High level of visual-spatial intelligence’s students in solving PISA geometry problems

Dwi Novitasari, Ahmad Nasrullah, Tabita Wahyu Triutami, Ratih Ayu Apsari and Dewi Silviana

1Universitas Mataram, Jl. Majapahit, Mataram, Nusa Tenggara Barat, Indonesia
2UIN Mataram, Jl. Gajah Mada 100 Jepang Baru, Nusa Tenggara Barat, Indonesia
3STKIP Bima, Jl. Pierre Tendean, Bima, Nusa Tenggara Barat, Indonesia

E-mail: ahmadnasrullah@uinmataram.ac.id

Abstract. Spatial reasoning and representation are two things that are often used in our daily life, especially in the field of mathematics and geometry. Therefore, this study aims to identify and describe the relationship between representation and spatial reasoning used by students who have high level of visual-spatial intelligence in solving PISA problems. This study is an explorative descriptive study through a qualitative approach. The subjects of this study were 2 third grade junior high school students with high level of visual-spatial intelligence. Data were collected by visual-spatial intelligence tests, mathematical problems related to PISA problems in the form of 3-dimensional and 2-dimensional geometry problems with two open-ended problems and interviews. The results showed that: (1) there is a relationship between representation and spatial reasoning, (2) students combine their representations and spatial reasoning in giving and proving their answers. The results also showed that students applied their spatial reasoning in the form of spatial visualization and mental rotation. While the most dominant representations used by students are visual representation (76.25%) and symbolic representation (20%).

1. Introduction

Spatial ability and spatial reasoning have contributed to abstract thinking and in solving problems especially in the field of mathematics to help people interpret mathematical symbols [1]. Spatial reasoning is related to the functioning of other high-level cognitive skills such as logical reasoning, memory retrieval, verbal skills, academic achievement and the development of expertise in skills and jobs in the fields of science, technology, engineering and mathematics (STEM) [2–6]. Some researchers even states that spatial reasoning and representation are important components in solving various problems related to daily life [7,8] for example when we read a map to find a location, there are representations, representational transformations, and spatial reasoning that we use unconsciously [9]. The National Council of Mathematics Teachers (NCTM) even states that mathematical reasoning and representation are two of the five mathematical abilities students must possess, [10].

Spatial reasoning is a general term that encompasses various abilities that involve mental representation and manipulation of spatial information, such as object rotation, mental folding,
perspective taking, and navigation [11,12]. Spatial reasoning also refers to the capacity to think about 2-dimensional and 3-dimensional objects and make conclusions about an object based on information that has been obtained [2].

Spatial reasoning has three components: visual spatialization, mental rotation and spatial orientation [13,14]. Spatial visualization is the ability to manipulate spatial patterned images using different perspectives or points of view. Mental rotation is the ability to mentally change 2-dimensional and 3-dimensional objects [15]. Spatial orientation is the ability to imagine an object from different angles [16]. Spatial visualization and mental rotation involve cognitive skills together in forming and manipulating mental images [8]. In algebra, strategies that are spatially related (such as representing graphically functional relationships, manipulating spatial representations and numeric numbers, and visualizing the structure of geometric patterns) are useful in generalizing functional relationships between quantitative variables [17,18].

One factor that influences individual differences in using spatial reasoning and representation in solving problems is intelligence. Gardner revealed that multiple intelligence has an important role in student learning. In the theory of multiple intelligences according to Gardner, intelligence is divided into 8 subcategories. That are verbal-linguistic intelligence, logical-mathematical intelligence, visual-spatial intelligence, kinesthetic-body intelligence, musical intelligence, interpersonal intelligence, intrapersonal intelligence and naturalistic intelligence. All humans have intelligence but individuals have special intelligence profiles [19]. In this research will be focused on spatial visual intelligence, especially on high visual-spatial intelligence level because visual spatial intelligence is one of the dominant intelligences used in mathematics, especially related to geometry.

Gardner & Hatch states that spatial intelligence refers to the capacity possessed by a person in using mental skills to solve various problems including those related to spatial navigation problems, visualizing objects from various angles and points at certain locations. Intelligence can be considered as an integrative and interactive thought process that encourages a person to absorb and process information efficiently. Spatial reasoning also activates spatial processing networks, and strategies to improve spatial reasoning abilities, such as visualizing objects from various perspectives and simulating object rotation mentally [20].

When solving mathematical problems, students with strong spatial skills can more easily solve the problems and can use a variety of problem solving strategies rarely used by other students especially students with weaker spatial skills [21].

