \( B^+ \rightarrow K^{*0} a^+_2 \). Similarly, \( p'_V \equiv P'_V - \frac{1}{3} P'^{Cl}_{EW,V} \) is determined in \( B^+ \rightarrow \rho^+ K^{*0}_2 \). (In fact, \( p'_V = 0 \) in factorization.) By comparing the branching ratios for these two modes measured in experiment, one can determine which contribution (i.e., \( p'_T \) or \( p'_V \)) is larger. The (additional penguin) SU(3) singlet amplitude \( S' \) is expected to be very small because of the Okubo-Zweig-Iizuka (OZI) suppression. As in \( \Delta S = 0 \) decays, there are certain processes whose amplitudes can be expressed by the SU(3) amplitudes, but are expected to vanish in factorization: For instance, \( A(B^+ \rightarrow \rho^+ K^{*0}_2) \) is given by Eq. (13) and

\[ A(B^0 \rightarrow \rho^- K^{*+}_2) = -(T'_V + P'_V + \frac{2}{3} P'^{Cl}_{EW,V}) . \]

Thus, in principle measurement of the rates for these decays can be used to test the factorization ansatz. We also note that the decay amplitudes for modes \( B^+ \rightarrow \rho^0 K^{*+}_2 \) and \( B^0 \rightarrow \rho^0 K^{*0}_2 \) can be respectively written as

\[ A(B^+ \rightarrow \rho^0 K^{*+}_2) = -\frac{1}{\sqrt{2}} (T'_V + C'_T + P'_V + P'^{Cl}_{EW,T} + \frac{2}{3} P'^{Cl}_{EW,V}) , \tag{16} \]
\[ A(B^0 \rightarrow \rho^0 K^{*0}_2) = -\frac{1}{\sqrt{2}} (C'_T - P'_V + P'^{Cl}_{EW,T} + \frac{1}{3} P'^{Cl}_{EW,V}) . \tag{17} \]

Since in factorization only the amplitudes having the subscript \( T \) does not vanish, we shall see that \( B(B^+ \rightarrow \rho^0 K^{*+}_2) = B(B^0 \rightarrow \rho^0 K^{*0}_2) \) in the factorization scheme, where \( B \) denotes the branching ratio. Thus, if \( T'_V \) or \( P'_V \) is (not zero and) not very suppressed compared to \( C'_T \), then there would be a sizable discrepancy in the relation \( B(B^+ \rightarrow \rho^0 K^{*+}_2) = B(B^0 \rightarrow \rho^0 K^{*0}_2) \), and in principle it can be tested in experiment.

From Tables I and II, we find some useful relations among the decay amplitudes. The equivalence relations are: for the \( \Delta S = 0 \) modes,

\[ \frac{1}{\sqrt{2}} A(B^+ \rightarrow \phi a^+_2) = A(B^0 \rightarrow \phi a^+_2) , \]
\[ = -\frac{1}{c} A(B^0 \rightarrow \phi f^+_2) = \frac{1}{s} A(B^0 \rightarrow \phi f^+_2) , \]
\[ A(B^+ \rightarrow K^{*+} \bar{K}^{*0}_2) = A(B^0 \rightarrow K^{*0} \bar{K}^{*0}_2) , \]
\[ A(B^+ \rightarrow \bar{K}^{*0} K^{*+}_2) = A(B^0 \rightarrow \bar{K}^{*0} K^{*0}_2) , \tag{18} \]

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and for the $|\Delta S| = 1$ modes,

$$A(B^+ \to \phi K^+_2) = A(B^0 \to \phi K^*_2).$$  \hfill (19)

The quadrangle relations are: for the $\Delta S = 0$ processes,

$$\frac{1}{c}A(B^+ \to \rho^+ f_2) - \frac{1}{s}A(B^+ \to \rho^+ f'_2) = \sqrt{2}\left[\frac{1}{c}A(B^0 \to \rho^0 f_2) - \frac{1}{s}A(B^0 \to \rho^0 f'_2)\right] = \sqrt{2}\left[\frac{1}{c}A(B^0 \to \omega f_2) - \frac{1}{s}A(B^0 \to \omega f'_2)\right],$$  \hfill (20)

and for the $|\Delta S| = 1$ processes,

$$A(B^+ \to K^{*0} a^+_2) + \sqrt{2}A(B^+ \to K^{*\pm} a^0_2) = \sqrt{2}A(B^0 \to K^{*0} a^0_2) + A(B^0 \to K^{*+} a^-_2),$$

$$\frac{1}{c}A(B^+ \to K^{*+} f_2) - \frac{1}{s}A(B^+ \to K^{*+} f'_2) = \frac{1}{c}A(B^0 \to K^{*0} f_2) - \frac{1}{s}A(B^0 \to K^{*0} f'_2),$$

$$A(B^+ \to \rho^+ K^{*0}) + \sqrt{2}A(B^+ \to \rho^0 K^{*+}) = A(B^0 \to \rho^- K^{*+}) + \sqrt{2}A(B^0 \to \rho^0 K^{*0}),$$  \hfill (21)

where $c \equiv \cos \phi_T$ and $s \equiv \sin \phi_T$. Note that the above relations are derived, purely based on flavor SU(3) symmetry. In the factorization scheme, (neglecting the SU(3) amplitudes with the subscript $V$) we would have in addition the approximate relations as follows\footnote{Considering SU(3) breaking effects, we use the symbol $\approx$ in the following relations instead of the equivalence symbol $=$.}

The following factorization relation would hold:

$$\sqrt{2}A(B^+ \to \rho^+ a^0_2) \approx A(B^0 \to \rho^+ a^-_2).$$  \hfill (22)

The quadrangle relations given in Eqs. (21, 21) would be divided into the following factorization relations: for the $\Delta S = 0$ processes,

$$\frac{1}{c}A(B^+ \to \rho^+ f_2) \approx \frac{1}{s}A(B^+ \to \rho^+ f'_2),$$

$$\frac{1}{c}A(B^0 \to \rho^0 f_2) \approx \frac{1}{s}A(B^0 \to \rho^0 f'_2),$$

$$\frac{1}{c}A(B^0 \to \omega f_2) \approx \frac{1}{s}A(B^0 \to \omega f'_2),$$  \hfill (23)

and for the $|\Delta S| = 1$ processes,
\[\sqrt{2}A(B^+ \rightarrow K^{*+}a_2^0) \approx A(B^0 \rightarrow K^{*+}a_2^-) ,\]
\[A(B^+ \rightarrow K^{*0}a_2^+) \approx \sqrt{2}A(B^0 \rightarrow K^{*0}a_2^0) ,\]
\[\frac{1}{c}A(B^+ \rightarrow K^{*+}f_2) \approx \frac{1}{s}A(B^+ \rightarrow K^{*+}f_2') ,\]
\[\frac{1}{c}A(B^0 \rightarrow K^{*0}f_2) \approx \frac{1}{s}A(B^0 \rightarrow K^{*0}f_2') ,\]
\[A(B^+ \rightarrow \rho^0K^{*+}_2) \approx A(B^0 \rightarrow \rho^0K^{*0}_2) ,\]
\[A(B^+ \rightarrow \omega K^{*+}_2) \approx A(B^0 \rightarrow \omega K^{*0}_2) .\] (24)

Therefore, in principle the above relations given in Eqs. (22, 23, 24) provide an interesting way to test the factorization scheme by measuring and comparing magnitudes of the decay amplitudes involved in the relations. In consideration of SU(3) breaking effects, the relation in Eq. (22) is best to use, because in fact the relation arises from isospin symmetry assuming \(C_V = P_V = P_{EW,V} = P_{EW,V}^C = 0.\) (However, if \(C_V\) is negligibly small (though not zero) compared to \(T_T\), Eq. (22) will approximately hold.)

