Constraining $\gamma$ from $K^*\pi$ and $\rho K$ Decays

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Abstract

We show that information on the weak phase $\gamma$ can be extracted from the $K^*\pi$ and $\rho K$ decays. Less hadronic uncertainty is involved when the observables of four of these modes are combined together. We further point out two approximate relations in these decay modes which can help determine whether there are new physics contributions in $\Delta I = 1$ transitions, as hinted in the $K\pi$ modes.

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1 Introduction

$B$ meson decays have been a rich source of useful information on the Cabibbo-Kobayashi-Maskawa (CKM) mechanism in the standard model (SM). It is now an active field to take advantage of recent and future data from $B$-factories to accurately fix the shape of the unitarity triangle of the CKM matrix. In particular, we have been able to determine in recent years one of the angles, $\beta$ (or $\phi_1$), to a high accuracy using mainly $b \rightarrow c \bar{c}s$ decays [1, 2]. It is therefore of great interest to find reliable methods for determining the other angles as well. Several methods have been proposed to determine the angle $\gamma$ (or $\phi_3$) using hadronic $B$ decays, such as the $DK$ modes [3], the $K\pi$ modes [4, 5], the $\eta\pi$ modes [6], the $K^{*\pm}\pi^\mp$ modes [7], and the $\pi\pi$ and $\bar{K}\pi$ modes [8], by combining the data of branching ratios and $CP$ asymmetries. However, recent data suggest possible new physics contributions to the $K\pi$ decays and therefore cast some doubt on the reliability of $\gamma$ thus obtained [9, 10, 11, 12, 13, 14].

With accumulating data it is now possible to consider an alternative method, which employs the $K^{*}\pi$ and $\rho K$ decays to constrain $\gamma$. These $VP$ decay modes are closely related to their $PP$ counterparts, the $K\pi$ modes, where $P$ and $V$ denote respectively pseudoscalar and vector mesons, because of their similar flavor structures. However, they do differ in that the final state mesons in the $VP$ decays contain different polarization components, whereas the $PP$ decays do not. Moreover, the $VP$ decay amplitudes can be divided into two types: those in which the spectator quark goes to the pseudoscalar meson in the final state and those in which the spectator quark goes to the vector meson.

Consequently, the $VP$ decays involve two disjoint sets of contributing topological amplitudes. One connection between these sets of amplitudes is by Lipkin’s assumption which states that the penguin amplitude in the $K^{*}\pi$ modes, $P_P'$, is equal to that in the $\rho K$ modes, $P_V'$, in magnitude but opposite in sign based on a parity argument [15]. This assumption can be used to readily explain the observed $K^{*}\eta$, $K^{*}\eta'$ branching ratios as a result of interference between these two types of penguin amplitudes. It is also verified in global $\chi^2$ fits to observed decay data in the framework of flavor SU(3) symmetry [16].

This paper is organized as follows. We provide the necessary formulas and current data for $B \rightarrow K^{*}\pi$ and $\rho K$ modes in Section 2. We first consider the constraints on $\gamma$ from individual sets of $K^{*}\pi$ and $\rho K$ decays in Sections 3 and 4, respectively. In Section 5, we show that information on $\gamma$ with less hadronic uncertainty can be obtained by combining the four $K^{*}\pi$ and $\rho K$ observables when Lipkin’s assumption is used. We discuss possible improvements in Section 6 and conclude our findings in Section 7.

2 Basics

We collect the latest $CP$-averaged branching ratio and $CP$ asymmetry data of the relevant decays in Table 1. The decay properties can be studied in the topological amplitude formalism [28, 29, 30, 31]. In Table 2, we list the topological amplitude decompositions of the $K^{*}\pi$ and $\rho K$ decays [32] along with the averaged decay strengths and $CP$ asymmetries compiled from Table 1. When computing each invariant decay amplitude from the corresponding branching ratio, we have first used the central values of the $B$ meson lifetimes: $\tau(B^+) = (1.653 \pm 0.014)$ ps and $\tau(B^0) = (1.534 \pm 0.013)$ ps [33].
The branching ratios are quoted in units of $10^{-6}$. The $C_P$ asymmetry of a decay mode, if measured, is quoted in the second line.

