Monte-Carlo Simulation-based Analysis of Bridge Structure Systems’ Reliability

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Abstract. Modern engineering systems, such as automobiles, should have reliability requirements. This paper focuses on the reliability of Systems with Complex Interconnection (SwCI). Most of the real complex systems in the modern vehicles are different sensors, as Bridge Structure Systems (BSSs). The BSSs are used as an example in the field of system reliability analysis. The proposed Monte-Carlo Simulation-based investigation can be used for the estimation of the Required Number of Spare Parts (RNSPs).

1. Introduction
The reliability analysis of technical systems and networks is a very important tool in analyzing and quantifying technical safety.

The objective of Labašova's paper was an overview to determine the reliability of Systems with Simple Interconnections (SwSI) [1]. Her aim was to analyze the reliability of the systems with parallel or serial arrangements too. According to Labašova, the Reliability Block Diagram (RBD) method of reliability is relatively simple to process. For less complex systems, it does not require software support, and is therefore relatively widely used. The systems that do not have so-called simple connections are named Systems with Complex Interconnection (SwCI) in other word: complex systems [2]. During the investigation of reliabilities of complex systems, they cannot be simplified to single equivalent blocks by a combination of series and/or parallel reductions. Therefore, the methods of RBD cannot be able to determine the reliability of the SwCI.

Most of the concrete complex systems in the modern vehicle industry are vehicle sensors, as a Bridge Structure System (BSS), and as sensory networks [3]. Nagy has investigated the electronic control network of a particular vehicle [4]. The mapped system can provide a base for evaluation of system reliability or for analysis of error-spreading on the network.

In addition to vehicle sensors, the capacity and safety of transport systems can also be analyzed as SwCI. Péter’s research work is aimed to develop applicable mathematical methods that accurately consider the characteristics of the trajectories. The aim of his paper [5] was to develop an efficient and fast algorithm that resulted in the determination of all the different trajectories.

In Cristescu’s publication, new techniques of approximation of the reliability of a two-terminal network are worked out based on the constructive theory of functions and few related methods [6].

Mathematical modeling is the representations of the process occurring in given modeled technical equipment by symbols or symbol systems of mathematics, from the point of view of the given investigation. Nowadays real engineering systems consist of a great number of interconnected
aggregates. The mathematical model is a simplified representation of a real physical system or technical process; thus, it can be imprecise and accordingly it has uncertainties.

If the modeler fails to reduce the model properly, the uncertainty is called epistemic.

The other type of uncertainty is approximated with the model-based investigated system or its neighbourhood and it is named parametric.

This parametric uncertainty can be investigated using the Monte-Carlo Simulation (MCS) [7]. MCS is one of the classical simulation techniques. In their paper, Metropolis and Ulam named the method Monte-Carlo [8]. This publication served as motivation and offered the general description of a method dealing with a class of problems in mathematical physics.

The main object of this paper is to work out a probabilistic uncertainty analysis method of BSSs’ reliability using MCS.

The paper is organized as follows: Section 2 shows the methodology of calculation of the reliability of BSS. Section 3 presents the MCS-based analysis of probability uncertainties of reliability of BSS, in the cases of different situations. Section 4 summarizes the paper and its conclusions, then outlines the future scientific work of the Author.

2. Reliability of the Bridge Structure System

The components of canonic systems have only two states – work (it performs its required function – designated as “+”) and wrong (it does not perform the required function – designated by “-”) [9].

A BSS is composed of five elements. We assumed that the individual blocks (and the system) are independent (the state of any block does not affect the state of other ones in any way) and two-state. Their availabilities can be characterized by probability of its failure $p_i$ and its reliability $r_i$. Their sum must be 1:

$$ r_i + p_i = 1 \quad i \in \{A; B; C; D; E\} $$

One of the applicable methods is to determine the reliability of BSS by the sum of the probabilities of an investigated system, system states.

**Figure 1 Bridge structure system**

The possible system states of the BSS are summarized in the form of a Truth Table, shown in Table 1, with each block assigned either a good or a faulty state. The $Q_i$ column includes the probabilities of each of the possible system states.

