CONSTRANTS ON THE SOURCE OF ULTRA-HIGH-ENERGY COSMIC RAYS USING ANISOTROPY VERSUS CHEMICAL COMPOSITION

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ABSTRACT

The joint analysis of anisotropy signals and chemical composition of ultra-high-energy cosmic rays offers strong potential for shedding light on the sources of these particles. Following up on an earlier idea, this paper studies the anisotropies produced by protons of energy > E/Z, assuming that anisotropies at energy > E have been produced by nuclei of charge Z, which share the same magnetic rigidity. We calculate the number of secondary protons produced through photodisintegration of the primary heavy nuclei. Making the extreme assumption that the source does not inject any proton, we find that the source(s) responsible for anisotropies such as reported by the Pierre Auger Observatory should lie closer than ~20–30, 80–100, and 180–200 Mpc if the anisotropy signal is mainly composed of oxygen, silicon, and iron nuclei, respectively. A violation of this constraint would otherwise result in the secondary protons forming a more significant anisotropy signal at lower energies. Even if the source were located closer than this distance, it would require an extraordinary metallicity >120, 1600, and 1100 times solar metallicity in the acceleration zone of the source, for oxygen, silicon, and iron, respectively, to ensure that the concomitantly injected protons do not produce a more significant low-energy anisotropy. This offers interesting prospects for constraining the nature and the source of ultra-high-energy cosmic rays with the increase in statistics expected from next-generation detectors.

Key word: cosmic rays

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1. INTRODUCTION

The origin of ultra-high-energy cosmic rays (UHECRs) is a long-standing puzzle of high-energy astrophysics and astroparticle physics. It is commonly believed that the source of these particles with energy > 10^19 eV are powerful extragalactic astrophysical objects. The candidates include active galactic nuclei (AGNs; e.g., Biermann & Strittmatter 1987; Takahara 1990; Rachen & Biermann 1993; Berezinsky et al. 2006; Dermer et al. 2009), gamma-ray bursts (e.g., Waxman 1995; Vietri 1995; Dermer & Atoyan 2006; Murase & Nagataki 2006), semi-relativistic hypernovae (e.g., Wang et al. 2007; Budnik et al. 2008; Chakraborti et al. 2011; Liu & Wang 2012), and extragalactic rotation-powered young pulsars (Arons 2003; Fang et al. 2012).

On their way to the detector, UHECRs suffer inevitable interactions with the cosmic microwave background (CMB) and the extragalactic background light (EBL) that permeate extragalactic space, in particular Bethe–Heitler pair production, photo-pion production, and photodisintegration. Photo-hadronic interactions inevitably introduce a high-energy cutoff in the UHECR spectrum beyond ~5 × 10^19 eV (Greisen 1966; Zatsepin & Kuz’m 1966; Puget et al. 1976; GZK), due to the rapid decrease of the attenuation length of UHECRs with increasing energy. Above these GZK energies, the typical horizon to which sources can be detected shrinks to values of the order of ~100–200 Mpc. This cutoff/suppression feature has now been observed by different experiments at a statistically significant level (Abbasi et al. 2008; Pierre Auger Collaboration et al. 2008; Abraham et al. 2010a), implying notably that the sources of the highest energy cosmic rays must be nearby luminous objects (e.g., Waxman 1995, 2005; Farrar & Gruzinov 2009; Piran 2010; Taylor et al. 2011).

The Pierre Auger Observatory (PAO), currently the largest UHECR observatory, has reported the detection of 69 events within the energy range 55–142 EeV between 2004 January and 2009 December (Pierre Auger Collaboration et al. 2010). A detailed analysis has shown that the fraction of these events correlating with nearby AGNs (~75 Mpc) in the Véron-Cetty and Véron catalog is (38 ± 7)%, above the isotropic expectation of 21%. Most of this excess is found around the direction of Centaurus A (Cen A) within a surrounding 18° window, in which 13 events in the energy range 55–84 EeV are observed while only 3.2 are expected (Pierre Auger Collaboration et al. 2010). However, both the intergalactic and the Galactic magnetic fields deflect the trajectories of cosmic rays, resulting in apparent correlations with objects which are not necessarily their true birthplaces. Furthermore, measurements on the maximum air shower elongations (X_{max}) and their rms (σ_{X_{max}}) by PAO suggest that the chemical composition of UHECRs is progressively dominated by heavier nuclei at energies above 4 EeV (Abraham et al. 2010b). If the cosmic rays are indeed intermediate-mass or heavy nuclei, the deviation of their arrival directions due to propagation in the intervening magnetic fields must be significant; hence, the observation of anisotropies appears slightly surprising. From a theoretical point of view, it may appear more favorable to accelerate heavy nuclei, as their
higher charge, comparatively to protons, reduces the energetic constraints placed on the source candidates (e.g., Lemoine & Waxman 2009). However, it also requires the acceleration site to be abundant in intermediate-mass or heavy elements, well beyond a typical galactic composition (Pierre Auger Collaboration et al. 2011; Liu et al. 2012). Finally, other experiments, such as High Resolution Fly’s Eye Experiment (HiRes) or the Telescope Array (TA), find that the composition at $\gtrsim 10^{19}$ eV remains dominated by light nuclei (Abbasi et al. 2010; Tsunesada et al. 2011).

One way to make progress is to use the pattern of anisotropies as a function of energy. This idea was first proposed in Lemoine & Waxman (2009): if a source produces an anisotropy signal at energy $E$ with cosmic-ray nuclei of charge $Z$, it should also produce a similar anisotropy pattern at energies $E/Z$ via the proton component that is emitted along with the nuclei, given that the trajectory of cosmic rays within a magnetic field is only rigidity-dependent. It is easy to show that the low-energy anisotropy should appear stronger, possibly much stronger than the high-energy anisotropy (assuming a chemical composition similar to that inferred at the source of Galactic cosmic rays), offering means to constrain the chemical composition of the source. This test has been applied on the PAO data set, and no significant anisotropy has been found at energies $E/Z$ with $E = 55$ eV and $Z = 6, 14,$ and $26$ (Pierre Auger Collaboration et al. 2011).