| Mode     | BaBar       | Belle       | CLEO        |
|----------|-------------|-------------|-------------|
| $B^+ \rightarrow K^{*0}\pi^+$ | $10.5 \pm 2.0 \pm 1.4 \ [17]$ | $9.83 \pm 0.90^{+1.06}_{-1.24} \ [18]$ | $7.6^{+3.3}_{-3.0} \pm 1.6 \ (< 16) \ [19]$ |
| $K^{*+}\pi^0$ | -           | -           | $7.1^{+11.4}_{-7.1} \pm 1.0 \ (< 31) \ [19]$ |
| $\rho^0 K^+$ | $5.2 \pm 1.2 \pm 0.7 \ [17]$ | $4.78 \pm 0.75^{+1.01}_{-0.97} \ [18]$ | $8.4^{+4.0}_{-3.4} \pm 1.8 \ (< 17) \ [19]$ |
| $\rho^+ K^0$ | -           | -           | $< 48 \ [20]$ |
| $B^0 \rightarrow K^{*+}\pi^-$ | $11.9 \pm 1.7 \pm 1.1 \ [21]$ | $14.8^{+4.0+2.8}_{-4.4-1.3} \ [22]$ | $16^{+6}_{-5} \pm 2 \ [23]$ |
| $K^{*0}\pi^0$ | $3.0 \pm 0.9 \pm 0.5 \ [25]$ | $0.4^{+1.9}_{-1.7} \pm 0.1 \ [22]$ | $0.0^{+1.3+0.5}_{-0.0-0.0} \ (< 3.6) \ [19]$ |
| $\rho^- K^+$ | $8.6 \pm 1.4 \pm 1.0 \ [25]$ | $15.1^{+3.3+2.4}_{-3.3-2.6} \ [22]$ | $16.0^{+7.6}_{-6.4} \pm 2.8 \ (< 32) \ [19]$ |
| $\rho^0 K^0$ | $5.1 \pm 1.0 \pm 1.2 \ [26]$ | $< 12.4 \ [27]$ | $< 39 \ [20]$ |

and the two-body $B$ decay formula

$$\Gamma(B \rightarrow M_1 M_2) = \frac{p_c}{8\pi m_B^2} |A(B \rightarrow M_1 M_2)|^2 ,$$

where $p_c$ is the magnitude of the 3-momentum of the final state meson in the rest frame of $B$, $m_B$ is the $B$ meson mass, and $M_1$ and $M_2$ can be either pseudoscalar or vector mesons.

In view of the much suppressed exchange, annihilation, and color-suppressed EW penguin amplitudes, we only keep the tree ($T'$), color-suppressed tree ($C'$), penguin ($P'$), and color-allowed electroweak (EW) penguin ($P'_{EW}$) amplitudes in the table. In our notation, the primes refer to the amplitudes of $|\Delta S| = 1$ decays while the unprimed ones are reserved for $\Delta S = 0$ decays. The subscript $P$ ($V$) associated with the amplitudes denotes that in the process the spectator quark in the $B$ meson ends up in the pseudoscalar (vector) particle in the final state. All these processes are dominated by penguin amplitudes.

Since the branching ratio of the $\rho^+ K^0$ mode has not been measured, we will take Lipkin’s assumption $P'_V = -P'_P$ to continue the analysis whenever necessary. However, such an assumption can be relaxed or corrected accordingly once the decay is observed.

