Joint optimization of imperfect condition-based maintenance and lot sizing via an availability-cost hybrid factor

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Abstract
This article develops a joint model of condition-based maintenance (CBM) and economic production quantity (EPQ) for a single-unit manufacturing system. The degradation process of the system is described by a random coefficient growth model, where the mean and variance may increase with time. The condition of the system is monitored after each lot size. Three types of actions are considered, preventive maintenance (PM), preventive replacement, and corrective maintenance (CM), which are performed according to multi-level thresholds. PM is imperfect, and the system lies between two states, that is, it is as good as new or as bad as old after PM with a certain probability. Different actions lead to different calculations to determine the optimal PM strategy based on the renewal reward theory. An availability-cost hybrid factor is proposed to balance the cost per unit time and the availability of the system. Numerical simulations are provided to illustrate the method, and then a sensitivity analysis is conducted by a real case study. A comparison with single-level threshold strategies demonstrates the effectiveness of the proposed approach.

KEYWORDS
condition-based maintenance, hybrid factor, imperfect, lot sizing

JEL CLASSIFICATION
Industrial engineering

1 | INTRODUCTION

As mechanization and automation widely spread, the labor and financial expenditures on maintenance actions have substantially grown. Increasing efforts have been applied to preventive maintenance (PM) and production plans within manufacturing in recent years. PM planning may improve the efficiency of equipment and meet the production requirements.¹ Traditional PM strategies can be classified into three types: corrective maintenance (CM), time-based maintenance (TBM), and condition-based maintenance (CBM).³ With the development of condition monitoring...
methods, CBM has become widely used in manufacturing industries and academic research. Compared to TBM, CBM is more realistic and practical. Optimal CBM strategies can avoid excessive maintenance and generate benefits for industrial enterprise competition. Mula et al. studied reviews of production planning under uncertainty in manufacturing systems. Studies of PM strategies and production plans are often performed separately, which leads to increasing failure and production delays. Thus, many researchers focus on modeling the trade-offs between PM and production scheduling. Nodem et al. presented an optimal method of production and PM for a degradation process subject to random breakdowns. The optimal production rate and PM rate of a single machine were obtained. The results showed that an optimal joint determination of PM and production scheduling may increase costs but reduce the frequency of maintenance, which had little impact on production plans. Xiao et al. proposed a joint optimization model to minimize the total cost for a series system. The optimal PM interval and the assignment of each job were obtained. It was concluded that the implementation and timing of PM influenced the system performance. Considering the minimum total expected weighted completion time of jobs as a single objective, Wang et al. suggested an integrated optimization model for a single machine. The method can solve optimally within reasonable computation times.

However, few conservative studies on joint modeling problems address CBM strategies. Xu et al. studied fixed and flexible maintenance cycles of single-machine scheduling. The results demonstrated that flexible maintenance could significantly improve the joint optimization of maintenance and production strategies. Thus, some studies have discussed joint problems based on CBM. Considering the minimum expected average total cost as a single objective, Wang et al. constructed an integrated model of economic production quantity (EPQ) and CBM. The system condition was described by a proportional hazards model, which is a dynamic procedure based on the maintenance threshold. The simulation-based iterative algorithm showed that the joint optimization policy can reduce the cost by over 20% compared to other conventional maintenance policies. Considering continuous-time and continuous-state degradation processes, Peng et al. developed a joint model to minimize the average long-run cost rate based on renewal theory. Experimental results showed that the optimization of the production time and the control threshold can achieve the lowest cost rate in the case of high-lost sales costs. Another study focused on integrated models of CBM and production planning problems from the aspect of the mean residual life (MRL), and it showed the practical applications of the results. Shamsaeei et al. demonstrated that a new method to solve joint problems can save up to 41% of the cost compared to cyclic schedules. The system condition monitored from the above studies considered the observations as the real value of the system condition. However, sometimes the reading values are higher than a preset PM threshold. In this case, a faulty PM or replacement may be implemented. Thus, Liu et al. developed a joint problem of CBM and EPQ based on the random coefficient regression model, which can divide the observation of the system condition into the actual condition item and the random error item. Gao et al. proposed an integrated model of production, inventory, and PM considering soft failures and hard failures. Furthermore, considering signal loss, Gao et al. studied the reliability modeling of linear multistate consecutively connected systems.

