Constraining equilateral-type primordial non-Gaussianities from imaging surveys

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We investigate expected constraints on equilateral-type primordial non-Gaussianities from future/ongoing imaging surveys, making use of the fact that they enhance the halo/galaxy bispectrum on large scales. As model parameters to be constrained, in addition to $f_{\text{NL}}^{\text{equil}}$, which is related to the primordial bispectrum, we consider $\delta_{\text{NL}}$, which is related to the primordial trispectrum that appeared in the effective field theory of inflation. After calculating the angular bispectra of the halo/galaxy clustering and weak gravitational lensing based on the integrated perturbation theory, we perform Fisher matrix analysis for three representative surveys. We find that among the three surveys, the tightest constraints come from Large Synoptic Survey Telescope; its expected 1σ errors on $f_{\text{NL}}^{\text{equil}}$ and $\delta_{\text{NL}}$ are respectively given by $7.0 \times 10^2$ and $4.9 \times 10^2$. Although this constraint is somewhat looser than the one from the current cosmic microwave background observation, since we obtain it independently, we can use this constraint as a cross check. We also evaluate the uncertainty with our results caused by using several approximations and discuss the possibility to obtain tighter constraint on $f_{\text{NL}}^{\text{equil}}$ and $\delta_{\text{NL}}$.

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I. INTRODUCTION

It has been widely known that the primordial non-Gaussianity is a powerful tool to understand the non-linearity and the interaction structure of inflationary era (for a review, see [1]). There are a large number of theoretical and observational studies for the primordial non-Gaussianity. Currently, the most stringent observational constraints on the primordial non-Gaussianity have been obtained from the analysis of the higher-order spectra of the cosmic microwave background (CMB) anisotropies performed by Planck Collaboration [2], and they show no evidence of the primordial non-Gaussianity, which is consistent with the standard single slow-roll inflation model. However, from the theoretical point of view the constraints are still somewhat weak, and it would be interesting and important to investigate the observational constraint independently from the CMB observations.

Alternative information which can be expected to probe the primordial non-Gaussian feature is obtained through large-scale structure observations. As the effect of the primordial non-Gaussianity, it has been known that the power spectrum of the biased tracers, such as haloes/galaxies, could be enhanced on large scales compared with the purely Gaussian case, which is called as a “scale-dependent bias” feature (e.g., [3–5]). However, such an enhancement could be realized for the so-called “local-type” primordial non-Gaussianity, which is consistent with the standard single slow-roll inflation model. On the other hand, there is another interesting class of primordial non-Gaussianity, so-called “equilateral-type” one, which would be large in non-canonical field driven inflation models [6–8]. It has been known that this type of primordial non-Gaussianity gives no distinct scale-dependence in the power spectrum, and hence it had seemed to be difficult to obtain a significant constraint on the equilateral-type non-Gaussianity from the large-scale structure observations.

Regardless of this, recently, there have been several works which discuss the possibility of probing the equilateral-type non-Gaussianity from the large-scale structure observations through the analysis of the higher-order spectra, e.g., bispectrum of the biased tracers. Actually, in the presence of the equilateral-type primordial bispectrum, it has been shown that the amplitude of the halo/galaxy bispectrum is enhanced on large scales [11–13], and making use of this fact, the future galaxy surveys could be expected to give a strong constraint on $f_{\text{NL}}^{\text{equil}}$, which is comparable to ones obtained by CMB observations [11].

In general, the equilateral-type primordial non-Gaussianity is characterized not only by $f_{\text{NL}}^{\text{equil}}$, but also by $\delta_{\text{NL}}$, which is related to the primordial trispectrum. Although the shapes of primordial trispectra of this class are very complicated, since they strongly depend on the theoretical models, obtaining the observational constraint on the primordial trispectrum is still important to clarify the interaction structure of the inflation model. In this respect, among the shapes of primordial trispectra, only three shapes of the primordial trispectra appeared in the effective field theory of inflation [14–17] and whose amplitudes are characterized by $g_{\text{NL}}$, $g_{\text{NL}}^{(\ell \tau)}$, and $\delta_{\text{NL}}$ have been constrained systematically by the CMB observations [18] (we will show the detailed forms of these trispectra later in Sec. II). For these equilateral-type primordial trispectra, in the previous paper [19], two of us have investigated their impacts on the halo/galaxy bispectrum and showed that $g_{\text{NL}}$, $g_{\text{NL}}^{(\ell \tau)}$, and $\delta_{\text{NL}}$ could be constrained from the halo/galaxy bispectrum observations, while we can focus only on $\delta_{\text{NL}}$, as these shapes are very close to each other and only one of the two can be used as the basis of the optimal...
analysis.

