Modeling of GeV-TeV gamma-ray emission of Cygnus Cocoon

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Abstract

OB-associations and superbubbles being energetically essential galactic powerhouses are likely to be the important acceleration sites of galactic cosmic rays (CRs). The emission profile of γ-ray sources related to superbubbles and stellar clusters indicates on continuous particle acceleration by winds of massive stars. One of the most luminous galactic γ-ray sources is Cygnus Cocoon superbubble, observed by multiple instruments, such as Fermi-LAT, ARGO, and, recently, HAWC. We discuss a model of particle acceleration and transport in a superbubble to explain GeV-TeV γ-ray spectrum of Cygnus Cocoon, which has a break at the energy of about 1 TeV. It is shown that the γ rays produced by hadronic interactions of high-energy protons accelerated by an ensemble of shocks from winds of massive stars and supernovae in the Cygnus Cocoon can explain the observations. The proton spectral shape at the highest energies depends on the MHD-fluctuation spectrum in the Cocoon. The viable solutions for Cygnus Cocoon may be applied to some other associations showing similar behaviour. We briefly discuss the similarity and differences of particle acceleration processes in extended superbubbles and compact clusters of young massive stars as represented by Westerlund 1 and 2 γ-ray sources.

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1. Introduction

The galactic star-forming regions are considered to be the likely sources of high-energy cosmic rays up to PeV, as suggested by Cesarsky & Montmerle (1983), Bykov (2014), Lin-genfelter (2018), Tolksdorf et al. (2019), Bykov et al. (2020), Morlino et al. (2021). Studies of the CR composition show that the volatile elements in CR material are likely accelerated from a plasma of temperature above ∼ 2 MK, which is typical for superbubbles produced by massive star winds and supernova explosions (see Tatischeff et al. 2021 for a recent discussion). The observed overabundance of 22Ne in galactic CRs can be understood with the models of CR acceleration in the powerful winds of Wolf-Rayet type stars in young massive star clusters (Binns et al. 2008; Kalyashova et al., 2019; Gupta et al., 2020).

Whether the massive stars associations and clusters can indeed efficiently accelerate CRs may be verified by modeling the spatial and spectral features of γ-ray emission from galactic OB-associations and compact clusters. Aharonian et al. (2019) using Fermi-LAT and H.E.S.S. data for OB-association Cygnus OB2 and compact clusters Westerlund 1 and Westerlund 2, examined the spatial distribution of γ-ray emission from these objects and found that the CR density behaves like r\textsuperscript{-1}. They concluded that relativistic particles are continuously injected into the interstellar medium, which may be an argument for combined action of multiple stellar winds. This supports the idea that massive stellar clusters and OB-associations are likely galactic CR accelerators. Both superbubbles and massive star clusters are likely dominating sources of very-high-energy γ rays from the starburst galaxies (see e.g. Ohm 2016).

Ackermann et al. (2011) presented Fermi-LAT GeV observations of the extended γ-ray source, associated with the superbubble surrounding the Cygnus OB2 region, known as Cygnus Cocoon. They pointed out that the Cocoon must be the site of
the active particle acceleration. Cygnus Cocoon is located at \( \sim 1.4 \) kpc from Solar system, with a size of \( \sim 55 \) pc and a total mechanical power exceeding \( 10^{38} \) erg/s (Aharonian et al. 2019; Ackermann et al. 2011; Hanson 2003). According to Fermi-LAT analysis, at GeV energies spectral energy distribution has a power-law form with a spectral index of -2.1. Studies of this region at TeV energies started with HEGRA (Aharonian et al. 2002) and continued with Milagro (Abdo et al., 2007a b, 2012), ARGO (Bartoli et al. 2014), VERITAS (Abeysekara et al. 2018) and HAWC (Hona et al., 2019, Abeysekara et al., 2021) observations, which showed that TeV spectrum of the Cocoon softens, changing its index from (-2.1) to (-2.6) at the transition energy \( \sim 1 \) TeV. It is worth mentioning that a few compact clusters (e.g. Westerlund 2, W40) also show the hard-soft behaviour of their γ-ray spectra (Yang et al. 2018; Sun et al., 2020).

