Using of Hamilton mechanics methods for modeling dynamic processes on the Earth surface

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Abstract. This paper presents the methods of mathematical modeling and managing on some dangerous dynamic processes on the Earth surface such as floods, wildfires, desertification, oil spills on the water surface and others. We call these processes by a common name - distribution processes. The Hamilton mechanics methods permit to describe uniformly wide class of dynamic processes - both the processes of control determined with the ordinary differential equations and the processes in the continuous beds. The basic model of the distributed processes was presented in form of Hamilton – Jacoby equation. The concept of the normal speed of the process front movement suggested. The indicatrix and figurotrise of spread, which determine the configuration of the distribution process, are introduced. A model of processes with dissipation is proposed. Classification of processes according to the degree of their mobility is given. Using the figurotrises a new approach to numerical modeling of the processes front propagation based on Godunov’s method of the movable grids proposed. Its essence is that the calculated grid is not given a priory, but it is determined by current solution of the problem. It moves and develops with the solution. The movable grid method can serve as a basis for creating agent models of propagation processes. A separate section of the paper is devoted to describing a method for managing the processes. In particular, the fundamentals of the so-called localization control, which consists in building an obstacle insurmountable for the process, are outlined. The proposed methods and algorithms are illustrated by numerical examples. The models presented in this paper was a theoretical basis for designing and developing some management systems of the distribution processes control.

1. Introduction
Due to global climate change, the likelihood of damage from natural and man-made disasters, which are often spontaneous distributed dynamic processes on the surface of the Earth, has increased. Floods, mudflows, landslides, wildfires, the spread of plant pests – these are examples of such processes. These processes are becoming less predictable in scope and impact. In addition, damage to environment also caused by human activities. So, when oil is produced on the shelf, oil spills on the surface of the water are possible, which leads to the destruction of aquatic ecosystems. Unreasonable management can lead to the gradual disappearance of vegetation – desertification of the territory. To support decision-making in the management of these processes, a large number of techniques, models and systems have been developed to predict their dynamics. These models and systems based on various construction principles (the physical nature of the process, statistical and experimental data, etc.) and have different degrees of purpose (strategic modeling, operational-tactical modeling, etc.).
In the future we will call these processes by a common name - distribution processes. From point of view of the control theory, they are objects with distributed parameters such as the moving non-stationary waves in an anisotropic medium on the surface of the Earth.

One of the basis for creation the ecology management systems are mathematical models of the processes under consideration. Among the mathematical models for decision-making on the process management, the most important and complex are models described their dynamics. Conventionally, such models can be analytical and experimental. Analytical models describe in detail the physical processes that occur in the processes under consideration [1 – 8]. With high accuracy parameters calculation of such processes, these models unfortunately have high computational complexity, however their implementation in real-time is impossible at present, even when using high-performance computing systems. Experimental models are based on a simplified representation of the process and use of the experimental data for description of burning wave propagation through a fuel bed. For example, wildfire models were suggested in the works [9 – 12], which is used in some forecasting systems, such as BEHAVE PLUS, FARSITE. These models, due to their simplicity, have high speed calculation parameters of fires, but they have typically a limited accuracy of forecasting.

All experimental models describe spatial waves on the Earth's surface. Despite the various computational methods used by the authors, the basis for all these models are Hamilton equations and Huygens' principle.

The Hamilton mechanics methods permit to describe uniformly wide class of dynamic processes - both the processes of control determined with the ordinary differential equations and the processes in the continuous beds [13 - 15]. In the work presented discusses some of the features of Hamiltonian mechanics, which using in experimental models of the distribution processes.

The Hamilton formalism can present the distribution process in one of three equivalent forms:

- in form of the propagated wave front, described with the Hamilton-Jacoby equations;
- in form of the diversity, determined on the trajectories ensemble of the appropriate dynamic system;
- in form of the movable sets, which have formed with the help of Huygens principle.

The peculiarity of some processes is the possibility of energetically unbalance appearance, which causes by heat energy dissipation to environment. In the present work the theory of generalized Hamilton-Jacoby equations, suggested by V. Maslow [15], is shown as an appropriate mathematical tool for the dissipative fronts’ description. On basis of the methods mentioned following problems linked with the process dynamics and suppression are developed:

- modeling of the process propagation;
- numerical simulation the complex-shaped process edges;
- modeling the fighter crews’ motion and process suppression.

