Thermal rho’s in the quark-gluon plasma

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I discuss different models which predict changes in the mass of the thermal $\rho$ field. I emphasize that while the predictions are strongly model dependent, nevertheless substantial shifts in the thermal $\rho$ mass are expected to occur at the point of phase transition. As long as the thermal $\rho$ peak does not become too broad, this should provide a striking signature of the existence of a phase transition.

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1. INTRODUCTION

In this note I examine various models for how the thermal $\rho$ peak might shift. While the predictions of these models are diametrically opposed, my intention herein, and in general, is not to insist the predictions of one model are necessarily superior to another. If the results of numerical simulations of lattice gauge theory have taught us anything, they have demonstrated that the phase diagram of QCD is rather complicated and intricate. Instead of insisting upon the virtues of a single model, it is perhaps more reasonable to explore different models, and the range of possible predictions. I try to stress what is most general in each case.

2. DECONFINEMENT DOMINATES

The simplest model of confinement is the bag model [1]. If $b$ is the bag constant, then the energy of a hadron is computed by carving out a cavity of radius $R$ from the vacuum. The energy is a balance between the volume energy of the cavity, versus the zero point energy which arises from confining quarks to the cavity: $E_i(R) = (4\pi/3)bR^3 + a_i/R$, where $a_i$ is a constant which depends upon the individual hadron of type $i$. Minimizing $E_i(R)$ with respect to $R$, $E_i \sim b^{1/4}$, and all hadron masses are proportional to $b^{1/4}$. This is really a trivial consequence of the model, since the only dimensional parameter is the bag constant, $b$.

To characterize the effects of nonzero temperature it is convenient to introduce a temperature dependent bag constant, $b(T)$ [2]. At zero temperature the bag constant is minus the pressure of the vacuum, pushing in upon the quarks and gluons confined to the bag. At nonzero temperature, then, we include the pressure of quarks and gluons pushing out on the bag, and pions pushing in on the bag, to obtain $b(T) = b - \pi^2T^4(8/45+7/60-1/30)$. Hence there is a deconfining phase transition when the effective bag constant vanishes,
$b(T_d) = 0$. Approaching the temperature of deconfinement from below, the effective bag constant vanishes linearly with temperature, $b(T) \sim T_d - T$. Plugging this back into the equation for the energy of a hadron and solving, the mass of a hadron — any hadron — vanishes like $m_i(T) \sim (T_d - T)^{1/4}$.

The conclusion that all hadron masses vanish at $T_d$ is really an inexorable consequence of the fact that there is only one dimensional parameter in the bag model. Once we characterize the phase transition by one of vanishing bag constant, of necessity the masses must vanish at $T_d$, since any mass can only be proportional to this single mass scale.

This was proposed as a general principle by Brown and Rho. Even if the phase transition is of first order, it is reminiscent of a second order transition. The striking difference is that according to Brown-Rho scaling, an infinite number of hadrons becomes massless at $T_d$, while in a second order transition, the number of massless fields is finite, equal to the rank of the representation for the the symmetry group appropriate for the transition.

Another model which gives similar behavior is that of sum rules. At zero temperature the vacuum is characterized by several condensates, including a gluon condensate, $\langle (G_{\mu\nu})^2 \rangle$, and a quark condensate, $\langle \bar{\psi}\psi \rangle$. At nonzero temperature, in the chiral limit the quark condensate $\langle \bar{\psi}\psi \rangle$ vanishes above the chiral transition, but the gluon condensate does not. Also, at nonzero temperature there are two gluonic condensates, for electric and magnetic fields, $\langle (E_i)^2 \rangle$ and $\langle (B_i)^2 \rangle$. Since even at the point of phase transition there are dimensional mass parameters about, typically hadronic masses are nonzero at $T_d$. Hadron masses do tend to fall, usually dramatically, because the gluonic condensates decrease with temperature (see, however, [2]).

I characterize these models in which hadron masses scale more or less uniformly with temperature as ones in which deconfinement dominates. This description is admittedly imprecise. In a theory without quarks, we know how to specify the phase transition, by the breaking of a global $Z(3)$ symmetry above $T_d$. For such a phase transition, the $Z(3)$ symmetry does not constrain how hadronic (here glueball) masses scale as $T_d$ is approached.

In these models the $\rho$ mass goes down with increasing temperature [2], along with every other hadron mass scale.

3. CHIRAL SYMMETRY RESTORATION DOMINATES

A different approach is to forget about how confinement arises in the first place, and concentrate upon the restoration of the global chiral symmetry at a temperature $T_\chi$. By its very nature, this approach will be limited to temperatures at or below $T_\chi$. For example, such models cannot, in any elementary manner, incorporate the large increase in entropy which numerical simulations of lattice gauge theory find is a universal feature of the phase transition, irrespective of values of the quark masses.

Unlike the previous section, with one dimensional parameter and an almost universal parametrization of the hadron spectrum, the characterization of chiral symmetry restoration requires a precise specification of all fields in the proper chiral multiplets. Assuming that effects of the axial anomaly are always significant, so that we need only classify parti-
cles according to \( SU(2) \times SU(2) \) multiplets, I introduce \( \Phi = \sigma t^0 + i \vec{\pi} \vec{t} \), where \( \sigma \) is an isosin-
glet \( 0^+ \) field. For the left and right handed vector fields I take \( A^{\mu}_{l,r} = (\omega^\mu \pm f^\mu_1) t^0 + (\vec{\rho}^\mu \pm \vec{a}_1^\mu) \vec{t} \) where \( \omega \) and \( \vec{\rho} \) are \( 1^- \) fields, and \( f_1 \) and \( \vec{a}_1 \) are \( 1^+ \) fields.

