Gravitational wave array - operation of the LIGO-Virgo network as a gravitational wave Hanbury Brown and Twiss interferometer

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Abstract. The current gravitational wave observatory network includes detectors at Hanford, Livingston, and Cascina. This network could be used as a gravitational wave Hanbury Brown and Twiss (HBT) interferometer. The gravitational wave HBT interferometer is an extension of conventional HBT intensity interferometers for Standard Model fields, since gravitational wave detectors measure amplitude instead of intensity. Glauber correlations in the GWA have a wide range of potential applications, such as determining the properties of a gravitational wave source. These correlations also have the potential of demonstrating and measuring non-classicality, i.e. the quantum nature, of the gravitational wave.

1. Introduction

Since the first detection of gravitational waves from the binary black hole inspiral of GW150914 \cite{1}, multiple detectors in the LIGO-Virgo network have been in operation during the detection process. Calculations of correlations in the detector outputs have been an essential element in the experimental methodology for identification of gravitational wave signals. The simultaneous operation of independent detectors might also be used as a gravitational wave array (GWA), as illustrated in Fig. 1, where the detectors of the network are used together as a gravitational wave Hanbury Brown and Twiss (HBT) interferometer. The detector correlations from the GWA would be the first and second order Glauber correlations \cite{2, 3}, which have proven broadly useful for classical and non-classical characterization of production and detection processes in astrophysics and particle physics \cite{4, 5}. One important distinction must be made between a gravitational wave HBT interferometer and all previous application of HBT interferometry. Gravitational wave detectors independently measure the amplitude of the gravitational wave and signal intensity must be calculated from these measurements. All previous detectors used as a HBT interferometer, have independently detected signal intensity.

Three recent papers \cite{6, 7, 8} have discussed the possible detection of squeezed states and non-classicality in the coherent primordial gravitational waves produced during inflation. The non-classicality would be demonstrated in the Glauber correlation functions, using gravitational wave detectors as a GWA \cite{2, 4, 9, 10, 11}. Squeezing in the production of gravitational waves might be seen in the second order correlation functions by showing that the correlation is consistent with a sub-Poissonian distribution. Detection of coherent gravitational waves from inflation will require the development of space based detectors. The transient gravitational waves detected by the LIGO-Virgo network to date are non-coherent. HBT interferometry might also be used to demonstrate non-classicality of non-coherent gravitational waves by operating the LIGO-Virgo network as a GWA. To understand the potential for determination of non-classicality with a GWA we will focus on the detection process as opposed to the production process considered in the previous work. One very important distinction should be made between the HBT effect associated with Standard Model fields (e.g.
Figure 1. Schematic of a gravitational wave array. LIGO and Virgo operations are associated with detectors at a, b, and c. Taking for example Hanford as detector a, Livingston detector b, and Virgo detector c.

photons, pions) and gravitational radiation. For Standard Model fields an independent detector measures intensity. In contrast for gravitational radiation, independent detectors measure gravitational wave amplitudes.

While the demonstration of non-classicality (quantum character) in the detection of gravitational waves is an intriguing possibility, there are other potentially important applications of the GWA. The baseline B in Fig. 1 is on the order of one wavelength, assuming a frequency of 100 Hz and baseline of 3,000 km, which should be well suited for the HBT effect [4]. However, improvement in the detector sensitivity or fortuitous future astrophysical events could make the GWA correlations useful in isolating the astrophysical event or even imaging of the gravitational wave source by reducing the ratio of the wavelength to the baseline. Correlated detectors and applications of the HBT effect have proven invaluable in astrophysics, quantum optics, and particle physics [4, 5]. The GWA should prove to be equally important for gravitational wave astronomy.

