Research Article

An Extended EPQ-Based Problem with a Discontinuous Delivery Policy, Scrap Rate, and Random Breakdown

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In real supply chain environments, the discontinuous multidelivery policy is often used when finished products need to be transported to retailers or customers outside the production units. To address this real-life production-shipment situation, this study extends recent work using an economic production quantity (EPQ)-based inventory model with a continuous inventory issuing policy, defective items, and machine breakdown by incorporating a multiple delivery policy into the model to replace the continuous policy and investigates the effect on the optimal run time decision for this specific EPQ model. Next, we further expand the scope of the problem to combine the retailer’s stock holding cost into our study. This enhanced EPQ-based model can be used to reflect the situation found in contemporary manufacturing firms in which finished products are delivered to the producer’s own retail stores and stocked there for sale. A second model is developed and studied. With the help of mathematical modeling and optimization techniques, the optimal run times that minimize the expected total system costs comprising costs incurred in production units, transportation, and retail stores are derived, for both models. Numerical examples are provided to demonstrate the applicability of our research results.

1. Introduction

This paper focuses on optimizing a producer-retailer integrated economic production quantity (EPQ)-based problem with a discontinuous delivery policy, scrap rate, and random breakdown. The EPQ model was first introduced by Taft [1] and its concept has since been frequently implemented by manufacturing firms to determine the most economic replenishment batch sizes for the products that need to be produced in-house [2, 3]. Although the traditional EPQ model assumes a perfect condition in each production run, in real manufacturing environments due to process deterioration or other uncontrollable factors generation of defective items and random breakdown are inevitable [4]. Widmer and Solot [5] applied a queuing network theory to the study of a breakdown and maintenance operation problem. A simple way of modeling these perturbations was proposed to take into account the performances evaluation of the flexible manufacturing system (FMS) (including the production rate and machine utilization). The analytical and simulation results were compared in order to demonstrate the accuracy of their modeling technique. Yu and Bricker [6] presented an informative application of Markov’s chain analysis to a multistage manufacturing problem. They also pointed out an error in the literature that had been undetected for many years. Groenevelt et al. [7] investigated the effects of breakdowns and corrective maintenance on the economic batch sizing decisions. Two inventory control policies were examined in the case of a breakdown, namely, the no-resumption (NR) and abort-resume (AR). The NR control policy assumes that after a breakdown situation is handled the production of the interrupted lots is not resumed while the AR control policy assumes that if the current on-hand inventory is below a certain threshold level, then the production is immediately resumed after a breakdown situation is taken care of. Their research results indicated that this
control structure is optimal among all stationary policies and
offered the exact optimal and closed form approximate lot
sizing formulas and bounds on average cost per unit time
for the approximations. Widyanada and Wei [8] developed
deteriorating items production inventory models with ran-
dom breakdown and stochastic machine repair time. The
repair time is assumed to be independent of the breakdown
rate. They applied the classical optimization technique to
the problem and derived an optimal solution. Through a
numerical example and sensitivity analysis, they showed that
the production and demand rates are the most sensitive
parameters to the optimal uptime, and the demand rate is the
most sensitive variable to the system costs for the stochastic
model with exponential distribution repair time. Chiu et
al. [9] determined the optimal replenishment run time for
an EPQ-based inventory model with nonconforming items
and breakdown. Their model assumes that after a Poisson
distributed breakdown occurs, the machine goes under repair
instantly and the production of the interrupted lot resumes
immediately when the machine is fixed and restored. The
system also considers a uniformly distributed scrap rate
related to the product process. A mathematical
model along with a recursive searching algorithm is used
in their study to derive the optimal replenishment policy
that minimizes the total system costs. A numerical example
was provided to demonstrate the practical application and
better cost efficiency of the proposed policy compared to
a breakdown that occurs under a no-resumption policy.
Additional studies relating to the issues of product quality,
unreliable production equipment, and their consequence
quality assurance can be found in [10–17].

Unlike the assumption of a continuous inventory issuing
policy of the traditional EPQ model, in real supply chain
environments, the discontinuous multidelivery policy is often
used when finished products need to be transported to
retailers or customers outside the production units. Schwarz
et al. [18] determined the fill-rate of a one-warehouse,
N-identical retailer distribution system. An approximation
model was adopted from a prior work to maximize the
system fill-rate subjected to a constraint on safety stock.
The properties of the fill-rate policy were used to provide
managerial insights into system optimization. Sarker and
Khan [19] examined a manufacturing system that procures
raw materials in a lot from suppliers and processes them
into the finished products that are subsequently shipped to
outside customers at fixed points in time. The system cost
function for the model was formulated by including both
raw materials and finished products. A solution procedure
was developed to determine an optimal ordering policy for
the procurement of raw materials and the production batch
size that minimizes the total system costs. Gómez et al. [20]
considered a centralized inventory sharing system of two
retailers that are replenished periodically. They assumed that,
between two replenishments, a unit can be transshipped to a
stocked-out retailer from the other retailer. Whenever there
is an absence of transshipments, the backorder costs are
incurred until the next replenishment. The objective of their
study is to minimize the long-run average costs, comprising
the replenishment, holding, backorder, and transshipment
costs. They discussed the challenges associated with positive
replenishment time and developed upper and lower bounds
of average costs in such situations. Other studies [21–29]
focused on various aspects of periodic or multiple delivery
issues in the vendor-buyer integrated supply chains.

With the purpose of addressing the real-life production-
shipment situation, this study extends a recent work [9] by
incorporating a multiple delivery policy into their model
to replace the continuous policy and investigates the effect
on the optimal run time decision for this specific EPQ
model. Next, we further expand the scope of the problem
to combine the retailer's stock holding cost into our study.
This enhanced EPQ-based model can be used to reflect the
situation found in contemporary manufacturing firms in
which finished products are delivered to the producer's own
retail stores and stocked there for sale. The objectives are to
determine the optimal run times that minimize the expected
total system costs comprising costs incurred in production
units, transportation, and retail stores for both models. As
little attention has been paid to this specific area, the present
study is intended to fill this gap.

