Low Complexity Hybrid Precoding and Channel Estimation Based on Hierarchical Multi-Beam Search for Millimeter-Wave MIMO Systems

Zhenyu Xiao, Member, IEEE, Pengfei Xia, Senior Member, IEEE and Xiang-Gen Xia, Fellow, IEEE

Abstract—In millimeter-wave (mmWave) MIMO systems, while a hybrid digital/analog precoding structure offers the potential to increase the achievable rate, it also faces the challenge of the need of a low-complexity design. In specific, the hybrid precoding may require matrix operations with a scale of antenna size, which is generally large in mmWave communication. Moreover, the channel estimation is also rather time consuming due to the large number of antennas at both Tx/Rx sides. In this paper, a low-complexity hybrid precoding and channel estimation approach is proposed. In the channel estimation phase, a hierarchical multi-beam search scheme is proposed to fast acquire \( N_S \) (the number of streams) multipath components (MPCs)/clusters with the highest powers. In the hybrid precoding phase, the analog and digital precodings are decoupled. The analog precoding is designed to steer along the hybrid precoding phase, the analog and digital precodings are decoupled. In the hybrid precoding may require matrix operations with a scale of antenna size, which is generally large in mmWave communication.

Index Terms—Hybrid precoding, millimeter-wave, mmWave, mmWave MIMO, beam search, hierarchical search.

I. INTRODUCTION

MILLIMETER-WAVE (mmWave) communication is a promising technology for next-generation wireless communication owing to its abundant frequency spectrum resource, which enables a much higher capacity than the current alternatives. In fact, mmWave communication has raised increasing attention as an important candidate technology in both the next-generation wireless local area network (WLAN) [1–5] and mobile cellular communication [6–12]. In general, mmWave communication faces the problem of high propagation loss due to the high carrier frequency. Thus, mmWave devices usually need large antenna arrays to compensate for the propagation loss. Fortunately, thanks to the short wavelength of the mmWave frequency, large antenna arrays are possible to be packed into small form factors.

Despite the possibility of using large arrays, the high power consumption of mixed signal components, as well as radio-frequency (RF) chains, makes it impractical, if not impossible, to realize a full-blown digital baseband beamforming as used in the conventional multiple-input multiple-output (MIMO) systems. In such a case, analog beamforming is considered for mmWave communication, where all the antennas share a single RF chain and generally have constant-amplitude (CA) constraint on their weights [13, 14]. As entry-wise estimation of channel status information (CSI) is time costly due to large arrays and subspace observations of the channel [15], the training approach is generally adopted, including the power iteration method by exploiting the directional feature of mmWave channel [16], and the switching beamforming which probes on a pre-defined codebook and finds the best codeword within the codebook [13, 14]. For switching beamforming, multiple-stage hierarchical search algorithms were proposed to reduce the number of measurements [3, 13–15, 17]. These schemes first probe with low-resolution codewords, i.e. codewords with larger beam widths, and then probe with high-resolution codewords, i.e. codewords with thinner beam widths. Although these analog beamforming schemes reduce search complexity, they generally share the disadvantage of steering towards only one communication beam, i.e., these schemes are not capable of achieving multiplexing gain in addition to array gain.

In order to achieve multiplexing gain, a hybrid analog/digital precoding structure was then proposed [7, 17–12], where a small number of RF chains are tied to a large antenna array. This structure enables parallel transmission, and thus provides the potential to approach the capacity bound that can be achieved via digital precoding. However, the large antenna size challenges the need of a low-complexity design of the hybrid precoding and channel estimation. In particular, the hybrid precoding may require matrix operations with a scale of antenna size, which is generally large in mmWave commu-

1Beamforming in the case of multi-stream transmission is called precoding here.
nication. Moreover, the channel estimation is also rather time consuming due to the large number of antennas at both Tx/Rx sides.

In [13], an overall approach was proposed for hybrid precoding and channel estimation in mmWave MIMO systems. In the channel estimation phase, a hierarchical codebook was designed by exploiting the hybrid structure, which is different from the former ones with only analog combining [13], [14]. Based on the codebook, a hierarchical multi-beam search method was proposed to acquire \( L_d \) (\( L_d \) is no less than the number of streams \( N_S \) and is generally equal to \( N_S^2 \)) multipath components (MPCs)/clusters with the highest powers. With these MPCs, the channel matrix was reconstructed. In the precoding phase [18], [19], the optimal precoding matrix is first obtained based on the estimated channel without considering the CA constraint, and then analog and digital precoding matrices are determined by minimizing the Frobenius distance between the product of the analog and digital precoding matrices and the unconstraint optimal one. By exploiting the sparse feature on the angle domain, the optimization problem is modeled to be a sparse reconstruction problem, and is solved by the orthogonal matching pursuit (OMP) approach.

Although this overall approach [13] is theoretically feasible, it may not achieve a promising performance when the number of RF chains or streams is small. Moreover, it has high computational complexity in the channel estimation of RF chains or streams. In the channel estimation phase, a hierarchical codebook was designed by exploiting the hybrid structure, which is different from the former ones with only analog combining [13], [14]. Based on the codebook, a hierarchical multi-beam search method was proposed to acquire \( L_d \) (\( L_d \) is no less than the number of streams \( N_S \) and is generally equal to \( N_S^2 \)) multipath components (MPCs)/clusters with the highest powers. With these MPCs, the channel matrix was reconstructed. In the precoding phase [18], [19], the optimal precoding matrix is first obtained based on the estimated channel without considering the CA constraint, and then analog and digital precoding matrices are determined by minimizing the Frobenius distance between the product of the analog and digital precoding matrices and the unconstraint optimal one. By exploiting the sparse feature on the angle domain, the optimization problem is modeled to be a sparse reconstruction problem, and is solved by the orthogonal matching pursuit (OMP) approach.

Although this overall approach [13] is theoretically feasible, it may not achieve a promising performance when the number of RF chains or streams is small. Moreover, it has high computational complexity in the channel estimation phase. The performance of achievable rate can be degraded by the codebook design in [13], which depends on both the number of RF chains and \( L_d \). In fact, only when both of them are large enough, good widebeam codewords can be shaped; otherwise there may be deep sinks within the coverage of a beam, which will result in high-rate miss detection of MPCs, and in turn poor achievable rate. Regarding the computational and time complexities, for the hybrid precoding, the singular value decomposition (SVD) and matrix multiplication of size the same as the antenna number are required, which may challenge its practical implementation. While for the channel estimation, the required number of time slots (or measurements) is proportional to \( L_d^2 M^2 \), which is basically too high for a practical mmWave system, where \( M \) is the hierarchical factor defined in Section IV.

In this paper, we propose a low-complexity overall hybrid precoding and channel estimation approach. The differences between this approach and the one proposed in [13] are:

- In the channel estimation phase, we propose a new hierarchical multi-beam search scheme, which uses a pre-designed analog hierarchical codebook, rather than the hybrid hierarchical codebook designed in [18]. The pre-designed analog hierarchical codebook is robust to the number of RF chains and \( N_S \), which guarantees a robust performance. Moreover, the proposed search scheme exploits the particular channel structure in mmWave communication and the hierarchical feature of the pre-designed codebook, which greatly reduces the required time slots.

