Real-Time Superresolution Interferometric Measurement Enabled by Structured Nonlinear Optics

Xin-Yu Zhang, Hai-Jun Wu, Bing-Shi Yu, Carmelo Rosales-Guzmán, Zhi-Han Zhu,* Xiao-Peng Hu,* Bao-Sen Shi,* and Shi-Ning Zhu

Optical interferometers are pillars of modern precision metrology, but their resolution is limited by the wavelength of the light source, which cannot be infinitely reduced. Magically, this limitation can be circumvented by using an entangled multiphoton source because interference produced by an N-photon amplitude features a reduced de Broglie wavelength $\lambda/N$. However, the extremely low efficiency in multiphoton state generation and coincidence counts actually negates the potential of using multiphoton states in practical measurements. Here, a novel interferometric technique based on structured nonlinear optics is demonstrated, i.e., parametric upconversion of a structured beam, capable of superresolution measurement in real time. The main principle relies in that the orbital angular momentum (OAM) state and associated intramodal phase within the structured beam are both continuously multiplied in cascading upconversion to mimic the superresolved phase evolution of a multiphoton amplitude. Owing to the use of bright sensing beams and OAM mode projection, up to a 12-photon de Broglie wavelength with almost perfect visibility is observed in real time and, importantly, by using only a low-cost detector. The results open the door to real-time superresolution interferometric metrology and provide a promising way toward multiphoton superiority in practical applications.

1. Introduction

In 1887, after six years of effort, Albert Michelson and Edward Morley announced their failure to observe the ether. This failure, however, provided the bases for Einstein to develop his special theory of relativity and began an era of optical interferometric measurement.[1] Since then, the optical interferometer has gradually become the primary toolkit of modern precision measurement. With the birth and evolution of laser techniques,[2] significant progress has been made in increasing the precision of interferometers over the past half-century. By estimating the phase displacement of a laser interferometer, one can measure a great variety of quantities, such as position, speed, acceleration, spectrum, and medium properties, with ultrahigh precision.[3–6] In particular, the two largest ever Michelson interferometers were constructed to search for ripples in space-time as the core part of the Laser Interferometer Gravitational Wave Observatory.[7] This giant instrument has unprecedented precision and successfully observed gravitational waves for the first time in 2015, 130 years after the Michelson–Morley experiment, finally confirming the space-time view in Einstein’s general theory of relativity. Despite hundreds of kilowatt lasers circulating within it, the performance of this giant interferometer has surprisingly already been limited by quantum theory.[10,11,8,9]

The performance of any interferometer depends heavily on its phase resolution and sensitivity, which are limited by the de Broglie wavelength and shot-noise limit of the light source, respectively.[12,13] Interestingly, both these limitations can be overcome by using N-photon entangled states as the light source, allowing N-fold superresolution and sensitivity in interferometric measurement. The underlying principle is, first, that the de Broglie wavelength of an N-photon wave packet, such as the reputed N00N state, depends on both the wavelength ($\lambda$) and number (N) of photons, i.e., $\lambda/N$. Second, the shot-noise limit originates from the uncertainty relation between the amplitude (i.e., number of photons) and phase of the light field, i.e., $\Delta \varphi \Delta n \geq 1$. In N00N states, the largest uncertainty is in the number of photons, corresponding to a minimal phase uncertainty, and these states can therefore be used to approach the Heisenberg limit in phase estimation, given by $\Delta \varphi = 1/n$.[14] Notably, pursuing supersensitivity in an infinitesimal area near the balance phase position, i.e., $\cos(x/2)$, of a SU(2)
interferometer is only meaningful if perfect transmission-detection efficiency and sufficient light power (such as kilowatt lasers) are attainable. Therefore, squeezed light (having no superresolution effect), not an N00N state, was chosen to further enhance the sensitivity of the detecting gravitational-wave detector. In contrast, realization of superresolution interferometric measurement featuring a reduced de Broglie wavelength with low-cost detectors and especially in a real-time manner would be beneficial in various applications. Thus, measuring a reduced de Broglie wavelength via recording multiphoton amplitude attracts constant interest in the optics community ranging from the most often demonstrated two-photon interference to the later reported 3-, 4-, and 8-photon experiments and the recently observed 10- and 12-photon entanglement. These demonstrations, on the one hand, are important benchmarks that show the considerable progress made in multiphoton manipulation, which is the physical basis for building photonic quantum information systems. On the other hand, the extremely low multiphoton coincidence rate (several photons per hour) and the poor interference visibility shown in results prevent the practical applications of multiphoton states in interferometric measurement.

