Constraining the density dependence of the symmetry energy with nuclear data and astronomical observations in the KIDS framework

Hana Gil,1,‡ Young-Min Kim,2,§ Panagiota Papakonstantinou,3,† and Chang Ho Hyun4,*

1Center for Extreme Nuclear Matters, Korea University, Seoul 02841, Korea
2Department of Physics, Ulsan National Institute of Science and Technology, Ulsan 44919, Korea
3Rare Isotope Science Project, Institute for Basic Science, Daejeon 34047, Korea
4Department of Physics Education, Daegu University, Gyeongsan 38553, Korea

(Dated: October 27, 2020)

Background: The properties of very neutron rich nuclear systems are largely determined by the density dependence of the nuclear symmetry energy. The KIDS framework for the nuclear equation of state (EoS) and energy density functional (EDF) offers the possibility to explore the symmetry-energy parameters such as \( J \) (value at saturation density), \( L \) (slope at saturation), \( K_{\text{sym}} \) (curvature at saturation) and higher-order ones independently of each other and independently of assumptions about the in-medium effective mass, as previously shown in the cases of closed-shell nuclei and neutron-star properties.

Purpose: We examine the performance of EoSs with different symmetry energy parameters on properties of nuclei and observations of neutron stars and gravitational waves in an effort to constrain in particular \( L \) and \( K_{\text{sym}} \) or the droplet-model counterpart \( K_\tau \).

Method: Assuming a standard EoS for symmetric nuclear matter, we explore several points on the hyperplane of \((J, L, K_{\text{sym}} \text{ or } K_\tau )\) values. For each point, the corresponding KIDS functional parameters and a pairing parameter are obtained for applications in spherical even-even nuclei. This is the first application of KIDS energy density functionals with pairing correlations in a spherical HFB computational code. The different EoSs are tested successively on the properties of closed-shell nuclei, along the Sn isotopic chain, and on astronomical observations, in a step-by-step procedure of elimination and correction.

Results: A small regime of best-performing parameters is determined and correlations between symmetry-energy parameters are critically discussed. The results strongly suggest that \( K_{\text{sym}} \) is negative and no lower than \(-200 \text{ MeV}\), that \( K_\tau \) lies between roughly \(-400 \text{ and } -300 \text{ MeV}\) and that \( L \) lies between 40 and 65 MeV, with \( L \lesssim 55 \text{ MeV} \) more likely. For the selected well-performing sets, corresponding predictions for the position of the neutron drip line and the neutron skin thickness of selected nuclei are reported. The results are only weakly affected by the choice of effective mass values. Parts of the drip line can be sensitive to the symmetry energy parameters.

Conclusion: Using KIDS EoSs for unpolarized homogeneous matter at zero temperature and KIDS EDFs with pairing correlations in spherical symmetry we have explored the hyperplane of symmetry-energy parameters. Using both nuclear-structure data and astronomical observations as a testing ground, a narrow regime of well-performing parameters has been determined, free of non-physical correlations and decoupled from constraints on the nucleon effective mass. The results underscore the role of \( K_\tau \) and of precise astronomical observations. More-precise constraints are possible with precise fits to nuclear energies and, in the future, more-precise input from astronomical observations.

I. INTRODUCTION

The number of measured or observed nuclei exceeds 3,000, providing firm ground to test models and theories of nuclear structure. At the same time, rare-isotope accelerators in operation or under construction around the world are expected to explore the nuclear landscape even further towards the drip lines, while astronomical observations including measurements of gravitational waves usher in a new epoch for the understanding of neutron stars. Such progress demands a more precise description of neutron-rich and/or high-density nuclear systems and a better understanding of the uncertainties and biases involved in the models. A key issue is the the density dependence of the nuclear symmetry energy \([1,2]\), typically represented by the parameters characterizing its value and derivatives at the saturation point,

\[
S(\rho) = J + Lx + \frac{1}{2} K_{\text{sym}} x^2 + \frac{1}{6} Q_{\text{sym}} x^3 + \cdots ,
\]

where \( S \) denotes the symmetry energy, \( \rho \) is the nucleon density, and \( x = (\rho - \rho_0) / 3 \rho_0 \) with \( \rho_0 \) the nuclear saturation density. Representative ways or observables to constrain the parameters include the neutron skin thickness of neutron-rich nuclei, isobaric analog states, heavy ion collisions, and information from astrophysics \([1,5]\). Uncertainties are large especially for the higher-order parameters. There are some overlapping acceptable ranges for \( J \) and \( L \) coming from diverse sources. For example, from the results reviewed in Ref. \([1]\), one can infer the ranges \(32 \text{ MeV} < J < 33 \text{ MeV} \), and \(55 \text{ MeV} < L < \)
for examining the role of all three parameters, because they force an almost analytical relation between the three EoS parameters owing to the insufficient number of corresponding EDF parameters. We believe this deficiency to belie paradoxical findings such that “realistic” EoS parameters cannot necessarily be applied in the description of nuclei with realistic results (see, e.g., [2]). For the selected well-performing sets, we also report predictions for the position of the neutron drip line and the neutron skin thickness of selected nuclei.

The paper is organized as follows. We begin in Sec. II with a reminder of the basic principles of the KIDS framework, now extended to include pairing, and some comments on correlations among EoS parameters in traditional Skyrme models. In Sec. III we undertake an exploration of the \((J, L, K)\) hyperplane centered around values of \((J, L)\) generally considered reasonable. We test the EoSs successively on basic properties of closed-shell nuclei, of open-shell nuclei and of neutron stars, in a step-by-step process of elimination and correction. In Sec. IV we compile our results, revisit the aforementioned correlations between EoS parameters and provide predictions for the neutron drip line and the neutron-skin thickness. We summarize our work and discuss future perspectives in Sec. V.

II. FROM EOS TO EDF AND NUCLEI

First, we summarize the basic features of the KIDS EoS and the KIDS EDF. Next, we introduce the application to a spherical HFB code. We will also comment on a correlation which has been observed between symmetry energy parameters in traditional EDF models and which we will revisit in this work.

A. KIDS framework

In infinite nuclear matter the energy per particle \(E\) at given isospin asymmetry \(\delta = (\rho_n - \rho_p)/\rho\) is expanded in powers of the cubic root of the density \(\rho = \rho_n + \rho_p\), with \(\rho_n\) and \(\rho_p\) the neutron and proton density, respectively. The form and most relevant terms were determined and discussed in Ref. [9]. If we assume in addition a quadratic dependence of the potential energy on \(\delta\), the energy per particle of infinite matter is written as

\[
E(\rho, \delta) = T(\rho, \delta) + \sum_{i=0}^{2} \alpha_i \rho^{1+i/3} + \delta^2 \sum_{i=0}^{3} \beta_i \rho^{1+i/3},
\]

where \(T(\rho, \delta) \propto \rho^{2/3}\) is the kinetic energy per particle of a free Fermi gas. The optimal number of terms was found to be 3 terms in the isospin symmetric part, \(\alpha_i\), and 4 terms in the isospin asymmetric part, \(\beta_i\) [9] [11].

