Description of nuclear systems within the relativistic Hartree-Fock method with zero range self-interactions of the scalar field

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Abstract

An exact method is suggested to treat the nonlinear self-interactions (NLSI) in the relativistic Hartree-Fock (RHF) approach for nuclear systems. We consider here the NLSI constructed from the relativistic scalar nucleon densities and including products of six and eight fermion fields. This type of NLSI corresponds to the zero range limit of the standard cubic and quartic self-interactions of the scalar field. The method to treat the NLSI uses the Fierz transformation, which enables one to express the exchange (Fock) components in terms of the direct (Hartree) ones. The method is applied to nuclear matter and finite nuclei. It is shown that, in the RHF formalism, the NLSI, which are explicitly isovector-independent, generate scalar, vector and tensor nucleon self-energies strongly density-dependent. This strong isovector structure of the self-energies is due to the exchange terms of the RHF method. Calculations are carried out with a parametrization containing five free parameters. The model allows a description of both types of systems compatible with experimental data.

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1 Introduction

The Relativistic Hartree-Fock (RHF) approach for finite nuclei has been developed in Refs. [1-8] (see also references therein) for the so-called linear models, which are characterized by linear field equations. In that approach, the contribution of the pion degrees of freedom (single pion exchange) is taken into account explicitly. In Ref. [2] it is shown, in particular, that the pseudovector coupling of pions to nucleons is more preferable than the pseudoscalar one in the nuclear structure context. In Ref. [9], certain important features concerning the spin-orbit interaction are directly related to the pion contribution. Thus, the incorporation of pions into the model represents one of the main advantages of the RHF method in comparison to the relativistic Hartree approach. However, in the papers mentioned above the nonlinear self-interactions (NLSI) of the mesonic fields have not been taken into account. Including nonlinear self-interaction terms corresponds to one of the possibilities to account for the three- and four-body forces in the nuclear structure calculations. Different types of self-interaction Lagrangians have been considered in literature up to now. Initially, the scalar field self-interactions have been introduced involving $\sigma^3$- and $\sigma^4$-terms, where $\sigma$ corresponds to the nuclear scalar field [11]. This type of self-interactions has been shown to play a very essential role in the relativistic Hartree calculations [12,13] to get, for example, the proper value of the compressibility modulus ($K$). One of the main problems of the RHF approach is that it brings about a high value of $K$. Thus, the inclusion of the NLSI terms in the RHF framework could solve this problem too. To work out the exact RHF equations with the NLSI is a complicated task not solved up to now. In Ref. [14], an approximate method to take into account self-interactions of the $\sigma^3$- and $\sigma^4$-type in the RHF procedure has been suggested (see Ref. [15] for other type of NLSI). This method involves a simple idea based on the inclusion of the nonlinear terms appearing in the equation for the $\sigma$ field, together with the scalar meson mass $m_\sigma$, into a scalar meson effective mass $m^*_\sigma$, which replaces $m_\sigma$ in the corresponding meson propagator in the nuclear medium. Let us mention also that in Ref. [10] the authors present a procedure which allows one to evaluate the contribution of the Fock terms in a truncation scheme, including self-interactions of the scalar meson field, and illustrate their method in the case of infinite nuclear matter.

In the present paper, we study the properties of a Lagrangian including, beside the exchange of $\sigma$, $\omega$, $\pi$ and $\rho$ mesons between nucleons, the NLSI of the scalar field in the zero range limit (ZRL), in the framework of the RHF approximation. The ZRL allows one to
express the exchange (Fock) terms for the NLSI explicitly via the direct (Hartree) terms in an exact way.

The paper is arranged as follows. In Sect. II, the general formalism is presented extensively: the Hartree-Fock contributions of the NLSI are calculated (in the ZRL without any approximation), both to the total energy of the system and to the nucleon self-energies, the expressions obtained being valid for nuclear matter (NM) and finite nuclei. For both cases, the results are shown in Sect. III. Finally, the conclusions are drawn in Sect. IV.

2 General formalism

A. The effective Lagrangian

The effective Lagrangian of the model considered in the present paper involves interactions of the nucleons via the exchange of mesons with the space-time transformation properties of the scalar \( \sigma \), vector (both isoscalar \( \omega \), and isovector \( \rho \)), and pseudoscalar \( \pi \) fields. It contains a "linear" part, which generates linear field equations identical to the corresponding linear part of Ref. [14], and it will not be reproduced here. Let us mention only that the pion field is coupled to nucleons through a pseudovector coupling [2]. The current Lagrangian involves, however, a self-interaction part that is somewhat different from its homologous part of Ref. [14]. It takes the form:

\[
U_{SI} = -\frac{1}{3} b \left( \frac{g_{\sigma}}{m_{\sigma}^2} \right)^3 (\bar{\psi}\psi)^3 + \frac{1}{4} c \left( \frac{g_{\sigma}}{m_{\sigma}^2} \right)^4 (\bar{\psi}\psi)^4, \tag{1}
\]

where \( \psi \) is the fermion field operator, \( b \) and \( c \) are the coupling constants of the nonlinear terms, \( g_{\sigma} \) and \( m_{\sigma} \) are, respectively, the scalar meson coupling constant and mass. We shall utilize also the dimensionless coupling constants \( \tilde{b} = \frac{b}{g_{\sigma}^2 M} \) and \( \tilde{c} = \frac{c}{g_{\sigma}^4} \), \( M \) being the nucleon mass.

Notice that the NLSI given in Eq. (1) coincide with the conventional scalar field NLSI used in Ref. [14] in the ZRL of the scalar field, i.e., when the term with \( m_{\sigma}^2 \) in the equation of motion of the scalar field dominates over the Laplacian and nonlinear terms. This is what happens for nuclear systems at small densities and with a smooth surface. Although, strictly speaking, real finite nuclei do not satisfy these conditions, for values of \( m_{\sigma} \gtrsim 500 \text{ MeV} \), we can consider that this approximation in effective Lagrangians is acceptable. The parameters \( b, c, \tilde{b} \) and \( \tilde{c} \) have the same units as in Ref. [14].

