Brane Bremsstrahlung in DBI Inflation

Philippe Brax‡ and Emeline Cluzel§

Institut de Physique Théorique, CEA, IPhT, CNRS, URA 2306, F-91191Gif/Yvette Cedex, France

Abstract. We consider the effect of trapped branes on the evolution of a test brane whose motion generates DBI inflation along a warped throat. The coupling between the inflationary brane and a trapped brane leads to the radiation of non-thermal particles on the trapped brane. We calculate the Gaussian spectrum of the radiated particles and their backreaction on the DBI motion of the inflationary brane. Radiation occurs either due to a parametric resonance when the interaction time is small compared to the Hubble time or a tachyonic resonance when the interaction time is large. In both cases the motion of the inflationary brane after the interaction is governed by a chameleonic potential, which tends to slow it down. We find that a single trapped brane can hardly slow down a DBI inflaton whose fluctuations lead to the Cosmic Microwave Background spectrum. A more drastic effect is obtained when the DBI brane encounters a tightly spaced stack of trapped branes.

PACS numbers: 98.80.Cq, 98.70.Vc

1. Introduction

D-brane inflation [1, 2, 3, 4, 5, 6, 7] has become a time honoured subject whose development has led to plausible tests of string theory in a cosmological setting. With the launch of the Planck satellite [8] and the advent of precision cosmology, this possibility has become even more relevant [9, 10, 11]. Along these lines, there are three typical observables which may have an impact on models of inflation inspired or derived from string theory. The first one is the spectral index. The second one is the production of gravitational waves [12, 13]. Another one is the possibility of primordial non-Gaussianities [14]. It is quite likely that Planck will reduce the uncertainty on the spectral index in such a way that different scenarios such as brane inflation or modular inflation [15, 16, 17] may be distinguished. For instance, modular inflation models of the race track type tend to favour a rather low spectral index [18, 19, 20, 21]. Similarly, the absence of detectable gravity waves in these low field inflation models is generic. In this paper we will consider a related family of brane inflation models which do not fall within the category of slow roll models. In DBI inflation [22], a test brane evolves due to the steepness of the warp factor. This is particularly well motivated in string

‡ philippe.brax@cea.fr
§ emeline.cluzel@cea.fr
compactification scenarios where a D3 brane evolves down a warped throat [23, 24] attracted towards an anti D3 brane. In general, potential terms such as a large mass prevent the existence of a slow roll period [6]. The brane may thus enter a regime of DBI inflation whose features are very different from the ones of slow roll inflation. In DBI inflation, the spectral index tends to be close to one [2, 14, 25] and non-gaussianities are large. Reheating in brane inflation [5, 26, 27, 28] is also quite different from reheating in standard inflation [29, 30, 31].

Recently, the issue of brane inflation in the $D3 - \bar{D}3$ system has received a host of new developments. It has been realised that the potential does not only comprise the Coulomb interaction and a mass term responsible for the $\eta$ problem [6]; there are also corrections coming from bulk effects in the compactification scheme [32, 33, 34]. These bulk effects may require a detailed knowledge of the full compactification manifold. It happens that the leading contributions may have two effects. One possibility is the existence of an inflection point [35, 36] in the inflation potential when a fractional power of the inflaton in $-c\phi^{3/2}$ is present. Slow roll inflation is tunable in this context and requires a choice of initial conditions to overcome the overshooting problem [37, 38]. Another possibility is that the bulk term contributes as another mass term to the potential. In this second case, DBI inflation may occur.

Our paper discusses brane models without attempting to realise their embedding in string theory. Hints along these lines can be found in [39]. In this paper, we will focus on DBI inflation as a field theoretical model corresponding to a warped throat with a quadratic potential. We will envisage the likely situation wherein the warped throat is not empty but contains a series of trapped branes [40]. These branes may be present due to fixed points in the compactification manifold. When the inflationary brane crosses these trapped branes, particles are created with a spectrum whose shape is Gaussian with a width related to the speed of the brane. The radiation of particles leads to the creation of a long range potential which might slow the motion down. The overall effect of the trapped brane is to induce a chameleonic term in the inflationary potential. Eventually this change of the potential implies that the Hubble rate is modified after the crossing of the stack. This effect decays rapidly as the radiated particles are diluted. As a result, the change of the Hubble rate is only effective on a few e-foldings after the interaction.

In this paper we concentrate on the detailed analysis of the creation of particles on the trapped branes and the subsequent slowing down of the inflationary brane in the DBI context. In section 2, we recall some of the salient characteristics of DBI inflation. In section 3, we consider the creation of particles on the trapped brane. In section 4, we study the backreaction of radiated particles on the inflationary brane and the induced change in the potential.
2. DBI Inflation

We are interested in brane models inspired from string theory. The main ingredients of our model will be the DBI nature of the inflaton dynamics and the quartic coupling of the inflaton $\phi$ to a scalar field $\chi$. The coupling is equivalent to the one between the inflaton and brane degrees of freedom on a trapped brane fixed along the inflationary valley [40]. In the slow roll regime, the same model has been considered in [39]. Here we analyse the case where the $\eta$ problem prevents slow roll inflation but allows DBI inflation.

The dynamics of the inflationary brane are described by the Dirac-Born-Infeld (DBI) action, where canonical kinetic terms have been replaced by a DBI term which can be expanded as a sum of higher order derivatives

$$ S = -\frac{1}{g_s} \int d^4x \sqrt{-g} \left( \frac{1}{f(\phi)} \sqrt{1 + f(\phi)g_{\mu\nu}\partial^\mu\phi \partial^\nu\phi} - \frac{1}{f(\phi)} + g_{\mu\nu}\partial^\mu\chi \partial^\nu\chi \right) + V(\phi) + \frac{g_2^2}{2} \chi^2 |\phi - \phi_1|^2 + \int d^4x \sqrt{-g} \frac{M_p^2}{2} R $$

(1)

where $R$ is the Einstein-Hilbert action is the Ricci scalar and $g_{\mu\nu}$ is the metric. The metric is of the FRW type with no curvature

$$ ds^2 = -dt^2 + a^2(t)dx^2 $$

(2)

in cosmic time. Conformal time is defined by $a(t)d\eta = dt$. We will exclusively focus on a warped case with an AdS throat where $f(\phi) = \frac{\lambda}{\phi^4}$. The 't Hooft coupling $\lambda = R^4/\alpha'^2 = R^4/l_s^4$ depends on the radius of compactification $R$. In this scheme, the inflaton represents the radial distance between the moving D3 brane and the $\bar{D}3$ inside the throat: $\phi = \sqrt{T_3}r$ where $T_3$ is the brane tension. The coupling constants $g_s$ and $g$ are respectively the string coupling and the Yukawa coupling where $g_s \approx g_2^2$.

The potential $V(\phi)$ consists of several terms. The Coulomb potential $V_{D3-\bar{D}3} = D \left( 1 - \frac{3D}{16\pi^2 g_s^3} \right)$ with $D = \frac{2}{f(\phi_{IR})}$, where $f$ is evaluated at the tip of the throat, corresponds to the attraction between the D3 and $\bar{D}3$ branes. There is also a mass term $V_2\phi^2$ coming from radiative and supergravity corrections to the potential. In general the mass term $V_2$ is positive apart from the case of a probe brane starting at the tip of the throat and moving towards the bulk [41, 42]. Corrections to the potential coming from bulk effects have to be added too. These corrections have integer and half-integer powers of $\phi$ and depend on the bulk of the compactification. The two leading corrections are proportional to $\phi^2$ and $\phi^{3/2}$. In the first case, the model is inflationary when the branes are sufficiently apart under the influence of the quadratic potential. In the second case, the mass term and a negative $\phi^{3/2}$ term imply the existence of an inflection point around which the potential becomes $V_0 + V_1(\phi - \phi_{\text{inflection}})$.

