On the (in)variance of the dust-to-metals ratio in galaxies

Lars Mattsson,1,2⋆ Annalisa De Cia,3 Anja C. Andersen1 and Tayyaba Zafar4

1Dark Cosmology Centre, Niels Bohr Institute, University of Copenhagen, Juliane Maries Vej 30, DK-2100 Copenhagen Ø, Denmark
2Nordita, KTH Royal Institute of Technology and Stockholm University, Roslagstullsbacken 23, SE-106 91 Stockholm, Sweden
3Department of Particle Physics and Astrophysics, Faculty of Physics, Weizmann Institute of Science, Rehovot 76100, Israel
4European Southern Observatory, Karl-Schwarzschildstrasse 2, D-85748 Garching bei München, Germany

Accepted 2014 February 22. Received 2014 February 21; in original form 2013 December 23

ABSTRACT

Recent works have demonstrated a surprisingly small variation of the dust-to-metals ratio in different environments and a correlation between dust extinction and the density of stars. Naively, one would interpret these findings as strong evidence of cosmic dust being produced mainly by stars. But other observational evidence suggest that there is a significant variation of the dust-to-metals ratio with metallicity. As we demonstrate in this paper, a simple star-dust scenario is problematic also in the sense that it requires that destruction of dust in the interstellar medium (e.g. due to passage of supernova shocks) must be highly inefficient. We suggest a model where stellar dust production is indeed efficient, but where interstellar dust growth is equally important and acts as a replenishment mechanism which can counteract the effects of dust destruction. This model appears to resolve the seemingly contradictory observations, given that the ratio of the effective (stellar) dust and metal yields is not universal and thus may change from one environment to another, depending on metallicity.

Key words: stars: AGB and post-AGB – supernovae: general – dust, extinction – galaxies: evolution – galaxies: spiral.

1 INTRODUCTION

The variation of the overall dust-to-metals ratios between galaxies of vastly different morphology, ages and metallicities appears surprisingly small in many cases, with a mean value close to the Galactic ratio (~0.5). The relatively tight correlation between the dust-to-gas ratio and the metallicity (yielding an almost invariant dust-to-metals ratio) in the Local Group galaxies has been known for quite a while (see Viallefond, Goss & Allen 1982; Isa, MacLaren & Wolfendale 1990; Whitsett 1991). Indirect evidence for a ‘universal’ mean value is also provided by the almost linear relation between B-band optical depth and stellar surface density in spiral galaxies (Grootes et al. 2013). But recent results based on gamma-ray burst (GRB) afterglows, quasar foreground damped Lyα absorbers (DLAs; Zafar & Watson 2013) and distant lens galaxies (see e.g. Dai & Kochanek 2009; Chen et al. 2013) now seem to extend this correlation beyond the local Universe and down to metallicities just a few per cent of the solar value. Zafar & Watson (2013) argue that this can only be explained by either rapid dust enrichment by supernovae (SNe) or very rapid interstellar grain growth by accretion of metals.

However, there can be significant variations within a galaxy (see e.g. Mattsson & Andersen 2012; Mattsson, Andersen & Munkhammar 2012), although the existence of dust-to-metals gradients is somewhat difficult to establish observationally with reliable independent methods. If, on the other hand, the dust-to-metals ratio does not vary much, in any environment, one may assume that dust grains as well as atomic metals are mainly produced by stars. Recent findings of large amounts of cold dust in SN remnants (Matsuura et al. 2011; Gomez et al. 2012) seem to support this hypothesis, although the exact numbers can be disputed (Temim & Dwek 2013; Mattsson et al. 2014). In other words, the overall picture is not consistent.

A new study by De Cia et al. (2013) seems to confirm the rising trend with metallicity of the dust-to-metals ratio in quasar DLAs found in previous studies (Vladilo 1998, 2004). Furthermore, Fisher et al. (2014, see also Herrera-Camus et al. 2012) derived a dust mass in the local starburst dwarf I Zw 18, as well as a high-redshift object of similar character, which clearly indicate a dust-to-metals ratio below the Galactic value. These results, together with the likely existence of dust-to-metals gradients along galaxy discs (Mattsson & Andersen 2012), suggest that the variance (or invariance) of the dust-to-metals ratio may depend on the environment. In such case, there may exist an equilibrium mechanism that keeps the dust-to-metals ratio close to constant if certain conditions are fulfilled, while metallicity dependence may occur as a result of deviations from those conditions in other environments.

Recently, Kuo, Hirashita & Zafar (2013) have tried to alleviate the tension between the results from the GRB afterglows of Zafar & Watson (2013) and other data (for local dwarf galaxies) by

⋆E-mail: mattsson@dark-cosmology.dk
The dust-to-metals ratio in galaxies

2 OBSERVATIONAL CLUES AND CONSTRAINTS ON THE DUST-TO-METALS RATIO

Recently, Grootes et al. (2013) derived a correlation between the optical depth in the B band $\tau_B$ and the stellar-mass surface density $\Sigma_*$ in nearby spiral galaxies selected from the Galaxy and Mass Assembly survey, which were detected in the far infrared (FIR)/submm in the Herschel-ATLAS field. They find a nearly linear relation

$$\log(\tau_B) = (1.12 \pm 0.11) \times \log\left(\frac{\Sigma_*}{M_\odot \text{kpc}^{-2}}\right) - 8.6 \pm 0.8,$$  

(1)

where the errors reflect the 1σ scatter in the data. The regression is marginally consistent with an exactly linear correlation between $\tau_B$ and $\Sigma_*$. If the dust density $\Sigma_d$ is proportional to the stellar-mass density $\Sigma_*$, there should exist a linear correlation between the optical depth $\tau_B$ and the corrected, deprojected surface density of a galaxy, much like the relation above, because of the connection with the dust abundance, i.e., $\tau_B \sim \Sigma_d$. Grootes et al. (2013) argue that their relation is evidence of efficient interstellar grain growth. This conclusion depends on whether stars, primarily massive stars, can make a significant contribution to the dust production and on the efficiency of dust destruction in the interstellar medium (ISM). In principle, the $\tau_B - \Sigma_*$ connection only says that dust and stars

‘go hand-in-hand’ in local spiral galaxies, which seems to suggest significant stellar dust production and that destruction of dust must be balanced by grain growth in the ISM. However, as we will go on to show later, this is not necessarily the case.