Let us mention also that the NLSI given by Eq. (1) have just the same structure as
the respective components of the Lagrangian appearing in the point coupling model and containing higher order terms in the fermion fields [17,18]. The inclusion of this kind of terms in the Lagrangian can be also justified by the necessity to introduce an extra density dependence in the Dirac-Brueckner-Hartree-Fock calculations to allow a simultaneous fit to the NN phase shifts and the nuclear matter equilibrium point [19].

B. The Hartree-Fock equations

To get the RHF equations corresponding to our effective Lagrangian, we closely follow the Refs. [3,14], restricting ourselves to the static approximation for the meson fields. The nucleon field \( \psi \) is expanded into a complete set of stationary single-particle spinors \( \{ \psi_\alpha(x)e^{-iE_\alpha t} \} \) and we consider the tree approximation.

The Dirac equation including exchange terms can be obtained by minimizing the total energy of the system, which is given by the expectation value of the total Hamiltonian in the space of Slater determinants \( \Pi a_\alpha^+|0> \), where \( a_\alpha^+ \) is a creation operator for a nucleon in the state \( \alpha \).

B.1 Contributions of the NLSI terms to the total energy of the system

The contribution to the energy of the linear part of the Lagrangian, of finite range, is taken directly from Ref. [3]. It includes, of course, the exchange terms. Here, we shall concentrate in the technique of calculating direct (Hartree) and exchange (Fock) components of the contribution to the energy of the self-interaction term \( U_{SI} \) given by Eq. (1).

In doing this, we obtain for the mean value of the cubic component of \( U_{SI} \)

\[
<0 | (\bar{\psi} \psi)^3 | 0 > = \langle 0 | \sum_{ijk} \bar{\psi}_i(1)\bar{\psi}_j(2)\psi_k(3)\psi_{j'}(2)\psi_{i'}(1)a_i^+a_j^+a_k^+a_{i'}a_{j'}a_{i'}|0 \rangle
\]

\[
= \sum_{ijk} \bar{\psi}_i(1)\psi_i(1)\bar{\psi}_j(2)\psi_j(2)\bar{\psi}_k(3)\psi_k(3)
\]

\[
-3 \sum_{ijk} \bar{\psi}_i(1)\psi_i(1)\bar{\psi}_j(2)\psi_j(2)\bar{\psi}_k(3)\psi_j(3)
\]

\[
+2 \sum_{ijk} \bar{\psi}_i(1)\psi_k(1)\bar{\psi}_j(2)\psi_i(2)\bar{\psi}_k(3)\psi_j(3),
\]

and for the mean value of the quartic component

\[
<0 | (\bar{\psi} \psi)^4 | 0 > = \langle 0 | \sum_{ijklm} \bar{\psi}_i(1)\bar{\psi}_j(2)\psi_k(3)\psi_{m'}(4)\psi_{m'}(4)\psi_{j'}(2)\psi_{i'}(1)
\]

\[
\times a_i^+a_j^+a_k^+a_m^+a_{m'}a_{k'}a_{j'}a_{i'}|0 \rangle
\]
\[\sum_{ijkkm} \bar{\psi}_i(1)\psi_i(1)\bar{\psi}_j(2)\psi_j(2)\bar{\psi}_k(3)\psi_k(3)\bar{\psi}_m(4)\psi_m(4) \]
\[-6\sum_{ikjm} \bar{\psi}_i(1)\psi_i(1)\bar{\psi}_m(2)\psi_m(2)\bar{\psi}_k(3)\psi_k(3)\bar{\psi}_m(4)\psi_m(4) \]
\[+8\sum_{ikjm} \bar{\psi}_i(1)\psi_i(1)\bar{\psi}_m(2)\psi_m(2)\bar{\psi}_j(3)\psi_j(3)\bar{\psi}_m(4)\psi_k(4) \]
\[-6\sum_{ikjm} \bar{\psi}_i(1)\psi_i(1)\bar{\psi}_m(2)\psi_m(2)\bar{\psi}_k(3)\psi_k(3)\bar{\psi}_m(4)\psi_k(4) \]
\[+3\sum_{ikjm} \bar{\psi}_i(1)\psi_i(1)\bar{\psi}_m(2)\psi_m(2)\bar{\psi}_k(3)\psi_k(3)\bar{\psi}_m(4)\psi_k(4). \]

Let us mention that the space coordinates in Eqs. (2) and (3) coincide for all spinors in the ZRL and that the subscripts \(i, j, k\) in the last three lines of Eq. (2) and \(i, j, k, m\) in the last four lines of Eq. (3) run over single-particle occupied states. Let us emphasize also that the sum of coefficients of the exchange (Fock) terms and of the direct (Hartree) term in Eqs. (2) and (3) is equal to zero, as it should be.

We define the scalar \((\rho_{sq})\), vector \((\rho_{vq})\) and tensor \((\rho_{Tq})\) densities for neutrons \((q = n)\) or protons \((q = p)\) in the usual way:

\[\rho_{sq}(1) = \sum_{i(q \text{ states})} \bar{\psi}_i(1)\psi_i(1),\]
\[\rho_{vq}(1) = \sum_{i(q \text{ states})} \bar{\psi}_i(1)\gamma^0\psi_i(1),\]
\[\rho_{Tq}(1) = \sum_{k=1}^{3} \sum_{i(q \text{ states})} \bar{\psi}_i(1)\sigma^{0k}\psi_i(1)n^k = i \sum_{i(q \text{ states})} \bar{\psi}_i(1)\gamma^0\vec{n} \cdot \vec{n}\psi_i(1),\]

where the subscript \(i\) runs over all the occupied states of a nucleon of type \(q\). Here, \(\vec{n}\) is the unit vector along the radial direction.

The scalar, vector and tensor total densities are \(\rho_s = \rho_{sn} + \rho_{sp}, \rho_v = \rho_{vn} + \rho_{vp}\), and \(\rho_T = \rho_{Tn} + \rho_{Tp}\), respectively.

