Pion Form Factor in the $k_T$ Factorization Formalism

Tao Huang\textsuperscript{1,2\ast}, Xing-Gang Wu\textsuperscript{2\dagger} and Xing-Hua Wu\textsuperscript{2\ddagger}

\textsuperscript{1}CCAST(World Laboratory), P.O.Box 8730, Beijing 100080, P.R.China,
\textsuperscript{2}Institute of High Energy Physics, Chinese Academy of Sciences,
P.O.Box 918(4), Beijing 100039, China.\textsuperscript{§}

Abstract

Based on the light-cone (LC) framework and the $k_T$ factorization formalism, the transverse momentum effects and the different helicity components’ contributions to the pion form factor $F_\pi(Q^2)$ are recalculated. In particular, the contribution to the pion form factor from the higher helicity components ($\lambda_1 + \lambda_2 = \pm 1$), which come from the spin-space Wigner rotation, are analyzed in the soft and hard energy regions respectively. Our results show that the right power behavior of the hard contribution from the higher helicity components can only be obtained by fully keeping the $k_T$ dependence in the hard amplitude, and that the $k_T$ dependence in LC wave function affects the hard and soft contributions substantially. As an example, we employ a model LC wave function to calculate the pion form factor and then compare the numerical predictions with the experimental data. It is shown that the soft contribution is less important at the intermediate energy region.

PACS numbers: 13.40.Gp, 12.38.Bx, 12.39.Ki

\textsuperscript{\ast} email: huangtao@mail.ihep.ac.cn
\textsuperscript{\dagger} email: wuxg@mail.ihep.ac.cn
\textsuperscript{\ddagger} email:xhwu@mail.ihep.ac.cn
\textsuperscript{§} Mailing address
I. INTRODUCTION

In the perturbative QCD (PQCD) theory, the hadronic distribution amplitudes and structure functions which enter exclusive and inclusive processes via the factorization theorems at high momentum transfer can be determined by the hadronic wave functions, and therefore they are the underlying links between hadronic phenomena in QCD at large distances (non-perturbative) and small distance (perturbative). However we require a conceptual framework within which the connection between the hadrons and their constituents can be made precise. A particularly convenient and intuitive framework is based upon the Fock state decomposition of hadronic states which arises naturally in the ‘light-cone quantization’ [1, 2]. A light-cone (LC) wave function is a localized (i.e. normalizable) stationary solution of the LC schrödinger equation

\[ i\partial |\Psi(\tau)\rangle = H_{LC} |\Psi(\tau)\rangle \]

that describes the evolution of a state \( |\Psi(\tau)\rangle \) on the LC time \( \tau \equiv x^+ = x^0 + x^3 \) in physical light cone gauge \( A^+ = A^0 + A^3 = 0 \), which is conjugate to the LC Hamiltonian \( H_{LC} \equiv P^- = P^0 - P^3 \). The LC wave functions are the amplitudes \( \Psi_n(x_i, k_{\perp i}, \lambda_i) \) to find \( n \) particles (quarks, antiquarks and gluons) with momenta \( k_i \) in a pion of momentum \( P \), i.e., \( \Psi(k_1, \cdots, k_n; P) \equiv \langle n|\pi\rangle = \langle k_1, \cdots, k_n|\pi(P)\rangle \), where \( x_i = k_i^+/P^+ \), with \( \sum_i x_i = 1 \), is the LC momentum fraction of the \( i \)-th quark or gluon in the \( n \)-particle Fock state.

An important issue, which has to be addressed when applying PQCD to exclusive processes, is how to implement factorization, i.e., separate perturbative contributions from those intrinsic to the bound-state wave function. Both collinear and \( k_T \) factorization are the fundamental tools of PQCD. If there is no end-point singularity developed in a hard amplitude, collinear factorization works. If such singularity occurs, indicating the breakdown of collinear factorization, (one may find a concrete example in the semi-leptonic decay \( B \to \pi l\bar{\nu} \) [3]), then \( k_T \) factorization should be employed. In the \( k_T \) factorization formula, by retaining the dependence on the parton transverse momentum \( k_T \) and resuming the resultant double logarithms \( \alpha_s \ln^2 k_T \) into a Sudakov form factor, such singularity does not exist [4]. Since the \( k_T \) factorization theorem has been proposed [1, 2, 5], it has been widely applied to various processes. Until recently, a better proof of the \( k_T \) factorization theorem for exclusive processes in PQCD has been provided by M. Nagashima and H.N. Li [6]. Their starting point is that the on-shell valence partons carry longitudinal momenta initially, and then acquire \( k_T \) through collinear gluon exchanges before participating in hard scattering. A hard
amplitude, derived from the parton level amplitudes with the gauge-invariant and infrared divergent meson wave function being subtracted, is then gauge-invariant and infrared-finite. Through this way, they demonstrated that all the physical quantities from the \( k_T \) factorization theorem are gauge-invariant. Therefore for the pion form factor, when in the energy region that PQCD is applicable, we can take the following factorization formula\[1, 5, 7, 8, 9\],

\[
F_{\pi}(Q^2) = \sum_{n,m,\lambda_i,\lambda_j} \int [dx_i dk_{i\perp}]_n [dy_j dl_{j\perp}]_m \psi^*_n(x_i, k_{i\perp}, \lambda_i; \mu) T_{nm}(x_i, k_{i\perp}; y_j, l_{j\perp}; q_{i\perp}; \mu) \psi_m(y_j, l_{j\perp}, \lambda_j; \mu)
\]

where summation over all helicities \((\lambda_i, \lambda_j)\) and \(n, m\) extends over the low momentum states only, and \(T_{n,m}\) are the partonic matrix elements of the effective current operator. \(\mu\) is the energy scale separating the perturbative from the non-perturbative region, and in order for the perturbative approach to make sense, \(\mu\) has to be much larger than \(\Lambda_{QCD}\) so that \(\alpha_s(\mu)\) is small.

We notice that although most of the calculations show that PQCD is self-consistent and applicable to the exclusive processes at currently experimental accessible energy region, the numerical predictions for the pion form factor are much smaller than the experimental data. There are two possible explanations: one is that the non-perturbative contribution will be important in this region; the other is that the non-leading order contribution in perturbative expansions may be also important in this region. To make choice between those two possible explanations one needs to analyze the non-leading contributions which come from higher-twist effect\[10\], higher order in \(\alpha_s\)\[11\], and higher Fock states\[7\] etc.

Employing the modified factorization expression for the pion form factor proposed by Li and Sterman\[5\], Refs.\[12\] considered the effect of the transverse momentum \((k_T)\) in the wave function and found that the transverse momentum in the wave function plays the role to suppress perturbative prediction. V.M. Braun et al.\[13\] gave a detailed quantitative analysis of the pion form factor in the region of intermediate momentum transfers in the LC sum rule approach and they observed a strong numerical cancellation between the soft contribution and the power suppressed hard contribution of higher twist and then the total non-perturbative correction to the usual PQCD result to be of order 30% for \(Q^2 \sim 1 GeV^2\).

