Non-zero $\theta_{13}$, CP-violation and Neutrinoless Double Beta Decay for Neutrino Mixing in the $A_4 \times Z_2 \times Z'_2$ Flavor Symmetry Model

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Abstract

We study the modification of the Altarelli-Feruglio $A_4$ flavor symmetry model by adding three singlet flavons $\xi'$, $\xi''$ and $\rho$ and the model is augmented with extra $Z_2 \times Z'_2$ symmetry to prevent the unwanted terms in our study. The addition of these three flavons lead to two higher order corrections in the form of two perturbation parameters $\epsilon$ and $\epsilon'$. These corrections yield the deviation from exact tri-bimaximal (TBM) neutrino mixing pattern by producing a non-zero $\theta_{13}$ and other neutrino oscillation parameters which are consistent with the latest experimental data.

In both the corrections, the neutrino masses are generated via Weinberg operator. The analysis of the perturbation parameters $\epsilon$ and $\epsilon'$, shows that normal hierarchy (NH) and inverted hierarchy (IH) for $\epsilon$ does not change much. However, as the values of $\epsilon'$ increases, $\theta_{23}$ occupies the lower octant for NH case. We further investigate the neutrinoless double beta decay parameter $m_{\beta\beta}$ using the parameter space of the model for both normal and inverted hierarchies of neutrino masses.

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I. INTRODUCTION

Though the particle physics experiments and observations have been successfully confirming the standard model (SM) of particle physics, the origin of flavor structure, strong CP problem, matter-antimatter asymmetry of the universe, dark matter, dark energy, non-zero tiny neutrino masses, presence of extra flavor of neutrinos, etc. are still open questions. The discovery of neutrino oscillations in 1998 by Super-Kamiokande (SK) Collaborators, Japan and Sudbury Neutrino Observatory Collaborators, Canada were the first proof of physics beyond the standard model. In the neutrino physics, we still do not know if the leptonic CP symmetry is violated or not, whether the neutrino masses are normal hierarchy (NH) or inverted hierarchy (IH), if the atmospheric mixing angle is maximal or not, are the neutrinos of the Dirac or Majorana type, what is the lightest absolute neutrino mass value etc. The neutrino oscillation experiments are only sensitive to the mass squared differences $\Delta m_{ij}^2$, and the leptonic mixing angles ($\theta_{ij}$).

The neutrino physics is an experimental driven dynamics field. It has made tremendous progress over the past twenty four years and attempts are underway to quantify the neutrino oscillation parameters more precisely. A few latest reviews on neutrino physics are placed in references [1–7].

Neutrino oscillation phenomenology is characterized by two large mixing angles, the solar angle $\theta_{12}$ and the atmospheric angle $\theta_{23}$ together with the relatively small reactor mixing angle $\theta_{13}$. In tribimaximal mixing (TBM), the reactor mixing angle $\theta_{13}$ is zero and the CP phase $\delta_{CP}$ is consequently undefined. However, in 2012 the Daya Bay Reactor Neutrino Experiment ($\sin^2 2\theta_{13} = 0.089 \pm 0.010 \pm 0.005$) [8] and RENO Experiment $\sin^2 2\theta_{13} = 0.113 \pm 0.013 \pm 0.019$ [9] showed that $\theta_{13} \simeq 9^\circ$. Also, several neutrino oscillation experiments like MINOS [10], Double Chooz [11], T2K [12], measured consistent nonzero values for $\theta_{13}$. Since TBM has been ruled out due to a non-zero reactor mixing angle, [9, 11] one of the admired ways to achieve realistic mixing is through either its extensions or through modifications.

