Quantum transport in magnetic topological insulators reveals a strong interplay between magnetism and topology of electronic band structures. A recent experiment on magnetically doped topological insulator Bi$_2$Se$_3$ thin films showed the anomalous temperature dependence of the magnetoconductivity while their field dependence presents a clear signature of weak antilocalization [Tkac et al., Phys. Rev. Lett. 123, 036406 (2019)]. Here, we demonstrate that the tiny mass of the surface electrons induced by the bulk magnetization leads to a temperature-dependent correction to the $\pi$ Berry phase and generates a decoherence mechanism to the phase coherence length of the surface electrons. As a consequence, the quantum correction to conductivity can exhibit nonmonotonic behavior by decreasing the temperature. This effect is attributed to the close relation of the Berry phase and quantum interference of the topological surface electrons in quantum topological materials.

**Anomalous Temperature Dependence of Quantum Correction to the Conductivity of Magnetic Topological Insulators**

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Quantum transport in magnetic topological insulators reveals a strong interplay between magnetism and topology of electronic band structures. A recent experiment on magnetically doped topological insulator Bi$_2$Se$_3$ thin films showed the anomalous temperature dependence of the magnetoconductivity while their field dependence presents a clear signature of weak antilocalization [Tkac et al., Phys. Rev. Lett. 123, 036406 (2019)]. Here, we demonstrate that the tiny mass of the surface electrons induced by the bulk magnetization leads to a temperature-dependent correction to the $\pi$ Berry phase and generates a decoherence mechanism to the phase coherence length of the surface electrons. As a consequence, the quantum correction to conductivity can exhibit nonmonotonic behavior by decreasing the temperature. This effect is attributed to the close relation of the Berry phase and quantum interference of the topological surface electrons in quantum topological materials.

**Introduction.**—Three-dimensional (3D) topological insulators (TIs) have stimulated intensive theoretical and experimental study in the past decade [1–6]. In the quantum diffusive regime, owing to the nontrivial $\pi$ Berry’s phase, the topological surface states are expected to experience a destructive quantum interference in the scattering process [7–10]. Accordingly, the magnetoconductivity shows a negative notch in a weak magnetic field ($B$) and is called weak antilocalization (WAL), which has been regarded as a significant transport signature for the topological surface states of TIs [11–16]. Besides, one anticipates that the conductivity correction from the WAL effect should decrease with increasing the temperature. However, the temperature dependence of conductivity usually shows an opposite tendency in experiments [17–22]. Such a dilemma in some pristine TIs can be resolved by further considering the electron-electron interaction effect at low temperatures [23–25]. Recently, Tkac et al. reported that the contradictory tendency between the temperature- and magnetic-field-dependent conductivity remains even after subtracting the interaction effect in the Mn-doped Bi$_2$Se$_3$ thin films [26]. As shown in Fig. 1, the magnetoconductivity $\delta \sigma(B)$ exhibits a monotonic temperature dependence for a non-doped Bi$_2$Se$_3$ sample, a typical behavior of WAL as expected theoretically, and a nonmonotonic temperature dependence for the doped ($x_{\text{Mn}} = 4\%$ and $x_{\text{Mn}} = 8\%$) samples, respectively, where $\delta \sigma(B) = \sigma(T, B) - \sigma(T, 0)$ with $\sigma(T, B)$ the temperature-dependent conductivity at a finite magnetic field $B$. At low temperatures, the doped and nondoped samples show opposite temperature dependence. Meanwhile, the magnetoconductivity for those samples always exhibit WAL correction as shown in Fig. 2 in Ref. [26]. The simple assumption of the monotonic temperature dependence of coherence length due to the electron-electron interaction effect [23,24,27] cannot account for these observations. Actually, the surface state in the magnetically doped TIs acquires a finite mass due to the time-reversal symmetry breaking accompanied with a small correction to the $\pi$ Berry phase [28–32]. The nearly $\pi$ Berry phase is capable of accounting for the WAL behavior for the magnetoconductivity but fails to explain the anomalous behavior.