A constant dust-to-metals ratio does not only seem to apply in the local Universe, however. Zafar & Watson (2013) combine extinction ($A_V$) values and abundance data from GRB afterglows with similar data from QSO (quasar) foreground absorbers and multiply imaged galaxy-lensed QSOs to determine the dust-to-metals ratios for a wide range of galaxy types and redshifts of $z = 0.1$–6.3, and almost three orders in metal abundance. The mean dust-to-metals ratio for their sample is very close to the Galactic value and the 1σ deviation is no more than 0.3 dex, suggesting that the dust-to-metals ratio may be fairly invariant throughout the observable Universe. Chandra X-ray observations of distant lens galaxies lend further support to this picture (Dai & Kochanek 2009; Chen et al. 2013). Taken at face value, these results would imply a very rapid dust-formation scenario that is roughly the same in any environment.

The number of data points at low metallicity is relatively small in the work by Zafar & Watson (2013). It is therefore not certain that the dust-to-metals ratio is nearly invariant also at low metallicities. A very recent study by De Cia et al. (2013) has shown, using a different method, that there is likely a turn-down in the dust-to-metals ratio at low metallicity. This is also consistent with the constraint on the dust-to-metals ratio derived for the local starburst galaxy I Zw 18 (Herrera-Camus et al. 2011; Fishler et al. 2014). De Cia et al. (2013) measure the degrees of depletion of gas-phase abundances in the ISM for various elements, particularly focusing on Fe and Zn, and infer the dust abundance from these depletions. The dust-depletion patterns are observed in UV/optical GRB afterglows and QSO spectra, associated with the ISM of the GRB host-galaxies and QSO–DLAs, and are derived assuming that the depletion is entirely due to dust condensation, regardless of its origin. In particular, the method used by De Cia et al. (2013) relies on the assumption that the observed [Zn/Fe] traces the overall dust content in the ISM, and thus that (1) the intrinsic relative abundance of Zn and Fe is solar and (2) a non-negligible amount of iron is present in the bulk of the dust. This is not obviously the case due to uncertainties in the origins of Zn and Fe, but investigating the reliability of these assumptions – and thus the exact slope of the trend of the dust-to-metals ratio with metallicity – goes beyond the scope of this paper.

What is particularly interesting about the new results by De Cia et al. (2013) is that the dust-to-metals ratio increases with increasing metallicity and, even more important, with increasing metal density. The latter is a clear indication of grain growth being an important part of the build-up of the dust mass. Further evidence from DLAs of a down-turn in the dust-to-metals ratio at low metallicity is seen in the works by, e.g. Vladilo (1998, 2004). A similar, although somewhat steeper, down-turn of the dust-to-gas ratio was also recently found by Rémy-Ruyer et al. (2014) for low-metallicity galaxies in the local Universe.

3 DUST PROCESSING IN THE ISM

3.1 Grain growth

In a gaseous medium of a given temperature and density, the rate of accretion of a gas-phase species $i$ on to a spherical dust grain is given by the surface area of the grain ($4\pi a^2$ where $a$ is the grain radius) and the sticking coefficient (probability) $f_i$ for that species...
The time-scale of dust destruction may not only be inversely proportional to the SN rate, but also the abundance of dust, since the rate of interactions (or collisions) is proportional to the number density \( n_i \). A reasonable modification to the dust-destruction time-scale would then be to introduce a dependence on the dust-to-gas ratio \( Z_\mathrm{d}/\rho_g \), i.e.

\[
\tau_\mathrm{d}^{-1} \approx \frac{\delta}{\rho_g} Z_\mathrm{d} \frac{d\rho_i}{dr},
\]

where \( Z_{\mathrm{d},G} \) is the present-day Galactic dust-to-gas ratio.

The dust-destruction efficiency \( \delta \) can be calibrated to the expected efficiency (time-scale) for the Galaxy, which we take to be roughly 0.7 Gyr (Jones, Tieless & Hoellenbach 1996). The effective Galactic gas-consumption rate is about \( 2 M_\odot \text{pc}^{-2} \text{Gyr}^{-1} \), and the gas density is \( \approx 10^3 \text{M}_\odot \text{pc}^{-2} \) (see e.g. Mattsson 2010, and references therein), which implies \( \delta \approx 5 \). Mattsson (2011) estimated \( \delta \approx 10 \) based on a Larson (1998) IMF and that stars of initial masses above \( 10 M_\odot \) become SNe. We can thus assume that \( \delta \approx 5 - 10 \) is a reasonable estimate of the expected range for \( \delta \).

4 SIMPLE MODELS OF DUST EVOLUTION

In Mattsson et al. (2012) and Mattsson & Andersen (2012), we showed that dust growth would be the most important mechanism for changing the dust-to-metals ratio \( \zeta \) in a galaxy throughout its course of evolution and/or create a dust-to-metals gradient along a galaxy disc. Since, in this work, we want to also consider the situations where \( \zeta \) is not changing much, we will focus on the two viable scenarios for dust production: (1) pure stellar dust production and inefficient dust destruction and (2) a scenario where dust destruction is balanced by dust growth in the ISM.

