Dynamic Connected Cooperative Coverage
Problem

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Abstract
We study the so-called dynamic coverage problem by agents located in some topological graph. The agents must visit all regions of interest but they also should stay connected to the base via multi-hop. We prove that the algorithmic complexity of this planning problem is PSPACE-complete. Furthermore we prove that the problem becomes NP-complete for bounded plans. We also prove the same complexities for the reachability problem of some positions. We also prove that complexities are maintained for a subclass of topological graphs.

Introduction
Unmanned autonomous vehicles (UAVs) are nowadays used in many applications (controlling wildlife, surveying dangerous areas, measuring pollution, etc.). For example, if a fire occurs, firefighters would send a fleet of UAVs from a base to measure pollution. The UAVs would have then to collaborate so they could map the entire area and always keep communication with the base.

As in Yan12, TNMP10 and BCQS18, we consider a geographical area, with a launch base and regions of interest to visit, and topological communication constraints. The big challenge is to synthesize a cooperative plan for the fleet of UAVs for visiting all the regions of interest at least once, always keeping communication with the base and coming back to the base at the end. Communication may be multi-hop (a UAV may communicate to the base via intermediate UAVs). The communication aspect is important in applications such as search-and-rescue.

In this paper, we formally define and study an abstract version of that problem we call it the dynamic connected cooperative coverage problem. A geographical area is modeled by a finite graph. The finite graph could be generated from triangulation of the continuous environment (see in Fig. 2, p. 2022 FKP05, and Fig. 3 of KGLR18).
Figure 1 shows an execution of an 11-step plan in such a graph: elementary possible moves are represented by solid lines; a dashed line between two regions means that communication is possible between them. At the first and final step, UAVs are at the base. At the end, all regions must have been visited. During the execution, UAVs cooperate to stay connected to the base (in other words, dashed lines forming a connected subgraph).

UAVs alternate between moving and performing tasks at all the nodes of the graph that require the UAVs to be stationary (taking high-quality photos, manipulating some objects, etc.). Humans (firemen, engineers, etc.) supervise the UAV mission at the base. Thus, it is required the UAVs to communicate huge amount of data to the base while they are performing tasks. That is why high-speed broadband communications is needed. Such technologies (e.g. laser) typically do not pass through buildings and therefore communication constraints are not trivial to handle. Note that UAVs do not need to communicate huge data to the base while they are moving.

The difficulty resides in the combinatorial when UAVs cooperate to keep communication with the base all along the plan. The plans of the UAVs are inter-dependent. Even if many UAVs have an “automated back to launch location” option, the planning must include the path to come back to launch location as we consider using the system in urban areas, where the UAVs would have to avoid any obstacle on their way.

Our problem can be seen as a variation of Multi-Agent Path Finding (MAPF). MAPF consists in finding plans of elementary moves of robots in a grid, starting from an initial situation where each robot has a designated initial cell to a final situation in which each robot has a designated goal cell. The robots should not collide. Finding an optimal plan in the context of MAPF has been proven to be NP-hard ([YL13], [MTS+16]). MAPF and the dynamic connected cooperative coverage problem differ mainly by their target applications, mainly warehouse or storage robots for the former [WDM07], search-and-rescue for the latter. Communication and connectivity of UAVs is a main ingredient to our problem compared to MAPF, and, as we will show, it makes our problem computationally more difficult, especially since we do not focus on finding optimal plans - not an optimisation problem - but just finding plans - the existence of a plan.

In this paper, we provide theoretical complexity results: we prove that the dynamic connected cooperative coverage problem is PSPACE-complete and that its bounded version is NP-complete. Upper bounds are trivial but lower bounds are proven by delightful reductions from tiling problems [Boa97]. It means that synthesizing plans for the dynamic connected cooperative coverage problem is as difficult as classical planning [Byl94]. We also prove that the reachability problem (reaching specific nodes in the graph) has the same complexities. We also prove that the lower bounds are the same even when restricting to a subclass of topological graph for which it is always possible to communicate between two nodes $v$ and $v'$ for which it is possible to move in one step from $v$ to $v'$ (the class of neighbor-communicable topologic graphs).

The rest of the paper is organized as follows. First we settle the definition.
Figure 1: Example of a mission execution.

Second, we provide the theoretical complexity upper bounds. Third, we recall basics about tiling problems and then provide the theoretical complexity lower bounds, we obtained via reduction from tiling problems. Finally we detail related work.

Definitions

Topologic graph

A geographical area is modeled by a topologic graph. Nodes are regions of interest where the launch base is a special region noted $B$. Relation $\rightarrow$ represents possible moves of UAVs: $v \rightarrow v'$ if a UAV can reach $v'$ from $v$ in one step. Relation $\cdot \cdot \cdot$ represents possible communications: $v \ldots v'$ if any UAV at $v$ can communicate with any UAV at $v'$. We say that $v$ communicates with $v'$. Formally:

**Definition 1 (Topological graph)** A topologic graph is a tuple $G=(V,\rightarrow,\ldots)$ where $V$ is a non-empty finite set of regions containing a specific element $B$ and $\rightarrow,\ldots \subseteq V \times V$ is such that $(V,\ldots)$ is a non-oriented graph, $B \rightarrow B$.

Definition imposes $B$ to be $\rightarrow$-reflexive since a UAV can stay at the base. Notice that it does not impose the relation $\rightarrow$ to be symmetric or other nodes to be reflexive so that we can capture windy environments, one-sided roads etc.

