Implications for the Cosmic Ray Spectrum of

a Negative Electron Neutrino (Mass)²

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Abstract

The hypothesis that the electron neutrino is a tachyon with $|m| \equiv \sqrt{-m^2} \approx 0.5\text{eV}/c^2$ is consistent with certain properties of the observed cosmic ray spectrum, including: the existence of a change in power law (the “knee”) at $E \approx 4 \times 10^{15}\text{eV}$, the $E^{-3}$ power law after the knee, another change in power law (the “ankle”) at $E \approx 10^{19}\text{eV}$, the changes in composition above the knee, the change in anisotropy at the knee, and the absence of a GZK cutoff above $E \approx 4 \times 10^{19}\text{eV}$. The hypothesis predicts a substantial flux of cosmic ray neutrons in a narrow energy region just above the knee of the spectrum.

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Introduction

Stimulated by the recent report of neutrino oscillations, which demonstrates that at least one type of neutrino has a non-zero mass squared, and persistently negative values for \(m_{\nu e}^2\) from tritium beta decay experiments, we conjecture that the electron neutrino is a tachyon with \(m_{\nu}^2 \approx -0.25eV^2/c^4\), or \(|m_{\nu}| \equiv \sqrt{-m_{\nu}^2} \approx 0.5eV/c^2\), and we consider the predicted consequences for the high energy cosmic ray spectrum:

1. The cosmic ray spectrum should have a discontinuity in power law (a "knee") at \(E \approx 4 \times 10^{15}eV\).

2. The power law for the spectrum should steepen by 0.33, i.e., go from \(E^{-2.67}\) to \(E^{-3.0}\) after the knee, and up to \(E \approx 10^{18}eV\)

3. The spectrum \((\times E^3)\) should have a dip at \(E \approx 10^{19}eV\), after which it should rise.

4. The average atomic mass of cosmic rays as a function of their energy should become increasingly heavy after the knee, but only up to \(E \approx 10^{18}eV\), after which it should become increasingly light over the ensuing decade, becoming almost pure protons at \(E \approx 10^{19}eV\)

5. The variation of anisotropy of the cosmic rays with energy should show a change in slope at \(10^{15}eV\).

6. Cosmic rays with energies in excess of \(E \approx 4 \times 10^{19}eV\) should be observed,
despite the existence of a conjectured GZK cut-off around this energy.

The model can be thought of as an example of how the hypothesis of “tachyonic” electron neutrino might provide an explanation of the preceding regularities, some of which have more conventional explanations. The hypothesis is consistent with other neutrino observations, and it makes a specific prediction: the existence of a cosmic ray neutron flux in a narrow range of energies just above the knee of the spectrum. Therefore, it should not be dismissed out of hand, even if it conflicts with much conventional wisdom about the cosmic rays, and the nonexistence of tachyons.

Tachyons

Tachyons, first postulated in 1967, by Bilaniuk, Deshpande, and Sudarshan, and later by Feinberg have not been taken seriously by most physicists, because of the paradoxes they create, and because all experiments specifically searching for tachyons have turned up negative. Thus, even though some of the equations of theoretical physics – especially in the field of string theory – have implied the existence of tachyons, they have generally been eliminated from most respectable theories.

Whatever one’s view of tachyons, their existence is clearly an experimental question. Those negative experiments to date merely rule out: (1) charged tachyons, (2) tachyons whose mass is too close to zero to be resolved in the experiment, or (3) tachyons not produced or detected in sufficient abundance.
to be observed. Weakly interacting neutral tachyons of low mass would have probably escaped detection, or else not be recognized as tachyons. In fact, Chodos, Hauser and Kostelecky suggested in 1985 that one or more of the three flavors of neutrinos is a tachyon. They based this idea on the fact that the best value of the square of the electron neutrino mass from the endpoint of the beta decay spectrum of tritium was at that time found to be negative by over two standard deviations.

