Two phase galaxy formation: The Gas Content of Normal Galaxies

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ABSTRACT

We investigate the atomic ($\text{H}_1$) and molecular ($\text{H}_2$) Hydrogen content of normal galaxies by combining observational studies linking galaxy stellar and gas budgets to their host dark matter (DM) properties, with a physically grounded galaxy formation model. This enables us to analyse empirical relationships between the virial, stellar, and gaseous masses of galaxies and explore their physical origins. Utilising a semi-analytic model (SAM) to study the evolution of baryonic material within evolving DM halos, we study the effects of baryonic infall and various star formation and feedback mechanisms on the properties of formed galaxies using the most up-to-date physical recipes. We find that in order to significantly improve agreement with observations of low-mass galaxies we must suppress the infall of baryonic material and exploit a two-phase interstellar medium (ISM), where the ratio of $\text{H}_1$ to $\text{H}_2$ is determined by the galactic disk structure. Modifying the standard Schmidt-Kennicutt star formation law, which acts upon the total cold gas in galaxy discs and includes a critical density threshold, and employing a star formation law which correlates with the $\text{H}_2$ gas mass results in a lower overall star formation rate. This in turn, allows us to simultaneously reproduce stellar, $\text{H}_1$ and $\text{H}_2$ mass functions of normal galaxies.

Key words: cosmology: theory - dark matter – galaxies: formation – galaxies: evolution.

1 INTRODUCTION

Neutral atomic hydrogen is the most abundant element in the Universe and plays a fundamental role in galaxy formation, principally as the raw material from which stars form. Within galaxies, the Interstellar Medium (ISM) acts as a temporally evolving baryonic component; competing processes cause the accumulation (through external infall from the intergalactic medium and stellar evolution) and depletion (through star formation and various feedback mechanisms) of hydrogen. Thus, observational determinations and theoretical predictions of the hydrogen budget within galaxies of various masses and morphologies is of central importance to constraining the physics of galaxy formation (see Kauffmann, White & Guideroni, 1993, Benson et al. 2003, Yang, Mo & van den Bosch, 2003, Mo et al. 2005, Kaufmann et al. 2009)

Moreover, within the ISM Hydrogen comprises the majority of the cold gas mass, and when non-ionized exists within two-phases, atomic $\text{H}_1$ and molecular $\text{H}_2$ . A large body of observational analysis has shown that within galaxies, $\text{H}_1$ generally follows a smooth, diffuse distribution whereas $\text{H}_2$ regions are typically dense, optically thick clouds which act as the birthplaces for newly formed stars (Drappatz & Zinnecker, 1984, Wong & Blitz, 2002, Krumholz & McKee, 2005, Blitz & Rosolowski, 2004, 2006, Wu et al., 2005). Due to the distinct differences in these phases, and the central importance of ISM physics to the evolution of galaxies, cosmological simulations have begun to include both phases (see Gnedin et al. 2000 & references therein), and observations have begun focusing on simultaneous measurements of both $\text{H}_1$ and $\text{H}_2$ (see Obreschkow & Rawlins, 2009).

The distinction between these two phases has recently been shown to be of crucial importance to constrain the physics of galaxy formation. In particular resolved spectroscopy using GALEX showing obscured star forming re-
motivated galaxy formation model, requiring an accurate description of all of the aforementioned processes (see Mo et al. 2005 for a detailed discussion). These issues manifest most clearly within the largely successful ΛCDM paradigm within the lowest mass systems, where it still remains unclear whether strongly non-linear feedback mechanisms, lower star formation efficiencies, or suppression of initial infall onto DM halos is the dominant driver for the suppression of luminous structure formation (Mo et al. 2005). It is more than likely that a combination of the above-mentioned effects will go a long way to alleviating current tensions between models and observations, since current semi analytical models (SAMs) incorporate several processes in order to generate a deficiency of stellar mass in DM halos; many of which operate most effectively at low masses (Benson et al. 2003, De Lucia et al., 2004).

Observationally, Zwaan et al. (2005) used the catalogue of 4315 extragalactic HI 21-cm emission line detections from the H I Parkes All Sky Survey (HIPASS, Barnes et al. 2001) and obtained the most accurate measurement of the HI mass function of galaxies to date. The HIMF is fitted with a Schechter function with a faint-end slope of $-1.37 \pm 0.03$. The sensitivity of this survey was so high that they were able to extend their analysis well down to HI masses of $10^{7.2} M_\odot$, hence this is most complete analysis so far. Using these statistical constraints, it has now become possible to make stringent comparisons between theoretical models and observations even in low mass galaxies.

The physics of cold gas becomes increasingly relevant for constraining galaxy formation models at relatively low masses (dominated by late-type galaxies), where the presence of gas becomes substantial and therefore may break the degeneracies between feedback, star formation, and infall processes. Moreover, within the ΛCDM scenario the HI and H$_2$ mass budgets in galaxies are determined by an intricate offset between several competing processes, all of which have strong mass dependencies. Thus the present HI and H$_2$ fractions are strong functions of host DM halo mass and the evolutionary history of each individual galaxy. More specifically, the fraction of gas which may be captured by the host DM halo, and in turn removed by feedback, is expected to depend strongly on the binding energy of the gas itself, which is principally determined by the DM halo virial mass and density distribution. Thus, under this framework the observational properties of galaxy populations are strongly influenced by their collective host DM halos (White & Rees, 1978, see Somerville et al. 2008 for a review).

Motivated by the above-mentioned observational advances and theoretical challenges, the primary aim of this work is to investigate the physical origins of the relationships between HI, stellar, and virial masses of galaxies. In order to do this we compare empirical galaxy relations derived from observational studies, with a physically motivated galaxy formation model. Observationally, we use a numerical approach (described in Shankar et al. 2006) that relies on the assumption of the existence of a one-to-one mapping between galaxy properties and host DM halo mass.$^1$ We interpret these results using a physically motivated SAM, (see Cook et al. 2009, hereafter C09, & references therein), which has been shown to reproduce many features of the local galaxy population.

SAMs provide a powerful theoretical framework within which we can explore the range of physical processes (e.g. accretion mechanisms, star formation, SN feedback, black hole growth and feedback etc.) that drive the formation and evolution of galaxies and determine their observable properties (see Somerville et al. 2008 and Baugh 2006 for extensive...

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$^1$ This approach is based also on the assumption on the completeness of the sample over which the mass functions has been obtained.
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In summary, we utilise the most up-to-date observations of the stellar and HI mass functions in order to derive relationships between the host DM halo and the galaxy stellar and gas properties, we interpret these results using a physically grounded SAM, and analyse the nature of the results by generating three model realisations, incorporating several contemporary recipes and highlighting how each helps alleviate previous model tensions. The outline of the paper is as follows: In §2 we present our SAM, outlining the basic framework from C09, and including the modifications and improvements, in §3 we use current observational results of the stellar and HI mass functions and use a numerical procedure in order to derive relationships between host DM halo mass and the galaxy stellar and HI components. In §4 we present the results of both the observational determinations and the theoretical model comparisons. We conclude in §5 by summarising the main outcomes of this work, discussing the implications, and the limitations.

Throughout the paper we adopt the standard ΛCDM concordance cosmology, as constrained by WMAP 5-year data (Spergel et al. 2007). Specifically, we adopt a flat cosmology with density parameters \( \Omega_M = 0.27 \) and \( \Omega_\Lambda = 0.73 \), and a Hubble constant \( H_0 = 70 \, \text{km s}^{-1} \, \text{Mpc}^{-1} \).

