Network analysis of a corpus of undeciphered Indus civilization inscriptions indicates syntactic organization

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Archaeological excavations in the sites of the Indus Valley civilization (2500 – 1900 BCE) in Pakistan and northwestern India have unearthed a large number of artifacts with inscriptions made up of hundreds of distinct signs. To date, there is no generally accepted decipherment of these sign sequences, and there have been suggestions that the signs could be non-linguistic. Here we apply complex network analysis techniques to a database of available Indus inscriptions, with the aim of detecting patterns indicative of syntactic organization. Our results show the presence of patterns, e.g., recursive structures in the segmentation trees of the sequences, that suggest the existence of a grammar underlying these inscriptions.

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INTRODUCTION

The last decade has seen a rising interest in the analysis and modeling of complex networks occurring in many different contexts [1], which includes networks defined in corpora of textual units [2]. Using the graph-theoretic paradigm to study a complex system has often revealed hitherto unsuspected patterns in it. While graph-based representation of texts has been used for some time in natural language processing tasks, such as, text parsing, disambiguation and clustering [3], the approach based on the new science of complex networks often asks questions from a different perspective that can shed new light on the organization of linguistic structure. For example, networks constructed on the basis of co-occurrence of words in sentences have been seen to exhibit (a) the small-world effect, i.e., a small average distance between any pair of arbitrarily chosen words, and (b) a scale-free distribution of the number of words a given word is connected to (i.e., its degree) [4]. These properties have been proposed as reflecting the evolutionary history of lexicons as well as the origin of their flexibility and combinatorial nature. A more recent study of a lexical network of words which are phonological neighbors has found that the degree distribution might be better fit by an exponential rather than a power-law function [5]. A theoretical model for such word co-occurrence network, which treats language as a self-organizing network of interacting words, has led to the suggestion that languages may have a core (the “kernel lexicon”) that does not vary as the language evolves [6].

However, even though text and speech are sequential, focusing exclusively on the local correlation between immediately consecutive words may not be a good strategy to describe natural languages. This is because of the presence of non-local relations between words that occur apart from each other in a sentence. Therefore, network analysis has been extended to syntactic dependency networks, where two words are connected if they have been related syntactically in a number of sentences [7]. The theory of complex networks has also been used to investigate the structure of meaningful concepts in the written texts of individual authors, which have been seen to have small-world, as well as, scale-free characteristics [8]. The conceptual network of a language has been explored by using the semantic relatedness of words as defined by a thesaurus, and this network too is seen to have small-world nature with scale-free degree distribution [9].

Almost all the network studies done on corpora of textual units so far have been confined to languages that are still in use. However, we have historical evidence of many extinct languages, the knowledge about which have come down to us in the form of written inscriptions. It is important to consider applying network analysis techniques to such texts and see whether it reveals new insights on the language as well as the writing system used for it. This is especially so, as the relation between a language and its writing system is neither simple nor unique [10]. While, on one hand, the same language can be written using multiple writing systems, on the other hand the same writing system can be used for writing many different languages. While most network studies have focused on alphabetic writing, there are many writing systems (including many that were used recording languages that are now extinct) that are based on other principles. These systems may differ remarkably in their ability to record the various aspects of speech: for example, logographic writing omits the phonemic structure of speech, while, phonographic writing may omit vowels and fail to distinguish various classes of consonants [11]. It is therefore intriguing to consider whether network analysis can reveal the similarities and differences between such distinct systems of writing, and moreover, if it can be used to distinguish structural features characterising writing (i.e., any system of recording language by visible or tactile marks [11]) from non-writing.

This is especially important, as it is clear from observing many of the earliest examples of writing that have been deciphered, that “no writing system was invented,
or used early on, to mimic spoken language or to perform spoken language’s function” [12]. Instead, writing was used to record information such as livestock or ration accounts, land grants, offering lists, lexical lists, divinations, etc., whose storage by verbal or spoken means was difficult and unreliable; thus the principal function of early writing was decontextualization and storage [13]. In almost all cases, a writing system became more or less capable of expressing spoken language only after centuries of development, a process of development that can be clearly seen in Sumerian cuneiform, Egyptian and Mayan writing [12]. It is therefore important to broaden the results of network analysis of linguistic corpora by applying such analytical techniques to inscriptions recorded using different writing systems, and, even to undeciphered inscriptions for which the underlying writing system is unknown.

In this article, we look at a corpus of inscriptions obtained through archaeological excavations carried out in the ruins of the Indus valley civilization. The inscriptions are in the form of short linear sequences of signs, of which there are a few hundred different types. Ever since their discovery in the early part of the 20th century, there have been attempts at deciphering these inscriptions. However, to date there has been no generally accepted method of interpreting them. We analyze a comprehensive database of these sequences using techniques inspired by complex network theory. Our aim is to see whether such methods can reveal the existence of patterns suggesting syntactic organization in the sign sequences. In the next section, we briefly introduce the historical context of the Indus inscriptions, while in Section 3, we discuss the data-set on which analysis has been carried out. Our results are reported in Section 4, and we finally conclude with a discussion of unresolved questions and further work that needs to be carried out.