To simplify our model, we make the same assumptions as in Mattsson et al. (2012) and Mattsson & Andersen (2012), i.e. a galaxy evolves effectively as a ‘closed box’ and the stellar dust/metals production can be described under the instantaneous recycling approximation. We also assume that the effects of the inevitably changing grain-size distribution are negligible on average, so that grain growth and destruction are functions of macroscopic properties only as described in the next subsection. Furthermore, we make the assumption that the fraction of condensable metals (metals that may end up in dust grains) \( Z_\mathrm{c} \) is essentially the same as the total metallicity, i.e. \( Z_\mathrm{c} \approx Z_\ast \). This assumption is quite reasonable as the observed depletion is surprisingly close to 100 per cent for many of the most abundant metals except oxygen and the noble gases (see e.g. Pinto

\[\text{\footnotesize Footnote: With the adaptations usually employed in chemical collision theory (Atkins & de Paula 2010), the collision frequency is } R_{\text{coll}} = \sigma_{\text{coll}} v_{\text{rel}} n_d, \text{where } \sigma_{\text{coll}} = 2\pi \alpha^2, \text{is the effective cross-section for grain–grain collisions, } n_d \text{ is the number density of dust grains and } v_{\text{rel}} \text{ is the typical relative velocity of two colliding grains. Using } R_{\text{coll}}, \text{we may define the collision density as } \frac{1}{2} R_{\text{coll}} n_d. \text{The factor } 1/2 \text{ has been introduced to avoid double-counting the collisions. Obviously, the collision density is proportional to } Z_\mathrm{d}^2 \text{ since } n_d \propto Z_\mathrm{d}. \text{The efficiency of dust destruction is roughly proportional to the shattering rate, since smaller fragments are more easily destroyed, and the shattering rate is to first order proportional to the collision rate, which sketchily motivates the modified model of dust destruction suggested above.} \]
et al. 2013). The equation for the evolution of the dust-to-metals ratio $\zeta = Z_d/Z$ is then (Mattsson et al. 2012)

$$ Z \frac{d\zeta}{dZ} = \frac{y_d}{y_z} + \frac{Z}{y_z} (G(Z) - D(Z)) - \zeta, \quad (9) $$

where $G$ is the rate of increase of the dust mass due to grain growth relative to the rate of gas consumption due to star formation, $D$ is the corresponding function for dust destruction and $y_d, y_z$ are the effective stellar dust and metal yields, respectively. The dust yield $y_d$ may have a significant dependence on the metallicity of the stellar population, which we will return to later. In terms of the time-scales for grain growth and destruction above, $G$ and $D$ can be defined as

$$ G(Z) = \epsilon Z \left[ 1 - \frac{Z_d(Z)}{Z} \right], \quad D = \delta \quad \text{or} \quad D(Z) = \delta' Z_d(Z), \quad (10) $$

where $\delta/\delta' = Z_d G$.

4.1 Pure stellar dust production

We first consider the case where we have only stellar dust production and no destruction of dust in the ISM ($\epsilon = \delta = 0$). For a ‘closed box’, the dust-to-gas ratio $Z_d$ is simply given by

$$ Z_d = y_d \ln \left( 1 + \frac{\Sigma_*}{\Sigma_{\text{gas}}} \right). \quad (11) $$

Note that by replacing $y_d$ with $y_z$, we would obtain the corresponding relations for metallicity. Series expansion around $\Sigma_*/\Sigma_{\text{gas}} = 0$ yields

$$ Z_d = y_d \frac{\Sigma_*}{\Sigma_{\text{gas}}} + \frac{y_d}{2} \left( \frac{\Sigma_*}{\Sigma_{\text{gas}}} \right)^2 - \cdots \quad (12) $$

from which we may conclude that $\Sigma_d \approx y_d \Sigma_*$ for an unevolved galaxy where $\Sigma_*/\Sigma_{\text{gas}} \ll 1$. Thus, the dust masses in young starbursts, like I Zw18, should give us a measure of the stellar dust yield $y_d$ (at least for low metallicities). This may also give a hint about the origin of the $\Sigma_*/\Sigma_{\text{gas}}$ connection seen in the results by Groote et al. (2013), i.e. that we should consider a model where a balance between growth and destruction leads to a similar $\Sigma_d \sim \Sigma_*$ relation for more evolved systems.

If we include interstellar dust destruction with a time-scale given by equation (7) ($D = \delta, G = 0$), the closed-box solution to equation (9) can be written in the form

$$ \frac{d\zeta}{dZ} = \frac{y_d}{y_z} \left[ 1 - \left( 1 + \frac{\Sigma_*}{\Sigma_{\text{gas}}} \right)^{-\delta} \right] \frac{\Sigma_{\text{gas}}}{\Sigma_*} \ln \left( 1 + \frac{\Sigma_*}{\Sigma_{\text{gas}}} \right)^{-1}. \quad (13) $$

Analysis of this solution shows that $\Sigma_*/\Sigma_{\text{gas}} \gg 1$ requires $\zeta \ll 1$ (Mattsson 2011). Using the time-scale given by equation (8), which is based on the suggested grain–grain interactions ($D = \delta' Z_d, G = 0$), gives a solution of the form

$$ \frac{d\zeta}{dZ} = \frac{1}{y_z} \frac{y_d}{\delta} \tanh \left[ \sqrt{y_d} \delta' \ln \left( 1 + \frac{\Sigma_*}{\Sigma_{\text{gas}}} \right) \right]. \quad (14) $$

which suggests the same asymptotic behaviour, i.e. $\Sigma_*/\Sigma_{\text{gas}} \gg 1$ requires $\zeta \ll 1$. This tells us that only stellar dust production cannot work if there is interstellar dust destruction on any level after the dust has become part of the diffuse ISM. The dust-to-metals ratio $\zeta$ will decrease monotonously unless the effective stellar dust yield $y_d$ increases in such a way that it compensates for the dust destruction. Otherwise, if we are to maintain a roughly constant $\zeta$, there cannot be any significant destruction of dust in the ISM.

4.2 Growth/destruction equilibrium model

With $G$ as in equation (10) and $D = \delta$ (the ‘canonical’ model of dust destruction), we have an equation for $\zeta$ which reads

$$ Z \frac{d\zeta}{dZ} = \frac{y_d}{y_z} + \frac{\zeta Z}{y_z} \left[ \epsilon (1 - \zeta) Z - \delta \right] - \zeta. \quad (15) $$

The equilibrium case $d\zeta/dZ = 0$ would correspond to $\epsilon (1 - \zeta) Z - \delta = 0$ and $\zeta = y_d/y_z$, which is equivalent to the criterion

$$ \frac{\epsilon}{\delta} = \frac{Z}{y_d} - \frac{y_z}{y_d}. \quad (16) $$

This is a problem, however, since $y_d, y_z$, as well as $\delta, \epsilon$ are constants by definition, while $Z$ cannot be constant, except under very special conditions. It is therefore virtually impossible to keep $\zeta$ more or less constant over a wide range of metallicities.