The widely accepted PMNS matrix encodes the mixing in between the neutrino flavour eigenstates and their mass eigenstates. In a three flavoured paradigm, three mixing angles and three CP phases are used to parameterize this PMNS matrix.
\[
U_{\text{PMNS}} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \cdot U_{\text{Maj}} \tag{1}
\]

where, \( c_{ij} = \cos \theta_{ij}, \ s_{ij} = \sin \theta_{ij} \). The diagonal matrix \( U_{\text{Maj}} = \text{diag}(1, e^{i\alpha}, e^{i(\beta + \gamma)}) \) contains the Majorana CP phases, \( \alpha, \beta \) which become observable in case the neutrinos behave as Majorana particles. To show that neutrinos are Majorana particles, it will most likely require neutrinoless double beta decay to be discovered. Such kind of decays are yet to be observed. To explain these issues, symmetry would play an important role. Wendell Furry [13] considered Majorana nature of particles, to study a kinetic process which was similar to double beta disintegraton without neutrino emission popularly known as neutrinoless double beta decay (NDBD) [14]. It can be expressed as \((A, Z) \rightarrow (A, Z + 2) + 2e^-\) which violates the lepton number by two units and creates a pair of electron, and Majorana neutrino masses are generated as electroweak symmetry is broken. The large value of the scale of lepton number violation (LNV), typically \( \Lambda \sim (10^{14} - 10^{15}) \) GeV, is generally linked to the observed smallness of neutrino masses. Since the neutrino mass is zero in standard model[15], we need to construct a model which is beyond the standard model by adopting a new symmetry and generate non-zero neutrino mass. One such model is the effective theories, which can generate neutrino masses through Weinberg operator [16].

There are other frameworks beyond the standard model (BSM) that can explain the origin of neutrino masses, for examples, the Seesaw Mechanism [17–22], Supersymmetry [23], Minimal Supersymmetric Standard Model (MSSM)[24], Next-to-Minimal Supersymmetric Standard Model (NMSSM)[25], String theory [26], models based on extra dimensions [27], Radiative Seesaw Mechanism [28] and also some other models. Now many neutrino experiments have proved without doubt that neutrino has tiny non-zero mass and indicate flavor mixing [29–32].

Many researchers have proposed as to what the lepton mixing pattern should look like. Tri-bimaximal (TBM) [33, 34], Trimaximal (TM1/TM2) [35–37], Quasi-degenerate neutrino mass models [38] and Bi-large mixing patterns [39–41] are examples of phenomenological neutrino mixing patterns. And also various models based on non-abelian discrete flavor symmetries [42] like \( A_4 \) [43–45], \( S_3 \) [46], \( S_4 \) [47–49], \( \Delta_{27} \) [50–52], \( \Delta_{54} \) [53, 54] etc. have been proposed to obtain tribimaximal mixing (TBM) and deviation from TBM are obtained by...


| Parameters | NH (3σ) | IH (3σ) |
|------------|---------|---------|
| $\Delta m^2_{21} [10^{-5} eV^2]$ | $6.82 \rightarrow 8.04$ | $6.82 \rightarrow 8.04$ |
| $\Delta m^2_{31} [10^{-3} eV^2]$ | $2.431 \rightarrow 2.599$ | $-2.584 \rightarrow -2.413$ |
| $\sin^2 \theta_{12}$ | $0.269 \rightarrow 0.343$ | $0.269 \rightarrow 0.343$ |
| $\sin^2 \theta_{13}$ | $0.02034 \rightarrow 0.02430$ | $0.02053 \rightarrow 0.02434$ |
| $\sin^2 \theta_{23}$ | $0.405 \rightarrow 0.620$ | $0.410 \rightarrow 0.623$ |
| $\delta_{CP}$ | $105 \rightarrow 405$ | $192 \rightarrow 361$ |

TABLE I: The 3σ ranges of neutrino oscillation parameters from NuFIT 5.1 (2021) [64]

adding extra flavons.

Our model is based on the Altarelli-Feruglio (A-F) $A_4$ discrete flavor symmetry model [55–57]. We have extended the flavon sector of A-F model by introducing extra flavons $\xi'$, $\xi''$ and $\rho$ which transform as $1'$, $1''$ and $1$ respectively under $A_4$ to get the deviation from exact TBM neutrino mixing pattern. We also introduced $Z_2 \times Z_2'$ symmetry in our model to prevent unwanted terms and this helps in constructing specific structure of the coupling matrices. And we have calculated higher dimension perturbative parameters $\epsilon$ and $\epsilon'$ from the Lagrangian and modified the neutrino mass matrix $M_\nu$ and realized symmetry based studies. However in a similar works [58, 59], they have been simply added arbitrarily perturbative terms to $M_\nu$ obtained from A-F model without calculations from the Lagrangian. But in these papers [60-63], perturbative term was calculated using Type-I seesaw mechanism and departed from the tri-bimaximal mixing pattern. Therefore, this gives us an opportunity to analyze $M_\nu$ obtained through Weinberg operator in detail and study the effect of two perturbative terms $\epsilon$ and $\epsilon'$ on neutrino oscillation parameters and NDBD parameter $m_{ee}$. In this way our work differs from others.

