Ultrahigh reflection from a medium with ultraslow group velocity

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We show that an incident wavepacket at the boundary to a medium with extremely slow group velocity, experiences enhanced reflection and a substantial spatial and temporal distortion of the transmitted wave packet. In the limit of vanishing group velocity, light cannot be transferred into the medium due to its perfect reflectivity.

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Currently there is a great deal of interest in slowing down light in dispersive media. It is triggered by a continuous experimental progress using electromagnetically induced transparency (EIT) in gaseous media. The attained group velocities are as small as 17 m/s for the medium being a Bose–Einstein condensate, 90 m/s in a hot rubidium gas, and even 8 m/s in a rubidium vapor at room temperature [1]. EIT provides here a large enhancement of the dispersion of medium while absorption is largely suppressed [2].

A theoretical description of optical radiation propagating with group velocities \(v_g \sim 10^{-3}c\) (where \(c\) is the speed of light in vacuum) or even slower, may require a careful reconsideration of some approximations routinely employed in optics. For example, the reflection of an optical beam at the boundary of the medium is usually considered to be determined by the refractive index of the medium. This is perfectly valid for a monochromatic beam, and serves as a perfect approximation for a wavepacket in a typical dielectric with weak dispersion, where each spectral component experiences essentially the same refractive index. However, when the dispersion steepens, a substantial variation of the refractive index over the spectrum of a wavepacket may be present. Then the usual approximation fails and new effects emerge that are dominated by the spectrally dependent refraction of the transmitted light. Besides a severe spatio-temporal distortion of the transmitted beam, one might expect difficulties in actually transferring the incident optical wavepacket into a medium of such strong dispersion.

In this Letter we show the significance of these effects for optical experiments with extremely slow group velocity. We point out a fundamental obstacle preventing one from infintely decreasing the speed of light inside a medium: The slowing down of light inside the medium is accompanied by an increase of the reflectivity of the boundary of the medium. Near the limit of stopping light, almost no energy can penetrate from outside (as well as leak from inside) and the boundary of the medium will behave as a perfect mirror.

Let us consider the reflection and transmission of an optical wavepacket at a plane boundary between vacuum and a linear dielectric with frequency dependent dielectric function \(\varepsilon(\omega)\). Since a relatively small absorption will have little effect on our conclusions we take \(\varepsilon(\omega)\) to be real valued [3]. Expanding the dielectric function of the medium around the center frequency \(\omega_0\) of the incident wavepacket and keeping only the leading term, we arrive at

\[
\varepsilon(\omega) = n_0^2 + \frac{2\alpha}{\omega_0} (\omega - \omega_0) + \ldots ,
\]

where \(n_0\) is the (absolute) refractive index of the medium at \(\omega = \omega_0\) and \(\alpha\) determines the steepness of the dispersion [4].

In the medium, the magnitude of the wave vector and the dielectric function are related by \(k(\omega) = \omega/c\sqrt{\varepsilon(\omega)}\), from which we obtain for the group velocity inside the medium,

\[
v_g = \frac{c}{n_0 + \alpha/n_0} .
\]

The transmittance at the boundary of the medium can be illustrated in more detail by considering the intensity reflection and transmission coefficients for the case of normal incidence [5]:

\[
R(\omega) = \left| \frac{1 - \sqrt{\varepsilon(\omega)}}{1 + \sqrt{\varepsilon(\omega)}} \right|^2 , \quad T(\omega) = \frac{2}{1 + \sqrt{\varepsilon(\omega)}} \left\| T(\omega) \right\|^2 ,
\]

As can be seen from Fig. [1], slowing down the group velocity brings forth two distinct features in the reflectivity: (a) For frequencies below the increasing cutoff frequency,

\[
\omega_c = \omega_0 \left( 1 - \frac{n_0^2}{2\alpha} \right) ,
\]

the low-frequency wing experiences total reflection, and (b) an overall increase of the reflectivity for the high-frequency wing. Usually these effects play no role in optical experiments. However, for a small ratio \(v_g/c\), the cutoff frequency may move into the spectrum of the incident optical wavepacket. One clearly observes from Fig. [1] that whereas for \(c/v_g = 5 \times 10^7\) and a wavepacket of 1 \(\mu\)s duration the reflectivity is still small, for lower group velocities the cutoff frequency approaches \(\omega_0\), while the reflectivity for the high-frequency tail also continuously increases. In the extreme limit \(v_g/c \rightarrow 0\) we then obtain overall reflection of the incident optical wavepacket for all frequencies apart from the center frequency.
The parameter \( r \) defined as the ratio of the characteristic spectral width of the incident wavepacket \( \Delta \omega \) and the pass-band,

\[
    r = \frac{\Delta \omega / 2}{\omega_0 - \omega_c} = \frac{\alpha}{n_0^2} \frac{\Delta \omega}{\omega_0} \approx \frac{c}{n_0 \gamma} \frac{\Delta \omega}{\omega_0},
\]

can be used as a measure for the relevance of the boundary effects. When \( r > 1 \) the cutoff frequency considerably moves into the spectrum of the incident wavepacket, so that the variation of the refractive index over the spectrum leads already to an enhanced reflection and substantial spectral (temporal) distortion of the wave packet. The spectral modifications are asymmetric with respect to the center frequency \( \omega_0 \). Since the low-frequency tail experiences a larger reflection compared to the higher frequencies, this results in an overall blue shift of the transmitted wavepacket.