**IV. ANALYSIS OF \(B \rightarrow VT\) IN THE ISGUR-SCORA-GRINSTEIN-WISE MODEL**

We present a set of relations between a flavor SU(3) amplitude involved in \(B \rightarrow VT\) decays and a corresponding amplitude in the generalized factorization, which bridge both approaches in \(B \rightarrow VT\) decays as follows [14]. (Note that all the SU(3) amplitudes with the subscript \(P\), such as \(T_P^{(i)}\) etc., vanish because those are proportional to the matrix element \(\langle T|j^\mu|0\rangle\).)

\[
T_T^{(i)} = i \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}(s) (m_V f_V \epsilon^{* \alpha \beta} F_{B \rightarrow T}^{B \rightarrow T}(m_V^2)) a_1 ,
\]
\[
C_T^{(i)} = i \frac{G_F}{\sqrt{2}} V_{ub} V_{ud}(s) (m_V f_V \epsilon^{* \alpha \beta} F_{B \rightarrow T}^{B \rightarrow T}(m_V^2)) a_2 ,
\]
\[
S_T^{(i)} = -i \frac{G_F}{\sqrt{2}} V_{tb} V_{td}(s) (m_V f_V \epsilon^{* \alpha \beta} F_{B \rightarrow T}^{B \rightarrow T}(m_V^2)) (a_3 + a_5) ,
\]
\[
P_T^{(i)} = -i \frac{G_F}{\sqrt{2}} V_{tb} V_{td}(s) (m_V f_V \epsilon^{* \alpha \beta} F_{B \rightarrow T}^{B \rightarrow T}(m_V^2)) a_4 ,
\]
\[
P_{EW,T}^{(i)} = -i \frac{G_F}{\sqrt{2}} V_{tb} V_{td}(s) (m_V f_V \epsilon^{* \alpha \beta} F_{B \rightarrow T}^{B \rightarrow T}(m_V^2)) \frac{3}{2} (a_7 + a_9) ,
\]
\[
P_{EW,T}^{C(0)} = -i \frac{G_F}{\sqrt{2}} V_{tb} V_{td}(s) (m_V f_V \epsilon^{* \alpha \beta} F_{B \rightarrow T}^{B \rightarrow T}(m_V^2)) \frac{3}{2} a_{10} ,
\] (25)
\[ F_{\alpha \beta}^{B \rightarrow T}(m_V^2) = \epsilon_{\mu}^{\ast}(p_B + p_T)_\rho [i h(m_V^2) \cdot \epsilon^{\mu \nu \rho \sigma} g_{\alpha \nu}(p_{\nu})_\beta(p_{\rho})_\sigma + k(m_V^2) \cdot \delta_{\alpha \rho} \delta_{\beta \rho}] \\
+ b_i(m_V^2) \cdot (p_{\nu})_\alpha(p_{\rho})_\beta g^{\mu \rho}. \]  

(26)

Here the effective coefficients \(a_i\) are defined as \(a_i = c_i^{eff} + \xi c_i^{eff} (i = \text{odd})\) and \(a_i = c_i^{eff} + \xi c_i^{eff} (i = \text{even})\) with the effective WC’s \(c_i^{eff}\) at the scale \(m_b\) \([14]\), and by treating \(\xi \equiv 1/N_c\ (N_c\) denotes the effective number of color) as an adjustable parameter.

With Tables I, II and the above relations \([23]\), one can easily write down in the factorization scheme the amplitude of any \(B \to VT\) mode shown in the tables. For example, from Table I and the relations \([23]\), the amplitude of the process \(B^+ \to \rho^+ a_2^0\) can be written as\(^2\)

\[ A(B^+ \to \rho^+ a_2^0) = -\frac{1}{\sqrt{2}} \left(T_T + C_V + P_T - P_V + P_{E,V} + \frac{2}{3} P_{E,V,T} + \frac{1}{3} P_{E,V,T} \right) \\
= \frac{G_F}{\sqrt{2}} (m_{\rho^+} f_{\rho^+} \epsilon^{\alpha \beta} F_{\alpha \beta}^{B \rightarrow T}(m_V^2))[V_{u b}^\ast V_{u d}(a_1 - V_{b d}^\ast V_{d s}(a_2 + a_{10}))]. \]  

(27)

Here we have used the fact that \(C_V, P_V, P_{E,V},\) and \(P_{E,V,T}\) with the subscript \(V\) all vanish in factorization. Expressions for all the amplitudes of \(B \to VT\) decays are given in Appendix as calculated in the factorization scheme.

The unpolarized decay rate for \(B \to VT\) is given by

\[ \Gamma(B \to VT) = \frac{G_F^2}{48\pi m^4_T} m_V f_V^2 \left| \{V_{u b}^\ast V_{u d(s)} \cdot (a_1 \text{ or } a_2) - V_{b d}^\ast V_{d s}(a_1' \text{ or } a_2') \} \right|^2 \]

\[ \cdot [\mathcal{X}|\vec{p}_V|^7 + \mathcal{Y}|\vec{p}_V|^5 + \mathcal{Z}|\vec{p}_V|^5], \]  

(28)

where \(|\vec{p}_V|\) is the magnitude of three-momentum of the final state particle \(V\) or \(T\) \((|\vec{p}_V| = |\vec{p}_T|)\) in the rest frame of the \(B\) meson. The effective coefficients \(a_i\) are defined as in Eq. \([23]\). The factors \(\mathcal{X}, \mathcal{Y},\) and \(\mathcal{Z},\) respectively, are given by

\[ \mathcal{X} = 8 m_b^4 b^2_+, \]

\[ \mathcal{Y} = 2 m_b^2 [6 m_b^2 m_T h^2 + 2(m_b^2 - m_T^2 - m_V^2) k b_+ + k^2], \]

\[ \mathcal{Z} = 5 m_b^2 m_V^2 k^2. \]  

(29)

Here we have summed over polarizations of the tensor meson \(T\) using the following formula \([13]\):

\(^2\)In the factorization scheme, we use the usual phase convention for the pseudoscalar and the tensor mesons as follows: \(\rho^0(a_2^0) = \frac{1}{\sqrt{2}}(u \bar{u} - d \bar{d}), \rho^-(a_2^-) = \bar{u} d, K^*-(K^*_{2^-}) = \bar{u} s.\)
\[
\sum_{\lambda} \epsilon_{\alpha\beta}(p_T, \lambda) \epsilon^*_{\mu\nu}(p_T, \lambda) = \frac{1}{2}(\theta_{\alpha\mu}\theta_{\beta\nu} + \theta_{\beta\mu}\theta_{\alpha\nu}) - \frac{1}{3}\theta_{\alpha\beta}\theta_{\mu\nu}, \tag{30}
\]

where \( \theta_{\alpha\beta} = -g_{\alpha\beta} + (p_T)_\alpha (p_T)_\beta/m_T^2 \).

The CP asymmetry, \( A_{CP} \), is defined by

\[
A_{CP} = \frac{B(B \to f) - B(\bar{B} \to \bar{f})}{B(B \to f) + B(\bar{B} \to f)}, \tag{31}
\]

where \( B \) and \( f \) denote \( b \) quark and a generic final state, respectively.

We calculate the branching ratios and CP asymmetries for \( B \to V T \) decay modes for various input parameter values. The predictions are sensitive to several input parameters, such as the form factors, the strange quark mass, the parameter \( \xi \equiv 1/N_c \), the CKM matrix elements and in particular, the weak phase \( \gamma \). In a recent work \cite{14} on charmless \( B \) decays to two light mesons such as \( PP \) and \( VP \), it has been shown that the favored values of the input parameters are

\[
\xi \approx 0.45, \quad m_s(m_b) \approx 85 \text{ MeV}, \quad \gamma \approx 110^0, \quad V_{cb} = 0.040, \quad \text{and} \quad |V_{ub}/V_{cb}| = 0.087
\]

in order to get the best fit to the recent experimental data from the CLEO collaboration. For our numerical calculations, we use the following values of the decay constants (in MeV units) \cite{13,17,18}:

\[
f_\rho = 216, \quad f_\omega = 216, \quad f_\phi = 236, \quad f_{K^*} = 222.
\]

