SDLCQ and String/Field Theory Correspondences*

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String/Field theory correspondences have been discussed heavily in recent years. Here, we describe a testing scenario involving a non-perturbative field theory calculation using the framework of supersymmetric discrete light-cone quantization (SDLCQ). We consider a Maldacena-type conjecture applied to the near horizon geometry of a D1-brane in the supergravity approximation. Numerical results of a test of this conjecture are presented with orders of magnitude more states than we previously considered. These results support the Maldacena conjecture and are within 10-15% of the predicted results. We present a method for using a “flavor” symmetry to greatly reduce the size of the Fock basis and discuss a numerical method that we use which is particularly well suited for this type of matrix element calculation. Our results are still not sufficient to demonstrate convergence, and, therefore, cannot be considered to be a numerical proof of the conjecture. We update our continuous efforts to improve on these results and present some results on the way to higher dimensional scenarios.

1. Introduction

The conjecture that certain field theories admit concrete realizations as string theories on particular backgrounds has caused a lot of excitement in the last years. Originally, the so-called Maldacena conjecture [1] assured that the \( \mathcal{N} = 4 \) supersymmetric Yang-Mills (SYM) theory in 3+1 dimensions is equivalent to Type IIB string theory on an \( \text{AdS}_5 \times S^5 \) background. Meanwhile, other string/field theory correspondences have been conjectured. Attempts to rigorously test these conjectures have met with limited success, because our understanding of both sides of the correspondences is usually insufficient. The main obstacle is that at the point of correspondence, we require two conditions to hold which are mutually exclusive. Namely, we want a situation where the curvature of the considered spacetime is small in order to be able to use the supergravity approximation to string theory. One also desires the corresponding field theory to be in a small coupling regime. So far it has been impossible to find such a scenario. We present a way out of this dilemma by relaxing the second requirement and performing a non-perturbative calculation on the field theory side. To create a manageable situation, we chose a special string/field theory correspondence in order to apply the non-perturbative method, namely SDLCQ, at its optimal working point.

SDLCQ, or Supersymmetric Discretized Light-Cone Quantization, is a non-perturbative method for solving bound-state problems that has been shown to have excellent convergence properties, in particular in low dimensions. Therefore, we are looking for a (preferably) two-dimensional field theory, which is conjectured to be equivalent to a string theory. It turns out that the Yang-Mills theory with 16 supercharges in two dimensions [2] has its corresponding string theory partner in a system of D1-branes in Type IIB string theory decoupling from gravity [5]. Since both systems have separately been studied in the literature already, this system is an optimal candidate to study the string/field theory correspondence.

The next step is to find an observable that can be computed relatively easy on both sides of the correspondence. It turns out that the correlation function of a gauge invariant operator is a well-behaved object in this sense. We chose the stress-energy tensor \( T^{\mu\nu} \) as this operator and will construct this observable in the supergravity approximation to string theory and perform a non-perturbative SDLCQ calculation of this correlator on the field theory side.

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2. Correlation functions from supergravity

It is instructive to take a closer look on the expected properties of $\mathcal{N} = (8,8)$ SYM, before we proceed to technical details on the string theory side. In the extreme ultra-violet (UV) this theory is conformally free and has a central charge $c_{UV} = N_c^2$. Perturbation theory in turn will be valid for small effective couplings $g = g_{YM} \sqrt{N_c} x$, where $x$ is a space coordinate. For large distances, in the far infra-red (IR), the theory becomes a conformal $\sigma$-model with target space $(R^8)^{N_c}/S_{N_c}$. The central charge is $c_{IR} = N_c$. It is a bit more involved to show that here perturbation theory breaks down when $x \sim \sqrt{N_c}/g_{YM}$, see e.g. Ref. [3].

