Research Article

Inflow Performance Analysis of a Horizontal Well Coupling Stress Sensitivity and Reservoir Pressure Change in a Fractured-Porous Reservoir

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Stress sensitivity has always been a research hotspot in fractured-porous reservoirs and shows huge impacts on well productivity during the depletion development. Due to the continuous reservoir pressure change, accurate evaluation of stress sensitivity and its influence on well productivity is of great significance to optimize well working system. Taking horizontal well trajectory as the research object, the principal focus of this work is on the analysis of inflow performance for a horizontal well coupling stress sensitivity and reservoir pressure change in a fractured-porous reservoir. Firstly, a relationship between permeability damage rate and stress sensitivity coefficient was established to quantitatively evaluate the influence of reservoir pressure and stress sensitivity on reservoir permeability. Secondly, considering stress sensitivity and reservoir pressure drop, a set of practical productivity equations were derived for a horizontal well in a fractured-porous reservoir by adopting the equivalent seepage resistance method. Finally, the influence of relevant important factors on the inflow performance of horizontal wells was discussed in depth. Results show that a positive correlation exists between stress sensitivity coefficient and maximum permeability damage rate. At the same maximum permeability damage rate, high initial reservoir pressure corresponds to low stress sensitivity coefficient. In general, stress sensitivity coefficient mainly ranges from 0 to 0.2. Reservoir pressure change drastically affects the production dynamic characteristics of horizontal wells, and both the inflow performance curve and the production index curve decline and shrink as reservoir pressure decreases. Stress sensitivity is negatively correlated with horizontal well productivity, and the inflow performance/production index curve bends closer to bottom-hole pressure axis, and an inflection point can be observed with the aggravation of stress sensitivity. In addition, horizontal wellbore length and initial reservoir permeability also show significant effects on the inflow performance and are positively correlated with well productivity. For water cut, it has little effect on the well production when bottom-hole pressure drawdown is low, but its effect gets stronger as the drawdown becomes higher. Meaningfully, depending on these newly established productivity equations, a reasonable production system can be quantitatively optimized and achieved for the horizontal wells in fractured-porous reservoirs.

1. Introduction

Under the influence of tectonism and diagenesis, natural fractures are often developed in various reservoirs, which are referred to as fractured-porous reservoirs, including sandstone fractured reservoirs, glacier fractured reservoirs, carbonate fractured reservoirs, and igneous fractured reservoirs [1, 2]. Fractured-porous reservoirs often show a degree of stress sensitivity during depletion development, leading to a decrease in reservoir permeability and well productivity [3–5]. Accurate evaluation of stress sensitivity and its influence on well productivity is of great significance to optimize...
the well working system in these kinds of reservoirs and thus can improve oil recovery and save production cost at the same time. As an ancient but important method, inflow performance analysis has been widely used for its simplicity and practicality to evaluate and predict well productivity in production and reservoir engineering [6–9], and it is also the main research method adopted in this study.

Oilfield development is a dynamic process coupling multiphase seepage and elastoplastic deformation of porous media. Changes in pore fluid pressure lead to deformation of rock skeleton, which is the origin of stress sensitivity [2, 4, 10, 11]. Hence, stress-sensitive reservoirs are also called deformable medium reservoirs. At present, there are two methods to study stress sensitivity: (1) direct laboratory experiment to evaluate the strength of stress sensitivity and the influence of rock deformation on porosity and permeability. Many laboratory tests show that for fractured-porous reservoirs, porosity and permeability of matrix and natural fracture decrease with the increase of overburden pressure and effective stress, and the permeability damage of fracture system is more serious than matrix system under the same effective stress increment [12–14]; (2) rigorous theoretical derivation by defining stress sensitivity coefficient to establish seepage theory of deformable medium reservoirs. Nur and Yilmaz [15] first defined stress sensitivity coefficient to quantify permeability variation. Depending on this newly defined parameter, Kikani and Pedrosa [16] derived a permeability formula with respect to pressure change in the form of exponential function:

\[ K = K_i e^{-\alpha dp} = K_i e^{-\alpha(p_i - p)} , \]

where \( \alpha \) is stress sensitivity coefficient; \( K_i \) and \( p_i \) are initial reservoir permeability and pressure; and \( K \) and \( p \) are testing permeability and testing pressure. This exponential expression is widely applied to investigate the fluid transport behavior of deformable medium [17–21]. In this study, we also use this exponential expression to incorporate stress sensitivity into horizontal well inflow performance. It can be seen from this expression that there is an obvious correlation between initial reservoir pressure and initial reservoir permeability. For a depleted fractured-porous reservoir, initial reservoir pressure is a key parameter affecting well productivity, but the relationship between initial reservoir pressure, stress sensitivity coefficient, and reservoir permeability remains unclear at present.

