ELASTIC AND INELASTIC SCATTERING OF DEUTERONS FROM $^{13}$C

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Abstract. The differential cross-sections of deuteron elastic and inelastic scattering on $^{13}$C target were measured at the cyclotron U-150M INP, Almaty, Kazakhstan. Accelerated deuteron ion energy was 14.5 MeV. The results of the study are new experimental data for the d + $^{13}$C process of elastic and inelastic scattering to the 3.68 (3/2$^-$), 6.86 (5/2$^+$) and 7.55 (5/2$^-$)MeV excited states of the $^{13}$C nucleus. The optical model with either a Woods–Saxon potential or a double folding model were used for analyzing the experimental results on elastic scattering. We performed theoretical calculations within the framework the coupled channel (CC) method for the indicated excited states. The optimal deformation parameters for the excited states of $^{13}$C nucleus were extracted.

1. Introduction

One of the main sources of obtaining knowledge about the properties of ground and low lying excited states of nuclei is investigation of elastic and inelastic scattering processes of light charged particles, such as deuterons and $\alpha$ particles. The nucleus $^{13}$C is interesting by reason of consisting of the following types of structure: the first excited state 3.09 MeV with neutron halo (1/2$^+$) [1-3], analogue 8.86 MeV (1/2$^-$) [3,4] of the Hoyle state in $^{12}$C and the recent discovery of over a compact size of the excited state 9.9 MeV (3/2$^-$) [3,5,6]. A substantial portion of this knowledge comes from analysis of experiments on elastic and inelastic scattering [7]. At present, there is not enough experimental data from deuteron interactions with the $^{13}$C nucleus. Very few works studied elastic and inelastic deuterons scattering processes on $^{13}$C [8-10]. Information about the mechanism of inelastic scattering is completely missing.

The differential cross sections of elastic and inelastic scattering of deuterons on $^{13}$C were measured at an incident energy of $E(d) = 14.5$ MeV. Analysis of the elastic-scattering data was implemented in the framework of the optical model in agreement with analyses of other $^{13}$C(d,d)$^{13}$C differential cross-section data in the energy range 13.7-18 MeV [8-10].

In this work, by using the coupled channels (CCs) method, from the theoretical description of the experimental data of inelastic scattering, we attempted to find the optimal deformation parameters for the excited states of $^{13}$C nucleus.
parameters for the $^{13}$C nucleus excited states (3.68 MeV ($3/2^-$), 6.86 MeV ($5/2^+$) and 7.55 MeV ($5/2^-$)).

2. Results and discussion
The experimental angular distributions of elastic and inelastic scattering of deuterons on the nucleus $^{13}$C were measured with an ion beam from the cyclotron U-150M at the Institute of Nuclear Physics (Almaty, Kazakhstan) at energy $E(d)=14.5$ MeV in the angular range of 10-100 degrees in laboratory system of coordinates.

![Energy spectrum of $^{13}$C(d,d)$^{13}$C at angle 36°.](image)

The collimation tube consisting of two diaphragms with diameter 2 mm were used for optimal focusing of the accelerated ions beam on the target and anti-scattering diaphragm at the end of the tube. The registration and identification of reaction products was performed by the ΔE-E method. As a target we used thin films of $^{13}$C (isotopic enrichment of about 80%) which were made using electron-beam sputtering. Several self-supporting films with a thickness of about 150 µg/cm$^2$ were used at the time of acquisition and observing of data in the experiment. The details and more comprehensive description of the experimental setup is reported in [11]. The spectrum for the d + $^{13}$C nuclear system with indicated levels at 3.09 MeV ($1/2^+$), 3.68 MeV ($3/2^-$), 6.86 MeV ($5/2^+$) and 7.55 MeV ($5/2^-$) of $^{13}$C excited states at angle 36° and at energy 14.5 MeV is shown in figure 1.

Calculations of differential cross-sections were carried out within the framework of the Optical model (OM) using FRESCO computer code [12]. The real and imaginary (with volume absorption) parts are included in the phenomenological Woods–Saxon (WS) potential. Our total real potential for these cases consists of the nuclear ($V_{nucl}$), spin-orbit ($V_{so}$) and the Coulomb ($V_C$) potentials, accordingly:

$$U(r) = V_{nucl}(r) + V_{so}(r)(l\ell) + V_C(r)$$

(1)
where, the nuclear potential is assumed to have a Wood-Saxon shape:

\[ V_{\text{nucl}}(r) = V_0 \left[ 1 + \exp\left(\frac{r - R}{a}\right) \right]^{-1} + iW \left[ 1 + \exp\left(\frac{r - R}{aw}\right) \right]^{-1}. \] (2)