Particle acceleration by shock waves and colliding flows in OB-associations was discussed in Bykov & Toptygin (2001); Bykov (2001); Parizot et al. (2004); Ferrand & Marcowith (2010); Bykov (2014); Vieu et al. (2020). We present here a possible explanation for a piecewise behaviour of the Cygnus Cocoon GeV-TeV spectrum through the modeling of particle acceleration in superbubbles based on the approach of Bykov & Toptygin (2001) and Bykov (2001). Assuming the hadronic origin of γ-rays we find the parameters of superbubble needed to provide the break in spectrum and estimate the efficiency of acceleration.

2. Proton acceleration model

The energy release of massive star clusters allows particle acceleration up to the hundreds of TeV. This can be estimated from the equation connecting the maximum energies of protons, accelerated in the source with the outflows with frozen-in magnetic fields and the mechanical luminosity of the source (see e.g. Lemoine & Waxman, 2009):

\[
E_{\text{max}} \approx 6 \times 10^{14} \frac{\beta_i^{1/2}}{\Gamma_i} \left( \frac{L_M}{10^{35} \text{erg s}^{-1}} \right)^{1/2} \text{eV},
\]

where \( \beta_i = u_i/c \) is the dimensionless velocity of the flow, \( c \) is the speed of light, \( \Gamma_i = 1/\sqrt{1-\beta_i^2} \). The magnetic luminosity \( L_M \) is about a few percent of the mechanical power. For example, the power of the compact cluster Westerlund 1 is \( \gtrsim 10^{39} \text{erg s}^{-1} \) (see e.g. Muno et al., 2006). Cygnus OB2 contains about 120 O stars (Knödlseder 2000) and it has the mechanical luminosity well above \( 10^{38} \) erg s\(^{-1} \) (see e.g. Aharonian et al., 2019).

Most of the star formation activity in Cygnus OB2 occurred between 1 and 7 Myr ago (Wright et al. 2015). Since that time the broad spectrum of magnetohydrodynamic (MHD) fluctuations has formed due to the presence of strong shocks from stellar winds and large-scale flows. Due to multiple strong shocks, particle propagation in superbubble can be highly intermittent. The particle acceleration processes in intermittent systems can be treated by using approximate kinetic equations, averaged over characteristic scales of the system. One characteristic scale is determined by the size of gradient of particle distribution function near a shock wave, while another scale is the energy containing scale of plasma motions, induced by fast winds and strong shocks. We use a parameterization for the CR mean free path of particles due to scattering by the quasi-resonant magnetic fluctuations of scales comparable to the CR gyroradius \( R_p(p) \) with the CR energy dependence as given by e.g. Toptygin (1985) and Strong et al. (2007):

\[
\Lambda(p) \approx l_{\text{corr}} \cdot \left( \frac{R_p(p)}{l_{\text{corr}}} \right)^{2-v}
\]

Here, \( v \) is the MHD turbulence power-law index. The parameter \( l_{\text{corr}} \), defined so, is of the order of mean distances between strong shocks.

There are two different regimes of CR particle transport in our model depending on the relation of characteristic size of gradient of particle distribution function near a shock wave \( l \approx \kappa(p)/u \) and the effective correlation length \( l_{\text{corr}} \). We introduce the momentum \( p_* \) where transition occurs: \( v \Lambda(p_*) \approx 3u l_{\text{corr}} \), where \( v \) is the particle velocity, and \( u \) is the amplitude of the bulk plasma speed.

