Effect on cosmic microwave background polarization of coupling of quintessence to pseudoscalar formed from the electromagnetic field and its dual

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Abstract

We present the full set of power spectra of cosmic microwave background (CMB) temperature and polarization anisotropies due to the coupling between quintessence and pseudoscalar of electromagnetism. This coupling induces the rotation of the polarization plane of CMB photons, thus resulting in non-vanishing $B$ mode and parity-violating $TB$ and $EB$ modes. Using the 2003 flight of BOOMERANG (B03) data, we give the most stringent constraint on the coupling strength. In some cases, the rotation-induced $B$ mode can confuse the hunting for the gravitational lensing-induced $B$ mode.

The existence of a dark component with an effective negative pressure, supported by several observations especially the Hubble diagram for the type-Ia supernovae (see, for example, \cite{1, 2}), is still one of the puzzles in cosmology. Cosmological constant is the simplest possibility for such dark component. However, the observed value of the cosmological constant is completely different from theoretical expectation \cite{3}. An alternative candidate, the so-called quintessence described by a dynamical scalar field $\phi$, is naturally considered. The dynamics of $\phi$ in general quintessence models is governed by a scalar potential $V(\phi)$ which makes the dark energy dominant in recent epoch. There are many different kinds of proposed potentials, for example, pseudo Nambu-Goldston boson, inverse power law, exponential, hyperbolic cosine, and tracking oscillating \cite{4}. To differentiate between the
models and finally reconstruct $V(\phi)$ would likely require next-generation observations.

The quintessential potential $V(\phi)$ and the field $\phi$ itself are difficult to be measured directly. What we can do is to investigate the dark energy density $\Omega_\phi$ and the time evolution for the equation of state (EOS) $w_\phi = P_\phi/\rho_\phi$, both of which are governed by the dynamics of $\phi$. Several observations have been made to give the answers: 157 supernovae in a redshift interval, $0.015 < z < 1.6$, in the “Gold Sample” obtained from a combination of ground-based data and the Hubble Space Telescope [2] as well as 115 supernovae with $0.015 < z < 1$ from the Supernova Legacy Survey provide information about the dark energy [5]. It is very difficult to determine whether the quintessence is more preferred than the cosmological constant by observations of the CMB temperature anisotropy spectrum only. Joint analysis of CMB data with supernovae or/and large scale structure survey such as SDSS or 2dFGRS can offer better constraints on quintessence models [6]. Recently, the study of the cross-correlation of maps between CMB and various tracers of matter through the integrated Sachs-Wolfe effect has also been carried out by several groups [7]. However, the intrinsic properties of dark energy are not well constrained so far from the investigations above, thus allowing a very wild range of the EOS, which is strongly model dependent.

An alternative way to study the quintessence is to consider the interaction of $\phi$ to ordinary matter. Coupling of $\phi$ to dark matter is considered as a possible solution for the late time coincidence problem [8]. It leaves distinct imprints on the CMB temperature anisotropy and the matter power spectrum due to an excess of cold dark matter at early epoch when compared to the standard cosmology model [9] (We define the “standard cosmology” as the model without coupling between quintessence and ordinary matter). Of particular interest in this Letter, we study the coupling of $\phi$ to the pseudoscalar of electromagnetism. Pseudoscalar couplings usually arise from the spontaneous breaking of a compact symmetry group, say, U(1) (see Frieman et al. 1995 at reference [4]). Carroll has argued that the coupling $\beta_{\phi F} \phi / \bar{M} F_{\mu \nu} \tilde{F}^{\mu \nu}$ leads to the rotation of the polarization vector of propagating photons if $\phi$ is varying with time. Here $\beta_{\phi F}$ is the coupling strength, $\bar{M}$ is the reduced Planck mass, and $\tilde{F}^{\mu \nu}$ is the dual of the electromagnetic tensor. This effect is called as the “cosmological birefringence” [10].

Measurements of the polarization of distant astronomical objects would provide information about the cosmological birefringence. Carroll used the rotation of polarization direction for distant radio sources to constrain the
Another proposed method is the CMB polarization from the last scattering surface \[11, 12\]. In this Letter, we study the effects of the cosmological birefringence on the power spectra of CMB temperature and polarization. Only assuming the shape of the potential \( V(\phi) \), we present the first CMB power spectra in the presence of the cosmological birefringence by the use of the full Boltzmann code.

