Optimal placement of Phasor Measurement Unit considering System Observability Redundancy Index: case study of the Kenya power transmission network

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ARTICLE INFO

Keywords:
Optimal PMU Placement
System Observability Redundancy Index
Depth First
Mixed Integer Linear Programming
Artificial Bee Colony
Zero Injection Bus

ABSTRACT

Modern power systems require advanced monitoring and control, a capability made possible by Phasor Measurement Units (PMUs) aided by synchrophasor technology. Because PMUs have significant costs, it is necessary to optimally place them in an electrical power network. This paper proposes the Optimal PMU Placement (OPP) on the Kenya Power Transmission Network (Nairobi Region 30-bus system) using the existing methods: The Depth-First method, the Mixed Integer Linear Programming (MILP) using intlinprog solver, and the Artificial Bee Colony (ABC) algorithm. The algorithms are first implemented on the IEEE-14 and 30 bus test systems for verification before implementing the Kenya Power Transmission Network (Nairobi Region 30-bus system). Finally, the results for the three methods are compared. A key consideration is the System Observability Redundancy Index (SORI) under normal baseload conditions, with and without the inclusion of Zero Injection Buses (ZIBs). A higher value of SORI increases the measurement redundancy of the PMUs installed at a given bus. This paper further proposes the modelling of ZIB with adjacent buses by considering their Observability Index (OI). The case studies are modelled in Power System Analysis Toolbox (PSAT), and the simulations are carried out in MATLAB. From the simulation results, the ABC algorithm gives the optimal solution with the highest SORI compared to the Depth-First method and MILP, with the exclusion of the ZIB. The Nairobi region 30-bus system requires 12 PMUs located at buses: 2, 5, 8, 9, 11, 13, 14, 16, 21, 24, 27 and 29 for complete power system observability with a SORI of 43.

1. Introduction

As of the beginning of 2021, Kenya has an installed electrical generation capacity of 2819 MW with a peak demand of 1938 MW. About 60% of the electrical energy generated comes from renewable sources, most coming from hydropower and geothermal power. The Kenya Power Transmission Network consists of a National Control Center (NCC) which provide decision for the electrical generation and transmission of the five interconnected regions, namely: Nairobi, Western, Central Rift, Mt. Kenya and Coast. In addition, each region has its own Regional Control Center (RCC). The NCC comprises the Supervisory Control and Data Acquisition (SCADA) system that controls and monitors the power system. The SCADA system is made up of the Remote Terminal Units (RTUs), which collects data of the power system from relays, current transformers (CTs) and voltage transformers (VTs). This information is conveyed to the Independent System Operators (ISOs), who are responsible for the control and monitoring of the power system [1, 2, 3]. Although the SCADA system is used to control and monitor the status of the power system, it provides measurements that are unsynchronized, resulting in inaccurate state estimations of the power system. Furthermore, they have a very low data transfer rate, preventing them from capturing small disturbances on the power system [4]. For example, the post-mortem report of the blackout that occurred in North American in 2003 indicated that the operating engineers were unable to undertake the control plan they had since the SCADA systems were unable to detect low-frequency oscillations, which resulted in a widespread blackout that affected fifty million people in eight states in the US and two Canadian provinces [4, 5].
To overcome the defaults in the SCADA system, modern power system use Phasor Measurement Unit (PMU) which provide advance control and monitoring using synchrophasor technology [1, 5, 6]. PMUs measurers the magnitude, phase angle, frequency, and the Rate of change of frequency (ROCOF) of currents and voltages using the Global Positioning Systems (GPS). The GPS provides a clocking signal synchronized with the voltage and current waveforms, providing highly time-stamped measurements at a higher data transfer rate. PMUs offer both offline and online applications in the power system. Some examples of offline applications are; modal analysis and post-disturbance analysis, while some of the online applications are; wide-area monitoring, frequency stability monitoring, protection systems and power quality analysis [7, 8, 9].

Even though PMUs are superior to SCADA, they are costly and cannot be installed at all the power system buses. Therefore, optimal PMU Placement (OPP) was introduced to guarantee complete power system observability by deploying fewer PMUs than the number of the buses in the network hence minimizing their installation costs [9, 10].

