Abstract

Paxos, Viewstamped Replication, and Zab are replication protocols that ensure high-availability in asynchronous environments with crash failures. Various claims have been made about similarities and differences between these protocols. But how does one determine whether two protocols are the same, and if not, how significant the differences are?

We propose to address these questions using refinement mappings, where protocols are expressed as succinct specifications that are progressively refined to executable implementations. Doing so enables a principled understanding of the correctness of the different design decisions that went into implementing the various protocols. Additionally, it allowed us to identify key differences that have a significant impact on performance.

1 Introduction

A protocol expressed in terms of a state transition specification $\Sigma$ refines another specification $\Sigma'$ if there exists a mapping of the state space of $\Sigma$ to the state space of $\Sigma'$ and each state transition in $\Sigma$ can be mapped to a state transition in $\Sigma'$ or to a no-op. This mapping between specifications is called refinement [17] or backward simulation [26]. If two protocols refine one another then we might argue that they are alike. But if they don’t, how does one qualify the similarities and differences between two protocols?

We became interested in this question while comparing three replication protocols for high availability in asynchronous environments with crash failures:

- **Paxos** [18] is a state machine replication protocol [15, 30]. We consider a version of Paxos that uses the multi-decree Synod consensus algorithm described in [18], sometimes called Multi-Paxos. Many implementations have been deployed, including in Google’s Chubby service [2, 4], in Microsoft’s Autopilot service [11] (used by Bing), and in the popular Ceph distributed file system [33], with interfaces now part of the standard Linux kernel.
- **Viewstamped Replication (VSR)** [27, 23] is a replication protocol originally targeted at replicating participants in a Two-Phase Commit (2PC) [20] protocol. VSR has also been used in the implementation of the Harp File System [24];
- **Zab** [12] (ZooKeeper Atomic Broadcast) is a replication protocol used for the popular ZooKeeper [10] configuration service. ZooKeeper has been in active use at Yahoo! and is now a popular open source product distributed by Apache.

Many claims have been made about the similarities and differences of these protocols. For example, citations [21, 3] claim that Paxos and Viewstamped Replication are “the same algorithm independently invented,” “equivalent,” or that “the view management protocols seem to be equivalent” [18].

In this paper, we approach the question of similarities and differences between these protocols using refinement mappings, as illustrated in Fig. 1. Refinement mappings induce an ordering relation on specifications, and the figure shows a Hasse diagram of a set of eight specifications of interest ordered by refinement. In this figure, we write $\Sigma' \rightarrow \Sigma$ if $\Sigma$ refines $\Sigma'$, that is, if there exists a refinement mapping of $\Sigma$ to $\Sigma'$.

At the same time, we have indicated informal levels of abstraction in this figure, ranging from a highly abstract specification of a linearizable service [9] to concrete, executable specifications. Active and passive replication are common approaches to replicate a service and ensure that behaviors are still linearizable. Multi-Consensus protocols use a form of rounds in order to refine active and passive replication. Finally, we obtain protocols such as Paxos, VSR, and Zab.

Each refinement corresponds to a design decision, and as can be seen from the figure it is possible to arrive at the
2 Masking Failures

To improve the availability of a service, a common technique is to replicate it onto a set of servers. A consistency criterion defines expected responses to clients for concurrent operations. Ideally, the replication protocol ensures linearizability [9]—the execution of concurrent client operations is equivalent to a sequential execution, where each operation is atomically performed at some point in time between its invocation and response.

2.1 Specification

We characterize linearizability by giving a state transition specification (see Specification 1). A specification defines states and gives legal transitions between states. A state is defined by a collection of variables and their current values. Transitions can involve parameters (listed in parentheses) that are bound within the defined scope. A transition definition gives a precondition and an action. If the precondition holds in a given state, then the transition is enabled in that state. The action relates the state after the transition to the state before. A transition is performed indivisibly starting in a state satisfying the precondition. No two transitions are performed concurrently, and if multiple transitions are enabled simultaneously, then the choice of which transition to perform is unspecified.

There are interface variables and internal variables. Interface variables are subscripted with the location of the variable, which is either a process name or the network, υ. Internal variables have no subscribers, and their value will be determined by a function on the state of the underlying implementation. Specification Linearizable Service has the following variables:

- \( \text{inputs}_ν \): a set that contains \((\text{clt}, \text{op})\) messages sent by process \(\text{clt}\). Here \(\text{op}\) is an operation invoked by \(\text{clt}\);
- \( \text{outputs}_ν \): a set of \((\text{clt}, \text{op}, \text{result})\) messages sent by the service, containing the results of client operations that have been executed;
- \( \text{appState} \): an internal variable containing the state of the application;
- \( \text{invoked}_{\text{clt}} \): the set of operations invoked by process \(\text{clt}\). This is a variable maintained by \(\text{clt}\) itself;
- \( \text{responded}_{\text{clt}} \): the set of completed operations, also maintained by \(\text{clt}\).

Similarly, there are interface transitions and internal transitions. Interface transitions model interactions with the
environment, which consists of a collection of processes connected by a network. An interface transition is performed by the process that is identified by the first parameter to the transition. Interface transitions are not allowed to access internal variables. Internal transitions are performed by the service, and we will have to demonstrate how this is done by implementing those transitions. The transitions of Specification Linearizable Service are:

- Interface transition \texttt{invoke(clt, op)} is performed when \texttt{clt} invokes operation \texttt{op}. Each operation is uniquely identified and can be invoked at most once by a client (enforced by the precondition). Adding \((\texttt{clt}, \texttt{op})\) to \texttt{inputs}_{\texttt{clt}}, models \texttt{clt} sending a message containing \texttt{op} to the service. The client maintains what operations it has invoked in \texttt{invoked}_{\texttt{clt}};

- Transition \texttt{execute(clt, op, result, newState)} is an internal transition that is performed when the replicated service executes \texttt{op} for client \texttt{clt}. The application-dependent deterministic function \texttt{nextState} relates an application state and an operation from a client to a new application state and a result. Adding \(((\texttt{clt}, \texttt{op}), \texttt{result})) to \texttt{outputs}_{\texttt{clt}} models the service sending a response to \texttt{clt}.

- Interface transition \texttt{response(clt, op, result)} is performed when \texttt{clt} receives the response. The client keeps track of which operations have completed in \texttt{responded}_{\texttt{clt}} to prevent this transition being performed more than once per operation.

From Specification Linearizable Service it is clear that it is not possible for a client to receive a response to an operation before it has been invoked and executed. However, the specification does allow each client operation to be executed an unbounded number of times. In an implementation, multiple execution could happen if the response to the client operation got lost by the network and the client retransmits its operation to the service. The client will only learn about at most one of these executions. In practice, replicated services will try to reduce or eliminate the probability of a client operation being executed more than once by keeping state about which operations have been executed. For example, a service could keep track of all its clients and eliminate duplicate operations using sequence numbers on client operations. In all the specifications that follow, we omit the logic to avoid operations from being executed multiple times to simplify the presentation.

