Quantum Support Vector Machine without Iteration

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Abstract—Quantum algorithms can enhance machine learning in different aspects. In 2014, Rebentrost et al. constructed a least squares quantum support vector machine (LS-QSVM), in which the Swap Test plays a crucial role in realizing the classification. However, as the output states of a previous test cannot be reused for a new test in the Swap Test, the quantum algorithm LS-QSVM has to be repeated in preparing qubits, manipulating operations, and carrying out the measurement. This paper proposes a QSVM based on the generalized quantum amplitude estimation (AE-QSVM) which gets rid of the constraint of repetitive processes and saves the quantum resources. At first, AE-QSVM is trained by using the quantum singular value decomposition. Then, a query sample is classified by using the generalized quantum amplitude estimation in which high accuracy can be achieved by adding auxiliary qubits instead of repeating the algorithm. The complexity of AE-QSVM is reduced to \(O(\kappa^3e^{-3}(\log(mn)+1))\) with an accuracy \(\epsilon\), where \(m\) is the number of training vectors, \(n\) is the dimension of the feature space, and \(\kappa\) is the condition number. Experiments demonstrate that AE-QSVM is advantageous in terms of training matrix, the number of iterations, space complexity, and time complexity.

Index Terms—Quantum support vector machine, quantum amplitude estimation, quantum singular value decomposition, quantum inner estimation, quantum computing.

1 INTRODUCTION

In recent years, machine learning techniques have become powerful tools for finding patterns in data, for instance image recognition [1], automated driven cars [2], network security [3], and etc. The development of machine learning has rapidly increased the demand for computing power in hardware. As the spacing of transistors approaches the physical limit of process manufacturing, it has been a bottleneck that researchers intend to improve the operating capability of the classical computer. More powerful ways of processing information are needed as the amount of data generated in our society is growing. Quantum computation is a promising new paradigm for performing fast computations, with the experimental demonstrations of quantum supremacy marked as the latest milestone [4], [5], [6], [7].

The study of quantum computation originated in the 1980s. In 1982, Feynman [8] was the first to proposed the concept of quantum computation. After that the integer factoring problem [9] and the database search algorithm [10] were essential evidences supporting the power of quantum computation. During the first decade of the 21st century, quantum computation appeared in various areas of computer science such as cryptography [11], [12], [13], signal processing [14], [15], [16], [17], and information theory [18].

Quantum machine learning (QML) is an emerging research area that attempts to harness the power of quantum information processing to obtain speedups for classical machine learning tasks. Despite the fact that quantum machine learning is a recently surging field, it already encompasses a rich set of quantum techniques and approaches, for example linear regression [19], [20], [21], clustering analysis [22], dimensionality reduction [23], [24], data classification [25], [26], [27], [28], and neural networks [29], [30], [31]. Besides, quantum machine learning algorithms have been applied to channel discrimination [7], vehicle classification [32], and image classification [33].

Support vector machine (SVM) is a supervised machine learning technique for solving classification. In recent years, there have been many studies about quantum SVM (QSVM) [34], [35], [36], [37], [38], [39], it provides quadratic speedup and was first proposed by Anguita et al. [40]. Moreover, the least squares QSVM (LS-QSVM) [41] given by Rebentrost et al. provide an exponential speedup compared with the classical algorithm.

In LS-QSVM and its descendants, the classification was realized by using the Swap Test [42], [43]. The output state is obtained by measuring the ancillary qubit in the Swap Test. See Fig. 1. \(|0\rangle, |\rho_1\rangle, \) and \(|\rho_2\rangle\) are the input states. For the outcome \(|0\rangle\), we have an entangled state \(|\rho_1\rangle|\rho_2\rangle + |\rho_2\rangle|\rho_1\rangle\sqrt{2}\) and for the outcome \(|1\rangle\), \(|\rho_1\rangle|\rho_2\rangle - |\rho_2\rangle|\rho_1\rangle\sqrt{2}\). In both cases, it is impossible to completely separate the input states for a second time. That is to say, Swap Test is destructive [43].

In LS-QSVM, however, the success probability \(P\) can be obtained to an accuracy \(\epsilon\) by iterating \(O(P(1-P)/\epsilon^2)\) times of the Swap Test [41]. Hence, if the accuracy \(\epsilon\) is achieved, the completed algorithm must be carried out repeatedly; thus, resulting in a high consumption in qubit and time.

To be specific, as shown in Fig. 2 LS-QSVM inherits the pattern of the classical least squares SVM (LS-SVM), i.e., the sample set was initially trained to obtain the parameter in LS-QSVM and the new sample was classified by using this.
parameter. In cases where classical sampling algorithms require polynomial time, an exponential speedup is obtained in LS-QSVM [41], [44], [45], [46], [47]. However, the speedup was achieved only once in training and classification. As analyzed above, the quantum state collapses due to the use of the Swap Test. Therefore, we have to repeatedly train the sample and carry out a classification to get the classification label with a high accuracy.

