Investigation of hydraulic fracturing mechanism by using a coupled continuous-discontinuous hydromechanical model

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Abstract. Fluid-induced fracture nucleation, propagation, and interaction are the essentials for a better understanding of the hydraulic fracturing process in unconventional reservoirs. In this study, a coupled continuous-discontinuous hydromechanical model was established to investigate the hydraulic fracturing propagation under varied conditions. The interactions between induced fractures and natural fractures are investigated and discussed by a series of hydraulic fracturing simulations. Also considered are the influences of bedding joints, in-situ stress ratios, and fluid injection rates on the patterns of hydraulic fractures and the stress field. It was found that hydraulic fracture propagation is controlled by both in-situ stress state and strength anisotropy of the reservoir rock. The simulations indicate that an increase in the fluid injection rate is favorable to the formation of a complex fracture network. More hydraulic fractures were developed when fracture fluids were injected into rock specimens with a faster pressurization rate than a quasi-static pressurization rate. However, higher fluid injection rates could result in higher breakdown pressures for fracture initiation and propagation. In addition, hydraulic fractures tend to extend along the direction of the maximum principal stress or approach this preferred path. Bedding joints are preferred locations and orientations for fracture initiation and propagation in laminated shale reservoirs.

1. Introduction

Hydraulic fracturing is commonly used to enhance the low permeability of shale reservoirs. Its successful application has resulted in the rapid increase in shale gas production in recent years [1]. Horizontal drilling and massive hydraulic fracturing are two important artificial measures to create a complex fracture network and enhance the low permeability of shale reservoirs. Hydraulic fracturing effectiveness has become the dominant factor that determines the development of low-permeability oilfields [2]. Consequently, fracture propagation behavior induced by the hydraulic fracturing treatment of shale reservoirs has been investigated intensively in recent years [3–6]. Fracture fluid seepage and flow in the fractured rock mass are mainly controlled by the pattern and connectivity of the hydraulic fracture network [7]. Thus, the quality of the artificially modified fracture network is an essential and vital factor in practical fracturing operations [8]. As things stand, there are no effective methods to check the real patterns of the fracture network although microseismic interpretation has been used in some gas fields [9]. Numerous numerical modeling methods have also been widely used to study the mechanism of hydraulic fracturing. However, there are still many intractable problems in the algorithms of numerical methods and models. In this study, a coupled...
hydromechanical model is introduced. Calculations using the model are based on a continuous-discontinuous element method, and we propose it to investigate the hydraulic fracturing mechanism and the effects of geological and operational parameters. The results show that fluid pressurization rates, in-situ ratios, and bedding joints have significant effects on the initiation, propagation, and damage of hydraulic fractures.

2. Modeling methodology

2.1. Brief introduction of the modeling method

Continuum-discontinuum element method (CDEM) is a dynamic, explicit algorithm that is based on element fracture under the Lagrange system [10]. This algorithmic method takes the advantages of the continuous model and the discrete element method (DEM). The algorithm is based on the time-dependent explicit iteration using dynamic relaxation. Deformation of blocks is calculated by employing the finite element method (FEM) while the interfaces between the blocks are calculated using the bond spring model or the contact spring model. The process of progressive failure of rocks from continuous deformation and crack growth leading to failure can be simulated with this numerical method noted by [11–14]. The basic concepts of this model are presented in Figure 1.

![Figure 1. Basic concept and models: (a) Geometrical model and (b) Interface model](image)

The blocks in the CDME are composed of one or more finite elements (FEs). The FEs can be simple tetrahedral, pentahedral, or hexahedral or even complex polyhedral. The interfaces between blocks are springs. Discontinuous deformation between blocks is mainly realized by spring deformation and fracture.

The governing equation of CDEM is established using the Lagrange equation. From the Lagrange equation, the kinetic equilibrium equation can be obtained as follows:

$$\sigma_{ij, j} + f_i - \rho \ddot{u}_i - \mu \dot{u}_i = 0$$

(1)

where $\sigma_{ij}$ is the stress tensor, $f_i$ is the body force; $\rho$ is the density; $\dot{u}_i$ represents the velocity vector; $\ddot{u}_i$ denotes the acceleration vector; and $\mu$ is the damping coefficient.

