Subgrid-scale cloud–radiation feedback for the Betts–Miller–Janjić convection scheme

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Cloud–radiation feedbacks play a crucial role in the climate system and continue to be a major source of uncertainty in global climate model projections. Despite its importance for regional and local circulations, these feedback processes are not included in subgrid-scale convective parametrizations used in regional weather and climate models, in particular in adjustment schemes that, as opposed to mass-flux schemes, do not deal with convective condensates. Here we present a cloud scheme developed for the Betts–Miller–Janjić (BMJ) cumulus scheme used in the Weather Research and Forecasting (WRF) model that is fully general and can easily be applied to any other convective scheme. We parametrize the convective cloud fraction as a function of the BMJ time-step precipitation rate with the vertical cloud profile given by a ‘top-heavy’ Poisson distribution, similar to observed profiles. The cloud condensates are defined based on the assumption that the mass of convective cloud per unit mass of water vapour in cloudy air is constant in the column. In this scheme there are two tunable parameters, δ, that determines the vertical structure of the convective cloud, and γ, that controls the amount of cloud mass. The performance of the scheme is evaluated in a 1-year run and WRF is found to give a much better representation of the observed cloudiness with smaller biases in the surface radiation fields with respect to observations and reanalysis.

Key Words: cloud–radiation feedback; BMJ scheme; WRF model

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1. Introduction

It has long been known that clouds play an important role in the climate system, and their accurate representation in climate models continues to be one of the major challenges that atmospheric scientists face today. Clouds affect the climate through the interaction with the Earth’s radiation budget and hydrological cycle on different time-scales (Schneider, 1972; Slingo, 1980, 1987; Stephens, 2005) and will therefore have a large impact on the surface hydrometeorological fields. In several regional climate and weather models, including in the Weather Research and Forecasting model (WRF: Skamarock et al., 2008), condensed phases of water at the grid resolution are allowed to interact with the radiative fields, but subgrid-scale cumulus clouds remain radiatively passive even though recent studies showed the cumulus cloud feedbacks to radiation to be important at regional weather and climate scales (Alapaty et al., 2012). In multi-year regional climate simulations focusing on the contiguous United States, Herwehe et al. (2014) showed that accounting for the cumulus cloud–radiation feedback leads to an improvement in the simulation of the surface heat fluxes, precipitation and extreme rainfall and temperature events. This would also be true in regions such as the maritime continent, an area with complex topography and land–sea contrasts leading to active cumulus convection activity which plays an important role in driving the large-scale atmospheric circulation (Ramage, 1968).

There are generally speaking three different types of parametrization schemes used in numerical models to predict cloudiness: diagnostic schemes, in which clouds are estimated empirically from model variables (Smagorinsky, 1960; Ricketts, 1973; Slingo, 1980, 1987); prognostic schemes, where clouds are computed explicitly using a prognostic equation (Sundqvist, 1978, 1988; Roeckner et al., 1987; Tiedtke et al., 1988; Le Treut and Li, 1991; Senior and Mitchell, 1993; Del Genio et al., 1996); and statistical schemes, which treat clouds as a result of random processes that are in statistical equilibrium with the grid-scale flow (Sesamori, 1975; Plant and Craig, 2008; Keane and Plant, 2012; Dorrestijn et al., 2013). Although prognostic schemes are physically more realistic, there are a number of reasons why their performance is limited, including (i) a restricted set of observations to quantitatively evaluate the schemes, (ii) a lack of a full understanding of some cloud processes, (iii) difficulties in...
Convective parametrization schemes fall into two main categories: mass-flux or moisture convergence schemes where the different cloud processes including the convective updrafts, downdrafts, entrainment and detrainment in clouds are modelled (e.g. Arakawa and Schubert, 1974; Kain and Fritsch, 1990; Emanuel, 2001); and adjustment schemes which relax the large-scale environment towards reference thermodynamic profiles (e.g. Betts, 1986; Betts and Miller, 1986; Janjić, 1994). Although subgrid-scale cloud–radiative feedbacks have been recently introduced in mass-flux schemes such as the Kain–Fritsch (Alapaty et al., 2012), to our knowledge no attempt has been made to develop a cumulus cloud–radiation feedback for convective adjustment schemes, such as the Betts–Miller scheme, that do not represent explicitly cloud condensates. In this work we use the WRF model, a fully compressible, non-hydrostatic numerical model that uses a terrain-following hydrostatic pressure-based coordinate in the vertical and the Arakawa-C grid staggering for horizontal discretization. WRF is a community model used in a wide variety of applications including coupled-model applications (Samala et al., 2013), idealized simulations (Steele et al., 2013), hurricane research (Davis et al., 2008) and regional climate research (Chotanonsak et al., 2011, 2012). In this article we develop and implement in WRF a diagnostic deep convective cloud scheme which accounts for subgrid-scale cloud feedbacks to radiation with the aim of improving the realism and performance of the model simulation. We apply the diagnostic cloud scheme to the Betts–Miller–Janjić (hereafter BMJ) convective adjustment scheme in this work but note that it can be easily extended to any other rain-producing adjustment scheme for deep convection.

In section 2 details about the model set-up and methods used are given. The convective cloud scheme is presented in section 3 while the results of the model experiments are discussed in section 4. The main conclusions are outlined in section 5.

2. Experimental set-up

In this study WRF version 3.3.1 is forced with the National Centers for Environmental Prediction (NCEP) Climate Forecast System Reanalysis (CFSR: Saha et al., 2010) 6-hourly data (horizontal resolution of 0.5° × 0.5°) and is run for multiple 3-day periods for the sensitivity experiments and for 1 year (March 2008 to April 2009, including a 1-month spin-up) for the evaluation of the scheme. The year of 2008 is chosen, as according to Ummenhofer et al. (2009) it is a neutral year with respect to both El Niño–Southern Oscillation (ENSO) and Indian Ocean Dipole (IOD). The physics parametrizations used include the WRF double-moment five-class microphysics scheme (Liu and Hong, 2010), the Rapid Radiative Transfer Model for Global (RRTMG) models for both short-wave and long-wave radiation (Iacono et al., 2008), the Yonsei University (YSU) Planetary Boundary Layer (PBL) scheme (Hong et al., 2006) with Monin–Obukhov surface layer parametrization (Monin and Obukhov, 1954), the four-layer Noah land surface model (Chen and Dudhia, 2001), and to parametrize cumulus convection, the BMJ scheme (Janjić, 1994) adapted for the Tropics (Fonseca et al., 2015). In all model experiments, seasonally dependent vegetation fraction and surface albedo are used. A simple interactive prognostic scheme for the sea-surface skin temperature based on Zeng and Beljaars (2005), which takes into account the effects of the sensible, latent and radiative fluxes as well as molecular and turbulent mixing, is added to the model to capture diurnal variations and allows SST feedback to the atmosphere. The surface-skin temperature (SST) that prescribes the lower boundary condition to the sea-surface skin temperature are the 6-hourly SSTs from CFSR that are linearly interpolated in time in order to have a temporally smooth forcing on the skin temperature. In addition, in all WRF simulations nudging is applied at the lateral boundaries using a ten grid-point buffer zone.

A tropical belt domain, which extends from about 40°S–40°N with a horizontal resolution of 30 km, is used in the model experiments. In the vertical 37 levels are considered, spaced more closely apart in the tropopause and PBL region, with the model top at 30 hPa. A time step of 1 min is used and the output from all runs is archived every 1 h. Analysis nudging is employed in all experiments with horizontal wind (u, v), potential temperature perturbation (Θ′) and water vapour mixing ratio (q_v) relaxed towards the CFSR values above 800 hPa and excluding the PBL on a time-scale of 1 h. This nudging configuration was found to give the best results for the Tropics (Fonseca et al., 2015).