The content material of our paper is organised as follows: In section 2, we give the overview of the framework of our model by specifying the fields involved and their transformation properties under the symmetries imposed. We give two types of corrections, analyse and study the impact of these correction terms on neutrino oscillation parameters. In section 3, we do numerical analysis and study the results for the neutrino phenomenology. We finally conclude our work in section 4.
II. FRAMEWORK OF THE MODEL

The non-Abelian discrete symmetry group $A_4$ is a group of even permutations of four objects and it has 12 elements ($12=\frac{4!}{2}$). It can describe the orientation-preserving symmetry of a regular tetrahedron, so this group is also known as tetrahedron group. It can be generated by two basic permutations $S$ and $T$ having properties $S^2 = T^3 = (ST)^3 = 1$. This group representations include three one-dimensional unitary representations $1$, $1'$, $1''$ with the generators $S$ and $T$ given, respectively as follows:

\[ 1 : S = 1, T = 1 \]
\[ 1' : S = 1, T = \omega^2 \]
\[ 1'' : S = 1, T = \omega \]

and a three dimensional unitary representation with the generators\(^1\)

\[ T = \begin{pmatrix}
 1 & 0 & 0 \\
 0 & \omega^2 & 0 \\
 0 & 0 & \omega
\end{pmatrix} \tag{2} \]
\[ S = \frac{1}{3} \begin{pmatrix}
 -1 & 2 & 2 \\
 2 & -1 & 2 \\
 2 & 2 & -1
\end{pmatrix} \tag{3} \]

Here $\omega$ is the cubic root of unity, $\omega = \exp(i2\pi)$, so that $1 + \omega + \omega^2 = 0$.

The multiplication rules corresponding to the specific basis of two generators $S$ and $T$ are as follows:

\[ 1 \times 1 = 1 \]
\[ 1'' \times 1' = 1 \]
\[ 1' \times 1'' = 1 \]
\[ 3 \times 3 = 3 + 3_A + 1 + 1' + 1'' \]

\(^1\) Here the generator $T$ has been chosen to be diagonal.
For two triplets

\[ a = (a_1, a_2, a_3) \]
\[ b = (b_1, b_2, b_3) \]

we can write

\[ 1 \equiv (ab) = a_1 b_1 + a_2 b_3 + a_3 b_2 \]
\[ 1' \equiv (ab)' = a_3 b_3 + a_1 b_2 + a_2 b_1 \]
\[ 1'' \equiv (ab)'' = a_2 b_2 + a_1 b_3 + a_3 b_1 \]

Here 1 is symmetric under the exchange of second and third elements of a and b, 1' is symmetric under the exchange of the first and second elements while 1'' is symmetric under the exchange of first and third elements.

\[ 3 \equiv (ab)_S = \frac{1}{3} (2a_1 b_1 - a_2 b_3 - a_3 b_2, 2a_3 b_3 - a_1 b_2 - a_2 b_1, 2a_2 b_2 - a_1 b_3 - a_3 b_1) \]

\[ 3_A \equiv (ab)_A = \frac{1}{3} (a_2 b_3 - a_3 b_2, a_1 b_2 - a_2 b_1, a_1 b_3 - a_3 b_1) \]

Here 3 is symmetric and 3_A is anti-symmetric. For the symmetric case, We notice that the first element here has 2-3 exchange symmetry, the second element has 1-2 exchange symmetry and the third element has 1-3 exchange symmetry.

