A study of the relativistic corrections to tritium $\beta$-decay

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Abstract

Forthcoming experiments like Project 8 and Ptolemy aim at investigating with high precision the end-point of the tritium $\beta$-decay spectrum sensitive to the neutrino mass. In light of this, using the standard parametrization in terms of nuclear polar form factors, we analyze the complete relativistic expression for the spectrum of the $\beta$-electron emitted by a tritium nucleus. Given the small parameters in the problem, we discuss the approximations that can be made, and present the first two corrections to the standard lowest order formula, improving on the few results available in the literature.

I. INTRODUCTION

The existence of a non-zero neutrino mass is now well established [1–6], and provides a compelling evidence for physics beyond the Standard Model. The differences between the squared masses of the three neutrinos is precisely measured [7–9], whereas the value of their absolute mass scale, their mass ordering, as well as their Dirac or Majorana nature are still unknown.

To this end, a number of existing and future experiments aim at determining the lightest neutrino mass. Specifically, this can be done by studying the spectrum of the electron emitted in $\beta$-decay close to its maximum allowed energy, the so-called end-point. Among these, the existing KATRIN [10,11] and the forthcoming Project 8 [13–15] and Ptolemy [16–18] experiments focus on the decay of tritium:

$^3\text{H} \rightarrow ^3\text{He}^+ + e^- + \nu_e$.

The current best laboratory bound on the effective neutrino mass, $m_{\nu}^2 \equiv \sum_i |U_{ei}|^2 m_i^2$, is given by $m_{\nu} \lesssim 0.8$ eV, as set by KATRIN [11,12]. Here $m_i$ are the neutrino mass eigenvalues and $U_{ei}$ the entries of the PMNS matrix [e.g., 19]. Ptolemy aims at the largest event rate, thanks to a high loading of tritium on, possibly, a flat graphene substrate, as well as an energy resolution as small as 100 meV [17,18]. In light of this, it is important to determine the expected $\beta$-decay spectrum with a theory uncertainty that is better than the experimental precision, which is usually employed. To the best of our knowledge, a discussion of this sort only appeared in [22]. However, we find it to be incorrect and leading to an overestimate of the corrections. In this work we solve this problem. We also show that, given the expected event rate in Ptolemy, the statistical uncertainty is far smaller than the theoretical error associated to the standard lowest order formula, which thus needs to be improved. Specifically, we argue that, for neutrino masses $m_{\nu} \gtrsim 100$ meV, the leading and next-to-leading order formulae have different shapes, potentially distinguishable by an experiment like Ptolemy.

II. REVIEW OF THE CALCULATION

In this work we follow the approach outlined in [22], where the relativistic matrix element is obtained from an hadronic model for the tritium nucleus [23]:

\[ M = \frac{G_F V_{ud}}{\sqrt{2}} \bar{u}(P_e) \gamma_\mu (1 - \gamma_5) v(P_e) \]
\[ \times u(P_f) \left[ G_V(q^2) \gamma_\mu + i \frac{G_M(q^2)}{2M_i} q^\mu q_\nu \gamma_\nu \right] u(P_i), \]

where $G_F$ is the Fermi constant and $V_{ud}$ the entry of the CKM matrix. Moreover, $P_i$ and $P_f$ are the 4-momenta of $^3$H and $^3$He$^+$ (with masses $M_i$ and $M_f$), $P_e$ and $P_\nu$ are those of the electron and antineutrino, and $q \equiv P_f - P_i$ is the momentum transferred to the nucleus. The form factors are [22]

\[ G_V = \frac{g_V}{(1 - q^2/M_V^2)^2}, \quad G_M = \frac{g_M}{(1 - q^2/M_V^2)^2}, \]
\[ G_A = \frac{g_A}{(1 - q^2/M_A^2)^2}, \quad G_P = \frac{2M_i}{m_e^2 - q^2} G_A, \]

