Three-dimensional simulations of accretion to stars with complex magnetic fields

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ABSTRACT
Disc accretion to rotating stars with complex magnetic fields is investigated using full 3D magnetohydrodynamic (MHD) simulations. The studied magnetic configurations include superpositions of misaligned dipole and quadrupole fields and off-centre dipoles. The simulations show that when the quadrupole component is comparable to the dipole component, the magnetic field has a complex structure with three major magnetic poles on the surface of the star and three sets of loops of field lines connecting them. A significant amount of matter flows to the quadrupole ‘belt’, forming a ring-like hotspot on the star. If the maximum strength of the magnetic field on the star is fixed, then we observe that the mass accretion rate, the torque on the star and the area covered by hotspots are several times smaller in the quadrupole-dominant cases than in the pure dipole cases. The influence of the quadrupole component on the shape of the hotspots becomes noticeable when the ratio of the quadrupole and dipole field strengths \( B_q/B_d \gtrsim 0.5 \). It becomes dominant in determining the shape of the hotspots when \( B_q/B_d \gtrsim 1 \). We conclude that if the quadrupole component is larger than the dipole one, then the shape of the hotspots is determined by the quadrupole field component. In the case of an off-centre dipole field, most of the matter flows through a one-armed accretion stream, forming a large hotspot on the surface, with a second much smaller secondary spot. The light curves may have simple, sinusoidal shapes, thus mimicking stars with pure dipole fields. Or, they may be complex and unusual. In some cases, the light curves may be indicators of a complex field, in particular if the inclination angle is known independently. We also note that in the case of complex fields, magnetospheric gaps are often not empty, and this may be important for the survival of close-in exosolar planets.

Key words: accretion, accretion discs – magnetic fields – MHD – stars: magnetic fields.

1 INTRODUCTION
The magnetic field of a rotating star can have a strong influence on the matter in an accretion disc. The field can disrupt the disc and channel the accreting matter to the star along the field lines. The associated magnetic activity can be observed through photometric and spectral measurements of different types of stars, like young solar-type Classical T Tauri stars (CTTs) (Hartmann, Hewett & Calvet 1994), X-ray pulsars and millisecond pulsars (Ghosh & Lamb 1978; Chakrabarty et al. 2003), cataclysmic variables (Wickramasinghe, Wu & Ferrario 1991; Warner 1995, 2000) and also brown dwarfs (e.g. Scholz & Ray 2006). The topology of the stellar magnetic field plays an important role in disc accretion and in the photometric and spectral appearance of the stars.

The dipole magnetic model has been studied since Ghosh & Lamb (1979a,b), both theoretically and with 2D and 3D magnetohydrodynamic (MHD) simulations. However, the intrinsic field of the star may be more complex than a dipole field. Safier (1998) argued that the magnetic field of CTTs may be strongly non-dipolar. Zeeman measurements of the magnetic field of CTTs based on photospheric spectral lines show that the strong (1–3 kG) magnetic field is probably not ordered, and indicate that the field is non-dipolar close to the star (Johns-Krull, Valenti & Koresko 1999; Johns-Krull 2007). Other magnetic field measurements with the Zeeman–Doppler imaging technique have shown that in a number of rapidly rotating low-mass stars, the magnetic field has a complicated multipolar topology close to the star (Donati & Cameron 1997; Donati et al. 1999; Jardine et al. 2002). Recently, observations from the ESPaDOnS/NAVAL spectropolarimeter (Donati et al. 2007a)
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2 THE NUMERICAL MODEL AND THE MAGNETIC CONFIGURATIONS

2.1 Model

In a series of previous papers (Koldoba et al. 2002; Romanova et al. 2004; Long, Romanova & Lovelace 2005; Ustyugova et al. 2006; Long et al. 2007), we described the model used in our 3D MHD simulations. The model used in this paper is similar to that of Long et al. (2007) and therefore it is only briefly summarized here.

We consider a rotating magnetized star surrounded by an accretion disc with a low density, high-temperature corona above and below the disc. The disc and the corona are initially in a quasi-equilibrium state. We solve the 3D MHD equations in a reference frame rotating with the star, with the z-axis aligned with the star’s rotation axis. The magnetic field is decomposed into a ‘main’ component \(B_0\) and a variable component \(B_1\). Here, \(B_0\) is time-independent in the rotating reference frame and consists of the dipole and quadrupole parts: \(B_0 = B_d + B_q\). Further, \(B_1\) is, in general, time-dependent due to currents in the simulation region and is calculated from the MHD equations in the rotating reference frame.

A special ‘cubed’ sphere grid with the advantages of both spherical and cartesian coordinate systems was developed (Koldoba et al. 2002). It consists of \(N_c\) concentric spheres, where each sphere represents an inflated cube. Each sphere consists of six sectors corresponding to the six sides of the cube, and an \(N \times N\) grid of curvilinear cartesian coordinates is introduced in each sector. Thus, the whole simulation region consists of six blocks with \(N_c \times N^2\) cells. In the current simulations, we chose a grid resolution of \(75 \times 31^2\). Other resolutions were also investigated for comparison. The coarser grids give satisfactory results for a pure dipole field. However, for a quadrupole field, the code requires a finer grid because of the higher magnetic field gradients. The code is a second order Godunov-type numerical scheme (see e.g. Toro 1999) developed earlier in our group (Koldoba et al. 2002). Viscosity is incorporated into the MHD equations in the interior of the disc so as to control the rate of matter inflow to the star. For the viscosity, we used an \(\alpha\)-prescription with \(\alpha = 0.04\) in all simulation runs, and with smaller/larger values for testing.