For sufficiently fine-grained topologic graphs, there are no obstacles between regions $v$ and $v'$ when $v \rightarrow v'$. Thus, communications between $v$ and $v'$ are not perturbed and $v \ldots v'$. In words, if one UAV can reach $v'$ from $v$ in one step then a communication between two UAVs in $v$ and $v'$ is possible. This hypothesis seems reasonable for many means of communication (lasers, etc.). That is why, we define the subclass of neighbor-communicable topologic graphs in which $v \rightarrow v'$ implies $v \ldots v'$.
Executions

Given a topologic graph and given $n$ UAVs, a configuration $c$ gives positions to each UAV such that they form a multi-hop system: they are all connected to the base. Furthermore, we suppose that at most one UAV is at a given region, except at $B$.

Definition 2 (Configuration) A configuration $c$ is an element of $V^n$ such that the graph $(V_{UAV}, \cdots \cap V_{UAV})$ is connected with $V_{UAV} = \{c_i / i \in \{1,..n\}\} \cup \{B\}$ and for all $i \neq j$, if $c_i \neq B$, then $c_i \neq c_j$. We note $c \rightarrow c'$ when $c_i \rightarrow c'_i \in G$ for all $i \in \{1,..n\}$.

Without loss of generality, as the UAVs are interchangeable, configurations are equivalent up to a permutation of UAVs. To avoid cumbersome notations in proofs, we consider equivalent configurations as equal. For instance, for $n = 5$, configurations $(B,B,v,v',v'')$ and $(B,v,v'',B,v')$ are equivalent.

Definition 3 (Execution) An execution in $G$ with $n$ UAVs of length $\ell$ is a sequence of configurations $(c_0,\ldots,c_\ell)$ such that $c_0 \rightarrow c_1 \rightarrow \ldots \rightarrow c_\ell$. A covering execution in $G$ with $n$ UAVs of length $\ell$ is an execution $(c_0,\ldots,c_\ell)$ such that $c_0 = c_\ell = (B,B,\ldots,B)$ (all UAVs are at $B$ at the start and at the end) and $\{c_t_i / t \in \{0,..\ell\}, i \in \{1,..n\}\} = V$ (all regions are visited at some point).

Notice that if we have $c_1 \rightarrow c'_1$ and $c_1$ and $c_2$ are equivalent then by taking $c'_2$ the same permutation of $c'_1$ compared to $c_1$ and $c_2$, then $c_2 \rightarrow c'_2$. Thus, it is always possible to transform an execution into an equivalent one by a permutation.

Example 1 Figure 1 shows a topologic graph with 11 regions. Here, the $\rightarrow$-relation is symmetric and is represented by solid black lines. The $\cdots$-relation is represented by dotted lines and is blue when not taken, orange when taken. Visited regions are represented by checked marks. The execution is read line by line. Notice that at each step, the subgraph $(V_{UAV}, \cdots \cap V_{UAV})$ is connected; it is the subgraph obtained by taking only the active communication lines in Figure 1. Although the topological graph has 11 nodes, it is sufficient to have 3 UAVs to map the topological graph.

Decision problems

We define the connected cooperative coverage problem and the connected cooperative reachability problem shortly denoted by Coverage and Reachability. The reachability problem essentially is introduced for pedagogical reasons, especially for making the lower bound results more diligent. We also define bounded versions of the two decision problems, namely bCoverage and bReachability. The bounded versions are inspired from the so-called polynomial-length planning problem [Tur02] in which we ask for the existence of a plan of length bounded by a polynomial in the size of the planning task. It is equivalent to add the length bound as an input to the decision problems in unary. Formally:
Definition 4 (Coverage problems)

Coverage:
- Input: a topologic graph $G$ and $n \in \mathbb{N}$;
- Output: yes if there is a covering execution in $G$ with $n$ UAVs; no otherwise.

$bCoverage$:
- Input: a topologic graph $G$, $n \in \mathbb{N}$ and $\ell \in \mathbb{N}$ in unary;
- Output: yes if there is a covering execution in $G$ with $n$ UAVs of length at most $\ell$; no otherwise.

Definition 5 (Reachability problems)

Reachability:
- Input: a topologic graph $G$ and a configuration $c$;
- Output: yes if there is an execution $(c^0, \ldots, c^\ell)$ in $G$ such that $c^0 = (B, \ldots, B)$ and $c^\ell = c$; no otherwise.

$bReachability$:
- Input: a topologic graph $G$, a configuration $c$ and $\ell \in \mathbb{N}$ in unary;
- Output: yes if there is an execution $(c^0, \ldots, c'^\ell)$ in $G$ such that $c^0 = (B, \ldots, B)$, $c'^\ell = c$ and $\ell' \leq \ell$; no otherwise.

We now establish upper bound complexities of Coverage, bCoverage, Reachability and bReachability.

Complexity: upper bounds

Proposition 1 Coverage and Reachability are in PSPACE.

Proof. In both cases, the straightforward non-deterministic guessing an execution runs in polynomial space: for Coverage, we only keep in memory the last configuration and the set of already visited regions. For Reachability, we only keep in memory the last configuration. By Savitch’s theorem (NPSPACE = PSPACE) [Sav70], the proposition is proven. $\square$

Proposition 2 bCoverage and bReachability are in NP.

