Enhancement of Persistent Currents by Hubbard Interactions In Disordered 1D Rings: Avoided Level Crossings Interpretation

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Abstract

We study effects of local electron interactions on the persistent current of one dimensional disordered rings. For different realizations of disorder we compute the current as a function of Aharonov-Bohm flux to zeroth and first orders in the Hubbard interaction. We find that the persistent current is enhanced by onsite interactions. Using an avoided level crossings approach, we derive analytic formulas which explain the numerical results at weak disorder. The same approach also explains the opposite effect (suppression) found for spinless fermion models with intersite interactions.

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I. INTRODUCTION

Recent experiments have found that small isolated metallic rings, threaded by magnetic flux, carry persistent currents. Although the existence of this effect had been anticipated theoretically for many years [1–4], the magnitude of the observed current was found to be much larger than expected.

Experiments performed on moderately disordered rings with small transverse width have measured the magnetic response of an ensemble of copper rings [5], of isolated gold loops [6], and of a clean, almost one-dimensional, sample [7].

The ensemble averaged persistent current was found to have an amplitude of order $10^{-2} ev_f/L$ and periodicity $\Phi_0/2$, (where $L$ is the circumference of the ring, $v_f$ is the Fermi velocity, $e$ is the electron charge, and $\Phi_0 = \hbar/e$ is the flux quantum). The single loops experiment reported finding an unexpectedly large current of order $ev_f/L$, the free electron value, and displaying periodicity in flux with period $\Phi_0$.

Theoretical approaches can be classified into non-interacting and interacting electrons approaches. Non interacting theories which take into account the effects of disorder and deviations from a perfect 1D geometry [8], predict values of the average persistent current that are smaller than observed, but still within one or two orders of magnitude of the experimentally determined value of the average current. More surprisingly, the theoretical estimates for the typical value of the persistent current, measured in the single loop experiment, are perhaps two or three orders of magnitude smaller than the reported values. It seems that agreement between theory and experiment worsens with increasing disorder. This suggests that perhaps interaction corrections are quantitatively important in the moderately disordered regime.

A first guess would be that interactions enhance the impurity scattering and further suppress the persistent current, as happens for the conductance of the disordered luttinger liquid [9]. However, this intuitive analogy may be quite misleading. While the conductance depends on the Fermi surface properties, i.e. velocity and scattering rates, the persistent
current is a sum of contributions from all occupied states. Also, by Galilean invariance, an interacting electron liquid without disorder carries the same current as the non-interacting gas, even though its Fermi velocity may be renormalized. Thus it is plausible that in the disordered system, interactions could work both ways, i.e. either suppress or enhance the persistent currents under conditions which need to be explored.

Exact diagonalization studies of spinless fermions with intersite interactions on small rings [10,11], have found that weak interactions reduce the persistent current below its non-interacting value. On the other hand, recent reports [12,13] for Hubbard interactions found the opposite effect.

In this paper [14] we clarify this seeming contradiction, and gain some insight into the puzzle of interaction corrections. In Section II we diagonalize the tight binding Hamiltonian numerically for different realizations of disorder, and compute the persistent current to zeroth and to first order in the Hubbard interactions. We find that interactions enhance the persistent current. In Section III we present the Avoided Levels Crossing (ALC) theory, which provides an analytic approximation to the numerical results. This is an expansion, at weak disorder, of nearly degenerate eigenstates at fluxes close to points of time reversal symmetry. The correlation between the zeroth and first order currents is explained by a common mechanism, i.e. the avoided level crossings. The opposite (suppression) effect for interacting spinless fermions, is also explained by the ALC theory in Section IV. We conclude by a summary and future directions.

II. NUMERICAL PERTURBATION THEORY

We consider a tight-binding Hamiltonian on a periodic chain with repulsive on-site Hubbard interaction:

\[ H = H_0 + U \sum_i n_{i\uparrow} n_{i\downarrow}, \]  
(1)

\[ H_0 = -\sum_{i,\sigma} \left[ e^{i \frac{2\pi}{L} \Phi} a_{i+1\sigma}^\dagger a_{i\sigma} + \text{h.c.} \right] + \sum_{i,\sigma} \epsilon_i a_{i\sigma}^\dagger a_{i\sigma}. \]  
(2)
where $a^\dagger_{i\sigma}$ creates an electron at site $i$ with spin $\sigma$. Our unit of energy is the hopping energy. $\epsilon_i$ is the dimensionless on-site disorder energy uniformly distributed in the interval $[-W/2, W/2]$. The chain has $L$ sites, $N_e$ electrons, the flux through its center is given by $\Phi$. With these conventions, the energy spectrum is periodic in the enclosed flux with period $\Phi_0$, and the current is given by

$$I(\Phi) = -\frac{\partial}{\partial \Phi} E(\Phi),$$  \hspace{1cm} (3)

where

$$E(\Phi) = E_0(\Phi) + E_1(\Phi) + O(U^2).$$  \hspace{1cm} (4)