THE INDUS INSCRIPTIONS

The Indus civilization, also known as the Mature Harappan civilization (2500 – 1900 BCE), was geographically spread over what is now Pakistan and northwestern India, covering approximately a million square kilometers [14]. It was marked by urbanization centered around large planned cities, as seen from the ruins of Harappa and Mohenjo-daro. Craft specialization and long-distance trade with Mesopotamia and Central Asia have been well-demonstrated. This civilization came to an end early in the 2nd millennium BC. There were no historical records of its existence until archaeological excavations in the late 19th and early 20th centuries uncovered artifacts and some of the ruined urban centers [15]. Among the artifacts uncovered during these discoveries were a variety of objects (especially seals) that were inscribed with a variety of signs arranged in sequences (Fig. 1). Although found primarily on seals and their impressions (sealings), inscriptions with similar signs have also been discovered on miniature tablets, pottery, copper tablets, bronze implements, etc. Unsurprisingly, given the high sophistication of the civilization and the level of social complexity it implies, with the concomitant requirements of coordination and communication, these inscriptions have been interpreted as corresponding to writing. However, despite periodic claims about decipherment of this writing system, there has as yet been no generally accepted interpretation of the signs. Despite the lack of success in understanding what the sequences represent, it has been generally agreed that the inscriptions were written from right to left in the vast majority of cases, although a few examples written from left to right or top to bottom or in boustrophedon are known [16–18].

The failure of decipherment is partly due to lack of knowledge about the language which the signs encode and the lack of any bilingual texts such as the Rosetta stone which was crucial in deciphering Egyptian hiero-
glyphs. While there is disagreement on the exact number of unique and distinct signs that occur in the inscriptions, there is overall agreement that they lie in the range of a few hundred. This rules out the possibility that the signs belong to (a) an alphabetic system, which contains on average about 25 letters (such as the Roman or Latin alphabet, used for writing most European languages including English), (b) a syllabic system consisting of around 100 signs (such as the kana system for writing Japanese) or (c) an ideographic writing system (e.g., Chinese), comprising more than 50,000 characters. However, the number of Indus signs is in the same range as the number of distinct signs used by logo-syllabic writing systems, such as Sumerian cuneiform for which the total number of independently occurring signs has been estimated to be around 900 \[10\]. The brevity of the Indus inscriptions (the longest single-line sequence has 13 signs) and the existence of a large number of signs that occur with very low frequency have led to some alternative suggestions regarding the meaning of the sign sequences. These include the possibilities that, e.g., (i) the signs correspond to a label specifying an individual and his belongings, in the manner of heraldic badges \[19\] and (ii) the signs are ritual or religious symbols which do not encode speech nor serve as mnemonic devices, much as the Vinca signs or emblems in Near Eastern artifacts \[20\]. The latter possibility implies the absence of any syntactic structure in the Indus inscriptions, a possibility that can be tested without making any a priori assumptions about the meaning of the signs.

DATA DESCRIPTION

Our earlier analysis \[21\] had been done on a corpus of Indus civilization inscriptions that had been compiled in the process of constructing the 1977 electronic concordance of Mahadevan \[16\] (see also Ref. \[22\]). More recently, Wells has compiled a larger and more comprehensive database (W09IMSc) of all available inscriptions associated with the Indus civilization \[13\]. This was done by exhaustive search of all available site reports of Indus excavations and the photographic corpus of Indus seals and inscriptions \[23, 24\]. It was supplemented by Internet searches for unpublished inscriptions and requests for unpublished material from individual researchers. Duplicate records of inscriptions from different sources were manually controlled. The W09IMSc database consists of sequences recorded from a total of 3896 artifacts and identifies 695 distinct signs. Each sign has been assigned a 3-digit code between 001 and 958. The sign list has been included as an Appendix to this paper.

To carry out our analysis, we have focused only on complete inscriptions, i.e., we have excluded all inscriptions which are only partially readable because of defaced or ambiguous signs or damage to the artifact. This results in a reduced set of 2393 artifacts. From this set, we only consider those sequences which occur on a single line. This is done in order to remove the ambiguity concerning the interpretation of sequences occurring as multiple lines of signs, namely, whether the different lines should be considered as independent sequences or whether it is one continuous sequence. Note that we do include artifacts which have sequences (in a single line) written on multiple sides. In this case, the sequence on each side is considered separately. Finally, each distinct sign sequence is considered only once. This series of operations gives us the Wells Unique Complete Single line text (WUCS) data-set, consisting of 1821 sequences comprising 593 unique signs. All sequences are standardized to read from right to left.

