The new extended rational SGEEM for construction of optical solitons to the (2+1)–dimensional Kundu–Mukherjee–Naskar model

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Abstract

This work proposes the new extended rational sinh-Gordon equation expansion technique (SGEEM). The computational approach is formulated based on the well-known sinh-Gordon equation. The proposed technique generalizes the sine-Gordon/sinh-Gordon expansion methods in a rational format. The efficiency of the suggested technique is tested on the (2+1)–dimensional Kundu–Mukherjee–Naskar (KMN) model. Various of optical soliton solutions have been obtained using this new method. The conditions which guarantee the existence of valid solitons are given.

Keywords: New extended rational SGEEM; KMN model; optical solitons.
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1 Introduction

Various complicated nonlinear physical phenomena can be expressed in the form of nonlinear partial differential equations (NPDEs). Nonlinear Schrödinger’s type equations (NLSEs) are unique types of nonlinear partial differential equations which are complex in nature. Such equations can be used to define various nonlinear physical aspects such as plasma physics, fluid dynamics, photonics, quantum electronics, and electromagnetism [1–5]. One of the important and hot subjects for investigating soliton propagation through nonlinear optical fibers is the theory of optical solitons [6]. The spread of ultrashort pulses of electromagnetic radiation in a nonlinear medium is a multidimensional phenomenon. The interaction between various physical elements such as material dispersion, diffraction and nonlinear reaction affect the dynamics of the pulse [6]. This area has drawn the attention of many scientists for more than two decades. Different computational methods have been used to reveal solutions of various type of NLEEs such as the modified \text{exp}(-\Psi(\eta))-expansion function method [7–9], the first integral method [10,11], the improved Bernoulli sub-equation function method [12,13], the trial solution
method \[14,15\], the new auxiliary equation method \[16\], the solitary wave ansatz method \[18\], the functional variable method \[19\] and several others \[20–39\].

However, a novel extended rational sinh-Gordon equation expansion technique is developed in this research. The new approach is based on the well-known sine-Gordon and sinh-Gordon equations. We employ the new approach to the (2+1) \(i\)imensional Kundu\textsuperscript{e}ukherjeek\textsuperscript{e}askar model \[40\] in generating various optical solitons.

The (2+1) \(i\)imensional KMN model is given by \[40\]

\[
i\Theta_t + \alpha \Theta_{xy} + i\beta \Theta(\Theta\Theta_x - \Theta^*\Theta_x) = 0.\tag{1.1}
\]

In Eq. (1.1), the unknown function \(\Theta(x,y,t)\) stands for the nonlinear wave envelope. The nonzero constants \(a\) and \(b\) are the coefficients of the dispersion term and the term that is different from conventional Kerr law nonlinearity or any known non-Kerr law media. The first term \(\Theta_t\) represents the temporal evolution of the wave. Eq. (1.1) describes the oceanic rogue waves as well as hole waves. It may also be used in describing optical wave propagation through coherently excited resonant wave guides that is doped with Erbium atoms \[40, 41\].

### 2 Analysis of the Method

In this section, we give the description of the novel extended rational sinh-Gordon equation expansion technique.

Consider the following sinh-Gordon equation \[42\]

\[
\Theta_{\alpha\alpha} = \gamma \sinh(\Theta).\tag{2.1}
\]

One may recall the following as the solutions of Eq. (2.1) \[43\]:

\[
\sinh(\Omega) = \pm \text{csch}(\Delta) \quad \text{or} \quad \sinh(\Omega) = \pm i \text{sech}(\Delta), \tag{2.2}
\]

\[
\cosh(\Omega) = \pm \text{coth}(\Delta) \quad \text{or} \quad \cosh(\Omega) = \pm \tanh(\Delta), \tag{2.3}
\]

\[
\sinh(\Omega) = \tan(\Delta) \quad \text{or} \quad \sinh(\Omega) = -\cot(\Delta) \tag{2.4}
\]

and

\[
\cosh(\Omega) = \pm \text{sec}(\Delta) \quad \text{or} \quad \cosh(\Omega) = \pm \tan(\Delta), \tag{2.5}
\]

where \(i = \sqrt{-1}\) and \(\Omega' = \sinh(\Omega)\), or \(\Omega' = \cosh(\Omega)\). For details, see \[43\].

