The supported landslide slopes’ operation simulation

L Panasiuk*, V Turina, D Stupina

1Don State Technical University, Gagarin, Sq., 1, Rostov-on-Don, 344000, Russia

E-mail: panasjuk.leonid@gmail.com

Abstract. The article describes the landslide processes formation processes modeling in free and reinforced slopes. Unlike many well-known approaches, the proposed approach does not introduce simplifying hypotheses. As, for example, hypotheses about the slip lines form or about the slope conventionally “solid” areas form. The slope stability problem is considered in the general formulation of a three-dimensional or two-dimensional (plane strain) plasticity theory. The limiting equilibrium approach and the continuation methods by the loading parameter are used. The resolving equations are constructed in the form of a finite element method (FEM) in displacements. It is shown how, in addition to physical, to take into account geometrical and constructive nonlinearity. The small fragment limiting state hypotheses can be assigned arbitrary. In the general formulation, a complex loading process is assumed — an external load can grow arbitrarily. In this case, the growth load linear nature is used within one external loading branch. Inside the branch, a “cycle in a cycle” process is organized in small increments of load. The process continues either until the limit state of the considered computational domain is reached, or until the load and the transition growth next branch end to the next one.

Introduction

The modern structural mechanics’ main task in a broad plan is formulated as follows: “... the structures calculation should give an exhaustive picture of their work at all stages of loading (including the stage of destruction)” [1]. Such a problem can be solved only when using hypotheses of materials with complicated physical and mechanical characteristics. One of the approaches to constructing complicated models is the theory of limiting equilibrium. Essential to the development of limiting equilibrium methods was the work of A.A. Gvozdev [2], in which the extremum principles were first rigorously proved for the first time, allowing to estimate the upper and lower bearing capacity of elastoplastic systems. The limiting equilibrium method further development in the form of concentrated deformations approach (for bar systems) was obtained in the works of A.R. Rzhanitsyn [3]. In this kinematic approach, systems with priori given destruction form were considered.

Despite the building technologies and structures rapid development, the modern building materials use is currently a significant number of accidents and collapses of buildings. One of the negative factors are the landslide processes. As a rule, the objects’ collapse due to landslide movements is associated with errors made in the process of conducting surveys and at the design stage. A frequent reason for the slopes’ stability erroneous evaluation is the improper use in the strength characteristics calculations of soils averaged over the depth [4].

A great influence on the calculation accuracy has the correct definition of the sliding surface, which requires engineering and geological studies of great accuracy. In this connection, the exploration wells diameters should be taken as large as possible.

The existing approaches analysis to assessing the landslide slopes stability allowed to identify the main drawbacks of the methods. The main ones are:

a) the need to a priori set a number of parameters and characteristics, which in essence should be the model implementation results, but not the input data. So, for example, a priori, the shape of the slip areas in a landslide slope is set;
b) normative models do not take into account the reinforcing effect of protective structures (retaining walls, sheet piles) and their deformability on the stress fields distribution and deformations in a landslide massif;
c) the protective structures calculation is carried out by transferring pressure from the side of the slope to the retaining walls’ structures. In this case, the pressure distribution on the wall is determined separately, without taking into account the joint work of the slope and the reinforcement structures;
d) regulatory procedures do not take into account the work stages on the construction of protective structures and earthworks;
e) In regulatory methods, the integral pressure value is determined from the landslide slope on the protective structures. At the same time, the retaining wall structural scheme that is complex in terms of its length and is not taken into account. For example, the multi-row pile cap and the nature of pressure transfer to the piles in different rows are not taken into account. This can lead either to excessively overestimated, or vice versa, significantly underestimated bending moments in piles. Eventually - to the wrong design decisions.

