Modified spectral method for optimal estimation in linear continuous-time stochastic systems

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Abstract. The spectral method to solve estimation problems for linear continuous-time stochastic systems with polynomial measurements is presented. It is based on both the spectral form of mathematical description (the representation of deterministic functions and random processes by orthogonal series) and the particle filter. The main goal of this work is to implement the continuous-time particle filter without a time discretization. The proposed spectral method provides the possibility to solve estimation problems such as filtering, smoothing and prediction.

1. Introduction
The paper considers the use of orthogonal expansions of random processes to solve estimation problems for linear continuous-time stochastic systems. The spectral method, or the spectral form of mathematical description for deterministic or stochastic signals and control systems, based on the theory of orthogonal expansions is applied [1, 2].

At present, to solve estimation problems, e.g. the filtering problem, particle filters (PFs) are used [3]. Such filters are based on the statistical modeling of a dynamical system, trajectories of which should be estimated. Mathematical models of a dynamical system and a sensor are used to obtain both the ensemble of dynamical system trajectories (or a set of states at fixed time) and the optimal or suboptimal estimate of the trajectory (or the state) given the measurements from the sensor.

Discrete-time PFs are most often applied, since many estimation problems have the discrete-time statement [4–8]. Continuous-time PFs are also used, but a time discretization is always required for its implementation [9–13]. In fact, the continuous-time PF with a time discretization is equivalent to the discrete-time PF. And the main goal of this paper is to implement the continuous-time PF without a time discretization. Such a filter can be constructed for linear stochastic systems with polynomial measurements [14] by using the theory of orthogonal expansions of random processes and the spectral method [15]. Similar models of dynamical systems and sensors may be used in estimation problems related to navigation data processing [7, 16–18] or in estimation problems related to the motion control [19–21].

We suppose that a mathematical model of the dynamical system, trajectories of which should be estimated, is described by a linear stochastic differential equation (SDE) with additive or multiplicative noise. The sensor is defined by a nonlinear SDE with additive noise, and the nonlinearity is given by a polynomial with an arbitrary degree.
The random process under certain conditions (the existence of the second-order moment and its continuity) can be represented by the series in terms of the eigenfunctions of its covariance operator (the Karhunen–Loève expansion [22, 23]). The set of eigenfunctions is the orthonormal basis for the space of square-integrable functions, and eigenfunctions can be expressed by orthogonal expansions with respect to an arbitrary orthonormal basis. This makes it possible to use some universal orthonormal basis, e.g. the Legendre polynomials or trigonometric functions, to represent the linear SDE solution [15].

The spectral form of mathematical description assumes that the linear SDE solution is represented by the corresponding spectral characteristic [14, 15], i.e. the infinite column matrix formed by ordered expansion coefficients, which are random variables. The algorithm for finding an exact solution is reduced to the matrix algebra operations such as addition, multiplication and inversion of infinite matrices (spectral characteristics of linear operators) and infinite column matrices (spectral characteristics of deterministic functions and random processes). For finding an approximate solution the infinite matrices should be truncated to finite ones, then the corresponding random process is approximately expressed by the linear combination of orthonormal functions with random coefficients. Such a representation preserves the continuous time in contrast to the use of numerical methods for solving SDEs [24]. The random processes such as the Brownian motion, the Ornstein–Uhlenbeck process, the Brownian bridge and the geometric Brownian motion have been simulated by this approach with different orthonormal functions mentioned above [25].

These results provide constructing the continuous-time PF without a time discretization [14]. A relation for the weight in the continuous-time PF includes the inner product of random processes, and the spectral transform preserves it. So, the inner product in the space of square-integrable functions is reduced to the inner product for its spectral characteristics in the space of square-summable sequences. In addition, the use of the special spectral characteristic represented by an infinite three-dimensional matrix allows one to consider the nonlinear transformation of random processes in the measurement equation, where the nonlinearity is given by a polynomial.

In this paper, modified relations to solve the estimation problem by the spectral method in comparison with results from [14] are presented. They ensure the correctness with respect to the initial conditions for the linear SDE solution when trigonometric functions are used as orthonormal basis. Moreover, modified relations reduce the computational cost due to fewer matrix multiplications in the estimation algorithm. The proposed method has been tested by the estimation of paths for the Ornstein–Uhlenbeck process [26] when the sensor is defined by a nonlinear SDE with the nonlinearity that is given by polynomials with various degrees.