In this work, we make progress for the challenge described above and propose a quantum support vector machine (QSVM) based on amplitude estimation (AE-QSVM). At first, the training process of AE-QSVM is realized by using the quantum singular value decomposition. Then, we present a method to calculate the inner product based on quantum amplitude estimation to classify the new sample. Compared with LS-QSVM whose complexity is $O(\kappa^3 \epsilon^{-3} \log(mn) + \log(n))$, the complexity of AE-QSVM is reduced to $O(\kappa^3 \epsilon^{-3} \log(mn) + 1)$ with an accuracy $\epsilon$, where $m$ is the number of training vectors, $n$ is the dimension of the feature space, and $\kappa$ is the condition number.

The remainder of the paper is organized as follows: The related works are given in Section 2. We present a general algorithm for quantum amplitude estimation in Section 3. Section 4 presents AE-QSVM. Simulations are presented in Section 5. In Section 6, we summarize this paper and discuss further research for AE-QSVM. Appendix reviews the basic concepts of quantum computation.

2 RELATED WORKS

2.1 Quantum support vector machine

As shown in [41], the QSVM is based on a least squares version of the classical SVM [48].

The task for the SVM is to classify a vector into one of two classes, given $m$ training data points of the form $\{(x_k, y_k) : x_k \in R^n, y_k = \pm 1\}_{k=1,...,m}$, where $y_k = 1$ or $-1$, depending on the class to which $x_k$ belongs. In LS-SVM, the problem after applying the optimization of the Lagrangian can be formulated as a linear equation:

$$F(b, \alpha) = \begin{pmatrix} 0 \\ 1 \\ K + \gamma^{-1}I_m \end{pmatrix} \begin{pmatrix} b \\ \alpha \end{pmatrix} = \begin{pmatrix} 0 \\ y \end{pmatrix}, \quad (1)$$

where $1 = (1, \ldots, 1)^T$, $I_m$ represents the $m \times m$ density matrix, $\gamma$ is a hyperparameter describing the ratio of the Lagrangian’s components, $\alpha = (\alpha_1, \ldots, \alpha_m)$ is the Lagrange multiplier, $y = (y_1, \ldots, y_m)$ is the label of training set, $b$ is the offset of the hyperplane, and $K$ is an $m \times m$ kernel matrix. According to the Eq. (1), for query data $x \in R^n$, the classifier can be determined by the following function:

$$f(x) = \text{sign} \left( \sum_{k=1}^{m} \alpha_k x_k^T x + b \right).$$

In the quantum case, Eq. (1) can be compactly rewritten as $F(b, \alpha) = |0, y\rangle$. When the matrix $F$ is well conditioned and has sparsity polylogarithmic in the dimension, the quantum state $|b, \alpha\rangle = F^{-1} |0, y\rangle$ is produced by using the HHL algorithm [49]. The desired LS-QSVM parameters can be represented as follows:

$$|b, \alpha\rangle = \frac{1}{\sqrt{C}} (b|0\rangle + \sum_{i=1}^{M} \alpha_k |k\rangle),$$

where $C = b^2 + \sum_{k=1}^{M} \alpha_k^2$.

For a classification task, the training data oracle is constructed as follows:

$$|\tilde{\mu}\rangle = \frac{1}{\sqrt{N_{\tilde{\mu}}}} (b|0\rangle|0\rangle + \sum_{k=1}^{m} \alpha_k |x_k||k\rangle),$$

with $N_{\tilde{\mu}} = b^2 + \sum_{k=1}^{m} \alpha_k^2 |x_k|^2$. In addition, the query state is constructed as follows:

$$|\tilde{z}\rangle = \frac{1}{\sqrt{N_{\tilde{z}}}} (|0\rangle|0\rangle + \sum_{k=1}^{m} |x||k\rangle|x\rangle),$$

with $N_{\tilde{z}} = m |x|^2 + 1$. For the physical implementation of the inner product, Swap Test can be utilized to project the ancillary qubit in state $1/\sqrt{2}(|\tilde{\mu}\rangle + |1\rangle|\tilde{z}\rangle)$ to state $1/\sqrt{2}(|0\rangle - |1\rangle)$. Then the success probability is $P = 1/2(1 - (\tilde{\mu} |\tilde{z}\rangle))$, which can be used to determine the label of $|x\rangle$ according to the sign of $1/2 - P$.

However, LS-QSVM only supports the binary classification problem by using all-pair technique [44] and one-against-all approach [45], respectively. Hou et al. [50] presented semi-supervised SVM, which exhibited a quadratic speed-up over classical algorithm. Feng et al. [47] present an improved QSVM model, exponentially improving the dependence on precision while keeping essentially the same dependence on other parameters. Besides, the SVM with the quantum kernel algorithm was also proposed to solve classification problems [51], [52].

