Combined effect of viscosity variation on rough porous rectangular plates with magneto hydrodynamic effect

E Sujatha and Sundarammal Kesavan
Department of Mathematics, SRM Institute of science and technology, Kattankulathur, Tamilnadu, India- 603203

suji_41181@yahoo.co.in

Abstract: The squeezing action between rough porous parallel rectangular plates under the effect of magnetohydrodynamic force and viscosity variation effect is considered in this paper. The modified Reynolds equation is derived. The load carrying capacity of the plate thus derived is found to increase with increase in the magnetic, roughness and viscosity variation parameter.

1. Introduction
Squeezing technology is applied in many engineering and medical fields. Squeezing produces a positive pressure which helps in the load carrying capacity of the system. The Stokes [1] is the simplest generalization of the classical theory of fluids which allows for polar effects like presence of couple stresses, body couples and non-symmetric tensors. Ramaniah, Gupta[2,3] and others analyzed the squeeze film action between finite plates of various geometries lubricated with couple stress fluid. Porous material allows the lubricant to seep in the pores and provides an cushioning effect to the lubricant. The effect of porosity and the boundary condition of a porous material was studied by Beavers, Joseph and others[4,5] who showed that the porosity helps in the load carrying capacity. Hartmann was the pioneer who studied the flow of an incompressible fluid between parallel plates with magnetic field acting normal to them. He investigated it both theoretically and experimentally. Hamza E.A[6] results show that the electromagnetic forces increases the load carrying capacity considerably. SundarammalKesavan et al[7] studied the effect of MHD on a porous rectangular plate and found that the MHD improves the performance. The lubricant which we choose must be suitable to function under various ranges of temperature. To enhance the performance of the fluid additives are added which makes the viscosity of the fluid to vary from layer to layer. Sinha et al.,[8] studied the effects of viscosity variation due to lubricant additives in journal bearing. Raghavendra Rao and Prasad studied the effects of velocity slip and viscosity variation in rolling and normal motion, roller bearings and journal bearings[9,10,11] using multi layer technique. Continuous working of the machinery leads to the wear of the surface. Also the additives reaction and the contamination of the lubricants contributes to the degradation of the surface resulting in roughness. Christensen and Tonder[12,13,14] extensively studied the effect of surface roughness and modeled it mathematically. N.B Naduvinamani, S.T Fathima[15,16,17] studied the effect of the surface roughness on porous parallel plates. The effect of the velocity and friction of porous elliptic plates lubricated with couple stress fluid considering the effects of slip velocity and effect of viscosity variation in porous parallel rectangular plates lubricated with couple stress fluids was carried out by Sujatha and Sundarammal.
In this paper we try to investigate the effect of surface roughness, magnetic effect and viscosity variation effect on a porous parallel rectangular plate.

2. Geometry
Let \( L_1 \) and \( L_2 \) be the dimensions of the rectangular plates in the \( x \) and \( y \) directions respectively. Then \( \beta = \frac{L_2}{L_1} \) be the aspect ratio. The upper plate approaches the lower rough porous plate with a constant velocity \( \frac{dH}{dt} \). The gap between the two plates is filled with an isothermal, incompressible electrically conducting fluid of thickness \( H \). A uniform magnetic field \( B_0 \) is applied in the \( z \) direction. It is assumed that the inertial force and the induced magnetic force are too small compared to the applied magnetic field.

\[ \begin{align*}
\frac{\partial p}{\partial x} &= \mu \frac{\partial^2 u}{\partial z^2} - \sigma^* B_0^2 u \\
\frac{\partial p}{\partial y} &= \mu \frac{\partial^2 v}{\partial z^2} - \sigma^* B_0^2 v \\
\frac{\partial p}{\partial z} &= 0 \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0
\end{align*} \]  

