Moduli from Cosmic Strings

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Abstract

Moduli fields are generally present in superstring-inspired models. They are typically characterized by masses of the order of the supersymmetry breaking scale and by interactions of gravitational strength to ordinary matter. If stable, they can easily overclose the Universe. If unstable, strong limits on their abundance have to be imposed not to disrupt the successful predictions of nucleosynthesis. We discuss the production of moduli quanta from loops of cosmic strings, which may form at phase transitions in the early history of the Universe. As an application, we then focus on strings formed at the end of hybrid inflation, showing that in this case the coupling of the moduli to the field which forms the string network has to be much weaker than the typical gravitational interactions not to exceed the bounds from nucleosynthesis. Alternatively, if the relevant coupling is of gravitational order, an upper bound of about $10^{13}$ GeV is imposed on the string energy scale.
1 Introduction

Most superstring–inspired models of particle physics predict the existence of moduli fields, that are generally characterized by a mass in the TeV range (that is, of the order of the expected scale of supersymmetry breaking) and by gravitationally suppressed couplings to ordinary matter. These properties are shared by their supersymmetric partners, the so-called modulinos, and by the partners of gravitons, the gravitinos, which are also present in these models. Due to their extremely weak couplings, such particles are not likely to be observed in the foreseeable accelerator experiments. However, they can play a relevant role in cosmology [1].

In standard cosmology, particles which interact only gravitationally with the thermal background are often denoted altogether as gravitational relics. They have a very early decoupling, and, as a consequence, they can be expected to have a significant abundance. If stable, gravitational relics with a mass above the keV range lead to the overclosure of the Universe. If unstable, they disrupt the successful predictions of big bang nucleosynthesis, unless their mass is above 20 TeV, so that nucleosynthesis can occur after they have decayed [1].

These limits do not hold in inflationary scenarios [2], where the primordial abundance (if any) is diluted away during inflation. However, in this case one has to check that gravitational relics are not (re)generated to a too high abundance after inflation [3]. For unstable relics with lifetimes $> 10^4$ sec, the most restrictive bound comes from photo-destruction of the light elements produced during nucleosynthesis [4]

$$Y_s \lesssim 10^{-14} \frac{(\text{TeV}/m_s)}{\frac{m_s}{m_s}}.$$  \hspace{1cm} (1)

The strength of this bound has led to investigate possible overproduction of gravitational relics from several different sources. The most standard studies concern the production from the thermal bath formed at reheating. It has been shown [2, 3, 4] (see also [5] for more recent discussions) that the bound (1) translates into the upper limit $T_{rh} \lesssim 10^9$ GeV on the reheating temperature. More recently, attention has been drawn to nonthermal production of gravitinos [2] and moduli [7] during the coherent oscillations of the inflaton field (preheating), while production of gravitinos from inflaton/inflatino decays has been discussed in [8].

In the present work we consider topological defects, and in particular cosmic strings, as possible source of gravitational relics. For a long time cosmic strings (for a review, see [9]) have represented a viable alternative to inflation in the explanation of the inhomogeneities we observe today in the Universe [10]. The recent accurate measurement of the CMB power
spectrum, however, seems to favor inflation as the main origin of primordial density perturbations. Nevertheless, the presence of a subdominant component of matter in the Universe in the form of a network of cosmic strings is not excluded [11]. Moreover, one of the most studied inflationary scenarios, hybrid inflation [12], predicts the formation of a network of cosmic strings at the beginning of reheating. Motivated by this observation, although the formalism we will use is fairly general, in the following analysis we will mainly refer to a cosmic string network that formed at the end of hybrid inflation, with a symmetry breaking scale of the order of $10^{15}$ GeV.

Cosmic strings can lead to particle production. Actually, they have already been considered as potential sources of ultra high energy cosmic rays [13] or of gamma–ray bursts [14], as well as a possible origin of nonthermal cold dark matter [15], or as source of the matter-antimatter asymmetry [16]. What is more relevant for the present discussion, it has also been suggested in reference [17] that string loops may lead to a significant production of gravitinos. Moreover, in ref. [18], bounds on the cosmic string scale have been obtained by analyzing string coupling to the dilaton of (super)string theory.

Here, we will consider loops of cosmic strings as possible sources of scalar moduli. Moduli degrees of freedom in general parameterize symmetries in superstring models. These symmetries are usually broken by supersymmetry breaking effects, so that moduli acquire a mass of the order of the supersymmetry breaking scale. In superstring–inspired models, the expectation value of the moduli fields determines the coupling constants and the masses of the theory. By expanding such coupling constants in powers of the moduli fluctuations, it is possible to see that matter fields can be generally expected to have a Planck mass suppressed coupling to moduli. In the present work, we assume that this is the case also for the field(s) responsible for the formation of cosmic strings.

