Research on Well Trajectory Optimization Design Based on
Ant Colony Algorithm

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Abstract. The intelligent optimization algorithm designed for the current drilling well trajectory has shortcomings such as slow convergence rate and easy to fall into local optimal solution. This paper proposes an optimization method based on ant colony algorithm for well trajectory optimized design. Firstly, the three-stage well trajectory optimization design is taken as an example. The shortest space curve length is the optimization target, and the vertical depth, well angle and azimuth angle are used as constraints to establish the optimal mathematical model of the well trajectory. Then the ant colony algorithm is used to find the optimal parameters of the well trajectory under the constraint conditions, so as to realize the optimization of the well trajectory design.

1. Introduction
In drilling engineering, well trajectory optimization is generally optimized with the shortest space curve length[1]. However, in actual engineering construction, only the inclination, azimuth and vertical depth of a certain point can be measured, which makes it difficult to determine the position of each point and optimize it[2]. In addition, due to the complicated geological conditions, the well trajectory often deviates from the designed well trajectory during actual drilling[3]. In order to ensure accurate drilling of the target area, the actual drilling plan usually needs to adjust the construction plan in time to prevent the risk of off-target. To this end, the well trajectory must be optimized to determine the best drilling route before drilling.

At present, the methods for optimizing well trajectory design mainly include particle swarm optimization algorithm and genetic algorithm[4]. However, the particle swarm optimization algorithm is easy to fall into the local optimal solution. The genetic algorithm has higher algorithm complexity and lower operating efficiency. In order to improve the operation efficiency of the well trajectory design problem and improve the optimization precision and speed of the well trajectory design. Using the ant colony algorithm to solve the complex discretization optimization problem[5], this paper proposes an ant colony algorithm based well trajectory optimization design research.

2. Well trajectory optimization design mathematical model
In the design of the drilling well trajectory, the three-stage well trajectory optimization design method is usually adopted according to the determined position of the current bottom hole target point and the direction of the well[6]. Fig.1 is a schematic diagram of the three-stage (ie straight-increasing-stable) well trajectory design. The east coordinate E, the north coordinate N, and the depth Z are used to establish a three-dimensional coordinate system ONEZ, and the well trajectory is divided into three sections of OA, AB, and BC.
Fig. 1 Schematic diagram of three-stage well trajectory design

There are four key points in Fig. 1: (1) wellhead position O; (2) making a slope point start A; (3) making a slope point end B; (4) target point C. There are also five key parameters: (1) slanting point depth $D_{kop}$; (2) building slope $k$; (3) maintain angle $\alpha_1$; (4) azimuth angle $\varphi_1$; (5) length of the inclined section $L_{bc}$.

\[
\min: L_{ult} = D_{kop} + L_{ab} + L_{bc}
\] (1)

In formula (1), $L_{ab}$ is the length of the inclined section, $L_{bc} = \left[D_c - D_{kop} - R_1 \cdot \sin \alpha_1 \cdot \cos \alpha_1 \right] / \cos \alpha_1$, $R_1$ is the radius of curvature of the well in the inclined section, and $D_c$ is vertical depth.

Simultaneously, the component values $N_c, E_c$ and $D_c$ in the three directions of N, E, and Z satisfy the following constraints:

\[
\begin{align*}
\Delta N_{ab} + \Delta N_{bc} &= N_c \\
\Delta E_{ab} + \Delta E_{bc} &= E_c \\
D_{kop} + \Delta D_{ab} + \Delta D_{bc} &= D_c
\end{align*}
\] (2)

The radius of curvature $R_1$, the well angle $\alpha_1$, and the azimuth angle $\varphi_1$ are brought into the formula (2) to obtain the following constraint equations:

\[
\begin{align*}
R_1 (1 - \cos \alpha_1) \cos \varphi_1 + L_{bc} \sin \alpha_1 \cos \varphi_1 &= N_c \\
R_1 (1 - \cos \alpha_1) \sin \varphi_1 + L_{bc} \sin \alpha_1 \sin \varphi_1 &= E_c \\
D_{kop} + R_1 \cdot \sin \alpha_1 + L_{bc} \cos \alpha_1 &= D_c
\end{align*}
\] (3)

Combining the two equations (2) and (3), the objective function in equation (1) is transformed into min $F$, and the optimized mathematical model of the three-stage well trajectory design is:

\[
\begin{align*}
\min F &= (\Delta N_{ab} + \Delta N_{bc} - N_c)^2 + (\Delta E_{ab} + \Delta E_{bc} - E_c)^2 + \\
&\left( D_{kop} + \Delta D_{ab} + \Delta D_{bc} - D_c \right)^2
\end{align*}
\] s. t. \[
\begin{align*}
D_{min} &< D_{kop} < D_{max} \\
R_{1min} &< R_1 < R_{1max} \\
\alpha_{1min} &< \alpha_1 < \alpha_{1max} \\
\varphi_{1min} &< \varphi_1 < \varphi_{1max} \\
L_{bcmin} &< L_{bc} < L_{bcmax}
\end{align*}
\] (4)

In formula (4), $D_{max}, D_{min}, R_{1max}, R_{1min}, \alpha_{1max}, \alpha_{1min}, \varphi_{1max}, \varphi_{1min}, L_{bcmax}, L_{bcmin}$ are the upper and lower limits of the depth of the slanting, the radius of curvature of the well, the maintain angle of the well, the azimuth angle of the well, and the length of the stable.

