A tunable charge qubit based on barrier-controlled triple quantum dots

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We present a theoretical proposal of a tunable charge qubit, hosted in triple quantum dots. The manipulation is solely performed by changing the heights of the two potential barriers between the three dots, while the energy of all three dots are fixed. We have found that when the relative height of the two barriers are changed, the direction of the axis of rotation in performing single-qubit gates can be varied. On the other hand, the corresponding rotation speed can be tuned by raising or lowering the two barriers at the same time. Our proposal therefore allows for tunability of both the rotation axis and rotating speed for a charge qubit via all-electrical control, which may facilitate realization of quantum algorithms in these devices.

I. INTRODUCTION

Electrons confined in semiconductor quantum dots are promising candidates for the physical realization of quantum computing. Either the charge [1, 2] or spin [3–6] states of electrons can be used to encode a qubit. Perhaps the most intuitive realization is the double-quantum-dot charge qubit, for which an electron is allowed to occupy either one dot or the other, serving as the two logical states. Universal single-qubit operation can be performed by alternating between zero and large detuning (“tilt control”) [1, 7–12], which achieves x and z-axis rotations, respectively. While this charge qubit has been demonstrated early-on to have very fast gate operations [1, 13], it at the same time strongly suffers from charge noises [12, 14, 15], which has limited its development. Spin qubits, on the other hand, are much less sensitive to charge noises [16]. The last decade has witnessed accomplishments of very high control fidelities and long coherence times in single-qubit operations based on various systems [17–26]. Nevertheless, spin qubits also come with a great deficiency that coherent two-qubit operations are challenging since couplings between two spin qubits [27–32] are typically weak. This and other considerations have called a revival of interests on charge qubits, with the expectation that the strong Coulomb interaction between them may be suitable for realizing fast two-qubit gates while the charge noises are reduced by improvements in experimental techniques [33–36]. A successful example is the microwave-driven double-dot charge qubit [37], for which the qubit is operated essentially at zero detuning (a “sweet-spot” at which the exchange interaction is first-order insensitive to charge noises and causes a constant z-rotation [38, 39]), and the relative phases of consecutive microwave bursts provide rotations around an arbitrary axis in the xy plane (“microwave control”).

Great tunability has also been recently demonstrated on certain types of “hybrid qubits” which are expected to combine advantages from manipulating both charge and spin states [40–49].

The microwave control provides flexibility of choosing axes of rotation for a single qubit in the rotating frame, which greatly simplifies the control: it has been shown that if a sequence of piecewise constant pulses are to be used to implement the control on adjustable axes, only two pieces are sufficient to achieve arbitrary single-qubit rotation [50]. Moreover, not only the rotation axes are flexible, the strength of the control field, i.e. the rotating speed, can also be modulated by the amplitude of the microwave. Together these advantages make the microwave control among the most viable control methods at present. Nevertheless, quantum gates based on microwave control are typically slow because the amplitude of the control fields are smaller as compared to energy level splitting in qubit devices [37]. Application of microwave or similar radio-frequency fields can heat up the sample and can be hard to localize to a given qubit in a scaled-up array. All-electrostatic operations are therefore still desired, because current technologies have achieved very high precision and short switching time in generating electrostatic control pulses, and that these pulse sequences are efficient in performing qubit control [15, 34]. These considerations have motivated us to find a qubit encoding scheme which can be controlled by all-electrical means that, at the same time, is flexible on both the direction of rotation axes and the rotating speed.

