Observation of superstrong coupling in circuit quantum electrodynamics

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Vacuum fluctuations fundamentally affect an atom by inducing a finite excited state lifetime along with a Lamb shift of its transition frequency [1]. Here we report the reverse effect: modification of vacuum modes by a single atom in circuit quantum electrodynamics [2]. Our one-dimensional vacuum is a long section of a high wave impedance (comparable to resistance quantum) superconducting transmission line [3]. It is directly wired to a transmon qubit circuit [4]. Owing to the combination of high impedance and galvanic connection, the transmon’s spontaneous emission linewidth can greatly exceed the discrete transmission line modes spacing [5]. This condition defines a previously unexplored superstrong coupling regime of quantum electrodynamics where many vacuum modes hybridize with each other through interactions with a single atom [6]. We explore this regime by spectroscopically measuring the positions of over 100 consecutive transmission line resonances. Spectroscopy data reveals a broad peak in the density of states (DOS) together with the dispersive interaction of photons at frequencies within the DOS peak. Both effects are well described by a Caldeira-Leggett Hamiltonian model of dissipation in our circuit [7, 8], with the transmon’s quartic anharmonicity treated as a perturbation. In particular, the position of the DOS peak follows the flux-tuned resonance of the bare transmon, and the peak width matches the calculated inverse spontaneous emission lifetime without adjustable parameters. Non-perturbative modifications of such a vacuum, including inelastic scattering of single photons [9], are expected upon replacing the transmon by more anharmonic circuits [10–14], with broad implications for simulating critical dynamics of quantum impurity models [15, 16].

The device consists of a split Josephson junction terminating a two-wire “telegraph” transmission line, which itself is made of a linear chain of $4 \times 10^4$ junctions (Fig. 1a). The opposite end of the line is connected to a dipole antenna for performing local spectroscopy. Recently it was shown that the collective electromagnetic modes of such chains are microwave photons with a velocity as low as $v \approx 10^6$ m/s and wave impedance as high as $Z_\infty \sim R_Q$, where $R_Q = h/(2e)^2 \approx 6.5$ kΩ is the superconducting resistance quantum [3]. A propagating photon corresponds to a plane wave of voltage across the two wires of the line. At low frequencies the dispersion is linear and there is a band edge at about $\omega_p/2\pi \approx 20 - 25$ GHz due to the Josephson plasma resonance of individual junctions. The split-junction differentiates itself from the rest when its Josephson energy $E_J$ and hence the plasma mode frequency $\omega_0$ is detuned down from the value of $\omega_p$ by an external flux \( \Phi \) through the loop. The absolute anharmonicity remains fixed by the charging energy $E_C = e^2/2C_J$ of the split-junction’s oxide capacitance $C_J$. We chose $E_C/h \approx 300$ MHz, such that for $\omega_0/2\pi \approx 5 - 10$ GHz the disconnected split-junction is no different from a conventional transmon.

The magnitude of light-matter coupling in our system becomes apparent in the infinite transmission line length limit. The line is then equivalent to a shunting resistance given by $Z_\infty$ [17] (Fig. 1b). By the correspondence principle, the first excited state lifetime is given by $T_1 = Z_\infty C_J$ (the “RC”-time of an electrical circuit). The finite lifetime results in the radiative broadening of the atomic transition by an amount $\Gamma = 1/2\pi T_1$. For a typical transmon with $E_C/h \approx 300$ MHz, we get $\Gamma \approx 2.5$ GHz/$Z_\infty$[kΩ]. Thus, a conventional 50 Ω-line would overdamp the split-junction [5]. The first novelty of our work consists of utilizing a high impedance,
Z_{∞} \gg 50 \, \Omega$, in order to suppress the galvanic coupling to the degree that $\omega_{0}T_{1} > 10$. The plasma resonance of the split junction is now well defined despite its direct incorporation into the chain.

