Compact stars in the pseudo-complex general relativity

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Abstract. The field equations of the pseudo-complex General Relativity (pc-GR) have an extra term, of repulsive character, which may halt the gravitational attractive collapse of matter distributions in the evolution process of compact stars. This additional extra term simulates the presence of dark energy in the Universe. On the first part of this contribution we consider the effect of this additional term of pc-GR on the structure of a white dwarf. Observations of type Ia supernova (SNe Ia) admit white dwarfs with masses as high as $2.3M_\odot - 2.6M_\odot$, surpassing this way the mass limit established by Chandrasekhar $(1.44M_\odot)$. In a more conventional theoretical description of a white dwarf, the electron degeneracy pressure ultimately stabilizes the star against gravitational collapse and establishes this way the Chandrasekhar maximum limit of the stellar mass; this mechanism can not explain however values of masses more expressive like those mentioned above. We investigate here a possible mechanism for carrying out such a hypothesis: the combination of the electron degeneracy and dark energy internal pressures. In this study we use a very simple model for the white dwarf mater which consists of nucleons and degenerate electrons, held together by the presence of the gravitational interaction and superimposed to a repulsive background of dark energy. In the pc-GR formalism, the corresponding modified Tolman-Oppenheimer-Volkoff (TOV) equations are solved and the mass-radius relations as well as the maximum mass of the white dwarf star are determined for different parameter configurations. In the second part of this contribution we explore the presence of this additional term and study the role of dark energy in the structure of neutron stars composed by nucleons, hyperons, mesons, and weakly interacting massive fermion dark matter particles (WIMPs) held together by the presence of the nuclear force and the gravitational interaction, superimposed to the repulsive background of dark energy. To describe the hadron-lepton sector we consider three different effective nuclear models, Zimanyi-Moszkowski, Boguta-Bodmer, and the analytic parametrized coupling model, which we extend to consider, in the baryonic sector, the presence of the whole fundamental baryon octet. By solving the corresponding pc-GR Tolman-Oppenheimer-Volkoff (TOV) equations we estimate the maximum gravitational mass of neutron stars.

1. Introduction

A white dwarf represents the final stage of the evolution process of stars with up to $\sim 10$ solar masses. After the hydrogen-fusing period which transforms its hydrogen content in heavier elements through different cycles of thermonuclear reactions, the star passes by a series of thermal pulses before definitively ejecting its outer layers. The remaining core is a white dwarf.

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with around 0.6 solar mass contained into a spherical volume with cents of the solar diameter of about two times the Earth diameter, one of the most dense and compact forms of matter. In a more conventional approach, white dwarfs are called degenerate stars, in the sense that by taking into account that the material in a white dwarf no longer undergoes fusion reactions, the star has no longer internal sources of energy, and as a result the electron degeneracy pressure mainly supports the star matter against gravitational collapse.

With respect to the internal structure of a conventional description of a white dwarf, it contains a core of electron-degenerate matter covered by a gaseous envelop, which has less than 1% of the total mass. It is the initial mass of the progenitor star that determines the dominant thermonuclear reaction cycles in each stage of its evolution and, therefore, the mass and composition of the core in the white dwarf stage. In a typical white dwarf, the core is composed by a mixture of carbon and oxygen above which lies a shell of helium with a outer thin layer of hydrogen. White dwarfs with helium core have lower masses ($M < 0.5M_\odot$) and most of them are the result of mass-transfer episodes during the evolution of binary systems or of strong mass-loss episodes during the red giant branch (RGB) evolution of single stars [1]. Additionally, more massive progenitors produce white dwarfs with cores with heavier elements, as iron, for example.

Although most of the mass of a white dwarf (99% or more) is concentrated in the core, the envelop corresponds to a significant part of the stellar volume. The envelope thickness depends on the relative concentration of H+He, on the surface temperature and gravity. In hot white dwarfs with low mass, for instance, the envelop thickness can reach around 1/3 of the stellar radius. For massive white dwarfs, the envelop is very thin and its thickness is independent of temperature.

