Probing features of the Lee-Wick quantum electrodynamics

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In this paper we discuss some aspects concerning the electromagnetic sector of the abelian Lee-Wick (LW) quantum electrodynamics (QED). Using the Dirac’s theory of constrained systems, the higher-order canonical quantization of the LW electromagnetism is performed. A quantum bound on the LW heavy mass is also estimated using the best known measurement of the anomalous magnetic moment of the electron. Finally it is shown that magnetic monopoles can coexist peacefully in the LW scenario.

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I. INTRODUCTION

Effective field theories (EFT) play a central role in modern physics. They cover almost all branches in physics such as nuclear systems derived from low-energy quantum chromodynamics [1], chiral perturbation theory [2–5], BCS theory formulated from conventional superconductivity [6], inflationary model in cosmology [7, 8], gravitationally induced decoherence [9], and so on. Even our most fundamental theories, General Relativity and the Standard Model, are thought as leading terms of some underlying theory [10, 11].

The ideas concerning EFT have began with a non-linear modification of Maxwell electromagnetism made in order to understand the photon-photon electromagnetic scattering process by E. Euler and W. Heisenberg [12] in the 1930s. At the same time, Fermi developed the theory of beta decay to describe the elementary process \( n \rightarrow p + e^- + \bar{\nu}_e \) in the framework of quantum field theory [13]. Even having some interesting features, Fermi and Euler-Heisenberg theories were not taken seriously since they were not renormalizable. Nevertheless, some years later, the development of the renormalisation and the group renormalisation techniques [14], along with the theorem derived by Appelquist and Carazzone [15] - which states that heavy mass particles can be decoupled from low energy dynamics under certain conditions - gave rise to current EFT programme [16].

Effective theories allow us to simplify the description of a given physical process by taking into account the appropriate variables at a given energy scale, i.e., we can regard only the relevant degrees of freedom at that energy range. It is basically a low energy dynamics valid below some energy scale and which does not depend on the behavior in the ultraviolet regime. EFT have the advantage of reducing the number of degrees of freedom, turning the description of the physical system under consideration easier to deal with. An appropriate choice of degrees of freedom is a crucial point in the understanding of the problem.

A possible way of finding an effective QED lagrangian is through the addition of a gauge invariant dimension-6 operator containing higher derivatives in the free Lagrangian of the \( U(1) \) sector, namely,

\[
\mathcal{L}_{\text{eff}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4M^2} F_{\mu\nu} \Box F^{\mu\nu},
\]

where \( M \) is the only parameter added to theory and is responsible for introducing a cutoff in the ultraviolet regime of QED. This modification is very similar to Pauli-Villars regularization procedure [17], Lee-Wick model [18, 19] at quantum level, and Podolsky electromagnetism [20, 21] at the classical context. The new degree of freedom in QED changes dramatically the behavior of the theory at short distances. Nevertheless, unitarity is not preserved at high energies.

Following up previous works [22, 23], the focus of this paper is on the electromagnetic sector of the LW quantum electrodynamics and it is organized as follows. In Section 2 we review some properties of the abelian Lee-Wick electrodynamics. In Section 3 the higher-order canonical quantization is considered in detail. In Section 4 we estimate a quantum bound in the LW heavy mass using the experimental value of the anomalous magnetic moment of the electron. Magnetic monopoles in LW theory and our conclusions are presented in Section 5.

In our conventions \( h = c = 1 \) and the signature of the metric is \((+1, -1, -1, -1)\).
II. OVERVIEW OF THE ABELIAN LEE-WICK MODEL

The Abelian LW model is defined by the following gauge-invariant Lagrangian

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4M^2} F_{\mu\nu} \Box F^{\mu\nu},$$  \hspace{1cm} (2)

where $F_{\mu\nu}(= \partial_\mu A_\nu - \partial_\nu A_\mu)$ is the field strength.

Let us then show that the above Lagrangian describes two independent (on-shell) spin-1 fields: massless one and massive one, with positive and negative norm, respectively. To do that it is appropriate to provide another massive one, with positive and negative norm, respectively. The field theory with real vectorial fields $A_\mu$ and $Z_\mu$ with Lagrangian

$$\mathcal{L} = \frac{1}{2} A_\mu \Box Z^\mu + \frac{1}{2} \partial_\mu A_\nu \partial_\nu Z^\mu - \frac{M^2}{8} A_\mu A^\mu$$