For a finite length $L$ line, the concept of spontaneous emission remains valid if it occurs faster than the time it takes the emitted radiation to discover the opposite end, i.e. $T_{1} \ll 2L/v$. In the frequency domain, this condition is equivalent to $\Gamma \rho \gg 1$, where $\rho = 2L/v$ is the density of standing wave resonances of the line. In other words, for a given $L$, the light-matter coupling must be strong enough, such that a large number of discrete internal modes of the transmission line simultaneously hybridize with the atom. Although it is difficult to imagine this so-called superstrong coupling regime with a conventional quantum optics system [6, 18], recently it has been addressed experimentally with circuits [19, 20], magnons [21], and quantum acoustics platforms [22, 23]. Adequate description of multi-mode circuit QED, especially the problems related to gauge-invariance and infinite series divergences, has been a subject of many recent theory works [24-27]. The main novelty of our work consists of reaching the condition $\Gamma \rho > 10$, such that the key physics is already given by the continuum limit while the experiment can still probe the vacuum modes one-by-one.

We start with constructing a Hamiltonian model of galvanic light-matter coupling in our circuit. One can show that an open section of a transmission line, as seen by the transmon, is identical to an infinite set of serial $L_{j}C_{j}$-circuits ($j = 1, 2, \ldots$) connected in parallel [8] (Fig. 1c). For a dispersionless line, the frequency and characteristic impedance of each oscillator are given by $\omega_{j} = 1/(L_{j}C_{j})^{1/2} = 2\pi(j - 1/2)/\rho$ and $z_{j} = (L_{j}/C_{j})^{1/2} = Z_{∞}\omega_{j}\rho/4$, respectively. A weak dispersion can be taken into account by introducing a slow frequency dependence to both $\rho$ and $Z_{∞}$.

We also introduce the bare transmon transition frequency $\omega_{0} = ((8E_{J}E_{C})^{1/2} - E_{C}/2)/\hbar$ and linearized impedance $z_{0} = R_{Q}(2E_{C}/\pi^{2}E_{J})^{1/2}$. Defining the annihilation (creation) operators $a_{0}(a_{0}^\dagger)$ for the bare transmon and $a_{j}(a_{j}^\dagger)$ for the uncoupled bath modes, the Hamiltonian reads:

$$H/h = \omega_{0}a_{0}^\dagger a_{0} - K(a_{0}^\dagger a_{0})^4 + \sum_{j>0} \omega_{j}a_{j}^\dagger a_{j} -$$

$$\left(a_{0} + a_{0}^\dagger\right) \sum_{j>0} g_{j}(a_{j} + a_{j}^\dagger) + \left(\sum_{j>0} g_{j}\omega_{j}^{-1/2}(a_{j} + a_{j}^\dagger)\right)^2,$$

where $K = E_{C}/2$ and individual couplings are given by $g_{j} = \omega_{0}(\omega_{j}/z_{j})^{1/2}/2$ [28].

The Hamiltonian (1) is a textbook Caldeira-Leggett model of a quantum degree of freedom interacting with an Ohmic bath [7]. The last term of (1) is the infamous $\chi^{2n}$-term of quantum optics, which here appears naturally in the circuit quantization procedure. Note that in our model $g_{j}^{2} \sim 1/j$, which automatically regularizes the often divergent perturbative series for the Lamb shifts and spontaneous emission linewidth appearing in more common models of multi-mode electrodynamics [29]. If the transmission line modes are insufficiently dense, $g_{j}\rho \ll 1$, only one mode hybridizes with the atom at a time. The vacuum Rabi splitting of the $j$-th mode is given by, in units of Hz, a frequency-independent quantity $2g$, where $g = g_{j}(\omega_{j} \rightarrow \omega_{0})/2\pi$. In the opposite case of interest here, $g_{j}\rho \gg 1$, the Fermi’s golden rule can be used to obtain the radiative linewidth of
The wave impedance $Z$ of the bare junctions plasma frequency $\omega_p/2\pi \approx 22.6 \text{ GHz}$. The DOS extracted for a device with $Z_\infty \approx 9.8 \text{ k}\Omega$. (c) DOS from (a) plotted vs flux. The two dashed lines indicate the fit to a transmon’s transition and a fixed width of the DOS peak.

The semi-classical result $\Gamma = 2\pi Z_\infty C_J$ validating our Hamiltonian (1) as:

$$H/h \approx \sum_j \left( \hat{\omega}_j \hat{c}_j^\dagger \hat{c}_j + K_j (\hat{c}_j^\dagger \hat{c}_j)^2 \right) + \sum_{i\neq j} \chi_{i,j} (\hat{c}_i^\dagger \hat{c}_j + \hat{c}_j^\dagger \hat{c}_i).$$

Here the $K_j$ is the induced Kerr shift of the $j$-mode frequency due to a single photon, which plays the role of the on-site repulsion in Bose-Hubbard models \[30\]. The $\chi_{i,j}$ is the induced cross-Kerr shift of the mode $i$ due to a single photon in the mode $j$ \[31\] (Methods).