In this Letter, we resolve the puzzle of the anomalous temperature dependence of quantum correction. The role of the magnetic doping is assumed to produce a finite gap for the surface states. Then, a magnetoconductivity formula of quantum interference is derived for massive Dirac fermions, which is simply characterized by the spin polarization $\eta$. The quantity is also associated to the correction to the $\pi$ Berry phase of surface electrons. The nearly $\pi$ Berry phase accounts for the WAL behavior for the magnetoconductivity. However, the temperature dependence of $\eta$...
Figure 2. Schematic diagram of the band structure and spin orientation for (a) massless and (b) massive Dirac fermions. The spin vectors at a certain Fermi energy are depicted by the red arrows. (c) and (d) show the corresponding Berry phase as the solid angle traced out the spin vectors on the Bloch sphere for (a) and (b), respectively. (e) and (f) show the trajectory of backscattering (solid line) and corresponding time-reversal trajectory (dashed line) for massless and massive Dirac fermions, respectively. The black arrow represents the momentum direction, and the red arrow denotes the spin orientation.

Model Hamiltonian and spin polarization.—Because of the hybridization of the top and bottom surface states or the time-reversal symmetry breaking caused by the magnetic doping, the surface electrons in the TI thin films can acquire a finite mass [28,33–35]; thus, it is proper to treat the surface states as massive Dirac fermions. Besides, in a TI thin film, the 3D bulk band is quantized into two-dimensional (2D) subbands owing to the quantum confinement effect. The 2D subbands have a similar low-energy Hamiltonian as the surface one but with a relatively large band gap [36]. We begin with the modified model of 2D massive Dirac fermions [5,34]:

$$H = \hbar v (\sigma_x k_x + \sigma_y k_y) + m(k) \sigma_z,$$

where $v$ is the effective velocity, $\hbar$ is the reduced Planck constant, $\sigma_{x,y,z}$ are the Pauli matrices, $k = (k_x, k_y)$ is the wave vector, $m(k) = mv^2 - \hbar^2 k^2 [k_x^2 + k_y^2]$ is the mass term, and $m$ and $b$ are the coefficients. The mass term gives the spin polarization $\eta = \langle \sigma_z \rangle = m(k_F)/\sqrt{\hbar^2 k_F^2 + [m(k_F)]^2}$ at the Fermi radii $k_F$, which is directly related to the Berry phase for Dirac fermions. As shown in Fig. 2, the spin lies in the plane of the Fermi circle for $\eta = 0$ and is tilted to the out of plane for $\eta \neq 0$. After the spin vector travels along the Fermi circle adiabatically, a Berry phase is acquired, $\phi_b = \frac{1}{2} \int_0^\pi \varphi \cos \eta \sin \theta d\theta d\varphi = \pi (1 - \eta)$. Furthermore, we mark the spin and momentum orientation in the trajectory of backscattering and corresponding time-reversal trajectory. For $\eta = 0$, the spins of incoming ($k$) and outgoing ($-k$) electrons are antiparallel to each other. The scattering sequences are accompanied by the coherent spin rotation which yields the WAL due to the $\pi$ Berry phase. For $\eta \neq 0$, the spin of the ($k$, $-k$) electron pair is partially tilted to the $z$ direction, and the spin-singlet and -triplet pairings mix together. Consequently, the accumulating Berry phase deviates from $\pi$, and, after taking the average of all the possible trajectories with different winding numbers, a new decoherence mechanism is introduced. When $\eta \rightarrow 1$, the spin is along the $z$ direction. The incoming and outgoing electrons form a triplet pairing and give rise to a WL correction.

