Strings and D-branes in holographic backgrounds

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We review recent progress in the study of non-rational (boundary) conformal field theories and their applications to describe exact holographic backgrounds in superstring theory. We focus mainly on the example of the supersymmetric coset $SL(2,\mathbb{R})/U(1)$, corresponding to the two-dimensional black hole, and its dual $\mathcal{N}=2$ Liouville. In particular we discuss the modular properties of their characters, their partition function as well as the exact boundary states for their various D-branes. Then these results are used to construct the corresponding quantities in the CFT of the NS5-brane background, with applications to Little String Theories.

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I. INTRODUCTION

Conformal field theories are the natural building blocks for the exact perturbative description of superstring theory around non-trivial backgrounds. It is possible to describe a large class of solutions, albeit without Ramond-Ramond fluxes. In addition to non-trivial compactifications and supersymmetric curved vacua, some exact time-dependent solutions are known. However for the two latter categories, the conformal field theories that we shall need are non-rational conformal field theories (NRCFTs). For technical and conceptual reasons they are far more difficult to handle than their rational cousins (RCFTs), and the simplest examples have been fairly understood only in the recent years.

This progress has been largely motivated by their relevance to describe backgrounds with an explicit implementation of holography. The only examples with only Neveu-Schwarz–Neveu-Schwarz fluxes are generated by fundamental strings and NS5-branes sources, the most famous one being the three-dimensional anti-de Sitter spacetime $\text{AdS}_4$, whose holographic nature is clearly established\textsuperscript{[3]}. However the holographic nature of five-branes solutions\textsuperscript{[4,5]} is less clear, mainly because of the non-trivial nature of the dual non-gravitational theory, called Little String Theory\textsuperscript{[6,7]}. This type of holography with a linear-dilaton background can be generalized e.g. to sub-critical superstrings models\textsuperscript{[8]}, in particular to the two-dimensional case, where the tools of matrix theory are very powerful to handle the theory\textsuperscript{[9]}.

These constructions can be recast as non-rational analogues of Gepner models\textsuperscript{[10]}, where the building blocks are, for the compact part, Kazama-Suzuki $\mathcal{N}=2$ cosets and for the non-compact part the super-cosets $SL(2,\mathbb{R})/U(1)$\textsuperscript{[11,12,13,14,15,16]}. Adding fundamental strings amounts to lift one of those non-compact cosets to $SL(2,\mathbb{R})$, i.e. three-dimensional anti-de Sitter space\textsuperscript{[17]}. Thus the main non-trivial building block for these constructions is the (super) coset $SL(2,\mathbb{R})/U(1)$\textsuperscript{[18,19,20]}, that we shall study in detail in this note. Below we shall begin with some general features of NRCFT that can be inferred from the solved theories of this kind. Section two deals with the the closed string sector of the super-coset $SL(2,\mathbb{R})/U(1)$ as well as its mirror $\mathcal{N}=2$ Liouville. In section three we consider the exact construction of D-branes in this model. Finally in section four we lift these results to the background of NS5-branes.

II. GENERAL ASPECTS OF NON-RATIONAL CONFORMAL FIELD THEORIES

The non-rational CFTs are of course defined in opposition to rational ones. The latter are characterized by having a finite number of primaries of their chiral algebra. Powerful algebraic techniques are available to solve them completely, with or without boundary\textsuperscript{1}. Famous examples of these models are minimal models of the (super)conformal algebra, WZW models on compact groups and their cosets.

In NRCFT the aforementioned techniques are no longer useful or need to be adapted. The complications arise first because the Hilbert space is now constructed out of an infinite number of primary fields, that may even form a continuum of states, since these models have non-compact target spaces. In general the spectrum of these theories splits into two categories of states. On the one hand the spectrum contains continuous representations, corresponding to asymptotic states propagating in the non-compact "radial" direction(s). On the other hand the states of the discrete representations correspond to (a finite set of) localized bound states. As we may infer from general considerations of scattering theory, these two kinds of states mix. It is known for example in the coset $SL(2,\mathbb{R})/U(1)$ that the reflection amplitude for continuous representations has poles whenever it is analytically continued to a discrete representation\textsuperscript{[21]}. The modular properties of the character of those theories exhibit a similar pattern. In the bosonic case\textsuperscript{[22]} and its supersymmetric version\textsuperscript{[23,24]} the modular transformations of the characters can be represented as contour integrals in the momentum plane of the non-

\textsuperscript{1} although only a subset of D-branes is known in general.

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compact directions, using a lemma proven in \[22, 23\]. For the transformations of the discrete characters, this integral has to be shifted in order to recast the integral as a contribution from continuous representations. The residues of the poles crossed during this process give characters of discrete representations. Thus, while the continuous representations modular transform onto continuous representations, the discrete representations give both discrete and continuous ones. These new qualitative features complicate the construction of partition functions and boundary states by a large amount.

