Towards a Model Independent Treatment of Future Solar Neutrino Data

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Abstract

Possibilities of a model independent treatment of the data from future real-time solar neutrino experiments (SNO, Super-Kamiokande and others) are discussed. It is shown that in the general case of transitions of the initial solar \(\nu_e\)'s into \(\nu_\mu\) and/or \(\nu_\tau\) the total flux of initial \(^8\text{B}\) neutrinos and the \(\nu_e\) survival probability can be determined directly from the experimental data. Relations between observable quantities are derived, which, if confronted with the experimental data, would allow to reveal the presence of sterile neutrinos in the solar neutrino flux on the earth. Neutrino transitions due to spin and resonant spin-flavour precession in the magnetic field of the sun are shortly discussed.

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1 Introduction

Solar neutrino experiments are very important for investigations of neutrino properties (masses and mixings, possible anomalously large magnetic moments, ...) and for explorations of the invisible central region of the sun in which solar energy is produced. At present, neutrinos from the sun are detected by three radiochemical detectors (Homestake [1], SAGE [3] and GALLEX [4]) and a real-time water Cherenkov detector (Kamiokande [2]). In all four experiments the detected flux of solar neutrinos is lower than the flux predicted by the Standard Solar Model (SSM) [5,6]. The most natural explanation of the possible deficit of solar neutrinos is provided by neutrino mixing [7] which produces resonant MSW [8] transitions of the initial $\nu_e$ into neutrinos of other type in the sun. In fact, all existing solar neutrino data can be described by the MSW mechanism. Two narrow regions were obtained (see Ref.[4]) for the parameters $\Delta m^2 = m_2^2 - m_1^2$ and $\sin^22\theta$ ($m_1$, $m_2$ are neutrino masses and $\theta$ is the vacuum mixing angle). However, this analysis is based on the assumption that the SSM prediction [5,6] of the neutrino fluxes from different reactions is correct. For example, the event rate in radiochemical detectors is given by

$$ N = \int_{E_{\text{th}}} \sigma(E) P_{\nu_e \rightarrow \nu_e}(E) \phi_{\nu_e}(E) \, dE, \quad (1) $$

where $\sigma(E)$ is the cross section for neutrino capture with energy threshold $E_{\text{th}}$, $P_{\nu_e \rightarrow \nu_e}(E)$ is the $\nu_e$ survival probability, $\phi_{\nu_e}(E)$ is the initial $\nu_e$ flux and $E$ is the neutrino energy. At present, in order to get informations about the survival probability $P_{\nu_e \rightarrow \nu_e}(E)$, which is determined by neutrino properties, it is necessary to use the neutrino flux $\phi_{\nu_e}(E)$ predicted by the SSM, including the neutrino flux from $^8\text{B}$ decay, which constitutes a very small part ($\sim 10^{-4}$ in the SSM) of the total flux and is strongly dependent from the temperature of the sun core, from the cross section of different nuclear reactions, especially $p^7\text{Be} \rightarrow ^8\text{B} \gamma$, etc... [3].

A new generation of real time solar neutrino experiments are under preparation and under development: SNO [10], Super-Kamiokande [11], BOREXINO [12], ICARUS [13], HELLAZ [14] and others.

In the present note we will consider the general case of transitions of the initial $\nu_e$’s into $\nu_\mu$ and/or $\nu_\tau$ due to neutrino mixing and matter effects. We will show that future solar neutrino experiments will allow to determine the survival probability $P_{\nu_e \rightarrow \nu_e}(E)$ without assumptions about the initial neutrino flux and to determine the initial neutrino flux independently from what happens to neutrinos on their way from the sun to the detector. We will discuss also possible transitions of solar $\nu_e$’s into sterile left-handed (anti)neutrinos due to Dirac and Majorana mixing [15] and transitions of solar $\nu_e$’s into sterile right-handed neutrinos (Dirac case) or active right-handed antineutrinos (Majorana case) due to spin [16] or spin-flavour [17,18] precession of neutrinos in the magnetic field of the sun. We will show that these possibilities can be tested in future solar neutrino experiments in a model independent way.
2 Transitions of solar $\nu_e$’s into $\nu_\mu$ and $\nu_\tau$