Proof. We define the same algorithms given in the Proof of Proposition [1] except that we stop the execution when the length is exceeded. Thus, the algorithms are non-deterministic and run in polynomial time. $\square$
Tiling problems

Tilings were introduced by Wang ([Wan61, Wan90]). As pointed out by van der Boas ([SvEB, Boa97]), tilings offer convenient decision problems for proving lower bound complexity. We also cite Levin’s work who invented the notion of NP-completeness independently from Cook and who introduced a bounded tiling problem [Lev73]. Some tiling problems are also addressed in some textbooks to characterize some complexity classes ([LP98], p. 262, 310; [HTK00], p. 58-63). We use tile types $t$ that are tuples $\langle \text{left}(t), \text{up}(t), \text{right}(t), \text{down}(t) \rangle \in \mathbb{N}^4$ giving colors (represented by integers) to the four sides of a tile $\square$. A tiling is represented by a function $\lambda$ that maps a tile type to each position $(i, j)$. Two horizontally or vertically adjacent tiles should match horizontally (constraints (h) and (v) in the following Definitions). The two decision problems introduced in this section are taken from [Boa97].

Square tiling problem

The square tiling problem consists in tiling a $k \times k$ square as depicted Figure 3a by using finite set of tile types and by respecting boundary color constraints along the edges. Figure 3b shows such a tiling. Note that tiles cannot be turned. Formally:

**Definition 6 (Square tiling problem)** The square tiling problem is the following decision problem:

- **Input**: a set $T \subseteq \mathbb{N}^4$ of tiles types and four sequences $\text{top}_1, \ldots, \text{top}_k \in T$, $\text{bot}_1, \ldots, \text{bot}_k \in T$, $\text{left}_1, \ldots, \text{left}_k \in T$, $\text{right}_1, \ldots, \text{right}_k \in T$ of length $k$

- **Output**: yes if there is a function $\lambda : \{1, \ldots, k\} \times \{1, \ldots, k\} \rightarrow T$ such that:
  
  (h) $\text{right}(\lambda(i, j)) = \text{left}(\lambda(i+1, j))$ for all $i \in \{1, \ldots, k-1\}$, for all $j \in \{1, \ldots, m\}$;

  (v) $\text{up}(\lambda(i, j)) = \text{down}(\lambda(i, j+1))$ for all $i \in \{1, \ldots, k\}$, for all $j \in \{1, \ldots, m-1\}$;

Figure 2: The square tiling problem.
1. \( up(\lambda(1,j)) = top_j \) for all \( j \in \{1..k\} \);
2. \( down(\lambda(k,j)) = bot_j \) for all \( j \in \{1..k\} \);
3. \( left(\lambda(i,1)) = left_i \) for all \( i \in \{1..k\} \);
4. \( right(\lambda(i,k)) = right_i \) for all \( i \in \{1..k\} \).

no otherwise

**Theorem 1** The square tiling problem is NP-complete [Boa97, SvEB].

**Corridor tiling problem**

Contrary to the square tiling problem, the corridor tiling problem consists in tiling a \( k \times m \)-rectangle, where \( m \) is arbitrary, by respecting the top and bottom edge constraints, left and right edges being all white. Formally:

**Definition 7 (Corridor tiling problem)** The corridor tiling problem is the following decision problem:

- input: A set \( T \subseteq \mathbb{N}^4 \) of tiles types and two sequences \( \text{top}_1, ..., \text{top}_k \) and \( \text{bot}_1, ..., \text{bot}_k \in T \) of length \( k \);
- output: yes if there exists an integer \( m \) and a function \( \lambda : \{1,..,m\} \times \{1,..,k\} \rightarrow T \) such that:

  (h) \( right(\lambda(i,j)) = left(\lambda(i+1,j)) \) for all \( i \in \{1..k-1\} \), for all \( j \in \{1,..m\} \);
  (v) \( up(\lambda(i,j)) = down(\lambda(i,j+1)) \) for all \( i \in \{1..k\} \), for all \( j \in \{1,..m-1\} \);

1. \( \lambda(1,j) = bot_j \) for all \( j \in \{1..k\} \);
2. \( \lambda(m,j) = top_j \) for all \( j \in \{1..k\} \);
3. \( left(\lambda(m,1)) = right(\lambda(m,k)) = \text{white} \).

**Theorem 2** The corridor tiling problem is PSPACE-complete [Boa97].

**Complexity: lower bounds**

**PSPACE lower bounds**

In this subsection, we reduce the corridor tiling problem that is PSPACE-complete (Theorem 2) to Reachability and Coverage. First we start with Reachability. Independently, a similar reachability problem, without a base, was proven PSPACE-hard in [TBR+18]. Their proof relies on Nondeterministic Constraint Logic [DH08].

**Theorem 3** Reachability is PSPACE-hard.
Proof. The proof is by polynomial time reduction from the corridor tiling problem. To do so, we map a desired corridor tiling instance $(T, \text{top}_1, \ldots, \text{top}_k, \text{bot}_1, \ldots, \text{bot}_k)$ to the Reachability instance $(G, k, c)$ described below.

**Description of $G$.** The topologic graph is shown in Figure 4. The set of nodes in $G$ contains the base $B$, a copy of $T$ with only tiles with white left-side, $k - 2$ copies of $T$ and a copy of $T$ with only tiles with white right-side. Copies of $T$ are represented by ellipses in Figure 4.

