Multi-objective optimization in determining the location of the transportation hub with generated data using nonparametric statistics

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Abstract. In this paper, we will discuss a mathematical modeling in determining the location of the transportation hub, so that the transportation hub can be placed optimally. In general, determining the location of the transportation hub is only considered the quantitative factors. However, in this paper will also consider the qualitative factor. Multi attribute group decision making (MAGDM) method will be used to bring qualitative forms into quantitative forms. This model is assumed that the number of passengers at a passenger generation point is unknown, therefore in this modeling there is a stochastic constraint. The distribution of the number of passengers in this paper is also unknown, therefore this modeling will be completed using nonparametric statistics estimation with generated data.

1. Introduction

Public transportation is a passenger transportation service that is available for public use. Public transportation is usually managed on schedule, operated on a designated route, and there are fees for each trip. There are many types of public transportation. One of the public transportation that is widely used by the public is Bus Rapid Transit (BRT). BRT is a bus system that is fast, convenient, safe and on time. In Indonesia, BRT is usually located in provincial capitals such as Jakarta, Yogyakarta, Semarang, etc. One of the facilities owned by BRT is a transportation hub. Transportation hub is a place to pick up or drop off passengers. For downtown, a transportation hub is placed at a distance of 300 m to 500 m with other transportation hub. Many factors effect in determining the location of the transportation hub. Some of the factors that effect it are the cost of building a bus stop, the costs paid by passengers in using transportation and whether the bus stop is effective if it is built at that location. These factors are included in the factor quantitative and qualitative factors. To get the optimal transportation hub location can be used a mathematical model.

In this paper, we will develop a mathematical model to determining the location of transportation hub based on the model of [1]. In [1], it has been described an optimization modeling to determine the location of transportation hub that have qualitative and quantitative objectives which is assumed that the number of passengers from a passenger generation point is unknown beforehand, so the number of passengers is a random variable which is assumed to be normally distributed. But, because in real life the distribution of the number of passengers cannot be known with certainty, so in this paper will be assumed that the distribution of the number of passengers is unknown beforehand.
In this paper also assumed that the number of passengers from a passenger generation point is unknown before, therefore there is a stochastic constraint in the modeling. This optimization modeling is also called stochastic programming. In order for the modeling can be completed, the stochastic constraint will be converted into a constraint that does not contain random variables using a theorem in [2]. In this paper, the optimization model also assumed that the distribution of the number of passengers is unknown, so an approach will be used, namely nonparameter statistics as in [3] to solve the optimization model. Because in real life it is too difficult to get data on the number of passengers, so that will be done a generated data of the number of passengers.

Furthermore, to get the optimal transportation hub location, it is necessary to evaluate all alternative transportation hubs to several attributes by decision makers. Less effective if only one decision maker evaluates, therefore several decision makers are needed. MAGDM (Multiple Attributes Group Decision Making) method will be used as in [4] to determine the evaluation value of each transportation hub. Because there are several attributes, it is necessary to determine the weight of the attribute. In determining the attribute weight we will use standard deviation method and average deviation method as in [5]. Here, decision makers can come from different backgrounds and knowledges, and also have different effects in making decisions. So, it is necessary to determine the weight of the decision makers according to their characteristics. The decision maker’s weight can be determined by the TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method as in [6].

The mathematical model of determining the location of the transportation hub has two objectives. The first objective is to minimize quantitative factors which is to minimize transportation costs and the costs of building the transportation hub. The other goal is to maximize qualitative factors which is to maximize the usefulness of the transportation hub. Therefore, the proposed model will be in the form of Multi-Objective Linear Programming. Weighting method will be used to bring this model into a single objective form as in [7]. Furthermore, this modeling will be completed numerically.

2. Model Formation

In this section we will present a model for determining the location of the transportation hub. Many factors can influence the determination of the location of the transportation hub, that are qualitative factors and quantitative factors.

