Tests of Higgs Boson Couplings at a $\mu^+\mu^-$ Collider†

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Abstract

We investigate the potential of a muon collider for testing the presence of anomalous Higgs boson couplings. We consider the case of a light (less than 160 GeV) Higgs boson and study the effects on the Higgs branching ratios and total width, which could be induced by the non standard couplings created by a class of $\text{dim} = 6 \ SU(3) \times SU(2) \times U(1)$ gauge invariant operators satisfying the constraints imposed by the present and future hadronic and $e^-e^+$ colliders. For each operator we give the minimal value of the $\mu^+\mu^-$ integrated luminosity needed for the muon collider ($\mu C$) to improve these constraints. Depending on the operator and the Higgs mass, this minimal $\mu C$ luminosity lies between 0.1 $fb^{-1}$ and 100 $fb^{-1}$.

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1 Introduction

One of the main goals of a future $\mu^+\mu^-$ Collider ($\mu C$), is to provide a Higgs boson factory [1]. It has been shown that with a good energy resolution (of the order of 0.003\%) and a reasonable integrated luminosity (of the order of the $fb^{-1}$), the mass and the total width and branching ratios of a light Higgs boson can be directly measured with a high accuracy [3]. In particular, the measurement of the mass and the total width, is the unique feature of a $\mu C$ [3]. In addition, the branching ratios can be more accurately measured than at a Linear $e^+e^-$ Collider (LC), as long as $m_H \lesssim 160$ GeV; i.e. as long as the total width $\Gamma_H$ is sufficiently small, so that the peak of the $\mu^+\mu^- \to H$ cross section is enhanced, and a large number of events is produced [2, 4].

The aim of the present paper is to discuss more precisely the potential of a $\mu^+\mu^-$ collider for the search of anomalous Higgs boson couplings. We assume that only a single light (i.e. $m_H \lesssim 160$ GeV) standard-like Higgs boson exists. Moreover, we assume that this Higgs boson may have anomalous couplings which can be generated by adding to the Standard Model (SM) lagrangian $L_{SM}$, a set of new physics (NP) terms associated to a high scale $\Lambda$, lying in the several TeV range. These NP terms are expressed in terms of all possible $dim = 6$ SU(3) $\times$ SU(2) $\times$ U(1) gauge invariant operators $O_i$ involving the various standard model fields and contributing with couplings $g_{eff}^i$. Thus, the contributions of each of these operators on the partial decay widths $\Gamma(H \to F)$, are determined by $g_{eff}^i$. Constraints on these coupling constants have already been established from the effects of the aforementioned operators in the gauge sector (at LEP1/SLC, LEP2 and TEVATRON)[8, 9, 10]. These constraints will be improved by further TEVATRON studies [11], as well as studies at LHC [12] and LC [13] (anticipated to run before the $\mu C$), in particular through the direct production of the Higgs boson. For each operator we collect the most stringent constraint on the associated coupling constant $g_{eff}^i$ that should be available by that time. We mention separately the constraints that could be obtained from the study of the $H\gamma\gamma$ couplings, if the $\gamma\gamma$ mode at a Linear Collider will also be available [14, 15].

Taking the accuracy at which a $\mu C$ can measure the Higgs total and partial widths, we subsequently determine the required integrated luminosity $\bar{L}(\mu\mu)$ needed in order the $\mu C$ to improved the above constraints. Hence, for each operator, we obtain the minimal value of $\bar{L}(\mu\mu)$ required for this improvement, as a function of the Higgs mass.

The contents of the paper is the following. In Section 2 we list the various $dim = 6$ operators affecting the Higgs couplings and give the most stringent constraints expected from studies at the colliders previous than $\mu C$. In Section 3, we describe the effects of these operators on the Higgs decay widths, and the luminosities required at a $\mu C$, for improving the previous constraints. For making the paper self-contained and avoiding normalization uncertainties, we have collected all the necessary analytic expressions in Appendix A. Finally, the results and their implications for the search of new physics, are summarized in Section 4.
2 The dim = 6 operators inducing anomalous Higgs couplings

The effective Lagrangian describing anomalous Higgs properties is written as:

$$\mathcal{L}_{NP} = \Sigma_i g^i_{eff} \mathcal{O}_i.$$  \hspace{1cm} (1)

We first consider purely bosonic operators. The full list\(^1\) has been given in \(\text[99]{[2, 3, 4]}\). Retaining only the operators affecting the Higgs boson couplings, this list includes the 8 CP-conserving operators:\(^2\):

$$\mathcal{O}_{\Phi i} = (D_{\mu} \Phi^{\dagger}) (\Phi^{\dagger} D^\mu \Phi), \quad g_{\Phi i}^{\Phi} = \frac{f_{\Phi_i}}{\Lambda^2} = \frac{f_{\Phi_i}}{v^2}$$  \hspace{1cm} (2)

$$\mathcal{O}_{BW} = \frac{1}{2} \Phi^{\dagger} B_{\mu\nu} \Phi \cdot \bar{\Phi}^{\dagger} \Phi = \frac{g g' f_{BW}}{2 \Lambda^2} \frac{f_{BW}}{v^2}$$  \hspace{1cm} (3)

$$\mathcal{O}_{BB} = (D_{\mu} \Phi^{\dagger}) B_{\mu\nu} (D_{\nu} \Phi), \quad g_{\Phi i}^{BB} = \frac{g' f_{B}}{2 \Lambda^2} \frac{f_{B}}{2 M_W^2} = \frac{g' \alpha_{BB}}{M_W^2}$$  \hspace{1cm} (4)

$$\mathcal{O}_{WW} = (\Phi^{\dagger} \Phi) \bar{\Phi}^{\dagger} \Phi = \frac{g f_{W}}{2 \Lambda^2} \frac{f_{W}}{2 M_W^2} = \frac{g \alpha_{WW}}{M_W^2}$$  \hspace{1cm} (5)

$$\mathcal{O}_{\Phi 2} = 4 \partial_{\mu} (\Phi^{\dagger} \Phi) \partial^\mu (\Phi^{\dagger} \Phi), \quad g_{\Phi 2}^{\Phi} = \frac{f_{\Phi 2}}{8 \Lambda^2} = \frac{f_{\Phi 2}}{v^2}$$  \hspace{1cm} (6)

