Magnetic-dipole transition probabilities in B-like and Be-like ions

I. I. Tupitsyn, A. V. Volotka, D. A. Glazov, V. M. Shabaev,
G. Plunien, J. R. Crespo López-Urrutia, A. Lapierre, and J. Ullrich

1 Department of Physics,
St. Petersburg State University,
Oulianovskaya 1, Petrodvorets,
198504 St. Petersburg, Russia
2 Institut für Theoretische Physik,
TU Dresden, Mommsenstraße 13,
D-01062 Dresden, Germany
3 Max-Planck Institut für Physik Komplexer Systeme,
Nöthnitzer Straße 38,
D-01187 Dresden, Germany
4 Max-Planck Institut für Kernphysik,
Saupfercheckweg 1,
D-69117 Heidelberg, Germany

Abstract

The magnetic-dipole transition probabilities between the fine-structure levels $(1s^22s^2p)\ ^2P_{1/2} - \ ^2P_{3/2}$ for B-like ions and $(1s^22s2p)\ ^3P_1 - \ ^3P_2$ for Be-like ions are calculated. The configuration-interaction method in the Dirac-Fock-Sturm basis is employed for the evaluation of the interelectronic-interaction correction with negative-continuum spectrum being taken into account. The $1/Z$ interelectronic-interaction contribution is derived within a rigorous QED approach employing the two-time Green function method. The one-electron QED correction is evaluated within framework of the anomalous magnetic-moment approximation. A comparison with the theoretical results of other authors and with available experimental data is presented.

PACS numbers: 32.70Cs
I. INTRODUCTION

During the last years, the precision of measurements of magnetic-dipole (M1) transitions between the fine-structure levels in highly charged ions has been continuously increased [1, 2, 3, 4, 5, 6, 7, 8]. Since in some cases the M1 transitions are sufficiently sensitive to relativistic-correlation and quantum-electrodynamic (QED) effects, this provides good prospects for probing their influences on atomic transition probabilities.

To date, a vast number of theoretical calculations of M1-transition probabilities between the fine-structure levels in highly charged ions has been performed (see, e.g., Refs. [9, 10, 11]). However, none of these works have provided a systematic analysis of various effects on the transition probability. Such an analysis for the \((1s^22s^22p)^2P_{1/2} - 2P_{3/2}\) transition in B-like ions and for the \((1s^22s2p)^3P_1 - 3P_2\) transition in Be-like ions is given in the present paper.

To calculate the decay rate one requires knowledge of the transition energy and the matrix element of the transition operator. Within this work we employ experimental values of the transition energy, which are measured accurately enough for the ions under consideration.

To analyse the influence of various effects, we decompose the transition probability \(W^{i\rightarrow f}\) into several terms,

\[
W^{i\rightarrow f} = W_{nr}^{i\rightarrow f} + \Delta W_D^{i\rightarrow f} + \Delta W_{CI}^{i\rightarrow f} + \Delta W_{neg}^{i\rightarrow f} + \Delta W_{QED}^{i\rightarrow f} + \Delta W_{freq}^{i\rightarrow f}.
\]

Here \(W_{nr}^{i\rightarrow f}\) represents the nonrelativistic M1-transition probability derived employing the LS-coupling scheme. Within the LS-coupling scheme, the amplitude of the magnetic-dipole transition is nonzero only between the fine-structure levels and depends on the quantum numbers \(L, S, \) and \(J\) of the initial and the final state [12]. This implies that the contribution of the interelectronic-interaction vanishes in the nonrelativistic limit. The explicit expression for \(W_{nr}^{i\rightarrow f}\) is presented in Section II.

The relativistic correction \(\Delta W_D^{i\rightarrow f}\) is obtained by employing the one-electron Dirac wave functions for the initial and the final state. For the relativistic case the interelectronic-interaction contribution is nonzero, but it is generally suppressed by a factor \((\alpha Z)^2/Z\). For instance, in case of B-like Ar it amounts to about 0.1%. The interelectronic-interaction correction is, however, rather important for the \((1s^22s2p)^3P_1 - 3P_2\) transition in Be-like ions, where the terms \(^3P_1\) and \(^1P_1\) are strongly mixed. In this investigation two approaches are employed for evaluating the interelectronic-interaction correction. The first one is based on the configuration-interaction (CI)
method in the Dirac-Fock-Sturm basis, whereas the second one employs perturbation theory with respect to $1/Z$. Utilizing the CI method the relativistic Hamiltonian is specified within the no-pair approximation \cite{13, 14, 15}. The corresponding contribution to the M1-transition probability is denoted by $\Delta W_{\text{CI}}^{i \rightarrow f}$. The evaluation of this term is described in Section \textbf{III}.

The no-pair Hamiltonian does not account for the negative-energy excitations in the many-electron wave function. However, this effect, being dependent on the choice of the one-electron basis, can become significant \cite{16, 17}. In Section \textbf{IV} the contribution due to the negative-spectrum $\Delta W_{\text{neg}}^{i \rightarrow f}$ is derived.

In Section \textbf{V} the interelectronic-interaction correction of first order in $1/Z$ is evaluated within a rigorous QED approach employing the two-time Green function method \cite{18}. Together with verifying the terms $\Delta W_{\text{CI}}^{i \rightarrow f}$ and $\Delta W_{\text{neg}}^{i \rightarrow f}$ to first order in $1/Z$, this provides the contribution $\Delta W_{\text{freq}}^{i \rightarrow f}$, which incorporates the $1/Z$ interelectronic-interaction corrections of higher orders in $\alpha Z$.

Finally, $\Delta W_{\text{QED}}^{i \rightarrow f}$ is the QED correction. The evaluation of this correction to the lowest orders in $\alpha$ and $\alpha Z$ is described in Section \textbf{VI}.

The main goal of the present work is to evaluate the lifetimes of the states $(2s^22p)^2P_{3/2}$ in B-like ions and $(2s2p)^3P_2$ in Be-like ions to utmost accuracy and to investigate the influence of various effects on the M1-transition probability. The corresponding analysis is presented in Section \textbf{VII}.

Atomic units ($\hbar = e = m = 1$) are used throughout the paper.