Possible moves for the UAVs are represented by the arrows $\rightarrow$ in Figure 4. Moreover, a UAV can also move from tile $t$ to tile $t'$ when $\text{up}(t) = \text{down}(t')$, i.e. $\square \rightarrow \square$, and $t$ and $t'$ belong to the same copy of $T$.

Communication links between regions are represented by $\ldots$ in Figure 4. Moreover, a UAV on tile $t$ in the $i$th ellipse can communicate with another UAV on tile $t'$ in the $i + 1$th ellipse if $\text{right}(t) = \text{left}(t')$, i.e. $\square \ldots \square$.

More formally $G = (V, \rightarrow, \ldots)$ is defined by:

- $V$ is the disjoint union of $\{B\}$, $\{(t, 1) \mid t \in T$ and $\text{left}(t) = \text{white}\}$, $\{(t, j) \mid t \in T$ and $j \in \{2, \ldots, k - 1\}\}$ and $\{(t, k) \mid t \in T$ and $\text{right}(t) = \text{white}\}$;
Figure 5: Topologic graph $G'$ of the Coverage-instance constructed from the Reachability-instance.

- $\rightarrow$ is the union of \{(B, (bot, i)) | i ∈ \{1, ..., k\}\} and \{(t, i), (t', i)) | up(t) = down(t')\} for all i ∈ \{1, ..., k\};

- $\Rightarrow$ is the union of \{(B, (t, 1)) | (t, 1) ∈ V\} and \{(t, i), (t', i + 1)) | right(t) = left(t')\} for all i ∈ \{1, ..., k - 1\}.

The formal definition $G$ sums up the informal explanation given above. Tile $t$ in the $i^{th}$ ellipse is denoted by $(t, i)$.

The goal configuration is $c = (top_1, ..., top_k)$.

**Intuition.** The intuition is that once all the UAVs left the base, a configuration corresponds to a row of $k$ tiles in the $k \times m$ rectangle in Figure 3. The position of the UAV in $i^{th}$ ellipse corresponds to the tile in column $i$. In such a row, tiles match horizontally by definition of $\rightarrow$. The second configuration $top_1, ..., top_k$ corresponds to the bottom row. Each time that the execution progresses, UAVs synchronously move in new tiles: it mimics a new row added to the tiling in construction. Tiles match vertically by definition of $\rightarrow$-transitions.

A tiling of the $k \times m$ rectangle whose top and bottom edges are $top_1, ..., top_k$ and $bot_1, ..., bot_k$ respectively exists if and only if the UAVs can reach the configuration $(top_1, ..., top_k)$. $\square$

**Theorem 4** Coverage is PSPACE-hard.

Proof. The proof is by reduction from Reachability. To do so we map an instance $(G, k, c)$ of Reachability to the instance $(G', k)$ of Coverage where $G'$ is depicted in Figure 5. $G'$ contains $G$ as a subgraph, plus fresh nodes $v_1, ..., v_k$ and $s_1, ..., s_k$. A UAV can move from any node of $G$ to $v_1$ and vice versa.

Node $s_1$ can communicate with the base $B$ and node $v_k$ can communicate with all nodes of $G'$. Now we prove that the $k$ UAVs can progress to the configuration $(c_1, ..., c_k)$ in $G$ if and only if there exists a covering execution in $G'$.

($\Rightarrow$) If the UAVs are in the configuration $(c_1, ..., c_k)$ then they can progress in one step to configuration $(s_1, ..., s_k)$. Then, they have no choice but progress to the configuration $(v_1, ..., v_k)$. Once in this configuration, the UAV placed on
the node $v_k$ can communicate with any UAV, placed on any node, and to the base $B$. Actually that UAV will stay at $v_k$. Meanwhile the UAV placed on the node $v_1$ will visit all unvisited nodes of $G$ and come back to $v_1$ while keeping communication to the base through the UAV placed on $v_k$. Meanwhile, UAVs placed on $v_2, \ldots, v_{k-1}$ come back to $B$. Finally, when all the nodes have been visited, both UAVs on $v_1$ and $v_k$ come back to $B$.

$(\Leftarrow)$ If there exists a covering execution of the whole graph $G'$, it means all nodes have been visited. In particular, node $s_k$ has been visited and let us consider the first time $t_{s_k}$ when $s_k$ is visited. Time $t_{s_k} - 1$ denotes the time just before $t_{s_k}$.

**Fact 1** At time $t_{s_k} - 1$, no node $v_i$ were visited and no node $s_i$ were visited.

Proof. Suppose by contradiction that a node $v_i$ was visited by some UAV before $t_{s_k}$, then the only possibility such a UAV to communicate to the base is that there is also a UAV at $v_k$ at time $t_{s_k}$. But then, it means that $s_k$ was visited strictly before $t_{s_k}$, leading to a contradiction. Thus, no node $v_i$ were visited at time $t_{s_k}$ (thus at time $t_{s_k} - 1$).

As no node $v_i$ are visited before $t_{s_k}$, no node $s_i$ are visited before $t_{s_k} - 1$. □

**Fact 2** At time $t_{s_k} - 1$, the configuration is $(c_1, \ldots, c_k)$.

Proof. At time $t_{s_k}$, as the UAV at $s_k$ needs to communicate, the only possibility is that the configuration is $(s_1, \ldots, s_k)$. Thus, the only possibility is that configuration is $(c_1, \ldots, c_k)$. □

Facts 1 implies that implies that the prefix from time 0 to time $t_{s_k} - 1$ of the covering execution is an execution in $G$. Fact 2 implies that subexecution reaches $(c_1, \ldots, c_k)$. □

**NP lower bound for bounded problems**

In this subsection, we reduce the square tiling problem which is NP-complete (Theorem 1) to **bReachability** and **bCoverage**.