The intermediate region, $1/g_{YM} \sqrt{N_c} < x < \sqrt{N_c}/g_{YM}$, where no perturbative field theoretical description is possible, is fortunately exactly the region which is accessible to string theory; or rather, to the supergravity (SUGRA) approximation to Type IIB string theory on a special background. It is that of the near horizon geometry of a D1-brane in the string frame, which has the metric

$$ds^2 = \alpha' g_{YM} \left( \frac{U^3}{g_{YM}^2} dx_\perp^2 + \frac{dU^2}{U^3} + U d\Omega_{8-p}^2 \right)$$
$$e^\phi = \frac{2 \pi g_{YM}^2}{U^3} g_{YM}.$$

(1)

where we defined $g_{YM} \equiv 8 \pi^{3/2} g_{YM} \sqrt{N_c}$. In the description of the computation of the two-point function we follow Ref. [3]. The correlator has been derived in Ref. [3], being itself a generalization of Refs. [4,5]. First, we need to know the action of the diagonal fluctuations around this background to the quadratic order. We would like to use the analogue of Ref. [3] for our background, Eq. (1), which is not (yet) available in the literature. However, we can identify some diagonal fluctuating degrees of freedom by following the work on black hole absorption cross-sections [4,5]. One can show that the fluctuations parameterized like

$$ds^2 = \left( 1 + f(x^0, U) + g(x^0, U) \right) g_{00}(dx^0)^2 + \left( 1 + 5 f(x^0, U) + g(x^0, U) \right) g_{11}(dx^1)^2 + \left( 1 + f(x^0, U) + g(x^0, U) \right) g_{UU} dU^2 + \left( 1 + f(x^0, U) - \frac{5}{7} g(x^0, U) \right) g_{00} d\Omega_7^2$$
$$e^\phi = \left( 1 + 3 f(x^0, U) - g(x^0, U) \right) e^{\phi_0},$$

(2)

satisfy the following equations of motion

$$f''(U) = -\frac{7}{U} f'(U) + \frac{g_{00} k^2}{U^3} f(U)$$
$$g''(U) = -\frac{7}{U} g'(U) + \frac{72}{U^2} g(U) + \frac{g_{00} k^2}{U^6} g(U).$$

Without loss of generality we have assumed here that these fluctuation vary only along the $x^0$ direction of the world volume coordinates, and behave like a plane wave. One can interpret a D1-brane as a black hole in nine dimensions. The fields $f(U)$ and $g(U)$ are then the minimal and the fixed scalars in this black hole geometry. In ten dimensions, however, we see that they are really part of the gravitational fluctuation. Consequently, we expect that they are associated with the stress-energy tensor in the operator field correspondence of Refs. [4,5,6]. In the case of the cor-

Figure 1. Phase diagram of two-dimensional $\mathcal{N} = (8,8)$ SYM: the theory flows from a CFT in the UV to a conformal $\sigma$-model in the IR. The SUGRA approximation is valid in the intermediate range of distances, $1/g_{YM} \sqrt{N_c} < x < \sqrt{N_c}/g_{YM}$.
Fourier transforming the latter yields the trivial (1/2) analytic contribution in this section. We expect to deviate from we have suppressed. We see that the leading non-
up to a numerical coefficient of order one which

\[ f(k^0, U) \text{ appears to be the one with the longest range since it is the lightest field.} \]

Eq. (3) for \( f(U) \) can be solved explicitly

\[ f(U) = U^{-3} K_{3/2} \left( \frac{\sqrt{g} \gamma M}{2k^2} k \right), \]

where \( K_{3/2}(x) \) is a modified Bessel function. If we take \( f(U) \) to be the analogue of the minimally coupled scalar, we can construct the flux factor

\[ F = \lim_{U_0 \to \infty} \frac{1}{2\kappa_1} \sqrt{g} U e^{-2(\phi - \phi_\infty)} \times \partial_U \log(f(U)) \bigg|_{U=U_0} \]

\[ = \frac{N U_0^2 k^2}{2g_\gamma M} - \frac{N^{3/2} k^3}{4g_Y M} + \ldots \]

up to a numerical coefficient of order one which we have suppressed. We see that the leading non-analytic contribution in \( k^2 \) is due to the \( k^3 \) term. Fourier transforming the latter yields

\[ \langle \mathcal{O}(x)\mathcal{O}(0) \rangle = \frac{N^2}{g_Y M x^5}. \]

