Wavefront of light passing through turbulent atmosphere gets distorted. This causes signal loss in free-space optical communication as the light beam spreads and wanders at the receiving end. Frequency and/or time division multiplexing adaptive optics (AO) techniques have been used to conjugate this kind of wavefront distortion. However, if the signal beam moves relative to the atmosphere, the AO system performance degrades due to higher Greenwood frequency. Here we solve this problem by adding a pioneer beacon that is spatially separated from the signal beam with time delay between spatially separated pulses. More importantly, our protocol works irrespective of the signal beam intensity and hence is also applicable to secret quantum communication. In particular, using semi-empirical atmospheric turbulence calculation, we show that for low earth orbit satellite-to-ground decoy state quantum key distribution, our method increases the secret key rate by about 27% over systems that use frequency and/or time division multiplexing. Finally, we propose a modification of existing wavelength division multiplexing systems as an effective alternative solution to this problem.

I. INTRODUCTION

Optical free-space communications and astronomical imaging are affected by atmospheric turbulence due to fluctuation of air density, pressure, and temperature. This turbulence induces a time-dependent inhomogeneous refractive index in air, distorting the wavefront of electromagnetic waves. Hence, light beam spreads and wanders at the detection end causing signal loss. High fidelity signal or image is obtained if one could adaptively and dynamically conjugate the optical path difference caused by the wavefront distortion. Adaptive optics (AO) is a well established method to achieve this goal [1–3]. In the most basic AO setup, a deformable mirror is used to collect light signal and a beam splitter is placed in front of the signal detector acting as a signal sampler to divert some signal light to a wavefront sensor. The detection result of this sensor is then used to estimate the wavefront distortion. Finally, one can adaptively conjugate this estimated distortion via fast feedback control of the deformable mirror through actuators to obtain a high fidelity signal or image [1–4].

Many variants of this basic setup have been proposed and used in the field. For instance, one may replace the beam splitter by a wavelength selector plus an additional beacon beam emitting light with a different wavelength from that of the signal beam. This wavelength-division multiplexing (WDM) setup is effective if the wavelengths of the two beams are close enough so that the wavefront distortion inferred from the beacon beam is close to that of the signal beam. And at the same time, the wavelength difference is big enough to avoid cross-talk between the two beams. Another variant is to use time-division multiplexing (TDM) method in which the beam splitter is replaced by an optical switch and a pulsed beacon beam [3]. We remark that in most WDM and TDM setups, the beacon and the signal beams are spatially coincided. In order to work, the WDM and TDM methods must use a sufficiently high intensity beacon beam so that the wavefront sensor can detect enough photons per unit time to estimate the wavefront distortion accurately. In contrast, the brightness of the signal source is irrelevant as far as AO correction is concerned. That is to say, both WDM and TDM methods work for low intensity signal sources, including most quantum signal sources used in secure quantum communication. In fact, WDM has been used in a few recent quantum communication experiments [5, 6].

A new challenge is faced if the signal source moves sufficiently fast relative to the atmosphere in the sense that its speed is much faster than the atmospheric wind speed. The faster the source, the faster the change in wavelength distortion. Thus, the higher the Greenwood frequency, namely the reciprocal of the beam wandering time [3, 7]. Our method to tackle this challenge is inspired by astronomical imaging of dim celestial objects. Recall that astronomers use an artificial high intensity laser guide star placed angularly close to the dim astronomical object as beacon source to replace the role of the diverted signal light [1, 2, 8]. With this inspiration, we solve the moving source problem by using two set of spatially separated artificial sources emitting at the same or nearly the same wavelength — a set of (pulsed) pioneer beacon source(s) to perform effective AO correction and another set of time-delayed (pulsed) signal source(s) for the actual optical communication. In essence, our proposal is a time-delayed spatial multiplexing protocol.

For concrete illustration, we consider the following prototype from now on although the general concept works in a much wider context. As shown in Fig. 1, we consider...
the satellite-to-ground communication setup with both the beacon and signal sources are fixed on a low earth orbit (LEO) satellite together with a stationary ground-based receiver telescope. By pioneer beacon source, we mean that the beacon beam is put in front of the signal beam along the direction of motion of the satellite relative to the ground. Furthermore, we fire each pulsed pioneer beacon beam shortly before firing the corresponding pulsed signal beam. In doing so, the beacon beam acts like a pioneer that probes the wavefront distortion of an optical path that will shortly be traveled by the signal beam. Specifically, if the two beams move sufficiently rapidly relative to the detector(s), the pulsed pioneer beacon beam and the corresponding delayed pulsed signal beam can be made to travel along essentially the same optical path by carefully tuning the delay time. Consequently, provided that the artificially imposed delay time between the two pulsed beams is shorter than the reciprocal of the Greenwood frequency, our technique should achieve almost the same level of AO correction for stationary sources without spatial multiplexing. As the two light sources are multiplexed spatially, their signals can be separated effectively by focusing the light beams provided that the angular separation of their images after applying AO correction is greater than the resolving power of the ground-based telescope and that the cross-talk between them is sufficiently small. In fact, a recent experiment using the 1 m telescope in Mount Stromlo Observatory plus AO imaging technique succeeded to image an artificial satellite down to 85 cm in size at 1000 km range [9]. This implies that using telescope with aperture larger than about 1 m, our prototype is able to resolve and separate the pioneer beacon and signal beams mounted on a typical-sized artificial satellite.

Note that we study this prototype because this is one of the most challenging situations in realistic applications. As the effectiveness of our approach does not depend on the nature of the signal light source, so by the same logic, we choose our signal source to be a phase-randomized weak coherent quantum source performing decoy state quantum key distribution (QKD). In this way, we could demonstrate the strength of our approach and compare it with existing ones. In fact, free-space channel is used in quantum communication because it has a lower attenuation rate than optical fibers of the same length [10]. No wonder why several pioneering demonstrations of long distance free-space QKD, including ground-to-ground and satellite-to-ground ones, have been reported [5, 6, 11, 12]. For free-space QKD, existing AO technologies are able to increase the key rate by reducing the widening effect and spatial noise of the signal so that the system can get a higher yield or coupling efficiency even in daytime [6, 13]. And to the best of our knowledge, all AO-based free-space QKD experiments to date use WDM [5, 6]. A drawback of this approach is that the different wavefront distortions experienced by the beacon and signal beams generally increases with communication distance. This could lower the yield and key rate when this distance is long. Alternatively, one may use TDM. The main disadvantage of TDM is that the signal rate has to be lowered as communication time is shared with the beacon beam.