(i) Initial conditions. The region considered consists of the star located in the centre of coordinate system, a dense disc located in the equatorial plane and a low-density corona which occupies the rest of the simulation region. Initially, the disc and corona are in rotational hydrodynamic equilibrium. That is, the sum of the gravitational, centrifugal and pressure gradient forces is zero at each point of the simulation region. The initial magnetic field is a combination of dipole and quadrupole field components which are force-free at \(t = 0\). The initial rotational velocity in the disc is close to, but not exactly, Keplerian (i.e. the pressure gradient is taken into account). The corona at different cylindrical radii \(r\) rotates with angular velocities corresponding to the Keplerian velocity of the disc at this distance \(r\). This initial rotation is assumed so as to avoid a strong initial discontinuity of the magnetic field at the boundary between the disc and corona. The distribution of density and pressure in the disc and corona, and the complete description of these initial conditions are given in Romanova et al. (2002) and Ustyugova et al. (2006).

The initial accretion disc extends inwards to an inner radius \(r_d\) and has a temperature \(T_d\) which is much less than the corona temperature \(T_q = 0.01 T_c\). The density of the corona is 100 times less than the density of the disc, \(\rho_c = 0.01 \rho_d\). These values of \(T_c, T_d, \rho_c, \rho_d\) are specified at the disc-corona boundary near the inner radius of the disc.

(ii) Boundary conditions. At the inner boundary \((r = R = R_s)\), where \(R_s\) is the radius of the star, boundary conditions are applied to the density \(\partial \rho / \partial r = 0\), pressure \(\partial p / \partial r = 0\), entropy \(\delta S / \delta r = 0\), velocity \(\partial (\rho - \Omega \times R) / \partial r = 0\) and the magnetic field, \(\partial B_r / \partial r = 0\), \(\partial B_\theta / \partial r = 0\). The \(r\)-component of the magnetic field satisfies \(\partial (r B_r) / \partial r = 0\). The matter flow is frozen to the strong magnetic field so that we have \((\rho - \Omega \times R) \parallel B\). At the outer boundary, free boundary conditions are taken for all variables with the additional condition that matter is not permitted to flow in through the outer...
boundary. The investigated numerical region is large, \( \sim 45 R_\star \), so
that the initial reservoir of matter in the disc is large and sufficient
for the performed simulations. Matter flows slowly inwards from
the external regions of the disc. During the simulation times studied
here, only a small fraction of the total disc matter accretes to the
star.

2.2 Reference units

We solve the MHD equations using dimensionless variables: dis-
tance \( \tilde{R} = R/R_0 \), velocity \( \tilde{v} = v/v_0 \), time \( \tilde{t} = t/t_0 \), etc. The
subscript ‘0’ denotes a set of reference (dimensional) values for
variables, which are chosen as follows: \( R_0 = R_\star /0.35 \), where \( R_\star \)
is the radius of the star; \( v_0 = (GM/R_0)^{1/2} \), time-scale \( t_0 = 2\pi R_0/v_0 \).
Other reference values are: angular velocity \( \Omega_0 = v_0/R_\star \); magnetic
field \( B_0 = B_\text{ref}(R_\star /R_0)^3 \), where \( B_\text{ref} \) is the reference magnetic field
on the surface of the star; dipole magnetic moment \( \mu_0 = B_\text{ref}R_\star^3 \);
quadrupole moment \( D_0 = B_\text{ref}R_\star^3 \); density \( \rho_0 = B_\text{ref}^2/v_0^2 \); pressure \( p_0 =
\rho_0 v_0^2/2 \); mass accretion rate \( \dot{M}_0 = \rho_0 v_0 R_\star^2 \); angular momentum flux
\( L_0 = \rho_0 v_0^3 R_\star^2 \); energy per unit time \( E_0 = \rho_0 v_0^3 R_\star^2 \) (the radiation flux
\( J \) is also in units of \( E_0 \), see Section 3.1); temperature \( T_0 = R_\star /t_0 \), where \( R \)
is the gas constant and the effective blackbody temperature
\( T_{\text{eff,0}} = (\rho_0 v_0^2/\sigma)^{1/4} \), where \( \sigma \) is the Stefan–Boltzmann constant.

In the subsequent sections and figures, we show dimensionless
values for all quantities and drop the tildes (\( \sim \)). To obtain the real
dimensional values of variables, one needs to multiply the dimen-
sionless values by the corresponding reference units. Our dimen-
sionless simulations are applicable to different astrophysical objects
with different scales. For convenience, we list the reference values
for typical CTTSSs, cataclysmic variables and millisecond pulsars in
Table 1.