This is in line with the discussion at the beginning of this section. We expect to deviate from the trivial \((1/x^4)\) scaling behavior of the correlator at \( x_1 = 1/g_Y M \sqrt{N_c} \) and \( x_2 = \sqrt{N_c} / g_Y M \). This yields the phase diagram in Fig. 1. It is interesting to note that the entire \( N_c \) hierarchy is consistent in the sense of Zamolodchikov’s c-theorem, which assures that the central charges obey \( c(x) > c(y) \), whenever \( x < y \) \cite{11}.

3. The correlator from SDLCQ

Discretized Light-Cone Quantization (DLCQ) preserves supersymmetry at every stage of the calculation if the supercharge rather than the Hamiltonian is diagonalized \cite{12}. The framework of supersymmetric DLCQ (SDLCQ) allows one to use the advantages of light-cone quantization \((e.g. \ a \ simpler \ vacuum) \) together with the excellent renormalization properties guaranteed by supersymmetry. Using SDLCQ, we can reproduce the SUGRA scaling relation, Eq. (6), fix the numerical coefficient, and calculate the cross-over behavior at \( 1/g_Y M \sqrt{N_c} < x < \sqrt{N_c} / g_Y M \). To exclude subtleties, \textit{nota bene} issues of zero modes, we checked our results against the free fermion and the ’t Hooft model and found consistent results.

The technique of (S)DLCQ was reviewed in Ref. \cite{12}, so we can be brief here. The basic idea of light-cone quantization is to parameterize space-time using light-cone coordinates

\[ x^\pm = \frac{1}{\sqrt{2}} (x^0 \pm x^1), \]

and to quantize the theory making \( x^+ \) play the role of time. In the discrete light-cone approach, we require the momentum \( p_- = p^+ \) along the \( x^- \) direction to take on discrete values in units of \( p^+ / K \) where \( p^+ \) is the conserved total momentum of the system. The integer \( K \) is the so-called harmonic resolution, and plays the role of a discretization parameter. One can think of this discretization as a consequence of compactifying the \( x^- \) coordinate on a circle with a period \( 2L = 2\pi K / p^+ \). The advantage of discretizing on the light cone is the fact that the dimension of the Hilbert space becomes finite. Therefore, the Hamiltonian is a finite-dimensional matrix, and its dynamics can be solved explicitly. In SDLCQ one makes the DLCQ approximation to the supercharges \( Q^i \). Surprisingly, also the discrete representations of \( Q_i \) satisfy the supersymmetry algebra. Therefore SDLCQ enjoys the improved renormalization properties of supersymmetric theories. To recover the continuum result, \( K \) has to go to infinity. Incidentally, what one finds is that SDLCQ usually converges faster than the naïve DLCQ towards the continuum limit.

Let us now return to the problem at hand. We would like to compute a general expression for the correlator of the form \( F(x^-, x^+) = \).
\[ (\mathcal{O}(x^-, x^+)\mathcal{O}(0, 0)) \text{ in DLCQ one fixes the total momentum in the } x^- \text{ direction, and it is natural to compute the Fourier transform and express it in a spectrally decomposed form} \]

\[
\tilde{F}(P_-, x^+) = \frac{1}{2L}(\mathcal{O}(P_-, x^+)\mathcal{O}(P_-, 0)) \\
= \sum_n \frac{1}{2L} \langle 0|\mathcal{O}(P_-)|n\rangle e^{-iP_+ x^+} \\
\times \langle n|\mathcal{O}(P_-, 0)|0\rangle. \tag{8}
\]