We use the values of the form factors for the \( B \to T \) transition calculated in the ISGW model \cite{6}. The strange quark mass \( m_s \) is in considerable doubt: \textit{i.e.}, QCD sum rules give \( m_s(1 \text{ GeV}) = (175 \pm 25) \text{ MeV} \) and lattice gauge theory gives \( m_s(2 \text{ GeV}) = (100 \pm 20 \pm 10) \text{ MeV} \) in the quenched lattice calculation \cite{19}. In this analysis we use two representative values of \( m_s = 100 \text{ MeV} \) and \( m_s = 85 \text{ MeV} \) at \( m_b \) scale. Current best estimates for CKM matrix elements are \( V_{cb} = 0.0381 \pm 0.0021 \) and \( |V_{ub}/V_{cb}| = 0.085 \pm 0.019 \). We use \( V_{cb} = 0.040 \) and \( |V_{ub}/V_{cb}| = 0.087 \). It has been known that there exists the discrepancy in values of \( \gamma \) extracted from CKM-fitting at \( \rho - \eta \) plane \cite{21} and from the \( \chi^2 \) analysis of non-leptonic decays of \( B \) mesons \cite{22,23}. The obtained value of \( \gamma \) is \( \gamma = 90^0 \sim 140^0 \). But in analysis of non-leptonic \( B \) decay, possibility of larger \( \gamma \) has been discussed by Deshpande \textit{et al.} \cite{22} and He \textit{et al.} \cite{23}. The obtained value of \( \gamma \) is \( \gamma = 90^0 \sim 140^0 \). In our calculations we use two representative values of \( \gamma = 110^0 \) and \( \gamma = 65^0 \).
In Tables III – VI, we show the branching ratios and the CP asymmetries for $B \to VT$ decays with either $\Delta S = 0$ or $|\Delta S| = 1$. In the tables the second and the third columns correspond to the sets of the input parameters,

$$\{\xi = 0.1, m_s = 85 \text{ MeV}, \gamma = 110^0\} \text{ and } \{\xi = 0.1, m_s = 100 \text{ MeV}, \gamma = 65^0\},$$

respectively. Similarly, the fourth and the fifth columns correspond to the cases,

$$\{\xi = 0.3, m_s = 85 \text{ MeV}, \gamma = 110^0\} \text{ and } \{\xi = 0.3, m_s = 100 \text{ MeV}, \gamma = 65^0\},$$

respectively. The sixth and the seventh columns correspond to the cases,

$$\{\xi = 0.5, m_s = 85 \text{ MeV}, \gamma = 110^0\} \text{ and } \{\xi = 0.5, m_s = 100 \text{ MeV}, \gamma = 65^0\},$$

respectively. Here $\xi \equiv 1/N_c = 0.3$ corresponds to the case of naive factorization ($N_c = 3$). It has been known that in $B \to D$ decays the generalized factorization has been successfully used with the favored value of $\xi \approx 0.5^{24}$. Also, as mentioned above, a recent analysis of charmless $B$ decays to two light mesons such as $PP$ and $VP^{14}$ shows that $\xi \approx 0.45$ is favored with certain values of other parameters for the best fit to the recent CLEO data.

The branching ratios for $B \to VT$ decay modes with $\Delta S = 0$ are shown in Table III. Among $\Delta S = 0$ modes, the decay modes $B^+ \to \rho^+ a_0^2$, $B^+ \to \rho^+ f_2$, and $B^0 \to \rho^+ a_2^-$ have relatively large branching ratios of a few times $10^{-7}$. The branching ratio for $B^+ \to \rho^+ f_2'$ is much smaller than that for $B^+ \to \rho^+ f_2$ by about two orders of magnitude, because the former decay rate is proportional to $\sin \phi_T = 0.13$, instead of $\cos \phi_T = 0.99$ which is a proportional factor of the latter decay rate. This prediction is consistent with that based on flavor SU(3) symmetry. We see that in the factorization scheme the following equality between the branching ratios holds for any set of the parameters given above: $2\mathcal{B}(B^+ \to \rho^+ a_2^0) \approx \mathcal{B}(B^0 \to \rho^+ a_2^-)$, as discussed in Eq. (22). (Little deviation from the exact equality arises from breaking of isospin symmetry.) We also see from Table III that $\mathcal{B}(B^+ \to \rho^0 a_2^+)$ is much smaller than $\mathcal{B}(B^+ \to \rho^+ a_2^0)$ by an order of magnitude or even three orders of magnitude depending on values of the input parameters. This is because in factorization the dominant contribution to the former mode arises from the color-suppressed tree diagram ($C_T$) and further the $C_T$ destructively interferes with $P_T$, while the dominant one to the latter mode arises from the color-favored tree diagram ($T_T$) and the $T_T$ constructively interferes with $P_T$. We note that $\mathcal{B}(B^+ \to \rho^0 a_2^{(0)}) \approx \mathcal{B}(B^+ \to \omega a_2^{(0)})$
and $\mathcal{B}(B^+ \to \rho^0 f_2^{(0)}) \approx \mathcal{B}(B^+ \to \omega f_2^{(0)})$, as is expected from the fact that $\rho^0$ and $\omega$ have the similar quark content and the decay amplitudes for the modes having $\rho^0$ in the final state are similar to those for the modes having $\omega$ in the final state (some differences appear only in the penguin diagram contributions which are small in $\Delta S = 0$ decays). The branching ratios of most processes are order of $10^{-8}$ or less. The CP asymmetries $A_{CP}$ in $\Delta S = 0$ decays are shown in Table IV. The CP asymmetries for $B^+ \to \rho a_2^+$ and $B^+ \to \omega a_2^+$ can be as large as 27% and 49%, respectively, with the branching ratio of $O(10^{-8})$ for $\xi = 0.5$.

In Table VII, we show the ratio $\mathcal{B}(B \to VT)/\mathcal{B}(B \to PT)$ for $\Delta S = 0$ decays, where quark contents of $V$ and $P$ are identical. For comparison, we choose the modes $B^+ \to \rho^0 a_2^0$ ($B^+ \to \pi^0 a_2^0$), $B^+ \to \rho^0 f_2$ ($B^+ \to \pi^0 f_2$), and $B^0 \to \rho^+ a_2^-$ ($B^0 \to \pi^+ a_2$) in $B \to VT$ ($B \to PT$) whose decay amplitudes have the dominant tree diagram contribution $T_T$. For these modes, the ratio $\mathcal{B}(B \to VT)/\mathcal{B}(B \to PT)$ can be written as

$$\frac{\mathcal{B}(B \to VT)}{\mathcal{B}(B \to PT)} \approx \frac{m_\psi f_\psi^2 [\mathcal{X}|\vec{p}_\psi|^7 + \mathcal{Y}|\vec{p}_\psi|^5 + \mathcal{Z}|\vec{p}_\psi|^3]}{2|m_p|^5 m^2_B f^2_\rho/f^{B \to T}(m^2_p)}.$$  \hspace{1cm} (32)

In the ratio, the dependence on $G_F$, the CKM matrix elements, and the effective coefficients $a_i$ does not appear. The ratio depends only on the form factors for $B \to T$ calculated in the ISGW model, in addition to masses of $P$, $V$ and $T$, and the decay constants $f_\rho$ and $f_\psi$. Thus, the ISGW model and the factorization scheme can be tested by measuring the above ratio for different modes, as shown in Table VII, in future experiment. Table VII shows that the ratio for $\Delta S = 0$ decays are indeed insensitive to different values of the input parameters, such as $\xi$ and the weak phase $\gamma$, and are in between 0.473 and 0.495.

The branching ratios and CP asymmetries for $|\Delta S| = 1$ decay processes are shown in Table V and VI, respectively. In $|\Delta S| = 1$ decays, the relevant penguin diagrams give dominant contribution to the decay rates. We see that the branching ratios for $|\Delta S| = 1$ decays are in range between $O(10^{-7})$ and $O(10^{-10})$, similar to those for $\Delta S = 0$ decays. The processes $B^+ \to K^{*+} a_2^0$, $K^{*+} f_2$, $K^{*0} a_2^+$, and $B^0 \to K^{*+} a_2^-$, $K^{*0} a_2^0$, $K^{*0} f_2$ have relatively large branching ratios of $O(10^{-7}) \sim O(10^{-8})$, since the amplitudes for these modes have the dominant penguin contribution $P_T$. We note that the branching ratios for $B \to \omega K^*_s$ and $B \to \phi K^*_s$ vary strongly depending on $\xi$. This is mainly because the amplitudes for these modes have the singlet penguin contribution $S'_T$ and the magnitude of $S'_T$ strongly depends on the value of $\xi$ in the factorization scheme. Unlike $\Delta S = 0$ decays such as $B \to \omega a_2$ and $B \to \phi a_2$, in $|\Delta S| = 1$
decays such as $B \to \omega K^*_2$ and $B \to \phi K^*_2$ the tree contribution is suppressed compared to the penguin contribution. Further, in the mode $B \to \omega K^*_2$, the amplitude $2S'_T$ is the only strong penguin contribution so that the branching ratio for this mode varies strongly depending on $\xi$ (even though $S'_T$ is expected to be small due to the OZI suppression). In $B \to \phi K^*_2$, the amplitude $P'_T + S'_T$ is the relevant strong penguin contribution, and in factorization $S'_T$ can become comparable (with the opposite sign) to $P'_T$ for certain values of $\xi$, say, $\xi = 0$ so that the branching ratio for this mode strongly depends on $\xi$. Table VI shows the CP asymmetries $A_{CP}$ in $|\Delta S| = 1$ decays. $A_{CP}$’s in most modes are expected to be small. In $B^+ \to K^{*+}a_2^0$, $B^+ \to K^{*+}f_2$, and $B^0 \to K^{*+}a_2^-$, $A_{CP}$ can be about $15\%-25\%$ with the branching ratios of $O(10^{-7}) - O(10^{-8})$.