Using the Fierz transformation [16], we can write for a fermion system:

\[\sum_{ab}(\bar{\psi}_a\psi_b)(\bar{\psi}_b\psi_a) = \frac{1}{4}\sum_{ab}[(\bar{\psi}_a\gamma_5\psi_a)(\bar{\psi}_b\gamma_5\psi_b) + (\bar{\psi}_a\gamma_\mu\psi_a)(\bar{\psi}_b\gamma_\mu\psi_b) \]
\[-(\bar{\psi}_a\gamma_5\gamma_\mu\psi_a)(\bar{\psi}_b\gamma_5\gamma_\mu\psi_b) + \frac{1}{2}(\bar{\psi}_a\sigma_{\mu\nu}\psi_a)(\bar{\psi}_b\sigma^{\mu\nu}\psi_b)].\] (5)

The quantities \(\sum_{abc} (\bar{\psi}_a\psi_b)(\bar{\psi}_c\psi_a)(\bar{\psi}_c\psi_c)\) and \(\sum_{abc} (\bar{\psi}_a\psi_b)(\bar{\psi}_c\psi_c)(\bar{\psi}_d\psi_d)(\bar{\psi}_c\psi_a)\) can also be written in a similar way, although they involve much more terms and will not be given here.
In order to write down the quantities $<0|{\bar{\psi}\psi}|0>$ and $<0|{\bar{\psi}\psi}|0>$ in terms of the nucleon densities defined in Eqs. (4), we take into account the following relation:

$$\sum_i \bar{\psi}_i \sigma^{\mu\nu} \psi_i = \sum_{k=1}^{3} (\delta_{\mu0} \delta_{\nu k} - \delta_{\mu k} \delta_{\nu 0}) \rho_T n^k. \quad (6)$$

From this relation, one gets two more useful equations

$$\sum_i \bar{\psi}_i \sigma^{\mu\nu} \psi_i \sum_i \bar{\psi}_i \sigma_{\mu\nu} \psi_i = -2 \rho_T^2 \quad (7)$$

and

$$\sum_i \bar{\psi}_i \sigma^{\mu\nu} \psi_i \sum_i \bar{\psi}_i \sigma_{\mu\nu} \psi_i = \rho_T^2 [(1 - \delta_{\mu0})(1 - \delta_{\nu0}) n^\mu n^\nu - \delta_{\mu0} \delta_{\nu0}]. \quad (8)$$

Finally, from this last equation one obtains

$$\sum_i \bar{\psi}_i \sigma^{\mu\nu} \psi_i \sum_i \bar{\psi}_i \sigma_{\mu\nu} \psi_i \sum_i \bar{\psi}_i \sigma_{\lambda\kappa} \psi_i \sum_i \bar{\psi}_i \sigma_{\lambda\kappa} \psi_i = 2 \rho_T^4. \quad (9)$$

Having in mind that the space coordinates in Eqs. (2) and (3) coincide for all spinors in the ZRL, we get from Eq. (2) for the cubic term

$$\langle 0|{\bar{\psi}\psi}|0 \rangle = \rho_s^3 - \frac{3}{4} \rho_s \left[ \rho_{sn}^2 + \rho_{sp}^2 + \rho_{vn}^2 + \rho_{vp}^2 - (\rho_{Tn}^2 + \rho_{Tp}^2) \right]$$

$$+ \frac{1}{8} \left[ \rho_{sn}^3 + \rho_{sp}^3 + 3(\rho_{sn}\rho_{vn}^2 + \rho_{sp}\rho_{vp}^2) - 3(\rho_{sn}\rho_{Tn}^2 + \rho_{sp}\rho_{Tp}^2) \right], \quad (10)$$

and from Eq. (3) for the quartic term

$$\langle 0|{\bar{\psi}\psi}|0 \rangle = \rho_s^4 - \frac{3}{2} \rho_s^2 \left[ \rho_{sn}^2 + \rho_{sp}^2 + \rho_{vn}^2 + \rho_{vp}^2 - (\rho_{Tn}^2 + \rho_{Tp}^2) \right]$$

$$+ \frac{1}{2} \rho_s \left[ \rho_{sn}^3 + \rho_{sp}^3 + 3(\rho_{sn}\rho_{vn}^2 + \rho_{sp}\rho_{vp}^2) - 3(\rho_{sn}\rho_{Tn}^2 + \rho_{sp}\rho_{Tp}^2) \right]$$

$$- \frac{3}{32} \left[ \rho_{sn}^4 + \rho_{sp}^4 + 6(\rho_{sn}\rho_{vn}^2 + \rho_{sp}\rho_{vp}^2) - 6(\rho_{sn}\rho_{Tn}^2 + \rho_{sp}\rho_{Tp}^2) \right]$$

$$+ \rho_{vn}^4 + \rho_{vp}^4 - 2(\rho_{vn}\rho_{Tn}^2 + \rho_{vp}\rho_{Tp}^2) + \rho_{Tn}^4 + \rho_{Tp}^4$$

$$+ \frac{3}{16} \left( \rho_{sn}^2 + \rho_{sp}^2 + \rho_{vn}^2 + \rho_{vp}^2 - \rho_{Tn}^2 - \rho_{Tp}^2 \right)^2. \quad (11)$$

Let us mention that the contributions given by Eqs. (10) and (11) include the densities $\rho_s$, $\rho_v$ and $\rho_T$ but they do not include the contributions of pseudoscalar $\rho_P$- and axial vector $\rho_A$-densities for parity reasons.
From Eqs. (10) and (11), it can be easily seen that the exchange (Fock) contributions to the energy of the system corresponding to both the cubic and quartic terms are very essential. Actually, only the \( \rho_3^s \)-term in Eq. (10) and the \( \rho_4^s \)-term in Eq. (11) originate from the direct (Hartree) contributions, all the other terms in Eqs. (10) and (11) arise from the Fock contributions, and they have a strong isovector structure (ISVS)\(^1\).

