Majorana neutrino mass constraints in the landscape of nuclear matrix elements

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We discuss up-to-date constraints on the Majorana neutrino mass $m_{\beta\beta}$ from neutrinoless double beta decay ($0\nu\beta\beta$) searches in experiments using different isotopes: KamLAND-Zen and EXO ($^{136}$Xe), GERDA and MAJORANA ($^{76}$Ge) and CUORE ($^{130}$Te). Best fits and upper bounds on $m_{\beta\beta}$ are explored in the general landscape of nuclear matrix elements (NME), as well as for specific NME values obtained in representative nuclear models. By approximating the likelihood of $0\nu\beta\beta$ signals through quadratic forms, the analysis of separate and combined isotope data becomes exceedingly simple, and allows to clarify various aspects of multi-isotope data combinations. In particular, we analyze the relative impact of different data in setting upper bounds on $m_{\beta\beta}$, as well as the conditions leading to nonzero $m_{\beta\beta}$ at best fit, for variable values of the NMEs. Detailed results on $m_{\beta\beta}$ from various combinations of data are reported in graphical and numerical form. Implications for future $0\nu\beta\beta$ data analyses and NME calculations are briefly discussed.

I. INTRODUCTION

The process of neutrinoless double beta decay ($0\nu\beta\beta$),

$$(Z, A) \rightarrow (Z + 2, A) + 2e^-,$$  

expected to occur for some candidate isotopes $(Z, A)$ if neutrinos are Majorana fermions, may be interpreted as a miniature event of leptonic matter creation or “Little Bang,” whose discovery would have profound implications for particle and nuclear physics and for cosmology [1].

In the standard three-neutrino paradigm [2], the process would be mediated by three Majorana neutrino mass states $\nu_i$ $(i = 1, 2, 3)$ mixed with the three known flavor states $\nu_{\alpha}$ $(\alpha = e, \mu, \tau)$ via a unitary mixing matrix $U_{\alpha i}$, parametrized in terms of three mixing angles $(\theta_{12}, \theta_{13}, \theta_{23})$, one (Dirac) phase $\delta$, and two (Majorana) phases $\varphi_{1,2}$. The relevant particle physics parameter is the effective Majorana neutrino mass $m_{\beta\beta}$, defined as

$$m_{\beta\beta} = |U_{ei}^2 m_1 + U_{e2}^2 m_2 + U_{e3}^2 m_3|,$$  

and related to the observable $0\nu\beta\beta$ decay half-life $T_i$ in each isotope $i = (Z, A)$ via

$$\frac{1}{T_i} = G_i M_i^2 m_{\beta\beta}^2,$$  

where $G_i$ is the phase space, and $M_i$ is the nuclear matrix element (NME) for the decay.

It is useful to contrast the Majorana $\nu$ mass $m_{\beta\beta}$ with the sum of neutrino masses

$$\Sigma = m_1 + m_2 + m_3,$$  

that, being a source of gravity, can produce observable cosmological effects [2]. Figure 1 shows the regions allowed in the $(\Sigma, m_{\beta\beta})$ plane at the $2\sigma$ level $(\Delta \chi^2 = 4)$ by a global analysis of neutrino oscillation data [3], for masses $m_i$ either in normal ordering (NO, $m_{1,2} < m_3$) or in inverted ordering (IO, $m_3 < m_{1,2}$). For a given value of $\Sigma$, the vertical spread of $m_{\beta\beta}$ is mostly due to the unknown relative phases of the $U_{ei}$ matrix elements in Eq. (2).

Current cosmological data provide typical upper bounds on $\Sigma$ at the level of $O(100)$ meV, that are more easily accommodated in NO than in IO [3]; see also the overview of recent constraints on $\Sigma$ and their impact on $\nu$ mass ordering in [4]. Several $0\nu\beta\beta$ decay searches are also exploring the $O(100)$ meV range for $m_{\beta\beta}$ [1, 2]; in particular, the latest constraints from KamLAND-Zen [5] ($^{136}$Xe) can plunge into the region $m_{\beta\beta} \sim$ few $\times 10$ meV for favorable values of the NME. Other very sensitive $0\nu\beta\beta$ searches, all probing half-lives $T_i > 10^{25}$ y at 90% C.L., have been performed by the experiments EXO [6] ($^{136}$Xe), GERDA [7] and MAJORANA [8] ($^{76}$Ge), and CUORE [9] ($^{130}$Te).

1 The range $m_{\beta\beta} \simeq 16–49$ meV, spanning the leftmost edge of the IO region in Fig. 1, is often (but improperly) dubbed in $0\nu\beta\beta$ jargon as “IO region,” despite being compatible with both IO and NO (as well as the quasi-degenerate region at higher $m_{\beta\beta}$). The misnomer may originate from plots of $m_{\beta\beta}$ versus the lightest $\nu$ mass $m_1$, where two elongated stripes for IO and NO appear in log scale as $m_1 \rightarrow 0$, see e.g. [1, 5]. However, the asymptotic separation of such stripes has no physical relevance, since $m_1$ is not directly measurable and cannot be resolved with an accuracy better than an observable such as $\Sigma$. Projecting away $m_1$ (as in Fig. 1) makes the point clear.
Building upon previous work [3], we discuss in detail how to combine current (Xe, Ge, Te) data for given NME values. The approach allows a do-it-yourself $\nu\beta\beta$ global analysis in terms of $\chi^2$ functions with (up to) quadratic dependence on the signal strength $1/T_i$, which is a good approximation to recent results [3][10][11]. In particular, we describe how to derive $m_{\beta\beta}$ constraints at a given confidence level, using both separate and combined (Xe, Ge, Te) data, for generic values of the nuclear matrix elements (the “NME landscape”), as well as for representative NME values from different nuclear models. Our approach clarifies interesting aspects of the $\nu\beta\beta$ data analysis, such as the relative importance of each isotope in determining (non)zero best fits and upper bounds for $m_{\beta\beta}$.