The sequences vary in length from 1 to 13 signs, the median length being 4 signs. Fig. 2 shows the distribution of the sequence lengths. Many of the single-sign inscriptions (i.e., sequences of length 1), are either a solo sign as defined in the next section, or, appear on one side of an artifact having inscriptions on multiple sides. A few of the single-sign inscriptions are graphically complex signs that appear to be ligatures of two or more relatively simpler signs.

RESULTS

Directed network of Indus signs

Using the WUCS data-set, a directed and weighted network of the 593 signs can be constructed, where a directed link from node \(i\) to node \(j\) implies that sign \(j\) occurs immediately to the left of sign \(i\) in at least one sequence. The link from \(i\) to \(j\) is weighted by the frequency of occurrence of the ordered pair in the entire data-set. A distinguishing property of a network having directed connections is the reciprocity of the links. This can be
measured as the ratio of the number of bidirectional links \( L_{bi} \) relative to the total number of links \( L \): \[ R = \frac{L_{bi}}{L}. \]

This measure represents the average probability that if a directed connection exists from one node to another, the connection in the reverse direction also exists. For a network where all the links are strictly one-way, \( R = 0 \), while, if there is complete absence of directionality, \( R = 1 \). In the context of a linguistic network, low values of \( R \) would indicate the presence of significant directional relations between signs, i.e., certain signs are more likely to appear before (after) certain other signs than after (before) them. Thus \( R = 0 \) would imply an extremely rigid relation between the signs: if a sign is seen to appear before another sign in one context, it will never appear after the other sign in any other context.

To measure the significance of the properties calculated for the empirical data-set we compare them against an ensemble of randomized data-sets. These are generated by randomly permuting the sign order in each of the 1821 sequences of the WUCS data-set individually and re-constructing the corresponding network of sign relations. This is done many times in order to generate a randomized ensemble of networks. Note that the sign frequency of each sign in a randomized corpus is unchanged from that in the empirical data-set by construction. Thus, properties which have almost identical value for the empirical data-set and the randomized ensemble can potentially be explained as the result of the sign frequency distribution and the fact that certain signs do not occur together in the same inscription, rather than being a result of the existence of syntactic structure, i.e., a set of rules which govern how the signs are consecutively strung together to form a sequence. For example, for the randomized data-set we obtain the reciprocity, \( R_{\text{rand}} = 0.338 \pm 0.007 \) (averaged over 100 realizations). As expected, the reciprocity for the randomized networks is much higher than that for the empirical network, as the shuffling of sign order in each seal disrupts any existing directional relations between the signs in a sequence.

An alternative randomization is also possible where all the different sequences are considered together, i.e., all the signs belonging to every sequence are mixed together and then reassembled into random sequences. However, this will generate many random sequences with signs that never co-occur in the same sequence in the empirical data (i.e., in the inscriptions). Thus, the first randomization method, by taking into account the context in which two signs co-occur, gives a much stricter criterion for deciding which of the features of the empirical network are significant (i.e., unlikely to appear by random chance given the frequency of occurrence of each sign).

A preliminary analysis of the data shows that 21 signs only appear as solo (or single-character) inscriptions and never in conjunction with another sign, viz., sign numbers 037, 039, 047, 110, 147, 281, 341, 386, 387, 699, 753, 780, 781, 782, 823, 841, 942, 945, 946, 956 and 957. In network terminology, the nodes corresponding to these signs have no in-coming links (i.e., in-degree= 0) or out-going links (i.e., out-degree= 0). Also, out of the 369 distinct signs that can appear at the start of a sequence of length \( \geq 2 \), 128 signs are only seen at the beginning and never in any other position in a sequence. In network terminology, these signs have no in-coming links (i.e., in-degree=0). We shall call these signs “beginners”: 025, 027, 028, 029, 041, 046, 051, 057, 058, 059, 069, 084, 098, 107, 112, 114, 117, 118, 119, 121, 126, 131, 133, 141, 144, 145, 146, 166, 178, 195, 201, 208, 209, 216, 229, 230, 261, 262, 266, 272, 276, 319, 323, 325, 327, 329, 361, 363, 370, 389, 403, 412, 428, 445, 451, 458, 473, 479, 490, 493, 501, 505, 513, 528, 541, 544, 545, 551, 563, 571, 573, 577, 579, 586, 591, 601, 620, 622, 625, 631, 634, 635, 640, 641, 678, 681, 683, 687, 688, 689, 693, 696, 698, 707, 710, 716, 728, 731, 736, 747, 751, 764, 768, 777, 795, 796, 799, 815, 818, 826, 827, 829, 843, 852, 859, 863, 864, 870, 871, 876, 878, 891, 896, 902, 903, 918, 922 and 950. Similarly, out of the 196 distinct signs that are seen to terminate a sequence in the WUCS database, 43 signs are seen only at the end of a sequence and never in any other position. In network terminology, these signs have no out-going links (i.e., out-degree=0) These signs will be referred to as “enders”: 045, 074, 105, 106, 129, 138, 157, 173, 200, 203, 224, 256, 289, 294, 303, 307, 324, 375, 394, 409, 410, 423, 427, 429, 430, 481, 512, 712, 749, 822, 842, 855, 860, 866, 869, 872, 875, 907, 909, 911, 930, 932 and 951. Note that among the remaining 401 signs (i.e., signs which do not belong to any of the above three classes) there are many that can occur at the beginning of a sequence, as well as elsewhere. Similarly several signs that can be observed to terminate some sequences may also be seen at other positions in a sequence. Indeed, there are 120 signs that are seen to begin some sequences or end other sequences. There are thus 127 signs which are seen to be always preceded as well as followed by other signs in any inscription that they occur in.

