THE FORMATION OF THE FIRST STARS I.
MASS INFALL RATES, ACCRETION DISK STRUCTURE AND PROTOSTELLAR EVOLUTION
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ABSTRACT

We present a theoretical model for primordial star formation. First we describe the structure of the initial gas cores as virialized, quasi-hydrostatic objects in accord with recent high resolution numerical studies. The accretion rate can then be related to characteristic densities and temperatures that are set by the cooling properties of molecular hydrogen. We allow for rotation of the gas core, assuming angular momentum conservation inside the sonic point of the flow. In the typical case, most mass then reaches the star via an accretion disk. The structure of the inner region of this disk is described with the standard theory of viscous disks, but with allowance for the substantial energies absorbed in ionizing and dissociating the gas. The size of the protostar and its luminosity depend upon the accretion rate, the energetics of the accreting gas, and the ability of the radiation to escape from the stellar accretion shock. We combine these models for the infall rate, inner disk structure, and protostellar evolution to predict the radiation field that is the basis for radiative feedback processes acting against infall (Paper II). The gross evolution of the ionizing luminosity suggests that the masses of the first generation of stars must have been at least as great as \( \sim 30M_\odot \).

Subject headings: cosmology: theory — early universe — galaxies: formation — stars: formation

1. INTRODUCTION

The formation of the first stars marks the beginning of the long saga of galaxy formation and evolution. These stars, if relatively massive, are thought to bring the Universe out of the “dark ages”, heating and reionizing the intergalactic medium, and enriching their surroundings with metals. The stellar mass determines whether or not supernovae occur, and if so the energy of the explosion, composition of the ejecta, and type of remnant. Very energetic supernovae may produce gamma-ray bursts (GRBs). The first stars may have had a profound influence on the formation of supermassive black holes, globular clusters, and proto-galactic fragments. For all these reasons, as well as simple curiosity about the very first stars in the Universe, we wish to understand the process of primordial star formation and how, in particular, the formation process may control the resulting stellar mass.

When and how quickly the universe was reionized depends on the lumpiness of the intergalactic medium (IGM), the time of first star formation, its efficiency, and the ionizing luminosity per stellar baryon (e.g. Madau, Haardt, & Rees 1999). The latter depends on the initial mass function (IMF) of the stellar population (Tumlinson & Shull 2000; Bromm, Kudritzki, & Loeb 2001; Ciardi et al. 2001; Schaerer 2002), although Bromm et al. (2001) showed that once the stars all have masses \( \gtrsim 300M_\odot \), the spectral luminosity per unit stellar mass becomes almost independent of the IMF. The hardness of the radiation field is also somewhat sensitive to the IMF (e.g. Tumlinson & Shull 2000; Schaerer 2002), so that He reionization can be affected. Significant reionization may also result from a population of supermassive black holes, making it more difficult to draw conclusions on the nature of the stellar population. The recent microwave background polarization results reported by Kogut et al. (2003) imply a relatively early epoch of reionization and a very top-heavy IMF for models invoking reionization due to stars.

As with contemporary stars, the bolometric luminosities and the total energy output of the first stars depend on their mass. Thus, along with the overall efficiency of star formation, the IMF has implications for the contribution the primordial stars make to the near infra-red background (Salvaterra & Ferrara 2003). Unfortunately the observational determination of this background is hampered by the zodiacal foreground emission. Clusters of primordial stars may potentially be detected by the next generation of space-based near infra-red observatories, perhaps showing up in the number count versus flux distribution of very faint sources, or in the fluctuations of the background (Magliocchetti, Salvaterra, & Ferrara 2003).