Here \( u \) and \( v \) are the velocity components in the \( x \) and \( y \) directions respectively, \( \mu \) is the viscosity of the lubricant, which in now replaced by the factor \( \mu \left( \frac{h}{h_1} \right)^\theta \) to add the effect of varying viscosity. \( \mu_1 \) is the known viscosity at the height \( h = h_1 \). We assume that relationship for the variation of viscosity with temperature can be replaced by viscosity film thickness relationship. The value of \( Q \) varies...
between 0 to 1 depending upon the nature of the lubricant. It takes the value 0 for perfect Newtonian fluids and 1 for perfect gases. We also assume that a thermal equilibrium exists. \( p \) represents the fluid pressure and \( \sigma^* \) the electrical conductivity.

The boundary conditions for the velocity components are

\[
u = v = w = 0 \text{ at } z = 0
\]

\[
u = 0 \text{ and } w = \frac{dH}{dt} \text{ at } z = H
\]

The flow in the porous medium is governed by the Darcy’s law given by

\[
u^* = -\frac{k}{\mu} \frac{\partial \hat{p}^*}{\partial x}
\]

Where \( \phi = \frac{k}{h_0^2} \) is the non dimensional porosity parameter, \( \beta = \frac{\eta/\mu}{k} \), the permeability of the porous region.

Solving equations (1) and (2) using (3) and boundary conditions (5) and (6) the velocity components in the \( x \) and \( y \) directions are given respectively by

\[
u = \frac{h_0^2}{\mu_1} \left( \frac{h}{h_0} \right)^{\frac{Q}{2}} M^2 \left[ \cos \left( \frac{Mz}{h_0} \right) - 1 - \left( \cosh \left( \frac{MH}{h_0} \right) - 1 \right) \frac{\sinh \left( \frac{Mz}{h_0} \right)}{\sinh \left( \frac{MH}{h_0} \right)} \right]
\]

\[
u = \frac{h_0^2}{\mu_1} \left( \frac{h}{h_0} \right)^{\frac{Q}{2}} M^2 \left[ \cos \left( \frac{Mz}{h_0} \right) - 1 - \left( \cosh \left( \frac{MH}{h_0} \right) - 1 \right) \frac{\sinh \left( \frac{Mz}{h_0} \right)}{\sinh \left( \frac{MH}{h_0} \right)} \right]
\]

\( h_0 \) is the film thickness at time \( t = 0 \) and \( M \) represents the Hartmann Number given by

\[
M = B_0 h_0 \left( \frac{\sigma^*}{\mu} \right)^{1/2}
\]

Substituting the velocity components obtained by equations (7) and (8) in the continuity equation (4) and integrating across the fluid film thickness using (5), (6), (7) we get the modified Reynolds equation as

\[
\frac{h_0^3}{\mu} \left( \frac{h}{h_0} \right)^{\frac{Q}{2}} M^3 \left[ \frac{\partial}{\partial x} \left( \frac{MH}{2h_0} - 2 \tanh \left( \frac{MH}{2h_0} \right) \sinh^2 \left( \frac{MH}{2h_0} \right) \frac{\partial \hat{p}^*}{\partial x} \right) \right] = -\frac{dH}{dt}
\]

Applying the surface roughness effect developed by Christensen and Tonder, who gave a stochastic approach to model the roughness governed by the probability density function
\[
f(h_i) = \begin{cases} 
\left(\frac{35}{32} - \frac{h_i^2}{c^2}\right), & -c \leq h_i \leq c \\
0, & \text{otherwise}
\end{cases}
\]  \hspace{1cm} (11)

to equation (9), where we take \( H = \bar{h} + h_i \), \( \bar{h} \) is the mean film thickness and \( h_i \) is the deviation in the mean film thickness, \( c \) is the maximum deviation from the mean film thickness, we get

\[
\frac{h_i^3}{\mu_i} \frac{\hat{h}}{h_i} E M^3 \left[ -2 \frac{\partial}{\partial x} \left( \frac{MH}{h_i} \right) \tanh \left( \frac{MH}{2h_i} \right) \sinh^2 \left( \frac{MH}{2h_i} \right) \frac{\partial p^*}{\partial x} \right] = -E \frac{dH}{dt}
\]  \hspace{1cm} (12)

