Error identification method of five-axis machine tool based on sample test method

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Abstract
In order to improve the predictive ability of the error model of the five-axis machine tool, an indirect error measurement (IEM) method of the five-axis machine tool based on the sample test method was proposed, and the optimized design of the stepped axis sample was completed. By inversely calculating the spatial error model of the machine tool, the function analysis relationship between the sample machining error and different error terms of the machine tool was established, and then the dynamic errors of the machine tool in different machining states were obtained. The reliability and practical effects of the method was verified through experiments. The results showed that the proposed IEM method was in good agreement with the laser measurement method. The predictive ability of the error model of the five-axis machine tool can be improved by using the IEM method, thereby more machining accuracy of the machine tool will be ensured.

Keywords Five-axis machine tool · Sample test method · Positioning error · Error model

1 Introduction
The precision and productivity of user demand for the five-axis machining of complex workpiece surfaces is gradually increasing [1]. Accuracy is a crucial consideration for evaluating the capability of multi-axis machine tools. Geometric error is one of the key contributors to the overall errors of machine tools. Geometric error compensation is currently a hot topic of research at home and abroad [2, 3]. There are two main methods for measuring errors in 5-axis machines as the direct measurement and the indirect measurement. Direct measurement means to measure the single error of the five-axis machine tool [4]. Due to too many single error of the five-axis machine tool, direct measurement efficiency is low. Indirect measurement is also known as error identification [5]. By detecting the position of the tool tip and separating the single error by using the machine tool motion model, multiple errors can be obtained in one measurement and the measurement efficiency has been greatly improved.

The traditional indirect measurement methods mainly include ball bar test, laser measurement, plane orthogonal gratting method, and R-Test method [6, 7]. These methods mainly have two types of problems. First, the measurement equipment is expensive and the operation process is complicated and time-consuming, which cannot truly reflect the motion error of the actual machine tool processing. Second, the objective of various measurement methods is to measure quickly and effectively, without considering the actual machining state, and the measurement results are static. In engineering practice, there is a coupling relationship in systems, and the static error can hardly solve the practical problems.

Sample test method [8] can solve the main problems existing in the traditional indirect measurement method and better simulate the actual machining process. Such as Japan put forward four a pyramid sample processing and testing, error motions that have relatively larger influence on circularity, and those that have negligibly small influence, are found out [9]. The sample processing test designed by Mayer et al. [10] identified not only the axis positioning error, but also the maximum number of motion errors of the five-axis machine tool. As shown in Fig. 1, such method indirectly measures the 5-axis machine tool error by simulating or actual cutting of different samples. Due to the full consideration of the cutting condition of the machine tool, the measurement result is dynamic and the measurement accuracy is higher. In this paper, the dynamic error measurement method of the five-axis machine tool was studied on the basis of considering the cutting state.
The dynamic error measurement method of the five-axis machine tool was studied on the premise of considering the cutting state. In order to measure the machine tool error in the actual cutting state, an indirect method of indirect error measurement (IEM) of the five-axis machine tool based on the sample machining was proposed based on the sample design. Through the actual cutting of the sample, the dynamic error of the five-axis machine tool was separated and identified indirectly by the actual cutting of the sample. Different from the general static error measurement in the cold state of the machine tool, the use of sample processing error of different machine tool error indirect separation and identification, does not affect the normal processing process, including the actual processing process of the impact of machine tool error, the measurement result is more close to the actual situation, the measurement accuracy is greatly improved.

2 Design of stepped shaft prototype

The prototype testing method is still in the research stage, and three problems exist. First, the prototype design lacks a uniform standard, such as the prototypes designed by Erkan [11] and Liebrich et al. [12], and the spatial error of the five-axis machine can be obtained by measuring the main ball spacing shown in Fig. 1b. Keaveney [13] and Kato [14] designed a circular table prototype, and by performing simulated cutting, the accuracy of machine motion can be checked. However, there is no optimal prototype structure that can characterize the accuracy of a 5-axis machine. Second, existing tests are still mainly simulating the tool trajectory and are still performed in the cold and no-cut condition of the machine. Ibaraki et al. [15] cut through the sample and separated part of the error of the five-axis machine tool from the machining error of the sample. Hong et al. [16] isolated part of the rotation axis error by processing the designed table. Although they performed actual machining of sample parts, they did not consider the effect of the cutting state on the dynamic error pattern of the 5-axis machine. Third, lack of sensitivity of the sample testing process to the machine error sources, which makes it difficult to separate and identify the machine errors. Ibaraki et al. [17] and Mchicht et al. [18] have explored this problem, but have not obtained an ideal identification model.