The metal enrichment of the IGM from core-collapse supernovae depends on the mass of the stellar progenitors. Massive primordial stars are thought to have much smaller mass-loss rates than their contemporary cousins (Baraffe, Heger, & Woosley 2001) so the mass at core collapse may be quite similar to the initial mass. For supernova progenitor masses \( \gtrsim 260M_\odot \), collapse is thought to proceed directly to a black hole, as the temperatures resulting from the pair-instability collapse are great enough to photodisintegrate nuclei, which counters explosive oxygen and silicon burning (Fryer, Woosley, & Heger 2001; Heger & Woosley 2002). There is little metal enrichment from such supernovae. For smaller masses, down to about \( \sim 140M_\odot \), explosive O and Si burning is able to completely disrupt the star, leaving no remnant and ejecting...
large quantities of heavy elements. Progenitors from $\sim 40$ to $140 M_\odot$ form black holes, with relatively inefficient metal ejection, while less massive supernovae form neutron stars, with more normal enrichment rates. In principle, metallicity determinations from high redshift absorption line systems or from very metal poor local stars (e.g. Aoki et al. 2002; Christlieb et al. 2002) can constrain the IMF of the early generations of stars.

A phenomenon that may be related to supernovae and hypernovae is the production of relativistic ejection and thereafter a gamma-ray burst (Woosley 1993; Paczynski 1998; MacFadyen & Woosley 1999; Tan, Matzner & McKee 2001; Matzner 2003). The amount of relativistic ejection accelerated by a blast wave in the stellar atmosphere is a sensitive function of the progenitor mass, density structure, and explosion energy (Tan et al. 2001). Massive stars that mostly collapse to a black hole and impart a large energy to the small fraction of ejected mass, are excellent accelerators of relativistic ejecta. However, for efficient gamma-ray production this ejecta should either interact with a relatively dense circumstellar envelope, or be subject to internal shocks. If GRBs and associated afterglows are found at high redshift they will provide excellent probes of the IGM, for example through the study of 21 cm absorption lines (Furlanetto & Loeb 2002).

The impact of the first stars on the heating and metal enrichment (and thus cooling) of the IGM may have important implications for the formation of other objects, such as supermassive black holes, globular clusters, and galaxies. If the first stars were sufficiently massive to collapse efficiently into black holes and their feedback on the surrounding gas was relatively weak, then they might become the seeds that grow to millions of solar masses and more. Other formation scenarios are possible (e.g. Rees 1984). One alternative requires direct collapse of gas to a black hole without a nuclear-burning stellar phase (e.g. Baumgarte & Shapiro 1999), and applies to “stars” with $m_* \gtrsim 10^6 M_\odot$ (Zeldovich & Novikov 1971). However, concentrating such a large mass of gas into a hydrostatic structure, without first forming less massive protostars, would be difficult for the very first generation of stars since molecular cooling acts to reduce the Jeans mass (e.g. Abel, Bryan, & Norman 2002, hereafter ABN). This scenario could perhaps be resuscitated for subsequent generations of star formation if the FUV background radiation produced by earlier generations of star formation were large enough to prevent H$_2$ formation. Other formation mechanisms have their origins in a very dense star cluster, composed either of nuclear-burning or compact stars. The ability of such clusters to form in the early universe is likely to be influenced by the nature of primordial stars; for example, if primordial stars are relatively massive, then the resulting feedback may inhibit further star formation in the vicinity. The same considerations apply to early globular cluster formation. The dominant feedback on cosmological scales that affects galaxy formation is likely to be via the intensity of the FUV background. If this is very high, then by suppressing the H$_2$ abundance, the collapse of baryons into “mini-halos” and the formation of dwarf galaxies would be inhibited.