We make the following assumptions about interface transitions:

- **Crash Failures**: A process follows its specification until it fails by crashing. Thereafter, it executes no transitions. Processes that never crash are called correct. A process that “shuts down” and later recovers using state from stable storage is considered correct albeit, temporarily slow. Processes are assumed to fail independently.

- **Failure Threshold**: There is a bound \(f\) on the maximum number of replica processes that may crash; the number of client processes that fail is unbounded.

- **Fairness**: Except for interface transitions at a crashed process, a transition that becomes continuously enabled is eventually executed.

- **Asynchrony**: There is no bound on the time before a continuously enabled transition is executed.

We will use refinement only to show that the design decisions that we introduce are safe. The intermediate specifications that we will produce will not necessarily guarantee liveness. It is important that the final executable implementations support liveness properties such as “an operation issued by a correct client is eventually executed,” but it is not necessary for our purposes that intermediate specifications have such liveness properties\footnote{State transition specifications may also include supplementary liveness conditions. If so, a specification \(\Sigma\) that refines a specification \(\Sigma'\) preserves both the safety and liveness properties of \(\Sigma'\).}. 

Specification 1 Linearizable Service

```
var inputs_{\texttt{clt}}, outputs_{\texttt{clt}}, appState, invoked_{\texttt{clt}}, responded_{\texttt{clt}}

initially: appState = \bot \land inputs_{\texttt{clt}} = outputs_{\texttt{clt}} = \emptyset \land
Vclt : invoked_{\texttt{clt}} = responded_{\texttt{clt}} = \emptyset

interface transition \texttt{invoke(clt, op)}:
precondition:
\(\texttt{op} \notin \texttt{invoked}_{\texttt{clt}}\)
action:
\texttt{invoked}_{\texttt{clt}} := \texttt{invoked}_{\texttt{clt}} \cup \{\texttt{op}\}
\texttt{inputs}_{\texttt{clt}} := \texttt{inputs}_{\texttt{clt}} \cup \{\{\texttt{clt}, \texttt{op}\}\}

internal transition \texttt{execute(clt, op, result, newState)}:
precondition:
\((\texttt{clt}, \texttt{op}) \in \texttt{inputs}_{\texttt{clt}} \land
\texttt{(result, newState)} = \texttt{nextState(appState, (\texttt{clt}, \texttt{op}))}\)
action:
\texttt{appState} := \texttt{newState}
\texttt{outputs}_{\texttt{clt}} := \texttt{outputs}_{\texttt{clt}} \cup \{\{(\texttt{clt}, \texttt{op}), \texttt{result})\}\)

interface transition \texttt{response(clt, op, result)}:
precondition:
\(((\texttt{clt}, \texttt{op}), \texttt{result}) \in \texttt{outputs}_{\texttt{clt}} \land \texttt{op} \notin \texttt{responded}_{\texttt{clt}}\)
action:
\texttt{responded}_{\texttt{clt}} := \texttt{responded}_{\texttt{clt}} \cup \{\texttt{op}\}\)
```
2.2 Active and Passive Replication

Specification[1] has internal variables and transitions that have to be implemented. There are two well-known approaches to replication:

- With active replication, also known as state machine replication [15, 30], each replica implements a deterministic state machine. All replicas process the same operations in the same order.

- With passive replication, also known as primary backup [1], a primary replica runs a deterministic state machine, while backups only store states. The primary computes a sequence of new application states by processing operations and forwards these states to each backup in order of generation.

Fig. 2a illustrates a failure-free execution of a service implemented using active replication:

1. Clients submit operations to the service (op1 and op2 in Fig. 2a).
2. Replicas, starting out in the same state, execute received client operations in the same order.
3. Replicas send responses to the clients. Clients ignore all but the first response they receive.

The tricky part of active replication is ensuring that replicas execute operations in the same order, despite replica failures, message loss, and unpredictable delivery and processing delays. A fault-tolerant consensus protocol [28] is typically employed for replicas to agree on the \( i \)th operation for each index \( i \) in a sequence of operations. Specifically, each replica proposes an operation that was received from one of the clients in instance \( i \) of the consensus protocol. Only one of the proposed operations can be decided. The service remains available as long as each instance of consensus eventually terminates.

Fig. 2b depicts a failure-free execution of passive replication:

1. Clients submit operations only to the primary.
2. The primary orders the operations and computes new states and responses.
3. The primary forwards the new states (so-called state updates) to each backup in the order generated.
4. The primary sends the response of an operation only after the corresponding state update has been successfully decided (this is made precise in Section 3).

Because two primaries may be competing to have their state updates applied at the backups, it is important that replicas apply a state update \( u \) on the same state used by the primary to compute \( u \). This is sometimes called the prefix order or primary order property [12, 13].

For example, consider a replicated integer variable with initial value 3. One client wants to increment the variable, while the other wants to double it. One primary receives both operations and submits state updates 4 followed by 8. Another primary receives the operations in the opposite order and submits updates 6 followed by 7. Without prefix ordering, it may happen that the decided states are 4 followed by 7, not corresponding to any sequential history of the two operations.

VSR and Zab employ passive replication; Paxos employs active replication. However, it is possible to implement one style on the other. The Harp file system [24], for example, uses VSR to implement a replicated message queue containing client operations—the Harp primary proposes state updates that backups apply to the state of the message queue. Replicas, running deterministic NFS state machines, then execute NFS operations in queue order. In other words, Harp uses an active replication protocol built using a message queue that is passively replicated using VSR.

2.3 Refinement

Below, we present a refinement of a linearizable service (Specification [1]) using the active replication approach. We then further refine active replication to obtain passive replication. The refinement of a linearizable service to passive replication follows transitively.

2.3.1 Active Replication

We omit interface transitions \( \text{invoke}(c lt, o p) \) and \( \text{response}(c lt, o p, r e s u l t) \), which are the same as in Specification [1]. Hereafter, a command \( c m d \) denotes a tuple \((c lt, o p)\).