In several papers Chodos et al. suggested that one could test whether neutrinos are tachyons using one of their strange properties, i.e., that particle decays producing tachyons which are energetically forbidden in one reference frame are allowed in another. Thus, consider the energetically forbidden “decay”: \( p \rightarrow n + e^+ + \nu_e \). For the decay to conserve energy in the rest frame of the proton, the energy of the neutrino obviously would need to be negative in that frame. (The positron or neutron could not have negative energy in this frame, because then they would have negative energy in all frames.) However, tachyons, unlike other particles, have \( E < p \) so they can change the sign of their energy when boosted to a sufficient velocity. Thus, the tachyon energy in the proton rest frame, \( E \) has the opposite sign from its energy in the lab \( E_{lab} = \gamma(E - \beta p \cos \theta) \) when \( \beta \) exceeds \( E/p \cos \theta < 1 \). At the threshold energy for proton beta decay the configuration of the three final state particles is a tachyon going directly forward in the lab with zero energy (but zero momentum \( p = |m| \)), and a neutron and positron going directly forward with a
common velocity.

The threshold energy for protons to decay is found by making \( E_\nu \) the least negative it can be in the CM frame, i.e., \(-E_\nu = m_n + m_e - m_p \equiv \Delta\). At threshold we take \( \cos \theta = 1 \), so for a relativistic neutrino, \( \beta_{th} = E_\nu/p_\nu \approx 1 + \frac{1}{2} m_\nu^2/E_\nu^2 \), and hence \( \gamma_{th} = (1 - \beta_{th}^2)^{-1/2} = \Delta/|m_\nu| \), so that

\[
E_{th} = \gamma_{th} m_p = \frac{m_p \Delta}{|m_\nu e|} = \frac{1.7 \times 10^{15}}{|m_\nu e|} \text{eV}
\]  

(1)

(For nuclei of mass number \( A \), \( m_p \) in the preceding formula is the mass of the parent nucleus, and \( \Delta \) is the mass difference: \( m(A, Z \pm 1) + m_e - m(A, Z) \).)

As various authors have explained, the idea of “stable” particles decaying is less paradoxical if one reinterprets the emitted (positive energy) neutrino in the lab frame to be an absorbed (negative energy) neutrino from a background sea in the proton rest frame – the so-called “reinterpretation principle.”

In any case, the net result is that protons or other “stable” nuclei at sufficiently high energies will undergo beta decay if the neutrino is a tachyon.

In order to test the prediction of Chodos et al. as applied to the cosmic ray spectrum, we need to calculate the phase space for proton or other stable nuclei to beta decay as a function of their energy. This is done by integrating that small region of phase space in the CM (the parent proton or stable nucleus rest frame) for which the neutrino energy changes sign between the CM and lab frames. The small fraction of the entire \( 4\pi \) solid angle that is available for any given (negative) neutrino energy in the CM is given by
\[
\frac{\Delta \Omega}{4\pi} = \frac{1}{2} \left( \frac{m_p}{E_{th}} \right)^2 \left( 1 - x^{-2} \right),
\]

where \( x = E/E_{th} \). The results of the phase space integration (done approximately using the relativistic approximation for neutrinos and electrons) give us the predicted mean free path before a proton undergoes beta decay. For large \( x \) the result is

\[
mfp = \frac{90eV^3ly}{|m_\nu|^3x^2}
\]

