Magnification of signatures of topological phase transition by quantum zero point motion

Pedro L. e S. Lopes¹ and Pouyan Ghaemi²

¹Instituto de Física Gleb Wataghin, Universidade Estadual de Campinas, Campinas, SP 13083-970, Brazil
²Physics Department, City College of the City University of New York, New York, NY 10031

In this letter we show that the zero-point motion of a vortex in superconducting doped topological insulators leads to significant changes in the electronic spectrum at the topological phase transition in this system. This topological phase transition is tuned by the doping level and the corresponding effects are manifest in the density of states at energies which are of the order of the vortex fluctuation frequency. This frequency could be much larger than the electronic energy gap in the spectrum generated by a stationary vortex. As a result the quantum zero-point motion can move the spectral signatures of the topological vortex phase transition to energies which are well within the resolution of scanning tunneling microscopy. Moreover, the phenomena studied in this letter present novel effects of Magnus forces on the vortex spectrum which are not present in the ordinary s-wave superconductors. Our results show that quantum zero point fluctuations can bring the fingerprints of topological phase transitions to more experimentally accessible grounds and point to the importance of study of quantum fluctuations in different candidates for realizing topological phase transitions.

Introduction. - Topologically distinct phases which cannot be classified by the classical Landau paradigm comprise some of the most recently discovered states of matter[1–3]. An important signature of these topological phases is the appearance of novel edge states; one such state being the so-called Majorana bound state on the edges of topological superconductors [4]. As ubiquitous signatures, their detection have been the main trend in the characterization of such topological phases. Although evidences of Majorana fermions have been identified in tunneling [5] and scanning tunneling microscopy (STM) measurements[6], the interpretation of their signatures is controversial in many cases, as the imprints from the phase transition are often mixed with signals from disorder and extra undesired quasiparticles.

While the aforementioned gapless edge states act as a signature of topological phases, the signatures of the transition from a topologically trivial to a topological phase present themselves in the bulk by the closing and re-opening of the excitation gap[7–9]. In many of the proposed systems which can be tuned through a topological phase transition (TPT), the excitation gap is very small compared with experimental resolutions. In this letter, we show that quantum fluctuations can shift the spectral weight before and after the TPT to further separated energies and as a result magnify the change of the spectrum resulting from this process. We discuss this effect in the context of the chemical potential induced topological phase transition in the vortices of superconducting doped topological insulators. In this particular situation, we also demonstrate how the effects of Magnus forces on the vortex dynamics[10] have a novel signature in the spectral change at this TPT, exposing the pumping of vortex modes responsible for the phase transition as described below. Our general results, however, could be extended to other types of topological phase transitions by identifying the corresponding quantum zero-point fluctuations.

The dynamics of vortices in superconductors (SCs) has been widely studied[11, 12]. Given the natural length scale of vortices, their different properties might display both classical and quantum phenomena. It is well known that the pinning of vortices is necessary for stability of type-II SCs. Within the BCS theory of superconductivity, an stationary vortex affects the spectrum of the superconductor by generating in-gap modes localized around the vortex core[13]. The energy of these discrete bound states, known as Caroli-de Gennes-Matricon (CdG) modes, is given by \( \epsilon_l = \Delta^2 \mu \left( l + \frac{1}{2} \right) \) where \( \Delta \) is the size of the gap, \( \mu \) is the fermi energy and \( l \) is an integer. The signatures of these in-gap states have been experimentally observed by STM measurements[14, 15]. In practice, however, even though the spatial resolution of STM is well within the size of the vortex modes[16], given the small size of their so-called mini-gap, \( \delta = \frac{\Delta^2}{\mu} \), the energy of each single mode is hard to be resolved and usually multiple modes are observed together[14]. Although a pinned vortex does not move across the sample, its quantum zero point motion cannot be ignored. Interestingly, it was shown that such quantum fluctuations will move part of the spectral weights of the in-gap vortex modes to the frequencies associated with vortex fluctuations[17–19].

The discovery of superconductivity in doped TIs triggered several studies, particularly because of the suggestions that doped TIs might realize topological superconductivity[20–24]. Theoretical studies of superconductivity in the surface states of TIs started before the experimental realization of bulk superconductivity in doped TIs, when it was shown that, theoretically, if superconductivity is induced in their helical surface states, the vortex modes will include a zero-energy Majorana bound state[25]. Motivated by the discovery of bulk superconductivity in doped TIs, it was shown later that the Majorana mode at the end of a vortex line persists up to
a critical value of doping in these systems as well[26–28]. At this critical doping level, the two Majorana modes at the two ends of the vortex hybridize and become gapped. The presence or absence of Majorana modes at the end of the vortex line differentiate a topologically trivial from a non-trivial phase, correspondingly. In fact, the vortex in doped superconducting TIs becomes effectively equivalent to a Kitaev chain, one of the pioneering theoretical models to realize topological phases and phase transitions with Majorana edge states[29].

The original mechanism for the TPT lies in the CdG modes, localized along the vortex core in the bulk of the system. The important property of these states is that they are gapped by the small energy scale of the mentioned mini-gap. This energy protects the surface Majorana zero modes, confining them to the surface of the sample. It was then shown that because of the strong spin-orbit coupling and the resulting band inversion in bulk bands of TIs[30], the Fermi surface of doped topological insulators has non-trivial topological properties which shows up as a non-zero Berry connection on the Fermi surface. This leads to a modification of the energy of CdG modes in superconducting doped TIs. The CdG Fermi surface. This leads to a modification of the energy which shows up as a non-zero Berry connection on the logical insulators has non-trivial topological properties which requires the formation of Majorana edge states[29].

The important property of these states is that they are gapped by the small energy scale of the mentioned mini-gap. This energy protects the surface Majorana zero modes, confining them to the surface of the sample. It was then shown that because of the strong spin-orbit coupling and the resulting band inversion in bulk bands of TIs[30], the Fermi surface of doped topological insulators has non-trivial topological properties which shows up as a non-zero Berry connection on the Fermi surface. This leads to a modification of the energy of CdG modes in superconducting doped TIs. The CdG modes are also separate in two sets due to the degenerate TI Fermi surfaces, as $E_{l}^{\pm} = \pm \frac{\Phi_{0}(\mu)}{2\pi} \left(1 \mp \frac{1}{2} \mp \frac{\Phi_{0}(\mu)}{2\pi}\right)$. Here $\Phi_{0}$ is the Berry phase around the curve on a Fermi surface defined by setting the wave-vector along the vortex line equal to zero. In this case, when $\Phi_{0} = \pi$, $E_{0}^{\pm} = 0$ and the zero energy surface Majorana modes at the ends of the vortex can merge through the gapless $l = 0$ mode which is now extended along the vortex.

