A Non-Critical String (Liouville) Approach to Brain Microtubules: State Vector Reduction, Memory Coding and Capacity

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Abstract

Microtubule (MT) networks, subneural paracrystalline cytoskeletal structures, seem to play a fundamental role in the neurons. We cast here the complicated MT dynamics in the form of a 1 + 1-dimensional non-critical string theory, thus enabling us to provide a consistent quantum treatment of MTs, including environment friction effects. We suggest, thus, that the MTs are the microsites, in the brain, for the emergence of stable, macroscopic quantum coherent states, identifiable with the preconscious states. Quantum space-time effects, as described by non-critical string theory, trigger then an organized collapse of the coherent states down to a specific or conscious state. The whole process we estimate to take $O(1 \text{sec})$, in excellent agreement with a plethora of experimental/observational findings. The microscopic arrow of time, endemic in non-critical string theory, and apparent here in the self-collapse process, provides a satisfactory and simple resolution to the age-old problem of how the, central to our feelings of awareness, sensation of the progression of time is generated. In addition, the complete integrability of the stringy model for MT we advocate in this work proves sufficient in providing a satisfactory solution to memory coding and capacity. Such features might turn out to be important for a model of the brain as a quantum computer.

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1 Introduction

The interior of living cells is structurally and dynamically organized by cytoskeletons, i.e. networks of protein polymers. Of these structures, MicroTubules (MT) appear to be the most fundamental. Their dynamics has been studied recently by a number of authors in connection with the mechanism responsible for dissipation-free energy transfer. Recently, Hameroff and Penrose have conjectured another fundamental rôle for the MT, namely being responsible for quantum computations in the human brain, and, thus, related to the consciousness of the human mind. The latter is argued to be associated with certain aspects of quantum theory that are believed to occur in the cytoskeleton MT, in particular quantum superposition and subsequent collapse of the wave function of coherent MT networks. While quantum superposition is a well-established and well-understood property of quantum physics, the collapse of the wave function has been always enigmatic. We propose here to use an explicit string-derived mechanism - in one interpretation of non-critical string theory - for the collapse of the wave function, involving quantum gravity in an essential way and solidifying previous intuitively plausible suggestions. It is an amazing surprise that quantum gravity effects, of order of magnitude $G_N^{1/2}m_p \sim 10^{-19}$, with $G_N$ Newton’s gravitational constant and $m_p$ the proton mass, can play a rôle in such low energies as the eV scales of the typical energy transfer that occurs in cytoskeletons. However, as we show in this article, the fine details of the MT characteristic structure indicate that not only is this conceivable, but such scenaria lead to order of magnitude estimates for the time scales entering conscious perception that are close enough to those conjectured/“observed” by neuroscientists, based on completely different grounds.

To understand how quantum space-time effects can affect conscious perception, we mention that it has long been suspected that large scale quantum coherent phenomena can occur in the interior of biological cells, as a result of the existence of ordered water molecules. Quantum mechanical vibrations of the latter are responsible for the appearance of ‘phonons’ similar in nature to those associated with superconductivity. In fact there is a close analogy between superconductivity and energy transfer in biological cells. In the former phenomenon electric current is transferred without dissipation in the surface of the superconductor. In biological cells, as we shall discuss later on, energy is transferred through the cell without loss, despite the existence of frictional forces that represent the interaction of the cell with the surrounding water molecules. Such large scale quantum coherent states can maintain themselves for up to $O(1\,\text{sec})$, without significant environmental entanglement. After that time, the state undergoes self-collapse, probably due to quantum gravity effects. Due to quantum transitions between the different states of the quantum system of MT in certain parts of the human brain, a sufficient distortion of the surrounding space-time occurs, so that a microscopic (Planck size) black hole is formed. Then collapse is induced, with a collapse time that depends on the order of magnitude of the number $N$ of coherent microtubulins. It is esti-
mated that, with an $N = O[10^{12}]$, the collapse time of $O(1 \text{ sec})$, which appears to be a typical time scale of conscious events, is achieved. Taking into account that experiments have shown that there exist $N = 10^8$ tubulins per neuron, and that there are $10^{11}$ neurons in the brain, it follows that that this order of magnitude for $N$ refers to a fraction $10^{-7}$ of the human brain, which is very close to the fraction believed responsible for human perception.

The self-collapse of the MT coherent state wave function is an essential step for the operation of the MT network as a quantum computer. In the past it has been suggested that MT networks processed information in a way similar to classical cellular automata (CCA) [10]. These are described by interacting Ising spin chains on the spatial plane obtained by filing open and flattening the MT cylindrical surface. Distortions in the configurations of individual parts of the spin chain can be influenced by the environmental spins, leading to information processing. In view of the suggestion [2] on viewing the conscious parts of the human mind as quantum computers, one might extend the concept of the CCA to a quantum cellular automaton (QCA), undergoing wave function self-collapses due to (quantum gravity) environmental entanglement.

An interesting and basic issue that arises in connection with the above role of the brain as a quantum computer is the emergence of a direction in the flow of time (arrow). The latter could be the result of successive self-collapses of the system’s wave function. In a recent series of papers [3] we have suggested a rather detailed mechanism by which an irreversible time variable has emerged in certain models of string quantum gravity. The model utilized string particles propagating in singular space-time backgrounds with event horizons. Consistency of the string approach requires conformal invariance of the associated $\sigma$-model, which in turn implies a coupling of the backgrounds for the propagating string modes to an infinity of global (quasi-topological) delocalized modes at higher (massive) string levels. The existence of such couplings is necessitated by specific coherence-preserving target space gauge symmetries that mix the string levels [3].

The specific model of ref. [3] is a completely integrable string theory, in the sense of being characterised by an infinity of conserved charges. This can be intuitively understood by the fact that the model is a (1+1)-dimensional Liouville string, and as such it can be mapped to a theory of essentially free fermions on a discretized world sheet (matrix model approach [11]). A system of free fermions in (1 + 1) dimensions is trivially completely integrable, the infinity of the conserved charges being provided by appropriate moments of the fermion energies above the Fermi surface. Of course, formally, the precise symmetries of the model used in ref. [3] are much more complicated [12], but the idea behind the model’s integrability is essentially the above. It is our belief that this quantum integrability is a very important feature of theories of space-time associated with the time arrow. In its presence, theories
with singular backgrounds appear consistent as far as maintainance of quantum coherence is concerned. This is due to the fact that the phase-space density of the field theory associated with the matter degrees of freedom evolves with time according to the conventional Liouville theorem\[5\]

\[ \partial_t \rho = -\{\rho, H\}_{PB} \] (1)
as a consequence of phase-space volume-preserving symmetries. In the two-dimensional example of ref. [5], these symmetries are known as $W_\infty$, and are associated with higher spin target-space states\[12\]. They are responsible for string-level mixing, and hence they are broken in any low-energy approximation. If the concept of ‘measurement by local scattering experiments’ is introduced \[5\], it becomes clear that the observable background cannot contain such global modes. The latter have to be integrated out in any effective low-energy theory. The result of this integration is a non-critical string theory, based on the propagating modes only. Its conformal invariance on the world sheet is restored by dressing the matter backgrounds by the Liouville mode $\phi$, which plays the role of the time coordinate. The $\phi$ mode is a dynamical local world-sheet scale \[5\], flowing irreversibly as a result of certain theorems of the renormalization group of unitary $\sigma$-models \[13\]. In this way time in target space has a natural arrow for very specific stringy reasons.

Given the suggestion of ref. [3] that space-time environmental entanglement could be responsible for conscious brain function, it is natural to examine the conditions under which our theory \[5\] can be applied. Our approach utilizes extra degrees of freedom, the $W_\infty$ global string modes, which are not directly accessible to local scattering ‘experiments’ that make use of propagating modes only. Such degrees of freedom carry information, in a similar spirit to the information loss suggested by Hawking\[14\] for the quantum-black-hole case. For us, such degrees of freedom are not exotic, as suggested in ref. \[15\], but appear already in the non-critical String Universe \[5, 4\], and as such they are considered as ‘purely stringy’. In this respect, we believe that the suggested model of consciousness, based on the non-critical-string formalism of ref. \[5\], is physically more concrete. The idea of using string theory instead of point-like quantum gravity is primarily associated with the fact that a consistent quantization of gravity is at present possible only within the framework of string theory. However, there are additional reasons that make advantageous a string formalism. These include the possibility of construction of a completely integrable model for MTs, and the Hamiltonian representation of dynamical problems with friction involved in the physics of MTs. This leads to the possibility of a consistent (mean field) quantization of certain soliton solutions associated with the energy transfer mechanism in biological cells.

According to our previous discussion emphasizing the importance of strings, it is imperative that we try to identify the completely integrable system underlying MT networks. Thus, it appears essential to review first the classical model for energy
transfer in biological systems associated with MT. This will allow the identification of the analogue of the (stringy) propagating degrees of freedom, which eventually couple to quantum (stringy) gravity and to global environmental modes. As we shall argue in subsequent sections, the relevant basic building blocks of the human brain are one-dimensional Ising spin chains, interacting among themselves in a way so as to create a large scale quantum coherent state, believed to be responsible for preconscious behaviour in the model of [2]. The system can be described in a world-sheet conformal invariant way and is unitary. Coupling to gravity generates deviations from conformal invariance which lead to time-dependence, by identifying time with the Liouville field on the world sheet. The situation is similar to the environmental entanglement of ref. [16, 17]. Due to this entanglement, the system of the propagating modes opens up as in Markov processes [18]. This leads to a dynamical self-collapse of the wave function of the MT quantum coherent network. In this way, the part of the human brain associated with consciousness generates, through successive collapses, an arrow of time. The interaction among the spin chains, then, provides a mechanism for quantum computation, resembling a planar cellular automaton. Such operations sustain the irreversible flow of time.

The structure of the article is as follows: in section 2 we discuss a model used for the physical description of a MT, and in particular for a simulation of the energy transfer mechanism. The model can be expressed in terms of a 1 + 1-dimensional classical field, the projection of the displacement field of the MT dimers along the tubulin axis. There exists friction due to interaction with the environment. However, the theory possesses travelling-wave solitonic states responsible for loss-free transfer of energy. In section 3, we give a formal representation of the above system as a 1 + 1-dimensional \( c = 1 \) Liouville (string) theory. The advantage of the method lies in that it allows for a canonical quantization of the friction problem, thereby yielding a model for a large-scale coherent state, argued to simulate the preconscious state. There is no time arrow in the above system. In section 4, we discuss our mechanism of introducing a dynamical time variable with an arrow into the system, by elevating the above \( c = 1 \) Liouville theory to a \( c = 26 \) non-critical string theory, incorporating quantum gravity effects. Such effects arise from the distortion of space-time due to abrupt conformational changes in the dimers. Such a coupling leads to a breakdown of the quantum coherence of the preconscious state. Estimates of the collapse times are given, with the result that in this approach conscious perception of a time scale of \( O(1 \text{ sec}) \), is due to a \( 10^{-7} \) part of the total brain. In section 5 we briefly discuss growth of a MT network in our framework, which would be the analogue of a non-critical string driven inflation for the effective one-dimensional universe of the MT dimer degrees of freedom. We view MT growth as an out-of-equilibrium one-dimensional spontaneous-symmetry breaking process and discuss the connection of our approach to some elementary theoretical models with driven diffusion that could serve as prototypes for the phenomenon. We also point out some other experiments, sensitive to weak fields - like gravitational ones, where our ideas about a quantum integrable model for MT may find some applications.
Section 6 deals with some interesting consequences of the complete integrability of the model, as far as memory coding and capacity are concerned. Both problems are resolved, as a result of huge ‘stringy’ symmetries, which appear naturally and are essential in maintaining quantum coherence of the full system, including back-reaction effects of matter on the effective two-dimensional space-time. Conclusions and outlook are presented in section 7. We discuss some technical aspects of our approach in two Appendices.

2 Physical Description of the Microtubules

2.1 The model and its parameters

In this section we review certain features of the MT that will be useful in subsequent parts of this work. MT are hollow cylinders (cf Fig 1) comprised of an exterior surface (of cross-section diameter 25 nm) with 13 arrays (protofilaments) of protein dimers called tubulins. The interior of the cylinder (of cross-section diameter 14 nm) contains ordered water molecules, which implies the existence of an electric dipole moment and an electric field. The arrangement of the dimers is such that, if one ignores their size, they resemble triangular lattices on the MT surface. Each dimer consists of two hydrophobic protein pockets, and has an unpaired electron. There are two possible positions of the electron, called $\alpha$ and $\beta$ conformations, which are depicted in Fig. 2. When the electron is in the $\beta$-conformation there is a $29^o$ distortion of the electric dipole moment as compared to the $\alpha$ conformation.

In standard models for the simulation of the MT dynamics, the ‘physical’ degree of freedom - relevant for the description of the energy transfer - is the projection of the electric dipole moment on the longitudinal symmetry axis (x-axis) of the MT cylinder. The $29^o$ distortion of the $\beta$-conformation leads to a displacement $u_n$ along the $x$-axis, which is thus the relevant physical degree of freedom. This way, the effective system is one-dimensional (spatial), and one has a first indication that quantum integrability might appear. We shall argue later on that this is indeed the case.

Information processing occurs via interactions among the MT protofilament chains. The system may be considered as similar to a model of interacting Ising chains on a triangular lattice, the latter being defined on the plane stemming from filing open and flatten the cylindrical surface of Fig. 1. Classically, the various dimers can occur in either $\alpha$ or $\beta$ conformations. Each dimer is influenced by the neighboring dimers resulting in the possibility of a transition. This is the basis for classical information processing, which constitutes the picture of a (classical) cellular automaton.

The quantum computer character of the MT network results from the assumption that each dimer finds itself in a superposition of $\alpha$ and $\beta$ conformations [2]. There
is a macroscopic coherent state among the various chains, which lasts for $O(1 \text{ sec})$ and constitutes the ‘preconscious’ state. The interaction of the chains with (stringy) quantum gravity, then, induces self-collapse of the wave function of the coherent MT network, resulting in quantum computation.

In what follows we shall assume that the collapse occurs mainly due to the interaction of each chain with quantum gravity, the interaction from neighboring chains being taken into account by including mean-field interaction terms in the dynamics of the displacement field of each chain. This amounts to a modification of the effective potential by anharmonic oscillator terms. Thus, the effective system under study is two-dimensional, possessing one space and one time coordinate. The precise meaning of ‘time’ in our model will be clarified when we discuss the ‘non-critical string’ representation of our system.

Let $u_n$ be the displacement field of the $n$-th dimer in a MT chain. The continuous approximation proves sufficient for the study of phenomena associated with energy transfer in biological cells, and this implies that one can make the replacement

$$u_n \rightarrow u(x,t)$$

with $x$ a spatial coordinate along the longitudinal symmetry axis of the MT. There is a time variable $t$ due to fluctuations of the displacements $u(x)$ as a result of the dipole oscillations in the dimers. At this stage, $t$ is viewed as a reversible variable. The effects of the neighboring dimers (including neighboring chains) can be phenomenologically accounted for by an effective double-well potential [19]

$$U(u) = -\frac{1}{2}Au^2(x,t) + \frac{1}{4}Bu^4(x,t)$$

with $B > 0$. The parameter $A$ is temperature dependent. The model of ferroelectric distortive spin chains of ref. [20] can be used to give a temperature dependence

$$A = -|\text{const}|(T - T_c)$$

where $T_c$ is a critical temperature of the system, and the constant is determined phenomenologically [19]. In realistic cases the temperature $T$ is very close to $T_c$, which for the human brain is taken to be the room temperature $T_c = 300K$. Thus, below $T_c$ $A > 0$. The important relative minus sign in the potential [19], then, guarantees the necessary degeneracy, which is necessary for the existence of classical solitonic solutions. These constitute the basis for our coherent-state description of the preconscious state.

Including a phenomenological kinetic term for the dimers, each having a mass $M$, one can write down a Hamiltonian [19]

$$H = kR_0^2(\partial_x u)^2 - M(\partial_t u)^2 - \frac{1}{2}Au^2 + \frac{1}{4}Bu^4 + qEu$$

(5)
where \( k \) is a stiffness parameter, \( R_0 \) is the equilibrium spacing between adjacent dimers, \( E \) is the electric field due to the ‘giant dipole’ representation of the MT cylinder, as suggested by the experimental results [19], and \( q = 18 \times 2e \) (\( e \) the electron charge) is a mobile charge. The spatial-derivative term in (5) is a continuous approximation of terms in the lattice Hamiltonian that express the effects of restoring strain forces between adjacent dimers in the chains [19].

The effects of the surrounding water molecules can be summarized by a viscous force term that damps out the dimer oscillations,

\[
F = -\gamma \partial_t u
\]

with \( \gamma \) determined phenomenologically at this stage. This friction should be viewed as an environmental effect, which however does not lead to energy dissipation, as a result of the non-trivial solitonic structure of the ground-state and the non-zero constant force due to the electric field. This is a well known result, directly relevant to energy transfer in biological systems [9]. The modified equations of motion, then, read

\[
M \partial_t^2 u - kR_0^2 \partial_x^2 u - Au +Bu^3 + \gamma \frac{\partial u}{\partial t} - qE = 0
\]

(7)

According to ref. [9] the importance of the force term \( qE \) lies in the fact that eq (7) admits displaced classical soliton solutions with no energy loss. The solution acquires the form of a travelling wave, and can be most easily exhibited by defining a normalized displacement field

\[
\psi(\xi) = \frac{u(\xi)}{\sqrt{A/B}}
\]

(8)

where,

\[
\xi \equiv \alpha(x-vt) \quad \alpha \equiv \sqrt{\frac{|A|}{M(v_0^2-v^2)}}
\]

(9)

with

\[
v_0 \equiv \sqrt{k/MR_0}
\]

(10)

the sound velocity, of order 1\( km/sec \), and \( v \) the propagation velocity to be determined later. In terms of the \( \psi(\xi) \) variable, equation (7) acquires the form of the equation of motion of an anharmonic oscillator in a frictional environment

\[
\psi'' + \rho \psi' - \psi^3 + \psi + \sigma = 0
\]

\[
\rho \equiv \gamma v|M(A|(v_0^2-v^2))^{-\frac{1}{2}}, \quad \sigma = q\sqrt{B}|A|^{-3/2}E
\]

(11)

which has a unique bounded solution [19]

\[
\psi(\xi) = a + \frac{b-a}{1 + e^{\frac{-\xi}{\gamma}}}
\]

(12)
with the parameters \( b, a \) and \( d \) satisfying:

\[
(\psi - a)(\psi - b)(\psi - d) = \psi^3 - \psi - \left( \frac{q\sqrt{B}}{|A|^{3/2}}E \right)
\]

Thus, the kink propagates along the protofilament axis with fixed velocity

\[
v = v_0[1 + \frac{2\gamma^2}{9d^2M|A|}]^{-\frac{1}{4}}
\]

This velocity depends on the strength of the electric field \( E \) through the dependence of \( d \) on \( E \) via (13). Notice that, due to friction, \( v \neq v_0 \), and this is essential for a non-trivial second derivative term in (11), necessary for wave propagation. For realistic biological systems \( v \approx 2 \text{ m/sec} \). With a velocity of this order, the travelling waves of kink-like excitations of the displacement field \( u(\xi) \) transfer energy along a moderately long microtubule of length \( L = 10^{-6} \text{m} \) in about

\[
t_T = 5 \times 10^{-7} \text{sec}
\]

This time is very close to Frohlich’s time for coherent phonons in biological system. We shall come back to this issue later on.

The total energy of the solution (12) is easily calculated to be [19]

\[
E = \frac{1}{R_0} \int_{-\infty}^{+\infty} dx H = \frac{2\sqrt{2}A^2}{3B} + \frac{\sqrt{2}}{3}kA + \frac{1}{2}M^*v^2 \equiv \Delta + \frac{1}{2}M^*v^2
\]

which is conserved in time. The ‘effective’ mass \( M^* \) of the kink is given by

\[
M^* = \frac{4}{3\sqrt{2}} \frac{MA\alpha}{R_0B}
\]

The first term of equation (16) expresses the binding energy of the kink and the second the resonant transfer energy. In realistic biological models the sum of these two terms dominate over the third term, being of order of \( 1eV \) [19]. On the other hand, the effective mass in (17) is [19] of order \( 5 \times 10^{-27} \text{kg} \), which is about the proton mass (\( 1GeV \)) (!). As we shall discuss later on, these values are essential in yielding the correct estimates for the time of collapse of the ‘preconscious’ state due to our quantum gravity environmental entangling. To make plausible a consistent study of such effects, we now discuss the possibility of representing the equations of motions (11) as being derived from string theory.

Before closing we mention that the above classical kink-like excitations (12) have been discussed so far in connection with physical mechanisms associated with the hydrolysis of GTP (Guanosine-ThreePhosphate) tubulin dimers to GDP (Guanosine-DiPhosphate) ones. Because the two forms of tubulins correspond to different conformations \( \alpha \) and \( \beta \) above, it is conceivable to speculate that the quantum mechanical
oscillations between these two forms of tubulin dimers might be associated with a quantum version of kink-like excitations in the MT network. This is the idea we put forward in the present work. The novelty of our approach is the use of Liouville (non-critical) string theory for the study of the dynamics involved. This is discussed in the next section.

Before doing so we consider it as useful to discuss the rôle of the surrounding water molecules as a medium providing us with the initial quantum coherent ‘preconscious’ state of mind. We shall briefly review existing works on the subject and outline the advantages of our approach, which uses completely integrable (non-critical string) theories to represent the MT dynamics.

2.2 The importance of the water environment
We start by first reviewing existing mechanisms conjecturing the importance of the surrounding water molecules for the proper functioning or even the existence of MT [21]. As a result of its electric dipole structure, the ordered water environment exhibits a laser-like behaviour [22]. Coherent modes emerge as a result of the interaction of the electric dipole moments of the water molecules with the quantized electromagnetic radiation. Such quanta can be understood [21] as Goldstone modes arising from the spontaneous breaking of the electric dipole symmetry, which in the work of ref. [21] was the only symmetry to be assumed spontaneously broken. In our string model, as we shall discuss later on, a more complicated (infinite-dimensional) symmetry breaks spontaneously, which incorporates the simple rotational symmetry of the point-like theory models. The emergence of coherent dipole quanta resembles the picture of Fröhlich coherent ‘phonons’ [8], emerging in biological systems for energy transfer without dissipation.

