G-STRUCTURE ON THE COHOMOLOGY OF HOPF ALGEBRAS

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Abstract. We prove that $\text{Ext}^\bullet(k, k)$ is a Gerstenhaber algebra, where $A$ is a Hopf algebra. In case $A = D(H)$ is the Drinfeld double of a finite-dimensional Hopf algebra $H$, our results imply the existence of a Gerstenhaber bracket on $H^\bullet_{GS}(H, H)$. This fact was conjectured by R. Taillefer. The method consists of identifying $H^\bullet_{GS}(H, H) \cong \text{Ext}^\bullet(k, k)$ as a Gerstenhaber subalgebra of $H^\bullet(A, A)$ (the Hochschild cohomology of $A$).

Introduction

The motivation of this paper is to prove that $H^\bullet_{GS}(H, H)$ has a structure of a G-algebra. The G-algebra structure is, roughly speaking, the existence of two products with compatibilities between them: one is associative graded commutative, and the other is a graded Lie bracket. We prove this result when $H$ is a finite-dimensional Hopf algebra (see Theorem 2.1 and Corollary 2.5). $H^\bullet_{GS}$ is the cohomology theory for Hopf algebras defined by Gerstenhaber and Schack in [4]. In order to obtain commutativity of the cup product we prove a general statement on Ext groups over Hopf algebras (without any finiteness assumption).

When $H$ is finite dimensional, the category of Hopf bimodules is isomorphic to a module category, over an algebra $X$ (also finite dimensional) defined by Cibils and Rosso (see [2]), and this category is also equivalent to the category of Yetter-Drinfeld modules, which is isomorphic to the category of modules over the Hopf algebra $D(H)$ (the Drinfeld double of $H$). In [11], Taillefer has defined a natural cup product in $H^\bullet_{GS}(H, H) = H^\bullet_b(H, H)$ (see [3] for the definition of $H^\bullet_b$). When $H$ is finite dimensional, she proved that $H^\bullet_b(H, H) \cong \text{Ext}^\bullet_X(H, H)$, and using this isomorphism she showed that it is (graded) commutative. In a later work [11] she extended the result of commutativity of the cup product to arbitrary-dimensional Hopf algebras, and she conjectured the existence (and a formula) of a Gerstenhaber bracket.

Our method for giving a Gerstenhaber bracket is the following: under the equivalence of categories $\mathcal{X}\text{-mod} \cong D(H)\text{-mod}$, the object $H$ corresponds to $H^{coH} = k$. So $\text{Ext}^\bullet_X(H, H) \cong \text{Ext}^\bullet_D(H)(k, k)$ (isomorphism of graded algebras); according to Ştefan [8] one knows that $\text{Ext}^\bullet_D(H)(k, k) \cong H^\bullet(D(H), k)$. In Theorem 1.8 we prove...
that, if $A$ is an arbitrary Hopf algebra, then $H^\bullet(A, k)$ is isomorphic to a subalgebra of $H^\bullet(A, A)$—in particular, it is graded commutative—and the morphisms are defined at the complex level. In Theorem 2.1 we prove that the image of $C^\bullet(A, k)$ in $C^\bullet(A, A)$ is stable under the brace operation (if $M$ is an $A$-bimodule, $C^\bullet(A, M)$ denotes the standard Hochschild complex whose homology is $H^\bullet(A, M)$); in particular, the image of $H^\bullet(A, k)$ is closed under the Gerstenhaber bracket of $H^\bullet(A, A)$. So, the existence of the Gerstenhaber bracket on $H_G\o H$ follows, at least in the finite-dimensional case, by taking $A = D(H)$. We did not know if this bracket coincides with the formula proposed in [11], but Taillefer, in a personal communication, told us that, using arguments as in [7], one can actually prove that the bracket given by us, in the finite-dimensional case, must agree with the bracket proposed by her. Nevertheless, the argument does not give a proof of existence in the infinite-dimensional case. So the problem, in that generality, remains open.