One of the other sources which may provide non-leading perturbative contribution is the higher helicity components in the LC wave function\[9, 15, 17\]. However, the results for the contribution coming from higher helicity components to the pion form factor in the
high energy region are conflicting in literature\cite{15, 17}. The hard scattering amplitude for
the higher helicity components of the pion form factor at the leading order of $\alpha_s(Q^2)$ was
given by Ref.\cite{9} in the LC framework. In present paper, we recalculate all the helicity
components’ contributions to the pion form factor within the LC PQCD framework, which
is consistent with the using of LC wave function. Our calculation keeps the transverse
momentum dependence fully in the hard scattering amplitude, i.e. such dependence is
kept in both the quark propagator and the gluon propagator, and the resultant expression
gives the right power behavior of the hard contribution from the higher helicity components
as $Q^2$ goes to large energy region. Furthermore, we carry out the numerical calculations
for the hard and the soft parts of all the helicity components’ contributions. In order to
explain our picture and to clarify the difference between Ref.\cite{17} and Ref.\cite{15}, we employ a
model LC wave function with reasonable constraints. We show that it is substantial to take
$k_T$ dependence in the wave function into account and to keep the transverse momentum
dependence fully in the hard scattering amplitude in the $k_T$ factorization formalism within
the LC framework.

The purpose of this paper is to reanalyze the effects coming from the higher-helicity
components of the pion wave function within the framework of LC PQCD and the $k_T$
factorization formalism, then give a comparative study on the contributions from different
helicity components within the soft and the hard region respectively. In section II, based on
the $k_T$ factorization formula, the hard scattering amplitude is given within the LC frame-
work. In section III, with a model LC wave function, the hard contributions from different
helicity components of pion are analyzed. Section IV is devoted to give a discussion of the
soft part contribution, especially on the contribution from different helicity components.
Conclusion and a brief summary are presented in the final section.

II. HARD SCATTERING AMPLITUDE WITH $k_T$ DEPENDENCE

In the light cone quantization, the pion form factor can generally be expressed by using
the Drell-Yan-West ($q^+ = 0$) frame\cite{16},

$$F_n(Q^2) = \hat{\Psi} \otimes \hat{\Psi} = \sum_{n, \lambda_i} \int [dx_i][dk_{i\perp}][n_x] \Psi_n^\ast(x_i, k_{i\perp}, \lambda_i)\Psi_n(x_i, k_{i\perp} + \delta_i q_{\perp}, \lambda_i), \quad (2)$$
where the summation extends over all quark/gluon Fock states which have a non-vanishing overlap with the pion, \( \Psi_n \) are the corresponding wave functions which describe both the low and the high momentum partons, \([dx][dk_{\perp}]\) is the relativistic measure within the \( n \)-particle sector and \( \delta_i = (1 - x_i) \) or \((-x_i)\) depending on whether \( i \) refers to the struck quark or a spectator, respectively. From Eq.\((2)\) and the \( k_T \) factorization formula Eq.\((1)\), at higher momentum transfer, the hard contribution to the pion form factor can be written as\([5, 8, 9, 17]\)

\[
F_\pi(Q^2) = \int [dx][dy][d^2k_\perp][d^2l_\perp]\psi_\ast(1-x)Q(x, k_\perp, \lambda)T_H(x, y, q_\perp, l_\perp, \lambda, \lambda')\psi(1-y)Q(y, l_\perp, \lambda') + \cdots,
\]

where the ellipses represent the higher Fock states’ contributions, \([dx] = dx_1dx_2\delta(1 - x_1 - x_2)\) and \([d^2k_\perp] = d^2k_\perp/16\pi^3\). \( \psi(1-x)Q(x, k_\perp, \lambda) \) is the valence Fock state LC wave function with helicity \( \lambda \) and with a cut-off on \(|k_\perp|\) that is of order \((1-x)Q\). Such a cut-off on \(|k_\perp|\) is necessary to insure that the wave function is only responsible for the lower momentum region. \( T_H \) contains all two-particle irreducible amplitudes for \( \gamma^* + q\bar{q} \rightarrow q\bar{q} \) and should be calculated from the time-ordered diagrams in LC PQCD. In the light cone gauge (\( A^+ = 0 \)), the nominal power law contribution to \( F_\pi(Q^2) \) as \( Q \rightarrow \infty \) is \( F_\pi(Q^2) \sim 1/(Q^2)^{n-1} \)\([18]\), under the condition that \( n \) quark or gluon constituents are forced to change direction. Thus only the \( q\bar{q} \) component of \( \psi(1-x)Q(x, k_\perp, \lambda) \) contributes at the leading \( 1/Q^2 \).

The lowest-order contribution for the hard scattering amplitude \( T_H \) comes from the one-gluon exchange shown in Fig.\([1]\). To simplicity our notations, we separate the spin-space wave function \( \chi^K(x, k_\perp, \lambda) \) out from the whole LC wave function, i.e., \( \psi(1-x)Q(x, k_\perp, \lambda) \rightarrow \chi^K(x, k_\perp, \lambda)\varphi(1-x)Q(x, k_\perp, \lambda) \) and then combined the spin-space wave function \( \chi^K(x, k_\perp, \lambda) \) into the original \( T_H \) to form a new one, i.e.,

\[
T_H = \xi_1T_H^{(\lambda_1+\lambda_2=0)}(\uparrow\downarrow \rightarrow \uparrow\downarrow) + \xi_1T_H^{(\lambda_1+\lambda_2=0)}(\downarrow\uparrow \rightarrow \downarrow\uparrow) + \xi_2T_H^{(\lambda_1+\lambda_2=1)}(\uparrow\uparrow \rightarrow \uparrow\uparrow) + \xi_2T_H^{(\lambda_1+\lambda_2=-1)}(\downarrow\downarrow \rightarrow \downarrow\downarrow),
\]

where \( \lambda_{1,2} \) are the helicities for the (initial or final) pion’s two constitute quarks respectively, \( \xi_1 = \frac{m^2}{2(m^2+k_{\perp}^2)^{1/2}[m^2+k_{\perp}^2]^{1/2}} \) and \( \xi_2 = \frac{k_{\perp}^2}{2(m^2+k_{\perp}^2)^{1/2}[m^2+k_{\perp}^2]^{1/2}} \) are two coefficients derived from \( \chi^K(x, k_\perp, \lambda) \). The spin space wave function \( \chi^K(x, k_\perp, \lambda) \) which comes from the spin space Wigner rotation can be found in Ref.\([14]\). Because both photon and gluon are vector particles, the quark helicity is conserved at each vertex in the limit of vanishing quark
FIG. 1: Six leading order time-ordered Feynman diagrams for the hard scattering amplitude, where $p_1 = (x_1, \mathbf{k}_\perp)$, $p_2 = (x_2, -\mathbf{k}_\perp)$, $p'_1 = (y_1, y_1 \mathbf{q}_\perp + \mathbf{l}_\perp)$, $p'_2 = (y_2, y_2 \mathbf{q}_\perp - \mathbf{l}_\perp)$.

Hence there is no hard-scattering amplitude with quark and antiquark helicities being changed.

