Phase Transition to a Hairy Black Hole in Asymptotically Flat Spacetime

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We discuss a phase transition of a Reissner-Nordström black hole to a hairy black hole in asymptotically flat spacetime. The hair is due to a massive charged scalar field. The no-hair theorem is evaded thanks to a derivative coupling of the scalar field to the Einstein tensor. The resulting hairy configuration is spherically symmetric. We solve the equations analytically near the transition temperature and show that the hair is concentrated near the horizon decaying exponentially away from it.
I. INTRODUCTION

Scalar-tensor theories belong to a class of theories that modify the Einstein’s theory of gravity and have been under intense investigation over the last few years. The most interesting class of scalar-tensor models are described by the Horndeski Lagrangian \[1\], which gives second-order field equations in four dimensions. It was soon realized that these scalar-tensor models of modified gravity share a classical Galilean symmetry \[2\]–\[4\], and most of these models represent a special case of Horndeski’s theory.

One of the terms appearing in the Horndeski Lagrangian is the derivative coupling of a scalar field to Einstein tensor. The cosmological implications of the derivative coupling to gravity was first discussed in \[5\], and subsequently it was shown that the presence of this coupling in the Lagrangian gives second-order field equations \[6\], in accordance with Horndeski’s theory. A cosmological model was discussed in \[7\]–\[8\], where the derivative coupling was introduced to explain early as well as late expansion of the Universe. Quintessence and phantom cosmology in the presence of this coupling was presented in \[9\]–\[10\], while accelerating expansion was discussed in \[11\]–\[12\]. The early inflationary phase was studied in \[13\], where it was found that the derivative coupling acted like a friction term. It was also found that this term had a Galilean symmetry \[14\].

Observational tests of inflation with a field coupled to the Einstein tensor was presented in \[15\], while in \[16\] it was shown that the derivative coupling to gravity provided a natural mechanism to suppress the overproduction of heavy particles after inflation. Particle production after the end of inflation in the presence of derivative couplings was also discussed in \[17\].

This interesting cosmological behavior of the derivative coupling of a scalar field to the Einstein tensor stems from the fact that this term introduces a scale in the theory and effectively acts as a cosmological constant \[18\]. As is well known, the presence of the cosmological constant alters the local properties of spacetime, allowing the evasion of the no-hair theorems. Hairy black hole solutions were found in the presence of a cosmological constant. Therefore, it is natural to ask whether black hole solutions exist in gravity theories with derivative couplings in the absence of a cosmological constant.

In the case of a positive cosmological constant with a minimally coupled scalar field with a self-interaction potential, black hole solutions were found in \[18\], and a numerical solution, albeit an unstable one, was presented in \[19\]. If the scalar field is non-minimally coupled, a solution exists with a quartic self-interaction potential \[20\]. However, this solution was also shown to be unstable \[21\]–\[22\]. In the case of a negative cosmological constant, stable solutions were found numerically for spherical geometries \[23\]–\[24\], and an exact solution in asymptotically AdS space with hyperbolic geometry was presented in \[25\], and later generalized to include charge \[26\], and further generalized to non-conformal solutions \[27\]. Also, an exact solution of a charged C-metric conformally coupled to a scalar field was presented in \[28\]–\[29\]. Further hairy solutions in the presence of a cosmological constant were reported in \[30\]–\[33\] with various properties.

Black hole solutions in the general Horndeski theory are not known. In theories resulting from the truncation of higher-dimensional theories, which in four dimensions give second-order field equations, black hole solutions were discussed in \[34\]. In a gravity model with a scalar field coupled to the Einstein tensor, an instability was found outside the horizon of a Reissner-Nordström black hole, by calculating the quasinormal spectrum of scalar perturbations \[35\]. It was shown that for higher angular momentum quantum numbers and derivative coupling constant larger than a certain critical value, the effective potential develops a negative gap near the black hole horizon. This indicates that a phase transition to a hairy black hole configuration can occur.

In our previous work \[36\], we investigated this effect in detail. We considered a gravity model consisting of an electromagnetic field and a scalar field coupled to the Einstein tensor with vanishing cosmological constant. We showed that a Reissner-Nordström black hole undergoes a second-order phase transition to a hairy black hole configuration of generally anisotropic hair at a critical temperature. Using perturbation theory, we calculated analytically the properties of the hairy black hole configuration near the critical temperature and showed that it is energetically favorable over the corresponding Reissner-Nordström black hole. Spherically symmetric black hole solutions were also discussed in \[37\].

The recent advances in holography, and in particular the application of the gauge/gravity duality to condensed matter systems (for a review, see \[38\]) has revived the interest on the dynamics of a scalar field outside a black hole horizon. The transition from a metallic state to a superconducting state in a strongly-coupled system can be described by its dual weakly-coupled gravitational system using the AdS/CFT correspondence \[39\]. The simplest holographic superconductor model \[39\] is described by an Einstein-Maxwell-scalar field theory with a negative cosmological constant. At high enough temperatures
where, the black hole-scalar system is decoupled and the black hole is stable. The boundary gauge theory describes a metallic state. When the temperature is lowered, the black hole becomes unstable and a new black hole configuration with scalar hair forms. The dual gauge theory describes a superconducting state (for a review, see [40] and references therein). An exact gravity dual of a gapless holographic superconductor was presented in [42, 43].

