D-branes in a plane wave from covariant open strings

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ABSTRACT: We derive boundary conditions for the covariant open string corresponding to D-branes in an Hpp-wave, by requiring kappa symmetry of its bulk action. Both half-supersymmetric and quarter-supersymmetric branes are seen to arise in this way, and the analysis furthermore agrees fully with the existing probe brane and supergravity computations. We elaborate on the origin of dynamical and kinematical supersymmetries from the covariant point of view. In particular we focus on the D-string which only preserves half of the dynamical supersymmetries and none of the kinematical ones. We discuss its origin in AdS$_5 \times S^5$ and its world-volume spectrum.

KEYWORDS: D-branes, pp-waves
1. Introduction

Several recent papers have investigated D-branes in an Hpp-wave (a homogeneous plane-wave solution of the type-IIB supergravity solution with a constant five-form flux). A classification of their embeddings and supersymmetries has been given by Skenderis and Taylor using a probe brane approach \cite{SkenderisTaylor}, while equivalent results have also been obtained by Bain et al. by deriving supergravity solutions of D-branes in Hpp-waves \cite{Bain}. In the present letter we would like to show that the analysis of boundary conditions for the covariant open string leads to the same classification. Apart from verifying the existing classification in an independent way, this also sheds light on some recent questions concerning the quantum analysis of these branes. In particular, we clarify the origin of dynamical and kinematical supersymmetries
from the covariant point of view\textsuperscript{1}. We restrict ourselves throughout to so-called longitudinal branes, i.e. branes for which the \(x^+\) and \(x^-\) directions are part of the world-volume.

The Hpp-wave admits three classes of longitudinal branes without world-volume fluxes. The first class consists of half-BPS branes, which have straight embeddings in Rosen coordinates (they “follow geodesics of point-particles”) and have world-volume dimensions \(p = 3, 5, 7\). The spectrum of these branes has been analysed by Dabholkar and Parvizi \textsuperscript{2} using the quadratic Green-Schwarz action in the light-cone gauge as derived by Metsaev \textsuperscript{3}. The analysis of Bergman et al. \textsuperscript{5} has shown these boundary conditions to be consistent with open/closed string duality. The second class of branes is obtained by moving these branes away from the origin, or formulated more precisely, by embedding them along straight coordinate lines in the Brinkman coordinate system. This breaks the dynamical supersymmetries, so that only the kinematical supersymmetries remain. They have been argued to be inconsistent with open/closed string duality \textsuperscript{5}. We will derive the corresponding open string boundary conditions for these two classes of D-branes directly from kappa symmetry requirements.

The third, and most interesting, class of branes consists of a single one, namely a D-string which preserves half of the dynamical and none of the kinematical supersymmetries. While it is at present not known whether the presence of only dynamical supersymmetries is enough for quantum consistency (the corresponding boundary state has not yet been constructed), there are several hints that suggest that this object should be taken seriously. Firstly, it has been found as a fully localised solution in supergravity \textsuperscript{2} (in contrast, the seemingly inconsistent branes mentioned in the previous paragraph only exist as smeared supergravity solutions). Secondly, we will show that the D-string can be traced back to an unstable object in the AdS\(_5 \times S^5\) geometry, and argued to become stable when the Penrose limit is taken. An important consequence of the existence of the D-string is that by a construction as in \textsuperscript{5}, it would imply the existence of a consistent D-instanton in the Hpp-wave.\textsuperscript{2}

We will start in the next section with a derivation of the D-brane boundary conditions from open string kappa symmetry. After establishing which supersymmetries and kappa symmetries are preserved by the various branes listed above in the covariant set-up, we explicitly construct their realisation on the physical states in the light-cone gauge in section \textsuperscript{4}. We show that there is a one-to-one relation between the branes we find and those obtained with other techniques, and moreover argue that all of them, including the D-string, can be seen in the light-cone gauge. The AdS origin of the D-string and its spectrum (in the Hpp-wave) is discussed in the last section.

\textsuperscript{1}Dynamical supersymmetries are those supersymmetries which commute with the Hamiltonian, while kinematical supersymmetries are those which do not.

\textsuperscript{2}We thank Matthias Gaberdiel and Michael Green for discussions about this issue.
2. D-brane boundary conditions from kappa symmetry

2.1 Generalities

The Green-Schwarz action is invariant under local $\kappa$-symmetry, which ensures that half of the fermionic degrees of freedom can be gauged to zero such that the resulting spectrum has the expected supersymmetry. For closed strings, the constraint of $\kappa$-symmetry ensures that the background fields satisfy their equations of motion. For open strings, $\kappa$-symmetry transformations result in boundary terms, which do not vanish without further constraints on the world-volume fields at the boundary. For a Minkowski background, these boundary terms have been examined by Lambert and West [6] and were shown to vanish when the standard boundary conditions for half-BPS branes are imposed (a similar type of analysis for membranes in eleven dimensions has been given by Ezawa et al. [7] and de Wit et al. [8]). In the present section we will extend the analysis to cover open Green-Schwarz strings in an Hpp-wave background.

Most of this calculation is rather technical, so we present here only the variations of the action and the resulting boundary conditions. At up to fourth order in the fermions, these boundary terms arise from variation of (A.8) and (A.9), labeled as $S_{1}^{WZ}$ and $S_{2}^{WZ}$ respectively. Full details can be found in the appendix.

2.2 Branes “at the origin”

Let us first discuss branes at the origin of the coordinate system. The terms in the kappa variation of the action (see equation (A.8) in the appendix) that survive in the flat space limit have appeared in the literature before [3]; they are given by

$$\delta_{\kappa}S_{1}^{WZ} \rightarrow \int_{\partial \Sigma} \left[ i(\bar{\theta}_{1}\Gamma_{r}\delta_{\kappa}\theta_{1} - \bar{\theta}_{2}\Gamma_{r}\delta_{\kappa}\theta_{2}) dX^{\mu}e_{\mu}^{r} + (\bar{\theta}_{1}\Gamma_{r}\delta_{\kappa}\bar{\theta}_{1}\bar{\theta}_{1}\Gamma^{r}d\theta_{1} - \bar{\theta}_{2}\Gamma_{r}\delta_{\kappa}\bar{\theta}_{2}\bar{\theta}_{2}\Gamma^{r}d\theta_{2}) \right].$$

(2.1)

Here we used kappa symmetry to express the variations of the bosons in terms of the variations of the fermions using (A.11). The terms above vanish by imposing the usual half-BPS boundary conditions,

$$\theta_{1} = P\theta_{2}, \quad \bar{\theta}_{1} = \bar{\theta}_{2}P\left(-\frac{1}{2(p-2)}\right)^{p-1}p$$

(2.2)

with

$$P = \Gamma^{+1\cdots(p-1)}, \quad P^{2} = \left(-\frac{1}{2}\right)^{(p-1)(p-2)}.$$  

(2.3)

