Abstract

We compute the differential and the total cross sections for $pp$ scattering using the QCD pomeron model proposed by Landshoff and Nachtmann. This model is quite dependent on the experimental electromagnetic form factor, and it is not totally clear why this form factor gives good results even at moderate transferred momentum. We exchange the electromagnetic form factor by the asymptotic QCD proton form factor determined by Brodsky and Lepage (BL) plus a prescription for its low energy behavior dictated by the existence of a dynamically generated gluon mass. We fit the data with this QCD inspired form factor and a value for the dynamical gluon mass consistent with the ones determined in the literature. Our results also provide a new determination of the proton wave function at the origin, which appears in the BL form factor.

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The increase of hadronic total cross sections was theoretically predicted many years ago [1] and this prediction has been accurately verified by experiment [2]. At present the main theoretical approaches to explain this behavior are the Regge pole model and the QCD-inspired models.

In the Regge pole model the increase of the total cross section with the energy is attributed to the exchange of a colorless state having the quantum numbers of the vacuum: the pomeron [3]. According to the Regge pole theory the hadronic total cross section at high energy \((s \gg |t|)\), where \(s\) and \(t\) are the Mandelstam variables of the process, behaves as \(\sigma_{\text{tot}} \propto s^{\alpha(0)-1}\). \(\alpha(0)\) is the intercept of the Regge trajectory \(\alpha(t) = \alpha_0 + \alpha' t\), where \(\alpha_0 = 1 + \epsilon_0\), \(\epsilon_0 \approx 0.08\) and \(\alpha' = 0.25\) GeV\(^{-2}\). This behavior leads to a violation of the Froissart bound, although it is believed that the exchange of more pomerons eventually will lead to the unitarisation of the scattering amplitude.

Within Regge theory Donnachie and Landshoff [4] predicted the differential elastic proton-proton scattering to be given by

\[
\frac{d\sigma}{dt} = \text{const.} \times F_1(t)^4(\alpha's)^{2(\alpha(t)-1)},
\]

where \(F_1(t)\) is the proton form factor measured in \(ep\) elastic scattering. This expression fits the data quite well at small \(t\) [5], and is reasonable even at moderate values of \(t\). It is clear that this cross section gives a non-trivial check for the functional form of \(F_1(t)\), as well as it not clear at all why the electromagnetic form factor works so well for the pomeron.

In this work we will consider a QCD pomeron model to compute Eq.(1). This model has already been used successfully, therefore we use the data in order to discuss its dependence on the proton form factor. The idea is to exchange the electromagnetic form factor by the asymptotic QCD proton form factor determined by Brodsky and Lepage (BL) [6]. Of course this last one is not expected to be suitable at the infrared (IR) region, but it is for this region that we introduce the concept of a dynamical gluon mass and one hypothesis about how the form factor should behave in the IR, matching it to the BL proton form factor at high energy. Our model for the proton form factor is used to obtain the differential and the total cross-sections for \(pp\) scattering, and it fits the data with a value for the dynamical gluon mass consistent with the ones determined in the literature. In this calculation we obtain a reasonable value for the proton wave function at the origin that appears in the BL form factor, which is used in many phenomenological applications, and turns out to be related to the dynamical gluon mass.

In the QCD framework the pomeron can be understood as the exchange of at least two gluons in a color singlet state [7]. A model for the Pomeron has been put forward by Landshoff and Nachtmann where it is evidenced the importance of the QCD...
non-perturbative vacuum [8]. One of the aspects of this non-perturbative physics appears as an infrared gluon mass scale which regulates the divergent behavior of the Pomeron exchange [9].

The Landshoff and Nachtmann (LN) model can explain diffractive scattering data quite successfully [9, 10], at the same time that it is a quite simple model. The calculation of hadronic cross sections in this model are straightforward, although there are two approximations that are usually performed in order to compare the model to the data. First, the pomeron exchange dependence on the energy has to be introduced in an \textit{ad hoc} way. This means that we multiply the scattering amplitude by a factor of the form \( Z = (s/w_0^2)^{0.08 + \alpha' t} \), \( w_0^2 \approx 1/\alpha' \) is an energy scale. We know that such behavior must come from the exchange of multiple gluon ladder diagrams described by the Balitski-Fadin-Kuraev-Lipatov (BFKL) equation [11]. However we also know that the exponent of the BFKL is not simply related to the data. Of course we expect that in the future the LN approach can be made compatible with the BFKL one. In the meantime we just follow as described above and introduce the factor \( Z \) by hand. Secondly, the cross sections depend on the square of hadronic wave functions, which are ultimately related to their respective form factors. As discussed above, in general it is the electromagnetic phenomenological form factor that is used in the actual diffractive cross section calculation [9]. The ideal case would be the use of a proton form factor totally derived from QCD, and this is the point that will deserve our attention.

