A novel tool for damage detection of an engineering structure based on the load path analysis using \( U^* \) index is developed with high sensitivity. \( U^* \) is a math index presentation of the internal stiffness distribution of a structure subjected to a certain load. The global variation of the stiffness distribution will occur due to the damage and can be measured by \( U^* \) index as a new damage detection indicator. \( U^* \) index theory is introduced first in the paper, and the damage detection mechanism and feasibility are then described, explained and studied through two sample structures by finite element analysis (FEA). Through the case studies, it is noted that good damage sensitivity can be realized even when the measurement position is far (30% of the whole structure size) away from the damage. The experimental study further validates the effectiveness and feasibility of \( U^* \) measurement in damage detection. The \( U^* \) index can be used as a new indicator for structural damage detection at an arbitrary measurement location with higher sensitivity than the stress analysis, which is only sensitive when the testing point is very close to the damage.

**INDEX TERMS** \( U^* \) index theory, load path analysis, static measurement, damage detection.

**I. INTRODUCTION**

Structural stiffness reduction due to damages (such as cracks and notches) and connection loss are inevitable over the serving lifespan of the engineering structures because of various loading conditions and environmental factors, like temperature change. It is crucial to develop reliable and effective damage detection techniques in order to inspect the structural health. The existing methods for damage detection can be generally categorized into dynamic identification methods and the static identification methods. Dynamic identification methods have been extensively studied by researchers over the past decades. Most of the dynamic approaches are based on the vibration data to estimate the damage-sensitive parameters, such as natural frequencies, mode shapes and modal strain energy [1]. Back in the 1970s, Cawley and Adams [2] came up with the idea of locating the structural defects from the change of natural frequencies. However, the change of vibration characteristics is typically too tiny to sense the structural damages. Smart materials, such as piezoelectric materials, have been applied to enrich the damage data. Islam and Craig [3] introduced the damage detection method in composite structures with the application of embedded piezoelectric sensors and actuators. Jiang et al. [4], [5] proposed another method to enlarge the damage-induced frequency-shift by using a tunable piezoelectric transducer circuitry. More recently, piezoelectric materials were employed by Zhao et al. [6], [7] to alter the vibration mode shapes of beams via feedback scan-tuning. Frequency-shift was significantly magnified while the piezo-patch at the location of the damage was activated, giving it the potential of locating the damage. In addition to the applications of smart materials, some other algorithms and approaches (e.g., transfer matrix method [8], [9], wavelet transform [10], [11], and entropy [12]–[14]) were also introduced to help extract the damage features from the dynamic signals.

While dynamic methods are catching plenty of attention, limitations of them have also emerged. Wang et al. [15] noticed some limitations in practical application by using dynamic approaches: (1) no damping or constant damping is assumed in most of the dynamic identification methods, which can affect the detection accuracy. (2) It is challenging...
to conduct modal analysis on complex structures as it requires numerous sensors to acquire the accurate mode shapes. (3) Dynamic data from higher-order vibration modes are difficult to be extracted from real-life measurements. Some researchers hence move forward to develop alternative methods based on the static test data. Sanayei and Nelson [16], [17] presented an analytical method to identify the damage-induced reduction of structural element stiffness by static displacement measurements. Banan et al. [18], [19] developed a recursive quadratic programming method to solve the optimization problem that minimizes the force-error estimator or displacement-error estimator. An adaptive parameter grouping scheme was then introduced by Hjelmstad and Shin [20] to localize the structural damage with sparse measurements. Yam et al. [21] found that deflection curvature is the most sensitive parameter in terms of static analysis in damage detection of plate-like structures. Chen et al. [22] utilized the grey system theory to successfully identify the damages from a cantilever beam by using limited static displacement data. Yang and Sun [23] proposed a static-based method by means of a new flexibility disassembly technique to estimate both the location and extent of the damage. Tian et al. [24] detected multiple damages in a wind turbine blade based on fiber Bragg gating. Andreans et al. [25] used wavelet transform and to process the static deflection response from image data. A simply supported beam with multiple cracks was used in the experiment to verify the method. Delamination of the beam was also spotted by Wang and Wu [10] based on the wavelet transform of the deflection data from the laser sensor. Liu et al. [26] detected the local damage and debonding in concrete structures simultaneously from noisy static measurements. Le et al. [27] developed another damage detection method for beam-type structures based on the static deflection change due to the damage without relying on refined finite element models. While the measurements of the static response in the structures are relatively simple and straightforward, there are still three major drawbacks in static-based methods [22]: (1) static response provides less information than the dynamic response which might produce unexpected difficulty and errors in damage detection; (2) the damage effect on the static response is insignificant in the elements that involve limited load transfer; (3) most of the current static testing methods and data only indicate local damage information, and information from the global view is missing.

