 Bounds on quantum gravity parameter from the SU(2) NJL effective model of QCD

K. Nozari, M. Khodadi and M. A. Gorji

Department of Physics, Faculty of Basic Sciences, University of Mazandaran - P. O. Box 47416-95447, Babolsar, Iran

received 10 November 2015; accepted in final form 21 December 2015
published online 11 January 2016

PACS 04.60.-m – Quantum gravity
PACS 04.60.Bc – Phenomenology of quantum gravity
PACS 12.39.-x – Phenomenological quark models

Abstract – The existence of a minimal measurable length, as an effective cutoff in the ultraviolet regime, is a common feature of all approaches to the quantum gravity proposal. It is widely believed that this length scale will be of the order of the Planck length \( \lambda = \lambda_0 \hbar \), where \( \lambda_0 \sim O(1) \) is a dimensionless parameter that should be fixed only by the experiments. This issue can be taken into account through the deformed momentum spaces with compact topologies. In this paper, we consider minimum length effects on the physical quantities related to three parameters of the SU(2) Nambu-Jona-Lasinio effective model of QCD by means of the deformed measure which is defined on the compact momentum space with \( S^3 \) topology. This measure is suggested by the doubly special relativity theories, Snyder deformed spaces, and the deformed algebra that is obtained in the light of the stability theory of Lie algebras. Using the current experimental data of the particle physics collaboration, we constrain the quantum gravity parameter \( \lambda_0 \) and we compare our results with bounds that are arisen from the other experimental setups.

Copyright © EPLA, 2015

Introduction. – Quantum gravity (QG) candidates such as string theory and loop quantum gravity strongly suggest the existence of a minimum length scale below which no other length can be observed [1,2]. It is then natural to expect that a nongravitational theory, which includes an invariant minimum length scale, arises at the weak gravity limit (but high energy regime) of the ultimate QG theory. Such an effective theory will be reduced to the standard well-known theories at the low energy regime in the light of the correspondence principle. In the absence of a full quantum theory of gravity, one may do in reverse: Starting from quantum mechanics or special relativity and deforming them in such a way that they include an invariant minimal length scale. The first attempt in this direction was taken by Snyder in 1947 who formulated a discrete Lorentz-invariant spacetime [3]. Quantum field theories turn out to be naturally ultraviolet-regularized in this setup [4]. Motivated by the string theories, generalized uncertainty relations are suggested that support the existence of a minimal length through the nonzero uncertainty in position measurement [5,6]. The polymer quantum mechanics is investigated in the symmetric sector of loop quantum gravity which also supports the existence of a minimal length scale known as the polymer length scale [7]. The relation between the generalized uncertainty relation and the polymer quantization scenario is also shown in ref. [8]. Furthermore, the doubly special relativity theories are formulated in order to take into account a minimal observer-independent length scale in the special relativity framework [9]. The noncommutative phase spaces are the other interesting framework to take into account a minimal length scale [10]. Apart from the details of the above-mentioned models, all of them are in agreement in the existence of a minimal measurable length. The question which then arises is: How much a minimal length would be small? The conclusive answer to this question will became clear just after formulating a full QG theory. Nevertheless, one can constrain the QG parameter by means of the correspondence principle. More precisely, the effective approaches are investigated by deforming the well-known theories to include a deformation parameter that signals the QG fundamental scale. Thus, one expects that the deformation parameter will disappear at low energy regimes since the QG effects are negligible at this regime. In other words, the effective theories should reduce to the standard undeformed ones at the low energy regime and also they would not destroy the prediction of the corresponding standard model at this regime.

