Discrimination of Quark Stars from Neutron Stars in Quadrupole Oscillations

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The frequencies and damping times due to gravitational radiation are calculated for self-bound quark star models. The results are compared with those for neutron star models. They are markedly contrasted in less relativistic cases. The distinction derived here from a simple model of quark stars may be relevant to the future theoretical and observational studies, since the oscillation properties essentially depend on the mass and radius of an equilibrium star.

§1. Introduction

Recently, Chandra X-ray Observations reveals a new aspect of a compact star. The isolated neutron star candidate RXJ1856.5-3754, which may be the closest to us, has the radiation radius 3.8-8.2 km. The distance is estimated as 60-130pc from the parallax and the interstellar column density. The ambiguity leads to the range of the estimated size, but the value is evidently much smaller than the canonical radius of a neutron star $\sim 10$ km.

The radiation radius $R_\infty$ is determined by the observed energy flux and black-body temperature. Assuming spherical symmetric equilibrium, $R_\infty$ is expressed by the radius $R$ in the Schwarzschild coordinate as $R_\infty = R(1 - 2GM/Rc^2)^{-1/2}$, where $M$ is the gravitational mass. It is clear that $R \leq R_\infty$. After some algebra, we also have $GM/c^2 \leq R_\infty/(3\sqrt{3})$. The gravitational mass must be less than 0.49-1.05 $M_\odot$ corresponding to the observed value of $R_\infty$. This limit of mass is significantly less than the canonical mass of a neutron star $\sim 1.4M_\odot$. In this way, the characteristic size and mass are unusual for any neutron star models.

There are however several attempts to reconcile with the standard neutron star models. For example, the atmosphere is metal dominated one, and/or the surface temperature distribution is inhomogeneous. At present, there is no positive evidence to support them even in the long term and high-resolution observation.

Taken at face value, RXJ1856.5-3754 is not a neutron star, but a different species of a compact star, which is likely to be a quark star. We would definitely judge the state of the compact star, if there is different information such as mass, which is not easily determined by the single star. Nakamura proposed a possible formation scenario of the quark star. In the standard scenario to neutron star after supernova, it is difficult to produce such light remnant $\sim 0.7M_\odot$ leading to the quark star. If the progenitor has significant rotation, small core is left through the centrifugal breakup. In that case, huge amount of energy can be radiated by gravitational waves. The quadrupole oscillation is likely to be driven by such a collapse, and the asymmetry of matter distribution is efficiently smoothed out by the gravitational radiation.
detection of such gravitational waves significantly depends on the number of the events. There is a big uncertainty about it, because we know just one candidate, RXJ1856.5-3754.

Motivating the observational suggestion, we will study the property of the quark star models. Our concern is not the equilibrium structure, which was already calculated in the literature, but the dynamical aspect, that is, non-radial oscillations resulting from small disturbances. In particular, we compute the characteristic frequency of the quadrupole f-mode oscillations, which is the most important for the gravitational radiation. Such calculations were extensively performed for several non-rotating neutron stars and slowly rotating polytropic models. By extending to the quark star models, the 'catalog' is enriched for the future gravitational wave astronomy. In addition to direct gravitational wave signal, oscillatory phenomena after some kinds of sudden bursts or flares may appear in X-ray and/or gamma-ray observation. The global properties such as mass and radius should be relevant to dynamical overall oscillations on the star. Non-spherical distortion efficiently decays due to gravitational wave emission within a few second as shown later. The typical oscillation frequency is kHz. These properties, irrespective of observational bands or tools, provide useful diagnosis for the interior of compact stars. Other variability except f-mode may also appear in different frequency range, but they are much reflected by details of the interior structure, which are not so clear at moment. Therefore, we do not consider them here. Microscopic equation of state for high-density matter is not yet established in neutron stars and quark stars. We use a simplest one, which may be helpful within our present knowledge of high-density matter to understand the contrasts between neutron stars and quark stars. The model and the numerical method are summarized in §2. The results are shown in §3. In §4, we compare the self-bound quark stars with neutron stars in their oscillation frequencies and the decay times. Finally in §4, concluding remarks are given.