If we instead consider our second equation of dust evolution, $Z \frac{d\zeta}{dZ} = \frac{y_d}{y_z} + \zeta \frac{Z^2}{y_z} \left[ (1 - \zeta) Z - \delta' \zeta \right] - \zeta,

(17)

for the case where the dust-destruction time-scale depends on the dust-to-gas ratio $Z_d$, i.e. $D(Z) = \delta' Z_d(Z)$, where $\delta' = \delta/\Sigma_0 G$, we obtain a more realistic equilibrium condition. More precisely, we have that $\epsilon (1 - \zeta) - \delta' \zeta = 0$, which leads to

$$ \frac{\delta'}{\epsilon} = \frac{y_z}{y_d} - 1. \quad (18) $$

This criterion is more useful than equation (16), since it does not involve any variable. If we adopt the Galactic dust-to-metals ratio, $\zeta_0 \approx 0.5$, we have $y_d/y_z \approx 0.5$ and thus $\epsilon \approx \delta'$. With $\delta \approx 5-10$ and $\delta' \approx 100$ (Galactic gas-to-dust ratio), we then have $\epsilon \approx 500-1000$, which suggests that a relatively high efficiency of grain growth is required to only maintain balance between growth and destruction. A parameter range $\epsilon \sim 500-1000$ is consistent with the results by Mattsson & Andersen (2012).

The special case $\epsilon = \delta'$ is worth some further consideration. Provided there is no dust if $Z = 0$, it follows directly from equation (17) that $\zeta(0) = y_d/y_z$ regardless of whether $\epsilon = \delta'$ or not. In the opposite limit (large $Z$), the dust-to-metals ratio $\zeta$ will approach its asymptotic value and thus be constant. Hence, equation (17) reduces to

$$ 0 = \frac{\epsilon}{y_z} \left[ 1 - 2 \zeta \right]. \quad (19) $$

which corresponds to $\zeta \rightarrow 1/2$ (the asymptotic value). Thus, if $\epsilon$ and $\delta'$ are similar, regardless of the actual value, we would have $\zeta \sim 0.5$. This result is particularly interesting since the dust-to-metals ratios in essentially all Local Group galaxies are close to $\zeta \approx 0.5$ (Inoue 2003; Draine et al. 2007). With the model suggested above, this ratio would be a universal ratio which all galaxies will evolve towards, while the dust-to-metals ratio at early times may be quite different. A similar idea is discussed in Inoue (2011).

The general solution to equation (17) for the initial condition $Z_d(0) = Z(0) = 0$ and $\epsilon > 0$ is

$$ \zeta = \frac{y_d}{y_z} \frac{M}{\epsilon} \left[ \sqrt{1 + \frac{y_z}{\epsilon} \left( 1 + \frac{\epsilon}{\delta'} \right)} - \frac{1}{2} \frac{1}{\sqrt{1 + \frac{y_z}{\epsilon} \left( 1 + \frac{\epsilon}{\delta'} \right)}} \right], \quad (20) $$

where $M(\epsilon, \delta')$ is the Kummer–Tricomi function of the first kind, which is identical to the confluent hypergeometric function $F_1(a, b; z)$ (see Mattsson et al. 2012, for proof that equations 15 and 17 can be transformed into Kummer’s equation). The growth/destruction equilibrium case, $\delta' / \epsilon = y_z/y_d - 1$, corresponds to $a = b$, where
we note that \( M(a, a; z) = e^a \). Consequently, \( \xi = y_d / y_Z \), as discussed above. In reality, one would expect deviations from an exactly constant dust-to-metals ratio to occur as a consequence of local variations of the yield ratio \( y_d / y_Z \) together with \( \delta ' \) and \( \epsilon \). The latter two parameters are clearly different for different dust compositions and may also have implicit dependences on the gas density and, perhaps most importantly, on the grain-size distribution, which can only be ‘universal on average’.

For the case \( \epsilon = \delta ' = 0 \) (neither dust growth nor destruction), we have the trivial solution \( \xi = y_d / y_Z \), which is of course identical to the equilibrium case above.

4.3 Metallicity-dependent stellar dust production

The effective stellar dust yield \( y_d \) has so far been treated as a constant. To first order, this is an acceptable approximation, but as we are here interested in dust production at very low metallicity, it is necessary to consider a scenario in which \( y_d \) is a function of the metallicity \( Z \). There are two reasons for this. First, some key elements for dust production (such as silicon) may be less abundant in low-metallicity stars. This is obviously the case for the massive, short-lived, asymptotic giant branch (AGB) stars which are producing mainly silicates, but has no (or very little) silicon production of their own. Secondly, dust condensation is strongly dependent on the absolute abundance/density of the relevant metals. That is, there may exist a critical metallicity below which dust condensation becomes inefficient due to low partial pressures for many metals, leading to less nucleation and slow accretion. It is already well established that such a critical metallicity exists for grain growth in the ISM (see e.g. Asano et al. 2013a). This can be the case also in massive stars which, despite that they produce significant amounts of metals, may have too low partial pressures of certain key elements to have efficient nucleation.

A very simple scenario would be the one where \( y_d \) is simply proportional to the metallicity \( Z \). Assuming that interstellar dust processing has no effect on the dust mass fraction of the ISM (\( G = D = 0 \) or \( G = D \neq 0 \) and \( y_d(Z) = y_{d,0} + kZ \), where \( y_{d,0}, k \) are constants, we have

\[
\frac{d\xi}{dZ} = \frac{y_{d,0} + kZ}{y_Z} - \xi,
\]

which has the simple solution [with initial condition \( \xi(0) = 0 \)]

\[
\xi(Z) = \frac{1}{2} \left( y_{d,0} + y_d(Z) \right) / y_Z.
\]

