Research on Iterative Calibration Method of Linear Model Parameters of Gravity Sensor

Shuhai Lu¹, Juliang Cao¹*, Shaokun Cai¹, Ruihang Yu¹, Bainan Yang¹

¹College of Intelligent Science, National University of Defense Technology, ChangSha, HuNan, 410073, China

*Corresponding author’s e-mail: cjvl@163.com

Abstract. In recent years, the calibration method of Strapdown gravimeter has become the bottleneck of relative gravity measurement accuracy. Based on this situation, this thesis studies the parameter iterative calibration method of SGA-WZ03 strapdown gravimeter to break the bottleneck of relative gravity measurement and further improve the measurement accuracy of relative gravimeter in flight experiment through the improvement of calibration method Degree. The parameter iterative estimation algorithm is based on the pulse output model of a single accelerometer to establish the linear pulse output model of the accelerometer components. Using the idea of parameter separation, the two-step iterative estimation of the model parameters and the tilt vector can complete the calibration of the linear model and the secondary model of the accelerometer. Compared with the traditional 24 bit calibration method, the parameter estimation iterative estimation calibration method has better stability than the traditional method.

1. Introduction
Gravity measurement can be divided into absolute and relative Gravity measurement [1]. Strapdown gravimeter used in relative gravity measurement has the advantages of small volume, light weight and good dynamics, and has always been an important field of relative gravity measurement [2].

The measurement error of Strapdown gravimeter mainly comes from the gyro and accelerometer components, and compared with the high-precision gyro components, the parameter stability of accelerometer components is poor. Therefore, the research on the calibration method of the related parameters of the accelerometer components and the compensation of the possible errors can improve the measurement accuracy of the gravimeter [3]. Therefore, it is very important to improve the accelerometer model and propose a new calibration method for improving the accuracy of gravity measurement [4].

2. Method and principle

2.1. Gravity vector measurement model
The mathematical model of gravity vector measurement can be obtained:

\[ \delta g^n = \dot{v}^n - C_b^n f^b + (2\omega^b_v + \omega^b_n) \times \dot{v}^b_v - \gamma^n \]  

(2.1)

In formula (2.1), each parameter on the right side can be obtained directly or indirectly [5]. The carrier’s acceleration \( \dot{v}^n \) and speed \( v^n \) can be measured directly by GPS. The direction cosine matrix \( C_b^n \) and specific force \( f^b \) can be measured by strapdown inertial navigation system. Projection of
angular velocity $\omega_{en}^s$ can be calculated indirectly by GPS and inertial navigation data. Both the earth's rotation speed $\omega_{ei}^s$ and normal gravity value $\gamma^n$ can be calculated directly according to the model [6].

In the measurement of gravity scalars, only the components of the gravity disturbance vector in the vertical direction need to be selected:

$$\delta g_U = v_U - f_U - (2\omega_{ei} \cdot \cos L + \frac{v_E}{R_N + h}) \cdot v_E - \frac{v_N^2}{R_M + h} + \gamma$$ (2.2)

As mentioned above, strapdown gravimeter adopts relative gravity measurement, so absolute gravity value of a known point is required before measurement [7]. Generally, the starting point of measurement test is regarded as a known point [8]. The absolute gravity value of all measurement points in the test can be obtained by knowing the absolute gravity value of the starting point and the measured relative gravity value.

2.2. Traditional laboratory calibration method
The traditional calibration principle of laboratory accelerometer does not consider the influence of the secondary term and cross coupling term of the accelerometer [9]. Since the calibration of the accelerometer needs the assistance of the laser gyro, before the experiment, the electronic level instrument is used to level the turntable mechanically to ensure that the outer frame axis rotates in strict plumb (coincides with the gravity vector) [11].

2.3. Parameter iterative estimation algorithm
The parameter iterative estimation algorithm is an iterative estimation algorithm which uses high-precision gravity value to estimate the parameters of the accelerometer component model. It aims to realize the unbiased estimation of the important parameters of the accelerometer component [10].