The existence of such coherent states in the surrounding water results in the friction term proportional to $\rho$ in (11). What we have argued above is that, because of this interaction, a kink soliton can be formed, provided that the MT are of sufficient length. Such solitons can then be themselves squeezed coherent states, being responsible for a preconscious state of the mind. This provides an explicit mechanism for the realization of the conjectures of [2]. Such coherent states extend over huge networks of MT covering approximately a fraction $10^{-7}$ of the human brain. Quantum gravity effects, then, are responsible for the collapse of such states in MT networks, leading to ‘conscious’ perception. Such effects are represented by the formation of virtual black hole states in (1+1)-dimensional non-critical string models. As we shall discuss in subsequent sections, this spoils the conformal invariance (criticality) of the effective $\sigma$-model based only on the displacement field $\psi$ of MT. To restore criticality, it is necessary to have time-dependent parts in the configuration of the dilaton field [3], in addition to the space-like linear dilaton backgrounds characterizing (1+1)-dimensional flat space-times. It is important to stress that space-like dilaton backgrounds are necessary for asymptotic flatness of space-time [3]. Such
backgrounds are provided by the friction ($\rho$) with the surrounding water. There lies the importance of the existence of effects originating from the interaction of the MT with the surrounding water molecules, which co-exist with quantum gravity effects that lead to a collapse of the preconscious state.

At this point we should stress that our suggested mechanism for conscious perception' in the framework of non-critical string (integrable) models has similarities with, but also important differences from, previous mechanisms of brain functioning, suggested in the context of local quantum field theory [24, 25, 21]. In the latter case, external stimuli are believed responsible for triggering spontaneous breaking of the electric dipole rotational symmetry of the water environment. The collective Goldstone modes of such a breaking (dipole quanta) are spin wave quanta and the system’s phases are macroscopically characterized by the value of the order parameter, which in this case is the polarization (coding). If the ground state of the system is considered as the memory state, then the above process is just memory printing. In this picture, memory recall corresponds to the excitation, under another external stimulus, of dipole quanta of similar nature to those leading to the printing. The brain, then, ‘consciously feels’ [25, 26] the pre-existing order in the ground state.

The usual problem with such mechanisms is associated with memory capacity. The set of code numbers associated with the breaking of just a limited number of symmetries is not sufficient [25] to explain the storage of an enormous amount of information in the human brain. We shall discuss these issues later on. At the moment we only mention the interesting suggestion of ref. [26] that dissipation can lead to an evasion of the problem of memory capacity, as a result of the doubling of the degrees of freedom of the dissipative system in order to achieve canonical quantization. In this case, memory states can be shown to be classified by infinite dimensional non-compact symmetries (in a thermodynamic limit), leading to the existence of ground state replicas. In this way overprinting is avoided. In our opinion, although such an approach is very appealing and certainly shares many things in common with our stringy picture for the brain MT, however it has the disadvantage of the non-hermiticity of the Hamiltonian [24, 27], thus leading to macroscopic energy dissipation. In order to make contact with energy-loss-free transport in biological cells [3, 4] one must find a model, where although dissipation effects exist (c.f. $\rho$ in (11)), and thus memory capacity problems are treated in a satisfactory way, however energy is conserved on the average, and thus Frohlich’s requirements are met. Moreover, as we shall discuss below, it would be desirable to have a unified framework that will incorporate realistic quantum-gravity entanglement, by means of an ‘environment’ formalism. All these requirements seem to be met in a non-critical string picture of MT dynamics [5], to which we now turn in some detail.
3 Non-Critical (Liouville) String Theory Representation of a MT

For this purpose, it is important to notice that the relative sign (+) between the second derivative and the linear term in \( \psi \) in equation (11) are such that this equation can be considered as corresponding to the tachyon \( \beta \)-function equation of a \((1 + 1)\)-dimensional string theory, in a flat space-time with a dilaton field \( \Phi \) linear in the space-like coordinate \( \xi \) [28].

\[ \Phi = - \rho \xi \] (18)

Indeed, the most general form of a ‘tachyon’ deformation in such a string theory, compatible with conformal invariance is that of a travelling wave [29] \( T(x') \), with

\[ x' = \gamma_{v_s}(x - v_s t) \quad ; \quad t' = \gamma_{v_s}(t - v_s x) \]
\[ \gamma_{v_s} \equiv (1 - v_s^2)^{-1/2} \] (19)

where \( v_s \) is the propagation velocity of the string ‘tachyon’ background. As argued in ref. [29] these translational invariant configurations are the most general backgrounds, consistent with a unique factorisation of the string \( \sigma \)-model theory on a Minkowski space-time \( G_{\mu\nu} = \eta_{\mu\nu} \)

\[ S = \frac{1}{4\pi\alpha'} \int d^2 z \left[ \partial X^\mu \partial X^\nu G_{\mu\nu}(X) + \Phi(X)R^{(2)} + T(X) \right] \] (20)

into two conformal field theories, for the \( t' \) and \( x' \) fields, corresponding to central charges

\[ c_{t'} = 1 - 24v_s^2 \gamma_{v_s}^2 \quad ; \quad c_{x'} = 1 + 24\gamma_{v_s}^2 \] (21)

In our case (14), the rôles of the space-like coordinate \( x' \) is played by \( \xi \) (9). The velocity of light in this effective string model is replaced by the sound velocity \( v_0 \) (10), and the velocity \( v_s \) is defined in terms of the velocity \( v \) of the kink (14) by expressing the friction coefficient \( \rho \) in terms of the central charge deficit (21) (for definitions and relevant notation see ref. [29] and also Appendix A)

\[ \rho = \sqrt{\frac{1}{6} (c(\xi) - 1)} = 2\gamma_{v_s} \] (22)

The space-like ‘boosted’ coordinate \( \xi \), thus, plays the rôles of space in this effective/Liouville mode string theory.

Notice that with the definition (22) the local-field-theory kink solution (12), propagating with a real velocity \( v \), is mapped to a non-critical string background which propagates with a velocity \( v_s \) that could be imaginary \((v_s^2 < 0)\). Indeed, the condition for reality of \( v_s \) can be easily found from (11,14,22) to be

\[ d^2 \geq 8/9 \] (23)
It can be easily seen that for the generic values of the parameters of the MT model, described in the previous section \[19\], (23) is not satisfied, which implies that, when formulated as a non-critical string theory, the MT system corresponds to a 1 + 1 dimensional non-critical string with Wick-rotated ‘time’ \( s = it \) \[29\], and, therefore, corresponds to a matter central charge that overcomes the \( c = 1 \) barrier

\[ 25 \geq c_s' = 1 + 24 |v_s|^2 (1 + |v_s|^2)^{-1} \geq 1 \quad (24) \]

In this regime, Liouville theory is poorly understood, but there is the belief that this range of matter central charges is characterized by polymerization properties of random surfaces. From our point of view this may be related to certain growth properties of the MT networks, which are briefly discussed in section 5.

An interesting question arises as to whether there are circumstances under which (23) is satisfied, in which case the matter content of the theory is characterized by the central charge \( c_s \leq 1 \) \[21\]. To this end, we note that the parameter \( d \) in (23) has a large uncertainty since, among others, it depends on \( A \). At present, there are no accurate experimental data for \( A \), which depends on temperature \([1]\). The parameter \( d \) is also sensitive to the order of magnitude of the electric field \( E \). The latter is non-uniform along the MT axis. There is a sharp increase of \( E \) towards the end points of the MT protofilament axis \[19\], and for such large \( E \), \( d \sim E^{1/3} \). Due to (14), an increase in \( d \) results in an increase in the kink velocity \( v \) at the end points of the MT. For realistic biological systems, under normal circumstances, the increase in \( v \) can be even up to two orders of magnitude, resulting in kink velocities of \( O[10^2 m/sec] \) \[19\] at the end points of the MT.

As we shall argue now, the parameter \( d \) can be drastically affected by an abrupt distortion of the environment due to the influence of an external stimulus. For instance, it may be possible for such abrupt distortions to cause a local disturbance among the dimers, so that the value of \( A \) is momentarily diminished significantly, \( d \) increases \[14\], and the condition (23) is met. However, such a distortion might affect the order of magnitude estimates of the effective mass scales involved in the problem, as we discussed in section 2, and this may have consequences for the decoherence time. So, we shall not consider it for the purposes of this work. More plausibly, an abrupt environmental distortion will lead to a sudden increase in the electric field \( E \), which, in turn, results in the formation of a fast kink of \( v \sim v_0 \), via an increase in \( d \). As we have seen above, this is not unreasonable for the range of the parameters pertaining to realistic biological systems. In such cases the (fast) kinks are represented as translational-invariant backgrounds of non-critical string theories, propagating with real velocities \( v_s \) \[22\]. This is the kind of structures that we shall mostly be interested in for the purposes of this work. Coupling them to quantum gravity will lead to the collapse of the preconscious state, as we shall discuss in the next section.
The important advantage of formulating the MT system as a $c = 1$ string theory, lies in the possibility of casting the friction problem in a Hamiltonian form. To this end, we now make some comments on the various non-derivative terms in the tachyon potential $V(T) \equiv U(\psi)$ in the target-space effective action [30]. Such terms contribute higher order non-derivative terms in the equations of motion [11]. The term linear in $\psi$ is fixed in string theory and normalized (with respect to the second derivative term) as in [11] [29]. The higher order terms are polynomials in $\psi$ and their coefficients can be varied according to renormalization prescription [30].

The general structure of the tachyon effective action in the target space of the string is, therefore, of the form [30]

$$L = e^{-\Phi} \sqrt{G} \left[ (T - c)^2 + \hat{L}(\nabla T, \nabla \Phi) \right]$$

where $G_{\mu\nu}$ is a target space metric field, and $c$ is a constant. The linear term is the only universal term, showing the impossibility of finding a stable tachyon background in bosonic string theory, unless it is time dependent.

In our specific model, we have seen that a term cubic in $\psi$ in the equation of motion [11], with a relative minus sign as compared to the linear term, was responsible for the appearance of a kink-like classical solution. Any change in the non-linear terms would obviously affect the structure of the solution, and we should understand the physical meaning of this in our biological system. To this end, we consider a general polynomial in $T$ equation of motion for a static tachyon in (1+1) string theory

$$T''(\xi) + \rho T'(\xi) = P(T)$$

where $\xi$ is some space-like co-ordinate and $P(T)$ is a polynomial of degree $n$, say. The ‘friction’ term $T'$ expresses a Liouville derivative, since the effective string theory of the displacement field $u$ is viewed as a $c = 1$ matter non-critical string. In our interpretation of the Liouville field as a local scale on the world sheet it is natural to assume that the single-derivative term expresses the non-critical string $\beta$-function, and hence is itself a polynomial $R$ of degree $m$

$$T'(\xi) = R(T)$$

Using Wilson’s exact renormalization group scheme, we may assume that $R(T) = a_2 T + a_4 T^2$, where $a_2, a_4$ are related to the anomalous dimension and operator product expansion coefficients for the tachyon couplings. The compatibility conditions for the existence of bounded solutions to the equation (26), then, imply the form [11] $P(T) = A_1 + A_2 T + A_3 T^2 + A_4 T^4$, with $A_i = f_i(a_2, a_3), i = 1, \ldots, 4$. Indeed such equations have been shown [31] to lead to kink-like solutions,

$$T(\xi) = \frac{1}{2a_4} \{ \text{sgn}(a_2 a_4) a_2 \tanh[\frac{1}{2} a_2 (x - ut)] - a_2 \}$$
where the velocity \( u = \frac{A_1 - 3a_0}{a_4} \). This fact expresses for us a sort of universal behaviour for biological systems. This shows the existence of at least one class of schemes which admit kink-like solutions of the same sort as the ones of Lal [9] for energy transfer without dissipation in cells.

The importance of solutions of the form (28) lies in the fact that they can be derived from a Hamiltonian and, thus, can be quantized canonically [32]. They are connected to the solitons (12) by a Renormalization Scheme change on the world-sheet of the effective string theory, reproducing (11). This amounts to the possibility of casting friction problems, due to the Liouville terms, into a Hamiltonian form. This is quite important for the quantization of the kink solution, which will provide one with a concrete example of a large-scale quantum coherent state for the preconscious state of the mind. In a pure field-theoretic setting, a quantization scheme has been discussed in ref. [33], using a variational approach by means of squeezed coherent states. There is a vast literature on soliton quantization using approximate WKB methods. We selected the method of ref. [33] for our purposes here, because it yields more accurate results than the usual WKB methods of soliton quantization [34], and it seems more appropriate for our purposes here, due to its direct link with coherent ground states. We shall not give details on the derivation but concentrate on the results. We refer the interested reader to the literature [33]. A brief description of the method is provided in Appendix B.

The result of such a quantization yields a modified soliton equation for the (quantum corrected) field \( C(x,t) \)

\[
\partial_t^2 C(x,t) - \partial_x^2 C(x,t) + M^{(1)}[C(x,t)] = 0 \tag{29}
\]

with the notation

\[
M^{(n)} = e^{\frac{1}{2}(G(x,x,t) - G_0(x,x))} \frac{\partial^2}{\partial x^2} U^{(n)}(z) \big|_{z=C(x,t)} \quad ; \quad U^{(n)} \equiv d^nU/dz^n \tag{30}
\]

Above, \( U \) denotes the potential of the original soliton Hamiltonian, and \( G(x,y,t) \) is a bilocal field that describes quantum corrections due to the modified boson field around the soliton. The quantities \( M^{(n)} \) carry information about the quantum corrections, and in this sense the above scheme is more accurate than the WKB approximation [33]. The whole scheme may be thought of as a mean-field-approach to quantum corrections to the soliton solutions. For the kink soliton (28) the quantum corrections (29) have been calculated explicitly in ref. [33], thereby providing us with a concrete example of a large-scale quantum coherent state.

The above results on a consistent quantization of the soliton solutions (28), derived from a Hamiltonian function, find a much more general and simpler application in our Liouville approach [35, 36]. To this end, we first note that the structure of the equation (26), which leads to (28), is generic for Liouville strings in non-trivial background space-times. If we view the Liouville mode as a local scale of
the renormalization group on the world-sheet, one can easily show that for
the coupling \( g^i \) of any non-marginal deformation \( V_i \) of the \( \sigma \)-model, the following
Liouville renormalization group equation holds

\[
\ddot{g}^i + Q \dot{g}^i = -\beta^i = -G^{ij} \partial_i C[g] \quad ; \quad Q^2 = \frac{C[g] - 25}{6} + \ldots
\]  

(31)

where the dot denotes differentiation with respect to the renormalization group
Liouville scale \( t \), and \( \ldots \) denote terms removable by redefinitions of the couplings
\( g^i \) (renormalization scheme changes). The tensor \( G^{ij} \) is an inverse metric in field
space\([13]\). Notice in (31) the rôle of the non-criticality of the string \((Q \neq 0)\) as
providing a source of friction\([3]\) in the space of fields \( g^i \). The non-vanishing renor-
malization group \( \beta \)-functions play the rôle of generalized forces. The functional
\( C[g] \) is the Zamolodchikov \( c \)-function, which is constructed out of a particular combination
of components of the world-sheet stress tensor of the deformed \( \sigma \)-model\([13, 5]\).

The existence of friction terms in (31) implies a statistical description of the
temporal evolution of the system using classical density matrices \( \rho(g^i, t) \)

\[
\partial_t \rho = -\{\rho, H\}_{PB} + \beta^i G_{ij} \frac{\partial \rho}{\partial p_j}
\]  

(32)

where \( p_i \) are conjugate momenta to \( g^i \), and \( G_{ij} \propto <V_i V_j> \) is a metric in the space
of fields \( \{g^i\} \)\([13]\). The non-Hamiltonian term in (32) leads to a violation of the
Liouville theorem (1) in the classical phase space \( \{g^i, p_j\} \), and constitutes the basis
for a modified (dissipative) quantum-mechanical description of the system\([5]\), upon
quantization.

In string theory, summation over world sheet surfaces will imply quantum fluctu-
atations of the string target-space background fields \( g^i \)\([5, 35]\). Canonical quantization
in the space \( \{g^i\} \) can be achieved, given that the necessary Helmholtz conditions
\([32]\) can be shown to hold in the string case\([35]\). The important feature of the
string-loop corrected (quantum) conformal invariance conditions is that they can be
derived from a target-space action\([39]\), which schematically can be represented as

\[
S = -\int dt \left( \int_0^1 d\tau g^i E_i(t, \tau g, \tau \dot{g}, \tau \ddot{g}) + \text{total derivatives} \right) ;
\]

\[
E_i(t, g, \dot{g}, \ddot{g}) \equiv G_{ij}(\ddot{g}^j + Q \dot{g}^j + \beta^j)
\]  

(33)

where the tensor \( G_{ij} \) is a (quantum) metric\([13]\) in theory space \( \{g^i\} \). It is charac-
terized by a specific behaviour\([33]\) under the action of the renormalization group
operator \( D \equiv \partial_t + \dot{g}^i \partial_i \),

\[
DG_{ij} = QG_{ij} = <V_i V_j> + \ldots
\]  

(34)

where the \( \ldots \) denote diffeomorphism terms in \( g \)-space, that can be removed by an
appropriate scheme choice\([36]\).

\(^{1}\) For a concise review of the formalism and relevant notation see Appendix A.
In this way, a friction problem (31) can be mapped non-trivially onto a canonically quantized Hamiltonian system, in similar spirit to the solitonic point-like field theory discussed in section 3. The quantum version of (32) reads

\[ \partial_t \hat{\rho} = i [\hat{\rho}, \hat{H}] + i \beta^i G_{ij} [\hat{g}^i, \hat{\rho}] \]  

(35)

where the hat notation denotes quantum operators, and appropriate quantum ordering is understood (see below). We note that the equation (35) implies that \( \partial_t \rho \) depends only on \( \rho(t) \) and not on a particular decomposition of \( \rho(t) \) in the projections corresponding to various results of a measurement process. This automatically implies the absence of faster-than-light signals during the evolution.

The analogy with the soliton case, discussed previously, goes even further if we recall the fact that energy is conserved in average in our approach of time as a renormalization scale. Indeed, it can be shown that the temporal change of the energy functional of the string particle, \( \mathcal{H} \), obeys the equation

\[ \partial_t \mathcal{H} \propto \mathcal{D} \Theta(z, \bar{z}), \mathcal{H}(0) = 0 \]  

(36)

where \( \Theta \) denotes the trace of the world-sheet stress tensor; the vanishing result is due to the renormalizability of the \( \sigma \)-model on the world-sheet surface, which, thus, replaces time-translation invariance in target space.

However, the quantum energy fluctuations \( \delta E \equiv [<< \mathcal{H}^2 >> - (<< \mathcal{H} >>)^2]^{1/2} \) are time-dependent:

\[ \partial_t (\delta E)^2 = - i [\beta^i, \mathcal{H}] [\beta^j G_{ji} >> = << \beta^i G_{ji} d_{dt} \beta^j >> \]  

(37)

Using the fact that \( \beta^i G_{ij} \beta^j \) is a renormalization-group invariant quantity, we can express (37) in the form

\[ \partial_t (\delta E)^2 = - << Q^2 \beta^i G_{ij} \beta^j >> = - << Q^2 \partial_i C >> \leq 0 \]  

(38)

We, thus, observe that, for \( Q^2 > 0 \) (supercritical strings), the energy fluctuations decrease with time for unitary string \( \sigma \)-models.

Before closing this section we wish to make an important remark concerning quantum ordering in (35). The quantum ordering is chosen in such a way so that energy and probability conservation, and positivity of the density matrix are preserved in the quantum case. Taking into account that in our string case \( \beta^i G_{ij} = \sum_n C_{ijk...j_k} g^{j_1} \cdots g^{j_n} \), with the expansion coefficients appropriate vertex operator correlation functions \( \Theta \), it is straightforward to cast the above equation into a Lindblad form

\[ \dot{\rho} \equiv \partial_t \rho = i[\rho, \mathcal{H}] - \sum_m \{ B^i_m B_m^i, \rho \} + 2 \sum_m B_m \rho B^i_m \]  

(39)
where the ‘environment’ operators $B_m, B^\dagger_m$ are defined appropriately as ‘squared roots’ of the various partitions of the operator $\beta^j G_{ij} \ldots g^i$. This form may have important consequences in the case one considers a wave-function representation of the density matrix. Indeed, as discussed in ref. [41], (39) implies a stochastic diffusion equation for a state vector, which has important consequences for the localization of the wave-function in a quantum theory of measurement. We shall review briefly this approach, and make a comparison with ours, in the next section. We stress, however, that our approach based on density matrices (39) and renormalizability of the string $\sigma$-model is more general than any approach assuming state vectors.

4 Quantum Gravity and Breakdown of Coherence in the String Picture of a Microtubule

4.1 Microscopic black holes in MT chains as an environmental phenomenon

4.1.1 Formation of black holes

Above we have established the conditions under which a large-scale coherent state appears in the MT network, which can be considered responsible for loss-free energy transfer along the tubulins. Suppose now that external stimuli produce sufficient distortion in the electric dipole moments of the water environment of the MT. As a result, conformational (quantum) transitions of the tubulin dimers occur. Such abrupt pulses may cause sufficient distortion of the space time surrounding the tubulin dimers, which in turn leads to the formation of virtual ‘black holes’ in the effective target two-dimensional space time. Formally this is expressed by coupling the $c = 1$ string theory to two-dimensional quantum gravity. This elevates the matter-gravity system to a critical $c = 26$ theory. Such a coupling, then, causes decoherence, due to induced instabilities of the kink quantum-coherent ‘preconscious state’, in a way that we shall discuss below. As the required collapse time of $\mathcal{O}(1 \text{ sec})$ of the wave function of the coherent MT network is several orders of magnitude bigger than the energy transfer time $t_T$ (13), the two mechanisms are compatible with each other. Energy is transferred during the quantum-coherent preconscious state, in $10^{-7} \text{ sec}$, and then collapse occurs to a certain (classical) conformational configuration. In this way, Frohlich’s frequency associated with coherent ‘phonons’ in biological cells is recovered, but in a rather different setting.

