General Analysis of $\bar{B} \to \bar{K}^{(*)}\ell^+\ell^-$ Decays at Low Recoil

Christoph Bobeth

Excellence Cluster Universe, Technische Universität München, D-85748 Garching, Germany

Gudrun Hiller

Institut für Physik, Technische Universität Dortmund, D-44221 Dortmund, Germany

Danny van Dyk

Institut für Physik, Technische Universität Dortmund, D-44221 Dortmund, Germany and Theoretische Physik 1, Naturwissenschaftlich-Technische Fakultät, Universität Siegen, Walter-Flex-Straße 3, D-57068 Siegen, Germany

We analyze the angular distributions of $\bar{B} \to \bar{K}^{(*)} \to \bar{K}\ell^+\ell^-$ and $\bar{B} \to \bar{K}\ell^+\ell^-$ decays in the region of low hadronic recoil in a model-independent way by taking into account the complete set of dimension-six operators $[s\Gamma b][\bar{\Gamma}^\prime \ell]$. We obtain several novel low-recoil observables with high sensitivity to non-standard-model Dirac structures, including CP-asymmetries which do not require flavor tagging. The transversity observables $H_T^{1,4,5}$ are found to be insensitive to hadronic matrix elements and their uncertainties even when considering the complete set of operators. In the most general scenario we show that the low recoil operator product expansion can be probed at the few-percent level using the angular observable $J$. Higher sensitivities are possible assuming no tensor contributions, specifically by testing the low-recoil relation $|H_T^i| = 1$. We explicitly demonstrate the gain in reach of the low-recoil observables in accessing the ratio $|C_9/C_{10}|$ compared to the forward-backward asymmetry, and probing CP-violating right-handed currents $\text{Im} C_{10}$. We give updated Standard Model predictions for key observables in $\bar{B} \to \bar{K}^{(*)}\ell^+\ell^-$ decays.

I. INTRODUCTION

Flavor Changing Neutral Current (FCNC) decays of beauty hadrons have a high sensitivity to New Physics (NP) since the corresponding Standard Model (SM) contributions are loop and flavor suppressed. In addition, the large value of the $b$-quark mass facilitates the control of power corrections.

The large number of complementary observables and the excellent accessibility at contemporary high energy experiments, in particular for muons, highlights the exclusive FCNC decays $\bar{B} \to \bar{K}^{(*)} \to \bar{K}\ell^+\ell^-$. In the kinematic region of low hadronic recoil, where the emitted $K^*$ is soft in the $B$-rest frame, a local Operator Product Expansion (OPE) can be performed [1][2]. Together with the improved Isgur-Wise relations [1][3][4], this results in a simple structure of the transversity decay amplitudes at leading order in $1/m_b$ [4]

$$A_i^{L,R} \propto C_i^{L,R} f_i, \quad i = L, R, 0,$$ \hspace{1cm} (1)

factorizing into universal short-distance coefficients $C_i^{L,R}$ and form factors $f_i$. This feature allows to extract short-distance couplings without long-distance pollution, and vice versa, as well as to test the performance of the OPE [1][5].

The opposite kinematical region of large recoil has been subjected to the question of optimized observables as well, e.g., [6][12]. Several proposals exploit specifically that QCD factorization (QCDF) [13][15] at leading order maintains universal short-distance coefficients for $A_{L,R}^{L,R}$, while Eq. (1) is broken for $i = 0$ at lowest order, and for all $i = L, R, 0$ at order $1/m_b$.

The additional benefit of the low recoil region is the strong parametric suppression of the subleading $1/m_b$ corrections to the decay amplitudes at the order of a few percent [1][4]. Together with an angular analysis [10] this enables a rich flavor physics program, complementing the large recoil region. One application is to extract form factor ratios $f_j/f_j$ from data, as has recently been demonstrated in [17][18].

The key questions addressed in this work are:

i) To which extent is Eq. (1) and its benefits preserved in the presence of operators beyond the SM ones?

ii) What are the optimal low recoil observables model-independently?

iii) What is their sensitivity to NP?

iv) What is the sensitivity to potential corrections to the OPE?

To answer the above questions we perform a most general, model-independent analysis of the decays $\bar{B} \to \bar{K}^{(*)} \to \bar{K}\ell^+\ell^-$ and $\bar{B} \to \bar{K}\ell^+\ell^-$. In terms of semileptonic dimension-six operators $[s\Gamma b][\bar{\Gamma}^\prime \ell]$ this concerns the chirality-flipped partners of the SM ones, (pseudo-)scalar and tensor operators. We compute various decay distributions and asymmetries.

The plan of the paper is as follows: The effective theory including the operator basis is given in Section [1]...
II. THE EFFECTIVE HAMILTONIAN

Rare semileptonic $|\Delta B| = |\Delta S| = 1$ decays are described by an effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^\ast \frac{\alpha_e}{4\pi} \sum_i C_i(\mu) O_i(\mu). \tag{2}$$

Here, $G_F$ denotes Fermi’s constant, $\alpha_e$ the fine structure constant and unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$ has been used. The subleading contribution proportional to $V_{tb} V_{ts}^\ast$ has been neglected.

The renormalization scale $\mu$, which appears in the short-distance couplings $C_i$ and in the matrix elements of the operators $O_i$, is of the order of the $b$-quark mass. In the following we suppress the dependence of the Wilson coefficients $C_i$ on the scale $\mu$.

In the SM $b \to s \ell^+ \ell^-$ processes are mainly governed by the operators $O_{7,9,10}$ which will be referred to as the SM operator basis. Beyond the SM chirality-flipped ones $O_{7,9,10}^\prime$, collectively denoted here by SM’, may appear. The SM and SM’ operators are written as \cite{6,8,19}

$$O_{7(9,10)} = \frac{m_b}{e} \left[ \bar{s} \sigma^{\mu\nu} P_L(\ell) b \right] F_{\mu\nu},$$

$$O_{9(10)'(s)} = \left[ \bar{s} \gamma_\mu P_R(\ell) b \right] \left[ \ell \gamma^\mu \right],$$

$$O_{10(10)'(p)} = \left[ \bar{s} \gamma_\mu P_R(\ell) b \right] \left[ \ell \gamma^\mu_5 \right]. \tag{3}$$

Furthermore, we allow for scalar and pseudo-scalar operators, referred to as S and P,

$$O_{S(s)} = \left[ \bar{s} P_L(\ell) b \right] \left[ \ell \right],$$

$$O_{P(p)} = \left[ \bar{s} P_R(\ell) b \right] \left[ \ell \gamma_5 \right], \tag{4}$$

which includes the chirality-flipped ones, as well as tensor operators, referred to as T and T5,

$$O_T = \left[ \bar{s} \sigma_{\mu\nu} b \right] \left[ \ell \sigma^{\mu\nu} \ell \right],$$

$$O_{T5} = \frac{i}{2} \epsilon^{\mu\nu\alpha\beta} \left[ \bar{s} \sigma_{\mu\nu} b \right] \left[ \ell \sigma_{\alpha\beta} \ell \right]. \tag{5}$$

Note that $O_{T5} = \left[ \bar{s} \sigma_{\mu\nu} b \right] \left[ \ell \sigma^{\mu\nu} \gamma_5 \ell \right] = T_{\text{TE}}/2$, see Eq. (C16), as commonly used in the literature \cite{19,21}. Current-current and QCD penguin operators $O_{1,5}$, as well as the chromo-magnetic dipole operator $O_8$, have to be included for a consistent description of $b \to s \ell^+ \ell^-$ decays, for definition see \cite{22}. The matrix elements of $O_{1,5,6}$ contribute to $b \to s + \{\gamma, g, \ell^+ \ell^-\}$ processes via quark-loop effects. The latter are taken into account by means of the effective Wilson coefficients $C_i^{\text{eff}}$. The effective Wilson coefficients are renormalization group invariant up to higher orders in the strong coupling constant $\alpha_s$. In the case of exclusive decays the $1/m_b$ corrections in the large- and low-recoil region from QCDF \cite{14,15,19} or SCET \cite{23,24} and the low-recoil OPE \cite{11,14,15}, respectively, should be included in the $C_i^{\text{eff}}$. We evaluate $\alpha_s$ at $\mu = \mu_b = O(m_b)$ which takes into account most of the NLO QED corrections \cite{25,26}.

III. $B \to K^* \ell^+ \ell^-$ AT LOW RECOIL

We study $B \to K^* \to K\pi \ell^+ \ell^-$ decays in the low-recoil region for a generalized operator basis and detail the relevant observables and their relations. In Section III A we give the results using SM operators only. In Section III B, III C, III D we include either SM’, S and P or T and 15 operators, respectively. Interference effects are worked out in Section III E.

The main results of this section are summarized in Table 1 where the low-recoil relations between the observables and the amount of their violations is given. Our results are based on the angular distribution presented in Appendix A and the angular observables in Appendix B.

A. SM operators

The amplitude of the exclusive decays $B \to K^* \ell^+ \ell^-$ can be treated at low recoil using an OPE and further matching onto HQET \cite{11}. After application of the improved Isgur-Wise relations \cite{11}, one finds for the transversity amplitudes \cite{14,15}, see also Eq. (1),

$$A_{0,\parallel}^{L,R} = -C_L^{L,R} f_0, \quad A_{\perp}^{L,R} = +C_L^{L,R} f_\perp. \tag{6}$$

The short-distance coefficients read

$$C_L^{L,R}(q^2) = C_7^{\text{eff}}(q^2) \mp C_{10},$$

$$C_7^{\text{eff}}(q^2) = C_7 + \kappa \frac{2m_b M_B}{q^2} C_7 + Y(q^2), \tag{7}$$

where $Y$ denotes the matrix elements of the 4-quark operators, see \cite{15} for details. Here, the matching correction

We present low recoil observables and relations from different operator sets in Section III and Section IV for $B \to K^* \to K\pi \ell^+ \ell^-$ and $B \to K\ell^+ \ell^-$, respectively. In Section IV we study the sensitivity of the low recoil observables to even small NP effects. The sensitivity to OPE corrections is worked out in Section VII as well as a brief discussion of S-wave backgrounds. We conclude in Section VII.

In several appendices we give formulae and subsidiary information. In Appendix A we discuss the full angular decay distribution in $B \to K^* \to K\pi \ell^+ \ell^-$ decays. In Appendix B we present the angular observables in terms of the transversity amplitudes for the complete set of semileptonic $|\Delta B| = |\Delta S| = 1$ operators. In Appendix C we detail the transversity amplitudes that parametrize the tensor contribution to the matrix element. An update of the SM predictions for the key observables in $B \to K^* \ell^+ \ell^-$ and $B \to K\ell^+ \ell^-$ decays is given in Appendix D.
The normalization factor \( N \) depends on the invariant mass squared of the lepton pair, \( q^2 \), and is given in Eq. (B21). The kinematical factor \( \lambda \equiv \lambda(M_{\ell^+\ell^-}^2, q^2) \) is given in Eq. (C4).

The factorization into short-distance coefficients and form factors, Eq. (6), allows to identify suitable combinations of the observables \( J_i \) appearing in the angular distribution of \( B \to K^*(\to K\pi)\ell^+\ell^- \), see Appendix A for details. The angular observables depend on two short-distance parameters \( \rho_{1,2} \) only,

\[
\begin{align*}
\frac{f_\perp}{N} &= \frac{\sqrt{2}\lambda}{M_B + M_K} V, \\
\frac{f_\parallel}{N} &= \frac{\sqrt{2}(M_B + M_K)}{A_1} \lambda A_1, \\
\frac{f_0}{N} &= \frac{(M_{\ell^+\ell^-}^2 - q^2)(M_B + M_K)^2 A_1 - \lambda A_2}{2M_K(M_B + M_K)\sqrt{q^2}},
\end{align*}
\]

as well as the long-distance free ratios

\[
H_T^{(2)} = \frac{\beta_5 J_5}{-2J_{2s}(2J_{2s} + J_3)}, \quad H_T^{(3)} = \frac{\beta_6 J_6}{2\sqrt{(2J_{2s})^2 - J_3^2}}.
\]

Here we point out a further nontrivial observable, which does depend neither on form factors nor on short-distance physics:

\[
H_T^{(1b)} = \frac{-J_{2s} J_6}{2J_3 J_5},
\]

and which equals one. Note that this observable can be obtained via \( H_T^{(1b)} = H_T^{(3)}/[H_T^{(1)} H_T^{(2)}] \). However, by using the definition Eq. (16) directly different \( J_i \) appear. This offers additional advantages in the experimental extraction from the angular distributions.