The paper is structured as follows: In Section II we describe the ingredients of our analysis in terms of notation, parametrization of experimental results for (Xe, Ge, Te), and associated NME’s. In Section III we discuss the main results of the analysis in terms of $m_{\beta\beta}$ constraints, by considering two qualitatively different situations: (1) cases where, a priori, $m_{\beta\beta} = 0$ is preferred, and (2) more general cases where the best fit may be at $m_{\beta\beta} > 0$. Upper bounds on $m_{\beta\beta}$ are explored both graphically and numerically in the NME landscape, by using separate and combined (Xe, Ge, Te) data. In Sec. IV we summarize our results and comment on further applications and perspectives.

II. INGREDIENTS OF THE ANALYSIS

In this Section we introduce the notation, the experimental results and their parametrization, the landscape of NME and the phase space related to the three isotopes Xe, Ge and Te.

A. Notation and units

Following [3], we introduce the inverse half-life

$$S_i = 1/T_i ,$$

that represents, up to a constant factor, the observable decay rate or signal strength in each $i = (Z, A)$ isotope. Equation [3] reads then

$$S_i = G_i M_i^2 m_{\beta\beta}^2 .$$

To keep the notation compact, we absorb in $G_i$ terms as $1/m_i^2$ and $g_A^i$ (where $g_A = 1.276$ [12] is the bare value of the axial-vector coupling), that are factorized out in other conventions. In particular, we can make contact with the notation of [1], where $1/T = G_0 g_A^i M_i^2 m_{\beta\beta}^2/m_i^2$, by identifying $G = G_0 g_A^i/m_i^2$ and $M = M_0$ for each isotope $i$. We also follow [1] by taking the $M_i$ as positive real numbers, referred to the bare value of $g_A$ (unless otherwise noticed). Qualitative effects of the so-called quenching of $g_A$ in nuclear matter [13] are separately commented below.

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2 For simplicity, we shall generally drop superscripts for the Xe, Ge, and Te isotopes.
Finally, the following units are adopted:

\[ m_{\beta\beta} = \text{meV}, \]
\[ T_i = 10^{26} \text{y}, \]
\[ S_i = 10^{-26} \text{y}^{-1}, \]
\[ G_i = 10^{-26} \text{y}^{-1} (\text{meV})^{-2}. \]

B. Experimental inputs and parametrizations

In principle, the 0νββ data analysis would be straightforward, if likelihood profiles were provided for the signal strength \( S_i \) (or for \( T_i \)) in each experiment, e.g., in terms of a function \( \chi_i^2(S_i) \). Barring error correlations among independent experiments, one should sum up the \( \chi_i^2 \) functions, express the \( S_i \) in terms of \( m_{\beta\beta} \) via Eq. (6) for a given set of \( M_i \), and map the resulting best fits and allowed regions for \( m_{\beta\beta} \). In practice, experimental papers often focus on a single point of the likelihood profile (e.g., the \( T_i \) bound at 90\% C.L.), whereas its shape has to be derived from supplementary information.

A useful empirical fact, first noted in [10] and further elaborated in [3], is that the functions \( \chi_i^2(S_i) \) are often well approximated by (up to) quadratic forms in \( S_i \); see also the recent results in [11]. Such forms cover 0νββ decay searches ranging from zero background (with poissonian, linear dependence on \( S_i \)) to large backgrounds (with gaussian, quadratic dependence on \( S_i \)) [3]. In particular, we have checked that the quadratic approximation works very well also for the latest KamLAND-Zen data [5] [14], up to 3σ level at least.

Each experimental result is thus parametrized through a function \( \Delta \chi^2_i(S_i) \) of the form:

\[ \Delta \chi^2_i(S_i) = a_i S_i^2 + b_i S_i + c_i, \]

where the offset \( c_i \) is set by the condition that the minimum value \( \Delta \chi^2_i = 0 \) is reached within the physical region \( S_i \geq 0 \), namely,

\[ c_i = \begin{cases} 0 & \text{for } b_i \geq 0, \\ b_i^2/4a_i & \text{for } b_i < 0. \end{cases} \]

For \( a_i > 0 \) the \( \Delta \chi^2_i \) functions are parabolic, with a vertex placed at either \( S_i = 0 \) (null result, \( b_i = 0 \)), or \( S_i < 0 \) (negative fluctuation in the unphysical region, \( b_i > 0 \)), or \( S_i > 0 \) (physical signal or positive fluctuation, \( b_i < 0 \)). In the latter case, the offset \( c_i \) guarantees \( \Delta \chi^2_i = 0 \) at \( S_i = -b_i/2a_i \). For \( a_i = 0 \), the \( \Delta \chi^2 \) functions are linear. For the same isotope, the results of independent experiments are combined by summing their \( \Delta \chi^2 \)'s, and readjusting the total offset as per Eq. (12). In all cases, 90\% C.L. bounds on the half-life (\( T_{90} = 1/S_{90} \)) are obtained by imposing \( \Delta \chi^2_i(S_{90}) = 2.706 \).

Table 1, updated from [3], with the inclusion of the latest KamLAND-Zen results [5], reports the coefficients of the parametrization in Eq. (11) and the \( T_{90} \) bounds for the most sensitive current experiments (\( T_{90} > 0.1 \)), as well as for combinations of experiments using the same isotope. For later purposes, we also consider hypothetical CUORE results for an exactly null signal, denoted as CUORE* (or Te*).

