Constraints on axionlike dark matter with masses down to $10^{-23} \text{ eV}/c^2$

W. A. Terrano, E. G. Adelberger, C. A. Hagedorn, and B. R. Heckel
Center for Experimental Nuclear Physics and Astrophysics,
Box 354290, University of Washington, Seattle, Washington 98195-4290

We analyzed an 6.7-year span of data from a rotating torsion-pendulum containing $\approx 10^{23}$ polarized electrons to search for the “wind” arising from ultralight, axionlike dark matter with masses between $10^{-23}$ and $10^{-18} \text{ eV}/c^2$. Over most of this range we obtain a 95% confidence limit $F_a/C_a \geq 1 \times 10^{15} \text{ eV}$ on the axionlike decay constant.

PACS numbers: 95.35+d,98.35Gi,14.80.Va

A wide variety of astrophysical observations provide compelling, but indirect, evidence for the existence of cold dark matter\cite{1}. Direct efforts to detect this dark matter typically assume that it consists of heavy, fermionic, supersymmetry-inspired particles called WIMPS, or else low-mass bosons called axions that would solve the strong CP problem. However, despite much effort, large-scale detectors have not found evidence for supersymmetry\cite{2} nor for dark-matter WIMPS\cite{3}. This has focused attention on low-mass bosonic dark matter, where sensitive instruments are now probing the expected coupling-strength and mass of the Peccei-Quinn axion\cite{4}.

Recent work\cite{5} has emphasized that bosons with masses anywhere between $10^{-22}$ and $100 \text{ eV}/c^2$ could be produced in the early universe with the properties required of the cold dark matter. If the bosons have masses below $1 \text{ eV}/c^2$ and comprise a significant fraction of the observed dark matter density $\rho_{DM}$, their number density must be so high that they behave as coherent waves rather than as particles. If the bosons are bound in our galaxy, they must be highly non-relativistic ($v_a/c \approx 10^{-3}$), and the bosonic waves would have a frequency

$$f_a = E/h = m_a c^2 [1 + (v_a/c)^2]/2,$$

(1)

corresponding to a central frequency $f_a = E/h = m_a c^2$ with a fractional spread $\delta f_a/f_a = (v_a/c)^2/2 \approx 10^{-6}$, and a de Broglie wavelength

$$\lambda_a = h/(m_a v_a).$$

(2)

Astrophysical observations may favor the very longest wavelength dark-matter candidates\cite{6,7}: conventional cold-dark-matter simulations predict density cusps at the centers of galaxies and a substantial abundance of low-mass dwarf galaxies, both of which disagree with observations\cite{8}. An ultralight boson (UB) with $m_a \approx 10^{-22} \text{ eV}/c^2$ would have $\lambda_a \approx 1.2 \times 10^{19} \text{ m}$ or 400 parsecs. In this case, the uncertainty principle (which states that the UB distribution cannot be localized to better than $\lambda_a$) could solve both of the above-mentioned problems with cold-dark-matter simulations, making this tiny mass an important target for experimental work.

Laboratory probes of axionic or axionlike dark-matter fall into three broad classes: coupling to highly sensitive electromagnetic circuits\cite{4}, oscillating atomic\cite{9} or neutron\cite{10} electric-dipole or parity-violating moments, and “wind”\cite{10} effects. The “wind” from the laboratory’s motion with respect to the axion wave acts on an electron like an oscillating “magnetic field”\cite{5} \cite{11} with

$$H_{eff} = C_e \tilde{a}_0 \sin(f_a t/h + \phi_a) \frac{(v_a \cdot \sigma_e)}{c}, \quad (3)$$

where the dimensionless factor $C_e$ characterizes the axion coupling to electrons and $\sigma_e$ is the orientation of the electron spin. $F_a$, $f_a$, and $\phi_a$ are the symmetry-breaking scale, oscillation frequency, and phase of the axionlike wave, respectively. If the local energy-density of dark matter ($0.45 \text{ GeV/cm}^3$\cite{4}) consists entirely of axionlike UBs, then $\tilde{a}_0 = \sqrt{2 \rho_{DM} (hc)^3} \approx 2.6 \times 10^{-3} \text{ eV}$, a value we assume throughout this work.

