Reversed-Spin Quasiparticles in Fractional Quantum Hall Systems and Their Effect on Photoluminescence

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The energy, interaction, and optical properties of reversed-spin quasielectrons (QE\textsubscript{R}'s) in fractional quantum Hall systems are studied. Based on the short range of the QE\textsubscript{R}–QE\textsubscript{R} repulsion, a partially unpolarized incompressible $\nu = \frac{4}{11}$ state is postulated within Haldane hierarchy scheme. To describe photoluminescence, a reversed-spin fractionally charged exciton $h$QE\textsubscript{R} (QE\textsubscript{R} bound to a valence hole $h$) is predicted. In contrast to its spin-polarized analog, $h$QE\textsubscript{R} is strongly bound and radiative.

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Laughlin quasielectrons (QE's) and quasiholes (QH's) of fractional quantum Hall (FQH) systems \cite{Laughlin1, Laughlin2} can be thought of as empty (QH) or filled (QE) states of the lowest or the first excited composite fermion (CF) Landau level (LL) \cite{Haldane1}, respectively. The energies of these Laughlin quasiparticles (QP's) and their interactions with one another and with a valence hole ($h$) have been studied quite thoroughly by means of exact diagonalization of small systems. The influence of QP's on photoluminescence (PL) of the Laughlin electron fluid is important in those systems in which the hole is spatially separated from the electrons by a distance $d$ that exceeds approximately one magnetic length ($\lambda$). In such systems, PL occurs from the radiative bound states of a hole and one or two QE's, called fractionally charged excitons (FCX's) \cite{Haldane2}. Reversed-spin QE's (denoted by QE\textsubscript{R}'s) \cite{Haldane3} are another type of elementary excitations of a Laughlin fluid, which can be thought of as particles in the reversed-spin lowest CF LL. As other QP's, the QE\textsubscript{R}'s carry fractional charge and have finite size, energy, and angular momentum.

In this note, the single-particle properties of the QE\textsubscript{R}'s, as well as the pseudopotentials defining their interaction with one another and with other QP's, are determined numerically and used to predict when the incompressible-fluid states with less than maximum polarization occur. The interaction of QE\textsubscript{R}'s with valence holes is also studied as a function of the layer separation $d$. In analogy to the FCX states, stable reversed-spin FCX's (denoted by FCX\textsubscript{R}) are predicted at a finite $d$ and small Zeeman splitting. It is shown that the FCX\textsubscript{R} optical selection rules are different from those of FCX's, because of the QE\textsubscript{R} angular momentum and spin being different from those of a QE. For example, the ground state of one QE\textsubscript{R} bound to a hole is optically active, in contrast to the nonradiative $h$–QE pair ground state. The stability of the radiative FCX and FCX\textsubscript{R} states, and thus also the PL spectrum of the Laughlin fluid, is shown to depend on the layer separation $d$, Zeeman splitting, and (critically) on the electron filling factor $\nu$.

In order to preserve the 2D translational symmetry of infinite systems, in a finite-size calculation we use Haldane spherical geometry \cite{Haldane4} in which the LL degeneracy $g = 2S + 1$ is controlled by the strength $2S$ of the magnetic monopole placed in the center of the sphere of radius $R$. The monopole strength $2S$ is defined in the units of flux quantum $\phi_0 = \hbar c/e$, so that $4\pi R^2 B = 2S\phi_0$ and $R^2 = S\lambda^2$. The many-body states on the
sphere are labeled by the set of good quantum numbers: electron spin ($J$) and its projection ($J_z$), hole spin ($\sigma_h$), and the length ($L$) and projection ($L_z$) of the total angular momentum of the electron-hole system.

The ground state of the 2DEG in the lowest LL at the Laughlin filling factor $\nu = \frac{1}{3}$ is completely polarized in the absence of the Zeeman splitting, $E_Z = 0$. There are two types of elementary charge-neutral excitation of these ground states, carrying spin $\Sigma = 0$ or 1, which in literature are referred to as the charge- and spin-density waves, respectively. The most important features of their dispersion curves $E_\Sigma (k)$ (k is the wave vector) are the magneto-roton minimum at the finite value of $k = 1.5 \lambda^{-1}$ in $E_0 (k)$, the finite gap $\Delta_0 \approx 0.076 \hbar^2 / \lambda$ at this minimum, and the vanishing of $E_1$ in the $k \to 0$ limit. Equally important is the similarity of the charge- and spin-density waves in the $\nu = \frac{1}{3}$ state to those at $\nu = 1$. The latter can be understood by means of Jain CF picture [3] where the excitations of the $\nu = 1/3$ electron state correspond to promoting one CF from a completely filled lowest ($n = 0$) spin-$\downarrow$ CF LL either to the first excited ($n = 1$) CF LL of the same spin ($\downarrow$) or to the same CF LL ($n = 0$) but with the reversed spin ($\uparrow$).

