A-dependence of weak nuclear structure functions

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Abstract

Effect of nuclear medium on the weak structure functions $F_2^A(x, Q^2)$ and $F_3^A(x, Q^2)$ have been studied using charged current (anti)neutrino deep inelastic scattering on various nuclear targets. Relativistic nuclear spectral function which incorporate Fermi motion, binding and nucleon correlations are used for the calculations. We also consider the pion and rho meson cloud contributions calculated from a microscopic model for meson-nucleus self-energies. Using these structure functions, $F_i^A/F_i^{proton}$ and $F_i^A/F_i^{deuteron}$ (i=2,3, $A=^{12}C, ^{16}O$, $CH$ and $H_2O$) are obtained.

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I. INTRODUCTION

MINERνA at Fermi lab is measuring neutrino nucleus cross sections in various nuclear targets in the energy region of 1-20 GeV. Among the various goals, one of the aim is to study structure functions in the deep inelastic scattering processes. In this work, we have studied nuclear medium effects on the weak structure functions $F_2^A(x,Q^2)$ and $F_3^A(x,Q^2)$, using a relativistic nucleon spectral function that describes the momentum distribution of nucleons in the nucleus. We define everything within a field-theoretical approach where nucleon propagators are written in terms of this spectral function. The spectral function has been calculated using Lehmann’s representation for the relativistic nucleon propagator and nuclear many-body theory is used to calculate it for an interacting Fermi sea in nuclear matter. A local-density approximation is then applied to translate these results to finite nuclei [1, 2]. We have also taken target mass correction and shadowing effects into account. The details of the model are given in Refs. [1–3]. This paper is aimed to study the nuclear dependence of the weak structure functions $F_2^A$ and $F_3^A$ in the charged current neutrino/antineutrino induced deep inelastic reactions using $^{12}C$, $^{16}O$, $CH$ and $H_2O$ nuclear targets. We have obtained proton and deuteron structure functions and the ratio $F_i^A/F_i^{proton}$ and $F_i^A/F_i^{deuteron}$($i=2,3$, $A=$nuclear target) to see the nuclear target dependence on the weak structure functions. In our numerical calculations the deuteron structure functions have been obtained using the same formalism as for the nuclear targets but performing the convolution with the deuteron wave function squared instead of the nuclear spectral function.

II. FORMALISM

In the nuclear medium the expression for the differential scattering cross section is written as

$$
\frac{d^2\sigma_{\nu,\bar{\nu}}^A}{d\Omega'dE'} = \frac{G_F^2}{(2\pi)^2} \left| \frac{m_W^2}{q^2 - m_W^2} \right|^2 |k'| L_{\alpha\beta}^{\nu,\bar{\nu}} W_{A\alpha\beta},
$$

where $L_{\alpha\beta}^{\nu,\bar{\nu}}$ is the leptonic tensor and is given by

$$
L_{\alpha\beta}^{\nu,\bar{\nu}} = k^\alpha k'^\beta + k^\beta k'^\alpha - k.k'g_{\alpha\beta} \mp i\epsilon_{\alpha\beta\rho\sigma}k_\rho k'_\sigma,
$$

where plus sign is for antineutrino and minus sign is for neutrino.

$W_{A\alpha\beta}^A$ is the nuclear hadronic tensor defined in terms of nuclear hadronic structure functions $W_i^A(x,Q^2)$:
\[ W_{\alpha\beta}^A = \left( \frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta} \right) W_{\alpha\beta}^1 + \frac{1}{M_A^2} \left( P_{A\alpha} - \frac{P_A q}{q^2} q_\alpha \right) \left( P_{A\beta} - \frac{P_A q}{q^2} q_\beta \right) W_{\alpha\beta}^2 - \frac{i}{2M_A^2} \epsilon_{\alpha\beta\rho\sigma} P_A^\rho q^\sigma W_{\alpha\beta}^3 \]  

(3)

\( M_A \) is the mass of the nucleus and \( P_A \) is the momentum of the nucleus.

In the local density approximation the nuclear hadronic tensor \( W_{\alpha\beta}^A \) is written as [1]:

\[ W_{\alpha\beta}^A = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \int_{-\infty}^{\mu} dp^0 \frac{M}{E(p)} S_h(p^0, p, \rho(r)) W_{\alpha\beta}^N(p, q), \]  

(4)

\( S_h \) is the spectral hole function and has been taken from Ref. [4]. \( \rho(r) \) is the baryon density for the nucleus. \( M \) and \( E \) is the mass and energy of the nucleon.