## 2 PHYSICAL MODEL

We follow the galaxy evolution using a semi-analytical approach, deriving analytical estimates wherever possible to aid simplicity. We track the evolution of dark matter, hot gas, cold gas and stellar mass using several parameterizations, based on the model described in C09 but with several important modifications.

Namely, we now trace the sequential infall of material onto both spheroid and disc structures and we include the effects of an ionizing UV background, adiabatic contraction of the DM halo and a two-phase ISM. Here we outline the basic framework for this model.

We initially describe the dark matter halo evolution using an extended Press-Schechter (EPS) algorithm based on that developed by Parkinson et al. (2008). These mergertrees have been tuned in order to reproduce the statistics of halo merger and accretion activity obtained from N-body simulations of structure formation (Springel et al. 2005). Within our model, we utilise these mergertrees by extracting the mass accretion history (MAH), i.e. the evolutionary path of a typical dark matter halo, obtained by tracking the most massive progenitor at each fragmentation event whilst moving from \( z = 0 \) to progressively higher redshifts (see van den Bosch, 2002, C09).

Specifying the DM halo properties, we define the virial radius \( r_{\text{vir}} \) as that of a spherical volume enclosing an overdensity in a standard way (Bryan & Norman, 1998). We compute the density profile for the dark matter halo using the fitting function of Navarro, Frenk & White (1997, hereafter NFW):

\[
\rho_{\text{NFW}}(r) = \rho_\ast \left( \frac{r}{r_\ast} \right)^{-1} \left( 1 + \frac{r}{r_\ast} \right)^{-2}.
\]

In order to define the scale radius \( r_\ast \), for our NFW profile we introduce the concentration parameter, which is defined to be \( c(z) \equiv r_{\text{vir}}/r_\ast \). This quantity has been studied by several authors (see Bullock et al. 2001, Zhao et al. 2003 (hereafter Z03), Macciò et al. 2007) who found large scatter for a fixed halo mass, but to scale generally with the MAH of the halo. We adopt here the \( z \)-evolution of Z03,

\[
[\ln(1 + c) - c/(1 + c)]c^{-3\alpha} \propto H(z)^{2\alpha} M_{\text{vir}}(z)^{1-\alpha},
\]

where \( \alpha \) is a piecewise function which can be found in Z03 and where the normalization can be fixed using the expression given by Macciò et al. 2007 at \( z = 0 \):

\[
\log c_0 = 1.071 - 0.098 \left[ \log \left( \frac{M_{\text{vir},0}}{M_\odot} \right) - 12 \right].
\]

From this, the scale density may be computed for a general profile to be \( \rho_\ast = M_{\text{vir}}(z)/(4\pi r_\ast^3 f(c)) \), with

\[
f(c) = \ln(1 + c) - \frac{c}{1 + c}.
\]

Finally, we specify the angular momentum properties...
of each halo through the spin parameter, defined to be \( \lambda = J_{\text{vir}} E_{\text{vir}}^{1/2} M_{\text{vir}}^{3/2} G^{-1} \), where \( E_{\text{vir}} \) and \( J_{\text{vir}} \) are the total energy and angular momentum of the halo. Assuming that a DM halo acquires its angular momentum through tidal torques with the surrounding medium, \( \lambda \) remains constant. It has been shown (Cole & Lacey, 1996) that the spin parameter varies little with cosmic epoch, halo mass, or environment and for a sample of haloes is well fitted by a log-normal distribution (where \( \bar{\lambda} = 0.04 \) and \( \sigma = 0.5 \)). However, for simplicity, here we assume that each halo has a parameter value of \( \lambda = 0.04 \) which remains constant throughout the DM halo evolution.

Several studies have shown the concentration evolution to be strongly correlated to the MAH of the halo (Z03, Li et al., 2006, Lu et al. 2006), finding that DM halos generally acquire their mass in two distinct phases; an initial phase characterised by rapid halo growth through major merger events, where the halo core structure forms, causing the gravitational potential to fluctuate rapidly, followed by a slower, more quiescent growth predominantly through accretion of material onto the outer regions of the halo. These two different modes are reflected in the evolution of the concentration parameter, which remains roughly constant during the 'fast accretion' phase and steadily increases during the 'slow accretion' phase. The transition redshift \( z_t \) between these two phases can therefore be calculated using the expression for the concentration parameter evolution given in Z03:

\[
\frac{\ln(1 + c_t) - c_t/(1 + c_t) e^{-\alpha c_t}}{\ln(1 + c_0) - c_0/(1 + c_0) e^{-\alpha c_0}} = \left[ \frac{H(z)}{H_0} \right]^{2\alpha} \left[ \frac{M_{\text{vir}}(z)}{M_{\text{vir},0}} \right]^{1-\alpha},
\]

where \( H(z) \) is the Hubble radius, '0' denotes quantities evaluated at the present cosmic time, and \( c_t = 4 \) and \( \alpha = 0.48 \) are scaled to match N-body simulations as in Z03.

By associating the two phases of DM evolution to two growth mechanisms for the baryonic sector, the fast and slow phases give rise to the formation of bulges and discs respectively (Mo & Mao, 2004, C09). Since within the 'fast accretion' phase, angular momentum may be readily lost by rapidly merging clumps of material, in an implicit merger scenario, resulting in the formation of a spheroidal structure, followed by the quiescent dissipationless infall of material in order to form discs (see C09 for a more detailed analysis and justifications of this). At difference with the prescriptions outlined in C09, here we expand our model to include the symbiotic infall of baryonic material as the DM halo evolves, thus

\[
\dot{M}_{\text{inf}} = f_{\text{coll}} \dot{M}_{\text{vir}}.
\]

We include the effects of an ionizing radiation background taking the prescriptions outlined in (Gnedin et al., 2004, Somerville et al., 2008), which is able to partially reduce the baryonic content in low-mass systems, thus

\[
f_{\text{coll}}(M_{\text{vir}}, z) = \frac{\Omega_b/\Omega_m}{(1 + 0.26M_f(z)/M_{\text{vir}})^2},
\]

where \( M_f(z) \) is the filtering mass at a given redshift, computed using the equations in (Kravtsov, Gnedin & Klypin, 2004, Appendix B). A second improvement over our previous model (C09) is to include the effects of cold accretion flows, shown to be the predominant mechanism leading to the formation of low-mass systems. Below a critical mass

\[
M_c = M_* \max[1, (1.3)^{z-z_c}],
\]

where \( M_* = 2 \times 10^{12} M_\odot \) and \( z_c = 3.2 \), we assume that all gas accreted onto DM halos is not shock heated to the virial temperature of the DM halo, but streams in on a dynamical time (see Dekel et al., 2008, 2009, Cattaneo et al. 2006). We note that below the shock heating mass scale it has been shown that rapid cooling does not allow for the formation of a stable virial shock (Keres et al., 2005) resulting in gas flowing unperturbed into the central regions of the DM halo. Thus, in halos below this mass the collapse happens on the dynamical timescale of the system \( t_{\text{coll}} = t_{\text{dyn}} \), whereas in halos above this mass \( t_{\text{coll}} = \max\{t_{\text{dyn}}, t_{\text{cool}}\} \), where the cool-
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The overall dynamical time of the forming halo. Therefore, within the ‘fast accretion’ phase we neglect the effects of angular momentum of cooling proto-galactic gas, which will result in the formation of a spheroidal gaseous system at a rate

$$M_{\text{cool}}(z) = 4\pi \int_0^{r_{\text{vir}}(z)} \frac{r^2 \rho_{\text{cool}}(r, z)}{t_{\text{cool}}(r, z)} \, dr,$$

where $t_{\text{cool}}(r, z)$ is determined by the cold-accretion recipe of Eqs. 8, 9 and 10. We denote this cold gas spherical component (‘gaseous bulge’), which act as a reservoir for star formation, by $M_{b, gas}(t)$, and stress that besides dissipative collapse, it can also grow through merger events and disc instabilities. Also, our model includes other two components: a spheroidal stellar component (‘stellar bulge’) $M_{b, star}(t)$ and a low angular momentum cold gas reservoir $M_{res}(t)$, which acts as a source of material eligible to accrete onto a central black hole.

