Ultra-compact binaries as gravitational wave sources

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Abstract. Ultra-compact binaries are among the most numerous sources in the millihertz gravitational wave band, and as such represent one of the primary sources for LISA-like detectors (gravitational wave interferometers in space). Already there are almost 60 ultra-compact binaries that have been detected by electromagnetic means, and more are being discovered in dedicated searches as time goes on. Prominent in this population is the doubly-degenerate white dwarf system J0651, whose orbital decay has been measured and shown to track accurately with the predicted evolution due to the emission of gravitational wave emission. This paper reviews the current understanding of the ultra-compact binary population, recent progress in electromagnetic studies, and prospects for multi-messenger astronomy of these systems once a LISA-like detector is operational.

1. Introduction

Like the electromagnetic (EM) spectrum, the gravitational wave (GW) spectrum covers an enormous range of frequencies, from the extremely low frequencies of cosmological gravitational waves (gravitational wavelengths on the order of the horizon scale), to ultra-high frequencies for tiny compact sources such as cosmic string cusps or primordial black holes (with appropriately short gravitational wavelengths). While all of the frequencies are allowable, whether or not they are astrophysically relevant is a matter of extrapolation from known sources, as well as intense speculation and debate.

Restricting attention to massive astrophysical binaries from stellar masses to supermassive black holes, the relevant parts of the gravitational wave spectrum are generally related to the astrophysical timescales of binary orbits. Thus, the most extensively studied part of the spectrum ranges from the ultra-low-frequency band at \( f_{gw} \sim 10^{-9}\text{Hz} \) (SMBH inspirals, detectable by pulsar timing arrays), to the high frequency band at \( f_{gw} \sim 1000\text{ Hz} \) (merging stellar mass black holes and neutron stars, detectable by ground-based interferometers like LIGO and Virgo).

There is no doubt that many of these prospective sources exist; they have all been detected by other observational means using traditional telescopes. But GW observations offer an altogether new and unique way to probe the universe, observing high-energy astrophysical systems not on
the interacting scales of atoms (where most electromagnetic ration originates from), but on the large system scales dominated by the bulk motion of matter. GW observatories will provide data that is wholly complementary to traditional EM observations, enabling new scientific investigations into the structure and evolution of systems that are currently poorly understood or poorly constrained.

As with all astronomical observations, it is exceedingly useful to have a large catalog of sources. In addition to providing an opportunity to conduct population studies, large catalogs also alleviate uncertainty in what is “normal” in a particular physical context by identifying outliers and odd events with respect to the entire catalog. This is often a particular worry in GW astronomy because current model predictions often forecast event rates as low as a handful per year.

In the millihertz band covered by LISA ($10^{-5} \text{Hz} \lesssim f_{gw} \lesssim 0.1 \text{Hz}$) the sources are expected to be more numerous because a wide variety of systems are that are long-lived spend long periods of their evolutionary time in this band, changing slowly under the steady emission of gravitational waves. Foremost among these are the ultra-compact binaries (UCBs): binary systems where the compact components are degenerate stellar remnants (white dwarfs, neutron stars, and stellar mass black holes).

The galaxy is replete with UCBs, with even conservative estimates suggesting there should be $\sim 10^6$ or more radiating in the band covered by LISA; most of these will be close white dwarf binaries.

1.1. Confusion Limit
The UCBs are persistent and long-lived sources in the millihertz band. If evolving purely under GW emission, their orbital frequency changes according to

$$\dot{f}_{orb} = \frac{48}{5} \frac{c^3}{G} \frac{f_{orb}}{M_c} \left(\frac{G}{c^3} \frac{2\pi f_{orb} M_c}{M_c}\right)^{8/3}, \quad (1)$$

where the mass parameter $M_c = (m_1 m_2)^{3/5} / (m_1 + m_2)^{1/5}$ is called the chirp mass. The ability to detect frequency evolution for a LISA-like interferometer is fixed by the total observing time, $T_{obs}$, giving resolving power of $\delta f \sim 1/T_{obs}$. For a 1 year observation, this is $\delta f \sim 3.17 \times 10^{-8}$ Hz. Integrating Eq. 1 one can find the time $t$ that a system takes to cross a frequency $\delta f$, equivalent to the time LISA needs to observe a system to see it evolve in frequency. For typical UCB masses of interest, with equal masses and total mass around the Chandresekhar mass, sources are stationary (they are monochromatic) in the LISA data below $f_{orb} \sim 2.5 \times 10^{-3}$ Hz.