From the literature survey, several methods have been proposed for OPP for complete power system observability. In [11], the Differential Evolution algorithm is proposed for OPP. An upgraded binary harmony algorithm for OPP is proposed in [12]. In [13], a two-step optimization method for OPP considering preinstalled PMUs is proposed. A particle swarm optimization algorithm for OPP considering practical cost implications is proposed in [14]. A Recursive Tabu Search algorithm using numerical method is presented in [15] for OPP. In [16], a Binary Imperialistic Competition Algorithm (BICA) is proposed for OPP considering single line outage and single PMU failure. A binary integer linear programming method is presented in [17] for the OPP problem. This research evaluated the possible failure of PMU/communication line and the impacts of both existing conventional power injection/flow measurements. In [18], three methods are proposed, the Depth-First method,

![Figure 1. A 10-bus network topology showing two sets of OPP locations.](image1)

![Figure 2. Merging of ZIB with an incident Radial bus.](image2)

![Figure 3. Merging of ZIB with the adjacent bus with the highest OI.](image3)

![Figure 4. The Depth-First flow chart for OPP problem.](image4)
Figure 5. Intlinprog flow chart for OPP problem.

Figure 6. The proposed ABC algorithm flow chart for the OPP problem.
The simulated annealing method, and the minimum spanning tree algorithm for the OPP problem. In [19], the Mixed Integer Linear Programming (MILP) is proposed for the OPP problem using the intlinprog, bintprog, cbc, scip, glpk, and lpsolve solver. In [20], the ABC algorithm is proposed for OPP for state estimation. The methods are tested on the IEEE standard buses and showed their effectiveness for the OPP problem formulation. The Nairobi region supplies most of Kenya’s electrical load demand; consequently, it is critical to install PMUs on the system to provide advanced monitoring and control to prevent power blackouts or contingencies that may arise due to voltage instability issues. Since no work on OPP has ever been done on the Kenya Power Transmission Network, this paper proposes OPP on the Nairobi region 30-bus system. The OPP problem is formulated using the existing methods; the Depth-First method in PSAT PMU Placement Toolbox, the Mixed Integer Linear Programming (MILP) using the Intlinprog solver in MATLAB and the Artificial Bee Colony (ABC) Algorithm. The methods are first applied on the IEEE standard buses for verification before implementing the Kenya Power Transmission Network. This research work extended the application of the existing methods by including the System Observability Redundancy Index (SORI) to determine the algorithm which gives the optimal output.

### Table 1. The ABC algorithm parameters for the OPP.

| Parameters                           | Values |
|--------------------------------------|--------|
| Colony size (CZ)                     | 100    |
| Number of food source – CZ/2         | 50     |
| The Search space (corresponds to the size of the network) | 14 or 30 |
| Limit for the occurrence of scout phase | 100    |
| Maximum iterations                   | 500    |

The randomly generated solutions undergo the following phases until an optimal output is achieved.

### Table 2. Results for optimal PMU placement using the Depth-First algorithm under normal baseload conditions without the ZIB.

| System        | Number of branches | Optimal PMU Location sets | Total number of buses with PMUs | SORI | CPU time      |
|---------------|--------------------|---------------------------|--------------------------------|------|---------------|
| IEEE-14 bus   | 20                 | 1, 4, 6, 8, 10, 14        | 6                               | 22   | 0.13194 s     |
| IEEE-30 bus   | 41                 | 3, 5, 6, 11, 12, 17, 18, 20, 21, 24, 26, 27 | 12 | 45 | 1.0789 s      |
| Nairobi 30-bus| 33                 | 1, 3, 5, 6, 7, 9, 13, 14, 17, 19, 21, 24, 29 | 13 | 42 | 0.25353 s     |

### Table 3. Results for OPP using Intlinprog algorithm under normal baseload condition without the ZIB.

| System        | Number of branches | Optimal PMU Location sets | Total number of buses with PMUs | SORI | CPU time |
|---------------|--------------------|---------------------------|--------------------------------|------|----------|
| IEEE-14 bus   | 20                 | 2, 8, 10, 13              | 4                               | 14   | 0.264 s  |
| IEEE-30 bus   | 41                 | 1, 5, 8, 10, 11, 12, 19, 23, 26, 29 | 10 | 35 | 0.823 s  |
| Nairobi 30-bus| 33                 | 2, 5, 6, 7, 10, 13, 15, 17, 21, 23, 25, 29 | 12 | 32 | 0.303 s  |

