Sign-tunable anomalous Hall effect induced by two-dimensional symmetry-protected nodal structures in ferromagnetic perovskite thin films

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Magnetism and spin–orbit coupling are two quintessential ingredients underlying topological transport phenomena in itinerant ferromagnets. When spin-polarized bands support nodal points/lines with band degeneracy that can be lifted by spin–orbit coupling, the nodal structures become a source of Berry curvature, leading to a large anomalous Hall effect. However, two-dimensional systems can possess stable nodal structures only when proper crystalline symmetry exists. Here we show that two-dimensional spin-polarized band structures of perovskite oxides generally support symmetry-protected nodal lines and points that govern both the sign and the magnitude of the anomalous Hall effect. To demonstrate this, we performed angle-resolved photoemission studies of ultrathin films of SrRuO3, a representative metallic ferromagnet with spin–orbit coupling. We show that the sign-changing anomalous Hall effect upon variation in the film thickness, magnetization and chemical potential can be well explained by theoretical models. Our work may facilitate new switchable devices based on ferromagnetic ultrathin films.

The interplay between magnetism and spin–orbit coupling (SOC) underlies the various topological transport phenomena seen in metallic ferromagnets 1–3. In three-dimensional (3D) ferromagnets, spin-polarized bands often accompany nodal points or nodal lines (NLs) 4–6. The lifting of the band degeneracy at a nodal structure due to SOC induces enhanced Berry curvature around it, which leads to various topological transport phenomena such as the anomalous Hall effect (AHE) 3–5. For instance, in the SrRuO3 (SRO) bulk, theoretical predictions have indicated that nodal structures with an SOC-induced gap play a role as magnetic monopoles in momentum space, which leads to a non-monotonic change in the AHE 6–8.

The 3D SRO bulk has been considered as a canonical system, in which the topological band structure induces a large AHE 6–9. However, the existence of magnetic monopoles in SRO has not yet been verified experimentally. Part of the reason for this is the lack of a suitable single crystal, as well as the difficulty in preparing clean, cleaved surfaces for angle-resolved photoemission spectroscopy (ARPES) study, due to its 3D structure. Meanwhile, it has recently been shown that the NLs of 3D spin-polarized bands can also induce a large AHE, as seen in a layered Fe3GeTe2 ferromagnet 8. The AHE driven by nodal points (so-called Weyl points) has also been reported in various 3D magnetic metals 2,9–12.

The relationship between the topological band structure and corresponding transport phenomena remains largely unexplored in two-dimensional (2D) metallic ferromagnets 1. The nodal structures in 2D ferromagnets are more fragile and unstable compared with 3D structures. Only when proper symmetry conditions are satisfied can nodal points or NLs manifest as symmetry-protected band degeneracy in 2D bands 13,14. Moreover, the increased correlations expected in the reduced dimensionality could alter both the quasi-particle energy spectrum and intrinsic AHE such that careful comparison between theoretical and experimental results is indispensable to verify the topological band structure in 2D magnets 15,16.

In this work, we demonstrate the relation between the symmetry-protected nodal structures of 2D spin-polarized bands and the AHE. For this purpose, we performed ARPES measurements on SRO ultrathin films, revealing their band structures. Based on tight-binding models, first-principles calculations and symmetry analysis, we propose that the spin-polarized bands derived from t2g orbitals in layered perovskite oxides generally support (1) nodal points with quadratic band crossing (QBC) protected by four-fold rotation symmetry and (2) NLs arising from the crossing between majority and minority spin bands. When SOC is included, these nodal points and NLs are gapped and generate a large Berry curvature in the surrounding area. Because the energies of nodal points and NLs are different, when the Berry curvatures that arise from them have opposite signs, the magnitude and sign of the AHE can generally be varied by changing the Fermi level. We demonstrate that the AHE of SRO thin films exhibits sign reversal, depending on the film thickness, temperature, magnetization and chemical potential, due to the symmetry-protected nodal structures of the 2D spin-polarized bands.
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Fig. 1 | FS of 2D ferromagnetic perovskites. a, Structures of layered perovskite oxides with the chemical formula An-xBnxO3n+1, where n denotes a natural number. As n increases, the electronic structure changes from 2D to 3D. Green, grey and red atoms represent A, B and O, respectively. b, Schematic FSs of SRO in the 2D limit, consisting of three bands (α, β and γ). When magnetization (M) and SOC are absent, all bands are spin degenerate. When M is finite, spin degeneracy is lifted, such that major (red) and minority (blue) bands form. On initiating SOC, the FSs become hybridized, inducing a finite Berry curvature.

As the 2D SrRuO3 is non-magnetic, it is not an appropriate system for our study. Therefore, as an alternative, we studied the quasi-2D limit of SRO by growing ultrathin films of SRO with a thickness of four unit cells (u.c.). Due to its quasi-2D nature, the Fermi surfaces (FSs) of SRO ultrathin films are expected to be somewhat similar to those of SrRuO3 (ref. 19). We confirm this through our ARPES measurements, as will be shown later. For SrRuO3, it is well known that the FSs consist of three bands, α, β and γ, where α and β FSs are composed of dxz orbitals, while the γ FS arises from the dxy orbital, as schematically shown in Fig. 1b (refs. 19,20). When ferromagnetism develops, spin-degenerate bands split into majority and minority bands, which results in six bands derived from t2g orbitals appearing at the Fermi level. With SOC, the majority and minority bands are hybridized at the points where they cross. This provides the general idea on how Berry curvature is generated in ferromagnetic 2D perovskites.

ARPES results

To experimentally verify the band structure of a 2D ferromagnetic perovskite, we performed ARPES measurements on SRO ultrathin films. Figure 2a shows a FS map as well as the energy–momentum (E–k) spectra of a 4 u.c. SRO thin film measured with 80 eV light with linear horizontal (LH) polarization. The α and β FSs are clearly visible and are similar to the corresponding FSs reported for SrRuO3 (refs. 19–22). Due to the rotational distortion of RuO6 octahedra in SRO ultrathin films grown on SrTiO3 (001) substrates23,24, the FS from the folded β band (β′) is also observed.

