Abstract

I sketch how long wavelength modes of the pion field can be amplified during the QCD phase transition. If nature had been kinder, and had made the pion mass significantly less than the critical temperature for the transition, then this phenomenon would have characterized the transition in thermal equilibrium. Instead, these long wavelength oscillations of the orientation of the chiral condensate can only arise out of equilibrium. There is a simple non-equilibrium mechanism, plausibly operational during heavy ion collisions, which naturally amplifies these oscillations. The characteristic signature of this phenomenon is large fluctuations in the ratio of the number of neutral pions to the total number of pions in regions of momentum space, that is in phase space in a detector. Detection in a heavy ion collision would imply an out of equilibrium chiral transition.

Shortly after the discovery that QCD is asymptotically free and that therefore quarks are weakly interacting at short distances, Collins and Perry noted that this means that at temperatures $T \gg \Lambda_{\text{QCD}}$, the theory describes a world of weakly interacting quarks and gluons very different from the familiar hadronic world. There are at least two qualitative differences between a plasma at $T \gg \Lambda_{\text{QCD}}$ and at $T \sim 0$. First, at low temperatures one has a plasma of hadrons, while at high temperatures the quarks and gluons are deconfined. Second, at low temperatures a $q\bar{q}$ condensate spontaneously breaks

---

1Talk given at the International Workshop on QCD Phase Transitions held in January, 1997 in Hirschegg, Austria.

2In this talk I will limit myself to discussing the physics of a QCD plasma with zero baryon number density. This should be a good approximation in the central rapidity regions of heavy ion collisions at RHIC energies and higher, where the number of pions per event is expected to be about two orders of magnitude larger than the baryon number per event.
chiral symmetry while at high temperature the interactions among quarks and anti-quarks are weak, no such condensate exists, and chiral symmetry is manifest.

In $SU(3)$ gauge theory with no quarks, there are analytic arguments \cite{3} confirmed by lattice simulation \cite{4} that the deconfinement transition is first order. While this conclusion presumably remains valid if all quarks are much heavier than $\sim \Lambda_{QCD}$, no deconfinement order parameter is known for $SU(N)$ gauge theory including dynamical quarks, and simulations with two light quarks \cite{5, 6} find that deconfinement occurs via a smooth crossover.

The presence of light quarks raises the possibility of a phase transition associated with the vanishing of the chiral order parameter. The Lagrangian for QCD with two massless quarks has a global $SU(2)_L \times SU(2)_R$ symmetry which, at low temperatures, is spontaneously broken to $SU(2)_{L+R}$ by a nonzero expectation value for the chiral order parameter, which can be written in terms of four real scalar fields $(\sigma, \vec{\pi})$ and the Pauli matrices according to

$$\langle \bar{q}_L q_R \rangle = \sigma \delta_{ij} + i \vec{\pi} \cdot \vec{\tau}_j .$$

In fact, the order parameter can be written as a four component scalar field

$$\phi^\alpha \equiv (\sigma, \vec{\pi}) ,$$

and the $SU(2)_L \times SU(2)_R$ transformations are simply $O(4)$ rotations. At low temperatures, $\langle \phi \rangle$ is nonzero. This picks a direction in $O(4)$ space defined to be the $\sigma$ direction, and spontaneously breaks the symmetry. The direction in which the expectation value points is defined as the $\sigma$ direction. Oscillations of $\phi$ in the $\vec{\pi}$ directions, that is oscillations of the orientation of the condensate, are massless Goldstone modes. As $T$ is increased, $\phi$ fluctuates more and more wildly until above some critical temperature $T_c$ the fluctuations are large enough that $\langle \phi \rangle = 0$ and the $O(4)$ symmetry is restored.

Since the order parameter appropriate for the chiral phase transition in QCD with two flavors of massless quarks has the symmetries of an $O(4)$ Heisenberg magnet, and since this model has a second order phase transition \cite{7} it is possible that the chiral phase transition is second order and is in the same universality class. \cite{5, 8, 9} At $T = T_c$ for a second order phase transition, the theory is at an infrared fixed point of the renormalization group and physics is scale invariant. This means that the order parameter fluctuates on all length

\footnote{This has been verified perturbatively in the coupling, perturbatively in $d - 4$ where $d$ is the number of dimensions of space, perturbatively in $1/N$ where $N = 4$ is the case of interest, nonperturbatively in numerical simulations, and by experiment for $N = 3$.}
scales, and in particular on arbitrarily long length scales. Long wavelength oscillations of the chiral order parameter are the defining feature of physics near $T_c$, and our goal later in this talk will be to discuss whether they may occur in a heavy ion collision and how they can leave an observable signature.

