Sign changes and resonance of intrinsic spin Hall effect in two-dimensional hole gas

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The intrinsic spin Hall conductance shows rich sign changes by applying a perpendicular magnetic field in a two-dimensional hole gas. Especially, a notable sign changes can be achieved by adjusting the characteristic length of the Rashba coupling and hole density at moderate magnetic fields. This sign issue may be easily realized in experiments. The oscillations of the intrinsic spin Hall conductance as a function of $1/B$ is nothing else but Shubnikov-de Haas oscillations, and the additional beatings can be quantitatively related to the value of the spin-orbit coupling parameter. The Zeeman splitting is too small to introduce effective degeneracy between different Landau levels in a two-dimensional hole gas, and the resonant intrinsic spin Hall conductance appears in high hole density and strong magnetic field due to the transition between mostly spin-$-\frac{1}{2}$ holes and spin-$\frac{1}{2}$ holes. Two likely ways to establish intrinsic spin Hall effect in experiments and a possible application are suggested.

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In the developing field of spintronics\textsuperscript{1}, which is believed to be a promising candidate for future information technology\textsuperscript{2}, the generation and efficient control of spin current is a key issue. So the intrinsic spin Hall effect (ISHE) predicted by Murakami \textit{et al.}\textsuperscript{3} and Sinova \textit{et al.}\textsuperscript{4} has generated intensive theoretical studies and this ISHE is associated with the dc-field-induced transitions between the spin-orbit-coupled (SOC) bands, and contributes from all occupied electron states below the Fermi energy. The ISHE is distinguished from the extrinsic spin Hall effect (ESHE), which is due to impurity scattering.\textsuperscript{5} On the experimental side, the spin Hall effect (SHE) reported in a two-dimensional hole gas (2DHG) is likely of the intrinsic origin\textsuperscript{6,7}, nevertheless, the intrinsic or the extrinsic origin of the SHE observed in a 3D electron film is still under debate\textsuperscript{8,9}.

The intrinsic spin Hall conductance (ISHC) of a 2DHG in a perpendicular magnetic field has been studied within $k$-cubic Rashba model by M. Zarea \textit{et al.}\textsuperscript{10} and a generalized Luttinger model by us\textsuperscript{11} most recently. The work by M. Zarea \textit{et al.}\textsuperscript{12} only focused on the low field regime, and a remarkable\textsuperscript{13} has been given by J. Schliemann on this aspect: in any case, influence of a magnetic field coupling to the orbital degrees of freedom should only be appreciable if the field is strong enough to produce typical cyclotron radii being of order of the system size or smaller; therefore, arbitrarily small fields cannot be expected to have an effect in real experiments. Within a more general band structure Hamiltonian, we have predicted that the “effective” energy crossing near Fermi energy between mostly spin-$-\frac{1}{2}$ holes and spin-$\frac{1}{2}$ holes at a typical magnetic field gave rise to a resonant ISHC\textsuperscript{12} with a certain hole density, however, a strong magnetic field region is required. Except resonance\textsuperscript{12,14,15}, the sign issue is another crucial aspect of ISHC\textsuperscript{16,17}, which can be modulating by external magnetic field\textsuperscript{12}. To be convenient for future experimental detection or possible applications, it is of course desirable to discuss the effect of magnetic field in detail, especially in moderate magnetic field region, which can be easily achieved in experiments or technologies.

In this paper, we will concentrate on the sensitivity of ISHC to magnetic field, the length scale of the Rashba coupling, and hole density, which shows rich sign changes. We predict that a more notable sign changes can be achieved by adjusting the length scale of the Rashba coupling at certain moderate magnetic fields for a fixed hole density. By discussing the effect of Zeeman splitting on energy levels, we clarify that the interplay between mostly spin-$-\frac{1}{2}$ holes and spin-$\frac{1}{2}$ holes is the origin of resonant ISHC in a 2DHG under a perpendicular magnetic field, which is different from the case\textsuperscript{18} in two-dimensional electron gas (2DEG). In addition, the oscillations of the intrinsic spin Hall conductance as a function of $1/B$ is nothing else but Shubnikov-de Haas (SdH) oscillations, and the additional beatings can be quantitatively related to the value of the SOC parameter. Two likely ways to establish ISHE in experiments and a possible application are suggested, which may be easily realized in experiments.

