Power Conservative Equivalent Circuit for DC Networks

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This work was supported by Companhia Energética de Brasília (CEB) as part of R&D program ANEEL 001/2016.

ABSTRACT This paper introduces a new equivalent circuit for linear direct current networks containing independent voltage sources and resistors, which represents all internal losses and can be used for the power and efficiency analysis of the actual circuit. It is shown that the internal losses of a direct current circuit have two components. One is variable and dependent on the internal resistances of the actual circuit and the power transferred to the component connected to the output terminals. The other is constant and dependent only on the internal sources and resistances of the original circuit. In addition, it is demonstrated that the Thévenin equivalent circuit is a particular case of the proposed circuit. Lastly, an alternative method for the determination of the resistance of the Thévenin equivalent circuit is introduced. The theoretical results are validated by numerical simulation.

INDEX TERMS Equivalent circuit, power conservation, Thévenin’s theorem, Thévenin equivalent circuit, efficiency of equivalent circuits.

I. INTRODUCTION
Equivalent circuits are commonly used in the analysis of electrical circuits because they provide simplicity in many situations. The concept of equivalent circuits is rooted in Ohm’s Law, Kirchhoff’s Laws and the Superposition Principle, which are different forms of the Principle of Energy Conservation, the Principle of Electrical Charge Conservation and the Principle of Least Action.

In 1883, Léon Charles Thévenin (1857-1926), an engineer at Postes and Télégraphes in France, published a paper which presents the theorem that would later become known as Thévenin’s Theorem. This gave rise to the Thévenin equivalent circuit for direct current networks [1].

It is known that despite the relevance of the Thévenin equivalent circuit, it is limited to representing the phenomena that occur in the actual circuit. It serves only to determine current and voltage at a pair of terminals. This originates from the fact that power is not preserved. In other words, the equivalent circuit does not allow the determination of the efficiency of the original circuit or of all power dissipated in the internal resistors. Thus, it can be stated that the Thévenin equivalent circuit is not conservative, as can also be said of the Norton equivalent circuit. Even between them, the Thévenin and the Norton equivalent circuits are not equivalent in the amount of internal power they consume within themselves [15]. In the literature, some authors have referred to these practical limitations of the Thévenin equivalent circuit [16].

A review of publications since 1883 that present the various theorems for analyzing circuits and networks [1-13] reveals that none of them addresses the analysis of input power, internal losses and efficiencies of equivalent circuits for DC networks.

This paper discusses an equivalent circuit that is an extension to and differs from that proposed by Thévenin with regard to its conservativeness in relation to the actual circuit. Thus, it serves not only to calculate the power transferred to a load resistor connected at the terminals, but also to calculate all internal dissipated power and efficiency of the actual circuit.

II. DERIVATION OF THE PROPOSED EQUIVALENT CIRCUIT
As mentioned above, the proposed equivalent circuit can be considered an extension of the Thévenin equivalent circuit. Therefore, the obtention of the new circuit begins with the determination of the parameters of the Thévenin equivalent circuit, as detailed below.
A. Thévenin Equivalent Circuit

A linear resistive network is represented by $N$ in Fig. 1, containing independent voltage sources and resistors, with two external terminals $ab$. The electrical resistance usually connected at the terminals $ab$ for the classical demonstration of Thévenin theorem is replaced by a voltage source denoted by $V_{ab}$, according to the substitution theorem.

The mesh current equations in matrix form are given by

$$
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n \\
V_o
\end{bmatrix}
= 
R
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n \\
I_o
\end{bmatrix},
$$

(1)

where $V_1, V_2, \ldots, V_n$ are the algebraic sum of all internal independent ideal voltage sources in the respective independent mesh of the network $N$ mesh. Any of these voltages can be zero. Therefore the number of independent voltage sources can be greater, lesser than or equal to the number of independent loops in the network $N$.

$V_o$ is the algebraic sum of all ideal independent source voltages of the external mesh, which is given by

$$
V_o = V_{ab} - V_a
$$

(2)

$V_o$ is the internal voltage source of the network $N$ that is in the mesh of $V_{ab}$.

$I_1, I_2, \ldots, I_n$ are the internal mesh currents of network $N$.

The resistance matrix includes all internal resistances of the network $N$.

