QCD Sum Rule Calculation for the Tensor Charge of the Nucleon

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Abstract

The nucleon’s tensor charges (isovector $g^T_v = \delta u - \delta d$ and isoscalar $g^T_s = \delta u + \delta d$) are calculated using the QCD sum rules in the presence of an external tensor field. In addition to the standard quark and gluon condensates, new condensates described by vacuum susceptibilities are induced by the external field. The latter contributions to $g^T_v$ and $g^T_s$ are estimated to be small. After deriving some simplifying formulas, a detailed sum rule analysis yields $g^T_v = 1.29 \pm 0.51$ and $g^T_s = 1.37 \pm 0.55$, or $\delta u = 1.33 \pm 0.53$ and $\delta d = 0.04 \pm 0.02$ at the scale of 1 GeV$^2$.

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I. INTRODUCTION

Studying nucleon charges such as the baryon charge, the axial charge, and the tensor charge from the underlying theory of the strong interactions, quantum chromodynamics (QCD), is obviously important to understand the properties of the nucleon as well as the QCD at low energies. On one hand, these nucleon charges, defined as matrix elements of bilinear fields ($\bar{\psi}\gamma^\mu \psi$ for baryon charge, $\bar{\psi}\gamma^\mu \gamma^5 \psi$ for axial charge, and $\bar{\psi}\sigma^{\mu\nu} \psi$ for tensor charge), are fundamental observables characterizing properties of the nucleon. On the other hand, these nucleon charges are connected through deep-inelastic sum rules to the leading-twist nucleon structure functions—the spin averaged structure function $f_1(x)$, the quark helicity distribution $g_1(x)$ [1] and the transversity distribution $h_1(x)$ [2, 3]. For example, the first moment of $g_1(x)$ is connected to the axial charge $g_A$ by the Bjorken sum rule [1]. Recently, Jaffe and Ji [3] showed that the first moment of $h_1(x)$ is related to the tensor charge $\delta\psi(\psi = u, d...)$ of the nucleon through

$$\int_{-1}^{1} h_1(x) dx = \int_{0}^{1} (h_1(x) - \bar{h}_1(x)) dx = \delta\psi .$$

(1)

Thus, the tensor charge, like the axial charge, is one of the important nucleon observables.

So far, there is yet no experimental data on $h_1(x)$ and on the tensor charge because $h_1(x)$ does not manifest itself in inclusive deep inelastic process and there are no fundamental probes which couple directly to the tensor current. A first measurement of $h_1(x)$ may be performed at HERA with semi-inclusive deep-inelastic scattering and at RHIC with transversely polarized Drell-Yan process [4], from which one may extract the value of the tensor charge. Therefore, theoretical study of the tensor charge of the nucleon in QCD is very important.

In Ref. [5], we sought to understand the physical significance of the tensor charge and made estimates in the MIT bag model and the QCD sum rule approach. The QCD sum rule approach [6] has been used extensively in the past as a useful tool to study hadron masses, coupling constants, form factors, etc. (see [7] for a review). To evaluate the tensor charge and other charges, one makes an extension of the standard QCD sum rules [6] to avoid infrared divergences arising from zero-momentum transfer in bilinear quark operators. There are several equivalent formulations of the extension: the two-point correlation function in an external field, i.e., the external field QCD sum rule approach [8], and the three-point function approach [9]. In Ref.[5], we used the three-point function approach to calculate the tensor charge, including operators up to dimension 6.

In the present paper we refine and extend our work in Ref. [5], in particular, we now include operators of dimension 8. To simply the calculation, we use instead the QCD sum
rule approach in external field [8]. The basic idea is this: Consider propagation of the nucleon current in the presence of an external tensor field $Z_{\mu\nu}$. The two-point correlation function of the nucleon current is calculated up to terms linearly proportional to $Z_{\mu\nu}$, and the coefficient of which is the linear response of the correlation function to the external $Z_{\mu\nu}$ field. The linear response is first calculated in terms of vacuum properties through an operator product expansion (OPE) [10]. In addition to the standard vacuum condensates [6], there are new condensates induced by the field $Z_{\mu\nu}$. The latter represents the response of the QCD vacuum to the external field and can be described by vacuum susceptibilities. The susceptibility terms are exactly equivalent to the bi-local contributions in the three-point correlation function approach [5]. On the other hand, the linear response can be expressed in terms of properties of physical intermediate hadron states, where the tensor charge enters. By matching the two calculations in the certain momentum range, we extract the tensor charge.

