Braneworld inflation with effective $\alpha$-attractor potential

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Abstract

In this paper, we study inflation with $\alpha$-attractor potential in the RS braneworlds where high energy corrections to the Friedmann equation facilitate slow roll. In this scenario, we numerically investigate the inflationary parameters and show that the high energy brane corrections have significant effect on the parameter $\alpha$, namely, the lower values of the parameter are preferred by observation in this limit. We find that the sub-Planckian values of the field displacement can be achieved by suitably constraining the brane tension.

1 Introduction

In the standard framework, a slowly rolling scalar field $a la$ a shallow field potential may account for inflation\cite{1}. Slow roll along a steep potential is also possible due to Hubble damping caused by high energy brane corrections. Indeed, in brane world scenario, our four dimensional space time dubbed brane is supposed to be embedded in a higher dimensional bulk\cite{2,3} such that the Einstein equations on the brane are modified. The corresponding Friedmann equation includes an extra term\cite{4,5} quadratic in density which facilitates slow roll in high energy regime at early times even in case of a steep potential\cite{6,7,8,9}. Thus, brane world scenario allows steep potential to support inflation\cite{10,11,12} which is not possible in standard case.

In the standard cosmology, an exponential potential\cite{13} does not give viable inflationary and post inflationary behavior. Situation changes significantly in the brane-world case\cite{10,11,12}. However, the steep braneworld inflation gives a ratio of tensor to scalar perturbation, $r$, around 0.4 for 60 e-folds of inflation\cite{12,14} which is not tenable observationally\cite{15,16}. Similar problem in the standard cosmology can successfully be addressed in the $\alpha$-attractor scenario\cite{17,18,19,20,21}. In this framework, the kinetic term in the Lagrangian has a specif non canonical form. This kind of kinetic term can be realized through a specific form of Kähler potential in supergravity\cite{17}. Canonicalization of such term gives rise to some flat regions or plateaus in the potential\cite{19,20,21} which are suitable for the study of inflation favored by observational data\cite{15,16}. This feature can also be suitable for the study of late time behavior, namely, quintessence\cite{20,22,23,24,25,26,27}.

It should be mention here that a super-Planckian displacement of the scalar field may spoil the flatness of quintessential region of the potential and may generate an wanted fifth force problem\cite{20,28}. On the other hand, it is impossible to evolve to quintessence starting from the inflationary region without invoking super-Planckian values of the field and not making the potential too curve during inflation\cite{29}. The $\alpha$-attractor solves this problem, namely, the canonicalization of the
potential makes it possible for the canonical scalar field to have a super-Planckian excursion while keeping its canonical counterpart under sub-Planckian.

In view of the aforesaid, we are led to consider the \( \alpha \)-attractor construct in the framework of RS brane worlds\(^3\) which might give new insights related to the sub-Planckian nature of non-canonical field. In section (2), we discuss how we obtain our effective \( \alpha \)-attractor potential and suggest approximations to check for analytical behavior. In section (3), we put the effective potential on brane and perform a full numerical study of different parameters related to inflation. Numerical analysis is done because of complexity in solving the problem analytically. In this section, we also show some important features our model exhibits. Next, in section (4) we show some approximated analytical results for our model. Next, in section (5) we compare our results with current observational bounds and with the results obtained, we constrain our different model parameters especially the parameter \( \alpha \). Next, in section (6) we briefly discuss the late time behavior followed by conclusions in section (7).