Predicting the Cosmic Ray Spectrum Above the Knee

We now describe how the assumption of a tachyonic neutrino can be used to make specific predictions about the cosmic ray spectrum beyond \( 10^{15} eV \). The input to the calculation are assumptions for: (1) the electron neutrino mass, (2) the energy spectrum and composition of cosmic rays at their source, and (3) the spatial distribution of sources. For the spatial distribution, we take an admixture of two source populations: “near” and “far” sources. Near sources are assumed to create cosmic rays that have path distances to Earth in the range \( 10^4 \) to \( 2 \times 10^6 \) ly, and far sources are assumed to have path distances to Earth in the range \( 2 \times 10^6 \) to \( 10^8 \) ly. (Clearly the terms “near” and “far” are something of a misnomer here: in view of the nonlinear paths of cosmic rays, their path lengths can greatly exceed the distances to sources except at the highest energies.)
For the source spectrum we use an $E^{-2.67}$ power law that fits the spectrum up to $10^{15}\text{eV}$ – see figure 2. Essentially, we assume that the source spectrum is $E^{-2.67}$ at all energies, and that any changes in the observed spectrum are due to particles in a given energy bin being shifted to lower energies as a result of beta decay. We have ignored the possibility that the spectra of sources might be distance dependent due to evolution. Since the composition of the cosmic ray spectrum above the knee is not known very well, we try various compositions and see which is in best agreement with the data. Again, however, we assume a fixed composition ratio independent of energy at the source, and assume that variations in composition with energy arise as the successive thresholds are reached for nuclei in the spectrum to decay. (Heavy nuclei tend to have higher thresholds than lighter ones because of the proportionality of $E_{th}$ on the nuclear mass.)

The case of the helium nucleus component in the cosmic rays is a special situation, because no beta decay has an alpha particle in the final state. The $E_{th}$ in the case of $He^4$ cosmic ray particles is the threshold energy at which the energetically forbidden decay: $He^4 \to 2p + 2n + \nu$ becomes possible for tachyonic neutrinos. (Equation 1 still applies here, with $\Delta = 2(m_p + m_n) - m_{He^4}$, and $m_p$ is the mass of the parent alpha particle.)

To derive the curves shown in the figures the Monte Carlo method was used. Protons and nuclei were generated at various distances from Earth according to the assumed $E^{-2.67}$ source spectrum, and the fate of all particles in a given
energy bin was considered to be the same, as their progress toward Earth was
followed. For protons leaving sources above the threshold energy for decay,
there is a chain of decays $p \rightarrow n \rightarrow p \rightarrow n \rightarrow p \cdots$ which stops when the
nucleon either reaches Earth or else has its energy reduced below threshold.
(We assume that at the source it was a matter of random choice as to whether
nucleons start out as neutrons or protons.) A similar decay chain occurs in
the case of $A > 1$ cosmic ray nuclei: $X_s(A, Z) \rightarrow Y_u(A, Z \pm 1) \rightarrow X_s(A, Z) \rightarrow
Y_u(A, Z \pm 1) \rightarrow X_s(A, Z) \cdots$, where the $s$ and $u$ subscripts denote stability or
instability to beta decay in the rest frame of the parent nucleus. After each
decay the daughter nucleus has less energy in the lab frame than the parent.

Calculating the loss in lab energy of the nucleon in a conventional beta decay
such as $n \rightarrow p + e^- + \bar{\nu}_e$ is straightforward. In the CM frame the proton has
very little energy following the decay, and hence in the lab frame the nucleon
loses a constant fraction $f \approx (1 - m_p/m_n)$ of its energy as a result of the decay.
For the energetically forbidden decay, such as $p \rightarrow n + e^+ + \nu_e$ the situation is
more complex. Here for proton lab energies much above threshold the neutrino
needs to have highly negative energies in CM in order that its energy in the
lab frame be positive, and hence the daughter nucleus energy can no longer
be ignored in the CM frame. The calculation can be done as a sequence of
two two-body decays: e.g., $p \rightarrow m(n, e^+) + \nu_e$ followed by $m(n, e^+) \rightarrow n + e^+$,
where in the first decay we choose only those events having neutrinos with
positive lab energy.
Figure 1 shows the $\log_{10}$ of the average fractional energy loss in the lab frame for the nucleon in the process $p \rightarrow n + e^+ + \nu_e$. As can be seen in the figure, the fractional energy loss in this decay is very small just above threshold, and it approaches an asymptotic value close to $2/3$ at very high energy, where the differences in mass between the three final state particles becomes unimportant.

Now, at virtually all energies above threshold, the nucleon spends most of its time en route from the source as a neutron, because the mean free path for neutrons before they decay is much greater than that for protons except quite close to the threshold energy, where the proton mfp before decay becomes infinite. Thus, the total number of steps in the decay chain of protons starting from the source with a given initial energy and source distance is basically determined only by the neutron mfp before decay. (This is fortunate because it means that the results of the calculation are quite insensitive to any approximations made in doing the phase space integral for the forbidden proton decay.)

