Liquid-Gas Phase Transition of Supernova Matter and Its Relation to Nucleosynthesis

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Abstract

We investigate the liquid-gas phase transition of dense matter in supernova explosion by the relativistic mean field approach and fragment based statistical model. The boiling temperature is found to be high ($T_{\text{boil}} \geq 0.7$ MeV for $\rho_B \geq 10^{-7}$ fm$^{-3}$), and adiabatic paths are shown to go across the boundary of coexisting region even with high entropy. This suggests that materials experienced phase transition can be ejected to outside. We calculated fragment mass and isotope distribution around the boiling point. We found that heavy elements at the iron, the first, second, and third peaks of r-process are abundantly formed at $\rho_B = 10^{-7}, 10^{-5}, 10^{-3}$ and $10^{-2}$ fm$^{-3}$, respectively.

Key words: Liquid-gas phase transition, Supernova explosion, Nucleosynthesis, Equation of state, Relativistic mean field, Nuclear statistical equilibrium

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1 Introduction

It is generally believed that there exist several phases in nuclear matter. Among the phase transitions between these phases, the nuclear liquid-gas phase transition has been extensively studied in these three decades [1]. It takes place in relatively cold ($T_{\text{boil}} = (5-8)$ MeV) and less dense ($\rho_B \sim \rho_0/3$) nuclear matter, and it causes multifragmentation in heavy-ion collisions. When the expanding nuclear matter cools down and goes across the boundary of coexisting region, it becomes unstable against small fluctuations of density or $np$ asymmetry, then various fragments are abundantly formed almost simultaneously. Especially at around the critical point, fragment distribution is
expected to follow the power law \[2\], \(Y_f \propto A^{-\tau}\), which is one of the characteristic features of critical phenomena. Recent theoretical model studies [3–7] have shown that it is very difficult to describe this fragment distribution in a picture of sequential binary decays of one big compound nucleus, which has been successfully applied to the decay of nuclei at low excitation. This finding suggests that it is necessary to consider statistical ensemble of various fragment configurations rather than one dominant configuration in describing fragment formation at around the boundary of coexisting region.

In the universe, the temperature and density of this liquid-gas phase transition would be probed during supernova explosion. In the collapse and bounce stages of supernova explosion, the density and temperature are high enough to keep statistical equilibrium [8,9]. At baryon densities of \(\rho_B \geq 10^{-5} \text{ fm}^{-3}\), since the density is too high for neutrinos to escape, neutrinos are trapped in dense matter. This leads to an approximate conservation of lepton fraction \(Y_L = L/B\) and entropy per baryon \(S/B\), where \(L\) and \(B\) denote the lepton and baryon number, respectively. After the core bounce, supernova matter, which is composed of nucleons and leptons, expands and cools down. As the baryon density and the temperature decrease, charged particle reactions become insufficient and the chemical equilibrium ceases to hold, namely the system freezes out at this point. If the supernova matter goes across the boundary of coexisting region and the boiling point \(T_{\text{boil}}\) of the liquid-gas phase transition is higher than the freeze-out temperature \(T_{\text{fo}}\), this matter will dissolve into fragments and form various nuclei in a critical manner. It further keeps equilibrium and expands to the freeze-out point. The statistical distribution of fragments at freeze-out would provide the initial condition for following nucleosynthesis such as the r-process. (See following references on r-process [10,11] and references therein.)

The importance of the nuclear liquid-gas phase transition in supernova explosion was already noticed and extensively studied before [12]. However, there are two more points which we should consider further. First, the main interest in the previous works was limited to the modification of the equation of state (EoS). The nuclear distribution as an initial condition for the r-process was not studied well. Secondly, in constructing the EoS of supernova matter, the mean field treatment was applied in which one assumed one kind of large nucleus surrounded by nucleon and alpha gas [13,14]. At temperature much above or below the boiling point, fragment mass distribution is narrow and the one species approximation works well. However, since fluctuation dominates at around the boiling point, it is necessary to take account of fragment mass and isotope distribution. This distribution of fragments can modify the following r-process nucleosynthesis provided that the freeze-out point is not far from the boiling point.

In this work, we study nuclear fragment formation through the nuclear
liquid-gas phase transition during supernova explosion. This process may lead to the production of medium mass nuclei as seed elements and serve as a pre-process of the r-process. We call this process as LG process [15]. In order to pursue this possibility quantitatively, it is necessary to determine the liquid-gas coexisting region. We find that the liquid-gas coexisting region extends down to very low density keeping the boiling point around $T_{\text{boil}} \sim 1$ MeV in a two-phase coexistence treatment of EoS with the Relativistic Mean Field (RMF) model [13,16–19]. In supernova explosion, it can happen that material with $S/B \geq 10$ is ejected to outside [20]. Adiabatic paths of ejecta are found to go through the calculated liquid-gas coexisting region even with high entropy. Having this finding of the passage through the coexisting region, we investigate the fragment distribution at around the boiling point in a statistical models of fragments [4–7], referred to as the Nuclear Statistical Equilibrium (NSE) in astrophysics. We show that heavy elements around the first, second, and third peaks of r-process are abundantly formed at $\rho_B = 10^{-5}, 10^{-3}$ and $10^{-2}$ fm$^{-3}$, respectively, with temperatures around and just below $T_{\text{boil}}$ in NSE. Furthermore, the isotope distribution of these elements are also well described in this model. We find that it is important to take account of the Coulomb energy reduction from the screening by electrons in supernova matter. Although nuclei formed at high densities $\rho_B \sim 10^{-2}$ fm$^{-3}$ having very small entropy at around $T_{\text{boil}}$ is not likely ejected to outside, heavy elements up to the r-process third peak are already formed statistically at these densities.

This paper is organized as follows. We describe the treatment of two-phase coexistence with RMF model in Sec. 2. The liquid-gas coexistence is shown in the $(\rho_B, T)$ diagram. We demonstrate that it would be possible for a part of ejecta in supernova explosion to experience the liquid-gas coexisting region. The effects of liquid-gas coexistence on EoS, proton fraction, and adiabatic path are also studied. In Sec. 3, we describe the nuclear statistical model of fragments at equilibrium (NSE) to study the production of elements. We take into account the Coulomb energy modification from electron screening. We evaluate the fragment distribution at around the boiling point and in the coexisting region within this statistical model. We found that fragments are formed abundantly even at very low densities if the temperature is around the boiling point. We compare the calculated mass and isotope distributions of fragments with the solar abundance [21]. In Sec. 4, we discuss the possibility of the ejection of nuclei synthesized in the coexisting region referring to a hydrodynamical calculation of supernova explosion [20]. We summarize our work in Sec. 5.
2 Relativistic Mean Field Approach

2.1 RMF Lagrangian and parameter set

Relativistic Mean Field (RMF) approach has been developed as an effective theory to describe the nuclear matter saturation in a simple way [16]. Having improvements to include meson self-coupling terms, it describes well the binding energies of neutron-rich unstable nuclei in addition to nuclear matter and stable nuclei [17].

In this work, we adopt an RMF parameter set TM1 [17]. It has been demonstrated that this parameter set TM1 can reproduce nuclear properties including proton- and neutron-rich unstable nuclei. In addition, the EoS table with TM1 has been successfully applied to neutron stars and supernova explosions [13,22–24]. Therefore, it is expected to be reliable also in describing two-phase coexistence in supernova matter, which contains asymmetric nuclear matter having proton-to-neutron ratio varying in a wide range.