For simplicity, we assume that the reservoir can be described by an exponential disc surface density

$$\Sigma_{\text{res}}(r, z) = \Sigma_0(z) e^{-r/r_{\text{res}}(z)},$$

with the scale radius $r_{\text{res}}$ being proportional to the influence radius of the SMBH $(r_{\text{res}} = \alpha GM_{\text{SMBH}}/V_{\text{vir}}^2)$, with $\alpha \approx 100$. Also, since we have no a priori information about the geometric distribution of baryonic matter within the bulge system, and since the dynamics and thus evolution of disc structure is correlated to the mass and geometry of the bulge structure, we assume that the bulge stellar and gaseous masses settle into a Hernquist density profile

$$\rho^*_b(r) = \frac{M_{\ast}^*}{2\pi r (r + r_b)^3}, \quad * = \text{stars, gas},$$

where the scale radius of this profile is related to the half light radius by $r_b = 1.8152 R_{\text{eff}}$. Using the fitting of Shen et al. (2003), we take the parametrization as a function of bulge mass to be

$$\log(R_{\text{eff}}) = \begin{cases} -5.54 + 0.56 \log(M_b) & \text{[log}(M_b) > 10.3] \\ -1.21 + 0.14 \log(M_b) & \text{[log}(M_b) \leq 10.3] \end{cases}$$

The star formation rate (SFR) per annulus in the gaseous bulge may be computed as

$$\frac{d\psi_b}{dr}(r, t) = 4\pi r^2 \rho_{b, gas}(r) t_{gas}(r),$$

where $t_{gas}(r)$ is the dynamical time for the gas in the bulge. Therefore in order to compute the total SFR we must integrate this expression over all radii.

Energetic feedback due to supernova events may transfer significant energy into the cold ISM, causing it to be re-heated and ejected from the system. Therefore, by considering energy balance in the ISM, supernovae feedback is able to remove gas from the bulge at a rate:

2 We need to make an assumption about the reservoir geometry because that is needed to calculate the velocity of the composite system $V_s$ [needed e.g. in Eqs. 27 and 32, the adiabatic halo contraction factor $\Gamma$ [Eqs. 28 and 29] and the gravitational potential $\phi^\prime$ of the composite system appearing in Eqs. 15 and 34. However, the geometry of the reservoir is not expected to have a major impact on our results, given its small size relative to the other components.
We note that, within this work we assume that $\tau_M$ at the rate (Granato et al., 2004)ating the reservoir of low-J material which fuels BH growthmomentum this gas may flow into the nuclear region, gener-

$$M_{b, gas}(t) = -\int_{\tau_s}^{\tau_f} \frac{\varepsilon_{SN}}{\phi(r)} \frac{\psi_b(r,t)}{\phi(r)} e^{E_{SN} \psi_b(r,t)} dr, \quad (15)$$

where $\eta_{SN}$ is the number of Type II supernovae expected per solar mass of stars formed, $E_{SN}$ is the kinetic energy released per supernova event, and $\varepsilon_{SN,b}$ is the efficiency of supernovae energy transfer used to remove the cold gas. Fi-

$$\dot{M}_{\text{visc}} = k_{acc} \frac{\varepsilon_{SN}}{G} \left( \frac{M_{res}}{M_{bh}} \right), \quad (20)$$

where $k_{acc} \approx 10^{-2}$, whilst the Eddington accretion rate is simply $\dot{M}_{\text{edd}} = L_{\text{edd}}/\eta c^2$, with $\eta \approx 0.15$ and $L_{\text{edd}} \approx 2.3 \times 10^{46} M_{BH}(t) / 10^8 M_\odot$ erg s$^{-1}$. (21)

QSO activity affects the interstellar medium of the host galaxy and also the surrounding intergalactic medium through both radiative heating and the kinetic energy input through gas outflows. Assuming that a fraction $f_h$ (which we treat as a free parameter) of the SMBH luminosity $L_h$ is transferred into the cold and hot gas phases, it is possible to compute the amount of cold and hot gas which is removed from the hot gas and gaseous bulge phases as in Granato et al., 2004:

$$M_{b, gas}^{QSO} = f_h \frac{2 L_h}{3 \sigma^3} \frac{M_{bh}}{M_{hot} + M_{b, gas}}, \quad (22)$$

$$M_{hot}^{QSO} = f_h \frac{2 L_h}{3 \sigma^3} \frac{M_{hot}}{M_{hot} + M_{b, gas}}, \quad (23)$$

where $\sigma = 0.65 V_{vir}$ is the velocity dispersion of the central SMBH. This expression is assumed to be ejected from the system.

For the chemical evolution of the cold bulge gas, we use the simple instant-recycling approximation (IRA), whereby a fraction of mass is instantly returned into the cold gas phase in the form of processed material. In particular, this implies that the effective SFR which enters the evolution equations for the gas and star bulge phases is given by

$$\dot{M}_{\text{SFR}}(t) = (1 - R) \int \frac{\psi_b(r,t)}{\phi(r)} \frac{\rho(r)}{\phi(r)} dr, \quad (24)$$

with $R = 0.25$. Also, we assume that $M_{\text{hydrogen}} = 0.71 M_{\text{cold}}$, which is a factor takes into account the contribution of Helium and other heavier elements.

### 2.2 Dissipationless gas collapse

By assuming that material may collapse to form a cool gaseous disc structure during the slow accretion phase, we

3 A Romano IMF $\phi(m_\star)$ is adopted: $\phi(m_\star) \propto m_\star^{-1.25}$ for $m_\star \geq M_\odot$ and $\phi(m_\star) \propto m_\star^{-0.4}$ for $m_\star \leq M_\odot$. This gives $\eta_{SN} = 5 \times 10^{-3} M_\odot^{-1}$.

