A recent theory of metaphysical indeterminacy says that metaphysical indeterminacy is multiple actuality: there is metaphysical indeterminacy when there are many 'complete precisifications of reality'. But it is possible for there to be metaphysical indeterminacy even when it is impossible to precisify reality completely. The orthodox interpretation of quantum mechanics illustrates this possibility. So this theory of metaphysical indeterminacy is not adequate.

I. SHALLOW AND DEEP METAPHYSICAL INDETERMINACY

Many questions of interest to philosophers invite the response that there is just no fact of the matter. Is the continuum hypothesis true? When someone undergoes fission, which of the resulting persons is him? Maybe the best thing to say is that it is indeterminate whether the continuum hypothesis is true, and that it is indeterminate which of the post-fission people is identical with the pre-fission person.

It may also be tempting to add that the source of this indeterminacy is neither semantic nor epistemic: the reason why it is indeterminate whether the continuum hypothesis is true is not that we have made too few 'semantic decisions' about what the language of set theory is to mean, or that the truth-value of the continuum hypothesis is something which (for special kinds of reasons) it is impossible for us to know. Instead, this is (perhaps) a case of metaphysical indeterminacy.

Some presentations of the orthodox interpretation of quantum mechanics make use of the notion of metaphysical indeterminacy. They portray reality itself as fuzzy or indefinite. Place a particle into a box with infinitely hard walls. Wait a while, and then look to see if the particle is in the left half or the right half of the box. According to the orthodox interpretation of quantum mechanics, the complete physical state of the particle before you look in the box does not determine whether you will see it in the left half or the right half when you look. Instead, the theory

\[1\] There is no canonically accepted statement of the orthodox interpretation. Some people use the language of determinacy when stating it; see, e.g., J.A. Barrett, *The Quantum Mechanics of Minds and Worlds* (Oxford UP, 1999), p. 37. Others do not, e.g., D.Z. Albert, *Quantum Mechanics and Experience* (Harvard UP, 1992), p. 38. I shall use 'the orthodox interpretation' to mean 'the orthodox interpretation, stated using the language of indeterminacy'.

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assigns a probability greater than 0 but less than 1 to your finding it in the left half, and a probability greater than 0 but less than 1 to your finding it in the right half. The orthodox interpretation says that if the complete physical state does not assign probability 1 to the particle’s being found on the left, and does not assign probability 1 to the particle’s being found on the right, then before we look, it is indeterminate whether the particle is on the left or on the right.

So maybe there is such a thing as metaphysical indeterminacy, and maybe quantum mechanics provides an example of it. But what is it? Can we get any kind of a grip on what metaphysical indeterminacy is?

One picture of metaphysical indeterminacy emerges from thinking about the picture that goes along with supervaluationist treatments of semantic indeterminacy. On the supervaluationist picture, our language suffers from semantic indeterminacy because it is poised between alternative interpretations. Each interpretation fits our language use equally well (and there are no other facts about the interpretations to break the tie). It is said that these interpretations are precisifications of our language.

Maybe a similar picture can make metaphysical indeterminacy intelligible. Perhaps reality itself is poised between alternatives. Someone is about to undergo fission. One of the post-fission people will leave by the left door; the other will leave by the right. Suppose we want to say that it is metaphysically indeterminate which post-fission person is identical with the pre-fission person. We can try to make this intelligible by saying that there is a precisification of reality in which the pre-fission person survives as the successor who leaves by the left door, and another precisification of reality in which the pre-fission person survives as the successor who leaves by the right door.

Each precisification of reality is, as it were, perfectly precise. That is, if there are many ‘aspects of’, or parts of, reality, a, b, ... and it is (metaphysically) indeterminate whether a is F, and it is (metaphysically) indeterminate whether b is G, ... then there is a precisification of reality at which a is F and b is G, and also a precisification of reality on which a is not F and b is not G, and so on.

But is this an adequate model for metaphysical indeterminacy? Maybe metaphysical indeterminacy runs so deep (or can run so deep) that reality cannot be completely precisified. I shall call this kind of metaphysical indeterminacy ‘deep metaphysical indeterminacy’.