**Discussion of Results**

The results of the Monte Carlo calculation are shown in figures 2, 3 and 4 along with the data. Following the usual practice, we show in figure 2 the $\log_{10}$ of the all particle flux multiplied by a power law ($E^3$), in order to emphasize departures from this power law. A reasonably good fit to the all particle
spectrum can be obtained for $|m_{\nu}| = 0.5eV/c^2$ (solid curve), assuming that 13% of sources are “near,” with elemental abundances: 70% A=1, 10% A=4, 10% A=14 (distributed between 5 to 19), 5% A = 24 (distributed between 20 to 40), and 5% A = 56 (distributed between 41 to 90). The solid curve also convolutes the Monte Carlo results with an assumed energy resolution $\Delta \log E = \pm 0.4$ (FWHM). As can be seen from the two dashed curves in figure 3, the goodness of fit worsens if the assumed energy resolution in the convolution is $\Delta \log E = \pm 0.2$ (long dashes) or zero (short dashes). In these two latter cases, the fits use an assumed tachyon mass of $|m_{\nu}| = 0.25eV/c^2$, and elemental abundances: 65% A=1, 10% A=4, 5% A=14, 5% A = 24, and 15% A = 56.

No decent fits can be obtained if $|m|$ is much greater than $0.5eV/c^2$. All three fits would also dramatically worsen if there were no near sources – since the curves would then fall off precipitously at $E \approx 10^{19}eV$. Thus, the flux beyond this energy appears in the model to come primarily from the 13% of sources that are “near.” The fits, however, are insensitive to the maximum path lengths of the far source component. If this is chosen as 10 Bly instead of 100 Mly, the fits are only slightly different from the curves in figure 2.

A convenient way to represent changes in the composition of the cosmic rays with energy is to plot the average $\log_e$ of the atomic mass – see figure 3. The model results showing $< \ln A >$ versus energy for two different energy resolutions are only in very rough agreement with the data in its essential
features: a rise of $<\ln A>$ from the knee of the spectrum to a maximum in the vicinity of $10^{17}$ to $10^{18}eV$ and a subsequent decline to a near zero value, i.e., almost pure protons, at $10^{19}eV$. Given the difficulty in making an experimental determination of the composition of cosmic rays above the knee, such rough agreement is probably all one could hope for.

Here is a simple explanation as to why the calculation predicts the specific features noted in the introduction:

- **The change in power law at** $\approx 4 \times 10^{15}eV$. This is the predicted threshold energy for cosmic ray protons to beta decay (given the assumed tachyon mass), and the proton component of the spectrum drops precipitously at this energy (to return at higher energies) – see figure 2. As the thresholds for heavier nuclei to beta decay are reached, they are also depleted from the overall spectrum.

- **The $E^{-3}$ power law between** $E = 10^{16}$ and $10^{18}eV$ observed in the data (near horizontal slope in figure 2) is reproduced by the model only with a proper choice of composition by atomic mass. The curves shown in figures 2 and 3 used elemental abundances noted previously.

- **The minimum at** $E \approx 10^{19}eV$ occurs because at this energy the threshold for the heaviest elements to beta decay is reached, and the spectrum becomes depleted. However the flux (multiplied by $E^3$) is restored after this minimum, because at higher energies an increasing fraction of
A=1 particles from the near source component can reach us, given their lengthened lifetime in the lab frame. As noted previously, the rise in the curves after $10^{19}eV$ depends on the 13% of sources that are “near.” Also, note that the position of the “ankle” (at $E \approx 10^{19}eV$) is shifted towards higher (lower) energies for lower (higher) assumed values of the neutrino mass.