Assuming there are $n$ passenger generation points, where the coordinates for point $i$ is $(x_i, y_i)$ and there are $m$ alternative transportation hubs, where the coordinates for the alternative transportation hub $j$ is $(x_j, y_j)$. The variables and parameters that will be used in the model are as follows:

- $H_j$: the maximum capacity of the alternative transportation hub $j$,
- $C_{ij}$: the transportation costs of passengers who choose passenger generation point $i$ to alternative transportation hub $j$,
- $x_{ij}$: passenger flow from point $i$ to point $j$,
- $d_{ij}$: distance from the passenger generation point $i$ to the alternative transportation hub $j$,
- $f_j$: the cost for building an alternative transportation hub $j$,
- $\delta_i$: the number of passengers at the passenger generation point $i$,
- $Z_j$: the evaluation value for alternative transportation hub $j$ which describes the influence of qualitative factors,
- $I_j$: \[ I_j = \begin{cases} 1 & \text{if alternative transportation hub } j \text{ is chosen}, \\ 0 & \text{otherwise}. \end{cases} \]

This modeling is assumed that the number of passengers at the passenger generation point $i$ is unknown beforehand. So that the passenger flow from point $i$ to point $j$ cannot be predicted. The purpose of this model is to determine the location of the transportation hub. By minimizing the quantitative factors and maximizing the qualitative factors, can be obtained the optimal location of the transportation hub. The quantitative factors that are considered in this model is the passenger cost from
the passenger generation point \( i \) to the alternative transportation hub \( j \) and the cost to build an alternative transportation hub \( j \). The qualitative factors that are considered in this model are how much the benefits from the establishment of alternative transportation hubs \( j \).

Liu et al. [1] has proposed a model for determining the location of the transportation hub as follows:

\[
\min f_1(x) = \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} x_{ij} d_{ij} + \sum_{j=1}^{m} f_j I_j
\]

\[
\max f_2(x) = \sum_{j=1}^{m} I_j Z_j
\]

Subject to:

\[
d_{ij} = |x_i - x_j| + |y_i - y_j|,
\]

\[
\sum_{j=1}^{m} x_{ij} \geq \delta_i, \quad i = 1, 2, 3, \ldots, n
\]

\[
\sum_{i=1}^{n} x_{ij} \leq H_j I_j, \quad j = 1, 2, 3, \ldots, m
\]

\[
x_{ij} \geq 0, \quad i = 1, 2, 3, \ldots, n, \quad j = 1, 2, 3, \ldots, m,
\]

\[
I_j = 1 \text{ or } 0, \quad j = 1, 2, 3, \ldots, m.
\]

Constraint 4 states that passenger flow from the passenger generation point \( i \) to all alternative transportation hubs must be greater or equal to the number of passengers at the passenger generation point \( i \). Constraints 5 states the capacity limits of each alternative transportation hub.

In [1], the number of passengers at the passenger generation point is assumed to be normally distributed, but in this paper it is assumed that the distribution from the number of passengers at the passenger generation point is unknown.

2.1. Model Transformation

Previously, it has explained that the number of passengers at the passenger generation point \( i \) was unknown beforehand, in other words \( \delta = \{\delta_1, \delta_2, \ldots, \delta_n\} \) is a stochastic vector [8]. Therefore, the model for determining the location of transportation hub above is a stochastic programming or more precisely is chance constrained programming.

Theorem 1. [2] Let \( x \) be a decision vector, \( \epsilon \) a stochastic vector, \( f(x, \epsilon) \) the objective function, and \( g_j(x, \epsilon)(j = 1, 2, \ldots, p) \) the stochastic constraint function; the chance constraint form will be as follows:

\[
P(g(x, \epsilon) \leq 0) \geq \alpha,
\]

where \( \alpha \) is the confidence level.

Suppose the stochastic vector \( \epsilon = \{\epsilon_1, \epsilon_2, \ldots, \epsilon_n\} \) degenerates to a stochastic variable \( \epsilon \), and \( \epsilon \)'s distribution function is \( \Phi \). If the form of function \( g(x, \epsilon) = h(x) - \epsilon \),

\[
P(g(x, \epsilon) \leq 0) \geq \alpha
\]

is true if and only if \( h(x) \leq K_\alpha \), where \( K_\alpha = \sup\{K|K = \Phi^{-1}(1 - \alpha)\} \).