The model’s surface and radiative fluxes over the oceans are evaluated against the monthly mean fluxes from the National Oceanography Centre Southampton (NOCS) version 2.0 dataset (Berry and Kent, 2009) while over land they are evaluated against the 6-hourly fluxes from CFSR as WRF has the same land surface model as the model used to produce the CFSR dataset (Saha et al., 2010). The precipitation rate is evaluated against that observed over the Tropical Rainfall Measuring Mission 3B42 version 6 (TRMM: Huffman et al., 2007). The model output is bilinearly interpolated to the CFSR, NOCS and TRMM grids for evaluation.

The model’s performance is assessed with different verification diagnostics including the model bias, normalized bias (μ), correlation (ρ), variance similarity (η), normalized error variance (α), root-mean-square error (RMSE) and normalized root-mean-square error (NRMSE). The bias is defined as the discrepancy between the model and observations while the normalized bias is given by the bias divided by the standard deviation of the discrepancy between the model and observations. The correlation is a measure of the phase agreement between the model and observations. The variance similarity is an indication of how the signal amplitude given by the model agrees with that observed and is defined as the ratio of the geometric mean to the arithmetic mean of the modelled and observed variances. The normalized error variance is the variance of the error arising from the disagreements in phase and amplitude, normalized by the combined modelled and observed signal variances. The root-mean-square error is the standard deviation of the discrepancy between the model and observations while the normalized root-mean-square error is defined as the root-mean-square error divided by the square root of the sum of the observed and modelled variances. The best model performance corresponds to zero bias and hence normalized bias μ, zero normalized error variance α, which requires that both correlation ρ and variance similarity η are equal to 1, with zero RMSE and hence NRMSE. These diagnostics are defined in Eqs (A1)–(A7) in appendix A.

3. Precipitating convective cloud scheme

The BMJ is a convective adjustment scheme where the precipitation is obtained from the relaxation of the temperature and humidity profiles towards reference thermodynamic profiles (Betts, 1986; Betts and Miller, 1986; Janjić, 1994). We developed a precipitating convective cloud (hereafter PCC) scheme which takes as inputs the outputs of precipitation, cloud top and cloud bottom from a convective adjustment scheme like the BMJ scheme. In the next subsections details of the PCC scheme are given: in section 3.1 the focus is on the convective cloud fraction, in section 3.2 it is on the convective cloud condensates and in section 3.3 is on the cloud–radiation feedback.
3.1. Cloud fraction

Regarding the deep convective cloud fraction, we make the following three simplifying assumptions:

1. The cloud fraction follows a top-heavy continuous Poisson distribution in the vertical with mean $\nu$;
2. There is maximum cloud overlap among all vertical levels;
3. Following Slingo (1987), the cloud fraction seen from the zenith is proportional to the logarithm of surface precipitation rate within bounds.

Based on assumption 1, the deep convective cloud fraction $C_{CU}(i)$ at the model level $i$ depends on the pressure $p_i$ at that level as follows:

$$C_{CU}(i) = A \frac{\nu(p_i)}{\Gamma[1 + j(p_i)]},$$  (1)

where $A$ is a normalization constant, $\Gamma$ is the gamma function and the index $j$ is a linear function of pressure such that $j = 0$ at the cloud top and increases downwards. It is desirable to define the convective cloud fraction, $C_{CD}$, as a direct function of pressure instead of the discrete model levels so that the vertical cloud profile is independent of the number and location of vertical levels in the model. From assumptions 2 and 3, the cloud fraction seen from the zenith, $C_{CD}$, is

$$C_{CD} = \max\{C_{CU}(i)\}_{v_i}$$

$$= \begin{cases} 
0 & ; RR < RR_{MIN} \\
C_{MAX} \frac{\ln(RR/RR_{MIN})}{\ln(\Gamma_{TOP}/\Gamma_{BOT})} & ; RR_{MIN} \leq RR \leq RR_{MAX} \\
C_{MAX} & ; RR > RR_{MAX}
\end{cases}$$  (2)

where $RR$ is the input precipitation rate, $RR_{MIN}$ is the minimum precipitation rate below which the cloud cover is too transient or sparse to matter so that the cloud fraction is put to zero, and $RR_{MAX}$ is the maximum precipitation rate above which the convective cloud fraction saturates at $C_{MAX}$. From the field campaign in the tropical Atlantic reported by Slingo (1987), $RR_{MIN} = 0.14$ mm day$^{-1}$, $RR_{MAX} = 85$ mm day$^{-1}$ and $C_{MAX} = 0.8$.

We can express $j(p_i)$ generally as

$$j(p_i) = \delta \left( \frac{p_i - p_{TOP}}{p_{BOT} - p_{TOP}} \right),$$  (3)

where $p_{TOP}$ and $p_{BOT}$ are respectively the pressure at the input cloud top and cloud bottom levels, $L_{TOP}$ and $L_{BOT}$, $\delta$ is a non-dimensional cloud thickness larger than one (to sample well the Poisson distribution).

Without loss of generality, the non-dimensional cloud thickness, $\delta$, can be related to the standard deviation, $\sqrt{\nu}$, of the Poisson distribution as

$$\delta = \psi \sqrt{\nu} \Rightarrow \nu = \frac{\delta^2}{\psi^2},$$  (4)

where $\psi$ is a positive number usually larger than 1. For simplicity, we define $\psi = 3$ and so the above equation allows the mean of the Poisson distribution, $\nu$, to be determined from the parameter $\delta$.

The basic reason for the simplification is to have a one-parameter profile for cloud fraction since global observations of profiles of cloud fractions are too limited to constrain well any more degrees of freedom in a proposed model profile. Since the peak in a discrete Poisson distribution occurs at the integer just smaller than the mean value, to be consistent with observed top-heavy profiles of tropical clouds (e.g. Alfonso et al., 1998), we require

$$\delta \gtrsim 2\nu = \frac{2\delta^2}{\psi^2}.$$

Therefore, the parameter $\delta$ must be chosen within the range

$$1 < \delta \lesssim \frac{\psi^2}{2}.$$  (5)

The optimal value for $\delta$ for tropical convection is determined by the numerical experiments discussed in section 4.1.

3.2. Cloud condensates

First, we make the following definitions: the mass of water vapour in the cloud column in Eq. (6), the loss of water vapour mass from the cloud column by precipitation in Eq. (7) and its minimum value in Eq. (8).

$$m_V = \sum_{i=TOP}^{L_{TOP}} [q_V(i) \rho_a(i) \Delta z(i)],$$  (6)

$$m_{RR} = \rho_w \tau_{BMJ} RR = \sum_{i=TOP}^{L_{TOP}} [q_{EF}(i) - q_V(i) \rho_a(i) \Delta z(i)],$$  (7)

$$m_{MIN} = \rho_w \tau_{BMJ} RR_{MIN}.$$  (8)

In the equations above, $q_V(i)$ and $\rho_a(i)$ are the specific humidity and the air density at the model level $i$ respectively, $\Delta z(i)$ is the thickness of the model level $i$, $q_{EF}(i)$ is the reference moisture profile from the convective adjustment scheme, $\rho_w$ is the density of liquid water (assumed constant), and $\tau_{BMJ}$ is the relaxation timescale from the BMJ or similar convective adjustment scheme.

