Phenomenological description of the microwave surface impedance and complex conductivity of high-\(T_c\) single crystals

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Measurements of the microwave surface impedance \(Z_s(T) = R_s(T) + iX_s(T)\) and of the complex conductivity \(\sigma_s(T)\) of high-quality, high-\(T_c\) single crystals of YBCO, BSCCO, TBCCO, and TBCO are analyzed. Experimental data of \(Z_s(T)\) and \(\sigma_s(T)\) are compared with calculations based on a modified two-fluid model which includes temperature-dependent quasiparticle scattering and a unique temperature variation of the density of superconducting carriers. We elucidate agreement as well as disagreement of our analysis with the salient features of the experimental data. Existing microscopic models are reviewed which are based on unconventional symmetry types of the order parameter and on novel mechanisms of quasiparticle relaxation.

I. INTRODUCTION

High-precision microwave measurements of the temperature dependence of the surface impedance \(Z_s(T) = R_s(T) + iX_s(T)\) of high-\(T_c\) superconductors (HTS’s) advance considerably our understanding of pairing of superconducting electrons in these materials. In particular, in 1993, the observed linear \(T\)-dependence of the penetration depth, \(\lambda(T) - \lambda(0) \propto \Delta X_s(T) \propto T\) below 25 K in the \(ab\)-plane of high quality \(\text{YBa}_2\text{Cu}_3\text{O}_6.95\) (YBCO) single crystals gave rise to productive investigations of the order parameter of HTS’s. Such linear variation of \(\lambda(T)\) at low \(T\) has by now been observed not only in orthorhombic YBCO single crystals [1,2] and films [3,4], but also in tetragonal \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_x\) (BSCCO) [5,6], \(\text{Ti}_2\text{Ba}_2\text{Cu}_2\text{O}_{y+\delta}\) (TBCCO) [7,8] and \(\text{Ti}_2\text{Ba}_2\text{CaCu}_2\text{O}_{8-y}\) (TBCO) [9,10] single crystals. This temperature dependence is not in accord with a nearly isotropic superconducting gap and it is now considered to provide strong evidence for \(d\)-wave pairing in these materials [11,12], in spite of the fact that the experimental data are not sensitive to the phase of the superconducting order parameter. Later research has shown that \(\Delta \lambda_{ab}(T)\) could be linear at low \(T\) for models invoking the proximity effect between normal and superconducting layers [13] or assuming anisotropic \(s\)-wave pairing [14]. However, none of these theories can explain substantially different slopes of \(\Delta \lambda_{ab}(T)\) at low \(T\) of YBCO samples grown by different methods [15] nor features, such as a bump [16] or a plateau [14], observed in the intermediate temperature range \(0.3T_c < T < 0.8T_c\). Models containing a mixed \((d+s)\) symmetry of the order parameter [11] hold some promise for a successful description of these experimental features, but this would require additional theoretical investigations.

Another important feature of the microwave response of HTS crystals is the linear variation with temperature of the surface resistance \(R_s(T)\) in the \(ab\)-plane at low temperatures. At frequencies of about 10 GHz and below the \(T\)-dependence of \(R_s(T)\) in BSCCO, TBCO, and TBCCO single crystals is linear over the range \(0 < T \lesssim T_c/2\) [14,17]. For YBCO crystals \(\Delta R_s(T) \propto T\) for \(T \lesssim T_c/3\) and \(R_s(T_0)\) displays a broad peak and valley at higher temperatures [14,10]. This peak can be understood in terms of a competition between an increase in the quasiparticle lifetime and a decrease in the quasiparticle density as the temperature is lowered. The fairly slow decrease in the quasiparticle density is indicative of a highly anisotropic or unconventional order parameter, resulting in a very small or vanishing energy gap, while the increase in the quasiparticle lifetime is attributed to the presence of inelastic scattering, which can be (i) due to the exchange of antiferromagnetic spin fluctuations [18] which would naturally lead to \(d\)-wave pairing, or (ii) due to strong electron-phonon interactions [19] within the anisotropic \(s\)-wave pairing model [20]. Moreover, there have been suggestions of unconventional states for describing the charge carriers in the \(\text{CuO}\) planes like the marginal Fermi liquid [21], the Luttinger liquid [22], and the Luttinger liquid [23]. However, to fit the data of YBCO, the inelastic scattering rate has to decrease with temperature much faster than any of these microscopic models would predict. Further, the \(d\)-wave model, with point scatterers, does predict a finite low temperature and low frequency limit, which is independent of the concentration and the strength of the scattering centers [24]. Therefore, the latter model does not explain the very different values of the observed residual surface resistance \(R_{\text{res}} \equiv R_s(T \to 0)\) on different samples [25]. Furthermore, the value of this universal surface resistance is much lower than the \(R_{\text{res}}\)-values obtained from experiments. There is no microscopic theory which explains the linear temperature dependence of \(\Delta R_s(T)\) up to \(T_c/2\) in the crystals with non-orthorhombic structure and the shoulder of \(R_s(T)\) observed on YBCO [26,27] for \(T > 40\) K.

In the absence of a generally accepted microscopic the-
ory a modified two-fluid model for calculating $Z_s(T)$ in HTS single crystals has been proposed independently in Refs. 23, 24 and then further developed in Refs. 25, 26. Our phenomenological model has two essential features different from the well-known Gorter-Casimir model.

The first is the introduction of the temperature dependence of the quasiparticle relaxation time $\tau(t) (t = T/T_c)$ described by the Grüneisen formula (electron–phonon interaction), and the second feature is the unique density of superconducting electrons $n_s(t)$ which gives rise to a linear temperature dependence of the penetration depth in the $ab$-plane at low temperatures

$$\lambda^2(0)/\lambda^2(t) = n_s(t)/n \simeq n(1 - \alpha t),$$

where $n = n_s + n_n$ is the total carrier density, and $\alpha$ is a numerical parameter in our model.