While the tilt control by changing the detuning is the most common “electrical” method to control a qubit, it has been recently realized, in double-quantum-dot spin qubit systems, that varying the barrier between the two dots (“barrier control”) [51–53] serves as a powerful alternative to other methods, having advantages in many ways [54–57]. In this paper, we apply the barrier control to a charge qubit encoded in a triple-quantum-dot system. We have found that, when the charge qubit is encoded using (0, 2, 1) and (1, 2, 0) states (entries in the brackets refer to electron occupancy in the respective dots),
includes the Coulomb interaction between three electrons

\[ \sum_{1 \leq i < j \leq 3} \frac{e^2}{4\pi\kappa |r_i - r_j|}. \]  

The confinement potential is defined by a sum of three parts,

\[ V(x, y) = V_0(x, y) + G_1(x, y, \xi_1) + G_2(x, y, \xi_2). \]  

The first part has a usual quadratic form for each dot,

\[ V_0(x, y) = \begin{cases} 
  v_1, & x \leq -a/2, \\
  v_2, & -a/2 < x \leq a/2, \\
  v_3, & x > a/2,
\end{cases} \]  

where

\[ v_i \equiv \frac{m^* \omega_0^2}{2} |r - R_i|^2 + \mu_i \]  

indicates the confinement potential of the \( i \)th quantum dot centering at \( R_i \). The coordinates of \( R_i \) are

\[ R_1 = (-a, 0), \quad R_2 = (0, 0), \quad R_3 = (a, 0). \]  

The remaining two terms of Eq. (4) are our control over the barriers between adjacent dots:

\[ G_1(x, y, \xi_1) = \xi_1 \exp \left\{- \frac{32 [(x + a/2)^2 + y^2]}{a^2} \right\}, \]  

\[ G_2(x, y, \xi_2) = \xi_2 \exp \left\{- \frac{32 [(x - a/2)^2 + y^2]}{a^2} \right\}. \]  

The height of the two barriers are controlled by parameters \( \xi_1 \) and \( \xi_2 \). We note that the two barriers are fixed at \( \pm a/2, 0 \) and for this purpose, the quadratic part of the confinement potential, Eq. (5), has discontinuities at \( x = \pm a/2 \). The barrier functions \( G_1 \) and \( G_2 \) override the cusps between two adjacent quadratic potentials, and we use this setup while we change the height of barriers we would like to minimize the other effects on the quantum-dot confinement potentials. It is also conceivable that experimentally one may change the barriers while leaving other factors characterizing the confinement potential, i.e. the location and energy of quantum dots, unchanged. Schematic diagrams showing the confinement potential and the barrier control are presented in Fig. 1. In Fig. 1(b) we have used \( \Delta \xi_1 \) and \( \Delta \xi_2 \) to denote the change in the barrier heights.

In this work we apply the Hund-Mulliken approximation to solve the problem. We approximate the ground states by those of a harmonic oscillator:

\[ \phi_i(r) = \frac{1}{a_B \sqrt{\pi}} \exp \left\{- \frac{1}{2a_B^2} |r - R_i|^2 \right\}. \]
where the $a_B$ is the Fock-Darwin radius $\sqrt{\hbar/(m^*\omega_0)}$, and $i = 1, 2, 3$ indicate the three quantum dots. We then orthogonalize the Fock-Darwin states in Eq. (9) to obtain approximated single-electron wave functions in the triple-quantum-dot system. The orthogonalization is performed by the transformation $[58, 59]$ \[
\{\psi_1, \psi_2, \psi_3\}^T = O^{-1/2}\{\phi_1, \phi_2, \phi_3\}^T,
\]
where $O$ is the overlap matrix (defined as $O_{l,l'} \equiv \langle \phi_l | \phi_{l'} \rangle$).

We consider three electrons (two spin-up and one spin-down) occupying the three quantum dots, and each dot allows a maximum of two electrons (i.e., only the lowest energy level is retained in the Hund-Mulliken approximation). Our complete bases contain the following 9 states:

\[
\begin{align*}
|\uparrow, \uparrow, \downarrow\rangle &= c_{1\uparrow}^\dagger c_{2\uparrow}^\dagger c_{3\downarrow}^\dagger |\text{vac}\rangle, \\
|\uparrow, \downarrow, \uparrow\rangle &= c_{1\uparrow}^\dagger c_{2\downarrow}^\dagger c_{3\uparrow}^\dagger |\text{vac}\rangle, \\
|\downarrow, \uparrow, \uparrow\rangle &= c_{1\downarrow}^\dagger c_{2\uparrow}^\dagger c_{3\uparrow}^\dagger |\text{vac}\rangle, \\
|\uparrow, \uparrow, 0\rangle &= c_{1\uparrow}^\dagger c_{2\uparrow}^\dagger c_{3\uparrow}^\dagger |\text{vac}\rangle, \\
|\uparrow, 0, \uparrow\rangle &= c_{1\uparrow}^\dagger c_{1\downarrow}^\dagger c_{3\uparrow}^\dagger |\text{vac}\rangle, \\
|0, \uparrow, \uparrow\rangle &= c_{2\uparrow}^\dagger c_{3\uparrow}^\dagger c_{3\uparrow}^\dagger |\text{vac}\rangle, \\
|0, \uparrow, 0\rangle &= c_{2\uparrow}^\dagger c_{1\downarrow}^\dagger c_{3\uparrow}^\dagger |\text{vac}\rangle, \\
|\uparrow, 0, 0\rangle &= c_{1\downarrow}^\dagger c_{3\uparrow}^\dagger c_{3\uparrow}^\dagger |\text{vac}\rangle, \\
|\uparrow, 0, \uparrow\rangle &= c_{1\downarrow}^\dagger c_{1\downarrow}^\dagger c_{3\uparrow}^\dagger |\text{vac}\rangle,
\end{align*}
\]

where $|\text{vac}\rangle$ refers to a vacuum state and $c_{i\sigma}^\dagger$ creates an electron on the $i$th dot with spin $\sigma$. Under these bases, the Hamiltonian can be expressed as a $9 \times 9$ matrix, the elements of which is then obtained from the configuration interaction calculation. Diagonalization of the matrix gives the energy spectra and eigenstates of the system.

III. RESULTS

In this work we take $\mu_1 = \mu_3 = 0$ while $\mu_2 < 0$, so the middle dot has lower energy than the other two. Two out of the nine states Eqs. (11a)-(11i) stand out as possible ground states, which we label as our qubit states as

\[
\begin{align*}
|0\rangle &= |\uparrow, \uparrow, 0\rangle, \\
|1\rangle &= |0, \uparrow, \uparrow\rangle.
\end{align*}
\]

This is essentially a charge qubit. In our calculation we have also found that hybridization between the qubit states and other states are very small, so we will only be concerned with the qubit states while discussing the results. In this work, we consider a situation where $\mu_1$, $\mu_2$, and $\mu_3$ are fixed so our control is exercised solely via the two barriers, $\xi_1$ and $\xi_2$. We shall show that the control of the two barriers allows for flexibilities in both the rotation axis and the rotating speed, which adds tunability to the charge qubit traditionally conceived.

Figure 2(a) shows the calculated energy spectra of our system as functions of the difference in the barrier heights, $\Delta \xi = \xi_1 - \xi_2$, while $\xi_1 + \xi_2$ is fixed at 4 meV. Only the lowest two energy levels are shown. (b) The energy level splitting between the two states shown in (a) as a function of $\Delta \xi$. Parameters: $\xi_1 + \xi_2 = 4$ meV. $a = 200$ nm, $\hbar \omega_0 = 60$ $\mu$eV, $\mu_1 = \mu_3 = 0$ and $\mu_2 = -4$ meV.

FIG. 2: (a) Calculated energy spectra of the triple-dot charge qubit system as functions of $\Delta \xi = \xi_1 - \xi_2$. Only the lowest two energy levels are shown. (b) The energy level splitting between the two states shown in (a) as a function of $\Delta \xi$. Parameters: $\xi_1 + \xi_2 = 4$ meV. $a = 200$ nm, $\hbar \omega_0 = 60$ $\mu$eV, $\mu_1 = \mu_3 = 0$ and $\mu_2 = -4$ meV.