Thomson scatterings of anisotropic radiation by free electrons give rise to the linear polarization, which is usually described by the Stokes parameters \( Q \) and \( U \) \[13\]. In standard cosmology, the time evolution of the polarization perturbation is governed by the Boltzmann equation \[14\]. If there is a physical mechanism which rotates the polarization plane, the evolution equations for the Fourier modes of the Stokes parameters are modified to

\[
\dot{\Delta}Q_{\pm iU}(k, \eta) + i k \mu \Delta Q_{\pm iU}(k, \eta) = n_e \sigma_T a(\eta) \left[ -\Delta Q_{\pm iU}(k, \eta) \right] \\
+ \sum_m \sqrt{\frac{6\pi}{5}} \pm 2Y_2^m(\hat{n}) S_P^{(m)}(k, \eta) + i 2\omega \Delta Q_{\pm iU}(k, \eta), \tag{1}
\]

where the derivatives are taken with respect to the conformal time \( \eta \), \( \mu = \hat{n} \cdot \hat{k} \) is the cosine of the angle between the CMB photon direction and the Fourier wave vector, \( n_e \) is the number density of free electrons, \( \sigma_T \) is the Thomson cross section, and \( a \) is the scale factor. \( Y_2^m \) is spherical harmonics with spin-weight \( s \) with \( m = 0, \pm 1, \pm 2 \) corresponding to scalar, vector, and tensor perturbations, respectively, if the axis of \( s Y_2^m \) is aligned with the wave vector \( k \). \( S_P^{(m)} \) is the source term of generating polarization, which is the composition of the quadrupole components of the temperature and polarization perturbations \( S_P^{(m)}(k, \eta) \equiv \Delta T_{2}^{(m)}(k, \eta) + 12\sqrt{6} \Delta_{\pm 2}^{(m)}(k, \eta) + 12\sqrt{6} \Delta_{\mp 2}^{(m)}(k, \eta) \).

We have followed the notation in Ref. [15] and expanded the perturbations in the spin-0 and spin-2 spherical harmonics [16] according to the scalar and tensor properties of the temperature anisotropy and polarization respectively. Thus, \( \Delta T_2^{(m)} \) and \( \Delta_{\pm 2}^{(m)} \) are the respective expansion coefficients for \( \Delta_T \) on basis \( Y_{l,m} \) and \( \Delta Q_{\pm iU} \) on basis \( \pm 2 Y_{l,m} \). The last term in Eq. (1) appears due to the rotation of the polarization plane. The dispersion relation for electromagnetic radiation coupling to the time varying quintessence field \( \phi \) is given by \( E^2 = k^2 \pm k \beta P F \dot{\phi}/(aM) \), where \( \pm \) refer to the right and left handed circular polarization, respectively. Therefore, the net angular
velocity of the polarization plane is [10]

\[
\omega = 2\beta \frac{\dot{\phi}}{a\dot{M}}.
\]  

We are used to decomposing the polarization on the sky into a divergence free component, the so-called \(E\) mode, and a curl component, the so-called \(B\) mode because the values for \(Q\) and \(U\) depend on the choice of a coordinate system. Whether \(B\) mode is generated depends on the existence of \(U\) for the local mode whose wave vector \(k\) parallels to the \(\hat{z}\) of the coordinates whereas \(Q\) is defined as the difference in intensity polarized in the \(\hat{\theta}\) and \(\hat{\psi}\) directions [17]. Mathematically, for \(m = 0\), only \(Q\) is generated in the local mode. The axisymmetry of the radiation field about the mode axis guarantees that no \(B\) mode can be generated by scalar mode perturbations.

In the presence of cosmological birefringence, we can find two important features in Eq. (1). Firstly, the rotation of the polarization plane generates \(U\) contributions to the local mode polarization. This converts the power from the \(E\) mode to \(B\) mode. The conversion depends on how much rotation there is from the epoch when the polarization is generated to today. Secondly, \(TB\) and \(EB\) cross correlations are expected to vanish due to the parity (\(T\) and \(E\) have parity \((-1)\ell\) while \(B\) has \((-1)^{\ell+1}\)). The cosmological birefringence violates the parity and thus generates the \(TB\) and \(EB\) power spectra whose magnitudes depend on the integrated rotation of polarization too. Substituting the angular velocity of the polarization plane in Eq. (2) into Eq. (1), we calculate the power spectra \(T, E, B, TE, TB,\) and \(EB\) modes.