If the transition is in the $O(4)$ universality class, the equilibrium physics of the order parameter near $T = T_c$ at wavelengths long compared to $1/T$ is classical and is described by the Ginzburg-Landau free energy

$$F = \int d^3x \left\{ \frac{1}{2} \partial^\alpha \phi^\alpha \partial_\alpha \phi^\alpha + \frac{\mu^2}{2} \phi^\alpha \phi^\alpha + \frac{\lambda}{4} (\phi^\alpha \phi^\alpha)^2 + H\sigma \right\}.$$  \hspace{1cm} (3)

Here $\mu^2$ and $\lambda > 0$ are temperature dependent and $T_c$ is the temperature at which $\mu^2 = 0$. We have introduced an explicit symmetry breaking term $H\sigma$ which tilts the potential, selects a $\sigma$ direction, and gives the Goldstone bosons a mass. If the underlying microscopic theory which under renormalization flows to (3) in the infrared were in fact that of a magnet, $H$ would be proportional to an externally imposed magnetic field. In QCD, $H$ is proportional to a common mass $m_q$ for the two light quarks. A nonzero $H$ turns the second order phase transition into a smooth crossover. If $H$ is small, the theory (3) yields universal, quantitative predictions for the behavior of the order parameter $\langle \phi \rangle$ as a function of $T$ for $T$ near $T_c$.\cite{8, 9, 10, 11, 12} The critical exponents describing physics at $H = 0$, $T \neq T_c$ and at $T = T_c$, $H = 0$ have been computed to high order in a perturbative expansion in $\lambda$,\cite{13} and by numerical simulation.\cite{14} The full equation of state — namely $\langle \phi \rangle$ as a function of $T$ and $H$ — has been obtained numerically by Toussaint.\cite{12}

Finite temperature lattice QCD simulations can test whether the QCD phase transition is in fact in the $O(4)$ universality class. Present simulations provide evidence that in two flavor QCD the phase transition is not first order, and is plausibly second order in the chiral limit.\cite{5, 6} As one example, let us consider the results of \cite{6}. They compute the order parameter as a function of $T$ over a range of temperatures near $T_c$ for two values of $m_q$, the lighter of which corresponds to $m_\sigma/m_\rho \sim 0.3$. They find that the order parameter decreases rapidly but smoothly over a range of temperatures about 10 MeV wide centered about a temperature $T_c \sim 140 - 160$ MeV. They then show that their results can be fitted equally well using the $O(4)$ equation of state\cite{12} and the mean field equation of state. However, the two fits make very different

---

\cite{4} Equilibrium finite temperature quantum field theory is formulated in 4 dimensional Euclidean space, where the fourth dimension is periodic with period $1/T$. Integrating out modes with energy $\sim 1/T$ and higher, yields an effective three dimensional classical field theory describing physics at wavelengths longer than $1/T$.\cite{3}
predictions for behavior at smaller quark masses. Thus, the present simulation is consistent with the hypothesis that the transition is in the $O(4)$ universality class, but until smaller quark masses are explored no stringent tests will be possible. To see the universal long wavelength physics one must use light enough quarks that $m_{\pi}(T_c) < T_c$, where $m_{\pi}(T_c)$, the pion inverse correlation length, is of order or slightly more than $m_{\pi}$. In the simulations of [6], $m_{\pi} > T_c$. This means that the correlation length is just not long enough for the physics to become effectively three dimensional and classical as in [3]. In present simulations done by a number of groups [5, 6] which have all had $m_{\pi}/m_{\rho} \sim 0.2$ or more, although a variety of methods of extracting critical exponents have been explored, it has not yet been possible to cleanly distinguish between, say, $O(4)$ and mean field exponents.

We have seen that the long wavelength oscillations of the orientation of the condensate are central to the physics of the chiral phase transition. Although

\[ m_{\pi}(T_c) < T_c \]

\[ m_{\pi} \sim T_c \]

It is worth enumerating the logical possibilities which could lead to a failure of the hypothesis that a second order phase transition with $O(4)$ exponents will be seen in lattice simulations. We only know that an infrared fixed point with the appropriate symmetry exists. We do not know that QCD is in fact in the basin of attraction of this fixed point. Renormalization toward the infrared starting from QCD may lead to the $O(4)$ fixed point, it may miss the fixed point and yield a first order transition, or, least likely, it may lead to some as yet undiscovered other fixed point, with different exponents. The first order option does in fact arise if the strange quark is lighter than a certain (tricritical) value. Simulations with two light and one heavier quark done by the Columbia group suggest that in nature, the strange quark is heavy enough that no first order transition occurs. It is very important to verify this conclusion in the next generation of lattice simulations, particularly as it has recently been questioned.