Most joint production and PM models consider only one PM level. Considering random failures, Salmasnia et al. proposed an integrated model with multilevel PM for a single machine to minimize the completion time and the costs. Compared to the no-PM situation and single-level PM, the joint model using multilevel PM spent less time and money renewing the system. Moreover, it may reduce the risk of unexpected failures. Furthermore, Duan et al. discussed joint problems in the case of imperfect PM based on the semi-Markov decision process to determine the optimal system-level threshold and unit-level thresholds. Nourelfath et al. developed an integrated model of production and PM for an imperfect process for a multiperiod and multiproduct manufacturing system. The numerical study analyzed and compared the performances of single-level PM and the situation of different PM levels. The problem can be solved heuristically in the case of multi-PM levels.

With increased applications in maintenance plans, optimization decision problems are complex problems with multiple conflicting criteria. In practice, manufacturing industries must make trade-offs in scheduling problems where multiple factors are considered to obtain the strategy. Almeida et al. proposed a review of different decision objectives to compare the results of single objectives and multiple objectives. The results showed that multiobjective problems can provide a better optimal strategy for PM and production scheduling. Considering reliability, cost, and downtime as constraints, Ebrahimipour et al. proposed a multiobjective model to make the optimal decision. Another study considered five major objectives. The results showed that it is significant to consider multiple objectives when solving the integrated problem of PM and production planning. Jin et al. established a multiobjective decision model by considering five objectives, including maintenance cost, completion time, the total weighted completion time of the task, the total weighted
delay time and machine availability. The optimal integrated decision was obtained by using a genetic algorithm. However, the PM action was perfect, and the optimization must be selected from a few decisions. Gao et al. developed an optimal interception strategy considering multiple targets.

In this article, a joint optimization model of imperfect CBM and lot sizing is proposed based on the random coefficient growth model, where measurement errors are considered. The coefficients are assumed to follow known distribution functions. The CBM actions are imperfect, where minimal maintenance and perfect maintenance are performed, that is, they bring the single-unit system as bad as the old state or as good as the new state with a certain probability. Multilevel PM thresholds are considered. If the observed equipment condition reaches the first PM level, an imperfect PM is carried out.

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The remainder of this article is organized as follows. Section 2 outlines the joint problem and maintenance strategy. Section 3 presents a numerical case study to demonstrate the applicability of the proposed model. Section 4 presents a sensitivity analysis and compares single-level and multilevel PM strategies. The result demonstrates that the proposed model is superior. Section 5 draws the conclusions.

2 PROBLEM DESCRIPTION AND CBM STRATEGY

To propose the approach, the problem is described to illustrate the degradation process of the single-unit manufacturing system. A CBM strategy based on a random coefficient growth model is developed subject to failure repair renewal and preventive replacement renewal. A dual-attribute criterion of CBM and EPQ optimizations for a single-unit system is considered instead of a single-attribute criterion.

2.1 Problem description

Considering a deteriorating process for a single-unit manufacturing system. The product rate and demand rate are constants and are denoted as p and d, and they satisfy $p > d$. Inspections are carried out at the end of each batch, and the single-unit manufacturing system can be repaired after each inspection or after a failure occurrence. Assume that the production time of each batch is $T$, and each production lot size is $Q$, which satisfies $T = Q/p$. The operation phase and outage phase of the single-unit equipment are shown in Figure 1. When the observed condition exceeds the first PM level,
denoted as $y(t_i) > C_1$, an imperfect PM is carried out. It is assumed that the probability of perfect PM is $a$, and the probability of minimum maintenance is $b$. When the observed condition exceeds the second PM level, denoted as $y(t_i) > C_2$, preventive replacement is conducted. When a shutdown occurs, denoted as $y(t_i) > D$, the equipment will be replaced immediately, where $C_1 < C_2 < D$. During the processes of production and maintenance, the situation of stockout can be avoided. There may be no inventory when a failure occurs, as shown in Figure 2.

