Cyclotron resonance lineshape in a Wigner crystal

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The cyclotron resonance absorption spectrum in a Wigner crystal is calculated. Effects of spin-splitting are modelled by substitutional disorder, and calculated in the coherent potential approximation. Due to the increasing strength of the dipole-dipole interaction, the results show a crossover from a double-peak spectrum at small filling factors to a single-peak spectrum at filling factors $\gtrsim 1/6$. Radiation damping and magnetophonon scattering can also influence the cyclotron resonance. The results are in very good agreement with experiments.

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Two-dimensional (2D) electron systems in strong magnetic fields have been studied intensively for the last 15 years. Most of the work has been concentrated on transport properties. Important information can also be extracted from measurements of high-frequency properties such as photoluminescence and the cyclotron resonance. A large number of experimental and theoretical studies have dealt with these phenomena. The interactions with the low-energy degrees of freedom influence the detailed structure of the high-energy resonance.

Experiments on 2D electron systems at small Landau level (LL) filling factors $\nu$ by Besson et al. and Summers et al. revealed intriguing changes in the cyclotron resonance absorption spectra as the filling factor $\nu$, and temperature were varied. For very small $\nu$ the spectra show two peaks. With increasing $\nu$, the absorption peak at the lower frequency increases in relative strength, and the peaks are shifted. Eventually, for $\nu \gtrsim 1/6$ the two peaks merge. The spectra also change in qualitatively different ways with increasing temperature for $\nu$ smaller and larger than $\approx 1/10$, respectively. It was speculated that this signaled a phase transition, for example the formation of a Wigner crystal.

In a recent paper, Cooper and Chalker introduced a model that explains the experimental results without invoking a phase transition. The idea behind the model is that, due to band-structure effects, spin-up and spin-down electrons have slightly different cyclotron resonance frequencies. At a finite temperature there are electrons with both spins, and the 2D system is a disordered mixture of the two spin species. In this model, the electrons (i.e. their guiding centers) occupy the lattice sites of a Wigner crystal (WC). The optical properties of this system near the cyclotron resonance frequency can be described in terms of excitations, “excitons”, from the lowest LL to the next LL. The excitons propagate on a triangular lattice with substitutional disorder. The propagation is caused by the electromagnetic dipole-dipole interaction, and its strength relative to the splitting between the spin-up and spin-down cyclotron frequencies determines the shape of the absorption spectrum. In Ref. the averaging over different disorder configurations was done numerically.

This paper presents a study of the effects of spin-splitting disorder treated within the coherent potential approximation (CPA) which makes it possible to obtain results that agree with experiment, with only a small calculational effort. We also present a thorough discussion of the physics behind the results. Moreover, we deal with two other processes that broaden the cyclotron resonance: radiation damping and thermal motion of the electron guiding centers treated in terms of magnetophonons (MP’s). Radiation damping gives a contribution to the resonance width comparable to experimental values once the electron density is large enough. In the case of a double-peak spectrum, the MP’s broaden the high-frequency peak more than the low-frequency peak.

The model that we will study describes excitons propagating in a WC with substitutional disorder. The WC is a convenient model system, but the results of the calculations should be similar also in an electron liquid. The Hamiltonian can be written

$$H = \sum_i \hbar \omega_i c_i \dagger c_i + \sum_{ij} t_{ij} c_i \dagger c_j,$$

where $c_i$ annihilates an exciton at site $i$. The site cyclotron frequency $\omega_i$ equals the bare cyclotron frequency $\omega_0 = eB/m^* \omega_c$ for the majority spins, and $\omega_c + \delta \omega$ for the minority spins. The exciton-hopping matrix elements due to the dipole-dipole interaction are given by

$$t_{ij} = -\frac{e^2}{8\pi \epsilon_0 \epsilon_r} |\mathbf{R}_i - \mathbf{R}_j|^3, \quad i \neq j,$$

while $t_{ii} = -\sum_{j \neq i} t_{ij}$. As will be seen, this fixes the cyclotron frequency at $eB/m^* \omega_c$ in a WC without disorder, in accordance with Kohn’s theorem. In Eq., $\mathbf{R}_i$ are lattice vectors of the WC, $\epsilon_0$ is the dielectric constant of vacuum, $\epsilon_r$ is the relative dielectric constant of the material (GaAs), and $l_c = (\hbar/eB)^{1/2}$ is the magnetic length. The Fourier transform $t_{\mathbf{q}}$ of the hopping matrix elements yields the exciton dispersion in a system without disorder. It is proportional to the trace of the dynamical matrix of a Wigner crystal in the absence of a magnetic field and can be related to the transverse and longitudinal phonon frequencies $\omega_{\mathbf{T}_\mathbf{q}}$ and $\omega_{L\mathbf{q}}$ by
$$t_q = \hbar(\omega_q^2 + \omega_L^2)/(2\omega_c).$$

For large magnetic fields and small filling factors Eq. (3) is, for all practical purposes, equivalent to the magnetoplasmon dispersion relation resulting from a lattice dynamics treatment.