A final logical possibility which would lead to nonobservation of $O(4)$ exponents is that the Ginzburg region may be too small to see the true critical behavior. Fluctuations of the order parameter are important only close to $T_c$, for $|t| \equiv |T - T_c|/T_c < t_G$ where $t_G$, like $T_c$, is not universal. Outside the Ginzburg region, that is for $|t| > t_G$, mean field theory is valid. If there is an appropriate small parameter in the theory, $t_G$ can be small. For example, in ordinary, low temperature, BCS superconductors, $t_G \sim (T_c/E_f)^4$ where the Fermi energy $E_f$ is $10^3$ to $10^4$ times bigger than $T_c$. As another example, numerical simulations of the $2+1$ dimensional Gross-Neveu model suggest that $t_G$ is small in this theory. Here, the small parameter is $1/N$, where $N$ is the number of fermions in the theory. In the large $N$ limit in this theory, the phase transition remains in the Ising universality class but $t_G$ goes to zero like $1/N$. (Note that in QCD, increasing either $N_c$ or $N_f$ has much more drastic effects like making the transition first order or changing the symmetry of the order parameter or destroying asymptotic freedom.) There is no parameter in QCD, small or not so small, which when taken to zero reduces $t_G$, leaving the transition otherwise unaffected. We can therefore expect $t_G$ to be of order one. To exclude the possibility that it is unexpectedly small, one needs lattice simulations with light enough quarks that $m_{\pi} \sim T_c$. This should suffice to distinguish between $O(4)$ and mean field exponents.
measuring critical exponents, and thus gaining a quantitative understanding of this physics, is possible on the lattice, it is extremely unlikely that such measurements can be done in heavy ion collision experiments. However, as I explain in the rest of this talk, classical long wavelength pion oscillations may arise in a heavy ion collision, and can leave a signature [20, 10, 21].

A long wavelength oscillation of the order parameter consists of large regions in which the chiral condensate points in directions other than \( (\sigma, \vec{0}) \). Let us first consider an idealized situation [22, 23, 24] in which there is a single large region in which \( \phi \) is uniform in space and misaligned. Because the pion mass is nonzero due to the explicit chiral symmetry breaking introduced by nonzero quark masses, in such a region of disoriented chiral condensate the \( \phi \) field would oscillate about the \( \sigma \) direction. That is, say,

\[
\phi = v(\cos \theta, 0, 0, \sin \theta) \text{ with } \theta(t) \sim \sin(m_\pi t). \tag{4}
\]

In this idealized case, \( \phi \) is independent of \( \vec{x} \). This can be thought of as an infinitely long wavelength oscillation of the orientation of the condensate, unpolluted by short wavelength clutter. (In a heavy ion collision, we will see that if long wavelength oscillations arise, they occur superposed with short wavelength “noise” and so a smooth region of disoriented chiral condensate is an idealization. Bjorken et al. have suggested that circumstances closer to the idealized case may actually arise in hadron-hadron collisions [24].)

An idealized disoriented region in which the disorientation is in the \( \pi_1 - \pi_2 \) plane corresponds to all charged pions (equally positive and negative since the fields are real), while if the disorientation is in the \( \pi_3 \) direction as in (4) it corresponds to all neutral pions with no charged pions. More generally if we define the ratio

\[
R \equiv \frac{n_{\pi^0}}{n_{\pi^0} + n_{\pi^+} + n_{\pi^-}} \tag{5}
\]

then each disoriented region will yield pions with some fixed \( R \). Under the assumption that all directions of disorientation are equally probable, an ensemble of “events” will yield a probability distribution for \( R \) [25, 22, 24, 20]

\[
P(R) = \frac{1}{2\sqrt{R}}. \tag{6}
\]

Note, for example, that the probability that the neutral pion fraction \( R \) is less than 0.01 is 0.1! In “events” with, say, 100 pions, the distribution (6) is very different than the binomial distribution obtained if each individual pion was independently randomly neutral or positive or negative. Rather than rolling the dice once per pion, and getting an \( R \) distribution looking like a Gaussian
about $R = 1/3$, in a population of idealized "events" just described, the dice are rolled once per event yielding the much broader, and skewed, distribution (3).

Now with a hint of how long wavelength oscillations of the order parameter could leave a signature, we return to the question of whether they arise. An equilibrium second order transition seems ideal, as near the critical temperature the correlation length is infinite and oscillations occur on arbitrarily long wavelengths. However, because the up and down quarks (and hence the pion) are not massless, we have seen that the putative second order transition is smoothed out, and the correlation length is not long compared to $1/T_c$, and classical oscillations do not arise. Although we are used to thinking of nature as being close to the chiral limit, because $m_\pi/m_\rho$ is small, it is unfortunately not close enough for $m_\pi/T_c$ to be small. On the lattice, one can envision using quark masses smaller than in nature, and so getting closer to having a second order transition. In an experiment, one does not have this option.

Fortunately, this is not the end of the story. There is no reason to assume that during a heavy ion collision the long wavelength modes of the order parameter stay in thermal equilibrium as the plasma expands and cools and chiral symmetry breaking occurs, even if local thermal equilibrium at a high temperature is achieved at early times. There is a simple non-equilibrium mechanism, which could plausibly operate in a heavy ion collision, which naturally leads to great amplification of long wavelength modes, relative to what would be seen in thermal equilibrium.