Whether ultrathin films indeed possess 2D electronic structures requires an examination of the kγ (that is, photon energy) dependence. The measured kγ dispersion is shown in Fig. 2b. Here kγ is calculated based on an inner potential of 14 eV obtained from experiments on SRO single crystals (Oh, J. S. et al., manuscript in preparation). Unlike 3D materials exhibiting kγ dispersion, the β band does not show any kγ dispersion as expected. This clearly illustrates that SRO ultrathin films have quasi-2D band structures, similar to SrRuO3 (refs. 20,21) and SrRuO3 (ref. 25).

In Fig. 2c, we reproduce the measured FSs. In addition to the α and β pockets shown by the dotted lines, there are more detailed structures. Superimposed on the map as thick lines are results obtained from an effective 2D tight-binding model fit (Methods for details). It is seen that the effective 2D model can explain the major features of the experimental dispersions. We also performed circular dichroism ARPES (CD-ARPES) to examine the Berry curvature, and the result is shown in the bottom-right corner of the figure (Supplementary Information for analysis of the CD-ARPES data). Moving from Γ to M, the intensity of CD data varies from zero to negative, and then to positive. A clear sign-changing behaviour is observed as a function of the electron momentum. Detailed discussions on the tight-binding fit and CD-ARPES results will be discussed later with Fig. 5.

Since ARPES measurement is sensitive to the surface condi-

tion, clearer band features can be resolved via in situ measurements. Figure 2d shows a FS map of the 4 u.c. SRO thin film in situ measured with linear vertical (LV) polarized He-I light (21.2 eV). Spin polarization of the bands was also obtained by performing spin-resolved ARPES (SARPES) measurements below and above the Curie temperature, Tc ≈ 110 K. Figure 2e shows the spin-resolved energy distribution curves and spin polarizations at the M point at 10 K (below Tc) and 125 K (above Tc). The energy distribution curves below Tc show a considerable difference between the majority and minority spins, whereas no difference was observed above Tc. The difference is also presented in the form of spin polarization, P = (I↑ − I↓)/(I↑ + I↓), where I↑ (I↓) is the intensity of the majority (minority) spin measured with SARPES, in the lower panels. Spin-resolved energy distribution curves along the high-symmetry lines (Γ–M and Γ–X) can be found in the Supplementary Information. These observations are consistent with itinerant ferromagnetism26,27.

The high-resolution FS map in Fig. 2d shows more detailed features in comparison with the result from ex situ films in Fig. 2a. An important feature visible in the high-resolution data is heavy bands observed at the Γ and M points. To investigate those heavy bands, the Γ–M cut of the data in Fig. 2d and its momentum distribution curve at the Fermi level are shown in Fig. 2f. To see the dispersion more clearly, we plot the 2D curvature28 of the data near the M point in the inset. Two hole-like bands are observed at the M point: one from the α band and the other, unknown and labelled as A, located within the α pocket. The latter does not exist in the case of SrRuO3. We fitted the momentum distribution curve with Lorentzian peaks as shown in the upper panel of Fig. 2f. Near the M point, three peaks are observed: two from the α band and one from the A band. We also find that the heavy bands near the Γ point are α and A replica bands due to the √2 × √2 rotational distortion of RuO6 octahedra26,27 (Supplementary Information for low-energy electron diffraction patterns).

Transport and magnetic properties

With the full experimental electronic structure identified, we turn our attention to the transport and magnetic properties of SRO ultrathin films, presented in Fig. 3. Figure 3a shows the

Image 80x529 to 293x705

Fig. 1 | FS of 2D ferromagnetic perovskites. a, Structures of layered Ruddlesden–Popper perovskite oxides with the chemical formula An-xBnxO3n+1, where n denotes a natural number. As n increases, the material takes on a more 3D character, interpolating between the 2D limit with n = 1 (A2BO4) and the 3D limit with n = ∞ (ABO3). Among the perovskite materials, SRO is a representative example of 3D ferromagnetic metals, with an AHE that has been attributed to the magnetic monopoles in momentum space of the topological band structure10.

Whether ultrathin films indeed possess 2D electronic structures requires an examination of the kγ (that is, photon energy) dependence. The measured kγ dispersion is shown in Fig. 2b. Here kγ is calculated based on an inner potential of 14 eV obtained from experiments on SRO single crystals (Oh, J. S. et al., manuscript in preparation). Unlike 3D materials exhibiting kγ dispersion, the β band does not show any kγ dispersion as expected. This clearly illustrates that SRO ultrathin films have quasi-2D band structures, similar to SrRuO3 (refs. 20,21) and SrRuO3 (ref. 25).

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Transport and magnetic properties

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thickness-dependent Hall resistivity measured at 10 K for 3.8–5.0 u.c. SRO ultrathin films under an out-of-plane magnetic field. Here, the thickness of the thin film was estimated from the corresponding in situ reflection high-energy electron diffraction intensity plot (Supplementary Information for details). In general, the ordinary Hall effect (OHE), AHE and the hump-like features that are observed near the coercive field can contribute to the Hall resistivity \( \rho_{xy} \), which is thus given by \( \rho_{xy} = \rho_{\text{OHE}} + \rho_{\text{AHE}} + \rho_{\text{hump}} \) (refs. 24,29,30). The OHE term, which is proportional to an applied magnetic field, was subtracted from all Hall resistivity data presented in this paper. As \( \rho_{\text{hump}} \) is non-zero only near the coercive field, \( \rho_{\text{AHE}} \) can be determined from the saturated \( \rho_{xy} \) in the high-field limit.