Table 1 Sample processing experiment plan

| Serial number | Factor A feed quantity (mm/min) | Factor B spindle speed (r/min) | Factor C radial cutting depth (mm) | Factor D axial cutting depth (mm) | Error of experimental results (mm) |
|---------------|---------------------------------|--------------------------------|-----------------------------------|----------------------------------|----------------------------------|
| 1             | 100                             | 1000                           | 20                                | 0.5                              | 0.0095                           |
| 2             | 100                             | 1500                           | 40                                | 1.0                              | 0.0087                           |
| 3             | 100                             | 2000                           | 60                                | 1.5                              | 0.0093                           |
| 4             | 150                             | 1000                           | 40                                | 1.5                              | 0.0085                           |
| 5             | 150                             | 1500                           | 60                                | 0.5                              | 0.0098                           |
| 6             | 150                             | 2000                           | 20                                | 1.0                              | 0.0086                           |
| 7             | 200                             | 1000                           | 60                                | 1.0                              | 0.0092                           |
| 8             | 200                             | 1500                           | 20                                | 1.5                              | 0.0082                           |
| 9             | 200                             | 2000                           | 40                                | 0.5                              | 0.0081                           |
| $K_1$         | 0.092                           | 0.091                          | 0.088                            | 0.091                            |
| $K_2$         | 0.090                           | 0.089                          | 0.084                            | 0.088                            |
| $K_3$         | 0.085                           | 0.087                          | 0.094                            | 0.087                            |
| $R$           | 0.007                           | 0.004                          | 0.010                            | 0.004                            |

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To test the prototype machining of a 5-axis machine, the stepped-axis prototype was designed as shown in Fig. 2 based on the machine error measurement method in circular trajectory motion. There is an angle of 15° between the table and the work surface. If there is no angle between the table and the work table, but upright on the work table, only need four-axis machining can complete the milling of the table, so it cannot achieve the accuracy of the five-axis machine tool detection, and when the table is tilted, the milling of the cone table is five-axis machining. The φ 16 mm carbide end milling cutter was used as the trial cutting tool. The cutting parameters were the cutting speed of 400 mm/min, the feeding rate of each tooth was about 0.05 ~ 0.1 mm, and the cutting depth was 0.2 mm.

In order to optimize the selection of the sample machining parameters, the author conducted an orthogonal experimental design. This experiment is a four-factor experiment. According to the main factors affecting the machining error (spindle speed, feed, radial depth of cut, and axial depth of cut), L9 (3^4) table is selected for the orthogonal design, and the material is 45 steel as an example. During the machining of the sample part, the radial depth of cut \( A_r \) was selected as 20 mm, 40 mm, and 60 mm, and the axial depth of cut \( A_p \) was set as 0.5 mm, 1.0 mm, and 1.5 mm, and the cutting speed and feed were set with reference to the tool description, and the preliminary experimental scheme is shown in Table 1. After completing a set of experiments with 45 steel samples, experiments were conducted in the same way with Inconel 600 and 6066 aluminum alloys to obtain the optimal processing parameters for different materials.

Table 2 Five-axis CNC machine tool translational axis geometric error elements

| Axis of motion | Roll-away nest error | Rolling error | Elevation error |
|----------------|----------------------|---------------|----------------|
| X \( \varepsilon_x \) | \( \varepsilon_y(x) \) | \( \varepsilon_z(x) \) |
| Y \( \varepsilon_y \) | \( \varepsilon_x(y) \) | \( \varepsilon_z(y) \) |
| Z \( \varepsilon_z \) | \( \varepsilon_x(z) \) | \( \varepsilon_y(z) \) |

Fig. 3 A axis and C axis offset scheme

A functional analytic relationship between sample machining error and 5-axis machine tool error

Determination of Different Circular Machining Trajectory on Sample

Preliminary design A and C axis offset combination

Optimized simulation of A and C axis bias

Whether the condition number of the error identification model Jacobi matrix is reduced?

