Electromagnetic cloaking devices for TE and TM polarizations

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New Journal of Physics 10 (2008) 115035 (15pp)
Received 1 July 2008
Published 27 November 2008
Online at http://www.njp.org/
doi:10.1088/1367-2630/10/11/115035

Abstract. In this paper, we present the design of an electromagnetic cloaking device working for both transverse electric (TE) and transverse magnetic (TM) polarizations. The theoretical approach to cloaking used here is inspired by the one presented by Alù and Engheta (2005 Phys. Rev. E 72 016623) for TM polarization. The case of TE polarization is firstly considered and, then, an actual inclusion-based cloak for TE polarization is also designed. In such a case, the cloak is made of a mu-near-zero (MNZ) metamaterial, as the dual counterpart of the epsilon-near-zero (ENZ) material that can be used for purely dielectric objects. The operation and the robustness of the cloaking device for the TE polarization is deeply investigated through a complete set of full-wave numerical simulations. Finally, the design and an application of a cloak operating for both TE and TM polarizations employing both magnetic inclusions and the parallel plate medium already used by Silveirinha \textit{et al} (Phys. Rev. E 75 036603) are presented.

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1. Introduction

Cloaking is one of the most attractive applications of metamaterials. The unusual interaction between the electromagnetic field and the micro/nano-scale inclusions constituting the artificial materials produces new interesting scattering phenomena, which can be useful to reduce the observability of a given object.

Different approaches to cloaking have been developed by different research groups worldwide [1]–[12]. Even if a proper comparison between all these approaches has not been developed so far, we may say that each of them, though based on different physics, clearly show advantages, but also undoubted limitations.

For instance, the theoretical approach based on the coordinate transformation [5]–[10], which is very elegant from the mathematical and physical points of view, works quite well even for large objects, and is independent of the object being cloaked, may find some problems at the fabrication stage, due to the employment of the reduced parameters and to the inhomogeneity of the cloak material. Nevertheless, some experiments have been already conducted with a certain degree of success [10]. As for any metamaterial design, anyway, losses play an important role, as these experiments clearly reveal. Apart from these difficulties, even from the fundamental point of view there are some problems, especially if we are interested in cloaking devices working not only at a single frequency. In the coordinate transformation approach, in fact, the paths of the electromagnetic field circumventing the object are covered with a phase velocity which is greater than the speed of light. Anyway, when a pulsed electromagnetic field impinges on the object covered with the cloak, since the group velocity does not exceed the speed of light, the resulting cloak cannot have the desired functionality over a broad range of frequencies, even if we are able to realize broadband metamaterials. On the other hand, as is well known from microwave circuit theory, losses are inherently related to any impedance transformation, even in the case of continuous transformation. For this reason, cloaks based on this approach must be quite large in order to have a very smooth variation of the parameters and reduce the losses as much as possible.

The approach based on plasmonic materials proposed in [1] is also characterized by advantages and disadvantages. One limitation is that this approach is, to some extent,
Object-dependent. Though some new results have been presented recently, showing that the shape of the object can be changed a little, while keeping the operation of the cloak [3], the object cannot be changed substantially. Another potential drawback of this approach resides in the dimensions of the objects that can be cloaked. Even if the approach as such works for electrically small objects, recently some results have been presented showing how it is possible to increase the object dimensions [4]. As to the losses, this approach has the important advantage of employing homogeneous materials (i.e. there is no need to synthesize an electric/magnetic profile) possibly having a real part of the permittivity close to zero (i.e. the so-called epsilon-near-zero (ENZ) metamaterials) [2]. Looking at the dispersion of the material, in fact, since the operation frequency is close to the plasma frequency and far away from the resonance of the inclusions, losses can be assumed to be rather low. In addition, the dispersion curve close to the plasma frequency has a slow slope and, thus, this approach is characterized by relatively good performance in terms of bandwidth. Finally, from the fabrication point of view, this approach leads to easier practical designs as compared to the one based on coordinate transformation.