To date, only one recent two-photon experiment has realized an acceptable coincidence rate and high visibility by using expensive superconducting detectors and photon pairs at a specific wavelength (coherence length matched). To avoid using the inefficient multiphoton state, scientists have also tried to directly extract multiphoton amplitude from a classical sensing beam (coherent state). But the efficiency of post-selecting multiphoton states from weak coherent light is still too low, as a result, an extremely low coincidence rate is still an insoluble problem. Therefore, an important question is how can the N00N-like superiority of this “entangled” bright beam be realized in more general interferometric measurement? Namely, without exploiting its $\ell$-fold angular sensitivity, how can the behavior of the N-photon amplitude and associated phase variation $N\Delta \varphi$ be built and recorded? We propose cascading second-harmonic generation (SHG) of the sensing structured beam to continuously double both the OAM and associated intramodal phase, mimicking an N00N-state injected interferometer. Specifically, we represent the SOC state using orthogonal linear polarizations $|\vec{\varphi}_\ell\rangle = \sqrt{1/2} (|\vec{\varphi}_+\rangle \pm |\vec{\varphi}_-\rangle)$, i.e., the mutually unbiased basis of circular polarizations, so that state (1) becomes $\sqrt{1/4} (|L_{\ell,0}^{00}\rangle + e^{i\varphi} |L_{\ell,0}^{0-}\rangle) |\vec{\varphi}_\ell\rangle + (|L_{\ell,0}^{0+}\rangle - e^{i\varphi} |L_{\ell,0}^{0-}\rangle) |\vec{\varphi}_\ell\rangle$. We then perform type-II SHG for this state, or rather, sum-frequency generation (SFG) between the state (1) $|\vec{\varphi}_\ell\rangle$ and components leading to, in addition to frequency doubling ($2\omega$), a 2nd-harmonic OAM mode:

$$|\psi_{\text{soc}}\rangle = \sqrt{1/2} \left( |L_{\ell,2}^{0+}\rangle - e^{i2\varphi} |L_{\ell,2}^{0-}\rangle \right)$$

Here, the $2\varphi$ phase-evolution behavior within the modal space spanned by conjugate OAM modes $L_{\ell,0}^{0\pm}$ is exactly the same as that in an SU(2) interferometer injected by the N00N state with $N = 2$. On this basis, we can further achieve three- or four-fold superresolution by performing SFG between the state (2) and state (1) $|\vec{\varphi}_\ell\rangle$-components or SHG of state (2), respectively. The corresponding harmonic OAM modes are:

$$|\psi_{\text{soc}}\rangle = \sqrt{1/4} \left( |L_{\ell,2}^{0+}\rangle + e^{i2\varphi} |L_{\ell,2}^{0-}\rangle \right)$$

$$- \sqrt{1/4} \left( e^{i\varphi} |L_{\ell,2}^{0+}\rangle + e^{i2\varphi} |L_{\ell,2}^{0-}\rangle \right)$$

(3)

$$|\psi_{\text{soc}}\rangle = \sqrt{1/6} \left( |L_{\ell,3}^{0+}\rangle + e^{i2\varphi} |L_{\ell,3}^{0-}\rangle \right) - \sqrt{2/3} e^{i2\varphi} |L_{\ell,0}^{0-}\rangle$$

(4)