$E_0$ is the exact ground state energy of $H_0$, whose single electron eigenstates are determined numerically. $E_1$ is the first order correction in $U$, which is given by

$$E_1 = U \sum_i n_{i\uparrow} n_{i\downarrow}$$

$$n_{i\sigma} = \langle \Psi_0 | a^\dagger_{i\sigma} a_{i\sigma} | \Psi_0 \rangle$$  \hspace{1cm} (5)

where $\Psi_0$ is the Fock ground state of $H_0$. Thus by diagonalizing $H_0$ we can readily obtain

$$I(\Phi) = I_0(\Phi) + I_1(\Phi) + O(U^2).$$  \hspace{1cm} (6)

The effects of disorder on $I_0(\Phi)$ can be seen in Fig. 1, where it is shown as a function of flux for a half filled lattice of six sites, for different disorder strengths $W$. Disorder smoothens and reduces the magnitude of $I_0$, and for $W >> 1$ it is dominated by the first harmonic $\sin(2\pi \Phi/\Phi_0)$.

In Fig. 2, $I_1(\Phi)$ is plotted. There are several features of the first order interaction correction that deserve comment. First, the most important observation is that it generally enhances the noninteracting current for all values of disorder strength and flux. That is to say: there is a positive correlation between the noninteracting current and the first order correction,

$$\langle I_0(\Phi) I_1(\Phi) \rangle_{\epsilon, \Phi} \geq 0.$$  \hspace{1cm} (7)
This result is found numerically to hold for all realizations of disorder which we have used. Second, in the limit of weak disorder, $I_1(\Phi)$ becomes singular at discontinuity points of $I_0(\Phi)$, which are at fluxes $(m + \frac{1}{2})\Phi_0$, $(m\Phi_0)$ for even (odd) number of filled orbitals, where $m$ are integers. Finally, we see the first order current also becomes dominated by its first harmonic at large disorder.

To study the scaling properties of the current with system size and disorder strength, it is convenient to characterize the strength of both $I_0$ and $I_1$ by their amplitude at $\Phi = \Phi_0/4$. And while it does not capture the singular nature of $I_1(\Phi)$ near the regime of very weak disorder, this characterization is still useful to study the scaling properties of the first order current, even in this limit. Previous studies have found both numerically and analytically, that the amplitude of the noninteracting current, averaged over disorder, behaves like:

$$|I_0(\Phi_0/4)| \simeq \frac{1}{2} \frac{e v_f}{L} \exp(-L/\xi).$$

By fitting to this functional form we extracted the behavior of the localization length $\xi$ for different system sizes and disorder values. Figure 3 shows the values of the localization length obtained for sizes $L=6,10,14,20$ and 25, at half-filling, averaged over many (up to four thousand) impurity configurations for each value of $W$. We find good agreement between our inferred value of the localization length and the known asymptotic form in the weak disorder limit: $\xi = 105/W^2$, valid when $W \ll 2\pi$ for large systems at half-filling.

There is less agreement in the strongly localized limit where the localization length is known to behave like: $\xi = (\ln(W/2) - 1)^{-1}$. We find a better fit taking $\xi = 1/\ln(W^{0.6}/2.5)$ for the $L = 25$ data. This discrepancy could be due to the small sizes considered, or a breakdown of Eq. (8) when $\xi$ is of order unity. We have calculated the amplitude $I_1(\Phi = \Phi_0/4)$ as function of strength of disorder, characterized by the scaling parameter $L/\xi$, for system sizes of $L=6,10,14,20,25$. At weak disorder (large $\xi$), the amplitude increases with the strength of disorder achieving its maximum value at some intermediate strength of disorder ($\xi \simeq L$). In this weakly disordered regime, a single scaling function could be used to describe the
This behavior is similar to that of the amplitude of the noninteracting current given by (8). Upon increasing the impurity scattering further, the first order current decreases with disorder. When the single particle wavefunctions are sufficiently localized, $I_1$ can be described by a different scaling form:

$$|I_1| \sim g(L/\xi),$$

(10)

where $g$ is a decreasing function of its argument. Eq. (10) suggests that in the localized regime $I_1$ dominates $I_0$ for large system sizes. However for localized doubly occupied single electron states

$$\langle \psi_{\alpha \uparrow} \psi_{\alpha \downarrow} | U \sum_i n_{i \uparrow} n_{i \downarrow} | \psi_{\alpha \uparrow} \psi_{\alpha \downarrow} \rangle \sim \frac{U}{\xi}$$

