Potential in the Schrodinger equation: estimation from empirical data

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Abstract

A recent model for the stock market calculates future price distributions of a stock as a wave function of a quantum particle confined in an infinite potential well. In such a model the question arose as to how to estimate the classical potential needed for solving the Schrodinger equation. In the present article the method used in that work for evaluating the potential is described, in the simplest version to implement, and more sophisticated implementations are suggested later.

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I. INTRODUCTION

The initial study on the quantum model for stock markets [1] suggested that the energy-matter interaction paradigm could be an excellent assumption for the modelling of investor interactions. On the other hand, the estimation of the classical potential would be possible by applying the radioactive decay paradigm. In what follows, we will describe a numerical method based on both paradigms that in its simplest version already provides good predictive results for future price distributions in a stock market. In addition, known patterns of market technical analysis are presented that could be elegantly explained from a quantum-mechanical point of view. The presentation will be completed with a short overview of market technical analysis, dedicated to the reader who is not familiar with this terminology.

II. THE MODEL IN A FIRST APPROACH

An investor is similar to an atom that can absorb (buy) or emit (sell) a quantum of energy (block of shares of a certain size). In the ”ground” state a investor does not own any stock. If he buys, such an investor goes into an ”excited” state. From this state, investor can sell, returning to a less excited state (reducing or underweighting investment) or to ground state (total sale of investment). If a market were made up of a few investors, variations in energy (money flow) would be discrete (see Fig. 1) and quantum nature of transactions would be obvious. In practice, a large number of investors and some kind of decoherence determine that prices take the form of pseudo-trajectories, although totally discontinuous and erratically evolving.

For modeling this process, we propose to take as a paradigm the radioactive decay. Thus, the mean life span of an excited state will be globally characterized by a disintegration constant. Thus, whether \( N_{0i} \) is the volume traded on \( i-th \) day, this means that a number \( N_{0i} \) of shares have changed hands on that day. Over time those shares gradually change hands again, so that after a time of \( t \) days, the number of remaining shares, not traded (held), will be \( N(t) \), approximately fulfilling the known relationship:

\[
N_i(t) = N_{0i}e^{-\lambda t}
\]

the disintegration constant mentioned above being \( \lambda \). Over time, daily trading volumes
FIG. 1. The flow of money appears as a pseudo-trajectory. Its quantum nature is not obvious.

![Graph showing money flow over trading days]

FIG. 2. Radioactive decay paradigm: global distribution is an intersection would evolve as in Fig. 2.

Intersections \( I \) are the number of non-traded (held) shares distributed by price and trading days. The probability of a share changing hands can be empirically estimated as the probability of trading shares belonging to the free float, i.e:

\[
P = \frac{v_m}{v_{ff}}
\]

with \( v_m \), \( v_{ff} \) being mean daily volume and free float of the company. The disintegration
FIG. 3. Global distribution of volumes with respect to price $s$

FIG. 4. Intraday distribution of volumes $v_i$ with respect to traded prices $p_j$. The total volume associated with $p_j$ is a summation as successive trading days overlap in price.

Constant can be deduced from (2):

$$\lambda(P) = -\ln(1 - P)$$

The width of potential well will constitute the variation domain of prices $p$, that is, $p_{\text{min}} \leq p \leq p_{\text{max}}$. Let us consider this domain divided into $k$ intervals with a width of $2\varepsilon$, centered on price values $p_j$, $j = 1...k$ (see Fig. 3).

In addition, traded volume $v_i$ corresponding to $i-th$ day is distributed between a maximum and minimum prices of that $i-th$ day by a proper distribution, being adjusted as shown in Fig. 4.

