Effectiveness of Nonuniform Heat Generation (Sink) and Thermal Characterization of a Carreau Fluid Flowing across a Nonlinear Elongating Cylinder: A Numerical Study

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ABSTRACT: During thermal radiation treatments, heat therapies, and examination procedures like scans and X-rays, the cylindrical blood vessels may get stretched; meanwhile, the blood flow through those blood vessels may get affected due to temperature variations around them. To overcome this issue, this work was framed to explore the impact of heat transmission in a Carreau fluid flow (CFP) through a stretching cylinder in terms of the nonlinear stretching rate and irregular heat source/sink. Temperature-dependent thermal conductivity and thermal radiation are taken into consideration in this study. To transform complicated partial differential equations into ordinary differential equations, appropriate similarity variables are used. For a limited set of instances, the derived series solutions are compared to previously published results. For linear and nonlinear stretching rates, graphs and tables are used to examine the influence of an irregular heat source/sink on fluid movement and heat transfer. The research outcomes demonstrate that the heat source and nonlinear stretching rate cause a disruption in the temperature distribution in the fluid region, which can alter the blood flow through the vessels. In all conditions except for the heat in an internal heat sink, the nonlinear stretching situation improves the velocity and heat profile. Furthermore, with the increase in the values of the Weissenberg number, the temperature profile shows opposing features in a shear-thickening fluid and shear-thinning fluid. For the former $n > 1$, the blood fluidity gets affected, restricting the free movement of blood. For the latter, $n < 1$, the phenomenon is reversed. Other industrial applications of this work are wire coating, plastic coverings, paper fabrication, fiber whirling, etc. In all of those processes, the fluid flow is manipulated by thermal conditions.

1. INTRODUCTION

The formation, utilization, conversion, and exchange of heat across physical arrangements are the theme of heat transfer (HT), a branch of current engineering. Heat conduction, thermal convection, thermal radiation, and energy removal through stage shifts are some of the techniques used to transmit heat. To achieve HT, engineers also examine the mass transfer of different chemical classes (physical transmission in the formula of advection), whether cold or hot. Despite the fact that these progressions are unique, they commonly occur in the same system in the same period. Recently, numerous examples of HT problems have been considered by researchers. Cai et al. described neural networks for HT problems. A review paper is presented by Mousa et al. on the enhancement techniques using HT. Different materials are suggested for HT in the study by Li et al. The efficiency of HT and electrical performance evaluation of materials is given in the work by Khodadadi et al. A modification of HT via a numerical study is performed by Sheikholeslami et al. A comprehensive review on HT is presented by Nguyen et al. HT in various types of geometries is investigated by Mohankumar et al. Extra applications of HT can be located in magnetic fields, solar studies, and controlling macroscopic HT.

A fluid material is a gas or (particularly) a liquid that has no set structure and quickly produces an exterior compression. Applications of fluid can be found in the polymer studies performed by Karakosta et al. Lamnatou et al. reported a deep study about heat systems characterized by the heat of the working fluid concentrating and nonconcentrating technologies. Sempere et al. evaluated recent research as well as ongoing