In this paper, we focus on the future/ongoing imaging surveys and investigate the expected constraints on the equilateral-type primordial non-Gaussianity from the analysis of the halo/galaxy bispectrum and estimate 1σ errors on $f_{NL}^{\text{equil}}$ and $g_{NL}^{(\delta \sigma)^4}$. In our analysis, as in the previous work [20], where two of us were involved, we also include the cross correlation between halo/galaxy density field and the weak gravitational lensing which directly traces the matter density field.

The paper is organized as follows. In Sec. II we present formulas for the three dimensional auto-/cross-bispectra of halo/galaxy and matter distribution in the presence of the equilateral-type primordial non-Gaussianity. We also derive angular bispectra which are observables of the photometric/imaging galaxy surveys. In Sec. III based on the Fisher matrix formalism, we quantitatively estimate the impact of the bispectra and weak-gravitational lensing effect on the detection of the equilateral-type primordial non-Gaussianity. Finally, Sec. IV is devoted to summary and discussion. Throughout this paper, unless specifically mentioned, we adopt the best fit cosmological parameters taken from Plank [21].

II. HALO/GALAXY AND WEAK LENSING BISPECTRA AND WITH EQUILATERAL-TYPE PRIMORDIAL NON-GAUSSIANITIES

A. Equilateral-type primordial non-Gaussianities

Here, we present the concrete form of the statistical quantities of primordial curvature perturbations as well as those for the linear density for the situations we are interested in.

We begin by defining the power spectrum of the primordial curvature perturbation, $P_{\Phi}$, where $k$ and $\delta_{D}^{(3)}$ are three-dimensional wave vector and the three-dimensional Dirac’s delta function, respectively. The bracket $\langle \rangle$ means the ensemble average. If the curvature perturbation is generated by a single-field slow-roll inflation, it can be shown that these perturbations almost obey Gaussian statistics, and thus the statistical properties are completely characterized by the power spectrum $[22]$.\n
$$\langle \Phi(\mathbf{k})\Phi(\mathbf{k'}) \rangle = (2\pi)^3 \delta_{D}^{(3)}(\mathbf{k} + \mathbf{k'})P_{\Phi}(k),$$ \hspace{1cm} (1)

where the subscript, c, means that we consider only the connected part of the correlation functions.

We begin by defining the power spectrum of the primordial curvature perturbation is well approximated by the following separable form in most cases $[23]$:

$$B_{\Phi}^{\text{equil}}(k_1, k_2, k_3) = 6f_{NL}^{\text{equil}}[-P_{\Phi}(k_1)P_{\Phi}(k_2) + 2 \text{ perms.}] - 2P_{\Phi}(k_1)^{2/3}P_{\Phi}(k_2)^{2/3}P_{\Phi}(k_3)^{2/3} + (f_{NL}^{\text{equil}}P_{\Phi}(k_1)^{1/3}P_{\Phi}(k_2)^{2/3}P_{\Phi}(k_3) + 5 \text{ perms.}),$$ \hspace{1cm} (4)

which is called the equilateral-type primordial bispectrum. Here $f_{NL}^{\text{equil}}$ is the nonlinearity parameter which characterizes the amplitude of the bispectrum.

On the other hand, the form of the primordial trispectrum generated by this class of inflation models strongly depends on the inflation models and it is very complicated in general. Recently, however, for relatively simple primordial trispectra in this class, the constraints on their amplitudes have been obtained by CMB observations [18]. The concrete forms of the primordial trispectra investigated in the work are given by

$$T_{\Phi}^{\sigma^2}(k_1, k_2, k_3, k_4) = \frac{221184}{25} S_{NL}^{\sigma^2} A_{\Phi}^3 \sigma^2 \langle \Phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \rangle,$$ \hspace{1cm} (5)

$$T_{\Phi}^{(\partial \sigma)^4}(k_1, k_2, k_3, k_4) = -\frac{27648}{325} S_{NL}^{(\partial \sigma)^4} A_{\Phi}^3 \sigma^2 \langle \Phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \rangle,$$ \hspace{1cm} (6)

