Topology optimization of fluid problems using genetic algorithm assisted by the Kriging model

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SUMMARY
A non-gradient-based approach for topology optimization using a genetic algorithm is proposed in this paper. The genetic algorithm used in this paper is assisted by the Kriging surrogate model to reduce computational cost required for function evaluation. To validate the non-gradient-based topology optimization method in flow problems, this research focuses on two single-objective optimization problems, where the objective functions are to minimize pressure loss and to maximize heat transfer of flow channels, and one multi-objective optimization problem, which combines earlier two single-objective optimization problems. The shape of flow channels is represented by the level set function. The pressure loss and the heat transfer performance of the channels are evaluated by the Building-Cube Method code, which is a Cartesian-mesh CFD solver. The proposed method resulted in an agreement with previous study in the single-objective problems in its topology and achieved global exploration of non-dominated solutions in the multi-objective problems. © 2016 The Authors International Journal for Numerical Methods in Engineering Published by John Wiley & Sons Ltd

Received 27 September 2015; Revised 15 April 2016; Accepted 30 April 2016

KEY WORDS: topology optimization; Kriging model; genetic algorithm; level-set representation; multi-objective optimization; Building-Cube Method

1. INTRODUCTION
Over the past three decades, structural optimization has been studied and matured to a level where it can be applied to many industrial problems. Structural optimization is classified into sizing, shape, and topology optimization. Sizing optimization sets width, length, or thickness of an object as variables. Shape optimization defines the boundary between fluid and solid regions, and it has been attracting much attention in flow problems. However, shape optimization cannot deal with the change of topology, for example, making new holes into an object. Topology optimization is the most flexible optimization method among these three approaches, which can not only modify the shape of an object but also allow the connectivity of the object to change.

Topology optimization has been applied to a variety of engineering optimization problems [1] such as structural mechanics problems [2], heat transfer problems [3], and acoustic problems [4] since Bendsøe and Kikuchi first proposed the so-called homogenization design method [5]. However, the first application to flow problems was later than the aforementioned applications. It was performed for the Stokes flow by Borrvall and Petersson [6].

The basic concept of topology optimization is replacement of the optimization problem with a material distribution problem in a fixed design domain using the characteristic function that indicates
whether material exists or not. However, conventional topology optimization tends to suffer from numerical instabilities such as checkerboard pattern, mesh dependency, and local minima [7]. Moreover, the conventional approaches often allow the existence of grayscales in the optimal shape [8]. The level set method [9–12] is one of the approaches that can avoid grayscales. The level set method introduces a signed scalar function called level set function and distinguishes solid and fluid regions according to the sign of the function. Thus, zero-contours of the function indicate the boundaries of the regions.

Conventional topology optimization generally explores the optimal solution by the gradient-based method according to the sensitivity of an objective function. However, the gradient-based method tends to get stuck to the local optima rather than the global optimum. On the other hand, evolutionary algorithm (EA) such as genetic algorithm (GA) is one of the metaheuristic optimization methods, which are more capable to explore the global optimum. However, EA requires numerous function evaluations to realize population-based multipoint simultaneous exploration. Thus, EA is not efficient to solve the optimization problems with expensive calculations (e.g., computational fluid dynamics (CFD)) for function evaluation if EA is employed solely. Moreover, topology optimization involves a large design space because of a high degree of freedom for shape and topology representation (i.e., the number of design variables is often equal to the number of elements in the finite element mesh). For instance, in the present study, a case with the lowest computational cost requires 10 min to evaluate one solution. To implement GA with the current population size and the number of generations, it takes more than 3 years if GA is employed solely.

Thus, as stated earlier, GA requires much expensive computational cost (i.e., large population and many generations) to obtain competitive solutions in expensive optimization problems. In this case, surrogate models are effective to reduce computational cost required for function evaluation. This model approximates the response of each objective or constraint function to design variables in an algebraic expression. This model is derived from several sample points with real values of the objective or constraint function given by expensive numerical simulations. Thus, it can promptly give estimates of function values at arbitrary design variable values. GA explores the optimum on the surrogate model that approximates the objective functions instead of using the objective functions directly.

In order to find a global optimum effectively, a non-gradient-based optimization method for topology optimization using a GA assisted by the Kriging surrogate model is proposed. To validate the non-gradient-based topology optimization methods applied to flow problems, this research focuses on two single-objective optimization problems, each of which is to minimize pressure loss or to maximize heat transfer, and one multi-objective optimization problem to minimize pressure loss and to maximize heat transfer of flow channels.

The rest of the paper is organized as follows. In Section 2, we introduce computational methods including flow channel representation method using level set function, Building-Cube Method employed to evaluate objective function of the channels, GA, and the Kriging-surrogate model. Section 3 presents two cases of single-objective optimization problem to minimize pressure loss. Section 4 presents two cases of single-objective optimization problem to maximize heat transfer performance. Section 5 presents two cases of multi-objective optimization problem, which combine the aforementioned two objective functions. Finally, concluding remarks are given in Section 6.