Specification [2] uses a sequence of slots. A replica executes transition \( \text{propose}(r e p l i c a, s l o t, c m d) \) to propose a command \( c m d \) for the given slot. We call the command a proposal. Transition \( \text{decide}(s l o t, c m d) \) guarantees that at most one proposal is decided for each slot. Transition \( \text{learn}(r e p l i c a, s l o t) \) models a replica learning a decision and assigning the decision to the corresponding slot of the \( \text{learn}_r e p l i c a \) array. Replicas update their state by executing a learned operation in increasing order of slot number with transition \( \text{update}(r e p l i c a, c m d, r e s, n e w S t a t e) \). The slot of the next operation to execute is denoted by \( v e r s i o n_{r e p l i c a} \).
**Specification 2** Specification Active Replication

- **var**
  - proposals<sub>replica</sub>[1…], decisions[1…], learned<sub>replica</sub>[1…]
  - appState<sub>replica</sub>, version<sub>replica</sub>, inputs<sub>replica</sub>, outputs<sub>replica</sub>

- **initially:**
  - \(\forall s \in \mathbb{N}^+ : \text{decisions}[s] = \bot \land \text{proposals<sub>replica</sub>[s]} = \emptyset\)
  - \(\forall \text{replica} :\)
    - \(\text{appState}_{\text{replica}} = \bot \land \text{version}_{\text{replica}} = 1 \land \forall s \in \mathbb{N}^+ : \text{decisions}[s] = \bot \land \text{proposals<sub>replica</sub>[s]} = \emptyset\)

- **interface transition** `propose(replica, slot, cmd)`:
  - **precondition:**
    - \(\text{cmd} \in \text{inputs}_{\text{replica}} \land \text{learned<sub>replica</sub>[slot]} = \bot\)
  - **action:**
    - \(\text{proposals<sub>replica</sub>[slot]} := \text{proposals<sub>replica</sub>[slot]} \cup \{\text{cmd}\}\)

- **internal transition** `decide(slot, cmd)`:
  - **precondition:**
    - \(\text{decisions}[slot] = \bot \land \exists r : \text{cmd} \in \text{decisions}[slot]\)
  - **action:**
    - \(\text{decisions}[slot] := \text{cmd}\)

- **internal transition** `learn(replica, slot)`:
  - **precondition:**
    - \(\text{learned<sub>replica</sub>[slot]} = \bot \land \text{decisions}[slot] \neq \bot\)
  - **action:**
    - \(\text{learned<sub>replica</sub>[slot]} := \text{decisions}[slot]\)

- **interface transition** `update(replica, cmd, res, newState)`:
  - **precondition:**
    - \(\text{cmd} = \text{learned<sub>replica</sub>[version}_{\text{replica}}] \land \text{cmd} \neq \bot \land \text{res} = \text{newState} \Rightarrow \text{appState}_{\text{replica}} = \text{newState}\)
  - **action:**
    - \(\text{outputs}_{\text{replica}} := \text{outputs}_{\text{replica}} \cup \{\text{cmd}, \text{res}\}\)
    - \(\text{appState}_{\text{replica}} := \text{newState}\)
    - \(\text{version}_{\text{replica}} := \text{version}_{\text{replica}} + 1\)

Note that `propose(replica, slot, cmd)` requires that `replica` has not yet learned a decision in `slot`. While not a necessary requirement for safety, proposing a command for a slot that is known to be decided is wasted effort. It would make sense to require that replicas propose in the first `slot` for which both `proposals<sub>replica</sub>[slot] = \emptyset` and `learned<sub>replica</sub>[slot] = \bot`. We do not require this at this level of specification to simplify the refinement mapping between active and passive replication.

To show that active replication refines Specification 1, we first show how the internal state of Specification 1 is derived from the state of Specification 2. The only internal state in Specification 1 is the appState variable. For our refinement mapping, its value is the application state maintained by the replica (or one of the replicas) with the highest version number.

To complete the refinement mapping we also have to show how transitions of active replication map onto enabled transitions of Specification 1 or onto stubs (no-ops with respect to Specification 1). The propose, decide, and learn transitions are always stubs because they do not update appState<sub>replica</sub> of any replica. An update(replica, cmd, res, newState) transition corresponds to execute(clt, op, res, newState), where cmd = (clt, op), in Specification 1 if replica is the first replica to apply op and thus leading to appState being updated. Transition update is a stutter if the replica is not the first replica to apply the update.

### 2.3.2 Passive Replication

Passive replication (Specification 3) also uses slots, and proposals are tuples \((\text{oldState}, (\text{cmd}, \text{res}, \text{newState}))\) consisting of the state prior to executing a command, a command, the output of executing the command, and a new state that results from applying the command. In an actual implementation, the old state and new state would respectively be represented by an identifier and a state update rather than the entire value of the state.

Any replica can act as primary. Primaries act speculatively, computing a sequence of states before they are decided. Because of this, primaries have to maintain a separate version of the application state. We call this the shadow state. Primaries may propose to apply different state updates for the same slot.

Transition `propose(replica, slot, cmd, res, newState)` is performed when a primary replica proposes applying cmd to shadowState<sub>replica</sub> in a certain slot, resulting in output res. State shadowState<sub>replica</sub> is what the primary calculated for the previous slot (even though that state...
is not necessarily decided as of yet, and may never be decided. Proposals for a slot are stored in a set since a primary may propose to apply different commands for the same slot due to repeated change of primaries.

Transition `decide(slot, cmd, res, newState)` specifies that only one of the proposed new states can be decided. Because `cmd` was performed speculatively, the `decide` transition checks that the state decided in the prior slot, if any, matches state `s` to which replica `r` applied `cmd`, thus ensuring prefix ordering.

Similarly to active replication, transition `learn` models a replica learning the decision of a slot. With the `update` transition, a replica updates its state based on what was learned for the slot. With active replication, each replica performs each client operation, while in passive replication only the primary performs client operations and backups simply obtain the resulting states.

Replicas wishing to act as primary perform transition `resetShadow` to update their speculative state and version, respectively denoted by variables `shadowStateReplica` and `shadowVersionReplica`. The new shadow state may itself be speculative and must be set to a version at least as recent as the latest learned state.

To show that passive replication refines active replication, we first note that all variables in passive replication have a one-to-one mapping with the ones in active replication, apart from variables `shadowVersionReplica` and `shadowStateReplica` which do not appear in active replication.

Transition `resetShadow` correspond to a stutter transition. Transitions `propose`, `decide`, and `learn` of passive replication have preconditions that are either equal to the corresponding ones in active replication or more restrictive. If these transitions are enabled in passive replication, they are therefore also enabled in active replication. We now show that this is also the case for transition `update`. For this to be true, (i) `cmd` must be different than `⊥`, and (ii) `(res, newState)` must be the result of applying command `cmd` on `appStateReplica`. Condition (i) is trivially satisfied from Specification 3. Condition (ii) holds for the following reasons. Since transition `update` is enabled, `cmd` is in `learnedReplica[versionReplica]`. From transition `learn`, `cmd` was decided for slot `versionReplica`. From transition `decide`, `cmd` was proposed in slot `versionReplica` and either `versionReplica = 1` or `versionReplica > 1` and the state update `cmd` was applied on the state decided in slot `versionReplica - 1`, that is, `appStateReplica`. If `versionReplica = 1`, `(res, newState) = nextState(appState, cmd)` trivially holds, otherwise it holds from the precondition of transition `propose`.