(11)

Thus even for weak interactions, the interaction corrections may be larger than the noninteracting level spacings which go as $1/L$. This invalidates perturbation theory in $U$ in the localized regime.

III. AVOIDED LEVEL CROSSINGS THEORY

Here we will discuss how the numerical results of the previous section can be understood in terms of avoided level crossings (ALC) at weak disorder. First, we derive the ALC approximation for the noninteracting current $I_0$.

A. ALC theory for $I_0$

In Fig. 4 we can see a typical spectrum for a tight-binding Hamiltonian of a 6 site ring, as a function of the applied flux. In the absence of disorder ($W = 0$) the eigenenergies are

$$\epsilon_n(\Phi) = -2 \cos \left( \frac{2\pi}{N} (n + \frac{\Phi}{\Phi_0}) \right),$$

(12)
with period $\Phi_0 = 2\pi$. At points of time reversal symmetry, i.e. where $\Phi$ is an integer multiple of $\Phi_0/2$, level crossings occurs between states of opposite angular momenta. The noninteracting persistent current is a sum over the currents carried by all occupied levels,

$$I_0(\Phi) = -\frac{\partial E_0}{\partial \Phi} = -\sum_{n,s}^{\text{occupied}} \frac{\partial}{\partial \Phi} \epsilon_n(\Phi), \quad (13)$$

where $n,s$ are the orbital and spin index, respectively. The noninteracting current $I_0(\Phi)$ will be a smooth function of $\Phi$ away from the points of level crossings. By symmetry of $\Phi \rightarrow -\Phi$, any pair of levels cross with opposite slopes, and thus if they are both fully occupied their contribution to the total current cancels. The only nonvanishing contribution comes from a topmost level which is not compensated by its partner. Then the current changes sign abruptly as the occupation moves from one branch to another. Thus for an odd number of fully occupied orbitals the current discontinuity occur at $\Phi = (m + 1/2)\Phi_0$, otherwise the discontinuities will occur at $\Phi = m\Phi_0$. In between the discontinuities the current varies linearly with the flux, which explains the periodic sawtooth shape for $I_0$ as seen in Fig. 1.

Introducing a small amount of disorder lifts the degeneracy at the crossings by opening small gaps. This reduces the persistent current $I_0$ since the occupied levels have smaller slopes near the former crossing points. This explains the behavior shown in Fig. 1: weak impurity scattering softens the discontinuities and leads to an overall reduction of the magnitude of current. We can quantify this observation by examining pairs of levels with momenta $\pm k$ which cross at $\Phi = 0$. We specialize to the case of an even number of filled levels with equal occupations for both spin directions. The unperturbed energies are given by

$$\epsilon_{\pm k}(\Phi) = -2 \cos \left( \pm k + \frac{2\pi}{L} \frac{\Phi}{\Phi_0} \right), \quad (14)$$

with $k = 2\pi m/L$, for a positive integer $m$. Consider the effect of a weak random potential $\epsilon_i$ which lifts the degeneracy in the $2 \times 2$ subspace of $k, -k$,

$$H_0 \rightarrow \begin{bmatrix} \epsilon_k(\Phi) & \tilde{V}_{k,-k} \\ \tilde{V}_{-k,k} & \epsilon_{-k}(\Phi) \end{bmatrix}, \quad (15)$$

where
\[ \tilde{V}_{k,-k} = \frac{1}{L} \sum_{i=1}^{L} \exp(-i2kx_i)\epsilon_i. \]  

(16)

From now on we will omit the subscripts \( k, -k \) off \( \tilde{V} \).