2.3 Magnetic configurations

(i) Combination of dipole and quadrupole fields. The intrinsic
magnetic field of the star is \( B = -\nabla \phi \), where the scalar potential
of the magnetic field is \( \phi(r) = \Sigma m_i/r - r_i \), \( m_i \) is the magnetic
‘charge’ analogous to the electric charge, and \( r \) and \( r_i \) are the positions
of the observer and the magnetic ‘charges’, respectively. The scalar
potential can be represented as a multipole expansion in pow-
ers of \( 1/r \), and the quadrupole term is
\[
\psi^{(2)} = D_{a\beta} \rho_a \rho_\beta /2r^3,
\]
where \( \rho_a = x_a/r, x_\alpha \) are the components of \( r, D_{a\beta} \) is the magnetic
quadrupole moment tensor, summation over repeated indices is im-
plicated and \( D_m = 0 \). In the axisymmetric case where \( D_{11} = D_{22} =
-D_{33}/2 \), let \( D = D_{33} \) be the value of the quadrupole moment and
refer to the axis of symmetry as the ‘direction’ of the quadrupole
moment, so we have the quadrupole moment \( D \). The combination
of dipole and quadrupole magnetic fields can be written as
\[
B(r) = \left( \frac{\mu}{r^3} - \frac{3D}{4r^4} \right) (\hat{\mathbf{r}} \cdot \hat{\mathbf{r}}) - \frac{1}{2} \left( \frac{\mu}{r^3} \right) \hat{\mathbf{D}} \cdot \hat{\mathbf{D}},
\]
where \( \hat{\mathbf{r}} \) and \( \hat{\mathbf{D}} \) are the unit vectors for the position and the
quadrupole moment, respectively. In general, the dipole and
quadrupole moments \( \mu \) and \( D \) are misaligned relative to the
rotational axis \( \Omega \), at angles \( \Theta \) and \( \Theta_\Omega \), respectively. In addition, they
can also be in different meridional planes with an angle \( \Phi \) between
the \( \Omega - \mu \) and \( \Omega - D \) planes.

The values of \( \mu \) and \( D \) determine the magnetic field strength
on the star’s surface. For example, if \( \mu \) and \( D \) are aligned with the
rotational axis \( \Omega \), and \( \mu = 0.5, D = 0.5 \), then the strength of
the dipole component on the north pole of the star is \( B_{d3} = 0.32 \) kG,
and the quadrupole one is \( B_{q3} = 0.68 \) kG.

(ii) Off-centre magnetic configurations. The magnetic field of a
star may deviate from the centre of the star. Such displacement
naturally follows from the dynamo mechanism of magnetic field
generation. If the dipole and quadrupole moments are displaced by
\( a \) and \( b \) from the centre of the star, we can replace \( r \) with \( (r - a) \)
and \( (r - b) \) in the corresponding terms in equation (1) to obtain the
magnetic field around the star. In this paper, we investigate accretion
to a displaced dipole and to a set of displaced dipoles.

3 ACCRETION TO STARS WITH DIFFERENT
COMPLEX MAGNETIC FIELD
CONFIGURATIONS

In Long et al. (2007), we have shown results of accretion to stars
with dipole plus quadrupole fields with aligned axes. In this section,
we present simulation results of accretion to a star with misaligned
dipole plus quadrupole magnetic fields. First, we consider the
configuration when the dipole magnetic moment \( \mu \) is aligned with the
stellar spin axis \( \Omega \), that is, \( \Theta = 0^\circ \), but the quadrupole magnetic
moment \( D \) is inclined relative to the spin axis at an angle \( \Theta_\Omega = 45^\circ \).
Next, we consider a more general configuration when \( \mu, D \) and
\( \Omega \) are all misaligned relative to each other, and there is an angle \( \Phi \)
between the \( (\Omega - \mu) \) and \( (\Omega - D) \) planes. To ensure that the case
is general enough to represent a possible situation in nature, we choose
\( \Theta = 30^\circ, \Theta_\Omega = 45^\circ \) and \( \Phi = 90^\circ \).

The strengths of the dipole and quadrupole moments are the
same in both cases, \( \mu = D = 0.5 \). Here, we choose a quite strong
quadrupole component to get an example of a complex magnetic
field, so that the disc is disrupted not only by the dipole component
but also by the superposition of the dipole and quadrupole fields.
Fig. 1 shows the distribution of magnetic fields for these two cases
in +z direction. We can see that the quadrupole component is still
comparable to the dipole component at \( r \sim 1 \) where the disc is

