Spectrum Leasing and Cooperative Resource Allocation in Cognitive OFDMA Networks

Meixia Tao and Yuan Liu

Abstract: This paper considers a cooperative orthogonal frequency division multiple access (OFDMA)-based cognitive radio network where the primary system leases some of its subchannels to the secondary system for a fraction of time in exchange for the secondary users (SUs) assisting the transmission of primary users (PUs) as relays. Our aim is to determine the cooperation strategies among the primary and secondary systems so as to maximize the sum-rate of SUs while maintaining quality-of-service (QoS) requirements of PUs. We formulate a joint optimization problem of PU transmission mode selection, SU (or relay) selection, subcarrier assignment, power control, and time allocation. By applying dual method, this mixed integer programming problem is decomposed into parallel per-subcarrier subproblems, with each determining the cooperation strategy between one PU and one SU. We show that, on each leased subcarrier, the optimal strategy is to let a SU exclusively act as a relay or transmit for itself. This result is fundamentally different from the conventional spectrum leasing in single-channel systems where a SU must transmit a fraction of time for itself if it helps the PU’s transmission. We then propose a subgradient-based algorithm to find the asymptotically optimal solution to the primal problem in polynomial time. Simulation results demonstrate that the proposed algorithm can significantly enhance the network performance.

Index Terms: Cognitive radio networks, cooperative communications, orthogonal frequency division multiple access (OFDMA), resource allocation, two-way relaying.

I. INTRODUCTION

Cognitive radio (CR), with its ability to sense unused frequency bands and adaptively adjust transmission parameters, has recently attracted considerable interest for solving the spectrum scarcity problem [1], [2]. A key concept in cognitive radio networks (CRNs) is opportunistic or dynamic spectrum access, which allows secondary users (SUs) to opportunistically access the bands licensed to primary users (PUs). Most of the works on dynamic spectrum access regard the secondary transmission as harmful interference and hence the SUs do not participate in the primary transmission. Recently, a new cooperation strategy between the primary system and the secondary system was proposed in [3] and further investigated in [4]. Therein, the PU link leases its channel to the SUs for a fraction of time to transmit secondary traffic in exchange for the SUs acting as relays to assist the transmission of primary traffic. The spectrum-leasing based cooperation can improve the performance of both the primary and secondary systems and result in a “win-win” situation.

The early works in [3] and [4] on spectrum leasing only investigated the time slot allocation in the single-PU, multi-SU, and single-channel scenario. More specifically, in [3], one PU targets at maximizing its rate while multiple SUs compete with each other to access the single channel. However, this scheme may result in an extreme case that the PU is aggressive and the SUs have no opportunity to access the channel. Recall that in CRNs, PUs are willing to share the spectrum resource with SUs if their quality-of-service (QoS) requirements are satisfied [1], [2]. In [4], the PU maximizes its utility in terms of rate and revenue while the SUs competitively make decisions based on their rates and payments. Nevertheless, the virtual payment and revenue may lead to another extreme case that the PU provides all of the transmission time to the SUs on the single channel, which is not practical in CRNs.

In this paper, we consider the general spectrum leasing and resource allocation problem in multi-channel multi-user CRN based on orthogonal frequency-division multiple-access (OFDMA). The motivation of using OFDMA is two-fold. First, OFDMA is not only adopted in many current and next generation wireless standards but also a strong candidate for CRNs [5]. The second is that OFDMA-based systems can flexibly incorporate dynamic resource allocations in CRNs (e.g., [6]–[8]). The primary system consists of multiple user pairs conducting bidirectional communication. The secondary system is a cellular network consisting of a base station (BS) and a set of SUs. The two systems operate in a cognitive and cooperative manner by allowing the SUs to occupy certain subcarriers given that the QoS of the PUs are satisfied with the assistance of SUs as cooperative relays.

As CRNs are typically hierarchical and heterogeneous, it is intuitive that if SUs can aggressively help PUs’ transmission, then less subcarriers will be needed by the PUs to satisfy their QoS requirements, and as a result the SUs can access more subcarriers for maximizing their own data rates. Meanwhile, as the communication in the primary system is bidirectional, the cooperation of SUs as relays can also bring network coding gain in the form of two-way relaying.1

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The authors are with the Department of Electronic Engineering at Shanghai Jiao Tong University, Shanghai, 200240, P. R. China, email: {mxtao, yuanliu}@sjtu.edu.cn.

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1 In two-way relay systems, a pair of nodes exchange information with the help of a relay node using physical layer network coding [9]–[11]. Two-way relaying can achieve much higher spectral efficiency than the traditional one-way relaying.
has to decide which cooperative transmission modes (one-way relaying and two-way relaying) to select and which set of SUs to choose, since it has higher priority in a CRN. Secondly, for the secondary system, it needs to schedule appropriate SUs to utilize the leased subcarriers for maximizing its total throughput. Moreover, for those SUs that not only be selected as relays but also be scheduled to transmit for themselves, the secondary system needs to balance their resource utilization. Thirdly, from the common perspective of the primary and secondary systems, it is crucial to determine which set of subcarriers to cooperate on together with how much power and time slots to transmit signals, in order to satisfy the QoS requirements of the primary system.

The main contributions of this paper are summarized as follows:

- We propose an optimization framework for joint bidirectional transmission mode selection, SU selection, subcarrier assignment, power control, and time slot allocation in the cooperative CRNs. The objective is to maximize the sum-rate of all SUs while satisfying the individual rate requirement for each of the PUs. There are three distinct features in our optimization framework. First, by subcarrier assignment and allocating time slot between PUs and SUs in cooperation sessions, multiuser diversity can be achieved in both frequency domain and time domain. Second, as the communication in the primary system is bidirectional, we can exploit network coding gain in the form of two-way relaying to improve spectral efficiency via the SUs’ assistance. Third, using the OFDMA-based relaying architecture, each PU pair can conduct the bidirectional communication by multiple transmission modes, namely direct transmission, one- and two-way relaying, each of them can take place on a different set of subcarriers.