Let us start our discussion with the SNO experiment, which will probably be one of the first new-generation experiments to be realized (data-taking is scheduled to start in 1995 \cite{ref}). SNO is a 1000 tons heavy water Cherenkov detector. In this experiment solar neutrinos will be detected through observation of the following three reactions:

$$\nu_e d \rightarrow e^- p p,$$  
(2)  

$$\nu_x e^- \rightarrow \nu_x e^-,$$  
(3)  

$$\nu_x d \rightarrow \nu_x p n.$$  
(4)

Let us consider first the charged-current (CC) reaction (2), which is expected to yield about 10 CC events/day (assuming 1/3 of the SSM flux). A measurement of the CC event rate $n^\text{CC}(E)$ as a function of neutrino energy $E (E = T_e + 1.44 \text{ MeV},$ where $T_e$ is the electron kinetic energy) will allow to determine the energy spectrum of $\nu_e$ on the earth:

$$\phi_{\nu_e}(E) = \frac{n^\text{CC}(E)}{\sigma_{\nu_e d}^\text{CC}(E)},$$  
(5)

where $\sigma_{\nu_e d}^\text{CC}(E)$ is the total cross section of process (2). This cross section was calculated in Ref.\cite{ref20}.

The electron energy threshold in the SNO experiment will be $\approx 5 \text{ MeV}$. This means that only $^8\text{B}$ and hep neutrinos can be detected. According to the SSM, in the region of high electron energy ($T_e \gtrsim 13 \text{ MeV}$) the contribution of hep neutrinos to the CC events is expected to be bigger than that of $^8\text{B}$ neutrinos. However, the contribution of hep neutrinos to the total number of CC events is expected \cite{ref3} to be much less than that of $^8\text{B}$ neutrinos ($\ll 1\%$). In the following we will neglect the contribution of hep neutrinos to the total counting rates.

The spectrum of initial solar $^8\text{B}$ neutrinos is given by

$$\phi_{^8\text{B}\nu_e}(E) = \Phi_{^8\text{B}\nu_e} X(E).$$  
(6)

The function $X(E)$ is the normalized ($\int X(E) \text{d}E = 1$) neutrino spectrum from the decay $^8\text{B} \rightarrow ^8\text{Be} + e^+ + \nu_e$, which is determined by the phase space factor (corrections due to forbidden transitions where calculated in Ref.\cite{ref24}). The factor $\Phi_{^8\text{B}\nu_e}$ in Eq. (6) is the total flux of initial $^8\text{B}$ solar $\nu_e$’s. The value of $\Phi_{^8\text{B}\nu_e}$ depends on the temperature of the sun’s core, the value of the cross sections of the nuclear reactions of the $pp$ cycle, etc... The distortions of the neutrino spectra are negligibly small under solar conditions \cite{ref22}.

A measurement of the energy spectrum of detected $\nu_e$’s will allow to determine the $\nu_e$ survival probability $P_{\nu_e \rightarrow \nu_e}(E)$ up to a constant. In fact, using Eq. (5), we have

$$P_{\nu_e \rightarrow \nu_e}(E) = \frac{1}{\Phi_{^8\text{B}\nu_e}} \frac{\phi_{\nu_e}(E)}{X(E)}.$$  
(7)
A dependence of $\frac{\phi_{\nu}(E)}{X(E)}$ on the energy $E$ would be an evidence in favor of resonant transitions of neutrinos in matter (or just-so vacuum neutrino oscillations). A detailed investigation of the dependence of the survival probability on $E$ will allow to distinguish different mechanisms of neutrino transitions (for a review of possible mechanisms see, for example, Ref.[23]).

It is clear that the detection of solar neutrinos only through the observation of the CC process (4) will not allow us to know if there are $\nu_\mu$ and/or $\nu_\tau$ in the solar neutrino flux on the earth. In order to get this information it is necessary to detect solar neutrinos also through the observation of neutral current (NC) induced processes [24].

From the detection of solar neutrinos through the NC process (4) it is possible to determine the total flux of initial $^8$B neutrinos $\Phi_{^8B}$. According to the SSM, the expected rate of NC events in the SNO detector is about 10/day. The detection threshold is equal to the deuterium binding energy $\simeq 2.2$ MeV. The rate of NC events is given by

$$N_{NC} = \int_{E_{th}} \sigma_{\nu d}^{NC}(E) \sum_{\ell=e,\mu,\tau} \phi_{\nu\ell}(E) dE ,$$

(8)

where $\sigma_{\nu d}^{NC}(E)$ is the cross section for the process $\nu_x d \rightarrow \nu_x n p$ (for the calculation of $\sigma_{\nu d}^{NC}(E)$ see Ref.[25] and references therein).