With three parameters in the symmetric part \(\alpha_i\), three EoS parameters of symmetric nuclear matter (SNM),
typically the saturation density \( \rho_0 \), the energy per particle at saturation \( E_0 \), and the compression modulus \( K_0 \) can be controlled independently. The relation between \((\rho_0, E_0, K_0)\) and \((\alpha_0, \alpha_1, \alpha_2)\) is analytical and straightforward; similarly, with four parameters \( \beta_i \), four symmetry-energy parameters can be controlled, i.e., of \( J, L, K_{\text{sym}}, \) and \( Q_{\text{sym}} \) in Eq. \( (1) \). The analytical relations between the KIDS-EoS parameters \((\alpha_i, \beta_i)\) and the characteristic EoS parameters \((\rho_0, E_0, K_0, J, L, \text{ etc.})\) can be found in Refs. \( \cite{11,13} \).

Given an EoS \((\rho_0, E_0, K_0, J, L, K_{\text{sym}}, Q_{\text{sym}})\) under investigation, a corresponding KIDS EDF can be obtained as explained in previous work for applications in nuclei \( \cite{10,11,13} \). The KIDS EDF is characterized by Skyrme-like parameters \((t_i, y_i, W_0)\), where \( i \) stands for the indices \((0, 1, 2, 31, 32, 33)\), \( y_i \) replaces the Skyrme notation \( t_i x_i \) for the exchange terms, and \( W_0 \) is a spin-orbit parameter. The indices \( 31, 32, 33 \) represent three density-dependent couplings. All KIDS-EDF parameters except \( W_0 \) and four linear combinations of \( t_1, y_1, t_2, y_2, t_{32}, y_{32} \) can be determined analytically from the KIDS-EoS parameters. Specific values for the isoscalar and isovector effective mass, \( \mu_s = m^*/m, \mu_v = m_{1V}/m \), may be specified at this stage, if desired, reducing the linear combinations of KIDS-EDF parameters to two. The analytical expressions connecting the KIDS-EDF parameters with the KIDS-EoS parameters \((\alpha_i, \beta_i)\) can be found in Refs. \( \cite{10,13} \).

Having found that the effective mass plays a minor role in predictions for bulk static properties (total energy and radius) of closed-shell nuclei, one can set \( y_1 = y_2 = 0 \) and use the simpler method of Ref. \( \cite{11,12} \) in such cases. On the other hand, when spectroscopic precision is required, as in the case of (two-) nucleon separation energies, assumptions about the effective mass can be of importance. We will return to this point.

For determining the remaining KIDS-EDF parameters which are not determined from the EoS, input from nuclear data is needed. In this work, the input data are the binding energies and charge radii of closed-shell nuclei and the cost function is

\[
\chi^2 = \frac{1}{N} \sum_{n=1}^{N} \left( \frac{O_{n}^{\text{calc}} - O_{n}^{\text{exp}}}{O_{n}^{\text{exp}}} \right)^2
\]

with \( N \) the number of input data. In the above, \( O \) represents the energy or charge radius and the superscript the calculated or experimental value.

The stand-out feature of the above fitting procedure is that the EoS parameters are not adjusted in the process but remain immutable. The purpose of the fitting is to optimize the gradient and spin-orbit terms, which are not active in homogeneous matter. Another important feature of the KIDS framework is that assumptions for the EoS parameters are independent of assumptions for the in-medium effective mass. The effective mass is decoupled from the EoS of homogeneous matter and cannot contaminate our constraints on the latter – as could be the case in more restricted ansätze \( \cite{14,15} \), see also Sec. \( \text{II}C \).

Once the KIDS EDF has been determined as above, its predictive power can be tested on more data. As a measure of predictive power we use the average deviation per datum (ADPD),

\[
\text{ADPD} = \frac{1}{N} \sum_{n=1}^{N} \left| \frac{O_n^{\text{calc}} - O_n^{\text{exp}}}{O_n^{\text{exp}}} \right|.
\]

In principle, the \( \chi^2 \) calculated on the new data set could also be used to the same effect.

These are the basic elements of the KIDS EoS and EDF formalism as has been applied in closed-shell nuclei as well as neutron stars \( \cite{11,13} \). We should note that in previous work \( N = 6 \) input data were used in Eq. \( (3) \) for determining an EDF from an EoS. The data were the binding energies and charge radii of \( ^{40,48}\text{Ca} \) and \( ^{208}\text{Pb} \). Next the the predictive power of each parameterization, which is interpreted as how realistic the starting EoS is, could be tested on basic properties of several other nuclei. Here we will use the same strategy in an initial broad-range search for well-performing regions of the symmetry-energy parameter space. Specifically, in Sec. \( \text{III} \text{A}1 \) we will perform fits \((\chi^2, \text{Eq. } (3))\) on the above 6 data and check the performance \((\text{ADPD, Eq. } (4))\) on additionally 7 data \((N = 13 \text{ in total})\). The additional data are the binding energies and radii of \( ^{160}\text{O}, ^{90}\text{Zr} \) and \( ^{132}\text{Sn} \) and the binding energy of \( ^{218}\text{U} \). Next, in Sec. \( \text{III} \text{A}2 \) we will refine the search by directly fitting \((\chi^2, \text{Eq. } (3))\) to \( N = 13 \text{ data} \). The performance of selected parameter sets \((\text{ADPD, Eq. } (4))\) will be examined in Sec. \( \text{III} \text{A}3 \) on the energies of Sn isotopes \((N = 20 \text{ data})\). Finally we will examine neutron-star properties \((\text{Sec. } \text{IV})\).

So far the KIDS framework has been applied only to closed-shell nuclei. We proceed to explain how the KIDS EDF can be applied to spherical, even-even open shell nuclei for the purposes of this work.

**B. Open-neutron-shell even-even nuclei**

Thus far the KIDS formalism has been applied in closed shell nuclei. It is rather straightforward to extend the applications to spherical, even-even open-shell nuclei. The KIDS EDF has the form of an extended Skyrme functional, i.e., a Skyrme functional with more than one density-dependent terms. Therefore, any computational code available for Skyrme functionals can be extended for KIDS applications quite easily. For spherical, even-even open-shell nuclei we have adopted the publically available HFB code HFB4RAD \( (\text{v}1.0) \) \( \cite{16} \) and extended it for use with KIDS functionals. Pairing correlations are accounted for by means of a two-nucleon pairing potential \( \cite{16} \)

\[
V_{\text{pair}} = t'^0 \left( 1 - \frac{\rho}{2\rho_0} \right) \delta(\vec{r}_1 - \vec{r}_2).
\]

In the notation of Ref. \( \cite{16} \) and for \( \rho_0 = 0.16 \text{ fm}^{-3} \) the above corresponds to \( \gamma' = 1 \) and \( t''_3 = -37.5t'_0 \text{ fm}^3 \) (mixed
pairing prescription). The pairing parameter $t'_0$ is chosen so as to reproduce the empirical mean neutron gap 1.342 MeV of $^{120}$Sn extracted from the 5-point formula \cite{17}. The angular momentum cut-off value is set to $19\hbar/2$.