60003-p1
In this respect, one can obtain an upper bound on the QG parameter in any well-tested low energy regime’s experiment. In recent years, many attempts have been done in this direction within the various experimental setups, see for instance refs. [11,12] in which the upper bounds are found on the QG parameter in the context of the generalized uncertainty principle (see also refs. [13,14] for the case of the noncommutative spaces). There are two determinant factors in these considerations: The energy scale of the experiment and the accuracy of the measurement in the proposed experiment. While the former is usually fixed for a particular experiment, the latter could be improved by upgrading the instruments of the experimental setups. Thus, it is plausible to expect that an experimental setup at the higher energy scales will constrain the QG parameter with more accuracy. In this sense, we have explored an upper bound on the QG length scale in the framework of Nambu-Jona-Lasinio (NJL) phenomenological model of quantum chromodynamics (QCD). The NJL model is nonrenormalizable when it is applied to the thermodynamics and it is then convenient to introduce a three-momentum cutoff Λ. Evidently, the three-momentum cutoff Λ in NJL model cannot exceed 1 GeV which is very small with respect to the expected energy scale of QG ($E_{Pl} \sim 10^{19}$ GeV). Although QG effects are very small in this energy regime, we are interested to answer the question that how much they would be small in order to respect the NJL model predictions?

The structure of the paper is as follows: In the second section, we briefly review the $SU(2)$ two-flavors NJL model. In the third section, we introduce a deformed measure that includes minimal length effects and is suggested by some effective theories of QG. Using the deformed measure, we find the QG corrections to the dependent parameters of the NJL model in the fourth section which allows us to constrain the QG dimensionless parameter λ₀. The last section is devoted to summary and conclusions.

**NJL model.** -- QCD is a theoretical framework of the strong nuclear force for hadrons in which quarks interact with non-Abelian $SU(3)$ gauge fields known as gluons. At the high energy regime, QCD has asymptotic freedom property, i.e. the running coupling of QCD decreases at short distances [15]. Therefore, the perturbation theory is applicable for the high energy phenomena with momentum transfer $q \gg \Lambda_{QCD}$, where $\Lambda_{QCD} \approx 100–200$ MeV is a typical energy scale of QCD. QCD has two important properties at low energy regime: The confinement and the spontaneous breaking of the chiral symmetry. The quarks and gluons are enclosed inside the packages that are called hadrons (quark-gluon bound states) through the confinement property. Beside, the large effective masses of the quarks and also the light masses of the pseudoscalar mesons originate from the spontaneous breaking of the chiral symmetry. Nevertheless, QCD is nonperturbative at the low energy regime since the strong coupling constant increases when the energy scale approaches to $\Lambda_{QCD}$. To remedy this problem, the effective phenomenological models of QCD are investigated. The so-called Lattice QCD (LQCD) is the most well-known candidate that is investigated to solve the nonperturbative feature at the confinement phase [16] (see ref. [17] for review). In particular, the dynamic generation of the fermion masses are explained by the chiral symmetry breaking in the NJL model with which we are interested in this paper. Evidently, the behavior of the mesons in the hot and dense matter are well understood in this setup [18]. Apart from the lack of the confining mechanism in this model, it is an appropriate phenomenological model to study the low energy aspects of the hadrons physics (see refs. [19] for details). The NJL effective phenomenological model of QCD is originally proposed before formulating the QCD to explain interactions of the nucleons with mesons [20]. Today, however, the model is defined by the Lagrangian formalism of QCD but with the important difference that now fermionic degrees of freedom are two (or three) flavors and three colors of the quark fields [21,22]. The simplest form of the Lagrangian for the $SU(2)$ NJL model with two quark flavors and a nonzero bare quark mass $m$ is given by [19]

$$\mathcal{L}_{NJL} = \bar{\psi}(i\gamma^a \partial^a - m)\psi + G \left[ (\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2 \right], \quad (1)$$

where quark fields $\bar{\psi}$ are Dirac spinors carrying colors and $\tau$ are the Pauli spin matrices. The linearized field equations for quark fields $\psi_u$ and $\psi_d$ now read as

$$i\gamma^a \partial^a - m_u \bar{\psi}_u + 2G \left[ \bar{\psi}_u \psi_u + \bar{\psi}_d \psi_d \right] \psi_u = 0,$$