$$T_{\Phi}^{(\partial \sigma)^4}(k_1, k_2, k_3, k_4) = -\frac{16588}{2575} S_{NL}^{(\partial \sigma)^4} A_{\Phi}^3 \sigma^2 \langle \Phi(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4) \rangle,$$ \hspace{1cm} (7)

with

$$S_{NL}^{\sigma^2}(k_1, k_2, k_3, k_4) = \frac{1}{(\Sigma_{i=1}^4 k_i)^3 \Pi_{i=1}^4 k_i},$$ \hspace{1cm} (8)

$$S_{NL}^{(\partial \sigma)^4}(k_1, k_2, k_3, k_4) = \frac{k_1^2 k_2^2 (k_3 \cdot k_4)}{\Pi_{i=1}^4 k_i^3} \left( \frac{1 + 3 k_1 k_2 + 12 k_1 k_2 k_3}{(\Sigma_{i=1}^4 k_i)^2} \right).$$
Here, the parameters \( g_{\text{NL}}^{(2)} \) and \( g_{\text{NL}}^{(3)} \) describe the strength of the primordial non-Gaussianity, \( A_{\Phi} \) is the amplitude of the primordial power spectrum, defined by \( A_{\Phi} = k^3 P_{\Phi} \) and the normalizations are the ones adopted in Ref. \[18\]. It is worth mentioning that these primordial trispectra are not only relatively simple, but also have natural theoretical origin in the sense that they are shown to be generated by the effective field theory of inflation \[16, 17\] as well as \( k \) inflation \[24\,26\].

From Eqs. \[11\] and \[12\], we can relate these spectra to those of the primordial curvature perturbations as

\[
P_L(k) = M(k)^2 p_{\Phi}(k),
\]

\[
B_L(k_1, k_2, k_3) = M(k_1)M(k_2)M(k_3)B_{\Phi}(k_1, k_2, k_3),
\]

\[
T_L(k_1, k_2, k_3, k_4) = M(k_1)M(k_2)M(k_3)M(k_4)T_{\Phi}(k_1, k_2, k_3, k_4).
\]

Equipped with the linear density field presented in this subsection, we will derive in the next subsection the observables of large-scale structure which are probed with future imaging surveys.

\section{Halo/galaxy and weak lensing bispectra based on the Integrated Perturbation Theory (iPT)}

In addition to the halo/galaxy bispectrum with primordial equilateral-type non-Gaussianities discussed in \[19\], here we consider the cross bispectra between the halo/galaxy density field and the weak gravitational lensing. Let us first define the three-dimensional bispectra \( B_{XYZ} \) of the observables:

\[
\frac{1}{3} \{ \langle \delta_X(k_1) \delta_Y(k_2) \delta_Z(k_3) \rangle + 2 \text{perms} \} = (2\pi)^3 \delta_D^2 \{ \langle \delta_X(k_1) \delta_Y(k_2) \delta_Z(k_3) \rangle \},
\]

where \( \delta_{X,Y,Z} \) is the three-dimensional density field, and \( X, Y, Z = h, m \) respectively represents the halo/galaxy density field and the matter fluctuation. Here, we employ the integrated Perturbation Theory (iPT) \[28\,29\] to obtain the analytic expression for the bispectra \( B_{XYZ} \) in terms of the primordial non-Gaussianities. In iPT, the statistical quantities such as power spectra and bispectra of the halo/galaxy density field and matter fluctuations are perturbatively constructed with the linear polyspectra and multi-point propagators which can be defined as \[28\,29\].
\[ \Phi_{XYZ}^{\text{grav}}(k_1, k_2, k_3) = \frac{1}{3} \left\{ \Gamma^{(1)}(k_1) \Gamma_Y^{(1)}(k_2) \Gamma_Z^{(2)}(-k_1, -k_2) P_L(k_1) P_L(k_2) + 2 \text{perms}(X \leftrightarrow Y \leftrightarrow Z) \right\} + 2 \text{perms}(k_1 \leftrightarrow k_2 \leftrightarrow k_3), \]  
\[ \Phi_{XYZ}^{\text{bis}}(k_1, k_2, k_3) = \Gamma^{(1)}(k_1) \Gamma_Y^{(1)}(k_2) \Gamma_Z^{(1)}(k_3) B_L(k_1, k_2, k_3), \]  
\[ \Phi_{XYZ}^{\text{tris}}(k_1, k_2, k_3) = \frac{1}{3} \left\{ \frac{1}{2} \Gamma^{(1)}(k_1) \Gamma_Y^{(1)}(k_2) \int \frac{d^3 p}{(2\pi)^3} \Gamma_Z^{(2)}(p, k_3 - p) T_L(k_1, k_2, p, k_3 - p) + 2 \text{perms}(X \leftrightarrow Y \leftrightarrow Z) \right\} + 2 \text{perms}(k_1 \leftrightarrow k_2 \leftrightarrow k_3). \]

