Confrontation of a Double Inflationary Cosmological Model with Observations

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Abstract

CDM models with non-scale-free step-like spectra of adiabatic perturbations produced in a realistic double inflationary model are compared with recent observational data. The model contains two additional free parameters relatively to the standard CDM model with the flat \((n = 1)\) initial spectrum. Results of the COBE experiment are used for the determination of a free overall spectrum normalization. Then predictions for the galaxy biasing parameter, the variance for ”counts in cells”, the galaxy angular correlation function, bulk flow peculiar velocities and the Mach number test are obtained. Also considered are conditions for galaxy and quasar formation. Observational data strongly restricts allowed values for the two remaining model parameters. However, a non-empty region for them satisfying all considered tests is found.

1 Introduction

Inflationary cosmological models (Starobinsky 1980, Guth 1981, Linde 1982, 1983) imply a density parameter \(\Omega_{\text{tot}} \approx 1 \ (|\Omega_{\text{tot}} - 1| < 10^{-4})\) within the observable part of the Universe. Combined with the value \(\Omega_{\text{bar}} \approx 0.017 h^{-2}\) following from the theory of primordial nucleosynthesis (see, e.g. Walker et al. 1991) this requires the most of matter in the Universe to be nonbaryonic \((h = H_0/100 \text{ km/s/Mpc})\). Second, the simplest inflationary models (with one slowly
rolling effective scalar field) predict density fluctuations with a spectrum of approximately Zeldovich-Harrison type (i.e., \((\frac{\delta \rho}{\rho})^2 \propto k^n\) with \(n \approx 1\)). This result was first consistently derived by Hawking 1982, Starobinsky 1982, and Guth & Pi 1982. The cold dark matter (CDM) model with this initial spectrum of perturbations and with biasing \(b_g \approx 1.5 - 2.5\) have most successfully explained the observed hierarchy of cosmic structures up to scales of approximately 10 \(h^{-1}\) Mpc (Davies et al. 1985). However, observations of such structures as the Great Attractor (Lynden-Bell et al. 1988, and Dressler 1991) and the Great Wall (de Lapparent et al. 1986), the large-scale clustering in the redshift survey of IRAS galaxies (Efstathiou et al. 1990a, Saunders et al. 1991), and the galaxy angular correlation function and "counts in cells" for the deep APM galaxy survey (Maddox et al. 1990, see also Loveday et al. 1992) imply that there is more power in the perturbation spectrum at scales larger than approximately 10\(h^{-1}\) than expected in the standard model (biased CDM plus \(n = 1\) initial spectrum of adiabatic perturbations). On the other hand, it is remarkable that the standard model is so close to these observational data: only a modest increase in amplitude of perturbations at large scales is required to fit them - no more than 2 - 3 times, and the latest results have the tendency to diminish this number (see e.g. Loveday et al. 1992 ). Finally, recent COBE measurements (Smoot et al. 1992) also imply the ratio of amplitude at scales \((10^3 - 10^4)h^{-1}\) Mpc to that at \((1 - 10)h^{-1}\) Mpc equal to \((1.1 \pm 0.2)b_g\) (1\(\sigma\) error bars). The earlier published positive RELICT - 1 result for \(\Delta T/T\) at large angles (Strukov et al. 1992a,b) is even larger but it was obtained using only one wavelength, so it may be at least partially non-primordial.

The spectrum of perturbations observed at the present time is a product of an initial (primordial) spectrum and some transfer function \(T(k)\). The latter results from a transition from the radiation-dominated era to the matter dominated one at redshifts \(z \sim 10^4\) and depends on the structure of dark matter. Thus, any deviation from the standard model may be explained either by changing the initial flat spectrum, or by having a different \(T(k)\) due to a more complicated matter content. The latter possibility arises, for example, in models with a mixture of hot and cold dark matter (Shafi & Stecker 1984, Holtzmann 1989, van Dalen & Schaefer 1992, and others) or with a cosmological constant and dark matter (Peebles 1984, Kofman & Starobinsky 1985, Efstathiou et al. 1990b, and Gorski et al. 1992). For a recent reanalysis of the former model see Pogosyan & Starobinsky 1993; the same for the latter model - see Bahcall et al. 1993.