\[\begin{array}{ccc}
M_\odot (M_\odot) & 0.8 & 1 \\
R_\odot & 2 R_\odot & 5000 \text{ km} \\
B_0 (G) & 10^3 & 10^6 \\
R_0 (\text{cm}) & 4 \times 10^{11} & 1.4 \times 10^9 \\
v_0 (\text{cm s}^{-1}) & 1.6 \times 10^7 & 3 \times 10^6 \\
\Omega_0 (s^{-1}) & 4 \times 10^{-5} & 0.2 \\
P_0 & 1.5 \times 10^{-3} & 2.2 \text{ ms} \\
B_0 (G) & 43 & 4.3 \times 10^8 \\
\rho_0 (\text{g cm}^{-3}) & 7 \times 10^{-12} & 2 \times 10^{-8} \\
p_0 (\text{g cm}^{-2}) & 1.8 \times 10^4 & 1.8 \times 10^5 \\
M_0 (\text{g s}^{-1}) & 1.8 \times 10^{19} & 1.9 \times 10^{18} \\
(\Omega_M \text{ yr}^{-1}) & 2.8 \times 10^{-7} & 1.9 \times 10^{-7} \\
L_0 (\text{g cm}^2 \text{s}^{-2}) & 1.15 \times 10^{18} & 4.9 \times 10^{16} \\
T_0 (K) & 1.6 \times 10^6 & 3.9 \times 10^{11} \\
E_0 (\text{erg s}^{-1}) & 4.8 \times 10^{33} & 1.2 \times 10^{38} \\
T_{\text{eff,0}} (K) & 4800 & 3.2 \times 10^5 \\
\end{array}\]
stopped by the magnetosphere (see Fig. 5). This means that both dipole and quadrupole contribute to stop the disc.

3.1 Misaligned dipole plus quadrupole configurations

Now we consider the configuration when the $\Omega = -\mu$ and $\Omega = -D$ planes coincide: $\mu = D = 0.5, \Theta = 0^\circ, \Theta_D = 45^\circ$. Fig. 2 shows the strength of the magnetic field on the surface of the star in different projections. One can see that the field is not symmetric and shows three strong magnetic poles and a much weaker magnetic pole with different polarities on the surface, due to the asymmetry of the misaligned dipole plus quadrupole configuration. The left-hand panel shows a strong positive magnetic pole (red) with $B = 67$ (dimensionless value) and a strong negative pole (dark blue) with $B = -41$. The middle panel shows a second strong positive pole (yellow) with $B = 35$, and in fact a very weak and small negative pole in the middle of the surface with $B = -7$ (dark green). We should note that the strength of the magnetic field around this weak pole is more negative than at the pole. The right-hand panel shows both the strong negative pole (dark blue) which is extended and the second strong positive pole (yellow).

Fig. 3 shows a 3D view of the structure of the magnetic field lines. The colour shows the different strengths and polarities of the magnetic field lines, from maximum positive (red) to maximum negative (blue) values. The magnetic field lines are more complex than in the aligned dipole plus quadrupole case. Now we see three sets of loops of closed field lines in the region near the star, connecting the three major poles and the weaker pole on the star, which is different from the one big and one small loops of field lines shown in the aligned dipole plus quadrupole case (Long et al. 2007).

Fig. 4 shows a 3D view of matter flow around the star. The disc matter is disrupted by the complex field and is lifted above the equatorial plane at the magnetospheric radius $r = r_m$, where the magnetic stress balances the matter stress, $\beta = (p + \rho v^2)/B^2/8\pi = 1$. We can see that most of the matter flows between the loops of field lines to the extended negative magnetic pole by choosing the shortest path which is energetically favourable, forming a modified quadrupole ‘belt’. This ‘belt’ has a different shape from that in the case of aligned dipole plus quadrupole configurations (see Long et al. 2007), which looks like a more ‘regular’ sheet perpendicular to the magnetic axis. Here, the ‘belt’ is twisted, although it is approximately perpendicular to the quadrupole axis. The bottom panel shows a slice in the equatorial plane, and we can see that in the $-x$ direction, the matter penetrates between field lines to the surface of the star.

Fig. 5 shows different projections of the accretion flow. One can see that the matter flow is not symmetric in the $xz$ and $xy$ planes. The $xz$ projection shows that in $-x$ direction, the magnetic stress is weak due to the combination of the dipole and tilted quadrupole components, so that the matter is not lifted, and flows almost directly to the surface of the star, and the equatorial plane does not have the magnetospheric gap which is typical for pure dipole cases. Because the quadrupole moment is inclined at $45^\circ$ relative to the $z$-axis in $xz$ plane, in the $+x$ direction, there is one set of closed loops of magnetic field lines, which stops the disc and forms a magnetospheric gap. Comparison of the dipole and quadrupole components in the $+x$ direction shows that in the region where the disc stops ($\beta = 1$), the dipole and quadrupole components are approximately equal, so that the quadrupole has a strong influence on the matter flow, which is clearly seen from the above figures. In the $yz$ projection, the matter flow is relatively symmetric because $\Omega$, $\mu$ and $D$ are all in the $xz$ plane. In the $xy$ projection, we can see that there is a gap in $+x$ direction where there is no direct matter flow, but there is no gap in the $-x$ direction.

Fig. 6 shows the hotspots on the surface of the star at $t = 8$. One can see that there is a ring-like hotspot and a small round hotspot nearby.
There is no clear hotspot near the positive part of the $D$ and $\Omega$ axes, due to the weak funnel stream in the northern hemisphere. The ring-like hotspot comes from accretion to the quadrupole ‘belt’, which is approximately symmetric relative to $D$ because the quadrupole magnetic component $B_q$ is stronger than the dipole component $B_D$ in the region close to the star. A small round spot forms as a result of the connection of field lines between the weak magnetic pole ($B = -7$) and the nearby region where $B$ is positive. Part of the matter flows to the south and forms this round hotspot.