specific assumptions. Moreover, in light of the increasing experimental precision, it is crucial to systematically understand the corrections to the lowest order formula, which is usually employed. To the best of our knowledge, a discussion of this sort only appeared in [22]. However, we find it to be incorrect and leading to an overestimate of the corrections. In this work we solve this problem. We also show that, given the expected event rate in Ptolemy, the statistical uncertainty is far smaller than the theoretical error associated to the standard lowest order formula, which thus needs to be improved. Specifically, we argue that, for neutrino masses $m_{\nu} \gtrsim 100$ meV, the leading and next-to-leading order formulae have different shapes, potentially distinguishable by an experiment like Ptolemy.
with $m_e$ the pion mass and $M_{A,V}$ two mass scales of the order of the GeV.

A version of the full relativistic spectrum has also been discussed in [21]. However, the matrix element employed there is not computed from first principles, but rather parametrized using Lorentz invariance together with some considerations regarding which terms should be the leading ones. While this approach reproduces well the lowest order formula, it is not the most suitable one for computing higher order corrections.

In the following we neglect the momentum dependence of the form factors. This approximation is correct up to relative corrections given by

$$\frac{g^2}{M_{A,V}^2} \simeq \frac{m_e^2}{M_{A,V}^2} \sim 10^{-7} \quad \text{for } G_V, G_M \text{ and } G_A,$$

$$\frac{q^2}{m_e^2} \sim 10^{-2} \quad \text{for } G_P.$$  

(4)

Given the above, one can compute the fully relativistic decay rate, differential in the electron energy, whose form in the lab frame is given by

$$\frac{d\Gamma}{dE_e} = \frac{(GF_{
u \nu})^2}{2\pi^3} F(Z, E_e)^2 \frac{M_e^2}{M_i^2} \sqrt{\Delta} \left( \Delta - 2m_{\nu} \frac{M_f}{M_i} \right) \times \left[ g_V^2 W_{VV} + g_A^2 W_{AA} + g_{\nu gV} W_{AV} + g_A^2 W_{AP} + g_{\nu g}^2 W_{PP} + g_{\nu g} W_{VM} + g_{\nu g m} W_{AM} + g_m^2 W_{MM} \right],$$

(5)

where $M_{i, 2}^2 = M_i^2 + m_e^2 - 2M_iE_e$, and $F(Z, E_e)$ is the Fermi factor accounting for the electromagnetic interaction between the outgoing electron and the decayed nucleus. Following Primakoff and Rosen [24], we approximate it as

$$F(Z, E_e) \simeq \frac{2\pi \alpha Z E_e}{1 - \exp \left( -2\pi \alpha Z E_e / p_e \right)},$$

(6)

with $Z = 2$ the atomic number of $^3\text{He}^+$. Finally, we define $\Delta \equiv E_e - E_{e, \text{max}}^\text{rel}$, with the relativistic end-point given by

$$E_{e, \text{max}}^\text{rel} = \frac{M_i^2 + m_e^2 - (M_i + m_\nu)^2}{2M_i}.$$  

(7)

The $W$'s appearing in Eq. (5) result from integrating the spin-averaged squared matrix element over the relativistic 3-body phase space, as outlined in [22]. Their expressions (again, in the approximation of constant form factors) are rather cumbersome. We report them fully in Appendix A.

### III. EXPANSION IN THE SMALL PARAMETERS

The dimensionful quantities entering the $W$ terms come with different sizes, resulting in a number of small parameters that one can use to approximate the full expression, Eq. (5). These parameters are chosen to be

$$\frac{m_e}{M_i} \sim 10^{-4}, \quad \frac{Q}{M_i} \sim 10^{-5}, \quad \frac{\Delta}{M_i} \sim \frac{m_\nu}{M_i} \sim 10^{-10},$$  

(8)

where $Q \equiv M_i - M_f - m_e$ is the $Q$-value. Since the spectrum is most sensitive to the neutrino mass close to the end-point, we also set $|\Delta| \sim m_\nu$, and assumed conservatively $m_\nu \sim 1$ eV. For everything else, we used the numerical values in Table I. We also report the current errors on the measured parameters. In fact, while this does not affect the present work, for higher order corrections, the uncertainties coming from the parameters can compete with the theoretical uncertainties. This is especially true for the masses, the $Q$-value and the $g$ couplings. The errors on overall factors are, instead, expected not to play a role, since a standard experimental analysis would fit the overall normalization anyway.