In addition, long-distance free CP asymmetries \( \delta_{T,7}^{CP} \) can be formed, which are related to the CP asymmetry of the decay rate, of the forward-backward asymmetry, and of \( H_T^{(2,3)} \), respectively [5].

Furthermore, several short-distance free ratios of form factors [9] can be obtained

\[
\begin{align*}
\frac{f_0}{f_\parallel} &= \frac{\sqrt{2}J_5}{J_6^s} = -\frac{J_{2s}}{2J_4}, \\
\frac{2\sqrt{2}J_4}{2J_{2s} - J_3} &= \sqrt{\frac{-J_{2c}}{2J_{2s} - J_3}}, \\
\frac{f_\perp}{f_\parallel} &= \sqrt{\frac{2J_{2s} + J_3}{2J_{2s} - J_3}} = \sqrt{\frac{-J_{2c}(2J_{2s} + J_3)}{2J_4}}, \\
\frac{f_0}{f_\perp} &= \sqrt{\frac{-J_{2c}}{2J_{2s} + J_3}}.
\end{align*}
\]

They allow to extract information on form factors directly from the data [17-19], providing a benchmark test for form factor determinations such as from lattice QCD. To sum up, using SM-type operators only – which may or may not receive contributions from beyond the SM – the low recoil OPE predicts at leading order in \( 1/m_b \)

\[
\frac{H_T^{(1)}}{\text{sgn}(f_0)} = H_T^{(1b)} = 1, \quad J_{7,8,9} = 0,
\]

and the observable form factor ratios given in Eqs. (17-19). As already stressed the subleading power corrections are parametrically suppressed and at the few percent level.

### B. Chirality-flipped operators

Taking into account the chirality flipped operators the universal structure of the transversity amplitudes [6] is
The angular observables \( J_i \) in (SM+SM') model read

\[
\frac{4}{3\beta^2} (2J_{2s} - J_3) = 2\rho_1^- f^2_\perp, \\
\frac{4\sqrt{2}}{3\beta^2} J_4 = 2\rho_1^- f_0 f_{\parallel},
\]

\[
\frac{2\sqrt{2}}{3\beta^2} J_5 = 4\text{Re}(\rho_2) f_0 f_{\perp}, \\
\frac{2}{3\beta^2} J_{6s} = 4\text{Re}(\rho_2) f_{\parallel} f_{\perp},
\]

\[
\frac{4\sqrt{2}}{3\beta^2} J_6 = 4\text{Im}(\rho_2) f_0 f_{\perp}, \\
\frac{4}{3\beta^2} J_9 = 4\text{Im}(\rho_2) f_{\parallel} f_{\perp},
\]

where \( J_7 = 0 \) still holds and \( \rho_1 \) and \( \rho_2 \) have been generalized to

\[
\rho_1^\pm = \frac{1}{2} (|C_R|^2 + |C_L|^2),
\]

\[
\rho_2 = \frac{1}{4} (C_R^L C^{R*} - C_L^L C^{L*}).
\]

Switching off the chirality flipped operators one recovers \( C'_L = C'_R = C_{9} \) (and analogously for \( L \rightarrow R \)), such that \( \rho_1^1 = \rho_1^1 = \rho_1 \).

In (SM+SM'), the asymmetries \( H_T^{(2,3)} \), defined in Eqs. (14)-(15), read

\[
H_T^{(2)} = 2 \frac{\text{Re}(\rho_2)}{\sqrt{\rho_1 \cdot \rho_1}}, \quad H_T^{(3)} = 2 \frac{\text{Re}(\rho_2)}{\sqrt{\rho_1 \cdot \rho_1}}.
\]

They remain long-distance free. Furthermore, the low recoil predictions obtained in the SM basis

\[
\frac{H_T^{(1)}}{\text{sgn}(f_0)} = H_T^{(1b)} = 1, \quad H_T^{(2)} = H_T^{(3)}, \quad J_7 = 0
\]

remain intact.

In Fig. 1 we show \( H_T^{(1)} \) and \( H_T^{(1b)} \). While both equal one at low recoil in SM+SM', at large recoil both observables exhibit a nontrivial \( q^2 \)-dependence and depend on short- and long-distance contributions. However, to lowest order form factors drop out in \( H_T^{(1)} \) cf. [12] and \( H_T^{(1b)} \). We show the residual uncertainty from the form factors and subleading \( 1/m_b \) corrections by the shaded bands. The SM is represented by the thin (blue) band, whereas the lighter shaded (gold) one corresponds to a scenario with \( C_{\tau,9'} = C_{7,9}^{SM}, C_{10'} = -C_{10}^{SM}, C_{7,9,10} = 0 \). For numerical input, see Appendix D.

Since in \( \text{(SM+SM')} J_{8,9} \neq 0 \) two additional long-distance free ratios

\[
H_T^{(4)} = \frac{2J_4}{\sqrt{-2J_{2c} (2J_{2s} + J_3)}}, \\
H_T^{(5)} = \frac{-J_9}{\sqrt{(2J_{2s})^2 - J_3^2}}
\]

can be constructed. They obey

\[
H_T^{(4)} = H_T^{(5)} = 2 \frac{\text{Im}(\rho_2)}{\sqrt{\rho_1 \cdot \rho_1}}.
\]
We point out a further nontrivial observable, which depends neither on form factors nor on short-distance physics:

\[ H_T^{(1c)} = 2 \frac{J_4 J_8}{J_{2c} J_9}, \]

(32)

where in (SM+SM')

\[ H_T^{(1c)} = 1. \]

(33)

For \( H_T^{(1c)} \) an analogous comment as on \( H_T^{(1b)} \) applies, see text below Eq. (16).

The transverse asymmetries \( H_T^{(2,3,4,5)} \) are driven by the real and imaginary part of \( \rho \), where \( \Im(\rho) \equiv \Im(H) \).

\[ \Re(\rho) = \Re(C_{10} - C_{10}^*), \]

(34)

\[ \Im(\rho) = \Im(C_{10} - C_{10}^*). \]

(35)

where \( \Im(\rho) \) vanishes for \( C_{10}^\prime = 0 \) including vanishing chirality-flipped four-quark operators. Real-valued SM+SM Wilson coefficients can still induce somewhat suppressed, finite values of \( H_T^{(4,5)} \) through the absorptive contributions in the matrix elements of the four-quark operators \( Y \) and \( Y' \), by \( \Im(\rho) = \Re(C) \). In any case, \( H_T^{(4,5)} \) are null tests of the SM. A SM background to right-handed currents arises at higher order in the OPE including and counting \( m_s/m_b \)-terms as such and enters \( H_T^{(4,5)} \) with additional parametric suppression by \( \alpha_s \) or \( C_7/C_8 \). Combinations result in further useful observables which do not depend on form factors either:

\[ \frac{H_T^{(4)}}{H_T^{(2)}} = \frac{2 J_8}{\beta_5 J_5}, \quad \frac{H_T^{(5)}}{H_T^{(3)}} = \frac{-2 \beta_5 J_9}{\beta_6 J_6}. \]

(36)

Both equal \( \Im(\rho) / \Re(\rho) \) in (SM+SM'). Since \( J_{8,9} \) are naive T-odd these angular observables give optimal access to CP violation in the presence of small strong phases. Since both \( J_{8,9} \) are also CP-odd, \( H_T^{(4,5)} \) can be measured from B-meson samples without tagging and give rise to a further long-distance free CP asymmetry defined as

\[ a_{CP}^{(4)} = \begin{cases} \frac{\sqrt{2} (J_8 - J_9)}{(J_{2c} + J_{2c}) [2 (J_{2a} + J_{2a}) + (J_3 + J_3)]} & J_9 - J_9 \\ \frac{1}{\sqrt{4 (J_{2a} + J_{2a})^2 - (J_3 + J_3)^2}} & \end{cases}, \]

(37)

for \( H_T^{(4)} \) and \( H_T^{(5)} \), respectively. Here, the barred quantities are obtained by conjugating the weak phases. In terms of the short-distance coefficients \( a_{CP}^{(4)} \) reads

\[ a_{CP}^{(4)} = 2 \frac{\Im(\rho) / \Re(\rho) = \frac{\rho_1 ^\prime - \rho_1}{\rho_1 ^\prime + \rho_1}}{\sqrt{(\rho_1 ^\prime + \rho_1 ^\prime) \cdot (\rho_1 ^\prime + \rho_1 ^\prime)}}. \]

(38)

The generalization of \( a_{CP}^{(3)} \) is given by

\[ a_{CP}^{(3)} = 2 \frac{\Re(\rho_2 - \rho_2)}{\sqrt{(\rho_1 ^\prime + \rho_1 ^\prime) \cdot (\rho_1 ^\prime + \rho_1 ^\prime)}}. \]

(39)

Due to the presence of \( \rho_1 ^\prime \) and \( \rho_1 ^\prime \), the generalization of the CP asymmetries \( a_{CP}^{(1)} \) and \( a_{CP}^{(2)} \) leads to a doubling

\[ a_{CP}^{(1, \pm)} = \frac{\rho_1 ^\prime - \rho_1}{\rho_1 ^\prime + \rho_1}, \quad a_{CP}^{(2, \pm)} = \frac{\rho_1 ^\prime - \rho_1}{\rho_1 ^\prime + \rho_1}. \]

(40)

In this case the CP asymmetry of the decay rate can not be related to any of the \( a_{CP}^{(1, \pm)} \) and is not long-distance free. However, from (25) it is straightforward to read off strategies to relate the \( a_{CP}^{(k, \pm)} \), \( k = 1, 2 \) to the \( J_i \). In particular \( a_{CP}^{(1, -)} \) can be extracted from ratios involving \( J_{2c}, (2 J_{2a} - J_3), J_4 \), whereas \( a_{CP}^{(1, +)} \) requires \( (2 J_{2a} + J_3) \). In analogy to Eq. (2.37) of Ref. [4], the set \( b \) has to be restricted to \( b = \{1, 3, 4\} \) for \( a_{CP}^{(2, -)} \) and to \( b = 2 \) for \( a_{CP}^{(2, +)} \).

In (SM+SM’) short-distance free ratios of angular observables \( J_i \) exist for \( f_0/f_\parallel \) as given in Eq. (17), and additionally

\[ \frac{f_0}{f_\parallel} = \frac{\sqrt{2} J_9}{-J_9}. \]

(41)

Due to \( (2 J_{2a} + J_3) \propto \rho_1 ^\prime \), however, no short-distance free ratios can be formed which involve \( f_\perp \). Hence, the observables \( F_L \) and \( A_T^{(2,3)} \) are no longer short-distance free either

\[ F_L = \frac{\rho_1 ^\prime f_\parallel}{\rho_1 ^\prime (f_\parallel ^2 + f_\parallel ^2)} + \rho_1 ^\prime f_\perp, \]

(42)

\[ A_T^{(2)} = \frac{\rho_1 ^\prime f_\perp}{\rho_1 ^\prime f_\parallel ^2 + \rho_1 ^\prime f_\perp ^2}, \quad A_T^{(3)} = \sqrt{\frac{\rho_1 ^\prime f_\parallel}{\rho_1 ^\prime f_\parallel ^2 + \rho_1 ^\prime f_\perp ^2}}. \]

(43)

and the method used in (17) to extract form factor ratios would yield \( \sqrt{\rho_1 ^\prime / \rho_1 ^\prime (f_\parallel / f_\parallel)} \). With current data the correction factor is within \( 0.7 \leq \sqrt{\rho_1 ^\prime / \rho_1 ^\prime} \leq 1.4 \) at 2 \( \sigma \). Furthermore, we obtain the relation in (SM+SM’)

\[ A_T^{(3)} = \sqrt{\left(1 - A_T^{(2)}\right) / \left(1 + A_T^{(2)}\right)}, \]

(44)

which can be checked experimentally.

