Late gas accretion on to primordial minihaloes: a model for Leo T, dark galaxies and extragalactic high-velocity clouds

Massimo Ricotti*

Department of Astronomy, University of Maryland, College Park, MD 20742, USA

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ABSTRACT
In this Letter, we revisit the idea of reionization feedback on dwarf galaxy formation. We show that primordial minihaloes with \( v_{\text{vir}} < 20 \text{ km s}^{-1} \) stop accreting gas after reionization, as it is usually assumed, but in virtue of their increasing concentration and the decreasing temperature of the intergalactic medium as redshift decreases below \( z = 3 \), they have a late phase of gas accretion and possibly star formation. We expect that pre-reionization fossils that evolved on the outskirts of the Milky Way or in isolation show a bimodal star formation history with 12- and <10-Gyr old population of stars. Leo T fits with this scenario. Another prediction of the model is the possible existence of a population of gas-rich minihaloes that never formed stars. More work is needed to understand whether a subset of compact high-velocity clouds can be identified as such objects or whether an undiscovered population of dark galaxies exists in the voids between galaxies.

Key words: galaxies: formation – cosmology: theory.

1 INTRODUCTION
It is widely assumed that reionization of the intergalactic medium (IGM) suppresses gas accretion and galaxy formation in small mass haloes with circular velocity \( v_{\text{cir}} \lesssim 20 \text{ km s}^{-1} \), corresponding to a virial temperature \( T_{\text{vir}} \lesssim 20000 \text{ K} \) and dark halo masses \( M \lesssim 10^6 \text{ M}_\odot \) (e.g. Babul & Rees 1992; Efstathiou 1992). Hereafter, we will refer to these small haloes affected by reionization feedback as ‘minihaloes’. The reason why gas accretion and star formation are suppressed in minihaloes is that, after reionization, the Jeans mass in the IGM exceeds the mass of these minihaloes and the IGM gas is unable to condense under the influence of their gravitational potential. Simulations of galaxy formation have confirmed this idea (e.g. Bullock, Kravtsov & Weinberg 2000; Gnedin 2000). However, a recently discovered dwarf galaxy – Leo T (Ryan-Weber et al. 2008) – despite having an estimated mass of about \( 7 \times 10^6 \text{ M}_\odot \) and having properties typical of other ultrafaint dwarf spheroidal (dSph) galaxies, contains gas and actively forming stars at the present day. This puzzling observation prompts taking a fresh look at the problem of gas accretion on to minihaloes after reionization. However, the model presented in this study stands independently of Leo T.

We show that the ability of dark minihaloes to accrete gas from the IGM depends not only on the ratio of their circular velocity to the IGM sound speed, but also on their concentration, \( c \). Typically, the concentration of a halo is \( c \sim 5 \) at the redshift of virialization but as the halo evolves in the expanding Universe, its concentration increases. The evolution of the halo concentration with redshift can be understood in the context of the theory of cosmological secondary infall of dark matter (Bertschinger 1985) and has been quantified using N-body simulations (Bullock et al. 2001; Wechsler et al. 2002). In this Letter, we show that primordial minihaloes with \( v_{\text{vir}} < 20 \text{ km s}^{-1} \) stop accreting gas after reionization, as expected, but in virtue of their increasing concentration and the decreasing temperature of the IGM at \( z < 3 \), they start accreting gas from the IGM at later times. As a result, we expect that pre-reionization fossils (Ricotti & Gnedin 2005, hereafter RG05) in the local group have a more complex star formation history than previously envisioned. A signature of this model is a bimodal star formation history with an old (\(~12 \text{ Gyr}\)) and a younger (\(<5–10 \text{ Gyr}\)) depending on the minihalo mass) population of stars. Another prediction of the model is the possible existence of minihaloes containing only gas but no stars. This would revive the suggestion that some compact high-velocity clouds (CHVCs) are extragalactic objects (Blitz et al. 1999). However, the bulk of these objects may be yet undiscovered due to their faint 21 cm and \( H\alpha \) fluxes. The ongoing survey ALFALFA (The Arecibo Legacy Fast ALFA Survey; Giovanelli et al. 2005, 2007) may be able to discover this new population of extragalactic objects. This Letter is structured as follows. In Sections 2 and 3, we describe our model for dark matter and gas accretion and in Section 4 we apply the model to the interpretation of Leo T and observations of CHVCs. Throughout the rest of the Letter, we use the following cosmological parameters (\( h = 0.72, \Omega_{\text{dm}} = 0.214, \Omega_k = 0.0438, \sigma_8 = 0.796, n_s = 0.963 \)) from Wilkinson Microwave Anisotropy Probe 5 (WMAP5) (Dunkley et al. 2008).