3Since the geometry is homogeneous, the coordinate origin is equivalent to any other point in the space; branes sitting at the origin can of course be located at any arbitrary other location, but this is manifest only in Rosen coordinates. For simplicity we will keep referring to these branes as “branes at the origin” when we really mean “branes which are flat in Rosen coordinates”.
Since the background five-form breaks the SO(8) symmetry to SO(4)×SO(4), we will label these gluing matrices \( P \) by two numbers \( n \) and \( m \), denoting the number of gamma matrices with indices in the first and second four transverse coordinates. The operators \( P^{(n,m)} \) satisfy the relations

\[
P^{(n,m)} I = IP^{(n,m)} \ (-)^n , \quad P^{(n,m)} \Gamma^r = \begin{cases} \Gamma^r P^{(n,m)} (-)^{p+1} & \text{when } r \in D, \\ \Gamma^r P^{(n,m)} (-)^p & \text{when } r \in N, \end{cases}
\]

(2.4)

(where \( N = \{ +, -, 1, \ldots, p - 1 \} \) and \( D = \{ p, \ldots, d - 2 \} \) and the operator \( I \) is defined in (A.5)). One can then derive that

\[
(\bar{\theta}_1 \Gamma_r \delta_s \theta_2) = \begin{cases} -(\bar{\theta}_2 \Gamma_r \delta_s \theta_2) & \text{when } r \in D, \\ + (\bar{\theta}_2 \Gamma_r \delta_s \theta_2) & \text{when } r \in N. \end{cases}
\]

(2.5)

These relations clearly make (2.1) vanish.

The new terms with respect to flat space arise from the five-form coupling in the Wess-Zumino term. We find that they lead to the boundary terms

\[
\delta_\kappa S_{WZ}^1 \bigg|_{\phi^2} \rightarrow 2 \mu \int_{\partial \Sigma} \left[ (\bar{\theta}_1 \Gamma_r \delta_s \theta_1 + \bar{\theta}_2 \Gamma_r \delta_s \theta_2) \right. \\
\left. \times (\bar{\theta}_1 \Gamma^r \Gamma^{-1} \Gamma^s \theta_2 + \bar{\theta}_2 \Gamma^r \Gamma^{-1} \Gamma^s \theta_1) \right] dX^\nu e_{\nu s},
\]

(2.6)

\[
\delta_\kappa S_{WZ}^1 \bigg|_{\phi^4} \rightarrow \frac{1}{2} \mu \int_{\partial \Sigma} \left[ (\bar{\theta}_1 \Gamma_r \delta_s \theta_1) (\bar{\theta}_2 \Gamma^r \Gamma^s \theta_1) + (1 \leftrightarrow 2) \right] dX^\nu e_{\nu s}.
\]

Note that these terms do not cancel against each other using only the half-BPS conditions (2.2) and the flip formula (2.3). Some of the terms that arise by varying \( S_{WZ}^2 \) in (A.4) are of a similar type, but they do not come with the right coefficient to cancel against the variation of \( S_{WZ}^1 \). Therefore, we need to cancel all of these contributions separately by imposing appropriate additional boundary conditions. Besides the relation (2.3) we now also need the new flip relation

\[
(\bar{\theta}_1 \Gamma^r \Gamma^{+1} \Gamma^a \theta_2) = \begin{cases} (-)^{(p-1)(p-2)} \frac{1}{2} + n + 1 (\bar{\theta}_2 \Gamma^r \Gamma^{+1} \Gamma^a \theta_1) & \text{when } r \in D, s \in N, \\ (-)^{(p-1)(p-2)} \frac{1}{2} + n (\bar{\theta}_2 \Gamma^r \Gamma^{+1} \Gamma^a \theta_1) & \text{when } r \in N, s \in N. \end{cases}
\]

(2.7)

The variations in (2.6) are then seen to cancel (separately for each line) when

\[
\frac{1}{2} (p - 1)(p - 2) + n = \text{odd}.
\]

(2.8)

This condition also makes the boundary terms arising from \( \delta_\kappa S_{WZ}^2 \) vanish. We see that, unlike in Minkowski space, kappa symmetry is preserved in the Hpp-wave only for particular orientations of the D-branes in the wave (given by \( n \) and \( p = m + n + 1 \)). Equation (2.8) holds true for the (2,0), (3,1) and (4,2) embeddings, which are the
half-BPS branes with \( p = 3, 5 \) and \( 7 \) respectively. These results are in agreement with the analysis of probe branes [1] and the light-cone open string analysis of [3]. In contrast, these boundary terms do not vanish for the \((0,0)\) embedding, which is the D-string.

For this \((0,0)\) embedding, the first line in (2.6) vanishes without further conditions because both \( r \) and \( s \) have to be Neumann directions, i.e. either plus or minus directions. However, canceling the second line leads to one of the constraints

\[
\Gamma^{- \theta_{1,2}} \bigg|_{\partial \Sigma} = 0 \quad \text{or} \quad \Gamma^{+ \theta_{1,2}} \bigg|_{\partial \Sigma} = 0 .
\]  

(2.9)

The latter leads, as expected and as we will see below, to no further problems when going to the light-cone gauge. The former condition is a bit special, as it shows that the \((0,0)\) D-string can be described in an alternative way. This condition seems to be sufficient to make all boundary terms at higher orders in theta vanish as well (see the appendix). It is, however, incompatible with the standard light-cone gauge.

With the above conditions, one can verify that the boundary terms arising from \( S_{WZ}^2 \) vanish as well; we will not discuss these in detail here.

2.3 Branes “outside the origin”

Let us now discuss the spin-connection dependent terms in the variation of the action. In contrast to the terms discussed above, these terms depend on the location of the end-point of the string in the target space. As we will see they automatically vanish for all D-branes sitting at the origin of the coordinate system. However, for branes outside the origin (and with flat embedding in Brinkman coordinates), their cancellation leads to additional boundary conditions. We find the following boundary terms:

\[
\delta_k S_{WZ}^1 \bigg|_{\theta^a} \to \int_{\partial \Sigma} \left[ (\bar{\theta}_1 \Gamma^n \delta_k \theta_1 + \bar{\theta}_2 \Gamma^n \delta_k \theta_2) \times (\bar{\theta}_1 \Gamma_n^{rs} \omega_{rs} \theta_1 - \bar{\theta}_2 \Gamma_n^{rs} \omega_{rs} \theta_2) \right] dX^\mu \epsilon^m_{\mu},
\]  

(2.10)

For the first line, we only have to consider the case where the \( m \) or \( n \) index on the gamma matrix in the second factor is not \(+\) or \(-\) (a lower minus sign on the gamma matrix is excluded because \( \omega_{++} \) is the only non-vanishing component of the spin connection, see (A.13); a lower plus sign on the gamma matrix would mean that both \( n \) and \( m \) are plus, and anti-symmetrisation then sets everything to zero). Using
the exchange property (2.5), the boundary terms then reduce to
\[ \delta_\kappa S_{WZ}^1 \rightarrow \mu \int_{\partial \Sigma} \left[ (\bar{\theta}_1 \Gamma^{+} \delta_\kappa \theta_1) dX^a - (\bar{\theta}_1 \Gamma^n \delta_\kappa \theta_1) dX^+ (\bar{\theta}_1 \Gamma^n + s' \theta_1 - \bar{\theta}_2 \Gamma^n + s' \theta_2) \partial_s S \right. \]
\[ \left. - \frac{1}{4} \left[ (\bar{\theta}_1 \Gamma^n \delta_\kappa \theta_1) (\bar{\theta}_2 \Gamma_n + s' \theta_2) - (1 \leftrightarrow 2) \right] dX^+ \partial_s S, \right. \]
where prime on the index \( s \) indicates that it no longer takes the values \(+\) or \(-\). We now need the exchange property
\[ (\bar{\theta}_1 \Gamma_{mn} \theta_1) = \begin{cases} 
(\bar{\theta}_2 \Gamma_{mn} \theta_2) & \text{when } (m,n) \in (D,D) \text{ or } (N,N), \\
-(\bar{\theta}_2 \Gamma_{mn} \theta_2) & \text{when } (m,n) \in (N,D) \text{ or } (D,N).
\end{cases} \]
This can be used to show that the above boundary terms (2.11) vanish only when the derivative on \( S \) is in a Neumann direction. When the derivative is in the Dirichlet direction, the above boundary terms are non-zero outside the origin. In this case we need to impose
\[ \Gamma^+ \theta_{1,2} \bigg|_{\partial \Sigma} = 0 \quad (2.13) \]
in order to make the boundary terms vanish. As in the previous section, these conditions also make the boundary terms arising from \( S_{WZ}^2 \) vanish. We will see the implications of these conditions for the remaining supersymmetry on physical states in the next section.

