STUDY OF COINCIDENCES BETWEEN RESONANT GRAVITATIONAL WAVE DETECTORS

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Abstract
Coincidences are searched with the cryogenic resonant gravitational wave detectors EXPLORER and NAUTILUS, during a period of about six months (2 June-14 December 1998) for a total measuring time of 94.5 days, with the purpose to study new algorithms of analysis, based on the physical characteristics of the detectors.
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1. Introduction
After the initial experiments with room temperature resonant detectors, the new generation of cryogenic gravitational wave (GW) antennas entered long term data taking operation in 1990 (EXPLORER [1]), in 1991 (ALLEGRO [2]), in 1993 (NIOBE [3]), in 1994 (NAUTILUS [4]) and in 1997 (AURIGA [5]).

Recently an analysis of the data taken in coincidence among all cryogenic resonant detectors in operation during the years 1997 and 1998 has been performed [6]. No coincidence excess was found above background using the event lists produced under the protocol of the International Gravitational Event Collaboration (IGEC), among the groups of ALLEGRO, AURIGA, EXPLORER / NAUTILUS and NIOBE. The coincidence search was done without any particular data selection. However one can consider the possibility to search for coincidences with events selected according to various possible criteria using all available information (we mention criteria based on: the event energy, the event duration, the applied threshold, the shape of the events, the coincidence window, the direction of possible GW, the noise).

Here we have used algorithms based on physical characteristics of the detectors, as the event energy (with a new algorithm) and the directionality. In this paper we explore their effect on the coincidence search.

For this purpose we shall use IGEC data obtained from 2 June 1998 when NAUTILUS, after a stop for instrumental improvements, resumed the operation. We search for coincidences between NAUTILUS and EXPLORER, whose apparatuses differ only in the operating temperatures (respectively 0.15 K and 2.6 K) and in particular have identical readout systems. Extension of the methods we develop here to other detectors in operation during the same period of time is envisaged.
Table 1: Main characteristics of the two detectors.

| detector   | latitude | longitude | orientation | mass | frequencies | temperature |
|------------|----------|-----------|-------------|------|-------------|-------------|
| EXPLORER   | 46.45 N  | 6.20 E    | 39° E       | 2270 | 904.7 Hz    | 2.6 K       |
| NAUTILUS   | 41.82 N  | 12.67 E   | 44° E       | 2270 | 906.97 Hz   | 0.15 K      |

We are well aware that any data selection jeopardizes the possibility to express the results by means of a probability that a coincidence excess, if any, had been accidental. With this proviso we shall still use parameters obtained from probability estimations for comparing different situations.

2. Events and signals

We now briefly describe how we obtain events from the measurements. For EXPLORER and NAUTILUS, whose main characteristics are given in table [1] the data are sampled at intervals of 4.54 ms and are filtered with a filter matched to short bursts [7] for the detection of delta-like signals. The filter makes use of power spectra obtained with off-line analysis. After the filtering of the raw-data, events are extracted as follows. Be \( x(t) \) the filtered output of the detector. This quantity is normalized, using the detector calibration, such that its square gives the energy innovation \( E \) of the oscillation for each sample, expressed in kelvin units. For well behaved noise due only to the thermal motion of the bar and to the electronic noise of the amplifier, the distribution of \( x(t) \) is normal with zero mean. The variance (average value of the square of \( x(t) \)) is called effective temperature and is indicated with \( T_{\text{eff}} \). The distribution of \( x(t) \) is

\[
f(x) = \frac{1}{\sqrt{2\pi T_{\text{eff}}}} e^{-\frac{x^2}{2T_{\text{eff}}}}
\]  

(1)

For extracting events we set a threshold in terms of a critical ratio defined by

\[
CR = \frac{|x| - \bar{|x|}}{\sigma(|x|)} = \frac{\sqrt{SNR} - \sqrt{\frac{2}{\pi}}}{\sqrt{1 - \frac{2}{\pi}}}
\]  

(2)

where \( \sigma(|x|) \) is the standard deviation of \( |x| \) (the moving averages \( \bar{|x|} \) are made over the preceding ten minutes) and

\[
SNR = \frac{E}{T_{\text{eff}}}
\]  

(3)

The threshold is set at \( CR=6 \) in order to obtain, in presence of thermal and electronic noise alone, about one hundred events per day, as agreed among the partners of the IGEC. This threshold corresponds to an energy \( E_t = 19.5 \ T_{\text{eff}} \). When \( |x| \) goes above the threshold, its time behaviour is considered until it goes below the threshold for more than ten seconds. The maximum amplitude and its occurrence time define the event.