- We show that in the multi-channel cooperative CRNs, the optimal strategy is to let a SU exclusively act as a relay for a PU or transmit data for itself on a cooperated channel. This result fundamentally differs from the conventional cooperation in the single-channel scenario where a SU must transmit a fraction of time for itself if it forwards the PU’s transmission on the channel.

- Using the Lagrange dual decomposition method, the joint optimization problem is decomposed into parallel subcarrier-based subproblems. An efficient algorithm is proposed to find the asymptotically optimal solution in polynomial time.

The remainder of this paper is organized as follows. Section II introduces the optimization framework, including system model and problem formulation. Section III presents the details of the Lagrange dual decomposition method for the joint resource-allocation problem. Section IV provides the simulation results. Finally, we conclude this paper in Section V.

II. OPTIMIZATION FRAMEWORK

We consider an OFDMA-based CRN where the primary system coexists with the secondary system as shown in Fig. 1. The primary system is an ad hoc network, consisting of multiple user pairs with each user pair conducting bidirectional communications. The secondary system of interest is the uplink of a single-cell network where a BS communicates a set of SUs. Note that the downlink can be analyzed in the same way. The proposed model can be justified in the IEEE 802.22 standard, where the CR systems are based on cellular basis. By taking advantage of the parallel OFDMA-based relaying architecture, each PU pair can conduct the bidirectional communication through three transmission modes, namely direct transmission, one- and two-way relaying, on different sets of subcarriers. As shown in Fig. 2, the PUs can transmit directly and the SUs can access the PUs’ residual subcarriers, or they transmit by a cooperation manner. On each cooperated subcarrier, a SU can assist a PU (or PU pair) using one- or two-way relaying. This setup can fully explore available diversities of the network, including user, channel, and transmission mode.

We model the wireless fading environment by large-scale path loss and shadowing, along with small-scale frequency-selective fading. The channels between different links experience independent fading and the network operates in slow fading environment, so that channel estimation is perfect. We assume that the two-hop transmission uses the same subcarrier for both links, i.e., the source→relay link and the relay→destination link. The time slot allocation between a PU and a SU on a cooperated subcarrier is illustrated in Fig. 2, in which we further assume that the two hops of the cooperative transmission use equal time slots. This is true for amplify-and-forward (AF) relaying strategy because AF needs equal time allocation, but more flexibility can be provided if the two hops pursue time adaptation for decode-and-forward (DF). Nevertheless, we still adopt equal time slot allocation between the two hops for simplicity.

Let $\mathcal{N} = \{1, 2, \cdots, N\}$ denote the set of subcarriers and
\( K = \{1, \cdots, k, \cdots, K\} \) denote the set of users, with the first \( K_P \) being the PU pairs and the remaining \( K_S = K - K_P \) being SUs. Here \( k \) represents PU pair index if \( 1 \leq k \leq K_P \) and represents SU index if \( K_P + 1 \leq k \leq K \). Denote \( k_1 \) and \( k_2 \) as the two users in the \( k \)th PU pair, \( 1 \leq k \leq K_P \). Denote \( P_n = [P_{n,k}, \cdots, P_{n,k_P}, \cdots, P_{n,K}]^T \) and \( R_n = [R_{n,k}, \cdots, R_{n,k_P}, \cdots, R_{n,K}]^T \) as the power and achievable rate vectors on subcarrier \( n \), respectively. When \( 1 \leq k \leq K_P \), \( P_{k,n} = [P_{k_1,n}, P_{k_2,n}]^T \) and \( R_{k,n} = [R_{k_1,n}, R_{k_2,n}]^T \). For interference avoidance, at most one PU (or PU pair) and one SU are active on each subcarrier. This exclusive subcarrier assignment and best relay selection can be implicitly involved in \( P_n \) and \( R_n \). Let \( P_{\text{max}} = [P_{k_1,\text{max}}, \cdots, P_{k_P,\text{max}}, \cdots, P_{K,\text{max}}]^T \) denote the peak power constraints vector. Again, if \( 1 \leq k \leq K_P \), \( P_{k,\text{max}} = [P_{k_1,\text{max}}, P_{k_2,\text{max}}]^T \). Let \( r = [r_1, \cdots, r_{k_1}, \cdots, r_{k_2}]^T \) (with \( r_k = [r_{k_1}, r_{k_2}]^T \)) be the rate requirements of the SUs. Denote \( R_{\text{max}} = [R_{k_1,\text{max}}, \cdots, R_{k_P,\text{max}}, \cdots, R_{K,\text{max}}]^T \), respectively. If \( 1 \leq k \leq K_P \), \( P_{n,k} \) and \( r_k \) are active on each subcarrier. This exclusive subcarrier assignment and best relay selection can be implicitly involved in \( P_n \) and \( R_n \).

Remark 1: In this paper, we assume that a central controller is available, so that the network channel state information and sensing results can be reliably gathered for centralized processing. Notice that the centralized CRNs are valid in IEEE 802.22 standard [12], where the cognitive systems operate on a cellular basis and the central controller can be embedded with a base station (BS). This assumption is also reasonable if a spectrum broker exists in CRNs for managing spectrum leasing and access [13], [14]. Such centralized approach is commonly used in a variety of CRNs (e.g., [6]–[8], [13]–[19]). Compared with distributed approaches (e.g., [3], [4], [20]), a CRN having a central manager that possesses detailed information about the wireless network enables highly efficient network configuration and better enforcement of a complex set of policies [13].

Remark 2: For CRNs, there is no single figure of QoS merit to measure the performance of the primary system. In this paper, we choose the rate requirement as the QoS metric. Other QoS metrics, like outage probability and signal-to-interference-plus-noise ratio (SINR), can be easily accommodated in the problem formulation. Moreover, the weighted sum-rate maximization for the SUs can be taken into account for the fairness issue, which does not affect the proposed algorithms in the sequel.