Where \( E(R) = \int \int Rf(h_i) \, ds \) \hspace{1cm} (13)

The mean value \( \alpha \), the standard deviation \( \sigma \) and the parameter \( \varepsilon \), used to measure the symmetry of the variable \( h_i \) are defined by

\[
\alpha = E(h_i); \sigma^2 = E(h_i - \alpha)^2; \varepsilon = E(h_i - \alpha)^3
\]  \hspace{1cm} (14)

Introducing the non dimensionless parameters

\[
\bar{x} = \frac{x}{L_4}; \bar{y} = \frac{y}{L_4}; \bar{H} = \frac{H}{h_0} = \frac{h_i + \bar{h}}{h_0} = \bar{h} + \bar{h}_i; \bar{p} = \frac{E(p)h_i^3}{L_4^3 \left( \frac{d\bar{h}}{dt} \right)}
\]

\[
\alpha = \frac{\alpha'}{h_0^3}; \sigma = \frac{\sigma'}{h_0^3}; \varepsilon = \frac{\varepsilon'}{h_0^3}
\]

The modified Reynolds equation takes the form

\[
\frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial y^2} = -M^3 \frac{1}{\mu_i} \left( \frac{\bar{H}}{h_i} \right)^4 E \left[ \frac{M\bar{H}}{2} \right] \tanh \left( \frac{M\bar{H}}{2} \right)
\]  \hspace{1cm} (15)

The suitable solution for equation (15) is given by

\[
\bar{p}(\bar{x}, \bar{y}) = \left( A \sin k'\bar{x} + B \cos k'\bar{x} \right) \left( C \sin k'\bar{y} + D \cos k'\bar{y} \right)
\]  \hspace{1cm} (16)

With boundary conditions \( \bar{p} = 0 \) at \( \bar{x} = 0, \bar{x} = 1, \bar{y} = 0, \bar{y} = \beta; \beta = \frac{L_2}{L_4} \) \hspace{1cm} (17)

The mean pressure is thus obtained as
\[
\bar{p}(x, y) = \sum_{m=1,3,5} \sum_{n=1,3,5} \left( \frac{16\beta^2 M^3}{m n \pi \mu (\beta^2 m^2 + n^2)} E(MH - 2 \tanh \left( \frac{MH}{2} \right)) \right) \left( \cos \left( \frac{m \pi x - \frac{n \pi y}{\beta}}{\beta} \right) - \cos \left( \frac{m \pi x + \frac{n \pi y}{\beta}}{\beta} \right) \right) \frac{2}{2}
\]

The load carrying capacity is given by
\[
E(W) = \int_{x=0}^{L_1} \int_{y=0}^{L_2} E(p) dx dy
\]

The non dimensional form of (17) is given by
\[
\bar{W} = \frac{E(W) h_0^3}{L_1 L_2 \frac{dh}{dt}}
\]

4. Results and discussions

Figure 1: Variation of load carrying capacity with Hartmann constant M.
Figure 2: Variation of load carrying capacity with Viscosity variation parameter $Q$.

Figure 3: Variation of load carrying capacity with $\alpha$. 
Figure 1 gives the changes in the load carrying capacity as the Hartmann constant M varies. We observe that there is an increase in the load carrying capacity as the Hartman constant M increases indicating that the magnetic field enhances the load carrying capacity.
Figure 2 gives the changes in the load carrying capacity for variation in the viscosity variation parameter $Q$. It is observed that the value of the load carrying capacity increase with an increase in the viscosity variation parameter $Q$.

Figure 3 gives the changes in the load carrying capacity with variation in the mean value $\alpha$. We observe that the load carrying capacity increases with an increase in $\alpha$.

