Mathematical Modeling of Overcoming the COVID-19 Pandemic and Restoring Economic Growth

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Abstract—A mathematical model is proposed that not only generates various scenarios of development, but also forms specific management measures aimed at suppressing the pandemic and restoring economic growth. The developed model of the mutual influence of the pandemic and the economy is not only a tool for effective and adequate forecasting, but is also capable of simulating various scenarios that may well correspond to real epidemiological processes. An advantage of the model is that the dynamics of the pandemic and GDP can be managed in practice in order to stabilize socioeconomic development.

Keywords: COVID-19, economic systems, governance models, restoring economic growth

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The development of mathematical models describing the spread of COVID-19 started nearly simultaneously with its first outbreak in China in January 2020. Such models are based on various approaches, an overview of which can be found, for example, in [1–3]. A serious disadvantage of most developed mathematical models is that they are unable to simulate cyclic processes characteristic of the wavy dynamics of COVID-19 incidence. Finally, there are no models capable of generating particular mathematically justified extents and periods of measures to be taken to overcome the pandemic and to ensure economic growth.

This paper concerns the mathematical modeling of overcoming the COVID-19 pandemic and restoring economic growth, as well as the modeling of necessary anti-crisis measures and effective timing of their implementation. As baseline economic-mathematical models, we used the Kermack–McKendrick model [4] and the Sanderson model [5, 6].

The model proposed by Kermack and McKendrick [4] is a classical SIR model for the number of infected people, and it consists of three differential equations. Some experts believe that SIR models are not appropriate for describing the COVID-19 pandemic [7]. Indeed, it is assumed in such models that the recovered individuals become immune to the virus, so they cannot be infected again. For the COVID-19 pandemic, this assumption does not hold. Moreover, a SIR model does not take into account vaccination against COVID-19. Accordingly, we added a fourth “vaccinal” equation to the SIR model and introduced correction components that take into account the possibility of repeated infection and other nuances. Additionally, since practical modeling deals with discrete statistical data, it is reasonable to pass from differential to difference equations. As a result, we obtained an epidemiological mathematical model in the form of a discrete system of equations:

\[
\begin{align*}
S_{j+1} &= S_j (1 - r I_j - q + a) \\
I_{j+1} &= I_j (1 + r S_j - v) + c R_j \\
R_{j+1} &= R_j (1 - c) + v I_j + d V_j \\
V_{j+1} &= V_j (1 - d) + q S_j + b R_j \\
&\text{for } j = 0, 1, 2, \ldots.
\end{align*}
\]

Here, \(S_j, I_j, R_j,\) and \(V_j\) are the numbers of susceptible (never infected, but unvaccinated), infected, recovered, and fully vaccinated (with two doses) people, respectively; \(r\) is the infection rate; \(v\) is the recovery rate; \(q\) and \(b\) are the vaccination rates for never infected and recovered, respectively; \(d\) is the revaccination rate; \(c\) is the repeated infection rate; and \(a\) is the population fluctuation coefficient, which takes into
account variations in the population size due to birth, death, and migration of people.

As an example, we consider COVID-19 statistics over Russia [8]. The highest peak of COVID-19 cases fell on February 2022, and the dynamics of morbidity had a sawtooth configuration. Figure 1 presents daily numbers of newly infected individuals over Russia in February. The dynamics of morbidity has a clearly seen cyclicity with a period of 7 days.

