Warped flux compactification and brane gravity

Shinji Mukohyama\textsuperscript{1,2}, Yuuiti Sendouda\textsuperscript{1}, Hiroyuki Yoshiguchi\textsuperscript{1} and Shunichiro Kinoshita\textsuperscript{1}

\textsuperscript{1} Department of Physics, The University of Tokyo, Tokyo 113-0033, Japan
\textsuperscript{2} Research Center for the Early Universe, The University of Tokyo, Tokyo 113-0033, Japan

E-mail: mukoyama@utap.phys.s.u-tokyo.ac.jp, sendouda@utap.phys.s.u-tokyo.ac.jp, hiroyuki@utap.phys.s.u-tokyo.ac.jp and kinoshita@utap.phys.s.u-tokyo.ac.jp

Received 16 June 2005
Accepted 30 June 2005
Published 21 July 2005

Abstract. We find a simple exact solution for a six-dimensional braneworld which captures some essential features of warped flux compactification, including a warped geometry, compactification, a magnetic flux and one or two 3-brane(s). In this set-up we analyse how the Hubble expansion rate on each brane changes when the brane tension changes. It is shown that the effective Newton’s constant resulting from this analysis agrees with that inferred by simply integrating extra dimensions out. On the basis of the result, a general formula for the effective Newton’s constant is conjectured and its application to cosmology with type IIB warped string compactification is discussed.

Keywords: extra dimensions, cosmology with extra dimensions, string theory and cosmology

ArXiv ePrint: hep-th/0506050
1. Introduction

Warped flux compactification is one of the essential parts of the construction of de Sitter vacua in string theory by Kachru, Kallosh, Linde and Trivedi \[1\]^3. A warped throat region \[11\] is smoothly attached to a Calabi–Yau manifold and all moduli except for the volume modulus are stabilized by fluxes \[12\]. The volume modulus is thought to be stabilized by non-perturbative effects such as D-instantons \[13,1\]. Another important ingredient of the construction is branes. Anti-D-branes are located at the bottom of the throat region to uplift the stable AdS vacua to metastable de Sitter vacua. This set-up provides a number of possible applications to cosmology \[14\]–\[18\].

So far, there is no known explicit form of the global geometry including the throat region, compactification, fluxes and the moduli stabilization. The lack of a known explicit form of the global geometry makes it difficult to analyse brane gravity in the context of warped flux compactification in detail from the higher dimensional point of view. Of course, even without the explicit global geometry, one could invoke that the four-dimensional Einstein gravity should be recovered as a low energy effective theory since all moduli are thought to be stabilized and made massive so that they can be integrated out. Nonetheless, it is perhaps fair to say that we have not yet had a complete understanding of how four-dimensional Einstein gravity is recovered in the warped flux compactification.

^3 See \[2,3\] for follow-up proposals. See also \[4,5\], \[6\]–\[9\] and \[10\] for other proposals of de Sitter, transiently accelerating and inflationary universes.
For example, suppose that a D-brane and an anti-D-brane annihilate somewhere in the warped throat region but that they are not coincident with a brane on which we are living. Following the idea of [19,20], it is suggested that this process may drive an inflation in our four-dimensional universe [14] and could leave cosmic superstrings as relics [17]. In this picture the inflaton and cosmic superstrings are not on our brane but living somewhere in the extra dimensions. It does not seem completely clear how they affect gravity on our brane. Indeed, a lot of questions would arise. ‘Does the inflaton living outside our world really inflate our brane?’ ‘Is there a deficit angle on our brane due to the cosmic superstring wandering somewhere in the extra dimensions?’ ‘If there is a deficit angle, then where on our brane and how much?’ Partial answers to these questions would be obtained if we completely integrated out extra degrees of freedom due to the extra dimensions, provided that all moduli are stabilized. On the other hand, in principle it should also be possible to answer these questions by analysing the higher dimensional theory directly. Evidently, it is important to use the two approaches complementarily and compare them towards obtaining a good understanding of brane gravity in the warped flux compactification.

Now let us heuristically remind ourselves how four-dimensional Einstein gravity is recovered in the Kaluza–Klein (KK) compactification and the Randall–Sundrum-type braneworlds.

In Kaluza–Klein compactification, zero modes and Kaluza–Klein modes are decoupled at the linearized level because of the momentum conservation along extra dimensions. This simple fact makes the recovery of the four-dimensional theory manifest since the standard model fields are supposed to consist of zero modes. Indeed, as long as moduli associated with compactification are made massive by a stabilization mechanism, they do not appear in the low energy physics and the Einstein theory is recovered as a low energy effective theory. This consideration also applies to cosmology as long as the energy scale is sufficiently lower than the compactification scale.

In the second Randall–Sundrum (RS2) braneworld with infinite extra dimension [21], the recovery of the four-dimensional Einstein theory is due to localization of zero modes [21]–[24]. Matter on our brane possibly excites not only zero modes but also KK modes since the momentum conservation does not prohibit couplings between the singular brane source and the KK modes⁴. However, the warped geometry localizes the zero mode to the vicinity of the brane and the coupling of the brane source to the zero mode is much stronger than that to KK modes. In this way, gravity on our brane at low energy is almost locally determined by the zero mode localized near the brane and the four-dimensional Einstein theory is recovered at low energy. (This point of ‘locally localized gravity’ has been made explicit in [25].) For FRW cosmology on the brane, there is no well-defined distinction between zero modes and KK modes because of the lack of enough symmetry. Nonetheless, the evolution of the brane is still determined locally since the unbroken symmetry, i.e. the homogeneity and the isotropy parallel to the brane, prevents waves from being generated and propagating in the bulk. In this way, gravity is still localized and the standard cosmological equation is recovered at low energy [26]–[31], [34,32,33].

On the other hand, in the warped compactification the recovery of four-dimensional theory seems more indirect and subtle. Unlike KK compactification but as in the RS2

⁴ Note that a delta function includes all momenta when it is Fourier transformed.
Warped flux compactification and brane gravity

A matter source on the brane can excite not only zero modes but also KK modes. However, unlike in the RS2 braneworld, gravity is not localized near the brane since the warp factor on the brane in the throat region is not larger but smaller than that in the bulk nearby. Hence, the evolution of matter on the brane changes the bulk geometry not only near the brane but possibly everywhere in the whole extra dimensions. Nonetheless, if all moduli are stabilized, the bulk geometry should quickly settle to a configuration which is determined by the boundary condition, i.e. the brane source(s), values of conserved quantities and the regularity of the other region of the extra dimensions. As a consequence of the change of the bulk geometry, the induced geometry on the brane responds to the evolution of the matter source on the brane. It is quite possible but must be checked that the four-dimensional Einstein theory is recovered as a rather indirect and subtle relation between the matter source on the brane and the response of the induced geometry. In this paper we shall support this picture in a simplified situation.

This picture is somewhat similar to that in the first Randall–Sundrum (RS1) scenario [35] with radion stabilization [36]. In the original RS1 brane model (without radion stabilization) the four-dimensional theory is not Einstein but Brans–Dicke theory [22, 23] because of the existence of a massless modulus called the radion. With the radion stabilization, there is no extra massless degree which could appear in the four-dimensional effective theory and the four-dimensional Einstein theory is recovered at energies sufficiently below the stabilization scale. The recovery of the four-dimensional Einstein theory is understood both from the low energy effective theory point of view and as a consequence of dynamics of bulk fields [37]–[43].

In the case of KK compactification and RS braneworlds, the recovery of the Einstein theory has been explicitly investigated from the higher dimensional point of view. One of the reasons that this was possible is that there are explicit background solutions (or at least explicit equations defining background solutions) around which we can analyse perturbations. On the other hand, in the warped flux compactification, the absence of an explicit global solution makes it less tractable to see the recovery of the four-dimensional Einstein gravity from the higher dimensional point of view as explicitly as in the KK compactification and the RS braneworlds.

Therefore, we would like to consider a simplified situation in which we can see the recovery of the four-dimensional Einstein theory in the warped flux compactification. The purpose of this paper is, as a first step, to consider a toy model which captures some essential features of the warped flux compactification and to see explicitly how the induced geometry on a brane responds to brane tension as a consequence of changes in the bulk geometry. In particular, we shall see that the relation between the change of brane tension and the response of the induced geometry is identical to that inferred from the four-dimensional Einstein theory.

The rest of this paper is organized as follows. In section 2 we describes a simple model of six-dimensional warped flux compactification with one or two 3-brane(s). In section 3 we argue that the four-dimensional Friedmann equation should be recovered on each brane at low energy and confirm the validity of a formula for the effective Newton’s constant in a simplified situation. Section 4 is devoted to a summary of this paper and discussion. In particular, on the basis of the result of this paper, a general formula for the effective Newton’s constant is conjectured and its application to cosmology with type IIB warped string compactification is discussed. In appendices A.1 and A.2 we show that two
seemingly singular limits of the solutions considered in this paper are actually regular. In appendix A.3 a higher order correction to the effective Friedmann equation on a brane is estimated and it is shown that higher order corrections can be ignored when the Hubble expansion rate on the brane is sufficiently lower than the bulk curvature scale.

2. Model description

In this paper we would like to consider a minimal set-up which captures essential features of the warped flux compactification and which is simple enough for analysing brane gravity from the higher dimensional viewpoint. The set-up must include at least a warped geometry, magnetic flux of an antisymmetric field along the extra dimensions and a brane. Since the simplest antisymmetric field is a $U(1)$ gauge field and the corresponding magnetic flux has two spatial indices, we need to consider at least two extra dimensions and, thus, at least six-dimensional spacetime. In this section we shall describe a model of a six-dimensional braneworld with warped flux compactification. In the next section we shall analyse this model in a situation where the tension of a 3-brane changes by a phase transition on the brane.

For the reason explained above, in this paper we consider a six-dimensional braneworld scenario with a $U(1)$ gauge field and a cosmological constant in the bulk. The bulk action is

$$I_6 = \frac{M_6^4}{2} \int d^6x \sqrt{-g} \left( R - 2\Lambda_6 - \frac{1}{2} F_{MN} F^{MN} \right),$$

(2.1)

where $M_6$ is the six-dimensional reduced Planck mass, $\Lambda_6$ is the bulk cosmological constant and $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field strength associated with the $U(1)$ gauge field $A_M$.

