Resonant feedback for axion and hidden sector dark matter searches

Edward. J. Daw
Department of Physics and Astronomy, The University of Sheffield, Hicks Building, Hounsfield Road, Sheffield S3 7RH, United Kingdom
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Resonant feedback circuits are proposed as an alternative to normal modes of conducting wall cavities or lumped circuits in searches for hidden sector particles. The proposed method offers several potential advantages over the most sensitive axion searches to date, that employ cavity resonators, including coverage of a wider range of axion masses, the ability to probe many axion masses simultaneously, and the elimination of experimentally troublesome mechanical tuning rod mechanisms. After an outline of the proposed method, we present a noise budget for a straw-man experiment configuration. We show that the proposed experiment has the potential to probe the axion mass range $2 - 40 \mu$eV with 38 days of integration time. Other existing and proposed resonant searches for hidden sector particles may also benefit from this approach to detection.

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I. INTRODUCTION

The identification of dark matter remains a central unsolved problem in astrophysics [1]. Dark matter candidates beyond the standard model include the QCD axion [2,3], which could be a significant component of dark matter and solve the strong CP problem [4,5], as well as other hidden sector fields [6,7]. In a tuned resonant cavity halo axion detector, first proposed by Sikivie [8], axions from our local halo undergo Primakoff conversion [9] in a static magnetic field $B$, converting to real photons at a rate enhanced by a factor of $Q$, the quality factor of a mode of a normal-conducting walled cavity [10].

Those cavity axion detectors which have published search results [11–15] have consisted of copper plated cylindrical cavities of unloaded $Q \sim 10^5$. The cavity volume encloses one or more moveable tuning rods used to control the TM mode frequencies. The TM010 mode has the largest form factor for photons converting from dark matter axions thermalized in the galactic halo. The energy of such photons is given by $\hbar \nu = m_a c^2 [1 + O(v^2/c^2)]$, where $\nu$ is the electromagnetic mode frequency and the kinetic energy term is of order $10^{-6}$ of the axion rest energy. Recently, the ADMX experiment has achieved sensitivity to the DFSZ [16,17] axion model for the first time [18], using RF SQUID amplifiers to achieve detector noise temperatures at the 150 nK level. Other hypothesized lightly coupled particles having a variety of spins and parities may be dark matter constituents. Such so-called hidden sector particles may play multiple significant roles in particle physics beyond the standard model, astrophysics, and cosmology. There have been many experiments and new proposals to probe a range of energy scales for new hidden sector fields [18–33]. In many of these experiments, resonant detection techniques, based on the same principles as the axion search described above, are used to enhance the rate of conversion to detectable electromagnetic signals. The necessity of tuning these detectors, dwelling at each tuning for sufficient time to achieve an acceptable signal to noise ratio, is a problematic limiting factor for all such resonant designs.

In this paper, we propose exploitation of a resonant feedback technique as an alternative to cavities, lumped circuits or other resonant structures. Feedback techniques have been considered previously by Rybka in the context of increasing the quality factor for a cavity resonance [34], but here we propose feedback as a technique to induce entirely new resonances suitable for axion searches. There are two components to the proposed scheme, the first being an electromagnetic structure with a high form factor for hidden sector fields to convert to electromagnetic fields, but not itself possessing a resonance at the frequency of interest. The second component is a filter circuit whose transfer function replicates that of a high Q resonator at the desired frequency. These two elements are connected together in a closed feedback loop, with open loop gain and phase shift just below the Nyquist stability limit [35]. We demonstrate that this detector has potential to yield similar resonant enhancement of the rate for axion conversion into photons as a conducting wall resonant cavity with the same resonant frequency and quality factor. We suggest that the same structures could be used in resonant searches for other hidden sector particles. We show further that suitable resonant filters can be realised using digital signal processing elements controlled parametrically through the filter coefficients. We derive expressions for the closed loop gain and quality factor of the system, the condition for stability of the circuit, and the quality factor of the resonance. We calculate a noise budget for an example configuration, showing that the performance requirements of each element shared with conventional designs are no more stringent than is the case in conventional receivers, and that the addition of room temperature digital electronics does not significantly increase the noise.
temperature of the apparatus.