In light of their simplicity, we first consider the following ratios

$$R(K^*\pi) \equiv \frac{\Gamma(K^{*+}\pi^-)}{\Gamma(K^{*0}\pi^+)} = \left| \frac{P'_P + T'_P}{P'_P} \right|^2 = 1.40 \pm 0.26 ,$$

$$R(\rho K) \equiv \frac{\Gamma(\rho^- K^+)}{\Gamma(\rho^+ K^0)} = \left| \frac{P'_V + T'_V}{P'_V} \right|^2 > 0.22$$

$$\to \frac{\Gamma(\rho^- K^+)}{\Gamma(K^{*0}\pi^+)} = \left| \frac{P'_V + T'_V}{P'_P} \right|^2 = 1.09 \pm 0.24 ,$$

(3)
Table 2: Topological decompositions and averaged experimental data for \(B \rightarrow K^*\pi\) and \(\rho K\) modes. Scale factors are given in the parentheses. Amplitude magnitudes \(|A_{\text{exp}}|\) extracted from experiments are quoted in units of eV. Exchange, annihilation, and color-suppressed EW penguin diagrams are neglected by assuming their dynamical suppression.

| Mode          | Amplitudes                                      | BR \((\times 10^{-6})\) | \(|A_{\text{exp}}|\) | \(A_{CP}\) |
|---------------|-------------------------------------------------|----------------------------|------------------------|------------|
| \(B^+ \rightarrow K^{*0}\pi^+\) | \(P'_P\)                                           | 9.76 ± 1.19               | 32.6 ± 2.0             | -          |
| \(K^{*+}\pi^0\) | \(-\frac{1}{\sqrt{2}}(P'_P + P'_{EW,V} + T'_P + C'_V)\) | < 31                     | < 58.1                 | -          |
| \(\rho^0\ K^+\) | \(-\frac{1}{\sqrt{2}}(P'_V + P'_{EW,P} + T'_V + C'_P)\) | 5.15 ± 0.90              | 23.7 ± 2.1             | -          |
| \(\rho^+\ K^0\) | \(P'_V\)                                          | < 48                     | < 72.3                 | -          |
| \(B^0 \rightarrow K^{*0}\pi^-\) | \(-(P'_P + T'_P)\)                                | 12.7 ± 1.8               | 38.6 ± 2.7             | −0.00 ± 0.12|
| \(K^{*0}\pi^0\) | \(\frac{1}{\sqrt{2}}(P'_P - P'_{EW,V} - C'_V)\)   | 1.69 ± 1.01 \((S = 1.34)\) | 14.1 ± 4.2            | −0.01 ± 0.26|
| \(\rho^-\ K^+\) | \(-(P'_V + T'_V)\)                                | 9.85 ± 1.85 \((S = 1.19)\) | 34.0 ± 3.2            | 0.17 ± 0.15|
| \(\rho^0\ K^0\) | \(\frac{1}{\sqrt{2}}(P'_V - P'_{EW,P} - C'_P)\)   | 5.1 ± 1.6                | 24.5 ± 3.7             | -          |

where \(\Gamma\) refers to the \(CP\)-averaged decay width and “\(\rightarrow\)” in the second line of Eq. \((3)\) indicates where Lipkin’s assumption is used. The numerical values are computed from Table 2. These ratios can be expressed, as will be seen later, in terms of relative strong and weak phases, along with the parameters

\[
\begin{align*}
  r_1 & \equiv \frac{|T'_P|}{|P'_P|} \\
  r_2 & \equiv \frac{|T'_V|}{|P'_V|} 
\end{align*}
\]

(4) (5)

Since factorization works well in the tree amplitudes, \(T'_{P,V}\) can be related to \(T'\) in the \(|\Delta S| = 1\) two-pseudoscalar \(B\) meson decays:

\[
\begin{align*}
  r_P & \equiv \frac{T'_P}{T'} = \frac{f_{K^*}F^B_{\pi}(m_{K^*}^2)}{f_KF^B_{\pi}(m_K^2)} \simeq 1.43 \\
  r_V & \equiv \frac{T'_V}{T'} = \frac{A^{B\rho}(m_K^2)}{F^B_{\pi}(m_K^2)} \simeq 0.85
\end{align*}
\]