As an example to understand the experimental limits for observing the signature of this TPT we use copper doped Bi$_{2}$Sr$_{2}$CaCu$_{2}$O$_{8}$+ which was the first topological insulator found to become superconducting upon doping, at 3.8 K[20]. In this material, spatially resolving the local density of states (LDOS) signature for the superconducting doped TI, the first term is a Bogoliubov-de Gennes (BdG) action for the superconducting doped TI,

$$S = \frac{1}{2} \int d^2 \mathbf{r} d\tau \bar{\Psi}^\dagger \left( \partial_\tau + H_{BdG} \right) \Psi$$

where

$$H_{BdG} = \left[ \begin{array}{cc} H_{TI} - \mu & \Delta \left( \mathbf{r} - \mathbf{R} (\tau) \right) \\ \Delta^\dagger \left( \mathbf{r} - \mathbf{R} (\tau) \right) & -H_{TI} + \mu \end{array} \right].$$

Here $\mu$ is the chemical potential and the Nambu spinor is $\Psi = (\psi, is_n \psi^\dagger)^T$, with $\psi = (\psi_{A\uparrow}, \psi_{A\downarrow}, \psi_{B\uparrow}, \psi_{B\downarrow}). A, B$ are orbital indices. The effective low energy, inversion symmetric, TI Hamiltonian is given by $H_{TI} = -iv_{D} t \tau \cdot \mathbf{R} \mathbf{\nabla} + \tau_z (m + \epsilon \mathbf{\nabla}^2)$ and $\tau$ and $s$ Pauli matrices acting on orbital and spin Hilbert spaces, respectively, and we define $\rho$ as the Pauli matrix acting on the Nambu spinors associated with the BdG equation. The superconducting pairing $\Delta \left( \mathbf{r} - \left( \mathbf{R} (\tau) \right) \right)$ can be a vortex profile centered at a fluctuating position $\mathbf{R} (\tau) = (x(\tau), y(\tau))^T$ whose dynamics is governed by $S_{\nu,\tau}^{\text{vortex}}[17]$

$$S_{\nu,\tau}^{\text{vortex}} = \frac{m_v}{2} \int \frac{d\omega}{2\pi} R^\dagger (i\omega) \left( \omega^2 + \omega_c^2 \begin{array}{cc} -\omega_c \omega & \omega_c \omega \\ -\omega_c \omega & \omega_c \omega \end{array} \right) R (i\omega).$$

Physically, the action (3) describes a particle of mass $m_v$ oscillating in an harmonic trap of frequency $\omega_c$ which depends on the properties of the trapping potential. This oscillator frequency dictates the qualitative features of the energy peak distribution of the LDOS. Finally, $\omega_c$ corresponds to a Magnus force acting on the vortex. The frequency $\omega_c$ will be shown to play an essential role. It breaks the $z$-mirror symmetry of $H_{TI}$, present at vanishing $z$-wavevector (see [26] or supplementary material for details), and leads to distinguished signatures of the VQPT in measurements of the LDOS.

To capture the coupling between electronic excitations and vortex fluctuations, we expand the superconducting pairing around the vortex rest position $\Delta (\mathbf{r} - \mathbf{R} (\tau)) \approx \Delta (\mathbf{r}) - \partial_\tau \Delta (\mathbf{r}) \cdot \mathbf{R} (\tau)$. This approximation is valid at weak-coupling[17]. Within this formalism, the full problem is described by a perturbative action $S = S_0 + S_{\nu,\tau}^{\text{vortex}} + S_{\text{int}}$, where $S_0$ is given by (1) with $\mathbf{R}(\tau) = 0$ and the interaction term is given by

$$S_{\text{int}} = -\int d^2 r d\tau \mathbf{R} (\tau) \cdot \bar{\Psi} \left( \begin{array}{cc} 0 & \partial_\tau \Delta \\ \partial_\tau \Delta^\dagger & 0 \end{array} \right) \Psi.$$  

The interaction between vortex modes and the fluctuations in the vortex position leads to a self-energy correction to the energy of the CdG modes.

Electronic self-energy.– Assuming a singlet intravalley pairing for doped TIs, the VQPT was found originally by an exact diagonalization of lattice toy models and a semi-classical study of the BdG mean-field
Hamiltonian [26], as well as numerically solving the self-consistent BdG equations [28]. In order to study the effects of vortex fluctuations, we first present the VQPT by an approximated real-space diagonalization of the BdG equation associated with the Hamiltonian (2) [31]. We expand the Grassmann fields in terms of eigenvectors of the BdG Hamiltonian for a stationary vortex configuration at \( r = 0 \), \( H_{BdG}^0 \), as \( \Psi = \sum_{q=1}^{\infty} \sum_n \chi_n^q (r) \psi_n^q (\tau) \) where \( H_{BdG}^0 \chi_n^q (r) = E_n^q \chi_n^q (r) \) and \( n \) labels conserved quantum numbers. At weak coupling, we further project into the two bands which cross the doubly degenerate Fermi surface of \( H_{TI} \). Labeling these states by \( \sigma = \pm \), we have \( \Psi \approx \sum_n \chi_n^+ (r) \psi_n^+ (\tau) + \chi_n^- (r) \psi_n^- (\tau) \). The numerical diagonalization may be done by replacing the infinite system with a disk of finite radius \( R \) with superconducting gap of the form \( \Delta_0 (r) = \Delta_0 \tanh (r/\xi) \) for the vortex and Fourier-Bessel expanding the components of \( \chi_n^\sigma (r) \) [17, 31], see supplementary material.

To study the VQPT we consider the lowest energy vortex modes. Then \( n \rightarrow l \) denotes eigenvalues of \( L_z = -i \partial_\tau - \frac{\mu}{2} \), which commutes with the Hamiltonian (see [26] or the supplementary material.) The two sectors (labeled by \( \sigma = \pm \)) are connected by particle-hole (PH) conjugation \( C = \rho_n s_n K \) operator (\( K \) is the complex conjugation operator) as \( C \chi_n^\sigma = \chi_{-n}^\sigma \). The energies of the CdG vortex modes in this case are the expected \( E_l^\pm = \frac{\Delta_0}{2} \left( 1 \mp \frac{\Delta_0}{2} + \frac{\mu}{2} \right) \), so that \( E_l^+ = -E_l^- \).

Here \( \Phi_0 (\mu) \) is the Berry phase calculated around the Fermi surface on the curve with zero wavevector along the vortex [26]. As the chemical potential increases, the Fermi surface enlarges and \( \Phi_0 (\mu) \) varies from 0 to \( 2\pi \), defining a critical chemical potential such that \( \Phi_0 (\mu_{cr}) = \pi \). Our results for the energies of the CdG modes, which are presented in Fig. 1, are consistent with the previous study of the phase transition in Refs. [26] and [28].

