A universal minimal mass scale for present-day central black holes

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The early stages of massive black hole growth are poorly understood\textsuperscript{1}. High-luminosity active galactic nuclei at very high redshift\textsuperscript{2} further imply rapid growth soon after the Big Bang. Suggested formation mechanisms typically rely on the extreme conditions found in the early Universe (very low metallicity, very high gas or star density). It is therefore plausible that these black hole seeds were formed in dense environments, at least a Hubble time ago ($z > 1.8$ for a look-back time of $t_\odot = 10$ Gyr\textsuperscript{3}). Intermediate-mass black holes (IMBHs) of mass $M_\bullet \approx 10^2 - 10^5$ solar masses, $M_\odot$, are the long-sought missing link\textsuperscript{4} between stellar black holes, born of supernovae\textsuperscript{5}, and massive black holes\textsuperscript{6}, tied to galaxy evolution by empirical scaling relations\textsuperscript{7,8}. The relation between black hole mass, $M_\bullet$, and stellar velocity dispersion, $\sigma$, that is observed in the local Universe over more than about three decades in massive black hole mass, correlates $M_\bullet$ and $\sigma$ on scales that are well outside the massive black hole's radius of dynamical influence\textsuperscript{6}, $r_\sigma = GM_\bullet/\sigma^2$. We show that low-mass black hole seeds that accrete stars from locally dense environments in galaxies following a universal $M_\bullet/\sigma$ relation\textsuperscript{9,10} grow over the age of the Universe to be above $M_\bullet \approx 3 \times 10^5 M_\odot$ (5\% lower limit), independent of the unknown seed masses and formation processes. The mass $M_\odot$ depends weakly on the uncertain formation redshift, and sets a universal minimal mass scale for present-day black holes. This can explain why no IMBHs have yet been found\textsuperscript{11}, and it implies that present-day galaxies with $\sigma < S_\odot \approx 40$ km s\textsuperscript{-1} lack a central black hole, or formed it only recently. A dearth of IMBHs at low redshifts has observable implications for tidal disruptions\textsuperscript{12} and gravitational wave mergers\textsuperscript{13,14}.

Recent analyses of large heterogeneous galaxy samples find that a universal $M_\bullet/\sigma$ relation $M_\bullet = M_\sigma(\sigma/L_\sigma)$ holds for all galaxy types\textsuperscript{15,16}, although the scope of this relation and its evolution with redshift remain controversial\textsuperscript{17}. Here we adopt the empirical fit\textsuperscript{18} $\log_{10}(M_\bullet/M_\odot) = 8.32 \pm 0.04 \pm 0.03 (5.35 \pm 0.23) \log_{10}(\sigma/200 \text{ km s}^{-1})$, where $\sigma = 0.49 \pm 0.03$ is the root mean square of the intrinsic scatter. We assume that this universal $M_\bullet/\sigma$ relation holds at all redshifts\textsuperscript{14}, and that the black hole seeds grow in a locally (within a few $r_\odot$) dense stellar environment. By fixing $r_\sigma$, the $M_\bullet/\sigma$ relation then imposes tight connections between the black hole and the dynamical properties of its stellar surroundings\textsuperscript{18}, and specifically the rate at which it consumes stars (see Methods section).

A central black hole grows by (1) the accretion of stars, compact remnants and dark matter particles that are deflected toward it on nearly radial orbits, and either fall whole above the event horizon or are tidally disrupted outside it, and then accreted; (2) viscosity-driven accretion of interstellar gas; and (3) mergers with other black holes. Of these growth channels, only the accretion of stars must follow from the existence of a central black hole in a stellar system. Moreover, the tidal disruption event (TDE) rate in steady state can be estimated from first principles, for given boundary conditions at $r_\odot$ (ref. \textsuperscript{19}).