3. Kinematical and dynamical supersymmetries

3.1 Symmetries, constraints and gauge fixing

Having derived D-brane boundary conditions for the open string variables, we now want to analyse their consequences in terms of the remaining supersymmetries on physical states. In order to make contact with results obtained using probe branes or supergravity solutions, we first need to understand the relation between the supersymmetry parameters appearing in the transformation rules of the closed string action and those appearing in the supergravity transformations. After that, we need to investigate the open string and determine which subset of these symmetries preserves the boundary conditions. Finally, we fix the kappa gauge freedom and obtain the action of the global symmetries on the physical states of the open string with D-brane boundary conditions.

Let us start by discussing the global supersymmetry invariance of the covariant action, i.e. before fixing a \( \kappa \)-symmetry gauge (see also [3]). In general the action is
by construction invariant under the transformations\footnote{These transformation rules are only correct to lowest order in theta. We are suppressing higher-order theta contributions here as they later become irrelevant anyway when we go to the light-cone gauge.}

\[ \delta \theta_{1,2} = \epsilon_{1,2}, \quad \delta X^\mu = i \bar{\theta}_1 \Gamma^\mu \epsilon_1 + i \bar{\theta}_2 \Gamma^\mu \epsilon_2, \quad \epsilon = \epsilon_1 + i \epsilon_2, \tag{3.1} \]

for an arbitrary (not necessarily constant) target space spinor $\epsilon$ evaluated at the world-volume, together with a transformation of all the background fields using the supergravity transformation rules with parameter $-\epsilon$. This is simply a reflection of the fact that superspace diffeomorphisms are equivalent to component supersymmetry transformations, and does not really constitute a non-trivial symmetry. From this observation it follows, however, that for the special situation in which $\epsilon$ is a Killing spinor (evaluated at the world-sheet) and the background is bosonic, the string action is invariant under

\[ \delta \theta_{1,2} = \epsilon_{1,2}^{\text{Killing}}, \quad \delta X^\mu = i \bar{\theta}_1 \Gamma^\mu \epsilon_{1}^{\text{Killing}} + i \bar{\theta}_2 \Gamma^\mu \epsilon_{2}^{\text{Killing}} \tag{3.2} \]

by itself. This follows because the transformations of the background fields become trivial in this case. In other words, the action is invariant under \textit{global} supersymmetries generated as shifts, with parameters which are identical to the target-space Killing spinors. We will call these supersymmetries \textit{shift symmetries}. We will in general suppress the label “Killing” on the associated $\epsilon$ parameters. Note that the shift parameter $\epsilon$ has a \textit{fixed}, but generically \textit{non-constant} dependence on the string world-sheet coordinates.

Not all of the shift symmetries survive in the open string, as some of them may be incompatible with the boundary conditions on the $\theta_{1,2}$ fields. The remaining global supersymmetries are easily seen when acting on physical fields (i.e. in the light-cone gauge, in which the kappa symmetry is eliminated). Let us recall how to go to the semi-light-cone gauge given by the condition $\Gamma^+ \theta_{1,2} = 0$.\footnote{It is rather simple to show, as we will do below, that this gauge is compatible with the boundary conditions. Note, however, that the D-string can be obtained with an alternative set of boundary conditions, namely $\Gamma^- \theta_{1,2} \big|_{\partial \Sigma} = 0$; see (2.9). These would pose problems for the standard semi-light-cone gauge.} For this we need to be more specific about the form of kappa transformations. These act on fermions according to

\[ \delta_{\kappa} \theta_{1,2} = \Gamma_r \Pi^r_j \kappa^j_{1,2}, \tag{3.3} \]

where $\Pi^r_i = \partial_i X^M E_{M}^r$. The two kappa parameters are self-dual and anti self-dual respectively,

\[ \kappa_1^\tau = -\kappa_2^\sigma, \quad \kappa_2^\tau = \kappa_2^\sigma. \tag{3.4} \]

One then easily obtains the kappa symmetry transformation necessary to bring the action into the $\Gamma^+ \theta_{1,2}$ gauge:

\[ \kappa^\tau_{1,2} = -\frac{1}{2 \Pi^+_1 + \Pi^+_2} \Gamma^+ \theta_{1,2}. \tag{3.5} \]
In the presence of a D-brane boundary condition
\[ \Gamma^+ \theta_{1,2} \big|_{\partial \Sigma} = 0, \quad (3.6) \]
the story is slightly changed, as this constraint fully removes kappa symmetry at the boundary. However, even with this “reduced” kappa symmetry (with \( \kappa \to 0 \big|_{\partial \Sigma} \)) we can still globally go to semi-light-cone gauge, since all fermions on the world sheet are also constrained by (3.6). In contrast, the light cone gauge cannot be reached for the first D-string condition in (2.9), as it would lead to discontinuous fermionic fields on the world-volume.

Once in the \( \Gamma^+ \theta_{1,2} = 0 \) gauge, half of the shift supersymmetries \( (\Gamma^+ \Gamma^- \epsilon) \) are such that they keep the system in this gauge. The other half of the shift symmetries \( (\Gamma^- \Gamma^+ \epsilon) \) moves the system out of the light-cone gauge. However, it is always possible to perform a compensating kappa transformation, such that the gauge condition is restored. The remaining global symmetries are then given by
\[
\delta \theta_{1,2} = \frac{1}{2} \Gamma^+ \Gamma^- \epsilon_{1,2} + \frac{1}{2} \Gamma^- \Gamma^+ \epsilon_{1,2} - \frac{1}{2} \Pi^+_\tau + \Pi^-_{\sigma} \Gamma'_r (\Pi'_{\tau} \mp \Pi'_{\sigma}) \Gamma^+ \epsilon_{1,2}
\]
\[= \frac{1}{2} \Gamma^+ \Gamma^- \epsilon_{1,2} - \frac{1}{2} \Pi^+_\tau + \Pi^-_{\sigma} \Gamma'_r (\Pi'_{\tau} \mp \Pi'_{\sigma}) \Gamma^+ \epsilon_{1,2}, \quad (3.7)\]
where primes indicate summation over transverse directions only.

All of these expressions are rather ugly unless a further bosonic light-cone choice is made. When \( X^+ = \tau \) the above results simplify because \( \Pi^+_\tau \mp \Pi^-_{\sigma} = 1 \). The remaining covariant momenta \( \Pi'_{\tau} \) also simplify to \( \partial_r X^r' \) in the Hpp-wave background.