There is also a trapped brane whose location is fixed at $\phi_1$ along the throat. Particles $\chi$ on the trapped brane are coupled to the inflationary brane with a quartic coupling at leading order. We will see that this coupling is responsible for the slowing down of the inflationary brane when crossing the trapped brane.
We define the equivalent of the Lorentz factor $\gamma$

$$
\gamma = \frac{1}{\sqrt{1 + f(\phi)g_{\mu\nu}\partial^\mu\phi\partial^\nu\phi}} = \frac{1}{\sqrt{1 - f\dot{\phi}^2}}.
$$

(3)

The inflationary dynamics are governed by the Klein Gordon equation for the spatially homogeneous inflaton field $\phi$

$$
\ddot{\phi} + 3H\dot{\phi} + \frac{\dot{\gamma}}{\gamma}\dot{\phi} + \frac{1}{\gamma} \frac{dV}{d\phi} + \frac{1}{\gamma f^2} \frac{df}{d\phi} - \frac{1}{\gamma^2 f^2} \frac{df}{d\phi} - \frac{\dot{\phi}^2}{2f} \frac{df}{d\phi} = 0
$$

(4)

or equivalently

$$
\ddot{\phi} + \frac{3H}{\gamma^2} \frac{\dot{\phi}}{\gamma} + \frac{1}{\gamma^3} \left( \frac{dV}{d\phi} + \frac{1}{f^2} \frac{df}{d\phi} \right) - \frac{1}{f^2} \frac{df}{d\phi} + \frac{3}{2} \frac{\dot{\phi}^2}{f} \frac{df}{d\phi} = 0
$$

(5)

In the ultrarelativistic limit, both the potential term and the friction term coming from the expansion of the universe are subdominant.

With the warped factor $f(\phi) = \lambda \phi^4$, equation (4) becomes :

$$
\ddot{\phi} + 3H\dot{\phi} + \frac{\dot{\gamma}}{\gamma}\dot{\phi} + \frac{1}{\gamma} \frac{dV}{d\phi} + \frac{4\dot{\phi}^3}{\lambda\gamma} + \frac{4\dot{\phi}^3}{\lambda\gamma^2} + \frac{2\dot{\phi}^2}{\phi} = 0
$$

(6)

It is easier to analyse the DBI dynamics using the Hamilton-Jacobi formalism. This formalism consists in eliminating the dependence in $t$ and using instead $\phi$-dependent functions. It is valid provided $\phi(t)$ does not oscillate. Using the Friedmann equation

$$
H^2 = \frac{\rho}{3M_P^2} = \frac{1}{3M_P^2 g_s} \left( \frac{\gamma - 1}{f} + V \right)
$$

(7)

where $M_P$ is the Planck mass, the potential can be written as

$$
V(\phi) = 3M_P^2 g_s H^2 - \frac{\gamma}{f} + \frac{1}{f}
$$

(8)

If we differentiate (7) and upon using the Klein-Gordon equation we can express $\frac{dV}{d\phi}$ and obtain

$$
\dot{\phi} = -2M_P^2 g_s \frac{1}{\gamma} \frac{dH}{d\phi}
$$

(9)

Similarly, the $\gamma$ factor can be written in terms of functions of $\phi$

$$
\gamma(\phi) = \sqrt{1 + 4M_P^2 g_s^2 f(\phi) \left( \frac{dH}{d\phi} \right)^2}
$$

(10)

For a given potential, the Hubble function of $\phi$ is the solution of a differential equation. Focusing on the limiting case where $\phi \to 0$ as $t \to \infty$ and expanding $H(\phi)$ in powers of $\phi$, the relativistic factor is dominated by

$$
\gamma(\phi) \approx 2M_P^2 g_s \sqrt{f(\phi)} \left| \frac{dH}{d\phi} \right|
$$

(11)

As a result, the inflaton follows a universal trajectory dictated by the coupling $f(\phi)$

$$
\dot{\phi} \approx -\frac{1}{\sqrt{f(\phi)}} \quad \text{and} \quad \phi(t) = \frac{\sqrt{\lambda}}{t}
$$

(12)
The potential term has an influence on the scale factor only.

We will exclusively focus on a quadratic potential where

\[ V(\phi) = m^2 \phi^2 \]  

(13)

In this case, there are co-dominant terms in (8) and one finds

\[ H = \frac{1}{3\sqrt{\lambda}} \left(1 + \sqrt{1 + \frac{3m^2 \lambda}{M_P^2 g_s}}\right) \phi \]  

(14)

The scale factor becomes

\[ \frac{a}{a_0} = \left(\frac{t}{t_0}\right)^{1/\epsilon} \]  

(15)

where

\[ \frac{1}{\epsilon} = \frac{1}{3} \left(1 + \sqrt{1 + \frac{3m^2 \lambda}{M_P^2 g_s}}\right) \approx \sqrt{\frac{\lambda}{3g_s M_P}}. \]  

(16)

Inflation occurs when \( \epsilon < 1 \). We will focus on the case where \( \epsilon \ll 1 \) which leads to interesting restrictions on the parameter space (see section 4). In all cases the coupling \( \frac{\lambda}{g_s} \) and \( \frac{m}{M_P} \) can be adjusted to verify \( \epsilon \ll 1 \) at strong coupling \( \lambda \gg 1 \). The \( \gamma \) factor becomes

\[ \gamma = \frac{2M_P^2 g_s}{\lambda} \frac{1}{\epsilon} t^2 \]  

(17)

In the following we shall use conformal time to study the creation of particles when the DBI brane crosses the trapped brane. In conformal time we find that

\[ a(\eta) \propto \left(\frac{1 - \epsilon}{\epsilon}\right)^{\frac{1}{1 - \epsilon}} (\eta/\eta_0)^{\frac{1}{1 - \epsilon}} \]  

(18)

in such a way that

\[ H \approx -\eta^{-1}, \quad \frac{a''}{a} \approx 2\eta^{-2} \]  

(19)

We deduce \( \phi \propto (\frac{\eta}{\eta_0})^{\frac{\epsilon}{1 - \epsilon}} \approx (\frac{\eta}{\eta_0})^\epsilon \) to leading order. We will make use of these identities in conformal time when discussing particle creation.

3. Particle Creation

3.1. WKB approximation

So far we have not taken into account the presence of the trapped brane. In fact, the trapped brane has an influence on the inflationary brane evolution. To determine its effect we need to study the quantum modes of the field \( \chi \). Let us expand the quantum field \( \chi \) in terms of creation and annihilation operators. For convenience sake we work in conformal time. The field reads

\[ \chi(\eta) = \int \frac{d^3k}{(2\pi)^3} (a_k \chi_k(\eta)e^{ikx} + a_k^* \chi_k^*(\eta)e^{-ikx}) \]  

(20)
and each mode satisfies the Klein-Gordon equation
\[ \chi''_k + 2\mathcal{H}\chi'_k + k^2\chi_k + a^2g^2|\phi - \phi_1|^2\chi_k = 0 \]  
(21)

We can put in a Schrödinger form by defining
\[ \Psi(\eta) = a\chi(\eta) \]
(22)

So \( \Psi \) obeys
\[ \Psi''_k + \omega^2_k\Psi_k = 0 \]
(23)

with a time dependent frequency
\[ \omega_k(\eta) = \sqrt{k^2 + A(\eta)}, \quad A(\eta) = a^2g^2|\phi - \phi_1|^2 - \frac{a''}{a} \]
(24)

where \( \phi \) is the unperturbed \( \phi(\eta) \) corresponding to the unperturbed motion of the inflationary brane. Here we have assumed \( \omega^2_k \) positive. When it is negative, the regime is tachyonic and the frequency will be written \( \Omega_k = \pm i\omega_k = \pm i\sqrt{|k^2 + A(\eta)|} \), so that \( \Omega^2_k = -\omega^2_k > 0 \).