The Lagrangian contains three meson fields; scalar-isoscalar $\sigma$ meson, vector-isoscalar $\omega$ meson, and vector-isovector $\rho$ meson. In this work, we limit the constituent particles as nucleons, electrons, electron-neutrinos, their anti-particles and photons. The explicit form of the Lagrangian including leptons is given as follows.

$$\mathcal{L} = \bar{\psi}_N \left(i \not\partial - M - g_\sigma \sigma - g_\omega \notomega - g_\rho \not\rho\right) \psi_N$$

$$+ \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4$$

$$- \frac{1}{4} W^{\mu \nu} W_{\mu \nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega_\mu - \frac{1}{4} R^{\mu \nu \rho \sigma} R_{\mu \nu \rho \sigma} + \frac{1}{2} m_\rho^2 \rho^\mu \rho_\mu + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2$$

$$+ \bar{\psi}_e (i \not\partial - m_e) \psi_e + \bar{\psi}_\nu i \not\partial \psi_\nu - \frac{1}{4} F_{\mu \nu} F^{\mu \nu},$$

$$W_{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu,$$

$$R^{\mu \nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu + g_\rho \epsilon^{abc} \rho^{ab} \rho_\mu,$$

$$F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$ (1)

In this section, photon contributions to pressure, energy and entropies are included, but we have dropped the photon-couplings to charged particles (See Sec. 3 on this point).

In a mean field approximation, three meson fields are replaced to their expectation values. Self-consistency condition for these values can be derived in a standard manner, as $\partial P/\partial x = 0$ where $x$ represents the meson field expectation value and $P$ denotes the pressure.
Supernova matter is composed of nucleons, electrons, neutrinos, their anti-particles and photons, characterized by a fixed lepton fraction (lepton-to-baryon ratio) $Y_L$, due to the neutrino trapping at high density. As a result, there are three conserved quantities; baryon number $B$, total charge $C$, and lepton number $L$. Then the chemical potentials of particles are represented by the corresponding three chemical potentials,

$$
\mu_i = B_i \mu_B + C_i \mu_C + L_i \mu_L ,
$$

(2)

where $\mu_B, \mu_C, \mu_L$ are baryon, charge and lepton chemical potentials, and $B_i, C_i, L_i$ are baryon, charge and lepton numbers of particle species $i$. The chemical equilibrium conditions for given conserved densities $\rho_k$ are expressed as $\partial P/\partial \mu_k = \rho_k$. We solve these conditions in a multi-dimensional Newton’s method by iteration.

2.2 Two-phase coexistence treatment in RMF

We apply the mean field approximation to the liquid and gas phases separately. In this treatment, we implicitly assume that two coexisting (liquid, gas) phases are uniform and have infinite size. We solve chemical and thermal equilibrium conditions between the liquid and gas phases.

In order to make liquid and gas phases coexist at equilibrium, we must apply the Gibbs conditions rather than the Maxwell construction, since the number of conserved quantity (= 3) is larger than one. The Gibbs conditions are given as follows,

$$
(1 - \alpha) \rho_{k, \text{Liq.}} + \alpha \rho_{k, \text{Gas}} = \rho_k , \quad \mu_{k, \text{Liq.}} = \mu_{k, \text{Gas}} , \quad P_{\text{Liq.}} = P_{\text{Gas}} ,
$$

(3)

where $k = B, C, L$. The quantities labeled by $\text{Liq.}$ and $\text{Gas}$ are those of liquid and gas phases, respectively. The gas volume fraction, $\alpha$, is a number between zero and unity. We solve these conditions by using multi-dimensional Newton’s method, in which the dimension is five in the asymmetric nuclear matter ($k = B, C$) and seven in the supernova matter ($k = B, C, L$), where the variables are $\rho_{k, \text{Liq.}}, \rho_{k, \text{Gas}}$ and $\alpha$. See Appendix A for the numerical technique at very low densities.

We show the liquid-gas coexisting region of supernova matter in Fig. 1. See Appendix B for the method to determine this boundary. The solid curves show the boundary of coexisting region, and two phases coexist below the boundary. We find that the critical temperature is very high $T_c \sim 14$ MeV for $Y_L = (0.3 - 0.4)$, which is the ratio expected in actual supernova explosions. These critical temperatures are much higher than those in neutrino-less supernova...
LG Coexistence Boundary

![Graph showing LG Coexistence Boundary with different phases and densities.](image)

**Fig. 1.** Boundary of liquid-gas coexisting region in supernova matter (solid curves) in comparison with symmetric nuclear matter \((Y_p = 0.5, \text{dotted line})\) and neutrino-less supernova matter (or finite temperature neutron star matter, \(\mu_L = 0, \text{dashed line})\).

matter (dashed line) and comparable to those in the symmetric nuclear matter (dotted line). The boiling points of supernova matter remain to be \(T_{\text{boil}} \sim 1\) MeV even at very low densities for all lepton fractions.

The large value of \(T_{\text{boil}}\) is due to symmetrization of nuclear matter by leptons. Since \(Y_L\) is kept constant but \(Y_p\) is not fixed, the supernova matter searches its minimum in free energy by changing \(Y_p\). In high density supernova matter, the lepton fraction is shared by electrons and neutrinos \((2/3Y_L < Y_e < Y_L)\), then the net neutrino fraction and the lepton chemical potential become positive \((\mu_L = \mu_\nu > 0)\). Compared to neutrino-less supernova matter \((\mu_L = 0)\), this positive \(\mu_L\) in supernova matter helps to enhance \(Y_e(= Y_p)\) \((\mu_e = -\mu_C + \mu_L)\), and to symmetrize nuclear matter. At very low baryon densities, electrons become non-degenerate, i.e. electron and anti-electron densities are much higher than the net electron densities because of small electron mass, and this also applies to neutrinos. Hence, electron and neutrino chemical potentials become small, and the charge chemical potential also becomes small. This causes nuclear matter symmetric.

In order to demonstrate this point, we show the density dependence of the asymmetry parameter of supernova matter in Fig. 2. Dotted, solid and dashed curves show the asymmetry parameter in uniform (homogeneous), two-phase coexisting, and the liquid part of coexisting matter, respectively. It is clear that, as the baryon density decreases, asymmetry parameter decreases and
this tendency is stronger in the coexisting region. In uniform matter, the asymmetry decreases rapidly at around $\rho_B < 10^{-7}$ fm$^{-3}$, where leptons dominate pressure and energy density, as shown later in Fig. 4. This is consistent with the above consideration on the lepton dominance. On the other hand, the asymmetry parameter start to decrease at much higher density in coexisting matter. This symmetrization is mainly due to the symmetry energy in nuclear matter. Since the baryon density in the liquid part of matter is around $\rho_0$, nucleons can gain symmetry energy by reducing the asymmetry. As shown by dashed lines, the asymmetry decreases quickly in the liquid part in the coexisting region.