The dynamics of a holographic superconductor depends crucially on the behavior of the scalar field near the horizon of the black hole. A model was presented in [42, 43] consisting of an electromagnetic field and a scalar field minimally coupled to gravity in the presence of a negative cosmological constant. It was shown that the effective mass of the scalar field becomes negative for large values of the charge of the background Reissner-Nordström black hole, thus breaking an Abelian gauge symmetry outside the horizon of the Reissner-Nordström black hole. In the language of gauge/gravity duality, this corresponds to the formation of a condensate in the boundary gauge theory, which below a certain critical temperature triggers the transition from a metallic state to a superconducting one. A heuristic way to explain this behavior of the scalar field outside the black hole horizon was presented in [43]. If the scalar particle is highly charged, then its gravitational attraction to the black hole is overcome by its electrostatic repulsion, and because the space has a boundary (being asymptotically AdS), it reflects back and condenses outside the black hole horizon. A generalization to higher dimensions and to other horizon topologies was presented in [44].

In this work, our main motivation is to study possible instabilities of a black hole near its horizon in asymptotically flat space (vanishing cosmological constant). We consider a gravitational system consisting of an electromagnetic field and a charged scalar field which, together with the standard minimal coupling to gravity, also couples to the Einstein tensor. As it was discussed in [42], in the absence of the derivative coupling to the Einstein tensor, we cannot have a “geometrical” breaking of the Abelian symmetry near the black hole horizon, since the space is asymptotically flat.

By turning on the derivative coupling to the Einstein tensor, and solving the resulting dynamical system of Einstein-Maxwell-scalar field equations, we will show that there is a critical temperature at which a phase transition to a hairy black hole configuration occurs. The dimensionful coupling constant of the derivative coupling to the Einstein tensor provides the scale for the confining potential, which is an effect similar to the AdS radius provided by the cosmological constant. We argue that the new hairy black hole configuration results from the breaking of the Abelian symmetry near the horizon by curvature effects. We obtain a hairy black hole which is spherically symmetric.

Our discussion is organized as follows. In Section II, we set up the theory and derive the field equations. In Section III, we solve the field equations perturbatively. We discuss the solution at leading as well as next-to-leading order. We present both analytic and numerical results. Finally in Section IV, we conclude.

II. THE FIELD EQUATIONS

Our system consists of a $U(1)$ gauge potential $A_\mu$, and a scalar field $\varphi$ of mass $m$ and charge $q$, in a dynamical gravitational background with vanishing cosmological constant. Thus, spacetime is asymptotically flat. The action is

$$I = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - (g^{\mu\nu} - \kappa G^{\mu\nu}) D_\mu \varphi (D_\nu \varphi)^* - m^2 |\varphi|^2 \right],$$

where $D_\mu = \nabla_\mu - iq A_\mu$, and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength. The action (1) contains a derivative coupling of the scalar field to Einstein tensor with coupling constant $\kappa$ of dimension length squared. For convenience, we introduce the notation

$$\Phi_{\mu\nu} = D_\mu \varphi (D_\nu \varphi)^* , \quad \Phi = g^{\mu\nu} \Phi_{\mu\nu} .$$

The field equations resulting from the variation of the action (1) are the Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \, , \quad T_{\mu\nu} = T_{\mu\nu}^{(\varphi)} + T_{\mu\nu}^{(EM)} - \kappa \Theta_{\mu\nu} ,$$

where,

$$T_{\mu\nu}^{(\varphi)} = \Phi_{\mu\nu} + \Phi_{\nu\mu} - g_{\mu\nu} (g^{ab} \Phi_{ab} + m^2 |\varphi|^2) ,$$

$$T_{\mu\nu}^{(EM)} = F_\alpha \mu F_{\nu\alpha} - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} ,$$

and

$$\Theta_{\mu\nu} = -\frac{1}{2} \left( \nabla_\mu \varphi \nabla_\nu \varphi - \frac{1}{4} g_{\mu\nu} (\nabla_\alpha \varphi \nabla^\alpha \varphi - m^2 |\varphi|^2) \right) .$$

[Note: Due to the complexity and length of the equations, only a subset of them is shown here. The full set is provided in the original document.]
\[ \Theta_{\mu\nu} = -g_{\mu\nu}R^{ab}_{\mu\nu\phi} + R_{\nu}^a(\Phi_{\mu a} + \Phi_{a\mu}) + R_{\mu}^a(\Phi_{\nu a} + \Phi_{a\nu}) - \frac{1}{2}R(\Phi_{\mu\nu} + \Phi_{\nu\mu}) \]

\[ -G_{\mu\nu}\Phi - \frac{1}{2}\nabla^a\nabla_\mu(\Phi_{\nu a} + \Phi_{a\nu}) - \frac{1}{2}\nabla^a\nabla_\nu(\Phi_{\mu a} + \Phi_{a\mu}) + \frac{1}{2}\Box(\Phi_{\mu\nu} + \Phi_{\nu\mu}) \]

\[ + \frac{1}{2}g_{\mu\nu}\nabla_a\nabla_b(\Phi^{ab} + \Phi^{ba}) + \frac{1}{2}(\nabla_\mu\nabla_\nu + \nabla_\nu\nabla_\mu)\Phi - g_{\mu\nu}\Box \Phi, \]

the Klein-Gordon equation

\[ (\partial_\mu - iqA_\mu) \left[ \sqrt{-g}(g^{\mu\nu} - \kappa G^{\mu\nu})(\partial_\nu - iqA_\nu)\varphi \right] = \sqrt{-g}m^2\varphi, \]

and the Maxwell equations

\[ \nabla_\nu F^{\mu\nu} + (g^{\mu\nu} - \kappa G^{\mu\nu}) \left[ 2q^2A_\nu|\varphi|^2 + iq(\varphi^*\nabla_\nu\varphi - \varphi\nabla_\nu\varphi^*) \right] = 0. \]

Our goal is to find spherically symmetric solutions of the coupled system of Einstein-Maxwell-scalar equations (3), (7) and (8). Apart from the complexity of the \( \Theta_{\mu\nu} \) contribution to the energy-momentum tensor \( T_{\mu\nu} \), there is another technical difficulty in solving the system of field equations. The presence of the dimensionful coupling \( \kappa \) does not allow the scalar field to be conformally coupled to gravity. In many of the existing hairy black hole solutions, the conformal symmetry in the scalar sector helps to find exact solutions [25, 26, 28].