| Isotope | Experiment or combination | \( a_i \) | \( b_i \) | \( c_i \) | \( T_{90}/10^{26} \text{y} \) | \( T_{90} \) (expt.) |
|---------|--------------------------|--------|--------|--------|----------------|-----------------|
| 136Xe   | KamLAND-Zen              | 5.157  | 3.978  | 0.000  | 2.300          | 2.3             |
| 136Xe   | EXO                      | 0.440  | -0.338 | 0.065  | 0.350          | 0.35            |
| 76Ge    | GERDA                    | 0.000  | 4.867  | 0.000  | 1.800          | 1.8             |
| 76Ge    | MAJORANA                | 0.000  | 0.731  | 0.000  | 0.270          | 0.27            |
| 130Te   | CUORE                    | 0.245  | -0.637 | 0.414  | 0.216          | 0.22            |
| 136Xe   | KamLAND-Zen + EXO       | 0.000  | 3.640  | 0.000  | 2.260          | 2.26            |
| 76Ge    | Ge (GERDA + MAJORANA)   | 0.000  | 5.598  | 0.000  | 2.070          | —               |
| 130Te   | Te (CUORE data as above)| 0.245  | -0.637 | 0.414  | 0.216          | 0.22            |
| 130Te   | Te* (CUORE*, sensitivity)| 0.245  | 0.000  | 0.000  | 0.301          | 0.28            |
A positive fluctuation, that partly cancel in their combination (denoted as $X_e$). As a result, the $X_e$ bound

experiments are characterized by parabolic functions. KamLAND-Zen and EXO report, respectively, a negative and
to separate experiments and to same-isotope combinations ($X_e$, Ge, Te). A few remarks about these graphs and the
of $0^\beta\beta$

consider representative $M_i$ before discussing the full combination of $X_e$, Ge, and Te constraints on

b this fluctuation is canceled by setting $b_i = 0$ (and thus also $c_i = 0$). The half-time limit for Te results ($T_{90} = 0.301$) is in
reasonable agreement with the median sensitivity quoted by the CUORE experiment for null result ($T_{90} = 0.28$). In
Sec. III the combination of $X_e$, Ge, and Te results (all with $\Delta \chi^2_i = 0$ at $S_i = 0$) will provide a simple starting point,
before discussing the full combination of $X_e$, Ge and Te combinations, it is $\Delta \chi^2_i = 0$ at $S_i = 0$.

Results for the Te isotope depend on the single CUORE experiment, which shows a positive fluctuation at the level of
$0.64\sigma = [\Delta \chi^2_i(0)]^{1/2}$. As anticipated we consider, besides the real Te results, also hypothetical Te results, where this
fluctuation is canceled by setting $b_i = 0$ and (thus also) $c_i = 0$. The half-time limit for Te results ($T_{90} = 0.301$) is in
reasonable agreement with the median sensitivity quoted by the CUORE experiment for null result ($T_{90} = 0.28$). In
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C. Landscape of nuclear matrix elements

In order to study the combination of $0\nu\beta\beta$ results in full generality, we consider unconstrained values of the
nuclear matrix elements $M_{X_e}$, $M_{Ge}$, $M_{Te}$ in the numerical range $M_i \in [0.2, 20]$. Within this landscape, we also
consider representative $M_i$ values from four different approaches to nuclear modeling, including the nuclear shell
model (SM), the quasi-particle random phase approximation (QRPA), the energy-density functional theory (EDF),
and the interacting boson model (IBM). The NME values are taken from a recent compilation of results [15–26] as
reported in [1] for the bare value of $g_A$ (see Tab. I therein), and are listed in Table II for the sake of completeness.

Figure 3 shows the NME landscape in each of the three planes charted by pairs $(M_i, M_j)$, together with the
representative $M_i$ values reported in Table II which refer to the bare $g_A$. The issue of the effective $g_A$ value to be
used in nuclear matter, either bare or quenched by a factor $q$ ($g_A \rightarrow q g_A$ with $q < 1$), is largely debated and
model-dependent [11]. For NMEs dominated by the axial-vector (Gamow-Teller) component (as it is often the
case), the leading quenching effect would amount to rescaling the product $G_i M_i^2$ by a factor $q^4 \sim g_A^4$, that can be
assumed to operate on $M_i$ only ($M_i \rightarrow q^2 M_i$) if $G_i$ is kept constant.

As a representative quenching effect one may consider, e.g., the typical case $q g_A \simeq 1$, namely, $q \sim 1/g_A$, leading
to an approximate rescaling $M_i \rightarrow M_i/g_A^2$, as shown in each panel of Fig. 3 by an arrow (applicable to any marked
point). Stronger quenching would be associated to longer arrows (not shown). On the other hand, quenching effects
would be weaker for sizeable NME vector (Fermi) components, not scaling with $q^2$. Moreover, some $M_i$ calculations
may exhibit a milder dependence on $q g_A$ for different reasons. In some QRPA calculations, e.g., $2\nu\beta\beta$ data are used to
adjust the particle-particle parameter $g_{pp}$, partly trading the effect of quenching $g_A$ from its bare value to unity
[27]. In the same approach, large statistical covariances are observed among the $M_i$ values for different isotopes,
inducing noticeable effects on $m_{\beta\beta}$ constraints [27], as recently discussed in [3]. The marked points in Fig. 3 also seem
to suggest an overall positive correlation (possibly enhanced by quenching effects) but, since they do not represent a
statistical distribution, their covariances (if any) will be ignored.

Figure 3 shows the numerical information of Table II in graphical form; the left and right panels refer, respectively,
to separate experiments and to same-isotope combinations ($X_e$, Ge, Te). A few remarks about these graphs and the
numerics are in order. The case of linear $\Delta \chi^2$ functions applies to current GERDA and MAJORANA results, whose
combination (denoted as Ge) sets a bound $T_{90} = 2.07$ stronger than for GERDA alone ($T_{90} = 1.8$). All the other
experiments are characterized by parabolic functions. KamLAND-Zen and EXO report, respectively, a negative and
a positive fluctuation, that partly cancel in their combination (denoted as $X_e$). As a result, the $X_e$ bound
is slightly weaker ($T_{90} = 2.26$) than for KamLAND-Zen alone ($T_{90} = 2.3$). Note that, for both the Ge and Xe
combinations, it is $\Delta \chi^2_i = 0$ at $S_i = 0$.