In summary, the optimization design problem of well trajectory can be understood as the solution to the optimal well trajectory design parameters.

3. Basic principles of ant colony algorithm

The ant colony algorithm is a new type of intelligent optimization algorithm whose basic principle is to simulate the foraging process of ants. At present, ant colony algorithm has been applied to many
combinatorial optimization problems such as TSP problem and vehicle scheduling problem. The basic idea of the ant colony algorithm is: the pheromone content on each path at the initial moment \( \tau_{ij} \) (\( \tau_{ij} \) is the pheromone content on the path \((i, j)\)), and \( \tau_{ij}[0] = C \) (\( C \) is a constant). The ant \( k (k = 1, 2, \ldots m) \) determines the direction of the transfer according to the amount of pheromone on each path during the motion. The state transition rule used by the ant system is called a random scale rule, which gives The probability that an ant \( k \) moves from \( i \) to \( j \) at time \( t \) is:

\[
P_{ij}^k(t) = \begin{cases} \frac{\tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)}{\sum_{j \in \text{allowed}_k} \tau_{ij}^{\alpha}(t)\eta_{ij}^{\beta}(t)}, & j \in \text{allowed}_k \\ 0, & \text{otherwise} \end{cases}
\]

(5)

The \( \text{allowed}_k = \{0, 1, \ldots n - 1\} \) in the formula indicates where the ant \( k \) next allows selection. From equation (5), it is known that the transition probability \( P_{ij}^k(t) \) is proportional to \( \tau_{ij}^{\alpha} \cdot \eta_{ij}^{\beta} \), \( \eta_{ij} \) is the visibility factor, and \( \alpha, \beta \) respectively reflect the information accumulated by the ant during the movement and the heuristic information is selected in the ant. The relative importance of the path. Unlike real ants, artificial ants have a memory function. In order to meet the constraints that ants must pass through all the different positions, a data structure is designed for each ant, called the taboo table. The taboo table records where the ant has passed at the moment \( t \) and does not allow the ant to pass through these locations in this cycle. When the loop ends, the taboo table is used to calculate the length of the path the ant has passed. After the taboo table is emptied, the ant is free to choose the path. After \( n \) moments, the ant completes a loop, and the pheromone on each path is adjusted according to the following formula:

\[
\tau_{ij}(t + 1) = \rho \cdot \tau_{ij}(t) + \Delta \tau_{ij}(t, t + 1)
\]

(6)

\[
\Delta \tau_{ij}(t, t + 1) = \sum_{k=1}^{m} \Delta \tau_{ij}^k(t, t + 1)
\]

(7)

In the formula, \( \Delta \tau_{ij}^k(t, t + 1) \) represents the amount of pheromone that the \( k \)th ant left on the path \((i, j)\) at the time \((t, t + 1)\), and its value depends on the degree of performance of the ant. The shorter the path, the more pheromone is released. \( \Delta \tau_{ij}^k(t, t + 1) \) represents the increment of the pheromone of the path \((i, j)\) in this cycle, usually setting the coefficient \( \rho < 1 \) to avoid an infinite accumulation of pheromones on the path.

4. Implementation of well trajectory optimization design

The implementation flow of the well trajectory optimization design using the ant colony algorithm is shown in Fig.2.

The specific implementation process is as follows:

(1) Initialize the ant colony \( A(t) \)

\( \text{Let } t = 0; \) (t is the time counter)

\( N_c := 0; \) (\( N_c \) is the cycle number counter)

\( \Delta \tau_{ij}(t) := C; \) (Set an initial value of the track strength for each path \((i, j)\))

\( \Delta \tau_{ij} = 0 \) (The initial value of the track strength increment is set to 0)

\( \eta_{ij} \) is determined by a heuristic algorithm; (In the well trajectory design, \( \eta_{ij} = 1 / d_{ij} \))

\( \text{tabu}_k = \phi; \) (In the initial stage, the taboo table is empty)

Place \( m \) ants randomly on \( n \) nodes;

Let \( s := 1 \) (s is the taboo table index, and the initial position of each ant is placed in the current taboo table.)

for \( k := 0 \) to \( n \) do

for \( k := 0 \) to \( b_{i}(t) \) do

\( \text{tabu}_k(s) = i \)