A description of primordial star formation requires relatively simple physics and chemistry, mainly because, by definition there are no complicated feedback processes from earlier stellar generations. For most early generations of stars, the only feedback variable will be the intensity of the FUV background, with perhaps some additional influence from hard X-rays (Cen 2003). In contrast to the present-day case, dynamically-important magnetic fields and dust grains are probably not present during the initial stages of collapse. The initial conditions for star formation are well-defined cosmological fluctuations. Given this relative simplicity, we have greater confidence in the applicability of the results of numerical simulations that have followed the collapse of cosmological scale perturbations down to almost stellar dimensions (ABN; Bromm, Coppi, & Larson 1999, hereafter BCL). This confidence is strengthened by the fact that it appears to be the microphysics of H$_2$ cooling that determines the types of baryonic structures that are formed, and not, for example, the details of the initial power spectrum of fluctuations in dark matter density. Molecule formation was predicted to occur in the presence of residual electrons left over from recombination via H$^-$ (McDowell 1961; Hirasawa, Aizu, & Taketani 1969). Including this physics, the results of the recent numerical simulations suggest that the initial gas cores out of which stars form are quite massive, $M_{\text{core}} \sim 100 - 1000 M_\odot$, in agreement with earlier theoretical expectations (Yonehama 1972; Hutchins 1976; Silk 1977; Carlberg 1981). However, at very high densities three-body molecule formation becomes efficient, and it was speculated that the increased cooling would lead to much smaller Jeans masses $\sim M_\odot$ (Palla, Salpeter, & Stahler 1983). The simulations indicate that this process does occur, but produces only a single initially sub-solar mass protostar at the center of the much larger core. Accretion then builds up the mass of the star (Omukai & Nishi 1998, Ripamonti et al. 2002).

The goal of this paper is to model the growth of the protostar from very small masses to large, and to determine the total energy output from the star. Here we do not consider how feedback processes may inhibit the accretion to the protostar, deferring this important question to other papers. In §2 we briefly review the stages of the collapse immediately prior to protostellar core formation. We then describe the accretion rate to and the structure of the infall envelope around the protostellar core and disk (§§3, 4). The structure and luminosity of the accretion is discussed in §5. We model the inner accretion disk in §6 and the evolution of the protostar in §7. The results for the bolometric and ionizing luminosities are presented in §8. Radiative feedback processes are considered in Paper II, while magnetic field generation and hydromagnetic winds have been modeled by Tan & Blackman (2003).

2. OVERVIEW OF BARYONIC STRUCTURE FORMATION: FROM COSMOLOGICAL FLUCTUATIONS TO STAR-FORMING CORES

A simple analytic picture of baryonic structure formation is the following (e.g. Peebles 1993; Madau 2002). Recombination at a redshift $z \sim 1200$ heralds the start of the “dark ages”. Thermal equilibration of matter and radiation is maintained via the residual free electrons until $z \sim 160$. Until this point the cosmological Jeans mass (gas plus dark matter), $M_J \propto (T/\rho^{1/3})^{3/2}$, is therefore independent of $z$ and has typical value $\sim 10^5 M_\odot$, similar to the scale of globular clusters. At lower redshifts, baryons cool adiabatically so that $T \propto (1+z)^2$ and $M_J \propto (1+z)^{3/2}$. Gas collects in dark
matter halos and cools further via H₂ line cooling. At some point the first luminous objects form, reionize the Universe to a temperature of \( \sim 10^4 \) K and raise the Jeans mass to galactic scales of \( \sim 10^{9-10}[(1 + 2\gamma_{\text{ion}})/10]^{-3/2} \). Numerical simulations (ABN, BCL) have largely confirmed this picture. Collapse is seen to proceed along filaments, with objects of characteristic mass \( M_f \) forming and rapidly merging to build-up somewhat larger masses. The simulations of ABN use adaptive mesh-refinement and follow cosmological perturbations in a relatively small volume from \( z = 100 \) to \( z \sim 20 \), where the first luminous object forms. These simulations are stopped when molecular lines, the main coolant at this stage, become optically thick. Spherically symmetric simulations (Omukai & Nishi 1998; Ripamonti et al. 2002) are able to follow the collapse all the way to the formation of a hydrostatic protostellar core.