The thickness dependence of \( \rho_{\text{AHE}} \) is shown in Fig. 3b. Here, \( \rho_{\text{AHE}} \) decreased as the thickness decreased and eventually took on a negative value with a sign change in between. We also performed intrinsic liquid gating of SRO ultrathin films to investigate how the change in the chemical potential affects \( \rho_{xy} \). Figure 3c shows the results of intrinsic gating experiments for 4 and 5 u.c. SRO thin films measured at 10 and 50 K, respectively. In both cases, the magnitude of \( \rho_{xy} \) changed substantially with the gate voltage. Figure 3d shows the out-of-plane magnetization of 4, 5 and 7 u.c. SRO thin films. Combining the thickness- and temperature-dependent Hall effect results with the magnetization (\( M \)) data, we can derive \( \rho_{\text{AHE}} \) versus \( M \) data, as plotted in Fig. 3e (Supplementary Information for details). It should be noted that the data points fall on a single line, implying that \( M \) may be the key parameter for \( \rho_{\text{AHE}} \). The sign reversal of \( \rho_{\text{AHE}} \) can also be clearly observed as the magnetization approaches 0.5 \( \mu_B \) per Ru, where \( \mu_B \) is the Bohr magneton.

The 2D symmetry-protected nodal structures
It is worth noting that the sign-switching behaviour of \( \rho_{\text{AHE}} \) is highly unusual\(^9\). To understand the origin of the unusual AHE,
we conducted a tight-binding model analysis combined with first-principles calculations. Figure 4a shows the band structure in the effective six-band tight-binding model (relevant to a 1 u.c. SRO), the parameters of which were adjusted to describe the FS in Fig. 2c with a magnetization $M=0.33 \mu_B$ per Ru. With magnetization and SOC, several bands are hybridized near the points at which they are crossed. We find that two different origins of symmetry-protected nodal structures, which we call a NL (white circle) and QBC (black crossed), are formed at the crossing points. The NLs and QBCs of spin-polarized bands, respectively, become the sources of the large nodal structures, which we call a NL (white circle) and QBC (black crossed). We find that two different origins of symmetry-protected SOC, several bands are hybridized near the points at which they are crossed. With magnetization and SOC, several bands are hybridized near the points at which they are crossed. We find that two different origins of symmetry-protected nodal structures, which we call a NL (white circle) and QBC (black crossed), are formed at the crossing points. The NLs and QBCs of spin-polarized bands, respectively, become the sources of the large Berry curvature when they are gapped due to SOC. We note that the lowest lying QBC band at the M point is the $A$ band that we focus on, these nodal structures are gapped and Berry curvature is thus generated, which leads to the AHE (Methods for details).

**Sign-changing AHE**

Multiple Berry curvature sources appearing near the Fermi level can induce a fluctuating Berry curvature distribution with alternating signs. The upper half of Fig. 5a shows that QBCs formed by minority bands induce negative Berry curvature while the others generate positive Berry curvature. For better visualization, we plot the Berry curvature distribution of a 1 u.c. SRO in Fig. 5b. The resulting Berry curvature distribution is similar to a previously reported one.

Since the sign-alternating Berry curvature distribution is generated by the topological band crossings, it would be desirable to experimentally show the Berry curvature behaviour in the momentum space. During the past decade, the connection between Berry curvature and orbital angular momentum (OAM) has been firmly established. In addition, measurement of OAM (and thus Berry curvature) with CD-ARPES has been well studied. In fact, CD-ARPES has been used to study chiral structures of pseudo-spins of topological bands. Therefore, the CD-ARPES in Fig. 2c: may be compared with the calculated OAM of our 1 u.c. tight-binding model. The upper half of Fig. 5c shows a clear match between the CD-ARPES data and OAM distribution. Positive CD intensity and OAM are observed near the M point, but they turn negative away from the M point. Eventually, both experimental and theoretical results turn to zero close to the $\Gamma$ point. Interestingly, we found that both the calculated OAM and Berry curvature flip their signs along the $\Gamma$–M high-symmetry line due to the intrinsic nature of QBC (Methods for the relationship between OAM and Berry curvature).

To demonstrate that the QBCs and NLs are present, and also that the associated sign-switching of the AHE signal is a generic property

**Fig. 3 | Non-monotonous AHE in SRO ultrathin films.**

a. Thickness-dependent $\rho_{xy}$ of SRO ultrathin films at 10 K as a function of magnetic field ($H$); $\rho_{xy}$ is defined from the saturated Hall resistivity $\rho_{H}$; $\rho_{xy}$ is the vacuum permeability. b. The $\rho_{xy}$ versus thickness curve for SRO ultrathin films at 10 K. The sign of $\rho_{xy}$ changes as the film thickness varies. c. The $\rho_{xy}$ of 4 and 5 u.c. SRO thin films with ionic liquid gating at 10 and 50 K, respectively. The sign and magnitude of $\rho_{xy}$ change with the bias voltage. d. Out-of-plane magnetization of 4, 5 and 7 u.c. SRO thin films. e. The $\rho_{xy}$ versus $M$ plot for 4, 5 and 7 u.c. SRO thin films at various temperatures. The sign of $\rho_{xy}$ changes near 0.5 $\mu_B$ per Ru.
of SRO ultrathin films independent of film thickness, we also constructed a tight-binding model for 4 u.c. SRO and compared it with the effective six-band model. The hopping parameters for the 4 u.c. tight-binding Hamiltonian were extracted from first-principles calculations and adjusted further by fitting the experimentally observed FSs. For realistic calculations, the atomic positions of 4 u.c. SRO were obtained from the experiment using coherent Bragg rod analysis. We believe that these results can support that the sign-changing AHE comes from competition between Berry curvature sources from multiple topological features as magnetization and chemical potential change. In particular, we found that as the chemical potential increases, the positive Berry curvatures from the NL and majority band QBC dominate the negative Berry curvature from the minority band QBC, which eventually leads to the sign reversal of the AHC.