Not surprisingly, the number of signs that appear as solo inscriptions in the randomized data-sets is the same as in the original data (= 21). This is because we randomize each sequence separately, and do not put together signs in a randomly generated sequence if they have never co-occurred in any of the empirical sequences. However, the number of beginners and enders will be different, as these depend on the underlying network relations which have been changed by randomizing the corpus. For example, calculating the number of beginners and enders
FIG. 3: The frequency distributions for (left) beginners, $f_b$, and (right) enders, $f_e$, are shown for the WUCS data-set. The corresponding distributions for randomized sequences, averaged over 100 different realizations, are shown for comparison.

in 100 randomized data-sets yields $n_b = 66.26 \pm 6.59$ and $n_e = 62.61 \pm 6.71$, respectively. It implies that, in the empirical data, there are significantly more beginners on one hand, and a significantly lower number of enders on the other, than would be expected purely on the basis of chance, given the frequency of occurrence of the individual signs. It is also possible to observe with what frequency beginners or enders in the empirical data-set appear in the same role in the randomized corpus. We observe that four beginners in the empirical set (signs 201, 216, 272 and 545) and two enders (signs 409 and 423) never occur as beginners and enders (respectively) in the 100 randomized trials we carried out. It implies that the occurrence of these signs always at the beginning or end of a sequence (and never in any other position) may be highly significant, and certainly not a result of simple chance.

We can now ask: in how many different sequences does a particular beginner or ender appear? In answer, we see that most beginners or enders occur only in very few distinct sequences. In Fig. 3 we have shown the frequency distribution of the beginners (left) and enders (right) in the empirical data and compared it with that observed from averaging over the distributions corresponding to 100 randomized trials. The difference between the empirical and random distributions is significant at low frequencies as it is much larger than the standard deviations for the randomized data. Note that there are many more beginner signs in each frequency class than would be expected had all the sequences in the corpus been randomly scrambled. The largest number of distinct sequences that a beginner can appear in is 8. Indeed, most of the beginner signs appear in only $1 - 3$ distinct sequences. This is even more so the case for enders, where a particular ender sign can appear at the end of a maximum of $4$ distinct inscriptions.

Degree and strength distribution

We now focus on the distribution of the number of links for each sign (i.e., the degree). If there had been a rigid relation between the signs, i.e., the occurrence of one particular sign was always preceded or followed by another particular sign, this would show up in the degree distribution. Thus, the occurrence of a sharply decaying
FIG. 5: The cumulative distribution function for in-strength, $s_{\text{in}}$, and out-strength, $s_{\text{out}}$, of the directed network of signs constructed from the WUCS data-set. The corresponding distributions for randomized sequences, averaged over 100 different realizations, are shown for comparison.

FIG. 6: The sub-network of the ten most frequently occurring signs in the WUCS data-set, with the ID numbers of each sign indicated alongside its image. Note that the connectivity in this subnetwork is substantially more dense ($\sim 0.49$), with about half of all the potential connections present, relative to the entire network whose connectivity is 0.0077.

degree distribution with an overall low number of links per sign would indicate that for most signs there is not much freedom of choice in deciding which sign will precede or follow it. Fig. 4 shows that both the in-degree (the number of incoming connections) and the out-degree (the number of outgoing connections) have long-tailed distributions, indicating that there is relatively a high degree of variation in the signs that a particular sign occurs adjacent to. However, this also does not correspond to the total freedom in choosing neighbors as is the case for a random sequence. The curves for the in-degree and out-degree distribution for the networks constructed from the randomized ensemble obtained by shuffling sign order in each of the sequences show a consistently higher probability for larger degrees. This indicates that the variation of sign relations is much more restricted in the empirical sequences than would be the case had each of the sequences been put together randomly.