We also provide a proof that the algebra $\Ext^\bullet_C(k, k)$ is graded commutative when $C$ is a braided monoidal category satisfying certain exactness hypotheses (see Theorem 1.4). This gives an alternative proof of the commutativity of the cup product in the arbitrary-dimensional case by taking $C = H\o YD$, the category of Yetter-Drinfeld modules.

In this paper $A$ will denote a Hopf algebra over a field $k$.

1. CUP PRODUCTS

This section has two parts. First we prove a generalization of the fact that the cup product on group cohomology $H^\bullet(G, k)$ is graded commutative. The general abstract setting is that of a braided (abelian) category with enough injectives satisfying an exactness condition (see Definition 1.2 below). The other part will concern the relation between self extensions of $k$ and Hochschild cohomology of $A$ with coefficients in $k$.

Let us recall the definition of a braided category:

Definition 1.1. The data $(C, \otimes, k, c)$ is called a braided category with unit element $k$ if

1. $C$ is an abelian category.
2. $- \otimes -$ is a bifunctor, bilinear, associative, and there are natural isomorphisms $k \otimes X \cong X \otimes k$ for all objects $X$ in $C$.
3. For all pair of objects $X$ and $Y$, $c_{X, Y} : X \otimes Y \rightarrow Y \otimes X$ is a natural isomorphism. The isomorphisms $c_{X,k} : X \otimes k \cong k \otimes X$ agree with the isomorphism of the unit axiom, and for all triples $X, Y, Z$ of objects in $C$, the Yang-Baxter equation is satisfied:

$$(\text{id}_Z \otimes c_{X,Y}) \circ (c_{X,Z} \otimes \text{id}_Y) \circ (\text{id}_X \otimes c_{Y,Z}) = (c_{Y,Z} \otimes \text{id}_X) \circ (\text{id}_Y \otimes c_{X,Z}) \circ (c_{X,Y} \otimes \text{id}_Z).$$

A data $(C, \otimes, k)$ satisfying axioms 1 and 2, but not necessarily axiom 3 is called a monoidal category.

We will use the notion of exact functor for a monoidal structure.

Definition 1.2. Let $(C, \otimes, k)$ be an abelian monoidal category. We say that $\otimes$ is exact if and only if the canonical morphism

$$H_*(X, d_X) \otimes H_*(Y, d_Y) \rightarrow H_*(X \otimes Y, d_{X \otimes Y})$$

is an isomorphism for all pairs of complexes in $C$. 

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Example 1.3. Let $H$ be a Hopf algebra over a field $k$. Then $C =_{H}\text{-mod}$ is a monoidal category with $\otimes = \otimes_k$, and this functor is clearly exact.

Theorem 1.4. Let $(C, \otimes, k, c)$ be a braided category with enough injectives and exact tensor product. Then $\text{Ext}_C^\bullet(k, k)$ is graded commutative.

Proof. We proceed as in the proof that $H^\bullet(G, k)$ is graded commutative (see for example [1], page 51, Vol. 1). The proof is based on two points: first a definition of a cup product using the bifunctor $\otimes$, and second a lemma relating this construction and the Yoneda product of extensions.

Let $0 \to M \to X_p \to \ldots X_1 \to N \to 0$ and $0 \to M' \to X'_q \to \ldots X'_1 \to N' \to 0$ be two extensions in $C$. Then $N_* := (0 \to M \to X_p \to \ldots X_1 \to 0)$ and $N'_* := (0 \to M' \to X'_q \to \ldots X'_1 \to 0)$ are two complexes, quasi-isomorphic to $N$ and $N'$ respectively. By the Künneth formula, $N_* \otimes N'_*$ is a complex quasi-isomorphic to $N \otimes N'$. So “completing” this complex with $N \otimes N'$ (more precisely considering the mapping cone of the chain map $N_* \otimes N'_* \to N \otimes N'$) one has an extension in $C$, beginning with $M \otimes M'$ and ending with $N \otimes N'$.