Following the expansion scheme outlined above, one finds the following leading order behaviors of the $W$'s:

$$W_{VV} = E_e (m_\nu - \Delta) - E_e m_\nu \frac{m_e}{M_i} - E_e m_\nu \frac{Q}{M_i} + O(m_\nu^2 Q \Delta / M_i^2),$$

(9a)

$$W_{AA} = 3E_e (m_\nu - \Delta) - 3E_e m_\nu \frac{m_e}{M_i} + E_e (m_\nu - 4\Delta) \frac{Q}{M_i} + O(m_\nu^2 Q \Delta / M_i^2),$$

(9b)

$$W_{AV} = 4E_e (m_\nu - \Delta) \frac{Q}{M_i} + O(m_\nu^2 Q \Delta / M_i^2),$$

(9c)

$$W_{AM} = - 4E_e (m_\nu - \Delta) \frac{Q}{M_i} + O(m_\nu^2 Q \Delta / M_i^2),$$

(9d)

$$W_{AP} = O(m_\nu^2 Q^2 \Delta / M_i m_e^2),$$

(9e)

$$W_{PP} = O(m_\nu^2 Q \Delta / m_e^2),$$

(9f)

$$W_{VM} = O(m_\nu^2 Q \Delta / M_i^2),$$

(9g)

$$W_{MM} = O(m_\nu^2 \Delta / M_i^2).$$

(9h)

Before computing the decay rate and its leading corrections, we briefly comment about the expansion presented in [22]. There, in order to estimate the relative sizes of the various terms, it is set $M_f = M_i$. However, this equality holds true only at lowest order, and receives corrections already at order $m_\nu / M_i$. Because of this, the scalings reported in Eq. (28) of [22] lead to an overestimate of the corrections (in all cases but one). In fact, when one keeps track of the fact that $M_i \neq M_f$, additional cancellations happen, reducing the size of the correction. We have verified that, starting from the full expressions reported in Appendix A, the correct estimates are those in Eqs. (9). Moreover, we report the presence of a wrong sign in the

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1 We note a typo in the denominator of Eq. (7) in [22], where $M_f$ should be replaced with $M_i$. 

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expression of $W_{AA, VV, AV}$, given in Eq. (29) of [22], for the first term in the parentheses proportional to $(g_V + g_A)^2$. The correct expression is given in Appendix A.

Now, in light of all this, the leading order expression for the differential rate is

$$\frac{d\Gamma}{dE_e} = \frac{(G_F V_{ud})^2}{2\pi^3} F(Z, E_e) \left( g_V^2 + 3g_A^2 \right) \times E_e P_e (m_\nu - \Delta) \sqrt{\Delta(\Delta - 2m_\nu)},$$

in agreement with the literature [e.g., 17, 21, 22]. The rate up to order $Q/M_i$, instead reads

$$\frac{d\Gamma}{dE_e} = \frac{(G_F V_{ud})^2}{2\pi^3} F(Z, E_e) E_e P_e \sqrt{\Delta} \left( \Delta - 2m_\nu \frac{M_i}{M_e} \right) \times \frac{M_i^2}{M_i^2} \left\{ \left( g_V^2 + 3g_A^2 \right) (m_\nu - \Delta - m_\nu \frac{m_e}{M_i}) 
+ \left[ \ - g_\nu^2 m_\nu + g_A^2 (m_\nu - 4\Delta) 
- 4g_A (g_M - g_V) (m_\nu - \Delta) \right] \frac{Q}{M_i} \right\}. $$