We turn our attention to one of the characteristic features of the liquid-gas coexisting region in supernova matter, which allows many fragments to be formed even at very low densities. First we define a new quantity, gas fraction $Y_g$, $Y_g = (\text{baryons in gas phase})/(\text{total baryons}) \equiv \alpha \rho_B^{\text{Gas}}/\rho_B$ as a measure of bulk fragment yield. Here, we can adopt the normal nuclear matter density $\rho_0$ for the baryon densities in the liquid phase. After we substitute these baryon densities $\rho_0$ and $\rho_B^{\text{Gas}}$ for the Gibbs condition, we can obtain the following equation,

$$\alpha \simeq \frac{\rho_0 - \rho_B}{\rho_0 - \rho_B^{\text{Gas}}}.$$  \hspace{1cm} (4)
For example, all baryons are bound in nuclei at $Y_g = 0$. Smaller $Y_g$ means larger amount of fragments. In Fig. 3, we show the gas fraction in nuclear matter composed of neutrons and protons ($np$ matter) and supernova matter as a function of the baryon density and the asymmetry parameter, calculated in the RMF model. We fix $Y_p$ for calculations of $np$ matter. The behavior of $Y_g$ in $np$ matter is smooth. In symmetric nuclear matter ($Y_p = 0.5$), both of the liquid and gas phases are symmetric, and the density in each phase is constant in the coexisting region. Then the gas fraction can be expressed by liquid, gas and the given average densities ($\rho_{Liq.}, \rho_{Gas}, \rho_B$) as

$$
Y_g \approx \frac{\rho_B^{Gas} (\rho_0 - \rho_B)}{\rho_B (\rho_0 - \rho_B^{Gas})}.
$$

This is a monotonically decreasing function of the baryon density and very small in the density range under consideration. When the asymmetry increases, the liquid phase loses the symmetry energy and nucleons are emitted to the gas phase, while $Y_g$ is still a decreasing function of $\rho_B$.

The behavior of $Y_g$ in supernova matter is very different from that in $np$ matter. As shown by the thick lines in Fig. 3, as the baryon density decreases, $Y_g$ first decreases then grows again. This behavior is determined by the proton fraction (proton-to-baryon ratio) $Y_p$. In the medium density region ($\rho_B = (10^{-7} - 10^{-2}) \text{ fm}^{-3}$), matter becomes symmetric as $\rho_B$ decreases, and the gas baryon fraction decreases to the symmetric nuclear matter value in Eq. (5). Therefore, baryons favor liquid state than nucleon gas in the coexisting region. After reaching symmetric matter, gas fraction increases again according to Eq. (5). It is also interesting to note that the proton fraction $Y_p$ of supernova
Fig. 4. Equation of state of supernova matter for lepton fractions $Y_L = 0.1, 0.2, 0.3$ and 0.4. Temperatures are $T = 0.1, 0.5, 1, 2, 5, 10, 15$ and 20 MeV, from top to down (from down to top) for $F/A$ ($E/A$ and $P/\rho_B$). Results for uniform supernova matter are shown by dotted curves, and those with liquid-gas coexistence are shown by solid curves. For energies per baryon ($E/A$) and free energies per baryon ($F/A$), nucleon mass is subtracted. The pressures are increased by a factor of 10 from low to high temperatures (from bottom to top).

matter shows clear $Y_L$ dependence at $\rho_B > 10^{-5}$ fm$^{-3}$, while $Y_p$ is almost independent on $Y_L$ at $\rho_B < 10^{-5}$ fm$^{-3}$. This density roughly corresponds to the neutrino-sphere, inside which neutrinos are trapped in supernova core.

### 2.3 Supernova Matter Equation of State

We show the EoS of supernova matter with (without) two-phase coexistence by solid (dotted) lines in Fig. 4. In both of the cases, energy and pressure increase at high densities above $\rho_0$ because of the vector meson contributions. At very low densities, lepton contributions become dominant, because lepton pressure and energy densities are finite even with $\rho_B = 0$ at finite temperatures. In uniform matter, the nuclear pressure and energy become close to free-gas values $P_N/\rho_B \to T, E_N/A \to 3T/2$. Between these two extremes of density, we can find the effects of two-phase coexistence.

In two-phase coexistence, by making liquid phase whose baryon density is around $\rho_0$, binding energy is gained. At low temperatures, since nuclear matter tends to be symmetric and liquid phase is dominant in a wide range of densities, liquid part of energy per baryon approaches toward $-16$ MeV.
Fig. 5. Adiabatic paths in supernova matter calculated in RMF. The thin solid (dashed) lines show the adiabatic paths of supernova matter with (without) the liquid-gas coexistence. Thick solid lines are the boundary of coexisting region. The each adiabat corresponds to entropy per baryon $S/B = 1, 2, ..., 10, 20, 100$. The panels are in case of lepton fraction $Y_L = 0.1, 0.15, ..., 0.5$ from upper-left to lower-right, respectively.

At large lepton fractions ($Y_L = 0.3 - 0.4$), the pressure behaves as in the case of Maxwell construction for volume instability; pressures in the coexisting region are lower (higher) at low (high) densities than in uniform matter during the coexistence. On the other hand, we can find double phase transition behavior in lepton deficient matter ($Y_L = 0.1 - 0.2$); while the pressure behaves as in the case of volume instability at higher densities ($\rho_B \geq 10^{-4} \text{ fm}^{-3}$, there is another overtaking in the density region of $\rho_B = 10^{-8} \sim 10^{-4} \text{ fm}^{-3}$. At these densities, the matter becomes unstable to the small fluctuations of proton fraction, and liquid part of the matter becomes rapidly symmetric as the density decreases. Thus the overtaking of the pressure in the low-density region may be suggesting the phase transition in the isospin degrees of freedom.

2.4 Adiabatic Paths

After the core bounce, some part of supernova matter expands almost adiabatically, and high entropy ($S/B \geq 10$) part of the matter would be ejected to
outside [20]. (See also Section 4.) Therefore, it is important to examine supernova matter along the adiabatic paths. In Fig. 5, we show the adiabatic paths in supernova matter with lepton fraction $Y_L = 0.10, 0.15, \ldots 0.50$. Entropy per baryon is taken to be $S/B = 1, 2, \ldots 10, 20$, and 100. At very high entropies such as $S/B = 100$, adiabatic paths are almost independent on $Y_L$ because of the lepton dominance. At lower entropies, we can clearly see nuclear and coexistence effects. While adiabatic paths in uniform matter (dashed lines) evolve very smoothly, those with two-phase coexistence (thin solid lines) bend closer to the boundary of coexisting region. This can be seen even at $S/B = 20$. This bending comes from the latent heat and suggests that significant amount of nuclei are formed around the boundary.

3 Fragment Distribution in Supernova Matter

Observations in the previous section tell us that it would be possible for supernova matter to experience the liquid-gas phase transition before it is ejected to outside. Therefore, it is interesting to study the composition of ejecta experienced the phase coexistence. However, we have assumed that the coexisting two phases are infinite. It is also to be noted that nuclear matter in the liquid phase is more symmetric than in the gas phase. As a result, the Coulomb energy is expected to be large, which we have neglected in the previous section, and the infinite matter in the liquid phase will fragment into finite nuclei. Although the effects of these nuclear formation on EoS may not be very large [12], fragment distribution at freeze-out would serve as the initial condition for the following r-process. Therefore, in this section, we evaluate the fragment yield in a fragment-based statistical model.

3.1 Statistical Model of Fragments

In order to describe distribution of finite nuclei, we utilize a fragment-based statistical model. This kind of statistical models have been widely used in heavy-ion collision studies [4–7] as well as in astrophysics, as referred to the nuclear statistical equilibrium (NSE) [11,25]. In NSE, we solve the statistical equilibrium condition among nucleons, fragments and leptons in fragment-based grand canonical ensemble. In this work, we have ignored relativistic corrections and anti-particle contributions of fragments, and fragments are assumed to follow the Boltzmann statistics, while leptons are treated relativistically. The ensemble averages can be generated from the grand potential,

$$\Omega = -PV = -VT \sum_i \rho_f - P_V - P_\gamma V,$$ 

(6)
\[ \rho_f = \zeta_f(T) \left( \frac{M_f T}{2\pi \hbar^2} \right)^{3/2} \exp \left( \frac{B_f + \mu_f}{T} \right), \]  
\[ \mu_f = Z_f (\mu_p - m_N) + N_f (\mu_n - m_N), \]

where \( \rho_f, M_f, B_f \) and \( \mu_f \) denote the density, mass, binding energy and the chemical potential of fragment \( f \), and \( P_\ell \) and \( P_\gamma \) are the lepton and photon pressures, respectively. The intrinsic partition function, \( \zeta_f(T) \), has been calculated by using the level density formula for fragments with \( A_f \geq 5 \) [6],

\[ \zeta_f(T) = \sum_i g_f^{(i)} \exp \left( -\frac{E^{(i)}_f}{T} \right) \]

\[ \simeq g_f^{(g.s.)} + \frac{c_1}{A_f^{8/3}} \int_0^\infty dE^* e^{-E^*/T} \exp(2\sqrt{a_f E^*}) , \]

where \( g_f^{(i)} = 2j_f^{(i)} + 1 \) is the spin degeneracy of the energy level at excitation energy \( E^{(i)}_f \) of the fragment species \( f \).