$$i\gamma^a \partial^a - m_d \bar{\psi}_d + 2G \left[ \bar{\psi}_d \psi_d + \bar{\psi}_u \psi_u \right] \psi_d = 0,$$

so that the dynamical quark masses are given by

$$M_u = m_u - 2G \left[ \bar{\psi}_u \psi_u + \bar{\psi}_d \psi_d \right],$$

$$M_d = m_d - 2G \left[ \bar{\psi}_d \psi_d + \bar{\psi}_u \psi_u \right].$$

The numerical value of the quark masses $M_u,d$ are known as constituent quark masses since they are almost equal to the common values in the nonrelativistic quark models. The NJL model defined in (1) includes three parameters: the bare quark mass $m$, the coupling strength (or quark-quark coupling) $G$ and the three-momentum cutoff $\Lambda$. Notice that the bare quark masses are usually taken to be $m_u = m_d = m$ to respect the isospin symmetry. These three NJL parameters are usually fixed to reproduce the chiral physics in the hadronic portion. The physical quantities quark (chiral) condensate $\langle \bar{\psi}\psi \rangle$ and pion decay constant $f_{\pi}$ are used to fix the two dependent parameters $\Lambda$ and $G$ of the NJL through the relations

$$\langle \bar{\psi}\psi \rangle_0 = \langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0 = -\frac{N_cM}{\pi^2} \int_0^\Lambda dp^2 \frac{p^2}{E}, \quad (2)$$

$$f_{\pi}^2 = \frac{N_cM^2}{2\pi^2} \int_0^\Lambda dp^2 \frac{E^3}{E^3}.$$
Table 1: Parameter set (the first and second rows) that are fixed by the physical quantities (the third and fourth rows). The constituent quark mass ends up with 

\[ N_f = 2 \text{ and } N_c = 3. \]

| \( \Lambda \) (GeV) | \( G \) (GeV\(^{-2}\)) | \( m \) (MeV) |
|------------------|------------------|------------|
| 0.651            | 10.08            | 5.5        |
| \(- (\bar{\psi} \psi)^{1/3} \) (MeV) | \( f_\pi \) (MeV) | \( m_\pi \) (MeV) |
| -251             | 92.3             | 139.3      |

where \( N_c = 3 \) denotes the color degrees of freedom and \( E = \sqrt{p^2 + M^2} \) is the energy of a quark and antiquark of flavor \( u \) or \( d \) with three-momentum \( p \). The bare quark mass \( m \) is determined by fixing the pion mass \( m_\pi \) through the dispersion relation

\[ 1 - G J_{pp} = 0, \quad (4) \]

with

\[ J_{pp} = 4(2I_1 + q_0^2 I_2), \quad (5) \]

and \( I_{1,2} \) are given by the integrals

\[ I_1 = \frac{N_c}{4\pi^2} \int_0^\Lambda \frac{p^2 dp}{E}, \quad (6) \]

\[ I_2 = -\frac{N_c}{4\pi^2} \int_0^\Lambda \frac{p^2 dp}{E} \left( \frac{1}{(E + q_0^2)^2 - E^2} + \frac{1}{(E - q_0^2)^2 - E^2} \right). \quad (7) \]

Substituting the integrals (6) and (7) into the relation (5) leads to the following condition for the pion mass:

\[ 2N_c G \int_0^\Lambda \frac{dp}{E} \left( 1 - \frac{q_0^2}{q_0^2 - 4E^2} \right) \bigg|_{q_0^2 = m_\pi^2} = 1, \quad (8) \]

through the dispersion relation (4). From the above relations, it is clear that the NJL model is nonrenormalizable and it is necessary to introduce a three-momentum cutoff \( \Lambda \) as

\[ \frac{1}{(2\pi)^3} \int d^3p = \frac{1}{2\pi^2} \int_0^\infty p^2 dp \rightarrow \text{NJL} \]

\[ = \frac{1}{2\pi^2} \int_0^\infty \Theta[\Lambda - p] p^2 dp, \quad (9) \]

where \( \Theta \) is the Heaviside step function which guarantees the fact that the interaction described by the NJL Lagrangian (1) would be valid just for the momenta smaller than the cutoff \( \Lambda \). Indeed, the interaction becomes weak for the large momenta and then reproduces the asymptotic freedom in this regime.