2.2 Quantum amplitude estimation

Quantum amplitude estimation (QAE) algorithm [53] is a fundamental quantum algorithm that allows a quantum computer to estimate the amplitude $U|0\rangle$ for a quantum circuit $U$. QAE [53], [54], [55] has attracted much attention as a fundamental subroutine of a wide range of application-oriented quantum algorithms, such as the Monte Carlo integration [56], [57], [58] and machine learning tasks [59], [60]. QAE was first introduced by Brassard et al. [53], in...
which a unitary operator $A_1$ acts on an initial state $|0\rangle^p$ and an ancillary $|0\rangle$ such that:

$$A_1 |0\rangle^p|0\rangle = \sqrt{a_1} |\psi_1\rangle |1\rangle + \sqrt{1-a_1} |\psi_0\rangle |0\rangle. \quad (6)$$

Let $|\varphi_1\rangle = \sqrt{a_1} |\psi_1\rangle |1\rangle$, $|\varphi_0\rangle = \sqrt{1-a_1} |\psi_0\rangle |0\rangle$, and $|\Phi\rangle = |0\rangle^p |0\rangle$. Then

$$|\varphi\rangle = A_1 |\Phi\rangle = |\varphi_1\rangle + |\varphi_0\rangle. \quad (7)$$

Following [53], $|\varphi_1\rangle$ and $|\varphi_0\rangle$ are respectively the $p$-qubit normalized good and bad states. QAE is to estimate the probability that the measuring $|\varphi\rangle$ yields a good state, i.e., $a_1 = \langle \varphi | \varphi_1 \rangle$. The probability to measure the good state can be amplified by applying the following unitary operator:

$$Q_1 = -A_1 S_0 A_1^{-1} S_{\varphi_1}, \quad (8)$$

where $A_1^{-1}$ is the inverse of $A_1$, $S_0 = I - 2 |0\rangle \langle 0|$, and $S_{\varphi_1} = I - 2 |\varphi_1\rangle \langle \varphi_1|$. 

In QAE, it uses $l$ ancillary qubits initialized in equal superposition to represent the final result. Then it applies the operator $Q_1$ controlled by the ancillary qubits. Eventually, it performs an inverse quantum Fourier transformation ($FT^{-1}$) on the ancillary qubits before they are measured. Subsequently, the measured integer $y \in 0, \ldots, L - 1 (L = 2^l)$ is mapped to an angle $\hat{\theta}_{a_1} = y\pi/L$. Thereafter, the resulting estimate of $a_1$ is defined as $\tilde{a}_1 = \sin^2 \hat{\theta}_{a_1}$.

### 3 The Generalized Quantum Amplitude Estimation

In QAE [53], it is assumed that the initial state of the interest problem is $|0\rangle^p$. In this section, we describe another method for the case where the initial state is arbitrary state $|\Phi\rangle$. Our method is the generalization of the quantum amplitude estimation (QAE).

Let $A$ be a quantum algorithm that acts on the initial state $|\Psi_1\rangle$ and an ancillary qubit $|0\rangle$ such that:

$$A |\Psi_1\rangle |0\rangle = |\Psi_1\rangle + |\Psi_0\rangle. \quad (9)$$

Let $|\Psi\rangle = A |\Phi\rangle = |\Psi_1\rangle + |\Psi_0\rangle$. \quad (10)

We will present how to estimate the probability $a$ when measuring $|\Psi\rangle$ produces a good state, where $a = \langle \Psi | \Psi_1 \rangle$.

The amplification amplitude operation Eq. (11) is hereby important for estimation.

$$Q = -A S_0 A^{-1} S_\chi, \quad (11)$$

where $S_\chi = I - 2 |\Psi_1\rangle \langle \Psi_1|$ conditionally changes the sign of the amplitudes of the good states,

$$|x\rangle \rightarrow \begin{cases} -|x\rangle & \text{if } x \text{ is good state}, \\ |x\rangle & \text{if } x \text{ is bad state}, \end{cases} \quad (12)$$

and the operator $S_0 = I - 2 |\Phi\rangle \langle \Phi|$ changes the sign of the amplitude if and only if the state is the initial state $|\Phi\rangle$.

In particular, it is the QAE proposed in [53], if $|\Phi\rangle = |0\rangle^s |0\rangle$, where $s$ is the number of qubits used to represent $|\Phi_1\rangle$.

We will show that the quantum state $|\Psi\rangle$ can still be written as a linear combination of $|\Psi_1\rangle$ and $|\Psi_0\rangle$ after performing $j$ times of operator $Q$.

#### Lemma 3.1. We have that

$$Q |\Psi_1\rangle = (1 - 2a) |\Psi_1\rangle - 2a |\Psi_0\rangle, \quad (13)$$

$$Q |\Psi_0\rangle = 2(1 - a) |\Psi_1\rangle + (1 - 2a) |\Psi_0\rangle, \quad (14)$$

where $a = \langle \Psi | \Psi_1 \rangle$.

**Proof.** First consider the action of the operator $Q$ on the vector $|\Psi_1\rangle$:

$$Q |\Psi_1\rangle = -AS_0 A^{-1} S_\chi |\Psi_0\rangle = -AS_0 A^{-1} |\Psi_0\rangle = A(I - 2 |\Phi\rangle \langle \Phi|) (|\Phi\rangle - A^{-1} |\Psi_0\rangle)$$

$$= \langle \Psi_1 | + (|\Psi_0\rangle - |\Psi_0\rangle) + 2(A |\Psi_1\rangle + |\Psi_0\rangle)$$

$$= -2 |\Psi_1\rangle - 2(1 - a) |\Psi_1\rangle + 2(1 - a) |\Psi_0\rangle$$

$$= (1 - 2a) |\Psi_0\rangle - 2a |\Psi_0\rangle, \quad (15)$$

where $a = \langle \Psi_1 | \Psi_1 \rangle$.