The goal of this paper is to demonstrate the power of our model to describe the general and distinctive features of superconducting electrons described by the Grünneisen formula (electron–phonon interaction), and the second feature is the unique density of superconducting electrons $n_s(t)$ which gives rise to a linear temperature dependence of the penetration depth in the $ab$-plane at low temperatures

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TABLE I. Surface impedance $Z_s(T) = R_s(T) + iX_s(T)$ in the $ab$-plane of high-$T_c$ single crystals at frequencies $\sim 10$ GHz

| HTS         | Superconducting state, $T < T_c$ | Intermediate temperatures $T \sim T_c/2$ | Normal state $T \to T_c$ |
|-------------|----------------------------------|---------------------------------------|-------------------------|
|             | Low temperatures $4$ $K < T < T_c$ | Broad peak in $R_s(T)$ at $25 < T < 45$ $K$ [4-14] | Different $R(T) = X(T)$ |
| Orthorhombic structure YBCO | $\Delta R_s(T) \propto T$, $\Delta X_s(T) \propto T$ at $T \gtrless T_c/4$; Essentially different slope of $\Delta \lambda(T) \propto T$ [1-14] | Peculiarities: 1. Shoulder [9,11] in $R_s(T)$ at $T \gtrsim 40$ $K$; 2. Bump [9] or plateau [8,10] on the curves of $X_s(T)$ at $50 < T < 80$ $K$ | Normal skin-effect |
| Tetragonal structure | | | |
| BSCCO $T_c \approx 90$ $K$ [19-22] | | | Rapid growth of $R_s(T)$ and $X_s(T)$ at $\omega\mu\rho(T)/2$ | |
| TBCO $T_c \approx 80$ $K$ [23,24] | | | |
| TBCCO $T_c \approx 110$ $K$ [10,12] | | | |

The reasons for such a discrepancy are still unclear. At frequencies of about $10$ GHz and below, the linear dependence $\Delta R_s(T) \propto T$ in BSCCO (Fig. 1), TBCCO, and TBCO single crystals may actually extend to temperatures of $\sim T_c/2$. This property, common for all HTS crystals with the tetragonal structure, is not characteristic of YBCO. As was noted previously, all microwave measurements on high-quality YBCO single crystals show a broad peak in the $R_s(T)$ curve centered near $30-40$ $K$ up to frequencies of $\sim 10$ GHz. The peak shifts to higher temperatures and diminishes in size as the frequency is increased. In YBCO crystals of higher quality the amplitude of the peak increases and $R_s(T)$ reaches its maximum at a lower temperature.

The underlying origin of this YBCO feature has remained unclear. The simplest idea is that the absence of this peak in crystals with tetragonal structure might be caused by their “poor” quality, as is the case in YBCO doped with Zn. However, this deduction is probably incorrect because, (i), there is a sufficiently large set of experimental data indicating that $R_s(T)$ is a linear function of $T$ for BSCCO, TBCO, and TBCCO, and (ii), the peak in $R_s(T)$ was also detected in such YBCO crystals with parameters $R_{res}$ and $\rho(T_c)$ which would characterize the quality of these crystal as “poor” compared to those of, for example, TBCCO or BSCCO. Results of the latter crystals are shown in Fig. 2. The more probable cause of the peak, however, is the presence of an additional component in the YBCO orthorhombic structure, namely CuO chains, which lead to a mixed ($d + s$) symmetry of the order parameter in YBCO. The electrons of the chains form an additional band, contributing to the observed $T$-dependence of $Z_s(T)$. This contribution seems to result in another

Temperature sections of these curves to $T = 0$ $K$ yields estimates of $R_{res} = 0.5$ $m\Omega$ and $\lambda_{ab}(0) = 2600$ $\AA$ for this crystal.

FIG. 1. Surface resistance $R_s(T)$ and reactance $X_s(T)$ in $ab$-plane of a BSCCO single crystal at 9.4 GHz. The insets show linear plots of $\lambda(T)$ and $R_s(T)$ at low temperatures.

The experimental $\Delta \lambda_{ab}(T)$ of YBCO, TBCO, and TBCCO crystals are also linear in the range $T < T_c/3$. It is important to notice the different slopes of the $\Delta \lambda(T) \propto T$ curves for $T \ll T_c$. In particular, in YBCO crystals, fabricated by different techniques, the slopes of $\Delta \lambda_{ab}(T)$ differs by almost one order of magnitude.
reproducible in the experiments. However, recent measurements of ∆σ(T) of BSCCO and YBCO single crystals grown in a high purity BaZrO$_3$ crucible, show no such features in the intermediate temperature range. The authors of Ref. 9 argue that the disagreement with the results of Ref. 11 arises from some problem connected with the surface of the crystal. The latter observation still lack a convincing explanation.

Finally, another feature in the T–dependence of the impedance of high-quality YBCO crystals was detected: a noticeable increase of $R_s(T)$ at temperatures larger than the peak temperature at 30 K. It turns out that this shoulder was reproducible in the experiments. Similarly, an explanation of this observation is lacking.

### B. Complex conductivity

Equations (2)–(4) allows us to express the real and imaginary parts of the complex conductivity $\sigma_s = \sigma_1 - i\sigma_2$ in terms of $R_s$ and $X_s$:

$$\sigma_1 = \frac{2\omega\mu_0 R_s X_s}{(R_s^2 + X_s^2)^2}, \quad \sigma_2 = \frac{\omega\mu_0 (X_s^2 - R_s^2)}{(R_s^2 + X_s^2)^2}. \quad (6)$$

Above the superconducting transition temperature, the mean free path $\ell$ of current carriers is shorter than the skin depth $\delta_n$ in the normal state (for $T \geq T_c$, $\ell \ll \delta_n$), which corresponds to the conditions of the normal skin effect. Equations (2)–(4), (6) also apply to the normal state of HTS’s, where $R_n(T) = X_n(T) = \sqrt{\omega\mu_0/2\sigma_n(T)}$ with $\sigma_n \equiv \sigma_1(T \geq T_c)$ and $\sigma_2 \ll \sigma_1$ at microwave frequencies.

The components $\sigma_1(T)$ and $\sigma_2(T)$ are not measured directly but derived from measurements of $R_s(T)$ and $X_s(T)$ using Eq. (6).

