Elastic instability in a straight channel of viscoelastic flow without prearranged perturbations

Yuke Li\textsuperscript{1} and Victor Steinberg\textsuperscript{1,2}

\textsuperscript{1}Department of Physics of Complex Systems, Weizmann Institute of Science, Rehovot 7610001, Israel
\textsuperscript{2}The Racah Institute of Physics, Hebrew University of Jerusalem, Jerusalem 91904, Israel

(Dated: January 20, 2022)

We report experimental results on elastic instability in a viscoelastic channel shear flow due to only a natural non-smoothed inlet and small holes along the channel for pressure measurements. We show that non-normal mode instability results in elastic waves and chaotic flow self-organized into periodically cycled stream-wise streaks synchronized by elastic wave frequency. The chaotic flow persists above the transition with increasing \( Wi \) further into elastic turbulence and drag reduction regimes. Thus, we resolve the recent puzzle whether strong prearranged perturbations are necessary to get an elastic instability in parallel shear viscoelastic flow. Moreover, flow resistance, velocity spectra decay, and elastic wave speed reveal the same scaling with \( Wi \) as obtained in the case of strong disturbances. The remarkable result is that the all scaling behavior and streaks are found in the entire channel with small attenuation, in a sharp contrast to the flow with strong prearranged perturbations.

In parallel shear Newtonian flows, turbulence emerges at the high Reynolds numbers (\( Re \)) due to inertial stresses, whereas for low \( Re \), they are dominated by viscous dissipation and remain laminar. In contrast, in viscoelastic fluid flows with curved streamlines, at \( Re \ll 1 \) and the high Weissenberg number, \( Wi \), a linear elastic instability due to a single fastest growing mode results in a single most stable mode, which leads to elastic turbulence (ET) at vanishing inertia, as \( Wi \) is increased further. ET is an inertia-less, chaotic flow driven purely by elastic stress generated by polymers stretched by the flow. The latter is modified by a feedback reaction of the elastic stress generated by polymers stretched along the curved streamlines, engenders a force towards the curvature center, which trig-...
shear viscoelastic flows, through a direct transition to a chaotic states with a large number of the excited modes observed experimentally in both parallel shear viscoelastic flows inconsistent with a normal mode instability [19–21].

Recently, a non-modal instability in a parallel shear channel flow is confirmed in a straight channel with strong perturbations at the inlet [23] [24], as suggested in Ref. 11 [14]. Furthermore, elastic waves and chaotic flow, observed only in a part of the channel, adjacent to the perturbation source, are self-organized into regularly cycled stream-wise vortices and streaks synchronized by elastic wave frequency in all three flows regimes [23]. Throughout the rest of a channel flow, elastic waves and CSs decay, with normalized perturbation intensity that reduces from roughly 10% down to 4% downstream until the outlet [24].

In this Letter, we present results that answer two questions, namely: (i) Whether three different chaotic flow regimes, occurred above an elastic instability as the result of finite-size perturbations due to a non-smoothed inlet and six small holes, exhibit the same scaling of the flow properties and elastic wave speed with $W_i$, as observed in the flow with strong prearranged perturbations [23] [24]; and (ii) Either these chaotic flows are self-organized into periodically cycled CSs, stream-wise rolls and streaks synchronized by elastic wave frequency or only into stream-wise streaks, similar to Ref. 25. The key observation is that the scaling behavior in three flow regimes and streaks are found in the entire channel with small attenuation, in a sharp contrast to Refs. [23] [24].

The experiments are conducted in a straight long channel, with dimensions $500(L) \times 3.5(w) \times 0.5(h) \text{ mm}^3$, shown in Fig.1. The only main possible source of flow disturbances is the channel’s non-smoothed inlet and six holes in the top plate for pressure measurements. As the working fluid, we use the polymer solution prepared viscous aqueous solvent with 64% sucrose with dissolved high-molecule-weight Polyacrylamide (PAAm, Polysciences, $M_w = 18MDa$) at a dilute concentration of 80ppm. The solution properties are the solution density $\rho = 1320 \text{ kg/m}^3$ and the solvent and solution viscosity $\eta_s = 0.13 \text{ Pa-s}$ and $\eta = \eta_s + \eta_p = 0.17 \text{ Pa-s}$, respectively, $\eta_p/(\eta_s + \eta_p) \approx 0.3$, where $\eta_p$ is the polymer contribution into the solution viscosity, and the longest polymer relaxation time $\lambda = 13 \text{ s}$ using the stress relaxation method [24].