\section{Magnetic-Dipole Transition Probability}

The spontaneous $L$-pole transition probability from the initial state $i$ to the final state $f$ reads \cite{19}

$$ W_{L}^{i \rightarrow f} = \frac{2\pi}{2J_i + 1} \sum_{M_i} \sum_{M_f} \sum_{M} |A_{LM}|^2, \hspace{2cm} (1) $$

where the initial state has the angular momentum $J_i$, its $z$-projection $M_i$, and the energy $E_i$, and $J_f$, $M_f$, $E_f$ denote the corresponding quantum numbers and the energy of the final state. The transition amplitude $A_{LM}$ is defined as

$$ A_{LM} = i^{L+1} \sqrt{\frac{\omega}{\pi c}} \sqrt{2L + 1} \langle f | T^L_M | i \rangle. \hspace{2cm} (2) $$

Here $T^L_M$ denote the components of the multipole transition operator $T^L$, which is a spherical tensor of rank $L$. In case of a magnetic transition, $T^L$ is proportional to the tensor product of the
Dirac-matrix vector $\alpha$ and the spherical tensor $C^L_M = \sqrt{4\pi/(2L+1)} Y_{LM}$ \[19\]

$$T^L_M = -i j_L(\omega r/c) (\alpha \otimes C^L_M),$$ \[3\]

where $j_L$ is the spherical Bessel function and $\omega = E_i - E_f$ is the frequency of the emitted photon.

The magnetic transition probability can be expressed in terms of the reduced matrix element of $T^L_M$

$$W^{i \rightarrow f}_L = \frac{2(2L+1)\omega}{2J_f+1} |\langle f \parallel T^L \parallel i \rangle|^2.$$ \[4\]

For the magnetic-dipole transition ($L = 1$), the tensor product can be written in terms of the vector product

$$T^1 = \frac{1}{\sqrt{2}} j_1(\omega r/c) \frac{[\alpha \times r]}{r} = \frac{\sqrt{2}}{r} j_1(\omega r/c) \mu,$$ \[5\]

where $\mu = -e [r \times \alpha]/2$ is the relativistic magnetic moment operator. Taking into account the first term in the expansion of $j_1(\omega r/c)$ only and turning into the nonrelativistic limit, one derives the following relation between the M1-transition operator $T^1_{nr}$ and the magnetic moment operator $\mu_{nr}$

$$T^1_{nr} = \frac{\sqrt{2}}{3} \frac{\omega}{c} \mu_{nr}.$$ \[6\]

The nonrelativistic magnetic moment operator is given by

$$\mu_{nr} = -\mu_B (L + 2S),$$ \[7\]

where $L$ and $S$ are the orbital and spin angular momentum operators, respectively, and $\mu_B = |e|\hbar/2mc$ denotes Bohr magneton.

In the $LS$-coupling scheme, which is realized in the nonrelativistic case, the magnetic-dipole transition probability is nonzero only between fine-structure levels with $\Delta J = \pm 1$ \[12\]. The reduced matrix element of $T^1_{nr}$ within the $LS$-coupling is given by

$$\langle J_f \parallel T^1_{nr} \parallel J_i \rangle = -\frac{\sqrt{2}}{3} \frac{\omega}{c} \mu_B \langle J_f \parallel (J + S) \parallel J_i \rangle = -\frac{\sqrt{2}}{3} \frac{\omega}{c} \mu_B \langle J_f \parallel S \parallel J_i \rangle.$$ \[8\]

Utilizing the general formula for the reduced matrix element of the spin operator \[20\] yields the corresponding expression for the transition probability

$$W^{i \rightarrow f}_{nr} = \frac{4\omega^3}{3c^3} \mu_B^2 \delta_{L_i,L_f} \delta_{S_i,S_f} S_i(S_i + 1)(2S_i + 1)(2J_f + 1) \left\{ \begin{array}{c} S_i \\ L_i, L_f, J_i \\ 1, S_i \end{array} \right\}^2.$$ \[9\]
In particular, for the $2s^22p_{3/2} \rightarrow 2s^22p_{1/2}$ transition one can easily find

$$W_{nr}^{i \rightarrow f} = \frac{4\omega^3}{9c^3} \mu_B = \frac{1}{3\lambda^3} \times 2.6973500 \times 10^{13} \, [\text{s}^{-1}],$$

(10)

where $\lambda$ is the transition wavelength, in Å. Thus, in the nonrelativistic limit the magnetic-dipole transition probability is completely determined by the quantum numbers of the initial and final states.

III. INTERELECTRONIC INTERACTION IN THE BREIT APPROXIMATION

To evaluate the interelectronic-interaction contributions, we start with the relativistic Hamiltonian in the no-pair approximation,

$$H_{\text{np}} = \Lambda_+ H \Lambda_+, \quad H = \sum_i h^D(i) + \sum_{i<j} V(i,j),$$

(11)

where $h^D(i)$ is the one-particle Dirac Hamiltonian and the index $i = 1, \ldots, N$ enumerates the electrons. The Coulomb-Breit interaction operator $V(i,j) = V_C(i,j) + V_B(i,j)$ is specified in coordinate space as

$$V_C(i,j) = \frac{1}{r_{ij}}, \quad V_B(i,j) = -\frac{\alpha_i \cdot \alpha_j}{r_{ij}} \cdot \frac{1}{2}(\alpha_i \cdot \nabla_i)(\alpha_j \cdot \nabla_j)r_{ij}.$$  

(12)

The frequency-dependent part of the full QED interaction operator, which is beyond the Breit approximation and gives rise to the terms of higher orders in $\alpha Z$, will be considered in Section IV. $\Lambda_+$ is the projector on the positive-energy states, which can be represented as the product of the one-electron projectors $\lambda_+(i)$ as

$$\Lambda_+ = \lambda_+(1) \cdots \lambda_+(N)$$

(13)

together with

$$\lambda_+(i) = \sum_n |u_n(i)\rangle \langle u_n(i)|.$$  

(14)

Here $u_n$ are the positive-energy eigenstates of an effective one-particle Hamiltonian $h^u$

$$h^u u_n = \varepsilon_n u_n,$$

(15)

which can be taken to be the Dirac Hamiltonian $h^D$, the Dirac Hamiltonian in an external field or the Hartree-Fock-Dirac Hamiltonian in an external field [13, 14, 15].
In order to determine the space of one-electron functions \( \{ \varphi_n \}_{n=1}^M \), we employed the combined Dirac-Fock (DF) and the Dirac-Fock-Sturm (DFS) basis set. Here the index \( n \) enumerates different occupied and vacant one-electron states. For the occupied atomic shells, the orbitals \( \varphi_n \) with \( n = 1, \ldots, M_0 \) were obtained by the standard restricted Dirac-Fock (RDF) method, based on a numerical solution of the radial RDF equations [21, 22]. Only the Coulomb part \( V_C(i,j) \) of the Coulomb-Breit interaction operator (12) was included in the RDF Hamiltonian \( h_{DF} \).

