Moisture safety in ventilated cathedral roofs

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Abstract. Cathedral roofs are commonly used when constructing small houses in Sweden. In contrast to roof constructions with a cold attic, where frequent moisture damage has been noted, the cathedral roof is difficult to access for inspection. Furthermore, Swedish building regulations sets high demands regarding moisture safety, although there are no clear guidelines for their compliance. Hence, designing a cathedral roof must be done with great care. Previous studies investigating moisture safety in cathedral roofs, applies a constant air exchange in the ventilated air cavity. In this study a cathedral roof, ventilated from eave to eave, was analysed by examining the relevance of considering the variation in cavity air flow when conducting coupled heat and moisture calculations. The varied cavity air flow was calculated in an air flow model, considering wind and thermal buoyancy as driving forces. The accuracy of moisture safety assessments using the MRD model via hygrothermal calculations in WUFI Pro were also studied. Comparing moisture calculations with measurements showed high similarity when using a model with constant cavity air flow, and even higher resemblance when using a model with varied air flow. When actual conditions are sought, the study indicated that pinpointing important parameters, such as initial moisture content and moisture related material properties, would further increase precision in moisture calculations.

1. Introduction

Increasing demands for energy efficiency has led to increased insulation thickness in roof constructions. For a cathedral roof this results in colder outer parts, similar to a roof with a cold (well insulated) attic, with an increased risk for moisture related problems. Another apparent similarity with cold attics, where frequent moisture damage has been noted [1], is the fact that models used for calculating moisture conditions in cold attics and cathedral roofs often are similar [2].

Measuring air exchange in cathedral roofs and exterior walls [3,4], demonstrates high variation in air flow both throughout day and year. This variation is relevant to consider when creating theoretical models of the problem. However, in Sweden there are no clear requirements or guidelines in regulations for considering the cavity air flow in moisture calculations. Simultaneously, uncertainties regarding the extent of air exchange in the cavity has led to moisture calculations, applying a standard value for constant air exchange [5,6]. The guiding publication RäknaF [6], provided by Swedish Moisture Research Centre, recommends 20 air exchanges per hour.

The similarity with cold attics and the associated moisture problems, the difficulty for inspection in addition to observed variations in measured cavity air flow - all supports the development of a hygrothermal calculation model for cathedral roofs, that considers the variation in cavity air flow,
conducted in this study. Moisture calculation results were validated against measurements in a reference roof construction. The study was limited to solely consider well-insulated cathedral roofs, which are ventilated from eave to eave by natural driving forces, having a material sensitive to moisture adjacent to the air cavity. The scope of the study was moisture safety, hence cavity air flow was not measured and could not be validated.

2. Reference Roof Construction
A cathedral roof of a small house located outside of Stockholm, Sweden, was used as a reference roof construction. The roof was highly insulated (450 mm for 45° inclination and 555 mm for 62° inclination respectively), with a painted metal sheet roofing, naturally ventilated with outdoor air in a 45 mm air cavity. Measurements of temperature and relative humidity in the cavity air and moisture ratio in the cavity separating battens were conducted in the cavity during consecutive 1.5 years (see figure 1).

The building geometry allowed for measurements at 4 different roof orientations within the length of 2 different air cavities, differentiating in roof inclination. Outdoor climate conditions were mainly retrieved from nearby weather stations where measurements are continuously conducted by the Swedish Metrological and Hydrological Institute [7].

Figure 1. Principle drawings, plan view and section of the reference roof construction.

3. Modelling the Cavity Air Flow

3.1. Air Flow Model
Variation in cavity air flow was considered by formulating a theoretical air flow model. Time variations was considered by assuming stationary conditions in-between timesteps of one hour. Falk [4] uses equation 1 for pressure differences over the cavity, where the flow rate is calculated relative to driving forces and air flow resistances. The driving forces are wind pressure and thermal buoyancy, and the resistances are pressure loss due to friction along the cavity length and local pressure losses at cavity entrance, exit and roof ridge.

\[
\Delta P_{\text{wind}} + \Delta P_{\text{thermal}} = \Delta P_{\text{friction}} + \sum \Delta P_{\text{local losses}} \quad [\text{Pa}]
\] (1)

Equation (1) was implemented into the air flow model to calculate hourly air flows deriving from variations in climate data.