- In the hybrid precoding phase, the analog and digital precodings are decoupled. The analog precoding is designed to be steer along the \( N_S \) acquired MPCs/clusters at both Tx/Rx sides, shaping an equivalent \( N_S \times N_S \) baseband channel, while the digital precoding operates on the \( N_S \times N_S \) equivalent channel, which greatly lowers the operation size of the matrices.

Performance evaluations show that, compared with the approach proposed in [13], the newly proposed approach achieves a close performance to the alternative, or even a better one when the number of RF chains or streams is small. Moreover, the computational complexity of the hybrid precoding and the time complexity of the channel estimation are greatly reduced. Specifically, antenna-size matrix operations are reduced to stream-size matrix operations for the hybrid precoding, while \( L_d^2 M^2 \)-proportional time slots are reduced to \( N_S^2 M \)-proportional time slots for the channel estimation.

The rest of this paper is organized as follows. In Section II, we introduce the system and channel models, and formulate the problem. In Sections III, we propose the hierarchical multi-beam search method for channel estimation. In Section IV, we present the hybrid precoding operation. In Section V, we show the performance evaluations. Lastly, we conclude the paper in Section VI.

Notation: \( a \), \( a \), \( A \), and \( A \) denote a scalar variable, a vector, a matrix, and a set, respectively. \( (\cdot)^* \), \( (\cdot)^T \), and \( (\cdot)^H \) denote conjugate, transpose and conjugate transpose, respectively. In addition, \([x_1, x_2, ..., x_M]\) denotes a row vector with its elements being \( x_i \). Some other operations used in this paper are defined as follows.

- \( \mathbb{E}\{\cdot\} \) \( \) Expectation operation.
- \( |x| \) \( \) Absolute value of scalar variable \( x \).
- \( ||x|| \) \( \) \( 2 \)-norm of vector \( x \).
- \( ||x||_0 \) \( \) \( 0 \)-norm of vector \( x \).
- \( ||X||_F \) \( \) Frobenius norm of \( X \).
- \( X_{i,j} \) \( \) \( i \)-th row and \( j \)-th column element of \( X \).
- \( X_{i,:} \) \( \) \( i \)-th column of \( X \).

II. SYSTEM AND CHANNEL MODELS

A. System Model

Without loss of generality, we consider a downlink point-to-point multiple-stream transmission in this paper, while the signal model and the proposed scheme are also applicable for an uplink transmission. An mmWave MIMO system with a hybrid digital/analog precoding structure is shown in Fig. 1 where relevant parameters are listed below.

- \( N_S \) \( \) Data streams transmitted from the base station (BS) to a mobile station (MS).
- \( N_R \) \( \) The number of RF chains at the BS.
- \( N_A \) \( \) The number of antennas at the BS.
- \( M_A \) \( \) The number of RF chains at the MS.
- \( M_R \) \( \) The number of RF chains at the MS.

Basically, we have \( N_R \leq N_A \) and \( M_R \leq M_A \), but in practical mmWave MIMO systems, \( N_R \) and \( M_R \) are far less than \( N_A \).
and $M_A$, respectively. Moreover, it is noted that the supported number of data streams, i.e., $N_S$, is constrained by the number of RF chains, which means $N_S \leq \min\{N_R, M_R\}$.

The BS performs digital precoding in the baseband and analog precoding in RF, respectively, while the MS performs analog combining in RF and digital combining in the baseband, respectively. Let $s_{N_S \times 1}$ denote the transmitted signal vector with normalized power, i.e., $E(ss^H) = I_{N_S}$, where $I$ is an identity matrix. Considering a narrow-band block-fading propagation channel as in [18], [19], the received signal vector at the MS writes

$$y = \sqrt{P}w_R^Hw_R^Hn + w_R^HF_BS + w_R^HF_Rn,$$  \hspace{1cm} (1)

where $P$ is the transmission power per stream, $F_B$ and $F_R$ are the $N_R \times N_S$ digital and $N_A \times N_R$ analog precoding matrices at the BS, respectively, $W_B$ and $W_R$ are the $M_R \times N_S$ digital and $M_A \times M_R$ analog precoding matrices at the MS, respectively, $n$ is a standard white Gaussian noise vector, i.e., $E(nn^H) = I_{N_S}$. In addition, we have the entrywise CA constraint for the RF precoding matrix $F_R$ and the RF combining matrix $W_R$, respectively. In addition, we have power normalization for the precoding at the BS, i.e., $\|F_RF_B\|^2 = N_S$.

B. Channel Model

Since mmWave channels are expected to have limited scattering [7], [11], [18], [20]–[22]. MPCs are mainly generated by reflection. Different MPCs have different physical transmit steering angles and receive steering angles, i.e., physical angles of departure (AoDs) and angles of arrival (AoAs). Consequently, mmWave channels are relevant to the geometry of antenna arrays. While the algorithms and results developed in this paper can be applied to arbitrary antenna arrays, we adopt uniform linear arrays (ULAs) with a half-wavelength antenna space in this paper. Consequently, the channel can be expressed as [15], [18], [19], [20]–[22]

$$H = \sqrt{N_A}M_A \sum_{\ell=1}^{L} \lambda_{\ell}g(M_A, \Omega_{\ell})g(N_A, \psi_{\ell})^H,$$ \hspace{1cm} (2)

where $\lambda_{\ell}$ is the complex coefficient of the $\ell$th path, $L$ is the number of MPCs and $L \geq N_S$, $g(\cdot)$ is the steering vector function, $\Omega$ and $\psi_{\ell}$ are cos AoD and AoA of the $\ell$th path, respectively. Let $\theta_{\ell}$ and $\varphi_{\ell}$ denote the physical AoD and AoA of the $\ell$th path, respectively; then we have $\Omega_{\ell} = \cos(\theta_{\ell})$ and $\psi_{\ell} = \cos(\varphi_{\ell})$. Therefore, $\Omega$ and $\psi_{\ell}$ are within the range $[-1, 1]$. For convenience, in the rest of this paper, $\Omega$ and $\psi_{\ell}$ are called AoAs and AoDs, respectively, as we design the hybrid precoding and multi-beam search schemes in the cosine angle domain. Similar to [18], [22], $\lambda_{\ell}$ can be modeled to be complex Gaussian distributed i.e., $\lambda_{\ell} \sim \mathcal{CN}(0, 1/L)$, while $\theta_{\ell}$ and $\varphi_{\ell}$ are modeled to be uniformly distributed within $[0, 2\pi]$. $g(\cdot)$ is a function of the number of antennas and AoD/AoA, and can be expressed as

$$g(N, \Omega) = \frac{1}{\sqrt{N}}[e^{j\pi 0\Omega}, e^{j\pi 1\Omega}, ..., e^{j\pi (N-1)\Omega}]^T,$$ \hspace{1cm} (3)

where $N$ is the number of antennas ($N$ is $N_A$ at the BS and $M_A$ at the MS), $\Omega$ is AoD or AoA. It is easy to find that $g(N, \Omega)$ is a periodical function which satisfies $g(N, \Omega) = g(N, \Omega + 2)$.