Research related to spatial reasoning and representations has previously been done. One of them was done by Fiantika and Setyawati [9] who described students’ spatial thinking processes in transforming 3-dimensional objects into 2-dimensional object representations between female and male students. Their research shows that boys and girls have different ideas and ways of finding the fastest solution to solving the problems using their representations in spatial thinking. The difference of our research with some previous studies is to describe how spatial reasoning and representation are used by students who have high visual-spatial intelligence level in solving mathematical problems.

2. Method
This research is a descriptive exploratory study with a qualitative approach. This study aims to determine and describe how the relationship between spatial reasoning and representation used by students with high levels of visual-spatial intelligence in solving mathematical problems related to PISA problems. The participants of the study are 32 third grade junior high school students in Mataram, Indonesia. Subjects were selected by purposive sampling. The data in this study were obtained by conducting visual-spatial intelligence tests, problem solving tests and interviews. The spatial visual intelligence instrument consisted of 18 questions that were completed in 20 minutes. Whereas the problem solving problem is a modification of the 2006 PISA problem which consists of 2 open ended questions relating to changing 3-dimensional objects to 2-dimensional objects and making 2-dimensional objects in accordance with the provisions.
After 32 participants completed the visual-spatial intelligence test questions, 2 participants who had high level of visual-spatial intelligence were selected as subjects. Both subjects were asked to solve PISA problems then their answers were explored and analyzed through in-depth interviews to find out what they thought, did, wrote, and said when solving the problems. Indicators of spatial reasoning can be seen in Table 1.

| Construct                | Spatial Reasoning Aspects                      | Spatial Reasoning Characteristics                                      |
|--------------------------|-----------------------------------------------|------------------------------------------------------------------------|
| Mental rotation          | Rotate 2-dimensional and 3-dimensional objects clockwise and counterclockwise | Rotating 2-dimensional and 3-dimensional objects both rotate clockwise and counterclockwise. |
| Spatial Visualization    | Symmetry, patterns, 2-dimensional and 3-dimensional forms and their relationship, reflection, symmetry | Visualize the results of folding or opening objects. Visually form and rearrange a form into another different form. Determine the symmetry of an object |

The indicators of representation in this study are (1) visual representations (in the form of images or graphics) consisting of (a) making visual representations of existing data or information and (b) using visual representations to solve the problems; (2) symbolic representations (mathematical equations or expressions) consist of (a) making mathematical expressions or expressions from existing data or information and (b) using mathematical equations or expressions to solve the problem [23].

The data that has been collected were analyzed using descriptive methods by Miles and Huberman which consists of five stages: 1) data collection, 2) transcribing the data that has been collected, 3) segmentation and categorization of the data, 4) describing the relationships and links between spatial reasoning and representation in solving problems and 5) making conclusions [24].

3. Result and Discussion
Based on the results of the visual-spatial intelligence test, the data obtained can be seen in Table 2.

| Category          | Frequency (Σ) | Percentage (%) |
|-------------------|---------------|----------------|
| High              | 2             | 5.41           |
| Average           | 31            | 83.78          |
| Low               | 4             | 10.81          |
| Total             | 37            | 100            |

Based on Table 2, it was found that there were 2 subjects who had high level of visual-spatial intelligence who were then used as subjects in this study. After the two subjects finished completing the problem solving test then an in-depth interview was conducted. The results of problem solving and interviews conducted on the two research subjects will be explained.

3.1 Solution to the Problem 1
From the results of solving the first problem and interviews that have been done, it is known that in solving problems, S1 subjects use spatial reasoning by observing the elements contained in the cube and then using the ability of spatial visualization by imagining opening the cube. so that the cube net is formed. After that, S1 uses a visual representation of the previously imagined cube net by drawing it on the answer sheet. Subject S1 checks that the answer of the cube nets he has drawn is correct by folding the cube net in his mind whether the image can form a cube or not. This process shows that subject S1 is using again his spatial visualization. The final step is to determine the position of the dots.
on each square in the cube net that if we add these dots then the result is equal to seven. In this case, the subject S1 first determines the pair of each dots where the sum of the dots is 7. The pair of dots are 1 with 6, 2 with 5 and 3 with 4, then determine the position of the dots on each square in the cube net in the opposite side. At this stage, visual representation and spatial visualization abilities are used by the subject to prove the truth of the visual images he has made. The same thing was done by subject S2 in producing the correct answer as shown in Figures 1 and 2.

![Figure 1](image1.png) ![Figure 2](image2.png)

**Figure 1.** The correct answer of the cube nets made by subject S1  
**Figure 2.** The correct answer of the cube nets made by subject S2

From Figures 1 and 2, it was found that subjects S1 and S2 could provide correct answers. The difference between the two subjects can be seen from the number of correct answers they were given. Subject S1 can provide more number of correct answers compared to subject S2 to solve the same problem. Based on the results of the interviews conducted, S1 produced other answers by modifying the first answer as shown in Figure 3.