At this point we should qualify our earlier statement about the role of the effective mass. We have seen that it hardly plays any role in bulk, static properties, like ground-state energies, of closed-shell nuclei \cite{10}. We will now examine also the two-neutron separation energy $S_{2n}$, which for a given isotope $(Z,N)$ is defined as the difference

$$S_{2n}(Z,N) = E_B(Z,N) - E_B(Z,N - 2), \quad (6)$$

where $E_B$ is the binding energy of the nucleus, a positive quantity for bound nuclei. Open-shell as well as closed-shell nuclei must be considered. We will use two different EoSs to demonstrate the possible effect of the effective mass assumptions.

For two EoS parameterizations and for different prescriptions for $\mu_s, \mu_v$ we determine the corresponding KIDS EDFs with the help of 13 input data (see Sec. \cite{11}) and a pairing parameter reproducing the $^{120}$Sn pairing gap. One EoS, labeled KIDS-P4 following Ref. \cite{11}, corresponds to $(J,L,K_{sym}/K_r,Q_{sym}) = (33,49,-156/-374,583)$ MeV (rounded values) and was fitted to the Akmal-Pandharipande-Ravenhall (APR) EoS \cite{18}. The second one corresponds to different symmetry energy parameters, $(J,L,K_r,Q_{sym}) = (30,51,-123.25/-350,650)$ MeV. The possible effect of the effective mass assumptions is illustrated . In Fig. 1 we show results for the binding energy per particle $E_B/A$ of the input closed-shell nuclei and for $S_{2n}$ along the Ca and Sn isotopic chains by 1) using the simplified procedure with $y_1 = y_2 = 0$ (resulting in high effective masses, $0.8 - 1.0$) and 2) with assuming two different values of the isoscalar and isovector effective mass $(\mu_s, \mu_v) = (0.7,0.7), (0.7,0.9), (0.9,0.7), (0.9,0.9)$. We compare with available data.

As we can see, the $E_B/A$ of the closed-shell nuclei is practically unaffected by the choice of the effective mass. (The differences in the total $E_B$ for each nucleus and EoS are of the order of 1 MeV.) The $S_{2n}$, however, and the corresponding predictions for the position of the drip line can be somewhat affected by the choice of effective mass. The effect can be attributed to the pairing energy being non-monotonic between shell closures and influenced by the single-particle level density, in turn determined by (mainly) $\mu_s$. Therefore, we will provide final results using $(\mu_s,\mu_v) = (0.7,0.7)$ and $(\mu_s,\mu_v) = (0.9,0.9)$ as representative values. Any spread in the final results will represent the uncertainty in our predictions.

C. Underdetermined EoS’s?

In Skyrme and relativistic mean-field models a certain interrelation among symmetry energy parameters has been revealed, namely an almost linear relation between $(3J-L)$ and $K_{sym}$ \cite{19}. The authors of \cite{19} point out that the relation holds “for a class of interactions with quadratic momentum dependence and a power-law density dependence” and indeed it is easy to demonstrate the origin of the relation in the case of traditional Skyrme functionals which belong in the above class. We will show...
that the limited number of free EDF parameters enforces a strong relationship among the symmetry energy parameters.

The Skyrme EoS for SNM can be written in shorthand as

\[ \mathcal{E}^{\text{Sk}}(\rho, 0) = f \rho^{2/3} + a_0\rho + a_1\rho^{1+\gamma} + a_2\rho^{5/3}. \]  

(7)

\( f \approx 75 \text{ MeV fm}^2 \) is shorthand for the usual kinetic factors.) In the above, we recognize four independent Skyrme-EoS parameters: \( a_0, a_1, \) and \( \gamma \) are directly related to \( t_0, t_3 \) and the exponent \( \gamma \); \( a_2 \) is related to the isoscalar effective mass \( \mu_s \) \cite{10 [20].

\[ a_2 = \frac{3}{10} \left( \frac{3\pi^2}{2} \right)^{2/3} \frac{\hbar^2}{m\rho_0}(\mu_s^{-1} - 1) \]

or approximately 37.5 \((\mu_s^{-1} - 1)\text{ MeV fm}^3/\rho_0\). Thus in principle four EoS parameters could be independently adjusted. Typically, a constraint must be imposed on the effective mass value \((0.6 \lesssim \mu_s \lesssim 1)\), so more precisely three EoS parameters can be adjusted. Those would be \( \rho_0, \tilde{E}_0, K_0 \). (Indeed, a need to adjust \( K_0 \) established the use of a power law \( \gamma < 1 \) historically \cite{21}.)

The Skyrme expression for the density dependence of the symmetry energy can be written as follows,

\[ \mathcal{E}^{\text{Sk}}(\rho) = gp^{2/3} + b_0\rho + b_1\rho^{1+\gamma} + b_2\rho^{5/3}, \]  

(8)

i.e., it involves 3 additional parameters \( b_i \), corresponding to \( x_0, x_3, \) and the isovector effective mass \( \mu_v \). \( g \approx 42 \text{ MeV fm}^2 \) is shorthand for the usual kinetic factors.) So up to three EoS parameters (such as \( J, L, K_{\text{sym}} \)) can be adjusted, again with the caveat that \( \mu_v \) cannot assume arbitrary values – so that only two of the parameters can be adjusted truly independently. In particular,

\[ b_2 = \frac{1}{6} \left( \frac{3\pi^2}{2} \right)^{2/3} \frac{\hbar^2}{m\rho_0} \left[ -3(\mu_v^{-1} - 1) + 4(\mu_s^{-1} - 1) \right] \]

or roughly 42 \([-3(\mu_v^{-1} - 1) + 4(\mu_s^{-1} - 1)]\text{ MeV fm}^2/\rho_0\). In addition, given the strong contribution of the kinetic terms \((a_2, b_2)\) to the density dependence, strong limitations must be considered for the effective mass to better control the EoS parameters and the neutron-star mass-radius relation \cite{14}.

The above analytical expressions and the definitions of \( J, L, K_{\text{sym}} \) lead to the following analytical relation between \( K_{\text{sym}} \) and \((3J - L)\),

\[ K_{\text{sym}}^{\text{Sk}} = -3(1+\gamma)(3J - L) + (1+3\gamma)g\rho_0^{2/3} + 2(2-3\gamma)b_2\rho_0^{5/3}, \]  

(9)

which is equivalent to Eq. (13) of Ref. \cite{19}. There, a regression analysis involving numerous EoS parameterizations gave the result (rounded up here for simplicity)

\[ K_{\text{sym}} \approx -5(3J - L) + 67 \text{ MeV}. \]  

(10)

If in Eq. \( \text{(9)} \) we set \( \gamma \approx 1/3 \) (keeping in mind that values between 1/6 and 2/3 are widely in use) and \( \rho_0 \approx 0.16 \text{ fm}^{-3} \) and assuming, on average, \( \mu_s \approx \mu_v \approx 0.8 \) \((b_2\rho_0 \approx 10.5 \text{ MeV fm}^2)\), we get

\[ K_{\text{sym}}^{\text{Sk}} \approx -4(3J - L) + 32 \text{ MeV} \]  

(11)

which does not deviate much from Eq. \text{(10)}. The two expressions are compared graphically in Fig. \text{[11]}.