We now proceed to describe the precise mechanism for the breakdown of coherence, once the system couples to quantum gravity. To discuss this issue, we first note that there is an exact conformal field theory [23], a Wess-Zumino model
over $SL(2, R)/O(1, 1)$ coset theory, whose target space has the metric of a two-dimensional black hole (to be defined below, c.f. (12)). Its Euclideanized version is better studied, and will be frequently used in this work, especially when one studies non-perturbative configurations of this field theory. As a conformal field theory, Euclidean two-dimensional black holes are described by a Wess-Zumino model over $SL(2, R)/U(1)$. The presence of a black hole in the $(1+1)$-dimensional effective string model corresponding to (11) leads to a non-trivial coupling with target space-time gravity, as well as the plethora of the $SL(2, R)$ global $W_\infty$ states characterising two-dimensional strings (3). First, we discuss an explicit way of dynamical creation of black holes in two-dimensional string theory [45, 23] through collapse of tachyonic matter $T(x,t)$. This is a procedure that can happen as a result of quantum fluctuations of various excitations.

Consider the equations of motion for the graviton and dilaton fields obtained by imposing conformal invariance in the model (20) to order $\alpha'$, ignoring the non-universal tachyon terms in the potential. It can be shown that a generic solution for the graviton deformation has the form [45]

$$ds^2 = -(1 + \int_{-\infty}^{t} dt' \partial_x T) dt^2 + \left(1 - 4 \frac{a^2 e^{-2x}}{3} \right)^{-1} dx^2$$

(40)

Consider now an incoming localized wavepacket of the form ($a \equiv \text{const}$)

$$T = e^{-x} a \frac{1}{\cosh[2(x + t)]}$$

(41)

It becomes clear from (40) that there will be an horizon, obtained as a solution of the equation derived by imposing the vanishing of the coefficient of the $dt^2$ term. Thus, for late times $t \to \infty$, the resulting metric configuration is a two-dimensional static black hole [23]

$$ds^2 = -(1 - \frac{4}{3} a^2 e^{-2x}) dt^2 + \left(1 - \frac{4}{3} a^2 e^{-2x} \right)^{-1} dx^2$$

(42)

and the whole process describes a dynamical collapse of matter. The energy of the collapsing wavepacket gives rise to the Arnowitt-Deser-Misner (ADM) mass $\frac{1}{2} a^2$ of

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2It is probably of interest to note that in two-dimensional string theories, the target space time gravitons are quasi-topological states, and as such the only non-trivial geometries are either flat space-times ($c = 1$ string theory) or the black hole one, represented in ref. [23] as a coset $SL(2, R)/O(1, 1)$ Wess-Zumino model. The physical states of these two models have been recently shown to be equivalent [12, 43], thereby proving in a rigorous way claims [23] that the $c = 1$ flat string theory can be considered as the asymptotic limit of the two-dimensional black hole. It is also important to notice that, for technical reasons, most of the computations and arguments, especially in the context of the world-sheet conformal field theory, refer to the Euclidean version $SL(2, R)/U(1)$ of the black hole. Analytic continuation is then assumed in order to transcribe the results in the Minkowski case. Note, however, that this procedure is not yet completely understood in the context of string theory [4].
the black hole \cite{23}. It is crucial for the argument that there is no part of the wave-packet reflected. Otherwise the resulting ADM mass of the black hole will be the part of the energy that was not reflected. In realistic situations, the black hole is only virtual, since low-energy matter pulses are always reflected in two-dimensional string theories, as suggested by matrix-model computations \cite{51}. Indeed, if one discusses pulses which undergo total reflection \cite{45},

\[ T(x, t) = e^{-x} \eta(x, t) \quad ; \quad \eta(x, t) = \frac{a}{\cosh(2(x + t))} + \frac{a}{\cosh(2(x - t))} \]  

(43)

then, it can be easily shown that there is a transitory period where the space time geometry looks like a black hole \cite{12}, but asymptotically in time one recovers the linear dilaton (flat) vacuum\cite{28}. The above example \cite{13} gives a generic way of a (virtual) dynamical matter collapse in a two-dimensional stringy space-time, of the type that we encounter in our model for the brain, as a result of abrupt quantum conformational changes of the dimers. In the case of MT, the replacement \( x \rightarrow \xi \), where \( \xi \) is the boosted coordinate in \cite{12}, is understood.

4.1.2 (Infinite) Symmetry Breaking and Massless Modes

Once a virtual black hole is formed in a MT chain, the subsystem of the displacement modes \( \psi(\xi) \) becomes open in a statistical mechanics sense. This subsystem is a (1+1)-dimensional non-critical (Liouville) theory. It is known that the singularity structure of black holes in such systems is described by a topological \( \sigma \)-model, obtained by twisting the \( N = 2 \) supersymmetric black hole \cite{46}. The field theory at the singularity is described by an enhanced topological \( W_{1+\infty} \otimes W_{1+\infty} \) symmetry. Away from the singularity, such symmetry is broken spontaneously down to a single \( W_{1+\infty} \) symmetry, as a result of the non-vanishing target-space gravitational condensate \cite{13}. Spontaneous breaking in a (1+1)-dimensional target string theory is allowed, in the sense that the usual infrared infinities that prevented it from happening in a local field theory setting are absent. In ref. \cite{46, 5} we have demonstrated this phenomenon explicitly by showing that, due to the (twisted) \( N = 2 \) supersymmetry associated with the topological nature of the singularity, there is a suppression of the tunneling effects, which in point-like theories would prevent the phenomenon from occurring. As a result, the appearance of massless states takes place \cite{1}, which are delocalized global states, belonging to the lowest-level of the string spectrum \cite{13}.

\footnote{The reader might find curious the appearance of massless excitations as a result of the breaking of gauge stringy symmetries, such as the target space \( W_{\infty} \otimes W_{\infty} \) in the two-dimensional string theory \cite{3}. However, as we have mentioned above and in ref. \cite{10}, these symmetries are topological, and the massless excitations are discrete non-propagating states, which correspond to longitudinal components of string states that cannot be gauged away \cite{17}. This situation should be contrasted to the case of conventional gauge (local) symmetries in point-like theories, where the massless Goldstone excitation is ‘eaten up’ (gauged away) by the longitudinal component of the (propagating) gauge boson. However, we mention that even in the case of point-like theories there are arguments for the existence of a Goldstone theorem for topological symmetries, such as the flux symmetry in (2+1)-dimensional QED, whose massless goldstone particle is the photon itself \cite{18}.}
These are the leg-poles that appear in the scattering amplitudes of $c = 1$ Liouville theory [17]. Their excitation in the mind, results in conscious perception, in a way similar to the one argued in the context of local field theory regarding the excitation of the dipole wave quanta [21]. Here, however, as we shall see, the mechanism of conscious perception appears formally much more complicated, due to the complicated nature of the enormous stringy symmetries that are spontaneously broken in this case.

4.1.3 Evaporation of the microscopic Black Holes

From a formal point of view, the formation of virtual black holes, with varying mass, can be modelled by the action of world-sheet instanton deformations [5]. The latter have the property of shifting (renormalizing) the Wess-Zumino level parameter $k$ (related to central charge) of the black hole conformal field theory of ref. [23]. From a conformal field theory point of view, instantons are associated with induced extra logarithmic divergencies (on the world sheet) in the presence of the matter leg-poles. In our approach to target time, $t$, as a dynamical world-sheet renormalization group scale [5], $\phi = -t$, such logarithmic divergencies, when regularized, lead to extra time dependences in the central charge of the theory, and hence to a time-dependent ‘effective’ $k$, as mentioned above. Some brief description of the formalism is given in Appendix A. For mode details we refer the reader to the literature [5, 49]. It can be shown [5] that in such a case the ADM mass of the black hole [50] depends on the scale (time)

$$M_{bh} \propto \frac{1}{\sqrt{k(t) - 2}} e^a$$

where $a$ is a constant that can be added to the dilaton field without affecting the conformal invariance of the black hole solution without matter. As shown in [5], $k(t)$ actually increases with time $t$, leading asymptotically to $\infty$, which corresponds to the flat space-time limit. In that case, the system keeps ‘memory’ of the dilaton constant $a$, which pre-existed the black hole formation. From a MT point of view, the constant $a$ corresponds to a spontaneously chosen vacuum string state as a result of spontaneous breaking mechanisms of, say, electric dipole rotational symmetries etc., in the ordered water environment [21]. As we shall argue later on, this is important for coding of memory states in this framework. From the above discussion it becomes clear that memory operation in our approach is a two step process: (i) formation of a black hole, of a fixed value for the dilaton vacuum expectation value (vev) $a$, related to spontaneous breaking occurring in the water environment, as a result of an external stimulus, and (ii) evaporation of the black hole, due to quantum instabilities, described by world sheet instanton effects in our completely integrable approach to MT dynamics. The latter effects drive the black hole to a vanishing-mass limit by shifting $k$ and not the dilaton vev, $a$. This implies storage of information, according to the general ideas of ref. [3]. Indeed, from a conformal field theory point of view, the constant $a$ can be shifted by exactly marginal deformations (moduli) of the black hole $\sigma$ model, whose couplings are arbitrary. Such an operator has been
constructed explicitly in ref. [52], and consists exclusively of combinations of global state deformations. In the terminology of ref. [52] is called $L_0^2 T_0$, and it is one of the (infinite number of) $W_\infty$ charges that has been conjectured to characterize a two-dimensional stringy black hole [5].

4.2 Decoherence mechanism

The important point to notice is that the system of $T(\xi)$ coupled to a black hole space-time [12], even if the latter is a virtual configuration, it cannot be critical (conformal invariant) non-perturbatively if the tachyon has a travelling wave form. The factorisation property of the world-sheet action [20] in the flat space-time case breaks down due to the non-trivial graviton structure [42]. Then a travelling wave cannot be compatible with conformal invariance, and renormalization scale dressing appears necessary. The gravity-matter system is viewed as a $c = 26$ string [23, 5], and hence the renormalization scale is time-like [28]. This implies time dependence in $T(\xi, t)$.

A natural question arises whether there exist a deformation that turns on the coupling $T(\xi)$ which is exactly marginal so as to maintain conformal invariance. The exactly marginal deformation of this black hole background that turns on matter, $L_0^2 L_1^0$ in the notation of ref. [22], couples necessarily the propagating tachyon $T(\xi)$ zero modes to an infinity of higher-level string states [52]. The latter are classified according to discrete representations of the $SL(2, R)$ isospin, and together with the propagating modes, form a target-space $W_\infty$-algebra [12]. This coupling of massive and massless modes is due to the non-vanishing Operator Product Expansion (O.P.E.) among the vertex operators of the $SL(2, R)/U(1)$ theory [52].

The possesses an infinity of conserved charges in target space [8] corresponding to the Cartan subalgebra of the infinite-dimensional $W_\infty$ [12]. At present, an explicit construction [32] of the physical black hole moduli has been restricted to the $L_0^2 T_0^0$ operator, mentioned above in connection with a shif in the ADM mass, as well as the operator $L_0^2 T_0^0$ turning on matter backgrounds. We believe, however, that the whole $W$-algebra of charges corresponds to exactly marginal deformations (moduli), leading to a quantum-coherence-preserving moduli hair for the stringy black hole [5]. In practice, such global charges, which contribute phase factors to the string universe wave function, are impossible to measure by localized scattering experiments in our world. This, as we shall explain immediately below, leads to the effective breakdown of quantum coherence in the low-energy world [3].

From the point of view of MT, such $W$ modes might be thought of as constituting the ‘consciousness degrees of freedom’ [4], which in this picture, are not exotic, as suggested in ref. [15]; but exist already in a string formalism, and they result in the complete integrability of the two-dimensional black hole Wess-Zumino model.
This integrability persists quantization \[53\], and it is very important for the quantum coherence of the string black hole space-time \[5\]. Due to the specific nature of the \(W_\infty\) symmetries, there is no information loss during a stringy black hole decay, the latter being associated with instabilities induced by higher-genus effects on the world-sheet \[54, 5\]. The phase-space volume of the effective field theory is preserved in time, only if the infinite set of the global string modes is taken into account. This is due to the string-level mixing property of the \(W_\infty\) - symmetries of the target space.

However, any local operation of measurement, based on local scattering of propagating matter, such as the functions performed by the human brain, will necessarily break this coherence, due to the truncation of the string deformation spectrum to the localized propagating modes \(T(\xi)\). The latter will, then, constitute a subsystem in interaction with an environment of global string modes. The quantum integrability of the full string system is crucial in providing the necessary couplings. This breaking of coherence results in an arrow of time/Liouville scale, in the way explained briefly above \[5\]. The black-hole \(\sigma\)-model is viewed as a \(c = 26\) critical string, while the travelling wave background is a non-conformal deformation. To restore criticality one has to dress \(T(\xi)\) with a Liouville time dependence \(T(\xi, t)\). From a \(\sigma\)-model point of view, to \(O(\alpha')\), a non-trivial consistency check of this approach for the black hole model of ref. \[23\] has been provided in ref. \[5\]. We stress, once more, that the Liouville renormalization scale now is time-like, in contrast with the previous string picture of a \(c = 1\) matter string theory, representing the displacement field \(u\) alone before coupling to gravity.

By viewing the time \(t\) as a local scale on the world sheet, a natural identification of \(t\) will be with the logarithm of the area \(A\) of the world sheet, at a fixed topology. As the non-critical string runs towards the infrared fixed point the area expands. In our approach to Liouville time \[3\], the actual flow of time is opposite to the world sheet renormalization group flow. This is favoured by a bounce interpretation of the Liouville flow due to specific regularization properties of Liouville correlation functions \[4, 7\]. This implies that we may set \(t \propto -\ln A\), with \(A\) flowing always towards the infrared \(A \to \infty\). In this way, a Time Arrow is implemented automatically in our approach, without requiring the imposition of time-asymmetric boundary conditions in the analogue of the Hartle-Hawking state \[57\]. In this respect, our theory has many similarities to models of conventional dissipative systems, whose mathematical formalism \[57, 58\] finds a natural application to our case.

With this in mind, one can examine the properties of the correlation functions of \(V_i, A_N = \langle V_{i_1} \ldots V_{i_N} \rangle\), and hence the issue of coherence breakdown. In critical string theory such correlators correspond to scattering amplitudes in the target-space theory. It is, therefore, essential to check on this interpretation in the present situation. Since the correlators are mathematically formulated on fixed area \(A\)
world-sheets, through the so-called fixed area constraint formalism \cite{59, 60}, it is interesting to look for possible $A$-dependences in their evolution. In such a case their interpretation as target-space scattering amplitudes would fail. Indeed, it has been shown \cite{36} that there is an induced target space $A$-dependence of the regularized correlator $A_N$, which, therefore, cannot be identified with a target space $S$-matrix element, as was the case of critical strings \cite{61}. Instead, one has non-factorisable contributions to a superscattering amplitude $S \neq SS^\dagger$, as is usually the case in open quantum mechanical systems, where the fundamental building blocks are density matrices and not pure quantum states\cite{6}. For completeness, we describe in Appendix A some formal aspects of this situation, based on results of our approach to non-critical strings \cite{5}.

It is this sort of coherence breakdown that we advocate as happening inside the part of the brain related to consciousness, whose operation is described by the dynamics of (the quantum version of) the model (5). The effective two-dimensional substructures, that we have identified above as the basic elements for the energy transfer in MT, provide the necessary framework for coupling the (integrable) stringy black hole space-time (42) to the displacement field $u(x, t)$. This allows for a qualitative description of the effects of quantum gravity on the coherent superposition of the preconscious states.

At this stage some comments are in order concerning the precise nature of the microscopic black holes. It is not clear whether such black hole states are related to real (four-dimensional) quantum gravity effects (e.g. spherically symmetric space-time singularities), which couple to the quantum MT chains, or they simply represent global environmental effects in such systems\cite{4}. In support of the former suggestion, we now proceed in an estimate of the collapse time, provided the latter is due to realistic quantum gravity effects.

4.3 Decoherence-time estimates

One can calculate in this approach the off-diagonal elements of the density matrix in the string theory space $u^i$, with now $u^i(t)$ representing the displacement field of the $i$-th dimer. In a $\sigma$-model representation (20), this is the tachyon deformation. The computation proceeds analogously \cite{3} to the Feynman-Vernon \cite{16} and Caldeira and Leggett \cite{17} model of environmental oscillators, using the influence functional

\footnote{It should be stressed that our environment-model-independent stringy approach to the MT is applicable even to cases when the virtual black hole creation in MT chains, discussed above, represents not real quantum gravity effects, but rather environmental entanglement with other biological structures in the brain. In such cases, the model (27) is still valid, but $\sqrt{\alpha'}$ is different from the Planck length, and it is of order of the characteristic scale of the interactions that cause the phenomenon.}
method, generalized properly to the string theory space
5
u
i
. The general theory of time as a world-sheet scale predicts \[5\] the following expression for the reduced density matrix\[16, 17\] of the observable states:

\[
\rho(u_I, u_F, t)/\rho_S(u_I, u_F, t) \approx e^{-N \int_0^t dt \int_{\tau - \epsilon}^{\tau + \epsilon} d\tau' \frac{(u(u(\tau')) - u(\tau))^2}{(\tau - \tau')^2}} \approx e^{-DNt(u_I - u_F)^2 + ...} \tag{45}
\]

where the subscript “S” denotes quantities evaluated in conventional Schrödinger quantum mechanics, and \(N\) is the number of the environment ‘atoms’\[7\] interacting with the background \(u_i\). For \((1, 1)\) operators, that we are interested in, the structure of the renormalization group \(\hat{\beta}\)-functions is \(\hat{\beta}^i = \epsilon u^i + \beta^{ui}\), where \(\beta^{ui} = O(u^2)\) and \(\epsilon \to 0\) is the anomalous dimension. Recalling \[5\] the pole structure of the Zamolodchikov metric, \(G_{ij} = \frac{1}{\epsilon} G^{(1)}_{ij} + \text{regular}\), one finds that the dominant contribution to the exponent \(K\) of the model \(45\) comes from the \(\epsilon\)-term in \(\hat{\beta}^i\) and the pole term in \(G_{ij}\):

\[
K = N \int_0^t \hat{\beta}^i G_{ij} \hat{\beta}^j d\tau \geq 2N \int_0^t u^i G^{(1)}_{ij} \beta^{uj} d\tau + O(\epsilon) \tag{46}
\]

Assuming slowly varying \(u^i\) and \(\beta^i\) over the time \(t\), this implies that the off-diagonal elements of the density matrix would decay exponentially to zero, within a collapse time of order \[7\]

\[
t_{coll} = \frac{1}{N} (O[\beta^{u^i} G^{(1)}_{ij} u^j])^{-1} t_s \tag{47}
\]

in fundamental string units \(t_s\) of time. The superscript \((1)\) in the theory space metric denotes the single residue in, say, dimensional regularization on the world sheet \[3, 37\]; \(N\) is the number of (coherent) tubulin dimers in interaction with the given dimer that undergoes the abrupt conformational change. Here the \(\beta^{u^i}\) -function is assumed to admit a perturbative expansion in powers of \(\lambda_s^2 \partial_X^2\), in target space, where the fundamental string unit of length is defined as

\[
\lambda_s = (\frac{\hbar c}{v_0})^{\frac{1}{2}} \tag{48}
\]

where \(v_0\) is the sound velocity \[14\], of order 1\(km/sec\ \[19\]. We work in a system of units where the light velocity is \(c = 1\), and we use as the scale where quantum gravity effects become important, the grand unified string scale, \(M_{gus} = 10^{18} GeV\), or in length \(10^{-32} cm\), which is 10 times the conventional Planck scale. This is so, because our model is supposed to be an effective description of quantum gravity effects in a stringy (and not point-like) space-time. This scale corresponds to a time scale of \(t_{gus} = 10^{-42} sec\). We now observe that, to leading order in the perturbative
\( \beta \) function expansion in (47), any dependence on the velocity \( v^2 \) disappears in favour of the scales \( M_{\text{gas}} \) and \( t_{\text{gas}} \). It is, then, straightforward to obtain a rough estimate for the collapse time

\[
t_{\text{col}} = O\left( \frac{M_{\text{gas}}}{E^2 N} \right)
\]

(49)

where \( E \) is a typical energy scale in the problem. Thus, we estimate that a collapse time of \( O(1 \text{ sec}) \) is compatible with a number of coherent tubulins of order

\[
N \simeq 10^{12}
\]

(50)

provided that the energy stored in the kink background is of the order of \( eV \). This is indeed the case of the (dominant) sum of binding and resonant transfer energies \( \Delta \simeq 1eV \) (16) at room temperature in the phenomenological model of ref. [19].

This number of tubulin dimers corresponds to a fraction of \( 10^{-7} \) of the total brain, which is pretty close to the fraction believed to be responsible for human perception on the basis of completely independent biological methods.

An independent estimate for the collapse time \( t_{\text{col}} \), can be given on the basis of point-like quantum gravity theory, assuming that the latter exists, either \textit{per se}, or as an approximation to some string theory. One incorporates quantum gravity effects by employing wormholes [62] in the structure of space-time, and then applies the calculus of ref. [7] to infer the estimates of the collapse time. In that case, one evaluates the off-diagonal elements of the density matrix in real configuration space \( x \), which should be compared to that in string theory space (45). The result of ref. [7] for the time of collapse induced by the interaction with a ‘measuring apparatus’ with \( \mathcal{N} \) units is

\[
t_{\text{col}}' \simeq \frac{1}{\mathcal{N}} (M_{\text{gas}}/m)^3 \frac{1}{m^3(\delta x)^2}
\]

(51)

where \( m \) is a typical mass unit in the problem. The fundamental unit of velocities in (51) is provided by the velocity of light \( c \), since the formula (51) refers to generic four-dimensional space-time effects. In the case of the tubulin dimers, it is reasonable to assume that the pertinent moving mass is the effective mass \( M^* \) (17) of the kink background (12). This will make contact with the microscopic model above. To be specific, (17) gives \( M^* \simeq 3m_p \), where \( m_p \) is the proton mass. This makes plausible the rather daring assumption that the nucleons (protons, neutrons) themselves inside the protein dimers are the most sensitive constituents to the effects of quantum gravity. This is a reasonable assumption if one takes into account that the nucleons are much heavier than the (conformational) electrons. If true, this would really imply, then, that elementary particle scales come into play in brain functioning. In this picture, then, \( \mathcal{N} \) is the number of tubulin dimers, and moreover in our case \( \delta x = O[4nm] \), since the relevant displacement length in the problem is of order of the longitudinal dimension of each conformational pocket in the tubulin dimer. Thus, substituting these in (51) one derives the result \( \mathcal{N} = N \simeq 10^{12} \), for the number of coherent dimers that induce a collapse within \( O(1 \text{ sec}) \).
It is remarkable that the final numbers agree between these two estimates. If one takes into account the distant methods involved in the derivation of the collapse times in the respective approaches, then one realizes that this agreement cannot be a coincidence. Our belief is that it reflects the fundamental rôle of quantum gravity in the brain function.