The proposed technique offers three significant advantages over standard resonant configurations. Firstly, parametric control of the mode parameters removes the need for cryogenic mechanical or electronic tuning elements such as tuning rods in resonant cavities. Secondly, the operation of many such digital filters in parallel means that the apparatus may cover many possible hidden sector particle masses in parallel. Thirdly, the explicit link between the length scale of the electromagnetic structure used as the detector and the wavelength of the conversion photons is broken, offering the possibility of searching a greater range of particle using the same magnet bore.

II. PROPOSED DETECTION SCHEME

In Sikivie’s resonant axion detector scheme, the axion field interacts with a static magnetic field, resulting in an oscillating electromagnetic field that excites a mode of an electromagnetic resonator. The conversion signal power is proportional to a form factor given by

$$C = \frac{|\int_V E \cdot B dV|^2}{\int_V \varepsilon |E|^2 dV \int_V |B|^2 dV'},$$  \hspace{1cm} (1)

where $E$ is the amplitude of the oscillating electric field from axion conversion. This form factor is integrated over the physical geometry of the electromagnetic configuration into which the axions convert. In the cavity detection scheme, the form factor is evaluated over the volume of the cavity, and the electric field is the position-dependent electric field of the mode used to sense axions. For cylindrical cavities containing off-center tuning rods, the form factor is evaluated from fields calculated using finite element modeling. Readout of cavity detectors is accomplished by coupling the cavity mode to an external amplifier using an electric field probe. Matching of the field probe to the external amplifier is accomplished by varying the insertion depth of the probe until the scattering parameter of the reflected signal off the probe, $s_{11}$, is at the desired coupling level. The resonant frequency is determined by the positions and geometries of the conducting walls of the cavity and the tuning rods for the mode in question.

Figure 1 shows the proposed alternative scheme, in which axions convert between the plates of a plate capacitor operated far below cutoff, with the applied magnetic field normal to the capacitor plates. The capacitor plates are coupled to transmission lines connecting them to an external circuit, with those capacitor plates matched to the resistive 50 Ω characteristic impedance of the transmission lines by discrete fixed resistors connecting the capacitor plates to the transmission line ground. At frequencies around 1 GHz, the magnitude of the reactance of a parallel plate capacitor having 1 cm plate separation and 50 cm diameter is 3.7 Ω, so the impedance mismatch due to the capacitor is not significant. The resonant filter, discussed further in the Appendix has transfer function

$$H(s) = \frac{\Gamma_s}{s^2 + \Gamma s + \omega_0^2},$$  \hspace{1cm} (2)

where $s = i\omega$ and $\omega = 2\pi f$, where $f$ is the frequency, $\Gamma/(2\pi)$ is the frequency full width at half maximum of the resonance, and $\omega_0/(2\pi)$ is the resonant frequency of the filter. Figure 2 shows an equivalent circuit relating the output signal to the axion-induced signal on the plate of the parallel-plate capacitor. The closed loop transfer function of this circuit is

$$H_C(s) = \frac{\tilde{v}_o(s)}{\tilde{v}_i(s)} = \frac{G_C G_W \Gamma_s}{s^2 + (1 + AG_C G_W)\Gamma s + \omega_0^2}. \hspace{1cm} (3)$$

Comparing with the open loop transfer function of Equation 2, we see that the width of the resonant peak is modified compared to the open loop case by a factor of
As for any feedback circuit, raising the round trip gain to be less than 1. We shall see in Section III that a large attenuation is also necessary to ensure that the noise budget in the readout circuit gives sufficient signal to noise ratio on the axion signal against noise contributions from the cold and warm readout components.