In NSE, the nuclear binding energy \( B_f \) plays an essential role. Fragment yields are sensitive to the binding energy modification due to, for example, the medium effects in supernova matter. In studies of heavy-ion collisions, since the density and its fluctuation are large, the repulsive interfragment Coulomb potentials are taken into account explicitly rather than in the form of mass modification. In supernova matter under consideration, attractive electron-fragment Coulomb potential effects are more important. Since electron density is not negligible and almost constant, we ignore inter-fragment Coulomb potentials and the electron effects on intrinsic fragment Coulomb energies are incorporated in the form of binding energy modification. We have used the Wigner-Seitz approximation in evaluating the Coulomb energy correction \[12\] to the binding energy of nuclei adopted in NSE. We assume that the electrons are distributed uniformly in a sphere with radius \( R_{ef} \) which is determined to cancel the charge of the fragment \( f \) at a given electron density \( \rho_e \).

\[ B_f(\rho_e) = B_f(0) - \Delta V_f^{\text{Coul}}(\rho_e) , \]

\[ \Delta V_f^{\text{Coul}} = -\frac{3}{5} \frac{Z_f^2 e^2}{R_0} \left( \frac{3}{2} \eta_f - \frac{1}{2} \eta_f^3 \right) , \quad \eta_f \equiv \frac{R_{0f}}{R_{ef}} = \left( \frac{\rho_e}{Z_f \rho_0 / A_f} \right)^{1/3} , \]

where \( B_f(0) \) is the nuclear binding energy in vacuum, and \( R_{0f} \) is the nuclear radius.

It is important to note that the Coulomb energy correction, \( \Delta V_f^{\text{Coul}}(\rho_e) \), contains the term proportional to \( \rho_e^{1/3} \). Because of this functional form, the
correction is meaningfully large even at $\rho_B = 10^{-6} \rho_0$. For example, when the electron fraction is $Y_e = 0.3$, the reduction of the Coulomb energy for heavy nuclei amounts to 90% of the total Coulomb energy at $\rho_B = \rho_0$, and the reduction is around 10 MeV even at $10^{-6} \rho_0$. The increase of binding energy acts to enhance heavy nuclei, and some nuclei beyond the dripline at vacuum or unstable against fission, can be stabilized in supernova matter. Finite gas nucleon density also plays a role to form nuclei beyond the dripline tentatively by the balance of nucleon absorption and emission. We have adopted the mass table of Myers and Swiatecki [26], which is based on the Thomas-Fermi model with shell correction for about 9000 kinds of nuclei.

Since nuclear binding energies depend on the electron density, we have to solve the chemical equilibrium condition of nuclei and leptons in supernova matter in a consistent way to satisfy $F_\mu = \mu_p + \mu_e - \mu_n - \mu_\nu = 0$. Provided that the baryon density and temperature are given, and that the charge and lepton densities are fixed as $(\rho_c, \rho_\ell) = (0, Y_L \rho_B)$, all the particle densities are determined if the average proton fraction $(Y_p)$ is given,

$$
(1 - Y_p) \rho_B = \sum_f N_f \rho_f(\mu_n, \mu_p, B_f(\rho_e)) \equiv \overline{\rho}_n(\mu_n, \mu_p, \rho_e),
$$

(12a)

$$
Y_p \rho_B = \sum_f Z_f \rho_f(\mu_n, \mu_p, B_f(\rho_e)) \equiv \overline{\rho}_p(\mu_n, \mu_p, \rho_e),
$$

(12b)

$$
Y_p \rho_B = \rho_c(\mu_e), \quad (Y_L - Y_p) \rho_B = \rho_\nu(\mu_\nu),
$$

(12c)

where $\overline{\rho}_n$ and $\overline{\rho}_p$ are the neutron and proton densities including those in nuclei. We can easily solve last two conditions in Eqs. (12c) numerically, and derivatives of $\mu_e$ and $\mu_\nu$ with respect to $Y_p$ can be obtained as $d\mu_e/dY_p = \rho_B (d\rho_e/d\mu_e)^{-1}$ and $d\mu_\nu/dY_p = -\rho_B (d\rho_\nu/d\mu_\nu)^{-1}$. Once $Y_p$ is given, first two equations are the same as usual conditions in fragment-based statistical models. By using the charge neutrality condition $\rho_c = Y_p \rho_B$, we can get the derivatives of $\mu_n$ and $\mu_p$ with respect to $Y_p$ as,

$$
\begin{pmatrix}
\partial \overline{\rho}_n/\partial \mu_n, \partial \overline{\rho}_n/\partial \mu_p \\
\partial \overline{\rho}_p/\partial \mu_n, \partial \overline{\rho}_p/\partial \mu_p
\end{pmatrix}
\begin{pmatrix}
 d\mu_n \\
 d\mu_p
\end{pmatrix}
= \rho_B
\begin{pmatrix}
-1 - \partial \overline{\rho}_n/\partial \rho_e \\
1 - \partial \overline{\rho}_p/\partial \rho_e
\end{pmatrix}
\begin{pmatrix}
 Y_p \\
 1
\end{pmatrix}
\frac{dY_p}{dY_p}.
$$

(13)

Therefore, we can solve the chemical equilibrium condition, $F_\mu = 0$, by the Newton’s method, $\delta F_\mu = -F_\mu/ (dF_\mu (Y_p)/dY_p)$.

### 3.2 Gas Fraction and Equation of State in a Statistical Model

Compared to RMF, there is no sharp phase transition in NSE because of the finite size of fragments. Although we can see kink-like behavior in the nuclear
Fig. 6. Left: Mass moment ratio (upper panel) and caloric curve (lower) of supernova matter with $Y_L = 0.35$ at $\rho_B = 10^{-7}$ and $10^{-3}$ fm$^{-3}$. In the upper panel, mass moment ratios, $M_2/M_1$ (thin solid lines), $\xi = \sigma^2(A_f)/\langle A_f \rangle$ (with $\alpha$, dotted lines), $\xi'$ (same as $\xi$ but without $\alpha$, thick solid lines) are shown. Right: $\xi'$ (upper panel) and caloric curve (lower) of supernova matter with $Y_L = 0.35$ at baryon densities $\rho_B = 10^{-7}, 10^{-5}, 10^{-3}$ and $10^{-2}$ fm$^{-3}$. Results with Coulomb correction (solid lines) and without Coulomb correction (dashed lines) are compared.

