Electromagnetic shape resonances of a dielectric sphere and radiation of portable telephones

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(Dated: March 31, 2022)

Abstract

The frequency band used by cellular telephones includes the eigenfrequencies of a dielectric sphere with physical characteristics close to those of a human head. Proceeding from the spatial features of the natural modes of such a sphere we propose an independent and clear evident accuracy test for the complicated numerical calculations which are conducted when estimating the potential hazard due to the use of cellular telephones, in particular, for the check of a proper handling of the electromagnetic shape resonances of a human head.

PACS numbers: 41.20.-q, 07.57.-c, 41.20.Jb

Keywords: quasi-normal modes, dielectric sphere, shape resonances, portable telephones, estimation of the health hazard
**Introduction.** Estimation of the potential health hazard due to the use of cellular telephones is now a problem of primary importance in view of extremely rapid development and very wide spread of this communication aid. The safety guidelines in this field are based on the findings from animal experiments that the biological hazards due to radio waves result mainly from the temperature rise in tissues\(^1\) and a whole-body-averaged specific absorption rate (SAR) below 0.4 W/kg is not hazardous to human health. This corresponds to a limits on the output from the cellular phones (0.6 W at 900 MHz frequency band and 0.27 W at 1.5 GHz frequency band). Obviously, the *local* absorption rate should be also considered especially in a human head\(^3\). Theoretical estimation of the temperature rise in tissues of a human body are accomplished in the following way. First the electric and magnetic fields inside the human body are calculated by solving the Maxwell equations with a given source (antenna of a portable telephone). The electric field gives rise to conduction currents with the energy dissipation rate \(\sigma E^2/2\), where \(\sigma\) is the conduction constant of respective tissues. In turn it leads to the temperature increase. The second step is the solution of the respective heat conduction equation (or more precisely, bioheat equation\(^3\)) with local heat sources \(\sigma E^2/2\) and with allowance for all the possible heat currents. Hence, for this method the distribution of electric field inside the human body (especially inside the head) is of primary importance.

In this letter we are going to argue that the knowledge of the properties of electromagnetic modes for a dielectric sphere with physical characteristics close to those for a human head can be useful, for example, when developing an independent accuracy test of complicated numerical calculations mentioned above. The point is the eigenfrequencies of such a dielectric sphere lie in the GSM 400 MHz frequency band which has been used in a first generation of mobile phone systems and now is considered for further use. Obviously, the natural modes of a human head belong to this band too. The natural modes of a dielectric sphere can be divided into two types, surface and volume modes. For the volume modes the electromagnetic energy is distributed in the whole volume of the sphere while in the case of surface modes this energy is located close by the sphere surface. All this holds for the natural modes of a human head also, however we have no respective analytic formulas in this case.

\(^1\) In principle, non-ionizing radiation can lead also to other effects in biological tissues\(^2\).
In order to be fully confident, that the pertinent numerical schemes handle the resonances of a human head in a proper way, we propose an independent accuracy test of these calculations. Without such a check it is not obvious because the routines of numerical solving the partial differential equations are local ones while the spatial behaviour of the relevant eigenfunctions characterizes the system as a whole.

**Shape resonances of a dielectric sphere.** Let us consider a sphere of radius \(a\), consisting of a material which is characterized by permittivity \(\varepsilon_1\) and permeability \(\mu_1\). The sphere is placed in an infinite medium with permittivity \(\varepsilon_2\) and permeability \(\mu_2\). It is assumed also that the electric currents are absent in both the media. The finite conductivity of the material inside a sphere will be taken into account below.

It is known that in the source-free case (\(j = 0, \rho = 0\)) the general solution of Maxwell’s equations are obtained from two scalar functions which may be chosen in different ways [4, 5]. In the case of spherical symmetry these functions are the scalar Debye potentials \(\psi\) (see, for example, the textbooks [6, 7]):

\[
\begin{align*}
E_{lm}^{\text{TM}} &= \nabla \times \nabla \times (r \psi_{lm}^{\text{TM}}), \\
H_{lm}^{\text{TM}} &= -i \omega \nabla \times (r \psi_{lm}^{\text{TM}}) \quad (\text{TM} - \text{modes}), \\
E_{lm}^{\text{TE}} &= i \omega \nabla \times (r \psi_{lm}^{\text{TE}}), \\
H_{lm}^{\text{TE}} &= \nabla \times \nabla \times (r \psi_{lm}^{\text{TE}}) \quad (\text{TE} - \text{modes}).
\end{align*}
\]

The time dependence factor \(e^{-i \omega t}\) is dropped for simplicity. These potentials obey the Helmholtz equation inside and outside the sphere \((r \neq a)\) and have the indicated angular dependence

\[
(\nabla^2 + k_i^2)\psi_{lm} = 0, \quad k_i^2 = \varepsilon_i \mu_i \frac{\omega^2}{c^2}, \quad i = 1, 2, \quad \psi_{lm}(r) = f_l(r)Y_{lm}(\Omega).
\]

