Ultra-Precision Rotating Reference Based on Self-acting External-pressure Complex Action Principle

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Abstract. In order to increase the loading capacity and stiffness of a gas lubricated rotating reference and its anti-disturbance capability, and further improve the rotating precision, this paper puts forward a kind of ultra-precision rotating reference based on a self-acting external-pressure complex action principle. First two important parameters were defined, the external pressurized bearing number, and the pressure ratio of the complex action gas bearing. On this basis, the analytical model of this rotating reference was set up completely. This model shows that the loading capacity function of the rotating reference is composed of six items. According to this analytical model, ignoring the self-acting effect, the loading capacity and stiffness increased to 30% higher than the usual in an external pressurized gas lubricated reference.

Keywords: gas lubrication, journal, gas bearing, hybrid bearing

1. Introduction

Gas bearings have gained extensive use because of their high precision, high efficiency, adaptive capacity to ambient temperature. But their main weaknesses, such as the lower load-carrying capacity, restricted its application field, especially in the field of precision work. The hybrid gas bearing could increase its load-carrying capacity greatly, especially the high-speed bearings. Many research institutes have engaged in some experimental researches and grow some fruits, even some applications and patents. But a kind of analytical model was needed to guide the practice, and find a better structure. In this paper, a novel structure was put forward, and under the incompressible condition, its analytical model was set up. Using this model the load-carrying capacity function was analyzed in some degree. The patents of this kind of complex gas bearing structure have been claimed.

2. Modeling

The structure of an infinite long flat hybrid gas bearing was shown as Fig.1. The shape of the groove section was shown as Fig.2. The structure of the throttling orifice was shown as Fig.3. The pressure-flux curves of different throttling principles, such as throttling gap, were alike. So the model built up with the throttling orifice could be used in other situations, only some coefficients should be changed.

2.1. Assumptions
Assume:
(1) The gas was incompressible;
(2) The pressure on the symmetry line of the gas bearing was equal, and equals $p_s$;
(3) The pressure is a linear function of $x$, $y$, in the groove area, the ridge area, and no-groove area;
When the throttling gap or the isopressing groove along the symmetry line of the gas bearing was used, the assumption (2) was more near the truth.

According to the symmetry of the flat gas bearing and the assumption (2), only take into account the half width of the flat gas bearing, and the performance of the whole infinite long flat gas bearing could be obtained.

The half width flat gas bearing was shown as Fig.4.

Use the $O$ as the origin, set up the rectangular coordinates system $\langle xOy \rangle$, by the lines of $y=L_1$ and $y=L_2$ the half width flat gas bearing was divided into three parts: (I) first grooved area, (II) second grooved area, (III) no groove area; Use the $O$ as the origin, set up the skew coordinates system $\langle xOs \rangle$.

The conversion of them should accord to equation (1) in area I, and equation (2) in area II.

$$
x_s = x - y tg \alpha; \quad y_s = y \quad (1)
$$

$$
x_s = x + (y - \eta \theta_1)tg \alpha; \quad y_s = y \quad (2)
$$

The angle of the groove line is $\alpha$ (rad). Define the groove width coefficient as $\beta$, the groove depth coefficient as $\gamma; \beta = b_2/b, \gamma = (\Delta + h)/h; b_2$ is width of groove, $b$ is width of ridge, $\Delta$ is depth of groove, and $h$ is thickness of gas film. Define the nondimensional coordinates of $x$, $y$, $x_s$, and $y_s$ as $X, Y, X_s$, and $Y_s$: $X=x/b, Y=y/b, X_s=x_s/b, Y_s=y_s/b$; Define the nondimensional width of the flat gas bearing as $\eta$: $\eta=L/b, L$ is width of bearing; Define the groove distribution coefficient as $\theta_1, \theta_2$: $\theta_1 = L_1/L, \theta_2 = L_2/L, L_1$ is width of first grooved area, $L_2$ is width of second grooved area. Define the nondimensional pressure, nondimensional inlet pressure, nondimensional pressure of gas supply as $P, P_s, \text{ and } P_o$: $P=p/p_a, P_s=ps/p_a, P_o=po/p_a$ and $p, ps, po$ are pressure, inlet pressure, pressure of gas supply, and ambient pressure.