Fig. 7. Liquid-gas coexisting region of supernova matter with $Y_L = 0.35$ calculated in NSE (solid line) in comparison with RMF results (dashed line). Results without Coulomb correction is shown by dotted curve. Boiling point in the statistical model has been determined as the maximum point of mass variance-to-average ratio, while there appear two local maxima in the fragment mass fluctuation when we include the contribution of $\alpha$ particle, as shown by dot-dashed curves.

part of energy per baryon as a function of the temperature (caloric curve), this kink is not clear enough to define the boiling point as seen in Fig. 6 . There are several definitions of $T_{boil}$ proposed in the literature. For example, $T_{boil}$ is proposed to be well defined at the peak of $M_2/M_1$ by Bauer [7].
Table 1
Boiling points in NSE at $Y_L = 0.35$ as a function of the baryon density. Results with (NSE) and without (NSE, nc) Coulomb corrections are shown.

| $\rho_B$(fm$^{-3}$) | $10^{-7}$ | $10^{-5}$ | $10^{-3}$ | $10^{-2}$ |
|---------------------|-----------|-----------|-----------|-----------|
| $T_{boil}$(NSE) (MeV) | 0.77      | 1.43      | 3.90      | 6.88      |
| $T_{boil}$(NSE,nc) (MeV) | 0.77      | 1.40      | 3.52      | 5.42      |

$n$-th moment of light fragments, $M_n$, is defined as $M_n \equiv \sum_f A_f^n \rho_f$. Another way is to define $T_{boil}$ by the peak of the light fragment mass variance-to-average ratio, $\xi \equiv \sigma^2(A_f)/\langle A_f \rangle$, which becomes small when one fragment (or nucleon) dominates, and becomes unity when the mass distribution is a Poissonian. We show these ratios in the upper-left panel of Fig. 6. At high densities, these definitions give reasonable boiling points, but at low densities, the peak of $M_2/M_1$ becomes dull, and $\xi$ shows two peaks. From fragment distributions, we find that the two peak structure appears due to the formation of $\alpha$ which can be comparable to neutrons at very low densities, where gas part becomes almost symmetric. In Fig. 7, we show the temperatures at local maxima of $\xi$ by dot-dashed lines. This formation of $\alpha$ makes also $M_2/M_1$ peak dull. Therefore, we here define the boiling point as the peak of the light fragment mass variance-to-average ratio $\xi'$, where we excluded $\alpha$ particle in the calculation of the light fragment mass average and variance. As shown in the solid lines in Fig. 6, the peak position of $\xi'$ is well defined at any density. In addition, this boiling point corresponds to the kink position in the caloric curve.

In Fig. 7, we show the density dependence of $T_{boil}$, in comparison with the RMF results. We find that the boiling points in NSE are lower than those in the two phase treatment of the RMF model by about a factor of two. The reduction of $T_{boil}$ is a natural consequence of finite size of nuclei. Because the intrinsic Coulomb energy cannot be completely removed by electrons at densities $\rho_B < \rho_0$, nuclei are limited to have finite size. Then nuclei loses surface energy in addition to the Coulomb energy, and gas nucleons are favored. However, it is worthwhile to note that the qualitative behavior of $T_{boil}$ is similar to that in RMF, and they are still high enough, $T_{boil} > 0.7$ MeV for $\rho_B > 10^{-7}$ fm$^{-3}$. We tabulate the boiling points at $\rho_B = 10^{-7}, 10^{-5}, 10^{-3}$ and $10^{-2}$ fm$^{-3}$ in Table 1.

In the RMF treatment, one of the most characteristic features in the coexisting region is the reduction of the gas fraction. This also applies to the NSE results. We have defined the gas fraction as the ratio of isolated nucleon density to the total baryon density, $Y_g \equiv (\rho_p + \rho_n)/\rho_B$. As shown in Fig. 8, gas fraction behaves similarly to that in RMF results; as the baryon density decreases, it quickly becomes very small until $\rho_B \sim 10^{-8}$ fm$^{-3}$, and gradually grows at lower densities again. In addition, it is interesting to note that $Y_g$ curves within the statistical model have the second minimum at $\rho_B \sim 10^{-3}$ fm$^{-3}$. Supernova
matter favors nuclear fragment (nucleus) state rather than nucleon gas at these densities. We call these minimum regions as the first ($\rho_B \sim 10^{-7} \text{ fm}^{-3}$) and the second ($\rho_B \sim 10^{-3} \text{ fm}^{-3}$) fragment windows, respectively. The first one is caused by the drastic isospin-symmetrization of supernova matter by leptons at low baryon densities. The second fragment window is specific to NSE. The mechanism of this appearance is not very clear, but we find that these two minima converges to one when we use the liquid drop mass formula for the nuclear binding energies and ignore the surface term.

In Fig. 8, we show the EoS in NSE at $T = 5$ and $1$ MeV with $Y_L = 0.35$, in comparison with those in RMF with the Thomas-Fermi approximation for a dominant configuration [13] (RMF+TF, dotted lines), RMF with two-phase coexistence (RMF(coex.), dashed lines), and homogeneous RMF (RMF(unif.), dot-dashed lines). We find that the finiteness of nuclei does not affect the EoS at high densities, $\rho_B \geq 10^{-2} \text{ fm}^{-3} (10^{-4} \text{ fm}^{-3})$ for $T = 5$ MeV (1 MeV), but modifies the density dependence of the pressure at lower densities, as seen in the difference between RMF(coex.) and RMF+TF. Compared with the EoS in RMF+TF, the present NSE results give very similar pressures, except for the density region $\rho_B \sim 10^{-3} \text{ fm}^{-3}(10^{-6} \text{ fm}^{-3})$ for $T = 5$ MeV (1 MeV), where the pressure are different by $10 \sim 20\%$. These densities correspond to the region where the given temperatures are close to the boiling points.

The agreement of EoS in NSE and RMF+TF is somewhat surprising. There are three large differences in NSE and RMF+TF: (1) Nuclear masses are taken from the table [26] in NSE, while masses are calculated in RMF+TF. (2) Interfragment and nucleon-fragment nuclear interactions are neglected in NSE, while they are included in RMF+TF. (3) One configuration is assumed in RMF+TF, while statistical ensemble is considered in NSE. Thus the above agreement might suggest that once nuclear masses are properly included, nuclear interactions between gas nucleon and fragments play a minor role in
Fig. 9. Equation of state of the supernova matter in NSE in comparison with those in RMF models at $T = 1$ MeV (lower curves) and $T = 5$ MeV (upper curves, scaled up by a factor of 10). The solid, dotted, dashed and dot-dashed line denote EoS in NSE, RMF with Thomas-Fermi approximation model (RMF+TF) [13], the liquid-gas coexisting RMF model (RMF(Coex.)) and homogeneous RMF model (RMF(Unif.)), respectively.

3.3 Fragment Distribution and Coulomb Correction Effects

In this subsection, we investigate fragment distribution in the density range $10^{-7} \leq \rho_B \leq 10^{-2}$ fm$^{-3}$, starting from the first fragment window to the density close to the critical point. We are most interested in the boundary of coexisting region at the temperatures around $T_{\text{boil}}(\rho_B)$. If the density is not very small, the freeze-out temperature $T_{fo}$ is expected to be around or just below the boiling point $T_{\text{boil}}$. Below the boiling points, fragments are formed abundantly by absorbing many of gas nucleons, and nuclear number density of particles (sum of nucleon and fragment densities) becomes quickly smaller, then the mean free path for each particle becomes much longer. This rapid fragment formation makes the typical interaction intervals longer, and is expected to help the system to freeze-out. The condition of freeze-out should be studied more carefully.

We can see common features of the temperature dependence of fragment mass distributions in each row of Fig. 10. When the temperature is higher than the boiling point, the fragments almost obey the exponential distribution. At the temperature near the boiling point, the distribution shows the power-law like behavior up to some mass. A similar trend, $Y(A_f) \sim A_f^{-\tau}$, also appears
in nuclear multifragmentation [1], where $Y$ and $A_f$ denote the fragment yield and mass. This power law was suggested by Fisher [2] for a mass distribution of droplets at around the critical point. As the temperature becomes lower than the boiling point, the distribution at each density becomes localized to some mass number. The localization is caused by the small entropy and shell effects.