2. COMPUTATIONAL METHODS

Figure 1 shows a flowchart of the optimization process in this study. Details of each step in the flowchart are described as follows in this section. In Sections 2.1 and 2.2, the representation method of flow channels and the BCM, which is a Cartesian-mesh CFD approach to evaluate objective functions, are stated. Section 2.3 presents non-dominated sorting genetic algorithm II (NSGAII), a GA employed in this study. Section 2.4 presents the Kriging model, a surrogate-model to approximate each objective function and to be evaluated by GA, and a criterion to decide additional sample points of the Kriging model.
2.1. Flow channel representation

The boundaries between fluid and solid regions are represented by the level set representation that introduces a signed scalar function (level set function) $\phi(x)$ where $x$ represents the location in the design domain. This research sets the range of $\phi(x)$ as $|\phi(x)| \leq 1$ and assumes that $x$ is in the fluid region if $\phi(x) > 0$ or in the solid region if $\phi(x) < 0$. In the present study, the objective functions are evaluated by a Cartesian-mesh CFD solver, and the boundary conditions are given by the immersed boundary method (IBM), which is stated in Section 2.2. Because the IBM uses neighboring information along the solid–fluid interface for interpolation, it needs to derive the normal vector on the interface. It is known that the unit normal vector $\mathbf{n}$ can be computed from the level set function using its geometric properties as shown in Equation (1).

$$
\mathbf{n} = \frac{\nabla \phi}{|\nabla \phi|}
$$

In order to describe a two-dimensional flow channel according to the sign of $\phi(x)$ and derive the normal vector by Equation (1), we need to obtain the three-dimensional distribution of $\phi(x)$. The example of the distribution of $\phi(x)$ is shown in Figure 2. As mentioned in Section 1, it is

Figure 1. Optimization flowchart.

Figure 2. Distribution of the level set function before and after solving the Laplace equation. (a) Initial distribution; and (b) Final distribution.
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required to reduce the number of the design variables to employ GA for topology optimization. For example, Guirguis et al. employed a Kriging-interpolated level set to obtain the distribution of $\phi(x)$ with a small number of the design variables and successfully applied GA to structural topology optimization [10]. In this study, the distribution of $\phi(x)$ is obtained by following procedure. At the outer boundary of the design domain, $\phi(x)$ is given as the step functions corresponding to the width of the inlet and outlet of the channel. Next, $\phi(x)$ is given at several discrete control points inside the design domain, which is treated as the current design variables and stated later. Then, setting $\phi(x)$ given at the outer boundary and at the control points as the boundary condition, the Laplace equation written in Equation (2) is solved in the entire design domain to interpolate $\phi(x)$ at the domain where $\phi(x)$ is not given and to obtain the distribution of $\phi(x)$.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

The Laplace Equation (2) is solved by the iterative method such as Jacobi method, Gauss–Seidel method, and successive over-relaxation. Each method is applicable to obtain the distribution of $\phi(x)$, and the present study employs Jacobi method. Because the solution of Equation (2) is independent of the mesh resolution, in this study, the design domain is discretized to the same mesh resolution as employed in CFD simulation. Note that the geometry of the design variables is independent of the meshing size for CFD simulations in present method, which reduces the number of the design variables and enables the application of GA for topology optimization problems.

The distribution of $\phi(x)$ derived from solving the Laplace equation is steep, and several shapes cannot be represented by any value of $\phi(x)$ given at the control points as the design variables as long as the control points are infinitesimal. For example, the channels whose boundary is exactly straight cannot be represented. This issue must be solved to compare the proposed method with the previous study reporting the optimum with straight boundaries [6]. This limitation of representation results from the distance between the adjacent control points where the value of $\phi(x)$ is given.

Thus, in this study, in order to alleviate this issue, the control point is considered as a circle with a certain radius, and the value of $\phi(x)$ is uniformly given in the circle. Then, the extrema in the distribution of $\phi(x)$ get closer, and the boundary can be drawn as a smoother curve that allows us to represent a straight channel. Such a treatment is applied to the optimization cases if necessary; the radius of the control point is fixed in the cases for the comparison with the previous study, otherwise the radius is allowed to change as an additional design variable.

A conventional level set method for topology optimization [11, 12] employs the partial differential equation to update the value of $\phi(x)$ for a modified shape. The partial differential equation is solved with design sensitivities derived from the sensitivity analysis. In this study, on the other hand, the partial differential equation is not solved, and the value of $\phi(x)$ is updated based on the GA stated in Section 2.3.

2.2. Building-Cube Method

In order to evaluate the objective function of the channels, CFD simulations are conducted by the BCM [13, 14], which is a Cartesian-mesh CFD approach. The BCM consists of block-structured isotropic Cartesian mesh, which has several advantages such as automated mesh generation, easy introduction of a high-order numerical scheme, and simple algorithm structure. The governing equations of the BCM are the two-dimensional continuity equation, incompressible Navier–Stokes equations for the incompressible unsteady-state flow, and the two-dimensional energy equation for unsteady state heat transfer. The non-dimensional forms of the governing equations are given as follows:

$$\nabla \cdot \mathbf{u} = 0$$

$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\nabla p + \frac{1}{Re} \nabla^2 \mathbf{u}$$

$$\frac{\partial T}{\partial t} + (\mathbf{u} \cdot \nabla) T = \frac{1}{Re Pr} \nabla^2 T$$
where the non-dimensional numbers; the Reynolds number $Re$, the Prandtl number $Pr$, and the non-dimensional variables; time $t$, the gradient operator $\nabla$, the velocity $u$, the pressure $p$, and the temperature $T$ are defined as follows:

$$
Re = \frac{\rho U_\infty L}{\mu}, \quad Pr = \frac{\mu C_p}{k}
$$

$$
t = \frac{\bar{t}}{L/U_\infty}, \quad \nabla = \bar{L}\bar{\nabla}, \quad u = \frac{\bar{u}}{U_\infty}, \quad p = \frac{\bar{p}}{\rho U_\infty^2}, \quad T = \frac{\bar{T} - T_\infty}{T_w - T_\infty}
$$