\section{The effective \( \alpha \)-attractor model}

Considering the formal \( \alpha \)-attractor Lagrangian density in 4-dimensional space-time\(^{17, 18, 19, 20, 21}\)

\[ L = \frac{1}{2} \left( \frac{\partial \phi}{\sqrt{1 - \frac{\phi^2}{6 \alpha m_p^2}}} \right)^2 + V_0 e^{-\kappa \phi / m_p}, \]  

where \( \alpha > 0 \) is a parameter featuring a pole in the kinetic energy, \( m_p = \frac{1}{\sqrt{8 \pi G}} \) is the 4-dimensional reduced Planck mass, \( G \) is the Newton’s constant, \( \kappa \) is the parameter determining the steepness of the potential, \( V_0 \) is a constant with the dimension of energy density. The modulus value of \( \phi \) will remain less than \( \sqrt{6 \alpha} m_p \) for any finite value of \( \alpha \) because the kinetic energy becomes singular at this value. This allows the scalar field to remain under sub-Planckian values as long as \( \alpha \lesssim \frac{1}{6} \). The same theory can be described in terms of a canonicalized inflaton field \( \varphi \) related to the non-canonical scalar field \( \phi \) via the transformation

\[ \phi = \sqrt{6 \alpha} m_p \tanh\left( \frac{\varphi}{\sqrt{6 \alpha} m_p} \right). \]  

From equation (2), it is clear the canonical field \( \varphi \) can take any value, keeping the non-canonical \( \phi \) sub-Planckian. By this transformation the potential given in equation (1) is described now in terms of the canonical field of the form

\[ V(\varphi) = V_0 e^{-\kappa \sqrt{6 \alpha} \tanh\left( \frac{\varphi}{\sqrt{6 \alpha} m_p} \right)}. \]  

This potential corresponds two plateaus, see Fig. 1. The inflationary regime is featured by a plateau corresponds to \( \phi \rightarrow -\sqrt{6 \alpha} m_p \), or equivalently by \( \varphi \rightarrow -\infty \), the other plateau is featured by \( \phi \rightarrow \sqrt{6 \alpha} m_p \) or equivalently by \( \varphi \rightarrow \infty \), featuring quintessence. For the inflationary limit potential (3) becomes
where $M^4 = V_0 e^{\kappa \sqrt{6 \alpha}}$, is a constant representing the energy scale for inflation, $M$ has dimension of mass, $M_{Pl} \equiv m_p/\sqrt{8 \pi}$ is the reduced 4-dimensional Planck mass and $n \equiv \kappa \sqrt{6 \alpha}$.

3 The effective $\alpha$-attractor potential in braneworld scenario

We place our effective potential on the Randall Sundrum II(RSII) brane\textsuperscript{[3]} to study the inflationary scenario. The matter fields are confined to the brane only for RSII model, so our scalar field will remain on the brane only. For a flat Friedmann-Lemaître-Robertson-Walker(WR) background on the brane with zero 4-dimensional cosmological constant, the Friedman equation becomes\textsuperscript{[4, 10, 11, 14]}

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8 \pi}{3 M_{Pl}^2} \rho \left( 1 + \frac{\rho}{2 \lambda} \right),$$

where $a$ is the scale factor, $H$ is the Hubble parameter, $\rho$ is energy density of matter field on the brane, $\lambda$ is 3-brane tension relating the 4-d Planck mass, $M_{Pl}$ with 5-d Planck mass $M_5$ via

$$\lambda = \frac{3}{4 \pi} \frac{M_5^6}{M_{Pl}^2}. \quad \text{(6)}$$

For high energies, $\rho^2$ term become significant and plays a crucial role in the dynamics of the scalar field hence of the universe. The scalar field or the inflaton field $\phi$, satisfies the Klein-Gordon equation

$$\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0. \quad \text{(7)}$$

$V(\phi)$ is the potential driving the inflation. The prime denotes a derivative with respect to $\phi$. The presence of quadratic term $\rho^2$ enhances the value of Hubble parameter (5) and hereby gives extra friction to the scalar field (7) and makes its evolution slower. Combining equations (6) and (7), one gets the evolution equation\textsuperscript{[6, 13]}