The crucial assumption which I then make is that of vector meson dominance. This severely constrains the dimensionless couplings of the model to be those which follow exclusively by promoting the global chiral symmetry to a local chiral symmetry. From the viewpoint of effective lagrangians, this is really an utterly remarkable principle \([7]\). The effective Lagrangian is then a sum of terms invariant under the local chiral symmetry,

\[
\mathcal{L}_{VDM} = tr \left( |D^\mu \Phi|^2 - \mu^2 |\Phi|^2 + \lambda (|\Phi|^2)^2 - 2ht^0 \Phi + \left( F^{\mu \nu}_l \right)^2/2 + \left( F^{\mu \nu}_r \right)^2/2 \right) ,
\]

and a single mass term which is solely responsible for the breaking of the local chiral symmetry,

\[
\mathcal{L}_{mass} = m^2 \left( (A^{\mu}_l)^2 + (A^{\mu}_r)^2 \right) .
\]

Here \( D^\mu \) and \( F^{\mu \nu}_{l,r} \) are the appropriate covariant derivative and field strengths for a local \( SU(2) \times SU(2) \) symmetry: for instance, \( D^\mu \Phi = \partial^\mu \Phi - ig(A^{\mu}_l \Phi - \Phi A^{\mu}_r) \), where \( g \) is the coupling of vector meson dominance.

This effective lagrangian involves several parameters, but for the physics of the \( \rho \) mass, all we need to know is that at zero temperature, \( m^2_\rho = m^2 \), and that the \( \rho - a_1 \) mass degeneracy is lifted after \( \sigma \) acquires a vacuum expectation value = \( \sigma_0 \), \( m^2_{a_1} = m^2 + g^2 \sigma_0^2 \).

Chiral symmetry predicts that since the \( \rho \) and \( a_1 \) are chiral partners under \( SU(2) \times SU(2) \), that their masses should be equal at the chiral transition (in the chiral limit). This is all chiral symmetry says: it does not predict where the \( \rho \) and \( a_1 \) masses meet.

If one looks at the expansion about zero temperature, the results are involved. This is due to the fact that there is mixing between the \( a_1 \) and \( \pi \) fields, which then mix the \( \rho \) and \( a_1 \). To order \( T^2 \), according to a general analysis by Eletsky and Ioffe \([8]\), the thermal \( \rho \) and \( a_1 \) masses don’t move at all; this is confirmed by studies in the linear and nonlinear sigma models \([4,9]\). In the linear model, to order \( T^4 \), the \( \rho \) mass goes down, and the \( a_1 \), up \([7]\)! This is not universal: by a dispersive technique, Eletsky and Ioffe find that both masses decrease, as in sum rules \([10]\).

At the point of phase transition, however, I would argue that the nature of the effective lagrangian, and especially the assumption of vector meson dominance, alone constrains the thermal \( \rho \) mass to be greater than that at zero temperature \([4]\). The point is really trivial: the mass term above, \( m^2 \), is by assumption independent of the scalar field, and so \( \sigma_0 \). So that part of the mass is fixed. In addition, there are thermal fluctuations, due to \( \pi \)'s, and, near the transition, \( \sigma \)'s, since those become degenerate with the \( \pi \)'s at the chiral transition. For scalar fields, the effect of these fluctuations is inevitably to push the thermal \( \rho \) mass up at the point of phase transition. For example, an elementary calculation to one loop order predicts that at \( T_\chi \),

\[
m^2_\rho(T_\chi)^2 = m^2_{a_1}(T_\chi)^2 = \frac{1}{3} \left( 2m^2_\rho + m^2_{a_1} \right) = (962 \text{ MeV})^2 .
\]

If one abandons vector meson dominance, there is no unique prediction. For example, if instead of the mass term above, suppose that one insists that the \( \rho \) mass arises from
spontaneous symmetry breaking. This can be done by setting $m = 0$ above, and adding the term

$$\mathcal{L}_\kappa = \kappa \, tr(|\Phi|^2) \, tr((A_\mu^a)^2 + (A_\mu^r)^2).$$

(4)

where $\kappa$ is a dimensionless coupling constant. At zero temperature, $m_\rho^2 = \kappa \sigma_0^2$. At nonzero temperature, the $\rho$ mass decreases as the condensate evaporates. Even so, it does not vanish entirely, since thermal fluctuations from $\pi$’s and $\sigma$’s still contribute. A simple calculation shows that for this term,

$$m_\rho^2(T_\chi) = m_{a_1}^2(T_\chi) = \frac{2}{3} m_\rho^2 = (629 \text{ MeV})^2.$$  

(5)

This is like the results from sum rules.

Consequently, and rather surprisingly, we see that the question of the position of the thermal $\rho$ mass at the point of phase transition provides a rather strong test of the applicability of the assumption of vector meson dominance at nonzero temperature. If vector meson dominance holds, then the thermal $\rho$ mass is greater at $T_\chi$ than at zero temperature; if it does not hold, there is no unique prediction, as the thermal $\rho$ mass could be either greater or less than that at zero temperature.

4. CONCLUSIONS

Wherever the $\rho$ goes, the really crucial question for experimentalists is how broad the thermal $\rho$ peak becomes. This is beyond the subject of present analysis; surely the thermal $\rho$ is broader than that at zero temperature. Pulling out the thermal $\rho$ peak will be extremely difficult. Nevertheless, if possible, it would provide truly dramatic evidence for a new state of matter.

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