Tunneling into 1D and quasi-1D conductors and
Luttinger-liquid behavior

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The paper addresses the problem whether and how it is possible to detect the Luttinger-liquid behavior from the IV curves for tunneling to 1D or quasi-1D conductors. The power-law non-ohmic IV curve, which is usually considered as a manifestation of the Luttinger-liquid behavior, can be also deduced from the theory of the Coulomb blockaded junction between 3D conductors affected by the environment effect. In both approaches the power-law exponents are determined by the ratio of the impedance of an effective electric circuit to the quantum resistance. Though two approaches predict different power-law exponents (because of a different choice of effective circuits), the difference becomes negligible for a large number of conductance channels.

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1. Introduction

One-dimensional and quasi-one-dimensional electron systems are attracting attention of theorists and experimentalists many years. From theorists' point of view they are interesting since in a number of cases one can obtain exact solutions for them taking into account many-body interactions without using the perturbation theory. Theoretically it has been well established that in a 1D electron gas with arbitrarily weak interaction Landau’s Fermi liquid (FL) theory breaks down, and the system is expected to behave as a Luttinger liquid (LL). The most important feature of the Luttinger liquid, in contrast to FL, is an absence of the fermion quasiparticle branch
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at low energies: excited states of the system must be described by the boson excitations which correspond to many-body electron states with a huge number of the electron-hole pairs (see, e.g., a recent review and references therein). This should have a pronounced effect on the tunneling into a LL conductor: the $IV$ curve of a tunnel junction between a normal FL and a LL conductor is expected to be non-ohmic described by a power law with an exponent depending on interaction strength.

Experimental evidences for LL behavior have been reported in quantum wires and single walled and multiwalled carbon nanotubes. However, it is not completely clear, to what extent these experimental evidences may be considered as an unambiguous proof of the LL behavior. Measurements of $IV$ curves for single walled and multiwalled carbon nanotubes have revealed a deviation from the LL behavior at high voltages. In Ref. they observed a crossover from a non-ohmic $IV$ curve to Ohm’s law with a Coulomb-blockade offset determined by the capacitance of the junction between a 3D metallic lead and a nanotube. This crossover is predicted by the theory of the Coulomb blockade in a junction between normal FL conductors. A crucial role in this theory belongs to the environment effect, which depends on the ratio of the real part of the circuit impedance to the quantum resistance $R_K = h/e^2 \approx 26 \, k\Omega$. The environment effect originates from the Nyquist-Johnson noise in an electric circuit. The noise results in fluctuations of the phase at the junction, which can suppress the Coulomb blockade. Further we call this approach the environment quantum fluctuation (EQF) theory.

The existing LL theory for tunneling takes into account accurately the Coulomb interaction inside the LL liquid conductor, but ignores the Coulomb energy of the junction charge, which is defined by the junction capacitance $C_T$. The fact that the junction Coulomb energy $e^2/C_T$ important not only at low voltages, as widely known, but also at high voltages, means that the high-voltage part of the tunnel-junction $IV$ is a bad probe of bulk properties of nanotubes, and one should look for evidences of LL behavior mostly at intermediate voltages characterized by the power-law $IV$ curve. However, both EQF and LL pictures predict a power-law dependence, and a serious problem arises, how, and whether it is possible at all, to discriminate these two pictures. This is the problem that the present paper addresses.

Section reminds the basic ideas of the EQF theory and defines the power-law exponents for the tunnel conductance and the function $P(E)$, which determines a probability of the transfer of the energy $E$ to the environment. In Sec. the results of the LL theory are discussed and compared with predictions of the EQF theory for an one-channel 1D conductor with spinless electrons. Section considers the same questions, but for a quasi-1D
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conductor with many channels. The results of the analysis are summarized in the concluding Sec. 5.

2. The Coulomb blockade and the environment quantum fluctuation (EQF) theory

We restrict our discussions by zero temperature. The standard tunneling theory based on the tunneling hamiltonian and the quantum-mechanical Golden Rule gives the following expression for the current through the junction:

\[ I = \frac{1}{e R_T} \int_0^{eV} dE_1 \int_0^{eV} \rho(E_2) dE_2 \delta(E_1 - E_2) . \]  

Here \( \rho(E) \) is the relative density of the state for the right conductor normalized to the constant DOS of the normal Fermi-liquid, the latter being included into the definition of the junction conductance \( 1/R_T \). The left conductor is supposed to be always a FL conductor and the relative DOS for him is unity. If the right conductor is also a FL conductor then \( \rho(E) = 1 \), and the IV curve is exactly ohmic.

