An Anatomy of Neutrino Oscillations

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Abstract

To understand neutrino oscillations with neutrinos treated as point-like Dirac particles, we describe how to use an off-diagonal (cross-generation) neutrino-Higgs (mass) interaction to simulate oscillations in a natural way. This results in an extra orthogonal $SU_f(3)$ family gauge theory, which cooperates with the Minimal Standard Model to form a unique extended Standard Model. Altogether, it may help us to resolve a few outstanding puzzles - the question of why there are only three generations, the question of why the masses of neutrinos are so tiny, the question of why neutrinos oscillate, and the question of why the dark-matter world is so huge (25%) as compared to the visible ordinary-matter world (5%).

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1 Introduction

What are neutrinos? Owing to the fact that they have masses, neutrinos may be point-like Dirac particles, to be regarded as the typical species as one kind of the building blocks of the Standard Model [1]. In this language, a non-zero mass means a non-zero vacuum expectation value (VEV) of certain form, that is, coupling to a scalar field of certain kind. We also know that neutrinos oscillate - a neutrino of certain flavor can suddenly change into that of different flavor. This is a miracle for a point-like particle. It might be a species belonging to the dark-matter world, which is known to be five times bigger than the ordinary-matter world (as described by the minimal Standard Model) - in the sense that they interact with dark-matter particles.

Let us realize [2] that neutrino oscillation is the sudden change of neutrino flavor - a certain violation of lepton flavor for these point-like Dirac particles. As illustrated by Fig. 1, there are many ”induced” lepton-flavor-violating decays or reactions among the charged leptons such as $\mu \to e + \gamma$, $\eta/\eta' \to e + \mu$, etc.

Certain LFV processes [3] such as $\mu \to e + \gamma$ [3] and $\mu + A \to A^* + e$ are, in fact, closely related to the most cited picture of neutrino oscillations. See Fig.1 for the illustration. The vertex for the $\nu_\mu \to \nu_e$ transition is in fact coupled to some family Higgs field with some VEV. Early on [2][4], it was realized that the cross-generation or off-diagonal neutrino-Higgs
interaction may in fact serve as the detailed mechanism of neutrino oscillations, with some vacuum expectation value of the new Higgs field(s), granting that neutrino are described as point-like Dirac particles. This case should be contrast with the oscillations in the composed systems such as the $K^0 - \bar{K}^0$ system.

![Feynman diagrams](image)

Figure 1: The leading diagrams for $\mu \rightarrow e + \gamma$, showing a close connection with neutrino oscillations.

We have so far very limited neutrino sources for experimentation, namely, reactor neutrinos, accelerator neutrinos, and solar neutrinos. Nuclear reactors manufacture mostly the electron-like antineutrinos ($\bar{\nu}_e$), while GeV accelerators produce a lot of muon-like neutrinos or antineutrinos, and solar neutrinos are electron-like. But this would be the case before the first neutrino oscillation takes place. When they are oscillating away, the energy consideration indicates that only GeV accelerator neutrinos could be seen in $\tau^\pm$ or in $e^\pm$. The experimental search should be given in this direction - to look for $\tau^\pm$ or $e^\pm$ in a GeV accelerator.

This is not the first example of oscillations, such as in the $K^0 - \bar{K}^0$ system, but this is indeed the first example of oscillations for "point-like" (Dirac) particles themselves. In the composite $K^0 - \bar{K}^0$ system, we could draw the Feynman diagrams that would illustrate the underlying mechanism in terms of the minimal Standard Model. But how do we do this for neutrino oscillations? It is a kind of neutrino-flavor changing interaction, at the level of Standard-Model building blocks, very fundamental indeed [2].

Neutrino oscillations call for a lepton-flavor-violation (LFV) interaction, which brings a neutrino of a given flavor into that of a different flavor [2]. Thus, neutrino masses and neutrino oscillations may be regarded as one of the most important experimental facts over the last thirty years [3].