At this stage, it is important to make some clarifying remarks concerning the kind of collapse that we advocate in the framework of non-critical string theory. In the usual model of collapse due to quantum gravity [7], one obtains an estimate of the collapse of the off-diagonal elements of the density matrix in configuration space, but no information is given for the diagonal elements. In the string theory framework of ref. [5] the collapse (45) also refers to off-diagonal elements of the string density matrix, but in this case the configuration space is the string background field space. In this case the off-diagonal element collapse suffices, because it implies localization in string background space, which means that the quantum string chooses to settle down in one of the classical backgrounds, which in the case at hand is the solitonic background discussed in section 2. This situation constitutes the stringy analogue of the emergence of almost classical states in an open quantum theory, as a result of decoherence effects induced by the environment [63]. Such states, termed ‘pointer-states’ in ref. [63], are minimum-entropy-producing states, which behave almost classical under time evolution. Due to the minimum-entropy production their time evolution can be characterized as ‘almost’ reversible. It should be stressed that this is a model dependent statement, and the very existence of ‘pointer states’ depends crucially on the type of interaction with the environment [64].

In ref. [65] we have discussed in detail the emergence of ‘pointer states’ in a matrix-model version of the model at hand, placed in a random environment, assumed to simulate the quantum gravity effects discussed in this section. The configuration space of the inverted-harmonic oscillator system, used to simulate the physics of the matrix model, is actually a field space, since the ‘position’ variable $q$ of the model [11, 65] is a ‘collective co-ordinate’ related through a canonical transformation to the ‘tachyon’ field (in our model this is the displacement field $u$). It has been argued in ref. [65], based on known results of the inverted-harmonic oscillator model [66], that the presence of an environment leads to the evolution of a pure quantum state, represented as a Gaussian wave-packet, at time $t = 0$, into a probability distribution for the particle, described by the diagonal elements of the density matrix, which retains the Gaussian shape but with a time- and temperature-dependent width. The temperature indicates the effects of the interaction of the system with a heat-bath, and as discussed in ref. [65] in our picture this ‘temperature’ is related to the deviations from the conformal invariance of the system. Such Gaussian states are distinct from Wigner coherent states, which in the case of the inverted harmonic oscillator are not square-integrable. These Gaussian wave-packets are square integrable, and constitute the proper ‘pointer states’ of the inverted harmonic oscillator
problem. In our stringy interpretation of this system \[11, 65\] such ‘pointer states’ are the closest analogues of classical ground states of the string, which arise as a result of quantum gravity environmental entanglement. Of course, the important question ‘which specific background is selected by the above process?’ cannot be answered unless a full string field theory dynamics is developed. However, we believe that our approach \[5\] of viewing the selection of a critical string ground state as a generalized ‘measurement’ process in string theory space might prove advantageous over other dynamical methods in this respect.

In the MT model, viewed as a completely integrable non-critical string theory, such classical ground states are the solitons discussed in section 2. The effects of quantum gravity in the one-dimensional chain, amount to induce a collapse of (squeezed) coherent states formed around such soliton solutions (c.f. Appendix B), thereby selecting a given classical ground state. This process is interpreted in this work as implying ‘conscious’ perception.

An important question may arise at this point concerning in connection with the passage from quantum to classical worlds. According to a more conventional belief, the latter should involve the collapse of the wave function of the system. Indeed, from a formal point of view, the density matrix refers to an ensemble of theories, whilst the wave function is a quantity that characterizes a single system. From the discussion above, we have shown how decoherence (i.e. the collapse of the off-diagonal elements of the density matrix in the space of string theories) induces the appearance of classical ground states of the string, but we have not properly addressed the question of how a single ground is chosen. As we mentioned previously, the question ‘which ground state is going to be selected’ is a very difficult one and at present cannot be answered without string field theory. However, we can provide arguments in favour of the localization of the state vector (if one insists on using such a formalism) based on specific properties of our approach to time as a Liouville field. The most important of them is the stochasticity that characterizes evolution in this picture to which we briefly turn now, for the benefit of the reader. For more details of this approach we refer the interested readers to the existing literature \[5, 56\].

4.4 Stochastic Nature of the Renormalization Group Flow and State-Vector Localization

To this end, we recall that in Liouville theory a correlation function of (1, 1) matter deformations $V_i$ is given by \[67\]

$$< V_{i_1} \ldots V_{i_n} >_\mu = \Gamma(-s) < V_{i_1} \ldots V_{i_n} >_{\mu=0}$$

\[52\]

\[\text{We note, for completeness, that in the harmonic oscillator case of ref. 63 the pointer states are Wigner coherent states.}\]
where $s$ is the sum of the appropriate Liouville energies, and $<\ldots>_\mu$ denotes a $\sigma$-model average in the presence of an appropriate cosmological constant $\mu$ deformation on the world-sheet\textsuperscript{7}. The important point for our discussion is the $\Gamma$-factor $\Gamma(-s)$. For the interesting case of matter scattering off a two-dimensional ($s$-wave four-dimensional) string black hole, the latter is excited to a ‘massive’ (topological) string state \[69\] corresponding to a positive integer value for $s = n^+ \in \mathbb{Z}^+$. In this case, the expression \[52\] needs regularization. By employing the ‘fixed area constraint’ \[59\] one can use an integral representation for $\Gamma(-s)$

$$\Gamma(-s) = \int dA e^{-A} A^{-s-1} \quad (53)$$

where $A$ is the covariant area of the world-sheet. In the case $s = n^+ \in \mathbb{Z}^+$ one can then employ a regularization by analytic continuation, replacing \[53\] by a contour integral as shown in fig. 4a \[56, 5\]. This is a well-known method of regularization in conventional field theory, where integrals of form similar to \[53\] appear in terms of Feynman parameters. We note that it is the same regularization which was also used to prove the equivalence of the Bogolubov-Parasiuk-Hepp-Zimmerman renormalization prescription to the dimensional regularization of ‘t Hooft \[70\]. One result of such an analytic continuation is the appearance of imaginary parts in the respective correlation functions, which in our case are interpreted \[56, 5\] as renormalization group instabilities of the system.

Interpreting the latter as an actual time flow, we then may consider the contour of fig. 4a as implying evolution of the world-sheet area in both (negative and positive) directions of time (c.f. fig. 4b), i.e.

*Infrared fixed point $\rightarrow$ Ultraviolet fixed point $\rightarrow$ Infrared fixed point* \quad (54)

In each half of the world-sheet diagram of fig. 4b the Zamolodchikov $C$-theorem tells us that we have an irreversible Markov process. According to the analysis of ref. \[3\] the physical system is time-irreversible, since the physical processes associated with the time transformations in each ‘branch’ of fig. 4b are inequivalent. Indeed, it has been argued in ref. \[3\] that a highly-symmetrical phase of the two-dimensional black hole occurs at the infrared fixed point of the world-sheet renormalization group flow. At that point, the associated $\sigma$-model is a topological theory constructed by twisting \[23, 12\] an appropriate $N = 2$ supersymmetric black-hole $SL(2, R)$ Wess-Zumino $\sigma$-model. The singularity of a stringy black hole, then, describes a topological degree of freedom. The highly-symmetric phase is interpreted as the state with the most ‘appropriate’ initial conditions, whose preparation requires finite entropy \[3\]. This in turn implies a ‘bounce’ interpretation of the renormalization group flow of fig. 4a, in which the infrared fixed point is a ‘bounce’ point, similar to the corresponding picture in point-like field theory. Thus, the “physical” flow of time is taken to be

\textsuperscript{7}In the case of a black-hole coset model this operator is a ‘modified cosmological constant’ involving some mixing with appropriate ghost fields parametrising the $SL(2, R)$ string \[68\].
opposite to the conventional renormalization group flow, i.e. from the infrared to the ultraviolet fixed point on the world sheet. This can be confirmed explicitly, by using world-sheet instanton calculus \[5\]. The instantons are semi-classical solutions, whose existence can be established rigorously in our black hole conformal field theory \[49\]. In fact the black hole model is the limiting case that admits (in its Euclideanized version) such solutions.

An important feature of this bounce picture is that it allows for a stochastic evolution in time. The quantum coupling constant space may be viewed as a stochastic manifold \[35\], where the probability distribution \( P(g, \tau | g_0) \) for the coupling/field \( g \) to evolve to this value from the initial value \( g_0 \) at RG time \( \tau = 0 \) obeys a Fokker-Planck equation (in the space of the string backgrounds)

\[
\partial_\tau P(\lambda, \tau) = \frac{1}{8\pi^2} \frac{\delta}{\delta \lambda} Q^2 \delta^{ij} \left( Q^2 P(\lambda, \tau) \right) + \frac{\delta}{\delta \lambda} \left[ \beta^i P(\lambda, \tau) \right] \tag{55}
\]

modulo ordering ambiguities for the \( \lambda \)-dependent diffusion coefficients. \( Q^2 \) in the above formula denotes the ‘running’ (along RG trajectories \[13, 5\]) central charge deficit of the effective theory. In the case of our non-critical black hole string, we identify \(-\tau\) with the real target space time \( t \). The diffusion coefficient decreases with time \( t \), and vanishes at the fixed point (equilibrium point) thereby leading to localization in string theory space (selection of a proper stable ground state as a result of the dissipation induced by the non-vanishing \( \beta^i \)).

The stochastic equation (55) plays an important rôle in the connection of the collapse of the off-diagonal elements of the density matrix, discussed above, pertaining to an ensemble of theories, and the localization of the state vector of a (single) system. Indeed, the Fokker-Planck equation for the probability distribution cannot be derived from the usual Schrödinger equation, which is associated with a conservation equation for the probability current rather than a stochastic diffusion equation. As has been shown in ref. \[41\], if one insists on the existence of a quantum state vector formalism, \( \{ |\Psi> \} \), defined in the presence of an environment via the density matrix \( \rho(\Psi) = \mathcal{M} |\Psi><\Psi| \), with \( \mathcal{M} \) a mean over the ensemble of theories, then, the environmental entanglement described by the Lindblad equation (39) corresponds to a stochastic differential Ito process for the state vector \( |\Psi> \),

\[
|d\Psi> = -\frac{i}{\hbar} H |\Psi> + \sum_m \left( <B_m^\dagger > \psi B_m - \frac{1}{2} B_m^\dagger B_m \right) dt + \sum_m \left( B_m - <B_m > \psi \right) d\xi_m \tag{56}
\]

where \( H \) is the Hamiltonian of the system, \( B_m^\dagger, B_m \) are the environment operators, \( < \ldots >_\psi \) denote averages with respect to the state vector \( |\Psi> \), and \( d\xi_m \) are complex differential random matrices, associated with white noise Wiener or Brownian
processes \[41\]. In our completely integrable non-critical string theory representation of the MT system the environment operators are related to the \(\beta\)-functions as explained previously (c.f. \[39\]).

As regards eq. \((56)\) attention is drawn to a recent formal derivation \[71\] of this equation from Umezawa’s thermo-field dynamics model \[27\], which is a quantum field theory model for open systems interacting with the environment. Note that, in our interpretation of the time evolution as a renormalization group evolution on the world sheet of the string, white noise distributions arise quite naturally as a result of the summation over world-sheet topologies \[72, 3, 35, 73, 74, 75, 76\]. Moreover, quantum-gravity entanglement in such a framework may be described by a canonically quantized formulation of string theory space \[35\], which makes the situation formally similar to thermo-field dynamics \[27\]. However, there are important differences in our approach, notably the absence of a doubling of the degrees of freedom in order to simulate the environment, and also the energy conservation on the average \((36)\), as a result of peculiar properties of the world-sheet renormalization group \[5\]. A brief review of the situation in the string case is given in Appendix A.

Within the stochastic framework implied by \((56)\) it can, then, be shown that under some not too restrictive assumptions for the Hamiltonian operator of the system, localization of \(|\Psi>\) within a channel will always occur, as a result of environmental entanglement. To prove this formally, one constructs a quantity that serves as a ‘measure’ for the delocalization of the state vector, and examines its temporal evolution. This is given by the so-called ‘quantum dispersion entropy’ \[41\] defined as

\[
K \equiv - \sum_k <P_k> \Psi \ln <P_k> \Psi
\]

where \(P_k\) is a projection operator in the ‘channel’ \(k\) of the state space of the system. Notice that from our point of view the state space is the string background theory space. This entropy is shown to decrease in situations where \((56)\) applies, under some assumptions about the commutativity of the Hamiltonian of the system with \(P_k\) \[41\], which implies that \(H\) can always be written in a block diagonal form. The result for the rate of change of the dispersion entropy is then

\[
\frac{d}{dt}(MK) = - \sum_k \frac{1 - <P_k> \Psi}{<P_k> \Psi} R_k \leq 0
\]

where \(R_k\) are the (positive semi-definite) effective interaction rates in channel \(k\), defined as \[41\]

\[
R_k \equiv \sum_j |<L_{kj}>\Psi|^2
\]

with \(L_{kj} \equiv P_k L_j P_k\) denoting the projection of the environment operators in a given channel. From a string theory point of view, the order of magnitude of the environment operators is that of the ‘square-root’ of the \(\beta\)-functions of the non-critical
string representing the system at hand (c.f. discussion below (39)). In the case of a system of $N$ test strings propagating in non-trivial (non-critical) backgrounds $g^I$, then, following ref. [7], one can argue that the strength of the environmental entanglement is enhanced by $N$, thereby leading to an enhancement of the effective interaction rate (59) by $N$. For the system of MT dimers at hand, this yields an estimate of the collapse-of-the-state-vector (or localization) time of order

$$t_{\text{local}} = \left[ \frac{\mathcal{K}(0)}{\sum_k <P_k|\Psi> \left(1 - <P_k|\Psi>\right)} \right] \frac{1}{N(\beta^{\mu}G^{(l)}_{ij}u_j)} t_s$$

(60)

in fundamental string units $t_s$ of time, for a quantum mechanical network of $N$ dimers interacting with the (gravitational) environment. The above expression is rather formal. We have made use of normalized non-singular interaction rates $R_k/|<P_k|\Psi>|^2$, to estimate the (non-critical-string) environmental gravitational entanglement, and used the fact that the square of the environment operators is of the order of the $\beta$-function of the non-critical string. The probabilities $<P_k|\Psi>$ are (normalized) probabilities of selecting a particular background for the MT configurations. They depend on time $t$ (Liouville mode in our approach) through the time-dependence of the state vectors $|\Psi>$. They should be computed within a string-field theory framework, and, hence, at present a detailed estimate of their magnitude is impossible. However, one can make the naturalness assumption that at $t = 0$

$$\frac{\mathcal{K}(0)}{\sum_k <P_k|\Psi> \left(1 - <P_k|\Psi>\right)} \simeq O[1]$$

(61)

Taking into account that in string theory the square of the $\beta$-functions decreases as the time flows (the string approaches its conformal point and, therefore, the $\beta$ functions tend to zero), one observes that (60) would yield an estimate of the delocalization time of the form form (47). This is the same result as the density matrix collapse we advocate in this work [7, 5]. The similarity should have been expected from the fact that equations (56) and (39) are supposed to describe the same physics. However, we should stress, once more, that the above argument made use of a naturalness assumption in order to infer the order of magnitude of a combination of probabilities of selecting certain backgrounds in string field theory, which are quantities that at present are impossible to be computed analytically.

Some comments are now in order, concerning the precise application of the above localization-of-state-vector theorem to our quantum gravity case. In our case, quantum gravitational effects have non-zero effective interaction rates because the latter represent the recoil of the space-time itself under matter scattering off an effective microscopic virtual black hole formed in the MT chains, as a result of abrupt conformational changes of the dimers. The latter phenomenon can be a result of an external stimulus, including realistic four-dimensional quantum gravity fluctuations. The very nature of the microscopic black hole, of having $ADM$ mass of order of $M_{\text{Planck}}$, which, then, evaporates in the foam, makes it impossible to be treated
semi-classically, by adding a hamiltonian term in the effective action. Hence, in the presence of such a black hole coupling of an MT chain to ‘environmental’ operators appears necessary, and the effective interaction rate $R_k$ is non-zero, thereby producing the localization effects on the state vector of the system.

The effective interaction rate vanishes when a *stable ground state* is produced, as is the case of dissipation (friction). Our stringy system exhibits ‘gravitational friction’ [5]. The permanent ground state is the critical string vacuum. Then localization stops. This is exactly the situation encountered in the ‘pointer states’ approach, discussed in ref. [63] and, in connection with the present problem, in ref. [65]. It is our point of view that the ‘pointer state’ approach, via decoherence, is more general and probably more suitable for describing the actual situation encountered in ‘conscious perception’ by the brain. The latter is associated with the emergence of *almost classical* states as a result of *decoherence*, which evolve in time almost as classical objects, thereby realizing - for all practical purposes - the transition from quantum to classical worlds. The estimated time of such a decoherence is argued in this work to be of $O(1 \text{sec})$. As we have discussed above, if one insists on using state vectors, which may not be necessary, then the localization time of the state vector can be of the same order, although the latter result relies on some naturalness assumptions in string field theory. We should stress that the state vector in this case *does not* satisfy a simple Schroedinger equation due to the environmental effects, which convert the latter to a stochastic equation of Ito type (56) [41]. This is a formal difference from the approach of ref. [63], but all the ideas in that work, concerning the effects of decoherence in the transition from the quantum to classical, are perfectly consistent with our approach here, and also in ref. [15].

The stochastic framework described above, which is inherent in our Liouville approach to the concept of time, leads also to some other interesting properties of ensembles of one-dimensional systems, which might be of relevance to properties of MT networks that have been observed experimentally. We now briefly turn to some of them.

### 4.5 Possible connection with conformal models of disorder

The appearance of extra logarithmic divergencies on the world sheet as a result of instantons, discussed above and in Appendix A, which cannot be accounted for by a standard renormalization group flow of a given $\sigma$-model, has been interpreted in ref. [5] as expressing a *change* in conformal field theory describing matter back-reaction effects on the space-time metric as the string propagates in a black hole background.

What we shall argue below is that, from a formal point of view, such effects may be thought of as being associated with certain operators of the underlying conformal field theory with zero conformal dimension, which play a non-trivial rôle in a black hole background. Such a situation has been recently argued to be the case...
of certain conformal models of disorder in condensed matter physics [77]. Indeed, it has been shown in ref. [77] that in certain (1+1)-dimensional conformal models with disorder there are operators with degenerating conformal dimension in the so-called replica limit (employed to deal effectively with disordered systems). In this limit the central charge of the theory vanishes. Such operators disappear from the spectrum of the theory in the O.P.E. of two primary field operators of the conformal model, but they leave behind unexpected logarithmic divergencies in correlation functions, which accompany extra operators termed logarithmic operators [77].

If we view the black hole model as a completely integrable critical string \( c_{tot} = 26 \), then including the ghost contributions arising from a gauge-fixing of the world-sheet reparametrization invariance [61], the total central of the theory vanishes, and the theory is conformal. The Hamiltonian of the string is given by the world-sheet Virasoro operator \( L_0 \). The conformal weight (dimension), \( h_\alpha \), of a string state \( |E_\alpha > \) is defined as

\[
L_0 |E_\alpha > = h_\alpha |E_\alpha >
\]

and similarly for the antiholomorphic part (denoted by a bar).

Given that effects associated with a change in conformal field theory, such as back-reaction of matter on the structure of space-time etc, are purely stringy effects, the most natural way of incorporated them in a \( \sigma \)-model language is to go beyond a fixed genus \( \sigma \)-model and consider the effects of resummation over world-sheet genera. This is a very difficult procedure to be carried out analytically. However, for our purposes a sufficient analysis, which describes the situation satisfactorily (at least at a qualitative level), is that of resumming one-loop (torus) world-sheets. Non-trivial effects arise from degenerate Riemann surfaces, namely from long-thin world-sheet tubes that are attached to a Riemann surface of lower genus (sphere in this case). Such effects are similar to ‘wormholes’ in a four-dimensional quantum gravity treatment [73].

From a formal point of view, such insertions in a Riemann sphere are described by adding bilocal world-sheet operators on the world-sheet sphere [74]

\[
\mathcal{B}_{ii} = \int d^2z V_i(z) \int d^2w V_i(w) \frac{1}{L_0 + L_0 - 2}
\]

where the last factor represents the string propagator on the world-sheet cylinder, with the \( L \)'s denoting Virasoro operators. By inserting a complete set of intermediate string states [74], \( E_\alpha \), we can express the world-sheet ‘tube’ in (63) as an integral over the radius \( q \) of the tube’s cross section

\[
\sum_\alpha \int dq \frac{1}{q^{1-h_\alpha} q^{1-h_\alpha}} \{ E_\alpha(z_1) \otimes (ghosts) \otimes E_\alpha(z_2) \} \Sigma \otimes \Sigma'
\]
where \( h_\alpha, \bar{h}_\alpha \) are conformal dimensions of the states/fields \( E_\alpha \), i.e. \( L_0 E_\alpha = h_\alpha E_\alpha \). The sum in (64) is over states propagating along the thin tube, connecting the world-sheet pieces \( \Sigma \) and \( \Sigma' \) (in the case of the degenerating torus handle of interest to us, \( \Sigma = \Sigma' \)). The terms ‘ghosts’ indicate appropriate insertions of ghost fields. This has to do with the fact that the total central charge of the ghost and matter theory vanishes in the critical string model of ref. [74].

One easily observes that extra logarithmic divergencies in (64), not included in the perturbative \( \beta \)-function analysis on \( \Sigma \), and thereby capable of expressing quantum effects associated with a change in conformal field theories, may come from states with \( h_\alpha = \bar{h}_\alpha = 0 \). If such states are discrete in the space of states, i.e. they bear non-trivial contributions to the sum-over-states in (64), then will lead to divergencies in (63). These properties are obviously background dependent.