Equations (2) should be supplemented by the boundary conditions at the origin, at the sphere surface and at infinity. In order for the fields to be finite at \(r = 0\) the Debye potentials should be regular there. Our goal is to find the eigenfrequencies and eigenfunctions in the problem at hand. Therefore at the spatial infinity, radiation conditions should be imposed [8, 9]

\[
\lim_{r \to \infty} r^{1/2} f_l(r) = \text{const}, \quad \lim_{r \to \infty} r^{1/2} \left( \frac{\partial f_l(r)}{\partial r} - i k_2 f_l(r) \right) = 0.
\]

At the sphere surface the standard matching conditions for electric and magnetic fields should be satisfied [6].
In view of all this the Helmholtz equation (2) becomes now the spectral problem for the Laplace operator multiplied by the discontinuous factor $-1/(\varepsilon_i \mu_i)$

$$-\frac{1}{\varepsilon_i \mu_i} \Delta \psi_{\omega lm}(r) = \frac{\omega^2}{c^2} \psi_{\omega lm}(r), \quad r \neq a, \quad i = 1, 2. \quad (4)$$

In this problem the spectral parameter is $\omega^2/c^2$. Due to the radiation conditions (3) this parameter is complex [9, 10]. Thus we are dealing here with shape resonances of a dielectric sphere and the respective eigenfunctions are the quasi-normal modes [9, 11, 12, 13].

In order to obey the boundary conditions at the origin and at spatial infinity formulated above, the solution to the spectral problem (4) should have the form

$$f_{\omega l}(r) = C_1 j_l(k_1 r), \quad r < a, \quad f_{\omega l}(r) = C_2 h^{(1)}_l(k_2 r), \quad r > a, \quad (5)$$

where $j_l(z)$ is the spherical Bessel function and $h^{(1)}_l(z)$ is the spherical Hankel function of the first kind [14], the latter obeys the radiation conditions (3).

At the sphere surface the tangential components of electric and magnetic fields (1) are continuous. As a result, the eigenfrequencies of electromagnetic field for this configuration are determined [6] by the frequency equation for the TE-modes

$$\sqrt{\varepsilon_1 \mu_2} \hat{j}_l(k_1 a) \hat{h}_l(k_2 a) - \sqrt{\varepsilon_2 \mu_1} \hat{j}_l(k_1 a) \hat{h}'_l(k_2 a) = 0 \quad (6)$$

and by the analogous equation for the TM-modes

$$\sqrt{\varepsilon_2 \mu_1} \hat{j}'_l(k_1 a) \hat{h}_l(k_2 a) - \sqrt{\varepsilon_1 \mu_2} \hat{j}'_l(k_1 a) \hat{h}'_l(k_2 a) = 0, \quad (7)$$

where $k_i = \sqrt{\varepsilon_i \mu_i} \omega/c, \quad i = 1, 2$ are the wave numbers inside and outside the sphere, respectively, and $\hat{j}_l(z)$ and $\hat{h}_l(z)$ are the Riccati-Bessel functions [14]

$$\hat{j}_l(z) = z j_l(z), \quad \hat{h}_l(z) = z h^{(1)}_l(z). \quad (8)$$

In equations (6) and (7) the orbital momentum $l$ assumes the values 1, 2, ..., and prime stands for the differentiation with respect of the arguments $k_1 a$ and $k_2 a$ of the Riccati-Bessel functions.

The frequency equations for a dielectric sphere of permittivity $\varepsilon$ placed in vacuum follow from (6) and (7) after putting there

$$\varepsilon_1 = \varepsilon, \quad \varepsilon_2 = \mu_1 = \mu_2 = 1. \quad (9)$$
The roots of these equations have been studied in the Debye paper [15] by making use of an approximate method. As the starting solution the eigenfrequencies of a perfectly conducting sphere were used. In this case the frequencies are different for electromagnetic oscillations inside and outside sphere. Namely, inside sphere they are given by the roots of the equations \( l \geq 1 \)

\[
\begin{align*}
  j_l \left( \frac{\omega}{c} a \right) &= 0 \quad \text{(TE-modes)}, \\
  d \frac{dr}{dr} \left( r j_l \left( \frac{\omega}{c} a \right) \right) &= 0, \quad r = a \quad \text{(TM-modes),}
\end{align*}
\]

while outside sphere these frequencies are determined by equations

\[
\begin{align*}
  h_{l}^{(1)} \left( \frac{\omega}{c} a \right) &= 0 \quad \text{(TE-modes)} \quad (12) \\
  d \frac{dr}{dr} \left( r h_{l}^{(1)} \left( \frac{\omega}{c} a \right) \right) &= 0, \quad r = a \quad \text{(TM-modes).}
\end{align*}
\]

The frequency equations for perfectly conducting sphere \((10), (11)\) and \((12), (13)\) can be formally derived by substituting \((9)\) into frequency equations \((6)\) and \((7)\) and taking there the limit \(\varepsilon \to \infty\).

