Observable Features of Tachyon-Dominated Cosmology

Audrey Claire Martin
Northern Arizona University
Department of Astronomy and Planetary Sciences
South San Francisco Street
Flagstaff, Arizona 86011 USA
acm586@nau.edu

Ian H. Redmount
Saint Louis University
Department of Physics
3511 Laclede Avenue
Saint Louis, Missouri 63103–2010 USA
ian.redmount@slu.edu

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ABSTRACT

A Friedmann-Robertson-Walker cosmological model dominated by tachyonic—faster-than-light—dark matter can exhibit features similar to those of a standard dark energy/dark matter or ΛCDM model. It can undergo expansion which decelerates to a minimum rate, passes through a “cosmic jerk,” then accelerates. But some features of a tachyon-dominated model are sufficiently distinct from those of the standard model that the two possibilities might be distinguished observationally. As a demonstration of concept, the distance-redshift relation of such a model is compared here with some observations of Type Ia supernovae. Other measures of the third time derivative of the cosmic scale factor—the true cosmic jerk—might be found to test a tachyonic-dark-matter hypothesis.

Subject headings: cosmology, dark energy, dark matter, tachyons, distance-redshift relation
1. Introduction

Over the past two decades it has become clear that the matter/energy content of the cosmos must be dominated by constituents quite distinct from the ordinary “luminous” matter of stars and planets. These are called “dark matter” if they obey equations of state similar to those of familiar matter and energy, and “dark energy” if they do not—the latter including cosmological-constant or vacuum-energy contributions. Despite considerable and ongoing efforts, however, specific dark-matter and dark-energy components have not yet been identified. Hence, it remains possible to consider even exotic candidates. For example, a gas of tachyons—faster-than-light particles, with spacelike four-momenta—can drive an open (spatially hyperbolic) cosmological model which expands from an initial singularity at a rate which decelerates to a minimum value, passes through a “cosmic jerk,” then accelerates (Starke & Redmount 2022). These are features that, in accord with current observations, characterize what has become the “standard” cosmological-constant/cold-dark-matter or ΛCDM cosmological model. The behavior of such a tachyon-dominated model differs sufficiently from a ΛCDM model that the two might be distinguished observationally. As a demonstration of concept, we present here some basic features of such a tachyon-dominated model to be tested against observations. A variety of more rigorous tests (Kramer & Redmount 2022; Gopal & Redmount 2022) should be possible.

A suitable tachyon-dominated model universe is described in Sec. 2. Numerical parameters sufficient to specify the model are obtained from a few basic cosmological measurements in Sec. 3. The distance-redshift relation for the model, which might be compared to a more extensive set of observations, is shown in Sec. 4. A tachyon-dominated model fitted to the same observational data, with its own predictions for the basic cosmological measurements, is presented in Sec. 5. Conclusions drawn from these results are described in Sec. 6.
2. Cosmological Model

The model in question is an open—spatially hyperbolic—Friedmann-Robertson-Walker spacetime geometry. It has line element

$$ds^2 = -c^2 dt^2 + a^2(t) \left[ d\chi^2 + \sinh^2 \chi \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right], \tag{1}$$

with comoving time coordinate $t$, angular coordinates $\chi$, $\theta$, and $\phi$, and scale factor or curvature radius $a(t)$. The Einstein Field Equations applied to this metric imply the Friedmann Equation

$$\left( \frac{da}{dt} \right)^2 - \frac{8\pi G}{3c^2} \rho a^2 = +c^2, \tag{2}$$

with $\rho$ the energy density of the cosmic fluid and $G$ Newton’s constant. The evolution of the model is determined by the form of the density $\rho$ as a function of $a$. A thermal ensemble of free, noninteracting (i.e., dark-matter) tachyons gives rise to density (Starke & Redmount 2022)

$$\rho(a) = \frac{(\rho_0 + M_0 c^2)a_0^4}{a^4} - \frac{M_0 c^2 a_0^3}{a^3}, \tag{3}$$

with $\rho_0$ the energy density and $M_0$ an invariant-mass density at some fiducial time $t_0$—e.g., the present time—at which the scale factor has value $a_0$. The Friedmann Equation (2) takes the form

$$\left( \frac{da}{dt} \right)^2 + \left( \frac{2B}{a} - \frac{A^2}{a^2} \right) = +c^2, \tag{4a}$$

with

$$A \equiv \left( \frac{8\pi G (\rho_0 + M_0 c^2) a_0^4}{3c^2} \right)^{1/2} \tag{4b}$$

and

$$B \equiv \frac{4\pi G M_0 a_0^3}{3}. \tag{4c}$$

The “potential energy” term on the left-hand side of Eq. (4a) is illustrated in Figure 1. For parameter values giving $B < Ac$, the model expands from an initial singularity ($a \to 0^+$) at

