Mean-field dynamo in partially ionized plasmas – I

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ABSTRACT

There are several astrophysical situations where one needs to study the dynamics of magnetic flux in partially ionized turbulent plasmas. In a partially ionized plasma, the magnetic induction is subjected to the ambipolar diffusion and the Hall effect in addition to the usual resistive dissipation. In this paper, we initiate the study of the kinematic dynamo in a partially ionized turbulent plasma. The Hall effect arises from the treatment of the electrons and the ions as two separate fluids and the ambipolar diffusion due to the inclusion of neutrals as the third fluid. It is shown that these non-ideal effects modify the so-called α effect and the turbulent diffusion coefficient β in a rather substantial way. The Hall effect may enhance or quench the dynamo action altogether. The ambipolar diffusion brings in an α which depends on the mean magnetic field. The new correlations embodying the coupling of the charged fluids and the neutral fluid appear in a decisive manner. The turbulence is necessarily magnetohydrodynamic with new spatial and time-scales. The nature of the new correlations is demonstrated by taking the Alfvénic turbulence as an example.

Key words: MHD – Sun: magnetic fields – stars: magnetic fields – ISM: magnetic fields.

1 INTRODUCTION

The kinematic dynamo has revealed many essential workings of an astrophysical dynamo for the generation of magnetic field in objects varying from stars to molecular clouds to accretion discs. The kinematic dynamo (Parker 1955; Steenbeck, Krause & Rädler 1966; Moffatt 1970; Stix 1972) is based on the possible generation of an electromotive force parallel to the mean magnetic field in a reflexion asymmetric turbulence, the so-called α effect. Here, α is a measure of the net kinetic helicity. The corresponding turbulent diffusion coefficient β becomes a function of the mean turbulent kinetic energy. The scale separation is an integral part of the kinematic dynamo. A weakly ionized plasma is defined by the condition (Alfvén & Fälthammer 1962) that the electron–neutral collision frequency $v_{\text{en}} \sim 10^{-15} n_e \sqrt{8 K_B T / [\pi m_e]}$ is much larger than the electron–ion collision frequency $v_{\text{ei}} \sim 6 \times 10^{-24} n_e A Z^2 (K_B T)^{3/2}$. This translates into the ionization fraction $n_i/n_e < 5 \times 10^{-11} T^2$ (Alfvén & Fälthammer 1962) where $n_s$ are the particle densities and $T$ is the temperature in Kelvin. Although, the ideal magnetohydrodynamics (MHD) is often used as a starting point of an astrophysical investigation, there are many systems with a rather low degree of ionization dominated by the charged particle–neutral collisions and the neutral particle dynamics. A major part of the solar photosphere (Leake & Arber 2006; Krishan & Varghese 2007), the protoplanetary discs (Krishan & Yoshida 2006) and the molecular clouds (Brandenburg & Zweibel 1994) are some of the examples of weakly ionized astrophysical plasmas. The dynamo action in such a plasma would be affected by the multifluid interactions. The issue of possible disconnection between the subsurface and the surface solar magnetic field, recently emphasized by Schüssler (2005), may have some bearing on the neglect of the neutral fluid-plasma coupling in the flux transport on the solar photosphere. Zweibel (1988) studied the dynamo process in a partially ionized plasma within a single fluid description. Including only the ambipolar diffusion, she determined the velocity, the density and the magnetic field fluctuations self-consistently in the form of MHD waves, and thus went beyond the kinematic dynamo. We develop a three-fluid framework for a kinematic dynamo including the Hall effect and the ambipolar diffusion in Section 2. The α effect of the kinematic dynamo is formulated in Section 3. The new correlations arising due to the coupling amongst different fluids are understood by taking the Alfvénic turbulence as an example, and we end the paper with a section on discussion and conclusion.