2.1. Bulk solution

We assume that the six-dimensional bulk geometry has the four-dimensional de Sitter symmetry and an additional axisymmetry. The former symmetry is to make the induced geometry on our brane a de Sitter spacetime, and the latter is imposed as a symmetry in the extra dimensions for simplicity. The general metric with these symmetries is locally written as

$$ds_6^2 = A(w)^2 ds_4^2 + dw^2 + R(w)^2 d\phi^2,$$

(2.2)

where $w$ is the proper distance along geodesics orthogonal to the brane world-volume and to the orbit of the axisymmetry, $A(w)$ and $R(w)$ are functions of $w$ only and $ds_4^2$ is the line element of the four-dimensional de Sitter spacetime with the unit curvature radius. For example, in the global chart $ds_4^2$ is written as

$$ds_4^2 = -dt^2 + \cosh^2 t d\Omega_3^2,$$

(2.3)

where $d\Omega_3^2$ is the line element of the unit 3-sphere.

If $A(w)$ is constant then the solution to the Einstein equation is of the ADD type [44]. This solution was already investigated in [45]–[47]. In appendix A.1, we shall see that this solution is a particular limit of the solutions below.
Warped flux compactification and brane gravity

Our interest is in the case of non-vanishing $\partial_w A$ since this corresponds to a warped geometry. (As already stated, we would like to consider a minimal set-up which includes at least a warped geometry, a flux and a brane.) In this case we can introduce a new coordinate $r$ via $r = A(w)$ at least locally. With the new coordinate, the line element is

$$ds^2_5 = r^2 \, ds^2_4 + g(r) \, dr^2 + f(r) \, d\phi^2,$$

where $f(r)$ and $g(r)$ are functions of $r$ only. It is of course possible and indeed straightforward to analyse the Einstein equation and find solutions with this ansatz. It is also possible to take double Wick rotation, find a family of solutions and then take inverse double Wick rotation. By the double Wick rotation

$$t \to i \left( \frac{\pi}{2} - \theta \right), \quad \phi \to iT,$$

the metric ansatz (2.4) is transformed to

$$ds^2_5 = -\tilde{f}(r) \, dT^2 + \tilde{g}(r) \, dr^2 + r^2 \, d\Omega_4^2,$$

where

$$d\Omega_4^2 = d\theta^2 + \sin^2 \theta \, d\Omega_3^2$$

is the line element of the unit 4-sphere. The metric (2.6) is nothing but a general ansatz for a spherically symmetric, static metric. There is a well-known family of solutions: RN–de Sitter (for $\Lambda_6 > 0$), RN (for $\Lambda_6 = 0$) and RN–AdS (for $\Lambda_6 < 0$) spacetime (‘RN’ stands for ‘Reissner–Nordström’). The RN–de Sitter, RN or RN–AdS solution is

$$\tilde{f}(r) = \frac{1}{\tilde{g}(r)} = 1 - \frac{\Lambda_6}{10} r^2 - \frac{\mu}{r^3} + \frac{e^2}{12r^6},$$

$$A_M \, dx^M = \frac{e}{3r^3} \, dT,$$

where $\mu$ is a constant of integration corresponding to the mass parameter and $e$ is the electric charge. Going back to the original ansatz by the inverse double Wick rotation

$$\frac{\pi}{2} - \theta \to -it, \quad T \to -i\phi,$$

we obtain the family of solutions

$$f(r) = \frac{1}{g(r)} = 1 - \frac{\Lambda_6}{10} r^2 - \frac{\mu}{r^3} - \frac{b^2}{12r^6},$$

$$A_M \, dx^M = \frac{b}{3r^3} \, d\phi,$$

where we have introduced the magnetic charge $b$ via $e \to ib$ so that $A_M \, dx^M$ remains real.

This set-up was considered in [48, 49] but, as far as we know, the family of explicit exact solutions of warped flux compactification presented below had not been found in the literature.
2.2. Brane sources

In the following we shall consider one or two 3-brane sources. For this purpose we shall use the well-known formula

$$\delta_{\pm} = \frac{\sigma_{\pm}}{M_6},$$

(2.11)

where $\delta_{\pm}$ is the deficit angle due to the tension $\sigma_{\pm}$ of the branes. We do, however, have to keep in mind that there is no simple general prescription, analogous to Israel’s junction condition [50], for obtaining physical characteristics of an arbitrary distributional source with more than one codimension [51, 52]. The difficulty is essentially due to the fact that Einstein equation is non-linear. The formula (2.11) is valid under axisymmetry if the radial stress is much smaller than the energy density [53]. That is, with these conditions, $\sigma_{\pm}$ in (2.11) can be effectively considered as the inertial mass per brane volume. On the other hand, if radial stress is not negligible, then the formula (2.11) should be considered as the definition of $\sigma_{\pm}$, which is in general different from inertial mass per brane volume.

Hereafter we assume that the function $f(r)$ given by (2.10) has two positive roots $r = r_{\pm}$ ($0 < r_- < r_+$) and is positive between them ($r_- < r < r_+$). This requires that the six-dimensional cosmological constant $\Lambda_6$ be positive. Thus, hereafter we assume that $\Lambda_6 > 0$. Since $f$ vanishes at $r = r_{\pm}$, the equation $r = r_{\pm}$ defines surfaces of codimension 2.

Near $r = r_{\pm}$ in the bulk ($r_- < r < r_+$),

$$\frac{dr^2}{f(r)} + f(r) d\phi^2 \simeq d\rho_{\pm}^2 + \kappa_{\pm}^2 \rho_{\pm}^2 d\phi^2,$$

(2.12)

where

$$\rho_{\pm} \equiv \sqrt{\frac{\mp 2(r - r_{\pm})}{\kappa_{\pm}}}, \quad \kappa_{\pm} \equiv \mp \frac{1}{2} f'(r_{\pm})(>0).$$

(2.13)

Hence, with the deficit angles $\delta_{\pm}$ at $r = r_{\pm}$, respectively, $\phi$ is identified as

$$\kappa_{\pm} \phi \sim \kappa_{\pm} \phi + (2\pi - \delta_{\pm}).$$

(2.14)

The identification at $r = r_+$ is consistent with that at $r = r_-$ if and only if

$$\frac{2\pi - \delta_{+}}{2\pi - \delta_-} = \frac{\kappa_+}{\kappa_-}.$$

(2.15)

This can be considered as a boundary condition since the lhs is specified by the brane sources and the rhs can be written in terms of the bulk parameters $\mu$ and $b$.

In this way, we can put 3-branes at $r = r_{\pm}$ and consider the region $r_- < r < r_+$ as the bulk spacetime. It is also possible to consider a solution with only one 3-brane at either $r = r_+$ or $r_-$ by setting $\delta_- = 0$ or $\delta_+ = 0$. The Hubble parameter $H_{\pm}$ on the brane at $r = r_{\pm}$ is given by

$$H_{\pm} = \frac{1}{r_{\pm}}.$$

(2.16)
3. Recovery of the Friedmann equation

Let us consider a \((4 + n)\)-dimensional, general warped compactification

\[
d s_{4+n}^2 = r^2 g_{\mu\nu}^{(4)} \, dx^\mu \, dx^\nu + \gamma_{ij} \, dy^i \, dy^j, \tag{3.1}
\]

where the four-dimensional metric \(g_{\mu\nu}^{(4)}\) depends on the four-dimensional coordinates \(x^\mu\) only, and the \(n\)-dimensional metric \(\gamma_{ij}\) and the warp factor \(r\) depend only on the coordinates \(y^i\) of the compact extra dimensions. With this warped metric, the \((4 + n)\)-dimensional Einstein–Hilbert action includes the four-dimensional Einstein term:

\[
(M_4)^{2+n} \int d^n y \sqrt{-g^{(4+n)}} R^{(4+n)} = (M_4)^{2+n} \int d^n y \sqrt{\gamma} r^2 \times \int d^4 x \sqrt{-g^{(4)}} R^{(4)} + \cdots, \tag{3.2}
\]

where dots represent terms including derivatives of \(r\), the curvature of \(\gamma_{ij}\) and so on. Therefore, if all moduli associated with the extra dimensions are stabilized and if their masses are large enough, then we expect the four-dimensional Einstein theory to be recovered at low energy and that Newton’s constant \(G_N\) on a brane at \(y^i = y_0^i\) should be given by

\[
\frac{1}{8\pi G_N} = (M_4)^{2+n} \int d^n y \sqrt{\gamma} \left[ \frac{r(y)}{r(y_0)} \right]^2, \tag{3.3}
\]

where the normalization factor \([r(y_0)]^{-2}\) has been included in order to take into account the fact that the induced metric on the brane is not \(g_{\mu\nu}^{(4)} \, dx^\mu \, dx^\nu|_{y=y_0}\) but \(r^2 g_{\mu\nu}^{(4)} \, dx^\mu \, dx^\nu|_{y=y_0}\).

This expectation is known to be correct for codimension 1 \((n = 1)\) braneworlds with radion stabilization \([37]–[43]\)\(^6\).

We expect the formula (3.3) to be correct also for codimension 2 or higher \((n \geq 2)\) braneworlds if all moduli are stabilized. In this paper, for simplicity, we consider the six-dimensional braneworld described in the previous section and a situation where the tension of one of the 3-branes at \(r = r_{\pm}\) changes by a phase transition on the brane. We suppose that the tension is almost constant deep inside the old and new phases. With this set-up, it is expected that the four-dimensional geometries on the brane deep inside the two phases are approximated by de Sitter spacetimes with different Hubble expansion rates. What we should see is the relation between the difference of tension in the two phases and the corresponding change of the Hubble expansion rate. Indeed, we shall see below that at low energy, the relation is identical to that inferred from the standard Friedmann equation, provided that the four-dimensional Newton’s constant is given by the formula (3.3). We shall also see what ‘low energy’ means exactly.