Y

Output optimized combination of A and C axis bias

N

Delete unqualified A and C axis offset combinations

Finalize shaft offset combination

The basic structure of the double-turntable five-axis machine is set as B0 body. The topological structure and low-order body array of the double-turntable five-axis machine tool can be obtained by numbering each body in turn along the direction away from the machine tool. When the 5-axis machine moves along X-, Y-, and Z-axes, and rotates around A- and C-axes, 6 error elements are generated, respectively, and 33 geometric error elements are added to the verticality error among X-, Y-, and Z-axes. Based on the above error elements, the ideal characteristic matrix and error characteristic matrix of 5-axis machine tool can be obtained by coordinate transformation of the error movement of each axis.
According to the basic principle that the tool coordinate system coincides with the workpiece coordinate system at the machining position, combined with the topological structure of the five-axis machine tool and the low-order body array, the ideal feature matrix and the error feature matrix obtained by solving can be used to obtain the ideal position of the tool forming point in the workpiece coordinate system, which coincides with the workpiece coordinate system. The actual position of the tool forming point in the workpiece coordinate system can be expressed as Eq. (1).

\[
P_{\text{ideal}} = \left[ \prod_{n=L_2(3)=0}^{n=1} T_{L_2(3)L_2^{-1}(3)} \right]^{-1} \left[ \prod_{n=L_2(7)=0}^{n=1} T_{L_2(7)L_2^{-1}(7)} \right] p_t
\]

The actual motion can be regarded as adding an error motion on the basis of the ideal motion, so the actual position of the tool forming point in the workpiece coordinate system can be expressed as Eq. (2):

\[
P_w = \left[ \prod_{n=L_2(\omega)=0}^{n=1} T_{L_2(\omega)L_2^{-1}(\omega)} \right]^{-1} \left[ \prod_{n=L_2(\tau)=0}^{n=1} T_{L_2(\tau)L_2^{-1}(\tau)} \right] p_t
\]

The integrated error model is the difference between the actual and ideal positions of the tool forming point in the workpiece coordinate system \( P_w - P_{\text{ideal}} \). The integrated error model of a five-axis machine along the \( X \)-axis was expressed in Eq. (3).

\[
\Delta x = \sin \gamma \cos \alpha \left\{ \begin{array}{l}
\cos \alpha \sin \gamma + \sin \alpha \cos \gamma + \\
+ \sin \alpha \cos \gamma [\varepsilon_x(z) - \\
\varepsilon_x(z) - \varepsilon_x(y)] - \cos \alpha \cdot \\
[\varepsilon_y(z) - \varepsilon_y(x) - \varepsilon_y(y)] + \\
+ \varepsilon_y S
\end{array} \right.
\]

\[
\sin \alpha \left\{ \begin{array}{l}
\sin \alpha \sin \gamma [\varepsilon_x(z) - \\
\varepsilon_x(x) - \varepsilon_x(y)] - \\
+ \varepsilon_x S \right.
\end{array} \right.
\]

where \( \Delta x \) represents the deviation of the tool forming point along the \( X \)-axis in the workpiece coordinate system. \( x, y, \) and \( z \) represent the position offset of \( X-, \ Y-, \) and \( Z \)-axes, respectively, \( \alpha \) and \( \gamma \) denote the directional shift of the \( A \) and \( C \)-axes, respectively, the remaining parameters related to \( C \),

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**Fig. 4** Five-axis CNC machine tool dynamic error indirect measurement scheme

- blank
- prototype workpiece
- Direct Measurement Method of Machine Tool Dynamic Errors Based on Sample Machining
- Coordinate Measurement Machine
- Evaluate the feasibility and advancement of indirect measurement methods
$S_i$ and $e$, such as $C_{ax}$, $C_{ce}$, and $e_{x}(y)$, all represent different error terms of the five-axis machine. Related parameters are shown in Table 2.

As can be seen from Eq. (3), after establishing a comprehensive error model, the mathematical relationship between the machining error of the sample and the error term of the machine tool is determined. The $x$, $y$, $z$, and $\alpha$ and $\gamma$ in Eq. (3) are regarded as known, and $\Delta x$ can be obtained by sample machining error. By inverting Eq. (3), relevant error terms of the five-axis machine tool can be indirectly identified.