In the LN model the elastic pp differential cross section can be obtained from

\[
\frac{d\sigma}{dt} = \frac{|A(s, t)|^2}{16\pi s^2} \tag{2}
\]

where the amplitude for elastic proton-proton scattering via two-gluon exchange can be written as

\[
A(s, t) = is8\alpha_s^2 [T_1 - T_2] \tag{3}
\]

with

\[
T_1 = \int d^2k D\left(\frac{q}{2} + k\right) D\left(\frac{q}{2} - k\right) |G_p(q, 0)|^2 \tag{4}
\]

\[
T_2 = \int d^2k D\left(\frac{q}{2} + k\right) D\left(\frac{q}{2} - k\right) G_p\left(q, k - \frac{q}{2}\right)
\times \left[2G_p(q, 0) - G_p\left(q, k - \frac{q}{2}\right)\right] \tag{5}
\]

where \( G_p(q, k) \) is a convolution of proton wave functions

\[
G_p(q, k) = \int d^2p d\kappa \psi^* (\kappa, p) \psi (\kappa, p - k - \kappa q). \tag{6}
\]
To estimate $G_p(q, k - q/2)$ it is usually assumed a proton wave function peaked at $\kappa = 1/3$ implying that

$$G_p(q, k - \frac{q}{2}) = F_1 \left( q^2 + 9 \left| k^2 - \frac{q^2}{4} \right| \right).$$  \hfill (7)

In the many calculations of this model up to now $G_p(q, 0)$ was given by the Dirac form factor of the proton

$$F_1(t) = G_p(q, 0) = \frac{4m^2 - 2.79t}{4m^2 - t} \frac{1}{(1 - t/0.71)^2}.$$  \hfill (8)

This means that the strongly interacting pomeron sees the proton in the same way as it is seen by a photon. This is not totally surprising, as discussed in the work of Landshoff and Nachtmann [8], since the pomeron formed by gluons that propagate in the vacuum up to a certain critical distance (the gluon mass in our case) lead naturally at high energies to a pomeron that interacts as an isoscalar photon.

In this work we would like to differ from the previous calculations exactly by the use of the asymptotic form factor expression determined by Lepage and Brodsky [12], which is given by

$$F_{QCD}(t) = G_p(q, 0) \approx \frac{C[\alpha_s(q^2)]^{2+4/3\beta}}{q^4},$$  \hfill (9)

where $\beta = 11 - (2/3)n_{flavor}$ is the coefficient of the QCD $\beta$ function. The $q^4$ behavior comes from the propagators of two gluons exchanged between the quarks forming the proton and $C$ is determined by the qqq wave function at the origin. Contrary to the electromagnetic form factor given by Eq.(8) the one of Eq.(9) is just the asymptotic one, and not normalized to 1 at $q^2 = 0$.

We know that the BL proton form factor should be reliable at asymptotic energies, i.e. it is just the tail of the actual form factor. How can we perform a full QCD calculation of the differential $pp$ elastic scattering without knowing the IR behavior of the form factor? To guess something about its IR functional form we must introduce in the form factor calculation the concept of a dynamically generated gluon mass, which is already going to modify the expression of Eq.(9).

Eq.(2) up to Eq.(7) were computed in Ref.[9, 10] using nonperturbative gluon propagators endowed with a dynamical gluon mass, whose existence also imply in an IR finite coupling constant [13]. The possibility that QCD generates a dynamical gluon mass has been put forward by Cornwall [14], and there are evidences for such behavior obtained from solutions of Schwinger-Dyson equations (SDE) [14, 15] and from lattice simulations (see Ref.[16] and references therein). These propagator and coupling constant have been used in many phenomenological calculations that are sensible to their infrared finite behavior [10, 17], and are given by:
\[ D_{\mu \nu}(q^2) = \left( \delta_{\mu \nu} - \frac{q_\mu q_\nu}{q^2} \right) D(q^2), \]  

where the expression for \( D(q^2) \), that was obtained by Cornwall as a fit to the numerical solution of a gauge invariant gluonic SDE, is equal to

\[ D^{-1}(q^2) = \left[ q^2 + M_g^2(q^2) \right] b g^2 \ln \left[ \frac{q^2 + 4M_g^2}{\Lambda^2} \right]. \]

where \( M_g(q^2) \) is a dynamical gluon mass described by,

\[ M_g^2(q^2) = m_g^2 \left[ \ln \left( \frac{q^2 + 4m_g^2}{\Lambda^2} \right) \right]^{-12/11}. \]

