Modeling and Verification of Stress Relaxation Behavior of Ti-6Al-4V
Xingzhen Zhang1, Ying Deng1, Tian Liang1, Shihan Jin2, Weidong Li2
1AVIC Manufacturing Technology Institute, Beijing, 100024, China
2Beihang University, Beijing, 100083, China

Abstract
The phenomenon of gradual decrease of internal stress with the deformation of material maintained under the precondition of certain temperature and initial stress or pre-strain is called stress relaxation. Due to that, the flow stress of the metal material falls rapidly when the hot forming process pauses, and then the required forming load. In this paper, the experiment was carried out to study the stress relaxation property of Ti-6Al-4V, in the temperature range of 1023K~1123K and with the pre-tension strain 0.7%, 4% and 10%. The quartic delay function was used to describe the stress relaxation behavior. The predicted value of stress relaxation equation is in good agreement with the experimental data, and the correlation coefficient is above 0.99. Arrhenius creep constitutive equation embedded in CAE software was derived. The finite element model of stress relaxation process of test bar was built, and the tensile-relaxation experiment was performed under the loading condition of 1:1 part forming process. The forming force results agree well, and the validity and accuracy of the constitutive model are verified, laying a foundation for the subsequent process simulation and optimization.

Keywords: Stress relaxation, Quartic delay function, Constitutive model, Experiment and FEA

1. Introduction
Under the condition of certain strain and temperature, the phenomenon that stress of metal gradually decreases along with the time is called stress relaxation[1], which is achieved through gradual transformation from elastic strain inner of material into the creep strain[2]. It is generally considered that the stress relaxation is creep behavior affected by continuous decreasing loading. The stress relaxation is one of important mechanical properties for titanium alloy, especially for the thermal-forming process, which is often used to reduce spring-back of parts and improve the forming precision[3]. In the process of great plastic deformation, by taking the multi-step forming measures, it can play the role of releasing the internal stress of the material and reducing the forming force, so that great plastic deformation can be realized under the impact of small forming force.

The constitutive model of material is the premise that numerical method is used to do characterization calculation for the process of metal forming process, which is usually divided into statistical constitutive equation coupled with macroscopic or microscopic, and phenomenological constitutive equation describing the relationship among of material flow stress, macro- strain, strain rate and temperature. For the stress relaxation and creep deformation behavior of titanium alloys, many scholars at home and abroad have conducted a lot of research. Through the standard testing for stress relaxation, Lin Zhaorong[4], Wang Chunyan[5] and Shen Falan[6] studied stress relaxation behavior of different titanium alloys, such as TA1, TA2, TC1, TA15 between the range from 723 to 1023K under the conditions of different initial stress and pre-strain. Based on the experimental data, Du sunyao[7] got relation of the short-term creep constitutive relationship by fitting which reflected the stress relaxation behavior of Ti-6Al-4V alloy sheet at high temperature was made finite element modelling. Elmer J.W. [8] studied the stress relaxation behavior of titanium alloy at the temperature of 723 and 873K and analyzed the time-correlated evolution of phase lattice parameters. The above researches mainly focus on the field of titanium alloy thermal forming/calibration, and the temperature is generally between 673 and 1023 K. In this paper, combined with the actual process of bending and torsion of a certain type of blade, the stress relaxation behavior and constitutive model of Ti-6Al-4V thick plate at 1073 K are tested with finite element modelling research, which lays a foundation for further research and optimization of the following process.

2. Testing for stress relaxation at high temperature
The material used for the test is taken from an annealed thick plate Ti-6Al-4V with the thickness of 25 mm and the thick plate is processed into a standard test bar along the direction of rolling. The gauge length is 100 mm and the diameter of gauge length is 10 mm. The specific shape and dimensions are shown in Fig. 1. The chemical constituents are shown in Table 1.
Table 1 Chemical constituents of Ti-6Al-4V

| Major Elements (%) | Others (%) |
|--------------------|------------|
| Ti 89.99           | Fe 0.19    |
| Al 5.56            | C 0.01     |
| V 4.07             | H 0.005    |
|                    | O 0.08     |
|                    | Other <0.1 |

The original structure of Ti-6Al-4V is shown in Fig. 2. There is a large number of equiaxed α grain, and the point-like is the β transformed phase. Due to the annealing treatment, the lamellar structure with dark contrast among the equiaxed α grain is β transformed.

