Reflections on the Strong CP Problem

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I discuss how anomalies affect classical symmetries and how, in turn, the non-trivial nature of the gauge theory vacuum makes these quantum corrections troublesome. Although no solution seems in sight for the cosmological constant problem, I examine three possible approaches to the strong CP problem involving vacuum dynamics, an additional chiral symmetry, and the possibility of spontaneous CP or P breaking. All of these “solutions” have their own problems and suggest that, at a deep level, we do not understand the nature of CP violation. Nevertheless, it remains extremely important to search for experimental signals predicted by these theoretical “solutions”, like invisible axions.

1. EFFECTIVE THEORIES AND THEIR FAILURES

The strong CP problem[1] is intimately connected with the failure of symmetries to survive quantum effects. Let me illustrate this point by a simple example. At the classical level, in general, complex mass terms for fermions are not necessarily a signal of time reversal (T) violation. In fact, offending complex terms in the Lagrangian

\[ L_{\text{mass}} = -me^{i\theta}\bar{\psi}_L\psi_R - me^{-i\theta}\bar{\psi}_R\psi_L \]  

(1)

can be rotated away provided that the fermion fields have chiral invariant interactions, such as those provided by gauge interactions. In this case, one may perform a chiral rotation

\[ \psi_L \rightarrow e^{i\theta/2}\psi_L ; \quad \psi_R \rightarrow e^{-i\theta/2}\psi_R \]  

(2)

which eliminates the phase \( \theta \) from \( L_{\text{mass}} \) altogether.

This pleasant situation, however, changes at the quantum level as a result of the existence of chiral anomalies.[2] Even though the transformation (2) is classically allowed (i.e. it does not alter the rest of the Lagrangian, besides \( L_{\text{mass}} \)), because the chiral current is not divergenceless at the quantum level, the transformation (2) induces an equivalent T-violating term. Specifically, if the field \( \psi \) interacts with some non-Abelian gauge field \( A_\mu^a \), so that the chiral current

\[ J_5^\mu = \bar{\psi}\gamma^\mu\gamma_5\psi \]  

(3)

has an anomaly[2]

\[ \partial_\mu J_5^\mu = \frac{g^2}{32\pi^2} F_\mu^a \tilde{F}^\mu_{a\nu} \]  

(4)

then the transformation (2) changes \( L_{\text{mass}} \) to

\[ L_{\text{mass}} \rightarrow -m(\bar{\psi}_L\psi_R + \bar{\psi}_R\psi_L) + \frac{\theta g^2}{32\pi^2} F_\mu^a \tilde{F}^\mu_{a\nu} \]  

(5)

The \( \theta F\tilde{F} \) term is C-conserving, but both P- and T-odd. So, at the quantum level, complex fermion mass terms are indeed signals of T-violation.

The Standard Model has two such classical symmetry failures. The first of these, which is due to the chiral anomaly, is at the root of the strong CP problem. Because of the chiral anomaly connected with the gluon field strength, \( G_\mu^{a\nu} \), the full Lagrangian of the Standard Model is augmented by the following effective interaction[3]

\[ L_{\text{eff}} = \frac{\theta}{8\pi} G_\mu^{a\nu} \tilde{G}_{a\mu\nu} \]  

(6)

Here \( \alpha_3 \) is, essentially, the square of the \( SU(3) \) coupling constant \( [\alpha_3 = g_3^2/4\pi] \) and \( \theta \) is a pa-

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[1] This work is supported in part by the Department of Energy under Grant No. DE-FG03-91ER40662, Task C.

[2] There is no equivalent \( \theta_{\text{weak}} WW \) interaction involving the \( SU(2) \) gauge fields, because the electroweak interactions possess an overall chiral symmetry.[4]
rameter containing both QCD and electroweak information:
\[ \bar{\theta} = \theta + \text{Arg} \det M . \] (7)

In the above \( \theta \) is the QCD vacuum angle, while \( M \) is the quark mass matrix which obtains after the breakdown of the electroweak symmetry.

The second manifestation of the violation of a classical symmetry in the Standard Model arises in the trace of the energy momentum tensor. This trace has an additional piece beyond the “classical” trace \( \bar{\theta} \), reflecting an anomaly in the dilatational current. One has, retaining only the QCD piece of the trace anomaly,\(^3\):

\[ T^\mu = \frac{\beta(g_3)}{2g_3} G^{\mu
u} G_{\alpha\mu\nu} + \theta^\mu , \] (8)

where \( \beta(g_3) \) is the \( \beta \)-function for QCD.