Here, we consider CDM models with non-scale-free primordial perturbation spectra. The scale invariance of the perturbation spectrum can be broken by different physical mechanisms during an inflationary stage (see, e.g., Kofman et al. 1985, Salopek et al., 1989). Scale-free but not scale-invariant spectra with \(n < 1\) seem not to be able both to provide enough power at \(L \sim (25 - 50)h^{-1}\) Mpc and to fit the COBE data (Polarski & Starobinsky 1992, Liddle et al. 1992, Adams et al. 1993; see, however, Cen et al. 1992). A non-scale-free initial spectrum with an effective step (compared to the \(n = 1\) spectrum) somewhere
between $1h^{-1}$ Mpc and $10h^{-1}$ Mpc works much better.

To obtain such a spectrum, it is necessary to abandon at least one of the main assumptions of the simplest version of the inflationary scenario, i.e. to assume either that the slow-rolling condition is temporarily violated at some moment of time (and then a step of an universal form in the spectrum arises, see Starobinsky 1992), or that there is more than one effective scalar field driving inflation. The latter possibility leads to double inflation, i.e., to a cosmological model with two subsequent inflationary stages (Kofman et al. 1985, Silk et al. 1987). These inflationary stages may be driven by the $R^2$ term and a scalar field (Gottlöber et al. 1991), where $R$ is the Ricci scalar or by two noninteracting scalar fields (Polarski & Starobinsky 1992). In this case, a step in the spectrum arises as a result of a rapid decrease of the Hubble parameter $H = \dot{a}/a$ in a period between two inflationary stages with slowly varying $H$ and $\dot{H}$. Further, we consider the first of the above mentioned double inflationary models, results for the second one are expected to be similar.

2 Perturbation spectrum and normalization

The Lagrangian density of the gravitational field including the $R^2$ term and a massive scalar field reads

$$L = \frac{1}{16\pi G}(-R + \frac{R^2}{6M^2}) + \frac{1}{2}(\varphi,_{\mu}\varphi^{,\mu} - m^2 \varphi^2).$$

(1)

The $R^2$ term is coupled via $M^2 \ll G^{-1}$ to General Relativity, $\varphi$ is the scalar field and $m^2 \ll G^{-1}$ is its mass ($c = \hbar = 1$). The model contains two more parameters than the standard model. They define the location and the relative height of an effective step in the perturbation spectrum. The height of the step depends on the ratio $M/m$ and the energy density of the scalar field at the onset of the second inflation. On the other hand, this energy density is responsible for the length of the second inflationary stage, i.e., for the physical scale at which a break in the spectrum appears. With $\varphi = \varphi_0 \approx 3G^{-1/2}$ at the end of the first inflationary stage, the break occurs just at the right place, and the height of the step is $\Delta \approx M/6.5m$. Note that due to the exponential dependence of the break location on the energy density, its shifting from 10 Mpc to 100 Mpc does not practically influence the height of the step.

We consider the rms value of the Fourier transform of a gravitational perturbation $\Phi(k)$. Here the conventions $\Phi(\vec{k}) = (2\pi)^{-3/2} \int \Phi(\vec{r}) e^{-i\vec{k}\vec{r}} d^3r$, $\langle \Phi(\vec{k}) \rangle = 0$, $\langle \Phi(\vec{k}) \Phi^*(\vec{k}') \rangle = \Phi^2(k) \delta^3(\vec{k} - \vec{k}')$, $k = |\vec{k}|$ for initially Gaussian perturbations are adopted. Provided the limit $k \rightarrow 0$ of $\Phi(k)$ coincides with a flat spectrum $\Phi(k)$. Then we can construct the two quantities $\Phi^1 = \lim_{k \rightarrow k_{br} - 0} \Phi(k)$ and $\Phi^2 = \lim_{k \rightarrow k_{br} + 0} \Phi(k)$. The height of the step $\Delta$ is given by $\Delta = \Phi^1/\Phi^2$. In this paper we consider spectra with steps $\Delta \approx M/6.5m = 2, 3, 4, 5$. The
Figure 1: Density perturbation spectra $P_i$ ($i = 1, 2, 3, 4$) for $M = 13m, M = 20m, M = 26m, M = 32.3m$ respectively, normalized to $\sigma_T(10^8)$ by the use of the COBE data. The dashed line shows the standard CDM model with the flat initial spectrum normalized in the same way.