In Table VII, we show the ratio $B(B \to VT)/B(B \to PT)$ for the modes $B^+ \to K^{*+}a_2^0$ ($B^+ \to K^{+}a_2$), $B^+ \to K^{*+}f_2$ ($B^+ \to K^{+}f_2$), and $B^0 \to K^{*+}a_2^-$ ($B^0 \to K^{+}a_2^-$) in $B \to VT (B \to PT)$ whose amplitudes have the dominant penguin contribution $P'_T$. For these modes, the ratio $B(B \to VT)/B(B \to PT)$ can be approximately expressed as Eq. (32), but unlike the $\Delta S = 0$ case, in this case, dependence of the ratio on the weak phase $\gamma$ and the strange quark mass $m_s$ remains, due to the effect of the suppressed tree diagram $T'_T$ and the $m_s$-dependence of $B(B \to PT)$. In the table, the second and the third columns correspond to the cases of sets of the parameters: $\{m_s = 85 \text{ MeV}, \gamma = 110^0\}$ and $\{m_s = 100 \text{ MeV}, \gamma = 65^0\}$, respectively. In both cases, the values of $\xi$ vary from 0.1 to 0.5. The result shows two different ranges of values of the ratio: in the former case (the second column), the ratio is about 2.5, while in the latter case (the third column), the ratio is about 1.0. Given values of $m_s$ and $\gamma$, the ratio is almost independent of the value of $\xi$.

V. CONCLUSION

We have analyzed exclusive charmless decays $B \to VT$ in the frameworks of both flavor SU(3) symmetry and generalized factorization. Using the flavor SU(3) symmetry, we have shown that certain decay modes, such as $B^+ \to \rho^+a_2^0$, $\rho^+f_2$ and $B^0 \to \rho^+a_2^-$ in $\Delta S = 0$ decays, and $B^+ \to K^{*+}f_2$, $K^{*0}a_2^+$ and $B^0 \to K^{*+}a_2^+$ in $|\Delta S| = 1$ decays, are expected to have the largest decay rates and so these modes can be preferable to find in future experiment. Certain ways to test validity of the factorization scheme have been presented by emphasizing interplay
between both approaches and carefully combining the predictions from both approaches. We have also shown that $B$ meson decays to two tensor mesons in the final state do not happen in the factorization scheme, which can be tested in future experiment.

We have calculated the branching ratios and CP asymmetries for $B \to VT$ decays, using the full effective Hamiltonian including all the penguin operators which are essential to analyze the $|\Delta S| = 1$ processes and to calculate CP asymmetries. We have also used the non-relativistic quark model proposed by Isgur, Scora, Grinstein, and Wise to obtain the form factors describing $B \to T$ transitions. As shown in Tables III and V, the branching ratios vary from $O(10^{-7})$ to $O(10^{-10})$. Consistent with the prediction from the flavor SU(3) analysis, the decay modes such as $B^+ \to \rho^+\rho^0_2$, $\rho^+ f_2$, $B^0 \to \rho^+\omega_2$ and $B^{+(0)} \to K_2^{*0(+)} a_2^{(+)(-)}$ have the branching ratios of order of $10^{-7}$. We have identified the decay modes where the CP asymmetries are expected to be large, such as $B^+ \to \rho^0 a_2^+$ and $B^+ \to \omega a_2^+$ in $\Delta S = 0$ decays, and $B^+ \to K^{*+} a_2^0$, $B^+ \to K^{*+} f_2$, and $B^0 \to K^{*+} a_2^-$ in $|\Delta S| = 1$ decays. Due to possible uncertainties in the hadronic form factors of $B \to VT$ and non-factorization effects, the predicted branching ratios could be increased. We have also presented the ratio $\mathcal{B}(B \to VT)/\mathcal{B}(B \to PT)$ for $\Delta S = 0$ and $|\Delta S| = 1$ decays, which primarily depends on the form factors for $B \to T$, especially in $\Delta S = 0$ case. Thus, measurement of this ratio for different modes in future experiment can test the ISGW modes and the factorization ansatz. Although experimentally challenging, the exclusive charmless decays, $B \to VT$, can probably be carried out in details at hadronic $B$ experiments such as BTeV and LHC-B, where more than $10^{12} B$-mesons will be produced per year, as well as at present asymmetric $B$ factories of Belle and Babar.

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APPENDIX

In this Appendix, we present expressions for all the decay amplitudes of $B \to VT$ modes shown in Tables I and II as calculated in the factorization scheme. Below we use $F_{\alpha \beta}^{B \to T}$ defined in Eqs. (20).

(1) $B \to VT$ ($\Delta S = 0$) decays.

\[
A(B^+ \to \rho^+ a_2^0) = \frac{G_F}{2} (m_{\rho^+} f_{\rho^+} \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to a_2^0} (m_{\rho^+}^2)) \left\{ V_{ub}^* V_{ud} a_1 - V_{tb}^* V_{td} (a_4 + a_{10}) \right\} \tag{33}
\]

\[
A(B^+ \to \rho^+ f_2) = \frac{G_F}{2} (m_{\rho^+} f_{\rho^+} \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to f_2} (m_{\rho^+}^2)) \left\{ V_{ub}^* V_{ud} c a_1 - V_{tb}^* V_{td} c (a_4 + a_{10}) \right\} \tag{34}
\]

\[
A(B^+ \to \rho^+ f'_2) = \frac{G_F}{2} (m_{\rho^+} f_{\rho^+} \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to f'_2} (m_{\rho^+}^2)) \left\{ V_{ub}^* V_{ud} s a_1 - V_{tb}^* V_{td} s (a_4 + a_{10}) \right\} \tag{35}
\]

\[
A(B^+ \to \rho^0 a_2^+) = \frac{G_F}{2} (m_\omega f_\omega \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to a_2^+} (m_\omega^2)) \left\{ V_{ub}^* V_{ud} a_2 \right. \right. \\
\left. \left. - V_{tb}^* V_{td} [-a_4 + \frac{3}{2} (a_7 + a_9) + \frac{1}{2} a_{10}] \right\} \tag{36}
\]

\[
A(B^+ \to \omega a_2^+) = \frac{G_F}{2} (m_\omega f_\omega \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to a_2^+} (m_\omega^2)) \left\{ V_{ub}^* V_{ud} a_2 \right. \right. \\
\left. \left. - V_{tb}^* V_{td} [2 (a_3 + a_5) + a_4 + \frac{1}{2} (a_7 + a_9) - \frac{1}{2} a_{10}] \right\} \tag{37}
\]

\[
A(B^+ \to \phi a_2^+) = \frac{G_F}{\sqrt{2}} (m_\phi f_\phi \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to a_2^+} (m_\phi^2)) \left\{ - V_{tb}^* V_{td} [(a_3 + a_5) - \frac{1}{2} (a_7 + a_9)] \right\} \tag{38}
\]

\[
A(B^+ \to \bar{K}^0 K^+_2) = \frac{G_F}{\sqrt{2}} (m_{\bar{K}^0} f_{\bar{K}^0} \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to K^+_2} (m_{\bar{K}^0}^2)) \left\{ - V_{tb}^* V_{td} [a_4 - \frac{1}{2} a_{10}] \right\} \tag{39}
\]

\[
A(B^+ \to \bar{K}^* K^+_2) = 0 \tag{40}
\]

\[
A(B^0 \to \rho^+ a_2^-) = \frac{G_F}{\sqrt{2}} (m_{\rho^+} f_{\rho^+} \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to a_2^-} (m_{\rho^+}^2)) [V_{ub}^* V_{ud} a_1 - V_{tb}^* V_{td} (a_4 + a_{10})] \tag{41}
\]