2. Statement and Optimization of
Proposed Model 1

In real supply chain environments, the discontinuous mul-
delivery policy is often used when finished products need
to be transported to retail stores or customers outside the
production units. To explicitly address this realistic situation,
the first proposed model in this study incorporates a multiple
delivery policy into an EPQ-based inventory model with
scrap items and breakdown [9] to replace the continuous
issuing policy and investigates the effect on the optimal
manufacturing run time decision.

Summary of assumptions (features) considered in the
proposed multi-item EPQ-based model are as follows: (1) a
random machine breakdown rate, (2) a random scrap rate in
production, and (3) a discontinuous multidelivery policy for
finished products. The details of the proposed model can be
described as follows. Suppose a product can be manufactured
at an annual rate $P_1$ and its demand is $\lambda$ units per year.
All items produced must pass a quality conformation check, and
the unit screening cost is included in the unit manufacturing
cost $C$. A random $x$ proportion of the products produced
is defective and will be scrapped at the end of the regular
production process. Hence, the production rate of scrap
is $d_1$ and $d_1 = P_1x$. Under regular operations (i.e., to
avoid a stock-out situation) $(P_1 - d_1 - \lambda) > 0$ must be
satisfied. Upon the completion of the production process,
the acceptable quality (finished) products are transported to
the outside retail store or customer, under a discontinuous
multidelivery policy, while fixed quantity $n$ installments of
the finished items are shipped to retail store at a fixed interval
of time in $t_2$ (Figure 1). The proposed model assumes that,
during the production uptime, a Poisson distributed machine
breakdown may occur, and an abort/resume (A/R) inventory
control policy is employed when a breakdown happens.
Under such an A/R policy, the machine goes under repair
immediately and a constant repair time is assumed. Upon
the completion of the repair, the interrupted lot is instantly
resumed (Figure 1).

Additional cost-related parameters used in this study
are the machine repairing cost $M$, setup cost per cycle $K$,
holding cost per item at the producer's side $h$, disposal cost
per scrapped item $C_s$, fixed delivery cost per shipment $K_1$,
variable delivery cost per item $C_T$, unit holding cost for safety
stock at the producer's side $h_3$, and holding cost per item at
the retailer's side $h_2$. Other notations used in the modeling
and analysis also include the following:

$t$: production time before a random machine break-
down takes place,
$H'_1$: on-hand inventory level in units when a random
machine breakdown takes place,
$\beta$: number of machine breakdowns per unit time (i.e.,
year), assumed to be a random variable that follows
the Poisson distribution,
$t_r$: machine repair time,
$t_1$: production uptime, the decision variable of the
proposed manufacturing run time model,
$H'$: maximum on-hand inventory level in units when
the regular production process ends (in the case of a
breakdown),
$t'_2$: time required to deliver all finished items produced
in a cycle (in the case of a breakdown),
$T'$: production cycle length (in the case of a break-
down),
$Q$: lot size for each production cycle,
$TC_1(t_1)$: total production-inventory-delivery costs
per cycle (in the case of a breakdown),
$E[TC_1(t_1)]$: the expected production-inventory-
delivery costs per cycle (in the case of a breakdown),
t_2$: time required to deliver all finished items produced
in a cycle (in the case of no breakdown),
$H$: on-hand inventory level in units when the regular
production process ends (in the case of no break-
down),
$T$: cycle length (in the case of no breakdown),
$I(t)$: on-hand inventory level of finished items at time $t$,
$I_s(t)$: on-hand inventory level of scrap items at time $t$,
$TC_2(t_1)$: total production-inventory-delivery costs
per cycle (in the case of no breakdown),
$E[TC_2(t_1)]$: the expected production-inventory-
delivery costs per cycle (in the case of no breakdown),
$TCU(t_1)$: total production-inventory-delivery costs
per unit time whether or not a breakdown takes place,
$E[TCU(t_1)]$: the long-run expected production-
inventory-delivery costs per unit time whether or not a
breakdown takes place,
$T$: the cycle length whether or not a machine break-
down takes place.

Since a machine breakdown may randomly take place at
production uptime $t_1$, the following two distinct cases must
be examined.

2.1. Case 1: A Random Machine Breakdown Takes Place at
Uptime $t_1$. In such a situation, $t < t_1$. Under the AR inventory
control policy, the machine goes under repair immediately,
and once it is fixed and restored, the interrupted lot is
instantly resumed (Figure 1). Since $x$ proportion of scrap
products is produced, the maximum number of scraps in a
cycle is \( xQ \) (or \( d_1 t_1 \)), and the on-hand inventory of scrap
items in the proposed manufacturing run time problem is as
illustrated in Figure 2.

The production cycle time \( T' \) can be seen as (1) from
Figure 1

\[
T' = t_1 + t_r + t_2' .
\]  

The total production-inventory-delivery cost per cycle,
\( TC_1(t_1) \), is comprised of (1) the variable production cost, (2)
the setup cost, (3) the disposal cost for scraps, (4) the machine
repair cost, (5) fixed and variable product delivery costs, (6)
holding cost for safety stocks, and (7) the producer’s inventory
holding costs in the entire production cycle. Thus, \( TC_1(t_1) \) is

\[
TC_1(t_1) = C(P_1 t_1) + K + C_S(t_1 P_1 x) + M + nK_1
+ C_T[t_1 P_1] + h_3 (\lambda t_r) T'
+ h \left[ \frac{H' + d_1 t_1}{2} t_1 + (H' + d_1 t) t_r + \frac{n-1}{2n} H' t_2' \right] .
\]  