(6) (7)

where the numerical results are evaluated using the BSWII form factor model \([34]\), \(f_\pi = 130.7\) MeV, \(f_K = 159.8\) MeV, and \(f_{K^*} = 217\) MeV \([35]\). Moreover, assuming flavor SU(3) symmetry, \(T'\) can be related to \(T\) extracted from the \(\Delta S = 0\) decays by simple CKM factor and decay constant ratio:

\[
\frac{T'}{T} \simeq \frac{\lambda f_K}{1 - \lambda^2/2} f_\pi = 0.284,
\]

(8)

where we have used \(\lambda = 0.2265\) \([33]\). A recent analysis \([36]\) of the semileptonic \(B \rightarrow \pi\ell\nu\) decay yields \(|T| = 24.4^{+3.9}_{-1.2}\) eV. In the current analysis, we take \(|T| = 24.4 \pm 7.6\) (the error taken to be 1.96 times the upper error of the previous number) eV as a conservative estimate to take into account
model dependence in the extraction of Ref. [16], later scaling, and the part of penguin contributions with the same CKM factor as the tree amplitude (although this part is not significant for explaining the data, as shown in Ref. [16]). We thus obtain \( |T'_{P}| = 10.0 \pm 3.1 \text{ eV}, |V'_{r}| = 5.9 \pm 1.9 \) and

\[
\begin{align*}
r_1 & \simeq 0.32 \pm 0.10, \\
r_2 & \simeq 0.19 \pm 0.06, \\
\end{align*}
\]

where \( |P'_{P}| \) is directly extracted from the branching ratio of \( B \to K^{*0}\pi^{+} \) and \( |P'_{r}| \) is temporarily using the same value. For comparison, \( r_1 = 0.37 \pm 0.03 \) and \( r_2 = 0.26 \pm 0.03 \) as determined from a global fit to the \( VP \) modes [16].

### 3 The \( K^{*}\pi \) Modes

Let’s concentrate on the \( K^{*}\pi \) decay modes in this section. Using the parameter \( r_1 \), we have the decay amplitude

\[
\mathcal{A}(K^{*+}\pi^{-}) = -P'_{P} [1 - r_1 e^{i(\delta_P + \gamma)}],
\]

where we fix the phase convention that \( P'_{P} = -|P'_{P}| \) and \( \delta_P \) is the strong phase of \( T'_{P} \) relative to the real axis. It is noted that a small penguin amplitude with the same weak phase as the tree amplitude is absorbed into it. This then gives

\[
\begin{align*}
R(K^{*+}\pi^{-}) &= 1 - 2r_1 \cos \delta_P \cos \gamma + r_1^2, \\
R(K^{*+}\pi^{-}) A_{CP}(K^{*+}\pi^{-}) &= -2r_1 \sin \delta_P \sin \gamma. \\
\end{align*}
\]

Eliminating the \( \delta_P \) dependence in Eqs. (12) and (13) gives

\[
\left( \frac{1 + r_1^2 - R(K^{*}\pi)}{2r_1 \cos \gamma} \right)^2 + \left( \frac{R(K^{*}\pi) A_{CP}(K^{*}\pi)}{2r_1 \sin \gamma} \right)^2 = 1. 
\]

Given fixed values of \( r_1 \) and \( A_{CP}(K^{*}\pi) \), one can obtain a curve that relates \( \gamma \) to \( R(K^{*}\pi) \). It is noted that the above equations are invariant under the transformations \((\gamma, \delta_P) \to (\pi \pm \gamma, \pi \pm \delta_P)\). Thus, there is a four-fold ambiguity in the extraction of \( \gamma \). Here we will restrict ourselves to the solution only in the first quadrant in view of the 39° – 80° range at 95% confidence level extracted from other observables [37]. Since only the absolute value of \( A_{CP}(K^{*}\pi) \) matters in Eq. (14), the allowed \( \gamma-R(K^{*}\pi) \) region shall be bounded in part by the curves corresponding to \( |A_{CP}(K^{*}\pi)|_{\text{min}} = 0 \) and \( |A_{CP}(K^{*}\pi)|_{\text{max}} = 0.13 \).