The eigenvalues and normalized eigenfunctions of (15) are:

\[ \epsilon_+(\Phi) : \psi_+ = \frac{1}{\sqrt{L}} \left( A_+ e^{ikx} + B_+ e^{-ikx} \right) \]
\[ \epsilon_-(\Phi) : \psi_- = \frac{1}{\sqrt{L}} \left( A_- e^{ikx} + B_- e^{-ikx} \right) \]  

(17)

where:

\[ A_- = e^{i\delta_-} \left[ 1 + \frac{4|\tilde{V}|^2}{(\epsilon_k - \epsilon_{-k})^2 + \sqrt{(\epsilon_{-k} - \epsilon_k)^2 + 4|\tilde{V}|^2}} \right]^{-1/2}, \]
\[ B_- = \frac{-2\tilde{V}^*}{\epsilon_{-k} - \epsilon_k + \sqrt{(\epsilon_{-k} - \epsilon_k)^2 + 4|\tilde{V}|^2}} A_- , \]  

(18)

and,

\[ \epsilon_\pm = \frac{\epsilon_k + \epsilon_{-k} \pm \sqrt{(\epsilon_k - \epsilon_{-k})^2 + 4|\tilde{V}|^2}}{2}, \]  

(19)

which satisfy:

\[ \epsilon_+(\Phi) + \epsilon_-(\Phi) = \epsilon_k(\Phi) + \epsilon_{-k}(\Phi). \]  

(20)

We see that the contribution of the occupied orbitals to \( I_0 \) is unchanged by weak disorder since:

\[ I_0^{k,-k} = -\frac{\partial(\epsilon_+ + \epsilon_-)}{\partial\Phi} = -\frac{\partial(\epsilon_k + \epsilon_{-k})}{\partial\Phi} \]  

(21)

Let us now use (13) to calculate \( I_0(\Phi) \) for free electrons with weak disorder by summing over occupied levels. We fill all levels up to \( k_f = 2\pi m_f/L \), plus two electrons at \( k_f \) so that an even number of orbital levels is filled. The total noninteracting current is given by the contribution from the filled level pairs, together with the contribution of lowest \( m = 0 \) (nondegenerate) level and the topmost orbital level at \( m_f \):
\[ I_0(\Phi) = \sum_m I_0^m + I_0^{m=0} + I_0^{m,f} \]
\[ = -2 \sum_{m \neq -1}^{m_f} \frac{4\pi}{L\Phi_0} \sin \frac{2\pi}{L} \left( m + \frac{\Phi}{\Phi_0} \right) - 2 \frac{\partial \epsilon_\Phi}{\partial \Phi} \]  

(22)

Neglecting corrections of order \(1/L^2\), we obtain the expression:

\[ I_0(\Phi) = \frac{8\pi}{L\Phi_0} \sin(k_f) \left[ -2 \frac{\Phi}{\Phi_0} + \left( \frac{\Phi}{\Phi_0} \right) \sqrt{\left( \frac{\Phi}{\Phi_0} \right)^2 + \frac{L|\tilde{V}|^2}{4\pi \sin(k_f)}} \right] , \]

(23)

for \( \Phi \in [-\Phi_0/2, \Phi_0/2] \). This expression for \( I_0 \) is valid as long as the energy scale of the disorder is much smaller than the level spacing at the Fermi level:

\[ |\tilde{V}| \ll 4\pi \sin(k_f)/L. \]  

(24)

Using the relation between \(|\tilde{V}|\) and the mean free path \(l_{el}\) defined by the one dimensional Born approximation,

\[ l_{el} \equiv \frac{2\pi \sin^2 k_f}{|\tilde{V}|^2 L} . \]

(25)

According to (24), the ALC approximation is valid for

\[ 8\pi l_{el}/L \geq 1, \]  

(26)

which is the ballistic, or delocalized regime.\(^1\) In terms of \(l_{el}\) one can write

\[ I_0(\Phi) = \frac{8\pi}{L\Phi_0} \sin(k_f) \left[ -2 \frac{\Phi}{\Phi_0} + \left( \frac{\Phi}{\Phi_0} \right) \sqrt{\left( \frac{\Phi}{\Phi_0} \right)^2 + \frac{L}{8\pi l_{el}}} \right] . \]

(27)

In order to compare (27) to the numerical result for Eq. (2), one needs to determine the parameter \(l_{el}\). Since fluctuations in \( I_0 \) for different disorder realizations, we determine \(l_{el}\) by fitting (27) to the numerical \( I_0 \). Once \(l_{el}\) is determined for a particular disorder realization, we use it to evaluate \( I_1 \) as shown below.

\(^1\)There is no intermediate “diffusive regime” between the localized and ballistic regimes in one dimension.\(^4\)
B. ALC theory for $I_1$

We shall now proceed to use the same approximation to explain the behavior of $I_1$.

