A Proposed Bio-Economic Model for the Prey-Predator Fishery Model with Harvesting: Toward Modeling Dynamics in Applied Mathematics

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Abstract--This study proposed and analyzed a mathematical model. The proposed model was used for studying the two-prey predator system dynamics in relation to the fishery model. One of the parameters that were considered involved the possible impact of harvesting on the target populations. In particular, ecological dynamics were studied relative to tilapia, cichlid, and the Nile perch as pre-predator systems. Through system non-dimensionalization, the study computed equilibrium points before obtaining the conditions for global and local instability. To achieve the local instability condition, the investigation used Routh-Hurwitz Criterion and eigenvalues approach. With appropriate Lyapunov function defined, the coexistence equilibrium was proved. In turn, there was the analysis of the bio-economic equilibrium before conducting numerical simulations to verify the analytical outcomes that the study obtained. From the numerical results, the study established that if tilapia and cichlid fishes are not overharvested, the three species are likely to coexist. The populations were observed to play a crucial role to the Nile perch population’s growth rate. Hence, the study’s proposed mathematical model proved informative in such a way that it paved the way for the fishery control management departments to avoid overharvesting tilapia and cichlid fishes.

1. Introduction

In the current world, the correlation between prey and predator forms one of the crucial topics of discussion, especially in ecology. As such, the prey-predator system has attracted growing research interest [1, 2]. In this study, a mathematical ecology aspect was applied in studying species interactions. In particular, a case study approach was employed. Whereas populations that were considered to be prey included tilapia fishes and cichlid fishes, the Nile perch was considered as the predator species. Imperative to note is that if harvesting is done without limitations, it could cause species extinction [3-5]. Hence, there has been a growing need to develop and apply mathematical models through which species sustainability could be maintained [6-8]. Hence, this study strived to develop and apply mathematical techniques through which species could be sustained without compromising their social, economic, and biological objectives for future and current generations' benefit. For studies that have examined, developed, and applied the prey-predator frameworks, most of the findings suggest that there is a population decrease with increasing harvesting efforts [7-12]. In response to this worrying trend, this study proposed a model for analyzing the bio-economic impact of fishery activities on the affected populations and regions.

2. Materials and Methods

One of the central assumptions was that the Nile perch depended on tilapia and cichlid fishes completely. The assumption held further that this sole dependence is due to the taste of the prey, as

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well as the easy with which they could be captured. Before constructing the model, additional assumptions were held. One of the assumptions was that tilapia and cichlid fishes have unlimited food supply. Another assumption was that there was an exploitative nature of inter-specific competition among cichlid and tilapia fishes. It was also assumed that if the predator is absent, the growth in species is logistical. The last assumption is that the functional response of the Nile perch’s predation followed Michaelis-Menten kinetics. As such, a Holling type II functional form was used to model the predation functional response.

Regarding computation, there was rescaling to ensure that the number of parameters was reduced. Hence, the system model that was used included:

\[
\frac{dx}{dt} = \lambda_1 x (1 - x) - \sigma_1 xy - \frac{P_1 xz}{1 + Q_1 x} - r_1 x \\
\frac{dy}{dt} = \lambda_2 y (1 - y) - \sigma_2 xy - \frac{P_2 yz}{1 + Q_2 y} - r_2 y \\
\frac{dz}{dt} = -wz + \frac{e_1 xz}{1 + Q_1 x} + \frac{e_2 yz}{1 + Q_2 y} - r_3 z
\]

In the absence of the predator, the equation would translate into:

\[
p_3 (x^*, y^*, 0) = p_3 \left( \frac{\lambda_1 (\lambda_1 - r_1) - \sigma_1 (\lambda_2 - r_3)}{\lambda_1 \lambda_2 - \sigma_1 \sigma_2}, \frac{\lambda_1 (\lambda_2 - r_2) + \sigma_2 (r_1 - \lambda_1)}{\lambda_1 \lambda_2 - \sigma_1 \sigma_2}, 0 \right)
\]

If tilapia fishes were absent, the equation would become:

\[
P_4 (x^*, 0, z^*) = P_4 \left( \frac{w + r_3}{e_1 - Q_1 (w - r_3)}, 0, \frac{-\lambda_1 Q_1 (w + r_3)^2}{P_1 (e_1 - Q_1 (w - r_3))^2} + \frac{(\lambda_1 Q_1 - r_1 Q_1 - \lambda_1) (w + r_3)}{P_1 (e_1 - Q_1 (w - r_3))} + \frac{\lambda_1 - r_1}{P_1} \right)
\]

