EGC Reception for FSO Systems Under Mixture-Gamma Fading Channels and Pointing Errors

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Abstract

The performance of equal gain combining reception at free-space optical communication systems is analytically studied and evaluated. We consider the case when the total received signal undergoes independent and not necessarily identically distributed channel fading, modeled by the versatile mixture-Gamma distribution. Also, the misalignment-induced fading due to the presence of pointing errors is jointly considered in the enclosed analysis. New closed-form expressions in terms of finite sum series of the Meijer’s $G$-function are derived regarding some key performance metrics of the considered system; namely, the scintillation index, outage probability, and average bit-error rate. Based on these results, some useful outcomes are manifested, such as the system diversity order, the crucial role of diversity branches, and the impact of composite channel fading/pointing error effect onto the system performance.

Index Terms

Equal gain combining (EGC), free-space optical (FSO) communications, mixture-Gamma distribution, pointing error misalignment.

I. INTRODUCTION

One of the main challenges in free-space optical (FSO) communications is the channel fading due to turbulence-induced scintillation and misalignment. The former is mainly caused by the

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random fluctuation of the refractive index, whereas the latter by time-varying pointing errors due to thermal expansion, dynamic wind loads, and internal vibrations \[1\]. Spatial diversity can be used as a means to effectively tackle this issue; e.g., by using multiple apertures at the transceiver. Some of the most popular spatial diversity schemes are the maximum ratio combining (MRC), equal gain combining (EGC), and selection combining (SC). MRC outperforms the other schemes at the cost of an increased computational complexity, which gets more emphatic in very high data-rate communications such as FSO systems. On the other hand, SC requires less computational burden at the cost of the poorest performance. Between these two extremes, EGC presents an efficient tradeoff between performance and complexity, which is suitable for practical applications.

To cope with the composite turbulence-induced and misalignment fading, several distribution models have been proposed. Among them, the most popular ones (due to their accuracy, versatility, generality, and mathematical tractability) are the Gamma-Gamma and Málaga distribution models. These models, combined with the pointing error effect, result to highly complicated probability density function (PDF)/cumulative distribution function (CDF) because the rather cumbersome high-order Meijer’s $G$-function is included \[2\]. Mostly due to this reason, the performance of diversity receivers has been only merely studied so far in FSO communication systems. Specifically, the performance of dual-branch diversity receivers was studied in \[3\], using a generalized pointing error model. In \[1\], the performance of diversity receivers with arbitrary branches was analyzed under log-normally atmospheric fading (which efficiently approaches weak-only turbulence conditions where large-scale fluctuations dominate). However, the aforementioned works provided asymptotic results, tightly approximating the exact performance only in the high signal-to-noise (SNR) regions (i.e., $\geq 25$dB). In addition, the average bit-error rate (ABER) of EGC under the composite Gamma-Gamma fading with pointing errors was studied in \[4\]. Yet, although the analysis in the latter work was considered for the entire SNR region of operation, it was in the form of a double infinite series representation.

In this paper, we study the performance of EGC reception for FSO systems with arbitrary diversity branches over composite fading channels and pointing errors. The mixture Gamma (MG) distribution model is used to capture the statistical properties of turbulence-induced channel fading. It was recently indicated in \[5\] that MG distribution sharply coincides to both the Gamma-Gamma and Málaga generalized distribution models. Thus, it serves as an effective model to accurately approach channel fading in weak-to-strong turbulence conditions. Further,
the misalignment-induced fading is also considered, by assuming the commonly adopted model of zero boresight pointing errors. New closed-form expressions are derived for key system performance metrics, i.e., the scintillation index, outage probability, and ABER. Moreover, some useful engineering insights are revealed, such as the system diversity order, the crucial role of diversity branches, and the impact of composite channel fading/pointing error effect onto the system performance.