We may be reasoning as follows: Using the language of the renormalizable quantum field theory as example, neutrino oscillations would correspond to an off-diagonal, cross-generation matrix in the three-neutrino space. It is bilinear in the Dirac fields, so that the coupling to the Higgs field, with some vacuum expectation value, is the only choice, if we insist on the requirement of renormalizability. This Higgs field is the "family" Higgs field, as it operates in the (lepton) family space [2].
2 Lepton-flavor-violating interactions

After these introductions, neutrinos are some objects difficult to understand. Neutrino oscillations would be the interactions in which neutrino flavors change in the transition. Taking into account neutrino oscillations is equivalent to introduction of lepton-flavor-violating interactions. Using renormalizable quantum field theory, it has to be off-diagonal bilinear in Dirac fields multiplied by the (family) Higgs field with some expectation vacuum values [2].

Using accelerators, we could produce the "high-energy" neutrino or antineutrino beam, in the energy greater than 2 GeV (i.e. the mass). Considering the $\bar{\nu}_\mu - p$ scattering (the only high-energy neutrino beam available to us, at this point), a fraction of $\bar{\nu}_\mu$ might convert to $\tau$, thus making it possible to produce $\tau^+$. Similarly, for production of $e^+$ via $\nu_\mu$.

To describe the reaction $\bar{\nu}_\mu(k) + p(p) \rightarrow \mu(k') + n(p')$, we introduce \[5\]

$$< n(p') | I_{\lambda}^{(-)}(0) | p(p) > = i \bar{\nu}(p') \{ \gamma_\lambda f_\nu(q^2) - \frac{\sigma_{\lambda \nu} q_\nu}{2m_p} f_M(q^2) \} u(p),$$

$$< n(p') | I_{\lambda}^{5(-)}(0) | p(p) > = i \bar{\nu}(p') \{ \gamma_\lambda \gamma_5 f_A(q^2) + \frac{i2Mq_\lambda \gamma_5}{m_\pi^2} f_F(q^2) \} u(p),$$

with $q_\lambda = (p' - p)_\lambda$ and $2M = m_p + m_\mu$. Here $f_\nu(q^2)$, $f_M(q^2)$, $f_A(q^2)$, and $f_F(q^2)$ are, respectively, the vector, weak magnetism, axial, pseudoscalar form factors \[5\]. The transition amplitude is given by

$$T(\bar{\nu}_\mu p \rightarrow \mu^+ n) = \frac{G_F \cos \theta_e}{\sqrt{2}} \bar{\nu}(k) \gamma_\lambda (1 + \gamma_5) \nu_\mu (k') < n(p') | \{ I_{\lambda}^{(-)}(0) + I_{\lambda}^{5(-)}(0) \} | p(p) > .$$

Thus, we obtain \[6\] \[1\]

$$\frac{d\sigma}{d\Omega_\mu} (\nu_\mu + p \rightarrow \mu^+ + n) = \frac{G_F^2 \cos^2 \theta_e E_\mu^2}{2\pi^2} \frac{E_\nu}{E_\nu} \left\{ \begin{array}{l} \{ f_\nu^2 f_M^2 \frac{q^2}{2m_p} + f_A^2 + m_\nu^2 (f_p^2 \frac{q^2}{m_\pi^2} - 2f_A f_F) \} \cos^2 \theta_\mu \\ + 2 (f_\nu + f_M)^2 \frac{q^2}{4m_p^2} + f_A^2 (1 + 4 \frac{q^2}{4m_p^2}) - 4 \frac{E_\mu}{m_p} (1 + \frac{E_\nu \sin^2 \theta_\mu}{2m_p}) f_A (f_\nu + f_M) \sin^2 \theta_\mu \right\}$$

$$= \frac{G_F^2 \cos^2 \theta_e E_\mu^2}{2\pi^2} \frac{E_\nu}{E_\nu} \cdot N_0$$

$$= (1.857 \times 10^{-38} cm^2) \cdot N_0; \quad (E_\mu, E_\nu \mbox{ in } 3 \mbox{ GeV}).$$