Here, we have neglected the higher-order contributions, which are expected to be small on large scales for the equilateral-type primordial non-Gaussianities. Substituting the expressions for \( B_L \) and \( T_L \) given by Eqs. (17) and (18) into the above expressions, in the large scale limit where the scale of interest \( \sim 1/k_i \) is much larger than the typical scale of the formation of the collapsed object \( \sim 1/p \), we have

\[ \Phi_{XYZ}^{\text{bis}}(k_1, k_2, k_3) = 6 e^{\text{equil}}_{\text{NL}} \Gamma^{(1)}(k_1) \Gamma_Y^{(1)}(k_2) \Gamma_Z^{(1)}(k_3) M(k_1) M(k_2) M(k_3) \left[ - (P_B(k_1) P_B(k_2) + 2 \text{perms}(k_1 \leftrightarrow k_2 \leftrightarrow k_3)) - 2 P_B(k_1)^{2/3} P_B(k_2)^{2/3} P_B(k_3)^{2/3} + \left( P_B(k_1)^{1/3} P_B(k_2)^{2/3} P_B(k_3)^{2/3} \right. + 5 \text{perms}(k_1 \leftrightarrow k_2 \leftrightarrow k_3) ) \right], \]

\[ \Phi_{XYZ}^{\text{tris}}(k_1, k_2, k_3) \simeq - \frac{4147}{6180} \delta^{(\sigma)} \left[ \Gamma^{(1)}(k_1) \Gamma_Y^{(1)}(k_2) M(k_1) M(k_2) P_B(k_1) P_B(k_2) \int \frac{d^3 p}{(2\pi)^3} \Gamma_Z^{(2)}(p, -p) P_L(p) \left( P_B(k_1)^{2/3} P_B(k_2)^{2/3} P_B(k_3) \right. + 5 \text{perms}(k_1 \leftrightarrow k_2 \leftrightarrow k_3) \left. \right) \right], \]

As shown in Ref. [19], the contributions of the equilateral-type primordial non-Gaussianities become larger on larger scales. Hence, according to Ref. [30], by employing the large-scale limit \( k_i \to 0 \), the multi-point propagators for halo/galaxy, which characterize the non-linear gravitational evolution and halo/galaxy bias properties, can be simply given by

\[ \Gamma^{(1)}(k) \simeq 1 + c_1(k), \]

\[ \Gamma^{(2)}(k_1, k_2) \simeq F_2(k_1, k_2) + \left( 1 + \frac{k_1 \cdot k_2}{k_2^2} \right) c_1(k_1) + \left( 1 + \frac{k_1 \cdot k_2}{k_1^2} \right) c_1(k_2) + c_2(k_1, k_2). \]

Here \( c_n \) represent the renormalized-bias function defined in

\[ c_n(k_1, k_2, \cdots, k_n) = (2\pi)^n \int \frac{d^3 k'}{(2\pi)^3} \left\langle \frac{\delta^n \delta_L(k')}{\delta \delta_L(k_1) \delta_L(k_2) \cdots \delta_L(k_n)} \right\rangle. \]

\( F_2 \) is the second-order kernel of standard perturbation theory, which is given by

\[ F_2(k_1, k_2) = \frac{10}{7} + \left( \frac{k_2}{k_1} + \frac{k_1}{k_2} \right) \left( \frac{k_1 \cdot k_2}{k_1 k_2} + \frac{4}{7} \left( \frac{k_1 \cdot k_2}{k_1 k_2} \right)^2 \right). \]

In order to obtain more concrete expressions for the renor-
where $W_a$ are the weight functions given by

$$ W_h(z) = \frac{n_h(z)}{\bar{n}_h}, \quad W_k(z) = \frac{4 \pi G \rho_m(z)}{H(1+z)^2 \bar{n}_h} \int_0^\infty dz' n_s(z') \frac{\chi(z') - \chi(z)}{\chi(z')}. $$