### 2.2 CBM strategy

We denote $y(t_i)$ as the observed condition of the system at time $t_i$. Since the deterioration process of the system is described by a random coefficient growth model, we have

$$y(t_i) = \eta(t_i; \theta, \xi) + \epsilon,$$  

(1)

where $\theta$ is a constant, and $\xi$ is a random parameter. The probability density function of $\xi$ is $g(\xi)$. $\epsilon$ is the error term, which obeys a normal distribution ($\epsilon \sim \mathcal{N}[0, \sigma]$). Random errors are time-independent and item-independent. The relationship between the detected condition and the real condition of the single-unit system is shown in Figure 3.
If the random parameter $\xi$ is given, we have

$$P(Y|\xi) = P(y(t) \leq Y|\xi) = P(\eta(t; \theta, \xi) + \epsilon \leq Y) = P(\epsilon \leq Y - \eta(t; \theta, \xi)) = \Phi \left( \frac{Y - \eta(t; \theta, \xi)}{\sigma} \right).$$

(2)

When the condition of the product equipment reaches failure level $D$, the system will immediately fail down. For all possible $\xi$, the failure distribution function can be expressed as

$$P(\tau \leq t) = F_\tau(t) = \int_A g(\xi)F_\tau(t|\xi)d\xi,$$  
(3)

where $\tau$ represents the failure time, and we have

$$F_\tau(t|\xi) = P(\tau \leq t|\xi) = \begin{cases} 1, & \tau \leq t \\ 0, & \tau > t. \end{cases}$$

(4)

That is, when the failure time occurs before $t$, the fault occurs; otherwise, no failure occurs. According to the discussion in Liu’s paper, the equipment condition satisfies $\eta = \xi t + \theta$. We can substitute it into Equation (3) to obtain

$$P(\tau \leq t) = P\left( \frac{D - \theta}{\xi} \leq t \right) = P\left( \frac{D - \theta}{t} \leq \xi \right) = \int_{-\infty}^{\infty} g(\xi)d\xi.$$  

(5)

The system can be renewed in three scenarios, which are discussed below. First, the situation of perfect PM renewal is discussed. The single-unit system is monitored at the end of the $k$th lot size, where $k$ is a positive integer. When the monitored condition meets $y(t_k) > C_1$, PM actions are carried out immediately. The probability of perfect PM is $a$, while the probability of minimal maintenance is $b$. Perfect PM may renew the system. A new production batch will start after the end of the PM action or the inventory decreases to zero. The probability of PM renewal is denoted as $P_p(t_k)$. For a given value of $\xi$, we have

$$P_p(t_k|C_1, C_2; \xi) = \begin{cases} 1 - F_\tau(t_k|\xi) \left[ \prod_{j=1}^{k-1} P(y(t_j) < C_1|\xi) + \prod_{j=1}^{k-1} P(C_1 \leq y(t_j) < C_2|\xi) b^{k-1} \right] + \sum_{j=1}^{k-2} \left[ \prod_{j=1}^{j} P(y(t_j) < C_1|\xi) \prod_{r=1}^{k-j-1} P(C_1 \leq y(t_j+r) < C_2|\xi) b^{k-j-1} \right] \times P(C_1 \leq y(t_j) < C_2|\xi) a, & k \geq 3 \\ 1 - F_\tau(t_k|\xi)[P(y(t_1) < C_1|\xi) P(C_1 \leq y(t_1) < C_2|\xi)] + \prod_{j=1}^{2} P(C_1 \leq y(t_j) < C_2|\xi) b a, & k = 2 \\ P(C_1 \leq y(t_1) < C_2|\xi) a, & k = 1 \end{cases}$$