What makes the quantum corrections (6) and (8) important is the nontrivial nature of the gauge vacuum. I begin by considering the \( \theta \bar{G} G \) term. Here, at first sight, it actually seems that this quantum correction may be ineffective, since the density \( \bar{G} G \) is a total derivative\(^3\):

\[ G^{\mu
u} \bar{G}_{\alpha\mu\nu} = \partial_\alpha K^\mu \] (9)

with

\[ K^\mu = \epsilon^{\alpha\beta\gamma} A_\alpha \left\{ G_{\alpha\beta\gamma} - \frac{g_3}{3} f_{ab\gamma} A_{b\beta} A_{c\gamma} \right\} . \] (10)

However, the non-trivial nature of the gauge theory vacuum does not allow one to throw out this total divergence. It turns out that amplitudes in which the vacuum gauge field configurations at \( t = \pm \infty \) differ by a, so-called, large gauge transformation\(^3\) are associated with non-trivial \( \bar{G} G \) configurations. In fact, the difference in the indices\(^3\) of the pure gauge fields at \( t = \pm \infty \) is given by\(^3\):

\[ \nu = n_+ - n_- = \frac{\alpha_s}{8\pi} \int d^4x G^{\mu
u} \bar{G}_{\alpha\mu\nu} . \] (11)

So, as long as amplitudes with \( \nu \neq 0 \) are important, one cannot ignore the quantum correction (6).

There is separate evidence that \( \nu \neq 0 \) configurations are important in QCD, related to the apparent non-existence of an approximate chiral \( U(1)_A \) symmetry of this theory. Since \( m_u \) and \( m_d \) are much smaller than the dynamical QCD scale, \( \Lambda_{\text{QCD}} \), the QCD Lagrangian should possess an approximate \( U(2)_A \) chiral symmetry. This symmetry is spontaneously broken by the formation of quark condensates

\[ \langle \bar{u}u \rangle = \langle \bar{d}d \rangle \sim \Lambda_{\text{QCD}}^3 , \] (12)

so one expects 4 near Nambu-Goldstone bosons. Although the pions behave as such, the \( \eta \) meson appears quite different\(^3\) and, in fact, \( m_\eta^2 \gg m_\pi^2 \). This can be understood if the \( U(1)_A \) subgroup of \( U(2)_A \) is not an (approximate) symmetry at all. Because of the chiral anomaly,\(^3\) even in the limit when \( m_u, d \to 0 \), one finds a violation of the associated \( U(1)_A \) chiral charge, with

\[ \Delta Q_5 = \frac{\int d^4x \partial_\mu J_5^\mu}{4\pi} = \frac{\alpha_s}{4\pi} \int d^4x G^{\mu
u} \bar{G}_{\alpha\mu\nu} = 2\nu . \] (13)

So, to understand why the \( \eta \) is not a Nambu-Goldstone boson it must be that \( \nu \neq 0 \) configurations are important in QCD. Whence, it follows that also the quantum corrections (6) must be significant, unless for some reason the parameter \( \bar{\theta} \) is very small or vanishes.

Not only does the QCD vacuum allow the formation of the quark condensates [cf Eq. (12)] which break chirality, it also permits the formation of a gluon condensate

\[ \langle G^{\mu\nu} G_{\alpha\mu\nu} \rangle \sim \Lambda_{\text{QCD}}^4 , \] (14)

This latter condensate, in view of Eq. (8), produces a significant vacuum energy density

\[ \langle T^\mu_\mu \rangle = \frac{\beta(g_3)}{2g_3} \langle G^{\mu\nu} G_{\alpha\mu\nu} \rangle + \theta^\mu \] (15)

One does not really know what the value of \( \langle \theta^\mu \rangle \) is. Naively, it should be at least of order \((100 \text{ GeV})^4\) due to the breakdown of the electroweak theory. However, it is possible that \( \langle \theta^\mu \rangle \) vanishes. At any rate, using the value for \( \langle G^{\mu\nu} G_{\alpha\mu\nu} \rangle \) deduced from QCD sum rules\(^3\), the first term

\[ \approx 2\nu . \] (13)
in Eq. (15) already produces a vacuum energy density of order
\[ \langle T^\mu_\mu \rangle \sim (350 \text{ MeV})^4. \] (16)
This value is about 45 orders of magnitude larger than the nominal bound for the cosmological constant.\(^4\)
\[ \Lambda \leq (3 \times 10^{-3} \text{ eV})^4. \] (17)
Because Λ measures the vacuum energy density, one expects that Λ = \( \langle T^\mu_\mu \rangle \). Obviously, the discrepancy between Eqs. (16) and (17) tells us that this is not the case. The reason for this flagrant violation of our intuition is a present day mystery.