Let us consider an idealization that is in some ways opposite to that of thermal equilibrium, namely the occurrence of a sudden quench from high to low temperatures in which the $(\sigma, \vec{\pi})$ fields are suddenly removed from contact with a high temperature heat bath and subsequently evolve according to zero temperature equations of motion. Unlike in the equilibrium case where universality was our guide, away from equilibrium we must make a non-universal choice of model Lagrangian to obtain equations of motion for the order parameter. The linear sigma model

$$\mathcal{L} = \int d^4x \left\{ \frac{1}{2} \partial^\mu \phi^\alpha \partial_\mu \phi_\alpha - \frac{\lambda}{4} \left( \phi^\alpha \phi_\alpha - v^2 \right)^2 + H\sigma \right\},$$

(7)

Quenching is an ad hoc assumption, not expected to be realized in a heavy ion collision in any detail, and so it is reasonable to consider different dynamical assumptions as has been done in [26, 27]. The hope is that qualitative behavior observed after a quench is representative of phenomena occurring in realistic conditions in heavy ion collisions in which the long wavelength modes are not in equilibrium.
is a reasonable choice\cite{20} although other models have also been studied.\cite{27} In\cite{21}, the nonlinear classical equations of motion derived from this Lagrangian were solved numerically on a lattice with spacing $a = (200 \text{MeV})^{-1}$, and with parameters in the potential chosen such that $f_\pi$ and $m_\pi$ have their zero temperature values and $m_\sigma = 600 \text{MeV}$. In the initial conditions, $\langle \phi \rangle \sim 0$ and the orientation of $\phi$ was chosen randomly at each lattice site, as appropriate for initial conditions above $T_c$ at a high enough temperature that the correlation length is shorter than $a$. We chose $\langle \phi^2 \rangle^{1/2} = v/2$, a choice which has since received some justification from the work of Randrup, as we discuss below.

Because of the explicit symmetry breaking, $\phi$ is soon oscillating about the $\sigma$ direction everywhere in space. Upon Fourier transforming, one finds that modes of each component of $\vec{\pi}$ with wave vector $\vec{k}$ oscillate about $\vec{\pi} = 0$ with $\omega \sim \sqrt{k^2 + m_\pi^2}$, as expected. The behavior of the amplitude of the low momentum pion modes as a function of time is striking. The initial conditions had a white noise power spectrum, with all modes having equal amplitudes. At very late times, things become boring once again, as the system approaches an equilibrium configuration in which equipartition of energy holds. Over a wide range of intermediate times of order 5 to 50 times $m_\pi^{-1}$, however, the long wavelength modes are greatly amplified. (This was confirmed in\cite{28, 29}.)

The affected modes are those with $|\vec{k}|$ less than about $m_\pi$. The longer the wavelength, the bigger the amplification; the longest wavelength mode not affected by the finite size of the box has its amplitude squared amplified by more than a factor of 1000 relative to that of the short wavelength modes which were not amplified at all, and by about a factor of 50 relative to its value at late times in equilibrium. For a visual image, think of long wavelength ocean swells superposed with lots of short wavelength chop. The essential qualitative phenomenon whose cause and consequences we now discuss is that long wavelength oscillations of the orientation of the condensate (pion oscillations) have been excited — the short wavelength modes have not been damped out. One does not see smooth domains, and the power spectrum is not well characterized by a single correlation length or domain size.\cite{10} This picture is more complicated than the idealized "smooth" region of disoriented chiral condensate we discussed above.

The emergence of long wavelength pion oscillations is a striking qualitative phenomenon, for which there is a simple qualitative explanation.\cite{20} One can linearize the equations of motion for the pion field to obtain

$$\frac{d^2}{dt^2} \vec{\pi}(\vec{k}, t) = -m_{\text{eff}}^2(k, t) \vec{\pi}(\vec{k}, t),$$

where $m_{\text{eff}}^2(k, t) = m_\pi^2 - k^2$.

where the time dependent “mass” is given by

\[ m_{\text{eff}}^2 \sim -\lambda v^2 + k^2 + \lambda \langle \phi^2 \rangle(t). \]  

(9)

Linearizing in this way constitutes an uncontrolled truncation, but it does yield some insight into the behavior found by numerical solution of the non-linear equations. One sees that the non-equilibrium dynamics leads to a time dependent \( m_{\text{eff}}^2 \) which can be negative for modes with small enough \( k \). This means that these modes will experience periods of time during which they are unstable to exponential growth. The longer the wavelength, the greater the amplification. In an equilibrium phase transition, explicit symmetry breaking keeps the correlation length at \( T_c \) too short to be of interest. This, then, is a simple non-equilibrium mechanism which leads to the amplification of arbitrarily long wavelength modes of the pion field even though the pion mass is nonzero.