A simple example related to the application we are discussing is given by the following simple (toy) model with a $D$–term

$$V \supset -\frac{1}{2} f^2 D^2 + D \left( \phi^+ q \phi - \xi \right),$$

(2)

where $\phi^T = (\phi_+, \phi_-)$ contains two scalar fields with opposite $U(1)$ charges, $q = \text{diag}(+1, -1)$, and $\xi$ is a Fayet–Iliopoulos term. The coupling constant $f$ of the gauge kinetic term is determined by the expectation value of a scalar modulus $\bar{s}$. Defining $s \equiv \bar{s} - \langle \bar{s} \rangle$, we can expand

$$f = f_0 (\langle \bar{s} \rangle) \left( 1 + \frac{\alpha \text{ Re } s}{8 M_P} + \ldots \right)$$

(3)

(here and in the following, dots denote higher order terms in $s/M_P$ which we neglect; moreover, we assume $f_0$ and $\alpha$ to be real). Hereafter, $\text{Re } s \equiv s$. Eliminating $D$ in eq. (2), and
making use of the expansion (3), we get
\[ V \supset \frac{1}{2} f_0^2 \left( 1 + \frac{\alpha s}{4 M_P} + \ldots \right) \left( |\phi_+|^2 - |\phi_-|^2 - \xi \right)^2. \] (4)

We can assume the vacuum expectation value of $\phi_-$ to vanish. Finally, renaming $f_0^2 \equiv \lambda/2$, $\phi_+ \equiv \phi$, $\xi = \eta^2$, the potential term is recast in the form
\[ V = \frac{\lambda}{4} \left( |\phi|^2 - \eta^2 \right) + \frac{\lambda}{4} \frac{s}{M_P} \left( |\phi|^2 - \eta^2 \right)^2 + \ldots \approx \frac{\lambda}{4} \left( |\phi|^2 - \eta^2 \right) + \lambda \alpha s \frac{\eta^2}{M_P} \left( |\phi| - \eta \right)^2 + \ldots \] (5)

The first term in the above potential leads to the spontaneous breaking of the $U(1)$ symmetry $\phi \to e^{i \theta} \phi$. The second term describes a cubic interaction between $s$ and $\phi$. We assume that in the early Universe the $U(1)$ symmetry is restored either due to finite temperature corrections to the potential or nonthermally. This second possibility is naturally implemented in hybrid inflation, where the field responsible for the $U(1)$ breaking is coupled to the inflaton $\psi$, and the symmetry gets restored at high values of $\psi$.

When the symmetry breaks, a string network is expected to form. Eventually, string loops are generated from the network. The oscillations of these loops act as a source of quanta of the modulus field. The rate of quanta emitted by a single loop has been estimated by Srednicki and Theisen [19], and we will briefly review this calculation in section 2. This computation applies if one can neglect the coupling between the loop and the thermal background. This turns out to be the case only at sufficiently late times, when the string network enters the so-called scaling regime (see below for details). In section 2 we then compute the total number density of quanta produced in this regime. Although we neglect the possible production at earlier times, we will see that the bound (1) can impose a significant upper limit on the coupling $\alpha$ between the string and the modulus field. In section 4, on the other hand, we assume $\alpha \sim 1$, such that an upper bound on $\eta$ is derived, by also taking into account effects related to cusp annihilations. We compare this bound with the corresponding one given in ref. [18]. Our conclusions are finally presented in section 5.

2 Rate of quanta emission by an oscillating loop

In this section we perform the calculation of the rate of emission of quanta of the scalar $s$ by an oscillating Nambu–Goto string loop. The interaction terms we consider are given in eq. (3). The calculation is analogous to the one presented in ref. [19] for the case of the coupling $s^2 |\phi|^2$. 

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We always consider a regime in which the string thickness can be neglected. In this approximation, a reparametrization invariant expression for the expectation value of \((|φ|−η)^n\) in the string state \(|S⟩\) is given by

\[
⟨S| (|φ(x^μ)| − η)^n |S⟩ \simeq η^n a^2 \int_0^L dσ \int dτ \sqrt{-\det g} δ^{(4)} (x^μ − s^μ(σ, τ)) ,
\]

where \(a \simeq (η \sqrt{λ})^{-1}\) is the string thickness, \(σ\) and \(τ\) are coordinates on the string worldsheet, and \(s^μ\) denotes the location of the core of the string. Since we are considering a closed string loop, \(σ\) ranges from 0 to \(L\). Finally, \(\det g\) is the determinant of the induced metric on the string worldsheet.