We calculated the light curves from the hotspots on the rotating star’s surface. Assuming the total energy of the inflowing matter is radiated isotropically as blackbody radiation, we obtain the flux of the radiation in a direction $\hat{k}$:

$$J = r^2 F_{\text{obs}} = \int I(R, \hat{k}) \cos \theta \, dS,$$

where $r$ is the distance between the star and the observer, $F_{\text{obs}}$ is the observed flux, $I(R, \hat{k})$ is the specific intensity of the radiation from a position $R$ on the star’s surface into a solid angle element $d\Omega$ in the direction $\hat{k}$, $\theta = \arccos(R \cdot \hat{k})$ and $dS$ is an element of the surface area. The specific intensity can be obtained as

$$I(R, \hat{k}) = \frac{1}{\pi} F_\epsilon(R) \cos \theta,$$

where $F_\epsilon(R)$ is the total energy flux of the inflowing matter. In our simulations, $J$ is the received energy per unit time, and the dimensionless value of $J$ is in units of $\dot{E}_0$ which is shown in Table 1 and discussed in Section 2.2. The hotspots constantly change their shape and location. However, our 3D simulations show that the changes are relatively small. So we choose the hotspots at some moment of time, fix them and rotate the star to obtain the light curves. The bottom panel of Fig. 6 shows the light curves at $t = 8$ for
different inclination angles $i = \arccos (\hat{\Omega} \cdot \hat{k})$. One can see that the light curve is approximately sinusoidal for small inclination angles. This is because the observer can only see a part of the ring-like hotspots at this time and the sinusoidal shape is determined by the rotation of the spots with the star. One can see that the shapes of the light curves at $i = 60^\circ$ and $90^\circ$ are unusual and do not correspond to any pure dipole field case (Romanova et al. 2004). When $i$ increases, the small round hotspot and more of the ring-like hotspots can be observed, and consequently the peak intensity and variability of the light curves are larger.

### 3.2 A more general case: $\mu$, $D$, $\Omega$ all misaligned

Next, we investigate the more general case when the dipole moment $\mu$, quadrupole moment $D$ and spin axis $\Omega$ of the star are all misaligned relative to each other: $\mu = 0.5, D = 0.5, \theta_\mu = 45^\circ, \theta_D = 30^\circ$ and $\phi = 90^\circ$. Fig. 7 shows the distribution of the magnetic field on the surface of the star. We can again see that there are three-dominant magnetic poles with magnetic strengths $B = 65, 37$ and $-44$, respectively. There is another weak magnetic pole in the middle region shown in the middle panel, but it shrinks and partially merges into the boundary of the nearby pole and looks smaller than that in the previous case. Fig. 8 shows 3D plots of matter flow at $t = 8$. In the top panel, one can see that again a significant amount of matter flows through the quadrupole ‘belt’ below the loops of field lines, because the quadrupole component dominates near the star and a single strong stream flows to the region nearby the north pole. So it could be expected that ring-like hotspots and round hotspots form on the surface of the star. The bottom panel shows a slice in the $xy$ plane. One can see that some matter can flow to the star directly without leaving the equatorial plane. We can also see three sets of loops of field lines.

Fig. 9 shows different projections of the matter flow. The matter flow is no longer symmetric in any projection which is expected due to the magnetic configuration. In the $xy$ and $yz$ planes, we can also see that there are three sets of loops of field lines which regulate the shape of the accretion flow. In the northern hemisphere, some matter is lifted from the equatorial plane and flows to the high-latitude area near the dipole axis. This is because the accretion disc is close to one of the major magnetic poles in this direction. In the southern hemisphere, matter penetrates the $\beta$ line between loops of field lines, and forms a modified quadrupole ‘belt’. The magnetosphere is not empty in the $xy$ plane, and matter can go directly to the star in some directions.

Fig. 10 shows the hotspots and the associated light curves. We can see there is one strong arc-like hotspot near the dipole magnetic axis, but not at the magnetic pole. This hotspot comes from the one-armed stream shown in Fig. 9. Another ring-like hotspot is below the equatorial plane but not symmetric with respect to either the $\Omega$, $\mu$ or $D$ axis. The light curve is sinusoidal for the small inclination angle $i = 15^\circ$. At larger $i$ beside the round north spot, part of the ring-like spot becomes visible to the observer, so the light curves depart from the sinusoidal shapes and have a larger amplitude.

### 3.3 Off-centre dipole fields

Until now, all we discussed are the cases in which the dipole and quadrupole magnetic moments are located at the centre of the star. The interior convective circulations may be displaced from the centre and lead to off-centre magnetic configurations. Therefore, we now consider disc accretion to a star with an off-centre pure dipole field, with $\mu = 0.5$. The magnetic moment is located at $x = 0.5R_\star, y = 0, z = 0$, that is, it is shifted from the centre of the star by half radius of the star. The misalignment angle is $\theta = 30^\circ$. Fig. 11 shows the 3D plot of disc accretion for this case. One can see that the north and south magnetic poles are not symmetric on the surface of the star. Due to the shift and the tilt of the dipole moment, the north magnetic pole is closer to the accretion disc than the south pole, so that most of the matter flows to the north magnetic pole. Fig. 12 shows the hotspots on the surface of the star and the corresponding light curves. The light curves are sinusoidal at $i = 15^\circ, 30^\circ$, and have unusual shapes at $i = 60^\circ, 90^\circ$.