We conclude that even if “nature prefers” to deviate from the above linear relation, the modeling may lack the flexibility to accommodate that preference. Then the Skyrme and similar models for the EoS are underdetermined. A similar conclusion was reached in Ref. \cite{22} based on an analysis of about 50 models and a Taylor expansion of the symmetry energy around saturation density. With the KIDS model, which can take the form of an extended Skyrme functional with an adequate number of independent parameters, the relevant EoS parameters can be explored independently \cite{9 [11]. Such is the purpose of this work in relation to the symmetry energy and nuclear structure. We will return to the above relation in discussing our results, see Sec. \text{IV A}.

III. PROCEDURE: SUCCESSIVE STEPS OF ELIMINATION AND CORRECTION

For the EoS of symmetric nuclear matter we adopt a fixed, three-term KIDS parameterization \cite{9} corresponding to a saturation density \( \rho_0 = 0.16 \text{ fm}^{-3} \), energy per particle at saturation \( \tilde{E}_0 = -16 \text{ MeV} \), and nuclear incompressibility at saturation \( K_0 = 240 \text{ MeV} \), while the skewness is \( Q_0 = -373 \text{ MeV} \). The focus of the present work is on the most important parameters characterizing the density dependence of the nuclear symmetry energy, namely \( J, L, K_{\text{sym}} \). The skewness parameter of the symmetry energy will be kept fixed to a given value \( Q_{\text{sym}} = 650 \text{ MeV} \), because we have found that moderate variations of \( Q_{\text{sym}} \) (within 100 – 200 MeV) have very little effect on nuclear structure and even a marginal effect on neutron-star properties \cite{11}. The above value for \( Q_{\text{sym}} \) comes from within the range 420 – 750 MeV determined from fitting to microscopic equations of state denoted by APR \cite{18} and QMC \cite{23} in \cite{11}.

When we are examining nuclear structure, instead of \( K_{\text{sym}} \) we explore the corresponding liquid-droplet parameter \( K_{\tau} \) given by \cite{2}

\[ K_{\tau} = K_{\text{sym}} - \left( \frac{6 + Q_0}{K_0} \right) L. \]  

(12)

Of course, switching between \( K_{\tau} \) and \( K_{\text{sym}} \) is straightforward.

With the SNM EoS parameters and \( Q_{\text{sym}} \) fixed, we proceed to explore the quantities of interest, \( J, L \) and \( K_{\text{sym}} \) or \( K_{\tau} \). For their values, we adopt ranges similar to or broader than those denoted by “CSKp” in \cite{20}. In particular, we begin by using \( J = 31, 32, 33, 34 \text{ MeV}, \) \( 30 \text{ MeV} \leq L \leq 70 \text{ MeV} \), and \(-550 \text{ MeV} \leq K_{\tau} \leq 0 \text{ MeV} \) or \( K_{\text{sym}} \) between \(-400 \) and \(+300 \text{ MeV} \).
A. Finite nuclei

1. Broad search: Best-performing $K_\tau$ values in closed-shell nuclei

As outlined in Sec. [II A] for each point on the $(J, L, K_\tau)$ hyperplane the corresponding KIDS functional parameters $(t_i, y_i, W_0)$ are obtained using as input the desired EOS parameters and 6 data points. The predictive power of each parameterization is tested at this stage on additional closed shell nuclei. Specifically, we inspect the ADPD, Eq. (4), where the $N = 13$ observables are presently the binding energies and charge radii of $^{16}$O, $^{40}$Ca, $^{48}$Ca, $^{90}$Zr, $^{132}$Sn, and $^{208}$Pb as well as the binding energy of $^{218}$U. Data for the energies are taken from the NNDC [24] and data for the radii from Ref. [25]. The purpose of using closed-shell nuclei as a primary filter is because they provide a more clean “EoS signal”, as it were: there is minimal uncertainty to the EDF predictions from approximations involving pairing correlations (see also Sec. [II B]) and, especially important for charge radii, neglected beyond-mean-field correlations which are necessary for reproducing isotopic shifts between shell closures [26, 27].

Figure 2 shows the ADPD for the indicated $J$ values as a function of $K_\tau$ (a) or as a function of $L$ (b). We observe that there is an optimal range of $K_\tau$ values roughly between $-400$ and $-300$ MeV (with some $J$ dependence). By comparison, the performance is quite flat with respect to $L$. Although the results shown in Fig. 2 were obtained with the simple procedure ($y_1 = y_2 = 0$), we have verified that the conclusions do not change when we perform fits with specific values of $(\mu_s, \mu_v)$.

Lower values of $J$ seem favorable, so in the following we will extend our search to lower values of $J$. We will examine three representative values of $K_\tau$ within the optimal range, namely $-400, -350, -300$ MeV.

2. Refined search: Best-performing $L$ for given $(J, K_\tau)$ in closed-shell nuclei

For the representative values of $K_\tau = -400, -350, -300$ MeV we obtain KIDS-EDF parameterizations this time using all the 13 data mentioned above and in Sec. [II A], i.e., the $\chi^2$ value is given by Eq. (3) with $N = 13$. We consider $J = 29, 30, 31, 32, 33$ MeV and $L$ up to $80$ MeV in steps of $1$ MeV. Results for the cost function $\chi^2$ are shown in Fig. 3 assuming $(\mu_s, \mu_v) = (0.9, 0.9)$. The choice $(\mu_s, \mu_v) = (0.7, 0.7)$ yields somewhat higher $\chi^2$ values, but similar trends. Based on the results for $\chi^2$ calculated on 13 data, we can determine the most-favored $L$ value for each $(J, K_\tau)$ set. The resulting $(J, L, K_\tau)_{CS}$ sets, where “CS” stands for closed-shell nuclei, are, in units of MeV: $(29, 31, -300)$, $(30, 55, -300)$, $(31, 72, -300)$, $(29, 33, -350)$, $(30, 51, -350)$, $(31, 68, -350)$, $(30, 43, -400)$, $(31, 57, -400)$, $(31, 70, -400)$. We could of course include the $(29, 33, -400)$ set which gives the lowest $\chi^2$. However, we found that it leads to unstable solutions for neutron stars, so we will not consider it further.

The choice of data inevitably introduces some bias in the above procedure. To control for the bias we will also consider variations of $L$ by $5$ MeV. In other words, next we examine the performance not only of the above $(J, L, K_\tau)_{CS}$ sets, but also of the $(J, L \pm 5$ MeV, $K_\tau)_{CS}$ sets.

3. Bias check: Sn isotopic chain

We now calculate the ADPD, Eq. (4) using the $N = 20$ measured binding energies of Sn isotopes. We have checked that the performance of an EoS on the energies along any isotopic chain like Sn or Pb is representative of the performance along the other isotopic chains as well. (In particular, we have examined the O, Ca, Ni, Zr, Sn, and Pb chains.) We test all 27
EoS with \((J, L, K_{\tau}) = (J, L \pm 5 \text{ MeV}, K_{\tau})_{\text{CS}}\) and assuming \((\mu_s, \mu_v) = (0.7, 0.7), (0, 0.9, 0.9)\) – that is, 54 EDFs in total. The results are tabulated in Table II. We use the notation \(L_{\pm} = L_{\text{CS}} \pm 5 \text{ MeV}\) for any given \((J, L, K_{\tau})_{\text{CS}}\) set. Two trends are apparent: First, higher effective masses are favored by this set of data. Nevertheless, we will use both pairs of effective mass values in our final results, Sec. IV. Second, lower (higher) \(L\) values are favored (disfavored). All \(L_{\text{CS}} + 5 \text{ MeV}\) sets give large ADPDs compared to the other sets, with the exception of the (29, 36, –300) MeV set. Therefore, we next discard all but one of the EoSs with \((J, L + 5 \text{ MeV}, K_{\tau})_{\text{CS}}\) and henceforth examine only the \((J, L, K_{\tau})_{\text{CS}}, (J, L - 5 \text{ MeV}, K_{\tau})_{\text{CS}}\) EoSs and in addition the (29, 36, –300) MeV EoS. We will denote the latter set of EoS’s with the subscript “CS+OS”, where OS stands for open-shell nuclei.