For a spherically symmetric solution, consider the metric ansatz

\[ \frac{ds^2}{\mu^2} = -e^{-\alpha(z)}dt^2 + \frac{l(z)e^{\alpha(z)}}{z^2} \left[ \frac{dz^2}{z^2} + d\Omega^2 \right], \]

where \( \mu \) is an arbitrary scale and all metric functions depend only on the radial coordinate \( z \) (no dependence on the angles \( \Omega = (\theta, \phi) \), or time \( t \)). We will place the horizon at \( z = 1 \), and choose coordinates so that \( le^\alpha \) remains finite at the horizon. Furthermore, we assume asymptotic flatness as \( z \to 0 \) (so \( \alpha \to 0 \), and \( l \to 1 \)). The arbitrary scale \( \mu \) can be thought of as the position of the horizon, by switching the radial coordinate to \( r \) defined by \( z = \frac{r}{\mu} \). After rescaling \( t \to t/\mu \), the metric no longer has an explicit dependence on the parameter \( \mu \) (depending on it only through the metric functions \( \alpha \) and \( l \)), and the horizon is placed at \( r = \mu \). In calculations, we may safely ignore \( \mu \), setting \( \mu = 1 \), but we need to restore it in dimensionful quantities (e.g., \( \kappa \to \kappa/\mu^2 \)).

The temperature of the black hole is (recall \( le^\alpha \) is finite at the horizon, by our choice of coordinates)

\[ T = \frac{1}{2\pi\mu l(1)e^{\alpha(1)} \sqrt{l'(1)^2/2}}. \]

Using the metric ansatz (4), the field equations reduce to the following equations.

The Einstein equations (3) are conveniently written as

\[ R^\nu_{\mu} = 8\pi G \left[ T^\nu_{\mu} - \frac{1}{2}g^\nu_{\alpha}T_{\alpha\mu} \right]. \]

Only the diagonal components are non-vanishing, and the two angular components are equal to each other by spherical symmetry. Therefore, there are three independent equations with corresponding components of the Ricci tensor,

\[ R^t_t = \frac{z^4 e^{-\alpha}}{4\mu^2 l} \left[ \frac{\nu'}{l} \alpha' + 2\alpha'' \right], \]

\[ R^z_z = \frac{z^4 e^{-\alpha}}{4\mu^2 l} \left[ -4 \frac{\nu^2}{l^2} + \frac{\nu'}{l} \left( \frac{4}{z} + \alpha' \right) + \frac{\nu''}{l} + 2\alpha'^2 + 2\alpha'' \right], \]

\[ R^\theta_\theta = R^\phi_\phi = \frac{z^4 e^{-\alpha}}{4\mu^2 l} \left[ \frac{\nu^2}{l^2} + \frac{\nu'}{l} \left( \frac{z}{2} + \alpha' \right) + 2\frac{\nu''}{l} + 2\alpha'' \right]. \]
The various contributions to the stress-energy tensor are

\[ T^{(EM)t}_t = T^{(EM)z}_z = -T^{(EM)\theta}_\theta = -T^{(EM)\phi}_\phi = \frac{\zeta^4}{2\mu^2l} A_t^2, \]

and

\[ \mu^2 T^{(\phi)t}_t = -\frac{1}{4} \left[ m^2 + q^2 e^\alpha A_t^2 \right] \varphi^2 - \frac{\zeta^4}{4l} e^{-\alpha} \varphi'^2, \]

\[ \mu^2 T^{(\phi)z}_z = -\frac{1}{4} \left[ m^2 - q^2 e^\alpha A_t^2 \right] \varphi^2 + \frac{\zeta^4}{4l} e^{-\alpha} \varphi'^2, \]

\[ \mu^2 T^{(\phi)\theta}_\theta = \mu^2 T^{(\phi)\phi}_\phi = -\frac{1}{4} \left[ m^2 - q^2 e^\alpha A_t^2 \right] \varphi^2 - \frac{\zeta^4}{4l} e^{-\alpha} \varphi'^2. \]

The Klein-Gordon equation (17) reads

\[ \frac{\zeta^4 e^{-\alpha}}{l^{3/2}} \left( \sqrt{\mathcal{A}} \varphi' \right)' - \mu^2 m^2_{\text{eff}} \varphi = 0, \]

where

\[ \mathcal{A} = 1 + \frac{k^2 e^{-\alpha}}{4\mu^2l} \left[ \frac{4\l''}{l} - \frac{z l^2}{l^2} + z \alpha^2 \right], \]

\[ m^2_{\text{eff}} = m^2 - q^2 e^\alpha A_t^2 \left[ 1 - \frac{k^2 e^{-\alpha}}{4\mu^2l^2} \left( \frac{3l^2}{l^2} + 2\alpha' l' + \frac{4l''}{l} \alpha^2 + 4\alpha'' \right) \right]. \]

Finally, the Maxwell equations reduce to Gauss’s Law,

\[ A_t'' + \left[ \frac{l''}{2l} + \alpha' \right] A_t' + \frac{q^2}{2} \left[ \frac{e^\alpha l}{z} \frac{\zeta^2}{4\mu^2l^2} \left( \frac{3l^2}{l^2} + 2\alpha' l' + \frac{4l''}{l} \alpha^2 + 4\alpha'' \right) \right] \varphi^2 A_t = 0. \]