### 3.2 Supersymmetries in the light-cone gauge

Let us now apply the general logic of the previous subsection to the D-brane boundary conditions found before. The Killing spinors of the Hpp-wave were constructed by Blau et al. \[10\] and are in our conventions given by
\[
\epsilon = \left( 1 + \sum_{m=1,2,3,4} \frac{i}{2} \mu X^m \Gamma^+ \Gamma_m I + \sum_{m=5,6,7,8} \frac{i}{2} \mu X^m \Gamma^+ \Gamma_m J \right) \times \left( \cos^2(\frac{1}{2} \mu X^+ )\mathbb{1} - \sin(\frac{1}{2} \mu X^+ )^2 I J - i \sin(\frac{1}{2} \mu X^+ ) \cos(\frac{1}{2} \mu X^+ ) (I + J) \right) (\lambda + i \eta).
\]
\[= \left( \cos^2(\frac{1}{2} \mu X^+ )\mathbb{1} - \sin(\frac{1}{2} \mu X^+ )^2 I J - i \sin(\frac{1}{2} \mu X^+ ) \cos(\frac{1}{2} \mu X^+ ) (I + J) \right) (\lambda + i \eta). \quad (3.8)\]
The spinors \( \lambda \) and \( \eta \) are constant. It is convenient to decompose this expression using
\[\lambda = \frac{1}{2} \Gamma^+ \Gamma^- \lambda + \frac{1}{2} \Gamma^- \Gamma^+ \lambda := \lambda^{(+)} + \lambda^{(-)}, \quad (3.9)\]
and similarly for the \( \eta \) spinor. As the spinors have positive chirality, one deduces
\[IJ (\lambda^{(\pm)} + i \eta^{(\pm)}) = \pm (\lambda^{(\pm)} + i \eta^{(\pm)}) \quad \text{and} \quad I (\lambda^{(\pm)} + i \eta^{(\pm)}) = \pm J (\lambda^{(\pm)} + i \eta^{(\pm)}). \quad (3.10)\]
We then obtain a decomposition of the Killing spinor into $X^+$ dependent and $X^+$ independent parts,

$$\epsilon = \left[ \cos^2\left(\frac{1}{2} \mu X^+\right) - \sin^2\left(\frac{1}{2} \mu X^+\right) - 2i \sin\left(\frac{1}{2} \mu X^+\right) \cos\left(\frac{1}{2} \mu X^+\right)\right] \left(\lambda^{(+)} + i \eta^{(+)\mp}ight)$$

$$+ \left[1 + \frac{\mu}{2} \left( \sum_{m=1,2,3,4} - \sum_{m=5,6,7,8} \right) i X^m \Gamma^+ \Gamma_m I \right] \left(\lambda^{(-)} + i \eta^{(-)}\right).$$

(3.11)

These two lines correspond to the “kinematical” and “dynamical” part of the Killing spinor. Note, however, that this decomposition does not correspond to the decomposition $\epsilon = \frac{1}{2} \Gamma^+ \Gamma^- \epsilon + \frac{1}{2} \Gamma^- \Gamma^+ \epsilon$.

Using expression (3.7) we can now immediately write down the global supersymmetry invariances of the closed Green-Schwarz action in the Hpp-wave$^6$:

$$\delta \theta_1 = \frac{1}{4} \Gamma^+ \Gamma^- \left[ \left( \cos^2\left(\frac{1}{2} \mu X^+\right) - \sin^2\left(\frac{1}{2} \mu X^+\right) \right) \lambda^{(+)} + 2 \sin\left(\frac{1}{2} \mu X^+\right) \cos\left(\frac{1}{2} \mu X^+\right) I \eta^{(+)\mp}\right]$$

$$- \frac{1}{2} \Gamma_{\sigma'} \left( \partial_{\sigma} - \partial_{\sigma'} \right) X^r \Gamma^+ \lambda^{(-)}$$

$$- \frac{1}{4} \Gamma^+ \Gamma^- \left[ \mu \left( \sum_{m=1,2,3,4} - \sum_{m=5,6,7,8} \right) X^m \Gamma^+ \Gamma_m I \eta^{(-)}\right],$$

(3.12)

$$\delta \theta_2 = \frac{1}{4} \Gamma^+ \Gamma^- \left[ \left( \cos^2\left(\frac{1}{2} \mu X^+\right) - \sin^2\left(\frac{1}{2} \mu X^+\right) \right) \eta^{(+)\mp} - 2 \sin\left(\frac{1}{2} \mu X^+\right) \cos\left(\frac{1}{2} \mu X^+\right) I \lambda^{(+)}\right]$$

$$- \frac{1}{4} \Gamma_{\sigma'} \left( \partial_{\sigma} + \partial_{\sigma'} \right) X^r \Gamma^+ \eta^{(-)}$$

$$+ \frac{1}{4} \Gamma^+ \Gamma^- \left[ \mu \left( \sum_{m=1,2,3,4} - \sum_{m=5,6,7,8} \right) X^m \Gamma^+ \Gamma_m I \lambda^{(-)}\right].$$

(3.13)

These are accompanied by a transformation of the transverse bosons, which get a contribution both from the shift symmetry as well as the compensating kappa transformation,

$$\delta X^{r'} = 2i \bar{\theta}_1 \Gamma^{r'} \lambda^{(-)} + 2i \bar{\theta}_2 \Gamma^{r'} \eta^{(-)}.$$  

(3.14)

Instead of relying on the covariant arguments given so far, one can of course also verify directly that the action of a closed string

$$S = \int d^2 \sigma \left[ - \frac{1}{2} \partial_+ X^{r'} \partial_- X_{r'} - \frac{1}{2} \mu^2 X^{r'} X_{r'}$$

$$+ i \bar{\theta}_1 \Gamma^- \partial_+ \theta_1 + i \bar{\theta}_2 \Gamma^- \partial_- \theta_2 - 2i \mu \bar{\theta}_1 \Gamma^- \Pi \theta_2 \right],$$

(3.15)

(with $\partial_\pm = \partial_r \pm \partial_\sigma$) is indeed invariant under the symmetries given above.

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$^6$Similar global transformation rules where given for the $d = 11$ supermembrane by Sugiyama and Yoshida [1]; their derivation, however, does not start from the covariant action.
As we discussed before, not all of these symmetries of the closed string survive for an open string, as the boundary conditions can be such that some of the global supersymmetries (3.12) and (3.13) do not survive. Let us, as an example, discuss the D-string boundary conditions. By imposing $\delta \theta_1 = \Gamma^+ \delta \theta_2$ at the boundary, we find from (3.12) and (3.13) that

$$\lambda = \Gamma^+ \eta, \quad \lambda^{(+)} = \eta^{(+)} = 0,$$

which together imply $\lambda^{(-)} = -\eta^{(-)}$. When this relation holds, one indeed verifies that the action (3.15) for an open string is invariant. This condition is precisely what has been obtained from probe branes as well. In the next subsection we will make this correspondence more precise for general branes.