The density $\rho_B = 10^{-7}$ fm$^{-3}$ corresponds to the the first fragment window, where supernova matter is symmetrized by leptons and nuclei are formed.
abundantly at low temperatures. As shown in Fig. 10 (first column), the main products near the boiling point \( (T = T_{\text{boil}} \sim 0.77 \text{ MeV}) \) are nucleon, \( \alpha \), iron peak nuclei, and they extend to the first peak nuclei of r-process in the distribution shoulder. The Coulomb correction effect is negligible at this density. Although the above distribution seems to be similar to the initial seed nuclear composition in a standard scenario of the r-process, we would like to point out that when the temperature rises to around \( T_{\text{boil}} \sim 0.8 \text{ MeV} \), the distribution become broad due to thermal fluctuation. Especially, just above the boiling point, \( T = 1.1 T_{\text{boil}} \) (top panel), the equilibrium nuclear distribution resembles to the solar abundance up to the iron peak.

Next, we show the results at \( \rho_B = 10^{-5} \text{ fm}^{-3} \) in Fig. 10 (second column), which corresponds to the density around the neutrino sphere. Most stable nuclei at this density are those at around the first peak of r-process, as can be seen in the distribution at low temperatures. As a result, these nuclei are formed easily also at around the boiling point \( (T = 1.0 T_{\text{boil}} = 1.43 \text{ MeV}) \), in addition to nucleons, \( \alpha \), and iron peak nuclei. The Coulomb correction effect is not large, but at around \( T_{\text{boil}} \), we can see small enhancement of heavy fragments with larger masses over the first peak of r-process.

In the second fragment window \( (\rho_B = 10^{-3} \text{ fm}^{-3}) \), power-law like behavior can be seen at around \( T_{\text{boil}} = 3.90 \text{ MeV} \) up to the second peak of r-process, as shown in the third column of Fig. 10. Coulomb correction effects are clearly seen at this density. The center of the distribution shifts from the first peak of r-process without Coulomb correction to the second peak of r-process with Coulomb correction.

As shown in the fourth column of Fig. 10, the distribution at \( \rho_B = 10^{-2} \text{ fm}^{-3} \) shows the same trend. It is interesting to find that the peak position in the NSE results is shifted downwards compared to the observed third peak of r-process. This shift mainly comes from the \( np \) ratio of formed nuclei. The observed third peak of r-process is a consequence of the neutron magic number \( N = 126 \) and the \( np \) ratio along the r-process path. In the present NSE model calculation, having a large \( np \) ratio, nuclei beyond the neutron dripline appear easily at equilibrium.

We also give an example of isotope distribution at \( Y_L = 0.35 \) in Fig. 11. Here, we choose a slightly lower temperature than the boiling point, \( T_{\text{fo}} = 0.9 T_{\text{boil}} \), for the freeze-out temperature, as discussed at the beginning of this subsection. The upper-left, upper-right, lower-left and lower-right panels of Fig. 11 show the isotope distributions at \( \rho_B = 10^{-2}, 10^{-3}, 10^{-5} \) and \( 10^{-7} \text{ fm}^{-3} \), respectively. At \( \rho_B = 10^{-2} \text{ fm}^{-3} \), while the observed isotope ratio are well explained in the calculation from \( Z = 20 \) (Ca) to \( Z = 92 \) (U) with one overall normalization factor, much more neutron rich nuclei appear at equilibrium. For isotones with \( N = 126 \), nuclei with \( Z = 54 \) to \( Z = 92 \) are formed. This large \( np \) ratio, which
comes from the large \( np \) asymmetry of the liquid phase as shown in Fig. 2, gives smaller mass number with \( N = 126 \).

At lower densities, similar trends can be seen in the mass number range produced at around \( T_{\text{boil}} \), except for \( \rho_B = 10^{-7} \text{ fm}^{-3} \). As already discussed, nuclear matter is symmetrized outside of the neutrino sphere, the calculated distribution at \( \rho_B = 10^{-7} \text{ fm}^{-3} \) is shifted toward proton rich side of the observed solar abundance for large \( Z \).

Fig. 11. Calculated even \( Z \) isotope distribution at \( \rho_B = 10^{-2}, 10^{-3}, 10^{-5} \) and \( 10^{-7} \text{ fm}^{-3} \) and \((T, Y_L) = (0.9 T_{\text{boil}}, 0.35)\) in comparison with the solar abundance by circle and triangle symbols. The boiling points are those for given densities. Yields are shifted, for clarify of plot, by a factor of 10 from \( Z = 20 \) (Ca) to larger \( Z \) nuclei (from top to bottom). One overall factor is chosen to get good global fit.

3.4 Fragment Distribution outside of Neutrino Sphere and in Neutrino-less Supernova Matter

While we have considered the \( \beta \)-equilibrium condition with fixed lepton fraction \( Y_L \), proton fraction \( Y_p \) is almost independent on \( Y_L \) outside of the neutrino sphere. Once \( Y_p, \rho_B, \) and \( T \) are given, neutron and proton chemical potentials are uniquely determined, thus boiling point and fragment distribution are also obtained uniquely. In RMF, this independence on \( Y_L \) outside of the neutrino sphere can be found in \( Y_p \) (Fig. 2) and in \( T_{\text{boil}} \) (Fig. 1). The same tendency
Fig. 12. The fragment distribution in neutrino-less supernova matter with $Y_p = 0.46$ (thick lines, $n, p, e^-$) and supernova matter (thin lines, $n, p, e^-, \nu_e$) at $\rho_B = 10^{-7} \text{fm}^{-3}$ in comparison with the solar abundance (filled circles). For supernova matter, we have chosen $Y_L = 0.1 \sim 0.45$. The boiling points are almost independent on $Y_L, T_{\text{boil}} \sim 0.77 \text{ MeV}$.

applies to fragment distribution in the NSE results of supernova matter and neutrino-less supernova (NS) matter in which neutrino chemical potential is zero ($\mu_\nu = \mu_L = 0$). In Fig. 12, we show the fragment mass distribution in NS matter at $\rho_B = 10^{-7} \text{fm}^{-3}$, in which $Y_p = 0.46$, in comparison with that in supernova matter ($Y_L = 0.1 \sim 0.45$). We find that the boiling points and nuclear distribution are almost independent on $Y_L$, and they are almost the same as those in NS matter.

The above observations tell us that the fragment distributions under thermal and charge equilibrium at around $T_{\text{boil}}$ are independent of degree of neutrino trapping. In addition, the consequent fragment distribution provides iron-group nuclei, which become seed elements in a standard r-process scenario. On the contrary, $Y_p$ clearly depends on $Y_L$ inside the neutrino sphere. The nuclear distribution formed in the neutrino sphere is sensitive to the dynamics of supernova explosion, especially on how much neutrinos escape before the neutrino trapping.

4 Relation to Nucleosynthesis in Supernovae

One of the key questions on the LG-process is whether the elements made through the liquid-gas phase transition are ejected to outside. If they are ejected, they contribute to the following nucleosynthesis such as the r-process.

As mentioned earlier, all trajectories of adiabatic path go across the boundary of coexisting region. If the material with $S/B \geq 10$ is ejected in super-
nova explosion, the present model calculation suggests that the material would go through the coexisting region before the ejection. This can happen if the freeze-out temperature of supernova matter is as low as $T = (0.5 - 2)\text{ MeV}$ at densities $\rho_B = (10^{-7} - 10^{-5})\text{ fm}^{-3}$, where $S/B \geq 8$ at the boiling points provided that the adiabatic paths go straight until the boundary. Since the fragment distribution is almost independent of the lepton fraction at these densities, the distribution shown in Fig. 10 would appear regardless of details of neutrino trapping.