2. Basic model

The next assumptions were taken in the work presented.

- The processes spreading on Earth surface are considered in projection on the horizontal plane (on the appropriate scaled forest map). The coordinate system \( X = [x^1, x^2]^T \) is connected with a map, \( X \in D \), where \( D \) is the region observed.
- The meteorological, topographical and physical parameters of the processes in any point of the map are known. The physical basis of a propagation process is described by an appropriate model.
Undoubtedly, the physical process and the front propagation is the common process. But on the tactical level of modeling it is useful methodically to divide this process on two sub-processes and describe them by means of two separate models. First of them describes physical process, and second one deals with the front of a spatial wave propagation. This work has been developed the last type of models.

The assumptions mentioned permit to describe process boundary in terms of the propagated fronts. Consider the Hamilton function [14]

$$H(P, X) = V^T P,$$  \hspace{1cm} (1)

where $V = V(X) = [v^1, v^2]^T$ is column vector of the process $t$ spreading rate, $P = [p^1, p^2]^T$ is any direction on the plane.

Function $H(P, X)$ raise to the system of differential equations

$$\dot{X} = H_p(P, X) = V,$$

$$\dot{P} = -H_x(P, X) = -V_x^T P,$$  \hspace{1cm} (2)

where $V_x = \left\| \frac{\partial v^i}{\partial x^j} \right\|$, $i, j = 1, 2$ is $2 \times 2$ matrix of the rate derivations.

System (2) is considered under the initial conditions:

$$X(0) = X(\alpha), \ P(0) = P(\alpha),$$  \hspace{1cm} (3)

where $\alpha \in A$ is numerical parameter, $X(\alpha)$ and $P(\alpha)$ are the parametric expressions for the initial front of process and the set of normal to this front, $A$ is reachability set of the parameters values.

Let $S(x, t)$ be the action function of the fire front. This function describes the process edge motion and satisfies to Hamilton – Jacoby equation

$$\frac{\partial S}{\partial t} + H\left( \frac{\partial S}{\partial x}, X \right) = 0$$  \hspace{1cm} (4)

with initial condition

$$S(X, 0) = S(X_0(\alpha)),$$  \hspace{1cm} (5)

which defines the process front at $t = 0$.

Solution of the problem (4), (5) is given by expression

$$S(X, t) = S(X_0(\alpha)) + \int_0^t L(X(\tau), P(\tau)) d\tau$$  \hspace{1cm} (6)

where integration leads along the trajectories of the system (2), (3);

$L(P, X) = P^T X' - H(P, X)$ is the Lagrange function of the problem (4), (5).

Considering the Hamilton function expression (1), the Hamilton – Jacoby equation may be transformed to the ordinary form of the front’s description.
\[
\frac{\partial S}{\partial t} + V^T \text{grad} S = 0 ,
\]  

where \( \text{grad} \ S = \left[ \frac{\partial S}{\partial x^1}, \frac{\partial S}{\partial x^2} \right]^T \) is vector gradient, which is normal to the front.

For some applications introduction of the \textit{propagation process} conception is useful [15]. This term designates the closed dynamical set \( X = X(X_0,t) \) bounded by fire front propagated from initial set \( X_0 \), determined by condition (5). The process \( X \) may be also determined as a set of the trajectories of the system (2), (3), where parameter \( \alpha \) runs all the points of set \( A \).

3. Normal rate

In some applications the normal rate of front spreading \( V_n \) is more appropriate to use. It may be expressed as

\[
V_n = V^T \text{grad} S ,
\]

where \( \text{grad} \ S \) is unity vector normal to the front. Then equation (7) transforms to the other form

\[
S_t + V_n |\text{grad} \ S| = 0 .
\]

Equations (7) and (9) are considered with initial condition (5).

