An Application of Stochastic Dominance with Truncated Normal Distribution on LQ45 Index

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Abstract. Return is the profit obtained by investors from the capital invested in a company. The movement of stock prices in an exchange affect the condition of stock returns. Stochastic dominance with truncated normal distribution is an analysis method of stock dominance with parametric approach where the stock returns is bounded between two values. The purpose of this research is to obtain the optimum stocks listed in the LQ45 index using stochastic dominance with truncated normal distribution. The assumption of a truncated normal distribution is derived from the normal distribution of monthly stock returns that symmetrically truncated on two sides. The dominance of stocks then obtained by comparing the truncated cumulative distribution function of returns based on first order stochastic dominance (FSD) and second order stochastic dominance (SSD). Based on the analysis, it can be concluded that there are 17 stock combinations that dominated by FSD and 14 stock combinations that dominated by SSD.

1. Introduction
Investment is the activity of infuse a number of funds with the aim of gaining profits in the future [1]. Invest in stocks is one of investment types that can be made in the capital market. The LQ45 is one of stock market index in the Indonesia Stock Exchange or IDX, which one indicator of stock prices movements, consists 45 combined stocks that have high liquidity and large market capitalization.

Fluctuations of stock prices in the exchange make the return face uncertainty. In other word, stocks are classified as risky assets because of the possibility of loss that can happen. Stochastic dominance (SD) is a method that suitable to use in uncertain condition, such as choosing the best stock based on diverse investor preferences. The fact is investors prefer strong stocks such as from companies listed in LQ45 index. The domination of stock that obtained by SD method can be used to form optimal portfolio as well as Multi Index Models [2]. Corresponding to calendar anomaly on IDX, which has effect to the condition of stock returns, the SD method give information that the returns on Wednesday and Friday are more optimum than the other days, also the return in the second week is the most optimum in a respective month [3].

The SD method provides a sequence of stocks that investors can choose according to their preferences [4]. In some cases, SD method need to be derive to give more alternative decision analysis. The almost stochastic dominance (ASD) can be used when the SD criterias have negative values or there is violation area based on SD criteria [5]. Moreover, with almost marginal condition stochastic dominance (AMCSD), the derivative method of SD, we also can choose efficient set of stocks and obtain an efficient portfolio [6].

The SD method shows the domination of a stock over other stock stochastically by comparing the cumulative distribution of the both stocks based on the utility function [7]. There are three criterias
representing the investor preferences for the desired stock, that are first order stochastic dominance (FSD), second order stochastic dominance (SSD), and third order stochastic dominance (TSD).

Although, the application of the SD method can be carried out without any specific assumptions, but sometimes a parametric approach will give more relevant results of dominance [8]. A parametric approach assuming a truncated normal distribution can be applied to the SD method [9]. Stock returns commonly have a range that is not so large and the stock price that forms a return is always positive. Therefore, the assumption of a truncated normal distribution is more appropriate than a normal distribution.

2. Research Method
This research uses monthly stock closing price data from stocks that listed in the LQ45 index over the period of August 2014 to July 2018 [10]. Those data are used to determine the amount of return for each stock based on the equation

\[ R_t(i) = \frac{P_t(i) - P_{t-1}(i)}{P_{t-1}(i)}, \quad (1) \]

where \( R_t(i) \) is the value of stock returns \( i \) in the period \( t \), \( P_t(i) \) is the closing price of stock \( i \) in the period \( t \), and \( P_{t-1}(i) \) is the closing price of stock \( i \) in the period \( t - 1 \) [11]. Furthermore, stock samples are selected by the Jarque Bera normality test on stock returns formed. Based on the mean value and standard deviation of the return it can be determined that two truncation points will truncate the normal return distribution symmetrically on the lower and upper tail. Finally, the analysis of stock dominance with the SD method with a truncated normal distribution can be done by the criteria of FSD and SSD.