The modulus \(s\), of mass \(m_s\), is coupled to the string via the trilinear interaction (5). As a consequence, the amplitude of transition from a string state \(|S⟩\) to a state \(|S', s(k^μ)⟩\) containing the string and a quantum of \(s\) of momentum \(k^μ\) is given, via LSZ reduction, by

\[
⟨S', s(k^μ) |S⟩ \simeq \int d^4x e^{i k^μ x^μ} ⟨S' | \left( \partial^μ \partial_μ + m_s^2 \right) s(x^μ) |S⟩ \simeq \lambda \alpha \frac{η^2}{M_P} \int d^4x e^{i k^μ x^μ} ⟨S' | (|φ| − η)^2 |S⟩ .
\]

To proceed, we assume that \(|S'⟩ \simeq |S⟩\) (that is, that the emission of a quantum of \(S\) does not alter substantially the string wave function). Then, in eq. (4), we choose a frame in which \(τ = t = s^0(σ, t)\), such that, after integration over \(d^3x\), we obtain

\[
⟨S', s(k^μ) |S⟩ \simeq \alpha \frac{η^2}{M_P} \int dt e^{iEt} \int_0^L dσ |\vec{s}'(σ, t)|^2 e^{-i\vec{k}•s(σ, t)} ,
\]

where \(E = k^0\) is the energy of the quantum of \(s\) emitted, and \(\vec{k}\) its momentum. We denote with a prime the derivative with respect to \(σ\) and with a dot the derivative with respect to \(t\).

The dynamical equations for the Nambu–Goto string read

\[
\ddot{\vec{s}} − \dot{s}'' = 0 , \quad \left(\dot{s} ± \dot{s}'\right)^2 = 1 ,
\]

and the periodicity of the loop requires \(\vec{s}(σ, t) = \vec{s}(σ + L, t)\). From eqs. (4), one can also show \([9]\) that \(\vec{s}(σ + L/2, t + L/2) = \vec{s}(σ, t)\), so that the motion of the string loop has actually period \(L/2\). Hence, we can decompose

\[
\int_0^L dσ |s'(σ, t)|^2 e^{-i\vec{k}•s(σ, t)} \equiv \sum_n a_n (k) e^{-4πint/L} .
\]
As a consequence, the power emitted by the string into quanta of $s$ can be expressed as

$$
\frac{dE_s}{dt} T = \int \frac{d^3k}{(2\pi)^3} \frac{E}{2E} |\langle S', s(k^\mu) | S \rangle|^2 = T \sum_{n=0}^{\infty} P_n , \quad (11)
$$

where $T = 2\pi \delta(0)$ is the total duration of the process, and

$$
P_n = 2\pi \left( \frac{\alpha \eta^2}{M_P} \right)^2 \int \frac{d^3k}{(2\pi)^3} \frac{E}{2E} |a_n|^2 \delta(E - 4\pi n/L) . \quad (12)
$$

The coefficients $a_n$

$$
a_n \equiv \frac{2}{L} \int_0^{L/2} dt \int_0^L d\sigma e^{iEt-ik\vec{s}} |\vec{s}'(\sigma, t)|^2 \quad (13)
$$
can be estimated via the stationary phase method [19], which consists in integrating the function $|\vec{s}'(\sigma, t)|^2$ only over the regions in which the phase $\psi \equiv Et - \vec{k} \cdot \vec{s}$ is nearly constant. Out of these regions, the phase $\psi$ rapidly oscillates, so that the integral there averages to negligible values. Requiring $\psi' = 0$ forces $\vec{s}' = 0$, which in turn implies $|\vec{s}| = 1$. Hence, at these points the string moves at the speed of light and, provided $\vec{s}''$ is nonvanishing, takes the appearance of a cusp.

Let us consider one of these points and choose coordinates such that $t = \sigma = 0$, $\vec{s} = \vec{0}$ there. For definiteness, the region of nearly constant phase will be considered the one for which $|\psi| < 1$. By expanding $\psi$ at small $t$ and $\sigma$, it is possible to show [19] that this region extends up to $|\sigma|, |t| \lesssim \sigma_{\text{max}} = L (LE)^{-1/3}$, and that the coefficients $a_n$ can be estimated to be $a_n \sim L (LE)^{-4/3}$. One can also show [19] that these regions are actually nonvanishing only provided that $\vec{k}$ lies in a cone centered around $\vec{s}$ and of amplitude $\epsilon \simeq (LE)^{-1/3}$, and provided that $E > E_{\text{min}} \equiv m_s^3/2 L^{1/2}$.