III. LAGRANGE DUAL DECOMPOSITION BASED OPTIMIZATION

The optimization problem in (1) is a mixed integer programming problem. In [21], the authors show that for the nonconvex resource optimization problems in OFDMA systems, the duality gap becomes zero under the time-sharing condition. It is also proved in [21] that the time-sharing condition is always satisfied as the number of OFDM subcarriers goes to infinity, regardless of the nonconvexity of the original problem. This means that solving the original problem and solving its dual problem are equivalent. Based on the result, the Lagrange dual decomposition method is recently applied to OFDMA-based cellular and cognitive radio networks in [19] and [22], respectively. In this section, we shall apply the result from [21] to solve our problem in (1). In particular, some valuable insights are obtained for multi-channel cooperative CRNs, which shows that the generalization from the single-channel case [3], [4] to the multi-channel case is nontrivial.

We first introduce two sets of dual variables, \( \lambda = [\lambda_1, \cdots, \lambda_K]^T \) and \( \beta = [\beta_1, \cdots, \beta_K]^T \) associated with constraints (1b) and (1c) respectively, where \( \lambda \geq 0 \) and \( \beta \geq 0 \). The Lagrange of the problem in (1) can be written as

\[
\begin{align*}
L(P, R, t, \lambda, \beta) &= \sum_{k=k_{P+1}}^{K} \sum_{n=1}^{N} R_{k,n} + \sum_{k=1}^{K_P} \left( \frac{P_{k,n}^2}{\sigma_{\text{BS}}^2} \right) \left( K_P + 1 \leq k \leq K \right) \\forall n \\forall t \\forall R_n \\forall H_n \\forall k \\forall \lambda_k \\forall \beta_k
\end{align*}
\]
Define $D$ as the set of all primary variables $\{P, R, t\}$ that satisfy constraints (1d)-(1f). The dual function is given by

$$g(\lambda, \beta) = \max_{\{P, R, t\} \in D} L(P, R, t, \lambda, \beta)$$

(3)

and the dual optimization problem can be expressed as

$$\min_{\lambda, \beta} g(\lambda, \beta)$$

s.t. \ $\lambda \geq 0, \beta \geq 0.$

(4a)

(4b)

The dual function (3) can be rewritten as

$$g(\lambda, \beta) = \sum_{n=1}^{N} g_n(\lambda, \beta) + \sum_{k=1}^{K_p} \lambda_k P_k^{\max} - \sum_{k=1}^{K_p} \beta_k r_k$$

(5)

where

$$g_n(\lambda, \beta) = \max_{\{P, R, t\} \in D} \left[ \sum_{k=K_p, n+1}^{K_p} R_{k, n} + \sum_{k=1}^{K_p} \beta_k R_{k, n} - \sum_{k=1}^{K_p} \lambda_k P_k \right]$$

(6)

are the $N$ independent per-subcarrier-based optimization sub-problems.

Since a dual function is always convex by definition [23], subgradient-based ellipsoid method [24] can be used to minimize $g(\lambda, \beta)$ by updating $\{\lambda, \beta\}$ simultaneously along with appropriate search directions, and it is guaranteed to converge to the optimal solution $\{\lambda^*, \beta^*\}$.

**Proposition 1:** For the dual problem defined in (4),

$$\Delta \lambda_k = P_k^{\max} - \sum_{n=1}^{N} P_{k, n}^*, 1 \leq k \leq K_p$$

(7)

and

$$\Delta \beta_k = \sum_{n=1}^{N} R_{k, n}^* - r_k, 1 \leq k \leq K_p$$

(8)

are subgradients of $g(\lambda, \beta)$, where $\{P_{k, n}^*, R_{k, n}^*\}$ are the optimal solutions of (6) for given $\{\lambda, \beta\}$.

**Proof:** Please see Appendix -A. \hfill \Box

As mentioned earlier, there are at most one PU (or PU pair), denoted as $P$, and one SU, denoted as $S$, active on a subcarrier. Here $P$ and $S$ also represent the best PU (or PU pair) and SU respectively, among all possible users, that maximize (6) for a given subcarrier $n$. This can be obtained by an exhaustive search. The complexity is detailed later. Specifically, it needs to first compute the optimal powers and rates for all users under all transmission modes, then let one PU and/or SU under one transmission mode that maximizes (6) active on each subcarrier. Therefore, the per-subcarrier problems in (6) can be alternatively expressed as

$$\max_{P_n, R_n, t_n} \quad \begin{cases} R_{S, n} + \beta P_{P, n} - \lambda P_{P, n} - \lambda_S P_{S, n} \\ R_{P, n} \in R(P_{P, n}, P_{S, n}, t_{S, n}, H_n) \end{cases}$$

s.t. \ $P_{P, n} \geq 0, P_{S, n} \geq 0, 0 \leq t_{S, n} \leq 1$ \quad (9a)

$$R_{P, n} \in R(P_{P, n}, P_{S, n}, t_{S, n}, H_n) \quad (9b)$$

$$R_{S, n} = t_{S, n} C \frac{P_{S, n} |h_{BS, S, n}^2|^2}{\sigma_{BS}^2}.$$ \quad (9c)

In what follows, for brevity of notation, the subscript $n$ in (9) is omitted due to all $N$ per-subcarrier-based subproblems having an identical structure. In addition, for direct transmission and one-way relaying, it is observed that the per-subcarrier optimization problem for the two links of a PU pair, i.e., $P_1 \rightarrow P_2$ and $P_2 \rightarrow P_1$ (with or without relaying), has the same structure and can be decoupled. Thus, for brevity, we only consider here the $P_1 \rightarrow P_2$ link as an example. In fact, the per-subcarrier optimization for direct transmission and one-way relaying needs to first compute the optimal values of the objective function in (9) for both links and then let one of them that has the maximum value active on the subcarrier. Moreover, we let $\gamma = |h_{P_1, P_2, n}^2|$, $\gamma_1 = |h_{P_1, S, n}^2|^2$, $\gamma_2 = |h_{S, P_2, n}^2|^2$, and $\gamma_n = |h_{S, BS, n}^2|^2$.