We parametrize the total mass of cloud condensates (whether liquid water or ice), $m_{CI}$, in a cloud column by the following equations:

$$\begin{cases} 
m_{CI} = 0 & \text{if } RR \leq RR_{MIN} \\
\ln m_{CI} = \gamma \ln (m_V) + (1 - \gamma) & \text{if } RR > RR_{MIN},
\end{cases}$$

(9)

$$\ln (m_{RR} - m_{MIN})$$

where $\gamma$ is a dimensionless parameter, dubbed ‘cloudiness’, such that

$$0 < \gamma < 1 \Rightarrow (m_{RR} - m_{MIN}) \ll m_{CI} \ll m_V.$$  (10)

For water budget considerations, $m_{CI}$ is already included in $(q_V)_\text{COL}$ and it comprises two parts: $(m_{RR} - m_{MIN})$ that would precipitate out and $m_{CI} - (m_{RR} - m_{MIN})$ that would re-evaporate at the end of time interval $\tau_{BMJ}$. Thus, no adjustment needs to be made to specific humidity beyond that which is already carried out by the BMJ or similar scheme.

To define the cloud condensates at each model level, we assume for simplicity that the ratio of cloud water/ice mass to water vapour mass in a convective cloud, $q_{CI}^*$, is a constant well-mixed value among all cloudy air parcels in the column so that

$$q_{CI}^* = \frac{m_{CI}}{\sum_{i=TOP}^{L_{TOP}} q_V(i) \rho_a(i) \Delta z(i) C_{CU}(i)}.$$  (11)

Thus, the ratio of cloud condensate mass to air mass at each vertical level $i$ is

$$q_{CI}(i) = q_{CI}^* q_V(i) C_{CU}(i),$$  (12)

with the mixing ratio against dry air given by

$$r_{CI}(i) = \frac{q_{CI}(i)}{1 - q_V(i)}.$$  (13)

We assume simply that the condensates are ice if $T(i) < 0^\circ\text{C}$ but liquid water otherwise.

As with the parameter $\delta$, the optimal value of $\gamma$ is determined by numerical experiments discussed in section 4.1. Further details about the PCC scheme are given in appendix B.
3.3. Cloud–radiation feedback

The convective cloud fraction given by Eq. (1) and condensates given by Eq. (12) are combined with the corresponding quantities from the microphysics scheme to obtain the total cloud fraction and total cloud condensates which are then used to compute the radiative fields. There is actually a small duplicity in having water vapour and cloud condensates represented separately at the same time by the PCC scheme for radiative calculations. But since \( m_{\text{CU}} \ll m_{\text{Q}} \) and water vapour has such a large absorption cross-section that the water vapour column mass is not the limiting factor, such duplicity makes no practical difference to the absorption of radiation by water vapour. An analogous argument holds for the small duplicity between cloud condensates and precipitated water if the latter partakes in the scattering and absorption of radiation in the model.

While cloud condensates are additive, cloud fractions are not, as there can be some overlap between convective clouds and large-scale clouds produced by the microphysics scheme. We estimate this overlap as the average of the maximum and minimum possible overlaps between the two. Hence, the total cloud condensates and cloud fraction are given by

\[
(q_{\text{C}}_{\text{TOT}}(\gamma)) = (q_{\text{C}})_{\text{CU}}(\gamma) + (q_{\text{C}})_{\text{MP}}(\gamma),
\]

\[
(q_{\text{I}}_{\text{TOT}}(\gamma)) = (q_{\text{I}})_{\text{CU}}(\gamma) + (q_{\text{I}})_{\text{MP}}(\gamma),
\]

\[
C_{\text{TOT}}(\gamma) = C_{\text{CU}}(\gamma) + C_{\text{MP}}(\gamma) - \frac{1}{2} (\gamma_{\text{MIN}} + \gamma_{\text{MAX}}),
\]

\[
\gamma_{\text{MIN}} = \text{MAX}(0, C_{\text{MP}}(\gamma) + C_{\text{CU}}(\gamma) - 1),
\]

\[
\gamma_{\text{MAX}} = \text{MIN}(C_{\text{MP}}(\gamma), C_{\text{CU}}(\gamma)),
\]

where the subscripts C and I refer to cloud and ice condensates and CU and MP refer to the contributions from the cumulus scheme, the microphysics scheme and their sum, respectively. The minimum and maximum overlap between the two cloud covers are denoted as \( \gamma_{\text{MIN}} \) and \( \gamma_{\text{MAX}} \). Note that in the absence of convective clouds, the total cloud fraction (condensate) is equal to the microphysics cloud fraction (condensate). The total cloud fraction is always bounded between 0 and 1.

4. Model experiments

4.1. Sensitivity experiments

In this section the optimal values of \( \delta \) and \( \gamma \) are determined in 3-day WRF runs performed during the spring equinox when most of the convective precipitation, associated with the ITCZ in 3-day WRF runs performed during the spring equinox when hence we test the cloud-mass parameter cloud column mass exerts only a minor influence on long-wave radiation but a major influence on short-wave radiation, and hence we test the cloud-mass parameter \( \gamma \) against the bulk extinction coefficient (BEC) for short-wave radiation, defined as the ratio of the net downward short-wave radiative flux at the surface to that at the top of the atmosphere. BEC lies between 0 for fully overcast skies and 1 for clear skies. Clouds with larger \( \gamma \) absorb and scatter more short-wave radiation from the Sun leading to smaller values of the BEC.

Appendix C proves that if (i) a physical variable has a Levy-stable distribution (Das, 2011), (ii) the probability density function (PDF) of the modelled variable is the same as the PDF of the observed variable, and (iii) the modelled and observed variables are well-sampled, the time mean of the observed variable has a compact linear relation with the modelled variable. Given this result, the optimal values of the two cloud parameters are the ones that maximize the coefficient of determination, \( R^2 \), of the linear regression of the corresponding radiative variables averaged over the last day of the WRF run and the observed climatology over all grid points. For both parameters, the optimization is done over the oceans where surface albedo and emissivity, in the absence of sea-ice as the domain lies in the Tropics, are practically uniform and constant for the observation and the model.

4.1.1. Optimization of \( \delta \) during spring equinox

From Eq. (5) with \( \psi = 3 \), we sample \( \delta \) from 1 to 4.5 in steps of 0.5. For each value of \( \delta \), a quantile-quantile (hereafter Q-Q) plot is generated, like the one in in Figure 1(a) for \( \delta = 2.5 \). From the 5th to the 95th percentile the plot is close to a straight line with the coefficient of determination \( R^2 \) shown in Figure 1(b). The fits are good for all \( \delta \) tested, lying in the range from 0.994 to 0.996 with the highest \( R^2 \) obtained for \( \delta = 2.5 \). This implies that the results are not so sensitive to precise choice of \( \delta \) and in practice we choose \( \delta = 2.5 \) to proceed.

In Figure 1(c) and (d) the domain-averaged vertical profiles of cloud fraction obtained in the last day of the 3-day run for different values of \( \delta \) are shown respectively for both deep (>12 levels) and less deep (<12 levels) convective clouds. As expected, as \( \delta \) increases the profiles are less ‘top-heavy’ and the profile for \( \delta = 2.5 \) is similar to observed cloud profiles like the right panel of Fig. 2 in Alfonso et al. (1998). In addition, it is also known that clouds associated with deep convection tend to be ‘top-heavy’ due to the presence of the anvil, cirrus and other high-level clouds.