**B.2 Contributions of the NLSI terms to the nucleon self-energy**

The contribution of the NLSI terms of the third (3) and forth (4) order to the self-energy of a nucleon of type \( q \) \([\hat{\Sigma}^{(3,4)}_q]\) can be extracted from the following equation:

\[
\hat{\Sigma}^{(3,4)}_q \psi_q = \frac{\delta}{\delta \psi_q} U^{(3,4)}_{SI} = [\Sigma^{(3,4)}_{sq} + \Sigma^{(3,4)}_{vq} \gamma^0 + i \Sigma^{(3,4)}_{Tq} \gamma^0 \vec{\gamma} \cdot \vec{n}] \psi_q.
\] (12)

From Eq. (12), it is seen that the NLSI terms \( U_{SI} \) give a contribution to the total nucleon self-energy of the same structure as the linear components of the Lagrangian. Thus, each component \( (\Sigma_{iq}) \) of the total self-energy can be written as

\[
\Sigma_{iq} = \Sigma_{iq}^{linear} + \Sigma_{iq}^{(3,4)},
\] (13)

where \( i = s, 0, T \) specifies the scalar, time component of the vector, and tensor self-energies. One can look in Refs. [3,14] for further details related to \( \Sigma_{iq}^{linear} \).

Taking into account that

\[
\frac{\delta}{\delta \psi_i^q} \rho_s = \psi_i^q, \quad \frac{\delta}{\delta \psi_i^q} \rho_v = \gamma^0 \psi_i^q, \quad \text{and} \quad \frac{\delta}{\delta \psi_i^q} \rho_T = i \gamma^0 \vec{\gamma} \cdot \vec{n} \psi_i^q,
\] (14)

one obtains for the contribution of the cubic terms of \( U_{SI} \) to the self-energy components

\[
\Sigma^{(3)}_{sq} = 3 \rho_s^2 - \frac{3}{4} (\rho_{sn}^2 + \rho_{sp}^2 + \rho_{vn}^2 + \rho_{vp}^2 - \rho_{Tn}^2 - \rho_{Tp}^2) - \frac{3}{8} (4 \rho_{sq} \rho_{sq} - \rho_{sq}^2 - \rho_{vq}^2 + \rho_{Tq}^2),
\] (15)

\[
\Sigma^{(3)}_{0q} = \frac{3}{4} \rho_{eq} (\rho_{sq} - 2 \rho_s),
\] (16)

\[
\Sigma^{(3)}_{Tq} = \frac{3}{4} \rho_{Tq} (2 \rho_s - \rho_{sq}).
\] (17)

For the contribution of the quartic terms of \( U_{SI} \) to the self-energy components, one has

\(^1\)We consider that a quantity has ISVS if it depends on the difference \( \rho_n - \rho_p \) (or \( \rho_{sn} - \rho_{sp} \)).
\[ \Sigma_{sq}^{(4)} = 4 \rho_{sq}^3 + \frac{3}{4} (\rho_{sq} - 4 \rho_{s}) \left( \rho_{sn}^2 + \rho_{sp}^2 + \rho_{vn}^2 + \rho_{vp}^2 - \rho_{Tn}^2 - \rho_{Tp}^2 \right) + \frac{1}{2} (\rho_{sn}^3 + \rho_{sp}^3) + \frac{3}{2} (\rho_{sn} \rho_{sn}^2 + \rho_{sp} \rho_{vp}^2) - \frac{3}{2} (\rho_{sn} \rho_{Tn}^2 + \rho_{sp} \rho_{Tp}^2) - 3 \rho_{s}^2 \rho_{sq} + \frac{3}{2} \rho_{s} (\rho_{sq}^2 + \rho_{Tq}^2) - \frac{3}{8} \rho_{sq} (\rho_{sq}^2 + 3 \rho_{vq}^2 - 3 \rho_{Tq}^2), \tag{18} \]

\[ \Sigma_{0q}^{(4)} = 3 \rho_{vq} \left[ \rho_{s} \rho_{sq} - \rho_{s}^2 - \frac{1}{8} (3 \rho_{sq}^2 + \rho_{vq}^2 - \rho_{Tq}^2) + \frac{1}{4} (\rho_{sn}^2 + \rho_{sp}^2 + \rho_{vn}^2 + \rho_{vp}^2 - \rho_{Tn}^2 - \rho_{Tp}^2) \right], \tag{19} \]

\[ \Sigma_{Tq}^{(4)} = 3 \rho_{Tq} \left[ \rho_{s}^2 - \rho_{s} \rho_{sq} + \frac{1}{8} (3 \rho_{sq}^2 + \rho_{vq}^2 - \rho_{Tq}^2) - \frac{1}{4} (\rho_{sn}^2 + \rho_{sp}^2 + \rho_{vn}^2 + \rho_{vp}^2 - \rho_{Tn}^2 - \rho_{Tp}^2) \right]. \tag{20} \]

From the previous publications [3,14], it is clear that the isovector structure of the Hartree-Fock solutions is strongly determined by the contribution of the \( \pi \)- and \( \rho \)-mesons. The present results, show that the isovector-independent NLSI also make essential contributions to the ISVS of the energy and self-energies of the system.

From Eqs. (12,13), it follows that the quantities \( \Sigma_{sq}^{(3,4)} \), \( \Sigma_{0q}^{(3,4)} \) and \( \Sigma_{Tq}^{(3,4)} \) enter the RHF Dirac equation in the same manner as the self-energies \( \Sigma_{sq} \), \( \Sigma_{0q} \) and \( \Sigma_{Tq} \), produced by linear interactions, do. However, the \( \Sigma_{iq}^{(3,4)} \)-components involve a strong density dependence, and, what is more surprising, a strong ISVS. The densities \( \rho_{sq} \), \( \rho_{vq} \) and \( \rho_{Tq} \) can be calculated in the following way

\[ \rho_{sq}(r) = \frac{1}{4 \pi r^2} \sum_{i(q \text{ states})} \left[ G_i^2(r) - F_i^2(r) \right], \]

\[ \rho_{vq}(r) = \frac{1}{4 \pi r^2} \sum_{i(q \text{ states})} \left[ G_i^2(r) + F_i^2(r) \right], \tag{21} \]

\[ \rho_{Tq}(r) = \frac{1}{2 \pi r^2} \sum_{i(q \text{ states})} G_i(r) \cdot F_i(r), \]

where \( G_i(r)/r \) and \( F_i(r)/r \) are the radial functions of the upper and lower components of the nucleon Dirac spinor. Let us mention that Eqs. (10, 11, 15-17) are valid for NM and finite nuclei.
3 Numerical results