**Notation:** $f_X(\cdot)$ and $F_X(\cdot)$ represent PDF and CDF of the random variable (RV) $X$, respectively. Moreover, $\Gamma(\cdot)$ denotes the Gamma function [6, Eq. (8.310.1)], $B(\cdot, \cdot)$ is the Beta function [6, Eq. (8.384.1)], $\Gamma(\cdot, \cdot)$ is the upper incomplete Gamma function [6, Eq. (8.350.2)], and $\text{erf}(\cdot)$ represents the error function [6, Eq. (8.253.1)]. Finally, $G[\cdot]$ represents the Meijer’s $G$-function [6, Eq. (9.301)].

**II. System Model**

Consider an FSO system with a single transmitter/aperture and $N$ apertures/diversity branches at the receiver. Hence, the received signal is modeled as $r = zs + \nu$, where $s \in \{0, 1\}$ denotes the transmitted symbol and $\nu$ is the additive white Gaussian noise at the receiver, modeled as a complex-Gaussian RV with zero-mean and unit-variance (i.e., assume a normalized received power so as to reflect the corresponding SNR). Also, it holds that $z \triangleq \sum_{j=1}^{N} x_j y_j$, standing for the joint channel fading (irradiance) and misalignment due to the possible pointing errors at the receiver. Specifically, $x_j$ and $y_j$ are the MG channel fading and pointing error effect, respectively, at the $j^{th}$ receive aperture. Moreover, assume that the RV sets $\{x_j\}_{j=1}^{N}$ and $\{y_j\}_{j=1}^{N}$ are independent and not necessarily identically distributed (i.n.n.i.d.) and independent and identically distributed (i.i.d.), correspondingly. Also, $x_j$ is independent of $y_j \; \forall j$. The corresponding PDFs are given by [5]

$$f_{x_j}(x) = \sum_{i=1}^{L} a_i x^{b_i - 1} \exp(-c_i x), \; x > 0,$$

and

$$f_{y_j}(y) = \frac{e^2}{A_0^2} y^{e^2 - 1}, \; 0 \leq y \leq A_0,$$

where $a_i$, $b_i$ and $c_i$ denote the parameters of MG distribution. As an illustrative example, to model Gamma-Gamma channel fading, these parameters are defined as [5, Eq. (5)]

$$b_i = \alpha, \quad a_i = \frac{\alpha \beta^{\alpha} w_i t_i^{\alpha + \beta - 1}}{\Gamma(\alpha) \Gamma(\beta)}, \quad c_i = \frac{\alpha \beta}{t_i^\beta},$$

$$\sum_{j=1}^{N} w_j t_j^{\beta - 1} \frac{1}{\Gamma(\beta)}.$$
where \( \alpha \) and \( \beta \) are the small- and large-scale fading parameters, respectively. Moreover, \( w_i \) and \( t_i \) stand for the weight factors (ensuring that \( \sum_{i=1}^{L} w_i = 1 \)) and abscissas of the MG distribution, respectively [7, p. 81]. A similar matching can be easily obtained when considering another generalized fading model, namely, the Málaga distribution [5, Eq. (9)]. The total number of sum terms in (1) determines the accuracy level of MG distribution. It was shown in [5, Figs. 2 and 3] that a condition of \( L \leq 10 \) satisfies an acceptable accuracy level for most practical applications, resulting to a rapidly converging series of (1). Further, \( A_0 \) is a constant term that defines the pointing loss given by

\[
A_0 = \left[ \text{erf}\left( \frac{\sqrt{\pi}r}{\sqrt{2}w_z} \right) \right]^2,
\]

while \( r \) is the radius of the detection aperture, and \( w_z \) is the beam waist. In addition, \( \xi \) denotes the ratio between the equivalent beam radius at the receiver and the pointing error displacement standard deviation (jitter) at the receiver (i.e., the condition when \( \xi \to +\infty \) reflects the non-pointing error case).

Upon the signal detection, the receiver applies EGC to enhance the communication quality, such that the corresponding SNR reads as

\[
\gamma = \bar{\gamma} z^2 = \bar{\gamma}(\sum_{j=1}^{N} x_j y_j)^2,
\]

where \( \bar{\gamma} \) is the (normalized) mean SNR with \( E_s, N_0 \), and \( \bar{I} \) denoting the transmit power, noise variance, and received mean irradiance, respectively.