The vacant orbitals \( \varphi_n \) with \( n = M_0 + 1, \ldots, M \) were obtained by solving the Dirac-Fock-Sturm equations

\[
\left[ h_{DF} - \varepsilon_{n_0} \right] \varphi_n = \xi_n W(r) \varphi_n ,
\]

which can be considered as a generalization of the method proposed in Ref. [23] to the relativistic Hamiltonian and to an arbitrary constant-sign weight function \( W(r) \). For every relativistic quantum number \( \kappa \) we choose an occupied DF function \( \varphi_{n_0} \), which we call as reference DF orbital and \( \varepsilon_{n_0} \) in (16) is the energy of this orbital. The parameter \( \xi_n \) in Eq. (16) can be considered as an eigenvalue of the Sturmian operator. Obviously, for \( \xi_n = 0 \) the Sturmian function coincides with the reference DF orbital \( \varphi_{n_0} \). If \( W(r) \to 0 \) at \( r \to \infty \), all Sturmian functions \( \varphi_n \) have the same exponential asymptotics at \( r \to \infty \). Therefore, the all set of eigenfunctions of the Dirac-Fock-Sturm operator forms a discrete set in the space of one-electron wave functions. The completeness of this basis in the nonrelativistic limit is well-known fact. In the relativistic case this problem is more complicated and we examined the completeness of the pure DFS basis, which we used in our many-electron atomic calculations, numerically, reproducing exact hydrogenlike wave functions for the same nuclear charge number \( Z \). It should be noted that the DFS orbitals are orthogonal with respect to the weight function \( W(r) \) and, therefore, form a linear independent basis set. The completeness and linear independence of the combined DF and DFS basis was also examined numerically.

In the nonrelativistic theory the widely used choice of the weight function is \( W(r) = 1/r \), which leads to the well-known “charge quantization”. In the relativistic case, however, this choice is not very suitable, since the behaviour of the Sturmian wave functions at the origin differs from that of the Dirac-Fock orbitals. In our calculations we employed the following weight function

\[
W(r) = \frac{1 - \exp[-(\alpha r)^2]}{(\alpha r)^2},
\]

which, unlike \( 1/r \), is regular at the origin.
To generate the one-electron wave functions $u_n$, we used the unrestricted DF (UDF) method in the joined DF and DFS basis,

$$u_n = \sum_m C_{mn} \varphi_m .$$  \hspace{1cm} (18)

The coefficients $C_{mn}$ were obtained by solving the HFD matrix equations

$$\hat{F} C_n = \varepsilon_n \hat{S} C_n ,$$  \hspace{1cm} (19)

where $\hat{F}$ is the Dirac-Fock matrix in the joined basis of DF and DFS orbitals of a free ion. If necessary, an arbitrary external field can be included in the $\hat{F}$ matrix. The matrix $\hat{S}$ is nonorthogonal, since the DFS orbitals are not orthogonal in the usual sense. The negative-energy DFS functions were included in the total basis set as well. Eq. (19) was used to generate the whole set of orthogonal one-electron wave functions $\{u_n\}_{n=1}^M$.

It should be noted that if even there is no external field in Eq. (19), the set of one-electron functions $\{u_n\}_{n=1}^M$ differs from the set of basis functions $\{\varphi_n\}_{n=1}^M$. For the occupied states, the UDF method accounts for core-polarization effects, in contrast to the RDF method. For the vacant states the difference is more significant, since the DF and DFS operators are inherently different.

The many-electron wave function $\Psi_+ (\gamma J M J)$ with quantum numbers $\gamma$, $J$, and $M_J$ is expanded in terms of a large set of configuration state functions (CSFs) $\Phi_\alpha (J M J)$

$$\Psi_+ (\gamma J M J) = \Lambda_+ \Psi (\gamma J M J) = \sum_\alpha c_\alpha \Phi_\alpha (J M J) .$$  \hspace{1cm} (20)

The standard configuration-interaction Dirac-Fock (CIDF) method is used to find the coefficients $c_\alpha$. The CSFs are constructed from the one-electron wave functions $u_n$ (18) as a linear combination of Slater determinants. The set of the CSFs is generated including all single, double, and triple excitations into one-electron states of the positive spectrum.

IV. NEGATIVE-CONTINUUM CONTRIBUTION

Due to some freedom in the choice of the wave function set $\{u_n\}$, the positive-energy subspace and the corresponding projector $\lambda_+$. Eq. (14) can be determined in different ways. This freedom can be used to find the optimum many-electron wave function $\Psi_{opt}$ within the variational method.

The energy determined by Hamiltonian (11) can be written as

$$E = \langle \Psi | H^{np} | \Psi \rangle = \langle \Psi_+ | H | \Psi_+ \rangle , \quad \Psi_+ = \Lambda_+ \Psi .$$  \hspace{1cm} (21)
The real orthogonal transformation (rotation) of the one-electron function space \( \{ u_n \} \) modifies the wave function \( \Psi_+ \) \[24\]

\[ \Psi' = \exp(T)\Psi_+ , \] (22)

where the operator \( T \) is antihermitian \( (T^\dagger = -T) \),

\[ T = \sum_{n<m} E_{nm} t_{nm} , \quad E_{nm} = a_{n}^\dagger a_{m} - a_{m}^\dagger a_{n} . \] (23)

Here \( a_{n}^\dagger \) and \( a_{n} \) are the creation and annihilation operators of electron in the \( u_{n} \) state. The matrix elements \( t_{nm} \) can be obtained from the variational principle. Then the wave function \( \Psi_{\text{opt}} \) satisfies the generalized Brillouin theorem \[25\]

\[ \langle \Psi_{\text{opt}} | [a_{n}^\dagger a_{m}, H] | \Psi_{\text{opt}} \rangle = 0 . \] (24)

This means that the optimum wave function \( \Psi_{\text{opt}} \) is invariable under the single excitations including negative-energy spectrum excitations. However, this does not hold for the wave function \( \Psi_+ \). Therefore, one should revise the calculation of the matrix element \( \langle \Psi_+ | A | \Psi_+ \rangle \) of any one-electron operator \( A \) by admixing the negative-energy spectrum excitations to \( \Psi_+ \). This is especially important for so-called “odd” operators, which mix the large and small components of the Dirac wave functions. The M1-transition operator \( T^{1} \) \[5\] is just of this kind. For this reason, the negative-continuum contribution can be significant and depends on the choice of the one-electron basis set \( \{ u_{n} \} \) \[16, 17\].

We consider two equivalent methods for evaluating the negative-continuum contribution to the matrix elements of a hermitian one-electron operator \( A \) with the wave functions \( \Psi_+ \). The first one is based on the Hellman-Feynman theorem whereas the second one employs the perturbation theory.

