Evolution of magnetic field and spin period in accreting neutron stars

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Abstract. Based on the accretion-induced magnetic field decay model, in which a frozen field and an incompressible fluid are assumed, we obtain the following results: (1) an analytic relation between the magnetic field and spin period, if the fastness parameter of the accretion disk is neglected (The evolutionary tracks of accreting neutron stars in the P-B diagram in our model are different from the equilibrium period lines when the influence of the fastness parameter is taken into account.); (2) the theoretical minimum spin period of an accreting neutron star is \(1.1 \text{ms} \left(\frac{\Delta M}{M_\odot}\right)^{-1} R_6^{-5/14} I_{45}^{1/2} \), \(1.1 \text{ms} \left(\frac{\Delta M}{M_\odot}\right)^{-1/2} R_6^{17/14} \), independent of the accretion rate (X-ray luminosity) but dependent on the total accretion mass, \(\Delta M\); however, the minimum magnetic field depends on the accretion rate; (3) the magnetic field strength decreases faster with time than does the period.

Key words: stars: magnetic fields – stars: neutron – stars: rotation

1. Introduction

The formation of the magnetic field of a neutron star has long been a complex issue and one which is as yet unsolved (Bhattacharya & van den Heuvel 1991; Chanmugam 1992; Phinney & Kulkarni 1994). There is not yet a commonly accepted model on the evolution of the magnetic field of a neutron star (for a general review cf. Bhattacharya & Srinivasan 1995). The currently popular idea seems to ascribe the field decay of a neutron star in an X-ray binary to the period during which accretion occurs. There is evidence that the magnetic fields of X-ray neutron stars and recycled pulsars are correlated with the duration of the mass accretion phase, or the total amount of matter accreted (Taam & van den Heuvel 1986; van den Heuvel et al. 1986).

In fact, Taam and van den Heuvel have already discovered a possible inverse correlation between the magnetic field and the estimated total mass of accreted matter for the binary X-ray sources. Later, Shibazaki et al. (1989) presented an assumed formula relating the decay of the magnetic field with accretion mass, which seems to reproduce the observed field-period relations of the recycled pulsars quite well.

Theoretically, for explaining the accretion induced field decay, some suggestions and models have been proposed (e.g. Bisnovatyi-Kogan & Komberg 1974; Romani 1990; Ruderman 1991a,b,c; Ding et al. 1993; Zhang et al. 1994; Urpin & Geppert 1995; Geppert et al. 1996; Urpin & Konenkov 1997; Zhang et al. 1997; Cheng & Dai 1997; Zhang 1998; Ruderman, Zhu & Chen 1998). Recently, van den Heuvel & Bitzaraki (1995a, 1995b), discovered a clear correlation between spin period and orbital period, as well as between the magnetic field and orbital period from the statistical analysis of 24 binary radio pulsars with nearly circular orbits and low mass companions. These relations strongly suggest that an increase in the amount of accreted mass leads to a decay of the magnetic field, and a “bottom” field strength of about \(10^8 \text{G} \) is also implied. White & Zhang (1997) discovered that the spin periods of LMXBs, implied by kilohertz X-ray QPO, constitute a homogenous group with a spin period of about 2 milliseconds, which has little correlation with X-ray luminosity. The above two recent observational statistics seem to place some constraints on the construction of a theoretical model of accretion induced magnetic field decay.

In this paper, we study the evolution of a magnetic field and spin period of accreting neutron stars according to an accretion induced magnetic field decay model (Zhang et al. 1995; Cheng & Zhang 1998), based on the idea of van den Heuvel & Bitzaraki (1995a, 1995b) and Romani (1990) for the mechanism of accretion induced field decay. In this model, the accretion matter starts to be channeled onto the two polar caps by the strong magnetic field near the Alfven radius. Part of the accreted matter flowing towards the equator pushes the field lines aside and thus dilutes the polar field strength. The bottom field should be reached when the polar cap extends over the entire stellar surface, which corresponds to the Alfven radius matching the star’s radius, and gives a stellar magnetic field of about \(10^8 \text{G} \). Since the spin period of an accreting X-ray neutron star depends sensitively on the magnetic field together with the influence of the fastness parameter, some interesting results can be obtained which are consistent with the recent observational data on low mass X-ray neutron stars by White & Zhang (1997) and on the millisecond pulsars by van den Heuvel & Bitzaraki (1995a, 1995b).
2. Models

