1. Editor’s note

The Third Workshop on Coverings, Selections, and Games in Topology is just around the corner. Visit
http://www.pmf.ni.ac.yu/spm2007/index.html

for details. We are looking forward to this opportunity to meet old and new friends and colleagues, and to exchange some new results and ideas.

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2. Research announcements

2.1. On the Pytkeev property in spaces of continuous functions (II). We prove that for each Polish space $X$, the space $C(X)$ of continuous real-valued functions on $X$ satisfies a strong version of the Pytkeev property, if endowed with the compact-open topology. (This shows that whereas it need not be metrizable, it is “very close” to that.)

We also consider the Pytkeev property in the case where $C(X)$ is endowed with the topology of pointwise convergence.

http://arxiv.org/math/math.GN/0701249

Boaz Tsaban and Lyubomyr Zdomskyy

2.2. Partial order embeddings with convex range. A careful study is made of embeddings of posets which have a convex range. We observe that such embeddings share nice properties with the homomorphisms of more restrictive categories; for example, we show that every order embedding between two lattices with convex range is a continuous lattice homomorphism. A number of posets are considered; for example, we prove that every product order embedding $\sigma : \mathbb{N}^\mathbb{N} \to \mathbb{N}^\mathbb{N}$ with convex range is of the form

\[
\sigma(x)(n) = \left( (x \circ g_\sigma) + y_\sigma \right)(n) \quad \text{if } n \in K_\sigma,
\]

and $\sigma(x)(n) = y_\sigma(n)$ otherwise, for all $x \in \mathbb{N}^\mathbb{N}$, where $K_\sigma \subseteq \mathbb{N}$, $g_\sigma : K_\sigma \to \mathbb{N}$ is a bijection and $y_\sigma \in \mathbb{N}^\mathbb{N}$. The most complex poset examined here is the quotient of the lattice of Baire measurable functions, with codomain of the form $\mathbb{N}^I$ for some index set $I$, modulo equality on a comeager subset of the domain, with its ‘natural’ ordering.

http://homepage.univie.ac.at/James.Hirschorn/research/embeddings/embeddings.html

James Hirschorn

2.3. Resolvability vs. almost resolvability. A space $X$ is $\kappa$-resolvable (resp. almost $\kappa$-resolvable) if it contains $\kappa$ dense sets that are pairwise disjoint (resp. almost disjoint over the ideal of nowhere dense subsets of $X$). Answering a problem raised by Juhasz, Soukup, and Szentmiklossy, and improving a consistency result of Comfort and Hu, we prove, in ZFC, that for every infinite cardinal $\kappa$ there is an almost $2^\kappa$-resolvable but not $\mathfrak{N}_1$-resolvable space of dispersion character $\kappa$.

http://arxiv.org/math/math.GN/0702296
2.4. **Lindelöf spaces of singular density.** A cardinal \( \lambda \) is called omega-inaccessible if for all \( \mu < \lambda \) we have \( \mu^\omega < \lambda \). We show that for every \( \omega \)-inaccessible cardinal \( \lambda \) there is a CCC (hence cardinality and cofinality preserving) forcing that adds a hereditarily Lindelöf regular space of density \( \lambda \). This extends an analogous earlier result of ours that only worked for regular \( \lambda \).

http://arxiv.org/math/math.LO/0702295

Istvan Juhasz and Saharon Shelah

2.5. **\( P(\omega)/\text{fin} \) and projections in the Calkin algebra.** We investigate the set-theoretic properties of the lattice of projections in the Calkin algebra of a separable infinite-dimensional Hilbert space in relation to those of the Boolean algebra \( P(\omega)/\text{fin} \), which is isomorphic to the sublattice of diagonal projections. In particular, we prove some basic consistency results about the possible cofinalities of well-ordered sequences of projections and the possible cardinalities of sets of mutually orthogonal projections that are analogous to well-known results about \( P(\omega)/\text{fin} \).

http://arxiv.org/math/math.LO/0702309

Eric Wofsey

2.6. **Everywhere meagre and everywhere null sets.** We introduce new classes of small subsets of the reals, having natural combinatorial definitions, namely everywhere meagre and everywhere null sets, which lie between the \( \sigma \)-ideal \( \mathcal{I}_0 \), introduced by Repicky in [M. Repicky, *Mycielski ideal and the perfect set theorem*, Proc. AMS 132 (2004), 2141–2150] and the \( \sigma \)-ideal \( \mathcal{B}_2 \), introduced by Roslanowski in [A. Roslanowski, *On game ideals*, Colloq. Math. 59 (1990), 159–168]. We investigate properties of these sets, in particular we show that these classes are closed under taking products and projections. We also prove several relations between these classes and other well-known classes of small subsets of the reals.