By the phase transition, the bulk geometry should change, since the boundary condition set by the brane tension changes. In particular, the bulk parameters \(\mu\) and \(b\) after the phase transition will in general be different from what they were before. Actually, for the following reason, the values of \(\mu\) and \(b\) after the phase transition should be uniquely determined by the brane tension. Indeed, the boundary condition (2.15) and the

\[^6\text{With the } Z_2 \text{ symmetry, the integration over the bulk must be multiplied by 2 to take into account the fact that there are two copies of the same bulk geometry.}\]
conservation of magnetic flux provide two independent conditions on the two independent parameters $\mu$ and $b$ and, thus, uniquely fix the parameters at least for a sufficiently small change.

When one of the brane tensions changes, the deficit angle at the position of the brane changes, according to the formula (2.11). This in general induces the change of the area of the extra dimensions, in particular the interval of $\phi$. Since the magnetic flux is nothing but the integral of the magnetic field over the extra dimensions, the change of the area and the flux conservation imply that the amplitude $b$ of the magnetic field should change. At the same time, the parameters $\mu$ and $b$ must satisfy the boundary condition (2.15) with the new deficit angle corresponding to the tension after the phase transition. Therefore, not only $b$ but also $\mu$ should change in general. In this way, the values of $b$ and $\mu$ after the phase transition are uniquely determined by the flux conservation and the boundary condition.

Accordingly, positions $r_\pm$ of branes are also determined uniquely since they are defined as roots of the function $f(r)$ in (2.10). In other words, the Hubble expansion rate $H_\pm = 1/r_\pm$ changes after the phase transition and there is a unique relation between the change of the brane tension and the change of the Hubble expansion rate. Since all relevant equations are invariant under the reflection $H_\pm \rightarrow -H_\pm$, the resulting physical relation must also be invariant under this reflection. Therefore, the relation must be even in $H_\pm$ and its Taylor expansion w.r.t. $\Delta H_\pm^2$ should start as

$$\Delta H_\pm^2 \propto \Delta \sigma_\pm + O((\Delta \sigma_\pm)^2),$$

(3.4)

where $\Delta X$ represents a small difference between the quantity $X$ before and after the phase transition. Finally, from this relation we obtain

$$H_\pm^2 \propto (\sigma_\pm - \sigma_\pm^{(0)}) + O((\sigma_\pm - \sigma_\pm^{(0)})^2)$$

(3.5)

for small $H_\pm^2$, where $\sigma_\pm^{(0)}$ is the value of $\sigma_\pm$ corresponding to $H_\pm^2 = 0$. In this argument, it has been implicitly assumed that the limit $H_\pm^2 \rightarrow 0$ is regular. In appendix A.2 we explicitly confirm that this limit is indeed regular.

The relation (3.5) is nothing but the standard Friedmann equation if the proportionality coefficient is $8\pi G_N/3$, where $G_N$ is Newton’s constant. In the following, we shall see both numerically and analytically that this is indeed the case, where Newton’s constant is given by the formula (3.3). To be more specific, we shall show that Newton’s constant $G_{N\pm}$ on the brane at $r = r_\pm$, respectively, is given by

$$\frac{1}{8\pi G_{N\pm}} = M_6^4 \int_{r_-}^{r_+} \left( \frac{r^2}{r_\pm^2} \right) \frac{1}{r_\pm} \frac{d^4 \phi}{d^2 \phi} = M_6^4 L^2 \Sigma \cdot \left( \frac{r_-}{r_+} \right)^{\pm 1},$$

(3.6)

where $L \equiv \sqrt{10/\Lambda_6},$

$$\Sigma \equiv \frac{1}{L^2} \int_{r_-}^{r_+} \frac{r^2}{r_+ r_-} \frac{d^4 \phi}{d^2 \phi} = \frac{\Delta \phi}{3L^2 r_+ r_-} (r_+^3 - r_-^3)$$

(3.7)

is the warped volume of the extra dimensions (in units of $L^2$) and

$$\Delta \phi = \frac{2\pi - \delta_+}{\kappa_+} = \frac{2\pi - \delta_-}{\kappa_-}$$

(3.8)

is the period of the angular coordinate $\phi$. 

Journal of Cosmology and Astroparticle Physics 07 (2005) 013 (stacks.iop.org/JCAP/2005/i=07/a=013) 9
For later convenience, here we define the magnetic flux \( \Phi \) in units of \( L \) as

\[
\Phi = \frac{1}{L} \int_{r_-}^{r_+} dr \int_0^{\Delta \phi} d\phi F_{\phi \phi} = -\frac{b \Delta \phi}{3L} \left( \frac{1}{r_-^2} - \frac{1}{r_+^2} \right).
\]  

All relevant equations including this are invariant under the reflection \((b, \Phi) \rightarrow (-b, -\Phi)\). Thus, we do not need to keep track of the sign of \( \Phi \) as long as its sign relative to \( b \) is correct. As already stated, the magnetic flux is conserved and, thus, must be fixed during the phase transition.

### 3.1. Reparametrization

As already explained, we expect that the Friedmann equation should be recovered at low energy as a consequence of the response of the bulk geometry to the evolution of the brane source. In this respect it is not physically relevant to keep track of the change of the bulk parameters \( \mu \) and \( b \). What is physically important is, instead, the brane tensions (or equivalently the deficit angles) and the Hubble expansion rates on the branes (or equivalently the positions \( r_\pm \) of the branes).

Hence, it is useful to express the metric in terms of \( h \equiv L \sqrt{H_+ H_-} = L/\sqrt{r_+ r_-} \) and \( \alpha \equiv H_+/H_- = r_-/r_+ \) instead of the original parameters \( \mu \) and \( b \). By the definition of \( r_\pm \), the new parameter \( \alpha \) satisfies \( 0 < \alpha \leq 1 \). (In appendix A.1, the \( \alpha \rightarrow 1 \) limit is shown to be regular.) By solving \( f(r_\pm) = 0 \) w.r.t. \( \mu \) and \( b^2 \), we obtain

\[
\begin{align*}
\frac{h^5}{L^3} &= -\frac{\beta_{\gamma/2}}{\beta_1} + (\alpha^{3/2} + \alpha^{-3/2})h^2, \\
\frac{h b^2}{L^5} &= \frac{12 \beta_2}{\beta_1} - 12 h^2,
\end{align*}
\]

where \( \beta_n = \sum_{i=0}^{2n} \alpha^{i-n} \). With this parametrization, (2.15) and (3.9) become

\[
\begin{align*}
\frac{2\pi - \delta_+}{2\pi - \delta_-} &= \frac{\gamma_+ - 3\beta_1^2 h^2}{\gamma_- - 3\beta_1^2 h^2} \alpha^4,
\end{align*}
\]

and

\[
\frac{\Phi^2}{(2\pi - \delta_+)(2\pi - \delta_-)} = \frac{16}{3} \frac{(\beta_2 - \beta_1 h^2)^2 \beta_1^4}{(\gamma_+ - 3\beta_1^2 h^2)(\gamma_- - 3\beta_1^2 h^2)}.
\]

respectively, where

\[
\begin{align*}
\gamma_+ &= 3\alpha^3 + 6\alpha^2 + 9\alpha + 12 + 15\alpha^{-1} + 10\alpha^{-2} + 5\alpha^{-3}, \\
\gamma_- &= 5\alpha^3 + 10\alpha^2 + 15\alpha + 12 + 9\alpha^{-1} + 6\alpha^{-2} + 3\alpha^{-3}.
\end{align*}
\]

By using \( \Delta \phi > 0 \), \( 0 < \alpha \leq 1 \) and \( b^2 \geq 0 \), it is shown that

\[
0 < \frac{2\pi - \delta_+}{2\pi - \delta_-} \leq 1,
\]

where the equality holds for \( \alpha = 1 \). The inequality (3.14) combined with the formula (2.11) implies that the tension of the brane at \( r = r_- \) must be smaller than the tension of the brane at \( r = r_+ \). If this condition is not satisfied then it is expected that the cosmological constant on the brane cannot be non-negative or/and the geometry of the extra dimensions...
becomes dynamical. The warped volume $\Sigma$ of extra dimensions defined in (3.7) is given by

$$\frac{\Sigma}{\sqrt{(2\pi - \delta_+)(2\pi - \delta_-)}} = \frac{2\beta_+^2}{3\sqrt{(\gamma_+ - 3\beta_+^2 h^2)(\gamma_- - 3\beta_+^2 h^2)}}. \quad (3.15)$$

3.2. Brane gravity on the IR brane

We now would like to see that the relation of the form (3.5) does indeed hold at low energy on each brane. Since the warp factor $r$ is smaller on the brane at $r = r_-$, we may call this brane an IR brane and the other brane at $r = r_+$ a UV brane. In this and the next subsections, we shall show the relation (3.5) on the IR and UV branes, respectively, by using numerical plots and determine the proportionality coefficient. In section 3.4 we show the same result analytically by low energy expansion.

Let us first consider the case where the phase transition takes place on the brane at $r = r_-$. In this case the deficit angle $\delta_+$ around the other brane at $r = r_+$ and the flux $\Phi$ must be fixed. It is convenient to define the following dimensionless quantities:

$$\eta_- \equiv \frac{2\pi - \delta_-}{2\pi - \delta_+},$$
$$h_- \equiv L^2 H_- = h \alpha^{-1/2},$$
$$\Phi_+ \equiv \frac{|\Phi|}{2\pi - \delta_+} = \left[\frac{\Phi^2}{(2\pi - \delta_+)(2\pi - \delta_-)}\right]^{1/2} \eta_-^{1/2},$$
$$g_- \equiv 8\pi G_{N-} M_6^4 L^2 (2\pi - \delta_+) = \frac{1}{3} \left[\frac{\Sigma}{\sqrt{(2\pi - \delta_+)(2\pi - \delta_-)}}\right]^{-1} \eta_-^{-1/2} \alpha, \quad \text{(3.16)}$$

where $G_{N-}$ is defined by (3.6). The formulae (3.11), (3.12) and (3.15) give expressions for $\eta_-, \Phi_+^2$ and $g_-$ in terms of $h_2$ and $\alpha$. The inequality (3.14) is written as $\eta_- \geq 1$, where the equality holds for $\alpha = 1$.