Next consider the action of the operator $Q$ on the vector $|\Psi_0\rangle$:

$$Q |\Psi_0\rangle = -AS_0 A^{-1} S_\chi |\Psi_0\rangle = -AS_0 A^{-1} |\Psi_0\rangle = A(I - 2 |\Phi\rangle \langle \Phi|) (|\Phi\rangle - A^{-1} |\Psi_1\rangle)$$

$$= A |\Psi_0\rangle + 2A |\Psi_1\rangle - 2A |\Psi_1\rangle (|\Psi_0\rangle - |\Psi_1\rangle)$$

$$= 2(1 - a) |\Psi_1\rangle + (1 - 2a) |\Psi_0\rangle. \quad (16)$$

$\square$

Following the previous study [53], the subspace $H_\Psi$ has an orthonormal basis consisting of two eigenvectors of $Q$ according to Theorem 3.1.

#### Theorem 3.1. For any initial quantum state $|\Phi\rangle$, $|\Psi\rangle = A |\Phi\rangle$ can be expressed as follows:

$$|\Psi\rangle = \frac{-i}{\sqrt{2}} (e^{i\theta_a} |\Psi_+\rangle - e^{-i\theta_a} |\Psi_-\rangle), \quad (17)$$

where $|\Psi_\pm\rangle = \frac{1}{\sqrt{2}} (|\Psi_1\rangle \pm |\Psi_0\rangle)$ represent two eigenvectors of $Q$. The corresponding eigenvalues are $\lambda_\pm = e^{\pm i\theta_a}$, where $i = \sqrt{-1}$ denotes the principal square root of $-1$, and the angle $\theta_a$ is defined so that $\sin^2 (\theta_a) = a$, and $0 \leq \theta_a \leq \pi/2$.

**Proof.** The angle $\theta_a$ is defined so that $\sin^2 (\theta_a) = a$. At first, we prove that $|\Psi\rangle$ can be represented as the linear
The generalized quantum amplitude estimation

**Algorithm 1**: The generalized quantum amplitude estimation

1. Prepare the quantum state $|0\rangle^\otimes h |\Phi_1\rangle |0\rangle$.
2. Apply Hadamard to the first register $|0\rangle^\otimes h$, where $h$ is an integer that relates to the precision.
3. Apply $A$ to the second register $|\Phi_1\rangle |0\rangle$.
4. Apply $Q^{2j}$ to the second register controlled by the first register.
5. Apply Fourier transformation in inverse to the first register.
6. Measure the first register and denote the outcome $|y\rangle$.
7. Output $\bar{a} = \sin^2(\pi \frac{y}{2^h})$.

| Combination of $|\Psi_+\rangle$ and $|\Psi_-\rangle$: |
|-----------------------------------------------|
| $|\Psi\rangle = |\Psi_1\rangle + |\Psi_0\rangle$ |
| $= \frac{2\sqrt{a}}{2\sqrt{1-a}} |\Psi_1\rangle + \frac{2\sqrt{1-a}}{2\sqrt{1-a}} |\Psi_0\rangle$ |
| $= -i \sqrt{2} (2i \sin \theta_a |\Psi_1\rangle + \frac{1}{2\sqrt{1-a}} e^{2\cos \theta_a} |\Psi_0\rangle)$ |
| $= -i \sqrt{2} (\cos \theta_a + i \sin \theta_a - \cos \theta_a + i \sin \theta_a) |\Psi_1\rangle$ |
| $+ \frac{1}{2\sqrt{1-a}} (\cos \theta_a + i \sin \theta_a + \cos \theta_a - i \sin \theta_a) |\Psi_0\rangle$ |
| $= -i \sqrt{2} (\cos \theta_a + i \sin \theta_a)(\frac{1}{\sqrt{2a}} |\Psi_1\rangle + i \frac{\sin \theta_a}{\sqrt{2(1-a)}} |\Psi_0\rangle)$ |
| $- (\cos \theta_a - i \sin \theta_a)(\frac{1}{\sqrt{2a}} |\Psi_1\rangle - i \frac{\sin \theta_a}{\sqrt{2(1-a)}} |\Psi_0\rangle)$ |

Let $|\Psi_\pm\rangle = \frac{1}{\sqrt{2}} (|\Psi_1\rangle \pm \frac{i}{\sqrt{2a}} |\Psi_0\rangle)$, we have

$$|\Psi\rangle = -i \sqrt{2} |e^{i\theta_a} |\Psi_+\rangle - e^{-i\theta_a} |\Psi_-\rangle)$$.

Next, we prove that $|\Psi_+\rangle$ is the eigenvector of $Q$. The corresponding eigenvalues are $\lambda_+ = e^{2\theta_a}$.

$$Q|\Psi_+\rangle = \frac{Q}{\sqrt{2a}} |\Psi_1\rangle + \frac{i}{\sqrt{2(1-a)}} Q |\Psi_0\rangle$$

$$= \frac{2\sqrt{2a}}{2\sqrt{1-a}} |\Psi_1\rangle + \frac{2\sqrt{2(1-a)}}{2\sqrt{1-a}} Q |\Psi_0\rangle$$

$$= \frac{1-2a + 2i(1-a)}{\sqrt{2(1-a)}} |\Psi_1\rangle + \frac{-2a + i(1-2a)}{\sqrt{2(1-a)}} |\Psi_0\rangle$$

$$= e^{2\theta_a} |\Psi_+\rangle$$.