Step is located at $L \simeq 2\pi k_{br}^{-1}$, where $k_{br}^{-1}$ denotes the wave number where the perturbation spectrum reaches the lower plateau at large $k$, the values $k_{br}^{-1} = (1, 3, 7, 10, 20, 30)$ Mpc$^{-1}$ were considered. Here and further throughout the paper, unless a scaling by $h$ is given, we assume $h = 0.5$. The length of a transition period between flat parts of the spectra is of the order of one magnitude of wave numbers.

Results of numerical calculations of the power spectra of density perturbations at the present epoch

$$P(k) \equiv P(k, z = 0) = \left(\frac{\delta \rho}{\rho}\right)^2 = \frac{1}{36} (kR_H)^4 \Phi^2(k)T^2(k)$$

are plotted in Fig.1. Here $T(k)$ is the transfer function for the standard CDM model (a review of best fits to it see in Liddle & Lyth 1993), for equations for perturbations see Gottl"ober et al. 1991, the overall normalization is explained below. We adopt $\Omega_{tot} = 1$ in accordance with inflation, thus $a(t) \propto t^{2/3}$ now. $R_H = 2H_0^{-1} \approx 6000h^{-1}$ Mpc is the present day Hubble radius sometimes non-rigorously called the cosmological horizon. For $\Delta \leq 3$, there is practically no oscillations in the spectrum. Then a very good analytical approximation for it may be obtained by assuming that the slow-roll condition is valid for both inflaton fields $R$ and $\phi$ in the region of the break (see, e.g., Gottl"ober et al.
Let us present this solution in a different form. The background equations for the model (1) in the slow-roll approximation

\[ 3H \dot{\phi} + m^2 \phi = 0, \]
\[ \frac{6H^2 \dot{H}}{M^2} = \frac{4\pi G m^2 \phi^2}{3} - H^2 \]

have an integral

\[ s \equiv -\ln \frac{a}{a_f} = 2\pi G \phi^2 + \frac{3H^2}{M^2}, \]

where \( a_f \) is the value of the scale factor at the end of inflation (Kofman et al. 1985, Starobinsky 1985a). Then, introducing \( f = H^2/s \), we arrive to the equation:

\[ \frac{df}{d \ln s} = \frac{1}{f} (f - f_1)(f_2 - f), \]
\[ f_1 = \frac{2}{3} m^2, \quad f_2 = \frac{1}{3} M^2, \quad f(s) < f_2. \]

The scalar field follows from the relation:

\[ 2\pi G \phi^2 = s \left( 1 - \frac{f(s)}{f_2} \right). \]

Let, e.g., \( M > m \sqrt{2} \). Then the solution of Eq.(6) is

\[ \frac{(f_2 - f)^{f_2/f_1}}{|f - f_1|^{f_2/f_1}} = \frac{s_1}{s} (f_2 - f_1). \]

\( f \) may be both larger and smaller than \( f_1 \). \( f = \frac{f_1 + f_2}{2} \) for \( s = 2s_1 \). For values of the quantities \( M/m \) and \( \phi \) used in our calculations, \( f_1 \ll f_2 \) and \( s_1 \gg 1 \). Using the general expression for adiabatic perturbations generated during a slow-roll multiple inflation (Starobinsky 1985a) we get the answer:

\[ k^3 \Phi^2(k) = \frac{9}{100} \cdot \frac{16\pi G s^2 f(1 - f/2f_2)}, \]

where \( s \) and \( f(s) \) are taken at the moment of horizon crossing at the inflationary stage \( k = a(t)H(t) \); so \( s = s(k) \approx \ln \frac{k_f}{k} \) in this expression, \( k_f = a_f H_f \).

It is useful to note here that the background dynamics of an inflationary model with two massive scalar fields (see Polarsky & Starobinsky 1992) in the slow-rolling regime may be reduced to the same equation (6) with \( f = H^2/s \).
and \( f_{1,2} = \frac{2}{3} m_{1,2}^2 \) (\( m_1 < m_2 \)). Thus, the replacement rule is \( M^2 \to 2m_2^2 \).