### Table 4. Results for OPP using ABC algorithm under normal baseload condition without the ZIB.

| System        | Number of branches | Optimal PMU Location sets | Total number of buses with PMUs | SORI | CPU time |
|---------------|--------------------|---------------------------|--------------------------------|------|----------|
| IEEE-14 bus   | 20                 | 2, 7, 11, 13              | 4                               | 16   | 7.439 s  |
|                |                    | 2, 6, 7, 9               | 4                               | 19   |          |
|                |                    | 2, 6, 8, 9               | 4                               | 17   |          |
| IEEE-30 bus   | 41                 | 3, 5, 6, 10, 11, 12, 15, 18, 25, 27 | 10 | 46 | 14.775 s  |
|                |                    | 1, 2, 6, 9, 10, 12, 18, 21, 25, 27 | 10 | 48 |          |
|                |                    | 2, 4, 6, 9, 10, 12, 15, 18, 25, 29 | 10 | 50 |          |
| Nairobi 30-bus| 33                 | 2, 3, 8, 9, 12, 14, 16, 19, 22, 25, 27, 29 | 12 | 40 | 14.218 s  |
|                |                    | 2, 5, 7, 9, 12, 14, 16, 20, 22, 24, 27, 29 | 12 | 41 |          |
|                |                    | 2, 5, 8, 9, 11, 13, 14, 16, 21, 24, 27, 29 | 12 | 43 |          |

The bold values represents the solution sets with the highest SORI.

### Table 5. Comparison of the ABC algorithm to the previous methods that considered SORI under normal baseload conditions without the ZIB.

| Method       | IEEE-14 bus | IEEE-30 bus |
|--------------|-------------|-------------|
|              | No. of PMUs | SORI | PMU locations | No. of PMUs | SORI | PMU locations |
| Proposed ABC | 4           | 19    | 2, 6, 7, 9    | 10           | 50   | 2, 3, 8, 9, 12, 14, 16, 19, 22, 25, 27, 29 |
| fmincon solver [32] | 4 | 19 | 2, 6, 7, 9 | 10 | 48 | 2, 4, 6, 10, 11, 12, 15, 19, 25, 29 |
| FPA [33]     | 4           | 19    | 2, 6, 7, 9    | 10           | 52   | 2, 4, 6, 9, 10, 12, 15, 18, 25, 27 |
| BGWO [34]    | 14          | 19    | 2, 6, 7, 9    | 10           | 52   | 2, 4, 6, 9, 10, 12, 15, 20, 25, 27 |
highest system observability. This paper further proposes modelling the Zero Injection Bus (ZIB) with adjacent buses by considering their Observability Index (OI).

The Nairobi region 30-bus system is an application of the validated methods based on the IEEE standard bus systems. Furthermore, the simulation results from the system depict the implication of the number of branches in a network to the number of optimally placed PMUs and their SORI values when compared to the IEEE standard buses using the three methods.

This paper is made up of six sections: Section 1 served as an introduction. Section 2 describes the Power System Observability and PMU placement rules, PMU problem formulation, System Observability Redundancy Index (SORI) and modelling of the Zero Injection Bus (ZIB). Section 3 presents the case studies used. The proposed methods are described in Section 4. The simulation results and discussion are covered in Section 5. Finally, the paper is concluded in Section 6 based on the results of the simulations and arguments presented in section 5.

Figure 7. Convergence of the ABC under normal baseload condition with the exclusion of the ZIB for the IEEE-14 bus.

Figure 8. Convergence of the ABC under normal baseload condition with the exclusion of the ZIB for the IEEE-30 bus.

Figure 9. Convergence of the ABC under normal baseload condition with the exclusion of the ZIB for the Nairobi region 30 bus system.

Figure 10. A comparative bar graph showing the Optimal number of PMUs using different algorithms under baseload conditions without the ZIB.

Figure 11. A comparative bar graph showing values of SORI using different algorithms under baseload conditions.
2. Power systems observability and PMU placement rules

Topological observability and Numerical observability are the methods applied to check for power system observability. The information about measurement types, locations, and network connectivity is based on logical operations in the topological observability method. The numerical observability method is based on the information of the measurement gain matrix or the numerical factorization of the measurement Jacobian. This paper used the topological observability method for the OPP problem [21, 22].