There is a strong dependence of the transition momentum \( p_* \) on the amplitude of the large-scale turbulent velocity \( u \) as \( p_* \propto u^{1/(2-v)} \) for \( v > 3/2 \). From the expressions above for \( p_* \) one can get the following estimation for the CR proton energy \( \epsilon_* \) at the regime transition point for \( v = 1.7 \)

\[
\epsilon_* \approx 20 \text{ GeV} \left[ \frac{B}{10 \mu \text{G}} \right] \left[ \frac{l_{\text{corr}}}{10 \text{ pc}} \right] \left[ \frac{u}{1,000 \text{ km s}^{-1}} \right]^{3.33}
\]

2.1. Spectrum of low-energy protons

For low enough energies when \( p < p_* \), the CR transport is determined mainly by turbulent advection, particles are tightly connected with plasma motions. This regime was studied in non-linear model of Bykov (2001), where the simplified description of long-wavelength turbulence was applied. The kinetic equation, applicable in intermittent systems with multiple shocks and long-wavelength turbulent motions, was developed by Bykov (2001) and has the form:

\[
\frac{\partial N}{\partial t} - \frac{\partial}{\partial r_i} \chi_{ij} \frac{\partial N}{\partial r_j} = G N + \\
+ \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D \frac{\partial N}{\partial p} + A \hat{L}^2 N + 2B \hat{L} \hat{P} N + Q(p).
\]

Here, \( N(r, p, t) \) is the CR distribution function, averaged over the ensemble of fluctuations of electric and magnetic fields generated by turbulent plasma motions. The source term \( Q(p) \) is determined by injection. The operators \( \hat{L} \) and \( \hat{P} \) are given by

\[
\hat{L} = \frac{1}{3p^2} \frac{\partial}{\partial p} p^2 \int_0^p \int_0^{p'} \frac{\partial^2}{\partial p'' \partial p'} \frac{\partial^2}{\partial p'' \partial p'}
\]

Turbulence and shock waves are characterised by the kinetic coefficients \( A, B, D, G \) and diffusion tensor \( \chi_{ij} \), which describe
correlations between shocks and long-wavelength plasma motions. The index $\gamma$ is determined by the shock ensemble properties (Bykov & Toptygin 1993). For the detailed description of Eq. (4) and definition of these coefficients, see Bykov (2014).

We assume the broad spectral range of magnetic fluctuations $dB_i^2/dk \propto k^{-\nu}$ where the turbulence power-law index $1 \leq \nu \leq 2$.

For the spectrum calculation the simplified form of Eq. (4) was used, where we neglected the cross-correlations coefficients $A$ and $B$. It was shown that the efficient conversion of turbulence energy to low-energy particles takes place. When $p < p_*$, but injected CRs are relativistic, which is the case for Cygnus Cocoon, proton spectrum demonstrates soft-hard-soft behavior with time, and arrives to asymptotic power-law momentum distribution with the index about $2$: $p^2N(p) \sim p^{-2}$.

In this context it is important to point out that contrary to the case of Cygnus Cocoon the recent thorough analysis of the multi-wavelength observations by Joubaud et al. (2020) did not reveal any significant departures of the measured $\gamma$-ray emissivity spectrum of the nearby Orion-Eridanus superbubble from the average spectrum measured in the solar neighbourhood. The apparent lack of any $\gamma$-ray emission excess could be understood from Eq. (5) where for the long-wavelength turbulent velocity amplitude below 1,000 $\text{km s}^{-1}$ (as it is expected in the Orion-Eridanus superbubble) $\epsilon_\nu$ can be well below GeV and the hard CR spectrum may not extend to the proton energies contributing to the photons detected by Fermi-LAT. It may indicate that small-scale MHD turbulence filled in the Orion-Eridanus superbubble has a spectral index $\nu \geq 5/3$.

2.2. Spectra of high-energy protons

At high energies ($p > p_*$) the characteristic diffusion scales of CRs are larger than $l_{\text{corr}}$, which means that particle interacts with several fronts on its free path. The particle propagation is described by the Fokker-Planck type equation (cf. Lemoine, 2019), which differs from the Eq. (4) as only the diffusive terms are left. Assuming spherical form of the superbubble:

$$\frac{\partial N}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial N}{\partial r} = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D(p) \frac{\partial N}{\partial p},$$  

Here, the diffusion in momentum space and spatial diffusion are only left. The spatial diffusion coefficient $\kappa(p) = v \Lambda(p)/3$ and diffusion coefficient in momentum space $D(p) = p^2/\tau_{\text{acc}} = p^2 u^2/9 \epsilon_\nu(p)$.

