Effect of Shape Imperfections on Collapse Moment of Strain Hardened Pipe Bends under in-plane Opening Bending Moment

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Abstract. The effect of ovality and variable wall thickness on the collapse load of pipe bends subjected to in-plane opening moment is investigated using Finite Element (FE) analysis for an elastic plastic material. The collapse load was estimated from the moment-rotation curves using Twice-Elastic-Slope method. The presence of ovality reduced the collapse load by 13.1% for a model having bend characteristic 0.1, thus making its effect critical. The presence of thinning had no significant effect on the collapse load. Based on the FE results, a mathematical equation is proposed to estimate the collapse load of strain-hardened pipe bends for a range of bend characteristic from 0.1 to 0.6, considering the effect of ovality.

Keywords: elastic-plastic; ovality; pipe bends; collapse load; thinning.

Nomenclature

| Symbol | Description                                      |
|--------|--------------------------------------------------|
| C₀     | Percentage of ovality                           |
| Cₘₜ   | Percentage of thickening                        |
| Cₜ     | Percentage of thinning                          |
| L      | Straight pipe length (mm)                       |
| D      | Pipe Outside diameter (mm)                      |
| Dₘₜ   | Pipe Maximum outside diameter (mm)              |
| Dₘᵡ   | Pipe Minimum outside diameter (mm)              |
| t      | Pipe Nominal thickness (mm)                     |
| tₘₜ   | Pipe Maximum thickness (mm)                     |
| tₘᵡ   | Pipe Minimum thickness (mm)                     |
| R      | Bend radius (mm)                                |
| r      | Mean radius of pipe (mm)                        |
| h      | Bend characteristic, \(=\frac{Rr}{r^2}\)         |
| M      | In-plane bending moment                         |
| M₀     | Straight pipe Collapse in-plane moment          |
| M₀ₛ   | Straight pipe Limit in-plane moment, \(=4rt^2\sigma_0\) |
| E      | Young’s modulus                                 |
| ν      | Poisson’s ratio                                  |
| σ₀     | Yield stress                                    |
1. Introduction
The most pivotal component in any piping system are the pipe bends. Pipe bends are the most economical means of changing the direction of flowing fluid within a piping network. They are pliant (than straight pipes) and lead to lowered reaction forces and moment due to their elastic deformations. But if the load exceeds the elastic limit, plastic deformation occurs, and the load bearing capability of the pipe is lost significantly. This being the real scenario, analysis must be performed taking into consideration the effects of strain hardening.

The study of pipe bends comprises of analytical [1-3], numerical [4-30] and experimental [31-36]. The three main types of loading considered in the above studies include: in-plane opening moment [35], in-plane closing moment [17] and out-of-plane moment [12,14]. Most of the earlier studies were for pipe bends in static conditions but later, studies were performed to include the effects of pressure [2,6,11], fatigue [31], shakedown [21] and cracks [9]. A recent study [24], the analysis of 180° pipe bend to investigate the effect on collapse load, by considering the effect of geometric nonlinearity and the effect of ovality. The study indicated a decrease in the collapse load for pipes with thin wall and small bend radius and minor deviations for pipes with thick walls and large bend radius, in comparison to 90° pipe bends. Additionally, another recent work performed by Sherif. S. Sorour [25-26], wherein the entire process of rotary pipe bending [RPB] was performed using Finite Element Method (FEM) to account for the actual change in cross section rather than assuming the cross section to be elliptic. An experimental and FE analysis performed by Chattopadhyay [20,32] on effect of crack initiation load and its growth on collapse load, concluded to reduce the collapse load. Sumesh et al. [27] performed Finite element Analysis on structurally distorted cracked pipe bends under in-plane closing bending to estimate limit moments. J. Li et al. performed FEA to compute the limit loads under combined bending and torsion [28], with circumferential through wall crack subjected to torsion [29] and compared cracked and un-cracked wall considering geometric non-linearities subjected to torsion [30].