http://www.math.uni.wroc.pl/~kraszew/files/papers.html

Jan Kraszewski

2.7. **Transversals for strongly almost disjoint families.** For a family of sets \( A \), and a set \( X \), \( X \) is said to be a transversal of \( A \) if \( X \subseteq \bigcup A \) and \( |a \cap X| = 1 \) for each \( a \in A \). \( X \) is said to be a Bernstein set for \( A \) if \( \emptyset \neq a \cap X \neq a \) for each \( a \in A \). When an almost disjoint family admits a set like a transversal or Bernstein set was first studied in [P. Erdős and A. Hajnal *On a property of families of sets*, Acta Math. Acad. Sci. Hung., 12 (1961) 87–124]. In this note we introduce the following notion: a family of sets \( A \) is said to admit a \( \sigma \)-transversal if \( A \) can be written as \( A = \bigcup \{ A_n : n \in \omega \} \) such that each \( A_n \) admits a transversal. We study the question when an almost disjoint family admits a \( \sigma \)-transversals and related questions.

http://www.ams.org/journal-getitem?pii=S0002-9939-07-08714-X

Paul J. Szeptycki
2.8. **A family of covering properties for forcing axioms and strongly compact cardinals.** This paper presents the main results in my Ph.D. thesis. Several proofs of SCH are presented introducing a family of covering properties which implies both SCH and the failure of various forms of square. These covering properties are also applied to investigate models of strongly compact cardinals or of strong forcing axioms like MM or PFA.

http://arxiv.org/math/math.LO/0703091
Matteo Viale

2.9. **Splitting families and forcing.** According to [M.S. Kurilić, Cohen-stable families of subsets of the integers, J. Symbolic Logic 66 (2001) 257–270], adding a Cohen real destroys a splitting family $S$ on $\mathbb{N}$ if and only if $S$ is isomorphic to a splitting family on the set of rationals, $\mathbb{Q}$, whose elements have nowhere dense boundaries. Consequently, $|S| < \text{cov}(\mathcal{M})$ implies the Cohen-indestructibility of $S$. Using the methods developed in [J. Brendle, S. Yatabe, Forcing indestructibility of MAD families, Ann. Pure Appl. Logic 132 (2005) 271–312], the stability of splitting families in several forcing extensions is characterized in a similar way (roughly speaking, destructible families have members with “small generalized boundaries” in the space of the reals). Also, it is proved that a splitting family is preserved by the Sacks (respectively: Miller, Laver) forcing if and only if it is preserved by some forcing which adds a new (respectively: an unbounded, a dominating) real. The corresponding hierarchy of splitting families is investigated.

http://dx.doi.org/10.1016/j.apal.2006.08.002
Miloš S. Kurilić

2.10. **A Sacks Real out of Nowhere.** There is a proper countable support iteration of length $\omega$ adding no new reals at finite stages and adding a Sacks real in the limit.

http://arxiv.org/math/math.LO/0703302
Jakob Kellner, Saharon Shelah

2.11. **Reflection principle characterizing groups in which unconditionally closed sets are algebraic.** We give a necessary and sufficient condition, in terms of a certain reflection principle, for every unconditionally closed subset of a group $G$ to be algebraic. As a corollary, we prove that this is always the case when $G$ is a direct product of an Abelian group with a direct product (sometimes also called a direct sum) of a family of countable groups. This is the widest class of groups known to date where the answer to the 63 years old problem of Markov turns out to be positive. We also prove that whether every unconditionally closed subset of $G$ is algebraic or not is completely determined by countable subgroups of $G$.

http://arxiv.org/math/math.GR/0703304
Dikran Dikranjan, Dmitri Shakhmatov
2.12. **Unconditionally \( \tau \)-closed and \( \tau \)-algebraic sets in groups.** Families of unconditionally \( \tau \)-closed and \( \tau \)-algebraic sets in a group are defined. It is proved that, if any element of a group \( G \) has at most \( \tau \) conjugates, then these families coincide in \( G \). It follows that any unconditionally closed set is algebraic in a group in which every element has at most countably many conjugates.

http://arxiv.org/math/math.GR/0703397
Ol’ga V. Sipacheva

2.13. **Stratifiability of \( C_k(X) \) for a class of separable metrizable \( X \).** Let \( X \) be a separable metrizable space. It is proved that the space \( C_k(X) \) of all continuous real-valued functions on \( X \) with the compact-open topology is stratifiable if and only if \( X \) is Polish.

http://erezn.andante.ru/topology/sckx3.ps
E. A. Reznichenko

2.14. **Dissipated Compacta.** The dissipated spaces form a class of compacta which contains both the scattered compacta and the compact LOTSes (linearly ordered topological spaces), and a number of theorems true for these latter two classes are true more generally for the dissipated spaces. For example, every regular Borel measure on a dissipated space is separable.