Our model is based on the Alterelli-Feruglio \( A_4 \) model \[55-57\]. We have added additional flavons \( \xi', \xi'' \) and \( \rho \) to get the deviation from exact TBM neutrino mixing pattern. We put extra symmetry \( Z_2 \times Z_2' \) to avoid unwanted terms. The particle content and their charge assignment under the symmetry group is given in Table II. The left-handed lepton doublets and right-handed charged leptons (\( e^c, \mu^c, \tau^c \)) are assigned to triplet and singlet (1, 1'', 1')

| Field | 1 | e^c | \( \mu^c \) | \( \tau^c \) | \( h_u \) | \( h_d \) | \( \Phi_S \) | \( \Phi_T \) | \( \xi \) | \( \xi' \) | \( \xi'' \) | \( \rho \) |
|-------|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| SU(2) | 2 | 1 | 1 | 1 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| \( A_4 \) | 3 | 1 | 1'' | 1' | 1 | 1 | 3 | 3 | 1 | 1'' | 1 |
| \( Z_2 \) | 1 | -1 | -1 | 1 | 1 | -1 | -1 | 1 | 1 | 1 | 1 | 1 |
| \( Z'_2 \) | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | -1 |

TABLE II: Full particle content of our model
representation under $A_4$ respectively and other particles transform as shown in Table-II. Here, $h_u$ and $h_d$ are the standard Higgs doublets which remain invariant under $A_4$. There are six $SU(2) \otimes U_Y(1)$ Higgs singlets, four ($\xi, \xi', \xi''$ and $\rho$) of which singlets under $A_4$ and two ($\Phi_T$ and $\Phi_S$) of which transform as triplets.

Consequently, the invariant Yukawa Lagrangian is as follows:

$$\mathcal{L} = y_e e^c(\Phi_T l) + y_\mu \mu^c(\Phi_T l)' + y_\tau \tau^c(\Phi_T l)'' + x_0 \xi(l) + x'_0 \xi'(l)'' + x''_0 \xi''(l) + x_b (\Phi_S l) + h.c. + ...$$  

(4)

where we have used the compact notation,

$$y_e e^c(\Phi_T l) \equiv y_e e^c(\Phi_T l) \frac{h_d}{\Lambda}$$  

(5)

$$x_0 \xi(l) \equiv x_0 \xi \frac{(h_u h_u)}{\Lambda^2}$$  

(6)

and so on and $\Lambda$ is the cut-off scale of the theory. The terms $y_e$, $y_\mu$, $y_\tau$, $x_0$, $x'_0$ and $x''_0$, $x_b$ are coupling constants. We assume $\Phi_T$ does not couple to the Majorana mass matrix and $\Phi_S$ does not couple to the charged leptons.

After spontaneous symmetry breaking of flavour and electroweak symmetry we obtain the mass matrices for the charged leptons and neutrinos. Assuming the vacuum alignment,

$$\langle \Phi_T \rangle = (v_T, 0, 0)$$

The charged lepton mass matrix is given as

$$M_l = \frac{v_d v_T}{\Lambda} \begin{pmatrix} y_e & 0 & 0 \\ 0 & y_\mu & 0 \\ 0 & 0 & y_\tau \end{pmatrix}$$  

(7)

where, $v_d$ and $v_T$ are the VEV of $h_d$ and $\Phi_T$ respectively. Now, considering higher dimension terms in the neutrino sector, we consider two types of corrections of the form $x'_0 \xi'(l)'' \frac{\rho \rho}{\Lambda^2}$ and $x''_0 \xi''(l)' \frac{\rho \rho}{\Lambda^2}$, where, $x'_0$ and $x''_0$ are coupling constants. These give rise to two cases and we will study the impact of these correction terms on neutrino oscillation parameters.

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2 Considering terms up to dimension-5.
A. Case I

With the additional higher dimension term $x''\xi''(ll)^I\frac{\rho\rho}{\Lambda^2}$ and using the VEVs $\langle \Phi_s \rangle = (v_s, v_s, v_s)$, $\langle \xi \rangle = 0$, $\langle \xi' \rangle = u'$, $\langle \xi'' \rangle = u''$, $\langle h_u \rangle = v_u$ and $\langle \rho \rangle = v_\rho$, we obtain the neutrino mass matrix which may be written as

$$M^{(I)}_\nu = m_0 \begin{pmatrix} \frac{2b}{3} & c - \frac{b}{3} + \epsilon & d - \frac{b}{3} \\ c - \frac{b}{3} + \epsilon & d + \frac{2b}{3} & -\frac{b}{3} \\ d - \frac{b}{3} & -\frac{b}{3} & c + \frac{2b}{3} + \epsilon \end{pmatrix}$$