To simplify the hard scattering amplitude, we adopt the standard momentum assignment at the “infinite-momentum” frame \cite{2},

$$P_\pi = \left( P^+, P^-, \mathbf{P}_\perp \right) = (1, 0, \mathbf{0}_\perp), \quad q = (0, q_\perp^2, \mathbf{q}_\perp),$$

where $P^+$ is arbitrary because of Lorentz invariance and the square of the momentum transfer $Q^2 = -q^2 = q_\perp^2$. Using $D$ to denote the “energy-denominator” in the 6 Feynman diagrams ($x^+$-ordered diagrams), all the needed “energy-denominators” are listed in the following \cite{3},

$$D_{11} = -\frac{(x_2 \mathbf{q}_\perp + \mathbf{k}_\perp)^2}{x_1 x_2} - \frac{[y_2(x_2 \mathbf{q}_\perp + \mathbf{k}_\perp) - x_2 \mathbf{l}_\perp]^2}{x_2 y_2(y_1 - x_1)},$$

$$D_{12} = -\frac{(x_2 \mathbf{q}_\perp + \mathbf{k}_\perp)^2}{x_1 x_2},$$

$$D_{21} = -\frac{l_\perp^2}{y_1 y_2} + \frac{[y_2(x_2 \mathbf{q}_\perp + \mathbf{k}_\perp) - x_2 \mathbf{l}_\perp]^2}{x_2 y_2(y_1 - x_1)}, \quad D_{22} = D_{12},$$

$$D_{32} = -\frac{k_\perp^2}{x_1 x_2} - \frac{[y_2(x_2 \mathbf{q}_\perp + \mathbf{k}_\perp) - x_2 \mathbf{l}_\perp]^2}{x_2 y_2(y_1 - x_1)}, \quad D_{31} = D_{11},$$

$$D_{i+3,j} = D_{i,j}(x \leftrightarrow y, \mathbf{k}_\perp \leftrightarrow -\mathbf{l}_\perp), \quad (i = 1, 2, 3; \ j = 1, 2),$$

where the last equation comes from the charge symmetry. With the help of the above
equations, the hard scattering amplitude can be shortly expressed as,

\[ T_{H}^{(\lambda_1+\lambda_2)} = g^2 C_F \left( T_a^{(\lambda_1+\lambda_2)} + T_b^{(\lambda_1+\lambda_2)} + T_c^{(\lambda_1+\lambda_2)} \right) + \begin{cases} x \leftrightarrow y \\ k_\perp \leftrightarrow -l_\perp \end{cases}, \]  

(11)

where the three terms in the parentheses, which correspond to Fig.1.a, Fig.1.b and Fig.1.c respectively, can be written as

\[ T_a^{(\lambda_1+\lambda_2)} = \frac{N^{(\lambda_1+\lambda_2)} \theta(y_1-x_1)}{D_{11} D_{12}} \frac{y_1-x_1}{y_1-x_1} + T_a^{in}, \quad T_a^{in} = \frac{-4 \theta(y_1-x_1)}{D_{12} (y_1-x_1)^2}, \]  

(12)

\[ T_b^{(\lambda_1+\lambda_2)} = \frac{N^{(\lambda_1+\lambda_2)} \theta(x_1-y_1)}{D_{21} D_{22}} \frac{x_1-y_1}{x_1-y_1} + T_b^{in}, \quad T_b^{in} = \frac{-4 \theta(x_1-y_1)}{D_{22} (x_1-y_1)^2}, \]  

(13)

\[ T_c^{(\lambda_1+\lambda_2)} = \frac{N^{(\lambda_1+\lambda_2)} \theta(y_1-x_1)}{D_{31} D_{32}} \frac{y_1-x_1}{y_1-x_1}. \]  

(14)

Here \( T_a^{in} \) and \( T_b^{in} \) represent the contributions from the instantaneous diagrams in the light-cone PQCD. The numerator \( N^{(\lambda_1+\lambda_2)} \) is the sum of some spinors and \( \gamma \) matrixes and for the usual helicity components \( (\lambda_1+\lambda_2 = 0) \),

\[ N^{(\lambda_1+\lambda_2=0)} = -q_\perp^2 \left( \frac{x_2(x_1 x_2 + y_1 y_2)}{x_1 (y_1 - x_1)^2} \right) - k_\perp^2 \left( \frac{(x_1 x_2 + y_1 y_2)}{x_1 (y_1 - x_1)^2} \right) - l_\perp^2 \left( \frac{(y_2 y_1 + y_1 y_2)}{y_1 (y_1 - x_1)^2} \right) - \left( 2q_\perp \cdot k_\perp \right) \left( \frac{(x_1 x_2 + y_1 y_2)}{x_1 (y_1 - x_1)^2} \right) + \left( q_\perp \cdot l_\perp \right) \left( \frac{(y_2 y_1 + y_1 y_2)}{x_1 (y_1 - x_1)^2} \right) \pm i \left( \frac{(x_2 - y_1)}{x_1 x_2 y_1 y_2} \right) \]  

where the plus sign corresponds to \( (\uparrow\downarrow \rightarrow \downarrow\uparrow) \) and the minus sign corresponds to \( (\downarrow\uparrow \rightarrow \uparrow\downarrow) \). And for the higher helicity components \( (\lambda_1+\lambda_2 = \pm 1) \),

\[ N^{(\lambda_1+\lambda_2=\pm 1)} = -q_\perp^2 \left( \frac{x_2(x_1 x_2 + y_1 y_2)}{x_1 (y_1 - x_1)^2} \right) - k_\perp^2 \left( \frac{(x_1 x_2 + y_1 y_2)}{x_1 (y_1 - x_1)^2} \right) - l_\perp^2 \left( \frac{(x_2 y_1 + x_1 y_2)}{y_1 (y_1 - x_1)^2} \right) - \left( 2q_\perp \cdot k_\perp \right) \left( \frac{(x_2 y_1 + x_1 y_2)}{x_1 (y_1 - x_1)^2} \right) + \left( q_\perp \cdot l_\perp \right) \left( \frac{(x_2 y_1 + x_1 y_2)}{x_1 (y_1 - x_1)^2} \right) \pm i \left( \frac{(l_\perp \times (k_\perp + x_2 q_\perp))}{x_1 x_2 y_1 y_2} \right), \]  

(15)

where the plus sign corresponds to \( \lambda_1 + \lambda_2 = 1 \ (\uparrow\uparrow \rightarrow \uparrow\uparrow) \) and the minus sign corresponds to \( \lambda_1 + \lambda_2 = -1 \ (\downarrow\downarrow \rightarrow \downarrow\downarrow) \).

In order to further simplify the hard scattering amplitude, we adopt the following two prescriptions: 1) It is pointed out in Ref. [8] that when one concerns with the effect from the intrinsic transverse momenta, the terms proportional to the “bound energies” of the pions...
in the initial and final states \( \sim k_\perp^2/(x_1x_2) \) and \( \sim l_\perp^2/(y_1y_2) \) can be ignored to avoid the involvement of the higher Fock states’ contributions [40, 2]. Notice that in the factorization expression for the pion form factor Eq. (3), we have \( k_\perp^2 \ll q_\perp^2 \) and \( l_\perp^2 \ll q_\perp^2 \). Hence when calculating \( T_H \) to the next-to-leading order in \( 1/Q \), we can safely neglect the terms such as \( k_\perp^2/q_\perp^2 \) and \( l_\perp^2/q_\perp^2 \) in both the “energy denominators” and the numerator \( N^{(\lambda_1+\lambda_2)} \).