In order to obtain the $x$, $y$, $z$, and $\alpha$, $\gamma$ values required to solve Eq. (3), the moving and rotating axis offsets of the five-axis machine are investigated in this paper. During machining, the $X$-, $Y$-, and $Z$-axis offsets are determined according to the spatial position of the forming point, and the number of offset pairs corresponds to the number of measurement points, which is determined according to the unknown quantity of the equation. Different from moving axes, there is no theoretical basis for the biasing of two rotating axes $A$ and $C$, so the biasing of two rotating axes $A$ and $C$ is mainly designed, and the specific scheme is shown in Fig. 3.

According to the analytical function between the machining error of the sample and the error of the 5-axis machine, different machining trajectories of the sample are determined, and the bias combinations of $A$ and $C$ axes are initially designed according to the different machining trajectories. On this basis, the Jacobi matrix of the error identification model is solved by optimizing the design and simulation processing of the $A$ and $C$ axis biases, and determining whether its condition number is reduced. If the number of conditions decreases, it means that the bias combination is good for machine error identification, otherwise, the bias combination should be deleted and the simulation process of the next bias combination should be optimized. The above process is repeated until the optimal $A$ and $C$ axis bias combination is finally determined.

Since the actual coordinates are measured by the coordinate measuring machine (CMM), the obtained coordinate values are relative to the CMM coordinate system and the $\Delta x$ in Eq. (3) is not the actual machining error of the sample. In order to obtain the actual coordinate values of the sample in the workpiece coordinate system, the spatial posture model between the coordinate system of the measuring machine and the workpiece coordinate system is established. The actual coordinates of the sample in the workpiece coordinate system are obtained and compared with its theoretical coordinates to obtain the required $\Delta x$.

### 3 Experimental verification

The IEM of the five-axis computer numerical control (CNC) machine tools verifies the feasibility and advancement of the indirect measurement method. Machining tests are performed on the prototype parts of the application design, and the machining errors of the prototype parts are measured, and the different error terms of the five-axis CNC machine tools are indirectly separated according to the measurement results. Meanwhile, the relevant error terms of the five-axis machine were measured directly to compare the difference between the direct and indirect measurement results and to verify the advancement of the indirect measurement method, and the measurement scheme is shown in Fig. 4. The coordinate measuring machine used in the experiment is Leitz PMM-C series, and the measuring range is $1200 \times 1000 \times 700$ mm. The laser tracking interferometer is Laser TRACER laser interferometer from Etalon Company.

The prototype machining test is performed on a 5-axis CNC machine, and then the prototype machining error is measured on a coordinate measuring machine, and the indirect measurement method of machine tool dynamic error based on the prototype machining is applied to indirectly
separate and identify the machine tool related error terms with a laser interferometer. Meanwhile, direct measurements are performed on the error terms related to the machine tool. Then, the results of the sample measurements and the direct measurements are compared and the results are shown in Fig. 5. The feed speed is 200 mm/min, and the spindle speed is 1500 r/min.

As can be seen from Fig. 5, the difference between the measurement results of the two methods for the rotational axis A-axis angular positioning error \( \varepsilon \leq 0.0075^\circ \), and the difference between the measurement results of C-axis \( \varepsilon \leq 0.0082^\circ \), it can be seen that there is not much difference between the identification results of the two methods. The positioning errors of X-axis, Y-axis, and Z-axis of the machine tool are measured directly by laser interferometer, and then measured by indirect measurement method. The results of the two methods are shown in Fig. 6, and it can be seen that the measurement results of the two methods match well, so the indirect measurement method is feasible in the measurement of the errors of the five-axis machine tool.

### 4 Conclusion

An IEM method of the five-axis machine tool error based on the sample test method is proposed to indirectly measure the dynamic error of a five-axis machine tool by inverting the spatial error model of the machine tool and establishing the analytic function between the sample machining error and different error terms of the machine tool. The method has the following characteristics: the method is inexpensive and simple to operate, the results of the measured machine tool errors are dynamic, and the measurement accuracy is high. An IEM method of 5-axis machine tool error based on sample test method can measure the dynamic error of machine tool more easily and improve the prediction ability of error model in actual manufacturing process.

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**Availability of data and material** Not applicable.

**Code availability** Not applicable.

**Declarations**

**Conflict of interest** The authors declare no competing interests.
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