Improved predictive current control of NPC multilevel inverters

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Abstract: The three-level neutral-point-clamped (NPC) inverter, which is a widely used topology of grid-connected multilevel inverters, suffers the drawback of the NP voltage drift. This paper introduced an improved predictive direct current control strategy to improve grid-connected current quality of the system. The proposed strategy uses the nearest-three-virtual-vector (NTV2) modulation strategy which can get rid of the NP voltage balance problem for any range of inverter output voltage. Since the disadvantages of a lot of calculation for NTV2 algorithm in real-time control, the vectors in other sextants are mapped into the first sextant by a simple coordinate transformation. The time derivatives of currents for each selected vectors are used to get the application times that minimize objective function. In addition, comparing with SVPWM approaches the proposed strategy have a constant switching frequency and better line current total harmonic distortion (THD), its validity is verified by the simulation and experiment.

Keywords: three-level inverter, neutral-point voltage balance, predictive current control, nearest-three-virtual-vector (NTV2)

Classification: Power devices and circuits

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1 Introduction

In recent years, multilevel inverters were extensively applied in grid connected dc-ac applications, among which neutral-point (NP)–clamped inverter is character by low switching frequency, low voltage du/dt rate and high output voltage quality compared to 2-level inverter [1, 2]. However, a major challenge for the NPC inverter is the deviation of its neutral point voltage away from its ideal value, a true balanced 3-level inverter should provide a balanced voltage across the dc-link capacitors, the neutral point unbalance will generate order harmonic in output [3], due to the capacitors charges while the other discharges, so a superior controller for inverters are needed, to reach the long-term stability of the neutral point voltage.

Many control strategies have been proposed, among them predictive controller have been studied extensively. The most popular are Space Vector Pulse Width Modulation (SVPWM). The three-level inverter SVPWM algorithm is derived from traditional algorithm of two level space vector control. In ref. [4], SVPWM strategies have been proposed as a NP controller toggles between redundant
switched vector states based on the NP error, by implementing a maximal positive or negative zero sequence signal injection. While this approach does not fully eliminate neutral point oscillation and fails at low power factors.

As an alternative to classic field-oriented vector control scheme, direct power control (DPC) strategy, was developed for the control of three-phase grid-connected dc-ac converters [5, 6, 7, 8]. The main disadvantage of DPC scheme is the varying switching frequency with different active/reactive power combinations, which generates an undesired broadband harmonic spectrum range and makes it pretty hard to design a line filter at converter’s ac side. Predictive direct power control (P-DPC) is a new approach that direct power control is mixed with predictive selection of a voltage-vectors’ sequences. The optimal duty cycles are calculated in order to minimize instantaneous active and reactive power errors so that high dynamics and a constant switching frequency are obtained. The aim of this paper is to propose an improved predictive DPC strategy for grid-connected dc-ac converters by reducing power oscillations and steady-state errors, and achieving stable neutral point voltage. Based on the idea in [9], this paper combines a nearest-three-virtual-vector (NTV^2) method in order to control the NP voltage balance over the full range of inverter output current [10], by synthesizing the new vectors by small vectors and middle vectors, which can produce zero average NP current.

This paper presents predictive current control which uses NTV^2 sequence in order to control NP voltage balance for grid connected applications. In section II, an accurate mathematical model is founded. Then this paper deals with the predictive current control of ac current and dc capacitor voltage regulation in NPC inverter by choosing the NTV^2 sequence. In section IV, the simulation and experiment are presented.

## 2 Modeling and control strategy

### 2.1 The operation of improved inverter

Fig. 1 shows a simplified circuit diagram of the grid-connected three-level NPC inverter [11]. It is composed of twelve IGBTs switches and six clamping diodes. On the DC-link side of inverter, the DC-link capacitor is split into two sources to provide a neutral-point “O”.

