Weibel, Firehose and Mirror mode Relations

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\section*{Abstract}
Excitation of Weibel magnetic fields in an initially non-magnetized though anisotropic plasma may trigger other low-frequency instabilities fed by pressure anisotropy. It is shown that under Weibel-like stable conditions the Weibel-like thermal fluctuation magnetic field allows for restricted Firehose-mode growth. In addition, low frequency Whistlers can also propagate in the plasma under certain anisotropic conditions. When the Weibel-like mode becomes unstable, Firehose instability ceases but Mirror modes take over. This will cause bubble structures in the Weibel-like field in addition to filamentation.

\section*{Keywords}
Waves and instabilities

\section*{1 Introduction}
Historically, there are just three celebrated fundamental very low frequency (electro-)magnetic instabilities in hot anisotropic plasmas, the well known firehose mode (Vedenov et al., 1961; Treumann & Baumjohann, 1997, for a plasma physics textbook), its complementary equivalent, the Mirror mode, and the Weibel instability (Weibel, 1959; Yoon & Davidson, 1987 and others). Recently, Schlickeiser et al. (2011) and Schlickeiser & Skoda (2011), applying a substantially more rigorous relativistic approach based on the theory of analytical functions, identified a much larger number of different electromagnetic low-frequency modes, weakly and strongly damped/unstable ones, which add additional dispersion channels to a magnetized plasma. Felten & Schlickeiser (2013a,b) and Felten et al. (2013) extended these calculations to relativistic Weibel-like (non-magnetic) conditions, again finding a large number of different dispersion channels which belong to damped or unstable modes including very low frequency and non-oscillating aperiodic modes.

Of the historical modes, the first is a general bulk plasma mode excited in an external magnetic field by a thermal anisotropy with larger magnetically parallel than perpendicular temperature, $T_\parallel > T_\perp$, resulting in Alfvén waves which radiate along the magnetic field. At the contrary, the latter Weibel-like mode acts in non-magnetized plasmas when, by some not further specified reason, the plasma exhibits a thermal (pressure) anisotropy with higher temperature (kinetic energy) in one than in the two other directions. In both the firehose and Weibel-like cases the cause of a higher equivalent temperature in one direction (and thus a temperature anisotropy) can also be a fast (relative) streaming (for physical mechanism of the Weibel-like streaming mode see Fried, 1959) of the plasma with kinetic energy exceeding the transverse thermal energy. This can be provided by beam or counter-streaming beam configurations (cf., e.g., Achterberg & Wiersma, 2007) and has been made responsible for the generation of magnetic fields under various non-dynamo conditions occurring, for instance, in shock waves (for a review see, e.g., Treumann, 2009), preferentially in relativistic shocks (for relativistic shocks see the review by Bykov & Treumann, 2011). The physical differences in the two modes are large. While Weibel-like modes provide a non-dynamo mechanism to produce quasi-stationary and thus non-propagating magnetic fields in an otherwise non-magnetized plasma, the firehose mode grows on an existing field and propagates along the field at Alfvén velocity $V_A$ thus transporting energy away from the region where it is excited, filling a large volume with magnetic fluctuations and contributing to turbulence and other effects. However, since both modes are generated by similar mechanisms though being different, having completely different properties, one expects that they will compete and possibly even act in tandem to generate magnetic fields and propagate them away from the source region.
The remaining Mirror mode, at the contrary, is an about stationary feature of the plasma with wave vector \( \mathbf{k} \) almost perpendicular to the ambient magnetic field. It generates plasma inhomogeneity at the lowest frequency of plasma turbulence. It is easily derived from anisotropic fluid theory, but its physical mechanism is attributed to trapping of particles in depleted magnetic field regions along the field. This mechanism is more complex and still not completely resolved (for a recent more complete account of the electron mirror mode which concerns us here, see, e.g., Pokhotelov et al. 2008, 2010, 2013, and references therein).

In the present note we briefly examine this situation at the example of the classical Weibel instability, showing that there indeed exists such a competition which may become important in limiting the growth of the Weibel-like modes, allowing other fluctuations to propagate on the magnetic Weibel-like background field which may contribute to distribution of magnetic fields in a larger volume. A similar analysis examining the newly identified electromagnetic modes (Schlickeiser et al., 2011; Schlickeiser & Skoda, 2011; Felten et al., 2013; Felten & Schlickeiser, 2013a,b,c), though being highly desirable, lies outside this brief communication.