4 This value of $\eta$ corresponds to rapidly spinning SMBH with spin parameter $a \approx 0.9$ (Bardeen, 1970).

5 We adopt a Romano et al., 2005 IMF, which has the standard Salpeter slope 1.25 in the high mass tail, and flattens to a slope 0.4 below 1 $M_\odot$. As shown in Romano et al. (2005), this performs better than the Salpeter one in reproducing the detailed chemical properties of elliptical galaxies.
may add material to the disc structure at a rate which is given, as in the spheroidal case, by

$$\dot{M}_{\text{coll}}(z) = 4\pi \int_0^{r_{\text{vir}}(z)} \frac{r^2 \rho_{\text{gas}}(r, z)}{t_{\text{coll}}(r, z)} dr,$$

where $t_{\text{coll}}(r, z)$ is determined again by the cold-accretion determinations (see Eqs. 8, 9 and 10). If we assume a dissipationless collapse of material upon cooling within the dark matter halo, we may relate the dark matter virial radius and spin parameter to the forming disc scale radius. In particular, if we assume an exponential disc surface density profile for the stellar and gaseous components,

$$\Sigma_d(r, z) = \Sigma_0(z)e^{-r/r_d(z)}, \quad * = \text{stars, gas},$$

the disc scale radius $r_d(z)$ evolves according to the scaling

$$r_d(z) = (2\pi)^{-1/2}\left(j_d/m_d\right)\lambda_{\text{vir}}(z)f(c)^{-1/2}f(\lambda, c, m_d j_d).$$

The function $f(\lambda, c, m_d j_d)$ may be exactly determined through (Mo, Mao & White, 1998)

$$f(\lambda, c, m_d j_d) = 2 \left[ \int_0^\infty e^{-u^2} V_c(r_u) \right]^{-1},$$

where $V_c(r)$ is the velocity profile of the composite system (bulge, reservoir, disc and DM) and where $m_d$ and $j_d$ are the ratios between the total mass and angular momentum of the disc component and the DM halo mass. More specifically, we take $m_d = (M_d^\text{star} + M_d^\text{gas})/\Sigma_{\text{vir}}$, and we assume $j_d = m_d$ (Mo, Mao & White, 1998). In order to account for adiabatic halo response, we take the standard prescription of Blumenthal (1986). In particular, denoting by $M_X(r)$ the mass of the component 'X' enclosed by a radius $r$, from the angular momentum conservation one obtains

$$M_i(r_i) r_i = M_f(r_f) r_f,$$

where $r_i$ and $r_f$ are respectively the initial and final radius of the shell under consideration, the initial mass distribution $M_i(r_i)$ is simply given by the NFW density profile, while $M_f(r_f)$ is the final mass distribution. Also, mass conservation easily gives

$$M_f(r_f) = M_d(r_f) + M_b(r_f) + M_{DM}(r_f) + M_{\text{gas}}(r_f) = M_d(r_f) + M_b(r_f) + M_{\text{gas}}(r_f) + (1 - f_{\text{gal}}) M_i(r_i), \quad (29)$$

where $f_{\text{gal}} = M_{\text{gal}}/M_{\text{vir}}$ (with $M_{\text{gal}} = M_d + M_b + M_{\text{gas}}$). By assuming spherical collapse without shell crossing, one can adopt the ansatz $r_f = \Gamma r_i$, with $\Gamma = \text{const}$ (Blumenthal, 1986), and Eqs. 28 and 29 may be solved numerically for the contraction factor $\Gamma$.

When the surface density of the gaseous disc increases, the cold gas becomes available to form stars. However, at present, star formation is poorly understood from both a microscopic, and large-scales. Therefore we parameterise the star formation using an empirical Schmidt law (Kennicutt, 1998) whereby the star formation rate is related to the surface density of cold disc gas:

$$\dot{\Sigma}_{\text{sf}}(r, z) = \epsilon_{\text{sf}} \frac{\Sigma_{\text{gas}}^4(r, z)}{M_\odot \text{pc}^2} n,$$

where $\epsilon_{\text{sf}} = 2.5 \times 10^{-4}$ controls the star formation efficiency and $n = 1.4$ is fixed to match the properties of spiral galaxies. This law is empirically proven to hold over many orders of magnitude of gas surface density, but was shown to break down at large radii (Kennicutt, 1998) where the surface density drops below a critical value, roughly corresponding to the Toomre (1964) stability criterion. We take this critical surface density to be

$$\Sigma_c(r) = \frac{\sigma_0 \kappa(r)}{3.36 G Q},$$

Toomre, 1964. Where $\sigma_0 = 6 \text{km s}^{-1}$ is the velocity dispersion of the gas, $Q = 1.5$ is a dimensionless constant and $\kappa(r)$ is the epicyclic frequency, given by

$$\kappa(r) = \sqrt{2 \frac{V_c(r)}{r} \left(1 + \frac{r}{V_c(r)} \frac{dV_c(r)}{dr} \right)^{1/2}}.$$  

Therefore, the conversion rate of gas mass to stellar mass is computed as

$$\psi_d(z) = 2 \pi \epsilon_{\text{sf}} \int_0^r r \Sigma_{\text{gas}}(r, z) dr,$$

where $r_c$ may be calculated by solving $\Sigma_d(r_c, z) = \Sigma_c(r_c)$ for $r_c$.

In order to account for feedback due to supernovae events, we may compute the amount of cold gas which is ejected from the system at each disc radius. In order to remove this cold gas from the disc, the supernovae feedback must be sufficient to unbind it, therefore we compare the amount of energy released through supernovae events at each disc radius with the binding energy at the same radius:

$$\dot{\Sigma}_{\text{SN}}(r, z) = - \frac{\epsilon_{\text{SN, d}} E_{\text{SN, d}} \Sigma_{\text{d}}(r, z)}{\phi(r, z)},$$

where $\phi(r, z)$ again, is the binding energy of the composite system (bulge, disc, reservoir and DM). The total amount of cold gas ejected from the system is then given by
where $\alpha$ with $M$ is given by enter the evolution equations for the gas and star disc masses (Blitz & Rosolowski, 2006, Dutton & van den Bosch, 2009), thus:

Assuming that star formation may only take place inside unstable, we assume they get disrupted in a dynamical time uncertainty due to the finite sample size of synthetic galaxies.

Finally, it is known that when discs become self-gravitating they are likely to develop bar instabilities, get disrupted and transfer material to the spheroidal component (Christodoulou, Shlosman & Tohline, 1995). We therefore assume that a stellar or gaseous disk is stable if

$$V_c(2.2r_d)/(GM_{\text{disc}}/r_d)^{1/2} > \alpha_{\text{crit}}^{\ast} = \text{stars, gas},$$

where $\alpha_{\text{crit}}^{\ast} = 1.1$ and $\alpha_{\text{crit}}^{\ast} = 0.9$ [see (Mo, Mao & White, 1998) and references therein. If we find that discs become unstable, we assume they get disrupted in a dynamical time and transfer their material (either stars or gas) to the bulge components.

### 2.3 Improved star formation law

Assuming that star formation may only take place inside dense molecular clouds several authors have shown that the Schmidt-Kennicutt star formation law (Eq. 30) may be reproduced within large mass systems by assuming that the star formation rate is proportional to the molecular cloud mass (Blitz & Rosolowski, 2006, Dutton & van den Bosch, 2009), thus:

$$\Sigma_{\text{SF}} = \tilde{\epsilon}_{\text{SF}} \Sigma_{\text{mol, HCN}}$$

Where $\tilde{\epsilon}_{\text{SF}} = 13 \text{Gyr}^{-1}$ and $\Sigma_{\text{mol, HCN}} = f_{\text{mol}} R_{\text{HCN}}$ is the molecular mass surface density as traced by HCN (see Gao & Solomon, 2004, Wu et al. 2005). Thus, calculating the ratio of molecular gas to atomic gas allows us to compute the star formation rate at all scales. The fraction of gas in discs which is molecular has been extensively analysed, and shown to be closely related to the mid-plane pressure within discs (Blitz & Rosolowski, 2006), given by:

$$P_{\text{up}} = \frac{\pi}{2} G \Sigma_{\text{gas}} (\Sigma_{\text{gas}} + (\sigma_g/\sigma_s)\Sigma_s)$$