The cumulative distribution function of \( \delta \) is denoted as \( F \), so that the quantile function of \( \delta \) will be denoted as \( F^{-1} \). Using Theorem 1, stochastic constraint (4) will be transformed into a constraint that do not have stochastic vector. Constraint 4 can be transformed to be

\[
\sum_{j=1}^{m} x_{ij} \geq F^{-1}_i(\beta_i), \quad i = 1, 2, \ldots, n.
\]
Because of the distribution of $\delta$ is unknown, so an approach will be used to find $F$ and $F^{-1}$ which is nonparameter statistics specifically empirical cumulative distribution function and empirical quantile function [9]. A theorem will be given which shows that empirical cumulative distribution function is very close to cumulative distribution function.

**Theorem 2.** [3] Let $X_1, X_2, \ldots, X_n$ be i.i.d. random variables with marginal distribution function $F : \mathbb{R} \rightarrow [0, 1]$. Define the empirical cumulative distribution function $\hat{F}_n : \mathbb{R} \rightarrow [0, 1]$ as

$$\hat{F}_n(x) = \frac{1}{n} \sum_{i=1}^{n} \mathbb{1}\{X_i \leq x\}.$$ 

The empirical distribution converges uniformly to $F(x)$, namely

$$\sup_{x \in \mathbb{R}} \left| \hat{F}_n(x) - F(x) \right| \xrightarrow{P} 0, \quad n \rightarrow \infty.$$ 

The following theorem asserts that empirical quantile function is strongly consistent for estimation of quantile function.

**Theorem 3.** [3] Let $0 < p < 1$. If $F^{-1}_n(x)$ is the unique solution $x$ of $F(x- \leq p \leq F(x)$, then

$$\sup_{x \in \mathbb{R}} \left| \hat{F}^{-1}_n(x) - F^{-1}(x) \right| \xrightarrow{P} 0, \quad n \rightarrow \infty.$$ 

### 2.2. Evaluation Value

MAGDM is a method for making decisions where decision makers evaluate several alternatives based on several attributes [4]. It would be more appropriate if the decision maker gave evaluate alternatives qualitatively rather than quantitatively. For example, when a decision maker wants to evaluate the comfort level of a car, terms such as extremely poor, very poor, poor, slightly poor, fair, slightly good, good, very good, extremely good can be used. It can be referred to as a linguistic term, which can be stated as follows

$$S = \{s_{-4} = \text{extremely poor}, s_{-3} = \text{very poor}, s_{-2} = \text{poor},$$
$$s_{-1} = \text{slightly poor}, s_0 = \text{fair}, s_1 = \text{slightly good}, s_2 = \text{good},$$
$$s_3 = \text{very good}, s_4 = \text{extremely good}\},$$

where $S = \{s_i| i = -t, \ldots, t\}$ is a finite discrete set, where $s_i$ represented a linguistic value. The discrete set $S$ is extended to a continuous set $\tilde{S} = \{s_\alpha| \alpha \in [-t, t]\}$ to ease the calculations.

The operational laws for set $S$ are given as follows, let $s_\alpha, s_\beta \in \tilde{S}$, dan $\lambda, \lambda_1, \lambda_2 \in [0, 1] :$

1. $s_\alpha \oplus s_\beta = s_{\alpha + \beta};$
2. $s_\alpha \oplus s_\beta = s_{\beta \oplus s_\alpha};$
3. $\lambda s_\alpha = s_{\lambda \alpha};$
4. $(s_\alpha)^\lambda = s_{\lambda \alpha};$
5. $\lambda(s_\alpha \oplus s_\beta) = \lambda s_\alpha \oplus \lambda s_\beta;$
6. $(\lambda_1 + \lambda_2)s_\alpha = \lambda_1 s_\alpha \oplus \lambda_2 s_\beta.$

**Definition 1.** Let $\tilde{S}$ be the extended continuous linguistic term set, and $s_i \in \tilde{S}$, then the subscript $i$ of $s_i$ can be obtained by the following function:

$$I(s_i) = i, \quad s_i \in \tilde{S}.$$
Definition 2. Let $s_\alpha, s_\beta \in \mathcal{S}$ be two linguistic variables, then
\[ d(s_\alpha, s_\beta) = \| s_\alpha - s_\beta \| = |\alpha - \beta|, \]
is the deviation between $s_\alpha$ and $s_\beta$.