The equation for the modes (23) can be approximately solved using the Wentzel-Kramers-Brillouin or WKB approximation. Far in the past, the solution is assumed to be in a Bunch-Davies vacuum where for \( \eta \to -\infty \) we have
\[ \Psi_k(\eta) = \frac{1}{\sqrt{2\omega_k(\eta)}} e^{-i\int^\eta\omega_k(\eta')d\eta'} \]
(25)

When the inflationary brane has passed through the trapped brane and \( \eta \to 0 \), it is a mixture of two possible modes
\[ \Psi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{-i\int^\eta\omega_k(\eta')d\eta'} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{i\int^\eta\omega_k(\eta')d\eta'} \]
(26)

where \( \alpha_k \) and \( \beta_k \) are the Bogoliubov coefficients. The WKB approximation is valid when \( |\frac{\omega'}{\omega^2}| \ll 1 \). We define
\[ R = \left| \frac{\omega'}{\omega^2} \right| \]
(27)

and find
\[ R = \left| \frac{\mathcal{H}g^2a^2|\phi - \phi_1|^2 + \phi'g^2a^2|\phi - \phi_1| - \frac{1}{2}\left( \frac{a''}{a} \right)'}{(k^2 + a^2g^2|\phi - \phi_1|^2 - \frac{a''}{a})^{3/2}} \right| \]
(28)

Our goal is to determine under which conditions the WKB approximation is violated. There are two physically different situations where the analysis can be easily carried out. They depend on
\[ \xi = \frac{H^2}{g|\phi|} \]
(29)

which is a constant in DBI inflation with a quadratic potential
\[ \xi = \frac{1}{ge^2\sqrt{\lambda}} \approx \frac{\sqrt{\Lambda}}{3g^3} \left( \frac{m}{M_P} \right)^2 \]
(30)

The creation of particles is very different for small or large \( \xi \).
3.2. Small \(\xi\) behaviour

In this region of the parameter space, the creation of particles occurs when the DBI brane is close to the trapped brane which corresponds to the regime where

\[
|\phi - \phi_1| \ll \frac{|\dot{\phi}|}{H} \tag{31}
\]

In this case the \(R\) factor becomes

\[
R \approx R_{\text{near}} = \frac{|\phi' g^2 a^2| |\phi - \phi_1| - \frac{1}{2} \left(\frac{a''}{a}\right)' |}{|k^2 + a^2 g^2| |\phi - \phi_1|^2 - \frac{a''}{a}|^{3/2}} \tag{32}
\]

and non-adiabaticity is mainly present in the region where \(g^2 a^2 |\phi - \phi_1|^2 \gg \frac{g''}{a}\) (or equivalently \(g^2 |\phi - \phi_1|^2 \gg 2H^2\)) for which we have

\[
R \approx \left| \frac{\phi' g^2 a^2 |\phi - \phi_1|}{(k^2 + a^2 g^2 |\phi - \phi_1|^2)^{3/2}} \right| \tag{33}
\]

From now on we define

\[
\mathcal{K} = \frac{k}{a} \tag{34}
\]

which corresponds to physical momenta,

The creation of particles arises when \(R > 1\) at its maximum which is located at \(g^2 a^2 |\phi - \phi_1|^2 = \frac{\mathcal{K}^2}{2}\) and is given by

\[
R_{\max} = \frac{2 |g| |\dot{\phi}|}{3^{3/2} \mathcal{K}^2} \tag{35}
\]

implying that the creation of particles occurs when \(\mathcal{K} \leq \frac{\sqrt{2}}{3^{3/4}} \sqrt{g |\dot{\phi}|}\). The maximal extension of the non-adiabatic region is

\[
\Delta \phi = \frac{1}{3^{3/4}} \sqrt{\frac{|\dot{\phi}|}{g}} \tag{36}
\]

Notice that \(\Delta \phi \gg \frac{H}{g}\) as \(\xi \ll 1\).

Inside a region of size \(|\phi - \phi_1| \leq H/g\) around the origin, there is a domain of non-adiabaticity when

\[
R \approx \frac{\left| \left(\frac{a''}{a}\right)' \right|}{2 |k^2 - \frac{a''}{a}|^{3/2}} \geq 1 \tag{37}
\]

corresponding to

\[
\sqrt{2 - 2^{2/3} H} \leq \mathcal{K} \leq \sqrt{2 + 2^{2/3} H} \tag{38}
\]

Due to the very small width of this zone in momentum space, virtually no particles are created in this interval.

Finally when \(\mathcal{K} \leq \sqrt{2} H\), there is a tachyonic instability. The size of the tachyonic region is much smaller than the size of the non-adiabaticity region where most particles are created. In fact, the creation of particles in the tachyonic region is negligible as it scales like \(\exp(H \Delta t_{\text{tachyon}}) \sim \exp(\xi) \sim 1\) where the time spent in the tachyonic region is \(\Delta t_{\text{tachyon}} \sim \frac{H}{g|\dot{\phi}|}\).
It is interesting to notice that the time spent by the brane in the interaction region is
\[ \Delta t \approx \frac{\sqrt{\xi}}{H} \]  
(39)

implying that the interaction lasts less than a Hubble time and therefore the interaction time corresponds to a number of e-folds
\[ \Delta N = \frac{\Delta a}{a} \approx \sqrt{\xi} \ll 1 \]  
(40)

The interaction is almost instantaneous.

So far we have used the fact that the Hubble rate is nearly constant in the interaction region. This can be ascertained as the variation of the Hubble rate in the interaction region is given by
\[ \left| \frac{\Delta H}{H} \right| \approx \frac{1}{(g\sqrt{\lambda})^{1/2}} \]  
(41)

Therefore we must impose that
\[ g \sqrt{\lambda} \gg 1. \]  

3.3. Large \( \xi \) behaviour

In this case, the ratio \( R \) can be simplified in two regimes depending on whether the DBI brane has moved far from the trapped brane or not. If the inflationary brane is far from the trapped brane then \( |\phi - \phi_1| \gg \dot{\phi}/H \) and
\[ R \approx R_{\text{far}} = \frac{|\mathcal{H}g^2 a^2|\phi - \phi_1|^2 - \frac{1}{2} \left( \frac{a''}{a} \right)'|}{|k^2 + a^2 g^2|\phi - \phi_1|^2 - \frac{a''}{a}|^{3/2}} \]  
(42)

On the contrary, if they are close, \( |\phi - \phi_1| \ll \dot{\phi}/H \) so
\[ R \approx R_{\text{near}} = \frac{|\phi' g^2 a^2|\phi - \phi_1| - \frac{1}{2} \left( \frac{a''}{a} \right)'|}{|k^2 + a^2 g^2|\phi - \phi_1|^2 - \frac{a''}{a}|^{3/2}} \approx \frac{|\left( \frac{a''}{a} \right)'|}{2|k^2 - \frac{a''}{a}|^{3/2}} \]  
(43)

These two regimes capture all the physics of the \( \chi \)-particle creation.