The steady-state solution of Eq. (6) (i.e. $\partial N/\partial t = 0$) depends on momentum like

$$N(p) = A_0 (p/p_*)^{-(\nu+1)/2} K_\nu((p/p_*)^{2-\nu} \Delta),$$  

where $K_\nu$ is the modified Bessel (Macdonald) function with the index $a = (\nu + 1)/[4 - 2\nu]$ (for $\nu \neq 2$). Here

$$\Delta = \frac{\pi}{2} \frac{c \Lambda(p_*)}{(2 - \nu) \mu u R}$$  

where $R$ is the radius of the system. The detailed derivation of this solution can be found in Appendix A.

The resulting proton spectrum in a superbubble is shown in Fig. 1. While matching the solutions for low-energy and high-energy regimes, one should have in mind that the transition region around $p_*$ has some uncertainty as the simplified description of the turbulence and CR propagation is used and the solutions are asymptotic.

The CR propagation both in the Galaxy and acceleration sources is governing by their scattering by MHD turbulence (e.g. Toptygin, 1985; Ptuskin 2011; Ptuskin et al. 2006; Aharonian et al., 2012; Malkov et al. 2013; Malkov, 2017; Moskalenko et al. 2019). In this study we use simplified models of the diffusive propagation in superbubbles, while the possible non-diffusive regimes are to be studied in a separate paper. The Kolmogorov-type turbulence model of the spectral index $\nu = 5/3$ is widely used in the global models of CR propagation while the Kraichnan-type ($\nu = 3/2$) turbulent model is discussed in the context of the low-energy CR reacceleration (Strong et al. 2007). Strong shocks produced by stellar winds and supernovae could be the dominant local source of the MHD turbulence at some evolution stages of superbubbles and compact stellar clusters. It was shown by Bykov & Toptygin (1987); Norman & Ferrara (1996) that multiple weak secondary shocks can be produced by interactions of the strong primary shocks provided by the sources of mechanic power with clouds, stellar winds or other types of strong density irregularities. The presence of multiple propagating weak shocks can provide the energy independent CR diffusion coefficient (see Bykov & Toptygin, 1987) in a rather wide energy range extended to the PeV regime which corresponds to $\nu = 2$. It is important to point out that such a mechanism gives the power-law proton spectrum as shown in Bykov & Toptygin (2001) and, therefore, power-law high-energy $\gamma$-ray spectrum as seen by HAWC. For the parameters of the Cygnus Cocoon, to provide the needed power-law index ($\sim 2.6$), the diffusion coefficient should be as small as $\approx 3 \times 10^{28} \text{cm}^2 \text{s}^{-1}$ in the PeV regime. The corresponding transition energy should be about 3.4 TeV. The alternative high-energy proton spectrum based on these assumptions is shown in Fig. 1.
The hadronic mechanism of γ-ray emission implies proton-proton interactions leading to η⁰ production and subsequent pion-decay radiation. We calculate the emission due to hadronic interactions using parameterizations of Kelner et al. (2006). For p-p total inelastic cross section we use the most recent parameterization by Kafexhiu et al. (2014). Note that Kelner’s approach is applicable for proton energies higher than 0.1 TeV, and photon energies higher than 0.1 GeV, which is sufficient for our modeling of GeV-TeV Cocoon spectrum.

The main parameters of our modeling are correlation length \( l_{\text{corr}} \), magnetic field \( B_0 \), and turbulent spectrum index \( \nu \). The other parameters are fixed: association size \( R = 55 \) pc (Ackermann et al., 2011 Aharonian et al. 2019) and the estimate of mean plasma flow velocity \( u = 1,500 \) km s\(^{-1}\) based on the measured values of the O- and B-stars wind velocity of 1,000-3,000 km s\(^{-1}\) (Seo et al. 2018) and estimated supernova remnant shocks velocity which in very rarefied plasma may be \( \sim 1,500 \) km s\(^{-1}\) for distances \( \geq 30 \) pc (see e.g. McKee & Ostriker 1977). While observing SNRs inside superbubbles is not an easy task, the shock velocity estimation of recently discovered SNR G116.6-26.1 in the hot rarefied halo of the Milky Way is consistent with the values discussed above (Churazov et al. 2021).

