Single Carrier-Frequency Division Multiple Access Radar: Waveform Design and Analysis

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ABSTRACT

This paper proposes Single Carrier-FDMA radar, which is not only a solution to the shortcomings of OFDM radars but also offers improved Ambiguity Function and autocorrelation properties. Unlike OFDM, the proposed SC-FDMA radar (with interleaved subcarrier mapping) does not suffer from high peak to average power ratio (PAPR) issue and hence does not affect the efficiency of the transmitter’s power amplifier. SC-FDMA utilizes all the benefits of single-carrier modulation, orthogonal frequency division multiplexing and frequency domain equalization. The wideband characteristics of SC-FDMA are beneficial in achieving high range resolution for radar applications. The paper presents a complete architecture of SC-FDMA radar with interleaved subcarrier mapping. The proposed radar system is analyzed for its ambiguity function, auto-correlation properties, and PAPR. The performance of the proposed radar waveform is compared with those of the notable existing radar waveforms including a standard OFDM radar, CAN, Hadamard, and PWcia. It is observed from the comparative analysis, that the proposed scheme outperforms OFDM and the other notable waveforms. The proposed work can be extended to develop multiple tasking radars as the different source signals can correspond to independent tasks to be performed simultaneously.

INDEX TERMS

Ambiguity function, OFDM, PAPR, SC-FDMA.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is a waveform modulation technique that has been commonly used in wireless communications such as LTE and WiMAX. After its successful implementation in wireless communications, OFDM waveform was suggested to be used in modern radar systems [1], [2].

OFDM being a multicarrier multiplexing scheme, not only improves range resolution and spectral efficiency but also provides frequency diversity to a radar system. It offers a range of performance improvements over the conventional radar systems including better discrimination between clutter and moving targets [3], [4]. The wideband characteristics of OFDM are desirable to achieve high range resolution for radar applications as narrowband subcarriers collectively make a large bandwidth signal.

OFDM provides flexibility in the selection of subcarriers and control over spectrum and bandwidth of the overall signal. This flexibility provides the opportunity to select OFDM as a suitable waveform candidate in radar systems [5]–[7]. Other spread spectrum techniques for radar waveform generation include CDMA that carry similar computational load. However, CDMA Radar does not offer the level of control on the spectral properties as does the OFDM based radar [8].

Despite several benefits of OFDM, it exhibits some weaknesses, like high peak to average power ratio (PAPR) and its vulnerability to frequency offsets. The high PAPR in OFDM systems reduces the power efficiency of the transmitter’s power amplifier. In literature, a number of techniques have been proposed to reduce the PAPR ratio, in general [9], [10]. There are techniques that reduce PAPR at the expense of distorting the transmitted OFDM signal. These techniques include clipping [11], R-OFDM [12], companding [13], peak cancellation [14] and peak windowing [15]. However, for radar applications, various techniques have been mentioned in [16] and the simplest among them is the clipping method. In this method, the peaks in the OFDM signal are deliberately clipped before amplification [17]. Although clipping action eliminates high peaks in OFDM signals; however, this is a nonlinear process that results in inband and out of band interference among the subcarriers destroying the orthogonality.
among them. The out of band interference can be removed by employing iterative filtering after clipping action. This imparts additional complexity to the radar system [10]. For achieving low PAPR, there are distortionless approaches as well, that maintain orthogonality between subcarriers and make it possible for the radar to process each subcarrier independently. One of such approaches is tone reservation method [18] but it only reduces the peaks, while ignoring the structure of the rest of the wave that is also littered with envelope fluctuations. The other distortionless techniques include selective mapping [19], partial transmit sequence [20], Interleaved OFDM (I-OFDM) [21], Interleaved Spread Spectrum OFDM [22], tone injection [18], active constellation extension [23] and constrained constellation shaping [24]. All of these techniques add up redundancy in the transmitted signal, increasing overhead to the transmitted signal and hence extending the complexity of the overall system.

Coding techniques are also used to reduce PAPR in OFDM signals. Levanon and Mozeson et al. used phase coded schemes like Newman, Narahashi and Schroeder for generating multicarrier radar signals to achieve low PAPR ratio. They achieved a reduction of PAPR up to 1.8 [2].

The other examples of coding techniques include Linear Block Coding, Golay sequences and Turbo coding. Golay codes have been investigated for PAPR reduction in OFDM Radar signals in [16]. OFDM subcarriers can be modulated with codewords made up of Golay Complementary Sequences that results in a reduction of PAPR up to 3 dB. The good performance of these coding techniques is achieved at the cost of coding rate and at the expense of high computation load to search the suitable codewords. Moreover, the OFDM systems even after using these coding techniques are still vulnerable to frequency offsets due to their multicarrier structure.