According to the accelerometer pulse output model, ignored the nonlinear error coefficient, and the accelerometer noise is assumed to satisfy the Gaussian distribution, then the linear pulse output model can be obtained by simplifying equation (2.2):

$$\begin{bmatrix}
N_x^a \\
N_y^a \\
N_z^a
\end{bmatrix}
= \begin{bmatrix}
N_x^0 \\
N_y^0 \\
N_z^0
\end{bmatrix}
+ \begin{bmatrix}
K_x^a & 0 & 0 \\
0 & K_y^a & 0 \\
0 & 0 & K_z^a
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 \\
\tau_{yx} & \tau_{yy} & 0 \\
\tau_{z} & \tau_{zy} & \tau_{zz}
\end{bmatrix}
\begin{bmatrix}
f_x^b \\
f_y^b \\
f_z^b
\end{bmatrix}
+ \begin{bmatrix}
\nabla_x \\
\nabla_y \\
\nabla_z
\end{bmatrix}$$ (2.3)

When the carrier is stationary relative to the local geographic coordinate system, the specific force measured by the accelerometer assembly is the projection of the gravity vector in the carrier coordinate system, and its mathematical expression is:

$$f^b = C_n^b f^n = -C_n^b g^n$$ (2.4)

Substituting the equation (2.4) into the formula (2.3) can obtain another form of the linear pulse output model of the accelerometer:
Suppose that there are \(m\) observation positions in the calibration, then there are \(m\) equations similar to equation (2.7). The difference lies in the tilt vector of different positions. Then the unknown parameters in the \(m\) equations can be divided into two categories [12]. One is the accelerometer related parameters independent of the position:

\[
\begin{bmatrix}
    N_x^a \\
    N_y^a \\
    N_z^a
\end{bmatrix}
= \begin{bmatrix}
    N_0^a \\
    N_0^a \\
    N_0^a
\end{bmatrix}
+ g \begin{bmatrix}
    K_{axx} & 0 & 0 \\
    K_{ayx} & K_{a} & 0 \\
    K_{azx} & K_{a0} & 0
\end{bmatrix}
\begin{bmatrix}
    I_1 \\
    I_2 \\
    I_3
\end{bmatrix}
+ \begin{bmatrix}
    \nabla_x \\
    \nabla_y \\
    \nabla_z
\end{bmatrix}
\tag{2.5}
\]

The other is the tilt vector according to the change of position:

\[
\begin{bmatrix}
    N_x^0 \\
    N_y^0 \\
    N_z^0
\end{bmatrix}
= \begin{bmatrix}
    N_0^0 \\
    N_0^0 \\
    N_0^0
\end{bmatrix}
+ g \begin{bmatrix}
    K_{axx} & 0 & 0 \\
    K_{ayx} & K_{a} & 0 \\
    K_{azx} & K_{a0} & 0
\end{bmatrix}
\begin{bmatrix}
    I_1 \\
    I_2 \\
    I_3
\end{bmatrix}
+ \begin{bmatrix}
    \nabla_x \\
    \nabla_y \\
    \nabla_z
\end{bmatrix}
\tag{2.6}
\]

According to the measurement principle of accelerometer, it can be seen that the pulse output of different observation positions does not affect each other, which is, \(N^a_i\) and \(N^a_j\) \((i \neq j)\) are independent of each other. The pulse output \(N^a\) in the whole calibration process also obeys the normal distribution, which is, \(N^a - N(\mu(x, y), \xi)\). It can be assumed that the measurement noise signal intensity of each observation position is the same but independent of each other: \(\xi = \sigma^2 I_{m \times m}\). Then the probability density function of the overall pulse output \(N^a\) can be expressed as:

\[
f(N^a) = \prod_{i=1,2,\cdots,m} f(N^a_i) = \frac{1}{(2\pi\sigma^2)^{\frac{m}{2}}} \exp \left( -\frac{1}{2\sigma^2} \sum_{i=1}^{m} \| N^a_i - \mu_i(x, y) \|^2 \right) \tag{2.8}
\]

Known the probability density function of pulse output \(N^a\), the model parameters and tilt vector can be estimated unbiased by maximum likelihood estimation (MLE).