In this paper, we consider this second approach, which exploits ENZ metamaterials and works for transverse magnetic (TM) polarization [2], for the case of transverse electric (TE) polarization, by using mu-near-zero (MNZ) metamaterials. At first, we recall the design principles of a cloak for TE polarization made of an ideal homogeneous material. Then, we present the design of the same cloak implemented through real-life magnetic inclusions at microwaves and, finally, we propose the design of an actual cloaking device working for both polarizations (TE and TM) by employing both magnetic inclusions for TE polarization and the parallel-plate medium, as proposed for TM polarization in [2].

2. Electromagnetic formulation of the problem for the TE polarization

The geometry under consideration is reported in figure 1. We have an infinite cylindrical object characterized by a radius \( a \) and a given permittivity \( \varepsilon \) and permeability \( \mu \). We cover the object with a cylindrical shell made of a material having permittivity \( \varepsilon_c \varepsilon_0 \) and permeability \( \mu_c \mu_0 \). The radius of the shell is \( b \). We consider a time harmonic variation of the kind \( \exp[j\omega t] \) and we
assume that a TE polarized plane wave impinges on the structure. The magnetic field of the impinging wave is directed along the axis of the cylinder.

The idea is to reduce the observability of the object, which means to reduce the reflection, the scattering and the absorption. A useful figure of merit for the reduced observability of the object can be, thus, the reduction of the total scattering cross section (SCS) of the structure with respect to that of the bare object. Since the total SCS of an object is given by the sum of the absorption cross section and the SCS and the forward scattering amplitude is related to the total cross section of the scatterer through the application of the optical theorem [13], it is straightforward to design the cloak just considering the reduction of the SCS of the covered object.

As already presented in [1, 2] for the case of TM polarization, the approach is based on the analytical derivation of the SCS of the covered cylinder of figure 1. In the case of TE polarization, the latter quantity can be written in the form:

\[
\sigma_{\text{TE}} = \frac{2\lambda}{\pi} \left| \sum_{n=0}^{\infty} \varepsilon_n c_n^{(\text{TE})} \cos n\varphi \right|^2,
\]

with

\[
\varepsilon_n = \begin{cases} 
1, & n = 0 \\
2, & n \neq 0
\end{cases},
\]

and \(c_n^{(\text{TE})}\) the scattering coefficients of integer order \(n\).

In order to reduce the observability of the cylindrical object in figure 1, this expression of the SCS has to be minimized. When the object is relatively large compared to the wavelength the minimization is usually performed numerically. On the other hand, following the procedure outlined in [1, 2] for the case of the TM polarization, under the assumption that the object is electrically small, the conditions to minimize the SCS of the infinitely long cylinder can be written in closed analytical form as [1]:

\[
\frac{b}{a} \approx \begin{cases} 
\frac{\sqrt{\varepsilon - 1}}{\sqrt{\varepsilon - 1}}, & n = 0, \\
\frac{2\sqrt{(\varepsilon - 1)(\varepsilon + 1)}}{\sqrt{(\varepsilon - 1)(\varepsilon + 1)}}, & n \neq 0.
\end{cases}
\]

These conditions relate the ratio between the radius of the object and the radius of the cylindrical shell with the constitutive parameters of both the object and the cloak.

The results presented so far for the infinitely long cylindrical structure of figure 1 are also valid in the case of a finite length cylinder. It is easy to show, in fact, that the SCS of the finite length cylinder of figure 2 is proportional in the far-field region to the one of the unbounded cylinder of figure 1 as:

\[
\sigma_{3D} \approx \frac{2L^2}{\lambda} \sigma_{2D} \sin^2 \theta_s \text{sinc}^2 \left[ \frac{kL}{2} \left( \cos \theta_i + \cos \theta_s \right) \right]
\]

with \(\text{sinc} (x) = \sin (x)/x\).
3. TE polarization cylindrical cloak made of an ideal MNZ metamaterial: analytical results

Let us consider now the following example. The object to cloak is a cylinder with radius \( a = 10 \text{ mm} \) and with constitutive parameters \( \varepsilon = 2\varepsilon_0, \mu = 2\mu_0, \varepsilon_0 \) and \( \mu_0 \) being the permittivity and the permeability of free-space, respectively. The design frequency is assumed to be \( f_0 = 3 \text{ GHz} \) and the shell radius \( b = 1.8 a \). Following the procedure outlined in the previous section, the numerical minimization of the SCS can be obtained using a cloak made of an ideal homogeneous material with the following set of constitutive parameters: \( \varepsilon_c = 1, \mu_c = 0.1 \).

