Transitions probabilities B(E2), B(M1) and B(E0) in $^{130}$Ba isotope

Mohsin Kadhim Muttaalb

University of Babylon, Science College, physics department

mam50_24@yahoo.com

Abstract:
Nuclear structure for even-even $^{130}$Ba isotope have been investigated. They examined carefully with IBM 1 and IBM 2. Low lying energy levels for even even positive parity states, reduced electric quadruple transitions probabilities B(E2), branching ratios, reduced magnetic dipole transitions probability B(M1), reduced electric monopole transitions probability B(E0), and mixing ratio X(E0/E2) have been studied.

In the framework of IBM1 noted the competition between the two parameters $(a_0$ and $a_2$) in $^{130}$Ba isotope was the increases of $a_0$ associated with decreases of $a_2$ which mean that the $\gamma$-unstable features in IBM2 $\chi_e$ and $\chi_0$ were (-1.3 and 0.7) which signified similarity with IBM1 expected. $^{130}$Ba isotope described to be transitional nucleus, transitional between SU(3) and O(6) features.

1. Introduction
Barium nuclei have 40 isotopes between mass number 114 to 153. They have always been of great interest the evidence of structure evolutions from weakly to well deformation or $\gamma$-soft shapes as a function of mass number. The interacting boson model (IBM) is based on the well-known shell model of the atomic nucleus, as well as a geometrical collective model of the nucleus[1-3]. It’s useful for explaining intermediate and heavy nuclei’s structure. IBM was created by Iachello and Arima [4-7]. According to (IBM) postulates, the nuclear pairs are represented by bosons with angular momentum $L = 0$ or $L = 2$, i.e. the number of bosons relies on the number of active nuclear particle number (or hole) of the pairs outside a closed shell. The complete boson number $(N)$ is estimated by adding the partial numbers, i.e. $N = N_p + N_n$, where $N_p$ and $N_n$ are the number of proton and neutron bosons, respectively. Many experiments have tried to clarify the behavior of the $^{130}$Ba nucleus using numerous approaches models[8-14]. In this research many properties of nuclear structure for even even $^{130}$Ba isotope has been investigated with IBM1 and IBM2 it has been examined carefully with benefit from continuously updating of nuclear decay schemes.

2. The Interacting Boson Model
The IBM assumes that the hamiltonian operator has only one body and two body terms, resulting in the implementation of the formation $(s^\dagger, d_m^\dagger)$ and annihilation $(s, d_m)$ operators where the index $m = 0, \pm 1, \pm 2$. The most general Hamiltonian can be written as [15]:

$$H = e_s (s^\dagger s) + e_d \sum_m d_m^\dagger d_m + \sum_{L=0,2,4} \frac{1}{2} (2L+1)^2 C_L [(d^\dagger d)^{(L)}(s^\dagger s)(0)] + \frac{1}{\sqrt{2}} \nu_2 [(d^\dagger d)^{(2)}(d^\dagger s)(2) + (d^\dagger s)(2)(d^\dagger d)^{(2)}] + \frac{1}{2} \nu_0 [(d^\dagger d)^{(0)}(s^\dagger s)(0) +...}$$
\[(s^+s^+_0)(dd)^0 + (dd)^0 \right]^{(0)} + u_2 \left[(d^+s^+_0)(ss^+_0)^0 + \frac{1}{2} u_0 \left[ (s^+s^+_0)(ss^+_0)^0 \right] \right]^{(0)}

(1)

Where \(C_L(L = 0,2,4), u_L(L = 0,2), u_L(L = 0,2)\) describe the boson interaction.

The most commonly used form of the IBM1 Hamiltonian is [16]:
\[H = \varepsilon \eta_d^L + a_q^L + a_1 L + a_2 Q + a_3 T_2 + a_4 T_4\]

(2)

Where \(\varepsilon = \varepsilon_d - \varepsilon_s\) is the boson energy, for simplicity \(\varepsilon_s\) was set equal to zero only \(\varepsilon = \varepsilon_d\) appears, and \(a_q, a_1, a_2, a_3, a_4\) designate the Quadrupole, Angular momentum, Pairing, Octupole, and Hexadecapole interactions between bosons. The d boson's five components and the s boson's single component spread over a six-dimensional space and for a fixed number of boson \(N\), the group structure of the problem is that of \(U(6)\). Considering the different reductions of \(U(6)\), three dynamical symmetries emerge, namely \(U(5)\), \(SU(3)\), and \(U(6)\); these symmetries are related to the geometrical idea of the spherical vibrator, deformed rotor and a symmetric \((\gamma - \delta)\) deformed rotor, respectively [17-19]. In order to calculate transition rates the simplest form of \(IBM1\) the one body transition operator has been given as follows [16-19]:

\[T_m = a_d^0 \delta^0 + \beta^0 \delta^0 \delta^0 \delta^0 + \gamma \delta^0 \delta^0 \delta^0 \delta^0 \]