The integral (7) is then changes to \( s = 2\pi G(\phi_1^2 + \phi_2^2) \). \( f_1 < f < f_2 \) in this case, so inflationary dynamics of the model with two massive scalar fields is isomorphous to only a finite part of possible inflationary regimes of the model (1). The expression for the power spectrum of adiabatic perturbations is slightly different from (9):

\[
k^3\Phi^2(k) = \frac{9}{100} \cdot 16\pi G s^2 f.
\]

Now let us turn to an overall normalization of the power spectra. It is chosen to fit the COBE data (Smoot et al. 1992) on large-angle \( \Delta T/T \) anisotropy \((2 \leq l < 30\), where \( l \) is the multipole number). If \( a_l \) is a rms multipole value summed over all \( m \) and averaged over the sky:

\[
a_l^2 = \frac{1}{4\pi} \sum_{m=-l}^{l} \left( \frac{\Delta T}{T} \right)_{lm}^2 = \frac{2l+1}{4\pi} \left( \frac{\Delta T}{T} \right)_{lm}^2,
\]

then the expected variance of the COBE data \( \sigma_{T}^2(\theta_{FWHM}) \) may be expressed as

\[
\sigma_{T}^2(\theta_{FWHM}) = \sum_{l \geq 2} a_l^2 \exp \left( -l(l+1)\theta_s^2 \right),
\]

where \( \theta_s = \theta_{FWHM}/2\sqrt{\ln 4} \) is the Gaussian angle characterizing smearing due to a finite antenna beam size, as well as due to an additional Gaussian smearing of raw data. We take \( \sigma_T(10^9) = (30 \pm 7.5)\mu K/2.735K \) (Smoot et al. 1992), so \( \theta_s = 4.25^\circ \approx 1/13.5 \).

We have numerically calculated the rms multipole values \( a_l \) for the perturbation spectra plotted in Fig. 2 (Gottlöber&Mücke 1993). In addition, it is possible to obtain an analytical expression for \( a_l \). Indeed, if \( f_1 < f < f_2 \), then Eq.(8) simplifies to \( f \approx f_2(s - s_1)/s_1 \) for \( 0 < s - s_1 < s_1 \). Thus, \( s_1 \approx \ln \frac{k_{br}}{k} \). Then it follows from Eq.(9) that

\[
k^3\Phi^2(k) \approx \frac{9}{100} \cdot 16\pi G f_2 s_1 (s - s_1) = \frac{9}{100} A^2 \ln \frac{k_{br}}{k} \quad (13)
\]

for \( k_{br} e^{-s_1} < k < k_{br} \) (here we introduce the quantity \( A \) used in Starobinsky 1983). Using the standard formula for the Sachs-Wolfe effect which is the only significant one for \( 2 \leq l < 30 \) and the case of adiabatic perturbations, it is straightforward to derive that

\[
a_l^2 = \frac{A^2(2l+1)}{400\pi} \int_0^\infty J_{l+1/2}(kR_H) \ln(k_{br}/k) k^2 R_H^2 dk = \frac{A^2}{400\pi^2} \frac{2l+1}{l(l+1)} \left( \ln(2l_0) - 1 - \Psi(l) - \frac{l + \frac{1}{2}}{l(l+1)} \right),
\]

(14)
where \( l_0 = k_{br} R_H \gg 1 \) is the multipole number corresponding to the step location (\( l \ll l_0 \)) and \( \Psi \) is the logarithmic derivative of the \( \Gamma \)-function, the difference between \( R_H \) and the radius of the last scattering surface may be neglected here. Substituting \( a_l^2 \) into Eq.(12) and choosing \( A \) to get the correct value for \( \sigma_T(10^\circ) \), we obtain properly normalized power spectra \( P(k) \) which are plotted in Fig.4.

It should be noted that, for the model considered, the contribution of primordial gravitational waves (GW) to large-angle \( \Delta T/T \) fluctuations is small as compared to the contribution from adiabatic perturbations (AP) and may be neglected. Really, using formulas derived in Starobinsky 1985a,b, one gets

\[
\frac{a_l(GW)}{a_l(AP)} \approx 2.5 \sqrt{\frac{3}{481 \ln \frac{k_{br}}{k_H}}} \approx 0.1
\]

for \( 1 \ll l < 30 \), the value of this ratio for \( l = 2 \) being \( \approx 6\% \) more, \( k_H = 2\pi R_H^{-1} \). Thus, the account of the GW contribution increases the total rms value of \( a_l(\text{tot}) = \sqrt{a_l^2(AP) + a_l^2(GW)} \) by less than 1%.