Compared with vertical wells, horizontal wells can traverse more natural fractures and enlarge the seepage area and then improve well productivity [22, 23]. Water channeling may occur along natural fractures because of the presence of edge-bottom water or injected water, and crude oil degassing also may happen near horizontal wellbore simultaneously, resulting in the multiphase inflow performance problem. At present, there are three methods to solve oil-gas-water three-phase flow: (1) Wiggins’ method, (2) Petrobras’ method, and (3) three-phase IPR general formula method, which form three typical productivity equations [9, 24, 25]. Wiggins’ productivity equation is only applicable to saturated reservoirs, and the maximum oil/water production must be calculated first. The general IPR productivity equation of three-phase flow is applicable to both saturated and unsaturated reservoirs, but flow efficiency should be calculated by graphic method or linear fitting method beforehand. Therefore, the productivity equation of Wiggins and three-phase IPR general formula are rather complicated when in use. Petrobras’ productivity equation is the weighted average of pure-oil phase productivity and pure-water phase productivity according to water cut, which is applicable to both saturated and unsaturated reservoirs. At present, there are many single-phase productivity equations and oil-gas two-phase productivity equations, and thus, the use of Petrobras’ method becomes simple and convenient. In this study, Petrobras’ method is also used to deal with the three-phase flow problem in fractured-porous reservoirs. Therefore, the key is to establish accurate pure-oil phase and oil-gas two-phase productivity equations of horizontal wells in stress-sensitive reservoirs.

There have been a lot of studies on pure-oil phase productivity equation of horizontal wells, mainly including the following three classical formulas: (1) Borisov’s equation; (2) Giger’s equation; and (3) Joshi’s equation [26–28]. Borisov [26] laid a foundation for single-phase steady-flow equation of horizontal wells, and subsequent productivity formulas of horizontal wells were modified from Borisov’s formula. Borisov’s equation is relatively simple, and the results calculated from this equation have not much difference with those from Giger’s equation and Joshi’s equation. The disadvantage of these single-phase productivity equations is that they do not take stress sensitivity and reservoir pressure change into account, and they also assume steady-state flow happens in the reservoirs, which is not consistent with the actual flow (pseudosteady flow). Similarly, there have been many oil-gas two-phase productivity equations of horizontal wells, mainly for dissolved gas drive reservoirs: (1) Bendakhila and Aziz’s equation; (2) Cheng’s equation; (3) Sun and Tian’s equation; and (4) Liu’s equation [29–32]. The main drawback of these four productivity equations lies in that they are only applicable to saturated reservoirs and ignore the possible water-phase flow at the bottom hole. Meaningfully, these single-phase or two-phase productivity equations have thrown some new insights into the derivation of more accurate productivity equations for horizontal wells, which is closer to the actual reservoir. Wang [4] considered stress sensitivity to derive the pure-oil phase and oil-gas two-phase productivity equations of horizontal wells in low-permeability deformable medium reservoirs by combining with the equivalent seepage resistance method and the conformal transformation method. However, this equation assumes that average reservoir pressure is approximately unchanged and steady flow happens in the reservoir during production, which is contrary to the actual production situation.

In general, due to the comprehensive influence of stress sensitivity, reservoir pressure change, and multiphase flow in fractured-porous reservoirs, the existing horizontal well inflow performance equations show obvious limitations and cause some error in predicting well productivity, which brings challenges to the optimization of well working system.
Therefore, the main purpose of this paper is to analyze the inflow performance for a horizontal well coupling stress sensitivity and reservoir pressure change in a fractured-porous reservoir. This paper is organized as follows: firstly, combining permeability damage rate and stress-sensitive expression, the relationship between reservoir permeability, stress sensitivity coefficient, and initial reservoir pressure was investigated. Secondly, by means of the equivalent seepage resistance method, a series of pseudosteady inflow dynamic equations for horizontal wells were derived considering stress sensitivity and reservoir pressure change. Thirdly, inflow performance curves and oil or liquid production index curves under different conditions were drawn out to discuss the effects of the related parameters on horizontal well productivity and the optimal bottom-hole pressure was also demonstrated.

2. Relationship between Reservoir Permeability, Stress Sensitivity Coefficient, and Initial Reservoir Pressure of Fractured-Porous Reservoir

The initial permeabilities of rock samples vary greatly, and the degree of stress sensitivity cannot be accurately reflected only from the change of permeability value. In general, permeability damage rate is defined to reflect reservoir stress sensitivity,

\[ D_K = \frac{K_i - K}{K_i} \times 100\% \]  \hspace{1cm} (2)

where \( D_K \) is permeability damage rate of rock sample, \%; \( K_i \) is initial permeability of rock sample, \( 10^{-3} \mu m^2 \); and \( K \) is testing permeability of rock sample, \( 10^{-3} \mu m^2 \). When \( K \) is the minimum permeability in the testing process, \( D_K \) corresponds to the maximum permeability damage rate of rock sample.