2. Methods and Results

2.1. Concept and Principle

The structured light involved in this work refers to a cylindrical vector (CV) beam with spatially varying polarization. The beam constitutes a nonseparable superposition state of orthogonal circular polarizations $\vec{\varphi}$ and spatial modes carrying opposite OAMs $\pm \ell \hbar$; the most common Laguerre–Gaussian (LG$^{\ell,p}$) modes with radial index $p = 0$ are considered here. This paraxial SOC state can be simply represented using the Dirac notation:

$$|\psi_{\text{soc}}\rangle = \sqrt{1/2} \left( |\vec{\varphi}_+\rangle \right. + e^{i\varphi} |\vec{\varphi}_-\rangle)$$

(1)

where $\varphi$ is the intramodal phase between the two polarization components. The mathematical form of the SOC state is similar to that of the path-number (i.e., N00N) state in an SU(2) interferometer. Thus, this classical “entangled” state has been widely used to mimic and study the quantum behavior of entanglement. In addition, the spatial polarization structure of state (1) is $2\pi \varphi$ periodic in the azimuthal direction, giving rise to an $\ell$-fold angular sensitivity in an image rotation operation. For this reason, state (1) has an N00N-like superiority in rotation angle measurements, i.e., the phase variation becomes $\ell \Delta \varphi$, but this principle is only valid in the scenario of image rotation.
where $\text{LG}_{p}^{i} = \sum_{\ell} a_{\ell} \text{LG}_{\ell}^{i}$ represents a $p$-index degenerate LG mode with the same OAM that can be calculated via the full-field selection rule for structured light in SFG. Specifically, in state (3) $|\text{LG}_{p}^{i}⟩ = \sqrt{7/3}|\text{LG}_{1}^{i}⟩ - \sqrt{1/3}|\text{LG}_{3}^{i}⟩$ and in state (4) $|\text{LG}_{p}^{i}⟩ = \sqrt{1/6}(|\text{LG}_{0}^{i}⟩ - 2|\text{LG}_{2}^{i}⟩ + |\text{LG}_{4}^{i}⟩)$, see Supporting Information for details. The 3$\varphi$ and 4$\varphi$ phase-evolution behaviors within the 3rd- and 4th-harmonic OAM modes (i.e., $\text{LG}_{1}^{p}$ and $\text{LG}_{3}^{p}$) provide an interface to observe the superresolved interference resulting from the three- and four-photon de Broglie wavelengths of the fundamental waves, respectively. Compared with the superresolved interference obtained from high-N00N states, the difference here, as well as its core advantage, is the use of a bright structured beam as the sensing consequence. There is no need for expensive and inefficient photon counters for multiphoton coincidence. In addition, similar to the issue encountered when using a high-N00N state with $N > 2$, unwanted OAM modes $|\varphi⟩ < N$, such as $|\text{LG}_{1}^{p}⟩$ and $|\text{LG}_{3}^{p}⟩$, contained in states (3) and (4), become noise and remain in the upconverted waves. However, by using OAM mode projection, we can easily extract the interference between OAM modes of interest (i.e., $\text{LG}_{3N}^{p}$) and thus achieve near-perfect interference visibility.

Note that we cannot attribute the superresolution obtained here to a reduction in the wavelength of upconverted waves despite this occurring. In contrast, shorter wavelengths make the signal hard to control, hindering the pursuit of higher-ratio superresolution. Luckily, this technique framework allows us to approach this problem straightforwardly, i.e., by exploiting parametric downconversion to roll back the wavelength of the signal, which will be demonstrated later with specific experiments in the paper.