The test equipment is an RDS-100 electronic creep testing machine, and the heating device is a heating furnace that the center is cylindrical, which is shown in Fig. 3.
The temperature control is used for the upper, middle and lower parts of heating furnace, and three sets of resistors are laid on the inner wall of the heating furnace. During testing, the asbestos rope is used to fix the thermo-couple at the lower end of the clamp of upper mold, the middle part of the test rod and the top end of the lower mold to control the temperature of upper, middle and lower parts of heating furnace in real time and ensure that the temperature deviation is within ±1 K. The test standard refers to the tensile stress relaxation method at high temperature specified in GB10120, and the RDS type electronic creep testing machine is used to exert fixed strain on the test bar. One end of the extensometer is inserted into the furnace and fixed on the lugs on both sides of the test bar to intercept the deformation of gauge section of the sample, the another end of the extensometer is located in the outside of the furnace, and the displacement of internal gauge length is respectively converted to the two grating scales on left and right sides by four springs and the elongation of the grating scale is the elongation of the gauge length of the test piece. The installation quality of the extensometer is checked with the appropriate loading force before the test. Through the corresponding adjustment, the difference between the average value of reading on both sides and the reading of either side is not more than 10% of the average value, and the accuracy of the grating scale is 0.001 mm. During the test, the pc end will record the changes of parameters such as tension, displacement and temperature along with change of time in real time. The test bar is heated to target temperature and kept warm for 5 minutes until the temperatures of the upper, middle and lower portions are uniform, then the test is started. The specific scheme of testing is shown in Table 2. The test results and the curve of corresponding stress varying along with time are shown in Fig. 4.

Table 2 Stress relaxation test scheme

| Test requirement | Parameters       |                |                |                |
|------------------|------------------|----------------|----------------|----------------|
| Temperature      | 1023K            | 1073K          | 1123K          |                |
| Pre-strain       | 0.7%             | 4%             | 10%            |                |
| Test time        | Until test force is under 50N |                |                |                |
| Specimen preparation | Coated with high-temperature antioxidant |                |                |                |
| Test times       | 3 times per test condition |                |                |                |
3. Constitutive Model

Scholars have proposed many empirical formulas to characterize the stress relaxation behavior of materials, including logarithmic equation, exponential equation[9,10]. After the comparison of different equations, we finally found that delay function has much flexibility for parameter fitting. Its fitting accuracy can be improved through the control of the order of delay function. Eventually quartic delay function is adopt to describe the stress relaxation behavior of Ti-6Al-4V at elevated temperature. The expression of quartic delay function is as follows:

$$\sigma = \sigma_0 + B_1 \exp\left(-\frac{t}{\tau_1}\right) + B_2 \exp\left(-\frac{t}{\tau_2}\right) + B_3 \exp\left(-\frac{t}{\tau_3}\right) + B_4 \exp\left(-\frac{t}{\tau_4}\right)$$  \hspace{1cm} (1)

In the formula: $\sigma$ is instantaneous stress, MPa; $\tau$ is relaxation time, s; $\sigma_0$ is stress relaxation limit, MPa; $B_1, B_2, B_3, B_4, \tau_1, \tau_2, \tau_3, \tau_4$ are relaxation parameters. The experimental data with 4% pre-strain are fitted, and the specific parameters are shown in Table 3. The comparison between the experimental data and the predicted data is shown in Fig. 5. R-square is the deterministic coefficient of the equation. Its value is between 0 and 1. The larger the value is, the higher the fitting degree of the equation.

| Initial specimen | 1023K 0.7% | 1023K 4% | 1023K 10% |
|------------------|------------|---------|----------|
| Initial specimen | 1073K 0.7% | 1073K 4% | 1073K 10% |
| Initial specimen | 1123K 0.7% | 1123K 4% | 1123K 10% |

Fig. 4 Test results

Table 3 Formula parameters at 4% pre-strain
In formula (1) the Explicit constitutive equation of Ti-6Al-4V is established, and the direct relationship between instantaneous stress and creep time in material during relaxation is described. In order to carry out the finite element simulation of stress relaxation process, it is necessary to transform the constitutive relationship displayed into the creep constitutive equation embedded in ABAQUS software. This transformation is based on the basic relationship between strain rate and stress, and creep strain rate is an important physical quantity in creep process.

