Do squarks have to be degenerate?
Constraining the mass splitting with $K-K$ and $D-D$ mixing

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We study the constraints on the mass-splitting of the first two generations of left-handed squarks obtained from $\Delta M_K$, $\epsilon_K$ and $D-D$ mixing. The different contributions from gluino, neutralino and chargino diagrams are examined in detail, concluding that it is not justified to neglect electroweak gaugino diagrams if the squark mass matrices contain flavor nondiagonal LL elements. We find that the constraints on the mass-splitting are very strong for light gluino masses. However, if the gluino is heavier than the squarks the constraints on the mass-splitting are much weaker. There are even large regions in parameter space where the different NP contributions cancel each other, leaving the mass-splitting nearly unconstrained.

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I. INTRODUCTION

Already in the early stages of minimal supersymmetric standard model (MSSM) analyses it was immediately noted, that a super GIM mechanism is needed in order to satisfy the bounds from flavor changing neutral currents (FCNCs) \cite{1}. Therefore, the mass matrix of the left-handed squarks should be (at least approximately) proportional to the unit matrix, since otherwise flavor off-diagonal entries are inevitably either in the up or in the down sector due to the SU(2) relation between the left-handed squark mass terms. The idea that nondegenerate squarks can still satisfy the FCNC constraints (K and D mixing) was first discussed in Ref. \cite{2} (an updated analysis can be found in Ref. \cite{3}) in the context of Abelian flavor symmetries \cite{4, 5}. In the meantime, there have been a lot of significant improvements both on the theoretical and on the experimental side: The mass difference in the D system was measured and the decay constants and bag factors were calculated to a high precision using lattice methods. A recent analysis of the constraints put on NP by Kaon and D mixing can be found in \cite{6}. In all MSSM analyses the main focus has been on the gluino contributions, while the chargino and neutralino contributions were usually neglected claiming that they are suppressed by a factor of $g^2/g^2_3$ \cite{2, 6, 8, 9, 10, 11, 12, 13}. However, it is no longer a good approximation to consider only the gluino contributions in the presence of off-diagonal elements in the LL block of the squark mass matrices because the winos couple to left-handed squarks with $g_2$. In addition, the gluino contributions suffer from cancellations between the crossed and uncrossed box-diagrams, especially if the gluino is heavier than the squarks. Therefore, the neutralino and chargino contributions can even be dominant if $M_2$ is light and the gluino is heavier than the squarks. This situation can occur in GUT-motivated scenarios in which the relation $M_2 \approx m_3 \alpha_3/\alpha_3$ holds. Therefore, we want to update the evaluation of the constraints from K and D mixing with focus on the mass splitting between the first two squark generations taking into account the weak contributions as well.

The squark spectrum is a hot topic concerning benchmark scenarios for the LHC. It is commonly assumed that the squarks are degenerate at some high scale and that nondegeneracies are introduced via the renormalization group \cite{12, 13}. In such scenarios, the nondegeneracies are proportional to Yukawa couplings and therefore only sizable for the third generation. However, flavor-off-diagonal entries in the squark mass matrix can also lead to nondegenerate squarks which can have an interesting impact on the expected decay and production rate of squarks \cite{14}. In principle, there remains the possibility that squarks have already different masses at some high scale. The question which we want to clarify in this article is which regions in parameter space with nondegenerate squarks are compatible with $D-D$ and $K-K$ mixing. We are going to discuss this issue in Sec. III after reviewing $K-K$ mixing and $D-D$ mixing in Sec. II. Finally we conclude in Sec. IV.

II. MESON MIXING BETWEEN THE FIRST TWO GENERATIONS

Measurements of flavor-changing neutral current (FCNC) processes put strong constraints on new physics at the TeV scale and provide a important guide for model building. In particular $D-D$ and $K-K$ mixing strongly constrain transitions between the first two generations and combining both is especially powerful to place bounds on new physics \cite{6}. In the down sector FCNCs between the first two generations are probed by the neutral Kaon system, the first observed example of meson-antimeson mixing. Kaon mixing was already discovered in the early 1950s and the CP violation was established in 1964. The up to date experimental values for the mass difference and the CP violating quantity $\epsilon_K$
are [15]:
\[
\Delta m_K/m_K = (7.01 \pm 0.01) \times 10^{-15}
\]
\[
\epsilon_K = (2.23 \pm 0.01) \times 10^{-3}
\] (1)