### 3. Experimental Section

To test the above principle, a series of proof-of-principle experiments was performed. Figure 1a shows the schematic setup of the superresolution interferometric measurement for $N = 2, 3, \text{and 4}$. By using a q-plane combined with a half-wave plate (birefringent phase shifter), a horizontally polarized Gaussian beam with a wavelength of 1560 nm was converted into state (1) with $\varphi = 1$ and adjustable intramodal phase $\varphi$. The prepared SOC state was first characterized using a spatial Stokes polarimeter to determine $\varphi$ and was then focused into three different quasi-phase-matching crystals to generate states (2), (3), and (4). Among the crystals, only that for the $N = 2$ experiment has a monoperiodic structure designed for solo type-II SHG (1560 nm $\hat{e}_{h} + 1560 \text{nm} \hat{e}_{v} \rightarrow 780 \text{nm} \hat{e}_{h}$), while the other two (for the $N = 3$ and 4 experiments) have quasiperiodic structures designed to be compatible with dual upconversion, i.e., type-II SHG cascading type-0 SFG (1560 nm $\hat{e}_{h} + 780 \text{nm} \hat{e}_{v} \rightarrow 520 \text{nm} \hat{e}_{j}$) or type-0 SHG (780 nm $\hat{e}_{h} \times 2 \rightarrow 390 \text{nm} \hat{e}_{j}$). The generated 2nd-, 3rd-, and 4th-harmonic waves were measured by OAM mode projection using spatial light modulation.
Figure 2. Experiments of superresolution interferometric measurements for \( N = 6, 8, \) and 12. a) Schematic of the experimental setup, where DC denotes the crystal for downconversion, and the other components are the same as those in Figure 1a. b) Measured superresolved interference between conjugate OAM modes \( \text{LG}^{±N} \) with \( N = 6, 8, \) and 12.

Spatial-mode transformation is

\[
\sqrt{\frac{1}{2}} \left( \langle \text{LG}_{2N}^0 \rangle \pm e^{iN\phi} \langle \text{LG}_{-N}^0 \rangle \right) \rightarrow \sqrt{\frac{1}{2}} \left( 1 \pm e^{iN\phi} \right) \langle \text{LG}_p^0 \rangle
\]  

(5)

Here the successive \( p \)-components within \( \langle \text{LG}_p^0 \rangle \) at the far-field are constructive interference (i.e., in phase), as a result, the superresolved signal carried by \( \langle \text{LG}_p^0 \rangle \) appears at the center of far-field patterns, surrounded by orange dashed circles shown in Figure 1a, that can be coupled into a single mode fiber and recorded by a photodiode. Here, to examine the theory visually, a CMOS camera was used to record the full projection patterns, and a variable neutral density filter attenuated the incident light below nW-level saturated power. More details on the experimental setup, nonlinear crystal parameters, and spatial mode transformation in the nonlinear interactions and projective measurements are introduced in the Supporting Information.

Figure 1b shows recorded interferometric signals, confirming the superresolution behavior of the intramodal phase within the \( N \)-harmonic OAM modes. The evolution of \( e^{iN\phi} \) (\( N = 2, 3, \) and 4), extracted from states (2), (3), and (4), compared with that in the original signal state (1) shown in the first row oscillates two-, three-, and fourfold faster, respectively. All the results exhibited near-perfect interference visibility (\( \nu \approx 1 \)), and more importantly, they were bright signals (\( \approx \mu W \)) recorded in real time using only a low-cost detector. The original signal at 1560 nm was a 1-W quasi-continuous beam with 500:1 duty cycle. The power of obtained superresolved signals with \( N = 2, 3, \) and 4 were around 16, 15, and 0.3 mW, respectively. Note that, for the case \( N = 2, \) only 10 mW input was enough to record a real-time signal via a fiber-coupled photodiode detector (nW-level saturated power); while for the case \( N = 4, \) using a 100-mW input can result in a blue signal visible to naked eye, see Supporting Information for more details.