Let us show that model (1) is capable of generating cyclical trajectories with the required period. It follows from what was said above that cycles with a period of 7 days are of interest. The following parameter values for system (1) were chosen in a computer experiment:

\[
\begin{align*}
\alpha &= 1.9459744565447; \\
\nu &= 1.80089137686731; \\
v &= 1.25507485843425; \\
r &= 2.14970486321458; \\
q &= 1.9459744565447; \\
b &= 0.0247039444045711; \\
c &= 0.0619984179153856; \\
d &= 0.00807089279296773.
\end{align*}
\]

Initial conditions under which system (1) yields a periodic orbit with a period of 7 days were found using approximate methods. Specifically,

\[
\begin{align*}
\bar{S}_0 &= 0.539753228690255; \\
\bar{I}_0 &= 0.946101872256248; \\
\bar{R}_0 &= 2.23329875094284; \\
\bar{V}_0 &= 3.89439991609052.
\end{align*}
\]

The periodic orbit produced by system (1) with initial conditions (2) was verified against the statistical data from [8]. As a result, we obtained predicted cycles of COVID-19 adapted to the actual epidemiological situation. Figure 1 presents the simulated short-term prediction of possible COVID-19 dynamics over March 2022.

The COVID-19 pandemic caused a global economic crisis. In Russia [9, 10] at the end of 2020, GDP per capita reduced by 2.07%, thus having violated the tendency of growth observed in the years preceding to the pandemic. For a mathematical interpretation of the mutual effect of the pandemic and the economy, we used two equations from Sanderson’s discrete model [5, 6], which is called the Wonderland model and describes the interrelations between economic, demographic, and ecological processes.

Eventually, the following mathematical model for the mutual effect of the pandemic and the economy was obtained:

\[
\begin{align*}
S_{j+1} &= S_j(1 - rI_j - q + a) \\
I_{j+1} &= I_j(1 + rS_j - v) + cR_j \\
R_{j+1} &= R_j(1 - c) + vI_j + dV_j \\
V_{j+1} &= V_j(1 - d) + qS_j + bR_j \\
z_{j+1} &= \frac{z_j \mu e^{(\rho - \omega)I_j}}{1 - z_j + z_j \mu e^{(\rho - \omega)I_j}} \\
y_{j+1} &= y_j(1 + \gamma - (\gamma + \eta)(1 - z_j)\gamma).
\end{align*}
\] (3)

Here, the coefficients \(\gamma, \eta, \lambda, \delta, \rho, \omega\) are constants and \(y_j\) is GDP per capita. Let \(z_j\) denote the level of overcoming the pandemic \((0 \leq z_j \leq 1)\). In the case \(z_j = 1\), we assume that the epidemiological situation is ideal and there are no infection cases. The value \(z_j = 0\) corresponds to the opposite limiting case, when the pandemic level is so high that the threat to human health and the economy is maximal.
The state is interested in overcoming the pandemic, restoring economic growth, and returning to a stable socioeconomic situation. Mathematically, a stable situation corresponds to a fixed point of system (3). To find a fixed point, the right-hand sides of (3) are set to $S_j$, $I_j$, $R_j$, $V_j$, $z_j$, $y_j$, respectively, and the coordinates of the fixed point are computed.

Since the morbidity and recovery characteristics are chaotic, to overcome the chaotic dynamics, we modify model (3) by applying results of the modern theory of chaos control. Specifically, after introducing the notation system (3) is rewritten in vector form:

$$
F(w(j)) = \begin{pmatrix}
    w_1(j)(1 - rw_2(j) - q + a) \\
    w_2(j)(1 + rw_1(j) - v) + cw_3(j) \\
    w_3(j)(1 - c) + vw_2(j) + dw_4(j) \\
    w_4(j)(1 - d) + qw_3(j) + bw_5(j) \\
    w_5(j)e^{\delta w_4(j) - \omega w_1(j)w_2(j)} \\
    1 - w_5(j) + w_3(j)e^{\delta w_4(j) - \omega w_1(j)w_2(j)} \\
    w_6(j)(1 + \gamma - (\gamma + \eta)(1 - w_5(j))^\beta)
\end{pmatrix}.
$$