First, we will study the case when the two branes are far from each other. The ratio \( R \) is maximal when \( g^2|\phi - \phi_1|^2 = 2K^2 + (2 - 7\epsilon)H^2 \) and its value is simply
\[ R_{\text{max}} = \frac{2 |\mathcal{H}K^2 - 4\epsilon H^3|}{|3K^2 - 6\epsilon H^2|^{3/2}} \]  
(44)

We find that there is a pole for \( K_{\text{pole}} = \sqrt{2\epsilon H} \). The value at the origin is very large: \( R_{\text{max}}(K = 0) \approx \epsilon^{-1/2} \gg 1 \) as \( \epsilon \ll 1 \). In all the interval between the origin and the pole, \( R_{\text{max}} \) is greater than one. We want to determine the physical momentum \( K_{\text{max}} \) for which \( R_{\text{max}} = 1 \) and then becomes smaller than unity for larger momenta. Expanding around the pole
\[ R_{\text{max}} = \frac{2}{3\sqrt{3}} \frac{H}{|K^2 - K_{\text{pole}}^2|^{1/2}} \]  
(45)
implying that
\[ K_{\text{max}}^2 \approx \frac{4H^2}{27}. \quad (46) \]
Hence we find that for physical momenta \( 0 < K < K_{\text{max}} \approx \frac{2H}{3\sqrt{3}} \), the WKB approximation is violated around \( g^2|\phi - \phi_1|^2 = 2(K^2 + H^2) \). Therefore there are two non-adiabatic regions far from the trapped brane: regions I and II centered respectively around \( \phi_A \) and \( \phi_B \). The approximation used here is valid as the maximal extension of the interaction zone is given by \( \Delta \phi \approx \frac{H}{g} \) implying that \( \Delta \phi \gg |\dot{\phi}|/H \) when \( \xi \gg 1 \).

We now consider when the inflationary brane and the trapped brane are close to each other. From (43), we find that
\[ R > 1 \quad \text{when} \quad \sqrt{2 - 2^{2/3}H} < K < \sqrt{2 + 2^{2/3}H} \quad (47) \]
The inequality (47) gives the range of physical momenta for which a non-adiabatic region appears in the immediate vicinity of the trapped brane. It is consistent with \( g^2|\phi - \phi_1|^2 \ll 2H^2 \) as \( \xi \gg 1 \).

On top of the non-adiabatic instability detailed above, there is a tachyonic resonance when \( \omega_k^2 < 0 \) corresponding to
\[ K^2 + g^2|\phi - \phi_1|^2 - 2H^2 < 0 \quad (48) \]
So for physical momenta larger than \( 2H^2 \), there is no tachyonic regime. We define \( \eta_- \) and \( \eta_+ \) as the two turning points such that \( \omega_k^2(\eta_-) = \omega_k^2(\eta_+) = 0 \). There is a physical momentum \( K^* \) for which the non-adiabatic and tachyonic regions intersect in just one point. For \( 0 < K < K^* \), the regions intersect and for \( K^* < K < K_{\text{max}} \) the tachyonic region and the non-adiabatic region are disconnected. In the tachyonic regime, we also use the WKB approximation. It is valid when \( \left| \Omega_k^{\prime \prime} \right| < 1 \) and the modes are
\[ \Psi_k(\eta) = \frac{a_k(\eta)}{\sqrt{2\Omega_k(\eta)}} e^{-\int^{\eta'}_\eta \omega_k(\eta')} d\eta' + \frac{b_k(\eta)}{\sqrt{2\Omega_k(\eta)}} e^{\int^{\eta'}_\eta \omega_k(\eta')} d\eta' \quad (49) \]
The Bogoliubov coefficients change after going through a non-adiabatic region.

The interaction time can be estimated and gives
\[ H\Delta t \approx \xi \quad (50) \]
leading to
\[ \Delta N = \frac{\Delta a}{a} \approx \xi \quad (51) \]
So the interaction region is spread out over a large number of efoldings. Moreover we must impose that the variation of the Hubble rate is small in the interaction region
\[ \frac{\Delta H}{H} \approx \frac{1}{g\epsilon \sqrt{\lambda}} \quad (52) \]
hence we must have \( g\epsilon \sqrt{\lambda} \gg 1 \).
3.4. Creation of particles

We are interested in the particles created when the inflationary brane crosses the trapped brane. This happens when the WKB approximation breaks down. Let us first concentrate on the $\xi \ll 1$ regime. The calculation of the number of particles in this region is well-known [40]. The Bogoliubov coefficient is obtained by expanding

$$\frac{\omega_k}{a} \approx \sqrt{K^2 + g^2|\phi - \phi_1|^2} \approx g|\dot{\phi}_1|\delta t \left(1 - \frac{K^2}{2g^2\dot{\phi}_1^2(\delta t)^2}\right)$$

around $\phi_1$ ($\delta t = t - t_1$) and integrating around a contour where the WKB approximation is still valid in the complex plane. The end result is that

$$|\beta_k|^2 = e^{-\frac{\kappa_1^2}{2h\eta_1}}$$

where $\kappa_1 = k/a_1$. As a result, the spectrum is Gaussian with a width determined by the speed of the brane.

In the $\xi \gg 1$ regime we have found different situations depending on the physical momentum $K$. The configuration where $0 < K < K_{\text{max}}$ is the most complex. There are two symmetric non-adiabatic regions far from $\phi_1$ and a tachyonic region in the proximity of $\phi_1$. The tachyonic region intersect with the non-adiabatic regions for $0 < K < K^* < K_{\text{max}}$. For $K_{\text{max}} < K < \sqrt{2 - 2^{2/3}H}$, the tachyonic region is still there but we no longer have any non-adiabatic region. For $\sqrt{2 - 2^{2/3}H} < K < \sqrt{2}H$, there is a tachyonic region and inside of it a non-adiabatic zone. For $\sqrt{2}H < K < \sqrt{2 + 2^{2/3}H}$, there is no tachyonic resonance but there is a non-adiabatic region around $\phi_1$. And finally for any $K > \sqrt{2 + 2^{2/3}H}$, there is no tachyonic resonance and the regime is always adiabatic. Notice that all these intervals are easy to interpret using the physical wave number $K$. The size of the physical intervals in $K$ is time-independent.

Let us first study the creation of particles for $0 < K^* < K < K_{\text{max}}$. This case is typical and will allow us to deduce the particle spectrum in the other intervals too.

**Figure 1.** Configuration of the interaction zone in the complex plane for $K^* < K < K_{\text{max}}$
Initially, for $\eta \to -\infty$, the modes are in a Bunch-Davies vacuum. Then the WKB approximation breaks down in the non-adiabatic region I, far enough from it the modes become

$$\Psi_k(\eta) = \frac{\alpha_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{-i \int^\eta \omega_k(\eta')d\eta'} + \frac{\beta_k(\eta)}{\sqrt{2\omega_k(\eta)}} e^{i \int^\eta \omega_k(\eta')d\eta'}$$