### A. Direct Transmission

In this transmission mode, either a PU or a SU occupies solely the given subcarrier. The per-subcarrier optimization problem in (9) can be expressed as

$$\max_{P_{P_1} \geq 0, P_{P_2} \geq 0} \quad \begin{cases} R_S + \beta P_{P_2} R_{P_2} - \lambda P_{P_1} R_{P_1} - \lambda_S P_S \\ R_S = C(P_S \gamma_2) \\ R_{P_2} = C(P_{P_2} \gamma). \end{cases}$$

(10a)

(10b)

(10c)

Since the problem in (10) is convex, by applying the Karush-Kuhn-Tucker (KKT) conditions [23], the optimal power allocations can be obtained as

$$P_{P_1}^* = \left( \frac{\beta_{P_2}}{a \lambda_{P_1}} - \frac{1}{\gamma} \right)^+$$

(11)

and

$$P_{P_2}^* = \left( \frac{1}{a \lambda_S} - \frac{1}{\gamma_n} \right)^+$$

(12)

where $a = \ln 2$ and $(x)^+ = \max(0, x)$. For a given subcarrier, the direct transmission further needs to compute the optimal values of the objective function in (10) over one of $P_{P_1}^*$ and $P_{P_2}^*$ with the other being zero, and then let one of them that has maximum value active. (11) and (12) show that the optimal power allocations are achieved by multi-level water-filling. In particular, the water level of each PU depends explicitly on its QoS requirement, and can differ from one another. On the other hand, the water levels of all SUs are the same.

### B. One-Way Relaying

If relaying is required on a given subcarrier, a fraction of $1 - t_S$ time is used by a PU to transmit the primary traffic with the help of a SU, while the rest $t_S$ time is assigned to the SU to transmit its own data. In this paper, we focus on DF only for
Similarly, if $P_{in}$ in (9) can be rewritten as
\begin{equation}
\max_{P_{1}, P_{2}, t_{S}} R_{S} + \beta P_{1} R_{P_{2}} - \lambda_{P_{1}} P_{1} - \lambda_{S} P_{S} \quad (13a)
\end{equation}
\begin{equation}
\text{s.t. } P_{1} \geq 0, P_{2} \geq 0, 0 \leq t_{S} \leq 1 \quad (13b)
\end{equation}
\begin{equation}
R_{S} = t_{S} C(P_{S} \gamma_{S}) \quad (13c)
\end{equation}
\begin{equation}
R_{P_{2}} = \frac{(1-t_{S})}{2} \min \{C(P_{P_{1}} \gamma_{1}), C(P_{P_{1}} \gamma + P_{S} \gamma_{2})\} \quad (13d)
\end{equation}

In (13d), the first term in the min-operation is the achievable rate of the $P_{1} \rightarrow S$ link, and the second term is the achievable rate by maximum ratio combining between the $S \rightarrow P_{1}$ link and $P_{1} \rightarrow P_{2}$ link. The following proposition is established for determining the optimal value of the time slot allocation variable $t_{S}$.

**Proposition 2:** For each cooperated subcarrier, a SU exclusively acts as a relay for cooperative transmission or transmits traffic for itself.

*Proof:* The proposition means that the time slot allocation variable $t_{S}$ is binary, i.e., $t_{S} \in \{0, 1\}$, which can be proved by contradiction.

Assume that the optimal solution of (13) is $(t_{S}^{*}, P_{1}^{*}, P_{S}^{*})$ with $0 < t_{S}^{*} < 1$. Next, we show that we can always find another better solution with $t_{S}$ being binary.

Let us rewrite the objective function (13a) as
\begin{equation}
f(t_{S}^{*}, P_{1}^{*}, P_{S}^{*}) = t_{S} \left[ C(P_{S}^{*} \gamma_{S}) \right. \\
\left. - \beta P_{1} \min \left\{ \frac{1}{2} C(P_{P_{1}} \gamma_{1}), \frac{1}{2} C(P_{P_{1}} \gamma + P_{S} \gamma_{2}) \right\} \right] \\
\left. + \beta P_{1} \min \left\{ \frac{1}{2} C(P_{P_{1}} \gamma_{1}), \frac{1}{2} C(P_{P_{1}} \gamma + P_{S} \gamma_{2}) \right\} \right] \\
- \lambda P_{1} P_{1}^{*} - \lambda_{S} P_{S}^{*}. \quad (14)
\end{equation}

If $C(P_{S}^{*} \gamma_{S}) < \beta P_{1} \min \left\{ \frac{1}{2} C(P_{P_{1}} \gamma_{1}), \frac{1}{2} C(P_{P_{1}} \gamma + P_{S} \gamma_{2}) \right\}$, we have
\begin{equation}
f(t_{S}^{*}, P_{1}^{*}, P_{S}^{*}) < \beta P_{1} \min \left\{ \frac{1}{2} C(P_{P_{1}} \gamma_{1}), \frac{1}{2} C(P_{P_{1}} \gamma + P_{S} \gamma_{2}) \right\} \\
- \lambda P_{1} P_{1}^{*} - \lambda_{S} P_{S}^{*} \\
= f(0, P_{1}^{*}, P_{S}^{*}). \quad (15)
\end{equation}

Similarly, if $C(P_{S}^{*} \gamma_{S}) \geq \frac{\beta P_{1}}{2} \min \{C(P_{P_{1}} \gamma_{1}), C(P_{P_{1}} \gamma + P_{S} \gamma_{2})\}$, we have
\begin{equation}
f(t_{S}^{*}, P_{1}^{*}, P_{S}^{*}) \leq C(P_{S}^{*} \gamma_{S}) - \lambda P_{1} P_{1}^{*} - \lambda_{S} P_{S}^{*} \\
< C(P_{S}^{*} \gamma_{S}) - \lambda_{S} P_{S}^{*} \\
= f(1, 0, P_{S}^{*}). \quad (16)
\end{equation}

These results contradict the assumption. This completes the proof. □

This proposition also holds for two-way relaying as discussed in the next subsection. The proof is similar and hence ignored.

Proposition 2 significantly simplifies the per-subcarrier optimization problem in (13) without loss of optimality by an exhaustive search over $t_{S}$. Specifically, we set $t_{S} = 0$ and $t_{S} = 1$ to compute the optimal values of $(13a)$, respectively, then follow the one that has the maximum value.