The pressure loss due to friction was calculated using the Darcy-Weisbach equation. Local pressure losses were decided by equations provided by Kronvall [8]. The local pressure loss at the roof ridge was presumed equal to a 90° change in flow path.

Driving forces were assumed to be mainly the sum of wind pressure and thermal buoyancy. The driving force from wind is the resulting pressure difference from pressure increase at the windward oriented eave, and pressure decrease at the leeward oriented eave. The pressure difference was calculated according to equation (2).
\[ \Delta P_{\text{wind}} = \Delta c_p \cdot \rho_{\text{out}} \cdot u_{\text{wind}}^2 \] (2)

Where:
- \( \Delta P_{\text{wind}} \) is the difference in pressure caused by wind as a driving force [Pa],
- \( \Delta c_p \) is the difference in wind pressure coefficient between the two eaves,
- \( \rho_{\text{out}} \) is the density of outdoor air [kg/m\(^3\)],
- \( u_{\text{wind}} \) is the wind speed in undisturbed flow, measured 10 meters above ground [m/s].

Pressure difference due to wind was calculated using wind speed measured at weather stations \([7]\) near the building where measurements were conducted. The density of outside air was calculated using the ideal gas law, using air temperature measured at nearby weather stations. Wind pressure coefficients, dependent on wind incidence angle, were derived from measurements with similar conditions, conducted by Gullbrekken, Uvsløkk, Kvande, Pettersson & Time \([9]\). Using the least square method to approximate their measured coefficients with a trigonometric equation resulted in equation (3).

\[ \Delta c_p(\varphi) = 0.48 \cdot \cos(1.3 \cdot \varphi) + 0.22 \] (3)

With \( \varphi \) as wind orientation, relative to a line perpendicular to the eave where air enters the cavity [°].

Driving force from thermal buoyancy was modelled based on previous research on thermal buoyancy in cathedral roofs \([10]\) and particularly on the formulation by Hagentoft \([11]\). The resulting system of equations for thermal buoyancy in cathedral roofs, ventilated from eave to eave, are displayed in equation (4).

\[
\begin{align*}
\{ \Delta P_{\text{thermal}} &= g \cdot \rho_{\text{out}} \cdot \sin(\Theta) \cdot T_{\text{out}} \int_0^L \frac{1}{T_{\text{cavity}}(l\Theta)} - \frac{1}{T_{\text{cavity}}(l\Theta)} \ d\theta \\
T_{\text{cavity}}(l\Theta) &= T_0 - (T_0 - T_{\text{in}}) \cdot e^{-\frac{l\Theta}{L_0}} \\
L_0 &= \rho \cdot c \cdot h_{\text{cavity}} \cdot u \cdot R_0 
\end{align*}
\] (4)

Where:
- \( \Delta P_{\text{thermal}} \) is the difference in pressure caused by thermal buoyancy [Pa],
- \( \rho_{\text{out}} \) is the density of outdoor air [kg/m\(^3\)],
- \( T_{\text{out}} \) is the temperature of outdoor air [K],
- \( \Theta \) is the roof inclination [°],
- \( L \) is the cavity length for each side of the roof (measured parallel to \( \Theta \)) [m],
- \( T_{\text{cavity}}(l\Theta) \) is the varying air temperature in cavity \( n \) as a function of cavity length (measured parallel to \( \Theta \)) [K],
- \( T_0 \) is the effective temperature, considering all heat exchange between the cavity and surrounding surfaces [K],
- \( T_{\text{in}} \) is the air temperature when entering cavity 1 or 2 respectively [K],
- \( L_0 \) is effective length [m], a quantity relating \( T_{\text{cavity}} \) to \( T_0 \),
- \( \rho, c \) is density [kg/m\(^3\)] and heat capacity [J/kg,K] for air,
- \( h_{\text{cavity}} \) is the air cavity height [m],
- \( u \) is the variable air velocity in the cavity [m/s],
- \( R_0 \) is the effective heat resistance for the roof [m\(^2\)K/W].
3.2. Model Outcomes
From the formulation of thermal buoyancy in equation (4) one can conclude that the contribution from thermal buoyancy is positive and adding to the air flow when the first part in the integral is greater than the second part. Since the first part of the integral is related to cavity 2 (the exit cavity) it was concluded that, for a calculated positive thermal driving force, the integrated air temperature must be greater in the entering, versus the exiting, cavity. In all other cases the contribution from thermal buoyancy is negative in the calculation, hence countering the air flow. Considering the expression for cavity temperature, several conclusions could be made regarding the outcomes of the model for thermal buoyancy:

- With no driving force from wind, air flow in the cavity will only be achieved when the effective temperature on one side of the roof is sufficiently greater than on the other side.
- Thermal buoyancy in combination with wind could result in a negative thermal contribution if the cavity air velocity is high enough (the effective length $L_0$ is sufficiently high).
- Negative thermal contribution to the air flow will be achieved in every case when the air is not cooled after passing the roof ridge.