In terms of the CdG eigenstates, equation (4) becomes

\[
S_{int} = - \sum_{l,l',\sigma} \int d\tau \tilde{\psi}_l^\sigma (\tau) \psi_{l'}^\sigma (\tau) R (\tau) \cdot M_{l,l'}^{\sigma},
\]

where

\[
M_{l,l'}^{\sigma} = \int d^2 r \chi_l^{\sigma} (r)^\dagger \begin{pmatrix} 0 & \partial_\tau - \frac{\omega}{2} \\ \partial_\tau + \frac{\omega}{2} & 0 \end{pmatrix} \chi_{l'}^{\sigma} (r).
\]

Vortex fluctuations then generate the following self-energy for CdG vortex modes which we calculate using the GW approximation [32] (see the supplementary material),

\[
\Sigma_l^\sigma (i\omega) = \sum_{l'} \sum_{\alpha = \pm} A_{l,l'}^{\alpha,\sigma} \frac{\omega - \text{sgn} (\Xi_l^\alpha) \omega_v + E_l^\alpha - \alpha \omega_c/2}{\omega^2 + \omega_v^2 + \omega_c^2/4}.
\]

Here \( A_{l,l'}^{\alpha,\sigma} \equiv \frac{|M_{l,l'}^{\alpha,\sigma}|^2}{m_v \omega_v} \) are reduced matrix elements with \( M_{m,m'}^{\alpha,\sigma} = \frac{1}{2} \left( M_{m,m'}^x + \alpha i M_{m,m'}^y \right) \) and \( \Xi_l^\sigma = E_l^\sigma + \alpha \omega_c/2 \). For unit vorticity, angular momentum conservation implies that \( l \) is connected only to \( l \pm 1 \) by such interactions. The energy scale introduced by \( \omega_v \equiv \sqrt{\omega_0^2 + \omega_c^2/4} \) (and dominated by \( \omega_0 \) as aforementioned), represents a “magneto-plasma” frequency in an Einstein model [17].

From the self-energy we can readily derive the density of states,

\[
\rho (r, \omega) = \sum_{\sigma, l} |\mathbf{u}_l^\sigma (r)|^2 \delta (\omega - E_l^\sigma - \Sigma_l^\sigma (\omega)).
\]

Here \( \mathbf{u}_l^\sigma (r) \) is the upper Nambu component of the wavefunctions \( \chi_l^\sigma (r) = \begin{pmatrix} \mathbf{u}_l^\sigma (r) \ T \mathbf{v}_l^\sigma (r) \end{pmatrix} \).

Through the perturbative interaction, the energy density profile of CdG modes is modified with part of the spectral weight from \( \omega = E_l^\sigma \) being transferred to new “satellite” peaks in the LDOS [17]. Both the spectrum \( E_l^\sigma \) and the profile of \( \mathbf{u}_l^\sigma (r) \) dramatically change the phenomenology described by (8) when the parent metallic state of the superconductor comes from doped TIs, as compared with ordinary metals. Given the atomic level resolution of STM, we can safely focus at the density of states at the vortex core \( r = 0 \). Generally, the wavefunction components may be expanded in terms of Bessel functions. In particular, at \( r = 0 \), only Bessel functions of order zero have non-zero amplitude while all the other Bessel functions vanish. In the Fourier-Bessel expansion of the CdG modes, only \( l = 0 \) and, as a result of spin-orbit
Figure 2. Differential conductivity for $\mu < \bar{\mu}_\pm$, and $\mu > \bar{\mu}_\pm$ in blue, and $\bar{\mu}_- < \mu < \bar{\mu}_+$, in red. The large central peaks correspond to $\omega \approx E_{l=0}^\pm$ and $\omega \approx E_{l=1}^\pm$ (the energies of the CdG modes for stationary vortex). The inset displays the effects of vortex fluctuations. The smaller satellite peaks appear at energies close to $\approx \pm \omega_c$.

coupling, $l = 1$ are the modes which have finite contribution for the zeroth Bessel function in $u_l^0(\mathbf{r})$ (see supplementary material). As $E_l^0 = \sigma \frac{2}{\xi_F} \left( -\frac{1}{2} + \frac{2\mu}{\pi} \right)$, these energy levels may be pumped from negative to positive values (and vice versa) by changing the chemical potential as the Berry phase evolves from 0 to $2\pi$. This novel feature leads to a change of sign of the factors of $\omega_c$ in the self-energy given in (7), which determines the energies of the satellite peaks. As a result, the TPT manifests itself by a discontinuous jump of the density of states from energies of order $\omega_c$ to energies of order $-\omega_c$. It is important to note that the Magnus force term associated with the vortex motion, whose amplitude is proportional to $\omega_c$, breaks the mirror symmetry which is connecting the PH sectors of the CdG modes. As a result, the discontinuous transition of energy of the CdG modes from particle to hole sector and the PH conjugate states from hole to particle sector does not happen simultaneously for both cases. This is essential for the change in the LDOS as it provides a energy window over which the density of states at the energy of vortex oscillations is remarkably modified by the TPT. It is also important to note that, for other CdG modes (such as the mode with $l = -1$ whose maximum amplitude is at $1/k_f \approx 10\AA$), the opposite transition will happen. Given the spatial resolution of STM, however, the different modes are well resolvable.

Vortex fluctuation modifies the critical chemical potential which can be determined by $\frac{\partial \langle \bar{\mu}_l \rangle}{\partial \sigma} = 1 + \eta \sigma$ where $\eta = \omega_c / \xi_F^2 < 1$ is a dimensionless parameter which depends on the magnus force (see the supplementary material). It is then clear that the two PH sectors (identified by $\sigma = \pm$) pass through the TPT at different chemical potentials. Figure 2 shows the differential conductivity at the vortex center $G(\mathbf{r} = 0, \omega) = -\frac{e^2}{h} \int d\omega' \rho \left( \mathbf{r} = 0, \omega + \omega' \right) f(\omega')$, where $G_0$ is the tunneling conductance, $\rho_0 = m_e/2\pi$ is the density of states of a free 2D electron gas and $f(\omega)$ is the Fermi-Dirac distribution (more details on numerical parameters are given in the supplementary material). Angular momentum conservation implies that each non-interacting energy level unfolds into a set of three peaks. We present the differential conductance for three ranges of chemical potential $\mu < \bar{\mu}_\pm$ in blue, $\bar{\mu}_- < \mu < \bar{\mu}_+$ in red, and $\mu > \bar{\mu}_\pm$ in blue again which appears to be identical to $\mu < \bar{\mu}_\pm$ as expected. The pattern should be of a central peak located at $E_l^\sigma$ with the two partners offset approximately by $\pm \Omega_l^\sigma$ with $\Omega_l^\sigma = \sqrt{\left(\omega_c + \delta + \omega_c / 2\right)^2 + 2A_{l=0}^\sigma}$. In our case, a total of 12 peaks is expected for each value of the chemical potential, not all of them being resolvable due to thermal effects. The large peaks closest to $\omega = 0$ correspond to $\omega \approx E_{l=-1}^0$. The strength of the respective satellite peaks is suppressed by a $\xi^{-5}$ factor, where $\xi$ is the coherence length [17]. An inset displays the position of these peaks.