### 3 A Generic Protocol

Specifications 2 and 3 contain internal variables and transitions that need to be refined for an executable implementation. We start with refining active replication. Multi-Consensus (Specification 4) refines active replication and contains no internal variables or transitions. As previously, the `invoke` and `response` transitions (and cor-
3.1 Certifiers and rounds

Multi-Consensus has two basic building blocks:

- A static set of \( n \) processes called certifiers. A minority of these may crash. So for tolerating at most \( f \) failures, we require that \( n \geq 2f + 1 \) holds.
- An unbounded number of rounds.

For ease of reference, Table 1 contains a translation between terminology used in this paper and those found in the papers describing the protocols under consideration.

In each round, a consensus protocol assigns to at most one certifier the role of sequencer. The sequencer of a round can certify at most one command for each slot. The other certifiers can copy the sequencer, certifying the same command for the same slot and round. Note that if two certifiers certify a command in the same slot and the same round, it must be the same command. Moreover, a certifier cannot retract a certification. Once a majority of certifiers certify the command within a round, the command is decided (and because certifications cannot be retracted the command will remain decided thereafter). In Section 3.4 we show why two rounds cannot decide different commands for the same slot. Each round has a round-id that uniquely identifies the round. Rounds are totally ordered by their round-ids. A round is in one of three modes: pending, operational, or wedged. One round is the first round (it has the smallest round-id), and initially only that round is operational. Other rounds start out pending. The two possible transitions on the mode of a round are as follows:

1. A pending round can become operational only if all rounds with lower round-id are wedged;
2. A pending or operational round can become wedged under any circumstance.

This implies that at any time at most one round is operational and that wedged rounds can never become un-wedged.

3.2 Tracking Progress

In Specification 4 each certifier \( cert \) maintains a progress indicator \( \text{progress}^c_{\text{cert}}[\text{slot}] \) for each slot, defined as:

**Progress Indicator:** A progress indicator is a pair \( \langle \text{round-id}, \text{cmd} \rangle \) where \( \text{round-id} \) is the identifier of a round and \( \text{cmd} \) is a proposed command or \( \perp \), satisfying:

- If \( \text{cmd} = \perp \), then the progress indicator guarantees that no round with an id less than \( \text{round-id} \) can ever decide, or have decided, a proposal for the slot.
- If \( \text{cmd} \neq \perp \), then the progress indicator guarantees that if a round with id \( \text{round-id}' \) such that \( \text{round-id}' \leq \text{round-id} \) decides (or has decided) a proposal \( \text{cmd}' \) for the slot, then \( \text{cmd} = \text{cmd}' \).
- Given two progress indicators \( \langle \text{round-id}, \text{cmd} \rangle \) and \( \langle \text{round-id}, \text{cmd}' \rangle \) for the same slot, if neither \( \text{cmd} \) nor \( \text{cmd}' \) equals \( \perp \), then \( \text{cmd} \leq \text{cmd}' \).

We define a total ordering \( \succ \) on progress indicators for the same slot as follows: \( \langle \text{round-id}', \text{cmd}' \rangle \succ \langle \text{round-id}, \text{cmd} \rangle \) iff

- \( \text{round-id}' \succ \text{round-id} \); or
- \( \text{round-id} = \text{round-id}' \) and \( \text{cmd}' \neq \perp \) and \( \text{cmd} = \perp \).

At any certifier, the progress indicator for a slot is monotonically non-decreasing.

3.3 Normal case processing

Each certifier \( cert \) supports exactly one round-id \( \text{round-id}^c_{\text{cert}} \), initially 0, the round-id of the first round. The normal case holds when a majority of certifiers support the
Specification 4 Multi-Consensus

var rid_cert, isSeq_cert, progress_cert[1...]
certifics_v, snapshots_v

initially: certifics_v = snapshots_v = Ø ∧ ∀cert: rid_cert = 0 ∧ isSeq_cert = false ∧ ∀slot ∈ N⁺ : progress_cert[slot] = ⟨0, ⊥⟩

interface transition certifySeq(cert, slot, ⟨rid, cmd⟩):
precondition:
   isSeq_cert ∧ rid = rid_cert ∧ progress_cert[slot] = ⟨rid, ⊥⟩ ∧ (∀s ∈ N⁺ : progress_cert[s] = ⟨rid, ⊥⟩ ⇒ s ≥ slot) ∧
   ∃replica : cmd ∈ proposals_replica[slot]
action:
   progress_cert[slot] := ⟨rid, cmd⟩
certifics_v := certifics_v ∪ {⟨cert, slot, ⟨rid, cmd⟩⟩}

interface transition observeDecision(replica, slot, cmd):
precondition:
   ∃cert' : ⟨cert', slot, ⟨rid, cmd⟩⟩ ∈ certifics_v ∧ rid_cert = rid ∧ ⟨rid, cmd⟩ > progress_cert[slot]
action:
   progress_cert[slot] := ⟨rid, cmd⟩
certifics_v := certifics_v ∪ {⟨cert, slot, ⟨rid, cmd⟩⟩}

interface transition supportRound(cert, rid, proseq):
precondition:
   rid > rid_cert
action:
   rid_cert := rid; isSeq_cert := false
   snapshots_v := snapshots_v ∪ {⟨cert, slot, ⟨rid, cmd⟩⟩ : ⟨cert, slot, ⟨rid, cmd⟩⟩ ∈ certifics_v}

interface transition recover(cert, rid, S):
precondition:
   rid_cert = rid ∧ isSeq_cert ∧ |S| > 2/3 ∧ S ⊆ \{⟨id, prog⟩ | (id, rid, cert, prog) ∈ snapshots_v\}
action:
   ∀s ∈ N⁺ : ⟨s, cmd⟩ := max｛prog[s] | (id, prog) ∈ S｝
   progress_cert[s] := ⟨rid, cmd⟩
   if cmd ≠ ⊥ then certifics_v := certifics_v ∪ {⟨cert, s, ⟨rid_cert, cmd⟩⟩}
   isSeq_cert := true

same round-id, and one of these certifiers is sequencer (its isSeq_cert flag is set to true).

Transition certifySeq(cert, slot, ⟨rid, cmd⟩) is performed when sequencer cert certifies command cmd for the given slot and round. The condition progress_cert[slot] = ⟨rid, ⊥⟩ holds only if no command can be decided in this slot by a round with an id lower than rid_cert. The transition requires that slot is the lowest empty slot of the sequencer. If the transition is performed, cert updates progress_cert[slot] to reflect that if a command is decided in its round, then it must be command cmd. Sequencer cert also notifies all other certifiers by adding ⟨cert, slot, ⟨rid, cmd⟩⟩ to set certifics_v (modeling a broadcast to the certifiers).

A certifier that receives such a message checks if the message contains the same round-id that it is currently supporting and that the progress indicator in the message exceeds its own progress indicator for the same slot. If so, then the certifier updates its own progress indicator and certifies the proposed command (transition certify(cert, slot, ⟨rid, cmd⟩)).

