A Mixed Optimization Method Based on Adaptive Kalman Filter and Wavelet Neural Network for INS/GPS During GPS Outages

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ABSTRACT To improve the navigation performance of the navigation system combining inertial navigation system (INS) and global positioning system (GPS) under complicated environments, especially GPS outages, a navigation method - wavelet neural network based on random forest regression (RFR-WNN) to assist adaptive Kalman filter (AKF) - is proposed. AKF is employed to correct INS errors, the Kalman filter is improved by introducing adaptive factor, to suppress the influence of the complex environment and random errors on the filtering accuracy; RFR-WNN is used to construct a high-precision prediction model when GPS works well, and to provide the required observations for AKF update when GPS outages. To solve the problem that the single neural network structure is easy to cause the overfitting, unstable and low prediction accuracy due to the lack of comprehensive training samples, RFR is introduced to optimize the single WNN, which can improve the generalization ability and prediction accuracy. In order to verify the effectiveness and advancement of the proposed method, vehicle navigation experiments were carried out, the results indicate that the proposed method has better navigation accuracy and performance than compared methods during GPS outages, and this advantage is more obvious in the case that fewer samples are collected.

INDEX TERMS Navigation technology, integrated navigation, wavelet neural network, adaptive Kalman filter, GPS outages, random forest regression.

I. INTRODUCTION

Navigation technology is widely used in military and civil engineering fields, how to provide a reliable and high-precision navigation method is the key to realize navigation applications [1], [2]. On the one hand, the development of applications in related fields have an increasingly demand on navigation technology.

With the development of navigation technology, inertial navigation system (INS) based on micro-electromechanical system (MEMS) can provide a wealth of motion information including velocity, position and attitude, and global positioning system (GPS) as a convenient velocity and position sensor is widely applied different kinds of fields. INS/GPS integrated navigation technology fully integrates the advantages of INS and GPS, with the advantages of small size, low cost, high navigation accuracy, high reliability and continuous all-weather work. Obviously, an INS/GPS integrated navigation system, fully integrates the advantages of INS and GPS, is much better than the single GPS or INS. INS/GPS integrated navigation system can provide low-cost and high-precision navigation solutions for small unmanned aerial vehicles, low-cost guided weapons and vehicles, so as to be widely used in both military and civilian areas [2]–[6]. However, in the actual application process, due to the inherent defects of GPS, such as the multipath effect and poor anti-interference ability, and the system will inevitably be interfered by the external environment (such as high buildings, mountains, trees, electromagnetic interference and various tunnels) during the operation of the carrier, resulting in GPS signal attenuation or even lock-lose, INS errors cannot be corrected. During GPS outages, GPS cannot provide...
observation information, the integrated system can only operate in pure INS mode, which will seriously affect the navigation accuracy, resulting in decline rapidly and even diverge without being suppressed [5]. With the continuous improvement of navigation accuracy requirements in various fields, the problem of GPS signal failure has been paid more and more attention. Therefore, it is of great significance how to effectively suppress INS errors and maintain the accuracy of navigation system when GPS is rejected.

In the past many years, many approaches have been proposed to improve the navigation performance of INS/GPS integrated system in case of GPS outages. The correlation filtering algorithms based on the classical Kalman filter (KF) is widely implemented in INS/GPS integrated system [7], [8]. However, the use of KF or extended Kalman filter (EKF) may cause divergence due to modeling errors, which can not be well applied to nonlinear system problems, especially for those inertial devices with low-cost and poor performance [9], [10]. Some researchers use unscented Kalman filter (UKF) to replace the above algorithms in order to better solve the problems of system uncertainty and nonlinearity. However, in the process of solving the UKF, it is necessary to obtain prior statistical information of the measurement and process noise, the complexity and operation time increase rapidly with the increase of dimension of state matrix [11], [12]. The noise of GPS and INS is usually considered as a synonym for white noise, while the actual ambient noise is ignored. In fact, the noise components of GPS are complex, including not only white noise, but also other random environmental noise caused by surrounding buildings and electromagnetic interference, and the model and noise of INS are also uncertain. In addition, the GPS / INS navigation system is unstable in the practical application and easily affected by external interference and equipment damage. Which bring some difficulties to information fusion and filtering estimation [13]–[15]. In order to solve these problems, adaptive control theory is widely used [12], [15], [16], and adaptive Kalman filter (AKF) is proposed.