(It is consistent to accept that there is deep metaphysical indeterminacy and to continue to believe that each aspect of reality that is indeterminate can be precisified. It is just that not all aspects of reality that are indeterminate can be precisified ‘at once’. It might turn out that there is no precisification of reality at which a is F and b is G, even though there is a precisification at which a is F, and a precisification at which b is G.)

Have we any reason to think that deep metaphysical indeterminacy is possible? I think we have. The kind of metaphysical indeterminacy that can be found in the orthodox interpretation of quantum mechanics is deep metaphysical indeterminacy. This may not show that deep metaphysical indeterminacy is actual. For one thing, there are presentations of the orthodox interpretation that avoid the language of indeterminacy. For another thing, the orthodox interpretation of quantum mechanics
DEEP METAPHYSICAL INDETERMINACY

suffers from all sorts of well known defects, and there are alternative interpretations of quantum mechanics that do not use the notion of indeterminacy at all. But the orthodox interpretation of quantum mechanics shows how and why deep metaphysical indeterminacy could arise. Whenever the set of basic physical properties has the structure seen in orthodox quantum mechanics, deep metaphysical indeterminacy may appear. Since this structure looks like a possible structure, we should think that deep metaphysical indeterminacy is at least possible.

What is this structure which the set of basic physical properties could have, and which would give rise to deep metaphysical indeterminacy? The best way to examine its nature is to explore in detail why the indeterminacy in the orthodox interpretation of quantum mechanics is inconsistent with the idea that reality can be completely precisified.

II. THE BARNES–WILLIAMS MODEL

Elizabeth Barnes and J.R.G. Williams have developed a model of metaphysical indeterminacy which implements the idea that (even when there is metaphysical indeterminacy) reality can be completely precisified. It will be convenient to work with their model as a standard version of a theory of shallow metaphysical indeterminacy.

Barnes and Williams model metaphysical indeterminacy by modifying a familiar model for modality. Suppose there are such things as possible worlds. A possible world, intuitively speaking, is a maximally specific way for reality to be. One of the possible worlds is the way reality is; this world is the actual world. Other possible worlds are merely possible.

To fit metaphysical indeterminacy into this picture, Barnes and Williams first restrict their attention to views about the nature of possible worlds other than modal realism. Instead, they focus on theories according to which possible worlds are sets of sentences, or are maximally specific properties, and other theories like these. Importantly, the possible worlds in these theories are perfectly precise. For example, the theory that possible worlds are sets of sentences also holds that these sentences belong to a language for which all needed ‘semantic decisions’ have been made.

Metaphysical indeterminacy can now be located. Metaphysical indeterminacy corresponds to multiple actuality. That is, it is metaphysically indeterminate whether \( p \) if and only if there is an actual world at which \( p \), and an actual world at which not-\( p \). That can happen only if there is more than one actual world. We usually assume that this is false. The actual world, we usually say, is the (unique) possible world that accurately represents reality. However, if there is metaphysical indeterminacy, if reality itself is ‘unsettled’, then it cannot be perfectly described using a precise

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2 The most detailed presentation of their view is in J.R.G. Williams and E. Barnes, ‘A Theory of Metaphysical Indeterminacy’, *Oxford Studies in Metaphysics* (forthcoming). See also Williams, ‘Multiple Actuality and Ontically Vague Identity’, *The Philosophical Quarterly*, 58 (2008), pp. 134–54, and ‘Ontic Vagueness and Metaphysical Indeterminacy’, *Philosophy Compass*, 3 (2008), pp. 763–88. There are some differences between the views in these papers which are not relevant here. The version of the view which I present is the one found in ‘Multiple Actuality and Ontically Vague Identity’.
language. No possible world represents reality with perfect accuracy. So we have to use a different definition of ‘actual world’: \( w \) is an actual world if and only if it is not the case that \( w \) determinately misrepresents reality. If reality itself is indeterminate, then there are many possible worlds which do not determinately misrepresent it; these are the actual worlds.

This model is not advanced as an analysis of metaphysical indeterminacy. As an analysis it would be circular, because ‘It is (metaphysically) indeterminate that …’ appears in the definition of ‘actual world’. But even though the model cannot be used to analyse the notion of metaphysical indeterminacy, it can still be useful. Metaphysical indeterminacy is an obscure notion which the model helps to make intelligible. It can also provide a guide to discovering the logical properties of the ‘It is metaphysically indeterminate that …’ operator. Furthermore, the model can clarify arguments which make use of (or directly concern) metaphysical indeterminacy.