In the present work, we push further and generalize this idea by considering the amount of secondary protons produced through photodisintegration interactions of the primary nuclei. We provide detailed analytical and numerical estimates of the ratio of significance of the anisotropy at $E/Z$ versus $E$ and derive the maximal distance to the source $D_{\text{max}}$ in order to avoid the formation of a stronger anisotropy pattern produced by the secondary protons at energy $E/Z$. This bound does not depend on the amount of protons produced by the source. We also discuss how the comparison of the anisotropy ratio constrains the metal abundance in the source, independently of the injection spectral index, and emphasize how large this metal abundance must be, if the anisotropies persist at high energies, but not at low energies. Finally, we briefly discuss the prospects for the detection of anisotropies at higher energies than $E$, with next-generation experiments, based on current reports of anisotropies at $E$.

The layout of this paper is arranged as follows. In Section 2, we discuss how the absence of a low-energy anisotropy signal could constrain the source distance and its metal abundance. In Section 3, we discuss the low-energy proton fraction and the possible anisotropy signal at higher energies, as well as their implication for the source. We draw some conclusions in Section 4.

2. ANISOTROPIES AT CONSTANT RIGIDITY

2.1. Low-energy Anisotropy Signal

We assume that some anisotropy is detected within a solid angle $\Delta \Omega$ in the energy range from $E_1$ to $E_2$. Following up on Lemoine & Waxman (2009), we quantify the significance of the anisotropy through its signal-to-noise ratio. Before doing so, we define the injected spectrum of an element with nuclear charge number $Z$ as

$$q_{Z, \text{inj}} = k_Z E^{-\gamma},$$

with $\gamma$ the power-law index and $k_Z$ the relative abundance of this element at a given energy. Provided that the maximum energy $E_{Z,\text{max}}$ and the minimum energy $E_{Z,\text{min}}$ of the accelerated spectrum are proportional to $Z$, i.e., scale with rigidity, the total mass of the element of charge $Z$ and mass $A_Z$ scales as

$$M_Z \propto A_Z \int q_{Z, \text{inj}} dE \propto k_Z A_Z Z^{1-\gamma}.$$  

Note that the above result does not depend on the magnitude of $s$, and the missing prefactor does not depend on $Z$. This implies in particular that the ratio of the relative abundance of a species at a given energy to that of hydrogen takes the form

$$\frac{k_Z}{k_H} = Z^{s-1} A_Z^{-1} M_Z M_H.$$  

We then denote, respectively, the number of injected and propagated primary cosmic rays with nuclear charge $Z$ in the energy range $[E_1, E_2]$ by $N_{Z, \text{inj}}(E_1; E_2)$ and $N_{Z, \text{prop}}(E_1; E_2)$. These two quantities are related through

$$N_{Z, \text{prop}}(E_1; E_2) = f_{Z, \text{surv}}(E_1; E_2) N_{Z, \text{inj}}(E_1; E_2),$$

where

$$f_{Z, \text{surv}}(E_1; E_2) \equiv \frac{\int_{E_1}^{E_2} q_{Z, \text{prop}}(E) dE}{\int_{E_1}^{E_2} q_{Z, \text{inj}}(E) dE}$$

is the surviving fraction of primaries after propagation.

As we do not know the precise composition of cosmic-ray events constituting the anisotropy signal, in a first scenario (A) we regard the fragments with less than $Z/4$ lost nucleons as primaries, i.e., $q_{Z, \text{prop}} = \sum_{i=3/4 Z}^{Z} q_{i, \text{prop}}$. This ad hoc choice guarantees that all arriving nuclei in the energy range $[E_1, E_2]$ which have suffered at most $Z/4$ photodisintegration interactions retain a similar rigidity, and thus follow a similar path in the intervening magnetic fields. Lighter nuclei, i.e., those that have suffered more than $Z/4$ interactions and arrive in $[E_1, E_2]$, carry higher rigidity. Depending on the intervening magnetic fields, such cosmic rays may or may not contribute to the anisotropies, since the magnetic fields may form a blurred image centered on the source (with higher rigidity cosmic rays clustering closer to the source direction), or impart a systematic shift in the arrival directions, in which case the higher rigidity particles might lie outside $\Delta \Omega$ (e.g., Waxman & Miralda-Escudé 1996; Kotera & Lemoine 2008). To account for this uncertainty, we will consider in the following an alternative scenario (B), in which $E_2 \rightarrow +\infty$ and as many photodisintegration interactions are allowed (i.e., $q_{Z, \text{prop}} = \sum_{i=0}^{Z} q_{i, \text{prop}}$), provided that the nucleus arrives with energy $> E_1$. In this scenario (B), we thus sum up over all rigidities in excess of $E_1/Z$.

We adopt the premise that anisotropy in the arrival distribution of UHECR nuclei has been detected at high energies between $E_1$ and $E_2$, i.e., at energies of the order of the GZK energy. Since protons with the same rigidity have energies between $E_1/Z$ and $E_2/Z$, one may safely neglect their subsequent energy losses given that their loss lengths at these energies are of the order of $\sim 1$ Gpc, considerably larger than the source distance considered here ($\lesssim 100$ Mpc). Photodisintegration interactions of nuclei with energy in the range $(A_Z/Z)[E_1, E_2] \approx [2E_1, 2E_2]$ produce secondary protons with energy in the range $[E_1/Z, E_2/Z]$. 

\[ \text{The Astrophysical Journal, 776:88 (10pp), 2013 October 20} \]

\[ \text{Liu et al.} \]
with number
\[ N_{p,\text{dis}}(E_1/Z; E_2/Z) = A_Z f_{Z,\text{loss}}(2E_1; 2E_2) N_{Z,\text{inj}}(2E_1; 2E_2) \]
\[ = 2^{1-s} A_Z f_{Z,\text{loss}}(2E_1; 2E_2) N_{Z,\text{inj}}(E_1; E_2), \]
where
\[ f_{Z,\text{loss}}(2E_1; 2E_2) \equiv \frac{\int_{E_1/Z}^{E_2/Z} q_{p,\text{dis}} dE}{A_Z \int_{E_1/Z}^{E_2/Z} q_{Z,\text{inj}} dE}. \]

In this expression, \( q_{p,\text{dis}} \) represents the spectrum of secondary protons produced during propagation.

