\textit{kT}-factorization is violated in production of high-transverse-momentum particles in hadron-hadron collisions

John Collins*  
Physics Department, Penn State University, 104 Davey Laboratory, University Park PA 16802, U.S.A.  
Jian-Wei Qiu†  
Department of Physics and Astronomy, Iowa State University, Ames IA 50011, U.S.A. and  
High Energy Physics Division, Argonne National Laboratory, Argonne IL 60439, U.S.A.  
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We show that hard-scattering factorization is violated in the production of high-$p_T$ hadrons in hadron-hadron collisions, in the case that the hadrons are back-to-back, so that \textit{kT} factorization is to be used. The explicit counterexample that we construct is for the single-spin asymmetry with one beam transversely polarized. The Sivers function needed here has particular sensitivity to the Wilson lines in the parton densities. We use a greatly simplified model theory to make the breakdown of factorization easy to check explicitly. But the counterexample implies that standard arguments for factorization fail not just for the single-spin asymmetry but for the unpolarized cross section for back-to-back hadron production in QCD in hadron-hadron collisions. This is unlike corresponding cases in $e^+e^-$ annihilation, Drell-Yan, and deeply inelastic scattering. Moreover, the result endangers factorization for more general hadroproduction processes.

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\section{I. INTRODUCTION}

The great importance of hard-scattering factorization in high-energy phenomenology hardly needs emphasis. Essential to its application and predictiveness is the universality of parton densities (and fragmentation functions, etc) between different reactions. However, as can be seen from \cite{1, 2, 3, 4}, process-dependent Wilson lines appear to be needed in the inclusive production of two high-transverse-momentum particles in hadron-hadron collisions, i.e., in the process

\begin{equation}
H_1 + H_2 \to H_3 + H_4 + X. \tag{1.1}
\end{equation}

In this paper we will show that this situation definitively leads to a breakdown of factorization.

The standard expectation is that the cross section is a convolution of a hard scattering coefficient $d\sigma$, parton densities, fragmentation functions and a possible soft function:

\begin{equation}
E_3E_4 \frac{d\sigma}{d^3p_i d^3p_l} = \sum \int dH_{i/H} d_{i/H} \int f_{i/H} f_{j/k} d_{3/4} d_{4/4} + \text{power-suppressed correction}. \tag{1.2}
\end{equation}

Here the sum and integral are over the flavors and momenta of the partons of the hard scattering, $f_{i/H}$ denotes a parton density, and $dH_{i/H}$ a fragmentation function.

It is noteworthy that the classical published proofs for factorization in hadron-hadron scattering \cite{6, 7} only concerned the Drell-Yan process. There are a number of difficult issues in the proof that are highly non-trivial to extend to other reactions in hadron-hadron collisions, even though Eq. (1.2) is a standard expectation.

We will examine the case that the produced hadrons are almost back-to-back. Then the appropriate factorization property is \textit{kT}–factorization, which entails \cite{7} the use of transverse-momentum dependent (TMD) parton densities and fragmentation functions. However, the issues raised by our counterexample to factorization are sufficiently general that they create a need to examine very carefully the arguments for factorization in hadroproduction of hadrons even in situations where ordinary collinear factorization with integrated densities is appropriate. In the case of \textit{kT}–factorization with TMD densities, the factorization formula needs the insertion of a soft factor $S$, not shown in Eq. (1.2).

The problems concern gluon exchanges between different kinds of collinear line, as in Fig. 2 below. To obtain factorization, the gluon attachments must be converted to Wilson lines in gauge-invariant definitions of the parton densities and fragmentation functions. This relies \cite{6} on the use of Ward identities applied to approximations to the amplitudes. But the approximations are only valid after certain contour deformations on the loop momenta.

Bacchetta, Bomhof, Mulders and Pijlman \cite{1, 2, 3, 4} argued that because of the complicated combination of initial- and final-state interactions, the Wilson lines must be modified. What is not so clear is the interpretation of their result. So in the present paper we present an argument to make fully explicit the failure of factorization.

Since the issue is one of factorization in general, and not just specifically in QCD, we clarify the issue by examining a particular process in a model field theory. The process is a transverse single-spin asymmetry of the kind controlled by a Sivers function. This is a case where prob-
lems in the contour deformation directly affect the value of
the cross section at the lowest possible order of pertur-
bation theory. Our model field theory is simple enough
that the calculations and their interpretation as imply-
ing factorization violation are unambiguous. But, as we
will explain in the final section, we expect the failure of
factorization to be more general: our particular process
and model simply make it very easy to see the failure.