A similar, but slightly less severe, puzzle is presented by Eq. (6). The \( \bar{G}G \) term, because it violates both P and T, can give rise to a sizeable electron dipole moment for the neutron, unless the angle parameter \( \theta \) is very small. To calculate the size of this dipole moment, it is useful to perform a chiral rotation which transform the \( \bar{G}G \) term into a complex quark mass term\(^1\)
\[ L_{\text{CP-viol.}} = i\bar{\theta}m_q \left[ \frac{a_2}{2} \bar{u} + \frac{d_{15}}{2} \bar{d} \right]. \] (18)
One can use the above effective Lagrangian directly to calculate the neutron dipole moment via the equation
\[ d_n \bar{n} \sigma_{\mu\nu} k^\nu \gamma_5 n = \langle n | T(J_\mu^{\text{em}} \times i \int d^4x L_{\text{CP-viol.}}) | n \rangle. \] (19)
To arrive at a result for \( d_n \), one insert a complete set of states \( |X\) in the matrix element above and tries to estimate which set of states dominates. In the literature there are two calculations along these lines. Baluni\(^{[10]}\) uses for \( |X\) the odd parity \( |N_{1/2}\) states which are coupled to the neutron by \( L_{\text{CP-viol.}} \). Crewther, et al.\(^{[11]}\) instead, do a soft pion calculation where, effectively, \( |X\) is \( |N_{\pi_{\text{soft}}}\)\). The result of these calculations are rather similar and lead to an expression for \( d_n \) whose form could have been guessed at. Namely,
\[ d_n \sim e \frac{m_q}{M_n} \left( \frac{m_q}{M_n} \right) \bar{\theta} \sim \begin{cases} 2.7 \times 10^{-16} \bar{\theta} & [10] \\ 5.2 \times 10^{-16} \bar{\theta} & [13] \end{cases} \] (20)
\( \bar{\theta} \) is very small. To calculate \( d_n \) one insert a complex quark mass term\(^1\)
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The present bound on \( d_n \)\(^{[12]}\) at the 95% C.L., is
\[ d_n < 1.1 \times 10^{-25} \text{ e cm}. \] (21)
Whence, to avoid contradiction with experiment, the parameter \( \bar{\theta} \) must be less than \( 2 \times 10^{-10} \). Why this parameter, which is the sum of two disparate terms, \( \bar{\theta} \) and \( \text{Arg} \det M \), should be so small is another mystery.

2. APPROACHES TO THE STRONG CP PROBLEM

There are no believable mechanisms to guarantee that the cosmological constant either vanishes or satisfies the bound (17). Obviously, the vacuum energy density induced by the gluon condensate and other VEVs from spontaneous symmetry breakdowns apparently either cancel among each other—something that is difficult to believe, given the different scales involved—or, somehow, do not end up by contributing to the cosmological constant. In this respect, the situation concerning the strong CP problem is better. Here, at least, there are some ideas on how perhaps to resolve the conundrum raised by the presence of the \( \bar{G}G \) term.

There are three distinct approaches to the strong CP problem. The first of these supposes that the vacuum dynamics itself selects \( \bar{\theta} \) to be zero, leading to no CP-violating effects. The second imposes an additional chiral symmetry on the theory\(^{[13]}\) which dynamically drives \( \bar{\theta} \rightarrow 0 \). The third approach supposes that CP (or perhaps P) is spontaneously broken, with the resulting theory producing naturally very small values for \( \bar{\theta} \), \( \bar{\theta} \leq 10^{-10} \)\(^{[14]}\). All three approaches leave a host of questions unanswered and are, in some sense, unsatisfactory. However, they do have some experimental consequences and, indeed, experiments may give us a hint of which of these approaches may be ultimately viable. In what follows, I want to briefly discuss and review these “solutions” to the strong CP problem, focusing particularly on their more troublesome features.

2.1. Vacuum Dynamics

There have been various attempts to solve the strong CP problem within QCD. Although I do
not believe that the solution of this problem is to be found in this direction, let me mention three such possibilities that have been raised at various times:

(i) One knows that the vacuum energy is periodic in $\bar{\theta}$

$$E_{\text{vac}}(\bar{\theta}) \sim (1 - \cos \bar{\theta}) . \quad (22)$$

Thus, if one were to assume that the correct theory has minimum vacuum energy, then $\bar{\theta} = 0$ would naturally ensue. Unfortunately, I know of no physical principle that demands that one should minimize $E_{\text{vac}}$.

(ii) A more interesting suggestion, perhaps, has been put forth by Schierholz. He argues that it is possible that QCD may not confine for $\bar{\theta} \neq 0$. Hence, since all indications are that QCD confines, it must be that $\bar{\theta} = 0$. Schierholz indeed finds evidence for a deconfining phase transition at finite vacuum angle in the CP$^N$ model. However, it is really difficult to extrapolate from this result to QCD. In fact, it is unclear to me what role, if any, the vacuum angle, or $\bar{\theta}$, plays for confinement in QCD. So I do not see how confinement could force $\bar{\theta} \to 0$.

(iii) Finally, there have also been suggestions that the $\theta$ vacuum is an artifact of the boundary condition imposed on the gauge transformation matrices at spatial infinity. If one does not impose such boundary conditions, the necessity for the $\theta$-vacuum disappears and so does the strong CP problem. However, then one is left again to understand why the $\eta$ meson does not behave like a Nambu-Goldstone boson. For this reason, I do not believe that the $\theta$-vacuum is an artifact.