At low energies, the primary protons also contribute to the anisotropy, with
\[ N_{p,\text{prop}}(E_1/Z; E_2/Z) \simeq N_{p,\text{inj}}(E_1/Z; E_2/Z) \]
\[ = \frac{M_H}{M_Z} A_Z N_{Z,\text{inj}}(E_1; E_2). \]

Note that the last equality is of particular interest. It shows that \( N_{p,\text{prop}}(E_1/Z; E_2/Z)/N_{Z,\text{inj}}(E_1; E_2) = (M_H/M_Z) A_Z \) controls the scaling of the signal-to-noise ratio (S/N) of the low-energy to high-energy anisotropy signals. This scaling factor does not depend on the injection spectrum index, but does depend on the metal abundance at the source. It remains valid for general injection spectra, provided that this spectrum is shaped by rigidity, i.e., \( q_{Z,\text{inj}}(E) \propto \phi(E/Z) \), with \( \phi \) an arbitrary function.

The noise is given by the square root of the number of events expected from the averaged all-sky spectrum of UHECRs in the same solid angle \( \Delta \Omega \). The observed spectrum of the isotropic background can be approximately described by a broken power law beyond \( \sim 10^{18} \) eV (Pierre Auger Collaboration et al. 2010), i.e.,
\[ \frac{dN_{\text{iso}}}{dE} \bigg|_{\text{iso}} = N_0 \times \begin{cases} (E/E_b)^{p_1} & E < E_b, \\ (E/E_b)^{p_2} & E > E_b, \end{cases} \]
where \( p_1 = 2.6, p_2 = 4.3 \), and \( E_b = 10^{19.46} \) eV. \( N_0 \) represents the overall amplitude, which cancels out in the following calculation. The noise counts in the energy range \( [E_1, E_2] \) then read
\[ N_{\text{iso}}(E_1; E_2) = \Delta \Omega \int_{E_1}^{E_2} \frac{dN_{\text{iso}}}{dE} dE, \]
and for the low-energy noise we have
\[ N_{\text{iso}}(E_1/Z; E_2/Z) = \Delta \Omega \int_{E_1/Z}^{E_2/Z} \frac{dN_{\text{iso}}}{dE} dE \]
\[ = \eta Z^{p_1-1} N_{\text{iso}}(E_1; E_2), \]
with \( \eta \equiv (1-p_2)(1-p_1)^{-1}(E_1/E_b)^{p_2} - (E_1/E_b)^{p_1} - (E_2/E_b)^{p_2} - (E_2/E_b)^{p_1}. \)

The above equation is valid for \( E_1 > E_2 \) and \( E_2/Z < E_b \), which is the case of the “Cen A excess.” If \( E_2 < E_b \) or \( E_1/Z > E_b \), Equation (11) will read \( Z^{p_1-1} N_{\text{iso}}(E_1; E_2) \) or \( Z^{p_1-1} N_{\text{iso}}(E_1; E_2) \), respectively.

Assuming that the anisotropy is mainly caused by cosmic-ray nuclei with charge \( Z \), the S/N in the energy range \( [E_1, E_2] \) can then be expressed as
\[ \Sigma_Z(E_1; E_2) = \frac{N_{Z,\text{prop}}(E_1; E_2)}{\sqrt{N_{\text{iso}}(E_1; E_2)}} \]
\[ = \frac{f_{Z,\text{surv}}(E_1; E_2) N_{Z,\text{inj}}(E_1; E_2)}{\sqrt{N_{\text{iso}}(E_1; E_2)}}, \]

while the S/N of the low-energy anisotropy produced by protons with the same rigidity is
\[ \Sigma_p(E_1/Z; E_2/Z) = \frac{N_{p,\text{prop}}(E_1/Z; E_2/Z)}{\sqrt{N_{\text{iso}}(E_1/Z; E_2/Z)}} \]
\[ = \frac{A_Z [M_H/M_Z + 2^{1-s} f_{Z,\text{loss}}(2E_1; 2E_2)] N_{Z,\text{inj}}(E_1; E_2)}{\sqrt{\eta Z^{p_1-1} N_{\text{iso}}(E_1; E_2)}}. \]

Consequently, the ratio of the S/Ns at low to high energy reads
\[ \frac{\Sigma_p}{\Sigma_Z} \]
\[ = \frac{2 M_H/M_Z + 2^{2-s} f_{Z,\text{loss}}(2E_1; 2E_2)}{f_{Z,\text{surv}}(E_1; E_2) \sqrt{\eta Z^{p_1-3}}}, \]

and if no anisotropy is recorded at low energies, one requires \( \Sigma_p/\Sigma_Z < 1 \). For reference, the PAO data indicate that, for the Cen A excess, \( \Sigma_p/\Sigma_Z \lesssim (2, 1.8, 0.8) \) at the 95% c.l., for \( Z = 6, 14, 26, \) corresponding to carbon, silicon, and iron, respectively (Pierre Auger Collaboration et al. 2011). The exact number depends on the statistics (which of course have increased since this analysis was carried out) and on the elements adopted in the analysis. In the following, we use the constraint \( \Sigma_p/\Sigma_Z < 1 \) to impose a limit on the maximum source distance. In terms of the (inverse) metal abundance, this constraint can be rewritten
\[ \frac{M_H}{M_Z} < \frac{1}{2} \left( \frac{\sqrt{\eta Z^{p_1-3}} f_{Z,\text{surv}}(E_1; E_2)}{2 - f_{Z,\text{loss}} (2E_1; 2E_2)} \right). \]

Again, if secondary protons are ignored, meaning \( f_{Z,\text{loss}} (2E_1; 2E_2) \rightarrow 0 \) in the above, then the non-detection of anisotropy at low energies imposes a lower limit of \( M_Z/M_H \) which does not depend on the spectral index. Note that this statement is not in contradiction with the statement in the Pierre Auger Collaboration et al. (2011) paper, that the limit on the quantity \( f_p/f_z \) used in that paper depends on the spectral index. This is due to the fact that \( f_p/f_z \), which is defined at a given energy, is not the “proton to heavy fraction in the source” or the “relative proton abundance”, as it is misleadingly referred to in the Auger paper (in the notation of that paper, the relative proton abundance is \( Z^{p_1-2} f_p/f_z \), which is equivalent to our \( M_p/M_Z \)).