This step can be simplified by considering the total volume $v_i$ associated with the closing
price \( p_{\text{close}} \) in \( i \) th trading day, but the probabilistic distribution is a better approximation. The numbers of (held) shares remaining after each trading day would be: \( v_0 e^{-\lambda T} \), \( v_1 e^{-\lambda(T-1)} \), ..., \( v_{i-2} e^{-2\lambda} \), \( v_{i-1} e^{-\lambda} \), \( v_i \) and the total number corresponding to price \( p_j \)

\[
N(p_j) = \sum_{i=0}^{T} v_i(p_j) e^{-\lambda(T-i)}
\]  

(4)

\( v_i(p_j) \) being the fraction of \( v_i \) traded at price \( p_j \). When quantum particle passes through a certain \( p_j \) position, the probability of interaction is proportional to the total number of remaining shares for price level \( p_j \), so we will assume that the potential field would be made up of \( V(p_j) \) values, such that:

\[
V(p_j) \sim \sum_{-\epsilon}^{+\epsilon} p_\epsilon N(p_\epsilon),
\]  

(5)

being \( p_\epsilon \in (p_j - \epsilon, p_j + \epsilon) \). As we do not know initial configuration \( \{V_0(p_j)\} \), we will assume that a flat configuration tends to real configuration for a long enough time. We will set this time frame \( T \) equal to the free float rotation period, that is, an elapsed time during which a total volume equal to the company’s free float is traded.

### III. SOME EXAMPLES

Under previous hypotheses we can obtain several examples that illustrate basic concepts used by technical analysts, namely support levels, resistance, sideway movement,...explained from a quantum-mechanical point of view.

#### A. Sideway trend

In particular, Fig. 5 is a configuration of potential in which two positions that act as support and resistance (see Appendix) can be clearly seen. Prices tend to move between \( P_0 \), \( P_1 \) in what analysts call a sideway trend, illustrated in Fig. 6.

In sideway trend an analyst’s interest is focused on predicting direction (up or down) in which prices will break the \( p_1 - p_0 \) channel. From our quantum-mechanical point of view, the highest probability of a breakout will be marked by a wave function. Thus, if wave
FIG. 5. Two potential peaks determine sideway trend

FIG. 6. price levels $p_0$, $p_1$ corresponding to the potential peaks

function is $f_1(p)$, the higher probability corresponds to an upward break, while if it is $f_2(p)$, a downward break is more likely.

**B. Trend lines**

An important concept widely used by analysts is *trend line* (see Appendix) which can be interpreted in quantum-mechanical terms. Thus, in Fig. 7, field of potential with a peak at $p_0$ is shown.
This potential would evolve over time in one way or another, depending on the position occupied by the wave function \( f(p) \). For example, if the maximum of wave function is located to the right of \( p_0 \), the potential would migrate to the right, determining the upward trend of Fig. 8.

Correlatively, if \( f(p) \) is found to the left of \( p_0 \) we would have the downward trend of Fig. 10.

Thus, the maximum \( f(p) \) would be "constructor" and the tail would be "destructor", which would determine the migration of the potential.
A figure well known by analysts is the intersection of prices trajectory with a trend line, which is either exceeded upwards or downwards, and which finally either bounces (Fig. 11) or breaks (Fig. 12) the trend.

A possible interpretation of the phenomenon in quantum-mechanical terms would be a hypothetical quantum tunneling effect in which the quantities of transmission and reflection factors would determine a rupture or rebound of quotes.
FIG. 11. Presumed quantum tunneling with a relatively high reflection factor

FIG. 12. Presumed quantum tunneling with a relatively high transmission factor

IV. A MORE ELABORATED MODEL

Daily variations in trading volume clearly indicate that $\lambda$ is not strictly constant, but oscillates around an average value. This average value would be used in the first approximation model described above. On the other hand, it is known that volume is related to volatility, which suggests a dependency such as $\lambda = G(1/\sigma)$, with $G$ probably being a linear function. Therefore, a more elaborate model would be one that contemplates the following dependence:
FIG. 13. Gaussian versus exponential model

\[ N_i(t) = N_0 e^{-G(1/\sigma)t} \]  \hspace{1cm} (6)

which would give rise to a family of daily curves dependent on parameter (daily volatility) \( \sigma \). This type of exponential model is characterized because the tangent in \( t = 0 \) has a pronounced slope (see Fig. 13), i.e., the assumption that investors start selling on next day after they have bought. If we wanted the model to reflect a tendency of investors to hold they block of shares for a certain period, we would have to use a Gaussian rather than an exponential model of type (7)

\[ \phi(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{- (t/(\sqrt{2}\sigma))^2} \]  \hspace{1cm} (7)

The tendency to hold blocks of shares is a characteristic that depends on type of market considered and, in particular, percentage of prevalent short-term trading.