We assume here that solar $\nu_e$’s can transform only into $\nu_\mu$ and $\nu_\tau$. In this case, we have

$$\sum_{\ell=e,\mu,\tau} P_{\nu_e \rightarrow \nu_\ell}(E) = 1 ,$$

(9)

where $P_{\nu_e \rightarrow \nu_\ell}(E)$ is the probability of the transition $\nu_e \rightarrow \nu_\ell$ ($\ell = e, \mu, \tau$). From Eq.(9), it follows that, independently from the mechanism of possible transitions of the initial $\nu_e$’s into $\nu_\mu$ and/or $\nu_\tau$, we have

$$\sum_{\ell=e,\mu,\tau} \phi_{\nu\ell}(E) = \phi_{\nu e}^{^8B}(E) .$$

(10)

With the help of Eqs.(3), (8) and (11), for the total flux of initial $^8$B $\nu_e$’s we obtain the following relation

$$\Phi_{^8B} = \frac{N_{NC}}{X_{\nu d}} ,$$

(11)

where

$$X_{\nu d} = \int_{E_{th}} \sigma_{\nu d}^{NC}(E) X(E) dE .$$

(12)

Thus, in the SNO experiment the total flux of $^8$B neutrinos will be measured. A comparison of the measured flux with that predicted by the SSM will be an important test of the model.

The detection of solar neutrinos through observation of CC and NC events will allow to determine the survival probability $P_{\nu_e \rightarrow \nu_e}(E)$ directly from the experimental data:

$$P_{\nu_e \rightarrow \nu_e}(E) = \beta(E) \frac{n_{NC}(E)}{N_{NC}} ,$$

(13)
where
\[ \beta(E) = \frac{X_{\nu_d}^{\text{NC}}}{\sigma_{\nu_d}^{\text{CC}}(E) X(E)} \] (14)
is a known quantity.

The sensitivity of the NC events to all types of active neutrinos is the same (\(\nu_e - \nu_\mu - \nu_\tau\) universality). For the relative contribution of \(\nu_\mu\) and/or \(\nu_\tau\) to NC events we have
\[ R_{\text{NC}} = 1 - \frac{\int_{E_{\text{th}}} \sigma_{\nu_d}^{\text{NC}}(E) P_{\nu_e \rightarrow \nu_e}(E) X(E) \, dE}{\times_{\nu_d}^{\text{NC}}} , \] (15)
where \(\phi_{\nu_e}(E)\) is the \(\nu_e\) flux obtained from the observation of CC events. Let us stress that the ratio \(R_{\text{NC}}\) does not depend on the total flux of initial \(^8\text{B}\) \(\nu_e\)’s.

If the ratio \(R_{\text{NC}} > 0\), it would mean that \(\nu_\mu\) and/or \(\nu_\tau\) are present in the flux of solar neutrinos on the earth. Notice that this remains valid if the \(\nu_e\) produced in the sun transform not only into \(\nu_\mu\) and \(\nu_\tau\) but also into sterile states.

