Exploration of students’ representation in solving Pythagorean theorem problems based on cognitive style

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Abstract. This study aims to explore the students’ representations in solving Pythagorean theorem problems. Participants of this study consisted of a female student with reflective style and a female student with impulsive cognitive style. They had an equivalent mathematical ability. This study used the qualitative method. The researcher decides on the methods of data collection based on mathematical problem-solving tasks and task-based interviews. To obtain valid data, the researcher performs the time triangulation. The findings of the study showed that: (1) the mathematical representations of reflective student were (a) recognizing and presenting known and unknown information visually by sketching the car’s path, (b) revealing then formulating problem-solving strategy using visual representation, (c) revealing mathematical model manipulation involve Pythagorean theorem, and (d) looking back the solution by using verbal representation; (2) the mathematical representations of impulsive student were (a) recognizing then presenting known and unknown information verbally, (b) revealing then formulating a problem-solving strategy using visual representation, (c) revealing mathematical model manipulation by saying that the distance between point A and B can be determined using the Pythagorean theorem, and (d) looking back the solutions by using verbal representations.

1. Introduction

Mathematics objects are abstract ideas. Students can comprehend and engage it only through representations. Representations can be used to communicate, and reason mathematical ideas. A representation is any configuration of signs, characters or objects that stand for something else [1]. Stylianou emphasized representation respect both to process and to the product i.e. both to the act of capturing a mathematical concept or relationship in a given form and to the form of this symbolization itself [2].

Yee and Bostic classify representation into symbolic and nonsymbolic. Symbolic representations include abstract ideas and numerical expressions; and nonsymbolic representations concrete model, pictorial, tabular, and mixed [3]. In this study, representations defined in the form of visual, verbal, and formal notation. Visual representations include pictures, diagrams, sketches, number lines, and graphs. Verbal representations include written and spoken words. Formal notation representations include expressions, formula, equation, and inequalities.
Research on representations showed that representation played a critical role in mathematics learning [4]. Students use representation as a tool to assist in mathematical understanding. This corroborates with the National Council of Teachers of Mathematics which recommended that students be given the opportunity to a variety of representations to foster mathematical understanding [5]. Furthermore, Lester and Kehle pointed out successful problem solving involves combining earlier experiences, knowledge, familiar representations, patterns of generalization, and intuition in a capacity to create new representation [6].

In the past several decades, representations have played an important role in learning mathematics especially in mathematical problem-solving [3,7–9]. Selling suggests the significance of giving all students entry to mathematics through representations [10]. Students need to monitoring and unpack specific patterns inside the problem while solving a mathematical application problem. Thus, students need to devise a concrete application problem into an abstract mathematical problem [11]. In the formulation process, students must employ multiple representations to cope with the same problem in different ways. Stylianou point out representations as tools towards the understanding, exploration, recording, and monitoring of problem-solving [8]. Furthermore, a study by Yee and Bostic [3] scrutinized that facility with various forms of representations was associated with higher problem-solving performance. Similarly, a quantitative study on the propositional logic problem emphasized the importance of various forms of representations in mathematics learning and assessment [12]. A study by Zahner and Corter suggested that the use of specific external visual representations was associated with greater rates of solution success in a probability problem [9].

In accomplish mathematical problems, every student has a dissimilar habitual approach. The individual habitual approach in perceiving, processing, organizing, remembering, and representing information to solve problems is called cognitive styles. When students have different cognitive styles, the manner to solve problems is also different, which subsequently affects the representation of students in solving the mathematical problem. There are students who are fast and some are slow to move from concrete to abstract mathematical ideas. In the problem solving process, there are students who tend to make a lot of mistakes and some of them who tend to be accurate. Therefore, there is still a great deal of work to be in this area.

One of the mathematical problems often encountered in everyday life is the problem of the application of the Pythagorean theorem. For example, a builder is examining an error, before making a building foundation. In examining this intellect, they used Pythagorean Triples 6, 8, 10, even though scientifically, the builder did not understand the reason why using it.

There has been less previous evidence for the student’s mathematical representation in solving the problem based on the difference of cognitive style especially impulsivity-reflectivity. Consequently, research is needed to describe the students’ representations based on impulsivity-reflectivity. Thus, the present study purposes to investigate how representations student with reflective and impulsive style in solving Pythagorean theorem problem.

2. Methods
This work used a qualitative approach to investigate students’ representations in solving Pythagorean theorem problems. There were two female participants in this study consisted of a student with reflective style and a student with impulsive style. They have the same mathematical ability. This study was conducted in one of a private junior high school in Surabaya, Indonesia.

In order to portray to how representations student’s with reflective and impulsive style in solving Pythagorean theorem problem, the researcher was designed mathematical problem-solving tasks and interview guidelines. The researcher performs the data triangulation to get convincing data. The analyzed data included: data reduction, data presentation, and conclusion.

3. Results and Discussion
The Pythagorean theorem problem which assigning to both participants as follows.
A car is initially at point A moving 200 m to the north, then moving 100 m to the east, and then moving 100 m to the south. Then the car moves 50 m to the east, then goes north as far as 100 m and stops at point B. What is the distance between point A and point B?
3.1 Description of Reflective Style (RS) Students’ Representations in Solving the Application of Pythagorean Theorem Problem

The interviews excerpts with the reflective student are presented as follows.