However, still today, in the age of the B-factories, the long
known neutral Kaon system still provides powerful
constraints on the flavor structure of any NP model. As
we see from Eq. (1) both the mass difference and the size
of the indirect CP violation are tiny and the numbers are
in agreement with the standard model (SM) prediction:
The SM contribution to the mass difference is small due
to a rather precise GIM suppression (the top contribution
is suppressed by small CKM elements) and also the CP
asymmetry is strongly suppressed because CP violation
necessarily involves the tiny CKM combination $V_{td}V_{ts}^*$
related to the third fermion generation. Therefore, Kaon
mixing puts very strong bounds on NP scenarios like the
to the third fermion generation. Therefore, Kaon
mixing was in the focus of many analyses [1, 2, 7, 8]. An
In contrast to the well-established Kaon mixing, it was
shortly after the experimental discov-
ery [10] and a recent update can be found in Ref. [11].
However, these studies did not consider the electroweak
contributions.

In summary, $D - \bar{D}$ and $K - \bar{K}$ mixing restrict FCNC
interactions between the first two generations in a strin-
gent way and one should expect the NP contributions to
the mass difference to be smaller than the experimental value [6]:
\[
\Delta m_{NP} \leq \Delta m_{exp}
\] (3)
CP violation associated with new physics is even more
restricted, especially in the $d$ sector:
\[
\epsilon_K^{NP} \leq 0.6 \epsilon_K^{exp}
\] (4)
Equations (3) and (4) summarize in a concise way the
allowed range for NP and we will use them to constrain
the NP contributions to $K$ and $D$ mixing in Sec. III.

III. CONSTRAINTS ON THE MASS SPLITTING
FROM KAON MIXING AND D MIXING.

In this section we want to discuss the constraints on the
mass splitting between the first two generations of
left-handed squark. Because of the $SU(2)$ relation be-
tween the left-handed up and down squark mass matrices,
$M_q^2 = V_{CKM}^† M_q^2 V_{CKM}$, in the super-CKM basis,
these mass matrices are not independent. The only way
to avoid flavor off-diagonal mass insertions in the up and
in the down sector simultaneously is to chose $M_q^2$ propor-
tional to products of CKM elements and Yukawa couplings.
Therefore, such scenar-
ios allow only sizable deviations from degeneracy with
respect to the third generation. However, even though
nondegeneracies with the third generation induce addi-
tional CP violation associated with $V_{ub}$ we find that this
mass splitting effectively cannot be constrained. This
finding is in agreement with Ref. [24] A bit more general
definition of MFV could be defined by stating that all flavor
change should be induced by CKM elements. This defini-
tion would also cover the case with a diagonal squark
mass matrix in one sector (either the up or the down sector)
but with off-diagonal elements, introduced by the
$SU(2)$ relation, in the other sector. This setup corres-
ponds to an exact alignment of the squark mass term
$m_q^2$ with the product of Yukawa matrices $Y_u^† Y_u$ (or with
$Y_d^† Y_d$ in the case of a diagonal down squark mass matrix).
The obvious way how off-diagonal elements of the
squark mass matrices enter meson mixing is via squark-
gluino diagrams. These contributions are commonly expected to be dominant since they involve the strong coupling constant. Also in our case under study, with flavor-violating LL elements, the gluino diagrams were assumed to be the most important SUSY contributions to the Wilson coefficient $C_1$ of the $\Delta F = 2$ effective Hamiltonian $H_{\Delta F=2}^{\mathrm{eff}} = \sum_{i=1}^{3} C_i O_i + \sum_{i=1}^{3} \tilde{C}_i \tilde{O}_i$: 

$$C_1^{\tilde{g}\tilde{g}} = -\frac{g_s^4}{16\pi^2} \sum_{s,t=1}^{6} \left[ \frac{11}{36} D_2 \left( m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2, M_2^2, M_2^2 \right) + \frac{1}{9} m_{\tilde{q}}^2 D_0 \left( m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2, M_2^2, M_2^2 \right) \right] V_s^{\mathrm{LL}} V_t^{\mathrm{LL}}$$

Our conventions for the loop-functions and the matrices in flavor space $V_{12}^{\mathrm{LL}}$ are given in the appendix of Ref. [27]. However, if we have flavor-changing LL elements it is no longer possible to concentrate on the gluino contributions for four reasons:

- The gluino contributions suffer from cancellations between the boxes with crossed and uncrossed gluino lines corresponding to the two terms in the square brackets in Eq. (5). The crossed box diagrams occur since the gluino is a majorana particle. This cancellation occurs approximately in the region where $m_{\tilde{g}} \approx 1.5 m_{\tilde{q}}$.
- In the SU(2) limit with unbroken SUSY the winos couple directly to left-handed particles with the weak coupling constant $g_2$. Therefore, flavor-changing LL elements can contribute without involving small left-right or gaugino mixing angles.
- Since charginos are Dirac fermions, there are no cancellations between different diagrams at the one-loop order.
- The wino mass $M_2$ is often assumed to be much lighter than the gluino mass. In most GUT models the relation $M_2 \approx m_{\tilde{q}} \alpha_2 / \alpha_3$ holds. Since the loop function is always dominated by the heaviest mass, one can expect large chargino and neutralino contributions if the squarks masses are similar to the lighter chargino masses.