$$\frac{\ddot{a}}{a} = \frac{8 \pi}{3 M_{Pl}^2} \left[ (V - \phi^2) + \frac{\dot{\phi}^2 + 2V}{8 \lambda} (2V - 5 \phi^2) \right]. \quad \text{(8)}$$
The inflationary condition $\dot{a} > 0$, is reduced to standard form $V > \dot{\varphi}^2$ for $\frac{\dot{\varphi}^2 + 2V}{8\lambda} \ll 1$. In the high energy scenario, the condition becomes $2V > 5\dot{\varphi}^2$. This condition may be used for characterizing end of inflation [14], $2V(\varphi_{end}) \simeq 5\dot{\varphi}_{end}^2$. Using the slow roll approximation ($V \gg \dot{\varphi}^2$, $\frac{3H^2}{\dot{\varphi}^2} \ll 1$) we can write equations (5) and (7) respectively as

$$H^2 = \frac{8\pi}{3M_{Pl}^2} V \left(1 + \frac{V}{2\lambda}\right) \quad (9)$$

and

$$3H\dot{\varphi} + V'(\varphi) = 0. \quad (10)$$

These two equations (9, 10), make the condition for inflation end to be

$$\frac{V^3(\varphi_{end})}{V'(\varphi_{end})} \simeq \frac{5\lambda M_{Pl}^2}{24\pi} \quad (11)$$

The amplitude of scalar and tensor perturbation in RSII inflationary scenario are given as [6, 14, 30]

$$A_S^2 = \frac{512}{75M_{Pl}^6} \left| \frac{V^3}{V'} \left(1 + \frac{V}{2\lambda}\right)^3 \right|_{k=aH} \quad (12)$$

$$A_T^2 = \frac{4}{25\pi M_{Pl}^2} F(x) \bigg|_{k=aH}, \quad (13)$$

where

$$x = H M_{Pl} \sqrt{3/(4\pi\lambda)} \simeq \sqrt{\frac{2V}{\lambda}(1 + \frac{V}{2\lambda})}, \quad (14)$$

$$F(x) = \left[ \sqrt{1 + x^2} - x^2 \sinh^{-1}(1/x) \right]^{-1/2}. \quad (15)$$

The ‘$\simeq$’ is used under slow-roll approximation. Amplitudes $A_S$ and $A_T$ are evaluated at horizon exit, $k = aH$, with $k$ being comoving wave number. The two slow roll parameters on the brane are given by

$$\epsilon \equiv \frac{M_{Pl}^2}{16\pi} \left(\frac{V'}{V}\right)^2 \frac{1 + \frac{V}{\lambda}}{(1 + \frac{V}{2\lambda})^2}, \quad (16)$$

$$\eta \equiv \frac{M_{Pl}^2}{8\pi} \frac{V''}{V} \frac{1}{1 + \frac{V}{2\lambda}}. \quad (17)$$

which indicates that in high energy regime($V/\lambda >> 1$), slow roll is possible even if the potential is steep. The spectral indices of scalar and tensor perturbations are

$$n_S - 1 \equiv \left. \frac{d\ln A_S^2}{d\ln k} \right|_{k=aH}, \quad (18)$$

$$n_T \equiv \left. \frac{d\ln A_T^2}{d\ln k} \right|_{k=aH}. \quad (19)$$

Under slow-roll conditions, we get

$$n_S = -6\epsilon + 2\eta. \quad (20)$$

The number of $e$ folds during inflation is given by

$$N \equiv \int_{t_*}^{t_{end}} H dt, \quad (21)$$

which under slow-roll condition can be written as

$$N \simeq -\frac{8\pi}{M_{Pl}^2} \int_{\phi_*}^{\phi_{end}} \frac{V}{V'} \left(1 + \frac{V}{2\lambda}\right) d\phi. \quad (22)$$
where $\ast$ denotes the value at the horizon exit. We define the ratio of tensor-to-scalar perturbation $r$ as [14]

$$r \equiv 16 \left( \frac{A_T}{A_S} \right).$$  \hspace{1cm} (22)

In the high energy limit $V/\lambda \gg 1$, one finds from equation (15) $F^2 \approx \frac{3V}{2\lambda}$, this together with the slow roll approximation ($\rho \sim V$), and using equations (9), (12), and (13) we get,

$$r = \frac{M^2_{Pl}}{2\pi} \left( \frac{V'}{V} \right)^2 \frac{V/\lambda}{(1 + V/2\lambda)^2} \approx 24\epsilon.$$  \hspace{1cm} (23)

One can easily show that in the low energy limit ($V/\lambda \ll 1$), $r = 16\epsilon$, which is the standard expression.