(8)

where, $m_0 = \frac{v^2_u}{\Lambda}$, $b = 2x_b v_u$, $c = 2x_a u''$, $d = 2x_a u'$ and $\epsilon = x'' \frac{v'' v^2}{\Lambda^2}$. We can take $c \simeq d$ and thus the neutrino mass matrix in equation (8) becomes

$$M^{(I)}_\nu = m_0 \begin{pmatrix} \frac{2b}{3} & d - \frac{b}{3} + \epsilon & d - \frac{b}{3} \\ d - \frac{b}{3} + \epsilon & d + \frac{2b}{3} & -\frac{b}{3} \\ d - \frac{b}{3} & -\frac{b}{3} & d + \frac{2b}{3} + \epsilon \end{pmatrix}$$

(9)

B. Case II

Here, we will take into consideration the correction term of the second type $x''\xi'(ll)^I\frac{\rho\rho}{\Lambda^2}$. The resulting neutrino mass matrix obtained in such a case is given as

$$M^{(II)}_\nu = m_0 \begin{pmatrix} \frac{2b}{3} & c - \frac{b}{3} + \epsilon' & d - \frac{b}{3} \\ c - \frac{b}{3} & d + \frac{2b}{3} + \epsilon' & -\frac{b}{3} \\ d - \frac{b}{3} + \epsilon' & -\frac{b}{3} & c + \frac{2b}{3} \end{pmatrix}$$

(10)

where $\epsilon' = x' \frac{v^2 u}{\Lambda^2}$, parameterizes the correction to the TBM neutrino mixing. Applying similar condition on $c$ and $d$ as in case I we obtain

$$M^{(II)}_\nu = m_0 \begin{pmatrix} \frac{2b}{3} & d - \frac{b}{3} + \epsilon' & d - \frac{b}{3} \\ d - \frac{b}{3} & d + \frac{2b}{3} + \epsilon' & -\frac{b}{3} \\ d - \frac{b}{3} + \epsilon' & -\frac{b}{3} & d + \frac{2b}{3} \end{pmatrix}$$

(11)

In section 3, we give the detailed phenomenological analysis for both the cases and discuss the effect of perturbations ($\epsilon$ and $\epsilon'$) on various neutrino oscillation parameters. Further,
we present a numerical study of neutrinoless double-beta decay considering the allowed parameter space of the model.

III. NUMERICAL ANALYSIS AND RESULTS

In the previous section, we have shown how Altarelli-Feruglio $A_4$ model could be modified by adding extra three singlet flavons and taking into consideration higher dimension terms. In this section, we perform a numerical analysis to study the capability of the perturbation parameters $\epsilon$ and $\epsilon'$ to produce the deviation of neutrino mixing from exact TBM. For each case, we will discuss the results for both normal as well as inverted hierarchies. Throughout the numerical analysis we have taken the value of $m_0$ to be in the range $[0.016 - 0.032]$ eV.

The neutrino mass matrix $M_{\nu}^{(I)}$ and $M_{\nu}^{(II)}$ can be diagonalized by the PMNS matrix $U$ as

$$U^\dagger M_{\nu}^{(i)} U^* = \text{diag}(m_1, m_2, m_3)$$ (12)

with $i = \{I, \ II\}$. We can numerically calculate $U$ using the relation $U^\dagger h U = \text{diag}(m_1^2, m_2^2, m_3^2)$, where $h = M_{\nu}^{(i)} M_{\nu}^{(i)\dagger}$. The neutrino oscillation parameters $\theta_{12}$, $\theta_{13}$, $\theta_{23}$ and $\delta_{CP}$ can be obtained from $U$ as

$$s_{12}^2 = \frac{|U_{12}|^2}{1 - |U_{13}|^2}, \quad s_{13}^2 = |U_{13}|^2, \quad s_{23}^2 = \frac{|U_{23}|^2}{1 - |U_{13}|^2},$$ (13)

and $\delta$ may be given by

$$\delta = \sin^{-1}\left(\frac{8 \text{Im}(h_{12} h_{23} h_{31})}{P}\right)$$ (14)

with

$$P = (m_2^2 - m_1^2)(m_3^2 - m_2^2)(m_3^2 - m_1^2) \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13}$$ (15)