Since \( x \) is assumed to be a random variable with a known
probability density function, the expected values of \( x \) are used
in our analysis to take the randomness of \( x \) into account. By
substituting all related system parameters into (2) [9], with
further derivations, \( E[TC_1(t_1)] \) becomes (see the appendix
for more details)

\[
E[TC_1(t_1)]
= K + nK_1 + M + hP_1 g
+ \left[ CP_1 + C_S E[x] P_1 + C_T P_1 (1 - E[x])
+ h_3 P_1 g (1 - E[x]) - \frac{hP_1 g (1 - E[x])}{2} \left( 1 - \frac{1}{n} \right) \right] t_1
+ \left[ \frac{hP_1 E[x]}{2} + \frac{hP_1^2}{2\lambda} (1 - E[x])^2 \left( 1 - \frac{1}{n} \right) \right] t_1^2,
\]  

2.2 Case 2: No Breakdown Takes Place at Uptime \( t_1 \). In such
a situation, \( t > t_1 \). The inventory level of finished items in
this case is depicted in Figure 3, and \( T = t_1 + t_2 \). The total
production-inventory-delivery cost per cycle \( TC_2(t_1) \) is as
displayed in

\[
TC_2(t_1) = C(P_1 t_1) + K + C_S (xt_1 P_1) + nK_1
+ C_T[t_1 P_1 (1 - x)] + h_3 (\lambda t_r) T
+ h \left[ \frac{H + d_1 t_1}{2} t_1 + \frac{n-1}{2n} H t_2' \right] .
\]  

Again, to take the randomness of \( x \) into account and
substitute all related parameters into (4), with further derivations,
\( E[TC_2(t_1)] \) becomes [12]

\[
E[TC_2(t_1)] = K + nK_1
+ \left[ CP_1 + C_S E[x] P_1 + C_T P_1 (1 - E[x])
+ h_3 P_1 g (1 - E[x]) \right] t_1
+ \left[ \frac{hP_1 E[x]}{2} + \frac{hP_1^2}{2\lambda} (1 - E[x])^2 \left( 1 - \frac{1}{n} \right) \right] t_1^2.
\]  

Figure 2: Inventory level of scrap items in the proposed manufacturing run time problem.
2.3. Integration of the Proposed Run Time Models with/without Breakdown. A machine breakdown may take place randomly and it follows a Poisson distribution with mean equal to $\beta$ per year. Let $f(t)$ be the probability density function of random time $t$ before a breakdown takes place, and $F(t)$ represents the cumulative density function of $t$. Hence, the long-run expected system costs per unit time $E[TCU(t_1)]$ are

$$E[TCU(t_1)] = \frac{\int_0^{t_1} E[TC_1(t_1)] f(t) dt + \int_{t_1}^{\infty} E[TC_2(t_1)] f(t) dt}{E[T]},$$

where

$$E[T] = \int_0^{t_1} E[T'] f(t) dt + \int_{t_1}^{\infty} E[T] f(t) dt$$

$$= \frac{t_1 P_1 (1 - E[x])}{\lambda}.$$ (7)

From Figures 1 and 3, it can be seen that $T'$ and $T$ are different in length ($T'$ is longer than $T$ since it contains machine repairing time) and because a breakdown can occur randomly, it is necessary to use the integration (i.e., equation (7)) to derive the expected cycle length.

It is also noted that the time between breakdowns obeys the exponential distribution with density function $f(t) = \beta e^{-\beta t}$ and cumulative density function $F(t) = 1 - e^{-\beta t}$. By substituting $E[TC_1(t_1)]$, $E[TC_2(t_1)]$, and $E[T]$ into (6) and solving the integration of the mean time to breakdown in $E[TCU(t_1)]$, we obtain

$$E[TCU(t_1)] = \frac{\lambda}{(1 - E[x])} \cdot \left\{ \frac{(K + nK_1)}{t_1 P_1} + \gamma_1 + \gamma_2 \frac{1}{2} \left( \frac{M}{P_1} + \frac{h g}{\beta} \right) \left( 1 - \frac{e^{-\beta t_1}}{t_1} \right) \right\} - \gamma_3 (e^{-\beta t_1}) - \gamma_4 \left( \frac{1 - E[x]}{1 - \frac{1}{n}} (1 - e^{-\beta t_1}) \right),$$

where

$$\gamma_1 = [C + C_T E[x] + C_{T'} (1 - E[x]) + h g (1 - E[x])],$$

$$\gamma_2 = \left[ \frac{h P_1}{\lambda} (1 - E[x])^2 \left( 1 - \frac{1}{n} \right) + h E[x] + \frac{h}{n} (1 - E[x]) \right].$$ (8)

2.4. Derivation of the Optimal Production Run Time. In order to derive the optimal production run time $t_1^*$, we first have to prove that $E[TCU(t_1)]$ is convex. Let $\xi(t_1)$ represent the following:

$$\xi(t_1) = \frac{2(K + nK_1) \beta + 2(1 - e^{-\beta t_1}) \gamma_4}{t_1^2 P_1 \beta^2 \gamma_3 + \gamma_4 (2 + \beta t_1)} \beta e^{-\beta t_1}.$$ (10)
Theorem 1 \((E[TCU(t_1)])\) is convex if \(0 < t_1 < \xi(t_1)\). The second derivative of \(E[TCU(t_1)]\) with respect to \(t_1\) is

\[
\frac{d^2 E [TCU (t_1)]}{d t_1^2} = \frac{\lambda}{(1 - E [x])} \cdot \left[ \frac{2 (K + nK_1)}{t_1^3 P_1} - hg \left[ \frac{1 - (1 - E [x])}{2} \left( \frac{1 - 1}{n} \right) \right] (\beta^2 e^{-\beta t_1}) \right. \\
+ \left. \left[ M + \frac{hg}{\beta} \right] \left( \frac{2 \left( 1 - e^{-\beta t_1} \right)}{t_1^3} - \frac{2\beta e^{-\beta t_1}}{t_1^2} - \frac{\beta^2 e^{-\beta t_1}}{t_1} \right) \right].
\]

(12)