2 EVOLUTION OF THE DARK MATTER CONCENTRATION
N-body simulations show that the concentration of dark haloes is, on average, \( c(z) \propto (1+z)^{-1} \) (Bullock et al. 2001; Wechsler

*E-mail: ricotti@astro.umd.edu
et al. 2002). Assuming a universal NFW density profile, the halo concentration at virialization, \( c_{\text{vir}} \), has a weak dependence on the halo mass (\( \propto M^{1/6} \)). If, instead, the halo profile has an inner slope that depends on the haloes mass (Ricotti 2003; Ricotti & Wilkinson 2004; Ricotti, Pontzen & Viel 2007), \( c_{\text{vir}} \) is a universal constant. Either way, although with some scatter and a dependence on the environment, the mean concentration of minihaloes is \( c(z) = c_{\text{vir}}(1 + z)/\langle 1 + z \rangle \), where \( c_{\text{vir}} \sim 5 \) and \( z_{\text{vir}} \) depends on \( v_{\text{vir}} \) and the cosmology. Using 5-yr WMAP cosmological parameters, we have

\[
v_{\text{vir}} \sim (17 \text{ km s}^{-1}) \left( \frac{M(\text{halo})}{10^8 M_\odot} \right)^{1/3} \left( \frac{1 + z_{\text{vir}}}{10} \right)^{1/2},
\]

where \( \sigma_8 \) is the variance of density perturbations at \( z = 0 \) in a sphere of 8 Mpc radius, and \( n \) is a number that expresses how rare is the initial density perturbation that produces the minihalo (i.e., the number, \( n_{\sigma} \), of standard deviations from the mean). A halo virialized from the collapse of an \( n_{\sigma} \) perturbation has a typical value of the concentration \( c(z, M) \sim 50 (n/3)(1 - \log \left( \frac{M(\text{halo})}{10^8 M_\odot} \right)^{-1/5})/\langle 1 + z \rangle \). In the rest of this Letter, we will use this equation as a rough estimate for the concentrations of minihaloes as a function of their mass and redshift. This expression does not take into account the dependences of the concentration on the local overdensity, but, qualitatively, the model is independent of the assumed value of the concentration.

Body simulations also show that present-day isolated dwarf haloes have a high median concentration of \( c \sim 35 \) (e.g., Colin et al. 2004), in agreement with the extrapolation of the fitting formula \( c(z) \propto (1 + z)/\langle 1 + z \rangle \), derived for more massive haloes (e.g., Bullock et al. 2001). However, high-resolution simulations able to resolve the smallest minihaloes (that are expected to have the highest concentrations) typically focus on Milky Way type systems and only a few focus on isolated minihaloes in the voids.

In summary, minihaloes that form at high redshift and do not merge into bigger haloes until redshift \( z < 1-2 \), are likely to achieve a large concentration and, as a result, have a late phase of gas accretion from the IGM. The evolution in relative isolation of the minihalo ensures that it is not tidally truncated and it does not evolve in the warm-hot intergalactic medium, a phase of the IGM that has been shock heated to millions of degrees and believed to be the reservoir of most of the baryons at \( z = 0 \). The model has observable effects on any minihalo surviving ‘undigested’ by a larger galaxy and, as a result, have a late phase of gas accretion. In particular, for relatively massive pre-ionization fossils the model predicts a bimodal star formation history. However, in very small mass haloes significant effects are only observed in the rare high-\( \sigma \) perturbations. The model that fits Leo T uses a 3\( \sigma \) perturbation. Future work will focus on estimating how many objects like Leo T may exist in the local group and explore whether and how many minihaloes smaller than Leo T could be detected as extragalactic CHVCs around the Milky Way.