The observeDecision(replica, slot, cmd) transition at replica is enabled if a majority of certifiers in the same round have certified cmd in slot. If so, the command is decided and, as explained in the next section, all replicas that undergo the observeDecision transition for this slot will decide on the same command.

3.4 Recovery

In this section, we show how Multi-Consensus deals with failures. The reason for having an unbounded number of rounds is to achieve liveness. When an operational round is no longer certifying proposals, perhaps because its sequencer has crashed or is slow, the round can be wedged and a round with a higher round-id can become operational.

Modes of rounds are implemented as follows: A certifier cert can transition to supporting a new round-id rid and prospective sequencer proseq (transition supportRound(cert, rid, proseq)). This transition can only increase rid_cert. Precondition rid > rid_cert ensures that a certifier supports a round and a prospective sequencer for the round at most once. The transition sends the certifier’s snapshot by adding it to the set snapshots_v. A snapshot is a four-tuple ⟨cert, rid, proseq, progress_cert⟩ containing the certifier’s identifier, its current round-id, the identifier of proseq, and the certifier’s list of progress indicators. Note that a certifier can send at most one snapshot for each round.

Round rid with sequencer proseq is operational, by definition, if a majority of certifiers support rid and added ⟨cert, rid, proseq, progress_cert⟩ to the set snapshots. Clearly, the majority requirement guarantees that there cannot be two rounds that are simultaneously operational, nor can there be operational rounds that do not have exactly one sequencer. Certifiers that support rid can no longer certify commands in rounds prior to rid. Consequently, if a majority of certifiers support a round-
\[ \text{decisions[slot]} = \begin{cases} \text{cmd} & \text{if } \exists \text{rid} : \{\{\text{cert}, (\text{slot}, (\text{rid}, \text{cmd})) \in \text{certifics}\} > n/2 \\ \bot & \text{otherwise} \end{cases} \]

Figure 3: Relation between the certifics variable of Specification 4 and the decisions variable of Specification 2. Here \( n \) is the number of certifiers.

id larger than \( x \), then all rounds with an id of \( x \) or lower are wedged.

Transition recover\((\text{cert}, \text{rid}, S)\) is enabled at \( \text{cert} \) if the set \( S \) contains snapshots for \( \text{rid} \) and sequencer \( \text{cert} \) from a majority of certifiers. The sequencer helps to ensure that the round does not decide commands inconsistent with prior rounds using the snapshots it has collected. For each slot, sequencer \( \text{cert} \) determines the maximum progress indicator \( \langle r, \text{cmd} \rangle \) for the slot in the snapshots contained in \( S \). It then sets its own progress indicator for the slot to \( \langle \text{rid}, \text{cmd} \rangle \). It is easy to see that \( \text{rid} \geq r \).

We argue that \( \langle \text{rid}, \text{cmd} \rangle \) satisfies the definition of progress indicator in Section 3.2. All certifiers in \( S \) support \( \text{rid} \) and form a majority. Thus, it is not possible for any round between \( r \) and \( \text{rid} \) to decide a command because none of these certifiers can certify a command in those rounds. There are two cases:

- If \( \text{cmd} = \bot \), no command can be decided before \( r \), so no command can be decided before \( \text{rid} \). Hence, \( \langle \text{rid}, \bot \rangle \) is a correct progress indicator.

- If \( \text{cmd} \neq \bot \), then if a command is decided by \( r \) or a round prior to \( r \), it must be \( \text{cmd} \). Since no command can be decided by rounds between \( r \) and \( \text{rid} \), \( \langle \text{rid}, \text{cmd} \rangle \) is a correct progress indicator.

The sequencer sets its isSeq\(_{\text{cert}}\) flag upon recovery. As a result, it is enabled to propose new commands. Normal case for the round begins and holds as long as a majority of certifiers support the corresponding round-id.

### 3.5 Refinement Mapping

Multi-Consensus refines active replication. We first show how the internal variables of Specification 2 are derived from the variables in Specification 4. Predicate decisions\([\text{slot}] = \text{cmd} \) holds if there exists any round that has decided the command (Section 3.4 argues why all rounds of a given slot can only decide the same command); otherwise decisions\([\text{slot}] = \bot \). This is captured formally in Fig. 3 where \( \text{cmd} \) is a tuple \((\text{clt}, \text{op})\).

The transition certifySeq\((\text{cert}, \text{slot}, (\text{rid}, \text{cmd}))\) of Multi-Consensus always corresponds to a stutter in active replication. The decide\((\text{slot}, \text{cmd})\) transition of Specification 2 is performed when, for the first time, a majority of certifiers in some round \( \text{rid} \) have certified command \( \text{cmd} \) in slot \( \text{slot} \), that is, when the last certifier \( \text{cert} \) in the majority performs transition certify\((\text{cert}, \text{slot}, (\text{rid}, \text{cmd}))\). The observeDecision transition of Multi-Consensus corresponds exactly to the learn transition of active replication. Both the supportRound and recover transitions are stutters with respect to Specification 2 as they do not affect any of its state variables.

### 3.6 Passive Replication

Section 2.2 showed how in passive replication a state update from a particular primary can only be decided in a slot if it corresponds to applying an operation on the state decided in the previous slot. We called this property prefix ordering. However, Specification 4 does not satisfy prefix ordering because any proposal can be decided in a slot, in particular one that does not correspond to a state decided in the prior slot. Thus Multi-Consensus does not refine Passive Replication. One way of implementing prefix ordering would be for the primary to wait with proposing a command for a slot until it knows the decisions for all prior slots. Doing so would be slow.

A better solution is to refine Multi-Consensus to obtain a specification that also refines Passive Replication and satisfies prefix ordering. We call this specification Multi-Consensus-PO. Multi-Consensus-PO guarantees that each decision is the result of an operation applied to the state decided in the prior slot (except for the first slot). We complete the refinement by adding two preconditions:

(i) In Multi-Consensus-PO, slots have to be decided in order. To guarantee this, we have each certifier certify commands in order by adding the following precondition to transition certify: \( \text{slot} > 1 \Rightarrow \exists \text{oldState} : \text{cmd} = (\text{oldState}, -) \land \text{progress}_{\text{cert}}[\text{slot} - 1] = (\text{rid}, \text{c}) \). Thus if, in some round, there exists a majority of certifiers that have certified a command in \( \text{slot} \), there also exists a majority of certifiers that have certified a command in the prior slot.

(ii) To guarantee that a decision in \( \text{slot} \) is based on the state decided in the prior slot, we add the following precondition to transition certifySeq: \( \text{slot} > 1 \Rightarrow \exists \text{oldState} : \text{cmd} = (\text{oldState}, -) \land \text{progress}_{\text{cert}}[\text{slot} - 1] = (\text{rid}, (-, -), \text{oldState}) \). This works because by the properties of progress
indicators, if a command has been or will be decided in round \( \text{rid} \) or a prior round, it is the command in \( \text{progress} \_\text{cert}[\text{slot} - 1] \). Therefore, if the sequencer’s proposal for \( \text{slot} \) is decided in round \( \text{rid} \), it is the command in \( \text{progress} \_\text{cert}[\text{slot} - 1] \). If the primary and the sequencer are co-located, as they usually are, this condition is satisfied automatically as the primary computes states in order.