2 Thermal fluctuation effects

In order to demonstrate the role of thermal magnetic fluctuations as a pathway for other modes, we restrict to the conventional Weibel-like thermal-anisotropy mode only. The following analysis could be made more complete by applying the expressions for the many dispersion channels identified in the above given references on the non-magnetized plasma modes. Here we refrain from such an extension of the present communication, not at least for the reason of the wealth of newly found modes but also for the complexity of the more precise expressions given there (Felten et al., 2013; Felten & Schlickeiser, 2013a,b,c) which would obscure our intendedly focussed discussion.

The thermal anisotropy-excited Weibel-like mode grows under the condition that the anisotropy is along direction \( z \). Writing \( T_z = T_{||} \), \( T_{x,y} = T_{\perp} \) the condition for growth is, in its simplest form, given by

\[
A \equiv \frac{T_{||}}{T_{\perp}} - 1 > 0
\]  

In this case it generates a magnetic field \( \mathbf{B}_W \) in the perpendicular direction, i.e. transverse to the direction of anisotropy. Under the opposite condition the Weibel mode is stable but still generates a zero-frequency perpendicular magnetic thermal fluctuation field (Yoon, 2007) of spectral energy density

\[
\langle b^2(k) \rangle = \frac{(A + 1)^2 k \lambda_e}{b_0^2 (A + 2) |k^2 \lambda_e^2 - A - \mu|^2}
\]  

written here for the electron-Weibel mode with skin depth \( \lambda_e \) and mass ratio \( \mu = m_e/m_i \). Its spectral density amplitude is given by

\[
\frac{b_0^2}{m_e c^2} = \frac{\mu_0}{m_e} \sqrt{\frac{\pi T_{\perp}}{T_{||}}}
\]

The thermal fluctuation level has a distinct dependence on wave number \( k \). For short wavelengths \( k \gg \lambda_e^{-1} \) it decays like \( \propto (k \lambda_e)^{-3} \) (confirming the result of Yoon, 2007). Due to the presence of the inert ion component it vanishes at \( k \lambda_e < 1 \), i.e. compared with the electron skin depth the fluctuation field is of longer scale, providing a weakly-oscillatory moderate-wavelength background magnetic field. One may note that the wavenumber is perpendicular to both, \( \mathbf{z} \), i.e. the direction of thermal pressure anisotropy, and the Weibel-like magnetic fluctuation field.

The magnetic fluctuation field is quasi-stationary in the sense that in the final step of calculation the real part of the spectral energy density \( \bar{b} \equiv b_0 \left[ \int dk \langle b^2(k) \rangle \right]^{1/2} \) that results from the above thermal spectral density when integrating over \( k \)-space, becomes

\[
\frac{\bar{b}}{b_0} = \left[ \frac{(A + 1)^2 (A + 1 + \mu)}{(A + 2)(A + \mu)(1 - A - \mu)} \right]^{1/2}
\]

Clearly, under the opposite condition the direction of anisotropy switches and the Weibel instability works in the orthogonal direction. It is thus universal, working at any thermal anisotropy in nonmagnetic plasma and disappears only for \( A = 0 \). Just for this reason it also makes sense to write the thermal Weibel level including a non-vanishing anisotropy, for the thermal level becomes itself anisotropic: it is thermal along the anisotropy and thermally excited transverse to it.
It serves as background magnetization on which electromagnetic waves at low frequency, much less than the electron plasma frequency $\omega \ll \omega_e$, can propagate in a nonmagnetic plasma. Propagation would be inhibited otherwise, allowing only for evanescent modes in the spatial range of the electron skin depth $\lambda_e$. Actually, for the same reason the unstable Weibel-like mode can penetrate just over the electron skin depth only from its generation site into the collisionless plasma perpendicular to the direction of anisotropy. This restriction necessarily causes a pronounced magnetic filamentation of the Weibel-like unstable plasma. On the other hand, once Weibel-like thermal fluctuations provide a weak magnetic background field, spreading across the plasma becomes possible for other electromagnetic modes.

3 Secondary instability

Once the thermal fluctuation spectrum of the Weibel mode is established, the plasma behaves weakly magnetized with the Weibel-like thermal fluctuation background field being structured at about the electron skin depth in the direction perpendicular to the anisotropy. This implies that the thermal magnetic background organizes into long magnetic filaments of typical transverse size of few electron skin depths with consequences for the propagation of any secondary electromagnetic modes.