This model produces a rising trend as seen in several observations, but is otherwise not very realistic. First, there is no ‘roof’ in the solution above. \( \xi \) can continue to grow even beyond the absolute upper limit \( \xi = 1 \). Secondly, it is expected that there is critical/threshold metallicity for efficient dust formation rather than a linear rise as above. Thus, a more realistic scenario is that in which stellar dust production becomes efficient at a certain metallicity, i.e. there is a smooth ‘jump’ in \( y_d \) at some metallicity \( Z_0 \). The transition from inefficient to efficient dust condensation is likely smooth, so it would be reasonable to adopt something of the form (see Fig. 1)

\[
y_d(Z) = y_{d,0} + \Delta y_d \exp \left( -\frac{Z}{Z_0} \right),
\]

where \( y_{d,0} \) is the minimum dust yield for inefficient dust condensation and \( y_{d,\text{max}} = y_{d,0} + \Delta y_d \) is the maximum dust yield obtained at high efficiency. Thus, we obtain the solution [with initial condition \( \xi(Z_0) = \xi_0 \)]

\[
\xi(Z) = \frac{y_{d,0} + \Delta y_d Z_e}{y_Z} + \frac{\Delta y_d Z_e}{y_Z} \left[ E_1 \left( \frac{Z_e}{Z_0} \right) - E_1 \left( \frac{Z_e}{Z} \right) \right] - \frac{Z_0}{Z_e} \exp \left( -\frac{Z_e}{Z_0} \right),
\]

where we have defined the so-called exponential integral \( E_n \) as

\[
E_n(x) = \int_1^\infty \frac{e^{-xt}}{t^n} \, dt.
\]

If the initial metallicity \( Z_0 \) is very small, or, more precisely, if \( Z_0 \ll Z_e \), we can simplify the above expression into

\[
\xi(Z) = \frac{y_{d,0} + \Delta y_d Z_e}{y_Z} - \frac{\Delta y_d Z_e}{y_Z} E_1 \left( \frac{Z_e}{Z} \right).
\]

5 RESULTS AND DISCUSSION

Below, we consider the dust-to-metals trends derived from optical depth, extinction magnitude and depletion levels of certain metals and compare them with the simplistic models described in the previous section. Furthermore, we present simple Monte Carlo simulations to demonstrate how such simple scenario would appear when allowing the model parameters to vary within a certain parameter space.

5.1 B-band optical depth and dust abundance: Do stars dominate cosmic dust production?

Due to the approximate proportionality between \( A_V \) and the dust-to-gas ratio, one would expect the \( B \)-band optical depth \( \tau_B \) to be a simple function of dust density. More precisely, \( \tau_B \sim \Sigma_d \). Given the result by Grootes et al. (2013), the dust mass density \( \Sigma_d \) is then simply proportional to the stellar-mass density \( \Sigma_* \). Theoretically, this proportionality is expected for all unevolved (gas-rich) galaxies (see Section 4.1). But for it to hold also for more evolved galaxies, a balance between growth and destruction of dust in the ISM is necessary.

The trend obtained by Grootes et al. (2013) is fundamentally an empirical result, and consistent with a simple model where dust is
produced by stars. Nevertheless, the nearly linear $\tau_B - \Sigma_\star$ is not answering the question whether dust is formed mainly in stars or grown in the ISM (growth and destruction can conspire to produce a $\Sigma_\star \sim \Sigma_\star$, relation), but seems to favour models with significant stellar dust production.

The linear relation discussed above may suggest that the dust-to-metals ratio is not showing large variations since the metal content of a galaxy is typically correlated with the stellar mass (see e.g. Lara-López et al. 2013; Pilyugin et al. 2013), but there is still plenty of room for scatter and the relation is derived for local spiral galaxies which may be in similar evolutionary states where the dust-to-metals ratio has reached an ‘equilibrium plateau’. A more diverse sample of objects would therefore provide a more useful statistical constraint.

5.2 Invariant dust-to-metals ratio

We have transformed the dust-to-metals ratios in Zafar & Watson (2013) from observational units to unitless ratios\(^2\) (as in the models discussed in previous sections) for those objects where all relevant quantities have been measured with sufficient accuracy (see Fig. 2). The relatively small variation of the dust-to-metals ratio ($\zeta = 0.47 \pm 0.13$) seen over such a wide range of redshifts and metallicities (and likely also galaxy types) in the work by Zafar & Watson (2013) is a relatively strong constraint on the dust formation scenario, provided it can be trusted despite the rather small number of reliable measurements. A dust-to-metals ratio $\zeta = 0.47$ would correspond to $y_d/y_2 = 0.47$ in an ‘equilibrium model’ where $\delta' = \epsilon (y_d/y_2 - 1)$ (see Section 4.2). As shown by the different models (analytic solutions to equation 15) overplotted in Fig. 2, variation of the yield ratio $y_d/y_2$ leads to a wide range of dust-to-metals ratios at low metallicity, but converges to the asymptotic value, which is $\zeta = 0.5$ for the special case $\delta' = \epsilon$. The 1σ scatter in the Zafar & Watson (2013) data suggests that $\zeta$ can vary at most about 30 per cent, but it should be noted the observed values cover a range $\zeta = 0.18$–0.67, which indicates that significant variations of $\zeta$ cannot be ruled out due to small number statistics.

The growth/destruction-equilibrium model suggested in Section 4.2 is attractive as it would explain the existence of a characteristic, essentially universal, dust-to-metals ratio $\zeta$, as suggested by Zafar & Watson (2013). Deviations from this ‘universal’ value could then be attributed to variations of the yield ratio $y_d/y_2$. As we have mentioned in Section 4.3, there could exist a critical metallicity (or, more precisely, number density of certain key elements) in stars below which dust condensation is inefficient. However, it could also be that $y_d/y_2$ is a ‘universal constant’ and that the limited variance in $\zeta$ could be explained by the fact that the growth and destruction parameters, $\epsilon$ and $\delta'$, respectively, can vary between different environments. Realistically, none of these parameters ($y_d$, $y_2$, $\epsilon$, $\delta'$) should be viewed as ‘universal constants’, of course.

We will return to this aspect of the variance in $\zeta$ in Section 5.4.