Because their possible worlds are perfectly precise, and those possible worlds play the role of precisifications of reality in their theory, Barnes and Williams have provided a model of shallow metaphysical indeterminacy. So it is incompatible with the deep metaphysical indeterminacy in the orthodox interpretation of quantum mechanics.

III. THE INCOMPATIBILITY WITH QUANTUM MECHANICS

I shall assume for the moment that the orthodox interpretation of quantum mechanics is true. When it is indeterminate whether the particle is on the left side of the box or on the right side of the box, Barnes and Williams say that there is an actual world at which the particle is on the left, and one at which the particle is on the right. This generalizes to features of things other than their positions. For many determinable properties of a quantum system (momentum, spin in a given spatial direction, …), there is a possible physical state of the system such that when the system is in that state, the orthodox interpretation says that it is indeterminate which value it has for at least one of those determinables. So in the Barnes and Williams model each actual world attributes to each quantum system a value for each determinable property, and all actual worlds agree on the values assigned to properties which have determinate values.

But there is a well known theorem, the Kochen–Specker theorem, which proves that these attributions cannot be made. My goal here is to outline the set-up and proof of this theorem so as to make it accessible to metaphysicians who have had little exposure to the mathematical formalism of quantum mechanics. The reasons why the theorem is true throw light on the circumstances in which there would be deep metaphysical indeterminacy.

Some of the quantum physics of spin-\( \frac{1}{2} \) particles, like electrons, is familiar to philosophers with little background in the philosophy of physics. We may set up an experiment to measure the spin of an electron along any spatial direction. Such a

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3 I mostly follow R.I.G. Hughes, *The Structure and Interpretation of Quantum Mechanics* (Harvard UP, 1989).
measurement will have one of two possible outcomes, ‘up’ (represented mathematically by $+1$) or ‘down’ (represented by $-1$). So for each direction in space $r$ there are two values the electron may have, $+1$ or $-1$, for the property spin-in-direction-$r$.

For a spin-$\frac{1}{2}$ particle, in contrast, there are three values the particle may have, $+1$, $0$ and $-1$, for the property spin-in-direction-$r$. I use ‘$S$’ as a name for this property. (Photons have spin-$1$.)

For each $r$ there is the property square-of-spin-in-direction-$r$, or $S^2$. One way to measure simultaneously a spin-and-spin-in-direction-$r$, say $S^2$, obviously, is to measure the value for $S$ and then square the result. So there are just two possible values for $S^2$, $1$ and $0$. Any (perfectly precise) possible world attributes to a spin-$\frac{1}{2}$ particle exactly one of these two values for each determinable property $S^2$.

Let $I$ be the property (of spin-$\frac{1}{2}$ particles) of having some value or other for spin in some direction or other. This is a ‘trivial’ property, with only one possible value ($+1$). The orthodox interpretation says that every spin-$\frac{1}{2}$ particle determinately has value $+1$ for $I$.

Now comes a crucial part of the set-up of the Kochen–Specker theorem. If $x$, $y$ and $z$ are three mutually perpendicular directions in space, then it is possible to measure simultaneously a spin-$\frac{1}{2}$ particle’s values for each of the properties $S_x^2$, $S_y^2$ and $S_z^2$. When there is a clause of properties of a system which are simul-

taneously measurable, the quantum mechanical formalism says that there is a property of the system corresponding to every algebraic combination of those properties. So there is a property $S_x^2+S_y^2+S_z^2$ for which a spin-$\frac{1}{2}$ particle may also have values; and in any circumstance in which the particle definitely has values (say) $a$, $b$ and $c$ for $S_x^2$, $S_y^2$ and $S_z^2$, by definition it definitely has value $a+b+c$ for $S_x^2+S_y^2+S_z^2$.