Approximate calculation of the eigenfrequencies of a dielectric sphere without using computer [15] did not allow one to reveal the characteristic features of the respective eigenfunctions (quasi-normal modes). The computer analysis of this spectral problem was accomplished in the work [16] where the experimental verification of the calculated frequencies was conducted also by making use of radio engineering measurements.

These studies enable one to separate all the dielectric sphere modes into the interior and exterior modes and, at the same time, into the volume and surface modes. It is worth noting that all the eigenfrequencies are complex

\[
\omega = \omega' - i \omega''.
\]

Thus we are dealing with "leaky modes". It is not surprising because we are considering here an open system [17] (a dielectric ball and outer unbounded space).

The classification of the modes as the interior and exterior ones relies on the investigation of the behaviour of a given eigenfrequency in the limit \(\varepsilon \to \infty\). The modes are called "interior" when the product \(k a = \sqrt{\varepsilon} \omega a/c\) remains finite in the limit \(\varepsilon \to \infty\), provided the imaginary part of the frequency \((\omega'')\) tends to zero. The modes are referred to as "exterior"
when the product $ka/\sqrt{\varepsilon} = \omega a/c$ remains finite with growing $\omega''$. In the first case the frequency equations for a dielectric sphere (6) and (7) tend to (10) and (11) and in the second case they tend to (12) and (13). The order of the root obtained will be denoted by the index $r$ for interior modes and by $r'$ for exterior modes. Thus $TE_{lr}$ and $TM_{lr}$ denote the interior TE- and TM-modes, respectively, while $TE_{lr'}$ and $TM_{lr'}$ stand for the exterior TE- and TM-modes.

For fixed $l$ the number of the modes of exterior type is limited because the frequency equations for exterior oscillations of a perfectly conducting sphere (12) and (13) have finite number of solutions [9]. In view of this, the number of exterior TE- and TM-modes is given by the following rule. For even $l$ there are $l/2$ exterior TE-modes and $l/2$ exterior TM-modes, for odd $l$ the number of the modes $TE_{lr'}$ is $(l + 1)/2$ and the number of the modes $TM_{lr'}$ equals $(l - 1)/2$.

An important parameter is the $Q$ factor

$$Q_{\text{rad}} = \frac{\omega'}{2\omega''} = 2\pi \frac{\text{stored energy}}{\text{radiated energy per cycle}}.$$  \hspace{1cm} (15)

For exterior modes the value of $Q_{\text{rad}}$ is always less than 1, hence these modes can never be observed as sharp resonances. At the same time for $\varepsilon$ greater than 5, the $Q_{\text{rad}}$ for interior modes is greater than 10 and it can reach very high values when $\varepsilon \to \infty$.

In the problem at hand the losses due to the radiation can be disregarded unlike the Ohmic losses. Indeed, the external source of electromagnetic energy (cellular telephone) compensates the radiation losses. While the Ohmic losses lead to the temperature rising in human tissues.

For physical implications more important is the classification in terms of volume or surface modes according to whether $r > l$ or $l > r$. For volume modes the electromagnetic energy is distributed in the whole volume of the sphere while in the case of surface modes the energy is located close by the sphere surface. The exterior modes are the first roots of the characteristic equations and it can be shown that they are always surface modes.

Figure 1 shows a typical spatial behaviour of the surface and volume modes of a dielectric sphere.

Thus a substantial part of the sphere modes (about one half) belong to the interior surface modes. It is important that respective frequencies are the first roots of the characteristic equations.
FIG. 1: Electric energy density $r^2 E_r^2$ for the surface (A) and volume (B) TE-modes of a dielectric sphere with $\varepsilon = 40$ placed in vacuum.

In order to escape the confusion, it is worth noting here that the surface modes in the problem in question obey the same boundary conditions at the sphere surface and when $r \to \infty$ as the volume modes do. Hence, these surface modes cannot be classified as the evanescent surface waves propagating along the interface between two media (propagating waves along dielectric waveguides [7], surface plasmon waves on the interface between metal bulk and adjacent vacuum [18, 19] and so on). When describing the evanescent waves one imposes the requirement of their exponential decaying away from interface between two media. In this respect the evanescent surface wave differ from the modes in the bulk.