Using the variation formulation, the equilibrium equation of momentum in Equation (9) can be transformed into the following matrix form in an element:

$$M \ddot{u}(t) + C \dot{u}(t) + K \dot{u}(t) = F(t)$$

(2)
where $M$, $C$, and $K$ represent the mass, damping, and stiffness matrices, respectively; $\ddot{u}(t)$, $\dot{u}(t)$, and $u(t)$ denote vectors containing the nodal accelerations, displacements, and velocities at time point $t$, respectively; and $F(t)$ is the external loading force. $F(t)$ can be expressed as follows:

$$F(t) = F_b + F_p + F_s + F_t$$

where $F_b$ is the body force; $F_p$ is the fluid pressure on the fracture surface; $F_s$ is the spring force; and $F_t$ is the force on the traction boundary.

The strain-displacement relationship is given as follows:

$$\varepsilon = \frac{1}{2} \left( u_{i,j} + u_{j,i} \right)$$

The effective stress tensor is given as follows:

$$\sigma_{ij} = \sigma_{ij} - \alpha p_n I$$

The constitutive relation law is given as follows:

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl}$$

The boundary conditions are given as follows:

$$u_i = \bar{u}_i, \quad \sigma_{ij} n_j = \bar{\sigma}_j$$

The initial conditions are given as follows:

$$u_i(x, y, z, 0) = \bar{u}_i^0(x, y, z)$$
$$\sigma_{ij}(x, y, z, 0) = \bar{\sigma}_{ij}^0(x, y, z)$$

In Equations (4–8), $\varepsilon$ and $\varepsilon_{kl}$ represent the strain tensor versus coordinate; $\sigma_{ij}$ is the stress tensor; $\sigma_{i,j}$ and $\sigma_{j,i}$ are both the first-order partial derivatives of displacement versus coordinate; $C_{ijkl}$ is the stress-strain tensor; $\bar{u}_i$ represents the displacement on the displacement boundary; $\bar{u}_i^0$ represents the displacement on the initial displacement boundary; $\sigma_i$ represents the confining stress on the external force boundary; and $\bar{\sigma}_i$ represents the confining stress on the initial external force boundary.

Darcy’s law for fluids is used in porous media and hydraulic fractures. In the CDEM, the velocity field of fluid can be obtained by the following equations:

$$v_m = \frac{k_m}{\mu} \nabla p_m$$
$$v_f = \frac{k_f}{\mu} \nabla p_f$$

where $v_m$ and $v_f$ represent the velocity field of fluid flow for porous media and hydraulic fractures, respectively; $k_m$ and $k_f$ represent the permeability of the porous media and hydraulic fractures, respectively; $\mu$ is the viscosity; $p_m$ and $p_f$ represent the pressure values for porous media and hydraulic fractures, respectively.

For pore seepage and fracture seepage, the pressure equation for single-phase flow can be written as follows:
\[-\nabla \cdot \mathbf{v}_m + q_m = \frac{1}{M_m} \frac{\partial \rho_m}{\partial t} \]
\[-\nabla \cdot \mathbf{v}_f + q_f = \frac{1}{M_f} \frac{\partial \rho_f}{\partial t} \quad (10)\]

where \( t \) denotes time; \( q_m \) and \( q_f \) denote the source and sink terms of pore flow and fracture flow, respectively; \( M_m \) and \( M_f \) are Biot modulus of the pore matrix and the fracture, which are related to the compressibility of the rock and soil and the pore fluid.

The permeability of hydraulic fractures according to Snow [15] can be written in the following form:

\[ K_f = \frac{b^2}{12} \quad (11) \]

where \( b \) denotes the aperture of the hydraulic fracture.