2. The Hanbury Brown and Twiss effect

The HBT effect is seen in correlations associated with independent detector responses. The only requirement for the effect is that the detectors are operated coincidently. To produce correlations in the detectors independently the correlation must be in observables of the signal. We will focus on the HBT effect associated with the correlation in detector responses due to the detection processes in physically distinct detectors. The observable used in existing studies of the Standard Model fields and the HBT effect is the signal intensity [4]. However, the HBT effect would also be present for amplitude measurements if the detectors can make independent measurements of the
Analysis of the HBT effect is based on normalized correlation functions. The first order normalized correlation function (first order Glauber correlation function or amplitude correlation) for general field amplitudes, $\psi$, is defined as $[2, 10, 11]$

$$g^{(1)}(\tau) = \frac{\langle \psi^* (t + \tau) \psi (t) \rangle}{\sqrt{\langle \psi^* (t + \tau) \psi (t + \tau) \rangle} \sqrt{\langle \psi^* (t) \psi (t) \rangle}}.$$  

(1)

For us $\psi$ will represent a gravitational wave amplitude. In an array the detectors are separated by a baseline $B$ as in Fig. 1, which is associated with a difference in the detector response time of $\tau$. If the amplitudes at the detectors are quantized the field amplitudes are promoted to operators, $\psi \rightarrow \hat{\psi}$ and $\psi^* \rightarrow \hat{\psi}^\dagger$. The classical second order normalized correlation function (second order Glauber correlation functions or intensity correlation) for field amplitudes, $\psi$, at the detectors is defined as,

$$g^{(2)}(\tau) = \frac{\langle \psi^* (t) \psi^* (t + \tau) \psi (t + \tau) \psi (t) \rangle}{\langle \psi^* (t) \psi (t) \rangle \langle \psi^* (t + \tau) \psi (t + \tau) \rangle}.$$  

(2)

The second order correlation can be expressed in terms of the first order correlation for a non-coherent source as $[11]$

$$g^{(2)}(0) = 1 + |g^{(1)}(\tau)|^2.$$  

(3)

Classical radiation will have values of $g^{(2)}(0) \geq 1$, which are Poissonian correlations (for the equal sign) or super-Poissonian correlations (for the greater sign). Squeezed states and anti-bunching $[11, 12]$ are associated with second order correlations, $g^{(2)}(0) < 1$, and are completely non-classical. A specific example of a non-coherent source with a given Gaussian distribution yields a first order normalized correlation of $[10, 11]$

$$|g^{(1)}(\tau)|^2 = e^{-\frac{\pi \tau^2}{\tau_c^2}},$$  

(4)

where $\tau_c$ is the coherence time. A coherent source has a single frequency $[10]$

$$\psi(z, t) = \psi_0 e^{ikz} e^{i\omega_0 t} e^{i\phi}$$

and $\psi^* (t + \tau) \psi (t + \tau) = \psi^* (t) \psi (t)$, which produces a constant second order correlation $[2]$

$$g^{(2)}(\tau) = 1.$$  

In general a coherent source produces no variation in the correlation of an HBT intensity interferometer. However, if you have squeezed coherent sources it is possible to have, $g^{(2)}(\tau) < 1$, which is completely non-classical $[11]$.

Existing applications of detector arrays make independent measurements of the field intensity and the HBT effect is calculated using the second order correlation $[2]$. However, gravitational wave detectors measure the amplitude directly and independently and the HBT effect in a GWA should be demonstrated in the first order correlation functions $[1]$. Unlike measurements of the Standard Model fields, independent intensity measurements are not practically possible for gravitational waves $[13]$. While it is not well established it might be possible to make direct measurements of field amplitudes of Standard Model fields comparable to gravitational wave strain amplitude measurements $[14]$ and demonstrate the HBT effect at first order.
3. The gravitational wave array and the HBT effect

The plane wave expansion in the transverse traceless gauge is \[15, 16, 17\],

\[
    h_{\alpha\beta}(u) = \sum_A \int_{-\infty}^{\infty} dk \int_{S^2} d\hat{\Omega} \ h_A(k, \hat{\Omega}) \ e^{iku} e^A_{\alpha\beta}(\hat{\Omega}),
\]

where \( \hat{\Omega} \) is the direction normal to \( S^2 \), \( u = t - z \) is the light cone coordinate, and \( z = \vec{x} \cdot \hat{\Omega}. \) The polarization vectors, \( e^A_{\alpha\beta}(\hat{\Omega}) \), are the plus and cross polarization states, \( A = +, \times \). Assuming a single polarization \( e^A_{\alpha\beta}(\hat{\Omega}) \to e_{\alpha\beta}(\hat{\Omega}) \),

\[
    h_{\alpha\beta}(u) = \int_{-\infty}^{\infty} dk \int_{S^2} d\hat{\Omega} \ h(k, \hat{\Omega}) \ e^{iku} e_{\alpha\beta}(\hat{\Omega}).
\]