The above system of non-linear equations need to be solved for the functions \( \alpha, l, A_t, \varphi \). We shall set \( 16\pi G = 1 \).
III. PERTURBATIVE SOLUTION

To solve the coupled non-linear system of equations (11), (16) and (18), we use perturbation theory. We expand the metric functions, and the scalar potential for small values of the scalar field $\varphi$. Introducing the bookkeeping parameter $\epsilon$, we have

$$\alpha = \alpha_0 + \epsilon \alpha_1 + \epsilon^2 \alpha_2 + \ldots$$
$$l = l_0 + \epsilon l_1 + \epsilon^2 l_2 + \ldots$$
$$A_t = A_{t0} + \epsilon A_{t1} + \epsilon^2 A_{t2} + \ldots$$
$$\varphi = \epsilon \varphi_0 + \epsilon^2 \varphi_1 + \epsilon^3 \varphi_2 + \ldots. \quad (19)$$

A. Leading order

At zeroth order, we obtain the system

$$\frac{l_0'}{l_0} \alpha_0' + 2 \alpha_0'' + \epsilon \alpha_0 A_{t0}'^2 = 0,$$
$$-4z \frac{l_0'}{l_0} + \frac{l_0'}{l_0} (4 + z \alpha_0') + 4z \frac{l_0''}{l_0} + 2z (\alpha_0'^2 + \alpha_0'') - z \epsilon \alpha_0 A_{t0}'^2 = 0,$$
$$-z \frac{l_0'}{l_0} + \frac{l_0'}{l_0} (-2 + z \alpha_0') + 2z \frac{l_0''}{l_0} + 2z \alpha_0'' + z \epsilon \alpha_0 A_{t0}'^2 = 0,$$
$$A_{t0}'' + \left[ \frac{l_0'}{2l_0} + \alpha_0' \right] A_{t0}' = 0,$$
$$\frac{z^4 e^{-\alpha_0}}{l_0^{3/2}} \left( \sqrt{l_0 A_{t0} \varphi_0} \right)' - \mu^2 m_{\text{eff},0}^2 \varphi_0 = 0 \quad (20).$$

where

$$A_0 = 1 + \frac{\kappa z^3 e^{-\alpha_0}}{4 \mu^2 l} \left[ \frac{4l_0'}{l_0} - \frac{z l_0'^2}{l_0} + z \alpha_0' \right],$$
$$m_{\text{eff},0}^2 = m^2 - q^2 e^{\alpha_0} A_{t0} \left[ 1 - \frac{\kappa z^3 e^{-\alpha_0}}{4 \mu^2 l} \left( \frac{3l_0'^2}{l_0^2} + 2 \alpha_0' \frac{l_0'}{l_0} + \frac{4l_0''}{l_0} + \alpha_0'' + 4 \alpha_0' \right) \right]. \quad (21)$$

If the scalar field vanishes (no hair), the Klein-Gordon equation is trivially satisfied, and the rest of the equations form a system of Einstein-Maxwell field equations whose solution is the Reissner-Nordström black hole written in Papapetrou coordinates,

$$\alpha_0 = \ln \left( \frac{1 + 2z \coth B + z^2}{l_0} \right)^2, \quad l_0 = (1 - z^2)^2, \quad A_{t0} = 2e^{-B} - \frac{4z}{(1 + z^2) \sinh B + 2z \cosh B} \quad (22).$$

The mass and the charge are found from the $O(z)$ terms in $\alpha$ and $A_t$. We obtain, respectively,

$$\frac{M}{\mu} = 8 \coth B, \quad \frac{Q}{\mu} = \frac{4}{\sinh B} \quad (23).$$

Notice that the ratio $\frac{Q}{M} = \frac{1}{2 \cosh B}$ is solely a function of $B$. As $B \to \infty$, $\frac{Q}{M} \to 0$, and we obtain the Schwarzschild black hole. For $B = 0$, the ratio attains its maximum value, $\frac{Q}{M} = \frac{1}{2}$, corresponding to the extremal Reissner-Nordström black hole.

The temperature is found from (10) to be

$$T = T_0 = \frac{e^{-2B} \sinh^2 B}{4\pi \mu} \quad (24).$$
FIG. 1. The effective mass $m_{\text{eff}}^2$ as a function of the radial coordinate $z$ outside the horizon (located at $z = 1$) for coupling constant (top to bottom) $\kappa = 100, 170, 500, 1000$, and mass and charge, respectively, of the scalar field (black hole), $m = 0.3, q = 0.5$ ($M = 3.2, Q = 1.0$).

For the Schwarzschild black hole ($B \to \infty$), $T = \frac{1}{16\pi\mu}$, whereas for the extremal Reissner-Nordström black hole ($B = 0$), $T = 0$, as expected.

For a hairy solution, we need a non-vanishing scalar field. At zeroth order, we must have $\phi_0 \neq 0$. Therefore, additionally, we must find the solution to the zeroth-order Klein-Gordon equation (last equation in (20); the first four equations are independent of $\phi_0$). This constrains the two-parameter family of solutions we just obtained (Reissner-Nordström black holes parametrized by $(M, Q)$, or equivalently $(\mu, B)$).