+ $\frac{M^2}{4} A_\mu Z^\mu - \frac{M^2}{8} Z_\mu Z^\mu$, \hspace{1cm} (3)

is equivalent to the field theory with the Lagrangian in Eq. (2). In fact, varying $Z_\mu$ gives

$$Z_\mu = A_\mu + \frac{2}{M^2} \Box A_\mu - \frac{2}{M^2} \partial_\mu \partial_\nu A^\nu,$$ \hspace{1cm} (4)

and the coupled second-order equations from (3) are fully equivalent to the fourth-order equations from (2). The system (3) now separates cleanly into the Lagrangians for two fields, when we make the change of variables

$$A_\mu = B_\mu + C_\mu,$$ \hspace{1cm} (5)

$$Z_\mu = B_\mu - C_\mu.$$ \hspace{1cm} (6)

In terms of $B_\mu$, $C_\mu$, $B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu$ and $C_{\mu\nu} \equiv \partial_\mu C_\nu - \partial_\nu C_\mu$, the Lagrangian now becomes

$$\mathcal{L} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \frac{1}{4} C_{\mu\nu} C^{\mu\nu} - \frac{M^2}{2} C_\mu C^\mu,$$ \hspace{1cm} (7)

which is nothing but the difference of the Maxwell Lagrangian for $B_\mu$ and the Proca Lagrangian for $C_\mu$.

The particle content of the theory can also be obtained directly from Eq. (2). To accomplish this goal we compute the residues at the simple poles of the saturated propagator (contraction of the propagator with conserved currents). Adding to (2) the gauge-fixing term $\mathcal{L}_\chi = -\frac{\lambda}{4} (\partial_\mu A^\mu)^2$, where as usual $\lambda$ plays the role of the gauge-fixing parameter, and noting that due to the structure of the theory and the choice of a linear gauge-fixing functional, no Faddeev-Popov ghosts are required in this case, we promptly get the propagator in momentum space, namely,

$$D_{\mu\nu}(k) = \frac{M^2}{k^2(k^2 - M^2)} \left\{ \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \left[ 1 + \lambda \left( \frac{k^2}{M^2} - 1 \right) \right] \right\}.$$ \hspace{1cm} (8)

Contracting (8) with conserved currents $J^\mu(k)$, yields

$$\mathcal{M} \equiv J^\mu D_{\mu\nu} J^\nu = -\frac{J^2}{k^2} + \frac{J^2}{k^2 - M^2},$$

which allows us to conclude, taking into account that $J^2 < 0$, that the signs of the residues of $\mathcal{M}$ at the poles $k^2 = 0$ and $k^2 = M^2$ are, respectively,

Res$\mathcal{M}(k^2 = 0) > 0$, Res$\mathcal{M}(k^2 = M^2) < 0$,

which confirms our previous result.

It is worth noticing that the wrong sign of the residue of the heavy photon indicates an instability of the theory at the classical level. From the quantum point of view it means that the theory is nonunitary. Luckily, these difficulties can be circumvented. Indeed, the classical instability can be removed by imposing a future boundary condition in order to prevent exponential growth of certain modes. However, this procedure leads to causality violation in the theory [21]; fortunately, this acausality is suppressed below the scales associated with the LW particles. On the other hand, Lee and Wick argued that despite the presence of the aforementioned degrees of freedom associated with a non-positive definite norm on the Hilbert space, the theory could nonetheless be unitary as long as the new LW particles obtain decay widths. There is no general proof of unitary at arbitrary loop order for the LW electrodynamics; nevertheless, there is no known example of unitarity violation. Accordingly, the LW electrodynamics is finite. Therefore, we need not be afraid of the massive spin-1 ghost.

In summary, we may say that the LW work consists essentially in the introduction of Pauli-Villars, wrong-sign propagator, fields as physical degrees of freedom which leads to amplitudes that are better behaved in the ultraviolet and render the logarithmically divergent QED finite.

We remark that for the sake of convenience we shall work in the representation of the gauge field $A_\mu$ as given in Eq. (2), with the propagator as in Eq. (8).

III. LEE-WICK CANONICAL QUANTIZATION

Our starting point for the higher-order canonical quantization is the following LW lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2M^2} \partial_\mu F_{\alpha\beta} \partial^\mu F^{\alpha\beta}. \hspace{1cm} (9)$$

We shall analyze the LW lagrangian (9) instead of (2). The reason for that is the lagrangian (9) - which is completely equivalent to lagrangian (2) - have third order derivative fields, which introduce additional complications in the canonical quantization. The pairs of canonically conjugate variables related to lagrangian (9) are $(A^\mu, \pi_\alpha)$ and $(\dot{A}^\alpha, \eta_\alpha)$ respectively, where $A^\alpha \equiv \dot{A}^\alpha$ is an independent variable. Since the gauge invariance
holds in the LW model, the second-order lagrangian \( \mathcal{L} \) is degenerate, i.e., the Hessian matrix is singular.

