Motion of a particle and the vacuum

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Abstract

We propose the deterministic dynamics of a free particle in a physical vacuum, which is considered as a discrete (quantum) medium. The motion of the particle is studied taking into account its interactions with the medium. It is assumed that this interaction results in the appearance of special virtual excitations, called "inertons," in the vacuum medium in the surroundings of the canonical particle. The solution of the equation of motion shows that a cloud of inertons oscillates around the particle with amplitude \( \Lambda = \lambda v_0/c \), where \( \lambda \) is the de Broglie wavelength, \( v_0 \) is the initial velocity of the particle, and \( c \) is the initial velocity of the inertons (velocity of light). This oscillating nature of motion is also applied to the particle, and the de Broglie wavelength \( \lambda \) becomes the amplitude of spatial oscillations. The oscillation frequency \( \nu \) is given by the relation \( E = h \nu \). The connection of the present model with orthodox nonrelativistic wave mechanics is analyzed.

Key words: polaron, elementary particles, physical vacuum, gravitation in microspace, geodesic equation, wave mechanics, quantum mechanics, harmonic oscillator, hidden dynamics of particle, physical constants, hypothetical particles

1 Introduction

The concepts of condensed media physics penetrate more and more deeply into the models of elementary particles [1,2] and into the models of the physical vacuum [1, 2-5]. Specifically, in Ref. 3 the vacuum is considered as an elastic medium; more fundamentally, the possibility of simulating the vacuum with the...
form of a quantum crystal is demonstrated in Ref. 5. A subquantum medium is also presented in papers studying the problem of the causal interpretation of quantum mechanics [6].

The present paper proposes a deterministic dynamics of a free elementary particle in a physical vacuum considered as a discrete (quantum) substance. The "molecules" of this substance, which may be called superparticles, form a peculiar elastic medium. The concept of the mass of the particle is introduced, and its motion is studied, taking into account its interaction with the vacuum medium. It is suggested that this interaction results in the appearance of special virtual excitations of the medium. The relationship between the present non-relativistic model and orthodox nonrelativistic quantum mechanics is analyzed.

2 A particle in a vacuum medium

A particle will be taken to mean one of multiple states of a superparticle corresponding to a lepton, for example, an electron. It is reasonable to assume that the size of a superparticle in the degenerate state is close to the value $10^{-28}$ cm (the size required by the grand unification of interactions). At such scales gravitational effects can become extremely substantial [4,5]. One of the manifestations of microgravitation can be an effect similar to the state of a polaron in a crystal, that is, deformation of the vacuum in the neighborhood of the particle. Clearly, the simplest of the reasons that can cause such deformation is a deviation of the typical dimension of the particle creation from the dimension of the initial degenerate superparticle. Figure 1 pictorially illustrates this. A vacuum medium is shown in Fig. 1, with each cell occupied by a superparticle.
in the degenerate state and interacting only with the nearest neighbors. If the superparticle possesses elastic properties, then the vacuum medium naturally should deform in the neighborhood of the particle at the creation of the particle (i.e., when the size of one of the superparticles varies from the initial value $R_0$ to the fixed value $R_{\text{part}}$, for example, $R_{\text{part}} < R_0$) [Fig. 1(b)].

In the initial state [Fig. 1(a)], the special position of the medium cells is characterized by Cartesian coordinates $X_i$, but with the creation of a particle, the new symmetry [Fig. 1(b)] is already described by generalized coordinates $Q_k$ which are functions of $X^i$. This distortion of boundary lines of the vacuum medium cells is similar to gravitational field lines. Therefore, it can be said that the particle is in a gravitational potential which defines the distortion of space.

In the general theory of relativity, the components of the metric tensor $g_{ij}$ act as the gravitational potential, but in our model, quantities $g_{ij}$ characterize the variation of the equilibrium position of superparticles. Identifying the deformation of the vacuum medium caused by the particle with gravitational potential of the particle, we can thus relate the stationary relative deviation of the dimension of the superparticle from the initial value $R_0$ to the appearance of mass in them. Taking into account the three dimensionality of space, the mass of the superparticles should be related to the relative variation of volume: $m(Q^k) = \text{const} R_0^3/R_{\text{part}}^3$ (but in the general case the ratios between volumes, i.e., between mass $m$ and $M$, are tensor quantities).

It should be noted that the vacuum may be regarded as a special crystal only within the deformation "coat" (or distance) formed in the discrete vacuum medium due to particle creation (superparticles are characterized by mass only within this region). This crystal should obviously possess some features typical of a solid crystal. The vacuum medium is found in a degenerate state beyond the limits of the "coat."