Under the assumption that the magnetic field lines of the neutron star are frozen in the entire crust (which has homogenous average mass density), we have obtained an analytical expression for the field evolution as follows (Cheng & Zhang 1998),

\[
B = \frac{B_f}{1 - C \exp\left(-\frac{\Delta M}{M_{cr}}\right)^{7/4}}\]  

(1)

where \( C = 1 - x_0^2 \) and \( x_0 = \left(\frac{B_0}{B_f}\right)^{2/7} \). \( B_0 \) is the initial magnetic field strength, \( M_{cr} \) is the crustal mass, \( \Delta M = M t \) and \( B_f \) is the magnetic field defined by the Alfvén radius matching the radius of the neutron star, i.e., \( R_A(B_f) = R \), which gives,

\[
B_f = 4.3 \times 10^8 \left(\frac{\dot{M}}{M_{Ed}}\right)^{1/2} \left(\frac{M}{M_\odot}\right)^{1/4} R_6^{-5/4} \ G,
\]  

(2)

where \( \dot{M}_{Ed} = 10^{18} \dot{M}_{Edd} \) is the Eddington accretion rate and \( R_6 \) is the radius of the neutron star in units of \( 10^6 \) cm.

Three consistent observational conclusions can be obtained, (1) the field decay is inversely related to the accreted mass, (2) the bottom field strength is about \( 10^5 \) Gauss, and (3) the bottom field strength is proportionally related to the X-ray luminosity. On the basis of the above solution, we will study the spin period evolution of accreting neutron stars.

To acquire the magnetic field versus period relation, we use the formula for the variation of the rotation due to accretion given by Ghosh & Lamb (1979, hereafter GL)

\[
-\dot{P} = 5.0 \times 10^{-5} \left(\frac{\dot{\chi}}{6}\right)^{3/7} R_6^{12/7} I_4^{12/7} \]  

(3)

where \( B_{f,12} \) is the surface field in units of \( 10^{12} \) G, \( I_45 \) is the moment of inertia in units of \( 10^{45} \) cm\(^2\), \( L_{37} \) is the X-ray brightness in units of \( 10^{37} \) erg s\(^{-1}\) and \( n(\omega_s) \) is a dimensionless function that depends primarily on the fastness parameter,

\[
\omega_s = 1.35 \left[ \left(\frac{\dot{M}}{M_\odot}\right)^{-2} R_6^{5/7} I_4^{12/7} B_{f,12}^{16/7} \right]^{-3/7} \]  

(4)

For a star rotating slowly in the same sense as the disk flow \((\omega_s < 1)\), GL found that \( n(\omega_s) \sim 1.4 \). They also found that the dimensionless function \( n(\omega_s) \) decreases with increasing \( \omega_s \) and becoming negative for \( \omega_s > \omega_c \). A simple expression for \( n(\omega_s) \) that agrees approximately with numerical results over the whole range of \( \omega_s \) is,

\[
n(\omega_s) = 1.4 \times \left(\frac{1 - \omega_s/\omega_c}{1 - \omega_s}\right).\]  

(5)

GL found \( \omega_c \sim 0.35 \) from their model, but stressed that the actual value of this critical fastness parameter was relatively uncertain. Subsequent work (Ghosh & Lamb 1991) indicates that \( \omega_c \) is unlikely to be less than 0.2, but it could be as large as 0.9. In the following subsections, we study the influence of the field decay on the evolution of the spin period of the neutron star in an X-ray binary.