\( \Lambda (\equiv \Lambda_{QCD}) \) is the QCD scale parameter. The infrared finite coupling constant obtained in the same procedure has the following expression

\[ \alpha_s(q^2) = \frac{4\pi}{\beta \ln \left( \frac{(q^2 + 4M_g^2(q^2))/\Lambda^2} \right)}, \]

A typical phenomenological value for the dynamical gluon mass is

\[ m_g = 500 \pm 200 \text{ MeV} \]

for \( \Lambda = 300 \text{ MeV} \). Finally, note that these solutions have been obtained in the Euclidean space, as well as are the momenta in the above equations.

In practice the SDE equations have to be solved with approximations and there are different functional forms proposed for the gluon propagator. In particular, a quite simple expression has been proposed in Ref.

\[ D^{-1}(q^2) = \left[ q^2 + M_g^2(q^2) \right], \]

and this expression, even neglecting the \( q^2 \) dependence in \( M_g^2(q^2) \), is the one that we shall use in our calculation. The same happens to the IR behavior of the running coupling constant. Its behavior at \( q^2 = 0 \) is around 1, and since the calculation is not strongly dependent on the ultraviolet logarithmic behavior of the coupling constant, we will just assume a constant value for this quantity. The effect of these approximations will be commented ahead.

Once we admit the idea of a dynamically generated gluon mass we can guess what could be expected for the proton form factor. Asymptotically, if we neglect the \( \alpha_s \) dependence in Eq.(9), we have

\[ G_p(q, 0) \big|_{q^2 \to \infty} \approx \frac{C}{(q^2 + m_g^2)^2}. \]
It could be argued that this approximation is rough due to the fact that we neglected the dependence on $\alpha_s$. Actually this is not so trivial, because Cornwall [14] points out that the product $g^2 D(q^2)$ is independent of the coupling constant. In Ref. [17] we have shown that for gluon masses in the range of Eq. (14) the values of the infrared coupling constant vary between 0.5 and 1. We will arbitrarily use the infrared value of 0.8. Unfortunately we still have a poor knowledge of the infrared QCD behavior, but if the Cornwall’s argument is correct it may happens that our approximation is much better than it looks like.

Since the gluon has its propagation limited by the value of the gluon mass ($m_g$), we expect that the IR proton form factor is approximately constant in a radius determined by this mass, and falls off vary fast after that. The simplest choice that we can make for this IR behavior is

$$G_p(q, 0)|_{q^2 \to 0} \approx \exp\left[-\frac{q^2}{2m_g^2}\right],$$

(17)

which is naturally normalized to 1 at the origin. Note that more complicated expressions could be used to describe the IR behavior, but they barely would reflect in such a simple way a proton that is formed by the exchange of dynamically massive gluons.

The full form factor must be a combination of Eq. (16) and Eq. (17), and must match at one intermediate scale, which we propose as being the dynamical gluon mass scale, with the Brodsky and Lepage proton form factor. Therefore our ansatz for the proton form factor for the full range of momenta is given by

$$G_p(q, 0) = \exp\left[-\frac{q^2}{2m_g^2}\right] \theta(2m_g^2 - q^2) + \frac{9m_g^4}{e} \frac{1}{(q^2 + m_g^2)^2} \theta(q^2 - 2m_g^2).$$

(18)

Eq. (18) is correctly normalized at the origin, match softly the low and high energy of the proton form factor, and, if compared to Eq. (16), gives a prediction for the proton wave function at the origin

$$C \equiv \frac{9m_g^4}{e},$$

(19)

whose value will be determined when we compare our cross section calculation to the experimental data.