For a bus in a network to be topological observable, one of the following rules must be fulfilled [12, 23, 24].

i. The voltage phasor at a given bus and the current phasor of all the incident branches to the bus are all known if a PMU is connected to that bus. (Direct measurement)

ii. The voltage phasor of one end of a branch can be obtained if the voltage phasor of the bus connected to it and the incident current phasor of the branch are known (Pseudo-measurement).

iii. The current phasor of a branch can be obtained if the bus voltage phasors of the ends of the branch are known (Pseudo-measurement).

2.1. PMU problem formulation

The objective function of OPP is minimizing the number of PMUs in a network to minimize their installation cost. The objective function is formulated as shown in Eq. (1).

\[
F = \min \sum_{i=1}^{N} P_i x_i
\]

(1)

where;

- \( F \) refers to the objective function for OPP;
- \( x_i \) refers to a binary decision variable associate with bus \( i \);
- \( N \) refers to the total bus number of the network;
- \( P_i \) refers to the PMU price at bus \( i \);

The objective function is subjected to the observability constraint given by Eq. (3) to ensure that each bus is observable.

\[
F_i = \sum_{i=1}^{N} a_i x_i \geq Y
\]

(3)

Figure 12. Graph representation of the IEEE-14 bus showing the optimal PMU locations highlighted in green under normal baseload conditions without ZIB.

Figure 13. Optimal PMU placement on the IEEE-14 bus under normal baseload condition without the ZIB using the ABC algorithm.
2.2. System Observability Redundancy Index (SORI)

System Observability Redundancy Index (SORI) is the total bus coverage that the optimally placed PMUs can see in the power system. A higher SORI is an indicator of the reliability of the PMU-based monitoring systems. In this paper, SORI is used to determine the algorithm that gives the highest measurement redundancy. The SORI is computed using Eq. (6) [19].

\[
\text{SORI} = \sum_{i=1}^{N} a_{ij}x_{\text{PMU}}
\]

Where:
- \( F_i \) is the observability constraint of each bus that is associated with the PMUs;
- \( F_i = \begin{cases} 1 & \text{if bus } i \text{ is observable} \\ 0 & \text{if bus } i \text{ is unobservable} \end{cases} \) (4)
- \( Y \) is a vector of size \( N \) whose elements are all 1;
- \( a_{ij} \) refers to the binary connectivity matrix of the network, whose elements are:
- \( a_{ij} = \begin{cases} 1 & \text{if } i = j \\ 1 & \text{if buses } i \text{ and } j \text{ are connected} \\ 0 & \text{if buses } i \text{ and } j \text{ are not connected} \end{cases} \) (5)
- \( X_{\text{PMU}} \) is the optimal location and number of PMUs that can observe buses \( N \);

Figure 1 shows a 10-bus network topology that can be used as an example for applying the OPP problem whose connectivity matrix is as shown in Eq. (7).

\[
A = \begin{bmatrix}
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{bmatrix}
\]

Eq. (8) show the observability constraints subjected to the objective function. From the constraints, at least one binary variable must be nonzero. Thus, for instance, the observability of bus 9 is obtained when a PMU is located in at least one of the buses 3, 6, 7, 8, and 9. Likewise, the observability of bus 10 is obtained by placing a PMU in at least one of the buses 2, 6 and 10.

\[
F_1 = x_3 + x_4 + x_9 \geq 1 \\
F_2 = x_3 + x_4 + x_6 + x_9 + x_{10} \geq 1 \\
F_3 = x_3 + x_4 + x_6 + x_7 + x_9 \geq 1 \\
F_4 = x_1 + x_3 + x_5 + x_6 \geq 1 \\
F_5 = x_1 + x_3 + x_5 + x_9 \geq 1 \\
F_6 = x_1 + x_5 + x_6 + x_{10} \geq 1 \\
F_7 = x_2 + x_5 + x_6 + x_9 + x_{10} \geq 1 \\
F_8 = x_3 + x_6 + x_7 + x_8 + x_9 \geq 1 \\
F_9 = x_2 + x_6 + x_{10} \geq 1
\] (8)