Main text

2.1 Basic provisions of the method of limiting equilibrium of continual systems

Let us consider a medium subject to the condition of Guber-Mises yield. The volume deformation is elastic, varies according to a linear law. Work on shift corresponds to the Prandtl diagram. Then the physical dependencies can be represented in the form (1) [5,10].

\[
\begin{align*}
\sigma_0 &= 3K_0 \varepsilon_0, \quad \sigma_i = 3G_i \varepsilon_i, \\
3\sigma_0 &= \sigma_x + \sigma_y + \sigma_z, \quad 3\varepsilon_0 = \varepsilon_x + \varepsilon_y + \varepsilon_z, \\
\sigma_i &= \frac{1}{\sqrt{2}} \left( (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6\left( \tau_{xy}^2 + \tau_{yx}^2 + \tau_{xz}^2 \tau_{zx}^2 \right) \right), \\
\varepsilon_i &= \frac{\sqrt{2}}{3} \left( (\varepsilon_x - \varepsilon_y)^2 + (\varepsilon_y - \varepsilon_z)^2 + (\varepsilon_z - \varepsilon_x)^2 + 1.5\left( \gamma_{xy}^2 + \gamma_{yx}^2 + \gamma_{xz}^2 + \gamma_{zx}^2 \right) \right), \\
K_0 &= \frac{E_0}{3(1-2\mu_0)}, \quad G_i = \frac{\sigma_i}{3\varepsilon_i}
\end{align*}
\]

In (1) \(\sigma_0\) и \(\varepsilon_0\) – define average normal stress and strain, \(\sigma_i\) и \(\varepsilon_i\) – define stress and strain intensity, \(K_0\) – is a bulk modulus, \(G_k\) – is a secant shear modulus.

In accordance with the accepted hypotheses, the dependence for the strain modules has the form (2).

\[
\begin{align*}
at \varepsilon_i \leq \varepsilon_T & \Rightarrow K_0 = \frac{E_0}{3(1-2\mu_0)}, \quad G_k = G_c = \frac{E_0}{2(1+\mu_0)}, \\
at \varepsilon_i > \varepsilon_T & \Rightarrow K_0 = \frac{E_0}{3(1-2\mu_0)}, \quad G_k = 0, \quad G_c = \frac{\sigma_i}{3\varepsilon_i}
\end{align*}
\]

When using the FEM and linearization of the problem by the Newton method, the basic formulas for stiffness matrices are constructed in the form (3) [9].
\[
k_k = \int \Phi^T H \Phi dv, \quad k_c = \int \Phi^T D_q \Phi dv, \tag{3}
\]

\[
\sigma = D_0 \varepsilon, \quad \Delta \sigma = H \Delta \varepsilon, \quad H = D_0 + D_2,
\]

\[
D_0 = \begin{bmatrix}
K_0 + \frac{4}{3} G_C & K_0 - \frac{2}{3} G_C & K_0 - \frac{2}{3} G_C & 0 & 0 & 0 \\
K_0 - \frac{2}{3} G_C & K_0 + \frac{4}{3} G_C & K_0 - \frac{2}{3} G_C & 0 & 0 & 0 \\
K_0 - \frac{2}{3} G_C & K_0 - \frac{2}{3} G_C & K_0 + \frac{4}{3} G_C & 0 & 0 & 0 \\
0 & 0 & 0 & G_C & 0 & 0 \\
0 & 0 & 0 & 0 & G_C & 0 \\
0 & 0 & 0 & 0 & 0 & G_C
\end{bmatrix},
\]

\[
D_2 = \frac{4}{3k_i^2} (G_K - G_C).
\]

In (3) \(e_q\) – is the deformation deviator components, \(k_k\) and \(k_c\) – are the “tangent” and “secant” local stiffness matrices of a separate finite element, \(\Phi\) – defines the coordinate functions matrix.

When solving an elastoplastic problem by the finite element method, and when linearizing by the continuation method with respect to the loading parameter, the resolving equations for an ensemble of finite elements are represented in the form (4).

\[
\begin{align*}
K_k^{(m)} \Delta q^{(m+1)} &= \Delta P^{(m+1)}, & \text{– step-by-step approach,} \\
K_k^{(m)} \Delta q^{(m+1)} &= P^{(m+1)} - K_c^{(m)} q^{(m)}, & \text{– self-correcting step-by-step approach}
\end{align*}
\tag{4}
\]

For the simplest simplex elements with a constant field of deformation (triangles, tetrahedrons), the limiting state zone appears immediately in the finite element entire volume. For finite elements in which the strain field is not constant over the volume, the integration in the finite element construction in (4) is performed numerically. Limit state zones occur sequentially in the group of computational nodes inside the element.

The algorithm for determining the maximum load does not differ from the algorithm for the frame systems [4, 5].