Let us consider now the process (3) of elastic scattering (ES) of neutrinos on electrons, which is due to CC and NC interactions. The rate of ES events expected in the SNO detector will be about 1 event/day (assuming 1/3 of the SSM flux) for an electron energy threshold \(T_{e_{\text{th}}} = 5\text{ MeV}\). The total rate of ES events is given by
\[ N_{\text{ES}} = \int \sigma_{\nu_e e}(E) \phi_{\nu_e}(E) \, dE + \int \sigma_{\nu_\ell e}(E) \sum_{\ell = \mu, \tau} \phi_{\nu_\ell}(E) \, dE , \] (16)
where \(\sigma_{\nu_e e}(E)\) is the cross section of the process \(\nu_e e \rightarrow \nu_e e\) (for \(\ell = e, \mu\)) integrated over the kinetic energy \(T_e\) of the recoil electron in the interval \(T_{e_{\text{th}}} \leq T_e \leq E/\left(1 + \frac{m_e^2}{2E}\right)\) and \(E_{\text{th}} = \frac{1}{2} T_{e_{\text{th}}} + \frac{1}{2} \sqrt{T_{e_{\text{th}}}^2 + 2m_e^2 T_{e_{\text{th}}}}\). Since
\[ \frac{1}{6} \sigma_{\nu_e e}(E) \lesssim \sigma_{\nu_\ell e}(E) \lesssim \frac{1}{10} \sigma_{\nu_e e}(E) , \] (17)
(depending on \(T_{e_{\text{th}}}^\text{th}\)) the contribution of \(\nu_\mu\) and/or \(\nu_\tau\) to the total rate of ES events is considerably smaller than the contribution of \(\nu_e\). However, from the MSW analysis of existing data it follows that the relative contribution of the second term in Eq.(16) could be as big as 40\% (see Fig.2). This means that, in spite of the suppression due to the cross section ratio (17), it could be possible to reveal the presence of \(\nu_\mu\) and/or \(\nu_\tau\) in the flux of solar neutrinos on the earth from the observation of ES events. The relative contribution of \(\nu_\mu\) and/or \(\nu_\tau\) to the total rate of ES events is given by
\[ R_{\text{ES}} = 1 - \frac{\int_{E_{\text{th}}} \sigma_{\nu_e e}(E) P_{\nu_e \rightarrow \nu_e}(E) X(E) \, dE}{\int_{E_{\text{th}}} \sigma_{\nu_e e}(E) P_{\nu_e \rightarrow \nu_e}(E) X(E) \, dE + \int_{E_{\text{th}}} \sigma_{\nu_\ell e}(E) (1 - P_{\nu_e \rightarrow \nu_e}(E)) X(E) \, dE} . \] (18)
A measurement of the value of $N^\text{ES}$ will also allow to determine the flux of initial $^8\text{B}$ $\nu_e$’s. In fact, from Eqs.(6) and (16), it is easy to obtain the following expression for the total flux of initial $^8\text{B}$ $\nu_e$’s:

$$\Phi_{^8\text{B}\nu_e} = \frac{\Sigma^\text{ES}}{X_{^8\text{B}\nu_e}}$$

(19)

where

$$\Sigma^\text{ES} = N^\text{ES} - \int_{E_{\text{th}}} \left( \sigma_{\nu_e e}(E) - \sigma_{\nu_\mu e}(E) \right) \phi_{\nu_e}(E) \, dE,$$

(20)

$$X_{^8\text{B}\nu_e} = \int_{E_{\text{th}}} \sigma_{\nu_\mu e}(E) \, X(E) \, dE.$$

(21)

So the total flux of initial $^8\text{B}$ $\nu_e$’s can be determined directly from the experimental data either through the observation of NC events (Eq.(11)) or through the observation of ES and CC events (Eq.(19)). Let us notice that the quantities $N^\text{NC}$ and $\Sigma^\text{ES}$, which can be determined independently through the observation of the processes (2)–(4), are not independent. In fact, independently on the flux of initial $^8\text{B}$ $\nu_e$’s we have the following relation:

$$N^\text{NC} = \frac{X^\text{NC}_{^8\text{B}\nu_e}}{X_{^8\text{B}\nu_e}} \Sigma^\text{ES}.$$  

(22)

As we will show below, this relation can be violated only if sterile neutrinos or active antineutrinos are present in the solar neutrino flux on the earth.

Up to now we considered the case in which only the total rate of ES events will be measured. Let us assume now that from the experimental data it will be possible also to obtain the ES event rate $n^\text{ES}(E)$ as a function of the neutrino energy $E$. The statistics of ES events in the SNO experiment possibly will not allow to determine $n^\text{ES}(E)$ with sufficient accuracy. However, a large number of ES events will be observed in Super-Kamiokande [11], ICARUS [13] and other future solar neutrino experiments.

A measurement of $n^\text{CC}(E)$ and $n^\text{ES}(E)$ will allow to determine the spectrum of $\nu_\mu$ and/or $\nu_\tau$ directly from the experimental data. In fact, we have

$$\sum_{\ell=\mu,\tau} \phi_{\nu_\ell}(E) = \frac{1}{\sigma_{\nu_e e}(E)} \left[ n^\text{ES}(E) - \sigma_{\nu_e e}(E) \phi_{\nu_e}(E) \right].$$

(23)

It is clear that a determination of the fluxes $\phi_{\nu_e}(E)$ and $\sum_{\ell=\mu,\tau} \phi_{\nu_\ell}(E)$ through the observation of CC and ES events will allow to predict the rate of NC events (see Eq.(8)). Let us stress that this prediction does not depend on the possible presence of sterile neutrinos in the solar neutrino flux on the earth.