$$\mathcal{O}_{GG} = (\Phi^{\dagger} \Phi) G^{\mu\nu} \cdot G_{\mu\nu}, \quad g_{\Phi 2}^{GG} = \frac{d_{G}}{v^2},$$  \hspace{1cm} (7)

where

$$\frac{g'}{g} = \frac{s_W}{c_W}, \quad \sqrt{2} G_F = \frac{1}{v^2} = \frac{4 M_W^2}{g^2},$$  \hspace{1cm} (8)

and 4 CP-violating ones

$$\widetilde{\mathcal{O}}_{BW} = \frac{1}{2} \Phi^{\dagger} B_{\mu\nu} \tau \cdot \bar{\Phi}^{\dagger} \Phi = \frac{\tilde{f}_{BW}}{v^2},$$  \hspace{1cm} (9)

$$\widetilde{\mathcal{O}}_{WW} = (\Phi^{\dagger} \Phi) W^{\mu\nu} \cdot \bar{\Phi}^{\dagger} \Phi = \frac{\tilde{f}_{WW}}{v^2},$$  \hspace{1cm} (10)

$$\widetilde{\mathcal{O}}_{BB} = (\Phi^{\dagger} \Phi) B^{\mu\nu} \cdot \bar{\Phi}^{\dagger} \Phi = \frac{\tilde{f}_{BB}}{v^2},$$  \hspace{1cm} (11)

\(^1\)We use the linear realization of the scalar sector, since we are investigating the case of a light Higgs particle.

\(^2\)The contribution of \[\mathcal{O}_{5, 6, 7}\] to the gauge kinetic energy is assumed to have been renormalized away.
\[
\widetilde{O}_{GG} = (\Phi^\dagger \Phi) G^{\mu\nu} \cdot \tilde{G}_{\mu\nu} \quad g_{eff}^{GG} = \frac{\tilde{G}}{v^2}, \quad (14)
\]

with\[\[\widetilde{V}^{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} (\partial_\alpha V_\beta - \partial_\beta V_\alpha).\]

We also consider possible modifications of the fermionic couplings of the Higgs boson.

A convenient \( dim = 6 \) operator for describing anomalous Higgs and heavy quark interactions was introduced in \( [6] \). In the case of the \( b \) quark it reads:

\[
O_{b1} = (\Phi^\dagger \Phi) [(\bar{f}_L \Phi) b_R + \bar{b}_R (\Phi^\dagger f_L)] \quad , \quad g_{eff}^{b1} = \frac{f_{b1}}{\Lambda^2} = \frac{\tilde{f}_{b1}}{v^2}, \quad (16)
\]

where \( f_L \) is the left handed doublet of the third family of quarks. The operator \( O_{b1} \) was motivated by the argument that if NP is associated to the origin of mass generation, it should also be characterized by a priority in generating anomalous couplings for the heavy particles (heavy quarks, possibly heavy leptons and Higgs bosons) and of course the gauge bosons \( [6] \).

In the present work we also generalize the operator \( O_{b1} \), to a convenient parametrization of anomalous \( Hff \) couplings for any fermion \( f \). For example for charged leptons we write

\[
O_{l1} = (\Phi^\dagger \Phi) [(\bar{l}_L \Phi) l_R + \bar{l}_R (\Phi^\dagger l_L)] \quad , \quad g_{eff}^{l1} = \frac{f_{l1}}{\Lambda^2} = \frac{\tilde{f}_{l1}}{v^2}, \quad (17)
\]

where \( l_L, l_R \) are the doublet, singlet of a given family of lepton.

### 2.1 Constraints on \( g_{eff}^{i} \) expected to be established before the Muon Collider run.

The coupling constants associated to each of these operators are submitted to constraints obtained or to be obtained, at present and future colliders expected to run before \( \mu C \); \( i.e. \) the LEP, SLC, TEVATRON, LHC, LC in its normal and \( \gamma \gamma \) mode. These arise from:

- virtual bosonic effects in \( e^+ e^- \rightarrow f \bar{f} \), which are already strongly constrained by existing precision measurements at the Z peak performed by LEP1/SLC \( [4] \), and will be further slightly improved by measurements at LEP2 and LC \( [8] \).

- direct effects in \( e^+ e^- \rightarrow W^+ W^- \) constrained by LEP2 and LC measurements \( [4] \); and also from \( W^+ W^-, W\gamma, WZ \) production at the hadron colliders TEVATRON and LHC \( [10] \).

- associate Higgs boson production processes through \( q\bar{q}' \rightarrow HW \) at the TEVATRON \( [11] \), and \( e^+ e^- \rightarrow HZ \) at LEP2 and LC \( [13] \).

\[^3\text{In \( [12] - [14] \), it is understood that instantonic contribution from the baryon number violating electroweak, as well as the QCD instantons, have been subtracted, so that only Higgs interactions are retained.}\]
• Higgs production through the process $gg \to H$ and $gg \to Hg$ at LHC [12].

• $\gamma\gamma \to H$, which should be constrained at an LC running in the $\gamma\gamma$ [7], [16].

For each bosonic operator, we have collected in Table 1a,b the most stringent constraint coming out of the above list of processes.

Table 1a: Upper limits on NP coupling constants: CP-conserving operators.

| $|f_{\phi 1}|$ | $|f_{BW}|$ | $|f_{B\Phi}|$ | $|f_{W\Phi}|$ | $|d_{W}|$ | $|d_{B}|$ | $|f_{\phi 2}|$ | $|d_{GG}|$ |
|-----------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|
| 0.002     | 0.0012    | 0.0056    | 0.002     | 0.001     | 0.0003    | 0.004     | 0.00015   |

Table 1b: Upper limits on NP coupling constants: CP-violating operators.

| $|f_{BW}|$ | $|d_{W}|$ | $|d_{B}|$ | $|d_{G}|$ |
|--------|--------|--------|--------|
| 0.005  | 0.004  | 0.0013 | 0.0007 |

The fermionic operators in (16, 17) induce purely Higgs-fermion anomalous interactions. No precise constraint have yet been set on the strength of this type of couplings. We express the coupling strength of these operators imposing $f_{f1} = 4\pi (f = l, b)$ and defining consequently the New Physics scale $\Lambda_{NP}$ through

$$|g_{eff}^{f_1}| = \frac{4\pi}{A_{NP}^2}.$$  \hspace{1cm} (18)

This scale can be compared to other fermionic scales, like e.g. those obtained from the four-fermion contact interactions in [17]. The measurement of the Higgs branching ratios at LC should then provide a lower bound on $\Lambda_{NP}$. For example, looking at $e^+e^- \to HZ$ through $B(H \to b\bar{b})$ at 250 GeV, with a luminosity of $\sim 100 fb^{-1}$, for $m_H = 130 GeV$, a lower limit on the scale $\Lambda_{NP}$ of the order of 40 TeV should be possible. We take this value as a starting point for looking at possible improvements with the muon collider.