(3)

Where \(a_d, \beta_d, \gamma_d\) are the coefficients of the various terms in the operators. The three, corresponding dynamical symmetries group chains of \(U(6)\) can be written as [16-19]:

\[I - U(6) \Rightarrow SU(5) \Rightarrow O(5) \Rightarrow O(3) \Rightarrow O(2)\]

\[II - U(6) \Rightarrow SU(3) \Rightarrow O(3) \Rightarrow O(2)\]

\[III - U(6) \Rightarrow O(6) \Rightarrow O(5) \Rightarrow O(3) \Rightarrow O(2)\]

(4)

The branching ratio for three, corresponding dynamical symmetries have been written as [15]:

\[R = \frac{B(E2; 2^+_1 \rightarrow 2^+_1)}{B(E2; 2^+_1 \rightarrow 2^+_1)}; R' = \frac{B(E2; 2^+_1 \rightarrow 0^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)}; R'' = \frac{B(E2; 2^+_1 \rightarrow 2^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)} \]

\[R = \frac{B(E2; 2^+_1 \rightarrow 2^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)} = \frac{10(N - 1)(2N + 5)}{10(2N + 3)} = \frac{10}{7}; \]

\[R' = \frac{B(E2; 2^+_1 \rightarrow 2^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)} = \frac{B(E2; 2^+_1 \rightarrow 2^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)} \approx \frac{10}{7}; \]

\[R'' = \frac{B(E2; 2^+_1 \rightarrow 2^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)} = \frac{B(E2; 2^+_1 \rightarrow 2^+_1)}{B(E2; 2^+_1 \rightarrow 0^+_1)} \approx \frac{10}{7}; \]

(5)

The multipole moments, in \(U(5)\), \(SU(3)\), and \(O(6)\), limits defined as

\[Q^2 \downarrow = \beta_d \sqrt{16 \pi / 5} \downarrow 2^+/7; Q^2 \downarrow = -a_d \sqrt{2 \pi / 5} \downarrow 2^+/7(4N + 3)\]

(6)

A simple schematic hamiltonian driven by microscopic consideration is given by [20,21]:

\[H = \varepsilon (n_{d_d} + n_{d_v}) + \kappa Q_d Q_v + V_d + M_{d_d} + M_{d_v}\]

(7)

\(\varepsilon_{d_d}, \varepsilon_{d_v}\) are proton and neutron energy, respectively, and are thought to be equal to \(\varepsilon_d = \varepsilon_v = \varepsilon\). The majorana operator \(M_{d_d}\) and it is typically used to eliminate states of mixed proton neutron symmetry. It is possible to write this expression as [22,23]:

\[M_{d_d} = \xi_d (s_d^d + d_d^d)^2 + \xi_v (s_v d_d^d + d_v s_d^d)^2 + \sum_{k=1,3} \zeta_k (d_d^d d_d^d)^2 - (d_d^d d_d^d)^2\]

(8)

If there is experimental evidence for a so called mixed symmetry condition, the majorana parameter is modified to correct the spectrum's position. The general angular momentum \(\ell\)
single boson transfer operator has the same shape as in IBM1 eq.(3) except that one has to consider $\pi, \nu$ degree of freedom and this can be written as[24-27]:

$$T^{(\ell)} = a_2 \delta_{\ell 2}[d^{\dagger}s + s^d d]^{(2)} + \beta_{\ell \rho}[d^{\dagger} d]^{(\ell)} + \gamma_{\ell \rho} \delta_{\ell 0}[s^d s]^{(0)} \ldots \rho = \pi \ or \ \nu$$  (9)

For E2 operator[15,26]

$$T^{E2} = e_\nu Q_\nu + e_\pi Q_\pi$$  (10)

Where $Q_\pi$ is the same as in eq.(7) and $e_\pi, e_\nu$ are boson effective charges depending on the boson number $N$ and they can take any value to fit the experimental result. The quadruple moment of $2^+_1$ can be in IBM2 defined as:-

$$Q^+_2 = (L,M = 2)[\hat{Q}][L,M = 2]$$  (11)

$$Q^+_2 = \sqrt{\frac{16\pi}{5}}(2L + 1)B(E2) \frac{1}{(2L + 1)(L + 1)(2L + 3)}$$  (12)