To fit our model spectrum (Eq. [7]) to the Fermi-LAT (Ackermann et al., 2011), ARGO (Bartoli et al., 2014) and HAWC (Abeysekara et al., 2021) GeV-TeV data and find corresponding parameters, we use Markov chain Monte Carlo (MCMC) methods based on the Python’s emcee package (Foreman-Mackey et al. 2013). Modeling shows that in the range of appropriate values of the mean magnetic field (5-30 \( \mu \)G), varying the magnetic field doesn’t affect much the resulting spectrum, so to prevent extra uncertainty we exclude it from the modeling and fix its value to 15 \( \mu \)G. For the other two parameters we find the following values with the errors at the 3-sigma confidence level:

\[
\nu = 1.61^{+0.02}_{-0.02}
\]

The fitted \( \nu \) value is between the indices for Kolmogorov (5/3) and Kraichnan (3/2) turbulence spectra. Note that for the higher plasma flow velocity \( u = 3,000 \) km s\(^{-1}\), we would get the value of \( \nu = 1.68 \) very close to the Kolmogorov spectrum index. The \( l_{\text{corr}} \), which is in fact the statistical characteristic separation between strong shocks, also has an appropriate value for the \( \sim 55 \) pc radius of superbubble. For a distance to the source of 1.4 kpc and gas density of 30 cm\(^{-3}\) (Abyesekara et al., 2021) we obtain the needed proton injection luminosity of the source to be \( \sim 1.2 \times 10^{37} \) erg s\(^{-1}\), so the acceleration efficiency is about 0.06 for the kinetic luminosity of \( 2 \times 10^{38} \) erg s\(^{-1}\) estimated in Ackermann et al. (2011).

3. Gamma-ray spectrum

The resulting \( \gamma \)-ray spectrum with the obtained parameters is shown in Fig. 3. A note of caution is in order here. The transition energy \( \epsilon_\star \) which matches the two CR transport regimes can not be exactly derived within the model and can only be estimated within a factor of few accuracy. Therefore the fitting parameters derived with the MCMC serve just to illustrate that the theory could reproduce the observed spectrum of the Cygnus Cocoon.

One can see that the hardening of the observed spectrum at the energies higher than 100 TeV is slightly different from our model spectrum. Abeysekara et al. (2021) claimed that pulsar wind nebulae (PWNe) powered by known sources (PSR J2021+4026, PSR J2032+4127) do not explain the extended Cocoon emission. A possible time variable behaviour of PSR J2032+4127 in very high energy regime (Bykov et al., 2021) could play a role however as well as a yet undiscovered PWN cannot be excluded as a source of the emission. Recently, HAWC Collaboration investigated similarities in hardening behaviour in several sources, including Cygnus (Abeysekara et al., 2021). In this respect, we constructed also the \( \gamma \)-ray spectrum for an alternative model of CRs propagation in superbubble.
where the MHD turbulence is dominated by the ensemble of weak shocks (as discussed in Section 2.2) leading to the power-law proton spectrum shown in Fig. 2. The corresponding spectrum of hadronic \( \gamma \)-ray emission of the system is shown in Fig. 4. It can successfully match the observed \( \gamma \)-ray spectrum of the Cygnus Cocoon. We also looked at the recent LHAASO data from Cygnus region with an approximate flux at \( \sim 100 \) TeV provided by (Cao et al., 2021c) to see if it matches the Cocoon data and our model. One can see that the only point available so far speaks in favour of our first model shown in Fig. 2. However, the authors cannot claim yet that the source of the \( \gamma \)-ray emission seen by LHAASO is Cygnus Cocoon. The thorough LHAASO spectral and morphological study of that source is expected in the near future.