Two criteria are used to determine the failure types of the element. The slipping and fracture behaviors that occur in the interface between elements are judged by the following tensile/shear failure criteria:

Tensile failure criteria are as follows:

\[ \sigma_n \geq \sigma_t \quad (12) \]

Shear failure criteria are given as follows:

\[ \tau \geq \sigma_n \tan \varphi + c \quad (13) \]

where \( \sigma_n \) is the normal stress; \( \sigma_t \) is the tensile strength; \( \tau \) is the tangential stress; \( \varphi \) is the internal friction angle; and \( c \) is the cohesion.

Iterative coupling solves the coupling of seepage and stress, while the calculation of the seepage field and stress field in the iterative calculation is performed separately. Specifically, the seepage and stress fields are calculated separately at each time step. The corresponding data are then exchanged to update the seepage and stress fields.

2.2. Model geometry

The aforementioned calculation method is applied to simulate the process of hydraulic fracturing. Figure 2 is a representation of a basic geometry and set-up. The model represents a 2D horizontal section of a shale gas reservoir. A vertical wellbore is excavated at the center of the model and served as the injection hole during the simulation. In the model with bedding joints, the bedding angles is 0°. The spacing of joints is twice the diameter of the injection hole. From Figure 2a, fixed displacement boundary conditions were set to mimic standard laboratory tests. In Figure 2c, constant stress boundary conditions were adopted to model the actual stress environment. In this case, the stresses in the \( x \) (H) directions on the model boundary were 15 MPa and those in the \( y \) (h) directions on the model boundary were 15 MPa and 10 MPa under different simulation conditions.
2.3. Input data

To eliminate the potential influence of heterogeneous pore structures, homogeneous and isotropic rocks were assumed as the model’s material. The mechanical parameters of the material used in the coupled model were calibrated through a series of trial-and-error experiments. The micro-parameters were calibrated to match the macro-properties of the Longmaxi shale rock. These include the elastic modulus, peak strength, and Poisson’s ratio. The physical and mechanical parameters of the model used in this study are respectively shown in Tables 1 and 2.

Table 1. Mechanical properties of rock matrix for the numerical models

| Index/unit             | Value |
|------------------------|-------|
| Elastic modulus /Pa    | 5e10  |
| Poisson ratio          | 0.25  |
| Density /m³/kg         | 2500  |
| Tensile strength /Pa   | 8e6   |
| Cohesive strength /Pa  | 3e6   |
| Friction angle /°      | 40    |
| Dilation angle /°      | 15    |

Table 2. Mechanical properties of bedding joints for the numerical models

| Index/unit             | Value |
|------------------------|-------|
| Normal stiffness /Pa   | 5e11  |
| Shear stiffness /Pa    | 5e11  |
| Friction angle /°      | 10    |
| Tensile strength /Pa   | 1e5   |
| Cohesive strength /Pa  | 1e6   |
| Tolerance /m           | 1e-6  |

2.4. Modeling scenarios
The foci of this study are fracture initiation and propagation under different pressurization rates. Thus, two completely different hydraulic loading methods were used during the simulation. For Case A, the initial hydraulic pressure $p_0$ was zero, and the pressurization rate was $0.1 \text{ MPa}/10,000 \text{ steps}$. Under these conditions, the fracturing process was studied and the breakdown pressure recorded. As the pressurization rate is insignificant, a quasi-static pressurization rate was assumed in this study. For Case B, the initial hydraulic pressure $p_0$ was zero, and the pressurization rate was $1 \times 10^{-6} \text{ m}^3/\text{s}$. Under these conditions, the fracturing process was studied, and the breakdown pressure was recorded. Since the pressurization rate is larger than that of Case A, a fast pressurization rate was assumed for Case B in this study.

3. Results and discussion
Fracture evolution of the rock specimens under different pressurization rates was simulated. The results show that both fracture initiation and propagation have different characteristics. For this simulation, after the injection of fluid into the wellbore, the fracture initiation depends on the local resistance of the wellbore and the stress state in the wellbore. The fluid pressure in the fracture is controlled by parameters such as fracture aperture, flow rate, pressurization duration, and fracture permeability. In addition, while the fracture aperture is influenced by the fluid pressure, the fracture directions are related to the stress field of the stimulated rock specimens.