**Remark 1.** Evenly sampling the cosine angle space $[-1, 1]$ with an interval length $2/N$ from $-1+1/N$ leads to $N$ steering vectors $g(N, -1 + (2k - 1)/N)$, $k = 1, 2, ..., N$. We say each vector of them represents a basis beam with a width $2/N$.

3Here the non-line-of-sight (NLOS) model is adopted for multi-stream transmission. For the LOS channel model in mmWave communications, the energy of the NLOS component is basically much lower than that of the LOS component [7], [18], [20]–[22]. Thus, one-stream transmission may be more preferable. However, we need to note that the proposed approach is also feasible in the LOS channel.
C. Problem Formulation

Our target is to maximize the total achievable rate for the downlink point-to-point transmission. Thus, the problem can be formulated as [26] Chapters 7 and 8, [22]

\[
\max_{F_B, F_R, W_B, W_R, Q} R = \log_2 \det \left( I + K_W^{-1/2} H_E Q H_E^H K_W^{-1/2} \right)
\]

subject to \[ \|F_R F_B\|^2_F = N_S, \]

\[ \text{Tr}(Q) = P, \]

(4)

where \( H_E = W_B^H W_R^H H F_Q F_B \), \( K_W = W_B^H W_R^H W_B \), and \( Q \) is the power allocation matrix. To solve this problem, we must solve the following two subproblems.

Subproblem 1: Multi-Beam Search. As we do not know the channel matrix, we need to estimate it first, which is the first subproblem. Apparently it would be rather time consuming to estimate \( H \) entry-wisely, as the antenna size is large. Fortunately, according to the structure of mmWave channel in [2], we only need to search the AoDs and AoAs of several most significant MPCs to capture the majority of the channel energy. The search efficiency depends on the design of the hierarchical codebook and the search scheme. Different from [18], we are going to propose an improved hierarchical multi-beam search scheme based on a pre-designed analog combining codebook.

Subproblem 2: Hybrid Precoding. Assuming the AoAs/AoDs of the most significant MPCs have been estimated, the remaining subproblem is the hybrid precoding, i.e., to obtain \( F_B, F_R, W_B, W_R \) and \( Q \) to maximize \( R \) under the CA constraint. Note that due to the CA constraint, it is hard to find a globally optimal solution to this subproblem. On the other hand, it may be not worthy at all to find the globally optimal solution at a cost of high computational complexity, since the channel estimation is already rather approximate. Hence, we focus on designing a low-complexity hybrid precoding.

III. HIERARCHICAL MULTI-BEAM SEARCH

In mmWave communication, channel estimation is generally realized by estimating the coefficients, AoDs and AoAs of several \( (N_S \text{ in the context of this paper}) \) most significant MPCs, as the conventional entry-wise estimation is time consuming due to the large antenna size. Before we introduce the proposed hierarchical multi-beam search method, let us start from the introduction of the brute force sequential search scheme for better understanding.

A. The Sequential Search Scheme

The sequential search scheme is straightforward, i.e., sequentially searching the whole Tx/Rx angle plane and finding the \( N_S \) (AoD AoA) pairs with the highest strengths. Therefore, the codebook for the sequential search consists of steering vectors with evenly sampled angles in the range of \([-1, 1]\), i.e., \( g(N, -1 + \frac{2i-1}{KN}), i = 1, 2, \ldots, KN \), where the sampling interval is \( \frac{2}{KN} \) and \( K \) is the over-sampling factor. The larger \( K \) is, the smaller the estimation errors of AoDs and AoAs are.

Regarding the considered system in Section II, sequential search is realized by sequentially transmitting training sequences from the BS with codewords \( g(N_A, -1 + \frac{2i-1}{KN}) \) and receiving the training sequences at the MS with codewords \( g(M_A, -1 + \frac{2j-1}{KM_A}) \) for \( i = 1, 2, \ldots, K M_A \) and \( j = 1, 2, \ldots, K N_A \). Consequently, we can obtain the angle-domain matrix \( G \):

\[
\begin{align*}
[G]_{i,j} &= g(M_A, -1 + \frac{2j-1}{MN_A}) H g(N_A, -1 + \frac{2i-1}{KM_A}), \\
&= 1, 2, \ldots, K M_A, \quad j = 1, 2, \ldots, K N_A.
\end{align*}
\]

(5)

Afterwards, it is straightforward to find the \( N_S \) most significant peaks with \( G \) on the Tx/Rx angle plane. If we denote the time period of a training sequence as a time slot (i.e., a measurement), we need \( K^2 M_A N_A \) time slots to estimate \( G \) with the sequential search approach. Considering that we have \( N_R \) and \( M_R \) RF chains at the BS and MS, respectively, in each time slot we can estimate \( N_R M_R \) elements of \( G \), by sending different orthogonal training sequences on the \( N_R \) RF chains with different steering vectors at the BS, and receiving also with different steering vectors on the \( M_R \) chains at the MS (similar to [17] and [18]). Thus, the total time cost to estimate \( G \) is

\[
T_{SS} = \frac{K^2 M_A N_A}{N_R M_R},
\]

(6)

which is proportional to \( M_A N_A \). While this method is feasible, the time cost would be significantly high for large-array devices.

B. The Hierarchical Multi-beam Search Scheme

To reduce the time cost for channel estimation, we propose the hierarchical multi-beam search scheme, where a corresponding hierarchical codebook needs to be designed in advance.

1) Hierarchical Codebook Design: A typical hierarchical codebook \( F \) is shown in Fig. [2] where \( F \) has \( S + 1 \) layers (not including the over-sampling layer). In the 0th layer, there are \( M^k \) codewords with the same beam width but different steering angles, \( k = 0, 1, \ldots, S \), where \( M \) is the hierarchical factor. The union of beam coverage of all the codewords in each layer equals \([-1, 1]\). The number of antennas is assumed to satisfy \( N = M^S \), where \( S \) is an integer. Let \( w(k, n) \) denote the \( n \)th codeword in the \( k \)th layer, \( n = 0, 1, \ldots, M^k \). Then the
beam coverage in the angle domain of \( w(k, n) \) is the union of those of \( \{ w(k + 1, (n - 1)M + m) \}_{m=1,2,...,M} \), which is the critical feature why the codebook is hierarchical. For convenience we say \( w(k, n) \) is a parent codeword of its \( M \) child codewords \( w(k + 1, (n - 1)M + m), m = 1, 2, ..., M \).