When the quasi 2DHG is sufficiently narrow and the density and the temperature are not too high, only the lowest heavy-hole subband is occupied. The effective Hamiltonian for a single heavy hole subjected a spin-orbit interaction due to structural-inversion asymmetry (SIA) can be written as\textsuperscript{18,19,20}

\[
H_0 = \frac{\hat{p}_x^2}{2m} + i \frac{\alpha}{\hbar} \left( p_y^3 \sigma_+ - p_x^3 \sigma_- \right),
\]

using the notations $p_{\pm} = p_x \pm ip_y$, $\sigma_{\pm} = (\sigma_x \pm i\sigma_y)/2$, where $\hat{p}_x$, $\sigma$ denote the hole momentum operator and Pauli matrices respectively. These Pauli matrices operate on the total angular momentum states with spin projection.
±3/2 along the growth direction, and we simply call it holes with spin-±3/2. In the above equation, $m$ is the heavy hole mass, and $\alpha$ is Rashba SOC coefficient due to SIA. The validity of above model\textsuperscript{18,19,20} given by the Hamiltonian (1) was restricted to sufficiently small wave numbers and densities.

We follow the method by M. Zarea\textsuperscript{11} et al. and use\textsuperscript{12} below. We impose a magnetic field $\vec{B} = B\hat{z}$ by choosing the Landau gauge $\vec{A} = -gy\vec{B}x$, and then $p_x = \hbar k_x + \frac{eB}{m} y$, where $-e$ is the electric charge. The destruction operator $a = \sqrt{2m\hbar}\left(p_x + i\gamma y\right)$, and the creation operator $a^\dagger = \frac{1}{\sqrt{2m\hbar}}\left(\hat{p}_x - i\gamma y\right)$ are introduced to describe Landau levels, where $\gamma = \frac{eB}{m\hbar}$. These operators have the commutation $[a, a^\dagger] = 1$. In terms of these operators, the Hamiltonian in the absence of Zeeman coupling is

$$H_0 = \hbar\omega\left[\left(a^\dagger a + \frac{1}{2}\right) + i\gamma\left(a^3\sigma^+ - a^3\sigma^-ight)\right],$$

(2)
in which the dimensionless parameter $\gamma$ is defined by $\gamma = \frac{2\alpha m}{\hbar^2}\sqrt{\frac{2eB}{\hbar c}}$. In units of $\hbar\omega$, the eigensystem can be expressed as

$$\psi_{ns} = \begin{pmatrix} -i \cos \theta_{ns} \phi_{n-3} \\ \sin \theta_{ns} \phi_n \end{pmatrix},$$

$$E_{ns} = (n - 1) + s \sqrt{\gamma^2 n(n-1)(n-2) + \frac{9}{4}},$$

(3)

with $s = \pm 1$ for $n \geq 3$, $s = 1$ for $n < 3$ and $\phi_n$ is the eigenstate of the $n$th Landau level in the absence of SOC. Besides, $\theta_{ns} = \arctan\left[\frac{2\gamma\sqrt{n(n-1)(n-2)}}{s\sqrt{4\gamma^2 n(n-1)(n-2)+9n+3}}\right]$, where $\theta_{ns} \in (0, \frac{\pi}{2})$ for $n > 3$, and $\theta = \frac{\pi}{4}$ when $n < 3$.

When the system is driven by a dc electric field in $\vec{y}$ direction, the most interesting case is the $z$-direction-polarized spin current along $x$ direction, and the corresponding spin current operator can be defined\textsuperscript{16,17} as $j_S = \frac{1}{\hbar}\left\{v_x, \sigma^z\right\}$ where $v_x$ is the velocity operator. We shall only focus on the first order in the perturbation term because $<n,s|j_S|n,s> > 0$, which is

$$\langle j_S \rangle_{ns} = \sum_{n', s'} \langle n, s|j_S|n', s'\rangle \langle n', s'|H'|n, s\rangle \epsilon_{ns} - \epsilon_{n's'},$$

