Numerical Analysis of Minimum Ground Temperature for Heat Extraction in Horizontal Ground Heat Exchangers

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Abstract: In this work, numerical simulation calculations were performed to investigate the minimum ground temperature that occurs when extracting thermal energy in a horizontal ground heat exchanger system in the Central European climate. The influence of ground thermal conductivity, heat flux extracted from the ground, periodic interruptions in the operation of the heat exchanger, periodic supply of heat energy to the ground, relative humidity of the ambient air, evaporation rate coefficient, and convective heat transfer coefficient on the ground minimum temperature were investigated. Based on the simulation, it was found that the high value of ground thermal conductivity favorably affects the operation of the installation with a ground heat exchanger. Both the reduction of the maximum heat flux taken from the ground, as well as periodic interruptions in the operation of the exchanger effectively protects the ground against excessive cooling. Further, it was found that heat supply to the ground in summer only slightly raises its minimum temperature, as well as the decrease of the relative humidity of the ambient air and evaporation rate coefficient. The change of the convective heat transfer coefficient has no significant impact on the minimum annual ground temperature.

Keywords: ground temperature; heat extraction; ground heat exchanger; heat pump

1. Introduction

Conventional methods of energy acquisition (mainly based on burning of fossil fuels) cause huge environmental problems and issues associated with the emission of harmful substances and greenhouse gases into the atmosphere. However, awareness of ecology and environmental protection increases every year, which contributes to the search for new and better solutions in the field of energy and fuels. Heat energy can be obtained from the ground, where it is accumulated, in a renewable manner.

Systems with heat pumps deserve special attention because they use the ground as a heat source, which is a solution friendly for the environment. They generate low power consumption due to the comparatively low operating costs in relation to conventional methods of energy acquisition.

The economic benefits of installing a heat pump coupled with a ground heat exchanger (GHE) are high enough so that such solutions are becoming more common both in private use and industrial use as well. In view of the manner of pipe arrangement, there are two main types of ground heat exchangers: horizontal and vertical ones. Horizontal exchangers are cheaper in terms of operation and assembly, which is why they are more often used [1].

Ground heat exchangers are the subject of numerous publications. The research based on theoretical relationships was the topic of the works by Pan et al. [2], Zhao et al. [3], Cai et al. [4], and Li et al. [5]. The results of experimental research are presented in the works by Cai et al. [4], Widiatmojo et al. [6], Li et al. [7], Kapıcıoğlu and Essen [8]. Numerous works relate to calculations based on numerical applications, these are, among others,
works by Garoosi et al. [9], Rashidi et al. [10], Li et al. [7], Li et al. [11], Monzó et al. [12] and Gan [13].

There are also many works concerning the temperature of the ground. The natural ground temperature (undisturbed by the presence of working GHE) was investigated, among others, by Krarti et al. [14], Khatry et al. [15], Givoni and Katz [16], and Michalakakou [17]. However, the presence of a working ground heat exchanger strongly affects the ground temperature in the vicinity of the heat exchanger’s pipes and this temperature is crucial from the point of view of extraction or supply of heat from/to the ground with the use of a GHE. The subject of ground temperature near the ground heat exchanger was raised in many publications. Zhang et al. [18] proposed a quasi-three-dimensional heat transfer model for the prediction of ground temperature near the pipes of the heat exchanger. Larwa and Kupiec [19] investigated the long-term influence of a horizontal GHE operation on the ground temperature.

When planning the construction and operation of a ground heat exchanger, it is necessary to analyze the physical properties of the ground, temperature profiles in it at various times of the year, and the impact of heat extraction or supply on changes in its temperature. It is crucial to apply mathematical modeling for this purpose. It allows one, among others, to specify the time needed for thermal regeneration of the ground after the heating season. Despite the existence of natural thermal regeneration in the summer season, too intensive exploitation of a heat exchanger during the heating season can cause excessive cooling of the ground. This phenomenon is disadvantageous from the point of view of heat extraction when using a ground exchanger, not only because of the effect of increasing the temperature difference between the heat sources but also due to the possibility of deterioration of the contact of the ground with the exchanger pipes caused by freezing of the ground near the GHE. Kayaci and Demir [20] investigated, among others, the decrease of ground temperature around the exchanger pipe after the 10-year heating period. Yang et al. [21] introduced a two-dimensional heat transfer model taking into account ground freezing.