3 NP effects on the Higgs total width and branching ratios

We now consider the effect of the $\text{dim} = 6$ operators on the partial decay widths $\Gamma(H \to F)$. Each operator is treated separately, and all necessary analytic expressions are given in the Appendix. For each interaction term $g_{eff}^{i}\mathcal{O}_i$, the relative NP effect on a partial width $\delta^{NP,i}(F)$ is defined through

$$\Gamma(H \to F) = \Gamma^{SM}(H \to F)[1 + \delta^{NP,i}(F)].$$  \hspace{1cm} (19)
The magnitude of $\delta^{NP,i}(F)$ is controlled by the constraints on the coupling constants $g_{eff}$ in Tables 1a,b. The corresponding relative effect on total Higgs width $\Gamma_H = \Sigma_F \Gamma(H \to F)$ is given by

$$\delta_H^{NP,i} = \Sigma_F [B(F)\delta^{NP,i}(F)],$$

while the one on the branching ratios $B(F) = \Gamma(H \to F)/\Gamma_H$ is

$$\delta_B^{NP,i}(F) = \delta^{NP,i}(F) - \Sigma_F [B(F)\delta^{NP,i}(F)].$$

It must be noticed that the NP effect on a given branching ratio $\delta_B^{NP,i}(F)$, may come either directly from the term $\delta^{NP,i}(F)$ in the channel considered, or indirectly from the NP effect in another channel contributing to the total width; (i.e. to the sum $\Sigma_F [B(F)\delta^{NP,i}(F)]$).

We next compare these effects with the experimental accuracies on the various Higgs branching ratios achievable at a muon collider. Following the procedure used in [2], we assume a gaussian $\mu^\pm$ beam energy resolution $\Delta \sim 2 \text{ MeV}(\sqrt{s}/100 \text{ GeV})$. Ignoring then initial state radiation effects, the peak cross section for the production of channel $F$ at $\sqrt{s} = m_H$ is given, [8], by

$$\bar{\sigma}(\mu^+\mu^- \to H \to F) \simeq \frac{4\pi}{m_H^2} \frac{B(H \to \mu^+\mu^-)B(H \to F)}{[1 + \frac{8\Delta^2}{\pi m_H^2}]}.$$  \hspace{1cm} (22)

while the total cross section is obtained by summing over all final states $F$ is

$$\bar{\sigma}_H \equiv \Sigma_F \bar{\sigma}(\mu^+\mu^- \to H \to F) \simeq \frac{4\pi}{m_H^2} \frac{B(H \to \mu^+\mu^-)}{[1 + \frac{8\Delta^2}{\pi m_H^2}]}.$$  \hspace{1cm} (23)

The statistical accuracies at which the measurements of $B(H \to \mu^+\mu^-)$ and $B(H \to F)$ can be achieved, are computed in terms of the number of events obtained from the cross sections of eq.(22), and the integrated luminosity $\bar{L}(\mu\mu)$. The main channels to study Higgs decay are the fermionic ones $f\bar{f}$ (with $f$ being either a $\mu$ or $\tau$ lepton, or a $c$ or $b$ quark), and $WW^*$, $ZZ^*$, $Z\gamma$, $\gamma\gamma$ and $gg$. The number of events setting the scale of the achievable accuracies for the various $B(H \to F)$ is evaluated from the SM predictions. Using the SM parts of the expressions given in Appendix A, with the QCD corrections defined in [19], we have reproduced the values of the branching ratios $B(H \to F)$ obtained in previous works.

We take into account the background, due to the $\mu^+\mu^- \to F$ annihilation through processes not involving a Higgs exchange in the $s$-channel. The main background processes (see [4]) are $\mu^+\mu^- \to b\bar{b}$ (due to $\gamma$ and $Z$ exchange), $\mu^+\mu^- \to WW^*$ (due to $\nu_\mu$, $\gamma$ and $Z$ exchanges), $\mu^+\mu^- \to ZZ^*$ ($\mu$ exchange), $\mu^+\mu^- \to \gamma\gamma$ ($\mu$ exchange) and $\mu^+\mu^- \to Z\gamma$ (due to $\mu$ exchange). Moreover, as a background for the gluon-gluon channel $\mu^+\mu^- \to H \to gg$, we consider the processes $\mu^+\mu^- \to q\bar{q}$ ($q = u, d, c, s$) due to $\gamma$ and $Z$ exchanges.

To be reasonably realistic, we also take into account some detection efficiencies. These amounts to reducing the number of events (due to the requirement of at least one leptonic decay) by the factors: 0.33 for $WW^*$, 0.098 for $ZZ^*$, and 0.067 for $Z\gamma$. This may be...
somewhat pessimistic, since we should keep in mind that some improvement could be obtained by using hadronic Z modes. For b quarks we use a detection efficiency of 50%. In the \( \gamma\gamma \) and \( Z\gamma \) channels we apply an angular cut of \( \cos\theta_{\text{em}} < 0.7 \).

Using these, we present in Tables 2 and 3 below the specific case of \( m_H = 130 \text{ GeV} \), for which \( \Gamma_H \simeq 4.67 \text{ MeV} \) and \( \bar{\sigma}_H \simeq 4 \times 10^4 \text{ fb} \), leading through (23) to a total number of about \( 4 \times 10^4 \) Higgs events, for an integrated luminosity \( \bar{L}(\mu\mu) \) in the range of \( 1 \text{ fb}^{-1} \).