The quantities $\alpha, h_-, \eta_-, \Phi_+$ and $g_-$ have the following physical meaning. The first quantity $\alpha \equiv r_-/r_+$ ($0 < \alpha \leq 1$) is the ratio of the warp factor so that a small (or large) value of $\alpha$ corresponds to a large (or small, respectively) hierarchy. The second quantity $h_-$ is the Hubble expansion rate on the brane at $r = r_-$ in units of the bulk curvature scale $L^{-1}$. Note that in this subsection, the phase transition is supposed to take place on this brane. Since the deficit angle $\delta_+$ around the other brane at $r = r_+$ is determined by the tension of the other brane at $r = r_+$ (see the formula (2.11)) and has nothing to do with the phase transition on the brane at $r = r_-$, $2\pi - \delta_+$ is constant during the phase transition. Hence, the change of tension of the brane at $r = r_-$ is proportional to the change of the third quantity $\eta_-$. The proportionality coefficient depends on $2\pi - \delta_+$ but it does not change by the phase transition. Similarly, specifying the magnetic flux up to its signature is equivalent to specifying the fourth quantity $\Phi_+$. As already stated, all relevant equations are invariant under reflection of the sign of the flux and, thus, we do not need to keep track of the sign. The normalization of $\Phi_+$ has been determined so that it does not change during the phase transition and that it absorbs the factor $(2\pi - \delta_+)^{-2}$ in the equation (3.12) divided by (3.11). The final quantity $g_-$ is proportional
Figure 1. Each line represents the relation between $h_+^2$ (vertical axis) and $\eta_-$ (horizontal axis) for a fixed value of $\Phi_+^2$ (=0.3, 0.4, 0.5, 0.6, 0.7 from left to right). Note that the value of $\eta_-$ is restricted to the region $\eta_- \geq 1$ as shown in (3.14). The domain shown in this figure can be joined to the domain shown in figure 3 by identifying the vertical line $\eta_+ = 1$ in this figure with the vertical line $\eta_+ = 1$ in figure 3.

to the quantity $G_N$ which is defined by (3.6) and which shall be identified as the effective Newton’s constant on the brane at $r = r_-$. The normalization of $g_-$ has been determined so that $g_-$ is dimensionless and that it eliminates explicit dependence on $(2\pi - \delta_+)$ from equation (3.19) below.

When the Hubble expansion rate on the brane vanishes, i.e. when $h^2 = 0$, the equations are simplified. Hereafter, a superscript (0) shows that the corresponding quantity is evaluated at $h^2 = 0$. In particular, it is easy to see that

\[
\frac{1}{9} \leq \Phi_+^{(0)2} < \infty,
\]

\[
\frac{10}{\pi} \geq g_-^{(0)} > 0,
\]

where each quantity (with $h = 0$) is a monotonic function of $\alpha$ and the left and right values are values at $\alpha = 1$ and 0, respectively.

Our aim in this paper is to show that, when a phase transition takes place on a brane, the Hubble expansion rate changes according to the standard Friedmann equation with Newton’s constant given by the formula (3.6). In the present context where the phase transition takes place on the brane at $r = r_-$, we would like to see a relation between $h^2$ and $\eta_-$ with $\Phi_+^2$ fixed. Actually, by using equations (3.11) and (3.12), it is fairly easy to plot the curve $(\eta_-, h_-^2)$ for various fixed values of $\Phi_+^2$. See figure 1. It is seen that the slope of each curve in figure 1 is negative near the horizontal axis, i.e. for small $h_-^2$. In other words, at low energy the Hubble expansion rate $h_-$ does indeed decrease as the brane tension $\sigma_-$ and the deficit angle $\delta_-$ decrease (see the formula (2.11)) by a phase transition and Newton’s constant on the brane is positive. The intersection of each curve with the horizontal axis defines the value $\eta_-^{(0)}(\Phi_+^2)$ of $\eta_-$ corresponding to $h_-^2 = 0$. Thus,

\[
\sigma_-^{(0)}(\Phi_+^2) \equiv [2\pi - (2\pi - \delta_-)\eta_-^{(0)}(\Phi_+^2)] \cdot M_6^4
\]
is the critical value of the tension $\sigma_-$ for which the effective cosmological constant on the brane vanishes. If $\sigma_- > \sigma_-^{(0)}(\Phi_+^2)$ (or $\sigma_- < \sigma_-^{(0)}(\Phi_+^2)$), then the effective cosmological constant on the brane is positive (or negative, respectively).

The positivity of the low energy effective Newton’s constant is highlighted in figure 2, where the vertical axis is again $h_+^2$ but the horizontal axis is now $\eta_-^{(0)}(\Phi_+^2) - \eta_-$. The positive slope of the curve near the origin clearly defines the low energy effective Newton’s constant since it determines how much the Hubble expansion rate changes as the brane tension changes. The dotted lines in figure 2 are straight lines passing through the origin with the slope $g_-^{(0)}(\Phi_+^2)$, where $g_-^{(0)}(\Phi_+^2)$ is the value of $g_-$ with $h_-^2 = 0$ for a given value of $\Phi_+^2$. It is easily seen that the solid lines and the dotted lines in figure 2 come in contact with each other at the origin. This implies that

$$h_+^2 = g_-^{(0)} \cdot (\eta_-^{(0)} - \eta_-) + O((\eta_-^{(0)} - \eta_-)^2).$$

By definition of $g_-$ and the formula (2.11), this is equivalent to

$$H_-^2 = \frac{8\pi G_{N-}}{3}(\sigma_- - \sigma_-^{(0)}) + O((\sigma_- - \sigma_-^{(0)})^2),$$

which confirms that $G_{N-}$ defined in (3.6) does indeed have the meaning of the four-dimensional Newton’s constant on the brane at $r = r_-$. The straight dotted lines in figure 2 are a good approximation to the solid lines in the regime where $h_-^2$ is sufficiently smaller than 1. This suggests that the Friedmann equation should be recovered at low energy where the Hubble expansion rate squared $H_-^2$ on the brane is sufficiently smaller than the bulk cosmological constant $\Lambda_6 (=10L^{-2})$. In section 3.4, we shall show the relation (3.20) analytically.

### 3.3. Brane gravity on the UV brane

Next let us consider the case where a phase transition takes place on the UV brane at $r = r_+$. In this subsection, we shall show the relation (3.5) on the UV brane by using numerical plots and determine the proportionality coefficient. In the next section 3.4 we show the same results analytically by means of low energy expansion.

In the present case where a phase transition takes place on the brane at $r = r_+$, the deficit angle $\delta_-$ around the brane at $r = r_-$ and the flux $\Phi$ must be fixed. Similarly to in the previous case, it is convenient to define the following dimensionless quantities:

$$\eta_+ \equiv \frac{2\pi - \delta_+}{2\pi - \delta_-},$$

$$h_+ \equiv LH_- = h_+ \alpha^{1/2},$$

$$\Phi_- \equiv \frac{|\Phi|}{2\pi - \delta_-} = \left[\frac{\Phi^2}{(2\pi - \delta_+)(2\pi - \delta_-)}\right]^{1/2} \eta_+^{1/2},$$

$$g_+ \equiv \frac{8\pi G_{N+}}{3}M_6^4L^2(2\pi - \delta_-) = \frac{1}{3} \left[\frac{\Sigma}{\sqrt{(2\pi - \delta_+)(2\pi - \delta_-)}}\right]^{-1} \eta_+^{-1/2} \alpha^{-1},$$

where $G_{N+}$ is defined by (3.6). The formulae (3.11), (3.12) and (3.15) give expressions for $\eta_+$, $\Phi^2$ and $g_+$ in terms of $h_+^2$ and $\alpha$. The inequality (3.14) is written as $0 < \eta_+ \leq 1$, where the equality holds for $\alpha = 1$. 

| Journal of Cosmology and Astroparticle Physics 07 (2005) 013 (stacks.iop.org/JCAP/2005/i=07/a=013) | 13 |
The quantities $h_+, \eta_+, \Phi_+$ and $g_+$ have physical meanings similar to those of $h_-, \eta_-, \Phi_-$ and $g_-$, which have already been explained in the third paragraph of the previous subsection.

When the Hubble expansion rate on the brane vanishes, i.e. when $h^2 = 0$, the equations are simplified. As before, a superscript $(0)$ shows that the corresponding quantity is evaluated at $h^2 = 0$. In particular, it is easy to see that

\begin{equation}
0 < \Phi_-^{(0)2} \leq \frac{1}{4}, \quad \infty > g_+^{(0)} \geq \frac{10}{3},
\end{equation}

where each quantity (with $h = 0$) is a monotonic function of $\alpha$ and the left and right values are values at $\alpha = 0$ and 1, respectively.

As already stated many times, our aim in this paper is to show that, when a phase transition takes place on a brane, the Hubble expansion rate changes according to the standard Friedmann equation with Newton’s constant given by the formula (3.6). In this subsection we are considering a situation where the phase transition takes place on the brane at $r = r_+$. Thus, we would like to see a relation between $h^2_+$ and $\eta_+$ with $\Phi^2_+$ fixed. Actually, by using equations (3.11) and (3.12), it is fairly easy to plot the curve $(\eta_+, h^2_+)$ for various fixed values of $\Phi^2_-$. See figure 3. It is seen that the slope of each curve in figure 3 is negative near the horizontal axis, i.e. for small $h^2_+$. In other words, at low energy the Hubble expansion rate $h_+$ does indeed decrease as the brane tension $\sigma_+$ and the deficit angle $\delta_+$ decrease (see the formula (2.11)) by a phase transition and Newton’s constant on the brane is positive.
At first sight, this appears to contradict the negativity of the slope of the curves in figure 1 near $h^2 = 0$: figure 3 indicates that $\eta_+$ decreases when $h^2$ increases from zero but figure 1 indicates that $\eta_- = 1/\eta_+$ also decreases when $h^2$ increases from zero. Actually, there is no contradiction. Along each curve in figure 1, what is fixed is not $\Phi^2_-$ but $\Phi^2_+$. If $h^2$ is increased from zero with $\Phi^2_-$ fixed, then figure 3 says that $\Phi^2_+ = \Phi^2_-/\eta^2_+$ should increase. In figure 1 this means that we have to move from a curve with the initial $\Phi^2_+$ to different curves with larger $\Phi^2_+$. Hence, if the value of $\eta_-$ were the same, then this would inevitably increase the value of $h^2$ from zero. Actually, since $\eta_-$ is the reciprocal of $\eta_+$, figure 3 says that $\eta_-$ should also increase. Therefore, when $h^2$ is increased from zero, there are two competitive effects which figure 3 implies: (i) the increase of $\Phi^2_+$; (ii) the increase of $\eta_-$. Effect (i) tends to increase $h^2$ from zero but effect (ii) tends to decrease $h^2$. The result of the competition is that effect (i) wins. Hence, the negativities of the slopes of the curves in figures 1 and 3 near the horizontal axis are consistent with each other.