Apart from the extended Cygnus Cocoon there might be some other particle acceleration sites in the Cygnus region producing PeV-energy protons and neutrinos like that discovered by Dzhappuev et al. (2021) which is likely associated with a flaring \( \gamma \)-ray binary (Bykov et al., 2021). Recently, Liu & Wang (2021) suggested that a significant fraction of sub-PeV gamma rays detected by the Tibet AS+MD array in the galactic disk in the regions \( 25^\circ < l < 100^\circ \) and \( 50^\circ < l < 200^\circ \) may originate from the source related to Cygnus region. The mentioned above very-high-energy emission detected by LHAASO may also be either of the Cocoon origin, or some other source in the region. Spectral energy distribution in TeV-PeV region from LHAASO is expected to provide the crucial information about spectral hardening and the need for additional hard-spectrum sources to explain the extended \( \gamma \)-ray emission in Cygnus region. In the case of the Cygnus Cocoon superbubble one cannot rule out however that some other source(s) with hard spectrum could contribute at the highest energies and account for the last two HAWC data points given in Abeysekara et al. (2021).

Discoveries of a few interesting ultrahigh-energy \( \gamma \)-ray sources in some other regions in the Milky Way were reported recently by LHAASO and HAWC. An unidentified extended \( \gamma \)-ray source LHAASO J0341+5258 located in the Galactic plane has a spectrum fitted with the power-law model of a photon index \( 2.98 \pm 0.19 \) in 10 - 200 TeV energy range (Cao et al., 2021a). However, the Fermi-LAT flux upper limit at 10 GeV would require much harder spectrum at lower energies. LHAASO J2108+5157 was significantly detected above 100 TeV with a power-law photon index of \( 2.83 \pm 0.18 \) between 20 and 200 TeV while again a harder spectrum is expected at lower energies to be consistent with the existed Fermi-LAT flux upper limits. The spatial extent of LHAASO J2108+5157 is not yet determined (Cao et al., 2021b). The source HAWC J1825-134 which is likely coincident with a giant molecular cloud has a \( \gamma \)-ray power-law spectrum of rather a hard index \( 2.28 \pm 0.12 \) which extends beyond 200 TeV without an apparent cut-off (Albert et al., 2021). Dedicated multi-wavelength studies of these sources are needed to understand whether the model described above is relevant to describe these \( \gamma \)-ray sources.

4. Conclusions

We discuss here a possible explanation for the piecewise behaviour of the Cygnus Cocoon GeV-TeV spectrum with the modeling of particle acceleration and propagation in a superbubble with multiple shocks of different strengths produced by powerful winds of massive stars and supernovae. The softening in the superbubble spectrum at TeV energies in the model indicates a change in the regime of CR propagation inside the accelerator.

We find fits for the model parameters and show that they have realistic values. The most sensitive parameter in the model is the shape of the spectrum of magnetic fluctuations. If the spectrum of MHD turbulence between the shocks can be approximated by a power-law of index \( \nu \) in the broad range of scales resonant to accelerated CRs then the most favorable values of the index are close to \( \nu \approx 5/3 \). We find a value for the Cygnus Cocoon \( \nu = 1.61 \).

In an alternative model the high-energy CR transport is governed by proton scatterings by an ensemble of multiple secondary weak shocks. The interaction of strong shocks produced by energetic outflows with density inhomogeneities is a source of multiple weak secondary shocks propagating through superbubble. The intermittent distribution of the multiple weak shocks results in energy independent CR diffusion coefficient. This CR propagation model as well provides the good fit to the \( \gamma \)-ray observations of the Cygnus Cocoon while the spectral shapes of \( \gamma \)-ray emission are somewhat different.