2.2. Boundary conditions and Pressure distribution

In a couple of groove and ridge areas, the pressure in the groove $P_g$ and the pressure at the ridge $P_r$ are linear functions of $X_s, Y_s$, the pressure in the no groove area $P_{II}$ is the linear function of $Y_s$ [1]. The first corner mark is 1, means that is the parameter in area I, The first corner mark is 2, means that is the parameter in area II:

$$
P_{Ig} = k_{10} + k_{11}X_s - k_2Y_s; \quad P_{rI} = k_{10} - \beta k_{11}[X_s - (\beta + 1)] - k_2Y_s; \quad P_{2g} = k_{20} + k_{21}X_s - k_3(Y_s - \eta \theta_1)
$$

$$
P_{2r} = k_{20} - \beta k_{21}[X_s - (\beta + 1)] - k_3(Y_s - \eta \theta_1); \quad P_{III} = 1 + k_4(\frac{1}{2}\eta - Y) \quad (3)
$$

The $k_{10}, k_{11}, k_{20}, k_{21}, k_2, k_3$ and $k_4$ are constants.

Boundary conditions:
(1) On the line of $Y_s=0$, the pressure average in area I $P_I$ equals $P_s$;
(2) On the line of $Y_s=\eta/2$, the pressure in no groove area $P_{III}$ equals $P_o$;
(3) On the line of $Y_s=\theta_1 \eta, P_I$ equals $P_{II}$;
(4) On the line of $Y_s=\theta_2 \eta, P_{II}$ equals $P_{III}$;
(5) On groove-line, the flux at the ridge and that in the groove should be equal [1];
(6) On the boundary between area I and II, the average flux along $y$ should be equal;
(7) On the boundary between area II and III, the average flux along $y$ should be equal;
(8) The flux of orifice is twice of the flux of flat gas bearing along $Y$ between two orifices.

Using the boundary conditions (1) to (7), the constants $k_2, k_3, k_4$ could be educed as equation (4), (5), (6), (7):

$$
\Delta_0 = \left( (\gamma^3 + \beta)(\beta \gamma^3 + 1) - \beta (\gamma^3 - 1)^2 \sin^2 \alpha \right)
$$

(4)
The is defined as \( \mathcal{I}_1 = 2 \frac{\pi^2}{(1 - 2\pi^2)} \) and the \( \mathcal{I}_2 \) is defined as \( \mathcal{I}_2 = 6\mu U/L (P_{a2} - 1) \) [1].

The pressure distribution of half width flat gas bearing is shown as Fig.5.

The relative average pressure \( \Delta P \) is:

\[
\Delta P = \frac{1}{2} N_{o1} \Delta E_{o1} + \frac{1}{2} N_{o2} \Delta E_{o2} + \frac{1}{2} E_g (P_s - 1) - \frac{1}{2} (P_s - 1) \]  

The \( N_{o1} \) is the first self-acting loading coefficient, the \( N_{o2} \) is the second self-acting loading coefficient, the \( E_{o1} \) is the first self-acting load-carrying capacity function, and the \( E_{o2} \) is the second self-acting load-carrying capacity function; the \( E_g \) is surface feature coefficient, see equation (9), (10), (11):

\[
N_{o1} = \frac{1}{2} (2\theta_1 - \theta_2 + 2\theta_1 (\theta_2 - \theta_1); \quad E_{o1} = \frac{2\beta (\gamma - 1)(\gamma - 1)\cos \alpha \sin \alpha}{(\Delta_0 + \epsilon (\beta + 1)(\gamma^3 + \beta))} \]
\[
N_{o2} = 2\theta_1 \epsilon (\theta_2 - \theta_1); \quad E_{o2} = \frac{2\beta (\gamma^3 - 1)(\gamma - 1)\cos \alpha \sin \alpha (\beta + 1)(\gamma^3 + \beta)}{\Delta_0 (\Delta_0 + \epsilon (\beta + 1)(\gamma^3 + \beta))} \]
\[
E_g = 2\theta_2 - \frac{\epsilon (\beta + 1)(\gamma^3 + \beta)}{\Delta_0 + \epsilon (\beta + 1)(\gamma^3 + \beta)} \]

Define external pressure gas bearing number \( \lambda \) as equation (12):

\[
\lambda = \frac{6C_jC_jL}{h^4 L_o} = \frac{6C_j d^4 L}{h^4 L_o} \; \; \; \; (P_s - 1) = \lambda \left( \frac{P_s - 1}{E_{o1}} + \Delta E_{o1}N_{o1} \right); \quad \frac{\theta_1 - 2\theta_2}{\theta_2} = \frac{2\theta_1 - 2\theta_2}{1 - 2\theta_2} \]