Spectroscopic observations of a white dwarf may reveal not only its surface chemical composition but also its surface temperature and gravity ($g = GM/R^2$). By assuming a model for its internal structure, it is possible to estimate the mass and radius of a white dwarf. However, the direct observational determination of the mass and radius of a white dwarf is only possible for binary systems.

White dwarf mass-radius relations are illustrated in Fig. 1. The points correspond to white dwarfs identified in the Data Release 4 of the Sloan Digital Sky Survey (SDSS) and spectroscopically classified as DAs, i.e., white dwarfs with hydrogen in the outer layer [2]. Their masses and radii were calculated assuming a C+O core at a ratio of 50% for each component. The continuous curves indicate the zero-temperature mass-radius relations for cores with different compositions calculated considering star models with a hydrogen envelope [3]. Sirius B, Procyon B and 40 Eri B are examples of white dwarfs in binary systems with well-determined distances [4] and with mass and radius directly measured, with no assumption on the core compositions. For the particular case of Procyon B, the mass-radius relation is not compatible with a typical C+O core, suggesting a core with a denser composition, such as iron.

Stars with masses four to eight times larger than the mass of the sun suffer a supernova explosion process which ejects their outer layers, leaving behind a small, dense core that continues to collapse. Gravity presses the material upon itself so strongly that protons and electrons combine to make neutrons and neutrinos (neutronization process), this being the main aspect that gave rise to the designation neutron stars to these compact objects: the neutrinos produced in the inverse beta decay process usually escape from the contracting core but the neutrons pack closer together until their density becomes equivalent to that of an atomic nucleus.

The composition of a neutron stars is however much more complex, involving different layers of nuclear matter with the presence of the fundamental baryon octet, mesons, leptons, kaon and pion condensates and, in the center of the star, a deconfined composition of quarks and gluons. If the remaining core of the star is less than about 3 solar masses, the particles of the system exert a quantum pressure which is capable of supporting the star. For masses larger than this
Figure 1. White dwarf mass-radius relations (in terms of solar mass and solar radius, respectively). The continuous curves are the zero-temperature mass-radius relations calculated for different core composition. The black points indicate the mass-radius of DA white dwarfs from the SDSS DR4, assuming a C+O core with an hydrogen envelope. The green points indicate mass-radius for three white dwarfs in binary systems.

amount, the internal pressure of the system of particles cannot support the star against gravity and the system collapses into a stellar black hole.

Neutron stars may be seen as a pulsar if their internal magnetic field are favorably aligned with their spin axis. Neutron stars moreover pack their mass inside a 20-kilometer diameter and on average, gravity on a neutron star is around 2 billion times stronger than gravity on Earth.

2. About the masses of white dwarfs and neutron stars

One of the most important and controversial topics related to white dwarfs and neutron stars is about their maximum masses.

Following the formulation of the Fermi-Dirac statistics, Fowler [5] treated the electron gas in the Sirius-B white dwarf as a degenerate non-relativistic gas and found no limiting mass for the star. Anderson [6] and Stoner [7] considered a relativistic treatment for the electron gas and found a limiting density although their treatment was heuristic. Chandrasekhar [8, 9, 10] obtained the limiting mass as $0.91M_\odot$ initially by using a relativistic treatment for the degenerate electron gas. Subsequently, he succeeded in formulating the theory of white dwarfs to full generality employing Newtonian gravity and an equation of state valid for the entire range of electron velocities (including relativistic velocities) for the degenerate Fermi gas. He has obtained this way the equations of hydrostatic equilibrium of the star, in the form of Lane-Emden equations with index 3. Chandrasekhar then solved the corresponding differential equations numerically to find the limiting mass of $1.44M_\odot$ for the white dwarf. Chandrasekhar [11] also considered the problem in the general relativistic framework to study the instability of a radially pulsating white dwarf star and obtained the critical mass as $1.4176M_\odot$. Anand [12] studied the
effect of rotation on a white dwarf star and showed that the value of the limiting mass increases to 1.704M⊙. Qualitative arguments given by Landau & Lifshitz [13] suggest that the inter-particle Coulomb interaction is negligible in a white dwarf star. Using the method of Bohm & Pines [14], Singh [15] showed that the correction to the electron density due to electron-electron interaction is small and can be treated as negligible. Salpeter [16] reconsidered the problem of taking into account Coulomb effects, Thomas-Fermi corrections, the contribution of exchange and correlation energies in white dwarfs and he showed that the corresponding equation of state departs measurably from the ideal degenerate case.

