The modification of classical dynamics of abelian confining theory by virtue of quantum Abrikosov-Nielsen-Olesen strings is discussed taking D=4 abelian Higgs model as an example. The form of string corrections to the Wilson loop correlators, gauge boson propagator, effective potential is presented and possible relations to abelian-projected QCD are outlined.

I. INTRODUCTION

The confining properties of gauge theories attract considerable attention for years. Despite quantum chromodynamics is undoubtedly the most physically interesting example, analysis of other models can also be instructive. Of particular interest is the scalar QED with the Higgs-type potential, the Abelian Higgs Model (AHM) \[1,2\] – the simplest abelian theory exhibiting the property of confinement. The dual charge and anticharge are confined in this model already on the classical level through the Abrikosov-Nielsen-Olesen (ANO) string formation.\[1\] It is numerically argued (see review \[3\] and references therein), that the dual version of this model might play the role of effective theory of abelian degrees of freedom for QCD after the abelian projection procedure. Vacuum is characterized by nonzero expectation value of the scalar field \[\langle |\Phi| \rangle\], vector gauge boson is massive with the mass \[m_\gamma = e \langle |\Phi| \rangle\], providing Meissner effect, and Goldstone mode does not interact with the gauge field. The Anderson–Higgs phenomenon is realized in this way on the classical level.

The quantum spectrum of this theory includes excitations of two different types. The vector gauge boson, acquiring nonzero mass \[m\] due to Higgs effect and scalar particle with the mass \[m_\rho\], corresponding to the excitation of the condensate are of perturbative type since their masses are proportional to the corresponding couplings. The second class of nonperturbative objects is represented by the ANO–strings and their excited states. Generally speaking, one could find two types of them: worldsheets of classical (but, probably, vibrating) strings bound by the external monopole loop (open strings) and pure quantum excitations characterized by the closed surfaces. Since the string tension of the ANO–strings nonanalytically depends on the coupling, these excitations have nonperturbative origin.

The consistent quantum theory of ANO–strings is unfortunately still absent (see \[4,5\] in this respect). In particular, it is not known, how to calculate the creation and annihilation probabilities for the closed strings and also the spectrum of quantum stringy states (which are in some sense analogous to glueballs in QCD) cannot be determined. Therefore exact dynamical calculations involving ANO strings beyond perturbation theory are impossible. Nevertheless one can extract some qualitative results concerning the role quantum ANO strings can play in the dynamics of the perturbative degrees of freedom. It is done in the present paper by "phenomenological" parametrization of the string contributions to different quantities by one unknown function – Gaussian string correlator. In some cases this contribution is exact.

The partition function we are going to consider reads

\[
Z = \int DA_\mu e^{-\frac{i}{4} \int F_{\mu\nu}^2 d^4x} e^{-S[A]} \quad (1.1)
\]

where

\[
e^{-S[A]} = \int D\Phi e^{-\int (\frac{i}{2} |D_\mu [eA|\Phi|] e^{-\frac{1}{2} (|\Phi|^2 - \eta^2)^2})d^4x} \quad (1.2)
\]

We are working in 4d Euclidean space in the paper and are not taking care of multiplicative field-independent factors in all formulas. The complex scalar field \(\Phi(x) = \phi(x)e^{i\theta(x)}\) couples with the gauge field \(A_\mu(x)\) via covariant derivative

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\[ D_\mu [eA] = \partial_\mu \Phi + ie A_\mu \Phi. \]  

The theory is assumed to be in the London limit \( \lambda/e^2 \gg 1 \). The radial part of the field \( \phi(x) \) fluctuates in the vicinity of \( \eta \): \( \Phi(x) = (\eta + \xi r(x)) e^{i\theta(x)} \), where \( \xi \equiv 1/\sqrt{\lambda} \). In the variables introduced eq. (1.2) becomes:

\[
e^{-S[A]} = \int \rho \mathcal{D} \rho \int \mathcal{D} \theta e^{-\frac{\pi}{\alpha} \int (\partial_\mu \rho)^2 \, d^4x} e^{i \int (\eta + \xi r(x))^2 (\partial_\mu \theta - e A_\mu)^2 \, d^4x - \int (4\eta^2 \rho^2 + 4\xi \eta \rho^2 + \xi^2 \rho^4) \, d^4x} \tag{1.3}
\]