In the field of structural design, the strength of the structure can be evaluated by load path analysis, which presents a global structural behavior under a static loading condition. Since the internal force is invisible in a structure, tracking the load transfer path has been a tough issue for decades [28]. While stress is usually used to express the local internal force, the load transfer route might be false due to the stress concentration [29]. To address this issue, the concept of internal stiffness was first introduced by Takahashi [30]. Based on this concept, the major force flow in the structure can be identified by the members with the highest internal stiffness to the loading point. Later on, Shinobu et al. introduced the $E^*$ index that can depict the load path and examine internal stiffness in the structures [31], followed by the introduction of $U^*$ index, which is a normalized derivation of $E^*$ index [32]. Since the load path analysis is finalized by iteratively reconstructing the finite element model, a significant amount of computational power and time is necessary. Hence Sakurai et al. developed a load inspection method that reduces over 90% of calculation time for $U^*$ distribution [33]. With the improvement of computer performance, $U^*$ index theory has been commonly applied in the structural design of aircraft and ground vehicles to improve their structural strength and weight efficiency [34]–[36].

It is noted that the load is mainly transferred to the supporting points through the path with higher stiffness in a structure. The $U^*$ index analysis provides a clear structural stiffness distribution relative to certain loading conditions and hence can indicate the corresponding load path. It brought us the idea that the $U^*$ index can be used to evaluate the structural stiffness reduction due to potential damages. While any structural damages appear on the main load path of the structure, they will cut off the original path and obviously affect the global internal stiffness distribution throughout the whole structure. Unlike local stress concentration induced by structural damage, when the main load path is changed due to the damage, the stiffness variation even at an arbitrary location far away from the damage can be noticeable. In this case, the $U^*$ index of an arbitrary test point can be changed obviously leading to a higher damage detection sensitivity compared with the traditional stress measurement. Furthermore, the $U^*$ based damage detection can be flexibly applied to structures with various loading or boundary conditions.

Inspired by $U^*$ index theory, this paper introduces a novel static damage detection method by load path analysis for the first time. In this paper, the $U^*$ index theory is briefly reviewed first. The mechanism of the proposed method is then explained in detail. The effectiveness of this method is examined by a cantilever beam and a rectangular plate based on the finite element method. By selecting a proper load path, the damage can be potentially detected from a testing point that is far from the damage location with over 10% change of $U^*$ index. An experimental study of a rectangular plate under in-plane loading is also conducted. The advantages, limitations, and potentials of the new damage detection method are concluded at the end of the paper.

II. $U^*$ INDEX THEORY

The concept of $U^*$ index theory was first introduced by Takahashi [32]. Since the applied forces tend to pass through the parts of a given structure with high internal stiffness, the distribution of $U^*$ index, which has a close relationship with the internal stiffness relative to the applied load, is capable to indicate how the forces transfer in the structure. The internal stiffness here is defined as an elastic constant between the loading point and any arbitrary point in the structure. A linear elastic structure is shown in Figure 1. (a) with load at point A
and restraint at point B, while point C is an arbitrary point in the structure. The structure can be then represented by three linear springs between any two points in the structures without considering rotational degrees of freedom (DOFs). The relationship between load and displacement of the system can be expressed as:

\[
\begin{bmatrix}
    P_A \\
    P_B \\
    P_C
\end{bmatrix} =
\begin{bmatrix}
    K_{AA} & K_{AB} & K_{AC} \\
    K_{BA} & K_{BB} & K_{BC} \\
    K_{CA} & K_{CB} & K_{CC}
\end{bmatrix}
\begin{bmatrix}
    d_A \\
    d_B \\
    d_C
\end{bmatrix},
\]