![NPC three-level inverter circuit prototype](image)
There are three kinds of switching states defined as follows in each phase:

- when \( S_x = 2 \), then \( V_x = \frac{1}{2} V_{dc} \);
- when \( S_x = 1 \), then \( V_x = 0 \);
- when \( S_x = 0 \), then \( V_x = -\frac{1}{2} V_{dc} \) \hspace{1cm} (1)

In the expression, \( x \) represents A-phase or B-phase or C-phase, the output voltage of each bridge arm is described as:

\[
U_{AN} = \left[ S_a - \frac{1}{3} (S_a + S_b + S_c) \right] \cdot \frac{V_{dc}}{2} \,
U_{BN} = \left[ S_b - \frac{1}{3} (S_a + S_b + S_c) \right] \cdot \frac{V_{dc}}{2} \,
U_{CN} = \left[ S_c - \frac{1}{3} (S_a + S_b + S_c) \right] \cdot \frac{V_{dc}}{2} \hspace{1cm} (2)
\]

Applying the Kirchhoff laws to the multilevel converter circuit, the dynamic equation of the ac current, \( i_a \), \( i_b \), and \( i_c \), equation (3) describes the converter behavior.

\[
L_a \frac{di_a}{dt} = U_{AN} - i_a R_a - U_{ga} \,
L_b \frac{di_b}{dt} = U_{BN} - i_b R_b - U_{gb} \,
L_c \frac{di_c}{dt} = U_{CN} - i_c R_c - U_{gc} \hspace{1cm} (3)
\]

The instantaneous current behaviors can be described by the vector equation

\[
U = Ri + L \frac{di}{dt} + U_g \hspace{1cm} (4)
\]

Where,

\[
U = \frac{2}{3} (U_{AN} + e^{j2\pi/3} U_{BN} + (e^{j2\pi/3})^2 U_{CN}), \hspace{1cm}
i = \frac{2}{3} (i_a + e^{j2\pi/3} i_b + (e^{j2\pi/3})^2 i_c), \hspace{1cm} U_g = \frac{2}{3} (U_{ga} + e^{j2\pi/3} U_{gb} + (e^{j2\pi/3})^2 U_{gc})
\]

Predictive model is the basic element of predictive current control, which used to predictive the system’s future output according to history information of the system. A discrete time form of the system model for sampling time \( T_s \) is used to predict the future grid current. Approximating the derivative \( dU/dt \) and \( di/dt \) by

\[
\frac{dU}{dt} \approx \frac{U(k) - U(k-1)}{T_s} ; \hspace{0.5cm} \frac{di}{dt} \approx \frac{i(k) - i(k-1)}{T_s} \hspace{1cm} (5)
\]

The grid current is obtained by replacing (5) in (3).

\[
i(k) = \frac{1}{RT_s + L} [Li(k-1) + T_s U(k) - T_s U_g(k)] \hspace{1cm} (6)
\]

Form the equation (6), the future load current can be determined by

\[
i(k + 1) = \frac{1}{RT_s + L} [Li(k) + T_s U(k + 1) - T_s U_g(k + 1)] \hspace{1cm} (7)
\]

Applying Park’s transformation, we can get the instantaneous current behavior which under static stationary coordinates is described as

\[
\begin{align*}
    i_a(k + 1) &= \frac{1}{RT_s + L} [Li_a(k) + T_s U_a(k + 1) - T_s U_{ga}(k + 1)] \\
    i_b(k + 1) &= \frac{1}{RT_s + L} [Li_b(k) + T_s U_b(k + 1) - T_s U_{gb}(k + 1)]
\end{align*} \hspace{1cm} (8)
\]
It is presuming that the grid voltage and output voltage of the inverter are constant during every sampling period. The active and reactive current slopes are described as
\[
\frac{di_{\alpha}}{dt} = \frac{i_{\alpha}(k) - i_{\alpha}(k-1)}{T_s}, \quad \frac{di_{\beta}}{dt} = \frac{i_{\beta}(k) - i_{\beta}(k-1)}{T_s}
\]  

(9)

By application a given voltage vector, the application time for the linear trajectories of active and reactive currents is
\[
i_{\alpha,i} = i_{\alpha,i-1} + \frac{di_{\alpha,i}}{dt} \cdot t_i, \quad i_{\beta,i} = i_{\beta,i-1} + \frac{di_{\beta,i}}{dt} \cdot t_i
\]  

(10)

Where, \( t_i \) is the application time for the \( i \)th vector, \( i_{\alpha,i}, i_{\beta,i} \) are the active and reactive current values after the application time \( t_i \), \( i_{\alpha,i-1}, i_{\beta,i-1} \) are the initial values of the active and reactive current.