The SM branching ratios presented in Table 2, indicate how these events are distributed among the various channels, and determine the achievable accuracies at \( \mu C \) indicated in Table 3. The corresponding accuracies for the more general case \( 0.1 \lesssim m_H \lesssim 0.18 \text{ TeV} \), are presented in Fig.1.

### Table 2: SM values of the Higgs branching ratios for \( m_H = 130 \text{ GeV} \)

| \( B(H \to F) \) | \( \mu^+\mu^- \) | \( \tau^+\tau^- \) | \( b\bar{b} \) | \( c\bar{c} \) | \( WW^* \) | \( ZZ^* \) | \( gg \) | \( \gamma\gamma \) | \( Z\gamma \) |
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.00020         | 0.057          | 0.52           | 0.024          | 0.28           | 0.035          | 0.073          | 0.0026         | 0.0021         |

### Table 3: Accuracies on \( B(H \to F) \) for \( m_H = 130 \text{ GeV} \)

\( \langle \delta_B(F) \rangle \) should be multiplied by \( \bar{L}(\mu\mu)^{-1/2} \), with \( \bar{L}(\mu\mu) \) measured in \( \text{fb}^{-1} \)

| \( \delta_B(F) \) | \( \mu^+\mu^- \) | \( b\bar{b} \) | \( WW^* \) | \( ZZ^* \) | \( gg \) | \( \gamma\gamma \) | \( Z\gamma \) |
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 0.015           | 0.020          | 0.022          | 0.084          | 0.085          | 1.2            | 4.8            |

We next compare the experimental accuracies \( \delta_B(F) \) to the corresponding relative shifts \( \delta^{\text{NP},i}_B(F) \) due to NP effects described by the various \textit{dim} = 6 operators in Eqs. (21) and the Appendix, and the results of Table 1a,b and Fig.1. Demanding \( \delta_B(F) < \delta^{\text{NP},i}_B(F) \) for each operator and each channel, we obtain the minimum value of \( \bar{L}(\mu\mu) \) required, so that \( \mu C \) provides an improvement of the results of the previous Colliders. These results are summarized in Fig.2a,b for the CP-conserving bosonic operators, in Fig.3 for CP-violating bosonic operators, and Fig.4 for the fermionic operators. In all cases only the most efficient channel id indicated. Finally, in Table 4a,b we repeat these results for the specific case \( m_H = 130 \text{ GeV} \); (the numbers in parenthesis refer to the improvements with respect to constraints expected from measurements in the \( \gamma\gamma \) mode of a LC).

### Table 4a: Required \( \mu^+\mu^- \) luminosity in \( \text{fb}^{-1} \) for \( m_H = 130 \text{ GeV} \):

**CP-conserving operators**

| \( \mathcal{O}_{\Phi 1} \) | \( \mathcal{O}_{BW} \) | \( \mathcal{O}_{B\Phi} \) | \( \mathcal{O}_{WW} \) | \( \mathcal{O}_{BB} \) | \( \mathcal{O}_{\Phi 2} \) | \( \mathcal{O}_{GG} \) |
|-----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| 413             | 12(27)         | 164            | 53             | 0.7(27)        | 0.3(32)        | 1.0            | 0.8 |
Table 4b: Required $\mu^+\mu^-$ luminosity in $fb^{-1}$ for $m_H = 130 GeV$:
CP-violating and fermionic operators

| $O_{BW}$ | $O_{WB}$ | $O_{BB}$ | $O_{GG}$ | $O_{b1}(\Lambda_{NP} = 50 TeV)$ | $O_{\mu1}(\Lambda_{NP} = 500 TeV)$ |
|----------|----------|----------|----------|----------------------------------|----------------------------------|
| 0.2(21)  | 15(36)   | 2(26)    | 3        | 0.7                              | 1                                |

A few comments about these results are now in order. The highest sensitivity arises from the Higgs decay channels $\gamma\gamma$, $Z\gamma$ and $gg$, for which the SM contribution is depressed by the loop factor $\alpha/\pi$ or $\alpha_s/\pi$. However, the $Z\gamma$ accuracy weakens because we require the use of the $Z$ leptonic branching ratio. So finally, the most stringent constraints arise for the operators contributing to the $\gamma\gamma$ and $gg$ channels. Below we comment separately on the various operators, assuming initially that LC will only work in its normal $e^-e^+$ mode.

$O_{\Phi1}$ leads to a wave function renormalization of the Higgs field which affects all modes, and to a direct $HZZ$ effect. Only this direct effect on the $ZZ$ channel is accessible through the study of the branching ratios, but the accuracy is reduced due to the small $Z$ leptonic branching ratio. Therefore, it will be difficult for the muon Collider to improve the previous constraints on this operator.

$O_{BW}$, $O_{BB}$, $O_{WW}$ and their CP-violating partners, affect directly the $HZZ$, $HZ\gamma$ and $H\gamma\gamma$ couplings. In these cases the existing constraints should easily be improved at $\mu C$.

$O_{B\Phi}$ only affects the $HZZ$ and $HZ\gamma$ couplings, and it will be more difficult to improve its constraint. The high value of the required luminosity could be reduced if one could use a better efficiency for the $Z\gamma$ channel. Here, we have pessimistically taken $\epsilon_{Z\gamma} = 0.067$, as given by the leptonic mode of the $Z$ only. For indication, if no reduction were applied ($\epsilon_{Z\gamma} = 1$), then the required luminosity would be 11 $fb^{-1}$.

Since $O_{WW}$ and $O_{WB}$ affect the $HWWW$ coupling, the $WW$ channel will allow to get better constraints for this operator. $O_{GG}$ and $O_{\tilde{G}G}$ affect the $Hgg$ mode and some improvement on the constraints to be set by LHC seems possible. On the other hand, $O_{\Phi2}$ only leads to a wave function renormalization of the $H$ field, so that no constraint can be obtained from branching ratios alone in this case.