I will return to analyzing the results of the simulation just described in terms of the ratio \( R \), but let me first sketch some improvements. The most important effect which has been left out above is expansion. The late time behavior with no expansion (equilibration of the system) is qualitatively different than that with expansion. Expansion causes the energy density to drop, and the dynamics linearizes as the modes stop interacting at late times. Hence, at late times the ratios of amplitudes between different modes stops changing. This is the classical analogue of “freeze out”. The hope in [20] was that once expansion is included, the modes would freeze out with amplitude ratios as found at times of order \( 5 - 50 m_\pi^{-1} \), namely with long wavelength modes amplified. This was in fact found in some, [30, 31, 32] but not all, [29] subsequent simulations which began with initial conditions akin to those above and which included effects of expansion in several different ways. (For treatments in other contexts which include effects of expansion, see [33].)

Another goal in implementing expansion, however, is to relax the quench assumption. This has been implemented by Randrup, [34] He begins with a linear sigma model configuration in thermal equilibrium at 400 MeV [3] and adds a damping term to the equations of motion which damps energy from the system as if the system is undergoing a three dimensional boost invariant expansion. Initially, \( \langle \phi \rangle \) is small, but because of the large initial temperature \( \langle \phi^2 \rangle \) is much larger than in the initial conditions of [20]. Randrup finds an

\footnote{Although this is certainly less ad hoc than the initial conditions in [20], it is not yet realistic. At high temperatures like these, at wavelengths of order \( 1/T \) there are presumably degrees of freedom other than those of the linear sigma model which are important.}
initial rapid decrease in $\langle \phi^2 \rangle$, during which $\langle \phi \rangle$ hardly changes and $\langle \phi^2 \rangle$ decreases to $\sim v^2/2!$ Thus, the expansion takes the system to a configuration similar to that configuration chosen in [20] as an initial condition for evolution after a quench, which in turn was supposed to model the effects of expansion. The subsequent evolution in Randrup’s simulation is similar to that in [20]; long wavelength modes grow, although because of the expansion they later freeze out as described above. Thus, Randrup’s result provides some support for some of the ad hoc choices made in [20].

A more ambitious project than the classical simulations described so far is to include the effects of quantum fluctuations (in addition to classical thermal fluctuations) on the dynamics of the order parameter. We have seen that with the pion mass nature has dealt us, the equilibrium correlation length is not long compared to $1/T_c$, and this suggests that quantum effects may matter. However, the nice thing is that the instability discussed above is an instability towards growth of long wavelength modes, which as they grow in amplitude become more classical. In equilibrium, the occupation number of a mode is $\sim T/\omega$ which is $\sim T/m_\pi$ at long wavelengths, so any growth relative to equilibrium pushes the mode into the classical regime. Simulations including quantum fluctuations have now been done by several groups,[31, 32] and they in fact find the same instability to growth of long wavelength modes found classically. However, in quantum treatments done to this point it has been necessary to make linearizing approximations. I argue in [10] that the simulations of [20] show that the growth of long wavelength modes in the full nonlinear theory is in fact greater than in the linearized theory. Hence, it is best to view the two treatments as complementary. The quantum treatment can be used to study under what circumstances the instability is present, but once the instability sets in and long wavelength modes begin to grow, a classical treatment becomes appropriate. Happily, the two approaches seem to yield approximately similar outcomes.

Because a non-equilibrium transition is required, the theoretical foundations of all the simulations are shaky. Using the linear sigma model to describe the dynamics seems reasonable for long wavelength modes, but cannot be justified in a controlled way. Either neglecting quantum effects or including them in a Hartree approximation risks missing some of the physics. Most important of all, we do not know what initial conditions to impose on the long wavelength modes of the chiral order parameter at the time when partonic language ceases to be appropriate and a description in terms of the linear sigma model becomes appropriate. Nobody can do a simulation which follows non-equilibrium dynamics beginning with cascading partons and ending with
long wavelength oscillations of the condensate. This makes it impossible to predict that long wavelength pion oscillations will be amplified in a particular experimental setting. The way to know that this phenomenon has occurred is to detect it, and to this we now turn.

The defining signature of the presence of long wavelength classical pion oscillations is large fluctuations in the ratio $R$. If a particular $\vec{k}$ mode ends up oscillating in the $\pi^3$ direction, then it will become neutral pions moving in the direction of $\vec{k}$. Similarly, oscillations in the $\pi^1 - \pi^2$ plane will become charged pions. If this were the only physics occurring, one could bin the data as a function of pseudorapidity $\eta$ and azimuth $\varphi$ and the bin by bin distribution of $R$ would be (3). The problem, of course, is the large background due to all the other pions in the event.

What is needed is a detector which covers as much of $\eta$ and $\varphi$ as possible, which counts both charged pions and photons, which accepts pions down to as low $p_T$ as possible (50 MeV or even lower; this may mean running with reduced magnetic field), and which accepts photons down to $p_T \sim m_\pi/2$ or so. The detector should be segmented in $\eta$ and $\varphi$ finely enough that the number of photons and charged pions in each segment can be accurately counted. Where should one be in $\eta$? Central rapidity is cleanest for a theorist, but at RHIC low $p_T$ photons are hard to count there, so perhaps somewhat off central rapidity may be better. Accurate measurement of $p_T$ and particle identification is not necessary, since at RHIC most charged particles are pions and most photons are from $\pi^0$'s. However, it would be very helpful to have enough $p_T$ information to make even a crude cut, keeping only pions with low $p_T$, say $p_T < 2 - 3 m_\pi$. The data, then, consists of an event by event catalogue of the positions in $(\eta, \varphi)$ of the photons and charged pions. From this, one can compute $R = n_\gamma/(n_\gamma + 2n_{ch})$ in bins in $(\eta, \varphi)$.