Population synthesis models [1], suggest that below $f_{orb} \sim 1 - 2 \times 10^{-3}$ Hz, there will be more than one UCB in the galaxy with orbital periods in any given frequency interval $f$ to $f + \delta f$. This means that in the LISA data there will be an accumulated signal at those frequencies that is the incoherent sum of the GW from multiple UCB sources located in different parts of the sky; the sources are “confused” and cannot be individually resolved. In the context of analyzing data, there will be some bright binaries whose signals stand out because they are intrinsically strong GW emitters or because they are close to Earth; data analysis studies suggest on the order of $\sim 10^4$ of these “resolvable binaries” will be detectable [2]. Below the bright binaries, the remaining confused binaries will merge to form an irreducible astrophysical noise, a confusion limited foreground. For most millihertz band, LISA-style interferometers, this irreducible foreground is the limiting source of noise in source analysis below $f_{gw} \sim 3 \times 10^{-3}$ Hz.

2. Verification binaries
Among the resolvable sources, there will be a subset of verification binaries. These are known UCBs that are detectable in both gravitational waves and using electromagnetic telescopes.
Many of these have already been discovered and are the subject of ongoing electromagnetic observing campaigns.

2.1. Evolutionary paradigm
The classically understood evolutionary pathway for isolated stars with masses $m \lesssim 8M_\odot$ is that when the core hydrogen fuel is expended, the core collapses to smaller size, increasing the core temperature and causing the star to expand into a red giant phase. Helium fusion begins in the core, leaving a core of carbon and oxygen. White dwarfs are the degenerate, Earth-sized remnants of that core. They are born at temperatures of $\sim 100,000$K, and are comprised largely of carbon and oxygen with thin atmosphere layers of helium and hydrogen (the atmosphere comprising only 2% of the total mass).

This picture is modified in binary systems. If the companion star is close enough during the red giant phase, the giant star can fill its Roche lobe and transfer mass to the companion. If the mass transfer is stable, then the result is a common envelope that contains both stars. The details of this process is complicated and poorly constrained, but simple prescriptions have been developed that conserve the energy or the angular momentum of the common envelope. Experimentally, common envelope binaries are common [3]. Observations suggest a new system forms every few years in the Milky Way. The process is brief, but can cause a star to lose most of its mass so that its remnant ends up as a low mass white dwarf. Extremely low mass white dwarfs require two phases of common envelope evolution and so are rare, but are the unambiguous signposts of compact binaries. Exactly the type of systems that make good gravitational wave sources.

2.2. Finding low mass white dwarfs
Recent surveys to find low-mass white dwarf binaries [4, 5] have been successful using optical spectroscopy to identify targets and measure radial velocities. Figure 1 shows typical white dwarf spectra, with absorption lines from hydrogen in the atmospheres. The width of the lines directly measures the surface gravity of these stars; broad lines are a signature of a white dwarf, which has a high surface gravity.

Another important feature in these spectra is the Balmer decrement, the ratio of line strengths between different lines in the Balmer sequence. Imaging passbands spanning this spectral feature will produce a measure of an object’s color, enabling the identification of low-mass white dwarf binaries.

Figure 2 shows a color-color diagram of stars in the Sloan Digital Sky Survey (SDSS). In this region, the g-r color reflects the effective temperature of these stars (brighter, more negative g-r value, are hotter) and u-g reflects their surface gravity (brighter, smaller u-g values have higher surface gravity). The entire plane is populated by sources (black dots); on the lower half of this diagram, at low surface gravity, the sources are normal A-type stars; at higher surface gravities the sources are normal WDs in the same temperature range.

The Extremely Low Mass (ELM) white dwarf survey targets the middle region of this color-color diagram, where low-mass white dwarfs are located due to their lower surface gravities. The first group of white dwarf binaries in this region were fortuitously discovered in other surveys, but targeted work over the last couple of years has more than tripled the number of known white dwarf binaries.