II. CONSTRUCTION OF MODEL

Since proofs of factorization apply to quantum field
theories in general (if they are renormalizable), the con-
struction of a counterexample, to demonstrate and to
understand a failure of the normal methods of proof, is
conveniently done in a simple model theory.

Our model resembles the one used by Brodsky, Hwang,
and Schmidt [8] in their discussion of single spin asym-
metries. It is defined as follows:

- The gauge group is abelian. This simplifies the
  graphs, and allows the next feature.
- The gluon is massive. This avoids the discussion
  being confused by actual infra-red divergences in
  the S-matrix.
- The initial-state particles correspond to Dirac fields
  that are neutral under the gauge group. We will
call them hadrons. The fields need to be Dirac
fields in order to have the single transverse spin
asymmetries that we will examine. We will use two
types of hadron.
- Each “hadron field” $H_i$ will have a coupling to a
  Dirac field $\psi_i$ and a scalar field $\phi_i$, which in [8]
  would be called a disquark field:

$$\lambda_i \left( \bar{H}_i \psi_i \phi_i^\dagger + \bar{\psi}_i H_i \phi_i \right). \tag{2.1}$$

The quark field $\psi_i$ has a coupling $g_i$ to the gauge
field, and the scalar field $\phi_i$ has the opposite
coupling.
- All the masses in the theory are comparable, to
  avoid confusing the calculation with logarithmic de-
  pendence on large ratios of masses.

In our analog of hadroproduction, Fig. 7, the two initial
state hadrons are of the two different types, and we use
this to simplify our argument for non-factorization. The
lines in the the lower part of graphs are chosen to be
those of gauge coupling $g_1$, while the lines in the upper
part of the graph are those with coupling $g_2$. But if
the attachments of the gluon to the upper lines were in
some way to correspond to a Wilson line in the parton
density in the hadron in the lower part of the graph, the
charge would have to be $g_1$. Since $g_1$ and $g_2$ are arbitrary,
there is no way to make a correspondence between the
graph and the Wilson line formalism, provided that the
contribution is non-zero, as we will demonstrate. This
will also insulate us against sign errors and the like.

We will also choose the detected outgoing particles $H_3$
and $H_4$ to correspond to the scalar fields. The sole pur-
pose here is to simplify the Dirac algebra slightly, thereby
making the calculations more transparent and elemen-
tary.

One feature of our counterexample appears to be very
special to an abelian gauge theory. This is that the two
couplings $g_1$ and $g_2$ need have no relation to each other:
there is a continuous infinity of representations of the
gauge group. In contrast, there is a single value of the
coupling $g$ for all the fields in a non-abelian theory. The
role of the ratio $g_2/g_1$ is now taken over by the represen-
tation matrices for the different fields in QCD (triplet,
antitriplet, octet), with the different couplings related
by factors of rational numbers. So in any particular ex-
ample there is a potential for a numerical coincidence
between the sizes of the numerical values of the graphs,
which could then appear to give consistency with factor-
zation. A counterexample to factorization would then
be more complicated, with a comparison of cases with
different kinds of partons (quarks, antiquarks or gluons),
cf. [1, 2, 3, 4].

III. REVIEW OF SIDIS AND DY

We now review how [8] a transverse single-spin asym-
metry (SSA) arises in semi-inclusive deep-inelastic scat-
ering (SIDIS), at the level of one-gluon-exchange, and
how it determines [9] the Wilson line that defines parton
densities. Then we review the differences that give factor-
ization with an exact sign reversal in the Sivers function
for the Drell-Yan process [9, 10]. This will give us meth-
ods of calculation that will give us a very elementary way
to obtain the SSA for the process [11].

A. SIDS

With the electromagnetic part of the scattering fac-
tored out, SIDIS is the process

$$\gamma^*(q) + H(p) \rightarrow H'(r) + X. \tag{3.1}$$

We use light-front coordinates in which the incoming mo-
menta are

$$p = \left( p^+, \frac{m_H^2}{2p^+}, 0_T \right), \quad q = \left( -xp^+, \frac{Q^2}{2xp^+}, 0_T \right). \tag{3.2}$$

The detected outgoing particle is defined by a longitudi-
dinal momentum fraction $z$ and a transverse momentum
$r_T$:

$$r = \left( xp^+, \frac{r_T^2 + m_H^2}{zQ^2}, \frac{zQ^2}{2xp^+}, r_T \right). \tag{3.3}$$
We will assume that $Q$ is large and that the detected transverse momentum $r_T$ is of order a hadronic mass scale $m$.