A wide range of mass distributions of a large number of white dwarfs stars, including low and large massive stars, were plotted by Bergeron et al. [17] et al. and Kepler et al. [18]. The most massive non-magnetic white dwarf observed was LHS4033 (see Bergeron et al. [17], Kepler et al. [18] and Dahn et al. [19]) which was predicted to have an oxygen-neon core with a mass in the range of $1.318M_\odot - 1.335M_\odot$.

Recent observations of type Ia supernova (SNe Ia) admit white dwarfs with masses as high as $2.3M_\odot - 2.6M_\odot$. Howell et al. [20] argued that the over-luminosity and low expansion velocities around the SN 2003fg white dwarf could be explained assuming the star to have a mass larger than $1.44M_\odot$. Hicken et al. [21] presented SN2006gz as a possible SNe Ia candidate that was identified with similar properties. Scalzo et al. [22] estimated the total mass of the SN 2007if progenitor to be in the range $2.2M_\odot - 2.6M_\odot$. Silverman et al. [23] suggested another member of SNe Ia class, SN2009dc, with similar peculiarities, possibly formed from the merger of two white dwarfs.

Similarly to white dwarfs, one of the most controversial and intriguing questions in modern astrophysics concerns the existence of a maximum mass of a neutron star. The establishment of this threshold and the controversies correlated with this conception followed historically a logical line of thought quite similar to the corresponding determination of the maximum mass of the white dwarfs.

In 1932, Lev Landau [24] mentioned the possible existence of dense stars which would look like giant nuclei and which can be regarded today as an early theoretical prediction or anticipation of the existence of neutron stars. He showed that hydrostatic equilibrium of these stars, in which the gravity contraction is supported by the pressure of degenerate electrons, can only be reached if the mass of the star do not exceed $1.5M_\odot$. His predictions in this regard occurred before the discovery of the neutron by James Chadwick [25]. Landau assumed that protons and electrons constitute atomic nuclei in stable configurations. After the discovery of the neutrons, an adaptation of his calculations provided a limit of $1.8M_\odot$ for the maximum mass of a neutron star.

In 2010, Paul Demorest and colleagues [26], using observational results for the Shapiro delay, measured the mass of the millisecond pulsar PSR J16142230. The result obtained was $1.97 \pm 0.04M_\odot$, a value substantially higher than any previously measured neutron star mass, placing this way strong constraints on the interior composition of neutron stars. Moreover, in 2013, John Antoniadis and colleagues [27] measured the mass of the millisecond pulsar PSR J0348+0432 to be $2.01 \pm 0.04M_\odot$, based on the spectroscopy of its white dwarf companion. Furthermore, this result allowed, for the first time, a test of general relativity predictions for compact stars.

In the following, as pointed out in the abstract of this contribution we consider a possible mechanism for the generation of such high masses, that is, we consider a treatment of the matter contained in white dwarfs and neutron stars in the scope of the pseudo complex general relativity.

3. The pseudo-complex general relativity

It was shown by Hess, Schäfer and Greiner [28, 29], within the pseudo-complex general relativity (pc-GR) theory, that the resulting field equations include an extra term associated to
the nature of spacetime itself, of repulsive character, that may sustain the collapse of matter distributions avoiding for instance the standard general relativity predictions of black hole creation as the final stage of the evolution of compact stars and space-time singularities. This additional term arises from micro-scale phenomena due to vacuum fluctuations which simulate the presence of dark energy in the Universe. In order to proceed within a familiar GR framework, similarly to the original formulation, we represent this term by an energy-momentum tensor $(T^{DE})_{\mu\nu}$:

$$G_{\mu\nu} = 8\pi(T^{DE})_{\mu\nu} + 8\pi(T^M)_{\mu\nu};$$  \hspace{1cm} (1)

development

thus, in this expression $(T^{DE})_{\mu\nu}$ represents the dark energy contribution and $(T^M)_{\mu\nu}$ is the energy-momentum tensor for the white dwarf or neutron star matter content. The corresponding equations in the pc-GR are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \mathcal{P}^0,$$

with $\mathcal{P}^0$ being a zero divisor quantity (for the details see Refs. [28, 29]); in this expression $G_{\mu\nu}$ is the pseudo-complex Einstein tensor, $R_{\mu\nu}$ is the Ricci tensor, defined in the same way as in the standard GR, with the difference that now it is pseudo-complex, and $R$ is the Riemann curvature (also known as scalar curvature).