The net outcome of the survey is dozens of white dwarf binaries with inferred merger times less than a Hubble time, and many with orbital periods that put them firmly in the millihertz gravitational wave band observable by LISA. Figure 3 shows the current detected population.
2.3. New LISA Verification Binaries

Two important systems have been discovered using these low-mass white dwarf surveys, both of them with short periods and expected to be strong sources in the LISA band: J0651, an eclipsing white dwarf system [6], and WD 0931, a face-on binary system [7]. Figure 4 shows these sources and several other ELM binaries plotted against an average LISA strain curve.

WD 0931 has a 20 minute orbital period, and shows only modest radial velocity variations, suggesting the system is close to face on, making it a strong gravitational wave source. Estimates suggest a strain of $h \sim 10^{-22}$, making it a strong verification source in the millihertz band.

J0651 is a particularly important system because the eclipses enable a precise measurement of the orbital ephemeris. Over just three years of observations, the time of eclipse in this system has shifted by 1 minute, about the same amount that the Hulse-Taylor pulsar has shifted in 40 years [8]. The shift is due to gravitational wave emission; this system is currently emitting more energy in gravitational wave radiation – 2.6 solar luminosities – than in light.

3. UCB Multi-Messenger Astronomy

The ultra-compact binary population of the galaxy is a preserved fossil record of binary evolution in the Milky Way [9, 10]. As such, detecting large numbers of UCBs in gravitational waves will provide important constraints on common envelope evolution, composition and distributions of SN Ia progenitor systems, probes of white dwarf structure, studies of tides between close stellar companions, and new precision tests of gravity. Some of this science can be accomplished using GW observations alone, but the synergy between GW and EM observations promises to enable
high-fidelity probes of the structure and interaction of close binary stars.

3.1. Multi-messenger population

Significant numbers of UCBs have been detected already in the EM spectrum, and more are being found in surveys that are now targeting them (a running collection of prospective sources that have been detected by EM means is maintained at [11]). These observations provide useful constraints on expectations for the GW emission, but GW data collection is still somewhat in the future. This begs an important question: given what we already know, how many systems will be simultaneously detectable in both gravitational waves and electromagnetic astronomy? Simultaneous observations in both GW and EM that will enable a maximal science return.

Despite being observable with telescopes, not all of the close white dwarf binaries found in surveys like ELM will be detectable by a LISA like mission. Many of them will radiate at too low a frequency, or will simply be lost in the confusion noise of the 10 million other binaries in the galaxy. Currently we are discovering systems with EM telescopes and predicting if they will be observable with LISA. Once LISA flies, we will experience a dramatic role-reversal where we will detect many with LISA first and then have to do an EM search to determine if they can
be seen in light. Until a GW observatory in space flies, we will not know precisely how many multi-messenger sources will exist, but good estimates can be made using population synthesis galaxies.

Population synthesis endeavours to simulate the entire galactic population of doubly degenerate stars by simulating the lives of millions of star systems at once, incorporating as much stellar evolution physics as possible. There are certain assumptions that have to be made, but existing synthesized galaxies have extremely high fidelity, producing not only distributions of masses and orbital periods, but galactic positions, EM brightnesses, and stellar temperatures. Figure 5 shows populations of multi-messenger binaries extracted from a population synthesis galaxy, plotted against possible future configurations of a LISA like detector [12]. From this one sees that for any plausible GW observatory in space, (1) the UCBs are an important and relevant source, and (2) there will be a sizable population of multi-messenger verification binaries.

Overall, the multi-messenger population will be dominated by white dwarfs (mostly detached, with a handful of interacting binaries), as well as a few accreting sources involving neutron stars and stellar mass black holes (though these will be rarer).

3.2. Multi-messenger binary data

The advantage of working with multi-messenger UCBs is that the EM and GW data provide highly complementary probes of the physical parameters that characterize a given binary. The precise parameters of a UCB define the evolutionary track that the binary is on, so measuring their values is of the utmost importance in mapping out the sequence and influence of different
phases of binary evolution. Many processes are known to play major roles in binary evolution but are poorly understood because the phases are either short-lived, or difficult to observe in action. As a case in point, consider common envelope evolution. This is a short lived phase in the binary evolution believed to be responsible for bringing the components from a wide orbit to a very tight orbit due to the orbital angular momentum loss due to the friction that this star suffers as it orbits. The efficiency with which this happens and the exact outcome of this is not understood. It could, however, be constrained if we had more accurate masses for the binary systems, something GW observations could provide.