If the front equation is presented in the evident form

\[
x^2 = f(x',t) ,
\]

then the equation of the fire front dynamics (8) transforms to the form

\[
\frac{\partial x^2}{\partial t} + V_n \sqrt{1 + \left( \frac{\partial x^2}{\partial x'} \right)^2} = 0
\]

with the initial condition:

\[
x^2(x',0) = f(x',0) = f_0(x') ,
\]

where function \( f_0(x') \) must be given.

4. Indicatrices and figuratrices of rate

Consider the problem of modeling the configuration of processes using wildfires as an example. For other types of distribution processes, similar reasoning can be done considering the physics of the process.

As the burning theory and wildfires observations show, the rate of fire spread depends on the wind speed and the slope of locality where fire front is propagated.

The normal rate of fire front spreading in any point of the fuel bed may be presented as a function of the several arguments:
\[ V = V(X, t, w, s, \beta, \gamma), \]
\[ V_n = V_n(X, t, w, s, \beta, \gamma), \]

where \( w \) is a wind speed, \( s \) is a slope of fuel bed, \( \beta \) is an angle between the wind vector direction and direction of the fire front normal, \( \gamma \) is an angle between the upslope vector and direction of the fire front normal.

Functions \( V \) and \( V_n \) may calculated with the help of special models. Detail discussion of problems mentioned is given in works [13 - 15].

As follows from above assumption 2, the rate of spread may be presented as a product of three functions:

in case of fool rate
\[ V(X, t, w, s, \beta, \gamma) = V_0(x, t, w, s)\chi_w(\beta, w)\chi_1(\gamma, s) \] (14)

and in case of normal rate
\[ V_n(X, t, w, s, \beta, \gamma) = V_0(x, t, w, s)\chi_{nw}(\beta, w)\chi_{sw}(\gamma, s) \] (15)

In formulas (14), (15) \( V_0 \) is the maximal rate of spread, when the direction of spreading coincides with the direction of the wind speed vector and with the upslope direction.

The multipliers \( \chi_w, \chi_s, \chi_{nw} \) and \( \chi_{sw} \) define local shape of the fire propagation. Functions \( \chi_w(\beta, w) \) and \( \chi_1(\gamma, s) \) named accordingly as wind and slope indicatrices of the full rate; functions \( \chi_{nw}(\beta, w) \) and \( \chi_{sw}(\gamma, s) \) named accordingly as wind and slope indicatrices of the normal rate. Following to H. Munkowski and H. Rund, the latter functions is named as figurotrices [13].

Let us consider some properties of the indicatrices and figurotrices. They are like the wind and slope functions. Therefore, the function \( \chi(P) \) and \( \chi_n(P) \) without wind and slope specification may be considered. In both cases \( P \) is the vector of direction.

- Functions \( \chi(P) \) and \( \chi_n(P) \) normalized to unity:
\[ \max_P \chi(P) = \max_P \chi_n(P) = 1. \] (16)

- A figurotrice always contains proper indicatrix (figure 1):
\[ \chi_n(P) \geq \chi(P). \] (17)

- Indicatrices \( \chi(P) \) are always the convex functions, but figurotrices \( \chi_n(P) \) may be non-convex.

- Let indicatrix be a vector:
\[ \chi(P) = R, |R| = 1. \]

It means that the fire propagation is possible only in single direction determined by vector \( R \). In this case, function \( \chi(R) \) is a circle \( S(R) \) constructed on vector \( R \) as on diameter (figure 2):
\[ \chi_n(P) = S(R), \quad \arg P \in [0, 2\pi]. \]  

Notice that this case defines the "minimal" figurotrix to which a real indicatrix corresponds.

- Let indicatrix \( \chi(P) \) be the convex closed set restricted by the continuous boundary \( \partial \chi(P) \). Then figurotrix \( \chi_n(P) \) is the limit shell of the circles \( s(R) \), every one of which is constructed on vector \( \chi(P) \) as on diameter:
  \[ \chi_n(P) = \lim_{R \to P} s(R), \quad \arg P \in [0, 2\pi]. \]  

- When the figurotrix \( \chi(P) \) is given, the indicatrix \( \chi(P) \) is determined as the closed set of points, restricted by a skirt line of the normal lines to rays \( \chi_n(P) \), \( P \in [0, 2\pi] \).

