Generate localized optical vortices by quadrupole transition and twisted light

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ABSTRACT

We explain how quadrupole-active atomic interactions be able to be enhanced once the neutral atom interacts with an evanescent winding light modes. The optical system presented is a semi-bounded space based on neutral atoms situated close to the evanescent Bessel-Gaussian modes. The quadrupole Rabi frequency (and hence the optical quadrupole potential) generated by such evanescent modes is large enough to be exploited in atoms manipulation. In particular, the atomic states in which the dipole transitions are prohibited but the quadrupole transitions are permissible create the optical quadrupole potential of accessible value to play an important role. It is also shown how such optical system acts to create of evanescent modes with $\ell > 0$ having an orbital angular momentum $\ell \hbar$ in which the intensity distribution is restricted in the vicinity of the dielectric surface.

1. Introduction

The active-quadrupole interaction is significant when the square of the matrix element dipole interaction is zero, which means the transition is active-dipole prohibited. If the square of the matrix element is non-zero, the quadrupole interaction leads to recognizable absorption and emission processes. Such interactions are considered to be electric-quadrupole permitted [1–6]. The standard quantum rules that clarify the situations under which the different kinds of interactions are permitted or prohibited are explained in detail in any basic book on atomic spectra [7].

Accordingly, it is expected that the main effects should be initially considered with reference to a two-level atom subject to a light mode is the optical forces. It is known that near resonance there are two types of forces in the atomic manipulation processes: gradient force and dissipative force. The gradient force acts to pull the atoms towards the centre of the focus while the dissipative force tends to push the atoms out of the focus [8–11]. These essential optical forces make it possible to achieve many important applications, including trapping and cooling of atoms, atomic optics, the formation of Bose-Einstein condensates, atomic lasers, atoms in optical lattices, and quantum simulators [12–14].

Some recent studies have established that quadrupole interactions due to traditional light modes can be avoided with acceptable values [15,16]. Other studies have confirmed that quadrupole interactions can be doubled using an evanescent light mode technique [1,5,6]. In more recent studies, replacing ordinary light modes with what are known as “winding light modes” has enabled new physical applications in atomic optics. The term winding light mode refers to a type of light that has a property of orbital angular momentum. Such light includes a number of configurations, the most used ones at present being freely propagating modes such as Bessel-Gaussian and Laguerre-Gaussian modes [17–25]. This is because with them, it is possible to create an optical vortex with a well-defined orbital angular momentum in the free space (unbounded space) [26] and near the dielectric surfaces (semi-bounded space) [27].

Here we carefully examine quadrupole effects in the interaction of atoms with the evanescent Bessel-Gaussian modes. The ultimate goal is to calculate the distribution of the quadrupole Rabi frequency which determines all characteristics of this interaction over the area of influence. Through the quadrupole Rabi frequency, the quadrupole optical forces, and hence the atom dynamics, can be evaluated. Due to the specificity of the light used because of its rotational properties, spiral influences on atom dynamics will emerge both in the trapping of atoms and in their motion.

2. Quadrupole interaction

First, we present the basic formulations that lead to the derivation of spatial dependence of the optical quadrupole potential. Consider a two-level atom interaction with a light mode propagating along a given direction (here, a positive $Z$ – axis). The full interaction
Hamiltonian is given by [28]

$$\hat{\mathcal{H}}_{\text{int}} = \hat{\mathcal{H}}_{\text{Dip}} + \hat{\mathcal{H}}_{\text{Quad}} + \ldots$$  \hspace{1cm} (1)

where $\hat{\mathcal{H}}_{\text{Dip}}$ represents the electric dipole interaction, which is given by

$$\hat{\mathcal{H}}_{\text{Dip}} = -\hat{\mu} \cdot \hat{\mathbf{E}}(\mathbf{R}) = -q \mathbf{r} \cdot \hat{\mathbf{E}}(\mathbf{R})$$  \hspace{1cm} (2)