Interestingly, the AHC is not proportional to the net magnetization, and the sign of the AHC can change with a small variation of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. The generic evolution pattern of magnetic monopoles in 3D SRO. 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Because the topological nodal structures (QBCs and NLs) are protected only by symmetries, similar nodal structures can appear in any 2D systems sharing the same space group symmetry as SRO. To confirm this idea, we also performed first-principles calculations on monolayer SrCoO$_3$, another ferromagnetic perovskite oxide, in which the sign-switching AHE was recently observed in thin films$^{41}$. Interestingly, the NLs and QBCs were also located close to the Fermi energy and generated Berry curvatures with opposite signs, as in SRO (Supplementary Information for details).

Let us briefly mention the stability of NLs and QBCs under rotation or tilting of oxygen octahedra. NLs are stable against rotation or tilting as they do not affect spins when SOC is absent. QBCs are stable against rotation distortion that preserves $C_3$ rotation. Although the tilting breaks the $C_3$ symmetry, the topological properties of QBCs should still remain intact as long as the SOC-induced gap is larger than the gap caused by tilting. We believe that a similar mechanism may be applicable to understanding the mysterious ferromagnetism in other ferromagnetic perovskite oxides and emergent interfacial ferromagnetism.

**Outlook**

In conclusion, by combining ARPES, transport measurements and theoretical analysis, we demonstrated the topological band structure of ferromagnetic SRO thin films. In particular, the band structures of a SRO film in the ultrathin 2D limit were observed and defined. Through theoretical analysis, we also showed that the spin-polarized bands of 2D ferromagnets generally possess nodal points and NLs that become the source of enhanced Berry curvature. Comparing the measured band structure with the tight-binding model, we identified the Berry curvature hot spots originating from multi-nodal structures, which led to the unconventional AHE. Competing contributions from different Berry curvature hot spots induce a sign-changing AHE, which can be controlled by varying the film thickness, temperature, magnetization and chemical potential. We believe that our findings will open up new avenues for investigating transport phenomena driven by symmetry-protected nodal structures of 2D magnetic systems, and facilitate the development of magnetic devices based on the engineering of magnetic topological band structures.

**Online content**

Any methods, additional references, Nature Research reporting summaries, source data, extended data, supplementary information, acknowledgements, peer review information; details of author contributions and competing interests; and statements of data and code availability are available at https://doi.org/10.1038/s41563-021-01101-4.
References

