Group Theory and Grammatical Description

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Abstract

This paper presents a model for linguistic description based on group theory. A grammar in this model, or G-grammar, is a collection of lexical expressions which are products of logical forms, phonological forms, and their inverses. Phrasal descriptions are obtained by forming products of lexical expressions and by cancelling contiguous elements which are inverses of each other. We show applications of this model to parsing and generation, long-distance movement, and quantifier scoping. We believe that by moving from the free monoid over a vocabulary V — standard in formal language studies — to the free group over V, deep affinities between linguistic phenomena and classical algebra come to the surface, and that the consequences of tapping the mathematical connections thus established could be considerable.

1 Introduction

There is currently much interest in bringing together the tradition of categorial grammar, and especially the Lambek calculus, with the more recent paradigm of linear logic to which it has strong ties. One active research area concerns the design of non-commutative versions of linear logic which can be sensitive to word order while retaining the hypothetical reasoning capabilities of standard (commutative) linear logic that make it so well-adapted to handling such phenomena as quantifier scoping.

Some connections between the Lambek calculus and group structure have long been known, and linear logic itself has some aspects strongly reminiscent of groups (the producer/consumer duality of a formula A with its
linear negation \( A^\perp \)), but no serious attempt has been made so far to base a theory of linguistic description solely on group structure.

This paper presents such a model, \( G\)-grammars (for “group grammars”), and argues that:

- The standard group-theoretic notion of conjugacy, which is central in \( G\)-grammars, is well-suited to a uniform description of commutative and non-commutative aspects of language;

- The use of conjugacy provides an elegant approach to long-distance dependency and scoping phenomena, both in parsing and in generation;

- \( G\)-grammars give a symmetrical account of the semantics-phonology relation, from which it is easy to extract, via simple group calculations, rewriting systems computing this relation for the parsing and generation modes.

The paper is organized as follows. In section 2 we introduce a “group computation” model, using standard algebraic tools such as free groups, conjugacy and normal subsets. The main deviation from traditional mathematical practice is in the focus given to the notions of compatible preorder and normal submonoid, whereas those of compatible equivalence relation and normal subgroup are more usual in algebra. Section 3 applies this model to linguistic description, and presents a \( G\)-grammar for a fragment of English involving quantification and relative pronouns. The next two sections are concerned with generation and parsing, which correspond to two ways of exploiting the relation of preorder associated with the \( G\)-grammar, one (generation) in which logical forms are iteratively rewritten as combinations of logical forms and phonological forms until only phonological forms are left, the other (parsing) in which phonological forms are rewritten as combinations of logical forms and inverses of those until, after cancellation of adjacent inverses, exactly one logical form is left. Section 6 briefly mentions certain aspects of group computation which cannot be detailed in the paper: the use of conjugacy for mixing commutative and non-commutative phenomena, the simulation of logic programs, decidability conditions for parsing and generation, and differences between \( G\)-grammars and categorial grammars.
2 Group Computation

A **monoid** $M$ is a set $M$ together with a product $M \times M \to M$, written $(a, b) \mapsto ab$, such that:

- This product is associative;
- There is an element $1 \in M$ (the neutral element) with $1a = a1 = a$ for all $a \in M$.

A **group** is a monoid in which every element $a$ has an inverse $a^{-1}$ such that $a^{-1}a = aa^{-1} = 1$.

A **preorder** on a set is a reflexive and transitive relation on this set.
When the relation is also symmetrical, that is, $R(x, y) \Rightarrow R(y, x)$, then the preorder is called an **equivalence relation**. When it is antisymmetrical, that is that is, $R(x, y) \land R(y, x) \Rightarrow x = y$, it is called a **partial order**.

A preorder $R$ on a group $G$ will be said to be compatible with the group product iff, whenever $R(x, y)$ and $R(x', y')$, then $R(xx', yy')$.

**Normal submonoids of a group.** We consider a compatible preorder notated $x \rightarrow y$ on a group $G$. The following properties, for any $x, y \in G$, are immediate:

\[
\begin{align*}
x \rightarrow y & \iff xy^{-1} \rightarrow 1; \\
x \rightarrow y & \iff y^{-1} \rightarrow x^{-1}; \\
x \rightarrow 1 & \iff 1 \rightarrow x^{-1}; \\
x \rightarrow 1 & \Rightarrow yxy^{-1} \rightarrow 1, \text{ for any } y \in G.
\end{align*}
\]

Two elements $x, x'$ in a group $G$ are said to be **conjugate** if there exists $y \in G$ such that $x' = yxy^{-1}$. The fourth property above says that the set $M$ of elements $x \in G$ such that $x \rightarrow 1$ is a set which contains along with an element all its conjugates, that is, a **normal subset** of $G$. As $M$ is clearly a submonoid of $G$, it will be called a **normal submonoid** of $G$.