\[
A(B^0 \to \rho^+ a_2^+) = 0 \tag{42}
\]

\[
A(B^0 \to \rho^0 a_2^-) = \frac{G_F}{2\sqrt{2}} (m_\rho f_\rho \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to a_2^-} (m_\rho^2)) \left\{ V_{ub}^* V_{ud} a_2 \right. \right. \\
\left. \left. - V_{tb}^* V_{td} [-a_4 + \frac{3}{2} (a_7 + a_9) + \frac{1}{2} a_{10}] \right\} \tag{43}
\]

\[
A(B^0 \to \rho^0 f_2) = \frac{G_F}{2\sqrt{2}} (m_\rho f_\rho \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to f_2} (m_\rho^2)) \left\{ V_{ub}^* V_{ud} c a_2 \right. \right. \\
\left. \left. - V_{tb}^* V_{td} c [-a_4 + \frac{3}{2} (a_7 + a_9) + \frac{1}{2} a_{10}] \right\} \tag{44}
\]

\[
A(B^0 \to \rho^0 f'_2) = \frac{G_F}{2\sqrt{2}} (m_\rho f_\rho \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to f'_2} (m_\rho^2)) \left\{ V_{ub}^* V_{ud} s a_2 \right. \right. \\
\left. \left. - V_{tb}^* V_{td} s [-a_4 + \frac{3}{2} (a_7 + a_9) + \frac{1}{2} a_{10}] \right\} \tag{45}
\]

\[
A(B^0 \to \omega a_2^-) = \frac{G_F}{2\sqrt{2}} (m_\omega f_\omega \epsilon^{* \alpha \beta} F_{\alpha \beta}^{B \to a_2^-} (m_\omega^2)) \left\{ V_{ub}^* V_{ud} a_2 \right. \right. \\
\left. \left. - V_{tb}^* V_{td} [-a_4 + \frac{3}{2} (a_7 + a_9) + \frac{1}{2} a_{10}] \right\} \tag{46}
\]
\[
A(B^0 \rightarrow \omega f_2) = \frac{G_F}{2\sqrt{2}} (m_\omega f_\omega e^{*a_3} F_{a_3}^{B \rightarrow f_2} (m_\omega^2)) \{ V_{ub}^* V_{ud} c a_2 - V_{tb}^* V_{td} c a_2 \}
\]
\[
\rightarrow 2 \left( a_3 + a_5 \right) + a_4 + \frac{1}{2} (a_7 + a_9) - \frac{1}{2} a_{10} \right\} \]  
(46) 
\]
\[
A(B^0 \rightarrow \omega f_2') = \frac{G_F}{2\sqrt{2}} (m_\omega f_\omega e^{*a_3} F_{a_3}^{B \rightarrow f_2} (m_\omega^2)) \{ V_{ub}^* V_{ud} s a_2 - V_{tb}^* V_{td} s a_2 \}
\]
\[
- V_{tb}^* V_{td} c \left( a_3 + a_5 \right) + a_4 + \frac{1}{2} (a_7 + a_9) - \frac{1}{2} a_{10} \right\} \]  
(47) 
\]
\[
A(B^0 \rightarrow \phi a_2^0) = \frac{G_F}{2} (m_\phi f_\phi e^{*a_3} F_{a_3}^{B \rightarrow a_2^0} (m_\phi^2)) \{ - V_{tb}^* V_{td} \left[ (a_3 + a_5) - \frac{1}{2} (a_7 + a_9) \right] \} \]
(49) 
\]
\[
A(B^0 \rightarrow \phi f_2) = \frac{G_F}{2} (m_\phi f_\phi e^{*a_3} F_{a_3}^{B \rightarrow f_2} (m_\phi^2)) \{ - V_{tb}^* V_{td} c \left[ (a_3 + a_5) - \frac{1}{2} (a_7 + a_9) \right] \} \]
(50) 
\]
\[
A(B^0 \rightarrow \phi f_2') = \frac{G_F}{2} (m_\phi f_\phi e^{*a_3} F_{a_3}^{B \rightarrow f_2} (m_\phi^2)) \{ - V_{tb}^* V_{td} s \left[ (a_3 + a_5) - \frac{1}{2} (a_7 + a_9) \right] \} \]
(51) 
\]
\[
A(B^0 \rightarrow K^{*0} K^{*0}) = \frac{G_F}{\sqrt{2}} (m_{K^{*0}} f_{K^{*0}} e^{*a_3} F_{a_3}^{B \rightarrow K^{*0} K^{*0}} (m_{K^{*0}}^2)) \{ - V_{tb}^* V_{td} a_4 - \frac{1}{2} a_{10} \} \]
(52) 
\]
\[
A(B^0 \rightarrow K^{*+} K^{*-}) = 0 \]
(53) 
\]

(2) \( B \rightarrow VT (|\Delta S| = 1) \) decays.
\[
A(B^+ \rightarrow K^{*+} a_2^0) = \frac{G_F}{2} (m_{K^{*+}} f_{K^{*+}} e^{*a_3} F_{a_3}^{B \rightarrow a_2^0} (m_{K^{*+}}^2)) \{ V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts} (a_4 + a_{10}) \}
\]
(54) 
\]
\[
A(B^+ \rightarrow K^{*+} f_2) = \frac{G_F}{2} (m_{K^{*+}} f_{K^{*+}} e^{*a_3} F_{a_3}^{B \rightarrow f_2} (m_{K^{*+}}^2)) \{ V_{ub}^* V_{us} c a_1 - V_{tb}^* V_{ts} c (a_4 + a_{10}) \}
\]
(55) 
\]
\[
A(B^+ \rightarrow K^{*+} f_2') = \frac{G_F}{2} (m_{K^{*+}} f_{K^{*+}} e^{*a_3} F_{a_3}^{B \rightarrow f_2} (m_{K^{*+}}^2)) \{ V_{ub}^* V_{us} s a_1 - V_{tb}^* V_{ts} s (a_4 + a_{10}) \}
\]
(56) 
\]
\[
A(B^+ \rightarrow K^{*0} a_2^+ + a_2^-) = \frac{G_F}{\sqrt{2}} (m_{K^{*0}} f_{K^{*0}} e^{*a_3} F_{a_3}^{B \rightarrow a_2^+} (m_{K^{*0}}^2)) \{ - V_{tb}^* V_{ts} (a_4 + \frac{1}{2} a_{10}) \}
\]
(57) 
\]
\[
A(B^+ \rightarrow \rho^+ K^{*0}) = 0 \]
(58) 
\]
\[
A(B^+ \rightarrow \rho^0 K^{*+}) = \frac{G_F}{2} (m_{\rho^0} f_{\rho^0} e^{*a_3} F_{a_3}^{B \rightarrow K^{*+}} (m_{\rho^0}^2)) \{ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} \frac{3}{2} (a_7 + a_9) \}
\]
(59) 
\]
\[
A(B^+ \rightarrow \omega K^{*+}) = \frac{G_F}{2} (m_\omega f_\omega e^{*a_3} F_{a_3}^{B \rightarrow K^{*+}} (m_\omega^2)) \{ V_{ub}^* V_{us} a_2 - V_{tb}^* V_{ts} \left[ (a_3 + a_5) + \frac{1}{2} (a_7 + a_9) \right] \}
\]
(60) 
\]
\[
A(B^+ \rightarrow \phi K^{*+}) = \frac{G_F}{\sqrt{2}} (m_\phi f_\phi e^{*a_3} F_{a_3}^{B \rightarrow K^{*+}} (m_\phi^2)) \{ - V_{tb}^* V_{ts} \left[ a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10}) \right] \}
\]
(61) 
\]
\[
A(B^0 \rightarrow K^{*+} a_2^-) = \frac{G_F}{\sqrt{2}} (m_{K^{*+}} f_{K^{*+}} e^{*a_3} F_{a_3}^{B \rightarrow a_2^-} (m_{K^{*+}}^2)) \{ V_{ub}^* V_{us} a_1 - V_{tb}^* V_{ts} (a_4 + a_{10}) \}
\]
(62) 
\]
\[
A(B^0 \rightarrow K^{*0} a_2^0) = \frac{G_F}{2} (m_{K^{*0}} f_{K^{*0}} e^{*a_3} F_{a_3}^{B \rightarrow a_2^0} (m_{K^{*0}}^2)) \{ - V_{tb}^* V_{ts} (a_4 + \frac{1}{2} a_{10}) \}
\]
(63) 
\]
\[
A(B^0 \rightarrow K^{*0} f_2) = \frac{G_F}{2} (m_{K^{*0}} f_{K^{*0}} e^{*a_3} F_{a_3}^{B \rightarrow f_2} (m_{K^{*0}}^2)) \{ - V_{tb}^* V_{ts} (a_4 + \frac{1}{2} a_{10}) \}
\]
(64) 
\]
\[ A(B^0 \to K^{*0} f'_2) = \frac{G_F}{2}(m_{K^{*0}} f_{K^{*0}} \varepsilon^{\alpha \beta} F_{\alpha \beta}^{B \to K^{*0}}(m_{K^{*0}}^2)) \left\{ -V^\ast_{tb} V_{ts} (a_4 - \frac{1}{2} a_{10}) \right\} \] (65)