It has been noted that typical steady-state TDE rates, $\Gamma$, around $10^{-4}$ yr\textsuperscript{-1} (Fig. 1), imply by simple dimensional analysis that massive black holes (MBHs) with low mass, $\lesssim 10^5 M_\odot$, may acquire a substantial fraction of their mass from TDEs over the Hubble time $t_\odot$, or equivalently, that linear growth by TDEs has a typical mass scale\textsuperscript{20–22}, $M_{\text{TDE}} \sim M_\bullet \Gamma t_\odot \sim 10^6 M_\odot$ (however, the growth equation is generally nonlinear, and therefore $M_{\text{TDE}}$ can significantly mis-estimate $M_\bullet$ ($t_0$; see Methods section). Previous studies have usually focused on the rates and prospects of TDE detection, and not on black hole growth. Although it was recently argued that $M_{\text{TDE}}$ arises as a minimal black hole mass in a specific formation scenario\textsuperscript{23}, the commonly held assumption remains that IMBHs with $M_\bullet \ll M_{\text{TDE}}$ do exist, and that this must constrain formation scenarios, or set an upper bound on the efficiency of TDE accretion, rather than a lower bound on IMBH masses\textsuperscript{24}.

Here, we argue that IMBHs are transient objects, which no longer exist in the present-day Universe. We derive a universal lower bound on the present-day mass scale of central black holes, $M_\odot$, that follows directly from the universal $M_\bullet/\sigma$ relation, and is independent of the unknown seed masses and their formation processes. We use the $M_\bullet/\sigma$ relation to set the boundary conditions, and show that the nonlinear growth equation for black holes can be bounded from below by a simple inequality that includes only growth by TDEs. We translate the intrinsic scatter in the $M_\bullet/\sigma$ relation to a probability distribution for the lower bounds $M_\odot$ and $S_\odot$, and show that $M_\odot$ lies just below the lightest MBHs yet discovered\textsuperscript{18}, $M_\odot \lesssim \text{min}(M_{\text{IMBH}}) \sim 10^6 M_\odot$.

Stars around a central black hole are constantly scattered in angular momentum to nearly radial orbits below a critical (loss-cone) value, $r_\odot = \sqrt{1 - e^2}$ (e is the orbital eccentricity), which approach the black hole closer than the tidal disruption radius, $r_T \approx (M_\bullet/M_\odot)^{1/3} R_\bullet$ ($M_\bullet$ and $R_\bullet$ are the stellar mass and radius), where they are destroyed. Main-sequence stars are disrupted outside an IMBH's event horizon, and a fraction $f_\odot$ of about 1/4 to 1/2 of their mass is ultimately accreted by the black hole\textsuperscript{25}. The TDE rate depends on the number of stars near the black hole, and on the competition between the two-body relaxation time $t_\odot$ (equation (12)) and the orbital time in supplying and draining loss-cone orbits. The integrated contribution in steady state from all radii is a function of $M_\odot$ and of the boundary conditions at $r_\odot$, fixed by $\sigma$. The TDE rate is well-approximated by a power-law $\Gamma \approx \Gamma_\odot(M_\bullet/M_\odot)^{\beta}$, whose index $\beta$ is a function of the $M_\bullet/\sigma$ index $\beta$, and changes across a critical mass scale $M_\bullet \sim 10^6 M_\odot$. (Fig. 1;

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see Methods section). The index $b \ll 1$ for the empirical range $4 \leq \beta < 6$ (refs 6,1).

Let us assume that a black hole seed forms with an initial mass $M_1$ large enough to dominate its radius of influence, in a centrosymmetric system that is massive enough to allow it to grow: that is, $M_{\text{sys}} \gg M_1 n_\ast (r_\ast) > M_1 \gg M_\ast$ at all times ($n_\ast$ is the stellar density). Consider first the case where the black hole grows only by accreting stars. The black-hole growth equation is

$$M_\ast = f_\ast M_\ast \Gamma_\ast (M_\ast / M_\ast) \equiv M_\ast^\bullet \quad M_\ast(0) = M_1,$$  \tag{1}

The solution for $b < 1$ in the $t \gg t_\ast = (M_1 / M_\ast)^{(1-b)/(1-b)} f_\ast \Gamma_\ast$ limit (equation (9)), $M_t(\ast) \approx [(1-b) f_\ast \Gamma_\ast t]^{1/(1-b)} M_\ast \equiv M_\ast^\bullet(t)$, is independent of $M_1$. Because $t_\ast \ll t_0$ for $M_1 \lesssim 10^3 M_\odot$, all seeds reach the same mass scale after $t_\ast = \mathcal{O}(t_0)$. (Fig. 2).