We begin by presenting the atmospheric model and system parameters used in our investigation in Sec. II. Then in Sec. III, we introduce our time-delayed spatial multiplexing method that uses a pulsed pioneer beacon beam plus a time-delayed pulse signal beam. We also analyze its effectiveness in transmitting information through the dynamical atmosphere. In Sec. IV, we show the schematic design of the spatial multiplexing system and discuss the cross-talk due to the pioneer beacon. With the above preparatory works, we study the performance of our scheme for the case when cross-talk between the pioneer beacon and the signal beams can be ignored in Sec V. Specifically, for the case of our concrete illustrative example, our scheme always gives higher Strehl ratio and transmission efficiency over those that use pure TDM or WDM. Besides, we analyze situation in which cross-talk cannot be ignored in Sec. VI. There we compute the secret key rate of decoy state BB84 QKD that is optimized over signal beam parameters. Again, we find that for our concrete illustration, our scheme always gives a higher provably secure key rate over pure TDM and WDM protocols. Actually, in some cases, our semi-empirical calculation shows that the improvement is as large as 27%. As our setup is new and its construction is engineering demanding, a compromise is to upgrade existing WDM systems to combine the beacon and signal beams. We report the performance of this modification in Sec. VII. We find that for zenith angle less than about 20°, the provably secure key rate of this modification is at least 95% of our time-delayed spatial multiplexing prototype reported in Sec. VI, making it an attractive practical alternative. Finally, we summarize our findings in Sec. VIII. The present work is based on our recent

FIG. 1. Satellite location and beams’ path at \( t = 0 \) and \( t = T_r \). Here \( \theta_s \) is the angular separation of the pioneer beacon and signal beams (as observed from the receiver), and \( \theta_1 \) is the angle traveled by the signal (pioneer beacon) beam’s path from \( t = 0 \) to \( t = T_r \) [see the solid (dash) lines]. In addition, \( \theta_2 \) is the angle between the pioneer beacon beam’s path (solid line at \( t = 0 \)) and the delayed signal beam’s path (dash line at \( t = T_r \)).
II. ATMOSPHERIC MODEL AND SYSTEM PARAMETERS OF THE FREE-SPACE COMMUNICATION CHANNEL

A. Atmospheric Model

One of the most important quantities that characterize the atmospheric coherence diameter due to turbulence is the Fried parameter \( r_0 \) [16]. In unit of meters, its value varies with the altitude and the zenith angle according to the equation

\[
r_0 = \left[ 0.423k^2 \sec(\zeta) \int_0^{h_{\text{max}}} C_n^2(h) \, dh \right]^{-3/5},
\]

where \( \zeta \) is zenith angle, \( k \) is the wavenumber of the light measured in m\(^{-1} \), \( h_{\text{max}} \) is the altitude of the moving source measured in meters, and \( C_n^2(h) \) is the refractive index structure parameter at altitude \( h \). Here we assume that \( C_n^2(h) \) follows the Hufnagel-Valley model [17], namely,

\[
C_n^2(h) = 0.00594 \left( \frac{w}{27} \right)^2 \left( \frac{h}{10^5} \right)^{10} \exp \left( -\frac{h}{1000} \right) + 2.7 \times 10^{-16} \exp \left( -\frac{h}{1500} \right) + 1.7 \times 10^{-14} \exp \left( -\frac{h}{100} \right)
\]

with \( h \) measured in meters. In most literature, \( w = 21 \) m/s is the pseudo-wind speed, taken to be the average wind speed of the jet stream.

Isoplanatic angle \( \theta_0 \) and Greenwood frequency \( f_G \) are two quantities that can be used to characterize the spatial and temporal limits in AO. At the receiving end, light rays coming from a cone with an angle much smaller than \( \theta_0 \) have about the same optical path length. And \( f_G \), which is the reciprocal of the beam wandering time, is an effective way to approximately quantify the rate of change of turbulence [3, 7]. Clearly, for a stationary source, AO is effective only if the angular separation between the (pulsed) beacon beam and the (pulsed) signal beam is much less than \( \theta_0 \). Moreover, the time delay between these two pulsed beams is much less than \( 1/f_G \). (Note that the situation is different from moving sources. We shall thoroughly analyze this situation in Sec. III and beyond.) These two quantities can be computed via \( C_n^2(h) \) through

\[
\theta_0 = \left[ 2.913k^2 \sec^{8/3}(\zeta) \int_0^{h_{\text{max}}} h^{5/3}C_n^2(h) \, dh \right]^{-3/5},
\]

and

\[
f_G = \left[ 0.1022k^2 \sec(\zeta) \int_0^{h_{\text{max}}} h^{5/3}C_n^2(h) \, dh \right]^{3/5},
\]

where \( v(h) \) is the wind speed at altitude \( h \).

For the case of moving source, \( v(h) \) used in Eq. (4) should be the natural wind speed plus \( v_{\text{wind}}(h) \) the apparent wind speed \( v_{\text{app}}(h) \) due to the movement of the source

\[
v(h) = v_{\text{wind}}(h) + v_{\text{app}}(h).
\]

This assumption of simply adding two scalar speeds is justified when the moving source is mounted on a LEO satellite because it moves at great angular speed so that \( v_{\text{app}} \gg v_{\text{wind}} \). We further assume that the natural wind speed follows the altitude-dependent Bufton wind profile [18]

\[
v_{\text{wind}}(h) = v_g + 30 \exp \left[ -\left( \frac{h - 9400}{4800} \right)^2 \right].
\]

Here \( v_g \) is the natural wind speed and is taken to be 5 m/s in our analysis.

B. LEO Satellite and Receiving End Telescope

LEO satellite-to-ground communication is considered in this paper because of its low aperture-to-aperture loss and high speed features. We set the altitude \( h_{\text{max}} \) of the satellite to 400 km. The distance between the transmitter and receiver can be expressed as [19],

\[
z_{\text{max}} = \sqrt{h_{\text{max}}^2 + 2h_{\text{max}}R_\oplus + R_\oplus^2 \cos^2(\zeta) - R_\oplus \cos(\zeta)},
\]

where \( R_\oplus \) is the Earth radius. For simplicity, we assume that the satellite is moving in a circular orbit. Thus, the angular slewing rate is equal to

\[
\omega_s = \left[ \frac{GM_\oplus}{h_{\text{max}}^2(h_{\text{max}} + R_\oplus)} \right]^{1/2} \cos^2(\zeta),
\]

where \( G \) is the universal gravitational constant and \( M_\oplus \) is the Earth mass. \( \omega_s \) is used to calculate the apparent wind speed

\[
v_{\text{app}}(h) = \omega_s h.
\]

At the receiving end, the aperture coupling efficiency can be approximated by using Gaussian beam equation [20]

\[
\eta_{\text{geo}} = 1 - \exp \left[ -\frac{1}{2} \frac{D^2}{\omega^2(\lambda, z)} \right],
\]
where $D$ is the diameter of the telescope aperture, $\lambda$ is the wavelength, $z$ is the propagation distance, and the $\omega(\lambda, z)$ is the waist function

$$\omega^2(\lambda, z) = \omega_0^2 \left[ 1 + \frac{z^2}{2z_R^2(\lambda)} \right], \quad (11)$$

with $z_R = \pi \omega_0^2 / \lambda$ being the Rayleigh range and $\omega_0 = 0.7 D T / 2$ being the beam waist. We take $D T = 0.05$ m as the diameter of the transmitter aperture. The telescope parameters used are based on a real telescope in the Lulin observatory [21]. It is a Cassegrain telescope with diameter $D = 1.03$ m, secondary mirror diameter $0.36$ m, and effective focal length $f = 8$ m.

