Extreme bursting events via pulse-shaped explosion in mixed Rayleigh-Liénard nonlinear oscillator

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Abstract We study the dynamics of a parametrically and externally driven Rayleigh-Liénard hybrid model and report the emergence of extreme bursting events due to a novel pulse-shaped explosion mechanism. The system exhibits complex periodic and chaotic bursting patterns amid small oscillations as a function of excitation frequencies. In particular, the advent of rare and recurrent chaotic bursts that emerged for certain parameter regions is characterized as extreme events. We have identified that the appearance of a sharp pulse-like transition that occurred in the equilibrium points of the system is the underlying mechanism for the development of bursting events. Further, the controlling aspect of extreme events is attempted by incorporating a linear damping term, and we show that for sufficiently strong damping strength, the extreme events are eliminated from the system, and only periodic bursting is feasible.

1 Introduction

The abrupt emergence of fast, repetitive large-amplitude peaks (spiking or active state) alternate with the slow, steady-state like small-amplitude oscillations (quiescent or rest state) at regular or irregular time intervals are characterized as bursting. Such complex and multiple-time scale oscillatory behavior is often appeared in biological systems, especially in neuronal systems [1–3], but also reported in physical, chemical, mechanical, and other systems [4–8]. Due to the universal occurrence of this phenomenon, the concept of bursting and its emerging dynamical mechanisms has been intensively studied in mathematical models [9–15], and in various real-world systems [16–20]. Accordingly, based on the multiple-scale analysis, especially, using fast-slow subsystem analysis [21–23], it has been demonstrated that bursting is often emerged in the dynamical systems due to various mechanisms such as singular Hopf bifurcation [24], canard phenomenon [25–27], the breakup of invariant torus [28, 29], via boundary crisis [30, 31], the blue-sky catastroph [32], through delayed bifurcation [33–35], speed escape of attractors [36], via pulse-shaped explosion [37–39], and few other dynamical mechanisms [40–43, 46]. In particular, very recently, an interesting sharp transition behavior called pulse-shaped explosion has been reported in Rayleigh’s type nonlinear dynamical system, in which the periodic complex bursting patterns occurred due to the appearance of pulse-shaped sharp quantitative changes in the branches of the equilibrium point and limit cycle attractor [37, 38]. In continuation with this, bursting oscillations induced by the bistable pulse-shaped explosion are also demonstrated in a nonlinear oscillatory system using multiple slow excitation frequencies [39]. Two different types of regular bursting patterns, that is, bursting types of point-point type and cycle-cycle type, have been observed in the Rayleigh system via pulse-shaped explosion [37]. In addition to that, the pulse-shaped explosion can also induce different bursting patterns in other dynamical systems as well [44, 45]. Even though considerable work has been done in recent times regarding the pulse-shaped explosion-induced bursting dynamics, this area still requires further investigation.

Recently, the emergence of extreme events (EE), defined as rare, recurrent, and large amplitude oscillations (events) that are significantly larger than the normal small amplitude oscillations, are manifested in state variables or observables of dynamical and real-world systems. This typical phenomenon has appeared in many natural and artificial systems without prior warning, but it greatly impacted life and society. Considering the widespread impact, EE have been documented in various forms of disasters in natural systems like floods [46], volcanic eruptions [47], tsunamis [48], droughts [49], regime shifts in ecosystems [50], epidemic spreading [51, 52], epileptic seizure in the human brain [53, 54], solar flares [55], and so on. Apart from that EE also appeared in the form of share market crashes [56], mass panics [57], jamming in computer and transportation networks [58, 59], large power black outs [60], collapse of large buildings due to extreme loading conditions [61, 62], and many more. These examples motivate researchers from different scientific domains to investigate such phenomena from statistical and dynamical system aspects.