We verified the presence of the induced Kerr effect using high-power spectroscopy. The modes inside the DOS peak clearly shift much more in response to a high-power driving compared to the modes outside of it (Fig. 4a). At some threshold power, we observe that the resonances within the DOS peak snap onto their uncoupled values. This effect can be likely explained as the split-junction effectively acquires an infinite inductance because the swing of its overdriven phase exceeds $2\pi$ \[32\].

The cross-Kerr interaction can be characterized more accurately. For a chosen flux bias, we drive a mode $j = 108$, corresponding to the peak in the DOS at a convenient atomic frequency around 7 GHz. The power is such that the mode is populated by an approximately one photon on average. The neighboring modes $i = j \pm 1, j \pm 2, ...$ are scanned with a low-power drive to obtain the resulting frequency shifts. This allows for a measurement of the dependence of $\chi_{i,j}$ on $i - j$ and comparison with our perturbative calculation. The $\chi_{i,j}$ is maximal for the DOS peak maximum, and drops rapidly as the $i$-modes leave the $\Gamma$-neighborhood of the atomic resonance. Although the measured shifts have some fluctuations of the presently unknown nature, the overall data matches the-
ory well using the photon number to drive power conversion as the only adjustable parameter (Methods).

In contrast with a capacitively-coupled transmon [19][21][33], the light-matter coupling constants $g_i$ in Hamiltonian (1) can be increased in our system without a fundamental limit. This can be readily achieved by either decreasing $Z_\infty$ towards 50 $\Omega$ or by increasing $z_0$ towards $R_Q$. The results of the former scenario can be understood as follows. A mode of the bare transmon $(a_0)$ can be approximated as a nearly equal superposition of $\Gamma_\rho \approx 10$ (in the current experiment) normal modes $(c_j)$ of the combined system. Normalization requires that each mode amplitude is approximately $1/(\Gamma_\rho)^{1/2} \sim 1/(gp)$, hence the induced Kerr and cross-Kerr constants scale as $E_C/(gp)^4 \sim E_C/100$ (Methods). This estimate matches with the maximal measured Kerr shift per photon using approximate photon number calibration (Methods). We thus arrive at a peculiar conclusion, that in the limit of large light-matter coupling the photon-photon interaction effects vanish. The quartic anharmonicity of the transmon in this limit rapidly dissolves over the modified vacuum modes and thus can be completely ignored in the Hamiltonian (1).

The latter scenario of increasing $z_0$, equivalent to reducing the split-junction’s overlap, is far more intriguing. Because of the enhanced quantum fluctuations of the superconducting phase-difference across the junction, the series expansion of the cosine Josephson non-linearity becomes invalid together with the normal mode analysis used above. At the same time, thanks to the galvanic connection, the detrimental effect of fluctuating offset charges typical of high $z_0$ junctions is neutralized by the large zero-frequency capacitance of the transmission line. In fact, now our superconducting circuit realizes the boundary sine-Gordon quantum impurity model, which is known to have a quantum critical point at $Z_\infty = R_Q$ [15][16]. The criticality originates from the relevant (in the renormalization group sense) interactions between one-dimensional photons through a cos local non-linearity. This model plays an important role in understanding a broad range of phenomena, from dissipative quantum phase transitions [35], to interacting electrons in one dimension [36], and to certain questions in quantum field theory [37]. Instead of a single small junction, the transmon can also be replaced by either charge or flux qubits, in which case one expects to simulate a related spin-1/2 Kondo impurity model in the spin-boson representation [9][13]. Exploring the modification of vacuum modes in the vicinity of a critical point would establish a potentially fruitful connection between circuit quantum electrodynamics and quantum impurity physics.

In summary, we have demonstrated an ideal superconducting resistor comparable to resistance quantum $h/(2e)^2$, with a special property that its internal modes are frequency resolved and individually accessible. A large number of these modes simultaneously hybridize with each other through a single transmon qubit, which establishes the superstrong coupling regime, relevant for exploring many-body states of photons.