(6)

where $\rho$, $\mu$, $C_p$, $k$, $\bar{L}$, $\bar{t}$, $\bar{u}$, $\bar{p}$, and $\bar{T}$ represent the density, the viscosity, specific heat, the thermal conductivity of the fluid, the gradient operator, the time, the fluid velocity, the pressure, and the temperature, respectively. $U_\infty$ is the reference velocity based on the mean velocity at the inlet. $L$ is the reference length based on the width on the inlet. $T_\infty$ is the inlet temperature. $T_w$ is the reference temperature based on the temperature of the solid. The convection terms are evaluated by a third-order upwind differencing [15], and the viscous terms are evaluated by a second-order central differencing. Time integration is conducted by the Crank–Nicolson method for the viscous terms and the Adams–Bashforth scheme for the convective terms, and the coupling of velocity and pressure is conducted by a fractional step method [16].

Because BCM employs a Cartesian-mesh CFD approach, it is easy to handle the complicated shapes of flow channels with topological change. However, in the Cartesian-mesh CFD approach, the object surface is represented by a staircase pattern, instead of smooth surface. For high-accuracy computation, the IBM [17] using ghost cell (GC) and image point (IP) is employed at the wall boundary. Figure 3 shows Cartesian-mesh near wall boundary and its treatment by the IBM. According to the approach by Mittal et al., wall cells adjacent to wall boundary are defined as GC. An IP is put in the normal direction to the wall using the unit normal vector obtained by Equation (1). The distance from GC to IP is often set to 1.5 times of the minimum size of the cell in two-dimensional case. Then, the quantities at IP are obtained from four fluid cells surrounding IP, which is illustrated in blue dashed line in Figure 3, based on the inverse distance weighting written in Equation (7). The quantities obtained at IP are used to give the boundary conditions at GC to reproduce precise wall geometry.

$$
q_{IP} = \sum_{i=1}^{4} w_i q_i \text{mask}(i)
$$

$$
\text{mask}(i) = 1 \quad : \quad \text{Fluid Cell}
$$

$$
\text{mask}(i) = 0 \quad : \quad \text{Wall Cell}
$$

$$
w_i = \frac{h_i^{-2}}{\sum_{i=1}^{4} h_i^{-2}}
$$

(7)

For velocity, the no-slip boundary condition is imposed. The fluid velocity at GC is given as Equation (8).

Figure 3. Wall boundary treatment by the immersed boundary method.
For pressure, the Neumann boundary condition is imposed. The pressure at GC is set to the same values at IP as written in Equation (9).

$$p_{GC} = p_{IP} \tag{9}$$

Finally, the temperature at the solid–fluid interface is expressed by the third type boundary condition described as Equation (10).

$$-k \left( \frac{dT}{dx} \right)_{wall} = h(T_w - T_{fluid}) \tag{10}$$

where $T_w$ is the temperature of the solid set to be 1, $T_{fluid}$ is the fluid temperature on the wall (solid), $k$ and $h$ correspond to the thermal conductivity and the heat transfer coefficient, respectively. In this study, $h$ is given by the Nusselt number: $Nu = h l / k$. Here, the reference length $l$ is the width of inlet set to be 1, and $k$ is also set to be 1. Thus, the heat transfer coefficient is equal to the Nusselt number. Equation (10) is rewritten with the temperatures at GC and IP as follows:

$$T_{GC} = T_{IP} + h(T_w - T_{IP}) \times 1.5\Delta x \tag{11}$$

Here, $1.5\Delta x$ indicates the distance between GC and IP.

2.3. Genetic algorithm

The GA mimics the evolution of organisms, which selects individuals from the current generation as parents, generates new individuals as children by the crossover and mutation of the parents, and inherits better individuals to the next generation. In this study, the non-dominated sorting genetic algorithm II [18] proposed by Deb et al. is employed for exploration because this algorithm is effective and widespread employed for many optimization problems [10, 19]. Initially, a parent population $P_t$ with the size of $N$ is created randomly. Here, $t$ indicates the number of generation. Each feasible solution is assigned a rank (the solution with lower rank is better) according to its objective function value. On the other hand, each infeasible solution is assigned a rank which is higher than the minimum rank for the feasible solutions. Between two infeasible solutions, the solution with a smaller constraint violation has a better rank. Then, after choosing $N$ solutions with lower rank in the parent population, recombination and mutation are conducted to create an offspring population $Q_t$ with the size of $N$. In order to introduce elitism, first, a combined population $R_t = P_t \cup Q_t$ with the size of $2N$ is formed. Then, the solutions in $R_t$ are sorted according to the ranks based on objective function values and constraint violation. Now, $N$ solutions are chosen from $R_t$ in the order of their ranks and make up a new population $P_{t+1}$. The procedure as described earlier is for one generation. The non-dominated solutions with the lowest rank are explored by repeating this procedure for a certain number of generations.