Far from the source the propagation direction is \( \hat{z}, \ h(k, \hat{\Omega}) \to h(k, \hat{z}), \) and the nonzero tensor components in the plane wave expansion (6) are the same, \( h_{\alpha\beta}(u) \to h(u), \)

\[
    h(u) = \int_{-\infty}^{\infty} dk h(k) \ e^{iku},
\]

where \( h(k) \) is the complex-valued Fourier amplitude of wave number \( k \) in the plane wave expansion \[17\]. The first order correlation can now be written in terms of (1) and (7) as

\[
    g^{(1)}(\tau) = \frac{\langle h^*(t + \tau) h(t) \rangle}{\sqrt{\langle h^*(t + \tau) h(t + \tau) \rangle} \sqrt{\langle h^*(t) h(t) \rangle}},
\]

where \( \tau \) is the delay time between detectors. The second order correlation function is

\[
    g^{(2)}(\tau) = \frac{\langle h^*(t) h^*(t + \tau) h(t + \tau) h(t) \rangle}{\langle h^*(t) h(t) \rangle \langle h^*(t + \tau) h(t + \tau) \rangle}. \tag{9}
\]

The correlation functions can also be developed non-classically by promoting the strain amplitude, \( h(t) \), to a quantum operator, \( \hat{h}(t) \). The non-classical first order correlation function for gravitons propagating in flat space-time is

\[
    g^{(1)}(\tau) = \frac{\langle \hat{h}^-(t + \tau) \hat{h}^+(t) \rangle}{\sqrt{\langle \hat{h}^-(t) \hat{h}^+(t) \rangle} \sqrt{\langle \hat{h}^-(t + \tau) \hat{h}^+(t + \tau) \rangle}}. \tag{10}
\]

The commutation properties of the amplitudes can be studied based on the assumption that the gravitational wave consists of gravitons. Following Giovannini \[7\] we will take the graviton quantum field operators to be the annihilation operator \( \hat{h}^+(x) \) and the creation operator \( h^-(x) \), assuming the scalar approximation for a single polarization. The second order correlation function can be written as,

\[
    g^{(2)}(\tau) = \frac{\langle \hat{h}^-(t) \hat{h}^-(t + \tau) \hat{h}^+(t + \tau) \hat{h}^+(t) \rangle}{\langle \hat{h}^-(t) \hat{h}^+(t) \rangle \langle \hat{h}^-(t + \tau) \hat{h}^+(t + \tau) \rangle}. \tag{11}
\]
The second order correlation (11) has been studied extensively for gravitational waves produced by cosmic inflation [6, 7, 8, 18]. However, these gravitational waves are not detectable with existing detectors. In this paper we consider astrophysical sources that are detectable by the existing LIGO-Virgo GWA.

The classical versus non-classical character of electromagnetic radiation is distinguishable through the second order correlation [11]. Non-classical radiation is characterized by anti-bunching, which gives $g^{(2)}(0) < g^{(2)}(\tau)$ and $g^{(2)}(0) < 1$. Classical radiation can exhibit bunching which gives $g^{(2)}(0) > 1$. This can be difficult to distinguish from classical noise. For non-coherent radiation other criteria would be required.

4. Correlations in coherent and non-coherent gravitational radiation

Assuming that the detector response at any given time is monochromatic, (7) can be expressed as

$$h(t - z_a) = h_a(t) e^{i\omega(t)t} e^{-i\phi(t)}, \quad (12)$$

where $h_a^2 = h_a^* h_a$ is the squared strain amplitude at detector $a$ at position $z_a$ and time $t$. Substituting (12) in (8) the first order correlation is

$$g^{(1)}(\tau) = \frac{\langle h_a(t + \tau) h_b(t) e^{-2i\omega(t)t} e^{i\Delta\phi(t)} e^{-i\omega(t+\tau)\tau} \rangle}{\sqrt{\langle h_a(t + \tau) h_a(t + \tau) \rangle} \sqrt{\langle h_b(t) h_b(t) \rangle}}. \quad (13)$$

The first order correlation greatly simplifies for constant amplitude and frequency [11],

$$g^{(1)}(\tau) = e^{-i\omega_0 \tau} \langle e^{i\Delta\phi(t)} \rangle, \quad (14)$$

and $|g^{(1)}(\tau)| = 1$ which is consistent with coherent radiation.