Where, following the detailed prescriptions of Dutton & van den Bosch, 2009, assuming a constant $\sigma_g/\sigma_s = 0.1$. Relating the mid-plane pressure to the formation of molecular clouds yields:

$$R_{\text{mol}} = \frac{\Sigma_{\text{mol}}}{\Sigma_{\text{atom}}} = \left( \frac{P_{\text{up}}/k}{4.3 \times 10^4} \right)^{0.92}$$

Thus, the molecular fraction is given by $f_{\text{mol}} = R_{\text{mol}}/(R_{\text{mol}} + 1)$, in order to relate this to the HCN fraction, we must further compute the fitting relation of Blitz & Rosolowski, 2006:

$$R_{\text{HCN}} = 0.1 \ast (1 + \Sigma_{\text{mol}}/(200 M_\odot \text{pc}^{-2}))^{0.4}$$

Therefore, we find that, in the high mass (and thus density) galaxies, where the molecular fraction is typically $\approx 1$, we recover the standard Schmidt-Kennicutt star formation power law, with exponent 1.4, whereas in the low-density galaxies we asymptote towards an exponent of 2.84, suppressing star formation in these systems, in accordance with observations (see Dutton & van den Bosch, 2009 and Blitz & Rosolowski, 2006 for a detailed description). Thus, throughout the evolution of each galaxy, we partition the ISM into $H_1$ and $H_2$ components using the above relations, and compute the star formation law (and therefore supernovae feedback) using equation 42 in order to self-consistently model each galaxy under this improved star formation law.

### 3 THE $H1$ MASS RELATIONSHIPS

In order to investigate the relationship between the stellar and the gas component in late-type galaxies, we follow the procedure already exploited by Shankar et al. (2006), and discussed in detail in Evoli et al., 2009. We defer the reader to these papers for a more detailed discussion and highlight the main results here. If two galaxy properties, $q$ and $p$, obey a one-to-one relationship, we can write

$$\phi(p) dp dq = \psi(q) dq$$

where $\psi(q)$ is the number density of galaxies with measured property between $q$ and $q + dq$ and $\phi(p)$ is the corresponding number density for the variable $p$. The solution is based on a numerical scheme that imposes that the number of galaxies with $q$ above a certain value $\bar{q}$ must be equal to the number of galaxies with $p$ above $\bar{p}$, i.e.,

$$\int_{\bar{p}}^{\infty} \phi(p) dp = \int_{\bar{q}}^{\infty} \psi(q) dq$$
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4 RESULTS

Throughout this section we present the results of model realisations, each comprising of a sample of $\approx 10^5$ galaxies in logarithmic virial mass increments in the mass range $9.5 < \log(M_\odot(z=0)/M_\odot) < 13.5$ in order to encompass the observational range of galaxy-hosting DM halos. In order to make statistical predictions with each generated galaxy sample at $z=0$ we weight each DM halo with the GHMF, which mirrors the Sheth & Tormen (2002) mass function within most galaxy sized DM haloes, but is derived in order to account for the increasing probability of multiple galaxy occupation in the highest mass haloes (see Shankar et al. 2006). Throughout this section we also plot the results for 'early' and 'late' type galaxies, by parting the populations into $[M_{\text{bulge}}/M_{\text{total}}]$ > 0.5 and $[M_{\text{bulge}}/M_{\text{total}}]$ < 0.5 respectively.

4.1 Component evolution

In order to illustrate the general behavior of our galaxy formation framework, Fig.3 highlights the evolution of each mass component from $z=8$ up to the present epoch. In the left-hand, center and right-hand panels we show a low-mass, intermediate mass and high-mass galaxy respectively, in order to highlight typical model outputs for each system. As can be seen, the evolution of each galaxy differs significantly due both to scatter at each mass through monte carlo selected mass accretion histories, and to the relative differences in efficiencies of the competing processes of infall, star formation and feedback on different mass-scales.

We find, in broad agreement with observations, that a typical low-mass galaxy (with $M_\odot(z=0) \approx 10^{10} M_\odot$) supports the growth of a disc structure from high redshifts showing extended star formation up to $z=0$ resulting in a gas-rich disc dominated galaxy with a negligible bulge component. In intermediate mass, $L_\odot$ galaxies, at high-redshifts gaseous collapse onto a spheroid system results in the co-evolution of SMBH’s and the spheroidal component resulting in a gas poor, ‘red and dead’ spheroidal stellar component which acts as the bulge within the resultant formed galaxy as the disc component grows steadily from $z \leq 1.5$. The high mass galaxy shows rapid early growth of the SMBH and spheroid component, followed by an epoch of dormancy in the star formation, no significant disc structure may form in these systems and ongoing star formation is prevented due...
to the presence of a large SMBH which acts to effectively expel any residual gas which may infall later, only allowing for extremely mild star formation following the main growth phase. The resulting galaxy spheroidal component thus comprises an old stellar population, with little gas and negligible star formation, and the disc component is over an order of magnitude smaller than the bulge, and gas rich, showing little star formation.

These results are consistent with the evolutionary histories tuned to match the chemical properties of local galaxies of different morphological type and different host halo masses (Cahura, Pipino & Matteucci, 2008 & references therein) which have also been shown to be consistent with photometry within the local Universe (Schurer et al. 2009) and are generally in agreement with observations of statistical samples of galaxies (Driver, et al. 2006), however, as can also be seen, due to the stochastic nature of the model, fluctuations in galaxy properties are expected, and thus we must revert to statistically representative samples of galaxy populations in order to make more robust comparisons.

Within these systems in-falling baryonic material is shock heated and therefore may form a static atmosphere of hot-gas. Feedback from a growing SMBH may thus halt the cooling of this gas, quenching star formation and sweeping out the ISM (See Granato et al., 2004). Within these systems in-falling baryonic material is shock heated and therefore cooling and feedback from a growing SMBH may quench star formation by sweeping out the ISM (see Granato et al. 2004)

4.2 Stellar and Baryon mass functions

The local stellar and baryonic mass functions provide a powerful constraint on theoretical models of galaxy formation: Encompassing much of the relevant physical processes which determine the assembly of baryons within DM halos.

In order for models to reproduce observational results, it has become clear that physical processes of gas accretion, supernovae feedback and photo-ionization are most effective in the lowest mass DM halos where the shallow potential wells are inefficient at trapping and holding baryonic material. Thus these processes drive the evolution of the faint-end slope of the mass functions (Benson et al., 2002), whereas the brightest galaxies (above $L_*$) are embedded within large DM halos which effectively trap baryonic material. Within these systems in-falling baryonic material is shock heated and therefore may form a static atmosphere of hot-gas. Feedback from a growing SMBH may thus halt the cooling of this gas, quenching star formation and sweeping out the ISM (See Granato et al., 2004). Coupled with the increasing subhalo contribution (through the cluster mass function) whereby in DM halos with $M_h(z=0) > 10^{13}$ the probability of a single galaxy occupation is low, gives rise to the relatively sharp cutoff of stellar mass within the largest DM halos (see Shankar et al. 2006 & Somerville et al. 2008, & contained references).