2. Optimal estimation problem

We define the state equation as the following linear SDE:

\[ \dot{x}(t) = a(t)x(t) + b(t)g(t) + (c(t)x(t) + d(t))v(t), \quad x(0) = x_0, \quad t \in \mathbb{T} = [0, T], \]  

(1)

or

\[ x(t) = x_0 + \int_0^t (a(\tau)x(\tau) + b(\tau)g(\tau))d\tau + \int_0^t (c(\tau)x(\tau) + d(\tau)) \circ dw(\tau), \]  

(2)

where \( x(t) \) is the unobservable random process, functions \( a(t), b(t), c(t), \) and \( d(t) \) are given, \( g(t) \) is an input signal or a control, \( v(t) \) is a standard Gaussian white noise, \( w(t) \) is the standard Wiener process corresponding to the white noise \( v(t) \), and \( x_0 \) is an initial state with a finite second-order moment, i.e. \( \mathbb{E} x_0^2 < \infty \) (\( \mathbb{E} \) denotes a mean).

The third term in the right-hand side of the equation (2) is the Stratonovich stochastic integral. Note that if the state equation is written in the Itô form, then it should be rewritten in the Stratonovich form using the well-known relations [26].
The measurement equation has the form

\[ z(t) = c(t, x(t)) + \zeta(t)n(t), \]  

(3)

or

\[ y(t) = \int_0^t c(t, x(t)) dt + \int_0^t \zeta(t)d\varpi(t), \]

where \( z(t) \) and \( y(t) \) are the observable random processes, \( c(t, x) \) is a polynomial of degree \( \gamma \) with given coefficients, i.e.

\[ c(t, x) = c_\gamma(t)x^\gamma + \ldots + c_1(t)x + c_0(t), \quad \gamma = \deg c(t, x), \]

a function \( \zeta(t) \neq 0 \) is also given, \( n(t) \) is a standard Gaussian white noise, \( \varpi(t) \) is the standard Wiener process corresponding to the white noise \( n(t) \). The white noises \( v(t) \) and \( n(t) \) are independent, consequently, the Wiener processes \( w(t) \) and \( \varpi(t) \) are also independent.

The optimal estimation problem is to find an estimate \( \hat{x}(t) \) given the measurements \( z_0^0 = \{ z(\tau), \tau \in [0, \theta] \} \) such that \( \hat{x}(t) = \psi(t, z_0^0) \) if the function \( \psi(t, \cdot) \) satisfies the condition

\[ E \Pi(\mathcal{E}(t)) \rightarrow \min_{\psi(t, \cdot)} \forall t, \theta \in \mathbb{T}, \]

where \( \mathcal{E}(t) = x(t) - \hat{x}(t) \) is the estimation error and \( \Pi(\varepsilon) \) is a loss function \([27]\). The estimate also depends on \( \theta \) but this dependence is not indicated for brevity. Similar conditions can be written using the measurements \( y_0^0 = \{ y(\tau), \tau \in [0, \theta] \} \).

For the quadratic loss function \( \Pi(\varepsilon) = \varepsilon^2 \) we have \( \hat{x}(t) = \psi(t, z_0^0) = E[x(t)|z_0^0] \), where \( \hat{x}(t) \) is an unbiased estimate with a minimum mean squared error. If \( t < \theta \) and \( t > \theta \), then we have the filtering problem, respectively; if \( t < \theta \) and \( t > \theta \), then we have the smoothing problem and the prediction problem, respectively \([28]\).

The optimal estimate \( \hat{x}(t) \) can be represented as the normalized weighted mean

\[ \hat{x}(t) = \frac{E \omega(\theta)x(t)}{E \omega(\theta)}. \]

In this relation, \( \omega(\theta) \) is the weight defined as follows

\[ \omega(\theta) = \exp \left\{ \int_0^\theta \frac{c(\tau, x(\tau))}{\zeta^2(\tau)} \left[ z(\tau) - \frac{1}{2} c(\tau, x(\tau)) \right] d\tau \right\}, \]

or

\[ \omega(\theta) = \exp \left\{ \int_0^\theta \frac{c(\tau, x(\tau))}{\zeta^2(\tau)} dy(\tau) - \frac{1}{2} \int_0^\theta \frac{c^2(\tau, x(\tau))}{\zeta^2(\tau)} \frac{d\tau}{\zeta^2(\tau)} \right\}. \]

Above relations underlie the continuous-time PF.