4. Equations of State

4.1. White dwarf sector

We describe the nucleons and electrons sector in our simple model of the white dwarf considering a massive star in the final stage of its evolutionary process, i.e., the stage of exhaustion of its nuclear fuel; in this stage the star begins to cool and contract [30]. A more complete description of a white dwarf may be found in the Ref. [31]. When the stellar temperature reaches a sufficient low level, the stellar electrons will occupy the lowest available energy levels. According to the Pauli Principle, there will be two electrons for each energy level when the temperature reaches a sufficient low level, the stellar electrons will occupy the lowest available energy levels. According to the Pauli Principle, there will be two electrons for each energy level. Considering a massive star in the final stage of its evolutionary process, i.e., the stage of exhaustion of its nuclear fuel; in this stage the star begins to cool and contract [30]. A more complete description of a white dwarf may be found in the Ref. [31]. When the stellar temperature reaches a sufficient low level, the stellar electrons will occupy the lowest available energy levels. According to the Pauli Principle, there will be two electrons for each energy level when the temperature reaches a sufficient low level, the stellar electrons will occupy the lowest available energy levels. According to the Pauli Principle, there will be two electrons for each energy level.

Each fermion of the system (electrons and nucleons) contribute to the energy density ($\varepsilon_i$) , pressure ($p_i$) and number density ($\rho_i$) (with $i = e, n$), according to (with $c = \hbar = 1$):

$$\varepsilon_i = \frac{\gamma_i}{2\pi^2} \int_0^{k_B} \frac{d^3k}{\sqrt{k^2 + m_i^2}},$$

$$p_i = \frac{1}{3} \frac{\gamma_i}{2\pi^2} \int_0^{k_B} \frac{k_i^2}{\sqrt{k_i^2 + m_i^2}} \frac{d^3k}{d^3k},$$

$$\rho_i = \frac{\gamma_i}{2\pi^2} \int_0^{k_B} k_i^2 d^3k,$$

where $\gamma_i = 2$ is the degeneracy of each momentum state due to the two spin projections of the fermions.

After integration of these equations we arrive to the following solutions for the equations of energy and pressure of the system in the low density regime $k_i << m_i$ (see Ref. [32]):

$$\varepsilon_i \approx \frac{m_i^4}{\pi^2} \left[ \frac{1}{3} \left( \frac{k_i}{m_i} \right)^3 + \frac{1}{10} \left( \frac{k_i}{m_i} \right)^5 - \frac{1}{56} \left( \frac{k_i}{m_i} \right)^7 + \frac{1}{144} \left( \frac{k_i}{m_i} \right)^9 \right] \approx \rho_i m_i + \frac{(3\pi^2 \rho_i)^{5/3}}{10\pi^2 m_i};$$

$$p_i \approx \frac{m_i^4}{3\pi^2} \left[ \frac{1}{3} \left( \frac{k_i}{m_i} \right)^5 - \frac{1}{14} \left( \frac{k_i}{m_i} \right)^7 + \frac{1}{24} \left( \frac{k_i}{m_i} \right)^9 \right] \approx \frac{(3\pi^2 \rho_i)^{5/3}}{15\pi^2 m_i}. \hspace{1cm} (5)$$
Since $m_N \gg m_e$, the nucleons and dark matter do not contribute appreciably to the pressure, so we can use the following approximation for the equation of state of the system:

$$\varepsilon_M = \rho_e \rho_N \nu,$$  \hspace{1cm} (6)

for the energy density; in this expression, $\nu = \frac{\rho_n + \rho_p}{\rho_e}$ is a parameter that represents the nucleon-electron ratio of the dominant nuclear species and which must be determined by the equilibrium condition for a neutral Fermi gas ($\rho_n = \rho_p (\nu - 1)$):

$$p_i = \frac{1}{3} \frac{\gamma_i}{2\pi^2} \int_0^{k_{Bi}} \frac{k^2}{\sqrt{k_i^2 + m_i^2}} \, dk,$$  \hspace{1cm} (7)

for the pressure of the system.