In another set of runs, we chose a superposition of several dipoles placed at different places in the star, and with different orientations of their axes. Fig. 13 illustrates the considered configurations of three off-centre dipoles of equal value $\mu = 0.5$, displaced by $0.4R_\star$ from the centre of the star in the equatorial plane and azimuthally separated by $120^\circ$. In configuration (a), all moments are misaligned.
Figure 9. Slices of density distribution for a more general case ($\mu = 0.5, D = 0.5, \theta_1 = 30^\circ, \theta_D = 45^\circ, \Phi = 90^\circ$). The left-hand panel, middle and right-hand panels show the projection in the $xz$, $yz$ and $xy$ planes, respectively. The colour background shows the density distribution varying from $\rho = 0.01$ (blue) to $\rho = 2.1$ (red). The magnetic field lines are shown in red. The yellow lines show where $\beta = 1$.

Figure 10. Hotspots from different angles at $t = 8$ for the case of $\mu = 0.5, D = 0.5, \theta = 45^\circ, \theta_D = 30^\circ$ and $\Phi = 90^\circ$. The left-hand panel, middle and right-hand panels represent edge-on, top and bottom views, respectively. The red colour corresponds to the densest region with maximum density $\rho = 2.3$. The bottom panel shows the light curves for different inclination angles.

Relative to the rotation axis at $\Theta = 30^\circ$, and in configuration (b), they are arranged in the equatorial plane in an anticlockwise manner.

Fig. 14 shows the distribution of the magnetic field on the surface of the star. One can see that there are six poles on the star. In case (a), three positive and three negative poles are in northern and southern hemispheres, respectively. In case (b), all poles are in the equatorial planes and poles with different polarities alternate.

Fig. 15 shows a 3D view of the matter flow for the cases (a) and (b). In case (a), most of the matter flows to the north poles which are closer to the accretion disc due to the inclination of the dipoles. The magnetospheric gap is empty in the equatorial plane. In case (b), all the matter flows between loops of the closed magnetic field lines in the equatorial plane and form some interesting equatorial funnels.

Fig. 16 shows the hotspots for the cases (a) and (b). There is a big triangular hotspot around the north pole for the case (a), which spans the area near the magnetic poles. Another hotspot is present near the south pole but with very low densities. This confirms the asymmetric matter flow shown in Fig. 15. We can also see that there are no polar hotspots for case (b). Only ring-like hotspots are present in the equatorial plane, which corresponds to what we observed in Fig. 15.

4 CONTRIBUTION OF THE QUADRUPOLE TO PROPERTIES OF MAGNETIZED STARS

In this section, we analyse properties of magnetized stars at different ratios between the dipole and the quadrupole fields. First, we consider different configurations at the same maximum value of the field on the surface of the star. Then, we analyse different properties such as the area covered by hotspots, mass accretion rate and spin torque. Next, we fix the dipole component but change the contribution of the quadrupole component with the main goal of understanding how strong the quadrupole should be compared with the dipole in order to change the shape of the hotspots from the pure dipole cases.

4.1 Area covered by hotspots, and torque

In this section, we compare properties of magnetized stars of different configurations under the condition that the maximum magnetic field on the surface of the star is same in all cases.

We choose a number of configurations ranging from purely dipole to purely quadrupole: (a) pure dipole field, with $\mu = 1.07, \theta = 45^\circ$; (b) pure quadrupole field, with $D = 0.5, \theta_D = 0^\circ$; (c) aligned dipole plus quadrupole field, with $\mu = 0.5, D = 0.27, \theta = \theta_D = 0^\circ$ and
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4.1.1 Where does the disc stop?

We calculated the magnetospheric radius \( r_m \) in +x direction in the equatorial plane and obtained the value of \( r_m \) for above four cases: \( r_{m,a} = 1.47, r_{m,b} = 0.62, r_{m,c} = 1.0 \) and \( r_{m,d} = 1.05 \), respectively. We can see that the disc stops closer to the star for the pure quadrupole case than for the pure dipole case, and \( r_m \) has intermediate values for the mixed cases. This result is expected because the quadrupole field decreases faster with distance than the dipole field, so the disc comes closer.

4.1.2 Area covered by hotspots

We also calculated the area covered by hotspots in the above cases (a)–(d). Like in Romanova et al. (2004), we chose some density \( \rho \) and calculated the area \( A(\rho) \) of the region where the density is larger than \( \rho \). Then, we obtain the fraction of the stellar surface covered by the spots, \( f(\rho) = A(\rho)/A_* \).