![Figure 3](image3.png)

**Figure 3.** Subject S1 switch one side of the cube net that has been made to get a new form of cube net that is different from before

Figure 3 shows that subject S1 uses spatial visualization and changes the representation of visual objects by switch the position of one side of the cube nets to form new cube nets, then using visual representations to draw the answer S1 made. The next step is to ascertain the correctness of the answer by folding the image whether it can form a cube or not while determining the position of the appropriate the dots of the cube net.

The process carried out by subjects S1 and S2 shows that in producing problem solving, the two subjects combine spatial reasoning (spatial visualization) and representation (visual representation). Spatial reasoning and representation complement each other in producing the correct solution. From this process it can be seen that spatial reasoning and representation complement each other in producing correct answers. However, there are several alternative answers given by the two wrong subjects, as shown in Figure 4.
Figure 4. The answers given by subject S1 (left) and subject S2 (right) are correct in making cube nets but wrong in placing the position of the dots of the cube net correctly.

Figure 4 shows that the two subjects experienced problems when ascertaining the visual representation that had been drawn, when folding cube nets back, the two subjects did not pay close attention to the whole eye of the dice on each side facing each other whether it was in accordance with the provisions given or not.

3.2 Solution to the Problem 2
Alternative answers given by subjects S1 and S2 can be seen in Figures 5 and 6 below.

Figure 5. Alternative answers given by subject S1 by combining spatial reasoning (spatial visualization) and representations

Figure 6. Alternative answers given by subject S2 by combining spatial reasoning (mental rotation and spatial visualization) and multiple representations in modifying answers so that different answers are obtained.
Figure 5 shows that subjects S1 and S2 first determine the design of the garden to be created by using visual representations in their minds. Next is to determine the size of each side in accordance with the design of the garden that was made before. Based on the information obtained in the questions, the two subjects concluded that the perimeter of the park they had to make should not be more than 32 meters. Therefore, after determining the design of the garden to be made, the next step is to use symbolic representation in mathematical calculations in determining the size of each side of the garden design to be made. The total number of all sides will not exceed 32.

Figure 5 also shows that the two subjects combined visual representation, symbolic representation and spatial reasoning (spatial visualization) in the process of producing correct answers. These three capabilities play a role in determining the suitability between garden designs that are made in accordance with the size of each side of the park to conform to specified conditions.

Figures 7 and 8 show that mistakes made by both subjects in providing alternative solutions to the problems. Based on the results of the interview, it is known that S1 subjects obtained the idea of completion as shown in Figure 7 by doing a mental rotation in the design in Figure 5 (a) clockwise 90° then determining the size of each side accordingly. Furthermore, mental calculations using symbolic representations are carried out to produce a garden design circumference of no more than 32 meters.

The results showed that both subjects represented the problems in visual form in accordance with the provisions given in the questions. The two subjects represent the problem in the form of a drawing first to make it easier for both subjects to understand the problem and look for problem solving strategies. This process shows that both subjects use visual representations. The use of visual representations (images) can provide the information needed to solve the problem [14].

In general, both subjects in this research that combine their various representations in solving the problems. The answers given by students, 76.25% were obtained using visual representations, 20% were obtained using symbolic representations, while 3.75% were obtained using other representations. The results also showed that subjects with high level of visual-spatial intelligence were quicker to encode stimuli and manipulate their visual representations so that the problem solving provided varied [25] by focusing on specific parts of the object [26].

4. Conclusion
Based on the results of the study, it can be concluded that subjects who have high level of visual-spatial intelligence in solving PISA problems: (1) use spatial reasoning in the form of spatial visualization and mental rotation both in understanding problems to the point of providing diverse problem solving strategies,(2) 76.25% of the answers obtained by students used visual representations, 20% were obtained using symbolic representations, and 3.75% were obtained using other representations.(3) spatial reasoning and representation are a unity that complements or complements
each other in producing various problem solving strategies. We hope that this research can provide contributions related to the relationship of spatial reasoning and representation in solving PISA problems for subjects who have high level of visual-spatial intelligence so that teachers in particular can develop appropriate learning models to improve their problem solving abilities.

Acknowledgments
The authors would like to thank the Head of the Department of Mathematics Education Universitas Mataram who has provided his support in writing this research paper. We also express our gratitude for the support from the Faculty of Teacher Training and Education, Universitas Mataram.