3 Comparison with observational data

3.1 Biasing parameter

By help of the density perturbation spectrum \( P(k) \) we are able to compute the variance of the mass fluctuation \( \sigma^2_M(R) \equiv \left( \frac{\delta M}{M} \right)^2 \) in a sphere of a given radius \( R \) (see e.g. Peebles 1980):

\[
\sigma^2_M(R) = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k) W(kR) dk,
\]

where the window function \( W(kR) \) is expressed as

\[
W(kR) = \frac{9}{(kR)^6} (\sin kR - kR \cos kR)^2
\]

The quantity \( \sigma^2_M \) gives the variance of total matter density perturbations. It becomes clear now that the distribution of galaxies is biased with respect to the distribution of total matter, so some biasing parameter \( b_g \) (assumed to be scale-independent for simplicity) has to be introduced:

\[
\xi_g(r) = b_g^2 \xi(r), \quad \sigma^2_{M_g}(R) = b_g^2 \sigma^2_M(R),
\]

where \( \xi(r) \) is the matter density correlation function. Usually it is adopted that \( \sigma^2_{M_g}(8h^{-1}\ \text{Mpc}) = 1 \), so \( b_g = \sigma_M^{-1}(8h^{-1}\ \text{Mpc}) \). Comparison of N-body simulations with the observed \( \xi_g(r) \) and with the mean pairwise velocity of galaxies at \( r = 1h^{-1}\ \text{Mpc} \) (Davies et al. 1985, for more recent calculations
TABLE 1: The bias factors and the velocity variances for the models. P1, P2, P3 and P4 denote models with $M = 13\,m; 20\,m; 26\,m; 32.2\,m$ corresponding to $\Delta = 2, 3, 4, 5$. The step location is given by $k_{br}^{-1}\,\text{Mpc}$.

| Spectrum $P_{\Delta}(k_{br}^{-1})$ | Bias factor $b_g$ | $\sigma_v(R)$ km/s | $\sigma_v(R)$ km/s |
|----------------------------------|------------------|-----------------|-----------------|
| depending on step location at $k_{br}^{-1}\,\text{Mpc}$ | $\Delta = 2, 3, 4, 5$ | $R = 40h^{-1}\,\text{Mpc}$ | $R = 60h^{-1}\,\text{Mpc}$ |
| P1(03) | 1.60 | 269 | 221 |
| P2(03) | 2.18 | 257 | 213 |
| P3(03) | 2.43 | 259 | 215 |
| P4(03) | 2.19 | 268 | 221 |
| P1(07) | 1.74 | 255 | 211 |
| P2(07) | 2.77 | 230 | 195 |
| P3(07) | 3.49 | 229 | 195 |
| P4(07) | 3.68 | 242 | 204 |
| P1(10) | 1.77 | 248 | 205 |
| P2(10) | 2.83 | 215 | 184 |
| P3(10) | 3.59 | 210 | 182 |
| P4(10) | 4.18 | 225 | 193 |
| P1(20) | 1.76 | 234 | 195 |
| P2(20) | 2.67 | 182 | 159 |
| P3(20) | 3.46 | 168 | 151 |
| P4(20) | 4.48 | 180 | 162 |

see e.g. Gelb 1993) shows that $b_g$ probably lies between 2 and 2.5 ($0.4 \leq \sigma_M(8h^{-1}) \leq 0.5$). We adopt rather conservative limits $1.5 < b_g < 3$ ($0.33 < \sigma_M(8h^{-1}) < 0.67$). Different bias factors obtained for the considered spectra are given in Table 1 (one should not forget about 25% error bars at $1.5\sigma$ level in all these quantities due to the error bars in $\sigma_T(10^\circ)$). This test excludes both the standard model and the $P(4)$ case (apart from the region $k_{br}^{-1} \approx 3\,\text{Mpc}$ for the latter). The $P(3)$ case is marginally admissible, and a broad range of $k_{br}$ remain allowed for the $P(1)$ and $P(2)$ cases.

3.2 Counts-in-cells

Further we compare the variance $\sigma^2(l)$ for the counts-in-cells analysis of large-scale clustering in the redshift survey of IRAS galaxies (Efstathiou et al. 1990) and in the Stromlo-APM survey (Loveday et al. 1992) with our predictions obtained for corresponding scales. The variance $\sigma^2(l)$ is related to the two-point galaxy correlation function according to
Figure 2: The variance $\sigma^2_c(l)$ plotted as a function of cell size $l$. Points obtained from the counts-in-cells analysis (Loveday et al. 1992) are plotted with $2\sigma$ error bars. Lines show predictions of $\sigma^2_c(l)$ computed from our spectra with the step location at $k^{-1}_{br} = 7$ Mpc. The solid lines correspond to spectra with different step height as indicated at the lines, the dashed line corresponds to the standard ("flat") CDM model. The IRAS data are plotted as quadratic symbols.