Consider next a realistic black hole that grows also by gas accretion and/or mergers, $M_\ast = M_\ast^\bullet + M_\ast^\star$, where $M_\ast^\bullet \geq 0$ is the accretion rate by the non-stellar channels. The full growth equation is

$$M_\ast = M_\ast^\bullet + M_\ast^\star \geq f_\ast M_\ast \Gamma_\ast (M_\ast / M_\ast)^\beta, \quad M_\ast(0) = M_1.$$  \tag{2}

The solution $M_\ast^\bullet(t)$ of the stars-only growth (equation (1)) then provides a lower limit on the actual mass $M_\ast(t)$ of the growing black hole. Note that $M_\ast^\bullet$ is not necessarily a lower limit on the actual stellar mass contribution $M_\ast^\star$ to $M_\ast$ for $b < 0$, but $M_\ast^\bullet < M_\ast^\star \ll M_1$ for $b < 0$. The universal minimal mass scale of a central black hole at $t_1$, and the corresponding minimal velocity dispersion scale are then

$$M_0 = M_\ast^\bullet(t_1) \approx [(1-b) f_\ast \Gamma_\ast t_1]^{1/(1-b)} M_\ast^\bullet, \quad \sigma_0 = (M_0 / M_\ast)^\beta \sigma_\ast,$$  \tag{3}

with the index $b$ for the $M_0 < M_1$ branch of equation (6). This implies that galaxies with $\sigma_\ast < \sigma_0$ do not have a central black hole, or have formed it only recently, for otherwise the coevolution of the black hole and nucleus over $t_\ast$ would have driven $\sigma_\ast$ to a much larger present-day value.

Assuming the universal $M_\ast / \sigma_\ast$ relation and solar-type stars, the range in $M_0$ values is due to the variety and uncertainty in the properties of galactic nuclei and of tidal disruption, and to the intrinsic scatter in the relation. The value $M_0$ increases with the index of the stellar cusp ($n_\ast \propto r^{-\alpha}$) inside $r_\ast$, and with the accreted mass fraction $f_\ast$ from $10^3 M_\odot$ for the shallowest possible cusp of unbound stars ($\alpha = 1/2, f_\ast = 1/4$) to $10^5 M_\odot$ for a steep isothermal cusp ($\alpha = 2, f_\ast = 1/2$). $M_0 = 10^5 M_\odot$ for the parameters adopted here: a dynamically relaxed cusp$^7$ where $\alpha = 7/4$, and an accreted mass fraction $f_\ast = 3/8$.

The scatter around the $M_\ast / \sigma_\ast$ relation probably reflects intrinsic physical differences between galaxies beyond the measurement errors on the $M_\ast / \sigma_\ast$ parameters$^{10}$. This induces roughly Gaussian probability distributions for $M_0$ and $\sigma_0$: $M_0 = (1.1 \pm 0.8) \times 10^5 M_\odot$ and $\sigma_0 = 79 \pm 35$ km s$^{-1}$. The lowest-$\sigma_\ast$ galaxies known to harbour active galactic nuclei$^{21}$ (and hence black holes, with estimated masses $10^5 \lesssim M_\ast \lesssim 10^6 M_\odot$) have $\sigma_\ast \approx 30–40$ km s$^{-1}$. This corresponds to the 5% lower limits $S_\ast \lesssim 40$ km s$^{-1}$ and $M_0 \lesssim 3 \times 10^5 M_\odot$, which we adopt here as representative lower limits. Lighter black holes are much rarer yet: for example, $M_\ast \lesssim 4 \times 10^4 M_\odot$ is below the 0.0001% limit.