The space of the wave functions used to find \( \Psi_{\text{opt}} \) is invariant under the transformation \( U = \exp(iA) \), if \( A \) is a one-particle operator. Therefore, one can employ the Hellman-Feynman theorem \[26\] to obtain the expectation value of \( A \)

\[ \overline{A} = \left. \frac{\partial}{\partial \mu} \langle \Psi_{\text{opt}}(\mu) | H(\mu) | \Psi_{\text{opt}}(\mu) \rangle \right|_{\mu=0} , \quad H(\mu) = H + \mu A , \] (25)

where it is implied that \( \mu A \) is included into the one-particle Hamiltonian, \( h^{u}(\mu) = h^{u} + \mu A \). Since the wave function correction

\[ \delta \Psi = \Psi_{\text{opt}} - \Psi_+ = [1 - \exp(-T)] \Psi_{\text{opt}} \simeq - \sum_{n<m} E_{nm} t_{nm} \Psi_{\text{opt}} \] (26)
accounts for single excitations only, the generalized Brillouin theorem (24) yields
\[ \langle \delta \Psi(\mu) \mid H(\mu) \mid \Psi_{\text{opt}}(\mu) \rangle + \langle \Psi_{\text{opt}}(\mu) \mid H(\mu) \mid \delta \Psi(\mu) \rangle = 0 \] (27)

and, therefore,
\[ \overline{A} = \frac{\partial}{\partial \mu} \left[ \langle \Psi_+(\mu) \mid H(\mu) \mid \Psi_+(\mu) \rangle - \langle \delta \Psi(\mu) \mid H(\mu) \mid \delta \Psi(\mu) \rangle \right]_{\mu=0}. \] (28)

Neglecting the second quadratic term in the equation above yields
\[ \overline{A} \simeq \frac{\partial}{\partial \mu} \left[ \langle \Psi_+(\mu) \mid H(\mu) \mid \Psi_+(\mu) \rangle \right]_{\mu=0}. \] (29)

Thus, the negative-continuum contribution can be evaluated by means of the formula
\[ \Delta \overline{A}_{\text{neg}} = \frac{\partial}{\partial \mu} \left[ \langle \Psi_+(\mu) \mid H(\mu) \mid \Psi_+(\mu) \rangle \right]_{\mu=0} - \langle \Psi_+ \mid A \mid \Psi_+ \rangle. \] (30)

Alternative expression for this contribution can be obtained employing the perturbation theory.
Using the equation for the derivative of \( u_n(\mu) \)
\[ \left. \frac{\partial}{\partial \mu} u_n(\mu) \right|_{\mu=0} = \sum_{m \neq n} \frac{\langle u_m(0) \mid A \mid u_n(0) \rangle}{\varepsilon_n - \varepsilon_m} u_m(0), \] (31)
we obtain
\[ \Delta \overline{A}_{\text{neg}} = 2 \sum_n^{(\text{pos})} \sum_m^{(\text{neg})} \frac{\langle u_m \mid A \mid u_n \rangle}{\varepsilon_n - \varepsilon_m} \langle a_n^+ a_n \Psi_+ \mid H \mid \Psi_+ \rangle. \] (32)

Here the indices \((\text{pos})\) and \((\text{neg})\) indicate that the summation is carried out over the positive- and negative-energy spectrum, respectively.

For the nondiagonal matrix elements, one can derive
\[ \Delta A_{\text{neg}}^{i \to f} = \frac{\partial}{\partial \mu} \left[ \langle \Psi_i^f(\mu) \mid H(\mu) \mid \Psi_i^i(\mu) \rangle \right]_{\mu=0} - \langle \Psi_i^f \mid A \mid \Psi_i^i \rangle \] (33)
and
\[ \Delta A_{\text{neg}}^{i \to f} = \sum_n^{(\text{pos})} \sum_m^{(\text{neg})} \frac{\langle u_m \mid A \mid u_n \rangle}{\varepsilon_n - \varepsilon_m} \times \left[ \langle a_m^+ a_n \Psi_i^f \mid H \mid \Psi_i^i \rangle + \langle \Psi_i^f \mid H \mid a_m^+ a_n \Psi_i^i \rangle \right]. \] (34)

These formulas were used in our calculations of the negative-continuum contribution to the M1-transition amplitude. It was found that the results obtained by means of Eqs. (33) and (34) are in a perfect agreement with each other.
V. HIGHER-ORDER INTERELECTRONIC-INTERACTION CORRECTIONS

The rigorous QED treatment of the interelectronic-interaction corrections to the transition probabilities can be carried out utilizing the two-time Green function method \[18\]. In Ref. \[27\] it was done for the \(1/Z\) interelectronic-interaction corrections in He-like ions. Here we perform the corresponding calculations for B-like ions. To simplify the derivation of formal expressions, we specify the formalism regarding the core electrons as belonging to a redefined vacuum (for details we refer to Refs. \[18, 28\]). This leads to merging the interelectronic-interaction corrections of order \(1/Z\) with the one-loop radiative corrections. The formulas for these corrections can easily be obtained from the corresponding expressions for the one-loop radiative corrections to the transition amplitude in a one-electron atom, derived in \[18\]. However, the standard electron propagator \(S(\varepsilon, \mathbf{x}, \mathbf{y})\), which enters the equations, must be replaced by

\[
\tilde{S}(\varepsilon, \mathbf{x}, \mathbf{y}) = S(\varepsilon, \mathbf{x}, \mathbf{y}) + 2\pi i \sum_c \overline{\psi_c} \left( \mathbf{x} \right) \psi_c \left( \mathbf{y} \right) \delta(\varepsilon - \varepsilon_c),
\]

(35)

where the summation runs over all occupied one-electron states referring to the closed shells. Accordingly, the total expression is represented by the sum of the pure QED and interelectronic-interaction contributions, which correspond to the first and second terms in the right-hand side of Eq. (35). As a result, the \(1/Z\) interelectronic-interaction correction to the M1-transition amplitude in a B-like ion between the initial state \(a\) and the final state \(b\) is

\[
\Delta A_{1M}^{\text{int}} = -\sqrt{\frac{\omega}{4\pi c}} \sqrt{3} \sum_e \left\{ \sum_{n \neq b} \langle bc | I(0) | nc \rangle \langle n | T^1_M | a \rangle \frac{\varepsilon_b - \varepsilon_n}{\varepsilon_a - \varepsilon_n} + \sum_{n \neq a} \langle b | T^1_M | n \rangle \langle cn | I(0) | ca \rangle \frac{\varepsilon_c - \varepsilon_n}{\varepsilon_a - \varepsilon_n} \right. \\
+ \sum_n \frac{\langle bc | I(\varepsilon_a - \varepsilon_b) | na \rangle \langle n | T^1_M | c \rangle}{\varepsilon_b + \varepsilon_c - \varepsilon_a - \varepsilon_n} + \sum_n \frac{\langle cb | T^1_M | n \rangle \langle nb | I(\varepsilon_a - \varepsilon_b) | ca \rangle}{\varepsilon_a + \varepsilon_c - \varepsilon_b - \varepsilon_n} \\
- \sum_{n \neq b} \frac{\langle bc | I(\varepsilon_a - \varepsilon_c) | an \rangle \langle n | T^1_M | c \rangle}{\varepsilon_b + \varepsilon_c - \varepsilon_a - \varepsilon_n} + \sum_{n \neq a} \frac{\langle cb | T^1_M | n \rangle \langle na | I(\varepsilon_a - \varepsilon_c) | ca \rangle}{\varepsilon_a + \varepsilon_c - \varepsilon_b - \varepsilon_n} \\
- \sum_n \langle bc | I'(\varepsilon_b - \varepsilon_c) | cb \rangle + \langle ac | I'(\varepsilon_a - \varepsilon_c) | ca \rangle \right\},
\]