The integration over \( \rho \) gives at the leading order in \( \xi \):

\[
e^{-S[A]} = \int \mathcal{D} \theta e^{\frac{-\pi}{2} \int (\partial_\mu \theta - e A_\mu)^2 \, d^4x + \frac{\eta^2}{4} \int (\partial_\mu \theta - e A_\mu)^4 \, d^4x} \tag{1.4}
\]

We denoted classical mass of the vector gauge boson \( m = e \eta \) in eq. (1.4). As it is well known, the \( \theta \)-dependence of the integrand in (1.4) cannot be omitted since for singular configurations of the phase \( \theta(x) \) it represents the interaction between ANO strings and the gauge field. The Stokes theorem leads to the following relation (3–8):

\[
\partial_\mu \partial_\nu \theta(x) - \partial_\nu \partial_\mu \theta(x) = \pi \epsilon_{\mu \nu \alpha \beta} \Sigma_{\alpha \beta}(x) \tag{1.5}
\]

where \( \Sigma_{\alpha \beta}(x) \) defines the vorticity tensor current \( \Sigma_{\alpha \beta}(x) = \int d\sigma \delta(4)(\xi(x) - x) \) and integral is taken over some 2-complex \( \Sigma \), which physically is the world-sheet of the ANO string(s). If there are no dynamical monopoles in the theory, these world-sheets are closed: \( \partial_\mu \Sigma_{\mu \nu}(x) = 0 \).

## II. CORRELATORS OF THE WILSON LOOPS

It is convenient to calculate Wilson loops correlators for electric and magnetic external currents, \( j_\mu(x) \) and \( J_\mu(x) \), respectively. The electric current is chosen to be \( j_\mu(x) = \int d\zeta \delta(z - x) \) while the magnetic one is introduced via the condition \( \partial_\mu \Sigma_{\mu \nu}(x) = J_\mu(x) \) where \( \Sigma_{\mu \nu}(x) \) is the vorticity tensor current corresponding to the ANO-string worldsheet. It reflects the fact, that each monopole in the condensate made of electrically charged particles must be accompanied by the ANO string. Since \( \partial \theta \Sigma = 0 \), correlators with total magnetic charge not equal to zero are automatically excluded, as it should be on the physical grounds.

The last term in the \( S[A] \) in (1.4), which is proportional to the fourth power of the gauge fields does not contribute at the leading order to the bilocal current-current interaction. It is suppressed by at least two powers of \( \xi \) and by additional factors coming from ANO-string correlation functions.

The Wilson loop for electric and magnetic currents is given by:

\[
\langle W(C) \rangle = \int \mathcal{D} A_\mu e^{\frac{i}{2} \int \mu \nu \, d^4x} e^{-S[A]} e^{i \epsilon C \int j_\mu A_\mu \, d^4x} = \int \mathcal{D} \theta e^{\int d^4x \int d^4y K(x,y)} \tag{2.1}
\]

where

\[
K(x,y) = \pi^2 \eta^2 \Sigma_{\mu \nu}(x) \Delta_m(x-y) \Sigma_{\mu \nu}(y) + \frac{\xi^2 e^2}{2} j_\mu(x) \Delta_m(x-y) j_\mu(y) + \frac{1}{2} \left( \frac{2\pi \epsilon}{e} \right)^2 J_\mu(x) \Delta_m(x-y) J_\mu(y) + i \pi \xi j_\mu(x) \epsilon_{\mu \nu \alpha \beta} \Sigma_{\alpha \beta}(y) \left( \frac{\partial}{\partial x_\mu} \left( \Delta_m(x-y) - \Delta_0(x-y) \right) \right) \tag{2.2}
\]

where the massive propagator \( \Delta_m(x-y) \) is given by

\[
\Delta_m(x) = \int \frac{d^4 p}{(2\pi)^4} \frac{e^{i p x}}{p^2 + m^2} \tag{2.3}
\]

and \( \Delta_0(x-y) = \Delta_{m=0}(x-y) \).