The agreement $M_0 \leq \min(M_{\text{obs}}^{\ast \odot} \sim 10^5 M_\odot$ follows directly from basic local physics (tidal disruption and loss-cone dynamics) and empirical global properties of the Universe (its age and a universal $M_\ast / \sigma_\ast$ relation). Our derivation of $M_0$ rests on four assumptions. (1) There is effective accretion of tidally disrupted stars ($f_\ast$ is a few $\times 0.1$)$.^20$ (2) Most black hole seeds were formed early, at a look-back time $t_\ast \approx \mathcal{O}(t_0)$. (3) Black hole growth is not typically mass or density limited; that is, the growing black hole is embedded in a stellar system with $M_\ast / \sigma_\ast > r_\ast M_\ast / \sigma_\ast$, for a substantial fraction of $t_\ast$. (4) The boundary conditions at $r_\ast$ are set by the universal $M_\ast / \sigma_\ast$ relation at all times.

An early start for black hole seeds, and the requirement that a system that can form and retain a seed black hole should be dense and massive enough, are both physically plausible and possibly even essential$^1$. Such a system can be approximated as embedded...
in an isothermal density distribution, and is dynamically relaxed (see Methods section). Furthermore, the accrretion rate of stars in a system with $N_\text{b}$ stars inside $r_{\text{c}}$, $\frac{dM_\text{b}}{dt} \approx f_\text{a} M_\text{d} N_\text{a} / (\log (1/\epsilon) T_b)$ (equation (16)), is slow enough to allow it to remain near equilibrium as it grows, as the timescale for growth by order of the stellar mass, $(dM_\text{d}/dt)/(M_\text{d} N_\text{a})$, is longer by a factor $\log (1/\epsilon) f_\text{a} \gg 1$ than the timescale to return to steady state, $T_b \approx T_b/\epsilon$ (ref. 1). The least secure assumption is that a universal $M_\text{c}/\sigma$ relation holds near its present-day value as the black hole grows. However, this is broadly consistent with observations of active galactic nuclei (such as cored dwarfs) or in very low-density galaxies (such as cored dwarfs). However, it is unlikely that such systems can form a black hole seed to start with, and therefore the black hole occupation fraction there is probably low. Candidate IMBHs have been reported in globular clusters and dwarf galaxies, including recently, but the evidence remains inconclusive.

Early TDE-driven growth and the suppression of the cosmic black hole mass function below $M_\text{d}$ have implications for black hole seed evolution, for the cosmic rates and properties of TDEs, and for gravitational waves from IMBH–IMBH mergers and intermediate-mass–ratio inspirals into IMBHs. We conclude by listing these briefly.

A high rate of TDEs can allow black hole seeds to continue growing despite the ejection of the ambient gas by supernovae feedback. The lack of IMBHs at low redshifts means that electromagnetic searches will have to reach very deep to detect TDEs from IMBHs (jetted TDEs may provide an opportunity). The prospects of detecting exotic processes related to IMBHs, such as tidal detonations of white dwarfs in the steep tidal field of a low-mass black hole, will be low. The mean observed TDE rate per galaxy, $\bar{f} \approx 10^{-7} \text{yr}^{-1} \text{gal}^{-1}$, is much lower than predicted rates. A dearth of black holes below $M_\text{d}$ may partially resolve the rate discrepancy.

IMBHs produce gravitational waves by intermediate-mass–ratio inspirals and by IMBH mergers. Detection of intermediate-mass–ratio inspirals by planned space-borne GW observatories is limited to redshifts below a few $\times 0.1$, and is therefore unlikely. However, IMBH mergers can be detected to very high redshifts. A gravitational wave search for IMBH mergers and intermediate-mass–ratio inspirals can reveal the formation history of black holes. We predict that black hole seeds are soon driven to higher mass by the accretion of stars, and therefore IMBHs are rare in the present-day Universe, but will be found near their high formation redshifts.

### Methods

This section summarizes results from loss–cone theory used to derive the equation for black hole growth by stellar disruptions, and discusses the properties of its solutions. We first present, without derivation, a recipe for the approximate power-law growth rate equation (equation (1)), which has the advantage of leading to simple analytical results. We then comment on the generic properties of its solutions, to clarify under what circumstances and to what extent simple dimensional analysis can be used to estimate the minimal mass limit, $M_\text{d}$. We then describe how the intrinsic scatter in the $M_\text{d}/\sigma$ relation is propagated through the growth equation to obtain the probability distributions for the lower limits $M_\text{d}$ and $\sigma_\text{d}$. Finally, we present for completeness and reproducibility an outline of the derivation of the full growth rate equation (used to verify our approximations; see Fig. 1) and of its power-law approximation.