In thinking about how to analyze real data, it is an instructive exercise to go back to the simulations of [20] and make explicit how the amplification of long wavelength modes is manifest in the number ratio $R$. For each of the three $\vec{\pi}$, and for each $\vec{k}$ on the momentum space lattice with spacing $2\pi/a$, define the “particle number”

$$n^i_{\vec{k}}(t) \equiv \frac{1}{\omega} \left| i \omega \pi^i(\vec{k}, t) + \dot{\pi}^i(\vec{k}, t) \right|^2,$$

(10)

8Other signatures which have been discussed include an excess in the total number of pions at low $p_T$ independent of their charge,[31, 32, 36, 37] various effects on pion pair correlations,[38] and various electromagnetic effects.[39, 37] Any of these would be telling if seen in conjunction with large fluctuations in $R$, but it is not clear that they would be unambiguous if seen alone.
where $\pi^i(\vec{k}, t)$ and $\dot{\pi}^i(\vec{k}, t)$ are the Fourier transforms of $\pi^i(\vec{x}, t)$ and its time derivative. If we had included expansion, had waited until the system linearized, and if we assumed that the classical fields correspond to quantum mechanical coherent states, these would be the mean particle numbers per mode at late time. Although the $n$’s should really be viewed just as time dependent quantities defined by (10), I will refer to them as numbers henceforth. The lattice has $64^3$ points, so we now have the number of neutral pions and the total number of pions at $64^3$ points in momentum space as a function of time. I now make a cut, keeping only those modes with $|\vec{k}| < 300$ MeV. For each remaining $\vec{k}$, I compute the polar angle $\theta$ and the azimuthal angle $\varphi$ describing the direction in momentum space. (In an experiment, one would use $\eta$. Here, without expansion, $\theta$ is the right variable.) I then divide momentum space into $10^\circ \times 10^\circ$ bins in $(\theta, \varphi)$, and discard the bins within $30^\circ$ of the north and south poles, as they each contain too few modes. For each of the remaining bins, I add up the number of neutral pions in all the modes with $\vec{k}$’s in the bin, do the same for the total number of pions, and compute $R$.

The results are shown in Figure 1, at two different times. The top panel is at a very late time, when the system has equilibrated and nothing interesting
is going on. The bottom panel is at an intermediate time when the long wavelength modes have large amplitudes. It is quite clear in the figures that the $R$ distribution is much broader in the lower panel. There are more high spots and low spots. Figure 2 shows histograms of the distribution of $R$ values in the 432 bins in Figure 1. The unshaded histogram corresponds to the top panel, and the shaded histogram corresponds to the bottom panel, in which long wavelength oscillations are present. We see that the effect of the long wavelength modes is to broaden the distribution (look at the tails), and skew it to the left (look at the peaks). This is exactly what one would expect from an admixture of a $1/\sqrt{R}$ distribution. Note that if the 300 MeV cut on $|\vec{k}|$ is pushed too high, the short wavelength modes swamp the long wavelength modes and both distributions look the same. If the cut is too low, there are very few modes per bin, and both distributions become wider. Eventually, if the cut is too low (or if the bin size is too small) and there is only one mode per bin, then both distributions are $1/\sqrt{R}$. Increasing the bin size above $10^\circ \times 10^\circ$ narrows both distributions by the same factor. The central limit theorem “acts” on both distributions as the bin size is increased, but the presence of large amplitude long wavelength modes “delays its action”. [11] The choice of
cuts and bin sizes in Figs. 1 and 2 was driven by the background (short wavelength modes) and statistical fluctuations (arising if there are too few modes per bin) in the simulation in [20]. In analyzing real data in an analogous fashion, such choices will be driven by the background (high p_T pions) and statistical fluctuations (arising if there are too few pions and photons per bin) and the fact that one counts photons rather than π^0's. Hence, the choices I have found convenient need not be the ones which are convenient in analyzing real data.

With real data, one should try bins of all sizes in both η and φ, ranging from bins which on average contain a few tens of pions, to bins in rapidity covering 2π in φ, to “bins” which consist of whole events, and construct R distributions as above for each bin size. (The data should also be analyzed using the wavelet formalism of [41].) It is also very important to look for singular bins far out on the tails of the various distributions. The simulations of [30] suggest that events of this sort may occur, although they are presumably rare. Broadened (and perhaps skewed) R distributions may be more common. Both should be looked for as functions of increasing beam energy and projectile size, and decreasing impact parameter.