We first look at the sum rules in which there are no contributions of vacuum susceptibilities. By eliminating the nucleon coupling constant through the sum rule for mass [11], we obtain the simplifying formulas of the tensor charges,

$$
\delta u = -\frac{4(2\pi)^2\langle \bar{q}q \rangle}{m_N^3} \left(1 - \frac{9m_o^2}{16m_N^2}\right),
$$

$$
\delta d = \frac{\langle g^2_G G^2 \rangle}{36m_N^4}
$$

\text{at the scale } \mu^2 = M_N^2. [The leading-log evolution of the tensor charge is}

$$
g^\alpha_T(\mu^2) = [\alpha(\mu^2)/\alpha(\mu_0^2)]^{3\alpha-2\alpha_L} g^\alpha_T(\mu_0^2)
$$

\text{where indices } \alpha = v, s, u, d \text{ specify } g^v_T = \delta u, g^d_T = \delta d, g^s_T = \delta u - \delta d, \text{ and } g^u_T = \delta u + \delta d, \text{ and } n_f \text{ is the flavor number.]} \text{ Substituting } \langle \bar{q}q \rangle = -(240 \text{MeV})^3, \langle g^2_G G^2 \rangle = 0.47 \text{GeV}^4, m_0^2 = -\langle \bar{q}q c \cdot Gq / \langle \bar{q}q \rangle \approx 0.8 \text{GeV}^2, \text{ and } m_N = 0.94 \text{GeV}, \text{ we get } \delta u = 1.29, \delta d \approx 0.02, g^v_T = 1.27 \text{ and } g^u_T = 1.31. \text{ The uncertainty from the vacuum condensates alone is at the level of 20%}. \text{ In order to extract these tensor charges with a better accuracy, we analyze the sum rules in a more careful way. The result is consistent with the simplifying analysis presented above.}

The paper is organized as follows. In Sec. II we derive the nucleon sum rules in the presence of an external tensor field. The sum rules are analysed in Sec. III, where we first derive the simplifying formulas for the tensor charges, then estimate the vacuum susceptibilities, and finally make a detailed sum rule analysis. The summary and conclusions are given in the last section.
II. QCD Sum Rules for the Nucleon in An External Tensor Field

To derive the QCD sum rules, we consider the two-point correlation function in the presence of an external constant tensor field $Z_{\mu\nu}$,

$$\Pi(Z_{\mu\nu}, p) = i \int d^4 x e^{i p x} \langle 0 | T(\eta(x)\bar{\eta}(0)) | 0 \rangle Z,$$

where $\eta$ is the interpolating current for a proton [11],

$$\eta(x) = \epsilon_{abc}[u_a^T(x)C\gamma_\mu u_b(x)]\gamma_5 \gamma_\mu d_c(x),$$

$$\langle 0 | \eta(0) | N(p) \rangle = \lambda_N v_N(p),$$

where $a$, $b$ and $c$ are color indices, superscript $T$ means transpose, $C$ is the usual charge-conjugation matrix, and $v_N(p)$ the nucleon spinor normalized to $\bar{v} v = 2m_N$. To construct the sum rule for tensor charges, we expand the correlation function (3) to the first order in $Z_{\mu\nu}$,

$$\Pi(Z_{\mu\nu}, p) = \Pi_0(p) + Z_{\mu\nu}\Pi_{\mu\nu}(p) + \ldots,$$

where $\Pi_0$ is the correlation function which can be used to construct the nucleon-mass sum rules. In the following, we focus on the linear response function $\Pi_{\mu\nu}(p)$.