Consider the nonlinear partial differential equation

\[
P(\Theta_{\alpha}, \Theta^2\Theta_{xx}, \Theta_{xxx} \ldots) = 0, \tag{2.6}
\]

where the subscript represents the partial derivative of \(\Theta\) with respect to \(x\).

Substituting the travelling wave transformation

\[
\Theta(x,t) = \Theta(\Delta), \quad \Delta = \omega(x - \mu t) \tag{2.7}
\]
into Eq. (2.6), the following nonlinear ordinary differential (NODE) is obtained:

\[ D(\Theta, \Theta', \Theta'', \ldots) = 0, \]  
(2.8)

where the superscript indicates the derivative of the function \( \Theta \) with respect to \( \Delta \).

The general steps of the new generalized rational sinh-Gordon equation expansion method are given as follows:

**Step-I:** Suppose that Eq. (2.6) adopts the following form of rational solution:

\[ \Theta(\Omega) = \sum_{j=1}^{m} \left[ \pm b_j \sinh(\Omega) + a_j \cosh(\Omega) \right] + a_0 \sum_{j=1}^{m} \left[ \pm c_j \sinh(\Omega) + d_j \cosh(\Omega) \right] + c_0. \]  
(2.9)

**Step II:** The unknown parameters involved are obtained by substituting Eq. (2.9) along with \( \Omega' = \sinh(\Omega) \) and/or \( \Omega' = \cosh(\Omega) \) into Eq. (2.8). This produces a polynomial in powers of hyperbolic functions. Summing the coefficients of these hyperbolic functions of the same power, provides a group of algebraic equations after equating each summation to zero.

**Step III:** The solutions of Eq. (2.6) are reached by inserting the values of the unknown parameters into the following rational solutions formed from Eqs. (2.2), (2.3), (2.4) and (2.5), respectively:

\[ \Theta(\Delta) = \sum_{j=1}^{m} \left[ \pm b_j \text{sech}(\Delta) \pm a_j \tanh(\Delta) \right] + a_0 \sum_{j=1}^{m} \left[ \pm c_j \text{sech}(\Delta) \pm d_j \tanh(\Delta) \right] + c_0. \]  
(2.10)

\[ \Theta(\Delta) = \sum_{j=1}^{m} \left[ \pm b_j \text{csch}(\Delta) \pm a_j \coth(\Delta) \right] + a_0 \sum_{j=1}^{m} \left[ \pm c_j \text{csch}(\Delta) \pm d_j \coth(\Delta) \right] + c_0. \]  
(2.11)

\[ \Theta(\Delta) = \sum_{j=1}^{m} \left[ b_j \tan(\Delta) \pm a_j \sec(\Delta) \right] + a_0 \sum_{j=1}^{m} \left[ c_j \tan(\Delta) \pm d_j \sec(\Delta) \right] + c_0. \]  
(2.12)

and

\[ \Theta(\Delta) = \sum_{j=1}^{m} \left[ -b_j \cot(\Delta) \pm a_j \tan(\Delta) \right] + a_0 \sum_{j=1}^{m} \left[ -c_j \cot(\Delta) \pm d_j \tan(\Delta) \right] + c_0. \]  
(2.13)

### 3 Applications

In this section, we give the applications of the new extended rational sinh-Gordon equation expansion method.

Consider the complex wave transformation

\[ \Theta(x, y, t) = \Theta(\Delta)e^{i\Theta}, \quad \Theta'(x, y, t) = \Theta(\Delta)e^{-i\Theta} \Delta = \omega(-\sigma_1 x - \sigma_2 y - \mu t), \]

\[ \Theta = -\gamma_1 x - \gamma_2 y + \kappa t + \phi. \]  
(3.1)
In Eq. (3.1), $\gamma_1$ and $\gamma_2$ are the frequencies of the solitons in $x$- and $y$-directions, respectively. The constant $\mu$ stands for the velocity of the soliton. The parameter $\kappa$ is the phase constant. The parameters $\sigma_1$ and $\sigma_2$ stand for the inverse width of the soliton along $x$- and $y$-directions, respectively [40].