Here, we denote the projected number density of halo by $\bar{n}_h$ and its redshift distribution per unit area by $n_s(z)$. These quantities are respectively given by

$$ \bar{n}_h = \int_0^\infty dz, \quad n_s(z) = \int_0^\infty dz' \frac{\chi(z')}{H(1+z)^2} \int_{M_{min}}^\infty dM_h n(M_h, z), $$

where $\chi$ is the comoving radial distance and $n(M_h, z)$ is the halo mass function. $M_{min}$ is the minimum mass of observed halos, where we set $M_{min}$ to $10^{13} h^{-1} M_\odot$ through this paper. For the redshift distribution of source galaxies for weak-gravitational lensing observations, denoted by $n_s(z)$, we adopt the following functional form (e.g., [32]):

$$ n_s(z)dz = \bar{n}_s \frac{3 z^2}{2(0.64 cm^2)} \exp \left(-\frac{z}{0.64 cm} \right)^{3/2} dz. $$

Employing the Limber approximation [34], which is valid in the flat-sky limit, we finally obtain

$$ B_{abc}(\ell_1, \ell_2, \ell_3) = \int \frac{d^2 k}{2 \pi^2} W_a(z) W_b(z) W_c(z) \times \mathcal{B}_{XYZ} \left( \ell_1, \ell_2, \ell_3, z \right). \quad (43) $$

### III. Forecast constraints on equilateral-type primordial non-Gaussianity

In this section, based on the Fisher matrix formalism, let us quantitatively estimate the impact of the bispectra and weak-gravitational lensing effect on the observational constraints on the equilateral-type primordial non-Gaussianity. Here, as representative future/ongoing imaging surveys, we shall consider three representative surveys: the Subaru Hyper Suprime-Cam (HSC) [35], the Dark Energy Survey (DES) [36], and the Large Synoptic Survey Telescope (LSST) [37]. Imaging surveys are characterized by the survey area $f_{sky} \equiv \Omega_s/4 \pi$, the mean source redshift $z_m$, and the mean number density of source galaxies per unit area $\bar{n}_a$. We take the values of these parameters for the representative surveys as the $f_{sky}$, $z_m$, $\bar{n}_a$ (arcmin$^{-2}$) = (0.0375 (1500 deg$^2$), 1.0, 35) for HSC [35], (0.125 (5000 deg$^2$), 0.5, 12) for DES [36], and (0.5 (20000 deg$^2$), 1.5, 100) for LSST [37].

1. It has been known that the Limber approximation becomes invalid at the large-angular scales. However, as shown in Ref. [33], in the case of a wide observed redshift range, the Limber approximation can be applied even at large-angular scales. In our analysis, we mainly investigate the cases with single-redshift bin, and hence our expression based on the Limber approximation should not be invalid.
Here, $p_0$ is a set of fiducial cosmological parameters, subscripts $i$ and $j$ run over all possible triangle configurations whose side lengths are within the range $[\ell_{\text{min}}, \ell_{\text{max}}]$. In our analysis, we set the minimum multipole to $\ell_{\text{min}} = \ell_f = \sqrt{\pi f_{\text{sky}}}$, and the maximum multipole $\ell_{\text{max}}$ is set to 150. Later, we will also discuss the $\ell_{\text{max}}$ dependence of the 1σ errors on the non-linearity parameters. In the above expression, $C_{ij}^B$ is the angular bispectra covariance matrix and $[\text{Cov}^B]^{-1}_{ij}$ is its inverse. Then, assuming the Gaussian covariance, which is based on the fact that large primordial non-Gaussianity is not the current observations, the covariance matrix of the angular bispectra for a given set of multipole bins $(\ell_i, \ell_j, \cdots)$ can be given by [38]