So, we have defined a cup product:

\[
\text{Ext}_C^p(N, M) \times \text{Ext}_C^p(N', M') \to \text{Ext}_C^{p+q}(N \otimes N', M \otimes M').
\]

We will denote this product by $\otimes$, and the Yoneda product by $\sim$. The lemma relating this product and the Yoneda one is the following:

Lemma 1.5. If $\eta \in \text{Ext}_C^p(M, N)$ and $\xi \in \text{Ext}_C^q(M', N')$, then

\[
\eta \otimes \xi = (\eta \otimes \text{id}_{N'}) \sim (\text{id}_M \otimes \xi).
\]

Proof of the Lemma. Interpreting the elements $\eta$ and $\xi$ as extensions, it is clear how to define a morphism of complexes $(\eta \otimes \text{id}_{N'}) \sim (\text{id}_M \otimes \xi) \otimes \xi$, and by the Künneth formula, it is a quasi-isomorphism.

In the particular case that $M = M' = N = N' = k$, the lemma implies that $\eta \otimes \xi = \eta \sim \xi$ for all $\eta$ and $\xi$ in $\text{Ext}_C^p(k, k)$. Now the theorem is a consequence of the isomorphism $(X_\ast \otimes Y_\ast, d_{X \otimes Y}) \cong (Y_\ast \otimes X_\ast, d_{Y \otimes X})$, valid for every pair of complexes in $C$, defined by

\[
(-1)^{pq}c_{X,Y} : X_p \otimes Y_q \to Y_q \otimes X_p.
\]

Note that the differentials are morphisms in the category $C$. So the map defined above commutes with the differentials because of the bifunctoriality of the braiding.

Example 1.6. Let $H$ be a cocommutative Hopf algebra. Then $\text{-mod}$ is braided with $c$ the usual flip. When $H = k[G]$ we recover that $H^\bullet(G, k)$ is graded commutative. The other typical example is $H = U(g)$, the enveloping algebra of a Lie algebra $g$. It is known that $\text{Ext}_{U(g)}(k, k) = \Lambda^\bullet(g)$, is graded commutative.

Example 1.7. Let $H$ be an arbitrary Hopf algebra with bijective antipode and $C = \text{mod}_H$ the category of Yetter-Drinfeld modules over $H$. It is well known (see [6], p. 214) that the map $M \otimes N \to N \otimes M$ defined by $m \otimes n \mapsto m_{-1}n \otimes m_0$ is a braiding on $\text{mod}_H$. So $\text{Ext}_{\text{mod}_H}(k, k)$ is graded commutative.

Theorem 1.8. If $A$ is a Hopf algebra, then $\text{Ext}_A^\bullet(k, k) \cong H^\bullet(A, k)$. Moreover, $H^\bullet(A, k)$ is isomorphic to a subalgebra of $H^\bullet(A, A)$. 

Proof. After Stefan [8], since $A$ is an $A$-Hopf Galois extension of $k$, $H^\bullet(A, M) \cong \operatorname{Ext}_{A}^\bullet(k, M_{\text{ad}})$ for all $A$-bimodules $M$.

Here, $M_{\text{ad}}$ denotes the left $H$-module with underlying vector space $M$, but with structure $h_{\text{ad}}m := h_{1}mS(h_{2})$. The notation $(S$ for the antipode, and the Sweedler-type summation) is the standard one.

In particular, $H^\bullet(A, k) = \operatorname{Ext}_{A}^\bullet(k, k)$. But one can give, for this particular case, an explicit morphism at the complex level. In order to do this, we will choose a specific resolution of $k$ as a left $A$-module. Notice that, in particular, our argument will give an alternative proof of Stefan's result for this case.