Let $M = \{1, 2, 3, \ldots, m\}$, $N = \{1, 2, 3, \ldots, n\}$, and $T = \{1, 2, 3, \ldots, t\}$, $i \in M$, $j \in N$, $k \in T$. Let $A = \{A_1, A_2, \ldots, A_m\}$ represent a set of $m$ feasible alternatives, let $U = \{u_1, u_2, \ldots, u_n\}$ represent a set of attributes, and let $D = \{d_1, d_2, \ldots, d_t\}$ be a set of DMs. $w = (w_1, w_2, \ldots, w_n)$ denotes the weight vector of attributes, where $w_j \geq 0$, $\sum_{j=1}^{n} w_j = 1$. $\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_t)$ denotes the weight vector of DMs, where $\lambda_j \geq 0$, $\sum_{k=1}^{n} \lambda_k = 1$.

Each of the alternatives given is evaluated based on attributes. The evaluation is carried out by decision makers. The evaluation value of the decision maker to $k$ is stated as follows
\[ X_k = (x_{ij}^{(k)})_{m \times n} = \begin{pmatrix} A_1 & \ldots & A_m \\ x_{11}^{(k)} & \ldots & x_{1n}^{(k)} \\ x_{21}^{(k)} & \ldots & x_{2n}^{(k)} \\ \vdots & \ddots & \vdots \\ x_{m1}^{(k)} & \ldots & x_{mn}^{(k)} \end{pmatrix}, \quad k \in T, \]

Then given the matrix $R_k$ which is the normalization matrix of the matrix $X_k$.
\[ R_k = (r_{ij}^{(k)})_{m \times n} = \begin{pmatrix} A_1 & \ldots & A_m \\ r_{11}^{(k)} & \ldots & r_{1n}^{(k)} \\ r_{21}^{(k)} & \ldots & r_{2n}^{(k)} \\ \vdots & \ddots & \vdots \\ r_{m1}^{(k)} & \ldots & r_{mn}^{(k)} \end{pmatrix}, \quad k \in T, \]

where
\[ r_{ij}^{(k)} = \frac{x_{ij}^{(k)}}{\sqrt{\sum_{i=1}^{m} (x_{ij}^{(k)})^2}}, \quad \text{for benefit attributes}, \]
and
\[ r_{ij}^{(k)} = 1 - \frac{x_{ij}^{(k)}}{\sqrt{\sum_{i=1}^{m} (x_{ij}^{(k)})^2}}, \quad \text{for cost attributes}. \]

Matrix $R_k$ matrix is multiplied by the weight vector of attributes $w$, obtained matrix $Y_k$ as follows
\[ Y_k = (y_{ij}^{(k)})_{m \times n} = (w_j r_{ij}^{(k)})_{m \times n} = \begin{pmatrix} A_1 & \ldots & A_m \\ y_{11}^{(k)} & \ldots & y_{1n}^{(k)} \\ y_{21}^{(k)} & \ldots & y_{2n}^{(k)} \\ \vdots & \ddots & \vdots \\ y_{m1}^{(k)} & \ldots & y_{mn}^{(k)} \end{pmatrix}, \quad k \in T. \]

Matrix $Y_k$ matrix is multiplied by the weight vector of DMs $w$, obtained decision matrix $Y$ as follows
\[ Y = \sum_{k=1}^{t} \lambda_k Y_k = (y_{ij})_{m \times n}. \]
where \( y_{ij} = \sum_{k=1}^{t} \lambda_k y_{ij}^{(k)} \).

In the MAGDM problem, it is necessary to compare all evaluation values given by decision makers to rank all alternatives. The greater the evaluation value, the better the alternative. If the evaluation value of all alternatives has a small difference that is judged based on an attribute, then in other words the attribute has a small role in determining the best alternative. Therefore, the attribute is given a small weight. Conversely, if the evaluation value of all alternatives has a large difference that is judged based on an attribute, then the attribute has an important role in determining the best alternative. Therefore, the attribute is given a large weight. If the evaluation value of each alternative has the same value based on an attribute, then the attribute is considered not important by decision makers. In other words, this attribute is given a weight equal to 0. The difference from the evaluation value of each alternative measured by an attribute can be calculated using the standard deviation or the average deviation method [5].

