σ-Particle in Production Processes

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Abstract. A $S$-wave $\pi\pi$ resonance below $1 \text{ GeV}/c^2$, sigma has been observed in analyses of the phase shift data of $\pi\pi$ scatterings. It is important to observe it in production processes. A huge events of $\pi^0\pi^0$ below $1 \text{ GeV}/c^2$ is seen in the central production of the GAMS experiment. Its mass spectrum is well described by the variant mass and width method (VMW) including $f_0(980)$, $f_2(1270)$ and a $S$-wave resonance which might be assigned to be a sigma. We report here that the angular distributions of the $\pi^0\pi^0$ system are also described with the $S$-wave resonance interfering with $f_0(980)$ and the tail of $f_2(1270)$ by VMW. The same method is applied to the $\pi^+\pi^-$ data of $J/\psi \rightarrow \omega \pi\pi$ decay, and is shown that not only $\pi^+\pi^-$ mass spectrum but also its $\cos\theta_{\pi}$ distributions are described well.

We have reported [1] that a $S$-wave $\pi\pi$ resonance is observed below $1 \text{ GeV}/c^2$ in the reanalysis of the phase shift data [2] of the $\pi\pi$ scatterings. Several analyses on the same data also have reported [3] an existence of a possible scalar resonance(s) of the $\pi\pi$ system. An existence of a sigma particle is expected in this region as a chiral partner of Nambu-Goldstone boson $\pi$. Its mass value is predicted to be twice of constituent mass of a $n$ quark in the extended Nambu-Jona Lasinio model [4] as the low energy effective theory of QCD. It is vitally important to observe the sigma in production processes as well as in the scattering process.

The GAMS group has observed huge $\pi^0\pi^0$ events below $1 \text{ GeV}/c^2$ in a proton proton central collision process at 450 GeV/c. Details of the experimental condition and data are described in reference [5]. It shows a resonant peak at 500-600 MeV/c$^2$, as shown in Fig.1(a). It can be described by a $S$-wave Breit-Wigner interfering with $f_0(980)$. The variant mass and width method (VMW) [6] is applied to the mass spectrum. Here we summarize formulae of the analysis, briefly. An amplitude, $\mathcal{M}$, is given by the product of production amplitude, $\mathcal{M}^{\text{prod}}(f_j)$, propagator $\Delta^j$, and coupling constant $g_{j\pi\pi}$. $j$ indicates $j$-th resonance. $f_0(980)$, $f_c$ and $f_2(1270)$ are supposed for $j$, where $f_c$ means a $\pi\pi$ resonance below $1 \text{ GeV}/c^2$. Helicity couplings, $\rho_h$ ($h = -2$ to $+2$, $\sum \rho_h^2 = 1$) is considered for $f_2$. An introduction of relative phases...
is an important and essential point in the VMW method. We define production amplitude, $\xi$, as $M_{\text{prod}}(f_{j}) \Delta^2 g_{j \pi \pi} \equiv \xi e^{i \theta_j}. \quad \theta_{f_0}$ is taken to be zero by definition.

The total amplitude is written as follows,

$$M = \frac{\xi_{f_0} e^{i \theta_{f_0}}}{(s - m_{f_0}^2) + i \sqrt{s} \Gamma_{f_0}^\text{tot}(s)} + \frac{\xi_{f_c} e^{i \theta_{f_c}}}{(s - m_{f_c}^2) + i \sqrt{s} \Gamma_{f_c}^\text{tot}(s)} + \frac{\xi_{f_2} e^{i \theta_{f_2}}}{(s - m_{f_2}^2) + i \sqrt{s} \Gamma_{f_2}^\text{tot}(s)} \sum_h \rho_h \epsilon_{\mu \nu}^h \frac{(p_1 - p_2)_\mu (p_1 - p_2)_\nu}{\sqrt{s}} \right). \quad (1)$$

$M$ is modified by $(s - m_{\pi}^2)/s \cdot M$ to take Adler zero into account. Differential Pomeron-Pomeron cross section and total cross section are

$$d\sigma_{PP} = \frac{|P_1|}{16\pi^2 \sqrt{s}} |M|^2 d\Omega_1 \quad (2)$$

$(P_1$ being momentum in the $2\pi$ rest frame) and

$$\sigma_{PP} = \frac{|P_1|}{4\pi \sqrt{s}} \left\{ \frac{\xi(f_0)}{(s - m_{f_0}^2)^2 + s(\Gamma_{f_0}^\text{tot}(s))^2} + \frac{\xi(f_2)}{(s - m_{f_2}^2)^2 + s(\Gamma_{f_2}^\text{tot}(s))^2} + \frac{\xi(f_2) \epsilon(f_0) e^{i \theta_{f_2}}}{(s - m_{f_2}^2 + i \sqrt{s} \Gamma_{f_2}^\text{tot}(s))(s - m_{f_0}^2 - i \sqrt{s} \Gamma_{f_0}^\text{tot}(s)) + \text{c.c.}} \right\}. \quad (3)$$

In the actual analyses for production processes, appropriate kinematical factor is multiplied to Eq. (3).