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The trademark characteristic properties of EE are unpredictability, and irregular occurrence, which is also the essential features of chaos. Thus, researchers are curious to observe the evidence of EE in nonlinear dynamical systems. Based on this, from the EE perspective, several motivations have emerged in the dynamical systems. Especially, understanding the development of extreme events in dynamical systems, their emerging mechanism, and controlling aspects are rigorously studied (both in isolated and coupled systems) in recent literature (For additional reading, see ref [63] and the references therein). In continuation with that, several novel routes to EE emergence have been demonstrated in dynamical systems. For example, the emergence of optical rogue waves in an optically injected laser system is experimentally demonstrated [64], in which EE emerged in the system via an external crisis-like route. Similarly, EE can also appear in the system via interior and boundary crisis as well [65–67]. Several studies reported that EE has also emerged in the system through the intermittency route [68–70]. Another route to the emergence of EE in systems with discontinuous boundaries is through sliding bifurcation [71], which has been reported recently. Extreme events originate in the systems due to instability in antiphase synchronization of the coupled systems via two different routes, intermittency and quasiperiodicity routes to complex dynamics for purely excitatory and inhibitory chemical synaptic coupling, respectively [72]. The authors in ref. [73] presented the origination of EE in coupled moving agents model due to the instability of the synchronization manifold. So far, we have discussed the emergence of EE in deterministic systems, which have a self-sustaining mechanism for generating EE under various circumstances. However, an important class of rare EE is induced by noise [74, 75]. But, the route the system takes during each transition to EE in stochastic systems is not as clear as in deterministic systems.

Despite a large number of recent publications reporting the emerging routes of EE, the subject is still young and contains interesting problems like the possibility of exploring new mechanisms and routes to EE in nonlinear dynamical systems is still open that demand the attention of researchers. In particular, the recently reported pulse-shaped explosion route to complex bursting is based on a specific Rayleigh oscillator. But a significant number of hybrid systems related to nonlinear oscillators of Rayleigh’s type are available in literature [76, 77], which are used to describe many physical phenomena and processes in electrical and mechanical systems [78–81]. In addition to this, it is also of interest to investigate how systems with aperiodic dynamics, especially chaotic systems, respond to this pulse-shaped explosion route. Further, in Sec.4, the controlling aspect of EE is investigated by presenting the statistical properties of complex periodic and chaotic bursting patterns in the hybrid model is demonstrated in Sec.3. Also, the fast-slow system theory is introduced to confirm the pulse-shaped explosion route. Further, in Sec.4, the controlling aspect of EE is investigated by presenting the linear damping term in the model equation. Finally, the results are summarized and concluded in Sec.5.

2 Model equation

To demonstrate the results, we consider a mathematical model of a mixed Rayleigh-Liénard oscillator with external and parametric periodic excitations. The equation of motion of the hybrid model is given by

\[ \ddot{x} + f(x, \dot{x}) + [1 - \mu \cos(\omega_1 t)](x + \gamma x^3) = F \cos(\omega_2 t). \] #1

here, the function \( f(x, \dot{x}) = -\alpha x \dot{x} + \beta x^3 \), in which \( \alpha \), and \( \beta \) indicates the quadratic and cubic nonlinear damping coefficients of Rayleigh-Liénard type, \( \gamma \) is the nonlinear stiffness constant, \( \mu \) and \( F \) are the amplitudes of the external and parametric excitations, and \( \omega_1 \) and \( \omega_2 \) are the frequencies of the external and parametric excitations, respectively. The class of mixed Rayleigh-Liénard system is widespread in the areas of science, and engineering. The dynamics of this hybrid system have been intensively studied in the literature,
and exciting results such as period-doubling bifurcation leading to chaotic dynamics, strange attractors, symmetry breaking, and so on have been reported using various excitation models [81–84]. Also, the quadratic nonlinear damping term $\alpha x \dot{x}$ appeared in many real-world models and was found to play a crucial role mostly in microelectromechanical, and nanoelectromechanical oscillators [85, 86], fluid mechanics [87], and have implications in mass and force sensing applications [88, 89], mechanical noise squeezing in laser cooling technology [90].