We also consider the temperature distribution on the surface of the star. In spite of the possible complex processes of radiation from the hotspots, we suggest that the total energy of the incoming stream is radiated approximately as a blackbody. The total energy flux carried by inflowing matter to the point \( R \) on the star is

\[
F_\nu(R) = \rho n \cdot v \left( \frac{1}{2} v^2 + w \right),
\]

where \( n = -\mathbf{\hat{r}} \), \( v \) is the matter velocity, and \( w = \gamma (p/\rho)/(\gamma - 1) \) is the specific enthalpy of the matter. So we have \( F_\nu(R) = \sigma T_{\text{eff}}^4 \) and the effective blackbody temperature:

\[
T_{\text{eff}} = \left[ \frac{\rho n \cdot v}{\sigma} \left( \frac{1}{2} v^2 + w \right) \right]^{1/4},
\]

where \( \sigma \) is the Stefan–Boltzmann constant. Similarly, we obtain the area \( A(T_{\text{eff}}) \) with effective temperature larger than \( T_{\text{eff}} \), and the fraction \( f(T_{\text{eff}}) = A(T_{\text{eff}})/A_* \).

Fig. 18 shows the distributions of \( f(\rho) \) and \( f(T_{\text{eff}}) \) for cases (a)–(b). We can see that the area covered by spots in the pure quadrupole case (b) is several times smaller than that in the dipole case (a). The area covered by spots in the mixed dipole plus quadrupole cases (b and c) is in between. We conclude that if most of the field is determined by the quadrupole, the expected area covered by spots is smaller. We do not know exactly the reason for the smaller fraction \( f \) in the cases of quadrupole field. One of the factors which we should mention is that the mass accretion rate to the surface of the star is also smaller in the case of quadrupole configurations (see Section 4.1.3 and Fig. 19). It seems that it is ‘harder’ for matter to penetrate between the quadrupolar field lines to the poles compared with the dipole case, and matter accretes through a narrow quadrupole ‘belt’ (Song et al. 2007), which leads to smaller accretion rates and smaller hotspots. Fig. 18 also shows that for equal \( f(\rho) \) or \( f(T_{\text{eff}}) \), the density and temperature of the spots in the pure quadrupole case are smaller than in the pure dipole case.

4.1.3 Mass accretion rate and torque

As we mentioned above, matter accretion rate to the surface of the star \( M = -\int dS \cdot \mathbf{\nu}_p \) is several times smaller for the quadrupole case than for the dipole case (\( \nu_p \) is the poloidal component of the velocity). The mixed cases (c) and (d) have intermediate \( M \) (see Fig. 19). We also calculated the angular momentum fluxes transported from the accretion disc and corona to stars for cases (a)–(d). The angular momentum flux carried by the matter
\[ \dot{L}_m = -\int dS \cdot \rho v_r \text{r}_p \] is about 10 times smaller than the angular momentum flux associated with the field \[ \dot{L}_f = \int dS \cdot \text{r}B_{\phi}B_p/(4\pi). \] So we only show \( \dot{L}_f \) in Fig. 19. When matter flows in, the magnetic field lines inflate from the initial configuration. Later, they reconnect, and the transport of angular momentum flux becomes strong. This process is longer in the pure dipole case (a). For corotation radius \( r_{\text{cor}} = 2 \) considered in the above cases, the star spins up mostly through the field lines connecting it to the inner part of the disc. Here, we choose slowly rotating stars (\( r_{\text{cor}} = 2 \)) for all magnetic configurations to make sure that the magnetospheric radius \( r_m \) is smaller than the corotation radius \( r_{\text{cor}} \), and the star spins up in all cases. So we can compare the positive spin torques for different magnetic geometries. Fig. 19 shows \( \dot{M} \) and \( \dot{L}_f \) for cases (a)–(d). One can see that the angular momentum flux is largest in the pure dipole case (a) and smallest in the pure quadrupole case (b). This is an interesting result because in the case of the quadrupole field, the disc comes closer to the star and it rotates faster, so that one would expect larger angular momentum flux in the case of the quadrupole field. But we see the opposite. We can speculate that stars with
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predominantly multipolar fields may not spin-up as fast as stars with a mainly dipole field. This can be explained by the smaller connectivity between closed multipolar field lines and the inner region of the disc (see also Long et al. 2007).

4.2 Hotspot shape

Next, we fix the dipole magnetic field at some relatively high value $\mu = 2$, the misalignment angle at some value $\Theta = 30^\circ$, and gradually increase the quadrupole component, $D = 0.5, 1, 1.5, 2$. In all cases, the quadrupole is aligned with the $\Omega$ axis. The main goal is to understand the ratio $D/\mu$ at which the properties of hotspots will depart from the dipole ones, and the ratio $D/\mu$ at which the quadrupole will strongly influence the shape of hotspots.

Fig. 20 shows that in the pure dipole case (left-hand column), the hotspots have a typical arc-like shape. With the increase of the quadrupole component, the southern hotspot is stretched. When the quadrupole is strong enough, the southern hotspot forms a ring. We conclude that the influence of the quadrupole component on the shape of the hotspots becomes notable when the ratio of the quadrupole and dipole moments $D/\mu > 0.25$, and becomes dominant in determining the hotspot shape when $D/\mu > 0.5$. The corresponding ratios of the magnetic field strengths on the surface of the star are $B_q/B_d = 0.5$ and $B_q/B_d = 1$, respectively.