The set of generalized canonical momenta from the lagrangian density \( \mathcal{L} \) are
\[
\pi^\nu = -F^{0\nu} + \frac{1}{M^2} \left( \Box F^{0\nu} + \partial_0 \partial_1 F^{1\nu} \right), \quad (10)
\]
\[
\eta^\nu = \frac{1}{M^2} \partial_0 F^{0\nu}. \quad (11)
\]

The relations \( (10) \) and \( (11) \) provide us the following primary constraints:
\[
\eta_0 \approx 0, \quad (12)
\]
\[
\pi_0 + \partial_1 \eta_1 \approx 0, \quad (13)
\]
where \( \approx \) means weak equations according to Dirac’s method \[28, 29\]. The consistency condition implies that equations \( (12) \) and \( (13) \) cannot evolve in time. So, applying the Poisson brackets, we obtain
\[
\{\eta_0, \pi_0, H_P\} \approx 0, \quad (14)
\]
\[
(\pi_0 + \partial_1 \eta_1) \{\pi_0 + \partial_1 \eta_1, H_P\} = \partial_0 \pi_0 \approx 0, \quad (15)
\]
which shows us the appearance of one secondary constraint. With this new constraint, the consistency condition is identically fulfilled. The lagrange multipliers \( \lambda_i \) are easily computed from Poisson brackets applied to the independent variables, which gives us \( \lambda_2 = \lambda_3 = 0 \) and \( \lambda_2 = A_0 \). So, the extended hamiltonian can be expressed as
\[
\mathcal{H}_E = \int d^4x \left\{ \pi_0 A_0 - \pi_i A_i + \dot{A}_0 (\partial_t \eta_1) - \frac{1}{2} M^2 \eta_1^2 \right. \\
+ (\partial_0 \eta_1) F_{ji} - \frac{1}{2} F_{0i}^2 + \frac{1}{4} F_{ij}^2 - \frac{1}{2 M^2} (\partial_0 F_{i0}) (\partial_j F_{j0}) \\
- \frac{1}{4 M^2} (\partial_0 F_{ij}) (\partial_0 F_{ij}) + \frac{1}{4 M^2} (\partial_0 F_{ij}) (\partial_0 F_{ij}) \\
\left. + \dot{A}_0 (\partial_0 + \partial_1 \eta_1) \right\}. \quad (16)
\]

The constraints obtained are all of the first class, a direct consequence of \( U(1) \) gauge invariance of the LW electromagnetism. The canonical quantization requires that we impose a gauge choice and remove the non-physical variables. The gauge condition necessary to change the set of the first class into the second class constraints is obtained by analysing the LW field equations in terms of potential \( A^\mu \), which can be expressed as
\[
\left(1 + \frac{\Box}{M^2}\right) \Box A^\nu - \partial^\nu \left(1 + \frac{\Box}{M^2}\right) \partial_\mu A^\mu = 0. \quad (17)
\]

The equation \( (17) \) suggests that a possible gauge choice is
\[
\left(1 + \frac{\Box}{M^2}\right) \partial_\mu A^\mu = C, \quad (18)
\]
where \( C \) is an arbitrary constant which can be chosen equal to zero, which provides us the following gauge conditions:
\[
\dot{A}_0 = 0, \quad \left(1 + \frac{\Box}{M^2}\right) \nabla \cdot \mathbf{A} = 0, \quad A_0 = 0. \quad (19)
\]

The LW field equation \( (17) \) is compatible with this particular gauge choice \[16\]. The gauge constraints also satisfy the consistency condition. All the constraints obtained are now of the second class and are given by
\[
\Omega_1 = \eta_0 \approx 0, \quad \Omega_2 = \pi_0 + \partial_1 \eta_1 \approx 0, \quad \Omega_3 = \partial_0 \pi_1 \approx 0, \quad \Omega_4 = \dot{A}_0 \approx 0, \quad \Omega_5 = \left(1 + \frac{\Box}{M^2}\right) \partial_0 A_i \approx 0, \quad \Omega_6 = A_0 \approx 0.
\]