In case without external monopoles, i.e. \( J_\mu(x) = 0 \), eq. (2.2) coincides with the result found in [5,6]. The correct factor for the charge carried by the monopole currents manifests the Dirac quantization condition. The factor \( \zeta \) is the ratio of the electric charge of the test particle to the electric charge of the condensed particles, which can be different from unity.

The integral over \( \theta \) is taken over the ensemble of all closed 2-complexes. The nonlocality of the string action in (2.2) makes the correspondence with known bosonic string models, characterized by the local actions far from being straightforward.
Nonperturbative effects in the model are taken into account via the following string correlator

$$\langle\langle \Sigma_{\mu}(z) \Sigma_{\rho\sigma}(t) \rangle\rangle = \frac{1}{Z} \int D\theta \Sigma_{\mu\nu}(z) \Sigma_{\rho\sigma}(t) e^{-\pi^2 \eta^2} \int d^4x \int d^4y \Sigma_{\mu\nu}(z) \Delta_{\mu\nu}(x-y) \Sigma_{\rho\sigma}(y)$$

with \(\Sigma_{\mu\nu}(x)\) defined according to (1.5). The Lorentz structure of the l.h.s. of (2.4) is fixed in the unique way by the condition of closeness of \(\Sigma\):

$$\langle\langle \Sigma_{\mu\nu}(z) \Sigma_{\rho\sigma}(t) \rangle\rangle = \epsilon_{\rho\mu\alpha\beta} \frac{\partial}{\partial z^\beta} \epsilon_{\sigma\rho\alpha\gamma} \frac{\partial}{\partial t^\gamma} d(z-t)$$

Such form of string correlator was explored in [7].

The expression (2.4) is physically the sum over all closed worldsheets of the ANO strings, which raises the question, to see, that the group of transformations, leaving eq.(2.4) invariant is larger than just reparametric one, and most of these symmetries are hidden in some way. In particular, any 2-complex in the r.h.s. of eq.(2.1) can be factorized into a linear superposition of 2d surfaces – the property, which presumably does not hold in the nonabelian case. It is not clear how one should incorporate this symmetry into the measure \(D\theta\) if one wishes to rewrite it in the conventional stringy form as integration over all embeddings and world-sheet geometries.

To construct the effective low-energy action for the abelian confining strings in the continuum would be in its own turn an interesting task. It is likely, that it would be local in the bulk, but not on the world-sheet (see, for example, [11], where Weyl–noninvariant string action is proposed and crumpling is argued to be prevented by introducing of higher order terms).

All this discussion is of academic interest, until it will help in practical calculations with eq.(2.4). The corresponding action is nonlocal and calculation of any nontrivial correlator starting directly from eq.(2.4) seems to be hopeless. However, to guess how this correlator looks like, one can use field theoretical language. Introducing vector field \(V_\mu\) via the definition \(\theta(x) = \int^x V_\mu dz_\mu\) and taking into account (1.5) one gets

$$\langle\langle \Sigma_{\mu\nu}(z) \Sigma_{\rho\sigma}(t) \rangle\rangle = -\frac{1}{Z} \frac{\delta^2}{\delta J_{\mu\nu}(z) \delta J_{\rho\sigma}(t)} \int D\theta V_\mu \Gamma[V_\mu] e^{-\frac{i\pi}{4}} \int d^4x \int d^4y G_{\mu\nu}(x) \Delta_{\mu\nu}(x-y) G_{\rho\sigma}(y) \cdot e^{i\pi \int d^4x J_{\mu\nu}(x) G_{\alpha\beta}(x) \epsilon_{\mu\nu\alpha\beta}}$$

where the field strength \(G_{\mu\nu}(x) = \partial_\mu V_\nu(x) - \partial_\nu V_\mu(x)\) was introduced. The Jacobian \(\Gamma[V_\mu]\) takes into account compactness of the corresponding \(U(1)\) and is proportional on the lattice to the sum over all closed 1-cycles with some weight:

$$\Gamma[V_\mu] = \sum_{\{C\}} e^{i\oint V_\mu(z) dz_\mu}$$

Since these currents are formed by the electrically charged quasiparticles, which are condensed, we expect the sum in (2.7) to be divergent and replaced in the continuum by