In general the event is due to a combination of a signal which, in absence of noise, has energy \( E_s \) (due to GW or other forces) and the noise. The theoretical probability to detect a signal with a given \( SNR_s = \frac{E_s}{T_{\text{eff}}} \), in presence of a well behaved Gaussian noise, is calculated as follows. We put \( y = (s + x)^2 \) where \( s \equiv \sqrt{SNR_s} \) is the signal we look for and \( x \) is the gaussian noise. We obtain easily [8]

\[
\text{probability}(SNR_s) = \int_{SNR_s}^{\infty} \frac{1}{\sqrt{2\pi y}} e^{-\frac{(SNR_s+y)^2}{2}} \cosh(\sqrt{y} \cdot SNR_s)dy
\]  

(4)
where we put $SNR_t = \frac{E_t}{T_{eff}} = 19.5$ for the present EXPLORER and NAUTILUS detectors.

The behaviour of the integrand is shown in fig. 1. This figure shows the spread of the event energy due to noise for a given $SNR_s$ of the applied signal. The distinction between the two concepts, signal and event, is essential for the analysis we propose in this paper.

### 3. Data selection

All the events which are in coincidence within a time window of ±5 s with events produced by a seismometer are eliminated, about 8% of the events.

It has been noticed that the experimental data are affected by noise which, in some cases, cannot be observed with any other auxiliary detector. Thus a strategy is needed for deciding when the measurements are considered to be good for the search of coincidences.

We are well aware that the selection of the experimental data must be done with great care and the safest strategy is to establish rules before even looking to the data. We have decided to take into consideration all the data recorded by the detectors (except those vetoed by the seismometer) and accept only the events for which the corresponding $T_{eff}$ is below a certain threshold. This threshold must be such that we are confident that no signal is being thrown away. All and only the events which have $T_{eff} \leq 100$ mK (over the preceding ten minutes) are taken into consideration. The events for which the corresponding $T_{eff}$ is greater than 100 mK are certainly generated at times the detector is not operating properly.

The following information is available on the IGEC Web page for each event:

- Time (UT) of the maximum of the event: YEAR, MONTH, DAY, MINUTE, SECOND.
- $H_o$: Bilateral Fourier amplitude at resonance of the maximum.
- SNR: Signal to noise ratio of amplitude.
- $T_{eff}$: Effective temperature [K] of the previous 10 minutes.
- Duration L of the event, in number of samples (4.54 ms).
- Time in seconds between the beginning and the maximum of the event.
The relationship between the Fourier transform $H_o$ of the event amplitude and the energy $E$ of the event is given by [9]

\[ H_o = 7.97 \times 10^{-21} \sqrt{E} \]  

(5)

with $H_o$ in units of $\frac{1}{Hz}$ and $E$ in kelvin.

Looking to these events we have noticed that some events occur during periods of high disturbance. Since we are elaborating here strategies for data analysis, we have thought convenient to select the data to be used in our analysis in various ways according to the noise. Thus another way of choosing the data to be analysed is to select periods with smaller noise. We apply two more data selections, only events with $T_{eff} \leq 50 \, mK$ for both EXPLORER and NAUTILUS and only events with $T_{eff} \leq 25 \, mK$. This data selection has been applied by us in a previously published paper [10]. In fig.2 we show the number of events per day and the hourly averages of $T_{eff}$ for EXPLORER and NAUTILUS for the case $T_{eff} \leq 25 \, mK$.

We notice large fluctuations, in spite of the stringent criteria for the data selection. Information on the various data selections are given in table 2. We notice that the number of available hours of measurement becomes rather smaller when lowering the threshold for $T_{eff}$, so that any possible result becomes statistically weaker at low $T_{eff}$.

4. Searching for coincidences

For the search of coincidences it is important to establish the time window. We have decided to adopt the same window used in past analyses, in particular that described in papers [11, 6], $w = \pm 1 \, s$. This is a reasonable choice considering the present detectors bandwidth (of the order of 1 Hz) and some time inaccuracy.