According to orthodox quantum mechanics, the spin properties of a spin-$\frac{1}{2}$ particle are not independent. Here is one constraint they satisfy: $S_x^2+S_y^2+S_z^2 = 2I$ (proving this is straightforward but beyond the scope of this paper). Focusing on one particular spin-$\frac{1}{2}$ particle, for each determinable property $P$, I shall write $i(P)$ for the value which this particle has for $P$. It follows from what I have said that

1. In every actual world, \( i(S_x^2)+i(S_y^2)+i(S_z^2) = i(2I) = 2 \), where $x$, $y$ and $z$ are any three mutually perpendicular directions in space. (That is, it is determinately the case that \( i(S_x^2)+i(S_y^2)+i(S_z^2) = 2 \).)

The argument for (1) is straightforward. Since it is determinately the case that the particle has value $1$ for property $I$, $i(I) = 1$ in every actual world. So $i(2I)$ should be $2$ in every actual world, since a particle’s value for $2I$ is just defined to be $2$ times its value for $I$. But why should $i(S_x^2+S_y^2+S_z^2)$ be equal to $i(S_x^2)+i(S_y^2)+i(S_z^2)$ in every actual world? It is (physically) necessary that if the particle has determinate values for $S_x^2$, $S_y^2$ and $S_z^2$, then it has a determinate value for $S_x^2+S_y^2+S_z^2$, and its value for the sum is just the sum of the individual values. (The converse is not true, but this is not needed for the proof.) But what is physically necessary is determinately true. So for any actual world $w$, if $S_x^2$, $S_y^2$ and $S_z^2$ have definite values in $w$, then $i(S_x^2+S_y^2+S_z^2) = i(S_x^2)+i(S_y^2)+i(S_z^2)$ in $w$. But $S_x^2$, $S_y^2$ and $S_z^2$ do have definite values in every actual world.
The Kochen–Specker theorem says that there is no possible world at which the equation in (i) is true. It follows that there are no actual worlds. Here is an outline of the proof:

Directions in space can be put in one-to-one correspondence with the points on the surface of a sphere (a direction corresponds to a point \( p \) if it is the direction from the centre of the sphere to \( p \)). So an attribution of exactly one of two values to \( S_r \) for each direction in space \( r \) corresponds to an assignment of either 0 or 1 to each point on a sphere. Each possible world determines such an assignment.

Any three mutually perpendicular directions in space determine three points on the sphere. Call such a triple of points an orthogonal triple. So every actual world determines an assignment of either 0 or 1 to each point on a sphere, subject to the constraint that exactly one point in each orthogonal triple is assigned 0.

It can be proved, using relatively simple geometry, that there are no assignments of 1s and 0s to points on a sphere that meet this constraint. QED.

So assuming that the orthodox interpretation of quantum mechanics is correct (and that the assumptions which went into the proof of the Kochen–Specker theorem are also correct), there are no actual worlds. This, of course, is an absurd conclusion. What has happened is that the Barnes–Williams model of metaphysical indeterminacy has broken down. It is not adequate for representing the kind of indeterminacy found in the orthodox interpretation of quantum mechanics.

I emphasize that it is not part of my argument that the orthodox interpretation of quantum mechanics is in fact correct. There are many other interpretations of quantum mechanics (Bohmian mechanics, for example, and the many Everettian interpretations) that make no use of the notion of metaphysical indeterminacy. If we reject the orthodox interpretation and accept one of those instead, then we do not have to say that there is actually any deep metaphysical indeterminacy. But it would still be true that the metaphysical indeterminacy in the orthodox interpretation of quantum mechanics is a possible kind of metaphysical indeterminacy. My conclusion is that the Barnes and Williams model fails to model all possible kinds of metaphysical indeterminacy.

(In a recent paper George Darby also discusses what the Kochen–Specker theorem can teach about metaphysical indeterminacy in quantum mechanics. He does not reach a definitive conclusion, but he does suggest that it can be used to show that the Barnes–Williams model is incorrect. However, his argument assumes that quantum mechanical indeterminacy is actual. So Barnes and Williams can avoid his conclusion by embracing interpretations of quantum mechanics that do not invoke indeterminacy. For the reasons I have given, they cannot avoid my own conclusion in this way.)

