Double gluon fragmentation to $J/\psi$ pairs at the Tevatron

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Abstract

It has been proposed that the large cross sections for prompt $\psi$, $\psi'$, and $\chi_c$ production at the Fermilab Tevatron $p\bar{p}$ collider can be explained by a dominant color-octet term in the fragmentation function for a gluon to split into quarkonium. We show that this mechanism makes testable predictions for double-quarkonium $\psi\psi$, $\psi\psi'$, $\psi\chi_c$, $\psi\Upsilon$ and $\psi\chi_{b}$ production, as well as for $W\psi$ production, using color-octet matrix elements previously determined from charmonium production data. The $\psi\psi$ signal would already be measurable at the Tevatron, while the $\psi\chi_c$ and $W\psi$ signals would be on the edge of present detectability.
Until recently the standard way to calculate $\psi$ production was the color-singlet model [1], where the $\psi$ is treated as a $c \bar{c}$ pair in a color-singlet $^3S_1$ state with vanishing relative velocity. Although the color-singlet model has been successful in some applications, it predicts rates that fall orders of magnitude below the data [2] when applied to prompt $\psi$ production at the Fermilab Tevatron via the hard scattering subprocess $g + g \rightarrow \psi + g$. A proposal to explain this discrepancy is that the dominant contribution to charmonium production at high transverse momentum $p_T$ comes from hard gluon production followed by fragmentation of the gluon to a $c \bar{c}$ pair in a color-octet state [3,4]. The conversion of the $c \bar{c}$ pair to $\psi$ is nonperturbative and involves the absorption or emission of two soft gluons. Because there are new, nonperturbative parameters associated with the color-octet mechanism [3,5,6], the rate for single quarkonium production cannot be immediately predicted, but rather the experimental data can be used to determine these parameters. In the present paper we show that the color-octet mechanism now makes testable predictions for double-quarkonium $\psi\psi$, $\psi\psi'$, $\psi\chi_c$, $\psi\Upsilon$, and $\psi\chi_b$ production, as well as for $W\psi$ production at the Tevatron (CM energy $\sqrt{s} = 1.8$ TeV).

Using the factorization approach developed in [4] the fragmentation function for a gluon to split into a $Q\bar{Q}$ quarkonium state $H$ is

$$D_{g\rightarrow H}(z, \mu^2) = \sum_n d_{g\rightarrow n}(z, \mu^2) \langle O_n^H \rangle,$$

where $z = (E_H + p_H)/(E_g + p_g)$ is the light-cone fraction, $\mu = 2m_Q$ is the renormalization scale, while $n$ denotes the angular-momentum quantum numbers $^{2S+1}L_J$ and the color quantum number 1 or 8. The nonperturbative matrix elements $\langle O_n^H \rangle$ represent the inclusive probability for forming the state $H$ from the $Q\bar{Q}$ pair [4]; color-singlet values may be calculated from potential models but color-octet values are unknown. The relative importance of the different matrix elements may however be determined by how they scale with $v$, the typical relative velocity of the heavy quarks in $H$. The short distance coefficients $d_{g\rightarrow n}$ can be calculated using perturbation theory in $\alpha_s$; they depend on $z$, $\mu$ and the quantum numbers indexed by $n$. Note that the dependence on the quarkonium state $H$ appears only in
the factor $\langle O_n^H \rangle$.

The leading-order perturbative calculation of the color-octet short-distance coefficient gives

$$d_{g\to\Delta^3S_1} = \frac{\pi\alpha_s(4m_Q^2)}{24m_Q^3}\delta(1-z) \equiv K(g\to\Delta^3S_1)\delta(1-z),$$

with numerical values $K(g\to\Delta^3S_1) = 0.01$ GeV$^{-3}$ for charmonium and $K(g\to\Delta^3S_1) = 2 \times 10^{-4}$ GeV$^{-3}$ for bottomonium. Here we put $\alpha_s(4m_c^2) = 0.26$ and $\alpha_s(4m_b^2) = 0.17$, with $m_c = 1.48$ GeV and $m_b = 4.88$ GeV; these masses are taken from the Buchmüller-Tye potential model calculations of Ref. [7]. The leading-order color-singlet term in the fragmentation function, which does not have a simple analytic form \[8\], is of order $\alpha_s^3$ and gives much smaller factors $K(g\to\Sigma^3S_1) = 4.3 \times 10^{-6}$ GeV$^{-3}$ (3.5 $\times$ 10$^{-8}$ GeV$^{-3}$) for charmonium (bottomonium) cases. It is consequently assumed that the color-octet term in the fragmentation function dominates over the color-singlet term, even though $\langle O_\psi \rangle$ is expected to be smaller than $\langle O_1^\psi \rangle$ by a factor of order $v^4$.