#### Low temperatures region ($T \ll T_c$)

When $R_s(T) \ll X_s(T)$, then Eq. (6) reduces to:

$$\sigma_1(T) = \frac{2\omega\mu_0 R_s(T)}{X_s^2(T)}, \quad \sigma_2(T) = \frac{\omega\mu_0}{X_s^2(T)}. \quad (7)$$

It then follows from Eq. (6) that the dominant changes of $\sigma_2(T)$ are determined mainly by the function $X_s(T)$, reflecting the $T$–dependence of the magnetic field penetration depth. The $T$–dependence of the real part of the conductivity, $\sigma_1(T)$, is determined by the competition between the increments of $\Delta R_s(T)$ and $\Delta X_s(T)$ relative to each other:

$$\Delta \sigma_1 \propto \left( \frac{\Delta R_s}{R_s} - \frac{3}{2} \frac{\Delta X_s}{X_s} \right), \quad \Delta \sigma_2 \propto -\frac{\Delta X_s}{X_s}. \quad (8)$$

It follows from Eq. (8) that the dominant changes of $\sigma_2(T)$ are determined mainly by the function $X_s(T) = \omega\mu_0\delta(T)\sqrt{\sigma_1(T)}$, reflecting the $T$–dependence of the magnetic field penetration depth. When $R_s(T) \ll X_s(T)$, the temperature dependence of $\Delta \sigma_1(T)$ predicted by the BCS theory: $\sigma_1 = 0$ at $T = 0$, and for $T \leq T_c/2$, $\sigma_1(T)$ shows an exponentially slow growth with increasing temperature. Note that the smallest value of $R_{res}$ detected in pure Nb is, at least, two order of magnitude smaller than the smallest value of $R_{res}$ measured in YBCO. The extremely small values of the surface resistance in Eq. (8) indicate that the increment of $\Delta \sigma_1(T)$, in classical superconductors is always positive ($\Delta \sigma_1(T) > 0$, at least in the temperature interval $T < 0.8 T_c$), before the maximum of BCS coherence peak is reached.

For HTS single crystals the $T$–dependence of $\sigma_1(T)$ is radically different from that predicted by theories of the microwave response of conventional superconductors. In the $T$–range $T < T_c$ the increments of $\Delta R_s(T)$ and $\Delta X_s(T)$ in HTS’s are not small, and, in addition, $\Delta X_s(T) \gg \Delta R_s(T)$. Although $R_s(T) < X_s(T)$, $\Delta R_s/R_s$ is not necessarily greater than $3\Delta X_s/X_s$ in Eq. (8) or positive at all temperatures. When that occurs, $\sigma_1(T)$ increases with decreasing temperature. The function $\sigma_1(T)$ is maximum at some $T = T_{\text{max}}$, and then $\sigma_1(T)$ becomes smaller with decreasing temperature. $\sigma_1(T)$ has a peak if the value of $R_{res}$ is sufficiently small when for $T \to 0$:  

![FIG. 2. Comparison of the temperature dependencies of surface resistance $R_s(T)$ of BSCCO and YBCO single crystals at 14.4 GHz. Experimental data are taken from Refs. 10, 11 (BSCCO at 14.4 GHz) and 12 (YBCO at 9.4 GHz, scaled by $\omega^2$ to 14.4 GHz). The inset shows the linear $-\sigma$-dependencies of $R_s(T)$ and $X_s(T)$ of BSCCO and YBCO single crystals.](image)
If inequality \( R_{res} < \frac{X_s(0) ΔR_s(T)}{3} \frac{ΔX_s(T)}{} \) is satisfied, \( T_{max} \) occurs at a finite temperature, while for \( R_{res} \) being equal to the right hand side of \( (9) \), \( T_{max} \) shifts to 0 K. If \( R_{res} \) is such that \( (9) \) is not satisfied, \( σ(T) \) decreases at low temperatures as the temperature is increased, which is quite different from what is observed with conventional superconductors.

Thus, the shape of the \( σ(T) \) for \( T \ll T_c \) depends on the value of the residual surface resistance \( R_{res} \), whose origin and accurate value are unknown. For this reason, the shapes of \( σ(T) \) curves are not determined unambiguously for \( T \leq T_c/2 \), unlike the functions \( R_s(T) \) and \( X_s(T) \), which are directly measured in experiments.

If we linearly extrapolate \( R_s(T) \) to \( T = 0 \) and attribute the resulting \( R_s(0) \) to the residual surface resistance, \( R_s(0) = R_{res} \), and then substitute the temperature dependent difference \( R_s(T) - R_{res} \) into the numerator of the first expression of Eq. \( (6) \), the result is that the \( σ(T) \) curve has a broad peak for HTS materials. Near \( T = 0, σ(T) \) increases linearly with \( T \) from zero, reaches a maximum at \( T_{max} \), and then decreases to \( σ(T_c) \). This procedure, however, ignores the possibility of intrinsic residual losses. Therefore, some authors (see, e.g., Refs. 14, 15, 16, 17) associate residual losses in HTS single crystals with a residual normal electron fluid. This implies that the source of the residual loss is in the bulk of the sample, although it is probably not intrinsic. If this contribution is excluded from the complex conductivity of the superconductor, one obtains \( σ(T = 0) \to 0 \), as can be seen in Fig. 3 from the measurements taken at 13.4, 22.7, and 75.3 GHz by the authors of Ref. 14. The peak of \( σ(T) \) shifts to higher temperatures and diminishes in size as the experimental frequency is increased. In YBCO single crystals the temperature \( T_{max} \) at which the maximum of \( σ \) occurs is close to the temperature at which the peak of \( R_s(T) \) occurs.

Finally, one may procure \( σ_1(T) \) from measurements of \( R_s(T) \) and \( X_s(T) \) for \( T > 0 \) without any concern about \( R_{res} \). In this case, \( σ_1(0) \) is not determined uniquely. Whether \( σ_1(T) \) has a peak or not depends on the validity of condition \( (9) \). The curves at 1 and 2 GHz in Fig. 3 have been obtained using Eq. \( (6) \) without subtracting any residual losses.

Temperatures close to \( T_c \) (\( T \to T_c \))

Equations \( (8) \) and \( (9) \) do not apply near \( T_c \). In this temperature range it is necessary to use the general local relationships \( (8), (9) \).

The conductivity \( σ_2(T) \) in the \( ab \)-plane of HTS crystals abruptly drops to very small values in the normal state. The expression \( (T_c/σ_2(0))dσ_2(T)/dT \) at \( T = T_c \), defining the slope of \( λ^2(0)/λ^2(T) \) at \( T = T_c \), varies between \(-2\) and \(-4\) for different crystals.

The real part of the conductivity, \( σ_1(T) \), does not show a coherence peak near \( T = 0.85 T_c \) as predicted by BCS. Usually, the real part of the conductivity, \( σ_1(T) \), of HTS single crystals has a narrow peak near \( T_c \) which increases with decreasing frequency. The width of the narrow peak of \( σ_1(T) \) coincides with the width of the \( R_0(T) \) transition at microwave frequencies. A possible explanation of the sharp peak just below \( T_c \) is inhomogeneous broadening of the superconducting transition, or fluctuation effects.