The natural variable to make a separation of perturbative contributions from those intrinsic to the bound-state wave function is the LC energy in the LC perturbative expansion [8, 20]. Under such condition, the two energy flow (\( -\frac{(x_2q_\perp+k_\perp)^2}{x_1x_2} \)) and (\( -\frac{(y_2(x_2q_\perp+k_\perp)-x_2l_\perp)^2}{x_2y_2(y_1-x_1)} \)) in the gluon propagator should be large, otherwise we can’t apply the PQCD, i.e.,

\[
(x_2q_\perp + k_\perp)^2 \gg \langle k_\perp^2 \rangle \sim \bar{\Lambda}^2
\]

and

\[
(y_2(x_2q_\perp + k_\perp) - x_2l_\perp)^2 \gg \langle k_\perp^2 \rangle, \quad \langle l_\perp^2 \rangle \sim \bar{\Lambda}^2
\]

are the conditions which make the PQCD applicable, where \( \bar{\Lambda} \), being of \( \mathcal{O}(\Lambda_{QCD}) \), represents a hadronic scale.

Applying the above two prescriptions, we finally obtain

\[
T_H = T_H^{(\lambda_1+\lambda_2=0)} + T_H^{(\lambda_1+\lambda_2=\pm1)} \tag{17}
\]

with

\[
T_H^{(\lambda_1+\lambda_2=0)} = \frac{16\xi_1 \pi C_F \alpha_s(Q^2)}{(1-x)(1-y)xy} \times (((x-1)q_\perp^2 - 2k_\perp \cdot q_\perp)(2l_\perp \cdot q_\perp + (y-1)q_\perp^2))^{-1} \times \nonumber
\]

\[
((x-1)(2l_\perp \cdot q_\perp + (y-1)q_\perp^2) - 2(y-1)k_\perp \cdot q_\perp)^{-1} \times \nonumber
\]

\[
(2(y-1)y(1-y + x(2y-1))(k_\perp \cdot q_\perp)^2 + (x-1)x(2l_\perp \cdot q_\perp + (y-1)q_\perp^2) \cdot \nonumber
\]

\[
((1-y + x(2y-1))(l_\perp \cdot q_\perp) + 2(x-1)(y-1)q_\perp^2) \nonumber
\]

\[
(x-1)(y-1)y(k_\perp \cdot q_\perp) \cdot (8x(l_\perp \cdot q_\perp) + (1-y + x(6y-5))q_\perp^2) \bigg), \tag{18}
\]

and

\[
T_H^{(\lambda_1+\lambda_2=\pm1)} = \frac{8(\xi_2 + \xi_2^*) \pi A^2 C_F \alpha_s(Q^2)}{(1-x)(1-y)xy} \times (((x-1)q_\perp^2 - 2k_\perp \cdot q_\perp)(2l_\perp \cdot q_\perp + (y-1)q_\perp^2))^{-1} \times \nonumber
\]

\[
((x-1)(2l_\perp \cdot q_\perp + (y-1)q_\perp^2) - 2(y-1)k_\perp \cdot q_\perp)^{-1} \bigg(2(x-1)x(l_\perp \cdot q_\perp)^2 + \nonumber
\]

\[
(y-1)(2y(k_\perp \cdot q_\perp)^2 + (x-1)(x(l_\perp \cdot q_\perp) - y(k_\perp \cdot q_\perp))q_\perp^2) \bigg). \tag{19}
\]
After doing a simple transformation, one may find that the obtained hard scattering amplitude for the higher helicity components \((\lambda_1 + \lambda_2 = \pm 1)\) coincides well with the one obtained in Ref. [9], while the hard scattering amplitude for the usual helicity components \((\lambda_1 + \lambda_2 = 0)\) is different from others after including all the \(k_T\) dependence in the LC PQCD framework. Due to the complicated integral in Eq. (3), Ref. [9] didn’t give the numerical results for the higher helicity contribution of the pion form factor. We will apply the VEGAS program [35] to evaluate the hard contribution in the next sections.

From Eqs. (18,19), ignoring the \(k_\perp\) dependence, we obtain

\[
T_H^{(\lambda_1 + \lambda_2 = 0)} = 2 T_H^{(\lambda_1 + \lambda_2 = 0)}(\uparrow \downarrow \rightarrow \uparrow \downarrow) + T_H^{(\lambda_1 + \lambda_2 = 0)}(\downarrow \uparrow \rightarrow \downarrow \uparrow) = \frac{16\pi C_F\alpha_s(Q^2)}{x_2 y_2 Q^4},
\]

\[
T_H^{(\lambda_1 + \lambda_2 = \pm 1)} = T_H^{(\lambda_1 + \lambda_2 = \pm 1)}(\uparrow \uparrow \rightarrow \uparrow \uparrow) + T_H^{(\lambda_1 + \lambda_2 = -1)}(\downarrow \downarrow \rightarrow \downarrow \downarrow) = 0 . \quad (20)
\]

It can be found from Eqs. (19,20) that the leading contribution from the higher helicity components is of order \(1/Q^4\), which is next-to-leading contribution compared to that of the ordinary helicity components.

### III. HARD CONTRIBUTION TO THE PION FORM FACTOR

In order to get the hard contribution for the pion form factor from Eq. (3), we need to know the soft hadronic wave function. Several important non-perturbative approaches have been developed to provide the theoretical predictions for the hadronic wave functions [13, 14, 20, 21, 22, 23, 24]. One useful way is to use the approximate bound state solution of a hadron in terms of the quark model as the starting point for modelling the hadronic valence wave function. The Brodsky-Huang-Lepage (BHL) prescription [20] of the hadronic wave function is in fact obtained in this way by connecting the equal-time wave function in the rest frame and the wave function in the infinite momentum frame. In Ref. [14], based on the BHL prescription, a revised LC quark model wave function has been raised that can give both the approximate asymptotic distribution amplitude and the reasonable valence state structure function which does not exceed the pion structure function data simultaneously. So in the present paper, we will use this revised LC quark model wave function for our latter discussions, i.e.

\[
\Psi(x, k_\perp) = \varphi_{BHL}(x, k_\perp)\chi^K(x, k_\perp) = A \exp\left[-\frac{k_\perp^2 + m^2}{8\beta^2 x(1-x)}\right]\chi^K(x, k_\perp), \quad (21)
\]
with the parameters, the normalization constant $A$, the harmonic scale $\beta$ and the quark mass $m$ to be determined. With the help of the model wave function, from Eq. (18), we can obtain the leading-twist hard part contribution to the pion form factor. From Eq. (18) we obtain the contribution from the usual helicity components ($\lambda_1 + \lambda_2 = 0$),

$$F_{\pi}^{(\lambda_1 + \lambda_2 = 0)}(Q^2) = \int dx dy [d^2 k_\perp][d^2 l_\perp] \frac{8\pi A^2 \xi_1 C_F \alpha_s(Q^2)}{(1 - x)(1 - y)xy} \times \exp \left( -\frac{m^2 + k_\perp^2}{8\beta^2} + \frac{m^2 + l_\perp^2}{y(1 - y)} \right) \times$$

$$\left( \frac{x(x + y - 2xy - 1)}{(1 - x)q_\perp^2 + 2q_\perp \cdot k_\perp} + \frac{y(x + y - 2xy - 1)}{(1 - y)q_\perp^2 - 2q_\perp \cdot l_\perp} + \frac{x + y - x^2 - y^2}{2(1 - y)q_\perp \cdot l_\perp - 2(1 - x)q_\perp \cdot l_\perp + (1 - x)(1 - y)q_\perp^2} \right),$$

(22)

and from Eq. (19) we obtain the contribution from higher helicity components ($\lambda_1 + \lambda_2 = \pm 1$),

$$F_{\pi}^{(\lambda_1 + \lambda_2 = \pm 1)}(Q^2) = \int dx dy [d^2 k_\perp][d^2 l_\perp] \frac{4\pi A^2 (\xi_2 + \xi_2^*) C_F \alpha_s(Q^2)}{(1 - x)(1 - y)xy} \times \exp \left( -\frac{m^2 + k_\perp^2 + m^2 + l_\perp^2}{8\beta^2} \right)$$

$$\times \left( \frac{-x}{(1 - x)q_\perp^2 + 2q_\perp \cdot k_\perp} + \frac{-y}{(1 - y)q_\perp^2 - 2q_\perp \cdot l_\perp} + \frac{x + y - 2xy}{2(1 - y)q_\perp \cdot l_\perp - 2(1 - x)q_\perp \cdot l_\perp + (1 - x)(1 - y)q_\perp^2} \right),$$

(23)

By integrating over the azimuth angles for $k_\perp$ and $l_\perp$ with the integration formula shown in the Appendix, the above six dimensional integration can be reduced to four dimensional integration, which can then be dealt with by numerical calculation with the help of the VEGAS program.