Therefore, we have to take into account the weak (and the mixed weak-strong) contributions to $C_1$:

$$C_1^{O_0} = -\frac{1}{128\pi^2} \frac{g_s^4}{4} \sum_{s,t=1}^{6} \left( D_2 \left( m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2, M_2^2, M_2^2 \right) + 2 M_2^2 D_0 \left( m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2, M_2^2, M_2^2 \right) \right) V_s^{\mathrm{LL}} V_t^{\mathrm{LL}}$$

$$C_1^{\tilde{g}\tilde{g}} = -\frac{1}{16\pi^2} \frac{g_s^2 g_2}{2} \sum_{s,t=1}^{6} \left( \frac{1}{6} D_2 \left( m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2, M_2^2, M_2^2 \right) + \frac{1}{3} m_{\tilde{q}}^2 M_2^2 D_0 \left( m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2, M_2^2, M_2^2 \right) \right) V_s^{\mathrm{LL}} V_t^{\mathrm{LL}}$$

$$C_1^{\tilde{\chi}^+ \tilde{\chi}^+} = -\frac{g_s^4}{128\pi^2} \sum_{s,t=1}^{6} \left( D_2 \left( m_{\tilde{q}_s}^2, m_{\tilde{q}_t}^2, M_2^2, M_2^2 \right) V_s^{\mathrm{LL}} V_t^{\mathrm{LL}} \right)$$

In Eq. (6) we have set all Yukawa couplings to zero and neglected small chargino and neutralino mixing. Because of the small Yukawa couplings of the first two generations and the suppressed bino-wino mixing the only sizable contribution of both the gluino and the electroweak diagrams is to the same operator $O_1 = \bar{s}\gamma^\mu P_L d \otimes \bar{s}\gamma_\mu P_L d$ as the SM contribution. Note that in all contribution the same combination of mixing matrices enters, since the CKM matrices in the chargino vertex cancels with the ones in the squark mass matrix. Reference [28] calculated all Wilson coefficients contributing to $\Delta F = 2$ processes in the MSSM and Ref. [29] included also the chargino and neutralino contributions into their numerical analysis. However, the main focus of Ref. [29] is not on the mass-splitting between the first two squark generations and the importance of the different contributions is not apparent from the scatter plots used in their analysis.

In Fig. 4 we show the size of the different contributions to $C_1$ as a function of the gluino mass. We have normalized all coefficients to $C_1^{\tilde{g}\tilde{g}}$ since only one box diagram contributes to it and therefore the coefficient depends only on one loop-function which is strictly negative. Note that for heavy gluino masses always the chargino and in some cases the mixed gluino-neutralino contribution are
As stated before, SU(2) symmetry links a mass split-
ing in the up (down) sector to flavor-changing LL ele-
ments in the down (up) sector. So, if one assumes a
"next-to minimal" setup in which one mass matrix is di-
agonal, one has to specify if this is the up or the down
squark mass matrix. If the down (up) squark mass ma-
trix is diagonal, which implies that it is aligned to $Y_u^T Y_d$
($Y_d^T Y_u$), one has contributions to $D - D$ ($K - K$) mixing.
Assuming a diagonal up-squark (down-squark) mass ma-
trix, the regions in the $m_\tilde{q}_1 - m_\tilde{q}$ plane compatible with
$K - K$ mixing ($D - D$ mixing) are shown in Figs. (2)
and (3). Note that there are large regions in parameter
space with nondegenerate squark still allowed by $K - K$
($D - D$) mixing due to the cancellations between the dif-
ferent contributions shown in Fig. (1). However, departing
from an exact alignment with either $Y_u^T Y_u$ or $Y_d^T Y_d$
there are points in parameter space which allow for an even
larger mass splitting due to an additional off-diagonal
element in the squark mass matrix. If this element is real
one can choose an appropriate value which maximizes the
allowed mass splitting [30]. Nevertheless, this additional
off-diagonal element now present in both sectors due to
the SU(2) relation could also carry a phase additional to the
CKM matrix. If this phase is maximal one obtains the
minimally allowed range for the mass splitting due to
the severe constraint from $\epsilon_K$. These minimally and
maximally allowed regions for the mass splittings are also
shown in Figs. (2) and (3).