The QCD sum rule is based on the principle of duality, which postulates the correspondence between a description of correlation functions in terms of hadronic degrees of freedom (phenomenological side) and that in quark and gluon degrees of freedom (QCD theory side). The phenomenological representation of the correlation function in an external $Z_{\mu\nu}$ field can be expressed in terms of physical intermediate states as

$$\Pi(Z_{\mu\nu}, p) = \frac{\lambda_N^2}{(p^2 - m_N^2)} g_T Z_{\mu\nu}[\hat{p}\sigma^{\mu\nu}\hat{p} + m_N^2 \sigma^{\mu\nu} + m_N \{\hat{p}, \sigma^{\mu\nu}\}] + \ldots,$$

where the nucleon’s tensor charge is defined by

$$\langle N(p, s) | J_{\mu\nu}^{v(s)}(0) | N(p, s) \rangle = g_T T^{v(s)}_{\mu\nu} u^\dagger \sigma_{\mu\nu} u + d^\dagger \sigma_{\mu\nu} d + \ldots,$$

where $J_{\mu\nu}^{v(s)} = \bar{\psi}_\alpha \sigma_{\mu\nu} \psi_\alpha$ with $\alpha = v, s, u, d$, specifying $J_{\mu\nu}^{v(s)} = \bar{u} \sigma_{\mu\nu} u$ or $d^\dagger \sigma_{\mu\nu} d$.

The theoretical side of the correlation function (3) can be computed in the deep Euclidean region using OPE. Matching the two results obtained above, we can extract the tensor charges in terms of QCD parameters and vacuum condensates.

A. Operator Product Expansion
We now compute the OPE for the nucleon-current correlation function in an external tensor field $Z_{\mu\nu}$. The coupling between quarks and the external antisymmetric tensor field $Z_{\mu\nu}$ is described by an additional term

$$\Delta \mathcal{L} = g_q \bar{q} \sigma^{\mu\nu} q Z_{\mu\nu}$$

in QCD Lagrangian, where the coupling constant $g_q$ depends on the quark type as well as the field $Z_{\mu\nu}$ ($g_q = g_u = -g_d$ for isovector type coupling, $g_q = g_u = g_d$ for isoscalar type). The equation of motion for quarks is now

$$(i\dot{D} + g_q Z_{\mu\nu} \sigma^{\mu\nu})q = 0,$$

where $D_\mu = \partial_\mu + ig_A \frac{\lambda^a}{2}, g_c$ is the QCD gauge coupling, $\lambda^a$ are the Gell-Mann matrices, and current masses of up and down quarks are neglected.

To calculate the Wilson coefficients in OPE [10], we construct the coordinate-space quark propagator in the presence of the external field $Z_{\mu\nu}$. Following the method of Refs.[8] and [12], we find

$$S_{ij}^{ab} = \langle 0 | T q_i^a(x) q_j^b(0) | 0 \rangle$$

$$= \langle 0 | T q_i^a(x) \hat{e}_j^b(0) | 0 \rangle$$

$$= \frac{i\delta^{ab}(\hat{e})_{ij}}{2\pi^2 x_i^4} - \frac{\delta^{ab}}{4\pi^2 x_i^4} g_q [x^2 Z_{\mu\nu}(\sigma^{\mu\nu})_{ij} - 4x^\nu x_\rho Z_{\mu\nu}^{ab}(\sigma^{\mu\nu})_{ij}]$$

$$+ \frac{i}{32\pi^2 x_i^4} g_c \frac{\lambda^a}{2} G_{\mu\nu}^m (\hat{x} \sigma^{\mu\nu} + \sigma^{\mu\nu} \hat{x})_{ij}$$

$$- \frac{1}{12} \delta^{ab} \delta_{ij} \langle 0 | \bar{q} q | 0 \rangle - \frac{\delta^{ab}}{24} Z_{\mu\nu} g_q \chi \langle \bar{q} q | (\sigma^{\mu\nu})_{ij} \rangle$$