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clinical trials to identify the most potential ways and barriers to translating these discoveries into feasible microRNA-based therapeutic tools for cancer therapy. Siafaka et al. examined the existing preparations for numerous viruses envisioned for use in pediatric patients, as well as a summary of different chronic and serious children’s ailments. This evaluation, given by the authors, can assist specialists working with pediatric preparations in developing additional well-organized and child-friendly drug distribution classifications. Pant et al. published a review study on fluid as an application in models. They have shown promise in the fight against drug resistance. By pointing to pathogens or detailed microenvironments, superficial func-tionalization with antimicrobial peptides recovers efficacy. Several fluid delivery techniques that have shown the ability to give antibiotics and anti-inflammatory drugs at the same time have been examined. The importance of fluid of additional adjuvant medications in sepsis care, such as antioxidants, antimicrobials, and extracorporeal blood purification, is also underlined. Wang et al. published a study on geological fluid describing the physical nonlinearity of fluid-rock classifications and the robust connection between flow and distortion in such equations, which run to absorbing predictions such as the extemporaneous development of fixed fluid flow in malleable stunts. Microthermometric findings for dolomite-hosted fluid inclusions are compatible with petrographic observations, indicating fluid entrapment prior to the commencement of dolomite reduction during exhumation, according to Peng et al. Model calculations indicate that aqua-rich fluids with comparatively large hydrogen fugacity may generate promising conditions for the reduction process, but this does not rule out the potential of carbonate methanation by hydrogen-rich fluids. The search for renewable and unpolluted energy supplies is fueled by the rise in global energy demand. Teimouri et al. reported that fluid transportation fuels made via Fischer–Tropsch synthesis from biomass-derived syngas provide an appealing, unpolluted, carbon-neutral, and long-term dynamism foundation. These applications make use of microfluidic systems’ typical characteristics, such as fluid manipulation precision. Furthermore, Ma et al. indicated that materials with rheology that can be changed constantly, swiftly, and reversibly can benefit a variety of applications. Fuentes and Gracia presented a potential application in power-to-gas technology using special types of fluid. Lastly, the technoeconomic gain is calculated using energy storage capacity and material prices. The study’s findings show that a one-step synthesis protocol for fluids linking the in situ generation of nanoparticles from low-cost extracts is a practical and operation-worthy methodology in manufacturing applications for increasing energy storing capacity and power rating while also extending the lifetime of tools. Gnanadurai et al. looked at how silver nanoparticles affected the use of a coconut oil-based cutting fluid in a negligible fluid presentation.

In physics, a Carreau fluid flow (CFF) is a form of generalized Newtonian fluid whose viscosity is proportional to the shear rate. Many studies and investigations are performed on this class of fluids. Khan et al. presented a detailed analysis of the theoretical and mathematical studies of non-Newtonian liquid movement over a heated rough rotating surface employing CFF, which is commonly used for pseudoplasticity and dilatant fluids. They determined, for the first time, the base flow profile under partial slip near a rough surface. Ahmad et al. employed CFF to analyze the blood flow. They demonstrated how to utilize the CFF to analyze blood flow through a stenosed artery. Under the assumption of minor stenosis, the flow-governing equations are calculated. The blood was analyzed mathematically using its non-Newtonian characteristics. Following that, the regular perturbation technique was used to analyze the analytical solution. This perturbation yields dimensionless Weissenberg number explanations up to the second order. Bhatti et al. examined the HT effects via CFF. They used a pair of rectangular plates to study heat transmission on the electro-magnetohydrodynamic CFF. Physical modeling is done using the Darcy–Brinkman–Forchheimer medium. The Lorentz force is induced by the existence of the extrinsic forced magnetic and electric fields, which causes the flow to occur. Hayat et al. introduced the entropy generation analysis of CFF. Maqbool et al. tested HT upon MHD CFF in a two-dimensional (2D) channel, while Riaz and Abbas did it in a three-dimensional (3D) channel.

Alsemiry et al. used the perturbation approach with appropriate small parameters to study the influence of a catheter on blood current and temperature transmission appearances of a CFF. Actually, this research is about how a catheter is inserted eccentrically into an artery during surgery. The model depicts the flow of fluid between eccentric tubes, with the inner tube representing a moving catheter and the outer tube indicating an artery with overlapping stenosis. Reedy et al. reported on unexpected characteristics of entropy formation in fully evolved CFF heat transport in a porous vertical microchannel. The thermal energy equation takes into account the possessions of current radiation and viscous heating.