In practice, the capacitor plates would be separated by insulating spacers and small microwave substrate circuit boards mounted on the back of the plates would be used to attach the connectors to coaxial transmission lines and the resistive terminations. It is also crucial that the reactive impedance of the capacitor decreases as its capacitance rises. This suggests that the volume of a magnet bore could be filled with a stack of capacitors connected in parallel, thereby decreasing the reactive impedance presented to the electronics by a factor of the number of capacitors.

Dark matter axions, having DeBroglie wavelengths of hundreds of metres produce a conversion signal coherent across the capacitor array, and in each capacitor the resulting oscillating electric field is parallel to the applied magnetic field inside the magnet bore. Transmission lines to the capacitors would be matched in physical length to ensure that the signals from axion conversion at the outputs of transmission lines from each capacitor in the array, and these transmission lines would be terminated on Wilkinson passive power combiners, the single output of which would be fed to the cryogenic amplifier, with the same arrangement in reverse leading from the cold attenuator output back to each of the capacitors. In this arrangement the form factor for axion to photon conversion is close to unity. The transmission lines connected to individual capacitor plates would be combined in-phase using Wilkinson power combiners; therefore the impedance mismatch of 3.7Ω due to the capacitative load does not get larger as more capacitors are combined.

III. NOISE BUDGET

The proposed feedback circuit incorporates both low noise cryogenic electronics and noisier room temperature digital electronics. In this section, we show that using existing amplifier technology it is possible nevertheless to maintain a good signal-to-noise ratio for mode oscillations around the feedback loop. Figure 3 shows a more realistic arrangement of amplification stages as implemented in a representative axion search. The temperature $T$ of a source of Johnson noise in a bandwidth $B$ emits noise power $P$ into a balanced sink given by

$$P = k_B T B.$$  

At a mode frequency of 700 MHz corresponding to an axion mass of 2.9μeV, assuming a magnetic field of 6.8T, and other experimental parameters corresponding to the ADMX2 experiment configuration [15], the signal power expected from a KSVZ [36,37] model axion converting to photons is $2 \times 10^{-22} \text{W}$ in a bandwidth of 750 Hz. This signal power, as well as the noise power in the same bandwidth may be calculated at different points around the feedback loop, labeled A-F. The attenuator must be cooled to as low a physical temperature as possible, probably by placing it in thermal contact with the refrigerator cold plate. If the temperature of the noise incident on the attenuator is $T_a$, the attenuation factor is $a$, and the attenuator temperature is $T_A$, then the noise temperature at the attenuator output is

$$T_O = a T_a + (1 - a) T_A.$$  

Using Equations 5 and 6, and taking the values for component noise temperatures and power gains assumed in Figure 3 we obtain the noise and signal powers at different points in the loop given in Table 4. The noise budget illustrates the fact that the high noise temperature of the digital electronics used to implement the resonant gain stage makes a negligible contribution to the noise budget in the cryogenic portion of the circuit, since this noise is attenuated by the large (roughly 120dB) attenuation of the cryogenically cooled attenuator that brings the open loop gain back to slightly less than 0dB to suppress spontaneous oscillation of the feedback circuit.

The signal to noise ratio around the loop is $-15$dB, which is a power ratio of 1/32. Integration is used to detect the power excess due to axions against the background of fluctuations about the average Johnson noise at surrounding frequencies, where the signal to noise ratio...
TABLE I. Noise budget around the closed loop resonant circuit shown schematically in Figure 3. The second column is total Johnson noise power into a 750Hz bandwidth, including both noise from the output of the previous stage, and noise contributed by the temperature noise at the input of the next component after the labelled location. The third column is noise power into the same bandwidth including only that due to the next component after the labelled location. The fourth column is the signal power. The attenuator will be set so that the open loop gain is in fact slightly less than 0dB to avoid the circuit going into oscillation, but here the difference between the actual open loop gain and 0dB is neglected.