The EQF theory assumes that both conductors, which form a junction, are 3D FL conductors, i.e., \( \rho(E) = 1 \), but takes into account fluctuations in the electric circuit (the Nyquist-Johnson noise). This results in the fluctuation of the phase of the tunnelling hamiltonian. Therefore the delta-function in Eq. (1) should be replaced by the function which is a Fourier transform of the phase correlator:

\[ P(E) = \frac{1}{2\pi \hbar} \int_{-\infty}^{\infty} dt e^{iEt/\hbar} \langle e^{i\hat{\varphi}(t)} e^{-i\hat{\varphi}(0)} \rangle , \]  

and Eq. (1) becomes

\[ I = \frac{1}{e R_T} \int_0^{eV} dE_1 \int_0^{eV} dE_2 P(E_1 - E_2) \int_0^{eV} dE (eV - E) P(E) . \]  

The phase \( \varphi \) is connected with the voltage \( V \) across the junction by the Josephson relation

\[ \hbar \frac{d\varphi}{dt} = eV . \]  

Since the phase \( \hat{\varphi} \) is an operator conjugate to the operator of the electron number, the operator \( e^{-i\hat{\varphi}(t)} \) is a creation operator of the electron in the circuit at the moment \( t \); or, more exactly, since \( \varphi \) is the phase difference across the junction, an operator of the electron annihilation in the left conductor and electron creation in the right conductor. The correlator in Eq. (3) may be presented as

\[ \langle e^{i\hat{\varphi}(t)} e^{i\hat{\varphi}(0)} \rangle = e^{J(t)} , \]  

where

\[ J(t) = \frac{1}{2\pi \hbar} \int_{-\infty}^{\infty} dt e^{iEt/\hbar} \langle e^{i\hat{\varphi}(t)} e^{-i\hat{\varphi}(0)} \rangle . \]
where at $T = 0$ the correlator $J(t)$ is
\[
J(t) = \langle [\hat{\phi}(t) - \hat{\phi}(0)]\hat{\phi}(0) \rangle = 2 \int_{0}^{\infty} \frac{d\omega}{\omega} \frac{\text{Re}Z(\omega)}{R_K} \left( e^{-i\omega t} - 1 \right).
\]  
(6)

This relation is a direct consequence of the fluctuation-dissipation theorem, which connects the correlator of a “coordinate” with a linear response to a conjugate “force”. A conjugate “force” for the “coordinate” $\varphi$ is $\bar{h}I/e$, and because of the Josephson relation (4), the imaginary part of the response is proportional to the real part of the impedance $Z(\omega) = V(\omega)/I(\omega)$:
\[
\text{Im}\{e\varphi(\omega)/\bar{h}I(\omega)\} = \text{Re}\{e^2Z(\omega)/\bar{h}\omega\}.
\]

The function $P(E)$ gives probability of the excitation of the environment mode after the electron tunneling. In the case of low-impedance environment ($\text{Re}Z/R_K \to 0$) this probability is negligible and $P(E)$ function reduces to the delta-function $\delta(E)$. This means that the environment fluctuations eliminate the Coulomb blockade. In the opposite limit of the high-impedance environment ($\text{Re}Z/R_K \to \infty$) the function $P(E)$ becomes $P(E) = \delta(E - E_c)$, where $E_c = e^2/2C_T$ is the Coulomb energy of the junction. The $IV$ curve of the junction in this case has a voltage offset $e/2C_T$, which is a manifestation of the Coulomb blockade. For finite $\text{Re}Z/R_K$ at $E$ much smaller than the Coulomb energy, $P(E)$ is a power-law function:
\[
P(E) \propto \frac{\tau}{\hbar} \left( \frac{E}{\bar{h}} \right) ^{\alpha_{E}-1},
\]  
(7)

where
\[
\alpha_{E} = \frac{2\text{Re}Z(0)}{R_K},
\]  
(8)

and $\tau = \text{Re}Z(0)C_T$ is the relaxation time in the effective electric circuit.

At zero temperature the probability $P(E)$ determines the second derivative of the $IV$ curve:
\[
\frac{d^2I}{dV^2} = \frac{e}{R_T} P(eV).
\]  
(9)

Thus the exponent $\beta_E$ of the conductance $dI/dV \propto V^{\beta_E}$ is equal to $\alpha_{E}$.