### 3 Sterile (anti)neutrinos

In the general case of neutrino mixing, (Dirac and Majorana mass term [15]) besides the active neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$, sterile neutrinos $\nu_\ell^S$, which do not take part in the standard
weak interactions, could exist. In this case, the neutrinos with definite mass are Majorana particles and the number of massive neutrinos is larger than the number of lepton flavours. A unique feature of the SNO and other future solar neutrino experiments is that directly from the data of these experiments it will be possible to get informations not only about the presence of $\nu_\mu$ and/or $\nu_\tau$ in the solar neutrino flux, but also about the presence of sterile (anti)neutrinos. In fact, from the unitarity of the mixing matrix, in the general case of Dirac and Majorana mixing we have

$$\sum_{\ell=e,\mu,\tau} P_{\nu_e\rightarrow\nu_\ell}(E) + \sum_{\ell} P_{\nu_e\rightarrow\nu_\ell^s}(E) = 1 , \quad (24)$$

where $\sum_{\ell} P_{\nu_e\rightarrow\nu_\ell^s}(E)$ is the total probability of transitions of $\nu_e$ into all possible sterile states.

From Eq.(24) we have

$$N^{NC} = \Phi^{sB}_{\nu_e} X^{NC}_{\nu_e} - \int_{E_{th}} \sigma^{NC}_{\nu_e}(E) \phi^{sB}_{\nu_e}(E) \, dE , \quad (25)$$

where $\phi^{sB}_{\nu_e}(E) = \sum_{\ell} P_{\nu_e\rightarrow\nu_\ell^s}(E) \Phi^{sB}_{\nu_\ell^s}$ is the flux of sterile neutrinos on the earth and $X^{NC}_{\nu_e}$ is given by Eq.(12). It is clear that without a definite assumption about $\Phi^{sB}_{\nu_e}$ it is impossible to make any conclusion about the existence of sterile neutrinos. However, we can obtain another relation for the total flux $\Phi^{sB}_{\nu_e}$.

$$\Sigma^{ES} = \Phi^{sB}_{\nu_e} X^{ES}_{\nu_e} - \int_{E_{th}} \sigma^{ES}_{\nu_e}(E) \phi^{sB}_{\nu_e}(E) \, dE , \quad (26)$$

where $\Sigma^{ES}$ and $X^{ES}_{\nu_e}$ are given in (20) and (21), respectively. Excluding $\Phi^{sB}_{\nu_e}$ from Eqs.(25) and (26) we obtain

$$\frac{N^{NC}}{X^{NC}_{\nu_e}} = \frac{\Sigma^{ES}}{X^{ES}_{\nu_e}} = \frac{\int_{E_{th}} \sigma_{\nu_e}(E) \phi^{sB}_{\nu_e}(E) \, dE}{\int_{E_{th}} \sigma^{NC}_{\nu_e}(E) \phi^{sB}_{\nu_e}(E) \, dE} . \quad (27)$$

The left hand side of Eq.(27) contains only quantities which can be measured through the observation of CC, NC and ES reactions. If it is found to be different from zero, it would follow that there are sterile particles in the flux of solar neutrinos on the earth.

If the rate $n^{ES}(E)$ will be measured, then the hypothesis of existence of sterile neutrinos could be tested from the investigation of only the CC process (2) and the ES process (3). In fact, with the help of Eq.(24), we obtain the following relation

$$\frac{1}{X(E)} \left[ \phi_{\nu_e}(E) + \sum_{\ell=\mu,\tau} \phi_{\nu_\ell}(E) \right] = \Phi^{sB}_{\nu_e} \left[ 1 - \sum_{\ell} P_{\nu_e\rightarrow\nu_\ell^s}(E) \right] . \quad (28)$$

1 According to our model calculations the two terms in the right-hand side of Eq.(27) can cancel each other. This cancelation depends on the value of the threshold energy of the recoil electron in the ES process. From our calculations it follows that higher threshold energies are preferable for the test of the presence of sterile neutrinos in the solar neutrino flux with the help of Eq.(25).
If it will occur that the left-hand side of this equation, which contains only measurable quantities, depends on energy, then it would mean that there are sterile neutrinos in the flux of solar neutrinos on the earth.