Integrating over the azimuth angles for $k_\perp$ and $l_\perp$, we obtain the contribution from the usual helicity components ($\lambda_1 + \lambda_2 = 0$),

$$F_{\pi}^{(\lambda_1 + \lambda_2 = 0)}(Q^2) = \int dx dy d\eta_1 d\eta_2 \frac{A^2 \xi_1 C_F \alpha_s(Q^2)}{32\pi^3 xy} \exp \left( -\frac{m^2 + |k_\perp|^2}{x(1 - x)} + \frac{m^2 + |l_\perp|^2}{y(1 - y)} \right) \times$$

$$\left( \frac{x(x + y - 1 - 2xy)}{(1 - x)\sqrt{1 - \eta_1^2}} + \frac{y(x + y - 1 - 2xy)}{(1 - y)\sqrt{1 - \eta_2^2}} + \frac{x + y - x^2 - y^2}{(1 - x)(1 - y)\sqrt{1 - \eta_1^2\eta_2^2}} \right),$$

(24)

and the contribution from the higher helicity components ($\lambda_1 + \lambda_2 = \pm 1$),

$$F_{\pi}^{(\lambda_1 + \lambda_2 = \pm 1)}(Q^2) = -\int dx dy d\eta_1 d\eta_2 \frac{A^2 \xi_3 C_F \alpha_s(Q^2)}{64\pi^3 xy} \exp \left( -\frac{m^2 + |k_\perp|^2}{x(1 - x)} + \frac{m^2 + |l_\perp|^2}{y(1 - y)} \right) \times$$

$$\left( \frac{(x + y - 2xy)}{(1 - x)(1 - y)\eta_1\eta_2\sqrt{1 - \eta_1^2\eta_2^2}} \right),$$

(25)
FIG. 2: The hard contribution to the pion form factor $Q^2 F_π(Q^2)$. The dotted line stands for the contribution from the usual helicity ($\lambda_1 + \lambda_2 = 0$) components, the dashed line stands for the contribution from the higher helicity ($\lambda_1 + \lambda_2 = \pm 1$) components and the solid line is the total hard contribution, which is the combined result for all the helicity components.

where $\xi_3 = \frac{|k_\perp||l_\perp|}{[m^2+k_\perp^2]^{1/2}[m^2+l_\perp^2]^{1/2}}$, $|k_\perp| = Q(1 - x)\eta_1/2$ and $|l_\perp| = Q(1 - y)\eta_2/2$, with $\eta_{1,2}$ in the range of $(0, 1)$. In Eq. (25), there is an overall minus sign and because the integrand is always positive, we can draw the conclusion that the higher helicity components will always suppress the contribution from the usual helicity components.

With the help of the LC wave function Eq. (21) and its parameter values shown in Eq. (36), we show the pion form factor with or without the higher helicity components in Fig. 2. One may observe a large suppression comes from the higher helicity components for the pion form factor as compared to the prediction obtained in the original hard scattering model [17]. This large suppression was obtained by Ref. [15] with a quite different picture. They argued that the transverse momentum in the quark propagator is of small contribution (about 15% [5, 37]). The hard-scattering amplitude, after neglecting the transverse momentum dependence in the quark propagator, was taken to be

$$T_H^{(\lambda_1+\lambda_2=\pm 1)} = -T_H^{(\lambda_1+\lambda_2=0)} = -\frac{4g^2C_F}{x_2y_2Q^2 + (k_\perp - l_\perp)^2} Q^2 \approx \infty \sim -\frac{4g^2C_F}{x_2y_2Q^2} + \frac{4g^2C_F(k_\perp - l_\perp)^2}{(x_2y_2Q^2)^2}. \quad (26)$$

It can be seen from Eq. (26) that the asymptotic ($Q^2 \to \infty$) behaviors of the two helicity states are directly with opposite signs and both states make the contribution at the order of $1/Q^2$. However it is not a right argument and it is this factor that causes the asymptotic
behavior of higher helicity contribution is of order $1/Q^4$ other than $1/Q^2$. In the present work, we have considered the $k_T$ dependence both in the wave function and in the hard scattering amplitude consistently within the LC PQCD approach, then our results have a right power behavior for the higher helicity components’ contributions.

IV. A DISCUSSION OF THE SOFT CONTRIBUTION TO THE PION FORM FACTOR

In the above sections, we have shown that the inclusion of the higher helicity components suppresses the hard scattering contribution at moderate $Q^2$. In order to compare our predictions with the present experimental data, we need to know the contribution from the soft part. Since this part is model dependent and is still under progress, as an example, we consider the soft contribution to the pion form factor with the model LC wave function shown in Eq. (21) and study the different helicity components’ soft contribution to the pion form factor separately.

For the soft part contribution, we have

$$F_s^{\pi^+}(Q^2) = \int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} \sum_{\lambda, \lambda'} \Psi^*(x, k_\perp, \lambda)\Psi(x, k'_\perp, \lambda') + \cdots,$$  

(27)

where $\lambda, \lambda'$ are the helicities of the wave function respectively, and the first term is the lowest order contribution from the minimal Fock-state and the ellipses represent those from higher Fock states, which are down by powers of $1/Q^2$ and by powers of $\alpha_s$.

Taking the LC wave function as is shown in Eq. (21), we obtain

$$F_s^{\pi^+}(Q^2) = \int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} \frac{m^2 + k_\perp \cdot k'_\perp}{\sqrt{m^2 + k_\perp^2} \sqrt{m^2 + k'_\perp^2}} \times A^2 \exp \left( -\frac{k_1^2 + m^2}{8\beta^2 x(1-x)} - \frac{k_1'^2 + m^2}{8\beta^2 x(1-x)} \right),$$  

(28)

where $k'_\perp = k_\perp + (1-x)q_\perp$ for the final state LC wave function when taking the Drell-Yan-West assignment. We proceed to integrate the transverse momentum $k_\perp$ in Eq. (28) with the help of the Schwinger $\alpha$–representation method,

$$\frac{1}{A^x} = \frac{1}{\Gamma(\kappa)} \int_0^\infty \alpha^{\kappa-1} e^{-\alpha A} d\alpha.$$  

(29)