The lowest-order graph Fig. 1 gives the following contribution to the differential structure tensor:

\[
\frac{dW^{\mu\nu}}{dz\,d^2r_T} = \frac{\lambda^2}{4\pi} \int \frac{dk^+\,dk^-}{(2\pi)^4} \delta\left(z - \frac{k^- + q^-}{q^-}\right) \frac{(2k^\mu + q^\mu)(2k^\nu + q^\nu)}{(k^2 - m_\rho^2)^2} (2\pi)^2 \delta((q + k)^2 - m_\pi^2) \delta((p - k)^2 - m_\pi^2) \times \frac{1}{2} \text{Tr}\left[(\not{p} + m_H)\left(1 + \gamma_5 \not{\sigma}\right)(\not{p} - \not{k} + m_\psi)\right].
\tag{3.4}
\]

The internal partons are all collinear to the target, i.e., $k^+ \sim p^+$, $k^- \sim m^2/p^+$, $k_T \sim m$, and to leading power in $Q$, parton model kinematics apply, so that $k^+ \simeq x p^+$ and $z \simeq 1$. We will assume throughout that the spin vector $s$ corresponds to a transverse spin (in the $(q,p)$ frame), and that it is normalized so that its extremal value obeys $s^2 = -1$. Since there is only one initial-state hadron, we do not bother with labels to indicate the kind of hadron (e.g., $\lambda_1$ or $\lambda_2$).

The above formula is simply related to a parton density:

\[
\frac{dW^{\mu\nu}}{dz\,d^2r_T} = \frac{(p^\mu - q^\mu p \cdot q/q^2)(p^\nu - q^\nu p \cdot q/q^2)}{p \cdot q} \delta(z - 1) P_{\phi/H}(x, r_T) + \text{power-suppressed correction},
\tag{3.5}
\]

where the parton density at lowest order is

\[
P_0(x, k_T) = \frac{\lambda^2 x (1 - x)}{16\pi^3} \frac{1}{k^2 + m_\pi^2(1 - x) + m_\psi^2 x - m_H^2 x(1 - x)}.
\tag{3.6}
\]

reason for the vanishing is that the trace is imaginary, while the rest of the graph is real, so that the contribution to a cross section must be zero.

The lowest-order graphs for a non-zero SSA are the one-gluon-exchange graphs in Fig. 2 which get an imaginary part from an intermediate state that can go on-shell; this state is made by the lines with momenta $q + k - l$ and $p - k + l$. Standard power-counting shows that the exchanged gluon can only be collinear to the target or soft. The minus momentum of the gluon is trapped in the region $l^- \sim m^2/p^+$ by the other target-collinear lines. The on-shell intermediate state corresponds to small angle elastic scattering, and so to very small $l^+$, of order $p^+ m^2/Q^2$. There the only significant dependence on $l^+$ is in the upper parton propagator. Multiplied by the neighboring gluon vertex this gives

\[
-\frac{g(2q^\mu + 2k^\mu - l^\mu)}{(q + k - l)^2 - m_\gamma^2 + i\epsilon} \simeq -\frac{-g\delta^\mu}{-l^+ + \text{other terms} + i\epsilon},
\tag{3.8}
\]

where the “other terms” are small or independent of $l^+$, and we have taken a leading power approximation for the momenta in the numerator. The contour of integration of $l^+$ can therefore be deformed into the lower half plane until $l$ is target-collinear. In that case the “other terms” in the numerator, shows that the gluon exchange correction is

![Graph](image-url)
FIG. 2: Virtual one-gluon-exchange corrections to Fig. 1 that give a SSA.

FIG. 3: Virtual one-gluon-exchange correction to parton density. The upper double line is the Wilson line, and the graph shown, together with its hermitian conjugate gives the first contribution to the Sivers function.

FIG. 4: Lowest-order graph for the Drell-Yan process.

equivalent to a contribution to the parton density with a suitable Wilson line, Fig. 2.