(36)

where \(I(\varepsilon) = \alpha^{\mu} \alpha^{\nu} D_{\mu\nu}(\varepsilon), I'(\varepsilon) = dI(\varepsilon)/d\varepsilon, \alpha^{\mu} = (1, \alpha), \) and \(D_{\mu\nu}(\varepsilon)\) is the photon propagator. In the Feynman gauge it reads

\[
D_{\mu\nu}(\varepsilon; \mathbf{x} - \mathbf{y}) = -4\pi g_{\mu\nu} \int \frac{d^3k}{(2\pi)^3} \frac{\exp(ik \cdot (\mathbf{x} - \mathbf{y}))}{\varepsilon^2 - k^2 + i0},
\]

(37)
where \( g_{\mu\nu} \) is the metric tensor. In the Coulomb gauge we have

\[
D_{00}(\varepsilon, x - y) = \frac{1}{|x - y|}, \quad D_{i0} = D_{0i} = 0, \quad (i = 1, 2, 3),
\]

\[
D_{ij}(\varepsilon, x - y) = 4\pi \int \frac{d^3k}{(2\pi)^3} \frac{\exp (i k \cdot (x - y))}{\varepsilon^2 - k^2 + i0} \left( \delta_{i,j} - \frac{k_i k_j}{k^2} \right), \quad (i, j = 1, 2, 3).
\]

Expression (36) incorporates the Coulomb-Breit part, which was taken into account by the CI method, together with terms of higher order in \( \alpha Z \), the so-called frequency-dependent correction. Specifying the operator \( I(\varepsilon) \) within the Coulomb gauge and setting \( \varepsilon = 0 \) in Eq. (36) yields the Coulomb-Breit interaction. In this way we can exclude the part, which has already been taken into account by the CI method, and obtain the frequency-dependent correction of order \( 1/Z \) as

\[
\Delta A_{1M}^{\text{freq}} = \sqrt{\frac{\omega}{\pi c}} \sqrt{3} \sum_c \left\{ \sum_{n \neq b} \frac{\langle bc | I_C(\varepsilon_b - \varepsilon_c) | cn \rangle \langle n | T_M^1 | a \rangle}{\varepsilon_b - \varepsilon_n} + \sum_{n \neq a} \frac{\langle b | T_M^1 | n \rangle \langle nc | I_C(\varepsilon_a - \varepsilon_c) | ca \rangle}{\varepsilon_a - \varepsilon_n} + \sum_n \frac{\langle bc | I_C(\varepsilon_a - \varepsilon_c) | na \rangle \langle n | T_M^1 | c \rangle}{\varepsilon_b + \varepsilon_c - \varepsilon_a - \varepsilon_n} + \sum_n \frac{\langle c | T_M^1 | n \rangle \langle bn | I_C(\varepsilon_b - \varepsilon_c) | ca \rangle}{\varepsilon_a + \varepsilon_c - \varepsilon_b - \varepsilon_n} - \sum_n \frac{\langle bc | I_C(\varepsilon_a - \varepsilon_b) | an \rangle \langle n | T_M^1 | c \rangle}{\varepsilon_b + \varepsilon_c - \varepsilon_a - \varepsilon_n} - \sum_n \frac{\langle c | T_M^1 | n \rangle \langle nb | I_C(\varepsilon_a - \varepsilon_b) | ca \rangle}{\varepsilon_a + \varepsilon_c - \varepsilon_b - \varepsilon_n} + \frac{1}{2} \langle b | T_M^1 | a \rangle \left[ \langle bc | I_C'(\varepsilon_b - \varepsilon_c) | cb \rangle + \langle ac | I_C'(\varepsilon_a - \varepsilon_c) | ca \rangle \right] \right\}, \tag{39}
\]

where \( I_C(\varepsilon_a - \varepsilon_b) = I_C(\varepsilon_a - \varepsilon_b) - I_C(0) \) and the subscript “C” refers to the Coulomb gauge.

It should be noted that the total \( 1/Z \) interelectronic-interaction correction given by equation (36) is gauge independent. This has been confirmed in our calculations to a very high accuracy. The calculations were performed employing the B-spline method for the Dirac equation [29].

VI. QED CORRECTION

QED effects modify the transition probability via the matrix element of the transition operator and via the transition energy. Since we employ the experimental value for the transition energy, we have to consider the QED effect on the transition amplitude only.

The lowest-order QED correction to the M1-transition amplitude can be derived by correcting the operator of the atomic magnetic moment for the anomalous magnetic moment of a free
electron. In the nonrelativistic limit it yields

$$\mu_{nr} \rightarrow \mu_a = -\mu_B [L + 2(1 + \kappa_e)S] = \mu_{nr} + \delta \mu_a,$$  \hspace{1cm} (40)

where

$$\delta \mu_a = -2\mu_B \kappa_e S,$$  \hspace{1cm} (41)

$$\kappa_e = \left[ \frac{\alpha}{2\pi} - 0.328 \frac{478.965 \ldots}{\pi} \right]^2 + \cdots \right].$$  \hspace{1cm} (42)

With the aid of the identity

$$\langle J_f \| J \| J_i \rangle = \langle J_f \| (L + S) \| J_i \rangle = \delta_{J_f,J_i} \sqrt{J_i(J_i + 1)(2J_i + 1)},$$  \hspace{1cm} (43)

one can easily find for the fine-structure level transition \((\Delta J = \pm 1)\)

$$\langle J_f \| \delta \mu_a \| J_i \rangle = 2\kappa_e \langle J_f \| \mu_{nr} \| J_i \rangle.$$  \hspace{1cm} (44)

Therefore, the QED correction to the M1-transition probability is given by

$$\Delta W_{\text{QED}}^{i \rightarrow f} = \frac{4\omega^3}{3c^3} \frac{1}{2J_i + 1} \left( |\langle J_f \| (\mu_{nr} + \delta \mu_a) \| J_i \rangle|^2 - |\langle J_f \| \mu_{nr} \| J_i \rangle|^2 \right),$$  \hspace{1cm} (45)

which yields

$$\Delta W_{\text{QED}}^{i \rightarrow f} \simeq 4\kappa_e \frac{4\omega^3}{3c^3} \frac{1}{2J_i + 1} |\langle J_f \| \mu_{nr} \| J_i \rangle|^2 \simeq 4\kappa_e W_{\text{nr}}^{i \rightarrow f}.$$  \hspace{1cm} (46)

QED corrections, which are not accounted for by this formula, are suppressed by a small factor \((\alpha Z)^2\).