The M1 operator obtained by letting $\ell = 1$ in eq. (9) is written [15,25] as

$$T^{(M1)} = \frac{3}{4}\pi \left[ (g_{\pi} L^{(1)}_{\pi} + g_{\nu} L^{(1)}_{\nu}) \right]$$  (13)

Where $g_{\pi}$, $g_{\nu}$ are the boson $g$ factor in unit of $\mu_0$ and $L^{(1)}_{\rho} = \sqrt{10}(d^{\dagger}d)^{(1)}_{\rho}$, this operator can be written alternatively as:

$$T^{(M1)} = \frac{3}{4}\pi \left[ (g_{\pi} + g_{\nu})(L^{(1)}_{\pi} + L^{(1)}_{\nu}) + \frac{1}{2}(g_{\pi} - g_{\nu})(L^{(1)}_{\nu} - L^{(1)}_{\pi}) \right]$$  (14)

The first term on the right hand side of this equation is diagonal; therefore, for M1 transition the previous equation may be written as:

$$T^{(M1)} = 0.77\left[ (d^{\dagger}d)^{(1)}_{\pi} - (d^{\dagger}d)^{(1)}_{\nu} \right]$$  (15)

The M1 strength may be expressed in terms of the multipole mixing

$$\delta^{(E0/E1)} = 0.835E_\pi(MeV). \Delta \quad \text{where} \quad \Delta = \frac{\langle j_E|\hat{B}(E2)|j_0 \rangle}{\langle j_E|\hat{B}(M1)|j_0 \rangle}$$  (16)

The E0 reduced transitions probability is expressed as [28].

$$B(E0; I_1 \rightarrow I_2) = e^2 R^2 \rho^2 E0$$  (17)

Where $e$ indicates the electronic charge, $R$ stands for the nuclear radius and $\rho$ (E0) is the matrix element transition which is gotten by [28].

$$\rho(E0) = Z/R^2 \sum_j\tilde{\beta}_{j0} \langle j_f|d^{\dagger}_j \times d_j|j_i \rangle, \rho = \pi, \nu$$  (18)

Where $\tilde{\beta}_{j0}, \tilde{\beta}_{av}$ represent the deformation parameters for (protons and neutrons)

The X(E0/E2) ratio can be calculated as follows [28]:

$$X(E0/E2) = \frac{B(E0; I_1 \rightarrow I_2)}{B(E2; I_1 \rightarrow I_2)}$$  (19)

Where $I_f = I_2$, for $I_1 = I_f \neq 0$, and $I_f = 2$ for $I_1 = I_f = 0$. This ratio is so essential as it mirrors to what extent the transition between $B(E2)$, and $B(E0)$ is strong.

3. Results and Discussion

The $^{130}$Ba isotope, has $Z = 56$, then 3 particle bosons the number of protons and neutrons lying between 50 and 82 magic shells, respectively, $^{130}$Ba has 74 neutrons which mean (4) hole neutron bosons, the parameters estimated for the calculations of the levels of low-lying excited energy for Barium isotope have been given in the table (1).

| Isotop | IBM1 parameters |
|--------|-----------------|
| $^{130}$Ba | $N$ | $\varepsilon$ | $a_0$ | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $\chi$ |
| 7 | 0.0 | 0.021 | 0.015 | -0.044 | 0.02 | 0.0 | -0.38 |

| $IBM2$ parameters $\chi_\pi = -1.3 \ , \ N_\pi = 3$ |
| $N_\nu$ | $\varepsilon_d$ | $\kappa$ | $\chi_\nu$ | $\zeta_2$ | $\zeta_{1,3}$ | $C_\nu^L$ | $C_\pi^L$ |
Experimental[29] and theoretical[15] energy ratios \((E4_{1}^{+}/E2_{1}^{+}), (E6_{1}^{+}/E2_{1}^{+})\) and \((EB_{1}^{+}/E2_{1}^{+})\) compared to the standard ratios for \(U(5), SU(3),\) and \(O(6)\) limits have been calculated for \(^{130}\text{Ba}\) isotope shown in table (2).