Cooperon gaps and weighting factors.—The quantum correction to the conductivity is evaluated by using the Feynman diagrammatic technique [37–42]. In the present calculation, we keep the matrix form for Green's functions and treat all possible Cooperon channels, correlators in the particle-particle pairing channels in electric conductivity of nonsuperconducting metals, on the same footing [43]. In the diffusion approximation, it is found that three out of four possible Cooperon channels contribute to the conductivity:

$$\sigma_{ij} = -\frac{4e^2}{h} \sum_q w_i w_j / C_6 \left( \epsilon_i^{-2} + q^2 \right),$$

where $i = s, t_+ , t_-$ contribute to the WL correction, and the channel $s$ contributes to the WAL correction according to the signs of their weighting factors $w_s > 0$ and $w_i < 0$. The original Cooperon structure factor $\Gamma(q)$ is in the basis of $\{\{\uparrow \uparrow \}, \{\uparrow \downarrow \}, \{\downarrow \uparrow \}, \{\downarrow \downarrow \}\}$. To diagonalize $\Gamma(q)$, we rotated the basis into the spin-singlet and -triplet basis $\{|s, s\rangle\}$, where $|s, s\rangle$ labels the total spin $s = 0, 1$ and its $z$ component $s_z$. The channels $\Delta = t_\pm$ correspond to two triplet pairings ($s = 1$) and result in the WL correction, while the channel $\Delta = s$ is the singlet pairing ($s = 0$) and gives out the WAL correction. $\epsilon_i^{-2}$ and $w_i$ are plotted in Fig. 3. When $\eta = 0$ and $\phi_b = \pi$, one finds a pure WAL correction from the channel $s$, which is consistent with the Hikami-Larkin-Nagaoka formula for the strong spin-orbit scattering [38]. When $\eta = 1$ ($\eta = -1$) and $\phi_b = 0 (2\pi)$, the channel $\Delta = s$ gives a pure WL correction as the conventional electron gas.

Temperature dependence of conductivity correction.—The integration over $q$ in Eq. (2) is logarithmically divergent in both the ultraviolet and infrared limit. To avoid the divergence, the two cutoffs have to be introduced to restrict $\epsilon^4_q \leq q \leq \epsilon^4_c$, where $\epsilon_c = \sqrt{D_0 \tau}$ is the mean free path and $\tau$ is the coherence length caused by the...
TABLE I. The components of four Cooperon channels $i = s, t_{0\pm}$ in the basis of spin-triplet and -singlet $|s, s_z\rangle$, the Cooperon gap $\epsilon_i^{-2}$ in units of the mean free path $\epsilon_e^{-2}$ and the weighting factors $w_i$.

| $i$ | Cooperon in $|s, s_z\rangle$ | $w_i$ | $\epsilon_i^{-2}/\epsilon_e^{-2}$ |
|-----|-----------------|-----------------|--------------------------|
| $s$ | $|0, 0\rangle$ | $-[(1-\eta^2)^2]/[2(1+3\eta^2)^2]$ | $[(1-\eta^2)^2]/[(1+\eta^2)^2]$ |
| $t_+$ | $|1, 1\rangle$ | $[4\eta^2(1+\eta^2)]/[1(1+3\eta^2)^2]$ | $[4(1-\eta^2)^2]/[(1+3\eta^2)(1+\eta^2)]$ |
| $t_0$ | $|1, 0\rangle$ | 0 | $\infty$ |
| $t_-$ | $|1, -1\rangle$ | $[4\eta^2(1+\eta^2)]/[1(1+3\eta^2)^2]$ | $[4(1+\eta^2)^2]/[(1+3\eta^2)(1-\eta^2)]$ |

inelastic scattering $[23,24,27]$. Consequently, Eq. (2) gives the quantum correction to the conductivity:

$$\sigma_{qi}(B = 0, T) = \frac{e^2}{\pi h} \sum_i w_i \ln \frac{\epsilon_i^{-2} + \epsilon_e^{-2}}{\epsilon_i^{-2} + \epsilon_e^{-2}}. \quad (3)$$