It is noted that because annual demand \(\lambda > 0\), the first term in the right-hand size (RHS) of (12) is positive. Hence, we obtain

\[
\frac{d^2 E [TCU (t_1)]}{d t_1^2} > 0
\]

if

\[
\frac{2 (K + nK_1)}{t_1^3 P_1} - hg \left[ \frac{1 - (1 - E [x])}{2} \left( \frac{1 - 1}{n} \right) \right] (\beta^2 e^{-\beta t_1}) \\
+ \left[ M + \frac{hg}{\beta} \right] \left( \frac{2 \left( 1 - e^{-\beta t_1} \right)}{t_1^3} - \frac{2\beta e^{-\beta t_1}}{t_1^2} - \frac{\beta^2 e^{-\beta t_1}}{t_1} \right) > 0.
\]

(13)

The RHS of (13) can be further derived as

\[
\frac{d^2 E [TCU (t_1)]}{d t_1^2} > 0
\]

if

\[
\left[ \frac{2 (K + nK_1)}{t_1^3 P_1} - hg \left[ \frac{1 - (1 - E [x])}{2} \left( \frac{1 - 1}{n} \right) \right] (\beta^2 e^{-\beta t_1}) \right. \\
+ \left. \left[ M + \frac{hg}{\beta} \right] \left( \frac{2 \left( 1 - e^{-\beta t_1} \right)}{t_1^3} - \frac{2\beta e^{-\beta t_1}}{t_1^2} - \frac{\beta^2 e^{-\beta t_1}}{t_1} \right) \right] > 0.
\]

(14)

Theorem 2 \((t^*_1 < t^*_1 < t^*_1 U)\). Because the proof of \(t^*_1\) falls within the upper and lower bounds, we can multiply the second term of (18) by \((2\beta P_1 t_1^2)\) and obtain

\[
\left\{ \left( P_1 \beta y_2 + 2 P_1 \beta^2 y_3 e^{-\beta t_1} \right) t_1 + \left( 2 \beta y_4 e^{-\beta t_1} \right) t_1 \right. \\
- \left[ \beta (K + nK_1) + y_4 (1 - e^{-\beta t_1}) \right]\} = 0.
\]

(21)
Thus
\[ t^*_1 = \text{the positive root of} \]
\[
\left\{ \begin{array}{l}
\left(-2\gamma_4\beta e^{-\beta t_1}\right) \\
\pm \left(2\gamma_4\beta e^{-\beta t_1}\right) \times \left[-2 \beta (K + nK_1) + \gamma_4 (1 - e^{-\beta t_1})\right]^{1/2} \\
\times \left(2 \left(2P_1\beta + 2P_1\beta^2\gamma_3 e^{-\beta t_1}\right)^{-1}\right) \}
\right.
\]

Equation (21) can be rearranged as
\[
2 \left[P_1\beta^2\gamma_3 t^*_1 + \gamma_4\beta t^*_1 + \gamma_4\right] \left(e^{-\beta t_1}\right) = 2 \left[\beta (K + nK_1) + \gamma_4\right] - \left(P_1\beta t^*_1\right)^2
\]
\[ = 2 \left[\beta (K + nK_1) + \gamma_4\right] - \left(P_1\beta t^*_1\right)^2
\]
\[ \text{or}
\]
e^{-\beta t_1} = \frac{2 \left[\beta (K + nK_1) + \gamma_4\right] - \left(P_1\beta t^*_1\right)^2}{2 \left[P_1\beta^2\gamma_3 t^*_1 + \gamma_4\beta t^*_1 + \gamma_4\right],
\]

where \( e^{-\beta t_1} \) is the complement of the cumulative density function \( F(t_1) = 1 - e^{-\beta t_1} \). As \( 0 \leq F(t_1) \leq 1 \), \( 0 \leq e^{-\beta t_1} \leq 1 \). Let \( e^{-\beta t_1} = 0 \) and \( e^{-\beta t_1} = 1 \) be the upper and lower bounds for \( e^{-\beta t_1} \), respectively. By substituting them into (22), we obtain
\[
\begin{align*}
t^*_{1U} &= \sqrt{\frac{2 \left[\beta (K + nK_1) + \gamma_4\right]}{\gamma_2 P_1 \beta}} \\
t^*_{1L} &= \text{the positive root of} \]
\[
\left\{ -\gamma_4 \pm \sqrt{\gamma_4^2 + 2P_1 \left(\gamma_2 + 2\beta P_1\right) (K + nK_1)} \right\} \]
\[ \left\{ \begin{array}{l}
P_1 \left(\gamma_2 + 2\beta P_1\right) \
\right.
\]
\[ \text{and} \ t^*_{1L} < t^*_1 < t^*_{1U}.
\]

It is noted that although the optimal production run time \( t^*_1 \) cannot be presented in a closed form, it does fall within the bounds. \( t^*_1 \) can be located with the use of a proposed recursive searching algorithm. Let
\[
\omega(t_1) = e^{-\beta t_1} = \frac{2 \left[\beta (K + nK_1) + \gamma_4\right] - \left(P_1\beta t^*_1\right)^2}{2 \left[P_1\beta^2\gamma_3 t^*_1 + \gamma_4\beta t^*_1 + \gamma_4\right],
\]
\[ : 0 \leq \omega(t_1) \leq 1.
\]

In order to locate \( t^*_1 \), we can use the following recursive searching algorithm.

1. Let \( \omega(t_1) = 0 \) and \( \omega(t_1) = 1 \) initially and calculate the upper and lower bounds for \( t^*_1 \), respectively (i.e., to obtain the initial values of \( t^*_{1U}, t^*_{1L} \)).
2. Substitute the current values of \( t^*_{1L}, t^*_{1U} \) into \( e^{-\beta t_1} \) and compute the new bounds, expressed as \( \omega_L \) and \( \omega_U \), for \( e^{-\beta t_1} \). Hence, \( \omega_L < \omega(t_1) < \omega_U \).
3. Let \( \omega(t_1) = \omega_L \) and \( \omega(t_1) = \omega_U \), and update the upper and lower bounds for \( t^*_1 \), respectively (i.e., to obtain the new values of \( t^*_{1L}, t^*_{1U} \)).
4. Repeat steps (2) and (3) until there is no significant difference between \( t^*_{1L}, t^*_{1U} \) (or there is no significant difference in terms of their effects on \( E[TCU(t^*_1)] \)).
5. Stop. \( t^*_1 \) is found.