### 2.2 Nearest-three-virtual-vector

In term of three-phase NPC inverter, there are three kinds of output level \( (V_{dc}/2, 0, -V_{dc}/2) \) for each phase, where \( V_{dc} \) is the dc-link voltage. So the three phases share 27 level outputs, which correspond to 27 space vectors, as shown in Fig. 2(a).

![27 space vector diagram](image)

(a) 27 space vector diagram  

![Diagram of the 1st sector](image)

(b) Diagram of the 1st sector  

Fig. 2. NPC inverter space vector diagram

The Nearest-three-virtual-vector (NTV²) strategy was first put forward in [12] and then many researchers develop this method [13, 14]. The main idea is to synthesize the virtual space vectors, which can produce zero average NP current. In Fig. 2(b), take the first sector as example, there are five triangles, the determination of the triangles is judged by three rules.

\[
\text{Rule1: } U_\alpha + U_\beta \leq \frac{2}{3}; \quad \text{Rule2: } \sqrt{3}U_\alpha + U_\beta \leq 1; \quad \text{Rule3: } U_\alpha > \frac{1}{\sqrt{3}}
\]  

(11)

The judgement of the triangles is showed in Table I. The number of the triangles is determined by the three rules.

NTV² strategy is to synthesize the new vectors by small vectors and middle vectors which have an uncontrollable influence on the neutral-point voltage, but it has the drawback of low dc voltage utilization. Thus, NTV² can control NP voltage balance for any load. As show in Fig. 2, the NTV² space-vector diagram in the first
The sextant, the zero \((V_{0\text{OOO}})\) and large vectors \((V_{L1(PNN)}, V_{L2(PPN)})\) do not affect the neutral-point current at all, the medium and small vectors affect the neutral-point current that may produce dc voltage imbalance. The sextant is divided into five triangles, where \(V_{Z0}\) represents the virtual zero vector, is defined as \(V_{Z0} = V_{0\text{OOO}}\) and \(V_{ZL1}\) and \(V_{ZL2}\) are virtual large vectors.

\[
\begin{align*}
V_{ZL1} &= V_{L1(PNN)} \\
V_{ZL2} &= V_{L2(PPN)}
\end{align*}
\]

\(V_{ZS1}\) and \(V_{ZS2}\) are virtual small vectors, are defined as

\[
\begin{align*}
V_{ZS1} &= \frac{1}{2} V_{S1(ONN)} + \frac{1}{2} V_{S1(POO)} \\
V_{ZS2} &= \frac{1}{2} V_{S2(OON)} + \frac{1}{2} V_{S2(PPO)}
\end{align*}
\]

The detailed relationship between virtual small vectors states and neutral-point current are

\[
\begin{align*}
\frac{1}{2}[i_a + (-i_o)] &= 0 \\
\frac{1}{2}[i_c + (-i_o)] &= 0
\end{align*}
\]

So the virtual small vectors produce zero NP current. The virtual middle vector is defined as

\[
V_{ZM1} = \frac{1}{3} V_{S1(ONN)} + \frac{1}{3} V_{M(POP)} + \frac{1}{3} V_{S2(PPO)}
\]

In the three-phase system without midline, the \(i_a, i_b,\) and \(i_c\) are produced by the switching states \(V_{S1(ONN)}, V_{M(POP)}, V_{S2(PPO)},\) respectively, the relationship of three-phase currents is

\[
i_a + i_b + i_c = 0
\]

So the NP current produced by the virtual middle vector is 0, the virtual middle vector produces zero NP current. When the reference vector is located in triangle 3, the voltage-second balance expression is as follows:

\[
\begin{align*}
V_{\text{ref}} &= V_{ZS1} \cdot d_{ZS1} + V_{ZM1} \cdot d_{ZM1} + V_{ZL1} \cdot d_{ZL1} \\
d_{ZS1} + d_{ZM1} + d_{ZL1} &= 1
\end{align*}
\]

Where, \(V_{\text{ref}}\) represents the reference voltage vector and \(d_{ZS1}, d_{ZM1},\) and \(d_{ZL1}\) represent the duty cycles of the vectors \(V_{ZS1}, V_{ZM1},\) and \(V_{ZL1},\) respectively. The other expressions are similar to that in triangle 3.