III. MODIFIED TWO-FLUID MODEL

As was shown in Ref. 24, high \( T_c \)-values (\( T_c \sim 100 \) K), the temperature dependence of the resistivity, the frequency dependence of the momentum relaxation time, and other properties of the normal state in optimally doped HTS’s are well described within the framework of the Fermi-liquid approach, including strong electron-phonon coupling (SC). The SC model also explains some of the features of the superconducting state of HTS’s. It follows from the Eliashberg theory that the distinctive component of superconductors with strong coupling is that the gap in the spectrum of electronic excitations is smeared. Strictly speaking, there is no gap, whatsoever, at \( T \neq 0 \). This leads to breaking of Cooper pairs, smearing of the peak in the density of states at \( hω = Δ(T) \) due to inelastic scattering of electrons by thermally excited phonons, and suppression of coherence effects. As a result, the amplitude of the coherence peak decreases and, according to Refs. 24, 25, virtually disappears at frequencies around 10 GHz if the electron-phonon coupling constant exceeds unity. Moreover, the mechanism of quasiparticle generation is radically different from that of the BCS model. The quasiparticles are generated without jumps across the energy gap and can be in states with all energies down to \( hω = 0 \). These states can be classified as gapless, and the quasiparticles can be treated as normal current carriers in the
two-fluid model. So it is not surprising that an important consequence of the SC model is the nonexponential behavior of $R_e(T)$ and $\lambda(T)$. Power-law temperature dependencies were also predicted by the two-fluid Gorter-Casimir (GC) model, and near $T_c$ they proved to be quite close to calculations performed by the SC model. In particular, the curves of $\lambda^2(0)/\lambda^2(T)$, calculated by the SC model, proved to be fairly close to the function $n_s(t)/n = 1 - n_n(t)/n = 1 - t^4$ in the GC model. The slopes of these curves at $T = T_c$ are in agreement with those measured with different YBCO single crystals and are equal to $-300$.

The fact that there is no BCS coherence peak in the conductivity of HTS crystals, indicates the necessity of taking into account strong coupling effects near $T_c$ and the feasibility of interpreting HTS properties at microwave frequencies in terms of a two-fluid model.

The complex conductivity $\sigma_s$ is a basic property of superconductors. In accordance with GC model, the expressions for the components of $\sigma_s = \sigma_1 - i\sigma_2$ are:

$$\sigma_1 = \frac{n_e e^2 \tau}{m} \left[ \frac{1}{1 + (\omega\tau)^2} \right],$$

$$\sigma_2 = \frac{n_e e^2}{m\omega} \left[ 1 + \frac{n_n}{n_s} \frac{(\omega\tau)^2}{1 + (\omega\tau)^2} \right].$$

(10)

At temperatures $T \leq T_c$ the total carrier concentration is $n = n_s + n_n$, where $n_{s,n}$ are the fractions of superconducting and normal carrier densities (both have the same charge $e$ and effective mass $m$). The real part of conductivity $\sigma_1$ is determined purely by the normal component, while both components, normal and superconducting, contribute to the imaginary part $\sigma_2$. The relaxation time $\tau$ of normal carriers in the GC model is independent of temperature. This is acceptable if we assume that the behavior of normal carriers in superconductors is similar to that of normal carriers in normal metals at low temperatures. Scattering of electrons at very low temperatures is due to impurities and independent of the temperature. Therefore, the temperature dependence of the real part of the conductivity $\sigma_1$ in the GC model is determined entirely by the function $n_n(T)$ with $n_s(T) = n - n_n(T)$ only.

For sufficiently low frequencies $(\omega\tau)^2 \ll 1$ the expressions of the conductivity components of Eq. (10) transform into simple relations

$$\sigma_1 = \frac{e^2\tau}{m} n_n, \quad \sigma_2 = \frac{e^2}{m\omega} n_s = \frac{1}{\mu_0 \omega \lambda^2},$$

(11)

where $\lambda = \sqrt{m/\mu_0 n_s e^2}$ is the London penetration depth of a static magnetic field.

Penetration of alternating fields into superconductors is controlled by the frequency-dependent skin depth. Based on results of the complex conductivity $\sigma_1$, one obtains the complex skin depth $\delta_s$ by generalizing the corresponding expression for a normal conductor:

$$\delta_s = \frac{\sqrt{2}\lambda}{\sqrt{\omega\tau(n_n/n_s) - 1}}.$$

(12)

With increasing angular frequency $\omega$ the skin depth $\text{Re}(\delta_s)$ decreases and, therefore, the London penetration depth $\lambda$ gives the upper bound for the penetration of the electromagnetic field into a superconductor. In GC model the $\lambda$ value diverges near $T_c$ as $\lambda(t) = \lambda/[2\sqrt{1 - t}]$ and the function $\sigma_2(t)/\sigma_2(0) = 4(1 - t)$ tends linearly to zero at $T = T_c$ with a slope equal to $-4$. At the same time, at $T = T_c$ the skin depth $\text{Re}(\delta_s)$, defined by Eq. (12), crosses over to the skin depth $\delta_n$ for a normal conductor.

A. Scattering and surface resistance of HTS single crystals

In conventional superconductivity one assumes that below $T_c$ the mean free path does not vary with temperature. In a normal metal, at much higher temperatures than the corresponding $T_c$ of a conventional superconductor, the electron scattering rate is proportional to $T^2$. Since the transition temperatures of HTS’s are much larger than those of conventional superconductors, it stands to reason that temperature will affect the electron scattering rate of the quasiparticles of HTS’s below $T_c$, but be limited to a constant rate at low temperatures. Therefore, if a two-fluid model is to be successful in explaining transport properties of HTS’s, then it is natural to include a temperature variation of $\tau$ into that model.

To obtain an expression for $\tau(T)$, we rely on the analogy between the ‘normal fluid’ component in the superconducting state and charge carriers in a normal metal. According to Mathiessen’s rule, the reciprocal relaxation time at temperatures below the Debye temperature $\Theta$ is

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{imp}}} + \frac{1}{\tau_{e-ph}}.$$  

(13)

The first term on the right is due to impurity scattering and is a constant of temperature, and the second is due to electron-phonon scattering and is proportional to $T^5$.

From Eq. (13) we express $\tau(T)$ as

$$\frac{1}{\tau(t)} = \frac{1}{\tau(T_c)} \frac{\beta + t^5}{\beta + 1},$$

(14)

where $\beta$ is a numerical parameter: $\beta \approx \tau(T_c)/\tau(0)$, provided this ratio is much less than unity. It should be pointed out, however, that this approximation is not always satisfied.