§2. Models and Calculations

Witten conjectured that three-flavor quark matter consisting of u, d, and s quarks may be actually be the ground state of the strong interaction. If this strange quark matter hypothesis is true, we expect pure quark matter star. The properties of the strange star are explored using a simple microphysical treatment of quarks. The quark matter is described by the fundamental theory of the strong interaction, QCD. The equation of state (EOS) for it can be derived in principle. However, there are not yet practical results due to the difficulty of the theory. Most important aspects of the strong interaction are asymptotic freedom and confinement of quarks. They are described by phenomenological treatment, the MIT bag model, i.e., free massless quarks are confined within the bag. Based on such a simple picture, the EOS can be written by a sum of the pressure by free quarks and the bag pressure.

\[ p = \frac{1}{3} \rho c^2 - \frac{4}{3} \frac{B}{(hc)^3}. \]  

The equilibrium stellar models can be constructed by solving the TOV equa-
tion. The self-bound star has a sharp edge at the surface, corresponding to \( p = 0 \), and therefore \( \rho c^2 = 4B/(hc)^3 \). The density is high enough, and the interior density distribution does not so drastically change in the magnitude from the center to the surface. The density distribution of self-bound stars and the comparison with neutron stars are fully explained elsewhere.\(^8\) For the simple EOS (1), the equilibrium models have a scaling law with the bag constant \( B \). In terms of the radii and masses of one sequence of stars with \( B \), those for any other choice of \( B' \) can be found from

\[
R(B') = \sqrt{B/B'} \ R(B), \quad M(B') = \sqrt{B/B'} \ M(B).
\]  

(2)

In this paper, we consider the spherical symmetric equilibrium models and the perturbations from them. The oscillations with small amplitude are described by the perturbation of the Einstein equation. By the spherical symmetry of the background model, the perturbation quantities may be decomposed into spherical harmonics. We consider the f-mode of quadrupole \( l = 2 \) only, which is the most important for gravitational radiation. The properties of such oscillation were extensively studied so far for non-rotating neutron stars constructed from various EOS\(^9\)\(^10\). The eigenfrequencies are determined by solving a fourth-order system of differential equations inside the star, and a second-order one outside the star. They are subject to appropriate boundary conditions. By imposing outgoing wave condition, the normal frequency becomes a complex number, that is, the real part \( \sigma_R \) represents the oscillation frequency and the imaginary part \( \sigma_I \) the damping rate. The equations and numerical methods adopted here are summarized in details elsewhere.\(^14\)

§3. Results

The oscillation frequency and the damping rate for the EOS (1) were previously calculated only for a few stellar models.\(^10\) We have here extended the similar kind of calculations to much larger range of stellar models in order to understand the general properties. The angular frequency \( \sigma_R \) is shown as a function of the compactness \( GM/Rc^2 \) in Fig.1. The frequency is normalized by \( \Omega_K = \sqrt{GM/R^3} \). In the limit of Newtonian case \( GM/Rc^2 \to 0 \), the frequency of stellar model with (1) is well approximated by \( \sigma_f = \sqrt{2l(l-1)/(2l+1)}\Omega_K \simeq 0.894\Omega_K \) for \( l = 2 \), which is the frequency of the Kelvin f-mode for the uniform density. The frequency may be approximated by that for the uniform density case, since the density distribution is almost constant in the self-bound quark star models. The similarity to the constant density case is much clear by comparing with polytropic EOS, \( p = K\rho^{1+1/n} \). The results for \( n = 1, 0.5 \) and 0.2\(^7\)\(^8\) are shown in Fig.1. The oscillation frequencies for the EOS (1) may be approximated like those of stellar models with smaller \( n \approx 0 \). We have also calculated for smaller value of \( n \). The agreement in the Newtonian limit becomes better as \( n \to 0 \), but the overall agreement in some range of \( GM/Rc^2 \)

\(^{10}\) A little bit careful treatment is necessary for the limit \( n = 0 \) in the relativistic polytropic models. The pressure and density perturbations are related as \( \delta p = C^2\delta \rho \) in general, but \( C^2 \to \infty \) and \( \delta \rho \to 0 \) for \( n \to 0 \). It is possible to calculate for \( n = 0 \) by modifying the relevant parts, but such a task would lead to only a tiny difference.
is impossible for any value of $n$. The treatment like $n \approx 0$ in the polytropic model is not exact, but just approximate one, as there is no reason to support.