2.1. Spin evolution with a constant fastness parameter

If the variation of the fastness parameter can be ignored, i.e. \( n(\omega_s) = 1 \), we can solve Eq.(3) analytically. First we can rewrite Eq.(3) as

\[
\dot{P} = -5.0 \times 10^{-5} \left(\frac{\dot{\chi}}{6}\right)^{3/7} R_6^{12/7} I_4^{12/7} \]  

where \( B_{f,12} = B_f / 10^{12} G \) and \( \dot{\chi} = B_f B_f \). From Eq.(1), we obtain

\[
d\tau = -\frac{4 M_{Ed}}{7 M_{Edd}} \frac{dt}{\tau} = \frac{dt}{\tau} = \frac{1}{2 \pi \sqrt{\Gamma_7} B_{f,12}^{5/7} R_6^{6/7} I_4^{6/7} z^{2/7}} \]  

where \( z = 2^{2/7} \). Using Eq.(2) to eliminate \( B_f \), assuming mass and radius as constants and the initial period condition \( P_0 = \infty \), we obtain the field-period relation (B-P) in the following analytic form,

\[
P = \frac{1.5 \text{ ms}}{\text{atanh}(x) - \text{atanh}(x_0)} \left(\frac{M}{M_\odot}\right)^{-1/2} R_6^{-5/14} I_4\left(M_{cr}/0.1 M_\odot\right)^{-1} \]  

(6)

where \( x = y^{-1} = (\frac{B_f}{B_f})^{2/7}, x_0 = (\frac{B_0}{B_f})^{2/7} \) and \( \text{atanh}(x) = [\ln(1 + x) - \ln(1 - x)]/2 \). There are two interesting limits for Eqs.(1) and (6). For \( \Delta M \ll M_{cr} \), Eqs. (1) and (6) can be approximated as

\[
B \approx B_f \frac{M_{cr}}{\Delta M} \propto t^{-1} \]  

(7)

and

\[
P \approx \frac{1.5 \text{ ms}}{x} \left(\frac{M}{M_\odot}\right)^{-1/2} R_6^{-5/14} I_4\left(M_{cr}/0.1 M_\odot\right)^{-1} \]  

\times t^{-2/7}. \]  

(8)

For \( \Delta M \gg M_{cr} \), Eqs. (1) and (6) can be approximated as

\[
P \approx 0.1 \text{ms} \left(\frac{\Delta M}{M_\odot}\right)^{-1/2} R_6^{-5/14} I_4\left(M_{cr}/0.1 M_\odot\right)^{-1/2} \]  

(9)

and

\[
P \approx 1.1 \text{ms} \left(\frac{\Delta M}{M_\odot}\right)^{-1} R_6^{-5/14} I_4\left(M_{cr}/0.1 M_\odot\right)^{-1/2}. \]  

(10)

We should note that the period of the neutron star cannot be shorter than the equilibrium spin-up line which represents the minimum period (Bhattacharyya & van den Heuvel 1991)

\[
P_{eq} = 2.4 \text{ms} B_9 \left(\frac{M}{M_\odot}\right)^{-3/7} \left(\frac{M}{M_\odot}\right)^{-5/7} R_6^{16/7}, \]  

(11)

to which such a spin-up may proceed at the Eddington accretion rate. If we substitute the minimum B field in Eq.(11), we obtain the minimum equilibrium period,

\[
P_{eq_{min}} = 1.1 \text{ms} \left(\frac{M}{M_\odot}\right)^{-1/2} R_6^{17/14}. \]  

(12)

However, we want to point out that Eq.(12) is valid only in cases where the neutron star has accreted a sufficient amount of
matter to spin-up to that period. For a given amount of accreted matter, the minimum period of the neutron star is

$$P_{\text{min}} = \max(1, \left(\frac{\Delta M}{M_\odot}\right)^{-1} R_6^{-5/14} I_{45}^{-1/2} $$

an expression which is independent of the accretion rate (X-ray luminosity) but depends on the total accretion mass $\Delta M$.