3. Extension to a Producer-Retailer Integrated EPQ-Based System (Model 2)

3.1. Enhanced Model Description and Formulation. In this section, we further extend the scope of the problem to incorporate the retailer's stock holding cost into our study. The new model can be considered a producer-retailer integrated system, because, in the present-day manufacturing sector, some producers of consumer goods may own and operate retail stores or regional sales offices to promote and sell their end products to customers (see Figure 4). With the intention of addressing such a real-life intrasupply chain situation, the second model of this study incorporates the retailer's stock holding cost into the first model and investigates its effect on the optimal production run time decision.

**Figure 4:** Extension to a producer-retailer integrated production-shipment system.
In the proposed study, the retailer’s stock holding positions are illustrated in Figure 5.

Extra parameters used in this enhanced model include the following.

\( h_2 \): holding cost per product stored on the retailer’s side,

\( I_c(t) \): on-hand inventory levels in units on the retailer’s side end at time \( t \),

\( D \): number of finished products (a fixed quantity) transported to the retail store per shipment,

\( I \): number of left-over products in \( t_n \) after satisfying the demand in \( t_n \),

\( T_C(t_1) \): total production-inventory-delivery costs per cycle of this enhanced model (in the case of a breakdown),

\( T_C(t_1) \): total production-inventory-delivery costs per cycle of this enhanced model (in the case of no breakdown),

\( E[T_C(t_1)] \): the expected production-inventory-delivery costs per cycle of this enhanced model (in the case of a breakdown),

\( E[T_C(t_1)] \): the expected production-inventory-delivery costs per cycle of this enhanced model (in the case of no breakdown),

\( E[T_C(t_1)] \): the long-run expected production-inventory-delivery costs per unit time in this enhanced model, whether or not a breakdown takes place.

Since the demand on the retailer’s side in time interval \( t_n \) is \( \lambda t_n \), after satisfying the demand, the number of left-over items (see Figure 5) in each \( t_n \) is

\[ I = D - \lambda t_n. \] (27)

Total inventory holding costs on retailer’s side with and without breakdown are shown, respectively, in

\[ h_2 \left[ \frac{n (D - I)}{2} t_n + \frac{n(n + 1)}{2} I t_n + \frac{n I}{2} (t_1 + t_r) \right], \] (28)

\[ h_2 \left[ \frac{n (D - I)}{2} t_n + \frac{n(n + 1)}{2} I t_n + \frac{n I}{2} (t_1) \right]. \] (29)

To incorporate the retailer’s holding costs into the original models with and without breakdown, respectively, we obtain

\[ T_C(t_1) = T_C(t_1) + h_2 \left[ \frac{n (D - I)}{2} t_n + \frac{n(n + 1)}{2} I t_n + \frac{n I}{2} (t_1 + t_r) \right], \]

\[ T_C(t_1) = T_C(t_1) + h_2 \left[ \frac{n (D - I)}{2} t_n + \frac{n(n + 1)}{2} I t_n + \frac{n I}{2} (t_1) \right]. \] (30)

To take the randomness of defective rate \( x \) into account and substitute all related variables into (30), with further derivations, \( E[T_C(t_1)] \) and \( E[T_C(t_1)] \) can be obtained as follows:

\[ E \left[ T_C(t_1) \right] = E \left[ T_C(t_1) \right] + h_2 \left( \frac{P_1 g(1 - E[x])}{2} \right) \cdot \left( 1 - \frac{1}{n} \right) \cdot \left( t_1 + h_2 \left( \frac{P_1 (1 - E[x])}{2} \right) \right) \cdot \left( \frac{P_1 (1 - E[x])}{\lambda n} + \left( 1 - \frac{1}{n} \right) \right) \cdot t_1^2, \] (31)

\[ E \left[ T_C(t_1) \right] = E \left[ T_C(t_1) \right] + h_2 \left( \frac{P_1 (1 - E[x])}{2} \right) \cdot \left( \frac{P_1 (1 - E[x])}{\lambda n} + \left( 1 - \frac{1}{n} \right) \right) \cdot t_1^2. \]
3.2. Integration of Enhanced Model with/without Breakdown. The mean time to breakdowns obeys the exponential distribution with \( f(t) = \beta e^{-\beta t} \). Therefore, \( E[TCU_2(t_1)] \) is

\[
E[TCU_2(t_1)] = \frac{\int_0^1 E[TC_3(t_1)] f(t) \, dt + \int_{t_1}^\infty E[TC_4(t_1)] f(t) \, dt}{E[T]},
\]

(32)

Substituting \( E[TCU_1(t_1)], E[TC_4(t_1)], \) and \( E[T] \) into (32) and resolving \( E[TCU_2(t_1)] \), we obtain

\[
E[TCU_2(t_1)] = \frac{\lambda}{(1 - E[x])} \cdot \left\{ \frac{K + nk_1}{t_1 P_1} + \frac{\gamma_2 t_1}{2} + \gamma_5 t_1 \right. \\
+ \left[ \frac{M}{P_1} + \frac{h g}{\beta} \right] \left[ 1 - e^{-\beta t_1} \right] - h g \left( e^{-\beta t_1} \right) \\
- \left( h - h_2 \right) \left[ \frac{g \left( 1 - E[x] \right)}{2} \left( 1 - \frac{1}{n} \right) \right] \left( 1 - e^{-\beta t_1} \right) \right\}.
\]