One can define three types of QP’s (elementary excitations) of the Laughlin $\nu = \frac{1}{3}$ fluid that constitute the charge- and spin-waves: Laughlin QH’s and QE’s and Rezayi QER’s. Each of the QP’s is characterized by such single-particle quantities as electric charge ($Q_{\text{QH}} = +\frac{1}{3}e$ and $Q_{\text{QE}} = Q_{\text{QER}} = -\frac{1}{3}e$), degeneracy $g_{\text{QP}}$ of the single-particle Hilbert space, and energy $\varepsilon_{\text{QP}}$. The charge-neutral excitations of Laughlin ground states are composed of a pair of QH and either QE ($\Sigma = 0$) or QER ($\Sigma = 1$).

In order to estimate the energies $\varepsilon_{\text{QP}}$ needed to create an isolated QP of each type, we applied the exact diagonalization procedure to systems of $N \leq 11$ electrons. By extrapolating the results to $N \to \infty$, we found the following values appropriate for an infinite system: $\varepsilon_{\text{QE}} = 0.0664 \hbar^2 / \lambda$ and $\varepsilon_{\text{QER}} = 0.0383 \hbar^2 / \lambda$. Our estimation of so-called “proper” QP energies (obtained by adding the term $Q_{\text{QP}}^2/2R$ to $\varepsilon_{\text{QP}}$) are: $\bar{\varepsilon}_{\text{QE}} = 0.0737 \hbar^2 / \lambda$, $\bar{\varepsilon}_{\text{QER}} = 0.0457 \hbar^2 / \lambda$, and $\bar{\varepsilon}_{\text{QH}} = 0.0258 \hbar^2 / \lambda$. Consequently, the energies of spatially separated QE–QH and QER–QH pairs are equal $E_0 = \bar{\varepsilon}_{\text{QE}} + \bar{\varepsilon}_{\text{QH}} = 0.0995 \hbar^2 / \lambda$ and $E_1 = \bar{\varepsilon}_{\text{QER}} + \bar{\varepsilon}_{\text{QH}} = 0.0715 \hbar^2 / \lambda$ (these are activation energies in transport experiments).

Which of the two negatively charged QP’s (QE or QER) occur at low energy in the particular system depends on the Zeeman term which is influenced not only by magnetic field, but also by many material parameters. Once the QP’s content is established, the correlations in the system can be understood by studying the appropriate interaction pseudopotentials defined as the dependence of pair interaction energy $V$ on the relative pair angular momentum $R$ (larger $R$ corresponds to larger separation) [8]. Here, $R_{\text{QE–QER}} = l_{\text{QE}} + l_{\text{QER}} - L$ and $R_{\text{QE–QE}} = l_{\text{QE}} + l_{\text{QE}} - L$, where $l_{\text{QP}}$ and $L$ denote the one- and two-QP angular momenta, respectively. The QH–QH, QE–QE, QE–QH, and QER–QH pseudopotentials can be found elsewhere [8,9], and therefore we will limit this discussion to $V_{\text{QER–QER}}$ and $V_{\text{QE–QER}}$ only.

Two QER’s can be formed in a $N$-electron system with at least two reversed spins at $2S = 3(N - 1) - 2$. An example of such spectrum is shown in Fig. (a) for $N = 8$ with $J = 2, 3$, and 4 corresponding to two, one and zero reversed spins, respectively. It is clear that the maximally spin-polarized system ($J = \frac{1}{2}N$) is unstable at filling factors not equal to the Laughlin value $\nu = \frac{1}{3}$. The $V_{\text{QER–QER}}$ is shown in Fig. (b). Although the obtained values depend on the system size (the repulsive character of inter-
action is restored only in the \( N \to \infty \) limit with \( V_{\text{QE-QER}}(1) \approx 0.01 e^2/\lambda \), the monotonicity of \( V_{\text{QE-QER}} \) seems to be independent of \( N \). Moreover, the super-linear shape of the curve indicates Laughlin correlations and thus incompressibility at \( \nu_{\text{QE}} = \frac{1}{L}, \frac{1}{2}, \ldots \) (in analogy to Haldane hierarchy picture of completely spin-polarized states \( \left\{ \frac{1}{2} \right\} \)). For example, Laughlin \( \nu_{\text{QE}} = \frac{1}{1} \) state occurs at the electron filling factor of \( \nu = \frac{4}{11} \) and corresponds to the 75% spin polarization (this state has also recently been proposed in the CF model \( \left\{ \frac{1}{2} \right\} \)). In view of the fact that the spin-polarized \( \nu_{\text{QE}} = \frac{1}{2} \) state is compressible \( \left\{ \frac{1}{2} \right\} \), the experimental observation \( \left\{ 1 \right\} \) of the FQH effect at \( \nu = \frac{4}{11} \) confirms the formation of QE-R’s in the \( \nu = \frac{1}{3} \) state.