V. CONCLUSIONS

This article describes a method for estimating the potential in the Schrodinger equation for a stock market. By inserting this potential into the infinite well quantum model, such a model provides a future price distribution as a wave function of a quantum particle confined...
FIG. 14. A typical trading day. Note that the volumes increase at the end of the day into well. The first results obtained suggest elegant quantum-mechanical explanations of various patterns belonging to the so-called technical analysis of the stock markets.

Appendix: Technical Analysis Basics

1. Data and variables

The fundamental variables used in technical analysis are price and volume, the latter being the number of shares traded during a trading day. The market, at the end of the day, offers the data in EOD (End Of Day) format as follows: date, opening price, maximum price, minimum price, closing price (see Fig. [14]).

All these data (historical data) are usually offered in the form of a CSV (Comma Separated Values) file on main financial websites, some of which are free of charge.

2. The trend

The trend is the predominant direction that prices take as a result of the evolution of supply and demand. If for a particular asset the balance between supply and demand were perfect, asset prices would be constant over time. Usually this is not the case and three types of trend appear, namely

- upward trend, when demand exceeds supply the buyers drive the price up.
FIG. 15. The three main trend categories: this suggests a fractal structure

- otherwise, in a downward trend sellers push the price down.

- sideways trend occurs when there is no predominant direction of price movement.

For some reason, the evolution of supply and demand is approximately linear over time, so it is possible to graphically represent trends using straight lines (see Fig. 15).

According to their duration the trends are classified in:

- Primary: more than 11 months
- Secondary: 2 to 11 months
- Tertiary: less than 2 months

The empirical observation that duration and slope of a trend are correlated is very interesting, and thus the greater the slope, the weaker the trend and the greater probability of being broken. The most stable trends are those between 30 and 45 degrees of inclination. The relative strength of a trend depends on its duration and number of bounces it has sustained. The primary trend is the one that defines the state of the market:

- Bull market, bullish (upward primary trend)
- Bear market, bearish (downward primary trend)
3. Supports and resistances

A trend line that passes through price trajectory highs is called support. Correlatively, a line resulting from joining lows is called resistance. Supports and resistances tend to act as barriers, preventing prices from move to a certain direction. Support or resistance level is also said to refer to a particular price that is difficult to drill down or break up. When a resistance is broken upwards, it becomes support (Fig. 16), and vice versa, a support in resistance (Fig. 17).

A very significant type of drilling or breakage is when it takes place with an upward or
downward gap. A gap is a price range in which no shares are traded and can only be seen well on bar charts (see Fig. 18).

The price path can be represented by a single line formed by the closing prices. But usually it is represented by bars that reflect the EOD (End Of Day) values opening, maximum, minimum and closing (see Fig. 19).
4. **Channels**

They occur when the price path moves between a parallel support and resistance (see Fig. [20]). In a sideways trend, horizontal channels are often formed (see Fig. [6]).

5. **Triangles and wedges**

They occur when support and resistance lines intersect. Whether the slope of the two lines has the same sign or not determines its designation. Usually triangles and wedges are convergent, but sometimes they are expanding or divergent (see Fig. [23]). Since the relative strength of the trend lines correlate with their slope, it is sometimes possible to predict how the break will be. Example: the horizontal line is always weaker than a line that has a 30-45 slope degrees, therefore, in see Fig. [24] one can expect the break to occur downwards.

[1] J. L. Subias, “Quantum model for price forecasting in financial markets,” e-print arXiv:q-fin.ST/1902.10502 (2018).

[2] O. Racorean, “Quantum tunneling of stock price in range bound market conditions,” e-print arXiv:q-fin.PR/1307.6727 (2013).
FIG. 21. Triangle: lines of support and resistance with opposite slopes

FIG. 22. Wedge: support and resistance with slopes of the same sign
FIG. 23. Expanding triangle

FIG. 24. The outcome of a triangle depends on the relative strength of its trend lines