A. Nuclear Matter

We shall start the discussion of our results for symmetric NM, where $\rho_T = 0$, $\rho_{s,p} = \rho_{s,n}$ and $\rho_{v,p} = \rho_{v,n}$. As is mentioned above, in the present paper we follow closely the Lagrangian of Ref. [14] corresponding to the set HFSI and, in our calculations, we have fixed the same parameters as in this reference. Thus, the bare nucleon mass and the $\pi$, $\omega$ and $\rho$ meson masses have been taken to be equal to their empirical values: $M = 939$ MeV, $m_\pi = 138$ MeV, $m_\omega = 783$ MeV, and $m_\rho = 770$ MeV. As for the $f_\pi$ and $g_\rho$ coupling constants, we have chosen the experimental values $f_\pi^2/4\pi = 0.08$ and $g_\rho^2/4\pi = 0.55$. The ratio $f_\rho/g_\rho = 3.7$ is taken in accordance with the vector dominance model. Then, we are left with the following five free parameters: $m_\sigma$, $g_\sigma$, $g_\omega$, $b$ and $c$, which are to be adjusted to reproduce some observables for NM and finite nuclei in a similar manner as in Ref. [14].

In Fig. 1, we present the result of the RHF calculation of the energy per particle ($E/A$) for NM as a function of $\rho_v$ for three cases: 1) The curve HFSI corresponds to the results obtained by the method suggested in Ref. [14] to take into account NLSI. 2) The curve ZRL$^*[b,c(HFSI)]$ corresponds to the results obtained with the Lagrangian and the method proposed in this paper, taking for the parameters $m_\sigma$, $b$ and $c$ the same values as in the HFSI approximation [14], $g_\sigma$ and $g_\omega$ being chosen to get the saturation point at the same density, $\rho_0 = 0.14 \text{ fm}^{-3}$, as in the HFSI set. We obtain in this case the compressibility modulus $K = 319$ MeV. 3) The curve ZRL$[g_\sigma, g_\omega, b,c(HFSI)]$ corresponds to the results obtained for the same set of parameters as in the HFSI set of Ref. [14], the NLSI being treated in the ZRL by the method suggested in the present paper (the saturation point is achieved at $\sim 0.113 \text{ fm}^{-3}$ in this case)$^2$. To check the reliability of the ZRL approximation, we have carried out calculations of the $E/A$ vs $\rho_v$-curves in the Hartree approach also. The results of the calculations are presented in Fig. 2. The ZRL approximation introduces appreciable differences with respect to the exact calculation for densities around the saturation value.

Considering $g_\sigma$, $g_\omega$, $b$ and $c$ as fitting parameters, it is easy to determine a set with a compressibility modulus $K$ equal to a given reasonable phenomenological value. In Fig. 3, the energy per particle $E/A$ as a function of $\rho_v$ is represented for the model considered in the present paper (ZRL) for three sets of parameters. In all sets the scalar mass and one coupling parameter are chosen with the same values as in the HFSI set of Ref. [14], whereas the other three coupling parameters are chosen to get the same values of $\rho_0$ and $E(\rho_0)/A$ in NM as in the HFSI set (i.e. $\rho_0=0.14 \text{ fm}^{-3}$, $E/A = -15.75 \text{ MeV}$) and $K=250$ or 275 MeV:

$^2$In the limit of small densities, where the contribution of the self-interaction terms becomes negligible, the HFSI- and ZRL-curves coincide.
1) ZRL$^1$ set: $\bar{c}$ is fixed to the value ($\bar{c} = -0.01461$) obtained in the HFSI set, while $\bar{b} = -0.011533$, $\frac{g^2}{4\pi} = 3.0621$, $\frac{g^2}{4\pi} = 7.849$ generate the required saturation conditions of NM with $K = 275$ MeV.

2) ZRL$^2$ set: $\bar{b}$ is fixed to the value ($\bar{b} = -0.006718$) obtained in the HFSI set, while $\bar{c} = 0.012475$, $\frac{g^2}{4\pi} = 3.3039$, $\frac{g^2}{4\pi} = 8.9346$ generate the required saturation point of NM with $K = 275$ MeV.

3) ZRL$^3$ set: $\bar{b}$ is fixed to the value ($\bar{b} = -0.006718$) obtained in the HFSI set, while $\bar{c} = 0.042561$, $\frac{g^2}{4\pi} = 3.1715$, $\frac{g^2}{4\pi} = 8.4777$ generate the required saturation point of NM with $K = 250$ MeV.

One could also try to get $K = 250$ MeV keeping the value of $c$ as in the HFSI set, however, we could not find solutions in this case. From Fig. 3, one can see that appreciable differences between the three models appear only at densities larger than the saturation one. Thus, a similar description of finite nuclei can be expected for these three sets.

Our next step is to find an adequate parameterization for NM and finite nuclei. To do that, we can follow the procedure used in Ref. [14]. In the present paper, the values of $g_\sigma$, $b$, $c$ are determined by reproducing the saturation conditions of symmetric NM for $\rho_0$, $E/A$ and a reasonable value of the compressibility modulus. In this work we have fitted two sets, ZRL1 and ZRL2, corresponding to the compressibility modulus 250 MeV and 275 MeV, respectively. Calculations for finite nuclei put some extra constraints on the values of $m_\sigma$ and $g_\omega$ ($m_\sigma$ is adjusted to get the experimental r.m.s. charge radius of the $^{16}O$ and the value of $g_\omega$ is chosen to get reasonable values of spin-orbit splittings). Alternatively, one can use finite nuclei data to fit directly the free parameters of the model.