4.2. Baryon-meson and lepton sector

The equation of state in the baryon-meson sector may be synthesized as $p_{NM} = p_{NM}(\varepsilon_{NM})$ where

$$\varepsilon_{NM} = \frac{1}{2} m^2_\sigma^2 \sigma^2_0 + \frac{1}{2} m^2_\sigma \sigma^2_0^2 + \frac{1}{2} m^2_\omega \omega^2_0 + \frac{1}{2} m^2_\rho \phi^2_0 + \frac{1}{2} m^2_\rho \phi^2_3 + \frac{1}{2} m^2_3 \delta^2_3 + \frac{bM}{3} \sigma_0^3 + \frac{c}{4} \sigma^4_0 +$$

$$+ \frac{1}{\pi^2} \sum_B \int_{0}^{k_{F,B}} k^2 dk \sqrt{k^2 + M^2_B},$$

$$+ \frac{1}{\pi^2} \sum_{B} \int_{0}^{k_{F,B}} k^2 dk \sqrt{k^2 + m^2_B},$$

which describes the internal energy density of nuclear matter (NM), and

$$p_{NM} = -\frac{1}{2} m^2_\sigma \sigma^2_0 - \frac{1}{2} m^2_\sigma \sigma^2_0^2 - \frac{1}{2} m^2_\omega \omega^2_0 - \frac{1}{2} m^2_\rho \phi^2_0 - \frac{1}{2} m^2_\rho \phi^2_3 - \frac{1}{2} m^2_3 \delta^2_3 - \frac{bM}{3} \sigma_0^3 + \frac{c}{4} \sigma^4_0 +$$

$$+ \frac{1}{3 \pi^2} \sum_{B} \int_{0}^{k_{F,B}} \frac{k^4 dk}{\sqrt{k^2 + M^2_B}},$$

$$+ \frac{1}{3 \pi^2} \sum_{B} \int_{0}^{k_{F,B}} \frac{k^4 dk}{\sqrt{k^2 + m^2_B}},$$

which represents the internal pressure of nuclear matter.

5. Dark matter-dark energy sector

Following Ref. [28, 29] we consider an isotropic fluid model for both the dark matter and the dark energy densities and we decouple the physical Higgs from the dark Higgs. This choice also decouples dark matter from ordinary matter and transforms the fermion-singlet system, with mass $m_\chi$, to a non-interacting Fermi gas of WIMPs. There remains a residual interaction of the $\chi$ singlet with the dark Higgs. However, after symmetry breaking in the dark sector, the $\chi$ singlet acquires a mass $m_\chi = m_{\chi_0} + g_\chi x_0$ (for the details see [33]). We shall treat $m_\chi$ as a free parameter, studying the impact of different $\chi$-mass values on the exotic stellar quantities.

The equation of state for a free gas of dark fermions at zero temperature $p_{DM}(\varepsilon_{DM})$ can be calculated via explicit expressions for the energy density

$$\varepsilon_{DM} = \frac{1}{\pi^2} \int_0^{k_F} dk k^2 \sqrt{m^2_\chi + k^2},$$

and pressure

$$p_{DM} = \frac{1}{3 \pi^2} \int_0^{k_F} dk \frac{k^4}{\sqrt{m^2_\chi + k^2}}.$$
Inside the star, the numerical implementation of the system of differential equations demands the specification of the dark matter (DM) equation of state and its relation to the dark energy (DE) sector. Following Ref. [28, 29] we consider an isotropic fluid model for both the dark matter and the dark energy densities and we assume a linear coupling between $\epsilon_{DM}$ and $\epsilon_{DE}$:

$$\epsilon_{DE} = \alpha \epsilon_{DM}.$$  

(12)

The central pressure of the DE-term is assumed to be related to the central energy density of the WIMPs, $\epsilon_0$, through $p_{DE_{C}} = |\alpha| \epsilon_0$, with a standard value $\epsilon_0 = 141 \text{ MeV/fm}^3$.