Also, Multi-Consensus-PO inherits transitions \( \text{invoke} \) and \( \text{response} \) from Specification 1 as well as transitions \( \text{propose}, \text{update}, \) and \( \text{resetShadow} \) from Specification 3. The variables contained in these transitions are inherited as well.

The passive replication protocols that we consider in this paper, VSR and Zab, share the following design decision in the recovery procedure: The sequencer broadcasts a single message containing its entire snapshot rather than sending separate certifications for each slot. Certifiers wait for this comprehensive snapshot, and overwrite their own snapshot with it, before they certify new commands in this round. As a result, at a certifier all \( \text{progress} \_\text{cert} \) slots have the same round identifier, and can thus be maintained as a separate variable.

### 3.6.1 Refinement Mappings

Below, we present a refinement between Multi-Consensus-PO and Multi-Consensus and then show that Multi-Consensus-PO refines Passive Replication.

### 3.6.2 Refining Multi-Consensus

Showing the existence of a refinement between Multi-Consensus-PO and Multi-Consensus is straightforward. Transitions and variables inherited from passive replication are mapped to variables and transitions of Multi-Consensus in the same way as they are mapped from passive replication to active replication (note that these variables and transitions are mapped to variables and transitions of Multi-Consensus that are themselves inherited from active replication). Since Multi-Consensus-PO only adds constraints to the transitions that are specific to Multi-Consensus, the refinement exists.

### 3.6.3 Refining Passive Replication

Passive replication has a single internal variable, \( \text{decisions} \), that is derived from \( \text{certifics} \), as in Fig. 3 that is, for any slot \( s \), \( \text{decisions}[s] = \text{cmd} \) holds if a majority of replicas have certified \( \text{cmd} \) for slot \( s \).

Transitions \( \text{certifySeq}, \text{supportRound}, \) and \( \text{recover} \) are stutters in passive replication, while an \( \text{observeDecision} \) transition of Multi-Consensus-PO corresponds to a \( \text{learn} \) transition in Specification 3.

Similarly to the refinement between active replication and Multi-Consensus, transition \( \text{decide}(\text{slot}, \text{cmd}, \text{res}, \text{newState}) \) is performed when, for the first time, a majority of certifiers in some round \( \text{rid} \) have certified command \( \text{cmd} \) in slot \( \text{slot} \), that is, when the last certifier \( \text{cert} \) in the majority performs transition \( \text{certify}(\text{cert}, \text{slot}, (\text{rid}, \text{cmd})) \).

In this case however, we must additionally show that when the last certifier of the majority undergoes the \( \text{certify} \) transition, the following condition holds in Specification 3: \( \exists r, \text{oldState} : (\text{oldState}, (\text{cmd}, \text{res}, \text{newState})) \in \text{proposals}_r[\text{slot}] \) and \( (\text{slot} > 1 \Rightarrow \text{decisions}[\text{slot} - 1] = (\text{oldState}) \in \text{newState}) \).

The first part of the condition holds since certifiers only certify commands that have been proposed. The second part of the condition holds for the following reason. From constraint (ii) of Multi-Consensus-PO, the sequencer only certifies a command if its corresponding \( \text{oldState} \) equals the \( \text{newState} \) of the command it stores in the previous slot. From the properties of progress indicators, if \( \text{progress} \_\text{cert}[\text{slot} - 1] = (\text{rid}, \text{cmd}) \), then if a command is decided in an earlier round than \( \text{rid} \), then it must be \( \text{cmd} \). From constraint (i) (commands are certified in order), if a command is decided for \( \text{slot} \), then \( \text{slot} - 1 \) decided on a command previously. Consequently, \( (\text{slot} > 1 \Rightarrow \text{decisions}[\text{slot} - 1] = (\text{oldState}) \).

### 4 Implementation

Specifications Multi-Consensus and Multi-Consensus-PO do not contain internal variables or transitions. However, they only specify which transitions are safe, not which transitions to perform and at what time. We show, informally, final refinements of these specifications to obtain Paxos, VSR, and Zab.

#### 4.1 Normal Case

We first turn to implementing the state and transitions of Multi-Consensus and Multi-Consensus-PO. The first question is how to implement the variables. Variables \( \text{inputs}_r, \text{outputs}_r, \text{certifics}_r, \) and \( \text{snapshots}_r \) are not per-process but global. They model messages that have been sent. In actual protocols, these are implemented by the network: a value in either set is implemented by a message on the network tagged with the appropriate type, such as \( \text{snapshot} \).

The remaining variables are all local to a process such as a client, a replica, or a certifier, and can be implemented...
as ordinary program variables. In Zab, the progress\textsubscript{cert} variable is implemented by a queue of commands. In VSR, the progress\textsubscript{cert} variable is replaced by the application state and a counter that counts the number of updates made to the state in this round. In Paxos, a progress indicator is simply a pair consisting of a round identifier and a command.

Fig. 5 illustrates the steps of normal case processing in the protocols. The figure shows three certifiers ($f = 1$). Upon receiving an operation from a client (not shown):

1. The sequencer cert proposes a command for the next open slot and sends a message to the other certifiers (maps to certifySeq(cert, slot, \{rid, cmd\}). In the case of VSR and Zab, the command is a state update that results from executing the client operation; with Paxos, the command is the operation itself.

2. Upon receipt by a certifier cert, if cert supports the round-id rid in the message, then cert updates its slot and replies to the sequencer (transition certify(cert, slot, \{rid, cmd\}). With VSR and Zab, prefix-ordering must be ensured, and cert only replies to the sequencer if its progress indicator for slot − 1 contains a non-empty command for rid.

3. If the sequencer receives successful responses from a majority of certifiers (transition observeDecision), then the sequencer learns the decision and broadcasts a decide message for the command to the replicas (resulting in learn transitions that update the replicas (see Specifications 2 and 3).

The various protocols make additional design decisions:

- In the case of VSR, a specific majority of certifiers is determined a priori and fixed for each round. We call this a designated majority. In Paxos and Zab, any certifier can certify proposals.

- In VSR, replicas are co-located with certifiers, and certifiers speculatively update their local replica as part of certification. A replica may well be updated before some proposed command is decided, so if another command is decided the state of the replica must be rolled back, as we shall see later. Upon learning that the command has been decided (Step 3), the sequencer responds to the client.