**B. Neutron star mass-radius relation and tidal deformability**

The maximum mass of a neutron star has long been a key issue to discuss the stiffness of EoSs at high densities. In the 1990’s, a widely accepted upper limit of mass from observations was about 1.5\(M_{\odot}\). Masses larger than 1.5\(M_{\odot}\) were reported, but the uncertainty was also large. In the new millenium, many new observations have been conducted, and now the largest mass of pulsars is increased to more than 2\(M_{\odot}\). This large mass tends to favor stiff EoSs. However, there are many models that produce a maximum mass larger than 2\(M_{\odot}\), so the large mass observation is not sufficient to constrain the EoS at high density. Moreover, the baryon density at the center of a star with mass 2\(M_{\odot}\) is more than 5\(\rho_0\), so it is uncertain or unlikely that nucleons will be the only or the major constituents of the matter at that high density. There are still many possibilities and large uncertainties in the EoS of neutron stars whose mass is larger than 1.4\(M_{\odot}\).

Detection of the gravitational waves and evaluation of the tidal deformability with them introduced more powerful and stringent constraints to the EoS of nuclear matter at high densities. Masses of neutron stars in the merger of GW170817 were estimated around 1.4\(M_{\odot}\) [29]. Density at the center of 1.4\(M_{\odot}\) star is estimated at about 3\(\rho_0\). This density is interesting because 3\(\rho_0\) is regarded as the upper limit of density below which nucleons are likely the main constituent of matter. At the same time, exotic degrees of freedom like strangeness, or a new phase of matter, e.g., deconfined quarks could appear around 3\(\rho_0\). Therefore measurement of the 1.4\(M_{\odot}\)-mass star will be more critical and informative in obtaining a reliable EoS at densities below 3\(\rho_0\).

In Fig. 4 we plot the neutron star mass-radius relation with the nine previously selected sets of \((J, L, K_{\text{sym}})_{\text{CS}}\) values. The bands around 2\(M_{\odot}\) denote the range of large masses from observations [30-32]. Interestingly, the predicted maximum masses are distributed in a narrow range \((2.1 \pm 2.2)M_{\odot}\). The irregular gray band represents the allowed mass-radius range obtained from an analysis of X-ray burst [33], and points with error bars are the result from the observation of a soft X-ray source in the NICER project [34, 35]. In the result of NICER, two analyses are performed independently, and they show slightly different results. Overlapping ranges of the two NICER results are \((1.3 - 1.5)M_{\odot}\) for mass and \((11.9 - 13.8)\) km for radius. NICER gives radius relatively larger than those from the X-ray burst.

Looking at the results of theory, three EoSs corresponding to \((J, L, K_{\text{sym}}) = (29, 31, -162), (29, 33, -203), (30, 43, -208)\) MeV are inconsistent with the NICER result. In addition, EoSs of \((29, 31, -162)\) and \((29, 33, -203)\) MeV can hardly satisfy the X-ray burst range around the mass 1.4\(M_{\odot}\). This may imply that an \(L\) value smaller than 30 MeV can be ruled out by the constraints from both NICER and X-ray burst. For \((30, 43, -208)\), it is within the range of X-ray burst,

**FIG. 3.** Fitting quality on 13 data as a function of \(L\) considering the shown values of \(J, K_{\tau}\) and \((\mu_s, \mu_v) = (0.9, 0.9)\).
but does not enter the uncertainty range of NICER. In the region of large radius, on the other hand, theory predictions are consistent with the range of NICER, but X-ray burst analysis rules out the $L$ values larger than 70 MeV. This result is consistent with the upper limit of the range denoted by CSkP in [29], where $L(\text{CSkP}) = 48.6 – 67.1$ MeV. Consequently, from the nine examined EoSs, the EoSs that satisfy both X-ray burst and the two NICER results simultaneously are reduced to $(J, L, K_{\text{sym}}) = (30, 55, -55), (30, 51, -123)$ and $(31, 57, -146)$ MeV. The ranges that satisfy all conditions from neutron stars (consistent with NICER data and with X-ray burst data) and closed-shell nuclei can be tentatively assigned as

$$(J, L)_{\text{CS+NS}} \approx (30 - 31, 50 - 65) \text{ MeV}. \quad (13)$$

The range is consistent with the range of CSkP. Among the three successful EoSs, the $(30, 55, -55)$ set predicts a radius slightly larger than those of the other two EoSs for the $1.4M_\odot$ mass stars. Tidal deformability is sensitive to the radius of a star, so it is worthwhile to consider what the theory predicts.

Table I presents the properties of the star with mass close to $1.4M_\odot$ in detail. Considered quantities are the tidal deformability $\Lambda$, the radius $R$ in units of km and the density at the center $\rho_{\text{cen}}$ in fm$^{-3}$. Observational value of $\Lambda$ for the $1.4M_\odot$ mass star from GW170817 was $\Lambda \leq 800$ in the first report [29]. An updated analysis provides a reduced range $190 - 120$ MeV. A new observation in 2019, GW190425 provides an upper limit $\Lambda \leq 600$ for the low-spin prior [37]. All the symmetry energy parameters in Tab. I satisfy the upper limit $\Lambda \approx 600$. However, the magnitude of $\Lambda$ is clearly dependent on and correlated to the value of $L$.

For $(J, L, K_{\text{sym}}) = (29, 31, -162), (29, 33, -203)$ and $(30, 43, -208)$ MeV, $\Lambda$ is roughly 200 – 250. In the opposite extreme $\Lambda$ is larger than 500 for $(31, 72, 20)$ and $(31, 68, -47)$ MeV. For the three EoSs that satisfy both X-ray burst and two NICER results simultaneously, $\Lambda$ is roughly 400 – 450. Aside from the $(32, 70, -88)$ set, $\Lambda$ appears correlated to $L$ in a way that $\Lambda \lesssim 300$ for $L \leq 40$, $\Lambda \sim 400$ for $L$ at the order of 50, and $\Lambda \gtrsim 500$ for $L$ larger than 60. Therefore, accurate measurement of the gravitational waves could play an essential role in reducing the uncertainty in the density dependence of the symmetry energy.

