Arbitrary orbital angular momentum addition in second harmonic generation

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Abstract
We demonstrate second harmonic generation performed with optical vortices with different topological charges imprinted on orthogonal polarizations. Besides the intuitive charge doubling, we implement arbitrary topological charge addition on the second harmonic field using polarization as an auxiliary parameter.

Keywords: optical vortices, orbital angular momentum, second harmonic generation

1. Introduction

Besides their intrinsic beauty, optical beams carrying orbital angular momentum (OAM) have proved to be a powerful tool for encoding and processing quantum information. First order Laguerre–Gaussian and Hermite–Gaussian (HG) modes carry the mathematical structure of a qubit [1] and allow for a two-qubit encoding when combined with polarization of a single photon. The interplay between the two degrees of freedom leads to interesting applications, including topological phases [2, 3], quantum cryptography [4–6], Bell inequalities [7–9], quantum logic gates [10–12], and quantum teleportation [13–15]. Polarization controlled spatial correlations between entangled photon pairs were first demonstrated in [16, 17]. Nowadays, a
rich technology is available for generation [18–21, 23], transformation [22] and measurement [23–25] of beams carrying OAM, as well as for the transfer of quantum information between polarization and spatial degrees of freedom [26–28].

The interaction of optical vortices with nonlinear media has led to many interesting observations, including OAM entanglement between photons created by spontaneous parametric down conversion [29–31]. The demonstration of topological charge doubling in second harmonic generation [32–34] is among the first interesting examples of the role played by OAM in nonlinear optical processes. After this seminal result, other works discussed OAM conservation and the corresponding topological charge addition and subtraction in several kinds of nonlinear wave mixing. We can quote the results on cavity free stimulated [35] and spontaneous parametric down conversion [29, 36], optical parametric oscillation [37, 38], non-collinear up conversion [39] and four wave mixing in atomic media [40, 41]. These experimental demonstrations were accompanied by a solid theoretical background [42–45]. In these previous works, polarization played a minor role in the topological charge conversion process. Here, we perform type II second harmonic generation with vortices created by the coherent superposition of orthogonally polarized OAM beams with the same optical frequency but different topological charges. Besides the well known charge doubling effect observed in [32, 33], we demonstrate arbitrary topological charge addition on the second harmonic field. Topological charge addition has also been demonstrated in moiré patterns of spiral zone plates [46] and nonlinear propagation of acoustic waves [47]. It can be a useful tool for wave computation in the optical domain. The paper is organized as follows. Section 2 provides a brief theoretical background for the topological charge addition based on the nonlinear wave equation for the second harmonic field. Section 3 describes the experimental setup, including the method for topological charge measurement. In section 4, the main results are presented and discussed. Finally, we summarize our conclusions in section 5.

2. Nonlinear spin–orbit coupling

We now provide a brief theoretical background in the same lines of [42, 43]. The propagation equation for the second harmonic field can be written as

$$\nabla^2 E_{2\omega} - \frac{n^2_{2\omega}}{c^2} \frac{\partial^2 E_{2\omega}}{\partial t^2} = \frac{\chi^{(2)}}{c^2} \frac{\partial^2 \left( E_{\omega}^H E_{\omega}^V \right)}{\partial t^2}. \quad (1)$$

We shall be concerned solely with the dynamics of the second harmonic field generated inside the crystal. This approach is justified by the non-depletion approximation for the fundamental frequency input fields. In order to capture the role played by the topological charges in the nonlinear process, we now write each interacting field in the Laguerre–Gaussian basis for paraxial beams

$$E_{\omega}^{\mu} = \sum_{lp} A_{lp}^{\mu} (z) u_{lp}(\mathbf{r}) e^{i(kz - \omega t)},$$

$$E_{2\omega} = \sum_{lp} B_{lp} (z) u_{lp}(\mathbf{r}) e^{2i(kz - \omega t)}, \quad (2)$$

where $\mu = H, V$. In cylindrical coordinates, the Laguerre–Gaussian solutions for the paraxial wave equation are
\[ u_{pl}(r, \phi, z) = \frac{\mathcal{N}_{pl}}{w(z)} \left( \frac{r \sqrt{2}}{w(z)} \right)^{|l|} L_p^{|l|} \left( \frac{2 r^2}{w^2(z)} \right) \times \exp \left[ -\frac{r^2}{w^2(z)} + i \varphi_{pl}(r, z) + i l \phi \right], \]

