The impact of rigorous mathematical thinking as learning method toward geometry understanding

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Abstract. To reach higher order thinking skill, needed to be mastered the conceptual understanding. RMT is a unique realization of the cognitive conceptual construction approach based on Mediated Learning Experience (MLE) theory by Feurstein and Vygotsky’s sociocultural theory. This was quasi experimental research which was comparing the experimental class that was given Rigorous Mathematical Thinking (RMT) as learning method and control class that was given Direct Learning (DL) as the conventional learning activity. This study examined whether there was different effect of two learning method toward conceptual understanding of Junior High School students. The data was analyzed by using Independent t-test and obtained a significant difference of mean value between experimental and control class on geometry conceptual understanding. Further, by semi-structure interview known that students taught by RMT had deeper conceptual understanding than students who were taught by conventional way. By these result known that Rigorous Mathematical Thinking (RMT) as learning method have positive impact toward Geometry conceptual understanding.

1. Introduction

Geometry arose independently in early cultures as a practical way for dealing with lengths, areas, and volumes. Geometry as a “mathematical science” with its logical structure—providing or affirmations—and the abstraction of the given object from its initial contents [1]. Geometry is an important part of mathematics curriculums [2]. On learning of geometry needed the ability of visualization, verbalization, picturing, logical and application. This fact was notion which caused most of students get difficulty on learning geometry.

Conceptual understanding is the comprehension of mathematical concept, operations, and relations [3]. Conceptual knowledge most clearly as knowledge that is rich of relationships [4]. Conceptual knowledge as a connected web of knowledge and this connection are particular as it consists of discrete information. This relationship concludes individual facts, definitions, theorems or propositions so that these are linked each other. By this fact, a unit of conceptual knowledge cannot be an isolated piece of information. Conceptual understanding concerned with mathematical concept, operation, and relation [5]. Based on these statements, geometry conceptual understanding was defined as an ability which is concerned with mathematical (i.e. geometry) conception, representation, and relation. The indicators of conceptual understanding in this research were (1) performing mathematical operations related to appropriate concepts; (2) classifying the objects based on whether the requirements to formulate the concept was fulfilled; (3) providing examples or non-examples of
the concept learned; (4) representing mathematical concepts into different ways; (5) linking interrelated concepts; (6) developing the necessary terms or adequately terms of a concept.

Based on researcher observation, most of Junior High School students in Ngawi stated that square was different with rectangular. They also did not remember the formula of planes area even they had gotten this material on elementary school. They also had difficulty on distinguishing the concept of area and perimeter. They sometimes used the concept of area to solve perimeter problems. It was indicated that junior high school students in Ngawi had low ability of understanding geometry concept especially on area concept. By this condition, this research focused on upgrading geometry conceptual understanding by implementing Rigorous Mathematics Thinking as learning method.

Rigorous Mathematical Thinking (RMT) as the learning method was invented by James Kinard [6]. RMT is learning theory which was developed by Kinard based on sociocultural theory by Vygotsky [7-9] about psychology tools [10] and Zone of Proximal Development (ZPD) [7] and mediated learning theory by Feurstein [11,12]. A method and apparatus are provided for teaching rigorous mathematical thinking to students. The method includes the steps of mediating the learner to appropriate a set of cognitive tasks as general psychological tools based upon their structure-function relationship, mediating the learner to perform the set of cognitive tasks through the use of the psychological tools to construct high-order cognitive process, mediating the learner to systemically build basic essential concepts needed in mathematics from everyday experiences and language, mediating the learner to discover and formulate the mathematical pattern and relationship in the cognitive process, mediating the learner to appropriate mathematically specific psychological tools to organize and orchestrate the use of cognitive functions and to construct mathematical conceptual understanding [6]. The practice of RMT focuses on mediating the learner in constructing robust cognitive process while concomitantly building mathematical concept using the three phrases (i.e. cognitive development, content as process development, cognitive conceptual construction practice).