Let us perform the $k^+$ and $k^-$ integrals by the on-shell conditions on the final state, and let us perform the $l^-$ integration by contour integration. Then the necessary imaginary part comes simply from the imaginary part of $(3.8)$ and thus from the replacement

$$-g(2q^\mu + 2k^\mu - l^\nu) \rightarrow ig\pi\delta^{\mu}_{\nu} \delta(l^+) .$$

This gives rise to an SSA with the aid of the trace

$$\frac{1}{2} \text{Tr} \left[ (\not{p} + m_H) \gamma_5 (\not{\bar{p}} - \not{k} + \not{l} + m_\psi) \gamma^+ (\not{\bar{p}} - \not{k} + m_\psi) \right]$$

$$\approx 2i\epsilon_{jk}s l^k p^+ [m_H (1 - x) + m_\psi] ,$$

(3.10)

where the approximation, good to leading power, arises from the neglect of the small components of $l$ with respect to the transverse components. The two-dimensional $\epsilon$ tensor obeys $\epsilon_{12} = 1$. Since the denominator in the integrand is not azimuthally symmetric in $l_T$, the integral over $l$ gives a non-zero result for the SSA from the whole graph.

The two graphs in Fig. 2 are related by hermitian conjugation and so they give equal contributions to the SSA.

B. Drell-Yan

The Drell-Yan (DY) process,

$$H_A(s) + H_B \rightarrow \gamma^+(q) + X ,$$

(3.11)

is treated quite similarly. We examine the cross section differential in $q_T$, and investigate a possible SSA, with $H_A$ having a transverse spin vector $s$.

In our model the lowest-order graph is Fig. 4. It is readily shown to be the (convolution) product of two transverse-momentum-dependent parton densities, and, just like the SIDIS process, it has no SSA at this order. Note that to have the process occur at the order shown within our model, the initial-state hadrons $H_A$ and $H_B$ must be antiparticles of each other.

The imaginary part in the amplitude needed to get an SSA with respect to the transverse spin $s$ of the lower hadron $H_A$ arises from graphs such as those in Fig. 5. Graphs (a) and (b) work just like those in Fig. 2 for SIDIS, except that the gluon couples to the incoming scalar antiparton instead an outgoing scalar parton, with the necessary reversal in sign of the coupling. Thus Eq. (3.8) is replaced by

$$g(2k_B^\mu + l^\mu) \rightarrow g\delta^{\mu}_{\nu} \delta(l^+) ,$$

(3.12)

which gives an imaginary part exactly opposite to that for SIDIS. The relative sign of the $l^+$ term and the $i\epsilon$ is now that for an initial-state interaction, so that the Wilson line in the operator definition of the parton density is now past-pointing instead of future-pointing. As shown in [9], an exact reversal of sign of the Sivers function between SIDIS and DY follows from the time-reversal symmetry of QCD.

No contribution to the SSA is given by graphs, like Fig. 5(c), in which the both ends gluons couple the the active...
FIG. 5: Virtual one-gluon-exchange corrections to Fig. 4 relevant for a SSA when the lower hadron has transverse spin $s$. Graph (a) and its hermitian conjugate (b) have imaginary parts (at the amplitude level) that give a non-zero SSA. Gluon exchange between the active partons, graph (c) and its not-shown conjugate, gives an imaginary part in the vertex correction that does not give a SSA. Spectator-spectator gluon exchange graphs, (d) and (e), do contribute individually to the SSA, but the two contributions cancel (at leading power).

IV. HADROPRODUCTION OF HADRONS

We now have the tools to make an extremely streamlined construction of a counterexample to factorization for the process of hadro-production of high transverse momentum hadrons, $H_1 + H_2 \rightarrow H_3 + H_4 + X$. We again use an SSA because non-factorization occurs with one gluon exchanged beyond the lowest order in which the reaction occurs at all. We choose $H_1$ to be the polarized hadron, and we choose the hadrons $H_1$ and $H_2$ to be of the two different flavors in our model. We also choose the detected final-state particles $H_3$ and $H_4$ to correspond to the two flavors of scalar parton. The high-transverse-momentum particles $H_3$ and $H_4$ are chosen to be almost back-to-back azimuthally (relative to the collision axis), so that transverse-momentum-dependent parton densities and fragmentation functions are needed for describing a factorized cross section.