$$\sigma^2_c(l) = \frac{1}{V^2} \int \int_{V=l^3} \xi_g(r_{12})dV_1dV_2. \quad (19)$$

In terms of the density perturbation spectrum $P(k)$, it can be written as

$$\sigma^2_c(l) = \frac{1}{2\pi^2} \int_0^\infty k^2 P(k) W_1(kl) dk, \quad (20)$$

where

$$W_1(\rho) = 8 \int_0^1 dx \int_0^1 dy \int_0^1 dz (1-x)(1-y)(1-z) \frac{\sin(\rho \sqrt{x^2+y^2+z^2})}{\rho \sqrt{x^2+y^2+z^2}}. \quad (21)$$

A comparison between $\sigma^2_c(l)$ for galaxies from the Stromlo-APM survey and IRAS galaxies (see Fig. 2) shows that the biasing parameter for IRAS galaxies $b_{IRAS} < b_{APM}$ for $l = 10h^{-1}$ Mpc and $b_{IRAS} \approx b_{APM}$ for larger scales (apart from the point $l = 40h^{-1}$ Mpc where $b_{IRAS}^2 \approx 1.5b_{APM}^2$ that is, however, inside $2\sigma$ error bars). Predictions for $\sigma^2_c(l)$ for the considered spectra normalized by the condition $\sigma_{Mg}(8h^{-1} \text{ Mpc})=1$ are shown in Fig. 2. It is seen that another
normalization condition $\sigma_c(12.5h^{-1}\text{Mpc})=1$ proposed in Pogosyan & Starobinsky 1993 works as well. The Kaiser correction (Kaiser 1987) was not taken into account. In any case, it is clear that this correction should be rather small that presents one more argument for a sufficiently large value of $b_g$.

In order to determine values of the model parameters which give the best fit to these data, we formally apply the $\chi^2$ test considering the error bars given in Loveday et al. 1992 as $2\sigma$ ones (in the logarithmic scale of $\sigma_c(l)$). The results are plotted in Fig.3. Taking into account the possibility of a change in the overall normalization, the estimated number of degrees of freedom is $N=7$, so we consider fits with $\chi^2 < 7$ as good and fits with $\chi^2 > 20$ as unacceptable. The best fit $\chi^2 \approx 2.2$ occurs for $\Delta = 3$ and $k_{br}^{-1} = 3\text{ Mpc}$.

### 3.3 Angular correlation function

Let us consider the predictions of our model for the galaxy angular correlation function $w(\theta)$. To this aim we put the Fourier transform of the 3-D correlation function $\xi_g(r)$ into the Limber equation and find

$$w(\theta) = \frac{\int_{0}^{\infty} kP(k)dk \int_{0}^{\infty} y^4 \varphi^2(y)J_0(k\theta y)dy}{2\pi^2 \int_{0}^{\infty} y^2 \varphi(y)dy}.$$  

(22)

We insert our power spectra into Eq.(22) and scale the angular correlation function to the Lick depth ($y^* \approx 240h^{-1}\text{Mpc}$) with the selection function
TABLE 2: The Mach number test (notation as in Table 1)

φ = \frac{1}{2} \exp \left( \frac{1}{2} \left( \frac{y}{y^*} \right)^2 \right). Results of the calculations along with the recent observational data for \( w(\theta) \) for the APM survey (provided by S. Maddox, see also Maddox et al. 1990) are shown in Fig. 4. The use of linear approximation in the calculation of \( w(\theta) \) is justified for \( \theta > 0.5^\circ \) only. This is the reason of a discrepancy between our theoretical curves and the data at small angles that should not be taken into account. At larger angles up to \( \theta \sim 10^\circ \), the agreement with the \( \Delta = 3 \) and \( \Delta = 4 \) cases is very good (and errors are too large to make any definite conclusion beyond this angle).