4.1.2. Optimization of \( \gamma \) (spring equinox)

A similar set of experiments is performed for \( \gamma \) in the range from 0.3 to 0.7 with a step of 0.1, avoiding unrealistically small and large values of cloud column mass. The Q-Q plot of observed versus modelled BEC obtained for \( \gamma = 0.5 \) (with \( \delta = 2.5 \)) is shown in Figure 1(e). Here we have two different linear regimes: a cloudier regime for BEC < 0.65 and a clearer regime for BEC > 0.65. The existence of one single regime for \( \delta \) (which affects primarily the long-wave radiation) and two distinct regimes for \( \gamma \) (which affects primarily the short-wave radiation) is consistent with the fact that clear air is practically opaque to long-wave radiation as clouds but is translucent to short-wave radiation unlike clouds. Data in the cloudier (clearer) regime comes mostly from cloudy (mostly clear) grid points. Figure 1(f) and (g) show the \( R^2 \) for each \( \gamma \) value and for the two regimes and in both regimes the maximum \( R^2 \) occurs at \( \gamma = 0.5 \).

4.1.3. Optimization for summer and winter solstices

Given that the zenith angle of the Sun changes during the year, the radiative impact of clouds varies with seasons. Hence, similar optimization runs have to be performed for the summer and winter solstices to further assess whether \( \delta = 2.5 \) and \( \gamma = 0.5 \) are indeed the best choices for the two parameters. In these runs, a narrower range of values, centred in their optimal values for the spring equinox, is considered to reduce computation costs. Given the north–south asymmetry about the Equator of the convective...
precipitation in these seasons, the linear fit is done for 40°N–30°S for the boreal summer and 30°N–40°S for the boreal winter. The values of $R^2$ for these runs are shown in Figure 2. As can be seen the fit is as good as it was for the spring equinox with high values of $R^2$ in the range 0.996–0.999 and with generally the same optimal values for both parameters. (NB At the winter solstice, for the cloudier regime the value of $R^2$ for $\gamma = 0.5$ and 0.6 are almost the same, but for the clearer regime the former has a relatively higher $R^2$ and so the optimal choice is for $\gamma = 0.5$.)

In conclusion, in 3-day runs performed for the 2008 spring equinox, summer solstice and winter solstice, it is found that higher $R$ the same, but for the clearer regime the former has a relatively higher $R^2$ for the optimization of $\delta$. Daily-averaged vertical profiles for (c) deeper (>12 vertical levels) and (d) shallower (<12 vertical levels) convective clouds. (e) is as (a) but for the bulk extinction coefficient and (f) and (g) are as (b) but for the optimization of $\gamma$ for the clearer and cloudier regimes, respectively. All the plots shown are for the 2008 spring equinox.

4.2. Evaluation of the convective cloud scheme

4.2.1. Surface energy fluxes

The PCC scheme is now evaluated against observations (TRMM and NOCSv2) and reanalysis (CFSR) using 1 year of model data taken from a run from 0000 UTC 1 March 2008 to 0000 UTC 1 April 2009, with the first month discarded as model spin-up. Details about the WRF configuration are given in section 2. Given that the frequency of the NOCSv2 data is monthly we compare it with the monthly-averaged WRF output whereas for TRMM (3-hourly data) and CFSR (6-hourly data) we process both model and observation time series in pentads (5-day averaged data) and assess the model’s performance for three periods of nearly equal length to ensure commensurate statistical sampling: boreal summer monsoon from June to September (JJAS), boreal winter monsoon from December to March (DJFM), and combined inter-monsoon seasons, April–May and October–November (AMON).

In order to illustrate the realism of the clouds produced by the model, in Figure 3 the WRF clouds with and without the PCC scheme are compared to satellite imagery for a day in the boreal summer over Asia and the west Pacific and boreal winter around the Americas. As can be seen, there is a much better representation of the observed cloud cover when the PCC scheme is used. The improvement is not just confined to tropical convection (in particular the intertropical convergence zone (ITCZ), monsoons and even tropical storms) but extends also to midlatitude frontal convective systems even in the winter hemisphere (e.g. in the North Atlantic in Figure 3(f)). Although not every observed cloud feature is captured by WRF (e.g. the shallow cumulus and stratocumulus clouds off the western coast of South America are not well simulated), it is fair to say that with the PCC scheme the model does a reasonably good job in simulating the observed cloud cover. Similar results are obtained for other regions and seasons (not shown).

Figure 4 shows the verification diagnostics with respect to NOCSv2 for the surface net short-wave and long-wave radiation fluxes and latent and sensible heat fluxes and for the experiments with and without the PCC scheme. Regarding the bias, only those for which the absolute value of the normalized bias $|\mu|$ is larger than 0.5 are shown, as for these the contribution of the bias to the root-mean-square error is greater than ~10%.

Regarding the surface net short-wave and long-wave fluxes, there is a clear improvement in the deep Tropics where deep convective clouds are dominant. In fact, in some regions
Huang et al. showed that WRF has problems in properly simulating them, Slingo et al. 1989; Klein and Hartmann, 1993) and recent studies (2013) tested a combination of five different PBL schemes with nine microphysics schemes, including the ones used in these experiments, and found that none of the model configurations gave a proper representation of these clouds. In particular, for the YSU PBL scheme used in this study, the shallow cumulus clouds were found to be too thin, a result that is consistent with the radiation biases shown in Figure 4. The best PBL schemes were found to be the Mellor–Yamada–Nakanishi–Niino (MYNN: Nakanishi and Niino, 2004, 2006) and the Total Energy/Mass-Flux (TEMF: Siebesma et al., 2007; Angevine et al., 2010) with a smaller sensitivity to the choice of the microphysics scheme. We could not get the TEMF scheme to work in our model implementation (it is known to be more unstable than other PBL schemes, cf. WRFV3.3 TEMF Release Notes) and so the experiment with the PCC scheme is repeated with the MYNN PBL scheme only. It is found that there is indeed an improvement of the surface radiation biases, in particular in the northeastern and southeastern Pacific but there is also a deterioration of other fields, including the surface precipitation (not shown). Given that the focus of this work is on deep tropical convection with an emphasis on precipitation, the YSU PBL scheme seems to be the best choice.

The other verification diagnostics, $\rho$, $\arccos(\eta)$ and $\alpha$, show very good performance for the net short-wave radiation although there is a slight deterioration of $\rho$ and hence $\alpha$ in the equatorial region when the convective cloud scheme is added to the model. However, the improvement in the bias is more important for the reduction in the normalized root-mean-square error (NRMSE) than the worsening in $\rho$ and $\alpha$, so overall there is an improvement in the model’s performance. For the surface net long-wave radiation the model has comparable performances in $\rho$, $\arccos(\eta)$ and $\alpha$ with or without the PCC scheme.