We have seen that due to the cancellations between the
different diagrams contributing to $D - D$ and $K - K$
mixing there are large allowed regions in parameter space
where the squarks are not degenerate (a mass splitting of
100% and more is well possible). This has also inter-
esting consequences for the LHC: While most benchmark
scenarios assume degenerate squark masses [12, 13] non-
degenerate masses can have interesting consequences on
the branching ratios [14]. The conclusion we can draw from
Figs. (2) and (3) is that there are regions in parameter
space, allowed by $K - K$ and $D - D$ mixing, with very
different masses for the first two squark generations.
Therefore, FCNC processes alone do not require the soft-
SUSY breaking parameter $M_2^2$ to be proportional to the
unit matrix at some high scale. This implicates that
there is more allowed parameter space for models with
Abelian flavor symmetries than without the inclusion of the
electroweak contributions to $D - D$ and $K - K$ mixing.

IV. CONCLUSIONS

In this article we have examined the constraints on
the mass splitting between the first two generations of
left-handed squarks from $K - K$ and $D - D$ mixing by
considering the gluino and the electroweak contribution.
While nearly all previous analyses focused on the gluino contributions to $K - \overline{K}$ and $D - \overline{D}$ mixing in the case of nonminimal flavor violation \cite{2, 3, 4, 5, 6, 7, 8, 9, 10}
Ref. \cite{29} included (but only numerically) the electroweak effects. However, the main focus of Ref. \cite{29} is not
on the mass splitting between the squarks and the importance of the different contributions is not apparent from
the scatter plots shown in their article. In our analysis we have examined in detail the size of the different contributions (neutralino, neutralino-gluino, gluino and chargino boxes) to $D - \overline{D}$ and $K - \overline{K}$ mixing in the presence of flavor off-diagonal mass-insertions in the LL sector of the squark mass matrices. It is found that gluino contributions suffer from a cancellation between the crossed and the uncrossed boxes for $m_\tilde{q} \approx 1.5 m_\tilde{g}$. In addition, winos couple directly to left-handed squark fields (without involving small gaugino or left-right mixing) and their contribution is not affected by such a cancellation. Therefore, we conclude that the (usually neglected) contributions from chargino, neutralino, and mixed neutralion-gluino diagrams can be of the same order as (or even dominant over) the gluino contribution especially if $M_2 \approx m_\tilde{q} < m_\tilde{g}$.

In the analysis of the allowed mass splitting between the first two generations we focused on the "minimal case" in which the up (down) squark mass matrix is diagonal in the super-CKM basis, but not proportional to the unit matrix. In this case flavor off-diagonal elements in the down (up) sector are induced via the SU(2) relation and are therefore contribute to $K - \overline{K}$ ($D - \overline{D}$) mixing. It is found that the constraints on the mass splitting are strong for light gluino masses. However, if the gluino is heavier than the squarks there are large regions in parameter space, allowed by $K - \overline{K}$ ($D - \overline{D}$) mixing, with highly nondegenerate squark masses. This has interesting consequences both for LHC benchmark scenarios (which usually assume degenerate squarks for the first two generations) and for models with Abelian flavor symmetries (which predict nondegenerate squark masses for the first two generation) because $K - \overline{K}$ and $D - \overline{D}$ mixing cannot exclude non-degenerate squark masses of the first two generations.

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FIG. 2: Allowed regions according to Eqs. (3) and (4) in the \( m_{\tilde{q}_1} - m_{\tilde{g}} \) plane with \( m_{\tilde{q}_{2,3}} = 500 \) GeV and \( m_{\tilde{q}_{2,3}} = 1000 \) GeV for different values of \( M_2 \). Yellow (lightest) corresponds to the maximally allowed mass splitting assuming an intermediate alignment of \( m_{\tilde{q}_1} \) with \( Y_u^T Y_u \) and \( Y_d^T Y_d \) [6]. The green (red) region is the allowed range assuming an diagonal up (down) squark mass matrix. The blue (darkest) area is the minimal region allowed for the mass splitting between the left-handed squarks, which corresponds to a scenario with equal diagonal entries in the down squark mass matrix but with an off-diagonal element carrying a maximal phase.
FIG. 3: Allowed mass splitting between the first two generations of left-handed squarks for different gluino masses. We assume the approximate GUT relation $M_2 = (\alpha_2/\alpha_s)m_{\tilde{g}} \approx 0.35$. The different colors correspond to the cases explained in the caption of Fig. 2. Note that the allowed mass splittings are large enough to permit the decay of the heavier squark into the lighter one plus a $W$ boson.