\[
\text{Cov}[B_{\alpha\beta}(\ell_i, \ell_j, \ell_k), B_{\alpha'\beta'}(\ell_m, \ell_n, \ell_p)] = \frac{\Omega_a}{9 N_{\text{trip}}(\ell_i, \ell_j, \ell_k)}
\times [\left\{ (C_{\alpha\beta}(\ell_i) + N_{\alpha\beta})(C_{\alpha'\beta'}(\ell_j) + N_{\alpha'\beta'}) (C_{\gamma\delta}(\ell_k) + N_{\gamma\delta}) \right.]
\times \left( \delta^K_{\ell_i+\ell_m} \delta^K_{\ell_j+\ell_n} \delta^K_{\ell_k+\ell_p} + \text{5 perms (} \ell_i \leftrightarrow \ell_m \leftrightarrow \ell_n \text{)} \right)
\times \left[ 2 \text{ perms (} a' \leftrightarrow b' \leftrightarrow c' \text{)} + \text{2 perms (} a \leftrightarrow b \leftrightarrow c \text{)} \right],
\]

with $\delta^K_{\ell_i+\ell_j}$ being the Kronecker delta. Here, $N_{ab}$ is the shot-noise contribution, given by $N_{ab} = 1/\bar{n}_b$ (ab = hh), $\sigma_f/\bar{n}_s$ (ab = $\kappa\kappa$) and 0 (otherwise). $C_{ab}$ in the above expression is the angular power spectra of halo/galaxy clustering and weak-gravitational lensing. Note that, here, we calculated the angular power spectra by using the Limber approximation, as is the case in Eq. [43]. The quantity $\sigma_f$ represents the dispersion of the intrinsic shape noise and we adopt $\sigma_f = 0.3$ [39].

2 In our analysis, we set the maximum multipole $\ell_{\text{max}}$ to be 150, and it was shown that for $\ell \lesssim 200$ the assumption of Gaussian covariance matrices is expected to be valid [18]. For $\ell \gtrsim 200$, the non-linear evolution of the matter density field does not become negligible, and we need to take into account the gravity-induced non-Gaussian contribution.

### A. Fisher matrix

Following Ref. [20], the Fisher matrix for the parameters $p$ which characterize the theoretical expression of the angular bispectra $B_i$ are defined by

\[ F_{\alpha\beta} = \sum_{\ell_i=\ell_{\text{min}}}^{\ell_{\text{max}}} \frac{\partial B_i(p)}{\partial p_{\alpha}} \text{Cov}_{ij}^B \frac{\partial B_j(p)}{\partial p_{\beta}} , \]

\[ B_i = \begin{pmatrix} (B_{hhh})_{ij} \\ (B_{hkh})_{ij} \\ (B_{hkk})_{ij} \end{pmatrix} \text{, Cov}_{ij}^B = \begin{pmatrix} \text{Cov}[(B_{hhh})_{ij}, (B_{hhh})_{ij}] & \text{Cov}[(B_{hhh})_{ij}, (B_{hkh})_{ij}] & \text{Cov}[(B_{hhh})_{ij}, (B_{hkk})_{ij}] \\ \text{Cov}[(B_{hkh})_{ij}, (B_{hhh})_{ij}] & \text{Cov}[(B_{hkh})_{ij}, (B_{hkh})_{ij}] & \text{Cov}[(B_{hkh})_{ij}, (B_{hkk})_{ij}] \\ \text{Cov}[(B_{hkk})_{ij}, (B_{hhh})_{ij}] & \text{Cov}[(B_{hkk})_{ij}, (B_{hkh})_{ij}] & \text{Cov}[(B_{hkk})_{ij}, (B_{hkk})_{ij}] \end{pmatrix} . \]

$N_{\text{trip}}(\ell_i, \ell_j, \ell_k)$ is the number of independent triplets that form a triangular configuration respectively within the $i$-, $j$-, and $k$-th bins. In the limit $\ell_i \gg \ell_k = \ell_{\text{min}}$, we obtain a simple analytical expression for $N_{\text{trip}}$ [38]:

\[ N_{\text{trip}}(\ell_i, \ell_j, \ell_k) \approx 2(2\pi f_{\ell_\text{eq}})(\ell_j \Delta \phi_{12} \Delta \ell_i) \]

where the angle $\Delta \phi_{12}$ is given by

\[ \Delta \phi_{12}(\ell_i, \ell_j, \ell_k) \approx (\sin \phi_{12})^{-1} \frac{\ell_k \Delta \ell_k}{\ell_i \ell_j} = \frac{2\ell_i \Delta \ell_k}{\sqrt{2\ell_i^2 \ell_j^2 + 2\ell_i^2 \ell_k^2 + 2\ell_j^2 \ell_k^2 - \ell_i^2 - \ell_j^2 - \ell_k^2}} . \]

Note that the width of the $i$-th multipole bin should be larger than the minimum multipole, i.e., $\Delta \ell_i > \ell_i$.