Thus, the three parameters \( \Lambda, G \) and \( m \) related to the NJL model should be fixed by the physical quantities that are defined in relations (2), (3) and (8). We will use the data that are reported in ref. [22] through this paper (see table 1). As a final remark in this section, the parameters that are listed in table 1 can reproduce an effective mass about \( M \approx 325 \text{ MeV} \) which corresponds approximately to the one-third of the mass of the nucleons. Therefore, this set of parameters of the model can explain the additional mass of the proton.

**Compact momentum spaces and deformed measure.** – In this section, we consider the effects of a minimal length scale to the relation (9) by which one can find QG corrections on the physical quantities (2), (3), and (8) in the NJL model. Indeed, the QG phenomenological approaches to the issue of minimal length suggest different deformations to the standard measure (9). But, interestingly, there is a criterion which is common between these alternative approaches: The topology of the momentum space will be compact in order to take into account an ultraviolet cutoff for the system under consideration, see ref. [23] for more details. For instance, the topology of the momentum part of a two-dimensional polymerized phase space is a circle \( S^1 \) rather than the standard \( R \) topology [24] and the radius of the circle is directly related to the maximal momentum (or minimal length). While the four-momentum space in special relativity has Minkowski geometry with \( R^4 \) topology, it has de Sitter geometry with \( R \times S^3 \) topology in doubly special relativity scenarios where \( R \) is identified with the space of energy and \( S^3 \) with the three-momentum space. The compact \( S^3 \) topology for the three-momentum space induces an ultraviolet cutoff and indeed the radius of the three-sphere \( S^3 \) determines the maximal three-momentum cutoff for the system under consideration [25]. Also, the three-momentum space of the nonrelativistic Snyder model is constructed on the three-sphere \( S^3 \) [26]. The constant curvature of maximally symmetric spaces, which determines an invariant QG scale, makes them to be relevant candidates for the three-momentum spaces in QG scenarios and, therefore, the three-sphere \( S^3 \) is the most relevant choice since it preserves the rotational invariance. Thus, in the semiclassical regime, the transition to the theories which deal with minimal length scale is possible just by replacing the standard measure of the three-momentum space by a measure that is defined on a compact topology such as \( S^3 \) as

\[ \int_{R^3} d\mu(p) \rightarrow \text{QG measure} = \int_{|p| < \frac{\Lambda}{2}} \frac{d^3p}{\sqrt{1 - (\lambda p)^2}} \quad (10) \]

where \( \lambda \) is the QG parameter with a dimension of length which signals the existence of a maximal momentum (or minimal length) in this setup. It is widely believed that this length scale will be of the order of the Planck length \( \lambda = \lambda_0 \ell_p \), where \( \lambda_0 = \mathcal{O}(1) \) is the dimensionless QG parameter which should be fixed by experiments. The deformed measure (10) is obtained in the context of the polymer quantum mechanics [27], doubly special relativity theories [28], nonrelativistic Snyder model [26] and also in the context of the theory of stability of the Lie algebras [29]. The natural appearance of
a three-momentum cutoff in deformed measure (10) suggests the identification of $1/\lambda$ with the three-momentum cutoff $\Lambda$ of the standard NJL model. In this respect, the NJL model becomes naturally ultraviolet-regularized. But, $1/\lambda$ should be of the order of the Planck energy $E_{\text{Pl}} \sim 10^{19}$ GeV (since we assume that it is a quantum gravitational cutoff) while $\Lambda_{QCD} \approx 100–200$ MeV. Therefore, $1/\lambda$ is no longer a QG cutoff if one identifies it with $\Lambda$. We would like, however, to interpret it as a QG parameter. Therefore, using (10), we consider the following QG-modified counterpart of (9) as

$$\frac{1}{2\pi^2} \int_0^\infty p^2 dp \to \text{QG} \frac{1}{2\pi^2} \int_0^{1/\lambda} \frac{p^2 dp}{\sqrt{1 - (\lambda p)^2}} \to \text{NJL}$$

$$\times \frac{1}{2\pi^2} \int_0^{1/\lambda} \Theta[\Lambda - p] p^2 dp \sqrt{1 - (\lambda p)^2}.$$