2.1. General Case

In the general case a BSS works if a matter, sign or energy “can cross” the system.

The state probabilities resulting in an operating system are included in the rows 1; 2; 3; 5; 6; 7; 9; 10; 11; 17; 18; 19; 21; 22; 25; and 27. The sum of the operating system state probabilities

$$ R_{sys} = Q_1 + Q_2 + Q_3 + Q_5 + Q_6 + Q_7 + Q_9 + Q_{10} + Q_{11} + Q_{17} + Q_{18} + Q_{19} + Q_{21} + Q_{22} + Q_{25} + Q_{27} $$

(2)
The reliability of the WLB can be calculated as the probability of the system state 1 (see Table I):

### 2.2. Wheatstone Like Bridge

In the automotive industry, sensors usually are Wheatstone Bridges or Wheatstone Like Bridges (WLB).

In the case of the working state of a sensor, its components should be in an operational state. So, the reliability of the WLB can be calculated as the probability of the system state 1 (see Table I):

The Table listing all possible states and their probabilities is called the Truth Table Method (TTM).

| $j$ | A | B | C | D | E | System | $Q_i$ |
|-----|---|---|---|---|---|--------|------|
| 1   | + | + | + | + | + | $r_A \ r_B \ r_C \ r_D \ r_E$ |
| 2   | - | + | + | + | + | $(1-r_A) \ r_B \ r_C \ r_D \ r_E$ |
| 3   | + | - | + | + | + | $r_A \ (1-r_B) \ r_C \ r_D \ r_E$ |
| 4   | - | - | + | + | - | $(1-r_A) \ (1-r_B) \ r_C \ r_D \ r_E$ |
| 5   | + | + | - | + | + | $r_A \ r_B \ (1-r_C) \ r_D \ r_E$ |
| 6   | - | + | - | + | + | $(1-r_A) \ r_B \ (1-r_C) \ r_D \ r_E$ |
| 7   | + | - | - | + | + | $r_A \ (1-r_B) \ (1-r_C) \ r_D \ r_E$ |
| 8   | - | - | - | + | + | $(1-r_A) \ (1-r_B) \ (1-r_C) \ r_D \ r_E$ |
| 9   | + | + | + | - | + | $r_A \ r_B \ r_C \ (1-r_D) \ r_E$ |
| 10  | - | + | + | - | + | $(1-r_A) \ r_B \ r_C \ (1-r_D) \ r_E$ |
| 11  | + | - | + | - | + | $r_A \ (1-r_B) \ r_C \ (1-r_D) \ r_E$ |
| 12  | - | - | + | - | + | $(1-r_A) \ (1-r_B) \ r_C \ (1-r_D) \ r_E$ |
| 13  | + | + | - | - | + | $r_A \ r_B \ (1-r_C) \ (1-r_D) \ r_E$ |
| 14  | - | + | - | - | + | $(1-r_A) \ r_B \ (1-r_C) \ (1-r_D) \ r_E$ |
| 15  | + | - | - | - | + | $r_A \ (1-r_B) \ (1-r_C) \ (1-r_D) \ r_E$ |
| 16  | - | - | - | - | + | $(1-r_A) \ (1-r_B) \ (1-r_C) \ (1-r_D) \ r_E$ |
| 17  | + | + | + | + | - | $r_A \ r_B \ r_C \ r_D \ (1-r_E)$ |
| 18  | - | + | + | + | + | $(1-r_A) \ r_B \ r_C \ r_D \ (1-r_E)$ |
| 19  | + | - | + | + | + | $r_A \ (1-r_B) \ r_C \ r_D \ (1-r_E)$ |
| 20  | - | - | + | + | + | $(1-r_A) \ (1-r_B) \ r_C \ r_D \ (1-r_E)$ |
| 21  | + | + | - | + | + | $r_A \ r_B \ (1-r_C) \ r_D \ (1-r_E)$ |
| 22  | - | + | - | + | + | $(1-r_A) \ r_B \ (1-r_C) \ r_D \ (1-r_E)$ |
| 23  | + | - | - | + | + | $r_A \ (1-r_B) \ (1-r_C) \ r_D \ (1-r_E)$ |
| 24  | - | - | - | + | + | $(1-r_A) \ (1-r_B) \ (1-r_C) \ r_D \ (1-r_E)$ |
| 25  | + | + | - | - | + | $r_A \ r_B \ r_C \ (1-r_D) \ (1-r_E)$ |
| 26  | - | + | - | - | + | $(1-r_A) \ r_B \ r_C \ (1-r_D) \ (1-r_E)$ |
| 27  | + | - | - | - | + | $r_A \ (1-r_B) \ r_C \ (1-r_D) \ (1-r_E)$ |
| 28  | - | - | - | - | + | $(1-r_A) \ (1-r_B) \ r_C \ (1-r_D) \ (1-r_E)$ |
| 29  | + | + | - | - | + | $r_A \ r_B \ (1-r_C) \ (1-r_D) \ (1-r_E)$ |
| 30  | - | + | - | - | + | $(1-r_A) \ r_B \ (1-r_C) \ (1-r_D) \ (1-r_E)$ |
| 31  | + | - | - | - | + | $r_A \ (1-r_B) \ (1-r_C) \ (1-r_D) \ (1-r_E)$ |
| 32  | - | - | - | - | + | $(1-r_A) \ (1-r_B) \ (1-r_C) \ (1-r_D) \ (1-r_E)$ |
3. The Monte-Carlo Simulation