From these results, the coefficients $P_n$ can be (somewhat crudely, neglecting all the numerical coefficients) estimated to be

$$
P_n \sim \alpha^2 \left( \frac{\eta^2}{M_P} \right)^2 \int k^2 dk d\Omega |a_n|^2 \delta(E - 4\pi n/L) \sim \alpha^2 \left( \frac{\eta^2}{M_P} \right)^2 E^2 \epsilon^2 |a_n|^2 \sim \alpha^2 \left( \frac{\eta^2}{M_P} \right)^2 \frac{1}{n^{4/3}}, \quad (14)
$$

where the behaviour $P_n \propto n^{-4/3}$ is in agreement with the analogous expression in [18].

The total power emitted by the string is obtained by summing over the modes $n$. We distinguish two cases. For $L > m_s^{-1}$, the sum starts from $n \simeq E_{\text{min}} L \simeq (m_s L)^{3/2}$ (notice that we have $E \geq E_{\text{min}} > m_S$ in this case). In the opposite case, $L \leq m_s^{-1}$, one simply
starts from $n = 1$ (which also ensures $E \geq L^{-1} > m_s$). By approximating the sum over $n$ with an integral, we get

\[ P_s \equiv \frac{dE_s}{dt} \sim \begin{cases} \alpha^2 \left( \frac{n^2}{M_P^2} \right)^2 \frac{1}{\sqrt{m_s L}}, & L \gg 1/m_s, \\ \alpha^2 \left( \frac{n^2}{M_P^2} \right)^2 \frac{1}{m_s m L}, & L \ll 1/m_s. \end{cases} \]  

(15)

In an analogous way, it is possible to estimate the number of quanta of $s$ emitted per unit time by the string oscillations

\[ \frac{dN_s}{dt} \sim \begin{cases} \alpha^2 \left( \frac{n^2}{M_P^2} \right)^2 \frac{1}{m_s m L}, & L \gg 1/m_s, \\ \alpha^2 \left( \frac{n^2}{M_P^2} \right)^2 \frac{1}{m_s} (m_s L), & L \ll 1/m_s. \end{cases} \]  

(16)

To compute the total number of quanta of $s$ emitted, this last expression must be integrated over the life of the loop. We do so for a regime in which the interaction of the loop with the thermal background can be neglected (that is, after the end of the so called “friction dominated regime”, see section 3 for more details). In this case, the evolution of the loops is determined only by their tension and by their decays.

Besides the production of quanta of $s$, the loop decays by emission of gravitons, with energy per unit time $P_g = \Gamma \eta^4/M_P^2$, where $\Gamma$ is a numerical factor (of the order of $50-100$) dependent on the shape of the loop [9]. Moreover, we consider the possibility that $\phi$ is coupled with some other scalar field $w$ (representing for example an MSSM field) via the interaction term $g \phi^2 w^2$. From this quartic coupling, the energy per unit time of quanta of $w$ produced by the loop is $P_w = \left( g^2 / \lambda^{3/2} \right) \eta / L$ [9]. The decay into quanta of $s$ is always subdominant with respect to the one into gravitons, provided that $\alpha^2 / \Gamma \ll 1$ (in the following, this relation will be always assumed to hold). By comparing $P_g$ and $P_w$, one instead sees that the loop mainly decays into gravitons as long as its radius satisfies

\[ L > \bar{L} \equiv \frac{g^2}{\lambda^{3/2}} \frac{1}{\Gamma} \frac{M_P^2}{\eta^3} \simeq 10^{-6} \text{ TeV}^{-1} \frac{g^2}{\lambda^{3/2}} \left( \frac{100}{\Gamma} \right) \left( \frac{10^{15} \text{ GeV}}{\eta} \right)^3, \]  

(17)

while the decay into quanta of $w$ dominates for smaller lengths. We conclude that, as long as $L \gtrsim \bar{L}$, the radius of the loop reduces according to

\[ L(t) \simeq L_F - \Gamma \frac{\eta^2}{M_P^2} (t - t_F) \quad \text{,} \quad \bar{L} < L < L_F, \]  

(18)

where $L_F = L(t_F)$ is the radius of the loop when it forms. From here on, we will assume that loops form with $L_F \gtrsim \bar{L}$ since this will be the case in the main application we will \footnote{Notice that, for $L \lesssim 1/m_s$, the following expression agrees, up to numerical factors, with the power emitted into gravitational waves $P_g$ (see below). Indeed, this is expected, since this regime corresponds to emission of one massless particle, as in the case of graviton production [9].}
consider in the next section. More precisely, since we will be interested in the production of quanta of $s$ with mass at the TeV scale, we assume here the hierarchy $L_F > m_s^{-1} > L$, the latter inequality being satisfied as long as $\eta$ is not much smaller than $10^{15}$ GeV, see eq. (17).