(1)

where \( P_i \) \((i = A, B, C)\) and \( d_i \) \((i = A, B, C)\) represent the applied loads and displacements at the corresponding points, and \( K_{ij} \) \((i = A, B, C, j = A, B, C)\) is the internal stiffness between denoted points. Since point B is fixed, the displacement of point B is defined as:

\[
d_B = 0.
\]

(2)

Therefore, the force-displacement relationship of point A can be written as:

\[
P_A = K_{AA}d_A + K_{AC}d_C,
\]

(3)

where \( K_{AA} \) can be described by the rigid translation (1995):

\[
K_{AA} = -(K_{AB} + K_{AC}).
\]

(4)

Due to the external load applied at point A, the total strain energy in the system can be expressed as:

\[
U = \frac{1}{2} P_A \cdot d_A.
\]

(5)

After substituting \( P_A \) from (3) into (5), the total strain energy will be:

\[
U = \frac{1}{2} (K_{AA}d_A + K_{AC}d_C) \cdot d_A.
\]

(6)

In the second case of Figure 1 (b), the arbitrary point C is restrained. The displacement of point C hence becomes zero. To result in the same displacement at point A, load \( P_A' \)

![FIGURE 1. Sample structure for \( U^* \) analysis (a) Original structure; (b) Modified structure [32].](image-url)

![FIGURE 2. Flow chart of \( U^* \) fast calculation.](image-url)

is imposed at point A. Thus, the total strain energy of the modified system will be:

\[
U' = \frac{1}{2} P_A' \cdot d_A.
\]

(7)

By using the value of \( U \) and \( U' \), the \( U^* \) index is defined as [32]:

\[
U^* = 1 - \frac{U}{U'} = [1 - \frac{U}{U - U'}]^{-1}.
\]

(8)

After substitution of (6) and (7), another expression of \( U^* \) related with \( K_{AC} \) can be obtained:

\[
U^* = [1 - \frac{2U}{(K_{AC}d_C) \cdot d_A}]^{-1}.
\]

(9)

(9) shows that \( U^* \) can be used as the index representing the internal stiffness between the loading point A and point C in the given structure. The flowchart of \( U^* \) calculation of the whole structure can be found in Figure 2. After calculating the \( U^* \) distribution, it can be drawn into a contour graph as shown in Figure 3. The value of \( U^* \) ranges from 1 at loading point A to 0 at fixed point B. The main load path with the smallest \( U^* \)
index gradient can be found by drawing the contour graph of the $U^*$ distribution. To define the main load path, a vector $\lambda$ called “stiffness decay vector” is defined as [34]:

$$
\lambda = -\text{grad } U^*.
$$

Then the main load path (the solid line in Figure 3) along the smallest $\lambda$ value can be obtained indicating the load transfer inside the structure.

## III. DAMAGE DETECTION BY $U^*$ INDEX

As the presence of structural damages (like cracks) leads to stiffness degradation, the overall $U^*$ distribution will be changed accordingly. It is noted that damages are most likely to occur on the main load path of a structure, where the greatest internal force is passing by. When a damage occurs on the main load path, the load path of the damaged structure will be shifted from the original route due to the local stiffness reduction and cut-off of the original load path by the damage. Meanwhile, the $U^*$ index on the original load path should change obviously even at the location far away from the damage position. Figure 4 shows the diagrams of the load paths shifting due to the damage. It is noted that with the shift of the entire load path, the damage will lead to the variation of the $U^*$ index along the whole original load path but not only at the damage location. This is the advantage of using $U^*$ index as a new damage detection tool compared with the stress/strain measurement, which provides the local structural behavior. To determine the existence of the damage at the critical locations of the structure (on the main load path), $U^*$ distribution of the healthy structure is calculated first to find out the original main load path. Following the route of the original main load path, the values of $U^*$ index on the same path of the damaged structure are then calculated. The structural damages are to be identified by comparison of the $U^*$ index on the selected load path between the healthy structure and damaged structure.