The values of the parameters chosen in the present paper for the ZRL1 and ZRL2 sets and for the HFSI set of Ref. [14], as well as some calculated NM properties, are given in Table I. One can appreciate that the scalar meson mass $m_\sigma$ needed to get a good description of the surface nuclear properties is considerably larger in the ZRL1 and ZRL2 sets than in the HFSI one. The parameters $\bar{b}$, $\bar{c}$ are quite different in the sets ZRL1 and ZRL2 as compared with those of the set HFSI. This is due to the strong Fock terms contribution of the NLSI in sets ZRL1 and ZRL2.

Fig. 4 shows for the ZRL1 set the values of $E/A$ as a function of $\rho_v$ for different values of the asymmetry parameter $\delta$ defined by the following equation:

$$
\delta = \frac{\rho_{v,n} - \rho_{v,p}}{\rho_{v,n} + \rho_{v,p}}
$$

(22)
It can be appreciated in this figure a strong shift of the equilibrium density $\rho_0(\delta)$ towards smaller values as $\delta$ increases and also that the pure neutron matter ($\delta = 1$) appears to be unbound. The difference between the E/A values corresponding to the symmetry parameters $\delta = 0$ and $\delta = 0.2$ remains almost constant for $\rho_\nu > 0.4$ fm$^{-3}$. This fact is reflected in a strong change in the trend of the symmetry energy parameter $a_4$ as a function of the density for $\rho_\nu > 0.4$ fm$^{-3}$. This can be seen clearly in Fig. 5, where we have represented the $a_4$ parameter in the ZRL1 set as a function of the density. The strong deviation of $a_4$ from the linear dependence at very high densities is related to the highly increasing role of the NLSI terms, especially the quartic ones, in this region. Anyway, this set generates reasonable solutions up to densities larger than the HFSI set does. In this latter case, to get a value of $K = 250$ MeV, one needs a contribution of the NLSI terms producing a rapid decreasing of $m^*_\sigma$ with the density for $\rho_\nu > \rho_0$ [14]. Thus, $m^*_\sigma$ becomes almost zero for densities larger than $\rho_\nu \approx 0.22$ fm$^{-3}$. That is why the quantities E/A and $a_4$ have only been represented up to this density in Figs. 1 and 5.

For comparison, we have also represented in Fig. 5 with the NLHF label the results for $a_4$ obtained in Ref. [10], within an approximate procedure to include exchange contributions of the NLSI terms of scalar type. One can see from this figure that the results corresponding to the NLHF approximation lie significantly below our results. However, the NLHF approximation does not include isovector mesons. Of course, the inclusion of these mesons would significantly increase the value of $a_4$.

We have mentioned above that the NLSI terms are explicitly isovector-independent [see Eq. (1)]. However, their contributions to the nucleon self-energies manifest a strong isovector structure due to the exchange part.

To illustrate this point, we show in Figs. 6-9 the density dependence of the quantities $\Sigma^{(3)}_{sE}$, $\Sigma^{(4)}_{sE}$, $\Sigma^{(3)}_{0E}$, $\Sigma^{(4)}_{0E}$, respectively. The first two quantities correspond to the exchange (Fock) contributions of the cubic and quartic terms, respectively, of the NLSI part of the Lagrangian to the scalar nucleon self-energy, whereas the last two quantities represent the exchange contributions to the (time component of the) vector nucleon self-energy. We have considered three values of the asymmetry parameter: $\delta = 0$, 0.5, 1. Notice that for the case $\delta = 1$, the proton self-energies correspond to a proton moving in pure neutron matter and that, in this case, $\Sigma^{(3,4)}_{0E}$ are equal to zero.

From Figs. 6-9, it is seen that the exchange (Fock) self-energies $\Sigma^{(3,4)}_{iE}$ ($i = s, 0$) represent a very important contribution to the respective total self-energies even at normal densities of NM (especially that of $\Sigma^{(3,4)}_{sE}$). This contribution is appreciably smaller for the ZRL2 set with a larger value of the compressibility modulus, due to the smaller values of
the parameters $\bar{b}$ and $\bar{c}$. It is also seen that $\Sigma_{iE}^{(3,4)} (i = s, 0)$ are essentially dependent on the neutron excess in the system (due to the Hartree-Fock framework used).

**B. Finite nuclei**

In order to make a more complete analysis of the properties of our nonlinear RHF model, we have carried out calculations for finite nuclei with the ZRL1 and the ZRL2 sets of Table I. Actually, as we have explained above, we have taken into account, besides the saturation NM properties, the experimental values of the binding energies, spin-orbit splittings and r.m.s. charge radii of finite nuclei in choosing the parameters of the ZRL1 and ZRL2 sets. We remind that the ZRL1 and ZRL2 sets contain five fitting parameters, as explained in subsection A dedicated to NM.

In Tables II and III, we present the RHF results of our ZRL1 set\(^3\), containing the exact exchange contribution of the NLSI, given by $U_{SI}$ in Eq. (1), for the ground state properties for the five doubly-magic nuclei: $^{16}O$, $^{40}Ca$, $^{48}Ca$, $^{90}Zr$, $^{208}Pb$. For comparison, we also present the results of our HFSI set of Ref. [14] (where the exchange terms of NLSI are treated in an approximate way) and experimental values. The C.M. values in Table II indicate an estimation for the center-of-mass correction to the total energy, which is not included in the ZRL1 and HFSI results.

From Tables II and III, one can see that the ZRL1 set allows a quite good description of binding energies, spin-orbit splittings and r.m.s. charge radii for spherical nuclei. The results are comparable to those of the HFSI set, although, as we have already indicated above, the ZRL1 set has reasonable solutions for NM up to much higher densities than the HFSI one has. As is common in the RHF approaches containing the exchange of pions [9], the ZRL1 set also predicts a strong reduction of the spin-orbit splitting as going from the $^{40}Ca$ nucleus to the $^{48}Ca$ one. We explained in Ref. [9] that this fact is a consequence of the small value of the pion mass.

Figs. 10-14 show the calculated charge distributions for the indicated nuclei and the corresponding experimental ones for comparison. It is remarkable that there is a very good agreement between the theoretical results and the experimental data, specially for the $^{16}O$ and $^{40}Ca$ nuclei.