Jian Li et al. proposed computationally efficient cyclic algorithms to design unimodular sequences that could be used for MIMO radars [25], [26]. These sequences have unit PAPR ratio as well as low auto-correlation sidelobes. In their work in [27], Ze Li et al. proposed Periodic correlation Weighted Cyclic Iteration Algorithm (PWCIA) for finite phase unimodular sequences to achieve lower integrated sidelobe levels as compared to [25] and [26]. These sequences also possess unit PAPR ratio because of their unimodular structure. However, in their work, there was still room for improving auto-correlation properties.

Most of the literature is focused on minimizing PAPR and improving auto-correlation properties of the multicarrier radar waveforms at the expense of either additional complexity or see-saw trade-off between a number of performance metrics but none of these researchers looked for an out-of-the-box solution. Nevertheless, such a solution was proposed by Yujiu Zhao et al. [28], in which the authors proposed piecewise nonlinear frequency modulated waveform with good auto-correlation properties providing higher degree of freedom, achieving high range resolution and PAPR up to 3 dB. Despite having achieved low PAPR, they still could not make it up to 0 dB. In a similar work, proposed by Mietzner [29], they used DFT-Spreaded OFDM waveform for radar application to reduce PAPR or crest factor. Their proposed waveform, however, suffered from energy leakage into range sidelobes. This leakage entails a certain power penalty regarding the mainlobe of the ambiguity function and turns out to cause blurred target images. These out-of-the-box solutions motivated the authors to present SC-FDMA waveform (also known as Linearly pre-coded OFDM or DFT-Spreaded OFDM), as a suitable candidate for a multicarrier based low-PAPR and multitasking radar system, proposing viable solutions for its shortcomings.

SC-FDMA, due to its intrinsic properties of no inter-channel interference, has been in use, in the uplink of LTE cellular communication systems. Since its advent, it has not only been regarded as a more robust technique in terms of PAPR characteristics as compared to its other multicarrier competitors but also has been considered more resilient to frequency offsets [30], [31]. In cellular communication, SC-FDMA offers very small PAPR that makes the user equipment more power efficient as compared to OFDM. In similar fashion, while not compromising its benefits, it could resolve PAPR issues in multicarrier radars. The contribution of this work can thus be summarized as follows:

- A complete architecture of SC-FDMA based radar is proposed.
- The proposed radar system is analyzed for its ambiguity function, auto-correlation properties and PAPR.
- The performance of the proposed radar waveform is compared with those of the notable existing multicarrier radar waveforms [25], [27], [32] and [28].

Rest of the paper is organized as follows: Section II covers the structure of proposed SC-FDMA waveform, its signal modeling. Section III presents analytical expressions for ambiguity function, auto-correlation, peak to sidelobe ratio, and peak to average power ratio; their simulations and discussion on the results. Section III also discusses an effective solution for the energy leakage issue resulting from the occurrence of recurrent lobes in the auto-correlation of the proposed waveform. Finally, section IV concludes the whole work presented in this paper.

II. THE PROPOSED SINGLE CARRIER-FDMA RADAR
A. SINGLE CARRIER FDMA ARCHITECTURE

SC-FDMA is a multiple access or modulation technique that uses single carrier modulation, orthogonal frequency division multiplexing and frequency domain equalization. It exhibits similar performance and the same overall system complexity as OFDM does. The architecture of generic SC-FDMA radar is shown in Fig. 1. In SC-FDMA radar, the symbols are transmitted sequentially, as compared to parallel transmission of OFDM symbols in OFDM radar, over multiple subcarriers. Multiple source signals are orthogonally multiplexed and demultiplexed in frequency domain by subcarrier mapping that provides SC-FDMA, an aspect of OFDM.
B. SC-FDMA Radar Signal Model

At transmitter, the input is most desirably a code taken from the family of phase coded waveforms. The code consists of $N$ complex symbols generated at a rate of $R_{sr}$ symbols/sec. $N$-point DFT is performed to create $N$ frequency domain symbols that modulate $N$ out of $M$ subcarriers occupying the entire bandwidth.

$$B = M \Delta f$$

where $\Delta f$ is the spacing between subcarrier frequencies and $B$ is the total bandwidth. The channel transmission rate is in symbols per second.

$$R_{ch} = \frac{M}{N} R_{sr}$$

where $R_{sr}$ is the rate at which radar source symbols are generated and $M$ is the total number of subcarriers. If we denote $K$ as the bandwidth spreading factor then

$$K = \frac{R_{ch}}{R_{sr}} = \frac{M}{N}$$

There are two ways of subcarrier mapping in SC-FDMA; interleaved and localized. In interleaved SC-FDMA (usually named as IFDMA), the data symbols of a single source are equally distributed over complete frequency band whereas in localized SC-FDMA (LFDMA) the data symbols of a single user are mapped on a consecutive set of frequencies.