When extracting heat from the ground, its temperature in the vicinity of the pipe surface decreases reaching a minimum value at a certain time of the year. This minimum temperature can reach different values depending on, among others, the intensity of heat extraction from the ground.

This work aims to carry out calculations to determine the minimum temperatures that can occur in the ground, where a ground heat exchanger coupled with a heat pump and with an installation for space heating is installed. In addition, it was investigated which parameters most strongly affect the value of the minimum ground temperature. The calculations use the ground heat transfer model described in [19].

2. Mathematical Model

The work [19] presents a one-dimensional mathematical model in which it was assumed that the ground is a semi-infinite body whose physical properties are homogeneous and immutable with temperature. The heat source placed in the ground (corresponding to ground heat exchanger piping) is flat and located at a depth $h_{GHE}$. Additionally, the only heat transport mechanism in the ground is thermal conduction.

Despite simplifications, the model seems to be a satisfying approximation when it is used for estimation of a long-term process in a semi-infinite body where the heat source is placed in a plane parallel to the surface, especially if the packing of the exchanger pipes is dense (for instance in a slinky configuration) [19].

2.1. Sol-Air Evaporation Temperature

Heat flux transferred between the surrounding and the surface of the ground which is also a heat flux conducted at the surface of the ground can be expressed as [22]:

$$q_{\text{cond}} = h_p e (T_{sae} - T_s)$$

(1)
Sol-air-evaporation temperature $T_{sae}$ is a fictitious air temperature which, when inserted into Formula (1), allows you to calculate the current heat flux transferred by various mechanisms between the environment and the ground surface. The balance of heat fluxes on the ground shows that:

$$T_{sae} = T_a + \frac{S - \varepsilon LW}{h_p} - \frac{C_{EV} f b (1 - RH)}{p_e} \tag{2}$$

Qualities $T_a$, $T_s$ and $S$ are time-dependent according to the following formulas:

$$T_a = T_{am} - A_a \cos(\omega t - P_a) \tag{3}$$

$$T_s = T_{sm} - A_s \cos(\omega t - P_s) \tag{4}$$

$$S = S_m - A_{sol} \cos(\omega t - P_{sol}) \tag{5}$$

Parameters $p_e$ and $p_r$ are defined by the relationships [14]:

$$p_e = 1 + C_{EV} f a \tag{6}$$

$$p_r = 1 + C_{EV} f a RH \tag{7}$$

Constants $C_{EV}$, $a$ and $b$ are equal to 0.0168 K/Pa, 103 Pa/K and 609 Pa, respectively.

2.2. Heat Flux Transferred in the Exchanger

The heat flux transferred in the exchanger is directly related to the thermal energy needed for heating and cooling the space. It is therefore strongly variable throughout the year. There are various combinations of quantities related to supplied and/or extracted of heat to/from the ground during the year. Time dependence of the heat flux transferred in the heat exchanger can be described by the following formula, which can be considered as an extended equation used by Lazzari et al. [23]:

$$q_{GHE} = q_{max} \left[ \left(1 - \frac{1}{2} |\kappa| \right) \cos(\omega t - P_a) - \frac{\kappa}{2} \cos(\omega t - P_a) \right] \tag{8}$$

In Equation (8), the parameter $\kappa$ characterizes the relationship between the amount of heat extracted and supplied from/to the ground. The definition of the $\kappa$ parameter is as follows:

$$\kappa = \begin{cases} 
(Q_c - Q_h) / Q_h & \text{for } Q_h > Q_c \\
(Q_c - Q_h) / Q_c & \text{for } Q_c > Q_h \\
0 & \text{for } Q_h = Q_c 
\end{cases} \tag{9}$$

When heat is only extracted from the ground ($Q_c = 0$), then $\kappa = -1$, when heat is only supplied to the ground ($Q_h = 0$), then $\kappa = 1$, whereas when the amount of heat extracted in winter and transferred in summer are equal ($Q_c = Q_h$), then $\kappa = 0$. Intermediate cases are also possible, e.g., for $Q_h = 2Q_c$ there is $\kappa = -0.5$.

Figure 1 presents three cases of time series of heat transferred in a ground heat exchanger:

- During the heating period, a heat flux whose maximum value is $q_{max} = 15 \text{ W/m}^2$ is extracted from the ground and during summer the heat is not supplied to the ground: $\kappa = -1$;
- During the heating period, a heat flux whose maximum value is $q_{max} = 10 \text{ W/m}^2$ is extracted from the ground and during summer the heat is not supplied to the ground: $\kappa = -1$;
- During the heating period, a heat flux whose maximum value is $q_{max} = 15 \text{ W/m}^2$ is extracted from the ground and during summer the heat is supplied to the ground in an amount corresponding to $\kappa = -0.5$. 