In order to show the results in a proper way, we use as a figure of merit for the cloak the ratio between the SCS of the bare object and of the object with the cloak put on it:

\[
\sigma_{\text{TE}} = \frac{\sigma_{\text{obj}}}{\sigma_{\text{obj+cloak}}}.
\]

In the following, we show the variation of the quantity \( \sigma_{\text{TE}} \) as a function of different parameters in order to outline the main features of the designed cloak, including the robustness to the change of the electrical and geometrical parameters, and the effect of the dispersion and the losses, which cannot be neglected in any metamaterial design.

In figure 3, we show the variation of \( \sigma_{\text{TE}} \) as a function of the relative permittivity of the cover material. When the permittivity of the object becomes larger and larger, the cross section of the covered cylinder is no longer electrically small compared to the wavelength and, thus, the SCS increases, exceeding the value of that of the bare object. Another interesting aspect to point out from figure 3 is that a slight variation of the cover permittivity from the design value does not affect the cloak performances too much.

In figure 4, we present the variation of \( \sigma_{\text{TE}} \) as a function of the frequency. The cover material is described as an ideal isotropic and homogeneous material having relative permittivity equal to one and a permeability following the Lorentz model, so that at the design frequency
Figure 3. Variation of $\sigma_{\text{TE}}$ as a function of the relative permittivity of the cover material at 3 GHz. The relative permeability of the cover material is set at the design value $\mu_c = 0.1$.

Figure 4. Variation of $\sigma_{\text{TE}}$ as a function of the frequency. The cover material has a relative permittivity equal to one and a relative permeability described by a Lorentz model such that the relative permeability at 3 GHz is $\mu_c = 0.1$. In this case, we have also some losses, due to the imaginary part of the permeability following the Lorentz model. Therefore, the peak of $\sigma_{\text{TE}}$ at the operating frequency is slightly lower with respect to the one in figure 3. It is worth noticing that, as expected, at the resonant frequency of the permeability (around 2 GHz) the SCS of the object with the cover is very much increased compared to that of the bare object.

In figure 5, we show the variation of $\sigma_{\text{TE}}$ with the losses. The relative permeability of the cover material is defined as $\mu_c = \mu'_c - j \mu''_c$ and the plot shows the variation of $\sigma_{\text{TE}}$ with $\mu''_c$, while $\mu'_c$ is kept unchanged and equal to the design value ($\mu'_c = 0.1$). We point out again that, since we are considering MNZ materials, the losses are expected to be anyway rather low.
Figure 5. Variation of $\sigma_{TE}$ as a function of the imaginary part of the relative permeability of the cover material at 3 GHz. The real part of the relative permeability is kept at the design value $\mu'_c = 0.1$.

Figure 6. Variation of $\sigma_{TE}$ as a function of the ratio $\alpha$ between the radius of the cylindrical shell $b$ and the radius of the cylinder $a$ at 3 GHz. The cover material parameters are $\varepsilon_c = 1, \mu_c = 0.1$ at 3 GHz.

Finally, in figure 6 we show the variation of $\sigma_{TE}$ as a function of the ratio $\alpha$ between the radius of the cylindrical shell $b$ and the radius of the cylindrical object $a$. This plot has been obtained keeping all the setup parameters to their design values and varying only the radius of the external cover. The robustness of the cloaking layout here proposed is similar to the one already proposed for TM polarization in [2]. Since the values of $\sigma_{TE}$ are still significantly high for slight changes of $\alpha$, in principle we may conclude that slight geometrical variations at the operating frequency do not affect the behavior of the cloak.
Figure 7. Maximum of the SCS versus frequency in arbitrary units in the case of (a) bare cylinder, (b) cylinder with cloak, for different lengths of the cylindrical structure. The geometrical parameters of the transverse section are \( b = 1.8a \), \( a = 10 \text{ mm} \), while the constitutive parameters of the object are \( \varepsilon = 2\varepsilon_0 \), \( \mu = 2\mu_0 \). The cover material has a relative permittivity equal to one and a relative permeability following the Lorentz model such that at the design frequency of 3 GHz it is \( \mu_c = 0.1 \). The two graphs are in the same scale.