These NRCFTs can still be solved by the conformal bootstrap, using the chiral symmetries of the model. Apart from the pattern described above, some features like the absence of identity representation in the closed string spectrum complicate the analysis of the factorization constraints. Generalizing the seminal works on Liouville theory \[26, 27, 28\], one uses some specific representations that are degenerate w.r.t. the chiral algebra. In these cases the operator product expansion (OPE) with any field is finite – in contrast with a generic OPE of these theories – and differential equations have to be obeyed. An important assumption is that the quantity we wish to compute can be analytically continued to the degenerate representations that are not continuous ones. Then, one can assumes on general grounds that some quantities involving these degenerate fields are perturbatively computable, and adding some assumption of strong/weak duality on the worldsheet gives the exact solution of the theory. However it is sometimes possible to use only the chiral symmetry of the model and the factorization constraints to solve the model \[29, 30, 31\]. In this case we are close to tell the same story as for rational theories.

A last important aspect of NRCFT that we wish to highlight is that, for specific points in the parameter space, these theories simplify and acquire some kind of rational behavior. The simplest example is the free NRCFT for a boson on a circle. When the radius squared turns out to be rational, i.e. \( R = \sqrt{2r/s} \), the theory has an extended chiral symmetry generated by \( J_0 = i\partial X \) and \( J_\pm = \exp(\pm i\sqrt{2rs}X) \). Then by summing over the orbits of this symmetry one gets a finite set of extended characters. Intuitively one may guess that this behavior generalizes to the (super) coset \( SL(2, \mathbb{R})/U(1) \) for rational level, since the target space asymptotes a cylinder of radius \( \sqrt{2k} \). It is indeed the case \[22, 23\] and the properties of the theories simplify. In particular in the supersymmetric case one obtain a finite set of rational N=2 R-charges, which is desirable to construct space-time supersymmetric vacua.

### III. The Supersymmetric Cigar and N=2 Liouville

As proposed in the introduction we shall focus now on the super-coset \( SL(2, \mathbb{R})/U(1) \), since it is a prototypic example of NRCFT and a basic building block of most of the non-trivial superstrings vacua. This theory is obtained by a straightforward application of the rules of coset construction\(^2\) to the supersymmetric WZW model \( SL(2, \mathbb{R}) \) at level \( k \). For the axial coset the sigma model is well defined because the action of the gauge field has no fixed point, and corresponds to an Euclidean two-dimensional black hole. The spectrum of primaries is obtained by descent from AdS\(_3\). It contains both discrete (real spin \( j \)) and continuous representations (imaginary spin \( j = 1/2 + is \), \( s \in \mathbb{R} \)) of the affine \( \mathfrak{sl}(2, \mathbb{R}) \) algebra, the spins of the former being restricted to the improved unitary range \( 1/2 < j < k+1/2 \). This is confirmed by a computation of the one-loop vacuum amplitude \[13, 33\], as for the bosonic coset \[32\] and the \( SL(2, \mathbb{R}) \) WZW model \[34\].

The worldsheet-supersymmetric partition function can be computed using the powerful techniques of marginal deformations of WZW models \[14\]. Indeed one can start with the supersymmetric \( SL(2, \mathbb{R}) \) model, for which the fermions are free, and deform with the truly marginal operator \( (j^3 + \psi^+ \psi^-)(j^3 - \psi^+ \psi^-) \) made with the total (left and right) elliptic currents of \( \mathfrak{sl}(2, \mathbb{R}) \). After analytic continuation to an Euclidean target space and an infinite deformation along the elliptic subgroup, one obtains the desired partition function of the supersymmetric coset \[15\] (see also \[33\]). This amplitude should split naturally into (non-minimal) characters of the N=2 superconformal algebra, since it is the largest chiral algebra of the model, and those characters appear naturally into the branching relations of the supersymmetric coset \[36, 43\].

An exact decomposition of the partition function has been carried out in \[15\]. We have found first a contribution of discrete representations filling the improved unitary range, with the correct multiplicities for all the descendants. For the continuous representations, the story is a little bit more complicated since their contribution is divergent, due to the infinite volume available for them. An infrared regularization of the partition function is possible (as in \[32, 35\]), leading to a finite non-trivial density of continuous representations, compatible with N=2 supersymmetry. However this regulator breaks (super)conformal symmetry, and there is a price to pay: as follows from our exact analysis, the partition function contains an extra non-universal contribution which is not related to the N=2 algebra.