(6)

where the first item $[1 - F_\tau(t_k|\xi)]$ represents the case of no failure before $t_k$. $P(C_1 \leq y(t_k) < C_2|\xi) a$ represents the probability that the observed system condition reaches the first PM level at the $k$th inspection, and perfect PM action is conducted. $\prod_{j=1}^{k-1} P(y(t_j) < C_1|\xi)$ represents the probability that the observed system condition is less than the first PM level before the $k$th inspection. $\prod_{j=1}^{k-1} P(C_1 \leq y(t_j) < C_2|\xi) b^{k-1}$ represents the probability that the observed value of the system conditions ranges from the first PM level to the second PM level before the $k$th inspection, and all the previous $k - 1$ times PM actions are minimally maintained. The remaining items in Equation (6) when $k \geq 3$ represent that there are $j$ times when the observed condition of the system is less than the first PM level $C_1$ and $k-j-1$ times when the observed conditions range from $C_1$ to $C_2$ among the previous $k-1$ inspections, and all the $k-j-1$ PM actions are minimally maintained. Furthermore, the inventory cost can be obtained by drawing the triangle area, which is $C_j kp(p - d)T^2/2d$. The cost of inspections is $kC_i$. The PM cost is the product of the imperfect PM cost per unit time $C_p$ and the expected number of PMs. The expected number of PMs during the previous $k-1$ inspections is
The expected cycle length of the perfect PM renewal is

$$E_c = p(C_1 \leq y(t_k) < C_2|\xi)[1 + k \prod_{i=1}^{k-1} p(C_1 \leq y(t_i) < C_2|\xi)]$$

$$+ \sum_{j=1}^{k-2} \left[ (k-j) \prod_{s=1}^{j} p(y(t_s) < C_1|\xi) \prod_{r=1}^{k-j-1} p(C_1 \leq y(t_{j+r}) < C_2|\xi) \right].$$

Thus, the expected total cost of PM renewal is

$$E(C_{pm}) = P_p(t_k) \left[ kC_i + (E_k + 1)C_P + C_t \frac{kp(p - d)T^2}{2d} \right].$$

The expected cycle length of the perfect PM renewal is given by

$$E(L_p) = P_p(t_k) \frac{kpT}{d}.$$

Second, the situation of preventive replacement renewal is discussed.

The probability of preventive replacement renewal is denoted by $P_r(t_k)$. For a given value of $\xi$, it is given by

$$P_r(t_k|C_1, C_2; \xi) = \begin{cases} \sum_{j=1}^{k-2} \left( \prod_{s=1}^{j} p(y(t_s) < C_1|\xi) \prod_{r=1}^{k-j-1} p(C_1 \leq y(t_{j+r}) < C_2|\xi) b^{k-j-1} \right), & k \geq 3 \\ \prod_{s=1}^{k-1} p(y(t_s) < C_1|\xi) + \prod_{s=1}^{k-1} p(C_1 \leq y(t_s) < C_2|\xi) b^{k-1} \right) \times [1 - F_r(t_k|\xi)] P(y(t_k) \geq C_2|\xi), & k = 2 \\ [1 - F_r(t_1|\xi)] P(y(t_1) \geq C_2|\xi), & k = 1 \end{cases}.$$.

Furthermore, the cost of inspections is $kC_i$. The inventory cost can be obtained by drawing the triangle area. The PM cost is the product of the imperfect PM cost per unit time $C_P$ and the expected number of PMs. The expected costs of PMs during the previous $k-1$ inspections is

$$E(C_{pr}) = P_r(t_k) \left[ kC_i + E_kC_P + C_r + C_t \frac{kp(p - d)T^2}{2d} \right].$$

The expected cycle length of preventive replacement renewal is given by

$$E(L_r) = P_r(t_k) \frac{kpT}{d}.$$

Finally, the situation of failure renewal is discussed. From the discussion above, the probability of failure renewal is given by

$$P_f(t_k|C_1, C_2; \xi) = \begin{cases} \sum_{j=1}^{k-2} \left( \prod_{s=1}^{j} p(y(t_s) < C_1|\xi) \prod_{r=1}^{k-j-1} p(C_1 \leq y(t_{j+r}) < C_2|\xi) b^{k-j-1} \right), & k \geq 3 \\ \prod_{s=1}^{k-1} p(y(t_s) < C_1|\xi) + \prod_{s=1}^{k-1} p(C_1 \leq y(t_s) < C_2|\xi) b^{k-1} \right) \times [F_r(t_k|\xi) - F_r(t_{k-1}|\xi)], & k = 2 \\ [F_r(t_2|\xi) - F_r(t_1|\xi)], & k = 1 \end{cases}.$$.
Furthermore, the expected running time of the equipment is

\[ E(\tau) = \int_{(k-1)T}^{kT} \frac{\partial P_f(\tau)}{\partial \tau} d\tau. \]  