(11)

The above relation allows us to find QG corrections to the quantities (2), (3), and (8) in the NJL model and then bounding the quantum gravity parameter $\lambda$.

**Quantum gravity corrections.**

**Quark condensate.** Using the QG-modified measure (11), the modification to the quark condensate (2) would be

$$\langle \bar{u}u \rangle = \langle \bar{d}d \rangle = -\frac{N_c M}{\pi^2} \int_0^{2\pi} \int_0^{\Lambda/\lambda} dp \frac{p^2 \Theta[\Lambda - p]}{E \sqrt{1 - (\lambda p)^2}}.$$

(12)

where we have substituted $\lambda = \lambda_0 p_{\text{Pl}}^{-1}$. At the Planck scale, the factor $\lambda_0(p/p_{\text{Pl}}) \sim 1$ and one then cannot expand the denominator in terms of the QG parameter $\lambda$. However, the existence of the step function guarantees that the momenta are bounded as $|p| \leq \Lambda$ and we have $\lambda_0(p/p_{\text{Pl}}) \sim 10^{-20} \ll 1$ and we then could always expand the denominator in this setup as

$$\langle \bar{u}u \rangle = \langle \bar{u}u \rangle_0 + \Delta (\langle \bar{u}u \rangle) + \mathcal{O} \left( \left( \frac{p}{p_{\text{Pl}}} \right)^4 \right),$$

(13)

where $\langle \bar{u}u \rangle_0$ is given by (2), that is the quark condensate in the absence of QG effects, and we have also defined

$$\Delta (\langle \bar{u}u \rangle) = -\lambda_0^2 \frac{N_c M}{2\pi^2} \int_0^\Lambda \frac{dp \ p^2 \Theta[\Lambda - p]}{E} \left( \left( \frac{p}{p_{\text{Pl}}} \right) \right)^2.$$

(14)

which is the QG correction to the quark condensate. Using the data presented in table 1 and the numerical solution for the standard quark condensate (14), we obtain the estimation for the QG correction to the quark condensate as

$$\left| \frac{\Delta (\langle \bar{u}u \rangle)}{\langle \bar{u}u \rangle_0} \right| = 7.84 \times 10^{-40} \lambda_0^2.$$

(15)

This result could be interpreted in two ways [11,12]. First considering $\lambda_0 = 1$, which guarantees that the QG effects become significant just at the Planck scale. So, the QG corrections are very small to be detected for the accessible energy scales (of the order of $10^{-30}$). The second interpretation is more interesting. We could obtain an upper bound on $\lambda_0$ to ensure the validity of the NJL estimation for the quark condensate up to the precision of the experiment under consideration. The empirical value derived from QCD sum rules for quark condensate has a precision of $\approx 10^{-2}$ [30,31]. Given that the QG correction (15) to the quark condensate (2) would be smaller than the precision, so we get the following upper bound on $\lambda_0$:

$$\lambda_0 < 8 \times 10^{17}. \quad (16)$$

**Pion decay constant.** By means of the deformed measure (11), the QG deformation to the integral equation of the pion decay constant (3) will be

$$f_{\pi}^2 = \frac{N_c M^2}{2\pi^2} \int_0^{\Lambda/\lambda} \frac{dp \ p^2 \Theta[\Lambda - p]}{E^3 \sqrt{1 - (\lambda p)^2}}$$

$$= f_{\pi 0}^2 + \Delta (f_{\pi}^2) + \mathcal{O} \left( \left( \frac{p}{p_{\text{Pl}}} \right)^4 \right),$$