From Figure 1, the network is fully observable using two PMUs placed at buses 2 and 6 or buses 4 and 8 which are the two possible optimal PMU location sets, whose solution vectors are as shown in Eq. (9).

\[
X = [0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 0\ 0\ 1\ 0\ 0\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 1\ 1\ 0\ 0
$$F_1 = x_1 + x_\text{radial bus} \geq 1$$

$$F_2 = x_1 + x_\text{radial bus} + x_4 \geq 1$$

$$F_3 = x_4 \geq 1$$

3. Identify the adjacent buses connected to ZIB which are not incident radial buses. Then, determine the Observability Index (OI) of the buses adjacent to the ZIB using Eq. (12).

$$OI = a_i y$$

(12)

Where:

- OI is the number of times a bus is observable by the adjacent buses;
- $a_i$ is a binary connectivity matrix obtained in Eq. (5);
- $y$ is an N size vector whose elements are all 1;
- The bus with the highest OI is merged with ZIB, as shown in Figure 3.

The observability constraint of the network before merging with the bus with the highest OI is as shown in Eq. (13),

$$F_1 = x_1 + x_\text{ZIB} \geq 1$$

$$F_2 = x_1 + x_\text{ZIB} + x_3 + x_4 \geq 1$$

$$F_3 = x_2\text{ZIB} + x_3 + x_4 \geq 1$$

$$F_4 = x_2\text{ZIB} + x_3 + x_4 \geq 1$$

(13)

The observability constraints after merging are reduced with the exclusion of the ZIB, as shown in Eq. (14).

$$F_1 = x_1 + x_\text{ZIB} \geq 1$$

$$F_2 = x_1 + x_\text{ZIB} + x_3 + x_4 \geq 1$$

$$F_3 = x_2\text{ZIB} + x_3 + x_4 \geq 1$$

$$F_4 = x_2\text{ZIB} + x_3 + x_4 \geq 1$$

(14)

3. Case studies

The case studies applied in this paper are the IEEE-14 bus, the IEEE-30 bus and the Nairobi region 30-bus system. The Nairobi Region is a 30 bus system made up of 132 KV and 220 KV transmission lines. The network consists of geothermal power, hydropower and thermal power. The data for the region was obtained from The Kenya Power and Lighting Company (KPLC). The networks are modelled in Figure 16.
PSAT (Power System Analysis Toolbox). The generation units are modelled as standard PV buses considering reactive power limits and the loads are modelled by constant PQ loads. Finally, the graph representation of the network is generated in PSAT before performing OPP.

4. Proposed methods

The Depth-First method [18], the Mixed Integer Linear Programming (MILP) using intlinprog solver [19], and the Artificial Bee Colony (ABC) algorithm [20] are applied for OPP considering the System Observability Redundancy Index (SORI). The methods are first used on the IEEE standard buses. The simulation results are verified with those in the literature before applying to the Nairobi region 30-bus system. The following subsection describes the procedure for the implementation of the OPP problem using the three methods.

4.1. The Depth-First method

The Depth First method is a stochastic optimization technique. The algorithm first explores the node linked with the most significant number of branches in the network. A PMU is then placed on this bus. When two or more nodes with the most significant number of branches are present, they are randomly selected for PMU placement. This procedure is applied to all the nodes in the network until the entire system is observable [18, 26]. The case studies are first modelled in Power System Analysis Toolbox (PSAT) in MATLAB software, as shown in Figures 9, 11, and 13. Then, the power flow analysis for running PMU placement is performed using the Newton Raphson method. Finally, the OPP problem is solved using the PMU placement toolbox of PSAT with the Depth-First method. Figure 4 shows the flowchart for using the Depth-First method for OPP in PSAT MATLAB.

4.2. Mixed Integer Linear Programming (MILP) using intlinprog solver

The intlinprog optimizer routine is a MILP solver implemented in MATLAB using the Branch and Bound Method (BBM) to achieve an optimal solution. This procedure is applied to all the nodes in the network until the entire system is observable [18, 26]. The case studies are first modelled in Power System Analysis Toolbox (PSAT) in MATLAB software, as shown in Figures 9, 11, and 13. Then, the power flow analysis for running PMU placement is performed using the Newton Raphson method. Finally, the OPP problem is solved using the PMU placement toolbox of PSAT with the Depth-First method. Figure 4 shows the flowchart for using the Depth-First method for OPP in PSAT MATLAB.

