Some Novel Contributions to Radiative B Decay in Supersymmetry without R-parity

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We present a systematic analysis at the leading log order of the influence of combination of bilinear and trilinear R-parity violating couplings on the decay $b \to s + \gamma$. Such contributions have never been explored in the context of $b \to s + \gamma$ decay. We show that influence of charged-slepton-Higgs mixing mediated loops can dominate the SM and MSSM contributions and hence can provide strong bounds on the combination of bilinear-trilinear $R$-parity violating couplings. Such contributions are also enhanced by large $\tan \beta$. With substantially extended basis of operators (28 operators), we provide illustrative analytical formulae of the major contributions to complement our complete numerical results which demonstrate the importance of QCD running effects.

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Introduction.— The minimal supersymmetric standard model (MSSM) has been the most popular candidate theory for physics beyond the Standard Model (SM) for the last couple of decades. With the recent accumulation of evidence for neutrino oscillations, it is clear that the lepton number conserving MSSM has to be amended. The simplest option is then to give up imposing $R$-parity and hence admit all gauge invariant terms in a (generic) supersymmetric SM. Other alternatives include incorporating any particular neutrino sector model such as the seesaw mechanism with extra gauge singlet superfields. We focus on the first option [1, 2]. The model has the special merit that the parameters that give rise to neutrino masses and mixings also have interesting phenomenological consequence in the quark and charged lepton sectors. Here in this letter, we report on some of the novel contributions to the $b \to s + \gamma$ decay from the model.

Within the SM, the flavor sector still needs more scrutiny from theory as well as experiments. In particular, flavor changing neutral current (FCNC) processes are widely considered to be the window of the physics beyond SM. Among the processes, $B \to X_s + \gamma$ is a particularly attractive candidate. The most up-to-date SM prediction [3] gives

$$Br[B \to X_s + \gamma \, (E_{\gamma} > 1.6 \text{ GeV})]_{\text{SM}} = (3.57 \pm 0.30) \times 10^{-4},$$

while the experimental number (world-average) is [4]

$$Br[B \to X_s + \gamma \, (E_{\gamma} > 1.6 \text{ GeV})]_{\text{EXP}} = (3.34 \pm 0.38) \times 10^{-4}.\tag{2}$$

It clearly leaves not much room for new physics contributions. Hence, can be used to obtain stringent constraints on flavor parameters of various new physics models.

There have been some studies on the process within the general framework of $R$-parity violation [5]. Ref. [6], fails to consider the additional 18 four-quark operators which, in fact, give the dominant contribution in most of the cases. The more recent work of ref. [7] has considered a complete operator basis. However, we find their formula for Wilson coefficient incomplete. In fact, the particular type of contributions — namely, one from a combination of a bilinear and a trilinear $R$-parity violating (RPV) parameters, we focus on here, has not been studied in any detail before.

It is not possible for us to give much analytical details of our study here in this short letter. We will only outline the major features of the full analysis given in a parallel report [7], to which interested readers are referred. We adopt an optimal phenomenological parametrization of the full model Lagrangian, dubbed the single-VEV parametrization (SVP), first explicitly advocated in ref. [8]. It is essentially about choosing a basis for Higgs and lepton superfields in which all the “sneutrino” VEVs vanish. The formulation gives the simplest expressions for all the mass matrices of the fermions and scalars without a priori assumption on the admissible form of $R$-parity violation. In particular, all the RPV effects to the fermion mass matrices are characterized by the three bilinear parameter $\mu_i$’s. Working under the formulation, it has been pointed out in ref. [9] that there are interesting contributions to the down squark and charged slepton mass matrices of the form ($\mu_i \lambda_{i\ell}$) and ($\mu_i \chi_{i\ell}$). These give explicit indications of the existence of bilinear-trilinear type contributions to fermion dipole moments at 1-loop order, including the transitional moment term to be identified with the $b \to s + \gamma$ decay. Detailed analytical and numerical studies have been performed on the case of neutron electric dipole moment [10] and $\mu \to e + \gamma$ [11]. Here, we report on the more difficult calculation of $B \to X_s + \gamma$. Background details on the model and the various mass matrices are given in ref. [1]. Based on the latter results, we implement our (1-loop) calculations using mass eigenstate expressions [7], hence free from the commonly adopted mass-insertion approximation. While a trilinear RPV parameter gives a coupling, a bilinear parameter now contributes only through mass mixing matrix elements characterizing the effective couplings of the mass eigenstate running inside the loop. The $\mu_i$’s are involved in fermion, as well as scalar mixings [1]. There are also the corresponding soft bilinear $B_i$ parameters involved only in scalar mixings [1]. Combinations of $\mu_i$’s and $B_i$’s with the trilinear $\chi_{i\ell}$ parameters are our major focus.