**Approximate power-law growth rate equation for black holes.** We focus here on a steady-state stellar system around a black hole, which has a density cusp $n_\text{c} \propto r^{-\frac{n}{2}}$ with $n_\approx \approx \frac{7}{4}$ inside the radius of influence $r_{\text{H}} = GM_\text{d}/\epsilon^2$. We further assume that the cusp is embedded in an external isothermal stellar distribution, $\rho(r) = \sigma^2 / (2\pi G^2 r^2)$, so that the stellar mass enclosed inside $r_{\text{H}}$ is twice the black hole mass $M_\text{d}$. Under the assumption of a universal $M_\text{c}/\sigma$ relation $M_\text{c} = M_\text{c}/\sigma^2 / \epsilon^2$, the dynamics leading to tidal disruption are characterized by a critical mass scale $M_\text{c} \approx 10^{12} M_\odot$. Tidal disruptions are dominated by stars originating from $r_\text{H} = 3 r_{\text{H}}$, and by stars originating from an inner critical radius $r_\text{H} = 3 r_{\text{H}}$ for $M_\text{c} < M_\text{d}$ (see below for more details). The TDE rate is well-approximated by a broken power law (Fig. 1)

$$\Gamma = \Gamma_{\text{TDE}}(M_\text{d}, M_\text{a})^{\frac{9}{4}}$$

whose index $b$ changes across $M_\text{d}$, which is given by (see equations (20–21) for the general case)

$$M_\text{d} = M_\text{c} \left( \frac{16}{3^5} \right)^{\Delta b + \frac{1}{2}}$$

in terms of the dimensionless velocity dispersion scale $\sigma_\text{d} = (M_\text{d}/M_\text{a})^{-\frac{1}{2}} / \epsilon$, where $\sigma_\text{a} = G M_\text{a} / r_\text{a}$ and $r_\text{a}$ and $M_\text{a}$ are the mass and radius of a typical star in the system, and where we approximated the logarithmic term (equation (14)) appearing in the general expressions by a typical value $\Lambda = 2$. The index $b$ is (see equation (22) for the general case).

$$b = \left( \frac{\left(105 - 23 \beta \right) / \Delta b}{1 - 2 \beta} \right)$$

Note that $b \ll 1$ for the empirically determined range of the $M_\text{c}/\sigma$ relation index ($\approx 4 \leq \beta \leq 6$). Defining $\tau_{\text{a}} = (\rho_\text{a}^2 / GM_\text{a})$, the rate factor is

$$\tau_{\text{a}} = \left( \frac{5}{105 - 23 \beta} \right) (M_\text{d}/M_\text{a})^{10 + 1/\Delta b}$$

To summarize, the approximate power-law TDE rate for a black hole with mass $M_\text{d}$ is calculated as follows. (1) Calculate the critical mass $M_\text{c}$ (equation (5)). (2) Calculate the power-law index $b$ (equation (6)) according to the low or high mass branch, depending on $M_\text{d}$, and similarly calculate the rate factor $\Gamma_{\text{TDE}}$ (equation (7)). (3) Use equation (4) to obtain the TDE rate from $\Gamma_{\text{TDE}}$ and $b$. Properties of the black hole growth solutions. The general solution of the growth equation (equation (8)) with the initial condition $M(t = 0) = M_0$ is

$$M(t) = \left( \frac{M_\text{c} / M_\text{a}^{1/2}}{t_{\text{a}} / t_{\text{f}}} \right)^{1/ \Delta b}$$

where $t_{\text{a}} = (\rho_\text{a}^2 / GM_\text{a})$ is the accretion timescale. The growth solution has three branches. The solution for $b > 1$ diverges exponentially to infinity in infinite time. When $b > 1$, $M_\text{c}$ diverges on a finite timescale

$$t_{\text{a}} = t_{\text{f}} (M_\text{d}/M_\text{a})^{1/ \Delta b}$$

and is supra-exponential. The $b < 1$ branch is sub-exponential and diverges slowly as a power law.