**Theorem 5** **bReachability** is NP-hard.

Proof. The proof is by polynomial reduction from the square tiling problem. From a instance $(T, \top, \bot, \lef, \rig, k)$ of the square tiling problem, we will construct a **bReachability**-instance $(G, c, k + 2)$. The topologic graph $G$ of Figure 6 looks like the one of Figure 4. It uses the same conventions for movements and communication. In this graph, the ellipses are now surrounded by nodes $bot_1, \ldots, bot_k, top_1, \ldots, top_k, \lef_1, \ldots, \lef_k, \rig_1, \ldots, \rig_k$ that represent the bottom, top, left and right edge colors of the $k \times k$-square. More precisely:

- $\lef_i$ (resp. $\rig_i$) is ****-connected to all tiles of the first (resp. $k^{th}$) copy whose left (resp. right) color is $\lef_i$ (resp. $\rig_i$);
Figure 6: Topologic graph of the bounded tiling problem reduction.

- $bot_i$ (resp. $top_i$) is $\rightarrow$-connected to (is $\rightarrow$-reachable from) all tiles of the $i^{th}$ copy whose bottom (resp. top) color is $bot_i$ (resp. $top_i$).

Actually the idea of the reduction is similar to the proof of Theorem 3 except that now, two extra UAVs runs on the $lef_i$- and $rig_i$-lanes to control both the left- and right- boundary color constraints and the vertical size of the square tiling. More formally $G = (V, \rightarrow, \cdots)$ is defined by:

- $V$ is the disjoint union of the sets $\{B\}$, $T \times \{1, \ldots, k\}$ and $\{(i, j) \in \{0, \ldots, k + 1\}^2 | \text{either } i = 0 \text{ or } i = k + 1 \text{ or } j = 0 \text{ or } j = k + 1\}$;

- $\rightarrow$ is the union of the sets $\{(B, (0, i)) | i \in \{0, \ldots, k + 1\}\}$, $\{((t, i), (t', i')) | up(t') = down(t')\}$, $\{((0, i), (t, i)) | bot_i = down(t)\}$, $\{((t, i), (k+1, i)) | top_i = up(t)\}$ for all $i \in \{1, \ldots, k\}$, $\{(i, 0), (i+1, 0), i \in \{0, \ldots, k\}\}$ and $\{((i, k+1), (i+1, k+1), i \in \{0, \ldots, k\}\}$;

- $\cdots$ is the union of the sets $\{(B, (i, 0)) | i \in \{0, \ldots, k+1\}\}$, and $\{((t, i), (t', i+1)) | right(t) = left(t')\}$ for all $i \in \{1, \ldots, k-1\}$, $\{((i, j), (i, j+1)) | (i = 0 \text{ or } i = k + 1) \text{ and } j \in \{0, \ldots, k\}\}$, $\{((i, 0), (t, 1)) | left_i = left(t)\}$ and $\{((i, k+1), (t, k)) | rig_i = right(t)\}$.

The $bReachability$ instance is $(G, c, k + 2)$ where $c$ is the configuration $((k+1,0), \ldots, (k+1, k+1))$ ($(lef_{top}, top_1, \ldots, top_k, rig_{top})$ in Figure 6). $\Box$

Theorem 6 $bCoverage$ is NP-hard.

Proof. The idea is similar than for Theorem 4. We proceed by polynomial time reduction from the square tiling problem. First we apply the reduction given in the proof of Theorem 5 from an instance $(T, \rightarrow, bot, lef, \rightarrow, \rightarrow, \rightarrow)$ we obtain an $bReachability$-instance of the form $(G, c, k + 2)$ (as depicted in Figure 6), where $c = ((k+1,0), \ldots, (k+1, k+1))$. Notice that these instances are such that all executions are of length at most $k + 2$. Therefore, the existence
of an execution is the same that the existence of an execution of length at most $k + 2$. Thus, the same construction of Figure 5 (just $k + 2$ instead of $k$) is sound. Indeed, from $(G, c, k + 2)$, we construct the instance $(G', k + 2, ℓ)$ where $ℓ$ is the sum of $k + 2$ (the number of steps to reach the configuration $c$), 2 (the two steps to reach the configuration $(v_1, \ldots, v_k, v_{k+2})$), $2 \times |G| + 1$ (an upper bound of the number of steps for the drone in $v_1$ to visit the unvisited node in the subgraph $G$ and to come back to the base). □

Restrictions to neighbor-communicable graphs

In this subsection, we prove that the lower bounds still hold for neighbor-communicable graphs.

Theorem 7 Reachability and Coverage are PSPACE-hard even when restricted to neighbor-communicable topologic graphs.

Proof. For Reachability, the proof is similar to the proof of Theorem 3: we just slightly modify graph $G$ of Figure 4 as follows. In order to prevent a UAV at $b_{ot_2}, \ldots, b_{ot_k}$ to communicate directly with $B$, we add an intermediate node $m_i$ between $B$ and each $b_{ot_i}$ ($B \rightarrow b_{ot_i}$ becomes $B \rightarrow m_i \rightarrow b_{ot_i}$) for $i = 1, \ldots, k$. We also add a communication edge $v \rightarrow v'$ whenever $v \rightarrow v'$. In rest of the proof, the new graph is still noted $G$.