According to permeability damage rate formula (2) and stress-sensitive permeability formula (1), the relationship between stress sensitivity coefficient and permeability damage rate can be established:

\[ \alpha = -\frac{\ln (1 - D_K)}{\Delta p} = -\frac{\ln (1 - D_K)}{p_i - p} \]  \hspace{1cm} (3)

When reservoir pressure approaches zero (namely, abandoned pressure), corresponding to the maximum permeability damage rate \( (D_{K_{\text{max}}} \) ), the above equation can be simplified as

\[ \alpha = -\frac{\ln (1 - D_{K_{\text{max}}})}{p_i} \]  \hspace{1cm} (4)

Taking maximum permeability damage rate as the evaluation index, stress sensitivity can be divided into three levels: weak \( (D_{K_{\text{max}}} \leq 30\%) \), intermediate \( (30\% < D_{K_{\text{max}}} \leq 70\%) \), and strong \( (D_{K_{\text{max}}} > 30\%) \). According to this classification standard and given different initial reservoir pressures, the stress sensitivity coefficients corresponding to the typical permeability damage rates at three stress-sensitive levels can be obtained by using Equation (4) (Table 1).

Further, Equation (4) can be used to calculate the stress sensitivity coefficients under different maximum permeability damage rates. Figure 1 presents the relationship between maximum permeability damage rate and stress sensitivity coefficient under different initial reservoir pressures \( (p_i = 20, 25, 30, \text{and} 35 \text{MPa}) \). Obviously, stress sensitivity coefficient and maximum permeability damage rate exist a significant positive correlation. At the same maximum permeability damage rate, the higher the initial reservoir pressure, the lower the stress sensitivity coefficient. Overall, stress sensitivity coefficient mainly ranges from 0 to 0.2, which provides a quantitative basis for investigating the influence of stress sensitivity on well productivity in the discussion part. Combining with the data presented in Table 1, the relationship curves between reservoir permeability and reservoir pressure under different initial reservoir pressures and stress sensitivities also can be obtained with Equation (4), as shown in Figure 2. Strong stress sensitivity leads to fast decrease of reservoir permeability during pressure depletion. Under the same initial reservoir permeability and stress sensitivity, high initial reservoir pressure corresponds to slow decrease of reservoir permeability.

3. Physical Model of Horizontal Well Coupling Stress Sensitivity and Reservoir Pressure Change in a Fractured-Porous Reservoir

In this study, fractured-porous reservoirs are regarded as classic double-porosity and single-permeability reservoirs, made up of matrix system and natural fracture system [33]. When reservoir pressure falls during the depletion development, natural fractures tend to close and then lead to stress sensitivity. Figure 3 is a schematic illustration of a horizontal well coupling stress sensitivity and reservoir pressure change in a fractured-porous reservoir. The natural fracture system provides the main flow ability together with the matrix system.

To simplify this physical model, some basic assumptions are made as follows:

(i) This reservoir is circular and has a closed outer boundary, and reservoir radius and thickness are \( r_e \) and \( h \), respectively.

(ii) The fractured-porous reservoir is homogeneous, and the initial permeability of the matrix and natural fracture system are \( k_{mi} \) and \( k_{fi} \), respectively. In this study, an equivalent initial reservoir permeability, \( k_{ei} \), is used to represent the flow capacity of the entire reservoir system, equal to the sum of \( k_{ni} \) and \( k_{mi} \), and reservoir stress sensitivity coefficient is \( \alpha_i \).

(iii) Initial reservoir pressure and bubble-point pressure are \( p_b \) and \( p_{bo} \), respectively. This reservoir may be unsaturated \( (p_0 > p_b) \) or saturated \( (p_0 \leq p_b) \). The average reservoir pressure during production is \( p_{\text{avg}} \).
(iv) Fluid in this reservoir is slightly compressible and has constant viscosity $\mu_0$ or $\mu_w$, volume factor $B_o$ or $B_w$, and total compressibility $C_t$, and flow in this model obeys Darcy's law.

(v) A horizontal well is located at the center of the circular reservoir, and its distance from the bottom boundary is $z_w$. Horizontal wellbore radius and half-length are $r_w$ and $L$, respectively, and bottom-hole flow pressure is $p_{wf}$.

(vi) Gravity and capillary effects are neglected

4. Productivity Equations of Horizontal Well Coupling Stress Sensitivity and Reservoir Pressure Change in a Fractured-Porous Reservoir

Regardless of water phase, single-phase flow (oil phase) appears first and then, two-phase flow (oil phase and gas phase) appears in the unsaturated reservoirs. The cut-off point of two scenarios is reservoir bubble-point pressure. When reservoir pressure depletes below bubble-point pressure, the dissolved gas starts to separate from crude oil, resulting in oil-gas two-phase flow. For saturated reservoirs, two-phase flow occurs from the beginning of depletion production. Therefore, the derivation of the productivity

| $\alpha$ | $p_i = 20$ MPa | $p_i = 25$ MPa | $p_i = 30$ MPa | $p_i = 35$ MPa |
| --- | --- | --- | --- | --- |
| Weak ($D_{k\text{max}} = 15\%$) | 0.0081 | 0.0065 | 0.0054 | 0.0046 |
| Intermediate ($D_{k\text{max}} = 50\%$) | 0.0347 | 0.0277 | 0.0231 | 0.0198 |
| Strong ($D_{k\text{max}} = 80\%$) | 0.0949 | 0.0759 | 0.0632 | 0.0542 |
4.1. Unsaturated Reservoirs. For unsaturated reservoirs, flow in the reservoir is single-phase oil flow when \( p_{wf} \geq p_b \), while \( p_{wf} < p_b \), oil-gas two-phase flow occurs near the wellbore.