3 The hypothesis of motion

The motion of a large polaron in a solid (see, e.g., Ref. 7) comprises the motion of the charge carrier and that of the polaron "coat." Similarly, in a vacuum medium a particle in a motion will also pull behind it the vacuum deformation "coat" (Fig. 1) will remain invariable at any point of its location (in order that this takes place, the rate of adjustment of superparticles naturally should be higher than the velocity of the particle). The particle moving in a superdensely packed medium naturally cannot move freely; it certainly should experience friction losing the velocity as this takes place. As a result of the interaction with the medium, the particle emits (absorbs) the elementary virtual excitations of this medium. It is reasonable to relate the initial speed of these excitations with a value on the order of or equal to the velocity of light $c$, but the velocity of the particle $v_0 < c$. This inequality and the availability of the deformation potential created by a particle in a vacuum medium allow one to assume in principle the possibility of nonstationary motion whose essence can
be reduced to the following.

Ejected (at an angle to the trajectory) excitations are ahead of the particle, but they are gradually retarded by its potential; then, having reached the boundary deformation coat-degenerated vacuum, return again to the particle (i.e., to the center of the potential maintaining them) and transmit to it the lost speed. However, to the moment of the return of an excitation to the particle, the latter has already shifted forward from the point where the excitation was emitted. Therefore, the excitations emitted by the particle ahead themselves will return to it from behind so that the father particle will be moved forward in a direction that is defined by the initial velocity vector $\vec{v}_0$ (Fig. 2). Let us study this motion mathematically under the assumption that external fields (gravitational, etc.) are absent.

4 Lagrangian and the equation of motion

Let us proceed from the Lagrangian

$$L = \frac{1}{2} g_{ij} \dot{X}^i(t) \dot{X}^j(t) + \frac{1}{2} \sum_{l=0}^{N-1} \tilde{g}_{ij}^{(l)} \dot{\delta}_{(l)}^{ij}(t(l)) \dot{x}_{(l)}^{ij}(t(l))$$

$$- \sum_{l=0}^{N-1} \frac{\varpi}{T} \delta_{t_{ij}, t(l)} \left\{ X^i(t) \sqrt{g_{tr}(\hat{A}^{-1} \tilde{g}_{ij}^{(l)})_0} \dot{x}_{(l)}^{ij}(t(l)) \right\} \right. + \dot{X}^i(t) \left|_{t=0} \sqrt{g_{tr}(\hat{A}^{-1} \tilde{g}_{ij}^{(l)})_0} \dot{x}_{(l)}^{ij}(t(l)) \right\}$$
The first term characterizes the particle, the second term characterizes the ensemble of $N$ excitations, emitted by the particle, and the third characterizes the contact interaction between the particle and the ensemble. $X^i$ and $\dot{X}^i$ are the $i$th components of the position and the velocity of the particle, respectively; $g_{ij}$ is a metric tensor generated by the particle in three-dimensional space. Indices $l$ correspond to the number of the respective excitation; they are enclosed in parentheses to distinguish them from indices $[i,j,\text{ and } r$ in Eq. (1)] describing components of vector and tensor quantities; $x_{(l)}^i$ and $\dot{x}_{(l)}^i$ are components of position and velocity of the $l$th excitation; $\tilde{g}_{rj}^{(l)}$ is the metric tensor generated by the $l$th excitation in three-dimensional space (it describes local deformation in the neighborhood of the excitation); $1/T_l$ is the frequency of collisions of the particle with the $l$th excitation; Kronecker’s symbol $\delta_{t-\Delta t(l),t(l)}$ provides the agreement of proper times of the particle and the $l$th excitation at the instant of their collision ($\Delta t(l)$ is the time interval after the expiry of which, measuring from the initial moment $t = 0$, the moving particle emits the $l$th excitation).

Operator $\hat{A}^{-1}$ appearing in the interaction energy characterizes the rotation of three-dimensional space around axis $X^i$; matrix $A$ belongs to the group of rotation $\text{SO}(3)$. This rotation of space eventually results in the motion of excitation metric $\tilde{g}_{rj}^{(l)} \rightarrow \tilde{g}_{r+\alpha,j}^{(l)}$ (with regard to the cyclic permutation index $\alpha$ in the same manner as indices $r$ and $j$ take on values $1, 2, 3$), that is, operator $\hat{A}^{-1}$ acts according to the following rule:

$$g_{ir}(\hat{A}^{-1}\tilde{g}_{rj}^{(l)})_0 = g_{ir}(A_{\frac{1}{A}} \frac{\partial z^k_l}{\partial x_{(l)}^i} \frac{\partial z^k_l}{\partial x_{(l)}^j})\Big|_{x(l)=0}$$

$$= g_{ir}(A_{\frac{1}{A}} \frac{\partial z^k_l}{\partial x_{(l)}^i} \frac{\partial z^k_l}{\partial x_{(l)}^j})\Big|_{x(l)=0} = g_{ir}(\tilde{g}_{r+\alpha,j}^{(l)})_0 (2)$$

the radius vector $x_{(l)}(x_{(l)}^j)$ of the $l$th excitation is measured from the radius vector of the particle $X(X^i)$. For example, if rotation takes place in Euclidean space around axis $X^1$, then (see Ref. 8)