3.1. Fracture initiation and propagation with quasi-static pressurization rate (Case A)
When the pressurization rate is small enough, the hydromechanical processes are similar to a steady water-rock coupling process. Two symmetrical hydraulic fractures were developed during the fracturing simulation process as can be seen in Figure 3.

![Figure 3](image-url)

**Figure 3.** Fracture pattern after quasi-static pressurization simulation: (a) Displacement distribution and (b) The maximum principal stress.

Breakdown pressure in shale formation is the key parameter designed for hydraulic fracturing in oil and gas exploration and production. The continuous injection results in an increase in the pore pressure around the injection hole. Some small cracks occur when the pore pressure exceeds the breakdown pressure of the rock specimens. Further injection also results in the gradual formation of two symmetrical hydraulic fractures around the injection hole because of the propagation and coalescence of the cracks. This, then, leads to the growth of the two hydraulic fractures, which extend quickly as the injection continues. At the steady propagation stage, the diameter of the borehole increases drastically and the hydraulic fractures propagate effectively. The flow rate and fluid pressure in the hydraulic fractures increase correspondingly with an increase in the aperture of the hydraulic fractures.
3.2. Fracture initiation and propagation with fast pressurization rate (case B)

When high-pressure fracture fluids are injected into rock specimens with a fast pressurization rate, which is similar to what is done in practical reservoir fracturing, there is a sharp increase in pore pressure. There is also an adjustment of the stress field that is induced during the injection operation. In this case, hydraulic injection and fracturing are regarded as a dynamic failure process that involves a complicated hydromechanical process. As illustrated in Figure 4, more fractures were developed than what was seen in the former case during the process of the fracturing simulation.

![Fracture pattern after fast pressurization simulation: (a) Displacement distribution and (b) The maximum principal stress.](image)

Hydraulic fracturing with fast pressurization rate can quickly result in an increase in the pore water pressure. This can generate significant changes in effective stress. The critical pressure needed for the initiation of a new crack depends on the state of stress in the rock. When the pore water pressure is greater than the breakdown pressure of the rock specimens, such as in the dynamic loading method, hydraulic fractures are initiated and propagated rapidly in different directions to form a fracture network. Upon further injection, four symmetrical hydraulic fractures gradually developed around the injection hole leading to the formation of a radial fracture network as the injection continues.

3.3. Effects of the in-situ ratios on fracture propagation

The in-situ stress ratio is a vital geological parameter in shale gas fracturing. A group of simulations was performed with stress boundary conditions and a quasi-static pressurization rate to investigate the effects of the in-situ stress ratios on hydraulic fracture initiation and propagation. The stress ratio defined as the ratio of $\sigma_H$ to $\sigma_b$ was studied to evaluate its effects on the response of fracturing injection. Simulated fracture distributions with stress ratios equal to 1.0 and 1.5 are illustrated in Figure 5. The hydraulic fractures caused by fluid injection have different orientations and patterns. Most importantly, when the stress ratio equals 1.0, the direction of the fracture zone is about 70° to $\sigma_H$. However, when the stress ratio equals 1.5, the direction of the fractures is about 20° to $\sigma_H$. The simulation results are also an indication that a large stress ratio is conducive to forming branching fractures in the direction of H. This results in an increase in the complexity of the fracture network morphology. In general, hydraulic fractures prefer to extend along the direction of the maximum principal stress or approach this preferred path, and a larger in-situ stress ratio could result in smaller breakdown pressures and faster fracture propagation.
Figure 5. Fracture distribution after fracturing with different in-situ ratios; the injection rate was consistent with the quasi-static pressurization rate: (a) In-situ stress ratio $\sigma_H/\sigma_h = 15:15$ and (b) In-situ stress ratio $\sigma_H/\sigma_h = 15:10$.