The goal of RMT as learning method is equip the learner with the capacity and motivation to construct and apply deep mathematical conceptual understanding. In other hand, considering that mathematics achievement of Indonesian students’ was in the lower grade than the other countries, it was necessary to conduct the research in purpose to upgrade students’ mathematics skill particularly in conceptual understanding and strategic competence by implementing Rigorous Mathematical Thinking (RMT) as learning method.

1.1. Research question
Is there the difference of mean value when consider on the mathematics conceptual understanding? How was the difference of student’s conceptual understanding between experimental class and control class?

2. Methods
This research used experimental method, which was comparing the effect of experimental class which used RMT learning method and control class which used DL as conventional learning method. The population was students of grade 7 in one of Junior High School in Ngawi, East Java, Indonesia. The population consisted of 154 students who were separated into 6 classes. The sample of this research was 27 students in experimental class and 27 students in control class. The sample was taken by using probability sampling and the technique used was cluster random sampling.

After obtained the experimental class and control class, conducted pre-test in each class to know whether these chosen class had the same ability of geometry conceptual understanding. To execute this stage (i.e. preface test), the data of pre-test was analyzed by using independent t-test. Then, learning activity conduct for 8 times meeting, and in the end of the meeting, students were given geometry scholastic task which was represented the indicator of conceptual understanding. The data obtained was analyzed by using independent t-test. It was used to know whether there was difference mean value when consider on geometric conceptual understanding between experimental class and control class.
To gain more information of students’ conceptual understanding on geometry used semi-structured interview. This stage aim to analyze and describe the students’ conceptual understanding based on the indicators that had been discussed before. In this stage, the whole geometry conceptual understanding score of each class were classified into 3 stages, based on interval level as shown in Table 1.

### Table 1. Grouping guidelines of score.

| No | Score Criteria                  | Category |
|----|---------------------------------|----------|
| 1. | $x_i \leq \bar{x} - s$         | Low      |
| 2. | $\bar{x} - s \leq x_i \leq \bar{x} + s$ | Medium   |
| 3. | $x_i \geq \bar{x} + s$         | High     |

Then, in each level score taken purposively 2 students to be informant who were given semi-structure interview then. By this stage, it was expected that the researcher could indicate the obvious differences of students’ geometry conceptual understanding based on learning method compared (i.e. RMT and DL).

### 3. Results and Discussion

This research aims to know the implementation result of RMT as learning method to conceptual understanding in geometry material. By the preface test obtained the result (see Table 2). Based on the table known that the mean value of experimental class was 45.83 and the mean value of control class was 46.06. Standard deviation indicated the gap between high score and low score. By Kolmogorov-Smirnov known that the data distribution was normal. From the Lavinne test known that the variance of two groups was equal (homogenous). From the significant 2-tails known that there was no difference mean value of two groups compared.

### Table 2. Mean value of conceptual understanding in preface test

| Class     | N  | Mean   | Std. Dev. | Prob. Value |
|-----------|----|--------|-----------|-------------|
| Experimental | 27 | 45.8333 | 14.502984 | 0.955       |
| Control   | 27 | 46.0648| 15.711123 | 0.955       |

On learning process, students in experimental class were treated by using RMT as learning method. The basic step was teacher had to mediate the students to construct their conceptual understanding of rectangular area as this concept would be used to construct the area concepts of others planes. In this step, teacher had to mediate students to analyze the relation between rectangular area and the red square (see Figure 1).

![Figure 1. Basic concept of rectangle area.](image)
Teacher mediated students to connect the amount of red square needed and the area of rectangular by asking “How many red square needed to cover the rectangular?” and “Is there any relation between rectangular area and the amount of red square used to cover the rectangular?” To develop the formula of rectangular area, teacher mediated students to connect the area of rectangular and the multiple of its sides, by asking “Is there any relation between the area of rectangular and the multiple of length and width side?” thus, what is the relationship?” By this step, students could build the basic essential concepts of rectangular area from everyday experience and language.