By combining eqs. (16) and (18), one sees that the quanta of $s$ are mainly produced when $L \sim m_s^{-1}$. For $L_F \gg m_s^{-1}$, their total number is approximatively given by

$$N_s(L_F) \simeq \frac{\alpha^2 \eta^2}{1 - m_s^2 \ln (L_F m_s)}.$$  

(19)

It is worth remarking that – even if we have assumed a gravitationally suppressed coupling between the string and the field $s$, see eq. (5) – the final amount $N_s$ of moduli emitted by a string loop is not Planck mass suppressed. This is due to the fact that, in the regime we are considering, the loop decays gravitationally, so its lifetime becomes infinite in the limit $M_P \to \infty$ [17].

In ref. [20] it has been noted that the results of [19] may be affected by nonperturbative processes leading to the annihilation of the cusps. Indeed, it was observed [20] that different portions of the loop in a region centered at the cusp and characterized by $|\sigma| \leq \sigma_c \equiv \eta^{-1/3} L^{2/3}$ have a mutual distance smaller than the width of the string. Thus, the approximation of a delta–like string breaks down in this region, and the simple Nambu–Goto dynamics cannot be trusted. This effectively puts an ultraviolet cut–off on the decomposition (10). To investigate the validity of the results (15) and (16), we have to ensure that the harmonics which dominate the production of the quanta of $s$ are below this cut-off, or, in other words, that the part of the loop responsible for the production is much longer than the part where nonperturbative phenomena are expected to occur. This is the case for $\sigma_{\text{max}} \gg \sigma_c$, which is in turn satisfied provided that the typical energy $E$ of the quanta at their emission is much smaller than the scale of the $U(1)$ symmetry breaking $\eta$. As we have seen in this section, the quanta of $s$ are mainly produced when the radius of the loop has the size of their inverse mass, $L \sim m_s^{-1}$. We have also seen that the typical energy of the quanta emitted at that time is $E \sim m_s^{3/2} L^{1/2} \sim m_s$, which is indeed much smaller than $\eta$. We remark that in the work [21] it has been shown that the Lorentz contraction of the thickness of the string reduces the portion of the loop where the overlapping occurs to $\sigma_c \sim \eta^{-1/2} L^{1/2}$. Thus, $a \text{ fortiori}$, $\sigma_{\text{max}} \gg \sigma_c$ also in this case.

This analysis confirms the validity of the results (15) and (16), also when nonperturbative effects are taken into account. The reason is that we have considered production of single quanta of $s$ via the trilinear coupling (5). We have seen that this production is dominated by the lowest harmonics $n$ in the sum (10). As we have discussed before eq. (13), for radii $L \sim m_s^{-1}$ the main production comes indeed from collective oscillations of the whole loop.
The production of two quanta from a quadrilinear coupling $\phi^2 s^2$ which was studied in [19] is instead dominated by the highest possible harmonics. In this case nonperturbative effects at small scales might be relevant [20].

To conclude this section, it is also useful to compute the total energy emitted into quanta of $w$. Assuming that these particles rapidly thermalize, it may (at least in principle) give a sizeable contribution to the entropy of the Universe. From the given $P_g$ and $P_w$, we find

$$E_w \simeq \frac{g^2 M_P^2}{\lambda^{3/2} \Gamma \eta} \ln \left( \frac{L_F}{L} \right).$$

(20)

We will see in the next section that, in the range of parameters we are considering, these quanta will give a negligible contribution to the background energy density and entropy of the Universe.

### 3 Quanta produced in the scaling dominated regime

Equation (19) gives an order of magnitude estimate of the number of moduli emitted by a loop that obeys the Nambu-Goto dynamics. In order to analyse the cosmological effects of the moduli that have been produced, we now need to estimate the number density of the loops.