An QE-QE-R pair can be formed in the system with at least one reversed spin. Another example of such spectrum is shown in Fig. 1(c) for \( N = 9 \) at \( 2S = 3(N - 1) - 2 = 22 \). The lowest energy states in the two considered subspaces, \( J = \frac{1}{2} N = \frac{9}{2} \) and \( J = \frac{1}{2} N - 1 = \frac{7}{2} \), contain a QE-QE and QE-QE-R pair, respectively. The pseudopotential \( V_{\text{QE-QER}}(R) \) was calculated for \( N \leq 10 \) and the results are presented in Fig. 1(d). The values of the pseudopotential depend on \( N \) and in the limit \( N \to \infty \) we found \( V_{\text{QE-QER}}(0) \to 0.015 e^2/\lambda \) and \( V_{\text{QE-QER}}(1) \to 0.01 e^2/\lambda \).

The behavior of \( V_{\text{QE-QER}}(R) \) is qualitatively different from that of \( V_{\text{QE-QER}}(R) \) and \( V_{\text{QE-QE}}(R) \). The most significant feature of the \( V_{\text{QE-QER}}(R) \) function is that it is super-linear in \( L(L + 1) \) only at \( 1 \leq R \leq 3 \) and sub-linear at \( 0 \leq R \leq 2 \) and at larger \( R \). As a result, \( m = 2 \) is the only possible Jastrow exponent in the many-body wave function that describes Laughlin correlations and thus yields incompressibility of the system.

If QE’s and QE-R’s could coexist in the \( \nu = \frac{1}{3} \) “parent” state (unlikely due to their sensitivity to Zeeman energy), one could apply a generalized CF picture \( \left\{ \frac{1}{1} \right\} \) to predict the allowed combinations of Jastrow exponents \( \left\{ m_{\text{QE}}, m_{\text{QER-QER}}, m_{\text{QER-QER}} \right\} \) describing the incompressible states of such two-component plasma. Based on the behavior of the three involved QP pseudopotentials for different values of \( R \), we reduced the number of possible exponent combinations to a few, from which only \( \left\{ 1, 1, 1 \right\} \) satisfies the incompressibility condition. Such hypothetical mixed QE/QE-R state corresponds to the \( \nu = \frac{1}{10} \) state with 80% spin polarization.

The PL spectra of a spin-polarized 2DEG can be understood in terms of QE’s and their interaction with one another and with a valence hole \( (h) \), where electron and hole layers are separated by a finite distance \( d \) (of the order of \( \lambda \)). It was shown \( \left\{ 1 \right\} \) that in the regime where the electron-electron repulsion is weak compared to the electron-hole attraction, a hole can bind one or two QE’s to create FCX (hQE or hQER). The optical selection rules following from the 2D translational
Because of strong $\text{QE}_R^*$–$\text{QE}_R$ repulsion, binding of more than one $\text{QE}_R$ by the valence hole $h$ should be more difficult. Similarly to the $\text{QE}$ case, the simplest FCX$_R$, $h\text{QE}_R$, is expected to occur in a system containing free $\text{QE}_R$’s at the values of $d$ at which the binding energies of $h\text{QE}$ and $h\text{QE}_R$ are smaller than the Laughlin gap to create additional QE-QH pairs. We have studied a good number of numerical spectra for different values of $d$. As expected, we found that both $h\text{QE}_R$ and $h\text{QE}$ ground states develop at $d$ larger than $\lambda$.

An example of such spectrum for a $7e$–$h$ system (with up to one reversed electron spin) at $d = 4\lambda$ and $2S = 17$ is shown in Fig. 2(a). In the CF picture this configuration corresponds to six CF’s filling the first CF LL and the seventh CF lying either in the second CF LL with parallel spin ($J = \frac{1}{2}N = \frac{7}{2}$) or in the first CF LL with reversed spin ($J = \frac{1}{2}N - 1 = \frac{5}{2}$). As shown in Fig. 2(a), at sufficiently large $d$ the lowest energy states contain well defined $h$–$\text{QE}$ or $h$–$\text{QE}_R$ pairs with all possible values of $L$ resulting from the rules of addition of angular momenta. Applying the appropriate orbital selection rule for radiative recombination [4,9] we determined that, unlike the dark $h\text{QE}$, the reversed-spin $h\text{QE}_R$ ground state is radiative. As shown in Fig. 2(b) for $d = 2\lambda$, the numerical spectra contain also larger FCX$_R$ complexes, $h(\text{QE}_R)_2$ and $h\text{QE}_R\text{QE}$. However, due to their weaker binding and because $h(\text{QE}_R)_2$ turns out dark and $h\text{QE}_R\text{QE}$ is very sensitive to Zeeman energy, the emission of $h\text{QE}_R$ is expected to dominate the PL spectrum of a partially unpolarized system at $\nu \approx \frac{1}{3}$.

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