- **Heavy to light transition** Just before the dip at $E \approx 10^{19}$ the composition is very heavy – see figure 3 – because only the heaviest elements are left in the spectrum at this point, since their thresholds have not yet been reached. However, at the highest energies the cosmic rays are found to be very light, because by around $E \approx 10^{19}$ the thresholds for all $A > 1$ nuclei have been reached – and they have all been depleted from the spectrum – while this energy is far enough above the threshold for $A = 1$ so that this component is coming back, owing to the greatly lengthened mfp’s due to the large dilation factor $\gamma$.

- **The abrupt change in the variation of anisotropy amplitude** (measured by the first harmonic of the arrival direction of air showers) observed to occur at the knee happens because above the threshold for proton beta decay, the A=1 component of cosmic rays is suddenly severely depleted, and hence the spectrum-average rigidity of the cosmic rays is reduced.
Cosmic rays with \( E > 4 \times 10^{19} \) are not blocked by interaction with the 2.7\(^0\) K cosmic background radiation (CBR), because as already noted, most of the flux beyond \( 10^{19}eV \) appears in our model to come from the 13 % of sources that are “near,” so the GZK cutoff would not apply. These ultrahigh energy particles primarily have A = 1, and as noted they spend nearly all of their journey as neutrons. But their neutrality is not a significant factor in preventing their interaction with the CBR, since neutrons would be expected to photoproduce pions with essentially the same cross section as protons.

Thus, in summary, our calculation of the cosmic ray spectrum above the knee follows naturally from the hypothesis that the electron neutrino is a tachyon with mass \( m_\nu \approx 0.5eV/c^2 \), and it is consistent with some other observations of the cosmic rays. However, any model has its problems and challenges, and we now turn to some of those.

Potential problems with the model

- conventional explanations exist for some of the regularities we have noted, and plausible mechanisms exist to account for the production of the component of the spectrum believed to be galactic in origin. However, few conventional explanations predict numerical values for the position of the knee and ankle, and many of the models have both ad hoc elements and many free parameters. Moreover, some of the features we have noted
have no conventional explanation, and some represent a very severe test of all conventional models – particularly the *abruptness* of the change in slope at the knee and ankle of the spectrum.

- **statistical significance** Some of the particular regularities our model predicts are not terribly significant individually. Moreover, we must consider the element of “informed choices,” i.e., choosing values of parameters that yield the best fit to the data, rather than making predictions before having seen the data. For this reason it is not possible to assign any probability of the model fitting the data simply on the basis of chance.

- **composition independent of energy** As noted previously, our model assumes an elemental composition at the source that is independent of energy, and then deduces the composition as seen on Earth, based on what gets depleted, or shoved down to lower energies through a chain of decays. It is highly unrealistic to suppose that the composition at the source is energy-independent, but by making this assumption we are merely limiting the number of free parameters in the model, making it more difficult to achieve a fit, not less.

- **Other models can account for the absence of a GZK cutoff** Various suggestions have been made to explain why cosmic rays with energies above the conjectured GZK cutoff \(E \approx 4 \times 10^{19} \text{eV}\) apparently fail to be significantly degraded in energy by interaction with the CBR. One
category of suggestions is that these cosmic rays are a new type of particle which interacts with CBR photons less strongly than nucleons or nuclei.\footnote{6} The hypothesis of remote sources (though not necessarily that of new types of particles) would receive additional support if it is found that they do point back to known distant radio quasars, as Farrar has suggested.\footnote{6} Another suggestion put forth by Coleman and Glashow is that Lorentz symmetry is slightly broken.\footnote{7} This would have the effect of kinematically forbidding interactions between cosmic ray protons and CBR photons, when the former have an energy above a certain value. Despite these alternative hypotheses for explaining the avoidance of a GZK cut-off, it would seem that the least exotic hypothesis is that the sources of cosmic rays with \(E > 4 \times 10^{19} \text{eV}\) simply are closer than a few dozen Mpc (as our model requires).