4.1.1. Single-Phase Oil Flow. By using the equivalent seepage resistance method, Borisov [26] decomposed the three-dimensional flow field of a horizontal well into two two-dimensional flow zones, including an inner zone (B-B: longitudinal plane) and outer zone (A-A horizontal plane) (Figure 4). Depending on the decomposition results, the productivity equations of horizontal well with stress sensitivity and reservoir pressure change can be derived by combining with the flow characteristics of two independent zones.

(i) Equivalent seepage in the outer zone: pseudosteady flow occurs in the outer zone, which can be regarded as follows: in the horizontal plane (A-A), crude oil flows from a circular closed zone, with a radius of \( r_e \) and an average reservoir pressure of \( p_{ave} \), to a vertical well with a diameter of \( L/2 \) and a bottom-hole pressure of \( p_m \) (corresponding to the reservoir pressure at the junction of two zones). Zhao et al. [34] derived out the pseudosteady productivity equation of a vertical well considering stress sensitivity and reservoir pressure change (Equation (17) in this reference):

\[
Q = \frac{542.87 k_o h}{\mu_o B_o (\ln (r_e/r_w) - (3/4))} \alpha_k \alpha e^{-e_2 k_p (p_e - p_m)} - e^{-e_2 k_p (p_e - p_{ave})}
\]

By referring to this equation, the productivity equation of the outer zone of the horizontal well with stress sensitivity and reservoir pressure change can be written as

\[
Q_1 = \frac{542.87 k_o h}{\alpha_k \mu_o B_o (\ln (r_e/(L/2)) - (3/4))} \alpha e^{-e_2 k_p (p_e - p_{ave})} - e^{-e_2 k_p (p_e - p_{ave})}
\]

The above equation can be rewritten into the form of inflow dynamic equation:

\[
p_{ave} - p_m = \frac{1}{\alpha_k} \ln \left\{ 1 + \frac{\alpha_k \mu_o B_o Q}{542.87 k_o h e^{-e_2 k_p (p_e - p_{ave})}} \right\} \left[ \ln \left( \frac{r_e}{L/2} \right) - \frac{3}{4} \right]
\]

(ii) Equivalent seepage in the inner zone: in the longitudinal plane (B-B), the horizontal well profile can be regarded as a vertical well with a radius of \( r_w \) and a bottom-hole pressure of \( p_{wf} \), and its seepage area is a cylindrical surface with a radius of \( h/2 \pi \), a height of \( 2L \), and a boundary pressure of \( p_m \). Pseudosteady flow also happens in this zone. Zhao et al. [34] also presented the formation pressure distribution of a vertical well considering stress sensitivity and reservoir pressure drop (Equation (9) in this reference):

\[
p(r) = p_{wf} + \frac{1}{\alpha_k} \ln \left[ 1 + \frac{\alpha_k \mu_o B_o Q}{542.87 k_o h e^{-e_2 k_p (p_e - p_{ave})} h} \right] \left( \ln \left( \frac{r}{r_w} \right) - \frac{r^2}{2r_w^2} \right)
\]

Similarly, referring to Equation (8) and combining with the equivalent scenario of the inner region, the pressure at the boundary of the inner zone for the horizontal well with stress sensitivity and reservoir pressure change can be expressed as follows:

\[
p_m = p_{wf} + \frac{1}{\alpha_k} \ln \left[ 1 + \frac{\alpha_k \mu_o B_o Q_2}{542.87 k_o h e^{-e_2 k_p (p_e - p_{ave})} (2L)} \right] \left( \ln \left( \frac{h/2 \pi}{r_w} \right) - \frac{(h/2 \pi)^2}{2(h/2 \pi)^2} \right)
\]

\[
= p_{wf} + \frac{1}{\alpha_k} \ln \left[ 1 + \frac{\alpha_k \mu_o B_o Q_2}{542.87 k_o h e^{-e_2 k_p (p_e - p_{ave})} (2L)} \right] \left( \ln \left( \frac{h/2 \pi}{r_w} \right) - \frac{1}{2} \right)
\]

Substitute the above equation into Equation (7) and eliminate \( p_m \). Considering \( Q_1 = Q_2 = Q_{ho} \), the pseudosteady
productivity equation of a horizontal well coupling stress sensitivity and reservoir pressure change can be obtained:

\[ p_{\text{avg}} - p_{\text{wf}} = -\frac{1}{\alpha_s} \left\{ \ln \left( 1 - \frac{\alpha_s k_r B_o Q_{ho}}{542.87 k_h h \varepsilon^a (p_i / p_{avg})} \ln \left( \frac{r_w}{L/2} \right) - \frac{3}{4} \right) \right\} + \frac{1}{\alpha_s} \ln \left( 1 + \frac{\alpha_s k_r B_o Q_{ho}}{542.87 k_h h \varepsilon^a (p_i / p_{avg}) (2L)} \left[ \ln \left( \frac{r_w}{h/2\pi} \right) - \frac{1}{2} \right] \right) \left( p_{\text{avg}} > p_{\text{wf}} \geq p_b \right) \] (10)