The Effective Hamiltonian Approach.— The partonic transition $b \to s + \gamma$ is described by the magnetic pen-
The branching fraction for $Br(b \to s + \gamma)$ is expressed through the semi-leptonic decay $b \to u c e \bar{v}$ so that the large bottom mass dependence ($\sim m_b^2$) and uncertainties in CKM elements cancel out.

$$Br(b \to s + \gamma) = \frac{\Gamma(b \to s + \gamma)}{\Gamma(b \to u|c e \bar{v}_e)} = Br_{\exp}(b \to u|c e \bar{v}_e),$$

where $Br_{\exp}(b \to u|c e \bar{v}_e) = 10.5\%$ and

$$\Gamma(b \to s \gamma) = \frac{\alpha m_b^5}{64 \pi^4} \left( |C_{1}^{\text{eff}}(\mu_b)|^2 + |\tilde{C}_{7}^{\text{eff}}(\mu_b)|^2 \right).$$

Note that we have also to include RPV contributions to the semi-leptonic rate for consistency.

**Analytical Appraisal of the Results.** — There are three kinds of bilinear RPV parameters, $\mu_i$, $B_i$, and $\tilde{m}_{ij\epsilon}^2$ related by the tadpole equation constraints.[1, 9] Without loss of generality, we choose $\mu_i$ and $B_i$ independent. The influence of a $|B_i|$ (or $|\mu_i|$), in conjunction with $|\chi_{ijk}^{\ell}|$ is felt through the lepton number violating mass mixings in (s)leptonic propagators of tree and penguin diagrams. $|B_i|$ insertions may have much stronger influence than the $|\lambda_i|$ as the former case is inversely proportional to light slepton mass-squared whereas the latter ones come with the inverse of heavier squark mass-squared. We focus our discussion here on the $|B_i|$ insertions to provide an analytical appraisal of the numerical results. The case for $\mu_i$ can be appreciated in a similar fashion.[12]

There are two kinds of $B_i \chi_i$ combinations that contribute to $b \to s + \gamma$ at 1-loop: (a) $B_i \chi_{ij2}^\epsilon$, and (b) $B_i \chi_{ij3}^\tau$. These involve quark-scalar loop diagrams. Case (a) leads to the $b_i \to s_0$ transition (where SM and MSSM contribution is extremely suppressed) whereas case (b) leads to SM-like $b_0 \to s_0$ transition. For the purpose of illustration, we will assume a degenerate sleptons spectrum and take the sleptonic index $i = 3$ as a representative. The $j = 3$ contributions for case (a) [(b)] with both sneutrino-Higgs mixings and charged-slepton-Higgs mixings are easy to appreciate. For the $j$ values, the charged loop contributions are still possible by invoking CKM mixings. Consider the contribution of case (a) with $|B_3^{\epsilon} \chi_{3|32}^{\epsilon}|$ to the Wilson coefficient $\tilde{C}_7$, for instance. Through the extraction of the bilinear mass mixing effect under a perturbative diagonalization of the mass matrices,[1] we obtain

$$\tilde{C}_7^{\epsilon} \approx -\frac{|V_{\text{CKM}}^{\epsilon b}|^2 |B_3^{\epsilon} \chi_{3|32}^{\epsilon}|}{M_2^2} \left\{ y_b \tan \beta \left[ F_2 \left( \frac{m_T^2}{M_2^2} \right) + Q_u F_1 \left( \frac{m_T^2}{M_2^2} \right) \right] + \frac{y_b m_l}{m_b} \left[ F_3 \left( \frac{m_T^2}{M_2^2} \right) + Q_u F_1 \left( \frac{m_T^2}{M_2^2} \right) \right] \right\}$$