The elastic strain, creep strain and pre-strain all satisfy the following equations during the whole stress relaxation process:

\[ \varepsilon = \varepsilon_e + \varepsilon_c \]  \hspace{1cm} (2)

In the formula, \( \varepsilon \) is the total strain, \( \varepsilon_e \) is the elastic strain and \( \varepsilon_c \) is the creep strain. Stress relaxation process is a process in which the elastic strain changes into the creep strain when the total strain is constant.

\[ \dot{\varepsilon}_c = 0 \]  \hspace{1cm} (3)

The relationship between creep strain rate and stress is obtained from equations (2) and (3):

\[ \dot{\varepsilon}_c = -\frac{d\sigma}{Ed\varepsilon} \]  \hspace{1cm} (4)

In the formula, creep strain rate, stress and modulus of elasticity are used. Substitute Formula (1) into Formula (4):

\[ \dot{\varepsilon} = \frac{1}{E} \left[ \frac{B_1}{r_1} \exp \left( -\frac{t}{r_1} \right) - \frac{B_2}{r_2} \exp \left( -\frac{t}{r_2} \right) + \frac{B_3}{r_3} \exp \left( -\frac{t}{r_3} \right) + \frac{B_4}{r_4} \exp \left( -\frac{t}{r_4} \right) \right] \]  \hspace{1cm} (5)

Equation (5) establishes the relationship between creep strain rate and relaxation time. Based on experimental data, the stress relaxation curve is transformed into the relationship between creep strain rate and stress as shown in Fig. 6, taking 4% pre-strain as an example.
Arrhenius-type constitutive equation is a phenomenological constitutive model related to strain rate, which can be used to express the relationship between strain rate, flow stress and temperature.

The effect of temperature and strain of Arrhenius equation on material deformation behavior can be expressed by Zener-Holloman factor:

\[
Z = \dot{\varepsilon} \exp\left[\frac{Q}{RT}\right]
\]

\[
\dot{\varepsilon} = A_1 \sigma^{n_1} \exp\left[-\frac{Q}{RT}\right] \quad \alpha \sigma < 0.8
\]

\[
\dot{\varepsilon} = A_1 \exp\left(\beta \sigma\right) \exp\left[-\frac{Q}{RT}\right] \quad \alpha \sigma > 1.2
\]

\[
\dot{\varepsilon} = A_1 \left[\sinh\left(\alpha \sigma\right)\right]^{n_2} \exp\left[-\frac{Q}{RT}\right] \quad \text{all } \sigma
\]

In the formula, \(\dot{\varepsilon}\) is the strain rate, \(R\) is the general gas constant, \(T\) is the absolute temperature, \(Q\) is the activation energy of thermal deformation, \(\sigma\) is the rheological stress, \(A_1, A_2, A_3, \alpha, n_1, n_2\) and \(\beta\) are the material constants independent of temperature and determined by experiments, and among them: \(\alpha = \beta n_1\)

Take the logarithms on both sides of equation (7), (8), (9) and we obtained:

\[
\ln \dot{\varepsilon} = \ln A_1 + n_1 \ln \sigma - \frac{Q}{RT}
\]

\[
\ln \dot{\varepsilon} = \ln A_1 + \beta \sigma - \frac{Q}{RT}
\]

\[
\ln \dot{\varepsilon} = n_1 \ln \sinh\left(\alpha \sigma\right) + \ln A_3 - \frac{Q}{RT}
\]

Make linear fitting at the same temperature with \(\ln \dot{\varepsilon}\) and \(1n \sigma\), \(\sigma\), the gradient is \(n_1\) and \(\beta\) respectively, and the values are obtained as shown in Table 4.

| Temperature/K | 1023K | 1073K | 1123K |
|---------------|-------|-------|-------|
| \(n_1\)      | 2.4821| 2.6701| 3.31471|
| \(\beta\)    | 0.00631| 0.00766| 0.00609|
| \(\alpha\)   | 0.00254| 0.00287| 0.000184|

Make linear fitting with \(1n \dot{\varepsilon}\) and \(1n \sinh(\alpha \sigma)\), the gradient is \(n_1\), the intercept is \(\ln A_3 - \frac{Q}{RT}\), and the values of \(n_2 A_3, Q\) is shown in Table 5.
Similarly, the values of the parameters of Arrhenius stress relaxation constitutive model at different temperatures can be calculated at pre-strain of 0.7% and 10%, as shown in Table 6.