We need to determine the two Bogoliubov coefficients. In fact they can be expressed as transmission and reflection coefficients $t$ and $r$

$$\alpha_k(\eta < \eta_A) = t \alpha_k(\eta > \eta_A)$$

$$\beta_k(\eta > \eta_A) = r \alpha_k(\eta > \eta_A)$$

where the conformal time $\eta$ increases. So with the chosen initial condition $\alpha_k(-\infty) = 1$, this gives

$$\alpha_k(\eta > \eta_A) = 1/t, \quad \beta_k(\eta > \eta_A) = r/t$$

The solution before the non-adiabatic region I is linked to the solution on the other side of the non-adiabatic region by an analytic continuation in the complex plane. We consider that the time variable $\eta$ is a complex variable. We draw a contour around $\eta = \eta_A$ in the complex plane (see figure 1). The radius of this semi-circle must be large enough for the WKB approximation to be valid before and after the non-adiabatic region. This is the case if

$$\omega_k(\eta_A)|\eta - \eta_A| \gtrsim R(\eta_A) = R_{\text{max}}$$

and we trust the expansion of $\omega_k^{-1}$ around $\eta_A$

$$\frac{1}{\omega_k(\eta)} = \frac{1}{\omega_k(\eta_A)} - \left(\frac{\omega_k'}{\omega_k^2}\right)(\eta_A)(\eta - \eta_A) - \sum_{n=2}^{\infty} \frac{R_A^{(n-1)}}{n!}(\eta - \eta_A)^n$$

where $R_A^{(n)}$ is the n-th derivative of $R$ at $\eta_A$. The condition on the size of the semi-circle implies that

$$\omega_k(\eta) \approx -\frac{1}{R_{\text{max}}(\eta - \eta_A)}$$

Using the fact that in the non-adiabatic region $\omega_k(\eta_A) = O(aH)$ and $R_A = O(\frac{H}{K}) = O(1)$, the condition on the contour is equivalent to $\delta t \gtrsim 1/H$. Now the duration of the interaction region is $\frac{H}{\delta \varphi}$ implying that the contour circling around the non-adiabatic region is much smaller than the interaction region. As a result we obtain that

$$\exp \left( \pm i \int \omega_k(\eta')d\eta' \right) \approx \exp \left( -i \int \frac{1}{R_{\text{max}}(\eta' - \eta_A)}d\eta' \right)$$

Notice that $\eta - \eta_A$ is negative before the non-adiabatic region and positive later on. A positive-frequency mode is changed in a negative-frequency mode when going through the non-adiabatic region (and vice versa). With $(\eta - \eta_A) = \rho e^{i\theta}$ and $d\eta = i\rho e^{i\theta}d\theta$, the contour integral is given by the residue theorem. A factor of $i$ also appears from $\eta - \eta_A \to e^{i\pi}(\eta_A - \eta)$ in $\frac{1}{\sqrt{2\omega_k}}$ so finally

$$\beta_k = -i \exp(-2i\theta^A) \exp \left( \frac{\pi}{R_{\text{max}}} \right) = -i \exp(-2i\theta^A) \exp \left( \frac{3\sqrt{3}\pi \mathcal{K}_A}{2H} \right)$$
with $K_A = k/a_A$ and the phase is

$$\theta^A = \int_{-\infty}^{\eta_A} \omega_k d\eta$$

(63)

Probability conservation imposes $|\alpha_k|^2 - |\beta_k|^2 = 1$, so we deduce

$$\alpha_k = e^{i\varphi} \sqrt{1 + \exp \left( 3\sqrt{3\pi} K_A \frac{H}{H} \right)}$$

(64)

where $\varphi$ is a random phase.

We now compute the Bogoliubov coefficient $b_k$ of the non-vanishing wave in the tachyonic region [43]. We neglect the decaying mode in this region depending on the coefficient $a_k$. We draw a contour around the turning point $\eta_-$ where $\omega_k^2(\eta_-) = 0$. We assume the radius of the semi-circle is large enough for the WKB approximation to be valid along the contour:

$$\omega_k^2 = \frac{d\omega_k^2}{d\eta}(\eta_-)(\eta - \eta_-)$$

(65)

After an analytic continuation we find

$$b_k = \alpha_k e^{-i(\theta^- + \frac{\pi}{4})} + \beta_k e^{i(\theta^- + \frac{\pi}{4})}$$

(66)

with the phase

$$\theta^- = \int_{-\infty}^{\eta_-} \omega_k d\eta$$

(67)

depending on the wave evolution before the turning point. Then passing around the second turning point at $\eta_+$, we obtian a new contribution to the Bogoliubov coefficients.

$$\tilde{\beta}_k = e^{\int_{\eta_+}^{\eta_+} \Omega d\eta} e^{-i(\theta^- + \frac{\pi}{4})} b_k = e^{\int_{\eta_-}^{\eta_+} \Omega d\eta} \left( \beta_k + \alpha_k e^{-2i(\theta^- + \frac{\pi}{4})} \right)$$

(68)

$$\tilde{\alpha}_k = e^{\int_{\eta_-}^{\eta_+} \Omega d\eta} e^{i(\theta^- + \frac{\pi}{4})} b_k = e^{\int_{\eta_-}^{\eta_+} \Omega d\eta} \left( \alpha_k + \beta_k e^{2i(\theta^- + \frac{\pi}{4})} \right)$$

(69)

The final contribution comes from the non-adiabatic region II where the wave appears to break into reflected and transmitted ones

$$\tilde{\alpha}_k \chi_- \rightarrow R_- \chi_+ + T_- \chi_-$$

(70)

$$\tilde{\beta}_k \chi_+ \rightarrow R_+ \chi_- + T_+ \chi_+$$

(71)

We are particularly interested in the the final Bogoliubov coefficient

$$\tilde{\beta}^f = R_- + T_+$$

(72)

The first situation (70) is the same as in (56) but with a phase $\theta^B$.

$$R_- = \frac{r}{t} \tilde{\alpha}_k = -i \exp(-2i\theta^B) \exp \left( 3\sqrt{3\pi} K_B \frac{H}{H} \right) \tilde{\alpha}_k$$

(73)

where $K_B = k/a_B$. The second situation (71) is the dual configuration for $\Psi^*$, where we find

$$|T_+|^2 = |	ilde{\beta}^f|^2 \left( 1 + \left| \frac{r^*}{t^*} \right|^2 \right)$$

(74)
Brane Bremsstrahlung in DBI Inflation

via current conservation. Therefore we can introduce another phase $\vartheta$ such that

$$T_+ = e^{i\vartheta} \tilde{\beta}_k \sqrt{1 + e^{3\sqrt{3}\pi \frac{K_B}{H}}}$$ \hspace{1cm} (75)

As a result we find the Bogoliubov coefficient

$$\beta^f_k = -i e^{\int_{\eta_+}^{\eta_-} \Omega d\eta} \left( e^{2i(\theta - \theta_A - \theta_B)} e^{3\sqrt{3}\pi \frac{K_A + K_B}{2H}} + e^{i\vartheta + i\vartheta - i2\theta} \sqrt{1 + e^{3\sqrt{3}\pi \frac{K_A}{H}}} \sqrt{1 + e^{3\sqrt{3}\pi \frac{K_B}{H}}} \right)$$ \hspace{1cm} (76)

Let us now examine the other regions. For $K_{\text{max}}^2 < K^2 < (2 - 2^{2/3})H^2$, there is a purely tachyonic contribution. For $\alpha_k(-\infty) = 1$, we know from (68) that

$$|\beta^f_k|^2 = e^{2 \int_{\eta_+}^{\eta_-} \Omega d\eta}$$ \hspace{1cm} (77)

The same behaviour is valid for $(2 - 2^{2/3})H^2 < K^2 < 2H^2$ because the non-adiabatic region has no importance when it is included in the tachyonic regime. Indeed the passage through the non-adiabatic region in a tachyonic region only changes the phase of the Bogoliubov coefficients.

When $2H^2 < K^2 < (2 + 2^{2/3})H^2$, there is a non-adiabatic region centered at $\phi_1$. In this region where $K$ is large, the duration of the interaction is less than $1/H$. We continue the wave function in the complex plane on a semi-circle of radius larger than $1/H$ such that

$$\frac{\omega_k}{a} \approx \sqrt{K^2 - 2H^2 + g^2 \dot{\phi}_1^2 \delta t^2} \approx g \dot{\phi}_1 \delta t + \frac{K^2 - 2H^2}{2g \dot{\phi}_1 \delta t}$$ \hspace{1cm} (78)

Notice that the WKB approximation is valid along this circle. The integral $\int \frac{\omega_k}{a} dt$ picks up an imaginary value due to the residue at the origin. As in (62) positive and negative-frequency modes are exchanged. And the coefficient $\beta_k$ is computed with the residue theorem

$$|\beta^f_k|^2 = e^{\frac{2H^2 - K^2}{g \dot{\phi}_1}}$$ \hspace{1cm} (79)

Notice that this is a small number in the region where $K \geq O(\sqrt{2}H)$.