2.2. The Chiral Solution to the Strong CP Problem

Because a chiral transformation can change the vacuum angle $\bar{\theta}$

$$e^{\lambda \tilde{Q}_L} \bar{\theta} = |\bar{\theta} + \alpha| , \quad (23)$$
a natural solution to the strong CP problem assumes the existence of some additional chiral symmetry in the Standard Model. Two suggestions have been put forth:

(i) The lightest quark, the $u$-quark, actually has zero mass, $m_u = 0$.\[13]\[13]

(ii) The Standard Model is invariant under an additional $U(1)$ chiral symmetry, $U(1)_{\text{PQ}}$.\[13]\[13]

The first of these possibilities is disfavored theoretically by a careful analysis of the low energy spectrum of QCD, which is inconsistent with having $m_u = 0$. In addition, in my view, by appealing to this "solution", one has just exchanged one problem for another. What is the origin of the chiral symmetry which makes $\det M = 0$?

I am, of course, prejudiced in favor of the second chiral solution! Imposing an additional, spontaneously broken, chiral symmetry $U(1)_{\text{PQ}}$ on the Standard Model replaces the static CP-violating parameter $\bar{\theta}$ by the dynamical CP-conserving field associated with the $U(1)_{\text{PQ}}$ pseudo Nambu-Goldstone boson—the axion.\[20]\[20]

This replacement

$$\bar{\theta} \to \frac{a(x)}{f} \quad (24)$$

introduces into the theory a new parameter $f$, the scale of the spontaneous breakdown of $U(1)_{\text{PQ}}$.

The axion field translates under a $U(1)_{\text{PQ}}$ transformation

$$a(x) \to a(x) + \alpha f . \quad (25)$$

Thus, in the effective low energy Lagrangian this field will always appear derivatively coupled, with the exception of a term needed to reproduce the chiral anomaly in the $U(1)_{\text{PQ}}$ current. Assuming the existence of an extra $U(1)_{\text{PQ}}$ symmetry, one finds

$$L_{\text{low energy}} = L_{\text{SM}} - \frac{1}{2} \partial_\mu a \partial^\mu a + L_{\text{int}}[\psi; \partial_\mu a; \bar{\psi} \gamma_5 \partial_\mu \bar{\psi}] + \frac{a}{f} \frac{\alpha_3}{8\pi} G^\mu_\nu \tilde{G}_{a\mu\nu}. \quad (26)$$

The last term in (26) insures that the $U(1)_{\text{PQ}}$ current indeed has the expected chiral anomaly

$$\partial_\mu J^\mu_{\text{PQ}} = \frac{\alpha_3}{8\pi} G^\mu_\nu \tilde{G}_{a\mu\nu}. \quad (27)$$
This term effectively also gives the axion field a non-trivial potential. The minimum of this effective potential

\[ 0 = \left\langle \frac{\partial V_{\text{eff}}}{\partial a} \right\rangle \bigg|_{(a)} = -\frac{\alpha_3}{8\pi f} \langle G^\mu_\nu \tilde{G}_{\alpha\mu\nu} \rangle \bigg|_{(a)} \]  

(28)

occurs at

\[ \langle a \rangle = -\bar{\theta} f, \]  

(29)

due to periodicity of the \( \langle G\tilde{G} \rangle \) vacuum expectation value in the relevant \( \theta \)-parameter: \( \bar{\theta} + \langle (a)/f \rangle \).\[13\] Obviously, Eq. (29) solves the strong CP problem, since the coupling of the physical axion field \( a_{\text{phys}} = a - \langle a \rangle \) to \( G\tilde{G} \) removes the \( \bar{\theta}G^2 \) term. Furthermore, the second derivative of the effective potential \( V_{\text{eff}} \) gives a small mass for the axion, of order \( \Lambda_{\text{QCD}}^2/f^2 \):

\[ m_a^2 = \left\langle \frac{\partial^2 V_{\text{eff}}}{\partial a^2} \right\rangle \bigg|_{(a)} = -\frac{\alpha_3}{8\pi f} \left. \frac{\partial}{\partial a} \langle G^\mu_\nu \tilde{G}_{\alpha\mu\nu} \rangle \right|_{(a)} \]

\[ \sim \frac{\Lambda_{\text{QCD}}^4}{f^2}. \]  

(30)

The above mechanism works for any value of the parameter \( f \) associated with the scale of \( U(1)_{\text{PQ}} \) breakdown. Because all interactions of the axion scale as \( 1/f \), as does its mass, the larger this scale is the more weakly coupled and lighter the axion is.\[21\] In our original paper, Helen Quinn and I chose quite naturally for the \( U(1)_{\text{PQ}} \) breaking scale \( f \), the weak scale \( v = (\sqrt{2} G_F)^{-1/2} \). Unfortunately, weak scale axions, with \( f \approx v \), are ruled out experimentally,\[24\] so our specific suggestion is no longer tenable. However, models where \( f \gg v \) are still perfectly consistent.