The same method can be used to prove that $Q|\Psi_0\rangle = e^{-2\theta_a} |\Psi_0\rangle$. The result is confirmed.

After $j$ applications of operator $Q$, the state is

$$Q^j |\Psi\rangle = \frac{-i}{\sqrt{2}} (e^{(2j+1)i\theta_a} |\Psi_+\rangle - e^{-(2j+1)i\theta_a} |\Psi_-\rangle)$$.

We can use the same method as introduced in [53] to estimate $a$ (Algorithm 1 in Table 1). The resulting estimate of $a$ is defined as $\bar{a}$. The quantum circuit to implement this algorithm is depicted in Fig. 3.

4 THE QUANTUM SUPPORT VECTOR MACHINE BASED ON GQAE

After introducing the tool we cast our gaze over the QSVM based on the GQAE (AE-QSVM), striving to address the problem arising in training and classification. At first, the training progress of the QSVM is realized based on quantum singular value decomposition. Then a query state was classified using GQAE to calculate the inner product.

4.1 Quantum support vector machine training

In LS-QSVM, the input matrix $F$ is not expected to be sparse and well structured, thus restricting the applications of HHL algorithm. An important method for solving linear equations is singular value decomposition. This paper use the quantum singular value decomposition to train AE-QSVM [61].

Singular value is equivalent to eigenvalue for the square matrix $F$. Therefore, the singular value decomposition of matrix $F$ is written as $F = \sum_{i} \lambda_i \nu_i \nu_i^T$, where $\lambda_i \geq 0$ are the eigenvalues and $\nu_i$ are the corresponding eigenvectors. If the matrix $F$ is singular, the solution $F^{-1} |0, y\rangle$ can be achieved for the Moore-Penrose pseudoinverse, that is, the nonzero eigenvalues are only inverted.

As in Algorithm 2 (Table 2), the quantum linear systems $F |b, \alpha\rangle = |0, y\rangle$ is solved by using the singular value decomposition algorithm [61]. We obtain the parameters of AE-QSVM:

$$|b, \alpha\rangle = \sum_i \beta_i \frac{1}{\lambda_i} |\nu_i\rangle$$.

In the basis of training set labels, the expansion coefficients of the ultimate state are the desired support vector machine parameters:

$$|b, \alpha\rangle = \frac{1}{\sqrt{C}} (b |0\rangle + \sum_{i=1}^M \alpha_i |k\rangle)$$.

4.2 Classification based on generalized quantum amplitude estimation

We have implement the training procedure of quantum support vector machine and would like to classify a query state $|x\rangle$.
Solve the quantum linear systems by using the improved singular value decomposition algorithm

**Algorithm 2**: Solve the quantum linear systems using the improved singular value decomposition algorithm [61]

Require: Matrix $F$ stored in the data structure, such that eigenvalues of $F$ lie in $[1/κ, 1]$. Input state $|0⟩|y⟩ = \sum_i \delta_i |i⟩ |y⟩$.
1. Perform singular value decomposition with precision $\varepsilon_1$ for $F$ on $|0⟩|y⟩$ to obtain $\sum_i \delta_i |i⟩ |λ_i⟩$.
2. Perform a conditional rotation and uncompute the SVE register to obtain the state $\sum_i \delta_i |i⟩ |λ_i⟩ (\frac{1}{√2} |0⟩ + γ |1⟩)$.
3. Perform the amplitude amplification to obtain $|b, α⟩ = \sum_i \delta_i |i⟩ |α⟩$.

Following Ref. [41], the training data oracle is constructed as follows by adding a register to the state $|b, α⟩$:

\[
|\tilde{μ}⟩ = \frac{1}{√N_\mu} (b|0⟩|0⟩ + \sum_{k=1}^{M} α_k |x_k⟩ |k⟩ |x_k⟩),
\]

and $N_\mu = b^2 + \sum_{k=1}^{M} α_k^2 |x_k|^2$. The query state is constructed:

\[
|\tilde{x}⟩ = \frac{1}{√N_\tilde{x}} ((|0⟩|0⟩ + \sum_{k=1}^{M} |x⟩ |k⟩ |x⟩),
\]

with $N_\tilde{x} = M |x|^2$.