The form of the correlation function in position space is then recovered by Fourier transforming with respect to \( P_- = K\pi/L \). We can continue to Euclidean space by taking \( r = \sqrt{2x^+x^-} \) to be real. The result for the correlator of the stress-energy tensor is

\[
F(x^-, x^+) = \sum_n \left| \frac{L}{\pi} \langle n|T^{++}(-K)|0\rangle \right|^2 \left( \frac{x^+}{x^-}\right)^2 \\
\times \frac{M^4}{8\pi^2K^3}K_4 \left( M\sqrt{2x^+x^-} \right), \tag{9}
\]

where \( M_i \) is a mass eigenvalue and \( K_4(x) \) is the modified Bessel function of order 4. Note that this quantity depends on the harmonic resolution \( K \), but involves no other unphysical quantities. In particular, the expression is independent of the box length \( L \).

The momentum operator \( T^{++}(x) \) of two-dimensional \( \mathcal{N} = 8 \) SYM is given by

\[
T^{++}(x) = \text{tr} \left[ (\partial_- X^I)^2 + \frac{1}{2} \langle iu^\alpha \partial_- u^\alpha - i(\partial_- u^\alpha)u^\alpha \rangle \right], \tag{10}
\]

with \( I, \alpha = 1, \ldots, 8 \). \( X \) and \( u \) are the physical adjoint scalars and fermions, respectively. When discretized, these operators have the mode expansions

\[
X^I_{i,j} = \frac{1}{\sqrt{A}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} a^I_{ij}(n)e^{-in\pi x^-/L} + a^{\dagger I}_{ij}(n)e^{+in\pi x^-/L}, \tag{11}
\]

\[
u^\alpha_{i,j} = \frac{1}{\sqrt{A}} \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} b^\alpha_{ij}(n)e^{-in\pi x^-/L} + b^{\dagger \alpha}_{ij}(-n)e^{+in\pi x^-/L}. \tag{12}
\]

The matrix element \( (L/\pi)(0)|T^{++}(K)|i \) can be substituted directly to give an explicit expression for the two-point function. We see immediately that the correlator has the correct small-\( r \) behavior, for in that limit, it asymptotes to

\[
\left( \frac{x^+}{x^-}\right)^2 F(x^-, x^+) = \frac{N^2(2n_b + n_f)}{4\pi^2r^4} \left( 1 - \frac{1}{K} \right),
\]

which we expect for the theory of \( n_b(n_f) \) free bosons (fermions) at large \( K \).

On the other hand, the contribution to the correlator from strictly massless states is given by

\[
\left( \frac{x^+}{x^-}\right)^2 F(x^-, x^+) = \frac{6}{K^3r^4} \times \sum_i \left| \frac{L}{\pi} (0)|T^{++}(K)|i \right|^2_{M_i=0}. \tag{13}
\]

It is important to notice that this \( 1/r^4 \) behavior at large \( r \) is not the one we are looking for at large \( r \). First of all, we do not expect any massless physical bound state in this theory, and, additionally, it has the wrong \( N_c \) dependence. Relative to the \( 1/r^4 \) behavior at small \( r \), the \( 1/r^4 \) behavior at large \( r \) that we expect is down by a factor of \( 1/N_c \). This behavior is suppressed because we are performing a large-\( N_c \) calculation. All we can hope is to see the transition from the \( 1/r^4 \) behavior at small \( r \) to the region where the correlator behaves like \( 1/r^5 \).

4. Symmetries and Numerics

In principle, we can now calculate the correlator numerically by evaluating Eq. (8). However, it turns out that even for very modest harmonic resolutions, we face a tremendous numerical task. At \( K = 2, 3, 4 \), the dimension of the associated Fock space is 256, 1632, and 29056, respectively. The usual procedure is to diagonalize the Hamiltonian \( P^- \) and then to evaluate the projection of each eigenfunction on the fundamental state \( T^{++}(-K)|0\rangle \). Since we are only interested in states which have nonzero value of such projection, we are able to significantly reduced our numerical efforts.