Results for the Te isotope depend on the single CUORE experiment, which shows a positive fluctuation at the level of
$0.64\sigma = [\Delta \chi^2_i(0)]^{1/2}$. As anticipated we consider, besides the real Te results, also hypothetical Teresults, where this
fluctuation is canceled by setting $b_i = 0$ (and thus also $c_i = 0$). The half-time limit for Te results ($T_{90} = 0.301$) is in
reasonable agreement with the median sensitivity quoted by the CUORE experiment for null result ($T_{90} = 0.28$). In
Sec. III the combination of $X_e$, Ge, and Te results (all with $\Delta \chi^2_i = 0$ at $S_i = 0$) will provide a simple starting point,
before discussing the full combination of $X_e$, Ge and Te combinations, it is $\Delta \chi^2_i = 0$ at $S_i = 0$.

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before discussing the full combination of $X_e$, Ge and Te combinations, it is $\Delta \chi^2_i = 0$ at $S_i = 0$.
TABLE II: Representative nuclear matrix elements $(M_{Xe}, M_{Ge}, M_{Te})$ computed within four different models (SM, QRPA, EDF, and IMB) for bare value of $g_A$.
Adapted from [1].

| Model | $M_{Xe}$ | $M_{Ge}$ | $M_{Te}$ | Reference |
|-------|----------|----------|----------|-----------|
| SM    | 1 2.28   | 2.89     | 2.76     | [15]      |
| QRPA  | 2 2.45   | 3.07     | 2.96     | [15]      |
| EDF   | 3 1.63   | 3.37     | 1.79     | [16]      |
| IBM   | 4 1.76   | 3.57     | 1.93     | [16]      |
| IBM   | 5 2.39   | 2.66     | 3.16     | [17]      |
| IBM   | 6 1.55   | 5.09     | 1.37     | [18]      |
| IBM   | 7 2.91   | 5.26     | 4.00     | [19]      |
| IBM   | 8 2.72   | 4.85     | 4.67     | [20]      |
| IBM   | 9 1.11   | 3.12     | 2.90     | [21]      |
| IBM   | 10 1.18  | 3.40     | 3.22     | [21]      |
| IBM   | 11 4.20  | 4.60     | 5.13     | [22]      |
| IBM   | 12 4.77  | 5.55     | 6.41     | [23]      |
| IBM   | 13 4.24  | 6.04     | 4.89     | [24]      |
| IBM   | 14 3.25  | 5.14     | 3.96     | [25]      |
| IBM   | 15 3.40  | 6.34     | 4.15     | [26]      |

Summarizing, the NME landscape in Fig. 3 is meant to cover a wide and continuous range of $M_i$ values, either unquenched or arbitrarily quenched. Graphical results will be shown for unconstrained $M_i$ values in this landscape. Marked points in Fig. 3 are meant to represent typical unquenched $M_i$ (central values) as taken from the literature (see Table II), while the arrows provide visual guidance for typical quenching effects ($q g_A \sim 1$).
D. Phase space

The last ingredient is represented by the phase space $G_i$ for $0\nu\beta\beta$ decay in $i = \text{Xe}, \text{Ge}$ and $\text{Te}$, that we take from the calculation in [26]. In our notation and units:

\begin{align*}
G_{\text{Xe}} &= 14.78 \times 10^{-6}, \\
G_{\text{Ge}} &= 2.40 \times 10^{-6}, \\
G_{\text{Te}} &= 14.42 \times 10^{-6}.
\end{align*}

The phase space uncertainties [28] are much smaller than those related to $0\nu\beta\beta$ data and are not considered herein.

III. CONSTRAINTS ON THE MAJORANA NEUTRINO MASS

The previously discussed functions $\Delta \chi_i^2(S_i)$ can be recast in terms of quadratic functions of $m_{\beta\beta}^2$ through Eq. (6):

$$\Delta \chi_i^2 = \alpha_i m_{\beta\beta}^4 + \beta_i m_{\beta\beta}^2 + \gamma_i,$$

where the offset $\gamma_i$ is set by

$$\gamma_i = \begin{cases} 0 & \text{for } \beta_i \geq 0, \\ \beta_i^2/4\alpha_i & \text{for } \beta_i < 0. \end{cases}$$

The best-fit value of $m_{\beta\beta}$ is set by:

$$\Delta \chi_i^2 = 0 \rightarrow m_{\beta\beta} = \begin{cases} 0 & \text{for } \beta_i \geq 0, \\ (-\beta_i/2\alpha_i)^{1/2} & \text{for } \beta_i < 0. \end{cases}$$

Table III reports the parametric coefficients ($\alpha_i, \beta_i, \gamma_i$) for the Xe, Ge, Te and $\text{Te}^*$ cases, with their explicit dependence on the matrix elements $M_{\text{Xe}}, M_{\text{Ge}}, M_{\text{Te}}$.

Constraints on $m_{\beta\beta}$ from two or more isotopes are obtained by summing the corresponding $\Delta \chi_i^2$, and by adjusting the offset so that it obeys Eq. (17), namely: $\Delta \chi^2 = \alpha_0 m_{\beta\beta}^4 + \beta_0 m_{\beta\beta}^2 + \gamma_0$, where $\alpha = \sum_i \alpha_i$, $\beta = \sum_i \beta_i$, and $\gamma_0 = 0$ for $\beta \geq 0$ ($\gamma = \beta^2/4\alpha$ otherwise). Bounds on $m_{\beta\beta}$ at a given confidence level are obtained by solving

$$\Delta \chi_i^2(m_{\beta\beta}) = \Delta_{\text{CL}},$$

where, e.g., $\Delta_{\text{CL}} = 2.706, 4$ and $9$ for limits at 90% C.L., $2\sigma$ and $3\sigma$, respectively.