A heat source model (HSM) has many modeling systems depending on its applications. Pyo et al. used welding HSM through a high degree of freedom. Malinari and Bokes studied the effect of an HSM on transient methods of diffusivity measurement. Aslam and Sahoo investigated a moving HSM for one objective deposited slight strengthen repair drip. For gas metal arc welding, Bjieli et al. developed calibration of a three-dimensional quasi-stationary heat transfer prototype. The five input parameters of the double-ellipsoid heat source employed in this model cannot be precisely determined. Therefore, they utilized the pareto search optimization technique to assess these values as part of a multiobjective calibration approach with two objective functions. Giudice et al. utilized HSM for heat ground forecast in laser beam joining. Kitano and Mikami constructed HSM for HT by employing shallow convolutional neural networks. More sensors and actuators were employed by Ragab et al. in active noise and vibration control. As a result, a fixed one-dimensional homogeneous thermoelastic piezoelectric rod exposed to a moving heat flow has been explored. Three fields, namely, thermal, mechanical, and electric potential, have an effect on the heating rod. For keyhole-type laser welding, Lorin et al. developed a novel HSM. HSM is typically a trial-and-error procedure, with the efficiency and accuracy of the process being heavily influenced by the researcher’s experience. Hongwei et al. proposed an optimization approach for estimating HSMs during welding that may be utilized to estimate the form parameters of different heat sources. Experimental data was successfully matched by finite-element simulations using the optimized parameters. The findings showed that utilizing this optimization strategy minimizes the cost of repetitive modeling to acquire the best HSM, as well as the impact of the researchers’ involvement on the productivity and exactness of welding heat models. Ab Aziz and Kasmani contributed to the study of radiant heat flow and impingement, which are the most common contributors to increasing the
severity of jet fire accidents. They looked at both free and impinging jet releases in their experiments.

Thermal radiation (TR) is a progression in which energy is unrestricted in the entire instructions by a heated shallow medium in the method of electromagnetic radiation (EMR) and actions straightforward to its location of preoccupation at the speed of light; it does not require an intermediate medium for its transference. Oliveira et al.\textsuperscript{40} developed a more appropriate methodology that employs HSM, such as radioactive heat transfer directional—spectral relations, to estimate the cutting process temperatures during metal face milling using the entire electrical response of commercial infrared cameras as an input. Kumar et al.\textsuperscript{41} studied the effect of TR on HT over an implausibly started perpendicular platter. Sudarsana and Sreedevi\textsuperscript{42} suggested entropy generation and HT with TR. Mesgarpour et al.\textsuperscript{43,44} considered TR in view of numerical optimization. They claimed that the optimization procedure might boost Nu while lowering calculation costs. This optimization methodology is 34 times faster than other optimization processes. The references\textsuperscript{45–54} include new additions that consider conventional and nanofluids with heat and mass transmission in a variety of physical circumstances.

The aim of this study is to evaluate the consequence of an irregular heat source/sink on the fluid flow and heat transfer of a Carreau fluid flow (CFF) toward a nonlinear stretching cylinder, which was inspired by a literature review. Non-Newtonian fluids, particularly a Carreau fluid passing via a nonlinear stretching cylinder, have received less attention in the literature. When the effects of irregular heat source/sink, thermal-dependent current conductivity, and thermal radiation are included, the studies are even fewer. To the best of our knowledge, the combined effort of studying heat transfer aspects of Carreau fluid flow (CFF) through a nonlinear stretching cylinder in the presence of radiation, heat-dependent current conductivity, and a nonuniform heat source/sink has not been made in the literature. This motivated us to work toward it. In this, the shooting method is employed to resolve the governing equations. Using the similarity transformation technique, partial differential governing equations are altered into nonlinear ordinary differential equations. The velocity and temperature variations in the fluid region are plotted and examined as a function of the various values of the relevant parameters. The behavior of physical limitations such as local skin friction and local Nusselt number is also explained and shown in a tabular format.