The early researches assumed pipe bends to have an ideal cross section i.e. circular. Kim et al. performed FE analysis of pipe bends, considering an Elastic Perfectly Plastic (EPP) material, imposed with internal pressure and in-plane loading with material non-linearity only [7], and considering geometric non-linearity [8]. Kim et al. [13] also proposed Twice Elastic Slope (TES) to estimate collapse moment. A mathematical equation to predict collapse load of bends imposed with internal pressure and in-plane moments was offered by Chattopadhyay et al. [19] for an Elastic Plastic (EP) material using the techniques of FE Analysis. Younan et al. [16] performed FEA for pipe bends exacted to out-of-plane moment, comparing EPP and EP material. These were some of the earliest researches considering the effects of strain hardened material although they were for ideal pipe bends.

However, the pipe bend’s cross section changes, from circular to oval with variations in wall thickness, during the pipe bending process. Thus, the influence of shape imperfections viz. ovality and thinning became crucial for the FE analysis. The ASME standards allow 8% for ovality and thinning in pipe bends. However, the analysis conducted incorporated up to 20% for ovality and thinning.

An equation was proposed, on performing FE analysis, to evaluate collapse load of pipe bend comprising shape imperfections exacted to internal pressure and in-plane moments, by Christo Michael et al. [4-5] Another analysis on the influence of shape imperfections on the collapse load of bends imposed with in-plane opening bending with and without pressure was performed by Buckshumiyan et al. [22] The above studies were for pipe bends including the influence of shape imperfections although it was for an EPP material.

In practical applications and experiments, the material is subjected to the effects of strain hardening and hence must be considered in FEA. The collapse moment of pipe bends considering the material to be EPP is very conservative and thus gives a scope to investigate the effects of both strain hardening and shape imperfections on collapse moment. This gives scope for this paper to explore the effects of shape imperfections namely ovality and thinning/thickening on collapse load of strain hardened pipe bends imposed with in-plane opening bending load.
2. Geometry Modelling and Material

2.1. Geometry
The geometry in analysis is a 90° pipe bend having two attached straight pipes at the ends. The straight pipe’s length is five times the mean pipe diameter i.e., \( L = 5 \times D \). The pipe’s mean radius is 50 mm. The thickness considered for the analysis are 2.5 mm, 5 mm and 10 mm and the bend radius considered for the analysis are 100 mm and 150 mm. From the above geometric dimensions, the bend characteristic (\( h = R_t/r^2 \)) for the pipe bend ranges from 0.1 to 0.6. The degree of ovality is defined as the ratio of difference between the maximum and minimum diameter to the nominal diameter. The bend is assumed to have shape imperfections with an assumed shape of an ellipse [4].

\[
C_O = \frac{D_{\text{max}} - D_{\text{min}}}{D} \times 100
\]

where nominal outside diameter: 
\[
D = \frac{D_{\text{max}} + D_{\text{min}}}{2}
\]

The thinning occurs at the extrados. It is defined as the ratio of difference between the nominal thickness and minimum thickness to the nominal thickness [4].

\[
C_t = \frac{t - t_{\text{min}}}{t} \times 100
\]

The thickening occurs at the intrados. It is defined as the ratio of the difference between the maximum thickness and nominal thickness to the nominal thickness [4].

\[
C_{th} = \frac{t_{\text{max}} - t}{t} \times 100
\]

The percentage rise in thickness in the intrados is equal to the percentage reduction in the thickness in the extrados.

The straight pipes are considered to have a circular cross section with the same thickness.

2.2. Material
The material is surmised as isotropic and elastic-plastic (strain-hardening) (Table 1)

The material used for the analysis is SS304, having properties as:
- Young’s modulus = 193 GPa
- Poisson’s Ratio = 0.2642
- Yield stress = 271.929 MPa

| Table 1. Plastic behaviour |
|---------------------------|
| Yield stress (MPa) | Plastic strain |
|---------------------|---------------|
| 271.929             | 0             |
| 345.91              | 0.00473       |
| 378.867             | 0.01264       |
| 403.619             | 0.02836       |
| 424.165             | 0.0491        |
| 527.587             | 0.105         |

3. Finite Element Modelling
FEA was performed using the ABAQUS software version 6.14 [37]. The geometry was also created using the same package.