Regarding the latent and sensible heat fluxes, the WRF biases are not as widespread as in the radiative fluxes but are still significant in some regions, in particular in the central and southeastern Pacific and southern Indian Ocean. What is more, all verification diagnostics show very little difference when the PCC scheme is employed. The reason for this is the absence of an interactive ocean model; WRF is run with a simple prognostic scheme for the sea-surface skin temperature based on Zeng and Beljaars (2005) that, although it takes into account the surface heat and radiative fluxes, only gives a small impact on the sea-surface skin temperature as this field is essentially controlled by the 6-hourly SSTs from CFSR. The SSTs provide the bottom boundary conditions for the sea-surface skin layer and are linearly interpolated in time to obtain a temporally smooth function. Given that the same sea-surface skin temperature is used to compute the surface heat fluxes, it is no surprise that there is very little difference in the verification diagnostics between the two experiments. Experimentation has shown that if WRF is coupled with a one-dimensional (1D) ocean-mixed layer model based on Pollard et al. (1973) the improvement in the representation of the

![Figure 2. Optimization of (a) $\delta$, and for the (b) clearer and (c) cloudier regimes of $\gamma$ for the 2008 summer solstice. (d)–(f) are as (a)–(c) respectively but for the 2008 winter solstice.](image-url)
surface heat fluxes is rather small. In fact, in some regions, there is even a deterioration of the model’s performance suggesting that the errors in the surface heat fluxes may not be easily corrected by using the interactive SSTs alone. More in-depth work beyond the scope of this article, such as coupling to a full regional ocean model, needs to be done to simulate the latent and sensible heat fluxes well (e.g. Wei et al., 2013).

Table 1 shows the verification diagnostics for the combined spatio-temporal dataset for the Tropics (25°N–25°S). The diagnostics shown include the model bias, \(\mu\), \(\rho\), \(\arccos(\eta)\), \(\alpha\) and NRMSE. The results are consistent with the findings from Figure 4: in general, when the PCC scheme is added to the model there is an improvement in the model performance for the surface radiative fluxes, but for the surface heat fluxes there is very little difference between the two experiments.

To further assess the improvement in the model’s performance when the convective cloud scheme is used, the corresponding Q-Q plots for the surface radiative and heat fluxes are shown in Figure 5. To construct these graphs the WRF fields are plotted against the observed ones given by NOCSv2 with a perfect match corresponding to all values being aligned along the diagonal shown by a solid black line. For the net short-wave radiation, WRF overestimates the observed fluxes as noted already, and there is clear improvement when the convective cloud scheme is used as the distribution approaches the diagonal. The fact that the slope of the Q-Q plot is also closer to 1 with the PCC scheme indicates that the WRF distribution has a similar shape to the one from NOCSv2. It is interesting to note that in the Q-Q plots for the BEC for the 3-day WRF runs, shown in Figure 1(e), we have two distinct regimes (clearer and cloudier) whereas in the Q-Q plots for net short-wave radiation in Figure 5 we have just one regime. The reason is that in the monthly data used to compute Figure 5, almost all grid-point values are always influenced by clouds in a period of a month and hence the clearer regime with a slope of less than one is absent.

For the net long-wave radiation there is a slight improvement when the PCC scheme is used, with the Q-Q plot being closer to the main diagonal. The model mostly underestimates the observed long-wave flux as noted before, with overestimation of the upper 1.0-percentile. As the long tail in the upper 1.0-percentile is present even without the PCC scheme, we can only speculate that the overestimation is more likely due to rare occurrences of extremely thick stratus clouds in eastern tropical ocean basins. For both experiments the model distribution is flatter (i.e. having a wider peak around the mean and longer tails) than the observed, consistent with low variance similarity \(\eta\), i.e. high \(\arccos(\eta)\).

The Q-Q plots of latent heat flux have slopes of nearly one with and without the PCC scheme, showing that WRF reproduces well the width of the distribution, consistent with the good variance similarity, i.e. low \(\arccos(\eta)\), in Table 1. The warm bias in latent heat flux is also evident in the Q-Q plots.

With the exception of the upper 2.0-percentile, WRF captures the observed surface sensible heat flux distribution reasonably well as the Q-Q plots are close to the main diagonal. But the slope is too steep signifying the model distribution is narrower than the observed, consistent again with low variance similarity \(\eta\) or high \(\arccos(\eta)\). As stated before, the PCC scheme brings little improvement in the model performance for these two fields which are controlled by underlying prescribed SSTs. The overestimation of the upper 2.0-percentile may be due to differences in extremely warm SSTs between the CFSR dataset used by WRF and the NOCSv2 dataset.

Figures 6–8 are as Figure 4 but comparing the WRF output with CFSR reanalysis for the two monsoon and one combined inter-monsoon periods. Table 2 shows the verification diagnostics for the combined spatio-temporal dataset for the Tropics, 25°N–25°S. The WRF radiative fields are compared with the ones from CFSR over land and not over the sea as (i) the global model in CFSR is an ocean–atmosphere coupled model whereas WRF uses prescribed SSTs and (ii) WRF uses the same land surface scheme, Noah LSM, and associated land surface datasets as the global model used in CFSR.

For all seasons there is an improvement in the representation of the net short-wave radiation in tropical areas where deep convective clouds are normally present, with very little difference elsewhere. As shown in Table 2, the averaged bias (normalized bias \(\mu\)) drops by nearly a factor of 2 from about 45–47 W m\(^{-2}\) (1.2–1.4) to around 22–27 W m\(^{-2}\) (0.6–0.8) with large improvement in variance similarity and hence normalized error variance as well, overall leading to improvements in NRMSE.

Figure 3. (a) Infrared satellite image from MTSAT (Japan) on 11 September at 0832 UTC and model clouds (units of kg m\(^{-2}\) from WRF experiments (b) without and (c) with the convective cloud scheme at 0900 UTC. (d)–(f) are as (a)–(c) but for the Americas on 3 December 2008 at 1800 UTC with the satellite image from GOES-12 taken at 1745 UTC. The satellite images were downloaded from the NOAA GIBBS website (http://www.ncdc.noaa.gov/gibbs/).
Figure 4. WRF biases (regions for which $|\mu| < 0.5$ are shaded in grey), correlation ($\rho$), arccosine of variance similarity ($\text{arccos}(\rho)$) and normalized error variance ($\sigma$) for the surface net surface short-wave and long-wave radiation fluxes (positive downwards, units of W m$^{-2}$) and latent and sensible heat fluxes (positive upwards, units of W m$^{-2}$) with respect to NOC5v2 for the period April 2008 to March 2009. In each pair of rows, the top (bottom) plots are for the WRF experiment without (with) the PCC scheme. In all plots the colour bar is linear with only the middle and end values shown. The land-sea mask is drawn in black.
Table 1. Combined spatio-temporal dataset bias, normalized bias ($\mu$), correlation ($\rho$), arccosine of variance similarity (arccos($\eta$)), normalized error variance ($\alpha$) and normalized root-mean-square error (NRMSE) for the two WRF runs with respect to NOCSv2 (corresponding plots in Figure 4) and for the Tropics (25°N–25°S) only.

| Fields                                | WRF experiment          | Bias     | $\mu$ | $\rho$ | arccos($\eta$) | $\alpha$ | NRMSE |
|---------------------------------------|--------------------------|----------|-------|--------|----------------|----------|-------|
| Net short-wave radiation flux         | No convective clouds     | 35.282   | 1.523 | 0.718  | 0.135          | 0.289    | 0.979 |
|                                       | With convective clouds   | 21.594   | 0.747 | 0.634  | 0.026          | 0.366    | 0.755 |
| Net long-wave radiation flux          | No convective clouds     | −9.593   | −1.072| 0.351  | 0.865          | 0.773    | 1.289 |
|                                       | With convective clouds   | −7.694   | −0.800| 0.367  | 0.901          | 0.772    | 1.125 |
| Latent heat flux                      | No convective clouds     | 12.423   | 0.302 | 0.350  | 0.293          | 0.665    | 0.852 |
|                                       | With convective clouds   | 11.341   | 0.278 | 0.350  | 0.297          | 0.666    | 0.847 |
| Sensible heat flux                    | No convective clouds     | −1.136   | −0.088| 0.037  | 1.112          | 0.984    | 0.996 |
|                                       | With convective clouds   | −1.426   | −0.113| 0.037  | 1.105          | 0.984    | 0.998 |

Figure 5. Q-Q plots (WRF vs. NOCSv2) for the monthly-mean surface net short-wave and long-wave radiation fluxes (positive downwards, units of W m$^{-2}$) and latent and sensible heat fluxes (positive upwards, units of W m$^{-2}$) without and with the PCC scheme, for the Tropics (25°N–25°S) only and for the period April 2008 to March 2009 (corresponding to left-column maps in Figure 4).