In the continuum limit, the result does not depend on which of the eight supercharges \( Q_\alpha \) one
chooses. In DLCQ, however, the situation is a bit subtler: while the spectrum of $(Q^-\alpha)^2$ is the same for all $\alpha$, the wave functions depend on the choice of supercharge $\mathcal{Q}$. This dependence is an artifact of the discretization and disappears in the continuum limit. What happens if we just pick one supercharge, say $Q^-_1$? Since the state $T^{++}(-K)|0\rangle$ is a singlet under R-symmetry acting on the “flavor” index of $Q^-\alpha$, the correlator $\langle\mathcal{P}\rangle$ does not depend on the choice of $\alpha$ even at finite resolution!

We can exploit this fact to simplify our calculations. Consider an operator $\mathcal{S}$ commuting with both $P^-$ and $T^{++}(-K)$, and such that $\mathcal{S}|i\rangle = s_0|i\rangle$. Then the Hamiltonian and $\mathcal{S}$ can be diagonalized simultaneously. We assume in the sequel that the set of states $|i\rangle$ is a result of such a diagonalization. In this case, only states satisfying the condition $\mathcal{S}|i\rangle = s_0|i\rangle$ contribute to the sum in (1), and we only need to diagonalize $P^-$ in this sector, which reduces the size of the problem immensely. We can deduce from the structure of the state $T^{++}(-K)|0\rangle$ that any transformation of the form

$$a_{ij}(k) \rightarrow f(I)a_{ij}(k), \quad f(I) = \pm 1$$

$$b_{ij}^\alpha(k) \rightarrow g(\alpha)b_{ij}^\alpha(k), \quad g(\alpha) = \pm 1$$

(14)

given arbitrary permutations $P$ and $Q$ of the 8 flavor indices, commutes with $T^{++}(-K)$. The vacuum will then be an eigenstate of this transformation with eigenvalue 1. The requirement for $P^- = (Q^-_1)^2$ to be invariant under $\mathcal{S}$ imposes some restrictions on the permutations. In fact, we will require that $Q^-_1$ be invariant under $\mathcal{S}$, in order to guarantee that $P^-$ is invariant.

The form of the supercharge from (1) is

$$Q^- = \int_0^\infty \ldots [\ldots]b_{ij}(k_3)Q_{ij}(k_1)Q_{ij}(k_2) + \ldots$$

(15)

$$+(\beta_1\beta_2^T - \beta_2\beta_1^T)_{\alpha\beta}b_{ij}(k_3)Q_{ij}(k_1)Q_{ij}(k_2) + \ldots$$

Here the $\beta_1$ are $8 \times 8$ real matrices satisfying $\{\beta_1, \beta_2^T\} = 2\delta_{jj'}$. We use the special representation for these matrices given in Ref. (16).

Let us consider the expression for $Q^-_1$, Eq. (13). The first part of the supercharge does not include $\beta$ matrices, and is therefore invariant under the transformation, Eq. (14), as long as $g(1) = 1$ and $Q[1] = 1$. We will consider only such transformations. The crucial observation for the analysis of the symmetries of the $\beta$ terms is that in the representation of the $\beta$ matrices we have chosen, the expression $B_{ij}^\alpha = \langle \beta_j\beta_i^T - \beta_i\beta_j^T \rangle_{\alpha\beta}$ may take only the values $\pm 2$ or zero. Besides, for any pair $(I,J)$ there is only one (or no) value of $\alpha$ corresponding to nonzero $\beta$. Using this information, we may represent $\mathcal{B}$ in a compact form. With the definition

$$\mu_{IJ} = \begin{cases} \alpha, & \mathcal{B}_{ij}^\alpha = 2 \\ -\alpha, & \mathcal{B}_{ij}^\alpha = -2 \\ 0, & \mathcal{B}_{ij}^\alpha = 0 \text{ for all } \alpha \end{cases}$$