Fig. 2 Basic flow chart of well trajectory optimization design

(2) According to the objective function designed by the well trajectory and the probability selection formula (5), m ants are set to move the search on the path between the nodes according to the transfer rule:

Let \( s := s + 1 \)
for \( k := 0 \) to \( n \) do
for \( k := 0 \) to \( b_k(t) \) do
First, position \( j \) is selected with probability \( p_{ij}(t) \), and ant \( k \) is moved to \( j \), and position \( j \) just selected is added to \( \text{tabu}_k \);

(3) When all ants have searched, select the best ant;
for \( k := 0 \) to \( m \) do
Calculate \( L_k \) according to the record of the taboo table;
for \( k := 0 \) to \( n-1 \) do (Search for ant \( k \)'s taboo table)
let \((h, l) := (\text{tabu}_k(s), \text{tabu}_k(s + 1))\) \((h, l)\) is the path of the connection position \((s, s+1)\) in the taboo table of ant \( k \)
for each path \((i, j)\), according to the state transfer formula (8)

\[
\tau_{ij}(t + n) = \rho_1 \cdot \tau_{ij}(t + n) + \tau_{ij}(t, t + n)
\]  

(8)
calculate \( \tau_{ij}(t + n) \)
let \( t := t + n \)
set \( \Delta \tau_{ij}(t, t + n) := 0 \) for each path \((i, j)\)

(4) According to the pheromone update rule, the pheromone is released according to a certain proportion of the path that the optimal ant passes, and the pheromone volatilizes with time, guiding the ant to search for the favorable path;
Record the shortest path so far.
If \( N_c < N_{cmax} \)
clear all taboo tables
let \( s := 1 \)
for \( k := 0 \) to \( n \) do
for \( k := 0 \) to \( b_l(t) \) do
\[ \text{tab}_u(s) = i \]

let \( t := t + 1 \)

set \( \Delta \tau_j(t, t + n) := 0 \) for each path (i, j)

return to step (2);

else

output optimal search results;

5. Instance verification

In order to verify the effectiveness of the well trajectory optimization design method proposed in this paper, a directional well in an oilfield is taken as an example. Table 1 shows the selection range of the well trajectory design parameters of the directional well.

| Parameter                     | Initial value selection range |
|-------------------------------|-------------------------------|
| Slanting point depth KOP(m)   | \( 300 \leq D_{kop} \leq 500 \) |
| Maintain angle (°)            | \( 30 \leq \alpha_1 \leq 45 \) |
| Azimuth angle (°)             | \( 0 \leq \phi_1 \leq 120 \)   |
| Building slope (°/30m)        | \( 2.10 \leq k \leq 2.40 \)   |
| Length of the inclined section(m) | \( 0 \leq L_{bc} \leq 2000 \) |

The initialization parameters of the optimization algorithm are set as follows: the number of ants \( m = 10, \alpha = 1, \beta = 0.5, \eta(0) = 1, \rho = 0.5 \), and the well trajectory parameters based on the conventional method and the algorithm of this paper are shown in Table 2.

| Parameter | Conventional method optimization parameters | This algorithm optimizes parameter values |
|-----------|--------------------------------------------|----------------------------------------|
| \( D_{kop} \) (m) | 315 | 309 |
| Maintain angle \( \alpha_1 \) (°) | 40.265 | 39.075 |
| Azimuth angle \( \phi_1 \) (°) | 30.9 | 30.6 |
| Building slope \( k \) (°/30m) | 2.1915 | 2.1795 |
| Length of the inclined section \( L_{bc} \) (m) | 457 | 450 |
| Well trajectory length \( L_{att} \) | 1213.46 | 1190.37 |

It can be seen from the results in Table 2 that the parameter optimization results of the proposed algorithm are significantly better than the conventional algorithm. The total length of the well trajectory obtained by the algorithm is also significantly shorter. The schematic diagram of the design parameters of Table 2 in the drilling engineering professional drawing software Landmark is shown in Figure 3.

![Fig.3 Schematic diagram of the well](image-url)
As shown in Fig. 3, the red line is the well trajectory obtained by the algorithm, and the blue line is the well trajectory obtained by the conventional method. The well trajectory based on the algorithm is smoother. In summary, the proposed algorithm can obtain a better solution in the well trajectory design problem, and the method is effective and feasible.

6. Conclusion
For the optimal design of the well trajectory in three-dimensional space, the ant colony algorithm can be used to obtain the well trajectory optimization parameters that meet the engineering requirements. In this paper, based on the ant colony algorithm, the well trajectory optimization design research is carried out, and the ant colony algorithm is used to solve the inference, and a better well trajectory design parameters can be obtained. Experiments show that the ant colony trajectory optimization design based on ant colony algorithm can meet the engineering requirements.

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