3.4 Large-scale bulk flows

Next we investigate our spectra with respect to large-scale bulk flows (peculiar velocities) of matter. The rms peculiar velocity inside a sphere of a radius \( R \) is given by the expression:

\[
s^2_v(R) = \frac{2}{\pi^2 R_H^2} \int_0^\infty P(k)W(kR) \exp \left( -k^2 R_S^2 \right) dk,
\]

(23)

where \( W(kR) \) is the window function given in eq. (17) and the length scale \( R_S \) characterizes the Gaussian smoothing of raw observational data. In the Table 1, the corresponding predictions for our model are shown. They may be compared with recent data from the POTENT reconstruction of the 3-D peculiar velocity.
field: $\sigma_v(40h^{-1}\text{Mpc}) = (405 \pm 60)\text{ km/s}$ and $\sigma_v(60h^{-1}\text{Mpc}) = (340 \pm 50)\text{ km/s}$ with $R_S = 12h^{-1}\text{ Mpc}$, error bars are $1\sigma$ (Dekel 1992, see also Bertschinger et al. 1990).

This test presents the most serious problem for the model because expected values of $\sigma_v$ are lower than in models having exactly flat spectrum at large scales, e.g. in the CDM+HDM $n = 1$ model or in the CDM model with a step in the initial spectrum produced by one scalar field (Starobinsky 1992), due to the logarithmic increase of $P(k)$ at $k \ll k_{br}$, see Eq. (13). On the other hand, for $k_{br}^{-1} \leq 7\text{ Mpc}$, they are higher than in tilted models with $n < 0.75$ and without significant contribution of gravitational waves to large-angle $\Delta T/T$ fluctuations. It should be mentioned here that bulk flow velocities obtained from observations contain large uncertainties. Second, it is fairly possible that actual values of bulk flow velocities at our location in the Universe are larger than average ones ("cosmic variance"). So, taking the data at their lower $2\sigma$ limit (this is $\approx 1.4$ times less than the average values that is permitted by cosmic variance) and pushing the overall normalization of the model to the upper $1.5\sigma$ limit of the COBE data, we find that the allowed region is $k_{br}^{-1} \leq 7\text{ Mpc}$.

### 3.5 Mach number test

Now we consider the Mach number test for our model (Ostriker & Suto 1990). The expression for the Mach number in terms of the quantities introduced above is given as

$$M^2(R_L) = \frac{\int_0^\infty P(k) \exp(-k^2 R_S^2) \exp(-k^2 R_L^2) dk}{\int_0^\infty P(k) \exp(-k^2 R_S^2)(1 - (1 + k^2 R_L^2/9) \exp(-k^2 R_L^2)) dk},$$

(24)

where $R_S = 5h^{-1}\text{ Mpc}$ characterizes the Gaussian smoothing. Ostriker & Suto (1990) found the Mach numbers $4.2 \pm 1.0, 2.2 \pm 0.5$ and $1.3 \pm 0.4$ at the scales $R_L = 4h^{-1}\text{ Mpc}, 8h^{-1}\text{ Mpc}, 18h^{-1}\text{ Mpc}$, correspondingly. The Mach numbers predicted for our spectra are given in Table 2. Here, once more, the values of $M^2$ obtained from observations in our vicinity may be larger than average ones due to the numerator of Eq. (24), i.e., due to local values of large-scale bulk velocities being larger than average. This test clearly shows that $\Delta > 2$.

### 3.6 Quasar and galaxy formation

Finally, we consider the compatibility of our model with the existence of a sufficient number of large galaxies at the redshift $z = 1$ and host galaxies for quasars at $z = 4$. The standard model of quasar formation assumes that they arise as a result of formation of massive black holes ($M \approx 10^9M_\odot$) in nuclei of galaxies with total masses $(10^{11} - 10^{12})M_\odot$ (including the dark matter component). We estimate $f(\geq M)$ - the fraction of matter in gravitationally bound objects with
Spectrum $P_i[k_{br}^{-1}]$ depending on step location at $k_{br}^{-1}$ Mpc

| Step | $\chi^2$ |
|------|--------|
| P0   | 18.00  |
| P1(03)| 13.41  |
| P2(03)| 9.59   |
| P3(03)| 11.08  |
| P4(03)| 10.17  |
| P1(07)| 14.44  |
| P2(07)| 11.18  |
| P3(07)| 13.69  |
| P4(07)| 28.06  |
| P1(10)| 15.80  |
| P2(10)| 14.27  |
| P3(10)| 13.69  |
| P4(10)| 20.04  |
| P1(20)| 19.56  |
| P2(20)| 49.82  |
| P3(20)| 24.17  |
| P4(20)| 17.27  |