3.1. Element and Mesh
A structured hexahedral mesh was adapted with the element type as C3D20R, a 20-node brick consisting of quadratic elements. The quantity of elements across thickness and length of the pipe are three and fifteen, respectively. The number of elements across the bend region and the cross section were varied for the mesh study. From the study it was concluded that 10 elements were required for
cross section and bend region. The final mesh count of the model is 3000 elements and will be adopted for further analysis.

3.1.1. Mesh study. A mesh study was executed and ideal mesh density at bend region and the cross section were identified to perform the analysis. The study was performed for $h = 0.2$ and $h = 0.4$. The number of elements varied from 1656 to 4176. The mesh study was conducted having the response as collapse moment. From Figure 2(b) for the model $h = 0.4$ the percentage decrease in the value from change of number of elements from 3000 to 3696 is 0.008% which is negligible. Similarly, from Figure 2(a) for the model $h = 0.2$ the percentage decrease is 0.032%.

Figure 1. Structured Mesh

(a) $h = 0.2$
3.2. Boundary conditions and Loading

The analysis has been simplified by considering a symmetric model. The fixed and rotation boundary conditions are specified using the Multi Point Constraint (MPC) as shown in Figure 3. The MPC is used to connect all the surface nodes to a single reference point so that any boundary condition applied to this reference point will be applied to all the surface nodes it is linked to.
The reference point 1 shown in Figure 4 is given a fixed boundary condition. The reference point 2 is given a rotation boundary condition along the y vector (anti-clockwise rotation) and constraining other degrees of freedom. The final boundary condition is symmetry about the XZ plane. The Static RIKS method with non-linear geometry option is employed to solve this model, since it is used for predicting the unstable, geometrical non-linear collapse of a structure.

3.3. Post-Processing
The post processing was done using the same software. The moment and the rotation were extracted from the reference point 2 (Figure 4). The collapse moment is evaluated using the TES method, the point where the straight line having two times the slope of initial elastic region, in moment rotation curve, intersects the curve. The graph is plotted in Excel and TES load was obtained.

3.4. Python Script
The setup of the case from material, mesh, Boundary conditions, constraints and Post-Processing remains the same for all the 150 cases analyzed. The geometry varies for each of the above case. Hence to simplify the task a Python script was generated which upon executing creates the input files for all the cases. Then a separate code was used to run the INP files and generate the ODB files. Finally, the data was tabulated in Excel.

4. Verification
The FE model and the analysis was verified using the literature of Goodall [2]. This model considers pipe bend geometry having an ideal cross section. The material used was SS304 considering the effects of strain hardening. The percentage deviation in the collapse moment for both models is less than 1% as visualized from Figure 5. Thus, the code and the FEA procedure adopted is verified and can be used for the underlying analysis.

![Figure 5. Verification of FEA procedure](image-url)
5. Comparison of Strain hardened material and EPP material

The effects of strain hardened material and an EPP material on the collapse moment has been investigated. From Figure 6(a) and (b), it is known that the collapse load for a strain hardened material or Elastic Plastic material is higher than that for an EPP. There is a 19.88% difference between the collapse loads for a model with \( h = 0.2 \) and 19.56% for the model with \( h = 0.4 \). The pipe bends are not designed for the maximum collapse load but are designed for a load higher than minimum required by incorporating a safety factor. Hence designing the pipe bends by considering the material to be EPP makes the design very conservative which may be unnecessary.

![Comparison of Strain hardened material and EPP material](image)

(a) \( h = 0.2 \)

(b) \( h = 0.4 \)

Figure 6. Comparison of EPP and EP material
6. Result and Discussion
The pipe bends usually are imposed with in-plane opening and closing and out-of-plane bending loads due to various dead and live loads namely wind load, seismic load, weight of the system and other loads acting on it. The pipe bends are further weakened due to geometric imperfections. Hence the effect of shape imperfections on collapse load is very crucial.

6.1. Effect of ovality on collapse moment
It is visualized from Figure 7; the collapse load reduces with rise in percent ovality. For the model having \( h = 0.6 \) the collapse load decreases from 26.62 kN-m to 24.99 kN-m with increase in percentage ovality from 0\% to 20\%.