The radius of the $1.4M_\odot$ mass star determined from GW170817 is $9.1 – 12.8$ km. All the sets of symmetry energy in Tab. I predict the radii within this range. A softer EoS (or a small $L$) gives a larger value of $\rho_{\text{cen}}$. Three $(J, L, K_{\text{sym}})$ sets that have $\Lambda$ at the order of 200 give the central density in the range $3.4 – 3.7\rho_0$. Two stiff EoSs corresponding to $(31, 72, 20)$ and $(31, 68, -350)$ have the center densities $2.5\rho_0$ and $2.6\rho_0$, respectively. For the three EoSs that are consistent with both X-ray burst and two NICER, theory predicts $\rho_{\text{cen}} = (2.7 – 2.9)\rho_0$. Similar to the tidal deformability, densities at the center are clearly classified according to the stiffness of the symmetry energy.

Figure 5 shows the particle fraction in the core of neutron stars with the EoSs determined by $(J, L, K_{\text{sym}})$ in Fig. 4. Fractions of neutrons and protons have a critical effect to the cooling of neutron stars. Large fraction of the proton satisfies the energy-momentum conservation in the direct Urca process, $n \to p + \nu$, and $p \to n + \nu$, and it leads to a hyper-fast cool down of the neutron star.
the simplest consideration, direct Urca is accessible when
the proton fraction is larger than 1/9. If direct Urca
happens in the core, neutron star cooling calculated with
any nuclear model cannot explain the observation data.[38, 39]. Therefore, direct Urca should be prohibited at
densities where nucleons are dominant constituents of the
matter, e.g. up to $(3 - 4)\rho_0$ which are center densities of the $1.4M_\odot$ neutron stars. Focusing on the densities
$0.5 - 0.6\text{ fm}^{-3}$ in Fig. 5, one can easily categorize the
proton fraction ($Y_p$) into three classes: a class with high
$Y_p$ values around 0.2, a middle class with $Y_p \sim 0.1$, and
the last one with protons highly depleted.

Cooling is beyond the scope of this work, so we don’t
have explicit results of the cooling curve. However, it
is evident that the energy will be emitted via the direct
Urca process in the neutron star of mass around $1.4\odot$,
for the high $Y_p$ EoSs ($J, L, K_{sym} = (31, 72, 20), (31, 68,
-470)$, and $(32, 70, -88)$. If symmetry energy is stiff (i.e.
large $L$), the cost to be asymmetric becomes expensive, so
the matter favors more neutron-proton symmetric states.
For this reason, $Y_p$ is relatively large for the EoSs with
large $L$ values. Since $Y_p \geq 1/9$ is easily satisfied with
large $L$ values, three large $L$ EoSs may not be allowed by
the consistency with the cooling data of neutron stars.
Thus we confirm again that the $L$ value is unlikely to
reach 70 MeV or higher.

So far we tested only the nine ($J, L, K_{sym}$)$_{CS}$
sets on neutron stars and selected three of them, ($J, L, K_{sym}$)$_{CS+NS}$. We have seen in Sec. IIIA 3 that the
energies of open-shell nuclei favor lower values of $L$ with
practically no deterioration in the description of closed-
shell nuclei. Such a trend will certainly have effects to
the properties of neutron stars, and acceptable ranges
of $J$ and $L$ that are compatible with the astronomical
observation. We will now examine the parameter sets
($J, L, K_{sym}$)$_{CS+OS}$.

Figure 6 shows the neutron star mass-radius relation
for the “CS+OS” sets. Compared to Fig. 2, the curves
for given $J, K_\tau$ are shifted to left as a whole. The three
leftmost curves lie outside the acceptable ranges and
therefore the corresponding EoSs can be discarded. The
$(29, 36, -300)$ curve is marginally consistent only with
the X-ray data and we discard it as well. We also discard
the rightmost curve as inconsistent with the X-ray burst
range. Given the current uncertainties, we tentatively
accept the remaining five sets of ($J, L, K_{sym}$) as reason-
able. The surviving sets tentatively suggest the following
ranges of parameters,

\begin{equation}
(J, L)_{CS+NS+OS} \approx (30 - 32, 45 - 65) \text{ MeV.} \end{equation}

slightly extending the range suggested in Eq. (13). The
new ranges are again consistent with the empirical range
and CSKp of Ref. 20.

Properties of $1.4M_\odot$ neutron stars for the
($J, L, K_{sym}$)$_{CS+OS}$ sets are shown in Tab. III. For
the ranges of Eq. (13), the tidal deformability of $1.4M_\odot$
neutron star has a range 400 – 460. With the maximal
ranges Eq. (14), the acceptable range of the tidal
deformability can be written as

\begin{equation}
A \approx 410 \pm 80. \end{equation}

The radius and the central density are distributed over
the ranges $(11.8 - 12.5)\text{ km}$, and $(0.43 - 0.49)\text{ fm}^{-3}$. These ranges are slightly broader than, and include the ranges corresponding to the optimal ranges of Eq. (13).

IV. SUMMARY OF CONSTRAINTS AND
RESULTS

A. Constraints on and correlations between
symmetry-energy parameters

Let us now return to correlations such as the one be-
tween $K_{sym}$ and $(3J - L)$ discussed in Sec. II C. That
and other correlations reported in the literature[19, 40 –
42] are shown in Fig. 7 (lines) along with our selected
KIDS EoSs (filled black points). Uncertainty bands of
the correlations are shown with dotted lines. The point
with errorbars labeled “Tsang et al.” represents the re-
sults reported in Ref. 8 based on a Taylor expansion
of the symmetry energy around the saturation density
and neutron-star properties. The line labelled “approx.”
corresponds to Eq. (11). The points representing the se-
lected KIDS EoSs are connected by dotted lines accord-
ing to the constraints they obey (CS: closed-shell nuclei;
OS: open-shell nuclei; NS: neutron stars). Numbers in
Fig. 7(a) are the respective values of $K_\tau$ in MeV. Crosses
represent EoSs which were discarded in Sec. IIIA 3 and
Sec. III B. Points inside magenta circles correspond to the
EoSs providing the best ADPD along the Sn chain. The
position of the KIDS-P4 EoS is also shown with a circle.
one should notice that, based on our results so far, it was not clear whether the ellipses should include the region around \((L, -K_{\text{sym}}) \approx (40 - 45, 100 - 150)\) MeV or not, because we had no points therein (accepted nor discarded). Therefore we have examined one more point representative of that region, indicated with an inverted triangle in Fig. 7. It turns out that the corresponding EoS, \((J, L, K_{\text{sym}}/K) = (29.5, 42, -113.3/ -300)\) MeV, satisfies the neutron-star constraints, namely it is consistent with the X-ray burst and NICER data and the maximum mass. In addition, the ADPD values along the Sn isotopic chain are close to the best ones tabulated in Table I, namely 0.338% and 0.269% for \((\mu_+, \mu_-) = (0.7, 0.7)\) and \((0.9, 0.9)\), respectively. Therefore we accept that EoS as well into our final set. This brought the lower bound for our estimate of \(L\) to about 40 MeV, down from about 45 MeV.