The absorption probability of a circularly polarized photon with frequency $\omega$ near the cyclotron frequency, impinging on the electron system at right angle, is

$$P(\omega) = -\frac{e^2 n_e \omega_r}{2m^* c_0 \sqrt{\epsilon_r c_0 \omega}} \text{Im} \left[ G(q=0,\omega) \right].$$

This result treats the $A_{\text{rad}} \cdot j$ interaction between the radiation and the electrons to lowest order (Fermi golden rule). In Eq. (4), $n_e$ is the electron density, $c_0$ is the speed of light in vacuum, and $G(q,\omega)$ is the Fourier transform of the retarded exciton Green’s function

$$G_{ij}(t) = -i \theta(t) \langle [c_i(t), c_j^\dagger(0)] \rangle.$$  

We use the CPA to calculate $G(q,\omega)$. The CPA is known to give correct results in a number of important limits, and Persson and Ryberg used it successfully to solve the formally equivalent problem of optical absorption in an isotopically mixed adsorbate layer. The disordered system, where the exciton frequency at a site is $\omega_c$ with probability $(1-c)$ and $\omega_c + \delta \omega$ with probability $c$, is replaced by a translationally invariant system (effective medium) where the exciton frequency is $\omega_c + \sigma(\omega)$ everywhere. The exciton self energy $\sigma(\omega)$, which must be determined self-consistently, is frequency-dependent and, in general, complex. The Green’s function can be written

$$G(q,\omega) = [\omega + i \gamma - \omega_c - \sigma(\omega) - t_q/h]^{-1},$$

where $\gamma$ represents exciton damping due to other mechanisms than disorder (see below). In the CPA, one real site (with frequency $\omega_c$ or $\omega_c + \delta \omega$) is placed in the effective medium of $\omega_c + \sigma(\omega)$ sites. Requiring that the scattering off the real site, treated to all orders, should vanish on the average yields an equation for $\sigma(\omega)$:

$$\sigma(\omega) = \frac{c_0 \delta \omega}{1 - [\delta \omega - \sigma(\omega)] G_{00}(\omega)}.$$  

The diagonal Green’s function in real space

$$G_{00}(\omega) = A_{\text{BZ}}^{-1} \int_{\text{BZ}} d^2 q G(q,\omega),$$

where $A_{\text{BZ}}$ is the area of the first Brillouin zone (BZ). In the calculations, we have evaluated Eq. (8) accurately, but the analytic approximation

$$G_{00}(\omega) = \frac{\hbar}{W} \left[ \frac{\hbar \alpha}{W} \ln \left( - \frac{\hbar \alpha}{\hbar \alpha - W} - 1 \right) + \frac{\hbar/2}{\hbar \alpha - W} \right],$$

where $\alpha = \omega + i \gamma - \omega_c - \sigma(\omega)$ and $W$ is the exciton bandwidth, gives almost the same results.

There are a number of processes that can cause exciton damping described by the constant $\gamma$ in Eq. (3). Radiative decay is one of the most important, and it can be calculated in a standard way. Excitons with a wave vector larger than $\sqrt{\omega_c/c_0}$ do not radiate. For smaller wave vectors the radiation damping is

$$\gamma_q = \frac{e^2 n_e}{4m^* c_0 \sqrt{\epsilon_r c_0 \omega}} \left[ \frac{\omega_c}{\sqrt{\omega_c^2 - q^2 c_0^2/\epsilon_r}} + \frac{\omega_c^2 - q^2 c_0^2/\epsilon_r}{\omega_c} \right],$$

(neglecting exciton dispersion). This result resembles those for the decay of free excitons in quantum wells. Numerically $h \gamma_0 = 0.013 \text{ meV}$ when $n_e = 10^{11} \text{ cm}^{-2}$.

It should be emphasized that Eq. (9) yields $\sigma(\omega)$ with radiation damping included, whereas neglecting it may give $P > 1$. On resonance $\omega = \omega_c$, and without disorder $\sigma(\omega) = 0$, setting $\gamma = \gamma_q$ in Eq. (3), and then using Eq. (9) yields $P = 1$. The present calculations suppress the $q$ dependence of the damping. The constant $\gamma = \gamma_q = 0 + \gamma_{ph}$, using $h \gamma_{ph} = 0.005 \text{ meV}$ for the phenomenological damping constant.