From (1), we obtain

$$
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n \\
V_o
\end{bmatrix}
= 
\begin{bmatrix}
R_{11} & R_{12} & \cdots & R_{1n} & R_{1o} \\
R_{21} & R_{22} & \cdots & R_{2n} & R_{2o} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
R_{n1} & R_{n2} & \cdots & R_{nn} & R_{no} \\
R_{o1} & R_{o2} & \cdots & R_{on} & R_{no}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n \\
I_o
\end{bmatrix}
$$

(3)

where

$$
R =
\begin{bmatrix}
R_{11} & R_{12} & \cdots & R_{1n} \\
R_{21} & R_{22} & \cdots & R_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
R_{n1} & R_{n2} & \cdots & R_{nn}
\end{bmatrix}.
$$

From (3), the internal current matrix is obtained as

$$
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n \\
I_o
\end{bmatrix}
= 
R^{-1}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n \\
V_o
\end{bmatrix} - 
R_{1o}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n \\
I_o
\end{bmatrix},
$$

(4)

The voltage $V_o$ is given by

$$
V_o = \begin{bmatrix} R_{1o} & R_{2o} & \cdots & R_{no} \end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
\vdots \\
I_n \\
I_o
\end{bmatrix} + R_{no}I_o.
$$

(5)

Substituting (4) into (5) gives

$$
V_o = \begin{bmatrix} R_{1o} & R_{2o} & \cdots & R_{no} \end{bmatrix}R^{-1}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n \\
V_o
\end{bmatrix} - \begin{bmatrix} R_{1o} & R_{2o} & \cdots & R_{no} \end{bmatrix}R^{-1}
\begin{bmatrix}
R_{1o} \\
R_{2o} \\
\vdots \\
R_{no}
\end{bmatrix}
I_o + R_{no}I_o.
$$

(6)

Thus,

$$
V_o = V_T - V_a
$$

(7)

Substitution of (6) into (7) gives

$$
V_T = \begin{bmatrix} R_{1o} & R_{2o} & \cdots & R_{no} \end{bmatrix}R^{-1}
\begin{bmatrix}
V_1 \\
V_2 \\
\vdots \\
V_n
\end{bmatrix} + V_a
$$

(8)

Let us next consider the condition that all internal sources of network $N$ are set equal to zero and the voltage source is connected at the terminals $ab$.

Voltages and currents are mathematically related by (9), where $I_{osc}$ and $V_{osc}$ represent the current and voltage at the terminals $ab$ for this condition.

$$
V_{osc} = R_{no}I_{osc} - \begin{bmatrix} R_{1o} & R_{2o} & \cdots & R_{no} \end{bmatrix}R^{-1}
\begin{bmatrix}
R_{1o} \\
R_{2o} \\
\vdots \\
R_{no}
\end{bmatrix}I_{osc}
$$

(9)

The resistance of the Thévenin equivalent circuit, the driving point resistance of the network at the terminals $ab$ when all internal sources are set equal to zero, is

$$
R_T = \frac{V_{osc}}{I_{osc}}.
$$

(10)

Therefore,

$$
R_T = R_{no} - \begin{bmatrix} R_{1o} & R_{2o} & \cdots & R_{no} \end{bmatrix}R^{-1}
\begin{bmatrix}
R_{1o} \\
R_{2o} \\
\vdots \\
R_{no}
\end{bmatrix}.
$$

(11)

Substituting the values of (2), (8) and (11) in (6) we find (12), which represents the classical Thévenin equivalent circuit, shown in Fig. 2.

$$
V_{ab} = V_T + R_TI_o
$$

(12)
B. INTERNAL POWER DISSIPATED IN THE NETWORK WITH DISCONNECTED EXTERNAL VOLTAGE SOURCE ($I_0 = 0\, A$)

Under the condition that all internal sources are connected except the $V_{ab}$ source, then $I_0 = 0\, A$ and the power dissipated internally in the network is

$$P_X = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix}.$$

(13)

The branch currents in the internal sources of the network are given by

$$\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} = R^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}.$$

(14)

Substituting (14) into (13) gives (15), which determines the power dissipated internally in the network when $I_0 = 0\, A$.

$$P_X = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix} R^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}.$$

(15)

This internally dissipated power is not represented by the Thévenin equivalent circuit because according to this circuit the input power is null when $I_0 = 0\, A$.