With the PCC scheme the surface net long-wave radiation flux is overcorrected in some regions and this explains the increase in the bias and $\mu$ although $\rho$ and $\eta$ show improvement for almost all seasons. For this field, the improvement in $\alpha$ compensates the deterioration in $\mu$ as the NRMSE shows no difference, or even a slight decrease, in the experiment with convective clouds.

Throughout all seasons in general, there is an improvement in the bias, $\mu$ and $\eta$ for surface latent and sensible heat fluxes, also shown in Table 2, while there is sometimes a slight deterioration in $\rho$ and $\alpha$. Regarding the NRMSE, there is little difference between the two experiments.

On the whole, the improvement of the model performance lies in the simulation of the surface radiation fluxes over land and sea for all seasons, with little net impact on simulation of surface sensible and latent heat fluxes.

4.2.2. Precipitation

Figure 9 shows the verification diagnostics for the model’s precipitation rate (with respect to TRMM). For the bias plots the model bias is shown with the solid black line corresponding to the $|\mu| = 0.3$ contour, corresponding to <5% contribution of the bias to the RMSE. A smaller $\mu$ threshold is used for the precipitation rate as for precipitation $\mu$ is in general always smaller than 0.5 for this field. As can be seen, WRF captures reasonably well the observed precipitation rate over all seasons with insignificant biases except over high terrain. The model is known to overestimate the observed precipitation (Teo et al., 2011) over high terrain where an accurate simulation requires a higher resolution to better resolve the orography and capture well mountain-valley breezes. In general, with the exception of regions where there is very little precipitation (e.g. deserts and eastern subtropical oceans), $\rho$ and $\eta$ are high and $\alpha$ is small further indicating good model performance in capturing temporal patterns in the rainfall pentads. A comparison of the plots for the two experiments indicates some regional improvement in the larger positive biases over tropical land areas such as the Amazon, Indochina and islands like Borneo: the convective clouds block the short-wave radiation from the Sun, cooling the surface temperature and so reduce the likelihood of convection.

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Figure 6. As Figure 4 but comparing the WRF output with CFSR over land only and for the boreal summer monsoon (JJAS 2008).
Figure 7. As Figure 6 but for the boreal winter monsoon (DJFM 2008/2009).
Figure 8. As Figure 6 but for the combined inter-monsoon seasons (AMON 2008).
The diagnostics for the combined spatio-temporal dataset for precipitation rate are given in Table 3. Overall, they also show good model performance with little difference whether the PCC scheme is used or not. Delving into the minor differences, one finds that in the boreal summer monsoon and the combined inter-monsoon seasons, i.e. 8 months of the year, the small biases and high variance similarities are slightly degraded. But this does not affect the improvement in correlation leading to smaller normalized error variance and hence NRMSE by the PCC scheme. In the 4 months of boreal winter monsoon, the use of the PCC scheme has the opposite but similarly small impact on model performance for rainfall. The model performance is such because the contribution of the bias to the RMSE does not exceed 0.2% and so is practically negligible. Moreover, the variance similarity is always higher than 0.97 (i.e. arccos(η) < 0.25) and hence to very good approximation, the normalized error variance is simply one minus the correlation: the correlation is about 0.5–0.6 and the normalized error variance is correspondingly about 0.4–0.5. Therefore, the model performance in RMSE is dominated by phase errors of spatio-temporal rainfall patterns which from Figure 9 mainly arise from drier regions. In other words, the minor differences in model performance arising from the use of the PCC scheme comes indirectly from regions of little precipitation which whole-Tropics rainfall diagnostics are sensitive to and so should not be emphasized.

In this section the PCC scheme is evaluated in a 1-year run and it is found to give an improvement in the simulation of the surface radiation fluxes with respect to both observations and reanalysis. As a result, in some land regions in the Tropics there is also a slight improvement in the simulation of the observed precipitation as given by TRMM. There are still large biases in the surface radiation fluxes outside the deep Tropics, in particular on eastern subtropical oceans where shallow cumulus and stratocumulus clouds are not well captured by the model. Very little improvement is seen in the representation of the surface heat fluxes as they are essentially controlled by prescribed SSTs from CFSR.

5. Conclusions

In most regional climate models resolved-scale clouds interact with the radiative fields, but this is not the case for subgrid-scale cumulus clouds even though cumulus cloud–radiation feedbacks have been shown to play an important role in the radiation budget (Alapaty et al., 2012) as well as in the simulation of precipitation and extreme rainfall and temperature events (Herwehe et al., 2014). In this article we present a precipitating convective cloud (PCC) scheme, applied to the BMJ, a convective adjustment scheme, in the WRF regional climate model to allow for these subgrid-scale cloud–radiation feedbacks. The PCC scheme is general and can easily be applied to other adjustment schemes for deep convection.

Following Slingo (1987), the two-dimensional (zenith-view) cloud fraction as seen is set to be proportional to the logarithm of surface precipitation rate with an upper limit of 0.8 based on results from a field campaign in the tropical Atlantic. We assume the cloud fraction follows a 'top-heavy' Poisson distribution in the vertical, similar to observed profiles, with a tunable 'cloudiness' parameter δ that determines the relative height of the cloud centroid. What is more, the vertical profile is defined in pressure coordinates so that its shape does not depend on the number and location of vertical levels set by the user, making it suitable for any model level configuration. Regarding the cloud condensates, as the amount of cloud has to be at least equal to the convective precipitation (as rain is formed from cloud condensates, as the amount of cloud has to be at least equal to the convective precipitation, as rain is formed from cloud water) and cannot exceed the amount of water vapour in the column (as cloud is condensed from water vapour), we define the cloud column mass as a linear interpolation between these two quantities on the logarithmic scale given their different orders of magnitude, with a tunable 'cloudiness' parameter γ that determines the relative amount of cloud condensate. A further assumption is made that the mass of cloud water per unit mass of water vapour in cloudy air is constant in a column for simplicity. The convective cloud fraction and condensates are combined with the corresponding quantities from the microphysics scheme and are then used in the radiation scheme to update the radiative fields.