2 THREE-COMPONENT MAGNETOFLUID

We begin with the three-component partially ionized plasma consisting of electrons (e), ions (i) of uniform mass density $\rho_i$ and neutral particles (n) of uniform mass density $\rho_n$. The equation of motion of the electrons can be written as

$$m_e n_e \left[ \frac{\partial V_e}{\partial t} + (V_e \cdot \nabla) V_e \right] = -\nabla p_e - e n_e \left( \frac{E + V_z \times B}{c} \right) - m_e n_e \nu_m (V_e - V_n), \quad (1)$$

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where the electron–ion collisions have been neglected since the ionized component is of low density. On neglecting the electron inertial force, the electric field \( \mathbf{E} \) is found to be

\[
\mathbf{E} = -\frac{\mathbf{V}_e \times \mathbf{B}}{c} - \nabla \frac{p_e}{en_e} = \frac{m_e}{e} \nu_{in} (\mathbf{V}_e - \mathbf{V}_n).
\]

(2)

This gives us Ohm’s law. For \( \delta = (\rho_i/\rho_n) \ll 1 \), the ion dynamics can be ignored. The ion force balance then becomes

\[
0 = -\nabla p_i + en_i \left( \frac{\mathbf{E} + \frac{\mathbf{V}_i \times \mathbf{B}}{c}}{\nu_{in}} \right) - \nu_{in} \rho_i (\mathbf{V}_i - \mathbf{V}_n),
\]

(3)

where \( \nu_{in} \) is the ion–neutral collision frequency, and the ion–electron collisions have been neglected for the low-density ionized component. Substituting for \( \mathbf{E} \) from equation (2), we get the relative velocity between the ions and the neutrals:

\[
\mathbf{V}_n - \mathbf{V}_i = \frac{\nabla(p_i + p_n)}{\nu_{in} \rho_i} = \frac{\mathbf{J} \times \mathbf{B}}{c \nu_{in} \rho_i},
\]

(4)

where

\[
\mathbf{J} = en_i(\mathbf{V}_i - \mathbf{V}_e).
\]

(5)

The equation of motion of the neutral fluid is

\[
\rho_n \left[ \frac{\partial \mathbf{V}_n}{\partial t} + (\mathbf{V}_n \cdot \nabla) \mathbf{V}_n \right] = -\nabla p - \nu_{in} \rho_i (\mathbf{V}_n - \mathbf{V}_i) - \nu_{in} \rho_i (\mathbf{V}_n - \mathbf{V}_e),
\]

(6)

where the viscosity of the neutral fluid has been neglected. Substituting for \( \mathbf{V}_n - \mathbf{V}_i \) from equation (4), and using \( \nu_{in} \rho_i = \nu_{in} \rho_n \), we find

\[
\rho_n \left[ \frac{\partial \mathbf{V}_n}{\partial t} + (\mathbf{V}_n \cdot \nabla) \mathbf{V}_n \right] = -\nabla p + \frac{\mathbf{J} \times \mathbf{B}}{c},
\]

(7)

where \( p = p_i + p_n + p_e \). Observe that the neutral fluid is subjected to the Lorentz force as a result of the strong ion–neutral coupling due to their collisions.

Consider Faraday’s law of induction:

\[
\frac{\partial \mathbf{B}}{\partial t} = -c \nabla \times \mathbf{E}.
\]

(8)

By substituting for the electric field from equation (2), we get

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V}_e \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},
\]

(9)

where the pressure gradient terms have been dropped for the incompressible case with constant temperature. Here, \( \eta = m_{e}n_{e}c^{2}/(4\pi \epsilon^{2}n_{e}) \) is the electrical resistivity predominantly due to electron-neutral collisions. Using the construction

\[
\mathbf{V}_e \times \mathbf{B} = [\mathbf{V}_n - (\mathbf{V}_n - \mathbf{V}_i) - (\mathbf{V}_i - \mathbf{V}_e)] \times \mathbf{B},
\]

(10)

and substituting for the relative velocity of the ion and the neutral fluid from equation (4), equation (9) becomes

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[ \mathbf{V}_n - \frac{\mathbf{J}}{en_i} + \frac{\mathbf{J} \times \mathbf{B}}{c \nu_{in} \rho_i} \right] \times \mathbf{B} + \eta \nabla^2 \mathbf{B}.
\]

(11)

One can easily identify the Hall term \( \mathbf{J}/en_i \), and the ambipolar diffusion term \( \mathbf{J} \times \mathbf{B} \) (Chitre & Krishan 2001). The Hall term is much larger than the ambipolar term for large neutral particle densities or for \( \nu_{in} \gg \omega_{ci} \), where \( \omega_{ci} \) is the ion cyclotron frequency. In this system, the magnetic field is not frozen to any of the fluids. Equations (7) and (11) along with the mass conservation

\[
\nabla \cdot \mathbf{V}_i = 0
\]

(12)

form the basis of our investigation.