The chirality-flip counterparts $\tilde{Q}_7$ and $\tilde{Q}_8$ of the standard (chromo)magnetic penguins $Q_7$ and $Q_8$, and a whole list of 18 new relevant four-quark operator of current-current type to be given as:

$$\tilde{Q}_{9-11} = (\bar{s}_L \gamma^\mu b_{iL}) (\bar{q}_{Ri} \gamma_\mu q_{Ra}) ; q = d, s, b;$$

$$\tilde{Q}_{3,4} = (\bar{s}_L \gamma^\mu b_{Ra,\beta}) \sum_{i = u, c, d, s, b} (\bar{q}_{Ri} \gamma_\mu q_{Ra,\alpha}) ;$$

$$\tilde{Q}_{5,6} = (\bar{s}_L \gamma^\mu b_{Ra,\beta}) \sum_{i = u, c, d, s, b} (\bar{q}_{Li} \gamma_\mu q_{Li,\alpha}) ;$$

$$\tilde{Q}_{9-13} = (\bar{s}_L \gamma^\mu b_{Ri}) (\bar{q}_{Li} \gamma_\mu q_{La}) ; q = d, s, b, u, c(6)$$

and six more operators from $\chi''$ couplings,[8] we skip here for brevity. The interplay among the full set of 28 operators is what makes the analysis complicated. The effect of the QCD corrections proved to be very significant even for the RPV parts.

We skip here the details involved in the evaluation of the various effective Wilson coefficients for the decay rate of $b \to s + \gamma$ and give only the numerical results from our leading log (LL) order analysis.[7]:

$$\tilde{C}_7^{\epsilon} (m_b) = -0.351 C_7^{\epsilon} (M_2^2) + 0.665 C_7^{\epsilon} (M_2^2) + 0.093 C_8^{\epsilon} (M_2^2) - 0.198 C_9^{\epsilon} (M_2^2) - 0.198 C_{10}^{\epsilon} (M_2^2) - 0.178 C_{11}^{\epsilon} (M_2^2),$$

$$\tilde{C}_7^{\epsilon} (m_b) = 0.381 C_1^{\epsilon} (M_2^2) + 0.665 C_7^{\epsilon} (M_2^2) + 0.093 C_8^{\epsilon} (M_2^2) - 0.198 C_9^{\epsilon} (M_2^2) - 0.198 C_{10}^{\epsilon} (M_2^2) - 0.178 C_{11}^{\epsilon} (M_2^2) + 0.510 C_{12}^{\epsilon} (M_2^2) + 0.510 C_{13}^{\epsilon} (M_2^2) + 0.381 C_{14}^{\epsilon} (M_2^2) - 0.213 C_{16}^{\epsilon} (M_2^2).$$
\[ C^0_{\tilde{\tau}} \approx -\frac{2Q_d y_b |B_3\lambda_{3\tilde{\tau}}^s| \tan \beta}{M^2} \left( \frac{m^2_s}{M^2} \right)^2 \]  