Here $\hat{\mu}$ is the electric dipole moment vector, $\mathbf{r}$ is the internal position vector, and $\hat{\mathbf{E}}(\mathbf{R})$ is the electric field vector. The second term in Equation (1), $\hat{\mathcal{H}}_{\text{Quad}}$, represents the electric quadrupole interaction, which is given by

$$\hat{\mathcal{H}}_{\text{Quad}} = -\frac{1}{2} \sum_{ij} \hat{Q}_{ij} \frac{\partial \hat{E}_i}{\partial R_j}$$  \hspace{1cm} (3)

where $\hat{Q}_{ij}$ represents the quadrupole moment tensor. In fact, quadrupole interaction takes place when dipole-prohibited and under this condition, the electric quadrupole interaction $\hat{\mathcal{H}}_{\text{Quad}}$ will dominate. Normally the electric quadrupole interaction $\hat{\mathcal{H}}_{\text{Quad}}$ reduces to

$$\hat{\mathcal{H}}_{\text{Quad}} = -\frac{1}{2} \sum_{ij} \hat{Q}_{ij} \frac{\partial \hat{E}_i}{\partial R_j}$$  \hspace{1cm} (4)

where the electric field is taken to be polarized along the $X$-direction. In this case, $\hat{Q}_{ij} = Q_{ij} (b + b^\dagger)$ represents the element’s tensor operator where $b$ and $b^\dagger$ are the raising and lowering operators of atomic level, respectively. Thus the quadrupole matrix element the two atomic levels is $Q_{ij} = \langle 1 | Q_{ij} | 2 \rangle$.

$$\hat{\mathbf{E}}(\mathbf{R}) = \hat{e}_x \mathcal{F}_{\{\eta\}}(\mathbf{R}) \hat{a}_{\{\eta\}} \exp[i\theta_{\{\eta\}}(\mathbf{R})] + H.c.$$  \hspace{1cm} (5)

In the above equation, $\hat{e}_x$ is the unit vector along $X$ direction and the subscript $\{\eta\}$ refers to the characteristics associated with each optical mode (its axial wavevector and winding number $\ell$), whereas $\mathcal{F}_{\{\eta\}}(\mathbf{R})$ and $\theta_{\{\eta\}}(\mathbf{R})$ are, respectively, the amplitude and the phase of the electric field. Finally, the operators $\hat{a}_{\{\eta\}}$ and $\hat{a}_{\{\eta\}}^\dagger$ are the destruction and creation operators of the field mode $\{\eta\}$, while $H.c.$ represents the Hermitian conjugate. Therefore, the desired expression for the quadrupole interaction Hamiltonian can be given as

$$\hat{\mathcal{H}}_{\text{Quad}} = \hat{h} \hat{a}_{\{\eta\}} \hat{a}_{\{\eta\}}^\dagger \mathcal{F}_{\{\eta\}}(\mathbf{R}) \exp[i\theta_{\{\eta\}}(\mathbf{R})] + H.c.$$  \hspace{1cm} (6)

where $\mathcal{F}_{\{\eta\}}(\mathbf{R})$ is the position-dependent quadrupole Rabi frequency that characterizes the interaction of an atom with an electric quadrupole moment with the electric field of the light. The particular details of any interaction process depend on the specific type of the evanescent mode.

### 3. Evanescent Bessel-Gaussian mode

The idea of using evanescent light modes to manipulate atoms was first put forward by Cooke and Hill in 1982 [29]. Since then, a large number of studies, both theoretical and experimental, have researched this topic with various types of light and dielectric structures. A significant share of these studies have been of the evanescent Bessel-Gaussian mode due to its distinctive angular momentum property. Such an optical system may be arranged, as depicted in Figure 1, to strike the internal planar surface of a dielectric medium in which it is propagating – that is, in contact with the vacuum. It is clear that when the interface with the vacuum occupies the plane $Z = 0$ and the angle of incidence $\phi$ exceeds the total internal reflection angle, an evanescent mode is generated above the outer surface. After some straightforward algebra with simple rotational transformations, the explicit form of the evanescent electric field of the Bessel-Gaussian mode plane polarized along the $\hat{y}$ direction such that its quantized electric field as a function of the centre-of-mass coordinate $\mathbf{R} = (X, Y, Z)$ and expressed in cylindrical coordinate $\mathbf{R} = (r, \theta, Z)$ has the form