Figure 1 shows the main results of this calculation. In Fig. 1(a) the magnetic field and temperature are kept constant, while the electron density varies. The qualitative behavior of the spectra as a function of $n_e$ is in close agreement with experiment (cf. Fig. 2 in Ref. 4). The bare cyclotron resonance lies at $h \omega_c = 27.64 \text{ meV}$ and the minority spin resonance is at $27.49 \text{ meV}$. If $n_e$ is chosen small enough, the absorption spectrum has two isolated resonances at these frequencies. However, all the spectra displayed in Fig. 1(a) are modified by the dipole-dipole interaction, since it is comparable in strength to the difference in resonance frequency between the two spin species. The exciton bandwidth $W \sim n_e^{3/2}$ increases from 0.10 meV at $\nu = 1/14.2$ to 0.67 meV for $\nu = 1/4.1$.

The physics behind the results of Fig. 1(a) can be understood in the following way (cf. Refs. 13 and 14): The transition between the two lowest LL’s at a site can be considered as a local oscillator. The high-frequency, majority oscillators respond in phase with the incident external radiation field when it has a frequency near the lower resonance. Then, the total field acting on the minority oscillators is enhanced due to anti-screening, and the low-frequency peak grows in strength with increasing $n_e$. At the same time, it is shifted towards higher frequencies because the majority-oscillator response is larger there. At high densities, only one peak centered at $\omega_m$
remains. This behavior is characteristic of motional narrowing. Once the hopping matrix elements become large enough, exciton hopping becomes so frequent that the excitons are not much affected by the disorder. In this regime, the disorder contribution to the linewidth behaves as $\sim \omega^2/W$, and decreases with increasing density. In an impurity-scattering language, the factor $1/W$ comes from the density of final states. The other contributions to the linewidth are not influenced by this narrowing, and in Fig. 3(a) the linewidth starts to grow at the highest densities due to radiation damping. This result is also in agreement with experiment.\footnote{Present address: European Synchrotron Radiation Facility, B. P. 220, F–38043 Grenoble, France. Email: johansso@esrf.fr}

Figure 3(b) shows absorption spectra at different temperatures and minority spin concentrations. At low enough temperature, there are only majority spins, and consequently only one peak. When the temperature is increased, this peak splits in qualitatively different ways depending on the density.

Consider now the effects of electron thermal and zero-point motion. The calculation of the exciton-hopping matrix elements in Eq. (2) assumes that the electrons are fixed at the lattice sites. In reality they perform vibrations so that the lattice site coordinates are time-dependent. To quantify this time-dependence, return to Eq. (2) and replace the lattice site coordinates $R_{ij}$ by $R_{ij} + u_{ij}$, where $u_{ij}$ is the electron displacement that can be described in terms of MP’s.\footnote{I have benefited from interesting discussions with Bo Persson on several occasions. I thank Alan Luther for useful comments on the manuscript.} Then expand the resulting expression to linear order in $u_{ij}$ recovering the static exciton-hopping matrix element plus a new term, describing exciton-MP interaction due to anharmonicity. This exciton-MP interaction Hamiltonian can be written

$$H_{\text{ex-mp}} = \sum_{q, q'} [M_{qq'} c_q^\dagger c_{q'} + H.c.],$$

where $a_q$ annihilates an MP of wave vector $q$.

The expression determining the matrix element $M_{qq'}$ is lengthy, however, $M_{qq'} = 0$ whenever $q$ or $q'$ or their difference vanishes. Thus, in a system without disorder, as a direct consequence of Kohn’s theorem, a $q = 0$ exciton is not scattered by the MP’s. But the exciton-MP interaction still influences the cyclotron resonance lineshape in the presence of disorder. The disorder scatters a long-wavelength exciton into an intermediate state with a finite $q$, from where the exciton can decay due to the interaction with the MP’s.

We include the effects of these processes on the exciton spectrum through a two-step calculation. First, the decay rate of an exciton with wave vector $q$, caused by $H_{\text{ex-mp}}$, is calculated in a system without disorder. The Fermi golden rule gives

$$\Gamma_q = \frac{2\pi}{\hbar} \sum_{q'} |M_{qq'}|^2 \left[ (1 + N_{q-q'}) \delta(t_q - t_{q'} - \hbar \omega_{q-q'}) + N_{q-q'} \delta(t_q - t_{q'} + \hbar \omega_{q-q'}) \right],$$

where $\omega_q$ is an MP frequency and $N_q$ is the thermal MP occupation. Then $i\gamma$ is replaced by $i(\gamma + \Gamma_q/2)$ in Eq. (3), and the same calculations as before yield the exciton self energy and Green’s function.