In the case of the fermionic operators we have expressed the required luminosity in terms of the scale $\Lambda_{NP}$. For $O_{b1}$, using either the $b\bar{b}$ or (indirectly) the $WW$ channels, a luminosity of 1 $fb^{-1}$ (10 $fb^{-1}$) allows to reach a scale $\Lambda_{NP}$ of 60 TeV (105 TeV). For $O_{\mu1}$, the sensitivity is much higher, because the SM coupling is reduced by the small value of the muon mass. Thus, a luminosity of 1 $fb^{-1}$ (10 $fb^{-1}$) allows to reach a scale $\Lambda_{NP}$ of 500 TeV (900 TeV) in this case. This should allow an important improvement as compared to the constraints expected from LC.

Finally we note that for $O_{BW}$, $O_{WW}$, $O_{BB}$ and their CP-violating partners, the results depend also on whether the $\gamma\gamma$ mode at LC will run before the $\mu^+\mu^-$ collider. In this later case, the required luminosities should lie in the range of 10 – 20 $fb^{-1}$. Otherwise a fraction of $fb^{-1}$ would be sufficient.
3.1 Additional tests with the total Higgs width.

We have also looked at the possible improvements brought by a measurement of the total Higgs width, taking a few points around $s = M_H^2$. In [3] a relative accuracy of 16% was quoted for a luminosity of 0.4 $fb^{-1}$; while in [20], a scan with 0.1 $fb^{-1}$ should give an accuracy of about 10%. We assume that the relative uncertainty in the total Higgs width varies statistically in terms of the number of events and write

$$\delta_\Gamma = \frac{\delta_\Gamma}{\sqrt{L(\mu\mu)}}, \quad (24)$$

where $\delta_\Gamma$ should be of the order of 0.03. We then directly compare $\delta_\Gamma$, to the relative NP effect $\delta_N^{NP,i}$ on the total width defined in eq.(21).

A priori, one could expect an improvement on the operators contributing to the main decay modes ($b\bar{b}, WW, ZZ$); as these channels would sensibly affect the total width, and were not much constrained by the study of the branching ratios. This is the case of $O_{\Phi,1}$, $O_{\Phi,2}$, $O_{BB}$ and $O_{b1}$. However with the accuracy assumed in eq.(24), it turns out that only $O_{\Phi,2}$ can be constrained, (which was not at all constrained by the branching ratios). Thus, an improvement of the constraint for this operator quoted in Table 1a, will appear as soon as $\bar{L}(\mu\mu) > 1 \, fb^{-1}$.

For $O_{\Phi,1}$, an improvement of the 400 $fb^{-1}$ luminosity level required by the study of the branching ratios, would only appear if the Higgs is light and $\delta_\Gamma \lesssim 0.02/\sqrt{L(\mu\mu)}$. For $m_H = 130$ GeV and $\bar{L}(\mu\mu) \approx 100 \, fb^{-1}$, this means an accuracy of about 0.01 MeV on the total width, which is probably impossible to achieve.

For $O_{b1}$ and $O_{BB}$, an accuracy of about $\delta_\Gamma \lesssim 0.01/\sqrt{L(\mu\mu)}$ is needed, in order to improve the results obtained from the study of the branching ratios. For other operators, like the CP-violating ones, an even smaller (rather unrealistic) $\delta_\Gamma$ is needed for an improvement from the total Higgs width measurement to arise.

4 Conclusions

We have studied under what conditions a $\mu^+\mu^-$ collider working as a Higgs factory, could improve the present and near future constraints on anomalous Higgs boson couplings.

These anomalous couplings are described by the set of $dim = 6 \, SU(3) \times SU(2) \times U(1)$ gauge invariant operators consisting of the 8 bosonic CP-conserving ones $O_{\Phi,1}$, $O_{BW}$, $O_{BB}$, $O_{W\Phi}$, $O_{WW}$, $O_{BB}$, $O_{GG}$; the 4 bosonic CP-violating ones $\tilde{O}_{BW}$, $\tilde{O}_{WW}$, $\tilde{O}_{BB}$, $\tilde{O}_{GG}$ and the fermionic operators $O_{b1}$, $O_{\mu1}$. For each of these operators we have taken the most stringent direct or indirect constraints expected from studies at the leptonic colliders, LEP, SLC, LC (in its $e^+e^-$ and $\gamma\gamma$ modes) and the hadronic colliders (TEVATRON and LHC), that will run before the muon collider.

We have then looked at the effects of these anomalous couplings on the Higgs branching ratios and the total width; and we have established the minimal integrated luminosity
needed for the µC to improve the constraints on each of the above operators, imposed by the Colliders expected to run previously. This analysis applies to Higgs boson masses below the WW threshold \((m_H \lesssim 2M_W)\), so that \(\Gamma_H\) is sufficiently small and the peak of the cross section sufficiently enhanced, to make the rare Higgs decay modes \(gg, \gamma\gamma\) and \(Z\gamma\) observable.

In this case one finds (see Fig.2-4) that for most of the operators (except for \(O_{\Phi,1}\), \(O_{B\Phi}\)), an integrated luminosity \(\bar{L}_{\mu\mu} \sim \text{few fb}^{-1}\) will be sufficient for improving the previous constraints. The special feature of the \(\mu^+\mu^-\) collider creating these improvements is that it provides good accuracies for the modes \(\mu^+\mu^-, b\bar{b}, \text{and } WW^*\). The first two modes should allow also to set unique constraints on the fermionic operators involving the Higgs field. For example, a luminosity of \(1 \text{ fb}^{-1}\) would allow to test fermionic scales of the order of 60, 500 TeV for \(O_{b1}, O_{\mu1}\). Such scales are higher than those accessible at LC, HERA, LHC for the four-fermion operators \(\text[n7]{}\). In addition these Higgs-fermion operators are of totally different nature and are perhaps more closely related to the role of NP in the mass generation mechanism.

The accuracy is worse for the \(gg\) mode and for the rare modes \(\gamma\gamma\) and \(Z\gamma\). Nevertheless, these channels are very sensitive to anomalous couplings, because the depressed SM contribution, and should provide very good constraints on the related NP couplings. In fact for most of the bosonic operators the best constraints come from these rare modes.

Another unique feature of the \(\mu^+\mu^-\) collider is to provide a good measurement of the total Higgs width, that cannot be obtained by any other means. This allows to constrain NP effects leading merely to a renormalisation of the Higgs couplings, which cannot be seen by solely studying the branching ratios; like \(\text{e.g.}\) the effects of \(O_{\Phi,2}\).