C. Scalar and pseudo-scalar operators

The (S+P) operators modify the angular observables \( J_{1,5,6,7} \) only. The respective NP contributions are
driven by $A_0 \Delta_{S,P}$, where $A_0$ denotes the $B \to K^*$ axialvector form factor and $\Delta_{S,P} \equiv C_{S,P} - C_{S',P}'$. We find that $J_{1c}$ only receives generically unsuppressed contributions,

$$ J_{1c} = \frac{3}{2} \rho_1 f_0^2 + 3N^2 (|\Delta_S|^2 + |\Delta_P|^2) \frac{\lambda}{m_b^2} A_0^2 + \mathcal{O}(m_t^2/q^2, m_s/m_b). \tag{45} $$

Helicity-suppressed ($\sim m_t/\sqrt{q^2}$) contributions from interference terms $SM \times S$ arise in $J_{5,6c,7}$. For the explicit expressions see Appendix [3]

We find that in the presence of $(S+P)$ operators the low recoil relations

$$ |H_T^{(1)}| = 1, \quad |H_T^{(2)}| = \frac{m_t}{\sqrt{q^2}}, \quad |H_T^{(3)}| = 1 + \mathcal{O}\left(\frac{m_t}{\sqrt{q^2}}\right), \quad |H_T^{(4)}| = \frac{m_t}{\sqrt{q^2}}, \quad |H_T^{(5)}| = \frac{m_t}{\sqrt{q^2}} \tag{46} $$

hold, and $H_T^{(3,4,5)}$ remain long-distance free. Since $J_{8,9}$ vanish in the considered scenario $H_T^{(4)} = H_T^{(5)} = 0$, and $H_T^{(1c)}$ is ill-defined in the SM-like scenario.

The helicity-suppressed contributions to $J_{5}$ break the relation $H_T^{(2)} = H_T^{(3)}$ at $O(m_t/\sqrt{q^2})$ through a finite $\Delta_S$. In this case $H_T^{(2)}$ ceases to be free of form factors, and rather depends on $A_0/f_0$. Moreover the relation $J_T = 0$ is broken at $O(m_t/\sqrt{q^2})$ if there is additionally CP violation beyond the SM. With the exception of using $J_{5}$, the ratio $f_0/f_\parallel$ can be extracted by means of the methods proposed in Eqs. (17) and (41).

The (pseudo-)scalar contributions to $J_{1c}$ break the relation $J_{1c} = -J_{2c}$, valid only in the $(SM + SM')$ basis for $m_t \to 0$, see also Appendix [13]. At the same time, contributions to the longitudinal polarization $F_L$ of $K^*$ mesons are induced, see Eq. (A9). These contributions prohibit that $F_L$ and $A_{FB}$, the lepton forward-backward asymmetry, can be extracted simultaneously from a fit to Eq. (A11), the angular distribution in $cos \theta_\ell$. Note that $F_L$ and $F_T = 1 - F_L$ can be extracted from Eq. (A8), the distribution in $cos \theta_K$, in a model-independent way. Discrepancies between the extracted values of $F_{L,T}$ from Eqs. (A8 and A11) would indicate BSM physics. (We assume here that S-wave contributions from $B \to K\pi\ell^+\ell^-$ have been removed from the data, see Section [7]). Note also that interference terms $(SM + SM') \times S$ contribute to $A_{FB}$ via $J_{6c}$ due to (A7).

### D. Tensor operators

The tensor operators $(T+T5)$ give rise to additional tensor transversity amplitudes $A_{ij}$. Here the labels $i$ and $j$ denote the transversity state $t, \perp, \|, 0$ of the polarization vectors which comprise the rank-two polarization tensor that was used in the computation. We obtain for pairs $(\|, 0), (0, \perp, \perp), (\|, \|, \perp)$ the total angular momenta $J = 0, 1, 2$, respectively. For the definition of the transversity amplitudes and their general results, see Appendix [C] and Eqs. (B18)-(B20), respectively.

At low recoil, after application of the improved Isgur-Wise relations, we obtain

$$ A_{\|,\|,0} = C_{T,T5} \frac{2k}{\sqrt{q^2}} M_B (1 + \tilde{\Lambda}_0) f_0, \quad A_{\|,\perp,0} = C_{T,T5} \frac{\sqrt{2k}}{\sqrt{q^2}} M_B (1 + \tilde{\Lambda}_\perp) f_\perp, \quad A_{0,\|,\|} = C_{T,T5} \frac{\sqrt{2k}}{\sqrt{q^2}} M_B (1 + \tilde{\Lambda}_\parallel) f_\parallel, \tag{47} $$

where $\tilde{\Lambda}_0,\|,\perp$ is $O(\Lambda_{QCD}/M_B)$.

In the presence of tensor operators $T$ and $T5$ in addition to $(SM + SM')$ the angular observables $J_i$ receive $i)$ contributions which do not interfere with other operators in $J_{1c,2c,2c,3c,4c}$, and $ii)$ helicity-suppressed interference contributions in $J_{1s,1c,5s,6s,7c}$, and $iii)$ no contributions in $J_{8,9}$ in the additional six transversity amplitudes $A_{2s,\|,\perp,\perp,\|,\perp,0,0}$. We find

$$ \frac{8}{3} J_{1s} = \left[ 3 \rho_1^0 + \rho_1^\perp (1 + \tilde{\Lambda}_\perp)^2 \right] f_\perp + \left[ 3 \rho_1^- + \rho_1^\perp (1 + \tilde{\Lambda}_\parallel)^2 \right] f_\parallel + \mathcal{O}\left(\frac{m_t}{\sqrt{q^2}}\right), \quad \frac{4}{3} J_{1c} = \left[ 2 \rho_1^- + \rho_1^\perp (1 + \tilde{\Lambda}_0)^2 \right] f_\perp + \mathcal{O}\left(\frac{m_t}{\sqrt{q^2}}\right), \quad \frac{4}{3\beta_f} (2 J, J) = 3 \left[ 2 \rho_1^- - \rho_1^\perp (1 + \tilde{\Lambda}_0)^2 \right] f_\perp, \quad \frac{4\sqrt{2}}{3\beta_f} J_5 = 4 \text{Re}(\rho_2) f_0 f_\perp + \mathcal{O}\left(\frac{m_t}{\sqrt{q^2}}\right), \quad \frac{2\sqrt{2}}{3\beta_f} J_6 = 4 \text{Re}(\rho_2) f_\perp f_\parallel + \mathcal{O}\left(\frac{m_t}{\sqrt{q^2}}\right), \tag{48} $$

$$ \frac{2\sqrt{2}}{3\beta_f} J_7 = 4 \text{Re}(\rho_2) f_\perp f_\parallel + \mathcal{O}\left(\frac{m_t}{\sqrt{q^2}}\right), \quad \frac{4\sqrt{2}}{3\beta_f} J_9 = 4 \text{Im}(\rho_2) f_0 f_\perp, \quad \frac{4\sqrt{2}}{3\beta_f} J_8 = 4 \text{Im}(\rho_2) f_\perp f_\parallel. $$

Here the additional short-distance combination reads

$$ \rho_1^\perp \equiv 16 \kappa^2 \frac{M_B^2}{q^2} \left[ |C_T|^2 + |C_{T5}|^2 \right]. \tag{49} $$

Without tensor operators the ratio $H_T^{(1)}$ is free of short- and long-distance contributions. In the presence of the
tensor operators we obtain
\[ H_T^{(1)} = \text{sgn}(f_0) \text{sgn}(\rho_2 - \rho_1^2) \times \left[ 1 + \frac{\rho_2 \rho_1^T}{2(\rho_1 - \rho_1^T)^2} (\Lambda_0 - \Lambda_\parallel)^2 \right] + \mathcal{O} \left( \Lambda_i^3 \right) \] (50)
and form factor factors still cancel. Deviations from \(|H_T^{(1)}| = 1\) arise at \(\mathcal{O}(\Lambda_i^3)\), while in \(H_T^{(1b,1c)}\) the suppression is only linear in \(\Lambda_i\). For instance,
\[ H_T^{(1b)} = 1 - \frac{\rho_1^T}{\rho_1 - \rho_1^T}(\Lambda_0 - \Lambda_\parallel) + \mathcal{O} \left( \Lambda_i^2 \right) . \] (51)
We further find that in scenarios with tensor operators \(H_T^{(3,4,5)}\) remain free of hadronic form factors, and the relations \(H_T^{(2)}/H_T^{(3)} = 1\) and \(H_T^{(4)}/H_T^{(5)} = 1\) hold up to helicity-suppressed and power-suppressed terms, respectively, see Table II and Table III. The relation \(J_{1c} + J_{2c} = 0\), valid in the (SM+SM') for \(m_\ell \to 0\), is broken by \(\rho_1^T\)
\[ J_{1c} + J_{2c} = 3 \rho_1^T (1 + \Lambda_0) f_0^2 + \mathcal{O} \left( \frac{m_\ell}{\sqrt{q^2}} \right) . \] (52)

E. Interference between operator sets

When considering the complete set of \(|\Delta B| = |\Delta S| = 1\) semileptonic operators, all of the previously presented low recoil relations are broken at some level, which can however be parametrically suppressed and small. For instance, the relations \(H_T^{(2)}/H_T^{(3)} = 1\) and \(J_7 = 0\) are broken at leading order by the simultaneous presence of tensors and scalars only, while \(H_T^{(1b)}/H_T^{(5)} = 1\) remains intact up to \(\mathcal{O}(\Lambda_i)\)-suppressed terms, see Table II for an overview.

The angular observables \(J_{1c}, J_5, J_{6c}\) and \(J_{1c}\) receive contributions from (pseudo-) scalar operators involving the form factor \(A_0\), see Section IIIIC. These terms modify the otherwise general structure
\[ J_a \sim \rho_1 f_2 f_3 . \] (53)

 Corrections to \(J_{1c}\) arise from operators S and P, while \(J_5, J_{6c}\) and \(J_{1c}\) are modified by interferences of S and P with tensor operators. Since the angular observables \(J_{2s,2c,3,4,6s,8,9}\) obey Eq. (53) it follows that \(H_T^{(1,3,4,5)}\) remain free of hadronic inputs in the complete operator basis. Our findings are summarized in Table III.

IV. \(B \to \bar{K} \ell^+ \ell^-\) at low recoil

The decay \(\bar{B} \to \bar{K} \ell^+ \ell^-\) is another accessible FCNC channel, which depends on the Wilson coefficients in a complementary way to \(B \to \bar{K} \ell^+ \ell^-\). The angular distribution of \(\bar{B} \to \bar{K} \ell^+ \ell^-\) can be written as
\[ \frac{d^2 \Gamma_K}{dq^2 d\cos \theta_\ell} = a + b \cos \theta_\ell + c \cos^2 \theta_\ell , \] (54)
where the angle \(\theta_\ell\) is defined as in \(\bar{B} \to \bar{K} \ell^+ \ell^-\) decays, see Appendix A. The \(q^2\)-dependent coefficients \(a, b\) and \(c\) are related to the decay rate, the lepton forward-backward asymmetry, \(A_{FB}^K\), and the flat term, \(F_H\), as follows
\[ \frac{d\Gamma_K}{dq^2} = 2 (a + c/3) , \] (55)
\[ A_{FB}^K = \frac{b}{d\Gamma/dq^2} , \] \[ F_H = \frac{2 (a + c)}{d\Gamma/dq^2} . \]
Here we label the \(\bar{B} \to \bar{K} \ell^+ \ell^-\) decay rate and forward-backward asymmetry by a superscript \('K'\) to distinguish them from the ones in \(\bar{B} \to \bar{K} \ell^+ \ell^-\) decays.