Linearizing system (4) about the fixed point and applying the Pyragas method [11], we obtain a modified system

$$
w(j + 1) = F(w(j)) + U(j); \quad j \in \{0, 1, 2, \ldots\},
$$

where $U(j)$ is the control function intended to stabilize the behavior of solutions to the system. Relying on the results of [12, 13] concerning the stabilization of discrete systems, we obtain a control function of the form

$$
U(j) = F(w_n) - F(w(j)) + A(w_n)[w(j) - w_n] + P(j)[w(j) - w(j - 1)].
$$

Here, $P(j)$ is a periodic matrix given by the formula

$$
P(j) = \begin{cases}
    (kE - A^2(w_n))(A(w_n) - E)^{-1}, & j = 2n, \\
    O, & j \neq 2n,
\end{cases}
$$

where $-1 < k < 1$, $E$ is the identity matrix, $O$ is the zero matrix, $A(w_n)$ is the Jacobian matrix for the vector function $F$, and $w_n$ is the fixed point.

Discrete system (5) is characterized by ultrahigh sensitivity to variations in the parameters. Thus, with varying coefficients, model (5) is capable of simulating
various scenarios corresponding to actual epidemiological and economic processes. In the simulation with the help of system (5), we used monthly statistics.

To produce predictions of overcoming the pandemic and restoring economic growth, system (5) was solved with initial conditions corresponding to the current situation. According to statistical data [8–10], at the end of February 2022, we had

$$
\begin{align*}
    w_1(0) &= 0.3748; \\
    w_2(0) &= 0.031; \\
    w_3(0) &= 0.0237; \\
    w_4(0) &= 0.5705; \\
    w_5(0) &= 0.4583; \\
    w_6(0) &= 894.4 \text{ thousand rubles.}
\end{align*}

The following parameter values for system (5) were chosen in a computer experiment:

$$
\begin{align*}
    a &= 0.001; \\
    b &= 0.5; \\
    c &= 0.3; \\
    q &= 0.0011; \\
    \gamma &= 0.00004; \\
    \eta &= 0.4; \\
    \omega &= -0.86; \\
    \rho &= 3.
\end{align*}
$$

According to numerous expert estimates, the GDP of Russia in 2022 can reduce by 7% and more because of the western sanctions directed against the Russian special military operation in Ukraine. In this context, the coefficients were specified as

$$
\begin{align*}
    d &= 0.0000066667; \\
    \lambda &= 1; \\
    r &= 0.7874015748; \\
    \nu &= 0.3393700787; \\
    \delta &= 0.1000958387.
\end{align*}
$$
for modeling over 2022 and as
\[
d = 0.0000052632; \quad \lambda = 2; \quad r = 1;
\]
\[
v = 0.35; \quad \delta = 0.0867999804
\]
for modeling over 2023 and 2024. System (5) was solved with initial conditions (7). As a result, we obtained predicted trajectories, which are plotted in Figs. 2–4.

Figures 2 and 3 clearly demonstrate that vaccination of the population is the main tool for the fight against COVID-19. Additionally, Figs. 3 and 4 indicate a significant relation between overcoming the pandemic and restoring economic growth. The correlation coefficient between the obtained values \( w_5(j) \) and \( w_6(j) \) is 0.79.

The developed model (5), (6) is not only a tool for effective and adequate prediction. The basic advantage of this model is that it additionally makes it possible to manage the dynamics of the pandemic and GDP in practice in order to stabilize socioeconomic development. With the use of the explicit analytical formula (6), it is possible to determine the range and timing of anticipatory correction measures.

The modeling results were used in computations and verification, taking into account the investment multiplier [14]. As a result, we obtained monthly values for the scale of vaccination and amounts of additional investments necessary for overcoming the pandemic and restoring economic growth. These results are presented in Table 1, which, in fact, is a graph of taking preventive anti-crisis measures.

Relying on the results presented above, we can conclude that anticipatory correction measures have to be taken at the right time to overcome the pandemic and restore economic growth. Only government institutions are capable of organizing and implementing administrative measures. According to the simulation results obtained in this work, the priorities of the state in the medium term are full vaccination and financial support of the economy in the required amounts and with proper timing.

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**CONFLICT OF INTEREST**

The authors declare that they have no conflicts of interest.

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