The remarkable behavior develops in the $l = 1$ satellite peaks. A novel signature is evidenced by the solitary blue peak at positive $\omega$ which corresponds to $\sigma = -$ sector when $\mu < \bar{\mu}_-$ and $\sigma = +$ sector for $\mu > \bar{\mu}_+$ but is absent when chemical potential is between the two critical values for the two PH sectors (i.e. $\bar{\mu}_- < \mu < \bar{\mu}_+$). The approximate positions of the $l = 1$ satellite peaks can be determined analytically and it may be shown that when $\mu$ moves above $\bar{\mu}_-$, the contribution from $\sigma = -$ jumps alone by an energy of approximately $-2\omega_c$ (see supplementary materials). Evidence of the jump also arises from the enhancement of the $l = 1$ negative frequency peak, as it gains contributions from the two $\sigma$ sectors. In fact, this peak does not jump alone, as one might expect. The partner jumping peak corresponds to $l = -1$ and has no weight at $r = 0$. As the chemical potential is increased further and reaches values larger than $\bar{\mu}_+$, the $\sigma = -$ contribution jumps by $2\omega_c$ and replaces the $\sigma = -$ contribution in positive frequencies. As a result the total density of states takes the same form for $\mu < \bar{\mu}_-$ and $\mu > \bar{\mu}_+$ but will be strongly modified for $\bar{\mu}_- < \mu < \bar{\mu}_+$. It is important to point out that the frequency of vortex oscillations can be indeed increased depending on the properties of the pinning potential. Recent developments in doping TIs with Niobium which leads to the formation of magnetic moments in the bulk superconducting TI, can provide stronger pinning and so larger frequencies for the vortex fluctuation [33].

Cryogenic STM measurements are fundamental to uncover these signatures. Situations with lighter and smaller vortices, whose zero-point motion effects would be stronger, could also be arranged as the vortex size is known to be strongly sensitive to temperature and magnetic field strength [34]. For vortices of too minute sizes
the Taylor expansion method to derive the interactions is not precise. In such cases, different approaches to the problem, such as used in [18], are necessary in order to obtain trustworthy predictions.

In summary we studied the effects of transverse quantum zero point motion of vortices on the electronic spectra of superconducting doped TIs. For a stationary vortex, the change in the spectrum at the topological vortex phase transition happens by shifting part of the spectral weight from the mini-gap $\Delta_0^+ \nu$ energy scale through zero energy. Such energy shift is much smaller than the resolution of STM. On the other hand, coupling to vortex fluctuations will bring the shift of spectrum to energies of order of vortex oscillation frequency which can be well resolvable. Topological phase transitions have been predicted in multiple different models and the study of relevant quantum fluctuations and their effect in enhancing the phase transition signatures in these models will be subject of future works.

**ACKNOWLEDGMENTS**

The authors acknowledge insightful discussions with V. L. Quito, S. Sachdev, S. Gopalakrishnan, V. P. Nair, M. Sarachik and A. P. Polychronakos. PLSL acknowledges support from FAPESP under grant 2009/18336-0.

**SUPPLEMENTARY MATERIAL**

A. Caroli-de Gennes modes

Here we present our numerical method to derive the spectrum of vortex modes for a stationary vortex in doped superconducting topological insulator [26], and compare the solutions with an analytical approximated symmetry given by $M = \rho_0 \tau_0 \sigma_0$. Noticing that $\{C, M\} = 0$ and naturally $\{C, H_{BDG}\} = 0$, we see that the eigenvalues of $M$ also label particle and hole partners. This allows one to separate $\phi_{l, n} (r)$ in four-spinors $\Phi_{l, n} (r)$, obeying corresponding Schrodinger’s equations with projected Hamiltonians $H^\pm [26]$.

$$H^\pm \phi_{l, n} = E_{l, n}^\pm \phi_{l, n}.$$  \hspace{1cm} (13)

We focus in $\phi_{l, n}^\pm$, noticing that $\phi_{l, n}^\pm = C \phi_{-l, n}^\pm$ with $E_{l, n} = -E_{-l, n}$. The 4 \times 4 reduced radial Hamiltonian reads

$$H^\pm = \rho_x \nu_y \left[ -i \partial_r + i \nu_z \frac{1}{r} \left( l - \rho_x + \nu_z \right) \right] - \mu \rho_z - \Delta_0 (r) \rho_x$$

$$+ \rho_x \nu_z \left[ m + \epsilon \left( \frac{1}{r} \partial_r r \partial_r - \frac{1}{r^2} \left( l - \rho_x + \nu_z \right)^2 \right) \right].$$  \hspace{1cm} (14)

Here $\nu$ Pauli-matrices represent a spin-orbital coupled space. Noticing that

$$a_l = \left( \partial_r + \frac{l}{r} \right)$$  \hspace{1cm} (15)

$$a_l^\dagger = - \left( \partial_r - \frac{l - 1}{r} \right)$$  \hspace{1cm} (16)

act as operators which lower and raise the level of Bessel functions (and $a_l^\dagger a_l$ gives the Bessel differential operator itself), it is easy to find a proper basis to expand the states. If $\Delta_0 = 0$, we recover a pair of topological insulator Hamiltonians with spectra given by $E_k^\pm = \pm \mu \pm \sqrt{k^2 + m_k^2}$, with $m_k = m - e k^2$ and $k$ a "radial
Schrodinger's equation reduces to
\[ \Phi^{\pm}_{n} = e^{\mp i\frac{l}{2}} \Phi^{\pm}_{n}(\mu) \Phi^{\pm}_{n} \]
where
\[ f_{l,k}(r) = \frac{1}{\sqrt{N_{k,l}^{+}}} \left( \frac{k J_{l-1}(kr)}{m_{k} - \sqrt{m_{k}^{2} + k^2}} J_{l}(kr) \right) \]
and
\[ g_{l,k}(r) = \frac{1}{\sqrt{N_{k,l}^{-}}} \left( \frac{(m_{k} + \sqrt{m_{k}^{2} + k^2}) J_{l}(kr)}{-k J_{l+1}(kr)} \right) \]
and the spinor is \[ \Phi^{\pm}_{n} = \{ c_{l,k}^{n}, d_{l,k}^{n} \}^{T} \].

In terms of our original variables, the wavefunctions are then written
\[ \chi^{\pm}_{n}(r) = \begin{pmatrix} u^{+}_{n}(r) \\ v^{+}_{n}(r) \end{pmatrix}, \]
where
\[ u^{+}_{n}(r) = \int \frac{dk}{2\pi N_{k}^{+}} e^{-il(l+1)\theta} k J_{l-1}(kr) \begin{pmatrix} 0 \\ -e^{il\theta} (m_{k} - \sqrt{m_{k}^{2} + k^2}) J_{l}(kr) \end{pmatrix} \]
\[ v^{+}_{n}(r) = \int \frac{dk}{2\pi N_{k}^{-}} e^{-il\theta} (m_{k} + \sqrt{m_{k}^{2} + k^2}) J_{l}(kr) \begin{pmatrix} 0 \\ k e^{-il(l+1)\theta} J_{l+1}(kr) \end{pmatrix} \]
and the mirror (particle-hole) partners are built from \[ \chi^{\pm}_{n}(r) = C \Psi^{\pm}_{n}(r) \].