The considered system model and corresponding analysis hereinafter can be easily extended to the case when \( M \geq 1 \) transmit apertures are applied, by simply substituting \( N \) with \( MN \) and setting \( \bar{\gamma} = E_s \bar{I}^2/(M^2 N N_0) \) [4, Eq. (3)].

III. STATISTICAL ANALYSIS

We commence by extracting some key statistical results regarding the received SNR, namely, its PDF, CDF and statistical moments. These results are quite useful for the overall performance evaluation of the considered system.

The PDF of SNR for EGC-enabled reception is obtained as

\[
f_{\gamma}(z) = \sum_{i_1=1}^{L} \cdots \sum_{i_N=1}^{L} \prod_{i=1}^{N} [a_{i_s}]^{N-1} \prod_{j=1}^{N-1} B \left( \sum_{l=1}^{j} b_{i_l}, b_{i_{j+1}} \right) \]

\[
\times \frac{\xi^{2N} \xi^{2-b_{i_1}} \Psi[A_0] z^2}{2 A_0^{N \xi^2} \xi^2} \frac{\sum_{i=2}^{N} b_{i_1+b_{i_2}}}{\xi^2} \Gamma \left( b_{i_1} - \xi^2, c \xi^2 \sqrt{z} \frac{\bar{\gamma} A_0}{A_0} \right),
\]

where \( \Psi[A_0] \triangleq \prod_{j=2}^{N} \frac{A_{0j}^{\xi^2-b_{i_j}}}{(\xi^2-b_{i_j})} \) for \( N \geq 2 \), while \( \Psi[A_0] \triangleq 1 \), for \( N = 1 \). Also, \( \xi^2 > b_{i_j}, 2 \leq j \leq N \) should hold in (3) to be a valid PDF. The proof is relegated in the Appendix.
Notice that the restriction $\xi^2 > b_{ij}$ reflects that $\xi^2 > \alpha$, according to (2). This implies the fact that the misalignment-induced parameter due to the pointing error effect should be greater than the corresponding turbulence-induced fading parameter. However, the latter inequality is a reasonable assumption for most practical applications, since $\xi^2 > \alpha$ holds for typical building sway-based pointing errors [8], [9].

The corresponding CDF of $\gamma$ reads as $F_{\gamma}(z) = \int_0^z f_{\gamma}(x) dx$. By using (3), transforming the upper incomplete Gamma function in terms of the Meijer’s G-function [10, Eq. (8.4.16.2)], while utilizing [10, Eq. (2.24.2.2)], we have that

$$F_{\gamma}(z) = \sum_{i_1=1}^L \cdots \sum_{i_N=1}^L \prod_{s=1}^N [a_{is}] \prod_{j=1}^{N-1} \left[ B \left( \sum_{l=1}^j b_{il}, b_{ij+1} \right) \right]$$

$$\times \xi^{2N} c_{i_1}^{\xi^2-b_{i_1}} \Psi[A_0] \frac{\int_{\gamma A_0}^{\sum_{l=2}^{N-2} b_{il} + \xi^2} 1 - (\sum_{l=2}^{N-2} b_{il} + \xi^2), 1}{\sum_{l=2}^{N-2} b_{il} + \xi^2}$$

$$\times G_{2,3}^{1,1} \left[ \frac{c_{i_1} \sqrt{z}}{\gamma A_0} \right] \left[ 0.0 - \xi^2, - (\sum_{l=2}^{N-2} b_{il} + \xi^2) \right].$$

(4)