the spin-orbit potential is:

\[ V_{\text{so}}(r) = V_{\text{so}}^0 \left( \frac{1}{r} \right) \frac{d}{dr} \left[ 1 + \exp\left(\frac{r - R_{\text{so}}}{a_{\text{so}}}\right) \right]^{-1}. \] (3)

and Coulomb potential of a uniform charged sphere

\[ V_C(r) = \frac{Z_pZ_t e^2}{2R_C} (3 - \frac{r^2}{R_C^2}), \quad \text{for} \quad r \leq R_C \]

\[ V_C(r) = \frac{Z_pZ_t e^2}{r}, \quad \text{for} \quad r > R_C \]

with radius

\[ R_i = r_i(A_i^{1/3} + A_i^{1/3}), \quad i = V, W, SO, C \]

where, \( V_0 \) is the Woods-Saxon potential depth, \( R \) the potential radius and \( a \) the diffuseness parameter which determines the sharpness of the potential surface. Larger values of \( a \) giving a softer surface. \( Z_p \) and \( Z_t \) are the proton numbers of the projectile and target of system, respectively.

The microscopic nuclear potential that we also used to analyze the experimental data for the d+\(^{13}\)C system was based on the Double folding (DF) model [13]. DF potential is calculated by using the nuclear matter distributions of both projectile and target nuclei together with an effective nucleon-nucleon interaction potential \( \nu_{\text{NN}} \). Thus, the DF potential is

\[ V^{\text{DF}}(R) = \int dr_1 \int dr_2 \rho_p(r_1)\rho_t(r_2)\nu_{\text{NN}}(r_12) \] (4)

\( \rho_p(r_1) \) and \( \rho_t(r_2) \) are the nuclear matter density distributions of both the projectile and target nuclei, respectively. Gaussian density distributions (GD) have been used for both nuclei [14] defined as:

\[ \rho(r) = \rho_0 \exp(-\beta r^2) \] (5)

where \( \beta \) is adjusted to reproduce the experimental values for the root-mean-square radii of d=2.11 fm and \(^{13}\)C=2.44 fm [15]. \( \rho_0 \) values can be obtained from the normalization condition

\[ \int \rho(r)r^2dr = \frac{A}{4\pi} \] (6)

where \( A \) is the mass number.

The effective nucleon-nucleon interaction, \( \nu_{\text{NN}} \), is integrated over both density distributions. Several nucleon-nucleon interaction expressions can be used for the folding model potentials. We have chosen the most common one, the M3Y (Michigan-3-Yukawa) realistic nucleon-nucleon interaction. The M3Y has two forms, one corresponds to M3Y-Reid [16] and another is based on the so-called M3Y-Paris interaction [17]. In the present work, we use the former form with the relevant exchange correction term due to the Pauli principle, given by

\[ \nu_{\text{NN}}(r) = 7999 \frac{\exp(-4r)}{4r} - 2134 \frac{\exp(-2.5r)}{2.5r} + J_{00}(E)\delta(r)\text{MeV}, \] (7)

where \( J_{00}(E) \) represents the exchange term, since nucleon exchange is possible between the projectile and the target.
In this case while the real part of the optical model has been obtained by using the above-described DF model, we have used the WS form for the imaginary potential. Therefore, for the nucleon–nucleon-DF potential case, the nuclear potential consists of a real and an imaginary part:

\[ U_{DF}(r) = N_r V_{DF}(r) + iW(r). \]  

(8)

where \( N_r \) is the normalization factor.

### Table 1. Potential parameters obtained for elastic scattering of deuterons from \(^{13}\text{C}\) at concerned energies.

| Energy (MeV) | Set | \( V \) (MeV) | \( r_V \) (fm) | \( a_V \) (fm) | \( N_r \) | \( W \) (MeV) | \( r_W \) (fm) | \( a_W \) (fm) |
|--------------|-----|--------------|--------------|--------------|---------|--------------|--------------|--------------|
|  13.7        | WS  | 105.0        |  0.8         |  1.2         |  8.4    |  1.8         |  0.6         |
|  14.5        | WS  | 106.0        |  0.91        |  1.2         | 12.4    |  1.8         |  0.6         |
|  17.7        | WS  | 111.5        |  0.91        |  1.02        |  9.4    |  1.8         |  0.6         |
|  18          | WS  | 113.5        |  0.91        |  1.2         | 12.4    |  1.8         |  0.6         |

The comparison between the experimental data and the theoretical calculations for \(^{13}\text{C}(d,d)^{13}\text{C}\) at the 13.7 [8], 14.5, 17.7 [9] and 18 MeV [10] energies are shown in figure 2 based on the potential parameters, which are recorded in table 1. In figure 2 the abbreviation WS indicates the calculations of the optical model with a Woods-Saxon potential. DF corresponds to the calculations of the optical model with the folding potential for the real part. The imaginary potential was taken from WS.

