Investigation of mistuning impact on vibration of rotor bladed disks

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Abstract. Mistuning often reduces the fatigue life of bladed disks. The objective of this study is to determine the degree of influence of various types of mistuning on bladed disk vibration. It is also important to determine how the position of the detuned blades in the bladed disk affects the vibrations. The results of experimental and numerical analysis of mistuned bladed disk vibration are presented. The authors investigated the effect of various types of mistuning (geometry, mass, etc.) on the free vibrations of the bladed disk. The worst cases with minimum mistuning and maximum localization were identified. The developed algorithms for calculating mistuned bladed disks vibration and obtained results can be used, when designing turbomachines rotors.

1. Introduction
The real bladed disks are not absolutely symmetrical constructions. There are always small differences in the blades in size, weight, their fixation in the disk. These differences are called mistuning. Mistuning arise due to inaccuracies in manufacturing, assembly, and due to operational defects. The mistuning leads to a change in the vibration spectrum and modes, to their localization, to stress increase and as a result to their fatigue life decreasing. Therefore, investigation of vibration behaviour of mistuned bladed disks is a very actual task.

Mistuning has been an active area of research for past more than 40 years. Analysis of mistuned bladed disks described in papers [1-17].

2. Finite analysis for cyclic symmetry systems with mistuning
Most vibration analyses of tuned bladed discs assume a model of only a single blade and it’s associated segment of the disc. Often attempts are made to minimise the computation effort required to solve the mistuned assembly case and have been found to be effective tools, capable of representing the dynamics of real assemblies once their tuned system properties are known. Such approach is used in papers by Griffin [12]. Analyzing the eigenvalue problem of the system with cyclic symmetry without mistuning may be proceeded by solving the next equation:

\[ K - \omega^2 M \delta = 0, \] (1)

where \( K \) is stiffness matrix, \( \delta \) is node displacements, \( \omega \) is eigenfrequency of vibration system.

For a cyclic symmetry system (CSS) the difference of deflection between the two boundaries of finite element [18-20] is given by the equation:

\[ K - \omega^2 M \delta = 0, \] (1)
\[ \delta_{s+1} = e^{\mu} , \quad \mu = i2 \pi m / N , \]  

where \( i = \sqrt{-1} , S = 1, ..., N, m = 0, 1, 2 ..., N/2 \) for evens \( N \) and \( m = 0, 1, 2 ..., (N-1)/2 \) for odds \( N \).

To analyze the CSS with a mistuning effect the modified perturbation method was suggested in the work [11]. In the free vibration case for this method, we have equation:

\[ \ddot{q} + \omega^2_B A q = 0 , \]

where \( \omega_B \) is nominal blade natural frequency without mistuning, \( \omega_{Bi} \) is blade natural frequency of \( i \)-th blade, \( \omega_c \) is coupling frequency, \( R^2 = \omega_c^2 / \omega_B^2 \) is dimensionless coupling, \( \Delta f_i = (\omega_{Bi}^2 - \omega_B^2) / \omega_B^2 \) is mistuning of \( i \)-th blade.

\[ \omega_i^* = \omega_i + \Delta \omega_i + \Delta^2 \omega_i . \]

Then eigenvalue of a mistuned system may be obtained as:

\[ \omega_i^* = \frac{\omega_i^2}{\omega_B^2} , \]

where \( i = 1, ..., N \), \( \omega_i \) - vector of blade frequency without mistuning (for an ideal structure), \( \Delta \omega_i \) - increasing vector of blade frequency with the first order mistuning, \( \Delta^2 \omega_i \) - increasing vector of blade frequency with the second order mistuning.

3. Numerical results

For case of geometrical mistuning we use blisk, presents on Figure 1 [16]. The length of one blade was changed.

For tuned blisk each group of modes has double modes with equal eigenfrequencies. For mistuned blisk these eigenfrequencies are splitted. One of these modes does not change. For other we can see changes of frequencies (Fig.2). We also obtain vibration localization even on 0.5% mistuning. With increase of mistuning strength the localization become more and more. And in case with 5% mistuning we have “worst case”, when one blade vibrate, but other blades stay immovable (Figure 2).
4. Experimental measurements of free vibration of mistuned blisk

An experiment on the vibration analysis of the mistuned blisk model was conducted. The experiment was conducted in the laboratory of the Institute of solid mechanics of TU Dresden. To measure vibration characteristics the laser scanning vibrometer PSV-400 was used. A flat impeller having a central hole was cut on high-precision laser equipment from sheet steel with a thickness of 2 mm. The model had 7 blades. Vibration measurements of the model without mistuning showed that the accuracy of its manufacture and fixation did not affect vibrations and model is almost symmetrical (Figure 4). The experimental results are in good agreement with the results of calculations in the programs ANSYS and BLADIS+ (Table 1).

Mistuning was introduced by joining the peripheral part of the blades of the segments of magnetic tape. The mass of each segment was measured on electronic scales to the nearest 0.001 g.

At the beginning, the additional mass was attached to only one of the blisk blade. The measurement results showed that even a small additional mass (0.1 g) introduces significant changes in the frequency spectrum and mode shapes of the model. For example, on the 2nd form there is a significant
localization of the oscillations when only one blade vibrates, and the remaining almost are stationary (Figure 5).