The fluid in the channel is driven by $N_2$ gas by pressurized up to 100 psi. During experiments, we use a PC-interfaced balance (BPS-1000-C2-V2, MRC) to measure the time-averaged fluid mass discharge rate at the channel exit, $Q = (\Delta m/\Delta t)$, where $m(t)$ is weight instantaneously fluid as a function of time. Then the mean velocity is calculated as $U = Q/\rho w h$ and $W_i = \lambda U/h$ and $Re = \rho U h / \eta$, which vary in the ranges (30, 6000) and (0.005, 0.9), respectively. We also measure pressure drops for flow resistance and fluctuations using high resolution differential pressure sensors of accuracy in various ranges: 5, 30, and 60 psi (HSC series, Honeywell).

We conduct measurements of the velocity field at various distances $l/h$ downstream from the inlet, using the particle image velocimetry (PIV) method. For that we illuminate small latex particles (3.2$\mu$m fluorescent tracers with concentration $\sim$0.67% w/w, Latex Microsphere, Thermo Scientific) by a laser sheet with $\sim$100$\mu$m thickness over the middle plane in the channel. Then we capture pairs of images of the tracers using high-speed (frame rate from 500 up to 8000) and high spatial resolution (up to 2048×2048 px$^2$) camera (Mini WX100, Photron FASTCAM). The OpenPIV software [27] is employed to analyze $u(x, z, t)$ and $w(x, z, t)$ in 2D $x$-$z$ plane. We typically record data for periods of $\sim O(15)$ minutes or $\sim O(50\lambda)$ for each $Wi$ to obtain sufficient statistics.

In Fig.2(a) we present the measurements of the frictional drag as a function of $Wi$ in high resolution presentation, where the friction factor, $f/f_{\text{lam}}$, is calculated as $f = 2D_h\Delta P/\rho u^2 h L_p$ and normalized by the laminar one $f_{\text{lam}} \sim Re^{-1}$. Here $D_h = 2wh/(w + h) = 0.875 \text{ mm}$ is the hydraulic length, $L_p = 270 \text{ mm}$ is the distance between two pressure measurement locations, and $\Delta P$ is the pressure difference on the length $L_p$. In Fig.2(b), we also measure pressure and velocity fluctuations to additionally characterize $Wi_c$ and different flow regimes.

Four flow regimes are identified by different dependencies of $f/f_{\text{lam}}$ on $Wi$ (Fig.2(a)). In a laminar flow the friction factor is independent on $Wi$ and equal unity up to the elastic instability onset at $Wi_c = 120 \pm 10$. First, $f/f_{\text{lam}}$ enhances with $Wi$ as a power-law with an exponent 0.10 $\pm$ 0.03, followed by ET where it grows slower with the exponent 0.06 $\pm$ 0.02. And finally in drag reduction (DR) regime, the friction factor decays with the exponent $-0.15 \pm 0.05$. Thus, the dependence of $f/f_{\text{lam}}$ on $Wi$ reveals four flow regimes separated by three grey dashed lines in Fig.2.

Pressure fluctuations are normalized by pressure fluctuations in laminar flow, as $P_{\text{rms}}/P_{\text{rms, lam}} - 1$, as well as velocity fluctuations reduced by velocity fluctuations at laminar regime normalized by mean flow velocity, as $(U_{\text{rms}} - U_{\text{r ms, lam}})/U$ (Fig.2(b)). The velocity fluctuations

FIG. 1. Schematics of experimental setup. Four instead of six holes are drawn for simplicity.
The velocity for each moment is measured at channel center. (b) Elastic wave speed ($c_{el}$) versus $Wi$ at $l/h=28$ and $80$ in lin-lin scales. The inset presents a peak of elastic wave in span-wise velocity energy spectrum ($E_{w}$) at $Wi = 2034$ at $l/h=330$ in lin-log scales.