For illustration purposes, we fix \( r_1 = 0.32 \) in Fig. 1. The solid and dash-dotted curves correspond respectively to \( |A_{CP}(K^{*}\pi)| = 0.13 \) and \( A_{CP}(K^{*}\pi) = 0 \). An upper bound of \( \gamma \lesssim 86^\circ \) is seen in the drawing. The lower bound is determined by the \( |A_{CP}(K^{*}\pi)|_{\text{min}} \) curve and shown to be trivial according to current data. The intersection vertex of the these curves at \( \gamma = 90^\circ \) rests at the lower end of or below the 1σ range of \( R(K^{*}\pi) \) for all possible \( r_1 \) within its 1σ limits of Eq. (9). Therefore, more conservative upper bounds on \( \gamma \) are obtained for larger values of \( r_1 \). However, certain values of \( \gamma \) in the middle range may be disfavored if the \( |A_{CP}(K^{*}\pi)|_{\text{max}} = 0.13 \) curve exceeds the upper boundary.
Figure 1: Relation between measured $R(K^*\pi)$ and the weak phase $\gamma$ for $r_1 = 0.32$. The solid curve corresponds to $|A_{CP}(K^*\pi)| = 0.13$, and the dash-dotted curve corresponds to $A_{CP}(K^*\pi) = 0$. The dashed lines represent the $1\sigma$ range of the observed $R(K^*\pi)$.

of $R(K^*\pi)$. For example, the range $18^\circ - 53^\circ$ is disfavored when $r_1 = 0.42$. If one uses $r_1 = 0.23$ as determined later in Section 5, one obtains an upper bound of $79^\circ$ on $\gamma$.

If $r_1$ is determined to be smaller, the vertex position will drop lower and both of the two intercept points of the $A_{CP}(K^*\pi) = 0$ curve will move toward unity, resulting in a stronger bound on $\gamma$. For example, $\gamma \lesssim 78^\circ$ if one takes the lower limit $r_1 = 0.22$ in Eq. (9).

4 The $\rho K$ Modes

A similar analysis can be done for the $\rho^-K^+$ and $\rho^+K^0$ modes. As mentioned before, the branching ratio of the $\rho^+K^0$ mode is yet to be measured. One can use Lipkin’s assumption instead to evaluate the experimental value of the ratio $R(\rho^-K^+)$, although this is completely unnecessary if the $\rho^+K^0$ decay is seen. In this case, we have the amplitude

$$A(\rho^-K^+) = P'_p \left[ 1 + r_2 e^{i(\delta_V + \gamma)} \right],$$

where $\delta_V$ is the strong phase of $T'_V$ relative to $P'_V$, and

$$R(\rho^-K^+) = 1 + 2r_2 \cos \delta_V \cos \gamma + r_2^2,$$

$$R(\rho^-K^+)A_{CP}(\rho^-K^+) = 2r_2 \sin \delta_V \sin \gamma.$$

One thus obtains the same type of equation as Eq. (14) for the $\rho K$ decays.

We draw in Fig. 2 the curves corresponding to $r_2 = 0.19$. As seen in the drawing, the intersection vertex of the asymmetry curves at $\gamma = 90^\circ$ falls within the $1\sigma$ limits of $R(\rho K)$, which is measured
Figure 2: Relation between measured $R(\rho K)$ and the weak phase $\gamma$ for $r_2 = 0.19$. The solid curve corresponds to $|A_{CP}(\rho K)| = 0.32$, and the dash-dotted curve corresponds to $|A_{CP}(\rho K)| = 0.02$. The dashed lines represent the 1\sigma range of the observed $R(\rho K)$.

to be around unity. No upper bound on $\gamma$ can be obtained in such cases. Since the $CP$ asymmetry is nonzero at 1\sigma level, a lower bound $\gamma \gtrsim 5^\circ$ is given by the $|A_{CP}(\rho K)| = 0.02$ curve. This is complementary to the information one learns from the $K^*\pi$ modes in the previous section, where no lower bound can be obtained due to the observed $CP$ asymmetry being consistent with zero. Varying $r_2$ does not change this lower bound much, and the most conservative bound on $\gamma$ is determined with the largest possible $r_2$. Moreover, decreasing $r_2$ shrinks the $|A_{CP}(\rho K)| = 0.31$ curve toward the vertex at $\gamma = 90^\circ$.