3. Preliminary results

The paper \([15]\) proposes the spectral method for modeling the linear SDE solution. In \([14]\), these results have been extended and applied to solve the optimal estimation problem. All the necessary relations for this are given below.

Firstly, denote the spectral characteristic of the unobservable random process \( x(t) \) by \( X \). Then

\[ X = (P - A - (V^\top)C)^{-1}(x_0\Delta_0 + BG + (V^\top)D), \]

(4)

where \( G \) and \( D \) are spectral characteristics of functions \( g(t) \) and \( d(t) \), respectively; \( V \) is the spectral characteristic of the standard Gaussian white noise \( v(t) \); \( A, B, \) and \( C \) are spectral
characteristics of multiplication operators with multipliers \( a(t), b(t), \) and \( c(t), \) respectively; \( P \) is the spectral characteristics of the differentiation operator taking into account the initial condition, and \( V \) is the spectral characteristic of the multiplier. These spectral characteristics should be defined with respect to the orthonormal basis \( \{q_i(t)\}_{i=0}^{\infty} \) of \( L_2(T) \) (the space of square-integrable functions on \( T \)), and \( \Delta_0 \) is the infinite column matrix with entries \( q_i(0). \)

Here \( G \) and \( D \) are infinite column matrices, \( V \) is the infinite column matrix with entries that are independent random values having a standard normal distribution; \( A, B, C, \) and \( P \) are infinite two-dimensional matrices, and \( V \) is the infinite three-dimensional matrix. Formulas for calculating the entries of these matrices are given in \([14, 15]\).

The relation (4) can be simplified if \( c(t) = 0 \) as

\[
X = (P - A)^{-1}(x_0\Delta_0 + BG + DV),
\]

where \( D \) is redefined as the spectral characteristic of the multiplication operator with the multiplier \( d(t) \), i.e. \( D \) is the infinite two-dimensional matrix instead of the infinite column matrix.

Secondly, denote the spectral characteristic of the observable random process \( z(t) \) by \( Z \). Then

\[
Z = C_\gamma(VX)^{\gamma-1}X + \ldots + C_1X + C_0 + \Xi N,
\]

where \( C_\gamma, \ldots, C_1, \) and \( \Xi \) are spectral characteristics of multiplication operators with multipliers \( c_\gamma(t), \ldots, c_1(t), \) and \( \zeta(t), \) respectively, \( C_0 \) is the spectral characteristic of the function \( c_0(t), \) and \( N \) is the spectral characteristic of the standard Gaussian white noise \( n(t). \) These spectral characteristics should also be defined using the orthonormal basis \( \{q_i(t)\}_{i=0}^{\infty} \) of \( L_2(T). \)

Here \( C_0 \) is the infinite column matrix, \( N \) is the infinite column matrix with entries that are independent random values having a standard normal distribution; \( C_\gamma, \ldots, C_1, \) and \( \Xi \) are infinite two-dimensional matrices.

Next, the weight can be expressed in the following way:

\[
\omega(\theta) = \exp \left\{ \Delta_\theta^T P^{-1}(V\zeta_t^t(X)) \left[ \Xi^{-1}Z - \frac{1}{2} C_\zeta(X) \right] \right\},
\]

where \( \Delta_\theta \) is the infinite column matrix with entries \( q_i(\theta), \) \( P^{-1} \) is the spectral characteristics of the integration operator, i.e. \( P^{-1} \) is the infinite two-dimensional matrix, and \( C_\zeta(X) = \Xi^{-1}(C_\gamma(VX)^{\gamma-1}X + \ldots + C_1X + C_0). \)

A detailed proof for relations (6) and (7) one can found in \([14]\).

4. Modified spectral relations for the weight and the estimation algorithm

The main disadvantage of the formula (7) for the weight is the use of the infinite column matrix \( \Delta_\theta. \) In fact, \( \Delta_\theta \) is the spectral characteristic of the Dirac delta function \( \delta(t - \theta) \) concentrated at \( \theta, \) and the formula (7) itself is the matrix form for the spectral inversion for the spectral characteristic of the random process

\[
\int_0^\theta \frac{c(\tau, x(\tau))}{\zeta^2(\tau)} \left[ z(\tau) - \frac{1}{2} c(\tau, x(\tau)) \right] d\tau.
\]

The spectral inversion is a representation of a function or a random process as the orthogonal series that converges in the norm of \( L_2(T), \) but the series may not converge pointwise. For example, there is the well-known fact that the trigonometric Fourier series converges to the average of the left and right limits of a function at the discontinuity point. The trigonometric
Fourier series may not converge to the corresponding function at the points \( t = 0 \) and \( t = T \). Moreover, for the orthonormal basis \( \{ q_i(t) \}_{i=1}^{\infty} \) such as the Legendre polynomials \( P_1 = \Delta_0 \), but for trigonometric functions \( (1/2) P_1 = \Delta_0 \) when \( \theta = 0 \), where \( 1_\theta \) is the spectral characteristic of the unit step function \( 1(t - \theta) \).