The single lowest-order graph for the process is shown in Fig. 6. Its hard scattering is just the gluon-exchange subgraph. The cross section is the convolution of the hard scattering with a transverse-momentum-dependent parton density in each hadron. The fragmentation func-
tions in this order are trivial delta functions. Although the longitudinal momenta of the incoming partons for the hard scattering are determined from the kinematics of $H_3$ and $H_4$, only a sum of their transverse momenta is determined. Hence a convolution over the transverse-momentum densities is needed. As before, there is no SSA at this order.

The graphs giving the lowest-order SSA are shown in Fig. 7. They have an extra gluon exchanged between the spectator line in the polarized hadron and one of the active partons. As with the DY process, a sum over cuts of graphs with a spectator-spectator interaction cancels, while exchanges purely between active partons give no SSA. An exchange purely between the spectator $(p_1 - k_1)$ and the active parton line ($k_1$) in the polarized hadron has no relevant intermediate on-shell state and therefore does not contribute to the SSA.

So only the three graphs shown in Fig. 7 contribute to the SSA, together with their conjugate graphs. Exactly as in our discussion of SIDIS and DY, the graphs are the same to the leading power except for eikonal factors from the parton lines connecting the upper end of the gluon to the hard scattering. The appropriate replacements for these lines are

\[
\frac{-g_2(2k_1^\mu - l^\mu)}{(k_1 + l)^2 - m_{g_1}^2 + i\epsilon} \mapsto ig_2 \pi \delta_{\mu\nu} \delta(l^+), \tag{4.1}
\]

\[
\frac{-g_2(2k_2^\mu + l^\mu)}{(k_2 + l)^2 - m_{g_2}^2 + i\epsilon} \mapsto ig_2 \pi \delta_{\mu\nu} \delta(l^+), \tag{4.2}
\]

\[
\frac{-g_1(2k_3^\mu - l^\mu)}{(k_3 - l)^2 - m_{g_1}^2 + i\epsilon} \mapsto ig_1 \pi \delta_{\mu\nu} \delta(l^+), \tag{4.3}
\]

for a total of $i\pi (g_1 + 2g_2) \delta(l^+)$.  

The $g_1$ term corresponds to a gluon coupling to a future-pointing Wilson line in the operator definition of the parton density, the same as for SIDIS. However, it is impossible for the contribution proportional to $g_2$ to be represented in terms of a Wilson line connecting the two parton fields for the distribution of partons of type $g_1$ in the hadron $H_3$. This is simply because the coupling for any such Wilson line has to correspond to the color charge of the parton, i.e., $g_1$, and not $g_2$. The full Wilson line, or some generalization, is needed because exchanges of multiple gluons also contribute. The quantity we are looking at is definitely associated with the hadron $H_3$ rather than being in some kind of exotic soft factor, since the attachment of lower end of the gluon line to the spectator parton is necessary for the non-zero SSA; the triviality of this observation is a special feature of our particular model.

The contribution to the SSA is therefore non-universal and does not correspond to a parton density. That is, factorization is broken.

Of course, the fact that the contribution is obtained from an eikonalized line indicates that it can be obtained from some kind of representation in terms of Wilson line operators. But the matrix element is for some more complicated and non-universal object $[1, 2, 3, 4]$ that cannot be treated as a parton density. It is allied to the objects used by Balitsky [15] to discuss scattering at high-energy and small angles. The eikonalization indicates that substantial simplifications are possible. But that situation would go well beyond normal factorization.

V. DISCUSSION

We should first emphasize that there is a large overlap between the present paper and the work in Refs. $[1, 2, 3, 4]$. What is not so clear from the earlier work is whether factorization in any standard sense continues to hold for in the process (1.1). For example, in [1], we read “We have assumed factorization to hold in this treatment of TMD effects although it is, at present, certainly not clear whether such a factorization holds for hadron-hadron scattering processes with explicitly TMD correlators.”

Our primary result is to show by a counterexample that hard scattering $k_T$-factorization with universal parton densities fails for the production of high $p_T$ hadrons in hadron-hadron collisions, when a pair of measured hadrons is close to back-to-back azimuthally. The overall issue is that in a gauge theory arbitrary exchanges of gauge fields between different collinear groups (“jets”) can occur without any power suppression. To obtain factorization it is necessary to show that the sum over these exchanges can be absorbed into the definitions of the parton densities and fragmentation functions, assisted by certain cancellations. A full proof will be quite general, applying to a general gauge theory and to many reactions. So one particular counterexample is sufficient to show that such a proof does not exist; we can then choose the counterexample for maximum clarity and simplicity.

