Theoretical Investigation of Spherical Aberration Coefficient Solenoid Magnetic Lens

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Abstract. In this study, a modulating model for solenoid magnetic field was presented. This model describes the on axis field through its hard edginess parameter as well as its axial half of half width at half peak field. The modulating model indicates to increasing the spherical aberration at the fields from real soft boundary to hard boundary modulating model. The investigation simply shows stay the peak field value constant at all order n, and the field sketch diverges to the hard boundary shape within high values of n > 60 and, as well the half of half width of field is (a) for all orders n > 6.

1.Introduction

In low energy sections of accelerators, as well as in other devices like electron microscope, charged particle beams are often focused by coil solenoid [1, 2]. The longitudinal magnetic field of coil solenoid has different densities. It peaks on the axis at the middle of the coil solenoid, declines towards the ends, and becomes approximately zero distant from coil solenoid. Radial magnetic field, in alternate, peaks at the ends of the solenoid coil. Solenoid magnetic lenses are usually used for transverse focusing of electron beams, but the aberration inevitably exists, which leads to the emittance growth and degrades performance in the system [3].

Based on a simple model, it can be assumed that the longitudinal magnetic field becomes as low as zero outside the solenoid, while it uniforms inside it. Charged particles hat transfers from the magnetic fieldless zone to a regular magnetic field, rotates with Larmour frequency. In a regular magnetic field, this frequency equalizes half of cyclotron frequency. It is easily to analyze the dynamics of rotating charged particles beam about the axis of coil solenoid in the Larmour frame. In this frame, the force of centrifuge will equal to 0.5 of Lorentz force, which implies that there is a net value of focusing force towards the solenoid axis. In fact, there is some ambiguity about why charged particles beam don’t revolve with the frequency of cyclotron within the coil solenoid, as a state in a regular magnetic field. Moreover, the focusing provided by a regular magnetic field is furthermore confounding because the path of helical trajectory behavior with a constant value of radius (Larmour radius) that assumed by charged particles beam in the regular magnetic field.

Electromagnetic Lenses without Poles We have designed a kind of electromagnetic lenses without poles. The coil is wound by a type of hollow wire which is square outside and the hollow inside is circular [3]. A mathematical field model for a coil solenoid of axial magnetic field (Bz) with the pivot along the axis (z) of coil solenoid which is given by the equation (1) [4].
\[ B_z(z) = \left( \frac{a^n}{\pi(2z - a)^n + a^n} \right) \]  

(1)

Where the symbol \( n \) larger than zero represents the value of order number, and \( a \) is the optimization parameter which represents the axial dimension of half half-width for any value of order \( n \) in this model. For the values of \( n \) are much greater than one, the model formulates to a hard boundary shape about value of \( z = \pm a \) [2]. From the suggested model equation, i.e., equation (1), it is noted that the distribution of the magnetic field will be shifted from the symmetry plan \( e \) (i.e., shifted away of \( z \) equal zero) by a distance approximately equivalent to half of half-width \( a \), this shift in field makes this distribution is asymmetric about the plane of symmetry. Therefore, some axial transformations have been proceeding to make this distribution symmetric about the symmetry plane \( z = 0 \) [4].

The off axis axial or radial fields can be calculated using potential-like extended chain of derivatives (either even or odd) of the equation (1) specified for on axis magnetic field as equations (2), and (3) by holding the first two expressions [5].

\[ B_z(r, z) = B_z(0, z) - \frac{r^2}{4} B_z^2(0, z) + \frac{r^4}{64} B_z^4(0, z) - \ldots \]

(2)

\[ B_z(r, z) = \left( \frac{a^n}{\pi(2z - a)^n + a^n} \right) - \left( \frac{2n^2a^n r^2 (2z - a)^2n - 2}{\pi((2z - a)^n + a^n)^3} - \frac{na^2r^2(n - 1)(2z - a)^{n - 2}}{\pi((2z - a)^n + a^n)^2} \right) \]