The duty cycle of vector \(V_{S1(ONN)}, V_{S1(POO)}, V_{S2(PPP)}, V_{M(POP)},\) and \(V_{L1(PPN)}\) are \(d_{S1(ONN)}, d_{S1(POO)}, d_{S2(PPP)}, d_{M(POP)},\) and \(d_{L1(PPN)}\). Based on the equation of (13) and (15) obtain the following expression:

| Triangle | Rule 1 | Rule 2 | Rule 3 |
|----------|--------|--------|--------|
| 1        |        | YES    |        |
| 2        | YES    | NO     | NO     |
| 3        | YES    | NO     | YES    |
| 4        | NO     | –      | YES    |
| 5        | NO     | –      | NO     |

Table I. Nearest-three-virtual-vectors’ sequence
\[
\begin{align*}
    d_{S1(ONN)} &= \frac{1}{2} d_{ZS} + \frac{1}{3} d_{ZM};
    d_{S1(POO)} &= \frac{1}{2} d_{ZS};
    \\
    d_{S2(PPO)} &= \frac{1}{3} d_{ZM};
    d_{M(PON)} &= \frac{1}{3} d_{ZM};
    d_{L1(PNN)} &= d_{ZL};
\end{align*}
\]  

(18)

From the Fig. 2(b), the first vector of the switching sequence in each triangle is ONN, and the last is PPO. Take the 1st region as example. The selection of basic vector in each small sector is list in Table II.

| Sector | Nearest-three-virtual-vectors’ sequence |
|--------|----------------------------------------|
| 1      | ONN → OON → OOO → POO → PPO           |
| 2      | ONN → OON → PON → POO → PPO           |
| 3      | ONN → PNN → PON → POO → PPO           |
| 4      | ONN → PNN → PON → PPN → PPO           |
| 5      | ONN → OON → PON → PPN → PPO           |

When the reference vector is located in triangle 3, the switching sequence in two sampling periods is show in Fig. 3.

![Fig. 3. Voltage sequence diagram](image)

The calculation of the dwell times for the space vectors is different. Considering the graph of the basic space vector is a hexagon, the vectors in other sextants can be mapped into the first sextant through vector-rotating conversion [15].

Assume that the used 60° coordinate system is the g-h coordinate system and g axis coincide with \(\alpha\) axis in \(\alpha - \beta\) coordinate, and then it counterclockwise rotates 60° as the h axis. The transformation between the two coordinate systems is shown in Table III.

\(V_{ref}(V_{refg}, V_{refh})\) is the coordinate projection in the g-h coordinates, \(V_g\), and \(V_h\) represent the coordinate projection of the reference vectors \(V_{ref}\) transformed to first sextant.

### 2.3 Application times

At the instant \(k_i\), assuming the reference vector \(V_{ref}\) located in \(i\)th triangle, the trajectory of active and reactive currents is showed for one switching period as
The equivalent vector expression in the first sector

| Sextant | Coordinates transformation |
|---------|----------------------------|
| I       | $V_g = V_{\text{refg}}$  
          | $V_h = V_{\text{refh}}$  |
| II      | $V_g = V_{\text{refg}} + V_{\text{refh}}$ 
          | $V_h = -V_{\text{refg}}$  |
| III     | $V_g = V_{\text{refg}}$  
          | $V_h = -V_{\text{refg}} - V_{\text{refh}}$  |
| IV      | $V_g = -V_{\text{refg}}$  
          | $V_h = -V_{\text{refg}}$  |
| V       | $V_g = -V_{\text{refg}} - V_{\text{refh}}$  
          | $V_h = V_{\text{refg}}$  |
| VI      | $V_g = -V_{\text{refg}} + V_{\text{refh}}$  
          | $V_h = V_{\text{refg}} + V_{\text{refh}}$  |