We finally note (using unitarity relations \(\text[c6, c21]\)), that the values of the NP scales to which these new constraints correspond, lie in the range of several tens of TeV. This is a domain where many theoretical models expect NP to show up.

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Note added in proof

When establishing the minimal integrated luminosity needed for the μC to improve the constraints on each of the considered operators, imposed by the Colliders expected to run previously; we have assumed that a luminosity of 100 $fb^{-1}$ could be accumulated at a linear $e^+e^-$ collider (LC). After completion of this work we were informed by H. Schreiber and P. Zerwas that the collider TESLA considered at DESY should be able to accumulate 1 $ab^{-1}$ in about 3 years of operation. Such a performance should allow to improve the determination of the $HZZ$ coupling and of the Higgs branching ratios through the process $e^+e^- \rightarrow HZ$. If the photon-photon mode could be put in operation, a similar improvement on the determination of the $H\gamma\gamma$ couplings will accordingly occur.

Such a factor 10 increase in the LC luminosity should reduce by a factor 3 the upper limits in the coupling constants $|f_{BW}|$, $|f_{B\Phi}|$, $|f_{W\Phi}|$, $|d_W|$, $|d_B|$, $|\bar{f}_{BW}|$, $|\bar{d}_W|$, $|\bar{d}_B|$ given in Table 1a,b; which in turn means that the minimum $\mu C$ luminosity required for the operators $O_{BW}$, $O_{B\Phi}$, $O_{W\Phi}$, $O_{WW}$, $O_{BB}$, $O_{\Phi 2}$, $\bar{O}_{BW}$, $\bar{O}_{WW}$, $\bar{O}_{BB}$ should also be increased by a factor of 10. For example the values lying between 0.1 and 10 $fb^{-1}$ for these operators in Fig.1,2 would now lie between 1 and 100 $fb^{-1}$.

The characteristic features of the $\mu^+\mu^-$ collider in providing a very good measurement of the total Higgs width, and of the $\mu^+\mu^-$ branching ratio and the Higgs partial widths, remain unchanged in this new comparison.
Appendix A: The partial decay widths of the Higgs boson

In this appendix we define the partial widths for the decay $H \rightarrow F$ of an off-shell Higgs particle by

$$\Gamma_{H \rightarrow F}(s) = \frac{(2\pi)^4}{2m_H} \left| T_{H \rightarrow F}(s) \right|^2 d\Phi_F ,$$

(A.1)

where $\Phi_F$ gives the usual definition of the invariant phase space [22]. Note that the off-shellness only appears in the invariant amplitude $T_{H \rightarrow F}(s)$.

A.1 $H \rightarrow \gamma\gamma$.

Contributions to this process arise from the SM at 1 loop [23], and from operators $O_{BW}$, $O_{WW}$, $O_{BB}$, their CP-violating partners, and also $O_{\Phi_1}$, $O_{\Phi_2}$ from $Z_H$. The result is

$$\Gamma_{H \rightarrow \gamma\gamma}(s) = \frac{\sqrt{2}G_F}{16\pi m_H} s^2 \left\{ \left| \frac{\alpha A}{4\pi} F_t + F_W \right| \sqrt{Z_H - 2d_W s_W^2 - 2d_B c_W^2 + \bar{f}_{BW} s_W c_W} \right|^2
+ \left| 2d_W s_W^2 + 2d_B c_W^2 - \bar{f}_{BW} s_W c_W \right|^2 \right\} ,$$

(A.2)

in which the Higgs wave function renormalization $Z_H$ is determined by the tree level NP contribution of $O_{\Phi_1}$, $O_{\Phi_2}$ and is given by

$$Z_H = [1 + 8\bar{f}_{\Phi_2} + \frac{1}{2}\bar{f}_{\Phi_1}]^{-1} ,$$

(A.3)

while the standard contribution arises from top and $W$ loops respectively determined by

$$F_t = -2t_t(1 + (1 - t_t)f(t_t)) ,$$

(A.4)

$$F_W = 2 + 3t_W + 3t_W(2 - t_W)f(t_W) ,$$

(A.5)

in terms of

$$f(t) = \left[ \sin^{-1}(1/\sqrt{t}) \right]^2 \quad \text{if} \quad t \geq 1 ,$$

$$f(t) = -\frac{1}{4} \left[ \ln \left( \frac{1 + \sqrt{1 - t}}{1 - \sqrt{1 - t}} \right) - i\pi \right]^2 \quad \text{if} \quad t < 1 ,$$

(A.6)

where $t_t = 4m_t^2/s$ and $t_W = 4m_W^2/s$.

A.2 $H \rightarrow \gamma Z$.

Contributions arise here also from the SM 1 loop top and $W$ contributions [23], as well as from $O_{BW}$, $O_{WW}$, $O_{BB}$, their CP violating analogs, and also from the operators $O_{B\Phi}$. 

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The result is
\[
\Gamma_{H\rightarrow \gamma Z}(s) = \frac{\sqrt{2}G_F s^2}{8\pi m_H^2}(1 - \frac{M_Z^2}{s})^3 \left( |\alpha| \frac{A_t + A_W}{4\pi} \sqrt{Z_H} + 2(d_W - d_B) s_W c_W \right.
\]
\[
- \frac{1}{2} (c_W^2 - s_W^2) \bar{f}_{BW} - \frac{s_W}{c_W} (\bar{f}_B - \bar{f}_W) \right|^2
\]
\[
+ \left[ 2(\bar{d}_W - \bar{d}_B) s_W c_W - \frac{1}{2} (c_W^2 - s_W^2) \bar{f}_{BW} \right]^2)
\]
\]
\] (A.7)

with
\[
A_t = \frac{(-6 + 16s_W^2)}{3s_W c_W} \left[ I_1(t_t, l_t) - I_2(t_t, l_t) \right]
\] (A.8)

\[
A_W = -\cot \theta_W [4(3 - \tan^2 \theta_W) I_2(t_W, l_W) + (1 + \frac{2}{t_W}) \tan^2 \theta_W - (5 + \frac{2}{t_W}) I_1(t_W, l_W)]
\] (A.9)

where \( t_t = 4m_t^2/s \), \( t_W = 4M_W^2/s \) as before, and \( l_t = 4m_t^2/M_Z^2 \), \( l_W = 4M_W^2/M_Z^2 \), and

\[
I_1(a, b) = \frac{ab}{2(a - b)} + \frac{a^2b^2}{2(a - b)^2} [f(a) - f(b)] + \frac{a^2b}{(a - b)^2} [g(a) - g(b)]
\] (A.10)

\[
I_2(a, b) = -\frac{ab}{2(a - b)} [f(a) - f(b)]
\] (A.11)