In recent years, the rapid development of artificial intelligence (AI) represented by neural network (NN) provides inspiration and new ideas for solving navigation problems [3], [4]. With the development of AI technology, various theoretical models and algorithms based on NN are introduced into integrated navigation and information fusion to improve the continuity of the integrated navigation system and enhance the navigation accuracy in GPS-denied environments [17], [18]. When the GPS signal is available, explore the inherent relationship between INS outputs and optimal estimation errors, and then use the well-trained NN to predict the parameters needed during GPS outages, thus making the integrated system work better. The above is the core idea of the NN-assisted optimization method [19]. Xu et al. [20] proposed a land-based vehicle positioning scheme in which the grey system model is mapped to the back-propagation NN and the grey NN is used to assist the UKF, which can bridge GPS outages simultaneously; Zhang [21] combined empirical modal decomposition threshold filtering and long short-term memory NN to provide GPS pseudo-range, so as to bridge GPS outages; Yao et al. [22] combined KF and improved multi-layer perceptron network, the pseudo-distance of GPS is predicted and estimated when GPS is defective; Chen and Fang [23] used radial basis function NN combined with time series analysis predict the KF measurement update for bridging GPS outages; Abdolkarimi et al. [24] optimized a new NN based on extreme learning machine for predicting and correcting INS errors when GPS signal is interrupted; Aggarwal et al. [25] combined D-S evidence theory with NN to reduce the complexity of neural network learning, thus improving the efficiency of learning and the effectiveness of prediction; Sharaf and Nourelldin [26] proposed a real-time data fusion technology based on radial basis function NN for GPS and inertial sensors integration, which enhanced the real-time prediction; Jwo et al. [27] introduced NN into the vector tracking loop to improve the GPS positioning performance when GPS signal is blocked. To some extent, the above methods have attained good results, but there are still some problems to be overcome. The above NN-based optimization methods often use only single neural network architecture for prediction, comprehensive and a large number of training samples is necessary when training NN. However, in the actual operation of the system under complex environment, the samples collected for NN training when GPS is available are few and not comprehensive. In this case, only a single-structure NN is used for training, which is prone to problems such as network training overfitting, local optimization, prediction models instability, resulting in unsatisfactory prediction results [17], [28]. Therefore, NN achieves better prediction results with limited training samples may be a challenging problem.

In view of the above-mentioned problem that the NN training samples are few and not comprehensive, a novel model combined with AKF and wavelet neural network (WNN) based on random forest regression (RFR) algorithms for INS errors compensation is proposed in this paper. By introducing the RFR based on the ensemble learning method to improve the single WNN, and to strengthen the WNN, so as to improve the dependability and prediction precision of the WNN, this is termed the RFR-WNN. In addition, for the information fusion in the INS / GPS integrated system, because of the complexity of actual noises, the small errors caused by the inaccurate filter design will accumulate to the next filtering step, which will affect the accuracy of error estimation, an improved AKF is introduced to enhance the robustness of information fusion. AKF is applied to estimate the optimal state of the system throughout the navigation process. RFR-WNN is trained online when GPS is available while RFR-WNN provides observation input for the AKF update process during GPS outages.

II. OVERVIEW OF THE PROPOSED SOLUTION

During the actual operation of GPS/INS integrated navigation system, due to the influence of surrounding environment,
GPS noise, and the uncertainty of low-cost INS model and noise, the system noise is complicated, which brings some definite difficulties to information fusion and estimation. In addition, the use of these small, incomplete, and poorly filtered samples to train NN when GPS is available will cause the NN used to predict observations produce overfitting, poor structural reliability and easy to fall into local optimal value during GPS outages, which seriously affects navigation accuracy. In this article, we studied the relationship between the parameters of INS and the observations of integrated navigation system. To provide good navigation information and enhance the stability of the system, we propose a method that is RFR-based WNN assisted AKF. Using the ensemble learning idea of RFR, we can effectively deal with the problems of few and incomplete samples, improve the structure of WNN, enhance the robustness of NN, and improve the modeling accuracy in training process. In addition, the introduction of an AKF can better filter the actual integrated navigation system and effectively suppress the influence of complex noise on the filtering effect.