5.3 Increasing dust-to-metals ratio

At first glance, an invariant dust-to-metals ratio in one context (e.g. Grootes et al. 2013; Zafar & Watson 2013) seems to be inconsistent with a clearly rising trend with metallicity in another (e.g. De Cia et al. 2013). But as we have already discussed, the growth/destruction-equilibrium model with $\epsilon = \delta'$ has an asymptotic dust-to-metals ratio $\zeta_\infty$, which is eventually reached regardless of what $\zeta$ is at early times. But if $\zeta$ shows a clear trend with metallicity, as in the results by De Cia et al. (2013), there cannot just be random variations of the yield ratio $y_d/y_2$. De Cia et al. (2013), as well as, e.g. Vladilo (1998), find a $\zeta$ increasing with metallicity, which is what one would expect in a scenario where the bulk of cosmic dust is grown in the ISM rather than produced directly by stars.

However, according to our simplistic model with a constant $y_2$, the expected trend without growth/destruction-equilibrium is a steep rise in $\zeta$ at some critical metallicity (see also Mattsson et al. 2012), which is not in agreement with the observed trend (see Fig. 3, models with $y_d/y_2 < 0.5$). The observed slower rise of the dust-to-metals ratio can thus be a result of a changing yield ratio. If $y_d$ increases at a certain metallicity, as described in Section 4.3, the observed trend could easily be explained. The analytic solutions for different $y_d/y_2$ (and $\epsilon = \delta'$) overplotted in Fig. 3 show that if the yield ratio changes from a few per cent at very low metallicity to $\sim 0.5$ at moderately low metallicity ($Z \sim 0.1 Z_\odot$), the correct rising trend would be obtained. Ultimately, this shows that we need to modify our model – a constant yield ratio $y_d/y_2$ fails to reproduce the trend.

The blue line in Fig. 3 is a numerical solution (fourth-order Runge–Kutta) using equation (23) with the parameter values plotted in Fig. 1 to describe $y_2(Z)$, which demonstrates exactly this point. At the same time, there is always a $y_d/y_2$ that will lead to a constant dust-to-metals ratio $\zeta$ for any given $\epsilon/\delta'$. We suggest

---

\(^2\)Defining the dust-to-metals ratio in observational units as $k/Z = \log(N_{\text{HI}}) + [X/H] + \log(A_V)$, where $N_{\text{HI}}$ is the column density of neutral hydrogen and [$X/H$] is the abundance of $X$ relative to the corresponding solar value, we adopted the Galactic value $k/Z_G = 21.3$ (Zafar & Watson 2013). The unitless dust-to-metals ratio is obtained as $\zeta = \xi_\star \xi_d (k/Z - (4/10))$, where $\xi_\star \approx 0.5$. Here, we adopt $\xi_\star = 0.47$ to maintain consistency between the data sets. But the exact value is not very important as long as the adopted value is the same for all data sets considered.
that this could be a good compromise in order to obtain a model that can explain why $\gamma$ in some cases show very little variation and in other cases a trend with metallicity. The case where interstellar dust processing has no effect on the dust mass fraction of the ISM ($G = D = 0$ or $G = D \neq 0$) is indicated by the dotted black line in Fig. 3 (corresponding to equation 26). The effect of interstellar grain growth is the difference between the solid blue and dotted black lines, where the critical metallicity (the point where the lines diverge) occurs at $Z/Z_\odot \approx 0.1$.

The most likely cause for a changing effective dust yield $y_d$ is the existence of a critical metallicity below which dust formation is significantly less efficient compared to the efficiency at higher metallicities. As we have already mentioned, a lower number density of key elements for dust condensation may be important in stars that do not produce much of these key elements themselves. But for most massive stars that undergo a core-collapse SN explosion, the amount of metals produced is significant even at $Z = 0$ (e.g. Nomoto, Kobayashi & Tominaga 2013, and references therein). However, gas opacities and cooling rates may be lower at very low metallicities, which in turn may affect the heating and cooling of existing dust grains. If the average grain temperature is high enough for sublimation to occur, the net efficiency of condensation may be low. Thus, it is not clear that very metal poor stars can be efficient dust producers even if raw material for dust formation is present.

AGB stars are probably not very important dust producers at low metallicity according to recent work in which a steep dependence on metallicity is found (Ventura et al. 2012). In addition, at really low metallicity of the interstellar gas, i.e. at very early times, low- and intermediate-mass stars have not had enough time to evolve into AGB stars either. For example, metal-poor halo stars in the Galaxy appear to have been formed from gas that is mainly enriched by massive stars (SNe with progenitor masses typically in the range 10–20$M_\odot$), although variations in the abundance patterns sometimes occur (Gilmore & Wyse 1998). Moreover, the destruction of dust in SNe is likely more efficient the more massive the progenitor star is (and the degree of dust condensation is likely lower), which means that a bias towards more massive stars at low metallicity may also lead to less stellar dust per unit stellar mass. Numerical models of SN dust production do indeed confirm that the most massive stars have less surviving dust in their ejecta (Bianchi & Schneider 2007). To summarize the above: oxygen-rich AGB stars (the more massive and short-lived ones) cannot produce very much dust at low metallicity since they do not produce the refractory elements needed for dust production, and the effective dust yield of massive stars is probably strongly metallicity dependent too. Thus, a $y_d/y_z$ increasing with metallicity seems reasonable.

The reason why the GRB and QSO–DLAs studied by Zafar & Watson (2013), as well as local galaxies, show so little variation in their dust-to-metals ratios (despite a wide range of metallicities) is still not obvious. But provided that the effective dust yield $y_d$ depends on the metallicity, this invariant ratio as well as the rising trend found in quasar DLAs by measuring depletions (Vladilo 1998, 2004; De Cia et al. 2013) could be ‘two sides of the same coin’. Statistical variations in the overall efficiencies of grain growth and destruction in the ISM, combined with some uncertainty in which metallicity $Z$, stellar dust production starts to become efficient, will allow for enough scatter in the dust-to-metals ratio as a function of metallicity to have one fundamental model which is consistent with both the flat and the rising trend. This will be explored in the next section. As an alternative hypothesis, one may consider the possibility that the $A_{16}$-based dust abundance estimates in Zafar & Watson (2013) are biased towards environments which have, relatively speaking, significant foreground contamination from intervening systems and therefore appear to have higher dust-to-metals ratios at low metallicity. This possibility should of course be investigated, but goes beyond the scope of this paper.