2. MATHEMATICAL FORMULATION

A steady 2D axisymmetric flow of an incompressible Carreau fluid along a nonlinear stretching cylinder with radius R is considered. As depicted in Figure 1, the r-axis is measured along the radial direction and the θ-axis is taken in the axial direction. The cylinder is assumed to be stretched by a nonlinear velocity \( U_s \), \( T_w \) is the constant temperature at the surface of the cylinder, and the temperature of the ambient fluid is fixed at \( T_w \). Thermal radiation and a nonuniform heat source/sink are presumed to act on the fluid flow. The governing boundary layer equations of the Carreau fluid flow and heat transfer over the nonlinear stretching cylinder are represented as follows

\[
(\nu u)_r + (\nu v)_r = 0
\]  

\[
(\nu u)_r + (\nu v)_r = u \left( \frac{1}{r} \right) + \frac{1}{r} \frac{3l^2(n-1)}{2} (u_r)^2 u_r + \frac{1}{2r} \left( \frac{1}{n-1} \right) (u_r)^3 \right)
\]

\[
u T_u + v T_v = \frac{1}{\rho c_p} \frac{1}{r} \frac{\partial}{\partial r} \left( r \alpha(T) T_v \right) - \frac{1}{\rho c_p} \left( q_s \right) + \frac{Q^*}{\rho c_p}
\]

where \( u = U_s, v = 0, T = T_w, r = R, u \to 0, T \to T_{w}, \) as \( r \to \infty \)

\[
U_s = U_0 \left( \frac{r}{R} \right)^m
\]

\[
\left( q_s \right) = \frac{-16\sigma^* T^4}{3k^*} T_{w}^3
\]

\[
Q^* = \frac{\alpha(T) U_w}{x_o} \left[ A(T_w - T_{w}) + B(T - T_{w}) \right]
\]
\[ uT_u + vT_v = \frac{1}{\rho_c \sigma} \frac{\partial}{\partial r}(r\alpha(T)T_r) - \frac{1}{\rho_c} \frac{\partial}{\partial r} \left( \frac{16\sigma}{3k}\right) T_r^3 \\
+ \frac{1}{\rho_c \sigma} \left( \frac{\alpha(T)U_u}{x_0} \right) A(T_r - T_\infty) f'(\eta) \\
+ B(T - T_\infty) \]  

(7)

The following similarity transformations are used to alter eqs 1, 2, and 7 into ordinary differential equations.

\[ \eta = \sqrt{\frac{U_p^{m-1}}{blm}} \left( \frac{r^2 - R^2}{2R} \right), \quad \theta = \frac{T - T_r}{T_r - T_\infty}, \psi = \frac{U_{r0}}{\Omega} Rf(\eta) \]  

(8)

Utilizing the similarity variables, eq 1 is automatically satisfied and eqs 2 and 7 are reduced to

\[ \frac{3(n - 1)}{2} \dot{S}^2 (1 + 2\eta)(\dot{f}'' + (1 + 2\eta)f''') + 2f'' \\
+ (1 + 2\eta)f'' + \frac{n - 1}{2} \dot{S}^2 \alpha(1 + 2\eta)(f'')^3 \\
+ \left( \frac{m + 1}{2} \right)f' - m(f')^2 \\
= 0 \]  

(9)\]

and boundary constraints eq 4 becomes

\[ f(\eta) = 0; \theta(\eta) = 1; f'(\eta) = 1, \text{ at } \eta = 0, f'(\eta) \rightarrow 0, \theta(\eta) \rightarrow 0, \text{ as } \eta \rightarrow \infty \]  

(11)

where the prime indicates differentiation concerning \( \eta \), Pr is the Prandtl number, Rd is the radiation parameter, \( \omega \) is the Weissenberg number, and \( \lambda \) is the curvature parameter. These parameters are defined as

\[ \lambda = \frac{1}{R} \sqrt{\frac{blm}{U_0^{m-1}}} \], \( \omega = \frac{\rho c_p}{\alpha_{\infty}} \), \( Rd = \frac{4\sigma}{k^* \alpha_{\infty}} \)  

(12)

The interesting physical properties of the skin friction coefficient \( C_f \) and the Nusselt number \( \frac{Nu}{Re_{\infty}^{1/2}} \) are represented as

\[ C_f = f''(0) + \frac{n - 1}{2} \dot{S}^2 (f'(0))^3 \]  

(13)

\[ \frac{Nu}{Re_{\infty}^{1/2}} = -\theta'(0) \left( 1 + \frac{4}{3} Rd \right) \]  

(14)

where \( x^m = x^{m+1-\frac{U_0}{blm}} \) is the Reynolds number.