The bilocal term (63) can be written as a local world-sheet effective action term, if one employs the well-known trick of wormhole calculus [73] by writing

\[
e^{B_{ii}} \propto \int d\alpha e^{\alpha^2 + \alpha_1} f V_i
\]

and this results in a ‘quantization’ of the theory space \( \{ g' \} \) of the string. The requirement for a canonical quantization are met by the non-critical (Liouville strings) as discussed in detail in ref. [35]. The highly non-trivial point we wish to make is that in case non-trivial logarithmic divergencies appear their effect in the quantized renormalization-group flow will be to induce, by their absorption, renormalization-group-scale-dependent ‘widths’ in the distribution functions of families of theories (Gaussian wave-packets) in quantum theory space [76]. Notably such distribution functions, with widths that are time-dependent (since in our approach target-time is a quantum RG scale), have arisen naturally as ‘pointer’ states [65] in our (world-sheet-genus resummed) matrix-model approach [11] to the black hole problem. As we discussed above, such states played an important rôle in the transition from the quantum to the classical world, and hence to the conscious perception of the brain.

To understand better the physical situation, and support the conjecture that the black hole model has such zero modes, we compare our system to that of (critical) string theories propagating in non-trivial backgrounds of extended objects in target space (e.g. solitons, instantons etc), where the appearance of such non-trivial zero modes appears to be a generic feature [75]. For instance, consider the case of matter (particle) scattering off a monopole background in the target space of the string [75]. In this case, the recoil of the scatterer (monopole) cannot be described by a single conformal field theory, but by a change in conformal field theories (this is similar to our back-reaction in the black hole case). In the example of ref. [73], where one linearizes around a target-space monopole background, there are (BRST-null) zero modes in the spectrum of \( L_0 \), associated with the position of the monopole center of mass in target space, which breaks translational invariance. The relevant marginal
operators therefore are constructed appropriately out of Noether currents. Such operators are total derivatives on the world sheet (θ-terms)

\[ V_k \propto \partial_\alpha J_\alpha^k \otimes \text{(ghosts)} \]  

where the ghost insertions ensure vanishing conformal dimension of \( V_k \), and \( J_\alpha^k \) (with \( \alpha \) a world-sheet index) is a target-space translation Noether current, represented as a world-sheet object, corresponding to the translation

\[ X^k \to X^k + u^k \]  

Circulation of the ‘zero modes’ (66) along the long tubes (64) of pinched world-sheet surfaces, then, produces logarithmic divergencies [75]. Restoration of conformal invariance induces a recoil of the monopole so that the total target momentum of the system ‘particle + monopole (scatterer)’ is preserved. This expresses the physical process of the recoil of the monopole scatterer, which in this way cancels the divergent effects appearing in the scattering of the particle off the monopole.

A similar thing happens in the black hole case [5]. Scattering of propagating matter off the black hole results in a change of state of the black hole background [69]. Such a change cannot be described by a single conformal field theory, because it is a quantum effect in target space. In the black hole case at hand there are conformal-dimension zero operators in the spectrum, associated with the \( W_\infty \) global modes [52, 5]. Such modes express the ‘recoil’ of the black hole scattering center, during matter propagation (scattering) in a black hole background. Propagation of such modes along the thin tubes of degenerate Riemann surfaces (64) will, then, lead to divergencies.

Semi-classically such effects can be represented qualitatively by the world-sheet instantons, which in this way represent higher-genus as well as global mode effects [5]. Especially in the topological (twisted supersymmetric) model of the black hole singularity [5], such instanton deformations may be expressed as total derivative terms in moduli space, and in particular as BRST \( Q \)-exact states [49, 5], where \( Q \) is the BRST charge in a string language[8]. This total-derivative \( Q \)-exact form of the instanton deformations in the black hole model, describing ‘recoil'/back-reaction of the space-time itself, is suggestive enough to look for an analogy with the monopole-string case of ref. [75] discussed above. It is worthy of pointing out that the similarity of the black hole problem to that of a string soliton is not unrelated to the connection between such systems as a result of (non-perturbative) duality symmetries [78]. A more detailed study of these considerations is in progress [36].

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8The ghost fields in this model originate from the fermion fields of the \( N = 2 \) supersymmetry whose twisted version produces zero central charge [49, 5]. For more details on the construction, as well as properties of the instanton, anti-instanton deformations we refer the reader to the literature [49, 5].
Contact with conformal field theories of disorder is, therefore, made upon the observation that the presence of operators of zero conformal dimension in string models in non-trivial space-time backgrounds, whose propagation along degenerate handles produce extra logarithmic divergencies, also characterizes certain models of disorder in condensed matter physics. Obviously such operators in the disordered systems are similar to the zero modes of the soliton or black-hole string, mentioned above. Circulation of such operators along degenerate Riemann tubes (wormholes) will produce logarithmic divergencies not accounted for by the conventional Renormalization Group flow at a fixed genus. In our opinion, considering the effects of logarithmic operators in resummed world-sheet might also have relevance for the theory of disorder itself, as describing the recoil of the impurities. From a target-space quantum gravity point of view, disorder may exhibit itself as a vanishing vacuum-expectation-value of the metric tensor in target space $< G_{MN} > = 0$, which would be a topological (highly-symmetrical) phase of string theory (‘disordered’ phase). As the energy goes down, diffeomorphism invariance is broken and a non-zero $< G_{MN} >$ is achieved (‘ordered’ phase). As we discussed in ref. and mentioned above, a situation like this has been argued to arise in the topological $N = 2$ \(\sigma\)-model that arguably simulates the singularity of the black hole. This may be interpreted as implying that formation of black holes contributes to disorder. Subtleties concerning the precise meaning of an order parameter in our two-dimensional target space time, of relevance to our present approach to MT dynamics, have been discussed in ref. and are related to a Kosterlitz-Thouless (or ‘topological’) order/disorder phase transition in this case. Of course all these are speculations. To fully understand the phase transition one has to resort to string field theory, or to a complete theory of quantum gravity, something which at present is not possible.

From our point of view in this work, a connection of our quantum-gravity situation with the conformal field theory of disorder as a conformal-field-theory changing situation, if formally established, will be a very important step towards a mathematically rigorous formulation of the process of conscious perception as a ‘disorder/order’ process. The rôle of the external stimuli inducing microscopic black hole formation, and thereby disorder (as a back-reaction/recoil process) in the MT chains, may, then, stand a good chance of being formulated in a rigorous way, by exploiting the conformal structure of our stringy MT model, stemming from its complete integrability. We believe that our work in this article provides sufficient motivation for such studies, either in the continuum formalism of our model of MT, or preferably in the truly non-perturbative lattice (random surface)-numerical simulation approach, using triangulations of the Euclidean world-sheet surfaces employed in the model. In connection with the latter analysis, it is worthy of mentioning that our approach to target time, viewing the latter as a local dynamical (Liouville) scale on the world sheet, employs Euclidean world-sheets, and the target space Minkowski signature is an induced phenomenon in the supercritical effective string model (\(C > 25\)) of the propagating matter in an evaporating black
hole background. This is essential for numerical simulations, which employ triangulations of euclidean world-sheets. We hope to turn to these issues in the near future.

5 Possible Experiments to check on the above ideas

It is the purpose of this section to consider possible applications of the above formalism of MT as (completely integrable) non-critical (Liouville) strings in explaining or predicting properties of these systems under some situations that can be met experimentally in the laboratory.

5.1 Growth (Dynamical Instability) of a Microtubule Network and Liouville Theory

The above considerations are valid for MT networks whose size of individual MT is larger than a certain critical size. Kink-like excitations, that were argued to be crucial for the physics of the conscious functions of the brain, cannot form for small microtubules. The question, therefore, arises whether the non-critical effective string theory framework described above is adequate for describing the growth process associated with the formation of a MT network. This phenomenon is physically and biologically very interesting since these structures are known to be the only ones so far that exhibit the so-called ‘dynamical instability growth’. This is an out-of-equilibrium process according to which an individual MT can switch randomly between an ‘assembly state’ (+), in which the MT grows with a speed $v_+$, and ‘diassembly state’ (−), in which the MT shrinks with velocity $v_-$. Recently there have been attempts to construct simple one-dimensional theoretical models with diffusion that can describe qualitatively the above phenomenon. An interesting feature of these models, relevant to our framework, is that for a certain range of their parameters exhibit a phase transition to an unbounded growth state. In the case of MT networks, it is known experimentally that a ‘sawtooth’ behaviour in the time-dependence of the size of a MT (Fig. 3), occurs as a result of hydrolysis of

\[ e^{-N}, \text{ where } N \text{ is the size of the system, thereby implying spontaneous symmetry breaking in the thermodynamic limit where } N \to \infty. \]
GTP nucleotides bound to the tubulin proteins (i.e. the transformation GTP $\rightarrow$ GDP $\rightarrow$ GDP), The hydrolysis is responsible for providing the necessary free energy for the conformational changes of the tubulin dimers: the dynamical instability phenomenon pertains to polymerization of the GTP tubulin, while the GDP tubulin stays essentially unpolymerized. In view of quantum oscillations between the two conformations of the tubulin, and the above different behaviour of polymerization, it is natural to conjecture that quantum effects may play a rôle in the MT growth process.

Thus, our Liouville theory representation of the effective degrees of freedom $u$ involved in the model of [19], invented to explain classical aspects of the hydrolysis of GTP $\rightarrow$ GDP, might, in principle, be able of explaining qualitatively the ‘sawtooth’ behaviour of Fig. 3, even before the formation of kinks. To this end, we remark that in Liouville dynamics, with the Liouville scale identified with the target time $\mathfrak{t}$, the inherent non-unitarity (in the world-sheet) of the Liouville mode implies that the central charge $C$ of the theory flows with the scale in such a way that near fixed points it oscillates a bit before settling down. Indeed, for a non-critical string with running central charge $C[g,t]$, $t$ is the Liouville scale/time, the following second order equation (local in target space-time) holds near a fixed point of the Renormalization Group Flow $^{38}$

$$\ddot{C}[g,t] + Q[g,t] \dot{C}[g,t] \leq 0 \text{ for } C \geq 25 ; \quad Q^2[g,t] = \frac{1}{3}(C[g,t] - 25) \quad (68)$$

This is a local phenomenon in target space-time. Globally, there is a preferred direction in time along which the entropy of the system increases $^{13}$. $^{5}$.

The small oscillations of $C$ in (68) may be attributed to the double direction of Liouville time that arises as a result of imaginary parts (dynamical instabilities) appearing due to world-sheet regularization by analytic continuation of non-critical string correlation functions $^{56}$. (Fig. 4) This point is discussed briefly in Appendix A.

Along each direction of Liouville time in Fig. 4 there is an associated variation of $Q[g,t]$ and $C[g,t]$, and the volume of the ‘one-dimensional universe’ of MT either increases or decreases $^{38}$. Thus, as a result of the oscillations of the Liouville dynamics $^{38}$ a ‘sawtooth’ behaviour of MT, which expresses the result of polymerization of GTP tubulin, can be qualitatively explained within the framework of non-critical Liouville dynamics. The analogy of the above situation to a stochastic expansion of the Universe (inflation) in non-critical string theory as discussed in ref. $^{35}$ should be pointed out.

It should be noted at this stage that in this effective framework the origin of non-criticality of the subsystem of tubulin dimers is left unspecified. Quantum Gravity
fluctuations appear on an equal footing with the environment of the nucleating solvant that surrounds the MT in their physical environment. The distinction can be made once a detailed description of the environment is given, which of course would specify the form of the target metric coupled to the matter system of tubulins. As we have discussed in previous sections, in the quantum Gravity case this is achieved by the exactly marginal operators of the \( SL(2, R)/U(1) \) conformal field theory that describes space-time singularities in one-dimensional strings \[^{[23]}\]. The latter involve global (non-propagating) string modes which cannot be detected by localized scattering experiments \[^{[5]}\]. On the other hand, it seems likely that an exact conformal field theory that describes the nucleating solvant does not exist. However, the effective Liouville string that describes the embedding of a MT in it, and the associated environmental entanglement, is obtained by a simple Liouville dressing of the model discussed in section 3. The information about the environment is hidden in the form of the target metric that couples to the system. The fact that both the formation of an MT via the dynamical instability phenomenon, and the quantum gravity effects on MT, involve conformational changes of the tubulin supports the above point of view. Of course the strength of the dynamical instability in case the latter is due to quantum gravity fluctuations will be much more suppressed as compared to the hydrolysis case. In that case, there will not be sufficient time for complete polymerization of the GTP tubulin conformation. In such a case one would probably expect ‘sudden’ ‘sawtooth’ peaks of the tip of the MT whose magnitude will be affected by the order of quantum gravity entanglement. The growth in this case will be bounded.

It should be mentioned that recently there have been some experiments claiming such length fluctuations \[^{[82]}\], in the case of carbon nanotubes. The authors of ref. \[^{[82]}\] claimed that they observed such changes in the length of the tubes, which they attributed to sudden jumps of the respective wavefunctions, according to the approach of Ghirardi Rimini and Weber \[^{[83]}\]. In this respect, one should think of repeating the tests but with isolated MT \[^{[84]}\]. We should stress, however, that the above discussion is at this stage highly speculative and even controversial, given that there appear to be conventional explanations of this phenomenon in carbon nanotubes \[^{[85]}\]. Of course such conventional explanations do not exclude the possibility of future observations of quantum-gravity induced bounded growth in MT, along the lines sketched above.

We close this section by mentioning that in realistic situations the growth process of an MT network is not unlimited. After a critical length is exceeded, the growth is saturated and eventually stops. The formation of kink excitations \[^{[12]}\] might be important for this auto-regulation of the MT growth \[^{[13]}\]. For more details on the conjectural rôle of the kinks in this growth-control mechanism we refer the reader to the literature \[^{[13, 80]}\].
The above situation should be compared with the limitations on the (stochastic) Universe expansion in the non-critical-string-driven inflationary scenario of ref. [35]. There, it can be shown that within our framework of identifying the Liouville field with the target time, the average density $\langle\delta_s\rangle \equiv Tr(\rho\delta_s)$ of the non-critical strings that drive the inflationary scenario obey an equation of the form [36, 35]

$$\partial_t \langle\delta_s\rangle = -aQ \langle\delta_s\rangle + bQ^3 \langle\delta_s\rangle$$

where $a, b$ are positive quantities, computable in principle in the Liouville-string framework [35]. The first term in (69) is due to the (exponential) expansion of the string-Universe volume, and the second term corresponds to the regeneration of strings via breaking of large strings whose size exceeds that of the Hubble horizon [35]. This second term comes from the diffusion due to the non-quantum mechanical terms in the equation (35). At early times the diffusion term balances the string depletion effects of the first term and the uniform density condition for inflation (universe exponential expansion) is satisfied. As the time elapses, however, the depletion term in (69) dominates, the Universe’s expansion is diminished gradually, and eventually stops. This is the case when the non-equilibrium non-critical string approaches its (critical-string) equilibrium state. Hence, in our case one may view (69) as an effective model for the temporal evolution of the density of tubulin dimers. Then, one can understand, at least qualitatively, the above limitations of the MT growth process by the presence of the kinks, since the latter can be associated with an equilibrium ground state of the effective string theory describing a MT.

5.2 Other Experiments sensitive to Gravity

The possibility that a (quantum) theory as weak as gravity affects the physics of low-energy systems, like MT networks, does not seem so remote, if we recall some recent experimental indications [87] about an appreciable sensitivity exhibited by MT structures to classical gravity effects, and more general to weak external fields. In such experiments, gravity effects can lead to a sort of symmetry breaking and pattern formation in assemblies of MT. Although theoretically the situation is still very vague, however the above phenomena appear consistent with predictions based on reaction-diffusion theories [88], involving out-of-equilibrium chemical reactions coupled to gravity. As we have argued in ref. [5] quantum gravity in non-critical string theory can be viewed as such an out-of-equilibrium theory, resulting in an irreversible flow of time and entropy production at a fundamental (string) energy scale. Whether MT systems, whose (quantum) physics scale is that of electroweak effects, are senstitive to quantum gravity effects, as opposed to classical ones, still remains to be seen. The idea does not seem so absurd if we draw an analogy with what happens in the neutral kaon system in particle physics [89]. There, violations of quantum mechanics, associated with quantum gravity effects of order $O(G_N^{1/3}m_K) \approx 10^{-19}$, could be on the verge of being observed experimentally in CPLEAR or DAφNE facilities [88].
In our picture, it becomes clear from the formal discussion in Appendix A (c.f. \cite{92,94}), that the effects that could lead to non-factorisability of the target $S$-matrix, and therefore to quantum gravity environmental entanglement, are suppressed by a single power of $M\text{String} = (\sqrt{\alpha'})^{-1}$, and hence the possibility of such terms having observable effects may not be negligible.

This prompted us to identify above the scale $\alpha'$, which appears as a typical scale of the MT system represented as a string (\cite{11}), with the quantum gravity scale, $M_{\text{Planck}} \simeq 10^{19} \text{ GeV}$ (up to an uncertainty factor of ten, see discussion above). This yields estimates of the collapse time of order $\mathcal{O}(1\text{sec})$, in brain MT networks consisting of $10^{12}$ MT dimers, which is in excellent agreement with estimates of times for conscious perception obtained by neurobiologists, based on completely different methods. This offers support to the above ideas, and to the ideas of ref. \cite{3,2,4}, that realistic quantum gravity effects may play an important role in conscious perception.

However, we should bear in mind that in the case of systems pertaining to the function of the brain things are by no means simple. The simple fact that the collapse time, calculated on the basis of string quantum gravity here, or conventional quantum gravity \cite{2} (provided that the latter exists as a mathematical theory), is in agreement with estimates of conscious perception time obtained by quite different methods, although a pleasant indication, however it by no means constitutes a proof of the relevance of quantum mechanics or quantum gravity on brain function. ¿From our point of view, such a proof could come from observations of fluctuations (‘quantum jumps’) of the length of isolated microtubules. This is correlated strongly with the stochastic growth or saw-tooth behaviour of the length of a MT assembly, discussed in the previous subsection. When applied to an individual MT this approach may lead to instabilities that could predict fluctuations of the tip of MT due to quantum gravity entanglement \cite{84}.

Whether experiments can be devised, which are sensitive enough to capture such microscopic fluctuations of isolated (cold) MT, is not known to us. We believe, however, that if they could be devised, they would constitute the best proof of the relevance of quantum (gravity) effects for brain functioning, provided of course that any other conventional source of mechanical instabilities \cite{85} is excluded. For instance, the stochastic/diffusive nature of Liouville gravity, advocated in ref. \cite{5}, encourages a comparison with the situation of ref. \cite{87} and, in general, with experiments testing predictions of reaction-diffusion theories. One is tempted to conjecture that the quantum fluctuations of the tip of MT structures, predicted here in the framework of Liouville theory, might also be seen in the pattern formation of the experiments of ref. \cite{87}, provided that the latter are repeated in ‘cold’ environment so as to minimize noise due to (thermal) dissipation or other mechanical instabilities \cite{85}, that could interfere with pure quantum gravity effects. Whether this is possible, or even conceivable as an idea for future research, is unknown to us at present, but we believe that such speculations deserve closer attention.
6 Memory Coding and Capacity of the brain as a (non-critical string) dissipative system

6.1 Existing local field theory models

Irrespective of the possibility of proving experimentally the possible effects of quantum gravity on brain function, the conjecture of this work that MT dynamics stems from one-dimensional Ising spin chains in the brain, that can be represented as a (completely integrable) non-critical string model admitting space-time singularities, implies certain peculiar but highly interesting properties of brain functioning, associated with non-equilibrium (dissipative) temporal evolution. The latter implies an irreversible arrow of time, evolution of pure states into mixed ones, and, more generally, what a particle field theorist would call \( CPT \) violation \(^5\).

In this respect, our model has many things in common with dissipative (local field theory) models of ref. \(^{24, 26}\) in an attempt to construct realistic models for memory capacity. Below we shall briefly review such models, and discuss the possible advantages of our (non-critical string) approach over such local field theory approaches to brain function, especially as far as memory coding and capacity are concerned.

In conventional brain models \(^{24}\), based on local field theories, the kind of symmetry assumed is that of rotational electric dipole symmetry of the surrounding water molecules. The quantum numbers associated with the latter constitute a certain class of code numbers. If the brain lies on a specific ground state, which implies spontaneous breaking of the dipole rotational symmetry, in order to reach any other ground state corresponding to a new code number it would require a sequence of phase transitions that would destroy the previously stored information, a procedure known as \textit{overprinting} \(^{24, 26}\).

A way out of this problem of \textit{memory capacity} would be to increase the symmetry of the problem to the one with huge dimensions \(^{23}\). In a local field theory this cannot be done without destroying the practical use of the model. The problem is analogous to that of how to incorporate the huge entropy of a macroscopic black hole in an information theory framework within a local field theory setting. This again would require an enormous amount of black hole degrees of freedom to account for the macroscopic entropy, which would be hard, if not impossible, to reconcile with the finite number of degrees of freedom existing in a local field theory.

An alternative approach, is that of ref. \(^{26}\), making use of \textit{dissipative} models for brain function, within a local field theory framework. As observed in ref. \(^{26}\), the
doubling of degrees of freedom which appears necessary for a canonical quantization of an open system in a dissipative environment [24], is essential in yielding [90] a non-compact $SU(1,1)$ symmetry for the system of damped harmonic oscillators, used as a toy example for simulating quantum brain physics. The quantum numbers of such a system are the $SU(1,1)$ isospin and its third component, $j \in \mathbb{Z} \frac{1}{2}, m \geq |j|$. The memory (ground) state corresponds to $j = 0$ and there is a huge degeneracy characterised by the various coexisting (infinite) eigenstates of the Casimir operator for the $SU(1,1)$ isospin. The open-character of the system introduces a time arrow which is associated with the memory printing process and is compatible with the ‘observation’ that ‘only the past can be recalled’ [26]. As far as we can see, the problem with this approach is that it necessarily introduces dissipation in the energy functional, through the non-hermitian terms in the interaction hamiltonian between the subsystem and the environment [24, 26]. Hence, it is not easy to see how to reconcile this with the above-mentioned property of biological systems to transfer energy without dissipation across the cells [8, 9]. Moreover, from our point of view, this approach cannot take into account realistic quantum gravity effects, which according to the hypothesis of the present work and of refs. [3, 2, 4] are considered responsible for conscious perception.