$$- \frac{i\delta^{ab}}{48} g_q \langle \bar{q} q | Z_{\mu\nu} (\hat{x} \sigma^{\mu\nu} + \sigma^{\mu\nu} \hat{x})_{ij} \rangle + \frac{\delta_{ab}}{192} \delta_{ij} x^2 \langle 0 | \bar{q} g_c \sigma \cdot G q | 0 \rangle$$

$$+ \frac{\delta^{ab}}{288} g_q \langle \bar{q} q | Z_{\mu\nu} [x^2 (\kappa - 2\zeta) (\sigma^{\mu\nu})_{ij} + 2x^\nu x_\rho (\kappa + \zeta) (\sigma^{\mu\nu})_{ij}] \rangle + \cdots.$$ \hspace{1cm} (11)

The first three terms in Eq.(11) are the perturbative propagator for a massless free quark and its interactions with the external $Z_{\mu\nu}$ field and the vacuum gluon field $G_{\mu\nu}^{a}$.

The next five terms are non-perturbative, arising from the quark and the mixed quark-gluon condensates, and the condensates induced by the external field $Z_{\mu\nu}$. In fact, due to breakdown of Lorentz invariance, the vacuum expectation values $\langle \bar{q} \sigma_{\mu\nu} q \rangle, \langle \bar{q} g_c \frac{\lambda^a}{2} G_{\mu\nu}^a q \rangle, \langle \bar{q} g_c \gamma_5 \tilde{G}_{\mu\nu} q \rangle$, where $\tilde{G}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma} \lambda^a \frac{1}{2}$, become non-zero. To characterize these vacuum response, we define the induced susceptibilities $\chi, \kappa, \zeta$ through

$$\langle 0 | \bar{q} \sigma_{\mu\nu} q | 0 \rangle z = g_q \chi Z_{\mu\nu} \langle \bar{q} q \rangle,$$

$$\langle 0 | \bar{q} g_c \frac{\lambda^a}{2} G_{\mu\nu}^a q | 0 \rangle z = g_q \kappa Z_{\mu\nu} \langle \bar{q} q \rangle,$$

$$\langle 0 | \bar{q} g_c \gamma_5 \tilde{G}_{\mu\nu} q | 0 \rangle z = -i g_q \zeta Z_{\mu\nu} \langle \bar{q} q \rangle.$$ \hspace{1cm} (12)
Using

\[ \langle 0 | T(\eta(x)\bar{\eta}(0)) | 0 \rangle = -2\epsilon^{abc}\epsilon^{\mu\nu\lambda\rho} Tr \{ S(x)^{b \mu}_u \gamma_\nu C S(x)^{a \nu}_v C \gamma_\mu \gamma_5 \gamma_5 S(x)^{c \rho}_w \gamma_5 \gamma_5 \}, \] (13)

and Eq. (11), it is straightforward to carry out the OPE. The corresponding Feynman diagrams are shown in Figs. 1 and 2. The results for the correlation function (3), including the contributions up to dimension 8, is

\[ \Pi(Z_{\mu\nu}, p) = Z_{\mu\nu} [ W_1 \hat{p} \sigma^{\mu\nu} \hat{p} + W_2 \sigma^{\mu\nu} + W_3 \{ \hat{p}, \sigma^{\mu\nu} \} + \cdots ] \] (14)

where

\[ W_1 = -\frac{g_d}{24\pi^2} \chi(\bar{q}q) \ln(-p^2) + \frac{g_d}{16\pi^2 p^2} \zeta(\bar{q}q) + \frac{g_d}{576\pi^2 p^4} \chi(\bar{q}q) \langle g_c^2 G^2 \rangle / \mu^2 \]

\[ + \frac{2g_u}{p^4} \langle \bar{q}q \rangle^2 - \frac{3g_u}{2p^2} \langle \bar{q}q \rangle \langle g_c \sigma Gq \rangle, \] (15)

\[ W_2 = \frac{g_d}{32\pi^4} p^4 \ln(-p^2) + \frac{g_d}{24\pi^2} (4\kappa + \zeta) \langle \bar{q}q \rangle \ln(-p^2) \]