C. Wavelength Selection

As pointed out in Ref. [20], a shorter wavelength gives better quantum channel performance due to the spatial filtering strategies, geometric coupling and size of focus spot. This conclusion is consistent with the dependence of $\eta_{\text{geo}}$ on $\lambda$ as shown in Eqs. (10) and (11). Moreover, we assume there is a field stop (FS) in front of the signal receiver to filter the background noise. The size of this FS is taken as the diffraction limit of the signal beam. In this configuration, the FS can filter most of the background light while 84% of the signal can pass through (if the signal is not distorted). Since the spot size of the beam is proportional to its wavelength, a longer wavelength increases the size of the FS and number of background photons that pass through.

Based on these factors and some site-specific conditions, Ref. [6] used $780$ nm as the wavelength of the signal beam and $808$ nm as the wavelength of the beacon. They successfully performed a daytime quantum communication. As our paper focuses more on quantum scenarios, we follow Ref. [6] by fixing the wavelengths of both the signal and beacon beams in our time-delayed spatial multiplexing prototype to $780$ nm. And for the WDM systems that we use for performance comparison, the beacon wavelength used is set to $808$ nm.

III. REDUCING THE APPARENT WIND SPEED BY USING A PIONEER BEACON AND A TIME-DELAYED SIGNAL PULSE

For source(s) moving relative to the detector(s), we can optimize the performance of our time-delayed spatial multiplexing prototype as follows. For simplicity, we assume that the sources and detector(s) are separately mounted on a rigid body. We place the beacon source ahead of the signal source(s) in the sense that $\vec{v}_t \cdot \vec{L} \geq 0$ at all time where $\vec{v}_t$ and $\vec{L}$ are the instantaneous tangential velocity vector of the source relative to the detector(s) and the instantaneous position vector of the signal source relative to the beacon source, respectively. More importantly, we set a delay time $T_r$ between the beacon and signal sources and adjust it possibly dynamically so as to reduce the angular separation between the optical paths of the advanced beacon source and the delayed signal source. (See Fig. 1 for an illustration.) In particular, suppose $\vec{v}_t \parallel \vec{L}$, then there is a delay time $T_r$ after which the advanced beacon source and the delayed signal source will propagate along the same optical path. Consequently, these two sources will have the same wavefront distortion provided that $T_r$ is much shorter than the atmospheric temporal fluctuation timescale characterized by $1 / f_G$. Under this condition, our method should greatly increase the fidelity of the signal source.

To quantify the improvement of this setup, we calculate the Greenwood frequency $f_G$ using Eq. (4). The major difference between stationary and moving sources is the apparent wind caused by the motion. Combining Eqs. (5), (6) and (9), we get the total wind speed [22],

$$v(h) = \omega_s h + v_g + 30 \exp \left[ -\left( \frac{h - 9400}{4800} \right)^2 \right]. \quad (12)$$

Since $v_{\text{app}} \gg v_{\text{wind}}$, the Greenwood frequency for the LEO satellite tracking case can be much higher than the intrinsic frequency of the atmospheric turbulence (for a stationary source). As shown in Fig. 2, when the zenith angle is $0^\circ$, $f_G$ intrinsic to the channel is about $42$ Hz while $f_G \approx 250$ Hz when slewing to track (but with $L = 0$ m) the LEO satellite is included.

Recall that we put the pioneer beacon beam ahead of the signal beam. We also set a delay time of $T_r$ between the beacon beam and the signal beam. For simplicity, we assume that $\vec{v}_t \parallel \vec{L}$ and the response time of the AO system is less than or equal to $T_r$. In this way, when the system receives the beacon signal at $t = 0$, it compensates the signal at $t = T_r$. Fig. 1 shows that if both beams are placed at the same physical location, the angle between the two timestamps is larger then the case which the beams are spatially separated. The apparent wind speed can therefore be reduced by a factor of $\theta_1 / \theta_2$. The equivalent angular slewing rate is

$$\omega'_s = \omega_s \frac{\theta_1}{\theta_2} = \omega_s \frac{\theta_2 - \theta_s}{\theta_2} = \omega_s \left| \frac{\omega_s T_r - \theta_s}{\omega_s T_r} \right| = \left| \omega_s - \frac{\theta_s}{T_r} \right|, \quad (13)$$

where $\theta_s = L / z_{\text{max}}$ is the angular separation between the beacon and the signal beam. Combined with Eqs. (4) and (12), it is clear that one can completely eliminate the effect of apparent wind speed and hence attain optimal performance for the AO system if $\theta_s / T_r = \omega_s$. Indeed this is what we observed in the $\zeta - f_G$ plot in Fig. 2.

Note that for $\theta_s / T_r < \omega_s$, the performance of our setup is worse than the case of a stationary source because the delay time $T_r$ is not short enough to allow the pulsed signal and beacon beams to travel through an almost identical optical path. Moreover, for fixed $L$ and $\zeta$, the Greenwood frequency $f_G$ increases with $T_r$. This is what
we expect from Eqs. (12) and (13). The more interesting case is when \( \theta_s/T_r > \omega_s \). In this case, the reduction in performance as reflected by the value of \( f_G \) is because the delay time \( T_r \) is too fast. Surely, we can artificially lengthening the delay time \( T_r \) or shortening the spatial separation \( L \) between the beacon and signal beams to fix \( f_G \) to its optimal value. Using this time delay adjustment trick, Eq. (13) can therefore be re-written as

\[
\omega'_s = \begin{cases} \omega_s - \theta_s/T_r & \text{for } \omega_s > \theta_s/T_r, \\ 0 & \text{for } \omega_s \leq \theta_s/T_r. \end{cases}
\]  

(14)

Last but not least, in Fig. 2, in essentially all zenith angles used in practice (more precisely whenever \( \zeta \lesssim 65^\circ \)), the Greenwood frequency curves calculated with \( L = 2 \text{ m} \) are lower than the curve calculated with no spatial separation. Since both \( \omega_s \) and \( \theta_s/T_r \) decrease when the zenith angle increases, the curves decrease and approach to the intrinsic frequency curve. Also, as \( \omega_s \) decreases more rapidly than \( \theta_s/T_r \) as \( \zeta \) increases, the curves with spatial separation intersect the curve without slewing. This means \( \theta_s/T_r = \omega_s \) at that point. For zenith angle larger than this point, \( \theta_s/T_r > \omega_s \), the delay time should be decreased to keep the frequency near the intrinsic frequency. Finally, we remark that the condition \( \vec{v}_L \parallel \vec{L} \) is essential in tuning \( \theta_1 \) to 0 and hence attaining the intrinsic Greenwood frequency. Nevertheless, as long as \( \vec{v}_L \cdot \vec{L} > 0 \), it is possible to improve the system performance by carefully adjusting \( T_r \) or \( \theta_s \).

