Stability Analysis of Systems With Time-Varying Delays via an Improved Integral Inequality

JUNKANG TIAN AND ZERONG REN
School of Mathematics, Zunyi Normal University, Zunyi 563006, China
Corresponding author: Junkang Tian (tianjunkang1980@163.com)

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I. INTRODUCTION

Over the past few decades, stability analysis has been one of hot issues for many dynamic systems such as delayed differential systems [1], nonlinear stochastic networked control systems [2], time-delay systems [3]. For the stability of time-delay systems, the LKF method has been widely used to get stability results by LMI [4]. Choosing LKF [5] and estimating the derivative [6]–[10] are the main factors in leading to conservatism. Therefore, how to establish effective integral inequality techniques for this estimation becomes an important task to get less conservative results for the systems with time-varying delays. The Jensen inequality [11] has been widely used to estimate the bound of the single integral term although it may introduce undesirable conservatism. Recently, to overcome the conservatism, the Wirtinger-based inequality was introduced in [12]. Then, further improvements were proposed by using a free matrix-based integral inequality [13]. More recently, based on Legendre polynomials, some new integral inequalities were derived in [14], which include Jensen and Wirtinger-based inequalities and also the recent inequalities [15], [16] as particular cases. However, these new inequalities were mainly used to the case of constant delays [14], [15]. Very recently, various improved inequalities were proposed to obtain stability criteria for systems with time-varying discrete delays, such as improved Jensen inequality [17], second-order Bessel-Legendre inequality [18], generalized reciprocally convex inequality [19], quadratic-partitioning based inequality [20], generalized free-weighting-matrix based inequality [21], generalized free-matrix-based integral inequality [22], [23]. Among all of the inequalities, the generalized free-matrix-based integral inequality [23] can reduce the conservatism effectively. The relationship between \( \int_c^d (t-c)\frac{k}{2} y(t)dt \) and \( \int_c^d y(t)dt, \int_c^d \int_{u_1}^t y(t)dtdu_1, \int_c^d \int_{u_1}^t \int_{u_2}^t y(t)dtdu_1du_2 \) was not considered in [17], which motivates further investigation.

This paper presents a generalized integral inequality which includes those in [11], [12], [17], [24] as special cases. Based on the generalized integral inequality, a new stability criterion is proposed. An example is introduced to show the superiority of the proposed criterion. The contributions of our paper are as follows:

- The integral \( \int_a^b y^T(s)\Psi(s)ds \) is estimated as \( \int_a^b y^T(s)P\Psi(s)ds \) based on the generalized integral inequality and the new LKF include fourth integrals, which may obtain more general results.

- In this paper, the relationship between \( \int_c^d (t-c)\frac{k}{2} y(t)dt \) \( \int_c^d y(t)dt, \int_c^d \int_{u_1}^t y(t)dtdu_1, \int_c^d \int_{u_1}^t \int_{u_2}^t y(t)dtdu_1du_2 \) is considered.

**Notation:** See TABLE 1.
II. PRELIMINARY

Consider the systems described by

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bx(t - h(t)) \\
\frac{dx(t)}{dt} &= \phi(t), \quad t \in [-h_2, 0]
\end{align*}
\]

where \(x(t) \in \mathbb{R}^n\) is the system state, \(A, B\) are \(n \times n\) constant matrices. The time-varying delay \(h(t)\) satisfies

\[
0 \leq h_1 \leq h(t) \leq h_2, \quad h_3 = h_2 - h_1
\]

Lemma 1 ([17]): For \(k = 0, 1, 2, \ldots\), define

\[
\varphi_{2k}(s) = \left(s - \frac{c + d}{2}\right)^{2k} + \sum_{i=0}^{k-1} a_{2k-i} \left(s - \frac{c + d}{2}\right)^{2i},
\]

\[
\varphi_{2k+1}(s) = \left(s - \frac{c + d}{2}\right)^{2k+1} + \sum_{i=0}^{k-1} b_{2k-i} \left(s - \frac{c + d}{2}\right)^{2i+1},
\]

\[
\forall i = 0, 1, 2, \ldots, k - 1,
\]

\[
\int_c^d \varphi_{2k}(s)\varphi_{2k}(s)ds = \int_c^d \varphi_{2k+1}(s)\varphi_{2k+1}(s)ds = 0
\]