When the output information of GPS receiver is available, the system operates in NN training mode. The training process under the condition of GPS working well as shown in Fig. 1. Observations input of AKF update process can be set as follows:

$$Z = \begin{bmatrix} \delta P \\ \delta V \end{bmatrix} = \begin{bmatrix} Pos_{INS} - Pos_{GPS} \\ Vel_{INS} - Vel_{GPS} \end{bmatrix}$$ (1)

That is the difference between the observations of the position and speed of INS and observations of the position and speed of GPS. We choose the measured values ($f_n$, $\omega_n$) of accelerometer and gyroscope as the input, and the observation values ($\delta P$, $\delta V$) of system output as the training target of RFR-WNN. Then the training samples of the training process can be described as:

$$RFR-WNN + AKF \rightarrow input : \left\{ f_n, \omega_n \right\},$$

$$output target : \begin{bmatrix} \delta P = Pos_{INS} - Pos_{GPS} \\ \delta V = Vel_{INS} - Vel_{GPS} \end{bmatrix}$$ (2)

where $f_n$ and $\omega_n$ denote the specific force of accelerometer output and the angular rate of gyroscope output respectively.

During the GPS outages, the prediction process for the integrated navigation system as shown in Fig. 2. At this time, the system works in NN prediction mode. The well-trained RFR-WNN, in Fig. 1, can predict the observations of integrated navigation system according to the input information well. The output results of RFR-WNN, that is, the prediction results, can be input as the observations during the AKF update process, thereby improving the navigation accuracy during GPS outages, which can be described as:

$$Z = nn_{output}$$ (3)

where $nn_{output}$ is the output of RFR-WNN.

It is noted that AKF applied throughout the whole navigation process, that is, AKF will continue to perform integrated filtering whether GPS signal is available or GPS signal is defective. RFR-WNN conducts online training when GPS works well; When GPS signal is blocked, RFR-WNN predicts the difference in speed and position between the INS and the GPS based on angular velocity and specific force information from the INS, so as to provide AKF with continuous observations update.

III. ALGORITHM DESCRIPTION

A. AKF ALGORITHM

The classic KF algorithm has excellent filtering capability for linear systems. However, in various circumstances, for nonlinear systems with significant random errors, it is unrealistic to obtain the precisely known model owing to the nonlinearity and nondeterminacy of the system noise and the complexity of the actual application environment noise, which greatly reduces the filtering ability of KF. The concept of self-adaptation is introduced into KF, an adaptive KF algorithm is formed, to overcome the shortcomings of traditional KF-like methods and solve the problem of state estimation for nonlinear systems with random noise [12], [16]. The mentioned AKF algorithm is described in detail below.

INS/GPS integrated navigation system can be described by state space model, which is mainly derived from state equation and observation equation. The equation of state and
measurement can be set as [12]:

\[
\begin{align*}
X_k &= \Phi_{k/k-1}X_{k-1} + \Gamma_{k/k-1}W_{k-1} \\
Z_k &= H_kX_k + V_k
\end{align*}
\]  

(4)

where \( k \) denotes the discrete time and \( X_k \) represents the state vector of system, which can be set as \( X = [\delta P^T \delta V^T \delta \varphi^T \mathbf{e} \ \nabla]^T \), in which \( \delta P \) is position vector; \( \delta V \) is velocity vector, \( \delta \varphi \) is attitude angle vector, \( \mathbf{e} \) and \( \nabla \) are respectively stand for gyro constant drifts and the accelerometer biases in INS; \( Z_t \) represents the observation vector of the system, for INS / GPS integrated navigation system, \( Z = [P_{GPS} - P_{INS} \ V_{GPS} - V_{INS}] \); \( \Phi_{k/k-1} \) and \( \Gamma_{k/k-1} \) and \( H_k \) are system structural parameters, which respectively represent the state transition matrix, the system noise matrix and the observation matrix, for simplicity, \( \Gamma_{k/k-1} \) and \( H_k \) can be simplified as \( \Gamma_{k/k-1} = \tilde{Z}_k \) and \( H_k = \tilde{V}_k \), where \( \tilde{Z}_k \) and \( \tilde{V}_k \) are covariance matrix of \( W_{k-1} \) and \( V_k \) respectively.

