Regge Trajectories Revisited
in the Gauge/String Correspondence

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Abstract

We attempt to obtain realistic glueball Regge trajectories from the gauge/string correspondence. To this end we study closed spinning string configurations in two supergravity backgrounds: Klebanov-Strassler (KS) and Maldacena-Nunez (MN) which are dual to confining gauge theories. These backgrounds represent two embeddings of $\mathcal{N} = 1$ SYM, in the large rank $N$ limit, in string theory. The classical configuration that we consider is that of a folded closed string spinning in a supergravity region with vanishing transverse radius ($\tau = 0$) which is dual to the IR of the gauge theory. Classically, a spinning string yields a linear Regge trajectory with zero intercept. By performing a semi-classical analysis we find that quantum effects alter both the linearity of the trajectory and the vanishing classical intercept: $J \equiv \alpha(t) = \alpha_0 + \alpha' t + \beta \sqrt{t}$. Two features of our Regge trajectories are compatible with the experimental Pomeron trajectory: positive intercept and positive curvature. The fact that both KS and MN string backgrounds give the same functional expression of the Regge trajectories suggests that in fact we are observing string states dual to $\mathcal{N} = 1$ SYM.
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1 Introduction

Regge theory is concerned with the particle spectrum, the forces between particles, and the high energy behavior of scattering amplitudes (for a comprehensive review see [1]). A first principle explanation of this theory remains an outstanding challenge for high energy particle theory. One of the most distinctive features of Regge theory are the Regge trajectories. A Regge trajectory is a line in a Chew-Frautschi [2] plot representing the spin of the lightest particles of that spin versus their mass $s = t$: $J = \alpha_0 + \alpha' t$.

The fact that Regge trajectories are well described by simple string models has been known almost since the experimental verification of the Regge trajectories. The string models used to describe Regge trajectories are generically related to strings in flat space and beyond this particular feature they do not provide a framework for describing hadronic physics. Moreover, there is no fundamental reason for a string in flat space to be useful in the description of some properties of hadronic states in a gauge theory.

Not long ago a precise duality between gauge theories and string theories has been uncovered [3]. In particular, in the context of the AdS/CFT correspondence backgrounds have been constructed that represent supergravity duals of confining gauge theories. In some cases the precise field theoretic content of the theory is known [4, 5]. The Klebanov-Strassler (KS) supergravity solution, for example, contains in a geometric way information about confinement, chiral symmetry breaking, duality cascade and instanton effects among others. Having a string theory that mathematically contains the same information as a confining gauge theory provides us, in principle, with the means to study the IR dynamics of confining gauge theories. Indeed, some of the hadronic states of confining gauge theories which admit a supergravity dual description have recently been described [6]. In this paper we use the gauge/gravity framework to revisit one of the trademark properties of confined matter – Regge trajectories.

A central question in our analysis is what are the precise features of the new description of confining theories that differ from the more naive approach of describing Regge trajectories via strings in flat space. We will show that at the classical level strings in confining backgrounds behave exactly as strings in flat space. It is only at one-loop level where the difference becomes significant. In particular, we will show that the intercept of Regge trajectories is a one-loop effect and therefore differs for superstrings in flat space and strings in confining backgrounds.
The traditional approach to obtaining Regge trajectories from string theories relies on identifying the spectrum of classical strings spinning in flat space with the spectrum of the corresponding particles. Recently, however, in the context of the gauge/gravity correspondence the role of classical solutions has been reexamined. It has been proposed that classical solutions of the string sigma model are in correspondence with sectors of large quantum numbers in the dual gauge theory \[7\]. As a concrete example, \[7\] considered a closed string spinning in AdS and established its correspondence with twist-two operators in \(\mathcal{N} = 4\) SYM. Namely, they obtained that the conserved quantities of the solitonic solution in the sigma model when translated in terms of gauge theory quantities imply that \(\Delta - S = (\sqrt{\lambda}/\pi) \ln S\) which is a prediction for the anomalous dimension of twist-two operators in \(\mathcal{N} = 4\) SYM at large 't Hooft coupling. Note that except for the dependence on the 't Hooft coupling \(\lambda\), this is the form of the anomalous dimension for twist-two operators in the perturbative regime. A very powerful feature of the correspondence \[7\] is that it allows a framework to go beyond the classical level. One can compute quantum corrections to the classical relation among the conserved quantities in the worldsheet approach and they should be in correspondence with quantum corrections on the gauge theory side. Indeed, Frolov and Tseytlin \[8\] performed the one-loop analysis of the classical solution proposed in \[7\] and found that to this level terms of the form \(\ln^2 S\) are absent, something that has been conjectured in the field theory approach to twist-two operators.

Under the point of view of \[7\], the large R-charge sector of \(\mathcal{N} = 4\) SYM described in \[9\] can be seen to be dual on the string theory side to a classical string shrunk to a point an orbiting along the great circle of \(S^5\) in the \(AdS_5 \times S^5\) background. In this case the description of \[9\] has the advantage, over the description of \[7\], of being exact on the string theory side. Using a limit similar to \[9\], an exact description of a set of hadrons with large flavor charged (annulons) was found in \[6\] (similar states were also found in nonsupersymmetric theories \[10,11\]). This sector can correspondingly be described as a classical configuration describing an extended string located near values of the radial direction in the Sugra background that correspond to the IR region of the gauge theory. This configuration also carries large angular momentum in the internal space perpendicular to the world volume. The density of states for these hadrons was computed in \[12\] and found to be of Hagedorn type with a coefficient determined by the quark-antiquark string tension and their flavor charge. The annulons are typical of embeddings of \(\mathcal{N} = 1\) SYM into string theory but their quantum numbers are not
shared by more realistic theories like QCD.

With this improved understanding of the role of classical solutions in the gauge/gravity correspondence and with the hope of describing states in the same universality class as the hadrons of QCD, we turn to the study of strings spinning in supergravity backgrounds dual to confining gauge theories. Our main motivation is to obtain a description of Regge trajectories in a situation where the relation between string theory and gauge theory has been established fairly rigorously. We study the classical configurations in general confining backgrounds and then proceed to explicitly consider the semiclassical quantization in the case of the KS and MN backgrounds.

Our analysis is semiclassical in nature, that is we compute quantum corrections to a given classical relation. This situation differs from the description of the annulons [6] in which by using an appropriate Penrose limit one obtains an exact string theory. Nevertheless, the robustness of our semiclassical approach is well justified. In fact, there is an extensive list of examples where a similar semiclassical approach has been used yielding very reliable results [8,13,14,15].

This paper is organized as follows. In section 2 we review some of the standard classical solutions that are relevant for the study of Regge trajectories, including open and closed strings spinning in flat space. We also consider closed strings in confining backgrounds (some classical aspects were considered in [16]). We begin by solving the classical equation of motion for strings in generic confining supergravity backgrounds: our string configuration describes a string spinning in the region dual to the IR gauge theory.

Section 3 contains an account of the quantization of strings spinning in flat space. In the phenomenological literature several effective string models have been proposed as possible sources of corrections to Regge trajectories. To our knowledge, we present the first such analysis in the context of IIB string theory. We find that in flat space the string spinning is a BPS configuration and therefore receives no corrections at the one loop level. For the Regge trajectories this implies that the intercept is that of the classical trajectories, that is zero. However, considering the bosonic contribution alone one finds a linear trajectory with positive intercept, something in qualitative agreement with experimental data.

In section 4 we consider quadratic fluctuations around a classical string spinning in the two trademark supergravity backgrounds dual to confining theories: KS and MN. Interestingly, we find that from the perspective of a semiclassical quantization
around the spinning string configuration, both theories behave exactly the same way. We therefore present in section 5 a unified analysis of the quantum corrected Regge trajectories.

Section 6 contains our attempts to compare our results with phenomenological data about Regge trajectories available from direct experiments or lattice calculations. In particular, we emphasize the relevance of our calculations for the soft Pomeron trajectory and the lattice results for glueballs. It is worth mentioning that although the supergravity theories that we considered are dual to embeddings of \( \mathcal{N} = 1 \) SYM into IIB string theory, the comparison with results of QCD shows that for questions pertaining to Regge trajectories these theories are certainly in the same universality class.

## 2 Classical Regge trajectories

Regge trajectories are very generic in hadronic physics. For example, they are very well established for mesons, baryons and the soft Pomeron [1] (see also section 6). It is remarkable that the gauge/gravity correspondence provides a framework in which each of these states can be shown to be described by a specific classical solution.

| Gauge Theory State          | String Theory Configuration                  |
|-----------------------------|---------------------------------------------|
| Glueballs                   | Spinning Folded Closed String                |
| Mesons of heavy quarks      | Spinning open strings                        |
| Baryons of heavy quarks     | Strings attached to a baryonic vertex         |
| Dibaryons                   | Strings attached to wrapped branes           |

Table 1: States in gauge theory and their corresponding classical configuration in the string theory.

An important point in the correspondence is the need to consider objects with angular momentum in Poincare coordinates. This implies that the conserved quantity conjugate to the Poincare time \( (E) \) measures the energy of a state in Minkowski space, that is, one is considering the energy of a state in the gauge theory side rather than the conformal dimension as corresponds to a configuration in global AdS time. The main feature of these classical solutions is that they are characterized (among other classical quantities) by their world volume angular momentum \( (J) \) which is the gravity dual to
spin in the gauge theory.

There is a distinctive feature that distinguishes between configurations of closed and open strings with angular momentum. Open strings with angular momentum are characterized by $E$, $J$ and $L$ (with $L$ the endpoints separation), whereas closed string with angular momentum are only characterized by $E$ and $J$.

2.1 Spinning open string in flat space

Let us briefly review the familiar story of the Regge trajectories associated with the spinning string in flat space. We begin with the Polyakov action restricted to the bosonic degrees of freedom of the string

$$S = \frac{T_s}{2} \int_0^\pi d\sigma \int d\tau \sqrt{\gamma} \gamma^{\alpha \beta} \partial_\alpha X^\mu \partial_\beta X_\mu = \frac{T_s}{2} \int_0^\pi d\sigma \int d\tau L(\sigma, \tau) \quad (2.1)$$

where the string tension is $T_s = 1/(2\pi \alpha')$. The spinning string classical configuration is described by

$$X^0 = e\tau, \quad X^1 = e \cos \sigma \cos \tau, \quad X^2 = e \cos \sigma \sin \tau \quad (2.2)$$

This solution satisfies the equations of motion of the coordinates $X^\mu(\sigma, \tau)$, the Virasoro constraints, and the Neumann boundary conditions $\partial_\sigma X^\mu|_{\sigma=0,\pi} = 0$.

The energy and angular momentum associated with this classical configuration are defined as

$$E = T_s \int_0^\pi d\sigma \frac{\partial X^0}{\partial \tau} = \pi T_s e$$

$$J = \int_0^\pi d\sigma (\dot{X}^2 X^1 - \dot{X}^1 X^2) = \frac{\pi}{2} T_s e^2 \quad (2.3)$$

The Regge trajectory is defined as the relationship $J = J(E^2)$

$$J = \frac{1}{2\pi T_s} E^2 = \alpha' E^2 \equiv \alpha' t \quad (2.4)$$

The slope of the Regge trajectory of a spinning open string is therefore $\alpha'$. Note that from the standpoint of a classical configuration the fermionic degrees are naturally set to zero and they do not affect the form of the Regge trajectory.
2.2 Spinning closed string in flat space

For a closed string, we have to require periodicity in $\sigma$ on the interval $[0, 2\pi]$. Therefore the classical spinning closed string configuration, solution to the equations of motion derived from the action (2.1), will be

$$X^0 = e\tau, \quad X^1 = e\sin(\sigma)\cos(\tau), \quad X^2 = e\sin(\sigma)\sin(\tau) \quad (2.5)$$

Note that the solution (2.5) obeys the Virasoro constraints as well. We chose the particular dependence on $\sigma$ such that the center of the string be at rest.

The energy and angular momentum of this configuration are

$$E = T_s \int_0^{2\pi} d\sigma \frac{\partial X^0}{\partial \tau} = 2\pi T_s e$$
$$J = T_s \int_0^{2\pi} d\sigma (\dot{X}^2 X^1 - \dot{X}^1 X^2) = \pi T_s e^2 \quad (2.6)$$

The Regge trajectory of a closed string

$$J = \frac{1}{4\pi T_s} E^2 = \frac{\alpha'}{2} E^2 \equiv \frac{1}{2} \alpha' t \quad (2.7)$$

will have a slope equal to a half the Regge slope of an open string. The difference between the slope of Regge trajectories for mesons and glueball has been noticed phenomenologically. In section (6) we comment on the string theory result predicting a ration of a half and the most common field theoretic approach based on effective descriptions.

2.3 Spinning Wilson line in flat space time

Some of the theories we consider do not admit “open strings” as microscopic degrees of freedom. However, they do admit open strings stretched between two fixed points in target space. In the context of the holographic duality these string configurations are the duals of the Wilson loops of the corresponding gauge theory [17]. Let us then consider such strings in flat space and in more general supergravity backgrounds dual to confining theories. Since we are considering classical aspects only we choose to work with the Nambu-Goto action describing a string in flat space

$$S_{NG} = \frac{1}{2\pi \alpha'} \int d^2 \sigma \sqrt{-det(\gamma_{\alpha\beta})} = \frac{1}{2\pi \alpha'} \int d^2 \sigma \sqrt{[(\dot{X}^0)^2 - \rho^2 (\dot{\phi})^2]} (\rho')^2, \quad (2.8)$$
where $\gamma_{\alpha\beta}$ is the induced worldsheet metric. For convenience we choose to work in polar coordinates which are related to the Cartesian coordinates by $X^1 = \rho \cos \phi$ and $X^2 = \rho \sin \phi$. A static Wilson loop is described by the configuration $X^0 = e\tau$, $\rho = \frac{L}{\pi} \sigma$, $\phi = \phi_0$ The space-time energy associated with this configuration

$$E_{\text{stat}} = \frac{1}{2\pi \alpha'} \int d\sigma \frac{\partial X^0}{\partial \tau} = \frac{1}{2\pi \alpha'} L,$$

(2.9)
since by the Virasoro constraint $e = \frac{L}{\pi}$. This linear potential obviously resembles a confining potential. For a spinning Wilson line we take the ansatz $X^0 = e\tau$, $\theta = \pi/2$, $\phi = e\omega \tau$, $\rho = \rho(\sigma)$. The space-time energy is given by

$$E = \frac{2}{2\pi \alpha'} \frac{1}{\omega} \arcsin(L\omega),$$

(2.10)
and the spin is

$$J = \frac{2}{2\pi \alpha'} \int d\sigma \rho^2 \frac{\phi}{\tau} = \frac{1}{2\pi \alpha'} \frac{1}{\omega^2} \left[ \arcsin L\omega - (L\omega) \sqrt{1 - (L\omega)^2} \right].$$

(2.11)
The two in the numerator has the same origin as in the previous subsection. For the special case of $\omega L = 1$ (when the ends of the strings move at the speed of light) we get

$$E = \frac{1}{2\alpha'} \frac{1}{\omega}, \quad J = \frac{1}{4\alpha'} \frac{1}{\omega^2},$$

(2.12)
such that we get the Regge behavior

$$J = \alpha' E^2 \equiv \alpha' t$$

(2.13)
In the regime where $\omega L \sim 0$ we get a correction to the linear term of the form

$$E \approx \frac{1}{\pi \alpha'} \left( L + \frac{3\pi^2}{2} \frac{\alpha'^2 J^2}{L^3} \right).$$

(2.14)
Notice the positive sign of the second term that indicates that the rotation of the Wilson loop increases the binding energy of the quark anti-quark system. It is important to bear in mind that the sub-leading term in expression (2.14) represents classical corrections to the standard confining result. In addition there are quantum corrections that will be discussed in section 4. One interesting feature of the above relations is the possibility to interpolate between a linear potential between a quark and an antiquark and for small $\omega L$ and Regge trajectory for $\omega L \rightarrow 1$. 
2.4 Closed spinning strings in supergravity backgrounds

Our starting point would be a supergravity solution of a form that naturally generalizes the $AdS_5$ metric in Poincare coordinates. Tacitly we assume that we are working with IIB SUGRA backgrounds and that the metrics we consider are appropriate deformations of $AdS_5$. We consider background metric that preserves Poincare invariance in the coordinates $(X^0, X^i)$. We will not dwell at this point in the specifics of the space transverse to the world volume of the “D3” brane and simply denote most of its structure by ellipsis in the background metric:

$$ds^2 = h(r)^{-1/2} \left[ -(dX^0)^2 + dX_1^2 + dX_2^2 + dX_3^2 \right] + h(r)^{1/2} dr^2 + \ldots \quad (2.15)$$