2.4. Kriging model

Although GA is capable of finding the global optimum, it requires numerous function evaluations to realize population-based multipoint simultaneous exploration. For efficient global optimization, the Krigeing surrogate model [20] is employed together with GA. The Kriging model is based on Bayesian statistics and can adapt well to nonlinear functions. In addition, the Kriging model estimates not only the function values themselves but also their uncertainties. Based on these uncertainties, the expected improvement (EI) of an objective function, which may be achieved on the Kriging model by adding a new sample point, is estimated. In a single-objective optimization problem where $y(x)$ is minimized, the improvement value $I(x)$ and its expected value $E[I(x)]$ are defined as Equations (12) and (13), respectively.

$$I(x) = \begin{cases} [y_{min} - y] & \text{if } y < y_{min} \\ 0 & \text{otherwise} \end{cases} \tag{12}$$
where $\phi$ is the probability density function denoted by $N[\hat{y}(x), s^2(x)]$. Here, $\hat{y}(x)$ is the estimation of $y(x)$, and $s^2(x)$ is the mean square error at point $x$ indicating the uncertainty of the estimated value. Maximizing the EI instead of the original objective function itself, the location of an additional sample point is determined for updating the Kriging model. Adding new samples to the Kriging model based on EI iteratively, these samples are expected to reach the global optima under the uncertainty of the Kriging model. This procedure is called efficient global optimization (EGO) proposed by Jones et al. and widely employed for optimization [21].

Because the present optimization is capable of topological change, the flow channels may often become unconnected depending on design variable values. Because such unconnected channels make it difficult to evaluate the objective function values, they should not be considered as an additional sample point for the Kriging model. Although GA has been successfully applied to topology optimization in several structural problems [10, 22, 23], it has not been accepted widely because of difficulty to ensure structural connectivity during the optimization procedure [24]. In order to deal with this issue (i.e., how to ensure structural connectivity), the original EI value of the objective function, Equation (13), is multiplied by the probability that the objective function value may be below a certain threshold estimated on the Kriging model. This probability $P(y(x) < a)$, where $a$ is a threshold, is formulated in

$$P(y(x) < a) = \frac{1}{s \sqrt{2\pi}} \int_{-\infty}^{a} \exp\left(-\frac{(y(x) - \hat{y}(x))^2}{2s^2(x)}\right) dy$$

Maximizing the product of Equations (13) and (14), the location of an additional sample point is determined for searching the global optima while assuring the connectivity of flow channels under the uncertainty of the Kriging model.

3. OPTIMIZATION PROBLEMS OF MINIMIZING PRESSURE LOSS

In this section, two cases of the single-objective optimization to minimize pressure loss, which is evaluated with Equation (15), are considered. Note that all parameters in Equation (15) are nondimensionalized with the parameters derived in Equation (6).

$$\text{pressure loss} = \left(\frac{1}{2}u^2 + p\right)_{in} - \left(\frac{1}{2}u^2 + p\right)_{out}$$ (15)

where $u$ is the velocity, and $p$ is the pressure at the inlet and outlet.

In this problem, for the sake of simplicity, the problems focus on the low Reynolds number cases that asymmetrical flow property cannot be observed. Thus, flow channels are considered to be vertically symmetric. At the outer boundary of the design domain, a fixed level set function value (0.5 or −0.5) is given to represent inlets and outlets of the channels. The wall boundaries of the channels are then obtained by solving the Laplace equation in the entire design domain. This domain is discretized as a 240 × 240 uniform Cartesian grid. The Reynolds number based on the width of inlet is 10. At the inlets, velocity is set as the Dirichlet boundary condition (parabolic profile given by Equation (16)), and pressure is set as the Neumann condition. At the outlets, on the other hand, velocity is set as the Dirichlet boundary condition (parabolic profile), and pressure is also set as the Dirichlet condition (zero pressure). Furthermore, no-slip boundary condition is given to the solid–fluid interface by Equation (8).

$$g_n = \tilde{g} \left(1 - \left(\frac{2s}{w}\right)^2\right) - \frac{w}{2} \leq s \leq \frac{w}{2}$$ (16)
where $\tilde{g}$ is the prescribed velocity at the center of the flow profile, $w$ is the width of the flow profile corresponding to the length of the inlet and the outlet, and $s$ is location within the flow profile.

3.1. Nozzle example (Case 1)

3.1.1. Problem definition. As the first example, a nozzle shown in Figure 4(a) is conducted. The current design variables are the values of $\phi(x)$ given at six red points 1–6 as shown in Figure 4(b). $\phi(x)$ values at the remaining points 7–9 are the same as those at the points 1–3 due to the vertical symmetry. Thus, the number of the design variables is six. The prescribed velocity $\tilde{g}$ is set to be 1 at the inlet whereas set to be 3 at the outlet.

The objective function is to minimize pressure loss. The following three constraints are considered: (1) the area of flow channels is less than 50% of the design domain; (2) flow channels go from inlets to outlets without dead ends; and (3) pressure loss is less than a threshold value. The third constraint aims to avoid aberrant flow channels with excessive pressure loss, which may make the estimation accuracy of the Kriging model worse. Moreover, in this case, because the boundary of flow channel is curved as in the previous work [25], infinitesimal control points are suitable to represent various topological changes.