An adaptive factor based on the innovation sequence is introduced to suppress the influence of sensor output outliers and various random noises on the filtering effect, and the prior prediction covariance is adjusted in real time to obtain the iterative formula of AKF. Rewrite the KF formula based on innovation adaptive form as follows [15]:

1) THE PREDICTION STAGE
State predicted equation:

\[
\hat{X}_{k/k-1} = \Phi_{k/k-1}\hat{X}_{k-1}
\]

(5)

Predicted state covariance:

\[
\begin{align*}
P_{k/k-1}^* &= \lambda_kP_{k/k-1} + \Gamma_{k+1}\Pi_{k+1}^T \\
P_{k/k-1}^* &= \Phi_{k/k-1}\Pi_{k-1}^* \Phi_{k/k-1}^T
\end{align*}
\]

(6)

(7)

2) THE UPDATE STAGE
Construct observation innovation sequence:

\[
r_k = Z_k - H_k\hat{X}_{k/k-1}
\]

(8)

Filter gain:

\[
K_k = P_{k/k-1}^*H_k^T(H_kP_{k/k-1}^*H_k^T + R_k)^{-1}
\]

(9)

Optimal state estimation:

\[
\hat{X}_k = \hat{X}_{k/k-1} + K_mr_k
\]

(10)

Update state covariance:

\[
P_k^* = (I - K_kH_k)P_{k/k-1}^*
\]

(11)

where \( \lambda_k \) denotes the adaptive factor. The following is a detailed introduction to the construction of \( \lambda_k \) according to the orthogonal principle of innovation sequence. Since the residual vector \( r_k \) contains new observation information and system model information, it can make a good description for process noise, the adaptive factor can be constructed by \( r_k \).

The error criterion statistics can be constructed as follows:

\[
\Delta r_k = \left( \frac{r_k^T r_k}{tr(C_{rk})} \right)^{\frac{1}{2}}
\]

(12)

where \( tr(C_{rk}) \) is the trace of the innovation covariance matrix, and the innovation covariance matrix can be determined by:

\[
C_{rk} = H_kP_{k/k-1}^*H_k^T + R_k
\]

(13)

then the adaptive factor can be constructed as:

\[
\lambda_k = \begin{cases} 
1 & \Delta r \leq d \\
\text{e}^{(\Delta r - d)^2} & \Delta r > d 
\end{cases}
\]

(14)

where \( d \) is an empirical constant, and we selected the value of \( d \) as 1 by referring to [15] and [16].

By introducing the above adaptive factor, the influence of varying observation noise and outlying observations on the filtering estimation results can be effectively suppressed, so as to improve the filtering ability of the system for various noise under complicated environment. AKF has advantages in the actual complex environment, compared with KF, such as lower sensitivity on system initial information, strong robustness to actual state changes and tracking ability to mutation status [9]. These advantages can make the improved KF algorithm have adaptive characteristic, and can provide matching robust updates for different types of errors, so that it cannot only suppress linear errors but also effectively suppress random errors.

B. PROPOSED RFR-WNN METHOD

On the basis of traditional NN, WNN uses wavelet function instead of Sigmoid function as activation function of hidden layer to establish the connection between the wavelet transform and the network coefficients. Which fully combines the advantages of wavelet analysis and NN, and can well model nonlinear and uncertain systems. WNN, compared with traditional NN, has higher prediction accuracy, fault-tolerant ability and faster convergence speed. It has been proved that WNN has exceptional performance in terms of data modeling and forecasting [28]–[31], widely used in the field of prediction.

Similarly to conventional NN, WNN is composed of three parts: input layer, hidden layer (wavelet layer) and output layer. The first layer (input layer) connects the input data to the network and gives the corresponding weight, then data are transmitted into the second layer (hidden layer); In the hidden layer, the nonlinear transformation from input space to hidden space is adopted, and the wavelet base function is used as the activation function of the hidden layer [28]. Fig. 3 shows the structure of a WNN model. Where, \( x_i(i = 1, 2, \ldots, I) \) denotes the input vectors; \( w_{ji}(j = 1, 2, \ldots, J; i = 1, 2, \ldots, I) \) denotes the connection weight of the \( i \)-th node in the input layer and the \( j \)-th node in the hidden layer; \( w_{kj}(k = 1, 2, \ldots, K; j = 1, 2, \ldots, J) \) is the connection weight of the \( j \)-th node in the hidden layer and the \( k \)-th node.
in the output layer; \( y_k(k = 1, 2, \ldots, K) \) is the output data; and \( I, J, K \) are the number of input nodes, hidden nodes and output nodes respectively.