The steps of MCSs are the following:

1) Determination or estimation of density functions \( f(x) \) of inputs.
2) Specified-distribution excitation of model by inputs based on the results of step 1).
3) Statistical investigation of output data collected from simulations conducted according to step 2).
4) Deduction of conclusions from the point of view of the engineering investigation.

3.1. Creation of Initial Data

As the first step, a statistical analysis was performed using available reliability data. The maximum, minimum, expected (mean) values and standard deviations, of block reliabilities were determined. The investigated time interval was 1 year (8760 hours). Due to the relatively small number of available data, the goodness-of-fit tests were left out. It was assumed that the measured parameters follow normal (Gauss) probability distribution.

| Number of samples | Mean    | St.Dev. | Minimum  | Maximum  |
|-------------------|---------|---------|----------|----------|
| \( r_A \)         | 21      | 0.940   | 0.00201  | 0.93032  | 0.94997  |
| \( r_B \)         | 19      | 0.9703  | 0.00071  | 0.96671  | 0.97360  |
| \( r_C \)         | 25      | 0.9398  | 0.00212  | 0.93047  | 0.94972  |
| \( r_D \)         | 20      | 0.9503  | 0.00039  | 0.94879  | 0.95136  |
| \( r_E \)         | 23      | 0.9104  | 0.01400  | 0.84079  | 0.97349  |

3.2. Simulation

During the simulation to generate actual values of reliability of blocks, the so-called Acceptance – Rejection method was used. Table 3 illustrates input data of the simulation based on the results of the statistical analysis.

| Mean | St.Dev |
|------|--------|
| \( r_A \) | 0.940 | 0.0020 |
| \( r_B \) | 0.970 | 0.0007 |
| \( r_C \) | 0.940 | 0.0021 |
| \( r_D \) | 0.950 | 0.0004 |
| \( r_E \) | 0.910 | 0.0140 |

Using the input data determined above, the reliability models as laid out in Section 2 were generated. During simulation the number of excitations was increased when the relative differences between actual and last result was more the 0.005. Figures 2 – 5 depict mean values and standard deviations of system reliabilities depend on the number of excitations. The results can be seen in Table 4 when the number of excitations is one million.

| Mean | St.Dev |
|------|--------|
| \( R_{sys} \) | 0.993685 | 1.5233 \( 10^{-4} \) |
| \( R_{WLB} \) | 0.740941 | 0.012050 |
Figure 2 Mean Values of System Reliability in General Case

Figure 3 Standard Deviations of System Reliability in General Case

Figure 4 Mean Values of Wheatstone Like Bridge Reliability
4. Determination of Requested Number for Spare parts

One of the most important questions from maintenance management point of view is the Required Number of Spare Parts (RNSP). Knowing the reliability of the system $R$, the RNSP can be determined by the equation

$$RNSP = \left\lceil \left( \frac{1}{R} - 1 \right)N \right\rceil,$$

where $N$ is the number of applied equipment (in this study: $N = 50000$).