The SC-FDMA radar can handle up to $K$ independent signal sources each carrying $N$ symbols. $K$ is referred to as number of users in communication systems; however, in case of radar, this concept can be used differently as the multiple tasks performed by a single radar. Nevertheless, in this research work, we take a single source scenario in which data from a single source is spreaded over the entire frequency band with equal frequency spacing. The mapping scheme used is interleaved subcarrier mapping which is more robust against fading and external interference by achieving maximum frequency diversity.

We assume $\{x_n : n = 0, 1, \ldots, N - 1\}$ as the modulation symbols we take from the family of phase coded waveforms. The waveform uses fixed carrier frequency but different phases that are switched among a total of $N$ different fixed values after regular intervals within a pulse duration. This waveform can be modeled as $N$ contiguous subpulses of duration $T_s$ also known as chips or elements. In this work, we are using 4-phase Frank code with 16 elements, 8-phase Frank code with 64 elements and P3 code with 40 elements. The phase coded symbols $\{x_n : n = 0, 1, \ldots, N - 1\}$ are generated by using following relation

$$x_n = \{\exp(j\phi_n) 0 \leq n \leq N - 1 0 \text{ elsewhere}\}$$

where $\phi_n$ is the $n$th phase state of the phase-coded sequence.

In the same way, $n$th symbol of $k$th source would be denoted by $x_{k,n}$ with $(k = 0, 1, \ldots, K - 1)$ and $(n = 0, 1, \ldots, N - 1)$. We assume data symbols from a single source and hence for simplicity, use the notation $x_n$ instead of $x_{k,n}$. These symbols are then converted to frequency domain samples $X_q$ by performing DFT.

$$X_q = \sum_{n=0}^{N-1} x_n \exp(-j2\pi nq/N)$$

where $q$ is the index representing the frequency domain symbol and $N$ is the length of DFT. After performing DFT, the frequency domain symbols $X_q$ are mapped to subcarriers.
according to one of the previously-mentioned subcarrier mapping schemes. We opted for the interleaved SC-FDMA and hence denoted the resulting frequency domain symbols as $Y_l$ with $(l = 0, 1, \ldots, N-1)$. These frequency domain symbols, are then subjected to IDFT operation and the resulting time domain symbol can be written as

$$y_m = \frac{1}{M} \sum_{l=0}^{M-1} Y_l \exp(j \frac{2\pi ml}{M}) \quad (7)$$

where $M$ being the length of IDFT is equal to the number of subcarriers such that $M \geq N$. The index $m$ is given as $m = n + kn \mod N$ where $(0 \leq n \leq N - 1)$ and $(0 \leq k \leq K - 1)$. In interleaved SC-FDMA, $\{y_m : m = 0, 1, \ldots, M - 1\}$ is equal to the source sequence $\{x_n : n = 0, 1, \ldots, N - 1\}$ divided by $K$, repeats $K$ times with a phase rotation of $\exp(j 2\pi rm/M)$ which is given as

$$y_m = \frac{1}{K} x_{m \mod N} \exp(j 2\pi rm/M) \quad (8)$$

where $r$ is the initial sequence number of the source symbol and $(0 \leq r \leq K - 1)$.

The transmitted interleaved SC-FDMA signal for passband is a complex signal represented as

$$y_c(t) = e^{j 2\pi f_c t} \sum_{m=0}^{M-1} y_m g(t - mT_s) \quad (9)$$

where $f_c$ is the carrier frequency of the system and $g(t)$ is the pulse shaping function. $y_c(t)$ can be represented in baseband form as

$$y(t) = \sum_{m=0}^{M-1} y_m g(t - mT_s) \quad (10)$$

Since $y(t)$ is a periodic signal in which the source signal is repeated $K$ times after $T$ interval. We consider the transmitted signal as a continuous wave for length $KT$ with a periodic complex envelope $y(t)$ with period $T$.

In (8), for a single source with $r = 0$, there is no phase rotation and we can write $y(t)$ as

$$y(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} \frac{1}{K} x_n g[t - (n + kn)T_s] \quad (11)$$

$y(t)$ is a periodic function repeating after $K$ intervals and can be written as

$$y(t) = \frac{1}{K} x(t \pm kT) \quad k = 0, 1, \ldots, K - 1 \quad (12)$$

From (12), it is obvious that the proposed waveform not only retains all the benefits of the original phase coded waveform but also improves range resolution by a factor $k$ through repetitions as the bandwidth of the proposed signal increases $k$ times.

### III. Simulation Results and Performance Analysis

Simulations are carried out to obtain a comparative performance analysis of the proposed SC-FDMA Radar waveform with other notable radar waveforms. For this purpose, various performance metrics are plotted by utilizing varying lengths of Frank and P3 codes as initial sequence mapped with subcarriers such that these subcarriers are integral multiples of the length of initial sequence (A requirement for effectively implementing interleaved SC-FDMA). Since Frank code generates only sequences which are perfect squares; therefore, for generating sequence lengths other than perfect squares, P3 code is used. All the simulations are performed in MATLAB®.