The intuition is that, on a cooperated subcarrier (see Fig. 2), if the subcarrier condition on the SU→BS link is good but on the cooperative link is poor, it is better that the PU leases the whole transmission time slot to the SU. Otherwise, the SU completely devotes itself as a relay to the PU. In other words, if a SU exclusively forwards a PU’s traffic on a subcarrier, the PU shall lease other subcarrier(s) to the SU as remuneration. This *channel-swap* based multi-channel cooperation fundamentally differs from the single-channel cooperation case [3], [4] where if a SU forwards a PU’s traffic, it must benefit from the PU on the channel. The spectral efficiency improvement brings more cooperation opportunities and leased subcarriers, and thus, the total throughput of the secondary system is increased. In the following we consider $t_{S} = 0$ and $t_{S} = 1$, respectively.

Case 1: $t_{S} = 0$. In this case, $S$ exclusively acts as a relay on a cooperated subcarrier. In DF one-way relaying, it is intuitive that $R_{P_{2}}$ is maximized when $C(P_{P_{1}} \gamma_{1}) = C(P_{P_{1}} \gamma + P_{S} \gamma_{2})$, which leads to
\begin{equation}
P_{S} = \gamma' P_{P_{1}} \quad (17)
\end{equation}
where $\gamma' = (\gamma_{1} - \gamma_{2}) / \gamma_{1}$. It is noted that DF occurs only if $\gamma_{1} > \gamma$. Substituting (17) to (13) and let $t_{S} = 0$, the problem can be rewritten as
\begin{equation}
\max_{P_{1} \geq 0} \beta P_{1} R_{P_{2}} - (\lambda_{P_{1}} + \lambda_{S} \gamma') P_{P_{1}} \quad (18a)
\end{equation}
\begin{equation}
\text{s.t. } R_{P_{2}} = \frac{1}{2} C(P_{P_{1}} \gamma_{1}). \quad (18b)
\end{equation}

The above is a convex problem. By applying the KKT conditions, the optimal power allocation is given by
\begin{equation}
P_{P_{1}}^{*} = \left[ \frac{\beta}{2(\lambda P_{1} + \lambda_{S} \gamma')} - \frac{1}{\gamma_{1}} \right]^{+}. \quad (19)
\end{equation}

$P_{S}^{*}$ can be obtained according to (17). The above optimal power allocation (19) shows that higher channel gain $\gamma_{1}$, meaning higher $\gamma'$, results in lower water level, which is the extra feature compared with the standard water-filling approach (e.g., (11) and (12) in direct transmission). One also observes that lower channel gain $\gamma_{2}$ leads to lower water level and vice versa.

Case 2: $t_{S} = 1$. In this case, $S$ uses a cooperated subcarrier solely for its own transmission. The optimal power allocation can be easily obtained and is the same as (12).

**C. Two-Way Relaying**

The two-way communication between $P_{1}$ and $P_{2}$ assisted by $S$ takes place in two phases. Specifically, in the first phase, also known as multiple-access (MAC) phase, $P_{1}$ and $P_{2}$ concurrently transmit signals to the assisting $S$. In the second phase, known as broadcast (BC) phase, $S$ broadcasts the processed signals to both $P_{1}$ and $P_{2}$. Different from direct transmission and one-way relaying, two-way relaying must occur in pair [25]–[30]. Thus
both the bidirectional links are taken into account together for two-way relaying. Here we only analyze the case of $t_S = 0$, and the case of $t_S = 1$ is omitted since the optimal power allocation is the same as (12) if $t_S = 1$.

The per-subcarrier problem in (9) can be expressed as (recall that $t_S = 0$)

$$\max_{P_{P} \geq 0, P_{S} \geq 0, R_P} \beta_{P_{1}} R_{P_{1}} + \beta_{P_{2}} R_{P_{2}} - \lambda_{P_{1}} P_{P_{1}} - \lambda_{P_{2}} P_{P_{2}} - \lambda_{S} P_{S}$$

subject to

$$R_{P} \in R(P_{P}, P_{S}, \gamma_{1}, \gamma_{2}) = C_{MAC} \cap C_{BC}$$

(20a)

and

$$C_{MAC} = \left\{ (R_{P_{1}}, R_{P_{2}}) \mid R_{P_{1}} \leq \frac{1}{2} C(P_{S} \gamma_{2}), R_{P_{2}} \leq \frac{1}{2} C(P_{P_{1}} \gamma_{1}), R_{P_{1}} + R_{P_{2}} \leq \frac{1}{2} C(P_{P_{2}} \gamma_{2} + P_{P_{1}} \gamma_{1}) \right\}$$

(20b)

and

$$C_{BC} = \left\{ (R_{P_{1}, R_{P_{2}}}) \mid R_{P_{1}} \leq \frac{1}{2} C(P_{S} \gamma_{1}), R_{P_{2}} \leq \frac{1}{2} C(P_{S} \gamma_{2}) \right\}$$

(21)

Note that the channel reciprocity is used in the BC phase, which is justified by the time-division duplex mode. Since both $C_{MAC}$ and $C_{BC}$ are convex sets, and so is their intersection, the problem in (20) is a convex problem and can be solved by convex techniques.