To summarize, our time-delayed signal pulse trick can always reduce the Greenwood frequency. More importantly, it could be reduced to the case without spatial separation of the beacon and signal beams whenever \( \omega_s \leq \theta_s/T_r \). Thus, we expect this time-delayed method to be effective in obtaining a high fidelity signal through AO. We are going to show that this is indeed the case in Sec. V.

**IV. THE ADVANTAGE OF SPATIAL MULTIPLEXING IN OUR SETUP**

From the discussion of the previous section, we expect that the time dependent feature in our setup is compatible with WDM and TDM systems in the sense that the pioneer beam setup can be built on top of them to reduce the apparent wind speed. However, we can further improve the AO system by using spatial multiplexing. As the beacon beam is physically separated from the signal beam, they can be distinguishable spatially. The setup itself is a spatial multiplexing system, which has two advantages compare to pure WDM and TDM systems. First, the wavelength of the sources can be the same so that chromat effects can be ignored. Second, there is no need to temporally interlace the signal pulses with beacon pulses. For our LEO satellite setup, the two beams can be spatially resolved using the technology that images a satellite at 1000 km range through a 1 m telescope with AO [9].

**A. Design Of The Beacon And Signal Beams**

Recall that the intuition of our improved method is that two physically nearby light beams of similar frequency pass through more or less the same air column at more or less the same time should be distorted in roughly the same way. Hence, a wavefront correction method based solely on the signal received by a wavefront sensing module that detects the pioneer beacon source beam should be able to correct both light beams at the same time with high fidelity. One may ask why we do not put the two beams together as time multiplexing technique should also work as long as the time interval of beam switching is much shorter than the change in atmospheric wavefront distortion. Our answer is that although pure wavelength division multiplexing works as demonstrated by recent experiments [5, 6], we are going to report in Sec. VI that our technique can attain a better secret key rate for moving sources. Specifically, by placing the pioneer beacon beam ahead of the signal beam along the direction of motion of the sources relative to the receiver, our method can better correct wavefront distortion. In fact, Eq. (13) in Sec. III gives the condition on the delay time \( T_r \) needed to make the two beams to travel through almost the same optical path. Consequently, if the atmospheric turbulence fluctuation time scale is much shorter than this delay time, the level of AO correction should
be equal to the situation of non-moving sources.

Fig. 3 shows the schematic of spatial multiplexing AO system. It consists of two physically nearby sources as well as a wavefront sensing module that detects the beacon source beam plus a nearby signal detection module that detects the signal source beam(s). To reduce photon loss in long distance communication, each of the beam source is placed at the focus of telescope on the satellite so that the emitted light beam close to the source can be well approximated by traveling plane wave. Our hope is that with this spatial configuration the optical paths of the two set of sources with the same or almost the same wavelength should experience more or less the same wavefront distortion. The wavefront correction then goes as follows. The beacon detection module estimates the atmospheric distortion and generates feedback signals to the control system. Then the control system drives the actuators of the deformable mirror or the spatial light modulator in the AO system. This should correct the wavefront distortion of the beacon beam as well as the possibly much weaker signal source beam simultaneously. Surely, in order to work, the two set of sources must be placed sufficiently close so that their optical paths are similar and at same time sufficiently far apart so that cross-talk between the beacon and signal source(s) due to effects such as diffraction and scattering is negligible.

The spatial configuration of our method is similar to the standard artificial guide star technique used in observational astronomy [23]. Note, however, that there are two major differences. First, all sources we used are artificial. Second, our beacon source is placed physically closed (and not just close in terms of apparent angular separation) to the signal source(s). We remark that this spatial configuration works not just for secure quantum communications. It is directly applicable to classical optical communication in free-space as well. And in this case, the intensity of the signal source(s) need not be low. In addition, our method is applicable to ground-based, air-to-ground as well as satellite-to-ground communications, stationary as well as moving sources relative to the sensing and detecting modules. Furthermore, a nice feature of our method is that the signal transmission rate will then be independent of the beacon source.

B. Minimum Physical Distance Between The Beacon And Signal Sources

The minimum possible distance of the beacon and signal sources is determined by both the resolving power of the optics and the level of cross-talk between the two set of sources. Note that upon successful AO correction, the center of the image of the beacon beam should be around the center of the optically sensitive surface of the wavefront sensing module. We put a field stop in the signal detection module to filter the noise spatially. Naturally, we set the radius of the field stop to the diffraction limit of the signal detection module [13]. The diffraction pattern depends on the structure of the telescope. In our concrete illustration, we use a 1.03 m Cassegrain telescope whose parameters are taken from a real telescope in Lulin Observatory [21]. The light intensity of the beacon beam at a distance $x$ away from the center equals

$$I_R(x) \approx I_R(0) \left( \frac{f \lambda}{\pi D x} \right)^2 \left[ J_1 \left( \frac{\pi D x}{f \lambda} \right) - b J_1 \left( \frac{b \pi D x}{f \lambda} \right) \right]^2,$$

(15)

where $f$ is the effective local length of the telescope, $b = 0.36/1.03$ is ratio of the diameters of the secondary to primary mirrors of the Cassegrain telescope used, $I_R(0) \approx 2\epsilon_0 c^3 \pi^2 (D/2)^4 / R^2$, and $J_1(\cdot)$ is the order one Bessel function of the first kind. Hence, the total light energy flux of the beacon beam imparted on the optically sensitive surface of the signal detection module is $\int_{FS} I_R(x) \, dA$ where the integral is over the area of the field stop of the signal detection module. For example, when $L = 2$ m, $\int_{FS} I_R(x) \, dA = 4.36 \times 10^{-15} I_R(0)$. The minimum distance should be set according to the required decay from the beam center. Otherwise, stray beacon beam photons will seriously affect the signal detection statistics.

C. Scattering Noise by the Strong Beacon Beam

The scattering caused by the strong beacon beam will affect the background noise of the system and hence in satellite-to-ground QKD application the final secret key rate. Some photons from the beacon may enter the signal receiving module and create errors. Here we estimate the scattering by the strong laser in the clear sky scenario. We use sky-scattering noise to get a rough estimate on the laser scattering noise. The equation for calculating the number of sky-noise photons entering the system is given by [13],

$$N_b = \frac{H_b(\lambda) \Omega_{FOV} \pi D_R^2 \lambda \Delta \lambda \Delta t}{4hc},$$

(16)

with $H_b$ in W m$^{-2}$ sr$\mu$m is the sky radiance, $\Omega_{FOV} = \pi \Delta \theta^2 / 4$ is the solid-angle field of view with a field stop,
\( D_R \) is diameter of the receiver primary optic, \( \Delta \lambda \) equals to the spectral filter bandpass in \( \mu \text{m} \), and \( \Delta t \) is the photon integration time of the receiver measured in meters. Furthermore, \( \Delta \theta \) is calculated by \( D_{WS}/f \) with \( D_{WS} \) being the diameter of the field stop. We assume \( \Delta \lambda = 1 \mu \text{m} \) as both beams use the same or nearly the same wavelength, the spectral filter is not able to block the photons from the beacon beam.