In the next phase, to construct the concept of others planes’ area, teacher mediated students to perform the learning activities sheet to gain the formula area of other planes by using rectangular area concept. In this article showed one example (see Figure 2). In this phase, teacher mediated students to construct the formula area of kite approached by rectangular area concept as psychological tools. Teacher mediated students by asking the relation between kite area (see Figure 2(a)) and the rectangular area (see Figure 2(b)). Teacher mediated students by asking “Is there any relation between kite area and rectangular area? What is the relation?” At first students hardly could state their opinion although they knew that the area of those planes was same. In this condition, teacher mediated and stimulated students to express their thoughts by making various devices e.g. by using color paper in form of kite (as in the Figure 2(a)) and cut it into various parts then reconstruct them in to rectangle (as in the Figure 2(b)). Thus, students understand that the kite’s area (see Figure 2(a)) was equal to the multiplication value of AB and OB side or the rectangle’s area (see Figure 2(b)). By this mediation, students could construct the planes area concepts (i.e. kite’s area formula) by using concept of rectangular area.

![Diagram](image_url)

**Figure 2.** The illustration of kite area formula which can be approached by rectangular area.

After finished the learning process (8 times meeting), conducted post-test in each class and obtained the result (see Table 3). By Kolmogorov-Smirnov test known that the data distribution was normal. By Lavinne test known that the variance of two groups were equal (homogenous). The significant 2-tails showed that there was significant difference mean value of two groups compared. This result showed that RMT as learning method gave more positive effect than DL (i.e. conventional learning method) toward geometry conceptual understanding.
The further stage examined how geometry conceptual understanding of each interval score in two classes compared. Based on the descriptive analysis (see Table 4) obtained the interval score in each class (see Table 5). Then, it was taken 2 students of each category to gain the natural phenomenon of geometry conceptual understanding. Those students were asked to re-perform the task of geometry conceptual understanding and given semi-structured interview afterward. The result of this stage consisted of students' natural phenomenon (i.e. conceptual understanding) in each score level of experimental class and control class.

**Table 3.** Mean value of conceptual understanding in post test.

| Class      | N   | Mean      | Prob. value |
|------------|-----|-----------|-------------|
| Experimental | 27  | 74.3056   | 0.001       |
| Control    | 27  | 58.7963   | 0.001       |

**Table 4.** Descriptive statistic of each group.

| Details       | Experimental Class | Control Class |
|---------------|--------------------|---------------|
| Mean          | 74.3056            | 58.7963       |
| Median        | 68.75              | 56.25         |
| Mode          | 68.75              | 43.75         |
| Std. Deviation | 14.64046          | 18.11914      |
| Variance      | 214.343            | 328.303       |
| Range         | 50.00              | 62.50         |
| Minimum       | 50.00              | 31.25         |
| Maximum       | 100.00             | 93.75         |

**Table 5.** The category of interval score of each class.

| Category | Score Criteria | Experimental Class | Control Class |
|----------|---------------|--------------------|---------------|
| Low      | $x_i < 59.66$ | $x_i < 40.68$      |               |
| Medium   | $59.66 < x_i < 88.95$ | $40.68 < x_i < 76.96$ |   |
| High     | $x_i > 88.95$ | $x_i > 76.96$      |               |

In experimental class, all chosen students could perform the operation related to appropriate concepts. They could classify the objects based on the properties required and provide the examples or non-example of the concept of plane’s area and perimeter. Students in high level score could master all indicators of geometry conceptual understanding. Students in medium score level likely performed the task incorrectly as they made mistake on computation and also they got difficult to develop the necessary terms or adequately terms of a concept (e.g. found the area of trapezoid with unknown height) as they did not remember the concept of Pythagoras theorem. Students in low score level were difficult to represent the concepts into different ways (e.g. when one angle of right-triangle turned into more than 900, the students did not think that the area remain the same). They also felt difficult to link interrelated concepts (e.g. the area of triangle is $\frac{1}{2}$ area parallelogram) and they hard to develop the necessary terms or adequately terms of a concept.