We are dealing with the large side of the weak-interaction cross sections but the neutrino beams cannot be "managed" in terms of the energy and the intensity. But the point which we are trying to get at is to try to see if $\tau^+$ or $e^+$ could be produced in a "high-energy" $\bar{\nu}_\mu$ beam - that might be relatively easy.
In reality, the Higgs field(s) must belong to some group (i.e., the family group) and the coupling is from a neutrino of a certain flavor to the neutrino of different flavor. Thus, it arises naturally the so-called family gauge theory [7], which we tried to introduce five years ago. We tried [8] to separate the left-handed and right-handed components (of the building blocks of matter) in constructing an overall extended Standard Model. Lately, we realized [9] that an extended Standard Model exists, that the Standard Higgs mechanism and its forced family Higgs mechanism work in an economical way (i.e., in a minimal fashion).

In the new Standard Model [9], we have

$$i\eta\overline{\Psi}_L(3,2) \times \Phi(3,2) \cdot \Psi_R(3,1) + h.c. \quad (5)$$

Here the first number refers to $SU_f(3)$ while the second number is for $SU_L(2)$. It is an off-diagonal matrix and an $SU(3)$ operation - three $SU_f(3)$ triplets combined to a singlet. Note that it is not a matrix product; it arises in $SU(3)$ and its importance is being unraveled.

In this operation, $\nu_e$ would transform into $\nu_\mu$ or into $\nu_\tau$, $\nu_\mu$ would into $\nu_\tau$ or $\nu_e$, and so on. This is interesting in view of neutrino oscillations, since it could be regarded as the underlying interaction (mechanism) for neutrino oscillations (which we are talking about [3]). An oscillation occurs in a way similar to the decay by way of creating a new species plus the vacuum expectation value (or, changing the vacuum). In quantum mechanics or in quantum field theory, this may be so far the only way how an oscillation between point-like Dirac particles can occur.

Now, consider the muon-like neutrino converting to a tau-like neutrino and then producing a tau, i.e. $(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau) + p \rightarrow \tau^+ + n$. Feynman’s rules yield

$$iT = \left(-\frac{G_F}{\sqrt{2}} \cdot \eta u(\nu_\mu \rightarrow \nu_\tau) \cdot \bar{v}(k) \cdot \frac{1}{i} \frac{m_2 - i\gamma \cdot k}{m_2^2 + k^2 - i\epsilon} \right) \cdot 
\gamma_\lambda(1 + \gamma_5)v(k') \cdot < n(p') | [I^{(-)}(0) + I^{5(-)}(0)] | p(p) > \quad (6)$$

Here the small off-diagonal coupling and the small propagator are "canceling" each other. Similarly for the process $(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) + p \rightarrow e^+ + n$.

Note that the irrelevant sign in the transition amplitude, such as in the last equation, could be ignored. The couplings as denoted by $\eta \cdot u(\nu_\mu \rightarrow \eta_\tau)$ and etc. could be all three different from zero.

So, we may try out to do the scattering $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$ at the energy of, say, $3 GeV$. It is interesting to see if $\tau^+$ gets to be produced or if $e^+$ could be produced. This would be a way to open up the door into the "family" world. Of course, we have to worry about the backgrounds; that makes it a high-precision experiment.

Remember that we have the outstanding question why there are three generations in the minimal Standard Model and, yet, another outstanding puzzle that the dark-matter world is about five times the visual ordinary-matter world (the latter described by the minimal Standard Model). Besides the role in the minimal Standard Model, neutrinos may be able
Here the conversion of the W-boson mass formulae (e.g. Ch. 10, Wu/Hwang [1]), especially if we drop the small masses compared to SU particles. We show in Figures 1(a), 1(b), and 1(c) three leading basic Feynman diagrams. Here the conversion of $\nu_\mu$ into $\nu_e$ is marked by a cross sign and it is a term from our off-diagonal interaction given above with the Higgs vacuum expectation values $u_+$ and $u_-$. Here the Higgs masses are assumed to be very large, i.e., greater than a few TeV, as in $SU_f(3)$. The only small number (coupling) is $\eta$, explaining the tiny masses of neutrinos.