Similar to \(\bar{B} \to \bar{K} \ell^+ \ell^-\) the low recoil OPE and the Isgur-Wise relations can be applied, allowing to trade the \(\bar{B} \to K\) tensor form factor \(f_2\) for the vector one \(f_1\) [22]. We obtain for the extended operator basis at low recoil to leading order in \(1/m_6\)
\[ \frac{4a}{\Gamma_0 \sqrt{\lambda_0}} f_+^3 = \rho_1^+ + \frac{f_0^2}{f_+^2} \rho_{S+P} \] (56)
\[ + \frac{m_\ell}{M_B} \rho_{T \times 79} + \frac{m_\ell}{\sqrt{q^2}} f_0^2 \rho_{P \times 10} , \]
\[ \frac{b}{\Gamma_0 \lambda_0 (M_B^2 - M_K^2) f_+ f_0} = \rho_{S \times T + P \times T^5} \] (57)
\[ + \frac{m_\ell}{\sqrt{q^2}} \rho_{T \times 10} + \frac{m_\ell}{m_b - m_s} \rho_{S \times 79} , \]
\[ \frac{4c}{\Gamma_0 \sqrt{\lambda_0}} f_+^2 = -\rho_1^+ + \rho_1^T , \] (58)
where \(\lambda_0 \equiv \lambda(M_B^2, M_K^2, q^2)\) and
\[ \rho_{S+P} = \frac{q^2 (M_B^2 - M_K^2)^2}{(m_b - m_s)^2 \lambda_0} \times \left( |C_S + C_{S'}|^2 + |C_P + C_{P'}|^2 \right) , \] (59)
\[ \rho_{P \times 10} = \frac{\sqrt{q^2}}{\lambda_0} \left( M_B^2 - M_K^2 \right)^2 \frac{m_b - m_s}{q^2} \times 4 \text{ Re} \left( (C_P + C_{P'}) (C_{10} + C_{10'}^*) \right) , \] (60)
\[ \rho_{S \times T + P \times T^5} = 16 \kappa \frac{M_B^2}{q^2} \text{ Re} \left( C_T \left( C_{79} + C_{79'}^* \right) \right) , \] (61)
\[ \rho_{T \times 10} = 4 \kappa \frac{M_B}{\sqrt{q^2}} \text{ Re} \left( (C_{10} + C_{10'}) C_{T5}^* \right) , \] (62)
\[ \rho_{S \times 79} = \text{ Re} \left( (C_S + C_{S'}) (C_{79} + C_{79'}^*) \right) . \] (63)
and the scalar one
\[ f_y \] yields a complementary constraint on \( \rho \) introduced in Section III.

At most corrections of order \( \alpha / m_b \) and \( C_7 / (C_5 m_b) \), while \( A_0 \) denotes breaking through terms involving the corresponding \( B \to K^* \) form factor. Observables marked with \( — \) vanish in the considered scenario.

Here we have neglected terms suppressed by \( m_t^2 / M_B^2 \), but kept those proportional to \( m_t / \sqrt{q^2} \). The form factor \( f_+ \) and the scalar one \( f_0 \), as well as the normalization \( \Gamma_0 \) are defined in \([19, 28]\), whereas \( \rho_1^+ \) and \( \rho_1^T \) have been introduced in Section III.

In the scenario (SM+SM') the differential decay rate
\[ \frac{d\Gamma^K}{dq^2} = \Gamma_0\sqrt{\lambda_0}\frac{3}{2}f_+^2\rho_1^+ , \]
yields a complementary constraint on \( \rho_1^+ \). The CP asymmetry of the rate, \( A_{CP} \), turns out to be free of long distance uncertainties, as \( f_+ \) cancels, and the CP asymmetries
\[ A_{CP}[B \to \bar{K}^+ \ell^-] = a_{CP}^{(1,+)}[B \to \bar{K}^+ \ell^-] \]
in \( B \to \bar{K}^+ \ell^- \) and \( \bar{B} \to K^+ \ell^- \) decays are identical at low recoil, see Eq. (69). The equality Eq. (69) allows to measure CP violation with the combined, larger data set. If the CP asymmetries turn out to be not equal it would imply contributions from outside of (SM+SM').

Furthermore, the decay \( \bar{B} \to K^+ \ell^- \) provides with \( F_H \) a powerful observable, which exhibits sensitivity to (S+P) and (T+T5) operators, whereas the (SM+SM') contributions are suppressed by \( m_t^2 / q^2 \) \([19]\).

While the terms of \( (m_t / M_B) \) in the denominator might be safely neglected in view of the numerically leading term \( \rho_1^T \), they could become of some relevance in the numerator of \( F_H \) if \( \rho_1^T \) and/or \( \rho_5^{S+P} \) are small and the interference of \( C_T \) and/or \( C_{T5} \) with the large (SM+SM') contributions can overcome the lepton mass suppression. The importance of the flat term in SM tests and NP searches becomes manifest from Eq. (67) since \( F_H \) is given directly by the magnitude of scalar and tensor Wilson coefficients and secondly it depends only on form factor ratios rather than the form factors themselves. For tensor contributions, this residual dependence drops out and \( F_H \) is free of hadronic uncertainties.

On the other hand, in \( A_{FB}^K \) tensor and (pseudo-)scalar operators need to be either simultaneously present or their contributions are lepton-mass suppressed, such as the interference terms between SM(‘) and T,T5,S, and P. Moreover, \( A_{FB}^K \) depends on the ratio of form factors \( f_0 / f_+ \). We obtain

\[ A_{FB}^K = \frac{3(M_B^2 - M_K^2)}{\sqrt{\lambda_0}} \frac{f_0}{f_+} \rho_5^{S+T5} \left( \frac{\rho_1^+}{2} + \frac{3}{2} \rho_1^T \right) \frac{\rho_5^{S+P}}{2} + O(m_t / M_B) . \]

### Table I: The low recoil relations in SM-like models (first row) and the leading terms that break them in SM extensions. A \( \checkmark \) denotes at most corrections of order \( \alpha / m_b \) and \( C_7 / (C_5 m_b) \). \( \Lambda \) \( \checkmark \) reminds that up to the latter corrections \( H_T^{(4,5)} = 0 \). Here \( \Lambda = \Lambda_{QCD} \), \( Q = O(m_b, \sqrt{q^2}) \) and \( \Delta_{S,P} \equiv c_{S,P} - c_{S,P} \), for details see text.

| Scenario | \( H_T^{(1)} \) | \( H_T^{(2)} \) | \( H_T^{(3)} \) | \( H_T^{(4)} \) | \( J_{6c} = 0 \) | \( J_7 = 0 \) | \( J_{8,9} = 0 \) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| SM       | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      |
| SM + (S+P)| \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      |
| SM + (T+T5)| \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      |
| SM + SM’| \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      |
| all      | \checkmark      | A_0             | \checkmark      | \checkmark      | \checkmark      | \checkmark      | \checkmark      |

### Table II: The low recoil observables \( H_T^{(i)} \), and the degree to which they remain free of hadronic input. A \( \checkmark \) denotes at most corrections of order \( \alpha / m_b \) and \( C_7 / (C_5 m_b) \), while \( A_0 \) denotes breaking through terms involving the corresponding \( B \to K^* \) form factor. Observables marked with \( — \) vanish in the considered scenario.

| Scenario | \( H_T^{(1)} \) | \( H_T^{(2)} \) | \( H_T^{(3)} \) | \( H_T^{(4)} \) |
|----------|-----------------|-----------------|-----------------|-----------------|
| SM       | \checkmark      | \checkmark      | \checkmark      | \checkmark      |
| SM + (S+P)| \checkmark      | \checkmark      | \checkmark      | \checkmark      |
| SM + (T+T5)| \checkmark      | \checkmark      | \checkmark      | \checkmark      |
| SM + SM’| \checkmark      | \checkmark      | \checkmark      | \checkmark      |
| all      | \checkmark      | A_0             | \checkmark      | \checkmark      |

### V. Sensitivity to New Physics

The fact that no order one NP signals have been observed in (semi)leptonic \( |\Delta B| = 1 \) processes to date suggests that NP effects with the exception of null tests are suppressed with respect to the SM contributions. Good control of theoretical uncertainties is therefore crucial to progress.
In Fig. 2, we show $H_T^{(2,3)}$ and $A_{FB}$ integrated in the low recoil bin $14.18 \text{GeV}^2 \leq q^2 \leq 19.21 \text{GeV}^2$ versus $|C_9/C_{10}|$ and normalized to their SM values within their respective theory uncertainties. As can be seen, a hypothetical measurement of $A_{FB}$ as good as $(90 \pm 5)\%$ (horizontal dashed black lines) would not be able to distinguish the SM from NP, due to the theory uncertainty. The latter is dominated by the one of the form factors, which only partially cancel within $A_{FB}$. Given our current understanding of the $B \to K^* \ell^+ \ell^-$ form factors, an experimental determination of $A_{FB}$ in the low recoil region that strives to indicate NP must exclude values larger than $0.85 \times A_{FB}^{\text{SM}}$ and lower than $1.18 \times A_{FB}^{\text{SM}}$. On the other hand, the $(90 \pm 5)\%$ measurement in $H_T^{(2)}$ or $H_T^{(3)}$ would suffice to establish NP due to the small, subpercent level theoretical uncertainty [4]. An advanced few percent-level measurement would probe $|C_9/C_{10}|$ up to a discrete ambiguity at similar, few percent level.

In Fig. 3, we show the $q^2$-integrated observables $A_{im}/A_{FB} = J_9/J_{10}, a_{CP}^{(4)}, -A_9^D$ and $-A_9$ for $14.18 \text{GeV}^2 \leq q^2 \leq 19.21 \text{GeV}^2$. For the definition of $A_9^D$, see [7]. All observables are shown as functions of the (imaginary part of the) NP coupling $\text{Im}(C_{10'})$. All other NP Wilson coefficients including $\text{Re}(C_{10'})$ are assumed to be zero. Since the observables are odd functions of $\text{Im}(C_{10'})$ to a very high degree the values for $\text{Im}(C_{10'}) < 0$ are not shown.

We find that $A_{im}/A_{FB}$ as well as $a_{CP}^{(4)}$ are better suited to probe small values of $\text{Im}(C_{10'})$ due to their steeper slope and smaller relative theoretical uncertainties. Approxi- mately (see Eq. (A2) for $\langle \rangle$-notation),

$$\langle A_9^D \rangle \simeq (+0.061 \pm 0.008) \text{Im}(C_{10'})\; , \quad (72)$$

$$\langle A_9 \rangle \simeq (+0.10 \pm 0.02) \text{Im}(C_{10'})\; , \quad (73)$$

$$\langle A_{im} \rangle / \langle A_{FB} \rangle \simeq (-0.140 \pm 0.004) \text{Im}(C_{10'})\; , \quad (74)$$

$$\langle a_{CP}^{(4)} \rangle \simeq (-0.240 \pm 0.005) \text{Im}(C_{10'})\; , \quad (75)$$

where we estimated the theory uncertainty from residual $1/m_b$ corrections and form factors similar to [5]. Note that both $A_{im}$ and $A_{FB}$ have been separately measured by CDF in two low recoil bins [29]. Due to the current experimental uncertainties and the absence of information on the correlation of the individual errors we refrain from calculating the ratio.