where \( \mathcal{N}_{pl} \) is a normalization constant, \( w_0 \) the beam waist, \( z_R = \pi w_0^2/\lambda \) the Rayleigh distance for wavelength \( \lambda \), \( l \) the topological charge and \( p \) the radial order. This mode decomposition and the corresponding orthonormality relations, \( \left( u_{pl}, u_{qm} \right) = \delta_{pq} \delta_{lm} \) are then used in equation (1) to provide the dynamical equation for the slowly varying second harmonic amplitude

\[ \frac{dB_{pl}}{dz} = \frac{2 i \omega \chi^{(2)}}{n_{2\omega} c} \sum_{qmn,sn} A_{lmm}^{pqr} A_{qmn}^H(z) A_{sn}^V(z). \]  

The topological charge addition is determined by the overlap integral

\[ A_{pqr}^{lmm} = \int \int u_{pl}^* \left( \mathbf{r} \right) u_{qm} \left( \mathbf{r} \right) u_{sn} \left( \mathbf{r} \right) r \, dr \, d\phi, \]

which imposes the selection rule \( l = m + n \) when the angular integral is trivially performed. We can intuitively understand this selection rule as the phase match condition for the transverse wave vector expressed in terms of the topological charges. Both input modes carry fundamental radial indexes \( q = s = 0 \) and their overlap with higher radial modes is negligible, so that \( B_{0, m+n}(z) \) is the dominant amplitude in the second harmonic field expansion. Therefore, it is possible to perform topological charge addition with a single frequency input by using the input polarization as an auxiliary parameter. It is worthwhile noting that the superposition of different paraxial modes with orthogonal polarization leads to nonseparable spin–orbit modes that can be used in a number of quantum information protocols.

### 3. Experiment

The experimental setup is sketched in figure 1. The TEM_{00} mode of an infrared laser (model Altechna ST-II-N–1064 nm) at 1064 nm wavelength is first set to horizontal polarization by a polarizing beam splitter (PBS-1). A half-wave plate (HWP-1) placed before PBS-1 is used for intensity control. The laser beam is then split in two by a 50/50 beam splitter. One half is sent to a spatial light modulator (SLM model Hamamatsu LCOS0500325) with a programmable holographic pattern, allowing for generation of an optical vortex \( \Psi_{SLM} \) with variable topological charge \( l_{SLM} \). The other half is sent to a holographic Mask that imprints a fixed topological charge \( l_{Mask} = 1 \) on its first diffraction order (\( \Psi_{Mask} \)). The desired modes are selected by two iris.
diaphragms and their waists are matched \((w_0 \approx 1.1 \text{ mm})\) by two pairs of collimating lenses \((1\text{ and } 2)\). A second half-wave plate (HWP-2), oriented at \(45^\circ\), turns into vertical the polarization of \(\Psi_{\text{SLM}}\). The two orthogonally polarized vortices are then recombined in a polarizing beam splitter (PBS-2) before the nonlinear interaction. Therefore, the input infrared field is described by the amplitude

\[
E_{\omega}(r, t) = \sqrt{\frac{I_0}{2}} \left[ u_{01}(r)\hat{e}_H + u_{0\text{SLM}}(r)\hat{e}_V \right] e^{i(kz-\omega t)},
\]

where \(I_0\) is the total input intensity, which corresponds to about 6 mW distributed over a beam diameter around 2 mm. This intensity is equally split between the two polarization modes, what corresponds to the initial condition \(A_{01}(0) = A_{0\text{SLM}}(0) = \sqrt{I_0/2}\) for equation (4). According to the selection rule that follows from equation (5), we expect the second harmonic field to be generated with topological charge \(l_{2\omega} = l_{\text{Mask}} + l_{\text{SLM}}\). In fact, we shall see shortly that the experimental results fairly confirm this expectation.