$$E_{\text{Evap}}^{\text{van}}(\mathbf{R})$$

$$\equiv \hat{\mathbf{Y}} \cdot \zeta_{\kappa_0} \left( -\frac{X \sin \phi}{Z_{\text{max}}} \right)^{\ell+1/2} \exp \left( -\frac{X^2 \sin^2 \phi}{Z_{\text{max}}^2} \right)$$

$$\times \exp(-\Phi) \times J_{\ell} \left( \frac{1}{W_0} \sqrt{\frac{X^2 \cos^2 \phi + Y^2}{1 + (\lambda Z / \pi W_0^2)^2}} \right) \exp(-\Theta)$$  \hspace{1cm} (7)

where the factor $\zeta_{\kappa_0} = \sqrt{8 \pi^2 I / \pi^2 \omega_0^2}$ is the amplitude of a corresponding plane wave of intensity $I$ propagating in the dielectric medium of refractive index $n$. The $\Phi$ factor represents the rotational nature and hence a well-defined orbital angular momentum due to azimuthal phase dependence. It can be easily shown that this term is responsible for generating the vortex property above the outer surface, given as

$$\Phi = \left( i \ell \tan^{-1} \left( \frac{Y}{X \cos \phi} \right) - \frac{2 \pi (2\ell - 1)}{8} \right)$$  \hspace{1cm} (8)

Meanwhile, $\Theta$ represents the factor that governs the penetration depth of the exponential decay along $Z$, which is essentially of the order of the wavelength. This
means that this term is responsible for determining the effect range of evanescent light above the outer surface. By increasing $\Theta$, it can easily overcome any obstacles caused by the surface such as different image interactions. Generally, this term is given by

$$\Theta = z k_0 \sqrt{\frac{\sin^2(\phi)}{n^2} - 1 + i k_0 n \sin \phi}$$  \hspace{1cm} (9)$$

In Equation (7) $J_\ell(\ldots)$ is the Bessel function of the order $\ell$ and $w_0$ is the input mode waist, while $Z_{\text{max}}$ is the typical ring spacing. It should be noted that the equations are only valid for the middle area of the Bessel-Gaussian mode. Although this formulation is approximate, it accurately fulfils the purpose of representing the spiral nature, as has been demonstrated in a number of studies [30,31].

4. Surface optical vortex

It is well-known that when light mode is on, an atom experiences a transition frequency $\omega < \omega_{\text{q}}$ interacting with the evanescent light mode and is subject to the effects of the optical force under any kind of interaction (dipole or quadrupole). This force plays a major role in mechanical action, especially in the absence of any influence of interactions images [32]. In addition, the gravitational force, which acts as the starting switch, is also very small compared to the optical force [33]. These two factors guarantee that the optical force dominates once the process begins. In cases where only the optical quadrupole transitions are permitted, the expression for the steady-state quadrupole optical force is given by [9–11]

$$F_{\text{Quad}}^{\text{q}}(\mathbf{r}, \mathbf{v}) = F_{\text{Diss}}^{\text{q}}(\mathbf{r}, \mathbf{v}) + F_{\text{Grad}}^{\text{q}}(\mathbf{r}, \mathbf{v})$$  \hspace{1cm} (10)$$

These two forces are well known in quantum optics, the former resulting from the absorption and spontaneous emission processes and called the dissipative force, whereas the other force is due to the heterogeneous distribution of the field and called the quadrupole force. These two forces are given respectively as follows.