Our results are roughly consistent with the correlations shown in Fig. 7(a) in that they remain within their combined uncertainty bands. On the other hand, the correlation reported in Ref. [19] and plotted in Fig. 7(b) appears overly restricting. We have already observed in Sec. II C that the dependence of \(K_{\text{sym}}\) on \(J\) and \(L\) in Skyrme models, Eq. (9), enforces a model-specific correlation reported in Ref. [19] and plotted in Fig. 7(b) that the analytical estimate of the correlation, Eqs. (9), labelled “approx.,” does not deviate much from the fitting result of Ref. [19]. Our numerical results reinforce the suggestion that this correlation is model-specific.

B. Neutron drip line and neutron skin thickness

We take the opportunity to report predictions for the position of the neutron drip line and for the neutron skin thickness of selected nuclei based on the six EoSs contained in the ellipses in Fig. 7.

We consider the drip nuclei focusing on even-even O, Ca, Ni, Zr, Sn, and Pb isotopes. For each isotopic chain the drip nucleus is defined as the last nucleus along the chain (highest neutron number, \(N_d\)) for which \(S_{2n}\) is positive and the neutron Fermi energy is negative. In Table IV we summarize results obtained with the four selected sets of \((J, L, K)\) values and with KIDS-P4.

The O drip nucleus is generally predicted to have neutron number \(N_d = 18 - 20\) by many EDFs. Current data suggest that the last bound isotope is \(^{24}\text{O}\), while \(^{26}\text{O}\) is only slightly unbound \(^{[14]}\). On the other hand, it has been shown that the inclusion of realistic three-nucleon forces (as well as proper treatment of continuum states) can play an important role in describing particularly the O drip nucleus \(^{[15]}\). It is likely that Oxygen isotopes are too light to be described accurately by mean-field approaches.
It is interesting also to comment on Ca. The discovery of $^{60}$Ca was published in Ref. [40]. As reported there, ab initio approaches wrongly predicted the drip line below $^{60}$Ca. EDF approaches which predicted the drip line closer to and beyond $^{70}$Ca agreed better with data. The present KIDS results also indicate that the drip nucleus is at $^{70}$Ca or beyond. Intrestingly, the KIDS-P4 EDF, which was based on the APR EoS, represents an intermediate prediction for $N_p$ (as does, e.g., the SLy4 functional [47] with $(J, L, K, \tau) \approx (32, 46, -323, 522)$). It is obvious from Fig. 1(b),(e) that KIDS-P4 provides a better description of the Ca two-neutron separation energy than the $(J, L, K, \tau) = (30, 51, -350)$ MeV EDF. Precise fits of the EoS parameters along isotopic chains within the KIDS framework could be pursued in the future to provide more confident predictions.

In Table IV the results for the neutron skin thickness is also reported for nuclei of current theoretical and experimental interest [2]. The variations among predictions of the various KIDS functionals are small considering the current experimental precision. Specifically, for given $J$ and $K$ the variation observed is less than 0.015 fm. This is not surprising given that the neutron skin thickness is understood to be correlated with $L$ and that the span in $L$ values considered here is less than 15 MeV. The results are consistent with various correlations reported in the literature, see, e.g., [2] and references therein. It is interesting to note the variations observed between the predictions with different $K$. The lower $K$ value ($-350$ MeV) consistently leads to thicker neutron skins. The trend becomes obvious when we consider the two EoSs in the third and sixth columns with equal $J$ and almost equal $L$ but different $K$. We conclude that the $K$ parameter is also important to consider in studies of the neutron skin thickness. A dedicated study would be worth pursuing in the future.

**V. SUMMARY AND PERSPECTIVES**

Using KIDS EoSs for unpolarized homogeneous nuclear matter at zero temperature and KIDS EDFs with pairing correlations in spherical symmetry we have explored the hyperplane of symmetry-energy parameters. Using both nuclear-structure data and astronomical observations as a testing ground, a narrow regime of well-performing parameters has been determined, free of non-physical correlations and constraints on the nucleon effective mass. Correlations reported in the literature between symmetry-energy parameters were critically discussed.

The results strongly suggest that $K_{\text{sym}}$ is negative and no lower than $-200$ MeV, that $K_\tau$ lies between $-400$ and $-300$ MeV and that $L$ lies between 40 and 65 MeV, with $L \lesssim 55$ MeV more likely. For $J$ we find most likely the values $31.0 \pm 1.5$ MeV. For the selected well-performing sets, we reported corresponding predictions for the position of the neutron drip line and the neutron skin thickness of selected nuclei of current interest. The results are found only weakly affected by the choice of effective mass values. The results rather underscore the role of $K_\tau$ and of precise astronomical observations.

The somewhat heuristic step-by-step process followed here was necessitated by the diversity of the data and their uncertainties. There are several refinements which could be pursued in the future in order to pin down a smaller domain of realistic $(J, L, K_{\text{sym}})$ values (the ultimate goal being a point, if one accepts that the nuclear EoS is unique). First, full advantage can be taken of nuclear data by fitting the EoS parameters to many measured masses along isotopic chains. Having already determined approximately a reasonably small regime of EoS parameters which describes well closed-shell nuclei and neutron stars, the task becomes feasible. In the process, we could also explore the role of $Q_{\text{sym}}$. However, based on a previous study [11], it is not expected to affect our results considerably. More-precise astronomical observations, leading to better constraints for the high-density EoS, might be required for constraining the $Q_{\text{sym}}$. The EoS of symmetric matter could also be explored. In brief, more-precise constraints are possible with precise fits to nuclear energies and, in the future, more-precise input from astronomical observations.

**ACKNOWLEDGMENTS**

The work of HG was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government No. 2018R1A5A1025563. YMK was supported by NRF grants funded by the Korea government (No. 2016R1A5A1013277 and No. 2019R1C1C1010571). The work of PP was supported by the Rare Isotope Science Project of the Institute for Basic Science funded by the Ministry of Science, ICT and Future Planning and the National Research Foundation (NRF) of Korea (2013M7A1A1075764). CHH was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government No. 2020R1F1A1052495.

[1] M. Baldo and G.F. Burgio, “The nuclear symmetry energy,” Progress in Particle and Nuclear Physics 91, 203 – 258 (2016)