For $0 < K < K^*$, the method used for $K^* < K < K_{\text{max}}$ is not reproducible as the semi-circles drawn around the non-adiabatic regions I and II intersect the semi-circles drawn around the turning points. We will see that the low energy part of the spectrum is dominated by the tachyonic instability and therefore does not really depend on the non-adiabatic region. As a result we will extend our results to the whole momentum range $0 < K < K_{\text{max}}$.

We have now obtained the Bogoliubov coefficients in both regimes for $\xi$. This is particularly important as one can define an adiabatic invariant for equation (23)

$$\mathcal{N}_k = \frac{\omega_k}{2} \left( \frac{|\Psi'_k|^2}{\omega_k^2} + |\Psi_k|^2 \right) - \frac{1}{2} = |\beta^f_k|^2$$ \hspace{1cm} (80)

As explained in [30], this is the comoving occupation number of particles with momentum $k$. 

4. A DBI Chameleon

4.1. The modified potential

We will compute the energy density of the created particles. In fact this energy density appears to add a contribution to the potential of the inflaton. This effect corresponds to the slowing down of the inflationary brane by the trapped brane. The emitted energy density is

$$\rho_\chi = \int \frac{d^3k}{(2\pi)^3} \tilde{\omega}_k N_k$$

(81)

where the integration is done over the physical momenta. This brings an extra factor of $1/a^3$ corresponding to the dilution of the created particles. The frequency $\tilde{\omega}_k = \omega_k/a$ is the rescaled frequency coming from the energy in conformal time $\frac{1}{2}\tilde{\omega}_k^2|\chi|^2 = \frac{1}{2}\omega_k^2|\Psi|^2$.

Far from the trapped brane, once particles have been created in the immediate vicinity of the brane,

$$\rho_\chi \approx g|\phi - \phi_1|a^{-3} \int |\beta_k|^2 \frac{d^3k}{(2\pi)^3}$$

(82)

This approximation is valid as long as $g|\phi - \phi_1| \gg \sqrt{2H^2 - K^2}$. When $\xi \ll 1$, the right hand side is at most $\sqrt{2g|\dot{\phi}|}$, which implies that $|\phi - \phi_1| \gg \frac{\dot{\phi}}{g}$, i.e. outside of the interaction region. In the $\xi \gg 1$, the same condition gives $|\phi - \phi_1| \gg \frac{H}{g}$ which is also the size of the interaction region. In the effective description of the inflationary brane motion, this energy density is equivalent to a linear potential and a constant force towards the trapped brane. In the absence of the inflationary potential, this force may pull the passing brane towards the trapped brane. We are in a position to determine the effective potential in both cases $\xi \ll 1$ and $\xi \gg 1$.

When $\xi \ll 1$ the potential can be easily calculated as the particle spectrum is gaussian

$$\rho_\chi \approx \frac{1}{(2\pi)^3} \frac{a_3^3}{a^3} (g|\dot{\phi}_1|)^{3/2} g|\phi - \phi_1|$$

(83)

where $a_3 \equiv a_1$ and the energy density is diluted before finally tending to zero rapidly.

In the $\xi \gg 1$ regime, the creation of particles is largely dominated by the tachyonic instability in the vicinity of the trapped brane and depends on $\int_{t_0}^{t_1} \frac{\Omega_k}{a} dt$. The behaviour of this integral is dominated by the region of small momenta. Let us concentrate on the low frequency part of the spectrum. In this case the integration region is maximal and the turning points are located at

$$\delta t_\pm \approx \pm \sqrt{2} \frac{H}{g|\dot{\phi}_1|}$$

(84)

whose norm is always less than $t_1$. When $K$ is small, we can expand $\Omega_k$

$$\frac{\Omega_k}{a} \approx \sqrt{2H^2 - g^2\dot{\phi}_1^2\delta t^2} - \frac{1}{2\sqrt{2H^2 - g^2\dot{\phi}_1^2\delta t^2}} \frac{k^2}{a^2}$$

(85)
Integrating between the two turning points we find
\[ \int_{t_-}^{t_+} \Omega_k dt \approx \frac{\pi}{2} \frac{2H^2 - K_S^2}{g|\dot{\phi}_1|} \]  
(86)

where $K_S = k/a_S$ and we have defined
\[ \frac{1}{a_S^2} = \frac{1}{a_1^2} \frac{1}{\pi} \int_{-1}^{1} \frac{du}{\sqrt{1 - u^2}} (1 + \frac{\sqrt{2} \epsilon}{g\sqrt{\lambda} u})^{2/\epsilon} \]  
(87)

where we recall that $g\epsilon \sqrt{\lambda} \gg 1$. This result is very important as it shows that the spectrum is approximately Gaussian with a width $\sqrt{g|\dot{\phi}_1|}$ which is much smaller than the band of integration $\Delta K = \sqrt{2}H$. Moreover the amplitude of the number of particles depends on $e^{2\pi H^2/g|\dot{\phi}_1|}$ which is an exponentially large number. Notice that the same factor is negligible in the $\xi \ll 1$ regime. Now that we know that the spectrum is dominated by the tachyonic instability, we can evaluate the energy density
\[ \rho_X \approx \frac{9}{(2\pi)^3 a_S^3 \epsilon} \frac{m^2}{a_3^3} H^3 \sqrt{\epsilon \sqrt{\lambda} a_S^3 g} \]  
(88)

Notice that the main difference with the $\xi \ll 1$ is the presence of a large exponential factor coming from the tachyonic instability close to the trapped brane. Here also, the energy is diluted after going out of the interaction region. The scale factor $a_S$ takes into account the fact that momenta are red-shifted in the integration region.

In both cases the effective potential after the interaction region becomes
\[ V \approx m^2 \phi^2 + \frac{1}{(2\pi)^3} y(\xi) \frac{a_S^3}{a_3^3} H^3 g|\dot{\phi}_1| \]  
(89)

where the coupling function depends on $\xi$ and reads
\[ y(\xi) \approx \xi^{-3/2} \]  
(90)

when $\xi \ll 1$ and
\[ y(\xi) \approx 9\xi^{-3/2} e^{2\pi \xi} \]  
(91)

when $\xi \gg 1$. This potential is similar to the ones used in chameleon models [44]. It has a moving minimum where
\[ \phi_{\text{min}} = \frac{1}{(2\pi)^3} y(\xi) g \frac{a_S^3}{a_3^3} \frac{H^3}{2m^2} \]  
(92)

This minimum goes to the origin as the scale factor increases. This implies that the effect of the trapped brane is only relevant for a few e-foldings after the passage through the interaction region. Immediately after the passage, the minimum is located at
\[ \frac{\phi_{\text{ini}}}{\phi_1} \approx \frac{1}{16\pi^3} \frac{y(\xi) \epsilon \sqrt{\lambda} H^2}{3g M_p^2} \]  
(93)

This gives a criterion for the influence of the minimum of the trapped brane on the motion of the inflationary brane. If $\phi_{\text{ini}} \ll \phi_1$, the trapped brane has no influence as the inflaton feels the $m^2 \phi^2$ branch of the potential. On the other hand, if $H$ is large enough
and $\phi_{\text{ini}} \geq \phi_1$, the inflationary brane feels the steep potential due to the trapped brane. In this case, the motion of the inflationary brane is affected for a few e-foldings.