3 THE ALPHA EFFECT IN THREE-COMPONENT MAGNETOFLUID

The \( \alpha \) effect along with its several variants is the key concept in the generation of large-scale magnetic fields from small-scale velocity and magnetic fields in the kinematic dynamo process (Krause & Rädler 1980). The magnetic induction equation (11) is written as

\[
\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V}_e \times \mathbf{B}) + \eta \nabla^2 \mathbf{B},
\]

(13)

where

\[
\mathbf{V}_e = \mathbf{V}_n + \mathbf{V}_i + \mathbf{V}_{Am},
\]

(14)

with

\[
\mathbf{V}_H = -\frac{\mathbf{J}}{en_i},
\]

(15)

as the Hall velocity and

\[
\mathbf{V}_{Am} = \frac{\mathbf{J} \times \mathbf{B}}{c \nu_{in} \rho_i},
\]

(16)

could be called ambipolar velocity. Following the standard procedure (Krause & Rädler 1980), the velocity \( \mathbf{V}_e \) and the magnetic field \( \mathbf{B} \) are split into their average large-scale parts and the fluctuating small-scale parts as

\[
\mathbf{V}_e = \mathbf{V}_e^0 + \mathbf{\tilde{V}}_e,
\]

(17)

\[
\mathbf{B} = \mathbf{B}^0 + \mathbf{\tilde{B}},
\]

(18)

such that

\[
\mathbf{V}_e^0 = 0, \quad \mathbf{B}^0 = 0.
\]

(19)

In the kinematic dynamo, the magnetic induction equation is solved for large- and small-scale fields. Substituting equations (17) and (18) into the induction equation (11), we find, in the first-order smoothing approximation,

\[
\mathbf{V}_e^0 = \mathbf{V}_e^0 + \frac{\mathbf{J}}{en_i} + \frac{\mathbf{J} \times \mathbf{B}}{c \nu_{in} \rho_i} + \frac{\mathbf{J} \times \mathbf{\tilde{B}}}{c \nu_{in} \rho_i}
\]

(20)

and the mean flow is found to be

\[
\mathbf{V}_e^0 = \mathbf{V}_e^0 - \frac{\mathbf{J}}{en_i} + \frac{\mathbf{J} \times \mathbf{B}}{c \nu_{in} \rho_i}.
\]

(21)

The turbulent electromotive force \( \mathbf{E} \) is defined as \( \mathbf{E} = \mathbf{V}_e^0 \times \mathbf{B}^0 \), and is a function of the mean magnetic induction \( \mathbf{B}^0 \) and mean quantities formed from the fluctuations. The fluctuations in turbulence have, generally, a correlation in spatial scale \( L_{cor} \) and time-scale \( \tau_{cor} \). In a two-scale turbulence, \( L_{cor} \ll L \) and \( \tau_{cor} \ll \tau \), where \( L \) and \( \tau \) represent the scales of the large-scale quantities. Thus, the fluctuations need to be determined in the immediate vicinity of the point at which the large-scale quantity is to be found. This enables us to employ Taylor’s expansion for \( \mathbf{B}^0 \) and express the turbulent electromotive force as (equation 5.4 of Krause & Rädler 1980), retaining only the first-order spatial derivatives and omitting all time derivatives,

\[
\mathbf{E} = \left( \mathbf{V}_e^0 \times \mathbf{B}^0 \right) = a_{ij} \mathbf{B}_j + b_{ijk} \frac{\partial \mathbf{B}_j}{\partial x_k}.
\]

(22)

For a zero mean flow (\( \mathbf{V}_e^0 = 0 \)), homogeneous, isotropic, steady and non-mirror symmetric turbulent velocity field \( \mathbf{V}_e^0 \), the coefficients \( a_{ij} \) and \( b_{ijk} \) become isotropic, and can be expressed as

\[
a_{ij} = \alpha b_{ij},
\]

(23)

\[
b_{ijk} = \beta \epsilon_{ijk}
\]

(24)
and the electromotive force can be expressed as
\[ \mathcal{E} = \alpha \mathbf{B} - \beta \nabla \times \mathbf{B}. \] (25)