$$F_{\text{Diss}}^{\text{q}}(\mathbf{r}, \mathbf{v}) = 2 \hbar \Gamma_{\text{q}} |\Omega_{\text{q}}^{\text{q}}(\mathbf{r})|^2 \left( \frac{\nabla \theta_{\text{q}}(\mathbf{r})}{\Delta_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v}) + 2 |\Omega_{\text{q}}^{\text{q}}(\mathbf{r})|^2 + \Gamma_{\text{q}}^2} \right)$$  \hspace{1cm} (11)$$

$$F_{\text{Grad}}^{\text{q}}(\mathbf{r}, \mathbf{v}) = -2 \hbar \nabla |\Omega_{\text{q}}^{\text{q}}(\mathbf{r})|^2 \left( \frac{\Delta_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v})}{\Delta_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v}) + 2 |\Omega_{\text{q}}^{\text{q}}(\mathbf{r})|^2 + \Gamma_{\text{q}}^2} \right)$$  \hspace{1cm} (12)$$

In the above two equations; $\Gamma_{\text{q}}$ represents the quadrupole emission rate and $\Delta_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v})$ refers to the dynamic detuning $\Delta_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v}) = (\omega_{\text{q}} - \omega) - \nabla \theta_{\text{q}}(\mathbf{r})$ while $\nabla \theta_{\text{q}}(\mathbf{r})$ denotes the gradient of the phase $\theta_{\text{q}}(\mathbf{r})$. It can easily be seen that the effective quadrupole potential $U_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v})$ can be derived from the second force as follows.

$$U_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v}) = -\frac{\hbar \Delta_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v})}{2} \ln \left( 1 + \frac{2 |\Omega_{\text{q}}^{\text{q}}(\mathbf{r})|^2}{\Delta_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v}) + \Gamma_{\text{q}}^2} \right)$$  \hspace{1cm} (13)$$

At this stage, it is necessary to make two points. First, there are two types of detuning, each of which has a different effect on the overall dynamic process. The first type, where $\omega_{\text{q}} < \omega$, is called negative detuning. Under its influence, the potential distribution $U_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v})$ is completely opposite in shape to the quadrupole Rabi frequency distribution $\Omega_{\text{q}}^{\text{q}}(\mathbf{R})$. The second type is the positive detuning which occurs when $\omega_{\text{q}} > \omega$ and potential distribution $U_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v})$ is similar in the shape to the quadrupole Rabi frequency distribution $\Omega_{\text{q}}^{\text{q}}(\mathbf{R})$. In summary, the determination of the quadrupole Rabi frequency distribution $\Omega_{\text{q}}^{\text{q}}(\mathbf{R})$ is fully sufficient to indicate the effective quadrupole potential distribution $U_{\text{q}}^{\text{q}}(\mathbf{r}, \mathbf{v})$.

Second, it should be assumed that the atom is constrained to move in the exact plane. For example, if the XY plane is taken, then in this case the elements of the quadrupole tensor are such that $Q_{xy} = Q_{xz} = 0$ while $Q_{xx} > 0$. Under this condition, the position-dependent Rabi frequency equation is given by [30]

$$\frac{\Omega_{\text{q}}^{\text{q}}(\mathbf{R})}{\Omega_0} = \left[ \left( -\frac{x \sin \phi}{Z_{\text{max}}} \right)^{1/2} \exp \left( -\frac{x^2 \sin^2 \phi}{Z_{\text{max}}^2} \right) \right] \times J_\ell(\zeta)$$

$$\left\{ \left( \frac{1}{J_\ell(\zeta)} = \frac{\partial J_\ell(\zeta)}{\partial X} - \frac{i \ell Y}{\ell^2} \right) \right\}$$  \hspace{1cm} (14)$$

where $\zeta$ is the Bessel’s function argument which can be written as

$$\zeta = \frac{1}{\sqrt{\omega_0}} \sqrt{\frac{x^2 \cos^2 \phi + y^2}{1 + (\lambda Z / \pi w_0^2)^2}}$$  \hspace{1cm} (15)$$

And the scaling $\Omega_0$ factors of the Rabi frequency is

$$\Omega_0 = \frac{\hat{Q}_{xx}}{\hbar} \sqrt{\frac{8 \pi^2 \ell^2}{m^2 e^2 c}}$$  \hspace{1cm} (16)$$

Finally, for further clarification, we point out that each type of detuning has its own applications in quantum optics depending on the kind of physical application, whether it acts as an attraction to atoms such as in an optical vortex, or repelling them such as in an atomic mirror.