Conversely, it is easy to show that with any normal submonoid $M$ of $G$ one can associate a preorder compatible with $G$. Indeed let’s define $x \rightarrow y$ as $xy^{-1} \in M$. The relation $\rightarrow$ is clearly reflexive and transitive, hence is a preorder. It is also compatible with $G$, for if $x_1 \rightarrow y_1$ and $x_2 \rightarrow y_2$, then $x_1y_1^{-1}, x_2y_2^{-1}$ and $y_1(x_2y_2^{-1})y_1^{-1}$ are in $M$; hence $x_1x_2y_2^{-1}y_1^{-1} = x_1y_1^{-1}y_1x_2y_2^{-1}y_1^{-1}$ is in $M$, implying that $x_1x_2 \rightarrow y_1y_2$, that is, that the preorder is compatible.
Remark. In general $M$ is not a subgroup of $G$. It is iff $x \rightarrow y$ implies $y \rightarrow x$, that is, if the compatible preorder $\rightarrow$ is an equivalence relation (and, therefore, a congruence) on $G$. When this is the case, $M$ is a normal subgroup of $G$. This notion plays a pivotal role in classical algebra. Its generalization to submonoids of $G$ is basic for the algebraic theory of computation presented here.

If $S$ is a subset of $G$, the intersection of all normal submonoids of $G$ containing $S$ (resp. of all subgroups of $G$ containing $S$) is a normal submonoid of $G$ (resp. a normal subgroup of $G$) and is called the normal submonoid closure $\text{NM}(S)$ of $S$ in $G$ (resp. the normal subgroup closure $\text{NG}(S)$ of $S$ in $G$).

The free group over $V$. We now consider an arbitrary set $V$, called the vocabulary, and we form the so-called set of atoms on $V$, which is notated $V \cup V^{-1}$ and is obtained by taking both elements $v$ in $V$ and the formal inverses $v^{-1}$ of these elements.

We now consider the set $F(V)$ consisting of the empty string, notated 1, and of strings of the form $x_1x_2\ldots x_n$, where $x_i$ is an atom on $V$. It is assumed that such a string is reduced, that is, never contains two consecutive atoms which are inverse of each other: no substring $vv^{-1}$ or $v^{-1}v$ is allowed to appear in a reduced string.

When $\alpha$ and $\beta$ are two reduced strings, their concatenation $\alpha\beta$ can be reduced by eliminating all substrings of the form $vv^{-1}$ or $v^{-1}v$. It can be proven that the reduced string $\gamma$ obtained in this way is independent of the order of such eliminations. In this way, a product on $F(V)$ is defined, and it is easily shown that $F(V)$ becomes a (non-commutative) group, called the free group over $V$.

Group computation. We will say that an ordered pair $\text{GCS} = (V, R)$ is a group computation structure if:

1. $V$ is a set, called the vocabulary, or the set of generators
2. $R$ is a subset of $F(V)$, called the lexicon, or the set of relators.

\footnote{For readers familiar with group theory, this terminology will evoke the classical notion of group presentation through generators and relators. The main difference with our definition is that, in the classical case, the set of relators is taken to be symmetrical, that is, to contain $r^{-1}$ if it contains $r$. When this additional assumption is made, our preorder becomes an equivalence relation.}
The submonoid closure $NM(R)$ of $R$ in $F(V)$ is called the result monoid of the group computation structure $GCS$. The elements of $NM(R)$ will be called computation results, or simply results.

If $r$ is a relator, and if $\alpha$ is an arbitrary element of $F(V)$, then $\alpha r \alpha^{-1}$ will be called a quasi-relator of the group computation structure. It is easily seen that the set $R_N$ of quasi-relators is equal to the normal subset closure of $R$ in $F(V)$, and that $NM(R_N)$ is equal to $NM(R)$.

A computation relative to $GCS$ is a finite sequence $c = (r_1, \ldots, r_n)$ of quasi-relators. The product $r_1 \cdots r_n$ in $F(V)$ is evidently a result, and is called the result of the computation $c$. It can be shown that the result monoid is entirely covered in this way: each result is the result of some computation. A computation can thus be seen as a “witness”, or as a “proof”, of the fact that a given element of $F(V)$ is a result of the computation structure.\footnote{The analogy with the view in constructive logics is clear. There what we call a result is called a formula or a type, and what we call a computation is called a proof.}

For specific computation tasks, one focusses on results of a certain sort, for instance results which express a relationship of input-output, where input and output are assumed to belong to certain object types. For example, in computational linguistics, one is often interested in results which express a relationship between a fixed semantic input and a possible textual output (generation mode) or conversely in results which express a relationship between a fixed textual input and a possible semantic output (parsing mode).