Considering that in the unknown quantity \(\mu_i(x, y)\), the model parameters \(x\) and tilt vector \(y\) are independent of each other, then the method of parameter separation estimation can be used to iteratively calculate \(x\) and \(y\).

3. Experiment

3.1. Design of calibration arrangement scheme for parameter iterative estimation algorithm

The problem of arranging the optimal observation position of accelerometer components is equivalent to solving the horizontal attitude angle when the model parameters have the maximum sensitivity to the gravity vector.

In this experiment, the three-axis turntable in the laboratory is used to simulate the infinite two axis rotating platform [13]. The actual figure is shown in Figure 1, and the gravimeter used in this chapter is shown in Figure 2:
3.2. Parameter estimation of linear model

Under normal temperature, a fiber optic gyro strapdown gravimeter is calibrated and tested. The gravimeter used is shown in Figure 2. There are 24 static positions in the calibration path, and each position is sampled for 100s [14].

Install the gravimeter on the high-precision three-axis rotating platform in the specified way, then keep the outer frame fixed, control the inner frame and the middle frame to rotate according to the calibration path designed above, and record the pulse output of each axis accelerometer, as shown in Figure 3.

The algorithm in Section 2.4 is used to process the accelerometer pulse, and the estimation results of each parameter are shown in Figure 4.

It can be seen from the above figuresthat after 100 times of iterative calculation, all parameters tend to be stable, indicating the convergence of the results of iterative calculation.
3.3. Comparison between iterative method and traditional calibration method of linear model

Select a section of static multi position measurement data, using the accelerometer parameters calibrated by this chapter calibration method and the scale factor and zero deviation calibrated by the traditional method to calculate the specific force of the accelerometer output respectively. There are 21 static positions in the verification experiment, and the gravity module values of the two calibration methods are shown in Figure 5:

![Figure 5. Comparison between linear model estimation method and traditional method](image)

|                      | Linear model | Traditional method |
|----------------------|--------------|-------------------|
| Mean                 | 9.7917       | 9.7927            |
| Variance             | 0.00006843   | 0.0002052         |
| Range                | 0.000267     | 0.000893          |

It can be seen from the chart that the calibration accuracy is 0.001 deviation from the average value of the traditional calibration method. However, the variance of the gravity modulus values of the linear model iterative method at 21 static positions is 0.00006843, which is one bit more accurate than the variance of the traditional calibration scheme. Similarly, we can also find that the range is also greatly reduced. Better guarantee the stability and reliability of relative gravity measurement at fixed points.

4. Conclusion

This paper studies the calibration of linear parameters of quartz flexible accelerometer components under the conditions of gravity as the observation. According to the pulse output model of a single accelerometer, a linear pulse output model of the accelerometer component is established. In order to achieve the optimal estimation of the model parameters, the optimal estimation conditions for each model parameter were determined by obtaining the extreme value of the sensitivity function of the gravity value relative to each model parameter, and a 23-position calibration arrangement scheme was designed based on this.

The experimental results show that the estimation accuracy of the linear model iterative method is equivalent to that of the traditional method, but the variance of the linear model iterative method is one order of magnitude smaller than that of the traditional method, which will be more advantageous in the application of relative gravity measurement. It is standard practice in the industry to judge the accuracy of relative gravity measurements by repeating line comparisons or inter-point differences. When we make gravity measurements on repeating lines or points, our proposed method can provide closer relative gravity values. From this conclusion, it can be seen that the linear model iterative scheme is more applicable in the field of relative gravity measurement.
We can infer that this method can effectively complete the gravity measurement tasks and reduce unnecessary geographical mapping times.

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