1. Burkov, A. A. Anomalous Hall effect in Weyl metals. Phys. Rev. Lett. 113, 187202 (2014).
2. Ye, L. et al. Massive Dirac fermions in a ferromagnetic kagome metal. Nature 555, 638–642 (2018).
3. Groenendijk, D. J. et al. Berry phase engineering at oxide interfaces. Phys. Rev. Res. 2, 023404 (2020).
4. Chang, G. et al. Room-temperature magnetic topological Weyl fermion and nodal line semimetal states in half-metallic Heusler Co2TiX (X = Si, Ge, or Sn). Sci. Rep. 6, 38839 (2016).
5. Chang, G. et al. Magnetic and noncentrosymmetric Weyl fermion semimetals in the RAlGe family of compounds (R = rare earth). Phys. Rev. B 97, 041104 (2018).
6. Kim, K. et al. Large anomalous Hall current induced by topological nodal lines in a ferromagnetic van der Waals semimetal. Nat. Mater. 17, 794–799 (2018).
7. Nagaosa, N., Sinova, J., Onoda, S., MacDonald, A. H. & Ong, N. P. Anomalous Hall effect. Rev. Mod. Phys. 82, 1539–1592 (2010).
8. Zeng, C., Yao, Y., Niu, Q. & Wetterling, H. H. Linear magnetization dependence of the intrinsic anomalous Hall effect. Phys. Rev. Lett. 96, 037204 (2006).
9. Wang, Q. et al. Large intrinsic anomalous Hall effect in half-metallic ferromagnet Co2Sn2S2 with magnetic Weyl fermions. Nat. Commun. 9, 3681 (2018).
10. Fang, Z. et al. The anomalous Hall effect and magnetic monopoles in momentum space. Science 320, 92–95 (2003).
11. Chen, Y., Bergman, D. & Burkow, A. Weyl fermions and the anomalous Hall effect in metallic magnetars. Phys. Rev. B 88, 125110 (2013).
12. Vazifeh, M. M. & Franz, M. Electromagnetic response of Weyl semimetals. Phys. Rev. Lett. 111, 027201 (2013).
13. Zyuzin, A. A. & Tiwari, R. P. Intrinsic anomalous Hall effect in type-II Weyl semimetals. JETP Lett. 103, 717–722 (2016).
14. Young, S. M. & Kane, C. L. Dirac semimetals in two dimensions. Phys. Rev. Lett. 115, 126803 (2015).
15. Niu, C. et al. Two-dimensional topological nodal line semimetal in layered X3Y (X = Ca, Sr, and Ba; Y = As, Sb, and Bi). Phys. Rev. B 95, 235138 (2017).
16. Zhang, H., Huang, H., Haule, K. & Vanderbilt, D. Quantum anomalous Hall phase in (001) double-perovskite monolayers via intersite spin-orbit coupling. Phys. Rev. B 90, 165143 (2014).
17. Wan, X., Turner, A. M., Vishwanath, A. & Savrasov, S. Y. Topological semimetal and Fermi-arc surface states in the electronic structure of pyrochlore iridates. Phys. Rev. B 83, 205101 (2011).
18. Neumeier, J. et al. Magnetic, thermal, transport, and structural properties of SrRuO3−δ enhanced charge-carrier mass in a nearly metallic oxide. Phys. Rev. B 50, 17910 (1994).
19. Puchkov, A., Shen, Z.-X., Kimura, T. & Tokura, Y. ARPES results on SrRuO3 Fermi surface revisited. Phys. Rev. B 58, R13322 (1998).
20. Damascelli, A. et al. Fermi surface, surface states, and surface reconstruction in SrRuO3. Phys. Rev. Lett. 85, 5194–5197 (2000).
21. Mackenzie, A. P. et al. Quantum oscillations in the layered superconductor Sr2RuO4. Phys. Rev. Lett. 76, 3786–3789 (1996).
22. Mackenzie, A. P. et al. The Fermi surface topography of Sr2RuO4. J. Phys. Soc. Jpn 67, 385–388 (1998).
23. Chang, S. H. et al. Thickness-dependent structural phase transition of strained SrRuO3 ultrathin films: the role of octahedral tilt. Phys. Rev. B 84, 104101 (2011).
24. Sohn, B. et al. Stable humplike Hall effect and noncoplanar spin textures in SrRuO3 ultrathin films. Phys. Rev. B 83, 032322 (2011).
25. Singh, D. & Mazin, I. Electronic structure and magnetism of Sr3Ru2O7. Phys. Rev. B 63, 165101 (2001).
26. Chang, Y. J. et al. Fundamental thickness limit of itinerant ferromagnetic SrRuO3. Phys. Rev. Lett. 103, 057201 (2009).
27. Jeong, D. W. et al. Temperature evolution of itinerant ferromagnetism in Sr2RuO4 probed by optical spectroscopy. Phys. Rev. Lett. 110, 247202 (2013).
28. Zhang, P. et al. A precise method for visualizing dispersive features in image plots. Rev. Sci. Instrum. 82, 043712 (2011).
29. Matsuno, J. et al. Interface-driven topological Hall effect in Sr2RuO4-SrRuO3 bilayer. Sci. Adv. 2, e1600304 (2016).
30. Sohn, B. et al. Hump-like structure in Hall signal from ultra-thin SrRuO3 films without inhomogeneous anomalous Hall effect. Curr. Appl. Phys. 20, 186–190 (2020).
31. Jin, L. et al. Two-dimensional Weyl nodal-line semimetal in a d4 ferromagnetic K3N monolayer with a high Curie temperature. Phys. Rev. B 102, 125118 (2020).
32. Jin, L. et al. Ferromagnetic two-dimensional metal-chlorides MCl (M = Sc, Y, and La); candidates for Weyl nodal line semimetals with small spin-orbit coupling gaps. Appl. Surf. Sci. 520, 146376 (2020).
33. Sun, K., Yao, H., Fradkin, E. & Kivelson, S. A. Topological insulators and nematic phases from spontaneous symmetry breaking in 2D Fermi systems with a quadratic band crossing. Phys. Rev. Lett. 103, 046811 (2009).
34. Chong, Y. D., Wen, X.-G. & Soljačić, M. Effective theory of quadratic degeneracies. Phys. Rev. B 77, 235125 (2008).
35. Xiao, D., Yao, W. & Niu, Q. Valley-contrastnig physics in graphene: magnetic moment and topological transport. Phys. Rev. Lett. 99, 236809 (2007).
36. Go, D., Jo, D., Kim, C. & Lee, H.-W. Intrinsic spin and orbital Hall effects from orbital texture. Phys. Rev. Lett. 121, 086602 (2018).
37. Cho, S. et al. Experimental observation of hidden Berry curvature in inversion-symmetric bulk 2H-WS2. Phys. Rev. Lett. 121, 186401 (2018).
38. Park, S. R. et al. Chiral orbital-angular momentum in the surface states of Bi2Se3. Phys. Rev. Lett. 108, 046805 (2012).
39. Schüler, M. et al. Local Berry curvature signatures in dichroic angle-resolved photoelectron spectroscopy from two-dimensional materials. Sci. Adv. 6, eaay2730 (2020).
40. Schultz, M., Levy, S., Reiner, J. W. & Klein, L. Magnetic and transport properties of epitaxial films of SrRuO3 in the ultrathin limit. Phys. Rev. B 79, 125444 (2009).
41. Mathieu, R. et al. Scaling of the anomalous Hall effect in Sr1−xCa,xRuO3. Phys. Rev. Lett. 93, 016002 (2004).
42. Zhang, D. et al. Origin of the anomalous Hall effect in Sr2CoO3 thin films. Phys. Rev. B 100, 060403 (2019).

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Methods

SRO thin-film fabrication. SRO ultrathin films were grown on TiO₂-terminated SrTiO₃ (STO) (001) single-crystal substrates using pulsed laser deposition. TiO₂-terminated STO substrates from Shinkosha were used for SRO thin-film growth. To dissolve the Sr compounds that can form on the surface of STO substrates, the SRO substrates were prepared by deionized water etching. The deionized water-treated STO substrates were preannealed in situ at 1,070 °C for 30 min with an oxygen partial pressure (PO₂) of 5 × 10⁻⁴ torr. We deposited an epitaxial SRO thin film in an oxygen partial pressure of PO₂ = 100 mtorr; the growth temperature of the STO substrate was 798 °C. A KrF excimer laser (wavelength, 248 nm) irradiated a stoichiometric SRO target with a fluence of 1.5 layers of SrCoO₃ (including an additional SrO layer at the surface) with a vacuum of 10 Å in which the experimental atomic positions of bulk SrCoO₃ were used49. The space group of the atomic structure including the vacuum was the matrix \( M \times N \), \( M \) is the number of the layers, \( N \) is the number of the unit cell. The SLATER–Koster method, the tight-binding Hamiltonian of ferromagnetic monolayer SRO is constructed as follows:

\[
H = \sum_k \left( \epsilon^{\text{soc}}_k \sigma_\uparrow \sigma_\downarrow \delta_{\sigma\sigma'} + f^{\text{soc}}_k \delta_{\sigma\sigma'} + \delta_{\sigma\sigma'} \delta_{\uparrow\downarrow} \right) \hat{d}^{\dagger \sigma}_{k\uparrow} \hat{d}^{\sigma}_{k\downarrow},
\]

where \( \epsilon^{\text{soc}}_k \) is the onsite energy; \( t_1 \) and \( t_2 \) are the amplitudes for nearest-neighbour interactions; \( t_1 \) and \( t_2 \) are the amplitudes for next-nearest-neighbour interactions; \( \delta_{\sigma\sigma'} \) denotes the amplitude of SOC; \( m \) is the amplitude of the Zeeman interaction; \( \sigma^\dagger \) and \( \sigma^\downarrow \) are the Pauli matrix. For example, \( \sigma^\dagger = \pm 1 \) if \( m = \pm 1 \). We fit tight-binding parameters with ARPES data to describe the AHC quantitatively (Table 1).

With the Hamiltonian, we can directly calculate the Berry curvature of each energy band \( \Omega(k) \) by applying the following formula:

\[
\Omega_k(k) = \sum_{\sigma \neq \sigma'} \left( \epsilon_{\sigma k} - \epsilon_{\sigma' k} \right) ^2 \nabla \epsilon_{\sigma k},
\]

where \( \epsilon_{\sigma k} \) is the energy of the \( \sigma \)th band represented by \( \sigma \) in the AHC \( \sigma \) is given by integrating the Berry curvature over the Brillouin zone below the Fermi energy:

\[
\sigma_{\text{ij}} = \frac{1}{\pi} \sum_{k} \int d\epsilon \epsilon \nabla \epsilon \Omega_k(k) \delta \left( \epsilon_{\uparrow k} - E_F \right),
\]

where \( \delta \) is step function that is 0 (1) when \( \epsilon_{\uparrow k} > (\leq) E_F \). Also, \( c = (R_0 \times \hbar)^{-1} \), where \( R_0 \) is the Hall resistance and \( \hbar \) is the thickness of the SRO thin film.

Tight-binding model for 4 u.c. SRO. We construct the 4 u.c. SRO tight-binding model where the parameters are extracted from the DFT calculation and adjusted further by fitting the experimentally observed FSs. The DFT calculation is performed based on the structure of 4 u.c. SRO ultrathin films, where the atomic structure information is obtained by coherent Bragg rod analysis that determines the layer-by-layer atomic positions of SRO ultrathin films. In the tight-binding Hamiltonian, only \( t_{\text{ij}} \) orbitals are considered since they account for most of the density of states near the Fermi level. Also, the tiny lattice rotation is neglected and the tetragonal structure is assumed. The Hamiltonian \( \tilde{H} = \psi \tilde{H} \psi^\dagger \) is described by the matrix \( \tilde{H} \) and the basis \( \psi \) and its complex conjugate \( \psi^\dagger \), where

\[
\psi = \left( \hat{d}_{\uparrow\uparrow}^{\dagger}, \hat{d}_{\downarrow\downarrow}^{\dagger}, \hat{d}_{\uparrow\downarrow}^{\dagger}, \hat{d}_{\downarrow\uparrow}^{\dagger} \right).
\]

Here \( E_i \) describes the inlayer part of the Hamiltonian \( E_i \rightarrow \tilde{H} \rightarrow \psi \rightarrow \psi^\dagger \), \( \pi \) denotes the index for the SRO layer. The \( \tau_{\uparrow\uparrow} \) and \( \tau_{\downarrow\downarrow} \) describe the hopping interaction between \( \tau_{\text{ij}} \) layer and \( \tau_{\text{ij}} \) layer. Let us note that \( t_{\text{ij}} = t_{\text{ij}} \), due to the Hermitian of the Hamiltonian.

The intralayer Hamiltonian \( \tilde{H}_{\text{intra}} \rightarrow \psi \rightarrow \psi^\dagger \) is written as

\[
\tilde{H}_{\text{intra}} = \sum_k \left( \epsilon^{\text{soc}}_k \right) \delta_{\sigma\sigma'} + f^{\text{soc}}_k \delta_{\sigma\sigma'} + \delta_{\sigma\sigma'} \delta_{\uparrow\downarrow} \hat{d}^{\dagger \sigma}_{k\uparrow} \hat{d}^{\sigma}_{k\downarrow},
\]

where \( \epsilon^{\text{soc}}_k \) is the onsite energy; \( t_1 \) and \( t_2 \) are the amplitudes for nearest-neighbour interactions; \( t_1 \) and \( t_2 \) are the amplitudes for next-nearest-neighbour interactions; \( \delta_{\sigma\sigma'} \) denotes the amplitude of SOC; \( m \) is the amplitude of the Zeeman interaction; \( \sigma^\dagger \) and \( \sigma^\downarrow \) are the Pauli matrix. For example, \( \sigma^\dagger = \pm 1 \) if \( m = \pm 1 \). We fit tight-binding parameters with ARPES data to describe the AHC quantitatively (Table 1).

Effective tight-binding model for monolayer SRO. In the main text, we assumed that the electronic structure of 4 u.c. SRO can be described well by effective 2D models, which are supported by \( \delta_{\text{ij}} \) dependent ARPES measurement. Following the
where \( t_{nm} \) and \( t_{n} \) are the amplitudes of the nearest-neighbour interactions and \( t_{nm} \) and \( t_{n} \) are the amplitudes of the nearest-neighbour interactions; \( U \) is the on-site energy difference between the \( d_{z^2} \) and \( d_{xz,yz} \) orbitals; and \( \lambda \) denotes the amplitude of SOC and \( m \) is the magnitude of the Zeeman interaction.