The relevant classical equations of motion for the string sigma model in this background are

$$\partial_a (h^{-1/2} \eta^{ab} \partial_b X^0) = 0,$$
$$\partial_a (h^{-1/2} \eta^{ab} \partial_b X^i) = 0,$$
$$\partial_a (h^{1/2} \eta^{ab} \partial_b r) = \frac{1}{2} \partial_\sigma (h^{-1/2}) \eta^{ab} \left[ - \partial_a X^0 \partial_b X^0 + \partial_a X_i \partial_b X^i \right]. \quad (2.16)$$

They are supplemented by the standard Virasoro constraints. We will attempt to construct spinning strings by taking the following ansatz

$$X^0 = e \tau, \quad X^1 = f_1(\tau) g_1(\sigma), \quad X^2 = f_2(\tau) g_2(\sigma), \quad X^3 = \text{constant}, \quad r = r(\sigma). \quad (2.17)$$

With this ansatz the equation of motion for $X^0$ is trivially satisfied. Let us first show that the form of the functions $f_i$ is fairly generic. Namely, considering the equation of motion for $X^i$ we obtain

$$-h^{-1/2} g_i \ddot{f}_i + f_i \partial_\sigma (h^{-1/2} g'_i) = 0, \quad (2.18)$$

where a dot denotes a derivative with respect to $\tau$ and a prime denotes a derivative with respect to $\sigma$. Enforcing a natural separation of variables we see that

$$\ddot{f}_i + (e \omega)^2 f_i = 0, \quad \partial_\sigma (h^{-1/2} g'_i) + (e \omega)^2 h^{-1/2} g_i = 0. \quad (2.19)$$
The radial equation of motion is
\[ \partial_{\sigma}(h^{1/2}\partial_{\sigma}r) = \frac{1}{2} \partial_{\tau}(h^{-1/2})[e^2 - g_i^2 f_i^2 + f_i^2 g_i^2]. \]
(2.20)

Finally the constraint becomes
\[ h^{1/2}r^2 + h^{-1/2}[-e^2 + g_i^2 f_i^2 + f_i^2 g_i^2] = 0. \]
(2.21)

The integrals of motion we would like to consider are
\[ E = \frac{e}{2\pi \alpha'} \int h^{-1/2} d\sigma, \]
(2.22)
\[ J = \frac{1}{2\pi \alpha'} \int h^{-1/2} [x_1 \partial_{\tau} x_2 - x_2 \partial_{\tau} x_1] d\sigma = \frac{1}{2\pi \alpha'} \int h^{-1/2} g_1 g_2 [f_1 \partial_{\tau} f_2 - f_2 \partial_{\tau} f_1] d\sigma \]
(2.23)

The above system can be greatly simplified by further taking
\[ f_1 = \cos e\omega \tau, \quad f_2 = \sin e\omega \tau, \quad \text{and} \quad g_1 = g_2. \]
(2.24)

Under these assumptions the equation of motion for \( r \) and the Virasoro constraint become
\[ \partial_{\sigma}(h^{1/2}\partial_{\sigma}r) - \frac{1}{2} \partial_{\tau}(h^{-1/2})[e^2 - (e\omega)^2 g^2 + g'^2] = 0, \]
\[ h^{1/2}r^2 + h^{-1/2}[-e^2 + (e\omega)^2 g^2 + g'^2] = 0. \]
(2.25)

The angular momentum is then
\[ J = \frac{e\omega}{2\pi \alpha'} \int h^{-1/2} g^2 d\sigma. \]
(2.26)

According to the gauge/gravity correspondence and in particular to the insight put forward in [7], this spinning string describes a state in the dual gauge theory with the same quantum numbers. Since we are working in Poincare coordinates the quantity canonically conjugate to time is the energy of the corresponding state in the four dimensional theory. The angular momentum of the string describes the spin of the corresponding state. Thus a spinning string in the Poincare coordinates is dual to a state of energy \( E \) and spin \( J \). In order for our semiclassical approximation to be valid we need the value of the action to be large, this imply that we are considering gauge theory states in the IR region of the gauge theory with large spin and large energy. We will show that, in the cases we study, the expressions (2.22) and (2.26) yield a dispersion relation that can be identified with Regge trajectories. Moreover, we will compute semiclassically quantum corrections to the Regge trajectories using the dual string theories.
Let us show that there exists a simple solution of the equations of motion (2.25) for any gravity background dual to a confining gauge theory. Recall that the conditions for a SUGRA background to be dual to a confining theory have been exhaustively explored \[18\]. The main idea is to translate the condition for the vev of the rectangular Wilson loop to display an area law into properties that the metric of the supergravity background must satisfy. It has been established that one of the sufficient conditions is for $g_{00}$ to have a nonzero minimum at some point $r_0$ usually known as the end of the space wall \[18\]. Note that precisely these two conditions ensure the existence of a solution of (2.25). Namely, since $g_{00} = \frac{h^{-1}}{2}$ we see that for a point $r = r_0 = \text{constant}$ is a solution if

$$\partial_r (g_{00})_{|r=r_0} = 0, \quad g_{00} |_{r=r_0} \neq 0.$$ (2.27)

The first condition solve the first equation in (2.25) and the second condition makes the second equation nontrivial. Interestingly the second condition can be interpreted as enforcing that the quark-antiquark string tension be nonvanishing. It is worth mentioning that due to the UV/IR correspondence in the gauge/gravity duality the radial direction is identified with the energy scale. In particular, $r \approx r_0$ is the gravity dual of the IR in the gauge theory. Thus, the string we are considering spins in the region dual to the IR of the gauge theory. Therefore we can conclude that it is dual to states in the field theory that are characteristic of the IR. Let us now explicitly display the Regge trajectories. The classical solution is given by (2.17) with $g(\sigma)$ solving the second equation from (2.25), that is, $g(\sigma) = (1/\omega) \sin(\omega \sigma)$. Imposing the periodicity $\sigma \rightarrow \sigma + 2\pi$ implies that $\epsilon \omega = 1$ and hence

$$X^0 = e \tau, \quad X^1 = e \cos \tau \sin \sigma, \quad X^2 = e \sin \tau \sin \sigma.$$ (2.28)

The expressions for the energy and angular momentum of the string states are:

$$E = 4 \frac{e g_{00}(r_0)}{2 \pi \alpha'} \int d\sigma = 2 \pi g_{00}(r_0) T_s e, \quad J = 4 \frac{g_{00}(r_0) e^2}{2 \pi \alpha'} \int \sin^2 \sigma d\sigma = \pi g_{00}(r_0) T_s e^2.$$ (2.29)

Defining now $T_{s, \text{eff}} = g_{00}(r_0)/(2 \pi \alpha')$ and $\alpha'_{\text{eff}} = \alpha' / g_{00}$ we find that the Regge trajectories take the form

$$J = \frac{1}{4 \pi T_{s, \text{eff}}} E^2 = \frac{1}{2} \alpha'_{\text{eff}} t.$$ (2.30)
Notice that the main difference with respect to the result in flat space is that the slope is modified to $\alpha'_{\text{eff}} = \alpha' / g_{00}$. It is expected that a confining background will have states that align themselves in Regge trajectories. The main purpose of our investigation is not the relation (2.30) itself but rather the corrections that it receives and that can be computed explicitly in the gauge/gravity correspondence for specific backgrounds. Of course the conditions (2.27) are necessary conditions for confinement but they are not sufficient. Namely, there are backgrounds satisfying (2.27) that are not dual to confining gauge theories. The most prominent example is perhaps flat space where the metric certainly satisfies (2.27) but there is no holographic argument in favor of identifying flat space with a confining gauge theory. We will nevertheless, devote some attention in section 3 to the quantum corrections to strings spinning in flat space for its historical and technical relevance to the topic of Regge trajectories.

3 Quantum corrected Regge trajectories for spinning strings in flat space

We begin with reviewing the general formalism of semiclassical quantization around a given classical string configuration. The quantization procedure depends on the string formulation used (Polyakov or Nambu-Goto) and on the gauge fixing. Since we are particularly interested in spacetime quantities like the energy and the angular momentum, it is important for us to develop a general expression relating the spacetime quantities to the worldsheet Hamiltonian (we follow the analysis of [8]). Let us consider the Polyakov formulation in the conformal gauge. In general, for each of the string coordinates we turn on quantum fluctuations such that

$$X^i(\sigma, \tau) = \bar{X}^i(\sigma, \tau) + \delta X^i(\sigma, \tau), \quad (3.1)$$

where $\bar{X}^i(\sigma, \tau)$ stands for the classical configuration. In particular, for the coordinates involved in defining energy and angular momentum we have

$$X^0 = e \tau + \delta X^0 \quad \phi = e \omega \tau + \delta \phi. \quad (3.2)$$

The Virasoro constraints:

$$g_{ij} \left[ \partial_\tau X^i \partial_\tau X^j + \partial_\sigma X^i \partial_\sigma X^j \right] = 0,$$

$$g_{ij} \partial_\tau X^i \partial_\sigma X^j = 0, \quad (3.3)$$
can be rewritten in the form of the requirement for the vanishing of the 2d Hamiltonian, namely

\[ H(X^i) = \frac{T_s}{2} g_{ij} [\partial_\tau X^i \partial_\tau X^j + \partial_\sigma X^i \partial_\sigma X^j] = 0. \] (3.4)

Now, upon substituting \( X^0 = e\tau + \delta X^0 \) we find that

\[ T_s g_{00} \partial_\tau X^0 = T_s \frac{e g_{00}}{2} + \frac{1}{e} H(\delta X^0, X^i), \] (3.5)

where we have replaced \( X^0 \) by \( \delta X^0 \) in the 2d Hamiltonian and \( X^i \) now denotes the rest of the coordinates. We can now substitute (3.5) into the expression for the space-time energy. If we also explicitly introduce the classical configuration of \( \phi \) and integrate we end up with the following expression for the space-time energy

\[ E = \bar{E} + \omega (J - \bar{J}) + \frac{1}{e} \int d\sigma H(\delta X^i) \] (3.6)

where \( \bar{E} \) and \( \bar{J} \) are the classical values of the energy and angular momentum respectively and \( H(\delta X^i) \) is the 2d Hamiltonian where the original dependence on \( X^i \) is now replaced by the dependence on the fluctuations \( \delta X^i \). What is left to be done is to eliminate the dependence on \( e \) and \( w \) by using the expression for the classical angular momentum. The Virasoro constraint implies that

\[ H = 0 = \dot{J} \phi + P_{\rho} \dot{\rho} - E \dot{X}^0 - L(P, q) + (d - 3 \text{ massless degrees of freedom}), \] (3.7)

where \( J = \int T_s \rho^2 \dot{\phi}, P_{\rho} = \int T_s \dot{\rho}, E = \int T_s \dot{X}^0 \). Explicitly, one can rearrange the Virasoro constraint as:

\[
e e - J = \int d\sigma \left( H(d - 3 \text{ massless dof}) + \frac{T_s}{2} (e^2 - (\dot{\delta X}^0)^2 - (\dot{\delta X}^0))^2 \right) \\
+ \frac{T_s}{2} (\dot{\rho} + \delta \rho)^2 (-1 + \delta \phi^2 + \delta \phi'^2) + \frac{T_s}{2} (\dot{\delta \rho}^2 + (\dot{\rho} + \delta \rho)^2) \\
= eE - J + \int d\sigma (H(d - 3 \text{ massless dof}) \\
+ T_s \left( -\frac{\delta X^{02}}{2} - \frac{(\delta X^{0r})^2}{2} + \frac{\dot{\delta \rho}^2}{2 (\dot{\delta \phi}^2 + \delta \phi'^2)} + \frac{\dot{\delta \rho}^2}{2} + \frac{\delta \phi'^2}{2} - \frac{1}{2} \delta \rho^2 \right) + \ldots \right) \] (3.8)

where the ellipsis denotes terms which are of higher order in fluctuations. On the right hand side of (3.8) we recognize the fluctuation Hamiltonian \( H(\delta X^0, \delta \phi, \delta \rho, \delta \vec{Z}) \), where \( \delta \vec{Z} \) denotes collectively the \( d - 3 \) massless degrees of freedom. The usefulness
of this particular rewriting of the Virasoro constraints will become transparent in the covariant quantization of the Polyakov string. By taking the expectation value of (3.8) on the ground state where $J|\Psi\rangle = \bar{J}|\Psi\rangle$ one obtains

$$e\Delta E = \int d\sigma <\Psi|H|\Psi>.$$  \hspace{1cm} (3.9)

The right hand side will compute the sum of the zero-point energies of all the degrees of freedom including the ghosts in covariant quantization. Thus, the right hand side of (3.9) is the sum of the zero-point energies of the physical degrees of freedom. This result formally obtained by assuming that the theory has been covariantly quantized turns out to be a physical one: it implies that in any quantization scheme the corrections to the spacetime energy will be given by the zero-point energies of the physical degrees of freedom on the worldsheet. In what follows in this section we will evaluate (3.9) by finding the spectrum of the physical degrees of freedom in a convenient gauge, instead of using covariant quantization.

### 3.1 Static open string: The Lüscher term as a quantum correction

Prior to tackling the quadratic fluctuations of a spinning string in flat space let us recall the analysis for a static string in this background. The relevant classical solution is a straight line along, let say along $x_1$, from $-L/2$ to $L/2$. In D flat dimensions the quantized action is simply that of $D-2$ massless fields associated with the $D-2$ transverse directions. Assuming a time interval of $T$ and demanding that the eigenfunctions vanish on the boundary, the eigenvalues of the corresponding Laplacian are

$$E_{n,m} = \left(\frac{n\pi}{L}\right)^2 + \left(\frac{m\pi}{T}\right)^2.$$  \hspace{1cm} (3.10)

Therefore the free energy $F_B$ is given by (see appendix (A))

$$-\frac{2}{D-2}F_B = -\frac{1}{12} \frac{T}{L},$$  \hspace{1cm} (3.11)

and we find the correction to the energy due to the bosonic fluctuations:

$$F_b = (D-2)\frac{\pi}{24} \frac{T}{L}, \quad \delta E = -\frac{1}{T} F_B = -(D-2)\frac{\pi}{24} \frac{1}{L}.$$  \hspace{1cm} (3.12)

This is an attractive Lüscher term and we can see explicitly that its appearance is a one-loop effect from the string theory point of view. Now, for a superstring one has to
add the fermionic fluctuations. For a GS formulation in 10 flat space-time dimensions
the fermionic action is
\[ S_{\text{fermion}}^{\text{flat}} = 2iT_s \int d\tau d\sigma \bar{\theta} \Gamma^{\alpha} \partial_{\alpha} \theta \] (3.13)
where \( \theta \) are 10-d Weyl Majorana spinor and \( \Gamma^{\alpha} \) are the \( SO(9,1) \) gamma matrices. Since
the square of the fermionic operator \( \Gamma^{\alpha} \partial_{\alpha} = \nabla \) and the eight transverse coordinate
match the eight components of the Weyl Majorana spinor one finds that the Lüscher
term is canceled out since
\[ F = 8 \times \left[ -\frac{1}{2} \log \det \nabla + \log \det(\Gamma^{\alpha} \partial_{\alpha}) \right] = 0. \] (3.14)
Thus, in flat space the static string does not receive quantum corrections. A simple
way to understand this situation is by verifying that a static open string in IIB is BPS.
The analysis of supersymmetry is directly related to kappa-symmetry whereby we have
\[ X'^{M} \dot{X}^{N} \Gamma^{\epsilon \epsilon^*} = \sqrt{-g} \epsilon. \] (3.15)
For the static solution \( X^0 = \tau, X^1 = \sigma = [-L/2, L/2] \). Therefore \( \sqrt{-g} = 1 \) and 3.15
reduces to \( \Gamma_{01} \epsilon^* = \epsilon \). Writing the complex spinor as \( \epsilon = \epsilon_1 + i \epsilon_2 \) and using the \( \Gamma^2_{01} = 1 \)
we see that we can arrange for \( (1 - \Gamma_{01}) \epsilon_1 = 0 \) which shows that the solution preserves
half the supersymmetries.