(14)

Assuming that the running time during the kth production cycle is \( \tau_k \), we have an expected running time during the kth production cycle of

\[ E(\tau_k) = \int_{(k-1)T}^{kT} (\tau - (k - 1)T) \frac{\partial P_f(\tau)}{\partial \tau} d\tau. \]  

(15)

The expected cost in the situation of inventory shortage is

\[ E(C_o) = C_o \max \{0, t_f P_f(t_k) - [(p - d)E(\tau_k)] / d\}. \]  

(16)

The inventory cost during the kth production cycle is given by

\[ E(C_I) = C_I \left[ \frac{[(k-1)p(p-d)T^2] P_f(t_k) + p(p-d)E(\tau_k)^2}{2d} \right]. \]  

(17)

The inspection cost of the previous k-1 times is \((k-1)C_i\). The total expected cost of CM is given by

\[ E(C_p) = C_o \max \{0, t_f P_f(t_k) - [(p - d)E(\tau_k)] / d\} + C_I \left[ \frac{[(k-1)p(p-d)T^2] P_f(t_k) + p(p-d)E(\tau_k)^2}{2d} \right] \]

\[ + \frac{p(p-d)E(\tau_k)^2}{2d} \left/ \left[ (k-1)C_i + C_f + E_k C_p \right] P_f(t_k) \right. \]. \]  

(18)

Furthermore, the expected cycle length of the failure renewal is given by

\[ E(L_f) = P_f(t_k) \left( \frac{(k-1)pT}{d} \right) + E(\tau_k) + \max \{ t_f P_f(t_k) \cdot [(p - d)E(\tau_k)] / d\}. \]  

(19)

The expected total cost per unit time based on the renewal reward theory is given by

\[ E(C_{Total}) = \frac{\sum_{k=2}^{\infty} (E(C_{pm}) + E(C_p) + E(C_{pr}))}{\sum_{k=2}^{\infty} (E(L_p) + E(L_f) + E(L_r))}. \]  

(20)

### 2.3 Availability of the single-unit system

By definition, the availability can be expressed as the ratio of the average uptime to the sum of the average uptime and the average downtime. Thus, the expression for the availability is

\[ A = \frac{\text{Uptime}}{\text{Uptime} + \text{Downtime}}. \]  

(21)

In the first two renewal cases, the availability of the kth production cycle is

\[ A_p(k) = \frac{kT}{kT_p/d + t_p}; \quad A_r(k) = \frac{kT}{kT_p/d + t_r}. \]  

(22)

In the case of failure renewal, the availability of the kth production cycle is given by

\[ A_f(k) = \frac{\tau_k}{\tau_k + \max \{(p - d)\tau_k/d, t_f\}}. \]  

(23)
Thus, the expected availability of the system is

\[ E(A) = \sum_{k=1}^{\infty} A_f(k)P_f(t_k) + \sum_{k=1}^{\infty} A_p(k)P_p(t_k) + \sum_{k=1}^{\infty} A_r(k)P_r(t_k). \]  

(24)

## 2.4 AVAILABILITY-COST HYBRID FACTOR

In this model, a dual-attribute criterion of CBM and EPQ optimizations for a single-unit system is considered instead of a single-attribute criterion. To combine the contributions of cost and availability, a hybrid factor \( U \) is employed. According to the method based on the linear scale transformation-max method, the hybrid factor \( U \) is given by

\[ \min U = w_1 \frac{\max E(A)}{E(A)} + (1 - w_1) \frac{E(C_{\text{total}})}{\min E(C_{\text{total}})}. \]  

(25)

where \( \max E(A) \) and \( \min E(C_{\text{total}}) \) represent the maximum system availability and the minimum cost per unit time, respectively. The availability-cost weight factor is denoted by \( w_1 \), \( 0 < w_1 < 1 \). The optimized \( C_1 \), \( C_2 \), and \( Q \) can be determined by the minimization of the hybrid evolution factor.