For the classification, we perform a quantum inner estimation for $|\tilde{μ}⟩$ and $|\tilde{x}⟩$ based on GQAE for the following reason:

\[
⟨\tilde{μ} | \tilde{x}⟩ = \frac{1}{√N_\mu N_\tilde{x}} (b + \sum_{k=1}^{M} α_k |x_k⟩ |x⟩ |x_k⟩).
\]

Following the previous studies [50], [62], [63], two ancillary qubits can be used to construct entangled states

\[
|φ_0⟩ = \frac{1}{√2} (|0⟩|\tilde{μ}⟩ - |1⟩|\tilde{x}⟩) |0⟩.
\]

Firstly, a Hadamard gate is acted on the first register (denoted as $H_1$) to obtain the following state:

\[
|φ_1⟩ = H_1 |φ_0⟩ = \frac{1}{√2} (|0⟩ + |1⟩) |\tilde{μ}⟩ - |0⟩ - |1⟩ |\tilde{x}⟩) |0⟩.
\]

Then, the Pauli-X gate is applied to the third register controlled by the first register with $|0⟩$ (denoted as $C-X$). That is, $|0⟩|i⟩|j⟩ → |0⟩|i⟩|j⟩ ⊕ 1$ and $|1⟩|i⟩|j⟩ → |1⟩|i⟩|j⟩$, where $⊕$ is modulo 2 addition.

\[
|ψ_2⟩ = (C-X) |φ_1⟩ = \frac{1}{2} (|0⟩|\tilde{μ}⟩ + |1⟩|\tilde{μ}⟩) |0⟩ - |0⟩ |\tilde{x}⟩ |1⟩ + |1⟩ |\tilde{x}⟩ |0⟩)
\]

\[
= \frac{1}{2} (|0⟩ |\tilde{μ}⟩ - |0⟩ |\tilde{x}⟩) |1⟩ + \frac{1}{2} (|1⟩ |\tilde{μ}⟩ + |1⟩ |\tilde{x}⟩) |0⟩.
\]

Let $A_1 = (H_1)(C-X) |ψ_1⟩ = \frac{1}{2} (|0⟩ |\tilde{μ}⟩ - |0⟩ |\tilde{x}⟩) |1⟩$, $ψ_0 = \frac{1}{2} (|1⟩ |\tilde{μ}⟩ + |1⟩ |\tilde{x}⟩) |0⟩$. The problem of interest is given by an operator $A_1$ acting on $|φ_0⟩$ such that

\[
A_1 : |φ_0⟩ → |ψ_1⟩ + |ψ_0⟩.
\]

Then we can use GQAE to calculate $⟨ψ_1 |ψ_1⟩$. Therefore, the inner product of $|\tilde{μ}⟩$ and $|\tilde{x}⟩$ can be obtained for the following equation:

\[
⟨ψ_1 |ψ_1⟩ = \frac{1}{2} (1 - (|\tilde{μ}⟩ |\tilde{x}⟩)).
\]

When we estimate that the inner product of $⟨ψ_1 |ψ_1⟩$ is greater than $\frac{1}{2}$ through GQAE, $|x⟩$ belongs to the $+1$; otherwise, $-1$.

Obviously, AE-QSVM is executed one time to obtain the classification result with an accuracy $ε$ when using GQAE to classify new samples, where $ε$ depends on the number of auxiliary qubits in the GQAE algorithm. The following will compare the algorithms LS-QSVM and AE-QSVM in terms of qubit consumption and complexity.

### 4.3 Space complexity analysis of AE-QSVM

The space complexity of AE-QSVM is analyzed in this subsection. That is we want to analyze the qubit consumption. For AE-QSVM, the qubit consumptions in training process is $3 + [\log(m+1)] + k + [\log(2 + \frac{1}{ε})] + 1$, where $k$ represents that the eigenvalues of $F$ are approximated to an accuracy of $2^{-k}$ [4]. [61].

Estimating the inner product in AE-QSVM, $h$ ancillary qubit is needed to have an accuracy of $ε$ [53].

\[
ε = \frac{2π√a(1-a)}{H} + \frac{π^2}{H^2},
\]

where $H = 2^h$. Solving $\frac{[\log(m+1)] + [\log(2 + \frac{1}{ε})]}{ε} > 0$, we have

\[
H > \frac{π(√a(1-a) + √a(1-a) + ε)}{ε}.
\]

Therefore,

\[
h ≤ [\log \frac{π + √3π}{ε}].
\]

Consequently, the qubit consumption of AE-QSVM is

\[
3 + [\log(m+1)] + [\log ε] + [\log(2 + \frac{1}{ε})] + 1 + [\log \frac{π + √3π}{ε}].
\]

Besides, for LS-QSVM, the qubit consumption in training process is also $3 + [\log(m+1)] + k + [\log(2 + \frac{1}{ε})] + 1$. We set the accuracy all to $ε$ in this paper; thus the qubit consumption of LS-QSVM is

\[
T × (3 + [\log(m+1)] + [\log ε] + [\log(2 + \frac{1}{ε})] + 1),
\]

where $T$ represents the number of the iterations.

Let $Q = 3 + [\log(m+1)] + [\log ε] + [\log(2 + \frac{1}{ε})] + 1$, the space complexity of LS-QSVM and AE-QSVM respectively are $Q + [\log \frac{π + √3π}{ε}]$ and $TQ$. That is to say, LS-QSVM have a higher space complexity when $T > 1 + \frac{[\log \frac{π + √3π}{ε}]}{Q}$ compared with AE-QSVM.

### 4.4 Time complexity analysis of AE-QSVM

We now analyze the time complexity for building AE-QSVM. In the stage of the training process, the time is dominated by the quantum singular value decomposition [61].