6.2 Advantages of a ‘stringy’ representation of brain models

String theory seems to provide a way out of these problems [5] due to the infinite-dimensional gauge stringy symmetries that mix the various levels. In the (completely integrable) black hole model of ref. [23], which is used to simulate the physics of the MT, there is an underlying world-sheet $SL(2,\mathbb{R})$ symmetry of the $\sigma$-model, according to which the various stringy states are classified. The various states of the model, including global string modes characterised by discrete values of (target) energy and momenta, are classified by the non-compact isospin $j$ and its third component $m$, which - unlike the compact isospin $SU(2)$ case - is not restricted by the value of $j$. Thus, for a given $j$, which in the case of string states plays the role of energy, one can have an infinity of states labelled by the value of the third isospin component $m$. All such states are characterised by a $W_{1+\infty}$ symmetry in target space. As we mentioned above, this symmetry is responsible for the maintenance of quantum coherence in the presence of a black hole background [3], in the sense of an area-preserving diffeomorphism in a matter phase-space of the two-dimensional target space theory. It should be noted that such area-preserving symmetries, as spectrum generating algebras, also appear in connection with the excitations of planar quantum Hall systems having non-degenerate ground states [24]. So, it should not be considered as a surprise that such symmetries appear in our two-dimensional spin chain model for the brain MT. In the particular case of two-dimensional string black holes there is even a formal analogy with quantum Hall models, as argued in ref. [92].
In ref. [5] it has been argued that such symmetries are responsible for an ‘infinite-dimensional’ quantum hair (W-hair) of the two-dimensional black hole, which consists of (conserved) quantum (global) charges, like the ADM mass \[23, 50\], characterising a black hole space-time even asymptotically, i.e. after evaporation. Such hair would induce a huge degeneracy in the ground state of the system that could lead to the solution of the problem of memory capacity. From a formal point of view, the rigorous existence proof of such conserved charges would be the explicit construction of exactly marginal deformations that correspond to turning on the above charges. The exactly marginal character of the deformations is required in order to maintain conformal invariance of the world-sheet \(\sigma\)-model and thus stable ground state of the string.

As we mentioned in section 4.1, for the black hole model of ref. \[23\], used to simulate also the physics of the MT dynamics, it is possible to construct \[52\] the exactly marginal deformation corresponding to the lowest non-trivial charge, which is the ADM mass of the black hole. From a \(W_\infty\) symmetry point of view, this would be the charge associated with the spin-two part of the target space spectrum, i.e. the stress-energy tensor of the black hole. There is a huge degeneracy of the ground state of the system which is due to the existence of known exactly marginal deformations that are responsible for changing continuously the ADM mass of the black hole, as discussed in section 4.1. In the notation of ref. \[52\], such deformations are denoted by \(L_0^2\), as we mentioned previously. Their coupling constant, which is a free parameter of the string model, shifts the ADM mass of the black hole space-time. The above operator turns on only backgrounds corresponding to the (discrete) higher-level string states that do not propagate in space-time. The ground state of such models consists of turning on backgrounds corresponding to matter propagating states. Such backgrounds are turned on by the other known exactly marginal deformation \(L_0^1\), which mixes the propagating states (belonging to the lowest string level) with an infinity of higher-level string global states. Both operators owe their existence to the target space \(W_{1+\infty}\)-spectrum-generating algebra of the black hole space-time \[3, 22\]. The latter is broken explicitly by ‘measurement’ by local scattering experiments or in general by operations that are performed within localised regions of space-time, such as those taking place in the conscious part of the brain.

Such a procedure will integrate out the global degrees of freedom, leaving only an effective (string) theory of propagating degrees of freedom in a black hole background space time. For each matter ground state of a propagating degree of freedom, say the zero mode of the massless field corresponding to the static “tachyon” background of ref. \[23\], with \(SL(2, R)\) quantum numbers \(j = -\frac{1}{2}, m = 0\), there will be an infinite degeneracy corresponding to a continuum of black hole space-time backgrounds with different ADM masses. These backgrounds are essentially generated by adding various constants to the configuration of the dilaton field in this two-dimensional string
theory \[23, 22\]. It should be noted here that the infinity of propagating “tachyon” states (lowest string mass-level (massless) states), corresponding to other values of \(m\), for continuous representations of \(j\), constitute excitations about the ground state(s), and, thus, they should not be considered as contributing to the ground state degeneracy. In principle, there may be an additional infinity of quantum numbers corresponding to higher-level \(W\)-hair charges of the black hole space time which are believed \[1\] responsible for quantum coherence at the full string theory level.

6.3 Memory Coding and capacity of the ‘stringy’ MT model

Taking into account the conjecture of the present work, that formation of virtual black holes can occur in brain MT models, which would correspond to different modes of collapse of pulses of the displacement field \(\psi\) defined in \(8\), one obtains a system of coding that is capable to solve in principle the problem of memory capacity. Information is stored in the brain in the following sense: every time there is an external stimulus that brings the brain out of equilibrium, one can imagine an abrupt conformational change of the MT dimers, leading to a collapse of the pulse pertaining to the displacement field. Then a (virtual) black hole is formed leading to a spontaneous collapse of the MT network to a ground state characterised by say a special configuration of the displacement field \((j, m)\). This ground state will be conformally invariant, and therefore a true vacuum of the string, only after complete evaporation of the black hole, which however would keep memory of the particular collapse mode in the ‘value’ of the constant added to the dilaton field, or other \(W\)-charges. This reflects the existence of additional exactly marginal deformations, consisting of global modes only, that are not directly accessible by local scattering experiments, in the context of the low energy theory of propagating modes (displacement field configurations \(\psi\)). In such a case, the resulting ground state will be infinitely degenerate, which would solve the problem of memory capacity\[10\].

Breaking of this degeneracy, can be achieved by means of an external stimulus, which is believed to be due to a weak field \[21\]. For instance, following the suggestion of ref. \[21\], we may imagine that an external weak field produces a spontaneous breaking of the electric dipole rotational symmetry in the water molecules, resulting

\[10\]It should be noted that the picture of the formation and evaporation of the two-dimensional black holes (foam) \[3\], that we advocate in this work in connection with concious brain processes, is not unrelated to the recently developed approach to space-time foam in the context of four-dimensional quantum string theory \[22\], employing duality symmetries. The latter help in making exactly solvable a strongly-coupled problem, like the formation and evaporation of a (virtual) space-time singularity in string theory, by mapping it to a weakly-coupled \(D\)-brane theory. In this picture, virtual black holes in string loops are believed responsible for the transition among string vacua, a process which in our two-dimensional case case would correspond to the evaporation of a black hole via world-sheet instanton effects, as discussed in section 4. However, we should stress that our approach employs time dynamically through the non-criticality of the strings involved, something which had not been considered so far in the approach of ref. \[22\].
in a ‘lasering’ \(^{22}\) of the environmental surroundings of the MT system (excitation of coherent dipole quanta). Such an excitation of collective modes results in a specific code characterising the ground state, as we mentioned above. The so selected ground state of the ordered water molecules affects the MT chains, due to the friction coupling \(\rho\) in (11), (18). As becomes clear from the analysis of section 2 ((11)-(14)), the effects of the environment are described by selecting a specific value for the vacuum expectation value (condensate) of the dilaton field in our suggested stringy approach to MT dynamics. The other \(W\)-charges (moduli) may also be selected this way, which we believe corresponds to the memory printing process, i.e. storage of information by a selection of a given ground state. A new information would then choose a different value of the dilaton field or other \(W\)-hair charges, etc. This provides a new and satisfactory mechanism of memory recall in the following sense: if a new pulse happens to correspond to the same set of (conserved) \(W\)-hair moduli configurations \(^{4}\), then the associated virtual black hole will be characterised by the same set of quantum hair, and then the same memory state is reached asymptotically (process of ‘memory recall’). The discussion we gave in the previous section about the rôle of world-sheet instanton deformations in shifting the \(ADM\) mass of a black hole \(^{14}\), while keeping memory of the dilaton v.e.v., finds a natural application in this coding process. Moreover, the irreversible arrow of time, endemic in Liouville string theory \(^{5}\) explains naturally why “only the past can be recalled” \(^{26}\).

To understand why the above process leads to a special coding, and how time reversal is spontaneously broken, as a result of this coding, it is sufficient to recall our discussion above, according to which in the presence of a space-time foamy environment, characterised by the virtual appearance and evaporation of black holes, there is a coupling of global modes to the propagating modes. As a result of the exactly marginal character of the deformations \(^{1,2}\), which thus respect conformal invariance at a string level, the environmental global modes match in a special way with the propagating mode \(j = -\frac{1}{2}\) \(m = 0\), which is the zero mode of the (massless) tachyon corresponding to the tachyon background of a two dimensional black hole which constitutes the ground state or memory state of our system. This is a special coding which were it not for the infinite degeneracy of the black hole space time would lead to a restricted memory capacity\(^{11}\).

We cannot resist in pointing out that the existence of such coded situations in memory cells bears an interesting resemblance with DNA coding, with the important difference, however, that here it occurs in the model’s state space. In this context, we note that the genetic diversity is not due to an infinite number of nucleotide types, since in nature there are only four of them, paired by two (\(A = T/G \equiv C\)), but rather to a macroscopically large number of existing combinations in the DNA

\(^{11}\)It should be noted, once more, that the various other (infinite) states corresponding to continuous representations of the \(SL(2, \mathbb{R})\) symmetry that pertain to various tachyon modes do not constitute memory states, because, as mentioned earlier, they are just excitations about the ground state.
helix. Similarly, for the extremely rich (macroscopic) memory capacity in our stringy MT model, we may not need the full (infinite) set of the $W$-hair charges, but just the dilaton vev $\langle \Phi \rangle$ may be sufficient as a ‘collective’ mode. The latter is, as we mentioned earlier, related to external stimuli through the equations (11)-(14).

Before closing, we consider it as useful to compare once more our results with those of ref. [26], using a dissipative system approach to the brain memory problem, but within a local field theory framework. In both models, the effect of the ‘environment’ was crucial in providing an infinity of degrees of freedom, capable of solving the memory capacity problem. The crucial difference of our string case is that the ground state of the string system, which is conformally invariant, is actually a state with given quantum numbers $j = -\frac{1}{2}, m = 0$ of the $SL(2, R)$ isospin in the asymptotically flat space time case. The degeneracy occurs, as we have already mentioned, as a result of the ‘existence of an environment’ of global modes, inaccessible by local scattering operations of the brain, which lead to exactly marginal deformations shifting the ground state value of the dilaton field, or in general leading to an infinity of $W$-hair charges. Moreover, the effects of the environment in the string case, as contrasted to that of ref. [21], are such that energy is conserved on the average, thereby making the connection with energy-loss-free energy transport across biological structures more apparent. The importance of the coupling between global and localized (propagating) modes of this (1+1)-dimensional string theory lies on the fact that it leads to a time arrow for specifically stringy reasons [5]. Essentially, time arrow emerges as a result of loss of information, carried by the global modes which are inaccessible to localized scattering processes, such as the ones taking place in the brain. From a target space-time point of view, such an information loss is manifested through entropy production and non factorisability (c.f. eq. (92) of Appendix A) of the superscattering operator [3].

7 Conclusions

In this work we have argued, following general ideas in refs. [3, 2], that in certain parts of the brain there is the possibility of the formation of quantum superpositions which extend over reasonably large distances and time scales. Such superpositions imply probably that brains cannot be understood as classical computers. We have made attempts to locate a possible microscopic mechanism for the formation of such superpositions, and argued that a natural place for their appearance are the networks of Microtubules (MT) that exist in the brain. Moreover we have considered in some detail the rôle of quantum gravity environmental effects in destroying such
quantum-coherent superpositions (collapse of the wave function), thereby leading to conscious perception.

We have presented an effective model for the simulation of the dynamics of the tubulin dimers in the brain. We have used an effective \((1 + 1)\)-dimensional string representation to study the dynamics of a detailed mechanism for energy transfer in the biological cells. We argued how it can give rise to a large-scale coherent state in the dimer lattice. Such a state is obtained from quantization of kink solitonic states that transfer energy through the cell without dissipation. The quantization became possible through the freedom that string theory offers, enabling one to cast dynamical problems with friction in a Hamiltonian form.

The collapse phenomenon in our approach does not require the existence of a wave-function, and it is induced by the formation of microscopic black holes (singularities) in the effective one-dimensional space-time of the tubulin chains. This is achieved by the dynamical collapse of pulses of the displacement field of the MT dimers. The pulses are a result of abrupt conformational changes \((\alpha \leftrightarrow \beta)\) that sufficiently distort the surrounding space-time. In this sense, the situation is similar but not identical, to the ‘sudden hits’ that a particle’s wave function suffers occasionally (fixed ‘by hand’ to occur every \(10^8\) years) in the model of quantum measurement of Girardi, Rimini and Weber \[83\]. However, contrary to these conventional theories, our stringy approach to gravity-induced collapse \[5\] incorporates automatically an irreversible flow of time for specifically stringy reasons, and energy conservation, while it provides a dynamical determination of the decoherence or collapse time, depending on Newton’s constant \(G_N\) and the ‘energy’ content of the system.

When applied to the model of MT, our approach implies a collapse time of \(\mathcal{O}(1\,\text{sec})\), which is obtained by the interaction of a tubulin dimer with a fraction of \(10^{-7}\) of the total number of tubulin dimers in the brain. This number is fairly close to the fraction of the brain that neuroscientists believe responsible for human perception. This is a very strong indication that the above ideas, although speculative at this stage, might be relevant for the discovery of a physical model for consciousness and its relation to the irreversible flow of time. In addition, our model predicts damped (microscopic) quantum-gravity-induced oscillations of the length of isolated MTs, which are due to the different properties of the two tubulin conformations under polymerization (phenomenon of bounded dynamical-instability growth).

There are certain formal aspects of our effective model, namely its two-dimensional structure and its complete (quantum) integrability, that might turn out to be important features for the construction of realistic soluble models for brain function. The quantum integrability is due to generalized infinite-dimensional symmetry structures (\(W\)-symmetries and its generalizations) which are strongly linked with issues
of quantum coherence and unitary evolution in phase-space. Moreover we have seen that due to such huge stringy symmetries there is an automatic solution of the memory coding and capacity problems, which some local field theories models of the brain are plagued with. Such symmetries are related to global non-propagating modes of the effective string theory, which do not decouple from the propagating (observed) modes in the presence of (microscopic) space-time singularities. It will be interesting to understand further the physical rôle of such structures in the models of MT. At present they appear as an environment of fundamental string modes that are physical at Planck scales. However, such structures, may admit a less-ambitious physical meaning, associated with fundamental biological structures of the brain. We should stress that the entire picture of non-critical string we have described above, which is a model-independent picture as far as environmental operators are concerned, could still apply in such cases, but simply describing purely biological environmental entanglement of the conscious part of the brain, the latter still being described as a completely integrable model.

In this respect, a possible connection of the brain MT system with conformal models exhibiting disorder has also been anticipated, which might be a key feature for a rigorous formulation of the conscious perception process from a conformal field theory point of view. This has been associated with the fact that the formation and evaporation of the microscopic black holes in the MT chains, which are the result of external stimuli, induce disorder due to back-reaction/recoil effects of matter (displacement field of MT) on the effective two-dimensional space time of the chains, expressing environmental entanglement. This picture is in general agreement with the recently developed picture of space time foam in four-dimensional string theory, employing duality symmetries to map the strongly-coupled system of (virtual) black hole singularities to well-behaved weakly coupled D-brane theories [78].

We believe that our work, although admittedly speculative at this stage, motivates further studies along the above directions which might prove useful towards a possible understanding of physical processes governing brain functions. In our opinion, the most realistic and profitable approach one could follow would be that of lattice simulations of such systems. This will allow for a rigorous study of finite size effects, which have been neglected in the present continuous formalism. Such effects may play an important rôle in determining the limitations of the systems as quantum computers, as far as properies such as: memory capacity, coding, and even the formation of MT structures themselves, are concerned. We hope to come back to such issues in the future.

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Appendix A

Extracts from Non-Critical (Liouville) String Theory and Time as the Liouville scale

In this appendix we comment on the non-factorizability of the induced target-space $S$-matrix for matter scattering in a non-conformal string background. This reflects information leakage as a result of the non-critical character of the string. Although for our purposes, primarily, we shall be interested in a specific string background, that of a stringy black hole, however in this section our discussion will be kept as general as possible with the aim of demonstrating the generality of our scheme.

Consider a conformal field theory on a two-dimensional world sheet, described by an action $S[g^*]$. The $\{g^*\}$ are a set of space-time backgrounds. The theory is perturbed by a deformation $V_i$, which is not conformal invariant

$$S[g] = S[g^*] + \int d^2z g^i V_i$$

(70)

The couplings $g^i$ correspond to world-sheet renormalization group $\beta$-functions

$$\beta^i = (h_i - 2)(g^i - (g^*)^i) + c^i_{jk}(g^j - (g^*)^j)(g^k - (g^*)^k) + \ldots$$

(71)

expressing the scale dependence of the non-conformal deformations. The operator product expansion coefficients are defined as usual by coincident limits in the product of two vertex operators $V_i$

$$\text{lim}_{\sigma \to 0} V_j(\sigma) V_k(0) \simeq c^i_{jk} V_i(\frac{\sigma}{2}) + \ldots$$

(72)

where the completeness of the set $\{V_i\}$ is assumed.

Coupling the theory (70) to two-dimensional quantum gravity restores the conformal invariance at a quantum level, by making the gravitationally-dressed operators $[V_i]_\phi$ exactly marginal, i.e. ensuring the absence of any covariant scale dependence with respect to the world-sheet metric $\gamma_{\alpha\beta}$. Below we simply outline the basic results, used in our approach here.

One rescales the world-sheet metric

$$\gamma_{\alpha\beta} = e^{\phi} \hat{\gamma}_{\alpha\beta}$$

(73)

with $\hat{\gamma}$ is kept fixed, and then one integrates over the Liouville mode $\phi$. The measure of such an integration can be expressed in terms of the fiducial metric $\hat{\gamma}$ by means of a determinant which is the exponential of the Liouville action. The final result for the gravitationally-dressed matter theory is then

$$S_{L-m} = S[g^*] + \frac{1}{4\pi \alpha'} \int d^2z \{\partial_\alpha \phi \partial^\alpha \phi - QR(2) + \lambda^i(\phi)V_i\}$$

(74)
where $\alpha'$ is the Regge slope for the world-sheet theory (inverse of the string tension). The gravitational dressing of the operators follows from the requirement of restoring the conformal invariance of the theory, at any given order in the coupling-constant expansion. For instance, to order $O(g^2)$ the gravitationally-dressed coupling $\lambda(\phi)$ are given by [59, 93]:

$$\lambda^i(\phi) = g^i e^{\alpha_i \phi} + \frac{\pi}{Q \pm 2\alpha_i} c^i_{jk} g^j g^k \phi e^{\alpha_i \phi} + \ldots$$

(75)

with

$$Q = \sqrt{\frac{|25 - c|}{3}}; \quad \alpha_i^2 + \alpha_i Q = \text{sqn}(25 - c)(h_i - 2)$$

(76)

and $c$ is the (constant) central charge of the non-critical string. From the quadratic equation for $\alpha_i$ only the solution

$$\alpha_i = -\frac{Q}{2} + \sqrt{\frac{Q^2}{4} - (h_i - 2)}$$

(77)

for $c \geq 25$, is kept due to the Liouville boundary conditions.

In ref. [5] we made an extra assumption, as compared to the above standard Liouville dynamics. We identified the field $\phi$ with a dynamical local scale on the world sheet. This induces extra counterterms in the world-sheet renormalized action. Consistency of the scheme required that the Liouville $\beta$ functions are identical with the flat space renormalization coefficients upon the replacement $g^i \rightarrow \lambda(\phi)^i$.

The type of operators that we are interested in this work, are such that $h_i = 2$ but $c^i_{jk} \neq 0$. In the language of conformal field theory this means that these operators are $(1, 1)$ but not exactly marginal. From (75), then, one obtains the simple relation

$$\frac{d\lambda^i(\phi)}{dt_p} = \beta^i$$

(78)

where the time $t_p$ is related to the Liouville mode $\phi$ as

$$t_p = -\frac{1}{\alpha Q} \ln A \quad ; \quad A \equiv \int d^2 z \sqrt{\gamma} e^{\alpha \phi(z, \bar{z})} \quad ; \quad \alpha = -\frac{Q}{2} + \frac{1}{2} \sqrt{Q^2 + 8}$$

(79)

with $A$ the world-sheet area. In the local scale formalism of ref. [5] $Q$ is given by

$$Q = \sqrt{\frac{|25 - C[g, \phi]|}{3}} + \frac{1}{2} \beta^i G_{ij} \beta^j$$

(80)

where $C[g, \phi]$ is the Zamolodchikov $C$-function [13], which reduces to the central charge $c$ at a fixed point of the flow. The extra terms in (80), as compared to (79), are due to the local character of the renormalization group scale [5]. Such terms may always be removed by non-standard redefinitions of $C[g, \phi]$. The quantity $G_{ij}$ is related to divergencies of the two-point functions $\langle V_i V_j \rangle$ and hence to Zamolodchikov metric [13, 5].
From the renormalization-group structure (75) one obtains close to a fixed point

\[ \ddot{\lambda} + Q \dot{\lambda} = -\beta = -G^{i\bar{j}} \partial_i C[\lambda, \phi] \]

(81)

where the dot denotes differentiation with respect to the Liouville local scale \( \phi \).

For the \( C[g, \phi] \) (local in target space-time) one obtains near a fixed point

\[ \dot{C}[g,t] + Q[g,t]C[g,t] \leq 0 \text{ for } C \geq 25 \; ; \; Q^2[g,t] = \frac{1}{3}(C[g,t] - 25) \]

(82)

The small oscillations of \( C[\lambda, \phi] \), before it settles down to a fixed point, are due to the ‘non-unitary’ world-sheet contributions of the Liouville mode \( \phi \); however globally in target space-time there is a monotonic change of the degrees of freedom of the system, as discussed in detail in [3].