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \varphi & \sin \varphi \\ 0 & -\sin \varphi & \cos \varphi \end{pmatrix}$$

where $\varphi$ is the angle of rotation. Thus operator $\hat{A}^{-1}$ acting according to rule (2) provides the change from the space of the particle motion to the space of excitation motion, that is, it shifts the excitations to trajectories different from the trajectory of the particle (see Fig. 2). In the subsequent discussion we shall use the notation

$$\hat{B}_{ij} = \sqrt{g_{ir}(\hat{A}^{-1}\tilde{g}_{rj}^{(l)})}_0 .$$

Let us write the Euler-Lagrange equations

$$d(\partial L/\partial \dot{Q}^k)/dt - \partial L/\partial Q^k = 0,$$
where for the particle
\[
Q^k = X^k(t_l) + \Delta t_l = X^k_{(l)}(t_l + \Delta t_l)
\]  
(5a)
and for excitation
\[
Q^k = x^k_{(l)}(t_l)
\]  
(5b)
we take into account the availability of the symbol \(\delta_{t_l-t_{l,t}}\) in the potential energy in Eq. (1). From Eq. (5), taking into account Eqs. (1) and (3), we obtain the equation of extremals:

\[
\ddot{X}^i_{(l)} + \dot{\Gamma}^i_{kl}X^l_{(l)}X^j_{(l)} + \frac{\pi}{T_l(g^{ks}\frac{\partial \hat{B}_{ij}^{(l)}}{\partial X^k_{(l)}})(X^i_{(l)}\ddot{X}^j_{(l)} + X^i_{(l)}\frac{\partial \hat{B}_{ij}^{(l)}}{\partial X^j_{(l)}} + X^j_{(l)}\frac{\partial \hat{B}_{ij}^{(l)}}{\partial X^i_{(l)}})} = 0;
\]  
(6)

\[
\ddot{x}^i_{(l)} + \dot{\Gamma}^i_{kl}x^l_{(l)} \ddot{x}^j_{(l)} + \frac{\pi}{T_l(g^{ks}\frac{\partial \hat{B}_{ij}^{(l)}}{\partial x^k_{(l)}})(x^i_{(l)}\ddot{x}^j_{(l)} + x^i_{(l)}\frac{\partial \hat{B}_{ij}^{(l)}}{\partial x^j_{(l)}} + x^j_{(l)}\frac{\partial \hat{B}_{ij}^{(l)}}{\partial x^i_{(l)}})} = 0;
\]  
(7)

here, \(\Gamma^i_{kl}\) and \(\hat{\Gamma}^{(l)}_{ij}\) are symmetrical connections (see, e.g., Ref. 8) for the particle and for the \(l\)th excitation, respectively; indices \(i,j\) and \(s\) take the values 1, 2, 3.

Let us analyze Eqs. (6) and (7). First of all, let us specify the form of tensors \(g_{ij}\) and \(\hat{g}^{(l)}_{ij}\) in three-dimensional space. We have noted that the deformation of the vacuum medium in the neighborhood of the particle can be related to the availability of the particle of deformation (gravitational) potential, which we shall designate as \(V(r)\). If we take \(V(r)\) to mean the potential of the particle related to its self-energy, then a nonparametrized metric tensor in this region of the space will take the form

\[
g_{ij} = \text{const } M \delta_{ij}(1 - V(M;r)),
\]
where the first term corresponds to the coefficient of metric of flat space, that is, of an undistorted vacuum medium \((g_{ij} = \delta_{ij})\). Similarly, for the \(l\)th excitation

\[
\hat{g}^{(l)}_{ij} = \text{const } m_{(l)} \delta_{ij}(1 - W_{(l)}(m_{(l)};r));
\]  
(9)

here, \(m_{(l)}\) is the mass of an \(l\)th excitation (the quantity \(m_{(l)}\), i.e., an additional deformation, is induced on the \(l\)th superparticle when the particle strikes it). Because the inequality \(m_{(l)} \ll M\) holds, \(W_{(l)}(M)\) is local, but it is identical by its nature to potential \(V\). Differences between them consists of the value of mass coefficient: for the particle, \(V \propto M\); for the elementary excitation, \(W_{(l)} \propto m\).

Clearly, from the physical point of view, the motion of the metric \(\hat{g}^{(l)}_{ij} \rightarrow \hat{g}^{(l)}_{ij} \rightarrow \hat{g}^{(l)}_{ij}\) under the effect of operator \(\hat{A}^{-1}\) should have a definite duration in time.
Then the process of "knocking out" by the particle from the vacuum medium of the next \( l \)th excitation can be conventionally subdivided into three stages:

1. The union of the particle with the \( l \)th oncoming superparticle and the formation of a common system.

2. As a result of stage (1), the intensification of internal processes in the \( l \)th superparticle occurs, which processes continue during a definite time interval.

3. Then the decay of the system occurs, that is, the further motion of the particle and the \( l \)th excitation emitted by the respective superparticle.