\begin{itemize}
\item **no known sources with a single power law.** No conventional mechanisms are known that have a \(E^{-2.67}\) power law spanning over ten decades, and it is not even clear what unconventional sources might be a candidate. Of course, there are no known sources in the conventional theory of cosmic rays at the highest energies either. Topological defects – including magnetic monopoles, cosmic strings, domain walls, etc. – have been suggested as sources for the highest energy cosmic rays.\footnote{8} But they have not been proposed to account for the lower energy region, which are believed to originate from shocks driven into the interstellar medium by supernova
explosions in the generally accepted conventional view. However, even for the lower energy region, the evidence supporting the conventional view of supernova shock waves as the source is only circumstantial, i.e., SN’s have enough energy for steady state cosmic ray production, and they would have a power law index at the source that is flatter than what is observed in the arriving flux, as expected. But, direct confirmation of the conventional theory of supernova shock sources does not yet exist, e.g., high energy gamma rays from pions produced during the acceleration process. One exotic possibility for sources has been proposed by Kuzmin and Tkachev: the decay of supermassive long-lived particles produced in the early universe from vacuum oscillations during inflation. One advantage of this possibility from the point of view of our model is that such sources could be a considerable fraction of cold dark matter, and hence could be prominent in the Milky Way galactic halo, and therefore relatively nearby. Yet, they would also be relatively isotropic, as seems to be the case for the limited number of events so far seen at the highest energies.

- **source spectrum.** Choosing the source spectrum to match the observed $E^{-2.67}$ power law below the knee probably is another unrealistic feature of the model. A more realistic model would have included a source spectrum less steep than $E^{-2.67}$ in order to include other energy loss processes besides those included here. Without such processes it would be difficult
to account for the observed primary to secondary ratios. However, the model is not intended to be an all-encompassing explanation of every aspect of the cosmic ray spectrum, but rather an example of how a tachyonic electron neutrino can account for many of its features.

- **no observed neutron component seen in the cosmic ray spectrum.** Although no neutron component has yet been observed in the cosmic ray flux, current techniques based on air shower measurements would not distinguish between proton-induced cascades and those initiated by neutrons. In fact, based on anisotropy data, Tkaczyk\textsuperscript{20} has estimated that the neutron component could be as high as 20\% in the $10^{16} - 10^{18}$ eV energy region if the neutrons come from sources in the galactic disk. (In our model, the chain of decays $p \rightarrow n \rightarrow p \rightarrow n \rightarrow p \cdots$ only occurs at energies above the threshold for proton decay, so neutrons would not be seen below this energy, given their mean free path before decay.)

**Possible Confirming Tests of the Hypothesis**

As already noted, most tritium beta decay experiments have consistently reported negative values for $m_{\nu e}^2$. The seven tritium beta decay experiments used by the Particle Data Group in 1998\textsuperscript{2} to find a value for $m_{\nu e}^2$ all report negative values. Two of these experiments report values that are negative by over four standard deviations, but they are inconsistent with each other:\textsuperscript{21, 22}
\[ m_{\nu e}^2 = -22 \pm 4.8 \] and \[ -130 \pm 20 \pm 15eV^2/c^4 \] Regretably, the value we have used here \(|m_{\nu e}| \approx 0.5eV/c^2\) is too small to be consistent with the negative values reported for \(m_{\nu e}^2\) in either of these experiments. Moreover, the tritium results have been explained in terms of either experimental anomalies, final state interactions, or new physics – though a few authors have attributed them to tachyonic neutrinos.