6. Modified TOV Equations

For simplicity we consider a spherically symmetric and static metric for the star:

$$ds^2 = -e^{\nu(r)}dt^2 + e^\mu(r)dr^2 + r^2(\sin^2(\theta) d\phi^2 + d\theta^2).$$

(13)

Moreover, in the following, both $(T^{DE})_{\mu\nu}$ and $(T^{M})_{\mu\nu}$ will be treated as isotropic perfect fluids, $T_{\mu\nu} = (\epsilon + p) u_{\mu} u_{\nu} + pg_{\mu\nu}$, where $p$ is the pressure and $\epsilon$ is the energy density of the fluid, and $u_{\mu}$ are the components of the four velocity of the fluid. Solving the field equations and assuming that the fluid components interact only gravitationally, the hydrostatic pc-GR TOV equilibrium equations can be written as (see Hess, Schäfer & Greiner [28, 29]):

$$\frac{dp_M}{dr} = -\frac{1}{r} \left( \frac{\epsilon_M(r) + p_M(r)}{r - 2m_M(r) + 2m_{DE}(r)} \right) \left( m_M(r) - m_{DE}(r) + 4\pi r^3 \left[ p_{DE}(r) + p_M(r) \right] \right);$$

(14)

$$\frac{dp_{DE}}{dr} = -\frac{1}{r} \left( \frac{\epsilon_{DE}(r) + p_{DE}(r)}{r - 2m_M(r) + 2m_{DE}(r)} \right) \left( m_M(r) - m_{DE}(r) + 4\pi r^3 \left[ p_{DE}(r) + p_M(r) \right] \right),$$

(15)

where:

$$m_M(r) = 4\pi \int_0^r r'^2 \epsilon_M(r') dr',$$

(16)

and

$$m_{DE}(r) = -4\pi \int_0^r r'^2 \epsilon_{DE}(r') dr',$$

(17)

represent the corresponding mass functions.

7. Results and Conclusions

In this work we have analyzed the role in the stellar stability of white dwarfs and neutron stars of the additional term of the pseudo-complex general relativity, associated to the nature of spacetime itself and which simulates, according to Hess & Greiner [28, 29], the presence of dark energy in the Universe.

Our main results for the mass-radius relations of white dwarfs are summarized in Fig. (2). For $\alpha = 0$ our predictions for the maximum mass of white dwarfs are in good agreement with the classical results of Subrahmanyan Chandrasekhar (see for instance [31]) using the conventional formulation of General Relativity. Our calculations for other values of the alpha parameter predict values of maximum masses of white dwarfs much larger than the results observed so far. Obviously, these results lack an observational confirmation.
The figures show the GR formalism. For neutron stars, our main results for the mass-radius relations are summarized in Figs. (3) - (6). For $\alpha = 0$, which characterizes a more conventional formalism of QHD + GR, i.e., without the presence of dark energy, and using Einstein’s equations of General Relativity, — although considering the presence of a less conventional ingredient in the interior of a neutron stars, i.e., dark matter—, our results are in very good agreement with the observational results of Refs. [26, 27]. However, for $\alpha \neq 0$, the presence of dark matter in the interior of a neutron star superimposed to a repulsive background of dark energy, due to the additional repulsion term of the field equations as predicted by pc-GR, causes the stellar mass to grow expressively, and even reach a value close to the mass limit predicted by Rhoades and Ruffini [34] of $2.9M_\odot$. It is
important to emphasize that our model for neutron stars considers the presence of hyperons that as is well known soften the equation of state of nuclear matter thus contributing to the decrease of the maximum mass of a neutron star. However, the effect of the additional repulsion due to the presence of vacuum fluctuations in the Universe, as predicted by pc-GR, are dominant when compared to the softening of the EoS by the presence of hyperons in the configuration of the maximum mass of a neutron star.

The results of mass-radius relations for white dwarfs and neutron stars indicate that pc-GR may represent a very interesting theoretical alternative in the determination of stable configurations of compact stars. Further work along this line is projected by our theoretical research group.

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