Let us evaluate the jump in the potential at the end of the interaction zone

$$\Delta V \approx \frac{9}{(2\pi)^3} \xi^{-3/2} e^{2\pi \xi} H^4$$

when $\xi \gg 1$ and

$$\Delta V \approx \frac{1}{(2\pi)^3} \frac{H^4}{\xi^2}$$

when $\xi \ll 1$. In both case it depends only on $H^4$ and $\xi$. For large enough $H$, this jump in $V$ can also change the Hubble rate due to the release of energy in the form of radiated particles.

As we have considered that $H$ is nearly constant and we have neglected any backreaction on the dynamics of the inflaton while the brane particles are created, we must impose that $\Delta V/V \leq 1$, which gives for $\xi \ll 1$

$$\frac{H^2}{M_p^2} \leq 3.(2\pi)^3 g^2 \xi^2$$

And for $\xi \gg 1$,

$$\frac{H^2}{M_p^2} \leq 3.(2\pi)^3 g^2 \xi^{3/2} e^{-2\pi \xi}$$

This condition gives an upper limit on $\xi$. In fact, if we require that the Hubble rate $H$ should be at least of order 1 GeV, $\xi$ must be at most equal to $\xi_{\text{lim}} \approx 14$ for $g \sim 10^{-1}$. For larger values of $\xi$, the tachyonic instability implies that there is a strong backreaction due to the explosive creation of particles. In this case, the inflaton loses all its energy very quickly and transfers it into radiated particles.

4.2. Discussion

Let us now make explicit some of the constraints on the parameters of the model. Using the condition $\epsilon \ll 1$, we have

$$m \gg \frac{g}{\sqrt{\lambda}} M_p$$

Hence the masses must be large enough and related to the Planck mass. This implies that

$$\xi \gg \frac{1}{g\sqrt{\lambda}}$$

guaranteeing the constancy of $H$ in the $\xi \ll 1$ region. Using this lower bound we obtain that

$$m \gg \frac{g^2}{\xi} M_p$$

For a fixed string coupling $g_s$, we see that $\xi$ determines the range of masses leading to inflation.
When $\xi$ is large, the mass $m$ can be smaller than the Planck scale. In the large $\xi$ regime we have

$$g^2 \gg \frac{m}{M_p}$$

and

$$\lambda \gg \frac{M_p}{m}$$

This regime tends to favour small masses and a large compactification radius.

So far we have not tried to connect DBI inflation to observations. Let us now assume that the DBI inflation regime we have analysed is responsible for the phase of inflation resulting in the quantum fluctuations leading to the CMB spectrum. In this case, the COBE normalisation determines the curvature perturbation

$$\mathcal{P}_s = \frac{H_{\text{inf}}^2\gamma}{4\pi^2M_p^2\epsilon} = \frac{g^4\xi^2}{2\pi^2} = \zeta^2 \sim 10^{-10}$$

So $\zeta \approx g^2\xi \sim 10^{-5}$. For a reasonable value of the string coupling $g_s \sim 10^{-2}$, we find that inflationary branes whose quantum fluctuations lead to the CMB anisotropies must be in the $\xi \ll 1$ regime. We have another observational constraint; since gravitational waves have not been detected yet, the ratio $r = \frac{P_t}{P_s}$ must be small. The tensor perturbations spectrum being

$$\mathcal{P}_t = \frac{4H_{\text{inf}}^2}{\pi M_p^2}$$

We deduce

$$r = \frac{16\epsilon}{\gamma} \leq 1$$

With both (103) and (105), we find

$$\frac{H}{M_p} \leq 10^{-5}$$

We note that this upper bound for the Hubble rate is smaller than the bound given by (96) and therefore the backreaction is always negligible in the $\xi \ll 1$ regime.

We have seen that in this regime the slowing down by a trapped brane can be effective if

$$\frac{H^2}{M_p^2} \geq \frac{3.16\pi^3g\xi^{3/2}}{\epsilon\sqrt{\lambda}}$$

or equivalently

$$\frac{H^2}{M_p^2} \geq 16\sqrt{3\pi^3}\xi^{3/2} \frac{m}{M_p}$$

Using (100), we find

$$\frac{H^2}{M_p^2} \geq 16\sqrt{3\pi^3}g\xi^{1/2}$$
This is not compatible with (106). So it is not possible to slow the motion of the inflationary brane drastically after hitting a single trapped brane. If we assume that there exists a stack of N closely packed branes in the interaction region, then equation (93) becomes

$$\frac{\phi_{\text{ini}}}{\phi_1} \approx \frac{1}{16\pi^3} \frac{g(\xi)\epsilon\sqrt{\lambda}}{3g} N \frac{H^2}{M_p^2}$$

(110)

And condition (109) becomes:

$$\frac{H^2}{M_p^2} \geq \frac{1}{N} \frac{1}{16\sqrt{3\pi^3}g{\zeta}^{1/2}}$$

(111)

Unless we have at least $N \sim 10^9$ branes in the stack, the slowing effects of the stack is not drastic. The motion of an inflationary brane leading to the CMB spectrum is hardly affected by trapped branes. On the contrary, branes in a regime $\xi \gg 1$ are efficiently slowed down because of radiated particles: a brane bremsstrahlung.

5. Conclusion

We have studied the slowing down of an inflationary brane in the DBI regime along an AdS throat when it hits a trapped brane. We have shown that the brane motion may be slowed down by the creation of particles on the trapped brane. This creation occurs either by parametric resonance or tachyonic instability. The latter case happens when the interaction region is large compared to the Hubble rate. Once the brane has left the interaction region, the effect of the radiated particles is to generate a linear potential whose slope is greatly enhanced in the tachyonic case. This is enough to stop the brane for a few e-foldings until the number of created particles has been diluted. Branes crossing the trapped brane in a time smaller than a Hubble time, are not drastically affected by the radiation of particles as they are slowed down in manner which does not alter their motion very much. A dramatic effect is only possible when a very large number of trapped branes are stuck in the interaction region.

An interesting consequence of our results is a selection mechanism for the motion of branes in a throat where trapped branes are also present. Indeed, as the branes go past the trapped branes, the ones leading to a tachyonic instability are successively slowed down. Only the ones with little interaction with the trapped branes can keep on moving. It happens that the ones with small curvature perturbations are the ones which would be little hampered by the presence of trapped branes. This could have an interesting effect selecting the motion of branes whose inflaton would generate small levels of curvature perturbations.

Another interesting possibilities is the modification of the perturbation spectra as the DBI brane crosses one or many trapped branes. Even if the motion of the DBI brane is not altered, the slowing down term in the potential having an effective relevance during a short time, one may envisage that the spectrum of primordial fluctuations might be affected by a jump in the derivative of the potential. This may even be the case when
the slowing down of the inflaton is negligible. A thorough study of this possibility as well as models with a jump in the potential energy is left for future work.

6. Acknowledgements

We would like to thank D. Easson for interesting suggestions and J. Martin for discussions and remarks. This work is partially funded by the "DarkPhys" ANR grant.