In Fig.2, we show the damping rate $\sigma_I$ of the mode. The characteristic decay rate can be evaluated by the energy loss rate by the quadrupole radiation formula, $\sigma_I = dE/dt/(2E) \propto (GM)^3/(R^4c^5)$. We use this scale as the normalization in Fig.2. The decay rate of the Newtonian star with uniform density is analytically calculated as $\sigma_I(R^4c^5)/(G^3M^3) = 32/625 = 5.12 \times 10^{-2}$. The decay rate of the star with (1) coincides with this value in the Newtonian limit, as the oscillation frequency does. The dependence of the compactness is also very similar to the model with small $n$. In this way, the general behavior of the frequency and the decay rate for the EOS (1) can be described by the results for the stellar models with small $n \approx 0$.

§4. Comparison among various models

The frequency of self-bound quark star model also has the scaling, $\sigma \propto \sqrt{M/R^3} \propto \sqrt{B}$. The bag constant $B$ is not exactly determined, but is usually chosen as $B^{1/4} = 145\text{-}165 \text{ MeV}$ [4]. The approximate form (1) itself would be meaningless by theoretically considering realistic effects such as the quark mass and gluon interaction. We use $B^{1/4} = 155 \text{ MeV}$, i.e., the energy density $B/(hc)^3 = 75 \text{MeV/fm}^3$ as a representative example. The energy density at the stellar surface $4B/(hc)^3$ corresponds to about twice the energy of normal nuclear matter $\rho_n c^2 = 141 \text{MeV/fm}^3$. 
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i.e. $\rho_n = 2.5 \times 10^{14}$ g/cm$^3$. We also show the results scaled to $B^{1/4} = 180$ MeV, i.e., $B/(\hbar c)^3 = 137$ MeV/fm$^3$. In Fig.3, we show the frequency $\nu = \sigma_R/2\pi$ of quadrupole oscillation along the sequence of quark stars with a fixed value of $B$. We also include the results for the various neutron star models\cite{9}. A lot of EOSs are proposed for the high-density neutron matter, but only selected EOSs are used. Our collection might be old-fashioned, but covers from soft to stiff EOSs. We omitted too soft ones, which cause the maximum mass of neutron star below 1.4 $M_\odot$. The four models are denoted by A\cite{13}, B\cite{14}, PPS-T and PPS-M\cite{17} in Fig.3. The stiffness is roughly in this order, A is the softest and PPS-M the stiffest. The description of these EOSs is also written elsewhere\cite{9}.

There is quite different dependence on the compactness $GM/Rc^2$ in the frequencies of gravitationally bound and self-bound stars. In the equilibrium sequence of neutron stars, the oscillation frequency increases with $GM/Rc^2$, because the mass increases and the radius decreases, and hence $\nu \propto \sqrt{GM/R^3}$ strongly increases. In order to understand this property, we labeled the average densities $3M/4\pi R^3$ along each equilibrium sequence in Fig.3. They are normalized by the nuclear density $\rho_n$. On the other hand, both mass and radius in the sequence of self-bound quark star models increase as $M \propto R^3$, and hence the oscillation frequency is almost constant, i.e., $\nu \propto \sqrt{GM/R^3} \approx \text{constant}$. Actually the average density of the model $B^{1/4} = 155$ MeV, changes within 2.1-3.2 $\rho_n$, and that of $B^{1/4} = 180$ MeV within 3.9-5.9 $\rho_n$. The frequency $\nu$ of a quark star is approximated as

$$\nu = \frac{\sigma_R}{2\pi} \approx 1.7 \times \left\{ 1 + 4 \left( \frac{GM}{Rc^2} \right)^2 \right\} \left( \frac{B^{1/4}}{155\text{MeV}} \right)^2 \text{kHz}. \quad (3)$$

The dependence on the bag constant $B$ is exact, but relativistic correction is introduced to be fitted for $GM/Rc^2 \leq 0.25$ in the braces. In general, the frequencies of quark stars are discriminated from those of neutron stars in less relativistic regime, $GM/Rc^2 \approx 0$. In that case, the radius of the quark star is much smaller, and the frequency is higher.