3.1.2. Results. For the Kriging surrogate model, 300 initial sample points, satisfying the constraints (1) and (2) are generated randomly, and their objective function (pressure loss) is evaluated by CFD. Then, a threshold of pressure loss in the constraint (3) is considered to remove sample points with excessive pressure loss from the initial sample points of the Kriging model. The Kriging model is constructed with sample points whose pressure loss is below the threshold. The appropriate threshold is chosen by observing the estimation accuracy of the Kriging model with different threshold.

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Int. J. Numer. Meth. Engng 2017; 109:514–532
DOI: 10.1002/nme
In this case, the threshold of the pressure loss is set to be 100, and the number of the initial sample points satisfying all constraints is 175. GA identifies the solution, which maximizes the product of Equations (13) and (14) on the Kriging model, as an additional sample point. The Kriging model is updated after adding this new sample point. In Case 1, the Kriging model is updated 15 times.

Figure 5(a) shows the optimal flow channel whose width is moderately narrowing as reported by the previous study [25]. Several other studies [6, 26] indicate that the inlet of the optimal shape gets narrowed. In this study, due to the representation method, the width of the inlet is fixed. To investigate the influence of the inlet length, we evaluate the pressure loss of a flow channel illustrated in Figure 5(b) with 0.9 times of the inlet length and the same design variables of Figure 5(a). The pressure loss of the optimal solution (Figure 5(a)) is 18.46, whereas the pressure loss of the channel with a narrowed inlet (Figure 5(b)) is 19.33. Reference [25] reports the similar result that the wider inlet indicates the lower pressure loss when finer mesh is employed. Although it is thought that this issue comes from the mesh resolution in previous study [25, 26], the fundamental cause is not clear because of several unclear factors in previous studies such as whether the width of the flow profile $w$ in Equation (16) is modified adaptively with the change of the inlet length during the optimization.

3.2. Double pipe example (Case 2)

3.2.1. Problem definition. Next, the design domain with two inlets and two outlets as shown in Figure 6 is considered. In this case, two types of the design domain with $\delta = 1$ (square domain) and $\delta = 1.5$ (rectangular domain) are considered to compare with the previous studies [6, 25, 26].

In the case of square domain, it is required to represent the straight boundaries with present representation method. The number, location, and radius of control points are investigated before conducting the optimization. Consequently, the control points are collocated as shown in Figure 7. Furthermore, for the square domain, each control point is treated as a circle to represent the boundary as a smooth curve. The radius of the circle is fixed to five cells so as not to interfere with the circles at adjacent control points. $\phi (x)$ values are assigned to the control points assuming vertical symmetry for each domain (i.e., first and sixth rows, second and fifth rows, and third and fourth
The number of the design variables is 18 in the square domain and 10 in the rectangular domain. In both domains, only a positive value of \( \phi(x) \) is given at the blue point to assure the flow channel connectivity. The flow velocity is given as a parabolic profile with the reference velocity of 1 for both inlets and outlets in both domains. This case employs the same Reynolds number, mesh resolution (a 240 × 240 uniform Cartesian mesh for the square domain and a 240 × 360 uniform Cartesian mesh for the rectangular domain), and boundary conditions as Case 1. The constraints considered here are (1) the volume fraction (i.e., the area of flow channels is less than 40% of the design domain) and other two constraints (2) and (3), which are the same as those of Case 1. In this case, the volume fraction is set to be less than 40% of the design domain, whereas it is set to be a constant value (33%) in most of the previous studies [6, 25]. This study employs a different constraint because of two reasons. First, it is difficult for the present method to keep the volume fraction as a constant value during the optimization because it needs to prepare many samples for population-based exploration of GA. Second, we think that the inequality constraint, the flow region is less than 40% of the design domain, can enable GA to explore the objective space more widely than the equality constraint (33%) because the equality constraint is thought to focus on the optimum of the square domain, two straight single pipes. Although GA can explore the entire objective space if no constraint is introduced, it requires much computational cost, and a broad flow channel is unlikely to be an optimum. Thus, we employed 40% as the volume fraction, which lets us explore the objective space more widely without requiring too much additional computational cost than the conventional studies using the equality constraint.

3.2.2. Results. In order to prepare the initial sample points of the surrogate model, 400 sample points satisfying the constraints (1) and (2) are generated randomly for each domain. Then, pressure loss is evaluated as the objective function for each sample point, and a threshold of pressure loss in the constraint (3) is set to be 50 for the square domain and 20 for the rectangular domain. Thus, the number of the initial sample points is reduced to 225 for the square domain and 244 for the rectangular domain. Figure 8(a) shows the optimal flow channel in the square domain obtained after updating the Kriging model 16 times, and Figure 8(b) shows the optimal flow channel in the rectangular domain obtained after updating the Kriging model 20 times.

The square domain yields two single straight channels from the inlets to the outlets, and the rectangular domain yields two channels joined together in the center of the domain. These results agree with the optimization results obtained in the past [6, 25, 26] regarding their topology.

4. OPTIMIZATION PROBLEMS OF MAXIMIZING HEAT TRANSFER

In this section, two cases of the single-objective optimization to maximize heat transfer are presented. Two cases with the same layout, which is shown in Figure 9, but different combinations of Reynolds number and Nusselt number are considered. The Prandtl number is set to be 6.78. Each dimensionless number is chosen to compare the present study with the previous study [27, 28].
In this case, in addition to the two-dimensional incompressible Navier–Stokes Equation (4), the two-dimensional energy Equation (5) is also solved to evaluate the temperature field. This case employs the same pressure and velocity boundary conditions and mesh resolutions as Case 2. The flow velocity is given as a parabolic profile with the reference velocity of 1 for the inlet and the outlet. Temperature is set to be 0 at the inlet, and given by the Neumann condition at the outlet. Furthermore, the temperature at the solid–fluid interface is expressed by the third type boundary condition as described in Equation (10).