**Features of dielectric sphere spectrum and their applications.** The parts of human body (for example, head) have the eigenfrequencies of electromagnetic oscillations like any compact body. In particular, one can anticipate that the eigenfrequencies of human head are close to those of a dielectric sphere with radius $a \approx 8$ cm and permittivity $\varepsilon \approx 40$ (for human brain $\varepsilon = 44.1$ for 900 MHz and $\varepsilon = 42.8$ for 1.5 GHz [3]). By making use of the results of calculations conducted in the work [16] one can easily obtain the eigenfrequencies of a dielectric sphere with the parameters mentioned above. For TE$_{l1}$ modes with $l = 1, 2, 3$ we have, respectively, the following frequencies: 280 MHz, 420 MHz, and 545 MHz. For TM$_{l1}$ modes with $l = 1, 2, 3$ the resonance frequencies are 425 MHz, 540 MHz, and 665 MHz. The imaginary parts of these eigenfrequencies are very small so the $Q$ factor in (15) responsible for radiation is greater than 100.

These eigenfrequencies belong to a new GSM 400 MHz frequency band which is now being standardized by the European Telecommunications Standards Institute. This band
was primarily used in Nordic countries, Eastern Europe, and Russia in a first generation of mobile phone system prior to the introduction of GSM.

Due to the Ohmic losses the resonances of a dielectric sphere in question are in fact broad and overlapping. Indeed, the electric conductance $\sigma$ of the human brain is rather substantial. According to the data presented in $\sigma \simeq 1.0$ S/m. The eigenfrequencies of a dielectric dissipative sphere with allowance for a finite conductance $\sigma$ can be found in the following way. As known $\varepsilon_{\text{diss}}$ the effects of $\sigma$ on electromagnetic processes in a medium possessing a common real dielectric constant $\varepsilon$ are described by a complex dielectric constant $\varepsilon_{\text{diss}}$ depending on frequency

$$\varepsilon_{\text{diss}} = \varepsilon + \frac{4\pi \sigma}{\omega}.$$  \hspace{1cm} (16)

The eigenfrequencies $\omega$, calculated for a real $\varepsilon$, are related to eigenfrequencies $\omega_{\text{diss}}$ for $\varepsilon_{\text{diss}}$ by the formula $\omega_{\text{diss}} = \left(\frac{\varepsilon}{\varepsilon_{\text{diss}}}\right)^{1/2} \omega \simeq \omega - 2\pi i \frac{\sigma}{\varepsilon}. \hspace{1cm} (17)$

The corresponding factor $Q_{\text{diss}}$ is

$$Q_{\text{diss}} = \frac{\omega'_{\text{diss}}}{2\omega''_{\text{diss}}} \simeq \frac{\varepsilon \omega}{4\pi \sigma}. \hspace{1cm} (18)$$

Substituting in this equation the values $\omega/2\pi = 0.5 \cdot 10^9$ Hz, $\varepsilon = 40$, $\sigma = 1$ S/m = $9 \cdot 10^9$ s$^{-1}$ one finds

$$Q_{\text{diss}} \simeq \frac{20}{18} \simeq 1. \hspace{1cm} (19)$$

In view of such substantial Ohmic losses the resonance enhancement of the oscillation amplitude inside a human head will not occur. However when the frequency of a mobile telephone coincides with the eigenfrequency of the head the distribution of electric and magnetic fields inside the head will be described by the corresponding normal mode which may be a surface mode or a volume one $\varepsilon_{\text{diss}}$.

Proceeding from this we propose the following test of numerical calculations used when estimating the potential hazard of cellular telephones. The test consists in simulation of the temperature distribution corresponding to the surface and volume modes in the framework of pertinent calculation schemes. For simplicity, the test calculations could be accomplished for a dielectric sphere (instead of a human head) with lower conductivity in comparison with that for a human brain (in order to enhance the effect). The distributions of electric and magnetic fields and the temperature distribution inside the sphere should be calculated
for two eigenfrequencies of the sphere, namely, one frequency corresponds to surface mode and another one belongs to volume mode. The distributions obtained should conform, at least qualitatively, to the spatial behaviour of respective electromagnetic normal modes (see Fig. 1).

**Conclusion.** Detailed analysis of electromagnetic spectra of a dielectric sphere enables us to propose an independent accuracy test of complicated numerical calculations conducted when estimating the potential health hazard due to use of cellular telephones. This test will permit one to make certain of a proper handling of the electromagnetic shape resonances of a human head in these studies.

This paper was completed during the visit of one of the authors (VVN) to Salerno University. It is his pleasant duty to thank G. Scarpetta and G. Lambiase for the kind hospitality extended to him. VVN was supported in part by the Russian Foundation for Basic Research (Grant No. 06-01-00120). The financial support of INFN is acknowledged. The authors are indebted to A.V. Nesterenko for preparing the figure.

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