(16)

together with the special choice of $\beta$ matrices we get the following expression for $\mu$

$$\mu = \begin{pmatrix} 0 & 5 & -7 & 2 & -6 & 3 & -4 & 8 \\ -5 & 0 & -3 & 6 & 2 & -7 & 8 & 4 \\ 7 & 3 & 0 & -8 & -4 & -5 & 6 & 2 \\ -2 & -6 & 8 & 0 & -5 & 4 & 3 & 7 \\ 6 & -2 & 4 & 5 & 0 & -8 & -7 & 3 \\ -3 & 7 & 5 & -4 & 8 & 0 & -2 & 6 \\ 4 & -8 & -6 & -3 & 7 & 2 & 0 & 5 \\ -8 & -4 & -2 & -7 & -3 & -6 & -5 & 0 \end{pmatrix}$$

The next step is to look for a subset of the transformations, Eq. (14), which satisfy the conditions $g(1) = 1$ and $Q[1] = 1$ and leave the matrix $\mu$ invariant. This invariance implies that $Q[\mu_I\mu_J\mu_{IJ}] = g(\mu_{IJ})f(I)f(J)\mu_{IJ}$. (17)

The subset of transformations we are looking for forms a subgroup $R$ of the permutation group $S_8 \times S_8$. Consequently, we will search for the elements of $R$ that square to one. Products of such elements generate the whole group in the case of $S_8 \times S_8$. We will show later that this remains true for $R$. Not all of the $Z_2$ symmetries satisfying (17) are independent. In particular, if $a$ and $b$ are two such symmetries then $aba$ is also a valid $Z_2$ symmetry. By going systematically through the different possibilities, we have found that there are 7 independent $Z_2$ symmetries in the group $R$. They are listed in Table 1. We explicitly constructed all the symmetries of the type, Eq. (14), which satisfy Eq. (17) using Mathematica. It turns out that the group of such transformations has 168 elements, and we have shown that all of them can be generated from the seven $Z_2$ symmetries mentioned above.
In our numerical algorithm we implemented the $Z_2$ symmetries as follows. We can group the Fock states in classes and treat the whole class as a new state, because all states relevant for the correlator are singlets under the symmetry group $R$. As an example consider the simplest non-trivial singlet $|1\rangle = \frac{1}{8} \sum_{I=1}^{8} \text{tr} (a^{I}(1,1) a^{L}(K-1,1)) |0\rangle$. (18)

Hence, if we encounter the state $a^{I}(1,1) a^{L}(K-1,1) |0\rangle$ while constructing the basis, we will replace it by the class representative; in this case, by the state $|1\rangle$. Such a procedure significantly decreases the size of the basis, while keeping all the information necessary for calculating the correlator. In summary, this use of the discrete flavor symmetry of the problem reduces the size of the Fock space by orders of magnitude.

In addition to these simplifications, one can further improve on the numerical efficiency by using Lanczos diagonalization techniques. Namely, we substitute the explicit diagonalization with an efficient approximation. The idea is to use a symmetry preserving (Lanczos) algorithm. If we start with a normalized vector $|u_1\rangle$ proportional to the fundamental state $T^{++}(-K)|0\rangle$, the Lanczos recursion will produce a tridiagonal representation of the Hamiltonian $H_{LC} = T^+ T^-$. Due to orthogonality of $\{|u_i\rangle\}$, only the $(1,1)$ element of the tridiagonal matrix, $H_{1,1}$, will contribute to the correlator. We exponentiate by diagonalizing $H_{LC} \vec{v}_1 = \lambda_i \vec{v}_1$ with eigenvalues $\lambda_i$ and get

$$F(P^+,x^+) = \frac{|N_0|^{-2}}{2L} \left( \frac{\pi}{L} \right)^2 \sum_{j=1}^{N_L} |\langle v_j |^2 e^{-\lambda_i \frac{L}{\pi r^2}}. $$

Finally, we Fourier transform to obtain

$$F(x^-,x^+) = \frac{1}{8\pi^2 K^2} \left( \frac{x^+}{x^-} \right)^2 \frac{1}{|N_0|^2} \sum_{j=1}^{N_L} |\langle v_j |^2 \lambda_j^2 K_d(\sqrt{2 x^+ x^-} - \lambda_j),$$

which is equivalent to Eq. (8). This algorithm is correct only if the number of Lanczos iterations $N_L$ runs up to the rank of of original matrix. But in praxi already a basis of about 20 vectors covers all leading contribution to correlator [13].