C. INTERNAL POWER DISSIPATED IN THE NETWORK WITH CONNECTED EXTERNAL VOLTAGE SOURCE ($I_0 \neq 0\, A$)

The power dissipated internally in the network when the external voltage source is connected at the terminals $ab$, making $I_0 \neq 0\, A$, is given by

$$P = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_n \end{bmatrix} + V_o I_o.$$

(16)

Substituting (4) into (16) gives

$$P = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix} R^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} + V_o I_o.$$

(17)

As previously shown, voltage $V_o$ is mathematically given by (6). Hence,

$$V_o I_o = \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} I_o$$

$$- \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} I_o^2$$

$$+ R_{oo} I_o^2.$$

(18)

Substituting (18) into (17) gives

$$P = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix} R^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

$$- \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} I_o$$

$$+ \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix}^{-1} \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix} I_o^2$$

$$+ R_{oo} I_o^2.$$

(19)

Thus,

$$P = \begin{bmatrix} V_1 & V_2 & \ldots & V_n \end{bmatrix} R^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix}$$

$$- \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix}^{-1} \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix} I_o$$

$$+ \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix}^{-1} \begin{bmatrix} R_1 & R_2 & \ldots & R_n \end{bmatrix} I_o^2$$

$$+ R_{oo} I_o^2.$$

(20)

Substituting (11) and (15) into (20) gives

$$P = P_X + R_T I_o^2$$

(21)

where $P$ represents the sum of all internal network losses while $P_X$ represents the internal losses for the disconnected external source, i.e., for $I_0 = 0\, A$.
The term \( R_T I_o^2 \) represents the internal power losses in the equivalent resistance \( R_T \) of the Thévenin equivalent circuit. We then conclude that the internal losses have two components. One is constant and independent of the power delivered to the external load, which is equal to the internal power dissipated with the terminals \( ab \) open. The other is variable and dependent on the current at the external terminals \( ab \). The constant losses \( P_X \) are not represented by the Thévenin equivalent circuit.

Expression (21) represents the internal power of the equivalent circuit shown in Fig. 3, where \( V_T \) and \( R_T \) are the parameters of the Thévenin equivalent circuit and \( R_X \) is the additional resistance given by (22).

We can also determine \( R_X \) using (23), which is found by substituting (8) and (15) into (22).

\[
R_X = \left( \begin{bmatrix} R_{1o} & R_{2o} & \ldots & R_{no} \end{bmatrix} R^{-1} \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{bmatrix} + V_0 \right)^2
\]

III. NUMERICAL EXAMPLE
This numerical example illustrates the determination of the parameters of the proposed equivalent circuit. Let us consider the circuit shown in Fig. 4, with \( R_1 = 4 \Omega, R_2 = 3 \Omega, R_3 = 1 \Omega, R_4 = 5 \Omega, R_5 = 3 \Omega, V_1 = 10 \text{V}, V_2 = 30 \text{V} \) and \( V_o = 15 \text{V} \).

The mesh current equations in matrix form are

\[
\begin{bmatrix} V_1 \\ V_2 \\ V_o \end{bmatrix} = \begin{bmatrix} (R_1 + R_2) & -R_2 & 0 \\ -R_2 & (R_2 + R_3 + R_4) & R_4 \\ 0 & R_4 & (R_3 + R_4) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_o \end{bmatrix}.
\]

Hence,

\[
R_{11} = R_1 + R_2 = 7 \Omega; \quad R_{12} = -R_2 = -3 \Omega; \quad R_{10} = 0 \Omega;
\]

Substitution of the values for the resistors and voltage sources yields \( V_T = 22.222 \text{V} \).

The resistance of Thévenin equivalent circuit is given by (26).

\[
R_T = R_{1o} - \left[ \begin{bmatrix} R_{1o} & R_{2o} \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} \\ R_{12} & R_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \right] (26)
\]

Substituting the values of the network resistances in (26), we find \( R_T = 4.759 \Omega \).

The internal dissipated power in resistor \( R_X \) is given by

\[
P_X = \left( \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} \\ R_{12} & R_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \right)^2 (27)
\]

Substituting the network parameter values in (27), we find \( P_X = 166.667 \text{W} \).

The resistance \( R_X \) is determined by (28).

\[
R_X = \left( \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} \\ R_{12} & R_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \right)^2 (28)
\]

Hence, \( P_T = R_T I_o^2 \).
The total internally dissipated power $P$ is the sum of $P_T$ and $P_X$. Therefore, it is given by

$$P = \frac{V_T^2}{R_X} + R_T I_o^2.$$  \hfill (31)

After appropriate numerical substitutions we find $P = 177.626 \text{ W}$.