In astrophotography, a bright star that is close to target can be used as a beacon to probe the channel. Therefore, the brightness of the beacon laser should be similar to a bright star. The sky radiance caused by the laser can be estimated by the sky radiance by the stars. Under moonless clear night condition, the typical sky radiance is about \( 1.5 \times 10^{-5} \text{ W m}^{-2} \text{ sr} \mu \text{m} \) [24]. Using the parameters mentioned above and let \( \Delta t = 1 \text{ ns} \), the probability of receiving a beacon photon will be in the order of \( 10^{-8} \), which is good enough in practice.

V. COMPARISON OF PERFORMANCE OF OUR PROTOTYPE WITH THOSE USING PURE TDM AND WDM

A. The Strehl Ratio

The Strehl ratio \( S \) is a well-known metric to determine the turbulence strength and performance of optical systems. It is defined as the ratio of the peak intensity of a distorted beam spot and the peak intensity of the beam with no distortion. If Strehl ratio equals to one, the wavefront is not aberrated. Without using AO, the Strehl ratio of the signal is [3]

\[
S_{\text{aber}} = \left[ 1 + \left( \frac{D}{r_0} \right)^{5/3} \right]^{-6/5}. \tag{17}
\]

When AO is used, the performance of the system can be estimated by [3]

\[
S_i = \exp(-\sigma_i^2), \tag{18}
\]

where \( S_i \) is the Strehl ratio of system \( i \), and \( \sigma_i^2 \) is the corresponding wavefront variance, which leads to system performance degradation. Here we compare three systems, namely, those using time-delayed spatial multiplexing, pure TDM, and pure WDM. Their wavefront variance can be written as

\[
\sigma_{SS}^2 = \sigma_{\text{band}}^2 + \sigma_{\text{iso}, SS}^2, \tag{19}
\]

\[
\sigma_{TDM}^2 = \sigma_{\text{band}}^2 + \sigma_{\text{iso}, TDM}^2, \tag{20}
\]

and

\[
\sigma_{WDM}^2 = \sigma_{\text{band}}^2 + \sigma_{\text{iso}, WDM}^2 + \sigma_\phi^2 + \sigma_{\text{ch}}^2 + \sigma_d^2, \tag{21}
\]

where \( SS \) (SS) indicates that the system is (is not) using the pioneer beacon setup, and the descriptions of the \( \sigma^2 \)'s are as follows [3, 25]

- \( \sigma_{\text{band}}^2 \): bandwidth limitation induced wavefront variance;
- \( \sigma_{\text{iso}}^2 \): temporal and spatial anisoplanatism induced wavefront variance;
- \( \sigma_d^2 \): chromatic effect on the diffraction pattern induced wavefront variance;
- \( \sigma_{\text{ch}}^2 \): chromatic path length error induced wavefront variance; and
- \( \sigma_\phi^2 \): chromatic anisoplanatism induced wavefront variance.

Details of the calculations and expressions of these \( \sigma^2 \)'s can be found in Appendix A from Eq. (A1) to Eq. (A13). As the aim of this Subsection is to compare different multiplexing methods, we ignore system degradation due to factors that are not related to temporal, chromatic or anisoplanatic effects in our calculation. Furthermore, for simplicity, we do not take interlacing into account for all TDM calculations in this paper.

Fig. 4 shows the results of the calculation. With AO compensation, \( S \) is improved by at least two orders of magnitude. Among the three types of AO systems, the time-delayed spatial multiplexing one has the highest overall \( S \). At large zenith angle \( \zeta \), performance of the pure WDM system drops significantly due to chromatic effects. On the other hand, other systems perform better at large \( \zeta \) due to slower slewing rate. This is also the
reason why the difference between the time-delayed spatial multiplexing system and the TDM decreases along \( \zeta \). Furthermore, as we have ignored the effect of interlacing beams, the Strehl ratio calculation for the pure TDM case is slightly overestimated. Thus, the actual performance of our time-delayed spatial multiplexing prototype is slightly better than the TDM method plotted in Fig. 4. More importantly, Fig. 4 shows that with a larger spatial separation \( L \), the system performs better. This is because the dominant factor of \( \sigma^2 \) is the temporal effect \( (f_c) \) but not the spatial effect \( (\theta_s) \).

### B. Channel Efficiency

The total efficiency \( \eta \) of the satellite-to-ground channel, which depends on the atmospheric condition, can be expressed as [20, 26]

\[
\eta = \eta_{\text{trans}}(\zeta)\eta_{\text{rec}}\eta_{\text{spec}}\eta_{\text{det}}\eta_{\text{geo}}\eta_{\text{FS}}. \tag{22}
\]

Here \( \eta_{\text{trans}}(\zeta) \) is the free-space transmission efficiency is the only factor in \( \eta \) that depends on the zenith angle \( \zeta \) although the dependence is rather weak. To simplify matter, we assume that \( \eta_{\text{trans}} \) is a linear function with of \( \zeta \) with \( \eta_{\text{trans}} = 0.92 \) at \( \zeta = 0^\circ \) and \( \eta_{\text{trans}} = 0.74 \) at \( \zeta = 75^\circ \). These two values are the results obtained by the MODTRAN simulation for clear sky conditions adopted in Ref. [26]. Actually, we have tried a few variations on \( \eta_{\text{trans}} \) and found that it does not change our results in any significant way. As for the other factors in Eq. (22), we follow Ref. [20] by picking the efficiency of the receiver \( \eta_{\text{rec}} = 0.5 \), the efficiency of the spectral filter \( \eta_{\text{spec}} = 0.9 \), the detector efficiency \( \eta_{\text{det}} = 0.8 \). Besides, the aperture-to-aperture coupling efficiency \( \eta_{\text{geo}} \) is given by Eq. (10), and the efficiency of the FS is given by [20]

\[
\eta_{\text{FS}} = 0.84S. \tag{23}
\]

In this framework, \( \eta_{\text{FS}} \) is the only factor that is related to the distortion loss. From Eq. (23), a higher \( S \) implies a higher total channel efficiency. Moreover, Fig. 4 shows that the values of \( S \) for our spatial multiplexing method and the pure TDM method trend to converge at large zenith angle \( \zeta \). Thus, we expect that their corresponding values of \( \eta \) tends to converge for large \( \zeta \). Indeed, these two effects are observed in Fig. 5. To summarize, provided that the cross-talk between the pioneer beacon and signal beams is not serious, Figs. 4 and 5 show the priority of using our protocol over pure TDM and WDM ones in signal communication.

