Production of good quality holograms by the THz pulsed vortex beams

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Abstract
In this paper, we discuss the quality of holograms based on the calculation of the modulation depth. It’s shown that the terahertz (THz) pulsed vortex beams play a vital role in holography filed, where two lasers with frequency difference have used. The THz vortex beams give new regions of larger frequency detuning and an important value of the modulation depth and fringe contrast (MDFC ratio) for obtaining the best holograms contrary to the Gaussian beams for which the best holograms are realized for small frequency detuning. The particular cases such as, Gaussian beam and single cycle pulses are deduced from our result. Numerical simulations are also presented to study the dependence of the MDFC ratio on the frequency detuning for THz vortex beams and Gaussian beams. This research could be beneficial in holographic interferometry, and it will firmly establish as a tool for scientific and engineering studies.

Keywords THz pulsed vortex beams · Modulation depth · Frequency detuning · Holograms

1 Introduction

THz technology is now widely used for extensive applications such as imaging with dispersive objects (Kulya et al. 2017), detection of packaged integrated circuits (Ahi et al. 2018), wireless communications (Nagatsuma et al. 2016; Semenova and Bespalov 2015), noninvasive diagnostics in biology and medicine (Sun et al. 2017; Yang et al. 2016; Balbekin et al. 2015a; Duka et al. 2014), and noninvasive quality control and testing in industry (Stoik et al. 2008; Ahi and Anwar 2016; Balbekin et al. 2015b). Recently, considerable interests have been paid to study light beams with helical wave front. In modern optics, this kind of light is called vortex beams. Nowadays, optical vortex beams have been generated in various spectral ranges as examples; we cite THz (He et al. 2013), x-ray (Peel et al. 2002), millimeter (Yan et al. 2014), extreme ultraviolet (Géneaux et al. 2016), visible (Cai et al. 2012), and radio (Yu et al. 2016) ranges.
In recent years, the optical vortices have known a growing interest from the laser researchers due to their applications in many domains such as micromanipulations of particles (Arlt et al. 2001), atomic optics (Ito et al. 1996), photons entanglement states (Mair et al. 2001), optical metrology (Gasvik 2003), binary optics and medical sciences (Gahagan and Swartzlander, 1996). The generation of optical vortex beams (Yin et al. 2003) has been developed using many different techniques, including computer-generated holograms (Vasara et al. 1989; Carpentier et al. 2008; Heckenberg et al. 1992), spatial light modulators (McGloin et al. 2003; Davis et al. 1996), optical coordinate transformations (Davidson et al. 1992), and fiber optics (Ramachandran et al. 2009).

Many vortex beams had been introduced for the cylindrical symmetry, as examples, we cite the widely known Hypergeometric–Gaussian beams (Kotlyar et al. 2007; Karimi et al. 2007; Skidanov et al. 2013), Bessel-Gaussian beams (Arlt and Dholakia 2000; McGloin and Dholakia 2005; Kotlyar et al. 2006; Vaity and Rusch 2015) and Laguerre–Gaussian beams (Allen et al. 1992; Kennedy et al. 2002; Vallon 2015) that are separable in the polar coordinates. While, the well–known rectangular laser beams, which are separable inside the orthogonal direction including Four-petal Gaussian beams (Duan and Li 2006), Finite airy beams (Siviloglou et al. 2007; Siviloglou and Christodoulides, 2007) and Hermite sinusoidal–Gaussian beams (Casperson and Tovar 1998; Tovar and Casperson 1998; Belaïhal and Ibnchaikh 2000; Hricha and Belaïhal 2005; Chib et al. 2020), are not originally vortex modes but they can be integrated by vortex phase amplitude.

On the other hand, the theory of holograms initially, called wave front reconstruction, since its invention by Gabor (1948), and the arrival of the laser at the beginning of the 1960s, has developed into a mature technology with a large variety of applications for which it is uniquely suited. In 1962, Leith and Upatnieks combined Gabor’s theory with their own work on side-reading radar and applied it to holography. In parallel with this work, Denisyuk (1962) combined the basic idea of reconstruction of the wave front with the color photography method. Benton and Bove (2008) discovered white-light transmission holography while researching holographic television at Polaroid Research Laboratories. Optical holography has attracted extensive attention, such as 3D display (Schnars and Jueptner 1994), optical metrology (Kreis 2005), medicine (Boyer et al. 1996), and many more. In the optical holography technique, the hologram is an essential part that contains the complete information of the object beam.