The optimal values for the two parameters are determined over 1 day in 3-day model runs performed during the spring equinox when most of the convective precipitation is confined to the Tropics. It is found that the optimal values are 2.5 for δ (corresponding to the cloud centroid within the top third

| Region          | WRF experiment | Bias | μ | ρ | arccos(η) | α | NRMSE |
|-----------------|----------------|------|---|---|-----------|---|-------|
| DJFM 2008/2009  | WRF experiment | Bias | μ | ρ | arccos(η) | α | NRMSE |
| Net short-wave radiation flux | No convective clouds | 46.062 | 1.214 | 0.650 | 0.097 | 0.353 | 0.934 |
| Net long-wave radiation flux   | No convective clouds | 1.516 | 0.093 | 0.887 | 0.064 | 0.115 | 0.341 |
| Latent heat flux            | No convective clouds | 4.311 | 0.270 | 0.897 | 0.004 | 0.104 | 0.334 |
| Sensible heat flux          | No convective clouds | 14.405 | 0.478 | 0.896 | 0.017 | 0.118 | 0.381 |
| AMON 2008                  | WRF experiment | Bias | μ | ρ | arccos(η) | α | NRMSE |
| Net short-wave radiation flux | No convective clouds | 45.111 | 1.292 | 0.671 | 0.146 | 0.337 | 0.948 |
| Net long-wave radiation flux | No convective clouds | 3.105 | 0.318 | 0.904 | 0.043 | 0.274 | 0.649 |
| Latent heat flux            | No convective clouds | 8.580 | 0.391 | 0.911 | 0.008 | 0.089 | 0.320 |
| Sensible heat flux          | No convective clouds | 15.516 | 0.563 | 0.884 | 0.174 | 0.130 | 0.414 |

Table 2. Combined spatio-temporal dataset bias, normalized bias (μ), correlation (ρ), arccosine of variance similarity (arccos(η)), normalized error variance (α) and normalized root-mean-square error (NRMSE) for the WRF runs with respect to CFSR (corresponding plots shown in Figures 5–8) and for the Tropics (25°N–25°S) only.
Figure 9. WRF biases (biases (shading) and the $|\mu| = 0.3$ contours plotted as a solid black line), correlation ($\rho$), arccosine of variance similarity (arccos($\eta$)) and normalized error variance ($\alpha$) for the precipitation rate (units of mm h$^{-1}$) with respect to TRMM for the boreal summer monsoon (JJAS 2008), boreal winter monsoon (DJFM 2008/2009) and combined inter-monsoon seasons (AMON 2008) only. For the last three diagnostics regions with very little precipitation (i.e. where the seasonal mean precipitation rate does not exceed 0.01 mm h$^{-1}$) are shaded in grey. In each pair of rows, the top (bottom) plots are for the WRF experiment without (with) the convective cloud scheme. In all plots the colour bar is linear with only the middle and end values shown.
of the cloud depth) and 0.5 for $\gamma$ (the mid-range cloud mass between water vapour and rainwater mass). Some of the runs are repeated for the winter and summer solstices, when there is a clear asymmetry in the convective precipitation and solar radiation about the Equator, and similar conclusions are reached. Hence, we were motivated to test these fixed values of $\delta$ and $\gamma$ for the whole year, otherwise one might have imagined seasonally dependent parameters which would have made the scheme more complicated.

The performance of the scheme is evaluated in a 1-year dataset generated by a run from 0000 UTC 1 March 2008 to 0000 UTC 1 April 2009 with the first month regarded as model spin-up. Comparison of the WRF clouds with satellite imagery shows good agreement for both monsoon seasons with model spin-up. Comparison of the WRF clouds with satellite imagery shows good agreement for both monsoon seasons with model spin-up. Comparison of the WRF clouds with satellite imagery shows good agreement for both monsoon seasons with model spin-up. Comparison of the WRF clouds with satellite imagery shows good agreement for both monsoon seasons with model spin-up. Comparison of the WRF clouds with satellite imagery shows good agreement for both monsoon seasons with model spin-up. Comparison of the WRF clouds with satellite imagery shows good agreement for both monsoon seasons with model spin-up.

Over the oceans, the surface radiation and heat fluxes are compared with those observed, as given by NOCSv2. It is concluded that when the PCC scheme is employed there is a reduction in the biases mainly in the warm pool region and the equatorial Tropics where deep convection is more prevalent. Similar improvements are also seen in other verification diagnostics, namely correlation ($\rho$), variance similarity ($\eta$), normalized error variance ($\alpha$) and normalized root-mean-square error (NRMSE) defined in Koh et al. (2012) and reviewed in appendix A. However, there remain significant biases in the subtropics as a result of a poor simulation of stratocumulus and shallow cumulus clouds unaddressed by our work of course. The surface heat fluxes have smaller domain-averaged biases in the first place and there is little improvement when the cloud scheme is used, as they are essentially controlled by prescribed SSTs from CFSR.

The WRF radiative fields are compared with the ones from CFSR but over land only, as the global model used to produce the reanalysis has a coupled ocean unlike WRF while both models share the same land surface scheme. There is an improvement of the model performance here for all seasons when the PCC scheme is used.

Consistent with the findings by Fonseca et al. (2015), WRF gives a good estimate of the observed precipitation, as given by TRMM, with the biases being significant mainly over the high terrain where the precipitation is mostly produced by the microphysics scheme and the model is known to have problems in simulating the observed precipitation arising from insufficient resolution of orography (Teo et al., 2011). Although the impact of the PCC scheme on the model’s precipitation is small, there is an improvement in particular over land in the Tropics where a better simulation of the surface radiation fields gives a better representation of the observed precipitation.

This is understandable as radiation and tropical convection are tightly coupled and our results further stress the importance of the radiative feedback on convection in order to successfully simulate the local and regional climate. We suspect the radiative feedback on oceanic convective precipitation is negligible in our model because our sea-surface skin temperature is controlled essentially by prescribed SSTs but its role is likely smaller than over land in any case because of the high thermal capacity of water.

It is important to note that there are other ways of accounting for subgrid-scale cloud–radiation interactions in a general-circulation model (GCM), e.g. the mosaic treatment where the GCM grid column is divided into a number of subcells so that an individual cloud layer within a subcell is either completely overcast or cloud-free (Liang and Wang, 1997; Liang and Wu, 2005; Wu and Liang, 2005) or the more complex ‘super-parametrization’ where a high-resolution cloud-resolving model (CRM) is embedded into each GCM grid column (e.g. Li et al., 2012). However, in this article the emphasis is not on the simulation of cloud-scale processes but on the mesoscale and large-scale effects of cumulus clouds–radiation feedback where the PCC scheme is most useful, given its light computational load compared to the above approaches.

The convective cloud scheme presented in this article allows for interaction between radiation and subgrid-scale deep convective clouds to be included in a climate model by complementing convective adjustment schemes which emphasize large-scale dynamics rather than detailed microphysics. The PCC scheme is designed to retain such an emphasis and is itself straightforward and computationally inexpensive. We anticipate that it will be of value to researchers working on tropical climate dynamics whether in global or regional modelling.

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**Appendix A: Verification diagnostics**

$$D = F - O,$$

$$\rho = \frac{1}{\sigma_D \sigma_O} \langle (F - \langle F \rangle) \cdot (O - \langle O \rangle) \rangle, \quad -1 \leq \rho \leq 1,$$

$$\eta = \frac{\sigma_D \sigma_O}{\sqrt{\langle O \rangle^2 + \sigma_O^2}}, \quad 0 \leq \eta \leq 1,$$

$$\alpha = \frac{\sigma_D}{\sqrt{\langle O \rangle^2 + \sigma_O^2}} \equiv 1 - \rho \eta, \quad 0 \leq \alpha \leq 2,$$
RMSE² = (D²) ≡ BIAS² + σ₂₁

NRMSE² = (D²) / σ₂₂ ≡ α(μ² + 1).

(A6) (A7)

In the equations above, D is the discrepancy between the model forecast F and the observations O; σₓ is the standard deviation of X; μ is the normalized bias; ρ is the correlation; η is the variance similarity; α is the normalized error variance; RMSE stands for root-mean-square error; NRMSE stands for normalized RMSE. In Eq. (A5), a random model has

\[ \alpha = \beta = 1 \] so that \( \alpha \leq 1 \) is recommended. In Eqs (A6) and (A7), the bias and normalized bias are seen to respectively increase RMSE and NRMSE by a fraction, \( (1 + \mu^2)1/2 - 1 \approx \mu^2/2 \) for small \( \mu \).