To study inflation, we start with the potential (4). The condition for inflation end (11) gives

$$3\alpha M^4 M^2_{Pl} \frac{64\pi n^2}{\exp \left[ -8\sqrt{\frac{\pi}{3}} \phi_{\text{end}} \sqrt{\alpha M^4_{Pl}} - 2ne \frac{4\sqrt{\frac{\pi}{3}} \phi_{\text{end}}}{\sqrt{\alpha M^4_{Pl}} + 2ne} \right]} \approx 5\frac{\lambda M^2_{Pl}}{24\pi}$$  \hspace{1cm} (24)

The total number of e-foldings of inflation is given by (21) for high energy limit ($\frac{V}{\lambda} \gg 1$)

$$N \approx \int_{\phi_*}^{\phi_{\text{end}}} \frac{\sqrt{3\pi} \sqrt{\alpha M^4}}{2n\lambda M^2_{Pl}} \exp \left[ -4\sqrt{\frac{\pi}{3}} \frac{\phi}{\sqrt{\alpha M^4_{Pl}}} - 2ne \frac{4\sqrt{\frac{\pi}{3}} \phi}{\sqrt{\alpha M^4_{Pl}} + 2ne} \right] d\phi.$$  \hspace{1cm} (25)

The two slow roll parameters in the high energy limit become

$$\epsilon \approx \frac{16n^2 \lambda}{3\alpha M^4} \exp \left[ \frac{8\sqrt{\frac{\pi}{3}} \phi}{\sqrt{\alpha M^4_{Pl}}} + 2ne \frac{4\sqrt{\frac{\pi}{3}} \phi}{\sqrt{\alpha M^4_{Pl}}} \right]$$  \hspace{1cm} (26)

$$\eta \approx \frac{8n \lambda}{3\alpha M^4} \left( 2ne \frac{4\sqrt{\frac{\pi}{3}} \phi}{\sqrt{\alpha M^4_{Pl}}} - 1 \right) \exp \left[ \frac{4\sqrt{\frac{\pi}{3}} \phi}{\sqrt{\alpha M^4_{Pl}}} + 2ne \frac{4\sqrt{\frac{\pi}{3}} \phi}{\sqrt{\alpha M^4_{Pl}}} \right]$$  \hspace{1cm} (27)

We do solve equations (24), (25) numerically and using (26) and (27) finally we compute $r$ and $n_S$ from the expression equation (23) and (20).

The value of the tensor-to-scalar ratio $r$, is found to be $24/N$, which is $0.4$ for $N = 60$ for the standard brane-world scenario[12, 14], our numerical results show a correction for the tensor-to-scalar ratio. We found that $r$, is depending on the ratio of potential strength $M^4$ to $\lambda$, i.e $\frac{M^4}{\lambda}$ and the parameter $\alpha$, it does not depend on the absolute value of $M^4$ and $\lambda$, which can also be seen in the crude analytical result(see equation(35)). The following numerical result will confirm the fact.

$r = 0.0990187$, for $\frac{M^4}{\lambda} = 100$, $\alpha = 1, \kappa = \sqrt{3}$ for both the value of $M$ equals 0.1 and 0.01 respectively.

Now we will discuss some important results of our analysis one by one.

(i) $N$ vs $r$ From the figure 2a we see that we have a clear improvement for the value of $r$ those compared to the case standard exponential potential on the brane.

(ii) asymptotic value for $\alpha$ In the limit $\alpha \to \infty$, $\alpha$-attractor correction becomes irrelevant[19, 20] and we get usual exponential potential. From the figure 2b it can be seen that $r$ approaches its asymptotic value as we increase the value of the parameter $\alpha$. 


exp. pot on brane

α

attractor pot. on brane

Figure 2: On left, figure (a) shows the variation of $r$ with $N$, the red (dashed) line for normal exponential potential on brane and the blue line (solid) for the exponential potential on brane with $\alpha$- correction for parameters value $M^4/\lambda = 100$ and $\alpha = 1$, on right, figure (b) shows the asymptotic behavior of $r$ as $\alpha$ increases, for $N = 55$ and $M^4/\lambda = 100$.