\[ + \frac{g_d}{384\pi^4} \langle g_c^2 G^2 \rangle \ln(-p^2) - \frac{2g_u}{3p^2} \langle \bar{q}q \rangle^2, \] (16)

\[ W_3 = \frac{g_u}{2\pi^2} \langle \bar{q}q \rangle \ln(-p^2) + \frac{g_u}{24\pi^2 p^2} \ln(-p^2/\Lambda^2) \langle \bar{q}q, \sigma Gq \rangle \]

\[ - \frac{g_u}{3p^2} \chi(\bar{q}q)^2 + \frac{g_u}{24p^4} \chi(\bar{q}q) \langle g_c \sigma Gq \rangle \]

\[ - \frac{g_u}{18p^4} (\kappa + 4\zeta) \langle \bar{q}q \rangle^2 - \frac{7g_u + g_d}{288\pi^2 p^4} \langle \bar{q}q \rangle \langle g_c^2 G^2 \rangle. \] (17)

### B. Sum Rules

The QCD sum rules are obtained by equating the OPE results in Eqs.(14)–(17) and the phenomenological description in Eq.(7), and performing Borel transformation. The renormalization scale dependence is also included properly. There are three different Dirac structures: chiral-odd $Z_{\mu\nu} \hat{p} \sigma^{\mu\nu} \hat{p}$ and $Z_{\mu\nu} \sigma^{\mu\nu}$, and chiral-even $Z_{\mu\nu} \{ \hat{p}, \sigma^{\mu\nu} \}$, each of which can be used to construct a sum rule and extract the tensor charges. Note that the phenomenological representation of $\Pi(Z_{\mu\nu}, p)$ in Eq.(7) contains the double-pole as well as various single-pole contributions. The latter terms can be treated in several ways. Here we eliminate them by multiplying both sides of the sum rules by $m_N^2 - p^2$ before Borel transformation. We find the sum rule for the structure $Z_{\mu\nu} \hat{p} \sigma^{\mu\nu} \hat{p}$,

\[ - \frac{g_d\chi a}{24\Lambda^4/9} M^2(m_N^2 E_0 - M^2 E_1) + \frac{g_u\zeta a}{16} m_N^2 L^{1/27} - \frac{g_d\chi b}{576\Lambda^4/9} (1 + \frac{m_N^2}{M^2}) \]

\[ + \frac{g_u a^2}{2} (1 + \frac{m_N^2}{M^2}) L^{16/27} - \frac{3g_u m_N^2 a^2}{8M^2} (1 + \frac{m_N^2}{2M^2}) L^{11/27} \]

\[ = g_d^2(\mu_0) \beta_N^2 e^{-m_N^2/M^2}, \] (18)
for $Z_{\mu\nu\sigma^\mu_\nu}$, 
\[
-\frac{g_d}{4} M^6 (m_N^2 E_2 - 3 M^2 E_3) + \frac{g_d}{24} (4\kappa + \zeta) a M^4 (m_N^2 E_1 - 2 M^2 E_2)
-\frac{g_d}{96} b M^2 (m_N^2 E_0 - M^2 E_1) + \frac{g_u}{6} a^2 m_N^2 
= g_T^\alpha (\mu_0) m_N^2 \beta_N^2 e^{-m_N^2/M^2},
\]
and for $Z_{\mu_\nu} \{\hat{p}, \sigma^\mu_\nu\}$, 
\[
g_u a \frac{1}{2} M^2 (m_N^2 E_0 - M^2 E_1) L^{4/27} + \frac{g_u m_N^2 m_0^2 a}{24 L^{8/27}} (\ln(m_N^2/\Lambda^2) - 1)
-\frac{g_u}{96 L^{4/27}} \chi a^2 (1 + \frac{m_N^2}{M^2}) - \frac{g_u}{72} (\kappa + 4\zeta) a^2 (1 + \frac{m_N^2}{M^2}) L^{16/27}
+\frac{g_u}{12} \chi a^2 + \frac{1}{288} (7 g_u + g_d) ab (1 + \frac{m_N^2}{M^2}) L^{4/27}
= g_T^\alpha (\mu_0) m_N^2 \beta_N^2 e^{-m_N^2/M^2}.
\]
Here $a = -(2\pi)^2 \langle \bar{q}q \rangle$, $b = \langle g_\sigma^2 G^2 \rangle$, $m_0^2 a = (2\pi)^2 \langle \bar{q} g_\sigma q \cdot G q \rangle$, $\beta_N^2 = (2\pi)^4 \lambda_N^2/4$, and $M$ is the Borel mass. The factors $E_n = 1 - \sum x^n e^{-x}$ with $x = s_0/M^2$ account for the sum of the contributions from excited states, where $s_0$ is an effective continuum threshold. The anomalous dimension of various operators, including that of the tensor current, is taken into account through the factor $L = \ln(M^2/\Lambda^2)/\ln(\mu_0^2/\Lambda^2)$ [6, 11], where $\mu_0$ is the initial renormalization scale (500 MeV) and $\Lambda = \Lambda_{QCD}$ is the QCD scale parameter (100 MeV). The tensor charges $g_T^\alpha$ in Eqs.(18)-(20) are renormalized at the scale $\mu_0$.