As mentioned above, owing to the diminishing wavelength of upconverted waves, the ratio \( N \) cannot be further improved by directly cascading upconversion more times. For instance, the wavelength of the signal carrying \( e^{i4\phi} \) obtained in the above experiment was 390 nm, which has already arrived at the edge of the ultraviolet spectrum. Therefore, using parametric downconversion to roll back the wavelength of upconverted waves was a straightforward but effective approach. Figure 2a shows the schematic setup of the approach (see Supporting Information for more details). Type-II degenerate downconversion was performed for the signal \( N = 4 \) at 390 nm to roll back its wavelength to 780 nm and then type-0 SHG to obtain a superresolved signal with a ratio \( N = 8 \) was cascaded. Similarly, for the signal \( N = 3 \) at 520 nm, nondegenerate downconversion was used to roll back its wavelength to 1560 nm and then type-0 SHG another two times, 1560 nm \( \rightarrow \) 780 nm and 780 nm \( \rightarrow \) 390 nm, obtaining superresolved signals with ratios \( N = 6 \) and \( N = 12 \), respectively was cascaded. The average power of final signals with \( N = 6, 8, \) and 12 were around 200, 0.3, and 0.4 \( \mu W \), respectively. In addition, to maintain the transverse structure of rolled back beams, a super Gaussian pump was used in the downconversion.[58] Details on involved spatial modes and associated mode transformation are given in Supporting Information.

Figure 2b shows the observed interferometric behavior within the \( N \)-harmonic OAM modes \( N = 6, 8, \) and 12, which oscillate...
six, eight, and even 12 times faster, respectively, than the original state (1); moreover, all the signals have near-perfect visibility. To clearly show the benefits of high-ratio superresolution, the phase variation range was chosen to be in the insensitive region of the original sensing signal, i.e., near \( \sin^2 \phi \). Compared with the almost constant amplitude of the original signal, all the superresolved signals oscillated for more than one cycle, and all the signal powers were visible to the naked eye. With respect to the data shown in the last row, in particular, this is the first observation of a 12-photon de Broglie wavelength in real time. In comparison, observing a reduced de Broglie wavelength of entanglement composed of 10 photons, even with state-of-the-art techniques, requires several hours to record each data point of the photon-coincidence interference and has poor visibility.\(^{[27–30]}\)

Another issue of concern is whether and how the OAM carried by state (1), set as \( \ell = \pm 1 \) for all the results in this paper, affects the measurement performance. First, unlike a superresolution interferometer fed with N00N states, the present scheme does not require the multiphoton amplitude to pass through the phase shifter but provides a highly efficient interface to amplify the phase variation in a classical sensing beam already output from the interferometer. If the phase variation to be measured comes from the interferometric apparatus, such as \( \psi \) loaded by the half-wave plate before the q-plate in Figure 1a, then the performance of the scheme is independent of the OAM. Second, this conclusion changes if the SOC state itself works as an interferometer and the measured parameter is sensitive to the OAM. For instance, if the birefringence shifter (HW) is replaced in Figure 1a with a Dove prism, then the resolution for measuring the rotation angle of the prism would be proportional to \( \ell \). As mentioned previously, the spatial polarization structure of state (1) has an \( \ell \)-fold angular sensitivity in an image rotation operation.\(^{[36]}\)

In this type application, using a higher OAM is preferred to pursue extreme performance.

4. Discussion and Conclusion

We have demonstrated a series of superresolution interferometric measurements using nonlinear interactions of structured light. Up to a dozen-fold superresolved interference, corresponding to a reduced de Broglie wavelength of \( \lambda/12 \), was successfully observed in real time and with near-perfect visibility. This scheme provides a novel roadmap for the development of advanced interferometric techniques with ultrahigh phase resolution. More importantly, this technique does not require the intractable multiphoton amplification to pass through the interferometer but provides a phase resolution amplifier that can amplify the phase variation in a classical sensing beam already output from an interferometer. That is to say, we can incorporate this method into existing interferometric techniques with only minor modifications, especially by using geometric phase elements and microstructured nonlinear media,\(^{[14,19–21]}\) and easily obtain a 2–4x resolution boosting.