VI. PROBING THE OPE

The relations between the low recoil observables can be used to quantitatively test the performance of the OPE. We employ the following ansatz:

$$A_i^{L,R} \propto C_i^{L,R} f_i(1 + \epsilon_i), \quad i = \perp, \parallel, 0. \quad (76)$$

Here the terms $\epsilon_i$ parametrize effects beyond Eq. (1) to each transversity state. These include higher order power corrections or contributions from even higher order the OPE.
such as duality violation. There are no separate corrections for the left- and right-handed lepton chiralities as the photon-current as a mediator of the considered effects couples vectorlike. Although not explicitly written the $\epsilon_i$ are in general $q^2$-dependent. Our ansatz parametrizes the most general situation within the SM neglecting lepton mass corrections of order $m_b^2/q^2$. The generic size of the subleading $1/m_b$ corrections imply $\epsilon_i$ of the order $\alpha_s \Lambda/m_b$ or $C_i/C_0 \Lambda/m_b$, about few percent. This is taken into account in current uncertainty budgets [3]. It is therefore desirable to have sensitivity to corrections at the (few) percent level. On the other hand, it is hard to quantify duality violation. While in a toy model duality violating contributions have been estimated to be very small [2] it is nevertheless useful to have experimental checks.

We find that the corrections enter the short-distance and form factor free relations quadratically

$$\begin{align*}
|H^{(1)}_T|,\ H^{(1b)}_T, \frac{H^{(2)}_T}{H^{(3)}_T} = 1 + O(\epsilon^2),
\end{align*}$$

and the form factor ratios $f_i/f_k$ linearly. The null tests depend linearly on the imaginary parts

$$J_{T,8,9} = O(\text{Im}(\epsilon)),$$

while the real parts enter at second order only. Note that the corrections Eq. (77) vanish for $\text{Im}(\epsilon_i) = 0$ since for all $\epsilon_i$ real the ansatz Eq. (76) would correspond to a mere rescaling of the form factors.$^1$

The double suppression in Eq. (77) makes these observables sensitive to somewhat sizable effects only or requires high experimental precision: For $\epsilon \sim 30 (10)\%$ the correction amounts to about 10 (1)\%. On the other hand, the respective background from the SM OPE is at the permille level.

Due to the linear dependence and the generic appearance of unsuppressed strong phases in the nonperturbative regime we find that the null tests Eq. (78) have potentially higher sensitivity to OPE corrections. Up to $O(\epsilon^2)$-corrections

$$\begin{align*}
J_T^{(1)} & = -3\sqrt{2} \beta_\ell \rho_2 f_0 f_\parallel \text{Im} (\epsilon_\parallel - \epsilon_\parallel^*) , \\
J_T^{(2)} & = \frac{3}{2\sqrt{2}} \beta_\ell^2 \rho_1 f_0 f_\perp \text{Im} (\epsilon_\perp - \epsilon_\perp^*) , \\
J_T^{(3)} & = \frac{3}{\beta_\ell^2} \rho_1 f_\parallel f_\perp \text{Im} (\epsilon_\perp - \epsilon_\parallel^*) .
\end{align*}$$

All coefficients in front of the imaginary parts are parametrically unsuppressed in the angular distribution. We investigate corrections from NP in Section IV A and comment on an experimental background from $K\pi$ in an S-wave with invariant mass around the $K^*(892)$ mass in Section IV B.

$^1$ We thank the unknown referee for emphasizing this point.

As summarized in Table [1] beyond the SM deviations from $|H^{(1)}_T| = 1$ arise from tensor operators only. Here we estimate their contributions allowing for complex Wilson coefficients. The currently strongest constraints stem from $B \to K\ell^+\ell^-$ decays, discussed in Section IV. We employ the recent LHCb measurements [30] of the branching ratio and the flat term $F_H$, Eq. (67), combined with branching ratio measurements from Belle [31], BaBar [32] and CDF [33]. In the scenario $(T+T5+\text{SM}+\text{SM}')$ we find at 95\% CL

$$|C_T|^2 + |C_{T5}|^2 \lesssim 0.5 .$$

The bound is dominated by the LHCb measurement of the two highest $q^2$-bins of $F_H$. We recall that in $F_H$ the form factor uncertainties drop out at leading order in this kinematic regime.

The outcome of our scan leading to Eq. (82) is shown in Fig. 4. Values of both $|C_T|, |C_{T5}|$ near zero are disfavored at 68\% CL. This follows from the current low recoil data on $F_H$ [30], which have central values at $\sim 1\sigma$ above the SM.

Using Eqs. (49), (50) and (82) we obtain

$$|H^{(1)}_T - 1| \lesssim 0.08 .$$

Scalar and pseudo-scalar operators contribute predominantly constructively to $F_H$, such that they do not invalidate an upper bound on the tensor contributions.

The relation $H^{(2)}_T/H^{(3)}_T = 1$ receives corrections by the simultaneous presence of tensor and scalar operators, see Table [I]. The latter are constrained by $B_s \to \mu^+\mu^-$ decays. Assuming that the branching ratio $\mathcal{B}(B_s \to \mu^+\mu^-)$

![Figure 4: The constraints on $|C_T|$ and $|C_{T5}|$ from $B \to K\ell^+\ell^-$ low recoil data at 68\% CL (inner gold area) and 95\% CL (outer blue areas).](image-url)
is saturated by scalar operators $O_{S(i)}$ we find $|\Delta s| \lesssim 0.59$ from the most recent upper bound $B(B_s \to \mu^+\mu^-) < 4.2 \times 10^{-9}$ at 95\% CL [33]. For this bound we also consider $B_s \to \mu X$ mixing effects pointed out in Ref. [33], allowing for $A_{\Delta T} = -1$ and use $\gamma_s = 0.088 \pm 0.014$. In combination with the bounds on the tensor couplings we obtain

$$|H_T^{(2)}/H_T^{(3)} - 1| \lesssim 0.12 \text{GeV} \times N \frac{f_{1f} A_0}{J\xi J_0} \simeq 0.06.$$  

(84)

Subleading effects from scalar operators alone are suppressed by $m_t/Q$ and do not exceed the percent level for muons.

The relation $J_T = 0$ receives corrections from NP with either $m_t/Q$-suppression or require tensor contributions. To estimate the sensitivity to OPE corrections vs. NP we compare the respective contributions

$$\frac{J_T^{NP}}{J_T} \simeq \frac{2}{\sqrt{q^2}} \frac{\sqrt{\lambda_T^{12}}}{\rho_2 \text{Im}(\epsilon_0 - \epsilon_\perp)} N \frac{A_0 f_{1f}}{f_0 J_0}$$

$$\lesssim \left( \frac{0.04}{\text{Im}(\epsilon_0 - \epsilon_\perp)} \right).$$

(85)

We learn that $J_T$ probes the OPE as good as at the few percent level, before tensor induced contributions can keep up. NP contributions to $J_T$ from scalar operators interfering with the SM are suppressed by $m_t/Q$, which do not exceed the percent level for muons.

We further find that both $J_8$ and $J_9$ are more sensitive to NP than $J_T$, with similar sensitivities given as

$$\frac{J_T^{NP}}{J_T} \simeq \frac{2 \text{Im}(\rho_2)}{\rho_1 \text{Im}(\epsilon_0 - \epsilon_\perp)},$$

(87)

$$\frac{J_T^{NP}}{J_T} \simeq \frac{-2 \text{Im}(\rho_2)}{\rho_1 \text{Im}(\epsilon_\perp - \epsilon_\parallel)},$$

(88)

both of which are generically order one in the presence of order one CP phases, see Eq. (85), unless the $\epsilon_i$ are $O(1)$ or larger. We conclude that $J_8$ and $J_9$ are likely to probe CP violating right-handed currents.

In general tensor operators are absent or small in most models; they arise e.g., from box-type matching conditions, can contribute to Wilson coefficients of dipole operators via renormalization group mixing [36,37] or arise in models with FCNC at tree-level, such as those with leptoquarks, as discussed in [19]. In this respect the OPE-based predictions of the short-distance independence of $H_T^{(1)}, H_T^{(2)}/H_T^{(3)}$ and $J_T$ are very clean.

**B. S-wave pollution**

We discuss the impact of the experimental background from $\bar{K}_\pi$ in an S-wave on the $\bar{B} \to \bar{K}^* \to K\pi\ell^+\ell^-$ angular analysis, recently addressed in Refs. [38,40] for the low $q^2$ region.

The resonant spin zero $\bar{K}_\pi$-contributions barely affect the low recoil region as the masses of the well established scalar kaons, notably $K_{\pi}(1430)$ cause their kinematical endpoint $q^2_{\text{max}} = 15 \text{GeV}^2$ to have barely overlap with the low recoil bin $q^2 \gtrsim (14 - 15) \text{GeV}^2$ [38]. In addition there is already phase space suppression $\propto \lambda_{0}^{3/2}$ below the endpoint. A nonresonant contribution with invariant mass around $m_K$ is, however, not suppressed by these arguments and can affect the required precision extraction of the angular observables $J_\ell$ [40] for SM or OPE tests. Fortunately all S-wave backgrounds can be controlled experimentally. The effect of an underlying spin zero component in the $\bar{B} \to \bar{K}^* \to K\pi\ell^+\ell^-$ angular distribution Eq. (A1) can be parametrized as follows:

i) The $J_\ell$ for $i = 3, 6, 9$ do not receive S-wave contributions.

ii) The terms $[J_i \sin 2\theta_K]$ for $i = 4, 5, 7, 8$ in Eq. (A1) need to be replaced by $[J_i \sin 2\theta_K + J_i \sin \theta_K]$. The $J_i$ denote interference terms; due to their different angular dependence they can be isolated.

iii) The terms $[J_{iS} \sin^2 \theta_K + J_{iC} \cos^2 \theta_K]$ for $i = 1, 2$ in Eq. (A1) need to be replaced by $[J_{iS} \sin^2 \theta_K + (J_{ic} + J_{isc}) \cos^2 \theta_K + J_{isc} \cos \theta_K]$. The interference terms $J_{isc}$ can be identified by angular analysis. The $J_{iS}, J_{ic}$ stem from S-waves only and can be measured at invariant $K\pi$ masses outside of the $K^*(892)$ peak, where the $J_{iS}, J_{ic}$ can be neglected.

Note that all $J_i$ depend in general on $q^2$; they incorporate resonant and nonresonant scalar contributions.

Procedure $iii)$ is required for all $H_T^{(k)}$ as well as observables which are normalized to the rate. The accuracy to which the $J_{ix}, i = 1, 2, x = s, c$ can be measured limits the experimental precision on the $J_{ix}$. Note that $J_{7,8,9}$ and the observables given in Eq. (36) can be extracted without side-band measurements. $J_9$ and the second observable in Eq. (36), which is proportional to $J_9/J_8$ do not receive contributions from S-waves at all.

**VII. CONCLUSIONS**

We analyze $\bar{B} \to \bar{K}^* \to K\pi\ell^+\ell^-$ and $\bar{B} \to \bar{K}\ell^+\ell^-$ decays, $\ell = e, \mu$, at low hadronic recoil in the most general basis of semileptonic dimension-six effective couplings. We investigate to which extent the beneficial features obtained from angular analysis and the OPE in the SM-like operator basis hold. We find:

The transversity observables $H_T^{(i)}, i = 1, 3, 4, 5$ remain in the general case free of hadronic matrix elements and are clean tests of the SM and beyond; for $H_T^{(2)}$ this is true if contributions from scalar operators are ignored, see Table III.

The form factor ratio $f_0/f_1$ can be extracted by means of Eqs. (17) and (18), excluding methods based on $J_5$ if
scalar operators are present. If no chirality-flipped operators contribute the ratios Eqs. 18 and 19 allow for a short-distance free extraction of form factor ratios involving $f_\perp$. There is a residual short-distance dependence from tensor operators in Eq. 17, which, however, is $\Lambda_{\text{QCD}}/M_B$ suppressed.