If $GCS = (V, R)$ is a group computation structure, and if $A$ is a given subset of $F(V)$, then we will call the pair $GCSA = (GCS, A)$ a group computation structure with acceptors. We will say that $A$ is the set of acceptors, or the public interface, of $GCSA$. A result of $GCS$ which belongs to the public interface will be called a public result of $GCSA$.

3 G-Grammars

We will now show how the formal concepts introduced above can be applied to the problems of grammatical description and computation. We start by introducing a grammar, which we will call a G-grammar (for “Group Grammar”), for a fragment of English (see Fig. 1).

A G-grammar is a group computation structure with acceptors over a vocabulary $V = V_{log} \cup V_{phon}$ consisting of a set of logical forms $V_{log}$ and a disjoint set of phonological elements (in the example, words) $V_{phon}$. Exam-
Examples of phonological elements are *john*, *saw*, *every*, examples of logical forms *j, s(j,l), ev(m,x,sm(w,y,s(x,y)))*; these logical forms can be glossed respectively as “*john*”, “*john saw louise*” and “for every man *x*, for some woman *y*, *x* saw *y*”.

The grammar lexicon, or set of relators, *R* is given as a list of “lexical schemes”. An example is given in Fig. 1. Each line is a lexical scheme and represents a set of relators in *F(V)*. The first line is a ground scheme, which corresponds to the single relator *j* *john*, and so are the next four lines. The fifth line is a non-ground scheme, which corresponds to an infinite set of relators, obtained by instanciating the *term meta-variable A* (notated in uppercase) to a logical form. So are the remaining lines. We use Greek letters for *expression meta-variables* such as α, which can be replaced by an arbitrary expression of *F(V)*; thus, whereas the term meta-variables *A, B, ...,* range over logical forms, the expression meta-variables α, β, ..., range over products of logical forms and phonological elements (or their inverses) in *F(V)*. 

The notation *P[x]* is employed to express the fact that a logical form containing an *argument identifier x* is equal to the application of the abstraction *P* to *x*. The meta-variable *X* in *P[X]* ranges over such identifiers (*x, y, z, ...*), which are notated in lower-case italics (and are always ground). The

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Expression meta-variables are employed in the grammar for forming the set of *conjuncts α exp α* of certain expressions *exp* (in our example, *exp* is *ev(N,X,P[X]) P[X]−1*, *sm(N,X,P[X]) P[X]−1*, or *X*). Conjugacy allows the enclosed material *exp* to move as a *block* in expressions of *F(V)*, see sections 3. and 4.
meta-variable \( P \) ranges over logical form abstractions missing one argument (for instance \( \lambda z \cdot s(j, z) \)). When matching meta-variables in logical forms, we will allow limited use of higher-order unification. For instance, one can match \( P[X] \) to \( s(j, x) \) by taking \( P = \lambda z \cdot s(j, z) \) and \( X = x \).

The vocabulary and the set of relators that we have just specified define a group computation structure \( GCS = (V, R) \). We will now describe a set of acceptors \( A \) for this computation structure. We take \( A \) to be the set of elements of \( F(V) \) which are products of the following form:

\[
SW_n^{-1}W_{n-1}^{-1} \ldots W_1^{-1}
\]

where \( S \) is a logical form (\( S \) stands for “semantics”), and where each \( W_i \) is a phonological element (\( W \) stands for “word”). The expression above is a way of encoding the ordered pair consisting of the logical form \( S \) and the phonological string \( W_1W_2 \ldots W_n \) (that is, the inverse of the product \( W_n^{-1}W_{n-1}^{-1} \ldots W_1^{-1} \)).

A public result \( SW_n^{-1}W_{n-1}^{-1} \ldots W_1^{-1} \) in the group computation structure with acceptors \( ((V, R), A) \) — the G-grammar — will be interpreted as meaning that the logical form \( S \) can be expressed as the phonological string \( W_1W_2 \ldots W_n \).

Let us give an example of a public result relative to the grammar of Fig. 1.