Let $\alpha_{1}$ and $\alpha_{2}$ be the two dual variables associated with the two rate constraints in (22). We first incorporate the two rate constraints in (22) into the objective function and rewrite the Lagrange dual problem of (20) as

$$\min_{\alpha_{1}, \alpha_{2}} \max_{(P_{P}, R_{P}) \in C_{DP}} \beta_{P_{1}} R_{P_{1}} + \beta_{P_{2}} R_{P_{2}} - \lambda_{P_{1}} P_{P_{1}} - \lambda_{P_{2}} P_{P_{2}} - \lambda_{S} P_{S}$$

$$- \alpha_{1} \left[ R_{P_{1}} - \frac{1}{2} C(P_{S} \gamma_{1}) \right]$$

$$- \alpha_{2} \left[ R_{P_{2}} - \frac{1}{2} C(P_{S} \gamma_{2}) \right]$$

subject to

$$\alpha_{1} \geq 0, \alpha_{2} \geq 0$$

(23a)

and

$$\max_{P_{P} \geq 0, P_{S} \geq 0} \frac{\alpha_{1}}{2} C(P_{S} \gamma_{1}) + \frac{\alpha_{2}}{2} C(P_{S} \gamma_{2}) - \lambda_{S} P_{S}$$

(25)

For brevity of notation, we let $\alpha_{1}^{*} = \beta_{P_{1}} - \alpha_{1}$ and $\alpha_{2}^{*} = \beta_{P_{2}} - \alpha_{2}$. It is noted that both $\alpha_{1}^{*}$ and $\alpha_{2}^{*}$ must be nonnegative, i.e., $\beta_{P_{1}} \geq \alpha_{1}$ and $\beta_{P_{2}} \geq \alpha_{2}$. In the following we present the solution to each subproblem.

The subproblem in (24) is a classic resource allocation problem in the Gaussian MAC [33], where the optimal power and rate allocations can be achieved by successive decoding. Specifically, users’ signals are decoded one by one in an increasing rate weight order [33]. Without loss of generality, we assume that $\alpha_{1}^{*} \geq \alpha_{2}^{*}$ (here $\alpha_{1}^{*}$ and $\alpha_{2}^{*}$ can be regarded as the rate weights for $P_{1}$ and $P_{2}$, respectively). Based on the polymatroid structure of the Gaussian MAC [33], we then incorporate the three rate constraints in (21) into the objective function of (24), the subproblem in (24) can be expressed as

$$\max_{P_{P} \geq 0} \frac{\alpha_{1}^{*}}{2} C(P_{P_{2}} \gamma_{2}) + \frac{\alpha_{2}^{*}}{2} \left[ C(P_{P_{1}} \gamma_{2} + P_{P_{1}} \gamma_{1}) - C(P_{P_{2}} \gamma_{2}) \right]$$

$$- \lambda_{P_{1}} P_{P_{1}} - \lambda_{P_{2}} P_{P_{2}}$$

(26)

It is easy to validate that the objective function of (26) is jointly concave in $P_{P_{1}}$ and $P_{P_{2}}$. By applying the KKT conditions, the optimal power allocations can be obtained as

$$P_{P_{2}}^{*} = \frac{\left( \frac{\alpha_{1}^{*} - \alpha_{2}^{*}}{2} \gamma_{1} \lambda_{P_{2}} - \gamma_{2} \lambda_{P_{1}} \right)}{1 - \frac{1}{\gamma_{2}}}$$

(27)

and

$$P_{P_{1}}^{*} = \frac{1}{2 a} \left[ \frac{\alpha_{2}^{*}}{\gamma_{1} \lambda_{P_{1}} - \gamma_{2} \lambda_{P_{1}}} - \frac{\left( \frac{\alpha_{1}^{*} - \alpha_{2}^{*}}{2} \gamma_{2} \right)}{1 - \frac{1}{\gamma_{2}}} \right]$$

(28)

It is observed that the optimal power allocations in the MAC phase have the form of water-filling. Moreover, the following proposition is provided according to the above closed-form power allocations.

**Proposition 3:** For $\alpha_{1}^{*} \geq \alpha_{2}^{*}$, a necessary condition for the occurrence of two-way relaying is $\gamma_{1} \lambda_{P_{2}} > \gamma_{2} \lambda_{P_{1}}$.

**Proof:** Please see Appendix B.

The subproblem in (25) is also convex since its objective function is concave in $P_{S}$. By applying the KKT conditions, the optimal power allocation is given by

$$P_{S}^{*} = \begin{cases} \theta_{1} + \sqrt{\theta_{1}^{2} + 4 \theta_{2} \theta_{3}} - 2 \theta_{2}, & \text{if } \lambda_{S} < \frac{\alpha_{1}^{*} \gamma_{1} + \alpha_{2}^{*} \gamma_{2}}{2 \theta_{2}} \\ 0, & \text{otherwise} \end{cases}$$

(29)

where $\theta_{1} = 2 a \lambda_{S} \gamma_{1} \gamma_{2}$, $\theta_{2} = 2 a \lambda_{S} (\gamma_{1} + \gamma_{2}) - \gamma_{1} \gamma_{2} (\alpha_{1} + \alpha_{2})$, and $\theta_{3} = 2 a \lambda_{S} - \alpha_{1} \gamma_{1} - \alpha_{2} \gamma_{2}$.

**Remark 3:** For both one- and two-way relaying, direct transmission mode is optimal if $P_{S}^{*} = 0$. In this case, the optimal power and rate allocations are the same as those obtained in the direct transmission mode in Section III-A. Moreover, it is noted that two-way relaying occurs if $P_{S}^{*}, P_{P_{1}}^{*}$, and $P_{P_{2}}^{*}$ are all positive. For two-way relaying, another interesting case is that $P_{S}^{*}$ is positive and one of $P_{P_{1}}^{*}$ and $P_{P_{2}}^{*}$ is equal to zero. In other words, one direction is inactive. In this case, one-way relaying must be optimal.

**Remark 4:** In our centralized framework, the cooperation is between the primary system and secondary system rather than
among individual users. Thus some SUs may not transmit their own traffic because they are the best option for primary traffic relaying. In this case, these SUs may be re-scheduled for transmission at the next transmission frame by a higher layer scheduler for long-term fairness. However, analysis on higher layer scheduling is beyond of the scope of this paper.

Remark 5: In this paper, we employ DF just for an illustration purpose. Other relaying strategies, like AF and compress-and-forward (CF), are generally applicable to our proposed framework and algorithms. However, the achievable rate expressions of AF and CF are not concave, for both one- and two-way relaying. To overcome the difficulty, some approximations can be adopted for AF and CF such that the achievable rate expressions are concave, and thus they can be solved using convex optimization techniques.