In the limit of $E \gg E_c$, $P(E)$ is a decreasing function of $E$:
\[
P(E) = \frac{2}{E} \frac{\text{Re}Z(E/\bar{h})}{R_K}.
\]  
(10)

Phase fluctuations at the junction are determined by the impedance of the effective electric circuit, which includes the capacitance of the junction, i.e.
\[
\frac{1}{Z(\omega)} = -i\omega C_T + \frac{1}{Z_0(\omega)},
\]  
(11)
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where \(Z_0(\omega)\) is the impedance of the circuit connected to the junction. For a pure ohmic environment, \(Z_0 = R\), the exponent \(\alpha_E = 2R/R_K\) determines also the IV curve and high voltages \(V \gg e/C_T, \hbar/e\tau\). High voltages probe the high-frequency impedance \(Z(\omega)\). At the relevant high frequency \(\omega = eV/\hbar\), the junction capacitance \(C_T\) short circuits the impedance, and the IV curve is obtained using Eqs. (3), (10), and (11):

\[
I \approx \frac{1}{R_T} \left[ V - \frac{e}{2C_T} + \frac{\alpha_E}{2\pi^2} \left( \frac{e}{2C_T} \right)^2 \frac{1}{V} \right].
\]

(12)

This high-voltage asymptotics, characterized by the Coulomb offset \(e/2C_T\) and the “tail” voltage \(\propto 1/V\) was experimentally studied and discussed by Wahlgren et al., and Penttilä et al. within the horizon picture.8, 9

For comparison with the LL theory important is the case of an infinite transmission \(LC\) line with the real impedance \(Z_0 = \sqrt{l_l/c_l}\), where \(l_l\) and \(c_l\) are the line inductance and the line capacitance per unit length. Then \(\alpha_E = 2Z_0/R_K\). A conductor can be treated as an infinite \(LC\) line, if its total ohmic resistance is much higher then the impedance \(\sqrt{l_l/c_l}\), but does not exceed, nevertheless, its total inductive resistance \(\omega l_l L\), where \(L\) is the transmission line length.

3. Tunneling into an one-channel 1D conductor (spinless electrons)

3.1. Tunneling in the Luttinger-liquid (LL) theory

For the sake of simplicity we start from the one-channel case of spinless electrons. In the LL theory the environment effects are absent, and we must return to Eq. (1), which after one integration is:

\[
I = \frac{1}{eR_T} \int_0^{eV} dE \rho(E)dE.
\]

(13)

Because of the electron-electron interaction the relative DOS \(\rho(E)\) essentially different from unity. It is determined by the Fourier transform of the electron Green function \(\langle \hat{\psi}(x,t)\psi^\dagger(x,0) \rangle\), which is determined by the boson degrees of freedom. For the end contact \(\rho(E)\) is given by the relation very similar to Eq. (7):

\[
\rho(E) \propto \left( \frac{E\tau_c}{\hbar} \right)^{\alpha_L-1},
\]

(14)

where

\[
\alpha_L = \frac{v_{pl}}{v_F}
\]

(15)
is the interaction parameter for the one-channel Luttinger liquid (spinless fermions), $v_F$ is the Fermi velocity, and

$$v_{pl} = \sqrt{v_F^2 + \frac{2e^2 v_F}{\pi \hbar} \ln \frac{r_g}{r_0}}$$

(16)

is the velocity of the boson collective mode in the Luttinger liquid with the long-range Coulomb interaction. Here $v_F = \pi \hbar n_1/m^*$ is the Fermi velocity, $n_1$ is the 1D electron density per unit wire length, $m^*$ is the effective electron mass, $r_0$ is the radius of the wire and $r_g$ is the distance from the metallic ground. The boson mode is nothing else but an 1D plasmon mode $\omega = v_{pl} k$. For small $\alpha_L - 1$ the correction to the constant Fermi-liquid DOS is small: $\rho(E) \approx 1 + (\alpha_L - 1) \ln(E/\tau_c)$. This correction can be obtained from the theory of Altshuler and Aronov using the perturbation theory.

The DOS $\rho(E)$ determines the first derivative of the IV curve (conductance):

$$\frac{dI}{dV} = \frac{1}{R_T} \rho(eV).$$

(17)

Thus the exponent of the conductance $\propto V^{\beta_L}$ is $\beta_L = \alpha_L - 1$.