Using Feynman rules from Wu and Hwang [1], we write, for Fig. 1(a),

$$
\frac{1}{(2\pi)^4} \int \frac{d^4 q \cdot \bar{u}(p', s')}{i m_2 - i\gamma \cdot q} \cdot \frac{1}{i m_2 + q^2 - i\epsilon} \cdot \frac{1}{i M_W^2 + (p - q)^2 - i\epsilon} \cdot \frac{1}{e(\sigma(k))} \cdot \frac{i \cdot 1}{2\sqrt{2} \sin \theta_W} \cdot \frac{i \gamma (1 + \gamma_5)}{i \cdot \frac{1}{M_W^2} \cdot \frac{1}{(p - q)^2 - i\epsilon}}
$$

with $\Delta_{\sigma \mu \nu} = (-i\epsilon)\{\delta_{\mu \nu}(-k - p - q)_\sigma + \delta_{\nu \sigma}(p - q + p - q - k)_\mu + \delta_{\sigma \mu}(-p + q + k)_\nu\}$.

On the other hand, Feynman rules yield, for Fig. 1(b),

$$
\frac{1}{(2\pi)^4} \int \frac{d^4 q \cdot \bar{u}(p', s')}{i m_2 - i\gamma \cdot q} \cdot \frac{1}{i m_2 + q^2 - i\epsilon} \cdot \frac{1}{i M_W^2 + (p - q)^2 - i\epsilon} \cdot \frac{1}{e(\sigma(k))} \cdot \frac{i \cdot 1}{2\sqrt{2} \sin \theta_W} \cdot \frac{i \gamma (1 + \gamma_5)}{i \cdot \frac{1}{M_W^2} \cdot \frac{1}{(p - q)^2 - i\epsilon}}
$$

and a similar result for Fig. 1(c).

The four-dimensional integrations can be carried out, via the dimensional integration formulae (e.g. Ch. 10, Wu/Hwang [1]), especially if we drop the small masses compared to the W-boson mass $M_W$ in the denominator. In this way, we obtain

$$
i T_a = \frac{G_F}{\sqrt{2}} \cdot \eta (u_+ + e u_-) \cdot (m_1 + m_2) \cdot (-2i) \cdot \frac{e}{(4\pi)^2} \cdot \bar{u}(p', s') \frac{\gamma \cdot e}{\sqrt{2} k_0} (1 + \gamma_5) u(p, s).
$$

(7)
It is interesting to note that the wave-function renormalization, as shown by Figs. 1(b) and 1(c), yields

\[ iT_{b+c} = \frac{G_F}{\sqrt{2}} \cdot \eta(u_+e_u_-)(m_1 + m_2) \cdot (-2i)\frac{e}{(4\pi)^2} \cdot \left\{ \frac{p'^2}{m_1^2 + p'^2} + \frac{p^2}{m_e^2 + p^2} \right\} \]

\[ \cdot \bar{u}(p', s') \gamma \cdot \epsilon \sqrt{2k_0} (1 + \gamma_5) u(p, s), \]  

noting that \( p'^2 = -m^2_\mu \) and \( p^2 = -m_e^2 \) would make the contribution from Figs. 1(b) and 1(c) to be the same as, but opposite in sign, that from Fig. 1(a).

In a normal treatment, one ignores the wave-function renormalization diagrams 1(b) and 1(c) in the treatment of the decays \( \mu \to e + \gamma \), \( \mu \to 3e \), and \( \mu + A \to e + A^* \). In that case, the cancelation would be there. This cancelation due to "gauge invariance" makes any observation of this decay mode virtually impossible.

Comparing this to the dominant mode \( \mu \to e + \gamma \) and the "golden" scattering \( \bar{\nu}_\mu \to \bar{\nu}_\tau + p \to \tau^+ + n \), one may consider the decay \( \pi^0 \to \mu + e \). We would move to a field that would be called "family physics" which deals with baryon-number-conserving lepton-flavor-changing reactions or decays.