It is reasonable to assume that at the first stage the interaction operator in Eqs. (6) and (7) is still not engaged, \( \hat{B}^{(l)}_{ij} = 0 \). Then the termwise difference between Eqs. (6) and (7) is reduced to the form

\[
(\ddot{X}_s^{(l)} - \ddot{X}_s^{(0)}) = 0, \quad (10)
\]

and Eq. (10) can be considered as the equation determining the point of intersection of geodesics particle and excitation. The particle and the excitation are united here to a common system, and because of this the acceleration that one of the partners of the system experiences coincides with the acceleration that the other partner experiences. Therefore, the difference in the first set of parentheses in Eq. (10) is equal to zero, and we obtain the relation

\[
\Gamma_{ij}^{s} \dot{X}_i^{(l)} \dot{X}_j^{(l)} = \tilde{\Gamma}_{ij}^{(l)s} \dot{X}_i^{(l)} \dot{X}_j^{(l)} . \quad (11)
\]

The structures of fields \( \Gamma_{ij}^{s} \) and \( \tilde{\Gamma}_{ij}^{(l)s} \) in the point of cross-geodesics are identical. However, the values of intensity of the fields are different; coefficients \( \Gamma_{ij}^{s} \) are generated by the mass \( M \) [according to (8)], \( \Gamma \propto \partial g/\partial X \propto V(M) \propto M \), but coefficients \( \tilde{\Gamma}_{ij}^{(l)s} \) are generated by the mass \( m^{(l)} \) [according to (9), \( \tilde{\Gamma}^{(l)s} \propto W^{(l)}(m^{(l)}) \propto m^{(l)} \)]. Hence the relation \( \Gamma_{ij}^{s}/\tilde{\Gamma}_{ij}^{(l)s} = M/m^{(l)} \) holds.

Let us designate the velocity of the particle at the point of emission of the \( l \)th excitation as \( v^2_{0l} \), and let us assume the rate of excitation at this point (initial rate) is equal to the velocity of light, \( |\dot{x}(l)|_0 = c \). Then, instead of expression (11), we obtain

\[
Mv^2_{0l} = m^{(l)}c^2 . \quad (12)
\]

As seen from relation (12), we face here an usual situation: the particle creates at each collision a virtual excitation in the vacuum with the kinetic energy equal to the kinetic energy of the particle itself.

In order that such solutions take place, we must postulate the following properties of the vacuum medium: first, the shape and size of superparticles can fluctuate, and the superparticles are less rigid as compared with the particle; second, by analogy with the behavior of atoms in a solid, let us assume that superparticles participate in collective vibrations at which they periodically shift from the initial equilibrium states.
Having accepted the present hypotheses, we come to the mechanism of emission of excitation. In fact, a moving particle at an inelastic contact with "soft" \( l \)th superparticle of course deforms the latter. At a farther displacement of the particle to the \((l + 1)\)st cell, the superparticle in the \(l\)th cell remains deformed (excited). However, deformation of the \(l\)th cell swiftly relaxes to the initial state as a result of fluctuations of the elastic energy stored in collective vibrations of the vacuum lattice. In this case, however, local deformation from the \(l\)th cell transferred by the lattice vibrations deep into the vacuum medium in a direction perpendicular to the trajectory of the particle (Fig. 3), that is, the migration of the deformation in this direction is due to elastic forces acting in the vacuum medium lattice, but not due to the transference of particle momentum. Thus the system in question is an open system; therefore, expression (12) could not be considered as an energy conservation law. The migration of this deformation can be identified with the emission of the \(l\)th excitation with mass \(m_{\{l\}}\).

Let us note that the motion of small polaron in a solid [7] (for the motion of the proton polaron, see Ref. 9), that of polariton excitations or a drift of ions in a polarizable medium can be analogous to the migration of the present elementary excitations. In the polaron potential well at the stationary state a charge carrier is on one of the lowest level. A shift of the particle to the next well takes place as a result of the fluctuating filling of the first well by the energy of the vibrating lattice, that is, it is activated by photons. But a directed drift of the particle is provided by the electric field, by the temperature gradient, by the medium deformation gradient, etc. In our case the drift of excitation in a direction perpendicular to the particle trajectory is caused by the initial condition – by the deformation of the \(l\)th superparticle in this direction (Fig. 3). The realization of the described mechanism assumes an emission of excitation at a right angle to the trajectory of the particle. Therefore, it is necessary to require that operator \(\hat{A}^{-1}\) (or \(\hat{B}^{(l)}_{ij}\)) in Eqs. (6) and (7) provides a rotation of the space through and angle \(\varphi = \pi/2\). In this case superimposed on the axes of the original coordinate system are the axes of a new coordinate system in which, however, the axes are redesignated; then the direction cosines in expression (3) are equal to one.