Another related property that is often observed in the case of weighted networks is the distribution of strength, i.e., the weighted sum of links for a node. To a certain extent, this is governed by the frequency of occurrence of individual signs, as a more common sign will have many more relations with other signs. Not surprisingly, in Fig. 5, we see that the in-strength and out-strength distributions of the networks constructed from empirical and randomized data match fairly well. In other words, the strength distribution of the WUCS network can be explained almost fully on the basis of individual sign frequencies and the fact that certain signs do not occur together in the same sequence.

Core-periphery organization

The connectivity, or average density of connections, in a network is measured as the ratio of non-zero entries in the adjacency matrix to the total number of matrix elements. The network of Indus sign relations is extremely sparse, with a connectivity of only 0.0077. This may be compared with the connectivity for the corresponding randomized corpus, $C_{\text{rand}} = 0.011$ (averaged over an ensemble of 100 different realizations, the standard deviation being less than 0.0004). Thus, the WUCS data-set shows that out of a possible $593 \times 593 = 351,649$ directed sign pairs, only 2719 sign pairs are actually observed (as compared to the $3827 \pm 20$ directed signs pairs observed when the corpus is randomized). This already suggests the existence of grammatical rules in the construction of the sequences that prevent the occurrence of a vast majority of the possible sign pairings.

If we graphically represent the sub-network of connections between nodes corresponding to the 10 most common signs in WUCS (i.e., the ones occurring with the highest frequency), we note that they are strongly interconnected (Fig. 6). In fact, almost half of all the possible sign pairs in this sub-set are actually observed to occur. This is partly an outcome of the inhomogeneous frequency distribution of the individual signs, with the most frequently occurring signs appearing in many different sequences and thereby having connections with a large variety of signs. This is indicated by a comparison of the connectivity of the sub-network of $q$ most frequent signs ($q$ ranging from 2 to 593) for the empirical network with that for the randomized ensemble. As Fig. 7 shows,
FIG. 7: The connectivity for the sub-network of \( q \) most frequently occurring signs in the WUCS data-set (\( q = 2, 3, \ldots, 593 \)) for the network constructed from the empirical data, compared to that for the networks constructed from randomized data-sets. The higher connectivity for the randomized case is an outcome of the long-tailed nature of the distribution of frequencies of individual signs. The values for the randomized data are averaged over 100 different realizations. The error bars are not indicated as they are smaller than the symbol size used for the randomized data.

the connection density for the sub-set of most frequently occurring signs is much higher than what would have been expected had the signs been placed in a sequence randomly (based only on their individual frequency of occurrence and the restriction that certain signs never co-occur in the same sequence). The lower sub-network density for the empirical network is a result of several possible sign relations (which have a very high probability of occurring in a randomized sequence) never appearing in the WUCS data-set. It suggests the existence of syntactic relations in constructing the sequences that prevent the occurrence of these highly probable sign relations.

The above analysis also indicates that there exists a core set of signs that appear together very frequently in a sequence. A natural question is whether the network generated from the WUCS data-set has a core-periphery organization. This would imply the existence of a densely connected central core (central in terms of network distance between the nodes) and a larger class of sparsely connected peripheral nodes, like that seen in the case of geographically embedded transportation networks \cite{26}. To obtain such a decomposition of the network we use a pruning algorithm that successively peels away the “outer layers” of peripheral nodes to reveal a subnetwork of a given core-order. The \( k \)-core of a network is defined as the subnetwork containing all nodes that have degree at least equal to \( k \). Thus, to obtain it, we have to iteratively remove all nodes having degree less than \( k \). In particular, the 2-core of a network is obtained by recursively eliminating all nodes that do not form part of a loop (i.e., a closed path through a sub-set of the connected nodes).

In a directed network, one can define a \( k \)-core either in terms of the in-degree (number of connections arriving at the node) or the out-degree (number of connections sent from the node). For the WUCS network, the innermost \( k \)-core turns out to have order 6 for out-degree and 8 for in-degree (Fig. 8). The corresponding core-size for the networks constructed from randomized sequences are also shown in Fig. 8. The values for the randomized data are consistently higher than those for the empirical network.

The intersection of the innermost out-degree core and the innermost in-degree core comprises 26 signs that are the ones most likely to occur at the medial positions of a given inscription: 001, 002, 003, 031, 032, 033, 140, 220, 231, 233, 235, 240, 368, 415, 590, 700, 705, 717, 740, 741, 798, 803, 820, 840 and 904. By observing which signs appear in the innermost out-degree core but not in the intersection set (i.e., taking the difference of these two sets), we obtain 12 signs that most frequently precede other signs in an inscription: 055, 060, 440, 575, 615, 692, 742, 745, 790, 806, 900 and 920. Similarly, by considering the signs which appear in the in-degree core of highest order but not in the intersection set, we obtain 16 signs that most frequently follow other signs in an inscription: 017, 090, 100, 125, 151, 176, 255, 350, 388, 390, 400, 455, 520, 550, 760 and 861. Together these 54 signs are the ones most likely to be used in an inscription.
indicate the standard deviations for the randomized data. Error bars changed (averaged over 100 different realizations). Error bars indicate the standard deviations for the randomized data.