5. Results and discussions

Usually, the scenario is completed in the theoretical model through numerical computations made from
variables close to those investigated experimentally. From this point of view, a cesium atom \((M = 2.20614 \times 10^{-25} \text{kg})\) will be taken, and in particular the quadrupole transition \(6^2S_{1/2} \rightarrow 5^2D_{5/2}\) corresponding to wavelength \(\lambda = 675 \text{nm}\) will be selected. The rest of the variables are optional and can be controlled down to the appropriate values. Accordingly, the intensity of the laser source is taken to be \(I = 5 \times 10^{10} \text{Wm}^{-2}\) and \(Q_{xx} = 10 e_0^2\). A waist of \(w_0 = \lambda/2\) was chosen for the Bessel-Gaussian mode and a typical ring spacing of \(Z_{\text{max}} = Z[34]\). The end with respect to the variables, is those related to the order of the Bessel-Gaussian modes \(\ell = 0, 1, 2, \ldots\), as each of them has its own distinctive characteristics that will certainly be reflected in the entire performance.

Before beginning to present the results, it may be more appropriate to give what can be considered a combined analysis of Bessel-Gaussian modes. Such an analysis will give initial indications of the expected results and will help in identifying the spatial features of the case under study. It is well established that laser light as a Bessel-Gaussian mode gives small diffraction, characteristically exhibiting a number of concentric high intensity rings separated by dark rings. In particular, the central peak is considered to be extraordinarily stable against diffraction, and it is this feature that underlies many recent applications. This, of course, is not exactly the same for all orders of modes. For example, the central spot of the fundamental mode \(\ell = 0\) is always bright (a central maximum) while that of all modes \(\ell > 0\) are always dark on the axis and are surrounded by concentric rings whose peak intensities decrease as \(r^{-1}\) [34].

On the other hand, it can easily be observed that the fundamental mode (although it has the least diffraction) does not have an azimuthal phase dependence on the mode axis. This means that the fundamental mode cannot afford orbital angular momentum. However, the situation will be completely different where all higher-order modes \(\ell > 0\) have this azimuthal dependence and thus are endowed with orbital angular momentum. Consequently, they have a non-diffracting dark core, thus they are showing the orbital angular momentum carried by these modes. We point out here that a non-diffracting dark core feature of all higher-order modes \(\ell > 0\) makes these modes appropriate for the creation of optical vortices [30,34].

Figure 2 shows the spatial distribution of the squared complex Rabi frequency, as given by Equation (14), for a Bessel-Gaussian mode with \(\ell = 0\). This figure (and all further figures) will plotted in the xy plane at \(z = 0\) and \(\varphi = 42^\circ\), the most appropriate incidence angle, as has been demonstrated in many studies. As the figure shows, regions of different maximum and minimum
Figure 3. Variations of the Rabi frequency distribution (in $\Omega_0$ unit) for $\ell = 1$ and its projection plot in the $xy$ plane.

Figure 4. Variations of the Rabi frequency distribution (in $\Omega_0$ unit) for $\ell = 2$ and its projection plot in the $xy$ plane.
Figure 5. Variations of the Rabi frequency distribution (in $\Omega_0$ unit) for $\ell = 10$ and its projection plot in the $xy$ plane.

intensities were obtained. Therefore, each atom having a transition frequency ($\omega < \omega_{\ell m}$) being close to touching the outer surface, it will undergo a different strength of interaction with the evanescent mode according to the friction region. Finally, the projection plot clearly confirms what was previously explained above about the fundamental mode regarding its lack of orbital angular momentum. Consequently, achieving the helical property of atomic motion through the fundamental mode is not possible.