To investigate the temperature dependence of $\sigma_{qi}(T)$, we assume $\epsilon_i = \epsilon_i^0(T/T_0)^{-p/2}$, where $p = 1$ for electron-electron interaction and $p = 3$ for electron-phonon interaction in 2D systems and $\epsilon_i^0$ is the coherence length at $T = T_0$ $[23,24]$. The characteristic parameter of the temperature-dependent conductivity is $[25]$

$$\kappa_{qi}^{(n)} = \frac{\pi h \partial \sigma_{qi}(B = 0, T)}{e^2} \frac{\partial \ln T}{1 + \epsilon_i^{-2}/\epsilon_e^{-2}} = \sum_i w_i p \frac{\partial \eta}{\partial \ln T} \quad (4)$$

if $\eta$ and $\epsilon_e$ are insensitive to the temperature. In this case, the presence of nonzero Cooperon gap $\epsilon_i^{-2}$ is highly nontrivial. As shown in Fig. 4(a), when $\eta = 0$, the conductivity correction is always logarithmically divergent and $\kappa_{qi}^{(n)} = -p/2$. However, once $0 < \eta < 1$, $\epsilon_i^{-2} \neq 0$, the conductivity correction saturates at lower temperatures, and $\kappa_{qi}^{(n)}$ would increase from some value $\in (-p/2, 0)$ to 0 gradually. In another limit of $\eta \sim 1$, as shown in Fig. 4(b), $\kappa_{qi}^{(n)} = p/2$ for $\eta = 1$, and $\kappa_{qi}^{(n)}$ decreases from some value $\epsilon_e^{-2}$ to 0 by lowering the temperature. Hence, the finite Cooperon gap leads to the saturation behavior of $\sigma_{qi}(T)$ at low temperatures.

In the magnetic TIs, the mass term is related to the magnetization; hence, $\eta$ is also a function of the temperature. Consequently, the slope $\kappa_{qi}$ has a correction term from $\partial \eta/\partial \ln T$:

$$\kappa_{qi}^{(m)} = \sum_i \left( \frac{\partial \eta}{\partial \ln T} + \frac{w_i p}{1 + \epsilon_i^{-2}/\epsilon_e^{-2}} \right) \quad (5)$$

with $g_i = \frac{\partial}{\partial \eta} \{w_i \ln[(\epsilon_i^{-2} + \epsilon_e^{-2})/(\epsilon_i^{-2} + \epsilon_e^{-2})]\}$. Here, we still assume that $\epsilon_e$ is insensitive to the temperature. We can have a qualitative analysis for the sign of $\kappa_{qi}^{(m)}$ for the case of $\eta \sim 0$. When $\eta \sim 0$, $\kappa_{qi}^{(m)} \approx -[\epsilon_i^{-2} \partial^2 \eta \partial / \partial \ln T + \epsilon_e^{-2} p / [2(\epsilon_i^{-2} + \epsilon_e^{-2})]]$. If $(\partial \eta/\partial \ln T) \geq 0$ and $\kappa_{qi} \leq 0$, the zero-field conductivity always decreases with increasing the temperature, indicating a WAL tendency as usual. However, if $(\partial \eta/\partial \ln T) < 0$, $\epsilon_e^{-2} (\partial^2 \eta/\partial \ln T) < 0$ and $\epsilon_e^{-2} p > 0$, $\kappa_{qi}$ may experience a sign change while decreasing the temperature, which implies an anomalous

FIG. 3. (a) The Cooperon gap $\epsilon_i^{-2}$ in units of square of the mean free path $\epsilon_e^{-2}$ and (b) the weighting factors as functions of spin polarization $\eta$, where $t_{0\pm}$ and $s$ represent the WL and WAL channels, respectively. The weighting factors for $t_+$ and $t_-$ channels are equal.

FIG. 4. Zero-field conductivity correction and slope $\kappa_{qi}^{(n)}$ as a function of the ratio of the mean free path to the coherence length $\epsilon_e/\epsilon_\phi$ for (a) WAL of spin polarization $\eta \sim 0$ and (b) WL of $\eta \sim 1$. Magnetococonductivity at different values of $\epsilon_e/\epsilon_\phi$ for (c) $\eta = 0.01$ and (d) $\eta = 0.9$. The calculation parameter $\epsilon_e^{-2} = 10 \text{ nm}$. 

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temperature dependence even in the case of the WAL correction. A similar analysis holds for $\eta \sim 1$.