Using the Reissner-Nordström metric functions (22), the functions (21) entering the zeroth-order Klein-Gordon equation read

$$A_0 = 1 + \frac{4\kappa}{\mu^2} \left[ \frac{z^4 \sinh^2 B}{(1 + z^2) \sinh B + 2z \cosh B} \right]^2,$$

$$m_{\text{eff},0}^2 = m^2 - 4q^2 e^{-2B} \left[ \frac{1}{1 + z^2} \right]^2 \frac{16q^2 \kappa e^{-2B} \sinh^2 B}{(1 + z)^2} \frac{z^4 (1 - z)^2}{(1 + z^2) \sinh B + 2z \cosh B}^2. \quad (25)$$

Notice that for $\kappa \geq 0$, we always have $A_0 > 0$. Thus, $m_{\text{eff},0}^2$ provides an effective potential which determines the existence of solutions to the scalar equation (“bound states”). For a solution, we need $m_{\text{eff},0}^2 < 0$ within a finite interval outside the horizon. Evidently, for sufficiently large charge of the scalar $q$, we have $m_{\text{eff},0}^2 < 0$ in a long enough interval that guarantees the existence of a solution, implying a potential instability of the black hole. In the absence of the derivative coupling of the scalar field to the Einstein tensor ($\kappa = 0$), this interval necessarily includes infinity ($z = 0$), attaining the asymptotic value $m_{\text{eff},0}^2 = \nu^2 < 0$, where

$$\nu^2 \equiv m^2 - 4q^2 e^{-2B}. \quad (26)$$

Consequently, a scalar particle flying away from the horizon (due to a repulsive electric force which exceeds the gravitational attraction) travels all the way to infinity and can never condense \[43\]. On the other hand, no instability occurs for $\nu^2 \geq 0$. This is in accord with the no-hair theorem in asymptotically flat spaces.

The above picture changes if $\kappa > 0$ \[1\]. While for $\nu^2 < 0$ we still have no condensation, for $\nu^2 > 0$ the additional terms proportional to $\kappa$ introduce a potential well which can trap scalar particles. For large

1 If the coupling constant $\kappa$ is negative, then the system of Einstein-Maxwell-Klein-Gordon equations \[11, 15, 16\] is unstable and no solutions can be found. We reached the same conclusion in our previous work \[13\]. The sign of the coupling constant $\kappa$ was also discussed in \[13\], where it was claimed that a negative $\kappa$ introduces ghosts into the theory. Finally, in \[16\] a very small window of negative $\kappa$ was shown to be allowed.
enough values of the coupling constant $\kappa$, this will be is deep enough for the scalar particles to condense, as we will show. Thus, the no-hair theorem is evaded and hair forms.

In figure 1 we plot $m_{\text{eff,0}}^2$ as a function of the radial coordinate $z$ for various (indicative) values of the parameters on which it depends, with $\nu^2 > 0$. We chose $m = 0.3$ and $q = 0.5$ for the scalar field. We also fixed the mass and charge of the Reissner-Nordström black hole to $M = 3.2$ and $Q = 1.0$, respectively. The four curves (top to bottom) correspond to the choices $\kappa = 100, 170, 500, 1000$, respectively. One can see the potential well forming in all cases, however for $\kappa = 100, 170$, the curve is above the axis, and the wave equation has no solution. For $\kappa = 500, 1000$, the potential well dips into negative values. This is a necessary but not sufficient condition for the existence of solutions. For the chosen values of mass and charge of the particle, a solution can be found corresponding to potential wells of the form depicted in fig. 1 for $\kappa = 1000$, and mass and charge of the black hole near the chosen values.

Next, we discuss the numerical solution of the zeroth-order wave equation (last equation in (20) with $A_0$ and $m_{\text{eff,0}}$ given in (25)). It is convenient to isolate the asymptotic behavior of the scalar field. As $z \to 0$ ($r = \frac{\kappa}{z} \to \infty$), we obtain

$$
\phi_0(z) \sim z^{1+2\mu}\xi e^{-\frac{\mu z}{2}} \sim \frac{1}{r^{1+2\mu}}e^{-\nu r}, \quad \xi \equiv \frac{\nu}{\sinh B} - \frac{m^2}{\nu},
$$

where $\nu$ is defined in (20). Notice that the field decays exponentially as $r \to \infty$, to be contrasted with the standard power law behavior is AdS space (in which hairy solutions have been shown to exist). Thus, if a solution exists in our case, the condensation occurs in the vicinity of the horizon and decays rapidly away from it.

Defining

$$
\phi_0(z) = z^{1+2\mu}\xi e^{-\frac{\mu z}{2}} \chi(z),
$$

we will solve the wave equation numerically for $\chi$. This function is regular at $z = 0$, and is arbitrarily normalized, since it obeys a linear equation. One may choose $\chi(0) = 1$ as one of the boundary conditions. The other boundary condition is determined from the analytic solution to the wave equation expanded around $z = 0$ (which is a regular point for $\chi$). We obtain

$$
\chi(z) = 1 + \frac{1}{2\mu \nu} \left(2\mu \xi(1 + 2\mu \xi) - 2\mu^2 \nu^2 (5\nu^2 + 2\nu^2 \coth^2 B - 8\nu^2 \coth B) + 8\mu^2 m^2 (2\coth B - 1)\right) z + O(z^2),
$$

from which we one can read off $\chi'(0)$, to be used as a second boundary condition. Thus, for a given set of parameters of the scalar field ($m, q$), one obtains a unique solution. However, these solutions are generally unacceptable, because they diverge at the horizon logarithmically ($\chi(z) \sim \ln(1 - z)$, as $z \to 1$). If $\kappa = 0$, all solutions diverge at the horizon, attesting to the validity of the no-hair theorem. For $\kappa > 0$, there are pairs of black hole parameters $(M, Q)$ for which regular solutions exist in the entire interval $z \in (0, 1]$. In the two-dimensional parameter space, they form a one-dimensional curve, separating the region of stability from the region of instability of a Reissner-Nordström black hole.