These six power spectra form a complete two-point statistics of CMB temperature and polarization anisotropies. To simplify the calculation, we only include the scalar perturbations. That is by setting \(m = 0\) in Eq. (1). Without showing the details, we just write down the power spectra which are obtained from the solutions for the line-of-sight integration as

\[
C_{\ell}^{(E,B)} = (4\pi)^2 \frac{9}{16} \frac{(\ell + 2)!}{(\ell - 2)!} \int k^2 dk \Delta_{(E,B)}(k, \eta_0) \Delta_{(E,B)}(k, \eta_0),
\]

\[
C_{\ell}^{EB} = (4\pi)^2 \frac{9}{16} \frac{(\ell + 2)!}{(\ell - 2)!} \int k^2 dk \Delta_{E}(k, \eta_0) \Delta_{B}(k, \eta_0),
\]

\[
C_{\ell}^{TE} = (4\pi)^2 \sqrt{\frac{9}{16}} \frac{(\ell + 2)!}{(\ell - 2)!} \int k^2 dk \Delta_{T}(k, \eta_0) \Delta_{E}(k, \eta_0),
\]

\[
C_{\ell}^{TB} = (4\pi)^2 \sqrt{\frac{9}{16}} \frac{(\ell + 2)!}{(\ell - 2)!} \int k^2 dk \Delta_{T}(k, \eta_0) \Delta_{B}(k, \eta_0),
\]
\[ \Delta_T(k, \eta_0) = \int_0^{\eta_0} d\eta g(\eta) S_T(k, \eta) j_\ell(kr), \]
\[ \Delta_E(k, \eta_0) + i\Delta_B(k, \eta_0) = \int_0^{\eta_0} d\eta g(\eta) S_P(k, \eta) \frac{j_\ell(kr)}{(kr)^2} e^{i2\alpha(\eta)}, \] (4)

where the visibility function \( g(\eta) \) describes the probability that a photon scattered at epoch \( \eta \) reaches the observer at the present time, \( \eta_0 \). Similar to \( S_P \equiv S_P^{(0)} \), \( S_T \) is the source term generating the temperature anisotropy. \( j_\ell \) is the spherical Bessel function and \( r = \eta_0 - \eta \). The rotation angle \( \alpha(\eta) = \int_{\eta}^{\eta_0} d\eta' \omega(\eta') \). We do not present the formula for the temperature anisotropy because it is unchanged under the rotation of the polarization plane.

We have modified the public code CMBFast [18] for our purpose. Here, we consider the potential \( V(\phi) = V_0 \exp(\lambda \phi^2/2M^2) \) for our quintessence model (the hyperbolic cosine potential is also considered, see below), where \( \lambda \) is a parameter determining how shallow the potential is. Hereafter we fix \( \lambda = 5 \) and we will obtain similar results by choosing other values of \( \lambda \). We plug the table of the EOS for this quintessence model into the modified CMBFast code and input the cosmological parameters from the best fit values of the WMAP three-year results [19]. The power spectra are then normalized to the first peak of the temperature anisotropy measured by the three-year WMAP observation [20]. The left panel of Fig. 1 shows the \( E \) and \( B \) mode power spectra with the coupling strength \( \beta_F \tilde{F} \) ranging from \( 10^{-5} \) to \( 10^{-3} \). The \( EB \) mode power spectrum is shown in the right panel. On small scales, increasing coupling strength results in a suppression of the \( E \) mode in the standard model and non-vanishing \( B \) and \( EB \) modes. Furthermore, the shapes of the \( B \) and \( EB \) mode power spectra basically follow the standard \( E \) mode except the reionization bump on large scales. To explain this, we may make a rough estimation in Eq. (3): \( C_{B\ell} \sim C_{E\ell} \sin^2 2\alpha_\ell \) and \( C_{EB\ell} \sim 0.5C_{E\ell} \sin 4\alpha_\ell \) with \( \alpha_\ell \) being the total rotated angle for certain angular scales \( \theta \sim \pi/\ell \) from last scattering epoch to today. This \( \alpha_\ell \), in general, is not constant for all the scales. The \( E \) mode power on small scales mainly comes from the recombination epoch at \( z \sim 1100 \). On the other hand, the boosting power on large scales comes from reionization epoch when the CMB photons are rescattered by free electrons at \( z \sim 10 \) [21]. From Eq. (2) and the evolution of \( \phi \), we find that the integrated rotation angle from the reionization epoch is much smaller than that from the recombination epoch. Therefore, there is much less power converted from \( E \) mode to \( B \) mode on
large scales than small scales.