## 3 NUMERICAL CASE STUDY

The applicability of the proposed joint model of CBM and EPQ is simulated based on the data from a steel factory. The values of the parameters are shown in Table 1.

From the inequality of PM levels \( 0 < C_1 < C_2 < D = 10 \), the ranges of \( C_1 \) and \( C_2 \) are \([0, 10]\). The condition of the system \( y(t_i) \) satisfies

\[ y(t_i) = \xi t_i + \epsilon, t_i \geq 0. \]  

(26)

The random variable is assumed to follow a Weibull distribution, which is given by

\[ F(\xi) = \begin{cases} 1 - e^{-((\alpha \xi)^n)}, & t \geq 0 \\ 0, & t < 0 \end{cases}. \]  

(27)

Optimization of the joint problem for the numerical case is implemented using a simulation process through MATLAB scripts with the following steps.

**Step 1:** Assign initial values for all variables in Equation (1) to (27) by using the parameters in Table 1 and setting the value ranges of the first and second PM levels. The random variables \( \xi \) and \( \epsilon \) are generated.

| Parameters | Values | Parameters | Values |
|------------|--------|------------|--------|
| \( a \)    | 0.6    | \( b \)    | 0.4    |
| \( p \)    | 10     | \( d \)    | 6      |
| \( t_f \)  | 0.2    | \( t_p, t_r \) | 0.05 |
| \( C_r \)  | 300    | \( C_1 \)  | 5      |
| \( C_1 \)  | 50     | \( C_p \)  | 200    |
| \( C_f \)  | 500    | \( C_o \)  | 50     |
| \( a \)    | 0.4    | \( \beta \) | 2.42   |
| \( \sigma \)| 0.0312 | \( D \)    | 10     |
Step 2: Calculate cycle lengths $L_p$, $L_r$, and $L_f$ according to Equation (9)–(19), respectively, and the probability of exceeding the three levels to determine the three system renewal cases.

Step 3: Calculate the total costs according to the three system renewal cases according to Equation (6)–(18), and determine the expected total cost per unit time by using Equation (20).

Step 4: Calculate $A_p(k)$, $A_r(k)$, and $A_f(k)$ by using Equation (22) and Equation (23). Using Equation (24), calculate the expected availability of the system.

Step 5: Calculate the maximum availability of the system $E(A)$ and the minimum cost $E(C_{total})$ to determine availability-cost hybrid factor $U$.

Step 6: For a given value of weight coefficient $w_1$, the optimality of $U$ is obtained from the optimized first and second PM levels $C_1$ and $C_2$, respectively, and the optimized lot size $Q$.

According to step 5, the optimal availability-cost hybrid factor $U$ can be determined after single-objective decisions of the total cost per unit time and the system availability are obtained. First, the total cost per unit time is considered the single objective, that is, $w_1 = 0$. The trend of the minimum total cost per unit time over production time is shown in Figure 4.

Figure 4 shows that the expected minimum total cost per unit time first decreases rapidly until it reaches the minimum value and then increases as the production time of each batch increases. It can be calculated by MATLAB. The optimal total cost per unit time is $C^* = 167.9$, and the corresponding production time of each batch is $T^* = 1.1$, $C_1^* = 9$, and $C_2^* = 10$. Second, the system availability is considered the single objective, that is, $w_1 = 1$. The trend of the maximum system availability over production time is shown in Figure 5.