For DM \( d_k \) and attribute \( u_j \), the standard deviation between alternative \( A_i \) and other alternatives is

\[
S_j^{(k)} = \sqrt{\frac{1}{m} \sum_{i=1}^{m} \left( w_j \left( x_{ij}^{(k)} - \frac{1}{m} \sum_{t=1}^{m} x_{tj}^{(k)} \right) \right)^2} = w_j^{(k)} \sigma_j^{(k)}, \quad j = 1, 2, \ldots, n,
\]

where

\[
\sigma_j^{(k)} = \sqrt{\frac{1}{m} \sum_{i=1}^{m} d\left( x_{ij}^{(k)}, x_j^{(k)} \right)^2}.
\]

For DM \( d_k \) and attribute \( u_j \), the average deviation between alternative \( A_i \) and other alternatives is

\[
V_j^{(k)} = \frac{1}{m} \sum_{i=1}^{m} \left( w_j \left( x_{ij}^{(k)} - \frac{1}{m} \sum_{t=1}^{m} x_{tj}^{(k)} \right) \right) = w_j^{(k)} \delta_j^{(k)}, \quad j = 1, 2, \ldots, n,
\]

where

\[
\delta_j^{(k)} = \frac{1}{m} \sum_{i=1}^{m} d\left( x_{ij}^{(k)}, x_j^{(k)} \right).
\]

In this case, \( x_j^{(k)} = \frac{1}{m} \sum_{t=1}^{m} x_{tj}^{(k)} \) (\( j = 1, 2, \ldots, n \)) states the average evaluation value of the \( u_j \) attribute given by DM \( d_k \) and \( d(x_{ij}^{(k)}, x_j^{(k)}) \) represents the deviation of the average value of \( x_j^{(k)} \) towards the evaluation value of \( x_{ij}^{(k)} \).

The standard deviation method and the average deviation method can both be used to determine the weight of an attribute objectively. Sometimes, decision makers use both methods simultaneously. Therefore, an optimization problem can be formed as follows

\[
\text{Max} \quad F(w) = \sum_{j=1}^{n} \left( uS_j^{(k)} + vV_j^{(k)} \right) = \sum_{j=1}^{n} w_j^{(k)} \left( u\sigma_j^{(k)} + v\delta_j^{(k)} \right)
\]

Subject to: \( \sum_{j=1}^{n} w_j^{(k)} = 1, \quad w_j^{(k)} \geq 0. \)
The standard deviation and average deviation of the attribute \( u_j \) are expressed as \( S_j(k) \) and \( V_j(k) \). While \( u \) and \( v \) state the choice of the DM. If \( u = 0 \) then the DM only calculates the average deviation, so if \( v = 0 \) then the DM only calculates the standard deviation, and if \( u \neq 0, v \neq 0 \) then the DM use both simultaneously.

Using Kuhn-Tucker condition [10], the solution of 8 can be obtained:

\[
 w_j^{(k)} = \frac{u \sigma_j^{(k)} + v \delta_j^{(k)}}{\sqrt{\sum_{j=1}^{n} \left( u \sigma_j^{(k)} + v \delta_j^{(k)} \right)^2}}.
\]

By normalizing \( w_j^{(k)} \), the weight of an attribute \( u_j \) is

\[
 w_j^{*(k)} = \frac{w_j^{(k)}}{\sum_{j=1}^{n} w_j^{(k)}} = \frac{u \sigma_j^{(k)} + v \delta_j^{(k)}}{\sqrt{\sum_{j=1}^{n} \left( u \sigma_j^{(k)} + v \delta_j^{(k)} \right)^2}}.
\]

In making a decision, less effective if only determined by one DM. Therefore, several DMs are needed. Usually DMs come from various fields of knowledge and each of the DMs has a variety of characteristics, abilities and experiences. In other words, a DM has different influences in determining decisions. Thus, these DMs need to be given the appropriate weight for their characteristics. To determine the weight of these DMs, the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) method will be used [6].