### VI. APPLICATION IN QUANTUM KEY DISTRIBUTION

We now consider the effect of cross-talk between the beacon and signal beams. To analyze the effectiveness of our protocol, we choose the most extreme setting that the signal beam is a weak coherent photon source used in decoy state BB84 QKD using three photon intensities, namely, the vacuum source and two phase randomized Poissonian distributed sources with intensities \( \mu \) and \( \nu \) [27–29]. In this setting, cross-talk noise could affect the system seriously by increasing the quantum bit error rate (QBER) and hence lowering secret key rate. In this regard, if our protocol works better than existing satellite-to-ground QKD setups, then it should also work in practically all realistic satellite-to-ground communication, both classical and quantum.

Recall that the background detection probability can be written as

\[
Y_0 = (N_b + N_{\text{cross}})\eta_{\text{spec}}\eta_{\text{rec}}\eta_{\text{det}} + 4f_{\text{dark}}\Delta t, \tag{24}
\]

FIG. 5. Total transmission efficiency \( \eta \) (plotted in scale of \( 10^{-3} \)) of different system configurations as a function of zenith angle \( \zeta \). The curves are labeled in the same way as in Fig. 4. Note that by turning off AO, the value of \( \eta \) is very close to 0.

| Quantity                          | Symbol | Value |
|-----------------------------------|--------|-------|
| Signal-state mean photon numbers  | \( \mu \) | 0.7   |
| Decoy-state mean photon numbers   | \( \nu \) | 0.1   |
| Repetition rate                   | \( f_{\text{source}} \) | 10 MHz |
| Sky radiance                      | \( H_b \) | 25 W m\(^{-2}\) sr \( \mu \)m |
| Dark count rate                   | \( f_{\text{dark}} \) | 10 Hz |
| Polarization cross-talk           | \( e_d \) | 0.01  |
| System noise error                | \( e_0 \) | 0.5   |
| Spectral filter bandpass          | \( \Delta \lambda \) | 0.2 nm |
| Detection time                    | \( \Delta t \) | 1 ns  |
| Error-correction efficiency       | \( f_c \) | 1.22  |

TABLE I. Parameters used for calculating the secret key rate of decoy state BB84 protocol used in Ref. [20].
where $N_b$, $N_{\text{cross}}$, $f_{\text{dark}}$, and $\Delta t$ are the sky photon noise, cross-talk noise due to the beacon, the dark count rate of the detectors and the detection time window, respectively. Moreover, $N_b$ is calculated using Eq. (16) with the parameters stated in Table I. We assume $N_{\text{cross}}$ is 5 times of the scattering noise calculated in Sec. IV C when spatial multiplexing is used. (We pick this number as a conservative estimate. In fact, the secret key rate does not change much even if we pick $N_{\text{cross}}$ to be 50 times of the scattering noise.) Furthermore, for WDM and TDM systems, we take the liberty to set $N_{\text{cross}} = 0$. The rest of the calculations are well known and can be found in Ref. [29]. We include them here for readers’ convenience. The QBER can be expressed as

$$E_\mu = \frac{e_0 Y_0 + e_d (1 - e^{\eta_\mu})}{Y_0 + 1 - e^{\eta_\mu}}, \quad (25)$$

where $e_0$ and $e_d$ are the system noise error rate and polarization cross-talk. The value of the parameters are based on Ref. [20] and are presented in Table I. These parameters are optimized to give the highest possible secret key rate for free-space photon transmission in the so-called asymptotic limit, namely, for the case of an arbitrarily large number of photon transfer. The secret key rate (more accurately, a provably secure lower bound of the number of secret key obtained at the end divided by the number of signal photon pulses emitted by the satellite) can be written as

$$R \geq q \{e^\mu \nu Y_1[1 - h_2(e_1)] - f_e Q_\mu h_2(E_\mu)\}, \quad (26)$$

where $h_2(x)$ is the binary entropy function, $Y_1$ is the single photon state yield

$$Y_1 = \frac{\mu}{\mu - \nu^2} \left( Q_\mu e^{\nu} - Q_\mu e^{\nu} \frac{\nu^2}{\mu^2} - \frac{\mu^2 - \nu^2}{\mu^2} Y_0 \right), \quad (27)$$

e_1$ is the single photon state error rate

$$e_1 = Q_\mu E_{\nu} e^{\nu} - e_0 Y_0 \frac{1}{Y_1 \nu}, \quad (28)$$

and $Q_\mu$ is the gain at intensity $\mu$

$$Q_\mu = Y_0 + 1 - e^{-\eta_\mu}. \quad (29)$$

Last but not least, $q$ is the probability that the trusted agents on the satellite and on the ground use the same basis for preparing and measuring their signal photons in their QKD experiment. In Ref. [20], $q$ is chosen to be 1/2. But in the asymptotic limit, the optimized key rate can be computed by taking the limit of $q \rightarrow 1$ using biased bases selection. Note however that as $q$ is a parameter independent of the channel and the AO setup used. It only appears as a multiplication factor in the R.H.S. of Eq. (26) as far as the key rate is concerned. Consequently, if the key rate of a certain method is higher than that of another method for a fixed $q > 0$, then the key rate of the former method is always higher than the later for all $q \in (0, 1]$. Therefore, we only need to compare the provably secure key rates of different methods by fixing, say, $q = 1/2$.

While theorists use a dimensionless key rate like the one in Eq. (26) as one of the effectiveness metric to study QKD protocols, from a practical point of view, here we use the “experimentalist” version of the key rate, namely, $f_{\text{source}} R$ in this study. It tells us the lower bound of the number of provably secure secret key bits generated per unit time. Figs. 6 and 7 show the final results of the
QBER and the “experimentalist” version of the key rate. When AO is not used, the QBER is high and no provably secure key can be generated. On the other hand, the QBERs of different AO systems are low and almost the same as each other when the zenith angle $\zeta$ is less than 60°. For $\zeta \gtrsim 65^\circ$, all but the pure WDM method give about the same QBER while the pure WDM method has a much higher QBER. For the key rate, the curves of the AO systems have the same pattern as the total channel efficiency. For all the distance $L$ between the pioneer beacon and signal beams used, our time-delayed spatial multiplexing prototype always gives a higher key rate than pure TDM method which is in turn always better than the pure WDM method. In fact, for $\zeta < 30^\circ$, ours is at least 4.9% better than the pure TDM method. (Note further that we have slightly over-estimated the key rate of the latter method here by ignoring the interlacing effect of the beacon and signal beams in TDM. Hence, the key rate improvement of our time-delayed spatial multiplexing prototype over the pure TDM method is actually slightly better than what is shown in Fig. 7.) Besides, the larger the value of $L$, the larger the key rate. By setting $L = 10$ m, our prototype increases the provably secure key rate by about 27%, which is very significant. Whereas for $\zeta \gtrsim 60^\circ$, the key rates of our prototype and the pure TDM method are roughly the same.