After obtaining the optimal solution in the dual domain, we now need to obtain the optimal solution to the original primal problem in (1). Due to the non-zero duality gap, the optimal solution obtained in the dual domain may not satisfy all the constraints in the original primal problem. To tackle this problem, we first obtain the optimal transmission mode selection and user-assignment for each subcarrier using the method in the dual domain, then the primal problem in (1) reduces to a pure power allocation problem and it is convex. By applying KKT conditions, the optimal power allocations follow the same expressions in the dual domain and the details are omitted here. This approach is asymptotically optimal due to the vanishing duality gap when the number of subcarriers is sufficiently large [21].

At the end of the section, we analyze the computational complexity of the proposed algorithm. The complexity of determining $P$ and $S$ on each subcarrier for direct transmission is $O(2K_P + K_S)$, and for one- and two-way relaying are $O(2K_P K_S)$ and $O(K_P K_S)$, respectively. Note that the complexity of the search over $t_S = 1$ is implicitly contained in the optimization of direct transmission mode. Therefore, the total complexity of solving all $N$ per-subcarrier problems is $O(N(2K_P + K_S + 3K_P K_S))$. Combining the complexity of the ellipsoid method, the total complexity of solving the dual problem is $O(N(2K_P + K_S + 3K_P K_S)(4K_P + K_S)^3)$, which is linear in $N$ and polynomial in $K_P$ and $K_S$.

IV. SIMULATION RESULTS

In this section, we evaluate the performance of the proposed cooperative scheme using simulation. The conventional scheme without cooperation is selected as a benchmark, which corresponds to the optimization of direct transmission in our proposed algorithm and its complexity is $O(N(2K_P + K_S)(4K_P + K_S)^3)$. As another benchmark, the performance of the fixed transmission mode (FTM) based allocation is also presented. In particular, the FTM scheme lets the transmission mode for each PU be pre-fixed according to nodes’ geographical information, and other optimizations are the same with the proposed optimal algorithm. This is attractive for practical systems where path loss dominates the performance of the network nodes. In specific, a PU is assigned to the direct transmission mode if the path loss (or distance) of the source-destination link is smaller than that of all source-relay links and the cooperative transmission modes are used otherwise. When it is assigned the cooperative transmission modes, two-way relaying is adopted if the path losses of the source-relay link and the relay-destination link is about the same, otherwise one-way relaying is used (in this case, $P_1 \rightarrow P_2$ direction is performed). Note that if a PU is assigned two-way relaying, the other PU in the same pair is also assigned two-way relaying. For those PUs who need SUs’ assistance, we assign the nearest SU to each PU, the search over the suitable SU for each PU is reduced to $O(1)$. Hence, the total complexity of this suboptimal algorithm is $O(N(5K_P + K_S)(4K_P + K_S)^2)$.

Fig. 3. Sum-rate of the secondary system versus transmit SNR per subcarrier, with $K_P = 2$ PU pairs, $K_S = 4$ SUs and rate requirement 5 bit/OFDM symbol for all PUs.
the different PU rate requirements in Fig. 5, where we fix transmit SNR 10 dB and $K_S = 4$ SUs. As expected, our proposed scheme outperforms the FTM and conventional schemes considerably over all ranges of PU rate requirements. This further demonstrates the effectiveness of the proposed scheme.

V. CONCLUSION

This paper studied the OFDMA-based bidirectional CRNs with cooperation between the primary and secondary systems, for supporting communication services with diverse QoS requirements. We proposed an optimization framework for joint optimization of bidirectional transmission mode selection, SU selection, subcarrier assignment, power control, and time slot allocation. We converted this mix integer programming problem with exponential complexity into a convex problem using the dual decomposition method and developed efficient algorithms with polynomial complexity.

A few important conclusions have been made throughout this paper. Firstly, the time slot allocation between a PU and a SU on a cooperated subcarrier is binary. Secondly, the proposed framework can greatly improve the total throughput of the secondary system by about 60%, compared with the non-cooperative scheme. Thirdly, choosing the appropriate transmission modes for the PUs is necessary. Last but not least, transmission mode adaptation and SU selection over different subcarriers can enhance the total throughput by about 20%.

The proposed algorithm can be used as the performance upper bound for suboptimal or distributed algorithms. In future work, it will be interesting to investigate incentive-based distributed schemes.

APPENDICES

A. Proof of Proposition 1

By definition of $g(\lambda, \beta)$ in (3), we have

$$g(\lambda', \beta') \geq \sum_{k=K_P+1}^{K} \sum_{n=1}^{N} R_{k,n} + \sum_{k=1}^{K} \lambda_k' \left( P_{k,n}^{\text{max}} - \sum_{n=1}^{N} P_{k,n}^* \right) + \sum_{k=1}^{K_P} \beta_k' \left( \sum_{n=1}^{N} R_{k,n} - r_k \right)$$

$$= g(\lambda, \beta) + \sum_{k=1}^{K} (\lambda_k' - \lambda_k) \left( P_{k,n}^{\text{max}} - \sum_{n=1}^{N} P_{k,n}^* \right) + \sum_{k=1}^{K_P} (\beta_k' - \beta_k) \left( \sum_{n=1}^{N} R_{k,n}^* - r_k \right).$$

Hence, Proposition 1 is proven by using the definition of subgradient.

B. Proof of Proposition 3

Both $P_{1}^n$ and $P_{3}^n$ must be positive if two-way relaying occurs, besides $P_{2}^n$ is positive. We first investigate $P_{3}^n$ in (27). It is easy to observe that $\gamma_1 \lambda_{1}^n$ must be greater than $\gamma_2 \lambda_{2}^n$, otherwise the first term in (27) is negative, and thus, $P_{3}^n = 0$. For
in (28), we let the first term is greater than that of the second term, i.e., \( \alpha_1^2 / \lambda > \left( \alpha_2^2 - \gamma \alpha \lambda P_2 - \gamma^2 \alpha P_1 \right) \). After some manipulations, we obtain \( \gamma \lambda P_2 / \gamma \alpha \lambda P_1 > \alpha_1^2 / \alpha_2^2 \). Combining the condition \( \alpha_1^2 / \alpha_2^2 \geq 1 \), we obtain \( \gamma \lambda P_2 > \gamma \alpha \lambda P_1 \). This completes the proof.