The low recoil relations among the $H_T^{(i)}$ and $J_{7,8,9} = 0$ receive corrections from both NP, see Table I, and contributions beyond the leading order OPE Eq. (1), as given in Eq. (76) and discussed in Section VII. Our analysis shows that with the present $|\Delta B| = |\Delta S| = 1$ constraints $J_7$ has model-independently the highest sensitivity to the latter corrections at the percent level, before a potential NP background kicks in. The sensitivity in $|H_T^{(1)}| = 1$ to OPE corrections becomes comparable or better if tensor operators are ignored. The interplay of constraints will evolve with future rare decay measurements, and the actual sensitivity to the OPE can increase.

The observables $J_{8,9}$ are sensitive to CP violating chirality-flipped contributions. We suggest to explore such scenarios with the observables $H_T^{(4,5)}$ and the respective CP- and T-odd CP asymmetry $a_{\text{CP}}$, all of which vanish in the SM-like basis. Further null tests are the ratios Eq. (36). Note that one of the latter, $J_9/J_{6s} = A_{\text{im}}/A_{FB}$ has already been experimentally accessed 29.

Our findings are of direct use to the high statistics studies at the LHC(b) experiments and forthcoming high luminosity flavor factories. We look forward to this application and future data.

Acknowledgments

DvD is grateful to Sebastian Hollenberg for comments on the manuscript, and to the particle theory group at the Excellence Cluster “Origin and Structure of the Universe” for kind hospitality during which parts of this work have been completed. GH gratefully acknowledges the kind hospitality of the theory group at DESY Hamburg, where parts of this work have been done.

Appendix A: Angular distribution

The differential decay rate of $B \to K^* (\to K\pi) \ell^+ \ell^-$ can, after summing over the lepton spins, assuming an on-shell $K^*$ of narrow width, and integrating over the $K\pi$-invariant mass, be written as

$$8\pi d^3\Gamma \over 3 \int dq^2 d\cos \theta_\ell d\cos \theta_K d\phi =$$

$$ (J_{1s} + J_{2s} \cos 2\phi + J_{6s} \cos \theta_\ell \sin^2 \theta_K)$$

$$+ (J_{1c} + J_{2c} \cos 2\phi + J_{6c} \cos \theta_\ell \cos^2 \theta_K) \cos \theta_\ell$$

$$+ (J_3 \cos \phi + J_9 \sin \phi \sin 2\phi) \sin^2 \theta_K \sin^2 \theta_\ell$$

$$+ (J_4 \cos \phi + J_7 \sin \phi \sin 2\phi) \sin \theta_K \sin 2\theta_\ell$$

$$+ (J_5 \cos \phi + J_7 \sin \phi) \sin \theta_K \sin \theta_\ell ,$$

with twelve angular coefficients $J_i = J_i(q^2)$ times the angular dependence. The angles are defined as $i$ the angle $\theta_\ell$ between $\ell^-$ and $\bar{B}$ in the $(\ell^+\ell^-)$ center of mass (cms), $ii$ the angle $\theta_K$ between $\bar{K}^-$ and $\bar{B}$ in the $(K^-\pi^0)$ cms and $iii$ the angle $\phi$ between the two decay planes spanned by the 3-momenta of the $(K^-\pi^0)$- and $(\ell^+\ell^-)$-systems, respectively 10 13. Within the (SM+SM') operator basis holds $J_{6s} = 0$. A nonvanishing $J_{6c}$ arises only from interference between the operator sets (SM+SM') and S 5. (SM+SM') and T, and P and T 21. The explicit expressions of the $J_i$ are given in Appendix B.

We denote by

$$\langle J_i \rangle = \int_{q^2_{\text{min}}}^{q^2_{\text{max}}} dq^2 J_i(q^2)$$

$q^2$-integrated angular observables $J_i$ in bins between $q^2_{\text{min}}$ and $q^2_{\text{max}}$. For composite observables $X$ we use $\langle X \rangle = X(\langle J_i \rangle)$. We assume in the following that an S-wave background from $K\pi$ around the $K^*(892)$ mass has been removed as discussed in Section VII.B.

Starting from the $q^2$-integrated decay distribution $d^3\Gamma/dq^2 dq^2 d\cos \theta_\ell d\phi$, one obtains the integrated decay rate and the three single-angular differential distributions

$$\langle \Gamma \rangle = 2\langle J_{1s} \rangle + \langle J_{1c} \rangle - \frac{1}{3} \langle 2J_{2s} \rangle + \langle J_{2c} \rangle ,$$

$$d \langle \Gamma \rangle / d\phi = \frac{\langle \Gamma \rangle}{2\pi} + \frac{2}{3\pi} \langle J_3 \rangle \cos 2\phi + \frac{1}{3\pi} \langle J_9 \rangle \sin 2\phi ,$$

$$d \langle \Gamma \rangle / d\cos \theta_\ell = \langle J_{1s} \rangle + \frac{\langle J_{1c} \rangle}{2} + \frac{\langle J_{6s} \rangle}{2} \cos \theta_\ell$$

$$+ \left( \langle J_{2s} \rangle + \frac{\langle J_{2c} \rangle}{2} \right) \cos 2\theta_\ell ,$$

$$d \langle \Gamma \rangle / d\cos \theta_K = \frac{3}{2} \left[ \left( \langle J_{1s} \rangle - \frac{1}{3} \langle J_{2s} \rangle \right) \sin^2 \theta_K$$

$$+ \left( \langle J_{1c} \rangle - \frac{1}{3} \langle J_{2c} \rangle \right) \cos^2 \theta_K \right]$$

after integration over either all or the remaining two angles, respectively.
The lepton forward-backward asymmetry $A_{FB}$ can be written as

$$\langle A_{FB} \rangle (\Gamma) = \langle J_{6s} \rangle + \frac{\langle J_{6c} \rangle}{2}, \quad (A7)$$

see Eq. (A5). The extraction of $J_{4,5,7,8}$ has been discussed in [7]. For alternative methods to obtain the $J_i$, see for example [8] [20] [39]. The longitudinal $K^*$ polarization fraction $F_L$ can model-independently be defined as

$$\begin{align*}
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_K} &= \frac{3}{4} (F_T \sin^2 \theta_K + \frac{3}{2} \langle F_L \rangle \cos^2 \theta_K). \quad (A8)
\end{align*}$$

From comparison with Eq. (A6) one can read off

$$\begin{align*}
\langle F_L \rangle &= \frac{1}{\Gamma} \left( \langle J_{1c} \rangle - \frac{1}{3} \langle J_{2c} \rangle \right), \quad (A9) \\
\langle F_T \rangle &= \frac{2}{\Gamma} \left( \langle J_{1s} \rangle - \frac{1}{3} \langle J_{2s} \rangle \right), \quad (A10)
\end{align*}$$

where $F_T + F_L = 1$.

In the experimental analyses by the collaborations Belle [31], BaBar [32], CDF [33] and LHCb [41] the distribution

$$\begin{align*}
\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_t} &= \frac{3}{4} \langle F_L \rangle (1 - \cos^2 \theta_t) + \frac{3}{8} \langle F_T \rangle (1 + \cos^2 \theta_t) + \langle A_{FB} \rangle \cos \theta_t \quad (A11)
\end{align*}$$

is at least partially employed. We stress that the latter is based on [cf. Eqs. (B1) - (B4)]

$$J_{1s} = 3J_{2s}, \quad J_{1c} = -J_{2c}, \quad (A12)$$

which is broken by $m_\ell \neq 0$ and/or in the presence of S, P, T or T5 contributions. Therefore, results for $F_L$ based on Eq. (A11) do not hold in full generality.

Note that in cases where Eq. (A12) holds, such as the SM with lepton masses neglected, $F_L = (|A_0^L|^2 + |A_0^R|^2) / \Gamma = -J_{2c} / \Gamma$. Furthermore, $\langle J_{2s} \rangle = 3/16 \langle \Gamma \rangle (1 - \langle F_L \rangle)$ and $\langle J_{2c} \rangle = -3/4 \langle \Gamma \rangle \langle F_L \rangle$.

---

**Appendix B: Angular observables**

The $J_i(q^2)$ of Eq. (A11) can be conveniently expressed within the (SM+SM') operator basis with the help of seven transversity amplitudes, $A_{0,1,\perp,\parallel}^{L,R}$ and $A_t$ [6]. The operators $S$ require an additional amplitude $A_S$, whereas the set $P$ can be absorbed into the amplitude $A_t$ [8]. In the presence of tensor operators $T$ and T5, six additional transversity amplitudes are needed to be introduced, with $ij = \{|\parallel, \perp, t, l, t, \parallel, 0, \perp, 0, \parallel\}$, see Appendix C. In the complete basis (SM+SM') + (S+P) + (T+T5) we obtain

\[
\begin{align*}
4/3 J_{1s} &= \frac{(2 + \beta_t^2)}{4} \left[ |A_{\perp}^L|^2 + |A_{\perp}^R|^2 + (L \rightarrow R) \right] + \frac{4m_t^2}{q^2} \text{Re} \left( A_{\perp}^L A_{\perp}^{R*} + A_{\parallel}^L A_{\parallel}^{R*} \right) \\
&+ 4 \beta_t^2 (|A_{0\perp}|^2 + |A_{0\parallel}|^2) + 4 (4 - 3\beta_t^2) (|A_{\perp}^L|^2 + |A_{\perp}^R|^2) \\
&+ 8 \sqrt{2} \frac{m_t}{q^2} \text{Re} \left[ (A_{\perp}^L + A_{\parallel}^R) A_{\parallel}^{\ast} + (A_{\perp}^L + A_{\parallel}^R) A_{\perp}^{\ast} \right], \\
4/3 J_{1c} &= |A_{0}^L|^2 + |A_{0}^R|^2 + \frac{4m_t^2}{q^2} |A_t|^2 + 2 \text{Re} (A_{0}^L A_{0}^{R*}) \bigg] + \beta_t^2 |A_S|^2 \\
&+ 8 (2 - \beta_t^2) |A_{10}|^2 + 8 \beta_t^2 |A_{\parallel}|^2 + 16 \frac{m_t}{q^2} \text{Re} \left[ (A_{0}^L + A_{0}^R) A_{10}^{\ast} \right], \\
4/3 J_{2s} &= \frac{\beta_t^2}{4} \left[ |A_{\perp}^L|^2 + |A_{\perp}^R|^2 + (L \rightarrow R) \right] - 16 (|A_{\perp}^L|^2 + |A_{\perp}^R|^2 + |A_{0\perp}|^2 + |A_{0\parallel}|^2) \bigg], \\
4/3 J_{2c} &= -\beta_t^2 \left[ |A_{0}^L|^2 + |A_{0}^R|^2 - 8 (|A_{0\perp}|^2 + |A_{\perp}^L|^2) \right], \\
4/3 J_{3} &= \frac{\beta_t^2}{2} \left[ |A_{\perp}^L|^2 - |A_{\perp}^R|^2 + (L \rightarrow R) \right] + 16 (|A_{\perp}^L|^2 - |A_{\perp}^R|^2 - |A_{0\perp}|^2 - |A_{0\parallel}|^2) \bigg], \\
4/3 J_{4} &= \frac{\beta_t^2}{\sqrt{2}} \text{Re} \left[ A_{\perp}^L A_{\perp}^{R*} + (L \rightarrow R) - 8 \sqrt{2} (A_{10} A_{\parallel}^{\ast} + A_{\perp} A_{\parallel}^{\ast}) \right],
\end{align*}
\]
\[
\frac{4}{3} J_5 = \sqrt{2} \beta_t \operatorname{Re} \left[ A_0^L A_0^{L*} - (L \rightarrow R) - 2 \sqrt{2} A_{0||} A_{0\parallel}^* - \frac{m_\ell}{\sqrt{q^2}} \left( |A_0^L + A_{0\parallel}^R| A_0^* - |A_0^{L*} + A_{0\parallel}^R| A_0^\parallel \right) \right] \]
\[
+ 4 \sqrt{2} A_{0||} A_0^\parallel - 4 \sqrt{2} [A_0^L - A_{0\parallel}^R] A_0^{\parallel*} - 4 [A_0^{L*} - A_{0\parallel}^R] A_0^\parallel \right), \] (B7)
\[
\frac{4}{3} J_{6s} = 2 \beta_t \operatorname{Re} \left[ A_0^L A_0^{L*} - (L \rightarrow R) + 4 \sqrt{2} \frac{m_\ell}{\sqrt{q^2}} \left( |A_0^L - A_{0\parallel}^R| A_0^* + |A_0^{L*} - A_{0\parallel}^R| A_0^\parallel \right) \right], \] (B8)
\[
\frac{4}{3} J_{6c} = 4 \beta_t \operatorname{Re} \left[ 2 A_{0||} A_0^\parallel + \frac{m_\ell}{\sqrt{q^2}} \left( (A_0^L + A_{0\parallel}^R) A_0^* + 4 A_{0||} A_0^\parallel \right) \right], \] (B9)
\[
\frac{4}{3} J_7 = \sqrt{2} \beta_t \operatorname{Im} \left[ A_0^L A_0^{L*} - (L \rightarrow R) + 2 \sqrt{2} A_{0||} A_0^* + \frac{m_\ell}{\sqrt{q^2}} \left( |A_0^L + A_{0\parallel}^R| A_0^* + |A_0^{L*} - A_{0\parallel}^R| A_0^\parallel \right) \right]
+ 4 \sqrt{2} A_{0||} A_0^\parallel + 4 \sqrt{2} [A_0^L - A_{0\parallel}^R] A_0^{\parallel*} - 4 [A_0^{L*} - A_{0\parallel}^R] A_0^\parallel \right), \] (B10)
\[
\frac{4}{3} J_8 = \frac{\beta_t^2}{\sqrt{2}} \operatorname{Im} \left[ A_0^L A_0^{L*} - (L \rightarrow R) \right], \] (B11)
\[
\frac{4}{3} J_9 = \beta_t^2 \operatorname{Im} \left[ A_0^L A_0^{L*} + (L \rightarrow R) \right], \] (B12)