(17)

where $f_{\pi 0}^2$ is given by the relation (3) and, in the same manner of the quark condensate, we have defined the QG correction

$$\Delta (f_{\pi}^2) = \lambda_0^2 \frac{N_c M^2}{4\pi^2} \int_0^\Lambda \frac{dp \ p^2}{E^3} \left( \left( \frac{p}{p_{\text{Pl}}} \right) \right)^2.$$

(18)

Substituting from the data in table 1, we get the estimation

$$\left| \frac{\Delta (f_{\pi}^2)}{f_{\pi 0}^2} \right| = 1.92 \times 10^{-42} \lambda_0^2.$$  \quad (19)

Again, we could set $\lambda_0 = 1$ that clearly leads to the very small QG correction to the pion decay constant of the order of $10^{-42}$. But, in the same manner for the quark condensate, we could find an upper bound on the QG dimensionless parameter $\lambda_0$ through the precision of the experiment. Indeed, the pion decay constant $f_{\pi}$ may be obtained from the decay rate $\pi^+ \to \mu^+ \nu$ so that its empirical value is reported with a precision of 3000 ppm (part per million) [32]. Despite the fact that the QG correction (19) should be less than the current accuracy of the precision of the pion decay constant measurement, we obtain the following upper bound on $\lambda_0$:

$$\lambda_0 < 2.1 \times 10^{18}.$$  \quad (20)

which is weaker than that we have obtained for the case of the quark condensate in relation (16).

**Pion mass.** Taking the QG effects by means of the deformed measure (11) into account, the integral relation for the pion mass (8) will be modified as

$$\frac{2N_c G}{\pi^2} \int_0^{\Lambda/\lambda} \frac{dp \ p^2 \Theta[\Lambda - p]}{E \sqrt{1 - (\lambda p)^2}} \left( 1 - \frac{q^2}{q_0^2 - 4E^2} \right) \bigg|_{q_0^2 = m^2_{\pi}} = 1.$$

(21)
Since we know that the QG effects are very small, we consider the ansatz $q = q_0 + \lambda \frac{\delta q}{q_0} = q_0(1 + \lambda^2\frac{\delta q}{q_0})$, where $q_0$ solves the nondeformed relation (8) and $\lambda^2(\delta q/q_0)$ is the dimensionless quantity which signals a very small QG effect. Substituting this ansatz into relation (21) and then using the nondeformed relation (8), it is easy to show that to first order of approximation $O(\lambda^2)$, we have

$$\delta q = \frac{1}{16} \left\{ 2\Lambda \sqrt{\Lambda^2 + M^2}(-3M^2 + 2\Lambda^2 + q_0^2) ight.$$ 

$$- (6M^4 - 6M^2q_0^2 + q_0^4)\ln \left[ \frac{M}{\Lambda + \sqrt{\Lambda^2 + M^2}} \right]$$

$$+ q_0(4M^2 - q_0^2)^{3/2} \tan^{-1} \left[ \frac{\Lambda}{\sqrt{(\Lambda^2 + M^2)(4M^2 - q_0^2)}} \right] \right\}$$

$$\times \left( \frac{2q_0\Lambda}{4M^2 + q_0} + q_0 \ln \left[ \frac{M}{\Lambda + \sqrt{\Lambda^2 + M^2}} \right] \right)$$

$$+ \frac{2M^2}{\sqrt{4M^2 - q_0^2}} \tan^{-1} \left[ \frac{\Lambda}{\sqrt{(\Lambda^2 + M^2)(4M^2 - q_0^2)}} \right] \right\}^{-1}. \quad (22)$$

The precision of the reported empirical value of the pion mass corresponds to 2.5 ppm [33]. Using the numerical value of $q_0 = m_\pi$ and also the data in table 1, we obtain the numerical estimation

$$\lambda^2 \left| \frac{\delta q}{q_0} \right| = 8.69 \times 10^{-39} \lambda_0^2,$$ \quad (23)

for the QG correction to the pion mass in this setup. As we have stated previously, while this correction is very small to be measured, it could be also smaller than the precision of the measurement. Taking this fact into account, we obtain an upper bound for $\lambda_0$ as

$$\lambda_0 < 1.4 \times 10^{16}, \quad (24)$$

which is stronger than those obtained for the two previous cases: Quark condensate and pion decay constant in relations (16) and (20), respectively.