We then fix a finite radius \( R \) for the cylinder size which forces us to discretized \( k \to \alpha_{l,j}/R \) where \( \alpha_{l,j} \) are the \( j \)-th Bessel zeroes at each \( l \) subspace. We fix a UV cutoff at some (large) \( N_{0} \)-th Bessel zero. Diagonalizing the resulting Hamiltonian leads to the spectrum shown in 3. One sees two in-gap modes, one corresponding to outer edge modes, which we neglect, while the other corresponds to our desired vortex modes, as can be checked by plotting their respective probability densities.

For the low energy states, \( n \equiv n_{CAG} \) (a label which we drop from now on), one may check that the spectrum follow the expected
\[ E^{\pm}_{l} = \frac{\Delta^{2}}{E_{F}} \left( l + \frac{1}{2} \pm \frac{\phi(\mu)}{2\pi} \right) \]
where \( \phi(\mu) \) is the chemical potential dependent Berry’s phase. At the critical chemical potential \( \phi(\mu_{c}) = \pi \). It grows monotonically from 0 to \( 2\pi \) with the chemical potential. Noticing that the values of the momentum in \( k \)-space are strongly localized at its Fermi value \( k_{F} \), as one might expect, it is easy to guess an analytical approximation for the wavefunctions which satisfies their desired asymptotic behaviors (see [26] for details). We have
\[ \chi^{+} = C e^{-\frac{i}{\xi} J_{l}(kr)} \begin{pmatrix} f(\theta, r) \\ g(\theta, r) \end{pmatrix}, \]
where
\[ f(\theta, r) = C e^{-i(l-1)\theta} k_{F} J_{l-1}(k_{F}r) \]
\[ g(\theta, r) = C e^{-i(l+1)\theta} k_{F} J_{l+1}(k_{F}r) \]
and the new normalizations read
\[ N_{k_{F}}^{\pm} = 2 \left( k_{F}^{2} + m_{k_{F}}^{2} \pm m_{k_{F}} \sqrt{m_{k_{F}}^{2} + k_{F}^{2}} \right). \]
Here, \( C \) is a normalization constant of order \( (k_{F}/\xi)^{1/2} \). In the main text we compare the analytical and numerical results for the wavefunction at \( l = 0 \) and \( \mu \approx \mu_{c} \), showing that the approximation indeed works.
which diagonalizes the Lagrangian density as
\[ S_{eff}^{vortex} = \int \frac{d\omega}{2\pi} \left( R^\dagger (i\omega), R_+ (i\omega)^\dagger \right) \mathbb{D}_0^\dagger (i\omega)^{-1} \left( R_- (i\omega), R_+ (i\omega) \right), \]

with
\[ \mathbb{D}_0^\dagger (i\omega) = \left( \begin{array}{cc} D^{-1} & 0 \\ 0 & D^{-1} \end{array} \right), \]

and the Green’s functions
\[ D_{\pm} (i\omega) = \frac{m_v}{2} (\omega \pm i\omega_c)^2 + \omega_0^2. \]

This sets the two important energy scales dictated by the vortex fluctuations as \( \omega_v \), from the Magnus force, and \( \omega = \sqrt{\omega_0^2 + \omega_v^2/4} \), from the harmonic trap.

As discussed in the former section, the low-energy modes divide into two Hilbert space sectors related by a \( z \)-mirror/particle-hole symmetry. Each sector is subject to an effective potential arising after the integration of the vortex 0D field theory. From equations 3 we may write:

\[ e^{-V_{eff}^\sigma} \left[ \hat{\psi}_\sigma^\dagger \hat{\psi}_\sigma \right] \propto \int \mathcal{D} [\mathbf{R}] e^{-S_{eff}^{vortex} + \int d\tau \sum_{l,t'} R(\tau) \mathbf{M}_{l,t'}^\sigma \hat{\psi}_\sigma^\dagger (\tau) \hat{\psi}_\sigma^\tau (\tau)}. \]

Define \( U^\sigma (i\omega) = \sum_{l,t'} \int \frac{d\nu}{2\pi} \mathbf{M}_{l,t'}^\sigma \hat{\psi}_\sigma^\nu (i\nu + i\omega) \hat{\psi}_\sigma^\nu (i\nu) \) and rewrite the scalar products in terms of the \( R_{\pm} (i\omega) \) coordinates and \( M^{\alpha;\sigma} = \frac{1}{2} (M_x + \alpha i M_y) \), with \( \alpha = \pm \). Then

\[ M_{l,t'}^{\sigma;\tau} = \left( M_{l',t}^{\tau;\sigma} \right)^* \int d^2r \left[ u^\tau_\sigma (r)^\dagger \partial_3 \Delta v^\nu_\sigma (r) + v^\nu_\sigma (r)^\dagger \partial_3 \Delta u^\tau_\sigma (r) \right]. \]
where $U_\alpha = \frac{1}{2} (U_x + \alpha i U_y)$. A tedious but straightforward simplification leads to the effective electronic self-interaction

\[
V_{\text{eff}}^\sigma \left[ \bar{\psi}^\sigma, \psi^\sigma \right] = \frac{1}{2} \sum_{l, l', n, n'} \int \frac{d\omega}{2\pi} \int \frac{d\tilde{\omega}}{2\pi} \int \frac{d\tilde{\omega}'}{2\pi} \times \nonumber \\
\bar{\psi}^\sigma_l (i\tilde{\omega} + i\tilde{\omega}') \psi^\sigma_{l'} (i\tilde{\omega}') \frac{V^\sigma_{l, l'; n, n'}}{(i\tilde{\omega})} (i\tilde{\omega}) \psi^\sigma_{n'} (i\tilde{\omega}')
\]

(37)

where

\[
V_{l, l', n, n'}^\sigma (i\tilde{\omega}) = -\frac{1}{m_v} \sum_{\alpha = \pm} \left[ \left( M_{l, l'}^{\alpha \sigma} \right)^\dagger M_{n, n'}^{\alpha \sigma} \right] \nonumber \\
\left[(\tilde{\omega} + \alpha i \omega_c/2)^2 + \omega_c^2 \right]
\]

(38)

From (23), the matrix elements have a simple form

\[
M_{m, m'}^{\alpha \sigma} = \int d^2 r \left[ u_m (r)^\dagger \partial_z \Delta v_m (r) + v_m (r)^\dagger \partial_z \Delta^1 u_m (r) \right]
\]

(39)

which also shows the convenient fact that $M_{l, l'}^{\pm \sigma} = M_{l', l}^{-\sigma}$.