According to (3), the first order energy is a sum of the density squared at all sites,

$$E_1(\Phi) = U \sum_i n_{i\uparrow} n_{i\downarrow}$$  \hspace{2cm} (28)

where for the filling of an even number of orbitals per spin, the single spin density is,

$$n_{is}(\Phi) = |\psi_0(i)|^2 + |\psi_{m,\sigma}(i)|^2 + \sum_{m=1}^{m_f-1} \sum_{\sigma=\pm} |\psi_{m,\sigma}(i)|^2.$$  \hspace{2cm} (29)

We will now show that the first order energy,

$$E_1(\Phi) = U \sum_i n_{i\uparrow} n_{i\downarrow}$$  \hspace{2cm} (30)

is enhanced near level crossings.

For a pair of fully occupied levels one has, by unitarity

$$|\psi_{m,+}(i)|^2 + |\psi_{m,-}(i)|^2 = |\psi_k(i)|^2 + |\psi_{-k}(i)|^2 = \frac{2}{L}.$$  \hspace{2cm} (31)

Consequently, for weak disorder, all occupied levels, except for the last one, contribute constant values to the total density:

$$n_{i\uparrow} = \frac{2m_f}{L} + |\psi_-(i)|^2$$  \hspace{2cm} (32)

or in terms of the coefficients $A, B$ \[17\]

$$n_{i\uparrow} = \frac{2m_f + 1}{L} + \frac{A^* B_- e^{i2kx_i}}{L} + \frac{A_- B^* e^{-i2kx_i}}{L}.$$  \hspace{2cm} (33)

The first order energy is thus given by \[32\],

$$E_1(\Phi)/U = \frac{(2m_f + 1)^2}{L} + \frac{2|A_-|^2|B_-|^2}{L}.$$  \hspace{2cm} (34)

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\[2\]This approach is similar to the discussion of Aharonov-Bohm oscillations of the participation ratio in disordered rings Ref. \[16\].
Using (17) and (25) we can write

\[ E_1(\Phi)/U \approx \left( \frac{2m_f + 1}{L} \right) + \frac{1}{2L} \left[ 1 + \left( \frac{8\pi l_{el}}{L} \right) \left( \frac{\Phi}{\Phi_0} \right)^2 \right]^{-1}, \]  

Differentiating (33) with respect to flux yields

\[ I_1(\Phi)/U = \frac{8\pi l_{el} \Phi}{(L\Phi_0)^2} \left[ 1 + \left( \frac{8\pi l_{el}}{L} \right) \left( \frac{\Phi}{\Phi_0} \right)^2 \right]^{-1}. \]  

In Fig. 5, (36) is compared to the numerical result for \( I_1 \) for various values of disorder. For each disorder realization, \( l_{el} \) is determined by fitting the numerical and ALC results for \( I_0 \). We see that for weak disorder there is a satisfying agreement between the ALC approximation and the numerical results. In Fig. 5c, the disorder is too large, and the ALC approximation fairs badly.

Eq. (36) explains both the positive correlation between \( I_0 \) and \( I_1 \) of (7). Since in one dimension the mean free path \( (l_{el}) \) and the localization length \( (\xi) \) differ by a proportionality factor of order unity [15], (36) agrees with the empirical scaling form (3) \( |I_1| \sim f(L/l_{el})/L \) which was found numerically at weak disorder.

\section*{IV. DIFFERENCE OF HUBBARD MODEL AND SPINLESS FERMIONS}

In recent papers [10,11], small disordered rings of spinless electrons have been exactly diagonalized. The persistent currents have been computed as a function of disorder and interaction strength. In contrast to our result for the Hubbard model, interactions have were seen to reduce\ the persistent current at weak disorder. Here we apply the ALC approach to explain this apparent difference between the models.

The spinless Hamiltonian \( H^s \) is given by

\[ H^s = H^s_0 + \sum_i U_{ij} n_i n_j, \]

\[ H^s_0 = - \sum_i [e^{i(\frac{2\pi}{L} \Phi_0)} a_{i+1}^\dagger a_i + \text{h.c.}] + \sum_i \epsilon_i a_{i}^\dagger a_i. \]  