3.4. Hydraulic fracture propagation in laminated rock

The presence of bedding planes in rocks affects the response of hydraulic fracturing. The preexisting bedding joints influence the fracture network morphology because the strength of bedding joints is usually lower than the strength of the rock matrix. During hydraulic fracturing, bedding planes are particularly vulnerable to damage and failure. In shale, tensile failure plays a vital role because the tensile strength of rocks is probably a tenth of the compression strength of rocks. Tensile failure of rock matrices usually shows up as initiation of small cracks and accumulation and coalescence of small tensile failures. These can lead to several macroscopic shear fractures of the rock mass. When the pore pressure exceeds the sum of the tensile strength of the rock and the minimum principal stress, tensile cracks occur and eventually propagate to form long hydraulic fractures.

Figure 6. Fracture distribution for bedding joints oriented at 0° after fracturing (injection rate: $1 \times 10^{-6} \text{m}^3/\text{s}$); the boundary conditions were consistent with the quasi-static pressurization rate: (a) Displacement distribution and (b) The maximum principal stress.

We investigated the effects of bedding joints on hydraulic fracture initiation and propagation. To this end, only a typical simulation was performed with fixed boundary conditions and a fast pressurization
rate. Figure 6 shows the simulated fracture distribution under the influence of preexisting horizontal bedding joints. In contrast with the other case model, the most distinct feature is that the position and orientation of the bedding joints play an important role in the propagation direction of the hydraulic fractures.

Figure 6. Fracture distribution under the influence of pre-existing horizontal bedding joints.

Figure 7. Fracture distribution for bedding joints oriented at 0° after fracturing (injection rate: $1 \times 10^{-5}$ m$^3$/s); the boundary conditions were consistent with the quasi-static pressurization rate: (a) Displacement distribution and (b) The maximum principal stress.

The simulation results show that bedding joints are conducive for forming hydraulic fracture networks. Under a constant injection rate, the bedding joints were activated and damaged to form the framework of the fracture network. When the injection rate increases 10 times, as illustrated in Figure 7, many more hydraulic fractures are developed in and around the injection well. However, higher fluid injection rates could result in higher breakdown pressures for fracture initiation and propagation. These results are in good agreement with most of the numerical results and laboratory experiments. Many transverse fractures developed in conjunction with the main hydraulic fractures. These can increase the complexity of fracture network morphology.

4. Conclusions

Hydraulic fracturing plays a vital role in enhancing permeability for shale gas development and enhanced geothermal systems using the high-pressure injection of a fracturing fluid into tight reservoir rocks. In this study, a coupled continuous-discontinuous hydromechanical model was used to investigate the hydraulic fracturing process under varied geological and operational conditions. The interactions between induced fractures and bedding joints, in-situ stress ratio, and fluid injection rate on the geometry of hydraulic fractures were investigated and discussed through a series of hydraulic fracturing simulations. It was found that hydraulic fracture propagation is controlled by both in-situ stress state and strength anisotropy of the reservoir rock. The simulations indicated that an increase in the fluid injection rate is favorable to the formation of a complex fracture network.

When fracture fluids are injected into rock specimens with a quasi-static pressurization rate, the hydromechanical process were observed to be similar to that of a steady water-rock coupling process, and only two symmetrical hydraulic fractures developed around the injection hole. When fracture fluids are injected into the rock specimens with fast pressurization rate, more fractures were developed than those developed in the former case during the fracturing simulation process. However, higher fluid injection rates could result in higher breakdown pressure for fracture initiation and propagation. In addition, hydraulic fractures tend to extend along the direction of the maximum principal stress or approach this preferred path. Larger in-situ stress ratios could result in smaller breakdown pressures and faster fracture propagation. Furthermore, bedding joints are preferred locations and directions for fracture initiation and propagation in laminated shale reservoirs. The outcome of this study highlights
important aspects of the hydraulic fracturing process, particularly at the micromechanical scale. Hence, the principles and concepts of the mechanics of hydraulic fracturing provide the much-needed guide and basis for further studies involving larger-scale reservoir modeling and more complex fracturing scenarios.

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