5. Results

To evaluate the expression for the correlator $F(r)$, we have to calculate the mass spectrum and insert it into Eq. (8). In the $\mathcal{N} = (8,8)$ supersymmetric Yang-Mills theory the contribution of massless states becomes a problem. These states exist in the SDLCQ calculation, but are unphysical. It has been shown that these states are not normalizable and that the number of partons in these states is even (odd) [2].

Because the correlator is only sensitive to two particle contributions, the curves $F(r)$ are different for even and odd $K$. Unfortunately, the unphysical states yield also the typical $1/r^4$ behavior, but have a wrong $N_c$ dependence. The regular $1/r^4$ contribution is down by $1/N_c$, so we cannot see this contribution at large $r$, because we are working in the large $N_c$ limit.

We can use this information about the unphysical states, however, to determine when our approximation breaks down. It is the region where
the unphysical massless states dominate the correlator sum. Unfortunately, this is also the region where we expect the true large-\(r\) behavior to dominate the correlator, if only the extra states were absent. In Fig. 2(a) for even resolution, the region where the correlator starts to behave like \(1/r^4\) at large \(r\) is clearly visible. In Fig. 2(b) we see that for even resolution the effect of the massless state on the derivative is felt at smaller values of \(r\) where the even resolution curves start to turn up. Another estimate of where this approximation breaks down, that gives consistent values, is the set of points where the even and odd resolution derivative curves cross. We do not expect these curves to cross on general grounds, based on work in [3], where we considered a number of other theories. Our calculation is consistent in the sense that this breakdown occurs at larger and larger \(r\) as \(K\) grows.

We expect to approach the line \(dF(r)/dr = -1\) line signaling the cross-over from the trivial \(1/r^4\) behavior to the characteristic \(1/r^5\) behavior of the supergravity correlator, Eq. (6). Indeed, the derivative curves in Fig. 2(b) are approaching \(-1\) as we increase the resolution and appear to be about 85 – 90\% of this value before the approximation breaks down. There is, however, no indication of convergence yet; therefore, we cannot claim a numerical proof of the Maldacena conjecture. A safe signature of equivalence of the field and string theories would be if the derivative curve would flatten out at \(-1\) before the approximation breaks down.

6. Conclusions

In this note we reported on progress in an attempt to rigorously test the conjectured equivalence of two-dimensional \(\mathcal{N} = (8, 8)\) supersymmetric Yang-Mills theory and a system of \(D1\)-branes in string theory. Within a well-defined non-perturbative calculation, we obtained results that are within 10-15\% of results expected from the Maldacena conjecture. The results are still not conclusive, but they definitely point in right direction. Compared to previous work [3], we included orders of magnitude more states in our calculation and thus greatly improved the test-
Figure 3. Log-Log plot of the three-dimensional correlation function $f \equiv r^5 \langle T^{++}(x)T^{++}(0) \rangle \left( \frac{r}{2\pi^2} \right)^2 \frac{16\pi^2 K^2}{105\sqrt{-i}}$ vs. $r$ for $g = g_{YM}^2 \sqrt{\frac{\alpha'}{2\pi}}/2\pi^{3/2} = 1.0$ for $K = 6$ and $T = 1$ to 5.

7. Outlook

It remains a challenge to rigorously test the conjectured string/field theory correspondences. Although the so-called Maldacena conjecture maybe the most exciting one, because it promises insight into full four-dimensional Yang-Mills theories in the strong coupling regime, there are other interesting scenarios. For instance, it was conjectured that the supergravity solutions correspond-
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