Let $C_\bullet(A, b')$ be the standard resolution of $A$ as an $A$-bimodule, namely $C_n(A, b') = A \otimes A^{\otimes n} \otimes A$ and $b'(a_0 \otimes \ldots \otimes a_{n+1}) = \sum_{i=0}^{n} (-1)^i a_0 \otimes \ldots \otimes a_i a_{i+1} \otimes \ldots \otimes a_{n+1} (a_i \in A)$. This resolution splits on the right. So $(C_\bullet(A) \otimes A k, b' \otimes id_k)$ is a resolution of $A \otimes_A k = k$ as a left $A$-module. Using this resolution, $\operatorname{Ext}_{A}^\bullet(k, k)$ is the cohomology of the complex $(\operatorname{Hom}_{A}(C_\bullet(A) \otimes_A k, (b' \otimes id_k))^*) \cong (\operatorname{Hom}(A^{\otimes*}, k), \partial)$. Under this isomorphism, the differential $\partial$ is given by

$$ \partial f(a_1 \otimes \ldots \otimes a_n) = \epsilon(a_1) f(a_2 \otimes \ldots \otimes a_n) + \sum_{i=1}^{n-1} (-1)^i f(a_1 \otimes \ldots \otimes a_i a_{i+1} \otimes \ldots \otimes a_n) + (-1)^n f(a_1 \otimes \ldots \otimes a_{n-1}) \epsilon(a_n), $$

which is precisely the formula of the differential of the standard Hochschild complex computing $H^\bullet(A, k)$.

One can easily check that the cup product on $\operatorname{Ext}_{A}^\bullet(k, k)$ which, by Lemma [12], equals the Yoneda product, corresponds to the cup product on $H^\bullet(A, k)$. So this isomorphism is an algebra isomorphism.

Now we will give two multiplicative maps $H^\bullet(A, k) \to H^\bullet(A, A)$ and $H^\bullet(A, A) \to H^\bullet(A, k)$. Consider the counit $\epsilon : A \to k$. It is an algebra map, and so the induced map $\epsilon_* : H^\bullet(A, A) \to H^\bullet(A, k)$ is multiplicative. We will define a multiplicative section of this map.

Let $f : A^{\otimes p} \to k$ be a Hochschild cocycle, and define $\widehat{f} : A^{\otimes p} \to A$ by the formula

$$ \widehat{f}(a_1^1 \otimes \ldots \otimes a_p^p) := a_1^1 \ldots a_p^p f(a_2^1 \otimes \ldots \otimes a_p^p) $$

where we have used the Sweedler-type notation with summation symbol omitted: $a_1^1 \otimes a_2^1 = \Delta(a^1)$, for $a^1 \in A$.

Let us check that $\widehat{f}$ is a Hochschild cocycle with values in $A$,

$$ \partial(\widehat{f})(a_0^0 \otimes \ldots \otimes a_p^p) = a_0^0 \widehat{f}(a_1^1 \otimes \ldots \otimes a_p^p) + \sum_{i=0}^{p-1} (-1)^i a_0^0 \ldots a_i^0 a_{i+1}^1 f(a_{i+2}^2 \otimes \ldots \otimes a_p^p) + (-1)^{p+1} a_0^0 f(a_0^0 \otimes \ldots \otimes a_p^{p-1})a_p^p $$

$$ = a_0^0 a_1^1 \ldots a_i^0 f(a_{i+2}^2 \otimes \ldots \otimes a_p^p) + (-1)^{p+1} a_0^0 \ldots a_i^0 f(a_{i+2}^2 \otimes \ldots \otimes a_p^{p-1})a_p^p $$