On the other hand, the absence of cichlid fishes gave:

\[
P_5 (0, y^*, z^*) = P_5 \left( 0, \frac{w + r_3}{e_2 - Q_2 (w - r_3)}, \frac{-\lambda_2 Q_2 (w + r_3)^2}{P_2 (e_2 - Q_2 (w - r_3))^2} + \frac{(\lambda_2 Q_2 - r_2 Q_2 - \lambda_2) (w + r_3)}{P_2 (e_2 - Q_2 (w - r_3))} + \frac{\lambda_2 - r_2}{P_2} \right)
\]
3. RESULTS AND DISCUSSION

From the numerical results, the simulation of the proposed model system involved inbuilt ODE solvers. Through Matlab figures and programming language, the solvers were coded. In turn, the figures were plotted through parameter values. The parameter values were summarized as follows.

| Parameter | Value |
|-----------|-------|
| $\lambda_1$ | 2.07 |
| $\lambda_2$ | 2.09 |
| $K_1$ | 200 |
| $K_2$ | 100 |
| $e_{12}$ | 0.001 |
| $e_{13}$ | 0.02 |
| $e_{21}$ | 0.002 |
| $e_{23}$ | 0.03 |
| $e_{31}$ | 1.5 |
| $h$ | 0.1 |
| $\gamma$ | 0.2 |
| $q_1$ | 0.14 |
| $q_2$ | 0.13 |
| $q_3$ | 0.125 |
| $\omega$ | 1 |

Table 1: A summary of the study’s parameter values obtained from numerical results

Another correlation that was investigated involved interactions among population dynamics relative to the aspect of time. Compared to other species, the results demonstrated that there is faster population growth in cichlid fishes. It is also notable that 3D technology was employed to visualize the dynamics. Similarly, the visualization gave room for further investigation of how harvesting cichlid fish could affect regional population dynamics. Findings demonstrated that when the harvesting effort increases, there is likely to be a significant decrease in the population of the Nile perch. Similar findings were obtained when tilapia fish population was investigated. Particularly, it was observed that with an increase in the harvesting effort for tilapia fish, there is likely to be a dramatic decrease in the population of the Nile perch fishes. However, the proposed model demonstrated that if the harvesting efforts for the Nile perch increase, the predator in this case, there is likely to be a significant increase in the population of cichlid and tilapia fishes. In the absence of Nile perch population, which would arise due to intensified harvesting efforts, the model revealed exponential growth in the population of cichlid and tilapia fishes.

Indeed, the proposed and analyzed mathematical model sought to determine or predict the two-prey-one predator dynamics in a given population system. The factor that played a moderating effect entailed the harvesting efforts occurring both on the part of the predator and that of the prey. The proposed model was applied towards examining the ecological dynamics among the Nile perch, cichlid, and tilapia fishes; which translated into a predator-prey interaction system. From the numerical results obtained after applying the mathematical model, it was observed that the rate of harvesting affected system stability. In particular, a situation where the rate of harvesting cichlid and tilapia fishes exceed their rate of intrinsic growth poses danger of specific extinction in the affected population(s). To ensure that the harvesting of tilapia and cichlid fishes is sustainable, the proposed model yielded new insight in such a way that it pointed to the criticality of keeping smaller rates of harvesting. However, another trend that was noted is that if the rate of harvesting the three species is much lower compared to their rate of intrinsic growth, based on the proposed model’s mathematical outcomes, the
system is likely to collapse. In particular, the model indicated that if the predator is absent, the remainder of the population constituting the prey would only coexist stably if a negligible or minimum level of inter-specific competition is maintained. Also, the model’s findings demonstrated that if the predator is absent, the two prey species would coexist only if they (the prey species) are harvested at a rate that does not exceed their rate of intrinsic growth.

The model yielded further insights regarding the factor of bio-economic equilibrium in a given population. Specifically, the model suggested that for this equilibrium (bio-economic) to exist, factors that play a determining role include the mortality rate of the predator, the intrinsic growth rate of the prey species, the harvesting effort, the catchability coefficient, the price per unity biomass, and the cost of fishing per unity effort for the selected species in the entirety. Specific results from the numerical analysis indicated that if a bio-economic equilibrium is to be achieved, the cost of fishing for each unity effort should not exceed the fishery revenue; outcomes that hold for all the selected species. It is also notable that the proposed mathematical model is insightful for decision-makers and policy formulators because it increases the understanding that when the rate of harvesting the prey species decreases, there is likely to be a significant increase in the density of the predator population. The eventuality is that if the population of the prey population increases gradually, based on the numerical simulation results obtained in this study, there is likely to be a significant increase in the predator population and vice versa.

4. Conclusion

In summary, it can be deduced from both the numerical and analytical results that if the rate of harvesting the prey population exceeds their rate of intrinsic growth, their population would become extinct, hence the predator population. On the other hand, the proposed model reveals that if the rate of harvesting both the prey population and the predator population is much lower than their rate of intrinsic growth, the entire system is unlikely to collapse. Overall, the model pointed to the importance of ensuring that the rate of harvesting the target species does not exceed their rate of growth. Thus, the framework proved important in terms of supporting sustainable development.

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