The objective function is to maximize the bulk mean temperature $T_m$ at the outlet given by

$$T_m = \frac{\int_{s_0} uTds}{\int_{s_0} uds}$$  \hspace{1cm} (17)

where $s_0$ indicates the area of the outlet, $u$ and $T$ are the velocity and the temperature at the outlet, respectively. In this problem, because the possible range of the objective function value is known thermodynamically (i.e., the maximum is not greater than the temperature of solid: 1 and the minimum is not less than the inlet temperature: 0). Thus, different from Cases 1 and 2, this case does not introduce a threshold into the objective function value because it can reduce the diversity of the population in GA. However, in order to ensure connectivity during the optimization and explore the global optimum efficiently, the original EI value of the objective function is multiplied by the probability that the objective function value may be above a certain value.

4.1. Low Reynolds number and high Nusselt number case (Case 3)

4.1.1. Problem definition. First, the Reynolds number and the Nusselt number are set to be 5 and 50, respectively. In this case, two different layouts of control points are considered. In the first

**Figure 9. Geometry of single pipe (Cases 3 and 4).**

**Figure 10. Layouts of control points (Case 3 (a) and (b)). (a) Regular collocation; and (b) Irregular collocation.**
Figure 11. Temperature distributions of the representative local optima in the initial and additional samples. Each of which has similar objective function value. (Case 3 (a)). (a) 9th sample (Maximum temperature); (b) 10th sample; and (c) A local optimum in the initial samples.

layout (Case 3 (a)), the control points are collocated regularly in the design domain as illustrated in Figure 10(a). In the second layout (Case 3 (b)), the control points are collocated as illustrated in Figure 10(b). In both layouts, the values of $\phi(\mathbf{x})$ are given at the control points vertically symmetric, and the control points are treated as circles whose radius can change in the range of $0 \leq r \leq 5$ as an additional design variable. Thus, the number of the design variables is seven in Case 3 (a) and 10 in Case 3 (b).

4.1.2. Results. First, the Case 3 (a) with seven design variables is discussed. In this case, 126 initial sample points satisfying the connectivity from the inlet to the outlet are used to construct the Kriging model. The Kriging model is updated 10 times. Figure 11 shows representative flow channels in the initial and the additional samples. In this case, several flow channels, each of which has similar objective function values but with different topology, are found as the local optima. This result indicates that the objective function is a multi-modal function, and it is required to employ a method of population-based multipoint simultaneous exploration such as GA. The result also indicates that, as the number of the solid islands in the channels increases, the size of each island becomes smaller. This is because the size of island is determined by the solid–fluid interface to achieve an equal amount of heat flux on the interface. Here, the number of the islands inside the channel depends on the number of the control points. Thus, we can expect to find flow channels with high heat transfer performance if more islands are put inside the channel. In order to investigate the influence of the number of the islands, Case 3 (b) is conducted.

Next, Case 3 (b) with 10 design variables is discussed. In this case, 235 initial sample points satisfying the connectivity from the inlet to the outlet are used to construct the Kriging model. The Kriging model is updated 14 times. Figure 12 shows representative flow channels in the additional samples. Also in this case, several flow channels with different topology are found. Moreover, different from Case 3 (a), this case finds the channels with a larger coupled island and more small isolated islands. The increase of the islands area leads to
the optimum with a better objective function value than Case 3 (a). However, because the bulk mean temperature reaches its upper limit thermodynamically in this case, several flow channels with different topology are still found as the local optima. Thus, it can be concluded that the objective function of this problem is a multi-modal function, and the length of the solid-fluid interface has a significant effect to increase the temperature.

4.2. High Reynolds number and low Nusselt number case (Case 4)

4.2.1. Problem definition. Next, the Reynolds number and the Nusselt number are set to be 50 and 10, respectively. The previous study [27, 28] reports the optimal solution has large islands inside the channel. Thus, it is required to collocate the control points so as to be able to represent large island. In this case, the control points are collocated as illustrated in Figure 13, and the values of $\phi(x)$
Figure 14. Temperature distributions and streamlines of the representative local optima in the additional samples (Case 4). (a) 1st sample; and (b) 12th sample (Maximum temperature).

Figure 15. Temperature distributions and streamlines of the representatives from initial solutions.

are given at the control points vertically symmetric. The control points are treated as circles whose radius can change in the range of $0 \leq r \leq 5$ as an additional design variable. Thus, the number of the design variables is 16 in Case 4.

4.2.2. Results. In this case, 345 initial sample points satisfying the connectivity from the inlet to the outlet are used to construct the Kriging model. The Kriging model is updated 18 times. Figure 14 shows representative flow channels with high temperature found in the additional samples. Also in this case, several flow channels, each of which has similar objective function values but with different topology, are found as the local optima.