\[
i_{a,1} = i_{a,0} + \frac{di_{a,1}}{dt} \cdot t_1, \quad i_{\beta,1} = u_{c\beta,0} + \frac{di_{\beta,1}}{dt} \cdot t_1; \quad i_{a,2} = i_{a,1} + \frac{di_{a,2}}{dt} \cdot t_2,
\]
\[
i_{\beta,2} = i_{\beta,1} + \frac{di_{\beta,2}}{dt} \cdot t_2; \quad i_{a,3} = i_{a,2} + \frac{di_{a,3}}{dt} \cdot t_3; \quad i_{\beta,3} = i_{\beta,2} + \frac{di_{\beta,3}}{dt} \cdot t_3;
\]
\[
i_{a,4} = i_{a,3} + \frac{di_{a,4}}{dt} \cdot t_4; \quad i_{\beta,4} = i_{\beta,3} + \frac{di_{\beta,4}}{dt} \cdot t_4; \quad i_{a,5} = i_{a,4} + \frac{di_{a,5}}{dt} \cdot t_5;
\]
\[
i_{\beta,5} = i_{\beta,4} + \frac{di_{\beta,5}}{dt} \cdot t_5
\]

\[
t_1 = \frac{1}{2} T_1 + \frac{1}{3} T_2, \quad t_2 = T_3, \quad t_3 = \frac{1}{3} T_2
\]
\[
t_4 = \frac{1}{2} T_1, \quad t_5 = \frac{1}{3} T_2, \quad T_s = 2T_1 + 2T_2 + 2T_3
\]

Where $T_1$, $T_2$ and $T_3$ are application times for three virtual voltage vectors, and $t_1 \sim t_5$ are application times for the actual vectors’ sequences shown in Table II.

To get optimized application times, the least-square optimization method is used. The objective function is selected as

\[
g = g_\alpha^2 + g_\beta^2
\]

Where,

\[
g_\alpha = [i_a^*(k + 1) - i_a(k)] - 2 \left[ \frac{di_{a1}}{dt} \cdot t_1 + \frac{di_{a2}}{dt} \cdot t_2 + \frac{di_{a3}}{dt} \cdot t_3 + \frac{di_{a4}}{dt} \cdot t_4 + \frac{di_{a5}}{dt} \cdot t_5 \right]
\]
\[
g_\beta = [i_\beta^*(k + 1) - i_\beta(k)] - 2 \left[ \frac{di_{\beta1}}{dt} \cdot t_1 + \frac{di_{\beta2}}{dt} \cdot t_2 + \frac{di_{\beta3}}{dt} \cdot t_3 + \frac{di_{\beta4}}{dt} \cdot t_4 + \frac{di_{\beta5}}{dt} \cdot t_5 \right]
\]

In this paper the second order Lagrange extrapolation formula is adopted to get the reference value, so the future reference $i_a^*(k + 1)$ and $i_\beta^*(k + 1)$ are shown in (23).

\[
i^*(k + 1) = 3i^*(k) - 3i^*(k - 1) + i^*(k - 2)
\]

In order to get the application times that minimizes $g$, we can solving the following two conditions.

\[
\frac{\partial g}{\partial T_2} = 0, \quad \frac{\partial g}{\partial T_3} = 0
\]

Solving the set of equation derived from (24). Resultant $T_1$, $T_2$, and $T_3$ are (25) in the Appendix.
3 Proposed control system

The block diagram of the proposed control strategy is shown in Fig. 4.

The main functions realized by Fig. 4 are listed as followings:

1) The grid voltages and current are measured and transformed in the predictive model, the prediction module of reference output forecasts output voltage of reference at next time, and the results are sent to the module of objective function.

2) The prediction module selects appropriate vectors and their sequence and then calculates the time derivatives of NTV² sequence. The vectors in other sextants mapped into the first sextant through vector-rotating conversion.

3) The switch states are utilized to compute the optimal set of application times that minimize tracking errors.

4 Results and discussion

This section presents simulation of the NPC inverter using three control strategies (SVPWM, proposed strategy) and experimental results on the proposed predictive current control. Table IV contains main simulation parameters used in the simulation and experimental results.

| Parameter               | Value   |
|-------------------------|---------|
| V<sub>dc</sub>          | 230 [V] |
| Grid voltage            | 150 [V] |
| Fundamental frequency   | 50 [Hz] |
| Switching frequency     | 10 KHz  |
| DC-link capacitances    | 235 [µF]|
| Inductance              | 5 [mH]  |
| Sampling time           | 5 [µS]  |

Fig. 5 shows SVPWM control of the three-phase current [7]. When the neutral-point voltage is unbalanced, the phase current is distorted due to low order harmonic components, the THD of phrase A is 6.3%.
However, the distortion of phase current is eliminated as shown in Fig. 6 after the proposed algorithm is applied. It can be seen that the symmetrical NTV2 sequence has much lower THD. The result of the proposed self-balancing strategy of the neutral point voltage can be observed that deviation is reduced, so the power quality is substantially improved. The simulation results suggest that the proposed control strategy obtain better effect than SVPWM strategy.