We can generalize the concept of $k$-core from the degree to the strength of a node, thereby defining a $s$-core. The $s$-core of a network is defined as the subnetwork containing all nodes that have strength at least equal to $s$. Thus, to obtain it, we have to iteratively remove all nodes having strength less than $s$. Fig. 9 shows the core size variation with core-order for both in-strength and out-strength. The innermost $s$-core for out-strength has order 31, and that for in-strength has order 29. As is clearly seen, the out-strength core size for the empirical network matches fairly well with that of the randomized networks, while the in-strength core for the empirical network is smaller than that for the randomized network at large core order, $s$. It implies that the set of mutually connected signs having high in-strength is significantly smaller than would be expected on the basis of chance had the signs been placed randomly in each sequence. This indicates the presence of certain context-based restrictions on the pairing of signs. In other words, the occurrence of a sign pair depends on what other signs occur in that sequence.

As in the case of degree, for strength also we can look at the intersection of the innermost in-strength and out-strength cores, which provides us with a set of 18 signs: 001, 002, 003, 032, 033, 100, 176, 220, 233, 235, 240, 390, 415, 590, 705, 740, 798 and 803. This is a subset of the group of signs obtained above by considering the intersection of the in-degree and out-degree cores of highest order, excepting for signs 100, 176 and 390. By considering the difference of the intersection set with the set corresponding to the innermost out-strength core, we obtain the signs that are most likely to precede the medial group of signs: 031, 060, 368, 690, 706, 741, 760, 806, 817, 820, 840, 861, 900 and 920. Similarly, from the difference of the intersection set with the set corresponding to the innermost in-strength core, the signs that are most likely to follow the medial group of signs is obtained: 090, 400 and 520. Together these 35 signs can be considered to constitute the “core lexicon” of the Indus inscriptions.

Network of significant links

So far we have placed all sign pairs that occur in the WUCS data-set on an equal footing. However, certain pairs may occur with high probability simply because the individual signs that make up the pair occur with high frequency. Fig. 10 shows that the frequency distribution of sign occurrences in the WUCS data-set has an approximately power-law distribution [27]. This implies that the most common signs will occur in a very large number of sequences (the most frequent sign “740” appearing as many as 831 times, which is more than 10% of the total of 8095 occurrences of the 593 signs in the WUCS data-set). By using the information about the probability of occurrence for individual signs in the data-set we can investigate significant sign relations, i.e., sign combinations that occur far more frequently than would be expected from the individual probabilities of the component signs. Thus, if sign $i$ occurs with a probability $p(i)$ and $j$ with $p(j)$ in the corpus, then the pair $ij$ is significant if it occurs with a probability $p(ij) \gg p(i)p(j)$. If $p(ij) \simeq p(i)p(j)$, we can conclude that the two signs...
are essentially independent of each other, and their joint occurrence is not indicative of any significant relation between them. To measure by how much $p(ij)$ has to be larger than the product of $p(i)$ and $p(j)$ in order to be significant, we need to compare the empirical joint occurrence probability against the corresponding value for the randomized ensemble. The randomized corpus is generated (as described earlier) by shuffling the order of signs in each sequence, such that the pair correlations in the original data are removed while keeping the individual sign frequencies unchanged. A sign pair $ij$ is considered significant if the empirical relative frequency of its occurrence, $P_{\text{emp}}(ij)$, is so large compared to the corresponding relative frequency in the randomized corpus, $P_{\text{rand}}(ij)$, that the pair can never occur with the observed frequency had the two signs been independent, i.e., had there been no dependency relation between them. This deviation of the empirical pair probability from that corresponding to the randomized corpus can be quantified by computing the $z$-score:

$$z_{ij} = \frac{P_{\text{emp}}(ij) - \langle P_{\text{rand}}(ij) \rangle}{\sigma_{\text{rand}}(ij)},$$

i.e., the difference between the relative frequencies of the sign pair for the empirical data and the mean for the randomized ensemble, divided by the standard deviation of the frequencies obtained for the different randomizations.

The cumulative probability distribution for the $z$-scores of all sign pairs are shown in Fig. 11. Had this distribution been a Gaussian, all sign pairs with $z$-scores higher than 3 could have been considered significant. The empirical distribution is observed to have a long tail and we can consider all pairs to be significant which have a $z$-score larger than a specified cut-off, $z_c$. We note that there are 377 sign pairs with $z$-score larger than $z_c = 3$, while the 31 significant pairs obtained when $z_c = 8$ are shown as a network of “most significant relations” in Fig. 12. This comprises 36 signs, containing 25 out of the 30 most frequent signs, indicating that some of the commonest signs have significant relations between them. While most significant pair relations are between such common signs, one notable exception is the significant relation between sign “017” (46th most common sign) and sign “585” (67th most common sign), both of which are relatively low-frequency signs. As this sign relation has a very high $z$-score, although the individual signs are themselves not very common, it is an intriguing sign pair and possibly has some functional significance in terms of interpreting the sequences.