(33)

where

\[
\gamma_5 = \frac{h_2 (1 - E[x])}{2} \left[ \frac{P_1 (1 - E[x])}{\lambda n} + \left( 1 - \frac{1}{n} \right) \right].
\]

(34)

3.3. Determining the Optimal Run Time. Let \( \psi(t_1) \) stand for the following:

\[
\psi(t_1) = \frac{2 (K + nk_1) \beta + 2 \gamma_4 (1 - e^{-\beta t_1})}{\left[ t_1^2 P_1 \beta^2 \gamma_6 + \gamma_4 \left( 2 + \beta t_1 \right) \right] \beta e^{-\beta t_1}}.
\]

(35)

Theorem 3 \( E[TCU_2(t_1)] \) is convex if \( 0 < t_1 < \psi(t_1) \). The second derivative of \( E[TCU_2(t_1)] \) with respect to \( t_1 \) is

\[
\frac{d^2 E[TCU_2(t_1)]}{d t_1^2} = \frac{\lambda}{(1 - E[x])} \left[ \frac{2(K + nk_1)}{t_1^2 P_1} + (h - h_2) \frac{g \left( 1 - E[x] \right)}{2} \right.
\\
\cdot \left( 1 - \frac{1}{n} \right) \left( \beta^2 e^{-\beta t_1} \right) - h g \left( \beta^2 e^{-\beta t_1} \right) \\
+ \left[ \frac{M}{P_1} + \frac{h g}{\beta} \right] \left( \frac{2 \left( 1 - e^{-\beta t_1} \right)}{t_1^2} - 2 \beta e^{-\beta t_1} - \frac{\beta^2 e^{-\beta t_1}}{t_1} \right) \left[ \beta e^{-\beta t_1} \right] \right].
\]

(36)

Since annual demand \( \lambda > 0 \), the first term in the RHS of (36) is positive, and

\[
if \left[ \frac{2(K + nk_1)}{t_1^2 P_1} + (h - h_2) \left( \frac{g \left( 1 - E[x] \right)}{2} \left( 1 - \frac{1}{n} \right) \right) \right.
\\
\cdot \left( \beta^2 e^{-\beta t_1} \right) - h g \left( \beta^2 e^{-\beta t_1} \right) + \left[ \frac{M}{P_1} + \frac{h g}{\beta} \right]
\\
\left. \cdot \left( \frac{2 \left( 1 - e^{-\beta t_1} \right)}{t_1^2} - 2 \beta e^{-\beta t_1} - \frac{\beta^2 e^{-\beta t_1}}{t_1} \right) \right] > 0.
\]

(37)

With further derivations, the left-hand side (LHS) of (37) becomes

\[
if \left[ \frac{2(K + nk_1)}{t_1^2 P_1} (h - h_2) \left( \frac{g \left( 1 - E[x] \right)}{2} \left( 1 - \frac{1}{n} \right) \right) \right.
\\
\cdot \left( \beta^2 e^{-\beta t_1} \right) - t_1^2 P_1 \beta h g \left( \beta^2 e^{-\beta t_1} \right)
\\
+ \left( M \beta + h g P_1 \right) \left[ 2 \left( 1 - e^{-\beta t_1} \right) - 2 t_1 \beta e^{-\beta t_1} - \beta^2 t_1^2 e^{-\beta t_1} \right] \right] > 0.
\]

(38)

Let

\[
\gamma_6 = \left( h - h_2 \right) \left[ \frac{g \left( 1 - E[x] \right)}{2} \left( 1 - \frac{1}{n} \right) \right] + h g;
\]

(39)

then, (37) becomes

\[
if \left[ \frac{2(K + nk_1)}{t_1^2 P_1} (h - h_2) \left( \frac{g \left( 1 - E[x] \right)}{2} \left( 1 - \frac{1}{n} \right) \right) \right.
\\
\cdot \left( \beta^2 e^{-\beta t_1} \right) - \beta t_1^2 P_1 \beta \gamma_6 \left( 2 + \beta t_1 \right) \beta e^{-\beta t_1} \right] > 0.
\]

(40)

or

\[
\frac{d^2 E[TCU_2(t_1)]}{d t_1^2} > 0
\]

if \( 0 < t_1 < \frac{2(K + nk_1) \beta + 2 \gamma_4 (1 - e^{-\beta t_1})}{t_1^2 P_1 \beta^2 \gamma_6 + \gamma_4 \left( 2 + \beta t_1 \right) \beta e^{-\beta t_1}} = \psi(t_1).
\]

(41)
Once \( E[T_{CU}(t_1)] \) is proven to be convex, the optimal run time \( t_1^* \) can be solved by setting the first derivative of \( E[T_{CU}(t_1)] \) to zero:

\[
\frac{dE\left[T_{CU}(t_1)\right]}{dt_1} = \lambda \left( \frac{-(K + nK_1)}{t_1^2} + \frac{y_2}{2} + y_5 + y_6(\beta e^{-\beta t_1}) \right) + \left( \frac{M}{P_1} + \frac{h g}{\beta} \right) \left( \frac{-(1 - e^{-\beta t_1})}{t_1^2} + \frac{\beta e^{-\beta t_1}}{t_1} \right) = 0. \tag{42}
\]

It can be seen that the first term in the RHS of (42) is positive, so the second term is equal to zero. In order to find the bounds for \( t_1^* \), let

\[
t_{1U} = \sqrt{\frac{2[\beta(K + nK_1) + y_4]}{P_1 \beta(y_2 + 2y_5)}}, \tag{43}
\]

\[
t_{1L} = \text{the positive root of}
\left\{ \begin{array}{l}
\frac{-y_4 \pm \sqrt{y_4^2 + 2P_1(K + nK_1)(y_2 + 2y_5 + 2y_6\beta)}}{P_1(y_2 + 2y_5 + 2y_6\beta)} \\
\end{array} \right. \tag{44}
\]

**Theorem 4** \((t_{1L} < t_1^* < t_{1U})\). For the proof of Theorem 4 please refer to the proof for Theorem 2 in Section 2.