The outputs of hidden layer and output layer of WNN shown in Fig. 3 can be expressed as:

\[
\begin{align*}
    y_j^{\text{hidden}} &= \varphi_j \left( \frac{\sum_{i=1}^{I} w_{ji} x_i - b_j}{a_j} \right) \\
    y_k &= \sigma \left( \sum_{j=1}^{J} w_{kj} y_j^{\text{hidden}} + d \right)
\end{align*}
\]  

(15)

where \( a_j, b_j \) denote dilation factor and translation factor respectively; \( \sigma(\cdot) \) stands for activation functions in output layer, the activation function \( \sigma(\cdot) \) in the output layer can be defined as \( \text{tanh}(\cdot) \), i.e.

\[
\sigma(z) = \tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}
\]

(16)

d is biases of the hidden layer; \( \varphi(\cdot) \) is wavelet basis function, as the activation function of hidden layer. In this paper, we select Morlet wavelet kernel function as \( \varphi(\cdot) \), which is a complex wavelet modulated by Gaussian function, for other wavelets, has the advantages of time-frequency regularity [32]. \( \varphi(\cdot) \) is expressed as follows:

\[
\varphi(t) = e^{j2\pi f_0 t} 
\]

(17)

where \( w_0 \) is central frequency. \( k \) and \( w_0 \) are taken as 2 and 1.75 generally to meet the admissibility condition. In the actual calculation, the real part of (17) is usually taken, concretely:

\[
\varphi(t) = \cos(1.75t)e^{-t^2/2}
\]

(18)

In the actual operation process of the integrated navigation system, the GPS signals may be lost soon due to the complex environment, which leads to few and incomplete samples collected for training WNN. In addition, the complexity of the application environment makes the noise generated by the system uncertain, all of these will make a single network unstable in structure, easy to over fitting, easy to fall into the local optimal, which will have a negative impact on the prediction accuracy and even cause prediction to diverge. A random forest regression (RFR) method is introduced for above problems to improve and optimize the single WNN.

The RFR consists of a set of regression trees which are trained using various bootstrap samples from the training data. Every tree is its own regression function, and the eventual output of RFR is taken as the average of the output of all the regression trees. RFR based on integrated learning ideas, using random sampling method to achieve the integration of multiple learners, through the combination of multiple learners to strengthen the prediction accuracy, making the structure is not easy to over fitting, and has good anti-noise ability. Moreover, it is easy to realize parallelization, with fast training speed and strong generalization ability [32]–[34]. The use of RFR to optimize single WNN can effectively enhance the generalization ability and prediction accuracy in the case of incomplete samples and complex noise.

Structure of the proposed RFR-WNN as shown in Fig. 4. In RFR-WNN, each regression tree of random forest is replaced by a WNN. When the GPS works well, the corresponding data collected by the integrated navigation system is used as the original overall training sample set. The Bootstraping sampling method is used to randomly extract the sub-training set with the same number of samples as the total training set from the total training sample set (the number of samples is \( N \)) as the training set for each tree, that is, each WNN [30]. Finally, each trained WNN is integrated into a strong WNN with good performance by using the integrated method to improve the shortcomings of single WNN in training phase, and then the original training sample set is used for testing to fine tune and optimize structure. The specific process of RFR-WNN construction is as follows:

**Step 1:** input the original training samples set \( D \) (the number of samples is \( N \)).

**Step 2:** the sample set with sample size of \( N \) is selected by Bootstrapping method from the original training samples set \( D \), a total of \( T \) times to generate \( T \) sub-training sets.

**Step 3:** for \( T \) sub training sets, \( T \) WNN models are trained respectively.
Step 4: the T trained sub-WNNs are integrated, the regression results obtained by T sub-WNNs are arithmetically averaged as the output after integration, and the original training samples set D is used as the test set to fine tune and optimize the integrated model.

Step 5: finally output the RFR-WNN model with good performance.

In the prediction process during GPS outages, the final prediction output of the RFR-WNN model is determined by the average of individual predictions of each WNN.

To sum up, the overall framework of the proposed method is shown in Fig. 5.