We consider the relators (instanciations of relator schemes):

\[
\begin{align*}
  r_1 &= j^{-1} \cdot s(j, l) \cdot l^{-1} \cdot saw^{-1} \\
  r_2 &= l \cdot louise^{-1} \\
  r_3 &= j \cdot john^{-1}
\end{align*}
\]

and the quasi-relators:

\[
\begin{align*}
  r_1' &= j \cdot r_1 \cdot j^{-1} \\
  r_2' &= (j \cdot saw) \cdot r_2 \cdot (j \cdot saw)^{-1} \\
  r_3' &= r_3
\end{align*}
\]

Then we have:

\[
\begin{align*}
  r_1' \cdot r_2' \cdot r_3' &= \\
  j \cdot j^{-1} \cdot s(j, l) \cdot l^{-1} \cdot saw^{-1} \cdot j^{-1} \cdot saw \cdot l \cdot louise^{-1} \cdot saw^{-1} \cdot j^{-1} \cdot saw \cdot l \cdot louise^{-1} \cdot saw^{-1} \cdot john^{-1}
\end{align*}
\]

which means that \( s(j, l) \cdot louise^{-1} \cdot saw^{-1} \cdot john^{-1} \) is the result of a computation \( (r_1', r_2', r_3') \). This result is obviously a public one, which means that the logical form \( s(j, l) \) can be verbalized as the phonological string \( john \cdot saw \cdot louise \).
Applying directly, as we have just done, the definition of a group computation structure in order to obtain public results can be somewhat unintuitive. It is often easier to use the preorder $\rightarrow$. If, for $a, b, c \in F(V)$, $abc$ is a relator, then $abc \rightarrow 1$, and therefore $b \rightarrow a^{-1}c^{-1}$. Taking this remark into account, it is possible to write the relators of our G-grammar as the “rewriting rules” of Fig. 2; we use the notation $\rightarrow$ instead of $\rightarrow$ to distinguish these rules from the parsing rules which will be introduced in the next section.

The rules of Fig. 2 have a systematic structure. The left-hand side of each rule consists of a single logical form, taken from the corresponding relator in the G-grammar; the right-hand side is obtained by “moving” all the remaining elements in the relator to the right of the arrow.

Because the rules of Fig. 2 privilege the rewriting of a logical form into an expression of $F(V)$, they are called generation-oriented rules associated with the G-grammar.

Using these rules, and the fact that the preorder $\rightarrow$ is compatible with the product of $F(V)$, the fact that $s(j,1) \quad \text{louise}^{-1}\text{saw}^{-1}\text{john}^{-1}$ is a public result can be obtained in a simpler way than previously. We have:

\[
\begin{align*}
\text{s}(j,1) & \rightarrow j \text{ saw } l \\
j & \rightarrow \text{john} \\
l & \rightarrow \text{louise}
\end{align*}
\]

by the seventh, first and second rules (properly instanciated), and therefore,
by transitivity and compatibility of the preorder:

\[s(j, l) \rightarrow j \text{ saw } l \rightarrow \text{ john saw l } \rightarrow \text{ john saw louise}\]

which proves that \(s(j, l) \rightarrow \text{ john saw louise}\), which is equivalent to saying that \(s(j, 1) \text{ louise}^{-1}\text{ saw}^{-1}\text{ john}^{-1}\) is a public result.

Some other generation examples are given in Fig. 3.

The first example is straightforward and works similarly to the one we have just seen: from the logical form \(i(s(j, 1), p)\) one can derive the phonological string \(\text{ john saw louise in paris}\).

**Long-distance movement and quantifiers** The second and third examples are parallel to each other and show the derivation of the same string \(\text{every man saw some woman}\) from two different logical forms. The penultimate and last steps of each example are the most interesting. In the penultimate step of the second example, \(\beta\) is instanciated to \(\text{ saw}^{-1}\text{ x}^{-1}\). This has the effect of “moving” as a whole the expression \(\text{ some woman } y^{-1}\) to the position just before \(y\), and therefore to allow for the cancellation of \(y^{-1}\) and \(y\). The net effect is thus to “replace” the identifier \(y\) by the string \(\text{ some woman}\); in the last step \(\alpha\) is instanciated to the neutral element \(1\), which has the effect of replacing \(x\) by \(\text{ every man}\). In the penultimate step of the third example, \(\alpha\) is instanciated to the neutral element, which has the effect of replacing \(x\) by \(\text{ every man}\); then \(\beta\) is instanciated to \(\text{ saw}^{-1}\text{ man}^{-1}\text{ every}^{-1}\), which has the effect of replacing \(y\) by \(\text{ some woman}\).