**III. Sum Rule Analysis**

Let us now analyse the sum rules derived in the previous sections and extract the tensor charges $g_T^v, g_T^s$ and $\delta u, \delta d$. In principle, one expects to obtain the same result from each of the sum rules in Eqs.(18)-(20). In practice, however, some sum rules work better than the others, as was evident in similar studies [8, 14, 15]. In our case, the sum rule in Eq. (18) from the structure $\hat{p} \sigma^\mu_\nu \hat{p}$ is preferred for the following reasons. Firstly, from Eqs.(14)-(17) the structure $\hat{p} \sigma^\mu_\nu \hat{p}$ contains extra powers of momentum in the numerator compared with structures $\sigma^\mu_\nu$ and $\{\hat{p}, \sigma^\mu_\nu\}$. Hence, the resulting sum rule converges better after Borel transformation. Secondly, the sum rule in Eq. (18) is known better phenomenologically, because the terms involving the poorly-known susceptibilities are numerically small compared with the dominant dimension-6 and 8 terms. Hereafter, we discard the sum rule in Eq. (19).

**A. Simple Formulas for the Tensor Charges**
Consider the sum rules (18) for the structure \( \hat{p}\sigma^{\mu\nu}\hat{p} \). As a first approximation, we disregard the anomalous dimensions, let \( M = m_N \), and use \( \beta_N^2e^{-m_N^2/M^2}|_{M=m_N^2} = -\pi^2m_N^3\langle \bar{u}u \rangle \) from the simplified mass sum rules [11]. In the case of \( g_u = 1, g_d = 0 \), we find a simple formula for the tensor charge \( \delta u \),

\[
\delta u = -\frac{4(2\pi)^2\langle \bar{q}q \rangle}{m_N^3}(1 - \frac{9m_0^2}{16m_N^2}).
\] (21)

Similarly, from the sum rule (20) with the structure \( \{\hat{p},\sigma^{\mu\nu}\} \), we get

\[
\delta d = \frac{(g_s^2G^2)}{36m_N^4}.
\] (22)

The result obtained above is independent of the susceptibilities \( \chi, \kappa \) and \( \zeta \). Thus, one can get a simple estimate of the tensor charges without knowing them. Substituting the vacuum condensates into the above equations, we find \( \delta u \approx 1.29, \delta d \approx 0.02, \) or \( g_v T \approx 1.27 \) and \( g_s T \approx 1.31 \) (at \( \mu^2 = m_N^2 \)). Interestingly, \( \delta d \) is much smaller than \( \delta u \), or isovector \( g_v T \) and isoscalar \( g_s T \) tensor charges have similar sizes.