The general form of Eqs. (6) and (7) is true for the particle and the \(l\)th excitation only at the moment of their interaction (the second stage), when metric tensors are influenced (also, an external gravitational field is available). After the interaction the particle and the \(l\)th excitation fly apart along their own trajectories, where the particle at each of its path is described by the metric tensor
\[
g_{ij} = \text{const} \, M \delta_{ij},
\]
and the excitation present in the field (8) of the particle in each point of its path is represented by the metric tensor
\[
\tilde{g}_{ij}^{(l)} = \text{const} \, m_{\{l\}} \delta_{ij}.
\]
Tensors (13) and (14) are constants; therefore, all their deviations along the
respective trajectories are equal to zero. As a result, only two terms remain in Eqs. (6) and (7) – the first and the last ones, the latter being transformed as follows:

\[ g^{\cdot\cdot} B^{(l)} \rightarrow \sqrt{m(l)/M} = v_{0l}/c, \]
\[ \tilde{g}^{(l)} \cdots \hat{B}^{(l)} \rightarrow \sqrt{M/m(l)} = c/v_{0l}. \]

Here we have taken into consideration relation (12) and the fact that by definition \( g^{sk} g_{kj} = \delta_j^s \). From now on we will omit the parentheses for indices \( l \).

### 5 Solution of the equations of motion

Let us assume that the particle moves along axis \( X^1 \equiv X \) in a Cartesian coordinate system. The reference point of excitation radius vector \( x_l(x_l) \) is associated with the particle. Let projections of these vectors on axes \( X^1, X^2, \) and \( X^3 \) be \( x_1^l, x_2^l, \) and \( x_3^l \). If we introduce a generalized coordinate \( x_\perp^l = \sqrt{(x_2^l)^2 + (x_3^l)^2} \) then the problem is reduced to a two-dimensional one: the particle moves along axis \( X \), and migrating excitations are characterized by projections \( x_\parallel^l \) and \( x_\perp^l \) on the axis and on the axis perpendicular to it, respectively. Thus the system of equations (6), (7) in Euclidean space with regard for formulas (15), (16) takes the form

\[ \ddot{X}_l + \frac{\pi v_{0l}}{T_l} c \dot{x}_\perp^l = 0, \]
\[ \dot{x}_\perp^l - \frac{\pi c}{T_l v_{0l}} (\dot{X}_l - v_{0l}) = 0, \]
Recall that indices by unknowns $X_l$ as well as by $x_l^\perp$ point to their dependence on proper time $t_l$ of the $l$th excitations. Differentiations of Eq. (18) with respect to $t_l$ yields

$$
\ddot{X}_l - \frac{\pi}{T_l} c \dot{x}_l = 0.
$$

(20)

Let us express $\ddot{X}_l$ in terms of Eq. (20), and let us substitute it into Eq. (17). As a result, we arrive at the equation of harmonic oscillator for $x_l^\perp$:

$$
\ddot{x}_l^\perp + \left(\frac{2\pi}{2T_l}\right)^2 x_l^\perp = C_{1l}.
$$

(21)

One can see from Eq. (21) that the excitation executes harmonic oscillations with period $2T_l$ along the axis perpendicular to the trajectory of the particle. Let us write initial conditions for transverse coordinate of excitations:

$$
x_l^\perp\bigg|_{t_l=0} = 0, \quad \dot{x}_l^\perp\bigg|_{t_l=0} = c.
$$

(22)

With regard to conditions (22), the solution of Eq. (21) takes the following form:

$$
x_l^\perp = \frac{\Lambda_l}{\pi} \sin(\pi t_l/T_l);
$$

(23)

$$
\dot{x}_l^\perp = c \cos(\pi t_l/T_l);
$$

(24)

here, amplitude $\Lambda_l/\pi$ corresponds to the point of maximum separation of the $l$th excitation from the particle (Fig. 2). In this case

$$
\Lambda_l/T_l = c.
$$

(25)

It should be noted that solutions $x_l^\perp$ and $\dot{x}_l^\perp$ in the form in which they are written in Eqs. (23), (24) are true only in the interval from $t_l = 0$ to $t_l = T_l$ (see below). Let us now find $X_l$ and $\dot{X}_l$ also in the time interval from $t_l = 0$ to $t_l = T_l$. We have from Eq. (17)

$$
\dot{X}_l = C_{2l} - \frac{\pi}{T_l} c x_l^\perp.
$$

(22)

Initial conditions for the particle are

$$
\dot{X}_l(t_l + \Delta t_l)|_{t_l=0} = \dot{X}(\Delta t_l) = v_0 t_l.
$$

(27)

Now, we obtain from Eq. (26) with regard to Eqs. (23) and (27):

$$
\dot{X}_l = v_0 t_l[1 - \sin(\pi t_l/T_l)],
$$

(28)

$$
X_l = v_0 t_l + \frac{\lambda_l}{\pi} [\cos(\pi t_l/T_l) - 1],
$$

(29)

where we have introduced the notation

$$
\lambda_l = v_0 T_l.
$$

(30)
Figure 4: Trajectories of motion of the $l$th excitation. It is emitted in time $\Delta t_l$ ($t$ and $t_l$ are proper times of the particle and the $l$th excitation, respectively, with $t = t_l + \Delta t_l$, where $\Delta t_l = Tl/2N$, $T_l = T - 2\Delta t_l \equiv T(1 - l/N)$, and $l = 0, N - 1$).