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METHODS

Device fabrication and measurement setup

The devices were fabricated on a highly resistive silicon substrate using the standard Dolan bridge technique. The process involves e-beam lithography on a MMA/PMMA bi-layer resist and the subsequent double-angle deposition of aluminum with the intermediate oxidation step. The spectroscopy measurements are done in a single-port reflection geometry using the wireless interface described in [3]. A global magnetic field was used to create a flux through the split junction loop.

The density of states

The density of states in our system can be expressed as the sum of two contributions

$$DOS(\omega) = \frac{1}{2\pi V (1 - (\omega/\omega_p)^2)^{1/2}} + \frac{1}{2E (\omega - \omega_0)^2 + \pi^2 \Gamma^2}$$

The first term is the DOS for photons in a bare "telegraph" transmission line. It is easily obtained from their dispersion relation $\omega(k) = vk/\sqrt{1 + (vk/\omega_p)^2}$. The second term takes into account another additional state coming
from the transmon. It is expressed as a Lorentzian function normalized by the double transmission line length so the integration over the full frequency range and the length results in exactly one. By fitting the measured DOS to the above expression we found the transmon’s resonance frequency \( \omega_0 \) and broadening \( \Gamma \) as well as the photons velocity \( v \) and cut-off frequency \( \omega_p \).

**Self- and cross-Kerr coefficients**

In order to find the expressions for \( K_j \) and \( \chi_{i,j} \) we start from the Hamiltonian (1) and limit the number of bath modes to \( N = 1000 \). Without the non-linear term the Hamiltonian (1) can be diagonalized with a unitary transformation given by a matrix \( U \). \( U \) is a block matrix, which ensures the commutation relations are preserved in the new basis \( c_i, c_i^\dagger \).

\[
\begin{pmatrix}
    a \\
    a^\dagger
\end{pmatrix} =
\begin{pmatrix}
    u & v \\
    -v & u
\end{pmatrix}
\begin{pmatrix}
    c \\
    c^\dagger
\end{pmatrix}
\]

Here \( a = (a_0, a_1, ..., a_{N-1}, a_N)^T, a^\dagger = (a_0^\dagger, a_1^\dagger, ..., a_{N-1}^\dagger, a_N^\dagger)^T \), etc., and \( u, v \) are \( N + 1 \) by \( N + 1 \) matrices. We can now express the quartic transmon’s non-linearity \( \sim (a_0 - a_0^\dagger)^4 \) in the new basis. Counting the terms \( (c_i^\dagger c_j)^2 \) and \( c_i^\dagger c_j c_j^\dagger c_i \) we arrive at the expressions

\[
K_j = \frac{1}{2} E C (v_{1j}^2 - u_{1j}^2)^2
\]

\[
\chi_{i,j} = 2 E C (v_{ii}^2 - u_{ii}^2) (v_{jj}^2 - u_{jj}^2)
\]

We checked that the same results can be obtained in the perturbative calculation only starting from the linear circuit Hamiltonian [31].

**Measurement of the cross-Kerr shifts**

The dispersive cross-Kerr interaction of photons caused by the Josephson junction non-linearity appears as a mode frequency shift which is linear in the photon population of any other mode. We picked up the mode \( j = 108 \) and changed its photon population with the second microwave tone. For every other mode in the vicinity of the DOS peak we performed a set of reflection measurements with the second tone being consecutively on or off. The mode frequency in every measurement was found from the fit of the reflection coefficient, both the real and imaginary parts, to the damped LC-oscillator model. The shift is then defined as an average mode frequency change between the on and off state of the second tone. The procedure was repeated for various second tone powers. To eliminate the detrimental effects of the flux jitter, we chose the shortest possible time between two consecutive measurements. Moreover the frequency of the second tone was modulated with a time scale much smaller than the measurement time and with an amplitude larger than the jitter amplitude.

This allowed us to create a constant population of the driving mode disregarding its frequency fluctuation. We performed the same measurements in the same frequency range but with the transmon’s resonance largely detuned. We found cross-Kerr shifts smaller by three orders of magnitude, which is consistent with our perturbative calculations of the bare transmission line non-linearity. This consistency made the rough photon calibration possible.
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