For Coverage, the construction given in Figure 5 with the new graph $G$ does not work. Indeed, all nodes may be visited although $c_1, \ldots, c_k$ was not reached: maybe $v_1$ and $v_k$ are reached by two lines of UAVs connected to the base, making the coverage of the full graph possible.

The corrected construction is given in Figure 7. When configuration $(c_1, \ldots, c_k)$ is reached, the UAVs go through a first layer of length $k + 1$ in which the first UAV can communicate with $B$. Then they go through another layer of length $k + 1$ in which the $k^{th}$ UAV can communicate with $B$. This way, it is mandatory that all UAVs move at the same time to visit $(v_1, \ldots, v_k)$. Once the $k^{th}$ UAV is at $v_k$, all UAVs can communicate with $B$ wherever they are, so they can visit remaining states in the copy of $G$. Now let us prove that $(c_1, \ldots, c_k)$ is reachable in $G$ if it is possible to cover all nodes in $G'$.

$(\Rightarrow)$ If $(c_1, \ldots, c_k)$ is reachable in $G$, then we extend the execution to reach $(v_1, \ldots, v_k)$ and by the same trick as in Figure 5, the UAV that reaches $v_1$ visits all the remaining unvisited nodes in $G$. Thus, we extend the execution for covering all nodes in $G'$.

$(\Leftarrow)$ Suppose all nodes are visited in $G'$. In particular, $v_1$ and $v_k$ are visited. Let us consider the first moment $t_{v_i}$ when a node $v_i$ is visited.

Fact 3 At that first moment, the configuration is $(v_1, \ldots, v_k)$.

Proof. Let us prove that there is a UAV at $v_k$. Suppose that at that moment there is no UAV at $v_k$. Due to the topological graph $G'$, the UAV at $v_i$ is disconnected from the base since nodes that communicate directly to $B$ are too
far from $v_i$; indeed, the top $k+1$-grid is too long and, for $i = 1$, the path on left between $v_1$ and the copy of $G$ is too long. Contradiction.

The UAV at $v_k$ came from the unique $2k+2$-long path from $c_k$ to $v_k$. Actually, $k+1$ steps before - let us call this moment $t_{s_k}$, she was on $s_k$. But at that time, due to the topological graph, there are $k$ UAVs on the row containing $s_k$, otherwise the UAV at $s_k$ would have been disconnected from the base (the bottom $k+1$-grid is too long).

So $k+1$ times later $t_{s_k}$, all the $k$ UAVs are at $(v_1, \ldots, v_k)$. □

Taking Fact 3 as granted, we consider time $t$ that is $2k+2$ steps before and we clearly have the following fact.

**Fact 4** At time $t$, the configuration is $(c_1, \ldots, c_k)$.

Moreover, the following fact holds.

**Fact 5** At time $t$, no node outside $G$ were visited.

Proof. By contradiction, if some node outside $G$ were visited, it means that some UAV went out the copy of $G$. By definition of $G'$, it would mean that a node $v_i$ would have been visited, before time $t$, hence strictly before $t_{v_i}$. Contradiction. □

To sum up, the prefix of the execution from $(B, \ldots, B)$ to $(c_1, \ldots, c_k)$ is fully inside the copy of $G$. So $(c_1, \ldots, c_k)$ is reachable in $G$. □

**Theorem 8** Both $b$Reachability and $b$Coverage are NP-hard when restricted to neighbor-communicable topological graphs.

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Figure 7: Topologic graph of the **Coverage**-instance constructed from the **Reachability**-instance for the case of neighbor-communicable topologic graph.
Proof. For \textbf{bReachability}, we use the same construction depicted in Figure 6 except that we add intermediate nodes (as in the proof of Theorem 7) between $B$ and $lef_{bot}$, $bot_1,\ldots, bot_k$, $rig_{bot}$ and edges $v\to v'$ are added whenever $v \to v'$. By still calling $G$ the obtained graph, the $\textbf{bReachability}$-instance is $(G, c, k+3)$, where $c = ((k+1, 0),\ldots, (k+1, k+1))$. The bound is now $k + 3$ instead of $k + 2$ because of the intermediate nodes.

For \textbf{bCoverage}, we use the same idea that in Theorem 5 but the construction given in Figure 7. The bound $\ell$ is the sum of $k + 3$ (the number of steps for reaching $c$ in $G$), $2 \times (k + 2)$ (the number of steps to reach the configuration $(v_1,\ldots, v_k)$) and $2 \times (k + 2) \times |G| + 1$ (the number of steps for the first UAV finishing the visit of all remaining unvisited nodes; $2 \times (2k + 2)$ corresponds to the number of steps in the two left-most paths in Figure 7 for the back and forth between $v_1$ and nodes of $G$). □

\section*{Related work}

As shown in the survey by Chen et al. [CZX14], many coverage problems have been addressed by using analytic techniques. For instance, in [Yan12] and [TNMP10], they also address UAVs that should cover an area while staying connected to the base, but solve this problem with specific path planning algorithms. The algorithms they provide are not proven formally but tested experimentally.

That is why we advocate for formal methods, that have already been applied to generate plans for robots and UAVs. For instance, model checking has been applied to robot planning (see [LPH14]) and to UAVs. Humphrey [Hum13] shows how to use LTL (linear-temporal logic) model checking for capturing response and fairness properties in cooperation (for instance, if a task is requested then it is eventually performed). Model checking has also been used to verify pre-programmed UAVs [WFCJ11].