3.1 Weibel-stable case

Let us assume that the Weibel mode has been stable (see footnote), which implies that $A \lesssim 0, 1 < |A| < 2$. Under this condition we have $T_\perp > T_\parallel$, with $T_\parallel$ being directed along the Weibel magnetic field, i.e. playing the role of a magnetically parallel anisotropy, while $T_\perp$ is a perpendicular anisotropy. Under this condition the Firehose mode could grow only under the condition

$$\frac{\beta_\perp}{\beta_\parallel} - 1 > \frac{2}{\beta_\parallel}, \quad \beta_\parallel,T_\perp = \frac{2\mu_0 N T_\parallel}{b^2}$$

(6)

with $b^2 = \int \frac{dk}{\beta_\parallel} \langle b^2(k) \rangle$ the thermal rms magnetic field (note that indexing is with respect to the direction of the initial thermal anisotropy where the directions with respect to the magnetic field are inverted!) existing in the otherwise nonmagnetic plasma. The plasma-$\beta$s refer to it in the present case. The left-hand side is positive in the Weibel-stable case and, hence, the Firehose instability can grow under the rewritten condition

$$\beta_\perp - \beta_\parallel > 2, \quad |A| < 2$$

(7)

where the second condition is imposed by the thermal fluctuation level requirement. The firehose mode can also propagate, because its propagation direction is along the thermal Weibel-like magnetic field which is along the direction of the Weibel-like magnetic channels and therefore allowed to propagate for a parallel mode like the Firehose-Alfvén wave. Checking for the electron Mirror mode (not including any more sophisticated effects) yields instability as long as

$$\beta_\parallel^2 - \beta_\perp > \beta_\perp \beta_\parallel \quad \text{and} \quad A = \beta_\parallel/\beta_\perp - 1 < 0$$

(8)

This implies a contradiction thus excluding the mirror mode. Weibel-stable plasmas are stable against Mirror but allow for Firehose modes. These propagate on the thermal level of the magnetic field in the direction perpendicular to the Weibel-stable direction of anisotropy. This is interesting to know as it shows that in an otherwise non-magnetic plasma which in one direction (this time the $\perp$-direction) the plasma is Weibel unstable the existence of thermal magnetic fluctuations in the Weibel-stable direction causes Firehose modes to radiate away from this region if only the above Firehose condition, based on the small thermal fluctuation level of the magnetic field, is satisfied. Since the thermal level is small this will, in praxis, always be the case if only $T_\perp > T_\parallel$.

In addition, if the plasma consists of a thermal background and a warm anisotropic component, the low-frequency Whistler (or Alfvén) mode can be destabilized as it only requires that for the anisotropic component $A < 0$ and hence $\beta_\perp < \beta_\parallel$ which is given by the stability condition of the Weibel mode (note again that the indices $\parallel,\perp$ refer to the anisotropy frame, not to the magnetic frame!). The spectral density of the magnetic fluctuations in this range with $\beta_\parallel,\beta_\perp < 1$ in the long wavelength range $k\lambda_e < 1$ is estimated as

$$\frac{\langle b^2(k) \rangle}{b_0^2} \approx \left( \frac{T_\parallel}{T_\perp} \right)^2 k\lambda_e$$

(9)

Using the Whistler dispersion relation for electrons in the low frequency range, i.e. $k^2\lambda_e^2 \approx \omega/\Omega_e - \omega$, with $\Omega_e = eb/m_e$ the electron cyclotron frequency in the quasi-stationary Weibel thermal fluctuation field, yields the relation

$$\frac{\langle b^2(k) \rangle}{b_0^2} \approx \left( \frac{T_\parallel}{T_\perp} \right)^2 \sqrt{\frac{\nu}{1-\nu}}, \quad \nu = \frac{\omega}{\Omega_e}$$

(10)

between the Whistler frequency range $\omega(k)$ and the magnetic spectral energy density. Since we know that the magnetic spectral density vanishes at $k = 0$, and using the dispersion relation for $k\lambda_e = 1$, we find that the frequency of Whistlers is in the range

$$0 \leq \nu < \frac{1}{2}, \quad \text{for} \quad 0 \leq k\lambda_e < 1$$

(11)

propagating along the Weibel thermal fluctuation field perpendicular to the Weibel anisotropy, i.e. in $\perp$-direction. These are indeed very low frequency Whistlers since $\Omega_e$ based on the thermal fluctuation level is small.

3.2 Weibel-unstable case

The interesting domain is that of unstable Weibel modes (for instance in view of application to relativistic shocks) in which case the magnetic field grows and may become quite
It is interesting to note that since the $\beta$s depend inversely on the growing magnetic field, starting from thermal level, they will decrease with Weibel-like mode growth, thereby ultimately stabilizing the second Mirror mode branch when $A \sim 1/\beta_\bot$, while the branch with the negative sign of the square root remains unaffected.