**Magnetococonductivity.**—Experimentally, the effect of quantum interference can be detected by measuring the variation of the conductivity in an external magnetic field. When the magnetic field is along the $z$ direction, $q_x$ and $q_y$ are quantized into a series of Landau levels as $q_x^2 + q_y^2 \rightarrow (n + \frac{1}{2})\hbar^2/2eB$ with $\epsilon_B = \sqrt{\hbar/4eB}$ the magnetic length and $n$ a non-negative integer. Consequently, the magnetococonductivity reads [43]

$$\delta\sigma_{q_i}(B) = \sum_{l=\pm l} w_l F \left( \frac{\epsilon^2_B}{\epsilon_{\phi}^2} + \frac{\epsilon^2_B}{\epsilon^2_l} \right), \quad (6)$$

where $F(x) \equiv (e^x/x - \ln x)$ with $\psi(x)$ the digamma function. Comparing with the previous theories, only one Cooperon channel was taken into account in the Hikami-Lukin-Nagaoka formula [38], which is valid only in two limits ($\eta = 0$ and $\eta = 1$). In the Lu-Shen formula, the two Cooperon channels of triple pairing $i = t_{\pm}$ were approximately treated as one for WL, which forms a competition against the Cooperon channel of singlet pairing ($i = s$) for WAL [25].

When $\eta \ll 1$, Eq. (6) is simplified as $\delta\sigma_{q_i}(B) \approx -\frac{1}{2} F(\epsilon_{\phi}^2/\epsilon^2_l) + 1 + 1/\epsilon^2_{\phi}$. The presence of $\eta^2/\epsilon^2_l$ means a new decoherence mechanism for the coherence length besides the interaction effect. It is closely related to the correction to the $\pi$ Berry phase and becomes dominant at lower temperatures as $1/\epsilon^2_{\phi} \rightarrow 0$. When $\eta$ is independent of the temperature, as shown in Fig. 4(c), the $\delta\sigma_{q_i}(B)$ gradually saturates when $\epsilon_{\phi}/\epsilon_{\phi} \rightarrow 0$, as the effective coherence length is approximately determined by $\epsilon_{\phi} = \epsilon_{\phi}/\eta$ instead of $\epsilon_{\phi}$ at low temperatures. Hence, even a small $\eta$ can generate an observable effect. When $0 < 1 - \eta \ll 1$, Eq. (6) is simplified as $\delta\sigma_{q_i}(B) \approx \frac{1}{2} F(\epsilon_{\phi}^2/\epsilon^2_{\phi_i})$, with $1 + 1/\epsilon^2_{\phi_i} = (1-\eta)^2/4\epsilon^2_{\phi_i} + 1/\epsilon^2_{\phi_i}$, where the new decoherence term leads to the saturation of $\delta\sigma_{q_i}(B)$ when $\epsilon_{\phi}/\epsilon_{\phi} \rightarrow 0$ [see Fig. 4(d)].

This decoherence mechanism corresponds to the decaying Berry phase of multiple scattering trajectories. The Berry phase contributes to the return probability as a phase factor $e^{i\theta} = e^{i\phi_{i}(1+2n)}$ after $n$ times of revolutions [44]. For $\eta \ll 1$, after averaging over $n$, we have $e^{i\theta} = e^{-\eta^2 i/2}$, where the minus sign stems from the $\pi$ Berry phase $(e^{i\theta_{1+2n}} = -1)$ and gives a WAL correction when $\phi_{\pi} \sim \pi$. The decaying factor can reproduce the effective coherence length $\epsilon_{\phi_i}$ in the magnetococonductivity formula for WAL [43]. Furthermore, in the magnetic TIs, $\eta$ can be a function of the temperature. $\epsilon_{\phi_i}$ or $\epsilon_{\phi_{1+2n}}$ can be a nonmonotonic function of the temperature and further leads to a nonmonotonic temperature dependence of magnetococonductivity. In addition, $\delta\sigma_{q_i}(B)$ is still a monotonous function of the magnetic field. Thus, a temperature-dependent $\eta$ can produce different temperatures and magnetic field dependences of magnetococonductivity.