VII. IMPROVING WDM SYSTEMS USING A PIONEER BEACON

As WDM is a popular method to combine the beacon and signal beams, it is easier to upgrade the system with a pioneer beacon than building a spatial multiplexing system. Here we present the improvement to WDM systems with our idea. To study that, we modify Eq. (21) to

$$
\sigma_{\text{WDM}}^2 = \sigma_{\text{band, SS}}^2 + \sigma_{\text{iso, SS}}^2 + \sigma_d^2 + \sigma_{\text{ch}}^2 + \sigma_{\phi}^2.
$$

(30)

Using the same set of parameters and following the same analysis in Secs. V and VI, the Strehl ratio, QBER, and key rate are shown in Figs. 8, 9, and 10 respectively. In Fig. 8, we observe that $S$ increases with increasing separation $L$ between the beacon and the signal while maintaining the same overall shape of the $\zeta$-$S$ curve. For zenith angle $\zeta \lesssim 60^\circ$, the improvement is up to about 26%, which is very significant. Whereas when $\zeta$ is close to 75°, the value of the Strehl ratio $S$ all drops to around 0.1 due to serious chromatic aberration. Fig. 9 depicts that adding spatial multiplexing does not improve the QBER. As for the secret key rate, Fig. 10 shows that it increases as $L$ increases. For $\zeta \lesssim 45^\circ$, the improvement is as large as about 27%. Nevertheless, the key rate drops to 0 Hz at $\zeta = 75^\circ$ for any value of $L$ used. Comparing with the QBERs of the pure TDM method and our time-delayed spatial multiplexing prototype in Fig. 6, we conclude that this drop is caused primarily by chromatic aberration. In summary, upgrading a pure WDM system with a pioneer beacon beam can significantly increase the secret key rate in decoy-state QKD using phase-randomized Poissonian source when the zenith angle $\zeta \lessgtr 45^\circ$. Furthermore, by comparing Fig. 7 with Fig. 10, for the same value of $L$, the two systems have comparable key rates for $\zeta < 20^\circ$ — the later is at least 95% of the former. These make upgrading existing WDM systems a very attractive alternative to building entirely new time-delayed spatial multiplexing ones.
FIG. 10. Key rate of WDM systems. The curves are labeled in the same way as in Fig. 8. Note that the key rate is 0 Hz when AO is turned off.

VIII. CONCLUSIONS

In this paper, we report a method to apply AO technologies to optical communication systems. The main ideas are the spatial separation of the beacon and the signal beam plus applying a suitable time delay between these two beams. Moreover, for fast-moving sources, our design can reduce the apparent wind speed caused by the movement of the object. This can reduce the Greenwood frequency of the turbulence. We estimate the cross-talk caused by the diffraction and the scattering of the beacon. As there is a field stop in the beacon receiving module and the power of the beacon is not high, the cross-talk by the beacon can be neglected. By semi-empirical study, we show that the key rate of our scheme in LEO satellite-to-ground QKD is better than the pure TDM and WDM methods. The improvement can be as high as about 27%. We also consider an alternative system that adds a pioneer beacon beam to existing pure WDM systems. We find that for zenith angle less than about 20°, this alternative setup performs QKD at a rate of at least 95% of our original proposal, making it an attractive engineering option in practice. Further analysis of the system performance can be found in Ref. [15]. Lastly, we stress that our method is also applicable to classical optical free-space communication as we do not any quantum property of the signal source.

ACKNOWLEDGMENTS

We would like to thank Hoi-Kwong Lo, Alan Pak Tao Lau, Chengqiu Hu, Wenyuan Wang, and Gai Zhou for the discussion in optics. This work is supported by the RGC grant 17302019 of the Hong Kong SAR Government.

Appendix A: Wavefront Variance Calculation

The calculation here is based on Refs. [3, 25]. First, we consider the variance caused by the limitation of the bandwidth of the AO system. It can be written as [3]

$$\sigma_{\text{band}}^{2} = \int_{0}^{\infty} |1 - H(f, f_c)|^2 F(f) \, df,$$  \hspace{1cm} (A1)

where $f_c = 500$ Hz is the bandwidth of the AO system, $f$ is a frequency variable,

$$H(f, f_c) = \left(1 + \frac{if}{f_c}\right)^{-1}$$  \hspace{1cm} (A2)

is the RC filter that used to model the system, and

$$F(f) = 0.0326 k^2 f^{-8/3} \int_{0}^{\tau_{\text{max}}} v^{5/3}(z) C_n^2(z) \, dz$$  \hspace{1cm} (A3)

is the power spectrum of the turbulence frequency. Here $v(z)$ is the wind speed calculated using Eq. (12). As discussed in Sec. III, the slewing rate can be reduced by using a pioneer beacon. Therefore, when computing $\sigma_{\text{band}}^{2}$, $\sigma^2_{\text{iso}}$, and $\sigma_{\text{band}, \text{SS}}^{2}$ the slewing rates used are Eq. (14) and Eq. (8), respectively. Furthermore, the minimum $T_r$ is taken as $1/f_c$. For $\sigma_{\text{iso}}^{2}$, its contribution comes from the combination of temporal anisoplanatism and spatial anisoplanatism. It can be expressed as [3]

$$\sigma_{\text{iso}}^{2} = \left(\frac{\theta'}{\theta_0}\right)^{5/3},$$  \hspace{1cm} (A4)

with $\theta_0$ is the isoplanatic angle calculated using Eq. (3) and

$$\theta' = \left\{ \int_{0}^{\infty} e^\tau \left[ \left(\theta_s + \frac{1.186 \theta_0 f_G \tau}{f_c} \right)^{5/3} - \frac{1}{2} \left(1.186 \theta_0 f_G \tau \right)^{5/3} \right] d\tau \right\}^{3/5},$$  \hspace{1cm} (A5)

where $\tau = 2\pi f_c \omega$. The same as the computation of $\sigma_{\text{band}}^{2}$, the slewing rate here is computed using $\omega / (\omega)$ when the spatial separation setup is (is not) used.

Next, we discuss the chromatic effects that appears in WDM systems. The first contribution of chromatic aberration is due to diffraction. The diffraction pattern of the beams at the receiving end depends on the wavelength [25]. When the beacon wavelength is $\lambda_b$, the variance on measuring the signal beam with wavelength $\lambda$ is [25]
\[ \sigma_d^2 = \frac{4.08}{\pi} k^2 \int_0^L \int_0^\infty K^{-8/3} \left\{ 1 - \left( \frac{4}{KD} \right)^2 \left[ J_1 \left( \frac{KD}{2} \right) \right] \right\} \left[ \cos \left( \frac{z K^2}{2 k_b} \right) - \cos \left( \frac{z K'^2}{2 k_b} \right) \right]^2 C_n^2(z) \, dz \, dK, \]  

(A6)

where \( K \) is the spatial frequency and \( k_b = 2\pi/\lambda_b \) is the wavenumber of the beacon. The second chromatic contribution comes from path length error between the beams. A DM is only able to compensate error perfectly at a single wavelength. This is because there is a path length difference between light beams with different wavelengths. The corresponding wavefront variance is [25]

\[ \sigma_{ch}^2 = 1.03 \left( \frac{D}{r_0} \right)^{5/3} \epsilon^2(\lambda, \lambda_b), \]  

(A7)

where

\[ \epsilon(\lambda, \lambda_b) = \frac{\lambda_b n_s(\lambda) - n_s(\lambda)}{\lambda} n_s(\lambda_b) - 1. \]  

(A8)

In the above equation, \( n_s \) is the refractive index and calculated at standard pressure and temperature base on the Ciddor’s model [30].