Assuming that the electron neutrino really were a tachyon, could future tritium beta decay experiments test for values of \(m_{\nu e}^2\) as small as \(0.25eV^2/c^4\)? The current systematic and statistical errors on \(m^2\) are over an order of magnitude smaller than that value, so without new types of instruments the answer is probably not. But, apart from the issue of the limits of experimental precision, and the need to better understand the basis of the negative \(m_{\nu e}^2\) values found in past experiments, it is also important that experimenters avoid the position that “the negative value for the best fit of \(m^2\) has no physical meaning.” Of course, experimentalists are not alone in believing negative mass\(^2\) particles are unphysical. For example, Hughes and Stephenson have argued that if the neutrino really were a tachyon, there would be no endpoint to the electron energy spectrum in tritium beta decay, and a large amount of phase space would exist for neutrinos of arbitrarily large negative energies. But, their result ignores the reinterpretation principle, according to which negative energy emitted tachyons are only part of the kinematically allowed region for tritium decay if they have positive energy in the lab frame.
If one is open to the idea that neutrinos could, in fact, be tachyons it is natural to ask why one should put any more faith in the mass value obtained from the fit to the cosmic ray spectrum ($|m_{\nu_e}| \approx 0.5\text{eV}/c^2$) than the much larger values found in tritium beta decay experiments? One answer is that the only statistically significant negative values found in tritium beta decay experiments are inconsistent with each other, and have been attributed to a number of plausible causes having nothing to do with tachyons. A second answer is that if any of the values from tritium beta decay experiments represent masses of real tachyons, then the knee of the cosmic ray spectrum would have to occur at an energy one or two decades lower than is observed, because the threshold energy for proton beta decay varies inversely with $|m_{\nu_e}|$. Alternatively, if the tachyon mass found from the cosmic ray spectrum fit is correct that only means that the values reported in the tritium experiments arise from causes other than real tachyons.

Are there other places one might look for confirmation of the hypothesis that the electron neutrino is a tachyon? Neutrino oscillation experiments, being sensitive to differences in $m^2$ cannot reveal whether any neutrino flavors have negative $m^2$. Caban, Rembielinski and Smolinski have suggested that the tachyonic neutrino hypothesis may explain the problem of the missing solar neutrinos. But, of course, the missing solar neutrino problem has already been explained in terms of neutrino oscillations and other less radical ways than the hypothesis of tachyonic neutrinos. The possibility of testing whether
$m_{\nu e}^2 < 0$ based on observing neutrinos from some future supernova remains a possibility, but it is uncertain if a value of $|m_{\nu e}|$ as small as $0.5eV/c^2$ could be clearly distinguished from zero when the current upper limit based on SN1987A is $|m_{\nu e}| < 15eV/c^2$.

There is, however, one unambiguous test of the tachyonic neutrino hypothesis involving a cosmic ray neutron flux. Graphs of cosmic ray flux times $E^3$ (see figure 2) may be good for spotting changes in the flux power law, but they can be extremely misleading in other respects. For example, there really is no “ultraviolet catastrophe,” as figure 2 seems to show, after one removes the $E^3$ multiplier. In addition, it might seem from the dotted curve in figure 2 that the $A = 1$ contribution to the flux drops precipitously right after the threshold for proton decay, and does not return until around $E \approx 10^{19}eV$. In fact, however, the small bump in figure 2 just before the precipitous drop is actually a rather large bump when one plots the flux without multiplying it by $E^3$, see figure 4, which shows both the neutron flux, and its fraction of the total plotted on a linear rather than a log scale.

Thus, a most distinctive prediction of the model is a spike of neutrons just above the threshold energy for proton beta decay. The position and height of the neutron spike does of course depend on the value assumed for the neutrino mass -- $|m| = 0.25eV/c^2$ in figure 4. Doubling that value would halve the energy at which the spike occurs, and increase its height eightfold. The accumulation of a spike of neutrons just above the threshold energy is a consequence of the
fractional energy loss of the nucleon becoming very small as the threshold is approached from above – see figure 1.

Experimentally, it may be impossible to distinguish individual cosmic ray neutrons from protons in the region of the knee of the spectrum, based on air shower measurements. But, there is one unambiguous difference: unlike protons or nuclei, neutrons point back to their sources in this energy region. Hence, multiple events coming from the same directions should be a clear indicator of neutrons. Moreover, given the neutron lifetime, the mean free path before decay at an energy of $10^{16} eV$ is only about 6000 ly – much too close for many sources in any conventional model. As figure 3 shows, neutrons should also be seen as a large component of the (much smaller) flux at energies above $10^{19} eV$. However, if neutrons were seen at these energies, their presence could well be the result of sources closer than 6 Mly, and they would, therefore, have little value in confirming the hypothesis of tachyonic neutrinos.