It is easily seen from Eq. (28) that the velocity of the particle is a periodic time function on interval $T_l$: at $t_l = 0$, $\dot{X}_l = v_{0l}$; at $t_l = T_l/2$, $\dot{X}_l = 0$; and at $t_l = T_l$, $\dot{X}_l = v_{0l}$ again.

Let us find the relationship between the initial velocity $v_{0l}$ of the particle (at the instant moment of time $t_l = 0$, where $l \neq 0$) and the initial velocity $v_0$ (at the instant of time $t = t_l = 0$, i.e., at $l = 0$). Assuming $l = 0$ in Eq. (28), we go from time $t_l$ to the proper time of the particle $t$:

$$\dot{X}_l(t_l + \Delta t_l) = \dot{X}(t) = v_0[1 - \sin(\pi t/T)].$$

(31)

For the instant of time $t_l = \Delta t_l$, it follows from Eq. (31) that

$$\dot{X}(\Delta t_l) = v_0[1 - \sin(\pi \Delta t_l/T_l)].$$

(32)

or, since $\Delta t_l = Tl/2N$ (see Fig. 4), we obtain instead of Eq. (32),

$$\dot{X}(\Delta t_l) = v_0[1 - \sin(\pi l/2N)].$$

(33)

But it is at the instant of time $t = \Delta_l$ that the $l$th excitation is emitted. Therefore, formula (33) should be compared with formula (27). As a result, we obtain for $v_{0l}$

$$v_{0l} = v_0[1 - \sin(\pi l/2N)].$$

(34)
Thus we have obtained harmonic solutions for the velocity and the coordinate of the particle, and in this case the motion of the particle is characterized not only by the time half period $T_l$ of the cycle, but also by the space period $2\lambda/2\pi = \lambda_l/\pi$. However, there is a time delay $2\Delta t_l$ (and also a spatial delay) between the instant of absorption of the $l$th excitation and the instant of its subsequent emission (see Fig. 4). Therefore, to extend solutions (28), (29) as well as (23), (24) to the following oscillations of the particle (Fig. 2), a particle transition from the $(n-1)$st to the $n$th oscillation should be included in these formulas, that is, a quasicyclicity in parameter $t_l$ should be introduced. Substitution $t_l \to t_{nl}$ meets this requirement, where

$$t_{nl} = t_l + 2(n-1)T_l, \quad n = 1, 2, 3, \ldots;$$

$$T_l = T(1 - 1/N), \quad 0 \leq t_l \leq T_l, \quad l = 0, N - 1. \quad (35)$$

Thus the complete solution of system of Eqs. (17), (18) is determined by Eqs. (23), (24) and (28), (29) in which, however, the quasicontinuous parameter $t_{nl}$ given by formula (35) takes the part of proper time.

We now turn our attention to the solution of Eq. (19). It follows from this solution that $\dot{x}_l^\parallel = \text{const}$. We can use the law of conservation of momentum to find $\dot{x}_l^\parallel$, since collisions of the particle with superparticles are inelastic. However, after a time $t_l = T_l$ has passed from the instant of emission of the $l$th excitation, the latter should again be absorbed by the particle. The path traversed by the particle in time $t_l = T_l$, according to Eq. (29) is equal to

$$X_l(T_l) = 3\pi v_0 l T_l/2. \quad (36)$$

Equating $X_l(T_l)$ to product $\dot{x}_l^\parallel$, we find

$$\dot{x}_l^\parallel = 3\pi v_0 l/2.$$

To take into account that function $\dot{x}_l^\parallel$ relates to the $n$th oscillation of the particle, it is sufficient to introduce time parameters

$$\tau_{nl} = (2n-1)\Delta t_l + (n-1)T_l, \quad n = 1, 2, 3, \ldots;$$

$$\Delta t_l = T_l/2N, \quad T_l = T(1 - 1/N), \quad l = 0, N - 1. \quad (38)$$

Then, instead of Eq. (37), we have for $\dot{x}_l^\parallel$ as for a function of proper time $t$ of the particle

$$\dot{x}_{nl}^\parallel = \frac{3\pi}{2} v_0 \Theta(t - \tau_{nl}) \Theta(\tau_{nl} + T_l - t); \quad (39)$$

here, the $\Theta$ function has been introduced [$\Theta(t) = 1$ when $t \geq 0$, and $\Theta(t) = 0$ when $t < 0$].
6 Orthodox wave mechanics

Let us write the Lagrangian (1) in two-dimensional Euclidean space in the form

\[ L = M \dot{\mathbf{X}}^2/2 + m[(\dot{\mathbf{x}}^\parallel)^2 + (\dot{\mathbf{x}}^\perp)^2]/2 \]

\[ -\frac{\pi}{T}\sqrt{mM} \left(\dot{x}^\perp X + v_0 x^\perp\right); \quad (40) \]

here, parameter \( m \) is the mass of the whole excitations cloud [the last three terms in expression (40) describe the motion of the center of mass of the cloud and its interaction with the particle]; \( 1/T \) is the frequency of collisions.