The $n^{th}$ statistical moment of $\gamma$ is defined as $\mu_{\gamma}^{(n)} = \int_0^\infty x^n f_{\gamma}(x) dx$. Using (3) and utilizing [6, Eq. (6.455.1)], $\mu_{\gamma}^{(n)}$ is obtained in a closed-form expression as

$$\mu_{\gamma}^{(n)} = \sum_{i_1=1}^L \cdots \sum_{i_N=1}^L \prod_{s=1}^N [a_{is}] \prod_{j=1}^{N-1} \left[ B \left( \sum_{l=1}^j b_{il}, b_{ij+1} \right) \right]$$

$$\times \xi^{2N} c_{i_1}^{\xi^2-b_{i_1}} \Psi[A_0] \Gamma \left( \sum_{l=1}^N b_{il} + n \right) \gamma^{-\sum_{l=2}^{N-2} b_{il} + \xi^2}$$

$$\times A_0^{N\xi^2} \left( \sum_{l=2}^{N-2} b_{il} + n + \xi^2 \right) \left( \frac{c_{i_1}}{\gamma A_0} \right)^{\left( \sum_{l=2}^{N-2} b_{il} + n + \xi^2 \right)}.$$

(5)

### IV. System Performance

Capitalizing on the previously derived statistics, some useful metrics that define the overall system performance are analytically evaluated.

#### A. Scintillation Index

The scintillation index (SI) is an important measure for evaluating the performance of FSO systems, which is defined as $\text{SI} \triangleq \mu_{\gamma}^{(2)} / (\mu_{\gamma}^{(1)})^2 - 1$. It can be directly extracted via the closed-form moments-function provided in (5).
B. Outage Probability

The outage probability is defined as the probability that the SNR falls below a certain threshold value, $\gamma_{th}$, such that $P_{out}(\gamma_{th}) = F_{\gamma}(\gamma_{th})$. With the aid of the closed-form expression of (4), the system outage performance can be easily computed.

C. Average Bit-Error Rate

The ABER is defined as \[ P_b \equiv Q \frac{\sum_{l=1}^{N} a_{i_l} \frac{\xi^{2N} \Psi[A_0]}{\sqrt{\pi} \Gamma(P) A_0^{N \xi^2}}}{4 Q \zeta^2 A_0^2} \left[ 1 - P - \frac{\sum_{l=1}^{N} b_{i_l} + \zeta^2}{\sum_{l=1}^{N} b_{i_l} + \xi^2}, \frac{\sum_{l=1}^{N} b_{i_l} + \xi^2}{\sum_{l=1}^{N} b_{i_l} + \xi^2} \right]. \]

D. Diversity Order

In the asymptotically high SNR region, $\bar{\gamma} \to +\infty$. Hence, expanding the Meijer’s $G$-function within (4) in terms of finite sum series [6, Eq. (9.303)] and utilizing [6, Eq. (9.100)], the system outage probability can be tightly approximated in the high SNR regime by

\[
P_{out|\bar{\gamma} \to +\infty}(\gamma_{th}) \approx \sum_{i_1=1}^{L} \ldots \sum_{i_N=1}^{L} \prod_{s=1}^{N} [a_{i_s}] \prod_{j=1}^{N-1} \left[ B \left( \sum_{l=1}^{j} b_{i_l}, b_{i_{j+1}} \right) \right] \frac{\xi^{2N} \zeta \Psi[A_0] \gamma_{th}}{\sqrt{\pi} \Gamma(P) A_0^{N \xi^2}} \frac{\sum_{l=1}^{N} b_{i_l} + \xi^2}{\sum_{l=1}^{N} b_{i_l} + \xi^2} \frac{\sum_{l=1}^{N} b_{i_l} + \xi^2 + \phi_i}{\sum_{l=1}^{N} b_{i_l} + \xi^2 + \phi_i}, \]