### 3 Cold Gas Accretion on to Primordial Minihaloes

For a given value of the parameter \( \Gamma = (v_{\text{vir}}/c_{\text{igm}})^2 \equiv T_{\text{vir}}/T_{\text{igm}} \), the density profile of the gas in a minihalo depends on its concentration and on the ability of the gas to recombine and cool. Fig. 1 shows the gas overdensity profile for an isothermal (solid curves) and an adiabatic (dotted curves) gas in hydrostatic equilibrium in NFW haloes with different values of the concentration (the derivation of the equations for the gas density profile is shown in the appendix). If the gas condenses in the minihalo adiabatically, the overdensity in the core does not exceed 100. However, if the gas condenses isothermally, the overdensity can reach values \( \sim 10^7 \), corresponding to gas densities of \( 1-10 \text{ cm}^{-3} \) at \( z = 0 \).

If the cooling/heating time of the gas inside the minihalo is longer than the Hubble time \( t_H \) (i.e. the time-scale for evolution of the minihalo concentration and gravitational potential), the gas is compressed and heated adiabatically (i.e. \( t_{\text{cool}} > t_{\text{heat}} > t_H \)). As shown in Fig. 2, after He II reionization at \( z \approx 3 \), due to the Hubble expansion, the mean temperature of the IGM decreases almost adiabatically because in the low-density IGM the heating time is \( t_{\text{heat}} \). Similarly to the IGM, during the initial phase of gas accretion on to a minihalo, the gas is compressed adiabatically and the pressure prevents the gas density in the halo core to increase substantially above the mean IGM density. However, if the core overdensity becomes greater than the Hubble time \( t_H \) could be detected as extragalactic CHVCs around the Milky Way.
be observed at 21 cm and HCHVCs. The isothermal density profile is well approximated by a
recombination lines if the gas is highly ionized by UV radiation
nearly isothermal. The gas over-density in the core of a minihalo can reach the large values shown in Fig. 1 by the solid curves for an isothermal gas.

The cooling time for a gas at $T \gtrsim 10^4$ K is always $t_{cool} < t_{rec}$, independently of the assumed ionization fraction, temperature and density of the gas. The cooling is due to hydrogen and helium recombination lines if the gas is highly ionized by UV radiation ($x_H \gtrsim 10^{-3}$), or hydrogen and helium Lyman $\alpha$ cooling. Thus, if the gas can recombine in a Hubble time, is also able to cool efficiently. Fig. 3 shows the evolution of the core gas density (top panel) and the ratio $t_{rec}/t_{H}$ (top panel) in minihaloes of circular velocity $v_{cir}$ for an isothermal (thick lines) and adiabatic (thin lines) gas. Only if $t_{rec}/t_{H} < 1$, the gas is able to collapse isothermally (thick lines) and condense substantially in the minihalo core.

The recombination time is $t_{rec} = (n_{H,core} \alpha(T))^{-1}$, where $\alpha(T)$ is the hydrogen recombination rate. Thus, at $z = 0$, we have $t_{rec}/t_{H} < 1$ if $n_{H,core} < 10^{-4}$ cm$^{-3}$. Using equation (A1), we also have $t_{rec}/t_{H} < 1$ if $\beta(z) < 1.5 [(14/(1+z) - 1]$. Writing $\beta$ as a function of $v_{vir}$, we find that minihaloes with $v_{vir} > v_{crit}$ condense isothermally. At redshift $z = 0$, $v_{crit} \approx c_{HII}^{0.5}(19.5/(4.4 + c/4))$. These minihaloes can be observed at 21 cm and Hz wavelengths and would resemble faint CHVCs. The isothermal density profile is well approximated by a $\beta$-model (see Fig. 1, above) with core radius $r_{core} \approx 0.2 r_{vir}$, where $r_{vir}$ is the virial radius at formation. After virialization, by definition, the halo radius at redshift $z < z_{vir}$ is $r_{halo}(z) \equiv r_{vir}(c/c_{vir})$.

The typical size of the gas core is

$$r_{core} \approx 140 pc \left( \frac{v_{vir}}{17 \ km \ s^{-1}} \right) \left( \frac{10}{1 + z_{vir}} \right)^{1.5},$$

although, for small values of the core overdensity, most of the gas extends out to $r_{halo} \approx 18 \ kpc \ (M/10^8 \ M_\odot)^{1/3}/(1+z)$. The circular velocity, $v_{vir}$ at $r_{core}$ is

$$v_{vir}(r_{core}) \approx 0.66 v_{vir}(r_{vir}) \approx 0.624 v_{vir}^{max},$$

where $v_{vir}^{max}$ is the maximum circular velocity.