Fisher and Glazman chose the cut-off time $\tau_c$ in Eq. (14) to be of order of the inverse Fermi energy. Another possible cut-off is related to the wave vector $k_c$ at which the Coulomb interaction becomes weak. According to Eq. (16) it is the inverse size of the 1D wire (nanotube): $k_c \sim 1/r_0$. Then the cut-off time in Eq. (14) is $\tau_c \sim r_0/v_F$. This cut-off is more relevant if the wire radius $r_0$ is essentially larger then the inverse Fermi wave vector.

### 3.2. Comparison the EQF and LL theories

Let us discuss similarity between the DOS $\rho(E)$ in the LL theory and the function $P(E)$ in the EQF theory. The former is defined as a Fourier transform of the averaged operator product $\langle \hat{\psi}(x,t)\hat{\psi}^\dagger(x,0) \rangle$ and the latter is a Fourier transform of the correlator $\langle e^{i\psi(t)}e^{-i\psi(0)} \rangle$. Both the operators $e^{-i\hat{\varphi}(t)}$ and $\hat{\psi}^\dagger(x,t)$ are creation operators of a single charge $e$. One should expect then similar functional dependencies for $\rho(E)$ and $P(E)$. This takes place when the voltage at the junction is less than the Coulomb gap $e/C_T$.

Indeed, the exponents $\alpha_E$ and $\alpha_L$ for the power laws predicted by the EQF theory, Eq. (8), and by the LL theory, Eq. (15), coincide, if one interprets the 1D plasmon mode as a wave along the lossless LC transmission line formed by a 1D conductor and a metallic ground. The capacitance per unit length of the transmission line is $C_l = 1/(2 \ln r_g/r_0)$, and the inductance per unit length is determined by the kinetic energy of electrons, i.e., is a kinetic inductance $l_l = m^*/(e^2 n_1) = R_K/2v_F$, which essentially exceeds the
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geometric inductance \( \ln(r_g/r_0)/c^2 \) because of low electron 1D density \( n_1 \) in 1D wires, compared to the density \( nS \) in 3D wires with 3D density \( n \) and the cross-section area \( S \). The transmission line with these parameters supports the sound-like wave with the velocity \( 1/\sqrt{l/c_1} \) which coincides with the plasmon velocity \( v_{pl} \) given by Eq. (16) in the limit of very strong Coulomb interaction \( v_{pl} \gg v_F \). Using the impedance \( Z = \sqrt{l/c_1} = l_1 v_{pl} \) of the infinite LC transmission line, Eq. \( (8) \) for \( \alpha_E \) becomes identical to Eq. \( (13) \) for \( \alpha_L \) in this limit.

But in fact similarity between \( \rho(E) \) and \( P(E) \) is not restricted by the limit of strong Coulomb interaction \( v_{pl} \gg v_F \). One can obtain the full expression for the plasmon velocity, Eq. (16), by taking into account the compressibility of the neutral Fermi gas in the derivation of the plasmon mode. Namely, equation of electron motion should be:

\[
\frac{\partial I}{\partial t} = \frac{e^2}{m} n_1 E_z - c_s^2 \nabla q .
\] (18)

In the 3D case this yields the spectrum of the 3D plasma waves with the dispersion: \( \omega^2 = \omega_{pl}^2 + c_s^2 k^2 \). Here \( c_s \) is the sound velocity, \( q \) is the charge per unit wire length, and \( E_z = \nabla \phi \) is the electric field along the wire. For the 1D Fermi gas \( c_s = v_F, \phi = 2q \ln(r_g/r_0) \), and Eq. \( (18) \) together with the continuity equation

\[
\frac{\partial q}{\partial t} + \nabla I = 0
\] (19)
yield the sound-like spectrum with the velocity given by Eq. \( (16) \). In order to take into account the compressibility of the neutral Fermi gas, the voltage \( V \) across the junction in the Josephson relation Eq. \( (4) \) should be defined as a difference of the electrochemical potential across the junction:

\[
V = \phi_1 - \phi_2 + \frac{\mu_1 - \mu_2}{e} .
\] (20)