### B. 1σ errors on the equilateral-type non-Gaussianities

Here, for free parameters $p$, we consider the two equilateral-type non-Gaussian parameters, i.e., $p = (p^p_{\text{NL}}, p^g_{\text{NL}})$, with the fiducial values of $p_0 = (0, 0)$. Note that we do not marginalize the uncertainty in the halo bias properties, since in iPT we can completely specify the halo bias by fixing the mass of observed halos.

The results of Fisher matrix analysis are summarized in Fig. [1] and Tab. [1]. The elliptic contours in Fig. [1] show the marginalized 1σ error constraints on $p^p_{\text{NL}}$ and $p^g_{\text{NL}}$. Light blue contours represent the constraints from the auto-angular bispectrum of halo/galaxy clustering, while the blue contours indicate the constraints when we add cross-angular bispectra between halos and weak lensing. The degeneracy of these two parameters is not very strong because the contributions from these parameters have different shape dependences [19]. Moreover, including the cross-angular bispectra partly breaks the parameters degeneracy, which makes the...
constraints tighter in each survey. In particular, the constraints by DES turn out to be most improved by taking into account the cross-angular bispectra. As we see in Tab. I, the tightest constraints in the three surveys come from LSST on both $\sigma(f^\text{equil}_{\NL})$ and $\sigma(\delta\sigma)^4$. Interestingly, the constraint on $\sigma(f^\text{equil}_{\NL})$ by DES is tighter than that by HSC, on the other hand, the constraint on $\sigma(\delta\sigma)^4$ is opposite. This result implies that deep imaging surveys are advantageous to give tighter constrains on the equilateral-type primordial trispectrum, denoted by $\sigma(\delta\sigma)^4$, while wide imaging surveys are advantageous for the equilateral-type primordial bispectrum, denoted by $f^\text{equil}_{\NL}$.

Before summarizing, we would like to mention the dependence of the resultant constraints on the maximum multipoles, $\ell_{\text{max}}$. While in the above result we have fixed $\ell_{\text{max}}$ to be 150, in Fig. 2 we plot the constraints on $f^\text{equil}_{\NL}$ and $\delta\sigma$ by HSC, DES and LSST as functions of $\ell_{\text{max}}$. As can be seen in this figure, the constraints coming from the bispectra continuously become improved as we increase $\ell_{\text{max}}$, because of the increased number of usable triangles. However, for a larger value of $\ell_{\text{max}}$, we need to take into account the late time non-linear evolution more seriously. We leave more precise forecasts including such a non-linear effect to future work.

### IV. SUMMARY AND DISCUSSION

In this paper, we have investigated the impact of angular bispectra from future imaging surveys to obtain constraints on the equilateral-type primordial non-Gaussianities. As non-linearity parameters characterizing such non-Gaussianities, we focus on $f^\text{equil}_{\NL}$ and $\delta\sigma^4$ and obtain simultaneous constraints on these two parameters. The parameter $\delta\sigma^4$ is related to one of the three equilateral-type trispectra, where the constraints were obtained by the CMB observations [18]. We have neglected the other two $f^\text{equil}_{\NL}$ parameters because it had been shown that one of them, $\delta\sigma^4$, could not be constrained from the bispectrum and the other, $\sigma^2(\delta\sigma)^2$, has similar shape dependence for $\delta\sigma^4$ in [19].