**VII. RESULTS AND DISCUSSION**

The individual contributions to the M1-transition probabilities and the corresponding lifetimes for B-like and Be-like ions are presented in Tables I and II, respectively. Due to the smallness of the E2 transition, which is also allowed, the lifetimes are essentially determined by the M1 transition. In case of B-like ions, the experimental values of the transition energy were taken from Ref. [30] for \(S^{11+}, \text{Cl}^{12+}, \text{K}^{14+}, \text{Ti}^{17+}\) and from Ref. [31] for \(\text{Ar}^{13+}\). As one can see from Table II the interelectronic-interaction correction \(\Delta W_{\text{CI}}\) turns out to be relatively small due to the smallness of the factor \((\alpha Z)^2/Z\). The most important contributions are given by the relativistic
correction $\Delta W_D$ and by the QED correction $\Delta W_{\text{QED}}$. For Be-like ions, the transition energies were taken from Ref. [32] for $\text{S}^{12+}$, $\text{Cl}^{13+}$, $\text{K}^{15+}$, $\text{Ti}^{18+}$ and from Ref. [81] for $\text{Ar}^{14+}$. In this case the interelectronic-interaction correction $\Delta W_{\text{CI}}$ provides an essential contribution to the total value of the transition probability. This is due to a strong mixing of the two terms $^3P_1$ and $^1P_1$. Except for $\text{Ar}^{13+}$ and $\text{Ar}^{14+}$, the uncertainties of the total transition probabilities are mainly determined by the experimental uncertainties of the transition energy. For argon ions, the uncertainty comes mainly from uncalculated higher-order QED corrections.

In Table III our results for the lifetime of the $(1s^22s^22p)^2P_{3/2}$ state are compared with other calculations and with experiment. It should be noted that the QED correction was taken into account in Refs. [10, 37] and in the present work only. Besides, different values of the transition energy $\omega$, indicated in Table III, were used in the different calculations. Since the M1-transition probability $W$ scales as $\omega^3$, a small deviation in $\omega$ can change $W$ significantly. For this reason, we recalculated the results of Cheng et al. [9] and Froese Fischer [10] for the $(1s^22s^22p)^2P_{3/2}$ state in B-like ions for those transition energies we have employed in our calculations. Table IV presents these values with $(\tau_{\text{pres}})$ and without $(\tau^0)$ the anomalous magnetic moment correction and the corresponding values $(\tau_{\text{pres}}^0)$ obtained in this work. As one can see from the table, there is an excellent agreement between our “non-QED” results $(\tau_{\text{pres}}^0)$ and those from Ref. [9] $(\tau^0)$. There is also a good agreement between our total results $(\tau_{\text{pres}})$ and those from Ref. [10] $(\tau)$. The comparison of our theoretical results with the experimental data shows generally a good agreement as well. However, in case of $\text{Ar}^{13+}$ there is a discrepancy between our $^2P_{3/2}$ lifetime value 9.538(2) ms and the most accurate experimental value 9.573(5) ms [7, 8].

Table V shows a fair agreement of our results for the lifetime of the $(1s^22s^22p)^3P_2$ state in Be-like ions with corresponding results obtained by other authors and with experimental data. We note that the QED correction has not been considered in the previous calculations cited in the table.

In conclusion, we have evaluated the magnetic-dipole transition probabilities between the fine-structure levels $(1s^22s^22p)^2P_{1/2} - 2P_{3/2}$ for B-like ions and $(1s^22s^22p)^3P_1 - 3P_2$ for Be-like ions. The relativistic, interelectronic-interaction, and radiative corrections to the transition probability have been considered. Except for a recent high-precision lifetime measurement on $\text{Ar}^{13+}$ [7, 8] with an accuracy level on the order of 0.1%, most experimental results have large error bars greater than 1.5% and, within these error bars, most of them are in a fair agreement with our theoretical predictions. In case of $\text{Ar}^{13+}$, the disagreement of our prediction with the high-precision experimental value amounts to 0.37% of the total transition probability, less than the value of the
corresponding QED correction. At present we have no explanation for this discrepancy.

Acknowledgments

Valuable conversations with O. Yu. Andreev are gratefully acknowledged. This work was supported in part by RFBR (Grant No. 04-02-17574), INTAS-GSI (Grant No. 03-54-3604), the Russian Ministry of Education. D.A.G. acknowledges financial support from the foundation “Dynasty”. A.V.V. and G.P. acknowledge financial support from the GSI F+E program, DFG, and BMBF. The work of A.V.V. was also supported by DAAD and “Dynasty” foundation.
[1] T. V. Back, H. S. Margolis, P. K. Oxley, J. D. Silver, and E. G. Myers, Hyperfine Int. 114, 203 (1998).
[2] D. P. Moehs and D. A. Church, Phys. Rev. A 58, 1111 (1998).
[3] E. Träbert, G. Gwinner, A. Wolf, X. Tordoir, and A. G. Calamai, Phys. Lett. A 264, 311 (1999).
[4] E. Träbert, P. Beiersdorfer, S. B. Utter, G. V. Brown, H. Chen, C. L. Harris, P. A. Neill, D. W. Savin, and A. J. Smith, Astrophys. J. 541, 506 (2000).
[5] E. Träbert, P. Beiersdorfer, G. V. Brown, H. Chen, E. H. Pinnington, and D. B. Thorn, Phys. Rev. A 64, 034501 (2001).
[6] E. Träbert, P. Beiersdorfer, G. Gwinner, E. H. Pinnington, and A. Wolf, Phys. Rev. A 66, 052507 (2002).
[7] J. R. Crespo López-Urrutia, A. N. Artemyev, J. Braun, G. Brenner, H. Bruhns, I. N. Draganić, A. J. González Martínez, A. Lapierre, V. Mironov, J. Scofield, R. Soria Orts, H. Tawara, M. Trinczek, I. I. Tupitsyn, and J. Ullrich, Nucl. Instr. Meth. Phys. Res. B 235, 85 (2005).
[8] A. Lapierre, U. D. Jentschura, J. R. Crespo López-Urrutia, J. Braun, G. Brenner, H. Bruhns, D. Fischer, A. J. González Martínez, Z. Harman, W. R. Johnson, C. H. Keitel, V. Mironov, C. J. Osborne, G. Sikler, R. Soria Orts, H. Tawara, I. I. Tupitsyn, J. Ullrich, and A. Volotka, accepted in Phys. Rev. Lett.
[9] K. T. Cheng, Y.-K. Kim, and J. P. Desclaux, At. Data Nucl. Data Tables 24, 111 (1979).
[10] C. F. Fischer, J. Phys. B 16, 157 (1983).
[11] E. Charro, S. López-Ferrero, and I. Martín, J. Phys. B 34, 4243 (2001).
[12] I. I. Sobelman, Atomic Spectra and Radiative Transitions, Springer, New York, 1979.
[13] J. Sucher, Phys. Rev. A 22, 348 (1980).
[14] M. H. Mittleman, Phys. Rev. A 24, 1167 (1981).
[15] D. A. Glazov, V. M. Shabaev, I. I. Tupitsyn, A. V. Volotka, V. A. Yerokhin, G. Plunien, and G. Soff, Phys. Rev. A 70, 062104 (2004).
[16] P. Indelicato, Phys. Rev. Lett. 77, 3323 (1996).
[17] A. Derevianko, I. M. Savukov, W. R. Johnson, and D. R. Plante, Phys. Rev. A 58, 4453 (1998).
[18] V. M. Shabaev, Phys. Rep. 356, 119 (2002).
[19] I. P. Grant, J. Phys. B 7, 1458 (1974).
[20] D. A. Varshalovich, A. N. Moskalev, V. K. Khersonskii, Quantum Theory of Angular Momentum, World Scientific, Singapore, 1988.
[21] I. P. Grant, Advances in Physics 19, 747 (1970).
[22] V. F. Bratzev, G. B. Deyneka, and I. I. Tupitsyn, Izv. Akad. Nauk SSSR 41, 2655 (1977) [Bull. Acad.
[23] P. F. Gruzdev, G. S. Soloveva, and A. I. Sherstyuk, Opt. Spektrosk. 42, 1198 (1977) [Opt. Spectrosc. 42, 690 (1977)].