At the first stage, the calculation is performed for a linearly elastic medium with a single external action. According to the obtained displacement vector in the design the nodes of deformation and stress are determined. The maximum load parameter at the first stage is determined from the condition that the stress intensity is equal to the limit value, from which the loading parameter for one point (or for one finite element) is determined. Then, from the loading parameters set, the minimum one is selected - [5, 9].
\[
\begin{align*}
K^{(0)}_0 \mathbf{q}^{(1)} &= \mathbf{F}, \Rightarrow \mathbf{q}^{(1)} = \left( K^{(0)}_0 \right)^{-1} \mathbf{P}, \\
\sigma^{(1)} &= \Phi \mathbf{q}^{(1)}, \quad \sigma^{(1)} = D^{(0)}_0 \mathbf{e}^{(1)}, \\
\beta^{(1)}_{i,r} \sigma^{(1)}_{i,r} &= \sigma_{T,r} \Rightarrow \beta^{(1)}_{i,r} = \frac{\sigma_{T,r}}{\sigma^{(1)}_{i,r}}, \\
\beta^{(1)} &= \min_{i,r} \left\{ \beta^{(1)}_{i,r} \right\}. 
\end{align*}
\]

In (5) the superscript in parentheses denotes the iteration number, \( r \) is the number of the next computational domain node (for the simplest elements, the number of the element).

In the second step, the increment of displacements, deformations and stresses from a unit load increment (6) is determined.

\[
\begin{align*}
K^{(1)}_x \Delta \mathbf{q}^{(2)} &= \Delta \mathbf{F}, \Rightarrow \Delta \mathbf{q}^{(2)} = \left( K^{(1)}_x \right)^{-1} \Delta \mathbf{P}, \\
\Delta \mathbf{e}^{(2)} &= \Phi \Delta \mathbf{q}^{(2)}, \quad \Delta \mathbf{e}^{(2)} = H^{(1)} \Delta \mathbf{e}^{(2)}.
\end{align*}
\]

Note that the strain and stress vectors increments are linear with respect to the load increment parameter \( \beta \), however, the stress and strain intensities increment are the nonlinear functions relative to \( \beta \). Therefore, unlike the load increment parameter definition when calculating the limit state of frames, here it is necessary to determine the load increment parameter for each computational domain node from the equation solution (7) that is nonlinear with respect to \( \beta \).

\[
\sigma^{(2)}_{i,r} = \sigma_i \left( \sigma^{(1)}_{i,r} + \beta^{(2)}_{i,r} \Delta \sigma^{(2)}_{i,r} \right) = \sigma_{T,r}
\]

2.2. The limiting equilibrium method modification for the landslide slopes operation modeling

The algorithm for determining the maximum load in continuous systems considered in paragraph (2.1) has a limited scope, since a rather simple medium, subject to the condition of Guber-Mises yield was considered. In fact, this condition is subjected to “noble” materials (for example, steel). For materials with complicated physical and mechanical properties, the limit state attainment is determined by more complex dependencies than the yield strength achievement by the stress intensity.

It is proposed for environments with complicated properties (soils, rock materials) to use in general the algorithm considered in (2.1), changing the condition for reaching the limit state to a dependence that better reflects the work of a particular material. For example, the von Mises yield condition does not take into account the tensile-compressive strength different limits, etc.

As a criterion for the limiting state onset, it is proposed to use the well-known limiting state hypotheses (strength hypotheses) for media with complicated properties.

In this paper, the following known limiting state hypotheses are considered [7].

Mohr Criterion (tensile-compressive strength limits are used) is:

\[
\sigma_i - \frac{\sigma_j}{\sigma_{\infty}} \sigma_i = \sigma_b
\]

Mohr Criterion (tensile shear strength limits are used) is:

\[
\sigma_i - \frac{\sigma_j - \tau_b}{\tau_b} \sigma_i = \sigma_b
\]

Pisarenko-Lebedev criterion (tensile-shear strength limits are used) is:
\[
\frac{\sigma_b}{\sigma_{\infty}} + \frac{\sigma_{\infty} - \sigma_b}{\sigma_b} \sigma_2 = \sigma_b
\]

Mises-Schleicher-Botkin criterion (for soils) is:
\[
\sigma_2 = \frac{6 \sin(\phi)}{3 - \sin(\phi)} \left[ -\sigma_0 + \frac{C}{\tan(\phi)} \right]
\]  \hspace{1cm} (11)