By using the integrated perturbation theory (iPT), we can systematically incorporate both the non-Gaussian mode-coupling from primordial polyspectra and non-linear halo biasing into theoretical template of bispectra. Therefore, we have employed this method to estimate the constraints on $f^\text{equil}_{\NL}$ and $\delta\sigma^4$ by the Fisher matrix analysis. As a result, we have shown that bispectra can give the constraints on $f^\text{equil}_{\NL}$.
and $g^{(\partial \sigma)^4}$ for the three representative surveys (HSC, DES and LSST), even though power spectra can not give the constraints on the equilateral-type primordial non-Gaussianity. We have also shown that by combining weak lensing data, the constraints on $\sigma_{\text{equil}}$ and $g^{(\partial \sigma)^4}$ are improved. In particular, in the case of DES, the constraint on $g^{(\partial \sigma)^4}$ becomes almost twice as tight by combining weak lensing data. The tightest constraints come from LSST, and its expected 1σ errors on $\sigma_{\text{equil}}$ and $\ell_{\text{max}}^{(\partial \sigma)^4}$ are respectively given by $7.0 \times 10^2$ and $4.9 \times 10^7$. The resultant constraints for all three surveys are somewhat looser than the ones from the current CMB observations [2, 18]. Regardless of this, they could be used for consistency check of CMB observations. In addition, the results forecast in this paper may be a bit conservative because we have considered only the large-angular scale of $\ell \leq \ell_{\text{max}} = 150$. As we have shown in Fig. 2, increasing maximum multipoles, $\ell_{\text{max}}$, makes the constraints tighter, due to the increased number of triangles which we can use. To roughly estimate the impact of using higher $\ell_{\text{max}}$, we calculated the constraints by using $\ell_{\text{max}} = 400$, and we obtained $\sigma(\sigma_{\text{equil}}) = 4.6 \times 10^2(3.7 \times 10^2)$ and $\sigma(g^{(\partial \sigma)^4}) = 2.9 \times 10^7(2.3 \times 10^7)$, as the results of 1σ marginalized (un-marginalized) errors from LSST. The constraints monotonically decrease for larger $\ell_{\text{max}}$ at least $\ell_{\text{max}} < 400$ in our analysis. However, on small-angular scales, the higher-order contribution from gravitational evolution and non-Gaussian feature of error covariance which we have neglected in this paper become important, and a more careful study is necessary. Also, tomographic techniques give us another chance to improve our constraints on the equilateral-type primordial non-Gaussianity. In a previous paper [20], two of us simply estimated the impact of tomographic techniques in the case of local-type primordial non-Gaussianity, and the constraints were improved by a factor of 1.4 to 3. We expect similar amount of improvement in the case of equilateral-type primordial non-Gaussianity.

Our analysis in this paper has been based on predictions with iPT, assuming a prior knowledge of halo bias properties. This treatment is consistent with previously known analytic treatment, and thus our results are qualitatively correct. However, for a practical application, further quantitative study is necessary, because the prediction of bispectrum based on iPT with non-Gaussian initial conditions has not been tested against the halo clustering in N-body simulations. In addition, for a proper comparison with observations, we need to incorporate nuisance parameters into the characterization of halo bias to reduce the impact of unknown systematics.

At the level of real observation, several sources of systematics (i.e., survey geometry and mask) can arise. It is known that such kind of systematics can mimic primordial non-Gaussianity in the case of power spectrum analysis [40]. Therefore, investigating the effect of such contaminations on bispectra and improving calibration schemes can be important future work for application of bispectrum analysis for photometric surveys.

Finally, we need to comment about the validity of the large-scale limit approximation used to calculate Eqs. (22), (25) and (26). Fig. 3 shows the relative errors between the exact and large-scale limit formulas of the auto-angular bispectrum of halo/galaxy clustering. Here, the redshift and halo mass are fixed at $z = 1$ and $M_h = 10^{13} M_\odot / h$, and the integration of the redshift and halo mass in Eq. (13) are removed for simplicity. The relative errors of the contributions from non-linear gravitational evolution and $\sigma_{\text{equil}}$ are smaller than 10% even at $\ell = 150$. However, even in low $\ell$, the relative error of the contribution from $g^{(\partial \sigma)^4}$ becomes large, and the exact formula is almost twice larger than the large-scale limit formula. Therefore our resultant constraints on $\sigma_{\text{equil}}$ could be modified by a factor. Since our analysis was ultimately not very strict as it contained many approximations (e.g., the Limber approximation and Gaussian covariance matrix), we think that the errors caused by the large-scale limit are comparable with the ones caused by using other approximations. Once the importance of tighter constraints on the equilateral-type primordial

FIG. 2. Sensitivity of the marginalized 1σ errors on $\sigma_{\text{equil}}$ (left), $g^{(\partial \sigma)^4}$ (right) to the parameters $\ell_{\text{max}}$. Solid (dashed) lines represent marginalized (un-marginalized) 1σ errors on equilateral type primordial non-Gaussian parameters for HSC (blue), DES (red), and LSST (green).
trispectrum through the halo/galaxy bispectrum is recognized in the future, we will come back to this topic again and explore it without relying the approximations mentioned above.

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