5.4 Monte Carlo simulation of the dust-to-metals ratio as a function of metallicity

We expect variations in not only the effective dust yield $y_d$, but also in the time-scales of grain growth and destruction ($\epsilon$ and $\delta$, in practice). To quantify the effects of such variations, to some extent, we have performed a couple of Monte Carlo simulations within which we vary the parameters $\delta$ and $\epsilon$ within reasonable ranges as well as setting them to zero (see Table 1). The yield ratio $y_d/y_z$ is not completely arbitrary either. On the one hand, the fraction of metals being injected into the ISM in the form of dust grains cannot be 100 per cent, since the degree of dust condensation must be limited by the physical conditions and the abundances of certain key elements (e.g. carbon or silicon) in the dust chemistry. On the other hand, this fraction cannot be too small either, since it is an observational fact that low- and intermediate-mass stars as well as massive stars in the local Universe produce significant amounts of dust. The fraction of dust that actually survive and eventually enrich the ISM is not known, but with the observed trend shown in Fig. 3 as reference we have calibrated the range of the effective yield ratio $y_d/y_z$ to approximately 0.02–0.44. Thus, two of the parameters of equation (23) are fixed: $y_d, 0 = 2.0 \times 10^{-4}$ and $\Delta y_d = 0.042$, while...
The dust-to-metals ratio in galaxies

Figure 4. Left-hand panel: Monte Carlo simulation with stellar dust production and no interstellar growth and/or destruction. Model parameters (random variables) according to Table 1. Right-hand panel: same as the left-hand panel but with interstellar growth and destruction included as well. The solid black line shows the same numerical solution as in Fig. 3. The overplotted observational data are taken from Zafar & Watson (2013).

Table 1. Random variables/parameters used for the Monte Carlo models.

| Model | Variable | Mean       | Range/std. dev. | Distribution |
|-------|----------|------------|-----------------|--------------|
| A:    | $\log (Z/Z_\odot)$ | $-2.5, \ldots, 0.5$ | Uniform |
|       | $y_d/y_Z$ | 0.5 | $\pm 0.1$ | Normal |
|       | $\epsilon$ | 0 | $\delta'$ | 0 | |
| B:    | $\log (Z/Z_\odot)$ | $-2.5, \ldots, 0.5$ | Uniform |
|       | $Z_e$ | $1.0 \times 10^{-4}$ | $(0.75, \ldots, 1.5) \times 10^{-4}$ | Uniform |
|       | $y_Z$ | 0.01 | 0.005, \ldots, 0.015 | Uniform |
|       | $\epsilon$ | 750 | 500, \ldots, 1000 | Uniform |
|       | $\delta'$ | 750 | 500, \ldots, 1000 | Uniform |

$Z_e$ remains as a random variable of the Monte Carlo simulation together with $\delta'$ and $\epsilon$.

In Fig. 4, we have plotted the resultant probability density functions (PDF) of our simulation results. To begin with, we performed a Monte Carlo simulation of the case of stellar dust production only, with the yield ratio $y_d/y_Z$ (with metallicity dependent) and the metallicity $Z$ as the only random variables. For this simulation, we assumed that $y_d/y_Z$ follows a normal distribution with standard deviation 0.1, centred at $y_d/y_Z = 0.5$ (model A in Table 1). The resultant PDF is consistent with data from Zafar & Watson (2013), as can be seen in the left-hand panel of Fig. 4. After establishing this ‘bench mark’, we then considered the case of a metallicity-dependent dust yield according to equation (23) with the parameter values given above and $Z_e = 0.75-1.5 \times 10^{-4}$. The $\epsilon$ and $\delta'$ ranges are difficult to define, but as we argued in Section 3.2, $\delta \sim 5-10$ ($\delta' \sim 500-1000$) is a reasonable estimate of the expected range for $\delta$. Under the assumption $\epsilon \approx \delta'$, we may then assume $\epsilon \sim 500-1000$ (see model B in Table 1). All random variables were in this case assumed to follow uniform distributions.

As we showed in Section 4.2, the dust-to-metals ratio converges to $\zeta = 0.5$ if $\epsilon = \delta'$, regardless of the value of $y_d/y_Z$ or whether $y_d$ is metallicity dependent or not. Clearly, this is the reason why the scatter in $\zeta$ becomes smaller at high metallicity when interstellar grain growth and destruction is included, compared to the case where $\epsilon = \delta' = 0$ in which the scatter is the same regardless of metallicity (cf. left- and right-hand panels in Fig. 4). This inherent property of the model suggests that one could, in principle, use the amount of scatter at approximately solar metallicity to constrain the width of the range of likely $\epsilon$ and $\delta'$ values. The observational data suggest a relatively small scatter (see Figs 2 and 3), albeit with large error bars on some data points. The parameter ranges that we have used in our simple Monte Carlo simulation appear to give a result that is consistent with the spread and uncertainty of the data at solar-like metallicities. Of course, one cannot draw very firm conclusions from a simplistic simulation like the present, but it seems that models which include interstellar grain growth and destruction is favoured by the fact that there appears to be significantly more scatter among the data points at low ($\sim 1/10$ of solar) metallicity than near solar metallicity. We therefore think that our growth/destruction equilibrium model is plausible and may provide guidance towards a more consistent picture of the origin and evolution of cosmic dust.

6 Conclusions

Several observational studies suggest a surprisingly small variation of the dust-to-metals ratio in vastly different environments. It is worth stressing that the ‘trivial solution’ to the problem, i.e. adopting a (constant) yield ratio of $y_d/y_Z \sim 0.5$, works for any model where there is a replenishment mechanism to counteract possible dust destruction (such as the model used by Kuo et al. 2013, for
example). But other observational evidence also suggest that there is a significant variation of the dust-to-metals ratio between different environments, and an invariant dust-to-metals ratio is problematic also in the sense that it requires fine-tuning and is pushing the limits of the ‘standard models’ of dust evolution in galaxies to explain all data (Kuo et al. 2013).