\(^3\)The results for finite nuclei with the ZRL2 set are almost identical to those obtained with the ZRL1 set. This is due to the fact that the properties of NM obtained with both parametrizations are very similar in the range of densities relevant in finite nuclei. Only for larger densities, the properties of these two sets are significantly different. Notice that the smaller values of the $\bar{b}$ and $\bar{c}$ coefficients in the ZRL2 set than in the ZRL1 one is related to the larger value of the compressibility modulus in the former case than in the second one.
4 Conclusions

In this work, we have considered a Lagrangian including, beside the exchange of $\sigma, \omega, \pi$ and $\rho$ mesons between nucleons, NLSI of zero range, associated with three- and four-body forces, in the framework of the RHF approximation. The Fierz transformation allows us to express the exchange (Fock) terms of the NLSI explicitly via the direct (Hartree) terms in an exact way. The model has been applied to NM and finite nuclei.

The NLSI $(\bar{\psi}\psi)^3$ and $(\bar{\psi}\psi)^4$ considered here are constructed from the scalar densities. However, the relativistic Hartree-Fock procedure generates, in this case, the nucleon self-energies (in the respective single-particle Dirac equation) with the space-time transformation properties of a relativistic scalar, vector and tensor (the pseudoscalar and axial components do not survive). It should be mentioned also that the NLSI generate strong density dependence of the respective nucleon self-energies $\Sigma^{(3,4)}_{sq}$, $\Sigma^{(3,4)}_{vq}$, and $\Sigma^{(3,4)}_{Tq}$. In the RHF framework, the method suggested manifests also a strongly developed ISVS of the self-energies generated by isovector-independent NLSI, this point being one of the basic features of the RHF framework (notice that in the relativistic Hartree approximation there is no ISVS due to the NLSI, neither, in the usual form $\sigma^3$- and $\sigma^4$ nor in the ZRL).

Let us mention that the same method, as suggested in the present paper, can be utilized to treat the self-interactions of the $\omega^4$-type ($\omega$ being the vector-isoscalar meson field) and also associated with four-body forces [15,17]. In this case, in the ZRL, the nonlinearities are constructed by mean of the relativistic vector densities $\bar{\psi}\gamma^\mu\psi$ and contain eight fermion operators. Self-interactions of the $\omega^4$-type will introduce an additional fitting parameter. We shall consider this case in more detail in a forthcoming paper. Actually, NLSI of different types ($\sigma\omega^2$, $\sigma^2\omega^2$, etc. [15]) can be treated in the ZRL in the same fashion as we did for the $U_{Sl}$ term.

The results given in sect. III show that our model with five parameters is flexible enough to generate NM properties, in the range of densities relevant for finite nuclei, compatible with data inferred from nuclear experiments. Although the present model allows to get reasonable results for NM until values of the density much higher than those of the HFSI set of Ref. [14], we cannot expect that our results, obtained with a parametrization that describes adequately finite nuclei properties, could be considered realistic for high densities, for example, larger than $2\rho_0$.

The calculations for doubly-magic nuclei have been also carried out within the ZRL1 set. Our results show a rather good agreement between theory and experiment for binding energies, spin-orbit splittings, r.m.s. charge radii and charge densities, especially, in light
and mid-weight nuclei.

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Table 1. Adjusted parameters and some properties of symmetric NM for the parameter sets ZRL1, ZRL2, corresponding to the zero range limit (ZRL) considered here, and HFSI from Ref. [14]. The value of the effective nucleon mass $M^*$ is defined in the conventional way and is given at the Fermi surface. $a_4$ is the symmetry energy parameter. The equilibrium density $\rho_0$ is given in $fm^{-3}$, the mass of the scalar meson $m_\sigma$, the energy per particle at the equilibrium $E/A$, the compressibility modulus $K$ and the symmetry energy parameter $a_4$ are given in MeV.

| Set   | $g_\sigma^2/4\pi$ | $g_\omega^2/4\pi$ | $m_\sigma$ | $b \times 10^3$ | $c \times 10^3$ | $\rho_0$ | $E/A$ | $K$ | $M^*/M$ | $a_4$ |
|-------|-------------------|-------------------|------------|-----------------|-----------------|---------|-------|-----|---------|-------|
| ZRL1  | 5.5743            | 11.732            | 497.8      | 2.9646          | 51.00           | 0.155   | -16.39| 250 | 0.58    | 35.0  |
| ZRL2  | 5.5121            | 11.565            | 496.8      | 2.4591          | 46.50           | 0.154   | -16.37| 275 | 0.58    | 35.0  |
| HFSI  | 4.005             | 10.4              | 412.0      | -6.718          | -14.61          | 0.140   | -15.75| 250 | 0.61    | 35.0  |

Table 2. Comparison of the results of the present calculation for the ZRL1 set in finite nuclei with the corresponding results of the HFSI set of Ref. [14] and the experimental ones. The total binding energy per particle $E/A$, the non-relativistic center-of-mass correction to $E/A$, and the proton spin-orbit splitting $\Delta_{LS}$ for the shells 1p of the $^{16}O$ nucleus and 2d of the $^{40}Ca$ and $^{48}Ca$ nuclei are given in MeV (the experimental values of $\Delta_{LS}$ in the Ca isotopes are not very well established [9]). The r.m.s. charge radii $r_c$ are given in fm.

| Set   | $^{16}O$ | $^{40}Ca$ | $^{48}Ca$ | $^{90}Zr$ | $^{208}Pb$ |
|-------|----------|-----------|-----------|-----------|-----------|
|       | $-E/A$  | $r_c$    | $\Delta_{LS}$ | $-E/A$  | $r_c$    | $\Delta_{LS}$ | $-E/A$  | $r_c$    | $\Delta_{LS}$ |
| ZRL1  | 7.37     | 2.71      | 6.3       | 8.33      | 3.44      | 7.1       | 8.51     | 3.49      | 2.6       | 8.67      | 4.25      | 7.85      | 5.49      |
| HFSI  | 7.43     | 2.73      | 6.4       | 8.33      | 3.48      | 7.05      | 8.45     | 3.48      | 3.27      | 8.58      | 4.26      | 7.78      | 5.52      |
| EXP   | 7.98     | 2.73      | 6.3       | 8.55      | 3.48      | 6-7.6     | 8.67     | 3.47      | 5         | 8.71      | 4.27      | 7.87      | 5.50      |
| C.M.  | 0.61     | 0.20      | 0.18      | 0.08      | 0.02      |           |          |           |           |           |           |           |           |
Table 3. The single particle energies (in MeV) for protons and neutrons in the $^{16}O$, $^{40}Ca$, and $^{48}Ca$ nuclei. For each state, the first and the second rows correspond to sets ZRL1 of the present paper and HFSI of Ref. [14], respectively. References to the experimental data are given in Refs. [9] and [14].