[24] E. Dalgaard and P. Jørgensen, J. Chem. Phys. 69, 3833 (1978).

[25] B. Levy and G. Berthier, Int. J. Quantum. Chem. 2, 307 (1968).

[26] S. T. Epstein, *Variation Method in Quantum Chemistry*, Academic Press, New York, 1974.

[27] P. Indelicato, V. M. Shabaev, and A. V. Volotka, Phys. Rev. A 69, 062506 (2004).

[28] M. B. Shabaeva and V. M. Shabaev, Phys. Rev. A 52, 2811 (1995).

[29] W. R. Johnson, S. A. Blundell, and J. Sapirstein, Phys. Rev. A 37, 307 (1988).

[30] B. Edlén, Phys. Scripta 28, 483 (1983).

[31] I. Draganić, J. R. Crespo López-Urrutia, R. DuBois, S. Fritzsche, V. M. Shabaev, R. Soria Orts, I. I. Tupitsyn, Y. Zou, and J. Ullrich, Phys. Rev. Lett. 91, 183001 (2003).

[32] B. Edlén, Phys. Scripta 28, 51 (1983).

[33] T. R. Verhey, B. P. Das, and W. F. Perger, J. Phys. B 20, 3639 (1987).

[34] M. E. Galavis, C. Mendoza, and C. J. Zeippen, Astron. Astrophys. Suppl. Ser. 131, 499 (1998).

[35] K. Koc, J. Phys. B 36, L93 (2003).

[36] C. Z. Dong, S. Fritzsche, B. Fricke, and W.-D. Sepp, Phys. Scripta T92, 294 (2001).

[37] W. R. Johnson, private communication.

[38] F. G. Serpa, J. D. Gillaspy, and E. Träbert, J. Phys. B 31, 3345 (1998).

[39] V. Kaufman and J. Sugar, J. Phys. Chem. Ref. Data 15, 321 (1986).

[40] U. I. Safronova, W. R. Johnson, and A. Derevianko, Phys. Scr. 60, 46 (1999).

[41] R. Glass, Astrophys. Space Sci. 91, 417 (1983).
TABLE I: The decay rates $W$ [s$^{-1}$] of the magnetic-dipole transition $(1s^22s^22p)^2P_{1/2} \rightarrow 2P_{3/2}$ and the lifetimes $\tau$ [ms] of the $(1s^22s^22p)^2P_{3/2}$ state in B-like ions. Numbers in the parentheses give the estimated error.

| Energy [cm$^{-1}$] | $S^{11+}$ | $Cl^{12+}$ | $Ar^{13+}$ | $K^{14+}$ | $Ti^{17+}$ |
|-------------------|-----------|-----------|-----------|-----------|-----------|
| $W_{nr}$          | 20.37538  | 47.43068  | 104.56308 | 219.4222  | 1599.635  |
| $\Delta W_D$      | -0.03542  | -0.09302  | -0.23145  | -0.5436   | -5.355    |
| $\Delta W_{CI}$   | 0.00637   | 0.01586   | 0.03723   | 0.0802    | 0.597     |
| $\Delta W_{neg}$  | -0.00159  | -0.00396  | -0.00929  | -0.0206   | -0.176    |
| $\Delta W_{QED}$  | 0.09451   | 0.22001   | 0.48502   | 1.0178    | 7.420     |
| $\Delta W_{freq}$ | 0.00007   | 0.00019   | 0.00049   | 0.0012    | 0.013     |
| $W_{total}$       | 20.439(5) | 47.57(16) | 104.85(2) | 220.0(6)  | 1602.1(5) |
| $\tau_{total}$    | 48.93(1)  | 21.02(7)  | 9.538(2)  | 4.546(12) | 0.6242(2) |
TABLE II: The decay rates \( W [s^{-1}] \) of the magnetic-dipole transition \((1s^22s2p)^3P_1 \rightarrow ^3P_2\) and the lifetimes \( \tau [\text{ms}] \) of the \((1s^22s2p)^3P_2\) state in Be-like ions. Numbers in the parentheses give the estimated error.

|                  | \( ^{12}\text{S} \) | \( ^{13}\text{Cl} \) | \( ^{14}\text{Ar} \) | \( ^{15}\text{K} \) | \( ^{18}\text{Ti} \) |
|------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
| \( W_{nr} \)     | 12.35488             | 29.03947             | 64.17056             | 135.36899            | 1045.4311            |
| \( \Delta W_D \) | -0.02017             | -0.05389             | -0.13242             | -0.31247             | -3.2611              |
| \( \Delta W_{CI} \) | -0.01302             | -0.04909             | -0.16457             | -0.50484             | -10.0481             |
| \( \Delta W_{neg} \) | -0.00053             | -0.00133             | -0.00313             | -0.00704             | -0.0649              |
| \( \Delta W_{QED} \) | 0.05731              | 0.13470              | 0.29766              | 0.62792              | 4.8493              |
| \( W_{total} \)  | 12.38(5)             | 29.07(11)            | 64.17(1)             | 135.2(4)             | 1036.9(4)            |
| \( \tau_{total} \) | 80.79(33)            | 34.40(13)            | 15.584(2)            | 7.398(22)            | 0.9645(4)            |
TABLE III: The lifetimes of the \( (1s^22s^22p)^2P_{3/2} \) level in B-like ions calculated in this work with \( (\tau_{\text{pres}}) \) and without \( (\tau_{\text{pres}}^0) \) the QED correction are compared with previous calculations \( (\tau_{\text{theor}}) \) and experiment \( (\tau_{\text{exp}}) \). The lifetime values are given in [ms]. The values of the transition energy [Energy] are presented in [cm\(^{-1}\)]. Numbers in the parentheses give the estimated error.