\( f(t) \) is given in (A.6) and

\[
g(t) = \sqrt{t - 1} \sin^{-1}\left(\frac{1}{\sqrt{t}}\right) \quad \text{if} \quad t \geq 1
\]

\[
g(t) = \frac{1}{2} \sqrt{1 - t} \left[ \ln \left(\frac{1 + \sqrt{1 - t}}{1 - \sqrt{1 - t}}\right) - i\pi \right] \quad \text{if} \quad t < 1
\] (A.12)

A.3 \( H \rightarrow gg \).

Contributions arise from the SM 1-loop top exchanges and from tree level contribution of the operators \( O_{GG} \) and \( O_{\Phi_1}, O_{\Phi_2} \). The result is

\[
\Gamma_{H\rightarrow gg}(s) = \frac{s^2}{8\pi m_H^2} \left[ 1 + \left( \frac{95}{4} - \frac{7N_F}{6} \right) \frac{s}{s} \right] \left| A_{SM} \sqrt{Z_H} - \frac{4d_G}{v} \right|^2 + \left| \frac{4\bar{d}_G}{v} \right|^2
\] (A.13)

where

\[
A_{SM} = -\frac{\alpha_s t_t}{2\pi v} (1 + (1 - t_t)f(t_t))
\] (A.14)

with \( t_t = 4m_t^2/s \) and \( f(t) \) given in (A.6). Note the presence of an important QCD correction factor which, (for the number of light quark flavours \( N_F = 5 \) is of the order of 65%.
A.4 \( H \to WW \).

Contributions arise from the SM at tree level and from operators \( O_{W\Phi}, O_{WW}, \tilde{O}_{WW}, \) as well as from \( O_{\Phi_1}, O_{\Phi_2} \) which induce a wave function renormalization of the Higgs field. For \( m_H > 2M_W \), we get

\[
\Gamma_{H \to W^+ W^-}(s) = \frac{\alpha \beta_W}{16\pi s_W M_W^2 m_H} \left[ \alpha \beta_W \right] \left[ 2 \left( 2M_W^2 \sqrt{Z_H} - 2d_W (s - 2M_W^2) - \tilde{f}_W s \right) \right]^2 \\
+ \left[ \sqrt{Z_H (s - 2M_W^2)} - 4d_W M_W^2 - \tilde{f}_W s \right]^2 \\
+ 8|\tilde{d}_W|^2 (s - 4M_W^2) \right) \] (A.15)

in which \( \beta_W = \sqrt{1 - 4M_W^2/s} \).

For \( M_W < m_H < 2M_W \), the Higgs decay width is computed with one virtual gauge boson decaying into a lepton or quark pair. The expression is \[23, 24, 12\]

\[
\Gamma_{H \to WW}(s) = \frac{3\alpha_s^2}{32\pi m_H s_W^{\frac{1}{2}}} \left[ (\sqrt{Z_H} - \tilde{f}_W 2 s) D_{SM}(x) + d_W \sqrt{Z_H} D_1(x) - \tilde{f}_W \sqrt{Z_H} D_4(x) \right] \\
+ 8d_W^2 D_2(x) + \tilde{f}_W^2 D_5(x) - d_W \bar{f}_W D_6(x) + 8|\tilde{d}_W|^2 D_3(x) \] (A.16)

where \( x = m_W^2/s \) and

\[
D_{SM}(x) = \frac{3(20x^2 - 8x + 1)}{\sqrt{4x - 1}} \cos^{-1} \left( \frac{3x - 1}{2x^{3/2}} \right) \\
- (1 - x) \left( \frac{47x}{2} - \frac{13}{2} + \frac{1}{x} \right) - 3(2x^2 - 3x + \frac{1}{2}) \ln(x) \ , \quad (A.17)
\]

\[
D_1(x) = \frac{24(14x^2 - 8x + 1)}{\sqrt{4x - 1}} \cos^{-1} \left( \frac{3x - 1}{2x^{3/2}} \right) \\
+ 12(x - 1)(9x - 5) - 12(2x^2 - 6x + 1) \ln(x) \ , \quad (A.18)
\]

\[
D_2(x) = \frac{54x^3 - 40x^2 + 11x - 1}{x \sqrt{4x - 1}} \cos^{-1} \left( \frac{3x - 1}{2x^{3/2}} \right) \\
+ \frac{(x - 1)(89x - 82 + \frac{17}{x})}{6} - (3x^2 - 15x + \frac{9}{2} - \frac{1}{2x}) \ln(x) \ , \quad (A.19)
\]

\[
D_3(x) = \frac{-28x^2 + 11x - 1}{x \sqrt{4x - 1}} \cos^{-1} \left( \frac{3x - 1}{2x^{3/2}} \right) \\
- \frac{x^2 - 21x}{6} + \frac{27}{2} - \frac{17}{6x} + \left( \frac{6x^2 - 9x + 1}{2x} \right) \ln(x) \ , \quad (A.20)
\]
\[ D_4(x) = -\sqrt{4x - 1} \frac{(10x - 4)}{x} \cos^{-1} \left( \frac{3x - 1}{2x^{3/2}} \right) \]

\[ - \left( \frac{x - 1}{3x^2} \right) (2x^3 + 50x^2 - 31x + 3) + \left( 6x - 9 + \frac{2}{x} \right) \ln(x) \quad , \quad (A.21) \]

\[ D_5(x) = \frac{3\sqrt{4x - 1}}{4x^2} \cos^{-1} \left( \frac{3x - 1}{2x^{3/2}} \right) \]

\[ + \frac{1}{16x^3} (1 - x)(x^4 - 7x^3 - 9x^2 - 25x + 4) - \frac{3}{4x^2} (x^2 + x - \frac{1}{2}) \ln(x) \quad (A.22) \]

\[ D_6(x) = \frac{4(14x^2 - 13x + 2)}{x\sqrt{4x - 1}} \cos^{-1} \left( \frac{3x - 1}{2x^{3/2}} \right) + \frac{2(1 - x)}{3x} (x^2 - 17x + 28) \]

\[ + 2(9 - \frac{2}{x}) \ln(x) \quad . \quad (A.23) \]