**Fitting the experiment.**—Armed with the formula of magnetococonductivity in Eq. (6), we are now ready to address the puzzle of the anomalous temperature dependence of the conductivity. In Fig. 1, the experimental data labeled by open squares are extracted from the temperature-dependent conductivity at a finite $B$ field in Figs. 4(a)–4(c) in Ref. [26]. Since the conductivity correction from the interaction effect is insensitive to the external magnetic field, the magnetococonductivity $\delta\sigma(B)$ can exclude the correction from the interaction effect and is mainly determined by the quantum interference effect: $\delta\sigma(B) \approx \delta\sigma_{q_i}(B)$. For the pristine Bi$_2$Se$_3$ of $x_{\text{Mn}} = 0\%$, the Fermi level intersects with both the surface band and bulk bands as clearly shown in the ARPES data in Ref. [26]. The $\delta\sigma$ data at different magnetic fields can be well fitted by considering one gapless surface states and two gapped bulk subbands [solid red lines in Fig. 1(a)] [36,41,48], and the fitting details can be found in Ref. [43].

The magnetococonductivities of the samples of $x_{\text{Mn}} = 4\%$ and $x_{\text{Mn}} = 8\%$ are similar and turn to increase with a decreasing temperature at low temperatures. The anomalous Hall resistivity in a ferromagnetic conductor has an empirical relation with the magnetic field $B$ and magnetization $M$: $\rho_{xy} = R_{xy}B + R_yM$ [49]. The magnetization is a function of the temperature below the Curie temperature $T_C$. Nonzero magnetization makes the surface states open a tiny gap. For the sample of $x_{\text{Mn}} = 8\%$, from the data of the anomalous Hall resistivity, it is found that $M$ is proportional to $1 - \sqrt{T}/T_C$ below the Curie temperature $T_C = 11.45$ K [50]. $\eta$ is assumed to obey the same behavior: $\eta(T) = \eta_0[1 - \sqrt{T}/T_C]\Theta(T_C - T)$ (see Sec. IIIIB in Ref. [43]), where $\eta_0$ is the spin polarization at the zero temperature and

![FIG. 5. (a) Magnetococonductivity at different temperatures for the Cr-doped Bi$_2$Se$_3$ thin film of $x = 0.23$. The open squares are the experimental data extracted from Fig. 2(j) of Ref. [31]. The solid red lines are the fitting results. (b) The temperature dependence of the fitted phase coherence length $\epsilon_{\phi}$ (open squares). The red line indicates $\epsilon_{\phi} \propto T^{-1.14}$.](https://example.com/fig5)
The path is estimated as $\ell_i \approx 14 \text{ nm}$ at $T = 2 \text{ K}$ and $\ell_i \approx 13.6 \text{ nm}$ at $T = 40 \text{ K}$ from the mobility and carrier density data, respectively. $\ell_i$ is insensitive to the temperature and is fixed as $14 \text{ nm}$ to reduce the number of fitting parameters. We further assume $\ell_\phi = \ell_0^2 T^{-\delta/2}$, where $\ell_0$ and $\delta$ are the fitting parameters and $T$ in units of Kelvin. In Fig. 1(c), the fitting curves show a good agreement with the experimental data and fitting curves, and the corresponding fitting parameters are listed in Table SII in Ref. [43]. As the fitting parameter $\eta \approx 0.2$, the weighting factors $w_i = 0.2, 0.5$, and $1$ T. The corresponding fitting parameters and $\ell_0$ are consistent at different temperatures.

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See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.124.206603 for details of (Sec. SI) the microscopic theory of quantum interference, (Sec. SII) the calculation of the Cooperon structure factor and Hikami box, (Sec. SIII) the experimental fitting for the samples of $x_{\text{Mn}} = 0\%$, $x_{\text{Mn}} = 4\%$, and $x_{\text{Mn}} = 8\%$, and (Sec. SIV) the experimental fitting for the sample of Cr-doped TI thin film, which includes Refs. [21,22,26,31,35,38,42,44–47].