Even for those cases where factorization does hold, the need to make suitable definitions of the parton densities,
Much of that work concerns the small region, diffractive processes is a case of modified universality: different parton densities are needed when the scale of the hard scattering is given a large increment. But there is an evolution equation for the scale-dependence, and this applies to an individual parton density. No details or specification of the hard scattering is needed to treat the evolution equation, either to derive it or to apply it. We should therefore refer in this case to “modified universality”, not to non-universality. Similarly the reversal of the sign of the Sivers function between SIDIS and DY processes is a case of modified universality.

At the upper end of the exchanged gluon in our counterexample, the interactions can be treated in the eikonal approximation. This is very similar to other discussions of partons passing through the gluon field of another hadron. A selection of relevant papers is [15, 17, 18, 19]. Much of that work concerns the small x region, diffractive scattering, etc, whereas our counterexample applies in the fully conventional region where normal parton-model concepts are generally considered as fully applicable, i.e., parton fractional momenta are moderate and the scale of the hard scattering is comparable to the center-of-mass energy rather than being much less.

Of course, interesting simplifications do occur, so that useful quantitative estimates can surely be obtained for the non-factorizing effects. However the methods are rather different than those for conventional factorization. Refs. [18,17,18,19] indicate that the effects of the eikonalized interactions are substantial, so that the numerical effects of non-factorization should be significant; in the present paper we did not estimate the numerical size of the non-factorization.

The gluon exchanges in our counterexample are clearly tied to the target hadron at their lower end. But the coupling at the upper end concerns some parton other than the one coming out of the lower hadron. The non-canceling terms are sufficiently tied to the color flow at the hard interaction that they are not universal in any normal sense. This is the clearest indication of non-universality.

The reader should not be misled by specific features of our counterexample into supposing that the failure of factorization is correspondingly restricted. These features include: the use of an SSA, the particular features of the model, and the particular order of perturbation theory. The use of the SSA is simply a way of getting the maximal conceptual sensitivity to problems in constructing a proof of factorization. For an unpolarized cross section, we would need an extra gluon to be exchanged in order for the nonfactorization issues to arise, from graphs such as those in Fig. 8. Evidently, to demonstrate nonfactorization explicitly in this case, the number of graphs would be much more lengthy. Standard power-counting arguments show that the contribution of this and related graphs is of leading power. It is very important to determine whether or not the sum of the potentially non-factorizing contributions actually does or does not cancel in the unpolarized cross section.

Similarly, the choice of quantum numbers of the parton fields, of the abelian gauge group, and of the quantum numbers for the detected particles was simply to provide maximum transparency and simplicity to the counterexample. The fact that non-factorization can only occur with at least two extra gluons in an unpolarized cross section might suggest that the non-factorization is at high order in the strong coupling and therefore substantially suppressed. However the region of interest is at low virtuality for the gluons, so that the appropriate coupling is for a low momentum scale, where QCD is a strongly coupled theory.

Even so, the number of extra gluons needed implies that the effects of non-factorization will only appear in quite high order in conventional perturbative QCD cal-
FIG. 8: The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.

FIG. 9: In a conventional perturbative QCD calculation for an unpolarized partonic cross section, non-factorization by the mechanisms discussed in this paper would first appear in graphs of this order.

culations. Normally one performs calculations with on-shell massless quarks and gluons, and extracts collinear divergences that are grouped with parton densities and fragmentation functions; any remaining divergences cancel between graphs. Non-factorization in the hadronic cross section corresponds to uncanceled divergences in quark-gluon calculations. The lowest order in which the mechanisms we have discussed could possibly give an uncanceled divergence in unpolarized partonic cross sections is next-to-next-to-next-to-leading-order (NNNLO), as in Fig. 9. The region for the uncanceled divergence is where the lower gluon is collinear to the lower incoming quark, and two of the exchanged gluons are soft. This graph is at least one order beyond all standard perturbative QCD calculations.