**IV. METHOD FOR EXPERIMENTAL DETERMINATION OF THE PROPOSED EQUIVALENT CIRCUIT PARAMETERS**

The parameters of the proposed equivalent circuit can be determined experimentally. A method is described below, using numerical simulation of the circuit shown in Fig. 4.

**A. MEASUREMENT OF VOLTAGE $V_T$**

The circuit with the terminals $ab$ open is shown in Fig. 5.

![Circuit with the terminals ab open for measuring voltage $V_T$.](image)

In the condition shown in Fig. 5, the simulation result yields $V_T = 22.22 \text{ V}$.

**B. MEASUREMENT OF RESISTANCE $R_T$**

For the determination of $R_T$, the voltages of sources $V_1$ and $V_2$ are set equal to zero and the electrical resistance is measured from the terminals $ab$, as shown in Fig. 6, where $R_T = V_a/I_a$. The value obtained by simulation is $R_T = 4.76 \Omega$.

![Measurement of the resistance $R_T$ from the terminals ab.](image)

**C. MEASUREMENT OF RESISTANCE $R_X$**

To determine the resistance $R_X$, the power consumed by the circuit with the terminals $ab$ open must be determined, as shown in Fig. 5. The values obtained by simulation are $V_1 = 10 \text{ V}$, $I_1 = 3.33 \text{ A}$, $V_2 = 30 \text{ V}$ and $I_2 = 4.44 \text{ A}$.

The internal power is defined by

$$P_X = V_1 I_1 + V_2 I_2.$$  \hfill (32)

Substituting the measured values for $V_1$, $I_1$, $V_2$ and $I_2$ into (32), we obtain $P_X = 166.66 \text{ W}$. The resistance is determined by

$$R_X = \frac{V_T^2}{P_X}.$$  \hfill (33)

After substituting the values of $P_X$ and $V_T$ into (33) yields $R_X = 2.96 \Omega$.

The obtained values for $V_T$, $R_T$ and $R_X$ are equal to the theoretical values calculated using (25), (26) and (28), respectively.

**V. ALTERNATIVE METHOD FOR DETERMINATION OF $R_T$**

The classic method for determining the resistance $R_T$ of the Thévenin equivalent circuit described above is to measure or calculate the resistance from the terminals $ab$ with the internal voltage sources replaced by a short circuit. An alternative method using experimentation, theoretical analysis or numerical simulation is presented below.

Fig. 7 shows the proposed equivalent circuit with the terminals $ab$ in short circuit.

![Proposed equivalent circuit with the terminals ab short-circuited.](image)

The power delivered to the circuit by the voltage source $V_T$ is

$$P_a = \frac{V_T^2}{R_a}.$$  \hfill (34)

where $R_a$ is the equivalent resistance of resistors $R_X$ and $R_T$ connected in parallel, given by

$$R_a = \frac{R_T R_X}{R_T + R_X}.$$  \hfill (35)

and

$$R_X = \frac{V_T^2}{P_X}.$$  \hfill (36)

After substituting (35) and (36) into (34), with proper algebraic manipulation we can write

$$R_T = \frac{V_T^2}{P_a - P_X}. $$ \hfill (37)

As previously demonstrated, the proposed equivalent circuit and the actual circuit dissipate the same internal power, regardless of the power transferred to the load connected at the terminals $ab$. Therefore, $P_a$ is the power measured at the input terminals of the actual circuit when the terminals $ab$ are short-circuited.
Thus, in order to determine \( R_T \), the voltage \( V_T \) of the Thévenin equivalent circuit, the power \( P_X \) consumed by the actual circuit with the terminals \( ab \) open, and the power \( P_a \) consumed by the actual circuit with the terminals \( ab \) in short circuit, must be measured or calculated.

In the numerical example shown above, numerical simulation yields \( V_T = 22.22 \text{ V} \), \( P_X = 166.66 \text{ W} \) and \( P_a = 270.43 \text{ W} \). Substituting these values in expression (37), we obtain \( R_T = 4.76 \Omega \), which is the same value obtained with the direct measurement of the terminals \( ab \). This method for determining the value of \( R_T \) can be used in any linear direct current network.