#### A. Ambiguity Function (AF)

In order to evaluate the performance of different radar waveforms, the Ambiguity Function (AF) is used. It is helpful when the effectiveness of the waveforms is investigated for different radar applications. It provides insight when dealing with range and Doppler resolution of any target. AF does not depend upon any specific target scenario despite it is determined by the pulse waveform specifications and the matched filter. Ambiguity function is a two dimensional correlation between a transmitted signal and its time delayed and frequency shifted signal. The time delay refers to as range resolution and the frequency shift, also termed as the Doppler shift, refers to the speed of the moving target.

For a complex baseband pulse signal $y(t)$, the ambiguity function [33] is given by

$$|A(\tau, f_d)| = \left| \int_{-\infty}^{\infty} y(t) y^*(t - \tau) \exp(j 2\pi f_d t) dt \right| \quad (13)$$

where $*$ denotes complex conjugate, $\tau$ represents time delay and $f_d$ represents Doppler shift. The function for a zero Doppler cut becomes the autocorrelation function of the waveform sequence as given below.

$$|A(\tau, 0)| = \left| \int_{-\infty}^{\infty} y(t) y^*(t - \tau) dt \right| \quad (14)$$

The single period ambiguity function of a finite energy interleaved SC-FDMA signal of length $T$ with periodic complex envelope is given as

$$A_T(\tau, f_d) = \frac{1}{T} \int_{0}^{T} y(t + \frac{\tau}{2}) y^*(t - \frac{\tau}{2}) \exp(j 2\pi f_d t) dt \quad (15)$$

As we know that the reference signal $y(t)$ is of duration $KT$, the response of the correlation receiver is the ambiguity function for $K$ periods which after normalization will be

$$A_{KT}(\tau, f_d) = \frac{1}{KT} \int_{0}^{KT} y(t + \frac{\tau}{2}) y^*(t - \frac{\tau}{2}) \exp(j 2\pi f_d t) dt \quad (16)$$
The interleaved SC-FDMA signal is a periodic sequence of original signal \( x(t) \) divided by \( K \), therefore

\[
A_T(\tau, f_d) = \frac{1}{K^2 T} \int_0^T x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) \exp(j 2\pi f_d t) dt
\]

Dividing the integral in equation (16) into \( K \) sections and substituting \( y(t) \)

\[
A_{K_T}(\tau, f_d) = \frac{1}{K^3 T} \sum_{k=1}^{K} \int_{(k-1)T}^{kT} x(t + \frac{\tau}{2}) x^*(t - \frac{\tau}{2}) \exp(j 2\pi f_d t) dt
\]

by substituting \( t = t' + (k-1)T \) in eq.18 we get

\[
A_{K_T}(\tau, f_d) = \frac{1}{K^3 T} \sum_{t=1}^{T} x(t' + (k-1)T + \frac{\tau}{2}) x^*(t' + (k-1)T - \frac{\tau}{2}) \exp(j 2\pi f_d (t' + (k-1)T)) dt'
\]

Since the transmitted signal is periodic i.e.

\[
x(t' + \frac{\tau}{2}) = x(t' + (k-1)T + \frac{\tau}{2})
\]

then

\[
A_{K_T}(\tau, f_d) = \frac{1}{K} \sum_{k=1}^{K} \exp(j 2\pi f_d (k-1)T)
\]

\[
\times \frac{1}{K^2 T} \int_0^T x(t' + \frac{\tau}{2}) x^*(t' - \frac{\tau}{2}) \exp(j 2\pi f_d t') dt'
\]

\[
A_{K_T}(\tau, f_d) = \frac{1}{K} A_T(\tau, f_d) \sum_{k=1}^{K} \exp(j 2\pi f_d (k-1)T)
\]

\[
A_{K_T}(\tau, f_d) = \frac{1}{K} A_T(\tau, f_d) \frac{1 - \exp(j 2\pi f_d KT)}{1 - \exp(j 2\pi f_d T)}
\]

\[
A_{K_T}(\tau, f_d) = A_T(\tau, f_d) \frac{\sin(\pi f_d KT)}{\sin(\pi f_d T)} \exp(j \pi f_d (K-1)T)
\]

Now we have

\[
|A_{K_T}(\tau, f_d)| = |A_T(\tau, f_d)| \left| \frac{\sin(\pi f_d KT)}{\sin(\pi f_d T)} \right|
\]

Simulations are performed to obtain ambiguity functions of OFDM and SC-FDMA waveforms to analyze and compare their performance in radar applications. The ambiguity function has been determined by numerical computations.