Using the substitution

\[ \dot{x}^\perp = \ddot{x}^\perp + \pi\sqrt{mM} X/T, \quad (41) \]

the Lagrangian (40) is reduced to the canonical form

\[ L = M \dot{\mathbf{X}}^2/2 - M(\pi/2T)^2 X^2/2 \]

\[ +m[(\ddot{x}^\perp)^2 + (\dot{\mathbf{x}}^\parallel)^2]/2 - \pi\sqrt{mM} v_0 x^\perp/T. \quad (42) \]

From this we obtain for the effective Lagrangian of the particle

\[ L_{\text{eff}} = M \dot{\mathbf{X}}^2/2 - M(\pi/2T)^2 X^2/2. \quad (43) \]

As is seen, function (43) describes a harmonic oscillator. It should be noted that this form of writing \( L_{\text{eff}} \) does not take into account the translational motion of the particle; therefore, the results based on expression (43) refer to the behavior of the particle in the system of the particle and excitations center of mass.

Other terms in expression (42) characterize the effective kinetic and potential energies of the cloud of excitations. Let us introduce the Hamiltonian corresponding to the function \( L_{\text{eff}} \):

\[ H_{\text{eff}} = p^2/2M + M(2\pi/2T)^2 X^2/2. \quad (44) \]

Solutions of the equations of motion given by function (44) are well known for different presentations (see, e.g., Refs. 10 and 11). Specifically, we can obtain from expression (44) the Hamilton-Jacobi equation

\[ \left(\partial S_1/\partial X\right)^2/2M + M(2\pi/2T)^2 X^2/2 = E, \quad (45) \]

from which we obtain the equation for a shortened action:

\[ S_1 = \int p \, dX = \int \frac{X}{\sqrt{2M[E - (2\pi/2T)^2 X^2/2]}} \, dX. \quad (46) \]

It is easily find [10,11] a solution \( X \) as a function of \( t \) for the harmonic potential:

\[ X = \frac{(2E/M)^{1/2}}{(2\pi/2T)} \sin(2\pi t/2T). \quad (47) \]
In action-angle variables, let us determine the increment of the action of the particle in period $2T$:

$$J = \oint pdX = \frac{2E}{(2\pi/2T)} \int_0^{2\pi} \cos^2 \vartheta d\vartheta = E \cdot 2T = E/\nu,$$  \hspace{1cm} (48)

here, the notation $\nu = 1/2T$ and the substitution $X = \sqrt{2E/\{M(2\pi/2T)^2\}} \sin \vartheta$ are used. Constant $E$ is initial energy of the particle, and so we obtain by substituting $E = Mv_0^2/2$ into Eq. (48)

$$J = Mv_0 \cdot Mv_0 = p_0 \lambda.$$  \hspace{1cm} (49)

Here, $\lambda = v_0 T$ can be called the reduced amplitude of the oscillating particle, since, according to Eq. (47), the amplitude of an oscillator is equal to $2Tv_0/2\pi$ [compare with expression (30), where $\lambda$ is the reduced spatial period of oscillations of a moving particle]. Thus the effective Hamiltonian (44) of the particle enables formulas (48) and (49) to be obtained, which, when we assume $J = h$ ($h$ is Planck’s constant), lead to two main relationships in quantum mechanics:

$$E = h\nu, \quad p_0 = h/\lambda.$$  \hspace{1cm} (50)

In formulas (50), $\nu$ is the frequency of oscillation of a particle with initial energy $E$, and $\lambda$ is the reduced amplitude of oscillations which we can identify with the de Broglie wavelength.

Let us write the complete action:

$$S = S_1 - Et = \int p \, dX - Et.$$  \hspace{1cm} (51)

Here, $S_1$ is defined in Eq. (46) and describes the particle in the finite region where the variable $X$ is restricted by amplitude $\lambda/\pi$. However, passing to configurational space, we can write the coordinate of the particle, taking into account its $n$th oscillation, in the form

$$X \rightarrow X_n + (n - 1)\lambda/\pi;$$

at $n \gg l$, it turns into a continual variable. Now, when we assume in Eq. (5), $p = \text{const} = p_0 = Mv_0$, we obtain an action that is transformed to a characteristic function of uniform infinite motion in configurational space:

$$\bar{S}_{\text{part}} = Mv_0X - Et.$$  \hspace{1cm} (52)

But if we assume in Eq. (51) according to expression (50) that $p = \text{const} = h/\lambda$ and $E = h\nu$, then we obtain the action in the form

$$\bar{S}_{\text{wave}} = h(X/\lambda - nt),$$  \hspace{1cm} (53)
and then $\tilde{S}_{\text{wave}}/\hbar$ can be related to the phase of a monochromatic wave propagating along the $X$-axis of configurational space. Both $S_{\text{part}}$ and $\tilde{S}_{\text{wave}}$ are obtained from one and the same Eq. (51) in one and the same approximation but in different representations: the former in terms of the initial momentum $p_0$ and energy $E = p_0^2/2M$ of the particle and the latter in terms of amplitude $\lambda$ and frequency $\nu$ of its oscillations. Both expressions (52) and (53) are identical, and this allows us to compare the particle moving in configurational space with the wave of the form

$$\psi = a \exp\left(2\pi \frac{\tilde{S}_{\text{wave}}}{\hbar}\right). \quad (54)$$

With regard to (30) and (50), a substitution of the wave function (54) into the wave equation

$$\Delta \psi - \frac{1}{(v_0/2)^2} \frac{\partial^2 \psi}{\partial t^2} = 0$$

results in Schrödinger’s equation (compare with Ref. 12, Chap. 1).