These considerations can be understood more easily if one looks at the correlation functions in the Liouville theory, viewing the Liouville field as a local scale on the world sheet. Standard computations [67] yield for an \( N \)-point correlation function among world-sheet integrated vertex operators \( V_i \equiv \int d^2 z V_i(z, \bar{z}) \):

\[ A_N \equiv \langle V_{i_1} \ldots V_{i_N} \rangle \mu = \Gamma(-s)\mu^s \langle \int d^2 z \sqrt{\gamma} e^{i\phi} \rangle^{s} V_{i_1} \ldots V_{i_N} \rangle_{\mu=0} \]

(83)

where the tilde denotes removal of the Liouville field \( \phi \) zero mode, which has been path-integrated out in (83). The world-sheet scale \( \mu \) is associated with cosmological constant terms on the world sheet, which are characteristic of the Liouville theory [59]. The quantity \( s \) is the sum of the Liouville anomalous dimensions of the operators \( V_i \)

\[ s = -\sum_{i=1}^{N} \frac{\alpha_i}{\alpha} - \frac{Q}{\alpha} \; ; \; \alpha = -\frac{Q}{2} + \frac{1}{2}\sqrt{Q^2 + 8} \]

(84)

The \( \Gamma \) function can be regularized [56, 5] (for negative-integer values of its argument) by analytic continuation to the complex-area plane using the the Saaschultz contour of Fig. 4. This yields the possibility of an increase of the running central charge due to the induced oscillations of the dynamical world sheet area (related to the Liouville zero mode). This is associated with the oscillatory solution (82) for the Liouville central charge. On the other hand, the bounce interpretation of the infrared fixed points of the flow, given in refs. [56, 5], provides an alternative picture of the overall monotonic change at a global level in target space-time.

The above formalism also allows for an explicit demonstration of the non-factorizability of the superscattering matrix associated with target-space interactions in non-critical string theory. This was very important for our purposes in the context of the collapse of the wave-function as a result of quantum entanglement due to quantum gravity fluctuations.

To this end, one expands the Liouville field in (normalized) eigenfunctions \( \{ \phi_n \} \) of the Laplacian \( \Delta \) on the world sheet

\[ \phi(z, \bar{z}) = \sum_n c_n \phi_n = c_0 \phi_0 + \sum_{n \neq 0} \phi_n \; ; \; \phi_0 \propto A^{-\frac{1}{2}} \]

(85)
with \( A \) the world-sheet area, and
\[
\Delta \phi_n = -\epsilon_n \phi_n \quad n = 0, 1, 2, \ldots, \quad \epsilon_0 = 0 \quad (\phi_n, \phi_m) = \delta_{nm} \tag{86}
\]
The result for the correlation functions (without the Liouville zero mode) appearing on the right-hand-side of eq. (83) is, then
\[
\tilde{A}_N \propto \int \Pi_n c_n \exp\left(-\frac{1}{8\pi} \sum_{n \neq 0} \epsilon_n c_n^2 - \frac{Q}{8\pi} \sum_{n \neq 0} R_n c_n + \sum_{n \neq 0} \alpha_i \phi_n(z_i) c_n \left( \int d^2 \xi \sqrt{\gamma} e^\alpha \sum_{n \neq 0} \phi_n c_n \right)^s \right) \tag{87}
\]
with \( R_n = \int d^2 \xi R(\xi) \phi_n \). We can compute (87) if we analytically continue \[67\] to a positive integer \( s \rightarrow n \in \mathbb{Z}^+ \). Denoting
\[
f(x, y) \equiv \sum_{n, m \neq 0} \frac{\phi_n(x) \phi_m(y)}{\epsilon_n} \tag{88}
\]
one observes that, as a result of the lack of the zero mode,
\[
\Delta f(x, y) = -4\pi \delta^{(2)}(x, y) - \frac{1}{\tilde{A}} \tag{89}
\]
We may choose the gauge condition \( \int d^2 \xi \sqrt{\gamma} \tilde{\phi} = 0 \). This determines the conformal properties of the function \( f \) as well as its ‘renormalized’ local limit\[94\]
\[
f_R(x, x) = \lim_{x \rightarrow y} (f(x, y) + \ln d^2(x, y)) \tag{90}
\]
where \( d^2(x, y) \) is the geodesic distance on the world sheet. Integrating over \( c_n \) one obtains
\[
\tilde{A}_{n+N} \propto \exp\left[ \frac{1}{2} \sum_{i, j} \alpha_i \alpha_j f(z_i, z_j) + \frac{Q^2}{128\pi^2} \int \int R(x) R(y) f(x, y) - \sum_i \frac{Q}{8\pi} \alpha_i \int \sqrt{\gamma} R(x) f(x, z_i) \right] \tag{91}
\]
We now consider infinitesimal Weyl shifts of the world-sheet metric, \( \gamma(x, y) \rightarrow \gamma(x, y)(1 - \sigma(x, y)) \), with \( x, y \) denoting world-sheet coordinates. Under these, the correlator \( A_N \) transforms as follows\[36\]
\[
\delta \tilde{A}_N \propto \left[ \sum_i h_i \sigma(z_i) + \frac{Q^2}{16\pi} \int d^2 x \sqrt{\gamma} \tilde{R} \sigma(x) + \frac{1}{\tilde{A}} \left\{ Q S \int d^2 x \sqrt{\gamma} \sigma(x) + (s)^2 \int d^2 x \sqrt{\gamma} \tilde{R} f_R(x, x) + Q S \int \int d^2 x d^2 y \sqrt{\gamma} R(x) \sigma(y) \tilde{G}(x, y) - s \sum_i \alpha_i \int d^2 x \sqrt{\gamma} \sigma(x) \tilde{G}(x, z_i) - \frac{1}{2} s \sum_i \alpha_i \tilde{f}_{R}(z_i, z_i) \int d^2 x \sqrt{\gamma} \sigma(x) - \frac{Q S}{16\pi} \int \int d^2 x d^2 y \sqrt{\gamma} \tilde{R} \tilde{R} f_R(x, x) \sigma(y) \right\} \right] \tilde{A}_N \tag{92}
\]
where the hat notation denotes transformed quantities, and the function \( G(x,y) \) is defined as
\[
G(z,\omega) \equiv f(z,\omega) - \frac{1}{2}(f_R(z,z) + f_R(\omega,\omega))
\]
and transforms simply under Weyl shifts. We observe from (92) that if the sum of the anomalous dimensions \( s \neq 0 \) (‘off-shell’ effect of non-critical strings), then there are non-covariant terms in (92), inversely proportional to the finite-size world-sheet area \( A \). In general, this is a feature of non-critical strings wherever the Liouville mode is viewed as a local scale of the world sheet. In such a case, the central charge of the theory flows continuously with time/scale \( t \), as a result of the Zamolodchikov \( c \)-theorem. In contrast, the screening operators yield quantized values. This induced time \((A-)\) dependence of the correlation function \( A_N \) implies the breakdown of their interpretation as factorisable \( \mathcal{F} \)-matrix elements.

In our framework, the effects of the quantum-gravity entanglement induce such \( A \)-dependences in correlation functions of the propagating matter vertex operators of the string, corresponding to the displacement field \( u(x,t) \) of the MTs. To this end, we first note that the physical states in such completely integrable models fall into representations of the \( SL(2, R) \) target symmetry, which are classified by the non-compact isospin \( j \) and its third component \( m \). There is a formal equivalence of the physical states between the flat-space time \((1+1)\)-dimensional string and the black-hole model, which confirms the point of view that the flat-space \((1+1)\)-dimensional string theory is the spatially- (and temporally-) asymptotic limit of the \( SL(2, R)/U(1) \) black hole. The existence of discrete (quasi-topological, non-propagating) Planckian modes in the two-dimensional string theory leads to selection rules in the number \( N \) of the scattered propagating degrees of freedom, according to the intermediate-exchange state:

Asymptotic correspondence \((\epsilon_\phi, p) \equiv \text{Liouville energy (momentum))}:
\[
j \to \epsilon_\phi, \; m \to \frac{3p}{2\sqrt{2}}
\]
Asymptotic Kinematics:
\[
\sum_{i=1}^{N-1} p_i = \frac{N-2}{\sqrt{2}} \quad ; \quad p_N = -\frac{N-2}{\sqrt{2}} \quad ; \quad N \geq 3
\]
Selection Rules:
\[
j = \frac{1}{3}m - 1 + \frac{1}{2}(N-2) \; , \; j \geq \frac{1}{4}(N-5)
\]
(94)

Such rules are obtained by imposing the Liouville energy \( \epsilon_\phi \) and momentum \( p \) conservation, leading to \( s = 0 \), with \( s \) the sum of Liouville anomalous dimensions as defined earlier. Obviously, if the exchange state is an (off-shell) propagating mode,\(^{12}\)

\(^{12}\)Originally, there were claims that there are extra states in the black hole models, as compared to the flat-space time string; however, later on it has been shown that such states can be either gauged away or boosted, and so they disappear from the physical spectrum.
belonging to the continuous representation of $SL(2, R)$, i.e. $j \in R$, $j \geq -\frac{1}{2}$, there are no restrictions on $N$, and a conventional $S$-matrix amplitude can be defined as the residue of the Liouville amplitudes with respect to the single poles in $s$ \([17]\). However, in two space-time dimensions graviton excitations are discrete, corresponding to string-level-one representations of $SL(2, R)$. Hence, once non-trivial quantum-gravity fluctuations are considered in our approach, which in two dimensions are black-hole backgrounds (12), one has to take into account discrete on-shell exchange modes in the Liouville correlation functions. Such states represent excited states of the (virtual) black-holes, created by the collapse of the propagating matter modes $u(x, t)$, as described in section 4 (12). In two-dimensional string theory black holes are like particles (5), the difference being their topological nature. Such modes constitute, in our case, ‘the consciousness degrees of freedom’, which cannot be measured by local scattering experiments. Integrating them out in the ‘mind’, implies a time arrow as described in ref. (5). Indeed, in the correlation functions (83), as a result of Liouville energy conservation (94), one of the modes is necessarily discrete. If we suppress such modes, and consider only external propagating modes, accessible to physical scattering processes, then it is evident that $s \neq 0$. According to our previous analysis (92), then, this implies world-sheet-area($A$) dependence of the correlation functions. In this picture, we also note that Quantum-Gravitational fluctuations of singular space-time form, corresponding to higher-genus world-sheet effects, have been argued (9) to be represented collectively by world-sheet instanton-anti-instanton deformations in the stringy $\sigma$-model. It is known (49) that such configurations are responsible for a non-perturbative breakdown of the conformal invariance of the $\sigma$-model. Using a dynamical (world-sheet) renormalization-group scale (Liouville mode) to represent all such non-conformal invariant effects (5), and identifying it with the target time, one, then, arrives at non-factorisable superscattering operators, as described above.

Notice that the precise microscopic nature of the environmental operators is not essential as long as the latter imply a conformal anomaly. There are general consequences of this conformal anomaly, including dynamical collapse of the string theory space to a certain configuration, as discussed in section 4. A similar situation occurs in ordinary quantum mechanics of open systems. Once a stochastic framework using state vectors is adopted (11) for the description of environmental effects, there will always be localization of the state vector in one of its channels, irrespective of the detailed form of the environment operators. It should be noted that stochasticity is a crucial feature of our approach too. This follows from the stochastic nature of the renormalization group in two-dimensions (72, 5). This stochasticity was argued to play an important rôle on the transition from quantum to classical worlds in the brain (conscious perception), discussed in section 4, as well as in the MT growth, discussed in section 5.

\[ \text{The on-shell condition imposes algebraic relations among } j \text{ and } m \text{ for such modes, involving the string-level number due to the Virasoro constraints (92). This, in turn, implies restrictions to the number } N \text{ of scattered particles in such cases.} \]
World-sheet Instanton Calculus and evaporation of a two-dimensional Stringy Black hole

In this part of the appendix we would like to discuss briefly some technical but important aspects of the specific two-dimensional black hole model of ref. [23], which is the exact conformal field theory used in the simulation of the dynamics of the MT chains in this work. In particular, we shall outline some details pertaining to world-sheet instantons effects in yielding extra logarithmic divergencies in correlation functions of discrete matter operators (‘tachyons’) of the two-dimensional theory. This justifies our assumption in the text that the effects of instantons are associated with topological (zero) modes of the black hole model, whose circulation along thin tudes of long handles connecting Riemann surfaces produces extra divergencies, expressed by the addition of $\theta$-terms in the effective action. We shall be very brief below, and concentrate only on giving the basic results. For details on the derivation we refer the interested reader to the literature [3, 43].

To this end, we remind the reader that the action of $SL(2,R)/U(1)$ coset Wess-Zumino model [23] describing a Euclidean black hole can be written in the form [23]

$$S = \frac{k}{4\pi} \int d^2z \frac{1}{1 + |w|^2} \partial_\mu \bar{w} \partial^\mu w + \ldots$$

(95)

where the conventional radial and angular coordinates $(r, \theta)$ are given by $w = sinh r e^{-i\theta}$ and the target space $(r, \theta)$ line element is

$$ds^2 = \frac{dwd\bar{w}}{1 + w\bar{w}} = dr^2 + tanh^2 r d\theta^2$$

(96)

The corresponding exactly-marginal deformation, which turns on matter backgrounds in this geometry is constructed by $W_\infty$ symmetry considerations, and is given by [52]

$$L_0^1 T_0^1 \propto \mathcal{F}_{\frac{1}{2},0,0}^{c-c} + i(\psi^{++} - \psi^{--}) + \ldots$$

(97)

where the $\psi$ denote higher-string-level operators [52], and the ‘tachyon’ operator is given by

$$\mathcal{F}_{\frac{1}{2},0,0}^{c-c}(r) = \frac{1}{\cosh r} F(\frac{1}{2}, \frac{1}{2}; 1, tanh^2 r)$$

(98)

with

$$F(\frac{1}{2}, \frac{1}{2}; 1, tanh^2 r) \approx \frac{1}{\Gamma^2(\frac{1}{2})} \sum_{n=0}^{\infty} \frac{(\frac{1}{2})_n (\frac{1}{2})_n}{(n!)^2} [2\psi(n+1) - 2\psi(n + \frac{1}{2}) +$$

$$+ ln(1 + |w|^2)](\sqrt{1 + |w|^2})^{-n}$$

(99)

There is an additional marginal deformation, dictated by the $SL(2,R)$ symmetry structure [52], which consists of topological string modes only. At large $k$, this
operator rescales the black hole metric, as can be seen from its contribution to the action of the deformed Wess-Zumino σ-model after the gauge field integration \[52\],

\[
g L_0^2 \mathcal{I}_0^2 \cong \int d^2 z \{ \partial r \partial r (1 - 2g \csc h^2 r - 2g \sech^2 r) + \partial \theta \partial \theta (\sinh^2 r + 2g - (\sinh^2 r + 2g)^2) / \cosh^2 r + 2g) \} \tag{100}
\]

Changing variables \(cosh^2 r + 2g \to cosh^2 r\) in (100) one finds that to \(O(g)\) the target space metric is rescaled by an overall constant.

Notice that (98) plays the rôle of the cosmological constant operator in flat spacetimes. In our two-dimensional black hole case, the analogous exactly marginal deformation is not simply the cosmological constant, but the operator (97). The latter consists of an infinite set of discrete topological \(W_\infty\) states, which, as argued in ref. [5], play an important rôle in preserving quantum coherence of the full string theory, due to information carried by them during a black hole evaporation/decay process [54]. Such states are physical, in that they affect the renormalization group structure of the theory, and in view of the interpretation of the RG scale as time in target space, they also affect the temporal evolution of our system.

To discuss, at least qualitatively, their effects in our theory we have argued in ref. [5] that one has to consider the effects of non-perturbative world-sheet configurations. As discussed in refs. [19, 3], the \(SL(2, R)/U(1)\) Wess-Zumino coset model describing a Euclidean black hole also has instantons given by the holomorphic function

\[
w(z) = \frac{\rho}{z - z_0} \tag{101}
\]

with topological charge

\[
Q = \frac{1}{\pi} \int d^2 z \frac{1}{1 + |w|^2} [\partial \phi w - h.c.] = -2ln(a) + \text{const} \tag{102}
\]

where \(a\) is an ultraviolet cut-off. The instanton action on the world-sheet also depends logarithmically on the ultraviolet cut-off. As in the case of the more familiar vortex configuration in the Kosterlitz-Thouless model, this logarithmic divergence does not prevent the instanton from having important dynamical effects. In the Bosonic σ-model, sufficient for our purposes here, the instanton-anti-instanton vertices take the form [19]

\[
V_{\text{IT}} \propto -\frac{d}{2\pi} \int d^2 z d^2 \rho e^{-S_0(\frac{|\phi(\phi + h.c. + ...)_{/f(|w|)}|}{f(|w|)} + e^{f(|w|)})} \tag{103}
\]

introducing a new term into the effective action. Making a derivative expansion of the instanton vertex and taking the large-\(k\) limit, i.e. restricting our attention to instanton sizes \(\rho \simeq a\), this new term has the same form as the kinetic term in (94),
and hence corresponds to a renormalization of the effective level parameter in the large \( k \) limit:

\[
k \to k - 2\pi k^2 d' \quad : \quad d' \equiv d \int \frac{d|\rho|}{|\rho|^3 \left[(\rho/a)^2 + 1\right]} d^2 \rho \quad (104)
\]

If other perturbations are ignored, the instantons are irrelevant deformations and conformal invariance is maintained. However, in the presence of “tachyon” deformations, \( T_0 \int d^2 z F_{\text{cc}}^{-c,c}(z, \bar{z}) \), there are extra logarithmic infinities in the shift (104), that are visible in the dilute gas and weak-“tachyon”-field approximations. In this case, there is a contribution to the effective action of the form

\[
T_0 \int d^2 z d^2 z' < F_{\text{cc}}^{-c,c}(z, \bar{z}) V_I(z', \bar{z'}) > \quad (105)
\]

Using the explicit form of the “tachyon” vertex \( F_{\text{cc}} \) given by \( SL(2, R) \) symmetry [52], it is straightforward to isolate a logarithmically-infinite contribution to the kinetic term in (95), associated with infrared infinities on the world-sheet expressible in terms of the world-sheet area \( \Omega/a^2 \) [3].

\[
g T_0 \int d^2 z' \left[ \frac{d}{\rho} \left( \frac{a^2}{a^2 + \rho^2} \right)^{\frac{\omega}{2}} \right] \int d^2 z \frac{1}{|z - z'|^2} \frac{1}{1 + |w|^2} \partial_z w(z') \partial_{\bar{z}} \bar{w}(z') + \ldots
\]

\[
\equiv g T_0 \ln \frac{\Omega}{a^2} \int d^2 z' \frac{1}{1 + |w|^2} \partial_z w(z') \partial_{\bar{z}} \bar{w}(z') \quad (106)
\]

Such covariant-scale-dependent contributions can be attributed to Liouville field dynamics, through the “fixed-area constraint” in the Liouville path integral [97]. The zero-mode part can be absorbed in a scale-dependent shift of \( k \) [3], which for large \( k >> 1 \) may be assumed to exponentiate:

\[
k_R \propto \left( \frac{\Omega}{a^2} \right)^{(\text{const}) \cdot \beta^I T_0} \quad (107)
\]

where \( \beta^I \) is the instanton \( \beta \)-function [19]. In ref. [3] we gave general arguments and verified to lowest order that instantons represent higher-string-level (global) mode effects, enabling us to identify \( \beta^I = -\beta^T \), where \( \beta^T \) is the renormalization-group \( \beta \)-function of a matter deformation of the black hole [14]. Notice that in (107) both the infrared and the ultraviolet cut-off scales enter. We do not distinguish between infrared and ultraviolet cut-offs in our framework. The physical scale of the system, which varies along a renormalization group trajectory, is the dimensionless ratio of the two, which is identified with the Liouville field.

The change in \( k \) and the associated change in the central charge \( c = \frac{36}{k^2} - 1 \) and the black-hole mass \( M_{bh} \propto (k - 2)^{-\frac{\omega}{2}} \) do not conflict with any general theorems. An analogous instanton renormalization of \( \theta \) (c.f. \( k \)) has been demonstrated [96] in related \( \sigma \)-models that describe the Integer Quantum Hall Effect (IQHE), discussed further in ref. [3].

\[\text{Notice that this implies that the matter } \beta \text{-function has to be computed in a non-perturbative way, which is consistent with the exact conformal field theory analysis of ref. [52].}\]
So far we have dealt with target-space Euclidean black holes. Although one could appeal to analytic continuation arguments \textit{a posteriori}, however it would be useful to have an understanding of the Minkowski case, which in the string framework of ref. 23 is obtained by analyzing the coset Wess-Zumino model over $SL(2, R)/O(1, 1)$. Instanton renormalization of $k$ can also be seen in this model. The $\sigma$-model action of such a theory contains \cite{19}, in addition to the action \cite{33}, a total-derivative $\theta$-term which can be thought of as a deformation of the black hole by an “antisymmetric tensor” background, which in two dimensions is a discrete mode as a result of the abelian gauge symmetry. Its Euclideanized version has also instanton solutions of the form \cite{101}, but with finite action, which induce “Liouville”-time-dependent shifts to $k$, prior to matter couplings. This situation is similar to what happens to the topological $N = 2$ theory, discussed in the text. The conclusions about the existence of extra divergences in correlation functions of discrete ‘tachyon’ operators persist in the Minkowski picture, thereby justifying the absorption of such divergencies by local renormalization scales (Liouville fields) on the world-sheet.

In our case, as we have seen, the instantons reflect a shift of the central charge between the matter and background sectors of a combined matter + black hole theory, in which the total central charge is unchanged. Qualitatively, their effects have been argued \cite{3} to correspond to a combination of world-sheet deformation operators in the Wess-Zumino model \cite{23}, pertaining to global modes at higher-string levels: the exactly marginal operator $L^2_0 T^2_0$ and the rest of the moduli representing the higher-string-level $W$-hair \cite{22}, and the irrelevant part of the exactly-marginal deformation $L^1_0 T^1_0$, which involves an infinite sum of massive (global) string operators \cite{22}. The fact that the $L^2_0 T^2_0$ operator rescales the target-space metric by an overall constant, implies that such perturbations have the same effect as the instanton. Thus the instanton represents the effects of global higher-level string modes that are related to each other and to massless excitations by a $W$ symmetry. Matrix elements of the full exactly marginal light matter + instanton operator have no dependence on the ultraviolet cut-off $a$, but the separate matter and instanton parts do depend on $a$, as we have seen above.