We plot the field-period relation in Fig. 1, which shows the evolutionary track curves of the magnetic field and spin period (curve 1 and curve 3). Solid diamonds in Fig. 1 are observed data summarized in Table 1. Initially, a small amount of mass is transferred, and the neutron star is spun-up from the death valley (Chen & Ruderman 1993) where it has a long period, which causes a modest field decay and produces systems such as PSR0655+64 and PSR1913+16. The binaries with longer-lived accretion phases, e.g. LMXB, will accept sufficient mass from their companions and yield a substantial field decay, as in the case of millisecond pulsars such as PSR1953+10 and PSR1620+21. Our analytic model B-P curves, which are obtained by assuming $n(\omega) = 1$, go beyond the spin-up line (the equilibrium period line). This results from the fact that the influence of the fastness parameter is neglected, therefore the spin-up torque still exists even when the spin of the neutron star is faster than the Keplerian angular velocity of the accretion disk at the inner radius. From the analytical P-B relation, we find that the minimum period can be obtained if the magnetic field reaches the bottom field strength (cf. Eq. 6). The maximum mass accreted in LMXB from the companion could be $\sim 1.0 M_\odot$ (van den Heuvel & B Zurzaki 1995a, 1995b), the mass of the neutron star after accretion could reach $\sim 2.4 M_\odot$ with a radius $R_6 \sim 1$ for realistic equations of state (cf. Table I of Cheng & Dai 1997) and $I_{45} \propto M$. The minimum spin period given by Eq. (10) is about $P_{\text{min}} \sim 1.7$ ms. Further, unlike the bottom field, the minimum period is independent of the accretion rate (X-ray luminosity) (cf. Eqs. (10) and (12)). Our expression seems to be supported by the recent work of White &

### Table 1. Parameters of binary neutron stars

| Source | $B (G)$ | $P (s)$ | $\Delta M (M_\odot)$ | Type | Refs |
|--------|---------|---------|----------------------|------|------|
| J034+05 | $1.1 \times 10^8$ | 0.8188 | 0.7 | L | a |
| J0347-47 | $4.9 \times 10^8$ | 0.8576 | 0.77 | L | a |
| B0655+64 | $1.2 \times 10^{10}$ | 0.196 | 0.003/0.003 | H | b,d |
| B0820+32 | $3 \times 10^{10}$ | 0.864 | 0.005/0.001 | H | b,d |
| J1004+52 | $3 \times 10^{10}$ | 0.0875 | 0.7 | L | a |
| B1239+26 | $3 \times 10^{10}$ | 0.0111 | 0.45 | L | a,c,e |
| J1712+67 | $2 \times 10^{9}$ | 0.08457 | 0.65 | L | a |
| B1718-19 | $1.28 \times 10^{12}$ | 1.04 | 0.7 | L | a |
| B1806-27 | $7.7 \times 10^{10}$ | 0.3344 | 0.18 | L | a |
| B1833+09 | $8.7 \times 10^{10}$ | 0.5209 | 0.8 | L | a |
| B1835+09 | $3 \times 10^{10}$ | 0.0656 | 0.73 | L | a |
| B1914+14 | $2 \times 10^{10}$ | 0.0859 | 0.003/0.003 | H | b,c,e |
| B1930+29 | $4.3 \times 10^{10}$ | 0.06613 | 0.03/0.039 | L | a,b,c,e |
| B1957+29 | $1.66 \times 10^{8}$ | 0.0161 | 0.65 | L | a,c,e |
| J2019+24 | $1.82 \times 10^{8}$ | 0.000393 | 0.65 | L | a,c,e |
| J2317+24 | $1.27 \times 10^{8}$ | 0.00345 | 0.79 | L | a,c,e |
| A0535-66 | $> 10^{11}$ | 0.069 | 0.002 | H | b |
| Her X-1 | $3.0 \times 10^{12}$ | 1.24 | 0.001 | L | H | b,f |
| 1E2259+59 | $5 \times 10^{11}$ | 7.3 | 0.1 | L | H | b,f |
| GX1+4 | $> 3 \times 10^{11}$ | 128 | 0.0001 | L | b |
| GX1+4 | $6 \times 10^{9}$ | 0.01 | 0.02/0.5 | L | b |
| Cen X-3 | $6 \times 10^{9}$ | 0.01 | 0.02/0.5 | L | b |
| Sco X-1 | $5 \times 10^{10}$ | 0.0082 | 0.05/0.8 | L | b |
| MXB1736-335 | $7 \times 10^{10}$ | 0.7 | 0.1/0.8 | L | b |

