Bloch-like oscillations induced by charge discreteness in quantum mesoscopic rings.

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We study the effect of charge discreteness in a quantum mesoscopic ring with inductance $L$. The ring is pierced by a time depending external magnetic field. When the external magnetic flux varies uniformly, the current induced in the ring oscillates with a frequency proportional to the charge discreteness and the flux variation. This phenomenon is very similar to the well known Bloch’s oscillation in crystals. The similitude is related to the charge discreteness in the charge-current representation, which plays the same role as the constant lattice in crystals.

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Recent advances in the development of mesoscopic physics, have allowed
an increasing degree of miniaturization and some parallel advances in nano-
electronics. On this respect, the quantization of mesoscopic electrical circuits
appears as a natural task to undertake. In this article we discuss the effects
of a time depending magnetic flux, $\phi_{\text{ext}}(t)$, acting on a mesoscopic ring (per-
fect conductor) with self-inductance $L$, producing in this way the equivalent
to a nondissipative circuit.

From the classical point of view, the motion equation for the current
can be obtained using energy balance for this nondissipative circuit. The
electrical power $P$ transferred to a mesoscopic ring, by an external magnetic
field $B_{\text{ext}}(t)$, is given by

$$P = I\varepsilon = -I \left( \frac{d\phi_{\text{ext}}}{dt} \right),$$

(1)

where $I$ is the induced current; but, in the slow time variation regime, this
power is used to overcome the electromotive force in the self-inductance $L$,
as the electric current $I$ is setting up, i.e.,

$$P = I\varepsilon = -I \left( L \frac{dI}{dt} \right).$$

(2)

In this way, from (1) and (2), we obtain the relationship:

$$L \frac{dI}{dt} = \left( \frac{d\phi_{\text{ext}}}{dt} \right).$$

(3)

Because of the similitude between electric circuits and particle dynam-
ics, the quantization of circuits seems straightforward [1]. Nevertheless, as
pointed out by Li and Chen [2], the charge discreteness must be considered
in the quantization process. Let $q_e$ be the elementary charge and consider
the charge operator $\hat{Q}$ as given by (spectral decomposition)

$$\hat{Q} = q_e \sum_n n \ket{n} \bra{n},$$

(4)

where $n$ is an integer. Following the references [2,3], and from the motion
equation (3), the Hamiltonian of the ring in the charge representation is given by
\[
\hat{H} = \frac{\hbar^2}{2q_e L} \sum_n \left\{ | n \rangle \langle n+1 | + | n+1 \rangle \langle n | - 2 | n \rangle \langle n | \right\}
\]

\[ - \frac{d\phi_{\text{ext}}}{dt} q_e \sum_n n \langle n | \langle n . \]

Moreover, the current operator \( \hat{I} = \frac{\hbar}{\sqrt{\hbar}} [\hat{H}, \hat{Q}] \) is given explicitly by
\[
\hat{I} = \frac{\hbar}{2i L q_e} \sum_n \left\{ | n \rangle \langle n+1 | - | n+1 \rangle \langle n | \right\} .
\]

The eigenstates and eigenvalues of the operator \( \hat{I} \) are easily found. In fact, the eigenstates are
\[
| I_k \rangle = \sum_n e^{ikn} | n \rangle ,
\]
where the quantum number \( k \) runs between 0 and \( 2\pi \). The current operator \( \hat{I} \) acting on the eigenstates (7) gives:
\[
\hat{I} | I_k \rangle = \frac{\hbar}{L q_e} \sin(k) | I_k \rangle ,
\]
that is, the eigenvalues \( I_k \) of the operator \( \hat{I} \) are
\[
I_k = \frac{\hbar}{L q_e} \sin(k) ,
\]
which are bounded since \( |I_k| \leq \hbar / L q_e \).