Then, we obtain

\[
\int_c^d x^T(t)Rx(t)dt \geq \sum_{i=0}^1 -\Omega_i^T(x)\Omega_i(x)
\]

where \(p_i = \int_c^d \varphi_i^2(s)ds > 0\) and \(\Omega_i(x) = \int_c^d \varphi_i(s)x(s)ds\)

Lemma 2 ([24]): For a continuously function \(F(y) : [c, d] \rightarrow \mathbb{R}^n\), and any \(\alpha \in (c, d)\), then we obtain

\[
\int_a^b dx_1 \int_a^b dx_2 \ldots \int_a^b F(x_{n+1})dx_{n+1} = \frac{1}{n!} \int_a^b (y - \alpha)^nF(y)dy
\]

Lemma 3 ([24]): For any matrix \(P \in \mathbb{S}^n_+\), and any continuously differentiable function \(y : [a, d] \rightarrow \mathbb{R}^n\), then we can obtain

\[
\int_a^d \dot{y}^T(t)Py(t)dt \geq \frac{1}{d-a} \sum_{i=0}^3 \Omega_i^T P \Omega_i
\]

where

\[
\begin{align*}
\Omega_0 &= y(d) - y(a) \\
\Omega_1 &= y(d) + y(a) - \frac{2}{d-a} \int_a^d y(t)dt \\
\Omega_2 &= y(d) - y(a) + \frac{6}{d-a} \int_a^d y(t)dt - \frac{12}{(d-a)^2} \int_a^d \int_a^d y(t)dtdu \\
\Omega_3 &= y(d) + y(a) - \frac{12}{d-a} \int_a^d y(t)dt + \frac{60}{(d-a)^2} \int_a^d \int_a^d y(t)dtdu
\end{align*}
\]

Lemma 4: For a matrix \(P \in \mathbb{S}^n_+\), and any continuously differentiable function \(y : [a, d] \rightarrow \mathbb{R}^n\), then we can obtain

\[
\int_a^d \dot{y}^T(t)Py(t)dt \geq \frac{1}{d-a} \sum_{i=0}^3 \Omega_i^T P \Omega_i
\]

where \(\Omega_i, i = 0, 1, 2, 3\) are the same as in Lemma 3.

Proof: Based on Lemma 1 and Lemma 3, we obtain

\[
\int_a^d \dot{y}^T(s)P\dot{y}(s)ds \geq \sum_{i=0}^3 \frac{1}{p_i} \Omega_i^T \Omega_i + \frac{1}{p_4} \Omega_4^T \Omega_4
\]

where \(\Omega_i, i = 0, 1, 2, 3\) are the same as in Lemma 3.

By Lemma 1, we have

\[
\tilde{\Omega}_4(y) = \int_a^d \varphi_4(s)\dot{y}(s)ds,
\]

\[
\varphi_4(s) = \left(s - \frac{a + d}{2}\right)^4 - \frac{(d-a)^2}{12},
\]

Then, we calculate the values of \(a_{20}\) and \(a_{21}\).

According to the following two equations

\[
\begin{align*}
\int_a^d \varphi_4(s)\varphi_4(s)ds &= 0 \\
\int_a^d \varphi_4(s)\varphi_2(s)ds &= 0
\end{align*}
\]

We have \(a_{20} = \frac{3}{560(d-a)^4}, a_{21} = -\frac{3}{14}(d-a)^2\),

\[
p_4 = \int_a^d \varphi_4^2(s)ds = \frac{1}{44100}(d-a)^9
\]

Then, by Lemma 2, we have

\[
\tilde{\Omega}_4(y) = \int_a^d \varphi_4(s)\dot{y}(s)ds
\]

\[
= \int_a^d \left[(s - a)^4 - 2(s-a)^3(d-a) + \frac{9}{7}(s-a)^2(d-a)^2 - \frac{2}{7}(s-a)(d-a)^3 + \frac{1}{70}(d-a)^4\right]\dot{y}(s)ds
\]
Substituting (10) and (11) into (8) yields (7). The proof is completed.