where $\zeta \triangleq [1 - (\sum_{l=2}^{N} b_{i_l} + \xi^2), 1]$, $\phi \triangleq [0, b_{i_1} - \xi^2, -(\sum_{l=2}^{N} b_{i_l} + \xi^2)]$, while $\zeta_i$ and $\phi_i$ denote the $i^{th}$ term of $\zeta$ and $\phi$, correspondingly. Note that (7) is a rather simple expression since it is given in terms of finite sum series including elementary-only functions. Thus, some useful
outcomes can be emerged, such as the system diversity order. In particular, it is well-known that the outage probability in the high SNR regime approaches the form of \( \bar{\gamma}^{-D} \), where \( D \) stands for the system diversity order. Noticing from (7) that the outage performance is dominated by \( \bar{\gamma}^{-(\sum_{i=2}^{N} b_i + \xi^2) + \min\{0, -(b_i - \xi^2)\}} \), we have that \( P_{\text{out}|\bar{\gamma} \to +\infty}(\gamma_{\text{th}}) \propto \bar{\gamma}^{-\left[\sum_{j=1}^{N} b_{ij}\right]} \). Thereby, the system diversity order is

\[
D = \left[\sum_{j=1}^{N} b_{ij}\right] = N \times \min\{\alpha, \beta\}. \tag{8}
\]

It is noteworthy from (8) that the rate of change of system performance is independent of the pointing error effect (in the high SNR regime), under the condition \( \xi^2 > \min\{\alpha, \beta\} \), which is in agreement with [1], [2]. Obviously, the pointing error effect plays a crucial role to the underlying coding (array) gain regardless of the number of receive apertures.

V. NUMERICAL RESULTS

In this Section, the accuracy of the proposed approach is numerically verified. For the sake of clarity and without loss of generality, the analytical results are cross-compared with numerical results modeled by simulating Gamma-Gamma faded channels. Thus, the MG parameters \( a_i, b_i, \) and \( c_i \) are set as per (2). Also, \( L = 10, r/w_z = 0.1, \) and \( [P, Q] = [0.5, 1] \) (i.e., the coherent BPSK modulation scheme is considered). The considered outage threshold is \( \gamma_{\text{th}} = 1 \) (i.e., assuming a minimum target data rate of 1 bps/Hz).

The outage and ABER performance is respectively presented in Figs. 1 and 2 with respect to the average received SNR. Obviously, the proposed analytical approach tightly approaches the corresponding simulation numerical results at all the SNR regions and for various fading conditions. Note that lower (higher) values of the parameters \( \alpha, \beta, \) and \( \xi \) indicate more (less) severe channel fading. It is also noteworthy that the asymptotic results remain efficient only in the high SNR regions (roughly, when \( \bar{\gamma} > 25 \text{dB} \)). This observation reflects on the beneficial role of the proposed scheme, which preserves the desired accuracy from low-to-high SNR values. In addition, the results of both Figs. 1 and 2 verify the remark of (8). Obviously, the system performance is emphatically enhanced for a higher number of apertures, even in severe turbulence- and/or misalignment-induced fading conditions.

\(^1\)The second equality of (8) arises from the identity [7, Eq. (10.30.2)] and the fact that the Gamma-Gamma distribution is asymptotically affected by the minimum of its two parameters.
Outage Probability vs. average SNR for various system parameters.

Fig. 1. Outage probability vs. average SNR for various system parameters.

ABER vs. average SNR for various system parameters, where $[\alpha, \beta, \xi] = [0.5, 2, 1]$.

Fig. 2. ABER vs. average SNR for various system parameters, where $[\alpha, \beta, \xi] = [0.5, 2, 1]$. 
VI. CONCLUSION

The performance of EGC-enabled receivers in FSO systems was analytically studied and evaluated. Important system performance metrics were derived in closed formulation in terms of finite sum series of the Meijer’s G-function, such as the scintillation index, outage probability, and ABER. A statistical analysis in the asymptotically high SNR regime revealed some impactful insights, such as the system diversity order, the crucial role of diversity branches, and the impact of the aforementioned fading parameters onto the overall system performance.