For the comparison of theoretical neutrino mixing parameters with the latest experimental data [64], the modified $A_4$ model is fitted to the experimental data by minimizing the following $\chi^2$ function:

$$\chi^2 = \sum_i \left(\frac{\lambda_i^{\text{model}} - \lambda_i^{\text{expt}}}{\Delta \lambda_i}\right)^2.$$ (16)

where $\lambda_i^{\text{model}}$ is the $i^{th}$ observable predicted by the model, $\lambda_i^{\text{expt}}$ stands for the $i^{th}$ experimental best-fit value and $\Delta \lambda_i$ is the 1$\sigma$ range of the $i^{th}$ observable.
First, we shall discuss the case I, for perturbation parameter $\epsilon$. In Fig. 1, we have shown the parameter space of the model for Case I, which is constrained using the 3$\sigma$ bound on neutrino oscillation data (Table I). For both normal and inverted hierarchies, one can see that there is a high correlation between different parameters of the model.

Fig. 2 shows the prediction of the various neutrino oscillation parameters for NH in case I. The calculated best fit values of $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ are (0.341, 0.023, 0.62) which are within the 3 $\sigma$ range of experimental values. Other parameters such as $\Delta m^2_{21}$, $\Delta m^2_{31}$ and $\delta_{CP}$ have their best-fit values, corresponding to $\chi^2$-minimum, at $(7.422 \times 10^{-5} eV^2, 2.556 \times 10^{-3} eV^2, -0.367\pi)$ respectively, which perfectly agreed with the latest observed neutrino oscillation experimental data. Similar variation of $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ with increase in $|\epsilon|$ is observed for IH.

Fig. 3 gives the neutrino oscillation parameters predicted by the model for IH. (0.341,
FIG. 2: Case I - Variation of the mixing angles, mass-squared differences and Dirac CP phase with the correction parameter $\epsilon$ in NH case. The color-bar represents the different values of the correction parameter $\epsilon$.

0.024, 0.61) are the best fit values of $\sin^2 \theta_{12}$, $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$, which are all within the 3 $\sigma$ range of experimental values. Also, $\Delta m^2_{21}$, $\Delta m^2_{31}$ and $\delta_{\text{CP}}$ have their best-fit values, corresponding to $\chi^2$-minimum, at $(7.465 \times 10^{-5}\text{eV}^2$, $2.492 \times 10^{-3}\text{eV}^2$, 0.39$\pi$). Besides, we determined that, as the correction parameter $|\epsilon|$ increases, the value of $\sin^2 \theta_{13}$ and $\sin^2 \theta_{23}$ moves away from 0 and $\frac{1}{2}$, respectively.

Thus, the model defined in Case I, clearly shows that the deviation from exact tribimaximal mixing, however, with the change in $|\epsilon|$ there is no observable preference for the octant of $\theta_{23}$.

Now, we discuss for the case II for the perturbation parameter $\epsilon'$. In Figs. 4 and 5 we show the results with second type of correction, described as Case II in section II B. The allowed ranges for the model parameters for both NH as well as IH are presented in Fig. 4.
FIG. 3: Case I - Variation of the mixing angles, mass-squared differences and Dirac CP phase with the correction parameter $\epsilon$ in IH case. The color-bar represents the different values of the correction parameter $\epsilon$.

Also the different correlation plots among the various neutrino oscillation parameters and their variations with $|\epsilon'|$ are shown in fig. 5. It is clear that the neutrino mixing deviates from exact TBM mixing as $|\epsilon'|$ increases from 0 in both NH and IH cases. The prediction of the atmospheric mixing angle $\theta_{23}$ in NH case shows slight preference towards lower octant for larger values of $\epsilon'$. Thus, the modification of Altarelli-Feruglio with additional term, $x'\xi' (\mathcal{H})^\dagger \beta \bar{\rho}$, allows us to deviate from TBM mixing. The best-fit values for the predictions of the various oscillation parameters is shown in Table III.