## IV. CASE STUDIES

In this section, two sample structures with notch damages are modeled with plane elements and analyzed on ANSYS Mechanical APDL R19.2. Material properties of aluminum alloy (Young’s modulus = 69 GPa, Poisson’s ratio = 0.3) are applied in the finite element models. The output data is transferred to Matlab 2018b for further processing and figure plotting. To better illustrate the advantages of the $U^*$ index method, von Mises stress distributions of these two sample structures are also provided for comparison.

### A. CANTILEVER BEAM WITH NOTCH DAMAGE

A cantilever beam, with a length of 200 mm, a width of 20 mm and a thickness of 5 mm is studied first. Two pre-defined notches are located at the fixed end and 75 mm from the fixed end as shown in Figure 5 (b) and Figure 5 (c), respectively. Both notches are 2-mm deep and 5-mm wide. An upward load of 100 N is applied at the free end of the beam. Nodal results of von Mises stress and $U^*$ index on the lower surface ($y = 0$ mm) of the beams are extracted for the following comparison.

In the first case, when the notch is located at the fixed end, the stress is clearly higher than the healthy beam at the position of the notch as shown in Figures 6 (a) and (b). However, there is no significant stress difference on the rest of the beam. For comparison, Figures 6 (c) and (d) show the distribution of $U^*$ on both healthy and damaged beams. It is noted that the values of $U^*$ of the whole damaged beam especially in the section closer to the beam fixed end are distinctly higher than the healthy beam. It implies that the relative internal stiffness between any point of the beam and the loading point increases with the reduced local stiffness at
FIGURE 6. Stress analysis and $U^*$ analysis of the healthy beam and the damaged beam with a notch at the fixed end. From Figure 6 (d), even at $x = 40$ mm, where is far away from the notch, the $U^*$ measurement can still sense the damage with a variation of 26% from the healthy one.

In the second case, while the notch is located at 75 mm from the fixed end, the change of von Mises stress at the notch is also noticeable as shown in Figures 7 (a) and (b), whereas the change is minor on other locations of the beam. Different
from the first case, as shown in Figures 7 (c) and (d), the values of $U^*$ are lower in the left half section of the beam compared to the results of the healthy one. However, the values of $U^*$ of the damaged beam surges from $x = 75$ mm, and it surpasses the healthy beam at $x = 80$ mm, which is the edge of the notch. The reason is that the internal stiffness of the left beam section to the loading point is decreased by the isolation of the notch, while the right beam section takes a more important role holding the load. As shown in Figure 7 (d), at the location of $x = 40$ mm, a $-9.32\%$ of $U^*$ shift induced by the notch can be noticed. In fact, $U^*$ at any point in the left beam-section drops by over 8% after the presence of the notch, which could be a prominent indicator for damage detection.

**B. RECTANGULAR PLATE WITH NOTCH DAMAGE**

The second sample structure is a 3 mm-thick rectangular plate with a length of 100 mm and a width of 60 mm as shown in Figure 8. A vertical notch with 40 mm-length, 2.5 mm-width, and 2 mm-depth is located at 15 mm away from the left side of the plate. The plate is constrained at two points on the left edge which are spaced 20 mm apart, while a tensile load is applied at the center of the right edge.

The von Mises stress distribution of healthy and damaged plates is shown in Figure 8 (a). The stress concentration can be clearly seen in the area of the notch, but stress concentration is not clear in the rest area of the plate. Instead, from Figure 8 (b), the global change of the $U^*$ distribution is more noticeable, as the internal forces disperse around the notch, which induces the main load path to deviate from the original route. To further study the exact damage effect on the plate behavior, the nodal results of both von Misses stress and $U^*$ index on the centerline of the plate ($y = 30$ mm), where is close to the original main load path of the healthy plate, are extracted.

As can be seen in Figures 9 (a) and (b), the nodal stress changes the most at the location of the notch by 182.7% compared with the healthy one, while the stress variation is very small at other locations with a change of only 1% at $x = 25$ mm for example. By contrast, as shown in Figures 9 (c) and (d), the $U^*$ distribution on the original load path shows a more obvious difference between the damaged plate and the healthy plate. It is worth noting that the signs of damage induced $U^*$ index change are altered from negative ($-6.05\%$ at $x = 5$ mm for example) on the left side of the notch to positive on the right side ($+7.34\%$ at $x = 25$ mm for example). This finding is similar to the results from the beam...
V. EXPERIMENT VALIDATION