There are \( N \) codewords in the \( S \)th layer of the codebook, and they are just the evenly sampled \( N \) basis beams mentioned in Remark 1. The angle resolution is \( 2/N \) in this layer. If a more accurate estimation of AoDs and AoAs is required, we need the over-sampling layer as shown in Fig. 2 where \( g_i = g(N, -1 + 2i - 1/KN), i = 1, 2, ..., KN \). All these \( KN \) codewords within this layer are the steering vectors to sample the angle domain with an interval of \( 2/(KN) \). Whether or not this over-sampling layer is needed depends on practical requirements.

The design of this hierarchical codebook is challenging due to the CA constraint. In our previous work \([23]\), analog beamforming was studied, and codewords with wide beams are designed by turning off part of the antenna elements. In \([18]\), hybrid analog/digital beamforming was studied, and the codebook design is formulated as a sparse problem based on the hybrid structure, and solved by the OMP algorithm. Note that in \([23]\) only analog combining was used for the codebook design, rather than the hybrid combining in \([18]\).

However, these two codebooks have separate drawbacks. For \([23]\), the number of active antennas is usually small when wide-beam codewords are configured. Because the transmit power per antenna is usually limited in mmWave communications, this would in turn limit the total transmit power in the beam search, and is undesired in general. For \([18]\), although multiple RF chains provide additional degrees of freedom, only when the number of RF chains is large enough, good wide-beam codewords can be generated; otherwise there will be deep sinks \([18\] Fig. 5) within the beam coverage, which degrades the beam search and the achievable-rate performance.

In this paper, we choose to use the joint sub-array and deactivation (JOINT) codebook design \([?]\) for hybrid beamforming. Note that in \([?]\) we only proposed the codebook design, while in this paper we further consider to use it in hierarchical multi-beam search. Since the codebook with \( M = 2 \) is widely used, we introduce the method to generate the codewords with the JOINT approach here for the case of \( M = 2 \). However, we must note that the approach can also be used for other cases where \( M \) has other values.

**Codeword Generation with JOINT:**

When \( k = S = \log_2(N) \), we have \( w(S, n) = g(N, -1 + 2n/N), n = 1, 2, ..., N \).

When \( k = S - \ell \), where \( \ell = 1, 2, ..., S \), we obey the following procedures to compute \( w(k, n) \):

- Separate \( w(k, 1) \) into \( m_S = 2^{\ell+1}/2 \) sub-arrays with \( f_m = [w(k, 1)]_{[m_S+1:m_S+1]} \), where \([x]\) is the flooring integer operation, \( n_S = N/m_S, m = 1, 2, ..., m_S \);
- Set \( f_m \) as \( 7 \), where \( N_A = m_S/2 \) if \( \ell \) is odd, and \( N_A = M \) if \( \ell \) is even;
- We have \( w(k, n) = w(k, 1) \circ \sqrt{N}g(N, 2(m-1)), n = 2, 3, ..., 2^k \), and \( \circ \) is the entry-wise product;
- Normalize \( w(k, n) \).

\[
\mathbf{f}_m = \begin{cases} 
  e^{jm\pi g(n_S,-1+\frac{2m-1}{n_S})}, & m = 1, 2, ..., n_A, \\
  0_{n_S \times 1}, & m = n_A+1, n_A+2, ..., M, 
\end{cases}
\]  

where \( n_A \) is the number of active sub-arrays. See \([?]\) for more details in the codebook design.

Fig. 3 shows the comparison of the beam patterns between JOINT and the approach in \([18\] termed as Sparse\], \( N = 32 \), and the codewords are in the second \((k = 2)\) layer. \( L_d = 1 \) for the Sparse approach.

Fig. 5 shows the comparison of the beam patterns between JOINT and the approach in \([18\] where we can find that when the number of RF chains is small, there are deep sinks within the beam coverage of the wide-beam codewords, and the sink is more severe when the number of RF chains is smaller, which are in accordance with the results in \([18\] (Fig. 5 therein). Clearly, if the AoD or AoA of an MPC is along the sink angle, it cannot be detected with the codeword, which results in miss detection and in turn degradation of the achievable-rate performance. Besides, the beam coverage of the codewords in \([18\] are sensitive to both \( L_d \) and the number of RF chains, i.e., different system settings need different codebooks. In contrast, there is no deep sinks within the beam coverage of the codewords in the JOINT codebook, and the JOINT approach is robust against the numbers of RF chains and data streams, because it uses analog combining.

2) Hierarchical Multi-Beam Search: Based on the pre-designed hierarchical codebook, we next introduce the proposed beam search algorithm to find the \( N_S \) most significant beams on the Tx/Rx angle plane. While it is natural to search the multi-beams one by one, there are two critical issues to consider when extending a one-beam search scheme to a multi-beam search scheme.

The first issue is how to cancel the effect of the already found beams in the on-going beam search. Here we propose to exploit the particular structure of the mmWave communication channel shown in \( 2 \), where each MPC/cluster has an AoD, AoA and path coefficient. Let
There are three phases to search a single MPC: process, and a single MPC will be searched in each of them. Measurements for the search in each layer at both sides [18]. In our scheme we adopt a more codewords and finds the best one. Thus, we only need the number of candidate child codewords becomes $M$. A straightforward method is to exhaustively measure all the already found MPCs. During the multi-beam search, we need to perform beamforming measurements over $H_{fd}$, which can be obtained as

$$y = w_{MS}^H(H_{fd}w_{BS} + n)$$

$$= w_{MS}^H(\sqrt{PH}w_{BS} + n) - w_{MS}^HH_{fd}w_{BS},$$

where the second component is just the contribution of the already found MPCs.

The second issue is how to perform layered search. According to the designed hierarchical codebook. In each layer there are $M$ candidate child codewords at both the BS and MS. A straightforward method is to exhaustively measure all the possible codeword pairs and find the best codeword pair [18], and requires $M^2$ measurements. In the case of multi-beam search, the number of measurements becomes $M^2 I_d^2$, since the number of candidate child codewords becomes $ML_d$ in each layer at both sides [18]. In our scheme we adopt a more efficient scheme, i.e., BS uses the parent codeword found in the last-layer search, while MS sequentially measures its $M$ child codewords and finds the best one; then, MS uses the found best codeword, BS sequentially measures its $M$ child codewords and finds the best one. Thus, we only need $2M$ measurements for the search in each layer.