3. NUMERICAL SOLUTIONS

Because eqs 9 and 10 are higher-order nonlinear ODEs, it is very hard to obtain an analytic solution; hence, a good numeric approach with excellent exactness of convergence for the present system must be chosen (Figure 2). Numerical results are found using a shooting approach.

![Flow chart of the shooting technique.](https://doi.org/10.1021/acsomega.2c02207)

To do this, frequently used ODEs were converted to first ODEs eqs 9 and 10, using the following changes.

\[ f = f(1), \quad f' = f(2), \quad f'' = f(3), \quad \theta = f(4), \quad \theta' = f(5) \]

\[ f' = f(2), \quad f'' = f(3) \]

\[ \frac{df}{d\eta}(3) = -\left\{ \left( \frac{3(n - 1)}{2} \right) \dot{S}^2 (1 + 2\eta)f'(3)^3 + 2Af'(3) \right. \\
\left. + (1 + 2\eta)f'(3) + \frac{n - 1}{2} \dot{S}^2 \alpha(1 + 2\eta)f'(3)^3 \\
+ \left( \frac{m + 1}{2} \right)f'(3) - \left( m(f')^2 \right) \right\} \\
\left( \frac{3(n - 1)}{2} \right) \dot{S}^2 (1 + 2\eta)^2 + (1 + 2\eta) \]  

\[ \theta' = f(5), \]

\[ \frac{d}{d\eta} \theta(5) = \left\{ 2\lambda + \left( \frac{PrPr(m + 1)f(1)}{2} \right) + 2\lambda f(4) \right. \\
\left. + \left( \frac{4\lambda Rd}{3} \right) f(5) + (1 + 2\eta) f(5)^2 e + (1 + f(4)) \times \left( Af(2) + Bf(4) \right) \right\} \left( \frac{3(n - 1)}{2} \right) \dot{S}^2 (1 + 2\eta)^2 + (1 + 2\eta) \]  

The changed boundary conditions are

\[ f_\alpha(1) = 0, \quad f_\alpha(2) = 1, \quad f_\alpha(4) = 1 \]  

\[ f_\beta(2) \rightarrow 1, \quad f_\beta(4) \rightarrow 0, \quad \text{as } \eta \rightarrow \infty \]

In this scenario, the presence of a continuous result stimulates the grid choice and error function. The margin remains fixed at
The value of $\eta \to \infty$ indicates that each number response reaches asymptotic properties perfectly under this strategy.

### 4. CODE VALIDATION

To validate the numerical scheme engaged in this work, a comparative study of the existing similar literature (Rangi and Naseem, Poply et al., Hashim et al., and Gangadhar et al.) has been done for the values $C_f Re_x^{1/2}$ for various values of $\lambda$. The outcomes of the above comparative study listed in Table 1 exhibit excellent agreement up to the level of $10^{-5}$. This error tolerance level of $10^{-5}$ was maintained for all parametrical calculations, and the CPU time was also similar irrespective of variations in physical constraints. This induces the confidence to proceed with this numerical scheme. Calculations with $\eta_\infty = 10$ take up CPU time on an Intel Core i3 processor.

### 5. RESULTS AND DISCUSSION

The results of the above-mentioned system were projected in the form of graphical displays. The parametrical behaviors of key factors of these studies like fluid flow, thermal dispersion, frictional effects, and thermal transference were explored.