R: From this information, is there a strategy to solve the problem?
RS: Already.
R: What is it like?
RS: From point A it will then be drawn, then north, east, then south, then east again, north and stop at point B.
R: What is the picture like?
RS: Draw a straight line up or 200 m north, continue to turn right 100 m, then drop half as far as 100 m, turn right again 50 meters, and go up 100 m until it stops at point B.
R: After sketching, what about the next step?
RS: Point A is connected to point B.
R: Then what are the points X, Y, C and E in the picture for what?
RS: To clarify the picture.
R: Oh, let’s be clear about that. What was asked then?
RS: Distance A to B.
R: To get this distance, how do you find it?
RS: When finished drawing, the path of the car forms a right triangle.
R: How do you know the right triangle?
RS: Because points A, X, and B form a right triangle.
R: Where is the 150 meter from?
RS: Oh it’s from a distance of XY 100 meters and under CE 50 meters. So, 100 + 50 = 150 meters.
R: Then, what does \(a^2 + b^2 = c^2\) mean?
RS: Pythagorean formula.

From solution and result of interviews above, it can be seen reflective student’s mathematical representations at the stage of understanding the problem of the application of the Pythagorean theorem were recognizing then presenting known information visually (sketch). She was sketching the car’s track then presenting known information visually in the form of a sketch that describes the car’s track. Furthermore, she was recognizing then presenting known information verbally by restating the problem orally, recognizing then presenting unknown visually, and recognizing then presenting unknown verbally.

At the stage of devising a plan, reflective student’s mathematical representations were revealing a problem-solving plan using visual representation. The reflective student doing a sketch based on the available information to visually represent the problem. Furthermore, she was formulating a plan by using visual representations in the form of pictures and verbal representations in the form of verbal words. Thus, the reflective student making a general plan and selecting relevant methods from known and unknown information on the problem.

Reflective student’s mathematical representations at the stage of carrying out the plan of the application of the Pythagorean theorem problem were revealing mathematical model manipulations that contain mathematical symbols and formulas according to the formal rules and expressing the interpretation of the results obtained verbally. From the excerpts of the interview, she was constructing
points and lines to represent the car’s path and clarify the problem situation. She was revealing that the car's path formed a right triangle so that the distance between point A and point B can be solved using the Pythagorean theorem. Thus, she was creating notation formal and visual representations using points, lines, and shape to represent the information on relationships that are involved in the problem. Moreover, reflective student keeping the track to obtain the answer.

Finally, at the stage of looking back, the reflective student was checking the correctness of the solutions by using verbal representations. Moreover, she was revealing orally the steps of completion from the beginning to the end and re-examining the scribble on the blurred paper.

The findings of the completion and description showed that the reflective students succeed to arrive a solution and writes detailly the solutions. The reflective student employs various representations (visual, verbal, and formal notation) in solving the Pythagorean theorem problem. This finding aligns with the previous study[5] that effective problem solvers perceive problem-solving as an opportunity to employ various representations. In addition, reflective students use long periods of time when solving Pythagorean theorem problems but the answers written are accurate. This corroborates with the findings of Kagan[13] reflective students have the characteristics of being slow in solving the problem but the solutions tend to be accurate.

3.2. Description of Impulsive Style (IS) Students’ Representations in Solving the Application of Pythagorean Theorem Problem

The interviews excerpts with the impulsive student are presented as follows.

R: From this information, is there a strategy to work on the problem?
IS: Yes.
R: What is it like?
IS: How do I draw it first.
R: What do you mean?
IS: The line is only though it is not 200 meters, it is like that.
R: Okay. Can you explain the next step?
IS: Firstly, I am observing. I use a helpline, if for example, it matches the formula, I use the formula.
R: What is the formula?
IS: If I can use the Pythagorean formula.
R: Oh yes. How to determine using the Pythagorean formula?
IS: From point A to point B, I connect it to form a right triangle.
R: Then, what was asked?
IS: The distance between points A and B.

Based on the solution and the results of interviews it can be seen that the impulsive student’s mathematical representations at the stage of understanding the problem were recognizing then presenting known information verbally and recognizing then presenting unknown verbally. She was rephrasing the original problem so that the statement of the problem became familiar and hence more accessible.

Impulsive student’s mathematical representations at the stage of devising a plan were revealing a plan or problem-solving strategy by using visual representation (pictures) then formulating a plan or
problem-solving strategy by using visual representations (pictures). She was sketching the car’s path then presenting known information visually in the form of a sketch that describes the car’s path. Moreover, she was formulating plans or problem-solving strategies using verbal representations in the form of words. She was revealing that she used a helpline to portray the known and unknown information on the problem but less accurate. She was constructing points and lines to portray the given information but less accurate and doing errors in representing the right triangle.

Impulsive student’s mathematical representation at the stage of carrying out the plan is was revealing mathematical model manipulations that contain mathematical symbols and formulas according to the formal rules and expressing verbally interpretations of the results. She was employing mathematical symbols or formal notation (i.e. Pythagorean theorem) to keep the track to arrive at the solution.

The impulsive student’s mathematical representation at the stage looking-back of the Pythagorean theorem problem was checking the correctness of the solutions using verbal representations. The impulsive student revealing verbally the steps of completion from the beginning to the end and checking the correctness of the calculations on the blurred paper.

The findings of the completion and description showed that impulsive students write the solutions concisely. The impulsive student employ multiple representations (visual, verbal, and formal notation) in solving the Pythagorean theorem problem. In addition, impulsive students solving briefly problems but the solutions are less accurate. This is in aligns with Kagan [13] that impulsive students use alternatives in a short and quick way to solve problems but less accurate.

4. Conclusion
Reflective students write details the solutions using various representations (visual, verbal, and formal notation) in solving Pythagorean theorem problem. In addition, reflective students use a long time when solving problems but the solution are precisely written. Meanwhile, impulsive students solving the Pythagorean theorem problem by using verbal and notation formal representations. In addition, impulsive students use a brief time to solve problems but the written answers are less accurate.

The findings of this study have clear implications for teaching Pythagorean theorem. Especially, teachers should have design learning and teaching strategies respect to the students' cognitive style. The teacher can be fostering student to create and use mathematical representation in accomplish problem.

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