Thick Dirac-Nambu-Goto branes on black hole backgrounds

Viktor G. Czinner

Abstract Thickness corrections to static, axisymmetric Dirac-Nambu-Goto branes embedded into spherically symmetric black hole spacetimes with arbitrary number of dimensions are studied. First, by applying a perturbative approximation, it is found that the thick solutions deviate significantly in their analytic properties from the thin ones near the axis of the system, and perturbative approaches around the thin configurations can not provide regular thick solutions above a certain dimension. For the general case, a non-perturbative, numerical approach is applied and regular solutions are obtained for arbitrary brane and bulk dimensions. As a special case, it has been found that 2-dimensional branes are exceptional, as they share their analytic properties with the thin branes rather than the thick solutions of all other dimensions.

1 Introduction

The study of higher dimensional black holes, branes and their interactions is an active field of research in several different areas of modern theoretical physics. One interesting direction, which has been first introduced by Frolov [1], is to consider a brane - black hole toy model for studying merger and topology changing transitions in higher dimensional classical general relativity [2], or in certain strongly coupled gauge theories [3] through the gauge/gravity correspondence. The model consists of a bulk $N$-dimensional black hole and a test $D$-dimensional brane in it ($D \leq N - 1$), called *brane-black hole* (BBH) system. The brane is infinitely thin, and it is described by the Dirac-Nambu-Goto [4] action.

Viktor G. Czinner$^{1,2}$
$^1$Centro de Matemática, Universidade do Minho, Campus de Gualtar, 4710-057 Braga, Portugal;
$^2$HAS Wigner Research Centre for Physics, Institute for Particle and Nuclear Physics, H-1525 Budapest, P.O.Box 49, Hungary.
e-mail: czinner.viktor@wigner.mta.hu
Due to the gravitational attraction of the black hole, the brane is deformed and there are two types of equilibrium configurations. The brane either crosses the black hole horizon (supercritical), or it lies totally outside of the black hole (subcritical), see Fig. 1.

**Fig. 1** A sequence of thin brane equilibrium configurations on Schwarzschild background. The different configurations belong to different boundary conditions. For simplicity, the horizon radius is put to be 1, and $R$ and $Z$ are standard cylindrical coordinates.

Generalizations of the BBH model by studying the effects of thickness corrections obtained from higher order curvature terms in the effective brane action, have also been studied by Frolov and Gorbonos [5] within a perturbative approach, near the critical solution and restricted to the Rindler zone. As an interesting result they found that when the spatial dimension of the brane is greater than 2, supercritical solutions behave quite differently from subcritical ones, and according to their numerical analysis, they did not find evidence for the existence of such solutions. They suggested that quantum corrections may cure this pathological behaviour.

As a different approach, in three consecutive papers [6, 7, 8], we have reconsidered the problem of thickness corrections to the BBH system within a more general framework than that of [5]. We did not restrict ourselves to work neither in the Rindler zone, nor in the near-critical solution region, and we also chose to follow a different path in obtaining the brane Euler-Lagrange equation. As a result we were able to provide the complete set of regular solutions of the thick-BBH problem for arbitrary brane and bulk dimensions, and also to clarify the question of phase transition in the system.

In this paper we present very briefly the outline of our main findings, and refer the reader to the works [6, 7, 8] for details and the complete results.

### 2 Perturbative approach

In [6], we applied a linear perturbation method to the thick BBH problem obtained from the curvature corrected Dirac-Nambu-Goto action:

$$S = \int d^D \xi \sqrt{-\text{det}g_{\mu\nu}} \left[ -\frac{8\mu^2}{3\ell} \left( 1 + C_1 R + C_2 K^2 \right) \right],$$  \hspace{1cm} (1)
and derived the general form of the perturbation equation for the thick branes. From the asymptotic behaviour of the perturbation equation, in accordance with the results of [5], we found that there is no regular solution of the perturbed problem in the Minkowski embedding case, unless the brane is a string, or a 2-dimensional sheet. This restriction, however, does not hold for the black hole embedding solutions, which are always regular within our perturbative approach.

From these results we concluded that the absence of regular solutions above the dimension $D = 3$ implies that the problem can not be solved within perturbative approaches around the thin solutions which are not smooth on the axis of the system. For a general discussion, one needs to find a new, non-perturbative solution of the problem, that is expected to behave differently from the thin solutions by being smooth on the axis.

After these conclusions, we provided the solution of the perturbation equation for various brane ($D$) and bulk ($N$) dimensions. The far distance equations were integrated analytically, while the near horizon solutions were obtained by numerical computations. The deformations of the perturbed brane configurations were plotted (see e.g. Fig. 2) and a comparison was made with the corresponding thin brane configurations with identical boundary conditions.

**Fig. 2** The picture shows two thick (red) brane configurations together with their thin (blue) counterparts in the case of an $N = 5, D = 4$ black hole embedding. The boundary conditions are $\theta_0 = \frac{\pi}{4}$ (bottom curves) and $\frac{\pi}{17}$ (top curves).

### 3 Non-perturbative solutions

In [7] we further studied the effects of curvature corrections to the BBH system. Since the results of [6] clearly showed that perturbative approaches fail to provide regular solutions near the axis of the system in Minkowski type topologies, we considered a different, exact, numerical approach to the problem. We analysed the asymptotic properties of the complete, 4th-order, highly non-linear Euler-Lagrange equation of the thick BBH system, presented its asymptotic solution for far distances, and obtained regularity conditions in the near horizon region for both Minkowski and black hole embeddings. We showed that the requirement of regularity for the thick solution defines almost completely the boundary conditions for the Euler-Lagrange equation in the Minkowski embedding case. The only exceptions
are the brane configurations with 1, 2 and 3 spacelike dimensions. In the cases of 1 and 3, regular solutions of the problem could be found, however, which was an unexpected result, the problem could not be solved with the applied, non-perturbative method for the case of 2 spacelike dimensions.

4 The 2-dimensional case

In [8] we further studied the problem of a topologically flat 2-brane in the thickness corrected BBH system. Despite the previously mentioned difficulties, we were able to find a regular, non-perturbative, numerical solution for this special case also, based on earlier perturbative considerations [6]. The main result here was the observation that the 2-dimensional case is special as being non-analytic at the axis of the system, just like the thin solutions. This property makes it unique in the family of thick solutions, as in all other dimensions both the Minkowski and black hole embedding solutions are analytic in their entire domain.

Acknowledgements The research leading to these results was supported in part by the Japan Society for the Promotion of Science contract No. P06816, the Hungarian National Research Fund, OTKA No. K67790 grant, and by the National Research Foundation of South Africa. It has also received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement n° PCOFUND-GA-2009-246542 and from the Foundation for Science and Technology of Portugal.

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