### 3.3 Comparison with probe brane and supergravity results

A nice consequence of the above analysis is that it makes it very simple to show that the global supersymmetries preserved by the open string with various D-brane boundary conditions indeed match the probe brane results of Skenderis and Taylor [1]. The condition for kappa symmetry of the probe brane embedding is written as

$$\epsilon = \gamma \epsilon, \quad \text{with} \quad \gamma = \gamma^{+mn}(*) \frac{\pm 1}{2} (-i),$$

where $\gamma$ is the kappa symmetry projector and $m$ and $n$ symbolically denote the number of indices in the first and second four transverse directions. This condition can be rewritten as

$$\epsilon_1 + i\epsilon_2 = \Gamma^{+mn}(\epsilon_2 - (-) \frac{\pm 1}{2} i\epsilon_1) \quad \Rightarrow \quad \epsilon_1 = \Gamma^{+mn} \epsilon_2,$$

(3.18)

(the real and imaginary part of the equation are equivalent because of the relation $(\Gamma^{+mn})^2 = (-)^{p(p+1)-1}$ which for odd $p$ equals $-(\Gamma^{+mn})^{p+1}$). Requiring a match between linearly independent terms of this equation is then identical to the conditions obtained from $\delta \theta_1 = \Gamma^{+mn} \delta \theta_2$, thereby completing the proof of the equivalence between probe brane and open string results.

A match is also found by comparing with the supergravity solutions of Bain et al. [4]. Here the comparison is necessarily less systematic, as the Killing spinors of the brane-in-wave backgrounds are not simply obtained from the Killing spinors of the Hpp-wave. The various branes have to be addressed case-by-case. For e.g. the D-string solution, we see from equation (3.18) of [4] that it requires $\gamma^+ \gamma^- \hat{\epsilon} = 0$, which in our notation corresponds to (3.16).

### 4. The “quarter BPS” D-string

#### 4.1 AdS$_5 \times S^5$ origin

Boundary conditions for open strings which are consistent at tree level do not necessarily have to be consistent for higher genus open string surfaces. This was demon-
strated by Bergman et al. \cite{5} for D$p$-branes with $p > 1$ located outside the origin of the Brinkman coordinates. Tree level boundary conditions used at one-loop in the open string genus expansion seem to be incompatible with the open/closed string duality. Having established consistent tree level open string boundary conditions for the quarter-supersymmetric D-string, we therefore now have to investigate whether this solution makes sense in the full theory.\footnote{Calling the D-string “quarter-BPS” is perhaps not quite correct, as BPS conditions can only be derived from supercharges that square to the Hamiltonian. Since the kinematical supercharges do not have this property, the BPS fraction strictly speaking only refers to the number of unbroken dynamical supersymmetries. In this sense the D-string could be called “half-BPS”, but to avoid confusion we will keep referring to all branes by the total number of unbroken supersymmetries.}

One way of attacking this question is to follow an analysis similar to \cite{5}. Another, less direct argument is to try to trace back the D-string to the AdS and D3-brane geometries. One expects that all branes which are consistent in the D3-brane geometry should also be consistent in the AdS geometry, and these should in turn lead to consistent D-branes in the Hpp-wave. Hence, if one can prove that the particular D-string in the D3-brane or AdS background which leads to the D-string in the Hpp-wave geometry (after the sequence of near-horizon and Penrose limits) is consistent in the initial space, then the D-string should also be a consistent solution in the Hpp-wave.

Although the D-string in the AdS space can not be quantised and analysed directly, the fact that there should be a dual gauge description might be useful in proving its existence. Unfortunately the dual gauge description of the string is at this moment not yet fully under control \cite{12}. However, we would like to mention several properties of this D-string.

It is obvious, from the way the Penrose limit is taken, that the D-string which wraps the big circle of the $S^5$ through which the limiting null geodesic passes leads to the D-string at the origin of the Hpp-wave space. This D-string is unstable against small perturbations, and will collapse to a point (i.e. a minimal $S^1$ on $S^5$) as we will now discuss.

Since the open string/CFT description of this D-string is lacking, we can use the effective action approach to study its (in)stability by looking at the spectrum of small quadratic fluctuations around the static position. Therefore, we start with the AdS$_5 \times S^5$ space written in global coordinates

$$\text{d}s^2 = R^2 (-\text{d}t^2 \cosh^2 \rho + \text{d}\rho^2 + \sinh^2 \rho \text{d}\Omega_5^2 + \text{d}\psi^2 \cos^2 \theta + \text{d}\theta^2 + \sin^2 \theta \text{d}\Omega_3^2) \quad (4.1)$$

and consider the D-string wrapping the equator of $S^5$. Choosing the static gauge $t = \tau$ and $\psi = \sigma$ with all the other coordinates set to zero, it is easy to see that this embedding is a solution of the Dirac-Born-Infeld equations of motion

$$\partial_\nu \left( \sqrt{-\det \hat{g}} \hat{g}^{ij} \partial_i X^\nu g_{\mu \rho} \right) - \frac{1}{2} \sqrt{-\det \hat{g}} \hat{g}^{ij} \partial_i X^\nu \partial_j X^\rho g_{\mu \rho} = 0 , \quad (4.2)$$
where $\hat{g}$ is the induced metric on the D-string world-sheet.

To investigate the issue of stability, it is enough to look at the fluctuations in the angular direction $\theta$. We will also consider the fluctuations in the AdS space direction $\rho$ to show that they do not contain unstable modes. We introduce arbitrary functions $\delta \theta$ and $\delta \rho$ of the world-volume coordinates and expand the DBI action to quadratic order in these fluctuations

$$S_{\text{quadr.}} \sim \int d^2 \sigma \left( -\delta \dot{\rho}^2 + \delta \rho^2 - \delta \theta^2 + \delta \dot{\theta}^2 - \delta \dot{\psi}^2 \right). \quad (4.3)$$

From this expression we conclude that the zero mode of $\delta \theta$ is indeed tachyonic, signaling the instability of the solution under small perturbations in this direction.

Actually, it is possible to describe in detail the behaviour of this perturbed D-string by searching for a time dependent solution $\theta(t)$ of the equations of motion (4.2). For $\mu = \psi$, this equation is satisfied as in the unperturbed case while for $\mu = t$, it leads to

$$\frac{d}{dt} \left( \frac{\cos \theta}{\sqrt{1 - \dot{\theta}^2}} \right) = 0 \quad (4.4)$$

and it is easy to check that the equation (4.2) for $\mu = \theta$ is a consequence of (4.4). Integrating one time (4.4) gives the first order differential equation

$$\frac{d\theta}{\sqrt{1 - c^2 \cos^2 \theta}} = dt, \quad (4.5)$$

where $c$ is a constant which is determined in terms of the initial position $\theta_0$ and velocity $v_0 \equiv \dot{\theta}_0 : c = \sqrt{1 - v_0^2} / \cos \theta_0$. The equation (4.5) can be integrated by using a function known as the incomplete elliptic integral in the mathematical literature and denoted $F(x|c)$:

$$F(\theta|c) - F(\theta_0|c) = t. \quad (4.6)$$

which defines implicitly the evolution of the D-string on the 5-sphere. When $c = 1$, which encompasses the case of the static D-string sitting at the equator, we can give a more explicit expression

$$\theta(t) = \arcsin \left( \frac{2ae^t}{1 + a^2 e^{2t}} \right) \quad \text{with} \quad a \equiv \sqrt{\frac{1 - \cos \theta_0}{1 + \cos \theta_0}}. \quad (4.7)$$

From this relation, one can write down the induced metric on the world-volume of the D-string

$$ds^2_{D1} = R^2(-dt^2 + d\theta^2 + \cos^2 \theta d\psi^2) = R^2 \left( \frac{1 - a^2 e^{2t}}{1 + a^2 e^{2t}} \right)^2 (-dt^2 + d\psi^2) \quad (4.8)$$

and compute, as a function of its initial position, the time it takes for the D-string to collapse to one of the poles of $S^5$: $t_s = - \log a = \arctanh (\cos \theta_0)$. As expected,
this time goes to infinity when the D-string is initially infinitesimally close to the equator ($\theta_0 \to 0$).