3. Stochastic Dominance with Truncated Normal Distribution
Stochastic dominance with a truncated normal distribution is a method of selecting risky investment assets with a parametric approach that compares the two functions of the cumulative distribution of returns [7]. The assumption of a truncated normal distribution is derived by bounding the normal distribution of returns at two points defined as

\[ A_i = \mu_i - \delta \sigma_i \quad \text{and} \quad B_i = \mu_i + \delta \sigma_i, \quad (2) \]

where \( A_i \) and \( B_i \) are the lower and upper truncated points respectively, \( \mu_i \) is the mean of return, \( \sigma_i \) is the standard deviation of the return, and \( \delta \) is a constant that shows the distance from the mean to the point of each truncation point.

Truncation in the normal distribution can be carried out symmetrically on two tails or non symmetrically that is only on one tail. The dominance rules for symmetrical truncation and non symmetrical truncation are the same. If \( F^* \) is the truncated cumulative distribution of stock I and \( G^* \) is the truncated cumulative distribution of stock II, then the dominance of stock I to stock II is given by rules based on the relationship of mean and standard deviation of both stocks [9].

There is a relationship between the mean and standard deviation of stock I and stock II, namely \( \mu_I > \mu_I \) and \( \sigma_I > \sigma_{II} \), so that by defining the truncation point of the upper tail of both stocks as \( B_I \) and \( B_{II} \) it can be clearly seen that \( B_I > B_{II} \). The relationship of the lower tail of the truncation point for both stocks can be identified \( A_I \gtrless A_{II} \) if and only if \( (\mu_I - \mu_I)/(\sigma_I - \sigma_{II}) \gtrless \delta \). Figure 1 shows that the cumulative distribution of stock I (F) will has a thicker left tail than the cumulative distribution of stock II (G) because \( \sigma_I > \sigma_{II} \) where F and G intersect at \( x_0 = [(\mu_I, \sigma_{II}) - (\mu_{II}, \sigma_I)]/\sigma_{II} - \sigma_I \).
The cumulative normal distribution for case $\mu_I > \mu_{II}$ and $\sigma_I > \sigma_{II}$

Based on Fig. 1 it is known if $A_I > A_{II}$ then $A_I > A_{II} > x_0$, so that $F^*$ dominates $G^*$ by FSD if and only if
\[
\frac{(\mu_I - \mu_{II})}{(\sigma_I - \sigma_{II})} > \delta.
\]

The case where the mean of stock I is greater than the mean of stock II which is $\mu_I > \mu_{II}$ and the standard deviation of stock I is greater than the standard deviation of stock II which is $\sigma_I < \sigma_{II}$, so this will have implications for $A_I > A_{II}$. The relation between $B_I$ and $B_{II}$ can be identified $B_I \geq B_{II}$ if and only if $(\mu_I - \mu_{II})/(\sigma_{II} - \sigma_I) \geq \delta$. Figure 2 shows that $F$ intersects $G$ from below at $x_0 = ([\mu_I, \sigma_{II}) - (\mu_{II}, \sigma_I)]/\sigma_{II} - \sigma_I$.

If the dominance of stock I over stock II by FSD with inequality (4) is met, then stock I will dominate stock II by SSD automatically. This property is derived from the dominance rule on stochastic dominance with a normal distribution where $F$ will dominate $G$ by SSD based on the mean variance rule, i.e. $\mu_I > \mu_{II}$ and $\sigma_I < \sigma_{II}$ [7].
4. Result and Discussion
There are 31 stocks included in the calculation of the LQ45 index over the period of August 2014 to July 2018 [10]. Then, the monthly stock closing price data is used to calculate the return of 31 stocks by equation (1). Jarque Bera normality test on stock return data is performed by the null hypothesis which is stock returns are normally distributed and the significance level used is 0.05. From the Jarque Bera normality test, a number of 22 stocks with normal distribution were obtained.