Rather than computing the spectral energy distribution (SED) assuming any mass-to-light ratio for each model galaxy (relying upon further model assumptions such as the dust-to-gas ratio, molecular cloud structure and optical depth etc) by comparing the stellar and total baryon budgets within each DM halo, we provide the most direct analysis of model outputs. However, we also note that uncertainties in the observational conversion of luminosity to stellar (and total baryonic) mass are systematically related to the spectral energy SED fitting methods used in order to extract physical quantities from the multi-wavelength observations, and on the quality of the observations themselves, this is known to have large uncertainties and we hope in a subsequent work, to utilize synthetic spectra using the detailed star formation histories, galaxy geometries, gas and dust content, in order to self-consistently model the multi-wavelength SED and make comparisons with the luminous properties of galaxies. In Fig.4 we show the Schecter function fit to the Bell et al. (2003a) estimate for the local stellar mass function, in order to generate this mass function the authors utilize a large sample of galaxies from the Two Micron All Sky Survey (2MASS) and the Sloan Digital Sky Survey (SDSS) converting galaxy luminosity into stellar mass using simple models to convert the optical and near infrared observations into stellar masses.

As we have shown in Fig.4, using three levels of sophistication we are able to highlight the differences between; a 'standard' model, whereby we ignore the effects of a photo-ionizing background and employ a standard Schmidt-Kennicutt (1998) star formation law, a 'UV' model whereby we include the suppression of mass flowing into galaxies through photo-ionization (see Eqn. 6 & 7), and a 'sophisticated' model, which combines the effects of a photo-ionizing background with a two-phase ISM and modified star formation law. We will refer to these model names throughout the next sections.

Figure 8. Deriving the relationships in §3 we show the baryon budget in the form of stellar material in DM halos of different mass. Only by exploiting the effects of a UV background and a two-phase ISM can we reproduce the global form of this relationship, finding good agreements to observations at all masses, with a slight over-prediction only in the lowest mass halos. As can be seen, 'standard' approaches significantly overproduce stellar material on all mass ranges, but the discrepancy is clearest in the low mass systems. The shaded region represents the 1 – σ observational uncertainty. Model error-bars represent poissonian uncertainties due to the finite sample size of synthetic galaxies.
More specifically, using a 'standard' model we significantly over-produce the number density of low mass galaxies. This occurs even when we increase supernovae feedback efficiencies to extremely high levels (see Mo et al. 2005 for a discussion), further pushing the parameter to higher values than \( s_{\text{sn}} = 0.7 \) would result in largely un-physical models, and it is clear that reproducing observations using this basic framework does not occur naturally. Secondly, increasing the supernovae feedback efficiencies whilst lowering the star formation efficiencies simply has the effect of reducing the stellar mass but not the gaseous mass, resulting in gas fractions which increase with increasing supernovae efficiency and thus are in disagreement with observations (see Mo et al., 2005 for a detailed discussion).

By suppressing the initial in-fall of material due to an ionizing UV background radiation, we are able to improve agreements with the mass function. However, we achieve the best reproduction of the stellar mass function through the additional reduction in star formation efficiency when employing the 'sophisticated' star formation law, which is determined by the amount of \( H_2 \) gas present in galaxy discs.

Alone, the stellar mass function provides a crucial observation for any physical galaxy formation model to reproduce, but by simultaneously comparing both the stellar and gaseous properties of galaxies, we are able to break degeneracies between gas infall and cooling, star formation and feedback processes, resulting in a significantly improved constraint on the theoretical framework. This has led previous authors to claim that the standard model, whereby low mass systems support efficient gas cooling leading to large gaseous rotationally supported discs, which then undergo mild star formation, significantly over-predicts galaxy \( H I \) masses when compared to observations (see Mo et al. 2005) causing serious tension between theory and observation.

In Fig.5 we compare model results with the Bell et al. (2003b) baryon mass function. We find that suppressing the infall of baryonic gas due to an ionizing UV background significantly improves agreement between observation and model output, since material is prevented from in-falling into the halos initially and thus requiring less feedback in order to gain agreement of the low stellar and gas fractions in these halos. A final, and further improvement between model and observation is achieved by partitioning the ISM into neutral and molecular hydrogen, which has the effect of reducing the SFR efficiency preferentially in the lowest mass DM halos. We find that the best agreement between model and observation naturally results from using the most sophisticated treatment of the ISM physics, and the initial infall of gas. We do however, still find discrepancies in the lowest mass halos (\( M_{\text{star}}(z = 0) < 10^9 \)), over-predicting the baryon and stellar content within these DM halos, and slightly over-producing the number density of the most massive galaxies. This issue is discussed in §5.

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6 We note also that we use a universal stellar initial mass function (IMF) which has been constrained in order to reproduce the chemical properties of galaxies accurately, increased supernovae rates are achievable using a more top-heavy IMF, however this has a strong chance to offset other properties of the formed galaxies with respect to observations (see Romano et al. 2005 for a discussion.)

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7 Taking \( M_{H_2} = f_{\text{mol}}M_{\text{hydrogen}} \) and \( M_{HI} = (1 - f_{\text{mol}})M_{\text{hydrogen}} \)
the $H_2$ mass function derived within the original Keres et al., 2005 work (using a constant $\chi$-factor). We refer the reader to their work for further details. We find that within observational errors, we reproduce this function over the entire observational range, however, we note that the large uncertainties within the determination of the precise value of the $\chi$-factor means that we are relatively loosely constrained, and we view this result as a general prediction of our model, rather than a constrained observational match.

Physically interpreting these results, we may conclude that utilising a two-phase ISM whereby star formation scales only indirectly with the total gas mass, but directly with the molecular $H_2$ mass, we find that we may accurately reproduce both the $HI$ and $H_2$ mass functions of galaxies apart from an over-prediction of neutral $HI$ within the lowest mass halos. This discrepancy also appears within the stellar mass function (Fig.4) and the baryonic mass function (Fig.5), although it is greatly improved using the ‘sophisticated’ ISM treatment.

4.4 Stellar-to-Dark matter properties

Exploiting the methods outlined in §3 we are able to make observational mappings between DM halo masses and their contained stellar mass. In Fig.8 we show the relative efficiency of conversion of baryonic material into stellar mass within DM halos relative to the cosmological baryon to dark matter ratio $f_b = \Omega_b/\Omega_m = 0.16$. These relationships provide direct observational constraints onto galaxy formation models and allow for a direct analysis of the relevant processes occurring over a range of mass scales (see Shankar et al. 2006). As can be seen, the lowest mass systems ($\log(M_*/M_\odot) < 11$) are strongly dark matter dominated, due to the inefficiency of weak gravitational potentials in these systems being able to capture and contain baryonic material and then efficiently form stars. Typically these systems are strongly disc-dominated, late-type galaxies.

Intermediate mass systems ($11 < \log(M_*/M_\odot) < 12$) generally contain a range of morphological types with significant spheroid and disc components. The stellar content in these systems is a strong function of the mass, whereby the larger systems become richer in stellar mass fraction reaching a ‘peak’ ratio of stellar to dark matter where there is a minimum efficiency in the combined effects of supernovae and nuclear feedback processes. When we approach the highest mass DM halos ($\log(M_*/M_\odot) > 12$), feedback through nuclear activity generated by an accreting SMBH is able to effectively remove the material. This effect becomes stronger in the higher mass systems and thus causes the flattening and slight downturn of the stellar conversion efficiency.

