SPIN EFFECTS AND AMPLITUDE STRUCTURE IN VECTOR MESON PHOTOPRODUCTION AT SMALL $x$

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Abstract

An analysis of light vector meson photoproduction at small $x$ on the basis of the generalized parton distribution (GPD) approach is presented. Our results on the cross section and spin density matrix elements (SDME) are in fair agreement with DESY experiments. The predicted double spin longitudinal $A_{LL}$ asymmetry is not small at HERMES energies.

This report is devoted to the study of the vector meson leptoproduction at large energies and small $x$-Bjorken ($x$) based on our results [1]. In this kinematic region the process factorizes [2] into a hard meson leptoproduction off gluons and GPD (Fig.1).

The cross section for longitudinally polarized virtual photons which dominate for asymptotically large photon virtualities exceeds the data by a large factor at small $x$ if calculated in the collinear approximation [3]. Moreover, in this approximation the amplitudes for transversally polarized photons exhibit infrared singularities [4] which signal the breakdown of factorization. Nevertheless the knowledge of these amplitudes is necessary to study spin effects in the vector meson production. Really, the analysis of SDME for light vector mesons, measured by H1 [5] and ZEUS [6], reveals that both the $\gamma^*_L \rightarrow V_L$ (TT) and $\gamma^*_L \rightarrow V_L$ (TL) transition amplitudes are non-negligible in the kinematic region accessible by the DESY experiments. It has been shown by us [7] that in the modified perturbative approach (MPA) [8] which includes the quark transverse degrees of freedom accompanied by Sudakov suppressions, one can solve both these problems. Reasonable agreement with H1 and ZEUS data [5, 6] has been obtained for electroproduced $\rho$ and $\phi$ mesons at small $x$ [1, 7].

In this report, we discuss the spin effects in the vector meson leptoproduction. Within the MPA the amplitudes for the three transitions $LL$, $TT$ and $TL$ can be calculated and afterwards cross sections and SDME. Note that the leading-twist wave function describes the longitudinally polarized vector mesons and cannot be applied to the transversally polarized light mesons. To study this case, we use the higher order $k$- dependent wave function proposed in [9].

The gluon contribution to the leptoproduction amplitudes for $t \sim 0$ and positive proton helicity reads as a convolution of the hard subprocess amplitude $\mathcal{H}^V$ and a large
distance gluon GPD $H^g$ (Fig.1):

$$\mathcal{M}_{\mu',+} = \frac{e}{2} C_V \int_0^1 \frac{d\tau}{(\bar{x} + \xi)(\bar{x} - \xi + i\varepsilon)} \times \left\{ \left[ \mathcal{H}^V_{\mu',+} + \mathcal{H}^V_{\mu',-} \right] H^g(\bar{x}, \xi, t) + \left[ \mathcal{H}^V_{\mu',+} - \mathcal{H}^V_{\mu',-} \right] \tilde{H}^g(\bar{x}, \xi, t) \right\}. \quad (1)$$

Here $\mu (\mu')$ denotes the helicity of the photon (meson), $\bar{x}$ is the momentum fraction of the transversely polarized gluons and the skewness $\xi$ is related to Bjorken-$x$ by $\xi \simeq x/2$. The flavor factors are $C_\rho = 1/\sqrt{2}$ and $C_\phi = -1/3$. The polarized GPD $\tilde{H}^g$ is much smaller than the GPD $H^g$ at small $\bar{x}$ and will be important in the $A_{LL}$ asymmetry only.

The subprocess amplitude $\mathcal{H}^V$ is represented as the contraction of the hard part $F$ and the non-perturbative meson wave function $\phi_V$

$$\mathcal{H}^V_{\mu',+} \pm \mathcal{H}^V_{\mu',-} = \frac{2\pi a_s (\mu_R)}{\sqrt{2N_c}} \int_0^1 d\tau \int \frac{d^2k_\perp}{16\pi^3} \phi_V(\tau, k_\perp^2) F^\pm_{\mu',\mu}. \quad (2)$$

The wave function is chosen in Gaussian form

$$\phi_V(k_\perp, \tau) = 8\pi^2 \sqrt{2N_c} f_V a_V^2 \exp \left[-a_V^2 \frac{k_\perp^2}{\tau \tilde{\tau}} \right]. \quad (3)$$

Here $\tilde{\tau} = 1 - \tau$ is the fraction of the meson momentum carried by the quark (antiquark), $f_V$ is the decay coupling constant and the $a_V$ parameter determines the value of average transverse momentum of the quark in the vector meson. Generally, values of $f_V, a_V$ may be different for the TT and LL amplitudes.