Figure 5 shows that the expected maximum system availability first increases rapidly and then levels off over the production time of each batch. MATLAB calculates the optimal system availability as $A^* = 0.854$ and the corresponding production time of each batch as $T^* = 5$, $C_1^* = 6$, and $C_2^* = 7$. Finally, considering $w_1 = 0.5$, we can obtain the minimum value of $U^*$ and corresponding values of $T^*$, $C_1$, and $C_2$, which are $T^* = 1.1$, $C_1^* = 7$, $C_2^* = 8$.

4 | SENSITIVITY ANALYSIS

The influences of the predetermined parameters are discussed by performing a sensitivity analysis. Several model parameters are used in this article. However, we only select the weight parameter $w_1$, the failure level $D$ and the degree of system restoration $a$ after maintenance actions as examples for sensitivity analysis. This is because the weight parameter is a determinant of the dual-attribute criterion optimizations; the failure level $D$ and the degree of system restoration $a$ directly affect the system failure state, which is catastrophic. The influence of the weight parameter $w_1$ is first discussed. The results are shown in Table 2.
FIGURE 5  Trend of the maximum system availability over production time

TABLE 2  Optimal PM levels with different values of weight parameter $w_1$

| Weight parameter $w_1$ | $w_1 = 0$ | $w_1 = 0.5$ | $w_1 = 1$ |
|------------------------|-----------|-------------|-----------|
| Optimal PM level $C_1^*$ | 9         | 7           | 6         |
| Optimal PM level $C_2^*$ | 10        | 8           | 7         |
| Optimal production time $T^*$ | 1.1      | 1.1         | 5         |

FIGURE 6  Results with different values of failure level $D$
TABLE 3  Optimal PM levels with different values of failure level $D$

| Failure levels | $D = 4$ | $D = 6$ | $D = 8$ | $D = 10$ | $D = 12$ | $D = 14$ |
|---------------|---------|---------|---------|---------|---------|---------|
| Optimal first PM level $C_1^*$ | 3       | 5       | 7       | 7       | 6       | 6       |
| Optimal second PM level $C_2^*$ | 4       | 6       | 8       | 8       | 8       | 8       |

TABLE 4  Values of parameters

| Probability of perfect PM | $a = 0$ | $a = 0.3$ | $a = 0.6$ | $a = 1$ |
|---------------------------|---------|-----------|-----------|---------|
| Optimal PM level $C_1^*$  | 6       | 7         | 7         | 7       |
| Optimal PM level $C_2^*$  | 7       | 8         | 8         | 10      |
| Optimal production time $T^*$ | 1       | 1         | 1.1       | 1.1     |

Considering the situation in which cost and system availability of equal importance, that is, $w_1 = 0.5$, the influence of the preset failure level $D$ is discussed. Figure 6 shows the results of different values of failure level $D$. Table 3 shows the results of optimal PM levels with different values of failure level.

Furthermore, the degree of system restoration after maintenance actions is discussed. Table 4 shows the results of the optimal production time for each batch with different values of perfect PM probability $a$.

5  | DISCUSSION

First, the optimal solutions subject to different weight parameters are obtained in Table 2. It shows that the optimal PM levels decrease while the weight parameter $w_1$ increases. This is because the system availability increases while failure frequency reduces since the mean downtime per corrective replacement is much larger than the mean downtime per PM action. More frequent PM actions can reduce system failure. This study showed that different preset weight parameters lead to different optimal solutions. The availability-cost hybrid factor becomes a single objective decision-making problem when $w_1 = 0$ or $w_1 = 1$. The parameters of the availability-cost hybrid factor are decided according to decision-maker preferences. This approach showed that the multicriteria decision objective is more consistent with the actual situation.

Second, Figure 6 shows that the optimal production time of each batch reaches its minimum value $T^* = 1$ when $D = 8$, and the corresponding optimal EPQ is $Q^* = 10$. Table 3 shows that the optimal second PM level is equal to the failure level when $D \geq 8$. This result demonstrates that the single-level and multilevel PM strategies are identical when the failure level of the system is more than or equal to 8. The multilevel PM strategy is effective for systems with low failure levels.