If \( f \gg v \), it is clear that what triggers the breaking of the \( U(1)_{\text{PQ}} \) symmetry must be the vacuum expectation value of some \( SU(2) \times U(1) \) singlet object \( \sigma \), with \( \langle \sigma \rangle = f \). Remarkably, astrophysics\[23\] and cosmology\[22\] put non-trivial constraints on \( f \) or, equivalently, the axion mass

\[ m_a \sim 6 \left[ \frac{10^6 \text{ GeV}}{f} \right] \text{ eV}. \]  

(31)

The axion is not stable since, through the electromagnetic anomaly, it can always decay into two photons. This lifetime scales as \( \tau(a \rightarrow 2\gamma) \sim f^5 \), so the axion becomes very long-lived for large values of \( f \).

These bounds, taken at face value, restrict \( f \) to a rather narrow range

\[ 5 \times 10^9 \text{ GeV} \leq f \leq 10^{12} \text{ GeV}. \]  

(32)

Although the above provides interesting phenomenological constraints for \( f \), the real issue is what physics causes the \( U(1)_{\text{PQ}} \) symmetry to break down precisely in the range of scales indicated by Eq. (32).

In superstring models, where axions arise naturally as fundamental fields,\[23\] the natural scale for \( f \) is somewhat above the bound (32). Typically\[24\]

\[ f \approx \frac{M_P}{16\pi^2} \sim 10^{16} - 10^{17} \text{ GeV}. \]  

(33)

How to reconcile this potentially natural scale for the \( U(1)_{\text{PQ}} \) breakdown with the bounds of Eq. (32) has led to a number of possible explanations. Typically, in these suggestions one removes the incompatibility between Eqs. (32) and (33) either by complicating the physics or by changing the relevant cosmology.

One way to get \( f \) into the observable range (32) is to associate the \( U(1)_{\text{PQ}} \) breakdown with some physical intermediate scale. A simple example is provided by some recent work of Murayama, Suzuki and Yanagida.\[25\] These authors achieve their goal by assuming that the \( U(1)_{\text{PQ}} \) symmetry results as a radiative effect in a supergravity theory with a flat potential. The relevant effective potential has a (negative) squared mass term for the singlet field \( \sigma \) which is radiatively generated and of order \( m_{\sigma/2} \sim M_W \). \( U(1)_{\text{PQ}} \) breaking occurs as a competition of this term with some gravitational induced non-renormalizable interactions for the singlet field \( \sigma \). The effective potential

\[ V = -m_{\sigma/2}^2 |\sigma|^2 + \frac{\lambda |\sigma|^4}{M_P^2} \]  

(34)

give a \( U(1)_{\text{PQ}} \) breaking VEV

\[ \langle \sigma \rangle = f \sim (m_{\sigma/2} M_P)^{1/2} \sim 10^{10} \text{ GeV} \]  

(35)

in the needed range of Eq. (32).

Alternatively, one can alter the cosmology, thereby allowing larger values for \( f \). The upper bound on \( f \), due to cosmology, more properly is
a bound on the product of $f$ and the square of an initial misalignment angle $\bar{\theta}_i$: \[ f \bar{\theta}_i^2 < 10^{12} \text{ GeV} . \] \[ (36) \]

The usual bound follows by assuming, rather naturally that $\bar{\theta}_i \sim O(1)$. However, Linde\cite{26} has suggested that inflationary models (and the anthropic principle) may well prefer initial misalignment angles $\bar{\theta}_i \ll 1$. In this case, $f$ values like those in Eq. (33) may well be allowed. Alternatively,\cite{23} one can again raise the bound on $f$ by arranging for a period of large entropy production for $T \sim \Lambda_{\text{QCD}}$. This effectively reduces the importance of axion oscillations to the energy density of the Universe and allows for values of $f > 10^{12}$ GeV, even if $\bar{\theta}_i \sim O(1)$. Of course, relaxing the cosmological bound on $f$ makes the observability of invisible axions, as the possible source of the dark matter in the Universe, questionable. Higher $f$'s implies smaller axion masses and hence lower frequencies to detect halo axions in resonance cavity experiments, as well as a smaller signal since this signal scales as $1/f^2$.\cite{28}

There is a second troublesome aspect of the $U(1)_{\text{PQ}}$ solution to the strong CP problem, connected with gravitational effects. One can make arguments which suggest that gravitational interactions do not allow exact global symmetries (like $U(1)_{\text{PQ}}$) to exist. Perhaps the simplest way to understand why this may be so is through the “No Hair” theorem for black holes. Basically, this theorem\cite{29} asserts that black holes are characterized only by a few fundamental quantities, like mass and spin, but possess otherwise no other quantum numbers. Because black holes can absorb particles which carry global charge, while carrying no global charge themselves, it appears that through these processes one can get an explicit violation of whatever symmetry is associated with the global charge. That is, global charge can be lost when particles carrying this charge are swallowed by a black hole.