Two qualitatively different cases arise from current results: (a) for Xe, Ge and $\text{Te}^*$, either separately or in combination, there is no offset $\gamma$ and the $\Delta \chi^2$ function is zeroed at $m_{\beta\beta} = 0$; (b) for Te results, characterized by $\gamma_i > 0$ (positive fluctuation), the combination with Xe or Ge (or both) may lead to $\gamma > 0$, implying a nonzero Majorana neutrino mass at best fit: $m_{\beta\beta} = [-(\beta/2\alpha)]^{1/2}$. We discuss separately these two cases below.

A. Combination of Xe, Ge, $\text{Te}^*$ constraints

In this section we consider current constraints from Xe, Ge, and from the pseudo-experiment $\text{Te}^*$ (corresponding to null signal in CUORE). In this case the analysis is straightforward, since the best fit is $m_{\beta\beta} = 0$ for all isotopes and their combinations (independently of the NME). The analysis including real Te data will bring forward cases with $m_{\beta\beta} > 0$ at best fit, as discussed in the next Section.

| Case | $\alpha_i$ | $\beta_i$ | $\gamma_i$ |
|------|------------|------------|------------|
| Xe   | $1.223 \times 10^{-7} M_{\text{Xe}}^4$ | $5.380 \times 10^{-7} M_{\text{Xe}}^2$ | 0          |
| Ge   | 0          | $1.344 \times 10^{-5} M_{\text{Ge}}^2$ | 0          |
| Te   | $5.094 \times 10^{-11} M_{\text{Te}}^4$ | $-9.186 \times 10^{-6} M_{\text{Te}}^2$ | 0.414      |
| $\text{Te}^*$ | $5.094 \times 10^{-11} M_{\text{Te}}^4$ | 0          | 0          |
TABLE IV: Bounds on $m_{\beta\beta}/\text{meV}$ at 2$\sigma$ level from Xe, Ge and Te* results, both separately and in combination, for each of the 15 representative NME calculations listed in Tab. II. The best-fit value is $m_{\beta\beta} = 0$ in all cases.

|       | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 | 11 | 12 | 13 | 14 | 15 |
|-------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Xe    | 86.9 | 80.9 | 121.6 | 112.6 | 82.9 | 127.9 | 68.1 | 72.9 | 178.6 | 168.0 | 47.2 | 41.6 | 46.7 | 61.0 | 58.3 |
| Ge    | 188.8 | 177.7 | 161.9 | 152.8 | 205.1 | 107.2 | 103.7 | 112.5 | 174.9 | 160.5 | 118.6 | 98.3 | 90.3 | 106.2 | 86.1 |
| Te*   | 191.8 | 178.8 | 295.7 | 274.3 | 167.5 | 386.4 | 113.4 | 123.1 | 182.5 | 164.4 | 102.3 | 83.6 | 74.5 | 90.2 | 77.3 |
| Xe+Ge | 81.0 | 75.5 | 101.9 | 95.0 | 75.3 | 86.8 | 59.3 | 63.7 | 132.0 | 122.7 | 44.8 | 39.2 | 42.8 | 54.8 | 50.4 |
| Xe+Te*| 85.7 | 79.8 | 120.4 | 111.6 | 81.3 | 127.4 | 66.6 | 69.2 | 147.2 | 135.3 | 46.5 | 40.7 | 46.2 | 60.1 | 57.5 |
| Ge+Te*| 149.7 | 140.2 | 155.6 | 146.5 | 122.2 | 106.9 | 91.3 | 88.8 | 140.7 | 127.8 | 85.8 | 69.5 | 77.5 | 92.9 | 79.4 |
| Xe+Ge+Te* | 80.1 | 74.7 | 101.3 | 94.4 | 77.2 | 86.7 | 58.4 | 61.5 | 120.4 | 110.7 | 44.2 | 38.5 | 42.4 | 54.2 | 49.9 |

We consider both separate and combined Xe, Ge and Te* constraints, as obtained by summing up the corresponding functions defined in Table III, $\Delta \chi^2 = \sum_i \Delta \chi^2_i = \alpha m_{\beta\beta}^4 + \beta m_{\beta\beta}^2$. For a given choice of $M_i$, upper limits on $m_{\beta\beta}$ are obtained by solving $\Delta \chi^2_{\text{CL}} = \alpha m_{\beta\beta}^4 + \beta m_{\beta\beta}^2$. For definiteness we set $\Delta \chi^2_{\text{CL}} = 4$ (2$\sigma$ bounds), unless otherwise specified.

Table IV reports the upper bounds on $m_{\beta\beta}$, for each of the representative $(M_{\text{Xe}}, M_{\text{Ge}}, M_{\text{Te}})$ calculations listed in Table II. Concerning constraints from single isotopes, in most cases Xe sets the strongest bound, followed by weaker ones from Ge and Te*. However, for the cases numbered as 9 and 10 (QRPA), the bounds from Xe, Ge and Te* are comparable to each other, and for case 6 (QRPA) the Ge bound actually prevails over the Xe (and Te*) bound.

Notice that such a hierarchy of $m_{\beta\beta}$ constraints may change at different confidence levels, since the $S_i$ constraints scale up at different rates (see Fig. 2). In Table IV the combination of pairs of constraints improves appreciably upon each separate constraint; the relative balance in each pair is highlighted below. Finally, the total combination Xe+Ge+Te* provides even stronger bounds on $m_{\beta\beta}$, that range from a minimum of 38.5 meV (case 12, EDF) to a maximum of 120.4 meV (case 9, QRPA) at 2$\sigma$.

Figure 4 shows isolines of the 2$\sigma$ bounds on $m_{\beta\beta}$, as derived by combining any two pairs among Xe, Ge and Te*, for unconstrained values of the NME. In each panel, single-isotope bounds are asymptotically recovered along each axis.

FIG. 4: Isolines of $m_{\beta\beta}$ bounds (at 2$\sigma$ level) in the landscape of nuclear matrix elements, obtained from the combination of any two results among Xe, Ge and Te*. In each panel, the bounds are dominated by the isotope on the y (x) axis, in the region above (below) the dashed line. See the text for details.
for vanishing matrix element on the other axis. When both matrix elements are sizeable, the joint bound improves upon separate ones. In particular, at each marked point, the bounds in Table IV are recovered for the corresponding NME and pair of isotopes.