where the lepton mass \(m_\ell\) has been kept and \(\beta_t = \sqrt{1 - 4 m_\ell^2/q^2}\).

Here the transversity amplitudes contain the contributions from the operators in Eqs. (6, 7) which are factorizable. Non-factorizable contributions from \(O_{\ell \leq 6,8}\) are taken into account by using effective Wilson coefficients \(C_{\ell6}^{\text{eff}}\). Within naive factorization the transversity amplitudes read

\[
A_\perp^{L,R} = \sqrt{2} N \sqrt{\lambda} \left\{ (C_9 + C_{9'}) \mp (C_{10} + C_{10'}) \right\} \frac{V}{M_B + M_{K^*}} + \frac{2 m_b}{q^2} (C_7 + C_{7'}) T_1, \] (B13)
\[
A_\parallel^{L,R} = - N \sqrt{2}(M_B^2 - M_{K^*}^2) \times \left\{ \frac{(C_9 - C_{9'}) \mp (C_{10} - C_{10'})}{M_B - M_{K^*}} \right\} \frac{A_1}{M_B + M_{K^*}} + \frac{2 m_b}{q^2} (C_7 - C_{7'}) T_2, \] (B14)
\[
A_0^{L,R} = - \frac{N}{2 M_{K^*} \sqrt{q^2}} \times \frac{1}{q^2} \left\{ \left( (C_9 - C_{9'}) \mp (C_{10} - C_{10'}) \right) \left[ (M_B^2 - M_{K^*}^2) - q^2 \right] (M_B + M_{K^*}) A_1 - \frac{\lambda}{M_B + M_{K^*}} A_2 \right\}
+ 2 m_b (C_7 - C_{7'}) \left[ (M_B^2 + 3 M_{K^*}^2 - q^2) T_2 - \frac{\lambda}{M_B^2 - M_{K^*}^2} T_3 \right]; \] (B15)
\[
A_t = N \frac{\sqrt{\lambda}}{\sqrt{q^2}} \left\{ 2 (C_{10} - C_{10'}) + \frac{q^2}{m_\ell} (C_P - C_{P'}) \right\} A_0, \] (B16)
\[
A_S = - 2 N \sqrt{\lambda} \frac{(C_S - C_{S'})}{(m_b + m_s)} A_0, \] (B17)
\[
A_{\perp\,(10)} = \frac{C_T^{(5)}}{M_{K^*}} \left[ (M_B^2 + 3 M_{K^*}^2 - q^2) T_2 \right]; \] (B18)
\[
A_{\perp\,(0,1)} = \frac{C_T^{(5)}}{\sqrt{q^2}} A_0, \] (B19)
\[ A_{0\parallel} (\ell\ell) = 2N \frac{(M_B^2 - M_{\bar{K}^*}^2)}{\sqrt{q^2}} C_{T(5)} T_2. \] (B20)

The normalization factor \( N \) is given as
\[ N = G_F \alpha_e V_{tb} V_{ts}^* \sqrt{\frac{\frac{q^2}{2} \beta_t \sqrt{\lambda}}{3 \cdot 2^{10} \pi^5 M_B^2}} \] (B21)
and the \( B \to K^* \) form factors \( V \), \( A_{0,1,2,3} \), \( T_{1,2,3} \) are defined as in Refs. 11, 6, 8, 14, 21, 42.

The (SM+SM') calculation of the 4-fold differential decay rate by Krüger and Matias 6 already includes the chirality-flipped operators of the SM' basis for \( m_t \neq 0 \). We reproduce their results. The complete set of operators was considered in the limit \( m_t = 0 \) by Kim and Yoshikawa 20. The extension to \( m_t \neq 0 \) for (S+P) operators has been performed by Altmanshofer et al. 8 within the transversity amplitude formalism. We agree with the arXiv v5 of this work 2.

The extension to \( m_t \neq 0 \) to include the tensor operators (T+15) has been performed by Alok et al. 21, 45. We agree with the arXiv v4 of reference (21) 3 for all expressions except for the sign of the \( A_{1,2} \) interference term in Eq. (B10).

Appendix C: \( B \to \bar{K}^* (\to \bar{K}\pi) \ell^+\ell^- \) Matrix Element

We present here the parametrization of the hadronic matrix element used to calculate the decay \( B \to \bar{K}^* (\to \bar{K}\pi) \ell^+\ell^- \),
\[ \mathcal{M} = \mathcal{F} \left( X_S \left[ \ell\ell \right] + X_P \left[ \ell\gamma_5\ell \right] + X_{\mu} \left[ \ell\gamma_{\mu}\ell \right] + X_A \left[ \ell\gamma_{\mu}\gamma_5\ell \right] + X_{\mu\nu} \left[ \ell\sigma_{\mu\nu}\ell \right] \right). \] (C1)

We define
\[ \mathcal{F} = \frac{G_F \alpha_e}{\sqrt{2}\pi} V_{tb} V_{ts}^* g_{K*K\pi} D V \left[ \bar{p}_K \right], \] (C2)
and use \( \bar{p}_K \), the three momentum of the \( \bar{K} \) in the \( \bar{K}\pi \) cms,
\[ |\bar{p}_K| = \frac{\sqrt{\lambda(M_{\bar{K}^*}, M_{\bar{K}}, M_\pi^2)}}{2 M_{\bar{K}}}. \] (C3)
and the kinematical function \( \lambda \) defined as usual
\[ \lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc). \] (C4)

### TABLE III: The numerical input used in our analysis. We neglect the mass of the strange quark. \( \tau_{B^0} (\tau_{B^+}) \) denotes the lifetime of the neutral (charged) \( B \) meson. Here, \( \lambda \) denotes the CKM parameter in the Wolfenstein parametrization.

| Observable | \[ B^0 \to \bar{K}^0 \ell^+\ell^- \] | \[ B^- \to K^- \ell^+\ell^- \] |
|---|---|---|
| \[ 10^4 \times \langle BR \rangle_{4\pi} \] | 6.44 $\pm$ 0.70 | 6.92 $\pm$ 0.22 |
| \[ 10^4 \times \langle BR \rangle_{2.0} \] | 7.50 $\pm$ 1.25 | 8.08 $\pm$ 1.35 |
| \[ 10^4 \times \langle BR \rangle_{4.3} \] | 1.38 $\pm$ 0.25 | 1.48 $\pm$ 0.27 |
| \[ 10^4 \times \langle BR \rangle_{10.0} \] | 1.63 $\pm$ 0.27 | 1.75 $\pm$ 0.29 |
| \[ 10^4 \times \langle BR \rangle_{14.18} \] | 3.40 $\pm$ 0.83 | 3.65 $\pm$ 0.99 |
| \[ 10^4 \times \langle BR \rangle_{16.0} \] | 3.09 $\pm$ 0.83 | 3.11 $\pm$ 0.89 |
| \[ 10^4 \times \langle BR \rangle_{18.0} \] | 3.18 $\pm$ 0.92 | 3.41 $\pm$ 0.98 |
| \[ 10^4 \times \langle BR \rangle_{16.0} \] | 6.34 $\pm$ 1.75 | 6.80 $\pm$ 1.88 |

### TABLE IV: The SM predictions for \( B^0 \to \bar{K}^0 \ell^+\ell^- \) and \( B^- \to K^- \ell^+\ell^- \) decays in \( q^2 \) bins. For the large recoil region \( q^2 \lesssim 8.68 \text{ GeV}^2 \), we use the QCDF results [14, 19], and include all known power-suppressed contributions [14]. For the low recoil region \( q^2 \gtrsim 14.18 \text{ GeV}^2 \) we use the OPE framework [28]. In both cases we employ \( B \to K \) form factors, or extrapolations thereof, from Ref. [19].

Using this parametrization, we obtain the hadronic tensors
\[ X_S = -i \frac{4\pi}{2} \frac{\cos \theta_K}{\cos \theta_K} A_S \],
\[ X_P = -i \frac{2\pi}{4\pi} \frac{\cos \theta_K}{\cos \theta_K} \frac{m_t}{\sqrt{q^2}} A_t \],
\[ X_{V,A} = i \frac{4\pi}{2} \frac{\cos \theta_K}{\cos \theta_K} \cos \theta_K (A_0^V \pm A_L^V) \]
\[ + \frac{i}{8\pi} \sin \theta_K \]
where the polarization vectors \(\varepsilon^\mu(\pm) = \frac{1}{\sqrt{2}}(0, 1, \mp i, 0)\),
\[\varepsilon^\mu(0) = \frac{1}{\sqrt{q^2}}(-q_2, 0, 0, -q_0)\) (C9)
\[\varepsilon^\mu(t) = \frac{1}{\sqrt{q^2}}(q_0, 0, 0, q_z)\) .