The proposed hierarchical multi-beam search scheme is shown in Algorithm 1. There are $N_S$ iterations in the search process, and a single MPC will be searched in each of them. There are three phases to search a single MPC:

- Search for the initial Tx/Rx codewords. As in mmWave communication the transmission power is generally limited, the beamforming gain cannot be too small. Thus, the beamforming training may not start from the 0th layer, where the codeword is omnidirectional and the gain is the lowest. Instead, the beamforming training may need to start from some layer, e.g., the $i_{LY}$ layer in Algorithm 1, to provide sufficient start-up beamforming gain. In this process, there are $M^{i_{LY}}$ candidate beamforming codeword pairs at both BS and MS. Thus, an exhaustive search over all the BS/MS codeword pairs is adopted to search the best

$$H = \sqrt{P}N_A M_A \sum_{\ell=1}^{L} \lambda_{d} g(M_{A}, \Omega_{\ell}) g(N_{A}, \psi_{\ell})^H$$

$$\triangleq \sum_{\ell=1}^{L} \beta_{\ell} g(M_{A}, \Omega_{\ell}) g(N_{A}, \psi_{\ell})^H.$$ Consequently, during the search process we can write $H$ as

$$\sqrt{PH} = \sum_{\ell \in I_{id}} \beta_{\ell} g(M_{A}, \Omega_{\ell}) g(N_{A}, \psi_{\ell})^H + \sum_{n \notin I_{id}} \beta_{n} g(M_{A}, \Omega_{n}) g(N_{A}, \psi_{n})^H \triangleq H_{fd} + H_{nfld},$$

where $I_{id}$ represents the indices of the already found MPCs, while $H_{fd}$ and $H_{nfld}$ represent the already found channel response and the not yet found channel response, respectively. During the multi-beam search, we need to perform beamforming measurements over $H_{nfld}$, which can be obtained as

$$y = w_{MS}^H(H_{fd}w_{BS} + n)$$

$$= w_{MS}^H(\sqrt{PH} w_{BS} + n) - w_{MS}^HH_{fd}w_{BS},$$

where the second component is just the contribution of the already found MPCs.

Algorithm 1: Hierarchical Multi-Beam Search Algorithm.

1) Initialization:

$$S = \max\{\log_{M_{A}}, \log_{M_{A}} M_A\}.$$ $i_{LY} = 2.$ /*The initial layer index. It can be other values depending on practical requirements.*/

$$H_{id} = 0.$$ /*The already found channel.*/

2) Iteration:

for $\ell = 1 : N_S$ do /*Search for the initial Tx/Rx codewords in the $i_{LY}$th layer.*/

for $m = 1 : M^{i_{LY}}$ do /*Hierarchical refinement.*/

for $n = 1 : M^{i_{LY}}$ do

$$y(m, n) = w_{BS}(i_{LY}, n)^H [\sqrt{PH} w_{MS}(i_{LY}, m) + n] - w_{BS}(i_{LY}, n)^H H_{fd} w_{MS}(i_{LY}, m)$$

$$= (m_{BS}, m_{MS}) = \arg \max \left| y(m, n) \right|$$

MS feeds back BS $m_{BS}$.

/*Hierarchical refinement.*/

for $s = i_{LY} + 1 : S$ do

for $n = 1 : M$ do

$$y_{MS}(n) =$$

$$w_{BS}(s - 1, m_{BS})^H [\sqrt{PH} w_{MS}(s, (m_{MS} - 1) M + n) + n] -$$

$$w_{BS}(s - 1, m_{BS})^H H_{fd} w_{MS}(s, (m_{MS} - 1) M + n)$$

$$= (n_{MS}, m_{BS}) = \arg \max \left| y_{MS}(n) \right|$$

$$m_{BS} = (m_{BS} - 1) M + n_{MS}$$

MS feeds back BS $m_{BS}$.

/*High-resolution refinement.*/

for $m = (m_{MS} - 1) K + 1 : M_{MS} K$ do

for $n = (m_{MS} - 1) K + 1 : M_{MS} K$ do

$$y(m, n) =$$

$$w_{BS}(i_{LY}, n)^H [\sqrt{PH} w_{MS}(i_{LY}, m) + n] -$$

$$w_{BS}(i_{LY}, n)^H H_{fd} w_{MS}(i_{LY}, m)$$

$$(I_{\ell}, J_{\ell}) = \arg \max \left| y(m, n) \right|$$

$$\beta_{\ell} = y(I_{\ell}, J_{\ell})$$

MS feeds back BS $J_{\ell}$.

/*Updating the already found channel response.*/

$$H_{id} = H_{id} + \beta_{\ell} (g_{I_{\ell}}^M) (g_{J_{\ell}}^B)^H$$

3) Result:

The $\ell$th ($\ell = 1, 2, ..., N_S$) index pair is $(I_{\ell}, J_{\ell})$ within the over-sampling layer.
Tx/Rx codeword pair, which are treated as the parent codewords for the following search.

- **Hierarchical refinement.** In this process, a staged search is performed to refine the beam angle step by step. The search process has \((S - i_{LY})\) stages, and begins from the \((i_{LY} + 1)\)th layer. In each stage, there are \(M\) candidate codewords at the BS and MS, which are the \(M\) child codewords of the found parent codeword in the last stage. At first, the BS uses the parent codeword, while the MS sequentially tests all the \(M\) child codewords and find the best one. Then the MS uses the best child codeword just found, while the BS sequentially tests all the \(M\) codewords and find the best one. The found best child codewords at the BS and MS are treated as the parent codewords in the next-stage search.

- **High-resolution refinement.** After the hierarchical refinement, the AoD and AoA of an MPC have been found with a resolution of \(2/N_A\) and \(2/M_A\). In practice \(K\) is generally small, because a promising performance can be achieved when \(K = 2\), and further increasing \(K\) does not significantly improve the performance, as will be shown in Section V. Hence, for each index pair \((m_{BS}, m_{BS})\) of the codewords found in the hierarchical refinement, we just need to test all the index pairs within \([m_{BS} - 1, K + 1, m_{BS}]\) at the MS and \([m_{BS} - 1, K + 1, m_{BS}]\) at the BS, and select the one with the greatest strength as \((I_t, J_t)\). The number of required measurements is \(K^2\).

### C. Time Complexity Comparison

Since there are multiple RF chains at the BS and MS, each time slot we can make multiple measurements by sending different orthogonal training sequences on different RF chains with different codewords at the BS, and receiving with different codewords at the MS. Thus, the total number of time slots for the hierarchical search method using **Algorithm 1** can be roughly computed as

\[
T_{HS} = N_S \left( \left\lceil \log_M(N_A) + \log_M(M_A - 2i_{LY}) \right\rceil \right) + \left[ \frac{M^{2i_{LY}}}{N_R M_R} \right] + \left[ \frac{K^2}{N_R M_R} \right],
\]

where \(\lceil \cdot \rceil\) is the ceiling integer operation, \(\log_M(N_A) + \log_M(M_A - 2i_{LY})\) and \(\frac{M^{2i_{LY}}}{N_R M_R}\) are the total number of stages and the number of measurements in each stage at the BS and MS in the hierarchical refinement, respectively, while \(\frac{K^2}{N_R M_R}\) are the numbers of measurements in the initial codeword search and the high-resolution refinement, respectively. Since \(\frac{M^{2i_{LY}}}{N_R M_R}\) and \(\frac{K^2}{N_R M_R}\) are irrelevant to the number of antennas, \(T_{HS}\) is roughly proportional to \(N_S^2 M\).

For a fair comparison, we assume the required number of MPCs \(L_d\) is equal to \(N_S\). Then the required number of measurements of the scheme in [18] is

\[
T_{SP} = MN_S^2 \left[ \frac{M}{N_R} \right] \log_M \left( \frac{M}{N_S} \right),
\]

which is roughly proportional to \(N_S^2 M^2\).