(4)

where $\langle j_S \rangle_{ns}$ is the current carried by a hole in the state $|n, s>$ of $H_0$, and $\epsilon_{ns} = \hbar\omega E_{ns}$. In addition, $n' = n \pm 1$ since the matrix element vanishes for other values of $n'$. The average current density of the $N_h$-hole system is given by $I_S = \frac{1}{2e}\sum_n \langle j_S \rangle_{ns}\epsilon_{ns}$ where $f$ is the Fermi distribution function, and $N_h = \sum_n f(\epsilon_n)$. The ISHC is then given by $G_S = I_S / EL_{xz}$, where the Landau degeneracy factor $eB/(hc)$ is also included. We can finally obtain the complete ISHC, and the 2DHG system we consider is confined in the $x - y$ plane of an area $L_x \times L_y$.

Experimental study\textsuperscript{18} has been performed for heavy-hole density $n_h = 2 \times 10^{16}$ m$^{-2}$, and the characteristic length $\alpha m/\hbar^2 = 0.25$nm of the Rashba coupling\textsuperscript{19}. By using these parameters, the ISHC as a function of $1/B$ is shown in Fig.1 (a). The ISHC oscillates as a result of the alternative occupation of mostly spin-$\uparrow$ holes and mostly spin-$\downarrow$ holes, which is consistent with our previous result\textsuperscript{12}. The oscillations of the ISHC as a function of $1/B$ are nothing else but another manifestation of SDH oscillations. Following the same argument of usual SDH\textsuperscript{12}, the periodicity of density of states with $1/B$ results from the equality of the area of two successive electron orbits in $k$-space and the oscillation frequency is given by $f = \frac{2n_n}{\hbar} = 41.36$, where $\phi_0 = \frac{\pi}{2}$ is the flux quantum. In relative weak magnetic field region $(1/B > 0.22/T$ in Fig.1 (a)), there are two independent oscillation frequencies originating from the two spin-splitting subbands, $n_h^+ = \frac{1}{4\pi} f_\uparrow = \frac{1}{4\pi} (k_{f\uparrow}^3)^2$, where $n_h^+$ are respectively the hole density in spin up (down) subbands and $k_{f\uparrow}$ are respectively their Fermi momenta. It is just the composition of these two frequencies produces the additional beating pattern appearing in our numerical result, and the additional beatings can be quantitatively related to the value of the SOC parameter\textsuperscript{19}.

$$k_{f\uparrow} = \frac{1}{4L}(1 - \sqrt{1 - 16\pi n_h L^2})$$

$$+ \left[ -\frac{1}{8L^2}(1 - \sqrt{1 - 16\pi n_h L^2}) + 3\pi n_h \right]^{1/2},$$

(5)

where $L = \alpha m/\hbar^2$. Fast Fourier transformation has been carried out separately on the strong- and weak magnetic field data shown in Fig.1 and frequencies obtained agree numerically with Eq. (5) and $f_\uparrow = \frac{1}{4\pi} (k_{f\uparrow}^3)^2$ rather well.

The importance of sign changes of ISHC has been emphasized by Yao et al.\textsuperscript{19} by applying first-principles calculations to study SHE in semiconductors and simple metals in the absence of a magnetic field. For the definition of sign, positive spin Hall conductance means that spin-up component flows to the positive $x$ direction. The sign of ESHE induced in the skew scattering, which dominates over the side jump process in the weak disorder limit, depends on the sign of the impurity potential, and does not change with the carrier density or the Rashba coupling strength. As shown in Fig.1(a), the sign of ISHC

![FIG. 1: (a) The ISHC as a function of 1/B for n_h = 2 × 10^{16} m^{-2}, and the characteristic length αm/ℏ^2 = 0.25nm of the Rashba coupling\textsuperscript{19}. (b) Same as Fig1.(a) but αm/ℏ^2 = 0.075nm.](image-url)
changes periodically between $\nu = 6$ and $\nu = 13$. Furthermore, this sign change is robust in this case, and a more notable sign change is sensitive to $\alpha m/\hbar^2$ for a fixed hole density, which can be achieved by modulating $\alpha m/\hbar^2$ at certain magnetic fields. Fig.1 (b) accounts for this. A more notable sign changes is shown for the same hole density as in Fig.1(a) at $\alpha m/\hbar^2 = 0.075nm$, where the magnetic field is in the range $2.07T \sim 3.60T$, especially when $2.50T < B < 2.85T$.