Fig. 3(b) plots $c_{el}$ vs $Wi$ in lin-log scales obtained by cross-correlation of velocity along stream-wise direction. By fitting the curve with $c_{el} = A \times (Wi - Wi_c)^{\delta}$, we obtain $A = 4.5 \pm 0.5 \times 10^{-3}$ m/s and $\delta = 0.72 \pm 0.02$, which are the same as found in earlier flows: flows between two widely separated obstacles [31], and a straight channel with an array of obstacles at the inlet to excite strong perturbations [23][24]. On the other hand, span-wise propagating elastic waves discovered in a straight channel with very weak perturbations generated by a small cavity in the top plate exhibit the coefficient $A$ of about three orders of magnitude less and the same scaling exponent that maybe attributed to the strong span-wise confinement [25]. This suggests that the stream-wise propagating elastic wave scaling relation with $Wi - Wi_c$ could be universal.

To gain better insights into coherent states, we broaden the PIV window to the whole channel and at the stream-wise length $\Delta l/h = 4$ to examine probable coherent states in this channel flow. As illustrated in Fig. 4 from left to right in a single elastic wave cycle, at the beginning streaks first are self-organized with slightly perturbed interface that further is perturbed more and finally destroyed leading to a random flow at the end of the cycle. Then the next cycle starts. The streak dynamics is synchronized by elastic waves period. To prove experimentally the periodicity of a cycled self-sustained process with the period of the elastic waves we utilize the approach developed by us early to study the interface dynamics [23][24] by examining a temporal evolution of $\Delta u' = u'_2 - u'_1$, the velocity fluctuation difference at two specific points across the interface, as shown in Fig. 5.
FIG. 4. Streak interface dynamics at $l/h=330$ and $Wi=2030$ in ET during a single cycle. The time-averaged stream-wise velocity profile is subtracted from the instant stream-wise velocity and normalized by time-averaged velocity $u'/U$. The black dashed line separates the low profile (blue) and high profile (red) streak interface. The moment $t^*=t_{fe}$ of each image from left to right is 0.20, 0.38, 0.65, and 0.92. The interfaces of streaks are gradually interfered and break into random at the end of the cycle without any secondary instability.

During each cycle, $Au'$ displays a non-monotonic temporal variation with the same peak values $Au'_{\text{max}}$ and cycle period. It should be pointed out that no secondary instability of the streaks such as the Kelvin-Helmholtz-like elastic instability is observed, which we attribute to lower intensity of the elastic waves compared to that found in Ref. [24].

To conclude, first of all, the main result of this study is resolving the recent puzzle whether strong prearranged perturbations are necessary to get an elastic instability in parallel shear visco-elastic flows. Indeed, we investigate in a straight planar channel visco-elastic flow the elastically induced instability. In contrast to previously published works, we remove obstacles in the channel, while the non-smoothed inlet and six holes in the top plate along the channel provide perturbations that lead to the intrinsic non-modal instability at $Wi_c=120$.

We find and study four flow regimes: laminar, transitional, ET, and DR, which are characterized by measuring the friction factor, pressure and velocity fluctuations, and stream-wise velocity power spectra scaling. The velocity of stream-wise-propagating elastic wave shows the universal scaling relation with $Wi-Wi_c$. Using the approach to quantify a cycling period of streak dynamics [23, 24], we show that the period of the streak cycling dynamics is equal to the elastic wave period.

The remarkable and surprising finding is the existence of the streaks and elastic waves over the entire channel length up to $l/h=980$, in a sharp contrast with the case of the strong prearranged perturbations at the inlet [23, 24], where the CSs and elastic waves retain only in the short range of the channel from $l/h=36$ up to $l/h\approx 200$. Such distinct difference between two types of external perturbations may be explained by the following arguments. Finite-size perturbations are necessary to excite an intrinsic elastic instability in a channel flow. The presented above results of various measurements show that the instability occurs due to non-normal mode bifurcation, which sensitively depends on initial conditions [15] including disturbances. It is revealed in different CSs observed in two cases: only streaks for small perturbations versus rolls and streaks for strong ones. Moreover, in the current case, besides the perturbations from inlet generated stream-wise elastic waves, six holes along the channel further support the elastic waves by pumping energy. It is clearly characterized by measurements of their attenuation along the channel, of which details
will be published elsewhere. On other hand, the current case has a reduced intensity of elastic waves that results in the absence of the secondary instability in streaks, namely the Kelvin-Helmholtz-like elastic instability discovered recently in a channel viscoelastic flow with strong perturbations [21].

We are grateful to Guy Han and Rostyslav Baron for their help with the experimental setup. This work was partially supported by grant from the Israel Scientific Foundation (ISF, grant #784/19.)

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