![Figure 1](image1.png)

**Figure 1.** Relationship between the indicatrix \( \chi(P) \) and the figurotrix \( \chi_n(P) \).

![Figure 2](image2.png)

**Figure 2.** «Minimal» real indicatrix \( \chi(P) \) and proper figurotrix \( \chi_n(P) \).

### 5. The processes with dissipation

Upset of the forest fire burning process energy balance is caused both by influence of the natural forest and meteorological factors and by efforts on the forest fire suppression.

On this reason fire propagation in some or in all directions may be stopped. From a mathematical point of view the equation of energy balance on steady-state propagation has no real roots but has a complex root. The complex spread rates appearance leads to a complex Hamiltonian. Following to V. Maslow theory \[15\] the dissipative processes are described with
the help of complex Hamilton function

\[ H(P, X) = H_R(P, X) + iH_I(P, X). \]  

(20)

Where \( H_R(P, X) = V_R^TP \) is real part and \( H_I(P, X) = V_I^TP \) is imaginary part of the complex Hamiltonian.

The real item of function \( H(P, X) \) rises the Lagrange diversity on the trajectories of the equation (2).

According to this theory the generalized Hamilton - Jacoby equations for this case are:

\[
\begin{align*}
\frac{\partial S_R}{\partial t} + H_R \left( \frac{\partial S_R}{\partial X}, X \right) - H'_R \frac{\partial S_I}{\partial X} &= 0, \\
\frac{\partial S_I}{\partial t} + H_R \left( \frac{\partial S_R}{\partial X}, X \right) + H'_R \frac{\partial S_I}{\partial X} &= 0
\end{align*}
\]

(21)

And considering expressions for the derivations \( H_R \) and \( H_I \):

\[
\begin{align*}
\frac{\partial S_R}{\partial t} + V_R^T \frac{\partial S_R}{\partial X} - V_I^T \frac{\partial S_I}{\partial X} &= 0, \\
\frac{\partial S_I}{\partial t} + V_I^T \frac{\partial S_R}{\partial X} + V_R^T \frac{\partial S_I}{\partial X} &= 0
\end{align*}
\]

(22)

under the initial condition:

\[ S_R(X, t, 0) = S_{R_0}(X(\alpha)), S_I(X, t, 0) = S_{I_0}(X(\alpha)). \]  

(23)

Solution of the system (22), (23) is given by the expressions

\[
\begin{align*}
S_R(X, t) &= S_{R_0}(\alpha) - \int_{\alpha}^{t} \left( P^T X - P^T V_R(X) + \xi^T V_I(X) \right) dt, \\
S_I(X, t) &= S_{I_0}(\alpha) - \int_{\alpha}^{t} P^T V_I(X) dt,
\end{align*}
\]

(24)

where \( P = \frac{\partial S_R}{\partial X}, \xi = \frac{\partial S_I}{\partial X} \).

Integration of the equations (24) leads along the trajectories of the system (2). Function \( S_R \) describes the wave front movement, and \( S_I \) describes the process dissipation. As it was shown in [16, 27], analog of the figuratri nes for dissipative processes is defined by the expression

\[
\gamma_n(P, \xi) = \text{sgn} \left( P^T V_R - \xi^T V_I \right),
\]

(25)

where \( P \in \sigma_0(X) \) \( \xi \in \sigma_0(X) \); \( \sigma_0(X) \) is a circle with it's center in \( X \) and unity radius; function \( \text{sgn} \) is defined as

\[ \text{sgn}(u) = \max \{0, u \}. \]

From formula (29) the next corollary follows. For some directions, determined by
appropriate combinations of the vectors $P$ and $\xi$ the normal front spread $V$ may be equal to zero. So in the case described dissipation expresses in narrowing of the possible propagation directions set.

Let us sum above results and determine different types of the fire configurations which are characterized by specific "degree of freedom" of the fire front propagation. The classification suggested may be interesting for estimation of the fire’s mobility and range of danger. As table (figure 3) shows, the degree of mobility rises in the following order:

1 - absence of propagation;
2 - extinguished propagation, this case corresponds to complex Hamiltonian;
3 - propagation on single direction, so called auto model (self-similar) front;
4, 5, 6 - propagation on two, three and four directions.