Once we are certain that \( t_1^* \) falls within the aforementioned upper and lower bounds, in order to find \( t_1^* \), we can first multiply the second term of (42) by \((2P_1 \beta t_1^*)\) and obtain the following:

\[
\left\{ \begin{array}{l}
\left[ P_1 \beta(y_2 + 2y_5) + 2P_1 \beta^2 y_6 e^{-\beta t_1} \right] t_1^2 + \left( 2y_4 \beta e^{-\beta t_1} \right) t_1^1 \\
-2 \left[ \beta(K + nK_1) + y_4 \left( 1 - e^{-\beta t_1} \right) \right] \\
\end{array} \right. \tag{45}
\]

\[
e^{-\beta t_1} = \frac{2 \left[ \beta(K + nK_1) + y_4 \right] - \left( P_1 \beta(y_2 + 2y_5) \right) t_1^2}{2 \left( P_1 \beta^2 y_6 t_1^1 + y_4(1 + \beta t_1) \right)}, \tag{46}
\]

where \( e^{-\beta t_1} \) is the complement of the cumulative density function \( F(t_1) = 1 - e^{-\beta t_1} \). As \( 0 \leq F(t_1) \leq 1, 0 \leq e^{-\beta t_1} \leq 1 \). Let \( e^{-\beta t_1} = 0 \) and \( e^{-\beta t_1} = 1 \) be the initial upper and lower bounds of \( e^{-\beta t_1} \), respectively. Then, by using the proposed recursive searching algorithm given at the end of Section 2, we can find the optimal production run time \( t_1^* \).

### 4. Numerical Example

In order to relieve the comparison efforts for readers, this section adopts the same numerical example as in [9]. For a demonstration of the proposed EPQ-based model 1, the following system parameters are used.

- \( P_1 \): production rate, 10,000 products per year;
- \( \lambda \): demand rate, 4,000 products per year;
- \( x \): random scrap rate, which follows uniformly distribution over the interval \([0, 0.2]\);
- \( \beta \): Poisson breakdown rate, 0.5 average times per year;
- \( g \): constant machine repair time \( t_r \), 0.018 year per repair;
- \( M \): machine repair cost, \$500 for each breakdown;
- \( K \): setup cost, \$450 per production run;
- \( C \): manufacturing cost, \$2 per item;
- \( C_D \): disposal cost, \$0.3 per scrap item;
- \( h \): holding cost, \$0.6 per item per unit time;
- \( K_1 \): fixed delivery cost, \$90 per shipment;
- \( n \): number of deliveries, 4 per cycle;
- \( C_T \): variable delivery cost, \$0.001 per item.

First, we use both upper and lower bounds of \( t_1^* \) to test for the convexity of \( E[T_{CU}(t_1)] \) (see Theorem 1). The computation results of (19), (20), and (11) indicate that \( t_{1L} = 0.5183 < \xi(t_{1L}) = 2.8723 \), and \( t_{1U} = 0.3411 < \xi(t_{1U}) = 2.6261 \). Hence, \([T_{CU}(t_1)]\) is convex (Figure 6).

In order to find the optimal \( t_1^* \), we first substitute the upper and lower bounds of \( t_1^* \) in (8) and obtain \( E[T_{CU}(t_{1L})] = \$11,601.63 \) and \( E[T_{CU}(t_{1U})] = \$11,014.50 \), respectively. Because the optimal run time \( t_1^* \) falls within the interval of \([t_{1L}, t_{1U}]\), we apply the proposed recursive searching algorithm stated at the end of Section 2 and find \( t_1^* = 0.3748 \). Afterwards, we find that \( t_{1L} = 0.3411 \) and \( t_{1U} = 0.5183 \). The step-by-step iterations of the algorithm are shown in Figure 6. Table 1 shows the step-by-step iterations of the algorithm.
The expected cost \( E[TCU(t_1^*)] \) on the optimal system cost \( E[TCU(t_1^*)] \) and on the optimal production run time \( t_1^* \) (see Table 2). It can be seen from Table 2 that as the ratio of \( K_1/K \) increases, the expected system costs per unit time \( E[TCU(t_1^*)] \) increase significantly. It is also noted that as \( K_1 \) increases, optimal production run time \( t_1^* \) also increases significantly.

4.1. Numerical Example for the Producer-Retailer Integrated EPQ System (Model 2). In order to demonstrate the research result of the producer-retailer integrated EPQ-based model, an additional system variable \( h_2 = $1.50 \) per item stored at the retailer's side is included.

Again, one can use the upper and lower bounds of \( t_1^* \) (equations (43) and (44)) to test for convexity of \( E[TCU(t_1^*)] \) (Theorem 3 and equation (35)). The results reveal that \( t_{1,U}^* = 0.3213 < \psi(t_{1,U}^*) = 2.6462 \) and \( t_{1,L}^* = 0.2186 < \psi(t_{1,L}^*) = 2.4876 \). Therefore, the expected cost \( [TCU(t_1^*)] \) is convex.

Next, by applying the proposed recursive searching algorithm we can calculate that the optimal run time \( t_1^* = 0.2314 \) years and the optimal \( E[TCU(t_1^*)] = $12,138.49 \). It is noted that the computation time for reaching the optimal \( t_1^* \) solution is 2.1 seconds (using Excel software in a desktop computer: Intel CPU G850 with 2.94 GB RAM and 2.89 GHz).

Figure 7 illustrates the behavior of \( E[TCU(t_1^*)] \) with regard to production run time. It is noted that, without the research result from the second model, the management of such a producer-retailer integrated system would probably use \( t_1 = 0.3748 \) years (from the result of model 1) for their run time decision. Further analysis (see Figure 7) shows cost savings of $351 (or 2.9% over the total system costs) simply by applying our research result.

The effects of the unit retailer's holding cost \( h_2 \) on the expected system cost \( E[TCU(t_1^*)] \) and on the optimal run time \( t_1^* \) are shown in Table 3, respectively.