As well known, the analysis in a coincidence search consists essentially in comparing the detected coincidences at zero time delay with the background, that is with coincidences occurring by chance. In order to measure the background due to the accidental coincidences, using a procedure adopted since
Table 2: Total number N of events, number of hours of data taking, average noise temperature $< T_{\text{eff}} >$ and hours in common, when both detectors were simultaneously operating.

| Detector   | $T_{\text{eff}}$ | N   | hours | $< T_{\text{eff}} >$ | hours in common | N   |
|------------|-------------------|-----|-------|-----------------------|-----------------|-----|
| EXPLORER   | $\leq 100$ mK     | 550 | 3415  | 40.6                  | 2271            | 37944 |
| NAUTILUS   |                   | 37734 | 3450  | 19.1                  | 24118           |     |
| EXPLORER   | $\leq 50$ mK      | 39211 | 2759  | 28.9                  | 1816            | 26481 |
| NAUTILUS   |                   | 34148 | 3371  | 14.0                  |                | 16677 |
| EXPLORER   | $\leq 25$ mK      | 16172 | 1498  | 18.7                  | 931             | 9765 |
| NAUTILUS   |                   | 27823 | 3168  | 9.3                   |                |      |

Table 3: Number $n_c$ of coincidences and average number $\bar{n}$ of accidentals. The total period of time in common when $T_{\text{eff}} \leq 100$ mK is 94.5 days.

| $T_{\text{eff}}$ | $n_c$ | $\bar{n}$ | hours |
|-------------------|-------|-----------|-------|
| $\leq 100$ mK     | 223   | 231.7     | 2271  |
| $\leq 50$ mK      | 137   | 139.8     | 1816  |
| $\leq 25$ mK      | 32    | 36.2      | 931   |

the beginning of the gravitational wave experiments [12], we have shifted the time of occurrence of the events of one of the two detectors 1,000 times in steps of 2 s, from -1,000 s to +1,000 s. For each time shift we get a number of coincidences. If the time shift is zero we get the number $n_c$ of real coincidences. The background is calculated from the average number of the $n_{\text{shift}}$ accidental coincidences obtained from the one thousand time shifts

$$\bar{n} = \frac{\sum_{1}^{1000} n_{\text{shift}}}{1000} \quad (6)$$

With this experimental procedure for the evaluation of the background we circumvent the problems arising from a non very stationary distribution of the events, provided we test properly the distribution of the shifted coincidences (see fig. 4 and reference [13]).

The result of our search for coincidences is given in table 3. There is no coincidence excess between EXPLORER and NAUTILUS, even for selected periods with smaller noise.

5. Data selection using the event energy

We want now to apply data selection algorithms based on the event energy. The most obvious one is to search for pairs of events which have (approximately) the same energy. In the past this energy criterion has been applied, requiring that the responses of the EXPLORER detector at the two resonance modes were within a factor of two one from each other [1]. Later we realized that the effect of the noise on signals near threshold is such that the event energies are only lightly correlated to the signal energies [14], and this reduces the efficiency of algorithms based on the event energy.

Recently an important result was found [15]. It has been seen that the distribution of the energy ratios of the event energies of two detectors, in the case of non gaussian noise, is different for real coincidences and accidental coincidences. This has pushed us to reconsider the importance to apply selection algorithms based on the event energies.

For making use of the event energy, in particular with detectors with different sensitivity, we must consider the result shown in fig. 4 which indicates the chance to have a certain event-energy for a given signal-energy. In principle, all event-energies are possible, from zero to infinity. Our procedure here is to consider only event-energies within $\pm$ one sigma from the signal energy (that is, we consider events
Table 4: Energy algorithm. Number \( n_c \) of coincidences, average number \( \bar{n} \) of accidentals and the covered time period for the three data selection.

| \( T_{\text{eff}} \) | \( n_c \) | \( \bar{n} \) | hours |
|----------------|----------|----------|-------|
| \( \leq 100 \ mK \) | 61       | 50.5     | 2271  |
| \( \leq 50 \ mK \)   | 45       | 37.7     | 1816  |
| \( \leq 25 \ mK \)   | 11       | 10.3     | 931   |

included in 68% of the area under the line in fig. [I].