Over-plotting model outputs using the ‘standard’, ‘UV’ and ‘sophisticated’ models, we may easily differentiate between outcomes due to the increased sensitivity of the plotting ranges: We see that the models which include the effects of a UV background show significant improvements over the entire mass range, and using the two-phase ISM physics and adopting the SFR dependent on $H_2$ mass, we are able to reproduce the observations to a high accuracy throughout the entire mass range due to the relative decrease in SF efficiency, with the only discrepancies at the lowest masses. It is important to note here that due to the sequential build-up of matter within halos from high redshifts to low, the importance of ionizing backgrounds and star formation on low mass systems manifests throughout the entire mass range at $z = 0$ (see Somerville et al. 2008 for further discussion of this). Under the standard $\Lambda$CDM scenario whereby small structures collapse first and form progressively larger systems, large structures at $z = 0$ are therefore significantly influenced by small scale processes at higher redshifts, where their constituent parts were forming and evolving, thus the low mass behavior is of global importance to galaxy formation theory.

4.5 Gas-to-Dark matter properties

In analogy to the previous section, in Fig.9 we plot the fraction of $HI$ to DM halo mass we are able to see that the gas fractions are highly sensitive to the different physical prescriptions, the scatter is attributed to the fact that the cold gas (and thus $HI$ mass) component at any time is controlled by the competing processes of infall, star formation, feedback and recycling. We also find that, due to the uncertainties and intrinsic dispersion in the relationship between the dark matter and gaseous matter, the observational trend is not as constrained for the gaseous as for the stellar component. Observationally, the lowest mass DM halos become progressively more depleted in $HI$, due to suppression of infall and increased efficiency in generating gaseous outflows, intermediate mass DM halos ($11 > \log(M_*/M_\odot) > 11.5$) possess the most $HI$ mass, due to the global maxima in trapping and containing baryonic material in DM halos (or conversely, a global minima in feedback processes), and at high masses we see a progressive loss of $HI$ abundance in halos, signifying an increase in removal of gaseous material through cooling inefficiencies and nuclear feedback processes.

In Fig.9 we show the outputs for each of the three re-
alisations, as can be clearly seen, the only successful reproduction of the majority of the observations is the 'sophisticated' model, whereby we model both the effects of a two-phase ISM and of an ionizing background radiation. Therefore modeling the star formation rate depending linearly on the surface density of molecular gas in the disc component results in a correct conversion of cold gas to stellar material (as can be seen from simultaneous fits to both Fig.8 & Fig.9), using the standard Schmidt-Kennicutt (1998) star formation law (and their constrained normalisation) we convert too much material from gas into stars, resulting in an over-production of stellar mass and an underproduction of gas mass in all DM halos, whereas the modification outlined in Dutton & van den Bosch (2009) allows for the correct fraction of virial mass in the $HI$ phase, apart from in the lowest mass systems ($11 > (log(M_*/M_\odot)) < 11$). This mismatch clearly suggests that despite our suppression of baryonic material onto low mass DM haloes, and improved star formation prescriptions we are still missing extra factors which dominate at $M_*(z = 0) < 10^{11} M_\odot$. We may attribute this effect with our lack of description of environmental effects, since the lowest mass halos are more probable to be located within larger over-densities and therefore subject to external forces (see the §5 and Mo et al. 2005 for more details).

4.6 Stellar-to-Gas properties

It is suspected that within the lowest mass systems, gas collapse, star formation and supernovae feedback result in a self-regulated conversion of gas into stars, thus these effects can be seen to produce precisely the correct fraction of stars to gas in Fig.10. Within our model the cold gas component fluctuates more than any other galaxy component since it is constantly being replenished due to infall and stellar recycling, exhausted through star formation and expelled through feedback processes. Therefore, directly comparing each galaxies gaseous and stellar properties provides a stringent comparison between observation and model. Interestingly we find that plotting these quantities, we reproduce a tight correlation with relatively little scatter.

We find an overall agreement to the data across the entire mass range under investigation using the 'sophisticated' model, showing that the $HI$ fraction becomes increasingly large with decreasing stellar mass, having roughly equal stellar and $HI$ masses at $M_* \approx 10^9 M_\odot$ increasing to a factor of $\approx 100$ times more $HI$ than stellar mass within systems with $M_* \approx 10^7 M_\odot$. Conversely, without using the effects of a two-phase ISM and UV background radiation, we find a general offset in all masses, with an over-efficient conversion of cold gas to stars (as may also be seen in Figs.8 & 9).

We also find that this result is relatively robust against parameter choices; only having a significant dependence on the star formation efficiency parameter (which is constrained observationally). We may attribute this to the fact that star formation is modeled as a function of the surface density of $H_2$ in galaxies and within the low mass disc dominated systems is relatively inefficient (due to the low surface densities and relatively high star formation thresholds) resulting in a well defined conversion of cold gas to stars and hence, $HI$ to stellar mass ratio. This conclusion also has important consequences when considering the overproduction of both stellar and $HI$ masses in the lowest mass DM halos, since we accurately reproduce the self-regulation properties of galaxies (as the correct balance between star formation and feedback is required to simultaneously reproduce the correct gas and stellar mass budgets in DM haloes). Overproduction in the low mass regions within Fig.8 & Fig.9 therefore, must be attributed to the accretion of too much material, a conclusion which is confirmed by several other authors (see Mo et al. 2005 & references therein).

We would like to plot also the $H_2$ mass counterparts for figures 8 & 9, however, due to the unconstrained mass functions, as discussed in §4.3, we are not able to construct conclusive observational constraints. Secondly, since partitioning the ISM within the single-phase ISM models is typically something relatively ad-hoc, we do not perform this analysis here.

4.7 Star formation properties

A final useful diagnostic to be used to constrain galaxy formation models is the instantaneous star formation rate (SFR) which is found to vary significantly within galaxies of different stellar mass. In Fig.11 we compare each model realisation with the observational SFR estimates as a function of stellar mass by Elbaz et al. 2007, who used a large sample of SDSS galaxies with spectroscopic data in order to accurately determine the SFR (see Brinchmann et al. 2004).

We find little difference between the SFR predicted by each model realisation as expected since each star formation rate prescription is constrained by $z = 0$ galaxy properties. Moreover, we generally find that within the low mass,
disc-dominated region \( (M_s < 10^{10} M_\odot) \) we obtain a good agreement between model and observation but we significantly over-predict the star formation rates in the largest galaxies. We may attribute this to a lack of AGN quenching within the disc component of the largest mass systems at late times, which has been applied in several models as a rather ad-hoc ‘radio-mode’ feedback (see Croton et al. 2006, Bower et al. 2006). Since, within our models we do not have any suppression of the growth of discs around large pre-formed bulges, aside from the late transition from the spheroid formation epoch to the disc formation epoch. We hope to investigate the effects of energetic feedback from a formed SMBH-spheroid system on the late properties of the disc since we hypothesise that; despite having a negligible effect on the growth of a pre-formed SMBH, star formation is prevented within the spheroid component at late times since even arbitrarily low accretion rates onto the central SMBH results in energetic feedback capable of heating the ISM, however, within our framework, during the quiescent disc growth phase no material is assumed to collapse onto the spheroid structure, even with a slight adjustment to our model we may allow for some material to collapse at late times onto the spheroid-SMBH system, resulting in the quenching of star formation in the larger systems and naturally generating Seyfert-type galaxies. However, for simplicity we have neglected this effect within this work, and hope to investigate the physical mechanisms capable of generating this self-consistently, within a subsequent work.

### 4.8 Two component ISM properties

As a main advance of this model over current SAMs, we detail the properties of the ISM by modeling the formation of molecular clouds \( (H_2 \text{ regions}) \) through pressure arguments within the disc component. This enables us to modify the star formation law, and thus allows us to gain insights into the more detailed gas-properties of normal galaxies under the semi-analytical framework.