3 Emergence of complex bursting patterns

The hybrid Rayleigh-Liénard system (1) exhibits periodic and chaotic behavior as a function of the frequencies of external and parametric excitations for fixed values of system parameters. In order to study the dynamics of the system, Eq. (1) is numerically integrated using the fourth-order Runge-Kutta method with the time step of 0.01. For numerical integration, the system parameters are fixed at $\alpha = -0.25$, $\beta = 0.05$, $\gamma = 0.1$, $\mu = 0.99$, $\omega_2 = 0.04$, and $F = 0.08$ throughout the manuscript until and otherwise mentioned.

The emergence of periodic and chaotic states of the system is quantified using the Lyapunov exponents ($\lambda$) by varying the parametric excitation frequency in the range of $\omega_1 \in (0.1, 0.3)$ by fixing the other parameters as given above. The estimated Lyapunov exponent is depicted in Fig. 1. The positive peaks ($\lambda > 0$) of the first Lyapunov exponent (black line) indicate the emergence of chaos, and $\lambda < 0$ (light gray line) shows the periodic dynamics. Notably, both chaos and periodic oscillations appeared in the form of bursting for the selected parameter region. The qualitative validation of those bursting can be seen from the time snaps of the system for different $\omega_1$ values.

The time evolution of the system (1) is depicted in the left panel of Fig. 2 for three different values of $\omega_1$ (by decreasing $\omega_1$ from higher to lower values) to demonstrate the bursting patterns. To be precise, for $\omega_1 = 0.2$, the system manifests chaos, in which intermittent, rare, and large-amplitude peaks have appeared at random intervals in the negative $x$ values amid small-amplitude oscillations, which are characterized as extreme chaotic bursts or named as EE. The reason for bursting in the negative $x$ values will be discussed later. The time snaps consist of rare and recurrent large-amplitude oscillations along with the small-amplitude oscillations are portrayed in Fig. 2a. The large excursions are clearly visible in the figure. Usually, those large excursions can be qualitatively distinguished from other peaks using the threshold height [63, 69] $H_T = \langle P_{\min} \rangle - N \sigma$, in which $\langle P_{\min} \rangle$ is the time average of the local minima of the time series, $\sigma$ is the standard deviation and $N$ is an arbitrary integer, which varies based on the dynamical system. For system (1), we choose the value of $N = 6$. The negative sign in the threshold height equation is due to the negative peaks that occurred in the system. In order to estimate the threshold height, we have used $2 \times 10^9$ data points after leaving adequate transient. The calculated value is marked as a horizontal dashed line in Fig. 1a. The negative peaks lower than $H_T$ are characterized as EE. A similar type of extreme dynamical state can also be observed in the system for several other values of $\omega_1$ when the system exhibits chaos. Figure 2b shows another example for the emergence of EE for $\omega_1 = 0.166$. Compare to Fig. 2a the size of the attractor is enlarged in Fig. 2b. Hence, larger peaks are observed in the system.

In contrast to the previous two cases, when we choose $\omega_1$ in the periodic regime, the system shows repetitive patterns of small amplitude oscillations, which are alternated by large-amplitude bursting (peaks) at regular intervals of time characterized as periodic bursting. Time snaps of one such periodic state are observed for $\omega_1 = 0.12$ and portrayed in Fig. 2c. When we estimate $H_T$ and plot it along with the time series, the prominent repetitive negative peaks are below the threshold height for the same value of $N$. However, they fail to prove EE’s randomness and unpredictability since they are repetitive patterns. Thus, the verification of EE using the estimation of threshold height $H_T$ is not sufficient for systems that exhibit periodic and chaotic bursting patterns in different parameter regions.