6. Numerical modeling of fire front propagation

Modeling of the fire front propagation is one of the most typical problems in the forest fire management [9 – 12]. The well-known grid procedure based on Dijkstra's algorithm [9, 10] uses the time delays of the fire spreading between the neighbor nodes of the grid. These time delays depend on direction and are verse proportional to spread in proper direction. So the procedure mentioned bases actually on using the wind (or slope) indicatrices.

Using of the figuratrices gives us a new approach to numerical modeling of the fire front propagation. The idea of algorithm is based on Godunov’s method of the movable grids [16]. Its essence is that the calculated grid set is not given a priory but it is determined by current solution of the problem. It moves and develops with the solution.

Let us consider the grid construction. The fire edge at time moment $t$ given as closed curve line passed the points $C_1, ..., C_n$ (see figure 4). Each point $C_i = C_i(t)$ determined by vector

$$C_i(t) = \{X_i, L(i), R(i), t_i\}, \ i = 1, ..., n$$

where $X_i = (x_i^1, x_i^2)$ are coordinates on the map, $L(i)$ and $R(i)$ are accordingly the numbers of neighbor points which lie on left and right side from $C_i$.

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| Character of the fire fronts motion | Possible directions of Indicatrix | Figuratrix |
|-----------------------------------|----------------------------------|------------|
|                                   |                                  |            |
Algorithm includes following stages [17].

- Creation of grid on the next time moment $t + \Delta t$ (first iteration). For each point $C_i(t)$ a new vector of coordinates is calculating

$$X(t + \Delta t) = X(t) + V_0(X, t)\mathcal{K}_n(P)\Delta t,$$

(26)

where $P$ is the normal vector to the front in $C_i(t)$. This vector is determined approximately as a normal in $C_i(t)$ to curve line, that passes through points $C_{i-1}(t), C_i(t), C_{i+1}(t)$. As it is shown in [14], expression (30) gives us a first order approximation of the equations (9) and (11).
Putting the grid in order. In process of step by step motion of the grid the distances between neighbor points may become very different and grid comes to disorder. For supporting the grid in order it is necessary to set a minimal and maximal distances between the neighbor points $L_{\text{min}}$ and $L_{\text{max}}$. On the second stage, if $|X_i - X_{i+1}| \geq L_{\text{max}}$ then a new point $C_j$ is placing between $C_i$ and $C_{i+1}$. In the other case, if $|X_i - X_{i+1}| \leq L_{\text{min}}$, point $C_i$ is excluding from set $C_1, ..., C_n$. In the both cases links between points determined by the proper indicators $R(i)$ and $L(i)$ are changed.

This algorithm permits to create rapid and effective computer programs for the fire front dynamics modeling. Figure 5 shows prognosis of wind-driven wildfire edge propagated through non-uniform forest fuel bed.
7. Model of wildfire localization
In this section some problems connected with the modeling of interaction between distribution processes and human being are considered. The most developed of such models are probably the model of forest fire localization and problems named as problems of localization control [17 - 21].

Localization of the distribution processes is direct suppression at edge of a distribution processes or it's surrounding by fire insuperable barrier. In conditions of real fighting a choice of the localization ways depends on many non-formalized factors. But it is useful to investigate some idealize cases that allows a mathematical modeling and can be used for planning purposes.

Other problem of this class connected with human safety in distribution processes contacts, is a problem of avoiding contact between distribution processes and moving crew [17]. Both problems mentioned bound with the theory of dynamic games [18].

Short review of the localization control problems is given below.
Let us consider motion of any tactical unit of distribution processes suppression force that constructed an insuperable barrier. It may be, for example, an emergency response crew, a bulldozer or a separate fireman. At the initial time moment \( t = t_0 \) this unit is in the point \( C \) with coordinates \( Y_c = (y_c^1, y_c^2) \), see figure 5. From this point the unit considered may move at any direction with the speed \( U(Y, Q, t) \), which depends on the point, vector of direction, and time. On small time period \( \Delta t \) the unit may move on distance \( \Delta L = \Delta L(Q) = U(Y, Q, t) \Delta t \), where \( Q \) is the selected direction. The geometrical place of the vectors \( \Delta L(Q) \), where \( \arg P \in [0, 2\pi] \), gives the attainable set of the unit at time moment \( t + \Delta t \).