**Syntactic tree generation**

We shall now attempt to reveal recursive structure indicative of the presence of syntactic rules for generating the inscriptions by “parsing” the longest sign sequences. We do this by generating segmentation trees of the sign sequences based on the statistical significance of sign pair occurrences. Given a inscription of length $n$, sign pairs are successively merged in decreasing order of their statistical significance, with the first merger being done for the sign pair with the highest $z$-score in that sequence. The next merger is done for the pair of signs having the next highest $z$-score and so on, until all the signs in the se-
sequence have been merged. In case of a tie between two or more pairs at any stage, the leftmost pair is chosen. This sequence of mergers is then “unfolded” to produce the resulting segmentation tree of the sign sequence which is shown schematically in Fig. 13. The height of the tree is an indicator of the presence of significant recursive structure in the sign sequence. In particular, if the signs are all independent of each other, then the segmentation tree has essentially the same height as the length of the sequence (Fig. 13 top). On the other hand, if for long sequences there exists subsequences that appear as an unit in the corpus several times, including as complete sequences in their own right, this is indicative of recursion. The corresponding tree height is substantially reduced as compared to the sequence length (Fig. 13 bottom).

We use this criterion to seek a signature of recursive, and hence syntactic, structure in the WUCS data-set. We have confined our attention to parsing the 33 inscriptions in WUCS data-set having 10 or more signs. Our earlier analysis of the EBUDS data-set [?] had shown that the average tree height for the longer sequences was around 5. We had concluded that the existence of such a characteristic length scale indicated that the longer sequences were actually composed of multiple smaller sequences that can occur independently in the corpus, and which have definite syntactic relations among their constituent signs. This is confirmed by the recent analysis done on the WUCS data-set.

Fig. 14 shows the segmentation trees of the three longest sequences, each comprising 13 signs. Two of these, M-0355 (sequence no. 1576 in the WUCS database) and M-0038 (sequence no. 1397 in the WUCS database) clearly indicate that they are made up of 3, or possibly 4, sub-sequences, while the third, H99-4064 (sequence no. 1261 in the WUCS database) appears to comprise two long sub-sequences. Let us consider the first of the sequences, M-0355. The 3-sign cluster “520-919-140” at the beginning of the sequence is the initial phrase, and is separated from the rest by sign 360. The medial sequence is broken into two parts “235-002-861” and “033-705-231”. The sequence ends with the terminal phrase “740-877-032”. We observe that each of these four sub-sequences obtained by this analysis also occur as units in other inscriptions in the WUCS data-set, thereby verifying the accuracy of the segmentation procedure. By breaking down long texts into (possibly meaningful) phrases that have independent existence, the method should help in identifying the grammatical rules by which the sequences are written.

DISCUSSION

In this paper we have used complex network analysis techniques on the sign network constructed from a sub-set of the corpus of inscriptions obtained in Indus civilization excavations. Our results suggest that though these sign sequences are yet to be deciphered, they have a highly structured arrangement which is suggestive of the existence of syntax. The inference of a set of rules (i.e., the grammar) for arranging these signs in a particular order, so as to be able to create pseudotexts that are indistinguishable from the excavated ones, is the eventual aim of the analysis described here. However, before we can successfully compile the “grammar” for these sequences, several open problems need to be addressed. One of the extensions of the present work has to do with looking beyond sign pairs to sign triplets, quadruplets, etc. Preliminary analysis of networks of “meta-signs” by us indicates that combinations beyond four signs may not have statistical significance. A detailed comparison between the sign network described here and the meta-sign network may provide clues about the possible hierarchical arrangement of subsequences in the longer sequences. Evidence of this is already seen from the construction of segmentation trees of individual sequences in the longer sequences. To analyze this, we need to redefine the links in the network as being connections between all signs that occur in the same inscription. Again, preliminary analysis seems to suggest that this does not provide substantially new results from those reported here. Based on the num-
FIG. 14: The segmentation trees for the three longest sequences in the WUCS data-set, viz., the inscriptions of Seal M-0355 from Mohenjo-Daro (top), M-0038 from Mohenjo-Daro (center) and H99-4064 from Harappa (bottom), obtained by using the z-scores of the 12 sign pairs comprising each of these sequences as described in the text. The thickness of the lines are proportional to the corresponding z-score values. At the left of each tree, an image of the corresponding seal is shown. The z-score computed for each pair, as well as the corresponding pair frequency in the W09IMSc data-set, are indicated below each pair. Also shown is the total frequency of occurrence of each constituent sign in the W09IMSc data-set.

ber of distinct signs (around 500 – 600) there have been several suggestions that, as the number is too high to be an alphabetic system but too small for an ideographic system, the inscriptions could well be written in a logo-syllabic system. Such a writing system combines both logograms (morphemic signs) and syllabic (or phonetic) signs without inherent meaning. In future work, we plan to investigate the differences that arise in the network structure of languages belonging to these very different systems, in order to make an inference on the nature of
the writing system used in the Indus inscriptions. One of the most controversial aspects of Indus decipherment is the question of how many distinct signs are there. Mahadevan [14] identified 417 signs, while Wells [18] has distinguished about 700 signs. Other researchers have come up with a wide range of different numbers. Therefore, an important open issue that needs to be settled is the robustness of these results with respect to the sign list being used.