We choose the \(z\)-axis in this frame along the \(K^*\) direction of flight and \(q_0, q_z\) denotes the timelike (spacelike) component of the four momentum \(q^\mu\). The polarization vectors fulfill the completeness relations
\[g_{n n'} = \varepsilon^\dagger_\mu(n) \varepsilon^\mu(n')\],
\[g_{\mu \nu} = \sum_{n, n'} \varepsilon^\dagger_\mu(n) \varepsilon^\nu(n) g_{n n'}\] (C10)

with \(g_{n n'} = \text{diag}(+\ldots,-\ldots,-)\) for \(n, n' = t, \pm, 0\). We use the relation Eq. (C10) to insert the full set of polarization vectors \(\varepsilon^\mu(n)\) between the hadronic and leptonic currents, and introduce the helicity amplitudes \(H_{a n_1...n_i}\) for arbitrary Dirac structures \(\Gamma^{\mu_1...\mu_i}\),
\[\langle K^*(k, \eta(a))|\bar{\psi}_{\mu_1...\mu_i}|\bar{B}(p)\rangle\] (C11)
\[= \sum_{n_i, n_i'} \langle K^*(k, \eta(a))|\bar{\psi}_{\mu_1...\mu_i}|\bar{B}(p)\rangle \prod_{i=1}^{t} \varepsilon^{\dagger}_{\mu_i}(n_i) g_{n_i n_i'} \varepsilon_{\mu_i}(n_i')\] (C12)

The tensorial transversity amplitudes \(A_{ij}\) are related to the helicity amplitudes \(H_{a n_1...n_i}\) by means of
\[A_{i0} = \frac{1}{\sqrt{2}}(H_{i+0} + H_{i-0})\]
\[A_{0i} = \frac{1}{\sqrt{2}}(H_{0+i} + H_{0-i})\]
\[A_{ij} = H_{0+i} - H_{0-i}\]
\[A_{i0} = H_{0i}^\Gamma\] (C13)
and
\[A_{ij} = \sum_{\Gamma=\ell, T5} A_{i j}^\Gamma\] (C14)

The polarization vectors of the \(K^*\) for polarizations \(a = \pm, 0\) in the \(\bar{B}\) cms read
\[\eta^\mu(\pm) = \frac{1}{\sqrt{2}}(0, 1, \pm i, 0)\),
\[\eta^\mu(0) = \frac{1}{\sqrt{M_{K^*}}}(0, 0, 0, M_{K^*} - q_0)\) .

This approach generalizes the concept of the transversity amplitudes, cf. e.g. Refs. [6, 8, 44], to which we also refer for the definition of the remaining transversity amplitudes \(A_{i}, i = 0, \perp, \parallel, \ell, T, S\).
We employ $\gamma_5 = i/(4!) \varepsilon_{\alpha\beta\mu\nu} \gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu$, such that

$$\text{Tr}[\gamma^\alpha \gamma^\beta \gamma^\mu \gamma^\nu \gamma_5] = 4 \varepsilon_{\alpha\beta\mu\nu},$$

$$\sigma^{\alpha\beta} \gamma_5 = i \partial_{\alpha\beta} \gamma^{\nu} \sigma_{\mu\nu}$$

with $\sigma_{\mu\nu} = i/2 [\gamma_\mu, \gamma_\nu]$, and $\varepsilon_{0123} = -e^{0123} = 1$.

**Appendix D: $B \rightarrow K^{(*)}\ell^+\ell^-$ SM predictions**

We update our SM predictions for $B \rightarrow K^{(*)}\ell^+\ell^-$ and $B \rightarrow K^*\ell^+\ell^-$ decays [28]. This includes the following improvements to the EOS [45] source code: Firstly, a common set of numerical input parameters is used, given in Table III. The resulting changes with respect to previous works are, however, marginal. Secondly, we improved the implementation of the subleading corrections within QCDF to the amplitudes in the region of large hadronic recoil. This concerns, in particular, the analytic expressions for the convolution integrals that involve the kaon light cone distribution amplitudes. The analytic results turn out to be more numerically stable than previous ones. In this process, we further switched from MS to the charm pole mass. Lastly, we implemented all numerically relevant power-suppressed hard scattering and weak annihilation contributions [15]. Subleading $V_{ud}V_{us}^\ast$ contributions are included in the numerical analysis; Non-factorizable effects at low $q^2$ estimated in [49] are not included. The results are presented in Table IV and V. We recall that the $q^2$-region of validity within QCDF is approximately within $(1 - 7) \text{GeV}^2$. Numerical predictions in the tables are extrapolations thereof and provided to match the experimental binning. Note that $\langle F_L \rangle + \langle F_T \rangle = 1$ [cf. Eq. (A5)], however, for convenience we give predictions for both observables. Since we calculated $\langle F_L, T \rangle$ individually the sum rule served as an independent check of the EOS code.

Besides the improvements mentioned above all details entering our low and large recoil predictions are given in [15, 28]. Detailed SM predictions for $H_T^{(1,2,3)}$ are given in [15, 18].

[1] B. Grinstein and D. Pirjol, Phys. Rev. D 70, 114005 (2004) [arXiv:hep-ph/0404250].
[2] M. Beylich, G. Buchalla and T. Feldmann, Eur. Phys. J. C 71, 1635 (2011) [arXiv:1101.5118 [hep-ph]].
[3] B. Grinstein and D. Pirjol, Phys. Lett. B 533, 8 (2002) [arXiv:hep-ph/0202198].
[4] C. Bobeth, G. Hiller and D. van Dyk, JHEP 1007, 098 (2010) [arXiv:1006.5013 [hep-ph]].
[5] C. Bobeth, G. Hiller and D. van Dyk, JHEP 1107, 067 (2011) [arXiv:1105.0376 [hep-ph]].
[6] F. Kruger and J. Matias, Phys. Rev. D 71, 094009 (2005) [arXiv:hep-ph/0502060].
[7] C. Bobeth, G. Hiller and G. Piranishvili, JHEP 0807, 106 (2008) [arXiv:0805.2525 [hep-ph]].
[8] W. Altmannshofer et al., JHEP 0901, 019 (2009) [arXiv:0811.1214 [hep-ph]].
[9] U. Egede, T. Hurth, J. Matias, M. Ramon and W. Reece, JHEP 0811, 032 (2008) [arXiv:0807.2589 [hep-ph]].
[10] E. Lunghi and A. Soni, JHEP 1011, 121 (2010) [arXiv:1007.4015 [hep-ph]].
[11] D. Becirevic and E. Schneider, Nucl. Phys. B 854, 321 (2012) [arXiv:1106.3283 [hep-ph]].
[12] J. Matias, F. Mescia, M. Ramon and J. Virto, JHEP 1204, 104 (2012) [arXiv:1202.4266 [hep-ph]].
[13] D. Das and R. Sinha, Phys. Rev. D 86, 056006 (2012) [arXiv:1205.1438 [hep-ph]].
[14] M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B 612, 25 (2001) [arXiv:hep-ph/0106067].
[15] M. Beneke, T. Feldmann and D. Seidel, Eur. Phys. J. C 41, 173 (2005) [arXiv:hep-ph/0412400].
[16] F. Kruger, L. M. Sehgal, N. Sinha and R. Sinha, Phys. Rev. D 61, 114028 (2000) [Erratum-ibid. D 63, 019901 (2001)] [arXiv:hep-ph/9907386].
[17] C. Hambrock and G. Hiller, Phys. Rev. Lett. 109, 091802 (2012) [arXiv:1204.4444 [hep-ph]].
[18] F. Beaujean, C. Bobeth, D. van Dyk and C. Wacker, JHEP 1208, 030 (2012) [arXiv:1205.1838 [hep-ph]].
[19] C. Bobeth, G. Hiller and G. Piranishvili, JHEP 0712, 040 (2007) [arXiv:0709.4174 [hep-ph]].
[20] C. S. Kim and T. Yoshikawa, arXiv:0711.3880 [hep-ph].
[21] A. K. Alok, A. Datta, A. Dighe, M. Duraisamy, D. Ghosh and D. London, JHEP 1111, 121 (2011) [arXiv:1008.2367 [hep-ph]].
[22] K. G. Chetyrkin, M. Misiak and M. Munz, Phys. Lett. B 400, 206 (1997) [Erratum-ibid. B 425, 414 (1998)] [hep-ph/9612133].
[23] A. Ali, G. Kramer and G. h. Zhu, Eur. Phys. J. C 47, 625 (2006) [arXiv:hep-ph/0601034].
[24] K. S. M. Lee, Z. Ligeti, I. W. Stewart and F. J. Tackmann, Phys. Rev. D 75, 034016 (2007) [arXiv:hep-ph/0612156].
[25] C. Bobeth, P. Gambino, M. Gorbahn and U. Haisch, JHEP 0404, 071 (2004) [hep-ph/0312090].
[26] T. Huber, E. Lunghi, M. Misiak and D. Wyler, Nucl. Phys. B 740, 105 (2006) [hep-ph/0512069].
[27] A. Bharucha, T. Feldmann and M. Wick, JHEP 1009, 090 (2010) [arXiv:1004.3249 [hep-ph]].
[28] C. Bobeth, G. Hiller, D. van Dyk and C. Wacker, JHEP 1201, 107 (2012) [arXiv:1111.2558 [hep-ph]].
[29] T. Aaltonen et al. [CDF Collaboration], Phys. Rev. Lett. 108, 081807 (2012) [arXiv:1108.6895 [hep-ex]].
[30] R. Anij et al. [LHCb Collaboration], arXiv:1209.4284 [hep-ex].
[31] J.-T. Wei et al. [BELLE Collaboration], Phys. Rev. Lett. 103, 171801 (2009) [arXiv:0904.0770 [hep-ex]].
[32] J. P. Lees et al. [BABAR Collaboration], Phys. Rev. D 86, 032012 (2012) [arXiv:1204.3933 [hep-ex]].
[33] H. Miyaki, “CDF results on the search for rare $B_{d,s} \rightarrow \mu^+\mu^-$ and $X_{d,s}\mu^+\mu^-$ decays”, Talk presented at “ICHEP 2012”, July 4-11, 2012, Melbourne, Australia. http://www-cdf.fnal.gov/physics/new/bottom/120628.blessed-b2smumu_96/
[34] ATLAS Collaboration, CMS Collaboration, LHCb Collaboration, ATLAS-CONF-2012-061, CMS-PAS-BPH-12-009, LHCb-CONF-2012-017, http://cdsweb.cern.ch/record/1452186/files/LHCb-CONF-2012-017.pdf

[35] K. De Bruyn et al. Phys. Rev. Lett. 109, 041801 (2012) arXiv:1204.1737 [hep-ph].

[36] F. Borzumati, C. Greub, T. Hurth and D. Wyler, Phys. Rev. D 62, 075005 (2000) hep-ph/9911245.

[37] C. Bobeth and U. Haisch, arXiv:1109.1826 [hep-ph].

[38] D. Becirevic and A. Tayduganov, arXiv:1207.4004 [hep-ph].

[39] J. Matias, Phys. Rev. D 86, 094024 (2012) arXiv:1209.1525 [hep-ph].

[40] T. Blake, U. Egede and A. Shires, arXiv:1210.5279 [hep-ph].

[41] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 108, 181806 (2012) arXiv:1112.3515 [hep-ex]].

[42] P. Ball and R. Zwicky, Phys. Rev. D 71, 014029 (2005) hep-ph/0412079.

[43] A. K. Alok, A. Dighe, D. Ghosh, D. London, J. Matias, M. Nagashima and A. Szynkman, JHEP 1002, 053 (2010) arXiv:0912.1382 [hep-ph].

[44] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner and V. E. Lyubovitskij, Eur. Phys. J. direct C 4, 18 (2002) arXiv:hep-ph/0205287.

[45] D. van Dyk et al. [EOS Collaboration], EOS: A HEP Program for Flavor Observables, http://project.het.physik.tu-dortmund.de/eos/

[46] J. Charles et al. [CKMfitter Group Collaboration], Eur. Phys. J. C41, 1-131 (2005). hep-ph/0406184. We use the numerical results as presented at ICHEP10.

[47] K. Nakamura et al. [Particle Data Group], J. Phys. G 37, 075021 (2010).

[48] [Tevatron Electroweak Working Group and CDF Collaboration and D0 Collab], arXiv:0903.2503 [hep-ex].

[49] A. Khodjamirian, T. Mannel, A. A. Pivovarov and Y. M. Wang, JHEP 1009, 089 (2010) arXiv:1006.4945 [hep-ph].