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1 Tachyons, never at rest, are not properly characterized by “rest mass.”
Fig. 1.— Friedmann-Equation “potential” functions for tachyon-dominated (solid curve) and dark-energy/dark-matter ΛCDM (dashed curve) models. Here “energy” $E$ refers to the terms of the Friedmann Equation [2], in units of $c^2$, and $a$ is the scale factor of the models, in gigaparsecs.
a decreasing rate, passes through a minimum expansion rate at the peak of the potential, then accelerates. Unlike the standard ΛCDM model, which ultimately continues to accelerate exponentially in time, this tachyon-dominated model asymptotically approaches finite expansion rate \( da/dt = c \). The tachyonic model illustrated in the figure has parameter values given in Sec. 3 following. The ΛCDM model shown has density

\[
\rho_{\Lambda \text{CDM}}(a) = \rho_\Lambda + \frac{\rho_{m0} a_0^3}{a^3}, \tag{5a}
\]

with constant vacuum-energy density

\[
\rho_\Lambda = 6.06 \times 10^{-10} \, \text{J/m}^3, \tag{5b}
\]

current-time matter density

\[
\rho_{m0} = 2.60 \times 10^{-10} \, \text{J/m}^3, \tag{5c}
\]

and the current-time scale factor from Eq. (10e), for purposes of comparison.

The Friedmann Equation (4a) can be solved in closed form. The model of interest here, satisfying \( B < Ac \), is described via scale factor and time

\[
a(\eta) = \frac{A}{c} \sinh \eta - \frac{B}{c^2} (\cosh \eta - 1) \tag{6a}
\]

and

\[
t(\eta) = \frac{A}{c^2} (\cosh \eta - 1) - \frac{B}{c^3} (\sinh \eta - \eta) \tag{6b}
\]
in terms of conformal time parameter \( \eta \in [0, +\infty) \).

3. Model Parameters

This model is specified by three parameters, e.g., the parameter \( A \) setting the overall scale of the model, the dimensionless ratio \( \beta \equiv B/(Ac) \), and the current conformal-time value \( \eta_0 \). These are to be determined by fitting predictions of the model to a suitable set of
observations. A simple example of such a procedure is shown in Sec. 5. But as a starting point, simply to test the feasibility of the model, the parameters can be set by matching the values of three well-known cosmological quantities, so that other consequences of the model can be compared to other data. For example, the current value of the Hubble parameter

\[ H_0 \equiv \frac{1}{a_0} \frac{da}{dt} \bigg|_{\eta=\eta_0} = \frac{c^2}{A} \frac{\cosh \eta_0 - \beta \sinh \eta_0}{[\sinh \eta_0 - \beta (\cosh \eta_0 - 1)]^2} , \]  

the current age of the universe

\[ t_0 = \frac{A}{c^2} [(cosh \eta_0 - 1) - \beta (sinh \eta_0 - \eta_0)] , \]  

and the redshift \( z_j \) of the “cosmic jerk,” at which the acceleration of cosmic expansion changes from negative to positive, given by

\[ 1 + z_j \equiv \frac{a(\eta_0)}{A/(\beta c)} = \beta \sinh \eta_0 - \beta^2 (\cosh \eta_0 - 1) \]  

can be used. Solving Eq. (7c) for \( \beta \), then solving the combined equation

\[ H_0 t_0 = \left( \frac{\beta}{1 + z_j} \right)^2 [(cosh \eta_0 - \beta sinh \eta_0) [(cosh \eta_0 - \beta sinh \eta_0) + (\beta \eta_0 - 1)] \]  

for \( \eta_0 \), then fixing \( A \) via Eq. (7b) determines all three model parameters.

Using the following sample values (Hinshaw et al. 2009; Lahav & Liddle 2010):

\[ H_0 = 71.7 \, \frac{\text{km/s}}{\text{Mpc}} \]

\[ = 2.32 \times 10^{-18} \, \text{s}^{-1} \]  

and

\[ t_0 = 1.37 \times 10^{10} \, \text{yr} \]

\[ = 4.32 \times 10^{17} \, \text{s} , \]
and estimate

\[ z_j = 0.500, \]  

this procedure yields parameters

\[ \beta = 0.993, \]  
\[ \eta_0 = 5.02, \]  

and

\[ A = (9.57 \times 10^{16} \text{ s}) c^2 \]
\[ = (0.930 \text{ Gpc}) c. \]  

These imply values

\[ B = (9.50 \times 10^{16} \text{ s}) c^3 \]
\[ = (0.924 \text{ Gpc}) c^2 \]

and present-time scale factor

\[ a_0 = 1.40 \text{ Gpc} \]

for this version of the model.