We find that a reasonable way to resolve this apparent contradiction, and avoiding fine-tuning and extreme model parameters, is to assume that stellar dust production can be efficient, but that interstellar dust growth is equally important and act as a replenishment mechanism which can almost exactly counteract the dust destruction in the ISM. In this scenario, the ratio of the effective (stellar) dust and metal yields is not likely a universal constant and may change due to some metallicity dependence of the stellar dust yield. We propose the existence of a critical stellar metallicity above which nucleation and condensation of dust in stars can be efficient.

We conclude that destruction and growth of grains in the ISM likely strives towards an equilibrium state, which mimics the general behaviour of the case of pure stellar dust production (and no destruction of grains). This explains the relatively small variation of the dust-to-metals ratio seen in several observational studies of local galaxies, but allows also for a significantly lower ratio at low metallicity if the effective stellar dust yield can vary with metallicity.

The suggested scenario has important implications for the rapid build-up of large dust masses at high redshifts. Instead of requiring an extreme efficiency of dust formation in massive stars (SNe) as suggested by, e.g. Dwek et al. (2007), the large dust masses seen in the quasar-host galaxy SDSS J1148+5251 (and other objects at high redshifts), follows naturally from the rapid production of metals that is expected in a massive starburst. Just as Valiante et al. (2011), we are led to conclude that, though massive stars must produce significant amounts of dust, dust masses of the order of $10^9-10^9 \, M_\odot$ (as in SDSS J1148+5251) are not likely a result of stellar dust sources only (as a consequence of interstellar dust destruction) and the resultant dust component must therefore be dominated by grain growth in molecular clouds.

ACKNOWLEDGEMENTS

The authors thank the anonymous reviewer for his/her constructive criticism which helped to improve this paper. Nordita is funded by the Nordic Council of Ministers, the Swedish Research Council and the two host universities: the Royal Institute of Technology (KTH) and Stockholm University. The Dark Cosmology Centre is funded by the Danish National Research Foundation. ADC acknowledges support by the Weizmann Institute of Science Dean of Physics Fellowship and the Kosland Center for Basic Research.

REFERENCES

Asano R. S., Takeuchi T. T., Hirashita H., Inoue A. K., 2013a, Earth Planets Space, 65, 213
Asano R. S., Takeuchi T. T., Hirashita H., Nozawa T., 2013b, MNRAS, 432, 637
Atkins P., de Paula J., 2010, Physical Chemistry. Oxford Univ. Press, Oxford, ch. 22, p. 832
Bianchi S., Schneider R., 2007, MNRAS, 378, 973
Chen B., Dai X., Kochanek C. S., Chartas G., 2013, ApJ, preprint (arXiv:1306.0008)
Dai X., Kochanek C. S., 2009, ApJ, 692, 677
De Cia A., Ledoux C., Savaglio S., Schady P., Vreeswijk P. M., 2013, A&A, 560, A88
Draine B. T. et al., 2007, ApJ, 665, 866
Dwek E., 1998, ApJ, 501, 643
Dwek E., Galliano F., Jones A. P., 2007, ApJ, 662, 927
Fisher D. B. et al., 2014, Nature, 505, 186
Gilmore G., Wyse R. F. G., 1998, ApJ, 116, 748
Gomez H. L. et al., 2012, ApJ, 760, 96
Grootes M. W. et al., 2013, ApJ, 766, 59
Herrera-Camus R. et al., 2012, ApJ, 752, 112
Hirashita H., Kuo T.-M., 2011, MNRAS, 416, 1340
Hirashita H., Yan H., 2009, MNRAS, 394, 1061
Inoue A. K., 2003, PASJ, 55, 901
Inoue A. K., 2011, Earth Planets Space, 63, 1027
Issa M. R., MacLaren I., Wolfendale A. W., 1990, A&A, 236, 237
Jones A. P., Tielens A. G. G. M., Hollenbach D. J., 1996, ApJ, 469, 740
Kuo T.-M., Hirashita H., 2012, MNRAS, 424, L34
Kuo T.-M., Hirashita H., Zafar T., 2013, MNRAS, 436, 1283
Lara-López M. A. et al., 2013, MNRAS, 434, 451
Larson R., 1998, MNRAS, 301, 569
McKee C. F., 1989, in Allamandola L. J., Tielens A. G. G. M., eds. Proc. IAU Symp. 135, Interstellar Dust. Kluwer, Dordrecht, p. 431
Matsura M. et al., 2011, Science, 333, 1258
Mattsson L., 2011, MNRAS, 414, 781
Mattsson L., Andersen A. C., 2012, MNRAS, 423, 38
Mattsson L., Andersen A. C., Munkhammar J. D., 2012, MNRAS, 423, 26
Mattsson L., Gomez H. L., Andersen A. C., Matsura M., 2014, MNRAS, submitted
Nomoto K., Kobayashi C., Tominaga N., 2013, ARA&A, 51, 457
Nozawa T., Kozasa T., Habe A., 2006, ApJ, 648, 435
Pilyugin L. S., Lara-López M. A., Grebel E. K., Kehrig C., Zinchenko I. A., López-Sánchez Á. R., Vílchez J. M., Mattsson L., 2013, MNRAS, 432, 1217
Pinto C., Kaastra J. S., Costantini E., de Vries C., 2013, A&A, 551, A25
Rémy-Ruyer A. et al., 2014, A&A, 563, A31
Slavin J. D., Jones A. P., Tielens A. G. G. M., 2004, ApJ, 614, 796
Temim T., Dwek E., 2013, ApJ, 774, 8
Valiante R., Schneider R., Salvadori S., Bianchi S., 2011, MNRAS, 416, 1916
Ventura P. et al., 2012, MNRAS, 424, 2345
Viallefond F., Goss W. M., Allen R. J., 1982, A&A, 115, 373
Vladilo G., 1998, ApJ, 493, 583
Vladilo G., 2004, A&A, 421, 479
Whittet D. C. B., 1991, Dust in the Galactic Environment. IoP Publishing, Bristol
Zafar T., Watson D., 2013, A&A, 560, A26

This paper has been typeset from a \TeX/B\TeX file prepared by the author.