In order to examine this possibility, we analyse the results of hydrodynamical calculation of core-collapse supernova [20]. In their calculation, the EoS table derived by the RMF-TF model with the TM1 interaction [13] has been used in the general relativistic hydrodynamics [27]. Note that this EoS table and the EoS in NSE give almost the same pressure as shown in Fig. 9. Among series of model calculations of adiabatic collapse, a case of iron core of $15M_\odot$ presupernova star [28] explode hydrodynamically with the fixed electron fraction, which were assumed to study the properties of EoS in model explosion. In this model explosion, the material from deep inside the core can be ejected. The entropy per baryon of this ejecta is about $S/B \sim 10$ determined by the shock passage. The trajectory of this ejecta passes through the coexisting region. Further inner material has lower entropies down to $S/B \sim 1$, which are favorable for the LG process, but the ejection becomes rather difficult. It is also to be noted that the model explosion is obtained by a simplified calculation without neutrino-transfer assuming large electron fraction. Further careful studies are necessary to examine quantitatively the mass ejection by performing the numerical simulations of hydrodynamics with neutrino-transfer of core-collapse.

Although hydrodynamical explosion (so-called prompt explosion) has been claimed to be limited for very small iron core and extreme cases of EoS and electron capture rates [8,9], the outcome of nucleosynthesis is extremely interesting if the mass ejection really occurs. A case of prompt explosion of the small iron core of $11M_\odot$ presupernova star has been studied as a site of successful r-process nucleosynthesis by the dynamical ejection of neutron-rich material [29]. In their studies, the material having entropy per baryon $S/B \sim 10$ is ejected. These ejecta may be affected by the LG-process to help the production of heavier r-process elements around actinides. If the ejection from relatively high density ($\rho_B \geq 10^{-5}\text{ fm}^{-3}$) takes place, the formed fragments are generally very neutron rich and some part of them are unstable against neutron emission. These nuclei provide a huge amount of neutrons, which help the following r-process to proceed. Mass ejection is also expected in asymmetric supernova explosion. The convection of material is generally seen in supernova simulations and the material deep inside may be ejected by hydrodynamical instability. (See [30] for example.) Jet-like ejection of material may occur in rotating collapse of stars as discussed in some literatures [31,32].
The ejection of material and its consequence on nucleosynthesis should be clarified together with the problem of supernova explosion mechanism and require further extensive studies.

5 Summary and discussion

In this paper, we have investigated the liquid-gas phase transition of supernova matter, and its effects on the fragment formation. We have used two models — the Relativistic Mean Field (RMF) model and the Nuclear Statistical Equilibrium (NSE) model.

In RMF, we have used the interaction TM1, which has been successfully applied to finite nuclei including neutron rich unstable nuclei, neutron stars, and supernova explosion [13,22–24]. Leptons are shown to play non-trivial roles such as the symmetrization of nuclear part of supernova matter. As a result, nuclear liquid gains symmetry energy, and the calculated boiling points in supernova matter ($T_{\text{boil}} > 1$ MeV for $\rho_B \geq 10^{-10} \text{ fm}^{-3}$) are comparable to those in symmetric nuclear matter at low densities. Adiabatic paths are shown to go across the boundary of coexisting region even at high entropy such as $S/B \geq 10$, which is expected to be enough for supernova matter to be ejected to outside. Clear concentration of adiabatic paths to the boundary of coexisting region have been found. All of these findings suggest that at least a part of ejecta in supernova explosion would experience the liquid-gas phase transition before freeze-out.

In NSE, we have used nuclear binding energies of Myers-Swiatecki model [26] with Coulomb correction due to electron screening as a medium effect [12]. Since larger species of nuclei become stable with this Coulomb energy correction, we have adopted the mass table of around 9000 nuclei constructed by Myers and Swiatecki [26]. Because of the finiteness of nuclei, they lose surface and Coulomb energy compared to the case of coexistence treatment of two infinite matter phases in RMF. The boiling points become slightly lower, but they are still high; $T_{\text{boil}} \geq 0.7$ MeV for $\rho_B \geq 10^{-7} \text{ fm}^{-3}$. Calculated fragment mass distributions around $T_{\text{boil}}(\rho_B)$ show enhancement of the iron peak elements, the first, second, and third peak r-process elements at $\rho_B = 10^{-7}, 10^{-5}, 10^{-3}$ and $10^{-2} \text{ fm}^{-3}$, respectively. In addition, calculated isotope distribution shows that very neutron rich nuclei around and beyond the neutron dripline may exist under thermal and chemical equilibrium in supernova matter with degenerate neutrinos. These unstable nuclei against neutron emission would provide a lot of neutrons after freeze-out, which may help the r-process to proceed.

From the present investigations, we can draw a new scenario for making seed nuclei before the r-process; fragments are abundantly formed through
the liquid-gas phase transition of supernova matter before the freeze-out, and this formation of fragments serve to produce the bulk structure of the seed elements. We call this process as the LG process as a pre-process of r-process [15].

It is interesting to note that our model based on the liquid-gas coexisting state of supernova matter can even provide the r-process nuclei or their seed in a simple manner based on the condition determined by the dynamics of supernova explosion such as $\rho_B$, $Y_L$, and $T$. One of the most promising conditions is $\rho_B = 10^{-5}$ fm$^{-3}$. This density roughly corresponds to the neutrino sphere. The entropy at $T_{\text{boil}}$ is a little smaller than the ejection criteria, $S/B \geq 10$ in one-dimensional hydrodynamical calculation of supernova explosion [20]. However, it would be possible that matter with small entropy can be ejected by convection and/or jet in asymmetric supernova explosion [30,32]. The most conservative freeze-out density for ejection would be $\rho_B = 10^{-7}$ fm$^{-3}$. The entropy at $T_{\text{boil}}$ is large enough, and the seed nuclei will be nucleons, $\alpha$, iron peak nuclei and a small amount of the first peak nuclei of r-process. Higher densities may not be relevant to ejection, but it may be closely related to the nuclear distribution on hot neutron star surface.

In this work, we have assumed equilibrium throughout this paper. One of the key questions is the freeze-out conditions of supernova matter, at which nuclear reactions become less frequent and supernova matter goes off equilibrium in the expansion time-scale. The seed nuclear distribution of the r-process will be given as the nuclear distribution on the freeze-out line in the $(\rho_B, T)$ diagram. It is important to determine the freeze-out condition in supernova dynamics. Another important direction is to construct a model which includes both of the mean field nature such as in RMF and the statistical nature in NSE. In a present NSE treatment, only the Coulomb correction is included as the medium effects, and medium effects from strong interactions are neglected. This neglect may lead to the overestimate of neutron rich nuclei, as discussed in recent statistical fragmentation models [33]. On the other hand, in the Thomas-Fermi treatment of heavy-nuclei with EoS derived using RMF, since statistical nature or fragment distribution is not taken care of, the treatment is not sufficient especially at around $T_{\text{boil}}$. Works in these directions are in progress.