### 3.2 Regge trajectories: The intercept as a quantum correction

We now proceed to study the quantum fluctuations up to quadratic order around the
classical solution describing a spinning string. For pedagogical reasons we derive the
spectrum of the quantum fluctuations by performing a semi-classical expansion around
the spinning string solution using both the Nambu-Goto action and the Polyakov ac-
tion. Our treatment allows to clearly visualize some of the shortcomings of these
methods. Let us begin by considering the Nambu-Goto action of a bosonic string in a
flat target-space background
\[ S = T_s \int \left[ - \left( \dot{X}^0 \right)^2 + \rho^2 + \rho^2 \dot{\phi}^2 + \dot{z} \dot{\bar{z}} \right] \left( - \left( X^{0'} \right)^2 + \rho'^2 + \rho^2 \phi'^2 + \dot{z}' \dot{\bar{z}}' \right) \]
\[ - \left( \dot{X}^0 X^{0'} + \dot{\rho} \rho' + \rho^2 \phi \phi' + \dot{z} \dot{z}' \right)^2 \bigg|^{\frac{1}{2}}. \] (3.16)
Denoting by \( \rho, \phi \) the polar coordinates which parametrize the the plane where the
string motion is confined \( ds^2 = -d(X^0)^2 + d\rho^2 + \rho^2 d\phi^2 + \ldots \), we expand around the
classical solution
\[
\bar{X}^0 = e\tau \quad ; \quad \bar{\rho} = e \sin(\sigma) \quad ; \quad \bar{\phi} = e\omega \tau.
\] (3.17)

In what follows for simplicity we will sometimes set \( e\omega = 1 \). By using diffeomorphism invariance to gauge-fix \( \bar{X}^0 = X^0, \bar{\rho} = \rho \), one obtains
\[
S_{NG} = Ts \left( \int \sqrt{e^4 \cos^4(\sigma)} + \frac{e^2 \tan^2(\sigma)}{2} (-\delta\dot{\phi}^2 + \delta\phi'^2) - \frac{1}{2}(\delta\ddot{z}\delta\dot{z} - \delta\dot{z}'\delta\dot{z}') \right),
\] (3.18)

where \( \delta\ddot{z}(\sigma, t) \) denote the other \( d - 3 \) fluctuations. To cast the action for the fluctuation \( \delta\phi \) in the form of a standard kinetic term plus a potential we make the field redefinition
\[
\delta\tilde{\phi} = \delta\phi \tan(\sigma),
\] (3.19)

and find
\[
S_{NG} = Ts \left( \int e^2 \cos^2(\sigma) + \frac{e^2}{2}(-\delta\dot{\tilde{\phi}}^2 + \delta\tilde{\phi}'^2) + \frac{e^2}{2\cos(\sigma)}(\delta\tilde{\phi})^2 - \frac{1}{2}(\delta\ddot{z}\delta\dot{z} - \delta\dot{z}'\delta\dot{z}') \right).
\] (3.20)

To conclude, the Nambu-Goto action when expanded around the spinning string solution describes one massive fluctuation \( \delta\tilde{\phi} \)
\[
\left( \partial^2_{\sigma} - \partial^2_{\tau} - \frac{2}{\cos^2(\sigma)} \right) \delta\tilde{\phi}(\sigma) = 0.
\] (3.21)

with a \( \sigma \)-dependent mass square \( 2/\cos^2(\sigma) \) and \( d - 3 \) massless fluctuations \( \delta\dot{z} \). One can show that the expansion of the \( \delta\tilde{\phi} \) fluctuation in eigenmodes is given by
\[
\delta\tilde{\phi}(\sigma, \tau) = \tilde{\phi}_{nm}\chi_{nm}(\sigma, \tau)
\]
\[
\equiv \sum_n \exp(\im\tau) \left( \sum_{m=\text{odd}} \frac{1}{\sqrt{\pi(m^2 - 1)}} (m\sin(m\sigma) - \cos(m\sigma) \tan \sigma) \right.
\]
\[
+ \sum_{m=\text{even}} \frac{1}{\sqrt{\pi(m^2 - 1)}} (m\cos(m\sigma) + \sin(m\sigma) \tan \sigma) \right)
\] (3.22)

where \( \chi_{nm} \) are normalized eigenfunctions of the differential operator
\[
(\partial^2_{\sigma} - \partial^2_{\tau} + \frac{2}{\cos^2(\sigma)})\chi_{nm}(\sigma, \tau) = (m^2 - n^2)\chi_{nm}(\sigma, \tau) \equiv \lambda_{nm}\chi_{nm}(\sigma, \tau),
\]
\[
\int_{-\pi}^{\pi} d\sigma \chi_{nm}\chi_{n'm'} = e^{\im(n+n')\tau}\delta_{m-m'}.
\] (3.23)

The partition function of this massive mode will therefore be very similar to that of a massless fluctuation. Certainly the eigenvalues as read off from (3.23) are the same as
for a massless mode with standard kinetic term and standard Fourier mode decomposition. However, there is one difference between the spectrum of the $\delta \tilde{\phi}$ fluctuation and the spectrum of a massless fluctuation. Namely, the former has one less mode than the latter since, as can be seen from (3.22), the eigenmode with $m = 1$ vanishes. In such cases one looks for a special solution for this mode. Indeed, it is easy to realize that the corresponding equation 

$$ (-\partial^2_\sigma + \frac{2}{\cos^2 \sigma}) \chi_{nm=1}(\sigma) = \chi_{nm=1}(\sigma) $$

However, this special eigenmode is not $\sigma$-normalizable and hence cannot be counted. This peculiar situation leads to a different answer for the energy. For instance, the zero mode energy calculation done at the beginning of this section becomes

$$ \sum_{n,m \in \mathbb{Z}} \log E_{n,m} = \sum_{n,m \in \mathbb{Z}} \log E_{n,m} - \sum_{n \in \mathbb{Z}} \log E_{n,1,m} = \sum_{n \in \mathbb{Z}} n - 1 = \zeta(-1) - 1 \quad (3.24) $$

In the above we have taken the simplified values $T = 2\pi$, $L = 2\pi$ since this is the periodicity that has been implicitly assumed in the Fourier mode decomposition (3.22). It is easy to restored the $T$ and $L$ dependence using the analysis of appendix (A). Let us see how the spacetime energy and angular momentum operators are expressed in terms of oscillators. The target-space energy is the canonical conjugate variable to $X^0$, while the angular momentum is the canonical conjugate variable to $\phi$. Using the Nambu-Goto action (3.16) we derive that the energy and angular momentum, respectively, are

$$ E = \int d\sigma \frac{\delta S}{\delta X^0} $$

$$ = T_s \int d\sigma \left[ \dot{X}^0 \left( (\dot{X}^0)^2 + \rho^2 + \rho^2 \dot{\phi}^2 + \ddot{z} \dddot{z} \right) - X^0 \left( -(\dot{X}^0)^2 + \rho^2 + \rho^2 \phi^2 + \dddot{z} \dddot{z} \right) \right] $$

$$ \times \left| (-\dot{X}^0)^2 + \rho^2 + \rho^2 \dot{\phi}^2 + \ddot{z} \dddot{z} \right| \left| -(\dot{X}^0)^2 + \rho^2 + \rho^2 \phi^2 + \ddot{z} \dddot{z} \right|^{\frac{1}{2}} \; , \quad (3.25) $$

$$ J = \int d\sigma \frac{\delta S}{\delta \phi} $$

$$ = T_s \int d\sigma \left[ \left( \rho^2 \dot{\phi} \left( -(\dot{X}^0)^2 + \rho^2 + \rho^2 \phi^2 + \ddot{z} \dddot{z} \right) + \rho^2 \phi' \left( -(\dot{X}^0)^2 + \rho^2 + \rho^2 \phi^2 + \ddot{z} \dddot{z} \right) \right) \right] $$

$$ \times \left| (-\dot{X}^0)^2 + \rho^2 + \rho^2 \dot{\phi}^2 + \ddot{z} \dddot{z} \right| \left| -(\dot{X}^0)^2 + \rho^2 + \rho^2 \phi^2 + \ddot{z} \dddot{z} \right|^{\frac{1}{2}} \; . \quad (3.26) $$
Substituting the gauge choice \( X^0 = e \tau, \rho = e \sin \sigma \) and expanding in fluctuations up to second order, one finds

\[
E = T_s \int d\sigma \left( \frac{1}{2e \cos^2 \sigma} \left( \dot{\sigma}^2 (3e^2 - 5e^2 \cos^2 \sigma + 2e^2 \cos^2 \sigma) + \phi'^2 (-e^2 + 3e^2 \cos^2 \sigma - 2e^2 \cos^4 \sigma) \\
+ \dot{z}^2 \cos^2 \sigma + \dot{z}'^2 \cos^2 \sigma \right) \right),
\]

\( 3.27 \)

\[
J = -T_s \int d\sigma \left( \frac{\sin^2 \sigma}{\cos^4 \sigma} \left( -3e^2 \sin^2 \sigma \dot{\phi}^2 - e^2 \sin^2 \sigma \phi'^2 - \dot{z}^2 \cos^2 \sigma - \dot{z}'^2 \cos^2 \sigma \right) \right).
\]

\( 3.28 \)

As announced in the introduction to the section (3), the difference \( eE - J \) (which in the Nambu-Goto bosonic string turns out to be equal to the Hamiltonian of the physical degrees of freedom) when evaluated on the ground state yields

\[
e(E - \bar{E}) = T_s \int d\sigma \langle \Psi \left| \frac{1}{2} (\dot{z}^2 + \dot{z}'^2 + e^2 \tan^2 \sigma (\dot{\phi}^2 + \phi'^2)) \right| \Psi \rangle \\
= \frac{\pi}{2} (D - 3) \sum_{n>0} n + \left( \sum_{n>0} n - 1 \right) \\
= \frac{\pi}{2} \left( -\frac{D - 2}{12} - 1 \right).
\]

\( 3.29 \)

Note that the quantum correction \( E - \bar{E} \) can be interpreted as a Luscher term since it is proportional to \( \frac{1}{e} \) where \( e \) is the length of the string. Using the classical values of \( J \) and \( E \) one obtains the quantum corrected Regge trajectory

\[
J \approx \frac{1}{2} \alpha' E^2 + \frac{\pi}{2} \left( \frac{D - 2}{12} + 1 \right) \equiv \frac{1}{2} \alpha' t + \frac{\pi}{2} \left( \frac{D - 2}{12} + 1 \right) + O(\frac{1}{\alpha't}).
\]

\( 3.30 \)

Let us discuss the validity of the above result. The Nambu-Goto action is not free even in flat space since the expansion of the square root contains terms higher order in fluctuations. We have, naturally used a semiclassical approximation by truncating the action to terms quadratic in fluctuations. The approximation relies on the fact that higher order interaction terms are suppressed by powers of the energy of the state \( 1/E \). This is the reason why we neglect terms that are order \( E^{-2} \) in our final expression for the quantum corrected Regge trajectories (3.30). The net effect of the one loop quantum corrections on the Regge trajectory is to provide a positive intercept. A nonvanishing and positive intercept is a generic characteristic of Regge trajectories for mesons, but more importantly for the soft Pomeron [31].
3.2.1 Fermions and the Regge intercept

The above discussion can be generalized to the case of 10-d Green-Schwarz superstring
\[
S = \frac{T_s}{2} \int d^2\sigma \sqrt{\gamma^{\alpha\beta}} \partial_\alpha X^\mu \partial_\beta X^\mu \\
+ iT_s \int d^2\sigma (\sqrt{\gamma^{\alpha\beta}} \delta_{IJ} - e^{\alpha\beta}(\tau_3)_{IJ}) \bar{\theta}^I \rho_\alpha \nabla_\beta \theta^J,
\]
(3.31)

where \(\alpha, \beta\) are worldsheet indices, \(\gamma^{\alpha\beta}\) is the worldsheet metric and \(\gamma = \det(-\gamma^{\alpha\beta})\).

The matrices \(\rho_\alpha\) are given by
\[
\rho_\alpha = e^m_\mu \partial_\alpha X^\mu \Gamma_m,
\]
(3.32)

where \(m\) is a flat target-space index and \(\Gamma^m\) are the ten dimensional Dirac matrices.

The derivatives \(\nabla_\alpha\) are defined as
\[
\nabla_\alpha = \partial_\alpha + \frac{1}{4} \Omega^{mn}_\mu \Gamma_{mn} \partial_\alpha X^\mu.
\]
(3.33)

Finally, \(\tau_3\) is one of the 2 dimensional Pauli matrices \(\tau_3 = \text{diag}(1, -1)\). Let us begin by gauge-fixing the kappa symmetry by choosing \(\theta^1 = \theta^2 \equiv \theta\), such that from the beginning the only fermionic degrees of freedom are physical. There are different ways to fix the conformal invariance but in accordance with our Nambu-Goto analysis we choose to integrate the world-sheet metric using its equation of motion. The effect of this algebraic operation is to remove the conformal invariance of the GS action, and the resulting action will be
\[
S = T_s \int d^2\sigma \sqrt{\det(\partial_\alpha X^\mu \nabla_\beta X^\mu + \bar{\theta} \rho_\alpha \nabla_\beta \theta)}.
\]
(3.34)

Next, by expanding to quadratic order in fluctuations around the classical configuration of a spinning string we find
\[
S = S_{NG} + i\frac{1}{2} T_s \int d^2\sigma \theta(e\Gamma^0 - \bar{\rho} \Gamma^0) \dot{\bar{\theta}} + \frac{1}{2} \bar{\theta}(e\Gamma^0 - \bar{\rho} \Gamma^0) \Gamma^\phi \theta + \bar{\theta}(\bar{\rho} \Gamma^\phi) \theta',
\]
(3.35)

where \(S_{NG}\) was given in (3.16) and the Dirac matrices \(\Gamma^0, \Gamma^\rho, \Gamma^\phi\) satisfy the usual Clifford algebra. The fermionic action can be rewritten with a standard kinetic term by making a unitary transformation similar to [19]
\[
\rho^0 = e\Gamma^0 - \bar{\rho} \Gamma^\phi = eU \Gamma^0 \cos \sigma U^{-1},
\]
\[
U = \exp(-\frac{1}{2} \text{arccosh}(\frac{1}{\cos \sigma}) \Gamma^0 \Gamma^\phi).
\]
Note that we can equally write
\[ \rho^1 = e U \Gamma^\rho e \cos \sigma U^{-1}. \] (3.36)

It is then natural to define new fermionic fluctuations
\[ \Psi(\sigma, \tau) = \sqrt{\cos \sigma U^{-1}}(\sigma) \theta(\sigma, \tau). \] (3.37)

In terms of these new variables the fermionic action becomes
\[ S_F = i T_s \int d^2 \sigma \left( \bar{\Psi}(\Gamma^0 \partial_\tau + \Gamma^\rho \partial_\sigma) \Psi + \bar{\Psi} \Gamma^\rho U \sqrt{\cos \sigma} \partial_\sigma(U^{-1} \frac{1}{\sqrt{\cos \sigma}}) \Psi \right. \]
\[ + \left. \frac{1}{2} \bar{\Psi} \Gamma^0 \Gamma^\rho U^2 \Psi \right) \]
\[ = i T_s \int d^2 \sigma \left( \bar{\Psi}(\Gamma^0 \partial_\tau + \Gamma^\rho \partial_\sigma) \Psi + \frac{1}{\cos \sigma} \bar{\Psi} \Gamma^0 \Gamma^\rho \Psi \right). \] (3.38)

Choosing a specific realization of the Clifford algebra for the Dirac matrices that appear in the action \( \Gamma^0 = i \tau_2 \otimes i d_2 \otimes i d_4, \Gamma^\rho = \tau_1 \otimes i d_2 \otimes i d_4, \Gamma^\phi = \tau_3 \otimes \tau_1 \otimes i d_4, \) and decomposing the 16-component spinor \( \Psi \) into four 4-component spinors \( \psi^i = (\psi^i_1, \psi^i_2, \psi^i_3, \psi^i_4), i = 1, \ldots 4, \) one observes that the Dirac equations obeyed by each of these four spinors split into two sets of coupled first order differential equations for the components \( \psi^i_2, \psi^i_3 \) and \( \psi^i_1, \psi^i_4 \) respectively. The equations satisfied by the pair \( (\psi_2, \psi_3) \) are
\[ (\partial_\tau + \partial_\sigma) \psi_2^i + \frac{1}{\cos \sigma}(-\psi_3^i) = 0, \] (3.39)
\[ (-\partial_\tau + \partial_\sigma) \psi_3^i + \frac{1}{\cos \sigma}(-\psi_2^i) = 0. \] (3.40)

Similar equations are satisfied by \( (\psi_1^i, \psi_4^i) \). Thus, the mass of the eight fermionic fluctuations is \( 1/\cos^2 \sigma \). Note that this mass term is identical to the one found for the bosonic fluctuation \( \delta \tilde{\phi} \) (3.21). However, in the bosonic sector only one mode is massive and the rest seven physical modes are massless, whereas there are eight massive fermionic modes. Altogether the contribution of the fermions to the intercept is \( -\frac{13}{3} \pi \). Since the bosonic contribution is \( \frac{5}{6} \pi \) the total intercept is \( -\frac{7}{3} \pi \). Notice that unlike for the bosonic string, the superstring admits a negative intercept. One remark is in order.