| Location | Total summed noise into 750Hz bandwidth [dBm] | Noise from local component into 750Hz bandwidth [dBm] | Signal power [dBm] |
|----------|-----------------------------------------------|--------------------------------------------------------|--------------------|
| A        | -175                                          | -178                                                   | -190               |
| B        | -155                                          | -166                                                   | -170               |
| C        | -135                                          | -166                                                   | -150               |
| D        | -115                                          | -150                                                   | -130               |
| E        | -55                                           | -127                                                   | -70                |
| F        | -55                                           | -178                                                   | -70                |
| G        | -175                                          | -178                                                   | -190               |

The key to speeding up a search for axions using the proposed detector is that the digital resonant filters, implemented digitally, can be built up in parallel sets, since they are realized as digital filters on field programmable gate arrays as discussed in the Appendix. It is reasonable to assume that we can implement 100 such resonators in parallel. There is easily enough space on a large FPGA for 100 parallel resonant circuits. The circuits must have resonances separated by many resonance widths, and with each resonance approximately 14kHz wide, allowing for 10 full widths separation between resonators, 100 resonators occupy 14MHz of bandwidth. This is well matched to the bandwidth of a single SQUID amplifier.

We can therefore calculate the total integration time to cover the mass range $2 - 40 \mu$eV. We can assume that the form factor, volume, and coupling of the axions to photons remain static. We assume 100 parallel resonators, with each resonator covering a bandwidth of approximately 15kHz per scan. The integration time per scan is 1120s to achieve DFSZ sensitivity. Therefore in 1120s we cover a frequency range of 1.5MHz. There is no degradation in the anticipated signal power with increasing frequency, and we assume the noise temperature of the electronics can be maintained at a level of roughly 150 mK. A target axion mass search range of $2 - 40 \mu$eV, corresponds to a range of mode frequencies of 480 MHz – 4.8 GHz, a bandwidth of 4.34 GHz, which is a factor of 2896 times the 1.5 MHz per 1120s integration needed for DFSZ sensitivity. Therefore the total search time for the targeted axion mass range is $1120 \times 2896$ s or 37.5 days. This experiment places a very large range of axion masses at the DFSZ model signal strength within reach with achievable integration times, the speed-up compared to that in previous searches being due to the availability of many high Q, high form factor resonators in parallel.

V. CONCLUSIONS

We have presented a concept for a resonant feedback variant on the Sikivie resonant axion detector. We have shown how such resonant feedback could be used with simple electromagnetic structures that can be stacked up to fill the bore of a high field magnet, and how such structures can be impedance matched to low noise electronics of the type used by previous and current axion searches. We have studied the noise contribution of the newly proposed digital electronics, and concluded that it does not adversely affect the signal-to-noise ratio in the chain. We have shown that with 100 parallel resonant filters, the detection rate for axions is sufficiently enhanced that a range of axion masses previously only just accessible to Sikivie-type detectors may be probed in tens of days rather than years. We very much hope that the proposed scheme can play a part in the detection of axions within a reasonable experiment lifetime. Other experimental groups have proposed resonant search experiments for axions and hidden sector particles proposing to utilize resonantly enhanced detection; see for example.
There are several areas in which further research connected to this idea is necessary. Though we believe the analysis presented here demonstrates that in the classical limit this arrangement will yield a sensitive axion detector, it is important to analyze this circuit in the quantum limit, since what we have essentially proposed here is a harmonic oscillator distributed between a ‘cold’ domain in which the excitation of the oscillator consists of a small number of quanta, and a ‘warm’ domain in which the same level of excitation is represented as a much amplified signal in the classical limit. Further work will be necessary to understand how this rather interesting variety of oscillator works in detail.