Substituting Eq. (3.1) into (1.1), gives

$$\omega^2 \alpha \sigma_1 \sigma_2 \Theta'' - (\kappa + \alpha \gamma_1 \gamma_2) \Theta - 2\beta \gamma_1 \Theta^3 = 0$$

(3.2)

from the real part, and

$$\mu = \alpha (\gamma_1 \sigma_2 + \gamma_2 \sigma_1)$$

(3.3)

from the real part.

Balancing the terms $\Theta''$ and $\Theta^3$, yields $m = 1$.

With $m = 1$, Eq. (2.9) turns to

$$\Theta(\Omega) = \frac{b_1 \sinh(\Omega) + a_1 \cosh(\Omega) + a_0}{c_1 \sinh(\Omega) + d_1 \cosh(\Omega) + c_0},$$

(3.4)

Substituting Eq. (3.4) along with $\Omega' = \sinh(\Omega)$ and/or $\Omega' = \cosh(\Omega)$ into Eq. (3.2), yields a polynomial in powers of hyperbolic functions. Collecting the coefficients of the hyperbolic functions of the same power and equating each summation to zero, yields a system of algebraic equations. Solving the system of algebraic equations, produces the values of $a_1$, $b_1$, $c_1$, $d_1$, and the other parameters involved. Substituting the values of the parameters into Eqs. (2.10)-(2.13) with fixed value of $m = 1$, gives some new results to Eq. (1.1).

Set-1: When

$$a_0 = d_1 \sqrt{\frac{(\kappa + \alpha \gamma_1 \gamma_2)}{2\beta \gamma_1}}, \quad c_0 = a_1 \sqrt{-\frac{2\beta \gamma_1}{(\kappa + \alpha \gamma_1 \gamma_2)}}, \quad \omega = -\sqrt{-\frac{2(\kappa + \alpha \gamma_1 \gamma_2)}{\alpha \sigma_1 \sigma_2}},$$

$$b_1 = -i \sqrt{\frac{(\kappa + \alpha \gamma_1 \gamma_2)}{2\beta \gamma_1}}$$

we have the following mixed dark-bright soliton:

$$\Theta_{1.1}(x,y,t) = \left( d_1 \sqrt{\frac{(\kappa + \alpha \gamma_1 \gamma_2)}{2\beta \gamma_1}} + \frac{ic_1 \text{sech}(|\Delta|) + d_1 \text{tanh}(|\Delta|)}{2\beta \gamma_1} \right) e^{i\theta},$$

(3.5)

and the mixed singular soliton

$$\Theta_{1.2}(x,y,t) = \left( c_1 \text{csch}(|\Delta|) + d_1 \text{coth}(|\Delta|) + a_1 \sqrt{-\frac{2\beta \gamma_1}{(\kappa + \alpha \gamma_1 \gamma_2)}} \right) e^{i\theta},$$

(3.6)
Set-2: When
\[ a_0 = d_1 \sqrt{\frac{(\kappa + \alpha \gamma \gamma_1)}{2\beta \gamma}}, \quad c_0 = a_1 \sqrt{\frac{-2\beta \gamma}{(\kappa + \alpha \gamma \gamma_2)}}, \quad b_1 = 0, \quad c_1 = 0, \quad \omega = \sqrt{\frac{(\kappa + \alpha \gamma \gamma_1}{2\alpha \sigma_1 \sigma_2}, \]
we get the following dark soliton
\[ \Theta_{2,1}(x,y,t) = \frac{\left(d_1 \sqrt{\frac{(\kappa + \alpha \gamma \gamma_1)}{2\beta \gamma}} + a_1 \tanh[\Delta]\right)}{a_1 \sqrt{\frac{-2\beta \gamma}{(\kappa + \alpha \gamma \gamma_2)} + d_1 \tanh[\Delta]}} e^{i \theta}, \quad (3.7) \]
and the singular soliton
\[ \Theta_{2,2}(x,y,t) = \frac{\left(a_1 \coth[\Delta] + d_1 \sqrt{\frac{(\kappa + \alpha \gamma \gamma_1)}{2\beta \gamma}}\right)}{a_1 \sqrt{\frac{-2\beta \gamma}{(\kappa + \alpha \gamma \gamma_2)} + d_1 \coth[\Delta]}} e^{i \theta}. \quad (3.8) \]