There are many parameters which are difficult to measure with EM or GW observations alone, either because they do not have a clear signature in the data, or because they are strongly correlated with other parameters. Consider the GW scaling amplitude, $A$, which can

Figure 4. Known low mass white dwarf binaries from the ELM Survey, plotted with expected GW strengths against eLISA sensitivity configurations (solid) and the expected confusion limit (dashed line). Filled circles represent systems that will merge in less than a Hubble time, and open circles correspond to systems that will merge in more than a Hubble time.
be expressed as a function of the system parameters as

$$A = \frac{4(G\mathcal{M}_c)^{5/3}}{c^4d}(\pi f_{gw})^{2/3},$$

where $d$ is the distance to the source. This is the amplitude for an optimally oriented system, but for unknown inclinations, $\iota$, the measured value of $A$ is uncertain by large factors, as shown by the long, elongated error ellipses in Figure 6. Parameter determination is further complicated by degeneracy in Eq. 2 between the distance $d$ and chirp mass $\mathcal{M}_c$, the effect of which cannot be isolated by measuring the amplitude and frequency of the gravitational waves alone. As a completely independent detection method, adding EM data can (for instance) greatly constrain the inclinations, which narrows the acceptable values of $A$.

The amplitude could be further constrained if distances are known. Once the GAIA survey is complete, it is expected that the known white dwarf population will increase by more than an order of magnitude, to more than $2 \times 10^5$ sources [13]. With a magnitude limit in the range of $m_V \approx 20 - 25$, estimates from population synthesis galaxies suggest as many as $10^3$ multi-messenger binaries may be detectable [12], meaning GAIA could provide a significant catalog of distances from astrometry.

The synergy between EM and GW observations of verification binaries extends to other parameters as well, most importantly to the sky position, which is obviously known for all verification binaries discovered in the pre-LISA era. Knowledge of the sky position greatly enhances our ability to model the GW waveforms in terms of their amplitude and modulation.

4. UCB Science Enabled by gravitational wave observations

4.1. Population studies

There have been many directed studies to examine how specific astrophysical effects can be characterized using GW data. For instance, the large number of of UCBs that LISA will detect provides a method for measuring the shape parameters of the galaxy. UCBs will be detectable across the entire Milky Way, allowing the radial and vertical scales heights of the disk to be mapped out to a few percent accuracy [14].
Figure 6. Left: Two-dimensional GW error ellipses of $A$ and $\cos \iota$ for AM CVn. The two thin black lines represent 1-$\sigma$ and 2-$\sigma$ ellipses and the black dashed lines represent the 1-$\sigma$ GW errors in $A$ and $\cos \iota$ with the cross at the true value of the parameters. The unphysical values (i.e. negative numbers along the amplitude axis) are caused by the Gaussian tails of the parameter uncertainty about their true values. The right panel is a zoom of the area that is constrained by the 1-$\sigma$ EM error of 4$^\circ$ shown in dash-dotted lines. The corresponding error ellipses for the reduced PDF are shown as thick black ellipses for 1-$\sigma$ and 2-$\sigma$.

LISA will also be an effective tool for identifying the galactic population of supernova Ia progenitors, as shown in Figure 7. Many of these systems are chirping (evolving in frequency), placing them in a part of the band where they are individually resolvable from the confused population. Furthermore, measuring the chirp breaks many of the parameter degeneracies that plague GW system characterization. LISA will have the ability to identify doubly degenerate systems, Type Ia progenitors, throughout the galaxy [15].

4.2. Tidal interactions

At higher frequencies, where the chirp $\dot{f}$ and chirp evolution, $\ddot{f}$ can be measured, the UCBs are extremely compact, raising the possibility that tidal effects could be measured. In system J0651, tidal distortions of the low-mass companion can already be detected in the EM data. How can the synergy of GW and EM data help understand tidal effects in this and other UCBs?