- Optionally, Paxos uses leases [8, 18] for read-only operations. Leases have the advantage that read-only operations can be served at a single replica while still guaranteeing linearizability. This method assumes synchronized clocks (or clocks with bounded drift) and has the sequencer obtain a lease for a certain time period. A sequencer that is holding a lease can thus forward read-only operations to any replica, inserting the operation in the ordered stream of commands sent to that replica.

- Zab (or rather ZooKeeper) offers the option to use leasing or to have any replica handle read-only operations individually, circumventing Zab. The latter is efficient, but a replica may not have learned the latest decided proposals and its clients receive results based on stale state (such reads satisfy sequential consistency [16]).

For replicas to learn about decisions, two options exist:

- Certifiers can respond back to the sequencer. The sequencer learns that its proposed command has been decided if the sequencer receives responses from a majority (counting itself). The sequencer then notifies the replicas.

- Certifiers can broadcast notifications to all replicas, and each replica can individually determine if a majority of the certifiers have certified a particular command.

There is a trade-off between the two options: with $n$ certifiers and $m$ replicas, the first approach requires $n + m$ messages and two network latencies. The second approach requires $n \times m$ messages but involves only one network latency. All implementations we know of use the first approach.

### 4.2 Recovery

Fig. 5 illustrates the recovery steps in the protocols. With Paxos, a certifier proseq unhappy with progress starts the following process to try to become sequencer itself (see Fig. 5a):

- Step 1: Prospective sequencer proseq supports a new round rid proposing itself as sequencer (transition supportRound(proseq, rid, proseq)), and queries at least a majority of certifiers.
Prospective sequencers in Zab are determined by a weak leader election protocol $\Omega$ [29]. When $\Omega$ determines that a sequencer has become unresponsive, it initiates a protocol (see Fig. 5b) in which a new sequencer is elected:

- Step 0: $\Omega$ proposes a prospective sequencer $proseq$ and notifies the certifiers. Upon receipt, a certifier sends a message containing the round-id it supports to $proseq$.

- Step 1: Upon receiving such messages from a majority of certifiers, prospective sequencer $proseq$ selects a round-id $rid$ that is one larger than the maximum it received, transitions to supporting it (transition $supportRound(proseq, rid, proseq)$), and broadcasts this to the other certifiers for approval.

- Step 2: Upon receipt, if certifier $cert$ can support $rid$ and has not agreed to a certifier other than $proseq$ becoming sequencer of the round, it performs transition $supportRound(cert, rid, proseq)$. Zab exploits prefix ordering to optimize the recovery protocol. Instead of sending its entire snapshot to the prospective sequencer, a certifier that transitions to supporting round $rid$ sends a round-stamp. A round-stamp is a lexicographically ordered pair consisting of the round-id in the snapshot (the same for all slots) and the number of slots in the round for which it has certified a command.

- Step 2.5: If $proseq$ receives responses from a majority, $proseq$ computes the maximum round-stamp and determines if it is missing commands. If it is the case, $proseq$ retrieves them from the certifier $cert_{max}$ with the highest received round-stamp. If $proseq$ is missing too many commands (e.g. if $proseq$ did not participate in the last round $cert_{max}$ participated in), $cert_{max}$ sends its entire snapshot to $proseq$.

- Step 3: After receiving the missing commands, $proseq$ broadcasts its snapshot, in practice a checkpoint of its state with a sequence of state updates, to the certifiers (transition $recover(proseq, rid, S)$). Certifiers acknowledge the reception of this snapshot and, upon receiving acknowledgments from a majority, $proseq$ learns that it is now the sequencer of $rid$ and broadcasts a commit message before resuming the normal case protocol (not shown in the picture).

In VSR, each round-id has a pre-assigned view manager $v$ that is not necessarily the sequencer. A round-id is a lexicographically ordered pair comprising a number and the process identifier of the view manager.

If unhappy with progress, the view manager $v$ of round-id starts the following recovery procedure (see Fig. 5c):

- Step 1: $v$ starts supporting round $rid$ (transition $supportRound(v, rid, v)$), and queries at least a majority of certifiers.

- Step 2: Upon receipt of such a query, a certifier $cert$ starts supporting $rid$ (transition $supportRound(cert, rid, v)$). Similarly to Zab, $cert$ sends its round-stamp to $v$.

- Step 2.5: Upon receiving round-stamps from a majority of certifiers, view manager $v$ uses the set of certifiers that responded as the designated majority for the round and assigns the sequencer role to the certifier $p$ that reported the highest round-stamp. The view manager then notifies certifier $p$, requesting it to become sequencer.
• Step 3: Sequencer $p$, having the latest state, broadcasts its snapshot (transition $\text{recover}(p, \text{rid}, S)$).

In the case of VSR, the state that the new sequencer sends is its application state rather than a snapshot.

4.3 Garbage Collection

Multi-Consensus has each certifier building up state about all slots, which does not scale. Unfortunately, little is written about this issue in the Paxos and Zab papers. In VSR, no garbage collection is required. Certifiers and replicas are co-located, and only store the most recent round-id they adopted and the application state that is updated upon certification of a command. During recovery the sequencer simply sends the application state to the replicas, and consequently, there is no need to replay any decided commands.

4.4 Liveness

All of the protocols require—in order to make progress—that at most a minority of certifiers experience crash failures. If the current round is no longer making progress, a new round must become operational. If certifiers are slow at this in the face of an actual failure, then performance may suffer. However, if certifiers are too aggressive, rounds will become wedged before being able to decide commands, even in the absence of failures.

To guarantee progress, some round with a correct sequencer must eventually not get preempted by a higher round [6, 5]. Such a guarantee is difficult or even impossible to make [7], but with careful failure detection a good trade-off can be achieved between rapid failure recovery and spurious wedging of rounds [14].

In this section, we will look at how the various protocols try optimizing progress.

4.4.1 Partial Memory Loss

If certifiers keep their state on stable storage (say, a disk), then a crash followed by a recovery is not treated as a failure but instead as the affected certifier being slow. Stable storage allows protocols like Paxos, VSR, and Zab to deal with such transients. Even if all machines crash, as long as a majority eventually recovers their state from before the crash, the service can continue operating.

4.4.2 Total Memory Loss

In Section 5.1 of “Paxos Made Live” [4], the developers of Google’s Chubby service describe a way for Paxos to deal with permanent memory loss of a certifier (due to disk corruption). The memory loss is total, so the recovering certifier starts in an initial state. It copies its state from another certifier and then waits until it has seen one decision before starting to participate fully in the Paxos protocol again. This optimization is flawed since it breaks the invariant that a certifier’s round-id can only increase over time (confirmed by the authors of [4]). By copying the state from another certifier, it may, as it were, go back in time, which can cause divergence.

Nonetheless, total memory loss can be tolerated by extending the protocols. The original Paxos paper [18] shows how the set of certifiers can be reconfigured to tolerate total memory loss, and this has been worked out in greater detail in Microsoft’s SMART project [25] and later for Viewstamped Replication as well [23]. Zab also supports reconfiguration [31].