The approach presented in this paper may be applied to other GeV-TeV sources related to associations of young massive stars, as some of compact clusters show similar behaviour in their \( \gamma \)-ray spectra (Yang et al., 2018; Sun et al., 2020). In Appendix B we show that the spectrum of the compact star cluster Westerlund 2 can be fitted successfully with our model. However, in some clusters we do not see the significant spectrum softening, e.g., detailed analysis of very-high-energy \( \gamma \)-ray observations of Westerlund 1 by Aharonian et al. (2019) revealed a hard \( \gamma \)-ray spectrum with a photon index \( \sim 2.3 \) up to
~ 400 TeV. Particle acceleration both in the extended superbubbles and the compact clusters of young massive stars is due to Fermi type mechanism on supersonic MHD flows with multiple shock waves. These type of sources may also have a comparable mechanical power. Therefore, the differences in their observed γ-ray spectra are likely determined by the differences in CR transport within the sources. The large energy density of accelerated CRs in the compact clusters with supernova remnants may result in the dominance of CR driven turbulence providing the Bohm diffusion inside the acceleration region. This may be important for Westerlund 1, explaining the proton spectrum extending up to PeV energies as follows from Aharonian et al. (2019) observations. Also, the apparent presence of a magnetar in Westerlund 1 indicates its likely interaction with a supernova remnant in the past.

We would like to stress the importance of precise measurements of the γ-ray spectra in the 100 TeV domain which is crucial to distinguish between the CR propagation regimes and therefore between the MHD turbulence models in superbubbles discussed above.

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Appendix A. The high-energy proton distribution function

\[ \frac{dN}{dr} - \frac{1}{r^2} \frac{d}{dr} r^2 k(p) \frac{dN}{dr} = \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D(p) \frac{dN}{dp}, \]  
(A.1)

The steady state equation is:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} r^2 k(p) \frac{dN}{dr} + \frac{1}{p^2} \frac{\partial}{\partial p} p^2 D(p) \frac{dN}{dp} = 0 \]  
(A.2)

The diffusion coefficient in momentum space \( D(p) = p^2 / \tau_{acc} = p^2 u^2 / 9k(p) \).

\[ \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial N}{\partial r} + \frac{1}{k(p)} \frac{\partial}{\partial p} p^4 \frac{u^2}{9k(p)} \frac{\partial N}{\partial p} = 0 \]  
(A.3)

The spatial diffusion coefficient \( k(p) = v \Lambda(p)/3 \), therefore

\[ k(p) = \frac{v}{3} \cdot l_{corr} \cdot \left[ \frac{R_{\gamma}(p)}{l_{corr}} \right]^{2-v}. \]

We are looking for the high-energy solution at \( p > p_{**} \) and in our case these energies are significantly higher than GeV, therefore \( v \approx c \). Let us introduce the new variable \( \eta = p / p_{**} \), then \( k(\eta) = k_0 \eta^{2-v} \), where \( k_0 = (c/3) \cdot l_{corr} \cdot (cp_{**} / eB_{corr})^{2-v} \). For \( \eta \) the equation takes the form:

\[ \frac{1}{r^2} \frac{\partial}{\partial r} r^2 \frac{\partial N}{\partial r} + \frac{1}{k(\eta)} \frac{\partial}{\partial \eta} \eta^{4} \frac{u^2}{9k(\eta)} \frac{\partial N}{\partial \eta} = 0 \]  
(A.4)

We separate the variables in the following way:

\[ N(r, \eta) = f(\eta) \chi(r)/r \]

For \( \chi(r) \) we get:

\[ \frac{1}{r} \frac{d}{dr} \left( r^2 \frac{d}{dr} \left( \frac{\chi}{r} \right) \right) = -\lambda \chi, \]  
(A.5)

and it has the solution

\[ \chi = \chi_0 \sin(\sqrt{\lambda} r) \]  
(A.6)

where the possible values of \( \lambda \) depend on the boundary conditions at the \( r = R \) where \( R \) is the radius of the system. The main difference between the internal and external regions is assumed to be the absence of significant velocity fluctuations in the external region, i.e. \( u = 0 \), while the spatial diffusion coefficient is close to the internal. Then, after joining the distribution functions and diffusive flows at \( r = R \) we obtain

\[ \cos(\sqrt{\lambda} R) = 0 \quad \rightarrow \quad \lambda_n = \left( \frac{\pi}{2R} (2n + 1) \right)^2, \quad n = 0, 1, ... \]  
(A.7)