Hence, any growing Weibel-like mode will readily self-consistently evolve into a chain of Mirror structures which on their own are filled with low-frequency Whistlers (for the observational evidence and theoretical arguments see, e.g., Baumjohann et al., 1999; Treumann et al., 2000) bouncing back and force in the mirrors with some of them possibly escaping to the environment. As a consequence, the Weibel-like instability does not only lead to magnetic and current filaments of perpendicular scale of the order of the electron skin depth. In addition the magnetic Weibel-like fields evolve into a sequence of mirror structures as the lowest frequency magnetically turbulent modes which can evolve according to their own nonlinear dynamics (cf., e.g., Pokhotelov et al., 2008, 2010, 2013, and references therein).

4 Conclusions

The Weibel-like instability has frequently been made responsible for the excitation of magnetic fields in otherwise initially non-magnetized plasmas. Actually, as has been known since long time (Landau & Lifschitz, 1959, 1960; Sitenko, 1967; Akhiezer et al., 1975, and followers), thermal fluctuations in the plasma readily lead to the spontaneous emission of magnetic fluctuations. At the lowest frequencies these form quasi-stationary magnetic fields (see, e.g., Yoon, 2007; Treumann and Baumjohann, 2012) with wavelength longer than the electron skin depth $\lambda_e$.

These fluctuations provide an initial weak magnetic background field which allows the Weibel-like magnetic field to penetrate over some distance into the otherwise magnetically nontransparent plasma which would inhibit penetration of any magnetic field over distances longer than $\lambda_e$, effectively exponentially screening the plasma from magnetic fields outside the Weibel-like mode source region. In the presence of an pressure anisotropy, however, Firehose, Mirror and Whistler modes can grow in the plasma and compete with the Weibel-like mode. A Weibel-stable plasma allows for Firehose and Whistler modes to grow, both propagating along the magnetic thermal fluctuation field. In a Weibel-unstable plasma the Firehose instability is stable. However, Whistler and Mirror modes can grow. The latter structure the unstably generated Weibel-like magnetic field into chains of magnetic bubbles or holes and trap low frequency Whistlers (Baumjohann et al., 1999; Treumann & Baumjohann, 2000; Treumann et al., 2000) thus contributing to magnetic structure and turbulence in addition to the known Weibel filamentation effect. This will have a profound effect on the self-consistent generation of magnetic plasma turbulence in regions of pressure anisotropy (or relative streaming) in an otherwise initially non-magnetized plasma. Not only may one expect that anisotropic or streaming plasmas will thus naturally be weakly magnetized, they will also be naturally turbulent thus providing plenty of scattering centers for energetic particles as required in all stochastic particle acceleration scenarios (for a recent review cf., e.g., Bykov & Treumann, 2011; Schure et al., 2012 and references therein).

The present investigation has been forcefully restricted to the investigation of one quite simple low-frequency electromagnetic mode only, the conventional thermally anisotropic Weibel instability. This instability belongs to a much wider class of low frequency electromagnetic modes which have been investigated in depth only
Several of these modes generate quasi-stationary magnetic fields in a similar way as the Weibel instability we were dealing with. It will be most interesting to investigate their effect on the excitation of other electromagnetic instabilities, the propagation of Alfvén and Whistler waves in the plasma that has become weakly re-magnetized by them. It should also be noted at this occasion that Simões et al. (2013) in a very recent paper performed particle-in-cell simulations of a thermally isotropic, quiescent and stable plasma to determine the electric and magnetic thermal fluctuation levels. As expected, both levels are different from zero. The electric fluctuations naturally map the Langmuir fluctuation branch for wavelength exceeding the Debye length, for shorter wavelengths they show a wide range of weak fluctuations extending even down below the plasma frequency. This result is very well known since long (cf., e.g., Lund et al., 1996, with and without a beam) and is nicely confirmed by these simulations. The magnetic fluctuations found in this case are also expected in the whole frequency range from fluctuation theory of any thermal system (Landau & Lifshitz, 1960; Sitenko, 1967). Their absence would have been very surprising. As they should, magnetic fluctuation amplitudes maximize at lowest frequencies and longest wavelengths. This was noted by Yoon (2007) and, for vanishing anisotropy, also in Treumann and Baumjohann (2012). Both the last two papers became in this respect completely independent of the Weibel mode indicating the general presence of magnetic fluctuations in thermally anisotropic and also thermally isotropic plasmas thereby ver softly magnetizing an initially non-magnetic plasma and permitting for propagation of low-frequency electromagnetic (mhd) modes.