References
[1] Mix K S 2019 Why are spatial skill and mathematics related? Child Dev. Perspect. 13 121–6
[2] King M J, Katz D P, Thompson L A and Macnamara B N 2019 Genetic and environmental influences on spatial reasoning: a meta-analysis of twin studies Intelligence 73 65–77
[3] Tam Y P, Wong T T, Wai W and Chan L 2019 The relation between spatial skills and mathematical abilities: the mediating role of mental number line representation Contemp. Educ. Psychol. 56 14–24
[4] Falomir Z and Olteanu A 2019 Special issue on problem-solving, creativity and spatial reasoning Cogn. Syst. Res. 58 31–4
[5] Hawes Z, Moss J, Caswell B, Seo J and Ansari D 2019 Relations between numerical, spatial, and executive function skills and mathematics achievement: a latent-variable approach Cogn. Psychol. 109 68–90
[6] Mix K S, Levine S C, Cheng Y-L, Young C, Hambrick D Z and Ping R 2016 Separate but correlated: the latent structure of space and mathematics across development J. Exp. Psychol. Gen. 145 1206–27
[7] Moreau D 2013 Differentiating two-from three-dimensional mental rotation training effects Q. J. Exp. Psychol. 66 37–41
[8] Hawes Z, Moss J, Caswell B, Naqvi S and MacKinnon S 2017 Enhancing children’s spatial and numerical skills through a dynamic spatial approach to early geometry instruction : effects of a 32-Week intervention Cogn. Instr. 1–29
[9] Fiantika F R and Setyawati S P 2019 Representation, representational transformation and spatial reasoning hierarchical in spatial thinking J. Phys. Conf. Ser. 1321 022056
[10] NCTM 2000 Principles and Standards for School Mathematics (United States of America: NCTM)
[11] Heyden V K M, Huizinga M, Kan K and Jolles J 2016 A developmental perspective on spatial reasoning: dissociating object transformation from viewer transformation ability Cogn. Dev. 38 63–74
[12] Hawes Z, Moss J, Caswell B and Poliszczuk D 2015 Effects of mental rotation training on children’s spatial and mathematics performance: a randomized controlled study Trends Neurosci. Educ. 1–9
[13] Ramful A, Lowrie T and Logan T 2016 Measurement of spatial ability: construction and validation of the spatial reasoning instrument for middle school students J. Psychoeduc. Assess. 35 709–27
[14] Utami C T P, Mardiyana and Triyanto 2019 The identification of visual representation ability of junior high school students in solving geometry problems J. Phys. Conf. Ser. 1180 012014
[15] Nazareth A, Killick R, Dick A S and Pruden S M 2018 Strategy selection versus flexibility: using eye-trackers to investigate strategy use during mental rotation J. Exp. Psychol. Learn. Mem. Cogn.
[16] Lutfitasari A, Amin S M and Masriyah 2018 Students’ spatial reasoning in solving geometrical problems based on personality types Proceedings of the Mathematics, Informatics, Science, and Education International Conference (MISEIC 2018) vol 157, ed R Ekawati (Atlantis
[17] Geary D C, Hoard M K, Nugent L and Rouder J N 2015 Journal of experimental child individual differences in algebraic cognition: relation to the approximate number and semantic memory systems J. Exp. Child Psychol. 140 211–27

[18] Lombardi C M, Casey B M, Pezaris E, Jong M, Mcpherran C, Casey B M and Pezaris E 2019 Longitudinal analysis of associations between 3-D mental rotation and mathematics reasoning skills during middle school: across and within genders J. Cogn. Dev. 1–23

[19] Niroo M, Nejad G H H and Haghani M 2012 The effect of gardner theory application on mathematical/logical intelligence and student’s mathematical functioning relationship Procedia - Soc. Behav. Sci. 47 2169–75

[20] Liang C, Liu Y, Chang Y and Liang C 2019 Differences in numeric, verbal, and spatial reasoning between engineering and literature students through a neurocognitive lens Cogn. Syst. Res. 1–20

[21] Casey B M, Lombardi C M, Pollock A, Fineman B and Pezaris E 2017 Girls’ spatial skills and arithmetic strategies in first grade as predictors of fifth-grade analytical math reasoning J. Cogn. Dev. 18 1–26

[22] Battista M T, Frazee L M and Winer M L 2018 Analyzing The Relation Between Spatial and Geometric Reasoning for Elementary and Middle School Students ed K S Mix and M T Battista (Springer, Cham)

[23] Hijriani L, Rahardjo S and Rahardi R 2018 Deskripsi representasi matematis siswa SMP dalam menyelesaikan soal PISA J. Pendidik. Teor. Penelitian, dan Pengemb. 3 603–7

[24] Miles M B and Huberman A M 1994 Qualitatif Data Analysis: An Expand Sourcebook Second Edition ed R Holland (London: SAGE Publication Ltd)

[25] Zhao B and Sala S Della 2017 Different representations and strategies in mental rotation Q. J. Exp. Psychol. 1–26

[26] Xu Y and Franconeri S L 2015 Capacity for visual features in mental rotation Psychol. Sci. 26 1241–51