Fig. 4 shows the comparison of time complexity, i.e., required time slots for beamforming training, between the sequential search, the proposed hierarchical multi-beam search and the scheme in [18], with the parameters listed in the caption. It can be observed that the proposed hierarchical search scheme has the lowest time complexity among the three. Compared with the scheme in [18], the proposed hierarchical search scheme achieves a further significant reduction on the required time slots.

### IV. LOW-COMPLEXITY HYBRID PRECODING

With the proposed hierarchical multi-beam search approach, we can obtain the index pairs of the \(N_S\) strongest MPCs, i.e., \((I_1, J_1), (I_2, J_2), \ldots, (I_{N_S}, J_{N_S})\). In this section, we perform low-complexity hybrid precoding based on the estimated channel information. In specific, we first perform analog precoding without considering the digital precoding, and then compute the digital precoding matrices in the baseband.

#### A. Analog Precoding

The target of analog precoding is to steer at the \(N_S\) most significant MPCs/clusters in the angle domain. Hence, the analog precoding and combining matrices are

\[
F_R = \left[ g(N_A, -1 + \frac{2J_1 - 1}{K N_A}), g(N_A, -1 + \frac{2J_2 - 1}{K N_A}), \ldots, g(N_A, -1 + \frac{2J_{N_S} - 1}{K N_A}) \right],
\]

and

\[
W_R = \left[ g(M_A, -1 + \frac{2I_1 - 1}{K M_A}), g(M_A, -1 + \frac{2I_2 - 1}{K M_A}), \ldots, g(M_A, -1 + \frac{2I_{N_S} - 1}{K M_A}) \right].
\]

In this formula it was assumed that \(N_A \geq M_A\).
B. Digital Precoding

While the analog precoding is to steer at the $N_s$ most significant MPCs/clusters in the angle domain, the digital precoding is designed to cancel interference between different streams and perform power allocation at the BS.

Provided that $F_R$ and $W_R$ has been designed, we get an equivalent $N_S \times N_S$ baseband channel

$$H_B = W_R^H F_R.$$  \hspace{1cm} (15)

Thus,

$$[H_B]_{i,j} = [W_R]_{i}^H [H] [F_R]_{j}.\hspace{1cm} (16)$$

Since we have obtained $I_k$ and $J_s$, $\ell = 1, 2, ..., L$, the estimation of $H_B$ can be easily realized within a single time slot as follows. Note that this can also be done in the phase of channel estimation after the multi-beam search.

The BS transmits orthogonal training sequences $s_j$ from the $j$th RF chain with codeword $g(N_A, -1 + 2jK_{MA})$, where $j = 1, 2, ..., N_S$. Then the MS receives $N_S$ RF chains simultaneously, where $g(M_A, -1 + 2jK_{MA})$ is adopted in the $i$th RF chain. Thus, at the $i$th RF chain of the MS, we observe

$$r_i = g(M_A, -1 + 2jK_{MA})^H H_j \sum_{j=1}^{N_S} g(N_A, -1 + 2jK_{MA}) s_j,$$

By multiplying with $s_k$ at the $i$th RF chain of the MS, where $i = 1, 2, ..., N_S$ and $k = 1, 2, ..., N_S$, we get

$$s_k^H r_i = \sum_{j=1}^{N_S} g(M_A, -1 + 2jK_{MA})^H H_j g(N_A, -1 + 2jK_{MA}) s_j,$$

where the noise component is neglected.

After the estimation of $H_B$ and the analog precoding, the received signal at the MS can be rewritten as

$$y = \sqrt{P}W_B^H H_B F_B s + W_B^H W_R^H n.$$  \hspace{1cm} (19)

Let $R_n = W_R^H W_R$, $W_B^H = W_B^H R_n^{-1/2}$, and $\tilde{H}_B = R_n^{-1/2} H_B$. The received signal is equivalent to

$$y = \sqrt{P} \tilde{W}_B^H \tilde{H}_B F_B s + \tilde{W}_B^H n.$$  \hspace{1cm} (20)

C. Computational Complexity Comparison

The proposed hybrid precoding consists of analog precoding and digital precoding. The computational complexity of the analog precoding is low, while the digital precoding requires $N_S \times N_S$ matrix operations, including matrix multiplication, SVD, etc. Thus, the proposed hybrid precoding scheme has an overall computational complexity $O(N_S^3)$ (It is known that the computational complexity of general matrix multiplication, matrix inversion, SVD are on the order of $O(M^3)$, where $M$ is the matrix size \(27\)).

In contrast, the hybrid precoding approach proposed in \(18\) and \(19\) also requires matrix operations, including matrix multiplication, SVD, etc. Since it jointly designs the analog and digital precodings, the involved matrices are with size $N_A \times M_A$. Hence, the hybrid precoding in \(18\) has an overall computational complexity $O(N_A^3)$ (assuming $M_A = N_A$).

In brief, the proposed low-complexity hybrid precoding scheme reduces the computational complexity from $O(N_A^3)$ to $O(N_S^3)$, where basically $N_S \ll N_A$.

V. Performance Evaluation

In this section, we evaluate the performance of the proposed low complexity hybrid precoding (LC-HPC) based on the hierarchical multi-beam search (HIBS). The channel model introduced in Section II is adopted, where the physical angles of the MPCs are randomly generated within $[0, 2\pi)$, and the average strengths of the MPCs are equal, i.e., an NLOS channel model is considered (see Footnote 3). Each performance curve is obtained by averaging $10^4$ instantaneous performances with randomly realized channel responses. In all the simulations, we set $N_A = M_A = 32$, but we note that we have also evaluated the performances with other numbers of antennas, and similar results can be observed. In addition, the other parameters are all set typical values for mmWave communication in the simulations, e.g., the numbers of MPCs and streams (i.e., $L$ and $N_S$) are basically small.

A. Hierarchical Multi-Beam Search

First, we demonstrate the performance of the HIBS scheme, for which the most critical figure of merit is the success rate to find the index pairs of multiple beams. According to \(2\), there are $L$ MPCs with different AoDs and AoAs, and HIBS finds
$N_S$ of them. We sequentially decide whether the found $N_S$ MPCs are among the original $L$ MPCs. The decision method is that if there is an $\ell$, $1 \leq \ell \leq L$, satisfying $|\psi_\ell - \alpha| < 1/N_A$ and $|\psi_\ell - \beta| < 1/M_A$, where $1/N_A$ and $1/M_A$ are the permitted AoD and AoA errors, $\alpha$ and $\beta$ are AoD and AoA of an estimated MPC, we say this MPC is successfully searched. Only when all the $N_S$ MPCs are successfully searched, the whole search process succeeds; otherwise it fails.