Doing the integration over $k_\perp$, we obtain

$$F_s^{\pi^+}(Q^2) = \int_0^1 dx \int_0^\infty d\lambda \frac{A^2}{128\pi^2(1+\lambda)^3} \exp \left( -\frac{8m^2(1+\lambda)^2 + Q^2(1-x)^2(2+2\lambda(4+\lambda))}{32(1-x)x\beta^2(1+\lambda)} \right),$$  

(28)
\[
\times \left( I_0 \left( \frac{Q^2(x-1)\lambda^2}{32x\beta^2(1+\lambda)} \right) \left( 32(1-x)x\beta^2(1+\lambda) - Q^2(1-x)^2(2+\lambda(4+\lambda)) \right) \\
+ 8m^2(1+\lambda)^2 \right) - I_1 \left( \frac{Q^2(x-1)\lambda^2}{32x\beta^2(1+\lambda)} \right) Q^2(1-x)^2\lambda^2 \right),
\]
where the \( I_n (n = 0, 1) \) is the modified Bessel function of the first kind. After taking the expansion in the small \( Q^2 \) limit, we obtain the probability,

\[
P_{qq} = F_{\pi^+}^s(Q^2)|_{Q^2=0} = \int dx \frac{d^2k_\perp}{(16\pi)^3} |\Psi(x, k_\perp)|^2
\]

\[
= \int_0^1 dx \int_0^\infty d\lambda \frac{A^2}{16\pi^2(1+\lambda)^2} \exp \left( \frac{m^2(1+\lambda)}{4(x-1)x\beta^2} \right) \times \left( m^2(1+\lambda) + 4x(1-x)\beta^2 \right)
\]

\[
\times \left( 8(1-x)x\beta^2 + m^2(1+\lambda) \right).
\]

In the above two equations, one may observe that the terms in the big parenthesis that are proportional to \( m^2 \) come from the ordinal helicity components, while the remaining terms in the big parenthesis are from the higher helicity components.

The parameters in the wave function can be determined by several reasonable constraints [14]. Two constraints can be derived from \( \pi \to \mu\nu \) and \( \pi^0 \to \gamma\gamma \) decay amplitude [20]:

\[
\int_0^1 dx \int d^2k_\perp \Psi(x, k_\perp) = f_{\pi}/(2\sqrt{3}),
\]

and

\[
\int_0^1 dx \Psi(x, k_\perp = 0) = \sqrt{3}/f_{\pi},
\]

where \( f_{\pi} \) is the pion decay constant: \( \langle 0|q(0)\gamma^+\gamma_5q(0)|P \rangle = if_{\pi}P^+ \), the experimental value of which is 92.4 \( \pm 0.25 MeV \) [25]. Experimentally the average quark transverse momentum of pion \( \langle k_\perp^2 \rangle_\pi \) is approximately of the order \( (300 MeV)^2 \) [26]. The quark transverse momentum of the valence state in the pion is defined as

\[
\langle k_\perp^2 \rangle_{qq} = \int dx \frac{d^2k_\perp}{(16\pi)^3} |k_\perp^2| \left| \frac{\Psi(x, k_\perp)}{P_{qq}} \right|^2,
\]

\[13\]
and it should be larger than $\langle k_\perp^2 \rangle_\pi$. We thus could require that $\sqrt{\langle k_\perp^2 \rangle_{q\bar{q}}}$ has the value of a few hundreds MeV, serving as the third constraint. Using the constraints and the model wave function Eq. (21), we obtain,

$$m = 310 MeV \ ; \ \beta = 396 MeV \ ; \ A = 0.050 MeV^{-1} ,$$  

(36)

for $\langle k_\perp^2 \rangle \approx (367 MeV)^2$. And by using the above parameters, we obtain

$$\langle r_{\pi^+}^2 \rangle^{q\bar{q}} = 0.216 fm^2 ,$$  

(37)

$$P_{q\bar{q}} = P_{q\bar{q}}^{(\lambda_1+\lambda_2=0)} + P_{q\bar{q}}^{(\lambda_1+\lambda_2=\pm 1)} = 0.744 .$$  

(38)

The value of $\langle r_{\pi^+}^2 \rangle^{q\bar{q}}$ is in nice agreement with the ones obtained in Refs. [31, 32]. In fact, we have used the same monopole ansatz see in Refs. [32, 34]. It is shown that the valence quark radius is smaller than the experimental value of the pion charged radius ($(0.671 \pm 0.008 fm)^2$). Therefore the valence portion of a hadron is more compact than the hadron radius. For the probability of finding the valance states in the pion, we have $(P_{q\bar{q}}^{(\lambda_1+\lambda_2=0)} = 0.398)$ for the usual helicity components and $(P_{q\bar{q}}^{(\lambda_1+\lambda_2=\pm 1)} = 0.346)$ for the higher helicity states, which show that the higher helicity components have the same importance as that of the usual helicity components. It has been shown that even though we have added the contributions from the higher helicity states, the probability of finding the minimal $q\bar{q}$ Fock state in pion is still less than unity, i.e. $(P_{q\bar{q}} = 0.744) < 1$. This is shown clearly in Fig.(a), so it is necessary to take the higher Fock states and the higher twist terms into consideration to give a full understanding of the pion form factor at the energy region $Q^2 \rightarrow 0$. It should be noticed that if one normalizes the valence Fock state to unity without including the higher helicity components, then the soft and hard contributions from the valence state can be enhanced and become important inadequately.

The result for the soft contribution to the pion form factor is shown in Fig.(b). From Fig.(b), one may observe a quite different behavior from that of the hard contribution for the higher helicity components $(\lambda_1+\lambda_2 = \pm 1)$. In the energy region $Q^2 \lesssim 1 GeV^2$, the higher helicity components give a large enhancement (the same order contribution) to usual helicity $(\lambda_1+\lambda_2 = 0)$ components and after that the higher helicity components’ contributions will decrease with the increasing $Q^2$. At about $Q^2 \sim 4 GeV^2$, the higher helicity components’ contributions become negative and as a result, the net soft contribution will then decrease fast with the increasing $Q^2$, which tends to zero at about $Q^2 \sim 16 GeV^2$. 

14
FIG. 3: The soft contribution to the pion form factor. Left is for the pion form factor $F_\pi(Q^2)$, while the right is for $Q^2F_\pi(Q^2)$, where the contribution comes from all the helicity components are shown in dashed line, the contribution from the ordinal helicity component $\lambda_1 + \lambda_2 = 0$ is shown in dotted line and the contribution from the higher helicity components $\lambda_1 + \lambda_2 = \pm 1$ is shown in dash-dot line. The experimental data is taken from [36].

FIG. 4: The combined results for the pion form factors $Q^2F_\pi(Q^2)$. The solid line stands for the contribution from the hard part, the dotted line stands for the contribution from the soft part, the dashed line is the total Pion form factors and the dash-dot line is the usual asymptotic result. The experimental data are taken from [36].
We show the combined results that come from the hard scattering part and from the soft part for the pion form factors $Q^2 F_π(Q^2)$ in Fig. 4 where for comparison, the experimental data [36] and the well-known asymptotic behavior for the leading twist pion form factor have also been shown. It is shown that the soft contribution is less important as $Q^2 > a \text{ few } GeV^2$, since we have taken the correct normalization condition Eq. (38) and considered the suppression effect from the higher helicity components. One may observe that our present result for the pion form factor is lower than the experimental data, it is reasonable since we have not taken the higher twist effects and the higher order corrections into consideration. The next-to-leading order correction will give about $\sim 20 - 30\%$ extra contribution to the pion form factor, while the twist-3 contribution is comparable with the leading twist contribution in a large intermediate energy region ($\sim 1 - 40 GeV$) [10][41].