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4 G. Darby, ‘Quantum Mechanics and Metaphysical Indeterminacy’, Australasian Journal of Philosophy, 88 (2010), pp. 227–45.
IV. COROLLARIES

The Kochen–Specker theorem was originally stated and proved out of an interest in hidden-variables theories. According to the orthodox interpretation of quantum mechanics, the entire physical state of a quantum system is encoded in its wavefunction. But the wavefunction does not determine a value for all of the system's determinable physical properties. In a hidden-variables theory, in contrast, a quantum system does have determinate values for each determinable physical property. But we are (necessarily) ignorant of the exact value a system has for each physical property. So when using the hidden-variables theory to make predictions, we have to 'average over our ignorance'. Since we cannot know the exact initial state of any given experimental set-up, we can only use our hidden-variables theory to assign probabilities to outcomes of experiments, even if the theory's underlying dynamics is deterministic. The hope was that a hidden-variables theory could be constructed which assigns the very same probabilities to outcomes of experiments as quantum mechanics assigns. The Kochen–Specker theorem was originally proved to show that no such hidden-variables theory can be constructed.

But there is a close connection between these two applications of the theorem. For a hidden-variables theory, a quantum system is under-described by its wavefunction, and a complete description specifies values for all of its physical properties. An attempt to model quantum indeterminacy using the Barnes–Williams model says that a quantum system is completely described by its wavefunction, so its values for some physical properties are unsettled; but there are many actual worlds, each of which specifies (possibly different) values for all of the system's physical properties. The state of a quantum system in a hidden-variables theory assigns values which (it says) are unknown to us, and an actual world in the Barnes–Williams model assigns values which are left indeterminate. The Kochen–Specker theorem shows that neither enterprise can succeed.

(Bohmian mechanics, for example, says that there are no such properties of spin-1 particles as the properties $S$. In Bohmian mechanics, spin becomes a feature of the wavefunction, rather than of particles. More generally, the Kochen–Specker theorem does not show that contextualist hidden-variables theories are impossible. A contextualist hidden-variables theory does not assign a value to each of the $S^2$ properties. Instead, for each orthogonal triple $x$, $y$ and $z$ it assigns a value to $S^x$ only relative to $S^y$ and $S^z$ (and there are infinitely many orthogonal triples containing $x$). So in these theories the fundamental features of spin-1 particles are relational in a complicated way.

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vagueness and indeterminacy. Absences of property independence resemble the existence of ‘penumbral connections’. Kit Fine introduced the notion of penumbral connections when arguing for a supervaluationist treatment of vagueness. He said that any precisification of our language must respect penumbral connections. A colour chip may be a borderline case of ‘blue’ and also a borderline case of ‘green’ while definitely being either blue or green. If a colour chip is a borderline case of both ‘blue’ and of ‘green’, then some precisifications place it in the extension of ‘blue’ and some do not, and similarly for ‘green’; but there is also the additional constraint that every precisification must place the colour chip in the extension of exactly one of the predicates. That the properties of a quantum mechanical system are not independent is something like a penumbral connection between them.

Fine assumed that there are precisifications of our language which respect the penumbral connections in our language. The Kochen–Specker theorem shows that there are not complete precisifications of reality which respect the dependencies among properties in orthodox quantum mechanics. I doubt, though, that the possibility of complete precisification marks a difference between semantic and metaphysical indeterminacy. Jamie Tappenden, for one, argues that it is not possible to precisify our language completely while respecting penumbral connections, and he develops a version of supervaluationism that uses partial precisifications.

The orthodox interpretation of quantum mechanics is compatible with the existence of partial precisifications of reality. But the prospects of using this notion to put together a model of metaphysical indeterminacy similar to Barnes and Williams’ model are dim. For suppose we keep their framework but replace perfectly precise possible worlds with imprecise possible worlds (sets of sentences from a language which suffers from semantic indeterminacy). Even when there is no metaphysical indeterminacy, we can expect it to happen that several imprecise possible worlds do not determinately misrepresent reality. So using imprecise worlds would give multiple actuality even when there is no metaphysical indeterminacy. How to model deep metaphysical indeterminacy remains an open question.8

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6 K. Fine, ‘Vagueness, Truth, and Logic’, Synthese, 30 (1975), pp. 265–300.
7 J. Tappenden, ‘The Liar and Sorites Paradoxes: Toward a Unified Treatment’, Journal of Philosophy, 96 (1999), pp. 551–77, at p. 567.
8 Thanks to Elizabeth Barnes, Robbie Williams, Steve Yablo, Agustin Rayo and an anonymous referee.