We set $κ$ as the condition number (the largest eigenvalue divided by the smallest eigenvalue), and only normalized
eigenvalues $\lambda_j$ in the interval $1/\kappa \leq |\lambda_j| \leq 1$ are taken into account. The kernel matrix $F^0$ with time $O(\log(mn))$ [41] is prepared. The running time of computer $e^{iKt_0}$ is $O(t_0^{-1})$, where $t_0$ is evolution time and $\varepsilon$ is the accuracy. Therefore, the phase estimation costs time $O(t_0^{-1}\log(mn))$ which presents the major time complexity. The probability of getting $\lambda_j^{-1}$ determines iteration times of quantum singular value decomposition, and $O(\kappa^2)$ iterations are needed to ensure the high success probability. However, only $O(\kappa)$ repetitions are performed to obtain the same success probability by amplitude amplification; thus the time complexity is $O(\kappa^2\varepsilon^{-1}\log(mn))$. For $t_0 = O(\kappa^{-1})$, the complexity can be written as $O(\kappa^2\varepsilon^{-5}\log(mn))$.

In the stage of classifying a new sample, the complexity is dominated by quantum amplitude estimation. The complexity of performing unitary operator $Q$ is $O(1)$ [44]; thus the complexity of classification is $O(\kappa^3\varepsilon^{-3})$.

Putting all the time together, the time complexity of AE-QSVM is $O(\kappa^3\varepsilon^{-3}(\log(mn) + 1))$.

5 Simulation

The main work of this paper is to solve the restriction problem of parameter matrix and the large resource consumption caused by quantum collapse when implementing the Swap Test. Therefore, our experiment is divided into two parts.

5.1 The simulation about the parameter matrix

In this subsection, we give an example to illustrate the problem of the matrix consisted of an input sample. In the LS-QSVM, the parameter matrix $F$ is a positive definite matrix or a positive semidefinite matrix. When $F$ is a positive semidefinite matrix, the inverse does not exist. To illustrate the validity of the algorithm used in subsection 4.1, consider the following matrix:

$$F_1 = \begin{pmatrix} 5 & -1 & 3 \\ -1 & 5 & -3 \\ 3 & -3 & 3 \end{pmatrix}.$$ (36)

The eigenvalues of the matrix $F_1$ is $\lambda = 0, 4, 9$, and their corresponding eigenvectors is

$$\nu_1 = (881/2158 \quad -881/2158 \quad -881/1079)^T,$$
$$\nu_2 = (985/1393 \quad 985/1393 \quad 0)^T,$$
$$\nu_3 = (-780/1351 \quad 780/1351 \quad -780/1351)^T.$$ (37)

The matrix $F_1$ is singular since the first eigenvalue $\lambda = 0$, i.e., the eigenvalue decomposition can not be used to solve the quantum linear systems $F_1x = b$. According to the analysis in subsection 4.1, we invert only the nonzero eigenvalues when the matrix $F_1$ is singular.

5.2 The simulation about the quantum resource consumption

Experiments are performed for the complete algorithm in this subsection. In theory, the high complexity in terms of space and time of LS-QSVM are caused by the Swap Test. Therefore, we first conduct experiments on the IBM platform for Swap Test. Then LS-QSVM and AE-QSVM are compared in both space and time.

At first, we conduct the experiment on the IBM qiskit platform. The quantum circuit of Swap Test for classification is shown in Fig. 4. The probability of getting $|1\rangle$ is $P$ when we measure the ancillary qubit. $P$ can be obtained to an accuracy $\varepsilon$ by iterating $O(P(1 - P)/\varepsilon^2)$ as shown in Ref. [41]. The number of iterations is identical for probability $P$ and $1 - P$ when achieving the same accuracy because of $0 < P < 1$. Therefore, we let $P = 0.1$, $P = 0.3$, and $P = 0.5$. It can be seen from Fig. 4 that the number of iterations increases as the accuracy increases. Specifically, the number of iterations will sharply increase as the error increases when $0 < \varepsilon < 0.1$.

Then, the qubit consumption is presented based on the above result. Fig. 4 shows that a large number of iterations are required when using LS-QSVM for classification. Besides, the quantum state of each iteration cannot be used for the next iteration. Therefore, a large number of qubits are needed to be consumed. AE-QSVM does not need iteration, and the number of qubits consumption is shown in Eq. (34). It can be seen from Fig. 4 that the number of iterations required is different for the different $P$. In this section, we take the average of three iterations for the same accuracy. Table 4 shows the number of qubits consumed by AE-QSVM and LS-QSVM for different sample sizes ($m$). It can be seen from Table 4 that the number of qubits to be consumed by both algorithms increases as $m$ increases. However, there are more qubits consumed by algorithm LS-QSVM than that of algorithm AE-QSVM. Especially, as the accuracy increases, the total amount of qubits consumed by algorithm LS-QSVM rapidly increases, while the total number of qubits consumed by algorithm AE-QSVM remains unchanged.