2.3. One-Dimensional Model of Heat Conduction in the Ground

The heat transport equation in the ground can be written as:

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} + \frac{q_v}{c_v} \]  

(10)

In this paper, Equation (10) is treated as a partial differential equation (PDE). It has been solved numerically with the use of the finite difference method and the Crank-Nicolson scheme. The applied boundary conditions are presented in the further part of this subsection.

The parameter \( q_v \) in Formula (10) expresses the rate of heat generation at the depth of the exchanger’s location related to the volume. This value is associated with the heat flux transferred in the exchanger. Simulation calculations were carried out numerically using the finite difference method in \( N \) points spaced from each other by \( \Delta x = h_{inf} / N \), so:

\[ q_v = \begin{cases} 
0 & \text{for } 0 < i < n \\
-\frac{q_{GHE}}{\Delta x} & \text{for } i = n \\
0 & \text{for } n < i < N 
\end{cases} 
\]

(11)

For heat extraction from the ground, \( q_{GHE} \) is positive, while \( q_v \) is negative.

On the ground surface, the boundary condition is:

\[ x = 0; \quad \frac{\partial T}{\partial x} = -\frac{h_p}{k} (T_{sae} - T_s) \]  

(12)

In the above boundary condition (12) \( T_{sae} \) is calculated from Equation (2) and \( T_s \) from Equation (4).

At a large depth, the temperature gradient disappears and therefore the boundary condition has the form (assuming that the heat flux associated with geothermal energy is negligibly small):

\[ x = h_{inf}; \quad \frac{\partial T}{\partial x} = 0 \]  

(13)

3. Results and Discussion

Simulation calculations were carried out for conditions corresponding to the Central European climate (Cracow, Poland). The values of individual qualities used in simulation calculations are presented in the Table 1.
Table 1. Summary of the values of quantities present in the model and used in simulation calculations.

| Quantity                                               | Quantity Symbol | Value                |
|--------------------------------------------------------|-----------------|----------------------|
| Annually averaged ambient temperature                  | $T_{am}$        | 8.3 °C               |
| Amplitude of ambient temperature                       | $A_a$           | 10.6 K               |
| Phase angle of ambient                                 | $P_a$           | 0.27 rad             |
| Annually averaged solar radiation flux absorbed by the ground | $S_m$           | 119 W/m²             |
| Amplitude of solar heat flux                           | $A_{sol}$       | 101 W/m²             |
| Phase angle of solar radiation                         | $P_{sol}$       | −0.153 rad           |
| Relative humidity of ambient air                       | $RH$            | 0.79                 |
| Long-wave radiation heat flux                           | $LW$            | 63 W/m²              |
| Evaporation rate coefficient                           | $f$             | 0.3                  |
| Distance between the heat exchanger and the surface of the ground | $h_{GHE}$       | 1.5 m                |
| Volumetric heat capacity of the ground                 | $c_v$           | $2.5 \times 10^6$ J/(m³K) |
| Convective heat transfer coefficient                    | $h$             | 15 W/(m²K)           |

3.1. Ground Temperature Profiles

Natural temperature profiles in the ground, i.e., when there is no ground heat exchanger (or when the heat exchanger is installed but not operating) are presented in Figure 2. The curves correspond to the following months of the year. From the graph, it can be seen that the ground temperature is fixed at the depth of about 14 m at a value of 9.7 °C. Furthermore, the profile is symmetrical relative to natural ground temperature values and on the surface temperature ranges from about −4 °C to about 23 °C.

![Figure 2. Natural ground temperature profiles for individual months of the year; $k = 1.4$ W/(mK).](image)

The profiles of the temperature in the ground, from which the heat is extracted with a maximum heat flux of 15 W/m² are shown in Figure 3. The graph shows the deformation of the profile toward lower temperatures compared to the profiles for the natural conditions (Figure 2). The temperature is fixed at the depth of about 25 m, i.e., well below the value for natural conditions. A significant reduction in the temperature of the ground in comparison with the natural conditions is due to heat extraction from the ground in the heat exchanger, uncompensated entirely by the heat transfer from the environment.
The temperature profiles in the ground in the day, when the ground reaches a minimum temperature (February 15th) are depicted in Figure 4. Four cases are presented:

- During the heating period, a heat flux whose maximum value is $q_{\text{max}} = 15 \text{ W/m}^2$ is extracted from the ground and during summer the heat is not supplied to the ground: $\kappa = -1$;
- During the heating period, a heat flux whose maximum value is $q_{\text{max}} = 10 \text{ W/m}^2$ is extracted from the ground and during summer the heat is not supplied to the ground: $\kappa = -1$;
- During the heating period, a heat flux whose maximum value is $q_{\text{max}} = 15 \text{ W/m}^2$ is extracted from the ground, and during summer the heat is supplied to the ground in an amount corresponding to $\kappa = -0.5$;
- No heat exchanger is installed in the ground.