Experimental tests of this idea are being planned. In a first test, a closed loop system could be built at room temperature and tested by using a network analyzer coupled through a pair of field probes to the space between the capacitor plates. The transfer function of such an arrangement in transmission should show the resonances generated by the digital filter, and by cooling the capacitor plates, it should be possible to see the resonant peaks in the Johnson noise spectrum of the electromagnetic radiation between the plates using a suitable low noise amplifier and spectrum analyzer.

Further testing could be carried out in existing cavity axion search experiments. An unused RF port can be used as an injection port for the feedback signal, and the existing cryogenic amplifier chain can be used as far as the output of the last amplifier before the mixer. A parallel set of receiver electronics implementing external resonance can be used to generate an artificial resonance far from the physical resonances of the cavity as a demonstration that the generation of such resonances by feedback is feasible.

Finally, the idea of filling the magnet bore with a stack of capacitors needs to be tested in finite element modeling, to ensure that the capacitor electric fields behave as expected, and to work out where the transmission lines couple to the capacitor plates. In particular it would be very convenient to couple the transmission lines to the plates at the edges, near to the wall of the magnet bore, rather than in the middle, since the other capacitors in the stack would then be an obstruction. It may also be necessary to have more than one path to each capacitor plate, driven in phase, and this idea can also be studied using finite element modeling tools.

Appendix: Digital resonant filter electronics

The resonant filter alluded to throughout this paper is here described in some detail. The filter is implemented digitally using a platform that allows continuous causal processing of the data, with field programmable gate arrays (FPGAs) providing a suitable architecture. The filter is an infinite impulse response type, and is fed by a stream of data, one sample at a time, the data being digitized from the output of the amplifier chain. There are several options for the practical aspects of this digitization. In recent cavity axion searches, the output of the amplifier chain is heterodyned to a lower frequency using an image reject mixer, so that digitization occurs with a sampling rate around 10 MHz. The proposed idea could utilize this technique, but would utilise a higher digitisation rate, around 64 MHz, and use two image-reject mixers in parallel, so that two out-of-phase IF quadratures are obtained from the RF signal stream. Both quadratures are digitised and the resulting data is passed through the resonant filter. Figure 4 shows a schematic of the digital electronics. In reality, a single local oscillator would be used, fed through a 90° hybrid to yield two out-of-phase local oscillator signals, each of which would be common to the corresponding pair of mixers before and after the digital electronics. The resonant filter itself utilises an iteration algorithm originally developed for the monitoring of sinusoidal backgrounds in gravitational wave detectors. If we represent the two quadratures of the \( n \)th sample of input data as the real and imaginary parts of a complex number \( x_n \), and similarly represent the \( n \)th sample of filter output as the real and imaginary parts of a second complex number \( y_n \), then the filter algorithm takes the following form,

\[
y_n = e^{i\Delta}(1 - w)y_{n-1} + wx_n.
\]

This filter resonates at a frequency of \( \Delta/(2\pi\tau_s) \) Hz, where \( \tau_s \) is the period between adjacent data samples, with a quality factor of \( Q = \Delta/(2\pi w) \), where \( w \) is a real number much less than unity. The filter can be implemented using only fixed point arithmetic such that the delay per sample is a single sampling period. The clock rate of 250 MHz common on modern FPGAs, with faster models also available, makes these calculations tractable at the proposed sampling rate, which provides adequate bandwidth to support 100 parallel resonances separated from each other by 10 full-widths in frequency space. The factor of \( e^{i\Delta} \) means that the sine and cosine of \( \Delta \) must
be calculated for each desired resonant frequency in each parallel filter; these values could be stored in the FPGA or calculated in parallel with filtering in preparation for the next frequency shift using a processor core on the FPGA alongside the filter circuits. Further speed-up in the calculations can be achieved by requiring that $w = 2^{-N}$ with integer $N$, in which case the multiplications by factors of $w$ can be implemented with $N$ bit-shifts to the right, avoiding some of the fixed point multiplies. There is the potential for the FPGA device to also carry out other aspects of the data analysis such as the Fourier transforms necessary as the first stage in the search for axions in the data.

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