**Neutrinoless double beta decay (NDBD):**

Till today, we do not know whether neutrinos are Dirac or Majorana type. If Majorana type, then the study of NDBD is very important. There are some ongoing NDBD experiments to determine Majorana nature of neutrino. The effective mass that governs the
FIG. 4: Case II - Correlation among the model parameters $|b|$, $\phi_b$ & $|d|$, $\phi_d$ in both NH (left column) as well as IH (right column).

| Parameter | NH         | IH        |
|-----------|------------|-----------|
| $\sin^2 \theta_{12}$ | 0.3407     | 0.341     |
| $\sin^2 \theta_{13}$ | 0.0217     | 0.0226    |
| $\sin^2 \theta_{23}$ | 0.576      | 0.547     |
| $\delta_{CP}$         | 0.061 $\pi$ | 0.145 $\pi$ |
| $\Delta m^2_{21}$     | $7.498 \times 10^{-5}$ | $7.492 \times 10^{-5}$ |
| $\Delta m^2_{31}$     | $2.517 \times 10^{-3}$ | $2.484 \times 10^{-5}$ |

TABLE III: Case II- Best-fit values for different parameters predicted by the model process is provided by,

$$m_{\beta\beta} = U^2_{Li} m_i$$

(17)
FIG. 5: Case II - Variation of the mixing angles, mass-squared differences and Dirac CP phase with the correction parameter $\epsilon'$ in NH (left) and IH (right) case. The color-bar represents the different values of the correction parameter $\epsilon'$. 
where \( U_{Li} \) are the elements of the first row of the neutrino mixing matrix \( U_{PMNS} \) (Eq. 1) which is dependent on known parameters \( \theta_{12}, \theta_{13} \) and the unknown Majorana phases \( \alpha \) and \( \beta \). \( U_{PMNS} \) is the diagonalizing matrix of the light neutrino mass matrix \( m_\nu \) so that,

\[
m_\nu = U_{PMNS} M_\nu^{(diag)} U_{PMNS}^T
\]

where, \( M_\nu^{(diag)} = \text{diag}(m_1, m_2, m_3) \). The effective Majorana mass can be parameterized using the diagonalizing matrix elements and the mass eigen values as follows:

\[
m_{\beta\beta} = m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i\alpha} + m_3 s_{13}^2 e^{2i\beta}
\]

Using the constrained parameter space we have evaluated the value of \( m_{\beta\beta} \) for case I and case II in both NH as well as IH cases. The variation of \( m_{\beta\beta} \) with lightest neutrino mass is shown in figure 6 for both the neutrino mass hierarchies. The sensitivity reach of NDBD experiments like KamLAND-Zen [65, 66], GERDA [67–69], LEGEND-1k [70] is also shown.
IV. CONCLUSIONS

We have constructed a flavon-symmetric $A_4 \times Z_2 \times Z'_2$ model to realize the latest neutrino oscillation experimental data which depart from Tribimaximal neutrino mixing pattern. The models have been checked for accuracy by adding three extra flavons $\xi', \xi'', \rho$ transforming under representations of $A_4$ to extend the A-F model. And the cyclic $Z_2 \times Z'_2$ symmetric term has been incorporated to eliminate unwanted terms in the calculations. The calculated two perturbation parameters $\epsilon$ and $\epsilon'$ clearly showed the deviation of neutrino mixing parameters from exact Tri-bimaximal neutrino mixing matrix. The resulting mass matrices give predictions for the neutrino oscillation parameters and their best-fit values are obtained using the $\chi^2$-analysis, which are consistent with the latest global neutrino oscillation experimental data. We found the magnitude of deviations from TBM is dominated by VEV of $\rho$. However, these quantities and corrections have little discriminative power, hence we supplement with observables related to neutrino mass. Therefore, we have also investigated NDBD in our model. The scatter plots of NDBD parameter ($m_{\beta\beta}$) and the lightest neutrino mass ($m_l$) parameter space are different in each model and allowed us to distinguish different models. The value of effective Majorana neutrino mass $|m_{\beta\beta}|$ is well within the sensitivity reach of the recent NDBD experiments like KamLAND-Zen, GERDA and LEGEND-1k. The determination of NDBD, cosmological mass and leptonic CP-violation phase $\delta_{CP}$ which are consistent with the latest experimental data will discriminate the neutrino mass models.

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