The coefficients \(a_0\) or \(\alpha\) and \(b_{ij}\) or \(\beta\) become functions of \(\mathbf{B}\) for large \(\mathbf{B}\) as we find here from the contribution of the ambipolar diffusion. The quantity \(\alpha\), a pseudoscalar, turns out to be the kinetic helicity of the turbulence and is defined as
\[ \alpha = -\frac{\tau_{\text{cor}}}{3} \frac{\nabla \cdot (\mathbf{V} \times \mathbf{V}_n)}{\eta}. \] (26)

Here, retaining terms quadratic in fluctuations:
\[ \alpha_n = \frac{\tau_{\text{cor}}}{3} \frac{\mathbf{V}_n \cdot \nabla \mathbf{V}_n}{\eta} \] (27)
are the measure of the average kinetic helicity of the neutral fluid in the turbulence possessing correlations over time \(\tau_{\text{cor}}\) and
\[ \alpha_{\text{H}} = \frac{2 \tau_{\text{cor}}}{3 \eta} \mathbf{J} \cdot \mathbf{V}_n \] (28)
represents the contribution of the Hall effect. The coupling of the charged components with the neutral fluid is clearly manifest through the possible correlation between the current density fluctuations and the vorticity fluctuations of the neutral fluid \(\mathbf{V}_n = \mathbf{\nabla} \times \mathbf{\Omega}_n\). The ambipolar term gives rise to
\[ \alpha_{\text{Am}} = \alpha_A \cdot \mathbf{B}, \] (29)
with
\[ \alpha_A = \frac{2 \tau_{\text{cor}}}{3 \eta \rho_{\text{in}}} \mathbf{J} \times \mathbf{\Omega}_n \] (30)
as the contribution from the ambipolar diffusion with its essential non-linear character manifest through its dependence on the average magnetic induction. One also observes that the Hall alpha (equation 28) requires a component of the fluctuating current density along the fluctuating vorticity of the neutral fluid whereas the ambipolar effect (equation 29) thrives on the component of the fluctuating current density perpendicular to the fluctuating vorticity. The turbulent dissipation is given by
\[ \beta = \frac{\tau_{\text{cor}}}{3} \frac{\mathbf{V}_n^2}{\eta} = \beta_n + \beta_{\text{H}} + \beta_{\text{Am}} \] (31)
with
\[ \beta_n = \frac{\tau_{\text{cor}}}{3} \frac{\mathbf{V}_n^2}{\eta} \] (32)
as the measure of the average turbulent kinetic energy of the neutral fluid in the turbulence possessing correlations over time \(\tau_{\text{cor}}\) and
\[ \beta_{\text{H}} = \frac{2 \tau_{\text{cor}}}{3 \eta \rho_{\text{in}}} \mathbf{J} \cdot \mathbf{V}_n \] (33)
represent the contribution of the Hall effect. The coupling of the charged components with the neutral fluid is clearly manifest through the possible correlation between the current density fluctuations and the velocity fluctuations of the neutral fluid. The ambipolar term furnishes
\[ \beta_{\text{Am}} = \beta_A \cdot \mathbf{B}, \] (34)
\[ \beta_A = \frac{2 \tau_{\text{cor}}}{3 \eta \rho_{\text{in}}} \mathbf{J} \times \mathbf{\Omega}_n \] (35)
with its essential non-linear character manifest through its dependence on the average magnetic induction. One also observes that the Hall \(\beta_n\) requires a component of the current density fluctuations along the velocity fluctuations of the neutral fluid whereas the ambipolar effect thrives on the component of the current density fluctuations perpendicular to the velocity fluctuations. We have used rigid or perfectly conducting boundary conditions (all surface contributions vanish) while determining the averages. Here, we consider what is known as the \(\alpha^2\) dynamo and take the mean flow \(\mathbf{V}_n = 0\). This actually determines the relative mean flow amongst the three fluids. The dynamo equation reduces to
\[ \frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\alpha \mathbf{B} - \beta \nabla \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}. \] (36)