Equation (10) is an implicit function of horizontal well productivity \( Q_{ho} \) with respect to bottom-hole pressure \( p_{\text{wf}} \), and the analytical expression of this productivity equation cannot be obtained directly. Moreover, the productivity equation is strongly nonlinear, but it can be programmed and solved by the Newton iterative method, and the corresponding oil production index can be obtained by dividing the bottom-hole pressure drawdown:

\[ f = \frac{Q_{ho}}{p_{\text{avg}}} \] (11)

4.1.2. Oil-Gas Two-Phase Flow. When \( p_{\text{wf}} < p_b \), fluid near the wellbore begins to degas and oil-gas two-phase flow appears in the reservoir. Vogel’s equation is established for vertical wells to describe their inflow performance curve, and thus, it is the most used inflow performance equation for horizontal wells in dissolved gas drive reservoirs: (1) Bendakhilia and Aziz’s equation [29], (2) Cheng’s equation [30], (3) Sun and Tian’s equation [32], and (4) Liu’s equation [31]. Cheng’s equation is consistent with Vogel’s equation in form and only requires a group of test points to obtain the inflow performance curve, and thus, it is the most used inflow performance equation for horizontal wells. The drawback of Cheng’s equation is that this equation does not normalize when bottom-hole pressure is equal to bubble-point pressure and zero, and break points appear on the inflow performance curves. To solve the nonnormalization problem of Cheng’s equation, Sun and Tian [32] used the reservoir numerical simulation data of Cheng to make regression and normalize the equation and then obtained a new dimensionless inflow performance equation of horizontal wells in dissolved gas drive reservoir, whose form is almost the same as Vogel’s equation:

\[ \frac{Q_{ho}}{Q_{ho \text{ max}}} = 1 - 0.0005 \frac{p_{\text{wf}}}{p_{\text{avg}}} - 0.6338 \left( \frac{p_{\text{wf}}}{p_{\text{avg}}} \right)^2 - 0.3657 \left( \frac{p_{\text{wf}}}{p_{\text{avg}}} \right)^3 \] (12)

Two-phase flow productivity equation of vertical wells can be established by Vogel’s equation [35]. Using this similar idea in vertical wells, two-phase flow productivity equation of horizontal wells also can be established by replacing \( p_{\text{avg}}, Q_{ho \text{ max}}, Q_{ho} \) with \( p_b, Q_c, \) and \( Q_{ho} - Q_b \) in Equation (12), respectively.

\[ \frac{Q_{ho} - Q_b}{Q_c} = 1 - 0.0005 \left( \frac{p_{\text{wf}}}{p_b} \right) - 0.6338 \left( \frac{p_{\text{wf}}}{p_b} \right)^2 - 0.3657 \left( \frac{p_{\text{wf}}}{p_b} \right)^3, \] (13)

where \( Q_{ho} \) is oil productivity during oil-gas two-phase flow; \( Q_b \) is oil productivity when bottom-hole pressure equals bubble-point pressure; and \( Q_c \) is maximum oil productivity during oil-gas two-phase flow.

According to Equation (10), at \( p_{\text{wf}} = p_b \), the derivative of oil productivity \( Q_{ho} \) with respect to bottom-hole pressure \( p_{\text{wf}} \) can be obtained, which is equal to the negative value of the corresponding production index during single-phase flow:

\[ \frac{dQ_{ho}}{dp_{\text{wf}}} \bigg|_{p_{\text{wf}}=p_b} = -J_b, \] (14)

The derivative of oil productivity \( Q_{ho} \) with respect to bottom-hole pressure \( p_{\text{wf}} \) at \( p_{\text{wf}} = p_b \) can be derived directly from Equation (13):

\[ \frac{dQ_{ho}}{dp_{\text{wf}}} \bigg|_{p_{\text{wf}}=p_b} = -Q_c \left( \frac{0.0005}{p_b} + \frac{1.2676 p_{\text{wf}}}{p_b^3} + \frac{1.0971 p_{\text{wf}}^2}{p_b^5} \right) \bigg|_{p_{\text{wf}}=p_b} = -\frac{2.3652 Q_c}{p_b}. \] (15)

Obviously, the two derivatives above are equal, and thus, we can get

\[ Q_c = \frac{J_b p_b}{2.3652}. \] (16)

Substituting the above equation into Equation (13), the pseudosteady productivity equation of horizontal wells happening oil-gas two-phase flow can be obtained as follows:

\[ Q_{ho} = Q_b + \frac{J_b p_b}{2.3652} \left[ 1 - 0.0005 \left( \frac{p_{\text{wf}}}{p_b} \right) - 0.6338 \left( \frac{p_{\text{wf}}}{p_b} \right)^2 - 0.3657 \left( \frac{p_{\text{wf}}}{p_b} \right)^3 \right] \left( p_{\text{avg}} > p_b > p_{\text{wf}} \right). \] (17)