In this paper, the modified spectral relations for both the linear SDE solution

\[
X = (P - A - (VV)^{-1})^{-1} (x_0 P 1_0 + BG + (VV)) D
\]

and the weight

\[
\omega(\theta) = \exp \left\{ \Upsilon_\theta^T (VC^T_\zeta(X)) \left[ \Xi^{-1} Z - \frac{1}{2} C_\zeta(X) \right] \right\},
\]

where \( \Upsilon_\theta = 1_0 - 1_\theta \) is the spectral characteristic of the indicator \( \chi_\theta(t) = 1(t) - 1(t - \theta) \) for the set \( [0, \theta] \subset T \), is proposed. The relation (8) ensures the correctness of the initial condition for the state equation (1) (in the sense of the space of square-integrable functions only but not pointwise) when trigonometric functions are used. It can also be simplified if \( c(t) = 0 \):

\[
X = (P - A)^{-1} (x_0 P 1_0 + BG + DV).
\]

The relation (9) is based on both the representation

\[
\int_0^\theta \frac{c(\tau, x(\tau))}{\zeta^2(\tau)} \left[ z(\tau) - \frac{1}{2} c(\tau, x(\tau)) \right] d\tau = \int_0^T \chi_\theta(\tau) \frac{c(\tau, x(\tau))}{\zeta^2(\tau)} \left[ z(\tau) - \frac{1}{2} c(\tau, x(\tau)) \right] d\tau
\]

and on the orthogonality property of the spectral transform \( S \) [14], i.e.

\[
S \left[ \int_0^T \chi_\theta(\tau) \frac{c(\tau, x(\tau))}{\zeta^2(\tau)} \left[ z(\tau) - \frac{1}{2} c(\tau, x(\tau)) \right] d\tau \right] = \Upsilon_\theta^T (VC^T_\zeta(X)) \left[ \Xi^{-1} Z - \frac{1}{2} C_\zeta(X) \right].
\]

Since relative values of weights but not their absolute values are used in the formula for the optimal estimate [14], the random process \( x(t) \) may be correctly estimated using the relation (7). However, the weight may be incorrect. The modified relation (9) eliminates this disadvantage. In addition, the use of the spectral characteristic \( \Upsilon_\theta \) reduces the computational cost due to fewer matrix multiplications.

Note that the weight for the case \( t = T \) should be calculated using a more simpler formula, which has been obtained in [14]. It is not given here for briefness, since any modification for that formula is not needed.

The estimation algorithm is follows [14]. For solving the optimal estimation problem approximately it is necessary to simulate \( M \) realizations \( X^j \) of the spectral characteristic \( X \) and the corresponding weights \( \omega^j \) with respect to the fixed spectral characteristic \( Z, j = 1, 2, \ldots, M \). Each realization \( X^j \) corresponds to a realization \( V^j \) of the spectral characteristic \( V \) according to relations (8) or (10). The weight \( \omega^j \) should be calculated by the modified relation (9) with the substitution \( X = X^j \). So,

\[
\hat{x}(t) \approx S^{-1}[\hat{X}] = \sum_{i=0}^\infty \hat{X}_i q_i(t), \quad t \in T,
\]

where \( S^{-1} \) is the spectral inversion, coefficients \( \hat{X}_i \) are entries of the spectral characteristic \( \hat{X} \) that can be expressed by a particle method as the normalized weighted mean

\[
\hat{X} = \frac{1}{\Omega} \sum_{j=1}^M \omega^j X^j, \quad \Omega = \sum_{j=1}^M \omega^j.
\]
To find an approximate solution of the optimal estimation problem for the unobservable random process defined by the state equation (1) given the measurements defined by the measurement equation (3), all the spectral characteristics should be truncated on all dimensions to a certain order $L$ (the transition from infinite matrices to finite ones, see [14] for details). Then

$$
\dot{x}(t) \approx S^{-1}[\dot{X}] \approx \sum_{i=0}^{L-1} \hat{X}_i q_i(t).
$$