[2] X. Roca-Maza and N. Paar, “Nuclear equation of state from ground and collective excited state properties of nuclei,” Progress in Particle and Nuclear Physics 101, 96 – 176 (2018)
[37] B. P. Abbott et al., “GW190425: Observation of a compact binary coalescence with total mass $\sim 3.4$ m⊙,” The Astrophysical Journal 892, L3 (2020)
[38] Yeunhwan Lim, Chang Ho Hyun, and Chang-Hwan Lee, “Nuclear equation of state and neutron star cooling,” Int. J. Mod. Phys. E 26, 1750015 (2017)
[39] Yeunhwan Lim, Chang Ho Hyun, and Chang-Hwan Lee, “Strangeness in neutron star cooling,” J. Korean Phys. Soc. 74, 547–554 (2019)
[40] Ingo Tews, James M. Lattimer, Akira Ohnishi, and Evgeni E. Kolomeitsev, “Symmetry parameter constraints from a lower bound on neutron-matter energy,” Astrophys. J. 848, 105 (2017)
[41] Jeremy W. Holt and Yeunhwan Lim, “Universal correlations in the nuclear symmetry energy, slope parameter, and curvature,” Physics Letters B 784, 77 – 81 (2018).
[42] Bao-An Li and Macon Magno, “Curvature-slope correlation of nuclear symmetry energy and its imprints on the crust-core transition, radius and tidal deformability of canonical neutron stars,” (2020), arXiv:2008.11338 [nucl-th].
[43] C. Drischler, K. Hebeler, and A. Schwenk, “Asymmetric nuclear matter based on chiral two- and three-nucleon interactions,” Phys. Rev. C 93, 054314 (2016)
[44] E. Lunderberg et al., “Evidence for the ground-state resonance of $^{26}$O,” Phys. Rev. Lett. 108, 142503 (2012)
[45] G. Hagen, M. Hjorth-Jensen, G. R. Jansen, R. Machleidt, and T. Papenbrock, “Continuum effects and three-nucleon forces in neutron-rich Oxygen isotopes,” Phys. Rev. Lett. 108, 242501 (2012)
[46] O. B. Tarasov, D. S. Ahn, D. Bazin, N. Fukuda, A. Gade, M. Hausmann, N. Inabe, S. Ishikawa, N. Iwasa, K. Kawata, T. Komatsubara, T. Kubo, K. Kusaka, D. J. Morrissey, M. Ohtake, H. Otsu, M. Portillo, T. Sakakibara, H. Sakurai, H. Sato, B. M. Sherrill, Y. Shimizu, A. Stolz, T. Sumikama, H. Suzuki, T. Takeda, M. Thoennessen, H. Ueno, Y. Yanagisawa, and K. Yoshida, “Discovery of $^{60}$Ca and implications for the stability of $^{70}$Ca,” Phys. Rev. Lett. 121, 022501 (2018)
[47] E. Chabanat, P. Bonche, P. Haensel, J. Meyer, and R. Schaeffer, “A Skyrme parametrization from subnuclear to neutron star densities part ii. nuclei far from stabilities,” Nuclear Physics A 635, 231 – 256 (1998)
| $K_\tau$ [MeV] | −300 | −350 | −400 |
|----------------|-------|-------|-------|
| $(J, L_+/L_+/L_+)$ | $(\text{resp. ADPDs}[%])$ | $(\text{resp. ADPDs}[%])$ | $(\text{resp. ADPDs}[%])$ |
| $(\mu_\pi, \mu_\nu) = (0.7, 0.7)$ | $(29.26/31/36):(0.264/0.297/0.314)$ | $(29.28/33/38):(0.368/0.426/0.485)$ | $(30.38/43/48):(0.462/0.525/0.590)$ |
| | | | |
| | $(30.50/55/60):(0.362/0.423/0.484)$ | $(30.46/51/56):(0.435/0.497/0.560)$ | $(31.52/57/62):(0.489/0.552/0.611)$ |
| | | | |
| | $(31.67/72/77):(0.428/0.490/0.822)$ | $(31.63/68/73):(0.501/0.564/0.624)$ | $(32.65/70/75):(0.505/0.565/1.085)$ |
| $(\mu_\pi, \mu_\nu) = (0.9, 0.9)$ | $(29.26/31/36):(0.159/0.216/0.278)$ | $(29.28/33/38):(0.305/0.368/0.433)$ | $(30.38/43/48):(0.460/0.468/0.536)$ |
| | | | |
| | $(30.50/55/60):(0.291/0.358/0.426)$ | $(30.46/51/56):(0.371/0.438/0.506)$ | $(31.52/57/62):(0.422/0.492/0.564)$ |
| | | | |
| | $(31.67/72/77):(0.355/0.425/0.495)$ | $(31.63/68/73):(0.501/0.564/0.624)$ | $(32.65/70/75):(0.434/0.506/0.580)$ |

**TABLE I.** ADPD results representing the performance of the $(J, L, K_\tau)_{\text{CS}}$ and the $(J, L_+, K_\tau)_{\text{CS}}$ EoSs on the energies of Sn isotopes. Two blocks of results, each spanning three rows, correspond to different effective masses as indicated. Three blocks of results, each spanning three columns, correspond to different $K_\tau$ as indicated. In each row of a $(\mu_\pi, \mu_\nu) \times K_\tau$ block we tabulate the values of a $(J, L_+/L_+/L_+)$ triad in MeV and the respective ADPD values.

| $K_\tau$ [MeV] | −300 | −350 | −400 |
|----------------|-------|-------|-------|
| $(J, L)$ [MeV] | $(30, 31)$ | $(30, 33)$ | $(30, 35)$ |
| $K_{sym}$ [MeV] | $(29, 31)$ | $(30, 33)$ | $(30, 35)$ |
| Mass [M$_\odot$] | 1.40 | 1.41 | 1.40 |
| $\Lambda$ | 234.7 | 455.0 | 604.9 |
| $R$ [km] | 11.2 | 12.4 | 12.9 |
| $\rho_{cen}$ [fm$^{-3}$] | 0.58 | 0.43 | 0.39 |

**TABLE II.** Tidal deformability $\Lambda$, radius $R$, and density at the center $\rho_{cen}$ of the 1.4$M_\odot$ neutron star with the $(J, L, K_{sym}$ or $K_\tau)_{CS+NS}$ values.

| $K_\tau$ [MeV] | −300 | −350 | −400 |
|----------------|-------|-------|-------|
| $(J, L)$ [MeV] | $(29, 26)$ | $(30, 28)$ | $(30, 30)$ |
| $K_{sym}$ [MeV] | $(29, 26)$ | $(30, 28)$ | $(30, 30)$ |
| Mass [M$_\odot$] | 1.41 | 1.41 | 1.40 |
| $\Lambda$ | 164.6 | 412.9 | 562.1 |
| $R$ [km] | 10.7 | 12.2 | 12.8 |
| $\rho_{cen}$ [fm$^{-3}$] | 0.66 | 0.45 | 0.41 |

**TABLE III.** The same as Tab. II but for the “CS+OS” parameter sets.

| EoS | $(\mu_\pi, \mu_\nu) = (0.7, 0.7)$ | $(0.9, 0.9)$ |
|-----|---------------------------------|---------------|
| $N_d$: O | $(29.5, 42, -300)$ | $(0.7, 0.7, 0.9, 0.9)$ |
| Ca | $(30.5, 50, -300)$ | $(0.7, 0.7, 0.9, 0.9)$ |
| Zr | $(30.55, -300)$ | $(0.7, 0.7, 0.9, 0.9)$ |
| Sn | $(30.46, -350)$ | $(0.7, 0.7, 0.9, 0.9)$ |
| Pb | $(30.51, -350)$ | $(0.7, 0.7, 0.9, 0.9)$ |
| KIDS-P4 | $(0.7, 0.7, 0.9, 0.9)$ | $(0.7, 0.7, 0.9, 0.9)$ |

**TABLE IV.** For the KIDS functionals corresponding to the indicated EoS sets are shown for the drip line neutron number $(N_d)$ of the shown isotopic chains and for the neutron skin thickness $(r_{np})$ of selected nuclei. The EoSs examined in this work (columns 2 – 6) are labelled by the values of $(J, L, K_\tau)$ in MeV. For all five, $Q_{sym} = 650$ MeV. KIDS-P4 has $(J, L, K_\tau, Q_{sym}) = (33, 49, -374, 583)$ MeV.