We note that the search of an arbitrary MPC may fail because of the noise, the effect of the previously found MPCs, and the mutual effect of MPCs, i.e., spatial fading caused by MPCs when measured with wide-beam codewords. In fact, we have simulated the performance with $L = 1$ and $N_S = 1$, i.e., there is only one MPC. The success rate can achieve 100% with high SNR. This is because there is no mutual effect of MPCs when $N$ is larger. This is mainly because the contributions of the already searched MPCs cannot be completely subtracted off, and they affect the search of the next MPC. The effect becomes more significant as $N_S$ increases. (iii) The success rate is higher when $K$ is larger. This is because larger $K$ means more-accurate estimation of the AoDs and AoAs, and further more-accurate contribution subtraction of the already searched MPCs. From Fig. 5 we can find that when $N_S = 3$, the improvement of success rate by increasing $K$ is more significant than the cases of $N_S = 2$ and $N_S = 1$. (iv) The success rate is higher when $i_{LY}$ is bigger. Indeed, to start from a higher layer not only raises the set-up SNR, but also reduce the possible mutual effect of MPCs when measured with low-layer codewords. However, a bigger $i_{LY}$ means a higher time complexity according to (11).

Fig. 7 depicts the comparison of success rates between the proposed HIBS scheme and the search scheme in (18) (termed with “Sparse”). From the comparison, we can find that the Sparse scheme is sensitive to the number of RF chains. Even with 4 RF chains, its performance of success rate is poorer than the proposed scheme, which requires only 1 RF chain for MPC estimation. This performance disadvantage is mainly due to the hierarchical codebook design in (18), where the wide-beam codewords may have deep sinks within the beam coverage when the number of RF chains is not large enough, which may easily result in miss-detection of MPCs.

Moreover, from these three figures we can observe that although the desired number of MPCs is $N_S$, the number of actual found MPCs with either HIBS or the Sparse scheme may be less than $N_S$, especially when $N_S$ is large. In such a case, the actual number of streams will be equal to the number of actual found MPCs, which is less than $N_S$, resulting a degradation of the achievable rate. However, in practical mmWave communications, the number of independent MPCs/clusters is not large [2], [20]. Thus, these two search schemes are basically suitable.

B. Low-Complexity Hybrid Precoding

Next, we evaluate the performance of achievable rate of the proposed LC-HPC scheme, and learn the effect of $K$ on the performance.

Fig. 8 shows the achievable rate of LC-HPC with varying $K$, where the HIBS scheme is exploited to estimate MPCs. The training sequence is assumed long enough to provide sufficiently high SNR for the MPC estimation. From this figure we find that LC-HPC achieves a promising performance. Specifically, compared with the rate bound, which is defined as the achievable rate without the CA constraint, LC-HPC has almost no loss of multiplexing gain, i.e., the slopes of the performance curves of LC-HPC are the same as those of the rate bounds. Although there is increasing SNR loss.
as $N_S$ increases, which results from the CA constraint, it is basically acceptable in practice where $N_S$ is generally small. Moreover, it is clear that the performance of LC-HPC with $K = 2$ is better than that with $K = 1$, but when $K \geq 2$, further increasing $K$ leads to little performance improvement. Thus, basically $K = 2$ is suitable for practical usage.

Fig. 7. Comparison of success rates between the proposed hierarchical multi-beam search scheme with the search scheme in [18] (termed with “Sparse”), where $M = 2$, $L = 4$, $N_R = M_R = 3$, $K = 2$, $i_{LY} = 2$. For the Sparse scheme, the required resolution is set as $K N_A$, the same as the proposed scheme.

![Fig. 7](image1.png)

Fig. 8. Achievable rate of LC-HPC with varying $K$, where the HIBS scheme is exploited to estimate MPCs. $M = 2$, $L = 4$, $N_R = M_R = 3$, $i_{LY} = 2$.

![Fig. 8](image2.png)

![Fig. 9](image3.png)

Fig. 9. Comparison of achievable rates of the proposed low-complexity hybrid precoding (LC-HPC) approach with the sparse precoding approach in [18] (termed as “SP-HPC”). $M = 2$, $L = 4$, $K = 2$, $i_{LY} = 2$. $M_R = N_R$ for all the curves.

as $N_S$ increases, which results from the CA constraint, it is basically acceptable in practice where $N_S$ is generally small. Moreover, it is clear that the performance of LC-HPC with $K = 2$ is better than that with $K = 1$, but when $K \geq 2$, further increasing $K$ leads to little performance improvement. Thus, basically $K = 2$ is suitable for practical usage.

Fig. 7 shows the comparison of achievable rates of the proposed LC-HPC approach with the sparse precoding approach in [18] (termed as “SP-HPC”). The HIBS scheme is exploited for the estimation of MPCs for LC-HPC, while the search method proposed in [18] is used for the SP-HPC approach. That is to say, this figure shows the performance comparison of the overall solutions proposed in this paper and [18]. The training sequence is assumed long enough to provide sufficiently high SNR for the MPC estimations in these two approaches. From this figure we find that LC-HPC achieves a close performance to SP-HPC. The performance of SP-HPC gets improved as $N_S$ and the number of RF chains increases, and is basically better than that of LC-HPC when $N_R$ and $N_S$ are not small. That is because SP-HPC selects the steering vectors directly from the optimization of the achievable rate, while LC-HPC just selects several estimated significant MPCs as the analog precoding matrix. However, when $N_S$ or the number of RF chains is small, LC-PHC behaves even better. For instance, when $N_S = 1$, LC-PHC with only 1 RF chain is even better than SP-HPC with 4 RF chains. This is again due to the hierarchical codebook design in [18], where the wide-beam codewords may have deep sinks within the beam coverage when the numbers of RF chains and $N_S$ are not large enough, which may easily result in miss-detection of MPCs.
VI. CONCLUSIONS

In this paper, a low-complexity overall hybrid precoding and channel estimation approach has been proposed. In the channel estimation phase, a new hierarchical multi-beam search scheme, which uses a pre-designed analog hierarchical codebook and the particular channel structure in mmWave communication, was proposed. While in the hybrid precoding phase, the analog precoding is designed to steer along the $N_S$ acquired MPCs/clusters at both Tx/Rx sides, and the digital precoding operates on the $N_S \times N_S$ equivalent baseband channel. Performance evaluations show that, compared with the approach proposed in [13], the newly proposed approach achieves a close performance to the alternative, or even a better one when the number of RF chains or streams is small. Moreover, the computational complexity of the hybrid precoding is reduced from $O(N_A^3)$ to $O(N_S^3)$, where basically $N_S \ll N_A$, while the required time slots for the multi-beam search is reduced from $N_S^3 M^2$-proportional to $N_S M$-proportional.

ACKNOWLEDGMENTS

The authors would like to thank the authors of [18] to share their source code online, and particularly thank Dr. Ahmed Alkhateeb for his kind help to explain how to use the source code.

REFERENCES

[1] R. C. Daniels, J. N. Murdock, T. S. Rappaport, and R. W. Heath, “60 GHz wireless: up close and personal,” IEEE Microwave Magazine, vol. 11, no. 7, pp. 44–50, Dec. 2010.