Set-3: When
\[ a_0 = c_1 \sqrt{\frac{(\kappa + \alpha \gamma \gamma_1)}{2\beta \gamma}}, \quad c_0 = -b_1 \sqrt{\frac{2\beta \gamma}{\kappa + \alpha \gamma \gamma_2}}, \quad \omega = -\sqrt{\frac{2(\kappa + \alpha \gamma \gamma_1}{\alpha \sigma_1 \sigma_2}, \]
a_1 = -\sqrt{\frac{2\beta \gamma + (c_1 - d_1(\kappa + \alpha \gamma \gamma_2)}{2\beta \gamma}}, \]
we get the following trigonometric functions solution:
\[ \Theta_{3,1}(x,y,t) = \frac{\left(b_1 \tan[\Delta] + d_1 \sqrt{\frac{2\beta \gamma + (c_1 - d_1(\kappa + \alpha \gamma \gamma_2)}{2\beta \gamma}}\right)}{d_1 \sec[\Delta] + c_1 \tan[\Delta] - b_1 \sqrt{\frac{2\beta \gamma}{\kappa + \alpha \gamma \gamma_2}}} e^{i \theta}. \quad (3.9) \]

Set-4: When
\[ a_0 = -c_1 \sqrt{\frac{(\kappa + \alpha \gamma \gamma_1)}{2\beta \gamma}}, \quad c_0 = b_1 \sqrt{\frac{2\beta \gamma}{\kappa + \alpha \gamma \gamma_2}}, \quad a_1 = 0, \quad d_1 = 0, \quad \omega = -\sqrt{\frac{(\kappa + \alpha \gamma \gamma_1}{2\alpha \sigma_1 \sigma_2}, \]
we get the following trigonometric function solution:
\[ \Theta_{4,1}(x,y,t) = \frac{\left(b_1 \tan[\Delta] - c_1 \sqrt{\frac{(\kappa + \alpha \gamma \gamma_1)}{2\beta \gamma}}\right)}{c_1 \tan[\Delta] + b_1 \sqrt{\frac{2\beta \gamma}{\kappa + \alpha \gamma \gamma_2}}} e^{i \theta}. \quad (3.10) \]

Set-5: When
\[ a_0 = c_1 \sqrt{\frac{(\kappa + \alpha \gamma \gamma_1}{2\beta \gamma}}, \quad c_0 = -b_1 \sqrt{\frac{2\beta \gamma}{\kappa + \alpha \gamma \gamma_2}}, \quad \omega = -\sqrt{\frac{(\kappa + \alpha \gamma \gamma_1}{\alpha \sigma_1 \sigma_2}, \]
a_1 = -\sqrt{\frac{2\beta \gamma + (c_1 - d_1(\kappa + \alpha \gamma \gamma_2)}{2\beta \gamma}}, \]
we get the following trigonometric functions solution:
constructed. Mixed dark-bright, singular solitons and trigonometric functions solutions are successfully con-
duced the following trial solution which was generated from the sine-Gordon equation \[45, 46\]:

$$\Theta_{5.1}(x,y,t) = \left( b_1 \cot[\Delta] - \frac{\sqrt{2b\beta_1 \gamma_1 + c_1^2 - d_1^2}(\kappa + \alpha \gamma_1 \gamma_2)}{2\beta_1} \csc[\Delta] + c_1 \sqrt{\frac{\kappa + \alpha \gamma_1 \gamma_2}{2\beta_1}} \right) e^{i\theta}. \tag{3.11}$$

Set-6: When

$$a_0 = -c_1 \sqrt{\frac{\kappa + \alpha \gamma_1 \gamma_2}{2\beta_1}}, \quad c_0 = b_1 \sqrt{\frac{2\beta_1 \gamma_1}{\kappa + \alpha \gamma_1 \gamma_2}}, \quad a_1 = 0, \quad d_1 = 0, \quad \omega = -\sqrt{\frac{2(\kappa + \alpha \gamma_1 \gamma_2)}{\alpha \sigma_1 \sigma_2}},$$

we get the following trigonometric functions solution:

$$\Theta_{6.1}(x,y,t) = -\left( \frac{b_1 \cot[\Delta] + c_1 \sqrt{\frac{\kappa + \alpha \gamma_1 \gamma_2}{2\beta_1}}}{{b_1} \sqrt{\frac{2\beta_1 \gamma_1}{\kappa + \alpha \gamma_1 \gamma_2}} - c_1 \cot[\Delta]} \right) e^{i\theta}. \tag{3.12}$$

Remarks: Solutions (3.5)-(3.8) are valid only for $\alpha \sigma_1 \sigma_2 (\kappa + \alpha \gamma_1 \gamma_2) < 0$, and solutions (3.9)-(3.12) are valid only for $\alpha \sigma_1 \sigma_2 (\kappa + \alpha \gamma_1 \gamma_2) > 0$.