| Ions  | \( \tau_{\text{pres}}^0 \) | \( \tau_{\text{pres}} \) [Energy] | \( \tau_{\text{theor}} \) [Energy] | Method & Ref. | \( \tau_{\text{exp}} \) & Ref. |
|-------|-----------------|----------------------------|----------------------------|---------------|----------------------|
| S\(^{11+}\) | 49.16 | 48.93(1) [13135] | 47.35 [13300] | MCDF [9] | |
|       | 49.07 [13115] | | MCBP [10] | | |
|       | 49.33 [13144] | | MCDF [33] | | |
|       | 49.07 [13136] | | SS [34] | | |
|       | 49.26 [13122] | | MRCD [35] | | |
|       | 49.60 | | RQDO [11] | | |
| Cl\(^{12+}\) | 21.12 | 21.02(7) [17408] | 20.55 [17565] | MCDF [9] | 21.2(6)[6] |
|       | 21.02 [17400] | | MCBP [10] | 21.1(5)[6] | |
|       | 21.19 [17421] | | MCDF [33] | | |
|       | 21.08 [17410] | | SS [34] | | |
|       | 21.19 [17386] | | MRCD [35] | | |
|       | 21.13 | | RQDO [11] | | |
| Ar\(^{13+}\) | 9.582 | 9.538(2) [22656] | 9.407 [22795] | MCDF [9] | 8.7(5)[38] |
|       | 9.515 [22660] | | MCBP [10] | 9.12(18)[2] | |
|       | 9.618 [22666] | | MCDF [33] | 9.70(15)[4] | |
|       | 9.569 [22653] | | SS [34] | 9.573(4)(5)[8] | |
|       | 9.588 [22657] | | RQDO [11] | | |
|       | 9.606 [22636] | | MCDF [36] | | |
|       | 9.615 [22619] | | MRCD [35] | | |
|       | 9.534 [22658] | | [37] | | |
| K\(^{14+}\) | 4.567 | 4.546(12) [29006] | 4.509 [29129] | MCDF [9] | 4.47(10)[5] |
|       | 4.521 [29044] | | MCBP [10] | | |
|       | 4.583 [29019] | | MCDF [33] | | |
|       | 4.558 [29004] | | SS [34] | | |
|       | 4.587 [28960] | | MRCD [35] | | |
|       | 4.577 | | RQDO [11] | | |
| Ti\(^{17+}\) | 0.6271 | 0.6242(2) [56243] | 0.6254 [56275] | MCDF [9] | 0.627(10)[3] |
|       | 0.6150 [56465] | | MCBP [10] | | |
|       | 0.6290 [56258] | | MCDF [33] | | |
|       | 0.6254 [56240] | | SS [34] | | |
|       | 0.6289 [56166] | | MRCD [35] | | |
|       | 0.6270 | | RQDO [11] | | |

MCDF - multiconfiguration Dirac-Fock method
MCBP - multiconfiguration Breit-Pauli method
SS - SUPERSTRUCTURE program
MRCI - multireference relativistic configuration interaction method
RQDO - relativistic quantum defect orbital method
TABLE IV: The lifetimes of the \( (1s^22s^22p)^2P_{3/2} \) level in B-like ions calculated in this work with \( (\tau_{\text{pres}}) \) and without \( (\tau_{\text{0 pres}}) \) the QED correction are compared with previous theoretical results, recalculated to the transition energy \( \text{[Energy[cm}^{-1}\text{]]} \) employed in this paper. The lifetime values are given in [ms].

| Ions   | Energy | \( \tau_{\text{0 pres}} \) | \( \tau_{\text{0 (Ref. [9])}} \) | \( \tau_{\text{pres}} \) | \( \tau \) (Ref. [10]) |
|--------|--------|-----------------|-----------------|-----------------|-----------------|
| S\(^{11+}\) | 13135  | 49.16           | 49.16           | 48.93           | 48.85           |
| Cl\(^{12+}\) | 17408  | 21.12           | 21.11           | 21.02           | 20.99           |
| Ar\(^{13+}\) | 22656  | 9.582           | 9.581           | 9.538           | 9.520           |
| K\(^{14+}\) | 29006  | 4.567           | 4.567           | 4.546           | 4.539           |
| Ti\(^{17+}\) | 56243  | 0.6271          | 0.6265          | 0.6242          | 0.6223          |

TABLE V: The lifetimes of the \( (1s^22s^22p)^3P_2 \) level in Be-like ions calculated in this work with \( (\tau_{\text{pres}}) \) and without \( (\tau_{\text{0 pres}}) \) the QED correction are compared with previous calculations \( (\tau_{\text{theor}}) \) and experiment \( (\tau_{\text{exp}}) \). The lifetime values are given in [ms]. The values of the transition energy \( \text{[Energy]} \) are presented in [cm\(^{-1}\)]. Numbers in the parentheses give the estimated error.

| Ions   | \( \tau_{\text{0 pres}} \) | \( \tau_{\text{pres[Energy]}} \) | \( \tau_{\text{theor[Energy]}} \) | Method & Ref. | \( \tau_{\text{exp & Ref.}} \) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|
| S\(^{12+}\) | 81.16           | 80.79(33) [9712] | 83.3 [9743]     | SHF [39]       | MBPT [40]       |
| Cl\(^{13+}\) | 34.56           | 34.40(13) [12913] | 35.7 [12893]    | SHF [39]       | MBPT [40]       |
| Ar\(^{14+}\) | 15.66           | 15.584(2) [16819] | 16.31 [16818]   | MCHF [41]      | MBPT [40]       |
| K\(^{15+}\) | 7.432           | 7.398(22) [21571] | 7.63 [21575]    | SHF [39]       | MBPT [40]       |
| Ti\(^{18+}\) | 0.9689          | 0.9645(4) [42638] | 0.990 [42653]   | SHF [39]       | MBPT [40]       |

SHF - scaled Hartree-Fock method  
MBPT - many-body perturbation theory  
MCHF - multiconfiguration Hartree-Fock method  
MCDF - multiconfiguration Dirac-Fock method