Using the boundary condition (8), the expression of \( (P_s - 1) \) be educed as equation (12). The \( E_{o1} \) is original external pressure load-carrying capacity function, and the \( E_{o2} \) is original coupling load-carrying capacity function, \( N_{o1} \) is original coupling load-carrying coefficient.

\[
E_{o1} = \frac{\lambda}{\left( \theta_1 + 2 \frac{2}{1 - 2\theta_2} \right)} \Delta - \frac{2 \frac{2}{1 - 2\theta_2} \epsilon (\beta + 1)(\gamma^3 + \beta)}{\left( \theta_1 + 2 \frac{2}{1 - 2\theta_2} \right)} \Delta - \frac{2 \left( \theta_1 - 2\theta_2 \right) \epsilon}{\theta_2} = \frac{2\theta_1 - 2\theta_2}{1 - 2\theta_2} \]

\[
E_{o2} = \frac{\beta (\gamma^3 - 1)(\gamma - 1)\cos \alpha \sin \alpha (\beta + 1)(\gamma^3 + \beta)}{\Delta_0 (\Delta_0 + \epsilon (\beta + 1)(\gamma^3 + \beta))} \]

\[
\Delta = \left( \gamma^3 + \beta \right) \left( \beta^3 + 1 + \epsilon (\beta + 1) - \beta (\gamma^3 + \gamma^2) \right) - \beta (\gamma^3 + \gamma^2) \sin^2 \alpha \]

The average pressure could be described as equation (16):

\[
\Delta \bar{P} = \frac{1}{2} N_{o1} \Delta E_{o1} + \frac{1}{2} N_{o2} \Delta E_{o2} + \frac{1}{2} \Delta N_{o1} E_{o1} + \frac{1}{2} \Delta N_{o2} E_{o2} + \frac{1}{2} \left( \frac{P_s - 1}{\lambda} \right) \Delta E_{o1} + \frac{1}{2} \left( \frac{P_s - 1}{\lambda} \right) \Delta E_{o2} \]
2.3. Load-carrying capacity

Load-carrying capacity is integration of the relative pressure \((p-p_a)\) in working area \(S\):

\[
F = 2 \int_0^{\theta_0} \frac{1}{\zeta} (p - p_a) dy dx = p_s S \Delta p = 2 F_0 \Delta p = F_0 \left( P_0 - 1 \right) \lambda E
\]

\(F_0\) is defined as \([1]\): \(F_0 = Sp_s/2\). The load-carrying capacity function \(E\) is:

\[
E = N_{a1} \zeta E_{a1} + N_{a2} \zeta E_{a2} + N_{oc} \zeta E_{oc} + N_{os} \zeta E_{os} + E_{gc} + E_{gs}
\]

\[(17)\]

The \(\zeta\) is pressure ratio of complex gas bearing, \(\zeta = \lambda / [(P_0 - 1)]\).

3. Numerical method analysis

Because the load-carrying capacity function is too complex to discuss the parameter’s effects directly from its analytical expression, in this section, the numerical method was used to analyze their effects.

Below is the situation that \(\alpha = 1.3, \beta = 1.86, \gamma = 2.34, \theta_0 = 0.15, \theta_1 = 0.3, \lambda = 20, \zeta = 1\), the load-carrying capacity components see Table.1; When \(\alpha = 1.3, \beta = 1.86, \gamma = 2.34, \theta_0 = 0.25, \theta_1 = 0.3, \lambda = 20, \zeta = 1\), the load-carrying capacity components see Table.2; When \(\alpha = 1.3, \beta = 1.86, \gamma = 2.34, \theta_0 = 0.3, \theta_1 = 0.3, \lambda = 20, \zeta = 1\), the load-carrying capacity components see Table.3;

When \(\alpha = 1.3, \gamma = 2.34, \theta_0 = 0.25, \theta_1 = 0.3, \lambda = 20\), and \(\zeta = 1, 5, 10, 20, 40\), the \(E-\beta\) curves were shown as Fig.6. When \(\beta = 1.86, \gamma = 2.34, \theta_0 = 0.25, \theta_1 = 0.3, \lambda = 20\), and \(\zeta = 1, 5, 10, 20, 40\), the \(E-\gamma\) curves were shown as Fig.7. When \(\alpha = 1.3, \beta = 1.86, \theta_0 = 0.25, \theta_1 = 0.3, \lambda = 20\), and \(\zeta = 1, 5, 10, 20, 40\), the \(E-\alpha\) curves were shown as Fig.8.