It can be seen that as \( h_2 \) or the ratio of \( h_2/h \) increases, the expected cost \( E[TCU(t_1^*)] \) increases, but the optimal production run time \( t_1^* \) decreases. In decision-making, these sensitivity analyses results can provide the management of a producer-retailer integrated system with valuable information and insights into the effects of various stock holding costs in different retailers' locations.

| \( \beta \) | Step # | \( t_{1,U}^* \) | \( \omega_U = e^{-\beta t_{1,U}^*} \) | \( t_{1,L}^* \) | \( \omega_L = e^{-\beta t_{1,L}^*} \) | Difference between \( t_{1,U}^* \) and \( t_{1,L}^* \) | \( [U] E[TCU(t_{1,U}^*)] \) | \( [L] E[TCU(t_{1,L}^*)] \) | Difference between \([U]\) and \([L]\) |
|---|---|---|---|---|---|---|---|---|---|
| 0.5 | Initial | 0.00000 | 1.00000 | | | | | |
| | 1st | 0.5183 | 0.7717 | 0.3411 | 0.8432 | 0.1772 | $11,601.63 | $11,014.50 | $587.13 |
| | 2nd | 0.3857 | 0.8246 | 0.3721 | 0.8302 | 0.0136 | $11,103.30 | $11,014.50 | $88.80 |
| | 3rd | 0.3756 | 0.8287 | 0.3746 | 0.8292 | 0.0010 | $11,006.42 | $11,006.41 | $0.01 |
| | 4th | 0.3749 | 0.8291 | 0.3748 | 0.8291 | 0.0001 | $11,006.41 | $11,006.41 | $0.00 |
| | 5th | 0.3748 | 0.8291 | 0.3748 | 0.8291 | 0.00000 | $11,006.41 | $11,006.41 | $0.00 |

| \( h_2/h \) | \( K_1/K \) | \( E[TCU(t_1^*)] \) | \( t_1^* \) |
|---|---|---|---|
| 0.5 | 0.2 | 0.4 | 0.6 | 0.8 | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 | 2.2 | \( h_2 = $1.50 \) per item stored at the retailer's side |
| | 0.5 | 0.75 | 1 | 1.25 | 1.5 | 1.75 | 2 | 2.25 | 2.5 | 2.75 | 3 | 3.25 | \( E[TCU(t_1^*)] \) in different retailers' locations. |
| | 0.3 | 0.45 | 0.6 | 0.75 | 0.9 | 1.05 | 1.2 | 1.35 | 1.5 | 1.65 | 1.8 | 1.95 | \( t_1^* \) in the proposed model 2. |
5. Concluding Remarks

Two exact models for an extended EPQ-based problem with a discontinuous delivery policy, scrap rate, and random breakdown are developed in this study. They specifically address different real-life situations in production, end-item delivery, and intrasupply chains such as a producer-retailer integrated system. Mathematical modeling along with optimization techniques is used to determine the optimal production run times that minimize the expected system costs per unit time. Without in-depth investigations on these separate models, the optimal production run time and other important information related to the system parameters cannot be revealed. The proposed real-life EPQ models with random machine breakdown, discontinuous product distribution policies, and quality assurance must be specifically studied in order to (1) obtain the joint effects of breakdown, discontinuous distribution policies, and quality assurance on the optimal production run time; (2) get to know the effects of different policy and scope of supply chains management on the optimal run time and overall system costs; and (3) gain the insight with regard to various system’s parameters of all particular EPQ-based models. Since little attention has been paid to the investigation of joint effects of these practical production situations on the optimal run time, this research is intended to bridge the gap. An interesting area for future study is the examination of the effect of variable production rates on these models.

Appendix

Derivations of (3) are as follows. Recall (2) as follows:

\[ \text{TC}_1 (t_1) = C (P_1 t_1) + K + C_s (t_1 P_s) + M + nK_1 + C_T [t_1 P_t] + h_3 \lambda t_r T' + h \left( \frac{H'}{2} + d_1 t_1 + (H'_1 + d_1) t_r + \frac{n-1}{2n} H_1' t_2 \right). \]  

(A.1)

Substituting all related system parameters into (2) (please refer to the basic formulations and solution process in [9]), the TC\(_1 (t_1)\) can be obtained as

\[ \text{TC}_1 (t_1) = K + M + \left[ CP_1 + C_s P_s + C_T P_T (1 - x) + h_3 P_1 g (1 - x) \right] t_1 + nK_1 + h p g t - \left[ \frac{h P_1 g (1 - x)}{2} - \frac{h P_1 g (1 - x)}{2n} \right] t_1 + t_1^2 \left[ \frac{h P_1^2}{2} + \frac{h P_1^2}{2\lambda} (1 - x)^2 - \frac{h P_1}{2} (1 - x) \right] - \frac{h P_1^2}{2\lambda n} (1 - x)^2 + \frac{h P_1}{2n} (1 - x) \].

(A.2)

To take the randomness of \(x\) into account by using the expected values of \(x\), with further derivations, \(E[\text{TC}_1 (t_1)]\) can be derived as follows (i.e., equation (3)):

\[ E \left[ \text{TC}_1 (t_1) \right] = K + nK_1 + M + h t P_1 g + \left[ CP_1 + C_s P_s E [x] + C_T P_T (1 - E [x]) \right] t_1 + h_3 P_1 g (1 - E [x]) - \frac{h P_1 g (1 - E [x])}{2} \left( 1 - \frac{1}{n} \right) t_1 + \frac{h P_1 E [x]}{2} + \frac{h P_1^2}{2\lambda} (1 - E [x])^2 \left( 1 - \frac{1}{n} \right) + \frac{h P_1}{2n} (1 - E [x]) t_1^2. \]  

(A.3)

Conflicts of Interest

The authors of the paper declare that there is no conflict of interests regarding the publication of this paper.

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