*Remark.* In all cases in which an expression similar to \(\alpha a_1 \ldots a_m \alpha^{-1}\) appears (with the \(a_i\) arbitrary vocabulary elements), it is easily seen that, by giving \(\alpha\) an appropriate value in \(F(V)\), the \(a_1 \ldots a_m\) can move arbitrarily to the left or to the right, *but only together in solidarity*; they can also freely permute cyclically, that is, by giving an appropriate value to \(\alpha\), the expression \(\alpha a_1 \ldots a_m \alpha^{-1}\) can take on the value \(a_k a_{k+1} \ldots a_m a_1 \ldots a_{k-1}\) (other permutations are in general not possible). The values given to the \(\alpha, \beta, \text{ etc.}\), in the examples of this paper can be understood intuitively in terms of these two properties.

We see that, by this mechanism of concerted movement, quantified noun phrases can move to whatever place is assigned to them after the expansion of their “scope” predicate, a place which was unpredictable at the time of the expansion of the quantified logical form. The identifiers act as “target markers” for the quantified noun phrase: the only way to “get rid” of
\[i(s(j,l), p)\]
\[\rightarrow s(j,l) \text{ in } p\]
\[\rightarrow j \text{ saw } l \text{ in } p\]
\[\rightarrow \text{John saw } l \text{ in } p\]
\[\rightarrow \text{John saw Louise in } p\]
\[\rightarrow \text{John saw Louise in Paris}\]

\[ev(m, x, sm(w, y, s(x, y)))\]
\[\rightarrow \alpha^{-1} \text{ every } m x^{-1} \alpha \text{ } sm(w, y, s(x, y))\]
\[\rightarrow \alpha^{-1} \text{ every } m x^{-1} \alpha \beta^{-1} \text{ some } w y^{-1} \beta s(x, y)\]
\[\rightarrow \alpha^{-1} \text{ every man } x^{-1} \alpha\]
\[\beta^{-1} \text{ some woman } y^{-1} \beta x \text{ saw } y\]
\[\rightarrow \alpha^{-1} \text{ every man } x^{-1} \alpha \text{ x saw y}\]
\[\text{(by taking } \beta = \text{saw}^{-1} x^{-1})\]
\[\rightarrow \text{every man saw some woman}\]
\[\text{(by taking } \alpha = 1)\]

\[sm(w, y, ev(m, x, s(x, y)))\]
\[\rightarrow \beta^{-1} \text{ some } w y^{-1} \beta ev(m, x, s(x, y))\]
\[\rightarrow \beta^{-1} \text{ some } w y^{-1} \beta \alpha^{-1} \text{ every } m x^{-1} \alpha s(x, y)\]
\[\rightarrow \beta^{-1} \text{ some woman } y^{-1} \beta\]
\[\alpha^{-1} \text{ every man } x^{-1} \alpha x \text{ saw } y\]
\[\rightarrow \beta^{-1} \text{ some woman } y^{-1} \beta \text{ every man saw } y\]
\[\text{(by taking } \alpha = 1)\]
\[\rightarrow \text{every man saw some woman}\]
\[\text{(by taking } \beta = \text{saw}^{-1} \text{ man}^{-1} \text{ every}^{-1})\]

\[r(tt(m, x, s(l, x)))\]
\[\rightarrow t(tt(m, x, s(l, x))) \text{ ran}\]
\[\rightarrow \text{the } tt(m, x, s(l, x)) \text{ ran}\]
\[\rightarrow \text{the } m \text{ that } \alpha^{-1} x^{-1} \alpha s(l, x) \text{ ran}\]
\[\rightarrow \text{the man that } \alpha^{-1} x^{-1} \alpha s(l, x) \text{ ran}\]
\[\rightarrow \text{the man that } \alpha^{-1} x^{-1} \alpha \text{ saw } x \text{ ran}\]
\[\rightarrow \text{the man that } \alpha^{-1} x^{-1} \alpha \text{ Louise saw } x \text{ ran}\]
\[\rightarrow \text{the man that Louise saw } \text{ ran}\]
\[\text{(by taking } \alpha = \text{saw}^{-1} \text{ Louise}^{-1})\]

Figure 3: Generation examples
an identifier $x$ is by moving $x^{-1}$, and therefore with it the corresponding quantified noun phrase, to a place where it can cancel with $x$.

The fourth example exploits a similar mechanism for handling relative clauses. At the time the relative pronoun is produced, an identifier inverse $x^{-1}$ is also produced which has the capability of moving to whatever position is finally assigned to the relative verb’s argument $x$.