For example, on the supposition that the uncertainty of system reliability follows a normal distribution; the probability of not having a spare part in the case of equipment failure is 0.05 and the following holds true:

$$R_{RNS} = m_R - 1.65s_R$$

The RNSPs were determined for the cases of different assessing uncertainty values. These results are shown in Table 5.

| Estimating Uncertainty | Number for Spare Part | WLB |
|------------------------|-----------------------|-----|
|                       | General Case          |     |
| 10 %                  | 328                   | 18928|
| 5 %                   | 331                   | 19343|
| 2 %                   | 334                   | 19821|
| 1 %                   | 336                   | 20152|
| 0,5 %                 | 338                   | 20438|
| 0,2 %                 | 340                   | 20798|
| 0,01 %                | 342                   | 21065|
5. Correlation Analysis

The correlation coefficient characterizes the strength of linear stochastic interdependencies of the random variables.

If one of the two parameters having a strong negative correlation should change in some direction, another one will change in the opposite direction most probably according to the correlation coefficient.

The correlation coefficient $r_{\eta\mu}$ can be estimated empirically by the equation

$$
r_{\eta\mu} = \frac{\sum_{i=1}^{n} \left( x_i - \bar{x} \right) \left( y_i - \bar{y} \right)}{\sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2 \sum_{j=1}^{n} (y_j - \bar{y})^2}}
$$

using the samples $x_1, x_2, \ldots, x_n$ and $y_1, y_2, \ldots, y_n$ which belong to the variables $\eta$ and $\mu$ [10].

The results of correlation analysis can be seen in Table 5 when the number of excitations is one million.

|       | $r_A$ | $r_B$ | $r_C$ | $r_D$ | $r_E$ |
|-------|-------|-------|-------|-------|-------|
| $R_{sys}$ | 0.4272 | 0.4668 | 0.6810 | 0.2299 | 0.2832 |
| $R_{WLB}$ | 0.1312 | 0.0443 | 0.1372 | 0.260146 | 0.9461 |

6. Conclusions

In this paper, a (n) MCS-based probabilistic uncertainty analysis method of BSSs’ reliability was presented. Its possibilities of implementation have been shown by a practical case study of the BSSs’ reliabilities.

The following conclusions can be deduced from the results of the analyses afore mentioned:

1) The proposed Monte-Carlo Simulation-based method can be used for investigating the uncertainty of system reliability and the Required Number for Spare Parts depending on the required estimating uncertainty.

2) The correlation analysis highlights

- the reliability of WLB has relative forceless correlations with the reliabilities of the elements A; B; C and D;
- the reliability of WLB has high positive correlations with reliability of block E, so improvement of E block reliability, results in increasing of reliability of WLB most significantly.

3) The disadvantage of the proposed TTM lies in the fact that the number of possible states of the system grows exponentially, depending on the number of blocks.

4) The drawback of MCS is that its elapsed time increases markedly, if the number of excitations increase (see Figure 8).

![Figure 7](image7.png)

*Figure 7 Correlation Coefficients of Wheatstone Like Bridge Reliability*

![Figure 8](image8.png)

*Figure 8 Elapsed Time of Monte-Carlo Simulation depends on Number of Excitations*

5) The shortcoming of the proposed parametric uncertainty analysis lies in the fact that its result can be impossible from technical point of view.

In the case of the reliability modeling, one may find theoretically a reliability which is greater than 1 or less than 0; because the probability distributions generally have infinite domains from a theoretical (mathematical) point of view. This drawback can be eliminated by Probability–Bounds Analysis [11].

The Author’s proposed prospective research direction in the future is the investigate of uncertainty analysis methodologies of SwCIs, such as vehicle sensory networks and sensors’ reliability based on the Probability-Bounds Analysis.
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