One can parametrize the effect of the breaking of global symmetries by gravitational interactions by adding to the low-energy Lagrangian non-renormalizable terms, scaled by inverse powers of the Planck mass $M_p$. These terms, of course, should be constructed so as to explicitly violate the symmetries in question, in our case $U(1)_{\text{PQ}}$. Schematically, therefore, the full Lagrangian of the theory, besides containing the usual Standard Model terms, should also include some effective non-renormalizable interactions containing various operators $O_n$, breaking explicitly $U(1)_{\text{PQ}}$

\[ \mathcal{L}_{\text{grav. int.}} = \sum_n \frac{1}{M_p^n} O_n . \] \[ (37) \]

Here the dimension of the operators $O_n$ is $n + 4$.

The addition of the non-renormalizable interactions (37) has a significant effect and, in general, may vitiate the $U(1)_{\text{PQ}}$ solution to the strong CP problem.\cite{30} Even though the interaction terms are scaled by inverse powers of the Planck mass, these terms both give an additional contribution to the axion mass and alter the QCD potential, so that $\bar{\theta}$ does not finally adjust to zero!

One can understand what is going on schematically by sketching the form of the effective axion potential in the absence and in the presence of the $U(1)_{\text{PQ}}$ breaking gravitational interactions.\cite{31} Without gravity, a useful parametrization for the physical axion effective potential, which follows from examining the contributions of instantons,\cite{13} is

\[ V_{\text{axion}} = -A^4_{\text{QCD}} \cos \frac{a_{\text{phys}}}{f} . \] \[ (38) \]

This potential displays the necessary periodicity in $a_{\text{phys}}/f$, has a minimum at $\langle a_{\text{phys}} \rangle = \bar{\theta}_{\text{eff}} = 0$, and leads to an axion mass $m_a = A^2_{\text{QCD}}/f$.

Including gravitational effects changes the above potential by adding a sequence of terms involving operators of different dimensions. Let us just consider one such term and examine the potential.\cite{31}

\[ \tilde{V}_{\text{axion}} = - A^4_{\text{QCD}} \cos \frac{a_{\text{phys}}}{f} \
\quad - \frac{c f d}{M_p^d} \cos \left[ \frac{a_{\text{phys}}}{f} + \delta \right] . \] \[ (39) \]

Here $c$ is some dimensionless constant and $\delta$ is a CP-violating phase which enters through the gravitational interactions. This potential modifies the formula for the axion mass, giving now

\[ m_a^2 \simeq \frac{A^4_{\text{QCD}}}{f^2} + \frac{c f^{d-2}}{M_p^{d-4}} . \] \[ (40) \]
For $f$ in the range of interest for invisible axions, the second term above coming from the gravitational effects dominates the QCD mass estimate for the axion, unless $c$ is extraordinarily small and/or the dimension $d$ is rather large. More troublesome still, $V_{\text{axion}}$ now no longer has a minimum at $(a_{\text{phys}}) = 0$. Rather one finds a minimum of $V_{\text{axion}}$ for values of

$$\theta_{\text{eff}} = (a_{\text{phys}})/f \simeq c \sin \delta \frac{f^d}{M_p^{d-4} \Lambda_{\text{QCD}}^4}.$$ (41)

That is, the gravitational effects (provided there is a CP violating phase associated with them) induce a non-zero $\theta$, even if the $U(1)_{PQ}$ symmetry is spontaneously broken by the vacuum. To satisfy the bound $\theta \lesssim 10^{-10}$ again necessitates that $d$ be large and/or that the constant $c$ be extraordinarily small.

To date there is no clear resolution to this problem and it could be that these considerations actually destroy the chiral solution to the strong CP problem. Because one does not really understand quantum gravity, one cannot be totally sure of the validity of the above arguments. Nevertheless, if one takes these arguments seriously, it is gratifying that various loopholes have emerged which preserve the $U(1)_{PQ}$ solution to the strong CP problem.

The simplest way to avoid any gravitational troubles is to arrange things in the theory, usually through the imposition of some discrete symmetries, so that gravity breaks $U(1)_{PQ}$ only through high dimension operators. If $d$ is sufficiently large, such that

$$\frac{f^d}{M_p^{d-4} \Lambda_{\text{QCD}}^4} < 10^{-10},$$ (42)

then there is no strong CP problem. If (42) holds, furthermore, it turns out that also the gravitational corrections to the axion mass are negligible and $m_a \sim \Lambda_{\text{QCD}}^4/f$.