Fig. 3 and 4 show AF plots for OFDM and the proposed SC-DMA for Frank-16 each with 32 subcarriers with pulsewidth and the chip width of 6.250 \( \times 10^{-5} \)s and 1.95 \( \times 10^{-8} \)s respectively. In the case of SC-FDMA, the bandwidth spreading factor is 2 i.e. the sequence of 16 coded symbols is repeated 2 times in time domain multiplied by a scaling factor of 1/2. The plots in Fig. 3 and 4 show diagonal ridges which are due to the selection of Frank code as the base signal in both of OFDM and SC-FDMA. The width of these ridges along delay axis represents range resolution. It is clear from Fig. 3 and 4 that the width of the major ridge in the proposed SC-FDMA signal is lesser than that of the OFDM plot. It is therefore inferred that the range resolution of SC-FDMA radar is higher than OFDM radar for same number of multiplexed subcarriers.

Fig. 5 shows AF plots for the proposed SC-FDMA for Frank-64 sequence multiplexed with 256 subcarriers. The pulsewidth and the chip width is 7.8 \( \times 10^{-6} \)s and 3.05 \( \times 10^{-8} \)s respectively. The bandwidth spreading factor is 4, i.e. the 64 code symbols are repeated 4 times in time domain. The plots in Fig. 5 show that the range resolution of the proposed SC-FDMA waveform increases as the number of subcarriers (mapped to the base signal) increases.

**B. AUTO-CORRELATION PROPERTIES**

In addition to orthogonality, good auto-correlation properties of a waveform are required for many radar applications. Good auto-correlation means that a transmitted waveform is uncorrelated to time shifted versions of itself. If a transmitted waveform exhibits good auto-correlation properties then the matched filter at the receiver can easily extract the backscattered signal for a given range bin and attenuates backscattered signals of other range bins. The auto-correlation function \( r \) of a complexed valued radar waveform \( y(t) \) is given in (14).
Fig. 6 shows the autocorrelation function of unimodular signal generated by using CAN (Cyclic Algorithm-New) proposed by He et al. [25] for a MIMO radar with 40 subsequences. It is compared with the proposed SC-FDMA signal with 40 elements generated by P3 code of 20 elements as initial sequence (40 subcarriers are used for mapping the 20 frequency domain elements of P3 code sequence). It is clear from the figure that the proposed signal exhibits better auto-correlation properties as compared to the signal in [25] except at recurring grating lobe points.

In Fig. 7, the autocorrelation properties of 100-element long waveform, PWCIA (Periodic Correlation Weighted Cyclic Iteration Algorithm) generated by a sequence of 8 phases proposed by Li et al. in [27] are compared with our proposed 128-element long, 8-phase SC-FDMA waveform initiated by 64-element Frank code; each with a chipwidth of 1 μs. The plots show that the autocorrelation of our waveform is better than that of the PWCIA waveform except at the recurring grating lobe points.

C. OCCURRENCE OF RECURRENT (GRATING) LOBES IN SC-FDMA RADAR AND THEIR REMOVAL

It is observed from Fig. 6, and 7, that the occurrence of recurring lobes is a serious issue in the autocorrelation of SC-FDMA waveform and thus needs a handful solution. Their position can be predicted intuitively by examining the AF of the proposed SC-FDMA waveform. These occur at the points where secondary diagonal ridges cross the zero Doppler axis in an AF diagram. The presence of secondary ridges is due to the periodicity in the waveform.
We remove these recurrent lobes by adopting the method proposed in [34]. The approach proposed by the authors is based on overlaying an orthogonal coding sequence over a pulse train in which a signal is repeated periodically after a fixed Pulse Repetition Interval (PRI). For a train of $K$ identical pulses of duration $KT$, the basic pulse of duration $T$ is divided into $N$ slices with a width $T_s = T/N$. In our case we are dealing with a long pulse of duration $KT$ in which a basic sequence of duration $T$ repeats $K$ times subsequently. We consider $T_s$ as the duration of a single element of the original initial sequence. Each slice is further encoded by the elements of an orthogonal phase-coded scheme represented by $K \times N$ matrix $A$. In this matrix $K$ rows represent coding sequence used for $K$ sub-pulses in the main pulse. The new overlaid signal is then given by

$$y(t) = \sum_{k=0}^{K-1} \sum_{n=0}^{N-1} a_{k,n} \frac{1}{K} x_n[t - (n + kN)T_s] \tag{25}$$

where $a_{k,n}$ represents an element of the matrix $A$. It is used to code the $n$th slice in the $k$th sequence of the original SC-FDMA pulse and is given by,

$$a_{k,n} = \exp(j\phi_{k,n}) \tag{26}$$

where $\phi_{k,n}$ is the $n$th phase state of the $k$th phase-coded sequence. The orthogonal phase-coded sequence which we opted for, is P4 code.