Calculating the air exchange in well insulated cathedral roofs by implementing the air flow model as described in section 2.1 results in high air flow rates. In this study, mean calculated air exchange was around 200 h$^{-1}$, tenfold the recommended constant air exchange [6] and likewise far greater than values for constant air exchange used in previous research [2,5]. The air flow model calculated air flow rates with large variations over days, and sometimes hours.

The air flow model was not validated against measured air exchanges in this study, since no such measurements were conducted. However, comparable measurements have previously been conducted by Gullbrekken, Kvande and Time [3] where apparent linear relationships between outdoor wind speed and cavity air velocity was found. Air exchanges calculated in the air flow model in section 2.1 was compared with the relationship - cavity air velocity being 20 % of outdoor wind speed [3] and resulted in high conformity. A typical period is displayed in figure 2.

![Figure 2. Calculated air exchanges in relation to linear relationship [3].](image_url)

The variation of the curve and the order of magnitude shows high resemblance between the two methods. The order of magnitude is also in agreement with measurements conducted for air cavities in walls [4]. Calculated air exchanges are therefore believed to be in the correct order of magnitude and reasonably climate dependent, despite not being validated against measurements of cavity air velocity or likewise.
4. Coupled Heat and Moisture Calculations

4.1. Moisture Calculation Model
The air flow model, as described in section 3.1, was integrated into hygrothermal calculations to evaluate the precision when calculating the moisture parameters measured in the reference roof construction. Analogous calculations were conducted using a constant air exchange of 20 h\(^{-1}\) [6] to examine the relevance of considering a varied air flow. The hygrothermal model was designed in the calculation program WUFI Pro [12] whilst considering relevant model parameters used in previous, comparable studies [2,5]. Outdoor climate conditions according to measurements at local weather stations [7] were presumed. Calculations were conducted both prior to and after obtaining measurement results, where the latter moisture calculation model was improved in relation to differences from measured data.

4.2. Comparison with Measurements
Results from hygrothermal calculations and measurements were analyzed qualitatively by evaluating time variations of temperature-, relative humidity- and moisture ratio. Results were also analyzed quantitatively i.e., by comparing peak in calculated MRD index [13] for each time series. The MRD index is a cumulative measure of current risk for mould growth in relation to temperature, relative humidity, duration of exposure and material properties [13]. Generally, results from initial calculations showed high resemblance with measured data for most parameters and for both calculation methods (constant and varied air flow respectively). The reliability of such a hygrothermal calculation model could therefore be legitimized, as previously shown by Mundt-Petersen [2]. The most important improvement implemented in the initial hygrothermal model was to pinpoint starting conditions for moisture content in wood. Degree of shading and values for moisture related material properties were also identified as important when actual (not critical) conditions were sought.

Results from the improved hygrothermal model likewise showed high resemblance with measured data for most parameters, especially in qualitative analysis of plotted time variations (see segments presented in figure 3-6 and related RMSE in figure captions). There was a higher compliance with measured data when implementing the improved model for the calculated moisture content in roof decking. The amplitude for variations in moisture content over time were still constantly lower than measured. Moisture ratio was the parameter where calculations deviated the most from measurements (see figure 3). For relative humidity and temperatures, calculations generally were very similar to measurement, the main difference being the amplitude (see relative humidity in figure 4). Calculations with implemented air flow model generally resulted in higher amplitude than measurements, especially for relative humidity. Constant air flow resulted in lower amplitude than measurements. The difference in amplitude could be interpreted as the calculated air flow being higher than the actual and accordingly the constant air flow being lower than the actual. With implemented air flow model, the general features of measurements time plots were mimicked roughly. Constant air flow seemed to follow a mean of measurement data with the exception for summer periods where the south oriented roof achieved higher temperatures and lower relative humidity than measured (see figure 5). This could be caused by underestimating the velocity of air flow. Both methods showed high correlation with measurements regarding temperature during winter, testifying to the temperature in climate data being correct for the period (see figure 6). The data in figure 3-6 is selected to be representative for the general result.
Figure 3. Moisture ratios for the northern roof during the first six months. Calculated RMSE (whole period) was 2.0 %–MR for Air Flow Model and 1.9 %–MR for Constant 20 h⁻¹.