Remark 1: The integral \( \int_{a}^{d} \dot{y}(s) R_{i}(s) ds \) is estimated as \( V_{ad}(\dot{y}(s)) \geq \frac{1}{\alpha-a} \Phi(\dot{y}(s)) \geq \frac{1}{\alpha-a} \sum_{i=0}^{\infty} R_{i} \), where \( \Phi(\dot{y}(s)) \) includes those in [11], [12], [17], [24], respectively. In this paper, the integral \( \int_{a}^{d} \dot{y}(s) R_{i}(s) ds \) is estimated as \( V_{ad}(\dot{y}(s)) \geq \frac{1}{\alpha-a} \sum_{i=0}^{\infty} R_{i} \), where \( \Phi(\dot{y}(s)) \) includes those in [11], [12], [17], [24] as special cases. This may yield less conservative results.

Lemma 5 ([19]): For any matrices \( Q \in S_{+}^{n}, M, N \in R^{m \times n}, \gamma \in R^{2n \times m} \), \( \forall \alpha \in (0, 1) \), the inequality

\[
-\gamma^{T} \begin{bmatrix}
\frac{1}{\alpha} Q & 0 \\
0 & 1-\alpha Q
\end{bmatrix} \gamma \\
\leq -\gamma^{T} \sum_{i=1}^{n} \tilde{\mu}_{i}^{2} \dot{y}(s) - \gamma^{T} \begin{bmatrix}
(1-\alpha) M_{1}^{T} \\
\alpha M Q^{-1} M^{T} + (1-\alpha) N Q^{-1} N^{T}
\end{bmatrix} \gamma
\]

holds, where

\[
\sum_{i=1}^{n} \tilde{\mu}_{i}^{2} \dot{y}(s) = \sum_{i=1}^{n} \alpha_{i}^{2} \dot{\mu}_{i}^{2}
\]

III. MAIN RESULTS

Theorem 1: For given scalars \( h_{1}, h_{2} \), system (1) is asymptotically stable if there exist matrices \( P \in S_{+}^{n}, Q_{1}, Q_{2}, Q_{3}, Q_{4} \in S_{+}^{n}, N_{1}, N_{2} \in R^{16n \times 5n} \), such that

\[
\Phi(\alpha) = \begin{bmatrix}
\phi_{1}(\alpha) + \phi_{2}(\alpha) \\
\alpha M_{1}^{T} + (1-\alpha) M_{2}^{T}
\end{bmatrix} < 0
\]

holds for \( \alpha = (0, 1) \), where

\[
\phi_{1}(\alpha) = He(\Sigma_{i=0}^{\infty} P \Sigma_{i+1} + \epsilon_{i}^{T} Q_{i} \epsilon_{i} - \epsilon_{i}^{T} Q_{i} \epsilon_{i+1} + \epsilon_{i}^{T} Q_{i} \epsilon_{i+2})
\]

\[
\phi_{2}(\alpha) = \sum_{i=0}^{4} (2i+1) \Sigma_{i+3}^{T} Q_{i} \Sigma_{i+3}
\]

\[
\Sigma(\alpha) = \begin{bmatrix}
\epsilon_{i}^{T} Q_{i} \epsilon_{i} + h_{1}^{2} \epsilon_{i}^{T} Q_{i} \epsilon_{i} + h_{2}^{2} \epsilon_{i}^{T} Q_{i} \epsilon_{i}
\end{bmatrix}
\]

and \( \epsilon_{i} \in R^{n \times 1} \) is defined as

\[
\epsilon_{i} = \begin{bmatrix}
0_{n \times (i-1)n} & I_{n} & 0_{n \times (16-i)n}
\end{bmatrix}
\]

for \( i = 1, 2, \ldots, 16 \).

