Vacuum effects in magnetic field with with account for fermion anomalous magnetic moment and axial-vector interaction

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Abstract. We study vacuum polarization effects in the model of Dirac fermions with additional interaction of an anomalous magnetic moment with an external magnetic field and fermion interaction with an axial-vector condensate. The proper time method is used to calculate the one-loop vacuum corrections with consideration for different configurations of the characteristic parameters of these interactions.

1. Introduction

Recently, much interest has been attracted to a possible violation of Lorentz symmetry. A field theoretical model, called Standard Model Extension, incorporating various possible terms with Lorentz violation (LV) was developed [1]. Such terms may appear due to many different reasons, e.g. as a consequence of as a nontrivial vacuum expectation values of some fundamental fields from higher dimensions, which could lead to space anisotropy in 4 dimensions at lower energies. Subject of a special interest is investigation of the proposed LV terms with an axial-vector interaction, which can be described by the following CPT–odd term in the Dirac Lagrangian: \( \bar{\psi} \gamma^5 \gamma^\mu b^\mu \psi \) (here \( b^\mu \) is a constant, that can be interpreted as a certain axial 4D-vector condensate of some more fundamental higher dimensional model) (see e.g.,[2]). On the other hand, appearance of such axial-vector interaction terms can be a consequence of a certain torsion field induced by fermions (see, e.g., [3]).

A charged fermion, due to the Dirac equation, has an intrinsic Dirac magnetic moment equal to the Bohr magneton \( \mu_0 = \frac{e \hbar}{2mc} \). In the framework of QED in the lowest order of the fine structure constant \( \alpha = \frac{e^2}{4\pi} \), the Dirac magnetic moment is added by a Schwinger term, or vacuum magnetic moment [8] \( \Delta \mu_{Sch} = \frac{e^2}{2\pi^2} \cdot \mu_0 \), so that the total magnetic moment becomes \( \mu_{Sch} = \mu_0 (1 + \frac{\Delta \mu_{Sch}}{\mu_0}) \).

In this report, we calculate the fermion one-loop corrections to the effective Lagrangian in a magnetic field due to the simultaneous presence of both anomalous magnetic moment and an axial–vector interaction terms\(^1\).

\(^1\) The vacuum effects in the presence of each of these two terms separately was considered in Refs. [4, 5, 2, 6, 7]
The modified Dirac equation for a fermion with an anomalous magnetic moment $^2 \mu = \kappa \mu_0$, in the presence of an additional interaction with a constant axial-vector field $b^\mu$, according to [1] can be written as follows:

$$\left(i\gamma^\alpha D_\alpha - m + \frac{\mu}{2} \sigma^{\alpha\beta} F_{\alpha\beta} - \gamma^5 \lambda^\alpha b_\alpha \right) \psi = 0, \quad D_\alpha = i\partial_\alpha - eA_\alpha.$$  \hspace{1cm} (1)

In what follows, we shall consider only a nonzero external magnetic field $\vec{H} = He_z$ and assume 3d-vector $\vec{b}$ is also parallel to the z-axis.

2. The Effective Lagrangian

The one-loop fermion contribution to the effective Lagrangian can be calculated with the help of the proper-time method [8]

$$\Delta \mathcal{L} = \frac{1}{4\sqrt{\pi}} \frac{eH}{(2\pi)^2} \int_{-\infty}^{\infty} dp \sum_{n,\zeta, \epsilon} \int_{\frac{2}{\Lambda^2}}^{\infty} \frac{ds}{s^{3/2}} e^{-sE^2}.$$

Here $E$ is the fermion energy, and the quantum numbers have the following meaning: $p$ is the $z$-component of the fermion momentum, $n = 0, 1, 2, \ldots$ is the Landau quantum number, $\zeta = \pm 1$ is the spin projection on the magnetic field direction, $\epsilon = \pm$ is the energy sign ($\Lambda$ is the cut-off parameter). The summation over all quantum numbers should take into account that the ground state has no degeneracy, i.e. $n = 0$, $\zeta = +1$ (for $e > 0$) and $n = 0$, $\zeta = -1$ (for $e < 0$).

In what follows, we shall consider various cases with special values of the parameters.

a) anomalous moment $\mu = 0$, axial condensate $b_0 = b \neq 0$, $\vec{b} = 0$:

$E^2 = m^2 + (\text{sgn}(p) \sqrt{p^2 + 2eHn} + \epsilon b)^2$ (for this energy spectrum, cf. e.g. [2], [4], [7], [5]).