Figure 4 shows the results of these calculations. The most interesting behavior occurs for an intermediate value of the electron density ($n_e = 3.2 \times 10^{10}$ cm$^{-2}$). The low-frequency peak is slightly broadened when MP scattering is included, but this is a minute change. The high-frequency peak, on the other hand, is substantially broadened, and only a weak shoulder remains. This seems to be in qualitative agreement with the experimental results of Ref. 3. There (see Fig. 1), for a similar filling factor, the high-frequency peak is more temperature-sensitive than the one at lower frequency. Similar results were also found in Ref. 3, where these thermal effects were modelled by frozen structural disorder. At lower densities, when the spectrum approaches that of two isolated resonances, both peaks are somewhat broadened by the MP scattering. No results for the motional narrowing regime are shown, since the MP scattering has little influence on the spectrum there.

The above results can be explained by very much the same reasoning as was used earlier. In the intermediate density regime and at frequencies near the first peak, the oscillators respond more or less in phase with each other. The small changes in the dipole-dipole interaction caused by the MP’s do not then perturb the spectrum appreciably. For higher frequencies, near the second peak, nearest-neighbor oscillators may be out of phase with each other. Changes in the coupling due to the MP’s are much more important in this case.

In conclusion, this paper has presented a model calculation of the cyclotron resonance lineshape in a Wigner crystal. The effects of spin-splitting of the cyclotron resonance has been treated within the CPA. The resulting absorption spectrum has two relatively independent peaks, one for each spin species, at low electron densities. When the density increases, more and more strength is transferred to the low-frequency peak due to anti-screening effects until, for filling factors $\nu \gtrsim 1/6$, only one peak remains. The radiation damping contribution to the linewidth was calculated and found comparable to experimental linewidths for $\nu \gtrsim 1/4$. Finally, a study of the effects of magnetophonon scattering showed that it has small effects on the spectrum, except under special circumstances. All of the results are in good agreement with experiments.

I have benefited from interesting discussions with Bo Persson on several occasions. I thank Alan Luther for useful comments on the manuscript.
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FIG. 1. Calculated absorption spectra. (a) The electron density is varied as indicated next to the curves, while the magnetic field and temperature are kept constant. The minority spin concentration is 10%. At the lowest density, the two spin species give one resonance peak each, but anti-screening effects transfer spectral strength from the high-frequency, to the low-frequency peak. Eventually, with increasing density, one motionally narrowed peak emerges. It starts to broaden at the highest densities because of radiation damping. (b) Spectra for two different electron densities $n_e$ (3.8 x $10^{10}$ cm$^{-2}$ and 2.7 x $10^{10}$ cm$^{-2}$), with varying temperature (and minority spin concentration) as indicated next to the curves. The spectra split in qualitatively different ways depending on $n_e$.

FIG. 2. Absorption spectra calculated with and without exciton-magnetophonon (MP) scattering. The MP's are not effective in scattering long-wavelength excitons. Thus, for the higher electron density, only the high-frequency peak is considerably affected by the MP’s. At the lower density, both peaks are slightly broadened.
Absorption (20%/div.)

\[
\begin{array}{cccc}
\text{Photon energy (meV)} & 27.4 & 27.5 & 27.6 & 27.7 & 27.8 & 27.9 \\
\hline
\text{\(n_c/10^{10}\) cm}^{-2} & 9.4 & 5.7 & 4.6 & 3.8 & 2.7 \\
\hline
\nu & 1/4.1 & 1/6.8 & 1/8.4 & 1/10.1 & 1/14.2 \\
\end{array}
\]

\(B = 16\ T\)
\(T = 2.2\ K\)
Absorption (20%/div.)

(b) $B = 14\,\text{T}$

$\nu = 1/8.9$

$\nu = 1/12.4$

$c$

$3.2 \times 10^{-5}$

$3.1 \times 10^{-2}$

$0.20$

$0.4\,\text{K}$

$1.2\,\text{K}$

$3.0\,\text{K}$

Photon energy (meV)
Absorption (10 %/div.)

\[ B = 14 \text{ T} \]
\[ T = 8 \text{ K} \]

Without MP's

With MP's

\[ 3.2 \times 10^{10} \text{ cm}^{-2} \]

\[ 1.8 \times 10^{10} \text{ cm}^{-2} \]