Earlier the EQF theory took into account only the electric potential difference \( \phi_1 - \phi_2 \), but neglected the chemical potential difference \( \mu_1 - \mu_2 \). This is usually well justified for 3D metals, but not for nanotubes. The effect of the neutral-gas compressibility on the plasmon velocity and the impedance can be incorporated by introducing a renormalized capacitance of the line,

\[
\frac{1}{c} = \frac{1}{c_1} + \frac{1}{c_0} \quad \frac{1}{c_0} = \frac{1}{e^2} \frac{\partial \mu}{\partial n} = \frac{R_K v_F}{2} .
\] (21)

While the geometric capacitance \( c_1 \) is related to the energy of the electric field between the wire and the metallic ground, the capacitance \( c_0 \) is related
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to the kinetic energy of the electron Fermi sea. Then the impedance for an infinite transmission line formed by a 1D wire is:

\[ Z = \frac{V}{I} = \sqrt{\frac{l_i}{c}} = l_i v_{pl} = \sqrt{\frac{l_i}{c_l} + \frac{R_K^2}{4}} = \frac{R_K}{2} v_{pl}. \]  

Finally we obtain that the probability \( P(E) \) at \( E < \hbar/\tau \) is described by the same power law as the DOS \( \rho(E) \) in the LL theory: \( \alpha_E = \alpha_L \). Thus both the EQF and the LL theory predict a suppression of conductance, but physical reasons for it look different. In the LL picture the current is suppressed because there are no single electron quasiparticles, and the charge is transported by bosonic modes (plasmons). In the Coulomb blockaded normal junction between 3D wires single-electron states are available, but at low voltage bias the tunneling electron has not enough energy to get into them.

Equation (22) shows that the impedance of a wire has a quantum limit like the d.c. resistance of a ballistic conductor. But the “quantum impedance” \( R_K/2 \) is two times smaller than the quantum resistance \( R_K \) found by Landauer for one channel (spinless electrons). This is because the d.c. energy dissipation is related to two contacts, whereas the quantum impedance presents losses at one contact. Another difference is that d.c. energy dissipation takes place in massive electrodes, which supply a current to a ballistic conductor. But a.c. energy dissipation given by the quantum impedance occurs inside the wire. Indeed, in order to be considered as an infinite transmission line (see discussion in the end of Sec. 2), the wire must have a small but finite ohmic resistance, which is able to suppress reflection of plasmons from another end of the wire (plasmon resonances).

Since \( P(E) \) is connected with the second derivative of the \( IV \) curve and \( \rho(E) \) is connected with the first derivative, the exponents for the conductance, \( \beta_E = \alpha_E \) and \( \beta_L = \alpha_L - 1 \), differ by unity. In the limit of the weak Coulomb interaction inside the conductor \( \alpha_E = \alpha_L = v_{pl}/v_F \approx 1 \) the difference is most pronounced: the LL theory predicts an ohmic \( IV \) curve, whereas the EQF theory predicts the Coulomb blockade regime with the power law \( I \propto V^2 \), if \( V \ll e/C_T \). This so-called “weak-Coulomb-interaction” limit is valid at the condition that \( v_F \gg \ln(r_g/r_0)/R_K \).

4. Tunneling into a multichannel quasi-1D conductor

One may expect that the difference between the EQF and the LL predictions should vanish if a number of channels (electron modes) grows. This was confirmed by the work of Matveev and Glazman. But some features of this crossover remain unclear. Matveev and Glazman have shown that
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in the limit of a large number of channels both theories predict the same power law for conductance: $\beta_E = \beta_L$. But this means that the exponents $\alpha_E$ and $\alpha_L$ for $P(E)$ in the EQF theory and for $\rho(E)$ in the LL theory differ by 1, in contrast to the one-channel case, in which the exponents for $\rho(E)$ and for $P(E)$ coincide, but the exponents for conductance are different. On the other hand, due to reasons explained in the previous section, $\rho(E)$ and $P(E)$ should have the same functional dependence on $E$ at small energies.