$$\Gamma[V_\mu] = \int D\chi e^{-\int(\frac{1}{2} |D_\mu[V]|^2 + L(\chi))d^4x}$$

where the first factor in the exponent came from the naive exponentiation of the corresponding determinant \([3,14]\) while the term \(L(\chi)\) provides the correct normalization of the original action [1,2]. Expression (2.8) does not contain any small dimensionless parameter. If \(L(\chi)\) is such, that the field \(V_\mu\) acquires mass, one gets in the infrared limit for the r.h.s. of (2.2)

$$d(p) \sim \frac{1}{p^2 + M^2}$$

One expects to have such behaviour in the Higgs phase of the theory, which was argued also in [11], where the mass \(M\) was associated with the mass of the lowest-lying 1 “glueball” of this theory. It is easy to verify, that in case \(M = 0\) the gauge field \(A_\mu\) becomes massless too – this situation corresponds to the Coulomb phase.
III. GAUGE BOSON PROPAGATOR AND EFFECTIVE POTENTIAL

It is known, that the concept of propagator is of limited use in the theories of the type considered here. The internal reason is the presence of nonlocal interacting objects – topological defects, or, in other words, the fact, that Dirac strings attached to the monopoles become dynamical (i.e. just ANO strings, see discussion in [12]) due to the presence of the condensate. The physical reason lies in impossibility to put into the vacuum of such theory the probing test charge small enough to neglect the corresponding change in the properties of the system. Born approximation does not work and the potential nonanalytically depends on the charges of interacting particles.

Therefore it is more instructive to consider gauge-invariant correlation functions from the beginning, as we did. Nevertheless there is a possibility to interpret the results (at least partly) in terms of gauge boson propagator. It happens if the external currents enter with a small parameter, the role of which is played by \( \zeta \) in (2.2).

One should stress, that in the theory considered here (and presumably in nonabelian theories too) confinement is due to formation of the confining string(s) and it is difficult (if not impossible) to describe this phenomenon in terms of propagation of anything (see, for example, [9], where the string formation was associated with the special singularities of the propagator, and references therein). On the other hand, one may consider only current-current terms of propagation of anything (see, for example, [15]), where the string formation was associated with the special is due to formation of the confining string(s) and it is difficult (if not impossible) to describe this phenomenon in terms of propagation of anything (see, for example, [9], where the string formation was associated with the special singularities of the propagator, and references therein). On the other hand, one may consider only current-current terms in the correlators like calculated above and extract the gauge boson propagator \( D_{\mu \nu}(x-y) \) from them according to the following definition

\[
\langle j_j \rangle = \langle \tilde{W} \rangle \cdot e^{-\kappa} \int d^4x \int d^4y \, j_\mu(x) D_{\mu \nu}(x-y) j_\nu(y)
\]

where the factor \( \kappa \) is equal to \( \zeta^2 e^2 \) or \( 4 \pi^2 / e^2 \) for \((jj)\) and \((JJ)\) cases, respectively and the factor \( \langle \tilde{W} \rangle \) accounts for the effects of confinement and will not interest us below. One gets from (2.2) taking into account (2.3):

\[
D_{\mu \nu}(p) = \frac{\delta_{\mu \nu}}{p^2 + m^2(1 - 4 \pi^2 \eta^2 d(p))} + D_{\mu \nu}(p)
\]

It is worth noting, that one can use either electric–electric or magnetic–magnetic correlators to derive (3.2). This could be expected from the beginning since despite there are two types of particles (electrically and magnetically charged) in the theory, there is only one photon. The quantum correction proportional to \( d(p) \) in (3.2) leads to the deviation of the static potential between electrical charges from the classical Yukawa–type one; the classical potential between external monopoles is also modified. Exact lattice measurements of the static potentials could shed some light on the relative importance of the discussed effects, in particular, beyond the London limit.