For each \((M_x, M_y)\) panel and \((x, y)\) isotope pair, the condition for the dominance of one isotope constraint over the other is easily derived. The two isotopes contribute equally to \(\Delta \chi^2\) when \(\Delta \chi^2_{x} = \Delta \chi^2_{y} = \Delta \chi^2_{z}\). The solutions to these equations read \(M_x m_{\beta3} = \xi_x\) and \(M_y m_{\beta3} = \xi_y\), where \(\xi_{x,y}\) are positive numbers. For \(\Delta \chi^2_{x} = 4\) it is \(M_{Te}/M_{Ge} = 1.154,\) \(M_{Ge}/M_{Xe} = 2.488,\) and \(M_{Te}/M_{Xe} = 2.871,\) shown as a dashed line in each panel of Fig. 2. Along the dashed line, the two isotopes contribute with equal strength to the 2\(\sigma\) upper bound; above the dashed line, the \(y\)-axis isotope dominates over the \(x\)-axis one, and viceversa. In this way one gets a graphical interpretation of the hierarchy of bounds for different NME, that was inferred from numerical inspection of Table IV.

B. Combination of Xe, Ge, Te constraints

In this section we consider real Te data, as opposed to the previous cases including Te\(*\) pseudo-data. The slight preference of Te data from CUORE for a nonzero signal (as compared with Xe, Ge and Te\(*\), see Fig. 2) brings forward new features of multi-isotope data constraints, although still at embryonic stages.

In general one may expect that, for relatively small values of \(M_{Te}\) (with respect to \(M_{Xe}\) and \(M_{Ge}\)), the Xe+Ge results will dominate over Te, keeping the best fit at \(m_{\beta3} = 0\). However, for increasing \(M_{Te}\), the Te results will eventually prevail and set \(m_{\beta3} > 0\) at \(\Delta \chi^2 = 0\), affecting also upper bounds at some value \(\Delta CL\).

This situation anticipates what could happen with future and more accurate 0\(\nu/\beta\beta\) data: their combination may (or may not) be consistent with some indications for nonzero \(m_{\beta3}\), depending on both the data and the NME values. A future preference for \(m_{\beta3} > 0\) might even lead to lower bounds on \(m_{\beta3}\), either separately or in combination, depending in part on (un)favorable values of the NME. Eventually, precise multi-isotope data might even test specific NME’s by selecting allowed ratios \(M_x/M_y [29, 30]\) namely, slanted allowed stripes in the NME landscape of Fig. 3.

In our approach, the occurrence of \(m_{\beta3} > 0\) at best fit is simply signaled, for a single isotope, by a coefficient \(\beta_i < 0\) in the \(\Delta \chi^2\) function (currently occurring only for Te) and, for any combination of multi-isotope data, by a negative coefficient \(\beta = \sum \beta_i < 0\). The best-fit value of \(m_{\beta3}\) is then \(m_{\beta3} = (-\beta/2\alpha)^{1/2}\) with \(\alpha = \sum \alpha_i\), and its specific value depends on the NME’s. For \(\beta < 0\), the offset must be taken as \(\gamma = \beta^2/4\alpha\). In all cases, upper bounds at a chosen confidence levels are set by \(\Delta \chi^2 = \Delta CL\). Table V shows the numerical results from current Xe, Ge and Te data, regarding the 2\(\sigma\) limits (lower part) and the best-fit values (lower half) of \(m_{\beta3}\). In the upper half, the rows corresponding the Xe, Ge and Ge+Xe are unchanged with respect to Table IV but are repeated for completeness.

Let us first comment on the \(m_{\beta3}\) best fits in the lower half of Table V. Of course, the Te row displays nonzero results, with \(m_{\beta3}\) scaling as \(1/M_{Te}\). In almost all cases including Te (combined with Xe or Ge or both), the positive contributions to \(\beta = \sum \beta_i\) from \(i = Xe\) and Ge are never erased by the negative contribution from Te, and \(m_{\beta3} = 0\) is preferred. Only for cases 9 and 10 (QRPA), it turns out that the large ratio \(M_{Te} \simeq 2.6 M_{Xe}\) makes Te prevail over Xe in the corresponding Xe+Te combinations, which show nonzero best fits.

| TABLE V: Upper half: Bounds on \(m_{\beta3}/\text{meV}\) at 2\(\sigma\) level from Xe, Ge and Te results, both separately and in combination, for each of the 15 representative NME calculations listed in Tab. IV | Lower half: Corresponding best-fit values of \(m_{\beta3}\). |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Xe | 86.9 | 80.9 | 121.6 | 112.6 | 82.9 | 127.9 | 68.1 | 72.9 | 178.6 | 168.0 | 47.2 | 41.6 | 46.7 | 61.0 | 58.3 |
| Ge | 188.8 | 177.7 | 161.9 | 152.8 | 205.1 | 107.2 | 103.7 | 112.5 | 174.9 | 160.5 | 118.6 | 98.3 | 90.3 | 106.2 | 86.1 |
| Te | 220.5 | 205.6 | 340.0 | 315.3 | 192.6 | 244.2 | 152.1 | 130.3 | 299.9 | 189.0 | 118.6 | 94.9 | 124.5 | 153.7 | 146.6 |
| Xe+Ge | 81.0 | 75.5 | 101.9 | 95.0 | 78.5 | 86.8 | 59.3 | 63.7 | 132.0 | 122.7 | 44.8 | 39.2 | 42.8 | 54.8 | 50.4 |
| Xe+Te | 89.6 | 83.4 | 124.9 | 115.7 | 85.6 | 130.4 | 70.4 | 75.0 | 167.5 | 154.5 | 48.6 | 42.9 | 48.1 | 62.9 | 60.1 |
| Ge+Te | 174.1 | 163.2 | 170.1 | 165.0 | 166.5 | 109.6 | 104.4 | 103.3 | 163.4 | 148.6 | 100.3 | 81.3 | 89.2 | 106.5 | 89.2 |
| Xe+Ge+Te | 83.5 | 77.9 | 104.5 | 97.4 | 81.2 | 88.0 | 61.4 | 66.2 | 135.0 | 124.7 | 46.2 | 40.5 | 44.1 | 56.5 | 51.9 |