To be convenient for future experimental detection, we have made an scan on the magnetic field, and the characterization length scale dependence on the most notable sign changes as a function of hole densities, and the optimal $\alpha m/\hbar^2$ for the most notable sign changes as a function of a hole density has been shown in Fig.2, beside, the corresponding most favorable range of magnetic field is also plotted as a function of hole densities, which denoted by shadow in Fig.2. Since the ISHC in $+x$ direction is larger than the ISHC in $-x$ direction for most case, the optimal $\alpha m/\hbar^2$ and the corresponding most favorable range of $B$ for the most notable sign changes means that when the ISHC in $-x$ direction is the largest for a fixed hole density.

Moreover, The calculated ISHC shows sign changes as hole density varies for a fixed magnetic field and the characteristic length scale of Rashba coupling, which differs from the ESHE too. As shown in Fig.3(a), the importance of this aspect is that, the required magnetic field ($B = 0.827T$) is easy to achieve in experiments, and the sign changes of ISHC may be detected expeditiously. It’s a likely way to establish the ISHE, and it also means a possible application in the future. If we can take the direction of the spin Hall current as a new sign, i.e., $+x$ direction means “0” and $-x$ direction means “1”, and these two states maybe the basic for a new logic electronic-device$^{12}$.

The resonant ISHC predicted by us$^{12}$ is caused by the energy crossing near the Fermi energy due to the transition between mostly spin-$-\frac{1}{2}$ holes and spin-$\frac{3}{2}$ holes.

However, for the 2D heavy-hole system described by a $k$-cubic Rashba model, we can deduce from Eq. (3) that

$$E_{ns} - E_{n\pm 1, \mp s} \neq 0$$

is steady for any $n$ and Rashba coupling constant in the absence of Zeeman splitting. Then there is NO resonant ISHC within Hamiltonian (2) in the presence of a dc field.

Besides, the resonant ISHC was also predicted in 2D quantum wells in a strong perpendicular magnetic field, where the Zeeman splitting plays a crucial role. The $k$-linear Rashba coupling, generated by spin-orbit interaction in wells lacking bulk inversion symmetry, competes with Zeeman splitting to introduce additional degeneracies between different Landau levels (i.e., two states $E_{n\mu}$, $E_{n\pm 1, \pm \mu}$) at certain magnetic fields. This degeneracy, if occurring at the Fermi level, gives rise to a resonant ISHC$^{14}$. The Hamiltonian for a 2DEG in a perpendicular magnetic field is

$$H_e = \frac{1}{2m}(\vec{p} + e\vec{A}/c)^2 + \frac{\lambda }{\hbar^2} \cdot (\vec{p} + e\vec{A}/c) \times \vec{\sigma} - \frac{1}{2} g'_{\mu} \mu_B \sigma_z,$$

(7)

where $m'$, $g'_\mu$ are the electron’s effective mass and Lande-g factor, respectively. In addition, $\mu_B$ is the Bohr magneton, $\lambda$ is the Rashba coupling. The eigen energy of $H_e$ is given by

$$\epsilon_{ns} = \hbar \omega_c \left(n + \frac{\mu}{2} \sqrt{(1 - g')^2 + 8n \eta^2} \right),$$

(8)

where $\omega_c = eB/m'c$ with $\mu = \pm 1$, for $n \geq 1$; and $\mu = 1$ for $n = 0$. Numerically, the degeneracies between different Landau levels ($E_{n\mu}$, $E_{n\pm 1, \pm \mu}$) request that

$$0 < g' < 1,$$

(9)

for a reasonable value of Rashba coupling $\lambda/\hbar$. To a turn, a set of parameters appropriate for In$_{0.53}$Ga$_{0.47}$As/In$_{0.52}$Al$_{0.48}$As$^{21}$ are in this range for $g' = 0.1$.