The subprocess amplitude is calculated within the MPA [8] where we keep the $k_\perp^2$ terms in denominators of the amplitudes and numerator of the TT amplitude. The gluonic corrections are treated in the form of the Sudakov factors which additionally suppress the end-point integration regions. The model leads to the following hierarchy of the amplitudes

$$LL : \mathcal{M}_{0\nu,0\nu}^{V(g)} \propto 1; \quad TL : \mathcal{M}_{0\nu,\nu}^{V(g)} \propto \sqrt{-t}; \quad TT : \mathcal{M}_{\nu,\nu}^{V(g)} \propto \frac{k_\perp^2}{QM_V}. \quad (4)$$

The other transitions are small and neglected in our analysis. Here $M_V$ is the scale which appears in the higher order wave function for transversely polarized meson and, respectively, in the $TT$ amplitude [1]. The $M_V$ should be about the meson mass $m_V$.

The GPD is complicated function which depends on three variables. For the small momentum transfer $t \sim 0$ we can use the double distribution form proposed in [10]

$$H^g(\bar{x}, \xi, t) = \left[ \Theta(0 \leq \bar{x} \leq \xi) \int_{\frac{\bar{x}}{1+\xi}}^{\frac{\xi}{1+\xi}} d\beta + \Theta(\xi \leq \bar{x} \leq 1) \int_{\frac{\xi}{1-\bar{x}}}^{\frac{\bar{x}}{1-\bar{x}}} d\beta \right] \frac{\beta}{\xi} f(\beta, \alpha = \frac{\bar{x} - \beta}{\xi}, t) \quad (5)$$

with the simple factorizing ansatz for the double distributions $f(\beta, \alpha, t)$

$$f(\beta, \alpha, t \simeq 0) = g(\beta) \frac{3}{4} \frac{[(1 - |\beta|)^2 - \alpha^2]}{(1 - |\beta|)^3}. \quad (6)$$
In this model, we calculate GPD \([1]\) through the gluon distribution \(g(\beta)\) and take the CTEQ5M results \([11]\) for it. Unfortunately, in the low \(x\) region the gluon distribution has an error which can exceed 15\%. This will cause large uncertainty in the calculated cross sections.

The \(t\)- dependence of the amplitudes is important in analyses of experimental data. For simplicity we parameterize it in the exponential form \(M_{ii}(t) = M_{ii}(0) \ e^{t \ B_{ii}/2}\) for \(LL, TT, TL\) transitions. Experimentally, only the slope of the \(\gamma^*p \rightarrow Vp\) cross section is measured whereas information about individual slopes \(B_{ii}\) is absent. We suppose that \(B_{LL} \sim B_{TT}\) but \(B_{TT}\) can not be equal to \(B_{LL}\). In the integrated cross section only the following combination \(|M_{TT}|^2 \propto (\frac{M_V}{M_V})^2 \ B_{TT}\) appears. Thus, by the corresponding choice of parameters we can obtain the same cross sections for a different \(B_{TT}\) slope. We test two different scenarios for the \(\rho\) production which will be discussed here.

1. \(B_{TT} \sim B_{LL}/2; \ M_V = m_V; \ f_{\rho T} = .250 \text{GeV}; \ a_{\rho T} = 0.65 \text{GeV}^{-1}\).
2. \(B_{TT} \sim B_{LL}; \ M_V = m_V/2; \ f_{\rho T} = .170 \text{GeV}; \ a_{\rho T} = 0.65 \text{GeV}^{-1}\).

This will result in the same integrated over \(t\) cross section. Differences can be found only in observables with the interference between different helicity amplitudes (SDME e.g.). For \(LL\) transition we use \(f_{\rho L} = .216 \text{GeV}, \ a_{\rho L} = 0.522 \text{GeV}^{-1}\).

![Figure 2a](image1.png)

**Figure 2a.** The cross section for \(\gamma^*p \rightarrow \rho^0p\) vs. \(Q^2\) for \(W = 75 \text{GeV}\) (full line). Dashed lines represent errors in the cross sections from uncertainty at gluon distribution.

![Figure 2b](image2.png)

**Figure 2b.** The ratio of longitudinal and transverse cross sections for the \(\rho\) production vs. \(Q^2\) at \(W = 75 \text{GeV}\) and \(t = -0.15 \text{GeV}^2\). The line - our results. Data are from H1 and ZEUS.