To what should one compare an R distribution obtained from data? One can use an R distribution obtained from an event generator in which pions are independently randomly neutral, positive, or negative. If there is no signal in the data, this should fit very well, because any badness of fit in each individual multiplicity distribution should cancel in the ratio R. However, it may be preferable to do the analysis without reference to an event generator. Steinberg[42] has suggested creating a data set by rotating the charged pions in each event in azimuth relative to the photons, say by 90°. One would look for broadening and skewing of the histograms from the real events relative to those from the rotated events.

Although there are fewer complications at RHIC than at lower energies, given the lack of theoretical certainty it is certainly wise to look today. WA98 at CERN is in the process of analyzing their event by event charged particle and photon data.[12] They are looking for, and so far have not found,[12] singular events which behave as if more than ∼ 30% of all the pions in the event come from a single idealized region of disoriented chiral condensate. They are in the process of binning the data in (η, φ) and looking for fluctuations in R using various bin sizes. Even before they finish their analyses, we can learn useful lessons for future experiments. Being able to cut on p_T, and get rid of the high momentum pions, would really help, as would having a charged particle veto in front of their photon multiplicity detector. NA49 is planning to search
for fluctuations in the number of charged pions only, as they do not count photons. PHENIX, PHOBOS, and STAR all plan to search for unusual fluctuations in $R$ at RHIC.

The conclusions from theory for experiment are qualitative. Consider a heavy ion collision which is energetic enough that there is a central rapidity region of high energy density and low baryon number. If, as this region cools through the chiral transition, long wavelength modes do not stay in equilibrium, then there is a robust mechanism which can lead to the amplification of long wavelength pion oscillations. This cannot happen in thermal equilibrium. One should not take the details of any of the theoretical simulations too seriously. All make dynamical assumptions and all rely on guesses for their initial conditions, since nobody can go from initial parton dynamics to the dynamics of the chiral order parameter at long wavelengths. What one should take seriously is that the place to look for signatures of the dynamics of the chiral order parameter during the phase transition is the low $p_T$ pions. We have discussed one effect, leading to such a signature, but perhaps the best outcome would be the detection of completely unexpected phenomena at low $p_T$. Long wavelength oscillations of the condensate would be observed by detecting unusual fluctuations in $R$, the number ratio of neutral pions, event by event as a function of rapidity and azimuth. One should look both for broadened and skewed $R$ distributions, and for singular events with particularly large fluctuations. Experimentalists are trying to disorder the chiral condensate and then detect its disoriented oscillations, as it returns toward its ordered ground state. They will continue at RHIC and the LHC, where sufficient energy should be available. Detection of unusual fluctuations in $R$ in a heavy ion collision would be a dramatic and definitive signature of an out of equilibrium chiral transition. The ball is in the experimentalists’ court, and we wish them well.

Acknowledgments — I am grateful to the organizers for inviting me to Hirschegg; the meeting was enjoyable and productive. I would like to thank everyone at the Trento conference on Disoriented Chiral Condensates. Many discussions I had there influenced this work. This work was supported in part by the Sherman Fairchild Foundation and by the Department of Energy under Grant No. DE-FG03-92-ER40701.

MINIMAX is looking for regions of disorientated chiral condensate at high rapidity in hadron-hadron collisions at Fermilab. The dynamics in these events is surely different from that we are discussing, but the signatures are similar and therefore their analysis techniques should prove useful when generalized for use in larger acceptance experiments.
References

[1] D. J. Gross and F. Wilczek, Phys. Rev. Lett. 30 (1973) 1343; H. D. Politzer, ibid, 1346.

[2] J. C. Collins and M. J. Perry, Phys. Rev. Lett. 34 (1975) 1353.

[3] B. Svetitsky and L. G. Yaffe, Nucl. Phys. B210 (1982) 423; B. Svetitsky, Phys. Reports 132 (1986) 1.

[4] For a review, see A. Ukawa, Nucl. Phy., B17 Proc. Suppl. (1990) 118.

[5] S. Gottlieb et al., Phys. Rev. Lett. 59 (1987) 1513; D35 (1987) 3972; D41 (1990) 622; D47 (1993) 315; M. Fukugita et al., Phys. Rev. Lett. 65 (1990) 816; Phys. Rev. D42 (1990) 2936; F. R. Brown et al., Phys. Rev. Lett. 65 (1990) 2491; F. Karsch, Phys. Rev. D49 (1993) 3791; F. Karsch and E. Laermann, Phys. Rev. D50 (1994) 6954; C. Bernard et al., Phys. Rev. D45 (1992) 3854; Phys. Rev. D54 (1996) 4585; G. Boyd et al., hep-lat/9607040; Y. Iwasaki et al., Phys. Rev. Lett. 78 (1997) 179; A. Ukawa, hep-lat/9612011.

[6] C. Bernard et al., hep-lat/9608020; hep-lat/9612025.

[7] R. Pisarski and F. Wilczek, Phys. Rev. D29 (1984) 338.

[8] F. Wilczek, Int. J. Mod. Phys. A7 (1992) 3911.