A third half wave plate (HWP-3) is used to control the polarization of the input field, which is then focused by a lens with focal length \(f_1 = 20 \text{ cm}\) inside a potassium titanyl phosphate crystal cut for type II phase match for second harmonic generation from 1064 nm to 532 nm. After the nonlinear interaction, the output fields are collimated by a second lens with focal length \(f_2 = 20 \text{ cm}\) and the infrared field is separated from the second harmonic by a dichroic mirror. The topological charge measurement on the second harmonic field was performed using the method developed in [25]. A tilted biconvex lens (BK7 glass), with focal length \(f_3 = 20 \text{ cm}\), was inserted on the path of the green output, introducing astigmatic mode conversion. For a suitable tilt angle, around \(19^\circ\), the second harmonic vortex was converted into a HG profile on the focal plane of the tilted lens. Finally, the amplified image of the HG profile was projected on a charge coupled device (CCD) camera by a lens with focal length \(f_4 = 5 \text{ cm}\). This method was particularly well suited for the low intensity generated in the second harmonic field. It introduces almost no power loss and allows for immediate identification of the topological charge through the HG order observed.

4. Results and discussion

In the first run of the experiment, the SLM was set to imprint the topological charge \(l_{\text{SLM}} = 2\) and HWP-3 was used to switch between three different processes. With the waveplate oriented
to turn the polarizations of $\psi_{\text{Mask}}$ and $\psi_{\text{SLM}}$ by 45°, we could block (B) either one of the incoming beams and observe the charge doubling of the vortex allowed to pass through the crystal. In figure 2(a), $\psi_{\text{SLM}}$ was blocked and we observed the double topological charge $l_{2\omega} = 2l_{\text{Mask}} = 2$ as demonstrated by the second order HG profile captured by the CCD camera. In figure 2(b), $\psi_{\text{Mask}}$ was blocked and we observed the fourth order HG profile as expected from $l_{2\omega} = 2l_{\text{SLM}} = 4$. In figure 2(c), HWP-3 was oriented to leave unchanged the incoming polarizations and both beams were allowed to pass through the crystal. Topological charge addition is clearly demonstrated by the third order HG profile displayed at the CCD camera, corresponding to $l_{2\omega} = l_{\text{Mask}} + l_{\text{SLM}} = 3$. This run of the experiment shows how polarization can be used as an auxiliary parameter to switch between different topological charge operations. Of course, the blocking mechanism can also be easily replaced by polarization control before each input of PBS-2. For example, by switching its polarization from horizontal (vertical) to vertical (horizontal) before PBS-2, one can set $\psi_{\text{Mask}}$ ($\psi_{\text{SLM}}$) in or out the second harmonic generation (SHG) process. Therefore, the topological charge operation (doubling or addition) can be completely controlled by polarization means. This is of practical relevance since polarization can be easily modulated at fast rates with electro-optical devices.

In the second run of the experiment, HWP-3 was set to 0° in order to test the topological charge addition for $-2 \leq l_{\text{SLM}} \leq 2$. The results displayed in figure 3 show that the topological charge produced in the second harmonic field is determined by the number of nodal lines and the orientation of the HG profiles. In all cases $l_{\text{Mask}} = 1$. (a) $l_{\text{SLM}} = -2$, $l_{2\omega} = -1$. (b) $l_{\text{SLM}} = -1$, $l_{2\omega} = 0$. (c) $l_{\text{SLM}} = 0$, $l_{2\omega} = 1$. (d) $l_{\text{SLM}} = 1$, $l_{2\omega} = 2$. (e) $l_{\text{SLM}} = 2$, $l_{2\omega} = 3$. The SHG topological charge is consistent with $l_{2\omega} = l_{\text{Mask}} + l_{\text{SLM}}$. 

Figure 2. Arbitrary topological charge addition on type II second harmonic generation. (a) $l_{2\omega} = 2 l_{\text{Mask}} = 2$, (b) $l_{2\omega} = 2 l_{\text{SLM}} = 4$, (c) $l_{2\omega} = l_{\text{Mask}} + l_{\text{SLM}} = 3$.

| $l_{\text{SLM}}$ | $l_{\text{Mask}}$ | -2 | -1 | 0 | 1 | 2 |
|-----------------|-----------------|----|----|---|---|---|
| 1               | a-)             | b-)| c-)| d-)| e-)|