Figure 3 discloses a retarding state of the fluid flow across the system influenced by the stretching index strictures ($m$). Nonlinear shear changes induced by the extending cylinder ($m$) restrict the axisymmetric fluidity of the Newtonian Carreau fluid to pass over it.

The thermal spreading controlled by the stretching index strictures ($m$) athwart the domain is revealed in Figure 4. The flow motion plays a vital role in thermal dispersal in the system. Since the fluidity is stopped for improving the stretching index, the thermal aspects also become slower than before. It is reflected in the declining trend in the thermal dissemination for higher stretching index strictures ($m$). Figures 5 and 6 unveil an altering capability of the power-law index ($n$) over the flow and thermal behaviors of the system. Although the impact looks graphically nominal, it possesses the nature of vital viscous alterations. Particularly, when used for such Newtonian Carreau

| $\lambda$ | $C_f Re_x^{1/2}$ |
|----------|------------------|
| 0        | 1.0000           |
| 0.1      | 1.036977         |
| 0.25     | 1.094378         |
| 0.3      | 1.111138         |
| 0.5      | 1.188715         |
| 0.7      | 1.257045         |
| 0.75     | 1.281833         |
| 1        | 1.459308         |
| $n$ = 0  | 1.000000         |
| $n$ = 0.2| 1.094373         |
| $n$ = 1  | 1.094386         |
| $n$ = 1.2| 1.094368         |
| $m$ = 0.1| 1.036979         |
| $m$ = 0.2| 1.111165         |
| $m$ = 0.3| 1.188727         |
| $m$ = 0.4| 1.257013         |
| $B$ = 0.3| 1.281819         |
| $B$ = 0.6| 1.281803         |
| $B$ = 1  | 1.453373         |
Fluids, the power-law index gets more priority. In view of flow motion (Figure 5), owing to the shear-thinning aspects, the fluidity gets better with the increasing value of the power-law index ($n$). Simultaneously, in Figure 6, the decreasing trend in thermal dispersal of the domain can be observed with the increasing power-law index ($n$). Accumulative power-law index ($n$) causes the thinning of the shear layer, resulting in the fluid passing more quickly than before, which in turn reduces the chance of absorbing more heat from the surface, which reflects in the diminution of thermal distribution. Enhanced values of the Weissenberg number ($We$) physically tend to elevate the viscoelastic nature of the Carreau fluid, which prevents it from flowing freely as before, as can be visualized in Figure 7.

The increasing trend of the thermal dispersion in Figure 8 is persuaded by improving the Weissenberg number ($We$), which exerts higher resistivity to the flow movement, which assists the thermal transference process and provides more time for heat transfer into the fluid.
Figure 9 depicts the significant role of the curvature ($\lambda$) of elongating cylinder in the fluidity of the system. The curvature of the cylinder is inversely correlated with its radius; therefore, the curvature tends to increase with the decrease in the cylinder radius, favoring better fluid movement in the domain, which can be noted through the increasing trend.

Interestingly, enhancement in the thermal dissemination can be spotted in Figure 10 for higher values of curvature ($\lambda$). The key aspect of this phenomenon is again the size of the cylinder. A smaller radius makes it easier for the flowing fluid to pass over it, which can absorb more heat from it and showcase such an escalation in thermal boundary layers. A classical comparison of the thermal state with the Prandtl variations ($Pr$) was performed under this specific environment. Figure 11 presents the fact that higher ($Pr$) reduces the thermal diffusivity, which makes the thermal dispersal process harder and exhibits the conventional retarding trend in the dispersion of domain temperature. Figure 12 unfolds the augmenting thermal dispersion for the thermal conductivity increment constraint ($\varepsilon$) of the flowing Carreau fluid. Improved thermal conductivity for a higher $\varepsilon$ tends to elevate the thermal distribution in the domain. Thermal radiation ($Rd$) plays a vital role in the heat dispersal in the domain. Increased heat from the distinct source makes the system hotter. To drive away that additive heat, the flowing fluid with effective thermophysical properties has to work hard, as reflected in Figure 13, which shows the escalation in thermal distribution for higher radiation values. Figures 14 and 15 respectively illustrate the state of thermal dispersal for both space-dependent ($A$) and temperature-dependent ($B$) heat source parameters. Additive heat was introduced into the system through such thermal sources. While both the parameters ($A$) and ($B$) tend to regulate such heat, as the system gets more heated, the major part of that heat is transferred to the fluid, thus boosting thermal dispersion. Comparatively, the space-dependent heat source variations were looks ahead of thermal-dependent constraints.