In the above expression \( W_{\alpha\beta}^N(p, q) \) is the nucleon hadronic tensor and, in the limit of lepton mass \( m_l \to 0 \), only three structure functions contribute to the cross section. Therefore, in this limit, we may write:

\[ W_{\alpha\beta}^N = \left( \frac{q_\alpha q_\beta}{q^2} - g_{\alpha\beta} \right) W_{\alpha\beta}^{N(\nu)} + \frac{1}{M^2} \left( p_\alpha - \frac{p.A q}{q^2} q_\alpha \right) \left( p_\beta - \frac{p.A q}{q^2} q_\beta \right) W_{\alpha\beta}^{N(\nu)} - \frac{i}{2M^2} \epsilon_{\alpha\beta\rho\sigma} p^\rho q^\sigma W_{\alpha\beta}^{N(\nu)} \]  

(5)

where \( W_{\alpha\beta}^N \) are the structure functions, which depend on the scalars \( q^2 \) and \( p.A q \).

\( W_i^A(x, Q^2) \) (i=1-3) given in Eq.(3) are redefined in terms of dimensionless structure functions \( F_i^A(x, Q^2) \) through

\[ M_A W_1^A(\nu, Q^2) = F_1^A(x, Q^2), \quad \nu W_2^A(\nu, Q^2) = F_2^A(x, Q^2), \quad \nu W_3^A(\nu, Q^2) = F_3^A(x, Q^2). \]  

(6)

The expressions for \( F_2^A(x, Q^2) \) and \( F_3^A(x, Q^2) \) in the nuclear medium are obtained as [1]:

\[ F_2^A(x_A, Q^2) = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_{-\infty}^{\mu} dp^0 S_h(p^0, p, \rho(r)) \frac{x}{x_N} \left( 1 + \frac{2x_N p^2}{M_N^2} \right) F_2^N(x_N, Q^2) \]  

(7)

\[ F_3^A(x_A, Q^2) = 4 \int d^3r \int \frac{d^3p}{(2\pi)^3} \frac{M}{E(p)} \int_{-\infty}^{\mu} dp^0 S_h(p^0, p, \rho(r)) \frac{p^0 - p_\gamma}{(p^0 - p_\gamma)^2} F_3^N(x_N, Q^2) \]  

(8)

where \( F_2^N \) and \( F_3^N \) are the nucleon structure functions written in terms of parton distribution functions, \( q \) is the four momentum transfer, \( q^2 = -Q^2 \), \( x \) is the Bjorken variable, \( \nu \) is the energy transfer,
FIG. 1: Ratio $R(x,Q^2) = \frac{F_2^{CH}}{F_2^{P}}, \frac{F_2^{CH}}{F_2^{D}}, \frac{F_2^{P}}{F_2^{i}}, \frac{F_2^{C}}{F_2^{P}}, \frac{F_2^{C}}{F_2^{D}}$ (i=2(left) and 3(right)) using full model at NLO.

All the nuclear information like Pauli blocking, Fermi motion, nucleon correlation are contained in the spectral function. The results obtained by using Eqs.(7) and (8) with target mass correction is our base result. When shadowing and anti-shadowing effects for $F_2^A$ and $F_3^A$, as well as pion and rho cloud contributions for $F_2^A$, are taken into account, results obtained by including all these effects, we call this as the results with full calculations. Using the results of weak structure functions $F_2^A$ and $F_3^A$, we have obtained the ratio of structure functions $\frac{F_i^A}{F_i^{\text{proton}}}$ and $\frac{F_i^A}{F_i^{\text{deuteron}}}$. For the numerical calculations, parton distribution functions for the nucleons have been taken from the parametrization of the Coordinated Theoretical-Experimental Project on QCD (CTEQ) Collaboration (CTEQ6.6) [5]. The Next-to-Leading-Order (NLO) evolution of the deep inelastic structure functions has been taken from the works of Vermaseren et al. [6].

III. RESULTS AND DISCUSSION

We have studied the medium effects on the weak structure functions $F_2^A$ and $F_3^A$ in the charged current anti(neutrino) induced deep inelastic reactions using carbon and oxygen targets for different values of $x$ at the next-to-leading order (NLO). We find that the difference between the base results and the full calculations is about 10-12% and 4-6% at low values of $x$ in the case of $F_2^A$ and $F_3^A$. 

\[ \gamma = \frac{q_z}{q_0} = \left(1 + \frac{4M^2x^2}{Q^2}\right)^{1/2} \quad \text{and} \quad x_N = \frac{Q^2}{2(p^0q_0 - p_zq_z)}. \]
respectively and this difference reduces to 1% at higher x for both the cases. We have presented the results for the ratio $F_i^A/F_i^{\text{proton}}$ and $F_i^A/F_i^{\text{deuteron}}(i=2,3)$ in Figs. 1 and observe that the nuclear medium effects are not the same in different nuclear targets, as well as the nature of medium effects in $F_2^A$ and $F_3^A$ are different. Furthermore, the ratio of $F_i^{\text{deuteron}}/F_i^{\text{proton}} (i=2,3)$ has also been shown in Figs. 1 and we find that the medium effects in deuteron are quite different from the medium effects in heavy nuclei. This study may be useful in the analysis of MINERνA and other proposed anti(neutrino) experiments in the energy region of few GeV using nuclear targets.

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