4. TE polarization cylindrical cloak made of an ideal MNZ metamaterial: full-wave simulations

In this section, we verify the analytical results presented in the previous section through full-wave numerical simulations performed through a numerical code based on the finite integration technique [14]. The cover material follows again the Lorentz dispersion, while the cylindrical object, although characterized by the same cross section and constitutive parameters as in the previous section, is assumed this time to have a finite length \( L \). The incident field is a TE plane wave, according to the definition given in figure 1.

In figure 7, we show the maximum SCS versus frequency of both the bare cylinder (figure 7(a)) and the cylinder with the cloak put on it (figure 7(b)). Of course, in this case, the observability of the cylinder is not reduced as much as in the infinite case presented in the previous section, but, as previously anticipated, the correct design of the cross section of the cylindrical structure (object + cloak) reduces the observability of the object for any length of the cylinder. From the results in figure 7(b), in fact, a reduction of the SCS around the design frequency of 3 GHz is clearly evident for any of the cylinder lengths considered.

Obviously, when we increase the length of the cylinder, the results should approach those presented in the previous section. In figure 8, we present the variation of the reduced observability figure of merit \( \sigma_{\text{TE}} \) with the length \( L \) of the cylinder.

5. TE polarization cylindrical cloak made of magnetic inclusions

In this section, we present a real life implementation of the MNZ cover using magnetic inclusions. In order to obtain the desired value of the permeability for the cover material,
we have used spiral resonators (SRs), following the design presented in [15, 16]. The object considered in this example is again a cylinder with radius $a = 10$ mm and length $L = 50$ mm, made of a material with constitutive parameters given by $\varepsilon = 2\varepsilon_0$, $\mu = 2\mu_0$. In order to design the MNZ cover we have considered four columns of SRs disposed as shown in figure 9(a). The single SR inclusion has two turns and an external length of $l_{SR} = 3.5$ mm, the separation between two adjacent turns is 0.6 mm, whereas the metallic strip has a width of 0.3 mm. The dimensions

*New Journal of Physics* **10** (2008) 115035 (http://www.njp.org/)
of the SRs have been designed not only to satisfy the permeability requirement at the cloaking frequency, but also to properly fit the cover thickness. Therefore, the SRs are radially placed exactly at the center of the ideal cylindrical shell.

We have excited the structure through the same plane wave as in the previous section and the full-wave simulations return the results shown in figure 9(b) and in figure 10. From figure 9(b), the reduction of the object observability around the desired frequency is clearly evident. In addition, as expected, at the resonant frequency of the SRs the covered structure scatters a lot and, thus, the observability of the object is indeed increased. Far from the resonance of the SRs and the design frequency, then, the SCS of the covered object approaches that of the bare cylinder. The pattern of the SCS in linear units (figure 10(a)) clearly shows that the dominant scattering term for the bare cylinder is the dipolar one, due to its electrically small dimensions. In figure 10(b) it is clearly evident how the cloak works: the dipolar term is almost suppressed and the higher order terms, which have a significantly lower amplitude, become the dominant ones.

For the sake of completeness, we report in figure 11 also the bi-dimensional plots showing the magnetic field pattern at the cloak frequency in the cases of bare and covered cylinders. In the case of the bare cylinder, the shadow effect is very evident, whereas in the covered case it is reduced, though the field is not perfectly uniform. This is mainly due to the fact that we have used only four columns of SRs here. As shown in the field map of figure 12, in fact, more uniform cloaking patterns can be obtained by increasing the number of SR columns. The reason for the choice to use only four columns will be more clear in the next section, when we will discuss how to implement a cloaking device that works for both polarizations.