We do not know the signal-energy. The new algorithm we propose is the following. We consider signals in a wide range, say: \( E_s \) from 20 mK to 2 K in steps of 20 mK. We find the coincident events, at zero delay (the real coincidences) and at shifted times (for the estimation of the accidentals). For each assumed signal with energy \( E_s \) we calculate the \( SNR_s \) different for each event, since the noise \( T_{\text{eff}} \) depends on the detected event and it is also different for the two detectors. We then verify if the \( SNR_{\text{event}} \) falls into \( SNR_s \pm 1 \sigma \), having calculated for each \( SNR_s \) the probability curve like that shown in fig. [I] for \( SNR_s = 20 \). If the two event energies are compatible with the event-energy expected for any of the assumed signals then we accept the coincidence (real or shifted).

The result of this analysis is given in table [II]. We notice that the use of the energy selection algorithm has reduced the number of the accidental coincidences by a factor of three.

6. Event selection according to the detector orientation with respect to the Galactic Centre

No extragalactic GW signals should be detected with the present detectors. Therefore we shall focus our attention on possible sources located in the Galaxy. If any of these sources exist we should expect a more favorable condition of detection when the detectors are oriented with their axes perpendicular to the direction toward the Galactic Centre (GC), since, the bar cross-section is proportional to \( \sin^4(\theta) \), where \( \theta \) is the angle between the detector axis and the direction to the GC.

After having applied the above energy algorithm, we search for coincidences considering only events obtained when the detectors were oriented with \( \theta \) greater than various given values and, according to the previous sections, for the various data selection.

The result is given in fig.[III] and shows a larger coincidence excess when the detector axes tend to be perpendicular to the direction towards the GC. Above \( \theta \sim 79^\circ \) the number \( n_c \) of coincidences drops quickly. If not instrumental, the quick drop could be taken as due to the width of the source. The time spent by the detectors when \( \theta \geq 79^\circ \) is 20% of the total time of 94.5 days.

We want now to verify that the evaluation of the background is properly done. We do this in the condition of the greatest coincidence excess, that is for \( \theta \geq 79^\circ \). We must consider that by selecting only times when the detectors had certain orientations we have several empty time regions. This makes it possible that in doing the shifting operation for evaluating the background one uses time periods of different duration. We have determined these time periods and found that they vary by a few percent, with a maximum of -10% for a time shift of +1000 s. In fig. [IV] we show, for the case \( \theta \geq 79^\circ \) and \( T_{\text{eff}} \leq 100 \ mK \), the delay histogram with no correction and the delay histogram corrected for the different periods of time for each shift. In this particular case the correction applies for a very small amount only for delays greater than about 700 s.

7. Conclusions

In order to make a first step to a complete analysis, we have selected the IGEC events of EXPLORER and NAUTILUS using algorithms based on known physical characteristics of the detectors. In particular a new algorithm which makes use of the event energy has been devised.
Fig. 3: The upper left figure shows, versus $\theta$ and for $T_{\text{eff}} \leq 100 \ mK$, the integral number of coincidences $n_c$ (indicated with asterisks) and the average number of accidentals $\bar{n}$ (calculated from the number of coincidences at zero delay and the average background $\bar{n}$ measured with 1000 delays). The right figure shows the Poisson probability that the observed number of coincidences $n_c$ was due to a background fluctuation. Similarly the second line of figures refers to the data selection $T_{\text{eff}} \leq 50 \ mK$ and the third line to the data selection $T_{\text{eff}} \leq 25 \ mK$.

Fig. 4: Data selection with $T_{\text{eff}} \leq 100 \ mK$. In the upper figure we show, for $\theta \geq 79^o$, the delay histogram. In the lower figure we show the same data normalized for the duration of the time period used for each time shift. In this particular case the normalization turns out to be very small, almost barely visible for delays above 700 s, but it is worth to remark that a possible effect due to different time coverage at various delays has been taken into account. The biggest asterisk indicate the nineteen coincidences at zero time delay.
With event selection based on this algorithm we find an excess of coincidences at zero time delay in the direction of the Galactic Centre. As well known in the scientific community, no g.w. signals are expected to be observed with the present detector sensitivity. Since this result would open new possibilities, a careful Bayesian approach suggests that, given the Poisson probabilities of a few percent, new data with other detectors are required before we can ensure that g.w. from the GC have been indeed observed. Thus, at present, we feel that the coincidence excess is not large enough to establish a claim for detection of true signals, but it is an important information to make available to the scientific community. We believe that the procedures adopted here might be useful for detecting gravitational waves with more or better data.

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