The evolution of a string network can roughly be divided into two stages. In the initial, friction dominated stage, the interaction of every string segment with the background plasma cannot be neglected. In this regime, the frictional drag of the background plasma smoothens out all structures on lengthscales below $L_{\text{min}} \sim \eta M_P^{1/2} / T^{5/2}$ [9]. The dynamics of the loops cannot be approximated as the one of Nambu-Goto strings, and therefore the analysis that led to eq. (19) cannot be trusted. Even if we expect that also in this epoch the decay of string loops will lead to production of moduli, we will make the conservative assumption of neglecting the moduli produced during the friction dominated regime.

The friction dominated stage comes to an end when the scale $L_{\text{min}}$ becomes comparable with the Hubble length, that is, when the temperature of the universe drops below $T_{\text{fr}} \sim \eta^2 / M_P$ [4]. In the subsequent regime the dynamics of loops smaller than the Hubble radius is well approximated by the Nambu-Goto dynamics. The network is then expected to reach a scaling regime, in which the energy in strings is a constant fraction of the background energy in the Universe [9].

The system we are considering consists of cosmic strings formed at the end of the inflationary phase, with a mass per unit length of about $\eta^2 \simeq (10^{15} \text{GeV})^2$. For those strings, the temperature at which the scaling regime begins is of the order of $\eta^2 / M_P \simeq 10^{12} \text{GeV}$.
However, due to thermal production of moduli, the reheating temperature is constrained to be smaller than about $10^9$ GeV. As a consequence, when radiation domination occurs, the string network is already in a regime in which the dynamics of the single loop is the Nambu–Goto dynamics.

For a string network evolved in a radiation dominated Universe, the number density of loops of strings created per unit time during the scaling regime is given by

$$\frac{dn_L}{dt} = \nu \left( \frac{\kappa - 1}{\Gamma (\eta^2/M_P^2)} \right)^{1/4} t, \tag{21}$$

where $\nu$ is a constant of order one and $\kappa$ is a constant in the range $2 < \kappa < 10$. In this regime, the typical length of a loop formed at the time $t$ amounts to

$$L_F(t) \simeq (\kappa - 1) \Gamma \frac{\eta^2}{M_P^2} t. \tag{22}$$

We have therefore all the elements to compute the total number of moduli produced by the string network. Notice, however, that the above formulae rely on the hypothesis that the string network evolved in a radiation-dominated, thermalized background. Actually, this is not the case for a network formed at the end of hybrid inflation, that evolved during the reheating period. Nevertheless, it is likely that a string network will be present after the end of reheating. In order to be able to estimate the abundance of moduli produced by the network, we will make the assumption that also in our case eqs. (21) and (22) become valid, once a thermal bath is formed and the radiation dominated period starts. Moreover, since the dynamics of the loops and of the network during reheating is unknown, we will conservatively neglect all the processes that could have led to production of quanta of $s$ during this period.

Loops that form at the temperature $T_{rh}$ have a typical radius given by eq. (22). This length is larger than $1/m_s$ provided that

$$\eta \gtrsim 10^{15} \text{GeV} \left( \frac{T_{rh}}{10^9 \text{ GeV}} \right) \left( \frac{\text{TeV}}{m_s} \right)^{1/2}, \tag{23}$$

where we have set $\nu = 1$, $(\kappa - 1) = 10$, $\Gamma = 100$. In order to simplify the discussion, we will assume that the above condition is satisfied. The number of particles emitted by every loop will be given, neglecting the logarithmic factor in eq. (19), by

$$N_s \sim \alpha^2 \frac{\eta^2}{\Gamma m_s^2}. \tag{24}$$

Different estimates can be found in the literature (see e.g. [23]) for the quantity $dn_L/dt$, with higher powers of $\eta/M_P$ at the denominator. Since this would enhance the production of quanta of $s$, in the following analysis we will conservatively assume the validity of eq. (21).
Hence, the number density of moduli produced in the time interval $t_F < t < t_F + dt_F$, and redshifted to the time $t_\ast = M_P/T_\ast^2 > t_F$, amounts to

$$dn_s \sim N_s \frac{dN_L}{dt} (t_F) dt_F \left( \frac{t_F}{t_\ast} \right)^{3/2} \sim \alpha^2 \nu \left( \frac{M_P^2}{m_s^2} \right) \frac{M_P^2}{M_P^{3/2} t_F^{5/2} t_\ast^3} dt_F t_\ast^{3/2} t_\ast^3. \quad (25)$$