**B. Estimates of Susceptibilities**

To make a more detailed analysis, we need the values of the susceptibilities \( \chi, \kappa \), and \( \zeta \) defined in Eq.(12). These susceptibilities correspond to the bi-local contributions in the three-point correlation function approach [5, 9]. In fact, using Eq. (9), it is easy to show

\[
\langle 0|\bar{q}\sigma^{\mu\nu}q|0\rangle_Z = g_qZ_{\mu\nu}\lim_{p_\lambda \rightarrow 0} i\frac{i}{6}\int d^4x e^{ipx}\langle 0|T(\bar{q}\sigma_{\alpha\beta}q(x), \bar{q}\sigma^{\alpha\beta}q)|0\rangle .
\] (23)

Combining with Eq.(12), we obtain

\[
\chi(\bar{q}q) = \lim_{p_\lambda \rightarrow 0} i\frac{i}{6}\int d^4x e^{ipx}\langle 0|T(\bar{q}\sigma_{\alpha\beta}q(x), \bar{q}\sigma^{\alpha\beta}q)|0\rangle = \frac{1}{6}\Pi_\chi(0) ,
\] (24)

where \( \Pi_\chi(0) \) is a bi-local correlator defined in the three-point correlation function approach (see Eq.(23) of Ref.[5]). Similarly, we have

\[
\kappa(\bar{q}q) = \frac{i}{6}\int d^4x\langle 0|T(\bar{q}g_{\gamma_5}\tilde{G}_{\alpha\beta}q, \bar{q}\sigma^{\alpha\beta}q)|0\rangle = \frac{1}{6}\Pi_\kappa(0) ,
\] (25)

\[
\zeta(\bar{q}q) = -\frac{1}{6}\int d^4x\langle 0|T(\bar{q}g_{\gamma_5}\tilde{G}_{\alpha\beta}q, \bar{q}\sigma^{\alpha\beta}q)|0\rangle = -\frac{1}{6}\Pi_\zeta(0) .
\] (26)

Substituting the above expressions for \( \chi, \kappa \) and \( \zeta \) into Eqs.(15)-(17) and (18)-(20), we obtain the same result obtained in the three-point correlation function approach [5].

Eq.(24)-(26) provide useful formulas to determine the susceptibilities \( \chi, \kappa \) and \( \zeta \). To find \( \chi \), we assume \( \rho(1^-) \) and \( B(1^{+-}) \) meson dominance in Eq. (23). Estimating the coupling
constants of relevant currents with the meson states again using the QCD sum rules, we get
\[ \Pi_\chi(0) \approx (0.15\text{GeV})^2, \]
or
\[ \chi a = -\frac{(2\pi)^2}{6}\Pi_\chi(0) \approx -0.15\text{GeV}^2. \]  (27)

As pointed out in Ref.[5], the small \( \Pi_\chi(0) \) comes from the cancellation of the two resonances. However, even if \( \rho \) meson does not couple to the current \( \bar{q}\sigma^{\mu\nu}q \) [13], we have \( \Pi_\chi(0) \approx (0.21\text{GeV})^2 \) and \( \chi a \approx -0.29\text{GeV}^2 \), which are still small.

The susceptibilities \( \kappa \) and \( \zeta \) can be estimated in a similar way. To get just an order of magnitude, we assume only B-meson couple to the current \( \bar{q}\sigma^{\mu\nu}q \). We find
\[ \Pi_\zeta(0) = -\Pi_\kappa(0) \approx e^{m_v^2/M^2} \left( \frac{2m_0^2}{m_v^2} \langle m\bar{q}q \rangle + \frac{M^2}{8\pi^2m_v^2} \langle g_c^2G^2 \rangle \right), \]  (28)

where \( M \) and \( m_v \) are the Borel mass and the vector meson mass, respectively. Substituting \( \langle m\bar{q}q \rangle \approx -(100\text{MeV})^4 \), \( \langle g_c^2G^2 \rangle \approx 0.47\text{GeV}^4 \), \( m_v \approx 1.2\text{GeV} \), \( M \approx 1\text{GeV} \), we get \( \Pi_\zeta(0) = -\Pi_\kappa(0) \approx 0.017\text{GeV}^4 \), or
\[ \zeta a = \kappa a \approx 0.10\text{GeV}^4. \]  (29)

The above estimates of \( \chi, \kappa \) and \( \zeta \) are admittedly crude. Fortunately, as we shall see from the anabelow, the effects of susceptibilities on the tensor charges \( g_T^v \) and \( g_T^s \) are small.