The intersection of each curve in figure 3 with the horizontal axis defines the value $\eta_+^{(0)}(\Phi^-_2)$ of $\eta_+$ corresponding to $h^2_+ = 0$. Thus,

$$\sigma_+^{(0)}(\Phi^-_2) \equiv \left[2\pi - (2\pi - \delta_-)\eta_+^{(0)}(\Phi^-_2)\right] \cdot M_6^4$$

(3.23)

is the critical value of the tension $\sigma_+$ for which the effective cosmological constant on the brane vanishes. If $\sigma_+ > \sigma_+^{(0)}(\Phi^-_2)$ (or $\sigma_+ < \sigma_+^{(0)}(\Phi^-_2)$), then the effective cosmological constant on the brane is positive (or negative, respectively).

The positivity of the low energy effective Newton’s constant is highlighted in figure 4, where the vertical axis is again $h^2_+$ but the horizontal axis is now $\eta_+^{(0)}(\Phi^-_2) - \eta_-$. The positive slope of the curve near the origin clearly defines the low energy effective Newton’s
Figure 4. Solid lines are curves shown in figure 3 in different coordinates. The vertical axis is again $h_+^2$ but the horizontal axis is now $\eta_+^{(0)}(\Phi_2^2) - \eta_+$, where $\eta_+^{(0)}(\Phi_2^2)$ is the value of $\eta_+$ corresponding to $h_+^2 = 0$ for a given value of $\Phi_2^2$. The values of $\Phi_2^2$ for the curves are 0.02, 0.08, 0.14, 0.2 from left to right. Dotted lines represent straight lines passing through the origin with the slope $g_+^{(0)}(\Phi_2^2)$ for each value of $\Phi_2^2$. These graphs confirm that $G_{N+}$ defined by (3.6) does indeed have the meaning of the four-dimensional Newton’s constant on the brane at $r = r_+$.

constant since it determines how much the Hubble expansion rate changes as the brane tension changes. The dotted lines in figure 4 are straight lines passing through the origin with the slope $g_+^{(0)}(\Phi_2^2)$, where $g_+^{(0)}(\Phi_2^2)$ is the value of $g_+$ with $h_+^2 = 0$ for a given value of $\Phi_2^2$. It is easily seen that the solid lines and the dotted lines in figure 4 come in contact with each other at the origin. This implies that

$$h_+^2 = g_+^{(0)} \cdot (\eta_+^{(0)} - \eta_+) + O((\eta_+^{(0)} - \eta_+)^2).$$

By definition of $g_+$ and the formula (2.11), this is equivalent to

$$H_+^2 = \frac{8\pi G_{N+}}{3} (\sigma_+ - \sigma_+^{(0)}) + O((\sigma_+ - \sigma_+^{(0)})^2),$$

which confirms that $G_{N+}$ defined in (3.6) does indeed have the meaning of the four-dimensional Newton’s constant on the brane at $r = r_+$. The straight dotted lines in figure 4 are a good approximation to the solid lines in the regime where $h_+^2$ is sufficiently smaller than 1. This suggests that the Friedmann equation should be recovered at low energy where the Hubble expansion rate squared $H_+^2$ on the brane is sufficiently smaller than the bulk cosmological constant $\Lambda_6 (=10L^{-2})$. In the next section 3.4, we shall show the relation (3.25) analytically.

3.4. Low energy expansion

In this subsection we show the low energy relations (3.20) and (3.25) analytically, where Newton’s constant $G_{N\pm}$ is given by (3.6). For this purpose we shall perform perturbative
expansion of relevant quantities and equations w.r.t. $h^2 (= h_+ h_-)$ and show that (3.19) and (3.24) do indeed hold in the lowest order in the perturbative expansion.

Let us first consider the case considered in section 3.2, where the phase transition takes place on the brane at $r = r_-$. In this case we expand $\alpha$ and $\eta_-$ as

$$\alpha = \sum_{n=0}^{\infty} \alpha_-(^n) (\Phi_+^2) h^{2n}, \quad \eta_- = \sum_{n=0}^{\infty} \eta_-(^n) (\Phi_+^2) h^{2n}. \quad (3.26)$$

Since $\Phi_+$ is fixed during the phase transition, we formally consider $\Phi_+$ as a quantity $O(h^0)$ ('O' standing for '(of) order'). It is straightforward to expand (3.12) and (3.11) w.r.t. $h^2$.

In $O(h^0)$, we obtain

$$\Phi_+^2 = \frac{16 \beta_1(0) \beta_1(0)}{3 \gamma_+ \alpha_-(0)^2}, \quad \eta_-(0) = \frac{\gamma_-(0)}{\gamma_+ \alpha_-(0)^2}, \quad (3.27)$$

where $\beta_n(0)$ and $\gamma_\pm(0)$ are $\beta_n$ and $\gamma_\pm$, respectively, with $\alpha$ replaced by $\alpha_-(0)$. On the other hand, the zeroth-order part of equation (3.15) gives the value $g_-(0) (\Phi_+^2)$ of $g_-$ with $h^2 = 0$ for a given value of $\Phi_+^2$ as

$$g_-(0) = \frac{\gamma_0(0) \gamma_-(0)^3}{2 \beta_1(0) \alpha_-(0)^2}. \quad (3.28)$$

In $O(h^2)$, equation (3.12) divided by (3.11) gives an expression of $\alpha_-(1)$ in terms of $\alpha_-(0)$ as

$$\frac{\alpha_-(1)}{\alpha_-(0)} = \frac{\beta_1(0) A_-(0)}{B_-(0)}, \quad (3.29)$$

where

$$A_-(0) = 3 \alpha_-(0)^3 + 6 \alpha_-(0)^2 + 9 \alpha_-(0) + 6 + 3 \alpha_-(0)^{-1} + 2 \alpha_-(0)^{-2} + \alpha_-(0)^{-3},$$

$$B_-(0) = 15 \alpha_-(0)^6 + 60 \alpha_-(0)^5 + 150 \alpha_-(0)^4 + 258 \alpha_-(0)^3 + 357 \alpha_-(0)^2 + 430 \alpha_-(0) + 460$$

$$+ 430 \alpha_-(0)^{-1} + 357 \alpha_-(0)^{-2} + 258 \alpha_-(0)^{-3} + 150 \alpha_-(0)^{-4} + 60 \alpha_-(0)^{-5} + 15 \alpha_-(0)^{-6}. \quad (3.30)$$

Here, as already explained, $\Phi_+$ was considered as a quantity $O(h^0)$ in the derivation of the above expression of $\alpha_-(0)$. On the other hand, the reciprocal of equation (3.11) in $O(h^2)$ gives

$$\eta_-(1) = - \frac{2 \beta_1(0) \alpha_-(0)^2}{\gamma_+ \alpha_-(0)^4} = - \frac{1}{g_-(0) \alpha_-(0)^4}. \quad (3.31)$$

Now, the definition of $\eta_-^{(0,1)}$

$$\eta_- = \eta_-(0) + \eta_-(1) h^2 + O(h^4) \quad (3.32)$$
is solved w.r.t. $h^2$ as
\begin{equation}
    h^2 = \frac{\eta_--\eta_-^{(0)}}{\eta_-^{(1)}} + O(h^4),
    \tag{3.33}
\end{equation}
or equivalently
\begin{equation}
    h_+^2 = \frac{h^2}{\alpha} = g_-^{(0)} \cdot (\eta_-^{(0)} - \eta_-) + O(h_-^4).
    \tag{3.34}
\end{equation}
This proves (3.19). By the definition of $g_-$ and the formula (2.11), this is equivalent to (3.20).

It is easy to perform a similar analysis for the other case considered in section 3.3, where the phase transition takes place on the brane at $r = r_+$, and to derive (3.19) and (3.25) analytically. In this case we expand $\alpha$ and $\eta_+$ as
\begin{equation}
    \alpha = \sum_{n=0}^{\infty} \alpha_{+}^{(n)} (\Phi_+^2) h^{2n}, \quad \eta_+ = \sum_{n=0}^{\infty} \eta_+^{(n)} (\Phi_+^2) h^{2n}.
    \tag{3.35}
\end{equation}
Since $\Phi_-$ is fixed during the phase transition, we formally consider $\Phi_-$ as a quantity $O(h^0)$. It is straightforward to expand (3.12) and (3.11) w.r.t. $h^2$. In $O(h^0)$, we obtain
\begin{equation}
    \Phi_-^2 = \frac{16 \beta_2^{(0)} \beta_1^{(0)3} \alpha_+^{(0)4}}{3 \gamma_-^{(0)2}},
    \eta_+^{(0)} = \frac{\gamma_+^{(0)} \alpha_+^{(0)4}}{\gamma_-^{(0)}},
    \tag{3.36}
\end{equation}
where $\beta_+^{(0)}$ and $\gamma_+^{(0)}$ are $\beta_+^{(0)}$ and $\gamma_+^{(0)}$, respectively, with $\alpha$ replaced by $\alpha_+^{(0)}$. On the other hand, the zeroth-order part of equation (3.15) gives the value $g_+^{(0)}(\Phi_+^2)$ of $g_+$ with $h_+^2 = 0$ for a given value of $\Phi_+^2$ as
\begin{equation}
    g_+^{(0)} = \frac{\gamma_-^{(0)}}{2 \beta_1^{(0)2} \alpha_+^{(0)3}}.
    \tag{3.37}
\end{equation}
In $O(h^2)$, equation (3.12) multiplied by (3.11) gives an expression of $\alpha_+^{(1)}$ in terms of $\alpha_+^{(0)}$ as
\begin{equation}
    \frac{\alpha_+^{(1)}}{\alpha_+^{(0)}} = \frac{\beta_1^{(0)2} A_+^{(0)}}{B_+^{(0)}},
    \tag{3.38}
\end{equation}
where
\begin{equation}
    A_+^{(0)} = 3 \alpha_+^{(0)3} - 6 \alpha_+^{(0)2} + 9 \alpha_+^{(0)} - 6 + 3 \alpha_+^{(0)2} + 2 \alpha_+^{(0)2} + \alpha_+^{(0)3},
    \tag{3.39}
\end{equation}
\begin{equation}
    B_+^{(0)} = 15 \alpha_+^{(0)6} + 60 \alpha_+^{(0)5} + 150 \alpha_+^{(0)4} + 258 \alpha_+^{(0)3} + 357 \alpha_+^{(0)2} - 430 \alpha_+^{(0)1} + 460
    + 430 \alpha_+^{(0)} + 357 \alpha_+^{(0)2} + 258 \alpha_+^{(0)3} + 150 \alpha_+^{(0)4} + 60 \alpha_+^{(0)5} + 15 \alpha_+^{(0)6}.  
\end{equation}
On the other hand, equation (3.11) in \( O(h^2) \) gives