If from some reason, which at present we cannot find, the \( m = 1 \) mode discussed above can be rescued, then all the bosonic and all the fermionic modes are massless and there is an exact cancellation between them which yields a vanishing intercept. As we will show in the next subsection, this is indeed the case in the covariant formulation.
3.3 Covariant treatment of fluctuations: No corrections to Regge trajectories

The analysis in the previous section gives us a very direct way to build a solid intuition for what the quantum corrections should be like. There is, however, a very subtle point that only the covariant approach can clarify. Namely, the implications of the field redefinitions used in both approaches on the quantum measure. Note that we have redefined both the bosonic field (3.19) and the fermionic fields (3.37). Normally, as has been the case for some static configurations in $AdS_5 \times S^5$ [21], the field redefinition is harmless as far as a quantum Jacobian for the measure is concerned. In particular, as explained in [19], some of the divergences are subleading and can be ignored. In our case, however, we will see that the situation is different. A careful analysis of the fluctuations that takes into consideration issues of the measure yields a slightly different result from the naive Nambu-Goto approach.

We will analyze the fluctuations in the context of the Polyakov action in both a path integral approach as well as a canonical quantization procedure. In both cases we use Cartesian coordinates.

3.3.1 Path integral quantization

The most transparent parametrization of the bosonic fluctuations uses the symmetries of the problem. Henceforth, we make the standard decomposition into background fields and quantum fluctuations:

$$X^m = \bar{X}^m + \delta X^m. \quad (3.41)$$

Next we choose the conformal gauge

$$\gamma_{\alpha\beta} = \partial_\alpha \bar{X}^m \partial_\beta \bar{X}^n \eta_{mn} = e^2 \cos^2 \sigma \eta_{\alpha\beta} = \sqrt{\gamma} \eta_{\alpha\beta}, \quad (3.42)$$

by identifying the world-sheet metric with the induced metric by the target-space background configuration. The bosonic fluctuations are world-sheet scalars, and their measure is given by

$$||\delta X^m||^2 = T_s \int d\sigma d\tau \sqrt{\gamma} \delta X^m \delta X^m. \quad (3.43)$$

In the Polyakov string formulation, the bosonic fluctuations when expressed in Cartesian coordinates are decoupled and their action is simply

$$S_B = \frac{T_s}{2} \int d\tau d\sigma \sqrt{-\gamma} \gamma^{\alpha\beta} \partial_\alpha \delta X^m \partial_\beta \delta X^m = \frac{T_s}{2} \int d\tau d\sigma \eta^{\alpha\beta} \partial_\alpha \delta X^m \partial_\beta \delta X^m. \quad (3.44)$$
Therefore, by integrating them out in path integral one obtains
\[
\int DX \ e^{S_{cl}s} \exp(T_s \int d\sigma d\tau \sqrt{\gamma} \gamma^{\alpha\beta} \partial_\alpha \delta X^m \partial_\beta \delta X_m) = e^{S_{cl}s} \det(\Delta_\gamma)^{-d/2},
\]
where \(d\) is the number of target-space dimensions, and
\[
\Delta_\gamma = \frac{1}{\sqrt{\gamma}} \partial_\alpha \gamma^{\alpha\beta} \sqrt{\gamma} \partial_\beta,
\]
is the two-dimensional world-sheet Laplace operator.

The contribution of the reparametrization ghosts to the partition function (omitting the zero-mode contributions) is
\[
\int DbDc \ exp(T_s \int d\sigma d\tau \sqrt{\gamma} b_{\alpha\beta} \gamma^{\alpha\gamma} \gamma^{\beta\delta} \nabla_\gamma c_\delta) = \det(P_1 P_1^\dagger)^{1/2}
\]
where \(P_1\) is the differential operator that maps vectors into symmetric traceless two-tensors. On a genus one world-sheet one has
\[
\det(P_1 P_1^\dagger)^{1/2} = \det(\Delta_\gamma)
\]
and thus the contribution of the two longitudinal bosonic degrees of freedom is canceled by ghosts.

Finally we are left with the fermionic degrees of freedom. The starting point is the Green-Schwarz superstring, with the fermionic degrees of freedom reduced to the physical ones by choice of kappa-gauge. As before, we choose to identify \(\theta^1 = \theta^2 \equiv \theta\). Expanding the Polyakov action to second order in the fermionic fluctuations \(\theta\) yields the following action:
\[
S_F = \frac{i}{2} T_s \int d\tau d\sigma \sqrt{\gamma} \gamma^{\alpha\beta} \partial_\alpha \bar{X}^\mu \bar{\theta} \Gamma_\mu \partial_\beta \theta
\]
\[
= \frac{i}{2} T_s \int d\tau d\sigma \left[ e \left( \bar{\theta} (\Gamma^0 + (\Gamma^1 \cos \tau - \Gamma^2 \sin \tau) \sin \sigma) \hat{\theta} \right. \right.
\]
\[
+ \bar{\theta} (\Gamma^1 \sin \tau + \Gamma^2 \cos \tau) \cos \sigma \theta' \left. \right) \right].
\]
(3.49)

Using that through unitary transformations
\[
U_{01} = \exp(-\frac{1}{2} \arccosh(\frac{1}{\cos \sigma}) \Gamma^0 \Gamma^1)
\]
\[
U_{12} = \exp(\frac{1}{2} \tau \Gamma^1 \Gamma^2)
\]
(3.50)
we can rewrite
\[ \Gamma^0 + (\Gamma^1 \cos \tau - \Gamma^2 \sin \tau) \sin \sigma = \cos \sigma U_{12} U_{01} \Gamma^0 U_{01}^{-1} U_{12}^{-1} \quad (3.52) \]
\[ (\Gamma^1 \sin \tau + \Gamma^2 \cos \tau) \cos \sigma = \cos \sigma U_{12} U_{01} \Gamma^2 U_{01}^{-1} U_{12}^{-1}, \quad (3.53) \]
the action simplifies, provided that we make the field redefinition \( \Psi = U_{01}^{-1} U_{12} \theta \), to
\[ S_F = \frac{i}{2} T_s \int d\tau d\sigma \cos \sigma \bar{\Psi}(\Gamma^0 \partial_\tau + \Gamma^2 \partial_\sigma) \Psi. \quad (3.54) \]
Integrating out the fermionic fluctuations, which are world-sheet scalars, with the path integral measure
\[ ||\bar{\theta}|| = T_s \int d\sigma d\tau \sqrt{\gamma} \bar{\theta} \theta, \quad (3.55) \]
one finds their contribution to the partition function
\[ \int D\theta \exp\left( \int d\sigma d\tau \sqrt{\gamma} \bar{\theta} \partial_\alpha \bar{X}^m \Gamma_m \partial_\beta \theta \right) = \det(\gamma^{\alpha\beta} \partial_\alpha \partial_\beta)^8 \]
\[ = \det(\gamma^{\alpha\beta} \partial_\alpha X^m \Gamma_m \partial_\beta(\gamma^{\gamma\delta} \partial_\gamma \bar{X}^n \Gamma_n \partial_\delta))^4 \]
\[ = \det(\gamma^{\alpha\beta} \partial_\alpha \partial_\beta)^4. \quad (3.56) \]
In the last step we used repeatedly equation (3.42) and the fact that partial derivatives commute. To explicitly evaluate (3.56) we use the fact that the determinants of two conformally related operators are also related by a simple relation [19] (see also [20]).
In particular the fermionic determinant (3.56) can be reexpressed as
\[ \ln(\det(\gamma^{-1/2} \delta^{\alpha\beta} \partial_\alpha \partial_\beta)) - \ln(\det(\delta^{\alpha\beta} \partial_\alpha \partial_\beta)) \]
\[ = -\frac{1}{4\pi} \int_0^{2\pi} d\sigma \int_0^T d\tau \frac{1}{12} \delta^{\alpha\beta} \partial_\alpha \ln \sqrt{\gamma} \partial_\beta \ln \sqrt{\gamma} \]
\[ = -\frac{T}{6} \quad (3.57) \]
The evaluation of the similar determinant for the bosons follows the same pattern:
\[ \ln(\det(\Delta_\gamma)) - \ln(\det(\delta^{\alpha\beta} \partial_\alpha \partial_\beta)) \]
\[ = \frac{1}{12\pi} \int_0^{2\pi} d\sigma \int_0^T d\tau \delta^{\alpha\beta} \partial_\alpha \ln \gamma^{1/4} \partial_\beta \ln \gamma^{1/4} \quad (3.58) \]
One crucial element in this analysis that differs from the naive Nambu-Goto approach is that the path integral approach captures some divergences embodied in the \(-T/6\) terms. If desired, we can explicitly evaluate all the determinants presented above (3.57) and (3.58) since the right hand side is completely explicit and the second term in the left hand sides has been explicitly evaluated in appendix A.
3.3.2 Canonical quantization

It is obvious that the $D - 3$ (seven in our case) $\vec{z}$ coordinates are decoupled from the classical configuration and hence in flat space time the fluctuations in these coordinates are free massless modes. So from here on we discuss only the rest of the coordinates. In the Cartesian coordinates basis the decomposition of the fields to the classical configurations and the quantum fluctuations take the form

$$
X^0 = e\tau + \delta X^0 \\
X^1 = e\cos \tau \sin \sigma + \delta X^1 \\
X^2 = e\sin \tau \sin \sigma + \delta X^2
$$

Upon inserting these configuration into the Polyakov action in the covariant gauge, we easily find from the equations of motions that the fluctuations are all massless, namely, obey

$$
(\partial_\sigma^2 - \partial_\tau^2) \delta X^i = 0
$$

in particular for $i = 0, 1, 2$. The Virasoro constraint takes the form of (3.6) where $\mathcal{H}(\delta X^i)$ is the 2d Hamiltonian density of the 10 free massless bosonic modes. In the covariant gauge, however, as was discussed above, one has to take into account the contribution of the reparametrization ghosts. It is well known that the latter eliminate the longitudinal modes and hence the final form of the bosonic part of the Virasoro constraint is

$$
e(E - \bar{E}) = -\pi\frac{D - 2}{24} = -\frac{\pi}{3}
$$

Note that in deriving this result in the Polyakov formulation we have not taken a quadratic approximation, but rather the full action. The only approximation made here is setting in the value of $J$ to be equal to $\bar{J}$.

Next we discuss the fermionic contribution to the energy $E$. It was shown above in (3.54) that the relevant operator for the fermionic determinant is $(\Gamma^0 \partial_\tau + \Gamma^2 \partial_\sigma)$. This means that unlike in the polar coordinates formulation, here there is no mass term and the fermionic modes, just as the bosonic ones, are free and massless. This of course implies that they contribute to the Virasoro constraint the same contribution as the bosonic modes but with an opposite sign namely $-\frac{\pi}{3}$. Hence, as for the path integral approach, also in the canonical quantization the bosonic and fermionic contributions to the intercept cancel each other.
The main conclusion of this section is upon using the Polyakov formulation in the Cartesian coordinates we observe that for the superstring spinning in flat space there are no quantum corrections to the intercept which therefore remains zero. Hence it is clear from both the path integral and the canonical quantization that the semiclassical partition function of the GS superstring whose classical configuration describes a string which spins in flat space is trivial, being identical with 1.

To summarize, we have presented two types of calculations of the quantum fluctuations around the classical spinning string in flat space-time. In the first we used the canonical quantization of the NG formulation in polar coordinates and in the second the Polyakov formulation in Cartesian coordinates both in a path-integral as well as a canonical quantizations. Recall that in the GS framework the NG and Polyakov fermionic actions are the same. In the NG formulation of the bosonic modes we have truncated the action to include only quadratic terms whereas in Polyakov’s formulation we used the exact expression with no truncation since it is quadratic. The outcome of the two types of evaluations is different. In the first we find a non-trivial intercept whereas in the second the intercept vanishes. Now since we do not see any loopholes in the second approach we believe that the correct result is that there is no intercept. As it stands the results that follow from the NG approach are in contradiction with this conclusion. At this point there are two options: (i) that indeed the NG approach leads to a different result (ii) that form a reason that we do not understand at present the \( m = 1 \) modes discussed above are not missing from the spectrum of eigenmodes and hence both the bosonic and fermionic modes admit a massless spectrum and the intercept vanishes. We leave the investigation of this open question for a future investigation.

### 3.4 Supersymmetry

In this subsection we show that, despite the one-loop cancellation just presented, the string configuration we are considering is not supersymmetric. The supersymmetry condition follows directly from the kappa-symmetry variation of the action (3.15). In
the particular background we are considering one has

\[ X'^M \dot{X}^N \Gamma_{MN} \epsilon^* = \sqrt{-\gamma} \epsilon \]
\[ \rho' \dot{X}^0 \Gamma_{\rho 0} + \rho' \dot{\phi} \Gamma_{\rho \phi} \epsilon^* = e^2 \cos^2 \sigma \epsilon \]
\[ \cos \sigma \Gamma_{\rho} (\Gamma_0 + \Gamma_{\rho}) \epsilon^* = \cos^2 \sigma \epsilon \]
\[ \Gamma^\phi \left( \Gamma^\phi - \sin \sigma \Gamma^\phi \right) \epsilon^* = -\cos \sigma \epsilon . \]

(3.62)

Since the Killing spinor of spacetime is constant (flat space), the above expression implies that no supersymmetry is preserved by this solution. The result is also intuitively clear from the string theory point of view. Since the configuration has finite extension it does not belong in the supergravity part of the spectrum which is the sector where supersymmetric configurations are most likely to be found. Moreover, we know that this classical configuration describes massive string states. Of course, some extended configurations are supersymmetric, like the straight string mentioned previously. However, spinning strings typically break all supersymmetries due to the fact that the spin does not enter as a central charge in the supersymmetry algebra and therefore can not support a BPS-like condition (see [22] for some recent considerations)

\footnote{1} \footnote{2}. The breaking of the supersymmetry makes the origin of the one-loop cancellation obtained in the previous section unclear to us. We will simply point out that other nonsupersymmetric configurations, like the circular Wilson loop\footnote{2}, enjoy very similar one-loop cancellations \footnote{10}.