As a countercheck of the effectiveness of the proposed design, we have extracted through a standard method based on the reflection and transmission coefficients the permeability function of the four columns of SRs when the same TE plane wave of the examples in figures 9–11 impinges on the structure. The results are shown in figure 13, where it is evident that at the cloak frequency the value of the effective relative permeability of the cover is close to zero.

**Figure 10.** SCS pattern in linear units of (a) the bare cylinder and (b) the covered cylinder at the cloak frequency 3.1 GHz.
Figure 11. Magnetic field amplitude at the cloak frequency 3.1 GHz in the case of (a) bare cylinder and (b) cylinder with cloak.

Figure 12. Time shots (same instant) of the magnetic field contour lines at the cloak frequency 3.1 GHz in the case of a covered cylinder with (a) four columns of SRs (b) eight columns of SRs.

In addition, in figure 14 it is shown that, when increasing the size of the SRs, the cloaking frequency shifts towards lower values, the lower frequencies being those at which the corresponding effective permeabilities exhibit values close to zero.

Finally, in order to check the robustness of the cloak against geometrical variations, we show in figure 15 the cloaking cover response to a reduction of the radius $a$ of the inner cylinder. The simulation has been performed keeping the cloak and all the setup parameters unchanged, while progressively reducing $a$.

6. TE and TM polarization cylindrical cloak

In order to obtain a cloaking device working for both TM and TE polarizations, we should mix the design we have here proposed for TE polarization with the one already proposed in [2]
Figure 13. Permeability function of the four columns of SRs extracted from the reflection and transmission coefficients.

Figure 14. Maximum of the SCS of the covered cylinder versus frequency for different values of the SRs size.

for TM polarization. The latter is based on the employment of a parallel plate medium [17], which consists of metallic plates extended along the axis of the cylinder and radially placed all around the object. In order to match the behavior of the required ENZ material, in [2] it has been shown that additional gaps are needed. In order to host the magnetic inclusions, working for TE
polarization, in the layout based on the parallel plate medium and working for TM polarization, it is necessary to reduce the number of plates to leave room for the SRs. The TE polarization layout we have proposed in the previous section (see figure 9(a)) can be easily merged with a TM polarization cloak made of just four plates, as shown in figure 16(a). Of course, in this case the reduced number of both plates (four) and SR columns (four) does not lead to an optimal design for either polarization. Anyway, the interesting advantage of this setup is that the cloaking device in figure 16(a) works now for both polarizations, as shown in figure 16(b).
Figure 17. Amplitude map of the electric field at the cloak frequency 3.25 GHz in the case of (a) bare cylinder and (b) cylinder with cloak when a TM polarized plane wave impinges on the structure. Amplitude map of the magnetic field at the cloak frequency 3.25 GHz in the case of (c) bare cylinder and (d) cylinder with cloak when a TE polarized plane wave impinges on the structure.

For sake of completeness, we report in figure 17 the maps of the field amplitudes at the cloaking frequency of 3.25 GHz for both polarizations in the case of the bare cylinder and in the case of the cylinder surrounded by the cloak.

7. Conclusions

In this paper, we have presented the design of electromagnetic cylindrical cloaks for TE polarization. At first, the cloak has been considered as an unbounded cylindrical shell made of an isotropic and homogenous ideal MNZ material. Then, the effectiveness of the design has been verified through full-wave simulations considering also cylindrical objects of finite lengths. Furthermore, a proper implementation of the MNZ cloak through SR magnetic inclusions has been proposed and verified through a set of numerical simulations. Finally, a possible layout working for both TE and TM polarizations at microwave frequencies has been suggested, by employing both the parallel plate medium layout already presented in the literature and working for TM polarization and the SR based design for TE polarization proposed in this paper.

Acknowledgments

We acknowledge the financial support of the following sources: FP-6 METAMORPHOSE Network of Excellence, FP-7 ECONAM, ESA Ariadna Program, PRIN 2006.

New Journal of Physics 10 (2008) 115035 (http://www.njp.org/)
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