Table.1 Load-carrying capacity components

\(\alpha = 1.3 \text{rad}, \beta = 1.86, \gamma = 2.34, \theta_0 = 0.15, \theta_1 = 0.3, \zeta = 1, \lambda = 20\)

| Load-carrying component | \(N_{a1}E_{a1}\) | \(N_{a2}E_{a2}\) | \(E_{oc}\) | \(E_{gc}\) | \(E_{os}\) | \(E_{gs}\) | \(E\) |
|------------------------|-----------------|-----------------|----------|----------|----------|----------|------|
| Value(10^{-2})         | 0.3578          | 0.1843          | 0        | 0        | 4.29184  | 1.11588  | 5.94982 |
| Percent (%)            | 6.01            | 3.1             | 0        | 0        | 72.13    | 18.75    | 100   |

Table.2 Load-carrying capacity components

\(\alpha = 1.3 \text{rad}, \beta = 1.86, \gamma = 2.34, \theta_0 = 0.25, \theta_1 = 0.3, \zeta = 1, \lambda = 20\)

| Load-carrying component | \(N_{a1}E_{a1}\) | \(N_{a2}E_{a2}\) | \(E_{oc}\) | \(E_{gc}\) | \(E_{os}\) | \(E_{gs}\) | \(E\) |
|------------------------|-----------------|-----------------|----------|----------|----------|----------|------|
| Value(10^{-2})         | 0.9939          | 0.1024          | -0.2735  | -0.0444  | 4.2918   | 1.1159   | 6.1862 |
| Percent (%)            | 16.07           | 0.166           | -0.442   | -0.72    | 69.38    | 18.04    | 100   |

Table.3 Load-carrying capacity components

\(\alpha = 1.3 \text{rad}, \beta = 1.86, \gamma = 2.34, \theta_0 = 0.3, \theta_1 = 0.3, \zeta = 1, \lambda = 20\)

| Load-carrying component | \(N_{a1}E_{a1}\) | \(N_{a2}E_{a2}\) | \(E_{oc}\) | \(E_{gc}\) | \(E_{os}\) | \(E_{gs}\) | \(E\) |
|------------------------|-----------------|-----------------|----------|----------|----------|----------|------|
| Value(10^{-2})         | 1.1927          | 0               | -0.4102  | -0.06655 | 4.29184  | 1.11588  | 6.1237 |
| Percent (%)            | 19.48           | 0               | -6.699   | -1.087   | 70.09    | 18.22    | 100   |
Fig. 1. Structure of infinite long flat complex gas bearing

$L$: flat width; $L_1$: width of I area; $L_2$: width of II area; $L_0$: distance between external pressure inlets; $P_a$: ambient pressure; $P_s$: pressure at the external pressure inlet; $b_g$: width of groove; $b$: width of ridge; $\alpha$: angle of spiral groove.

Fig. 2. The shape of groove section

$h$: film thickness; $\Delta$: groove depth;

$U$: surface moving speed.

Fig. 3. Orifice sketch

$d$: Diameter of orifice; $L_i$: Depth of orifice;

$p_o$: Pressure of gas supply.

Fig. 4. Half width flat gas bearing

I, II: grooved area; III: no groove area.

Fig. 5. Pressure distribution of half width flat gas bearing along Y

Fig. 6. The $E-\beta$ curve, $\zeta=1, 5, 10, 20, 40$

$\alpha=1.3, \gamma=2.34, \theta_1=0.15, \theta_2=0.3, \beta=20$
4. Conclusion

(1) The load carrying capacity of this complex gas bearing is composed of six parts: the first self-acting item, the second self-acting item, the original external pressurized item, the original coupling item, the surface character external pressurized item, and the surface character coupling item;

(2) The external pressure gas bearing number $\lambda$ is the main parameter that represents the effect of external pressure; The pressure ratio $\zeta$ is the main parameter that represents the combination property of hybrid gas bearing;

(3) In the load-carrying capacity of this complex gas bearing, there is an apparent coupling effect and surface feature effect, the surface feature effect could increase the loading capacity greatly, and in properly choosing the parameters the coupling effect could be zero

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