The stringy corrections were discussed in [9] and also in [10] for the propagator of the dual vector boson (which mediates interaction between electric charges in the magnetically charged monopole condensate), while we are interested in the exchange between electrically charged particles in the condensate of electric charges. To some extent the dynamical corrections to the minimal string action

\[
S_{\text{min}} = \int d^4p \pi^2 \eta^2 \Sigma_{\mu \nu}(p) \Delta_m(p) \Sigma_{\mu \nu}(-p)
\]

are analogous to the quantities studied in [11]. The string fluctuations modify \( \Delta_m(p) \) as

\[
\Delta_m(p) \rightarrow \frac{1}{p^2 + m^2 + 4 \pi^2 \eta^2 d(p)}.
\]

In the infrared region this correction is damped as compared with (3.2). It is simple consequence of the fact, that the effective vertex of the Kalb–Ramond field (propagating according to (3.3) interaction contains additional power of momentum. The signs of the corrections to (3.2) and (3.3) are different. For positive \( d(p) \) it means, that string fluctuations make the vector boson lighter while the string tension smaller with respect the the classical values.

In the strict sense the Gaussian approximation adopted in (3.3) is not justified contrary to (2.2), where it is controlled by the small parameter \( \zeta \). There are special cases however where Gaussian correlator provides exact answer. One example of such sort is the effective potential. Perturbative equation on extremum of the effective potential reads (see, for example, [13])

\[2\text{The dependence of the results from [1] on the auxiliary vector } n_\mu \text{ inevitable in the theory with Higgs effect considered in the Zwanziger formalism makes it difficult to compare (3.3) to the results from [1].} \]
\[ \frac{\lambda_R}{6} \rho^2 - m_R^2 = \frac{3 e^4 \rho^2}{16 \pi^2} \ln \frac{\mu^2}{e^2 \rho^2} \] (3.4)

where \( m_R \) and \( \lambda_R \) are renormalized mass and coupling constant respectively and \( \mu \) – normalization point. The string contribution has the following form:

\[ \frac{\delta S_{\text{eff}}}{\delta \rho} = \frac{\lambda_R}{6} \rho^2 - m_R^2 - \frac{3 e^4 \rho^2}{16 \pi^2} \ln \frac{\mu^2}{e^2 \rho^2} + 12 \pi^2 \int \frac{d^4 p}{(2\pi)^4} \frac{p^4 d(p)}{(p^2 + e^2 \rho^2)^2} = 0 \] (3.5)

One expects, that the integral in the last term produces quadratically divergent correction to the mass \( m_R \), logarithmically divergent contribution to \( \lambda_R \) and finite term, having the same structure as the r.h.s. of (3.4). The resulting effective potential could have minimum therefore at the point \( \rho \neq 0 \) even if the strings are present. The selfconsistent picture implies the string correlator \( d(p) \) corresponding to the string solutions to the equations following from the exact effective potential (3.5).

Having in mind the AHM as an effective theory for abelian–projected QCD, one should mention, that there are numerical evidences [16] that the parameters of the effective AHM correspond the Bogomolny bound \( m = m_H \) rather than in the London limit. In particular, the QCD vacuum correlation length \( T_g \) (which is an analog of \( m^{-1} \)) is about 1 Gev while the lowest excitation in gluodynamics is 0^{++} glueball, with the typical mass 1.5 Gev. It means, that one \emph{a priori} has no reason to expect some kind of decoupling here since there are no degrees of freedom, which are really "heavy".

Therefore numerically observed facts, supporting just \emph{the classical} picture of confinement in terms of AHM (for example, exponential profile of the chromoelectric field inside the QCD string) should look rather unexpected if \emph{quantum} corrections are taken into account. In other words, the classical picture is protected in the London limit and not beyond it. It should also be noted, that one cannot simply omit all quantum corrections and consider classical equations of motion, following from AHM as the only fingerprint of the original nonabelian theory after abelian projection. In particular, absence of integration over worldsheets in (2.1) for the case of single monopole loop \( J \) would lead to the unphysical dependence of the Wilson loop on the arbitrary chosen surface bound by \( J \).

Another point is related with the fact, that the string effective action has only one mass parameter – mass of the auxiliary Kalb-Ramond field (which coincides with the vector boson mass). The situation in QCD is much more complicated: there is the tower of states, contributing to the gauge–invariant 2-point field strength correlator. It is an open question to what extent the quantum dynamics of the abelian projected theory can mimic this essentially nonabelian feature.

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