The super-coset \( SL(2, \mathbb{R})/U(1) \) has been conjectured to be dual to another CFT with N=2 superconformal symmetry, the N=2 Liouville theory \[36\]: this statement is the supersymmetric version of the duality between the bosonic coset \( SL(2, \mathbb{R})/U(1) \) and sine-Liouville theory \[39\]. In both cases it amounts to a strong/weak duality on the worldsheet. Evidence for this equivalence comes from a sigma-model mirror symmetry \[37\], and from the agreement between perturbative computations assuming that the duality holds \[40\] and the conformal bootstrap results \[29, 30\].\(^3\) However we would like to argue that these results come from a more fundamental structure of these theories. Indeed, both theories pos-

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\(^2\) We consider the gauging of the elliptic subgroup giving the Euclidean black hole.

\(^3\) Another method is given in \[38\].
sessed the same chiral algebra, which is the (non-minimal) N=2 SCA. This algebra can be decomposed into the bosonic coset $SL(2, \mathbb{R})/U(1)$ and a free boson, or from a complementary point of view, a non-minimal N=2 algebra supplemented by a free time-like boson is lifted to the supersymmetric $SL(2, \mathbb{R})$ current algebra \cite{22, 41, 42}. The conformal bootstrap results for the Euclidean AdS$_3$ \cite{29, 30} have been obtained using only the chiral symmetries of the model, without an explicit reference to a specific action. Thus they can be applied to the supersymmetric $SL(2, \mathbb{R})/U(1)$ coset by the coset construction, and to the N=2 Liouville theory as well, because they both lift to the same current algebra \cite{43}. Therefore the theories can differ only by the way the left and right representations of the algebra are glued in the closed string spectrum. The vector coset $SL(2, \mathbb{R})/U(1)$ has a singular sigma-model, therefore it receives substantial corrections; as shown by marginal deformation techniques \cite{34}, its single cover is given by a $\mathbb{Z}_k$ orbifold of the cigar. It is likely that the N=2 Liouville theory describes this vector coset, as it is suggested by using mirror symmetry \cite{37}. In the following we shall see that all this reasoning about the duality extends straightforwardly when one adds a boundary to the CFT and construct exact D-branes.

### IV. Boundary N=2 Liouville from Boundary AdS$_3$

The study of D-branes in these exact superstrings backgrounds is essential, in order to understand the non-perturbative dynamics in these non-compact manifolds. Moreover it may give some indications about the holographic degrees of freedom we are looking for. The construction of the exact boundary states, which contain all the information about the couplings of the D-branes to the closed string states, follow the same logic as before. Indeed, after the boundary bosonic Liouville theory has been solved \cite{44, 45, 46}, the conformal bootstrap methods have been employed successfully to construct the D-branes in Euclidean AdS$_3$ \cite{48}. These results have been used later \cite{49} to study the bosonic coset $SL(2, \mathbb{R})/U(1)$.

In \cite{48} we constructed the D-branes in the super-coset $SL(2, \mathbb{R})/U(1)$ using similar methods. A very important aspect of this analysis follows from the fact that the conformal bootstrap of \cite{48} uses only the chiral symmetries of the models. Therefore the arguments we gave in the previous section about the duality super-coset $SL(2, \mathbb{R})/U(1)$ / N=2 Liouville extend straightforwardly in the presence of a boundary. However one should be careful about the way left and right representations are glued, in order to construct the basis of Ishibashi states \cite{51}. The various boundary states of the theory can be constructed by descent from D-branes of Euclidean AdS$_3$. Indeed the BRST formalism allows to rewrite the coset theory as the constrained product $SL(2, \mathbb{R})_{k+2} \times$ Fermions $\times U(1)_k \times$ Ghosts $\times$ Superghosts. The boundary state will be a tensor product of boundary states for each of these factors, whose boundary conditions are correlated through the preserved BRST current. In particular, there is a direct connection between the boundary conditions for the currents of the $sl(2, \mathbb{R})$ algebra and the gluing conditions of type A or B defined in \cite{54}. The D-branes that we obtained satisfy by construction the factorization constraints, since they descend from consistent D-branes in Euclidean AdS$_3$. However we needed to check that the Cardy condition \cite{52} — the consistency of the annulus diagram in the open string channel — held, since our D-branes are constructed out of a non-unitary theory. The D-branes of the cigar are depicted in fig. 1.

![FIG. 1: The consistent D-branes in the cigar: D0-, D1- and D2-branes (left to right)](image)

#### a. A-type branes

They correspond to the A-type gluing conditions of the N=2 SCA. In the cigar they are D1-branes, extending to infinity in the radial direction, with Dirichlet boundary conditions for the compact $U(1)$. They couple only to closed strings of the continuous representations. The annulus amplitude gives a continuous spectrum of open strings, albeit with a divergent density since the D-brane is non-compact. We encountered previously the same problem with the closed strings partition function. However in the present case it can be solved by considering as in \cite{48, 49} the relative partition function w.r.t. a reference brane. The non-universal part of the density is finite and related to the boundary reflection amplitude.