Proof: Introduce a LKF given by

\[
V(x_{i}) = \eta^{T}(t) P_{i}(t) + \int_{t_{i-\delta_{i}}}^{t_{i}} x^{T}(s) Q_{1} x(s) ds + \int_{t_{i-\delta_{i}}}^{t_{i}} x^{T}(s) Q_{2} x(s) ds
\]

\[
+ \int_{t_{i-\delta_{i}}}^{t_{i}} x^{T}(s) Q_{3} x(s) ds + \int_{t_{i-\delta_{i}}}^{t_{i}} x^{T}(s) Q_{4} x(s) ds
\]

where

\[
\eta(t) = \begin{bmatrix}
\epsilon_{1}^{T}(t) & \int_{t_{i-\delta_{i}}}^{t_{i}} x^{T}(s) ds & \int_{t_{i-\delta_{i}}}^{t_{i}} x^{T}(s) ds & v_{i}^{T}(t)
\end{bmatrix}
\]

\[
v_{i}^{T}(t) = \begin{bmatrix}
\int_{t_{i-\delta_{i}}}^{t_{i}} x^{T}(s) ds & v_{i}^{T}(t)
\end{bmatrix}
\]
\[
\begin{align*}
v_1(t) &= \int_{t-h_1}^{t} \int_{t-h_2}^{t} x(s)ds du_v \\
v_2(t) &= \int_{t-h_1}^{t} \int_{t-h_2}^{t} y(s)ds du_{v_2} du_1 \\
v_3(t) &= \int_{t-h_1}^{t} \int_{t-h_2}^{t} \int_{t-h_3}^{t} y(s)ds du_{v_3} du_2 du_1
\end{align*}
\]

The derivative of \(V(x(t))\) is
\[
\dot{V}(x(t)) = 2\eta^T(t)P\dot{y}(t) + \eta^T(t)Q_1x(t) - x^T(t)(t-h_1)Q_1x(t-h_1) + x^T(t-h_1)Q_2x(t-h_1) - x^T(t-h_2)Q_2x(t-h_2) + h_1\dot{x}(t)Q_3\dot{x}(t) + h_2\dot{x}(t)Q_4\dot{x}(t)
\]
\[
- h_1 \int_{t-h_1}^{t} \hat{x}(t)Q_3\hat{x}(t)ds \\
- h_1 \int_{t-h_2}^{t} \hat{x}(t)Q_4\hat{x}(t)ds
\]

Then, we obtain
\[
\begin{align*}
\dot{V}(x(t)) &= \xi^T(t) \left[ H(e^{1}\Sigma^1 P \Sigma_2) + e^{2}_1 Q_1 e_1 - e^{2}_2 Q_1 e_2 + e^{2}_2 Q_2 e_2 \\
&\quad - e^{2}_2 Q_2 e_4 + h_1^2 + e^{2}_0 Q_3 e_0 + h_2^2 + e^{2}_0 Q_4 e_0 \right] \xi(t) \\
&\quad - h_1 \int_{t-h_1}^{t} \hat{x}(t)Q_3\hat{x}(t)ds \\
&\quad - h_1 \int_{t-h_2}^{t} \hat{x}(t)Q_4\hat{x}(t)ds
\end{align*}
\]

where
\[
\xi(t) = \begin{bmatrix}
x^T(t) & x^T(t-h_1) & x^T(t-h_2)
\end{bmatrix}^T
\]
\[
\rho_1(t) = h(t) - h_1, \quad \rho_2(t) = h_2 - h(t)
\]
\[
\phi_1(t) = \begin{bmatrix}
\frac{1}{\rho_1(t)} \int_{t-h_1}^{t-h_1} x^T(s)ds & \frac{1}{\rho_2(t)} \int_{t-h_2}^{t-h_2} x^T(s)ds & 0 & 0
\end{bmatrix}^T
\]
\[
\phi_2(t) = \begin{bmatrix}
\frac{1}{\rho_1(t)} \int_{t-h_1}^{t-h_1} x^T(s)dsdu_v & \frac{1}{\rho_2(t)} \int_{t-h_2}^{t-h_2} x^T(s)dsdu_v & 0 & 0
\end{bmatrix}^T
\]
\[
\phi_3(t) = \begin{bmatrix}
\frac{1}{\rho_1(t)} \int_{t-h_1}^{t-h_1} \int_{t-h_1}^{t-h_1} y^T(s)dsdu_v & \frac{1}{\rho_2(t)} \int_{t-h_2}^{t-h_2} \int_{t-h_2}^{t-h_2} y^T(s)dsdu_v & 0 & 0
\end{bmatrix}^T
\]
\[
\phi_4(t) = \begin{bmatrix}
\frac{1}{\rho_1(t)} \int_{t-h_1}^{t-h_1} \int_{t-h_1}^{t-h_1} \int_{t-h_1}^{t-h_1} y^T(s)dsdu_v & \frac{1}{\rho_2(t)} \int_{t-h_2}^{t-h_2} \int_{t-h_2}^{t-h_2} \int_{t-h_2}^{t-h_2} y^T(s)dsdu_v & 0 & 0
\end{bmatrix}^T
\]