A better determination of $A_{CP}(\rho^-K^+)$ will be able to provide stronger bounds. For example, if the upper limit of $A_{CP}(\rho^-K^+)$ is lowered or $r_2$ turns out to be larger, a middle range of $\gamma$ will be excluded. On the other hand, a stronger lower bound on $\gamma$ can be deduced from a larger lower limit on $A_{CP}(\rho^-K^+)$ or a smaller $r_2$.

5 The $K^*\pi$ and $\rho K$ Modes Combined

Instead of treating $r_1$ and $r_2$ independently, one may relate one to the other by employing the relation $|P'_V| = |P'_p|$: 

$$r_{VP} \equiv \frac{r_2}{r_1} = \frac{|T_V|}{|T'_p|} = \frac{f_K A_{\rho K}^B(m_{K^*}^2)}{f_{K^*} F_{1\pi}(m_{K^*}^2)} \approx 0.6 ,$$

(18) where the same numerical factors are used as before. This can be compared with the result of $0.7\pm0.1$ obtained from a global fit [16]. Therefore, $r_1$ and $r_2$ are related to each other by a simple numerical factor. We will take the former as the independent parameter.

6
There are then four parameters \((\gamma, r_1, \delta_P, \text{and} \delta_V)\) for the four observables in Eqs. \((12), (13), (16), \text{and} (17)\). One can readily find a solution \(\gamma = 42^\circ, r_1 = 0.23, \delta_P = 179^\circ, \text{and} \delta_V = 81^\circ\) from the central values of the observables, where we have again imposed the requirement that \(\gamma\) has to fall within the favored region \((39^\circ - 80^\circ)\) determined by Ref. \([37]\). (The other solutions correspond to the transformations \((\gamma, \delta_P, \delta_V) \rightarrow (\gamma, \pi \pm \delta_P, \delta_V)\) without changing \(r_1\).)

It is seen that the value of \(\gamma\) extracted using this method rests on the lower end of the currently preferred range. The value of \(r_1\) is found to be consistent with our previous estimate. Note that we do not assume any knowledge about the size of \(T'_P\) or \(T'_V\) here and avoid the somewhat far-reaching relation between them and the tree amplitude in the \(\pi\pi\) decays by employing flavor SU(3) symmetry. Varying \(r_{VP}\) between 0.5 and 0.7 does not change the solutions of \(\delta_P\) and \(\delta_V\) much, but \(\gamma\) and \(r_1\) decrease from \(48^\circ\) to \(37^\circ\) and from 0.25 to 0.22, respectively. The strong phase \(\delta_P = 179^\circ\) means that \(T'_P\) lies almost in line with \(P'_P\). This can be readily understood as the result of that the central value of \(A_{CP}(K^+\pi^-)\) is zero and that there is a constructive interference between the two amplitudes.

Unfortunately, the uncertainties on the current data are still too large (in contrast to the \(K\pi\) case) to obtain a restricted 1\(\sigma\) range for the weak phase \(\gamma\). If the data precision can be improved by, for example, a factor of two, a 1\(\sigma\) range of \(24^\circ - 50^\circ\) can be obtained assuming the same central values as the present ones.

It should be emphasized that the assumption \(|P'_P| = |P'_V|\) can be relaxed once \(B(\rho^+K^0)\) is measured. As seen in Eq. \((18)\), what is essential is the factorization of the tree amplitudes. In that case, \(r_{VP}\) should be further scaled by the factor \(|P'_P|/|P'_V|\). Note that the information on the relative strong phase between \(P'_P\) and \(P'_V\) is not crucial here.