Note that in the expansion in powers of the small parameter $b$, the linear term will be absent due to the oddness of the expression, so that expansion in $b$ of expression (2) gives two terms of the $b^2$ order:

$$\Delta \mathcal{L}^{b^2}_{21} = -2 \frac{eHb^2}{4(2\pi)^2} \int_{-\infty}^{\infty} dp \sum_{n,\zeta, \epsilon} \int_{\frac{1}{\Lambda^2}}^{\infty} \frac{ds}{s^{3/2}} \coth(eHs) e^{-s(m^2 + p^2 + 2eHn)} 2s^2 (p^2 + 2eHn).$$

As a result, the regularized expression for the one loop effective Lagrangian is obtained:

$$\Delta \mathcal{L}^{b^2} = \Delta \mathcal{L}^{b^2}_{21} + \Delta \mathcal{L}^{b^2}_{22} = \frac{b^2}{4\pi^2} \left[-m^2eH \int_0^{\infty} ds e^{-sm^2} \left(\coth(eHs) - \frac{1}{eHs}\right) + \Lambda^2 - m^2 \log \Lambda^2/m^2 \right].$$

Note that terms with the cut-off $\Lambda$ do not actually depend on the magnetic field, and thus can be eliminated by subtraction, yielding the final result

$$\Delta \mathcal{L}^{b^2} = -b^2 \frac{(eH)m^2}{4\pi^2} \int_0^{\infty} ds e^{-sm^2} \left(\coth(eHs) - \frac{1}{eHs}\right).$$  \hspace{1cm} (3)

This result could be easily generalized to a more general case with the anomalous moment still equal to zero, $\mu = 0$, but with an axial condensate given in a more general form.

$^2$ Assuming possible extension of the Standard model, some additional terms may appear in the anomalous magnetic moment (see, e.g., [9]), besides the Schwinger term $\Delta \mu_{\text{Sch}}$, and this is included in the general coefficient $\kappa$. 

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b = (b_0, 0, 0, b_3), where b_0 \neq 0, b_3 \neq 0. Then, by introducing a vector \( \beta^\mu = \frac{1}{2} \epsilon^{\mu\rho\alpha\beta} b_\rho F_{\alpha\beta} \), with nonzero components \( \beta^0 = \epsilon^{0312} b_3 F_{12} = b_3 H \) \( \beta^3 = \epsilon^{3012} b_0 F_{12} = -b_0 H \) we shall have to deal with the same intermediate expressions as in the previous situation, and the unified result can then be presented by Eq. (3) where now b is the 4-vector \( b = (b_0, 0, 0, b_3) \).

For a weak field \( eH \ll m^2 \) after the integration one obtains

\[
\Delta L^{b_0} = \frac{1}{12\pi^2} b^2 (eH)^2 \int_0^{\infty} dx e^{-x} \frac{x}{m^2} = \frac{b^2 (eH)^2}{12\pi^2 m^2}.
\]

For a strong field \( eH \gg m^2 \), using a substitution \( x = sm^2 \), after approximate integration one obtains

\[
\Delta L^{b_0} \approx \frac{b^2 eH}{4\pi^2}.
\]

More explicit approximation can be made using Euler psi-function (see e.g. [5]).

b) anomalous moment \( \mu \neq 0 \), axial condensate \( b_0 = 0, \vec{b} = 0 \):

\[
E^2 = p^2 + (\sqrt{2eHn + m^2 + \zeta \mu H})^2.
\]

(for this energy spectrum, cf. e.g. [10], [2], [6]).

Assuming \( \mu H \) to be small let us consider terms of order \( (\mu H)^2 \). After substraction and regularization procedures similar to the previous case one obtains

\[
\Delta L^{\mu} = \frac{(\mu H)^2}{2(2\pi)^2} \left( \Lambda^2 + m^2 \log \frac{\Lambda^2}{m^2} \right) - \frac{(\mu H)^2 eH}{2(2\pi)^2} \int_0^{\infty} ds \frac{e^{-s m^2}}{s} \left( \coth(eHs) - \frac{1}{eHs} \right).
\]

The second term converges but the first term diverges so it could be considered with the linear in \( \mu H \) term

\[
\Delta L^{\mu} = \frac{eH (\mu H) m}{(2\pi)^2} \int_0^{\infty} ds \frac{e^{-s m^2}}{s} = \frac{eH (\mu H) m}{(2\pi)^2} \log \frac{\Lambda^2}{m^2}.
\]  