Small minihaloes that can accrete gas from the IGM may not be able to form stars because the gas cannot cool below $T \sim 10^4$ K if the gas has low metallicity or is metal free. These haloes may be stellar less as CHVCs or may be pre-reionization fossils with a 12-Gyr old stellar population, having gas but no young stars. In order to form stars, the gas in the minihalo must cool below 10$^4$ K and develop a multi-phase ISM. If the gas is metal free, H$_2$ formation and cooling is unlikely to be large enough to support star formation at $z = 0$. This is because, in a gas of primordial composition, the absence of dust grains that typically are the main catalyst for H$_2$ formation, requires that H$_2$ forms though a chemical reaction that is very slow and that involves the ion H$^-$ as a catalyst. At $z = 0$, the flux of the H$_2$ photo-dissociating background (with energies 11.3 $< h\nu < 13.6$ eV) that destroys very efficiently H$_2$, is expected to be much larger than during the dark ages at $z \sim 30$. Positive feedback processes that may be important at high $z$ in increasing the formation rate of H$^-$ and H$_2$ are absent at $z = 0$ (Ricotti, Gnedin & Shull 2002). Thus, it is unlikely that dark minihaloes that were never able to form stars will be able to do that at $z = 0$ for the first time. However, due to the interesting possibility of Pop III formation at $z < 1$, this scenario should be explored in more detail in order to assess quantitatively what is the probability of Pop III star formation in isolated minihaloes at $z = 0$.

Let us now estimate which level of metal pre-enrichment is necessary for star formation in gas-rich minihaloes. The cooling function from hyperfine transitions of oxygen and carbon depends on the gas metallicity, $Z$, roughly as $\Delta_3 \sim 10^{-3} (Z/Z_\odot)$, where $\Delta_3 \sim 10^{-23}$ erg s$^{-1}$ cm$^{-3}$. Thus, a necessary condition for star formation is $t_{cool} \approx (0.7 \ yr) T/(n_{H,core} \Delta_3) < t_{H}$, that can be written as $n_{H,core} > n_{H,core}^{max} \approx 0.03$ cm$^{-3} (Z/10^{-2} Z_\odot)^{-1}$. The left-hand panel in Fig. 4 shows $n_{H,core}$ and $N_H$ in minihaloes that evolve isothermally at $T \sim 10^4$ K but that do not form stars (i.e. candidates for extragalactic CHVCs). The horizontal lines show the requirement for metal cooling and star formation assuming gas metallicity $Z = 0.1$ and 0.01 $Z_\odot$. The right-hand panel in Fig. 4 shows $n_{H,core}$, $N_H$ and $M_{H_2}/M_{gas}$ (the dynamical mass to gas-mass ratio) in the core of minihaloes that are able to cool to $T = 5000$ K (the temperature of Leo T ISM), thus, able to form stars (i.e. $n_{H,core} > n_{H,core}^{max}$).

The symbols show the observed value for Leo T. For $v_{vir}(r_{vir}) = 7$ km s$^{-1}$, corresponding to Leo T circular velocity, the model gives a core radius $r_c \sim 80$ pc, that is slightly smaller than 100 pc measured for Leo T. The dark matter and gas mass within the core are $4 \times 10^6$ and $2 \times 10^6 M_\odot$, respectively, in reasonable agreement with observations of Leo T.

4 CONCLUSIONS AND DISCUSSION

Although Leo T contains gas, its properties are as similar to those of other ultrafaint dSph galaxies and quite different from those of more massive dwarf irregular galaxies (Bovill & Ricotti 2008). Its properties are consistent with those of pre-ionization fossils but, in the context of this model, it is difficult to understand how it could hold on to its gas.