Figure 1: $E$, $B$ (left panel; the lower three thick curves are $B$ modes) and $EB$ (right panel) mode power spectra from the cosmological birefringence with different coupling strength.

If the coupling strength is large enough, the $B$ mode induced by the cosmological birefringence will mix up with the gravitational lensing-induced $B$ mode. Gravitational lensing by large scale structures modifies slightly the primary $E$ mode power spectrum. Most noticeably it generates, through mode coupling, $B$ mode polarization out of pure $E$ mode signal [22]. The lensing-induced $B$ power spectrum, which peaks around $\ell \sim 1000$, has the roughly similar shape with that from the birefringence. We also show the power spectrum of the lensing-induced $B$ mode in Fig. 1 by a thin solid curve for comparison. The birefringence-induced $B$ mode is indeed compatible with the lensing-induced $B$ mode for $\beta F_{\tilde{F}} \sim 10^{-4}$.

Figure 2: $TE$ (left panel) and $TB$ (right panel) mode power spectra from the cosmological birefringence with different coupling strength.

Fig. 2 shows the $TE$ and $TB$ power spectra for different coupling strength.
Having the complete set of power spectra, we can constrain the coupling strength from observed data. In order to focus on the cosmological birefringence, we do not make the global fit to all the cosmological parameters. It is debatable that $\beta_{F\tilde{F}}$ is degenerate with other cosmological parameters due to the decrement of the $TE$ and $E$ power spectra. As we will see later, the upper limit on the coupling strength prevents it from making a significant change on the $TE$ and $E$ modes.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{likelihood_function.png}
\caption{Likelihood function of the coupling strength.}
\end{figure}

We use the North American data from the 2003 Antarctic flight of BOOMERANG [23] to calculate the $\chi^2 = \sum_{i,j} (D^b_i - T^b_i)C^{-1}_{ij}(D^b_j - T^b_j)$ fitting for $\beta_{F\tilde{F}}$ while fixing other cosmological parameters, where $D^b_i$ is the $i$th band-power, $C$ is the covariance matrix, and $T^b_i = \sum_{\ell} C_{\ell} W_{i\ell}/\ell$ is obtained by the theoretical prediction multiplied by the band-power window function $W_{i\ell}/\ell$. Those band-power data, including covariance matrices and window functions, are available; see [24]. In practice, we only fit $\beta_{F\tilde{F}}$ to the $EB$ and $TB$ power spectra which come from the parity violation. We do not use the $B$ mode power spectrum because it is contaminated by lensing-induced signal. If the lensing-induced $B$ mode can be successfully cleaned by appropriate techniques such as those proposed by Seljak and Hirata [25], we expect that including the $B$ mode will give a stronger constraint. We convert $\chi^2$ to the likelihood by $L = e^{-\chi^2/2}$ and normalize the maximum likelihood value to unity. The result is shown in Fig. 3. The upper limit on the coupling strength is found to be $|\beta_{F\tilde{F}}\Delta\phi|/M < 8.32 \times 10^{-4}$ at 95% confidence level, where $\Delta\phi$ is the total change of $\phi$ till today. This small value of the coupling strength gives an insignificant change on $TE$ and $E$ modes and thus will not affect the determination of the cosmological parameters. We also use the hyperbolic cosine potential $V(\phi) = V_0 \cosh(\lambda\phi/M)$ for the testing. Even though the quintessence evolution in this potential is different
from that in the exponential case, the upper limit value of $|\beta_{\rho}\Delta\phi|/\bar{M}$ does not change much. This value is much smaller than the result in Ref. [10], $3 \times 10^{-2}$, where $\Delta\phi$ is only from $z = 0.425$ to today. It is remarkable that the rotation-induced $B$ mode with the upper limit value of the coupling strength exceeds the lensing-induced $B$ mode. Therefore, careful measurements of $TB$ and $EB$ are necessary for separating the two effects.

Several authors have studied the effect of parity violation on the CMB power spectra by assuming a constant rotation angle $\alpha$ [11, 12]. They obtained a new set of rotated power spectra from combining the power spectra in the standard model with the sine or cosine function of $\alpha$. Furthermore, taking the rotation angle as a free parameter, Feng et al. [12] constrained the rotation angle using the data made by the first year WMAP and BOOMERANG observations. However, the time-varying scalar field $\phi$ in their work is constrained such that the integrated rotation angle should be very small from the recombination to the reionization epoch. Therefore, it is less supported by general quintessence models.

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