For further discussion of this problem, I rederive the result of Matveev and Glazman in terms of the transmission-line modes. A quantum wire with $N$ electron channels can be considered as a collection of $N$ parallel transmission lines. We assume that the electrostatic energy in this circuit is

$$E_{el} = \sum_{i=1}^{N} \frac{q_i^2}{2c_i} + \frac{1}{2c_l} \left( \sum_{i=1}^{N} q_i \right)^2,$$  \hspace{1cm} (23)

where $c_l = 1/(2 \ln r_g/r_0)$ is the capacitance per unit length between each channel and the ground, and the capacitances $c_i$ are related with the electron kinetic energies, which can be different for various channels in general. The voltages, which are defined as electrochemical potentials with respect to the ground, are:

$$V_i = \frac{\partial E_{el}}{\partial q_i} = \frac{q_i}{c_i} + \frac{1}{c_l} \sum_{j=1}^{N} q_j.$$  \hspace{1cm} (24)

Using the continuity equations,

$$\frac{\partial q_i}{\partial t} + \frac{\partial I_i}{\partial x} = 0,$$  \hspace{1cm} (25)

and the equations of motion for electrons in all channels,

$$l_i \frac{\partial I_i}{\partial t} = \frac{\partial V_i}{\partial x},$$  \hspace{1cm} (26)

we obtain $N$ equations for charge densities $q_i$. Here $l_i$ is the inductance per unit length of the $i$th channel. For the plane wave solutions $q_i \propto e^{ikx-i\omega t}$, these $N$ equations are:

$$s^2 l_i q_i = \frac{q_i}{c_i} + \frac{1}{c_l} \sum_{j=1}^{N} q_j,$$  \hspace{1cm} (27)

where $s = \omega/k$ is the velocity of the sound-like modes. Equations (27) are identical to Eqs. (11) of Matveev and Glazman. In terms of parameters, used by Matveev and Glazman, the interaction potential $V$ and the Fermi
velocity \( v_i \) of the \( i \)th channel, the inductance and the Fermi velocity of the \( i \)th channel are \( l_i = c_l V / v_i \) and \( v_i = 1/\sqrt{l_i c_i} \).

One must find \( N \) eigenmodes of the system. Further we assume that all channels are identical: \( c_i = c_0, l_i = l_l \). Then there are only two eigenvalues:

\[
s_1^2 = \frac{1}{l_l c_l} + \frac{N}{l_l c_l} \tag{28}
\]

and \( s_2^2 = 1/l_l c_0 \), which is \((N-1)\)-degenerate. The first eigenvector is \( q_i^{(1)} \propto 1 \). This is a plasmon mode, in which the total charge \( \sum_j q_j \) oscillates and the velocity \( s_1 \) is the plasmon velocity \( v_{pl} \). The other \( N - 1 \) modes are neutral, and therefore their velocity \( s_2 \) coincides with the Fermi velocity \( v_F = 1/\sqrt{l_l c_0} \). The \( N - 1 \) eigenvectors for the eigenvalue \( s_2^2 \) can be chosen as \((j \neq 1)\)

\[
q_i^{(j)} \propto \delta_{ij} - \frac{1}{N}. \tag{29}
\]

Now we want to calculate the impedance. In the \( N \)-channel system the impedance is a matrix: \( Z_{ij} = V_i / I_j \). In order to find it, one must calculate voltages \( V_i \) of various channels assuming that the current is present only in the \( j \)th channel. The densities of state for various channels, which are considered in the LL model, are associated with the diagonal elements of the impedance matrix. In our case all channels are equivalent, and one can calculate the impedance \( Z_{11} \) for the first channel. One should find a superposition of \( N \) eigenvectors, which determines the charge

\[
q_i = \sum_{j=1}^{N} A_j q_i^{(j)} = A_1 + A_i (1 - \delta_{1i}) - \frac{1}{N} \sum_{j=2}^{N} A_j , \tag{30}
\]

the current

\[
I_i = \sum_{j=1}^{N} A_j s_j q_i^{(j)} , \tag{31}
\]

and the voltage

\[
V_i = \sum_{j=1}^{N} A_j l_j s_j^2 q_i^{(j)} \tag{32}
\]

in any channel. The coefficients \( A_j \), which define the superposition, should be found from the conditions that the total current comes only into the 1st channel. The condition for the 1st channel is:

\[
I = A_1 s_1 - \sum_{j=2}^{N} A_j s_2 \frac{1}{N} . \tag{33}
\]
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For the other $N - 1$ channels ($i \neq 1$):

$$0 = A_1 s_1 + A_i s_2 - \sum_{j=2}^{N} \frac{A_j s_2}{N}.$$  \hfill (34)