Fig. 21 shows the light curves for the above cases. It is interesting to note that at $i = 30^\circ$ the amplitude is relatively small for the strong quadrupole case. This can be explained by the fact that part of the southern ring-like hotspot is not visible to the observers in this orientation. When the quadrupole component becomes more important, the intensity and amplitude of the light curves become smaller. The shape of the light curves is more irregular than in the pure dipole configuration. Light curves are complex but may provide information about the magnetic configuration of the star, if the inclination angle is known independently.

![Figure 20. The hotspots for different cases at $t = 8$. The top row shows one side of the star, and the bottom row the other. Each column represents different configurations $D = 0, 0.5, 1.0, 1.5$ and 2.0 from the left- to the right-hand side, respectively.]

![Figure 21. Light curves for inclination angle $i = 30^\circ$ (top panel) and $i = 60^\circ$ (bottom panel) at $t = 8$. The solid, dashed, dash-dotted, dotted and dash-dot-dotted lines represent the configurations $D = 0, 0.5, 1.0, 1.5$ and 2.0, respectively.]

5 SUMMARY AND DISCUSSION

We investigated disc accretion to rotating stars with different complex magnetic fields using full 3D MHD simulations. The main results of this work are the following.

(i) We investigated accretion to a star with a dipole plus quadrupole field for general conditions where the moments of the dipole and quadrupole components are misaligned and also where they are in different meridional planes. We concentrated on the case where both dipole and quadrupole fields are strong so that the total field is complex. In this case, there are three major magnetic poles with different polarities on the surface of the star, and three sets of loops of closed magnetic field lines connecting these poles. The matter flow is not symmetric; it tends to flow to the star along the shortest path to the nearest magnetic pole. Most of the matter flows to the star through a narrow quadrupole ‘belt’ between loops of field lines. Thus, arc- and ring-like hotspots typically form on the star.

(ii) We investigated cases with first one and then a set of off-centre dipoles. In the case of one displaced dipole, the matter flow is not symmetric. More matter flows through one stream than the others, and one hotspot is larger than the others. In the case of three displaced dipoles, there are three positive and three negative poles on the star. Depending on the directions of the dipole moments, matter flows through multiple streams and forms strong hotspots at the north pole or in the equatorial plane.

(iii) The mass accretion rate, the area covered by hotspots and the torque on the star are several times smaller in the quadrupole-dominated case than in the dipole cases (for conditions where the maximum strength of the magnetic field on the star’s surface is the same).

(iv) The quadrupole component has a notable influence on the shape of hotspots if the magnetic field of the quadrupole
component $B_q \gtrsim 0.5 B_d (D/\mu \gtrsim 0.25)$. The shape of hotspots is chiefly determined by the quadrupole field if $B_q \gtrsim B_d (D/\mu \gtrsim 0.5)$.

(v) The light curves from hotspots of rotating stars with complex magnetic fields are often nonisoidal in the case of small inclination angles, $i \lesssim 30^\circ$, but are more complex for $i \gtrsim 60^\circ$. Even for $i \gtrsim 60^\circ$, the light curves often resemble those of misaligned dipoles at large $i$. However, in some cases they are very unusual. We conclude that (i) very unusual, non-isoidal light curves may be sign of the complex fields; (ii) simple sinusoidal light curves do not rule out complex fields and (iii) light curves may be a tool for analysing the complexity of the field, if the inclination angle is determined independently.

Many CTTSs show highly variable light curves. From our simulations, we conclude that hotspots do not change their shape or position significantly and thus may not be responsible for such strong variability. The variability might be explained by some other mechanism, such as a highly variable accretion rate or unstable accretion (Kulkarni & Romanova 2008; Romanova et al. 2008).

Results of 3D MHD simulations of accretion to stars with complex fields can be used to compare models with observations. For example, recently, Donati et al. (2007b) derived the magnetic topology on the surface of CTTS V2129 Oph, and found that it has a dominant octupole component with $B_{oct} \sim 1.2$ kG and a weaker dipole component with $B_d \sim 0.35$ kG. Most of the accretion luminosity is concentrated in a quite large high-latitude spot (5 per cent of the total stellar surface) close to the magnetic pole. We have not modelled accretion to stars with octupole fields, but with different contributions from dipole and quadrupole components. This gives information on how matter flows to the star. Our analysis shows that the quadrupole dominantly determines the matter flow close to the star and the shape of hotspots if $B_q > B_d$. Thus, we would expect that in a star with a dominant octupole, the accretion spot would be determined by one of octupole belts, or part of the belt in the case of tilted octupole fields. So from our point of view, for such a strong octupole field, the single round spot in the high-latitude region is an unexpected feature, which requires additional analysis in the future.

The results of our simulations are also important for analysis of magnetospheric gaps in young stars. In stars with multipole magnetic field, for example, in the case of the dipole plus quadrupole fields with different orientations of the axes, the probability is high that one accretion stream crosses the equatorial plane, thus filling it with matter. This factor may be important in determining the survival of close-in exosolar planets, which have a peak in their spatial distribution at a few stellar radii. In the case of a dipole field, a large low-density magnetospheric gap may be formed in the equatorial plane, which may halt subsequent migration of planets (Lin, Bodenheimer & Richardson 1996; Romanova & Lovelace 2006). However, if a star has a complex magnetic field with several poles, one of streams may flow through the equatorial plane and the magnetospheric gap may be not empty.

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