It is easy to see that dynamics of the attainable set may be described with the help of Hamilton mechanics methods if introduce the Hamilton function

\[
H(Q, Y) = U^T Q.
\] (27)

In particular, the equations of trajectories (2), (3) of propagated front (7), (9), (11) and proper numerical methods are correct for the forest fire localization modeling.

The initial front in this case must be presented as a circle of small radius with center in \( C_0 \). Set of all trajectories \( y(t) \), that are issued from initial set \( Y_0 \), may be named as process of attainability \( Y(Y_0, t) \). This process is analogues to the stated above process of distribution process propagation \( X(X_0, t) \). Difference between the processes \( X \) and \( Y \) is that the process of distribution propagation \( X \) realizes at same time all the trajectories but process of attainability \( Y \) realizes only one trajectory \( L = y(t) \), which named as localization line.

Let us consider interaction of the processes \( X \) and \( Y \). Both are determined on the same set \( D \) and have common time scale. They propagate undependably until coming to contact. When processes \( X \) and \( Y \) propagate in contact, they exert the mutual influence on the rates of spread:
\[ \hat{V}(P, X, t) = \begin{cases} V(P, X, t), & X \notin Y(Y_0, t), \\ 0, & X \in Y(Y_0, t) \end{cases} \]

\[ \hat{U}(Q, Y, t) = \begin{cases} U(Q, Y, t), & Y \notin X(X_0, t), \\ 0, & Y \in X(X_0, t) \end{cases} \]  \hspace{1cm} (28)

There \( \hat{V} \) and \( \hat{U} \) are rates of spread when the processes \( X \) and \( Y \) are in interaction. The interacted processes are designed \( X_R \) and \( Y_R \).

When localization realizes by several crews it is important to define their interaction. Let a set of centers \( C_0 = \{ C^k \} \) is given; \( C^k \in D, \ k = 1, \ldots, p \). The localization lines \( L_j(t) \) start simultaneously from all centers \( C_s = \{ C^l \}, \ C_s \in C_0 \), and finish in the centers \( C_s = \{ C^l \}, \ j = 1, \ldots, n \). Each of localization lines have its own direction. If line \( L_j(t) \) runs clockwise relative to process \( X(X_0, t) \) then its orientation is \( \beta_j = +1 \); if line \( L_j(t) \) runs counter clockwise relative to this process then orientation is \( \beta_j = -1 \). Set of orientations is noted as \( B = \{ \beta_j \}, \ j = 1, \ldots, n \).

Then each localization problem is characterized by three of sets:
\[ \Lambda = \{ C_s, C_F, B \} \]
and named as localization scheme (figure 6).

![Localization scheme](image)

| \( C_s \) | \( C_F \) | \( B \) |
|---|---|---|
| \( L_1 \) | \( C^1 \) | \( C^2 \) | +1 |
| \( L_2 \) | \( C^1 \) | \( C^2 \) | -1 |
| \( L_3 \) | \( C^3 \) | \( C^4 \) | +1 |
| \( L_4 \) | \( C^3 \) | \( C^4 \) | -1 |

**Figure 6.** Localization scheme.

From the set of localization lines satisfying to given conditions, there may be selected some subset that satisfies to additional cost criterions. Among these criterions may be indices following:

- minimal number of lines in localization scheme (minimal number of crews);
- minimal time of localization process finishing;
- minimal area occupied by process \( X_R \).
As example, let us consider problem on one side surrounding of the process \( X \) when number of localization lines must be minimal.

There are given:

- the process \( X(X_0, t) \);
- partially determined scheme of localization;
- initial \( C_s^1 \) and finish \( C_F^n \) centers;
- orientation of localization lines \( B = \{ \beta_j \} \); all \( \beta \) are equal;
- number \( n \) is unknown a priory.

It is necessary:

- to complete the localization scheme, which is defined by all centers \( C_s^2 = C_F^1, C_s^3 = C_F^2, \ldots, C_s^n = C_F^{n-1} \);
- to construct the localization lines \( \{ L_j(t) \} \); \( j = 1, \ldots, n \).