Bodin et al. [BCQS18] treat a similar problem except that the UAVs cover the graph without returning to the base. If we remove the return to the base constraint, we claim that all our complexity results still hold. They provide an implementation by describing the problem in PDDL (Planning Domain Description Language) and then run the planner FS (Functional Strips) [FRLG17]. Both \textbf{Reachability} and \textbf{Coverage} may be expressed in MA-STRIPS [BD08], that is a multi-agent variant of STRIPS (Stanford Research Institute Problem Solver) in which actions for each agent can be described independently. The representation in multi-agent planning languages is especially efficient when actions of the different agents are independent and when they required to coordinate not so often. However, as the agents should maintain connection, it requires a lot of coordination.

Interestingly Murano et al. [MPRL15] advocate for a graph-theoretic representations of states, that is, by giving locations to agents as we do in Definition 2. Aminof et al. (AMRZ16, Rub15) propose a very general formalism to specify LTL and MSO (monadic second-order logic) properties which is expressive
enough to express connectivity between agents with an MSO formula. Indeed, linear temporal operators enable to express that any vertex should be visited in the future and the connectivity invariant. MSO on the topological graph enables to express the connectivity as a fix point (the subgraph made up of the UAVs and the base is connected). They provide an algorithm for parametrized verification in the sense that they check a temporal property in a class of graphs. This is relevant for partially-known environments. The algorithm described is non elementary and therefore not usable in practice. We nevertheless claim that studying fragments of it is relevant, and our paper seems to be a relevant fragment.

Conclusion

On the theoretical side, we introduced the multi-agent planning problems decision problems - namely bCoverage, bReachability, Coverage, Reachability- that could become standard problems for proving that other multi-agent decision problems are NP-hard or PSPACE-hard. In some sense, this paper could be the starting point of a theory of multi-agent problems in complexity theory as constraint logic [DH08] is for games.

Up to now, it is unknown whether our decision problems remain hard when the \( \rightarrow \)-relations become symmetric. We think this open issue is important since symmetric \( \rightarrow \)-relations (if UAVs can go from \( v \) to \( v' \), they can also come back from \( v' \) to \( v \)) are relevant for practical applications. We also plan to study the parametrized complexity [DF99] of our problems - parameters could be the treewidth of the topological graph, the number of UAVs.

Interestingly, we plan to generalize to decentralized versions of our problems and to dynamic environments. Instead of generating sequences of actions, we will have to generate strategies as in ATL (alternating-time temporal logic) [DEG10]. As UAVs stay connected, we may suppose that when information is gained, it is common knowledge and that all actions, especially sensing actions, are public [BLMR17]. We also aim at using a high-level dedicated formal logic to express objectives, such as the language proposed in [Rub15] and [AMRZ16].

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References

[AMRZ16] Benjamin Aminof, Aniello Murano, Sasha Rubin, and Florian Zuleger. Automatic verification of multi-agent systems in parameterised grid-environments. In Proceedings of the 2016 International
François Bodin, Tristan Charrier, Arthur Queffelec, and François Schwarzentruber. Generating plans for cooperative connected uavs (demo). In Proceedings of the 27th International Joint Conference on Artificial Intelligence (IJCAI) and the 23rd European Conference on Artificial Intelligence (ECAI), Stockholm, 13-19 July 2018, 2018.

Ronen I. Brafman and Carmel Domshlak. From one to many: Planning for loosely coupled multi-agent systems. In Proceedings of the Eighteenth International Conference on Automated Planning and Scheduling, ICAPS 2008, Sydney, Australia, September 14-18, 2008, pages 28–35, 2008.

Francesco Belardinelli, Alessio Lomuscio, Aniello Murano, and Sasha Rubin. Verification of multi-agent systems with imperfect information and public actions. In Proceedings of the 16th Conference on Autonomous Agents and MultiAgent Systems, AAMAS 2017, São Paulo, Brazil, May 8-12, 2017, pages 1268–1276, 2017.

Peter Van Emde Boas. The convenience of tilings. In In Complexity, Logic, and Recursion Theory, pages 331–363. Marcel Dekker Inc, 1997.

Tom Bylander. The computational complexity of propositional STRIPS planning. Artif. Intell., 69(1-2):165–204, 1994.

Y. Chen, H. Zhang, and M. Xu. The coverage problem in uav network: A survey. In Fifth International Conference on Computing, Communications and Networking Technologies (ICCCNT), pages 1–5, July 2014.

Catalin Dima, Constantin Enea, and Dimitar P. Guelev. Model-checking an alternating-time temporal logic with knowledge, imperfect information, perfect recall and communicating coalitions. In Proceedings First Symposium on Games, Automata, Logic, and Formal Verification, GANDALF 2010, Minori (Amalfi Coast), Italy, 17-18th June 2010., pages 103–117, 2010.

Rodney G. Downey and Michael R. Fellows. Parameterized Complexity. Monographs in Computer Science. Springer, 1999.

Erik D. Demaine and Robert A. Hearn. Constraint logic: A uniform framework for modeling computation as games. In Proceedings of the 23rd Annual IEEE Conference on Computational Complexity, CCC 2008, 23-26 June 2008, College Park, Maryland, USA, pages 149–162, 2008.
[FKP05] Georgios E. Fainekos, Hadas Kress-Gazit, and George J. Pappas. Temporal logic motion planning for mobile robots. In Proceedings of the 2005 IEEE International Conference on Robotics and Automation, ICRA 2005, April 18-22, 2005, Barcelona, Spain, pages 2020–2025, 2005.