4 Quadratic fluctuations in confining backgrounds

Quadratic fluctuations of classical configurations in confining string backgrounds were analyzed for the static Wilson line configurations \footnote{24}. The quantum fluctuations result in a typical Lüscher term which introduces \( \frac{1}{L} \) corrections to the linear potential. These corrections carry information about the spectrum of the theory and can, in principle, be measured on the lattice. In this paper we compute quantum corrections which are definitely measurable as the intercept of Regge trajectories. Our aim is to determine the universal features of such corrections. To clarify the universality we conduct our analysis of the quadratic fluctuations for the strings spinning in Sugra backgrounds dual to confining gauge theories. We consider the KS and MN backgrounds explicitly.
4.1 The Klebanov-Strassler background

We begin by reviewing the KS background, which is obtained by considering a collection of $N$ regular and $M$ fractional D3-branes in the geometry of the deformed conifold \[1\]. The 10-d metric is of the form:

$$ds_{10}^2 = h^{-1/2}(\tau) dX_\mu dX^\mu + h^{1/2}(\tau) ds_6^2,$$

where $ds_6^2$ is the metric of the deformed conifold \[20, 27\]:

$$ds_6^2 = \frac{1}{2} \varepsilon^{4/3} K(\tau) \left[ \frac{1}{3K^3(\tau)} \left( d\tau^2 + (g^5)^2 \right) + \cosh^2 \left( \frac{\tau}{2} \right) \left[ (g^3)^2 + (g^4)^2 \right] + \sinh^2 \left( \frac{\tau}{2} \right) \left[ (g^1)^2 + (g^2)^2 \right] \right].$$

where

$$K(\tau) = \frac{(\sinh(2\tau) - 2\tau)^{1/3}}{2^{1/3} \sinh \tau},$$

and

$$g^1 = \frac{1}{\sqrt{2}} \left[ - \sin \theta_1 d\phi_1 - \cos \psi \sin \theta_2 d\phi_2 + \sin \psi d\theta_2 \right],$$
$$g^2 = \frac{1}{\sqrt{2}} \left[ d\theta_1 - \sin \psi \sin \theta_2 d\phi_2 - \cos \psi d\theta_2 \right],$$
$$g^3 = \frac{1}{\sqrt{2}} \left[ - \sin \theta_1 d\phi_1 + \cos \psi \sin \theta_2 d\phi_2 - \sin \psi d\theta_2 \right],$$
$$g^4 = \frac{1}{\sqrt{2}} \left[ d\theta_1 + \sin \psi \sin \theta_2 d\phi_2 + \cos \psi d\theta_2 \right],$$
$$g^5 = d\psi + \cos \theta_1 d\phi_1 + \cos \theta_2 d\phi_2.$$ (4.4)

The 3-form fields are:

$$F_3 = \frac{M\alpha'}{2} \left\{ g^5 \wedge g^3 \wedge g^4 + d[F(\tau)](g^1 \wedge g^3 + g^2 \wedge g^4) \right\}$$
$$= \frac{M\alpha'}{2} \left\{ g^5 \wedge g^3 \wedge g^4(1 - F) + g^5 \wedge g^1 \wedge g^2 F$$
$$+ F' d\tau \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \right\},$$

and

$$B_2 = \frac{g_s M\alpha'}{2} \left[ f(\tau) g^1 \wedge g^2 + k(\tau) g^3 \wedge g^4 \right],$$

$$H_3 = dB_2 = \frac{g_s M\alpha'}{2} \left[ d\tau \wedge (f' g^1 \wedge g^2 + k' g^3 \wedge g^4)$$
$$+ \frac{1}{2} (k - f) g^5 \wedge (g^1 \wedge g^3 + g^2 \wedge g^4) \right].$$
The self-dual 5-form field strength is decomposed as $\tilde{F}_5 = F_5 + \star F_5$, with

$$F_5 = B_2 \wedge F_3 = \frac{g_s M^2(\alpha')^2}{4} \ell(\tau) g^1 \wedge g^2 \wedge g^3 \wedge g^4 \wedge g^5 ,$$

(4.7)

where

$$\ell = f(1 - F) + kF ,$$

(4.8)

and

$$\star F_5 = 4g_s M^2(\alpha')^2 \varepsilon^{-8/3} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\tau \frac{\ell(\tau)}{K^2 h^2 \sinh^2(\tau)} .$$

(4.9)

The functions introduced in defining the form fields are:

$$F(\tau) = \frac{\sinh \tau - \tau}{2 \sinh \tau} ,$$

$$f(\tau) = \frac{\tau \coth \tau - 1}{2 \sinh \tau} (\cosh \tau - 1) ,$$

$$k(\tau) = \frac{\tau \coth \tau - 1}{2 \sinh \tau} (\cosh \tau + 1) .$$

(4.10)

The equation for the warp factor is

$$h' = -\alpha f(1 - F) + kF$$

where

$$\alpha = 4(g_s M \alpha')^2 \varepsilon^{-8/3} .$$

(4.11)

(4.12)

For large $\tau$ we impose the boundary condition that $h$ vanishes. The resulting integral expression for $h$ is

$$h(\tau) = \alpha \frac{2^{2/3}}{4} I(\tau) = (g_s M \alpha')^2 2^{2/3} \varepsilon^{-8/3} I(\tau) ,$$

(4.13)

where

$$I(\tau) \equiv \int_{\tau}^{\infty} dx \frac{x \coth x - 1}{\sinh^2 x} (\sinh(2x) - 2x)^{1/3} .$$

(4.14)

The above integral has the following expansion in the IR:

$$I(\tau \to 0) \to a_0 - a_1 \tau^2 + O(\tau^4) ,$$

(4.15)

where $a_0 \approx 0.71805$ and $a_1 = 2^{2/3} 3^{2/3}/18$. The absence of a linear term in $\tau$ reassures us that we are really expanding around the end of space, where the Wilson loop will find it more favorable to arrange itself.
4.2 Quadratic fluctuations of the KS model

We consider the quadratic fluctuations and their influence on the Regge trajectories \(2.30\). The full string theory in such backgrounds is not known. However, for a semiclassical treatment the sigma model action is needed only up to quadratic terms and it is given by (we follow [25]):

\[
S = \frac{1}{4\pi\alpha'} \int d\sigma_1 d\sigma_2 \sqrt{\gamma} \left[ (g_{\mu\nu} \gamma^{\alpha\beta} + b_{\mu\nu} \epsilon^{\alpha\beta}) \partial_\alpha X^\mu \partial_\beta X^\nu \\
+ i(\gamma^{\alpha\beta} \delta_{IJ} - \epsilon^{\alpha\beta}(\rho_3)_{IJ}) \partial_\alpha X^m \bar{\theta}^I \Gamma_m \partial_\beta \theta^J \right],
\]

(4.16)

where \(\theta^I (I=1,2)\) are the two real positive chirality 10-d MW spinors and \(D_b\) is the pullback to the world-sheet of the supergravity covariant derivative in the variation of the gravitino:

\[
D_\alpha = \partial_\alpha + \frac{1}{4} \partial_\alpha X^\mu \left[ (\omega_{mn\mu} - \frac{1}{2} H_{mn\mu} \rho_3) \Gamma^{mn} + \frac{1}{3!} F_{mnp} \Gamma^{mpq} \kappa_1 + \frac{1}{2 \cdot 5!} F_{mnpq} \Gamma^{mpq} \rho_0 \right] \Gamma^\mu
\]

(4.17)

where the \(\rho_s\)-matrices in the \(I, J\) space are the Pauli matrices \(\kappa_1 = \sigma_1, \rho_0 = i\sigma_2, \rho_3 = \sigma_3\). Let us first consider the metric. The part of the metric perpendicular to the world volume, which is the deformed conifold metric, does not enter in the classical solution which involves only world volume fields. Noting that the value \(r_0\) of section \(2.4.1\) is \(\tau = 0\), we expand the deformed conifold up to quadratic terms in the coordinates:

\[
ds_6^2 = \frac{\epsilon^{4/3}}{2^{2/3} 3^{1/3}} \left[ \frac{1}{2} g_5^2 + g_3^2 + g_1^2 + \frac{1}{2} d\tau^2 + \frac{\tau^2}{4} (g_1^2 + g_2^2) \right].
\]

(4.18)

Let us further discuss the structure of this metric. It is known on very general grounds that the deformed conifold is a cone over a space that is topologically \(S^3 \times S^2\) [26].

We can see that the \(S^3\) roughly spanned by \((g_3, g_4, g_5)\) has finite size, while the \(S^2\) spanned by \((g_1, g_2)\) shrinks to zero size at the apex of the deformed conifold. Note that the radius of the \(S^3\) is given by the deformation parameter \(\epsilon\). What is less known is that up to the level of accuracy of \(4.18\), \((g_3, g_4, g_5)\) spanned a space that is not only topologically but also geometrically an \(S^3\) of radius \(\sqrt{2}\) [27]. This fact follows from the explicit construction of a matrix \(T \in SU(2)\) such that

\[
\frac{1}{2} \text{Tr} dT^\dagger dT = \frac{1}{2} g_5^2 + g_3^2 + g_1^2.
\]

(4.19)
Because $T \in SU(2)$ the resulting metric is a round $S^3$ with radius $\sqrt{2}$. Thus, we introduce a standard parametrization of $S^3$ in terms of Euler angles $(\theta, \phi, \psi)$ and identify

$$ \frac{1}{2} g_5^2 + g_3^2 + g_4^2 = \frac{1}{2} (d\theta^2 + d\phi^2 + d\psi^2 + 2 \cos \theta \ d\phi \ d\psi). $$

(4.20)

Expanding around the classical value of these coordinates $\phi = \psi = 0$, $\theta = \pi/2$ we obtain essentially an $\mathbb{R}^3$ which we parametrize by $(y_1, y_2, y_3)$. We end up with

$$ \frac{1}{2} g_5^2 + g_3^2 + g_4^2 \rightarrow \frac{1}{2} [dy_1^2 + dy_2^2 + dy_3^2]. $$

(4.21)

It must be the case then that $(g_1, g_2)$ spanned a space that is topologically an $S^2$. Because this $S^2$ space is fibered over $S^3$ the combination $g_1^2 + g_2^2$ will necessarily contain some of the coordinates $(\theta, \phi, \psi)$. However, up to quadratic terms we can neglect them$^3$.

Thus, to this level of accuracy, we can confidently identify $g_1^2 + g_2^2$ with a round $S^2$ with a standard parametrization $(\bar{\theta}, \bar{\phi})$. This round $S^2$ has coefficient $\tau^2$ and combines with the $\tau$ direction to give an $\mathbb{R}^3$, which we choose to parametrize by $(\tau_1, \tau_2, \tau_3)$:

$$ \frac{1}{2} [d\tau^2 + \frac{\tau^2}{2} (g_1^2 + g_2^2)] \rightarrow \frac{1}{2} [d\tau_1^2 + d\tau_2^2 + d\tau_3^2]. $$

(4.22)

To complete the bosonic part of the string action we need to consider the B-field (4.6) in this limit, that is, up to quadratic terms in the action (4.16). The B-field contribution to the action at this order vanishes. A simple way to confirm this is by analyzing the structure of the NSNS 3-form field strength (4.7). Let us consider the first term in (4.7). Since near $\tau = 0$ we have the following expansion $f' \approx \tau^2/4 + O(\tau^4)$ we see that this term is proportional to the volume element of $\mathbb{R}^3$ defined by $(\tau, g_1, g_2)$, thus in the new coordinates this term of the NSNS 3-form field strength is proportional to $d\tau_1 \wedge d\tau_2 \wedge \tau_3$. In the action this term contributes in the form

$$ \frac{1}{2} \frac{g_s M \alpha'}{2} \tau_1 \partial_\alpha \tau_2 \partial_\beta \tau_3 \epsilon^{\alpha\beta}, $$

(4.23)

which is third order in the action. Similarly, one can check that all other terms in the B-field contribute terms that are third and higher order and can, therefore, be neglected in the approximation we are working. Note, that the situation is, again, different from [6] where the B-field contributed to the quadratic string action. The reason being that if we take the light-cone gauge involving any of the coordinates $y_i$,

$^3$This situation is completely different from the discussion of [6] where the precise structure of the fibration is crucial.
then the above term becomes quadratic. Here, we do not anticipate taking a light-cone
gauge that involves any of the six directions of the deformed conifold. Interestingly,
there is an effective $\sigma$-dependent mass term for the $\tau$ directions. It arises as we expand
in the GS action the warp factor up to quadratic order in $\tau$

$$\frac{1}{2} \int \sqrt{\gamma} \gamma^\alpha{}^\beta (g_{\tau\tau}(\bar{X})) \left|_{\tau=0} \right. \partial_\alpha \tau^i \partial_\beta \tau_i + \frac{1}{2} \frac{\partial^2 g_{\mu\nu}(\bar{X})}{\partial \tau^2} \left|_{\tau=0} \right. \partial_\alpha \bar{X}^\mu \partial_\beta \bar{X}^\nu \tau^i \tau_i$$ (4.24)

Using the conformal gauge $\gamma_{\alpha\beta} = \eta_{\alpha\beta} \sqrt{\gamma}$ with the choice of the Weyl factor such that
the worldsheet metric is equal to the target-space induced metric

$$\gamma_{\alpha\beta} = \partial_\alpha \bar{X}^\mu \partial_\beta \bar{X}^\nu g_{\mu\nu}(\bar{X}) \left|_{\tau=0} \right. = \frac{\varepsilon^4}{2^{1/3} a_0^{1/2}} g_s M \alpha' e^2 \cos^2 \sigma$$ (4.25)

we find the action for the three massive $\tau$ fluctuations

$$\frac{1}{2} \int \sqrt{\gamma} \left( \sqrt{-g} g_s M \alpha' a_0^{1/2} \eta^{\alpha\beta} \partial_\alpha \tau^i \partial_\beta \tau_i + \frac{a_1}{2 a_0} \tau^i \tau_i \right)$$ (4.26)

If one is willing to forget about the subtle issues of the path integral measure, then one
would conclude that the $\tau^i$ fluctuations have a $\sigma$-dependent "mass term"

$$m_B^2 = \frac{2 \cdot 3^{1/3} a_1}{a_0^2} \frac{\varepsilon^{4/3}}{g_s M \alpha'} \frac{e^2}{g_s M \alpha'} \cos^2 \sigma \equiv 2 m_0^2 e^2 \cos^2 \sigma.$$ (4.27)

Note that $m_0$ is proportional to the glueball mass deduced from the dilaton spectrum
in the KS background. Recall that the latter corresponds to a glueball of zero angular
momentum.

**Fermionic sector**

Since the fermions are already quadratic in fluctuations given that their classical value
is zero we need to worry only about the value of the field strength at the point $\tau = 0$.
This means that effectively $H_{NS}$ vanishes and the RR 3-form $F_3$ is completely directed
along the $S^3$ directions:

$$F_3 = \frac{M \alpha'}{2} g_5 \wedge g_3 \wedge g_4$$ (4.28)

### 4.3 The Maldacena-Núñez background

The MN background whose IR regime is associated with $\mathcal{N} = 1$ SYM theory is that
of a large number of D5 branes wrapping an $S^2$. To be more precise: (i) the dual field
theory to this SUGRA background is the $\mathcal{N} = 1$ SYM contaminated with KK modes which cannot be de-coupled from the IR dynamics, (ii) the IR regime is described by the SUGRA in the vicinity of the origin where the $S^2$ shrinks to zero size. The full MN SUGRA background includes the metric, the dilaton and the RR three-form. It can also be interpreted as an uplifting to ten dimensions a solution of seven dimensional gauged supergravity \[28\]. The metric and dilaton of the background are

\[
\begin{align*}
 ds^2 &= e^\phi \left[ dX^a dX_a + \alpha' g_s N (d\tau^2 + e^{2g(\tau)} (e_1^2 + e_2^2) + \frac{1}{4} (e_3^2 + e_4^2 + e_5^2)) \right], \\
 e^{2\phi} &= e^{-2\phi_0} \frac{\sinh 2\tau}{2e^{g(\tau)}}, \\
 e^{2g(\tau)} &= \tau \coth 2\tau - \frac{\tau^2}{\sinh^2 2\tau} - \frac{1}{4}. 
\end{align*}
\]

(4.29)

where,

\[
\begin{align*}
 e_1 &= d\theta_1, & e_2 &= \sin \theta_1 d\phi_1, \\
 e_3 &= \cos \psi d\theta_2 + \sin \psi \sin \theta_2 d\phi_2 - a(\tau) d\theta_1, \\
 e_4 &= -\sin \psi d\theta_2 + \cos \psi \sin \theta_2 d\phi_2 - a(\tau) \sin \theta_1 d\phi_1, \\
 e_5 &= d\psi + \cos \theta_2 d\phi_2 - \cos \theta_1 d\phi_1, & a(\tau) &= \frac{\tau^2}{\sinh^2 2\tau}. 
\end{align*}
\]

(4.30)

where $\mu = 0, 1, 2, 3$, we set the integration constant $e^{\phi_0} = \sqrt{g_s N}$ The 3-form can be obtained as

\[
\begin{align*}
 H^{RR} &= g_s N \left[ -\frac{1}{4} (w^1 - A^1) \wedge (w^2 - A^2) \wedge (w^3 - A^3) + \frac{1}{4} \sum_a F^a \wedge (w^a - A^a) \right], \\
 A &= \frac{1}{2} \left[ \sigma_1 a(\rho) d\theta_1 + \sigma^2 a(\rho) \sin \theta_1 d\phi_1 + \sigma^3 \cos \theta_1 d\phi_1 \right] 
\end{align*}
\]

(4.31)

and the one-forms $w^a$ are given by:

\[
 w^1 + i w^2 = e^{-i\psi} (d\theta_2 + i \sin \theta_2 d\phi_2), \quad w^3 = d\psi + \cos \theta_2 d\phi_2 \quad (4.32)
\]