Figure 4 gives the changes in the load carrying capacity with variation in the standard deviation $\sigma$. We notice that the load carrying capacity increases with an increase in $\sigma$.

Figure 5 gives the changes in the load carrying capacity with changes in the $\varepsilon$. With increase in $\varepsilon$ we notice an increase in the load carrying capacity.

5. Conclusion
The investigation shows that the magnetic effect, the viscosity variation parameter and the roughness effect parameter increases that the load carrying capacity of the model under consideration. Thus it enhances the life of the bearing with such a configuration.

References
[1] Stokes V K1966 Couple stresses in fluids Phys.Fluids9pp1709-1715
[2] Ramanaiah G1979 Squeeze films between finite plates lubricated by fluids with couple stress Wear 54(2)p315
[3] Gupta P S and Gupta A S1977 Squeezing flow between parallel plates Wear 45(2)p177
[4] Beavers G S and Joseph D D 1967 Boundary conditions at a naturally permeable wall J.Fluid mech., 30(1)p197-207
[5] Beavers G S, Sparrow E M and Magnuson R A 1970 Experiments on coupled parallel flows in a channel and a boundary porous medium J.Basic Eng., 92Dpp843-848
[6] Hamza E A 1998 The magneto hydrodynamic squeeze film J.Tribol. 110(2)p375
[7] SundarammalKesavan, Ali Chamkha and Santhana Krishnan N 2014 Magnetohydrodynamic squeeze film characteristics between finite porous parallel rectangular plates with surface roughness effect International journal of numerical methods for heat and fluid flow 24(7)pp 1595-1609
[8] Sinha P, Singh C and Prasad K R1981 Effect of viscosity variation due to lubricant additives in journal bearings Wear 6pp175-188
[9] Raghavendra Rao R and Prasad K R2002 Effects of velocity slip and viscosity variation in rolling and normal motion Journal of Aeronautical Society of India 54(4)pp399-407
[10] Raghavendra Rao R and Prasad K R2003 Effects of velocity slip and viscosity variation for lubrication of roller bearings Defence Science Journal 53(4)pp431-442
[11] Raghavendra Rao R and Prasad K R2004 Effects of velocity slip and viscosity variation on journal bearings ANZIAM Journal 46pp143-155
[12] Christensen H and Tonder K C 1969 Tribology of rough surfaces: stochastic models of hydrodynamic lubrication SINTEF report 10/69-18
[13] Christensen H and Tonder K C 1970 The hydrodynamic lubrication of rough bearing surfaces of finite width ASME-ASLE Lubrication Conference Cincinnati Ohio USA Paper no.70-Lub-7
[14] Christensen H and Tonder K C 1969 Tribology of rough surfaces: parametric study and comparison of lubrication models SINTEF report 22/69-18
[15] Naduvninamani N B, Fathima S T and Hiremath P S 2003 Effect of surface roughness on characteristics of couple stresses squeeze film between anisotropic porous rectangular plates Fluid Dyn. Res. 32(5) p217
[16] Naduvinamani N B, Fathima S T and Hiremath P S 2004 On the squeeze effect of lubricants with additives between rough porous rectangular plates *ZAMM* **84**(12)p825

[17] Fathima S T, Naduvinamani N B and Marulappa S H 2012 Hydromagnetic squeeze films between porous rectangular plates with couple stress fluids *Japanese society of TribologistsTribology online* **7**(4)pp258-266

[18] Sujatha E and Sundarammal Kesavan 2016 Effect of viscosity variation in Porous parallel rectangular plates lubricated with couple stress fluids *Global Journal of Pure and Applied Mathematics* **12**(1)pp 99-103

[19] Sujatha E and Sundarammal Kesavan 2015 Velocity and frictional effects of porous elliptical plates lubricated with couple stress fluid considering the effects of slip velocity *International Journal of Scientific & Engineering Research* **6**(3)