[FRLG17] Guillem Francès, Miquel Ramírez, Nir Lipovetzky, and Hector Geffner. Purely declarative action descriptions are overrated: Classical planning with simulators. In Proceedings of the Twenty-Sixth International Joint Conference on Artificial Intelligence, IJCAI 2017, Melbourne, Australia, August 19-25, 2017, pages 4294–4301, 2017.

[HTK00] David Harel, Jerzy Tiuryn, and Dexter Kozen. Dynamic Logic. MIT Press, Cambridge, MA, USA, 2000.

[Hum13] Laura R. Humphrey. Model Checking for Verification in UAV Cooperative Control Applications, pages 69–117. Springer Berlin Heidelberg, Berlin, Heidelberg, 2013.

[KGLR18] Hadas Kress-Gazit, Morteza Lahijanian, and Vasumathi Raman. Synthesis for robots: Guarantees and feedback for robot behavior. Annual Review of Control, Robotics, and Autonomous Systems, 1(1):null, 2018.

[Lev73] L. A. Levin. Universal sequential search problems. Problems of Information Transmission, 9(3):265–266, 1973.

[LP98] Harry R. Lewis and Christos H. Papadimitriou. Elements of the theory of computation, 2nd Edition. Prentice Hall, 1998.

[LPH14] Bruno Lacerda, David Parker, and Nick Hawes. Optimal and dynamic planning for markov decision processes with co-safe LTL specifications. In 2014 IEEE/RSJ International Conference on Intelligent Robots and Systems, Chicago, IL, USA, September 14-18, 2014, pages 1511–1516, 2014.

[MPR15] Aniello Murano, Giuseppe Perelli, and Sasha Rubin. Multi-agent path planning in known dynamic environments. In PRIMA 2015: Principles and Practice of Multi-Agent Systems - 18th International Conference, Bertinoro, Italy, October 26-30, 2015, Proceedings, pages 218–231, 2015.

[MTS+16] Hang Ma, Craig A. Tovey, Guni Sharon, T. K. Satish Kumar, and Sven Koenig. Multi-agent path finding with payload transfers and the package-exchange robot-routing problem. In Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence, February 12-17, 2016, Phoenix, Arizona, USA., pages 3166–3173, 2016.
[Rub15] Sasha Rubin. Parameterised verification of autonomous mobile-agents in static but unknown environments. In Proceedings of the 2015 International Conference on Autonomous Agents and Multiagent Systems, AAMAS 2015, Istanbul, Turkey, May 4-8, 2015, pages 199–208, 2015.

[Sav70] Walter J. Savitch. Relationships between nondeterministic and deterministic tape complexities. J. Comput. Syst. Sci., 4(2):177–192, 1970.

[SvEB] M.W.P. Savelsbergh and Peter van Emde Boas. Bounded tiling, an alternative to satisfiability. In G. Wechsung (ed.), proc. 2nd Frege Memorial Conference, Schwerin, Sep 1984, Akademie Verlag, Mathematische Forschung vol. 20, 1984, pp. 401-407.

[TBR+18] Davide Tateo, Jacopo Banfi, Alessandro Riva, Francesco Amigoni, and Andrea Bonarini. Multiagent connected path planning: Pspace-completeness and how to deal with it. In Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence, New Orleans, Louisiana, USA, February 2-7, 2018, 2018.

[TNMP10] WT Luke Teacy, Jing Nie, Sally McClean, and Gerard Parr. Maintaining connectivity in uav swarm sensing. In GLOBECOM Workshops (GC Wkshps), 2010 IEEE, pages 1771–1776. IEEE, 2010.

[Tur02] Hudson Turner. Polynomial-length planning spans the polynomial hierarchy. In Logics in Artificial Intelligence, European Conference, JELIA 2002, Cosenza, Italy, September, 23-26, Proceedings, pages 111–124, 2002.

[Wan61] Hao Wang. Proving Theorems by Pattern Recognition, II, pages 1–41. AT&T, 1961.

[Wan90] Hao Wang. Proving Theorems by Pattern Recognition, II, pages 159–192. Springer Netherlands, Dordrecht, 1990.

[WDMA07] Peter R. Wurman, Raffaello D’Andrea, and Mick Mountz. Coordinating hundreds of cooperative, autonomous vehicles in warehouses. In Proceedings of the Twenty-Second AAAI Conference on Artificial Intelligence, July 22-26, 2007, Vancouver, British Columbia, Canada, pages 1752–1760, 2007.

[WFCJ11] Matt Webster, Michael Fisher, Neil Cameron, and Mike Jump. Formal methods for the certification of autonomous unmanned aircraft systems. Computer Safety, Reliability, and Security, pages 228–242, 2011.

[Yan12] Evsen Yanmaz. Connectivity versus area coverage in unmanned aerial vehicle networks. In Proceedings of IEEE International Conference on Communications, ICC 2012, Ottawa, ON, Canada, June 10-15, 2012, pages 719–723, 2012.
[YL13] Jingjin Yu and Steven M. LaValle. Structure and intractability of optimal multi-robot path planning on graphs. In *Proceedings of the Twenty-Seventh AAAI Conference on Artificial Intelligence, July 14-18, 2013, Bellevue, Washington, USA.*, 2013.