Table 2: Experimental[29] and standard [15] energy ratios for \(^{130}\text{Ba}\) isotope.

| Ratio             | \(^{130}\text{Ba}\) | \(U(5)\) limit | \(SU(3)\) limit | \(O(6)\) limit |
|-------------------|----------------------|-----------------|-----------------|----------------|
| \(E4_{1}^{+}/E2_{1}^{+}\) | 2.52                | 2               | 3.33            | 2.5            |
| \(E6_{1}^{+}/E2_{1}^{+}\) | 4.45                | 3               | 7               | 4.5            |
| \(EB_{1}^{+}/E2_{1}^{+}\) | 6.7                 | 4               | 12              | 7              |

The calculated energy levels comparison with experimental data [29] for \(^{130}\text{Ba}\) isotope illustrated in figures (1).

![Figure 1](image_url)

Figure 1: The levels of the calculated energy in comparison with experimental [29] for \(^{130}\text{Ba}\) isotope.

Effective boson charges used in \(IBM1\) and \(IBM2\) for electric quadruple transitions and calculated branching ratios listed in Table (3) have been compared with available experimental data [29]. Therefore calculated reduced electric quadruple transitions probability \(B(E2)\), and electric quadrupole moment of \(2^{+}\) state comparison with the experimental values [29-31] for \(^{130}\text{Ba}\) isotope listed in Tables (4).

Table 3: The effective boson charges used in \(IBM1\) and \(IBM2\) to calculate \(B(E2)\) transition and comparisons between calculated branching ratios with experimental data[29-31] for even-even \(^{130}\text{Ba}\).

| \(^{130}\text{Ba}\) | The effective boson charges \((eb)\) |
|-------------------|----------------------------------|
| \(IBM1\)          | \(IBM2\)                         |
| \(E2SD\)          | \(E2DD\)                         | \(e_{v}\)  | \(e_{\pi}\) |
| 0.106              | -0.123                           | 0.022     | 0.22        |
Table 4: Calculated reduced electric quadruple transitions probability $B(E2)$ and electric quadrupole moment of $2^+_1$ state comparison with the experimental values [29-31] for $^{130}$Ba isotope.

| Isotope | $B(E2)$ values in $(e^2b^2)$ $^{130}_{56}$Ba |
|---------|------------------------------------------------|
| $J^+_i \rightarrow J^+_f$ | Expe. | IBM-1 | IBM-2 | $J^+_i \rightarrow J^+_f$ | Expe. | IBM-1 | IBM-2 |
| $2_1 \rightarrow 0_1$ | 0.226 | 0.227 | 0.23 | $3_1 \rightarrow 4_3$ | -- | 0.083 | 0.0737 |
| $4_1 \rightarrow 2_1$ | 0.3036 | 0.315 | 0.3045 | $5_1 \rightarrow 3_1$ | -- | 0.13 | 0.156 |
| $6_1 \rightarrow 4_1$ | 0.3126 | 0.323 | 0.339 | $7_1 \rightarrow 5_1$ | -- | 0.149 | 0.144 |
| $8_1 \rightarrow 6_1$ | 0.299 | 0.297 | 0.291 | $9_1 \rightarrow 7_1$ | -- | 0.123 | 0.121 |
| $10_1 \rightarrow 8_1$ | 0.21123 | 0.248 | 0.249 | $4_2 \rightarrow 4_1$ | -- | 0.0233 | 0.032 |
| $0_2 \rightarrow 2_1$ | -- | 0.001 | 0.002 | $3_1 \rightarrow 1_1$ | -- | 0.0129 | -- |
| $2_2 \rightarrow 0_2$ | -- | 0.0288 | 0.04 | $1_1 \rightarrow 2_1$ | -- | 0.036 | -- |
| $2_2 \rightarrow 2_1$ | -- | 0.0301 | 0.034 | $1_1 \rightarrow 2_1$ | -- | 0.0044 | -- |
| $4_2 \rightarrow 2_2$ | -- | 0.1015 | 0.14 | $Q_{21}(eb)$ | -1, -0.86 | -0.3 | -1.229 | -0.99 |
| $3_1 \rightarrow 2_2$ | -- | 0.219 | 0.299 | |

To calculate $B(M1)$ transition probability the effective $g$ – factors for proton $g_p$ and neutron $g_n$ calculated for $^{130}$Ba isotope were $g_p = (0.4\mu_p)$ and $g_n = (0.34\mu_n)$ equations (15) have been used in IBM2 to calculate the $B(M1)$ transition probabilities as it is shown in table (5). The calculation values for $B(M1)$ where no available experiments data [29,31].