(4)

One may note that

\[
\frac{m^2 (\mu H)^2}{2(2\pi)^2} \log \frac{\Lambda^2}{m^2} = \left( \frac{\kappa}{4(2\pi)} \right)^2 (eH)^2 \log \frac{\Lambda^2}{m^2},
\]

\[
\frac{\Lambda^2 (\mu H)^2}{2(2\pi)^2} = \left( \frac{\kappa}{4(2\pi)} \right)^2 (eH)^2 \Lambda^2 / m^2.
\]

At the same time

\[
\frac{eH (\mu H) m}{(2\pi)^2} \log \frac{\Lambda^2}{m^2} = \frac{\kappa}{4(2\pi)^2} (eH)^2 \log \frac{\Lambda^2}{m^2}.
\]

Thus, the divergences in the effective Lagrangian \( \Delta L^{\mu H} = \Delta L^{\mu^2} + \Delta L^{\mu} \) could be eliminated by making renormalization for the factor in the anomalous magnetic moment expression, yielding the final result

\[
\Delta L^{\mu^2} = -\frac{(\mu H)^2 eH}{2(2\pi)^2} \int_0^{\infty} ds \frac{e^{-s m^2}}{s} \left( \coth(eHs) - \frac{1}{eHs} \right).
\]
c) anomalous moment $\mu \neq 0$, axial condensate, $b_0 = b \neq 0$, $\vec{b} = 0$:

$$E^2 = m^2 + p^2 + 2eHn + (\mu H)^2 + b^2 \pm 2\sqrt{m(\mu H) + bp}^2 + 2eHn[(\mu H)^2 + b^2]$$  \hspace{1cm} (5)

(for the energy spectrum, cf. [2]).

In this more general case we shall follow [2] and introduce, for convenience, a mixing angle $\Theta$:

$$\Theta = \arctan \frac{b}{\mu H}, \quad \mu H = \tilde{\mu} H \cos \Theta, \quad b = \tilde{\mu} H \sin \Theta, \quad \tilde{\mu} H = \sqrt{(\mu H)^2 + b^2}.$$  

We shall name the quantity $\tilde{\mu}$ an effective anomalous magnetic moment. Using the angle $\Theta$, we go over to the effective mass and momentum:

$$\tilde{m} = m \cos \Theta + p \sin \Theta \hspace{1cm} (6)$$

$$\tilde{p} = -m \sin \Theta + p \cos \Theta \hspace{1cm} (7)$$

The energy spectrum (5) can be written in these new effective variables as follows:

$$E^2 = (\sqrt{\tilde{m}^2 + 2eHn + \tilde{\mu} H \zeta})^2 + \tilde{p}^2.$$  \hspace{1cm} (8)

The resulting renormalized expression for the one-loop effective Lagrangian can be written in the second order in the anomalous magnetic moment $\mu$ and Lorentz-violating parameter $b$ in terms of a parameter $\tilde{\mu} H$ as follows:

$$\Delta \tilde{\mathcal{L}}^{\tilde{\mu}^2} = -\left(\tilde{\mu} H\right)^2 \frac{eH}{(2\pi)^2} \int_0^{+\infty} ds e^{-m^2s} \left(m^2 \sin^2 \Theta + \frac{1}{2s} \cos^2 \Theta\right) \left(\coth(eHs) - \frac{1}{eHs}\right).$$

It is clear that this general result can be easily transformed into the special cases a) and b)

- $\Theta = 0$ implements the case $b = 0$, then $\tilde{\mu} H = \mu H$ and we have $\Delta \mathcal{L} = \Delta \mathcal{L}^{\mu^2}$;
- $\Theta = \frac{\pi}{2}$ implements the case $\mu = 0$, then $\tilde{\mu} H = b_0$ and we have $\Delta \mathcal{L} = \Delta \mathcal{L}^{b^2}$.

In this work we calculated the effective Lagrangian for the case, when both condensates $\mu$ and $b_\mu$ are nonvanishing, generalizing the previously considered particular assumptions that only one of the two possible terms, $b_\mu$ or $\mu$, is nonvanishing. In the corresponding limiting cases $\tilde{\mu} \to \mu$ or $\tilde{\mu} \to b$ our general expression goes over to results of [6] and [5, 4] respectively.

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