### 6 Other Modes

Let’s now turn to the following quantities derived from a larger set of decay modes:

\[
R_c(K^*\pi) \equiv \frac{2\Gamma(K^+\pi^0)}{\Gamma(K^0\pi^0)} = \left| \frac{P'_P + P'_{EW,V} + T'_P + C'_V}{P'_P} \right|^2 < 6.35, \\
R_n(K^*\pi) \equiv \frac{\Gamma(K^{*+}\pi^-)}{2\Gamma(K^{*0}\pi^0)} = \left| \frac{P'_P + T'_P}{P'_P - P'_{EW,V} - C'_V} \right|^2 = 3.77 \pm 2.32, \\
R_c(\rho K) \equiv \frac{2\Gamma(\rho^0K^+)}{\Gamma(\rho^+K^0)} = \left| \frac{P'_V + P'_{EW,P} + T'_V + C'_P}{P'_V} \right|^2 > 0.21, \\
\rightarrow \frac{2\Gamma(\rho^0K^+)}{\Gamma(\rho^0K^0)} = \left| \frac{P'_V + P'_{EW,P} + T'_V + C'_P}{P'_V} \right|^2 = 1.06 \pm 0.23, \\
R_n(\rho K) \equiv \frac{\Gamma(\rho^-K^+)}{2\Gamma(\rho^0K^0)} = \left| \frac{P'_V + T'_V}{P'_V - P'_{EW,P} - C'_P} \right|^2 = 0.97 \pm 0.35.
\]

Again, “\(\rightarrow\)” in Eq. \((21)\) indicates the use of Lipkin’s assumption. In principle, one can also obtain information on \(\gamma\) from the combination of \(R_c(K^*\pi)\) and \(A_{CP}(K^{*+}\pi^0)\) and the combination of \(R_n(\rho K)\) and \(A_{CP}(\rho^0K^+)\). Currently, we are still lacking in data of the \(CP\) asymmetries and two branching
ratios. Moreover, they require the determination of the corresponding ratios

\[
\begin{align*}
    r_{c1} & \equiv \frac{T'_V + C'_V + P'_{EW,V}}{P'_P}, \\
    r_{c2} & \equiv \frac{T'_V + C'_P + P'_{EW,P}}{P'_P}.
\end{align*}
\]  

(23)  

(24)

This is more involved than the case of $K\pi$ because although $T' + C' + P'_{EW}$ can be deduced from $\pi^+\pi^0$ using SU(3), the $VP$ counterparts $\rho^+\pi^0$ and $\rho^0\pi^+$ contains additional contributions from the penguin amplitudes $P'_P$ and $P'_V$ that interfere with each other constructively. Extra assumptions need to be imposed in order to extract the required information [16].

Recently, it is pointed out that current experimental data indicate some discrepancy between $R_c(K\pi) = 2\Gamma(K^+\pi^0)/\Gamma(K^0\pi^+)$ and $R_n(K\pi) = \Gamma(K^+\pi^-)/2\Gamma(K^0\pi^0)$ that should be equal to each other at the leading-order expansion. Such a discrepancy can be resulted from two possibilities: either the $\pi^0$ detection efficiency in experiments is systematically underestimated, or it calls for contributions of $\Delta I = 1$ amplitudes beyond the SM [9, 10, 11, 12, 13, 14]. The $R_c-R_n$ comparison in the cases of $K^*\pi$ and $\rho K$ decays is somewhat analogous. However, a direct experimental comparison is not yet available because of insufficient data in the $K^{*\pm}\pi^0$ and $\rho^+ K^0$ modes. Employing the Lipkin relation again for the $\rho^+ K^0$ mode, the current data show an approximate agreement between Eq. (21) and Eq. (22).

If the discrepancy between $R_c$ and $R_n$ in the $K\pi$ system is due to the underestimated $\pi^0$ detection efficiency, we expect a similar pattern in the $K^*\pi$ system but not in the $\rho K$ modes. This seems to be partly favored by the rough agreement between Eq. (21) and Eq. (22), although a mild assumption is used here and the $K^*\pi$ counterparts still await to be seen. Suppose the interpretation of new physics, which is short-distance in nature, is correct for the $K\pi$ modes, then one should expect to see its effects in the $VP$ (both $K^*\pi$ and $\rho K$) and $VV$ ($K^*\rho$) modes too. However, a detailed analysis is required because the latter modes involve different polarizations.