The dynamic responses are compared to check the validity of the real-time performance of two strategies, at 0.3 seconds, the reference current increase from 5 A to 12 A, the dynamic characteristics result of the two strategies as Fig. 7(a) and Fig. 7(b) shows.

From the simulation results, the current response time of the SVPWM is 10 ms, while the proposed controller only need 5 ms to reach stable states, the dynamical
response is approved effectively, and the proposed strategy has fairly good robustness.

In order to verify the performance of the proposed, an experimental setup was developed using a DSP model TMS320C6713, ACM10YE13H IPM is used to build the NPC inverter. The capacitor in dc-link is 235 µF, for a sampling time $T_s = 5 \mu S$, and tested with an load active power with values $P = 0.5$ KW, reactive power values $Q = 0.5$ KW. The experiment results are shown in Fig. 8, (a) is the steady output current waveform by SVPWM strategy, THD = 6.41%, (b) is the steady output current waveform by the proposed strategy, THD = 1.14%. So the

![Graphs showing experiment results](image)

**Fig. 8.** Experiment results
proposed strategy is capable of eliminating such current distortions and reactive power oscillations. The result of neutral point voltage control is given in (c). To further illustrate and validate such improvements, Fig. 8. (d), (e) shown the SVPWM results and proposed control results, when the active power changes from 0.5 KW to 1 KW.

The experimental results show that with the proposed control strategy, the fast current response is gained and the current harmonic is eliminated in the condition of the low fluctuations neutral point voltage.

5 Conclusion

In this paper, a predictive current controller which uses a NTV² voltage vectors’ sequence is presented, it can reduce the NP ripple amplitude and provide high quality of line current compared with SVPWM strategy, when the system exists harmonic current. The proposed advanced current control scheme based on the 60° coordinate system is easy to get the basic vectors and effective time of them, the disadvantages of a lot of calculation of the trigonometric functions and the estimation of the sectors for the traditional SVPWM algorithm are avoided after transforming the whole space vector diagram into the first sextant. Thus the fluctuations of the NP voltage can be balanced through calculating the effective time of the output voltage vectors. The feasibility and effectiveness of control strategy are verified by experiment results.

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Appendix

\[
T_2 = \left[ \left( \frac{dl_{q1}}{dt} - \frac{dl_{q2}}{dt} \right) \cdot \left( i_a^*(k+1) - i_a(k) \right) + \left( \frac{dl_{d1}}{dt} - \frac{dl_{d2}}{dt} \right) \cdot \left( i_d^*(k+1) - i_d(k) \right) + \left( \frac{dl_{d1}}{dt} - \frac{dl_{d2}}{dt} \right) \cdot \frac{dl_{d1}}{dt} \right] \frac{T_s}{2}
\]

\[
T_3 = \left[ \left( \frac{dl_{d1}}{dt} - \frac{dl_{d2}}{dt} \right) \cdot \left( i_a^*(k+1) - i_a(k) \right) + \left( \frac{dl_{d1}}{dt} - \frac{dl_{d2}}{dt} \right) \cdot \frac{dl_{d1}}{dt} \right] \frac{T_s}{2}
\]

\[
\frac{dl_{d1}}{dt} = \frac{1}{2} \frac{dl_{d1}}{dt} + \frac{1}{2} \frac{dl_{d1}}{dt} = \frac{1}{3} \frac{dl_{d1}}{dt} + \frac{1}{3} \frac{dl_{d1}}{dt} = \frac{1}{5} \frac{dl_{d1}}{dt} + \frac{1}{5} \frac{dl_{d1}}{dt} = \frac{1}{5} \frac{dl_{d1}}{dt}
\]

\[
T_1 = T_s/2 - T_2 - T_3
\]

(25)