\[
\eta^{(1)}_+ = -\frac{2\beta_1^{(0)} \alpha_+^{(0)} \gamma_-^{(0)}}{g_+^{(0)}} = -\frac{\alpha_+^{(0)}}{g_+^{(0)}}. \tag{3.40}
\]

Now, the definition of \( \eta^{(0,1)}_+ \)

\[
\eta_+ = \eta_+^{(0)} + \eta_+^{(1)} h^2 + O(h^4) \tag{3.41}
\]

is solved w.r.t. \( h^2 \) as

\[
h^2 = \frac{\eta_+ - \eta_+^{(0)}}{\eta_+^{(1)}} + O(h^4), \tag{3.42}
\]

or equivalently

\[
h^2_+ = h^2 \alpha = g_+^{(0)} \cdot \left( \eta_+^{(0)} - \eta_+ \right) + O(h_+^4). \tag{3.43}
\]

This proves (3.24). By the definition of \( g_+ \) and the formula (2.11), this is equivalent to (3.25).

It is straightforward to extend the analysis in this subsection to higher order in the expansion w.r.t. \( h^2 \) and to interpret the result as higher order corrections to the effective Friedmann equation. In appendix A.3 the results in \( O(h^4) \) is summarized and it is shown that higher order corrections can be ignored when the Hubble expansion rate \( H_\pm \) on a brane is sufficiently lower than the bulk curvature scale \( L^{-1} \).

4. Summary and discussion

We have considered a six-dimensional model of warped flux compactification and analysed gravity sourced by a 3-brane in a simplified set-up. This set-up includes a warped geometry, compactification, a flux and one or two 3-brane(s). We have considered a situation where the tension of a 3-brane changes by a phase transition on the brane. Assuming that the tension is almost constant deep inside the old and new phases, we have investigated the relation between the change of tension and the change of the Hubble expansion rate on the brane. The relation is the same as that inferred from the four-dimensional Einstein equation, provided that Newton’s constant is given by the formula (3.6).

We have explicitly seen how the induced geometry changes according to a change of brane source. With the static ansatz for the bulk, the bulk geometry is uniquely determined by the brane tensions and the value of the conserved flux. As a consequence, the Hubble expansion rate induced on the brane is determined. Provided that the solutions considered in this paper are dynamically stable\(^7\), this result implies that, when a brane tension changes, the bulk geometry should evolve toward the corresponding unique configuration. This process determines the evolution of the bulk geometry and, thus, the induced geometry on the brane. In this way, the induced geometry responds to the brane source.

\(^7\) Work in progress. Preliminary consideration suggests stability at least near \( \eta_\pm = 1 \) at low energy, but detailed analysis remains to be done.
Although the situation we have considered is rather restrictive, the physical picture we have obtained seems more general. Provided that all moduli are stabilized, when a brane source changes slowly compared to the timescale of the moduli dynamics, the bulk geometry should quickly settle to a configuration which is determined by the boundary condition, i.e. the brane source(s), values of conserved quantities and the regularity of the other region of the extra dimensions. As a consequence of the change of the bulk geometry, the induced geometry on the brane responds to the evolution of the matter source on the brane.

Suppose that all moduli are stabilized in a \( (4 + n) \)-dimensional warped compactification. From the viewpoint of the effective field theory with massive moduli integrated out, it is in general expected that the Einstein gravity should be recovered at low energy and that the four-dimensional Newton’s constant should be given by formula (3.3):

\[
\frac{1}{8\pi G_N} = \left( M_{4+n} \right)^{2+n} \int d^n y \sqrt{\gamma} \left[ \frac{r(y)}{r(y_0)} \right]^2.
\]  

(4.1)  

(See the first paragraph of section 3 for the reasoning leading to this expectation.) Here, \( M_{4+n} \) is the \( (4+n) \)-dimensional Planck mass, \( r(y) \) is the warp factor of mass dimension \(-2\) depending on the coordinates of compact extra dimensions \( y' \), \( \gamma_{ij} \text{d}y^i \text{d}y^j \) is the metric of the extra dimensions (see equation (3.1)) and the brane source is supposed to be localized at \( y^i = y^i_0 \). Although the effective field theory approach is usually a powerful and useful tool for obtaining a correct answer quickly, the result must be checked by independent methods. We expect that this formula should hold at low energy in a wide class of warped compactification if all moduli are stabilized and made massive.

The formula (3.3) was already proved to be correct in Randall–Sundrum-type, codimension 1 braneworlds with radion stabilization [37]–[43] (see footnote 6). The result of the present paper confirms the same formula for codimension 2 braneworlds in a simplified situation.

Future subjects include the analysis of dynamical stability (see footnote 7), the recovery of the four-dimensional linearized Einstein equation for inhomogeneous perturbations, evolution of the FRW universe on the brane, extensions to systems with a dilaton and so on.

The simple six-dimensional set-up in this paper at the very least provides non-trivial evidence for the validity of the expected formula (3.3) for the effective Newton’s constant in warped flux compactification. Thus, let us now briefly discuss application of the formula (3.3) to cosmology with type IIB warped flux compactification considered in [1], [14]–[18]. The ten-dimensional geometry is of the form

\[
dS_{10}^2 = h^{-1/2}(\tau) g^{(4)}_{\mu\nu} \text{d}x^\mu \text{d}x^\nu + h^{1/2}(\tau) \gamma_{mn} \text{d}\psi^m \text{d}\psi^n,
\]  

where \( \gamma_{mn} \) is the metric of a six-dimensional compact geometry (Calabi–Yau space) depending on the coordinates \( \psi^m \) \((m = 5, \ldots, 10)\) of extra dimensions, \( h(\tau) \) is a
function of the radial coordinate $\tau \in \{\psi, m\}$ of the compact geometry and $g_{\mu\nu}^{(4)}$ is a four-dimensional metric. It is supposed that the function $h(\tau)$ is of order 1 in the bulk of the compact geometry but that there is a region called a warped throat where $h(\tau)$ becomes exponentially large. In the warped throat region the warp factor $h^{-1/2}(\tau)$ becomes exponentially small.

In most of the references [14]–[18] it was (implicitly) assumed that what drives (or at least affects) the four-dimensional cosmology is a brane in the throat region but that our four-dimensional world is somewhere in the Calabi–Yau space where the warp factor $h^{-1/2}(\tau)$ is of order unity. In other words, the physical metric of our four-dimensional world is $g_{\mu\nu}^{(4)}$ up to a normalization constant of order unity, while the induced metric on the brane is not $g_{\mu\nu}^{(4)}$ but

$$g_{\mu\nu}^{(b)} = h^{-1/2}(\tau_0) \cdot g_{\mu\nu}^{(4)}, \quad (4.3)$$

where $\tau_0$ is the value of the radial coordinate $\tau$ at the position of the brane. At first sight, how the brane at $\tau = \tau_0$ drives the cosmology of our four-dimensional world might seem non-trivial since the warp factor $h^{-1/2}(\tau_0)$ is exponentially small. The approach adopted in the literature is to write down the brane action in terms of $g_{\mu\nu}^{(4)}$ and couple it to the Einstein gravity in the frame $g_{\mu\nu}^{(4)}$, assuming that Newton’s constant $G_N^{(4)}$ is given by the Kaluza–Klein-like formula

$$\frac{1}{8\pi G_N^{(4)}} = M_{10}^8 V_6, \quad (4.4)$$

where $V_6$ is the volume of the Calabi–Yau space.