Let us now consider the Penrose limit of this shrinking D-string. The time $t$ and active angular coordinates $\psi$ and $\theta$ are rescaled as

$$t = x^+ + \frac{x^-}{R^2}, \quad \psi = x^+ - \frac{x^-}{R^2} \quad \text{and} \quad \theta = \frac{y}{R} \quad \text{for} \quad R \to \infty. \quad (4.9)$$

Therefore, we observe that, in order to survive under the Penrose limit, the D-string defined by the equation (4.7) must sit initially at an angle $\theta_0$ such that $a \sim 1/R$. In other words, it must be at a distance $\theta_0$ of order $1/R$ from the equator. This analysis can be repeated in the general case $c \neq 1$ and one can see that the D-strings which survive must again be infinitesimally close (in initial position and velocity) to the static configuration; in more quantitative words, one should also have $c - 1 \sim 1/R^2$. The Penrose limit of such D-strings correspond to stable solutions of the DBI action in the plane wave background. In particular, the tachyonic mode along the direction $\theta$ is washed out under this process. This can be seen by evaluating the action on the Penrose limit of the solution; in contrast to the action of the shrinking D-string in the AdS background, the action is now constant. Hence, through the Penrose limit, the unstable (static or moving) D-string on $S^5$ gets mapped into a susy and hence stable (static or moving) configuration in the Hpp-wave geometry.

Finally, let us conclude this section by observing that the D-string in the AdS space which is wrapping the equator of the 5-sphere originates from an unstable circular D-string in the geometry of the D3-branes, which lies in the two-plane that intersects the D3-brane over a point, times the time. This is obvious from the isometries which are preserved by these two configurations. Note, however, that while the circular string in the D3-brane geometry is always a non-static configuration, this is not true any longer in the near horizon geometry, which admits a static configuration.

### 4.2 World-volume spectrum

In order to find the world-volume spectrum we have to determine the mode expansion of the bosonic and fermionic fields on the open string. The equations of motion obtained from the light-cone action (3.15) are

$$\partial_+ \partial_- X^r + \mu^2 X^r = 0,$$

$$\partial_+ \theta^1 - \mu I \theta^2 = 0,$$

$$\partial_- \theta^2 + \mu I \theta^1 = 0. \quad (4.10)$$

The mode expansions for the closed string can be found in [4] and these have been used to construct the mode expansions subject to half-supersymmetric boundary
conditions, see [3]. For boundary conditions that remove all of the kinematical supersymmetries the story is slightly different, as we will now show. We will for simplicity only consider strings starting and ending on a D-string, i.e. with boundary conditions

\[\theta^1 = \Gamma^+ \theta^2 \bigg|_{\sigma=0,\pi}.\]  

(4.11)

The \(\tau\)-independent parts of the fermionic mode expansion are easily found. Using the ansatz

\[\theta^1 = (1 + \Omega I) \theta^+ e^{\mu \sigma} + (1 - \Omega I) \theta^- e^{-\mu \sigma} + \sum_{n=-\infty}^{+\infty} c_n \left( \theta^1_n e^{-i\omega_n \tau - i\sigma} + \tilde{\theta}^1_n e^{-i\omega_n \tau + i\sigma} \right),\]

\[\theta^2 = (\Omega + I) \theta^+ e^{\mu \sigma} + (\Omega - I) \theta^- e^{-\mu \sigma} + \sum_{n=-\infty}^{+\infty} c_n \left( \theta^2_n e^{-i\omega_n \tau - i\sigma} + \tilde{\theta}^2_n e^{-i\omega_n \tau + i\sigma} \right),\]  

(4.12)

(where \(\Omega = \Gamma^{+-}\)) we find that, for the D-string boundary conditions, the Fourier coefficients \(\theta^1_n, \tilde{\theta}^1_n, \theta^2_n\) and \(\tilde{\theta}^2_n\) can all be expressed in terms of a single mode \(\theta_n\) as

\[
\theta^1_n = [n - \omega_n + i\mu \Omega I] \theta_n, \\
\tilde{\theta}^1_n = [n + \omega_n - i\mu \Omega I] \theta_n, \\
\theta^2_n = [n + \omega_n + i\mu \Omega I] \Omega \theta_n, \\
\tilde{\theta}^2_n = [n - \omega_n - i\mu \Omega I] \Omega \theta_n,
\]

(4.13)

where \(\omega_n = \text{sgn}(n) \sqrt{\mu^2 + n^2}\) and, for convenience, we have introduced the factors \(c_n = 1/\sqrt{1 + (\omega_n - n)^2/\mu^2}\). Note in particular that for \(n = 0\) the above implies \(\theta_0^1 + \tilde{\theta}_0^1 = 0\) and \(\theta_0^2 + \tilde{\theta}_0^2 = 0\) which eliminates the zero modes from the expansions (4.12) (consistent with the fact that these D-branes do not have any kinematical supersymmetries). Finally, reality of the coordinates implies that \(\theta^+ = -\theta^-\).

The bosonic mode expansions have been given in [13],

\[X^r = \frac{x^{r_0}_0 \sinh \mu (\pi - \sigma) + x^{r_1}_0 \sinh \mu \sigma}{\sinh \mu \pi} + i \sum_{n \neq 0} \frac{1}{\sqrt{2|\omega_n|}} a^n_0 e^{-i\omega_n \pi} \sin n\sigma \]  

(4.14)

for an open string stretched between two D-strings located at the transverse positions \(x_0\) and \(x_1\).

Therefore, we see that the open strings attached to the D-string do not have zero modes in the Dirichlet direction (defined, strictly speaking, as the \(\sigma\)-independent part of their Fourier expansion). However, this property does not mean that the D-string lacks the zero modes which allow it to move in the transverse directions. The latter correspond to the fields \(x_{0/1}^r\) appearing in the expansion (4.14).

Using the standard Poisson/Dirac brackets between the bosonic/fermionic coordinates, one can perform the canonical quantization of the open string ending on
a D-string; for the quarter-BPS D-string, the commutation relations between the oscillators are
\[
\forall m, n \neq 0, \quad [a^a_m, a^b_n] = \text{sgn}(n) \delta_{m+n,0} \delta^{rs} \quad \text{and} \quad \{\theta^1_m, \theta^1_n\} = \frac{1}{4} (\Gamma^+)_{ab} \delta_{m+n,0} \quad (4.15)
\]
where we have arbitrarily chosen the Fourier modes $\theta^1_n$ as the independent fermionic variables. These commutation relations can in principle be used as a starting point to construct the open string spectrum but we will refrain from addressing that problem here.