Equation (14) corresponds to the low-temperature limit of the Bloch-Grüneisen formula, which includes impurity scattering and can be presented over a wide temperature range by the expression

$$\frac{1}{\tau(T)} = \frac{1}{\tau(T_c)} \frac{\beta + t^5}{\beta + 1},$$

(14)
\[
\frac{1}{\tau(T)} = \frac{1}{\tau(T_c)} \beta + \frac{\beta}{\tau(T_c)} \frac{\beta}{\beta + 1} \int_0^{\kappa/T} z^2 e^z dz, \tag{15}
\]

where \(\kappa = \Theta/T_c\). For \(T < \Theta/10\ (\kappa > 10\tau)\), Eq. (15) approaches the form of Eq. (14). For \(T > \Theta/5\ (\kappa < 5\tau)\), we obtain from Eq. (14) the linear \(T\)-dependence of \(1/\tau(T) \propto t\). Examples of \(1/\tau(T)\) for different parameters of \(\beta\), \(\kappa\), and \(\tau(T_c)\) are shown in Fig. 4.

For \(\omega \tau(T_c) \ll 1\), which is normally satisfied at microwave frequencies, the parameter \(\omega \tau(T_c)\) is obtained from measurements of \(R_s(T_c)\) and \(X_s(0)\):

\[
\omega \tau(T_c) = \frac{\lambda^2(0)}{2R_s^2(T_c)} = \frac{\sigma_1(T_c)}{\sigma_2(0)}. \tag{16}
\]

At frequencies \(\sim 10\) GHz, the value of \(\omega \tau(T_c)\) for the best HTS crystals is of the order of \(10^{-3}\) at \(T = T_c\) and remains less than unity at all temperatures \(T < T_c\), as will be discussed below. Therefore, the expressions of the conductivity components in Eq. (14) in the two-fluid model turn into the simple form (11).

All experimental data of \(R_s(T)\) of high-quality HTS single crystals can be elucidated by our two-fluid model with \(\tau(T)\) given by Eqs. (14) or (15).

Measurements of \(R_s(T)\) of YBCO single crystals at frequencies of order or less than \(10\) GHz are analyzed first. Values of \(\sigma_2(T)/\sigma_2(0) = \lambda^2(0)/\lambda^2(T) = n_s(T)/n\), measured in the same experiments, and \(\sigma_1(T)/\sigma_1(T_c)\) obtained from Eq. (13) are substituted into the Eq. (3). Use is made of the relation \(n_s(T)/n = 1 - \sigma_2(T)/\sigma_2(0)\), which is obtained from the experimental data, and \(\tau(T)\), employing Eqs. (14) or (15).

Setting \(\beta = 0.005\) and \(\kappa = 9\) in Eq. (13) and taking the experimental values \(\sigma_2(T)/\sigma_2(0)\) from Fig. 11 (see below) and \(\omega \tau(T_c) = 7.5 \times 10^{-4}\) at \(1.14\) GHz we find from Eqs. (11) and (3) the \(T\)-dependencies of \(R_s(T)\), shown by the curves in Fig. 5. These curves match the data of Ref. 11 over the entire temperature range. The same result is obtained using Eq. (14) instead of Eq. (13), with \(\beta = 0.005\). For at \(\kappa \gg 1\) and \(T \lesssim T_c\), Eqs. (14) and (15) are identical.

From Eqs. (3) and (11) it follows that for \(\alpha t \ll 1\) [see Eq. (9)] a rough estimate of the temperature \(t_m\) at which \(R_s(T)\) is maximum is obtained from the relation \(\beta \approx 4t_m^3\). The value of \(\tau(0)\) is found from the slopes \(dR_s/dT\) and \(d\lambda/dT\) of the experimental data of \(R_s(T)\) and \(\lambda(T)\) at \(T \to 0\) (\(\omega \tau(0) < 1\)):

\[
\omega \tau(0) = \frac{1}{\mu_0 \lambda} \frac{dR_s}{dT}. \tag{17}
\]

With Eq. (10) and (17) the parameter \(\beta \approx \tau(T_c)/\tau(0)\) is determined from the surface impedance data. As \(\beta\) increases the maximum and minimum of \(R_s(T)\) change into an inflection point with a horizontal tangent and
for larger $\omega$ values the maximum of $R_s(T)$ disappears completely.

The linear $T$-increase of $R_s(T)$ at low temperatures (inset in Fig. 5) is a direct consequence of the linear change of $\lambda(T)$ near $T = 0$, proportional to the coefficient $\alpha$ in Eq. (1), and due to a constant scattering rate at low temperatures, as shown in Fig. 4.

The dashed and dotted curves in Fig. 5, are calculated $R_s(T)$ values at 1.14 GHz, with $t^5$ replaced by $t^4$ (dashed curve) and by $t^6$ (dotted curve) in Eq. (14). The best fit of the experimental data is $1/\tau(t) \propto t^4$. Moreover, Eq. (15) enables us to incorporate the shoulder of $R_s(T)$ obtained with YBCO single crystals in Refs. [14-15]. This is shown in Fig. 6, which contains measurements (squares) of $R_s(T)$ at 10 GHz taken from Ref. [16], and calculations (solid line) of $R_s(T)$ using Eqs. (11) and (3) with $\omega \tau(T_c) = 4 \times 10^{-3}$, $\sigma_2(T)/\sigma_2(0)$ obtained from the same experimental data. $\beta = 0.02$ and $\kappa = 4$ in Eq. (13).

The calculated curves in Figs. 5 and 6 are very close to the experimental data and display the common and unique features of $R_s(T)$ for $T < T_c$ and $\omega \tau < 1$ of high-quality YBCO single crystals fabricated by different methods, namely: (i) the linear temperature dependence of surface resistance, $\Delta R_s(T) \propto T$, caused by the linear variation of $\Delta X_s(T) \propto \Delta \lambda_s(T) \propto T$ at temperatures $T \ll T_c$ and by $\tau(T) \rightarrow$ const at low temperatures; (ii) the broad peak of $R_s(T)$ in the intermediate temperature range due to the rapid decrease of the relaxation time $\tau(T) \propto T^{-5}$, with increasing temperature; and (iii) the increase in $R_s(T)$ in the range $T_c/2 < T < T_c$ (Fig. 6) caused by the crossover from $T^{-5}$ to $T^{-1}$ of $\tau(T)$ in Eq. (13), which occurs in Fig. 6 at a lower temperature than in Fig. 5. The behavior of $1/\tau(T)$ for these two YBCO crystals is shown in Fig. 4.