To conclude the discussion of tests of the possible existence of sterile neutrinos, let us make the following remark: Assume that

\[ R^{ES} = 0 \quad \text{and} \quad R^{NC} = 0 \]  \hspace{1cm} (29)

but \( \phi_{\nu_e}(E) / X(E) \) depends on the neutrino energy. This would mean that in the flux of solar neutrinos on the earth there are no \( \nu_\mu \) and \( \nu_\tau \) but the spectrum of \( \nu_e \) is distorted with respect to the initial spectrum of \( ^8B \) neutrinos. This is possible only in the case of transitions of \( \nu_e \) into sterile neutrinos.

4 Spin and spin-flavour precession

Up to now we have considered only effects due to neutrino mixing. In the recent years many papers have discussed possible effects of large neutrino magnetic moments \( (\simeq 10^{-11} - 10^{-10} \mu_B, \text{where } \mu_B \text{ is the Bohr magneton}) \) for solar neutrinos [16,17,18]. The future real time solar neutrino experiments undoubtedly will allow to test this hypothesis in detail. We will limit ourselves only to a few remarks.

In the general case of Dirac diagonal and transition magnetic moments and neutrino mixing, the flux of solar neutrinos on the earth contains left-handed active neutrinos \( \nu_\ell \) (with \( \ell = e, \ldots \)) and right-handed sterile neutrinos \( \nu_{\ell R} \) (with \( \ell = e, \ldots \)) produced by spin and resonant spin-flavour precessions in the magnetic field of the sun. In this case, as in the case of oscillations of \( \nu_e \) into left-handed sterile (anti)neutrinos considered above, the relations (27) will be satisfied (where \( \phi_{\nu_s}(E) \) is the total flux of right-handed sterile neutrinos). So if the left hand side of Eq.(27), in which only measurable quantities enter, is different from zero, this will be an evidence that in the flux of solar neutrinos on the earth there are left-handed sterile (anti)neutrinos (Dirac and Majorana mixing) or right-handed sterile neutrinos (Dirac magnetic moments). It is possible to distinguish these two cases only if the CC, NC and ES rates depend on time (which will be an evidence in favour of magnetic moments).

In the case of Majorana transition magnetic moments, the flux of solar neutrinos on the earth contains left-handed neutrinos, right-handed antineutrinos \( \bar{\nu}_\mu \) and \( \bar{\nu}_\tau \) and also possibly \( \bar{\nu}_e \) [17,28]. It is possible to show that a detection of solar neutrinos through the simultaneous observation of CC, NC and ES events will allow to reveal the existence of antineutrinos in the flux of solar neutrinos on the earth. In fact, using only the fact that the total flux of neutrinos and antineutrinos on the earth must be equal to the initial flux
of $^8$B neutrinos, we obtain the following relation:

$$\frac{N^{NC}_{\nu d}}{X^{NC}_{\nu d}} - \frac{\Sigma^{ES}}{X^{ES}_{\nu e}} = \frac{1}{X^{NC}_{\nu d}} \int_{E_{th}} \sigma^{NC}_{\nu d}(E) (1 - r_{\nu d}(E)) \sum_{\ell=e,\mu,\tau} \phi_{\bar{\nu}_\ell}(E) dE$$

$$- \frac{1}{X^{ES}_{\nu e}} \left[ \int_{E_{th}} \sigma_{\bar{\nu}_e}(E) (1 - r_{\bar{\nu}_e}(E)) \phi_{\bar{\nu}_e}(E) dE \right]$$

$$+ \int_{E_{th}} \sigma_{\bar{\nu}_e}(E) \left( 1 - r_{\bar{\nu}_e}(E) \right) \sum_{\ell=e,\mu,\tau} \phi_{\bar{\nu}_\ell}(E) dE,$$