Now we can compute the constraint matrix \( \mathbf{C} = [\Omega_1, \Omega_2] \), which is no singular (det\( \mathbf{C} \neq 0 \)) since all constraints are of the second class. According to Dirac’s procedure, we must replace the Poisson brackets by the Dirac brackets, which allow us to replace the constraints \( \Omega_3, \Omega_4 \) and \( \Omega_5 \) as strong equations. Thus, the LW canonical commutators are given by
\[
[A_i (x,t), \pi_j (x,t)] = i \delta_{ij} \delta^3(\mathbf{x} - \mathbf{x}'),
\]
\[
+ [A_i (x,t), \eta_j (x',t)] = i \delta_{ij} \delta^3(\mathbf{x} - \mathbf{x}'). \quad (20)
\]

It is worth noting that in the absence of higher-order derivative terms, the commutator \( (20) \) reproduces the Maxwell commutation relation. To end up, is important to note that the Poincaré algebra is also satisfied.

The history of the quantization of higher-order electromagnetic theories began a long time ago. The first who tried to perform the higher-order quantization in the electromagnetism was B. Podolsky and Kikuchi in 1944 \[21\] and some years later B. Podolsky and P. Schwed \[30\] using the Gliptia-Bleuer method; the results obtained however are dubious due to the fact that in that epoch there was not an easy method to deal with quantization methods of gauge theories, a procedure not yet complete clear up to now. In 1950s, G. R. Pitman \[31\] and R. E. Martin \[32\] probed some aspects of Podolsky electrodynamics in their Ph.D. thesis. Again, the results found are not correct due the inability of treating the electron self-energy. Canonical quantization by Dirac formalism began in early 1960’s. Since then, some authors tried to achieve the quantization of higher-order theories \[34, 37\] via Dirac’s method; on the other hand, canonical quantization of higher-order electromagnetism starting from the determination of primary and secondary constraints have been performed in references \[38–41\]. The lagrangian \[49\] used for us is not the same as the previous authors used. Nevertheless, the physical content is completely equivalent.
IV. ANOMALOUS MAGNETIC MOMENT OF THE ELECTRON AND THE LW HEAVY MASS

Taking into account that QED predicts the anomalous magnetic moment of the electron correctly to ten decimal places, a quantum bound on the mass \( M \) of the LW heavy particle can be found by computing the anomalous electron magnetic moment in the context of the LW electrodynamics and comparing afterward the result obtained with that of QED. To accomplish this goal, we recall that the anomalous magnetic moment stems from the vertex correction for the scattering of the electron by an external field, as it is shown in Fig. 1.

\[
F(0) = \frac{\alpha}{\pi} \int_0^1 da_1 \int_0^{1-a_1} da_2 \frac{\alpha_1}{\alpha_2 + \alpha_3} \left[ \frac{\alpha_1}{\alpha_2 + \alpha_3} - \frac{\alpha_1^2 (\alpha_2 + \alpha_3)}{(\alpha_2 + \alpha_3)^2 + \frac{\alpha_1}{\pi}} \right],
\]

where \( \varepsilon \equiv \frac{m^2}{M^2} \), \( m \) being the electron mass. We call attention to the fact that the term \(-\frac{\varepsilon}{\pi\sqrt{1 - M^2/k^2}} \left[ 1 + \lambda \left( \frac{k^2}{M^2} - 1 \right) \right]\) that appears in Eq. [8] makes no contribution to the term factor \( F(0) \) because the propagator always occurs coupled to conserved currents.

Integrating the above expression first with respect to \( \alpha_3 \) and subsequently with respect to \( \alpha_2 \), gives

\[
F_2(0) = \frac{\alpha}{\pi} \int_0^1 da_1 \int_0^{1-a_1} da_2 \left[ \frac{\alpha_1}{\alpha_2 + \alpha_3} \right] = \frac{\alpha}{\pi} \int_0^1 da_1 \frac{\alpha_1^2}{\alpha_1 + \varepsilon(1 - \alpha_1)^2}.
\]

(24)

Computing \( F_2(0) \), we arrive at the conclusion that

\[
F_2(0) \approx \frac{\alpha}{2\pi} \left[ 1 - \frac{2}{3} \left( \frac{m}{M} \right)^2 - 2 \left( \frac{25}{12} + \ln \left( \frac{m}{M} \right) \right) \left( \frac{m}{M} \right)^4 + \mathcal{O} \left( \left( \frac{m}{M} \right)^6 \right) \right].
\]

(25)