The minimum required metallicity of the element responsible for the observed anisotropy thus depends on the values of \( f_{Z,\text{surv}} \) and \( f_{Z,\text{loss}} \), which are directly determined by the source distance. A larger source distance will result in a smaller \( f_{Z,\text{surv}} \) and a larger \( f_{Z,\text{loss}} \) as more nuclei are photodisintegrated. There exist therefore a critical distance, beyond which the abundance of hydrogen in the source relative to metals becomes negative. This happens when \( f_{Z,\text{loss}} (2E_1; 2E_2) > f_{Z,\text{surv}}(E_1; E_2) \geq 2^{-2-s} \sqrt{\eta Z^{p_1-3}}, \) meaning that even if the source injects no primary protons, secondary protons produced during propagation cause a stronger anisotropy at low energies. Therefore, the critical distance, which we denote by \( D_{\text{max}} \), is the upper limit of the distance of the source responsible for the anisotropy signal in \( [E_1, E_2] \). The value of \( D_{\text{max}} \) is also related to the primary cosmic-ray species adopted and the injection spectrum used. Once these parameters are given, we can uniquely determine \( D_{\text{max}} \) by finding the distance for which \( \Sigma_p(E_1/Z; E_2/Z)/\Sigma_Z(E_1; E_2) = 1 \) using the method outlined above.
2.2. Photodisintegration of Nuclei

UHECRs interact with the CMB and EBL photons while they propagate through extragalactic space. For nuclei, energy losses due to the photodisintegration process and the Bethe–Heitler process (pair production) by CMB photons are comparable around 55 EeV. As energy increases, the photodisintegration process plays a more and more dominant role in the energy loss process. Photodisintegration does not change the Lorentz factor of the cosmic-ray nucleus, but does lead to the nucleus losing one or several nucleons as well as α particles through the giant dipole resonance (GDR) or quasi-deuteron (QD) process. These secondary nuclei can be further disintegrated to protons. On average, the mass number of a nucleus evolves as (Stecker 1969)

$$\frac{dA}{dx} = \frac{1}{2r_A^2} \sum_i \Delta A_i \int_{\epsilon_{th,i}}^{\infty} d\epsilon \sigma_{\text{dis},i}(\epsilon) \epsilon \int_{\epsilon/2\gamma_A}^{2\gamma_A} d\epsilon' \frac{n_{\gamma}(\epsilon', z)}{\epsilon'^2},$$

(16)

with $\sigma_{\text{dis},i}$ the cross-section for photodisintegration through the $i$th channel (e.g., single-nucleon emission, deuteron emission, α particle emission, and so on) and $\epsilon_{th,i}$ the threshold energy of the $i$th channel, which is $\sim 10$–20 MeV for all species of nuclei for the GDR process and $\sim 30$ MeV for the QD process. $\Delta A_i$ is the number of nuclei lost through the $i$th channel (e.g., $\Delta A = 1, 2, 4$ for single nucleon emission, deuteron and α particle emission, and so on). $\epsilon_{\gamma}$ and $n_{\gamma}(\epsilon_{\gamma}, z)$ are, respectively, the photon energy and the number density of the target photon field in the lab frame at redshift $z$, while $\epsilon$ is the photon energy in the rest frame of the nucleus. The physics of ultra-high-energy nuclei transport through the radiation backgrounds has been discussed by a number of authors, e.g., Puget et al. (1976), Bertone et al. (2002), Khan et al. (2005), Hooper et al. (2008), and Aloisio et al. (2012). In this work, we will adopt the tabulated cross-section data generated by the code TALYS and implement them into the Monte Carlo framework along with other energy loss processes, as described in Hooper et al. (2007), to obtain the propagated spectra.

As Monte Carlo simulations of nuclei propagation remain somewhat costly in computing, it is useful to have a simple analytical estimate of the photodisintegration process. Detailed treatments are discussed in Hooper et al. (2008) and Aloisio et al. (2013a, 2013b). Here, we adopt an even simpler approximation, which provides a sufficiently reliable approximation for the integrations that follow. In this analytical treatment, only the photodisintegration process is taken into account, with all other energy loss processes being neglected. A phenomenological fit of the nucleon loss rate $dA/dx$ for a nucleus with initial mass $A_0$ and Lorentz factor $\gamma$ is

$$\frac{dA}{dx} = c_1(\gamma_{\gamma_0})A^2 + c_2(\gamma_{\gamma_0})A \text{ Mpc}^{-1},$$

(17)

where $c_1(\gamma_{\gamma_0})$ and $c_2(\gamma_{\gamma_0})$ are functions of the Lorentz factor of cosmic-ray nuclei in units of $10^{10}$, which can be written in the form of $a_1(\gamma_{\gamma_0}) + a_2 \exp(-a_3/\gamma_{\gamma_0})$.

Table 1 presents the results from a Markov Chain Monte Carlo exploration of the parameter space, and Figure 1 shows
the phenomenologically fit and numerical nucleon loss rate for several cosmic-ray species. From Equation (17) we can derive the average mass number of a nucleus of initial mass number \( A_0 \) and Lorentz factor \( \gamma_{10} \) after propagation over a distance \( x \):

\[
A(x, \gamma_{10}) = \frac{A_0 c^2 e^{-c^2 x}}{A_0 c^2 (1 - e^{-c^2 x}) + c^2}.
\]

For Poisson statistics with mean rate \( dA/dx \), the probability that a nucleus undergoes at most \( N \) interactions reads

\[
P_N(A_x, x, E) = \frac{\Gamma[N + 1, x | dA/dx]}{\Gamma [N + 1]}.
\]

Strictly speaking, Equation (17) leads to modified Poisson statistics, because the rate \( dA/dx \) depends on \( A \), which evolves as photodisintegration interactions occur. It is possible to derive the generalized probability law for \( P_N \), at the expense of tedious calculation; however, as we demonstrate in the following, the above form for \( P_N \) provides a sufficient approximation for our case of interest.