This last expression has then to be integrated over the time $t_F$ at which loops form. It is clear that the earliest possible times $t_\ast$ dominate the integral. According to the above discussion, we set $t_\ast$ to be the time at which reheating ends. Therefore, dividing the total number $n_s$ of moduli produced by the entropy $s \sim T_\ast^3$, we get

$$Y_s \equiv \frac{n_s}{s} \sim \frac{\alpha^2 \nu}{(k-1)^2} \frac{T_\ast^3}{m_s^2 M_P} \sim 10^{-2} \alpha^2 \left( \frac{T_{\text{rh}}}{10^9 \text{GeV}} \right)^3 \left( \frac{\text{TeV}}{m_s} \right)^2. \quad (27)$$

Therefore, we see that for the typical values $T_{\text{rh}} \simeq 10^9 \text{GeV}$, $m_s \simeq \text{TeV}$, $\eta \simeq 10^{15} \text{GeV}$, the coupling $\alpha$ is constrained to be smaller than about $10^{-6}$ (whereas it could be expected to be of the order of unity) if we do not want to violate the nucleosynthesis bound (1).

\section{Bounds on the string scale}

As we have seen in the previous section, if the scale $\eta$ of the string network is the natural scale of hybrid inflation, i.e. above about $10^{15} \text{GeV}$, a strong constraint is imposed either on the reheating temperature or on the coupling $\alpha$. In the present section, on the other hand, we will assume that the coupling $\alpha$ gets its “natural” value of the order of unity and that $T_{\text{rh}} \simeq 10^9 \text{GeV}$, and we will therefore derive a constraint on the scale $\eta$. An estimate in this sense has been carried out in \cite{18}, where, for modulus mass of the order of the TeV, the constraint $\eta \lesssim 10^{11} \text{GeV}$ was obtained. In obtaining this result, the authors of ref. \cite{18} referred to a regime in which the length of the string loops is much smaller than the Compton wavelength of the modulus $\sim m_s^{-1}$. However, it may be possible that other decay channels are relevant for such small loops, so that the time evolution of the loop length is not given anymore by the expression (18). This will in turn affect the total number of moduli emitted by a single string loop during its life, eventually modifying the bounds obtained in \cite{18}.

\footnote{With the analogous computation, one can show that – starting from eq. (20) – the energy density in quanta of $w$ produced by the loops and redshifted to the time $t_\ast$ amounts to

$$\rho_w \simeq \frac{g^2 \nu}{\lambda^{3/2} (k-1)} \frac{M_P T_{\text{rh}}^2}{\eta^3} T_\ast^4 \sim 10^{-14} \frac{g^2}{\lambda^{3/2}} \left( \frac{10^{15} \text{GeV}}{\eta} \right)^3 \left( \frac{T_{\text{rh}}}{10^9 \text{GeV}} \right)^2 T_\ast^4. \quad (26)$$

Thus, in the application we are considering, we can neglect the increase of entropy due to the decay of the loops.}
Indeed, a new decay channel that can be expected for the small loops is given by cusp annihilation. As we have discussed in section 2, the description of a delta–like string breaks down when we get close to cusps, where two portions of the strings overlap. As a consequence, the cusp is generically expected to decay into a burst of quanta of the field $\phi$, although the details of this process can be hardly described analytically, and the power emitted by cusp annihilation can be only roughly estimated \[20, 21\]. In the following analysis we will assume the validity of the more conservative estimate given in \[21\]

$$P_c \sim \eta^2 \frac{1}{\sqrt{\eta L}}.$$  \hspace{1cm} (28)

When comparing $P_c$ with $P_g$, we see that the dominant decay channel for the string loop is given by cusp annihilation for loops whose length satisfies

$$L \lesssim L_c = \frac{M_p^4}{\Gamma^2 \eta^5}.$$ \hspace{1cm} (29)

For $m_s \sim \text{TeV}$, the length $L_c$ is larger than $m_s^{-1}$ only if $\eta \lesssim 10^{15} \text{GeV}$. We thus see that the annihilation of the cusps does not affect the results presented in the previous section, where $\eta$ was taken to be slightly above this value. However, for smaller $\eta$, loops smaller than about $1/m_s$ will evolve as

$$\eta^2 \frac{dL}{dt} \sim \eta^2 \frac{1}{\sqrt{\eta L}}.$$ \hspace{1cm} (30)

rather than according to eq. (18). As a consequence, in this regime, the total number of moduli emitted by loops formed with a length $L_F$ will be given by

$$N_s \simeq \begin{cases} \alpha^2 m_s^{-1} \frac{m_s^{9/2} L_F^{5/2}}{m_s^2}, & m_s^{-1} \lesssim L_F \lesssim L_c, \\ \alpha^2 \frac{M_p^4}{\Gamma^2 \eta^9/2} L_F^{5/2}, & L_F \lesssim m_s^{-1} \lesssim L_c. \end{cases}$$ \hspace{1cm} (31)