This case has two differences from Case 3. First, because the Reynolds number in this case is higher than that in Case 3, flow separation occurs easily, and several vortices are generated. Second, because these vortices affect heat transfer significantly, the shapes of islands inside the channels cannot be designed as arbitrarily as in Case 3. In Case 3, the flow channels with one or more islands have high temperature. In Case 4, on the other hand, putting islands arbitrarily does not always lead to high temperature. Figure 15 shows the streamlines and temperature distributions of flow channels with relatively large island(s) but have low temperature because of the location of vortices. The flow channels shown in Figure 15 have vortices on the most part of islands. On the other hand, the local optima with high temperature shown in Figure 14 also have vortices, but the length of the solid-fluid interface is longer than that of channels in Figure 15. Moreover, the aft island of the optimum (Figure 14(b)) is convex in order to prevent flow separation.

As described in Case 3, the objective function of this problem is likely to be a multi-modal function. In Case 4, GA found not only the local optima which have similar objective function values to the optimum but also several local optima shown in Figure 16, which has about 10% lower objective function values than the optimum. These local optima are suggestive of important
Figure 16. Temperature distributions and streamlines of the representative local optima with 10% lower objective function values than that of the optimum in the additional samples. (a) 9th sample; and (b) 18th sample.

solutions in a multi-objective optimization problem stated in Section 5.2. In both channels, vortices do not appear around an island. The low temperature of these channels is due to the smaller island compared with the island of channels shown in Figure 14. However, as stated in Section 5.2, the pressure loss of the channels in Figure 16 is quite small than that in Figure 14.

Therefore, in this case, flow separation is the most significant factor to increase temperature. The flow channels with several scattered small islands as seen in Figure 12(b) cannot be a local optimum because wake flow of those blunt bodies are separated easily and generate several vortices, which interfere with efficient heat transfer. Thus, although the objective function of this problem is a multi-modal and the length of the fluid and solid interface has a significant effect to increase the temperature as in Case 3, the shape of an island cannot be arbitrary. The shape of islands should be designed to prevent flow separation, and a convex shape is favorable for the aft part.

Finally, because the objective function considered in this section is a multi-modal function and the previous study [27, 28] employed the gradient-based method for exploration, it seems not so meaningful to compare the results of present method with those of previous study. However, note that GA found the solutions as local optima with the same topology of the results reported in previous study.

5. MULTI-OBJECTIVE OPTIMIZATION PROBLEM

Finally, two cases of multi-objective optimization problem to minimize pressure loss and maximize heat transfer of flow channels are considered. The design domain is the same as that shown in Figure 9. These cases employ the same velocity, pressure, temperature boundary conditions, and mesh resolution as Cases 3 and 4. Cases 5 and 6 employ the same combinations of dimensionless numbers as Cases 3 and 4, respectively.

There are a number of non-dominated solutions, which are not worse than any other solution regarding all objective functions, in a multi-objective optimization problem, whereas there is only one optimal solution in a single-objective optimization problem. Thus, it is important to ensure the diversity of the solutions in GA and capture the trade-off among objective functions. Therefore, it should be careful to introduce a threshold of the pressure loss while keeping the diversity of the solutions in GA. However, without a threshold, initial sample points including excessive pressure loss are used to construct the Kriging model, which may make the estimation accuracy of the Kriging model worse. Because the width of the flow channels with excessive pressure loss is very small, these flow channels can be regarded as unconnected. Moreover, the bulk mean temperature of such unconnected or nearly unconnected flow channels is evaluated to be 0 in Cases 3 and 4. Thus, such solutions are hardly able to be the non-dominated solutions, and it will not lose the diversity of solutions even if such solutions are removed from initial sample points. Multi-objective optimization employs several additional sample points every time the Kriging model is updated in contrast to a single-objective optimization employs one additional sample point for every update. This study
performs cluster analysis using the k-means method [29] to select representative sample points from many non-dominated solutions obtained by maximizing the EI value of each objective function on the Kriging model. In both cases stated later, four additional sample points are chosen for every update.

5.1. Low Reynolds number and high Nusselt number case (Case 5)

5.1.1. Problem definition. First, this case employs the same Reynolds number, the Nusselt number, and the layout of the control points as those in Case 3 (a). The design variables are also treated the same as Case 3 (a) with seven design variables (i.e., $\phi(x)$ values are given at the control points assuming vertical symmetry in the range of $|\phi(x)| \leq 1$, and the radius of the control points changes in the range of $0 \leq r \leq 5$).

5.1.2. Results. First, in order to prepare the initial sample points of the Kriging model, 400 sample points satisfying connectivity from the inlet to the outlet are generated randomly. In this case, a threshold of the pressure loss is set to be 13. Thus, the number of the initial samples is reduced to 215.

Figure 17 indicates the initial sample points, the non-dominated solutions among them, and the non-dominated solutions obtained after the 19th update of the Kriging model in the objective space. The front of non-dominated solutions obtained after the 19th update is more convex to the optimization direction than that of the initial non-dominated solution. The non-dominated solutions in the 19th update can be classified into four groups according to their characteristics.

First, in the yellow group, the flow channels have low pressure loss and low bulk mean temperature. These flow channels do not include any solid island inside the channels. This is consistent with the results of the single-objective problems as shown in the previous sections that there are no islands if the pressure loss is low, and there is at least one island if the bulk mean temperature is high.

Second, the non-dominated front is discontinuous between the red group and the yellow group. The solutions in the red group get higher bulk mean temperature, whereas their pressure loss does not increase drastically. All these solutions contain one island in the channel. In this group, as the island gets larger, the pressure loss and the temperature go higher. Thus, it is revealed that putting an island into a channel contributes to better performance of heat transfer.