Although the UBs and the solar system are gravitationally bound in the galaxy and originally had similar velocities this will not be the case today. The traditional isothermal isotropic equilibrium halo model\cite{15} predicts that dissipation and angular momentum conservation (the “ballerina effect”) combined to give the present-day solar system a circular velocity $v_\odot$ which is an order of magnitude larger than that of the dark matter. In this equilibrium model $v_a \approx v_\odot$. However, this simple model ignores the possibility that recent mergers between the Milky Way and its satellite galaxies have not fully equilibrated, leaving debris streams with large velocities. Recent analyses of Gaia-2 data\cite{12} \cite{14} suggest that 10-50% of the dark matter in our galaxy is in such streams. If the solar system is currently in a debris stream, $v_a$ could point in any direction with a magnitude less than the local escape velocity.

Motivated by these considerations, we searched for the frequency-dependent signatures of an axion wind in Eöt-Wash rotating torsion-balance data previously taken with a pendulum containing $N_e \approx 9.8 \times 10^{22}$ polarized electrons. This remarkable pendulum\cite{17} (shown in Fig. 1) was formed from closed magnetic circuits containing two different permanent magnetic materials with high (Alnico) and low (Sm Co5) spin densities at the same internal

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Motivated by these considerations, we searched for the frequency-dependent signatures of an axion wind in Eöt-Wash rotating torsion-balance data previously taken with a pendulum containing $N_e \approx 9.8 \times 10^{22}$ polarized electrons. This remarkable pendulum\cite{17} (shown in Fig. 1) was formed from closed magnetic circuits containing two different permanent magnetic materials with high (Alnico) and low (Sm Co5) spin densities at the same internal
FIG. 1. (color online) Spin pendulum. The light green and darker blue volumes are Alnico and Sm Co$_5$, respectively. Upper left: top view of a single magnetized “puck”; its spin moment points to the right. Lower right: the assembled pendulum with its magnetic shield shown cut-away to reveal the 4 pucks inside. Two of the 4 mirrors (light gold) used to monitor the pendulum twist are prominent. Arrows with filled heads show the relative densities and directions of the electron spins, open-headed arrows show the directions of $\mathbf{B}$. The pucks are arranged so that the spin dipole is centered on the pendulum, and the different materials have a vanishing composition-dipole moment. The 8 tabs on the shield held small screws that were used to tune out the pendulum’s residual gravitational moment. The 103 g pendulum had a rotational inertia of 169 g cm$^2$.

magnetizations. The resulting device had a negligible external magnetic field but carried both net spin $S$, orbital angular momentum $L$ and a total angular momentum $J = -S$. The net spin dipole was calibrated using the Coriolis force from the earth’s rotation on the electron-spin “gyroscope”. The device was sufficiently sensitive to yield an upper limit of $\approx 2 \times 10^{-22}$ eV on the energy required to invert an electron spin about a direction fixed in inertial space; this is equal to the electrostatic energy of 2 electrons separated by 48 astronomical units. However that analysis, which searched for a preferred frame, would have averaged away the time-varying signal of Eq. 3.

Here we analyze a larger data set (taken for Refs. [17–19]) that spanned 2438 days in 241 subsets. During each subset the experimental conditions (for example the turntable rotation rate, the angle of the spin dipole in the turntable frame[17,19] and the positions of external sources[18,19] remained constant. The subsets had durations between 0.8 and 10 days. Our data consisted of 15588 individual measurements. Each measurement typically contained exactly 2 full turntable revolutions and lasted for $\sim 2800$ s, a duration long compared to the pendulum’s free-oscillation period of $\approx 200$ s. Following Ref. [17], we assumed that during an individual measurement the pendulum’s energy as a function of turntable angle $\phi_{tt}$ was

$$E(\phi_{tt}) = -N_e \sigma \cdot \beta = -N_e \beta \cos \phi$$

(4)

where $N_e$ is the number of polarized electrons in the pendulum, $\sigma$ is the direction of the spin dipole, $\beta$ is a vector assumed to be approximately fixed in the lab during an individual measurement, and $\phi = \phi_{tt} - \phi_0$ was the instantaneous angle between the rotating spin dipole and $\beta$. Therefore the pendulum experienced a torque

$$T(\phi_{tt}) = -\partial E/\partial \phi = N_e (\beta_W \cos \phi_{tt} - \beta_N \sin \phi_{tt})$$

(5)

that was inferred by correcting the measured pendulum twist angle in the rotating frame for pendulum inertia plus electronic and digital time constants. Each measurement yielded independent determinations of $\beta_N$ and $\beta_W$, where N and W are local North and West directions. Measurement uncertainties in each data subset were deduced from the scatter of the points in that subset. We suppressed lab-fixed signals (arising from the Coriolis force as well as from many other less interesting effects) by setting to zero the average $\beta_N$ and $\beta_W$ values in each of the 241 subsets. Figure 2 displays the $\beta_N$ and $\beta_W$ values of our measurements.