Number \( n \) must be as small as possible.

When \( n = 1 \) is given a priory, the indicatrices of the processes \( X \) and \( Y \) are simple functions and these processes propagate on uniform plane, then the localization line may be constructed as a solution of some variation problem \([17 - 20]\). In more complex cases it is necessary to use step-by-step numerical algorithm \([17]\).
Figure 7. Sample of the localization problem solving.

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isochrones of process X;
isochrones of processes Y;
localization lines L.

This algorithm consists two stages (see fig. 7). On the first one the boundaries $B^i$ of interacted processes $X_R^j(X_0,t)$ and $Y_R^j(Y_0,t)$, $j = 1,2,...$, are constructed, where $j$ is number of step (number of localization line). The initial set $X_0$ is the same for each step. The initial set $Y_0^1$ is $C_s^1$ and when $j > 1$, then $Y_0^j$ is a set, bounded by $B^j$. This procedure finishes when $Y_R^n$ captures the $C_F^n$. On second stage the localization lines are constructed by backwards motion from finish point $C_F^n$ to initial one $C_s^1$. This lines $L_j(t)$ in any point are normal to boundary of
The procedure described is realized with the help of the special computer graphical program.

Currently, many mathematical models in the considered was worked out. However, most of them do not describe processes at the same time the spread of distributed processes and its liquidation. The authors for this purpose proposed agent-based approach. Agent based modeling (agent-based modeling, ABM) is a new approach to simulation, which is aimed at simulation of complex dynamic systems by studying the behavior and interactions of autonomous and independent entities (agents) in a medium [18]. To simulate forest fire situations are of interest as geographic information systems (GIS), which have the capacity of storage and visualization of spatial information. Sharing of agent-based modeling and GIS allows you to combine the advantages of both directions, apply advanced tools for modeling the processes occurring in the real landscape.

8. Discussion
Modeling of wildfires is now a wide field of scientific and engineering works. There are many approaches to this problem which depend on aim of modeling and required details of describing. Let’s compare the theory presented in our work with the physically grounded models. Probably the most detailed model of this class is developed by professor Anatoly Grishin and his colleagues from Tomsk University [11]. In this model a burning forest fire considered as a polyphase, multi-layer porous three-dimensional heterogeneous medium, consisting of dry organic substance (forest fuel), dispersed water, solid product of pyrolysis, stationary char combustion product, gas phase, dispersed ash particles, soot particles and drops of rain.

The model contains set of equations of mass, energy and heat balance and appropriate boundaries and initial conditions. Common number of the 3-dimensional equations, boundary and initial conditions reaches some dozen. Of course, such model seems to be very detail and physically well-founded. But it’s not suitable for practical use. In the first place, this model requires big amount of the physical and chemical constants characterized forest fuel and products of their burning. In the second, numerical solution of such problems is very expensive and complex even for modern supercomputers.

The model presented in our paper, of course, is not such physically argued as one described before. But it has some advantages for practical prediction of the forest fires propagation. This model requires a few input parameters; it gives acceptable precision and may be used with help of a personal computer or notebook. And what is more, the same mathematical and software environment may be used for developing the planes of wildfire suppression. This model may be used as a theoretical basis for developing and designing wildfire management systems.

9. Conclusion
The model presented in this paper may be used as a theoretical basis for designing and developing distribution processes management systems of different application.

Let’s consider some works in this field which have been realized in Siberian Technological University in last years.

- Software package “Forest Fire Prognosis” was developed as a subsystem of Automation System of Scientific Forest Investigation (ASNI-LES) in Suchachev Forest Institute of Russian Academy of Sciences (Krasnoyarsk) [17]. This package contains some dozen of program modules which help to calculate parameters of forest fuel and forest fire dynamics.
Training system “TAJGA” was developed for teaching the personnel of forest service and the students of forestry specialty [18, 23]. With help of this system the users study to reach decision on tactics of forest fire suppression and to distribute the available resources for fire putting out.

Computer system “SPLAT” was developed for managing the resources of aviation forest protection service [22]. This system permits to choose the optimal type of airplane or helicopter, the set of technical devices and fire crews for effective forest fire suppression.

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