Similarly, the leading and next-to-leading order expressions for the electron end-point read,

$$E_{e,(0)}^{\text{max}} = m_e + Q - m_\nu,$$  
(12a)

$$E_{e,(1)}^{\text{max}} = m_e + Q - m_\nu - m_e \frac{Q}{M_i}.$$  
(12b)

**IV. RELEVANCE FOR PTOLEMY**

Thanks to the storage of atomic tritium on a solid state substrate, Ptolemy aims at employing a large mass of tritium, possibly up to 100 g [17], which would also allow to hunt for the cosmic neutrino background. In Figure 1 we show the relative correction to the leading order rate obtained from Eq. (11), i.e.

$$\Delta \tilde{\Gamma} \equiv \frac{d\tilde{\Gamma}_{(1)}/dE_e - d\tilde{\Gamma}_{(0)}/dE_e}{d\tilde{\Gamma}_{(0)}/dE_e},$$

where $\tilde{\Gamma}$ is the theoretical rate convoluted with the experimental resolution, take to be a Gaussian of width $\sigma = 100$ meV:

$$\frac{d\tilde{\Gamma}}{dE_e} = \int_{-\infty}^{\infty} dE' \frac{e^{-\left(E_e-E'(2\sigma^2)\right)}}{\sqrt{2\pi}\sigma} \frac{d\Gamma}{dE'}.$$  
(14)

Importantly, since in a realistic analysis one would also fit the position of the end-point [17], we use the expression for the exact relativistic $E_{e,(0)}^{\text{max}}$, Eq. (7), in both the leading and next-to-leading order rates.

Considering (conservatively) events in the range $\Delta \in [-2, 0]$ eV, Eq. (10) returns an expectation of $N \sim 10^{12}$ events/year. Assuming a Poisson distribution, this corresponds to a relative statistical uncertainty of order $10^{-6}$ after a year of data taking. As one can see from Figure 1, for sufficiently heavy neutrinos, this is considerably smaller than the relative difference introduced by the next-to-leading order correction to the rate reported in Eq. (11). The shape of the latter, in fact, varies by a fraction of the order of $10^{-4} \div 10^{-5}$ over a large range of energies. Such corrections must then be included in order to avoid theoretical biases on the parameter estimation.

**V. CONCLUSION**

In the coming years a host of existing and future experiments should place increasingly more stringent bounds on the neutrino mass. Among these, Ptolemy might reach unprecedented rates, and correspondingly low statistical uncertainties. This improvement on the experimental side demands precise theoretical prediction.
In this note we discussed the corrections due to relativistic effects in the nuclear decay of tritium. Specifically, by systematically keeping track of the approximations made, we revisit some results available in the literature, and present a revised compendium of what we identify as the most relevant corrections to the $\beta$-decay spectrum for the experimental setup as envisioned by Ptolemy. We show that, given the expected number of events, these correction are important to avoid theory biases.

This is a first step towards the development of theory predictions which are accurate enough to face the forthcoming experimental challenges. In particular, we note that experiments like the ones mentioned above use (or plan to use) tritium bound either in a molecule or to a solid state substrate. The presence of a spectator system comes with a number of difficulties and further correction, which have been discussed in [28–35]. Consequently, several more effects will then need to be taken into account to achieve the required precision as, for example, atomic, molecular and solid state ones.