We compute the differential pp scattering cross section in the LN model, as performed in Ref. [9], with the help of the proton form factor of Eq. (18). In Fig. (1) the result is compared to the experimental data from et. al. [5]. The data are well fitted assuming a dynamical gluon mass of $m_g = 460$ MeV, for $\Lambda = 300$ MeV. The calculations have some dependence on the gluon mass. We also present in Fig. (1) the results obtained using $m_g = 400$ and 550 MeV. We also compute the total cross section obtained from Eq. (3) (after multiplication by the factor $Z$). It is clear
that the LN model calculation, which turns out to be a function of $m_g$ and $C$, can reproduce only the Pomeron term (the one that increases with $s$) of the fit for the total cross sections proposed by Donnachie and Landshoff [20], where it is shown that the high energy behavior of $pp$ ($p\bar{p}$) scattering is proportional to $21.7s^{0.0808}$.

The high energy data on $pp$ scattering from Ref. [21] is also shown in Fig.(2) and compared with our calculations, using the same values of the dynamical gluon mass, which turns out to be, together with the Donnachie and Landshoff fit, just another check for our results.

From Fig. (1) and (2) we see that the data shows a preferred value $m_g = 460$ MeV, that is compatible with previous determinations of the dynamical gluon mass. With this value we obtain

$$C = 0.15 \text{ GeV}^4.$$  \hspace{1cm} (20)

It is interesting that in Ref. [9] the data of elastic proton-proton scattering is well fitted assuming a dynamical gluon mass of $m_g = 370$ MeV for $\Lambda = 300$ MeV, while our recent calculation in Ref. [22] shows excellent agreement with the data for $m_g = 400 (-100) (+350)$ MeV. Our result is in agreement with such calculations and provides a new determination of the proton wave function at the origin.

The value of $C$ is quite compatible with the results obtained from the branching ratio $\frac{\Gamma(\psi \rightarrow pp)}{\Gamma(\psi \rightarrow \text{hadrons})}$, whose experimental value is 0.0022. In this calculation we use the expression for the branching ratio given by [23]:

$$\frac{\Gamma(\psi \rightarrow pp)}{\Gamma(\psi \rightarrow \text{hadrons})} = 3.2 \times 10^6 \frac{\alpha_s^3(s)(|P_{c.m.}| < T^2)}{s^4}$$  \hspace{1cm} (21)
Fig. (2) Proton-proton total cross section. The experimental data \((p\bar{p})\) are from Ref. [21].

It is also shown the curve given by the Donnachie and Landshoff fit.

where \(|\vec{P}_{c.m.}|/\sqrt{s} \simeq 0.4\), \(s = 9.6\) GeV\(^2\) and

\[
<T> \equiv \int_0^1 [dx] [dy] \frac{\phi^* (y_1, s)}{y_1 y_2 y_3} \left[ x_1 (1 - y_1) + y_1 (1 - x_1) \right] \frac{x_1 y_3 + x_3 y_1}{x_3 (1 - y_3) + y_3 (1 - x_3)} \frac{\phi^* (x_i, s)}{x_1 x_2 x_3}
\]

We performed the above calculation using

\[
\phi(x) = \phi(x)_{ass} = C \ x_1 x_2 x_3 \left( \log \frac{q^2}{\Lambda^2} \right)^{-2/3\beta}
\]

Thus we find \(C \simeq 0.13\) GeV\(^4\). This value is consistent with the one of Eq. (20), and shows that our simple approach to the proton form factor is quite sensible.

In conclusion, we introduced in the proton-proton elastic scattering calculation within the LN model the asymptotic behavior of the QCD proton form factor, supplied by a prescription about its infrared behavior in terms of a dynamically generated gluon mass. The high energy behavior of the \(pp\) diffractive cross section is determined as a function of the factor \(C\), which is related to the qqq wave function at the origin, and turns out to be obtained as a function of the gluon mass. We obtained \(C = 0.15\) GeV\(^4\) after fitting the elastic pp scattering data with \(m_g = 460\) MeV for \(\Lambda = 300\) MeV. Our result provide a desirable connection of the data with the perturbative calculation of the proton form factor. Besides that we believe that the form factor expression presented in Eq. (18) can be further studied in different
processes. Finally, the approximations that we have performed are related to the poor knowledge of the gluon propagator and coupling constant infrared behavior, and any improvement in the calculation could be only performed after an equivalent improvement in the expression of the QCD infrared Green functions.

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