Note that we use notation where $x^0, x^i$ have dimension of length whereas $\rho$ and the angles $\theta_1, \phi_1, \theta_2, \phi_2, \psi$ are dimensionless and hence the appearance of the $\alpha'$ in front of the transverse part of the metric. Moreover, following the notation of \[29\] a factor of $g_s N$ is multiplying the $\alpha'$ instead of $N$ that appears in \[5\].
4.4 Quadratic fluctuations in the MN background

The position referred to as \( r_0 \) in section (2.4.1) is \( \tau = 0 \). Therefore, we will expand the metric around that value. Let us first identify some structures in the metric that are similar to the deformed conifold considered in the previous section. Notice that \( e_1^2 + e_2^2 \) is precisely an \( S^2 \). Moreover, near \( \tau = 0 \) we have that \( e^2 g \approx \tau^2 + \mathcal{O}(\tau^4) \). Thus \( (\tau, e_1, e_2) \) span \( \mathbb{R}^3 \) which we parametrize as \( (\tau_1, \tau_2, \tau_3) \).

\[
d\tau^2 + e^{2g(\tau)}(e_1^2 + e_2^2) \longrightarrow d\tau_1^2 + d\tau_2^2 + d\tau_3^2. \tag{4.33}
\]

Certainly \( e_3^2 + e_4^2 + e_5^2 \) parametrizes a space that is topologically a three sphere fibered over the \( S^2 \) spanned by \( (e_1, e_2) \). However, near \( \tau = 0 \) we have a situation very similar to the structure of the metric in the deformed conifold. Namely, at \( \tau = 0 \) there we have that: \( e_5 \to g_5, e_3 \to \sqrt{2}g_4, e_4 \to \sqrt{2}g_3 \) (up to a trivial identification \( \theta_1 \to -\theta_1, \phi_1 \to -\phi_1 \)). This allows us to identify this combination as a round \( S^3 \) of radius 2. Subsequently expanding around the classical values of this round \( S^3 \) we obtain::

\[
e_5^2 + e_3^2 + e_4^2 \longrightarrow dy_1^2 + dy_2^2 + dy_3^2. \tag{4.34}
\]

In complete analogy with the KS model the \( \tau \) directions receive a mass term from the classical classical solution

\[
\gamma^{\tau\tau} g_{00} \partial_\tau X^0 \partial_\tau X^0 + \gamma^{\alpha\beta} \partial_\alpha X^i \partial_\beta X^i g_{ii} = \left( \frac{8e^{\phi_0}}{g} \alpha' g_s N e^2 \cos^2 \sigma \right) \tau^i \tau_i. \tag{4.35}
\]

The B-field in this background is zero. We thus have obtained the same bosonic spectrum for the quadratic fluctuations around the Regge trajectories in both the KS and MN backgrounds.

Fermionic sector

The treatment of the fermionic sector is very similar to the situation with the KS metric. Indeed, at the classical level, without including any fluctuation the RR 3-form flux is all directed along the \( S^3 \) directions and equals\(^4\)

\[
H^{RR} = -\frac{1}{4} g_s N w^1 \wedge w^2 \wedge w^3. \tag{4.36}
\]

This implies that that the effective action for the fermion is precisely of the same type as in the KS case except that the value of \( \ell \) is different in this case and depends on the specific parameters of the MN solution.

\(^4\)To bring the RR 3-from field strength to this form we use the fact that near the origin \( (\tau = 0) \) the field \( A \) in (4.31) is a pure gauge.
5 Quantum corrected Regge trajectories for spinning strings in confining backgrounds

The quantization can be done together for both backgrounds since their bosonic and fermionic content is identical. It will be useful to trade the parameters of the theories $\varepsilon$ (KS) and $\phi_0, g_sN$ (MN) by the gauge theoretic quantities: String tension, KK masses, etc.

5.1 Bosonic corrections

As can be inferred from the analysis of the expansion in the KS and MN backgrounds around the spinning string the main new modification consist on the appearance of a massive term for the radial direction $\tau$. The equation of motion for the radial fluctuations is

\[\left[\partial^2_r - \partial^2_\sigma + 2m_0^2 e^2 \cos^2(\sigma)\right] \delta \tau_i = 0, \quad (5.1)\]

where $i = 1, 2, 3$ and $m_0^2 = \frac{4}{9} \frac{1}{g_s N a}$ for the MN solution and $m_0^2 = \frac{3^{1/3} a_1}{a_0^2} \frac{\varepsilon^{1/3}}{g_s M a^2} \frac{1}{g_s M a^2}$ for the KS solution. Assuming that the fluctuations take the standard form $\delta \tau_i = e^{in\tau} \delta \tau_i(\sigma)$ we obtain:

\[\left[\frac{\partial^2}{\partial \sigma^2} + \left(n^2 - \frac{2m_0^2 e^2}{2}\right) - m_0^2 e^2 \cos 2\sigma\right] \delta \tau_i(\sigma) = 0. \quad (5.2)\]

The solution to this equations are the Mathieu functions. One of them is $C(a, q, z)$ which is even and the other $S(a, q, z)$ which is odd. It is known that there is no simple analytical presentation for the Mathieu functions. However, they are very well studied numerically and series expansions near $q = 0$ are known (see appendix B). Note that for $q = 0$ the Mathieu functions are simply $\cos \sqrt{a} u$ and $\sin \sqrt{a} u$. For nonzero $q$ the Mathieu functions are periodic only for certain values of $a$. To extract some useful physical information we will content ourselves with analyzing the effect of the mass term in these solutions. Basically, taking $m_0 = 0$ in (5.1) reduces the problem to flat space. Thus for practical reasons we will consider how the small but nonzero $m_0$ modifies the flat space result. In principle we could go up to higher and higher order in $m_0$ since we know the values of the eigenfunction and eigenvalues of the Mathieu equation in a series expansion in $q = m_0^2/4$ (see appendix B). The eigenvalues of the operator (5.1) are

\[\lambda_{r,n} = n^2 + m_0^2 e^2 + r^2 + \frac{1}{2(r^2 - 1)} \frac{m_0^4 e^4}{4} + \mathcal{O}(m_0^8), \quad (5.3)\]
where \( r \) and \( n \) are integers. The quantity we are interested is

\[
\sum_{r,n \in \mathbb{Z}} \lambda_{r,n} \approx \sum_{r,n \in \mathbb{Z}} r^2 + n^2 + m_0^2 e^2 = \ldots
\] (5.4)

The sum of this type of eigenvalues is similar to the situation for a massive scalar field yielding the following expression for the zero point energy \([30]\):

\[
\gamma_0(\mu) = \frac{\mu}{2} + \sum_{n=1}^{\infty} \sqrt{n^2 + \mu^2} = \frac{\mu}{2} + \left[ -\frac{1}{12} + \frac{\mu^2}{2} \ln(4\pi e^{-\gamma}) + \sum_{n=2}^{\infty} (-1)^n \frac{\Gamma(n - \frac{1}{2})}{n!\Gamma(-\frac{1}{2})} \xi(2n - 1) \mu^{2n} \right].
\] (5.5)

In our particular case \( \mu = m_0 e \) and we should trust the above expression up to the first nontrivial term in \( m_0 \).

Let us now investigate the implications of these bosonic modes on the Regge trajectories. Altogether there are five massless bosonic modes and three “massive” ones. Substituting their contributions into (3.6) we get

\[
e(\tilde{E} - \bar{E}) = \frac{\pi}{2} \left( -\frac{8}{12} + 3m_0 e \right)
\] (5.6)

By substituting in the previous equation that \( \bar{E} = e/(\alpha') \), \( J = e^2/(2\alpha') \), we find that the bosonic fluctuations lead to a nonlinear Regge trajectory

\[
J \approx \frac{1}{2} \alpha'(\tilde{E} - \frac{3\pi}{2}m_0)^2 + \Delta_b = \frac{1}{2} \alpha'E^2 + \alpha_0 - \frac{3\pi}{2} \alpha'E m_0 + \Delta_b
\] (5.7)

Several remarks are in order: (i) The term \( \Delta_b \), which would have been the quantum correction had the bosonic modes been massless, will be canceled out by an equal term from the fermionic fluctuations. (ii) The intercept is given by \( \alpha_0' = \frac{\alpha'}{2} \left( \frac{3\pi}{2} m_0 \right)^2 \).

Recall that \( m_0 \) is proportional to the mass of the glueball of vanishing \( J \). This is in agreement with the form of the Regge trajectory where one can interpret the intercept as \( \alpha_0' = \frac{1}{2} \alpha'm_{J=0}^2 \). (iii) The deviation between our “Regge” trajectory and the usual one is the \( -\frac{3\pi}{2} \alpha'E m_0 \) term.

We have not been completely rigorous in the treatment of the \( \delta\tau_i \) fluctuations. The complete contribution of these modes to the path integral can be computed as in section \([\ldots]\). Namely, consider the path integral

\[
\int D\delta\tau_i \exp \left[ -\frac{T_s}{2} \int d\sigma d\tau \sqrt{\gamma} \gamma^{\alpha\beta} \left( \partial_\alpha \delta\tau_i \partial_\beta \delta\tau^i + m_0^2 \cos^2 \sigma \delta\tau_i \delta\tau^i \right) \right]
\]

\[
= \det \left( \Delta_\gamma - \frac{1}{\sqrt{\gamma}} m_0^2 e^2 \cos^2 \sigma \right)^{-3/2},
\] (5.8)
where the determinant is as usual de 2-d Laplace operator:

$$\Delta_{\gamma} = \frac{1}{\sqrt{\gamma}} \partial_{\alpha} \gamma^{\alpha \beta} \sqrt{\gamma} \partial_{\beta}.$$  \hspace{1cm} (5.9)

In the particular conformal gauge where we work, with the worldsheet metric equal to the induced target space metric, we can relate the determinant of the above expression (5.8) to the determinant computed in (5.5). Explicitly we have

$$\det \left( \Delta_{\gamma} - \frac{1}{\sqrt{\gamma}} m_0^2 \cos^2 \sigma \right) = \det \left[ \gamma^{-1/2} \left( \eta^{\alpha \beta} \partial_{\alpha} \partial_{\beta} - m_0^2 \cos^2 \sigma \right) \right].$$  \hspace{1cm} (5.10)

The contribution of the conformal factor was calculated in section 3.3 following the methods of [19,20] and amounts to a shift in the expression of the determinant by $-T/6$ (see equations (3.57) and (3.58)).

### 5.2 Fermionic corrections

To complete our treatment of Regge trajectories in confining backgrounds we now turn to fermions. We will fix, as in the flat space case, the kappa-symmetry by identifying $\theta_1 = \theta_2 = \theta$. The fermionic action, to terms quadratic in the fluctuations, is given by

$$S_F \approx \frac{i}{2} T_s \int \sqrt{\gamma} \gamma^{\alpha \beta} \left( \bar{\theta} \partial_{\alpha} \bar{\theta} \Gamma^\mu \partial_{\beta} \theta + \frac{1}{4} \partial_{\alpha} \bar{\theta} \partial_{\beta} \Gamma^\mu \partial_{\beta} \bar{\theta} \right) \hat{f}_{\gamma}.$$ \hspace{1cm} (5.11)

where $\hat{f}$ is the contraction of the 3-form field strength with ten dimensional Dirac gamma matrices. In particular, we have

$$\hat{f}^2 = -\frac{3}{2} g_0^2 M a_0^3/2.$$ \hspace{1cm} (5.12)

In the conformal gauge the action becomes

$$S_F \approx \frac{i}{2} T_s \int \left( \eta^{\alpha \beta} \partial_{\alpha} \bar{\theta} \Gamma^\mu \partial_{\beta} \theta - \frac{1}{2} \sqrt{\gamma} \hat{f}_{\theta} \right)$$

$$= \frac{i}{2} T_s \int \left[ \sqrt{g_{00}} \left( \bar{\theta} \left( \Gamma^0 - \left( \Gamma^1 \cos \tau - \Gamma^2 \sin \tau \right) \sin \sigma \right) \hat{\theta} \right.$$ 

$$+ \left. \bar{\theta} \left( \Gamma^1 \sin \tau + \Gamma^2 \cos \tau \right) \cos \sigma \theta \right) - \frac{1}{2} \sqrt{\gamma} \hat{f}_{\theta} \right].$$ \hspace{1cm} (5.13)

The kinetic term can be simplified further by using the same type of unitary transformation as in the flat space case (see (3.36)), at the expense of obtaining a more complicated mass term. In the end, one obtains

$$S_F \approx \frac{i}{2} T_s \int \cos \sigma \left[ \sqrt{g_{00}} \left( \bar{\Psi} \Gamma^0 \Psi + \bar{\Psi} \Gamma^2 \Psi' \right) + \Psi \theta \mathcal{M} \Psi \right].$$ \hspace{1cm} (5.14)
where the mass term is

\[
M = -\frac{1}{2\cos \sigma} \sqrt{\gamma} \hat{f} + e\sqrt{g_{00}} \left( \Gamma U_{01}^{-1} U_{12}^{-1} \partial_\tau U_{12} U_{01} + \Gamma^2 U_{01}^{-1} \partial_\sigma U_{01} \right).
\] (5.15)

The ten dimensional rotations implemented by

\[
U_{12} = e^{\frac{i}{2} \Gamma_1 \Gamma_2} \quad U_{01} = \exp\left(-\frac{1}{2} \text{arccosh}\left(\frac{1}{\cos \sigma}\right) \Gamma_0 \Gamma_1 \right)
\] (5.16)

are used to rewrite the Dirac matrices that appear in the kinetic term as

\[
\Gamma_1 \cos \tau - \Gamma_2 \sin \tau = U_{12} U_{01} \Gamma_1 U_{12}^{-1} U_{01}^{-1} \\
U_{12} U_{01} \Gamma_0 \cos \sigma U_{01}^{-1} U_{12}^{-1} = (\Gamma_0 + (\Gamma_1 \cos \tau - \Gamma_2 \sin \tau) \sin \sigma = U_{12} U_{01} \Gamma_0 \cos \sigma U_{01}^{-1} U_{12}^{-1}.
\] (5.17)

The fermionic determinant

More rigorously and to avoid issues with the field redefinition of the fermions as in (3.37) we turn to the path integral evaluation of the partition function. The path integral we want to consider is similar to the one for fermions in flat space with the notable difference that the RR 3-form field strength is nonvanishing and contributes what we could think of as an extra mass term for the fermions

\[
\int [D\theta] \exp \left[ \frac{i T_s}{2} \int d\sigma d\tau \sqrt{\gamma} \theta \gamma^{\alpha \beta} \partial_\alpha \bar{X}^m \Gamma_m \left( \partial_\beta + \frac{1}{2} \partial_\beta \bar{X}^n \hat{f} \Gamma_n \right) \right]
\]

are calculated as

\[
= \left[ \det (\gamma^{\alpha \beta} \partial_\alpha \bar{X}^m \Gamma_m \left( \partial_\beta + \frac{1}{2} \partial_\beta \bar{X}^n \hat{f} \Gamma_n \right) ) \right]^8
\]

\[
= \left[ \det (\gamma^{\alpha \beta} \partial_\alpha \bar{X}^m \Gamma_m \left( \partial_\beta + \frac{1}{2} \partial_\beta \bar{X}^n \hat{f} \Gamma_n \right) ) \right]^4
\]

\[
= \left[ \det \left( \gamma^{\alpha \beta} \partial_\alpha \partial_\beta + \hat{f}^2 \right) \right]^4
\]

\[
= \left[ \det \gamma^{-1/2} (\eta^{\alpha \beta} \partial_\alpha \partial_\beta - m_F^2) \right]^4. \quad (5.18)
\]

where the fermionic mass term \(m_F\) can be expressed as

\[
m_F^2 = \frac{3\varepsilon^{4/3}}{2^{4/3} a_0^3 (g_s^4 M^2 \alpha'^2)} e^2 \cos^2 \sigma \equiv 2\ell^2 e^2 \cos^2 \sigma \quad (5.19)
\]