5 Parsing

To the compatible preorder $\rightarrow$ on $F(V)$ there corresponds a “reverse” compatible preorder $\leftarrow$ defined as $a \leftarrow b$ iff $b \rightarrow a$, or, equivalently, $a^{-1} \rightarrow b^{-1}$. The normal submonoid $M'$ in $F(V)$ associated with $\rightarrow$ is the inverse monoid of the normal submonoid $M$ associated with $\rightarrow$, that is, $M'$ contains $a$ iff $M$ contains $a^{-1}$.

It is then clear that one can present the relations:

\begin{align*}
\text{j} & \rightarrow 1 \\
\text{A}^{-1}\text{r(A)} & \rightarrow 1 \\
\alpha & \leftarrow 1 \\
\text{some}^{-1} & \rightarrow 1
\end{align*}

in the equivalent way:

\begin{align*}
\text{john} & \rightarrow 1 \\
\text{ran} & \rightarrow 1 \\
\text{some} & \rightarrow 1
\end{align*}

Suppose now that we move to the right of the $\rightarrow$ arrow all elements appearing on the left of it, but for the single phonological element of each relator. We obtain the rules of Fig. 4 which we call the “parsing-oriented” rules associated with the G-grammar.

By the same reasoning as in the generation case, it is easy to show that any derivation using these rules and leading to the relation $PS \rightarrow LF$, where $PS$ is a phonological string and $LF$ a logical form, corresponds to a public result $LF PS^{-1}$ in the G-grammar.

A few parsing examples are given in Fig. 5; they are the converses of the generation examples given earlier.

In the first example, we first rewrite each of the phonological elements into the expression appearing on the right-hand side of the rules (and where the meta-variables have been renamed in the standard way to avoid name
Figure 4: Parsing-oriented rules

john $\rightarrow$ j
louise $\rightarrow$ l
paris $\rightarrow$ p
man $\rightarrow$ m
woman $\rightarrow$ w
ran $\rightarrow$ $A^{-1} r(A)$
saw $\rightarrow$ $A^{-1} s(A,B) B^{-1}$
in $\rightarrow$ $E^{-1} i(E,A) A^{-1}$
the $\rightarrow$ $t(N) N^{-1}$
every $\rightarrow$ $\alpha ev(N,X,P[X]) P[X]^{-1} \alpha^{-1} X N^{-1}$
some $\rightarrow$ $\alpha sm(N,X,P[X]) P[X]^{-1} \alpha^{-1} X N^{-1}$
that $\rightarrow$ $N^{-1} tt(N,X,P[X]) P[X]^{-1} \alpha^{-1} X \alpha$

clashes). The rewriting has taken place in parallel, which is of course permitted (we could have obtained the same result by rewriting the words one by one). We then perform certain unifications: A is unified with j, C with p; then B is unified to l. Finally E is unified with $s(j,l)$, and we obtain the logical form $i(s(j,l),p)$. In this last step, it might seem feasible to unify E to $i(E,p)$ instead, but that is in fact forbidden for it would mean that the logical form $i(E,p)$ is not a finite tree, as we do require. This condition prevents “self-cancellation” of a logical form with a logical form that it strictly contains.

Quantifier scoping In the second example, we start by unifying m with N and w with M; then we “move” $P[x]^{-1}$ next to $s(A,B)$ by taking $\alpha = xA^{-1}$.

$\beta = B$ then again we “move” $Q[y]^{-1}$ next to $s(A,B)$ by taking $\beta = B sm(w,y,Q[y])^{-1}$; $x$ is then unified with A and y with B. This leads to the

$\alpha ev(N,X,P[X]) P[X]^{-1} \alpha^{-1} X N^{-1}$

$\alpha sm(N,X,P[X]) P[X]^{-1} \alpha^{-1} X N^{-1}$

$\alpha sm(w,y,Q[y])^{-1}$

$X \alpha$

4Another possibility at this point would be to unify l with E rather than with B. This would lead to the construction of the logical form $i(1,p)$, and, after unification of E with that logical form, would conduct to the output $s(j,i(1,p))$. If one wants to prevent this output, several approaches are possible. The first one consists in typing the logical form with syntactic categories. The second one is to have some notion of logical-form well-formedness (or perhaps interpretability) disallowing the logical forms $i(1,p)$ [louise in paris] or $i(t(w),p)$ [(the woman) in paris], although it might allow the form $t(i(w,p))$ [the (woman in paris)].