Figure 3(a) shows the energies of the qubit states as functions of $\xi_1$, where keeping $\xi_1 = \xi_2$. Both energies are changed as the barrier heights are varied (in this case, the two barriers have to be varied simultaneously). The difference between them, $\Delta E$ as a function of $\xi_1$, is shown in Fig. 3(b). When $\xi_1$ and $\xi_2$ are raised from 0 to 4 meV, $\Delta E$ is reduced from around 0.02 meV to 0.005 meV, namely by a factor of four. $\Delta E$ is smallest at $\Delta \xi = 0$, and we have found that it is even better tunable at more positive and negative $\Delta \xi$ values.

Figure 4 shows how the rotation axis is varied with $\Delta \xi$. The axis is found by projecting the eigenstates $(|\psi\rangle$
corresponding to a specific $\Delta \xi$ value to the qubit bases, i.e.

$$|\psi\rangle = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} |1\rangle. \quad (13)$$

Therefore the rotation axis is in the $xz$ plane apart from $\hat{z}$ by an angle \[60\]

$$\theta = 2 \arctan \left\{ \frac{\langle 1| \psi \rangle}{\langle 0| \psi \rangle} \right\}. \quad (14)$$

Fig. 4(a) schematically depicts how the relative height of the two barriers affects the direction of the rotation axis. When the two barriers are vastly different in height, the rotation axis is close to $\hat{z}$; on the other hand, when the two barriers are leveled, the rotation axis is $\hat{x}$. The detailed dependence of $\theta$ on $\Delta \xi$ is shown in Fig. 4(b). When $\Delta \xi < 0$, $\theta$ increases from zero and approaches $\pi/2$. $\theta = \pi/2$ for $\Delta \xi = 0$. $\theta$ increases to around $\pi$ when $\Delta \xi$ further increases.

Fig. 5 is a pseudocolor plot of the angle between the rotation axis and $\hat{z}$, $\theta$, as a function of both $\xi_1$ and $\xi_2$. In this figure, $\xi_1 + \xi_2 = 4$ meV has been marked as the green/gray dashed line, corresponding to the parameter range used in Figs. 2 and 4. Along this direction, the rotation axis can be varied. The double arrow along $\xi_1 = \xi_2$ indicates the directions along which the amplitude of the qubit energy level splitting can be changed, offering additional tunability to the qubit system.

As two examples of the qubit operation, we show the
rotation axes $\hat{x} + \hat{z}$ and $\hat{x} - \hat{z}$ ($\theta = \pi/4, 3\pi/4$) and the corresponding confinement potential in Fig. 6. Fig. 6(a) and (b) show the case for rotation axis $\hat{x} + \hat{z}$. Fig. 6(a) is a schematic plot showing the axis on the Bloch sphere, while Fig. 6(b) shows the shape of the confinement potential on the $xz$ plane along with the parameters $\xi_1 = 1.64$ meV and $\xi_2 = 2.36$ meV used. Note that the barrier at $x = a$ is higher than the one at $x = -a$. Fig. 6(c) depicts the rotation axis $\hat{x} - \hat{z}$ on the Bloch sphere, while Fig. 6(d) shows the confinement potential on $xz$ plane with $\xi_1 = 2.36$ meV and $\xi_2 = 1.64$ meV. In this case, the barrier at $x = -a$ is higher than the one at $x = a$.

IV. CONCLUSIONS

In this paper, we have demonstrated a tunable charge qubit based on triple quantum dots. While the energy of all three dots are fixed, the manipulation is performed using the two barriers between the three dots. When the relative height of the two barriers are changed, the rotation axis for single-qubit operation is varied so as to offer flexibility in performing quantum algorithms. Moreover, when both barriers are raised or lowered together, the amplitude of the qubit energy level splitting is altered, and so as the rotating speed. Therefore our proposal allows for tunability of both the rotation axis and rotating speed via all electrical control, which may facilitate realization of quantum algorithms in these devices.

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