4. Distance-Redshift Relation

The distance-redshift relation is a feature of the model that might be tested against astronomical observations. Since incoming light rays in a spacetime geometry with line element (\( \Pi \)) travel on trajectories satisfying \( d\chi = -d\eta \), the metric distance \( \ell \) to a source is given by

\[ \ell = a_0(\eta_0 - \eta_s), \]
with $\eta_*$ the conformal-time parameter at emission of the signal, and subscript zero denoting present-time values. The redshift $z$ of the source is related to the scale factor $a(\eta)$ thus:

$$1 + z = \frac{a_0}{a(\eta_*)}. \quad (12)$$

These can be combined with the model’s scale factor (6a) to obtain distance-redshift relation

$$\ell(z) = a_0 \left[ \eta_0 - \alpha - \sinh^{-1}\left( \frac{a_0 \cosh \alpha}{(A/c)(1 + z)} - \sinh \alpha \right) \right]$$

$$= (1.40 \text{ Gpc}) \left\{ 2.19 - \sinh^{-1}\left[ 8.53 \left( \frac{1.51}{1 + z} - 0.993 \right) \right] \right\}. \quad (13)$$

Here the parameter $\alpha \equiv \tanh^{-1} \beta$ is introduced. Numerical values are taken from results (10a)-(10e); these imply value $\alpha = 2.83$.

Metric distance $\ell(z)$ is not measured directly. Observations provide luminosity distances (Misner, Thorne, & Wheeler 1973; Lightman, Press, Price, & Teukolsky 1973), e.g.,

$$D_L(z) \equiv \left( \frac{\mathcal{L}}{4\pi S} \right)^{1/2}$$

$$= (1 \times 10^{-7} \text{ Gpc}) \exp\left( \frac{\ln(10) (m - M)}{5} \right)$$

$$= a_0 (1 + z) \sinh[\ell(z)/a_0]$$

$$= a_0 (1 + z) \sinh \left[ \eta_0 - \alpha - \sinh^{-1}\left( \frac{a_0 \cosh \alpha}{(A/c)(1 + z)} - \sinh \alpha \right) \right]$$

$$= (1.40 \text{ Gpc}) (1 + z) \sinh \left\{ 2.19 - \sinh^{-1}\left[ 8.53 \left( \frac{1.51}{1 + z} - 0.993 \right) \right] \right\}, \quad (14)$$

for a source with intensity $S$, luminosity $\mathcal{L}$, apparent magnitude $m$, and absolute magnitude $M$. The overall factor $1 + z$ incorporates redshift and time-dilation effects, and the sinh function incorporates the curvature correction appropriate to an open model with line element (1).

While this relation may appear opaque, it is actually not so unreasonable. It can be compared to measurements of Type Ia supernovae, e.g., from Garnavich et al. (1998). Data
adapted from Figure 3 of that work are shown in Table 1 which displays redshift $z$, distance moduli $m - M$ for apparent magnitudes $m$ and absolute magnitudes $M$, distances $D_L$

Table 1: Redshift/Distance Data for Type Ia Supernovae from Garnavich et al. (1998)

| $z$   | $m - M$ | $D_L$, Gpc | $\sigma_D$, Gpc |
|-------|---------|------------|-----------------|
| 0.014 | 34.00   | 0.0629     | 0.006           |
| 0.020 | 34.80   | 0.0904     | 0.008           |
| 0.050 | 37.20   | 0.2654     | 0.024           |
| 0.100 | 38.30   | 0.4205     | 0.039           |
| 0.440 | 41.83   | 1.6342     | 0.151           |
| 0.480 | 42.40   | 2.0677     | 0.191           |
| 0.500 | 42.59   | 2.2268     | 0.205           |
| 0.970 | 44.15   | 3.4796     | 0.641           |

obtained via the second line of Eq. (14), and uncertainties $\sigma_D$ estimated from the error bars shown in Garnavich et al. (1998) and that equation. The comparison of model relation and data is shown in Figure 2. Here the model relation fits the low-redshift data points reasonably well, but yields greater distances for the larger redshift values. As a demonstration of concept, however, this shows that the tachyon-dominated model could be tested against such observations. The parameters $A$, $B$, and $\eta_0$—that is, the invariant mass, density, and temperature of the tachyon gas, as well as the distance and time scales of the model—could be fixed, as is done to evaluate the features of more familiar models (Hinshaw et al. 2009), by fitting the model to the data.
Fig. 2.— Distance-redshift relation for the open (spatially hyperbolic) tachyon-dominated model with parameters from Eqs (10a)–(10e). Data points are adapted from Garnavich et al. (1998), as displayed in Table 1.
5. Tachyon-Dominated Model Fitted to Data