IV. EXPERIMENT AND COMPARISON

A. EXPERIMENTAL PLATFORM AND SCHEMES

In order to test the effectiveness and advancedness of the proposed method, we carried out on-land vehicle experiment. A self-developed INS/GPS integrated navigation system as the experimental system, and collected INS/GPS data from a moving vehicle platform as shown in Fig. 6 and Fig. 7. In the INS/GPS integrated navigation system of experimental platform, the INS contains three sets of accelerometers (MS9010) and gyroscopes (STIM202), a Ublox NEO-M8T single-chip receiver was used as the GPS receiver in the experiment. In addition, the tactical grade inertial measurement unit NovAtel SPAN-LCI, a high-precision navigation instrument, was used as a reference. The parameters of relevant navigation instruments are shown in Table 1.

To better evaluate the performance of the proposed method in the case of sufficient training samples and few training samples, the following two experimental schemes are designed:

1) SCHEME 1
Under the condition that the INS/GPS integrated navigation system works well, the vehicle platform performs integrated navigation algorithm and runs for 2000 s to ensure that sufficient training samples are provided to the neural network, and then the GPS signal is interrupted for 300 s. That is, the vehicle running for 2300 s with 300 s GPS outages.

2) SCHEME 2
Under the condition that the INS/GPS integrated navigation system works well, the vehicle platform performs integrated navigation algorithm and runs for 400 s. In this case where the training samples collected for the neural network are fewer, the GPS signal is interrupted for 300 s. That is, the vehicle running for 700 s with 300 s GPS outages.
TABLE 2. Time allocation of two experimental schemes.

| Scheme | Total duration | GPS on (training period) | GPS outages (prediction period) |
|--------|----------------|--------------------------|--------------------------------|
| Scheme 1 | 0s–2300s      | 0s–2000s                 | 2000s–2300s                    |
| Scheme 2 | 0s–700s       | 0s–400s                  | 400s–700s                      |

roads, and driving operation including going straight, lane-changing, speeding up and down suddenly and swerving.

In the above two experimental schemes, the combination of RFR-WNN and AKF (termed RFR-WNN+AKF) are proposed as the data processing method for GPS/INS navigation system. According to the overall structure of the proposed RFR-WNN+AKF, measured values of accelerometer and gyroscope are selected as the input, the corresponding position and velocity differences between the GPS and the INS are collected as the output for the training mode when GPS is available, and observations required for AKF is obtained continuously from the RFR-WNN of INS/GPS integrated system when GPS signal is defective. To verify the advantage of the proposed RFR-WNN+AKF method more clearly, we compared the proposed method with following other methods: 1) pure INS (i.e., KF only executes time update process); 2) the method of back propagation neural network (BPNN) assisted Kalman filter in [6] (i.e., combining BPNN and KF to overcome the GPS outages for INS/GPS system, termed the BPNN+KF); 3) the method of single WNN assisted AKF (i.e., combining signal WNN and AKF to overcome the GPS outages for INS/GPS system, termed the WNN+AKF). All the algorithms were implemented using a same single-core i7-6700 processor with 3.4 GHz, 16 GB RAM and the Windows 7 operating system.

B. COMPARISON RESULTS BETWEEN DIFFERENT ALGORITHMS

1) PERFORMANCE COMPARISON WITH DIFFERENT ALGORITHMS WHEN TRAINING SAMPLES COLLECTED ARE SUFFICIENT

The vehicle platform with INS/GPS integrated navigation system run according to experimental scheme 1 after initial alignment.

Fig. 8 presents the east and north velocity errors of pure INS, BPNN+KF, WNN+AKF and RFR-WNN+AKF structures after GPS loss. The position errors of different algorithms in the northern and eastern during GPS outages as shown in Fig. 9. It is worth noting that the time “0” is the start of the prediction phase. So the time “0” in Figs. 8 and 9 are the 2000 s in the actual experiment. We used the standard deviation (Std) and root mean square error (RMSE) of errors to compare the performance of the different algorithms. In this paper, RMSE can be defined as follows:

\[
RMSE = \sqrt{\frac{1}{N} \sum_{k=1}^{N} (P_k - \hat{P}_k)^2}
\]  