In case of heat extraction from the ground in conditions $q_{\text{max}} = 15 \text{ W/m}^2$ and $\kappa = -1$ minimum temperature of the ground at the depth of 1.5 m is $-7.7 \, ^\circ\text{C}$. If, with the same heat extraction intensity during the heating season, heat supply to the ground corresponding to $\kappa = -0.5$ is used during summer, then the minimum ground temperature is $-6.9 \, ^\circ\text{C}$. Therefore, the influence of heat supply during the summer season affects marginal minimum temperature rise. The minimum ground temperature at the heat exchanger installation depth for conditions: $q_{\text{max}} = 10 \text{ W/m}^2$, $\kappa = -1$ is higher than for conditions: $q_{\text{max}} = 15 \text{ W/m}^2$, $\kappa = -0.5$. In the first case it is $-4.2 \, ^\circ\text{C}$ and in the second it is...
−6.9 °C. It follows that in order to avoid excessively low ground temperatures, it is better to reduce the maximum heat extracted from the ground than to increase the amount of heat supplied to the ground out of the heating season.

3.2. Time Series of Ground Temperature

The temporary changes in ground temperature at the depth of heat exchanger installation (1.5 m) for conditions analogous to those considered in Figure 4 are shown in Figure 5.

The results of the simulation show that in conditions where heat is only extracted from the ground (κ = −1), for q_{max} = 15 W/m² minimum ground temperature is −7.7 °C, and for q_{max} = 10 W/m² the minimum temperature is −4.2 °C. The difference between these minimum temperatures equal to 3.5 K confirms the significant impact of heat extraction intensity on the ground temperatures in the immediate vicinity of the heat exchanger.

The influence of the ground thermal conductivity on its temperature at the depth of the heat exchanger’s installation is presented in Figure 6. Calculations were carried out for k = 1.4 W/(mK) and k = 2.0 W/(mK). The first value of the thermal conductivity corresponds approximately to a ground with a moisture content of 13% [24], loam and loamy clay (EN 12524). The k value equal to 2 W/(mK) corresponds approximately to the sand, gravel (EN 12524), and sandstone (VDI, 1974). The graph shows two cases: natural ground temperatures (q_{max} = 0 W/m²) and ground temperatures that occur if the exchanger extracts heat with a maximum heat flux q_{max} = 10 W/m². In the first case, the minimum ground temperature is lower for a higher value of thermal conductivity k = 2.0 W/(mK). However, when heat is extracted from the ground, then the lower minimum ground temperature applies to the lower value of thermal conductivity k = 1.4 W/(mK). Due to the practical importance of the latter case, the viewpoint of prevention of excessively low temperatures of the ground it is preferred to install the heat exchanger in the ground with higher thermal conductivity.
Influence of ground thermal conductivity on the ground temperature at the depth of heat exchanger’s installation for various conditions of heat extraction from the ground.

In addition, the influence of the relative humidity of ambient air, RH, evaporation rate coefficient, f, and convective heat transfer coefficient, h, on the minimum ground temperature in the vicinity of its pipes were investigated. For the part of calculations concerning the abovementioned parameters, the values of quantities presented in Table 2 were used.