Figures 16–18 exemplify the Nusselt number impact with the thermal radiation ($Rd$) and both space-dependent ($A$) and temperature-dependent ($B$) heat source parameters and thermal
dispersion ($\varepsilon$). All such constraints work in favor of the heat transmission process. On the other hand, increasing the parameters $A$, $B$, and $\varepsilon$ results in a decrease in the Nusselt number as they act against it.

Table 2 displays the key aspects of studies done on frictional coefficient and thermal transference rates. Especially, for problems involving Carreau fluids, which are Newtonian in nature and also have heat transfer ability, both the above-mentioned constraints are crucial. As the values of the Weissenberg number ($\text{We}$) and power-law index ($n$) increase, the value of the frictional coefficient increases whereas that of the stretching index ($m$) is reduced. This shows that the friction factor becomes dominant as the cylinder stretches further. On the other hand, as the viscous nature of the fluid reduces with improving Weissenberg number ($\text{We}$) and fluidity power-law index ($n$), the frictional influence becomes nominal. In view of thermal transference, the Weissenberg number ($\text{We}$) stands against it, whereas both stretching index ($m$) and power-law index ($n$) assist the process by aiding the fluidity to drive more heat than before.

The influence of the Prandtl number ($\text{Pr}$), thermal radiation ($R_d$), and space-dependent ($A$) and temperature-dependent ($B$) dependent heat source parameters over the thermal transference rate is tabulated in Table 3. Both the Prandtl variation and increase in heat from a distinct source do not affect the heat transfer process, which is evident through the results of the parametric study with the increasing trends found in Table 3. Conversely, the heat source affects the thermal transference rate in both space- and thermal-dependent aspects, which can be observed from the declining values of thermal transfer rate for those parameters.

**6. CONCLUSIONS**

Considering 2D, steady axisymmetric flow of an incompressible Carreau fluid along a cylinder with radius $R$ which is stretching nonlinearly, parametrical studies were performed for the crucial parameters. The numerical solution to transform governing equations by the similarity variable was obtained using the
Table 2. Values of \( f^*(0) + \left( \frac{n-1}{2} \right) We^2 (f^*(0))^3 \) and \(-\theta(0)\left( 1 + \frac{2}{3}Rd \right)\) with Various Values of \( m, We, n \) and \( n \) and Other Parameters That Are Fixed, i.e., \( \lambda = 0.2, m = 0.2, \epsilon = 0.2, Rd = 1, Pr = 6.8, A = 0.3, B = 0.3 \)

| \( m \) | \( We \) | \( n \) | \( f^*(0) + \left( \frac{n-1}{2} \right) We^2 (f^*(0))^3 \) | \(-\theta(0)\left( 1 + \frac{2}{3}Rd \right)\) | CPU time (sec) |
|---|---|---|---|---|---|
| 0.1 | 0.1 | 0.2 | -0.596090 | 1.388809 | 1.006499 |
| 0.5 | | | -0.843136 | 1.744996 | 1.013205 |
| 1.0 | | | -1.072648 | 2.114902 | 1.007747 |
| 0.1 | 0.3 | 0.2 | -0.597664 | 1.386978 | 1.017418 |
| 0.5 | | | -0.588925 | 1.383087 | 1.011327 |
| 1.0 | | | -0.560993 | 1.358253 | 1.007588 |
| 0.3 | | | -0.596126 | 1.388837 | 1.008981 |
| 0.5 | | | -0.596197 | 1.388892 | 1.029734 |