Up to this point, our analysis has not taken into account the residual surface resistance $R_{res}$ of the samples. In the YBCO crystals whose data are plotted in Figs. 5 and 6, scaled to the same frequency of 10 GHz, the resistance $R_{res} < 50 \mu \Omega$. $R_{res}/R(T_c) < 10^{-3}$ is so small that $R_{res}$ was neglected even at $T \ll T_c$. In most HTS crystals which were investigated, however, $R_{res}/R(T_c) > 10^{-3}$ (see, e.g., Figs. 1 and 2). Therefore, it is important that $R_{res}$ is added to the calculated $R_s(T)$ values when comparing the latter with the experimental data.

Figure 7 compares the measured $R_s(T)$ and $X_s(T)$ of BSCCO, plotted in Fig. 1, with calculations obtained from Eqs. (3) and (4). In this case, we have added to the calculated values of $R_s(T)$ a constant $R_{res} = 0.5 \text{ m}\Omega$. The calculation is based on measurements of $\sigma_2(T)/\sigma_2(0)$ obtained in the same experiment and plotted in the inset to Fig. 13 (see below), with parameter $\omega \tau(T_c) = 0.9 \times 10^{-2}$, $\beta = 2$ and $\kappa = 3$ in Eq. (13). It is clear that the agreement between the calculated and experimental curves is good throughout the temperature interval $5 \leq T \leq 120 \text{ K}$.

Another reason for including $R_{res}$ is that the ratio $R_{res}/R(T_c)$, $\omega^{3/2}$. Fig. 8 is based on the experimental data of BSCCO single crystal measured in Ref. [17] at three frequencies: 14.4 GHz ($\omega \tau(T_c) = 0.7 \times 10^{-2}$), 24.6 GHz, and 34.7 GHz. The solid curves are calculations at these frequencies obtained from Eqs. (13) and (4) using $\tau(T)$ from Eq. (13) with $\beta = 0.1$ and $\kappa = 4$. The comparison procedure is different from that discussed above for YBCO crystals since $R_{res} \propto \omega^2$ is added to the calculated $R_s(T)$ values. The inset of Fig. 8 shows a linear plot of the measured and calculated surface resistance at low temperatures. We emphasize that at temperatures below $T_c/2$ the value of $\Delta R_s(T)$ changes proportional to $T$. 
In the millimeter and shorter wavelength bands, the condition $\omega\tau < 1$ may not be satisfied in the superconducting state of the purest HTS single crystals due to the fast growth of $\tau(T)$ with decreasing $T < T_c$. Therefore, it is natural not only to take $R_{res}$ into account in analyzing the experimental data of $Z_s(T)$ and $\sigma_s(T)$ but also the more general Eq. (11) of the two-fluid model should replace Eq. (1). The $R_s(T)$ data of Ref. 4 at frequencies of 13.4, 22.7, and 75.3 GHz, are shown in Fig. 9 with the calculated $R_s(T)$ values (obtained on the same YBCO crystal as was used in Fig. 5). We used $\tau(T_c)/\tau(0) \approx \beta = 5 \times 10^{-3}$ in Eq. (14) for all curves shown in Fig. 9 (same as previously used in Fig. 5), and added $R_{res} = 0.3 \text{ m}\Omega$ to $R_s(T)$ [Eq. (3)] at 75.3 GHz only. The conductivity components $\sigma_1(T)$ and $\sigma_2(T)$ which are contained in Eq. (3) are obtained from the experimental data of $\sigma_2(T)/\sigma_2(0)$ at 1.14 GHz, \cite{4} (shown in Fig. 11), and from Eq. (10).

Figure 10 shows another example. The experimental $R_s(T)$ data (squares) of TBCO single crystal ($T_c = 78.5 \text{ K}$) are compared with calculations based on Eqs. (3), (10), and (15). The curve representing the theoretical $R_s(T) + R_{res}$ was plotted using $\beta = 0.1$, $\kappa = 5.5$, $\omega\tau(T_c) = 1.7 \times 10^{-2}$, $R_{res} = 0.8 \text{ m}\Omega$, and with $\sigma_2(T)/\sigma_2(0)$, shown in the inset (circles) of Fig. 10.

B. Temperature dependence of the superconducting electron density

Our phenomenological model would be incomplete if simple formulas were not available that describe correctly the measurements of $\Delta\lambda_{ab}(T)$. Figures 10 (inset), 11 and 12 show $\sigma_2(T)/\sigma_2(0) = \lambda^2(0)/\lambda^2(T) = n_s(T)/n$ in the $ab$-plane of YBCO, YBCO, and BSCCO single crystals from Refs. \cite{23, 31, 32}, respectively. All of these quantities change linearly with temperature at low-temperatures and can be approximated by the function $n_s/n = (1 - t)^\alpha$, (18)

where $\alpha$ is a numerical parameter. For $t \ll 1$, Eq. (18) follows from Eq. (18). For the cited experiments, the values of $\alpha$ fall into the range $0.4 < \alpha \leq 0.9$. Near $T_c$ we obtain $\lambda(t) \propto n_s(t)^{-1/2} \propto (1 - t)^{-\alpha/2}$, which is also
in fairly good agreement with experimental data. However, equation 19 yields an infinite value of derivative \(d\sigma_2(t)/dt \propto (1-t)^{\alpha-1}\) at \(t = 1\) for \(\alpha < 1\).

The above functions for \(n_s(t)\), however, in their simplest forms 18 and 19, cannot account for all features in \(\lambda^2(0)/\lambda^2(T)\) detected recently in YBCO crystals (see Table I) in the intermediate temperature range. Moreover, the slope of these curves at \(T \ll T_c\) requires that \(\alpha > 1\) in Eq. 18, which would lead to zero slope of the \(\sigma_2(T)/\sigma_2(0)\) curve at \(T = T_c\). Therefore we have added an additional empirical term to the right-hand side of Eq. 18 without violating the condition of particle conservation, \(n_s + n_n = n\),

\[
n_s/n = (1-t)^{\alpha}(1-\delta) + \delta(1-t^{3/\delta}),
\]

where \(0 < \delta < 1\) is the weight factor. For \(\delta \ll 1\) and \(\alpha > 1\) the dominant contribution to \(\sigma_2(T)\) throughout the relevant temperature range is still due to the first term on the right of Eq. 20, while the second is responsible for the finite slope of \(\sigma_2(T)/\sigma_2(0)\) at \(T = T_c\), equal to \(-4\), in accordance with the GC model. As \(\delta\) increases, the second term on the right side of Eq. 20 becomes more essential. The experimental curve of \(\sigma_2(T)/\sigma_2(0)\), derived from \(B_{c1}(T)\) and \(X_c(T)\) measurements of YBCO crystal in Ref. 1, is properly described by Eq. 20 with \(\delta = 0.5\) and \(\alpha = 5.5\) (Fig. 13). This calculation reflects the characteristic features of the experimental data, namely, the linear section of \(n_s\) and the positive second derivative (\(\alpha > 1\)) in the low-temperature region, the plateau in the intermediate temperature range, and the correct value of the slope near \(T_c\).