for the charged and neutral scalar loop, respectively. In the above equations, proportionality to \( \tan \beta \) shows the importance of these contributions in the large \( \tan \beta \) limit. The \( M^2, \bar{M}^2, \bar{M}^2 \), are all scalar (slepion/Higgs) mass parameters. The term proportional to \( y_b \) above has chirality flip into the loop. Thinking in terms of the electroweak states, it is easy to appreciate that the loop diagram giving a corresponding term for \( \tilde{C}_{\tilde{\tau}}^0 \) (cf. involving \( \Delta m_{3\tilde{\tau}}^{\nu}, \Delta m_{3\tilde{\tau}}^{\nu,\nu} \)) requires a Majorana-like scalar mass insertion, which has to arrive from other RPV couplings. In the limit of perfect mass degeneracy between the scalar and pseudoscalar part (with no mixing) of multiplet, it vanishes. Dropping this smaller contribution, together with the difference among the Inami-Lim loop functions, which has to arrive from other RPV couplings. Note that we always keep the results on a specific combination of RPV parameters. The term proportional to \( y_b \) above has much of an effect on the qualitative dependence of the results on a specific combination of RPV parameters. Note that we always keep \( R \)-parity conserving flavor violating squark and slepton mixings vanishing, to focus on the RPV effects. We take non-vanishing values for relevant combinations of a bilinear and a trilinear RPV parameters one at a time, and stick to real values only. Our model choice is (with all mass dimensions given in GeV): squark masses 300, down-type Higgs mass 300, \( \mu_0 = -300 \) sleptons mass 150 and gaugino mass \( M_2 = 200 \) (with \( M_1 = 0.5 M_2 \) and \( M_3 = 3.5 M_2 \)), \( \tan \beta = 37 \) and \( A \) parameter 300. The mass for \( H_u \) and soft bilinear parameter \( B_0 \) are determined by electroweak symmetry breaking conditions which are modified in the presence of RPV parameters \[ \textbf{[12]}. \]

Under the scenario discussed, we impose the experimental number to obtain bounds for each combination of RPV parameters independently (given in Table I). We address here a couple of cases in a bit more detail. Consider, for instance, the case (b) combination \( |B_3\lambda_{3\tilde{\tau}}^s| \). We obtain a bound of \( 5.0 \times 10^{-5} \), when normalized by a factor of \( \mu_\tau^3 \). Since this is a \( b \to s \) transition, the RPV contribution interferes with the SM as well as the MSSM contribution. In Fig. \[ \textbf{[1]} \] we have plotted the relevant Wilson coefficients. Over and above the loop contributions we see that there are contributions coming from four-quark operator with Wilson coefficients \( C_{11} \approx 1 \) (with \( y_b \)) which is stronger than the other two four-quark quark coefficients \( C_{10,13} \approx y_s \) (not shown in the graph). Since the neutral scalar loop contribution is proportional to the loop function \( F_1 \) (which is of order \( .01 \)), it is suppressed compared to current current contributions. Also here the charged scalar contribution comes only with chirality flip inside the loop and has a CKM suppression. So the current-current is dominant. It has a more subtle role to play when one writes the regulariziation scheme-independent \( C_{\tilde{\tau}}^{\text{eff}} = C_{\tilde{\tau}} - C_{11} \) at scale \( M_W \). Due to dominant and negative sign chargino contribution (\( A_{\tau}, \mu_\tau < 0 \)), the positive sign \( C_{11} \) interferes constructively with \( C_{\tilde{\tau}} \) and enhances the rate.

The case (b) combination \( |B_3\lambda_{3\tilde{\tau}}^s| \) is a different story, as it leads to \( b \to s \) transition and hence RPV does not interfere with SM or MSSM contribution. This leads to a

![FIG. 1: Various Wilson coefficients versus |B_3\lambda_{3\tilde{\tau}}^s|. ‘+’ sign stands for MSSM chargino contribution, ‘×’ stands for the MSSM charged Higgs contribution, ‘*’ stands for the neutralino contribution, all contributing to \( C_{\tilde{\tau}} \). ‘Empty square’ stands for \( C_{11}(M_W) \), ‘filled square’ for \( C_{\tilde{\tau}}(M_W) \) and ‘empty circle’ for \( C_{\tilde{\tau}}(m_\tilde{\tau}) \).](image1)

![FIG. 2: Various Wilson coefficients versus |B_3\lambda_{3\tilde{\tau}}^s|. ‘+’ sign stands for charged slepton contribution, ‘×’ stands for sneutrino contribution, ‘*’ stands for MSSM chargino contribution, all contributing to \( C_{\tilde{\tau}} \). ‘Empty square’ stands for MSSM charged Higgs contribution, ‘filled square’ for \( C_{11}(M_W) \), ‘empty circle’ for \( C_{\tilde{\tau}}(M_W) \), ‘filled circle’ for \( C_{\tilde{\tau}}(M_W) \), ‘empty triangle’ for \( C_{\tilde{\tau}}(m_\tilde{\tau}) \), and ‘filled triangle’ for \( C_{\tilde{\tau}}(m_\tilde{\tau}) \).](image2)
Again, the current-current contribution due to £C11 has a very subtle role to play here. The regularization scheme-independent effective Wilson coefficient $C_{\gamma}^{\text{eff}} = C_{\gamma} - C_{11}$ at the scale $M_{\mu}$. The negative sign leads to cancellations and hence weakens the bound.