5 Discussion

Table summarizes differences between Paxos, VSR, and Zab. We believe that these differences are important because they both demonstrate that the protocols do not refine one another, and the differences have pragmatic consequences as discussed below. The comparisons are based on published algorithms; actual implementations may vary. We organize the discussion around normal case processing and recovery overheads.

5.1 Normal Case

Passive vs. Active Replication

In active replication, there are at least $f + 1$ replicas that each have to execute operations. In passive replication, only the sequencer executes operations, but has to propagate state updates to the backups. Depending on the overheads of executing operations and the size of state update messages, one or the other may perform better. Passive replication has the advantage that execution at the sequencer does not have to be deterministic and can take advantage of parallel processing on multiple cores.

Read-only Optimizations

Paxos and Zookeeper support leasing for read-only operations, but there is no reason why leasing could not be added to VSR as well. Indeed, Harp (built on VSR) uses leasing. A lease improves latency of read-only operations in the normal case, but delays recovery in case of a failure. ZooKeeper offers the option whether leases should be used. Without leases, clients read any replica at any time. Doing so compromises consistency since replicas may have stale state.
### Table 2: Overview of important differences between the various protocols.

| What                        | Section | Paxos          | VSR            | Zab            |
|-----------------------------|---------|----------------|----------------|----------------|
| replication style           |         | active         | passive        | passive        |
| read-only operations        |         | leasing        | certification  | yes            |
| designated majority         |         | upon decision  | upon decision  | yes            |
| time of execution           |         | majority vote  | view manager assigned | depends on role |
| sequencer selection         |         | two-way        | from sequencer | majority vote  |
| recovery direction          |         | slot-at-a-time | application state | two-way/from sequencer |
| recovery granularity        |         | reconfigure    | partial        | reconfigure    |
| tolerates memory loss       |         |                |                |                |

**Designated Majority**  VSR uses designated majorities. This has the advantage that the other (typically \( f \)) certifiers and replicas are not employed during normal operation, and they play only a small role during recovery, saving almost half of overhead. There are two disadvantages: (1) if the designated majority contains the slowest certifier the protocol will run at the rate of that slowest certifier, as opposed to the “median” certifier; and (2) if one of the certifiers in the designated majority crashes or becomes unresponsive or slow, then a recovery is necessary. In Paxos and Zab, recovery is necessary only if the sequencer crashes. A middle ground can be achieved by using \( 2f + 1 \) certifiers and \( f + 1 \) replicas.

**Time of Command Execution**  In VSR, replicas apply state updates speculatively at the same time that they are certified, possibly before they are decided. Commands are forgotten as soon as they are applied to the state. This means that no garbage collection is necessary. A disadvantage is that the response to a client operation must be delayed until all replicas in the designated majority have updated their application state. In other protocols, only one replica has to have updated its state and computed a response, because in case the replica fails another deterministic replica is guaranteed to compute the same response. Note that at this time each command that led to this state and response has been certified by a majority and therefore the state and response are recoverable even if this one replica crashes.

In Zab, the primary also speculatively applies client operations to compute state updates before they are decided. However, replicas only apply those state updates until after they have been decided. In Paxos, replicas only execute a command after it has been decided and there is no speculative execution.

### 5.2 Recovery

**Sequencer Selection**  VSR provide an advantage in selecting a sequencer that has the most up-to-date state (taking advantage of prefix ordering), and thus it does not have to recover this state from the other certifiers, simplifying and streamlining recovery.

**Recovery Direction**  Paxos allows the prospective sequencer to recover the state of previous slots and, at the same time, propose new commands for slots for which it already has retrieved sufficient state. However, all certifiers must send their certification state to the prospective sequencer before it re-proposes commands for slots. With VSR, the sequencer is the certifier with the highest round-stamp and it does not need to recover state from the other certifiers. A similar optimization is sketched in the description of the Zab protocol (and implemented in the Zookeeper service).

With Paxos and Zab, garbage collection of the certification state is important to ensure that the amount of state that has to be exchanged on recovery does not become too large. It is often faster to bring a recovering replica up to date by replaying decided commands that it missed rather than by copying state.

With VSR, the selected sequencer pushes its snapshot to the other certifiers. The snapshot has to be transferred and processed before new certification requests, possibly resulting in a performance hiccup.

**Recovery Granularity**  In VSR, state sent from the sequencer to the backups is the entire application state. For VSR, this state is transaction manager state and is small, but in general such an approach does not scale. However, in some cases that cost is unavoidable, even in the other protocols. For example, if a replica has a disk failure, replaying all commands from day 0 is not scalable either, and the recovering replica instead will have to seed its state from another one. In such a case, the replica will load a checkpoint and then replay missing commands to bring the checkpoint up-to-date—this technique is used in Zab (and in Harp as well). With passive replication protocols, replaying missing commands simply means applying state updates; with active replication protocols, replaying commands entails re-executing commands. Depending on the overheads of executing operations and the size of state up-
date messages, one or the other approach may perform better.

**Tolerating Memory Loss**  An option suggested by VSR is to keep only a round-id on disk; the remaining of the state is in memory. This technique works only in restricted situations where at least one certifier has the most up-to-date state in memory.

### 6 A Bit of History

Based on the discussion so far one may think that rounds and sequencers were first introduced by protocols that implement Multi-Consensus. However, we believe the first consensus protocol to use rounds and sequencers is due to Dwork, Lynch, and Stockmeyer (DLS) [6]. Rounds in DLS are countable and round \( b + 1 \) cannot start until round \( b \) has run its course. Thus, DLS does not refine Multi-Consensus.

Chandra and Toueg’s work on consensus [5] formalized the conditions under which consensus protocols terminate by encapsulating synchrony assumptions in the form of failure detectors. Their consensus protocol resembles the Paxos single-decree Synod protocol and refines Multi-Consensus.

To the best of our knowledge, the idea of using majority intersection to avoid potential inconsistencies first appears in Thomas [32]. Quorum replication [32] supports only storage objects with read and write operations (or, equivalently, get and put operations in the case of a Key-Value Store).

### 7 Conclusion

Paxos, VSR, and Zab are well-known replication protocols for an asynchronous environment that admits a bounded number of crash failures. The paper describes a specification for Multi-Consensus, a generic specification that contains important design features that the protocols share. These features include an unbounded number of totally ordered rounds, a static set of certifiers, and at most one sequencer per round.

The protocols differ in how they refine Multi-Consensus. We were able to disentangle fundamentally different design decisions in the three protocols and consider their impact on performance. Most importantly, compute-intensive services are better off with a passive replication strategy such as used in VSR and Zab (provided that state updates are of a reasonable size). To achieve predictable low-delay performance for short operations during both normal case execution and recovery, an active replication strategy without designated majorities, such as used in Paxos, is the best option.

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