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References

Achterberg, A. & Wiersma, J.: The Weibel instability in relativistic plasmas. I. Linear theory. II. Nonlinear theory and stabilization mechanism. Astron. Astrophys. 475, 1-18, doi: 10.1051/0004-6361:20065365, 2007.

Akhiiezer, A. I., Akhiiezer, I. A., Sitenko, R. V. & Stepanov, K. N.: Nonlinear Theory and Fluctuations, in Plasma Electrodynamics, vol 2, pp. 116-142, Pergamon Press, Oxford, 1975.

Baumjohann, W. & Treumann, R. A.: Basic Space Plasma Physics, Revised Edition, Imperial College Press at World Scientific, London & Singapore, 2012.

Baumjohann, W., Treumann, R. A., Georgescu, E., Haerendel, G., Fornacon, K.-H. & Auster, U.: Waveform and packet structure of ion roars, Ann. Geophys. 17, 1528-1534, doi:10.1007/s00382-999-1528-9, 1999.

Bykov, A. M. & Treumann, R. A.: Fundamentals of collisionless shocks for astrophysical application. 2. Relativistic shocks, Astron. Astrophys. Rev. 19, 42, doi: 10.1007/s00159-011-0042-8, arXiv:1105.3221, 2011.

Felten T., Schlickeiser R., Yoon P. H. & Lazar M.: Spontaneous electromagnetic fluctuations in unmagnetized plasmas. II. Relativistic form factors of aperiodic thermal modes, Phys. Plasmas 20, 052113, doi: 10.1063/1.4804402, 2013.

Felten T. & Schlickeiser R.: Spontaneous electromagnetic fluctuations in unmagnetized plasmas. IV. Relativistic form factors of aperiodic Lorentzian modes, Phys. Plasmas 20, 082117, doi:10.1063/1.4817805, 2013b.

Felten T. & Schlickeiser R.: Spontaneous electromagnetic fluctuations in unmagnetized plasmas. V. Transverse, collective mode for arbitrary distribution functions, Phys. Plasmas 20, 104502, doi:10.1063/1.4824114, 2013c.

Fried B. D.: Mechanism for instability of transverse plasma waves, Phys. Fluids 2, 337-, doi: 10.1063/1.1705933, 1959.

Landau L. D. & Lifshitz E. M., Fluid Mechanics, Pergamon Press, Course of Theoretical Physics, vol. 6, New York, 1959.

Landau L. D. & Lifshitz E. M., Electrodynamics of Continuous Media, Pergamon Press, Oxford, 1960.

Lund, E. J., Treumann, R. A. & Labelle, J.: Quasi-thermal fluctuations in a beam-plasma system, Phys. Plasmas 3, 1234-1240, 1996, doi:10.1063/1.871747.

Pokhotelov, O. A. & Amariutei, O. A.: Quasi-linear dynamics of Weibel instability, Ann. Geophys. 29, 1997-2001, doi:10.5194/angeo-29-1997-2011, 2011.

Pokhotelov, O. A., Sagdeev R.Z., Balikhin M. A., Fedun V. N. & Dudnikova G. I.: Nonlinear mirror and Weibel modes: peculiarities of quasi-linear dynamics, Ann. Geophys. 28, 2161-2167, doi:10.5194/angeo-28-2161-2010, 2010.

Pokhotelov, O. A., Onishchenko. O. G. & Stenflo, L.: Physical mechanisms for electron mirror and field swelling modes, Phys. Scripta 87, ID 065303, doi:10.1088/0031-8949/87/06/065303, 2013.

Pokhotelov, O. A., Sagdeev, R. Z., Balikhin, M. A., Onishchenko. O. G. & Fedun, V. N.: Nonlinear mirror waves in non-Maxwellian space plasma, J. Geophys. Res. 113, ID A04225, doi:10.1029/2007JA012642, 2008.

Schlickeiser R., Lazarian A. & Skoda T.: Spontaneous electromagnetic fluctuations in anisotropic magnetized thermal plasma., Phys. Plasmas 18, 012103, doi:10.1063/1.3532787, 2011.

Schlickeiser R. & Skoda T.: Linear theory of weakly amplified, parallel propagating, transverse temperature-anisotropy instabilities in magnetized thermal plasmas, Astrophys. J. 716, 1596-1606, doi:10.1088/0004-637X/716/2/1596, 2011.

Schure, K. M., Bell, A. R., O’C Drury, L. & Bykov, A. M.: Diffusive shock acceleration and magnetic field amplification, Space Sci. Rev. 173, 491-519, doi: 10.1007/s11214-012-9871-7, 2012.