Because our calculations directly concern cross sections that use transverse-momentum-dependent parton densities, a certain amount of care is needed in interpreting the results. The natural direction for the Wilson lines is light-like, as from Eq. (3.8). However light-like Wilson lines give divergences in transverse-momentum-dependent densities [11]. These are due to rapidity divergences [20] in integrals over gluon momentum; they cancel [7] in conventional parton densities only because of an integral over all transverse momentum in integrated parton densities. The solution adopted by Collins, Soper and Sterman [7] (CSS) was to define parton densities without Wilson lines but in a non-light-like axial gauge. The gauge-fixing vector introduces a cut-off on gluon rapidity, and then an evolution equation with respect to the cut-off was derived. The non-perturbative functions involved in this CSS evolution equation have been measured (e.g., [21]) in fits to DY cross sections, and would be an essential ingredient in testing non-factorization.

However, there are some unsatisfactory features of the use of axial gauges, which are made particularly evident in polarized cross sections. This includes complications concerning gauge links at infinity [22], when a Wilson line formalism is used. A much better definition is to use a non-light-like Wilson line. This again obeys an equation of the CSS form. It is also possible to use a subtractive formalism [20, 23] with light-like Wilson lines but with generalized renormalization factors involving vacuum expectation values of Wilson lines, which also implement a rapidity cutoff, and lead to a CSS equation.

To test the predicted non-factorization, we simply need predictions of high-\(p_T\) hadrons in hadron-hadron collisions, made on the basis of fits to parton densities in DIS and DY and to fragmentation functions in \(e^+e^-\) and SIDIS [24]. Probing the SSA would be particularly interesting, and such measurements are underway at Relativistic Heavy Ion Collider (RHIC) [23, 26]. The same physics is probed in the transverse shape of jets, and would be worth investigating.

Our counterexample applies in a kinematic region where the normal intuitive ideas of the parton model appear quite appropriate, even with a generalization to \(k_T\)-factorization. Therefore it forces us to question under what conditions factorization is actually valid and to what extent it has actually been demonstrated. It cannot be assumed that naive extensions of apparently established results are applicable beyond the cases to which the actual proofs explicitly apply.

For hadron-hadron collisions, factorization has been proved [5, 6] for the Drell-Yan process integrated over transverse momentum or at large transverse momentum (of order \(Q\)). These proofs apply in the presence of gluon exchanges of the kind that we discuss in the present paper. But these papers do not go beyond this, to the production of hadrons. Because factorization is important to all aspects of hadron-collider phenomenology, it is critical to solve this problem for the hadroproduction of high-\(p_T\) hadrons. Given our counterexample to \(k_T\)-factorization, a proof of factorization can only succeed in a situation where conventional collinear factorization is appropriate. For dihadron production this is when the hadron-pair has itself large transverse momentum or when the pair’s out-of-plane transverse momentum is integrated over a wide range.

In fact, Nayak, Qiu and Sterman [27] have recently given strong arguments that collinear factorization does indeed hold in such a situations. The graphs examined...
are similar to ours. They apply Ward identities to prove an eikonalization generalizing our specific calculations. Then they observe that a unitarity cancellation occurs of a kind endemic in factorization proofs \[6, 23\]. This concerns graphs that are related by different placements of the final-state cut. In our model, one example is given by Fig. 2(a) and (b), and another is Fig. 17(a) and its conjugate. Such cancellations fail in our examples, because the final-states of the related graphs have different transverse momentum, and the cross section is not sufficiently inclusive in transverse momentum.

Mechanisms that cause \(k_T\)-factorization to fail in back-to-back hadron production also tend to cause resummation methods to fail. They will also tend to break factorization or cause large perturbative corrections when detailed distributions of final-state hadrons are examined. Since many such cases are implicit in the analysis of complicated multi-jet cross sections, and of jet shapes and the like, encountered in searches for new physics, further understanding is essential as are quantitative estimates of the effects. They can have a particularly important effect in the extrapolation to the LHC of quantitatively measured distributions at the Tevatron, for example as embodied in Monte-Carlo event generators. The methods of \[15, 17, 18\] will be important. Probably some of these effects have already been modeled in some approximation and in at least some Monte-Carlo event generators, for example by the soft color interaction model \[29\].

Troublesome though it may be for phenomenology, breaking of factorization should be viewed not as some kind of failure, but as an opportunity. Examination of the distribution of high-transverse-momentum hadrons in hadron-hadron collisions will lead to interesting non-trivial phenomena.

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