The matrix $A$ represents element of P4 code that is given as

$$A = \begin{bmatrix}
    a_{0,0} & a_{0,1} & \cdots & a_{0,N-1} \\
    a_{1,0} & a_{1,1} & \cdots & a_{1,N-1} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{K-1,0} & \cdots & a_{K-1,N-1}
\end{bmatrix} \tag{27}$$

Fig. 8 shows the same CAN autocorrelation signal plotted in Fig. 6 compared with the modified proposed waveform after removal of its grating lobes. The recurrent grating lobes are removed from the autocorrelation of the signal by using equation (25), in a very elegant way such that autocorrelation properties of the proposed waveform are improved. Likewise, in Fig. 9, the PWCIA waveform is compared with modified proposed waveform after resolving grating lobes issue.

Fig. 10, shows the autocorrelation of OFDM waveform, the proposed SC-FDMA waveform and the proposed SC-FDMA waveform after removing grating lobes. Each waveform contains 64 element in initial Frank phase-coded sequence which are multiplexed with 128 subcarriers ($N=64$, $M=128$) and the final pulse is 128 elements long. It is clear from the figure that the autocorrelation properties of our proposed waveform are better than that of OFDM signal.
FIGURE 10. Autocorrelation of OFDM waveform and the proposed SC-FDMA waveform before and after the removal of grating lobes; each waveform contains $N=64$, and $M=128$.

D. PEAK TO SIDELOBE RATIO

In an auto-correlated signal, peak to sidelobe ratio (PSR) is an important parameter when analyzing a waveform. This relation is given by

$$\text{PSR} = 10 \log_{10} \left| \frac{r_o}{r_{SL}} \right|^2$$  \hspace{1cm} (28)

where $r_o$ is the peak value and $r_{SL}$ is the sidelobe value of autocorrelation function of the waveform $x(t)$ and is given as,

$$r_{SL} = \max\{r_m\}_{m=1}^{M-1} \quad m \neq 0$$  \hspace{1cm} (29)

where

$$r_m = \left| \frac{1}{K^2 T} \int_0^T x(t + \frac{mT_s}{2})x^*(t - \frac{mT_s}{2}) \exp(j2\pi f_s t) dt \right|$$  \hspace{1cm} (30)

In Fig. 11, peak to sidelobe ratios of the proposed SC-FDMA waveform are plotted against the number of waveform elements for $M = 128, 256, 512, 1024, 2048, 4096$. The behavior of these peaks is then compared with the trend shown by the peak to sidelobe ratios of CAN [25] and Hadamard sequence [32]. For these plots, the Hadamard sequence is scrambled with PN (pseudo-noise) sequence to reduce its correlation sidelobes. It is clear from the comparison, that our proposed waveform exhibits very high PSR values as compared to the other two waveform sequences.

E. PEAK TO AVERAGE POWER RATIO

In multicarrier systems, the spectral spreading causes frequency offsets that destroy the orthogonality in subcarriers and the resulting signal distortion degrades the performance of the overall system. The measure of the envelope variations are given by Peak to Average Power Ratio (PAPR) which is a relation between the instantaneous peak power and the average signal power as discussed in section I. PAPR is mathematically given by

$$\text{PAPR} = \frac{\max(|y_c(t)|^2)}{\frac{1}{T} \int_0^T |y_c(t)|^2 dt}$$  \hspace{1cm} (31)

where $y_c(t)$ and $y(t)$ are given in (9) and (10) respectively. In multicarrier systems, like OFDM, the high PAPR produces signal excursions into nonlinear region of operation of transmitter amplifier that results in nonlinear distortions and spectral spreading [10]. It is therefore, necessary to use linear amplifiers in the transmitters [35] which are difficult to be employed in radars especially in airborne or vehicular applications where power is a constraint. Therefore, the peak to average power ratio must be controlled in these systems. The proposed SC-FDMA Radar offers low PAPR due to its inherent single carrier structure.

We have calculated PAPR ratio of OFDM and the proposed interleaved SC-FDMA for different phase coded waveforms with 32 and 256 subcarriers given in Table. 1 and 2. The phase coded waveforms include Pseudo Random Number (PRN), Frank, Zadoff-Chu, Barker, P1, P3 and Px with different code lengths.

The results given in Table 1 show that each of the proposed SC-FDMA signals, generated by using different initial sequences, offers better PAPR values as compared to their respective OFDM signals.

Table. 2 presents the values of PAPR of the proposed SC-FDMA waveforms and their comparison with unimodular sequences proposed in [25] and [27], and with nonlinear piecewise frequency modulated signals proposed in [28]. Although the autocorrelation properties of the waveform in [28] are good, however, its PAPR is much higher than the proposed waveform. PAPR of unimodular sequences of [25] and [27] is 0 dB due to their unimodular structure of constant amplitude. However, the proposed waveform exhibits better auto-correlation properties as compared to these waveforms alongewith exploiting all the benifits of multicarrier phase-coded waveforms.
In this paper, SC-FDMA (with interleaved subcarrier mapping), has been proposed as a radar waveform. The proposed SC-FDMA signal being a constant envelope periodic sequence, exploits all the benefits of periodic waveforms and offers null PAPR with very good autocorrelation properties. Analytical and simulation results of the proposed SC-FDMA radar have been compared with those of OFDM and the other waveforms by achieving higher range resolution, better autocorrelation properties and lower PAPR. The proposed SC-FDMA signal being a constant envelope periodic waveform can be used to develop multiple tasking radars as the radar waveform to solve Doppler ambiguity.