\[ A(B^0 \to \rho^{-} K^{*+}) = 0 \] (66)

\[ A(B^0 \to \rho^{0} K^{*0}) = \frac{G_F}{2}(m_{\rho^0} f_{\rho^0} \varepsilon^{\alpha \beta} F_{\alpha \beta}^{B \to K^{*0}}(m_{\rho^0}^2)) \left\{ V^\ast_{ub} V_{us} a_2 - V^\ast_{tb} V_{ts} \frac{3}{2} (a_9 + a_7) \right\} \] (67)

\[ A(B^0 \to \omega K^{*0}) = \frac{G_F}{2}(m_{\omega} f_{\omega} \varepsilon^{\alpha \beta} F_{\alpha \beta}^{B \to K^{*0}}(m_{\omega}^2)) \left\{ V^\ast_{ub} V_{us} a_2 - V^\ast_{tb} V_{ts} \frac{3}{2} (a_9 + a_7) \right\} \] (68)

\[ A(B^0 \to \phi K^{*0}) = \frac{G_F}{\sqrt{2}}(m_{\phi} f_{\phi} \varepsilon^{\alpha \beta} F_{\alpha \beta}^{B \to K^{*0}}(m_{\phi}^2)) \left\{ -V^\ast_{tb} V_{ts} (a_3 + a_4 + a_5 - \frac{1}{2} (a_7 + a_9 + a_{10})) \right\} \] (69)
TABLE I. Coefficients of SU(3) amplitudes in $B \to VT$ ($\Delta S = 0$). The coefficients of the SU(3) amplitudes with the subscript $V$ are expressed in square brackets. As explained in Sec. II, the contributions of the SU(3) amplitudes with the subscript $V$ vanish in the framework of factorization, because those contributions contain the matrix element $\langle T | J_{\mu}^{\text{weak}} | 0 \rangle$, which is zero. Here $c$ and $s$ denote $\cos\phi_r$ and $\sin\phi_r$, respectively.

| $B \to VT$ | factor | $T_T[T_V]$ | $C_T[C_V]$ | $S_T[S_V]$ | $P_T[P_V]$ | $P_{EW,T}[P_{EW,V}]$ | $P_{EW,T}[P_{EW,V}]$ |
|------------|--------|------------|------------|------------|-------------|------------------|------------------|
| $B^+ \to \rho^+ a_2^0$ | $-\frac{1}{\sqrt{2}}$ | 1 [1] | 0 | 1, [-1] | [1] | $\frac{2}{3}$, $[\frac{1}{3}]$ |
| $B^+ \to \rho^+ f_2$ | $\frac{1}{\sqrt{2}}$ | c [c] | $[2c + \sqrt{2}s]$ | c, [c] | $[\frac{c - \sqrt{2}s}{3}]$ | $\frac{2c}{3}$, $[-\frac{s}{3}]$ |
| $B^+ \to \rho^+ f_2'$ | $\frac{1}{\sqrt{2}}$ | s [s] | $[2s - \sqrt{2}c]$ | s, [s] | $[\frac{\sqrt{2}c + s}{3}]$ | $\frac{2s}{3}$, $[-\frac{c}{3}]$ |
| $B^+ \to \rho^0 a_2^+$ | $-\frac{1}{\sqrt{2}}$ | [1] | 1 | 0 | -1, [1] | 1 | $\frac{1}{3}$, $[\frac{2}{3}]$ |
| $B^+ \to \omega a_2^+$ | $\frac{1}{\sqrt{2}}$ | [1] | 2 | 1, [1] | $\frac{1}{3}$ | $-\frac{1}{3}$, $\frac{2}{3}$ |
| $B^+ \to \phi a_2^+$ | 1 | 0 | 0 | 1 | 0 | $-\frac{1}{3}$ | 0 |
| $B^+ \to K^{*+} \bar{K}_2^{*0}$ | 1 | 0 | 0 | 0 | [1] | 0 | $-\frac{1}{3}$ |
| $B^+ \to \bar{K}^{*0} K_2^{*+}$ | 1 | 0 | 0 | 0 | 1 | 0 | $-\frac{1}{3}$ |
| $B^0 \to \rho^+ a_{2}^{-}$ | -1 | 1 | 0 | 0 | 1 | 0 | $\frac{2}{3}$ |
| $B^0 \to \rho^{-} a_{2}^{+}$ | -1 | [1] | 0 | 0 | [1] | 0 | $\frac{2}{3}$ |
| $B^0 \to \rho^0 a_{2}^{0}$ | $-\frac{1}{\sqrt{2}}$ | 0 | 1, [1] | 0 | -1, [-1] | 1, [1] | $\frac{1}{3}$, $[\frac{1}{3}]$ |
| $B^0 \to \rho^0 f_2$ | $-\frac{1}{\sqrt{2}}$ | 0 | c, [-c] | $[-(2c + \sqrt{2}s)]$ | -c, [-c] | c, $[\frac{c + \sqrt{2}s}{3}]$ | $\frac{c}{3}$, $[\frac{s}{3}]$ |
| $B^0 \to \rho^0 f_2'$ | $-\frac{1}{\sqrt{2}}$ | 0 | s, [-s] | $[-(2s - \sqrt{2}c)]$ | -s, [-s] | s, $[\frac{-\sqrt{2}c + s}{3}]$ | $\frac{s}{3}$, $[\frac{s}{3}]$ |
| $B^0 \to \omega a_{2}^{0}$ | $\frac{1}{\sqrt{2}}$ | 0 | 1, [-1] | 2 | 1, [1] | $\frac{1}{3}$, [-1] | $-\frac{1}{3}$, $[-\frac{1}{3}]$ |
| $B^0 \to \omega f_2$ | $\frac{1}{\sqrt{2}}$ | 0 | c, [c] | $[2c + \sqrt{2}s]$ | c, [c] | $\frac{c + \sqrt{2}s}{3}$ | $\frac{c}{3}$, $[-\frac{s}{3}]$ |
| $B^0 \to \omega f_2'$ | $\frac{1}{\sqrt{2}}$ | 0 | s, [s] | $[2s - \sqrt{2}c]$ | s, [s] | $\frac{s + \sqrt{2}c}{3}$ | $\frac{s}{3}$, $[-\frac{c}{3}]$ |
| $B^0 \to \phi a_{2}^{0}$ | $\frac{1}{\sqrt{2}}$ | 0 | 0 | 1 | 0 | $-\frac{1}{3}$ | 0 |
| $B^0 \to \phi f_2$ | $\frac{1}{\sqrt{2}}$ | 0 | 0 | c | 0 | $-\frac{c}{3}$ | 0 |
| $B^0 \to \phi f_2'$ | $\frac{1}{\sqrt{2}}$ | 0 | 0 | s | 0 | $-\frac{s}{3}$ | 0 |
| $B^0 \to K^{*0} \bar{K}_2^{*0}$ | 1 | 0 | 0 | 0 | [1] | 0 | $-\frac{1}{3}$ |
| $B^0 \to \bar{K}^{*0} K_2^{*0}$ | 1 | 0 | 0 | 0 | 1 | 0 | $-\frac{1}{3}$ |
| $B \to VT$ | factor | $T'_T[T'_P]$ | $C'_T[C'_V]$ | $S'_T[S'_V]$ | $P'_T[P'_V]$ | $P'_{EW,T}$ | $P'_{EW,V}$ | $P^C_{EW,T}[P^C_{EW,V}]$ |
|----------------|--------|---------------|---------------|---------------|---------------|---------------|---------------|-----------------|
| $B^+ \to K^{*+}a^0_2$ | $\frac{-1}{\sqrt{2}}$ | 1 | [1] | 0 | 1 | [1] | [2/3] |
| $B^+ \to K^{*+}f_2$ | $\frac{1}{\sqrt{2}}$ | c | [c] | $[2c + \sqrt{2}s]$ | c, $[\sqrt{2}s]$ | $[c - \sqrt{2}c]$ | $\frac{2}{3}c$, $[-\sqrt{2}s]$ |
| $B^+ \to K^{*+}f'_2$ | $\frac{1}{\sqrt{2}}$ | s | [s] | $[2s - \sqrt{2}c]$ | s, $[-\sqrt{2}c]$ | $[s + \sqrt{2}c]$ | $\frac{2}{3}s$, $[\sqrt{2}c]$ |
| $B^+ \to K^{*0}a^+_2$ | 1 | 0 | 0 | 0 | 1 | 0 | $-\frac{1}{3}$ |
| $B^+ \to K^{*0}a^+_2$ | 1 | 0 | 0 | 0 | [1] | 0 | $[\frac{-1}{3}]$ |
| $B^+ \to \rho^+K^{*0}_2$ | $\frac{-1}{\sqrt{2}}$ | [1] | 1 | 0 | [1] | 1 | $[\frac{2}{3}]$ |
| $B^+ \to \rho^0K^{*0}_2$ | $\frac{1}{\sqrt{2}}$ | [1] | 1 | 2 | [1] | $\frac{1}{3}$ | $[\frac{2}{3}]$ |
| $B^+ \to \phi K^{*0}_2$ | 1 | 0 | 0 | 1 | 1 | $-\frac{1}{3}$ | $-\frac{1}{3}$ |
| $B^0 \to K^{*+}a^-_2$ | $-1$ | 1 | 0 | 0 | 1 | 0 | $\frac{2}{3}$ |
| $B^0 \to K^{*0}a^0_2$ | $\frac{1}{\sqrt{2}}$ | 0 | $[-1]$ | 0 | 1 | $[-1]$ | $-\frac{1}{3}$ |
| $B^0 \to K^{*0}f_2$ | $\frac{1}{\sqrt{2}}$ | 0 | [c] | $[2c + \sqrt{2}s]$ | c, $[\sqrt{2}s]$ | $[c - \sqrt{2}c]$ | $-\frac{c}{3}$, $[\sqrt{2}c]$ |
| $B^0 \to K^{*0}f'_2$ | $\frac{1}{\sqrt{2}}$ | 0 | [s] | $[2s - \sqrt{2}c]$ | s, $[-\sqrt{2}c]$ | $[s + \sqrt{2}c]$ | $-\frac{s}{3}$, $[\sqrt{2}c]$ |
| $B^0 \to \rho^-K^{*0}_2$ | $-1$ | [1] | 0 | 0 | [1] | 0 | $\frac{2}{3}$ |
| $B^0 \to \rho^0K^{*0}_2$ | $\frac{-1}{\sqrt{2}}$ | 0 | 1 | 0 | $[-1]$ | 1 | $[\frac{1}{3}]$ |
| $B^0 \to \omega K^{*0}_2$ | $\frac{1}{\sqrt{2}}$ | 0 | 1 | 2 | [1] | $\frac{1}{3}$ | $[-\frac{1}{3}]$ |
| $B^0 \to \phi K^{*0}_2$ | 1 | 0 | 0 | 1 | 1 | $\frac{-1}{3}$ | $\frac{-1}{3}$ |
TABLE III. The branching ratios for $B \rightarrow V T$ decay modes with $\Delta S = 0$. The second and the third columns correspond to the cases of sets of the parameters: $\{\xi = 0.1, m_s = 85 \text{ MeV, } \gamma = 110^0\}$ and $\{\xi = 0.1, m_s = 100 \text{ MeV, } \gamma = 65^0\}$, respectively. Similarly, the fourth and the fifth columns correspond to the cases: $\{\xi = 0.3, m_s = 85 \text{ MeV, } \gamma = 110^0\}$ and $\{\xi = 0.3, m_s = 100 \text{ MeV, } \gamma = 65^0\}$, respectively. The sixth and the seventh columns correspond to the cases: $\{\xi = 0.5, m_s = 85 \text{ MeV, } \gamma = 110^0\}$ and $\{\xi = 0.5, m_s = 100 \text{ MeV, } \gamma = 65^0\}$, respectively.