### Table 6 parameters fitting results of creep constitutive equation

| Strain | Temperature /K | Material parameter $A_3$/s⁻¹ | Street level exponential $\beta$/MPa | Stress parameter $n_2$ | Creep activation energy $Q$/J·mol⁻¹·K⁻¹ |
|--------|----------------|-------------------------------|-------------------------------------|-----------------------|-----------------------------------------|
| 0.7%   | 1023           | 0.00359                       | 0.00582                             | 1.95867               | 15987.5                                |
|        | 1073           | 0.00468                       | 0.00695                             | 2.09586               | 19586.4                                |
|        | 1123           | 0.00549                       | 0.00765                             | 2.11579               | 16625.9                                |
| 4%     | 1023           | 0.00476                       | 0.00631                             | 2.07578               | 18159.2                                |
|        | 1073           | 0.00541                       | 0.00766                             | 2.14042               | 10665.1                                |
|        | 1123           | 0.70253                       | 0.00061                             | 2.48939               | 23456.5                                |
| 10%    | 1023           | 0.00725                       | 0.00654                             | 2.29865               | 26652.1                                |
|        | 1073           | 0.00685                       | 0.00896                             | 2.59872               | 18851.2                                |
|        | 1123           | 0.00821                       | 0.00052                             | 2.68542               | 14321.5                                |

### 4. Test Validation

In order to verify the accuracy of the creep constitutive equation for the description of stress relaxation process of Ti-6Al-4V, a finite element model of segmented tensile stress relaxation process is established by using ABAQUS finite element software, taking the standard test stick as the object, and the loading process is in agreement with the actual process (the loading trajectory $\circ_1$ is shown in Fig.7). The tensile-stress relaxation verification test of standard Test rod was carried out. As a control, a one-way tensile test of the standard test rod (no stress relaxation, loading trajectory $\circ_2$ as shown in Fig.7) is carried out, and the loading speed and final tensile amount are consistent with the stress relaxation tensile test. A two-dimensional geometric model of one quarter of stress relaxation specimen is established considering symmetry, as shown in Fig.8. The tensile capacity is 0.8mm, so the constitutive model is selected under the condition that the pre-strain is 0.7%. The comparison between the finite element simulation results and the test results is shown in Fig.9, taking the tensile force as the control criterion. It can be seen that the results of finite element analysis are close to the test results, which shows that the stress relaxation finite element model of Ti-6Al-4V Titanium alloy established in this paper can accurately analyze the stress relaxation forming process of unidirectional tensile.
Fig. 7 Two different loading paths

Fig. 8 Two-dimensional geometric model of one quarter of stress relaxation specimen
Fig. 9 shows the change of tensile load in the process by loading mode ①. According to the figure: the maximum tensile force of loading mode ① at three different temperatures is about 9.7kN, 8.5kN and 7.8kN respectively, located at the maximum deformation amount, and the maximum tensile force of loading mode ② is about 6.1kN, 5.8kN and 4.5kN respectively, at the maximum displacement of each segment load. Because the titanium alloy has a certain softening trend in the tensile process, the maximum tensile force required is not 1/3 of the total tensile force, although the loading mode ② each stretch is 1/3 of the loading mode ①. However, it can still be found that under the same amount of deformation, the use of ladder loading-stress relaxation loading method can effectively reduce the load required for the forming process.
5. Conclusion

The stress relaxation characteristics of Ti-6Al-4V under the condition of 0.7%, 4% and 10% in the 1023K~1123K temperature range and the pre-tensile strain were studied by high temperature stress relaxation test, and the test data were fitted by quartic delay function, and the Arrhenius constitutive model was derived from the transformation. In order to lay a foundation for the subsequent finite element simulation, the finite element simulation of the actual process is carried out based on the test rod, and compared with the results of the forming force Test, and the non-stress relaxation unidirectional tensile control test is designed and implemented, and the following conclusions are drawn:

1) using quartic delay function to describe the direct relationship between instantaneous stress and creep time in the relaxation process material, the fitting precision is high, and the finite element simulation and the comparison of the test results show that the Arrhenius constitutive model derived from quartic delay function can be used to accurately describe the stress relaxation forming process of unidirectional tensile;

2) The maximum tensile force required for step loading mode (stress relaxation) is less than the loading mode without stress relaxation stage, which indicates that stress relaxation can effectively reduce the stress in the forming process, and based on this characteristic, it can be applied to the practical forming process of Ti-6Al-4V parts, reducing the demand for equipment and energy.

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