However, the situation is completely different with the higher modes $\ell > 0$, as they generally give the spiral effect with a wide diversity of distribution according to the mode order. Figures 3–5 display the same variable presented in Figure 1 with the same parameters for three modes; $\ell = 1$, $\ell = 2$, and $\ell = 10$, respectively. It turns out that the spatial distributions tend to become more complex as the order of the mode increases. Accordingly, the expected atomic motion to be achieved is also complex. Nevertheless, it can easily be observed from their projections that they retain the characteristic of orbital angular momentum. It emerges from these considerations that all Bessel-Gaussian modes except the fundamental mode can generally be exploited in the creation of optical vortices.

Optical vortices created with semi-bounded space will certainly be better than those produced in unbounded space. Because they are tightly bound to the surface as an evanescent mode, they are generated in the vacuum region, with their intensity in vacuum depending on the Bessel order and displaying interesting spatial variations, while still decaying exponentially with distance away from the surface. The important consequence is that this kind of optical system further enables active-quadrupole interactions to be exploitable in atom dynamics. Furthermore, this arrangement also has the advantage of limiting the effect to a definite location, which makes it more stable in space.

6. Conclusion

This study has been concerned with the coupling of pure optical vortices, in particular the evanescent Bessel-Gaussian modes, to dipole-prohibited but quadrupole-allowed atomic transitions. Accordingly, the work focused on the interaction of atoms with evanescent winding light, which has additional importance in the emergence of the orbital angular momentum property. Although novel features of this property have been much discussed in the past two decades [26], it has remained limited to electric dipole transitions. The limitation on this interaction did not help to further the prospect of engaging orbital angular momentum with electronic transitions that has been an ultimate
goal of many theoretical and experimental studies. Its importance is due to the fact that the orbital angular momentum can be transferred to the internal degrees of freedom of the atom [35]. In fact, researchers have realized that this could be easily achieved using electric quadrupole interaction, but the main problem was that this interaction is too weak to be exploited.

Recently, a number of physical studies have demonstrated some methods by which the value of the quadrupole interaction can be made acceptable for some applications in quantum optics. Perhaps one of the most effective and efficient of these methods places the atoms near evanescent light, which enhances quadrupole interaction by two orders of magnitude over other methods [4–6]. For this reason, it has recently received concerted attention in the context of atomic mirrors with more conventional forms of laser light or surface optical vortices with unconventional forms of laser light such as Laguerre-Gaussian and Bessel-Gaussian modes [36].

In addition, in order to get close to practical reality, some typical experimentally accessible parameters for atomic transitions should be chosen. Some recent experiments have succeeded in achieving these quadrupole atomic transitions in a number of atoms of such as sodium and cesium. Here, the cesium atom with quadrupole transition $6^2S_{1/2} \rightarrow 5^2D_{5/2}$ is selected, and we have explained how the moments of electric quadrupole couple to the electric field of the evanescent winding light, leading to the appropriate value of the quadrupole potential [26]. We have pointed out that the outcome of the interaction is certainly significantly improved, especially in the case of a relatively large evanescent winding light. We believe that further investigations should use the optical quadrupole potential produced by two counter-propagating and co-propagating modes in the same context that was done in the dipole interaction. It is well-known that each of them has its own distinct characteristics and thus its own applications, and it should be expected that a more complex intensity distribution of the quadrupole potential due to a wide range of values available for excitable modes will provide further utility to these applications. It may be possible for these techniques to be utilized in a better way than that achieved by dipole interactions, especially for optical surface tweezers and spanners, which have been of significant interest in recent years [36].

Disclosure statement

No potential conflict of interest was reported by the author(s).

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