Based on the testing equipment available in our lab, it will be much easier to prepare the small fixtures applying considerable tensile load on the plate structure as studied in the simulation. Considering that the theory behind the load path-based damage detection on different structures is the same, only the test of the plate structure under tensile load was performed to prove the feasibility of the proposed method. Hence, a pair of polylactic acid (PLA) plates are fabricated by 3D printing as shown in Figure 10. One of the plates has a notch with a thickness of 2 mm to simulate the damaged plate in section 4.2, while the other one is intact. Four through holes at Test No.1-4 are drilled on both the damaged and the healthy plates for adding constraints with bolts and nuts. To avoid material failure around the constrained and loading points, washers are employed to distribute the load of the nuts to the plate material. Compared with the simulated plates in section 4.2, the fixed points and the loading point of both plates are strengthened with additional materials to avoid material failure during the tensile tests. A set of stainless-steel fixtures is also designed and built to properly hold and apply loads on the plates.

Theoretically, the $U^*$ value at a specific point can be calculated by (8), where $U$ is the original strain energy in the system, and $U'$ is the strain energy of the modified structure with an extra constraint at the selected point, while the same displacement is applied on the loading point. However, in real-world testing, with the same displacement applied to the loading point, the materials have a higher chance to yield. To simplify the experiment, the expression of $U^*$ can be written as (11), while the structure is under perfectly elastic deformation,

$$U^* = 1 - \frac{U}{U'} = 1 - \frac{1}{2}k\cdot d^2_{A} = 1 - \frac{k}{k'}, \quad (11)$$

where $k$ and $k'$ are the stiffness of the original structure and the modified structure with constrained test/measure point.
as described by (7). The stiffness can be obtained from the linear regression of the force-displacement curves at the loading point. The force-displacement data are captured by the MTS Insight electromechanical test system as displayed in Figure 11. The stiffness of the plate with original boundary conditions is tested first to be the benchmark for the following tests. Then, the 4 holes (Test No.1-4) are constrained in turns for the measurements of the stiffness of the modified structures so as to obtain the $U^*$ values at different test points. For example, Figure 12 (a) depicts a modified healthy plate with Test No.2 fixed. To avoid the material failure of the tested specimen, the maximum load is limited at 400 N. The values of $U^*$ from the tests are provided in Figure 13 (a). Since the notch is located between Test No.1 and No.2, $U^*$ of the damaged plate at Test No.1 is lower than the one of the healthy plate, but it surpasses the healthy plate at Test No.2, which matches the simulation results in Figure 9 (c). Figure 13 (b) shows the $U^*$ differences between the notched plate and the healthy plate, where differences of -13.61% and +27.03% appear at Test No.1 and Test No.2 respectively. Even at the position of Test No.4, which is 30 mm away from the notch, there is still 4.6% $U^*$ variation due to the notch. Overall, the feasibility and effectiveness of the load path analysis and $U^*$ measurement in damage detection are validated by the experiments, regardless of some differences in $U^*$ values compared with the simulation, which could result from the profile modification of the 3D printed specimens, non-linearity of the PLA materials and the errors from the load cell of the tensile test machine.

Other than the experimental process conducted in our research, on an even larger mechanical part, a much faster testing process can be done by restricting the structural boundaries as well as the loading point and applying unit load on arbitrary locations of the structure measuring the $U^*$ index [33]. However, with the limitation of current available tensile testing machine and materials, we do not have a large enough base area and fixture to handle this test. The further
convenient and practical design of the experimental test can be an important and meaningful future work.

VI. CONCLUSION
This work introduces $U^*$ index as a novel and powerful damage detection tool with higher sensitivity compared with traditional stress analysis when the measurement location is away from the damage. The mechanism of this new damage detection tool is illustrated in detail first, followed by case studies on two different structures based on FEA. The results from the case studies reveal that good damage sensitivity can be realized (with >5% $U^*$ index change compared with the healthy case) even if the measurement position is far (30% of the whole structure size) from the damage. Its feasibility and effectiveness are further proven by the experiment of a rectangular plate with notch damage. In summary, two major advantages of the proposed damage detection tool are: (1) the global stiffness reduction due to the structural damages can be revealed by the change of $U^*$, which can be easily acquired from static measurement; (2) this tool is applicable to beam-type structures and plate-type structures. Future studies of the $U^*$-based damage detection tool includes multi-crack detection and simplification of the measurements in practical applications.