(2)

\[ B_r(r, z) = -\frac{r}{2} B_z'(z) + \frac{3}{16} B_z^3(z) + \ldots \]

\[ B_r(r, z) = \left( \frac{nra^n(2z - a)^{n - 1}}{\pi((2z - a)^n + a^n)^2} - \frac{3n^3a^n r^3 (2z - a)^3n - 3}{\pi((2z - a)^n + a^n)^4} + \frac{2a^n r^2 n^2 r^2 (n - 1)(2z - a)^2n - 3}{2\pi((2z - a)^n + a^n)^3} \right) \]

(3)

\[ \frac{na^2r^2(n - 1)(2z - a)^{n - 3}}{2\pi((2z - a)^n + a^n)^2} \]

(3)

Where \( r \) represents the radial distance obtained from the axis of coil solenoid, while the prime indicates a derivative regarding to \( z \). The spherical aberration \( C_s \) expresses the measure for an increment of image radius \( (r) \) at the plane of image for certain \( r_o \) radius of the object, given by \( r_i = C_s r_o^3 \). For a coil solenoid having on-axis field \( B_z \) with field gradient \( B_z' \), in the paraxial limit, the spherical aberration coefficient \( C_s \) is assigned by equation (4) as in [6]. Therefore, (ideally \( C_s = 0 \), when the image radius \( r_i = 0 \) will be at the focus;

\[ C_s = \frac{1}{2} \int_{-\infty}^{+\infty} B_z^2 \text{d}z \left/ \int_{-\infty}^{+\infty} B_z^2 \text{d}z \right. = \frac{N}{D} \]  

(4)

From the physical point of view, the \( C_s \) entails that an off-axis particle incident at a certain dimension \( r \) away from axis of coil solenoid, the focal length \( f \) fractional reduction will be \( (C_s f)^2 \) due to third order spherical aberration [5]. In equation (4), the factor \( 0.5 \) was absorbed in the Numerator-\( N \) in order that the Denominator-\( D \) represent the focusing power \( (P) \) (which inversely mentioned to \( f \)) [5]. To find universal formula of the spherical aberration (\( C_s \)) for the
modulating field model, was substituted Bz(z) in equation (1) with (n=2) in equation (4) which indicate in N-term, and D-term of equations (5) and (6) as follows;

\[
N = \frac{1}{2} \left[ \frac{\pi}{2} \int_{-\infty}^{+\infty} B_z^2 \, dz = \frac{8a^4}{\pi^2} \int_{-\infty}^{+\infty} \frac{(2z-a)^2 \, dz}{((2z-a)^2 + a^2)^4} \right]
\]

\[
N = \frac{1}{4\pi^2a} \left[ \arctan \left( \frac{2z-a}{a} \right) + \frac{2a^2(2z-a)}{4\pi^2(2z-a)^2 + a^2} + \frac{a^2(2z-a)}{6\pi^2((2z-a)^2 + a^2)^2} + \frac{2a^4(2z-a)}{3\pi^2((2z-a)^2 + a^2)^3} \right]_{-\infty}^{+\infty}
\]

\[
N = \frac{1}{4\pi a} \quad \text{(5)}
\]

\[
D = \frac{a^4}{\pi^2} \left[ \frac{dz}{2} \frac{2z-a}{(2z-a)^2 + a^2} \right]_{-\infty}^{+\infty}
\]

\[
D = \frac{a}{4\pi^2} \left[ \arctan \left( \frac{2z-a}{a} \right) + \frac{a^2(2z-a)}{4\pi^2((2z-a)^2 + a^2)^2} \right]_{-\infty}^{+\infty}
\]

\[
D = \frac{a}{4\pi} \quad \text{(6)}
\]

Their ratio between N-term, and D-term is equivalent to spherical aberration coefficient for mathematical model field of the lens at n = 2 as illustrate in equation (7).

\[
C_{s,n=2} = \left| \frac{N}{D} \right|_{-\infty}^{+\infty} = \frac{1}{a^2} = \frac{(2+1)}{3a^2}
\]

\[
C_{s,n} = \frac{(n+1)}{3a^2} \quad \text{(7)}
\]

The approximation of thin-lens can also simplify the analysis of the particle movement of charge (e), and mass (m), enabling compact expression for P of the magnetic lens which is defined in equation (8) [7].