Combining Equations (10) and (17), the two-phase flow productivity equation for horizontal well with stress...
sensitivity and reservoir pressure change in unsaturated reservoirs can be established as follows:

\[
p_{\text{avg}} - p_{\text{wf}} = -\frac{1}{\alpha_k} \ln \left\{ 1 - \frac{\alpha_k \mu_{B_o} Q_{b_0}}{542.87 k_{ch} h e^{-\alpha_k (P_{\text{ref}} - P_{\text{avg}})}} \left[ \ln \left( \frac{r_e}{L/2} \right) - \frac{3}{4} \right] \right\}
+ \frac{1}{\alpha_k} \ln \left\{ 1 + \frac{\alpha_k \mu_{B_o} Q_{b_0}}{542.87 k_{ch} e^{-\alpha_k (P_{\text{ref}} - P_{\text{avg}})/(2L)}} \left[ \ln \left( \frac{h/2\pi}{r_w} \right) - \frac{1}{2} \right] \right\},
\]

where \( Q_{b_0} \) is water productivity of horizontal well. Since there is no degassing problem in water phase, water productivity can be calculated using this equation whether bottom-hole pressure is above or below bubble-point pressure.

The respective oil productivity for \( p_{\text{ref}} \geq p_h \) and \( p_{\text{ref}} < p_h \) can be calculated from Equation (10) and Equation (18). Further, according to the weighted average of water cut, the three-phase inflow dynamic equations (total liquid production) for a horizontal well with stress sensitivity and reservoir pressure change can be established for \( p_{\text{ref}} \geq p_h \) and \( p_{\text{ref}} < p_h \), respectively, and the corresponding liquid production index also can be obtained easily.

\[
Q_{\text{th}} = (1 - f_w) Q_{b_0} + f_w Q_{l_{\text{avg}}}. 
\]

4.1.3. Oil-Gas-Water Three-Phase Flow. According to the Petrobras' method of establishing three-phase inflow performance curve for vertical wells, the three-phase inflow performance curve for horizontal wells can be obtained by taking the weighted average of oil and water productivity with respect to water cut [35]. Water phase and oil phase have different physical properties, like viscosity and volume factor, but their flow behaviors are similar in the formation. Hence, referring to Equation (10), the inflow dynamic equation of water phase can be written as

4.2. Saturated Reservoirs. The initial pressure of saturated reservoir is lower than the bubble-point pressure, and the fluid near the wellbore is degassed, and oil-gas two-phase flow occurs as soon as horizontal well begins to produce. Three-phase flow also occurs if water production is considered.

4.2.1. Oil-Gas Two-Phase Flow. For saturated reservoirs, the modified Cheng equation can still be used to determine the pseudosteady productivity equation, but note that the well productivity is zero when \( p_{\text{ref}} = p_h \). Replacing \( p_{\text{avg}} \) with \( p_h \) in Equation (18), the pseudosteady productivity equation of horizontal well with oil-gas two-phase flow in saturated reservoir considering stress sensitivity and reservoir pressure change can be obtained as follows:

\[
p_{\text{avg}} - p_{\text{wf}} = -\frac{1}{\alpha_k} \ln \left\{ 1 - \frac{\alpha_k \mu_{B_o} Q_{b_0}}{542.87 k_{ch} h e^{-\alpha_k (P_{\text{ref}} - P_{\text{avg}})}} \left[ \ln \left( \frac{r_e}{L/2} \right) - \frac{3}{4} \right] \right\}
+ \frac{1}{\alpha_k} \ln \left\{ 1 + \frac{\alpha_k \mu_{B_o} Q_{b_0}}{542.87 k_{ch} e^{-\alpha_k (P_{\text{ref}} - P_{\text{avg}})/(2L)}} \left[ \ln \left( \frac{h/2\pi}{r_w} \right) - \frac{1}{2} \right] \right\},
\]

(21)
Equation (21) can be programmed and solved by the Newton iterative method, and then, the oil production index under different bottom-hole flow pressures can also be obtained.

4.2.2. Oil-Gas-Water Three-Phase Flow. Referring to the previous derivation results, water- and oil-phase productivity equations in this flow pattern can be expressed by Equations (19) and (21), respectively. Similarly, using the Petrobras’ method, liquid production and production index can be obtained when three-phase flow occurs in saturated reservoir considering stress sensitivity and reservoir pressure change.

5. Results and Discussion

Depending on the above productivity equations established in fractured-porous reservoirs, various inflow performance relationship curves can be drawn out to investigate the productivity change of horizontal wells incorporating stress sensitivity and reservoir pressure change. The influences of stress sensitivity and reservoir pressure change on inflow performance and production index were discussed in detail. The influences of other parameters on inflow performance characteristic, including horizontal wellbore length, initial reservoir permeability, and water cut, were investigated too. The parameters like reservoir thickness, viscosity and volume factor of crude oil, skin factor, and others also can be discussed by using the above productivity equations. However, considering the focus of this study and avoiding excessive and tedious discussion, these factors will not be discussed here. The relevant data of North Truva Oilfield in Kazakhstan (Table 2), a typical fractured-porous carbonate reservoir, was chosen to carry out the specialization of horizontal well production system. Note that when there is no special explanation, the basic values of the relevant parameters, as shown in Table 2, are selected during the discussion.