Figure 2 depicts a sample profile of the scalar field $\phi_0$ obtained by numerically solving the wave equation as outlined above. We chose the scalar field parameters $m = 0.30$ and $q = 0.50$ A regular solution was found for the black hole parameters $M = 3.20$ and $Q = 1.05$. The scalar field is concentrated within $z \lesssim 0.2$, i.e., within about five times the horizon radius (in radial coordinate $r = \frac{\kappa}{z}$), beyond which it decays exponentially ($\phi_0 \sim e^{-\nu r}$, for $r \gtrsim 5\mu$).

Finally in figure 3 we plot critical lines for various values of the coupling constant $\kappa$ ($\kappa = 1000, 3000, 5000, 10000$, from top to bottom). Below each critical line, the Reissner-Nordström black hole is stable, whereas above it, an instability arises. We chose fixed values of the mass and charge of the scalar, $m = 1.0$, $q = 0.5$, respectively. Notice that as $\kappa$ decreases, the region of instability shrinks toward the extremal value $Q/M = 0.5$ (zero temperature). This is expected, because the wave equation has no regular solutions for small $\kappa$ (for $\kappa = 0$, such solutions are forbidden by the no-hair theorem).

### B. Next-to-leading order

Having obtained the zeroth-order solution to the field equations, we now turn to the derivation of the next-to-leading order corrections to the metric, the electrostatic potential, and the scalar field. Since
FIG. 2. The profile of the scalar field $\varphi_0$ outside the horizon ($0 < z \leq 1$) for coupling constant $\kappa = 1000$, and mass and charge, respectively, of the scalar field (black hole), $m = 0.30$, $q = 0.50$ ($M = 3.187$, $Q = 1.048$).

FIG. 3. Critical curves separating the region of instability (above each curve) from the region of stability (below the curve) for coupling constant (top to bottom) $\kappa = 1000$, 3000, 5000, 10000, and mass and charge of the scalar field $m = 1.0$, $q = 0.5$, respectively. The horizontal axis is $M$ and the vertical axis is $Q/M$, where $M$ and $Q$ are the mass and charge of the black hole, respectively.
the scalar field enters the Einstein-Maxwell equations quadratically (through the stress-energy tensor), it follows that the $O(\epsilon)$ corrections vanish ($\alpha_1 = l_1 = A_{l_1} = 0$). Therefore, the next-to-leading order corrections are given by the $O(\epsilon^2)$ terms.

From the sum of the $tt$ and $\theta\theta$ components of the Einstein equations, we obtain at $O(\epsilon^2)$,

$$l_2'' - \frac{1 - 5z^2}{z(1 - z^2)}l_2' + \frac{8z^2}{(1 - z^2)^2}l_2 = \mathcal{L},$$

(30)

where the right-hand side consists of zeroth-order functions which have been already calculated,

$$\mathcal{L} = -\kappa \left[ q^2 e^{3\alpha_0}l_0 A_{t_0}^2 \varphi_0 + z^3 e^{-\alpha_0} \left( -1 + \frac{z l_0'}{2l_0} \right) \varphi_0' \right] \varphi_0''$$

$$+ \kappa e^{-\alpha_0} \left[ -q^2 e^{2\alpha_0}l_0 A_{t_0}^2 + \frac{z^4 l_0''}{8l_0} + \frac{z^2}{4} \left( 4 - 2z\alpha_0' + z^2 \alpha_0'^2 \right) + \frac{z^3}{4} \left( -1 + z\alpha_0' \frac{l_0'}{l_0} - \frac{l_0''}{l_0} \right) \varphi_0' \right] \varphi_0''$$

$$- q^2 \kappa e^{\alpha_0}l_0 A_{t_0} \varphi_0 \varphi_0' \left[ 4A_{t_0}' + A_{t_0} \left( \frac{1}{z} + 2\alpha_0' + \frac{l_0'}{l_0} \right) \right]$$

$$+ \frac{e^{\alpha_0} l_0''}{z^4} \left[ -m^2 + q^2 e^{\alpha_0} A_{t_0}^2 \right] \varphi_0^2 - \frac{q^2 \kappa e^{\alpha_0} l_0}{8z} \varphi_0^2 \left[ 8z A_{t_0}^2 \right]$$

$$+ 8A_{t_0} \left( \frac{l_0'}{l_0} A_{t_0}' + (-1 + 2z\alpha_0')A_{t_0}' + zA_{t_0}'' \right)$$

$$+ A_{t_0}^2 \left( -z \frac{l_0''}{l_0} + 2(-1 + 2z\alpha_0') \frac{l_0'}{l_0} + 2z \frac{l_0''}{l_0} - 4\alpha_0' + 4z\alpha_0'^2 + 4z\alpha_0'' \right).$$

(31)

Notice that $\mathcal{L}$ vanishes both at the horizon and at infinity (at $z = 0, 1$). This equation can be solved for $l_2$ analytically. We obtain

$$l_2(z) = -z^2(1 - z^2) \int_1^z dw \frac{\mathcal{L}(w)}{2w} - (1 - z^2)^2 \int_0^z dw \frac{w\mathcal{L}(w)}{2(1 - w^2)},$$

(32)

where we fixed the integration constants by demanding $l_2(0) = l_2(1) = 0$ ($l$ must have a double zero at the horizon, $z = 1$). It is plotted in fig. [I] for the same values of the various parameters that were used for the plot of the zeroth-order scalar field (fig. [2]).