| Decay mode       | $\mathcal{B}(10^{-8})$ | $\mathcal{B}(10^{-8})$ | $\mathcal{B}(10^{-8})$ | $\mathcal{B}(10^{-8})$ | $\mathcal{B}(10^{-8})$ | $\mathcal{B}(10^{-8})$ |
|------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| $B^+ \rightarrow \rho^+ a^0_2$ | 21.93                   | 22.17                   | 19.46                   | 19.70                   | 17.13                   | 17.37                   |
| $B^+ \rightarrow \rho^+ f_2$  | 23.33                   | 23.58                   | 20.70                   | 20.95                   | 18.23                   | 18.48                   |
| $B^+ \rightarrow \rho^+ f'_2$ | 0.26                    | 0.26                    | 0.23                    | 0.23                    | 0.20                    | 0.20                    |
| $B^+ \rightarrow \rho^0 a^+_2$ | 0.84                    | 0.78                    | 0.046                   | 0.033                   | 1.10                    | 1.16                    |
| $B^+ \rightarrow \omega a^+_2$ | 0.77                    | 0.77                    | 0.039                   | 0.034                   | 1.18                    | 1.28                    |
| $B^+ \rightarrow \phi a^+_2$  | 0.064                   | 0.053                   | 0.006                   | 0.006                   | 0.022                   | 0.012                   |
| $B^+ \rightarrow \bar{K}^* K^*$ | 0.062                   | 0.041                   | 0.053                   | 0.033                   | 0.045                   | 0.027                   |
| $B^0 \rightarrow \rho^+ a^-_2$ | 40.72                   | 41.16                   | 36.13                   | 36.57                   | 31.81                   | 32.26                   |
| $B^0 \rightarrow \rho^0 a^0_2$ | 0.39                    | 0.36                    | 0.022                   | 0.015                   | 0.51                    | 0.54                    |
| $B^0 \rightarrow \rho^0 f_2$  | 0.42                    | 0.38                    | 0.023                   | 0.016                   | 0.55                    | 0.57                    |
| $B^0 \rightarrow \rho^0 f'_2$ | 0.005                   | 0.004                   | 0.0003                  | 0.0002                  | 0.006                   | 0.006                   |
| $B^0 \rightarrow \omega a^0_2$ | 0.36                    | 0.36                    | 0.018                   | 0.016                   | 0.55                    | 0.60                    |
| $B^0 \rightarrow \omega f_2$  | 0.38                    | 0.38                    | 0.019                   | 0.017                   | 0.58                    | 0.63                    |
| $B^0 \rightarrow \omega f'_2$ | 0.004                   | 0.004                   | 0.0002                  | 0.0002                  | 0.006                   | 0.007                   |
| $B^0 \rightarrow \phi a^0_2$  | 0.030                   | 0.025                   | 0.003                   | 0.003                   | 0.010                   | 0.006                   |
| $B^0 \rightarrow \phi f_2$  | 0.030                   | 0.025                   | 0.003                   | 0.003                   | 0.010                   | 0.006                   |
| $B^0 \rightarrow \phi f'_2$ | 0.0004                  | 0.0003                  | 0                   | 0                       | 0.0001                  | 0                       |
| $B^0 \rightarrow \bar{K}^* K^{*0}$ | 0.12                   | 0.076                   | 0.098                   | 0.062                   | 0.082                   | 0.050                   |
TABLE IV. The CP asymmetries for $B \to V T$ decay modes with $\Delta S = 0$. The definitions for the columns are the same as those in Table III.