### 5.3 Quantum corrected Regge trajectories from string theory

Putting together all the partial results of this section we obtain that the corrections to the classical Regge trajectories is given by the zero point energy:

\[
\ln Z = \pi \left( 4\ell - \frac{3}{2} m_0 \right). \quad (5.20)
\]
This explicit formula has been obtained in the limit in which both $m_0$ and $\ell$ are small. This is the limit in which the eigenvalues of the Mathieu equation have explicit analytical expressions. In a more general situation there is no reason for us to restrict ourselves to the small $m_0$ and $\ell$ limit. In fact, as mentioned before, general formulas for the eigenvalues of the Mathieu equations are available. According to the expression

$$K\text{lebanov-Strassler} \quad M\text{aldacena-Núñez}$$

$$m_0 = \frac{3^{1/3} a_1^{1/3}}{a_0} \frac{e^{2/3}}{g_s M a^\alpha}$$

$$\ell = \frac{3^{1/3}}{2^{7/6} a_0} \frac{g_s}{g_s N a^\alpha}$$

Table 2: Parameters determining the Regge intercept for the KS and MN solution.

the effect of these quantum corrections on the Regge trajectories is

$$e(E - \bar{E}) = \left(\frac{3}{2} m_0 - 4\ell \right) \pi e \equiv z_0 e \quad (5.21)$$

Then, by substituting in the previous equation that $\bar{E} = e/(\alpha')$, $J = e^2/(2\alpha')$, we derive a nonlinear Regge trajectory

$$J = \frac{1}{2} \alpha'_e f E^2 - \alpha'_e f z_0 E + \frac{1}{2} \alpha'_e f z_0^2. \quad (5.22)$$

Based on table (2) and the expression (5.20), it seems natural to consider the small $\ell$ limit since for both backgrounds the dimensionless effective parameter is $\ell e$. This combination is proportional to the classical string energy and therefore controls the backreaction of the spinning string on the supergravity background. In the case of the KS background there is another ratio: the deformation parameter to the effective radius of the background, which can be taken to be small. The supergravity limit requires the effective radius of the background in string units $g_s N$ to be large. There are factors of $g_s$ in both expressions which can in principle be taken arbitrary and thus allows us to explore other regimes but this might require considering higher loops. Nevertheless, the case can be made for generic values of $\ell$ and $m_0$ resulting in a positive value of the expression (5.20). This implies that the second term in (5.22) is negative and that the intercept of the corresponding Regge trajectory is positive.
6 Phenomenology

In this section we attempt to elucidate how our results compare to the phenomenological data. Let us begin by clarifying the regime of validity of our string theory calculations. The models we considered are expected to be dual to $\mathcal{N} = 1$ SYM plus extra matter coming from the KK supergravity modes. Evidently this is not QCD, which will constitutes our main source of experimental and lattice results. In the semiclassical analysis that we use, we assume that the supergravity approximation is valid. The validity of SUGRA has two implications. The first one is that the extra matter fields and the pure glue fields will have approximately similar masses and we will necessarily end up with mixing of these two sectors. The second implication is that the validity of the supergravity approximation requires low curvatures which translates into large $N$ in the gauge theory. We will nevertheless, see remarkable qualitative and even quantitative similarities. Our last cautionary point involves the difference among Regge trajectories for mesons, baryons and glueballs. For mesons and baryons Regge trajectories have been experimentally confirmed since the 60’s. The precise experimental value of the slope $\alpha'$ depends on the flavor content of the states lying on the corresponding trajectory. However, a universal value could be taken to be $\alpha' \approx 0.85 \text{GeV}^{-2}$. For the purpose of this paper we will consider some of the trajectories presented in [1] but will use the current (2002) particle data book for accuracy. The soft Pomeron trajectory is qualitatively different from the Regge trajectories for mesons and baryons. Its slope is flatter. The phenomenological parameters of the soft Pomeron are: $J = 1.08 + 0.25t$, that is $\alpha' = 0.25 \text{ GeV}^{-2}$ [21]. The identification of the Pomeron with glueballs seems very plausible and a strong pushed toward its demonstration is being made using lattice [32] and other semi-analytical techniques [33]. The identification of the soft Pomeron trajectory with a trajectory of glueballs does not provide direct information about the value of the glueballs and we therefore turn to the more complete data provided by lattice results [34, 35]. There are many models attempting to explain the difference between the Pomeron slope and the slope for baryons and mesons. Most of the arguments are based on the universal idea that the flux tube between a quark and an antiquark has flux in the fundamental representation while for glueballs the flux is in the adjoint representation. It then follows that the ratio of the slopes is given by the Casimir operators in the fundamental and adjoint representations:

$$\frac{\alpha_{gg}}{\alpha_{q\bar{q}}} = \frac{C_{\text{fund.}}}{C_{\text{adjoint}}} = \frac{N^2 - 1}{2N^2}$$

(6.1)
In the large N limit the ratio goes to 1/2. This is precisely the ratio of slopes obtained by considering the trajectories resulting from classical solutions of closed and open strings spinning in flat space (see eqn (2.4) and (2.7)). Since, as shown in section 2 at the classical level the net effect of putting the strings to spin in actual confining backgrounds rather than in flat space, is a rescaling of the string tension we conclude that this 1/2 is precisely the ratio of the slopes of the Regge trajectories for closed/open (glueball/mesons) predicted by the gauge/gravity correspondence.

6.1 A theoretical value of the Regge slope for glueballs

In this subsection we will show that the low-lying glueball masses calculated in the KS model by Cáceres and Hernández [36] provide an impressive numerical match for the slope of the soft Pomeron trajectory. A similar analysis was carried out for the MN background in [37], it results turn out to be less conclusive. The values obtained in [36] are presented in table 3. The mass is measured in units of the conifold deformation

| State | (Mass)$^2$/$\varepsilon^{4/3}$ |
|-------|---------------------------------|
| 0++   | 9.78                           |
| 0++*  | 33.17                          |
| 1−−   | 14.05                          |
| 1−−*  | 42.90                          |

Table 3: Mass$^2$ in units of the conifold deformation for the low-lying glueballs in the KS model [36].

$\varepsilon^{2/3}$. In the KS background the conifold deformation naturally sets the mass scale of the four dimensional gauge theory. However, there is also a large number $g_s M$ which sets a hierarchy of scales scales between the glueball masses and the string tension [4]. Since we do not reliably know how to fix the value of $\varepsilon$ it makes little sense to find the full Regge trajectory $J = \alpha_0 + \alpha t$. However, since the combination $\alpha' t$ is independent of the units used to measure mass, it makes sense to compute it. Using, as customary, the lightest glueballs for a given spin we find

$$J = 0.234 t + \alpha_0. \quad (6.2)$$

The first term is remarkably close to the experimental value for the soft Pomeron of 0.25 $t$. 

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6.2 Glueball Regge trajectories from the lattice

Since definite experimental evidence for the existence of glueballs remain elusive we will limit our analysis to the lattice data. The lattice results we will use are extracted from [34, 35]. The analysis of [34] was performed for QCD \((N = 3)\), the prediction for the lowest lying glueball at each spin are given in table 4. The analysis of [35] is summarized in table 5. It contain extrapolations of the values of the mass for \(N \rightarrow \infty\) assuming that the difference between the masses at finite and infinite \(N\) is of the form \(\text{const}. N^{-2}\). The best linear fit for the data of [34] and [35] is given by

\[
\begin{array}{c|c|c}
\text{State} & \text{Mass (GeV)} & \text{Mass (GeV) for } SU(N \rightarrow \infty) \\
\hline
0^{++} & 1.73 & 1.60 \\
2^{++} & 2.40 & 2.16 \\
3^{++} & 3.64 & \\
\end{array}
\]

Table 4: Continuum limit for glueball masses from Morningstar and Peardon [34].

\[
\begin{array}{c|c|c}
\text{State} & \text{Mass(GeV) for SU(3)} & \text{Mass(GeV) for SU}(N \rightarrow \infty) \\
\hline
0^{++} & 1.64 & 1.60 \\
2^{++} & 2.33 & 2.16 \\
3^{++} & 4.00 & \\
\end{array}
\]

Table 5: Glueball masses from Teper [35].

\[J_{MP} = -0.234 + 0.259 \, t_{MP}, \quad J_{T} = 0.148 + 0.189 \, t_{T}.\] (6.3)

As we have seen in previous sections the intercept is a crucial parameter that result from quantum corrections on the string side. We are thus particularly interested in comparing our theoretical result against the lattice date. Unfortunately, the evidence is inconclusive since the data of [34] yields a negative intercept while [35] yields a positive intercept. If we assume that all the points have roughly the same error we could combine all the data, this results in

\[J = 0.0265 + 0.213 \, t.\] (6.4)
This trajectory has a positive slope and the value of $\alpha'$ is closer to the one observed for the soft nonperturbative Pomeron. Another argument in favor of a positive intercept can be given by considering the best fit without including the zero spin states. The reason for excluding the zero spin states arises because the semiclassical analysis requires large spin.

6.3 Pomeron phenomenology

6.3.1 The intercept

There are other considerations leading to a positive intercept for the soft Pomeron trajectory. A given Regge trajectory $\alpha(t)$ contributes a term proportional to $s^{\alpha(0)-1}$ to the total cross section $\sigma^{ab}$. It is observed that many total cross sections in elastic scattering ($\bar{p}p$, $pp$, $\pi^-p$, $\pi^+p$) rise with $s$ at high energy [31], which is why a soft-pomeron-exchange term is needed in addition to the experimentally observed trajectories of $(\rho, \omega, \ldots)$. One of the implications of Regge theory is that the total cross-section has, to leading order in $(s/s_0)$, a simple power behavior: $\sigma^{tot} \sim \left(\frac{s}{s_0}\right)^{\alpha(0)-1}$, where $s_0$ is empirically around 1 Gev$^2$. The total cross-sections for various states are remarkably constant over a large range of $s$. Constancy of $\sigma^{tot}(s)$ requires $\alpha_0 \approx 1$. The fact that most mesonic and baryonic trajectories have $\alpha_0$ different from 1 prompted introduction of a new trajectory with $\alpha_0 \approx 1$. In [31] a parametrization of the intercept for the Pomeron trajectory as $\epsilon = \alpha(0) - 1$ has been shown to fit all data for total cross sections ($\bar{p}p$, $pp$, $\pi^-p$, $\pi^+p$, $K^-p$, $K^+p$, $\bar{p}n$, $pn$) with $\epsilon = 0.081$ and $\epsilon = 0.096$. This value of the intercept which imply a positive power of $s$ in the total cross section will naively violate the Froissart bound $\sigma^{ab} \leq (\pi/m_\pi^2) \log^2(s/s_0)$ when the energy becomes extremely large. It is natural to assume then, that the power of $s$ is only an effective power which must reduce as $s$ increases. There are various proposal as to the concrete mechanism for achieving this variation in the power of $s$ including different couplings of the pomeron and or multipomeron exchange [31][38]. However, the explanation is that the total cross section takes this form only in the range of energies discussed, for higher energies there should be two or more pomerons being exchanged. It is, of course, far-fetched for us to claim that our formula (5.20) coincides with the experimental data. What is natural to assume, given that the metric and the three form are related by the Sugra equations of motion, is that $m_0$ and $\ell$ in (5.20) are of the same order. Under this natural assumption we obtain a positive intercept for the Regge trajectories describing glueballs.
6.3.2 Nonlinearity of the Pomeron Regge trajectory from string theory

There are various experimental and theoretical reasons why the Regge trajectory must be nonlinear. Strong experimental evidence for the nonlinearity of the Pomeron Regge trajectory was presented by the UA8 collaboration [39]. The standard linear trajectory of the Pomeron is too small in the $1-2 \, \text{GeV}^{-2}$ $|t|$ - region to adequately describe the data and it is suggested experimentally that:

$$\alpha(t) = 1.10 + 0.25t + \alpha'' t^2,$$

where $\alpha'' = 0.079 \pm 0.012 \, \text{GeV}^{-4}$. It is, of course a challenge to provide a theoretical foundation for such trajectory. Some phenomenological attempts have been put forward in, for example, [40]. We find it very encouraging that the nonlinear trajectories we obtain share some of the properties of (6.5) at the qualitative level. In particular (5.22) has a positive intercept and positive curvature ($\alpha(t)'' > 0$) which are completely compatible with (6.5).

6.4 Regge trajectories for mesons

The data for mesons is more abundant. Some of the trajectories are

- $\rho(770) \,(1^{-})$, $a_2(1320) \,(2^{++})$, $\rho_3(1690) \,(3^{-})$, $a_4(2040) \,(4^{++})$
- $\omega(782) \,(1^{-})$, $f_2(1420) \,(2^{++})$, $\omega_3(1670) \,(3^{-})$, $f_4(2050) \,(4^{++})$
- $K^*(892) \,(1^{-})$, $K^*_3(1430) \,(2^{+})$, $K^*_3(1780) \,(3^{-})$, $K^*_4(2045) \,(4^{+})$, $K^*_5(2380) \,(5^{-})$
- $\pi^0(135) \,(0^{-})$, $b_1(1235) \,(1^{-})$, $a_2(1700) \,(2^{++})$.

In the above list the masses are given in MeV but the Regge trajectories are usually written in GeV units. The main conclusion we would like to draw is that the intercept depends on the particular Regge trajectory but it is generically small and positive. It can also be seen that the slope is practically universal and considerably larger than that of the glueballs. Our main analysis applies strictly to Regge trajectories made of glueballs since we consider closed strings. However, the treatment of open strings is similar. The technical elements follow closely those described in this paper which yield a positive value for the intercept.
6.4.1 Nonlinear Regge trajectories for mesons

Realistic Regge trajectories extracted from data are nonlinear. For example, the straight line that crosses the $\rho$ and $\rho_3$ squared masses corresponds to an intercept of $\alpha_{\rho}(0) = 0.48$, whereas the physical intercept is located at 0.55. Similarly, the straight line which crosses the $K_2^*$ and $K_4^*$ squared masses corresponds to an intercept of 0.1, whereas the physical intercept is 0.4. A more detailed studied was carried out in [41].

The corrections computed using strings spinning in sugra backgrounds dual to confining gauge theories have a similar qualitative effect on linear Regge trajectories.

Comments on the 2+1 Glueball trajectories

It is interesting to note that the existence of Regge trajectories for glueballs in 2+1 dimensions has been shown explicitly in [42]. In this case, as opposed to the cases considered in 3+1, the intercept is unambiguously negative. It would be very interesting to extend the analysis of this paper to 2+1 where lattice data is more abundant. In the context of the gauge/gravity correspondence there are supergravity backgrounds dual to confining 2+1 gauge theories. It has recently been shown that these Sugra backgrounds dual to confining gauge theories in 2+1 admit a set of hadronic states similar to the annulons [43]. This fact encourages one to believe that more generic hadronic states similar to the large spin hadrons considered in this paper ought to exists in these 2+1 confining theories.

7 Conclusions

In this paper we have studied Regge trajectories from the string theory point of view. Classically, Regge trajectories in string theory are linear with zero intercept. We have shown that for strings spinning in flat space the one-loop corrected Regge trajectory receives an positive intercept in the bosonic case whereas in the supersymmetric case the trajectory remains a straight line through the origin even at one loop order.

Our main results are related to the analysis of strings spinning in supergravity backgrounds dual to confining gauge theories. By explicitly considering the KS and MN backgrounds we obtained that for configurations of spinning string they are both qualitatively identical. A unified analysis of the quantum correction was carried out in section 5. Generically the trajectories are nonlinear at the one loop level. We found that for the full background the resulting Regge trajectory generically has a
positive intercept and the has $\alpha(t)'' > 0$. Since we consider spinning closed strings, our results should be compared to glueball trajectories. The most relevant experimental information comes from the soft Pomeron trajectory. The Pomeron trajectory has a positive intercept. However we find that the modifications to the Regge trajectories are different for the bosonic string and the superstring. Motivated by an attempt to see if our analysis contains in principle a qualitative match we considered only the bosonic contribution at one loop. Interestingly we find that two distinctive features of our trajectory coincide with properties of the best experimental fit to the UA8 collaboration data \[39\], that is a Regge trajectory of the form: $J = \alpha(t) = \alpha(0) + \alpha' t + \alpha'' t^2$. Our trajectories have a positive intercept and a positive curvature ($\alpha(t)'' > 0$).