REFERENCES

[1] S. Haykin, “Cognitive radios: Brain-empowered wireless communica-
tions,” IEEE J. Sel. Areas Commun., vol. 23, no. 2, pp. 201–220, 2005.

[2] I. F. Akyildiz, W.-Y. Lee, M. C. Vuran, and S. Mohanty, “Next genera-
tion/dynamic spectrum access/cognitive radio wireless networks: A sur-
vey,” Comput. Netw., vol. 50, no. 13, pp. 2127–2159, Sep. 2006.

[3] S. Boyd and L. Vandenberghe, Convex Optimization. U.K.: Cam-
bridge Univ. Press, 2004.

[4] R. G. Bland, D. Goldfarb, and M. J. Todd, “The ellipsoid method: A sur-
ev,” Oper. Res., vol. 29, no. 6, pp. 1039–1091, Nov.-Dec., 1981.

[5] Y. Liu, M. Tao, B. Li, and H. Shen, “Optimization framework and
graph-based approach for relay-assisted bidirectional OFDMA cellular
networks,” IEEE Trans. Wireless Commun., vol. 9, no. 6, pp. 3490–3500,
Nov. 2010.

[6] K. Jivanichphaiya, R. Zhang, and Y. C. Liang, “Optimal resource alloca-
tion for two-way relay-assisted OFDMA,” IEEE Trans. Veh. Technol.,
vol. 58, no. 7, pp. 3311–3321, Sept. 2009.

[7] Y. Liu and M. Tao, “Optimal channel and relay assignment in OFDM-
based multi-relay multi-pair two-way communication networks,” IEEE
Trans. Commun., vol. 60, no. 2, pp. 317–321, Feb. 2012.

[8] W. Zhang, Y. Liu, and M. Tao, “Resource allocation with subcarrier pair-
ing in OFDMA two-way relay networks,” IEEE Wireless Commun. Lett.,
vol. 1, no. 2, pp. 61–64, Apr. 2012.

[9] Y. Liu, J. Mo, and M. Tao, “QoS-aware policies for OFDM bidirec-
tional transmission with decode-and-forward relaying,” in Proc. IEEE GLOBE-
COM, 2012.

[10] C. Lin, Y. Liu, and M. Tao, “Cross-layer optimization of two-way relaying
for statistical QoS guarantees,” IEEE J. Sel. Areas Commun., to appear.

[11] L.-L. Xie, “Network coding and random binning for multi-user channels,”
in Proc. IEEE CICIT, 2007.

[12] T. J. Oechtering, C. Schnurr, I. Bjelakovic, and H. Boche, “Broadcast ca-
pacity region of two-phase bidirectional relaying,” IEEE Trans. Inf. The-
ory, vol. 54, no. 1, pp. 454–458, Jan. 2008.

[13] D. N. C. Tse and S. V. Hanly, “Multiaccess fading channels-Part I: Poly-
matroid structure, optimal resource allocation and throughput capacities,”
IEEE Trans. Inf. Theory, vol. 44, no. 7, pp. 2796–2815, Nov. 1998.

[14] V. Erceg, L. Greenstein, S. Tjandra, S. Parkoff, A. Gupta, B. Kulic, A. Julius, and R. Jastrzab, “An empirically based path loss model for wire-
less channels in suburban environments,” IEEE J. Sel. Areas Commun.,
vol. 17, no. 7, pp. 1205–1211, July. 1999.

Meixia Tao received the B.S. degree in Electronic En-
gineering from Fudan University, Shanghai, China, in 1999, and the Ph.D. degree in Electrical and Elec-
tronic Engineering from Hong Kong University of Science and Technology in 2003. She is currently an
Associate Professor with the Department of Electronic Engineering, Shanghai Jiao Tong University, China.
From August 2003 to August 2004, she was a Member of Professional Staff at Hong Kong Applied Science
and Technology Research Institute Co. Ltd. From August 2004 to December 2007, she was with the De-
partment of Electrical and Computer Engineering, National University of Singa-
pore, as an Assistant Professor. Her current research interests include coopera-
tive transmission, physical layer network coding, resource allocation of OFDM
networks, and MIMO techniques. She is an Editor for the IEEE TRANSA-
CTIONS ON COMMUNICATIONS and the IEEE WIRELESS COMMUNICATIONS
 LETTERS. She was on the Editorial Board of the IEEE TRANSACTIONS ON
WIRELESS COMMUNICATIONS from 2007 to 2011 and the IEEE COMMUNI-
CATIONS LETTERS from 2009 to 2012. She also served as Guest Editor for the IEEE COMMUNICATIONS MAGAZINE with feature topic on LTE-Advanced
and 4G Wireless Communications in 2012, and Guest Editor for EURASIP JWCN with special issue on Physical Layer Network Coding for Wireless Co-
operative Networks in 2010. She was in the Technical Program Committee for various conferences, including IEEE INFOCOM, IEEE GLOBECOM, IEEE ICC, IEEE WCNC, and IEEE VTC. She is the recipient of the IEEE ComSoc
Asia-Pacific Outstanding Young Researcher Award in 2009.

Yuan Liu received the B.S. degree from Hunan Uni-
versity of Science and Technology, Xiangtan, China, in 2006, and the M.S. degree from Guangdong Univer-
sity of Technology, Guangzhou, China, in 2009, both in Communications Engineering and with the highest
honors. He is currently pursuing his Ph.D. degree at the Department of Electrical Engineering in Shang-
hai Jiao Tong University. His current research in-
terests include cooperative communications, network
coding, resource allocation, physical layer security,
MIMO and OFDM techniques. He is the recipient of
the Guangdong Province Excellent Master Theses Award in 2010. He has been honored as an Exemplary Reviewer of the IEEE COMMUNICATIONS LET-
TERS. He is also awarded the IEEE Student Travel Grant for IEEE ICC 2012. He is a student member of the IEEE.