Within this work we highlight the importance of modeling the ISM in two-phases advocating it as a simple, yet important advance over current frameworks; whereby the formation of \( H_2 \) regions is determined by the planar pressure with-in the gaseous disc structure. This added ingredient is important for two main reasons: Firstly the decline in \( H_2 \) regions within low mass systems results in a higher fraction of cold gas in the form of neutral \( H I \) which is thus detectable through conventional 21-cm line surveys (see Barnes et al. 2001), whereas \( H_2 \) mass estimates prove to be significantly more difficult, relying on uncertain conversion factors between \( H_2 \) and CO-lines, therefore, assuming a single conversion between total cold gas and \( H I \), as is commonly done in SAMs provides inaccurate outputs. Secondly, the star formation properties of galaxies have been shown to relate explicitly to the detailed properties of the internal structure of the ISM (see Krumholz et al. 2009, Gnedin et al. 2009, Obruchkov & Rawlings, 2009), and therefore this added layer of complexity should now be embedded within current SAMs.

In Fig.12 we show the fraction of cold gas which is in the form of \( H_2 \) within the disc component as a function of total stellar mass (disc and bulge masses). As can be seen, there is a tight relationship with little scatter in the lowest mass systems due to the overall dominance of disc components and the molecular fraction decreases steadily with decreasing stellar mass, we find a peak in molecular fraction corresponding to approximately 80% \( H_2 \) at \( M_s \approx 10^{11} M_\odot \) where the disc mass reaches a maximum. Above this mass the spheroid component dominates and the disc surface density thus drops, lowering the efficiency of molecular cloud formation rapidly.

![Figure 12](image-url)  
**Figure 12.** The molecular fraction of gas within galaxy disks. Showing that in low surface density galaxies, the formation of molecular clouds is suppressed, and thus star forming regions are diminished, and within the high-mass systems, the spheroid component dominates reducing the surface density in disks and therefore the molecular gas fraction rapidly declines. We hope to compare this result to observational studies as they become available. Model error-bars represent poissonian uncertainties due to the finite sample size of synthetic galaxies.

### 5 CONCLUSIONS

Prompted by several theoretical attempts to model the low-mass end of the stellar mass function (Somerville et al. 2008 & references therein) and significant observational effort to constrain the \( H I \) mass function using large surveys of galaxies (Barnes et al. 2001, Zwaan et al. 2005), within this work we have developed a physically motivated model in order to explain the inefficiencies of low-mass DM halos in trapping baryonic material and forming stars. Motivated primarily by the importance of physical descriptions within small scale systems which form the building blocks of larger systems within a hierarchically clustered Universe, also because theoretical models have either neglected, or find significant troubles in simultaneously matching both the stellar and \( H I \) mass functions within the low mass end (see Mo et al. 2005).

Therefore, within this paper we have utilized well constrained observations of the stellar (Bell et al. 2003a, 2003b) and \( H I \) (Zwaan et al. 2005) mass functions and employed a numerical technique (Shankar et al. 2006) in order to derive relationships between galaxy properties and their host DM halos. Assuming a one-to-one mapping of these systems, we are thus able to make detailed comparisons between models and observations.

In order to interpret these phenomenological relation-
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ships physically, we develop a cosmologically grounded galaxy formation model outlined in C09 but with several significant modifications. Under this framework we follow the development of baryonic material as it accumulates and evolves within growing DM halos and is subject to cooling, heating, possible angular momentum losses, star formation and recycling, and feedback through both supernovae events and the growth of a central supermassive black hole. By comparing three model realisations with varying sophistication; one using a ‘standard’ approach, whereby the cosmological baryonic fraction \( f_b = \Omega_b/\Omega_m \) is allowed to accrete within halos throughout their lifetimes and the star formation in discs (and thus the vast majority of low mass systems) is given by the Schmidt-Kennicutt (1998) star formation law, determined by the total cold gas mass, a ‘UV’ model, whereby the infall of baryonic material within low mass DM halos is suppressed due to the presence of an ionizing UV background (see Eqn.6, 7), and finally a ‘sophisticated’ model with a modified infall due to UV radiation and a modified star formation law which requires a two-phase ISM, comprising of neutral \( \text{H}\text{I} \) and molecular \( \text{H}_2 \) and typically results in a lower star formation rate within low mass systems at early times.

These realisations clearly demonstrate that using a ‘standard’ approach, the most simple case is not able to simultaneously match both the stellar and \( \text{H}\text{I} \) mass functions, significantly over-producing low mass galaxies (see Figs.4, 5, 6, 8 & 9), whereas the use of a ‘sophisticated’ approach, we are able to match observations reasonably accurately to relatively low masses, finding discrepancies within the lowest mass systems. As an additional consequence of the modified star formation law in the ‘sophisticated’ model realisation, we naturally partition the cold gaseous ISM into \( \text{H}\text{I} \) and \( \text{H}_2 \) components, allowing for predictions of both of these quantities without the need for further assumptions, finding that we are able to match both components simultaneously to a good accuracy over the entire mass range when comparing to the \( \text{H}_2 \) results of (Keres et al. 2003, Obreschkow & Rawlings, 2009, Fig.7), and only over-producing the amount of \( \text{H}\text{I} \) mass in the lowest mass galaxies (see Zwaan et al. 2005 & Fig.6).

Finally, analysing several properties of galaxies against their stellar masses, we find that, unlike the simple approaches, the \( \text{H}\text{I} \)-to-stellar mass ratio is accurately reproduced using the sophisticated treatment (see Fig.10), indicating that the self-regulation of star formation allows for the correct conversion of material, is relatively insensitive to the infall and supernovae feedback rates within physical limits. Comparisons of the star formation rates within these models however shows little difference at \( z = 0 \) as expected, and indicates a secondary problem with our simple physical model, over-predicting the SFR in the largest systems (with \( M_*>10^{12}M_\odot \)), however, these systems are typically spheroid-dominated and are therefore relatively insensitive to the details of the disc formation recipes.

It is clear to assess the limitations of our framework, we conclude that further suppression of infall onto the lowest mass systems would allow for a further reduction in the need for strong supernovae feedback and should further ease tensions between models and observations, this could only come through environmental effects such as tidal shocks or gravitational pre-heating (Mo et al. 2005), however this effect has not been studied in detail through hydrodynamic simulations and remains to be fully investigated. By adding a channel whereby even small amounts of gaseous material may be transferred to the spheroid component during late times, small amounts of ‘radio mode’ AGN activity may be triggered, little affecting the spheroid or the SMBH masses, but significantly lowering the SFR in the discs, preferentially at large masses, naturally resulting in Seyfert-type active galaxies and reducing the SFR in these large discs, hopefully bringing Fig.11 into better agreement with observations, we also hope to investigate the pan-redshift galaxy population under this framework (Cook et al., 2009b submitted).
Interestingly however, within this relatively simplistic framework we are able to self-consistently reproduce several of the key observations, it is therefore clear that mergers, to some degree, are not the dominant driver for the global evolution of the galaxy population. It will therefore provide a useful exercise to mount our physical prescriptions onto a full merger-tree DM background which should allow us to model environmental effects consistently. The main results from this paper indicate however, that using a relatively simple framework, we find an reasonable agreement to the stellar, $H_1$ and $H_2$ mass functions of galaxies arising naturally, thus, we advocate all current SAMs to begin to incorporate two-phase ISM physics into their frameworks.

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