Returning to a time varying amplitude and substituting (12) into (9) the second order correlation is

$$g^{(2)}(\tau) = \frac{\langle h_a^2(t) h_b^2(t + \tau) \rangle}{\langle h_a^2(t) \rangle \langle h_b^2(t + \tau) \rangle}, \quad (15)$$

which is in general not equal to one and demonstrates that the gravitational radiation is non-coherent. Because of the substantive difference in optical intensity interferometry and gravitational wave amplitude correlations we examine the $\tau = 0$ correlation and coherence conditions. The square amplitude can be expanded [11] at $\tau = 0$ as,

$$h_a^2(t) = h_b^2(t) = \langle h^2 \rangle + \Delta h^2(t),$$

where $\langle \Delta h^2(t) \rangle = 0$, and (15) simplifies to

$$g^{(2)}(0) = 1 + \frac{\langle [\Delta h^2(t)]^2 \rangle}{\langle h^2 \rangle^2}, \quad (16)$$

which is consistent with non-coherence. If the amplitude were constant, $[\Delta h^2(t)]^2 = 0$ then $g^{(2)}(0) = 1$, which is again consistent with coherent radiation.
5. Non-classicality in gravitational waves

One concrete signature of non-classicality (i.e. quantum character) in radiation is $g^2(0) < 1$, which is associated with anti-bunching. This criteria cannot be applied to gravitational waves detected by the LIGO-Virgo GWA, due to the non-coherence of the detected signal and the absence of anti-bunching. Kanno and Soda [6] discuss several potential alternative criteria for detecting non-classicality in gravitational radiation, such as: entanglement entropy, entanglement negativity, quantum discord, and the Bell inequality. Since for Standard Model fields, independent detectors measure intensity, these criteria have only been studied (effectively) for second order correlations. In contrast the LIGO-Virgo detectors directly measure amplitude, thus opening up the possibility for criteria of non-classicality in first order Glauber correlation functions, $g^1(\tau)$, in [13].

The LIGO-Virgo network does use correlation functions to help distinguish signal from noise in the detection and for matched filtering for characterization of the gravitational wave source [19, 20]. However, these are not detector correlations in the sense used in this paper. Determination of any HBT effect will require using templates similar to matched filtering in calculating detector correlation functions. While criteria for second order correlations from gravitational waves produced by inflation have been proposed [6] there does not appear to be any previous theoretical work on non-classicality for binary inspirals, and such criteria needs to be developed.

Compared to the distinct criteria for non-classicality of Standard Model fields associated with anti-bunching, further theoretical work is required to develop a comparable criteria for first order correlations. Among the more promising possibilities for development of a concrete criteria for non-classicality is something like “quantum discord” [21, 22]. Applying this criteria to matched filtering in gravitational wave detections might be possible through calculations of quantum discord for amplitude measurements. Quantum discord has been applied to HBT intensity interferometry for light [23]. The theory of quantum discord would need to be expanded beyond the Standard Model fields to include the possibility of gravitons. While we are not aware of any previous research on quantum discord and curved space time there has been research on quantum discord in the Unruh effect [24].

6. Potential applications of the gravitational wave array

HBT interferometers have proven to be invaluable for characterization of production and detection processes in the study of a wide range of Standard Model fields [4]. Correlated gravitational wave detectors operating as a gravitational wave HBT interferometer may prove equally important in the study of gravitational wave production and detection. In addition to the possible demonstration of non-classicality in gravitational radiation, as discussed above, here we outline a few additional interesting possibilities for future GWAs.
6.1. Effective baseline variation

The rotation of the Earth will change the relative orientation of the detectors and the signal, which changes the effective baseline or time separation between detectors [15]. It should be possible to use variations in the Glauber correlations due to changes in the effective baseline to help isolate the source direction, Fig. 1. Variations in the detector correlations could determine the least path difference in the detectors and help to isolate the direction of the source signal. The effectiveness of this application would depend on the time resolution of the detections and in particular on the coherence time, “the time duration over which the phase of the wave train remains stable” [11]. This variation might be useful with future detectors where the observed signal is more persistent than current detections.