![Figure 7. Collapse load vs Percent Ovality for various bend characteristic](image-url)

To truly evaluate the effect of ovality, percentage difference in collapse moment must be investigated. The percentage difference in ovality is defined as the ratio of difference in the collapse moment of a model not considering the effect of shape imperfections (reference model) to the model with consideration of shape imperfections to the reference model. From Table 2, for the model having bend characteristic 0.1 the collapse load decreases by 13.09\% with increase in percent ovality to 20\%. This implies that the percentage of ovality in a model must be carefully interpreted for design of pipe bends.
Table 2. Effect of ovality on Collapse moment

| COLLAPSE_LOAD | OVALITY% | 0.1  | 0.15 | 0.2  | 0.3  | 0.4  | 0.6  |
|---------------|---------|------|------|------|------|------|------|
| 0  | 0  | 0  | 0  | 0  | 0  | 0  |
| 5  | 2.771789 | 2.819055 | 2.527388 | 2.396501 | 1.978124 | 1.266013 |
| 10 | 6.057354 | 5.810865 | 5.190525 | 4.893265 | 4.112683 | 2.753184 |
| 15 | 9.528685 | 8.875936 | 7.970253 | 7.469431 | 6.447588 | 4.313444 |
| 20 | 13.09255 | 11.96861 | 10.84699 | 10.17345 | 8.921972 | 6.11378 |

6.2. The effect of thickness and bend radius on collapse load

From Figure 8, we can visualize that the trends for collapse moment with change in thickness for various bend radii. The collapse load reduces with decrease in thickness. For model having bend radius 100 mm the collapse load decreases from 22.4 kN-m to 3.7 kN-m with decrease in thickness from 10 mm to 2.5 mm for percentage ovality of 20%. Also, the collapse moment decreases with decrease in bend radius. For the model having thickness 5 mm, the collapse load decreases from 9.7 kN-m to 8.5 kN-m with decrease in bend radius from 150 mm to 100 mm for percentage ovality of 20%.

Figure 8. Collapse load vs Thickness for various bend radii

From Figure 9, we can understand the interaction of thickness of pipe, bend radii and percent ovality on percent collapse moment. With increase in thickness of the pipe, percent deviation in collapse moment reduces. Also, with increase in bend radius, percent deviation in collapse moment reduces. Thus, it is very crucial to design thin pipe bends subjected to ovality due to pipe bending process.
(a) $t = 2.5 \text{ mm}$

(b) $t = 5 \text{ mm}$
6.3. Effect of thinning on collapse moment

From the Figure 10, it is observed that thinning has no effect on the collapse load. The percentage difference between the values differs by less than 2% which is insignificant for all the models considered. Thus, we can neglect the effect of thinning for the determination of collapse moment.
6.4. Collapse load equation considering the effects of ovality
A mathematical expression is presented to determine the collapse moment of pipe bends including ovality for a strain hardened pipe bends using Data Fit [38] for a range of \( h = 0.1 \) to 0.6.

\[
\frac{M_o}{M_o^{0.35}} = \left( \frac{R}{100r(72.8+1.46C_0)} \right)^{0.24} \left( \frac{t}{3.04} \right)^{0.36}
\]  

(4)

The average deviation is 1.3% and the Coefficient of Multiple Determination (\( R^2 \)) is 0.998. Thus, from Figure 11, the above equation is compatible with the FEA results for \( h \in [0.1,0.6] \), with the maximum difference of 5.65%.

The previous equations proposed to estimate collapse load either included the effects of strain hardening only [8,19] or shape imperfections with EPP material [4,5]. However, the current study proposes an equation to estimate collapse load with both shape imperfections and strain hardening effects included.

![Figure 11. Comparison of proposed mathematical equation with FEA results](image)

7. Conclusion
The following conclusions are deduced from the nonlinear analyses on pipe bends with shape imperfections under in-plane opening moment for a strain hardened material:

- The effect of thinning on collapse load is negligible and its effects are not considered for further study.
- The collapse load of pipe bends is significantly influenced by ovality. As the percentage of ovality increases the collapse moment reduces, hence considered during the design of pipe bends.
- It is also noted that the collapse moment increases with increase in bend radius and thickness.
- A mathematical expression is presented to determine the collapse moment of strain hardened pipe bends including the effect of ovality in the range of bend characteristics between 0.1 and 0.6.

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