4.3. The ABC algorithm

Finally, the ABC algorithm is used for OPP on the three systems. The ABC algorithm was motivated by the foraging and waggle dance behaviours of honey bee colonies. The colony comprises three groups: employed bees, onlooker bees, and scout bees. First, the employed bees search for a food source (which represents the solution) around the locality they had visited earlier. Next, they disseminate the nature of the food source to other bees by performing a wangle dance. The onlooker bees then wait around the beehive to choose the employed bees to follow. Finally, the scout bee randomly searches for a search space to find a new food source. The scout bee, after finding a suitable solution, becomes an employed bee [28, 29, 30].

The number of food sources is equal to the number of employed bees, representing a possible solution. Therefore, in the OPP problem, each food source represents a viable solution for PMU placement. The following subsection describes the procedure for the application of the ABC algorithm for OPP [20]. The flowchart is also represented in Figure 6.

4.3.1. Initialization phase

In this phase, the ABC parameters for the OPP problem are defined. The parameters used for the study are as shown in Table 1. The food
sources \( (x_i) \) represent the possible solution for OPP and are randomly generated using Eq. (15). \( x_i^{\text{upper}} \) and \( x_i^{\text{lower}} \) are the upper and lower bounds of the food source. \( \text{rand} \) represents a random number chosen between 0 to 1 that controls the generation of \( x_i \).

\[
x_i = x_i^{\text{lower}} + \text{rand} (x_i^{\text{upper}} - x_i^{\text{lower}})
\]

(15)

4.3.2. The employed bee phase

In this phase, a food source is allocated to an employed bee to exploit. The new position of the food source \( (x_{ij}^{\text{new}}) \) which represents new PMU location is obtained using Eq. (16). \( \emptyset \) represents a random number between -1 and 1 that controls the occurrence of neighbour solutions. \( y \) must not be equal to \( i \) to facilitate the generation of a new solution for PMU location.

\[
x_{ij}^{\text{new}} = x_{ij} + \emptyset (x_i - x_{ij})
\]

(16)

The fitness value \( (f_i) \) of the PMU location generated are obtained using Eq. (17) through a greedy selection process. Finally, the solutions with the best fitness are used to update new PMU locations. \( f_i \) represents the OPP objective function of each food source.

\[
f_i = \begin{cases} 
1 & f_i \geq 0, \\
1 + \text{abs}(f_i) & f_i < 0,
\end{cases}
\]

(17)

4.3.3. The onlooker bee phase

In this phase, an onlooker bee selects a food source depending on its probability \( (p_i) \) obtained by Eq. (14). The selected food source then undergoes the same procedure as captured in the employed bee phase to generate a new solution.

\[
p_i = \frac{f_i}{\sum_{j=1}^{N} f_j},
\]

(18)

4.3.4. The scout bee phase

A bee turns into a scout bee in this phase if the food source fails to generate better solutions, even after attaining the set limit. Thus, the food source is neglected and a new one is obtained randomly using Eq. (15).

All the phases described above are executed until all the conditions are achieved. Finally, the best food source representing the best PMU locations is obtained as the output. The SORI value of each optimal location is then calculated using Eq. (6).

5. Simulation results and discussion

This section presents the simulation result for the OPP on the case studies, using the proposed methods. The first subsection gives the simulation result under normal baseload conditions without the inclusion of the ZIB and compares the results achieved using the techniques. The last section presents the results for the OPP on the standard IEEE bus systems using the ABC algorithm with the inclusion of the ZIB.

5.1. Normal baseload condition without ZIB

5.1.1. The Depth-First method

Table 2 shows the simulation results for the OPP using the Depth-First method. From the table, The IEEE-14 bus test system is completely observable with 6 PMUs whose locations are, 1, 4, 6, 8, 10 and 14, with a SORI of 22. The IEEE-30 bus is completely observable using 12 PMUs whose locations are; 3, 5, 6, 11, 12, 17, 18, 20, 21, 24, 26, and 27, with a SORI of 45. The results for the IEEE buses using the Depth-First method are first validated with those presented in [18] before applying the method to the Nairobi region 30-bus system. Finally, The Nairobi region 30-bus system is completely observable with 13 PMUs whose locations are; 1, 3, 5, 6, 7, 9, 13, 14, 17, 19, 21, 24 and 29 with a SORI of 42.