Let \(\alpha = \frac{h(t)-h_1}{h_2}\), applying Lemma 4, we can get
\[
\begin{align*}
\int_{t-h_1}^{t} \hat{x}(t)Q_3\hat{x}(t)ds & \leq -\xi^T(t)\left( \sum_{i=0}^{4} (2i+1)\Sigma_{i+3}^{T} Q_3 \Sigma_{i+3} \right) \xi(t) \\
&\quad - h_1 \int_{t-h_1}^{t} \hat{x}(t)Q_3\hat{x}(t)ds \\
&\quad - h_1 \int_{t-h_1}^{t} \hat{x}(t)Q_4\hat{x}(t)ds
\end{align*}
\]

By Lemma 5, we can obtain
\[
\begin{align*}
-\gamma^T &\begin{bmatrix}
\frac{1}{\alpha} & 0 \\
0 & 1 - \alpha
\end{bmatrix} \gamma \\
&\leq \phi_2(\alpha) + \alpha M_1 \Theta^{-1} M_1^T + (1 - \alpha) M_2 \Theta^{-1} M_2^T \\
&= \eta(\alpha)
\end{align*}
\]

From (15)-(18), the \(V(x(t))\) can be estimated as
\[
\dot{V}(x(t)) \leq -\xi^T(t)(\phi_1(\alpha) + \eta(\alpha))\xi(t)
\]

If the LMI (13) is verified for \(\alpha = 0, 1\), then the inequality \(\phi_1(\alpha) + \eta(\alpha) < 0\) holds for all \(\alpha \in (0, 1)\). This completes the proof.

**Remark 2:** A new LKF which contains a fourth integral is chosen to derive novel stability results. The \(\int_{t-h_1}^{t} \int_{t-h_2}^{t} \int_{t-h_3}^{t} y^T(s)dsdu_v du_1 du_2 du_3\) is added as a state vector, which may obtain more general results.

**IV. A NUMERICAL EXAMPLE**

A numerical example is given to demonstrate advantages of the proposed criterion.

**Example 1:** Consider system (1) with:
\[
A = \begin{bmatrix}
-2.0 & 0 \\
0 & -0.9
\end{bmatrix}, \quad B = \begin{bmatrix}
-1.0 & 0.0 \\
-1.0 & -1.0
\end{bmatrix}
\]

Table 1 presents the admissible upper bound of \(h_2\) for different \(h_1\). From table 1, one can conclude that the theorem 1 is
less conservative than those in [3], [14]–[16], [18], [23]. For $h_2 = 2.44$, $x(0) = (0.01, -0.01)^T$, the state responses of the system (1) are given in Figure 1.

V. CONCLUSION

This paper focuses on the stability of systems with time-varying delays. By using a new augmented LKF and combined with a generalized integral inequality, a new stability criterion is obtained. Both the generalized integral inequality and the new augmented LKF include fourth integrals, which may obtain more general results. A numerical example is given to show the effectiveness of the proposed criterion. In the future work, the proposed stability approach can be applied to other dynamic systems such as a singular system and a neural network system.

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**Junkang Tian** received the B.S. degree from Leshan Normal University, Leshan, China, in 2004, and the M.S. and Ph.D. degrees from the University of Electronic Science and Technology of China, Chengdu, China, in 2007 and 2013, respectively. He is currently an Associate Professor with Zunyi Normal University, Zunyi, China. His current research interests include system and control theory, networked control systems, robust control, and nonlinear systems.

**Zerong Ren** received the B.S. degree from Leshan Normal University, Leshan, China, in 2004, and the M.S. degree from Southwest Petroleum University, Chengdu, China, in 2014. She is currently a Lecturer with Zunyi Normal University, Zunyi, China. Her current research interests include time-delay systems, impulsive systems, and complex networks.

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