Interaction (38) shows a screened Coulomb-like retarded interaction. The self-energy in the GW approximation comes now from a simple 1-loop calculation

\[
\Sigma^\sigma_l (i\tilde{\omega}) = -\sum_{l'} V_{l, l'; l'}^\sigma (0) \int \omega \nonumber \\
+ \sum_{l'} \int \omega V_{l, l'; l'} (i\tilde{\omega} - i\omega) G_{l'}^{0\sigma} (i\omega).
\]

(40)

The first term vanishes. The second must be considered with care as the pole structure is sensitive to the structure of the energy levels. An integration over the complex plane gives the self-energy of the main text

\[
\Sigma^\sigma_l (i\tilde{\omega}) = \sum_{l'} \sum_{\alpha = \pm} A_{l, l'}^{\alpha \sigma} \left( i\tilde{\omega} - \left( \text{sgn} (\Xi^\alpha_{l'} \omega_v + E^\sigma_l) - \alpha \omega_c/2 \right) \right)
\]

(41)

with

\[
K_{l, j, j'}^{\pm} = \text{sgn} (l + 1/2) (-1)^{j-j'} \frac{\alpha_j \alpha_{j'+1}}{R \left( \alpha_j^2 \alpha_{j+1}^2 \right)} \frac{M_{j, j+1}^\pm M_{j'+1, j}^\pm + \left( \frac{\alpha_{j'+1}}{R} \right)^2}{\sqrt{\left( \frac{\alpha_j}{R} \right)^2 + M_{j, j+1}^\pm}} \nonumber \\
+ \frac{1}{2} \sum_{j, j'} \frac{d_{l, j, j'} \left( E_{l, j} - E_{l+1, j} \right) K_{l, j, j'}^{\pm} - L_{l, j, j'}^{\pm} c_{l+1, j'} + L_{l, j, j'}^{\pm} c_{l, j+1'}}{d_{l, j+1, j'} - d_{l, j, j'} + 1}
\]

(42)

To calculate the matrix elements one may make use of the Feynman-Hellman relations, adapted to our Hamiltonian and in a finite cylinder. A long calculation making full use of Bessel function relations gives finally

\[
M_{l, l'}^{\pm} = \sum_{j, j'} \sum_{\alpha = \pm} \frac{A_{l, l'}^{\alpha \sigma}}{\sqrt{\left( \frac{\alpha_j}{R} \right)^2 + M_{j, j+1}^\pm}} \frac{M_{j, j+1}^\pm M_{j'+1, j}^\pm + \left( \frac{\alpha_{j'+1}}{R} \right)^2}{\sqrt{\left( \frac{\alpha_j}{R} \right)^2 + M_{j, j+1}^\pm}}.
\]

(43)
and
\[
\mathcal{L}_{j,l'}^{\pm} = \text{sgn} \left( l + 1/2 \right) \left( -1 \right)^{j-j'} \frac{2\epsilon_{jl} \alpha_{jl'} + 1}{R^3} \frac{\left( \frac{(l+1)(l+1)}{R^2} + M_{jl}^\pm M_{jl'}^\pm \right)}{\sqrt{\left( \frac{\alpha_{jl}}{R} \right)^2 + M_{jl}^\pm \left( \frac{\alpha_{jl'}^\pm}{R} \right)^2 + M_{jl'}^\pm}}.
\]

Here, \( R \) is the cylinder finite radius, \( \alpha_{jl} \) is the \( j \)-th zero of the \( l \)-th Bessel function and \( M_{jl}^\pm = m_{jl}^2 \mp m_{jl} \sqrt{m_{jl}^2 + \left( \frac{\alpha_{jl}}{R} \right)^2} \) with \( m_{jl} = m - \epsilon \alpha_{jl}/R \). The other matrix elements may be found from
\[
M_{jl}^{\pm;\sigma} = \left( M_{jl}^{\pm;\sigma} \right)^\dagger = \left( M_{jl}^{\pm;\sigma} \right)^\dagger.
\]

These expressions are very similar to Bartosch’s, corrected for spin-orbit coupled states.

\section{Peak Analysis}

Here we describe in detail the determination the relative positions of the differential conductance peaks. Starting from the LDOS definition as
\[
\rho (r, \omega) = \sum_{m, \sigma, \tau} \left| \langle \epsilon_m | \psi_{\sigma, \tau}^\dagger (r) | N_0 \rangle \right|^2 \delta (\omega - \epsilon_m),
\]
where \(|N_0\rangle\) is a \( N_0\)-particle ground-state, \(|\epsilon_m\rangle\) is an \((N_0+1)\)-particle excited state and \(\psi_{\sigma, \tau}^\dagger (r)\) is an electronic state creation operator, it is easy to show that
\[
\rho (r, \omega) = \sum_{\sigma = \pm} \rho_\sigma (r, \omega)
\]
\[
\rho_\sigma (r, \omega) = -\frac{1}{\pi} Im \sum_{m} \frac{|u_m^\sigma (r)|^2}{\omega - E_m - \Sigma_\sigma^\pm + i\epsilon},
\]

using the vortex-modes eigenbasis. STM measurements probe the differential conductance
\[
G (r, \omega) = \frac{G_0}{\rho_0} \int d\omega' \rho \left( r, \omega + \omega' \right) f' \left( \omega' \right),
\]
where \( f(\omega) \) is the Fermi distribution.

At zero-temperature this reduces simply to the LDOS, up to a constant. At finite temperature we may write
\[
G (r, \omega) / G_0 = -\frac{1}{\rho_0} \sum_{l, \sigma = \pm} \sum_{i} \frac{|u_l^\pm (r)|^2}{1 - \left( \frac{\delta M_{jl}^\pm (\omega)}{\delta \omega} \right) \Sigma_\sigma^\pm + \omega}
\]
\[
\times f' \left( \omega_{l, \sigma, 0} - \omega \right),
\]
where \( \omega_{l, \sigma, 0} \) is the \( i \)-th solution to
\[
\omega - E_l^\sigma - \Sigma_\sigma^\pm (\omega) = 0.
\]

In our case this generally represents a cubic equation with three solutions. While (53) determines where are the relative positions of the peaks in energy space, the derivatives \( \frac{\partial \Sigma_\sigma^\pm (\omega_{l, \sigma, 0})}{\partial \omega} \) will fix the peaks relative sizes.