In recent years, Odoulov et al. (2015) have reported the recording of permanent holographic arrays using laser beams with a small frequency difference. In 2018, Malik and Escarquiel (2018) have proved that the best holograms can be obtained with significant frequency detuning. In 2020, Malik and Escarquiel (2020) showed also that dark hollow beams give more exact results than Gaussian beams for holography.

The present work aims to present, in a simple way that THz vortex beams present more precise results than Gaussian beams for holography. However, to the best of our knowledge, the modulation depth and fringe contrast ratio of THz vortex beams have not been studied elsewhere. The remainder of this manuscript is organized as follows: the theoretical calculation details for the MDFC ratio is developed in Sect. 2. Section 3 focuses on exploring the special cases deduced from the laser beam. Then, Sect. 4 is devoted to the discussion of our results with numerical examples. Finally, the main results are outlined in the conclusion part.
2 Modulation depth of THz pulsed vortex pulses

In this section, we will evaluate the modulation depth of the considered pulses with two different colors. For that, we consider the interference of two THz pulsed vortex beams with two colors. Their electric fields are given by the following expression

\[
E_1(r, t) = A_1 f(t) \cos(\omega_1 t - \vec{k}_1 \cdot \vec{r}), \quad (1a)
\]

and

\[
E_2(r, t) = A_2 f(t) \cos(\omega_2 t - \vec{k}_2 \cdot \vec{r}), \quad (1b)
\]

where \( A_1 \) and \( A_2 \) are the amplitudes, \( \omega_1 \) and \( \omega_2 \) designate the angular frequencies, \( \vec{k}_1 \) and \( \vec{k}_2 \) represent the wave-vectors of the electric fields and \( f(t) \) is the temporal profile given by Kapoyko et al. (2015)

\[
f^2(t) = \left( \frac{\delta t}{\tau} \right)^2 e^{-\vec{r}^2 / \tau^2}, \quad (2)
\]

with \( \tau \) is the pulse duration, \( n \) is an integer and \( \delta \) is the controller parameter of the central dark spot size of the THz vortex pulse.

In time and space, from Eq. (1) we deduce the intensity distribution of the electric resulting field

\[
I(\vec{r}, t) = |E_1(\vec{r}, t) + E_2(\vec{r}, t)|^2
\]

\[
= E_1(\vec{r}, t) E_1^*(\vec{r}, t) + E_2(\vec{r}, t) E_2^*(\vec{r}, t) + E_1(\vec{r}, t) E_2^*(\vec{r}, t) + E_2(\vec{r}, t) E_1^*(\vec{r}, t)
\]

\[
= f^2(t) \left[ A_1^2 + A_2^2 + 2A_1 A_2 \cos(\Omega t - \vec{k} \cdot \vec{r}) \right]. \quad (3)
\]

where \( \Omega = \omega_2 - \omega_1 \) and \( \vec{k} = \vec{k}_2 - \vec{k}_1 \) are the frequency detuning and the wave vectors of the fringe pattern, respectively. Equation (3) can be written, in terms of the fringe contrast defined by \( m = \frac{2A_1 A_2}{A_1^2 + A_2^2} \), as

\[
I(\vec{r}, t) = (A_1^2 + A_2^2 f^2(t)) \left[ 1 + m \cos(\Omega t - \vec{k} \cdot \vec{r}) \right]. \quad (4)
\]

By using Eq. (4), one evaluates a time-integrated pattern of the energy density per unit area by

\[
\varepsilon(\vec{r}) = \int_{-\infty}^{+\infty} I(\vec{r}, t) dt. \quad (5)
\]

Substituting in Eq. (5) the expression of the intensity distribution given by Eq. (4) and the temporal \( f^2(t) \), one obtains

\[
\varepsilon(\vec{r}) = (I_1 + I_2) \left\{ 1 + M_d \cos(\vec{k} \cdot \vec{r}) \right\}, \quad (6)
\]

which can be rearranged as

\[
\varepsilon(\vec{r}) = \left( A_1^2 + A_2^2 \right) \int_{-\infty}^{+\infty} f^2(t) dt \left\{ 1 + \frac{m}{2} \frac{e^{-ikr} \int_{-\infty}^{+\infty} f^2(t) e^{i\Omega t} dt + e^{ikr} \int_{-\infty}^{+\infty} f^2(t) e^{-i\Omega t} dt}{\int_{-\infty}^{+\infty} f^2(t) dt} \right\}. \quad (7)
\]
So, the modulation depth is given by.

\[ M_d = \frac{m}{2 \cos(kr)} \frac{e^{-ikr} K^+ + e^{ikr} K^-}{K}, \]  

(8)

where.