Table 3. Values of \(-\theta(0)\left( 1 + \frac{2}{3}Rd \right)\) with Various Values of \( Pr, Rd, A, B \) and the Other Parameters That Are Fixed, i.e., \( n = 0.2, We = 0.1, \lambda = 0.2, m = 0.1, \epsilon = 0.2 \)

| \( Pr \) | \( Rd \) | \( A \) | \( B \) | \(-\theta(0)\left( 1 + \frac{2}{3}Rd \right)\) | CPU time (sec) |
|---|---|---|---|---|---|
| 3 | 1 | 0.3 | 0.3 | 0.264584 | 1.010932 |
| 6.8 | | | | 1.388809 | 1.015686 |
| 7 | 2 | | | 1.430511 | 1.007600 |
| 3 | 1 | | | 1.603729 | 1.019074 |
| 3 | 3 | | | 1.752820 | 1.017067 |
| 4 | 1 | | | 1.893894 | 1.003206 |
| 0 | 0.2 | | | 1.500242 | 1.008116 |
| | 0 | | | 1.721061 | 1.004597 |
| 0.3 | 0.2 | | | 1.473194 | 1.005410 |
| 0 | | | | 1.629949 | 1.012281 |

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NOMENCLATURE

\( \epsilon_p \) specific heat (Jkg\(^{-1}\) K\(^{-1}\))
\( C_f \) skin friction
\( c_1, c_2, \ldots, c_7 \) arbitrary constants
\( f \) nondimensional stream function
\( f_0 \) initial guess of nondimensional stream function
solution of m-order of nondimensional stream function

\( f_m \)

porosity parameter

\( K \)

Weissenberg number

\( m \)

stretching index

\( n \)

power-law index

\( \frac{m}{Nu} \)

return flow number

\( T_w \)

temperature of the wall (K)

\( p \)

embedding parameter

\( A, B \)

nonuniform heat source/sink parameter

\( q_r \)

thermal radiation (Wm\(^{-2}\))

\( R_d \)

radiation parameter

\( R \)

radius of the cylinder (m)

\( U_0 \)

reference velocity (ms\(^{-1}\))

\( Re_r \)

local Reynolds number

\( T_\infty \)

ambient temperature (K)

\( (u,v) \)

fluid velocity in the \((x,y)\)-axis direction (ms\(^{-1}\))

\( T \)

temperature (K)

\( Pr \)

Prandtl number

Greek symbols

\( \alpha_\infty \)

thermal conductivity away from the cylinder (Wm\(^{-1}\)K\(^{-1}\))

\( \eta \)

similarity variable

\( \alpha(T) \)

temperature-dependent thermal conductivity (Wm\(^{-1}\)K\(^{-1}\))

\( \theta_0 \)

initial guess of dimensionless temperature

\( \gamma \)

shear rate (s\(^{-1}\))

\( \beta_\psi \)

volumetric coefficient of thermal expansion (K\(^{-1}\))

\( \theta \)

dimensionless temperature

\( \theta_m \)

solution of m-order of nondimensional temperature

\( \varepsilon \)

small perturbation number

\( \Gamma \)

relaxation time (s)

\( \rho \)

density (kgm\(^{-3}\))

\( \sigma^* \)

Stefan–Boltzmann constant (Wm\(^{-2}\)K\(^{-4}\))

\( \psi \)

kinematic viscosity (m\(^2\)s\(^{-1}\))

\( \psi \)

stream function (m\(^2\)s\(^{-1}\))

\( \theta_0 \)

nonzero control convergence parameters for heat equation

\( \lambda \)

curvature parameter

\( \theta_0 \)

nonzero control convergence parameters for momentum equation

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