Distance-redshift relation (14) can be fitted to the data of Table 1 by adjusting the three model parameters $a_0$, $\eta_0$, and $\alpha$. The results of a Levenberg-Marquardt $\chi^2$-minimization calculation (Press, Flannery, Teukolsky, & Vetterling 1986a) are shown in Figure 3. The parameters of the model shown are those obtained from a simple Monte Carlo calculation: Ten iterations of the fitting program are carried out, with normally-distributed variations introduced into the data. (Press et al. 1986b) The means and standard deviations of the resulting parameter values (Press et al. 1986c) yield

$$a_0 = (2.97 \pm 0.71) \text{ Gpc} , \quad (15a)$$

$$\eta_0 = 3.33 \pm 0.41 , \quad (15b)$$

and

$$\alpha = 1.28 \pm 0.35 , \quad (15c)$$

and model features

$$\beta = 0.856 \pm 0.093 , \quad (15d)$$

$$A = [(1.07 \pm 0.26) \times 10^{17} \text{ s}] c^2 \quad (15e)$$

$$B = [(0.896 \pm 0.28) \times 10^{17} \text{ s}] c^3 \quad (15f)$$

present-time Hubble parameter

$$H_0 = (71.7 \pm 3.9) \text{ km/s/Mpc} , \quad (15g)$$

current age

$$t_0 = (1.31 \pm 0.15) \times 10^{10} \text{ yr} \quad (15h)$$

$$= (4.13 \pm 0.46) \times 10^{17} \text{ s} ,$$
Fig. 3.— Distance-redshift relation for the open tachyon-dominated model with best-fit parameters given by Eqs. (15a)–(15i). Data points are as displayed in Table 1 and Fig. 2.
and cosmic-jerk redshift

\[ z_j = 1.62 \pm 1.40 . \]  

(15i)

The fit shown in Figure 3 has \( \chi^2 = 100 \), or \( \chi^2 \) per degree of freedom 20.0; the iterations of the Monte Carlo calculation have average \( \chi^2 \) value 144., or \( \chi^2 \) per degree of freedom 28.8. These suggest, as the graph does, at best a modest fit to the data. It may be that the small errors of the low-\( z \) distance values force a close fit to these points, leaving larger discrepancies at the higher \( z \) values. Clearly a larger, more carefully curated data set is needed for a more thoroughgoing test of the tachyonic model; one such study is in progress. (Kramer & Redmount 2022)

But results (15g) and (15h), in particular, show that the tachyon-dominated model can yield cosmological features close, but not identical, to those obtained from more standard models—such as Eqs. (9a) and (9b). Hence, these results serve as a “demonstration of concept,” indicating that the tachyonic dark-matter model might ultimately serve as a novel alternative to the now standard ΛCDM cosmology.

6. Conclusions

Despite their somewhat fanciful nature, tachyons might actually be a viable dark-matter candidate. An open Friedmann-Robertson-Walker spacetime with mass/energy content dominated by a tachyon gas can show the same general features—expansion from an initial singularity decelerating to a minimum rate, then subsequently accelerating—as a standard ΛCDM model. More specific features of such a tachyon-dominated model can be compared with observations—sufficient perhaps to justify more careful and thorough investigation.

One important distinction between the tachyonic and ΛCDM models is that the latter continues expanding exponentially in time, while the former asymptotically approaches
constant expansion rate $da/dt = c$. Hence, determination of not just the first and second
time derivatives of the scale factor $a$, but also its third derivative—the true “cosmic
ejerk”—might be crucial to distinguishing between the two possibilities. Furthermore,
the standard model must asymptotically approach the geometry of de Sitter (de Sitter
1917a,b) spacetime, while the tachyonic model must approach the Milne (Milne 1932)
universe, which is flat spacetime. This might imply that the quantum-state spaces for
elementary-particle fields in the two models are different (Redmount 1999), hence, that the
two possible cosmologies might even be distinguished by close examination of physics at the
very smallest scales.
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