V. SUMMARY AND CONCLUSION

In this paper, the transverse momentum effects and the higher helicity components' contributions to the pion form factor are systematically studied based on the LC framework and the $k_T$ factorization formalism. Both collinear and $k_T$ factorization are the fundamental tools for applying PQCD to the pion form factor since they can separate the calculable perturbative contributions from the non-perturbative parts that can be absorbed into the bound-state wave functions. The $k_T$ factorization theorem has been widely applied to various processes and the $k_T$ factorization theorem for exclusive processes in PQCD has been proved by M. Nagashima and H.N. Li. Thus it provides a scheme to take the dependence of the parton transverse momentum $k_T$ into account. Ref. [5] shows that the end-point singularity can be cured by resuming the resultant double logarithms $\alpha_s \ln^2 k_T$ into a Sudakov form factor and then the PQCD analysis can make sense. In fact, the Sudakov effects have a small effects for the pion form factor in the region where experimental results are available. We note that there are $k_T$ dependence in the wave function in the $k_T$ factorization and it generates much larger effects than the Sudakov suppression to the hard scattering amplitude in the present experimental $Q^2$ region. Our results show that it is substantial to take $k_T$ dependence in the wave function into account.

The light cone formalism provides a convenient framework for the relativistic description of the hadron in terms of quark and gluon degrees of freedom, and the application of PQCD
to exclusive processes has mainly been developed in this formalism. In the present paper, we have given a consistent treatment of the pion form factor within the LC PQCD framework, i.e. both the wave function and the hard interaction kernel are treated within the framework of LC PQCD. Taking into account the spin space Wigner rotation, one may find that there are higher-helicity components \( (\lambda_1 + \lambda_2 = \pm 1) \) in the LC spin-space wave function besides the usual-helicity components \( (\lambda_1 + \lambda_2 = 0) \). We have studied the higher helicity components’ contributions to the hard part and the soft part of the pion form factor by using the light cone PQCD approach with the parton’s transverse momentum \( k_T \) included. We find that the asymptotic behavior of the hard-scattering amplitude for the higher-helicity components including the transverse momentum in the quark propagator is of order \( 1/Q^4 \) which is the next to leading order contribution compared with the contribution coming from the ordinary helicity component, but it can give sizable contribution to the pion form factor at the intermediate energies.

In order to compare our predictions with the experimental data, we need to know the contribution from the soft part. As an example, we have considered the soft contribution to the pion form factor with a reasonable wave function in the LC framework. Our results show that the soft contribution from the higher helicity components has a quite different behavior from that of the hard scattering part and has the same order contribution as that of the usual helicity \( (\lambda_1 + \lambda_2 = 0) \) components in the energy region \( (Q^2 \lesssim 1 GeV^2) \). As \( Q^2 > 1 GeV^2 \), the higher helicity components’ contributions will decrease with the increasing \( Q^2 \). At about \( Q^2 \sim 4 GeV^2 \), the higher helicity components’ contributions become negative and as a result the net soft contribution to the pion form factor will then decrease with the increasing \( Q^2 \), which tends to zero at about \( Q^2 \sim 16 GeV^2 \). Thus the soft contribution is less important in the intermediate energy region. Although the soft contribution is purely non-perturbative and model-dependent, our results show that the calculated prediction for the pion form factor should take the \( k_T \) dependence in the soft and hard parts into account beside including the higher order contribution. Therefore one needs to keep the transverse momentum in the next leading order corrections and to construct a realistic \( k_T \) dependence in the hadronic wave function in order to derive more exact prediction to the pion form factor in \( k_T \) factorization.
Acknowledgements

We would like to thank Drs F.G. Cao, J. Cao, H.N. Li and B.Q. Ma for useful discussions. This work was supported in part by the Natural Science Foundation of China (NSFC).

APPENDIX A: INTEGRATION FORMULA

The error function is defined as

$$Erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$ \hfill (A1)

An important property for the error function is \(\lim_{x \to \infty} Erf(x) = 1\).

Second we list some useful formula that are needed for integrating over the azimuth angle of the momenta \(k_\perp\) and \(l_\perp\) and then reduce the integration dimension from six to four. And the remaining four dimensional integration can be done numerically.

By using polarization coordinate, we have

$$[dk_\perp^2][dl_\perp^2] = kldkd\theta d\rho/(16\pi^3)^2,$$ \hfill (A2)

where \(k\), \(l\) and \(\theta\), \(\rho\) are the module and azimuth angle of \(k_\perp\) and \(l_\perp\) respectively. By using the following formula, the integration over the azimuth angle can be done analytically.

$$f_1(A, B) = \int_0^{2\pi} \frac{d\theta}{A + B \cos(\theta)} = \frac{2\pi}{\sqrt{(A + B)(A - B)}},$$ \hfill (A3)

$$f_2(A, B) = \int_0^{2\pi} \frac{\cos(\theta)d\theta}{A + B \cos(\theta)} = \frac{2\pi}{B} \left(1 - \frac{A}{\sqrt{(A + B)(A - B)}}\right),$$ \hfill (A4)

$$f_3(A, B) = \int_0^{2\pi} \frac{\sin(\theta)d\theta}{A + B \cos(\theta)} = 0,$$ \hfill (A5)

$$f_4(A, B) = \int_0^{2\pi} \int_0^{2\pi} \frac{\cos(\theta - \rho)d\theta d\rho}{A + B \cos(\theta)} = 0,$$ \hfill (A6)

$$f_5(A, B, C) = \int_0^{2\pi} \int_0^{2\pi} \frac{\cos(\theta - \rho)d\theta d\rho}{A + B \cos(\theta) + C \cos(\rho)},$$ \hfill (A7)

where \(A\), \(B\) and \(C\) are functions that are free from \(\theta\), \(\rho\). The result for the function \(f_5\) is very complicated and for simplicity its explicitly form will not be listed here. However, by adding a small component \((BC \cos(\theta) \cos(\rho))\) (for the integration we need to deal with, we have \(BC << A\), which corresponding to \(k_\perp \cdot l_\perp << q_\perp^2\)), it can be solved approximately,

$$f_5(A, B, C) \approx \int_0^{2\pi} \int_0^{2\pi} \frac{\cos(\theta - \rho)d\theta d\rho}{(A + B \cos(\theta))(A + C \cos(\rho))} = f_2(A, B)f_2(A, C).$$ \hfill (A8)
There one may notice that under the present approximation, the actual azimuth angle, i.e. \( \alpha \), for \( q_\perp \) will not affect the final integrated results due to the fact that after integration over \( \theta \) and \( \rho \), it will always accompanied by a factor \((\cos(\alpha)^2 + \sin(\alpha)^2) \equiv 1\).

After integrating over the azimuth angle, we can change the integration over the radius of \( k_\perp \) and \( l_\perp \) to two dimensionless variables \( \eta_1 \) and \( \eta_2 \) that are within the range of \((0, 1)\) through the relation

\[
|k_\perp| = Q(1 - x)\eta_1/2, \quad |l_\perp| = Q(1 - y)\eta_2/2.
\] (A9)

The relation is so choosing as to insure that all the quantities in the radical sign obtained by doing the azimuth angle integration are always positive.