5. Discussion and open issues

In the present letter we have shown how to obtain boundary conditions for the covariant open Green-Schwarz string which correspond to D-branes in an Hpp-wave. We have shown that this analysis reproduces all D-branes known from previous probe brane and supergravity computations. The advantage of our approach is that, since we start from a covariant system, we are able to produce directly the global supersymmetry invariances of the action after light-cone gauge fixing.

We have also discussed the properties of the curious quarter-supersymmetric D-string. This object preserves dynamical supersymmetries (only) and may therefore be consistent with open/closed string duality. It is interesting to observe that similar states appear in other backgrounds as well, for instance the Nappi-Witten background with two planes, and it would be interesting to study their quantum consistency as well. In view of the original motivation to study Hpp-waves, it is also important to understand the corresponding states in the dual gauge theory. Progress on this issue will be reported elsewhere.

As a side result, we have mentioned that there seem to be two different ways to obtain the D-string from boundary conditions on the open string (see equation (2.9)). It is at present not clear to us whether these boundary conditions correspond to the same physical object.

Finally, the present analysis and evidence in favour of the presence of a D-string is likely to be an argument that supports the existence of a D-instanton in the Hpp-wave background. The present literature on the gauge dual of D-instanton induced vertices in the Hpp-wave background is not sufficient yet to rule out such an object, and further progress along the lines of [5] would be welcome.

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A. Appendix

A.1 Details of the Green-Schwarz string in the Hpp-wave

This appendix contains the details of the covariant action of the Green-Schwarz string in the Hpp-wave. This mostly follows Metsaev [4] though notation is slightly different. From an analysis of the kappa symmetry transformation $s$ in superspace, one can see that the only boundary terms arise from the variation of the Wess-Zumino part of the action (see footnote 8 below). This part of the action is given by

$$S_{WZ} = -2i \int_0^1 dt \int d^2 \sigma \, e^{ij} E_{it}^r (\Theta^r C \Gamma_r E_{jt})$$

(A.1)

where the super-vielbeine are expanded as [4]

$$E = \frac{\sinh \mathcal{M}}{\mathcal{M}} D \Theta,$$

$$E^r = e^r - 2i \bar{\Theta} \Gamma^r \frac{\cosh \mathcal{M} - 1}{\mathcal{M}^2} D \Theta.$$  

(A.2)

The matrix $\mathcal{M}^2$ is given by

$$\mathcal{M}^2 = \begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix},$$

(A.3)

where $(i, j$ run over $1, 2, 3, 4$ only)

$$A^a_b = \sum_{i=1}^{5} (L_i \theta)^a (\bar{\theta} R_i)_b$$

$$= -(\Gamma^+ \Gamma^r)^a (\bar{\theta} \Gamma_i)_{b} - (\Gamma'^+ \theta)^a (\theta \Gamma^i)_{b} + \frac{1}{2} (\Gamma^{ij} \theta)^a (\bar{\theta} \Gamma^+ \Gamma^{ij} I)_{b}$$

$$+ \{1, 2, 3, 4\} \rightarrow \{5, 6, 7, 8\},$$

(A.4)

$$B^a_b = \sum_{i=1}^{5} (L_i \theta)^a (\theta R_i)_b$$

$$= -(\Gamma^+ \Gamma^r)^a (\theta \Gamma^i)_{b} + (\Gamma'^+ \theta)^a (\theta \Gamma^i)_{b} - \frac{1}{2} (\Gamma^{ij} \theta)^a (\theta \Gamma^+ \Gamma^{ij} I)_{b}$$

$$+ \{1, 2, 3, 4\} \rightarrow \{5, 6, 7, 8\}.$$

The matrix $I$ appearing above (denoted $\Pi$ in [4]) and the associated $J$ are given by

$$I = \Gamma^{1234}, \quad J = \Gamma^{5678}.$$  

(A.5)

The two real fermions are grouped according to

$$\Theta = \frac{1}{\sqrt{2}} \begin{pmatrix} \theta_1 + i \theta_2 \\ \theta_1 - i \theta_2 \end{pmatrix}, \quad \bar{\Theta} = \frac{1}{\sqrt{2}} \begin{pmatrix} \theta_1 - i \theta_2 \end{pmatrix}^T C,$$

(A.6)
Inserting these expressions, keeping all the terms which lead to variations up to and including fourth order in Θ and taking care of the t-integral, one obtains

\[
S_{WZ} = -i \int d^2 \sigma \epsilon^{ij} \left[ e^i_i (\Theta^T \Gamma_r D_j \Theta) - \frac{1}{2} i (\Theta \Gamma^r \Gamma^s_1 \Theta) (\Theta^T \Gamma_r D_j \Theta) + \frac{1}{12} e^i_i (\Theta^T \Gamma_r \mathcal{M}^2 D_j \Theta) \right].
\]  

(A.7)

Note that, due to the t-integral which leads to a different coefficient for every power of \( \theta \), the first line above does not factorise as \( E_i (\ldots) \). To this order in the fermions, one thus finds two parts: one which is obtained from the flat-space action by replacing the normal derivative with a covariant derivative, and one which arises from a \( \mathcal{M}^2 \) term in the expansion of \( E^a \). These are, in terms of \( \theta_{1,2} \), given by respectively

\[
S_{WZ}^1 = \int d^2 \sigma \left[ -i \epsilon^{ij} e^i_i (\bar{\theta}_1 \Gamma_r \bar{D}_j \theta_1 - \bar{\theta}_2 \Gamma_r \bar{D}_j \theta_2^2) + \epsilon^{ij} (\bar{\theta}_1 \Gamma^r \bar{D}_j \theta_1) (\bar{\theta}_2 \Gamma_r \bar{D}_j \theta_2^2) \right],
\]

(A.8)

and

\[
S_{WZ}^2 = -\frac{1}{12} \int d^2 \sigma \epsilon^{ij} \left[ (\bar{\theta}_1 \Gamma^r \Gamma^+ \Gamma^s \theta_2 + \bar{\theta}_2 \Gamma^r \Gamma^+ \Gamma^s \theta_1) (\bar{\theta}_1 \Gamma_m \partial_i \theta_1 + \bar{\theta}_1 \Gamma_m \partial_i \theta_2) \right.
\]

\[
+ (\bar{\theta}_1 \Gamma^r \Gamma^{+m} \theta_1 - \bar{\theta}_2 \Gamma^r \Gamma^{+m} \theta_2) (\bar{\theta}_1 \Gamma_m \partial_i \theta_2 - \bar{\theta}_2 \Gamma_m \partial_i \theta_1)
\]

\[
- \frac{1}{2} (\bar{\theta}_1 \Gamma^r \Gamma^{mn} \theta_1 - \bar{\theta}_2 \Gamma^r \Gamma^{mn} \theta_2) (\bar{\theta}_1 \Gamma^+ \Gamma_m \partial_i \theta_2 - \bar{\theta}_2 \Gamma^+ \Gamma_m \partial_i \theta_1)
\]

\[
+ \{1, 2, 3, 4\} \rightarrow \{5, 6, 7, 8\} \right] \partial_j X^\mu e_{\mu r}.
\]

(A.9)

Here \( m, n \) run over \( 1 \ldots 4 \) only. We do not have to consider here the terms in \( \bar{D}_j \theta \) that are proportional to \( \partial_j X^\mu \), as these will produce variations of order \( \theta^6 \). In these expressions the hatted covariant derivatives are given by

\[
\bar{D}_j \theta_{1,2} = \partial_j \theta_{1,2} + \frac{1}{4} \omega_{\mu \nu \sigma} \Gamma^{\nu \sigma} \theta_{1,2} \partial_j X^\mu \mp \frac{1}{2} e_\mu^r \Gamma^+ \Gamma_r \theta_{1,2} \partial_j X^\mu.
\]

(A.10)

There are additional four-fermi terms in the action, but these will not contribute to the boundary terms of the kappa variation at second and fourth order in the fermions.