Table 5: Calculated magnetic dipole transitions $B(M1)$ in($\mu^2$) where no available experimental data [29,31] for $^{130}$Ba isotope.

| Isotope | $^{130}_{56}$Ba |
|---------|----------------|
| $J^+_i \rightarrow J^+_f$ | $B(M1)\mu^2$ | $J^+_i \rightarrow J^+_f$ | $B(M1)\mu^2$ |
| Expe. | IBM2 | Expe. | IBM2 |
| $1_1 \rightarrow 2_1$ | -- | 0.00112 | $3_1 \rightarrow 2_1$ | -- | 0.00045 |
| $1_1 \rightarrow 2_2$ | -- | 0.00535 | $3_1 \rightarrow 2_2$ | -- | 0.00281 |
| $1_1 \rightarrow 2_3$ | -- | 0.00077 | $4_1 \rightarrow 3_1$ | -- | 0.00186 |
| $2_2 \rightarrow 2_1$ | -- | 0.00216 | $4_2 \rightarrow 4_1$ | -- | 0.00709 |
| $2_2 \rightarrow 2_3$ | -- | 0.00013 | $5_1 \rightarrow 4_1$ | -- | 0.00097 |
| $2_1 \rightarrow 2_3$ | -- | 0.00775 | |

The deformation parameters for protons and neutrons have been used to calculate monopole transition matrix elements $B(E0)$ for $^{130}$Ba isotope are ($\beta_{2\nu} = 0.053f m^2, \beta_{ov} =$
The monopole transition matrix element $B(E0)$ and mixing ratio $X(E0/E2)$ have been calculated using eq.(18,19). Calculation values of monopole transition matrix element $B(E0) e^2b^2$ and $X(E0/E2)$ listed in Table (6).

Table 6: Calculated monopole transition matrix element $B(E0) e^2b^2$ and $X(E0/E2)$ where no evaluable experimental data [29,32] for $^{130}$Ba isotope.

| Isotope |  $^{130}$Ba  |
|---------|--------------|
|         | $J_i^+ \rightarrow J_f^+$ | $B(E0)e^2b^2$ | $X(E0/E2)$ | $J_i^+ \rightarrow J_f^+$ | $B(E0)e^2b^2$ | $X(E0/E2)$ |
|         | IBM2 | IBM2 | Expe. | IBM2 | IBM2 | Expe. |
| $0_2 \rightarrow 0_1$ | 0.00114 | 0.0048 | -- | 2_3 | 0.027 | 6.74 | -- |
| $0_3 \rightarrow 0_1$ | $1.\times10^{-6}$ | $8.14\times10^{-6}$ | -- | 2_2 | 0.008 | 2.51 | -- |
| $0_2 \rightarrow 0_2$ | 0.0005 | 0.00214 | -- | 4_2 | 0.0034 | 0.107 | -- |
| $2_2 \rightarrow 2_1$ | 0.16 | 4.72 | -- | 4_1 | 0.0034 | 0.107 | -- |

4. Conclusion

The interacting boson model has been applied to calculated barium isotope due to flexibility of the model to describe not only the exact symmetries corresponding to different geometrical models but also transitional cases within the same framework. The continuous updating of the decay schemes and the experimental information related to the barium nucleus has been presented in restudying $^{130}$Ba isotope and rearranging it according to the dynamic symmetry, $^{130}$Ba isotope where the researchers disagreed about their characteristics, some of them describe them as transitional characteristics between vibrational $U(5)$ and $γ$–unstable $O(6)$, and the others described as pure $γ$–unstable nuclei. In the framework of $IBM1$ the competition is noted between the two parameters ($a_0$ and $a_2$) where the increases of $a_0$ associated with decreases of $a_2$. This mean that the $γ$–unstable features are continuous increases with opposite of rotational properties, in $IBM2$, $χ_0$ and $χ_0$ are (-1.3 and around 0.7), signifying the similarity with $IBM1$ expected as tabulated in Table(1).The experimental energy ratios ($E4_i^+/E2_i^+$),($E6_i^+/E2_i^+$ and $E8_i^+/E2_i^+$) have been sloped from SU(3) to O(6) features which seemed as transitions between rotational and $γ$- soft nuclei as shown in Table (2).

In Tables (3, 6) listed the calculated branching ratios, $B(E2)$, $B(M1)$ and $B(E0)$ depended on comparisons with fewer experimental data; however, the results are in accept agreement. The electric quadrupole moment $Q_{2+}$ in $IBM1$ and $IBM2$ were (-1.229 and -0.99) (eb) indicated a reduced rotational index.

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