Exponential growth describes, for example, radiation-pressure-regulated accretion of gas at the Eddington limit. Supra-exponential growth describes the disc accretion of gas, or the Bondi accretion if the accretion disc is resolved. The exponential and supra-exponential solutions ($b > 1$) are functions of $M_\text{d}$ on all timescales, and $M_\text{d}$ plays a role there related to the exponential or
Intrinsic $M_*/\sigma_*$ scatter and distribution of lower limits. The observed intrinsic scatter in the $M_*/\sigma_*$ relation at $z \approx 0$, with r.m.s. $\delta_\sigma$, can be interpreted as reflecting a variance in the initial conditions of individual galaxies at their formation, a Hubble time ago, or a variance that developed gradually over their individual evolutionary histories and reached the observed r.m.s. value at $t_0$.

We assume that the estimation errors in the parameters $\alpha$ and $\beta$ of the $M_*/\sigma_*$ relation, $\log(M_*/M_*) = (\delta \sigma / \sigma_*) + (\delta \log(\sigma_*) / \sigma_*)$, can be approximated by a correlated bi-Gaussian distribution, $(\alpha, \beta) \sim N(\delta \sigma, \delta \log(\sigma_*), \delta \log(\sigma_*), \sigma_*)$, for an arbitrarily chosen low reference velocity dispersion $\sigma_*$, the correlation coefficient $\rho_{\log(\sigma_*)\alpha} = -1$ (ref. 17), and that the intrinsic scatter is drawn from a Gaussian distribution, $\sigma_{\log(\sigma_*)} = (G(0,0))$.

We approximate the evolution of the scatter by assuming a discrete time steps of duration $\Delta t = \eta \zeta$, where the accumulated change in $\alpha$ due to scatter, $\langle \Delta \alpha \rangle$, is modified in a random walk fashion by $\Delta t = \alpha + \epsilon / \langle \Delta t \rangle$. We then evolve the black hole mass over time $\Delta t$ by the growth equation (equation 8), and repeat until $t = t_0$. The joint and marginal probability distributions for $M_*$ and $S_0$ at $t_0$ are obtained by Monte Carlo simulations over randomly drawn values of $\alpha$ and $\beta$.

The limit $n_1$ corresponds to that which is determined by the galaxy's initial conditions, whereas $n_2$ corresponds to scatter that is determined by the galaxy's evolution. We find that the probability distributions for $M_*$ and $S_0$ do not depend strongly on the choice of $n_1$, and that they converge rapidly for $n_2 > 3$ to an asymptotic form. The values quoted in this study, 5% lower limits of $M_*=2.8 \times 10^8 M_\odot$ and $S_0 = 38 \text{ km s}^{-1}$, correspond to the asymptotic evolutionary scatter case ($n_2 = 5$), whereas the initial scatter case ($n_2 = 1$) differs only slightly, with 5% lower limits of $M_*=1.9 \times 10^8 M_\odot$ and $S_0 = 36 \text{ km s}^{-1}$.

Full growth rate equation for black holes. The tidal disruption (plunge) rate can be approximated by the flux of stars into the black hole from the boundary between the inner region, where stars slowly diffuse into the loss-cone (the empty loss-cone) and the outer region, where stellar scattering is strong enough that the loss-cone is effectively full (the full loss-cone) 19. The boundary is at a critical radius, $a_*$, that satisfies

$$ \frac{P(\alpha, \beta)}{P(\alpha_0, \beta_0)} = 1 $$

where $P = 2\pi \sqrt{GM_*/a^3}$ is the orbital period, $L = \sqrt{GM_*/a}$ is the circular angular momentum at $a$, $J_a = \sqrt{GM_*/a}$ is the circular angular momentum of the loss-cone ($J_a \approx 1.2GM_*/R_*$) for tidal disruption, so $J_a / J_{\odot} \approx (a/r_{\odot})^2$, where $Q = M_*/M_\odot$. $J_a$ is the two-body (non-resonant) relaxation timescale:

$$ T_a(a) = \frac{5}{8} \frac{Q^2(a)}{N_a(a)\log(Q)} $$

where $N_a(a) = \mu_{aQ} / J_a r_{\odot}^{3-a}$ is the number of stars enclosed in $r$, and $r_{\odot} = \eta GM_*/\sigma_*$ is the radius of influence. The numeric prefactors are conventionally assumed to be $\mu_a = 2$ and $\eta = 1$. We further assume that $a < r_{\odot}/4$.