Therefore, it is necessary to characterize and confirm EE using other statistical techniques. Hence, we have estimated the probability distribution function (PDF) of the corresponding time series data shown in Fig. 2a–c to confirm and distinguish EE from periodic bursting states and plotted in Fig. 2d–f, respectively. The PDF of the time series data provided in Fig. 2a, b show the non-Gaussian, continuous long-tail distribution corroborating the occurrence of EE. The vertical dashed line in Fig. 2d, e indicates $H_T$. On the other hand, the PDF of the time series data respective to the periodic oscillations (Fig. 2c) did not show the continuous long-tail distribution. Still, it displayed a single peak of the repetitive bursting.

![Fig. 1](image_url) The Lyapunov exponents of the system (1) is plotted as a function of $\omega_1$ shows the chaotic dynamics of the system for various values of $\omega_1$. The other system parameters are fixed at $\alpha = -0.25$, $\beta = 0.05$, $\gamma = 0.1$, $\omega_2 = 0.04$, $F = 0.08$, and $\mu = 0.99$.
Further, the rarity of the events is quantified by the distribution of inter-event intervals \( IE_I = t_{m+1} - t_m \), \( m = 1, 2, \cdots, (M - 1) \) which follows Poisson-like distribution when the system exhibits EE. Here, \( t_m \) is the time of the occurrence of \( m^{th} \) event in a set of \( M \) events. Figure 2g, h depicts the semi-log scale histogram plots of the estimated IEI for the time series of Fig. 2a, b, respectively, satisfying the Poisson-like distribution \( P(x) = \Lambda e^{-\Lambda x} \) with the scaling parameter \( \Lambda = 0.0004248 \), and \( \Lambda = 0.0003625 \) (dashed line), respectively. Nevertheless, the histogram of inter-event interval calculated for the periodic bursting failed to show the Poisson-like distribution due to the periodic nature of the system, which is evident from Fig. 2i. Hence, we propose that estimating the threshold height alone is ineffective in recognizing EE in a system that exhibits periodic and chaotic bursting. Additionally, one inevitably evaluates other statistical techniques like calculating PDF and IEI helps to distinguish EE better from other normal events.

The global picture of the regions of periodic states and chaotic bursting in the form of EE is identified in the parameter space of two excitations frequencies \( \omega_1 \in (0.16, 0.28) \) and \( \omega_2 \in (0.03, 0.08) \), respectively, which is depicted in Fig. 3. As mentioned earlier, EE is exhibited when the system manifests chaos. Therefore, we assume (and confirm) that simultaneously estimating the Lyapunov exponent and the threshold height, \( HT \), helps us distinguish the EE region from the periodic bursting region as a function of \( \omega_1 \) and \( \omega_2 \). If the system shows chaos with a positive Lyapunov exponent, and when the minima of the \( x \)-variable are beneath \( HT \), then the dynamics that occurred at the specific frequencies are marked as EE (dark gray points in Fig. 3). Otherwise, the parameter values to which the Lyapunov exponent becomes negative are considered periodic bursting regions (the white region in Fig. 3). We also double-checked the emergence of those two bursting states in the specified parameter range.

It is interesting and crucial to understand the mechanism responsible for the emergence of such bursting events. To this purpose, we transform Eq. (1) into slow-fast system with one single slow variable \( \delta(t) = \cos(\omega t) \) [91]. For our study, the frequency ratio between the parametric and external excitations is fixed as \( 1 : n \). That is, \( \omega_1 = n\omega_2 \), in which \( n \) is an odd integer. The modified equation for the fast subsystem of Rayleigh-Liénard system can be given as

\[
\ddot{x} + f(x, \dot{x}) + [1 - \mu k_\delta(\delta)](x + \gamma x^3) = A\delta,
\]  