Table 1. The list of stocks which have normally distributed returns.

| Number | Stock | Number | Stock | Number | Stock | Number | Stock |
|--------|-------|--------|-------|--------|-------|--------|-------|
| 1      | ADHI  | 7      | BMRI  | 13     | JSMR  | 19     | SMGR  |
| 2      | AKRA  | 8      | BSDE  | 14     | KLBF  | 20     | TLKM  |
| 3      | ASII  | 9      | GGRM  | 15     | LPKR  | 21     | UNTR  |
| 4      | BBCA  | 10     | INCO  | 16     | LPPF  | 22     |       |
| 5      | BBRI  | 11     | INDF  | 17     | PTPP  |       |       |
| 6      | BBTN  | 12     | INTP  | 18     | POWN  |       |       |

Using a significance level ($\alpha$) of 0.05, the truncation points for each tail corresponds to $2\sigma$ from the mean. The illustration of the normal distribution of INTP stock returns which are truncated symmetrically based on equation (2) represents in Fig. 3.

Figure 3. (a) The density of normal distribution (b) The density of truncated normal distribution

All stocks are then combined and identified to choose the rule of dominance inequality (3) or (4) to be used. From the analysis for stock combinations with the case of $\mu_I > \mu_{II}$ and $\sigma_I > \sigma_{II}$, there are 17 stock combinations which have dominance by FSD.

Table 2. The results of stock dominance by FSD.

| Stock | Domination | Total |
|-------|------------|-------|
| ADHI  | LPKR       | 1     |
| BBRI  | AKRA, INDF, JSMR, SCMA | 4 |
| BBTN  | LPPF       | 1     |
| GGRM  | KLBF, TLKM | 2     |
| INTP  | ADHI, LPKR | 2     |
| PTPP  | LPKR       | 1     |
| PWON  | LPPF       | 1     |
| TLKM  | KLBF       | 1     |
| UNTR  | AKRA, BSDE, JSMR, SCMA | 4 |
Based on Table 2, we can see the stock ranking based on the number of domination. The first rank is BBRI and UNTR which both dominates four stocks and three of them are the same stocks namely AKRA, JSMR, and SCMA. The difference is, BBRI also dominates INDF while UNTR dominates BSDE. The second rank are GGRM and INTP which are GGRM dominates KLBF and TLKM while INTP dominates ADHI and LPKR. Furthermore, there are ADHI and PTPP which both dominates LPKR, BBTN and also PWON which dominates LPPF. Finally the last rank is TLKM that dominates KLBF.

The domination analysis for stock combinations with the case of $\mu_I > \mu_{II}$ and $\sigma_I < \sigma_{II}$ give 14 stock combinations that have dominance in FSD and SSD. The results of the analysis are given in Table 3.

| Stock | Domination       | Total |
|-------|------------------|-------|
| ASII  | SCMA             | 1     |
| BBRI  | BSDE, LPPF, SMGR, UNTR | 4     |
| BBTN  | ADHI, INTP, LPKR, PTPP | 4     |
| BMRI  | KLBF             | 1     |
| INDF  | SCMA             | 1     |
| PWON  | LPKR             | 1     |
| UNTR  | LPPF, SMGR       | 2     |

Based on Table 3, BBRI and BBTN were ranked first with both dominating four stocks. Where BBRI dominates BSDE, LPPF, SMGR, and UNTR, while BBTN dominates ADHI, INTP, LPKR, and PTPP. Furthermore, in the second rank there were UNTR which had two dominance, namely towards LPPF and SMGR. In the next rank, there are ASII and INDF which both have dominance over SCMA, BMRI which dominates KLBF, and PWON which dominates LPKR.

5. Conclusion

Based on the discussion, it can be concluded that from 12 stocks that have dominance, there are 17 combinations of stocks that dominate by FSD and 14 stocks combinations that dominate by SSD. Also based on the total number of dominance both FSD and SSD, it can be obtained the ranking represent the optimum stocks, namely BBRI with 8 dominance, UNTR with 6 dominance, BBTN with 5 dominance, INTP, GGRM, PTPP, and PWON with 2 dominance, ADHI, ASII, BMRI, INDF, and TLKM with 1 dominance.

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