Numerical values of the nonperturbative matrix elements are given in Table I for the $S$-wave states $\psi, \psi'$ and $\Upsilon$, and in Table II for the $P$-wave states $\chi_c$ and $\chi_b$. The color-octet matrix elements $\langle O_8^\psi(3S_1) \rangle, \langle O_8^{\psi'}(3S_1) \rangle$ and $\langle O_8^{\chi_J}(3S_1) \rangle$ are determined empirically from Tevatron data \[3\][5][6], based on calculations using both color-octet and predicted color-singlet contributions in lowest order. The bottomonium matrix elements $\langle O_8^{T(1S)}(3S_1) \rangle, \langle O_8^{T(2S)}(3S_1) \rangle$ and $\langle O_8^{\chi_J(1P)}(3S_1) \rangle$ were estimated from the charmonium values by scaling arguments \[5\]. All our color-octet values follow Ref. \[3\] (but we have corrected a misprint in the value of $\langle O_8^{\chi_J(1P)}(3S_1) \rangle$). The color-singlet matrix elements, shown here for comparison, are determined from potential model calculations using a Buchmüller-Tye potential \[7\]; they are related to the radial wave function $R$ and its derivative at the origin by

$$\langle O_1^H(3S_1) \rangle = (2J + 1)\frac{N_c}{2\pi}|R(0)|^2, \quad H = \psi, \psi', \Upsilon,$$

$$\langle O_1^H(3P_J) \rangle = (2J + 1)\frac{3N_c}{2\pi}|R'(0)|^2, \quad H = \chi_c, \chi_b,$$
where \( N_c = 3 \) is the number of colors.

Given these results, predictions can immediately be made for other processes involving hard gluon production with the gluon fragmenting to quarkonium. Two recent papers have considered prompt \( \psi \) and \( \Upsilon \) production at the LEP \( e^+e^- \) collider via \( Z^0 \to q\bar{q}g \) decays [9,10]. In this paper we address tests of the octet fragmentation mechanism that can be made at the Tevatron \( p\bar{p} \) collider, based on multiple quarkonium production or on quarkonium production in association with a \( W \)-boson.

In our calculations we shall neglect the evolution with \( \mu^2 \) of the gluon fragmentation function, which greatly simplifies the calculation. This introduces some error, but including evolution would not necessarily be an improvement, since naive Altarelli-Parisi evolution does not respect the phase space constraint \( D_{g\to\psi}(z,\mu^2) = 0 \) for \( z < M_{\psi}^2/\mu^2 \) [11].

**Double-\( \psi \) production**

The \( p_T \) spectrum of prompt single \( \psi \) production is presumed to be dominated by the color-octet term in the \( g \to \psi \) fragmentation function [4] given in Eq. (1). Assuming that \( q\bar{q}, gg \to gg \) are the dominant contributing hard subprocesses, we can predict double-\( \psi \) production, with both gluons fragmenting to \( \psi \), through the ratio

\[
\frac{d\sigma(p\bar{p} \to \psi\psi X)}{dp_T(\psi_1)dp_T(\psi_2)} \approx \frac{1}{2} K(g \to \bar{s}3S_1)(\mathcal{O}_g^S(\bar{s}3S_1)) \frac{d\sigma(p\bar{p} \to \psi g X)}{dp_T(\psi)} \delta(p_T(\psi_1) - p_T(\psi_2)), \tag{5}
\]

in the absence of cuts, where the \( \frac{1}{2} \) is a combinatorial factor. This relation remains true in the presence of \( p_T \) and angle cuts, provided they are applied equally to \( g \) and \( \psi \).