(iii) It is worth noting that value of $r$ is insensitive to $\kappa$ in the original exponential potential, i.e $n$ for the potential (11) which we found to be same from the result we obtained in analytical approximation.

4 Approximated analytical results

An oversimplified crude approximation for the potential (4) can help us to get an approximate analytical result which we can use as a reference. To do so, we further simplify the potential in the limit $\phi \rightarrow -\infty$ as

$$V(\phi) \simeq M^4 \left[ 1 - 2n \exp \left( \frac{2\sqrt{8\pi}}{\sqrt{6\alpha M_P}} \phi \right) \right].$$

The condition for end of inflation (11) gives

$$\frac{3 \alpha M^4 M_P^2 e^{\frac{s\sqrt{\frac{\phi}{M_P}}}{\sqrt{6\alpha M_P}}}}{64\pi n^2} \simeq \frac{5\lambda M_P^4}{24\pi}.$$  \hspace{1cm} (29)

The Slow roll parameters (26,27) under this approximation become

$$\epsilon \simeq \frac{16\lambda n^2 e^{\frac{s\sqrt{\frac{\phi}{M_P}}}{\sqrt{6\alpha M_P}}}}{3 \alpha M^4 \left( 1 - 2n e^{\frac{s\sqrt{\frac{\phi}{M_P}}}{\sqrt{6\alpha M_P}}} \right)^3} \simeq \frac{16\lambda n^2 e^{\frac{s\sqrt{\frac{\phi}{M_P}}}{\sqrt{6\alpha M_P}}}}{3 \alpha M^4},$$

$$\eta \simeq -\frac{8\lambda n e^{\frac{s\sqrt{\frac{\phi}{M_P}}}{\sqrt{6\alpha M_P}}}}{3 \alpha M^4 \left( 1 - 2n e^{\frac{s\sqrt{\frac{\phi}{M_P}}}{\sqrt{6\alpha M_P}}} \right)^2} \simeq -\frac{8\lambda n e^{\frac{s\sqrt{\frac{\phi}{M_P}}}{\sqrt{6\alpha M_P}}}}{3 \alpha M^4}.$$  \hspace{1cm} (31)
Figure 3: allowed region for $\alpha$ and $M^4/\lambda$ for $r \leq 0.1$

The amplitude of the scalar perturbation\cite{12} is given by

$$A_S^2 \approx \frac{M^{16} \alpha}{25 \lambda^3 M_{Pl}^4 n^2} e^{-\frac{8\sqrt{\frac{\varphi}{\lambda}}}{\sqrt{3} M_{Pl}}} \left(1 - 2n e^{\frac{4\sqrt{\frac{\varphi}{\lambda}}}{\sqrt{3} M_{Pl}}} \right)^6 \left| \frac{M^{16} \alpha e^{-\frac{8\sqrt{\frac{\varphi}{\lambda}}}{\sqrt{3} M_{Pl}}}}{25 \lambda^3 M_{Pl}^4 n^2} \right|_{k=\alpha H} \right). \tag{32}$$

The number of $e$-foldings under this approximation is evaluated to be

$$N \approx \int_{\varphi_{end}}^{\varphi_{end}} \frac{\sqrt{3\pi} \sqrt{\alpha} M^4}{2n \lambda M_{Pl}} e^{-\frac{4\sqrt{\frac{\varphi}{\lambda}}}{\sqrt{3} M_{Pl}}} \left(1 - 2n e^{\frac{4\sqrt{\frac{\varphi}{\lambda}}}{\sqrt{3} M_{Pl}}} \right)^2 d\varphi \approx \int_{\varphi_{end}}^{\varphi_{end}} \frac{\sqrt{3\pi} \sqrt{\alpha} M^4 e^{-\frac{4\sqrt{\frac{\varphi}{\lambda}}}{\sqrt{3} M_{Pl}}}}{2n \lambda M_{Pl}} d\varphi. \tag{33}$$