Compared with previous work,\(^{[16–18,31–34]}\) our results show two significant advances: real-time measurement (high-power signal) and near-perfect visibility (\( \nu \approx 1 \)), enabled by the use of bright sensing beams and OAM mode projection, respectively. Notably, the latter gives rise to a natural question: does the phase sensitivity in this scheme also exceed the shot-noise limit? The answer is no, because we need to consider the energy loss of the final signal relative to the original sensing beam passing through the phase shifter. Regarding the energy loss in this scheme, first, the weight of \( e^{iN\phi} \)-carrying modes (i.e., \( LG_{\ell,N}^+ \)) in the upconverted wave becomes less than one when \( N > 2 \) and continuously decreases with \( N \); second, the upper bound of the SHG efficiency in theory is also less than one (see Supporting Information). Therefore, the detectable signal power of \( LG_{\ell,N}^+ \) extracted from the final upconverted wave is much lower than that of \( LG_{\ell,0}^+ \) in the original sensing beam, especially for a high-\( N \) case. Despite this, on the premise of having perfect visibility, the actual power level of detectable signal is more important for practical applications. This decides the “absolute” phase sensitivity of an interferometer and whether enables the real-time measurement. To date, only one recent study using state-of-the-art superconducting nanowire single-photon detectors has demonstrated an N00N-state interferometer that can provide a near real-time measurement with a detectable photon beam energy still below the picowatt level with superresolution with \( N = 2 \).\(^{[33]}\) Another noteworthy approach for superresolution with classical light, reported in ref.\(^{[34]}\), was realized by directly extracting (or filtering out) multiphoton amplitude from a coherent state. However, the efficiency in this passive post-selection is still too low to provide an applicable signal power. In comparison, owing to constructing the multiphoton amplitude actively via stimulated parametric interactions in this work, even for \( N = 12 \) and an equivalent de Broglie wavelength of 130 nm, the final detectable energy in the sensing beam is still visible to the naked eye (microwatt level) and can be further increased exponentially with the laser power. Therefore, we can easily determine a subtle phase displacement that occurs in the insensitive region of the original sensing signal, as shown in Figure 2b, via recording the bright superresolved signals with only a low-cost detector. Achieving the same performance using the present interferometric technique would require the use of a deep-ultraviolet laser as the source, which is impossible for most applications, or to build an unrealistic 12-photon interferometer that has a perfect visibility and microwatt level photon-coincidence signal.

Finally, how can the phase resolution in this scheme be further improved, and what are its limits? According to the above discussion, by adding more upconversion times and using downconversion to push the shortwave limit, we can achieve a higher resolution, i.e., a shorter equivalent de Broglie wavelength \( \lambda/N \). However, more parametric interactions mean lower finally detectable signal power. On the one hand, using a high-power laser source, such as Q-switched or mode-locking pulsed laser, is better. On the other hand, optimizing the upconversion strategy to deliver a more favorable OAM mode transformation in parametric processes is crucial. For instance, although we do not have the corresponding crystals for this, cascading another SHG for the \( N = 8 \) result in Figure 2b to obtain a super resolution with \( N = 16 \) is a wiser strategy than the strategy for \( N = 12 \). This is because the modal weight of \( LG_{\ell,16}^+ \) in the final harmonic wave for \( N = 16 \) is higher than that for \( N = 12 \) (see Supporting Information). Notably, using SOC states with type-II SHG can avoid the generation of unwanted mode noise, such as that arising from the transformation from state (1) to state (2), but the latter becomes a pure scalar mode. This has inspired us to design a device that can convert the latter scalar mode, such as state (2), into an associated
SOC mode with its intramodal phase unchanged, and in Supporting Information we show a proposed geometric device for this issue. In this way, unwanted mode noise can be completely eliminated in the following upconversion, greatly boosting the detectable energy of the final interference signal. In addition to crystal-based nonlinearity, atomic four-wave mixing can offer a much larger parametric gain and is thus another promising approach worth exploring. In the near future, using this scheme with appropriate technical improvements to achieve a superresolution interferometric measurement with $N > 100$, corresponding to an extreme-ultraviolet de Broglie wavelength, is expected to be an attainable goal.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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