We can justify this approach by using the formula (3.3). First, the induced geometry on the brane is, as already stated, the conformally transformed metric $g_{\mu\nu}^{(b)}$ given in (4.3). Thus, the stress–energy tensor $T_{\mu\nu}^{(b)}$ in the brane frame $g_{\mu\nu}^{(b)}$ and the stress–energy tensor $T_{\mu\nu}^{(4)}$ in the frame $g_{\mu\nu}^{(4)}$ are related to each other as follows:

$$T_{\mu\nu}^{(b)} = h(\tau_0) \cdot T_{\mu\nu}^{(4)}, \quad (4.5)$$

where $\tau_0$ is again the value of the radial coordinate $\tau$ at the position of the brane and it has been assumed that the value of $h(\tau)$ at the position of our four-dimensional world is normalized to 1. Now, the formula (3.3) applied to this set-up says that we can use the effective Einstein equation of the form

$$G_{\mu\nu}^{(b)} = 8\pi G_N^{(b)} T_{\mu\nu}^{(b)}, \quad (4.6)$$

where $G_{\mu\nu}^{(b)}$ is the Einstein tensor of the brane metric $g_{\mu\nu}^{(b)}$ and Newton’s constant $G_N^{(b)}$ on the brane is given by

$$\frac{1}{8\pi G_N^{(b)}} = M_{10}^8 \int d^6 \psi \, h^{3/2}(\tau) \sqrt{\gamma} \cdot \left(\frac{h(\tau)}{h(\tau_0)}\right)^{-1/2}. \quad (4.7)$$

By the assumption that the function $h(\tau)$ is of order unity in the bulk of extra dimensions, this is reduced to

$$\frac{1}{8\pi G_N^{(b)}} \simeq h^{1/2}(\tau_0) M_{10}^8 V_6 \simeq \frac{h^{1/2}(\tau_0)}{8\pi G_N^{(4)}}. \quad (4.8)$$
where $V_6$ is again the volume of the Calabi–Yau space. This result implies that the effective Einstein equation on the brane (4.6) is equivalent to
\[ G^{(4)}_{\mu\nu} = 8\pi G_N T^{(4)}_{\mu\nu}, \]
where we have used the conformal transformation
\[ G^{(b)}_{\mu\nu} = h^{1/2}(\tau_0)G^{(4)}_{\mu\nu} \]
between Einstein tensors $G^{(b)}_{\mu\nu}$ and $G^{(4)}_{\mu\nu}$ of $g^{(b)}_{\mu\nu}$ and $g^{(4)}_{\mu\nu}$, respectively. Therefore, we have justified the approach adopted in the literature by using the formula (3.3). The four-dimensional fields resulting from the brane action do indeed couple to the four-dimensional metric $g^{(4)}_{\mu\nu}$ with Newton’s constant $G_N$ given by the Kaluza–Klein-like formula (4.4).

**Acknowledgments**

We would like to thank Andrei Frolov, Yoshiaki Himemoto, Lev Kofman, Kazuya Koyama, Hideaki Kudoh, Tetsuya Shiromizu, Jiro Soda, Keitaro Takahashi and Takahiro Tanaka for useful discussions and/or comments. We are grateful to Katsuhiko Sato for continuing encouragement. YS and HY are supported by JSPS.

**Appendix**

A1. The $\alpha \to 1$ limit

In the limit $\alpha \equiv r_-/r_+ \to 1$, the coordinate distance between two branes vanishes and, thus, the bulk geometry appears to collapse. However, as we shall see below, the proper distance between $r = r_-$ and $r_+$ does not vanish and the geometry of extra dimensions remains regular.

When $\alpha$ is sufficiently close to 1, the function $f(r)$ between $r_\pm$ is approximated by
\[ f(r) \simeq a^2 - b^2(r - r_0)^2, \]
where $r_0$, $a$ and $b$ are positive constants and the $\alpha \to 1$ limit corresponds to sending $a \to +0$ with $b$ finite. The bulk coordinate $r$ is restricted to the interval $-a/b \leq r - r_0 \leq a/b$. With the new coordinate system $(\theta, \varphi)$ defined by
\[ \theta = \cos^{-1}\left[ \frac{b}{a}(r - r_0) \right], \quad \varphi = ab\phi, \]
the metric of the extra dimension becomes that of a round sphere with radius $1/b$:
\[ \frac{dr^2}{f(r)} + f(r) d\varphi^2 \simeq \frac{1}{b^2}(d\theta^2 + \sin^2 \theta d\varphi^2). \]

Evidently, the coordinate $\theta$ runs over the full interval $[0, \pi]$. On the other hand, the period of the coordinate $\varphi$ appears to collapse since the coefficient $ab$ in the definition (A2) of $\varphi$ vanishes in the $\alpha \to 1$ limit. Actually, this is not the case. The ‘surface gravity’ $\kappa_\pm$ defined in (2.13) is $\kappa_+ \simeq \kappa_- \simeq ab$ and the period of the old coordinate is, thus, $\Delta \phi \simeq (2\pi - \delta_\pm)/(ab)$. Therefore, the new coordinate $\varphi$ has the period
\[ \Delta \varphi = ab\Delta \phi \simeq 2\pi - \delta_+ \simeq 2\pi - \delta_- , \]
which is indeed finite. Thus, the geometry of extra dimensions is nothing but a round sphere with a deficit angle $\delta_+ \simeq \delta_-$, i.e. the football-shaped extra dimensions considered in [45]–[47]. In those papers a $Z_2$ symmetry is assumed so that tensions of two 3-branes are identical. With the $Z_2$ symmetry, $\eta_\pm$ is always 1 but $\Phi_\pm = \Phi/(2\pi - \delta_\pm)$ changes when the brane tension changes. Accordingly, as can be seen from figures 1 and 3, the Hubble expansion rate on the brane changes when the brane tension changes.

The warp factor $r$ becomes constant in the $\alpha \to 1$ limit and, thus, the six-dimensional solution in the $\alpha \to 1$ limit corresponds to an unwarped flux compactification of the type considered by Arkani-Hamed, Dimopoulos and Dvali [44].

### A.2. The $h \to 0$ limit

The limit $h \to 0$ appears to be singular, but it is not. Indeed, with the coordinate change

$$r \to \frac{\bar{r}}{h}, \quad \phi \to h\bar{\phi}, \quad ds^2_6 \to h^2 ds^2_4, \quad \mu \to \frac{\bar{\mu}}{h\bar{r}}, \quad b \to \frac{\bar{b}}{h^4}, \quad (A.5)$$

the $h \to 0$ limit of the bulk solution becomes a locally regular form:

$$ds^2_6 = \bar{r}^2 ds^2_4 + \frac{d\bar{r}^2}{f(\bar{r})} + \bar{f}(\bar{r}) d\bar{\phi}^2, \quad (A.6)$$

where

$$f(\bar{r}) = \frac{\Lambda_6}{10 \bar{r}^2} - \frac{\bar{\mu}}{\bar{r}^3} - \frac{\bar{b}^2}{12 \bar{r}^6}, \quad (A.7)$$

and $ds^2_4$ is the metric of the four-dimensional Minkowski spacetime. This configuration of course satisfies the six-dimensional Einstein equation and is related by a double Wick rotation to a topological black hole with the horizon topology $R^4$. The new coordinate $\bar{\phi}$ is identified as

$$\bar{k}_\pm \bar{\phi} \sim \bar{k}_\pm \bar{\phi} + (2\pi - \delta_\pm), \quad (A.8)$$

where

$$\bar{k}_\pm \equiv \mp \frac{1}{2} \bar{f}'(\bar{r}_\pm). \quad (A.9)$$

This means that the period of $\bar{\phi}$ is finite if the brane tensions are finite. Therefore, the geometry remains regular in the $h \to 0$ limit.

### A.3. Higher order correction

It is straightforward to extend the analysis in section 3.4 to higher order in the expansion w.r.t. $h^2$ and to interpret the result as higher order corrections to the effective Friedmann equation. In this appendix we just summarize the result up to $O(h^4)$.

The result up to $O(h^4)$ is summarized as

$$\left[1 - \frac{h^2}{h^2_{\pm}} + O(h^4)\right] h^2_{\pm} = g^{(0)}_{\pm} \cdot (\eta^{(0)}_{\pm} - \eta_{\pm}), \quad (A.10)$$
Warped flux compactification and brane gravity

where

\[ h^2_{\pm} = \frac{B^{(0)}_{\pm}}{\beta^2 A^{(0)}_{\pm}} \alpha^{(0)}_{\pm}. \]  

(A.11)

This is equivalent to

\[ \left[ 1 - \frac{H^2_{\pm}}{H^2_{\ast\pm}} + O((H^2_{\pm}/\Lambda_6)^2) \right] H^2_{\pm} = \frac{8\pi G N_{\pm}}{3} (\sigma_{\pm} - \sigma^{(0)}_{\pm}), \]  

(A.12)

where \( H^2_{\ast\pm} = h^2_{\ast\pm}/L^2 \) and includes a higher order correction to the effective Friedmann equation suppressed by \( H_{\ast\pm} \).

This means that higher order corrections can be ignored when the Hubble expansion rate \( H_{\pm} \) on a brane is sufficiently lower than the bulk curvature scale \( L^{-1} \).

References

[1] Kachru S, Kallosh R, Linde A and Trivedi S P, 2003 Phys. Rev. D 68 046005 [SPIRES] [hep-th/0301240]

[2] Escoda C, Gomez-Reino M and Quevedo F, 2003 J. High Energy Phys. JHEP11(2003)065 [SPIRES] [hep-th/0307160]

[3] Burgess C P, Kallosh R and Quevedo F, 2003 J. High Energy Phys. JHEP10(2003)056 [SPIRES] [hep-th/0309187]

[4] Silverstein E, (A)dS backgrounds from asymmetric orientifolds, 2001 Preprint hep-th/0106209

Maloney A, Silverstein E and Strominger A, de Sitter space in noncritical string theory, 2002 Preprint hep-th/0205316

[5] Becker M, Curio G and Krause A, 2004 Nucl. Phys. B 693 223 [SPIRES] [hep-th/0403027]

[6] Townsend P K and Wohlfarth M N R, 2003 Phys. Rev. Lett. 91 061302 [SPIRES] [hep-th/0303097]

[7] Ohta N, 2003 Phys. Rev. Lett. 91 061303 [SPIRES] [hep-th/0303238]

Ohta N, 2003 Prog. Theor. Phys. 110 269 [SPIRES] [hep-th/0304172]

Ohta N, 2005 Int. J. Mod. Phys. A 20 1 [SPIRES] [hep-th/0411230]

[8] Roy S, Accelerating cosmologies from M/String theory compactifications, 2003 Phys. Lett. B 567 322 [SPIRES] [hep-th/0304084]

[9] Neupane I P and Wiltshire D L, Accelerating cosmologies from compactification with a twist, 2005 Preprint hep-th/0502003

Neupane I P and Wiltshire D L, Cosmic acceleration from M-theory on twisted spaces, 2005 Preprint hep-th/0504135

[10] Becker K, Becker M and Krause A, 2005 Nucl. Phys. B 715 349 [SPIRES] [hep-th/0501130]