Table 2. Summary of the values of quantities present in the model and used in simulation calculations concerning the influence of RH, f and h on the minimum ground temperature.

| Quantity                                               | Quantity Symbol | Value                          |
|--------------------------------------------------------|-----------------|--------------------------------|
| Annually averaged ambient temperature                  | T_{am}          | 8.3 °C                         |
| Amplitude of ambient temperature                        | A_{a}           | 10.6 K                         |
| Phase angle of ambient                                 | P_{a}           | 0.27 rad                       |
| Annually averaged solar radiation flux absorbed by the ground | S_{m}          | 119 W/m²                       |
| Amplitude of solar heat flux                           | A_{sol}         | 101 W/m²                       |
| Phase angle of solar radiation                         | P_{sol}         | −0.153 rad                     |
| Ground thermal conductivity                            | k               | 1.4 W/(mK)                     |
| Long-wave radiation heat flux                          | LW              | 63 W/m²                        |
| Parameter characterizing the relationship between the amount of heat extracted and supplied from/to the ground | \kappa          | −1                             |
| Distance between the heat exchanger and the surface of the ground | h_{GHE}        | 1.5 m                          |
| Volumetric heat capacity of the ground                 | c_v             | 2.5·10^6 J/(m³K)               |
| Maximum heat flux transferred in the ground heat exchanger | q_{max}        | 15 W/m²                        |

The influence of the relative humidity of ambient air on the ground temperature at the depth of the heat exchanger’s installation is shown in Figure 7. As presented in the work by Wypych and Piotrowicz [25], the mean relative air humidity in Cracow, Poland, varied in the range of about 0.70–0.85, which is typical for Central European climate, hence, the values from this interval were considered. It can be seen, that, in general, the changes in values of the relative humidity of ambient air insignificantly influence the ground temperature. Its minimum annual temperature reaches −8.1 °C for the case of relative air humidity equal to 0.85 and it is only 0.5 °C higher for relative air humidity equal to 0.70. Therefore, it can be said, that it is better to extract heat in a ground heat exchanger coupled with a heat pump when the relative humidity of ambient air is lower, however, it does not make a significant difference.
Figure 7. Influence of the relative humidity of ambient air on the ground temperature at the depth of the heat exchanger’s installation.

Figure 8 shows the influence of the evaporation rate coefficient on the ground temperature. This parameter varies in the range of 0.1–0.5 and it depends on the ground humidity, which in turn is the function of the amount of precipitation and groundwater permeability [14,26]. Minimum ground temperatures differing by 0.8 °C were achieved for the adopted limit values of the evaporation rate coefficient. The highest value of the minimum annual ground temperature was −7.4 °C for the evaporation rate coefficient equal to 0.1. In the case of the evaporation rate coefficient equal to 0.5, the minimum annual ground temperature reaches −8.2 °C. Therefore, preferably, the ground should have a low evaporation rate coefficient from the point of view of the protection against excessive cooling, however, the difference is not significant.

Figure 8. Influence of the evaporation rate coefficient on the ground temperature at the depth of the heat exchanger’s installation.

The dependence of the ground temperature as a function of the convective heat transfer coefficient is presented in Figure 9. The convective heat transfer coefficient is uneasy to calculate, there are also few experimental works concerning this parameter in the field of heat transfer between ground and air. In practice, it is often calculated with the use of empirical correlations that take into account wind velocities, such as the McAdams formula [27] and the formula presented by Allen et al. [28], which was used in this paper. The average annual wind velocity in the Central European climate often takes values
close to 2–3 m/s. The values of the convective heat transfer coefficient in the range of 12–18 W/(m²K) correspond to the abovementioned values of the wind velocity. Hence, this range of the convective heat transfer coefficient was considered. For the value of the convective heat transfer coefficient equal to 12 W/(m²K), the temperature reached −7.2 °C, while for the value of the convective heat transfer coefficient of 18 W/(m²K), the ground temperature was −7.3 °C. Thus, it can be claimed, that the change of the convective heat transfer coefficient has no significant impact on the minimum annual ground temperature.

![Figure 9. Influence of the convective heat transfer coefficient on the ground temperature at the depth of the heat exchanger’s installation.](image)

3.3. Investigation of the Impact of Interruptions in Exchanger Operation

Simulations of ground temperature changes in case of periodic cessation of heat withdrawal from it were also carried out. Figure 10 shows time changes of ground temperature for two modes of exchanger operation:

- The cycle: 1 year of heat extraction, then 1 year of interruption;
- No annual interruptions in heat extraction from the ground.

![Figure 10. Time series of ground temperature at the depth of heat exchanger’s installation at different operation modes: $k = 1.4 \text{ W/(mK)}$, $q_{\text{max}} = 10 \text{ W/m}^2$, $\kappa = -1$, $RH = 0.79$, $f = 0.3$, $h = 15 \text{ W/(m}^2\text{K)}$.](image)

The calculations concern the parameters: $k = 1.4 \text{ W/(mK)}$, $q_{\text{max}} = 10 \text{ W/m}^2$, $\kappa = -1$, $RH = 0.79$, $f = 0.3$ and $h = 15 \text{ W/(m}^2\text{K)}$.