Finally, we would like to comment on possible branching ratios of the $K^{*\pm}\pi^0$ and $\rho^+ K^0$ decays by noting two approximate sum rules:

\[
\begin{align*}
    \Gamma(K^{*0}\pi^+) + \Gamma(K^{*+}\pi^-) & \approx 2 \left[ \Gamma(K^{*+}\pi^0) + \Gamma(K^{*0}\pi^0) \right], \\
    \Gamma(\rho^+ K^0) + \Gamma(\rho^- K^0) & \approx 2 \left[ \Gamma(\rho^0 K^+) + \Gamma(\rho^0 K^0) \right].
\end{align*}
\]  

(25)  

(26)

They hold only when the terms $|C'_{V(P)} + P'_{EW,V(P)}|^2 + 2\text{Re}[T^\ast_{P(V)}(C'_{V(P)} + P'_{EW,V(P)})]$ are negligible in comparison with the dominant penguin contributions. It is noticed from a global fit to $VP$ data [16] that the contributions due to $C'_V$ and $P'_{EW,V}$ are quite sizeable, thus raising doubt on the sum rule (25) while those due to $C'_P$ and $P'_{EW,P}$ are less significant. Assuming these sum rules, one can deduce from current data that $\mathcal{B}(K^{*+}\pi^0) = (9.9 \pm 1.6) \times 10^{-6}$ and $\mathcal{B}(\rho^+ K^0) = (10.7 \pm 4.3) \times 10^{-6}$. These numbers are consistent with the current upper bounds. In particular, $\mathcal{B}(\rho^+ K^0)$ thus obtained and the measured $\mathcal{B}(K^{*0}\pi^+)$ are about the same, which is another indication of the equality $|P'_P| = |P'_V|$. At any rate, a precise determination of the rates of $K^{*+}\pi^0$ and $\rho^+ K^0$ decays will be very helpful in checking the $R_c-R_n$ relations and the above sum rules.
7 Conclusions

We have shown that the branching ratios and $CP$ asymmetries of the $K^*\pi$ and $\rho K$ modes can help us constraining the weak phase $\gamma$. In particular, we emphasize the uses of the $K^{*0}\pi^+, K^{*+}\pi^-, \rho^+K^0$, and $\rho^-K^+$ decays. With the choice of $r_1 = 0.32$ and $r_2 = 0.19$, we show that the existing data give us the bounds $\gamma \lesssim 86^\circ$ from the $K^*\pi$ modes and $\gamma \gtrsim 5^\circ$ from the $\rho K$ modes. Although currently very loose, these bounds are expected to improve with higher data precision in the coming years.

By relating the two types of tree amplitudes, $T_P'$ and $T_V'$, in the $VP$ decays, one can reduce the number of parameters in the problem and solve for $\gamma$. This method is free from SU(3) breaking uncertainties. It is found that the solution of $\gamma$ thus obtained is consistent with those obtained from other observables. However, a better constraint on $\gamma$ from these modes is not possible until the data precision is further improved. The solution also tells us the tree-to-penguin ratio, which we find to agree with that estimated using flavor symmetry.

We also point out that it is interesting to check whether $R_c = R_n$ for both $K^*\pi$ and $\rho K$ decays. As shown above, the equality roughly holds for the $\rho K$ modes when Lipkin’s assumption is used. If the equality turns out to hold in the case of $\rho K$ but to be violated for $K^*\pi$ with a similarly puzzling difference as the $K\pi$ decays, then it is likely that the underestimated $\pi^0$ detection efficiency explanation is favored. On the other hand, if the data tell us that both equalities are violated in these $VP$ modes, then the new physics explanation is more plausible. However, one then has to work out the new physics contributions to different polarization components. We therefore stress the importance of measuring the yet unseen modes in order for us to validate the use of the relation $P'_V = -P'_P$ and to see any further hints on new physics.

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