The exact solution for the critical radius can be written by the implicit equation

$$ a_* / r_{\odot} = \frac{1}{\Lambda_0 \left(1-4\alpha_0\right)} Q^{1/12-3\alpha_0} $$

where $\Lambda_0 = 5/(4\mu_a \Lambda_0 r_{\odot})$ and

$$ \Lambda_0(Q, a) = \log Q / \log(J_a(a) / J_{\odot}) $$

where the last equality in equation (13) assumes the $M_*/\sigma_*$ relation in terms of the dimensionless velocity dispersion scale $s = (M_*/M_\odot)^{1/\alpha_0} / \sigma_*$, where $\sigma_a = \sqrt{GM_*/R_\odot}$.

When $a_0 > r_{\odot}$, the rate is estimated at $r_{\odot}$. The transition occurs above a critical black hole mass such that $a_0(R_\odot) = r_{\odot}$.

$$ M_\odot = M_0 \left(\frac{\sigma_a^2}{\sigma^2} \right)^{-3/2} = M_0(A_\odot \sigma^2)^{-3/2} $$

where $A_\odot$ is the orbital period, the black hole mass at any finite time is generally unrelated to either $M_0$ or $M_\odot$. The asymptotic exponential solution ($b < 1$) can be written as $M_0 = M_\odot (1 - b^{1/2})^{-1}$.

In this case, $M_\odot$ provides a reasonable approximation for $M_0$ as long as $b \ll 1$. This is indeed the case for the empirical universal $M_*/\sigma_*$ relation, where $b = (\alpha/4) / (\beta + 1) = 0.125$. However, other combinations of cusp and $M_*/\sigma_*$ indices can lead to arbitrarily large disparities: for example $b = (\alpha/3) / (\beta + 0.3) = 0.6$, results in $M_0 \approx 0.1M_\odot$. It should be emphasized that the solution branch that describes the black hole mass is not determined solely by the assumed growth channel — tidal disruptions in this case — but also by the choice of boundary conditions, which here are determined by an empirical relation. Other possible values of $b$ and $M_*/\sigma_*$ would imply very different relations between $M_0$ and $M_\odot$ for the transition to the exponential and super-exponential solutions ($b > 1$) occurs for $\beta \leq 2.1$ (for $a = 1/4$) or for $\beta \leq 3$ (for $a = 1/2$). Therefore, it is not generally true that $M_\odot$ estimates the black hole mass. Its relevance depends on the specific solution and on the adopted boundary conditions, and cannot be assumed a priori.

A simpler power-law approximation can be obtained by choosing this typical value for the logarithmic term $\Lambda_0 \approx 2$. Then, $\gamma = \gamma(Q^2)$ (see above), where the normalization $\gamma(Q^2) = \gamma(Q^2) = \int_0^\infty \frac{1}{Q^2} \frac{Q^2}{N_\odot} \log(Q)$

where $N_\odot$ denotes the weak functional dependence on $Q$ via the logarithmic term $\Lambda_0 \approx 2$. The index is

$$ b = \frac{7(3-a)}{3(4-a)} \left(15/2 - a/3\right) / \beta $$

$\gamma(Q^2)$ is the Eddington limit) at $L_{\odot} = 10^3 M_\odot$. For $\beta = 5$, the luminosity $L_{\odot} = 10^3 M_\odot$. Depending on the exact value of the logarithmic slope of the $M_*/\sigma_*$ relation, $\beta$, the TDE rate on the low-mass branch can either rise or fall with $M_\odot$.

Data availability. The numerical results that support the plots within this paper and other findings of this study are available from the corresponding authors upon reasonable request.

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**Author contributions**

T.A. and B.B.-O. developed the ideas presented in this paper together and collaborated in its writing.

**Additional information**

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**Competing interests**

The authors declare no competing financial interests.