(2)
μ branch, which approaches infinity when the simple pulse-shaped explosion as showing a single peak. The black line in Fig. 4a shows the bifurcation of a stable equilibrium system exhibits pulse-shaped explosion both in positive and negative values of $x$ when $\delta = 0$ and several occurrences of this transition are based on the value of $\delta$ and disappearance of equilibrium points in the form of a sharp pulse-shaped transition occurred in the system as a function of time, the phase space and exhibiting large oscillations. The above mentioned case is shown in Fig. 4af or represents the rest area of bursting. Therefore, the system trajectory approaches near the steep region, making large excursions in of $\delta$ and leads to large-amplitude irregular bursting when the trajectory goes near the steep region. Also, the amplitude of the large oscillations depends on how near the trajectory goes to the steep region, which occurs completely at random intervals of time due to the chaotic nature of the system. The small-amplitude oscillations appear in the flat region, which is also evident from Fig. 4b.

This critical transition drives the attractors to infinity in a narrow parameter space near the critical escape line at $\mu = 1$. The birth and disappearance of equilibrium points in the form of a sharp pulse-shaped transition occurred in the system as a function of time, and several occurrences of this transition are based on the value of $n$, named as pulse-shaped explosion [37]. For an illustration, when $n = 2$, Eq. (2) exhibits relatively trivial periodic bursting patterns. Nevertheless, taking $n = 3$ as an example, we interpret the simple pulse-shaped explosion as showing a single peak. The black line in Fig. 4a shows the bifurcation of a stable equilibrium branch, which approaches infinity when $\mu$ is slightly less than 1 ($\mu = 0.99$) at $\delta = \delta_c = -0.5$. Beyond this critical value of $\delta$, the two fragmented steep branches coalesce together and create a stable equilibrium branch again as a function of $\delta$. This process is repeated in the time domain. The establishment of such steep branches forms pulse-like sharp quantitative changes as a function of $\delta$, a pulse-shaped explosion. In other words, the steep region of the curve indicates the active area of bursting, and the flat area represents the rest area of bursting. Therefore, the system trajectory approaches near the steep region, making large excursions in the phase space and exhibiting large oscillations. The above mentioned case is shown in Fig. 4a for $n = 3$. The red (gray) line depicts the trajectory of the fast-slow subsystem and shows the emergence of repetitive periodic bursting equivalent to Fig. 2c. When we increase the $n$ value, the number of explosions is increased in the fast subsystem. We choose $n = 5$ as an example of two peaks appearing in the pulse-shaped explosion for different $\delta$ values. The system exhibits chaos for the chosen parameter values and leads to large-amplitude irregular bursting when the trajectory goes near the steep region. Also, the amplitude of the large oscillations depends on how near the trajectory goes to the steep region, which occurs completely at random intervals of time due to the chaotic nature of the system. The small-amplitude oscillations appear in the flat region, which is also evident from Fig. 4b.

One can notice that the periodic or chaotic bursting has occurred only in the negative region of $x$-variable even though the system exhibits pulse-shaped explosion both in positive and negative values of $x$-variable [Fig. 4]. In order to clarify this, we have
Fig. 5 Bifurcation of the equilibrium points resulting in complex bursting patterns as a function of $\cos(0.04t)$ for $a \ n = 3$, and $b \ n = 5$, respectively. Red (dark gray) and green (light gray) filled solid circles indicate stable equilibrium, and stable limit cycles, respectively. Black, and blue (dark gray) open circles denote unstable equilibrium and unstable limit cycle. The continuous gray line represents the pulse-shaped explosion curve and the right side values of the y-axis indicating the amplitude of the pulse-shaped explosion.

determined the stability analysis of the equilibrium point using the XPPAUT as a function of $\delta(t) = \cos(0.04t)$ for two different cases of $n=3$, and 5, which are plotted in Fig. 5. In particular, Fig. 5a shows the bifurcation of equilibrium points for the case of $n = 3$. When we look into the figure from right to left by decreasing the control parameter $\delta$ from 1 to -1, we observe the stable equilibrium point, marked as red (dark gray) filled circles, lost its stability at $\delta = 0$ via a supercritical Hopf bifurcation. After that, the stable limit-cycle solution can be obtained for $\delta < 0$, depicted by green (light gray) filled circles. One can note here that the stable limit-cycle always revolves around the unstable equilibrium point (marked as open black circles) and undergo a sharp transition in the negative $x$-direction at $\delta = -0.5$, which is the reason for the appearance of bursting in the negative $x$-direction.