Using the above formulae, it is possible to analyze systematically the production of moduli that can have been occurred during the various cosmological eras. As in the previous one, also in the present section we will neglect particle production that could have been occurred during reheating and during the friction dominated era (notice that the same conservative hypothesis is made in \[13\]). In this case, the analysis of the total number of moduli emitted by the string network shows that they are mainly produced when the typical length of loops formed by the network is of the order of $m_s^{-1}$, that is when the temperature of the Universe is about $T \sim 10^{-7} \eta$ (for the usual values $m_s \sim \text{TeV}$, $(\kappa - 1) \sim 10$, $\Gamma \sim 100$). Then, the abundance of moduli produced for $\eta \lesssim 10^{14} \text{GeV}$ turns then out to be given by

$$Y_s \sim 10^{-5} \alpha^2 \left( \frac{\eta}{10^{15} \text{GeV}} \right)^{11/2}.$$ \hspace{1cm} (32)
This result, for $\alpha \sim 1$, implies a bound

$$\eta \lesssim 10^{13} \text{ GeV}$$

(33)

for a cosmic string network that is coupled via $1/M_P$ coupling to scalar moduli of mass of the order of the TeV. Notice that this bound is actually weaker by a couple of orders of magnitude than the analogous one presented in ref. [18]. The difference can be simply accounted for by the total number of moduli emitted by a string loop: if cusp annihilation effects are neglected, one obtains that, for loops of length $\sim m_s^{-1}$, $N_s \sim \alpha^2 \eta^2/m_s^2$ [18], whereas, if cusp annihilation is assumed to be effective, eq. (31) gives $N_s \sim \alpha^2 \eta^{9/2}/(M_P^2 m_s^{5/2})$ that, for the values of $\eta$ we are interested in, leads to a much less efficient production of moduli.

5 Conclusions

We have considered the production of scalar moduli by a network of cosmic strings. In particular, we have analyzed the simple case of local, nonsuperconducting strings that obey the Nambu-Goto dynamics. We considered only the production that took place in cosmological epochs in which the interaction with the background plasma did not significantly affect loop dynamics, and during which we disposed of a reliable estimate of the number density of loops present in the Universe. Despite the above conservative assumptions, the production can be rather efficient. In particular, a cosmic string network formed at the end of hybrid inflation at a scale of about $10^{15}$ GeV can lead to a moduli abundance that exceeds by several orders of magnitude the Big Bang Nucleosynthesis bound. Such conclusions can be avoided either by assuming a low reheating temperature ($T_{\text{rh}} \lesssim 10^6$ GeV), or by requiring that the string coupling to moduli is much smaller than the typical gravitational coupling $1/M_P$. This second option is certainly a possibility, since the coupling of moduli to the fields which form the strings is a model dependent quantity, although it is hard to expect such a small coupling to arise naturally. Actually, if we assume $\alpha \sim 1$ (and take $T_{\text{rh}} \simeq 10^9$ GeV), the energy scale of the string has to be smaller than about $10^{13}$ GeV not to conflict with Big Bang Nucleosynthesis. This bound is of few orders of magnitude weaker than the one reported in the previous literature [18], due to the fact that cusp annihilation effects can lead to a faster decay of loops of strings, thus decreasing the amount of moduli produced.

In our computation, we have made an important distinction between processes characterized by the production of a single modulus (from a trilinear vertex $\phi^2 s$) and processes with the production of two quanta (from quartic vertices $\phi^2 s^2$). In the former case, the production is mainly due to collective oscillations of the whole loop, while in the latter high
frequency and small scale oscillations dominate. In this second case, the computation may be affected by nonperturbative phenomena leading to cusp annihilation which occur when high frequency oscillations lead different portions of the loop to be closer to each other than the width of the loop itself. The bounds we have derived come from single quanta emissions, which are insensitive to these small scale effects.

As we have stressed in the Introduction, moduli fields are not the only gravitational relics that one should be worried about. In particular, as was also noted in [7], production of gravitinos by the oscillating loops may turn out very efficient, since in this case we know the coupling to be of gravitational strength. The production of gravitinos is technically more involved than the one for scalar particles presented here. Moreover, being a two particles production, nonperturbative effects may influence the final result.

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