#### b. B-type branes

They correspond to the B-type boundary conditions, and can be of two kinds depending of the D-brane of Euclidean AdS$_3$ they come from. The first class of B-branes are point-like D0-branes, sitting at the tip of the cigar. Their boundary states contain couplings both to the discrete and continuous representations. They carry an integer label corresponding in the open string channel to the spin of a finite representation of the $sl(2, \mathbb{R})$ algebra. These representations are non-unitary except the trivial representation, thus only the corresponding D0-brane is physical. The second kind of B-brane is much more involved. They are D2-branes covering all the cigar with a magnetic field on their worldvolume. On general grounds we expect that they carry a D0-brane charge. There is indeed a relative quantization condition on the magnetic field \cite{49} allowing to induce a D0-like contribution for the open string partition function. However we found that they appear with negative multiplicities, hence only the D2-brane without magnetic field seems to be physical.

In related works \cite{23, 53} the D-branes of the N=2 Liouville theory and the bosonic $SL(2, \mathbb{R})/U(1)$ were studied using the method called “modular bootstrap”, which assumes that the Cardy condition is stringent enough to find the correct boundary states, and that the Cardy ansatz is valid for the NRCFTs. While this method gives the same D0-brane, it cannot give the D1-branes of the cigar which are not of the same type as the D0-brane associated to the trivial representation. However it gives other types of extended D-branes. For instance, different D1-branes but they don’t seem to be consistent with the conformal bootstrap and the semi-classical limit \cite{53}. A new kind of D2-branes descending from dS$_2$ D-branes of AdS$_3$...
has been found, and it would be interesting to check their compatibility with the factorization constraints. Some other works\cite{24,54,55} also apply directly the conformal bootstrap method to the N=2 SCA.

V. CLOSED AND OPEN STRINGS NEAR NS5-BRANES

The results given above can be lifted to various superstring setups. Here we will focus on the background created by NS5-branes distributed on a topologically trivial circle, known to be T-dual to an exact CFT\cite{56}. When all the NS5-branes are separated from each other, the background is expected to be perturbative and one can take a double scaling limit\cite{21} where gravity decouples and this perturbative nature holds. We have shown in\cite{15} that the complicated solution for the ring of five-branes in this limit can be obtained as a null gauging of the super-wzW model $SL(2,\mathbb{R}) \times SU(2)$. This CFT has an N=4 SCA on the worldsheet hence there are no perturbative corrections to the effective action. However it is likely that instantons corrections show up, and they are indeed captured by the supergravity solution. Computing them from the worldsheet, following\cite{57}, is a challenging task. Using the BRST construction, the null coset can be recast as a $Z_k$ orbifold of $SL(2,\mathbb{R})/U(1) \times SU(2)/U(1)$. It is indeed under this form that the double scaled little string theory has been known, using duality\cite{58} and holographic arguments\cite{21}. The $Z_k$ orbifold can be thought as coming from the GSO projection generalizing Gepner models. However we would like to stress that this orbifold changes deeply the background of the effective theory, because in the semi-classical limit its twisted sectors become very light; hence the correct geometry is not given by the sum of the two coset factors but by the metric of $\sqrt{\rho}$.

Once this identification has been understood it has been possible to write the one-loop amplitude for this NS5-brane background\cite{15} (see also\cite{23}). The various BPS D-branes in this background are now under study\cite{4}. In particular there are new non-factorizable D-branes that can be constructed out of the coset D-branes\cite{59}. The compact D1-branes of type IIB are also especially interesting since as we shown they can be related to the matrix model of Little String Theory in the large $k$ limit. In type IIA we have also D4-branes stretched between the NS5-branes on which a D=4, N=2 SYM theory lives\cite{61}. Quite remarkably, the one-point function for these D4-branes can be related to the beta-function of the gauge theory.

VI. CONCLUSIONS

It seems now that these new results will allow shortly to solve the non-rational conformal field theory as much as their rational cousins. Some quantities like the boundary three-point functions are not known in general but it is only a matter of computational complexity. Hence a huge class of space-time supersymmetric non-compact backgrounds can be studied in great detail in the string theory regime, much like the Gepner models some years ago. These non-compact models are also related to Lorentzian backgrounds such as black holes and cosmological models, and the methods presented here can in principle be extended to those cases by some appropriate analytic continuation. However it leads to important problems related to the choice of the vacuum, and the stability of the Wick-rotated theory is not guaranteed. Solving these issues will shed a new light on space-like singularities in string theory which are so far not understood.

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