In the TOPSIS method, there are two ideal solutions, namely PIS (Positive Ideal Solution) and NIS (Negative Ideal Solution). These two ideal solutions can be used in determining rankings from alternative transportation hubs.

Define PIS is the average matrix of all decision matrices. It is assumed that PIS is the final decision in MAGDM. For example, in a competition that is attended by several people if there are \( k \) DMs, then the final value of each competitor is the average value given by \( k \) DMs. NIS represents disappointment from decision makers.

The average matrix of all decision matrices or PIS is stated as follows

\[
 Y^* = (y_{ij}^*)_{m \times n} = \begin{pmatrix}
 u_1 & u_2 & \cdots & u_n \\
 y_{11}^* & y_{12}^* & \cdots & y_{1n}^* \\
 y_{21}^* & y_{22}^* & \cdots & y_{2n}^* \\
 \vdots & \vdots & \ddots & \vdots \\
 y_{m1}^* & y_{m2}^* & \cdots & y_{mn}^*
\end{pmatrix}
\]

where \( Y^* = \frac{1}{t} \sum_{k=1}^{t} Y_k \) and \( y_{ij}^* = \frac{1}{t} \sum_{k=1}^{t} y_{ij}^{(k)} \) for \( i \in M \) dan \( j \in N \).

The distance from the decision matrix \( Y_k \) with \( Y^* \) is stated as follows

\[
 S^+_k = \| Y_k - Y^* \| = \left( \sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij}^{(k)} - y_{ij}^*)^2 \right)^{\frac{1}{2}}, \quad k \in T.
\]

The smaller the distance \( Y_k \) with \( Y^* \), the better the decision \( Y_k \) of \( k \)th DM.

NIS is divided into two parts, namely \( LNIS \) (Left Negative Ideal Solution) and \( RNIS \) (Right Negative Ideal Solution). \( LNIS \) is expressed as \( Y_l^- \) and \( RNIS \) is expressed as \( Y_r^- \) which is defined as follows
The distance from the decision matrix $Y_k$ with LNIS is stated as follows

$$S_{l}^{-k} = \| Y_k - Y_l^- \| = \left( \sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij}^{(k)} - y_{ij}^-)^2 \right)^{\frac{1}{2}}, \quad k \in T.$$ 

The distance from the decision matrix $Y_k$ with RNIS is stated as follows

$$S_{r}^{-k} = \| Y_k - Y_r^- \| = \left( \sum_{i=1}^{m} \sum_{j=1}^{n} (y_{ij}^{(k)} - y_{ij}^r)^2 \right)^{\frac{1}{2}}, \quad k \in T.$$ 

The larger the value of $S_{l}^{-k}$ and $S_{r}^{-k}$, the better the decision $Y_k$ of $k$th DM.

The closeness coefficient of the $k$th DM with respect to $Y^*$ is defined as follows

$$C_k = \frac{S_{l}^{+k} + S_{r}^{-k}}{S_{l}^{+k} + S_{l}^{-k} + S_{r}^{-k}}, \quad k \in T.$$ 

By normalizing $w_{j}^{(k)}$, the weight of a DM $d_k$ is

$$\lambda_k = \frac{C_k}{\sum_{k=1}^{t} C_k}, \quad k \in T.$$ 

After determining the weight of the attributes and the weight of the DMs, then it can be determined the evaluation value of each alternative transportation hub $j$ which is $Z_j$ as follows

$$Z_j = \sum_{k=1}^{t} \lambda_k \sum_{j=1}^{n} w_{j}^{*} I(x_{ij}^{(k)}).$$ (9)
3. Simulation

In this section, we will discuss a simulation of a previously formed model that has two objective functions. It can be said that the model is a multi-objective linear programming problem. To get the optimal solution, the model needs to be transformed into a linear programming problem that has a single objective [11]. The weighting method can be used to transform the model. [7].