The influence of $|\mu_{i} \lambda_{ijk}^{\prime}|$ is less stronger than the $B_{i}$ insertions. The $\mu_{i}$ insertions affect the MSSM chargino and the neutralino type of diagrams by mixing them with the charged and the neutral leptons. Since such contributions are suppressed by the heavier squark masses, the influence is not as strong as slepton loops. However there exist gluino mediated loop diagrams with a flavour violating chirality flip in the down-squark propagator ($\propto |\mu_{i} \lambda_{ijk}^{\prime}|$) which gives non-negligible contributions and indeed lead to good bounds.

Conclusions. — To conclude we have systematically studied the influence of the combination of bilinear-trilinear RPV parameters on the decay $b \to s + \gamma$ analytically as well as numerically. Such a study has not been attempted before. We demonstrate the plausible dominance of the RPV contributions over conventional SM and MSSM contributions in some parameter space regions. These contributions are enhanced by large $\tan\beta$. It is shown that charged-slepton-Higgs mixing mediated loop typically dominates over the sneutrino-Higgs mixing mediated loop. Our study has consistently incorporated all the QCD corrections at the leading log order, with a whole list of extra operators and their Wilson coefficients arising from RPV couplings. We have shown that, through the formulation of scheme-independent effective Wilson coefficients, the new current-current operators can considerably influence the decay rate. Under a typical and compatible model parameter choice, we obtain strong bounds on several combinations of RPV parameters. Bounds on such bilinear-trilinear parameter combinations are not available before. Various $b \to s + \gamma$ contributions over different parameter space regions may complicate the story and partial cancellation among them are a likely possibility. And our leading log calculation bears relatively large certainty. Nevertheless, the bounds show values of the RPV parameter combinations that will play a major role in endangering the compatibility of the theoretical $b \to s + \gamma$ result with the experimental limits. This interpretation of our results is quite robust.

Our analytical formulae include all RPV contributions at 1-loop level. Numerical study has also been performed on combinations of trilinear parameters. We quote here a few exciting bounds under a similar sparticle spectrum. For instance $|\lambda_{133}^{\prime} \cdot \lambda_{223}^{\prime}|$ for $i = 2, 3$ should be less than $1.6 \times 10^{-3}$ to be compared with rescaled existing bound of $2 \times 10^{-2}$.

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| Product | Our bound | Wilson Coef. |
|---------|-----------|--------------|
| $\mu_{i} \lambda_{ijk}$ | $5.0 \times 10^{-3}$ | $C_{7,8}, C_{7,8}, C_{11}, C_{10}, C_{13}$ |
| $\mu_{i} \lambda_{iij}$ | $7.4 \times 10^{-3}$ | $C_{7,8}, C_{7,8}, C_{10}, C_{11}$ |
| $\mu_{i} \lambda_{iij}$ | $2.3 \times 10^{-3}$ | $C_{7,8}$ |
| $\mu_{i} \lambda_{ijj}$ | $6.5 \times 10^{-2}$ | $C_{7,8}$ |
| $\mu_{i} \lambda_{ijj}$ | $8.0 \times 10^{-2}$ | $C_{7,8}$ |
| $\mu_{i} \lambda_{ijj}$ | $4.5 \times 10^{-2}$ | $C_{7,8}$ |
| $\mu_{i} \lambda_{ijj}$ | $2.2 \times 10^{-3}$ | $C_{7,8}$ |
| $\mu_{i} \lambda_{ijj}$ | $1.0 \times 10^{-2}$ | $C_{7,8}$ |
| $\mu_{i} \lambda_{ijj}$ | $8.0 \times 10^{-2}$ | $C_{7,8}$ |

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[13] See also a parallel discussion for the $\mu_{i}$ insertions fermion mixings in case of quark dipole moment given in Ref. [9].