We note that the \( gq \to gq \) and \( g\bar{q} \to g\bar{q} \) subprocesses are not in fact completely negligible and contribute about 20% of single-\( \psi \) production with \( p_T(\psi) > 4 \) GeV at Tevatron energies; this correction reduces our prediction in Eq. (5) by about 20% and we take it into account below. We note also that the differential cross section on the right-hand side of Eq. (5) refers to all single prompt-\( \psi \) production (including both direct \( g \to \psi \) fragmentation and indirect \( g \to (\chi_{cJ}, \psi') \to \psi \) contributions but excluding \( \psi \)'s from the decay of \( B \)-mesons); however, the second \( \psi \) on the left-hand side is produced directly (excluding \( \chi_{cJ} \), \( \psi' \) and \( B \)-meson decays). To obtain the total prompt double-\( \psi \) rate, we must add similar contributions from \( \chi_{cJ} \),
and $\psi'$ production (see Table III) multiplied by the corresponding $\chi_{cJ} \to \psi \gamma$ and $\psi' \to \psi X$ branching ratios, that give approximately the same $p_T(\psi)$ distributions and increase the total rate by a further factor $\approx 2.0$. In practice $\psi$ is usually detected via $\psi \to \mu^+\mu^-$ decay, with branching fraction $B(\psi \to \mu^+\mu^-) = 0.0597(25)$; for this decay mode (indicated by the notation $\psi_{\mu\mu}$) we obtain the differential cross section for total prompt double-$\psi$ production as a function of $p_T(\psi)$ shown in Figure 1. The curve was generated from $q\bar{q}, gg \to gg$ subprocesses using the MRSD0 parton distribution functions [12] with renormalization scale and factorization scale both chosen equal to the transverse momentum of the fragmenting gluon $p_T(g) \simeq p_T(\psi)$; it includes the enhancements from indirect contributions. A pseudorapidity cut of $|\eta| < 0.6$ was imposed on the $\psi$’s produced.

Integrating Eq. (5) and including the factor 2 enhancement from indirect $g \to (\chi_{cJ}, \psi') \to \psi$ contributions, we obtain

$$\frac{\sigma(p\bar{p} \to \psi_{\mu\mu}\psi_{\mu\mu}X)}{\sigma(p\bar{p} \to \psi_{\mu\mu}gX)} \approx K(g \to \bar{S}\,^3S_1)\langle O_8^\psi(\bar{S}^3S_1)\rangle B(\psi \to \mu^+\mu^-) = 7.5 \times 10^{-6},$$

(6)

where the cross sections are defined with a minimum $p_T$ requirement on all $\psi$. This equation remains true after pseudorapidity cuts, provided the same cuts are applied equally to $\psi$ and to the recoil gluon jet $g$ in $p\bar{p} \to \psi gX$. The observed cross section for single prompt $\psi$ production at the Tevatron with $p_T(\psi) > 4$ GeV and $|\eta| < 0.6$ is $\sigma(p\bar{p} \to \psi_{\mu\mu}gX) \approx 24$ nb [2], from which we infer that the cross section arising from $gg$ final states is about 19 nb and hence $\sigma(p\bar{p} \to \psi_{\mu\mu}\psi_{\mu\mu}X) \approx 0.14$ pb. Thus Eq. (6) predicts that of order 10-20 double-$\psi_{\mu\mu}$ production events should be detectable already with $p_T(\psi) > 4$ GeV and $|\eta(\psi)| < 0.6$, for integrated luminosity of order 100 pb$^{-1}$ now accumulated by each of the Tevatron detectors.

We note that experimental factors such as detector efficiency could reduce this number considerably. On the other hand, if $\psi \to e^+e^-$ decays are also detected (there should be good efficiency for this in events that are already tagged by one $\psi \to \mu\mu$), the event rate will be increased by a factor 3 or 4, depending on whether double-$\psi_{ee}$ final states are included.

**$\psi +$ quarkonium production**

All the dependence on the $\psi$ state in Eq.(5) appears only in the matrix element $\langle O_8^\psi(\bar{S}^3S_1)\rangle$. 


It is therefore trivial to generalize this equation to the case where one gluon fragments to a $\psi$ while the other gluon fragments to some other quarkonium state. We have tabulated the results in Table III. Note that the $\chi_{cJ}$ and $\chi_{bJ}$ detection modes and branching fractions remain unspecified, and that the $g \rightarrow H$ fragmentation picture is applicable only for $p_T(g) > m_H$. The other branching fractions used are: $B(\psi' \rightarrow \mu^+\mu^-) = 0.0077(17)$, $B(\Upsilon(1S) \rightarrow \mu^+\mu^-) = 0.0248(07)$, and $B(\Upsilon(2S) \rightarrow \mu^+\mu^-) = 0.0131(21)$. The $\psi + (\chi_c \rightarrow \psi\gamma)$ channels account for about 33% of the prompt double-$\psi$ production rate, as already noted above, so they predict about 5 $\psi_{\mu\mu}\psi_{\mu\mu}\gamma$ events for 100 pb$^{-1}$ luminosity; there is some extra loss of efficiency from the need to measure the photon and reconstruct the $\chi_c$ invariant mass, but these signals would appear to be approaching detectability. Similarly, the $\psi + (\psi' \rightarrow \psi\pi\pi, \psi\eta)$ channels account for about 17% of the prompt double-$\psi$ production rate, with a few events at present luminosities, but it may well be impracticable to reconstruct $\psi'$ in these modes because of backgrounds. The predicted rates for $\psi_{\mu\mu}\psi'_{\mu\mu}$, $\psi_{\mu\mu}\Upsilon_{\mu\mu}$ and $\psi_{\mu\mu}\chi_b$ production are all too small to be observable at present. However, we note that the indirect process $g \rightarrow \chi_{bJ}(1P) \rightarrow \Upsilon(1S)$ is predicted to dominate by a factor 3 over direct $g \rightarrow \Upsilon(1S)$ fragmentation, when the relevant $\chi_{bJ}(1P) \rightarrow \Upsilon(1S)\gamma$ branching fractions are folded in, so the net $\Upsilon(1S)$ rate is 4 times larger than the third row of Table III would suggest.