We searched our $\beta_N$ and $\beta_W$ values for signals from axions with $v_a$ in an arbitrary direction in the equatorial (X,Y) plane. (Signals for $v_a$ along Z were not considered here as they have no sidereal modulation.

![Graph](image-url)
and are more difficult to distinguish from mundane lab-
fixed effects. We made evenly-spaced scans over
67,200 values of $f_a$. At each $f_a$ we computed 8 ba-
sis states: $b_{iXNcos}, b_{iXWcos}, b_{iXNsin}, b_{iXWsin}$, plus cor-
responding states for $a_i$ along $Y$. Here, for example,
$K_{iXN} = K_{XN} \eta_i \cos \omega t$ where $K_{XN}$ transforms equatorial ($X,Y$) to local ($N,W$) coordinates and
varies at the sidereal frequency; $\eta_i = \sin(\omega \tau_i^f)/2)/\omega \tau_i^f$ accounts for the attenuation of $f_a$ by the finite length of
the measurements: $\tau_i^f$ and $\tau_i^s$ are the mean time and du-
ration of the $i$th measurement, and $\omega = 2 \pi f_a$. We zeroed
the average values of the 8 basis states in each of the 241
subsets as well. This procedure ensured that the effects
of zeroing the mean values of the $\beta$ data subsets, the
varying lengths and uncertainties of the subsets, as well
as the gaps between the subsets were handled correctly.
Our constraints for axion velocities along $X$ were derived
from a linear fit that yielded quadrature amplitudes $a_{Xcos}$
and $a_{Xsin}$

$$\begin{align*}
\beta^a_i &= b_{iXNcos} a_{Xcos} + b_{iXNsin} a_{Xsin} \\
\beta^b_i &= b_{iXWcos} a_{Xcos} + b_{iXWsin} a_{Xsin},
\end{align*}$$

(6)

with similar expressions for $a_{Ycos}$ and $a_{Ysin}$.

We first analyzed the frequency range of our highest
sensitivity ($1 \times 10^{-8} \leq f_a \leq 1 \times 10^{-4}$ Hz); here the
maximum $f_a$ was small compared to the inverse dura-
tions of the measurements so that the basis states were not appreciably attenuated. For each $f_a$, we determined
4 quadrature amplitudes and their uncertainties. Re-
sults are shown in Figure 3. All 4 quadrature am-
plitudes are characterized by Gaussians with zero mean and
$\sigma = 0.154$ zeV. The distributions of the quadrature am-
plitudes divided by their uncertainties are also zero-mean Gaussians but with $\sigma = 1.0$. We marginalised over the
uninteresting phase $\phi_a$ by computing radial amplitudes
$a_X = \sqrt{[a_{Xcos}]^2 + [a_{Xsin}]^2}$ and $a_Y = \sqrt{[a_{Ycos}]^2 + [a_{Ysin}]^2}$. As expected, $a_X$ and $a_Y$ are well modeled by the Rayleigh
distribution whose only free parameter is the Gaussian $\sigma$.

The spectral distribution of $a_X$ is shown in Fig. 4. The
small bump centered at $f_a \approx 11.6 \mu$Hz occurs when $f_a$ and
the sidereal frequency coincide. In that case our pro-
cedure of zeroing the average value of the basis states
reduced their mean magnitudes by a factor $\sqrt{2}$. This
automatically increased the fitted amplitudes (and their uncertainties) by the same factor. A careful examination
revealed that the central values in the bump region ex-
ceed the expected increase by an additional 40%, which
accounts for the observation that 5.2%, rather than 5.0%,
of the points in Fig. 4 lie above the lower blue curve. We
checked that the excess on the bump was not an arti-
fact of our analysis using a simulation where we kept
the actual uncertainties and times of our $\beta$ data, but re-
placed the central values of the $\beta$s with values normally-
distributed around 0 by the actual uncertainties. The
simulated data showed no additional excess, and we con-
clude that the excess arose from a subtle systematic with
a characteristic period of about 1 day. Binning the data
as function of time of data did reveal a systematic effect:
the scatter of the points in night-time data was less than
that of the day-time data, but correcting for this had no
significant effect on the results.