One may then derive the propagated spectra, surviving fractions, and secondary proton spectra used in the previous subsection, as follows. Consider first the simpler scenario (B) in which one sums up over all fragments with rigidities in excess of \( E_1/Z \). Writing \( Q_{Z, \text{prop}}(E) = E q_{Z, \text{prop}}(E) \) the number of particles per log interval, and neglecting losses other than photodisintegration, one finds

\[
Q_{Z, \text{prop}}(E) = \sum_{i=0}^{i=A_x-1} p_i(A_x, x, E) \times Q_{Z, \text{inj}}\left(\frac{E}{1 - i/A_x}\right),
\]

with

\[
p_i(A_x, x, E) \equiv (x | dA/dx|^i) \exp(-x | dA/dx|)/i! \text{ the probability to undergo } i \text{ photodisintegration interactions over a distance } x, \text{ thereby decreasing the injection energy from } E/(1 - i/A_x) \text{ down to } E.
\]

Then, the fraction of surviving fragments with rigidity \( > E_1/Z \) can be obtained as

\[
f_{Z, \text{surf}}(> E_1) = \frac{1}{f_{E_1}} \int_{q_{Z, \text{inj}}(E)} q_{Z, \text{inj}}(E) dE \int_{E_1}^{\infty} d\ln E Q_{Z, \text{prop}}(E)
\]

\[
= \frac{1}{f_{E_1}} \int_{E_1}^{\infty} q_{Z, \text{inj}}(E) dE \int_{E_1}^{\infty} dE q_{Z, \text{inj}}(E)
\times P_j(A_x, x, E),
\]

with \( j = \min \{A_x - 1, \text{Int}[A_x/(1 - E_1/E)]\} \). The latter equality is obtained by inverting the summation interval between the integral and the discrete sum, changing variables in the integration from \( E \rightarrow E/(1 - i/A_x) \), and then permuting the order of integration. Note that \( P_j(A_x, x, E) = \sum_{i=0}^{i=0} p_i(A_x, x, E) \).

The fraction of photodisintegrated nuclei with energy more than \( 2E \) is given by \( f_{Z, \text{loss}}(> 2E) = 1 - f_{Z, \text{surf}}(> 2E) \), and the number of secondary protons is easily evaluated using Equation (6).

In scenario (A), one considers only the fragments with energy in the range \([E_1, E_2]\), which have suffered at most \( Z/4 \) photodisintegration interactions, so as to study a group of nuclei with similar rigidities. Equation (20) remains valid, if the sum over \( i \) runs from \( i = 0 \) to \( i = Z/4 \), therefore one finds

\[
f_{Z, \text{surv}}(E_1; E_2) = \frac{1}{f_{E_1}} \int_{q_{Z, \text{inj}}(E_1)}^{q_{Z, \text{inj}}(E_2)} dE \times \left\{ \int_{E_1}^{\infty} dE q_{Z, \text{inj}}(E) P_j(A_x, x, E) \right\} \times \left\{ \int_{E_1}^{\infty} dE q_{Z, \text{inj}}(E) P_j(A_x, x, E) \right\},
\]

with \( j_1 = \min \{Z/4, \text{Int}[A_x/(1 - E_1/E)]\} \) and \( j_2 = \min \{Z/4, \text{Int}[A_x/(1 - E_2/E)]\} \). Here as well, one defines \( f_{Z, \text{loss}}(> 2E) = 1 - f_{Z, \text{surv}}(> 2E) \), and the number of secondary protons is easily evaluated using Equation (6).

2.3. Results

So far, our treatment has remained quite general. Here, we apply it to the specific case of the “Cen A excess” reported by the Pierre Auger Collaboration. We thus use \( E_1 = 55 \text{ EeV}, E_2 = 84 \text{ EeV} \) and assume for simplicity that the source injects a pure oxygen, silicon, or iron composition. In Figure 2 we show both the analytical and the numerical results of the ratio of anisotropy significance at low to high energies as a function of the distance to the source that is responsible for the anisotropy. In this figure, we do not assume any proton component in the source composition, so that \( N_{p, \text{prop}} \rightarrow 0 \), \( M_H/M_Z \rightarrow 0 \) in Equations (13) and (14). We adopt an exponential cutoff power-law spectrum, as generally expected, with a cutoff energy \( E_{\text{max}} \propto Z \). The four panels correspond to different injection spectral indices and maximal energies. As can be seen, these results share the following common features.

At small source distances, the anisotropy signal produced by secondary protons is less prominent than the high-energy one, because only a few primary nuclei photodisintegrate on this short path length. As more and more secondary protons are produced with increasing source distance, this ratio grows and eventually exceeds unity. In the case \( s = 2 \), \( E_{\text{max}} = (Z/26) \times 10^{21} \text{ eV} \), both the analytical treatment and the numerical treatment result in a maximum source distance of \( \sim 15 \text{ Mpc}, \sim 60 \text{ Mpc}, \) and \( \sim 180 \text{ Mpc} \) for oxygen nuclei, silicon nuclei, and iron nuclei, respectively. Note that for \( E_{\text{max}} = (Z/26) \times 10^{21} \text{ eV} \), the source produces protons of energy \( > 40 \text{ EeV} \); such protons would presumably produce a strong anisotropy, though its magnitude would depend strongly on the distribution and characteristics of intervening magnetic fields. We show therefore the case with a high \( E_{\text{max}} \) for the sake of generality in order to illustrate the dependence of the results on the maximal energy.

From lighter to heavier nuclei, the constraint on the source distance becomes weaker, since at a given energy lighter nuclei carry a comparatively larger Lorentz factor, and as a consequence, their energy lies further beyond the photodisintegration interaction threshold (see also Figure 1). The small differences of \( D_{\text{max}} \) for the same species among the four panels can be interpreted as follows: protons at \( E/Z \) all come from primary nuclei at \( 2E \), so a smaller cutoff energy or a steeper power-law slope will decrease the amount of primary nuclei at \( 2E \), leading to less secondary protons produced at \( E/Z \), so that the values of maximum source distances in these cases are larger.

Another way to plot these results is to consider the minimum metal abundance \( M_Z/M_H \) that is required at the source in order to satisfy the bound \( \Sigma_j/\Sigma_Z < 1 \). The results are shown in Figure 3 as a function of the distance to the source. All four panels indicate that a mass ratio of nuclei to proton \( > 1:1 \) is
Figure 2. Ratio of anisotropy significance at low to high energies as a function of the distance to the sources responsible for the anisotropy. Solid lines represent the numerical results, while dashed lines represent the analytical results; thick solid line: scenario (A), in which one sums up over fragments of similar rigidity, in interval $[E_1, E_2]$, with at most $Z/4$ photodisintegration interactions; thin solid lines: scenario (B), in which one sums up over all fragments with rigidities in excess of $E_1/Z$. The source is assumed to inject pure O, Si, or Fe composition as indicated.