5.1.2. intlinprog solver

Table 3 shows the OPP results using the intlinprog solver in MATLAB. From the table, the IEEE-14 bus is completely observable with 4 PMUs whose locations are 2, 8, 10 and 13 with a SORI of 14. The IEEE-30 bus is completely observable with 10 PMUs whose locations are; 1, 5, 8, 10, 11, 12, 19, 23, 26, and 29 with a SORI of 35. The results for the IEEE buses using intlinprog are first validated with those presented in [19] before applying the method to the Nairobi region 30-bus system. Finally, the Nairobi Region 30-bus system is completely observable with 12 PMUs whose locations are; 2, 5, 6, 7, 10, 13, 15, 17, 21, 23, 25 and 29 with a SORI of 32.

5.1.3. The ABC algorithm

Finally, the ABC algorithm is applied for OPP on the IEEE-14 bus, IEEE-30 bus and the Nairobi region 30-bus system and the results are presented in Table 4. The algorithm gives multiple optimal solutions. Only the first three solution sets with the highest SORI values are presented in this paper. From the table, the IEEE-14 bus is completely observable with 4 PMUs, with three different Optimal PMU location sets, i.e. (2, 7, 11, 13), (2, 6, 7, 9) and (2, 6, 8, 9) with SORI values of 16, 19 and 17, respectively. The second set gives the highest SORI value of 19, with the highest measurement redundancy. The IEEE-30 bus is completely observable with 10 PMUs, with three different Optimal PMU locations sets i.e. (3, 5, 6, 10, 11, 12, 15, 18, 25, 27), (1, 2, 6, 9, 10, 12, 18, 21, 25, 27) and (2, 4, 6, 9, 10, 12, 15, 18, 25, 29) with SORI values of 46, 48 and 50, respectively. The third set gives the highest SORI value of 50, with the highest measurement redundancy. The Nairobi region 30-bus system is completely observable with 12 PMUs, with three different Optimal PMU locations sets i.e. (2, 3, 8, 9, 12, 14, 16, 19, 22, 25, 27, 29), (2, 5, 7, 9, 12, 14, 16, 20, 22, 24, 27, 29) and (2, 5, 6, 9, 11, 13, 14, 16, 21, 24, 27, 29) with SORI values of 40, 41 and 43, respectively.

5.2. The OPP algorithm with ZIB inclusion

Table 5 shows the OPP results using the ABC algorithm with the inclusion of the ZIB. The bold values represents the solution sets with the highest SORI.
The third set gives the highest SORI value of 43, with the highest measurement redundancy. The sets highlighted in grey show the optimal PMU placement with the highest SORI value for the three systems.

From Tables 2, 3, and 4, the Depth-First method and the intlinprog solver take the shortest time to achieve the optimal solution for all the case studies compared to the ABC algorithm that takes the highest computation time for all the case studies since it employs the randomization and local search technique.

Table 5 shows the comparison of the ABC algorithm results with those in the literature considering the SORI. From the table, all the methods give the same number, location and SORI value for the IEEE 14 bus system. For the IEEE 30 bus system, the methods provide the same number of PMU with different optimal location sets. The FPA (Flower Pollination Algorithm) and the BGWO (Binary Grey Wolf Optimization) algorithm provide the highest SORI values, followed closely by the ABC and the fmincon solver.

5.1.4. Convergence properties of the ABC algorithm

The simulations are performed on an HP computer of intel processor core i5 @2.30 HZ and 4 GB RAM. Figures 7, 8, and 9 show the convergence properties of the three case studies under normal base load conditions without the inclusion of the ZIB. From Figure 7, the algorithm converges in 5 iterations to achieve the optimal solution (best fitness function) for the IEEE-14 bus system. From Figure 8, the algorithm converges in 25 iterations to achieve the optimal solution (best fitness function) for the IEEE-30 bus system. From Figure 9, the algorithm converges in about 24 iterations to achieve the optimal solution (best fitness function) for the Nairobi region 30-bus system. From the simulation results, a larger system takes more time to converge than a smaller system. The IEEE-14 bus takes the shortest time to converge compared to the IEEE-30 bus and the Nairobi region 30 bus system.