When \( \eta \approx 1 \), the variance similarity is not sensitive to the \( \sigma_O : \sigma_F \) ratio and so arccos(\( \eta \)) is plotted instead where arccos(\( \eta \)) is approximately the fractional difference between the modelled and observed variances:

\[ \text{arccos}(\eta) \propto \frac{\sigma_O - \sigma_F}{\sigma_O} \text{ or } \frac{\sigma_O - \sigma_F}{\sigma_F}, \quad \eta \lesssim 0.1. \]  

(A8)

So, it is often useful to plot \text{arccos}(\( \eta \)) to have better resolution in the performance metric.

More information about these diagnostics can be found in Koh et al. (2012).

Appendix B: Further Insight into the PCC Scheme

Further insight into the PCC scheme can be obtained by substituting for \( m_{\text{CI}} \) in Eq. (9) with the help of Eq. (11), and expressing \( q_{\text{CI}}^* \) in terms of RR so that for \( RR > RR_{\text{MIN}} \):

\[ q_{\text{CI}}^* = \frac{(m_{\text{MIN}})^{1-\gamma}(m_V)^\gamma}{\sum_{i=L_{\text{BOT}}}^{L_{\text{TOP}}} q_V(i) \rho_d(i) \Delta z(i) C_{\text{CU}}(i)} (\chi - 1)^{1-\gamma}, \]

where \( \chi \) is the normalized rain rate defined by \( \chi = RR / RR_{\text{MIN}} \).

From Eqs (1) and (2), the deep convective clouds fraction can be rewritten as

\[ C_{\text{CU}}(i) = C_{\text{CD}} \cdot \phi(i) = \frac{C_{\text{MAX}}}{\ln(\chi_{\text{MAX}})} \ln(\chi) \cdot \phi(i), \]

where \( \chi_{\text{MAX}} = RR_{\text{MAX}} / RR_{\text{MIN}} \) and \( \phi(i) \) is a function bearing the shape of the Poisson distribution in the vertical such that \( 0 < \phi \leq 1 \). So we can make explicit the dependence of \( q_{\text{CI}}^* \) on the normalized rain rate as:

\[ q_{\text{CI}}^* = q_{0} g(\chi), \]

with

\[ q_0 = \frac{(m_{\text{MIN}})^{1-\gamma}(m_V)^\gamma}{C_{\text{MAX}} \sum_{i=L_{\text{BOT}}}^{L_{\text{TOP}}} q_V(i) \rho_d(i) \Delta z(i) \phi(i)}, \]

\[ g(\chi) = \frac{(\chi - 1)^{1-\gamma}}{\ln(\chi)}. \]

(B1)

We next examine the asymptotic behaviour of the function \( g(\chi) \). As \( 0 < \gamma < 1 \),

\[ \lim_{\chi \to 0} g(\chi) \propto \frac{(\chi - 1)^{1-\gamma}}{\ln(\chi)} \approx \frac{(\chi - 1)^{1-\gamma}}{1 - \frac{\chi}{2} + \frac{\chi^2}{3}} = \frac{\chi^{1-\gamma}}{\frac{2}{3} + \frac{3}{5}}. \]

(B2)

Hence, near 1, instead of using Eqs (11) and (12), \( q_{\text{CD}}(i) \) is approximated as follows:

\[ q_{\text{CD}}(i) = Q_{\text{CD}}^* C_{\text{CD}} \cdot q_V(i) \phi(i), \]

(B3)

with

\[ Q_{\text{CD}}^* = \frac{q_0}{2} = \frac{1 - \frac{2}{3} + \frac{3}{5}}{2}. \]

(B4)

\[ C_{\text{CD}} = \frac{\chi^{-\gamma}}{\ln(\chi_{\text{MAX}})} \left( \chi^{1-\gamma} - \frac{\chi^{2-\gamma}}{2} + \frac{\chi^{3-\gamma}}{3} \right). \]

(B5)

Figure B1 shows a schematic plot of the functions \( g(\chi) \) and \( \chi / g(\chi) \) where \( \chi \) is the normalized precipitation rate. There

Figure B1. Schematic plot of \( g(\chi) \) (a) and \( \chi / g(\chi) \) (b) as a function of the normalized precipitation rate \( \chi \).
is a maximum in $\chi I(\chi)$ at $\chi = \chi_0$ that corresponds to the maximum efficiency in the formation of rain from cloud water. We may interpret the PCC scheme as qualitatively coding for the competing mechanisms below:

- falling raindrops sweep up cloud droplets generating more rain so that an increase in the precipitation rate leads to an increase in the efficiency of forming rain;
- collision of raindrops tend to break them up sometimes forming cloud droplets and so as the precipitation rate goes up the efficiency of forming rain goes down.

For normalized precipitation rates smaller (larger) than $\chi_0$, raindrop growth by falling through clouds dominates (is dominated by) raindrop break-up into cloud droplets. Such an interpretation is consistent with observations.

Appendix C: Relationship between observed and modelled variables

Consider a random variable $\omega$ with a Levy-stable distribution (Das, 2011) such that

$$\omega = a\xi + b,$$  \hspace{1cm} (C1)

where $\xi$ is a random variable with $\langle \xi \rangle = 0$ and $\langle \xi^2 \rangle = 1$, and $a$ and $b$ are constants. In our case, $\omega$ is the one-day observation sampled over all grid points in the domain. Levy-stability implies that the monthly mean of $\omega$ has the form below:

$$\bar{\omega} = a\bar{\xi} + b = A\tilde{\xi} + B,$$  \hspace{1cm} (C2)

where $A$ and $B$ are constants. Suppose the modelled variable $\tilde{\omega}$ has the same distribution as the observed variable $\omega$:

$$\tilde{\omega} = a\tilde{\xi} + b.$$  \hspace{1cm} (C3)

Equations (C2) and (C3) imply that $(\bar{\omega} - B)/A$ and $(\tilde{\omega} - b)/a$ have the same distribution of $\xi$. We define a random variable, $\zeta$, as follows:

$$\zeta = \frac{A(\bar{\omega} - B)}{a} - \frac{B}{a}.$$  \hspace{1cm} (C4)

From Eq. (C1), the distribution of $\xi$ must also be Levy-stable and so $\zeta$ takes the form

$$\zeta = \xi \sqrt{2(1 - \rho)},$$  \hspace{1cm} (C5)

where $\rho$ is the correlation of $\bar{\omega}$ and $\bar{\xi}$ over all grid points and is bounded between $-1$ and $1$. The RHS of Eq. (C5) can be verified easily by checking that its mean and variance agree with that of the RHS of Eq. (C4). Combining Eqs (C4) and (C5),

$$\frac{\bar{\omega}}{A} = \frac{\bar{\omega}}{a} + \left(\frac{B}{a} - \frac{b}{a} + \xi \sqrt{2(1 - \rho)}\right).$$  \hspace{1cm} (C6)

Therefore, the modelled one-day variable, $\tilde{\omega}$, and the observed monthly mean, $\bar{\omega}$, have a compact linear relation as long as $b/a > 0$ or $A > 0$, i.e. the standard deviation of the observation across the domain is much less than the mean. A departure from such a compact linear relation between $\bar{\omega}$ and $\bar{\xi}$ denotes that the assumption behind Eq. (C3) is false, i.e. the model fails to capture the same distribution as the observation.

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