Balandin criterion (rocks and stone materials) is:
\[
J_2 \leq \frac{1}{3} \left[ \sigma_{pp} \sigma_b - (\sigma_{pp} - \sigma_b) J_1 \right]
\]  \hspace{1cm} (12)

Genieva's criterion (for concrete) is:
\[
J_2 \leq \frac{1}{3} \left[ \sigma_f \sigma_b - (\sigma_{\infty} - \sigma_b) J_1 \right] \left[ 1 - \left( 1 - \frac{3 \tau^2_b}{\sigma_{pp} \sigma_b} \right) \left( 1 - \frac{J_1}{2} \left( \frac{J_1}{3} \right)^2 \right) \right]
\]  \hspace{1cm} (13)

In the limit states formulas (2.24) - (2.29) it is accepted as:
\( J_1, J_2, J_3 \) - stress tensor invariants,
\( \sigma_i \) - stress intensity, \( \sigma_0 \) - average atress,
\( \sigma_b \) - material strength on uniaxial tension,
\( \sigma_{pp} \) - material strength for uniaxial compression,
\( \tau_b \) - material shear strength at pure shear,
\( C \) - adhesion,
\( \psi \) - internal friction angle.

The limiting state onset in an elementary volume should be differentiated into compression and tension. From that, the limiting state is reached by the prevailing compression or stretching, the modeling of the area depends on the next loading steps.

So, if the limiting state is reached at the prevailing compression (\( J_1 < 0 \)), then the elementary volume further behavior is modeled in the same way as in paragraph (2.1): \( G_K = 0 \).

However, if the limiting state occurs at the prevailing tension (\( J_1 > 0 \)), then this practically means a break at a computational domain given point, a discontinuity. Then it can be assumed that the material in this area has lost the ability to perceive not only the deformations increment, but also to perceive the achieved VAT level. All the material modules should be zeroed, which corresponds to the exclusion of this area (neighborhood of the node or the entire finite element) from the calculated finite element model.

Due to the fact that the volume does not work not only for the deformations increment, but also has ceased to resist the achieved level of deformations, the stresses redistribution occurs. This effect must be taken into account by an additional calculation, which is realized in the internal iterative process of solving a nonlinear problem by the Newton method:
\[
K^{(m,n)}_C \Delta q^{(m,n+1)} = F^{(m)} - K^{(m,n)}_C q^{(m,n)}
\]  \hspace{1cm} (14)

In (14), \( m \) is the load increment outer iteration number; \( n \) is the inner iteration number to account for the internal efforts’ redistribution at the load growth stage \( m \).

After the redistribution of forces, there may be an effect of progressive destruction — an avalanche-like growth of the marginal state zone with a constant external load.

2.3 Experimental research

In the experimental part, a series of numerical experiments was performed to determine the landslide slopes limiting state. The real landslide slope in the Taganrog city with new buildings was considered. The slope modeling was performed for different variants of its strengthening. The hypotheses of the limiting state
of a small volume of soil varied. In the course of the experiments, the slope limiting state zones formation evolution was obtained. These zones' expansion leads to the distinct slip areas formation, having a complex shape in cross section. Strengthening the slope leads to a change in slip areas and an increase in the maximum load on the slope surface.

Different options for taking soaking only reinforce the picture of the stresses' distribution non-uniformity, their concentration zones and the sign change. Also, there is a sign change in displacements, which causes the local breaks possibility in the soil mass. At the same time, the soil layers can lose connectivity with each other, which will lead to sliding of the individual layers. The strength test showed that for all the plots the strength condition of the elements in the concentration zones was violated both by the Coulomb-More hypothesis and by the Mises-Schleicher-Botkin hypothesis.

Summary
A method for determining the landslide slopes limiting state is constructed. The technique uses the following approaches and methods of modern construction mechanics and soil mechanics:

1. The finite element method (FEM) is used as the main spatial domain discretization method, which allows to take into account the complex form of the system topology and arbitrary boundary conditions.
2. We describe the non-linear work of the soil using the elementary volumes limiting equilibrium hypotheses.
3. The problem evolution on the loading parameter will allow, in the framework of the limit state hypothesis, to determine the maximum limit load.

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