In situations where it is not possible to short the terminals \( ab \), the power can be measured with a resistance \( R_o \) connected to the terminals and the resistance \( R_T \) is determined by

\[
R_T = \frac{V_T^2}{P_a - P_X} - R_o. \tag{38}
\]

The method presented herein shows that the resistance \( R_T \) of the equivalent circuit can also be measured from the input terminals of the internal voltage sources of the actual circuit, and not only from the output terminals (the usual approach).

VI. POWER AND EFFICIENCY ANALYSIS

The Thévenin equivalent circuit, the proposed equivalent circuit and the actual circuit are shown in Fig. 8, where the reference current direction of \( I_o \) corresponds to the actual current direction.

![Figure 8](image-url)

**Figure 8.** (a) Thévenin equivalent circuit, (b) proposed equivalent circuit and (c) actual circuit.

The powers at the inputs of the Thévenin and proposed equivalent circuits are, respectively, given by

\[
P_{1T} = V_T I_0 \tag{39}
\]

and

\[
P_{1X} = \frac{V_T^2}{R_X} + V_T I_o. \tag{40}
\]

The power supplied to the actual circuit by voltage sources \( V_1 \) and \( V_2 \) is given by

\[
P_{11} = V_1 I_1 + V_2 I_2. \tag{41}
\]

Currents \( I_1 \) and \( I_2 \), obtained by analysis, are given by (42) and (43), respectively.

\[
I_1 = \frac{R_2 + R_3 + R_4}{\Delta} V_1 + \frac{R_2}{\Delta} V_2 + \frac{R_2 R_4}{\Delta} I_o \tag{42}
\]

\[
I_2 = \frac{R_2}{\Delta} V_1 + \frac{R_1 + R_2}{\Delta} V_2 + \frac{R_4 (R_1 + R_2)}{\Delta} I_o \tag{43}
\]

where

\[
\Delta = R_1 (R_2 + R_3 + R_4) + R_2 (R_3 + R_4) \tag{44}
\]

The powers obtained by (39), (40) and (41) are represented in Fig. 9, as a function of the current \( I_o \). The results show that the input power of the actual circuit given by (41) is equal to the input power of the proposed equivalent circuit, given by (40). In both circuits the input power is \( P = 166.66 \text{ W} \) when \( I_o = 0 \), while the input power of the Thévenin equivalent circuit is \( P_T = 0 \) for this condition.

![Figure 9](image-url)

**Figure 9.** Input powers for the circuits shown in Fig. 7: (a) Thévenin equivalent circuit \( P_{1T} \), (b) proposed equivalent circuit \( P_{1X} \) and (c) actual circuit \( P_{11} \).

The internal losses of the three circuits are given by (45), (46), and (47), respectively.

\[
\Delta P_T = R_T I_o^2 \tag{45}
\]

\[
\Delta P_X = \frac{V_T^2}{R_X} + R_T I_o^2 \tag{46}
\]

\[
\Delta P_1 = R_1 I_1^2 + R_3 I_2^2 + R_5 I_o^2 + R_2 (I_1 - I_2)^2 + R_4 (I_2 - I_o)^2 \tag{47}
\]

The currents \( I_1 \) and \( I_2 \) are given by (42) and (43), respectively.

These losses given by (45), (46) and (47) are represented in Fig. 10 as a function of the current \( I_o \). The results show that
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FIGURE 10. Internal losses: (a) Thevenin equivalent circuit ($\Delta P_T$), (b) proposed equivalent circuit ($\Delta P_X$) and (c) original circuit ($\Delta P_1$).

The proposed circuit represents all internal losses of the original circuit, while the Thevenin equivalent circuit considers only the internal losses that occur in the resistor $R_T$.

The efficiencies of the three circuits are defined by (48), (49) and (50), for the Thévenin equivalent circuit, proposed equivalent circuit and actual circuit, respectively.

$$\eta_T = \frac{P_{1T} - \Delta P_T}{P_{1T}}$$ (48)

$$\eta_X = \frac{P_{1X} - \Delta P_X}{P_{1X}}$$ (49)

$$\eta_1 = \frac{P_{11} - \Delta P_1}{P_{11}}$$ (50)

The efficiency curves of the original circuit and the proposed equivalent circuit are the same and different from the efficiency curve of the Thévenin equivalent circuit, as shown in Fig. 11.