(19)

where \(N\) denotes the total sample number, \(P_k\) and \(\hat{P}_k\) represent the reference value provided by reference system and the calculated value solved by each algorithm of velocity and position at time \(k\), respectively. RMSE can reflect the deviation between the calculated value of each method and the reference value, and it is sensitive to the large error in a set of data. The smaller the RMSE, the higher the navigation accuracy, which means the less affected by outliers. The Std can reflect the dispersion degree of a dataset. Therefore, RMSE can be used to evaluate the accuracy and robustness of different methods, and Std can be used to evaluate the navigation accuracy and stability of different algorithms. The Std and RMSE of the velocity and position errors for northern and eastern of different algorithms are shown in Table 3. It can be clearly seen from Figs. 8 and 9 that the performance of the proposed RFR-WNN+AKF method is generally superior to other methods. Moreover, the errors increase rapidly due to the accuracy of pure inertia solution diverges rapidly during GPS outages, and the three algorithms can suppress the velocity and position errors of INS in different degrees. The three methods all show good performance during GPS outage when the training samples collected are sufficient. However,
it can be seen from Fig. 8 and Fig. 9 that RFR-WNN+AKF method has better robustness and anti-disturbance than other methods because of strong ability to estimate system uncertainties. Due to the uncertainty of the system, BPNN+KF method may even cause large error fluctuations. As seen from the Table 3, three methods effectively reduce the Std and RMSE and higher precision error compensation results be achieved by using RFR-WNN+AKF. Using the proposed RFR-WNN+AKF method, RMSE of the velocity at the eastern reduces from 1.192 m/s for pure INS solution to 0.101 m/s for the proposed RFR-WNN+AKF, improving by 90%. In addition, the computation cost of the four methods in experiment scheme are compared, which is embodied in the running time of each method as shown in Table 4.

It can be seen from Table 4 that the operation time of the proposed method is the longest. This is because the structure of the proposed method is more complex than other methods and the training samples are fully combined and trained. On the other hand, the proposed method does not sacrifice too much computation cost to improve accuracy. In fact, during the training phase, the training of RFR-WNN and the operation of AKF are carried out simultaneously. At this time, the training of neural network is a separate item, and does not take up the time of integrated navigation operation. During the prediction process, RFR-WNN has been trained well, which can predict the observations of integrated navigation system directly. Powerful modern navigation computer makes the disadvantage of sacrificing little computation cost less obvious. It is worth adding a small amount of

![FIGURE 9. Position errors comparison of different methods in scheme 1: (a) east position error; (b) north position error.](image)

![FIGURE 10. Radar map for RMSE of navigation parameter error and computation time of the four methods in experiment scheme 1.](image)
computation time in exchange for a significant increase in navigation accuracy.

Furthermore, the radar map as shown in Fig. 10 visually compares the statistical data (RMSE) of navigation parameter error and computation time of the each method in experimental scheme 1. The above comparison and analysis can be clearly verified from Fig. 10, that is, the proposed method can significantly improve the navigation by adding little computational load during GPS outages.

2) PERFORMANCE COMPARISON WITH DIFFERENT ALGORITHMS IN THE CASE OF FEW TRAINING SAMPLES COLLECTED

The vehicle started navigation according to experimental scheme 2 after the alignment course. Figs. 11 and 12 illustrate the performance of the velocity errors and the position errors generated by different algorithms after GPS loss in the case of fewer training samples collected. Note that the time 0 in Figs. 11-12 are the 400 s in the actual experiment. Table 5 compares the Std and RMSE of the velocity and position errors at northern and eastern of different methods in scheme 2.

As can be seen plainly from Figs. 11 and 12, the performance of the RFR-WNN+AKF method proposed in this paper is superior to the other two methods. The advantages of the RFR-WNN+AKF method in terms of error compensation, prediction accuracy, anti-disturbance and robustness are more obvious during GPS outage in the case of fewer training samples collected. While the other two methods are not ideal for INS error suppression, and even tend to diverge. As can be seen from Table 5 that with the proposed RFR-WNN+AKF method, the RMSE of the velocity at eastern can be reduced from 1.208m/s of the pure INS strategy to 0.125m/s, which is an improvement of 90%; RMSE of the velocity at the northern position reduces from 1.192m/s to 0.165 m/s, which is an improvement of 89%; the RMSE of eastern position is reduced from 23.498 m to 1.226m; and the RMSE of northern position is reduced from 45.825m to 1.223m. Table 6 compares the computation time of different methods in experiment scheme 2. From the comparison between Table 5 and Table 6, it can be seen that the more data collected by the INS/GPS integrated navigation system, the longer the computation time of each algorithm. When using neural network to solve the GPS outages problem,
TABLE 5. Std and RMSE of velocity and position errors of different methods in scheme 2.