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Appendix A: Full expression for the $W$ factors

We report the full expression for the factors $W$ entering the rate in Eq. (5). When approximating the rate following the scheme described in Section III, most of them will not contribute to the lowest orders, considerably simplifying the result. We also notice that, if a precision larger than $\mathcal{O}(10^{-7})$ is required, it is also necessary to re-compute the $W$'s including the momentum dependence of the form factors in Eq. (2). We first define,

$$A_1 \equiv m_\nu \frac{M_f + m_\nu}{M_i} - \Delta, \quad A_2 \equiv M_f \frac{M_f + m_\nu}{M_i} - \Delta,$$

$$A_3 \equiv m_\nu \frac{M_f}{M_i} - \Delta, \quad B_1 \equiv \frac{M_i E_e - m^2_e}{M^2_{12}},$$

$$B_2 \equiv \frac{M_i (M_i - E_e)}{M^2_{12}}, \quad C \equiv \Delta \left( \Delta - 2m_\nu \frac{M_f}{M_i} \right),$$

$$D \equiv \frac{M^2_i (E^2_e - m^2_e)}{3M^4_{12}}.$$ (A1)

The $W$'s are then given by

\begin{align*}
W_{VV} &= A_1 B_1 A_2 B_2 - M_f A_1 B_1 + E_e A_3 - CD, \quad \text{(A2a)}
W_{AA} &= A_1 B_1 A_2 B_2 + M_f A_1 B_1 + E_e A_3 - CD, \quad \text{(A2b)}
W_{AV} &= 2 (A_1 B_1 A_2 B_2 - E_e A_3 - CD), \quad \text{(A2c)}
W_{AP} &= 2 \left[ (m^2_e + m^2_\nu) M_i^2 (A_1 B_1 A_2 B_2 - CD) + 2m^2_e m^2_\nu M_i A_2 B_2 - (m^2_e + m^2_\nu) M_i^2 M_f A_1 B_1 \right.
\quad - 2m^2_e m^2_\nu M_i M_f \right]/m^2_\nu, \quad \text{(A2d)}
W_{VM} &= \frac{M_f + M_i}{Mi} (CD - A_1 B_1 A_2 B_2) + \frac{M_f (M_i + M_f)}{2Mi} A_1 B_1 - \frac{M_f + M_i}{2Mi} E_e A_3 + \frac{M_f}{Mi} E_e A_1 B_2 \quad \text{(A2e)}
\quad + \frac{M_f}{M^2_{12}} (A_1 A_2 + C) A_2 B_1,
W_{AM} &= (M_f + M_i) (CD - A_1 B_1 A_2 B_2 + E_e A_3)/M_i, \quad \text{(A2f)}
W_{MM} &= -m^2_\nu m^2_\nu 4M^3_i - (m^2_e + m^2_\nu) 3M^2 f (A_1 B_1) - \frac{m^2_\nu m^2_\nu}{4M^2 i} A_2 B_2 - \frac{m^2_e + m^2_\nu}{4M^2 i} (A_1 B_1 A_2 B_2 - CD) + \frac{m^2_e}{6Mi} E_e \frac{C}{M^2_{12}} \quad \text{(A2g)}
\quad - \frac{M_f}{2Mi} \left[ \frac{m^2_\nu}{3} + \left( \frac{m^2_\nu}{2} + \frac{M^2 f A^2 i}{M^2_{12}} \right) + \left( \frac{4M^2 f A^2 i}{M^2_{12}} - m^2_\nu \right) \frac{(M_i E_e - m^2_e)^2}{3M^2_{12}} \right] + \frac{m^2_\nu}{2Mi} E_e A_1 B_1 \quad \text{(A2h)}
\quad + \frac{1}{4Mi} E_e \left[ \frac{3A_1 A_2}{4} + C \right] \frac{(M_i E_e - m^2_e)^2}{M^2_{12}} + \frac{m^2_e}{2} (A_1 A_2 + C) A_1 M_i - E_e \quad \text{(A2i)}
\quad + \frac{1}{4Mi} (A_1 A_2 + C) \left[ \frac{M_i E_e}{3M^2_{12}} \left( \frac{m^2_\nu - M^2 f A^2 i}{M^2_{12}} \right) + \left( \frac{m^2_\nu}{3} + \frac{M^2 f A^2 i}{M^2_{12}} \right) M_i (M_i - E_e) \left( M_i E_e - m^2_e \right) \right].
\end{align*}
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