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3 An overall NME rescaling factor \(\lambda (M_{x,y} \rightarrow \lambda M_{x,y})\) is degenerate with an inverse rescaling of the Majorana mass \((m_{\beta3} \rightarrow m_{\beta3}/\lambda)\).
FIG. 5: Isolines of best-fit values of $m_{\beta\beta}$/meV in the planes charted by (Xe, Te) and (Ge, Te) nuclear matrix elements. The best fit is zero in the lower-right parts of each panel, as well as in the whole (Xe, Ge) panel (not shown).

Concerning the upper half of Table IV, the combination of Te with Xe (or Ge) does not necessarily improve upon the separate 2σ bounds. Roughly speaking, when the best-fit value of $m_{\beta\beta}$ in Te is comparable or larger than the upper bound from Xe (Ge) alone, the joint bound from Xe+Te (Ge+Te) is weakened, as a result of the slight tension between the two isotopic data. A slight weakening also occurs whenever Te is added to Xe+Ge in the global combination Xe+Ge+Te. The 2σ bounds on $m_{\beta\beta}$ compiled in Table IV are contained in the following range:

$$m_{\beta\beta} \in [40.5, 135.0] \text{ meV (Xe + Ge + Te)}.$$  (20)

The lowest (most optimistic) edge of this range would significantly cut from above the IO and NO allowed regions in Fig. 1 setting also an upper limit on $\Sigma$ at the level of $\sim 400$ meV.

Figure 6 shows isolines of the best-fit value of $m_{\beta\beta}$ in the left and right panels charted by (Xe, Te) and (Ge, Te), respectively. In the left panel, the condition $\sum_i \beta_i > 0$ for $m_{\beta\beta} > 0$ implies $M_{\text{Te}}/M_{\text{Xe}} > 2.42$, satisfied only by two marked points (corresponding to the QRPA cases 9 and 10 in Table II) close to the isoline at $m_{\beta\beta} = 30$ meV. In the right panel, none of the marked points falls in the analogous region $M_{\text{Te}}/M_{\text{Ge}} > 1.21$, although three of them are very close to its border. If future $0\nu\beta\beta$ experiments will show some indications for a signal, plots like these will help to

FIG. 6: Isolines of $m_{\beta\beta}$ bounds (at 2σ level) in the planes charted by (Xe, Te) and (Ge, Te) nuclear matrix elements. The (Xe, Ge) panel (not shown) is unchanged with respect to Fig. 5.

4 These effects are analogous to those noted in Sec. II B for the combination of KamLAND-Zen and EXO data, leading to a $T_{90}$ bound slightly weaker than from KamLAND-Zen alone, as a result of two opposite fluctuations.
TABLE VI: Upper bounds on \( m_{\beta\beta} / \text{meV} \) at 90% C.L., 2\( \sigma \), and 3\( \sigma \), from the combination of current Xe+Ge+Te data, for the representative NME calculations considered in this work.

| Model | 90%  | 2\( \sigma \) | 3\( \sigma \) |
|-------|------|---------------|---------------|
| 1     | 72.7 | 83.5          | 109.4         |
| 2     | 67.8 | 77.9          | 101.9         |
| 3     | 89.3 | 104.5         | 141.5         | SM          |
| 4     | 83.3 | 97.4          | 131.7         |
| 5     | 71.0 | 81.2          | 105.4         |
| 6     | 73.6 | 88.0          | 125.3         |
| 7     | 53.9 | 61.4          | 81.6          |
| 8     | 57.6 | 66.2          | 86.7          | QRPA        |
| 9     | 117.7| 135.0         | 176.3         |
| 10    | 108.9| 124.7         | 162.3         |
| 11    | 40.3 | 46.2          | 60.1          |
| 12    | 35.4 | 40.5          | 52.6          | EDF         |
| 13    | 38.2 | 44.1          | 58.1          |
| 14    | 48.8 | 56.5          | 74.7          |
| 15    | 44.6 | 51.9          | 69.5          | IBM         |

locate the best-fit values of \( m_{\beta\beta} \) as a function of the NME, for each isotope pair. The (in)consistency of the best fits in different pairs will provide interesting clues about the interpretation of data in terms of light Majorana neutrinos.

Figure 6 shows isolines of the \( m_{\beta\beta} \) upper bounds at 2\( \sigma \), in the same planes of Fig. 5. In comparison with the lower panels of Fig. 4, a slight weakening of the bounds can be appreciated. Note that, in the presence of subregions where \( m_{\beta\beta} > 0 \) at best fit, the offset \( \gamma \) depends on information coming from both isotopes, whose \( \chi^2 \) contributions cannot be separated in the combination. The condition of equal contributions to \( \Delta \text{CL} \) cannot be defined in general terms, and the dashed lines of Fig. 4 are thus absent in Fig. 6.

We conclude this Section by discussing the constraints on \( m_{\beta\beta} \) at various confidence levels, as derived from the global combination of current (Xe+Ge+Te) data, using the representative NME values in Table II. Since the best fit is \( m_{\beta\beta} = 0 \) in all Xe+Ge+Te cases (see Tab. V), only upper bounds need to be quoted.

Table VI reports the 90% C.L., 2\( \sigma \), and 3\( \sigma \) upper limits on \( m_{\beta\beta} \)(in meV). Figure 7 shows the \( N_{\sigma} = (\Delta \chi^2)^{1/2} \) bounds as continuous functions of \( m_{\beta\beta} \). Qualitatively, the strongest limits in are obtained using NMEs from the EDF and IBM models, followed by QRPA and SM cases in mixed order. These results can be generalized to any other choice of NME calculations, using the information provided in this paper.