Similarly, in the case of $n = 5$, the stable equilibrium point produces a sharp transition in the positive $x$-direction for a specific value of $\delta > 0$. Hence, the chaotic trajectory crossing this $\delta$ region is slightly perturbed. However, it does not produce significant large oscillations in the positive $x$-direction. This is evident from Fig. 4b. When we decrease $\delta$ further, the stable equilibrium is bifurcated into a stable limit-cycle orbit via a sub-critical Hopf bifurcation at $\delta = 0$ (marked as $HB_{sub}$ in Fig. 5b). The emerged limit-cycle has a steep transition in the negative $x$-direction due to the pulse-shaped explosion, which leads the chaotic trajectory to exhibit large excursions in the negative direction as manifested in Fig. 4b. The continuous gray line in Fig. 5a, b are pulse-shaped explosion plotted for comparison.

Thus, we confirm that the system exhibits critical transition in the form of pulse shaped explosion, which is the mechanism for the emergence of bursting. Moreover, the emergence of chaotic oscillations is the reason for the emergence of EE in the Rayleigh-Liénard hybrid system. Further, the reason for the bursting in the negative $x$-direction is also explained. In the next section, we study the influence of linear damping on EE in the hybrid system.

4 Influence of linear damping

Controlling of EE is attempted by incorporating the linear damping term $\xi \dot{x}$ in Eq. (1) so that the equation can be rewritten as

$$\ddot{x} + f(x, \dot{x}) + \xi \dot{x} + [1 - \mu \cos(\omega_1 t)](x + \gamma x^3) = F \cos(\omega_2 t),$$

(3)

here $\xi$ is the strength of linear damping. We observe that for a sufficient linear damping strength, the chaotic nature of the system is tamed, and the chaotic oscillations have vanished from the system. Thereby the chaotic bursting in the form of EE is eliminated from the system dynamics. In order to confirm the results we have estimated the emergence of EE as a function of $\omega_1$, and $\omega_2$ for two different values of $\xi$, which are depicted in Fig. 6a, b for $\xi = 0.03$, and $\xi = 0.05$, respectively. The gray points indicate the parameter values to which EE emerged in the system, and the white region indicates the periodic bursting oscillations. One can note that compared to Fig. 3, in Fig. 6a, the gray shaded EE region is significantly reduced, confirming the elimination of EE. Further, increase in the linear damping to $\xi = 0.05$, the region of EE is even more reduced, which is manifested in Fig. 6b. Finally, when $\xi > 0.065$, the system exhibits no EE, and only periodic oscillations are feasible.
5 Conclusion

We have systematically investigated the emerging bursting dynamics in the Rayleigh-Liénard hybrid system driven by parametric and external excitations to consolidate the results.

We found the emergence of periodic and chaotic bursting oscillations as a function of excitation frequencies. We have pointed out that bursting patterns occurred in the system due to the emergence of a sharp pulse-like explosion that appeared in the equilibrium points of the system, named the pulse-shaped explosion. Notably, during the emergence of chaos, large amplitude oscillations appeared in the system infrequently at random intervals. Those large-amplitude oscillations are characterized as EE. Even though the development of periodic bursting via the pulse-shaped explosion was reported earlier, this is the first study to document the development of EE via pulse-shaped explosion. Thus, a novel route to EE emergence is reported in a dynamical system. We also proposed that finding EE based on the threshold height is inadequate for the systems exhibiting periodic and chaotic oscillations. Therefore, additional statistical techniques are required to differentiate the rare events from the repetitive large amplitude oscillations. The rarity of the EE is verified using the inter-event interval histogram, which satisfies the Poisson-like distribution.