With the weighting method, the objective function of the model can be transformed as follows

$$\min f(x) = w_1 \left( \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} x_{ij} d_{ij} + \sum_{j=1}^{m} f_j I_j \right) - w_2 \left( \sum_{j=1}^{m} I_j Z_j \right),$$

where $w_1$ and $w_2$ are weights for each objective functions. The higher the value of $w_i$, $i = 1, 2$, then the higher the priority of the objective function.

Let, there are 4 alternative transportation hub $A = \{A_1, A_2, A_3, A_4\}$ and 3 DMs $D = \{d_1, d_2, d_3\}$. There are 5 attributes that will be used to evaluate the transportation hub, namely: $u_1$ is commercial potential, $u_2$ is Passenger distributed strength, $u_3$ is effective connection degree, $u_4$ is policy environment and $u_5$ is environmental impact. The following set of linguistic terms is given

$$S = \{s_{-4} = \text{extremely poor}, s_{-3} = \text{very poor}, s_{-2} = \text{poor},$$

$$s_{-1} = \text{slightly poor}, s_0 = \text{fair}, s_1 = \text{slightly good}, s_2 = \text{good},$$

$$s_3 = \text{very good}, s_4 = \text{extremely good}\}.$$  

Evaluation values of 4 alternative transportation hubs evaluated by 3 DMs are given in Table 1, 2 and 3.

Table 1. The evaluation values given by $d_1$

|       | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|-------|-------|-------|-------|-------|-------|
| $A_1$ | $s_1$ | $s_2$ | $s_2$ | $s_3$ | $s_1$ |
| $A_2$ | $s_2$ | $s_2$ | $s_0$ | $s_2$ | $s_2$ |
| $A_3$ | $s_3$ | $s_2$ | $s_3$ | $s_1$ | $s_3$ |
| $A_4$ | $s_3$ | $s_1$ | $s_{-1}$ | $s_2$ | $s_1$ |

Table 2. The evaluation values given by $d_2$

|       | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|-------|-------|-------|-------|-------|-------|
| $A_1$ | $s_0$ | $s_2$ | $s_2$ | $s_3$ | $s_2$ |
| $A_2$ | $s_2$ | $s_1$ | $s_2$ | $s_0$ | $s_2$ |
| $A_3$ | $s_2$ | $s_0$ | $s_1$ | $s_2$ | $s_3$ |
| $A_4$ | $s_0$ | $s_1$ | $s_{-1}$ | $s_0$ | $s_0$ |
Table 3. The evaluation values given by $d_3$

|     | $u_1$ | $u_2$ | $u_3$ | $u_4$ | $u_5$ |
|-----|-------|-------|-------|-------|-------|
| $A_1$ | $s_1$ | $s_2$ | $s_2$ | $s_3$ | $s_2$ |
| $A_2$ | $s_0$ | $s_1$ | $s_2$ | $s_0$ | $s_3$ |
| $A_3$ | $s_3$ | $s_1$ | $s_3$ | $s_2$ | $s_{-1}$ |
| $A_4$ | $s_0$ | $s_1$ | $s_0$ | $s_0$ | $s_1$ |

Evaluation values of alternative transportation hub $j$ i.e. $Z_j$ can be calculated using Equation 9. With $u = v = 0.5$, evaluation values is obtained as follows

\[
Z_1 = 0.5029 \\
Z_2 = 0.3545 \\
Z_3 = 0.5316 \\
Z_4 = 0.0788
\]

Let there are 5 passenger generation points, namely $K_1$, $K_2$, $K_3$, $K_4$ and $K_5$. Coordinate of the passenger generation points and alternative transportation hubs is given in Table 4 and 5.