The first term of the above equation is equal to that calculated by Schwinger in 1948 [43]. Since then \( F_2(0) \) has been calculated to order \( \alpha^{10} \) for QED. The second term of Eq. (24) is the most important correction related to the parameter \( M \) of the LW electrodynamics. Recent calculations concerning \( F_2(0) \) in the framework of QED give for the electron [44]

\[ F_2(0) = 1.159 \times 10^{-12}, \]

where the uncertainty comes mostly from that of the best non-QED value of the fine structure constant \( \alpha \). The current experimental value for the anomalous magnetic moment is, in turn, [45]

\[ F_2(0) = 1.156 \times 10^{-12}. \]

Comparison of the theoretical value predicted by QED with the experimental one shows that these results agree in 1 part in \( 10^{12} \). As a consequence,

\[ \frac{2}{3} \left( \frac{m}{M} \right)^2 < 10^{-12}. \]

(26)

Consequently, a lower limit on the heavy particle Lee and Wick hypothesized the existence is \( M \approx 409 \) GeV.

V. FINAL REMARKS

An important question concerning the LW finite QED is whether or not the LW heavy photon and magnetic charge can live in peace in its context. To answer this question we introduce a magnetic current \( k^\mu = (\sigma, k) \) in the LW dual field equations. It is fairly straightforward to show that the resulting system of modified higher-order field equations, namely,

\[
\left( 1 + \frac{\Box}{M^2} \right) \partial_\mu F^{\mu\nu} = J^\nu
\]

(27)

\[
\partial_\mu F^{\mu\nu} = k^\nu
\]

(28)

where \( F^{\mu\nu} = \frac{1}{4} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \) \((\epsilon^{0123} = +1)\), describes the existence of a magnetic charge. In fact, assuming the absence of electric fields, charges, and currents as well as
the absence of magnetic current, we are left essentially with two equations for the magnetic field which have the familiar Dirac monopole solution $B = \frac{q}{4\pi r^2} \hat{r}$, where $q$ is the magnetic charge. Using the usual methods, the Dirac quantization condition $\frac{q}{4\pi} = \frac{n}{\pi}$, where $n$ is an integer, can be promptly recovered. We have thus succeeded in finding a consistent system of Maxwell + vectorial boson mass + magnetic charge equations. We remark that the Dirac monopole and the massive vectorial boson cannot coexist in the context of Proca massive electrodynamics [39] because the latter, unlike the LW QED, is not gauge invariant. The very existence of the Dirac monopole is undoubtedly linked to the existence of the gauge invariance of the corresponding theory. Interestingly enough the system formed by Eqs. (27) and (28) is not symmetric under the duality transformation $E_\mu \to \tilde{F}_\mu$, $\tilde{F}_\mu 	o -F_\mu$, augmented by $J^\mu \to k^\mu$, $k^\mu \to -J^\mu$. This fact raises an interesting question: Would it be possible to accommodate simultaneously magnetic charge and duality transformations in the framework of a higher-order electromagnetic model? A good attempt in this direction might be, for instance, the model defined by the field equations

$$
\left(1 + \frac{\Box}{M^2}\right) \partial_\mu F^{\mu
u} = J^\nu \tag{29}
$$

$$
\left(1 + \frac{\Box}{M^2}\right) \partial_\mu F^{\mu
u} = k^{\nu} \tag{30}
$$

since it is symmetric under duality transformations. It is worth noticing that $(1 + \frac{\Box}{M^2}) \partial_\mu \tilde{F}^{\mu
u} = k^{\nu}$ is identically zero in the absence of the magnetic current. Let us see then whether this model admits a monopole-like solution. For a magnetostatic charge of strength $g$ and duality transformations cannot be accommodated in the context of the Dirac monopole. Now, if the distances are neither too large nor much small, the potential vector cannot exist everywhere in the domain bounded by $S$ because $F^{\mu
u}$ satisfies Eq. (30) rather than Eq. (28). Unlucky we cannot overcome this difficulty by introducing the concept of a string as Dirac did since in this case $\nabla \cdot B = \frac{q}{4\pi} \tilde{F}(r)$ does not vanishes anywhere in the aforementioned domain. The preceding analysis leads us to conjecture that Dirac-like monopoles and duality transformations cannot be accommodated in the context of one and same higher-order electromagnetic model.

To conclude, the bound we have found on the LW heavy mass was obtained using the most accurate experimental data currently available as input for the anomalous magnetic moment of the electron. As far as the truly (loop) quantum effects are concerned, a quick glance at equation (26) clearly shows that a better agreement between theory and experiment concerning the anomalous magnetic moment of the electron would lead to an improvement of the quantum bound. Consequently, there is a great probability of setting a better quantum bound on the LW heavy mass in the foreseeable future.

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