Third, Table 4 shows that the optimal values of $T^*$, $C_1^*$, and $C_2^*$ increase as the value of the perfect PM probability increases. The results suggest that the result is the same as that of the single-level PM strategy when $a = 1$. This is because the PM action is a preventive replacement when $a = 1$, which is the same as the PM action assumption of the single-level PM strategy.

6  | CONCLUSION

This article proposes a joint optimization model of imperfect CBM and lot sizing for a single-unit manufacturing system. The degradation process is described by a random coefficient growth model, where the mean and variance increase with time and the coefficients are assumed to follow known distribution functions. The random coefficient growth model can reflect the monitoring error since the monitoring system condition is expressed by the actual condition and the random error. Considering the random error, this approach has advantages in describing the degradation process compared to previous studies that model the joint problem without detection errors.
Two mutually exclusive renewal scenarios are considered: perfect PM renewal and failure renewal. The system is monitored at the end of each lot size. CBM actions are imperfect, where minimal maintenance and perfect maintenance are performed with a certain probability. If the observed equipment condition reaches the first PM level and is less than the second level, an imperfect PM is carried out. If the equipment state reaches the second PM level, a preventive replacement is performed. If the system breaks down, it exceeds the failure level, and a CM action is immediately performed. Our objective is to optimize the multilevel thresholds and the product quantity. To combine and quantify multiple objectives for decision-makers, an availability-cost hybrid factor is developed to obtain the optimal PM levels and product quantity. Numerical simulations are provided to illustrate the method. Through sensitivity analysis, the result shows that the optimal PM levels decrease while the weight parameter $w_1$ increases. The optimal values of PM levels and product quantity increase as the probability of perfect PM increases. This result demonstrates that different objectives lead to different optimal strategies for the joint model of PM and lot sizing. The availability-cost hybrid factor makes it possible for decision-makers to address multiple objectives simultaneously for manufacturing enterprises. Furthermore, the results show that the multilevel PM strategy is effective for systems with low failure levels. The single-level and multilevel PM strategies are identical when the failure level of the system is more than or equal to 8. It can be concluded that multilevel PM strategies are effective in avoiding failures for these production systems with low reliability or some core components with low fatigue resistance in bad working environments.

Some of the extensions to this study can be investigated in the authors’ further works: (1) A modified joint model of PM and EPQ can change fixed productivity and demand rates into dynamic productivity and demand rates. (2) Additionally, some external factors can be considered time-varying covariates to describe the degradation process. (3) The product quality and rework process can be studied in the joint model.

CONFLICT OF INTEREST
The author declares no potential conflict of interest.

AUTHOR CONTRIBUTIONS
Jianhua Yang: Conceptualization (lead); project administration (lead); supervision (lead); writing – original draft (supporting); writing – review and editing (equal). Xiao Zhao: Data curation (lead); formal analysis (lead); software (lead); writing – original draft (lead). Mengying Han: Resources (equal).

DATA AVAILABILITY STATEMENT
Data available on request from the authors.

NOTATIONS
$t_i$ The time of the $i$th monitoring point, $i = 1, 2, \ldots$
$y(t_i)$ The observed condition of the system at time $t_i$
$T$ Production time of each batch
$p$ Production rate of the single-unit system
$d$ Demand rate
$C_1$ The first condition level of the system, that is, PM level
$C_2$ The second condition level of the system, that is, preventive placement level
$D$ The failure level of the system
$a$ The probability of perfect PM when the observed system condition reaches the first condition level
$b$ The probability of minimal maintenance when the observed system condition reaches the first condition level
$t_p$ The mean downtime per PM replacement
$t_r$ The mean downtime per preventive replacement
$t_f$ The mean downtime per corrective replacement
$C_i$ Monitoring cost per unit time
$C_{C}$ CM cost per unit time
$C_p$ PM cost per unit time
$C_r$ Preventive replacement cost per unit time
$C_o$ Shortage cost per unit time
$C_I$ Inventory cost per unit time
**ABBREVIATIONS**

CBM  Condition-based maintenance  
CM  Corrective maintenance  
EPQ  Economic production quantity  
MRL  Mean residual life  
PM  Preventive maintenance  
TBM  Time-based maintenance

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