Figure 3. Topological charge addition in type II SHG. To turn the polarizations of $\psi_{\text{Mask}}$ and $\psi_{\text{SLM}}$ by 45°, we could block (B) either one of the incoming beams and observe the charge doubling of the vortex allowed to pass through the crystal. In figure 2(a), $\psi_{\text{SLM}}$ was blocked and we observed the double topological charge $l_{2\omega} = 2l_{\text{Mask}} = 2$ as demonstrated by the second order HG profile captured by the CCD camera. In figure 2(b), $\psi_{\text{Mask}}$ was blocked and we observed the fourth order HG profile as expected from $l_{2\omega} = 2l_{\text{SLM}} = 4$. In figure 2(c), HWP-3 was oriented to leave unchanged the incoming polarizations and both beams were allowed to pass through the crystal. Topological charge addition is clearly demonstrated by the third order HG profile displayed at the CCD camera, corresponding to $l_{2\omega} = l_{\text{Mask}} + l_{\text{SLM}} = 3$. This run of the experiment shows how polarization can be used as an auxiliary parameter to switch between different topological charge operations. Of course, the blocking mechanism can also be easily replaced by polarization control before each input of PBS-2. For example, by switching its polarization from horizontal (vertical) to vertical (horizontal) before PBS-2, one can set $\psi_{\text{Mask}}$ ($\psi_{\text{SLM}}$) in or out the second harmonic generation (SHG) process. Therefore, the topological charge operation (doubling or addition) can be completely controlled by polarization means. This is of practical relevance since polarization can be easily modulated at fast rates with electro-optical devices.

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| $l_{2\omega}$ | -1 | 0 | 1 | 2 | 3 |
|--------------|----|---|---|---|---|
| a-)          | b-)| c-)| d-)| e-)|

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Figure 3. Topological charge addition in type II SHG. The value of the topological charge produced in the second harmonic field is determined by the number of nodal lines and the orientation of the HG profiles. In all cases $l_{\text{Mask}} = 1$. (a) $l_{\text{SLM}} = -2$, $l_{2\omega} = -1$. (b) $l_{\text{SLM}} = -1$, $l_{2\omega} = 0$. (c) $l_{\text{SLM}} = 0$, $l_{2\omega} = 1$. (d) $l_{\text{SLM}} = 1$, $l_{2\omega} = 2$. (e) $l_{\text{SLM}} = 2$, $l_{2\omega} = 3$. The SHG topological charge is consistent with $l_{2\omega} = l_{\text{Mask}} + l_{\text{SLM}}$. 

| $l_{\text{SLM}}$ | $l_{\text{Mask}}$ | -2 | -1 | 0 | 1 | 2 |
|-----------------|-----------------|----|----|---|---|---|
| 1               | a-)             | b-)| c-)| d-)| e-)|

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charges of the orthogonally polarized vortices are added in the second harmonic. The sign of the topological charge determines the orientation of the HG profile created by the tilted lens, depending on the sign of the tilt angle. In our case, positive topological charges were converted into HG profiles oriented at $-45^\circ$. In Figure 3(a) we observe a first order HG profile oriented at $+45^\circ$, corresponding to the up converted topological charge $l_{2\omega} = -1$, which is to be compared with the image displayed in Figure 3(c), showing a first order HG profile oriented at $-45^\circ$, corresponding to $l_{2\omega} = +1$. The image displayed in Figure 3(b) does not exhibit the nodal line characteristic of high order HG profiles, it corresponds to $l_{2\omega} = 0$. Finally, Figures 3(d) and (e) show the results corresponding to $l_{2\omega} = 2$ and 3, respectively. In all cases, the relation $l_{2\omega} = l_{\text{Mask}} + l_{\text{SLM}}$ is verified.

5. Conclusion

In conclusion, we used polarization as an auxiliary parameter to combine different optical vortices in second harmonic generation and achieve arbitrary topological charge addition. The well known charge doubling effect occurs as a particular instance of this nonlinear process. The use of polarization allows for topological charge addition with a single frequency input in collinear geometry. The choice between charge doubling or addition can be fully controlled by polarization means, what provides an important simplification with respect to other setups since polarization can be easily modulated at high rates with electro-optical devices. For example, it can be useful for wave computation with discrete variables, as suggested in [47].

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