Table 4. Coordinate of the passenger generation points

| Passenger generation point | Coordinate X | Coordinate Y |
|---------------------------|--------------|--------------|
| 1                         | 33.2         | 5.8          |
| 2                         | 11.9         | 6.3          |
| 3                         | 25.4         | 23.3         |
| 4                         | 15.6         | 31.6         |
| 5                         | 7.0          | 20.5         |

Table 5. Coordinate of the alternative transportation hubs

| Alternative transportation hub | Coordinate X | Coordinate Y |
|-------------------------------|--------------|--------------|
| 1                             | 12.1         | 24.4         |
| 2                             | 20.5         | 15.8         |
| 3                             | 15.4         | 11.3         |
| 4                             | 24.7         | 6.4          |

Transportation costs of passengers who choose the passenger generation point $i$ to the alternative transportation hub $j$ or $C_{ij}$ provided in Table 6. Table 7 states the expected value $\mu$, standard deviation $\gamma$, scale parameters $\rho$, shape parameters $k$ and confidence level of $\delta_i$ with $i = 1, 2, 3, 4, 5$. The maximum capacity of all alternative transportation hubs is 350 passengers and the cost to build an alternative transportation hub $j$ is $f_j = [20, 19, 18, 22]$ (unit: ten million rupiahs).
It is assumed that the distribution of random variables $\delta$ is unknown, therefore calculation of $F^{-1}$ can be done using an approach namely nonparametric statistics, specifically EQF. With generated data of the number of passengers in a passenger generation point in 200 days, using Weibull distribution and normal distribution and $w_1 = w_2 = 0.5$, a solution of the model can be seen in table 8. This paper will compare the results if the distribution of $\delta$ is known and unknown.

Table 6. Transportation cost (unit: a hundred rupiahs)

| $C_{ij}$ | $j = 1$ | $j = 2$ | $j = 3$ | $j = 4$ |
|----------|---------|---------|---------|---------|
| $i = 1$  | 33      | 27      | 25      | 32      |
| $i = 2$  | 31      | 38      | 42      | 34      |
| $i = 3$  | 38      | 48      | 44      | 40      |
| $i = 4$  | 47      | 42      | 32      | 46      |
| $i = 5$  | 37      | 29      | 36      | 31      |

Table 7. The number of passengers

| $\delta_1$ | $\delta_2$ | $\delta_3$ | $\delta_4$ | $\delta_5$ |
|------------|------------|------------|------------|------------|
| $\mu_i$    | 110        | 80         | 120        | 100        | 130        |
| $\gamma_i$ | 20         | 20         | 40         | 15         | 10         |
| $\rho_i$   | 110.5      | 81         | 121        | 101        | 131        |
| $\kappa_i$ | 126        | 46         | 68.5       | 57         | 74         |
| $\beta_i$  | 0.95       | 0.95       | 0.95       | 0.95       | 0.95       |

Table 8. Comparison of results

| Distribution | Weibull | Normal |
|--------------|---------|--------|
| $I_1$        | Unknown | Known  |
| $I_2$        | 1       | 1      |
| $I_3$        | 1       | 1      |
| $I_4$        | 1       | 1      |
| $x_{14}$     | 113     | 114    |
| $x_{25}$     | 87      | 87     |
| $x_{31}$     | 108     | 106    |
| $x_{32}$     | 19      | 21     |
| $x_{41}$     | 106     | 107    |
| $x_{51}$     | 136     | 137    |
Next, a probability distribution function graph will be given from the Weibull distribution and normal distribution using parameter $\delta_1$ in Table 7.

![Weibull Distribution](image1)

**Figure 1.** Weibull Distribution

![Normal Distribution](image2)

**Figure 2.** Normal Distribution

Table 8 shows that the results of the simulation with the distribution of the number of passengers is unknown will approach the results of the simulation with the distribution of the number of passengers is known. So a simulation of determining the location of a transportation hub can still be done even though the distribution of the number of passengers is unknown.

4. Conclusion

In this paper a model has been formed to determine the location of the transportation hub. Not only quantitative factors are considered, this model also takes into account qualitative factors, so it has two objectives. With MAGDM qualitative factors can be brought to quantitative factors. The number of passengers in a passenger generation points cannot be predicted so that this model has a stochastic constraint, which can be called stochastic programming. By using a theorem in stochastic programming, the model can be brought into a model without stochastic constraint. So that, it becomes a multi-objective
linear programming problem. The model in this paper used the assumption that distribution of the number of passengers is unknown, so it is solved by nonparametric statistics with generated data of the number of passengers.

Acknowledgements
This research was supported by the project FMIPA-UGM Research Grant, 119/J01.1.28/PL.06.02//2019.

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