We have identified that the simultaneous existence of chaos and the pulse-shaped explosion are critical mechanisms for the emergence of EE. The transformed fast-slow system with a single slow variable is used to characterize and validate the pulse-shaped explosion. Finally, the influence of linear damping on EE is studied. We found that the chaotic nature of the system is attenuated while increasing the damping strength, thereby eliminating EE from the system, and only periodic busting is feasible.

This paper focuses on the emergence of extreme events via the pulse-shaped explosion in the Rayleigh-Liénard hybrid model related to the odd excitation frequency ratios. Exploring the pulse-shaped explosion route to EE in other dynamical systems with multiple-frequency slow excitations is also interesting. We are progressing our research in this direction.

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Author contributions All the authors contributed equally to the preparation of this manuscript.

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References

1. D. Terman, J. Nonlin, Sci 2, 135–182 (1992)
2. P.F. Rowat, R.C. Elson, J. Comput. Neuroci 16, 87–112 (2004)
3. C.R. Laing, B. Doiron, A. Longtin, L. Noonan, R.W. Turner, L. Maler, J. Comput. Neuroci 14, 329–42 (2003)
4. H. Simo, P. Woafo, Mech. Res. Commun 38, 537–41 (2011)
5. F. Fetzer, J. Laser Appl 30, 012009 (2018)
6. M.A. Nahmias, H-T. Peng, T. F. de Lima, C. Huang, A. N. Tait, B. J. Shastrl, P.R. Prucnal, arXiv preprint arXiv:2012.08516 (2020)
7. J. Nowacki, H.M. Osinga, K. Tsaneva-Atanasova, J. Math. Neurosci 2, 7 (2012)
8. M. Brons, K. Bar-Eli, J. Phys. Chem 25, 8706–8713 (1991)
9. B. Deng B, Math. Biosci 119, 241–250 (1994)
10. R.E. Plant, J. Math. Biol 11, 15–32 (1981)
11. G. de Veries, Phys. Rev. E 64, 051914 (2001)
12. G.S. Medvedev, Phys. Rev. Lett 97, 048102 (2006)
13. P. Channell, G. Cymbalyuk, A. Shilnikov, A Phys. Rev. Lett. 98, 134101 (2007)
14. C. Hens, P. Pal, S.K. Dana, Phys. Rev. E 92, 022915 (2015)
15. A. Ghosh, D. Roy, V.K. Jirsa, Phys. Rev. E 80, 041930 (2009)
83. A. Maccari, Nonlin. Dyn. 25, 293–316 (2001)
84. Y.J.F. Kpomahou, L.A. Hinvi, J.A. Adéchinan, C.H. Miwadinou, Complexity, 6631094 (2021)
85. A. Eichler, J. Moser, J. Chaste, M. Zrdojek, I. Wilson-Rae, A. Bachtold, Nat. Nanotechnol. 6, 339–342 (2011)
86. S. Zaitsev, O. Shtempluck, E. Buks, O. Gottlieb, Nonlin. Dyn. 67, 859–883 (2012)
87. Z. Ran, Appl. Fluid. Mech. 5, 41–67 (2009)
88. K.L. Ekinci, Y.T. Yang, M.L. Roukes, J. Appl. Phys. 95, 2682–2689 (2004)
89. L. Papariello, O. Zilberberg, A. Eichler, A. Chitra, Phys. Rev. E 94, 022201 (2016)
90. N. Akerman, S. Kooler, Y. Glickman, Y. Dallas, A. Keselman, R. Ozier, Phys. Rev. A 82, 061402(R) (2010)
91. X. Han, Y. Zhang, Q. Bi, J. Kurths, Chaos 28, 043111 (2018)