**ψ + gauge boson production**

Another test of the dominance of the color-octet term in the gluon fragmentation function is high-$p_T$ $\psi +$ gauge boson production. Requiring the $\psi$ and gauge boson to have large relative $p_T$ severely restricts the number of possible production mechanisms.

High-$p_T$ $\psi + \gamma$ production at the Tevatron, via gluon-fusion and color-singlet-fragmentation mechanisms, was investigated in Ref. [13]; it was found that gluon-fusion was the dominant production mechanism for all $p_T$ accessible to experiment. We have investigated the additional contribution from the subprocess $q\bar{q} \rightarrow \gamma g$, with the gluon fragmenting to $\psi$ via the color-octet mechanism. We find that this color-octet contribution falls well below the gluon-fusion contribution for $p_T < 20$ GeV, essentially because there are many
more gluons than quarks in the proton at the relevant $x$-values for Tevatron energies. Hence high-$p_T$ $\psi + \gamma$ production is not sensitive to the color-octet contribution, at $p_T$ values where most events will be seen.

In contrast, high-$p_T$ production of $\psi + W$ offers much cleaner tests of the color-octet mechanism. It has no gluon-fusion contribution, and the dominant direct production sub-process is $q\bar{q} \rightarrow Wg$ with the gluon fragmenting to $\psi$ via the color-octet mechanism. This mechanism contributes the differential cross section

$$\frac{d\sigma}{dp_T}(p\bar{p} \rightarrow W\psi_{\mu\mu}X) \approx K(g \rightarrow S^3S_1)(O^\psi_S(3S_1))B(\psi \rightarrow \mu^+\mu^-)\frac{d\sigma}{dp_T}(p\bar{p} \rightarrow WgX),$$

where $p_T$ refers on the left to $\psi$ and on the right to $g$, for sufficiently large $p_T$. Rapidity cuts must be applied equally to $\psi$ and to the recoil gluon jet. Equation (7) describes only the direct $g \rightarrow \psi$ contribution; indirect $g \rightarrow (\chi_cJ, \psi') \rightarrow \psi$ contributions will enhance the rate by a factor 2 as discussed above. Integrating Eq. (7) for $p_T > 5$ GeV and including this enhancement factor, we obtain

$$\sigma(p\bar{p} \rightarrow \psi_{\mu\mu}W_{e\nu}X; p_T(\psi) > 5$ GeV) \approx 10$ fb ,

where both $\psi \rightarrow \mu\mu$ and $W \rightarrow e\nu$ branching fractions are included. The value for $\sigma(p\bar{p} \rightarrow W_{e\nu}g; p_T(g) > 5$ GeV) on the right-hand-side of Eq. (8) cannot be taken directly from experiment, since $W$+jet production has substantial contributions from $gq(\bar{q}) \rightarrow Wq'(\bar{q}')$ subprocesses; it was calculated at tree level for $\sqrt{s} = 1.8$ TeV (divide the rate by 2(4) if $p_T > 10(15)$ GeV instead). This cross section implies one event per 100 pb$^{-1}$ and is therefore on the edge of detectability at the Tevatron at present; however, including the $\psi \rightarrow ee$ and $W \rightarrow \mu\nu$ decay modes would increase the cross section by a factor 4, and future increases in luminosity could make it measurable.

**Summary**

The proposal that prompt charmonium production at the Tevatron is dominated by a color-octet term in the gluon fragmentation function introduces new parameters that
can however be determined from data \cite{3,4,5}. We have shown that this mechanism can now be tested through its predictions for double-charmonium and charmonium+$W$ production. It is remarkable that indirect $g \rightarrow (\chi_{cJ}, \psi') \rightarrow \psi$ contributions enhance $\psi$ signals by about a factor 2 and analogous $g \rightarrow \chi_{bJ} \rightarrow \Upsilon$ contributions, inferred by scaling arguments \cite{5}, enhance $\Upsilon(1S)$ production by a factor 4. We have found that the predicted rate for $\psi\psi$ production should already be observable at the Tevatron, with the $p_T$ distribution shown in Fig.1; the rates for $\psi\psi'$, $\psi\chi_{c}$, $\psi\Upsilon$, and $\psi\chi_{b}$ production (summarized in Table III) are smaller but the $\psi + (\chi_{c} \rightarrow \psi \gamma)$ signal could already be detectable too. We have also investigated $\psi$ plus gauge boson production at high $p_T$; $\psi + \gamma$ production proves to be insensitive to color-octet contributions, but $\psi + W$ production offers a clean test of the color-octet mechanism, with a predicted rate on the edge of present observability at the Tevatron.