Lastly, we calculate the wavefront variance the caused by chromatic anisoplanatism. Light waves with different wavelength travel different paths because of dispersion. The isoplanatic error induced by this can be written as [25]

\[ \sigma_p^2 = \left[ \frac{\sin(\Delta n)}{\cos^2(\Delta n)} \right] T_{5/3}, \]  

(A9)

where

\[ \Delta n = |n_s(\lambda) - n_s(\lambda_b)| \]  

is the difference in refractive index and

\[ T_{5/3} = 2.91 k_b^2 \sec(\Delta) \int_0^{h_{\text{max}}} I^{5/3}(h) C_n^2(h) \, dh, \]  

(A11)

with \( I(h) \) equals to the integral of the air density normalized to the value at sea level

\[ I(h) = \int_0^h \alpha(z) \, dz. \]  

(A12)

For simplicity, we only take integral of the troposphere in this paper as this layer contributes most to \( I(h) \). Specifically, we follow Ref. [31] by using the air density model

\[ h(\rho) = 44330.8 - 42266.5 \rho^{0.234969} \]  

(A13)

for \( h \leq 1.1 \times 10^4 \) m, where \( \rho \) is measured in unit of kg/m^3. Clearly, \( h(\rho) \) is an invertible function. By denoting its inverse function by \( \rho(\alpha) \), then \( \alpha(h) \) is simply \( \rho(h)/\rho(0) \).

[1] F. Roddier, ed., *Principles Of Adaptive Optics* (CUP, Cambridge, 2009).
[2] O. Guyon, Anna. Rev. Astron. Astrophys. 56, 315 (2018).
[3] R. K. Tyson and B. W. Frazier, *Principles Of Adaptive Optics*, 5th ed. (CRC Press, New York, 2022).
[4] Y. Wang, H. Xu, D. Li, R. Wang, C. Jin, X. Yin, S. Gao, Q. Mu, L. Xuan, and Z. Cao, Sci. Rep. 8, 1124 (2018).
[5] Y. Cao, Y.-H. Li, K.-X. Yang, Y.-F. Jiang, S.-L. Li, X.- L. Hu, M. Abulizi, C.-L. Li, W. Zhang, Q.-C. Sun, W.-Y. Liu, X. Jiang, S.-K. Liao, J.-G. Ren, H. Li, L. You, Z. Wang, J. Yin, C.-Y. Lu, X.-B. Wang, Q. Zhang, C.-Z. Peng, and J.-W. Pan, Phys. Rev. Lett. 125, 260503 (2020).
[6] M. T. Gruneisen, M. L. Eickhoff, S. C. Newey, K. E. Stoltenberg, J. F. Morris, M. Barcian, M. A. Harris, D. W. Oesch, M. D. Oikler, M. B. Flanagan, B. T. Kay, J. D. Schlitter, and R. N. Lanning, Phys. Rev. Appl. 16, 10.1103/physrevapplied.16.014067 (2021).
[7] D. P. Greenwood, J. Opt. Soc. Am. 67, 390 (1977).
[8] P. L. Wizinowich, D. E. Mignard, A. B. Bouchez, R. D. Campbell, J. C. Y. Chin, A. R. Contos, M. A. van Dam, S. K. Hartman, E. M. Johansson, and R. E. Lafon, Publ. Astron. Soc. Pac. 118, 297 (2006).
[9] F. Bennet, I. Price, F. Rigaut, and M. Copeland, in *Advanced Maui Optical And Space Surveillance Technologies Conference* (2016) poster presentation, source available in https://www.semanticscholar.org/paper/Satellite-Imaging-with-Adaptive-Optics-on-a-1-M-Bennet-Price/21541a3038bec1a04e668afd29135282053e263e.
[10] S. Pirandola, U. L. Andersen, L. Banchi, M. Berta, D. Bunandar, R. Colbeck, D. Englund, T. Gehring, C. Lupo, C. Ottaviani, J. L. Pereira, M. Razavi, J. S. Shaeiri, M. Tomamichel, V. C. Usenko, G. Vallone, P. Villoresi, and P. Wallden, Adv. Opt. Photonics 12, 1012 (2020).
[11] S.-K. Liao, W.-Q. Cai, W.-Y. Liu, L. Zhang, Y. Li, J.-G. Ren, J. Yin, Q. Shen, Y. Cao, Z.-P. Li, F.-Z. Li, X.-W. Chen, L.-H. Sun, J.-J. Jia, J.-C. Wu, X.-J. Jiang, J.-F. Wang, Y.-M. Huang, Q. Wang, Y.-L. Zhou, L. Deng, T. Xi, L. Ma, T. Hu, Q. Zhang, Y.-A. Chen, N.-L. Liu, X.-B. Wang, Z.-C. Zhu, C.-Y. Lu, R. Shu, C.-Z. Peng, J.-Y. Wang, and J.-W. Pan, Nature (London) 549, 43 (2017).
[12] F. Xu, X. Ma, Q. Zhang, H.-K. Lo, and J.-W. Pan, Rev. Mod. Phys. 92, 10.1103/revmodphys.92.025002 (2020).
[13] M. T. Gruneisen, B. A. Sickmiller, M. B. Flanagan, J. P. Black, K. E. Stoltenberg, and A. W. Duchane, in *Emerging Technologies in Security and Defence II; and Quantum-Physics-based Information Security III*, Society of Photo-Optical Instrumentation Engineers (SPIE)
K. S. Chan and H. F. Chau, Improving classical and quantum free-space communication by adaptive optics and by separating the reference and signal beams with time delay for source(s) moving relative to the detector(s) (2022), patent Application PCT/CN2021/096100.

K. S. Chan, Improving Quantum Key Distribution By Adaptive Optics, Master’s thesis, University of Hong Kong (2022).

D. L. Fried, J. Opt. Soc. Am. 55, 1427 (1965).

R. E. Hufnagel and N. R. Stanley, J. Opt. Soc. Am. 54, 52 (1964).

R. J. Sasiela, Electromagnetic Wave Propagation in Turbulence (SPIE, Bellingham, 2007).

D. Vasylyev, W. Vogel, and F. Moll, Phys. Rev. A 99, 053830 (2019).

R. N. Lanning, M. A. Harris, D. W. Oesch, M. D. Oliker, and M. T. Gruneisen, Quantum communication over atmospheric channels: A framework for optimizing wavelength and filtering (2021), arXiv:2104.10276.

Lulin Observatory website, http://www.lulin.ncu.edu.tw/instrument/LOT/ [Accessed: 28 April 2022].