5.1. Inflow Performance Comparison between Horizontal Well and Vertical Well with Stress Sensitivity and Reservoir Pressure Change. To our knowledge, there is no horizontal well productivity equation that considers both stress sensitivity and reservoir pressure change in the literature. Zhao et al. [34] deduced the vertical well productivity equation under this kind of complex situation. Therefore, the comparison between the productivity results calculated from Zhao et al.’s model and our model can deepen the understanding and prove the novelty of this newly proposed model.

Figures 5 and 6 reveal the difference of the inflow performance and oil or liquid production index between horizontal well and vertical well in a fractured-porous reservoir, both of which couple stress sensitivity and reservoir pressure change, respectively. Under the same reservoir condition and two-phase flow (or three-phase flow), the inflow performance of two kinds of wells is greatly different, and the productivity and production index of vertical well are much lower than those of horizontal well. It is worth noting that horizontal well requires lower drawdown pressure to achieve the maximum productivity and shows a significant inflection point (blue dot). Similar to vertical well, this kind of characteristic is also caused by stress sensitivity and reservoir pressure drop [34]. In addition, the production index curve of vertical well is steeper than that of horizontal well. All these
characteristics directly demonstrate the advantage of horizontal wells over vertical wells in the oilfield development.

5.2. Influence of Key Parameters on Inflow Performance of Horizontal Well Coupling Stress Sensitivity and Reservoir Pressure Change in a Fractured-Porous Reservoir

5.2.1. Reservoir Pressure Change. In this study, reservoir pressure level is defined to reflect the reservoir pressure change during depleted development. It represents the ratio of average reservoir pressure to initial reservoir pressure. Figures 7 and 8 show the inflow performance and production index of a horizontal well in the fractured-porous reservoir under varied reservoir pressure level (100%, 85%, 70%, 55%, and 50%), respectively. Obviously, reservoir pressure has a positive correlation with horizontal well productivity and production index. As reservoir pressure gradually decreases, both the inflow performance and production index curves shrink towards the coordinate origin, and the maximum productivity and the corresponding bottom-hole pressure also decrease, showing that the inflection points move down to the lower left. Therefore, efficient and economical reservoir development requires adjusting drawdown pressure in response to reservoir pressure changes to optimize well production. When reservoir pressure level drops to 50%, fluid flow in the formation is very weak, and thus, horizontal well productivity is rather low and nearly approaches zero. Additionally, with the decrease of bottom-hole pressure, the production index reduces linearly when reservoir pressure level is higher than 85%. However, the production index curve bends towards the bottom-hole pressure axis and occurs significant inflection points (first increase and then decrease) when reservoir pressure level is lower than 85%. This phenomenon can be determined by the degassing of crude oil near the bubble-point pressure (equal to 20.54 MPa). For the reservoir whose average pressure is lower than bubble-point pressure, degassing is weak and dissolved gas displacement enhances the production index when drawdown pressure is low, but degassing becomes serious and the fluidity of crude oil also becomes worse when drawdown pressure gets large, and then, extracted gas flows into well bottom hole first, leading to the reduction of production index.

Figure 6: Comparison of oil or liquid production index between horizontal well and vertical well with stress sensitivity and reservoir pressure change in a fractured-porous reservoir.

Figure 7: Effect of reservoir pressure change on inflow performance of a horizontal well with stress sensitivity in a fractured-porous reservoir.

Figure 8: Effect of reservoir pressure change on oil production index of a horizontal well with stress sensitivity in a fractured-porous reservoir.
5.2.2. Stress Sensitivity. Stress sensitivity is one of the focuses of this study. Figures 9 and 10 display the inflow performance curve and production index curve of horizontal well under different reservoir stress sensitivities ($\alpha_k = 0, 0.001, 0.005, 0.025, 0.05, \text{and} 0.075$), respectively.

As shown in Figure 9, stress sensitivity has little effect on the horizontal well productivity under low drawdown pressure; however, it shows a strong influence on the inflow performance and is negatively correlated with well productivity when bottom-hole pressure drops to lower than 20 MPa. With the intensification of stress sensitivity, the inflow performance curves bend to the bottom-hole pressure axis, and a series of inflection points (blue dot) appear when stress sensitivity increases to a certain degree, which corresponds to the maximum productivity and the optimal bottom-hole pressure. In addition, the inflection point (blue dot) moves up to the upper left as stress sensitivity goes up. Figure 10 indicates that when stress sensitivity gets serious, the production index decreases as a whole, appearing as these curves move to the left. Meanwhile, the shape of the production index curve changes from linear to concave, as a result of the joint action of the increase of seepage resistance caused by stress sensitivity and the decrease of bottom-hole pressure.