\[ K^+ = \left( \frac{\delta}{\tau} \right)^{2n} \int_{-\infty}^{+\infty} e^{-r^2/\tau^2} r^{2n} e^{i\Omega t} dt, \]  

(9a)

\[ K^- = \left( \frac{\delta}{\tau} \right)^{2n} \int_{-\infty}^{+\infty} e^{-r^2/\tau^2} r^{2n} e^{-i\Omega t} dt, \]  

(9b)

and.

\[ K = \left( \frac{\delta}{\tau} \right)^{2n} \int_{-\infty}^{+\infty} e^{-r^2/\tau^2} r^{2n} dt. \]  

(9c)

By using the following identities (Gradshteyn and Ryzhik 2007).

\[ \int_{-\infty}^{+\infty} x^m e^{\beta x} dx = \frac{\Gamma(\gamma)}{p^{\gamma}}, \]  

(10)

with \( \gamma = \frac{m+1}{p} \), \( \text{Re}\beta > 0 \) and.

\[ \Gamma(n + \frac{1}{2}) = \frac{(2n)! \sqrt{\pi}}{n! 4^n}, \]  

(11)

where \( \Gamma(.) \) is the gamma function, Eq. (9.c) can be written as follows.

\[ K = \tau \delta^{2n} \frac{(2n)! \sqrt{\pi}}{n! 4^n}. \]  

(12)

For the evaluation of the integrals \( K^+ \) and \( K^- \), we will use the following identity (Belafhal et al. 2020).

\[ \int_{-\infty}^{+\infty} t^m e^{-pt^2+2qt} dt = \frac{q^2}{(2i\sqrt{p})^m} \sqrt{\frac{\pi}{p}} H_m \left( \frac{iq}{\sqrt{p}} \right), \]  

(13)

where \( H_m \) is the Hermite polynomial.

After tedious algebraic calculations, Eqs. (9.a) and (9.b) can be expressed as.

\[ K^+ = e^{-\frac{\Omega^2 r^2}{4}} \tau \sqrt{\pi} \left( \frac{\delta^2}{4} \right)^n (-1)^n H_{2n} \left( \frac{\Omega \tau}{2} \right), \]  

(14)

and.

\[ K^- = e^{-\frac{\Omega^2 r^2}{4}} \tau \sqrt{\pi} \left( \frac{\delta^2}{4} \right)^n (-1)^n H_{2n} \left( \frac{\Omega \tau}{2} \right). \]  

(15)
Finally, the use of Eqs. (8), (12), (14) and (15) yields the expression of the modulation depth established as.

$$M_d = me^{-\frac{\Omega^2 \tau^2}{4} \left[ \frac{n!}{(2n)!} \right]} \left[ (-1)^n H_{2n} \left( \frac{\Omega \tau}{2} \right) \right].$$  \hspace{1cm} (16)

This last equation is a closed-form of the modulation depth of the THz pulsed vortex beams. In the following, we will investigate some particular cases of the considered beams and we will show that our result can generalized some previous works.

3 Particular cases

3.1 Case of Gaussian pulses

We examine now the particular case corresponding to $n = 0$ and from Eq. (16) one deduces the modulation depth of these pulses given by.

$$M_d^G = me^{-\frac{\Omega^2 \tau^2}{4}}. \hspace{1cm} (17)$$

This result corresponds at that of the Gaussian pulse beams evaluated by Odoulov et al. (Odoulov et al. 2015; Malik and Escarquel 2018, 2020).

4 Case of single cycle pulses

The temporal profile of these pulses is given by Kapoyko et al. (2015)

$$f^2(t) = \left( \frac{\delta t}{\tau} \right)^2 e^{-\tau^2 / \tau^2}. \hspace{1cm} (18)$$

So, Eq. (16) can be written for $n = 1$, as.

$$M_d^{SCP} = m e^{-\frac{\Omega^2 \tau^2}{4}} \left[ \left( \frac{\Omega^2 \tau^2}{4} - 1 \right) \right]. \hspace{1cm} (19)$$

This modulation depth depends on the following parameters: the fringe contrast, the frequency detuning and the pulse duration.

5 Numerical simulations and discussions

In this part, we study the modulation depth and fringe contrast for the THz vortex beams and the Gaussian beams, of the temporal profile given by Eq. (2). So by using the main result established by Eq. (16), some numerical examples are performed to illustrate the MDFC ratio for the THz pulsed vortex beams in some plots in two views 3D or 2D, and we investigate some particular cases of the considered beams.
We illustrate in Fig. 1 the MDFC ratio for the particular case of the Gaussian pulses given by Eq. (17) as a function of the pulse duration $\tau$ and the frequency detuning $\Omega$ in a three-dimensional (x–y) plot.