As it will be seen below, increasing \( n \) leads to the fast fall of the momentum distribution function \( f(\eta) \) so we take only one solution with \( n = 0 \). The equation for \( f(\eta) \) has the form

\[ \frac{1}{k_0 \eta^{2-v}} \cdot \frac{1}{\eta^2} \frac{\partial}{\partial \eta} \eta^4 \frac{u^2}{9k(\eta)} \frac{\partial f}{\partial \eta} = \lambda f \]  
(A.8)

We search \( f(\eta) \) as

\[ f(\eta) = \eta^{-(v+1)/2} \Phi(\eta), \]

which leads us to the modified Bessel equation for \( \Phi(\eta) \). Throwing away the exponentially growing with energy solution, we obtain:

\[ f(\eta) = A_0 \eta^{-(v+1)/2} K_a \left( \frac{\eta^{2-v} 3k_0 \sqrt{\lambda_0}}{2 - v} \right), \]  
(A.9)

where \( K_a \) is the Macdonald function with the index \( a = (v + 1)/(4 - 2v) \) (for \( v \neq 2 \)). With the values of \( k_0 \) and \( \lambda_0 \) defined above we get the final particle distribution function in the momentum space (Eq. [?]).

Appendix B. The case of Westerlund 2 young massive star cluster

In order to test our model on a system where the MHD energy source is likely dominated by powerful stellar winds, we apply the approach described in the paper to the compact cluster of young massive stars Westerlund 2. The source shows the similar to the Cygnus Cocoon hard-soft behaviour of its γ-ray spectrum (Yang et al. 2018, Mestre et al. 2021) while no apparent supernova remnants neither extended nor compact were reported so far in Westerlund 2. We use the observational data by Fermi in the GeV region (Ackermann et al. 2017) and H.E.S.S. in the TeV region (H. E. S. S. Collaboration et al. 2011). For
high energies we employ the solution obtained in Eq. (7), i.e. the model with particle diffusion between strong shocks. The main difference between Cygnus Cocoon and Westerlund 2 is the radius of the cluster. Although the majority of massive stars are expected to be concentrated in ≤ 1 pc circle (Vargas Álvarez et al. 2013), we take for our calculations the 2 pc radius as the region containing massive stellar winds. We also need to increase the mean magnetic field $B_0$ to the value ~ 50 mGkS, which can be achieved in compact clusters. As before, we take the mean plasma flow velocity as 1,500 km s$^{-1}$.

Within the same approach as before, we get the following results with the errors at the 3-sigma confidence level:

\[
\begin{align*}
\nu &= 1.53^{+0.07}_{-0.12} \\
\lambda_{\text{core}} &= 0.51^{+0.45}_{-0.11} \text{ pc}
\end{align*}
\]

We find that parameters' values are reasonable, although both model spectrum and the observed flux values have sizeable errors. Same as for the Cygnus Cocoon, for the higher mean plasma flow velocity (3,000 km s$^{-1}$) we would get turbulence index $\nu \sim 1.63$, which is closer to Kolmogorov index. To calculate the CR luminosity we use the distance to the source to 3-sigma error of the fitting parameters.

\[ L_{\text{CR}} = \frac{k}{e} \]

The typical value of $B_0$ is 50 mGkS and the cluster radius $R = 2$ pc. Blue region corresponds to 3-sigma error of the fitting parameters.

![Fig. B.5. The γ-ray spectral energy distribution for Westerlund 2 compact cluster for the parameters $\nu = 1.53$, $\lambda_{\text{core}} = 0.51$ pc. The model implies the magnetic field $B_0 = 50$ mGkS and the cluster radius $R = 2$ pc. Blue region corresponds to 3-sigma error of the fitting parameters.](image_url)

Obtain we that our model can successfully explain the Westerlund 2 observations, which implies that the suggested mechanism of particle acceleration and propagation can be realized in compact clusters as well as in loose OB-associations. The case of a very interesting source Westerlund 1 which is the most massive young stellar cluster in the Milky Way and, unlike Westerlund 2, has compact supernova remnants, is briefly discussed in Section 4.

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