As it was said before, we will study a mesoscopic ring with self-inductance \( L \) which is pierced by a magnetic field \( B_{\text{ext}}(t) \) producing a time depending flux \( \phi_{\text{ext}}(t) \). In fact, we will show that, if we begin with one eigenstate of the current operator then, the dynamic evolution is related to a series of states with index \( k \). Explicitly, \( k(t) = \phi_{\text{ext}}(t) + k_0 \), and then, for a homogeneously increasing magnetic flux, an oscillating behavior of the current exists. The frequency of the oscillations depends on the external flux variation, and is given by \( \omega = \frac{q_e}{\hbar} \left( \frac{d\phi_{\text{ext}}}{dt} \right) \), which is a constant, if \( \left( \frac{d\phi_{\text{ext}}}{dt} \right) \) is a constant. We stress the similitude between this behavior and Bloch’s oscillations in crystals under an external dc field [4-7]. In our case, it is the charge discreteness which plays a role equivalent to the lattice constant.
In order to find the oscillations, we proceed as follows: let \( |k(t)\rangle\) be the state at time \( t \) which is assumed as an eigenstate of the current operator \( \hat{I} \). Let \( |k(t + \Delta t)\rangle\) be the state of the systems at time \( t + \Delta t \). To show that this state is also an eigenstate of the current operator, we use the first order evolution equation

\[
|k(t + \Delta t)\rangle = |k(t)\rangle + \frac{\Delta t}{i\hbar} \hat{H} |k(t)\rangle.
\] (10)

Using the commutator

\[
[\hat{I}, \hat{H}] = -\frac{\hbar}{2iL} \left( \frac{d\phi_{\text{ext}}}{dt} \right) \sum_n \{ |n\rangle \langle n + 1| + |n + 1\rangle \langle n| \},
\] (11)

and neglecting the second order terms \((\Delta t^2)\), we obtain

\[
\hat{I} |k(t + \Delta t)\rangle = \left( I_k + \frac{\Delta t}{L} \frac{d\phi_{\text{ext}}}{dt} \cos k \right) |k(t + \Delta t)\rangle.
\] (12)

That is, \( |k(t + \Delta t)\rangle\) is an eigenstate of the current operator with eigenvalue \( \left( I_k + \frac{\Delta t}{L} \frac{d\phi_{\text{ext}}}{dt} \cos k \right) \). So, if the state of the system is initially an eigenstate of the current operator, then it always evolves toward a state of the current with quantum number \( k(t) \). Clearly, from (12) and going to the limit \( \Delta t \to 0 \), we obtain the evolution equation for the quantum number \( k \) (acceleration theorem [8,9]):

\[
\frac{dk}{dt} = \frac{q_e}{\hbar} \frac{d\phi_{\text{ext}}}{dt}.
\] (13)

In this way, \( k \) has a linear behavior with respect to the external flux

\[
k(t) = \frac{q_e}{\hbar} \phi_{\text{ext}}(t) + k_o,
\] (14)

assuming that \( \phi_{\text{ext}}(0) = 0 \).

If we consider that the magnetic flux varies uniformly with time:

\[
\phi_{\text{ext}}(t) = \alpha t,
\] (15)

the quantum number \( k \) becomes uniformly accelerated and then, the current \( \langle k(t) | \hat{I} | k(t)\rangle \) oscillates with a frequency

\[
\omega = \frac{q_e}{\hbar} \alpha.
\] (16)
As it was said before, these oscillations in the current, and the charge, are equivalent to Bloch’s oscillations in crystals under an external dc electric field. This analogy is very much related to charge quantization (4), which plays a role similar to the constant lattice in a crystal. Finally we note that a Hamiltonian like that described by equation (5), under an external dc electric field has been extensively studied in solid state physics (tight-binding Hamiltonian). All eigenstates are factorial localized and the spectrum is discrete [10,11]. Also, we want to emphasize here that, as showed in [12], the discretization process related to a Hamiltonian like (5) is not univocal.

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