where $X^{NC}_{\nu d}$, $\Sigma^{ES}$, and $X^{ES}_{\nu e}$ are given in Eqs. (13), (20) and (21), respectively, $r_{\nu d}(E) = \sigma^{NC}_{\nu d}(E)/\sigma^{NC}_{\bar{\nu}_d}(E)$, $r_{\bar{\nu}_e}(E) = \sigma_{\nu e}(E)/\sigma_{\bar{\nu}_e}(E)$, $r_{\bar{\nu}_e}(E) = \sigma_{\nu e}(E)/\sigma_{\bar{\nu}_e}(E)$ and $\phi_{\bar{\nu}_\ell}$ are the fluxes of antineutrinos $\bar{\nu}_\ell$, with $\ell = e, \mu, \tau$. In SNO and other future solar neutrino experiments it is planned to search for $\bar{\nu}_e$ from the sun through the observation of the reaction $\bar{\nu}_e p \rightarrow e^+ n$. Therefore, with the help of Eq.(30), the left hand side of which contains only quantities measurable through the observation of CC, NC and ES reactions, it will be possible to check whether $\bar{\nu}_\mu$ and/or $\bar{\nu}_\tau$ are present in the solar neutrino flux on the earth (investigations of the time dependence of $N^{NC}$ and $\Sigma^{ES}$ are also necessary). In the energy region discussed here $r_{\nu d}(E)$ differs from 1 by only 2–3% [25]. This means that the possible contribution of the first term in the right-hand side of Eq.(30) is strongly suppressed. We have also $-1 \lesssim 1 - r_{\bar{\nu}_e}(E) \lesssim 0.4$ and $1 - r_{\bar{\nu}_e}(E) \approx -0.3$.

Notice that the relation (11) that connects the total flux of initial $^8$B neutrinos with the NC event rate $N^{NC}_{\nu d}$ is valid also in the case of transitions of $\nu_e$ into active neutrinos and antineutrinos. In order to see this it is necessary to take into account that $\sigma^{NC}_{\nu d}(E) \simeq \sigma^{NC}_{\bar{\nu}_d}(E)$.

5 Conclusions

The experiments on the detection of solar neutrinos have exceptional significance for the investigation of neutrino properties (masses, mixings, magnetic moments, ...) and for the investigations of the sun. The analysis of the existing solar neutrino data is based on the assumption that the standard solar model correctly predicts the neutrino fluxes from the different reactions of the $pp$ and CNO cycles. In the present note it was shown that future real-time solar neutrino experiments (SNO, Super-Kamiokande and others) will allow to determine directly from the experimental data the $\nu_e$ survival probability as well as the flux of initial $^8$B neutrinos. Different tests of the presence in the solar neutrino flux on the earth of sterile neutrinos (due to Dirac and Majorana mixing or spin precession of neutrinos with Dirac magnetic moments) or active antineutrinos (due to resonance spin-flavour precession of neutrinos with Majorana magnetic moments) are proposed.

The realization of the program of determination of the initial $^8$B neutrino flux and the survival probability $P_{\nu_e \rightarrow \nu_e}(E)$, that we have discussed here, will allow to determine the parameters that characterize neutrino properties directly from the experimental data.
The ratios $R_{\text{NC}}$ and $R_{\text{ES}}$ which give the relative contribution of $\nu_\mu$ and/or $\nu_\tau$ to the rate of NC and ES events, respectively, depend only on the survival probabilities and do not depend on the initial flux of $^8\text{B}\nu_e$'s (see Eqs. (15) and (18)). In Fig. 1 and Fig. 2 the results of the calculations of $R_{\text{NC}}$ and $R_{\text{ES}}$ in the framework of the two-flavour MSW mechanism are presented. In this case the survival probability depends only on the two parameters $\Delta m^2$ and $\sin^2 2\vartheta$. The curves in Fig. 1 and Fig. 2 correspond to fixed values of $R_{\text{NC}}$ and $R_{\text{ES}}$, respectively. The two regions of values of $\Delta m^2$ and $\sin^2 2\vartheta$ which were obtained from the analysis of the existing data (under the assumption that the fluxes of initial solar neutrinos are given by the SSM) are also plotted.

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Figure Captions

Fig.1 The ratio $R^{NC}$, which gives the relative contribution of $\nu_\mu$ and/or $\nu_\tau$ to the rate of NC events. The curves correspond to the values of $R^{NC}$ equal to 0.1, 0.2, ..., 0.9. The two regions of values of $\Delta m^2$ and $\sin^2 2\vartheta$ which were obtained from the analysis of the existing data \cite{9} are also plotted.

Fig.2 The ratio $R^{ES}$, which gives the relative contribution of $\nu_\mu$ and/or $\nu_\tau$ to the rate of ES events. The curves correspond to the values of $R^{ES}$ equal to 0.1, 0.2, ..., 0.9. The two regions of values of $\Delta m^2$ and $\sin^2 2\vartheta$ which were obtained from the analysis of the existing data \cite{9} are also plotted.