(12)

5. Numerical experiments

The Ornstein–Uhlenbeck process is described by the linear SDE with additive noise

$$
\dot{x}(t) = \mu x(t) + \sigma v(t), \quad x(0) = x_0, \quad t \in \mathbb{T},
$$

(13)

where $\mu$, $\sigma$ and $x_0$ are given parameters, i.e. $a(t) = \mu$, $b(t) = 0$, $c(t) = 0$, $d(t) = \sigma$ (an input signal $q(t)$ is not given due to $b(t) = 0$).

The relation (10) expresses the spectral characteristic $X$ of the Ornstein–Uhlenbeck process as follows

$$
X = (P - \mu E)^{-1}(x_0 P_{10} + \sigma V),
$$

(14)

i.e. $A = \mu E$, $B = O$, $C = O$, $D = \sigma E$ ($E$ and $O$ are identity and zero matrices, respectively) because the spectral characteristic of the multiplication operator with a multiplier $\alpha \in \mathbb{R}$ is $\alpha E$. Here $P$ is the spectral characteristic of the differentiation operator taking into account the initial condition, $1_{0}$ is the spectral characteristic of the unit step function $1(t)$, $V$ is the spectral characteristic of the random process $v(t)$.

The measurement equation is

$$
z(t) = \lambda x(t) + \zeta n(t),
$$

where $\lambda$ and $\zeta$ are given parameters, i.e. $c(t, x) = \lambda x(t)$, $\zeta(t) = \zeta$. We consider three cases: $\gamma = 1$ (linear sensor), $\gamma = 2$ (quadratic sensor), and $\gamma = 3$ (cubic sensor).

Using results from [14] we can write

$$
Z = \lambda (V X)^{\gamma - 1} X + \zeta N, \quad C_\zeta(X) = \frac{\lambda}{\zeta} (V X)^{\gamma - 1} X.
$$

i.e. $C_\gamma = \lambda E$, $C_{\gamma - 1} = C_1 = O$, $\Xi = \zeta E$, and $C_0 = \hat{O}$, where $\hat{O}$ is the zero column matrix. Here $V$ is the spectral characteristic of the multiplier and $N$ is the spectral characteristic of the random process $n(t)$.

The estimate of the path for the Ornstein–Uhlenbeck process can be expressed by the formula (11), and its approximation can be expressed by the formula (12) taking into account that all the spectral characteristics are truncated.

Let $\{q_i(t)\}_{i=0}^\infty$ be the trigonometric functions defined on the time interval $\mathbb{T}$. The sample path for the Ornstein–Uhlenbeck process and its estimates, i.e. $x(t)$ and $\hat{x}(t)$, are presented in figure 1 for $T = 1$ ($T = [0, 1]$) and $\theta = 0.5$; $\mu = -1$, $\sigma = 0.5$, $x_0 = 1$ (parameters and the initial condition for the state equation), $\lambda = 5$, $\zeta = 0.2$, $\gamma = 1$; its approximation problem approximately), $M = 10^3$ (sample size for solving the optimal estimation problem approximately), $L = 100$ (truncation order). Note that in [14] the sample path for the Ornstein–Uhlenbeck process has been estimated in the simplest case, i.e. $\theta = T$. Here $\theta < T$ and the estimate $\dot{x}(t)$ gives the filtering solution for $t = \theta$, the smoothing solution for $t < \theta$, and the prediction solution for $t > \theta$.

The figure 1 clearly shows that the quality of the estimation is worse for $t > \theta = 0.5$ because it is the prediction solution. These numerical results also show the modeling and estimation
error when \( t = 0 \) and \( t = T \). This error has been described in the previous section. Its reason is that the trigonometric Fourier series does not converge pointwise.

Additionally, the sample path for the Ornstein–Uhlenbeck process and its estimates are presented in figure 2 when \( \theta = 0.75 \) (other parameters are the same). Here we can see that the quality of the estimation is worse for \( t > \theta = 0.75 \).

**Figure 1.** The sample path for the Ornstein–Uhlenbeck process and its estimates (\( \theta = 0.5 \)).

**Figure 2.** The sample path for the Ornstein–Uhlenbeck process and its estimates (\( \theta = 0.75 \)).

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