The solid line in Fig. 1(a) shows the result of fit by Eq. 3). Following parameters for mass and width have been obtained for $f_c$.

1) In ref. [5] $i m \Gamma(s)$ was used instead of $i \sqrt{s} \Gamma(s)$ in Eq. (1), and gives a heavier $\sigma$ mass.
FIGURE 2. (a) The fit for effective mass distribution by VMW-method shown by a solid line. The dashed line is $D$-wave contribution coming from $f_2(D_{f_2})$ and $b_1\pi$ decay ($D_{BG}$). Dotted line shows $D_{BG}$ contribution. (b) The fit for $\cos \theta$ distribution by VMW-method shown by a solid line. The dashed lines are $D$-wave contribution $|D_{f_2}|^2 + |D_{BG}|^2$, while dotted lines are interference of $f_2$ and $S$-wave state, $|S \cdot D_{f_2}|$. Data are taken from ref. 10).

$$m_c = 590 \pm 10 \text{ MeV}/c^2, \quad \Gamma_{c\pi\pi} = 710 \pm 30 \text{ MeV}/c^2.$$  

The $\cos \theta_{GJ}$ angular distributions of the $\pi^0\pi^0$ system are also fitted with Eqs. (1) and (2), which are shown in fig. 1(b) for every 100 MeV/c$^2$ mass interval. Dotted curves in the figures are results of the fit. Parameters for masses and others are common to those obtained above. The values of $\theta_{f_c}$ and $\theta_{f_2}$ are 210 and 235 in degrees, respectively. The $\pi^0\pi^0$ data are described excellently by VMW not only for the mass distribution, but also for angular distributions.  

We have performed an analysis on $\pi^+\pi^-$ data of $J/\psi \rightarrow \omega\pi\pi$ obtained by the DM2 collaboration, which also shows a huge events in low mass region. It is reported that they obtained $m_{\text{low mass}} = 414 \pm 20 \text{ MeV}/c^2$ and $\Gamma_{\text{low mass}} = 494 \pm 58 \text{ MeV}/c^2$ for the low mass $S$-wave by fit with two BW’s + polynomials, ignoring their mutual interferences. The fit could not reproduce the $\cos \theta_{\pi}$ distributions in the mass region between 550 and 750 MeV/c$^2$, as is mentioned in their report.

An estimation of a contribution of $\pi^+\pi^-$ background events coming from $J/\psi \rightarrow b_1(1235)\pi$ and $b_1(1235) \rightarrow \pi^+\pi^-\pi^-\pi^0$, which shows a $D$-wave like behavior, is

\[2\) It is to be noted that M. Pennington claimed at the Hadron’95 [7] that our reported result of the analysis of the $\pi^0\pi^0$ mass spectrum produced in the $pp$ central collision is not acceptable. This claim was caused from inappropriate application of “pole-universality” in his (their) analysis of phase shifts of the $\pi\pi$ scattering data [8]. Detailed discussion on the “universality” has been given at this conference [9].
important in the analysis. We will take this effect $D_{BG}$ as follows;

$$|\mathcal{M}|^2 = |S + D_{f_2}|^2 + |D_{BG}|^2,$$

$$|D_{BG}|^2 = a_0^2 p_1^4 (Y(0))^2 + 2a_1^2 p_1^4 (Y(1))^2 + 2a_2^2 p_1^4 (Y(2))^2,$$  \hspace{1cm} (4)

with parameters $a_i (i=0,1,2)$. Fig. 2(a) shows the experimental data of the $\pi^+\pi^-$ mass distribution, where a solid curve is the result of fit of the VMW. $m_{\text{low mass}} = 482 \pm 3$ MeV/$c^2$ and $\Gamma_{\text{low mass}} = 325 \pm 10$ MeV/$c^2$ are obtained. The dashed curve in the figure indicates the contribution from the $D$-waves of both $f_2(1270)$ and $b_1\pi$. A dotted curve indicates that of $b_1\pi$.

The $\cos\theta_{\pi}$ distributions are also fitted, as shown in fig. 2(b) in every 100 MeV/$c^2$ mass interval. Solid curves in the figures show results. Dashed curves in the figure indicate the contribution from the $D$-waves. $\theta_{\text{low mass}} = 214$ and $\theta_{f_2} = 157$ in degrees are obtained for relative phases between the low mass and $f_2$, respectively. The qualitative features of data are reproduced well by VMW, by including interferences between the $S$-wave and $S$ tail of $D$-wave from $f_2(1270)$, whose contributions are shown by dotted lines in the figures.

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