We focus most of our analysis at \(|r| = 0\), which, from (23), means that only the states with \( l = 0, 1 \) give non-vanishing contributions. From angular momentum conservation, \( M_{l,l'}^{\pm;\sigma} = \delta_{l',l+1} M_{l,l+1}^{\pm;\sigma} \) and from \( A_{l,l'}^{\pm;\sigma} = A_{l',l}^{\pm;\sigma} \) we can read the corresponding result for \( \alpha = - \). These simplifications allow us to reduce the self-energy to just a couple relevant pieces,

\[
\Sigma_0^\pm (\omega) = \frac{A_{0,1}^{+;\sigma} + A_{0,1}^{-;\sigma}}{\omega - \text{sgn} \left( \Xi_{0,1}^{\pm;\sigma} \right) \omega_c - E_0^\pm + \omega_c/2} + \frac{A_{0,1}^{+;\sigma} - A_{0,1}^{-;\sigma}}{\omega - \text{sgn} \left( \Xi_{0,1}^{-;\sigma} \right) \omega_c - E_0^\pm + \omega_c/2},
\]

and
\[
\Sigma_1^\pm (\omega) = \frac{A_{1,2}^{+;\sigma} + A_{1,2}^{-;\sigma}}{\omega - \text{sgn} \left( \Xi_{1,2}^{+;\sigma} \right) \omega_c - E_2^\pm + \omega_c/2} + \frac{A_{1,2}^{+;\sigma} - A_{1,2}^{-;\sigma}}{\omega - \text{sgn} \left( \Xi_{1,2}^{-;\sigma} \right) \omega_c - E_2^\pm + \omega_c/2}.
\]

It is a simple job to notice that \( \text{sgn} \left( \Xi_{0,1}^{+;\sigma} \right) = \text{sgn} \left( \Xi_{1,2}^{+;\sigma} \right) = + \) and \( \text{sgn} \left( \Xi_{0,1}^{-;\sigma} \right) = - \) for whichever values of the chemical potential. The sign of \( \Xi_{0,1}^{+;\sigma} \), however, does depend on \( \mu \). To be definite, let us take \( \omega_c = \eta \Delta^2 \mu \), with \( 0 < \eta < 1 \). This allows one to define a value \( \bar{\mu}_\sigma \) at which \( \text{sgn} \left( \Xi_{0,1}^{-;\sigma} \right) \) changes. Plugging (26), we have
explicitly

\[ -\sigma + \left( \frac{\sigma (\bar{\mu}_\sigma)}{\pi} - \eta \right) = 0 \]

\[ \Rightarrow \frac{\sigma (\bar{\mu}_\sigma)}{\pi} = 1 + \eta \sigma, \]

where \( \phi (\mu) \) is the Berry phase. As it grow monotonically from 0 to \( 2\pi \), it is clear that the sector \( \sigma = + \) has a sign change at larger values of \( \mu \) than the \( \sigma = - \) sector, as long as \( \eta \neq 0 \).

To determine the relative sizes and positions of the peaks, we examine the derivatives of the self-energy, as well as equation (53) explicitly.

1. Peak sizes

The derivatives of the self-energies read, after some simplification

\[ \frac{d\Sigma^\sigma_1 (\omega)}{d\omega} = - \frac{A_{1,1}^{\sigma+}}{(\Delta\omega_1^\sigma - \delta - \omega_c/2 - \omega_v)^2} \]

\[ \frac{d\Sigma^\sigma_0 (\omega)}{d\omega} = - \frac{A_{0,1}^{\sigma+}}{(\Delta\omega_0^\sigma - \delta - \omega_c/2 - \omega_v)^2} \]

where \( \Delta\omega_\sigma^\sigma = \omega - E_\sigma^\sigma \) and \( \delta \) is the mini-gap.

The matrix elements are much smaller than the other physical quantities. Dimensional analysis and explicit manipulation of (35) shows that, at constant \( \omega_c/\Delta_0 \), these overlaps sizes depend on the coherence length as \( \xi^{-3} \) [17]. The peak sizes, nevertheless, are going to be sensitive to \( A_{1,1}^{\sigma+} \). As will be seen in the next subsection, the satellite peaks positions are dominated by the vortex oscillation frequency \( \omega_v \). Plugging \( \Delta\omega_\sigma^\sigma \approx 0 \) or \( \Delta\omega_\sigma^\sigma \approx \pm \omega_v \), one sees that \( d\Sigma^\sigma_1 (\omega)/d\omega \) is small (concretely it is \( A_{1,1}^{\sigma+}/\omega_v^2 \ll 1 \)) at \( \Delta\omega_0^\sigma \approx 0 \) while it may be larger at \( \Delta\omega_0^\sigma \approx \pm \omega_v \), going as \( \sim -A_{0,1}^{\sigma+}[\frac{1}{s^2}] \), where \( s = \frac{\pm \omega_v/2}{2\omega_v} \). The latter case reduces greatly the size of the satellite peaks from \( l = 0 \), similarly as pointed by Bartosch et al. [17].

2. peak positions

Our last goal is to explain the positions of the peaks as function of the chemical potential, demonstrating that they are much less sensitive to the matrix elements than the peak sizes and that they are mainly fixed by the vortex fluctuation frequency, which might be much larger than the other energy scales of the problem.

Simplifying the self-energy and plugging into (53), shows that independent of chemical potential, for \( l = 0 \) we have

\[ \Delta\omega_0^\sigma \left[ (\Delta\omega_0^\sigma)^2 - (\delta + \omega_c/2 + \omega_v)^2 + (A_{0,1}^{\sigma+} + A_{-1,0}^{\sigma+}) \right] + (\omega_v + \delta + \omega_c/2) \left( A_{0,1}^{\sigma+} - A_{-1,0}^{\sigma+} \right) = 0. \]

Using \( A_{0,1}^{\sigma+} \approx A_{-1,0}^{\sigma+} \) we get results similar to reference [17] for ordinary s-wave superconductor. Since the ma-

\[ \Delta\omega_0^{\sigma+} = 0 \]
For $\mu > \mu_+$ the role of $+$ and $-$ in the sum equations are exchanged. Since the total density is the sum of contributions form both $\sigma = \pm$ sectors, the LDOS in the two regions of $\mu < \mu_-$ and $\mu > \mu_+$ are the same.

We now get to the most important regime of $\mu_+ < \mu < \mu_-$. The position of the peaks for both $\sigma = \pm$ sectors are at

$$
\Delta \omega_1^\sigma = (\omega_v + \delta + \omega_c/2) \quad (73)
$$

Clearly, as $\mu$ crossed $\mu_-$ the third peak for $\sigma = -$ sector is shifted by $-2\omega_v$ and this to a clear modification of LDOS which persists up the $\mu = \mu_+$ at which the peak form the $\sigma = +$ sector moves by $2\omega_v$ and recovers the original LDOS.

The “creation” of a satellite peak at positive energy should not happen without an accompanying compensation of a positive energy peak jumping into negative energies. Indeed, such a compensation does occur for the contribution of $l = -1$ (which exchanging angular momentum with the vortex motion is connected to $l = -2$ and $l = 0$, the latter giving the jump.) It just turns out that, since the spatial dependence of the LDOS is determined by $V_l^\sigma(r)$, as can be seen from (51), the peaks from $l = -1$ do not contribute to the LDOS at the center of the vortex, $r = 0$. The peaks from $l = -1$ should contribute to the LDOS at a distance $\sim k_F^{-1}$ from the vortex center, which should be of the order of ten Angstroms in a superconducting TI. This can be resolved with the current STM technology.