The phase of the wave (54) carries reliable information only about the extremum values of the parameters. Therefore, it is clear that the formalism of the $\psi$ function cannot successfully describe only stationary states. But for dynamic variables the $\psi$ function gives only a probabilistic estimate which is also clear, since this formalism does not assume a mechanism describing the system dynamics. At the same time, if the vacuum is really a medium, then the need for a supplement or modification of action $\tilde{S}_{\text{wave}}$ is obvious. In our interpretation of the hidden dynamics of the particle, this is the transition $\tilde{S}_{\text{wave}} \rightarrow S$ and then to the equations of motion (17) to (19) in real space. In the considered dynamics, the motion of a particle is determined by solutions (29) and (30) with regard to formula (35). It comprises the time modulation of coordinate $X(t)$ with frequency $1/T = v_0/\lambda$, and this is why such motion can be associated with the traveling wave [some similarity with the traveling wave is especially obvious in coordinates $\tilde{X}, \tilde{t}$, Fig. 5].

7 Conclusion

In the present paper we postulated the availability of a discrete vacuum medium and provided it with the properties that would enable the particle to move between superdensely packed "molecules" of a vacuum.

In addition, the frequency of collisions of the particle with excitations of this discrete medium can be written in two ways. On the one hand, the frequency of each $l$th collision for a particle $1/T_l = v_0/\lambda$, according to expression (30). On the other hand, according to expression (25), the frequency of collisions with a particle for each $l$th excitation is $1/T_l = c/\Lambda_l$. At $l = 0$, parameters of a particle and a cloud of excitation attain extremum values, so, by comparing expressions (30) and (25) at $l = 0$, we obtain

$$v_0/\lambda = c/\Lambda. \quad (56)$$
Figure 5: Graph in dimensionless coordinates $\pi X_n/\lambda$ as a function of its proper dimensionless time $\pi t_n/T$, plotted according to Eq. (29) with regard to formula (35), that is, the graph of solution of equation $\pi X_n/\lambda = \pi t_n/T + \cos(\pi t_n/T)$. 
Relation (56) connects the spatial period of oscillations of a particle $\lambda$ (i.e., the de Broglie wavelength) to amplitude of the cloud $\Lambda$. For example, in the case of a free electron with velocity $v_0 = 10^5 \text{ cm/s}$ and a de Broglie wavelength $\lambda = \hbar/Mv_0 \approx 6 \times 10^{-5} \text{ cm}$, we obtain for the characteristic dimension of excitation cloud according to expression (56), $\Lambda = \lambda c/v_0 \cong 2 \text{ cm}$, that is, $\Lambda$ is a macroscopic value.

Let us also evaluate the number of excitations $N$ in the cloud of this electron. For this purpose, let us divide the de Broglie wavelength $\lambda$ by the dimension of superparticle $R_0$. If we assume $R_0 \sim 10^{-28} \text{ cm}$, then we obtain $N \sim \lambda/R_0 \sim 10^{22}$. In our model, the cloud of excitations surrounding a particle "feels" obstacles at a distance $\sim \Lambda$ from the particle, and those excitations transmit the respective information to the particle. Such motion is close in principle to de Broglie's "motion by guidance," which he associated with a constant intervention of subquantum medium [6,12].

Spatial oscillations of a particle are apparently the result of its adiabatic motion at which the particle does not leave behind faults in the vacuum medium structure. At the same time, the oscillator is characterized by the adiabatic invariant $J = E \nu$. In mechanics the mass $M$ of the particle and by the system elasticity constant $\gamma$: $\nu = \sqrt{\gamma/M/2\pi}$. Parameters $M$ and $E$ are the characteristics of the particle. Hence, assuming $J = \hbar$, we automatically determine Planck's constant as an adiabatic invariant of oscillator whose elasticity constant is defined by elastic properties of the vacuum medium. Let us note that such an interpretation of the constant $\hbar$ is not contradictory to that by Lochak [13], whose studies are devoted to the physical nature of $\hbar$.

In this paper we correlated the deformation of the vacuum medium with gravitational potential. Therefore, excitations of the vacuum medium could be called gravitons, but they basically differ from gravitons of the general theory of relativity (where they have polarization 2). In view of this, we have called the present excitations, "inertons." This name seems quite acceptable, since it reflects a connection with the motion of the particle, that is, with inertia.

The experimental detection of inertons would be a direct confirmation of the existence of a physical vacuum in the form of a peculiar quantum medium. This, in turn, would be a positive argument in favor of the concept of the oscillating motion of elementary particles set forth in this paper.

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