From now onwards, we omit the bar on the large-scale magnetic field. Assuming one-dimensional space dependence, we assume the magnetic induction \(\mathbf{B} = (0, B, \partial A/\partial x)\) in Cartesian coordinates \((x, y, z)\). In the corresponding spherical configuration, one identifies the Cartesian coordinates \((x, y, z)\) with the polar coordinates \((\theta, \phi, r)\). Thus, \(B\) and \(\partial A/\partial x\) represent the toroidal and the poloidal parts of the field, respectively (Stix 1972). The boundary conditions then turn out to be the vanishing of \(B\) and \(A\) at the endpoints of a finite \(x\)-interval, say \(x = 0\) and \(x = \pi R\) corresponding to the poles of the sphere. It is convenient to put the induction equation in a dimensionless form using a normalizing magnetic field \(B_0\), a spatial scale \(R\), a time-scale \(R^2/\eta_1\) and writing \(A = A' R\). It leads to
\[ \frac{\partial B_y}{\partial t} = -R_a \frac{\partial^2 A'}{\partial x^2} + \frac{\partial^2 A' B}{\partial x^2} - R_{\text{H}A} \frac{\partial}{\partial x} \left[ B + a \frac{\partial A'}{\partial x} \right] \frac{\partial A'}{\partial x} \] (37)
and
\[ \frac{\partial A'}{\partial t} = R_a B + \frac{\partial^2 A'}{\partial x^2} + R_{\text{H}A} \left( B + a \frac{\partial A'}{\partial x} \right) B \] (38)
where
\[ R_a = \frac{\alpha_1 R}{\eta_1}, \] (39)
\[ R_{\text{H}A} = \frac{\alpha_{\text{H}} R B_0}{\eta_1}, \] (40)
\[ R_{\text{H}A} = \frac{\alpha_{\text{H}} B_0}{\eta_1}, \] (41)
\[ a = \frac{\alpha_1}{\alpha_{\text{H}}}, \] (42)
\[ b = \frac{\beta_{\text{H}}}{\beta_{\text{H}}}, \] (43)
\[ \eta_1 = \eta + \beta_n + \beta_{\text{H}}, \] (44)
\[ \alpha_1 = \alpha_n + \alpha_{\text{H}}. \] (45)

Here, one observes that since the Hall effect contributes linearly, it can be combined with the standard \(\alpha_1\) effect. The ambipolar effect is non-linear and appears separately in the induction equation. The Hall effect can completely quench or enhance the standard \(\alpha_1\) contribution to the dynamo for \(\mathbf{V}_n = \pm \mathbf{J}'/(\epsilon \eta_1)\). In the absence of the ambipolar effect, one recovers the standard \(\alpha_1^2\) effect with an exponential growth rate of the magnetic induction.
It is instructive to examine the new correlations for the case of say Alfvénic turbulence. Now in the weakly ionized case, the relation between the velocity and the magnetic-field fluctuations for the Alfvén waves is given by $V_A = \pm 3B_0 / \sqrt{4\pi\rho_0}$ (Krishan & Varghese 2007) with $\delta = \rho_i / \rho_n$. Substituting these results in the expression for $\alpha_H$, we find

$$\alpha_H = \pm 2\lambda_H^2 \alpha_v,$$

where $\lambda_H = c/\omega_{pi}$ is the ion-inertial scale and $\omega_{pi} = \sqrt{4\pi n_e e^2/m_i}$ is the ion-plasma frequency and

$$\lambda_v = \frac{\Omega_c V_A}{\Omega_i}$$

is the ratio of the average kinetic helicity and the average enstrophy of the neutral fluid turbulence. It is interesting to note that the ambipolar $\alpha$ effect vanishes for the Alfvénic turbulence. In the absence of the ambipolar contributions, equations become linear with a solution of the form $\exp[i(kx + \gamma t)]$, where $\gamma = -k^2 \pm |R_\alpha|$ and $k$ is the dimensionless wavenumber. Then, the field grows for $k < R_\alpha$, that is, for large values of the effective $\alpha$ at large spatial scale.