a: Van den Heuvel & B Zurzaki 1995
b: Taam and van den Heuvel 1986
c: Lorimer et al. 1996, d: Camilo et al. 1996
e: Taylor et al. 1991, f: Mikishima 1992
g: Bhattacharya et al. 1994, h: Bell et al. 1997, i: Corbet et al. 1995
Zhang (1997). They find that the luminosities in the ten samples of QPO LMXB vary by two order of magnitude from \( L_{36} = 1 \) to \( L_{36} = 1 \), but the spin periods of the sources diffuse into a narrow region from 2.76 ms to 3.8 ms.

2.2. Spin evolution with a non-constant fastness parameter

Numerical solutions (curve 2 and curve 4) for Eqs. (1), (3), (4) and (5), where the critical fastness parameter is set at 0.9, are plotted in Fig. 1. However, we find that the influence of the fastness parameter has little effect in the low magnetic field region. The main effect of the fastness parameter is to force the evolution curves Back or below the equilibrium period line. The evolutionary curves in the P-B diagram, however, are not sensitive to the fastness parameter near the bottom field and/or near the minimum period. In Fig. 1, these curves show that the field decay time scale is longer than the spin-up time scale because the field reaches the bottom value first. The neutron star then evolves towards the minimum spin period horizontally in the P-B diagram. The numerical solution are consistent with our analytic expressions in Eqs. (7) and (8).

However, the fastness parameter effect is important when the evolutionary track is close to the equilibrium period line, which means that the spin angular velocity of the star matches the Keplerian angular velocity at the inner edge of the accretion disk, and the accretion torque produced by magnetic lines immersed in the accretion disk tends to produce the negative (spin-down) torque. This effect ensures that the evolution track cannot go beyond the equilibrium period line. It is interesting to note that none of the model B-P curves go along the equilibrium period line, which may seem confusing. However, the equilibrium period line only represents the final position of the evolution track in the P-B diagram. Therefore it is not surprising that the real evolutionary track deviates from the equilibrium period line. Physically, the accretion induced field decay arises from the contraction of the co-rotation radius of the magnetosphere during the accretion spin-up phase, and the evolution track has little chance to meet the equilibrium line if the field decay time scale is shorter than the spin-up time scale at the early stage of the accretion phase. Some X-ray sources in HMXB such as Her X-1 and Vela X-1 should be very close to the equilibrium period line.

3. Conclusion

We have presented a simple model for the evolution of the magnetic field and spin period of accreting neutron stars. Analytic formulae for evolution trajectories in B-P are derived. The theoretical minimum period of the neutron star does not depend on the accretion rate but instead depends on the total amount of accreted matter, and the stellar parameters (including the moment of inertia, stellar mass and radius, which depend on the equations of state). Our model results seem to be supported by the observed data (White & Zhang 1997). However, in this paper we have ignored the fact that the stellar parameters, i.e. \( M \), \( R \) and \( I \), are all time dependent. The exact evolution curves must take this factor into account. On the other hand, the minimum period should not depend on the details of the evolution trajectories; it only depends on the final values of the stellar parameters which are equations of state dependent. Observing the minimum period in LMXB may provide useful constraints on the equation of state for high density matter. For \( \Delta M < 1 M_\odot \), the minimum period is longer than 1.7 ms for a wide range of realistic equations of state (Wirninga et al. 1988).

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