Besides the golden decays \( \mu \to e + \gamma \) (much too small), \( \pi^0 \to \mu + e \), and neutrino oscillations (already observed), we have to decide, to what extent, violation of the \( \tau - \mu - e \) universality. As the baryon asymmetry is sometime attributed to the lepton-antilepton asymmetry, the current scenario for neutrino oscillations [3] appears to be inadequate. If we take the hints from neutrinos rather seriously, there are so much to discover, even though the minimal Standard Model for the ordinary-matter world remains to be pretty much intact.

3 Episode

After the first version of this article appeared, there are two major breakthroughs - first, Hwang and Yan [8] realized that three lepton \( SU_L(2) \) doublets could form an \( SU_f(3) \) triplet and to make a consistent theory one should start from the so-called "basic units" made up from the right-handed Dirac components (of Dirac fields) or from the left-handed Dirac components. And, secondly, the Higgs mechanisms are not independent but, instead, are cooperative in the Standard-Model Higgs \( \Phi(1, 2) \), purely family Higgs \( \Phi(3, 1) \), and the mixed Higgs \( \Phi(3, 2) \) in an extended Standard Model [9].

Under the assumption that, in the dark-matter world, the dark-matter particles are also species in the extended Standard Model, most of reactions happening among dark-matter particles, even involving neutrinos, cannot be detected directly in the ordinary-matter world. We could argue that the "minimum" extended Standard Model would be the extended Standard Model to be based on the group \( SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3) \) [9], provided that why there are three generations of fermions could be explained. This "minimal extended" Standard Model would be the most natural choice of all the models.
How about the right-handed options [10]? If we adopt the "basic units" as Hwang and Yan [8] propose, the models would not belong since each basic unit is associated with one kinetic term, one-to-one. Of course, the projection operators could be deviated from the "right-handed" or "left-handed" in the starting point; or, we do not start from those basic units. In any case, the door seems to be shut up but, after different thoughts, there is another door.

In a slightly different context [11], I proposed to work with two working rules: "Dirac similarity principle", based on eighty years of experience, and "minimum Higgs hypothesis", from the last forty years of experience. By "Dirac similarity principle", all the fermions of the extended Standard Model are point-like Dirac particles and this has important implication for neutrinos. Using these two working rules, the extended model mentioned above becomes rather unique - so, it is so much easier to check it against the experiments.

We would be curious about how the dark-matter world looks like, though it is difficult to verify experimentally. The first question would be if the dark-matter world, 25 % of the current Universe (in comparison, only 5 % in the ordinary matter), would clusterize to form the dark-matter galaxies, maybe even before the ordinary-matter galaxies. The dark-matter galaxies would then play the hosts of (visible) ordinary-matter galaxies, like our own galaxy, the Milky Way. Note that a dark-matter galaxy is by our definition a galaxy that does not "feel" any ordinary strong and electromagnetic interactions (as in our visible ordinary-matter world). This fundamental quest deserves some thoughts, for the structural formation of our Universe.

Of course, we should remind ourselves that, in our ordinary-matter world, those quarks can aggregate in no time, to hadrons, including nuclei, and the electrons serve to neutralize the charges also in no time; then atoms, molecules, complex molecules, and so on. The early stage of "aggregation" does not involve the gravitational force, and so much faster than those by the gravitational force. This early stage of aggregation serves as the "seeds" for the clusters, and then stars, and then galaxies, maybe in a time span of 1 Gyr (i.e., the age of our young Universe). The aggregation caused by strong and electromagnetic forces is fast enough to help giving rise to galaxies in a time span of 1 Gyr. On the other hand, the seeded clusterings might proceed with abundance of extra-heavy dark-matter particles such as familons and family Higgs, all greater than a few TeV and with relatively long lifetimes (owing to very limited decay channels). So, further simulations on galactic formation and evolution may yield clues on our problem.

Finally, coming back to the fronts of particle physics, neutrinos, especially the right-handed neutrinos, might couple to the particles in the dark-matter world. Neutrino oscillations and the family "symmetry" are hints for us to enter the new world. Any further investigation along this direction would be of utmost importance. In addition, it may shed light on the nature of the dark-matter world.

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