IV. CONCLUSION

In this paper, SC-FDMA (with interleaved subcarrier mapping), has been proposed as a radar waveform. The proposed SC-FDMA signal being a constant envelope periodic sequence, exploits all the benefits of periodic waveforms and offers null PAPR with very good autocorrelation properties. Analytical and simulation results of the proposed SC-FDMA radar have been compared with those of OFDM and the notable radar waveforms. The comparative analysis showed that the proposed radar waveform outperformed OFDM and other waveforms by achieving higher range resolution, better autocorrelation properties and lower PAPR. The proposed waveform can be used to develop multiple tasking radars as different source signals may correspond to independent tasks to be performed simultaneously. The proposed work can thus be extended to develop an SC-FDMA radar framework allowing multiple tasks to be carried out simultaneously by translating “users” into radar tasks and assigning each task, a subset of subcarriers. In such cases, the performance analysis would also need cross-correlation investigation, in addition to the performance metrics, mentioned in this paper.

REFERENCES

[1] N. Levanon, “Multifrequency complementary phase-coded radar signal,” IEEE Proc.-Radar, Sonar Navigat., vol. 147, no. 6, pp. 276–284, 2000.
[2] N. Levanon and E. Mozeson, Radar Signals, Hoboken, NJ, USA: Wiley, 2004, ch. 11, pp. 327–372.
[3] S. Sen and A. Nehorai, “Target detection in clutter using adaptive OFDM radar,” IEEE Signal Process. Lett., vol. 16, no. 7, pp. 592–595, Jul. 2009.
[4] Y. Kim and S. S. Sekhon, “Detection of moving target and localization of clutter using Doppler radar on mobile platform,” IEEE Geosci. Remote Sens. Lett., vol. 12, no. 5, pp. 1156–1160, May 2015.
[5] M. A. Sebt, Y. Norouzi, A. Sheikhi, and M. M. Nayebi, “OFDM radar signal design with optimized ambiguity function,” in Proc. IEEE Radar Conf., May 2008, pp. 1–5.
[6] S. Sen, M. Hurtado, and A. Nehorai, “Adaptive OFDM radar for detecting a moving target in urban scenarios,” in Proc. Int. Waveform Diversity Design Conf., Feb. 2009, pp. 268–272.
[7] R. F. Tigrek, W. J. A. De Heij, and P. Van Genderen, “OFDM signals as the radar waveform to solve Doppler ambiguity,” IEEE Trans. Aerosp. Electron. Syst., vol. 48, no. 1, pp. 130–143, Jan. 2012.
[8] H. Wang, J. Wang, and H. Li, “Target detection using CDMA based passive bistatic radar,” J. Syst. Eng. Electron., vol. 23, no. 6, pp. 858–865, Dec. 2012.
[9] T. Jiang and Y. Wu, “An overview: Peak-to-average power ratio reduction techniques for OFDM signals,” IEEE Trans. Broadcast., vol. 54, no. 2, pp. 257–268, Jun. 2008.
[10] Y. Ralmatallah and S. Mohan, “Peak-to-average power ratio reduction in OFDM systems: A survey and taxonomy,” IEEE Commun. Surveys Tuts., vol. 15, no. 4, pp. 1567–1592, 4th Quart., 2013.
[11] X. Li and L. J. Cimini, “Effects of clipping and filtering on the performance of OFDM,” in Proc. IEEE 47th Veh. Technol. Conf. Technol. Motion, vol. 3, May 1997, pp. 1634–1638.
[12] L. Kong, W. Xu, H. Zhang, and C. Zhao, “R–OFDM transmission scheme for visible light communication using RGBA-LED,” in Proc. IEEE 84th Veh. Technol. Conf. (VTC-Fall), Sep. 2016, pp. 1–6.
[13] X. Wang, T. T. Tjhung, and C. S. Ng, “Reduction of peak-to-average power ratio of OFDM system using a companding technique,” IEEE Trans. Broadcast., vol. 45, no. 3, pp. 303–307, Sep. 1999.
[14] P. O. Börjesson, H. G. Feichtinger, N. Gripp, M. Isaksson, N. Kaiblinger, P. Ödling, and L.-E. Persson, “A low-complexity PAR-reduction method for DMT-VDLS,” in Proc. Int. Sympl. Digit. Signal Process. Commun. Syst., 1999, pp. 164–169.
[15] S. Cha, M. Park, S. Lee, K.-J. Bang, and D. Hong, “A new PAPR reduction technique for OFDM systems using advanced peak windowing method,” IEEE Trans. Consum. Electron., vol. 54, no. 2, pp. 405–410, May 2008.
[16] R. F. Tigrek, “A processing technique for OFDM-modulated wideband radar signals,” Ph.D. dissertation, Delft Univ. Technol., Delft, The Netherlands, 2010.
[17] H. Ochiai and H. Imai, “Performance analysis of deliberately clipped OFDM signals,” IEEE Trans. Consum. Electron., vol. 50, no. 1, pp. 89–101, Jan. 2004.
[18] S. Watanasuwakull and W. Benjapalokalul, “PAPR reduction for OFDM transmission by using a method of tone reservation and tone injection,” in Proc. 5th Int. Conf. Inf. Commun. Signal Process., Dec. 2005, pp. 273–277.
[19] R. W. Bäuml, R. F. H. Fischer, and J. B. Huber, “Reducing the peak-to-average power ratio of multicarrier modulation by selected mapping,” Electron. Lett., vol. 32, no. 22, p. 2056, 1996.
[20] S. H. Müller and J. B. Huber, “OFDM with reduced peak-to-average power ratio by optimum combination of partial transmit sequences,” Electron. Lett., vol. 33, no. 5, pp. 368–369, 1997.
[21] A. D. S. Jayalath and C. Tellambura, “Reducing the peak-to-average power ratio of orthogonal frequency division multiplexing signal through bit or symbol interleaving,” Electron. Lett., vol. 36, no. 13, pp. 1161–1163, 2000.
[22] P. Tu, X. Huang, and E. Dutkiewicz, “Peak-to-average power ratio performance of interleaved spread spectrum OFDM signals,” in Proc. Int. Symp. Commun. Inf. Technol., Oct. 2007, pp. 82–86.
[23] B. S. Krongold and D. L. Jones, “Par reduction in ofdm via active constellation extension,” IEEE Trans. Broadcast., vol. 49, no. 3, pp. 258–268, Sep. 2003.
[24] A. Aggarwal and T. H. Meng, “Minimizing the Peak-to-Average power ratio of OFDM signals using convex optimization,” IEEE Trans. Signal Process., vol. 54, no. 8, pp. 3099–3110, Aug. 2006.
[25] H. He, P. Stoica, and J. Li, “Designing unimodular sequence sets with good Correlations—including an application to MIMO radar,” IEEE Trans. Signal Process., vol. 57, no. 11, pp. 4391–4405, Nov. 2009.
[26] P. Stoica, H. He, and J. Li, “New algorithms for designing unimodular sequences with good correlation properties,” IEEE Trans. Signal Process., vol. 57, no. 4, pp. 1415–1425, Apr. 2009.