The non interacting current, \( I^s_0 \), is given by half the value of Eq. (13). In the ALC approximation, it is given by half the value of Eq. (27).
Following the analogous derivation of $I_1$, we use (33) to obtain:

$$E_{\Phi}^\ast = (2m_f + 1)^2 \bar{U}(0) + 2|A_-|^2 |B_-|^2 \bar{U}(2k_f),$$

(38)

where

$$\bar{U}(k) = \frac{1}{L} \sum_{j \neq i} \exp(-ikx_j)U_{ij},$$

(39)

which implies

$$E_{\Phi}^\ast = (2m_f + 1)^2 \bar{U}(0) + \bar{U}(2k_f) \left[ 1 + \left( \frac{8\pi l_e}{L} \right) \left( \frac{\Phi}{\Phi_0} \right)^2 \right]^{-1}$$

$$I_{\Phi}^\ast = \frac{16\pi l_e \Phi}{\Phi_0^2 L} \bar{U}(2k_f) \left[ 1 + \left( \frac{8\pi l_e}{L} \right) \left( \frac{\Phi}{\Phi_0} \right)^2 \right]^{-2}$$

(40)

For the spinless nearest neighbor case studied in Ref. [11]:

$$E_{\Phi} = U \sum_i n_in_{i+1},$$

(41)

the corresponding Fourier coefficient is

$$\bar{U}(2k_f) = 2U \cos(2k_f).$$

(42)

Above a quarter filling, $k_f \geq \pi/4$, $\bar{U}$ is negative. Consequently $I_{\Phi}^\ast$ has the opposite sign to that of $I_1$ in Eq. (36), and to $I_{\Phi}^\ast$. That is to say, the persistent current of the spinless fermion model is suppressed by the interactions.

The difference between the models can be attributed to the different effects of intersite interactions (in the spinless model), versus local interactions (in the Hubbard model).

V. REMARKS AND CONCLUSIONS

We have investigated the first order effect of Hubbard interactions on the persistent current in one dimensional disordered rings.

The findings can be summarized as follows. $I_1$ was found to correlate in sign with $I_0$, a fact which goes contrary to the naive intuition that interactions suppress currents (e.g. as they do for the conductivity of the Luttinger model with an impurity [9]).
We can understand this result by observing that $I_1$ depends on charge fluctuations which vary strongly with flux near the degeneracy points, which also determine the sign and magnitude of $I_0$. Using the avoided level crossings theory, we obtain analytical expressions which fit the numerical results for both $I_0$ and $I_1$. We can use the same theory to explain why spinless fermions with intersite interactions exhibit suppression of currents rather than enhancement.

We have found numerically that $I_1(L/\xi)$ scales differently with the localization length than $I_0(L/\xi)$, it seems that effects of electron-electron interactions grow as disorder is increased.

However, first order perturbation theory in the weakly disordered case is expected to hold as long as the interaction matrix elements do not exceed the single particle level spacings. Thus we are restricted to the regime $U \leq 1$. We can draw on the exact diagonalization results [10, 11] where for weak disorder, the numerical currents are found to vary linearly with interaction strength in a sizeable regime. Thus we believe that first order perturbation theory should be valid for physically interesting interaction parameters.

It would be very satisfying if a similar analysis could be applied to the experimentally relevant case of three dimensional rings. Preliminary results yield positive correlations between $I_0$ and $I_1$, but where a simple minded application of the ALC approach cannot describe the diffusive regime of $l_{ed} << L$. It would also be important to understand the effects of true long range Coulomb interactions with the screening and exchange effects which are absent in the Hubbard model.

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FIGURES

FIG. 1. The noninteracting current ($I_0$) as a function of the applied flux for several strengths of disordered potential: $W=0$, 0.5, 1.0, 2.0 and 4.0. All curves are for a half-filled lattice of six sites.

FIG. 2. The first order interacting current ($I_1$) as a function of the applied flux for disordered potential values: $W=1.0$, 2.0 and 4.0. As before, all curves are for a half-filled lattice of six sites.

FIG. 3. The logarithm of the localization length ($\ln \xi$), implied by Eq. (8), is plotted as a function of strength of disorder ($W$) for half-filled rings with $N=6$, 10, 14, 20 and 25 sites. Data for the different system sizes collapse well onto a single curve.

FIG. 4. Energy level spectrum of the tight-binding Hamiltonian as a function of applied flux for a $N=6$ ring in the absence of disorder (solid lines) and for weak disorder (dotted lines).

FIG. 5. Comparison between numerically determined interactions correction $I_1$ (solid lines), and the analytic ALC result (dashed lines). Figs. (a) – (c) show results for three values of disorder strength $W$. $I_{el}(W)$ are determined by fitting Eq. (27) to the numerical disorder averaged $I_0(\Phi)$. Error bars depict fluctuations of numerical $I_1$ for different disorder realizations.