Next, we look at the $tt$ component of the Einstein equations together with Gauss’s Law. At $O(\epsilon^2)$, they form a system of coupled linear equations to be solved for $\{\alpha_2, A_{t_2}\}$,

$$\alpha_2'' + \frac{l_0'}{2l_0} \alpha_2' - \left[ \alpha_0'' + \frac{l_0'}{2l_0} \alpha_0' \right] \alpha_2 + e^{\alpha_0} A_{t_0} A_{t_2}' = \mathcal{P},$$

(33)

$$A_{t_2}'' + \left[ \frac{l_0'}{2l_0} + \alpha_0' \right] A_{t_2}' + A_{t_0} A_{t_2}' = \mathcal{Q},$$

(34)
where the right-hand sides consist of terms dependent on zeroth-order functions only,

\[ P = \frac{\kappa}{4\mu^2 l_0} 2^3 e^{-\alpha_0} \left[ \frac{z l_0}{l_0} - 2 + z \alpha_0 \right] \varphi_0 + \frac{1}{8} \varphi_0'' + \frac{e^{\alpha_0} l_0}{8 z^2} \left[ m^2 + e^{\alpha_0} q^2 A_0 \right] \varphi_0^2 \]

\[ - \frac{\kappa z^2 e^{-\alpha_0}}{32 \mu^2 l_0} \left[ 7 z^2 l_0^2 + 6 z l_0 \right] \left( -4 + z \alpha'_0 \right) - \frac{4 z^2 l_0''}{l_0} + 24 - 24 z \alpha_0' + 3 z^2 \alpha_0'' - 4 z^2 \alpha_0'' \varphi_0'' \]

\[ - \frac{q^2 e^{\alpha_0} A_0^2}{32} \left[-3 \varphi_0^2 \right] + \frac{q^2 \kappa A_0^2}{8} \left[-3 \varphi_0^2 \right] + \frac{4 l_0'}{l_0} + \alpha''_0 + 4 \alpha_0'' \varphi_0'' \]

\[ + \frac{l_2}{l_0} \left[-9 \frac{l_0'}{l_0} + 12 \frac{l_0'}{l_0} + 8 \frac{l_0''}{l_0} \right], \]

\[ Q = - \frac{q^2 A_0^2 e^{\alpha_0} l_0}{2z^4} \varphi_0^2 + \frac{q^2 \kappa A_0^2}{8} \left[ -3 \varphi_0^2 \right] + \frac{4 l_0'}{l_0} + \alpha''_0 + 4 \alpha_0'' \varphi_0'' \]

\[ - \frac{A_0 l_0'}{2} + \frac{l_2}{l_0} \left[ A_0 \left( \frac{l_0'}{l_0} + \alpha' \right) + A_0'' \right]. \]  

Equation \( (34) \) is of first-order in \( A_0' \), and yields

\[ A_1 z = \frac{4(1 - z^2) \sinh B}{\left[ (1 + z^2) \sinh B + 2z \cosh B \right]^2} \times \left[ 4 \sinh B 02(z) + C + \int_0^2 \frac{[1 + w^2] \sinh B + 2w \cosh B Q(w)}{1 - w^2} \right], \]

where \( C \) is an arbitrary integration constant. The correction to the electrostatic potential is deduced by integrating and using \( A_1(1) = 0 \),

\[ A_1(z) = - \int_1^2 dw A_1(w). \]  

It provides a correction to the charge \( Q \) of the black hole,

\[ \delta Q = - e^2 A_1(0) = - \frac{4 e^2 C}{\sinh B} = - C e^2 Q \mu, \]

where we used \( \alpha_2(0) = 0 \).

Then using \( (36) \), equation \( (33) \) becomes a second-order equation in \( \alpha_2 \),

\[ \alpha_2'' = \frac{2z}{1 - z^2} \alpha_2' - \frac{8}{\left[ (1 + z^2) \sinh B + 2z \cosh B \right]^2} \alpha_2 = P', \]

where once again the right-hand side is a zeroth-order (known) function,

\[ P' = P + \frac{16 \sinh B}{\left[ (1 + z^2) \sinh B + 2z \cosh B \right]^2} \left[ C + \int_0^2 \frac{[1 + w^2] \sinh B + 2w \cosh B Q(w)}{1 - w^2} \right], \]

to be solved for \( \alpha_2 \) subject to the boundary conditions \( \alpha_2(0) = 0 \) and \( \alpha_2(1) < \infty \). This equation can be solved numerically.

The solution thus obtained involves an arbitrary parameter \( C \). This parameter will be fixed by solving the scalar equation at \( O(\epsilon^2) \). The latter reads

\[ \frac{z^4 e^{-\alpha_0}}{l_0^3/2} \left( \sqrt{l_0 A_0} \varphi_0' \right) - \mu^2 m_{\text{eff,0}}^2 \varphi_0 = \frac{z^4 e^{-\alpha_0} l_0^3}{2 l_0^3/2} \left( \sqrt{l_0 A_0} \varphi_0' \right) - \frac{z^4 e^{-\alpha_0}}{l_0^3/2} \left( \frac{1}{\sqrt{l_0}} \left( - \frac{\kappa z^3 e^{-\alpha_0}}{\mu^2} A_1 + \frac{1}{2} A_0 \right) \varphi_0 \right) + \mu^2 m_{\text{eff,1}}^2 \varphi_0, \]  