A product of two compact LOTSes is usually not dissipated, but it may satisfy a weakening of that property. In fact, the degree of dissipation of a space can be used to distinguish topologically a product of \( n \) LOTSes from a product of \( m \) LOTSes.

http://arxiv.org/math/math.GN/0703429
Kenneth Kunen

2.15. **A note on strong negative partition relations.** We analyze a natural function definable from a scale at a singular cardinal, and using this function we are able to obtain quite strong negative square-brackets partition relations at successors of singular cardinals. The proof of our main result makes use of club-guessing, and as a corollary we obtain a fairly easy proof of a difficult result of Shelah connecting weak saturation of a certain club-guessing ideal with strong failures of square-brackets partition relations. We then investigate the strength of weak saturation of such ideals and obtain some results on stationary reflection.

http://arxiv.org/math/math.LO/0703647
Todd Eisworth

2.16. **First countable spaces without point-countable \( \pi \)-base.** We answer several questions of V. Tkačuk from [Point-countable \( \pi \)-bases in first countable and similar spaces, Fund. Math. 186 (2005), pp. 55–69.] by showing that

1. There is a ZFC example of a first countable, 0-dimensional Hausdorff space with no point-countable \( \pi \)-base (in fact, the order of any \( \pi \)-base of the space is at least \( \aleph_\omega \)).
(2) If there is a $\kappa$-Suslin line then there is a first countable GO space of cardinality $\kappa^+$ in which the order of any $\pi$-base is at least $\kappa$;

(3) It is consistent to have a first countable, hereditarily Lindel"of regular space having uncountable $\pi$-weight and $\omega_1$ as a caliber (of course, such a space cannot have a point-countable $\pi$-base).

http://arxiv.org/math/math.GN/0703728
Istvan Juhasz, Lajos Soukup, and Zoltan Szentmiklossy

2.17. Less than continuum many translates of a compact nullset may cover any infinite profinite group. We show that it is consistent with the axioms of set theory that every infinite profinite group $G$ possesses a closed subset $X$ of Haar measure zero such that less than continuum many translates of $X$ cover $G$. This answers a question of Elekes and Toth and by their work settles the problem for all infinite compact topological groups.

http://arxiv.org/math/math.GR/0703726
Miklos Abert

2.18. Projective $\pi$-character bounds the order of a $\pi$-base. All spaces below are Tychonov. We define the projective $\pi$-character $p(X)$ of a space $X$ as the supremum of the values $\pi\chi(Y)$ where $Y$ ranges over all continuous images of $X$. Our main result says that every space $X$ has a $\pi$-base whose order is at most $p(X)$, that is every point in $X$ is contained in at most $p(X)$-many members of the $\pi$-base. Since $p(X)$ is at most $t(X)$ for compact $X$, this provides a significant generalization of a celebrated result of Shapirovskii.

http://arxiv.org/math/math.GN/0703835
Istvan Juhasz and Zoltan Szentmiklossy

3. Problem of the Issue

We recall from [2] that a topological space $X$ is a $QN$-space (resp $wQN$-space), if every pointwise convergent to 0 sequence $(f_n : X \to \mathbb{R})_{n \in \mathbb{N}}$ of continuous functions (contains a subsequence which) converges quasi-normally, which means that there exists a convergent to 0 sequence $(\epsilon_n)_{n \in \mathbb{N}}$ of positive reals, such that for every $x \in X$ the inequality $|f_n(x)| < \epsilon_n$ holds for all but finitely many $n \in \mathbb{N}$. Of course, every QN-space is a wQN-space. In addition, every wQN-space has the Hurewicz property $U_{fin}(\mathcal{O}, \Gamma)$, see [2, Corollary 2.2]. It is also known [6, 1] that $X$ is a wQN-space if, and only if, it satisfies the selection hypothesis $S_1(C_{\Gamma}, C_{\Gamma})$, and being a QN-spaces is equivalent [1] to a certain selection principle stronger than $S_1(\Gamma, \Gamma)$. Combining these results we conclude that the above properties are related as follows:

$$QN \Rightarrow S_1(\Gamma, \Gamma) \Rightarrow S_1(C_{\Gamma}, C_{\Gamma}) \Rightarrow U_{fin}(\mathcal{O}, \Gamma)$$
Neither the first nor the third implication can be reversed in ZFC, even for sets of reals without perfect subsets, see [2] and [5] respectively. The question whether the second implication can be reversed was posed in [7] and is still open.