Most of the energy in these systems is dissipated in gravitational waves, but the tidal distortion also bleeds additional energy out of the orbit, owing to the misalignment of the white dwarfs. Looking at the overall contributions to the frequency evolution, there are both gravitational wave and tidal terms that contribute to $\dot{f}$. The question we would like to answer with multi-messenger data is what is the relative size of the contributions to $\dot{f}$? The existence of non-GW contributions to the frequency evolution of a UCB can be determined from a well known observable constructed from the frequency evolution [16]

$$\left( \frac{\ddot{f} f}{\dot{f}^2} \right)_{GW} = \frac{11}{3},$$

though it is expected to be difficult to measure for most systems [17].

The combination of EM and GW observations, especially for well characterized systems like J0651, can allow for greatly improved measurements of the system parameters that are
Figure 7. A population synthesis realization shows clear populations in the \( \{m_1, m_2\} \) plane. All systems that have a total mass \( m_1 + m_2 \) greater than the Chandrasekhar mass (nominally, Type Ia progenitors) are colored purple and located in the upper right. LISA resolvable binaries are colored green (sub-Chandrasekhar mass) and blue (Chandrasekhar mass or greater).

degenerate when explaining tidal energy losses in UCBs [18], though for most UCBs the expected parameter errors suggest the constraints may not be useful even in joint observations; an extended EM observing campaign (e.g. \( \sim 10 \) year) could mitigate this concern. For gravitational wave observations to be highly effective in constraining \( f \) and \( f' \), systems will need to be extremely massive and at high frequencies (\( f \gtrsim 6 \) mHz), where LISA-like detectors are most sensitive and the confusion foreground has died away [19].

4.3. Structure probes
Another role that multi-messenger observations can play in the observation of UCBs is in probing the internal structure and dynamics, such as characterizing the accretion disks in cataclysmic variable systems.

Compact interacting binaries spill mass through the inner Lagrange point, and the resulting stream impacts on the accretion disk creating a bright spot. This spot emits \( \sim 30\% \) of the total light in the system, and manifests in the light curve as orbital humps. The phase of the light curve is related to the profile of the bright spot with respect to the observer. By contrast, the phase fronts of the gravitational wave signal arrive depending on the orientation of the binary
axis with respect to the observer, allowing a geometrical orientation of the components of the binary to be known. Knowing the location of the individual components allows the measurement of the bright spot impact location angle, $\alpha$, which is a proxy for the accretion disk radius as shown in Figure 8 [20]. The expected errors are comparable to the current existing errors from systems where the accretion disk is eclipsed, but the method will not be restricted to eclipsing systems.

**Figure 8.** Matter streaming through the inner Lagrange point impacts on the accretion disk, making a bright spot that shows as a feature in the EM light curve. GW observations locate the binary axis, giving a reference point to measure the angle $\alpha$.

5. Summary
In much of the LISA literature, the confusion background is characterized as a source of noise which must be accounted for when considering observations of other sources of gravitational waves. It is a sad fact that the UCBs are most often thought of as noise, a nuisance which interferes with seeing more interesting sources of gravitational waves (like supermassive black hole binaries or extreme mass ratio inspirals). In reality, however, the galactic binary foreground is an important astrophysical signal which will be easily visible to LISA and have profound implications for our understanding of the galaxy.

As tracers of galactic astrophysics, binary populations will provide observational access to many different components of the galaxy, including the disk and bulge [21], the dark galactic halo [22, 23], and even globular cluster systems [24]. Observations of the galactic binary foreground in gravitational waves provide a key way to measure the properties of an assumed but unseen component of the galactic stellar population. The total population of highly evolved compact binaries provides a unique record of galactic stellar evolution which should be easily accessible to an instrument like LISA. Near the frequencies where LISA is most sensitive, it is also plausible that galactic binary signals from the Milky Way’s nearest companions might also be detected, and that there will be a low level confusion background of all the galactic binaries in the local Universe [25], which would form an ultimate noise floor in the low frequency gravitational wave band.

As a prospective source population, ultra-compact galactic binaries promise to be one of the richest areas for discovery as gravitational wave astronomy becomes a more prominent part of our observing toolbox.
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