5We have assumed that the meta-variables corresponding to identifiers in P and Q have been instanciated to arbitrary, but different, values x and y. See [4] for a discussion of this point.
john saw louise in paris
→ j A⁻¹ s(A,B) B⁻¹ l E⁻¹ i(E,C) C⁻¹ p
→ s(j,B) B⁻¹ l E⁻¹ i(E,p)
→ s(j,l) E⁻¹ i(E,p)
→ i(s(j,l),p)

every man saw some woman
→ α ev(N, x, P[x]) P[x]⁻¹ α⁻¹ x N⁻¹ m A⁻¹ s(A,B) B⁻¹
   β sm(M, y, Q[y]) Q[y]⁻¹ β⁻¹ y M⁻¹ w
→ α ev(m, x, P[x]) P[x]⁻¹ α⁻¹ x A⁻¹ s(A,B) B⁻¹ β sm(w, y, Q[y]) Q[y]⁻¹ β⁻¹ y
→ x A⁻¹ ev(m, x, P[x]) P[x]⁻¹ s(A,B) B⁻¹ β sm(w, y, Q[y]) Q[y]⁻¹ β⁻¹ y
→ x A⁻¹ ev(m, x, P[x]) P[x]⁻¹ s(A,B) Q[y]⁻¹ sm(w, y, Q[y]) B⁻¹ y
→ ev(m, x, P[x]) P[x]⁻¹ s(x,y) Q[y]⁻¹ sm(w, y, Q[y])
and then either:
→ ev(m, x, P[x]) P[x]⁻¹ sm(w, y, s(x,y))
→ ev(m, x, sm(w, y, s(x,y))

or:
→ ev(m, x, s(x,y)) Q[y]⁻¹ sm(w, y, Q[y])
→ sm(w, y, ev(m, x, s(x,y))

the man that louise saw ran
→ t(N) N⁻¹ m M⁻¹ tt(M, x, P[x]) P[x]⁻¹ α⁻¹ x α l A⁻¹ s(A,B) B⁻¹ C⁻¹ r(C)
→ t(m) M⁻¹ tt(M, x, P[x]) P[x]⁻¹ α⁻¹ x α s(l,B) B⁻¹ C⁻¹ r(C)
→ t(m) M⁻¹ tt(M, x, P[x]) P[x]⁻¹ s(l,B) x B⁻¹ C⁻¹ r(C)
→ t(m) M⁻¹ tt(M, x, P[x]) P[x]⁻¹ s(l, x) C⁻¹ r(C)
→ t(m) M⁻¹ tt(M, x, s(1,x)) C⁻¹ r(C)
→ tt(t(m), x, s(1,x)) C⁻¹ r(C)
→ r(tt(t(m), x, s(1,x)))

Figure 5: Parsing examples
expression:
\[ ev(m, x, P[x]) P[x]^{-1} s(x, y) Q[y]^{-1} sm(w, y, Q[y]) \]
where we now have a choice. We can either unify \( s(x, y) \) with \( Q[y] \), or with \( P[x] \). In the first case, we continue by now unifying \( P[x] \) with \( sm(u, y, s(x, y)) \), leading to the output \( ev(m, x, sm(w, y, s(x, y))) \). In the second case, we continue by now unifying \( Q[y] \) with \( ev(m, x, s(x, y)) \), leading to the output \( sm(w, y, ev(m, x, s(x, y)) \). The two possible quantifier scopings for the input string are thus obtained, each corresponding to a certain order of performing the unifications.

In the last example, the most interesting step is the one (third step) in which \( \alpha \) is instanciated to \( s(l, B)^{-1} \), which has the effect of “moving” \( x \) close to the “missing” argument \( B^{-1} \) of “louise saw”, to cancel it by unification with \( B \) and consequently to fill the second argument position in the logical form headed by \( s \). After this step, \( P[x] \) is ready to be unified with \( s(1, x) \), finally leading to the expected logical form output for the sentence.

6 Final remarks

Mixing commutative and non-commutative phenomena

We have already seen examples where commutative and non-commutative aspects are both present in a lexical entry. Thus, the presence of \( \alpha, \alpha^{-1} \) in the entry for ‘every’ in Fig. 2 allows the expression \( every N X^{-1} \) to move as a block to the position where the argument \( X \) of the verb is eventually found; in this movement, it is however impossible for \( every \) and \( N \) to exchange their relative positions, and it can be shown that, for the input logical forms of Fig. 3, only the four phonological strings listed can be obtained.