$$ + \sum_{i=0}^{p-1} (-1)^{i+1} a_0^0 \ldots a_i^0 a_{i+1}^1 f(a_{i+2}^2 \otimes \ldots \otimes a_p^p) a_p^p.$$
Using that $f$ is a Hochschild coycle with values in $k$, we know that
\[
0 = \epsilon(a^0)f(a^1 \otimes \ldots \otimes a^p) + \sum_{i=0}^{p-1} (-1)^{i+1} f(a^0 \otimes \ldots \otimes a^i a^{i+1} \otimes \ldots \otimes a^p) + (-1)^{p+1} f(a^0 \otimes \ldots \otimes a^{p-1}) \epsilon(a^p).
\]
So, the summation term in $\partial(\widehat{f})$ can be replaced using the equality
\[
\sum_{i=0}^{p-1} (-1)^{i+1} a^0 a_1^i a_1^{i+1} \ldots a_1^p f(a_2^0 \otimes \ldots \otimes a_2^i, a_2^{i+1} \otimes \ldots \otimes a_2^p) = -a^0 a_1^i a_1^{i+1} \ldots a_1^p f(a_2^0 \otimes \ldots \otimes a_2^i) + (-1)^{p+1} f(a_2^0 \otimes \ldots \otimes a_2^{p-1}) \epsilon(a_2^p)
\]
and this finishes the computation of $\partial(\widehat{f})$.

Clearly $\epsilon \widehat{f} = f$; so $\epsilon_*$ is a split epimorphism. To check that $f \mapsto \widehat{f}$ is multiplicative is straightforward:

Let $g : A^{\otimes q} \to k$ be a cocycle and $\widehat{g} : A^{\otimes q} \to A$ the cocycle with values in $A$ corresponding to $g$. We can check the following:

\[
\widehat{\widehat{f} \circ g}(a^1 \otimes \ldots \otimes a^{p+q}) = a^0 a_1^i a_1^{i+1} \ldots a_1^p f(a_2^0 \otimes \ldots \otimes a_2^i, a_2^{i+1} \otimes \ldots \otimes a_2^p) = (\widehat{\widehat{f} \circ g})(a^1 \otimes \ldots \otimes a^{p+q}).
\]

2. Braces operations

In this section we prove our main theorem, stating that the map $H^*(A,k) \to H^*(A,A)$ is “compatible” with the brace operations, and as a consequence with the Gerstenhaber bracket. Note that the map $H^*(A,k) \to H^*(A,A)$ is defined at the standard complex level. Let us define $C^p(A,M) := \text{Hom}(A^{\otimes p}, M)$.

**Theorem 2.1.** The image of the map $C^*(A,k) \to C^*(A,A)$ is stable under the brace operation. Moreover, if $\widehat{f}$ and $\widehat{g}$ are the images in $C^*(A,A)$ of two elements $f$ and $g$ belonging to $C^*(A,k)$, then $\widehat{f} \circ \widehat{g} = \widehat{\widehat{f} \circ g}$.

**Proof.** Let us recall the definition of the brace operations (see [3]). If $F : A^{\otimes p} \to M$ and $G : A^{\otimes q} \to A$ and $1 \leq i \leq p$, then $F \circ_i G : A^{\otimes p+q-1} \to M$ is defined by

\[
(F \circ_i G)(a^1 \otimes \ldots \otimes a^i \otimes b^1 \otimes \ldots \otimes b^q \otimes a^{i+1} \otimes \ldots \otimes a^p) = F(a^1 \otimes \ldots \otimes a^i \otimes G(b^1 \otimes \ldots \otimes b^q) \otimes a^{i+1} \otimes \ldots \otimes a^p).
\]

Assume now that $f : A^{\otimes p} \to k$, $g : A^{\otimes q} \to k$ and $F = \widehat{f}$ and $G = \widehat{g}$, namely