![Image](image-url)

**Figure 4.** (a) - first mode of tuned blisk vibration; (b) - localization on second mode of mistuned blisk vibration.

Was also investigated 2 variants, where the additional mass was added to all blades of the blisk. In the first case, the magnitude of the additional mass uniformly increased from 0.2 to 1.2 g in the circumferential direction from blade to blade (0.2-0.4-0.6-0.8-1.0-1.2). In the second case, the mass in the circumferential direction was varied "sawtooth" (0.2-1.0-0.6-1.2-0.4-0.8). Comparison of results showed that in the first case, there is a significant localization of vibrations in contrast to the second case. Also for the first case the difference in frequency for the pair of mode shapes is more significant than for the second (Table 2).

| Mode | Experiment, Hz | ANSYS, Hz | BLADIS+, Hz |
|------|----------------|-----------|-------------|
| $f_1$ | 85.219         | 88.220    | 86.710      |
| $f_2$ | 86.975         | 88.274    | 86.726      |
| $f_3$ | 87.250         | 88.274    | 86.726      |
| $f_4$ | 92.375         | 90.268    | 89.007      |
| $f_5$ | 92.094         | 90.268    | 89.007      |

**Table 1.** Natural frequencies of tuned model.

| Mode | Variant 1, Hz | Variant 2, Hz |
|------|---------------|---------------|
| $f_1$ | 79.844        | 80.031        |
| $f_2$ | 81.750        | 82.032        |
| $f_3$ | 83.063        | 82.344        |
| $f_4$ | 88.719        | 89.219        |
| $f_5$ | 90.313        | 90.094        |

**Table 2.** Natural frequencies of mistuned model.

The actual continuation of this work is the sensitivity analysis and the optimal arrangement of the detuned blades in the axial bladed disk (Rolls-Royce plc. [17]), shown in Figure 5.
5. Conclusions
The mentioned results of the calculations and measurements demonstrate, that even very small mistuning (0.5%) lead to splitted modes and localization, when distortion of harmonic distribution law for circumferential amplitudes of blades is observed. Nodal diameters take broken line form. In case of the set of mistuned blades, minimal mistuning impact on blisk vibration is achieved by “saw-tooth” placement of blades in the disk.

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References
[1] Yang M T and Griffin J H 2001 A Reduced-Order Model of Mistuning Using a Subset of Nominal System Modes Journal of Engineering for Gas Turbines and Power 123 pp 893-900
[2] Repetckii O V and Buy Manh Cuong 2010 To a question of the choice of a numerical method of the analysis of stresses at an assessment of a multi-cycle fatigue of blades of transport turbomachines ISEA News 6 pp 153-158
[3] Repetckii O V Do Manh Tung 2014 Investigation of the characteristics of mistuned turbomachinery bladed discs vibration based on the reduced-order modeling by finite element method Bulletin SibSAU 1 (53) pp 60-66
[4] Repetckii O V Phan Van Tuan 2012 Construction of mathematical model for analysis of friction dampers influences on vibration of gas turbine engines blades BSU News 1 pp 200-205
[5] Elovenko D A Repetckii O V 2011 Analysis of thermophysical properties of heat insulation materials for new constructions of cylindrical walls in high-pressure autoclaves BSU News 6 pp 201-206
[6] Elovenko D A Repetckii O V 2011 Analysis of stress state of elastic half-plane loaded with constant pressure at limited intermediate sections with specified period by finite element method based on MSC Mars software BSU News 5 pp 171-175
[7] Repetckii O V Ryzhikov I N Nguyen Tien Quyet 2015 An approach to turbomachinery bladed disc durability estimation Bulletin ISTU 5 (100) pp 22-28
[8] Heiman B Gerdt V Popp K Repetckii O V 2010 Mechatronics: components, methods, examples (Novosibirsk: Publishing House of the SB RAS) p 602
[9] Irretier H Repetckii O V 1998 Vibration and Life Estimation of Rotor Structure IFToMM Conf. on Rotor Dynamics (Darmstadt)
[10] Repetckii O V Ryzhikov I N Nguyen Tien Quyet 2016 Dynamics of gas turbine engines rotors taking into account non-linear effects Vibroengineering PROCEdia 8 pp 361–365
[11] Bladh R Castanier M P Pierre C 1999 Reduced Order Modelling and Vibration Analysis of Mistuned Bladed Disk Assemblies With Shrouds *Journal of Engineering for Gas Turbines and Power* 121 July pp 515-522

[12] Griffin J H 1988 On Predicting the Resonant Response of Bladed Disk Assemblies *Journal of Engineering for Gas Turbines and Power* 110 pp 45-50

[13] Irretier H and Schmidt K J 1982 Mistuned Bladed Disks – Dynamical Behaviour and Computation *Proc. IFToMM Conference of Rotordynamics Problems in Power Plants* (Rome, Italy) pp 215-226

[14] Rao J S 1991 Turbomachine Blade Vibration (New Delhi: Wiley Eastern Limited)

[15] Sinha A 1986 Calculating the Statistics of Forced Response of a Mistuned Bladed Disk Assembly *AIAA Journal* 24 11 pp 1797-1801

[16] Repetckii O V Ryzhikov I N Schmidt R 2010 Investigations of the influence of various types of mistuning on the vibrations and fatigue life of turbomachine bladed disks *Letter of the Baikal DAAD Fellowship Union* 1 pp 18–23

[17] Repetckii O V and Beirow B 2016 Application of the analysis of sensitivity for the research of systems with mistuning *Baikal Letter DAAD 1* pp 88-97

[18] Repetckii O V 1990 Automation of strength calculations of turbomachines (Irkutsk: publishing house) p103

[19] Repetckii O V 1990 Computer analysis of the dynamics and strength of machines (Irkutsk: Irkutsk State Technical University Publishing House) p 301

[20] Repetckii O V Do Manh Tung 2012 Mathematical model operation and numerical analysis of vibrations of ideal cyclic and symmetric systems finite element method *ISEA News* 3 pp 149-153