Interestingly, there is one other hypothesis that has been suggested to account for the knee of the cosmic ray spectrum that also predicts that neutrons should be a component of the cosmic ray flux beginning just above the knee of the spectrum. Wigmans has recently suggested that massive relic neutrinos might be gravitationally captured and concentrated around cosmic ray sources such as neutron stars. If the relic neutrinos could be concentrated to a sufficient degree, cosmic ray protons passing through these stellar “neutrino atmospheres” would lose energy through the inverse beta decay process:
\[ p + \bar{\nu} \rightarrow n + e^+ \]. If the relic neutrino has very little energy and a very small mass, the energy threshold for the reaction is almost identical for the threshold for \( p \rightarrow n + e^+ + \nu \), where \( \nu \) is a tachyon.\(^\text{32}\) But, of course, if Wigmans’ hypothesis were correct, few of the neutrons produced by cosmic ray protons just above the knee of the spectrum could reach us, given a mean free path before decay of only 6000 ly. In contrast, with the tachyonic neutrino hypothesis, neutrons can reach us even if the sources are at much greater distances, since a chain of decays occurs: \( p \rightarrow n \rightarrow p \rightarrow n \rightarrow p \cdots \) resulting in an observed flux of neutrons piling up just above the threshold for proton decay.

This is not the place to attempt to convince the reader that the paradoxical properties of tachyons – especially their apparent violation of causality – does not make their existence absurd. Let it only be said that the plausibility of tachyons should hinge more on whether they are capable of absurd possibilities, e.g., sending messages back in time, than on any theoretical formulation of a definition of causality, and that the various absurd possibilities sometimes attributed to tachyons can be dismissed by reinterpreting emitted (negative energy) tachyons as being absorbed positive energy ones, as various authors have demonstrated.\(^\text{4, 5, 10}\) If tachyons do really exist, either nature would have found a way of eliminating absurd possibilities, or else physicists would have to readjust their thinking as to the boundaries of the absurd.
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Figure 1: $\log_{10}$ of the average fractional energy loss of the nucleon in the decay $p \to n + e^+ + \nu_e$, assuming a neutrino mass $|m| = 0.5eV/c^2$ for proton energies above the threshold.
Figure 2: Solid curve shows the prediction of the model for the cosmic ray flux \((\times E^3)\) assuming a tachyon mass \(|m| = 0.5\text{eV/c}^2\), with convolution, assuming an energy resolution of \(\Delta \log E = \pm 0.4\). The two dashed curves show fits with a tachyon mass of \(|m| = 0.25\text{eV/c}^2\): short dashed curves assumes no convolution to account for energy resolution, and the long dashed curve uses an energy resolution \(\Delta \log E = \pm 0.2\). The dotted curve shows the spectrum with \(A = 1\) only using \(|m| = 0.25\text{eV/c}^2\). All curves assume 13\% near sources with mass compositions noted in the text. Points are the data from: JAYCEE (diamonds), AGASA (with error bars), Aoyama-Hirosaki (squares), Tibet (crosses), Akeno 1km\(^2\) array (diamonds), Proton Satellite (asterisks).
Figure 3: Prediction of the model for the cosmic ray composition \(< \ln A >\) as a function of particle energy. Solid and dashed curves makes the same assumptions of values of the tachyon mass, composition, and the percentage of “far” cosmic ray sources as the solid and dashed curves shown in figure 3. Data points with a horizontal line segment are from JACEE (1995), squares are BASJE (1994), and crosses are Fly’s Eye (1993). For the data from the stereo Fly’s Eye (squares), we have made a linear interpolation in \(< \ln A >\) using their elongation rate data to deduce values for \(< \ln A >\)
Figure 4: Solid curve shows the prediction of the model for the cosmic ray flux of neutrons, assuming a tachyon mass $|m| = 0.25 \text{eV}/c^2$, 13% near sources, with no convolution to account for finite energy resolution. The dotted curve shows the fraction the neutron flux is of the total flux. The flux has been expressed in units that allow the same vertical scale to be used for both curves. Doubling the tachyon mass would shift the neutron peak to half the energy, or shift log $E$ downward by 0.3, and increase the peak height eightfold.