However, despite these limitations, based on the results reported here it seems fair to conclude that the inscriptions do have an underlying syntactic organization. By comparing with a randomized ensemble of sequences that maintain the original sign frequency and restrictions on the co-occurrence of signs in the same inscriptions, but which otherwise lack any of the empirical correlations between sign pairs, we have established beyond reasonable doubt that the sequences cannot be just random juxtaposition of signs. It appears to rule out the possibility put forward by one group that the inscriptions are merely a set of magical or mystical symbols without any inherent meaning [20]. However, further analysis is needed to conclude whether the sequences represent writing in a formal sense. This is particularly difficult as there is no consensus about the definition of writing. As a standard textbook on the subject mentions, “every attempt at a single universal definition of writing runs the risk of being either ad hoc or anachronistic, or informed by cultural bias” [11]. While it can be broadly defined as a system of intercommunication by means of conventional visible marks [28, 29], often writing tends to be narrowly defined as a means of efficiently encoding speech even though there is no writing system that can record the entire linguistic structure of speech [11]. In fact, a conception of what constitutes writing is critically contingent upon the historical and cultural circumstances which gives rise to the assumptions underlying such a conception [10].

As Coe has pointed out, the refusal to recognize Mayan glyphs as writing because of pre-conceived notions about what “writing” should be, proved to be one of the biggest obstacles to its eventual decipherment [30]. Similarly, Ventris’ decipherment of Linear B was challenged for a long time because a writing system that leaves out endings and includes only word stems seems strange from the point of view of modern alphabetic writing. However, it was primarily used for “recording accounts, inventories and similar brief notes; there is no example of continuous prose, … the script is appropriate to its actual use, which is no more than an elaborate kind of mnemonic device.” [31].

This brings us again to the point mentioned earlier in the paper that early writing was never used for recording spoken language. One of the objections sometimes put forward to the notion of Indus inscriptions being a form of writing is that there are a large number of sequences of short length. However, many early writing systems exhibit such brevity. For example, the written language of early Sumerian documents is very restricted and there are no sign sequences that can be interpreted as expressions larger than individual words. Another example is early Egyptian writing seen in inscriptions obtained from artifacts in royal burials dating from the late predynastic period (c. 3200 BCE) [32]. A few hundred tags made of bone and ivory which bear around forty different inscription types have been found in the tombs. The majority of these tags have two hieroglyphs. Also, more than a hundred ceramic jars have been discovered which bear large single or paired signs painted on their outer surfaces. Yet another example comes from the earliest examples of Chinese writing, viz., the very brief inscriptions on bronze ritual vessels from the Anyang period, belonging to the last two centuries of the 2nd millennium BC [33]. The majority of these inscriptions comprise only a clan sign and an ancestor dedication. Indeed, brevity seems to be a common feature of most examples of early writing. This could be because the main use of writing was as a means of maintaining accounts, lists and other economic records. For example, only about 15% of the old Sumerian inscriptions of the late Uruk and Jemdet Nasr periods (3300 – 2900 BCE), the period during which the Sumerian writing system took shape, have non-economic subject matter [10].

In fact, even today it is possible to see examples of such use of writing that can result in extremely short sequences. For example, a package may be marked by one or few signs (e.g., a numeral or an initial consisting of alphabetic characters) in order to distinguish it from others in terms of content, ownership, origin or destination. Another example is that of an inventory of a group of commodities using a set of tally marks. Thus, very short sequences may suggest the application of a writing system in a specialized or restricted context, most possibly economic. One should bear in mind that many of the shortest Indus inscriptions contain signs comprising multiple vertical bars resembling tally marks and which have sometimes been hypothesised to represent numerals. Indeed, the possibly economic nature of the Indus inscriptions have been independently suggested by evidence that many of the seals were used to impress clay-tag sealings that were affixed to packages [18]. This suggests that the Indus inscriptions share several characteristic features with early writing systems rather than being an anomaly.

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Appendix
The Appendix comprising four sheets at the end of the paper contains a sign list of the W09IMSc data-set. Each sign has been numbered with a 3 digit ID between 001 and 958. Below the ID number, the frequency of occurrence of the sign in the W09IMSc data is indicated.

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