FIGURE 11. Efficiencies of the circuits: (a) Thevenin equivalent circuit, (b) proposed equivalent circuit and actual circuit.

VII. ON THE VALUE OF THE RESISTANCE $R_X$

The resistance $R_T$ of the Thévenin equivalent circuit is dependent only on the internal resistances of the actual network and the way they are associated, with the voltages of all internal voltage sources set equal to zero, as given by (11).

On the other hand, according to the generic (23), the value of resistance $R_X$ is dependent on the resistances and internal voltages of the original circuit. However, as demonstrated below, there are also particular situations in which the value of this resistance is also dependent only on the internal resistances.

Let the matrix of conductance be

$$G = \begin{bmatrix} G_{11} & G_{12} & \cdots & G_{1n} \\ G_{21} & G_{22} & \cdots & G_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ G_{n1} & G_{n2} & \cdots & G_{nn} \end{bmatrix}$$ (51)

Substituting (51) into (23) yields

$$R_X = \frac{\left[ R_{1o} \ R_{2o} \ \cdots \ R_{no} \right] G \left[ V_1 \ V_2 \ \cdots \ V_n \right]^2}{\left[ V_1 \ V_2 \ \cdots \ V_n \right] G \left[ V_1 \ V_2 \ \cdots \ V_n \right]}.$$ (52)

Let us consider the particular case where $V_1 \neq 0V$ and $V_2 = V_3 = \cdots = V_n = 0V$. Substituting these values in (52), we find

$$R_X = \frac{\left[ R_{1o}G_{11} + R_{20}G_{12} + \cdots + R_{no}G_{1n} \right]^2}{G_{11}}.$$ (53)

Equation (53) demonstrates that the resistance $R_X$ is independent of the value of the internal voltage $V_1$, and dependent only on the internal resistances of the original network, as occurs with the resistance $R_T$. One can generalize the result and state that in any network with only one internal source, the value $R_X$ is dependent only on the internal resistances of that network.

VIII. THEOREM AND COROLLARIES

In the previous sections, the proof of the theorem formulated below was presented. This theorem can be considered an extension of the Thevenin Theorem, with corollaries.

Theorem: Any linear DC network consisting of resistors and independent voltage sources, with two accessible terminals can be replaced by an equivalent circuit with a DC voltage source $V_T$ and two resistors $R_T$ and $R_X$. The voltage of the voltage source $V_T$ is the voltage measured at the open terminals. The resistance $R_T$, associated in series with the voltage source $V_T$, is that measured at the terminals with all internal ideal voltage sources replaced by a short circuit. The resistance $R_X$, associated in parallel with the $V_T$ source,
is given by the equation $R_X = V_T^2/P_X$, where $P_X$ is the power supplied by the internal sources and dissipated internally in the circuit with the terminals open.

Corollary 1: In any direct current network with a pair of terminals, the power dissipated internally has two components, one being constant and dependent only on the internal sources and resistances and the other variable and dependent on the internal resistances and the power transferred to the load connected at the terminals.

Corollary 2: The efficiency of the Thevenin equivalent circuit is always larger than the efficiency of the actual network.

Corollary 3: In the equivalent circuit of any linear DC network with two accessible terminals, the resistance $R_T$ associated in series with the DC voltage source $V_T$ is given by $R_T = V_T^2/(P_u - P_X)$ where $P_X$ is the power dissipated in the circuit with the terminals open and $P_u$ is the power dissipated in the circuit with the terminals in short circuit.

IX. CONCLUSION

This paper proposes a new equivalent circuit for DC networks that is power-conservative and can be considered an extension of the Thevenin equivalent circuit.

It is demonstrated that in a network formed by voltage sources and resistors, with two accessible terminals, the power dissipated internally has two components, namely (a) one constant and independent of the power transferred to the external terminals, and (b) one variable and dependent on the power transferred to the device connected to the terminals.

A new method for the analytical or experimental determination of the resistance of the Thevenin equivalent circuit is also described.

The proposed equivalent circuit can be used in the power and efficiency analysis of DC networks.

ACKNOWLEDGMENT

Acknowledgment is made for the work of Victor Borges, Engineer with the Brazilian Institute of Power Electronics and Renewable Energy (IBEPE), and Ygor Marca, graduate student at the Electrical Engineering Department of Federal University of Santa Catarina, who collaborated in editing this manuscript.

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