Satisfied that the statistical properties of $a_X$ and $a_Y$
were well described by the Rayleigh distribution, we
widened our scan to cover frequencies between $10^{-9}$
Hz and $3.2 \times 10^{-4}$ Hz. Now the frequency interval
in our 67,200 point scan nearly equaled the inverse of
the 2438-day span our our data and the high-frequency
signals were appreciably attenuated by the finite dura-
tions of the measurements. Our $C_c/F_a$ constraints from

FIG. 3. (color online) Upper 2 panels: histograms of $a_{Xcos}$
and $a_{Xsin}$ coefficients for $10^{-8} \leq f_a \leq 10^{-4}$ Hz. Results for
$a_{Ycos}$ and $a_{Ysin}$ are very similar. All 4 quadrature am-
plitudes are characterized by zero-mean Gaussians with $\sigma = 0.154$ zeV.
Lower panel: histogram of corresponding $a_X$ amplitudes. The results follow the expected Rayleigh distribution, the 95%-confidence upper-limit on $a_X$ (as well as on $a_Y$) is 0.38 zeV.
FIG. 4. (color online) Spectral distribution of $a_X$. The lighter (orange) jagged line shows the fit amplitudes. The 2 smooth lines show the sensitivities of the analysis. The lower (blue) smooth curve shows the 95% C.L. Rayleigh uncertainties in the individual fit amplitudes while the upper red curve contains a trials penalty. Absent a signal, the Rayleigh distribution predicts that 5% of the amplitudes will lie above the lower curve and that there is a 5% probability that a single amplitude anywhere in our frequency range will lie above the upper curve; as expected no amplitude exceeds the upper curve. The darker (blue) jagged line shows the result of adding to our $\beta_N$ and $\beta_W$ data a synthetic $1 \mu$Hz $X_{\cos}$ signal with an amplitude of 2.5 eV. The extracted amplitude, $2.498 \pm 0.144$ eV, is resolved at $17\sigma$. Small satellite peaks in the synthetic signal arise from gaps in our time-series data.

This scan on axion velocities lying in an arbitrary direction in the XY plane are shown in Fig. 5 We assume that $|v_a| = 240\text{km/s} \approx |v_\odot|$ [21], this is approximately the virial velocity and roughly half the local escape velocity [22]. We neglected the earth’s orbital velocity around the sun which is an order of magnitude smaller than $|v_\odot|$. Our constraints based on the simple equilibrium halo model, where $v_a$ is the XY component of $v_\odot$, are shown in Fig. 5 as well.

This analysis probes axionlike dark-matter coupled to electrons over 4 of the 22 decades of possible masses for coherent axionlike dark-matter [5]. Were it of sufficient interest, one could re-fit the raw pendulum-twist data with combined turntable, dark-matter and sidereal basis-functions, extending constraints up to the pendulum’s free-oscillation frequency of $\approx 5 \text{mHz}$ and cover an additional decade of mass range.

Our results exclude ultralight galactic-halo axions with $F_a/C_e \lesssim \text{few PeV}$, significantly above the $F_a/C_e \lesssim 100 \text{TeV}$ scale probed by experiments that rely on both sourcing and detecting axions, such as fifth-force searches [19, 24] and light-shining-through-walls tests [25]. Astrophysical considerations disfavor $F_a/C_e \lesssim 1 \text{EeV}$ axions as they would provide a channel for excessive cooling [26]. Axions in the parameter space we probed would require a mechanism to suppress their production inside stars, such as chameleon self-interactions [27]. It is interesting that an EeV-scale axion would slightly improve the consistency between cooling models and observations for some of the well-studied astrophysical systems [28, 29].

We expect that by replacing the tungsten torsion fibers used in this work with ones made from fused silica, and
employing advanced twist-angle readout and turntable control, the constraints reported here could be improved by up to a factor 100. An additional factor of 10 would be needed to probe the coupling strengths hinted at by the cooling data. We thank J. Detwiler, V. Flambaum, L. Hui, M. Lisanti, Y. Stadnik and T. Quinn for helpful conversations. Ted Cook constructed the spin pendulum and Claire Cramer took some of our data. This work was supported in part by National Science Foundation Grants PHY-1305726 and PHY-1607391.