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A Low Density Approximation

In the calculation of liquid-gas coexistence at low temperatures, it becomes necessary to solve the chemical equilibrium of liquid and very low density gas. For example, the gas baryon density becomes around $\rho^G_b = 10^{-74}$ fm$^{-3}$ at the coexisting condition ($\rho_b, T = (10^{-10}$ fm$^{-3}, 0.1$ MeV). In order to efficiently obtain the derivative matrix $\partial \mu_i / \partial \rho_j \ (i, j = B, C, L)$, which are required in solving coexistence, we take the low temperature and low density approximation for nucleons at low baryon densities ($\rho_b \ll 10^{-5}$ fm$^{-3}$) around and below the boiling point in the mean field calculation.

In the mean field approximation, nucleon distribution is a function of the effective mass $M^* = M + g_\sigma \sigma_0$ and effective chemical potentials $\nu_i = \mu_i - g_\omega \omega_0 - g_\rho \tau_i \rho_{30}$ ($i = n$ or $p$), where $\sigma_0, \omega_0$ and $\rho_{30}$ are the expectation values of the meson fields of $\sigma, \omega$ and the neutral $\rho$ mesons, respectively. For a given baryon density $\rho_b$ and proton fraction $Y_p$, the vector meson expectation values are uniquely deterimined as $\omega_0 = \omega_0(\rho_b), \rho_{30} = \rho_{30}(\rho_T)$, where $\rho_T \equiv \rho_p - \rho_n$. In a non-relativistic limit [18], we can take energy as $E^* = M^* + p^2/2M^*$. At low densities ($\rho_b \ll 10^{-5}$ fm$^{-3}$), the nucleon fugacity $f_i \equiv \exp (- (M_i^* - \nu_i)/T)$ becomes much smaller than unity, then we can safely ignore the second and higher order terms in the fugacity $f_i$. In this approximation, the Fermi distribution is approximated to be the Boltzmann distribution, then the baryon number density $\rho_i$ and the scalar density $\rho_s$ are analytically obtained for a given value of $\sigma_0$ as,

$$\rho_i(\nu_i, \sigma_0) = G_-(\nu_i, M^*(\sigma_0), T) + O(f_i^2),$$  \hspace{1cm} (A.1)
$$\rho_s(\nu_n, \nu_p, \sigma_0) = G_+(\nu_n, M^*(\sigma_0), T) + G_+(\nu_p, M^*(\sigma_0), T) + O(f_i^2),$$  \hspace{1cm} (A.2)
$$G_\pm(\nu, M^*, T) = g \left( \frac{M^* T}{2\pi \hbar^2} \right)^{\frac{3}{2}} \left( e^{\nu/T} \pm e^{-\nu/T} \right) e^{-M^*/T}. \hspace{1cm} (A.3)$$

The self-consistent condition $\sigma_0 = \sigma_0(\rho_s(\nu_n, \nu_p, \sigma_0))$ can be solved by iteration using (A.1-A.3), which converges in a few steps at low densities. All of the above densities are represented by three variables, $\nu_n, \nu_p$ and $\sigma_0$ for a given temperature $T$, and we can eliminate the $\sigma_0$ dependence by using the total derivative of the above self-consistent condition.

$$d\sigma_0 = \frac{d\sigma_0}{d\rho_s} \left( \frac{\partial \rho_s}{\partial \nu_n} d\nu_n + \frac{\partial \rho_s}{\partial \nu_p} d\nu_p + \frac{\partial \rho_s}{\partial \sigma_0} d\sigma_0 \right), \hspace{1cm} (A.4)$$
$$d\rho_i = \frac{\partial \rho_i}{\partial \nu_i} d\nu_i + \frac{\partial \rho_i}{\partial \sigma_0} d\sigma_0, \hspace{1cm} (A.5)$$
$$d\mu_i = d\nu_i + g_{\omega_0} \frac{d\omega_0}{d\rho_b} d\rho_b + g_\rho \tau_i \frac{d\rho_{30}}{d\rho_T} d\rho_T. \hspace{1cm} (A.6)$$
By solving the first equation (A.4) in $d\sigma_0$ and substituting it in the second equation (A.5), we obtain $\partial \rho_i / \partial \nu_j$ and then $\partial \nu_i / \partial \rho_j$. Finally, the third equation (A.6) gives the partial derivatives $\partial \mu_i / \partial \rho_j$ by eliminating $d\nu$. Having the (relativistic) lepton integrals and the above derivatives in nucleons, it is straightforward to construct the matrix $\partial \mu_i / \partial \rho_j$ ($i, j = B, C, L$).

### B Procedures to obtain coexisting region of supernova matter in RMF

The boundary of coexisting region of supernova matter has been determined in three steps; symmetric nuclear matter, asymmetric nuclear matter, and supernova matter.

First, for a given temperature, we solve the coexisting condition in symmetric nuclear matter ($Y_p^{\text{Liq.}} = Y_p^{\text{Gas}} = 0.5$). If there is a density region where pressure is decreasing for increasing $\rho_B$, liquid and gas phases can coexist, and we can find coexisting densities, $\rho_B^{\text{Liq.}}$ and $\rho_B^{\text{Gas}}$, by using the Maxwell construction. Secondly, we solve the coexisting condition for ($\rho_B^{\text{Liq.}}, Y_p^{\text{Liq.}}$) and ($\rho_B^{\text{Gas}}, Y_p^{\text{Gas}}$) in asymmetric nuclear matter. Having the coexisting condition at a given $Y_p^{\text{Liq.}}$, it is easy to find coexisting condition for slightly different $Y_p^{\text{Liq.}}$ by using the multi-dimensional Newton’s method. Starting from symmetric nuclear matter, three variables ($\rho_B^{\text{Liq.}}, \rho_B^{\text{Gas}}, Y_p^{\text{Gas}}$) are determined for a given $Y_p^{\text{Liq.}}$ which is slightly different from that in the previously solved condition by requiring the condition, $\mu_B^{\text{Liq.}} = \mu_B^{\text{Gas}}, \mu_C^{\text{Liq.}} = \mu_C^{\text{Gas}}$, and $P^{\text{Liq.}} = P^{\text{Gas}}$. We show the boundary of coexisting region of ($\rho_B, Y_p$) by the thick solid line in Fig. B.1. Filled circles show the point where two phase become uniform, ($\rho_B^{\text{Liq.}}, Y_p^{\text{Liq.}}$) = ($\rho_B^{\text{Gas}}, Y_p^{\text{Gas}}$). The density gap $\rho_B^{\text{Liq.}} - \rho_B^{\text{Gas}}$ generally decreases as the liquid becomes more asymmetric, because of the symmetry energy loss. At lower temperature, the coexisting pressure and thus the coexisting gas baryon density become small, then larger density gap appears. Thirdly, proton fraction $Y_p$ is determined as a function of $\rho_B$ in uniform (homogeneous) supernova matter at a given $Y_L$, by using the charge neutrality condition, $\rho_e = \rho_p = Y_p \rho_B$, and the chemical equilibrium condition, $\mu_\nu = \mu_L = \mu_e + \mu_p - \mu_n$. When $Y_p$ is in the coexisting region of nuclear matter, liquid and gas phases can coexist in supernova matter. The boundary of coexisting region for a given $T$ is determined by the crossing point of these two lines. Since $Y_p$ becomes smaller for smaller $Y_L$ as shown in Fig. B.1, the coexisting density region becomes narrower for smaller $Y_L$. This is the reason why the boiling temperatures decrease for smaller lepton fraction as shown in Fig. 1.

For neutrino-less supernova matter (NS), the procedure is almost the same, except that the chemical equilibrium condition is modified to $\mu_\nu = 0$. 

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Fig. B.1. The boundary of liquid-gas coexisting region in nuclear matter (thick solid lines), and proton fraction as a function of the baryon density in supernova matter without (thin solid lines) and with (dotted line) coexistence. Filled circles show the points where liquid and gas phases converges to the uniform matter.

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