### A.5 \( H \to ZZ \).

Contributions arise from the SM at tree level and from operators \( O_{BW}, O_{B\Phi}, O_{W\Phi}, O_{WW}, O_{BB}, \bar{O}_{BW}, \bar{O}_{WW}, \bar{O}_{BB} \) as well as \( O_{\Phi_1}, O_{\Phi_2} \) through the wave function renormalization of the Higgs field. For \( m_H > 2M_Z \), we get

\[
\Gamma_{H \to ZZ}(s) = \frac{\alpha \beta_Z}{32 s_W^2 M_W m_H} \left( 2 \left[ 2M_Z^2 (\sqrt{Z_H} + \tilde{f}_{\Phi_1}) - (2d_B s_W^2 + 2d_W c_W + \bar{f}_{BW} s_W c_W)(s - 2M_Z^2) - (\tilde{f}_W + \tilde{f}_B s_W^2) s \right]^2 \right.

\[ - \left[ (\sqrt{Z_H} + \tilde{f}_{\Phi_1})(s - 2M_Z^2) - 2M_Z^2 (2d_B s_W^2 + d_W c_W^2) \right]^2 + 3d_B s_W^2 + 2d_W c_W^2 + \bar{f}_{BW} s_W c_W \right] M_H^2 (s - 4M_Z^2) \bigg) , \quad (A.24) \]

with \( \sqrt{Z_H} \) given by eq.(A.3) and \( \beta_Z = \sqrt{1 - 4M_Z^2/s} \).

For \( M_Z < m_H < 2M_Z \), we get

\[
\Gamma_{H \to ZZ^*}(s) = \frac{\alpha^2 s}{128 \pi m_H s_W^4 c_W^4} \left( 7 - \frac{40 s_W^2}{3} + \frac{160 s_W^4}{9} \right) \cdot \left[ \left( \sqrt{Z_H} + \tilde{f}_{\Phi_1} - \frac{1}{2x} (\tilde{f}_W + \tilde{f}_B s_W^2 c_W^2) \right)^2 D_{SM}(x) \right.

\[ + (\sqrt{Z_H} + \tilde{f}_{\Phi_1}) \left( d_B s_W^2 + d_W c_W^2 + \frac{\bar{f}_{BW}}{2} c_W s_W \right) D_1(x) \bigg) \]

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\[-(\sqrt{Z_H} + \tilde{f}_{\Phi,1})(\tilde{f}_W + \tilde{f}_{B}\frac{s_{W}^{2}}{c_{W}^{2}})D_4(x)\]
\[+2(2d_{Bs}^{2} + 2d_{c}^{2} + \tilde{f}_{BW}c_{W}c_{W})^{2}D_2(x) + \left(\tilde{f}_W + \tilde{f}_{B}\frac{s_{W}^{2}}{c_{W}^{2}}\right)^{2}D_5(x)\]
\[+\frac{1}{2}(2d_{Bs}^{2} + 2d_{c}^{2} + \tilde{f}_{BW}c_{W}c_{W})\left(\tilde{f}_W + \tilde{f}_{B}\frac{s_{W}^{2}}{c_{W}^{2}}\right)D_6(x)\]
\[+2|((2\tilde{d}_{Bs}^{2} + 2\tilde{d}_{c}^{2} + \tilde{f}_{BW}c_{W}c_{W})|^{2}D_3(x)|^{2}\]
(A.25)

where \(x = \frac{M_{\tilde{H}}^{2}}{s}\).

**A.6 \(H \to f\bar{f}\).**

Contributions arise from the SM at tree level, from operators \(O_{\Phi,1}, O_{\Phi,2}\) through the wave function renormalization of the Higgs field \(Z_H\), (compare (A.3)), and from the fermionic operator \(O_{f1}\). We find

\[\Gamma_{H \to f\bar{f}}(s) = \frac{N_{c}s}{8\pi m_{H}}\frac{m_{f}}{v}\left|\frac{\sqrt{Z_H} + \frac{3}{2\sqrt{2}}\tilde{f}_{f1}}{2\sqrt{2}}\right|^{2}\]
(A.26)

with the colour factor

\[N_{c} = 1\quad\text{for leptons ,}\]
(A.27)
\[N_{c} = 3\left(1+5.67\frac{\alpha_{s}}{\pi}\right)\quad\text{for quarks ,}\]
(A.28)

and \(\beta_{f} = \sqrt{1 - 4m_{f}^{2}/s}\). In the case of quarks \((f = q)\), the mass \(m_{f}\) is the running mass \(m_{q}(m_{H})\) computed with the expression given in ref. [19].
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Figure 1: Accuracies on the various branching ratios $B(H \rightarrow F)$ versus the Higgs mass. (The indicated values should be multiplied by $\bar{L}(\mu\mu)^{-\frac{1}{2}}$, the integrated luminosity $\bar{L}(\mu\mu)$ being measured in $fb^{-1}$).
Figure 2: $\mu^+\mu^-$ luminosity needed to improve the constraints on the CP-conserving operators obtained from an LC used in the $e^+e^-$ (solid) or Laser backscattering (dash) mode. Only the most efficient decay channel is indicated.
Figure 3: $\mu^+\mu^-$ luminosity needed to improve the constraints on the CP-violating operators obtained from an LC used in the $e^+e^-$ (solid) or Laser backscattering (dash) mode. Only the $\gamma\gamma$ decay channels is indicated, which is the most efficient. The dash line contributions of are for $O_{BB}$, $O_{BW}$ and $O_{WW}$ and have the same relative ordering as the solid-line results.
Figure 4: $\mu^+\mu^-$ luminosity needed to reach the new physics scale $\Lambda_{NP}$ corresponding to the operator $O_{b1}$ (a), $O_{\mu 1}$ (b), for the indicated decay channel.