This system of equations has a solution: $A_1 = I/N s_1$ and for $i \neq 1$ $A_i = -I/s_2$. Now the voltage at the first channel is:

$$V_1 = l_I \left( A_1 s_1^2 - \sum_{j=2}^{N} \frac{A_j s_2^2}{N} \right) = l_I \left( \frac{1}{N} s_1 + \frac{N-1}{N} s_2 \right).$$  \hfill (35)

Finally the impedance is

$$Z_{11} = \frac{V_1}{I} = \frac{1}{N} \sqrt{\frac{l_I}{c_0} + \frac{N l_I}{c_l} + \frac{N-1}{N} \sqrt{\frac{l_I}{c_0} + \frac{R_K^2}{4 N^2} + \frac{l_I}{N c_l} + \frac{N-1}{N} \frac{R_K}{2}}}.$$  \hfill (36)

The “electric” part of this impedance, $\sqrt{l_I/N c_l}$, is related to a simple fact that the inductance of $N$ parallel transmission lines is in $N$ times less than the inductance $l_I$ of one transmission line.

In terms of the impedance $Z_{11}$ the exponent $\alpha_L$ is

$$\alpha_L = \frac{2 Z_{11}}{R_K} = \frac{1}{N} \frac{v_{pl}}{v_F} + \frac{N-1}{N}.$$  \hfill (37)

One can compare it with the EQF exponent for a multichannel system:

$$\alpha_E = \frac{2 Z}{R_K} = \frac{1}{N} \frac{v_{pl}}{v_F}.$$  \hfill (38)

This directly follows from the one-channel value, bearing in mind that the multichannel impedance is an impedance of $N$ parallel transmission lines. Thus the exponents $\alpha_L$ and $\alpha_E$ are connected with the impedances by the same relations, which follow from the fluctuation-dissipation theorem, but the impedances are different in the EQF and the LL approaches. The EQF theory ignores the neutral modes, which are important in the LL approach. In order to obtain the EQF impedance one should assume that the current $I$ is distributed uniformly between channels, and therefore the neutral modes are not excited. On the other hand, the EQF approach takes into account the single-particle excitations, which are absent in the LL approach, but the neutral modes play the same role, and eventually the exponents for conductance are the same for multichannel conductors.

The LL and EQF conductance exponents, which follow from Eqs. (37) and (38), are very close each other for a large number $N$ of channels:

$$\beta_L = \alpha_L - 1 = \frac{1}{N} \left( \frac{v_{pl}}{v_F} - 1 \right), \quad \beta_E = \alpha_E = \frac{1}{N} \frac{v_{pl}}{v_F}.$$  \hfill (39)
However, the EQF theory predicts the conductance exponent, which is similar to that in the LL theory, only in the Coulomb blockade regime $V \ll e/C_T$. At higher voltages the junction capacitance $C_T$ shunts the environment impedance, and the $IV$ curve is given by the asymptotic expression Eq. (12). This corresponds to constant conductance: $\beta_E = \alpha_E = 0$. If in analogy with the EQF theory one tries to introduce the capacitance $C_T$ into the effective circuit in the LL picture, this yields a dramatically different result. The capacitance $C_T$ short circuits the impedance $Z_{11}$, like in the EQF picture, and $\alpha_L \to 0$. But now the conductance exponent $\beta_L \to -1$, and the LL theory predicts the voltage-independent current at high voltages. This contradicts to experiment, and it is unclear what is a proper way to take into account the junction Coulomb energy in the LL theory.

5. Discussion and summary

The analysis has shown that despite a conceptual difference between the EQF and the LL approaches, their predictions for the power law exponents of the conductance are the same for multichannel conductors. Therefore one cannot discriminate between two approaches in studying the power law exponents for a quasi-1D conductor with a large number of conduction channels.

This means that it is difficult to detect the LL behavior in multiwall nanotubes. For single-wall nanotubes the difference between the LL and EQF predictions is more pronounced, but also not very large. Every nanotube wall has four channels (two due to spin, and two due to two bands). This means that for a single-wall nanotube the difference between the conductance exponents $\beta_E$ and $\beta_L$ is only 0.25 for the same ratio $v_{pl}/v_F$. In order to reveal this difference and thus to discriminate between two pictures, it would be useful to measure not only the $IV$ curve, but also the plasmon velocity for the same nanotube using contactless methods.

Luttinger liquid is a well established and irrefutable concept of the condensed-matter physics, which has been confirmed by many years of theoretical studies. However, its experimental verification remains to be a challenging problem, which requires further efforts both in experiment and theory.

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