Finally, the complexity of AE-QSVM and LS-QSVM are compared. It can be seen from the above experiment that extensive iterations are needed to achieve a high degree of accuracy for LS-QSVM. Although the complexity of inner estimation is higher than that of the Swap Test, the measurement is not needed in AE-QSVM. Therefore, we will compare the complexity of the two algorithms. In order to compare LS-QSVM and AE-QSVM, we use expectation in

Fig. 4. The number of iterations along error for Swap Test

![Fig. 4. The number of iterations along error for Swap Test](image-url)
TABLE 3
LS-QSVM compared with AE-QSVM about the number of qubit consumption

| Algorithm | Qubits | Accuracy |
|-----------|--------|----------|
| AE-QSVM   | m=10   | 0.70     |
| LS-QSVM   |        | 14       |
| AE-QSVM   | m=100  | 0.80     |
| LS-QSVM   |        | 14       |
| AE-QSVM   | m=1000 | 0.90     |
| LS-QSVM   |        | 14       |
| AE-QSVM   | m=10000| 0.95     |
| LS-QSVM   |        | 14       |

Mathematics to describe the complexity with an accuracy \( \varepsilon \), i.e.,:

\[
O\left(\int_0^1 (\kappa^3 \varepsilon^{-3} \log(mn) + \log(n)) \frac{P(1-P)}{\varepsilon^2} PdP\right) = O\left(\frac{\kappa^3 \varepsilon^{-3} \log(mn) + \log(n)}{12 \varepsilon^2}\right). \tag{37}
\]

The complexity of AE-QSVM is

\[
O(\kappa^3 \varepsilon^{-3} (\log(mn) + 1)) \tag{38}
\]
as analyzed in the above section.

Experiments are designed based on the six datasets: Tic tac toe (m = 958, n = 9), Haberman (m = 306, n = 3), Ionosphere (m = 351, n = 34), Heart statlog (m = 270, n = 13), Liver disorders (m = 345, n = 6), and Bupa (m = 345, n = 6). We first calculated the condition numbers of the six datasets and then calculated the complexity of the two methods according to Eq. 6 and Eq. 7 respectively. It can be seen from Table 4 that the complexity of AE-QLSM is significantly lower than that of LS-QSVM when the accuracy is above 0.2, and the complexity of the two algorithms differs little when the accuracy is 0.3. However, the algorithm is meaningless if the error is too large. Therefore, our algorithm is superior than the LS-QSVM algorithm when the error is in the acceptable range.

6 Discussion and Conclusion

The quantum support vector machine based on quantum amplitude estimation is introduced in this paper. At first, the quantum singular value decomposition was used to train AE-QSVM which excludes the constraints of sparsity and is well structured for input matrix. Then, we used the quantum amplitude estimation instead of Swap Test to realize the classification so that the support vector machine was not needed to be repetitively performed. In AE-QSVM, auxiliary qubits are used to transform the probability amplitude into basis state; thus high accuracy can be achieved by adding the number of auxiliary qubits. Therefore, the algorithm AE-QSVM essentially reflects the superiority of quantum algorithms.

The results from this study can extended towards the following directions in future work:

(1) AE-QSVM can be extended to the nonlinear classifier. One of the most powerful uses of support vector machines is to perform the nonlinear classification [64]. One can use our proposed algorithm RN-QSVM to find a nonlinear classifier based on the Gaussian kernel [65].

\[
(K(A, B))_{ij} = e^{-\delta \|A_i - B_j\|^2}, \quad i = 1, \ldots, m, j = 1, \ldots k
\]

where \( A \in R^{m \times n} \) and \( B \in R^{n \times l} \) \( \delta \) is a positive constant, and the kernel \( K(A, B) \) maps \( R^{m \times n} \times R^{n \times l} \) into \( R^{m \times l} \). A quantum version of the Gaussian kernel was proposed by Bishwas et al. [51]. The running time complexity of the quantum Gaussian kernel is significantly shorter compared with its classical version.

(2) The inner estimation based on amplitude estimation has low complexity compared with the Swap Test. Therefore, this generalized amplitude estimation can be used in other machine learning methods, for instance, neural networks and clustering.

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| Dataset            | Algorithm  | LS-QSVM | AE-QSVM |
|--------------------|------------|---------|---------|
| Liver disorders    | LS-QSVM    | $4.89 \times 10^{13}$ | $1.65 \times 10^{13}$ |
|                    | AE-QSVM    | $3.85 \times 10^{13}$ | $2.98 \times 10^{13}$ |
| Heart failure      | LS-QSVM    | $3.66 \times 10^{12}$ | $2.97 \times 10^{12}$ |
|                    | AE-QSVM    | $3.35 \times 10^{12}$ | $2.93 \times 10^{12}$ |
| Hart failure       | LS-QSVM    | $5.79 \times 10^{12}$ | $1.82 \times 10^{12}$ |
|                    | AE-QSVM    | $5.89 \times 10^{12}$ | $3.00 \times 10^{12}$ |
| Bupa               | LS-QSVM    | $1.45 \times 10^{12}$ | $1.07 \times 10^{12}$ |
|                    | AE-QSVM    | $1.58 \times 10^{12}$ | $1.09 \times 10^{12}$ |

TABLE 4: LS-QSVM compared with AE-QSVM on six datasets.

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