(A color version of this figure is available in the online journal.)

needed. Of course, as the distance increases, so does the minimum $M_Z/M_{HI}$, in order to compensate for the greater number of secondary protons produced during propagation. The distance where $M_Z/M_{HI} \rightarrow +\infty$ corresponds to $D_{\text{max}}$. Conversely, the asymptote as $D \rightarrow 0$ indicates the minimum $M_Z/M_{HI}$ amount when secondary protons can be safely neglected.

3. DISCUSSION

We emphasize that the method that we have presented remains quite general and could be applied to data sets of next generation experiments. Nevertheless, the results obtained in Figures 2 and 3 assume tacitly that the heavy chemical composition and the anisotropy signal reported by PAO are not artifacts. It is fair to say that these two results remain disputed. The significance level of the anisotropy, for instance, is not comfortably high and deserves to be improved with extended data sets. The measurements of the chemical composition by HiRes and TA differ appreciably from that of PAO. In particular, their data of $(X_{\text{max}})$ and rms $\sigma_{X_{\text{max}}}$ show a proton-dominated spectrum at all energies $>10^{18}$ eV (Abbasi et al. 2010; Tsunesada et al. 2011). One should not expect to detect anisotropies at low energies if the composition were pure proton, as the low-energy protons have a much smaller rigidity than the high-energy ones. On the other hand, the analysis of the chemical composition depends on the details of the hadronic interaction model, such as the cross-sections, multiplicities, and so on. The fact that these parameters are poorly constrained at present prevents one from drawing firm conclusions. As for the apparent anisotropy, Clay et al. (2010) have shown that there is no significant difference between the energy distribution of the events inside and outside the 25° window of Cen A using a K-S test, implying that events around Cen A do not have any special origin; such an analysis, however, cannot provide a conclusive answer, given the limited event statistics currently available. Additionally, two recent papers suggest that at most five to six events around Cen A can originate from it by backtracing the events’ trajectories in the intervening magnetic field (Farrar et al. 2013; Sushchov et al. 2012). We should, however, be cautious with such strong conclusions given that they depend on the magnetic field model adopted, which still carries a large degree of uncertainty.

3.1. Source Metallicity

With the above caveats in mind, it is interesting to discuss where the previous results lead us. The constraints derived from Figure 3 are indeed quite strong. For reference, the solar composition (Lodders & Palme 2009) corresponds to $M_{HI}/M_{\text{CNO}} \sim 70$, $M_{HI}/M_{\text{Si}} \sim 900$, and $M_{HI}/M_{\text{Fe}} \sim 550$. Consequently, the minimum metallicities required to match $\Sigma_p/\Sigma_Z < 1$,
notwithstanding the secondary protons, are $\sim 120 Z_{\odot}$ for CNO, $\sim 1600 Z_{\odot}$ for Si, and $\sim 1100 Z_{\odot}$ for iron-like nuclei. The comparison to $Z_{\odot}$ is less severe for oxygen, but this nucleus is also more fragile and the minimum metallicity diverges rapidly beyond some 20–30 Mpc. Conversely, the production of secondary protons is less severe for iron nuclei, but for such nuclei, the minimum requirements on the source metallicity are already quite extraordinary.

The observables $\langle X_{\text{max}} \rangle$ and $\sigma X_{\text{max}}$, as reported by PAO, suggest that the all-sky-averaged composition of arriving UHECRs may be oxygen-like (Hooper & Taylor 2010). If the anisotropy signal observed by PAO mainly consists of oxygen-like nuclei, our calculations indicate that the source responsible for the anisotropy should lie within 20–30 Mpc. There are only a limited number of known powerful radio galaxies within this distance, such as Cen A, and M87. Such radio galaxies are relatively weak, in terms of jet power and magnetic luminosity, which implies that they cannot accelerate particles beyond $E_{\text{max}} \sim 2 \times 10^{18} - 10^{19}$ eV (see the discussion in Lemoine & Waxman 2009). Even assuming that these sources accelerate oxygen nuclei to the highest energies, the minimum metallicity required by the above arguments lies well above what is measured in the central parts of such radio galaxies (Hamann & Ferland 1999). The situation becomes even worse if one considers silicon or heavier nuclei. Consequently, and as already emphasized in Lemoine & Waxman (2009), the current data set of PAO, in particular the clustering toward Cen A, does not provide support for acceleration of UHECRs in this object.

![Figure 3](https://example.com/figure3.png)

**Figure 3.** Minimum metal mass relative to hydrogen in the source, assuming that pure O, Si, or Fe compositions are injected. Thick solid lines and thin solid lines respectively represent results in scenario (A) and (B), which are the same as in Figure 2. (A color version of this figure is available in the online journal.)

If future data sets confirm the existence of anisotropies at high energies and the absence of anisotropies at low energies, then the present work provides strong constraints on the nature and the source of UHECRs: either protons exist at ultra-high energies, and some of them are responsible for the observed anisotropies (in which case no anisotropy is indeed expected at lower energies); or, a close-by source with rather extraordinarily high metallicity produces these anisotropies. The only physically motivated scenario for such a source so far is acceleration at the external shock of a semi-relativistic hypernova inside the wind of the progenitor (Wang et al. 2007; Budnik et al. 2008; Chakraborti et al. 2011; Liu & Wang 2012).

### 3.2. Composition Close to the Ankle

Provided that the same source population produces UHECRs with energy both $> E_1$ and $> E_1/Z$, the proton fraction at $E_1/Z$ becomes an interesting aspect of the problem. The key point indeed is that if $M_Z \gtrsim M_{\text{HI}}$ inside the sources, as suggested by the above discussion, and all sources are alike, then the chemical composition at $E_1/Z$ must contain a significant heavy component. More specifically, the fraction of protons at low energies is given by

$$x_p(E_1/Z; E_2/Z) = \frac{\dot{N}_p(E_1/Z; E_2/Z)}{\dot{N}_{Z,\text{prev}}(E_1/Z; E_2/Z) + \dot{N}_p(E_1/Z; E_2/Z)}$$

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8 We thank S. Nagataki for suggesting this to us.
where \( \tilde{N}_p = \tilde{N}_{p, \text{prop}} + \tilde{N}_{p, \text{dis}} \) is the total proton number, including the contribution from secondary protons and primary protons, as integrated over all sources, and similarly for \( \tilde{N}_{Z, \text{prop}} \). Here we neglect the partially disintegrated fragments. Assuming that every source has equal emissivity and the same injection spectrum, we have

\[
\tilde{N}_{Z, \text{prop}}(E_1; E_2) \approx \int_{(1+z)E_1/Z}^{(1+z)E_2/Z} q_{z, \text{inj}}(E) dE \\
\times \int_0^{l_{Z, \text{loss}}(E)} n(z) f_{Z, \text{surv}}(E) dD_c(z)
\]