We point out that dark minihaloes with large concentration are able to accrete gas from the IGM more efficiently than lower concentration haloes of the same mass. The concentration of a halo depends on the redshift of virialization as $c(z) \propto (1+z_{vir}^3)/(1+z)$. Thus, as a result of their growing concentration, early forming minihaloes,
although affected by reionization feedback, may have a late phase of cold gas accretion from the IGM and form stars (perhaps for the first time) at \( z < 1 \). This model explains all the observed properties of Leo T. Having a distance of 420 kpc from the Milky Way, Leo T is just starting to fall into the Milky Way Halo, thus it is likely to have evolved in isolation (fulfilling a requirement of the model) and unlikely to have been tidally stripped. Recent observations suggest that Leo T has a bimodal SFH (star formation history; de Jong et al. 2008) with a >12- and 9-Gyr population [with star formation (SF) continuing until few Myr ago], in agreement with our expectation of a late phase of accretion from the IGM. The bimodality of the SFH should be most pronounced in minihaloes with the lowest \( v_{\text{esc}} \), gradually disappearing as \( v_{\text{esc}} \) increases towards the 20 km s\(^{-1}\) threshold for pre-reionization fossils (RG05). Leo T has a gas velocity dispersion \( \sim 7 \) km s\(^{-1}\) that, according to our model, implies a dark halo with maximum circular velocity of 11 km s\(^{-1}\) (equation 4). Leo T has a core radius \( r_c \sim 100 \) pc, \( M_{\text{dyn}}/M_\odot \sim 8–10 \) (Simon & Geha 2007; Ryan-Weber et al. 2008) and maximum H\(_i\) column density \( N_{\text{H}} = 7 \times 10^{20} \) cm\(^{-2}\). These values are all in agreement with our model as shown in Section 3 and in the right-hand panel of Fig. 4. If future observations will show that the dark halo of Leo T has \( v_{\text{esc}} > 20 \) km s\(^{-1}\) or its SFH does not show any decline after 12 Gyr ago, our model for Leo T will be disproved.

Minihaloes smaller than the one hosting Leo T can also accrete gas from the IGM at \( z < 1 \), but may not be able to form stars. Sufficiently small minihaloes are likely to be dark and, if they evolved in isolation, the IGM around them would be nearly metal free. Cosmological simulations show that the fraction of primordial minihaloes that host a luminous galaxy decrease steeply with decreasing \( v_{\text{esc}} \); \( f(\text{lum}) \sim \text{min} [1, 50 \text{ per cent} (v_{\text{esc}}/10 \text{ km s}^{-1})^4]\) (Ricotti, Gnedin & Shull 2008). A fraction of dark minihaloes may be able to accrete gas from the IGM but, if the gas density in their core is below the threshold required for star formation, these objects will resemble CHVCs. Additional work is needed to determine whether our model is fully consistent with the properties of known CHVCs (Blitz et al. 1999; Braun & Burton 1999; Robishaw, Simon & Blitz 2002); however, the typical column densities we predict in Fig. 4 (left-hand panel) are in agreement with a subset of existing observations. The typical size and size distribution of CHVCs may agree with the model assuming a mean distance of minihaloes of 1 Mpc. We estimate that realistic extension of the diffuse gas in dark minihaloes is \( \lesssim 5–10 \) kpc. This may exclude several known CHVC and/or set an upper limit for their distance from the Milky Way. More extended extragalactic CHVCs are possible for minihalo masses \( M > 10^8 M_\odot \) that are likely to be luminous. In addition, this scenario for more massive minihaloes has already been investigated by previous studies and found not fully satisfactory (Sternberg, McKee & Wolfire 2002; Maloney & Putman 2003; Putman et al. 2003). More work in this area is also motivated by the ongoing survey ALFALFA (Giovanelli et al. 2005, 2007) that may be able to discover a even fainter population of extragalactic CHVCs. Hence, future work will focus on estimating the number, flux and size distribution of gas-rich minihaloes in the local volume and the local voids, producing synthetic maps for 21 cm and Hz wavelengths.

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Figure 4. Left-hand panel: the gas density, \( n_{g, \text{core}} \) (solid curves), and hydrogen column density, \( N_{\text{H}} = 2 \pi r n_{g, \text{core}} \) (dashed curves), within the core, \( r_c \), of a minihalo as a function its circular velocity at \( r_c \), at \( z = 0 \). The curves from the bottom to the top panel refer to 2, 3 and 4\( \sigma \) perturbations. The minimum \( v_{\text{esc}} \) in each curve is determined by the condition \( t_{\text{cool}}/t_{\text{HII}} < 1 \), necessary for cooling to \( T_{g, \text{core}} \sim 10^4 \) K. The horizontal lines with arrows show the requirement for cooling to temperatures below \( 10^4 \) K, necessary for initiate star formation, gas metallicity \( Z = 0.1 \) (lower line) and 0.01 \( Z_\odot \) (higher line). Right-hand panel: same as in the left-hand panel but for minihaloes forming from a 3\( \sigma \) perturbation and able to form stars due to a gas pre-enrichment to metallicity \( Z = 0.1 Z_\odot \) and reaching a gas temperature \( T_{g, \text{core}} = 5000 \) K. The thick curves and thin curves refer to haloes observed at redshift \( z = 0 \) and 2, respectively.
APPENDIX A: GAS DENSITY PROFILE IN A NFW POTENTIAL