6.2. Very long baseline interferometer

A future GWA could also be operated as a very long baseline interferometer [25]. An image of the source is recovered from an interferometer by taking the inverse Fourier transform of the magnitude of the first order correlation function, the “visibility” [25, 26]. The resolution of a signal source is proportional to the wavelength, \( \theta \approx \frac{\lambda}{B} \), where the baseline B is the distance between detectors a and b in Fig. 1. The LIGO detections to date would have a resolution on the order of one. This value for the resolution is too large for effective imaging of gravitational wave sources. Gravitational wave imaging using a GWA might require a space based or Moon based detector c to provide a baseline B sufficient to image a gravitational wave source.

6.3. Future detector networks

The baseline B in Fig. 1 for a Moon-Earth based GWA would be on the order of 400,000 km. If we assume a signal frequency of 1 kHz the resolution of a gravitational wave source would be on the order of \( 10^{-3} \) rad. This resolution would be useful in determination of the source direction but not sufficient for source imaging.

Using a GWA to image gravitational wave sources will require operating one of the detectors of the array in space. For a single space based detector the frequency range must be within the range of the Earth based detector and sufficiently high to offer useful angular resolution. One of the many planned future gravitational wave detectors that could be operated in a GWA is the Earth based underground Einstein Telescope (ET) [27]. The improved measurement sensitivity for strain amplitude, at frequencies on the order of 1 kHz, would make the ET a promising candidate in a future GWA. Using the ET as part of a GWA for imaging of gravitational wave sources would require pairing with a space based detector, operating in the same frequency range, to permit a sufficiently long baseline, Fig. 1.

The ET and space based detector GWA would offer a unique method of imaging astrophysical events. Detection of gravitational waves produced during core-collapse
supernova, with a GWA of sufficient baseline, could provide a method of imaging the star core. This would only be possible with detector frequencies in the range of the ET, to permit reasonable resolution of the source. The GWA could also be used to image neutron stars, by detection of gravitational waves from star-quakes or from spinning non-spherical stars. This method of imaging would be especially important for the 100 millions of galactic neutron stars that are not currently detectable.

One of the considerations in the planning and design of any future gravitational wave detectors should be the potential use of the detector in a GWA. This makes it particularly important to explore the operation of the LIGO-Virgo network as a GWA. It also adds weight to the need for the development of a complete theory of gravitational wave HBT interferometry via second and first order correlations.

7. Conclusions and discussion

The LIGO-Virgo network of gravitational wave observatories has been operating as a GWA since the first observed binary inspiral event. It is possible that normalized correlation function calculations can demonstrate non-classicality of the gravitational wave in both past and future detection processes. This will require better understanding of the first and second order correlation functions for a GWA. There is also the possibility of using the GWA for better characterization of gravitational wave sources. Matched filtering will be needed for either classical or non-classical applications of the GWA and theoretical models of quantum gravity and graviton production from binary inspirals is required to make this possible.

One of the most promising methods for determination of non-classicality in gravitational wave detection is the development of a theory of quantum discord for amplitude measurements and gravitons. This approach is particularly promising based on applications to light were the non-classical correlations are most pronounced with very weak signals [28, 29]. There has been speculation of a first order correlation between the Livingston and Hanford gravitational wave detections that might not be accounted for in matched filtering [30].

While our discussion of classical and non-classical applications of a GWA have focused on the present LIGO-Virgo network, future detector networks could also operate as a GWA. Improved detector sensitivities for both strain amplitude and frequency would make it possible to detect coherent signals as well as improving detector response for non-coherent processes. Determination of non-classicality should be much easier with coherent processes that include anti-bunching [6, 7, 8].

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