(41)
where the right-hand side consists of known functions (zeroth-order as well as next-to leading order that we have already calculated),

\[ A_1 = \frac{l_2}{l_0^2} \left[ -1 + \frac{z l_0^0}{2 l_0} + \frac{l_2}{4 l_0} \left[ 8 l_0^0 - 3 z l_0^{2/2} - z \alpha_0^2 \right] + \frac{z}{2} \alpha_0^0 \alpha_2^0 + \alpha_2 \left[ \frac{\nu^0}{l_0} - \frac{z l_0^{2/2}}{4 l_0} + \frac{z}{4} \alpha_0^2 \right] \right], \]

\[ m_{\text{eff},1}^2 = -m^2 \left[ \frac{l_2}{l_0} + \alpha_2 \right] + q^2 A_{10} e^{\alpha_0} \left[ 2 A_{12} + \frac{l_2}{l_0} A_{20} \right] \]

\[ + \frac{\kappa}{4 \mu^2} q^2 z \frac{z}{l_0} A_{10} A_{20} \left[ -3 l_0^0 - 2 \alpha_0^0 l_0 + \frac{4 l_0^0}{l_0} + \alpha_0^2 + 4 \alpha_0^2 \right] \]

\[ + \frac{\kappa}{4 \mu^2} q^2 z \frac{z}{l_0} A_{10} A_{20} \left[ \alpha_2^0 + \frac{\alpha_0^2}{2} \left( l_0^0 + \alpha_0^0 \right) + \alpha_2^0 \left( -3 l_0^0 + 2 \alpha_0^0 l_0 + \frac{4 l_0^0}{ l_0} + \alpha_0^2 + 4 \alpha_0^2 \right) \right] \]

\[ + \frac{\kappa}{4 \mu^2} q^2 z \frac{z}{l_0} A_{10} A_{20} \left[ \alpha_2^0 + \frac{\alpha_0^2}{2} \left( l_0^0 + \alpha_0^0 \right) + \alpha_2^0 \left( -3 l_0^0 + 2 \alpha_0^0 l_0 + \frac{4 l_0^0}{l_0} + \alpha_0^2 + 4 \alpha_0^2 \right) \right] \] \quad \text{(42)}

After multiplying both sides of (41) by \( e^{\alpha_0} \nu^{1/2} \varphi_0 \), and integrating over the interval \([0, 1]\), the left-hand side vanishes on account of (11) and (25). We deduce

\[ 0 = \int_0^1 dz \left( 1 - z^2 \right) \left[ \left( \frac{l_2}{2 l_0} \right)^2 A_{10} \varphi_0 \varphi_0^0 - \frac{\kappa e^{-\alpha_0}}{\mu^2 l_0^2} A_{10} \varphi^2_0 - \mu^2 e^{\alpha_0} l_0^2 \varphi^2_0 \right]. \] \quad \text{(43)}

Despite appearances, this is a simple linear algebraic equation in the unknown parameter \( C \). It is easily solved analytically for \( C \).

For the choice of parameters in figure \[2\], we obtain from (43), \( C = 8.0 \).

Using this value, we can then solve (39) for \( \alpha_2 \) numerically. Using the boundary condition \( \alpha_2(0) = 0 \) and fixing \( \alpha_2'(0) \) arbitrarily, we obtain a solution which, in general, diverges logarithmically at the horizon. By varying the value of \( \alpha_2'(0) \), we pinpoint the regular solution. It is depicted in figure \[5\].

Next, we use the numerical solution for \( \alpha_2 \) to determine the correction to the electrostatic potential \( A_{12} \). The result is depicted in figure \[6\].

This completes the determination of all metric functions as well as the electrostatic potential at next-to-leading order.

The resulting hairy black hole has the following global properties. Its mass is

\[ M = -2 \mu \alpha'(0) = 8 \mu \coth B - 2 \mu e^2 \alpha_0'(0) + O(e^4), \] \quad \text{(44)}
FIG. 5. Next-to-leading-order metric function $\alpha_2(z)$ for choice of parameters as in figure 2. In addition we have chosen $\alpha'_2(0) = -0.0000105$.

FIG. 6. Next-to-leading-order metric function $A_{t2}(z)$ for choice of parameters as in figure 2.

where $\alpha'_2(0)$ is determined from the solution of Eq. (39), its charge is

$$Q = -\mu A'_t(0) = \left[1 - C \epsilon^2 \right] \frac{4\mu}{\sinh B} + O(\epsilon^4),$$

where $C$ is determined from Eq. (43), and its temperature can be found from (10) to be

$$\frac{T}{T_0} = 1 - \epsilon^2 \tau + O(\epsilon^4), \quad \tau = \left. \left( \alpha_2 + \frac{l_2}{l_0} - \frac{\rho'_2}{2\rho_0} \right) \right|_{z=1} = \alpha_2(1) + \int_0^1 dz \frac{z\mathcal{L}(z)}{4(1-z^2)},$$

where $T_0$ is given by the zeroth-order approximation (24), $\alpha_2(1)$ is determined from the solution of Eq. (39), and $\mathcal{L}$ is defined in (31).

IV. CONCLUSION

We considered a gravitating system without a cosmological constant term (therefore living in asymptotically flat space) consisting of an electromagnetic field, and a massive charged scalar field which had a
derivative coupling to the Einstein tensor of strength $\kappa$. For small values of the scalar field we solved the coupled system of Einstein-Maxwell-scalar field equations perturbatively.

We found that for sufficiently large values of the coupling constant $\kappa$, an Abelian $U(1)$ gauge symmetry was broken in the vicinity of the horizon of the black hole, leading to a new black hole configuration with isotropic hair. A non-vanishing $\kappa$ allowed us to evade the no-hair theorem. The scale introduced by the dimensionful coupling constant $\kappa$ acted similarly to the scale of the negative cosmological constant in AdS space, producing a potential well that could trap scalar particles near the horizon. Unlike in AdS space, which yields a power law asymptotic behavior for the scalar field, in our case we showed that the scalar field decayed exponentially away from the horizon.

It would be interesting to calculate correlation functions in this background and explore the possibility of a correspondence to a gauge theory, similar to the gauge theory / gravity duality in AdS space. It would also be interesting to derive a full numerical solution of the field equations down to zero temperature, which would enable us to probe the ground state of the system. Work in this direction is in progress.

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