5.1.5. Comparison of the results of the three methods

The comparative bar graph in Figure 10 compares the number of PMUs against the three algorithms for OPP under normal base load conditions without the ZIB. From the graph, the Depth-first method gives 6 PMUs for the IEEE-14 bus test system, 12 PMUs for the IEEE-30 bus and 13 PMUs for the Nairobi region 30-bus system. On the other hand, intlinprog and the ABC algorithm give the same number of PMUs for the three systems, i.e., 4 PMUs for IEEE-14 bus, 10 PMUs for IEEE-30 bus and 12 PMUs for the Nairobi region 30-bus system.

The graph in Figure 11 compares the SORI values for the three systems using the three algorithms. From the graph, For the IEEE-14 bus, the Depth-First method, the intlinprog solver and the ABC algorithm give SORI values of 22, 14 and 19, respectively. For the IEEE-30 bus, the Depth-First method, the intlinprog solver and the ABC algorithm give SORI values of 45, 35 and 50, respectively. Finally, for the Nairobi region 30-bus system, the Depth-First method, the intlinprog solver and the ABC algorithm give SORI values of 42, 32 and 43, respectively.

From the two graphs of Figures 10 and 11, the ABC algorithm gives the lowest number of PMUs for OPP for the three cases with the highest SORI. Therefore, the results for the ABC algorithm are taken as the best among the three methods. Figures 13, 15, and 17 show the OPP locations for the IEEE-14 bus, the IEEE-30 bus and the Nairobi Region 30-bus system using the ABC algorithm with their respective graph representations shown in Figures 12, 14, and 16.

5.2. Inclusion of ZIB

The proposed method for modelling the ZIB was implemented on the IEEE-14 bus and 30 bus systems using the ABC algorithm. Table 6 shows the location of the ZIBs and the incident radial buses.

Table 7 shows the results for OPP using the ABC algorithm with the inclusion of ZIB. From the table, it can be deduced that the ZIBs and the incident radial buses are excluded from PMU placement, which further reduces the number of PMUs to guarantee full power system observability. The results were then compared with those in the literature considering the inclusion of ZIB. The results are as shown in Table 8. From the table, the proposed ZIB approach gives the same optimal number of PMUs as those in the literature for the IEEE-14 bus with the same bus locations. In addition, the optimal number of PMUs for the IEEE-30 bus is the same but with different bus locations.

6. Conclusion

This paper verifies the performance of the IEEE standard buses and the real power system of the Nairobi Region 30-bus system using the existing methods; the Depth- First method, the MILP using the intlinprog solver, and the ABC algorithm, considering the System Observability Redundancy Index (SORI). The methods are first tested on the IEEE-standard buses to validate the results with those in the literature before implementing them on the real power system of the Nairobi region 30-bus system. From the simulation results, the methods’ performances on the IEEE-standard buses and the Nairobi region-30 bus system are slightly different. However, The ABC algorithm gives the optimal number and location of the PMUs on the three systems with the highest SORI. In addition, the simulation results of the real power system of the Nairobi Region 30 - bus system and the IEEE-30 bus system are slightly different, even though they have the same number of buses. The IEEE -30 bus system with the highest number of branches than the Nairobi Region -30 bus system gives the lowest number of PMU placement for complete power system observability. Therefore, the number of branches in a power system plays a critical role in determining the optimal number of PMUs for complete power systems observability and the SORI.

The Nairobi region 30-bus system results show the strategic positions and locations where PMUs should be placed to increase the monitoring and control of the system.

The study also proposed the modelling of the ZIB by considering the Observability Index (OI). The proposed approach is tested on the IEEE standard buses and the simulation results validated its effectiveness with those methods in the literature.
Declarations

Author contribution statement

Edwin Otieno Okendo: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Cyrus Wabuge Wekesa: Performed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.

Michael Juma Saulo: Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data.

Funding statement

This work was supported by the Pan African University Institute of Basic Sciences Technology and Innovation (PAUSTI).

Data availability statement

The authors do not have permission to share data.

Declaration of interests statement

The authors declare no conflict of interest.

Additional information

No additional information is available for this paper.

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