FIG. 7: Significance of upper limits on \( m_{\beta\beta} \) in terms of \( N_{\sigma} = (\Delta \chi^2)^{1/2} \), from the combination of current Xe+Ge+Te data, for the representative NME calculations considered in this work.
In Fig. 7 one of the $N_\sigma$ curves labelled as QRPA shows a a markedly different (almost linear) slope, intersecting three SM curves. This peculiar curve corresponds to the lowest QRPA point in both panels of Fig. 6 (case 6 in Table II), that is characterized by a rather large value of $M_{\text{three SM}}$ curves. This peculiar curve corresponds to the lowest QRPA point in both panels of Fig. 6 (case 6 in $\chi^2_T$ as the 90% C.L. limit on $m_{\beta\beta}$) implying a roughly linear $N_\sigma(m_{\beta\beta})$ function. Once more, this observation shows the importance of considering the full likelihood profile of the experimental results (e.g., in terms of $S = 1/T$), rather than pointlike information (such as the 90% C.L. limit on $T$).

As a final step, one could include a joint probability distribution or a $\Delta \chi^2$ penalty defined over the NME landscape ($M_{\text{Xe}}, M_{\text{Ge}}, M_{\text{Te}}$), and numerically minimize the total $\Delta \chi^2$ function. This exercise was performed in [3] by assuming a conservative characterization of the NME and their correlated uncertainties, derived within QRPA calculations [27]. A limit $m_{\beta\beta} < 110$ meV was obtained at 2$\sigma$ [3]. By repeating the same exercise with the updated (Xe+Ge+Te) combination considered herein, we get the following marginalized bounds (in meV): $m_{\beta\beta} < 79.5$ at 90% C.L., $m_{\beta\beta} < 99.8$ at 2$\sigma$, and $m_{\beta\beta} < 169$ at 3$\sigma$. Roughly speaking, from these results and from the summary in Eq. (20) one can state that the combination of current $0\nu\beta\beta$ experiments sets 2$\sigma$ upper bounds on $m_{\beta\beta}$ at the level of $\sim 90$–100 meV for “average” NME values, possibly lowered to $\sim 40$–50 meV for favorable NME values.

A final remark is in order. In principle, one should replace the QRPA input from [27] with more general and up-to-date estimates of the NME’s and their uncertainties, characterizing also the spread among different models and calculations. However, no consensus estimates exist yet for NME fiducial values and covariances, although relevant work is in progress toward this goal [1, 31]. Part of the planned strategy involves benchmarking nuclear models for $0\nu\beta\beta$ decay against a variety of data, coming from related electroweak and strong interaction processes or from nuclear structure [32–35].

IV. CONCLUSIONS AND PERSPECTIVES

We have discussed an approach to the analysis of neutrinoless double beta decay experiments, in terms of $\Delta \chi^2$ profiles for the signal strength $S_i$ (inverse of the half-life $T_i$) in the isotopes $i = \text{Xe, Ge and Te}$, building upon previous work [3]. The approach becomes exceedingly simple for quadratic approximations to such profiles, implying quadratic (in)equality in the landscape of nuclear matrix elements $M_i$ that connect the $S_i$ to the Majorana mass $m_{\beta\beta}$. For convenience, some results have been discussed in terms of pseudo data for null signal in Te (dubbed Te∗) for “average” NME values, possibly lowered to $\sim 40$–50 meV for favorable NME values.

Simple relations among the $M_i$ have been derived to gauge the relative contributions of different isotopic data in setting upper limits to $m_{\beta\beta}$ (for null best fits in Xe, Ge and Te∗), and to identify the conditions leading to a preference for nonzero $m_{\beta\beta}$ (for generic Xe, Ge and Te data). Using the latest available $0\nu\beta\beta$ data, as well as representative values of the NME from different models, we have discussed current constraints on $m_{\beta\beta}$ at several confidence levels and in various combinations, both numerically and graphically. Global 2$\sigma$ upper limits on $m_{\beta\beta}$ are found in the range from 40.5 to 135 meV, depending on the NME.

The approach can be easily extended to nonstandard processes for $0\nu\beta\beta$ decay [1, 36, 37] by replacing the relation $S_i = G_i M_i^2 m_{\beta\beta}^2$ with the appropriate phase space, NME and particle physics parameter characterizing the process. Also, the approach can be extended to generic $\Delta \chi^2(S_i)$ functions, with a modest price to pay in terms of numerical (rather than analytical) solutions. We invite the experimental collaborations involved in $0\nu\beta\beta$ decay searches to publicly provide such $\Delta \chi^2(S_i)$ functions or equivalent ones, as they contain much more information than the usually quoted 90% C.L. limits on $T_i$. Indeed, the relative impact of such limits and of the resulting bounds on $m_{\beta\beta}$ in a multi-isotope combination depend sensitively on the likelihood profiles of $S_i$ (or $T_i$), and not only on the relative size of the $M_i$.

Our approach to the multi-isotope data analysis would be complete if one could also assign joint probability densities to the $M_i$, whose variations could then be treated as nuisance parameters and marginalized. So far, detailed results for the NME central values and covariances, including $g_3$ quenching uncertainties, have been obtained in a specific (QRPA) model [27]. In perspective, it would be important to extend such investigations to other nuclear models, eventually reaching consensus values for the $M_i$ and for their correlated (and possibly reduced) uncertainties.

In this sense, the combined analysis of $0\nu\beta\beta$ results is proceeding through to the same steps that have characterized similar fields (e.g., solar neutrinos) in the past: from low-statistics data and theoretical models with large uncertainties, to a wealth of accurate experimental results interpreted in increasingly refined and constrained models. Our work aims at providing one methodological step along this path.
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