The cross section for the \(\gamma^*p \rightarrow \rho p\) production integrated over \(t\) is shown in Fig. 2a (full line). Good agreement with DESY experiments \([5, 6]\) is to be observed. As mentioned before, the gluon GPD has quite large errors. The dashed lines in Fig. 2a reflect uncertainty in our results caused by the gluon distribution \(g(\beta)\) in \((6)\). It can be seen that the obtained uncertainty exceeds essentially experimental errors in the cross section at small \(Q^2\).

The ratio of the cross section with longitudinal and transverse photon polarization \(R\) was calculated too. The model results for the ratio \(R\) of the \(\rho\) production is shown in Fig. 2b and in consistent with H1 and ZEUS experiments \([5, 6]\).
Figure 3 Three left figures: the $Q^2$ dependence of SDME on the $\rho$ production at $t = -0.15$ GeV$^2$ and $W = 75$ GeV. Right figure: The $t$-dependence of SDME on the $\rho$ production at $Q^2 = 5$ GeV$^2$ and $W = 75$ GeV. Data are taken from H1 and ZEUS. The solid (dashed) lines- our results for $B_{TT} = B_{LL}/2$ ($B_{LL}$).

In Fig.3, we present our results for SDME at DESY energy range. The three left figures show the $Q^2$ dependencies of SDME. Description of experimental data is reasonable. In the right figure of Fig. 3 we show the $t$-dependence of $r_{00}$ SDME for different scenarios 1,2. They give quite different predictions for the momentum dependence of $r_{00}$ but both the models agree with the known H1 experimental data [5] at small momentum transfer.

The $A_{LL}$ asymmetry for a longitudinally polarized beam and target is sensitive to the polarized GPD. Really, the leading term in $A_{LL}$ asymmetry integrated over the azimuthal angle is determined through the interference between the $H^g$ and the $\tilde{H}^g$ distributions

$$A_{LL}[ep \to epV] = 2\sqrt{1 - \varepsilon^2} \frac{\text{Re} [M^H_{++}M^{\tilde{H}^*}_{++}]}{\varepsilon |M^H_{0+}|^2 + |M^{\tilde{H}^*}_{++}|^2}.$$  

We expect a small value for $A_{LL}$ asymmetry at high energies because it is of the order of the ratio $\langle \tilde{H}^g \rangle/\langle H^g \rangle$ which is small at low $x$.

At COMPASS(SMC) energies $W = 10$ GeV(15 GeV) the $A_{LL}$ asymmetry of the $\phi$ production is expected to be small (Fig.4). At HERMES energies $W = 5$ GeV the major contribution to the amplitudes comes from the region $0.1 \leq \varpi \leq 0.2$ where $\Delta g/g$ is not small, which leads to a large value of $A_{LL}$. The asymmetry is shown in Fig.4 together with the uncertainties in our predictions due to the error in the polarized gluon distribution. Note that for the $\rho$ production the polarized quark GPD should be essential in the $A_{LL}$ asymmetry.

In summary: Light vector meson electroproduction at small $x$ was analyzed here within the GPD approach. The amplitudes were calculated using the MPA with the wave function dependent on the transverse quark momentum. By including the higher order $k^2_{\perp}/Q^2$ effects in the denominators of hard subprocess (2) we regularize the endpoint singularities in the amplitudes with transversally polarized photons. It was pointed out that the diffraction peak slopes of the $TT$ and $TL$ amplitudes are not well defined. However, within scenarios 1 and 2 we can get the same accurate description of the $Q^2$
dependence of the cross section, $R$ ratio and SDME for the $\rho$ meson production. Our results for the $\phi$ production can be found in [1]. Some comparison with the two-gluon exchange model [12] known as $\ln(1/x)$ approximation can be found in [1] too.

The knowledge of the $B_{TT}$ and $B_{TL}$ slopes is essential in analyses of the momentum transfer dependence of the SDME. We found reasonable results for SDME in the GPD model for both the scenarios. This means that now we have two possibilities for the $t$-dependence of the scattering amplitudes with similar and different $B_{LL}$ and $B_{TT}$ slopes which agree with existing experimental data. Unfortunately, the data on spin observables have large experimental errors. This does not permit one to determine which model is relevant to experiment. To clarify situation, additional theoretical study of the momentum dependence of the $LL$, $TT$ and $TL$ transition amplitudes is needed. An experimental investigation to reduce errors in SDME is extremely important.

Thus, we can conclude that the vector meson photoproduction at small $x$ is a good tool to probe the gluon GPD. Study of the $t$ dependence of SDME can give important information on the structure of different helicity amplitudes in the vector meson production.

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