(24)

and

\[
\tilde{N}_{p, \text{dis}}(E_1; E_2) \approx A_Z \int_{(1+z)E_1/Z}^{(1+z)E_2/Z} q_{z, \text{inj}}(E) dE \\
\times \int_0^{l_{p, \text{loss}}(E/A_Z)} n(z) f_{Z, \text{loss}}(E) dD_c(z).
\]

(25)

Here \( n(z) \) is the source density as a function of redshift \( z \) and \( D_c(z) \) is the comoving distance to the light cone at redshift \( z \); \( l_{Z, \text{loss}} \) and \( l_{p, \text{loss}} \) represent the energy loss lengths \( \langle E/(dE/ds) \rangle \) for nuclei and protons, respectively. The second equation assumes that photodisintegration takes place on short distance scales compared to \( l_{p, \text{loss}}(E/A_Z) \), which is a very good approximation. The energy losses to be considered here include all processes besides photodisintegration, such as pair production, adiabatic cooling, etc. Since the energy loss distance of protons with energy \((1+z)E/Z\) is much larger than the energy loss distance of nuclei at energy \(2(1+z)E\), \( f_{Z, \text{loss}}[2(1+z)E] \to 1 \) for most sources. On the other hand, \( f_{Z, \text{surv}}[(1+z)E/Z] \approx \exp[-D_c/(l_{Z, \text{loss}}[(1+z)E/Z])] \), and given that \( l_{Z, \text{loss}}[(1+z)E/Z] \) is the upper limit of integration, we have \( e^{-1} \approx f_{Z, \text{surv}} \approx 1 \). As an estimation here we take \( f_{Z, \text{surv}} = 1 \). Then, the above two equations can be written as

\[
\tilde{N}_{Z, \text{prop}}(E_1; E_2) \approx k_Z Z^{-1} \tilde{E}^{1-s} \int_0^{l_{Z, \text{loss}}(E/Z)} (1+z)^{1-s} n(z) dD_c
\]

(26)

\[
\tilde{N}_{p, \text{dis}}(E_1; E_2) \approx k_Z Z^{-1} \tilde{E}^{1-s} A_Z \int_0^{l_{p, \text{loss}}(E/Z)} (1+z)^{1-s} n(z) dD_c.
\]

(27)

Since \([E_1, E_2]\) is a narrow energy range, we denote the average energy in this range by \( \tilde{E} \). Considering that \( n(z) \) usually evolves with redshift \( z \), we make here a further approximation that the term \((1+z)^{1-s}\) cancels the evolution in \( n(z) \) to some extent and the integrand is reduced to a constant. Then one can find that

\[
x_{p}(E_1; E_2) \approx \frac{1 + 2^{1-s}M_H/M_Z}{1 + 2^{1-s}M_H/M_Z + A_Z^{-2}l_{Z, \text{loss}}(E/Z)/l_{p, \text{loss}}(E_{\text{dis}}/Z)}.
\]

(28)

In the local universe, for oxygen nuclei, \([E_1/Z, E_2/Z] \sim [7, 10] \text{ EeV}\) hence the energy loss in this energy range is comparably caused by photodisintegration on EBL photons and pair production on CMB photons, leading to an energy loss length of \(\sim 2-3 \text{ Gpc}\). For silicon and iron nuclei, \([E_1/Z, E_2/Z] \sim [4, 6] \text{ EeV}\) and \([2, 3] \text{ EeV}\), respectively, in which energy range the dominant cooling process is adiabatic cooling with an energy loss length \(\sim 4 \text{ Gpc}\). For protons, however, the dominant energy loss process in the corresponding energy range is caused by pair production on CMB photons with an energy loss length \(\sim 1-2 \text{ Gpc}\). Therefore, typically, the energy loss length for nuclei is larger than that for protons by a factor of \(2-3\). If \(2^{-1} M_H/M_Z \lesssim 1\), as suggested by the previous discussion, this implies in turn that the composition \(Z_{(E_1/Z, E_2/Z)}\) should comprise less than \(\sim 50\%\) protons, in potential conflict with the claims of a light composition close to the ankle of the cosmic-ray spectrum.

3.3. Trans-GZK Anisotropies

Another interesting aspect is the possible anisotropy signal that one may expect at higher energies, given the reported anisotropies at \(>55 \text{ EeV}\). The detection of such anisotropies provides a strong motivation for next-generation experiments such as JEM-EUSO (Casolino et al. 2011), which will provide a substantially larger amount of statistics.

Here we start by assuming that the currently observed anisotropy mainly consists of nuclei with charge number \(Z\) and that their source also accelerates heavier nuclei with nuclear charge number \(Z'\). These heavier nuclei will produce a similar anisotropy pattern at higher energies \(Z'E_1/Z\); we define \(E_1' = Z'E_1/Z\) for clarity. The ratio of significance between these two anisotropy signals then reads

\[
\frac{\Sigma_Z(>E_1')}{\Sigma_Z(>E_1)} \approx \frac{M_Z}{M_{Z'}} \frac{f_{Z', \text{surv}}(>E_1')}{f_{Z, \text{surv}}(>E_1)} \left( \frac{Z'}{Z} \right)^{(p_s-3)/2},
\]

(29)

where \(f_{Z', \text{surv}} \approx \exp(-x/l_{Z', \text{loss}})\). With the approximation \(A_Z \simeq 2Z\), the two species of nuclei with the same rigidity share approximately the same Lorentz factor. At the same Lorentz factor, heavier nuclei lose energy faster than lighter nuclei, but the differences between the energy loss lengths of different species such as O, Si, Fe with rigidity \(E/Z\) are at most a factor of a few; furthermore, the energy loss lengths are larger than the maximum source distance \(D_{\text{max}}\) that we have obtained in Section 3. So we expect \(0.1 \lesssim f_{Z, \text{surv}}(>E_1)/f_{Z', \text{surv}}(>E_1) \lesssim 1\). Also, \(p_s = 4.3\), and hence \((Z'/Z)^{(p_s-3)/2} \lesssim 1\). Therefore, a stronger anisotropy signal is expected at higher energies if the source is more abundant in nuclei \(Z'\) than nuclei \(Z\). In this case, however, some accompanying effects will occur and one should also check whether these effects already cause violations against current measurements or lead to self-contradiction. We consider here the following three aspects.