The second way to avoid problems is if, indeed, the strength of the non-renormalizable interactions, $c$, is extremely small. This apparently is possible in some string theories which have a large compactification radius, where one can obtain parameters $c < 10^{-56}$. In that case, again, the gravitational correction to $\theta < 10^{-10}$ and the changes to the axion mass are totally irrelevant. Of course, it gives one pause to imagine that the understanding of why $\theta < 10^{-10}$ should be through a global symmetry, $U(1)_{PQ}$, whose violation by gravitational interactions are under control because of the presence of an even smaller parameter $c$, $c < 10^{-56}$!

A third possibility, which is perhaps the most interesting from my point of view, is that there is no CP-violating phase in the effective interaction (39) (i.e. $\sin \delta = 0$). That is, the gravitationally induced terms that violate $U(1)_{PQ}$ do not also violate CP. This is quite an interesting possibility phenomenologically, because $\theta_{\text{eff}} = 0$, but the interrelation between the axion mass $m_a$ and the scale of $U(1)_{PQ}$ breaking, $f$, is changed. Now, in addition to a term proportional to $1/f$ the axion mass gets a direct term proportional to the gravitational breaking of $U(1)_{PQ}$:

$$m_a = \frac{\Lambda_{\text{QCD}}^2}{f} + (m_a)_{\text{gravity}}.$$ (43)

In this case, the cosmological and astrophysical properties of axions may in fact be quite different from those in standard invisible axion models.

2.3. Spontaneous Breaking of CP/P

It may be possible, perhaps, to resolve the strong CP problem by imagining that CP (or perhaps even P) is a symmetry of nature, but one which is spontaneously broken by the vacuum. In this case, there is no QCD vacuum angle $\theta$ and, at the Lagrangian level, $\text{Arg det } M = 0$. Nevertheless, because CP is spontaneously broken, eventually at the loop level one induces an effective angle $\theta$. However, one can perhaps arrange the theory so that the resulting $\theta < 10^{-10}$. This, generally speaking, requires that also at one-loop level $\bar{\theta} = 0$.

One can distinguish two different classes of models which try to resolve the strong CP problem in this fashion, with their distinction being related principally to the scale at which CP is spontaneously broken. In the first class of models the breaking of CP occurs at the weak scale, while in the second class this breaking occurs at

\[ \text{Note that for } d = 5 \text{ the induced term is enormous since } f^5/M_p \Lambda_{\text{QCD}}^4 \sim 10^{46}. \]
a scale close to the Planck mass. Both types of models, however, have generic problems. In what follows, I again briefly focus on these problems.

To break CP spontaneously in the Standard Model requires having a more complicated Higgs sector with two or more Higgs fields, each of which acquires some complex VEV. However, this more complicated Higgs sector, with its CP-violating phases

$$\langle \Phi_i \rangle = v_\ell e^{i\delta_i}$$

and with scales $v_\ell \sim 0$ (100 GeV) is problematic, since it leads to flavor changing neutral currents at an unacceptable level, unless one imposes some extra constraints. [36]

Although this is a troublesome feature of these kind of models, it is probably not their worse aspect. The spontaneous breaking of CP leads to the formation of domains with different CP values in the early Universe. These domains are separated from each other by walls where considerable energy is stored. As the Universe cools to its present temperature, the energy density associated with these domain walls dissipates very slowly. For VEVs of the order of the weak scale, the energy density which would be associated with these domain walls in the present Universe is enormous, far exceeding the Universe’s critical density.

$$\rho_{\text{wall}} \sim \langle \Phi_i \rangle^3 T^2 \sim 10^{-7}\text{GeV}^4$$

$$\gg \rho_c \sim 10^{-46}\text{GeV}^4.$$ [45]

Hence, it is really not tenable imagining that CP is spontaneously broken at scales of the order of the weak scale.

Because of this cosmological problem, it has been suggested that perhaps the spontaneous breaking of CP occurs at scales so large that inflation has not yet taken place. In this case, the domain wall problem disappears because our observable Universe after inflation just occupies one of these CP domains. Obviously having such large VEVs associated with spontaneous CP violation also eliminates the FCNC problem since $v_{\text{CP-viol.}} \gg 100$ GeV. However, the difficulty in these models resides in transmitting the CP-violating phase generated at these high scales to the low energy sector, so that one can actually generate the observed CP-violation in the neutral Kaon complex.