[11] Klebanov I R and Strassler M J, 2000 J. High Energy Phys. JHEP08(2000)052 [SPIRES] [hep-th/0007191]

[12] Giddings S B, Kachru S and Polchinski J, 2002 Phys. Rev. D 66 106006 [SPIRES] [hep-th/0105097]

[13] Witten E, 1996 Nucl. Phys. B 474 343 [SPIRES] [hep-th/9604030]

[14] Kachru S, Kallosh R, Linde A, Maldacena J, McAllister L and Trivedi S P, 2003 J. Cosmol. Astropart. Phys. JCAP10(2003)013 [SPIRES] [hep-th/0308055]

[15] Blanco-Pillado J J, Burgess C P, Cline J M, Escoda C, Gomez-Reino M, Kallosh R, Linde A and Quevedo F, 2004 J. High Energy Phys. JHEP11(2004)063 [SPIRES] [hep-th/0404230]

[16] Buchel A and Roiban R, 2004 Phys. Lett. B 590 284 [SPIRES] [hep-th/0311154]

Berg M and Haack M, 2005 Phys. Rev. D 71 026005 [SPIRES] [hep-th/0404877]

Burgess C P, Cline J M, Stoica H and Quevedo F, 2004 J. High Energy Phys. JHEP09(2004)033 [SPIRES] [hep-th/0403119]

[17] DeWolfe O, Kachru S and Verlinde H, 2004 J. High Energy Phys. JHEP05(2004)017 [SPIRES] [hep-th/0403123]

Iizuka N and Trivedi S P, 2004 Phys. Rev. D 70 043519 [SPIRES] [hep-th/0403203]

Buchel A and Ghodsi A, 2004 Phys. Rev. D 70 126008 [SPIRES] [hep-th/0404151]

Firouzjahi H and Tye S H, 2005 J. Cosmol. Astropart. Phys. JCAP03(2005)009 [SPIRES] [hep-th/0501099]
Hsu J P, Kallosh R and Prokushkin S, 2003 J. Cosmol. Astropart. Phys. JCAP12(2003)009 [SPIRES]
[hep-th/0310177]
Firouzjahi H and Tye S H, 2004 Phys. Lett. B 584 147 [SPIRES] [hep-th/0312020]
Hsu J P and Kallosh R, 2004 J. High Energy Phys. JHEP04(2004)042 [SPIRES] [hep-th/0402047]
Koyama F, Tachikawa Y and Watari T, 2004 Phys. Rev. D 69 106001 [SPIRES] [hep-th/0311191]
Koyama F, Tachikawa Y and Watari T, 2004 Phys. Rev. D 70 129907 [SPIRES] (erratum)
McAllister L, An inflation mass problem in string inflation from threshold corrections to volume
stabilization, 2005 Preprint hep-th/0502001
Dasgupta K, Hsu J P, Kallosh R, Linde A and Zagermann M, 2004 J. High Energy Phys.
JHEP08(2004)030 [SPIRES] [hep-th/0405247]
Chen P, Dasgupta K, Narayan K, Shimakova M and Zagermann M, Brane inflation, solitons and
cosmological solutions: I, 2005 Preprint hep-th/0501185
Piao Y-S, D/antiD dark energy in string compactification, 2005 Preprint gr-qc/0502006
Cremades D, Quevedo F and Sinha A, Warped tachyonic inflation in type IIB flux compactifications
and the open-string completeness conjecture, 2005 Preprint hep-th/0505252
[17] Polchinski J, Introduction to cosmic F- and D-strings, 2004 Preprint hep-th/0412244 and references therein
[18] Mukohyama S, D-brane as dark matter in warped string compactification, 2005 Preprint hep-th/0505042
[19] Dvali G R and Tye S H, 1999 Phys. Lett. B 450 72 [SPIRES] [hep-ph/9812483]
[20] Quevedo F, 2002 Class. Quantum Grav. 19 5721 [SPIRES] [hep-th/0210292] and references therein
[21] Randall L and Sundrum R, 1999 Phys. Rev. Lett. 83 4690 [SPIRES] [hep-th/9906064]
[22] Garriga J and Tanaka T, 2000 Phys. Rev. Lett. 84 2778 [SPIRES] [hep-th/9911055]
[23] Sasaki M, Shiiromizu T and Maeda K, 2000 Phys. Rev. D 62 024008 [SPIRES] [hep-th/9912233]
[24] Giddings S B, Katz E and Randall L, 2000 J. High Energy Phys. JHEP03(2000)023 [SPIRES]
[hep-th/0002091]
[25] Karch A and Randall L, 2001 J. High Energy Phys. JHEP05(2001)008 [SPIRES] [hep-th/0111156]
[26] Shiiromizu T, Maeda K and Sasaki M, 2000 Phys. Rev. D 62 024012 [SPIRES] [gr-qc/9910076]
[27] Csaki C, Graesser M, Kolda C F and Terning J, 1999 Phys. Lett. B 463 34 [SPIRES] [hep-ph/9906513]
[28] Cline J M, Grojean C and Servant G, 1999 Phys. Rev. Lett. 83 4245 [SPIRES] [hep-ph/9906523]
[29] Flanagan E E, Tye S H and Wasserman I, 2000 Phys. Rev. D 62 044039 [SPIRES] [hep-th/9910498]
[30] Binétruy P, Deffayet C, Ellwanger U and Langlois D, 2000 Phys. Lett. B 477 285 [SPIRES]
[hep-th/9910219]
[31] Mukohyama S, 2000 Phys. Lett. B 473 241 [SPIRES] [hep-th/9911165]
[32] Kraus P, 1999 J. High Energy Phys. JHEP12(1999)011 [SPIRES] [hep-th/9910149]
[33] Ida D, 2000 J. High Energy Phys. JHEP09(2000)014 [SPIRES] [gr-qc/9912020]
[34] Mukohyama S, Shiiromizu T and Maeda K, 2000 Phys. Rev. D 62 024028 [SPIRES] [hep-th/9912287]
Mukohyama S, Shiiromizu T and Maeda K, 2001 Phys. Rev. D 63 029901 [SPIRES] (erratum)
[35] Randall L and Sundrum R, 1999 Phys. Rev. Lett. 83 3370 [SPIRES] [hep-ph/9905221]
[36] Goldberger W D and Wise M B, 1999 Phys. Rev. Lett. 83 4922 [SPIRES] [hep-th/9907065]
[37] Tanaka T and Montes X, 2000 Nucl. Phys. B 582 259 [SPIRES] [hep-th/0001092]
[38] Csaki C, Graesser M L and Kribs G D, 2001 Phys. Rev. D 63 065002 [SPIRES] [hep-th/0008151]
[39] Mukohyama S and Kofman L, 2002 Phys. Rev. D 65 124025 [SPIRES] [hep-th/0112115]
[40] Mukohyama S, 2002 Phys. Rev. D 65 084036 [SPIRES] [hep-th/0112205]
[41] Kudoh H and Tanaka T, 2002 Phys. Rev. D 65 104034 [SPIRES] [hep-th/0112013]
Kudoh H and Tanaka T, 2003 Phys. Rev. D 67 044011 [SPIRES] [hep-th/0205041]
[42] Csaki C, Graesser M and Randall L, 2000 Phys. Rev. D 62 045015 [SPIRES] [hep-th/9911406]
[43] Kanti P, Kogan I I, Olive K A and Pospelov M, 1999 Phys. Lett. B 468 31 [SPIRES] [hep-ph/9909481]
Kanti P, Olive K A and Pospelov M, 2000 Phys. Rev. D 62 126004 [SPIRES] [hep-ph/0005146]
[44] Arkani-Hamed N, Dimopoulos S and Dvali G R, 1998 Phys. Lett. B 429 263 [SPIRES] [hep-ph/9803315]
Antoniadis I, Arkani-Hamed N, Dimopoulos S and Dvali G R, 1998 Phys. Lett. B 436 257 [SPIRES]
[hep-ph/9804398]
Arkani-Hamed N, Dimopoulos S and Dvali G R, 1999 Phys. Rev. D 59 086004 [SPIRES] [hep-ph/9807344]
[45] Carroll S M and Guica M M, Sidestepping the cosmological constant with football shaped extra dimensions,
2003 Preprint hep-th/0302067
[46] Garriga J and Porrati M, 2004 J. High Energy Phys. JHEP08(2004)028 [SPIRES] [hep-th/0406158]
[47] Navarro I, 2003 Class. Quantum Grav. 20 3603 [SPIRES] [hep-th/0305014]
Navarro I, 2003 J. Cosmol. Astropart. Phys. JCAP09(2003)004 [SPIRES] [hep-th/0302129]
[48] Vinet J and Cline J M, 2005 Phys. Rev. D 71 064011 [SPIRES] [hep-th/0501098]
Vinet J and Cline J M, 2004 Phys. Rev. D 70 083514 [SPIRES] [hep-th/0406141]
[49] Cline J M, Descheneau J, Giovannini M and Vinet J, 2003 J. High Energy Phys. JHEP06(2003)048
[SPIRES] [hep-th/0304147]
[50] Israel W, 1966 Nuovo Cim. B 44 1
Israel W, 1967 Nuovo Cim. B 48 463 (erratum)
[51] Israel W, 1975 Bull. Am. Phys. Soc. 20 98
Israel W, 1977 Phys. Rev. D 15 935 [SPIRES]
[52] Geroch R and Traschen J, 1987 Phys. Rev. D 36 1017 [SPIRES]
[53] Frolov V P, Israel W and Unruh W G, 1989 Phys. Rev. D 39 1084 [SPIRES]
[54] Lanczos C, 1922 Phys. Zeits 23 539
Lanczos C, 1924 Ann. Phys., Lpz. 74 518
[55] Shiromizu T, Himemoto Y and Takahashi K, 2004 Phys. Rev. D 70 107303 [SPIRES] [hep-th/0405071]
Shiromizu T, Takahashi K, Himemoto Y and Yamamoto S, 2004 Phys. Rev. D 70 123524 [SPIRES]
[hep-th/0407268]
[56] Koyama K and Koyama K, Gravitational backreaction of anti-D branes in the warped compactification, 2005 Preprint hep-th/0505256