APPENDIX

DERIVATION OF (3)

Define $h_j \triangleq x_j y_j$, such that $z = \sum_{j=1}^{N} h_j$. Conditioning on $y_j$, it holds that $f_{h_j|y_j}(x|y_j) = \sum_{i=1}^{L} \frac{a_{i1} a_{i2}}{y_j^{b_{i1}} b_{i2}^{b_{i2}}} \exp\left(-\frac{c_{i1}}{y_j} x\right)$. First, we study the case where $z = x_1 y_1 + x_2 y_2 = h_1 + h_2$ and the respective PDF of $z$ is obtained as $f_z(z|y_1, y_2) = \int_{0}^{A_0} f_{h_1|y_1}(x|y_1) f_{h_2|y_2}(z - x|y_2) dx$. Utilizing \[6, Eq. (3.191.1)\], the latter integral can be evaluated as

$$f_{z|y_1,y_2}(z|y_1,y_2) = \sum_{i_1=1}^{L} \sum_{i_2=1}^{L} a_{i_1} a_{i_2} B \left(\frac{b_{i_1}}{y_1}, \frac{b_{i_2}}{y_2}\right) z^{b_{i_1} + b_{i_2} - 1} \exp\left(-\frac{c_{i_1}}{y_1} x\right). \tag{A.1}$$

Following a similar procedure for $N$ steps, the generalized formulation for the PDF of $z$ is derived as

$$f_{z|\{y_j\}_{j=1}^{N}}(z|\{y_j\}_{j=1}^{N}) = \sum_{i_1=1}^{L} \cdots \sum_{i_N=1}^{L} \prod_{s=1}^{N} \left[ a_{i_s} \right] \prod_{j=1}^{N-1} \left[ B \left( \sum_{l=1}^{j} b_{i_l}, b_{i_{j+1}} + 1 \right) \right] z^{\sum_{l=1}^{N} b_{i_l} - 1} \exp\left(-\frac{c_{i_1}}{y_1} x\right). \tag{A.2}$$

We proceed on the unconditional PDF of $z$. Capitalizing on the independence amongst the involved RVs, it holds that

$$f_z(z) = \int_{0}^{A_0} \cdots \int_{0}^{A_0} f_{z|\{y_j\}_{j=1}^{N}}(z|\{y_j\}_{j=1}^{N}) \times f_{y_1}(y_1) \cdots f_{y_N}(y_N) dy_1 \cdots dy_N. \tag{A.3}$$

Integrating the latter expression with respect to $y_1$ with the aid of \[6, Eq. 3.351.2\], we get

$$f_{z|\{y_j\}_{j=2}^{N}}(z|\{y_j\}_{j=2}^{N}) =$$
\[
\sum_{i_1=1}^{L} \cdots \sum_{i_N=1}^{L} \prod_{s=1}^{N} \prod_{j=1}^{N-1} \left[ B \left( \sum_{l=1}^{j} b_{i_l}, b_{i_{j+1}} \right) \right] \\
\times \frac{\xi^2_c x^{\xi^2-1}}{A_0^2} \prod_{j=1}^{N-1} \left( b_{i_1} - \xi^2 c_i z \right) \prod_{j=2}^{N} \left( y_{j-1}^{b_{i_j}} \right) . \tag{A.4}
\]

Next, integrating the latter PDF with respect to \( y_2 \), an integral of the following form appears as
\[
\int_{0}^{A_0} y_2^{\xi^2-\xi^2+1} dy_2 = \frac{A_0^{\xi^2-\xi^2+1}}{\xi^2_1}, \quad \xi^2 > b_{i_2}.
\]
By following the above procedure for \( N-1 \) steps, the total (unconditional) PDF of \( z \) reads as
\[
f_z(z) = \sum_{i_1=1}^{L} \cdots \sum_{i_N=1}^{L} \prod_{s=1}^{N} \prod_{j=1}^{N-1} \left[ B \left( \sum_{l=1}^{j} b_{i_l}, b_{i_{j+1}} \right) \right] \\
\times \frac{\xi^2 c_i^{\xi^2-1}}{A_0^{\xi^2}} \prod_{j=1}^{N-1} \left( b_{i_1} - \xi^2 c_i z \right) \prod_{j=2}^{N} \left( y_{j-1}^{b_{i_j}} \right) , \tag{A.5}
\]
where \( \Psi[A_0] \) is defined back in (3). Hence, using (A.5), the PDF of SNR for EGC-enabled reception is provided in (3).

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