Passing to hereditary versions of these properties, we have the following implications:

\[ QN \subseteq [4] hQN \Rightarrow hS_1(Γ, Γ) \subseteq [3] hS_1(C_Γ, C_Γ) \Rightarrow hU_{fin}(O, Γ). \]

It is open whether any of the two displayed implications can be reversed. The question whether every hereditarily Hurewicz space a QN-space is particularly interesting.

As it was recently showed in [9], a space \( X \) is a QN-space if, and only if, it has the property \( S_1(\mathcal{B}_Γ, \mathcal{B}_Γ) \), which is equivalent [8] to the Hurewicz property applied to the family of all countable Borel covers of \( X \), and to the property that all Borel images of \( X \) in \( \mathbb{N}^\mathbb{N} \) are bounded [8]. In light of this equivalence the above question can be reformulated as follows:

**Problem 3.1. Does every hereditarily Hurewicz space satisfy** \( S_1(\mathcal{B}_Γ, \mathcal{B}_Γ) \)?

*Lyubomyr Zdomskyy*

**References**

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[8] M. Scheepers, B. Tsaban, The combinatorics of Borel covers, Topology and its Applications 121 (2002), 357–382.

[9] B. Tsaban, L. Zdomskyy, Hereditarily Hurewicz spaces, in preparation.
4. Unsolved problems from earlier issues

Issue 1. Is \((\Omega^\Gamma) = (\Omega^\Gamma)\)?

Issue 2. Is \(\cup_{\text{fin}}(\mathcal{O}, \Omega) = S_{\text{fin}}(\Gamma, \Omega)\)? And if not, does \(\cup_{\text{fin}}(\mathcal{O}, \Gamma) \) imply \(S_{\text{fin}}(\Gamma, \Omega)\)?

Issue 4. Does \(S_1(\Omega, \Gamma)\) imply \(S_{\text{fin}}(\Gamma, \Gamma)\)?

Issue 5. Is \(p = p^*\)? (See the definition of \(p^*\) in that issue.)

Issue 6. Does there exist (in ZFC) an uncountable set satisfying \(S_1(\mathcal{B}_\Gamma, \mathcal{B})\)?

Issue 8. Does \(X \notin \text{NON}(\mathcal{M})\) and \(Y \notin \text{D}\) imply that \(X \cup Y \notin \text{COF}(\mathcal{M})\)?

Issue 9 (CH). Is \(\text{Split}(\Lambda, \Lambda)\) preserved under finite unions?

Issue 10. Is \(\text{cov}(\mathcal{M}) = \text{o\#}\)? (See the definition of \(\text{o\#}\) in that issue.)

Issue 11. Does \(S_1(\Gamma, \Gamma)\) always contain an element of cardinality \(b\)?

Issue 12. Could there be a Baire metric space \(M\) of weight \(\aleph_1\) and a partition \(\mathcal{U}\) of \(M\) into \(\aleph_1\) meager sets where for each \(\mathcal{U}' \subseteq \mathcal{U}\), \(\bigcup \mathcal{U}'\) has the Baire property in \(M\)?

Issue 14. Does there exist (in ZFC) a set of reals \(X\) of cardinality \(d\) such that all finite powers of \(X\) have Menger’s property \(S_{\text{fin}}(\mathcal{O}, \mathcal{O})\)?

Issue 15. Can a Borel non-\(\sigma\)-compact group be generated by a Hurewicz subspace?

Issue 16 (MA). Is there an uncountable \(X \subseteq \mathbb{R}\) satisfying \(S_1(\mathcal{B}_\Omega, \mathcal{B}_\Gamma)\)?

Issue 17 (CH). Is there a totally imperfect \(X\) satisfying \(\cup_{\text{fin}}(\mathcal{O}, \Gamma)\) that can be mapped continuously onto \(\{0, 1\}^\mathbb{N}\)?

Issue 18 (CH). Is there a Hurewicz \(X\) such that \(X^2\) is Menger but not Hurewicz?

Issue 19. Does the Pytkeev property of \(C_p(X)\) imply the Menger property of \(X\)?

Issue 20. Does every hereditarily Hurewicz space satisfy \(S_1(\mathcal{B}_\Gamma, \mathcal{B}_\Gamma)\)?

Previous issues. The previous issues of this bulletin, which contain general information (first issue), basic definitions, research announcements, and open problems (all issues) are available online, at http://front.math.ucdavis.edu/search?&t=%22SPM+Bulletin%22

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