Using Eq. 20 with \(\alpha = 2\) and \(\delta = 0.2\), one can also describe the \(T^2\)-dependence of \(\sigma_2(T)\) of BSCCO crystals (Figs. 1 and 7), plotted in the inset to Fig. 13.
C. Real part of conductivity

Since the measurements and calculations of $R_s(T)$, $X_s(T)$, and $\sigma_2(T)$ are in good agreement and consistent with $\sigma_1(T)$ in the range $T < T_c$, it is proposed that the modified two-fluid model is a powerful tool for describing the electrodynamic properties of HTS's. The only feature that has not been investigated by this model is the behavior of $Z_e(T)$ and $\sigma_s(T)$ in the temperature range near $T_c$. A spectacular display is the narrow peak in the real part of the conductivity (see Fig. 3).

$\sigma_1(T)$ of YBCO crystals, obtained from measurements at 1.14 GHz \cite{Ref.14} is plotted (circles) in Figs. 3 and 14.

\[
\sigma_1(T) (10^7 \, \Omega^{-1} \, m^{-1})
\]

\[T (K)
\]

FIG. 14. Comparison of the experimental $T$–dependence of $\sigma_1(T)$ (open circles in Fig. 3) of YBCO single crystal at 1.14 GHz (Ref.\cite{14}) with that calculated using the modified two-fluid model (solid line), taking into account the inhomogeneous broadening of the superconducting transition ($\delta T_c = 0.4 \, K$ in Eq. (21)).

The narrow peak near $T_c$ can be described by an effective medium model \cite{Ref.14} which takes into account inhomogeneous broadening of the superconducting transition. Assume that different regions of a given specimen experience transitions to the superconducting state at different temperatures within the $T$–range $\delta T_c$. If the dimension of each of these regions is smaller than the magnetic field penetration depth (microscopic-scale disorder), the distribution of the microwave currents over the sample is uniform, and the calculation of the effective impedance $Z_{\text{eff}}$ of the sample reduces to two operations: first, the impedances $Z_s$ of all regions in the specimen (with different $T_c$) that are connected in series along a current path are added, and, second, averaging over the sample volume is performed. As a result, we obtain

\[
Z_{s}^{\text{eff}}(T) = R_{s}^{\text{eff}}(T) + iX_{s}^{\text{eff}}(T) = \int_{\delta T_c} Z_s(T, T_c) f(T_c) dT_c ,
\]

where the distribution function $f(T_c)$ is such that the fraction of the sample volume with critical temperatures in the range $T_c < T < T_c + dT_c$ equals $f(T_c)dT_c$. In the simplest case $f(T_c)$ is a Gaussian function. In the experiments of Ref.\cite{14}, the width of the superconducting resistive transition was approximately 0.4 K, which we equate to the width of the Gaussian distribution $f(T_c)$. Using the general relations \cite{Ref.14}, with the effective impedance components obtained from Eq. (21), $\sigma_{1}^{\text{eff}}(T)$ is calculated near $T_c$ and is plotted with the experimental data in Fig. 14 for YBCO\cite{14}. The overall agreement is good.

In the framework of the discussed approach, $\sigma_{1}^{\text{eff}}(T)$ displays a narrow peak at $T^* = T_c - \delta T_c$. It is easy to check that the relative peak amplitude is approximately equal to

\[
\frac{\sigma_1(T^*) - \sigma(T_c)}{\sigma(T_c)} \approx \begin{cases} \gamma, & \text{if } \gamma > 1 \\ \gamma^2, & \text{if } \gamma < 0.1 \end{cases}
\]

where $\gamma = \delta T_c/[T_c \omega \tau(T_c)]$, implying, the narrower the superconducting resistive transition, the smaller the peak amplitude. Usually, experiments yield $\gamma > 1$ (e.g., the data of Ref.\cite{14} gives $\gamma \approx 7$ at 1.14 GHz) and, therefore, the peak amplitude should be inversely proportional to frequency.

We applied the above procedure to other specimens to incorporate corrections into the calculations of the $\sigma_1(T)$ curves, caused by inhomogeneous broadening of the superconducting transition. We adjusted the previous calculations of $R_s(T)$ (Figs. 7, 8, and 10) and $\sigma_2(T)$ (Figs. 10, 12, and inset to Fig. 13) by substituting the resulting $Z_s^{\text{eff}}(T)$ into the general equation \cite{14} for the conductivity $\sigma_1$. The resulting curves for BSCCO and TBCO are shown in Figs. 15–17.

\[
\sigma_1(T) (10^7 \, \Omega^{-1} \, m^{-1})
\]

\[T (K)
\]

FIG. 15. Experimental data of $\sigma_1(T)$ at 14.4 and 34.7 GHz of BSCCO single crystal \cite{14} and calculations of $\sigma_1(T)$ using Eqs. (\ref{eq:14}), (\ref{eq:16}), (\ref{eq:21}) and (\ref{eq:10}), taking into account sample inhomogeneities ($\delta T_c = 2 \, K$).
materials are the linear temperature dependence of surface frequencies in the superconducting and normal states (Table I). For high-quality YBCO, BSCCO, TBCO and TBCCO crystals, this broad peak at low temperatures is due to superposition of a number of normal carriers as the temperature decreases, for each individual crystal. In other words, the value of $\beta$ at low temperatures is smaller for YBCO crystals than for BSCCO, TBCO or TBCCO. For the latter crystals the residual losses $R_{\text{res}}$ are usually large and they have to be taken into account.

(i) we introduce a temperature dependence of the relaxation time of the quasiparticles in accordance with the Bloch-Grüneisen law. We find that the $R_s(T)$ curves in different HTS crystals are well described using Eqs. (14) or (15) for $1/T \approx \tau(T)$. In the latter approach there is only one fitting parameter, $\kappa = \Theta/T_c$, while the other parameter $\beta = \tau(T_c)/\tau(0) \ll 1$ can be estimated directly from the experimental data with the help of Eqs. (10) and (17). The absence of the broad peak of $R_s(T)$ in tetragonal HTS single crystals is due to a less rapid increase of $\tau(T)$ with decreasing temperature. In other words, the value of $\beta$ is smaller for YBCO crystals than for BSCCO, TBCO or TBCCO. For the latter crystals the residual losses $R_{\text{res}}$ are usually large and they have to be taken into account.