4 Results and Discussion

In this study, we succeeded in formulating the extended rational sinh-Gordon equation expansion technique. The developed method is employed to the (2+1) dimensional Kundu equation introduced the rational sine-Gordon expansion method. The authors introduced the following trial solution which was generated from the sine-Gordon equation [45, 46]:

$$\Theta(\Delta) = \sum_{j=1}^{m} \tanh^{-1}(\Delta) \left[ b_j \sech(\Delta) + a_j \tanh(\Delta) \right] + a_0 \sum_{j=1}^{m} \tanh^{-1}(\Delta) \left[ c_j \sech(\Delta) + d_j \tanh(\Delta) \right] + c_0. \tag{4.1}$$

In this study, we come up with the following sets of trial solutions that were generated from the sinh-Gordon equation [42]:

$$\Theta(\Delta) = \sum_{j=1}^{m} \left[ \pm b_j \sech(\Delta) \pm a_j \tanh(\Delta) \right] \left[ \pm c_j \sech(\Delta) \pm d_j \tanh(\Delta) \right] + a_0 \sum_{j=1}^{m} \left[ \pm c_j \sech(\Delta) \pm d_j \tanh(\Delta) \right] + c_0. \tag{4.2}$$

$$\Theta(\Delta) = \sum_{j=1}^{m} \left[ \pm b_j \csch(\Delta) \pm a_j \coth(\Delta) \right] \left[ \pm c_j \csch(\Delta) \pm d_j \coth(\Delta) \right] + a_0 \sum_{j=1}^{m} \left[ \pm b_j \csch(\Delta) \pm a_j \coth(\Delta) \right] + c_0. \tag{4.3}$$

$$\Theta(\Delta) = \sum_{j=1}^{m} \left[ \pm b_j \tan(\Delta) \pm a_j \sec(\Delta) \right] \left[ \pm c_j \tan(\Delta) \pm d_j \sec(\Delta) \right] + a_0 \sum_{j=1}^{m} \left[ \pm b_j \tan(\Delta) \pm a_j \sec(\Delta) \right] + c_0. \tag{4.4}$$

and

$$\Theta(\Delta) = \sum_{j=1}^{m} \left[ \pm b_j \cot(\Delta) \pm a_j \tan(\Delta) \right] \left[ \pm c_j \cot(\Delta) \pm d_j \tan(\Delta) \right] + a_0 \sum_{j=1}^{m} \left[ \pm b_j \cot(\Delta) \pm a_j \tan(\Delta) \right] + c_0. \tag{4.5}$$
These newly introduced trial solutions generalize all the kind of solutions that may be obtained by using the sine- and sinh-Gordon expansion methods in rational format.

Figure 1 The (a) 2-, 3-dimensional and (b) contour surfaces of Eq. (3.5).

Figure 2 The (a) 2-, 3-dimensional and (b) contour surfaces of Eq. (3.7).

5 Conclusion

In this research, the extended rational sinh-Gordon equation expansion method is developed. The newly developed technique gives variety of wave solutions when tested on the (2+1)= dimensional Kundu equation askar model. Dark, mixed dark-bright, singular, mixed singular solitons and trigonometric functions solutions are successfully constructed. The conditions which guarantee the existence of the valid solutions to this model are given. The 2-, 3-dimensional and contour graphs to this model are plotted. It is known that dark soliton describes the solitary waves with lower intensity than the background, bright soliton describes the solitary waves whose peak intensity is larger than the background [47]. The singular soliton solutions is a solitary wave with discontinuous derivatives; instances of such solitary waves are compactons, which have finite (compact) assistance, and peakons, whose peaks have a discontinuous first derivative [48, 49]. The generalized sinh-Gordon equation expansion method is efficient and powerful mathematical tool which may be used in generating varieties of wave solutions to different kind of nonlinear wave equations.
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