In control class, the students of low level score used the formula of rectangular area when performed the question of perimeter area. It showed that they could not perform the operation related
to appropriate concepts. It was the basic concept of area and perimeter, but it seemed that they had not mastered it yet. In other hand, they were hard to provide the examples or non-example of plane area and perimeter concept indeed. They also could not represent the concepts of triangle area into different ways. They were hard to link interrelated concepts and also develop the necessary terms or adequately terms of a concept. Nevertheless, they could classify the objects based on the properties required. The students in medium level were likely hard to represent the concepts into different ways (e.g. when one angle of right-triangle turned into more than 90°, the students did not think that the area remain the same), but some of them could link interrelated concepts and develop the necessary terms or adequately terms of a concept. The students in high level almost mastered all indicator of conceptual understanding, but they seemed difficult to represent the area concepts into different ways. They were likely just remember not understand about the formula of plane area and perimeter. It caused they felt difficult to link interrelated concepts such as the area of rectangular and other planes.

By this stage, known that students who taught by RMT had deeper conceptual understanding than students who taught by conventional ways. It was in line with the previous research [13]. The students who taught by conventional learning was notion just memorizing the concept of planes area and perimeter. It was differ from the students who taught using RMT, they had an insight into the learning process. It was in line with the goal of RMT as learning method that invented by Kinard [6].

4. Conclusion

Based on the result described in the previous section, RMT as learning method gave better effect than the conventional method. By semi-structured interview students who taught by RMT had deeper geometry conceptual understanding than students who taught by conventional ways. This might be caused that in RMT teacher gave more attention by mediating the students to perform the cognitive task as general psychology tools.

By this finding RMT might tune with the culture and condition of Indonesian Junior High School students’. However, this research limited on implementing RMT as learning method toward conceptual understanding. Therefore, we recommended other researchers to explore the impact of RMT to problem solving and others mathematical cognitive skills.

5. References

[1] Alexandrov A 1974 Mathematik und Dialektik Matematik über die Matematik, ed M Otte et al (Berlin: Springer) 47
[2] Regina M M 2000 Adolescence 35 138 (San Diego: Libra Publisher).
[3] Kilpatrick J Swafford J and Findell B (ed) 2001 Adding it up: Helping Children Learn Mathematics (Washington DC: National Academy Press)
[4] Hiebert J, editor 2013 Conceptual and procedural knowledge: The case of mathematics. Routledge;
[5] Samuelsson J The impact of teaching approaches on students’ mathematical proficiency in Sweden International Electronic Journal of Mathematics Education 5 2 61-78.
[6] Kinard J 2007 Method and apparatus for creating rigorous mathematical thinking. United States patent application US 11/584,367.
[7] Albert L R 2012 Rhetorical ways of thinking: Vygotskian theory and mathematical learning Springer Science & Business Media
[8] Konzulin A (ed) Vygotsky's 2003 Educational Theory in Cultural Context, ed Alex K et all (United Kingdom: Cambridge University Press) 15-20
[9] Cobb P 2006 New Mathematics Education Research and Practice, ed J Maasz and WSchloeglmann (Rotterdam: Sense Publishers).
[10] Kozulin A Psychological tools: A sociocultural approach to education Harvard University Press 1998
[11] Hakim W A R and Budiarto M T 2015 MATHedunesa (electronic version) 1(4) (Surabaya: University of Surabaya).
[12] Feuerstein R 2000 Redefining The Standarts of Care for Infants Children and Families with Special Needs (Washington DC: ICDL Press) 557-578

[13] Hendrayana A. *Pengaruh pembelajaran pendekatan rigorous mathematical thinking (RMT) terhadap pemahaman konseptual, kompetensi strategis, dan beban kognitif matematis siswa smp boarding school: Sekolah Berasrama* (Badung: Universitas Pendidikan Indonesia).

Acknowledgments
This research could not be separated from the support of various parts. Thereby, author would like to thank to family, friends, lecturer and communities of Mathematics Education Department, Postgraduate Program, Universitas Sebelas Maret also others parties who support this research.