| Error               | Algorithm     | Std  | RMSE  |
|---------------------|---------------|------|-------|
| East velocity error | Pure INS      | 0.801| 1.208 |
| (m/s)               | BPNN+KF       | 0.378| 0.489 |
|                     | WNN+AKF       | 0.215| 0.267 |
|                     | RFR-WNN+AKF   | 0.079| 0.125 |
| North velocity error| Pure INS      | 0.705| 1.192 |
| (m/s)               | BPNN+KF       | 0.364| 0.476 |
|                     | WNN+AKF       | 0.233| 0.283 |
|                     | RFR-WNN+AKF   | 0.131| 0.165 |
| East position error | Pure INS      | 15.895| 23.498|
| (m)                 | BPNN+KF       | 6.056| 6.483 |
|                     | WNN+AKF       | 2.634| 2.895 |
|                     | RFR-WNN+AKF   | 0.871| 1.226 |
| North position error| Pure INS      | 31.135| 45.825|
| (m)                 | BPNN+KF       | 6.852| 6.976 |
|                     | WNN+AKF       | 2.887| 3.121 |
|                     | RFR-WNN+AKF   | 1.160| 1.223 |

TABLE 6. Computation time of different methods in experiment scheme 2.

| Method           | Running time (s) |
|------------------|------------------|
| Pure INS         | 40               |
| BPNN+KF          | 46               |
| WNN+AKF          | 50               |
| RFR-WNN+AKF      | 53               |

FIGURE 13. Radar map for RMSE of navigation parameter error and computation time of the four methods in experiment scheme 2.

FIGURE 14. Radar maps for statistical data (RMSE) of navigation parameter error and computation time of two methods in different experimental schemes: (a) parameter of BPNN+KF method in two experimental schemes; (b) parameter of RFR-WNN+AKF method in two experimental schemes.

more training data can improve the navigation accuracy more effectively while also increases the computation cost.

In order to compare performance of four methods in experimental scheme 2 more intuitively, the radar map for statistical data (RMSE) of navigation parameter errors and computation time of each method is drawn as shown in Fig. 13. It can be seen from Fig. 13 that the proposed method can still maintain good navigation performance during GPS outages under the condition of limited training samples.

Moreover, we made comparative analysis about the BPNN+KF method in [6] and proposed RFR-WNN+AKF method on their navigation performance during GPS outages in different experimental schemes from the longitudinal aspect, as shown in Fig. 14.

As can be seen from Fig. 14, when the training samples when there are small training samples collected for neural network, the navigation performance of the BPNN+KF method decreases significantly. In contrast, the proposed
RFR-WNN+AKF still maintains a good navigation performance, with only a small reduction in the navigation performance. This is due to the proposed RFR-WNN structure makes full use of these small amount of training samples, which reduces the negative effect of less samples on neural network training to a certain extent.

The proposed RFR-WNN+AKF can provide higher accuracy of navigation than the other methods because the AKF to estimate system uncertainties and nonlinear noise is better than that of the KF, and the stability and prediction accuracy of RFR-WNN structure is better than that of single WNN because of the optimization of WNN by RFR.

V. CONCLUSION

In this paper, we proposed a novel method that is WNN based on RFR to assist AKF, to provide a reliable and high performance navigation solution for INS/GPS integrated navigation system during GPS outages. A RFR-WNN model is presented to overcome the problem of over fitting and instability of the single neural network caused by few and incomplete training samples. RFR based on integrated learning idea is used to enhance the generalization ability and stability of WNN, and improve the prediction performance; a proposed AKF is mainly aimed at the complexity and uncertainty of system noise, and the adaptive factor is introduced to improve the robustness and accuracy of integrated filtering. When GPS signal is available, RFR-WNN is trained to obtain appropriate internal structure and applied during GPS outages, the finely trained RFR-WNN model is used to predict the measurement input of the AKF update process, while the AKF is applied throughout the navigation process. Thus, a whole navigation measurement with high performance can be obtained after applying the proposed RFR-WNN+AKF method. The land vehicle navigation experiment demonstrated the superiority of the proposed method over other methods during GPS outages under complex environment. Especially in the case of limited training samples, compared with the existing BPNN+KF method, the proposed RFR-WNN+AKF method reduces the RMSE of east velocity, north velocity, east position and north position by about 53%, 42%, 58% and 61%, respectively. Of course, how to improve the navigation performance is worth further study in the case that the GPS is interrupted at the beginning of the system operation and there are hardly any neural network training samples. The real-time implementation in applications will also be our focus in the future.

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