From this figure, one observes that the MDFC ratio increases from 0 to 1 as the color changes from black to white (see the rectangular bar). It can also be seen that the value of the MDFC ratio decreases when the pulse duration increases and the frequency detuning is larger for the Gaussian profile. Consequently, the appropriate value of the MDFC ratio is obtained for higher pulse duration and frequency difference. So, we find these results to be in excellent agreement with the published experimental studies hitherto elaborated (Odoulov et al. 2015).

In Fig. 2 the temporal profile of the THz vortex beams is depicted for different values of the beam order $n$ by varying the controlled parameter $\delta$. As be seen from this figure, the beam family keeps a similar shape profile and the same comportment when the beam order is changed. It is also observed from this plot, that the central dark spot of THz vortex beams increases and the dark region becomes wider with increasing $n$, and the width of the lobes begins to get smaller with the decreases of the skew parameter $\delta$. Also, it can be seen that the intensity increases when the beam orders increase.

To show the influence of the beam order and the skew parameter on the MDFC ratio, we give in Fig. 3 the variation of the MDFC ratio of THz vortex pulses as a function of the frequency detuning for three values of the pulse duration ($\tau = 0.21$ ps, 0.30 ps and 0.35 ps) and with different beam orders ($n = 1, 2, 3$ and 4). From this figure, we can see clearly that the increasing of the beam order $n$ leads to the addition of the side lobes. For example, when $n = 2$ and for $\tau = 0.21$ ps (see Fig. 3a), we obtain two lobes, when $n = 3$, we observe the apparition of another lobe (see Fig. 3b). We can also note that when the pulse duration increases a slight increase occurs in the width of the lobes.

The variation of the MDFC ratio as a function of the frequency detuning for the THz vortex beams profile (dashed curve) and the Gaussian beams profile (solid curve) is presented in Fig. 4 for different values of $n$ (1, 2, 3 and 4) and two values of the pulse duration $\tau$. It is clear from this figure that for the same frequency detuning, the MDFC ratio of the THz vortex beams is much higher compared to that one of the Gaussian beams (red hatched area). We can also observe the appearance of side lobes for the THz vortex beams which increase with the increase of the beam order $n$. We note that the total

![Fig. 1 The MDFC ratio as a function of the pulse duration $\tau$ and frequency detuning $\Omega$ of the Gaussian pulses](image-url)
number of side lobes is proportional to the order of the studied profile. However, the MDFC ratio presents a slight increase when the pulse duration $\tau$ increases.

In Fig. 5, we illustrate the MDFC ratio for THz pulsed vortex beams as a function of the pulse duration $\tau$ and the frequency detuning $\Omega$ for and different values of the beam order $n$. 

Fig. 2 Temporal profile of THz vortex pulses for different skew parameters and for: a $n = 1$, b $n = 2$, c $n = 3$, d $n = 4$, e $n = 5$, and f $n = 6$
The rectangular bar shows the increase of the MDFC ratio from 0 to 1 as the color changes from black to white. It’s noted that the MDFC ratio gradually decreases as the frequency detuning increases and then a thin strip is formed while the MDFC ratio keeps null. In the end and as Ω increases further, the MDFC ratio begins to enhance and gradually decreases to zero, which results the appearance of two null regions (Fig. 5a) favoring the formation of the holograms. We can observe that by increasing the beam order n, the new thin strips appear (Fig. 5b–d). Hence, the MDFC ratio of the THz pulsed vortex beams presents discontinuities compared to that one of the Gaussian profile, due to the formation of null region (MDCD ratio = 0). The increasing of the beam order n leads to increase the discontinuities and a better quality of the hologram can be obtained for this beams family.

6 Conclusion

In this paper, we have investigated a THz vortex beam profile showing its important role in holography where two lasers with frequency difference are used. Based on the expression of the modulation depth, we have revealed that a hologram quality can be improved by adjusting the beam profile parameter. We can conclude that the THz vortex beams profile,
Fig. 4 The MDFC ratio as a function of the frequency detuning for two values of the pulse duration set as $\tau = 0.21$ ps and $\tau = 0.35$ ps and $a_n = 1$, $b_n = 2$, $c_n = 3$ and $d_n = 4$
contrary to the Gaussian beams profile, offers a new area where best quality holograms can be produced for higher frequency detuning. We believed that the present work is valuable for investigations of THz vortex beams in the holographic interferometry field.

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