For the ideal case of EIT with purely radiative transitions of rate \( \gamma \), negligible Doppler effect, and long-living coherence, one finds \( n_0 \approx 1 \) and \( r \approx 3/(8\pi^2) N \lambda^3 \gamma \Delta \omega / \Omega^2 \) with \( N \) and \( \Omega \) being the atomic density and the Rabi frequency of the driving field, respectively. The relevant spectral width of the wavepacket may be taken as the EIT transparency window \( \Delta \omega \approx \Omega^2 / \gamma \), and we immediately obtain the estimation \( r \approx 3N \lambda^3 / (8\pi^2) \). In conclusion, for optical fields, say of wavelength \( \lambda = 800 \text{ nm} \), \( r \) may already approach unity for rather modest densities of \( N \approx 5 \times 10^{13} \text{ cm}^{-3} \).

In general, the importance of the boundary effects can be demonstrated by the energy transfer into the medium, characterized by the ratio: \( T_E = E_t / E_i \). Here \( E_t \) and \( E_i \) are the total energies of the transmitted and incident wavepackets, respectively, defined as the normal component of the Poynting vector in and outside the medium, integrated over time and the transverse area of the beam. Fig. 2 shows a typical example of how the energy-transfer ratio \( T_E \) changes with group velocity for an incident Gaussian wavepacket. The continuous decrease of \( T_E \) with decreasing group velocity indicates a vanishing energy flow through the boundary for \( v_g / c \rightarrow 0 \). The above result remains conceptually valid for a dielectric function of a more general form than Eq. (1). Given that any additional term appearing in Eq. (1) is small compared to the first-order term, we expect only a minor modification of the cutoff frequency and of the overall index of refraction.

Analogous effects arise when the sharp boundary is replaced by a layer of finite width \( L \), where the dielectric function varies continuously along the direction of propagation. Such a regime is readily available in EIT with the continuous variation of \( \varepsilon \) being provided by the externally controllable driving field. Despite arbitrary intermediate modifications, the reflection properties depend only on the difference between the initial and final values of the dielectric function, \( \varepsilon(\omega, z = 0) \) and \( \varepsilon(\omega, z = L) \), respectively [3]. Then our results are recovered upon replacement of \( \varepsilon(\omega) \) by \( \varepsilon(\omega, z = L) \) in Eq. (1).

For the more general situation where the incident beam has some spread, we expect along with temporal also spatial distortions. Typically the spread of the transmitted beam will increase with increasing refractive index. Approaching the extreme case of very large dispersion, when the cutoff frequency appears inside the spectrum, the lower frequency components are totally reflected, whereas the other components are refracted over a wide range of angles.
The enhanced reflectivity from the highly dispersive medium draws attention also to the importance of backscattering effects in the field dynamics in inhomogeneous/nonlinear media. Whenever the parameter $r$ (or $1/T_E$) gains appreciable values, the widely used approximation of unidirectional propagation may fail. Instead, the full wave equation must be used for a proper inclusion of the counter-propagating waves.

The occurrence of a cutoff frequency where wave propagation is inhibited is a well known phenomenon in the ionosphere and waveguides [5]. However, compared to these cases we deal here with a dielectric function of a conceptually different form. To illustrate this, we compare in Fig. 3 the frequency dependence of the group velocity for these cases. Whereas, in the case of the ionosphere or waveguides the group velocity is approaching $c$ for increasing frequency, in our case a maximum can be observed, after which the group velocity is continuously falling to zero. That is, in our case a suppression of wave propagation is present also for higher frequency.

In conclusion, an incident wavepacket at the boundary to a highly dispersive medium with extremely slow group velocity experiences enhanced reflection and a substantial spatio-temporal distortion of the transmitted wavepacket. In the case of EIT, the frequency selective reflectivity may be utilized for generating pulses (in reflection) revealing spectral shapes sensitively controllable by the external driving field. One may also expect that spectrally resolved polarization measurements of the reflected beam can provide an efficient tool for probing the dispersive properties of such media. On the other hand, the transmission losses due to the enhanced reflectivity pose a serious restriction on possible applications of slow light for quantum-optical purposes. Finally, we have shown, that in the extreme limit of vanishing group velocity, light cannot penetrate the boundary due to the perfect reflectivity of the medium.

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[1] L.V. Hau, S.E. Harris, Z. Dutton, and C.H. Behroozi, Nature (London) 397, 594 (1999); M.M. Kash, V.A. Sautenkov, A.S. Zibrov, L. Hollberg, G.R. Welch, M.D. Lukin, Y. Rostovtsev, E.S. Fry, and M.O. Scully, Phys. Rev. Lett. 82, 5229 (1999); D. Budker, D.F. Kimball, S.M. Rochester, and Y.Y. Yashchuk, ibid 83, 1767 (1999).

[2] For a review of EIT see S.E. Harris, Phys. Today 50 (7), 36 (1997) and references therein.

[3] For a dispersive medium the imaginary and real parts of $\varepsilon(\omega)$ are linked via a Kramers-Kronig relation.

[4] Without loss of generality, we consider here the case $\alpha \geq 0$. The opposite case is easily obtained by exchanging the low and high frequency tails of the wavepacket.

[5] J.D. Jackson, Classical Electrodynamics (Wiley, New York, 1975), 2nd edition.

[6] M. Born and E. Wolf, Principles of Optics (Cambridge University Press, Cambridge, 1980), 6th edition.