Next, the solutions in the blue group have two isolated islands. In this group, the bulk mean temperature increases by inserting another island. However, the pressure loss also increases.

Finally, the solutions in the purple group have a coupled island. In this group, two significant results are revealed. First, these solutions keep the bulk mean temperature almost equal to the optimum found in Case 3 (a), whereas the pressure loss is larger than any other solution in the other groups. Second, the trade-off between each objective function becomes weak. This result indicates that the bulk mean temperature reaches its upper limit thermodynamically in this case, whereas the
pressure loss can change drastically according to a subtle change of island’s shape. Thus, recalling that the objective function of temperature is a multi-modal function as seen in Cases 3 and 4, this result indicates the possibility that if we can find the points where the pressure loss increases drastically and the heat transfer is saturated, it is able to design the flow channel that satisfies both high temperature and low pressure loss.

5.2. High Reynolds number and low Nusselt number case (Case 6)

5.2.1. Problem definition. Next, this case employs the same Reynolds number, the Nusselt number, and the layout of the control points as those in Case 4. The design variables are also treated the same as Case 4 with 16 design variables.

5.2.2. Results. First, 450 sample points satisfying connectivity from the inlet to the outlet are generated randomly to prepare the initial sample points of the Kriging model. In this case, a threshold of the pressure loss is set to be 6. Thus, the number of the initial samples is reduced to 397.

Figure 18 indicates the initial sample points, the non-dominated solutions among them, and the non-dominated solutions obtained after the 15th update of the Kriging model in the objective space. In this case, the non-dominated solutions in the 15th update cannot be classified based on its topological characteristics as in Case 5. As stated in Case 4, the flow separation and generation of vortices are not negligible in the current Reynolds number. Based on these characteristics, the non-dominated solutions can be classified into four groups.

First, in the yellow group, the solutions have low pressure loss and low bulk mean temperature. The topological characteristic of this group is that the solutions have two or three tiny islands or do not have any islands. This result is consistent with the result of Case 4 that putting islands does not always lead to high temperature and putting islands or expanding the area of the channel lead to generate vortices that incur high pressure loss and inefficient heat transfer.

Second, the solutions in the red group can be classified into two types based on their topology; a channel has one large island and several tiny islands or has one or two relatively small coupled islands. In this case, it is important to note that the flow separation does not occur around the islands in all channels and no vortices are generated. Thus, the trade-off relation can be explained in the same way as Case 5; putting island and increasing the length of the solid-fluid interface are important factors to increase temperature, while they also let the pressure loss increase.

Next, the solutions in the blue group have large pressure loss and high temperature. All solutions in this group contain one large coupled island, which does not appear in the aforementioned two groups. Thus, flow separation and generation of vortices on the aft island are inevitable in most solutions. However, the solution shown in Figure 18 as the representative of this group has no flow

Figure 18. The solution set in the multi-objective problem (Case 6).
separation and vortices in the channel. As stated in Case 4, this solution can achieve low pressure loss and high temperature simultaneously.

Finally, the solutions in the purple group have one huge island. The orange-colored points are explored during the optimization, but they are not added to update the Kriging model because they make the estimation accuracy worse. In this group, although the aft island is convex, flow separation occurs in all solutions, and the temperature of these solutions is similar to those found in Case 4. Moreover, the temperature reaches its upper limit thermodynamically, and the trade-off between each objective function becomes weak.

In this case, in order to achieve both high temperature and low pressure loss, we can also exploit the characteristic of this problem; the objective function of temperature is a multi-modal function. The solutions in the purple group have high temperature, but their pressure loss is much larger than the other groups. Thus, the solutions in the blue group are considered to be industrially useful.

6. CONCLUSION

A non-gradient-based topology optimization was conducted in flow channel design problems that minimize pressure loss and/or maximize heat transfer using a Kriging-surrogate-based GA.

In the single-objective optimization to minimize pressure loss, two cases with different layout were conducted, and the results agreed with the previous study in its topology.

In the single-objective optimization to maximize the bulk mean temperature, two cases with different dimensionless numbers were conducted. The GA found not only the optimal shape but also several shapes that have quite similar objective function values but with different topologies from each other. In both cases, the objective function of temperature seems to be a multi-modal function, and putting a solid island in the fluid region and increasing the solid–fluid interface have significant effects to increase the temperature. However, in the case at high Reynolds number, the flow separation severely affects heat transfer performance, and the shape of islands should be considered so as not to provoke flow separation.

Finally, combining aforementioned two problems, multi-objective optimization problems to minimize pressure loss and to maximize heat transfer were conducted in two cases with different dimensionless numbers. As a result, it was revealed that the size and the shape of a solid island inserted in a flow channel are the most important factors to increase the temperature and determine the pressure loss for both cases. However, with a certain size of island, the temperature reaches its upper limit, and the trade-off between pressure loss and heat transfer becomes weak. Thus, it is significant in the multi-objective optimization to capture an entire trade-off between these objective functions and find the points where the pressure loss increases drastically and the heat transfer is saturated. The present result is expected to help us design the flow channels that satisfy both high temperature and low pressure loss with further investigation.

Thus, the proposed method showed its capability to conduct global exploration for both single-objective and multi-objective topology optimization in flow problems.

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