References:

[1] X. G. Wen, Int. J. Mod. Phys. B 04, 239 (1990)
[2] M. A. Levin and X.-G. Wen, Phys. Rev. B 71, 045110 (Jan 2005), http://link.aps.org/doi/10.1103/PhysRevB.71.045110
[3] F. Burnell and S. H. Simon, Annals of Physics 325, 2550 (2010), ISSN 0003-4916, http://www.sciencedirect.com/science/article/pii/S0003491610001107
[4] F. Wilczek, Nature Physics 5, 614 (2009)
[5] V. Mourik, K. Zuo, S. M. Frolov, S. R. Plissard, E. P. A. M. Bakkers, and L. P. Kouwenhoven, Science 336, 1003 (2012), http://www.sciencemag.org/content/336/6084/1003.full.pdf, http://www.sciencemag.org/content/336/6084/1003.abstract
[6] S. Nadj-Perge, I. K. Drozdov, J. Li, H. Chen, S. Jeon, J. Seo, A. H. MacDonald, B. A. Bernevig, and A. Yazdani, Science 346, 602 (2014), http://www.sciencemag.org/content/346/6209/602.full.pdf, http://www.sciencemag.org/content/346/6209/602.abstract
[7] B. A. Bernevig, T. L. Hughes, and S.-C. Zhang, Science 314, 1757 (2006), http://www.sciencemag.org/content/314/5806/1757.full.pdf, http://www.sciencemag.org/content/314/5806/1757.abs.tract
[8] R. M. Lutchyn, J. D. Sau, and S. Das Sarma, Phys. Rev. Lett. 105, 077001 (Aug 2010), http://link.aps.org/doi/10.1103/PhysRevLett.105.077001
[9] P. Ghaemi, S. Gopalakrishnan, and T. L. Hughes, Phys. Rev. B 86, 201406 (Nov 2012), http://link.aps.org/doi/10.1103/PhysRevB.86.201406
[10] P. Ao and D. J. Thouless, Phys. Rev. Lett. 70, 2158 (Apr 1993), http://link.aps.org/doi/10.1103/PhysRevLett.70.2158
[11] P. G. DE Gennes and J. MATRICON, Rev. Mod. Phys. 36, 45 (Jan 1964), http://link.aps.org/doi/10.1103/RevModPhys.36.45
[12] J. Bardeen and M. J. Stephen, Phys. Rev. 140, A1197 (Nov 1965), http://link.aps.org/doi/10.1103/PhysRev.140.A1197
[13] C. Caroli, P. D. Gennes, and J. Matricon, Physics Letters 9, 307 (May 1964)
[14] H. F. Hess, R. B. Robinson, and J. V. Waszczak, Phys. Rev. Lett. 64, 2711 (May 1990), http://link.aps.org/doi/10.1103/PhysRevLett.64.2711
[15] A. Maldonado, S. Vieira, and H. Suderow, Phys. Rev. B 88, 064518 (Aug 2013), http://link.aps.org/doi/10.1103/PhysRevB.88.064518
[16] C. Chen, Introduction to scanning tunneling microscopy (Oxford University Press, 1993)
[17] L. Boris and S. Sachdev, Phys. Rev. B 74, 144515 (Oct 2006), http://link.aps.org/doi/10.1103/PhysRevB.74.144515
[18] P. Nikolić and S. Sachdev, Phys. Rev. B 73, 134511 (Apr 2006), http://link.aps.org/doi/10.1103/PhysRevB.73.134511
[19] P. Nikolić, S. Sachdev, and L. Boris, Phys. Rev. B 74, 144516 (Oct 2006), http://link.aps.org/doi/10.1103/PhysRevB.74.144516
[20] Y. S. Hor, A. J. Williams, J. G. Checkelsky, P. Roushan, J. Seo, Q. Xu, H. W. Zandbergen, A. Yazdani, N. P. Ong, and R. J. Cava, Phys. Rev. Lett. 104, 057001 (Feb 2010), http://link.aps.org/doi/10.1103/PhysRevLett.104.057001
[21] L. A. Wray, S.-Y. Xu, Y. Xia, Y. S. Hor, D. Qian, A. V. Fedorov, H. Lin, A. Bansil, R. J. Cava, and M. Z. Hasan, Nature Physics 6, 855 (2010)
[22] T. V. Bay, T. Naka, Y. K. Huang, H. Luigjes, M. S. Golden, and A. de Visser, Phys. Rev. Lett. 108, 057001 (Jan 2012), http://link.aps.org/doi/10.1103/PhysRevLett.108.057001
[23] S. Sasaki, M. Kriener, K. Segawa, K. Yada, Y. Tanaka, M. Sato, and Y. Ando, Phys. Rev. Lett. 107, 217001 (Nov 2011), http://link.aps.org/doi/10.1103/PhysRevLett.107.217001
[24] L. Fu and E. Berg, Phys. Rev. Lett. 105, 097001 (Aug 2010), http://link.aps.org/doi/10.1103/PhysRevLett.105.097001
[25] L. Fu and C. L. Kane, Phys. Rev. Lett. 100, 096407 (Mar 2008), http://link.aps.org/doi/10.1103/PhysRevLett.100.096407
[26] P. Hosur, P. Ghaemi, R. S. K. Mong, and A. Vishwanath, Phys. Rev. Lett. 107, 097001 (Aug 2011), http://link.aps.org/doi/10.1103/PhysRevLett.107.097001
[27] C.-K. Chiu, P. Ghaemi, and T. L. Hughes, Phys. Rev. Lett. 109, 237009 (Dec 2012), http://link.aps.org/
[28] H.-H. Hung, P. Ghaemi, T. L. Hughes, and M. J. Gilbert, Phys. Rev. B 87, 035401 (Jan 2013), http://link.aps.org/doi/10.1103/PhysRevB.87.035401

[29] A. Y. Kitaev, Physics-Uspekhi 44, 131 (2001)

[30] H. Zhang, C.-X. Liu, X.-L. Qi, X. Dai, Z. Fang, and S.-C. Zhang, Nature Physics 5, 438 (2009)

[31] F. m. c. Gygi and M. Schlüter, Phys. Rev. B 43, 7609 (Apr 1991), http://link.aps.org/doi/10.1103/PhysRevB.43.7609

[32] L. Hedin, Phys. Rev. 139, A796 (Aug 1965), http://link.aps.org/doi/10.1103/PhysRev.139.A796

[33] Y. Qiu, N. K. Sandersa, J. Dai, J. E. Medvedeva, W. Wu, P. Ghaemi, T. Vojta, and Y. S. Hor, “Symbiosis of ferromagnetism and supercurrent in topological insulators,”

[34] J. E. Sonier, Journal of Physics: Condensed Matter 16, S4499 (2004), http://stacks.iop.org/0953-8984/16/i=40/a=006