C. Sum Rule Analysis

Now we perform a standard sum rule analysis for Eqs. (18) and (20). Multiplying both sides of equations by \( L^{-4/27} \), we study \( g_T^c(\mu) \) as function of Borel mass \( M^2 \). In our calculation, \( s_0 \) is taken to be \( 2.3\text{GeV}^2 \), \( \beta_N^2 = 0.26\text{GeV}^6 \), and other parameters are the same as those following Eq. (2). The solutions for \( \delta u(\mu^2) \) and \( \delta d(\mu^2) \) are plotted as function of \( M^2 \) in Fig.3. \( \delta u \) from the two sum rules are shown in solid and dashed lines, \( \delta d \) are shown in dot-dashed and short-dashed lines. We take the spread of the two sum rule predictions as our theoretical uncertainty. Averaging the two results, we have
\[ \delta u = 1.33 \pm 0.53 , \quad \delta d = 0.04 \pm 0.02 \]  (30)
at the scale \( \mu^2 = M^2 = (1\text{GeV})^2 \). Correspondingly,
\[ g_T^v = 1.29 \pm 0.51 , \quad g_T^s = 1.37 \pm 0.55, \]  (31)

which are consistent with the simplified results early.
IV. Summary and Conclusions

In this paper we studied the nucleon’s tensor charges (isovector one $g^v_T$, isoscalar one $g^s_T$, and $\delta u$ and $\delta d$) by means of QCD sum rule approach. The calculations include terms up to dimension 8, which presumably gives more reliable predictions than our previous result. Our final numbers for the proton’s tensor charges are $g^v_T = 1.29 \pm 0.51$, $g^s_T = 1.37 \pm 0.51$, $\delta u = 1.33 \pm 0.53$, $\delta d = 0.04 \pm 0.02$ at the scale of 1 GeV$^2$.

In obtaining the above results, we used different methods to analyze the sum rules for $g^\alpha_T$. The first one is to derive the simplifying formulas for the tensor charges at $\mu^2 = m_N^2$. The second method is to analyze the sum rules with different chiral structure, from which we obtained the tensor charge at the scale of 1 GeV$^2$. Both analyses seem to be consistent.

Our QCD sum rule calculation shows that $\delta u \gg \delta d$, which means that the up quark dominates the contribution in a transversely-polarized proton. Furthermore, the isovector tensor charge $g^v_T$ of the proton is comparable in magnitude with the isovector axial charge $g_A$. To the contrary, the isoscalar tensor charge is markedly different from the isoscalar axial charge. We hope that the future experimental measurement will test the present prediction.

At last, we wish to point out that since $\delta d$ is small, the precise value of which, including its sign, is difficult to calculate. Furthermore, the sum rules presented here have no stable solutions, which may reflect that some relevant physics is missing in the OPE. For instance, instanton physics, important in the case of the nucleon mass [16], might stabilize these sum rules and improve the present prediction.

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Figure Captions

**Fig.1.** Diagrams for the calculation of the Wilson coefficients of the correlation function corresponding to the structures $\hat{p}\sigma^{\mu\nu}\hat{p}$ and $\sigma^{\mu\nu}$.

**Fig.2.** Diagrams for the calculation of the Wilson coefficients of the correlation function corresponding to the structure $\{\hat{p}, \sigma^{\mu\nu}\}$.

**Fig.3.** The solutions of tensor charges from Eqs. (18) and (20). The solid and dashed lines represent $\delta u$, and the dot-dashed and short-dashed lines represent $\delta d$, with upper curves from Eq. (18).
This figure "fig1-1.png" is available in "png" format from:

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