[2] K.-C. Huang and Z. Wang, Millimeter Wave Communication Systems. Hoboken, New Jersey, USA: Wiley-IEEE Press, 2011.

[3] E. Perahia, C. Cordeiro, M. Park, and L. L. Yang, “IEEE 802.11 ad: defining the next generation multi-Gbps Wi-Fi,” in IEEE Consumer Communications and Networking Conference (CCNC). Las Vegas, NV: IEEE, Jan. 2010, pp. 1–5.

[4] S. K. Yong, P. Xia, and A. Valdez-Garcia, 60GHz Technology for Gbps WLAN and WPAN: From Theory to Practice. West Sussex, UK: Wiley.

[5] Z. Xiao, “Suboptimal spatial diversity scheme for 60 GHz millimeter-wave WLAN,” IEEE Communications Letters, vol. 17, no. 9, pp. 1790–1793, Sept. 2013.

[6] F. Khan and J. Pi, “Millimeter-wave mobile broadband: unleashing 3–300GHz spectrum,” in IEEE Wireless Commun. Netw. Conf., Cancun, Mexico, March 2011.

[7] A. Alkhateeb, J. Mo, N. González-Prelcic, and R. Heath, “MIMO precoding and combining solutions for millimeter-wave systems,” IEEE Communications Magazine, vol. 52, no. 12, pp. 122–131, Dec. 2014.

[8] J. Choi, “On coding and beamforming for large antenna arrays in mm-wave systems,” IEEE Wireless Communications Letters, vol. 3, no. 2, pp. 193–196, April 2014.

[9] S. Han, I. Chih-Lin, Z. Xu, and C. Rowell, “Large-scale antenna systems with hybrid analog and digital beamforming for millimeter wave 5G,” IEEE Communications Magazine, vol. 53, no. 1, pp. 186–194, Jan. 2015.

[10] W. Roh, J.-Y. Seol, J. Park, B. Lee, J. Lee, Y. Kim, J. Cho, K. Cheun, and F. Aryanfar, “Millimeter-wave beamforming as an enabling technology for 5G cellular communications: theoretical feasibility and prototype results,” IEEE Communications Magazine, vol. 52, no. 2, pp. 106–113, Feb. 2014.

[11] S. Sun, T. S. Rappaport, R. Heath, A. Nix, and S. Rangan, “MIMO for millimeter-wave wireless communications: beamforming, spatial multiplexing, or both?” IEEE Communications Magazine, vol. 52, no. 12, pp. 110–121, Dec. 2014.

[12] Y. Niu, Y. Li, D. Jin, L. Su, and A. V. Vasilakos, “A survey of millimeter wave communications (mmwave) for 5g: opportunities and challenges,” Wireless Networks, vol. 21, no. 8, pp. 2657–2676, 2015.

[13] J. Wang, Z. Lan, C. Pyo, T. Baykas, C. Sum, M. Rahman, I. Gao, R. Funada, F. Kojima, and H. Harada, “Beam codebook based beamforming protocol for multi-Gbps millimeter-wave WPAN systems,” IEEE Journal on Selected Areas in Communications, vol. 27, no. 8, pp. 1390–1399, Oct. 2009.

[14] J. Wang, Z. Lan, C. Sum, C. Pyo, J. Gao, T. Baykas, A. Rahman, R. Funada, F. Kojima, and I. Lukkis, “Beamforming codebook design and performance evaluation for 60GHz wideband WPANs,” in IEEE Vehicular Technology Conference Fall (VTC 2009-Fall). Anchorage, AK: IEEE, Sept. 2009, pp. 1–6.

[15] S. Hur, T. Kim, D. J. Love, J. V. Krogmeier, T. A. Thomas, and A. Ghosh, “Millimeter wave beamforming for wireless backhaul and access in small cell networks,” IEEE Transactions on Communications, vol. 61, no. 10, pp. 4391–4403, Oct. 2013.

[16] Z. Xiao, L. Bai, and J. Choi, “Iterative joint beamforming training with constant-amplitude phased arrays in millimeter-wave communications,” IEEE Communications Letters, vol. 18, no. 5, pp. 829–832, May 2014.

[17] L. Chen, Y. Yang, X. Chen, and W. Wang, “Multi-stage beamforming codebook for 60GHz WPAN,” in 2011 6th International ICST Conference on Communications and Networking in China (CHINACOM). Harbin, China: IEEE, Aug. 2011, pp. 361–365.

[18] A. Alkhateeb, O. El Ayach, G. Leus, and R. Heath, “Channel estimation and hybrid precoding for millimeter wave cellular systems,” IEEE Journal of Selected Topics in Signal Processing, vol. 8, no. 5, pp. 831–846, Oct. 2014.

[19] O. El Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. Heath, “Spatially sparse precoding in millimeter wave MIMO systems,” IEEE Transactions on Wireless Communications, vol. 13, no. 3, pp. 1499–1513, March 2014.

[20] T. S. Rappaport, Y. Qiao, J. I. Tamir, J. N. Murdock, and E. Ben-Dor, “Cellular broadband millimeter wave systems,” in IEEE Journal of Selected Topics in Signal Processing, vol. 1, no. 1, pp. 156–166, June 2007.

[21] Z. Xiao, X.-G. Xia, D. Jin, and N. Ge, “Iterative eigenvalue decomposition and multipath-grouping Tx/Rx joint beamformings for millimeter-wave communications,” IEEE Transactions on Wireless Communications, vol. 14, no. 3, pp. 1595–1607, March 2015.

[22] T. He and Z. Xiao, “Suboptimal beam search algorithm and codebook design for millimeter-wave communications,” Mobile Networks and Applications, vol. 20, no. 1, pp. 86–97, Jan. 2015.

[23] J. Nsenga, W. Van Thillo, F. Horlin, V. Ramon, A. Bourdoux, and R. Lauwereins, “Joint transmit and receive analog beamforming in 60 GHz MIMO multipath channels,” in IEEE International Conference on Communications (ICC). Dresden, Germany: IEEE, June 2009, pp. 1–5.

[24] V. Acero, D. Tse, and P. Viswanath, Fundamentals of Wireless Communication. New York, USA: Cambridge University Press, 2005.

[25] A. Goldsmith, S. A. Jafar, N. Jindal, and S. Vishwanath, “Capacity limits of MIMO channels,” in IEEE Communications Magazine, vol. 41, no. 10, pp. 5–24, Oct. 2003.

[26] S. Hur, T. Kim, D. J. Love, J. V. Krogmeier, T. A. Thomas, and A. Ghosh, “Millimeter wave beamforming for wireless backhaul and access in small cell networks,” IEEE Transactions on Communications, vol. 61, no. 10, pp. 4391–4403, Oct. 2013.

[27] Z. Xiao, L. Bai, and J. Choi, “Iterative joint beamforming training with constant-amplitude phased arrays in millimeter-wave communications,” IEEE Communications Letters, vol. 18, no. 5, pp. 829–832, May 2014.