On the other hand, the interlayer hopping interaction is written as

\[
\tau_{nm}^{\text{dd}_{z^2},\text{dd}_{xz,yz}} = \sum_{k} \left( e_{n,k}^{\text{dd}_{z^2}} \delta_{k^0} \delta_{n,k}^{\text{dd}_{xz,yz}} + e_{n,k}^{\text{dd}_{xz,yz}} \delta_{k^0} \delta_{n,k}^{\text{dd}_{z^2}} \right)
\]

(9)

where

\[
\begin{align*}
\epsilon_{n,k}^{\text{dd}_{z^2}} &= -u_{i,n} - u_{i,n} \cos k_y, \\
\epsilon_{n,k}^{\text{dd}_{xz,yz}} &= -u_{i,n} - u_{i,n} \cos k_x, \\
\epsilon_{n,k}^{\text{dd}_{z^2},\text{dd}_{xz,yz}} &= -t_{n}.
\end{align*}
\]

(10)

Here we assume \( n > m \) without losing generality; \( u_{i,n} \) is the amplitude of the nearest-neighbour interaction and \( u_{i,n} \) is the amplitude of the next-nearest-neighbour interaction. The parameters of the effective tight-binding Hamiltonian are summarized in Table 2.

**Table 2** Parameters for a 4 u.c. effective tight-binding model Hamiltonian

| \( n \) | \( t_{n} \) (eV) | \( t_{n} \) (eV) | \( t_{n} \) (eV) | \( s_{n} \) (eV) | \( u_{n} \) (eV) | \( u_{n} \) (eV) | \( f \) (eV) | \( \lambda \) (eV) | \( E \) (eV) |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.376 | -0.03 | 0.35 | 0.12 | 0.320 | 0.15 | 0.253 | 0.15 | -0.03 | 0.06 | 0.75 |
| 2 | 0.348 | -0.03 | 0.35 | 0.12 | 0.2 | 0.253 | 0.15 | 0.213 | 0.15 |
| 3 | 0.290 | -0.03 | 0.30 | 0.10 | 0.253 | 0.15 | 0.213 | 0.15 |
| 4 | 0.250 | -0.03 | 0.25 | 0.01 | 0.253 | 0.15 | 0.213 | 0.15 |

QBC. The negative AHC under small magnetization in SRO can be explained by the k-p Hamiltonian expanded near QBCs. Neglecting rotation and tilting, a perovskite thin film has a square lattice structure. Without SOC, space group generators are four-fold rotation about the z axis (C4) and three mirrors M1, M2, and M3, where M3 changes the sign of the \( x \) coordinate. The \( k \cdot p \) Hamiltonian valid near the \( \Gamma \) or \( M \) points is written as

\[
H_{\text{QBC}} = E_{x}(k) + \beta(k_{x}^{2} - k_{y}^{2})\tau_{x} + 2\beta k_{x} k_{y} \tau_{y},
\]

(11)

where \( E_{x}(k) \) is part of the Hamiltonian proportional to identity and \( \beta \) is the coefficient of the \( k \cdot p \) Hamiltonian.

\[
E_{x}(k) = |d_{x^2-y^2}|^{2} \equiv |1+1| \quad \text{and} \quad E_{y}(k) = |d_{x^2-y^2}|^{2} \equiv |-1|
\]

are the basis near the QBC. We now discuss the influence of SOC. The on-site SOC term is expressed as \( \lambda L \cdot S = \lambda(L_{x}S_{z} + L_{y}S_{z} + L_{z}S_{z}) \), where \( S \) is the spin operator. Since

\[
(1+1, |L_{x}S_{z} |+1, \sigma) = (-1, |L_{y}S_{z} |-1, \sigma) = \lambda \sigma_{x},
\]

the effective Hamiltonian with SOC \( H \) is written as follows:

\[
H = H_{\text{QBC}} + H_{\text{SOC}} = E_{x}(k) + \beta(k_{x}^{2} - k_{y}^{2})\tau_{x} + 2\beta k_{x} k_{y} \tau_{y} + \lambda \sigma_{x},
\]

(12)

where \( \sigma_{x} \) is \( +1 \) (\(-1\) for majority (minority) bands. Thus the gap is opened and the relevant Berry curvature is given by

\[
\Omega(k) = \frac{\lambda \sigma_{x}}{\beta} \left( \frac{2k^{2}}{(k^{2} + L^{2}j^{5})^{3/2}} \right),
\]

where the total Berry curvature \( \Omega(k) \) is quantized to \( 2\pi \). Unlike the Berry curvature induced by linear band crossing, which is maximum at \( |k| = 0 \), Berry curvature induced by QBC is zero at \( |k| = 0 \) and maximum at \( |k| = (2\lambda^{2}/\beta^{2})^{1/3} \).

In SRO, although QBC formed by majority bands is closer to the Fermi level than that of minority bands, the former induces smaller Berry curvature. Since the bands at the \( M \) point are all hole-like, the Fermi level is between the majority bands for \( k_{1} < |k| < k_{5} \) and minority bands for \( k_{5} < |k| < k_{1} \), where \( \Delta_{1} \) and \( \Delta_{3} \) are positive numbers. Then, the Berry curvature that contributes to the AHE is

\[
\Omega_{\text{minority}}(k) \approx \frac{\lambda}{2} \frac{\tau_{x} / L^{2}j^{5}}{\tau_{x}}.
\]

Here \( \Omega_{\text{majority}}(k) \) is the Berry curvature from the majority (minority) band. For small magnetization, Hall conductance can be negligible, whereas for large magnetization, Berry curvature induced by minority bands becomes negligible and Hall conductance becomes positive.