Summary and conclusions. – It is widely believed that quantum gravity (QG) effects would become significant at the Planck scale, where the energy scale of the system is comparable with the Planck energy $E_{Pl} \sim 10^{19}$ GeV. Although a full quantum theory of gravity has not been formulated yet, QG candidates such as string theory and loop quantum gravity suggest the existence of a minimal length scale. Taking a minimal length into account immediately leads to the deformation of the algebraic structure of the quantum mechanics. In this respect, effective models such as the generalized uncertainty principle, polymer quantum mechanics and noncommutative quantum mechanics have been suggested which support the existence of a minimum length as an ultraviolet cutoff for the system under consideration. The doubly special relativity theories are also investigated which take into account an invariant observer-independent length scale in special relativity. It is natural to expect that the minimum length scale is of the order of the Planck length as $\lambda = \lambda_0 l_{Pl}$, where $l_{Pl} \sim 10^{-35}$ m is the Planck length and $\lambda_0 = O(1)$ is a dimensionless parameter which should be fixed by experiment. However, a definite value for $\lambda_0$ should be determined through very high energy regime experiments. Nevertheless, one can obtain upper bounds on $\lambda_0$ (or equivalently on $\lambda$) through the accuracy of the measurement in some well-known low energy experiments. For instance, the QG parameter is constrained as $\lambda < 10^{17}$ in the noncommutative geometry framework [14]. The upper bound $\lambda < 10^{11}$ is obtained in the generalized uncertainty principle framework [11] (see also refs. [12] where the weaker bounds are obtained). In the context of the polymer quantum mechanics, the upper bounds $\lambda < 10^{27}$ and $\lambda < 10^{22}$ are obtained in refs. [34] and [35], respectively. Apart from the details of the these effective theories, the common feature between all of them is the deformation of the momentum space such that it defines on the compact topology. In this paper, considering a compact momentum space with three-sphere $S^3$ topology, which is suggested by the doubly special relativity theories, Snyder model and noncommutative spaces, we have studied the effects of the minimal length on the determination of the physical quantities in the SU(2) Nambu-Jona-Lasinio model of QCD. In particular, we found QG corrections to the three physical quantities including the chiral condensate, the pion decay constant and the pion mass. QG corrections would be less than the current accuracy of the measurement and we then obtained the upper bound $\lambda_0 < 1.4 \times 10^{16}$ for the QG dimensionless parameter in this setup. Stronger bounds can be obtained by improving the precision of the measurements in this setup in future.

***

We would like to thank an anonymous referee for very insightful comments.

REFERENCES

[1] Gross D. J. and Mende P. F., Nucl. Phys. B, 303 (1988) 407; Amati D., Ciafaloni M. and Veneziano G., Phys. Lett. B, 216 (1989) 41; Garay L., Int. J. Mod. Phys. A, 10 (1995) 145.

[2] Rovelli C. and Smolin L., Nucl. Phys. B, 442 (1995) 593; Ashtekar A. and Lewandowski J., Class. Quantum Grav., 14 (1997) A55.

[3] Snyder H., Phys. Rev., 71 (1947) 38.

[4] Kadsheevsky V. G., Nucl. Phys. B, 141 (1979) 477; Kadsheevsky V. G. and Fursaev D. V., Teor. Mat. Fiz., 83 (1990) 474.

[5] Konishi K., Paffuti G. and Provero P., Phys. Lett. B, 234 (1990) 276; Maggiore M., Phys. Lett. B, 304 (1993) 65; 319 (1993) 83; Phys. Rev. D, 49 (1994) 5182.