[9] K. Rajagopal and F. Wilczek, Nucl. Phys. B399 (1993) 395.

[10] K. Rajagopal, in Quark Gluon Plasma 2, R. Hwa, ed. (1995) 484, hep-ph/9504310.

[11] C. Detar, in Quark Gluon Plasma 2, R. Hwa, ed. (1995) 1.

[12] D. Toussaint, Phys. Rev. D55 (1997) 362.

[13] G. Baker, B. Nickel and D. Meiron, Phys. Rev. B17 (1978) 1365; and an unpublished University of Guelph report (1977).

[14] K. Kanaya and S. Kaya, Phys. Rev. D51 (1995) 2404.

[15] F. R. Brown et al. in [3].

[16] Y. Iwasaki et al., Phys. Rev. D54 (1996) 7010.

[17] E. M. Lifschitz and L. P. Pitaevskii, Statistical Physics Part 2, (Pergamon, Oxford, 1980).

[18] A. Kocic and J. Kogut, Phys. Rev. Lett. 74 (1995) 3109.

[19] R. Pisarski and M. Stephanov, private communication.
[20] K. Rajagopal and F. Wilczek, Nucl. Phys. B404 (1993) 577.
[21] For a beautiful review, see J.-P. Blaizot and A. Krzywicki, Acta Phys. Polon. 27 (1996) 1687.
[22] A. Anselm, Phys. Lett. 217B (1988) 169; A. Anselm and M. G. Ryskin, Phys. Lett. 266B (1991) 482.
[23] J.-P. Blaizot and A. Krzywicki, Phys. Rev. D46 (1992) 246.
[24] J. D. Bjorken, Int. J. Mod. Phys. A7 (1992) 4189; Acta Phys. Pol. B23 (1992) 561; K. L. Kowalski and C. C. Taylor, hep-ph/9211282; J. D. Bjorken, K. L. Kowalski and C. C. Taylor, hep-ph/9309235.
[25] I. V. Andreev, JETP Lett. 33 (1981) 367.
[26] A. Krzywicki, Phys. Rev. D48 (1993) 5190; S. Gavin and B. Müller, Phys. Lett. B329 (1994) 486; S. Mrowczynski and B. Müller, Phys. Lett. B363 (1995) 1.
[27] P. F. Bedaque and A. Das, Mod. Phys. Lett. A8 (1993) 3151; A. Barducci et al., Phys. Lett. B369 (1996) 23; J. I. Kapusta and A. P. Vischer, nucl-th/9605023; A. Abada and M. C. Birse, hep-ph/9612231.
[28] S. Gavin, A. Gocksch and R. Pisarski, Phys. Rev. Lett. 72 (1994) 2143.
[29] A. Bialas, W. Czyz and M. Gmyrek, Phys. Rev. D51 (1995) 3739.
[30] Z. Huang and X. Wang, Phys. Rev. D49 (1994) 442; M. Asakawa, Z. Huang, and X. Wang, Phys. Rev. Lett. 74 (1995) 3126.
[31] F. Cooper et al. Phys. Rev. D50 (1994) 2848; D51 (1995) 2377; C54 (1996) 3298; Y. Kluger, hep-ph/9405279; hep-ph/9408286; hep-ph/9503205.
[32] M. A. Lampert et al., Phys. Rev. D54 (1996) 2213.
[33] J.-P. Blaizot and A. Krzywicki, Phys. Rev. D50 (1994) 442; S. Yu. Khlebnikov, Mod. Phys. Lett. A8 (1993) 3971; Z. Huang and M. Suzuki, Phys. Rev. D53 (1996) 891; H. Davoudiasl, hep-ph/9611263.
[34] J. Randrup, Phys. Rev. D55 (1997) 1188; Phys. Rev. Lett. 77 (1996) 1226; hep-ph/9612453.
[35] D. Boyanovsky et al., Phys. Rev. D51 (1995) 734; Phys. Rev. D54 (1996) 1748.
[36] S. Gavin, Nucl. Phys. A590 (1995) 163; hep-ph/9407368.
[37] Z. Huang, hep-ph/9501366.
[38] J. P. Blaizot and D. Diakonov, Phys. Lett. B315 (1993) 226; C. Greiner, C. Gong and B. Müller, Phys. Lett. B316 (1993) 226.

[39] Z. Huang, M. Suzuki, and X. Wang, Phys. Rev. D50 (1994) 2277; D52 (1995) 2610; D. Boyanovsky et al., hep-ph/9701360.

[40] K. Rajagopal, work in preparation sketched here.

[41] Z. Huang et al., Phys. Rev. D54 (1996) 750.

[42] T. Peitzmann, talk at this meeting; talks by T. Nayak, P. Steinberg, J. Urbahn and B. Wyslouch at Trento meeting on Disoriented Chiral Condensates, 1996.

[43] G. Roland, talk at this meeting.

[44] T. C. Brooks et al., hep-ph/9609375; J. D. Bjorken et al., hep-ph/9610379.