Let us briefly indicate how this commutative/non-commutative partnership could be used for error correction purposes. Suppose that a relator of the form \( \alpha Error^{-1} Repair \alpha^{-1} Report \) is added to the grammar, where \( Error \) is some erroneous input (for instance it could be the word ‘principle’ improperly used in a situation where ‘principal’ is needed), \( Repair \) is what the input should have been (in our example, the word ‘principal’), and \( Report \) is a report which tells us how the error was corrected. Then, the expression \( Error^{-1} Repair \) can move in block to the spot where the error occurs, replacing the erroneous word by the correction and allowing normal processing to continue. The report can then be used for warning or evaluation purposes.

Fully commutative structures and logic programs

Suppose that one wants to have a group computation structure which is completely commutative, that is, one in which elements can move freely relative to each
other. This property could be stipulated by introducing a notion of “commutative group computation” using the free commutative group $FC(V)$ rather than $F(V)$. Another possibility, which illustrates the flexibility of the group computation approach, is just to add a relator scheme $\alpha \beta \alpha^{-1} \beta^{-1}$ to $R$, where $\alpha$ and $\beta$ are expression meta-variables. This expression is called a \textit{commutator} of $\alpha$ and $\beta$ because when multiplied by $\beta \alpha$ it yields $\alpha \beta$. This single relator scheme permits to permute elements in any expression, and has the same effect as using $FC(V)$.

An example where commutative structures are useful is the case of logic programs. It can be shown that if one encodes a clause of a logic program $P_0 \leftarrow P_1 \ldots P_n$ as a relator scheme $P_0 P_n^{-1} \ldots P_1^{-1}$ in a commutative structure, and defines public results to consist of a single ground predicate $P$, then public results in the group computation structure coincide with consequences of the program.

**G-grammars, rewriting, and computability**  

In the discussion of parsing and generation, we saw how a derivation according to the rewriting rules of Figs. 2 and 4 is always “sound” with respect to the group computation structure. We did not consider the opposite question, namely whether it is “complete” with respect to it: can any public result relative to the GCS be obtained by such rewritings? The answer to this question is given by a theorem demonstrated in [4], which states that such a rewriting system is complete relative to the GCS if the system does not contain “ground cycles”, that is situations where a ground term $T$ can derive $\ldots T \ldots$. This condition is true of both the rewriting systems of Figs 2 and 4. For instance, in the generation case, it can be checked that any ground logical form that appears on the right-hand side of a rule of Fig. 2 is \textit{strictly smaller} than the ground logical form on the left-hand side, therefore precluding ground cycles. This condition is related to the \textit{computability} of generation and parsing. Thus it is obvious that the decreasing size property of generation rules implies that any derivation from a ground logical form is bounded in length. This can be used to prove that generation is decidable and can only produce a finite number of results for a fixed input, and a similar argument can be made for parsing. In the terminology of $\mathfrak{B}$ a grammar with this properties is said to be \textit{inherently reversible}; this property is difficult to guarantee in formalisms relying on empty categories for long-distance dependencies, a problem which does not appear when using conjugacy for the same purpose.

**G-grammars, DCGs and categorial grammars**  

In a similar way to what was done in the case of a logic program, it is possible to simulate a definite clause grammar with a group computation structure. The translation of a clause $A_0 \rightarrow A_1 \ldots A_n$ becomes the relator scheme $A_0 A_n^{-1} \ldots A_1^{-1}$.
The only difference is that (i) the product is non-commutative, and (2) the $A_i$'s, for $1 \leq i \leq n$, can be nonterminals or words. The group computation structure obtained computes the same derivations as the DCG under the condition that the DCG is not ground-cyclic. This is always the case if the nonterminal on the left-hand side contains more linguistic material than each nonterminal on the right-hand side, a natural condition to have in most cases.

There is however one situation in which this condition is not natural, namely that of context-free grammars, the limiting case of DCGs where nonterminals do not contain any linguistic material. Thus, in a CFG, rules such as: $VP \rightarrow oftenVP$ are standard. Here we have a ground cycle in the rule itself. Its translation as a relator is: $VPVP^{-1} often^{-1}$, which simplifies to $often^{-1}$. The consequence is that, in the GCS, the word ‘often’ can be added freely in any string generated from $S$! In order to make the CFG simulable by a CGS, we have first to enrich it into a DCG which is not ground-cyclic. This can be easily done by including terms that memorize derivations or the length of terminal strings.

The context-free rule just discussed can be used to illustrate an important difference between G-grammars and categorial grammars. In a categorial grammar, an expression such as $VP/VP$ is not a neutral element: it can only combine with a VP to give a VP, not with an NP to give an NP. By contrast, in a group computation structure, an expression such as $VPVP^{-1}$ is indistinguishable from the neutral element, a feature that is required if one wishes to gain access to many standard mathematical tools.
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