Let’s discuss the effect of Zeeman splitting on the ISHC of a 2DHG. The 2D heavy-hole described by $k$-cubic
Rashba model in a perpendicular magnetic field including Zeeman splitting can be written as

\[ H_z = \hbar \omega [(a^1 a + \frac{1}{2}) + \gamma (a^3 \sigma^+ - a^3 \sigma^-)] + \frac{3}{2} g_s \mu_B B \sigma_z \]  

(10)

and the energy levels in units of \( \hbar \omega \) can be expressed as

\[ E_{n,s} = (n - 1) + \frac{s}{2} \sqrt{(3 - 3g)^2 + 4\gamma^2 n(n - 1)(n - 2)} \]  

(11)

where \( g = g_s m/2m_e \). The key difference between \( k \)-linear and \( k \)-cubic Rashba model is the constant along with \( g \), and “3” in \((3 - 3g)^2\) is due to \( k \)-cubic Rashba term, which is similar as “1” in \((1 - g')^2\) arising from \( k \)-linear Rashba term. From Eq.(12), we can deduce that if the energy crossing between \( E_{n,s} \) and \( E_{n\pm 1,s} \) occurs, which may give rise to a resonant ISHC, requests that \( \frac{4}{3} < g < 1 \) for a reasonable value of \( am/\hbar^2 \). In calculation, we take the effective mass \( m = 0.27m_e (\text{Ref.}^{22,19}) \), and then the corresponding effective gyromagnetic factor \( g_s \) is in the range of 4.93 \( \sim \) 7.4. A typical value of \( g_s \) has been given by Winkler\(^{22}\), and \( g_s = -0.44 \). The requested \( g_s \) is far away from the typical value of \( g_s = -0.44 \), which means that the Zeeman splitting is too small to introduce "effective" degeneracy between different Landau levels. So the resonance effect stems from energy crossing of different Landau levels near the Fermi level due to the competition of Zeeman splitting and \( k \)-cubic Rashba SOC is out of reach for real materials or in experiments.

In addition, the Zeeman splitting plays a crucial role on the ISHC within \( k \)-linear Rashba model. When Zeeman energy is neglected, the zero ISHC persists in a magnetic field\(^{22}\). Within the perturbation method in this paper, we can also achieve this conclusion. However, the Zeeman splitting has much less effect on the ISHC of a 2DHG in magnetic field. To demonstrate this, a resonant ISHC within Luttinger model including linear Zeeman splitting has been shown in Fig.3 (b), and parameter used are taken from Ref\(^{22,12}\) except \( n_h = 2.9 \times 10^{10} m^{-2} \), and \( g_s = -0.44 \). The resonance appears at \( B = 29.98 T \). Therefore, the resonant ISHC of a 2DHG in a perpendicular magnetic field contributed from the interplay between mostly spin-\( \frac{1}{2} \) holes and spin-\( \frac{3}{2} \) holes is confirmed.

To compare with the previous results\(^{18}\) in the absence of magnetic field, we calculate the interband contribution to the ISHC as a function of the length scale \( am/\hbar^2 \) of the Rashba coupling at a hole density of \( n_h = 3 \times 10^{14} m^{-2} \) for a fixed magnetic field. Our numerical result has been shown in Fig.4. The circle red dots indicates \( \nu = 100 \) and the blue triangle indicate \( \nu = 1000 \). The ISHC in the absence of magnetic field as a function of \( am/\hbar^2 \) at the same hole density has been plotted in this fig too, which is denoted by the solid black line. To take a overview expeditiously, it has also been shown in the fig inset solely and data are obtained within the method in Ref\(^{18}\). As seen in Fig.4, the interband contribution to the ISHC increases with increasing Rashba coupling, and it approaches the value in absence of magnetic field in weak magnetic limit.

In conclusion, two likely ways to establish ISHE in experiments shall be suggested. (1) A resonant ISHC shall appear in a 2DHG for a high hole density and in a strong perpendicular magnetic field, and this phenomenon can be used to distinguish the ISHE from the extrinsic one. (2) The ISHC shows rich sign changes, and a notable sign changes can be achieved for a lower hole density and moderate magnetic field. The sign of ESHE depends on the sign of scattering potential\(^{16,24}\), and such difference may be used to distinguish ESHE from intrinsic contributions and determine the intrinsic origin of the effect. The sign issue should be regarded as a very important aspect of SHE, and can be used in future experiments or application.

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