1. Secondary protons produced by nuclei \(Z'\) above energy \(E_1/Z\). Since nuclei \(Z'\) at \(E_1\) have the same rigidity as the nuclei \(Z\) at \(E_1\), the secondary protons emitted by nuclei \(Z'\) will fall well within the energy range of interest. According to Equation (6), we can write the secondary protons above

\[9\] If we also consider the slightly disintegrated fragments as surviving primaries, \(f_{Z, \text{surv}}\) will be closer to unity.
2. The chemical composition of UHECRs at energy \( E_1 \): as the UHECR background decreases rapidly with increasing energy, the composition of cosmic rays emitted by the source can strongly influence the composition measurement at higher energies, provided that the source accelerates a larger fraction of nuclei \( Z' \) than nuclei \( Z \). Although it is difficult to find a quantitative relation between the composition of the source and that of the all-sky averaged composition, one might naively expect the all-sky averaged composition above a given energy \( E \) (denoted as \( \xi_Z(>E) \)) to be positively related to \( A_z(N_{Z,prop}(>E)/N_{iso}(>E)) \), and we find

\[
\frac{\xi_Z(>E_1')}{\xi_Z(>E_1)} = \frac{M_Z f_{Z,\text{surv}}(>E_1')}{M_Z f_{Z,\text{surv}}(>E_1)} \left( \frac{Z'}{Z} \right)^{\mu Z - 1} \left( \frac{E_1'}{E_1} \right) \left( \frac{\mu Z}{\Sigma Z} \right) \frac{\Sigma Z'}{\Sigma Z}.
\]

As one can see, since \( Z' > Z \), if stronger anisotropy signal is detected at higher energies (\( \Sigma Z' > \Sigma Z \)), the UHECR composition is expected to be heavier.

3. The surviving nuclei \( Z' \) in the energy range between \( E_1 \) and \( E_2 \). Assuming that the source is more abundant in nuclei \( Z' \) than nuclei \( Z \), the source should emit a larger amount of nuclei \( Z' \) in the energy range between \( E_1 \) and \( E_2 \). So after propagation, the number of surviving nuclei in a fixed energy range is \( f_{Z,\text{surv}} \int k_Z E^{-\alpha} dE \propto f_{Z,\text{surv}} M_Z Z^{-2} \).

With the fact that heavier nuclei lose neutrons slower than lighter nuclei at the same energy (not the same Lorentz factor), we expect the ratio of nuclei \( Z' \) and nuclei \( Z \) emitted by the source in the energy range to be

\[
\frac{N_{Z,\text{prop}}(E_1; E_2)}{N_{Z,\text{prop}}(E_1; E_2)} = \frac{M_{Z'} f_{Z,\text{surv}}(E_1; E_2)}{M_Z f_{Z,\text{surv}}(E_1; E_2)} \left( \frac{Z'}{Z} \right)^{s-2} > 1.
\]

It does not mean, however, that these \( Z' \) nuclei would contribute to the anisotropy pattern seen in the range \([E_1, E_2]\), because they have smaller rigidity than the \( Z \) nuclei.

If the source does not accelerate nuclei beyond charge \( Z \), then the anisotropy at higher energies is produced by nuclei of charge \( Z \). To derive the corresponding ratio of significances, make the substitution \( M_{Z'}/M_Z \to 1 \), \( Z'/Z \to E_1'/E_1 \) in Equation (29).

Then, with \((p_2 - 3)/2 \approx 0.65\), one expects the ratio to increase slightly up to the energy at which the distance to the source matches the energy loss distances, and then to drop sharply beyond this distance. The detection of such a feature would provide useful constraints on \( Z \) and \( D \).

4. CONCLUSION

In this work, we have generalized a test of the chemical composition of UHECRs, which proposes to use the anisotropy pattern measured as a function of energy. The basic principle is that if anisotropies are observed at high energies \( E \gtrsim 6 \times 10^{19} \) eV, and if one assumes that these anisotropies are caused by heavy nuclei of charge \( Z \), then one should observe a strong anisotropy signal at energies \( E/Z \) close to the ankle, due to the proton component (Lemoine & Waxman 2009). In the present paper, we have accounted for the production of secondary protons through the photodisintegration interactions of nuclei. Assuming that no anisotropy signal is detected at low energies, we derive an upper bound on the distance to the source.

Our numerical estimates are based on the report of PAO of an excess in the direction to Cen A. At present, the significance of this detection is not well established and one must await future data to confirm or invalidate it. Nevertheless, the method presented here remains general and might well be applied to future, more extended data sets. Taking the results of PAO at face value, we derive a maximal distance to the source of the order of 20–30 Mpc, 80–100 Mpc, and 180–200 Mpc if the nuclei responsible for the anisotropies are oxygen, silicon, and iron, respectively. The differences between these estimates of the maximal distance are directly related to the energy loss lengths of these nuclei at GZK energies. Our results are summarized in Figure 3, which shows the minimum mass of metals relatively to hydrogen required in the source, in order to produce a weaker anisotropy at \( E/Z \) than at \( E \). At distances exceeding the above estimates, this amount diverges, meaning that even if the source does not accelerate any protons, the amount of secondary protons produced during propagation is sufficient to cause a secondary anisotropy at \( E/Z \) larger than that observed at \( E \). At small source distances, where photodisintegration effects are negligible, one nevertheless finds a minimum mass \( M_{Z}/M_{\text{He}} \sim 1 \). When measured relatively to the solar composition, this indicates that the metallicity inside the source should exceed \( \sim 120 Z_{\odot} \), \( \sim 1600 Z_{\odot} \), or \( \sim 1100 Z_{\odot} \) if oxygen, silicon, or iron nuclei, respectively, are responsible for the high-energy anisotropy. This result does not depend on the spectral index, or on the details of the injection spectrum, as long as the latter is shaped by rigidity. When combined, these bounds on the distance and metallicity bring in quite stringent constraints on the source of these particles. Additionally, these constraints imply that if the heavy nuclei at GZK energies are silicon or iron, the proton fraction in the all-sky composition at ankle energies should be less than \( \sim 50\% \), in potential conflict with measured data.

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