| State   | $^{16}O$  |        |        | $^{40}Ca$  |        |        | $^{48}Ca$  |        |
|---------|-----------|--------|--------|-----------|--------|--------|-----------|--------|
|         | protons   | neutrons |       | protons   | neutrons |       | protons   | neutrons |       |
|         | calc.     | exp.    | calc.  | exp.      | calc.   | exp.   | calc.    | exp.    | calc.   |
| $1s_{1/2}$ | 39.51 40±8 | 43.89 47 | 48.71 50±11 | 57.22 | 54.22 55±9 | 59.01 |
|         | 38.96     | 43.15   | 48.64  | 56.83     | 55.3   | 58.8   |
| $1p_{3/2}$ | 19.28 18.4 | 23.36 21.8 | 33.39 34±6 | 41.53 | 39.33 35±7 | 43.08 |
|         | 18.80     | 22.73   | 32.2   | 40.08     | 39.7   | 41.95  |
| $1p_{1/2}$ | 12.98 12.1 | 16.95 15.7 | 29.43 34±6 | 37.47 | 38.24 35±7 | 41.24 |
|         | 12.40     | 16.22   | 27.8   | 35.56     | 38.0   | 39.60  |
| $1d_{5/2}$ |           | 17.22 14.3-16 | 24.98 21.9 | 23.04 | 20.5 | 26.20 | 16 |
|         |           | 16.20   | 23.77   | 23.2     | 25.0   |        |
| $2s_{1/2}$ | 8.64 10.9 | 16.36 18.2 | 15.30 15.8 | 18.42 | 12.4 |
|         | 9.83      | 17.23   | 15.8   | 18.5     |        |        |
| $1d_{3/2}$ | 10.10 8.3 | 17.71 15.6 | 20.45 15.5 | 21.74 | 12.4 |
|         | 9.19      | 16.55   | 19.9   | 20.2     |        |        |
| $1f_{7/2}$ |           |         |        | 9.98 9.9 |        |        | 9.21      |
Fig. 1: The E/A values for symmetric NM in the RHF approach in three cases: The HFSI (dashed) curve corresponds to the results obtained with the HFSI set of Ref. [14]; the ZRL\( [g_\sigma, g_\omega, b, c (\text{HFSI})] \) (solid) curve corresponds to the results obtained by the method suggested in the present paper, with the same set of parameters as in Ref. [14]; and the ZRL\( ^* [b, c (\text{HFSI})] \) (dash-dotted) curve corresponds to the results with the parameters \( g_\sigma \) and \( g_\omega \) chosen to get saturation at the same point as in the HFSI set.

Fig. 2. The E/A values for symmetric NM in the Hartree approach in two cases: The NL3 (dashed) curve represents the results obtained with the NL3 parametrization of Ref. [20] and the ZRL\( / \text{H:NL3} \) (solid) curve are the results obtained with the NL3 parametrization in the ZRL.
Fig. 3. The E/A values for symmetric NM, with the Lagrangian considered in sect. I (which contains the zero range NLSI given in Eq. (1)) for different choices of the parameters fitted to the same values of NM $\rho_0$ and $E(\rho_0)/A$ as in the set HFSI of Ref. [14] (see the text), and generating two values of the compressibility modulus: The ZRL$^{c1}$ (solid) curve, with $\bar{c} < 0$ taken from the HFSI set, corresponds to $K = 275$ MeV; the ZRL$^{b1}$ (dashed) curve, with $\bar{b}$ taken from the HFSI set, corresponds to $K = 275$ MeV; and the ZRL$^{b2}$ (dash-dotted) curve, with $\bar{b}$ as for the ZRL$^{b1}$ case, corresponds to $K = 250$ MeV.

Fig. 4. The E/A values in asymmetric NM, for the ZRL1 set, with different values of the asymmetry parameter $\delta$.
Fig. 5. The symmetry energy parameter $a_4$ as a function of the NM density for sets ZRL1, HFSI [14] and NLHF [10].

Fig. 6. The density dependence of the exchange (Fock) contribution $\Sigma_{sE}^{(3)}$ of the $(\bar{\psi}\psi)^3$ term to the scalar nucleon self-energy for different values of the asymmetry parameter $\delta$. 

Fig. 7. The same as in Fig. 6 but for the density dependence of the exchange (Fock) contribution $\Sigma_{sE}^{(4)}$ of the $(\bar{\psi}\psi)^4$ term.

Fig. 8. The density dependence of the exchange (Fock) contribution $\Sigma_{0E}^{(3)}$ of the $(\bar{\psi}\psi)^3$ term to the (time component of) the vector self-energy.
Fig. 9. The same as in Fig. 8 but for the density dependence of the exchange (Fock) contribution $\Sigma^{(4)}_{0E}$ of the $(\bar{\psi}\psi)^4$ term.

Fig. 10. The charge distribution (dashed line) for the $^{16}\text{O}$ nucleus, with the ZRL1 set. The experimental charge distribution (solid line [21]) is also shown for comparison.
Fig. 11. The same as in Fig. 10 but for the $^{40}Ca$ nucleus.

Fig. 12. The same as in Fig. 10 but for the $^{48}Ca$ nucleus.
Fig. 13. The same as in Fig. 10 but for the $^{90}$Zr nucleus.

Fig. 14. The same as in Fig. 10 but for the $^{208}$Pb nucleus.