In Fig.4, we show the decay time $\tau = 1/\sigma_I$ for self-bound quark star and neutron stars. The decay times of both stellar models strongly depend on the compactness $GM/Rc^2$. There is not so much clear difference unlike the oscillation frequency. The decay time $\tau$ for the EOS\cite{11} is approximated as

$$\tau = \frac{1}{\sigma_I} \approx 1.7 \times 10^{-3} \left\{ 1 + 12 \left( \frac{GM}{Rc^2} \right)^{3/2} \right\} \left( \frac{GM}{Rc^2} \right)^{-5/2} \left( \frac{B^{1/4}}{155\text{MeV}} \right)^{-2} \text{s}. \quad (4)$$

The formula provides exact dependence on the bag constant $B$ and the value in the Newtonian limit. The expression in the braces is the relativistic correction to be fitted for $GM/Rc^2 \leq 0.25$. In Figs.3 and 4, we only show the results for two choices of $B$, but one can easily calculate for the other choices by eqs. (3) and (4).

We now apply our results to the parameters inferred from the quark star candidate RXJ1856.5-3754. The compact star has small radiation radius, 3.8-8.2 km as mentioned in §1. This information provides one constraint between the mass $M$ and
radius $R$. Assuming the canonical value $B^{1/4} = 155$ MeV, we have $M = 0.068M_\odot$, $R = 3.8$ km, for $R_\infty = 4.0$ km and $M = 0.46M_\odot$, $R = 7.2$ km, for $R_\infty = 8.0$ km. Corresponding frequencies and decay times are marked as 'a' and 'b' in Figs. 3 and 4. Expected frequency and decay time should be within these points. These values are shifted to $M = 0.12M_\odot$, $R = 3.8$ km, for $R_\infty = 4.0$ km, and $M = 0.70M_\odot$, and $R = 6.6$ km, for $R_\infty = 8.0$ km, if $B^{1/4} = 180$ MeV. These points are labeled as 'a*' and 'b*' in Figs. 3 and 4. From these figures, we can discriminate the quark stars from neutron stars by the frequency and decay time of their oscillations. Suppose that we have the information about $\nu = \sigma R/2\pi$ and $\tau = 1/\sigma_I$ at the same time. As for a definite example, we assume the frequency $\nu \approx 1.7$ kHz. Since the oscillation frequency determines the bag constant $B$ in the self-bound quark star model, we have $B^{1/4} \approx 155$ MeV. Alternatively some neutron star models with $GM/Rc^2 \geq 0.1$ are also possible from Fig. 3. In other words, the average density is inferred as 1-2 $\rho_n$. The decay times are quite different as seen from Fig. 4. The decay times corresponding to the neutron stars are 0.1-0.8 s, while the longer range such as 1-10 s is possible for the quark stars. This is a speculation of the possibility and a detailed argument requires further study.
§ 5. Concluding remarks

In this paper, we have presented suggestive results for the pulsations associated with gravitational radiation. The gravitational waves carry key information of the sources. We can in principle use them as probes of compact stars. The oscillation frequency scales with the average density of a star \( \nu \propto \sqrt{M/R^3} \), since the f-mode has no radial node. The decay time due to gravitational radiation significantly depends on the relativistic factor \( M/R \). We may utilize these properties to discriminate self-bound quark stars from gravitationally bound neutron stars. If self-bound quark stars are realized in less relativistic regime, the oscillation frequency is high enough due to high density, but decay time is longer than that of neutron star. In that case, the mass and size are quite different in the equilibrium sequences of two compact stars. It is clear that present self-bound quark model is too crude like the bag model. The model is used to calculate explicitly, but the resultant properties of the oscillations roughly depend on the macroscopic values such as the mass and radius. We therefore expect that such discriminated properties should be involved in any detailed models.

Finally, we will comment on the astrophysical relevance of the quadrupole f-mode oscillation. The driving mechanism like Cepheid variables is not yet known, so that the persistent oscillations would not be observed. Rather, abrupt changes of the structure like the formation\(^5\) and/or some kinds of bursts may be relevant. The origin and the nature of them are also unclear at moment. However, it is evident that the gravitational wave emission is crucial in such dynamical process. The most efficient mode is the quadrupole \((l = 2)\) f-mode, which was calculated here. We should await for the fully relativistic numerical calculation to simulate the violent events, but the oscillation frequency may be used even for checking the constructing numerical code\(^8\).

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