We would like to comment of the regime of validity of our calculations. Since we use a classical string in supergravity backgrounds we start by requiring that the supergravity approximation to string theory be valid. That implies small curvatures which in the field theory side means large rank of the gauge group $N$. The fact that we work at large $N$ implies that some finer structure of Regge trajectories (cuts, etc.) is hidden. Making contact with this finer structure is one of the most interesting directions of future development. Equally worth pursuing is the precise relation of our work to the study of Polchinski and Strassler on Regge physics \[44\] (Regge physics in the context of the AdS/CFT has also been discussed in \[45\]). Another important point in our approximation is that we treat the string semiclassically. This implies that we want its energy to be large so that the fluctuations are reasonably small but we certainly do not want to make it so heavy as to invalidate treating it as a probe in the supergravity background. This is easily achieved in the models we consider and implies a hierarchy between the glueball mass and the curvature of the supergravity background. Let us also point out that the models which we discussed are dual to gauge theories with no fundamental matter and this limits further the comparison to QCD. It would very interesting to improve our discussion by considering models with fundamental matter.

It is worth mentioning that our result is not only compatible with some of the aspects of the best experimental fit to the Pomeron trajectory but seems to be compatible with some of the phenomenological models introduced in the literature, in particular, models that contain square roots.
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A Evaluation of the free energy for the static open string

We include here the details of the evaluation of the free energy for the static open string. Our purpose is to show explicitly how the regularization works. We follow [46] who uses analytic regularization in which

\[
\ln x = -\frac{\partial}{\partial \beta} x^{-\beta}\big|_{\beta=0}.
\]  

(A.1)

We want to evaluate

\[
\det \left( -\partial^2_{\tau} - \partial^2_{\sigma} \right) = \exp \text{Tr} \ln(-\partial^2_{\tau} - \partial^2_{\sigma}).
\]  

(A.2)

The fluctuations have to vanish at the boundary and therefore the eigenfunctions are of the form \(\eta(\tau, \sigma) = \sin(n\pi \sigma/L) \sin(m\pi \tau/T)\) with eigenvalues

\[
\lambda_{m,n} = \left(\frac{m\pi}{T}\right)^2 + \left(\frac{n\pi}{L}\right)^2.
\]  

(A.3)

Thus,

\[
\text{Tr} \ln(-\partial^2_{\tau} - \partial^2_{\sigma}) = \sum_{m,n} \ln \lambda_{m,n}
\]

\[
= \sum_{m,n} -\frac{\partial}{\partial \beta} (\lambda_{m,n})^{-\beta}\big|_{\beta=0}
\]

\[
= -\frac{\partial}{\partial \beta} \sum_{m,n} \left(\frac{m\pi}{T}\right)^2 + \left(\frac{n\pi}{L}\right)^2 \right)^{-\beta}\big|_{\beta=0}.
\]  

(A.4)
We now trade the sum over $m$ by an integral, given that we are interested in the limit $T \to \infty$

\[- \frac{\partial}{\partial \beta} \frac{1}{2\pi} \sum_{n=1}^{\infty} \frac{T}{(n\pi/L)^2} \int d\omega \left( \omega^2 + (n\pi/L)^2 \right)^{-\beta} |_{\beta=0} \]

\[= - \frac{\partial}{\partial \beta} \frac{T}{2\pi} \sum_{n=1}^{\infty} \left( \frac{n^2\pi^2}{L^2} \right)^{1/2-\beta} \sqrt{\frac{\Gamma(\beta-1/2)}{\Gamma(\beta)}} |_{\beta=0} \]

\[= - \frac{T}{(4\pi)^{1/2}} \frac{\partial}{\partial \beta} \frac{\Gamma(\beta-1/2)}{\Gamma(\beta)} \xi(2\beta-1) \left( \frac{L^2}{\pi^2} \right)^{\beta-1/2} |_{\beta=0} \]

\[= \frac{\pi T}{12L} \]

**B Some properties of Mathieu functions**

In this appendix we collect some known facts about Mathieu functions that are relevant for the concrete use made in the main text. The Mathieu differential equation is

\[ \frac{d^2}{dz^2} y + [a - 2q \cos(2z)] y = 0. \]  

(B.1)

Its general solution is of the form

\[ y = c_1 C(a,q,z) + c_2 S(a,q,z), \]  

(B.2)

where $C(a,q,z)$ and $S(a,q,z)$ are Mathieu functions denoting the even and odd solutions. As mentioned in the main text, there is no analytic simple form for these functions. However series expansions for the functions $C(a,q,z)$ and $S(a,q,z)$ near small $q$ are well known.

\[ C(a_r(q),q,z) \approx \cos(rz) + \frac{1}{4} \left( \frac{\cos((r-2)z)}{r-1} - \frac{\cos((r+2)z)}{r+1} \right) q \]

\[ + \frac{1}{32} \left( \frac{\cos((r-4)z)}{(r-2)(r-1)} - \frac{2(r^2+1)\cos(rz)}{(r^2-1)^2} + \frac{\cos((r+4)z)}{(r+1)(r+2)} \right) q^2 + \ldots \]

(B.3)

Here $r$ is an integer number known as the characteristic exponent of the Mathieu function which allows to write the Mathieu function as $\exp(irz) f(z)$ where $f(z)$ is $2\pi$ periodic with characteristic value $a$. There are also well established series expansions for the Mathieu characteristics, that is, for the values of $a$ such that the Mathieu
function is periodic.

\[
a_r(q) \approx r^2 + \frac{1}{2(r^2 - 1)} q^2 + \frac{5r^2 + 7}{32(r^2 - 4)(r^2 - 1)^2} q^4
+ \frac{9r^4 + 58r^2 + 29}{64(r^2 - 9)(r^2 - 4)(r^2 - 1)^5} q^6 + \ldots
\]

(B.4)

References

[1] P. D. Collins, “An Introduction To Regge Theory And High-Energy Physics,” Cambridge University Press, 1977.

[2] G. F. Chew and S. C. Frautschi, “Regge Trajectories And The Principle Of Maximum Strength For Strong Interactions,” Phys. Rev. Lett. 8 (1962) 41.

G. F. Chew and S. C. Frautschi, “Principle Of Equivalence For All Strongly Interacting Particles Within The S Matrix Framework,” Phys. Rev. Lett. 7 (1961) 394.

[3] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2 (1998) 231 [Int. J. Theor. Phys. 38 (1999) 1113] [arXiv:hep-th/9711200].

[4] I. R. Klebanov and M. J. Strassler, “Supergravity and a confining gauge theory: Duality cascades and chSB-resolution of naked singularities,” JHEP 0008 (2000) 052 [arXiv:hep-th/0007191].

[5] J. M. Maldacena and C. Nunez, “Towards the large N limit of pure N = 1 super Yang Mills,” Phys. Rev. Lett. 86 (2001) 588 [arXiv:hep-th/0008001].

[6] E. G. Gimon, L. A. Pando Zayas, J. Sonnenschein and M. J. Strassler, “A soluble string theory of hadrons,” JHEP 0305 (2003) 039 [arXiv:hep-th/0212061].

[7] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “A semi-classical limit of the gauge/string correspondence,” Nucl. Phys. B 636 (2002) 99 [arXiv:hep-th/0204051].

[8] S. Frolov and A. A. Tseytlin, “Semiclassical quantization of rotating superstring in AdS(5) x S**5,” JHEP 0206 (2002) 007 [arXiv:hep-th/0204226].
[9] D. Berenstein, J. M. Maldacena and H. Nastase, “Strings in flat space and pp waves from N = 4 super Yang Mills,” JHEP 0204 (2002) 013 [arXiv:hep-th/0202021].

[10] R. Apreda, F. Bigazzi and A. L. Cotrone, “Strings on pp-waves and hadrons in (softly broken) N = 1 gauge theories,” arXiv:hep-th/0307055.

[11] S. Kuperstein and J. Sonnenschein, “Analytic non-supersymmetric background dual of a confining gauge theory and the corresponding plane wave theory of hadrons,” arXiv:hep-th/0309011; S. Kuperstein, “Non-supersymmetric deformation of the Klebanov-Strassler model and the related plane wave theory,” arXiv:hep-th/0311138.

[12] L. A. Pando Zayas and D. Vaman, “Hadronic density of states from string theory,” Phys. Rev. Lett. 91 (2003) 111602 [arXiv:hep-th/0306107].

[13] A. A. Tseytlin, “On semiclassical approximation and spinning string vertex operators in AdS(5) x S**5,” Nucl. Phys. B 664 (2003) 247 [arXiv:hep-th/0304139].

[14] S. Frolov and A. A. Tseytlin, “Multi-spin string solutions in AdS(5) x S**5,” Nucl. Phys. B 668 (2003) 77 [arXiv:hep-th/0304255].
S. Frolov and A. A. Tseytlin, “Quantizing three-spin string solution in AdS(5) x S**5,” JHEP 0307 (2003) 016 [arXiv:hep-th/0306130].
S. Frolov and A. A. Tseytlin, “Rotating string solutions: AdS/CFT duality in non-supersymmetric sectors,” Phys. Lett. B 570 (2003) 96 [arXiv:hep-th/0306143].
G. Arutyunov, S. Frolov, J. Russo and A. A. Tseytlin, “Spinning strings in AdS(5) x S**5 and integrable systems,” arXiv:hep-th/0307191.
N. Beisert, S. Frolov, M. Staudacher and A. A. Tseytlin, “Precision spectroscopy of AdS/CFT,” JHEP 0310 (2003) 037 [arXiv:hep-th/0308117].

[15] G. Arutyunov, J. Russo and A. A. Tseytlin, “Spinning strings in AdS(5) x S**5: New integrable system relations,” [arXiv:hep-th/0311004].

[16] A. Armoni, J. L. F. Barbon and A. C. Petkou, “Rotating strings in confining AdS/CFT backgrounds,” JHEP 0210 (2002) 069 [arXiv:hep-th/0209224].

[17] J. M. Maldacena, “Wilson loops in large N field theories,” Phys. Rev. Lett. 80 (1998) 4859 [arXiv:hep-th/9803002].
S. J. Rey and J. T. Yee, “Macroscopic strings as heavy quarks in large N
gauge theory and anti-de Sitter supergravity,” Eur. Phys. J. C 22 (2001) 379
\[\text{arXiv:hep-th/9803001}.\]

[18] J. Sonnenschein, “Stringy confining Wilson loops,” \text{arXiv:hep-th/0009146}
Y. Kinar, E. Schreiber and J. Sonnenschein, “Q anti-Q potential from strings in curved spacetime: Classical results,” Nucl. Phys. B 566 (2000) 103
\[\text{arXiv:hep-th/9811192}.\]

[19] N. Drukker, D. J. Gross and A. A. Tseytlin, “Green-Schwarz string in AdS(5) x S(5): Semiclassical partition function,” JHEP 0004 (2000) 021
\[\text{arXiv:hep-th/0001204}.\]

[20] A. S. Schwarz and A. A. Tseytlin, “Dilaton shift under duality and torsion of elliptic complex,” Nucl. Phys. B 399 (1993) 691 \[\text{arXiv:hep-th/9210015}.\]
A. S. Schwarz, “The Partition Function Of A Degenerate Functional,” Commun. Math. Phys. 67 (1979) 1.

[21] S. Forste, D. Ghoshal and S. Theisen, “Stringy corrections to the Wilson loop in N = 4 super Yang-Mills theory,” JHEP 9908 (1999) 013 \[\text{arXiv:hep-th/9903042}.\]
S. Forste, D. Ghoshal and S. Theisen, “Wilson loop via AdS/CFT duality,” \[\text{arXiv:hep-th/0003068}.\]

[22] D. Mateos, T. Mateos and P. K. Townsend, “Supersymmetry of tensionless rotating strings in AdS(5) x S**5, and nearly-BPS operators,” \text{arXiv:hep-th/0309114}

[23] M. Bianchi, M. B. Green and S. Kovacs, “Instanton corrections to circular Wilson loops in N = 4 supersymmetric Yang-Mills,” JHEP 0204 (2002) 040
\[\text{arXiv:hep-th/0202003}.\]

[24] Y. Kinar, E. Schreiber, J. Sonnenschein and N. Weiss, Nucl. Phys. B 583, 76 (2000) \[\text{arXiv:hep-th/9911123}.\]

[25] J. G. Russo and A. A. Tseytlin, “On solvable models of type IIB superstring in NS-NS and R-R plane wave backgrounds,” JHEP 0204 (2002) 021
\[\text{arXiv:hep-th/0202179}.\]

[26] P. Candelas and X. C. de la Ossa, “Comments On Conifolds,” Nucl. Phys. B 342 (1990) 246.
[27] R. Minasian and D. Tsimpis, Nucl. Phys. B 572 (2000) 499 [arXiv:hep-th/9911042].

[28] A. H. Chamseddine and M. S. Volkov, “Non-Abelian BPS monopoles in N = 4 gauged supergravity,” Phys. Rev. Lett. 79 (1997) 3343 [arXiv:hep-th/9707176].

[29] A. Loewy and J. Sonnenschein, “On the holographic duals of N = 1 gauge dynamics,” JHEP 0108, 007 (2001) [arXiv:hep-th/0103163].

[30] L. A. Pando Zayas and D. Vaman, “Strings in RR plane wave background at finite temperature,” Phys. Rev. D 67 (2003) 106006 [arXiv:hep-th/0208066].

[31] S. Donnachie, G. Dosch, O. Nachtmann and P. Landshoff, “Pomeron Physics And QCD,” Cambridge University Press, 2002.

[32] H. B. Meyer and M. J. Teper, “Glueballs and the pomeron,” [arXiv:hep-lat/0308035].

[33] F. J. Llanes-Estrada, S. R. Cotanch, P. J. de A. Bicudo, J. E. Ribeiro and A. P. Szczepaniak, “QCD glueball Regge trajectories and the Pomeron,” Nucl. Phys. A 710 (2002) 45 [arXiv:hep-ph/0008212].

[34] C. J. Morningstar and M. J. Peardon, “The glueball spectrum from an anisotropic lattice study,” Phys. Rev. D 60 (1999) 034509 [arXiv:hep-lat/9901004].

[35] M. J. Teper, “Glueball masses and other physical properties of SU(N) gauge theories in D = 3+1: A review of lattice results for theorists,” [arXiv:hep-th/9812187].

[36] E. Caceres and R. Hernandez, “Glueball masses for the deformed conifold theory,” Phys. Lett. B 504 (2001) 64 [arXiv:hep-th/0011204].

[37] L. Ametller, J. M. Pons and P. Talavera, “On the consistency of the N = 1 SYM spectra from wrapped five-branes,” Nucl. Phys. B 674 (2003) 231 [arXiv:hep-th/0305075].

[38] P. V. Landshoff, “Pomeron physics: An update,” Nucl. Phys. Proc. Suppl. 99A (2001) 311 [arXiv:hep-ph/0010315].

[39] A. Brandt et al. [UA8 Collaboration], “Measurements of single diffraction at $s^{**}(1/2) = 630$-GeV: Evidence for a non-linear alpha(t) of the pomeron,” Nucl. Phys. B 514 (1998) 3 [arXiv:hep-ex/9710004].
[40] M. M. Brisudova, L. Burakovsky, T. Goldman and A. Szczepaniak, “Non-linear Regge trajectories and glueballs,” Phys. Rev. D 67 (2003) 094016 arXiv:nucl-th/0303012.

[41] M. M. Brisudova, L. Burakovsky and T. Goldman, “Effective functional form of Regge trajectories,” Phys. Rev. D 61 (2000) 054013 arXiv:hep-ph/9906293.

[42] H. B. Meyer and M. J. Teper, “Glueball Regge trajectories in (2+1) dimensional gauge theories,” Nucl. Phys. B 668 (2003) 111 arXiv:hep-lat/0306019.

[43] G. Bertoldi, F. Bigazzi, A.L. Cotrone, C. Núñez and L.A. Pando Zayas, “On the Universality Class of Certain String Theory Hadrons,” to appear.

[44] J. Polchinski and M. J. Strassler, “Hard scattering and gauge/string duality,” Phys. Rev. Lett. 88 (2002) 031601 arXiv:hep-th/0109174.

J. Polchinski and M. J. Strassler, “Deep inelastic scattering and gauge/string duality,” JHEP 0305 (2003) 012 arXiv:hep-th/0209211.

[45] R. A. Janik and R. Peschanski, “Reggeon exchange from AdS/CFT,” Nucl. Phys. B 625 (2002) 279 arXiv:hep-th/0110024.

R. A. Janik and R. Peschanski, “Minimal surfaces and Reggeization in the AdS/CFT correspondence,” Nucl. Phys. B 586 (2000) 163 arXiv:hep-th/0003059.

[46] O. Alvarez, “The Static Potential In String Models,” Phys. Rev. D 24 (1981) 440.