Figure 4. Relative humidity for the northern roof during a typical week in winter. Calculated RMSE (whole period) was 11 %–RH for Air Flow Model and 7.1 %–RH for Constant 20 h⁻¹.

Figure 5. Temperatures for the southern roof during a typical week in summer. Calculated RMSE (whole period) was 4.5 °C for Air Flow Model and 7.0 °C for Constant 20 h⁻¹.

Figure 6. Temperatures for the eastern roof during a typical week in winter. Calculated RMSE (whole period) was 5.0 °C for Air Flow Model and 9.3 °C for Constant 20 h⁻¹.

A quantitative analysis of the validity of the calculations were conducted by calculating risk for moisture related damage for both calculated and measured values (relative humidity and temperature) according to the MRD model [13]. The resulting MRD index time plots showed high resemblance between measurements and calculations, particularly for calculations with implemented air flow model. The MRD time plot for the model with constant air flow also showed resemblance with measurements apart from always being lower than measurements. Greatest compliance for the model with constant air flow was achieved for the northernly oriented roof. Calculated RMSE was improved when considering MRD-index (giving higher weight to periods with increased risk for moisture damage in relation to previous
parameters). In all instances implementing the air flow model resulted in higher calculated risk for moisture damage, for south and north orientations also being higher than measurements (hence a conservative calculation). East and west orientation resulted in lower calculated MRD index than for measurements which in some level could be attributed to underestimated degree of shading. Implementing the air flow model could, however, correctly identify the critical roof orientation (constant air flow inaccurately identified north as critical). MRD index time plots for roof orientation north and east are presented in figure 7 and 8, respectively.

**Figure 7.** MRD-index for the northern roof. Calculated RMSE (whole period) was 0.064 for Air Flow Model and 0.027 for Constant 20 h\(^{-1}\).

**Figure 8.** MRD-index for the eastern roof. Calculated RMSE (whole period) was 0.037 for Air Flow Model and 0.061 for Constant 20 h\(^{-1}\).

The relevance of considering the varying air flow was examined by analyzing the differences in calculation results between constant air flow and implemented air flow model. Although the mean air flow in the air flow model was drastically higher than the constant air exchange, the difference in calculated moisture levels was not as large. Constant air flow showed higher resemblance with measurements for the normally critical roof orientation, north. Even though the air flow model resulted in higher resemblance with the measured values for the other orientations and showed higher compliance with variations in the measured data in qualitative assessments, hygrothermal calculations with a constant air flow was, as previously been suggested [2,5], useful for calculating critical conditions. However, considering the varying air flow resulted in greater overall compliance with measurements i.e. identifying the critical roof orientation, resembling time plots for MRD, relative humidity etc. and displaying a conservative assessment of MRD index. Furthermore, implementing varying air flow resulted in robustness for leakage of internal humid air (although leakage did not improve resemblance with measured data). Constant air flow was very sensitive for the same leakage. The model with constant air flow was also sensitive to the applied value for air exchange which could be interpreted as adaptability. Since internal leakage is common (due to insufficient air tightness), the sensitivity for leakage and applied air exchange, in hygrothermal models with constant air flow speaks against the credibility of such models.
5. Conclusions
From conducted studies, several conclusions can be summarized as follows:

- An air flow model for the air cavity can be implemented in hygrothermal calculations and provide high resemblance with measurements of temperature, relative humidity, moisture ratio and related moisture safety assessment [13]. The resemblance was improved, especially for moisture ratio, when pinpointing initial moisture content in wood.
- Hygrothermal calculations with a constant air exchange can show high resemblance with measurements, given an appropriately chosen value of air exchange.
- Hygrothermal calculations with an implemented air flow model, in most cases, resulted in higher resemblance with measured data than calculations with the constant air exchange recommended in the Swedish hygrothermal calculation guidelines RäknaF [6].
- When actual conditions are sought in hygrothermal calculations, this study identified the starting conditions for moisture content in wood, values for moisture related material properties and degree of shading as important parameters.

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