---

[1] G.P. Lepage, S.J. Brodsky, T. Huang and P.B. Mackenzie, in *Particles and Fields-2*, page 83, Invited talk presented at the Banff summer Institute on Particle Physics, Banff, Alberta, Canada, 1981.

[2] G.P. Lepage and S.J. Brodsky, Phys.Rev.D22, 2157(1980), *ibid*. 24, 1808(1981).

[3] A. Szczepaniak, E.M. Henley and S. Brodsky, Phys.Lett. B 243, 287 (1990); M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys.Rev. Lett. 83, 1914(1999); Nucl.Phys. B591, 313(2000).

[4] H.N. Li and H.L. Yu, Phys.Rev. Lett.74, 4388(1995); Phys.Lett. B353, 301(1995); Phys.Rev. D 53, 2480(1996).

[5] H.N. Li and G. Sterman, Nucl.Phys. B325, 129(1992); J. Botts and G. Sterman, Nucl.Phys. B225, 62(1989).

[6] M. Nagashima and H.N. Li, Phys. Rev. D 67, 014019 (2003).

[7] A. Szczepaniak, C.R. Ji and A. Radyushkin, Phys. Rev. D 57, 2813 (1998).

[8] C.R. Ji, A. Pang and A. Szczepaniak, Phys.Rev. D52, 4038(1995).

[9] F.G. Cao, J. Cao, T. Huang and B.Q. Ma, Phys.Rev. D55, 7107(1997).

[10] F.G. Cao, Y.B. Dai and C.S. Huang, Euro.Phys.J. C 11, 501(1999); A. Szczepaniak, A.G. Williams, Phys.Lett. B302, 87(1993); Z.T. Wei and M.Z. Yang, Phys.Rev. D67, 094013(2003).

[11] R.D. Field, R. Gupta, S. Otto and L. Chang, Nucl.Phys. B186, 429(1981); E. Braaten and S.M. Tse, Phys.Rev. D35, 2255(1987); E.P. Kadantseva, S.V. Mikhailov and A.V. Radyushkin,
Yad.Fiz. 44, 507(1986); Sov.J.Nucl.Phys. 44, 326(1986); N.G. Stefanis, W. Schroers and H.C. Kim, Eur.Phys.J. C18, 137(2000).

[12] R. Jakob, P. Kroll, Phys.Lett. B315, 463(1993); Phys. Lett. B319, 545(E)(1993); F.G. Cao, T. Huang, Mod.Phys.lett. A13, 253(1998); Commun.Theor.Phys. 27, 217(1997).

[13] V.M. Braun, A. Khodjamirian and M. Maul, Phys.Rev. D61, 073004(2000).

[14] T. Huang, B.Q. Ma and Q.X. Shen, Phys.Rev.D49, 1490(1994).

[15] S.W. Wang and L.S. Kisslinger, Phys.Rev. D54, 5890(1996).

[16] S.D. Drell and T.M. Yan, Phys.Rev.Lett.24, 181(1970).

[17] B.Q. Ma and T. Huang, J.Phys. G21, 765(1995).

[18] S.J. Brodsky, SLAC-PUB-2447, 1979; G.P. Lapage and S.J. Brodsky, Phys.Lett. B87, 359(1979).

[19] G.P. Lepage and S.J. Brodsky, Phys.Rev.D24, 2848(1981)

[20] S.J. Brodsky, T. Huang and G.P. Lepage, in Particles and Fields-2, Proceedings of the Banff Summer Institute, Banff, Alberta, 1981, edited by A.Z. Capri and A.N. Kamal (Plenum, New York, 1983), P143; G.P. Lepage, S.J. Brodskyyk T.Huang, and P.B. Mackenize, ibid., p83; T. Huang, in Proceedings of XXth International Conference on High Energy Physics, Madison, Wisconsin, 1980, edited by L.Durand and L.G. Pondrom, AIP Conf.Proc.No. 69(AIP, New York, 1981), p1000.

[21] V.L. Chernyak and A.R.Zhitnitsky, Nucl.Phys.B201, 492 (1982); Phys.Rep.112, 173(1984); Nucl.Phys.B246, 52(1984); T. Huang, X.D. Xiang, and X.N. Wang, Chin.Phys.Lett.2, 76(1985); Phys.Rev.D35, 1013(1987).

[22] S. Gottlieb and A.S. Kronfeld, Phys.Rev.Lett.55, 2531(1985); Phys.Rev.D33, 227(1986); G.Martinelli and C.T.Sachrajda, Phys.Lett.B 217, 319(1989); D.Daniel, R.Gupta and D.G. Richards, Phys.Rev.D43, 3715(1991).

[23] P. Korll and M. Raulfs, Phys.Lett. B387, 848(1996).

[24] V.M. Braun and I.E. Filyanov, Z.Phys. C44, 157(1989); T. Huang and Q.X. Shen, Z.Phys.C50, 139(1991); C.E. Carlson and F. Gross, Phys.Rev.D36, 2060(1987).

[25] Particle Data Group, E.J. Weinberg, etal., Phys.Rev. D66, 1(2002).

[26] See, e.g., W.J. Metcalf etal., Phys.Lett. B91, 275(1980).

[27] O.C. Jacob and L.S. Kisslinger, Phys.Rev.Lett.56, 225(1986); Phys.Lett. B243, 323(1990); L.S. Kisslinger and S.W. Wang, Nucl.Phys. B399, 63(1993).
[28] L.S. Kisslinger, H.M. Choi and C.R. Ji, Phys.Rev. D63, 113005(2001).

[29] W. Schweiger Nucl.Phys.Proc.Suppl. 108, 242(2002).

[30] V. Anisovich, D. Melikhov and V. Nikonov, Phys.Rev. D52, 5295(1995).

[31] T. Huang, Nucl.Phys.B (Proc. Suppl.) 7B, 320,1989.

[32] F. Cardarelli etal., Phys.Rev. D53, 6682(1996).

[33] S.J. Brodsky and S.D. Drell, Phys.Rev. D22, 2236(1980); S.J. Brodsky, D.S. Hwang, B.Q. Ma and I. Schmidt, Nucl.Phys.B593, 311(2001).

[34] F. Cardarelli etal., Phys.Lett. B332, 1(1994).

[35] G.P. Lepage, J. Comp. Phys 27(1978) 192.

[36] L.J. Bebek etal., Phys.Rev. D9, 1229(1974); 17, 1693(1978); C.N.Brown etal., ibid. 8, 92(1973); Phys.Rev. Lett. 86, 1714(2001).

[37] H.N. Li, Phys.Rev. D48, 4243(1993); J. botts and G. Sterman, Nucl.Phys. B381, 129(1989).

[38] T. Huang and Q.X. Sheng, Z. Phys. C50, 139(1990).

[39] T. Huang and X.G. Wu, in preparation.

[40] As the transverse momenta $k_{\perp}$ and $l_{\perp}$ are included, it is necessary to take into account the contributions from higher Fock states to satisfy the gauge-invariance, since the covariant derivative $D_\mu = \partial_\mu + igA_\mu$ makes both transverse momenta $k_{\perp}$, $l_{\perp}$ and the transverse gauge degree $gA_{\perp}$ be of the same order [8].

[41] The twist-3 contribution is model dependent, if we take the wave function with a better end point behavior, then the twist-3 contribution can be greatly suppressed [39].