In this appendix, we derive the equations describing the density profile of intergalactic gas inside a dark minihalo. Although, the IGM density, temperature and dark halo potential evolve with redshift, the assumption of quasi-hydrostatic equilibrium is a good approximation within the halo radius. Expressing the halo radius as \( r_{\text{halo}} = v_{\text{cir}} t_{\text{H}} / (4 \Delta_{\text{dm}})^{1/2} \), where \( \Delta_{\text{dm}} \sim 178 \) is the halo overdensity and \( t_{\text{H}} \) is the Hubble time, we find that \( t_{\text{cros}} < t_{\text{H}} \) (where \( t_{\text{cros}} = r_{\text{halo}} / v_{\text{cir}} \) is the halo crossing time) if \( \Gamma = v_{\text{cir}} / \sqrt{c_{\text{igm}}} > (4 \Delta_{\text{dm}}) \sim 800 \). In this Letter, we are interested in haloes with \( \Gamma \lesssim 1 \), thus \( t_{\text{cros}} < t_{\text{H}} \) and we can safely assume quasi-hydrostatic equilibrium in order to calculate the gas density profile. We assume a NFW density profile, \( \rho \propto 1/[c x (1+cx)^2] \), with \( x \equiv r / r_{\text{halo}} \), halo concentration \( c \equiv r_{\text{halo}} / r_s \), where \( r_s \) is the core radius of the dark matter profile. The overdensity profile of a gas with EOS \( P = K \rho \gamma \), in hydrostatic equilibrium in a NFW halo of mass \( M \), concentration \( c \), and circular velocity, \( v_{\text{cir}} \), is

\[
1 + \delta(x) = \frac{\rho}{\rho_b} = \left[ 1 + (\gamma - 1) \ln \left( 1 + \delta_{\text{iso}}^\beta \right) \right]^{1/(\gamma - 1)},
\]

where \( \rho_b \) is the mean IGM gas density and \( 1 + \delta_{\text{iso}}^\beta = (1 + cx)^\beta \) is the overdensity for \( \gamma = 1 \) (isothermal EOS). In equation (A1), we have normalized the density profile so that \( \delta_b = 0 \) at \( x \to \infty \). The parameter \( \beta = \Gamma c / f(c) \sim \Gamma (4.4 + 0.25c) \) determines the core overdensity (at \( x = 0 \)). The gas density profile has a core with overdensity \( 1 + \delta_{\text{iso}}^\beta = [1 + (\gamma - 1)\beta]^{1/(\gamma - 1)} \), that for \( \gamma = 1 \) is \( 1 + \delta_{\text{iso}}^\beta = e^\beta \). When \( \delta_{\text{iso}}^\beta \gtrsim 10^6 - 10^7 \), the normalization of the density profile becomes inaccurate because the mass inside the halo exceeds the cosmic value within the turnaround radius. In this regime, the density profile must be normalized by imposing that the total gas mass within the halo equals a given fraction of the dark halo mass (this is necessary because the calculation of the gas density profile does not take into account the Hubble flow and would overestimate the gas accretion from the IGM in this regime). The \( \beta \)-profile \( 1 + \delta(x) = \delta_b \left[ 1 + (5cx)^2 \right]^{-b/2} \), where \( b = 0.2 \beta / 3 \), provides a good fit to the isothermal density profile and has the advantage of being relatively easy to normalize to a constant mass \( M_{\text{gas}} = 4\pi / 3 \Delta_b \rho_b r_b^3 \), where \( \Delta_b \sim 5 \) is the mean gas overdensity inside the halo. The core density for the \( \beta \)-profile with the aforementioned normalization is \( \delta_b = \Delta_b (1 - b) \alpha_1 / (\alpha_1^{1-b} - b) \), with \( \alpha_1 = (c/0.2)^3 \).

We will use this expression in our calculations of the core density for \( \gamma = 1 \).

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