TABLE 3: $\chi^2$-values for the models

masses beginning from $M$ and higher - using a very simple though rather crude formula by Press & Schechter (1974):

$$ f(\geq M) = 1 - \text{erfc} \left( \frac{\delta_c}{\sqrt{2} \sigma_M(R, z)} \right), \quad (25) $$

where $M = \frac{4}{3} \pi R^3 \rho_0$, $\sigma_M(R, z) = \sigma_M(R)/(1 + z)$, $\sigma_M(R)$ is the rms value of the mass fluctuation in the linear approximation at the present moment, it is defined in Eqs. (16,17). The choice of the quantity $\delta_c$ that is equal to the value of $\frac{\Delta \rho}{\rho}$ in the linear approximation at the moment when the considered region collapses in the course of a fully non-linear evolution is critical for this approach. For the spherical model, $\delta_c = 3(12\pi)^{2/3}/20 \approx 1.69$, but attempts to fit results of N-body simulations to the Press-Schechter formula did not produce a unique answer. It seems that the best fit for $\delta_c$ lies between 1.33 and 2.

If we accept the most recent estimates of the mass fraction in hosts of quasars at $z = 4$ (see, e.g. Haehnelt 1993): $f(\geq 10^{11}M_{\odot}) \approx 10^{-4}$, when it follows from Eq. (25) that the corresponding rms linear mass fluctuation at the present moment

$$ \sigma_M(10^{11}M_{\odot}) \approx 1.3\delta_c \approx 2.2 \pm 0.5 \quad (26) $$

for $\delta_c$ lying in the range mentioned above. Clearly, this is only a lower limit on $\sigma_M$ because quasars may form not in all host galaxies and the presently observed...
Figure 4: Predicted angular correlation functions compared with the observationa l data from the APM survey (the data were kindly provided by S. Maddox). The solid lines correspond to a spectrum with $k_{br}^{-1} = 7$ Mpc, the dashed line shows the prediction for the standard CDM model quasar density at $z = 4$ may be less than the real one. However, due to extreme sensitivity of the expression (25) to a change in $\sigma_M$, the actual value of the latter cannot be significantly larger than that given in Eq. (26). A similar estimate was presented in recent paper by Blanchard et al. (1993) (it is 30% higher for their value $\alpha = 1.7$).

Another estimate may be obtained from the fact that large galaxies (or, at least, a significant part of them) seem to be already existent at $z = 1$. Then, assuming $f(\geq 10^{12} M_\odot) \geq 0.1$ at $z = 1$, we get

$$\sigma_M(10^{12} M_\odot) \geq 1.2\delta_c \approx 2.0 \pm 0.4.$$  \hspace{1cm} (27)

Note also that the estimate of Haehnelt (1993): $f(\geq 10^{12} M_\odot) \geq 10^{-5}$ at $z = 4$ leads to almost the same result: $\sigma_M(10^{12} M_\odot) \geq 1.1\delta_c$.

Now the same quantities may be calculated for our spectra. Analysis of the results shows that the predicted values of $\sigma_M(10^{11} M_\odot)$ and $\sigma_M(10^{12} M_\odot)$ for $\Delta = 3$ and $1$ Mpc $\leq k_{br}^{-1} \leq 20$ Mpc lie just inside the required ranges. For $\Delta = 2$ they are probably too high, for $\Delta = 4$ - too low (apart from the points $k_{br}^{-1} = 1$ Mpc and 20 Mpc).
4 Conclusions

We have compared the double inflationary model (1) characterized by two additional free parameters as compared to the standard model with a number of observational tests and have found that there is a reasonable agreement with all considered tests for the following region of the parameters: $1 \text{ Mpc} < k_{br}^{-1} < 10 \text{ Mpc}$ (that corresponds to real length scales $L_{br} \approx (6 - 60) \text{ Mpc}$) and $2 < \Delta < 4$. The best fit seems to be given by $k_{br}^{-1} = (3 - 7) \text{ Mpc}$, $\Delta \approx 3$. Generally, there is a correlation between a step size and a break location giving a good fit to the data, i.e., for a break at larger scales also a higher step is necessary. However, due to a complicated structure of the obtained spectra (e.g., oscillations throughout the range of the break for $\Delta > 2$) this connection is not so straightforward.

The most restricting for the model are large scale bulk flows. To obtain even a marginal agreement we have to assume that their actual rms values are at least $\approx 1.4$ times less than the average values of Dekel’s (1992) data (due to the cosmic variance) and that the rms value of large-angle CMB temperature fluctuations is not less than the average result of COBE. These assumptions are certainly crucial tests for the considered model.

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