The starting potential was the global potential chosen from Lohr’s work [18]. Generally, the global potential was used by researchers, but in our case we changed some parameters of potential. The normal phenomenological spin-orbit potential is also given in (3), which permitted a better description of data at large angles using the next parameters: \( V_{so} = 9.3 \text{ MeV}, \ r_{so} = 0.9 \text{ fm} \) and \( a_{so} = 0.9 \text{ fm} \). The radii of the nuclear density distribution for the real \( (r_V) \), and imaginary \( (r_W) \) parts, and diffusions \( (a_V \text{ and } a_W) \) of the potential were fixed to remove the discrete ambiguity in the optical potential (OP). Minimization of \( \chi^2 \) with a variation of the remaining 2 parameters of OP \( (V, W) \) helped to achieve fitting the theory to the experiment. The Coulomb radius was fixed as \( r_c = 1.28 \text{ fm} \) in the OM calculations.

The calculated values of the elastic scattering cross sections are in good agreement with the existing experimental data [8-10] which are presented in Fig. 2.

Analyses of the angular distributions of excited states 3.68 MeV \((3/2^-)\), 6.86 MeV \((5/2^+)\) and 7.55 MeV \((5/2^-)\) of \(^{13}\text{C}\) nuclei were performed within the coupled channel method (CC). Our calculations were exclusively based on the work [19], which was performed with good resolution at energies \( E(\alpha) = 388 \text{ MeV} \) on \(^{13}\text{C}\) nuclei, where it was shown that the states of 3.68 MeV \((3/2^-)\) and 7.55 MeV \((5/2^-)\) are mainly excited, i.e. with \( L = 2 \). The Woods-Saxon (CC(WS)) and double folding (CC (DF)) potential parameters were used in the calculations. The comparison between the experimental data and theoretical predictions for the states 3.68 MeV \((3/2^-)\), 6.86
Figure 2. Comparison between the experimental data and the calculated differential cross section for elastic scattering of deuterons from $^{13}$C at energies 13.7, 14.5, 17.7 and 18 MeV using Woods-Saxon (WS) and double folding (DF) potentials.

MeV ($\frac{5}{2}^+$) and 7.55 MeV ($\frac{5}{2}^-$) at energy $E(d)=14.5$ MeV are shown in figure 3. The CC (DF) provides the best description of the experimental data.

The parameters with information about the relevant state, such as spin, parity and excitation energy should be provided in the frame of coupled channel calculations, as we mentioned above. In addition the deformation parameter is used as an adjustable quantity. Calculations allowed us to estimate the value of the quadrupole deformation parameter of the target nucleus: $\beta_2 = 0.51$, which agrees with the data from other sources. The deformation parameter of the $^{13}$C nucleus is in the range 0.45 - 0.6, as mentioned in [7,20].
3. Summary

New experimental data were presented for the processes of elastic and inelastic scattering leading to the 3.68 MeV (3/2−), 6.86 MeV (5/2+) and 7.55 MeV (5/2−) excited state in $^{13}$C using deuteron beams at an energy of 14.5 MeV incident on a $^{13}$C target. Optical model (phenomenological) and double folding (semi-microscopic), as two of the most suitable approaches were used for analyzing the mechanism of deuteron elastic scattering. Additionally, we applied and compared our data analyzes with the previous measurements at higher energies. The data for the excited states were analyzed within the framework of coupled channel method with two different potentials of Woods-Saxon and double folding. The obtained deformation parameters for the $^{13}$C nucleus at 14.5 MeV are close to the results obtained in [7,20].

In the nearest future we would consider presentation and analysis of data on inelastic scattering of other states: 3.09 MeV(1/2+) - state with neutron halo, 8.86 MeV(1/2−) - analogue of the Hoyle state in the $^{12}$C and 9.90 MeV (3/2−) – possible compact cluster state, which were

Figure 3. Comparison between the experimental data and the calculated differential cross section for inelastic scattering of deuterons from $^{13}$C (3.68 MeV (3/2−); 6.86 MeV (5/2+); 7.55 MeV (5/2−)) at energy 14.5 MeV using different potentials in coupled channel method: Woods-Saxon (WS) and double folding (DF).
observed but have not been analyzed yet. It is planned to complete and finish analysis of this data using the parameters of optical potentials found in this study in future works.

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