Simulation calculations were carried out for 12 years of operation of the installation since its start-up. The obtained results show that for annual heat extraction the ground
reaches a minimum temperature of about $-4.5^\circ\text{C}$ and for a cycle with yearly intervals occurring every second year the minimum temperature is about $-3^\circ\text{C}$. Hence, the use of the cycle allows a significant increase in the minimum ground temperature.

4. Conclusions

The carried out simulations show that the ground thermal conductivity significantly affects the value of the minimum temperature occurring at the depth of the heat exchanger’s installation. The ground heat exchanger will therefore be best protected against the occurrence of excessively low temperatures if it is installed in the ground of high thermal conductivity.

An equally important parameter is the amount of heat extracted from the ground during the heating season (characterized by the value of $q_{\text{max}}$). The simulation results confirmed that reduction of heat extraction flux from the ground during the heating season is the best way to avoid undercooling of the ground. The analysis of heat supply during the summer shows that from the point of view of protection against undercooling of the ground it is profitable but does not guarantee the elimination of excessive cooling due to too high heat flux extracted during the winter season.

Studying the impact of interruptions in the exchanger operation indicated the certain benefits of such action. However, it should be assumed that frequent annual breaks are unacceptable for heat pump users.

As can be seen from the calculations, the decrease of the relative humidity of the ambient air and evaporation rate coefficient very slightly raises the minimum annual ground temperature, improving the operation of the ground heat exchanger to a small extent.

The calculations also showed that the change of the convective heat transfer coefficient has no significant impact on the minimum annual ground temperature.

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Nomenclature

Symbols:
- $a$: constant (=103) Pa/K
- $A_a$: amplitude of ambient temperature, K
- $A_s$: amplitude of ground surface temperature, K
- $A_{\text{sol}}$: amplitude of solar heat flux, W/m$^2$
- $b$: constant (=609), Pa
\( c_v \) volumetric heat capacity of the ground, J/(m\(^3\)K)
\( C_{EV} \) constant (=0.0168), K/Pa
\( f \) evaporation rate coefficient
\( h \) convective heat transfer coefficient, W/(m\(^2\)K)
\( h_{GHE} \) distance between the heat exchanger and the surface of the ground, m
\( h_{inf} \) maximum ground depth at which calculations are performed (=30), m
\( i \) non-negative number
\( k \) ground thermal conductivity, W/(mK)
\( LW \) long-wave radiation heat flux, W/m\(^2\)
\( n \) number corresponding to the depth at which the ground heat exchanger is installed (=\( \text{Int}(h_{GHE}/\Delta x) \))
\( N \) number of points for which numerical calculations were performed
\( p_e \) parameter in definition equation of sol-air-evaporation temperature
\( p_r \) parameter in definition equation of sol-air-evaporation temperature
\( P_a \) phase angle of ambient, rad
\( P_s \) phase angle of the ground surface, rad
\( P_{sol} \) phase angle of solar radiation, rad
\( q_{cond} \) heat flux transferred between the surrounding and the surface of the ground, W/m\(^2\)
\( q_{GHE} \) heat flux transferred in the ground heat exchanger, W/m\(^2\)
\( q_{max} \) maximum heat flux transferred in the ground heat exchanger, W/m\(^2\)
\( q_v \) heat generation rate per unit volume, W/m\(^3\)
\( Q_c \) amount of heat supplied to the ground in the summer period, J
\( Q_h \) amount of heat extracted from the ground during the heating season, J
\( RH \) relative humidity of ambient air
\( S \) solar radiation flux absorbed by the ground, W/m\(^2\)
\( S_m \) annually-averaged solar radiation flux absorbed by the ground, W/m\(^2\)
\( t \) time, s
\( T \) temperature, °C
\( T_a \) ambient temperature, °C
\( T_{am} \) annually-averaged ambient temperature, °C
\( T_s \) ground surface temperature, °C
\( T_{sm} \) annually-averaged ground surface temperature, °C
\( T_{sae} \) sol-air-evaporation temperature, °C
\( x \) position coordinate in the ground, m

**Greek symbols:**
\( \alpha \) thermal diffusivity of the ground, m\(^2\)/s
\( \varepsilon \) emissivity of the ground surface
\( \kappa \) parameter characterizing the relationship between the amount of heat extracted and supplied from/to the ground
\( \omega \) frequency (=\( 2\pi/365 \)), 1/day

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