The variation \( \delta_\kappa X^\mu \) can be expressed in terms of \( \delta_\kappa \theta_{1,2} \) through the defining relation in superspace,

\[
\delta_\kappa X^M E_M^r = 0 \quad \Rightarrow \quad \delta_\kappa X^\mu = -i \bar{\theta}_1 \Gamma^\mu \delta_\kappa \theta_1 - i \bar{\theta}_2 \Gamma^\mu \delta_\kappa \theta_2 + \mathcal{O}(\theta^4).
\]

(A.11)

Upon variation of the action all bulk terms vanish because the background is on-shell,
and the remaining boundary terms are given by \(^8\).

\[
\delta_\kappa S^1_{WZ} = \int_{\partial\Sigma} \left[ i \delta_\kappa X^\mu (\bar{\theta}_1 \Gamma_\mu \hat{D} \theta_1 - \bar{\theta}_2 \Gamma_\mu \hat{D} \theta_2) - i dX^\mu \left( \bar{\theta}_1 \Gamma_\mu \delta_\kappa \theta_1 + \frac{1}{2} \bar{\theta}_1 \Gamma_\mu \Gamma^r \omega_{\nu rs} \theta_1 \delta_\kappa X^\nu + \frac{1}{2} \bar{\theta}_1 \Gamma_\mu \Gamma^+ \nu \theta_2 \delta_\kappa X^\nu \right) \right. \\
- \bar{\theta}_2 \Gamma_\mu \delta_\kappa \theta_2 + \frac{1}{2} \bar{\theta}_2 \Gamma_\mu \Gamma^r \omega_{\nu rs} \theta_2 \delta_\kappa X^\nu + \frac{1}{2} \bar{\theta}_2 \Gamma_\mu \Gamma^+ \nu \theta_1 \delta_\kappa X^\nu \left. \right) - (\bar{\theta}_1 \Gamma^r \delta_\kappa \theta_1) (\bar{\theta}_2 \Gamma_r \hat{D} \theta_2) + (\bar{\theta}_2 \Gamma^r \delta_\kappa \theta_2) (\bar{\theta}_1 \Gamma_r \hat{D} \theta_1) \right].
\]

(A.13)

### A.2 Kappa symmetry boundary terms at higher orders in theta

With the boundary conditions (2.2) for generic branes and (2.9) for the D-string, we have found that

\[
(\Theta^T C \Gamma_r D_j \Theta) = 0 \text{ if } r \in N.
\]

(A.14)

This is very useful for higher order calculations: it means that in the variation of the first term in (A.7) we do not have to consider variation of the \(e_i^r\) factor at all. However, there are still other variations.

Just as in (A.9), the higher order terms in \(\mathcal{M}^2\) will reduce to products of \(\theta\)-bilinears, each factor of which is a sum or difference of two terms obtained by interchange of \(\theta_1\) and \(\theta_2\). We need an efficient way to figure out the relative signs between these two terms. One finds the following general expression:

\[
\Theta^T C \Gamma_r (\mathcal{M}^2)^n D \Theta = \sum_{R_i, L_i} \left[ (\theta \Gamma_r L_n \theta) \mp_n (\bar{\theta} \Gamma_r L_n \bar{\theta}) \right] \times \left[ (\bar{\theta} R_n L_{n-1} \bar{\theta}) \pm_{n-1} (\bar{\theta} R_n L_{n-1} \bar{\theta}) \right] \times \cdots \times \left[ (\theta R_1 D \bar{\theta}) \pm_1 (\theta R_1 D \bar{\theta}) \right],
\]

\[
\bar{\Theta} \Gamma_r (\mathcal{M}^2)^n D \Theta = \sum_{R_i, L_i} \left[ (\bar{\theta} \Gamma_r L_n \bar{\theta}) \mp_n (\theta \Gamma_r L_n \theta) \right] \times \left[ (\theta R_n L_{n-1} \theta) \pm_{n-1} (\theta R_n L_{n-1} \theta) \right] \times \cdots \times \left[ (\theta R_1 D \bar{\theta}) \pm_1 (\theta R_1 D \bar{\theta}) \right].
\]

(A.15)

\(^8\) From (A.11) one can now show that the only boundary terms come from the Wess-Zumino part of the action: in the kinetic terms, all variations of the fields inside a derivative are of the form

\[
\partial_i (\delta_\kappa X^M) E_{M^r} = -\delta_\kappa X^M \partial_i E_{M^r},
\]

(A.12)

there is therefore no need for partial integration, and no boundary terms are generated.
The \( L_i \) and \( R_i \) are defined in (A.4). The signs \( \pm_n \) are the relative signs between the \( n \)-th term in \( A \) and the \( n \)-th term in \( B \), i.e. \( \pm_n = \{+,-,-,-,-\} \). The expression (A.9) is a special case of the general formula above.

Using these expressions, one can easily write down the expression for the WZ part of the action, (A.1). An analysis of these seems to suggest that even at higher order in \( \theta \), the resulting boundary terms in the variation under kappa symmetry all vanish when the D-brane boundary conditions derived in the main text are satisfied. This is in particular true for the D-string with either one of the boundary conditions in (2.9).

### A.3 Conventions and notation

We follow the conventions of Metsaev [4]. The \( X^\pm \) coordinates are defined as \( X^\pm = (X^0 \pm X^9)/\sqrt{2} \). The metric for the Hpp-wave is

\[
ds^2 = 2 \, dX^+ dX^- - S(dX^+)^2 + (dX^i)^2. \tag{A.16}
\]

The vielbeine for the PP-wave are

\[
e^+ = dX^+, \quad e^- = dX^- - \frac{S}{2} dX^+, \quad e^r = \delta^r_\mu dX^\mu, \tag{A.17}
\]

with \( \eta_{+-} = 1 \), from which one computes the only non-zero component of the spin connection,

\[
\omega_{+r} = -\frac{1}{2} \partial_r S \, dX^+. \tag{A.18}
\]

For spinors \( \theta \), which are positive chirality Majorana-Weyl, we use a 32-component notation everywhere. The chirality projector and Dirac bar are

\[
\Gamma := \Gamma_0^{0-9}, \quad \bar{\theta} = \theta^T \mathcal{C}, \tag{A.19}
\]

where \( \mathcal{C} \) is the charge conjugation matrix satisfying \( \mathcal{C}^T = -\mathcal{C} \) and \( \mathcal{C}^{-1}(\Gamma^\mu)^T \mathcal{C} = -\Gamma^\mu \) in ten dimensions. Our index conventions are summarised in the following table:

| coordinate basis | tangent basis |
|------------------|--------------|
| \( M, N, P, \ldots \) \( A, B, C, \ldots \) |
| \( \mu, \nu, \rho, \ldots \) \( r, s, t, \ldots \) |
| \( \alpha, \beta, \gamma, \ldots \) \( a, b, c, \ldots \) |
| \( i, j, k \) |

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