Using condition of inflation end\cite{29}, we express $N$ in terms of field value at the horizon exit,

$$N \approx \frac{3 \alpha M^4 \left(1 - e^{-\frac{4\sqrt{\frac{\varphi}{\lambda}}}{\sqrt{3} M_{Pl}}} \right)}{8 \lambda n}. \tag{34}$$

Using equation (34) and the fact $r = 24\epsilon$, we find the tensor-to-scalar ratio from equation (30),

$$r \approx \frac{288 \alpha \lambda M^4}{\left(\sqrt{10} \sqrt{\alpha \lambda} M^2 + 4N \lambda \right)^2} \approx \frac{288}{10 \left(1 + \frac{4N}{\sqrt{10} \alpha \sqrt{M^4} M_{Pl}} \right)^2} \tag{35}$$

The amplitude of scalar perturbation\cite{32} is found to be

$$A_S^2 \approx \frac{4M^8 \left(\sqrt{10} M^2 \sqrt{\alpha \lambda} + 4N \lambda \right)^2}{225 \alpha \lambda^3 M_{Pl}^4} \approx \frac{8 \left(\frac{M^4}{\lambda} \right)^2 M^4}{45 M_{Pl}^2} \left(1 + \frac{4N}{\sqrt{10} \alpha \sqrt{M^4 / \lambda}} \right)^2. \tag{36}$$

It should be mentioned here that the relation given by the equations from \cite{28} to \cite{30} do not represent correct dependencies for the respective quantity, we basically keep them as a first hand reference.
Figure 4: 68% (black) and 95% (red) contour regions ($n_S - r$ plane) taken from Planck 2015 results (Planck TT+lowP+BKP+BAO) \cite{15}, we overlay our model’s result on it. We obtain our results by varying $\alpha$ from 0.15 to 10. For both the figure black dot and green dot correspond $\alpha = 5$ and 0.5 respectively and dotted blue line is for $N = 50$ and solid blue line is for $N = 60$. For the left figure, $\frac{M^4}{\lambda} = 100$ and for the right figure, $\frac{M^4}{\lambda} = 20$.

| $\alpha$ | $M$(GeV) |
|---|---|
| 0.16 | $1.86 \times 10^{15}$ |
| 1 | $3.6 \times 10^{15}$ |
| 10 | $1.01 \times 10^{16}$ |

Table 1: Values of $M$ for different $\alpha$, $\lambda$ is taken as allowed by $r$

5 Constraining model parameters from observations

In order to constrain the parameters of our model we stick to our numerical results. Firstly, we find that $r$ is independent of $M$, $\kappa$ and depends only on the ratio $\frac{M^4}{\lambda}$ and $\alpha$.

The observational constraint on the parameter, $r \lesssim 0.1$ \cite{15}, allows us to constrain the parameter $\alpha$ and the ratio $M^4/\lambda$. Theoretically, the high energy limit corresponds $\frac{M^4}{\lambda} \gg 1$, but this ratio is highly dependent on the other parameter $\alpha$ under observational bound. A higher value of the parameter $\alpha$ limits the parameter $M^4/\lambda$ to a lower value. In other words, a decent limit of the assumption that during inflation $M^4/\lambda \gg 1$ pushes $\alpha$ to a lower value. The bound $\alpha \leq 39.6$, in the reference \cite{20} is reduced to $\alpha \lesssim 10$ for a value of $M^4/\lambda \gtrsim 7$. In figure 3 we show the allowed values of $\alpha$ against $M^4/\lambda$. In figure 4 we compare our results for different parameters’ value with Planck 2015 results.