[27] Z. Li, P. Li, X. Hao, and X. Yan, “Optimal unimodular sequences design method for active sensing systems,” Math. Problems Eng., vol. 2018, May 2018, Art. no. 2860809, doi: 10.1155/2018/2860809.

[28] Y. Zhao, X. Lu, J. Yang, W. Su, and H. Gu, “OFDM waveforms designed with piecewise nonlinear frequency modulation pulse for MIMO radar,” Int. J. Remote Sens., vol. 39, no. 23, pp. 8746–8765, Jul. 2018.

[29] J. Mietzner, “DFT-spread OFDM MIMO-radar—An alternative for reduced crest factors,” in Proc. 20th Int. Radar Symp. (IRS), Jun. 2019, pp. 1–10.

[30] H. Myung, J. Lim, and D. Goodman, “Single carrier FDMA for uplink wireless transmission,” IEEE Veh. Technol. Mag., vol. 1, no. 3, pp. 30–38, Sep. 2006.

[31] Z. Souad and B. Ridha, “SCFDMA in uplink via tone reservation,” Int. J. Comput. Netw. Commun., vol. 3, no. 6, pp. 157–168, Nov. 2011.

[32] J. Seberry and M. Yamada, “Hadamard matrices, sequences, and block designs,” in Contemporary Design Theory: A Collection of Surveys. Hoboken, NJ, USA: Wiley, 1992.

[33] M. A. Richards, Fundamentals of Radar Signal Processing. New York, NY, USA: McGraw-Hill, 2005, ch. 4, pp. 169–198.

[34] E. Mozeson and N. Levanon, “Removing autocorrelation sidelobes by overlaying orthogonal coding on any train of identical pulses,” IEEE Trans. Aerosp. Electron. Syst., vol. 39, no. 2, pp. 583–603, Apr. 2003.

[35] A. De Maio, Y. Huang, M. Piezzo, S. Zhang, and A. Farina, “Radar code design with a peak to average power ratio constraint: A randomized approximate approach,” in Proc. 19th Eur. Signal Process. Conf., 2011, pp. 436–440.

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