\[
p = \frac{1}{f}
\]

\[
p = \frac{e^2}{8mE} \left[ \frac{\pi}{2} \int_{-\infty}^{+\infty} B_z^2 \, dz = \frac{e^2}{8mE} \right]
\]

\[
p = \frac{ae^2}{32\pi mE} \quad \text{(9)}
\]
Where $E$ is a kinetic energy of electrons that passes through lens. This energy is measured in Joule unit and given by the form $E = (e)(V_r)$, as $V_r$ accounts for the potential difference employed for the acceleration of an electrons from rest. When explaining the magnetic field action, it obvious that electrons which passes through a lens exert a spiral movement. Accordingly, the plane involving the exit ray will be rotated about an angle ($\Theta$) in relation to the plane including the incoming electrons [7, 8]. Thus, by still making a thin lens approximation ($a$ is much less than $f$), and substituting field model equation (1) into $\Theta$ formula will introduce in the following equation (10), it can be find rotation angle will illustrate in equation (11).

$$\theta = \frac{e}{\sqrt{8mE}} \int_{-\infty}^{+\infty} B \, dz$$

(10)

$$\theta = \frac{e\alpha^2}{\pi \sqrt{8mE}} \int_{-\infty}^{+\infty} \frac{dz}{((2z-a)^2 + a^2)} = \frac{e\alpha^3}{2\pi \sqrt{8mE}} \tan^{-1} \left( \frac{2z-a}{a} \right)$$

$$\theta = \frac{e\alpha^3}{2\sqrt{8mE}}$$

(11)

Based on equations (8), (9) and (11), we can be obtaining a relationship between $f$ of thin lens and rotation angle along on axis of solenoid, equation (12).

$$\theta = \sqrt{\frac{na^5}{f}}$$

(12)

2. Results and Discussion

Regarding the equation (1) one can plot axial magnetic field of modulating model for various orders $n$ against the half-half width of solenoid field in figure 1. It will be seen from the figure, half-half width of solenoid field remains constant up to $n = 6$ at the assumed value $a = 0.03$ (m), and then increases rapidly with the increase of order $n$, see figure 2. The peak field of modulating field model of solenoid is constant in one value equal about 0.32 (T), regardless increasing the half-half width with order $n$, and note the field sketch diverges to the hard boundary shape at $n$ values higher than 60, and also the half-half width of field ($a$) for an $n$ values higher than 6.
Figure 1. The coil solenoid modulating model equation (1) field on axis for diverse values of order ($n$) illustrating soft boundary to hard boundary transformation for ($n > 60$).

Figure 2. Half-half width ($a$), of solenoid field with order $n$ in equation (1) solenoid model.

According to the equation (7), the one can sketched spherical aberration coefficient for a coil solenoid with order $n$. It is worth to mention, the spherical aberration increases highly with order $n$, figure 3, while decreasing inversely with square of half-half width of solenoid field, see figure 4.
The result of equations (8), and (9) plotted in figures 5, and 6. We have observed from figure 5, the \( P \) increasing linearly with increasing the half-half width \( a \), while \( f \) of a thin lens decreases inversely with half-half width \( a \), figure 6, as well as result equation (12) illustrated in figure 7, its noticeable that the value of \( f \) is weakest when the rotation angle increases.
Figure 5. Focusing power ($P$) of the lens with half-half width of field, at $V_r=5\times10^4$ (V).

Figure 6. Focal length ($f$) of the lens with half-half width of field, at $V_r=5\times10^4$ (V).
Figure 7. Analytic focal length ($f$) as a function rotation angle.

Figures 8, shows rotation angle of electrons around the optical axis of a solenoid, at point of ends $z_{-\alpha}$ to $z_{+\alpha}$ for different values of half-half width of field $a$, this figure schemed as formed in equation (11). It will be indicated that rotation angle growing with increasing half-half width of solenoid model field. The values of rotation angle extending from $0.513^\circ$ to $3.126^\circ$. 
4. Conclusions

In the current research, some conclusions were reached and we include them in the following points;
1. A modulating field model giving a constant peak field in spite of changing the order of solenoid field.
2. The half-half width of solenoid field stay in the same value until order n = 6, and then increasing rabidly when order n increasing.
3. The field sketch diverges to the hard boundary shape at n values higher than > 60 and, also the half-half width of field is (a) for all values of orders n higher than 6.

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