REFERENCES
[1] S. Das, P. Saha, and S. K. Patro, “Vibration-based damage detection techniques used for health monitoring of structures: A review,” J. Civil Struct. Health Monitor, vol. 6, no. 3, pp. 477–507, Jul. 2016.
[2] P. Cawley and R. D. Adams, “The location of defects in structures from measurements of natural frequencies,” J. Strain Anal. Eng. Design, vol. 14, no. 2, pp. 49–57, Apr. 1979.
[3] S. Zhao, N. Wu, and Q. Wang, “Detection of the delamination location of a beam with a wavelet transform: An experimental study,” Smart Mater. Struct., vol. 20, no. 1, Jan. 2011, Art. no. 012002.
[4] T. Jiang, Q. Kong, D. Patil, Z. Luo, L. Huo, and G. Song, “Detection of debonding between fiber reinforced polymer bar and concrete structure using piezoceramic transducers and wavelet packet analysis,” IEEE Sensors J., vol. 17, no. 7, pp. 548–561, Sep. 2017.
[5] B. Wimashana, N. Wu, and C. Wu, “Application of entropy in identification of breathing cracks in a beam structure: Simulations and experimental studies,” Struct. Health Monitor., vol. 17, no. 3, pp. 549–564, May 2018.
[6] X. Wang and N. Wu, “Crack identification at welding joint with a new smart coating sensor and entropy,” Mech. Syst. Signal Process., vol. 124, pp. 85–92, Jun. 2019.

FIGURE 13. Experiment results of $U^*$ analysis: (a) $U^*$ distributions on the selected test points of the healthy plate and damaged plate; (b) difference of the $U^*$ values between the healthy plate and the damaged plate.
[32] T. Sakurai, J. Tanaka, A. Otani, C. Zhang, and K. Takahashi, “Load path optimization and U structural analysis for passenger car compartments under frontal collision,” in Proc. SAE Tech. Paper Ser., Oct. 2003, pp. 148–191.

[33] T. Sakurai, K. Takahashi, H. Kawakami, and M. Abe, “Reduction of calculation time for load path U* analysis of structures,” J. Solid Mech. Mater. Eng., vol. 1, no. 11, pp. 1322–1330, 2007.

[34] E. Wang, Y. Yoshikuni, Q. Guo, T. Nohara, H. Ishii, H. Hoshino, and K. Takahashi, “Load transfer in truck cab structures under initial phase of frontal collision,” Int. J. Vehicle Struct. Syst., vol. 2, no. 2, pp. 60–68, 2010.

[35] E. Y. Wang, T. Nohara, H. Ishii, H. Hoshino, and K. Takahashi, “Load transfer analysis using indexes U and U for truck cab structures in initial phase of frontal collision,” Adv. Mater. Res., vols. 156–157, pp. 1129–1140, Oct. 2010.

[36] Q. Wang, G. Zhang, C. Sun, and N. Wu, “High efficient load paths analysis with U* index generated by deep learning,” Comput. Methods Appl. Mech. Eng., vol. 344, pp. 499–511, Feb. 2019.

SHENGJIE ZHAO received the M.Sc. degree from the University of Manitoba, in 2016, where he is currently pursuing the Ph.D. degree. He worked as an Engineer-in-Training at Kingsman Industries, from 2017 to 2018. His research interests include structural health monitoring, load path analysis, and machine learning in engineering applications.

NAN WU received the Ph.D. degree from the University of Manitoba, in 2012. He is serving as an Associate Professor of mechanical engineering with the University of Manitoba. His research interests include mechanical vibrations, smart materials, and structural health monitoring and enhancement.

QUAN WANG received the Ph.D. degree in solid mechanics from Beijing University, in 1994. He is currently serving as a Chair Professor with Shantou University. His research interests and expertise are in the fields of damage detection, smart materials and structures, energy harvesting, and nano-technology. He is a Fellow of the Royal Society of Canada and the Canadian Academy of Engineering.

* * *