With the inclusion of the ambipolar contributions, equations become non-linear. In Fig. 1, we present a case with the Hall and the ambipolar contributions. The comparable values of the coefficients of the linear and the non-linear $\alpha$ terms with $R_\alpha = 1.6$, $R_{\beta\alpha} = 1.7$, $R_{BA} = 0.1$, $a = 1$ and $b = 0.7$ lead to a near constant toroidal field near $x = 2$, fast decaying solution at $x = 0.4$ and growing solution at $x = 0.75$ beyond $t \sim 0.5$. The poloidal field, however, grows at all values of $x$. The panels c and d demonstrate the expected formation of spatially sharp magnetic structures due to the non-linearity of the ambipolar diffusion (Brandenburg & Zweibel 1994). The toroidal field in addition undergoes a reversal at $x \sim 0.5$. Fig. 2 shows the dominant effect of the ambipolar term with $R_\alpha = 0.2$, $R_{\alpha\alpha} = 3.5$, $R_{BA} = 1.5$, $a = 1$ and $b = 1$. Both the components of the magnetic field, after an initial near-steady state, grow rather fast and again the formation of small spatial scale structures is evident. Thus, the inclusion of the Hall and the ambipolar effects opens up a range of possible profiles of the magnetic field.

In this first attempt, a framework and some instructive examples of the dynamo solutions in a three-component magnetofluid have been given. In subsequent work, we plan to investigate the role of the Hall and the ambipolar terms in some realistic situations such as the solar surface, molecular clouds and the accretion discs. In order to deal with these systems, the differential rotation in the objects must be included. In the three-component system, one would have to specify the rotation profile of all the components since the system can afford to carry a net current density. The inclusion of the ion–neutral collisions introduces an additional time-scale with which the turbulence correlation time needs to be contrasted. The inclusion of the Hall effect brings in the physics at the ion-inertial spatial scale and ion gyration time-scale. The possibilities are many and varied, and should be explored in a system specific manner.

**4 DISCUSSION AND CONCLUSION**

We solve the field equations (37) and (38) demonstrating the linear and the non-linear $\alpha$ effect for the initial conditions given by Stix (1972): $A'(x, 0) = 0$, $B(x, 0) = \sin x$. In the absence of the ambipolar contribution ($R_{\alpha\alpha} = 0$, $R_{BA} = 0$), the equations (37) and (38) become linear with a solution of the form $\exp[i(kx + \gamma t)]$, where $\gamma = -k^2 \pm |R_\alpha|$ and $k$ is the dimensionless wavenumber. Then, the field grows for $k < R_\alpha$, that is, for large values of the effective $\alpha$ at large spatial scale.

With the inclusion of the ambipolar contributions, equations become non-linear. In Fig. 1, we present a case with the Hall and the ambipolar contributions. The comparable values of the coefficients of the linear and the non-linear $\alpha$ terms with $R_\alpha = 1.6$, $R_{\alpha\alpha} = 1.7$, $R_{BA} = 0.1$, $a = 1$ and $b = 0.7$ lead to a near constant toroidal field near $x = 2$, fast decaying solution at $x = 0.4$ and growing solution at $x = 0.75$ beyond $t \sim 0.5$. The poloidal field, however, grows at all values of $x$. The panels c and d demonstrate the expected formation of spatially sharp magnetic structures due to the non-linearity of the ambipolar diffusion (Brandenburg & Zweibel 1994). The toroidal field in addition undergoes a reversal at $x \sim 0.5$. Fig. 2 shows the dominant effect of the ambipolar term with $R_\alpha = 0.2$, $R_{\alpha\alpha} = 3.5$, $R_{BA} = 1.5$, $a = 1$ and $b = 1$. Both the components of the magnetic field, after an initial near-steady state, grow rather fast and again the formation of small spatial scale structures is evident. Thus, the inclusion of the Hall and the ambipolar effects opens up a range of possible profiles of the magnetic field.

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![Figure 1](https://example.com/f1.png)

**Figure 1.** Fields $B$ and $A'$ as functions of $t$ and $x$. Panels (a) and (b) show their time variations at fixed $x$ values, while the panels (c) and (d) show the spatial variations at some fixed $t$ values. Chosen $R_\alpha = 1.6$, $R_{\alpha\alpha} = 1.7$, $R_{BA} = 0.1$, $a = 1$ and $b = 0.7$. 
Figure 2. Fields $B$ and $A'$ as functions of $t$ and $x$. Panels (a) and (b) show their time variations at fixed $x$ values, while the panels (c) and (d) show the spatial variations at some fixed $t$ values. Chosen $R_\alpha = 0.2$, $R_{\alpha A} = 3.5$, $R_{\beta A} = 1.5$, $a = 1$ and $b = 1$.

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