**NL.** Without SOC, the crossing between spin-up and spin-down bands form a NL. The corresponding Hamiltonian is written as

\[
H_{\text{NL}} = \begin{pmatrix}
E_{\uparrow}(k) & 0 \\
0 & E_{\downarrow}(k)
\end{pmatrix} = E_{0}(k) + \Delta E(k)\sigma_{x},
\]

(13)

where \( \Delta E(k) = \Omega_{\text{Majority}}(k) \) and \( \Omega_{\text{Majority}}(k) \) is the Berry curvature of the majority (minority) band. The Fermi energy lies between two energy bands, the Berry curvature is given by

\[
\Omega(k) = \frac{i}{\hbar} \left( \frac{\partial H(k)}{\partial k_{x}} \right) |+\rangle + \frac{\partial H(k)}{\partial k_{y}} |\rangle + \text{c.c.} \left( e^{-i\frac{\pi}{4}} \right),
\]

(17)

where \( |\rangle \) is a higher (lower) energy state, \( e \) is its energy eigenvalue, and c.c. represents complex conjugation. Similar to the calculation for QBC, integration of the Berry curvature is quantized to \( 2\pi \).

Let us note, however, that some NLs are intact under SOC. This is due to the Fermi energy lies between two energy bands, the Berry curvature is given by

\[
\Omega(k) = \frac{i}{\hbar} \left( \frac{\partial H(k)}{\partial k_{x}} \right) |+\rangle + \frac{\partial H(k)}{\partial k_{y}} |\rangle + \text{c.c.} \left( e^{-i\frac{\pi}{4}} \right),
\]

(17)

where \( \Delta_{1} \) and \( \Delta_{3} \) are the Fermi energy. Let the Fermi energy lies between two energy bands, the Berry curvature is given by

\[
\Omega(k) = \frac{i}{\hbar} \left( \frac{\partial H(k)}{\partial k_{x}} \right) |+\rangle + \frac{\partial H(k)}{\partial k_{y}} |\rangle + \text{c.c.} \left( e^{-i\frac{\pi}{4}} \right),
\]

(17)

where \( |\rangle \) is a higher (lower) energy state, \( e \) is its energy eigenvalue, and c.c. represents complex conjugation. Similar to the calculation for QBC, integration of the Berry curvature is quantized to \( 2\pi \).

Two bands with different mirror eigenvalues do not hybridize.

**Relationship between OAM and Berry curvature.** Here, we explain that OAM and Berry curvature show similar distributions. For simplicity, let us consider a two-band \( k \cdot p \) Hamiltonian where the gap is opened by the SOC.
The Hamiltonian is written as

$$H = E_0(k) + \beta k_x^2 - \beta k_y^2 + 2\beta k_x k_y + \lambda \sigma_z,$$

where $\sigma_z = \pm 1$, which represents the direction of spin polarization. The energy spectrum is given as $E = E_0 \pm \sqrt{k^2 + \beta^2 k^4}$ where the gap due to the SOC is proportional to $\lambda$.

The Berry curvature $\Omega(k)$ of the occupied band is

$$\Omega(k) = 2\lambda \frac{\beta k_y^2}{(\lambda^2 + \beta^2 k^4)^{3/2}} \sigma_z,$$

On the other hand, the $z$ component of the OAM of the occupied band $|occ\rangle$ is defined as $\langle L_z \rangle = \langle occ | \tau_z | occ \rangle$, where

$$L_z(k) = 2\lambda \frac{\lambda + \sqrt{k^2 + \beta^2 k^4} - \sqrt{k^2 + \beta^2 k^4} + \beta k^4}{(\lambda + \sqrt{k^2 + \beta^2 k^4})^2 + \beta^2 k^4} \sigma_z.$$

The Berry curvature and OAM show a similar tendency since both of them are induced by the SOC that opens the gap at QBC. In particular, we note that the Berry curvature and OAM have the same sign, determined by $\lambda$.

Data availability
The data that support the findings of this study are available from the corresponding authors on request.

References
43. Kresse, G. & Furthmüller, J. Efficient iterative schemes for ab initio total-energy calculations using a plane-wave basis set. Phys. Rev. B 54, 11169 (1996).
44. Perdew, J. P., Burke, K. & Ernzerhof, M. Generalized gradient approximation made simple. Phys. Rev. Lett. 77, 3865–3868 (1996).
45. Blöchl, P. E. Projector augmented-wave method. Phys. Rev. B 50, 17953–17979 (1994).
46. Pizzi, G. et al. Wannier90 as a community code: new features and applications. J. Phys. Condens. Matter 32, 165902 (2020).
47. Liechtenstein, A. I., Anisimov, V. I. & Zaanen, J. Density-functional theory of correlated electron systems: orbital ordering in Mott-Hubbard insulators. Phys. Rev. B 52, R5467–R5470 (1995).
48. Vaugier, L., Jiang, H. & Biermann, S. Hubbard $U$ and Hund exchange $J$ in transition metal oxides: screening versus localization trends from constrained random phase approximation. Phys. Rev. B 86, 165105 (2012).
49. Bezdicka, P., Watiaux, A., Grenier, J., Pouchard, M. & Hagenmüller, P. Preparation and characterization of fully stoichiometric SrCoO$_3$ by electrochemical oxidation. Z. Anorg. Allg. Chem. 619, 7–12 (1993).

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Author contributions
B.S., E.L., B.-J.Y. and C.K. conceived the project. S.Y.P., B.-J.Y. and C.K. led the project. B.S. synthesized and characterized the materials with support from B.K. and T.W.N.; B.S., W.K. and J.H. conducted ARPES measurements with support from J.D.D., J.S.O., J.K.J., D.O. and Y.K.; B.S., H.R. and S.H. conducted spin-resolved ARPES measurements. B.S. performed transport measurements. B.S. and B.K. performed magnetic measurements. M.K. and D.K. performed ionic liquid gating. B.S. analysed the experimental data. E.L. conducted tight-binding calculations and symmetry analysis. S.Y.P. conducted first-principles calculations. B.S., E.L., B.-J.Y. and C.K. wrote the paper with contributions from other authors. All authors participated in the discussions and commented on the manuscript.

Competing interests
The authors declare no competing interests.

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