5.2.3. Horizontal Wellbore Length. Horizontal wellbore length is an important factor affecting well productivity. To be consistent with the physical model, the half-length ($L$) of horizontal wellbore is taken as the evaluation index. Figures 11 and 12 demonstrate the effect of horizontal wellbore half-length on the inflow performance and production index curves of a horizontal well with stress sensitivity and reservoir pressure change ($L = 150, 200, 250, 300, \text{and} 350$ m), respectively. It can be concluded that wellbore length also has a positive correlation with well production. Under the same bottom-hole pressure, the well productivity and production index enhance with the increase of wellbore length, and also, both the inflow performance and production index curves gradually deviate from the bottom-hole pressure axis. As the horizontal wellbore lengthens, the radian of the inflow performance curve gradually increases, but the slope of the production index curve decreases. Similarly, it is easy to obtain the maximum well productivity and optimal bottom-hole pressure for each wellbore length, but different from stress sensitivity, the optimal bottom-hole pressure keeps almost unchanged when wellbore length increases.

5.2.4. Initial Reservoir Permeability. There is no doubt that for the stress-sensitive reservoirs, initial reservoir permeability...
will have an important impact on well productivity. Figures 13 and 14 present the influence of initial reservoir permeability on inflow performance and production index curves of a horizontal well with stress sensitivity and reservoir pressure change ($k_{ei} = 4 \times 10^{-3}$ μm$^2$, $8 \times 10^{-3}$ μm$^2$, $12 \times 10^{-3}$ μm$^2$, $16 \times 10^{-3}$ μm$^2$, and $20 \times 10^{-3}$ μm$^2$), respectively. On the whole, its influence law is very similar to the influence of horizontal wellbore length on the inflow performance. Initial reservoir permeability is positively correlated with the horizontal well productivity and production index. With the increase of initial reservoir permeability, the well productivity and production index under the same bottom-hole pressure increase obviously, and both the inflow performance and production index curves also gradually deviate from the vertical axis. Similarly, the optimal bottom-hole pressure to achieve the maximum productivity does not change much when initial reservoir permeability increases or decreases.

5.2.5. Water Cut. For a horizontal well occurring oil-gas-water three-phase flow, water cut is a nonnegligible factor affecting well productivity. Figures 15 and 16 demonstrate the inflow performance relationship and liquid production index curves of a horizontal well with stress sensitivity and reservoir pressure change in a fractured-porous reservoir.
As reservoir pressure gets large, water cut becomes a significant factor. As stress sensitivity increases, high initial reservoir pressure corresponds to slow decrease of reservoir permeability. Stress sensitivity is negatively correlated with horizontal well productivity and production index, and a series of inflection points can be observed with the intensification of stress sensitivity. As stress sensitivity goes up, the optimal production point moves up to the upper left, and the shape of the production index curve changes from linear to concave.

Both horizontal wellbore length and initial reservoir permeability are positively correlated with horizontal well productivity but have little effect on the optimal bottom-hole pressure. The influence of water cut on horizontal well inflow performance becomes dramatic when drawdown pressure gets large, and with the increase of water cut, both the inflow performance and liquid production index are enhanced.

The inflow performance curves drawn from the derived productivity equations can help to quantitatively optimize the horizontal well’s production system in the fractured-porous reservoirs and improve oil recovery efficiency and economy.

**Nomenclature**

| Symbol | Description |
|--------|-------------|
| \(K_f\) | Initial permeability of rock sample, \(10^{-3} \mu m^2\) |
| \(K_t\) | Testing permeability of rock sample, \(10^{-3} \mu m^2\) |
| \(k_m\) | Initial permeability of matrix system, \(10^{-3} \mu m^2\) |
| \(k_i\) | Initial permeability of natural fracture system, \(10^{-3} \mu m^2\) |
| \(k_e\) | Initial equivalent permeability throughout the reservoir, \(10^{-3} \mu m^2\) |
| \(D_k\) | Permeability damage rate of rock sample (%) |
| \(D_{k_{max}}\) | Maximum permeability damage rate of rock sample (%) |
| \(r_e\) | Reservoir radius (m) |
| \(h\) | Reservoir thickness (m) |
| \(r_w\) | Horizontal wellbore radius, m |
| \(L\) | Horizontal wellbore half-length (m) |
| \(a_s\) | Stress sensitivity coefficient throughout the reservoir, dimensionless |
| \(p_0\) | Initial reservoir average pressure (MPa) |
| \(p_b\) | Reservoir bubble-point pressure (MPa) |
| \(p_{avg}\) | Average reservoir pressure (MPa) |
| \(p_{ah}\) | Bottom-hole flow pressure (MPa) |
| \(P_m\) | Reservoir pressure at the junction of two zones (MPa) |
| \(\mu\) | Fluid viscosity (cp) |
| \(B\) | Volume factor (m^3/m^3) |
| \(f_w\) | Water cut (%) |
| \(Q_i\) | Outer zone oil productivity of horizontal well (m^3/d) |
| \(Q_z\) | Inner zone oil productivity of horizontal well (m^3/d) |
| \(Q_h\) | Horizontal well oil productivity (m^3/d) |
| \(Q_{hw}\) | Horizontal well water productivity (m^3/d) |

**Data Availability**

The relevant data of the paper is calculated by programming according to the equation derived in the paper.




Conflicts of Interest

The authors declare that they have no conflicts of interest.

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