(ii) we replace the well-known temperature dependence of the density of superconducting carriers in the GC model, $n_s = n(1 - t^4)$, by one of the functions proposed by Eqs. (13), (14) or (15). All of these functions change linearly with temperature at $t \ll 1$ (see Eq. (8)). This permits one to extract the common and distinctive features of $X_s(T)$ and $\sigma_2(T)$ from different HTS crystals.

It also follows from the equations of the modified two-fluid model, that at low temperatures, $t \ll 1$, and low frequencies ($\omega \tau(0) < 1$), all curves of $Z_s(T)$ and $\sigma_s(T)$ have linear regions: $\sigma_1 \propto \alpha t / \beta$, since $n_s/n \approx \alpha t$ and $\tau \approx \tau(0) = \tau(T_c) / \beta$. Furthermore, $\Delta \sigma_2 \propto -\alpha t$. Then, in accordance with Eq. (8), $R_s \propto \alpha t / \beta$ and $\Delta X_s \propto \Delta \lambda \propto \alpha t / 2$. As the temperature increases, the curve of $\sigma_1(t)$ passes through a maximum at $t \approx 0.5$ if the inequality (4) is valid. This peak is due to superposition of two competing effects, namely, the decrease in the number of normal carriers as the temperature decreases, for $t < 1$, and the increase in the relaxation time, which saturates at $t \propto \beta^{1/3}$, at which point the impurity scattering starts to dominate. The features in the $X_s(T)$ and $\sigma_2(T)$ curves for YBCO single crystals in the intermediate temperature range (plateau or bump) can also be described within the framework of our modified two-fluid model, if we take into account the modification of $n_s(t)$, described by Eq. (20) with $0 < \delta \leq 0.5$. The nar-
row peak in the real part of the conductivity $\sigma_1(T)$, near $T_c$, in HTS single crystals can be explained in terms of an effective medium model, taking into account strong electron-phonon coupling of the quasiparticles and inhomogeneous broadening of the superconducting transition.

It is natural to compare the tenets of our phenomenological model with the results of microscopic theories. As was shown in Refs. 5 and 7, the simple formula (8), which defines the linear low temperature dependence of the magnetic field penetration depth in the $ab$-plane of HTS crystals, is consistent with the $d$-wave model of Ref. 8 in the limit of strong (unitary) scattering. Besides, there is nothing foreign in introducing the function $1/T^5$ for the purpose of characterizing scattering in the superconducting state of HTS. A similar temperature dependence of the relaxation rate of quasi-particles follows from the SC model if the phonon corrections to the electromagnetic vertex are taken into account.

In the framework of our modified two-fluid model, the linear low $T$–dependence of the real part of conductivity $\sigma_1(T)$ is consistent with a constant scattering rate, as it is in a normal metal. While the assumption of a Drude form of the conductivity is supported by the $d$-wave microscopic analysis, it was shown that pair correlations in the usual impurity scattering models lead to strong $T$–dependence of the scattering rate (neglecting vertex corrections), namely, $\tau(T) \propto T$ in unitary limit, or $\tau(T) \propto 1/T$ in the Born limit. An attempt to resolve this problem in Ref. 9 by choosing an intermediate scattering rate has not provided satisfactory results yet. Very recently the authors of Refs. 9 and 10 argue that experimental observation $\sigma_1(T) \propto T$ could be explained by the generalized Drude formula $\sigma_1(T) \propto n_{qp}(T) \tau(T)$ if the quasiparticle density varies as $n_{qp}(T) \propto T$ (as indeed happens for $d$-wave pairing) and if the effective quasiparticle scattering time $\tau(T)$ saturates at low $T$. Various possible physical mechanisms of the temperature and energy dependence of $\tau$ are discussed in Ref. 10, scattering from the "holes" of the order parameter at impurity sites, and scattering from extended defects. These mechanisms may provide the required saturation of $\tau(T)$ at low $T$.

As was discussed recently in Ref. 11, the vertex corrections can also modify the low temperature conductivity. However, the temperature dependence has not been investigated yet.

Nevertheless, the microscopic models, which have been investigating the microwave response based on a pure $d$-wave order parameter symmetry, cannot account for the linear section of the $R_\alpha(T)$ curves extending to $T_c/2$ (at frequencies of 10 GHz and below) in tetragonal HTS single crystals, observation of radically different values of the slopes of $\sigma_2(T)$ for $T \ll T_c$ (corresponding to $\alpha > 1$ in Eq. (20)), observed on YBCO crystals, and unusual features of $\sigma_2(T)$ in the intermediate temperatures range.

Recently, observations of unusual microwave properties of HTS materials have caught the attention of a number of researchers. These observations are tentatively attributed to mixed $(d+s)$-wave symmetry of the order parameter. Most studies deal with the low temperature variation of the London penetration depth and its relation to an order parameter of mixed symmetry. In particular, it was shown in Ref. 12 that the low temperature properties of $\lambda(T)$ can be used to distinguish between a pure $d$-wave order parameter and one with $(s+id)$ symmetry, having a small subdominant $s$-wave contribution in systems connected with a tetragonal lattice. Moreover, additions of impurities suppress the $d$-wave symmetric part to the benefit of the $s$-wave part. As a result, a variety of low-temperature dependencies of $\lambda(T)$ is possible for various impurity concentrations, which allows one, in principle, to determine whether or not the order parameter of a superconductor with an orthorhombic lattice is of $(s+id)$ or $(s+d)$ symmetry. In Ref. 12 the $(d+s)$ model was generalized to take into account the normal state anisotropy. This is the realistic approach to high-$T_c$ cuprates with an orthorhombic distortion, since recent microwave conductivity data suggest that a substantial part of the $ab$-anisotropy of $\lambda(T)$ is a normal state effect. It was shown that such an anisotropy affects not only the $ab$-anisotropy of the transport coefficients, but also the density of states and other thermodynamic quantities. The possible temperature variation of the penetration depth $\lambda(T)$ was analyzed recently in Ref. 13 in the framework of the $(d+s)$ model of hybrid pairing. The slope of the $\Delta\lambda(T) \propto T$ for $T \ll T_c$ and its dependence on the $\Delta_s/\Delta_d$ admixture in the gap function was analyzed quantitatively, taking into account the impurity scattering. However, the quantitative comparison of the latter calculation with the experimental data has not been performed yet. More interesting discoveries in this field of research can be expected in the near future.

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