* Present Address: Department of Physics, Princeton University, Princeton NJ 08550 USA; wter-rano@princeton.edu

[1] G. Bertone, Particle Dark Matter: Observations, Models and Searches. (Cambridge University Press, Cambridge, UK, 2010).

[2] C. Autermann, Prog. Part. Nucl. Phys. 90, 125 (2016).

[3] G. Arcadi, M. Dutra, P. Ghosh, M. Lindner, Y. Mambrini, M. Pierre, S. Profumo and F.S. Queiroz, arXiv:1703.07364 (2017).

[4] N. Du et al. (ADMX collaboration), Phys. Rev. Lett. 120, 151301 (2018).

[5] P.W. Graham and S. Rajendran, Phys. Rev. D 88, 035023 (2013).

[6] W. Hu, R. Barkana and A. Gruzinov, Phys. Rev. Lett. 85, 1158 (2000).

[7] L. Hui, J.P. Ostriker, S. Tremaine and E. Witten, Phys. Rev. D 95, 043541 (2017).

[8] D.H. Weinberg, J.S. Bullock, F. Governato, R.K. de Nary, and A.H.G. Peter, PNAS 112, 12249 (2015).

[9] B.M. Roberts, Y.V. Stadnik, V.A. Dzuba, V.V. Flambaum, N. Leefer and D. Budker, Phys. Rev. Lett. 113, 081601 (2014).

[10] C. Abel et al., Phys. Rev. X 7, 041034 (2017).

[11] Y.V. Stadnik and V.V. Flambaum, Phys. Rev. D 89, 043522 (2014) and private communication (2015); which corrected a factor of 2 error in Eq. 8 of Ref. [11].

[12] N. W Evans, C. A. J. O’Hare and C. McCabe, arXiv:1810.11468 (2018).

[13] A. Fattahi, V. Belokurov, A. J. Deason, C. S. Frenk, F. A. Gomez, R. J. J. Grand, F. Marinacci, R. Pakmor and V. Springel, arXiv:1810.07779 (2018).

[14] L. Necib, M. Lisanti, S. Garrison-Kimmel, A. Wetzel, R. Sanderson, P.F. Hopkins, C.A. Faucher-Giguère and D. Kereš, arXiv:1810.12301 (2018).

[15] J.P. Ostriker, P. J.E. Peebles, and A. Yahil, Ap. J. 193, L1, (1974).

[16] L. Necib, M. Lisanti, S. Garrison-Kimmel, A. Wetzel, R. Sanderson, P.F. Hopkins, C. Faucher-Giguère, D. Kereš, arXiv:1810.12301 (2018).

[17] B. R. Heckel, E. G. Adelberger, C. E. Cramer, T. S. Cook, S. Schlamminger, U. Schmidt, Phys. Rev. D 78, 092006 (2008).

[18] W. A. Terrano, B. R. Heckel and E. G. Adelberger, Class. Quant. Grav. 28, 145011 (2011).

[19] B. R. Heckel, W. A. Terrano and E. G. Adelberger, Phys. Rev. Lett. 111, 151802 (2013).

[20] Ref. [17] obtained a constraint along Z, albeit with lower precision, by taking data with 4 orientations of the spin pendulum with respect to the torsion fiber; however, this was not done for the data taken for Refs. [18, 19].

[21] J. Bovy et al., Astrophys. J. 759, 131 (2012).

[22] T. Piffl et al., Astron. Astrophys. 562, A91, (2014).

[23] The Rice distribution applies when the quadrature amplitudes are Gaussians with non-zero means: see https://en.wikipedia.org/wiki/Rice_distribution.

[24] W.A. Terrano, E.G. Adelberger, J.G. Lee, and B.R. Heckel, Phys. Rev. Lett. 115, 201801 (2015).

[25] R. Ballou et al. (OSQAR collaboration), Phys. Rev. D 92, 092002 (2015).

[26] M. M. Miller Bertolamia, B. E. Melendez, L. G. Althaus, and J. Isern, arXiv:1406.7712v2 (2014).

[27] P. Jain and S. Mandal, Int. J. Mod. Phys. D 15, 12, 2095 (2006).

[28] A.H. Córsico et al. J. Cosmology and Astroparticle Phys. 2016 07:036 (2016).

[29] N. Viaux, M. Catelan, P. B. Stetson, G. G. Raffelt, J. Redondo, A. A. R. Valcarce and A. Weiss, Phys. Rev. Lett. 111, 231301 (2013).