\[
F(a^1 \otimes \ldots \otimes a^p) = a_1^1 a_1^2 \ldots a_1^p f(a_2^1 \otimes \ldots \otimes a_2^p).
\]
and similarly for $G$ and $g$. Then (denoting $(a \otimes b)$ by $(a, b)$),
\[
(F \circ_i G)(a^1, \ldots, a^i, b^1, \ldots, b^p, a^i+1, \ldots, a^p) = F(a^1, \ldots, a^i, G(b^1, \ldots, b^p), a^{i+1}, \ldots, a^p) = F(a^1, \ldots, a^i, b^1 \ldots b^q, g(b_2, \ldots, b_2^p), a^{i+1}, \ldots, a^p) = \tilde{a}_1 \ldots \tilde{a}_i b_1^i \ldots b_1^q, a_{i+1}^i, \ldots, \tilde{a}_i^p, f(a_2^1, \ldots, a_2^i, b_2^1 \ldots b_2^q, g(b_3^1, \ldots, b_3^p), a_{i+1}^i, \ldots, \tilde{a}_i^p) = \tilde{f} \circ_i G(a^1, \ldots, a^i, b^1, \ldots, b^p, a_{i+1}^i, \ldots, a^p).
\]

Recall that the brace operations define a “composition” operation $F \circ G = \sum_{i=1}^p (-1)^{q(i-1)} F \circ_i G$, where $F \in C^p(A, A)$ and $G \in C^q(A, A)$. The Gerstenhaber bracket is defined as the commutator of this composition. So we have the desired corollary:

**Corollary 2.2.** If $A$ is a Hopf algebra, then $H^\bullet(A, k)$ is a Gerstenhaber subalgebra of $H^\bullet(A, A)$.

**Example 2.3.** Let $A$ be a Hopf algebra. Then $\text{Ext}^1_A(k, k) \cong \text{Der}(A, k) = \text{Prim}(A^*)$, where $\text{Prim}(A^*) = \{x \in A^* \text{ such that } m^*(x) = x \otimes 1 + 1 \otimes x \}$. It is easy to check that the Lie bracket given in the above theorem coincides with the commutator of the convolution product, viewing $\text{Der}(A, k)$ as a subset of $A^*$.

**Example 2.4.** Let $G$ be a connected affine algebraic group and $\mathfrak{g} := \text{Ker}(\epsilon)/\text{Ker}(\epsilon)^2$ its tangent Lie algebra. One has that $HH^\bullet(\mathcal{O}(G), \mathcal{O}(G)) = \Lambda^\bullet(\mathcal{O}(G))\text{Der}(\mathcal{O}(G)) \cong \mathcal{O}(G) \otimes \Lambda^\bullet \mathfrak{g}$, where the Gerstenhaber structure here is the Schouten-Nijenhuis bracket. Also $\text{Ext}^\bullet_{\mathcal{O}(G)}(k, k) = \Lambda^\bullet \mathfrak{g}$, and it is generated (as an algebra) in degree one. So the bracket is determined by its values on $\text{Ext}^1_{\mathcal{O}(G)}(k, k) = \mathfrak{g}$, which is the bracket of $\mathfrak{g}$ as a Lie algebra. This $G$-algebra structure is also well known.

Consider $H$ a finite-dimensional Hopf algebra and $X = X(H)$ the algebra defined by Cibils and Rosso (see [2]). We can prove, at least in the finite-dimensional case, the conjecture of [11] that $H_{GS}(H, H)$ is a Gerstenhaber algebra:

**Corollary 2.5.** Let $H$ be a finite-dimensional Hopf algebra. Then $H^\bullet_{GS}(H, H)$ is a Gerstenhaber algebra.

**Proof.** The isomorphism $H^\bullet_{GS}(H, H) \cong \text{Ext}^\bullet_X(H, H)$ was proved in [10].

Let $A$ denote $D(H)$, the Drinfeld double of $H$. One knows that $H^\bullet_{GS}(H, H) \cong A^\bullet_{GS}(M^\text{co}H, M^\text{co}H) = \text{Ext}^\bullet_A(M^\text{co}H, M^\text{co}H)$. Then $X = X(H, H) \cong \text{Ext}^\bullet_A(M^\text{co}H, M^\text{co}H) = \text{Ext}^\bullet_A(k, k)$, and this a Gerstenhaber subalgebra of $H^\bullet(A, A)$.

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