References

[1] G. R. Dvali and S. H. Tye, “Brane inflation,” Phys. Lett. B 450 (1999) 72 [arXiv:hep-ph/9812483].
[2] G. R. Dvali, Q. Shaﬁ and S. Solganik, “D-brane inflation,” arXiv:hep-th/0105203.
[3] C. P. Burgess, M. Majumdar, D. Nolte, F. Quevedo, G. Rajesh and R. J. Zhang, “The Inﬂationary Brane-Antibrane Universe,” JHEP 0107 (2001) 047 [arXiv:hep-th/0105204].
[4] S. H. S. Alexander, “Inﬂation from D-anti-D brane annihilation,” Phys. Rev. D 65 (2002) 023507 [arXiv:hep-th/0105032].
[5] J. H. Brodie and D. A. Easson, “Brane inﬂation and reheating,” JCAP 0312 (2003) 004 [arXiv:hep-th/031138].
[6] S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, “Towards inﬂation in string theory,” JCAP 0310 (2003) 013 [arXiv:hep-th/0308055].
[7] D. Baumann, “TASI Lectures on Inﬂation,” arXiv:0907.5424 [hep-th] (review on inﬂation).
[8] [Planck Collaboration], “Planck: The scientiﬁc programme,” arXiv:astro-ph/0604069
[9] L. Lorenz, J. Martin and C. Ringeval, “Brane inﬂation and the WMAP data: a Bayesian analysis,” JCAP 0804 (2008) 001 [arXiv:0709.3758 [hep-th]].
[10] L. Lorenz, J. Martin and C. Ringeval, “Constraints on Kinetically Modiﬁed Inﬂation from WMAP5,” Phys. Rev. D 78 (2008) 063543 [arXiv:0807.2414 [astro-ph]].
[11] L. Lorenz, J. Martin and C. Ringeval, “K-inﬂationary Power Spectra in the Uniform Approximation,” Phys. Rev. D 78 (2008) 083513 [arXiv:0807.3037 [astro-ph]].
[12] J. E. Lidsey and I. Huston, “Gravitational wave constraints on Dirac-Born-Infeld inﬂation,” JCAP 0707 (2007) 002 [arXiv:hep-th/0705.0240].
[13] R. Kallosh, “On Inﬂation in String Theory,” Lect. Notes Phys. 738 (2008) 119 [arXiv:hep-th/0702059].
[14] M. Alishahiha, E. Silverstein and D. Tong, “DBI in the sky,” Phys. Rev. D 70 (2004) 123505 [arXiv:hep-th/0404084].
[15] J. P. Conlon and F. Quevedo, “Kaehler moduli inﬂation,” JHEP 0601 (2006) 146 [arXiv:hep-th/0509012].
[16] J. J. Blanco-Pillado, D. Buck, E. J. Copeland, M. Gomez-Reino and N. J. Nunes, “Kaehler Moduli Inﬂation Revisited,” arXiv:0906.3711 [hep-th].
[17] N. Barnaby, J. R. Bond, Z. Huang and L. Kofman, “Preheating After Modular Inﬂation,” arXiv:0909.5053 [hep-th].
[18] J. J. Blanco-Pillado et al., “Racetrack inﬂation,” JHEP 0411 (2004) 063 [arXiv:hep-th/0406230].
[19] J. J. Blanco-Pillado et al., “Inﬂating in a better racetrack,” JHEP 0609 (2006) 002 [arXiv:hep-th/0603129].
[20] P. Brax, A. C. Davis, S. C. Davis, R. Jeannerot and M. Postma, “D-term Uplifted Racetrack Inﬂation,” JCAP 0801 (2008) 008 [arXiv:0710.4876 [hep-th]].
[21] Ph. Brax, S. C. Davis and M. Postma, “The Robustness of n_s < 0.95 in Racetrack Inﬂation,” JCAP 0802 (2008) 020 [arXiv:0712.0535 [hep-th]].
[22] E. Silverstein and D. Tong, “Scalar Speed Limits and Cosmology: Acceleration from D-cceleration,” Phys. Rev. D 70 (2004) 103505 [arXiv:hep-th/0310221].
[23] S. Kachru, R. Kallosh, A. D. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” Phys. Rev. D 68 (2003) 046005 [arXiv:hep-th/0301240].
[24] I. R. Klebanov and M. J. Strassler, “Supergravity and a confining gauge theory: Duality cascades and chiSB-resolution of naked singularities,” JHEP 0008 (2000) 052 [arXiv:hep-th/0007191].
[25] D. A. Easson, S. Mukohyama and B. A. Powell, “Observational Signatures of Gravitational Couplings in DBI Inflation,” arXiv:0910.1353 [astro-ph.CO].
[26] N. Barnaby, C. P. Burgess and J. M. Cline, “Warped reheating in brane-antibrane inflation,” JCAP 0504 (2005) 007 [arXiv:hep-th/0412040].
[27] J. Lachapelle and R. H. Brandenberger, “Preheating with Non-Standard Kinetic Term,” JCAP 0904 (2009) 020 [arXiv:0808.0936 [hep-th]].
[28] A. C. Davis and R. H. Ribeiro, “Enhanced (p)reheating in DBI Inflation,” arXiv:0908.4217 [hep-th].
[29] A. H. Guth Phys. Rev. D 23, 347 (1981), A. D. Linde Phys. Rev. Lett. B 108, 389 (1982), A. Albrecht and P. J. Steinhardt Phys. Rev. Lett 48, 1220 (1982)
[30] L. Kofman, A. D. Linde and A. A. Starobinsky, “Towards the theory of reheating after inflation,” Phys. Rev. D 56 (1997) 3258 [arXiv:hep-ph/9704452].
[31] G. N. Felder, L. Kofman and A. D. Linde, “Instant preheating,” Phys. Rev. D 59 (1999) 123523 [arXiv:hep-ph/9812289].
[32] D. Baumann and L. McAllister, “Advances in Inflation in String Theory,” arXiv:0901.0265 [hep-th].
[33] D. Baumann, A. Dymarsky, S. Kachru, I. R. Klebanov and L. McAllister, “Holographic Systematics of D-brane Inflation,” JHEP 0903 (2009) 093 [arXiv:0808.2811 [hep-th]].
[34] D. Baumann, A. Dymarsky, I. R. Klebanov, J. M. Maldacena, L. P. McAllister and A. Murugan, “On D3-brane potentials in compactifications with fluxes and wrapped D-branes,” JHEP 0611 (2006) 031 [arXiv:hep-th/0607050].
[35] D. A. Easson and R. Gregory, “Circumventing the eta problem,” arXiv:0902.1798 [hep-th].
[36] D. Baumann, A. Dymarsky, I. R. Klebanov, L. McAllister and P. J. Steinhardt, “A Delicate Universe,” Phys. Rev. Lett. 99 (2007) 141601 [arXiv:0705.3837 [hep-th]].
[37] B. Underwood, “Brane Inflation is Attractive,” Phys. Rev. D 78 (2008) 023509 [arXiv:0802.2117 [hep-th]].
[38] S. Bird, H. V. Peiris and D. Baumann, “Brane Inflation and the Overshoot Problem,” Phys. Rev. D 80 (2009) 023534 [arXiv:0905.2412 [hep-th]].
[39] D. Green, B. Horn, L. Senatore and E. Silverstein, “Trapped Inflation,” arXiv:0902.1006 [hep-th].
[40] L. Kofman, A. D. Linde, X. Liu, A. Maloney, L. McAllister and E. Silverstein, “Beauty is attractive: Moduli trapping at enhanced symmetry points,” JHEP 0405 (2004) 030 [arXiv:hep-th/0403001].
[41] X. Chen, “Inflation from warped space,” JHEP 0508 (2005) 045 [arXiv:hep-th/0501184].
[42] S. Kecskemeti, J. Maiden, G. Shiu and B. Underwood, “DBI inflation in the tip region of a warped throat,” JHEP 0609 (2006) 076 [arXiv:hep-th/0605189].
[43] J. F. Dufaux, G. N. Felder, L. Kofman, M. Peloso and D. Podolsky, “Preheating with Trilinear Interactions: Tachyonic Resonance,” JCAP 0607 (2006) 006 [arXiv:hep-ph/0602144].
[44] P. Brax, C. van de Bruck, A. C. Davis, J. Khoury and A. Weltman, “Detecting dark energy in orbit: The cosmological chameleon,” Phys. Rev. D 70 (2004) 123518 [arXiv:astro-ph/0408415].