Because $v_{\text{CP-viol.}} \gg v \sim 250$ GeV, it is obvious that whatever fields $\sigma_i$ are responsible for this VEV, these fields again must all be $SU(2) \times U(1)$ singlets. Thus, very naturally, these kind of models have no direct coupling of these fields to quarks, leading to $\text{Arg} \det M = 0$ at tree level. As a result, the prevalent form of CP violation at low energy for these models occurs through the mixing of these $SU(2) \times U(1)$ singlet fields with other Higgs fields in the theory. In particular, often the phases associated with the “high scale” $SU(2) \times U(1)$ singlet VEVs are transferred to the coupling of triplet fields $\chi$, which can mediate directly $\Delta S = 2$ transitions like $ss \leftrightarrow dd$. [38]

Hence, it is quite natural for models of spontaneous CP violation at large scales to lead to superweak models [39] of CP violation at low energy. Furthermore, in these models [40] it is relatively easy to eliminate CP violating contributions to $\text{Arg Det} M$ at one loop level, so that non-zero values of $\theta$ do not appear until 2 loops. Obviously, we shall know relatively soon whether superweak models for CP violation are tenable. This may be made clear by the next round of the $e'/e$ experiments [41], but probably will most clearly emerge from B mesons CP-violation studies at the, soon to be operational, B factories.

I should remark that it is possible to construct models where, even though CP is broken spontaneously at a high scale, the low energy observable CP-violation is indistinguishable from the standard CKM model. [42] These types of models were first constructed by Nelson [43] and Barr [44] and are quite interesting. What mitigates against them, however, is that they are rather recondite and to make them work requires new physics at different scales. Basically these models have a contribution to $\theta$ already at one-loop and to control this they need to invoke quite different scales. I have discussed in some detail the structure of these Nelson-Barr models [44] and do not want to repeat this discussion here. Suffice to say that, typically, the one-loop contribution to $\text{Arg det} M$...
is of the form

$$\text{Arg det} M \sim \left( \frac{M_I}{M_X} \right)^2 \times \text{phases} \quad (46)$$

where $M_X$ is a GUT scale and $M_I$ is the mass of some new fermions. One can guarantee $\bar{\theta} < 10^{-10}$ by assuming $M_I \ll M_X$. Solving the strong CP problem in this way, by assuming a significant hierarchy in an obscure sector of the theory, which is mostly decoupled from low energy physics, is clearly rather unsatisfactory—at least to me!

3. CONCLUDING REMARKS

The strong CP problem (why $\bar{\theta} < 10^{-10}$ rather than of $O(1)$) is perhaps not as serious an issue as the cosmological constant problem. After all QCD predicts $\langle T^{\mu}_{\mu} \rangle \sim 10^{-2} \text{GeV}^4$, which is more than forty orders of magnitude larger than what cosmology informs us, $\langle T^{\mu}_{\mu} \rangle < 10^{-46} \text{GeV}^4$! Nevertheless, in my view, the strong CP problem is a fairly clear indication that, at a deep level, we really do not understand the nature of CP breaking.

I do not believe that the solution to the strong CP problem will come from QCD itself. Rather its solution must come from a better understanding of the whole theory. It is just possible that the key to the resolution of the strong CP problem will be found when we garner a better understanding of low energy CP-violation. In particular, if the observed CP-violation in the Kaon system were due to some superweak interactions, then it is possible that $\bar{\theta}$ is indeed calculable and small. We will know experimentally soon, both from the next round of $\epsilon'/\epsilon$ experiments and from studies of B meson CP violation at the B factories, whether low energy CP violation is described on the main by the CKM model or not. If the CKM picture holds, which is my expectation, then the strong CP problem will remain a problem.

In my view, it is likely that the solution to the strong CP problem really is related to the existence of an effective global chiral symmetry which makes $\bar{\theta}$ a dynamical parameter. So, I am a believer in axions and in a new dynamical scale $f$, related to the breakdown of this over-all chiral symmetry. I am, however, less certain that the scale $f$ is in the invisible axion range $[5 \times 10^9 \text{ GeV} < f < 10^{12} \text{ GeV}]$, although I believe it is crucially important to search for axions in this range. My skepticism here is connected to the perceived wiggle-room which both alternative dynamics and cosmology provide to the determination of $f$. Nevertheless, if $f$ is much greater than the weak scale, $f \gg v$, as it surely is, one cannot really escape asking what is its relation to the Planck mass, $M_P$.

More generally, my sense is that it is very important to try to understand the compatibility of a global chiral symmetry, like $U(1)_{\text{PQ}}$ with gravity. Does $U(1)_{\text{PQ}}$ survive gravitational effects, or not? My hunch is that it does and that when we will understand things better we will find that the strong CP problem and the cosmological constant problem are deeply related. At a deep level, the solution of these problems probably lies in string theory and supersymmetry. In fact, in supersymmetric theories scale and chiral transformations are naturally related, with the dilaton and the axion both making up the scalar components of a Nambu-Goldstone chiral superfield, and with the trace and chiral anomalies being similarly twinned. The big question, however, is whether these musings can ever be turned into a proper understanding of these problems!

Acknowledgments

I am very grateful to Pierre Sikivie for having invited both Helen Quinn and me to participate in this 20th anniversary celebration of the PQ symmetry and of axions. Both the meeting and the hospitality were splendid!

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