The non-canonical scalar degrees of freedom, $\phi$ remains sub-Planckian as long as $\alpha \lesssim \frac{1}{6}$ \cite{2}. We can obtain this bound in a more compelling way in our model if we consider $\frac{M^4}{\lambda} \gtrsim 350$ along with the observational bound $r \lesssim 0.1$. In other words, the value of $\frac{M^4}{\lambda}$, nearly bigger than 350 will always keep the non-canonical scalar degree of freedom within a sub-Planckian value.

The COBE normalization corresponds to the amplitude of the scalar perturbations (see equation \cite{12} $A_S \simeq 2 \times 10^{-10}$ \cite{31}), which along with the bound on $\alpha$ determines the energy scale of inflation. We found that (Table 1) it is near the grand unification scale, almost same as the one in standard inflationary cosmology. Consequently, after inflation ends, the field will have large overshoot below the background freezing itself for a long time; only at late times it will evolve mimicking cosmological constant like behavior.
Late time behavior

Let us briefly comment on the post inflationary features of the model. First, the brane corrections to Freidmann equation are insignificant in the post inflationary era. Secondly, it is interesting that irrespective of the nature of original exponential potential, the $\alpha$ attractor effective potential, see Fig.1 has a generic form, namely, it has plateau followed by a sharp steep behavior like a water fall settling fast to a constant value thereafter. In this case, the tracker\cite{33, 34, 35, 36} behavior is inherently absent which makes the dynamics of scalar field sensitive to its initial conditions. Actually, the important features of dynamics are encoded in a quantity dubbed $\Gamma$ which for the potential (3) is given by,

$$\Gamma(\phi) \equiv \frac{V''(\phi)}{V(\phi)^2} = 1 + \frac{\sinh \left( \frac{4\sqrt{\frac{\pi}{3}} \phi}{\sqrt{\alpha M_{Pl}}} \right)}{n}$$

Eq.37 tells us that $\Gamma$ increases fast with $\phi$ and crosses one and keeps increasing thereafter facilitating slow roll, see Fig.5. On the other hand, to realize tracker behavior, it is necessary that $\Gamma$ being greater than unity stays close to one for long time such that the field approximately mimics the background. In this case the slope of the potential, starting from a large value, gradually diminishes pushing the system to slow roll regime at late times. In the present case owing to the behavior of $\Gamma$ in Fig.5, the field energy density would witness the large overshoot with respect to the background in a short span of time freezing the field on its potential(in the flat region) due to large Hubble damping. Field evolution would commence only at late stages when background energy density becomes comparable to field energy density allowing the slow roll of field giving rise to late time acceleration. Hence, the present scenario gives rise to thawing\cite{33, 34, 35} behavior.

Conclusion

In this paper, we have considered an inflationary scenario with $\alpha$-attractor for an exponential potential on RS brane. We have carried out full numerical analysis and presented approximated analytical results. We have found that our results pass the observational constraint for suitable parameter values. The observational bound on the parameter scalar-to-tensor ratio, $r \leq 0.1$\cite{15} is easily satisfied by our model for a range of parameters -- $\alpha \lesssim 10$ with $M^4/\lambda \gtrsim 7$. We found that a lower values of the parameter $\alpha$ gives rise to a large range of the parameter $M^4/\lambda$, falling within the window allowed by observations(Figur3). The lower bound on $\alpha$, related to the inflation scale, $\alpha \gtrsim 10^{-7}$\cite{20}, is not considered here to compare with observational consistency. The significance of the brane correction underlies with the assumption that $V/\lambda \gg 1$ or equivalently $M^4/\lambda \gg 1$ during inflation which automatically pushes $\alpha$ toward a lower values in order to meet observational
constraints. We numerically found that for consistency with observation, \( M^4/\Lambda \gtrsim 350 \) corresponds to \( \alpha < 1/6 \) necessary which keeps the non-canonical scalar field to be sub-Planckian. We also find that our inflation is scale is near the grand unification scale, same as the case for standard inflationary models.

The Present work, with high numerical precision, can be extended to obtain more accurate bounds on the parameters. The other aspects associated with inflation like reheating can also be investigated for the model under consideration. The investigation of alternative reheating suitable to the the present framework is left for future work.

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