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TABLES

TABLE I. Values of the nonperturbative matrix elements used to calculate $\psi$, $\psi'$ and $\Upsilon$ production at the Tevatron. The color-octet matrix elements for charmonium were determined empirically from CDF data; the color-octet matrix elements for bottomonium were determined by re-scaling the corresponding charmonium matrix elements [5]. The color-singlet matrix elements, shown here for comparison, were determined from potential model calculations [7].

| Matrix Element | $\psi$  | $\psi'$ | $\Upsilon(1S)$ | $\Upsilon(2S)$ |
|----------------|---------|---------|----------------|----------------|
| $\langle O_1(3^3S_1) \rangle$ | 1.2 GeV$^3$ | 0.75 GeV$^3$ | 9.3 GeV$^3$ | 4.5 GeV$^3$ |
| $\langle O_8(3^3S_1) \rangle$ | 0.012 GeV$^3$ | 0.0073 GeV$^3$ | 0.01 GeV$^3$ | 0.006 GeV$^3$ |

TABLE II. Values of the nonperturbative matrix elements used to calculate $\chi_c$ and $\chi_b$ production at the Tevatron; different J-values differ simply through a factor $(2J+1)$. The color-octet matrix element for $\chi_c$ was determined empirically from CDF data; the color-octet matrix element for $\chi_b(1P)$ was found by re-scaling the $\chi_c$ matrix element [5]. The color-singlet matrix elements, shown for comparison, were determined from potential model calculations [7].

| Matrix Element | $\chi_{cJ}$ | $\chi_{bJ}(1P)$ |
|----------------|-------------|-----------------|
| $\langle O_1(3^3P_0)/(2J+1) \rangle$ | 0.11 GeV$^5$ | 2.0 GeV$^5$ |
| $\langle O_8(3^3S_1)/(2J+1) \rangle$ | 0.005 GeV$^3$ | 0.014 GeV$^3$ |

TABLE III. The ratio of the cross section for prompt $\psi + H$ production to the cross section for prompt $\psi + g$ production with $H = \psi', \chi_{cJ}, \Upsilon(1S), \Upsilon(2S), \chi_{bJ}$, based on direct $g \to H$ fragmentation contributions only.

| $H$ | $\sigma(p\bar{p} \to \psi_{\mu\mu} H X) / \sigma(p\bar{p} \to \psi_{\mu\mu} g X)$ |
|-----|--------------------------------------------------------------------------------
| $\psi'$ | $K(g \to \overline{S}^3 S_1) B(\psi' \to \mu^+ \mu^-) \langle O_8^{\psi'}(3^3S_1) \rangle = 6 \times 10^{-7}$ |
| $\chi_{cJ}$ | $K(g \to \overline{S}^3 S_1) \langle O_8^{\chi_{cJ}}(3^3S_1) \rangle = (2J+1) 5 \times 10^{-5}$ |
| $\Upsilon_{\mu\mu}(1S)$ | $K(g \to \overline{S}^3 S_1) B(\Upsilon(1S) \to \mu^+ \mu^-) \langle O_8^{\Upsilon(1S)}(3^3S_1) \rangle = 5 \times 10^{-8}$ |
| $\Upsilon_{\mu\mu}(2S)$ | $K(g \to \overline{S}^3 S_1) B(\Upsilon(2S) \to \mu^+ \mu^-) \langle O_8^{\Upsilon(2S)}(3^3S_1) \rangle = 1 \times 10^{-8}$ |
| $\chi_{bJ}(1S)$ | $K(g \to \overline{S}^3 S_1) \langle O_8^{\chi_{bJ}}(3^3S_1) \rangle = (2J+1) 3 \times 10^{-5}$ |
FIGURES

FIG. 1. The differential cross section for $p\bar{p} \rightarrow \psi\psi X$ versus $p_T(\psi)$ at the Tevatron.
$\text{BR}^2(\psi \rightarrow \mu^+ \mu^-) \frac{d\sigma}{dp_T} (\text{nb/GeV})$