Since instantons rescale the target-space metric and the black hole mass, they may also be used to represent black hole decay. This is higher-genus effect in string theory \cite{54}, so one should expect that instantons could reflect the contributions of higher genera. This expectation is indeed supported by an explicit computation of instanton effects in a dilute-gas approximation in the presence of dilatons. This point has been briefly discussed in the text, and for more details we refer the reader to the literature \cite{3, 54}. 
Appendix B

Variational Approach to Soliton Quantization via Squeezed Coherent States

It is the purpose of this appendix to discuss briefly the formalism leading to the quantization of the solitonic states discussed in section 2.

One assumes the existence of a canonical second quantized formalism for the (1 + 1)-dimensional scalar field \( u(x, t) \), based on creation and annihilation operators \( a_k^\dagger, a_k \). One then constructs a squeezed vacuum state \[ \Psi(t) = N(t) e^{T(t)} |0\rangle \quad ; \quad T(t) = \frac{1}{2} \int \int dxdyu(x)\Omega(x, y, t)u(y) \] where \( |0\rangle \) is the ordinary vacuum state annihilated by \( a_k \), and \( N(t) \) is a normalization factor to be determined. \( \Omega(x, y, t) \) is a complex function, which can be splitted in real and imaginary parts as

\[
\Omega(x, y) = \frac{1}{2} [G_0^{-1}(x, y) - G^{-1}(x, y, t)] + 2i\Pi(x, y, t)
\]

\[
G_0(x, y) = \langle 0 | u(x)u(y) | 0 \rangle
\]

The squeezed coherent state for this system can be then defined as

\[
|\Phi(t)\rangle \equiv e^{iS(t)}|\Psi(t)\rangle \quad ; \quad S(t) = \int_{-\infty}^{+\infty} dx [D(x, t)u(x) - C(x, t)\pi(x)]
\]

with \( \pi(x) \) the momentum conjugate to \( u(x) \), and \( D(x, t), C(x, t) \) real functions. With respect to this state \( \Pi(x, t) \) can be considered as a momentum canonically conjugate to \( G(x, y, t) \) in the following sense

\[
< \Phi(t)| - i \frac{\delta}{\delta \Pi(x, y, t)} |\Phi(t)\rangle = -G(x, y, t)
\]

The quantity \( G(x, y, t) \) represents the modified boson field around the soliton.

To determine \( C, D, \) and \( \Omega \) one applies the Time-Dependent Variational Approach (TDVA) according to which

\[
\delta \int_{t_1}^{t_2} dt < \Phi(t)|(i\partial_t - H)|\Phi(t)\rangle = 0
\]

where \( H \) is the canonical Hamiltonian of the system. This leads to a canonical set of (quantum) Hamilton equations

\[
\dot{D}(x, t) = -\frac{\delta H}{\delta C(x, t)} \quad \dot{C}(x, t) = \frac{\delta H}{\delta D(x, t)}
\]

\[
\dot{G}(x, y, t) = \frac{\delta H}{\delta \Pi(x, y, t)} \quad \dot{\Pi}(x, y, t) = \frac{\delta H}{\delta G(x, y, t)}
\]
where the quantum energy functional $\mathcal{H}$ is given by

$$\mathcal{H} \equiv \langle \Phi(t)|H|\Phi(t)\rangle = \int_{-\infty}^{\infty} dx \mathcal{E}(x)$$

(114)

with

$$\mathcal{E}(x) = \frac{1}{2}D^2(x,t) + \frac{1}{2}(\partial_x C(x,t))^2 + M^{(0)}[C(x,t)] +$$

$$\frac{1}{8} < x|G^{-1}(t)|y > + 2 < x|\Pi(t)G(t)]\Pi(t)|y > + \frac{1}{2}lim_{x\rightarrow y}\nabla_x \nabla_y < x|G(t)|y > -$$

$$\frac{1}{8} < x|G_0^{-1}|y > - \frac{1}{2}lim_{x\rightarrow y}\nabla_x \nabla_y < x|G_0(t)|y >$$

(115)

where we use the following operator notation in coordinate representation $A(x,y,t) \equiv < x|A(t)|y >$, and

$$M^{(n)} = e^\frac{1}{2}(G(x,x,t)-G_0(x,x))\frac{\partial^2}{\partial z^2}U^{(n)}(z)|_{z=C(x,t)} , \quad U^{(n)} \equiv d^nU/dz^n$$

(116)

Above, $U$ denotes the potential of the original soliton Hamiltonian, $H$. Notice that the quantum energy functional is conserved in time, despite the various time dependences of the quantum fluctuations. This is a consequence of the canonical form (113) of the Hamilton equations.

Performing the functional derivations in (113) one can get

$$\dot{D}(x,t) = \frac{\partial^2}{\partial x^2}C(x,t) - \mathcal{M}^{(1)}[C(x,t)]$$

$$\dot{C}(x,t) = D(x,t)$$

(117)

which after elimination of $D(x,t)$, yields the modified (quantum) soliton equation (29). We note that the quantities $\mathcal{M}^{(n)}$ carry information about the quantum corrections, and in this sense the above scheme is more accurate than the WKB approximation [34]. The whole scheme may be thought of as a mean-field-approach to quantum corrections to the soliton solutions.

In our string framework, then, these point-like quantum solitons can be viewed as a low-energy approximation to some more general ground state solutions of a non-critical string theory, formulated in higher genera on the world sheet to account for the quantum corrections. We have not worked out in this work the full string-theory representation of the relevant quantum coherent state. This will be an interesting topic to be studied in the future, which will allow for a rigorous study of the effects of the global string modes on the collapse of the quantum-coherent preconscious state.
References

[1] P. Dustin, *MicroTubules* (Springer, Berlin 1984);
    Y. Engleborghs, *Nanobiology* 1 (1992), 97.

[2] R. Penrose and S. Hameroff, *Orchestrated Reduction of Quantum Coherence in Brain Microtubules: a Model of Consciousness*, to appear.

[3] R. Penrose, *The Emperor’s New Mind* (Oxford Univ. Press 1989); *Shadows of the Mind* (Oxford Univ. Press 1994).

[4] D.V. Nanopoulos, *Theory of Brain Function, Quantum Mechanics and Superstrings*, ACT-08/95, CERN-TH/95-128, CTP-TAMU-22/95, hep-ph/9505374, based on talk presented at the “XV Brazilian National Meeting on Particles and Fields”, Angra dos Reis, Brazil (October 4-8 1994).

[5] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 37;
    CERN, ENS-LAPP and Texas A & M Univ. preprints, CERN-TH.6896/93, ENS-LAPP-A426-93, CTP-TAMU-29/93; ACT-09/93 (1993); hep-th/9305116, to appear in Mod. Phys. Lett. A;
    CERN-TH.6897/93, ENS-LAPP-A427-93, CTP-TAMU-30/93; ACT-10/93 (1993); hep-th/9305117;
    CERN-TH.7195/94, ENS-LAPP-A-463/94, ACT-5/94, CTP-TAMU-13/94, lectures presented at the Erice Summer School, 31st Course: From Supersymmetry to the Origin of Space-Time, Ettore Majorana Centre, Erice, July 4-12 1993; hep-th/9403133, to be published in the proceedings (World Sci.);
    For a pedagogical review of this approach see: D.V. Nanopoulos, Riv. Nuov. Cim. Vol. 17, No. 10 (1994), 1.

[6] J. Ellis, J.S. Hagelin, D.V. Nanopoulos and M. Srednicki, Nucl. Phys. B241 (1984), 381.

[7] J. Ellis, S. Mohanty and D.V. Nanopoulos, Phys. Lett. B221 (1989), 113.

[8] H. Fröhlich, *Bioelectrochemistry*, ed. by F. Guttman and H. Keyzer (Plenum, New York 1986).

[9] P. Lal, Physics Letters 111A (1985), 389.

[10] S. Hameroff, S. A. Smith, R.C. Watt, Ann. N.Y. Acad. Sci. 466 (1986), 949;
    S. Hameroff, *Ultimate Computing* (Elsevier North-Holland, Amsterdam 1987).

[11] E. Brézin and V.A. Kazakov, Phys. Lett. B236 (1990), 144;
    M. Douglas and A. Shenker, Nucl. Phys. B335 (1990), 635;
    D. Gross and A.A. Migdal, Phys. Rev. Lett. 64 (1990), 127;
For a recent review see, e.g., I. Klebanov, in *String Theory and Quantum Gravity*, Proc. Trieste Spring School 1991, ed. by J. Harvey et al. (World Scientific, Singapore, 1991), and references therein.

[12] I. Bakas and E. Kiritsis, Int. J. Mod. Phys. A7 (Suppl. 1A) (1992), 55.

[13] A.B. Zamolodchikov, JETP Lett. 43 (1986), 730; Sov. J. Nucl. Phys. 46 (1987), 1090.

[14] S. Hawking, Phys. Rev. D14 (1976), 2460.

[15] D. Page, *Information Loss in Black Hole and/or Conscious Beings?*, Alberta preprint, hep-th/9411193, to be published in *Heat Kernel Techniques and Quantum Gravity*, eds. S.A. Fulling (Texas A&M University 1995)).

[16] R.P. Feynman and F.L. Vernon Jr., Ann. Phys. (NY) 94 (1963), 118.

[17] A.O. Caldeira and A.J. Leggett, Ann. Phys. 149 (1983), 374.

[18] Y.R. Shen, Phys. Rev. 155 (1967), 921;  
A.S. Davydov and A. A. Serikov, Phys. Stat. Sol. B51 (1972), 57;  
B.Ya. Zel’dovich, A.M. Perelomov, and V.S. Popov, Sov. Phys. JETP 28 (1969), 308;  
For a review see : V. Gorini *et al.*, Rep. Math. Phys. Vol. 13 (1978), 149.

[19] M.V. Satarić, J.A. Tuszyński, R.B. Zakula, Phys. Rev. E48 (1993), 589.

[20] M.A. Collins, A. Blumen, J.F. Currie, and J. Ross, Phys. Rev. B19 (1978), 3630.

[21] E. Del Giudice, S. Doglia, M. Milani and G. Vitiello, Nucl. Phys. B251 (FS 13) (1985), 375; *ibid* B275 (FS 17) (1986), 185.

[22] E. Del Giudice, G. Preparata and G. Vitiello, Phys. Rev. Lett. 61 (1988), 1085.

[23] E. Witten, Phys. Rev. D44 (1991), 344.

[24] L.M. Ricciardi and H. Umezawa, *Kibernetik* 4, (1967) 44.

[25] C.I.J. Stuart, Y. Takahashi and H. Umezawa, J. Theor. Biol. 71 (1978) 605;  
*Found. Phys.* 9 (1979) 301.

[26] G. Vitiello, Int. J. Mod. Phys. B9, 973 (1995).

[27] For a review see H. Umezawa, *Advanced Field Theory : micro, macro and thermal concepts* (American Inst. of Physics, N.Y. 1993).
[28] I. Antoniadis, C. Bachas, J. Ellis and D.V. Nanopoulos, Phys. Lett. B211 (1988), 393; Nucl. Phys. B328 (1989), 117; Phys. Lett. B257 (1991), 278;
   D.V. Nanopoulos, in Proc. International School of Astroparticle Physics, HARC (Houston) (World Scientific, Singapore, 1991), p. 183.

[29] D. Minic, J. Polchinski and Z. Yang, Nucl. Phys. B369 (1992), 324.

[30] T. Banks, Nucl. Phys. B361 (1991), 166.

[31] M. Otwinowski, R. Paul, and W. Laidlaw, Phys. Lett. A128 (1988), 483.

[32] S.A. Hojman and L.C. Shepley, J. Math. Phys. 32 (1991), 142;
   F. Pardo, J. Math. Phys. 30 (1989), 2054.

[33] Y. Tsue, Y. Fujiwara, Progr. Theor. Physics 86 (1991), 443; ibid 469.

[34] For a sampling of references see: R.F. Dashen, B. Hasslacher and A. Neveu, Phys. Rev. D10 (1974), 4114, 4130; ibid D12 (1975), 2443;
   N.H. Christ and T.D. Lee, Phys. Rev. D12 (1975), 1606;
   E. Tomboulis, Phys. Rev. D12 (1975), 1678;
   J-L. Gervais and A. Jevicki, Nucl. Phys. B110 (1976), 93, 113;
   R. Jackiw, Rev. Mod. Phys. 49 (1977), 681.

[35] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, preprint ACT-03/95, CERN-TH.7480/94, CTP-TAMU-12/95, ENSLAPP-A-492/94, hep-th/9503162.

[36] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, to appear.

[37] G. Shore, Nucl. Phys. B286 (1987), 349;
   H. Osborn, Nucl. Phys. B294 (1987), 595; ibid B308 (1988), 629; Phys. Lett. B222 (1989), 97.

[38] C. Schmidhuber and A.A. Tseytlin, Nucl. Phys. B426 (1994), 187;
   H. Dorn, HU-Berlin-IEP-94-21 (1994) preprint; hep-th/9410084.

[39] N.E. Mavromatos and J.L. Miramontes, Phys. Lett. B226 (1989), 291.

[40] G. Lindblad, Comm. Math. Phys. 48 (1976), 119.

[41] N. Gisin, Helv. Phys. Acta, Vol. 62 (1989), 363, and references therein;
   N. Gisin and I. Percival, J. Phys. A 26 (1993), 2233.

[42] T. Eguchi, Phys. Lett. B316 (1993), 74.

[43] M. Alimohammadi and F. Ardalan, 2-d Gravity as a limit of the $SL(2,R)$ black hole, UCTP-103-95 preprint, hep-th/9505033.
[44] J. Distler and P. Nelson, Nucl. Phys. B374 (1991), 123.
[45] J. Russo, Phys. Rev. D47 (1993), R4188.
[46] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B288 (1992), 23.
[47] A.M. Polyakov, Mod. Phys. Lett. A6 (1991), 635.
[48] A. Kovner, B. Rosenstein and D. Elieser, Nucl. Phys. B350 (1991), 325; 
A. Kovner and B. Rosenstein, Phys. Lett. B266 (1991), 443.
[49] A.V. Yung, Int. J. Mod. Phys. A9 (1994), 591; 
SWAT 94/22 (1994), hepth/9401124; SWAT 95/62 (1995), hepth/9502000.
[50] See for instance: *Gravitation*, C.W. Misner, K.S. Thorne and J.A. Wheeler 
(W.H. Freeman and Co., San Fransisco (1973)).
[51] J. Polchinski, Nucl. Phys. B362 (1991), 125.
[52] S. Chaudhuri and J. Lykken, Nucl. Phys B396 (1993), 270.
[53] F. Yu and Y.S. Wu, J. Math. Phys. 34 (1993), 5872.
[54] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B276 (1992), 56;
[55] J. Hartle and S. Hawking, Phys. Rev. D28 (1983), 2960.
[56] I. Kogan, Phys. Lett. B265 (1991), 269.
[57] B. Misra, I. Prigogine and M. Courbage, *Physica* A98 (1979), 1;
I. Prigogine, *Entropy, Time, and Kinetic Description*, in *Order and Fluctuations in Equilibrium and Non-Equilibrium Statistical Mechanics*, ed G. Nicolis et al. (Wiley, New York, 1981);
B. Misra and I. Prigogine, *Time, Probability and Dynamics*, in *Long-Time Prediction in Dynamics*, ed G. W. Horton, L. E. Reichl and A.G. Szebehely (Wiley, New York, 1983);
B. Misra, *Proc. Nat. Acad. Sci. U.S.A.* 75 (1978), 1627.
[58] R. M. Santilli, Hadronic J. 1, 223, 574 and 1279 (1978); *Foundations of Theoretical Mechanics*, Vol. I (1978) and II (1983) (Springer-Verlag, Heidelberg-New York);
For an application of this approach to dissipative statistical systems see: J. Fronteau, A. Tellez-Arenas and R.M. Santilli, Hadronic J. 3 (1979), 130;
J. Fronteau, Hadronic J. 4 (1981), 742.
[59] F. David, Mod. Phys. Lett. A3 (1988), 1651;
J. Distler and H. Kawai, Nucl. Phys. B321 (1989), 509.
[60] N.E. Mavromatos and J.L. Miramontes, Mod. Phys. Lett. A4 (1989), 1847.

[61] M.B. Green, J.H. Schwarz and E. Witten, String Theory, Vol. I and II (Cambridge Univ. Press 1986).

[62] S. Coleman, Nucl. Phys. B307 (1988), 867.

[63] W.H. Zurek, Phys. Rev. D26 (1982), 1862; Physics Today 44 (October 1991), 33.

[64] A. Albrecht, Phys. Rev. D46 (1992), 5504.

[65] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Some Physical Aspects of Liouville String Dynamics, contributions by J.E. and N.E.M. in Phenomenology of Unification from Present to Future, Roma 23-26 March 1994, p. 187 (World. Sci. (1994)); hept-th/9405196.

[66] G. Barton, Ann. Phys. 166 (1986), 322.

[67] M. Goulian and M. Li, Phys. Rev. Lett. 66 (1991), 2051.

[68] M. Bershadsky and D. Kutasov, Phys. Lett. B266 (1991), 345.

[69] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B284 (1992), 27; ibid 43.

[70] T. Roy and A. Roy Chowdhuri, Phys. Rev. D15 (1977), 3768.

[71] T. Saito and T. Arimitsu, Mod. Phys. Lett. B7 (1993), 1951.

[72] D. Friedan, unpublished as quoted by T. Banks and E. Martinec, Nucl. Phys. B294 (1987), 733; S. Das, G. Mandal and S. Wadia, Mod. Phys. Lett. A4 (1989), 745.

[73] S. Coleman, Nucl. Phys. B310 (1988), 643; T. Banks, I. Klebanov and L. Susskind, SLAC-PUB-4705 (1988).

[74] J. Polchinski, Nucl. Phys. B307 (1988), 61; ibid B357 (1995), 241.

[75] W. Fischler, S. Paban and M. Rozali, Phys. Lett. B352 (1995), 298, hep-th/9503072.

[76] C. Schmidhuber, Nucl. Phys. B435 (1995), 156.

[77] J.S. Caux, I.I. Kogan and A. Tsvelik, Oxford preprint (1995), OUTP-95-62S, hep-th/9511134.

[78] For a recent review see: A. Strominger, Santa Barbara preprint, UCSBTH-95-29 (1995), hep-th/9510207, and references therein.
[79] T. Mitchison and M.W. Kirschner, Nature (London) 312 (1984), 232; ibid 237.
[80] M. Dogterom and S. Leibler, Phys. Rev. Lett. 70 (1993), 1347.
[81] M.R. Evans, D.P. Foster, C. Godrèche and D. Mukamel, Phys. Rev. Lett. 74 (1995), 208.
[82] A.Yu. Kasumov, N.A. Kislov and I.I. Khodos, Vibration of cantilever of a supersmall mass, unpublished (1992).
[83] G.C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D34 (1986), 470.
[84] H. Rosu, IFUG-19-1994 Guanajato U. preprint, bulletin gr-qc:9409007, and also at Pedestrian Notes in Quantum Fundamentals Guanajato U. preprint, IFUG-24-1994, bulletin gr-qc: 9411035 (revised May 1995).
[85] B.G. Sumpter and D.W. Noid, J. Chem. Phys. 102 (1995), 6619.
[86] N. Turok, Phys. Rev. Lett. 60 (1988), 549.
[87] J. Tabony and D. Job, Proc. Nat. Acad. Sci. (USA), Vo. 89 (1992), 6948.
[88] P. Glandsdorff and I. Prigogine, Thermodynamic Theory of Structure, Stability and Fluctuation (Wiley, New York 1971);
G. Nikolis and I. Prigogine, Self Organization in non-equilibrium systems (Wiley, New York 1977);
A. Babloyantz, Molecules, Dynamics and Life (Wiley, New York 1986);
C. Vidal and H. Lemarchand, La Réaction Créatrice (Hermann, Paris 1988).
[89] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B293 (1992), 142; CERN-TH.6755/92, hep-th/9212057, Int. J. Mod. Phys. A, in press.
[90] E. Celeghini, M. Rasetti and G. Vitiello, Ann. Phys. (N.Y.) 215 (1992), 156.
[91] A. Cappelli, C. Trugenberger and G. Zemba, Nucl. Phys. B396 (1993), 465; S. Iso, D. Karabali and B. Sakita, Phys. Lett. B296 (1992), 143.
[92] J. Ellis, N.E. Mavromatos and D.V. Nanopoulos, Phys. Lett. B296 (1992), 40.
[93] C. Schmidhuber, Nucl. Phys. B404 (1993), 342 ; I. Klebanov, I. Kogan and A.M. Polyakov, Phys. Rev. Lett. 71 (1993), 3243.
[94] L. Alvarez-Gaumé, unpublished notes on two-dimensional gravity and Liouville theory (1991).
[95] J. Distler and P. Nelson, Nucl. Phys. B374 (1992), 123.
[96] A. Pruisken, Nucl. Phys. B290[FS20] (1987), 61.
**Figure Captions**

**Figure 1** - Microtubular Arrangement: (a) the structure of a Microtubule (MT), (b) cross section of a MT, (c) two neighboring dimers along the direction of a MT axis.

**Figure 2** - The two conformations $\alpha$ and $\beta$ of a MT dimer. Transition (switching) between these two conformational states can be viewed as a quantum-mechanical effect. Quantum-Gravity entanglement can cause the collapse of quantum-coherent states of such conformations, which might arise in a MT network modelling the preconscious state of the human brain.

**Figure 3** - Illustration of the phenomenon of ‘dynamical instability’ of a MT network: (a) unbounded ‘sawtooth’ growth (b) bounded ‘sawtooth’ growth. Dotted lines show the average over many MT with the same dynamical parameters.

**Figure 4** - (a) Contour of integration in the analytically-continued (regularized) version of $\Gamma(-s)$ for $s \in \mathbb{Z}^+$. This is known in the literature as the Saalschutz contour, and has been used in conventional quantum field theory to relate dimensional regularization to the Bogoliubov-Parasiuk-Hepp-Zimmermann renormalization method, (b) schematic representation of the evolution of the world-sheet area as the renormalization group scale moves along the contour of fig. 4(a).
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