| Decay mode          | $A_{CP}$ | $A_{CP}$ | $A_{CP}$ | $A_{CP}$ | $A_{CP}$ | $A_{CP}$ |
|---------------------|----------|----------|----------|----------|----------|----------|
| $B^+ \to \rho^+ a_2^0$ | -0.073   | -0.070   | -0.072   | -0.069   | -0.071   | -0.068   |
| $B^+ \to \rho^+ f_2$  | -0.073   | -0.070   | -0.072   | -0.069   | -0.071   | -0.068   |
| $B^+ \to \rho^+ f'_2$ | -0.073   | -0.070   | -0.072   | -0.069   | -0.071   | -0.068   |
| $B^+ \to \rho^0 a_2^+$ | -0.34    | -0.36    | 0.66     | 0.91     | 0.27     | 0.25     |
| $B^+ \to \omega a_2^+$ | 0.017    | 0.016    | -0.72    | -0.79    | -0.49    | -0.44    |
| $B^+ \to \phi a_2^+$  | 0        | 0        | 0        | 0        | 0        | 0        |
| $B^+ \to K^{*0} K_2^{*+}$ | 0        | 0        | 0        | 0        | 0        | 0        |
| $B^0 \to \rho^+ a_2^-$ | -0.073   | -0.070   | -0.072   | -0.069   | -0.071   | -0.068   |
| $B^0 \to \rho^0 a_2^0$ | -0.34    | -0.36    | 0.66     | 0.91     | 0.27     | 0.25     |
| $B^0 \to \rho^0 f_2$  | -0.34    | -0.36    | 0.66     | 0.91     | 0.27     | 0.25     |
| $B^0 \to \rho^0 f'_2$ | -0.34    | -0.36    | 0.66     | 0.91     | 0.27     | 0.25     |
| $B^0 \to \omega a_2^0$ | 0.017    | 0.016    | -0.72    | -0.79    | -0.49    | -0.44    |
| $B^0 \to \omega f_2$  | 0.017    | 0.016    | -0.72    | -0.79    | -0.49    | -0.44    |
| $B^0 \to \omega f'_2$ | 0.017    | 0.016    | -0.72    | -0.79    | -0.49    | -0.44    |
| $B^0 \to \phi a_2^0$  | 0        | 0        | 0        | 0        | 0        | 0        |
| $B^0 \to \phi f_2$    | 0        | 0        | 0        | 0        | 0        | 0        |
| $B^0 \to \phi f'_2$   | 0        | 0        | 0        | 0        | 0        | 0        |
| $B^0 \to K^{*0} K_2^{*0}$ | 0        | 0        | 0        | 0        | 0        | 0        |
TABLE V. The branching ratios for $B \to VT$ decay modes with $|\Delta S| = 1$. The definitions for the columns are the same as those in Table III.

| Decay mode          | $B(10^{-8})$ | $B(10^{-8})$ | $B(10^{-8})$ | $B(10^{-8})$ | $B(10^{-8})$ | $B(10^{-8})$ |
|---------------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $B^+ \to K^{*+}a_2^0$ | 10.78        | 5.97         | 9.74         | 5.40         | 8.75         | 4.88         |
| $B^+ \to K^{*+}f_2$  | 11.20        | 6.19         | 10.11        | 5.61         | 9.09         | 5.06         |
| $B^+ \to K^{*+}f'_2$ | 0.14         | 0.078        | 0.13         | 0.070        | 0.11         | 0.064        |
| $B^+ \to K^{*0}a_2^+$ | 16.45        | 16.45        | 12.97        | 12.97        | 9.91         | 9.91         |
| $B^+ \to \rho^0K_2^{*+}$ | 0.59        | 0.81         | 0.57         | 0.55         | 0.62         | 0.39         |
| $B^+ \to \omega K_2^{*+}$ | 5.30        | 4.70         | 0.029        | 0.035        | 3.91         | 3.28         |
| $B^+ \to \phi K_2^{*+}$ | 2.52         | 2.52         | 10.39        | 10.39        | 23.66        | 23.66        |
| $B^0 \to K^{*+}a_2^-$ | 20.48        | 11.33        | 18.50        | 10.27        | 16.62        | 9.26         |
| $B^0 \to K^{*0}a_2^0$ | 7.65         | 7.65         | 6.03         | 6.03         | 4.61         | 4.61         |
| $B^0 \to K^{*0}f_2$  | 7.94         | 7.94         | 6.26         | 6.26         | 4.78         | 4.78         |
| $B^0 \to K^{*0}f'_2$ | 0.10         | 0.10         | 0.079        | 0.079        | 0.060        | 0.060        |
| $B^0 \to \rho^0K_2^{*0}$ | 0.54        | 0.75         | 0.53         | 0.50         | 0.57         | 0.36         |
| $B^0 \to \omega K_2^{*0}$ | 4.87         | 4.32         | 0.027        | 0.032        | 3.60         | 3.02         |
| $B^0 \to \phi K_2^{*0}$ | 2.34         | 2.34         | 9.64         | 9.64         | 21.96        | 21.96        |
TABLE VI. The CP asymmetries for $B \to VT$ decay modes with $|\Delta S| = 1$. The definitions for the columns are the same as those in Table III.

| Decay mode | $A_{CP}$ | $A_{CP}$ | $A_{CP}$ | $A_{CP}$ | $A_{CP}$ | $A_{CP}$ |
|-------------|----------|----------|----------|----------|----------|----------|
| $B^+ \to K^{*+}a_2^0$ | -0.15 | -0.26 | -0.14 | -0.25 | -0.14 | -0.24 |
| $B^+ \to K^{*+}f_2$ | -0.15 | -0.26 | -0.14 | -0.25 | -0.14 | -0.24 |
| $B^+ \to K^{*+}f'_2$ | -0.15 | -0.26 | -0.14 | -0.25 | -0.14 | -0.24 |
| $B^+ \to K^{*0}a_2^+$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $B^+ \to \rho^0K_2^+$ | -0.006 | -0.004 | 0.001 | 0.001 | 0.007 | 0.010 |
| $B^+ \to \omega K_2^+$ | -0.035 | -0.038 | 0.107 | 0.088 | -0.041 | -0.047 |
| $B^+ \to \phi K_2^+$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $B^0 \to K^{*+}a_2^-$ | -0.15 | -0.26 | -0.14 | -0.25 | -0.14 | -0.24 |
| $B^0 \to K^{*0}a_2^0$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $B^0 \to K^{*0}f_2$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $B^0 \to K^{*0}f'_2$ | 0 | 0 | 0 | 0 | 0 | 0 |
| $B^0 \to \rho^0K_2^0$ | -0.006 | -0.004 | 0.001 | 0.001 | 0.007 | 0.010 |
| $B^0 \to \omega K_2^0$ | -0.035 | -0.038 | 0.107 | 0.088 | -0.041 | -0.047 |
| $B^0 \to \phi K_2^0$ | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE VII. Ratios of the branching ratios for $B \to VT$ and for $B \to PT$ decay modes, where $V$ and $P$ have identical quark content. The second and the third columns correspond to the cases of sets of the parameters: $\{m_s = 85 \text{ MeV}, \gamma = 110^0\}$ and $\{m_s = 100 \text{ MeV}, \gamma = 65^0\}$, respectively. In both cases, the values of $\xi$ vary from 0.1 to 0.5.

| Ratio | $m_s = 85 \text{ MeV}, \gamma = 110^0$ | $m_s = 100 \text{ MeV}, \gamma = 65^0$ |
|-------|----------------------------------|----------------------------------|
| $\mathcal{B}(B^+ \to \rho^0a_2^0) / \mathcal{B}(B^+ \to \pi^0a_2^0)$ | 0.482–0.483 | 0.495 |
| $\mathcal{B}(B^+ \to \rho^0f_2) / \mathcal{B}(B^+ \to \pi^0f_2)$ | 0.472–0.473 | 0.484–0.485 |
| $\mathcal{B}(B^0 \to \rho^0a_2^-) / \mathcal{B}(B^0 \to \pi^0a_2^-)$ | 0.473–0.474 | 0.485–0.486 |
| $\mathcal{B}(B^+ \to K^{*0}a_2^0) / \mathcal{B}(B^+ \to K^{*0}a_2^0)$ | 2.50–2.55 | 1.03–1.10 |
| $\mathcal{B}(B^+ \to K^{*0}f_2) / \mathcal{B}(B^+ \to K^{*0}f_2)$ | 2.39–2.50 | 0.99–1.05 |
| $\mathcal{B}(B^0 \to K^{*0}a_2^-) / \mathcal{B}(B^0 \to K^{*0}a_2^-)$ | 2.51–2.63 | 1.04–1.10 |