Summarizing the above numerical results: a pre-galactic halo (gas plus dark matter) of \( \sim 10^{5-6} M_\odot \) forms at the intersection of several filaments; a quasi-hydrostatic, gas-dominated core forms with mass \( \sim 4000 M_\odot \), \( r \sim 10 \) pc, \( T \sim 300 \) K (set by molecular cooling, with the mass fraction of molecules \( \sim 10^{-3} \) and their formation catalysed by free electrons via \( \text{H}^- \)): gradual contraction of the core is driven by cooling in the dense interior; rapid 3-body \( \text{H}_2 \) formation occurs at \( n_{\text{He}} \gtrsim 10^{14} \) cm\(^{-3} \), followed by strong cooling and supersonic inflow; and a hydrostatic core of mass \( \sim 5 \times 10^{-3} M_\odot \) and radius \( \sim 14 R_\odot \) forms when the gas becomes optically thick to the cooling radiation (continuum cooling from collision-induced absorption).

3. The accretion rate: isentropic accretion model

We consider primordial gas with \( X = 0.76 \) and \( Y = 0.24 \) so that in the atomic phase \( \phi = 1.22 n_{\text{H}} \) and \( n_{\text{He}} = 0.079 n_{\text{H}} \). The isothermal sound speed is \( c_{\text{th}} = \sqrt{kT/\mu} = 1.425(T/300) \) km s\(^{-1} \). The accretion rate to the protostar and any associated accretion disk can be expressed as

\[
\dot{m}_{\text{sd}} = \phi_\ast \frac{m_*}{t_{\text{ff}}},
\]

(Stahler, Shu, & Taam 1980; McKee & Tan 2002, 2003), where \( \phi_\ast \) is a numerical parameter of order unity and \( t_{\text{ff}} = (3\pi/32Gn_\rho)^{1/2} \) is the free-fall time measured at the location with interior baryonic mass \( M = m_{\text{sd}}/\epsilon_{\text{sd}} \); i.e. \( \epsilon_{\text{sd}} \) is the fraction of the mass that manages to collapse to the star or disk. We assume that the mass fraction of dark matter is small, which is valid \( \sim 0.3 \) pc from the core center for \( M \) up to several hundred solar masses (ABN, however their results assume a baryon to dark matter ratio of 0.064, compared to 0.21 derived from the results reported by Spergel et al. 2003). On larger scales, the effect of dark matter is to reduce the free-fall time, or equivalently increase the value of \( \phi_\ast \) by factors of order unity relative to the purely baryonic case. McKee & Tan (2002) showed that if the core out of which the star is forming is in a self-similar virial equilibrium with a density \( \rho(r) \propto r^{-k_p} \), then the core undergoes an inside-out collapse (Shu 1977) with \( \phi_\ast \simeq 1.62 - 0.48 k_p \). This accretion rate is substantially less than that which occurs in the collapse of a uniform core (Larson 1969; Penston 1969); for the isothermal case, the inside-out accretion rate is 48 times less (Hunter 1977). The simulations of ABN suggest that primordial star formation proceeds in a manner intermediate between these two extremes, but closer to the inside-out collapse: at the time of protostar formation the envelope is contracting at about a third of the sound speed, as in the maximally sub-sonic Hunter (1977) settling solution. The accretion rate in this case is about 2.6 times that of the Shu value.

For a self-similar equilibrium, the pressure is also a power-law in radius \( P \propto r^{-k_p} \), so that the pressure obeys the polytropic relation

\[
P = K \rho^{\gamma_p},
\]

with \( \gamma_p = k_p/k_p \). In hydrostatic equilibrium, \( k_p = 2(k_p - 1) \), so that \( \gamma_p = 2(1 - 1/k_p) \). For an isentropic gas, in which \( \gamma_p \) equals the adiabatic index \( \gamma \) that describes temporal changes in the gas, the parameter \( K \) is a function of the entropy, and it is therefore termed the “entropy parameter” (McKee & Holliman 1999).

McKee & Tan (2002, 2003) expressed the protostellar accretion rate in terms of the mass of the star and the pressure at the surface of the core out of which the star is forming. However, the accretion rate can equally well be expressed in terms of the entropy parameter \( K \) (Yahil 1983). From the equation of hydrostatic equilibrium, one can readily show that

\[
\rho = \left[ \frac{(3 - k_p)K^3}{4\pi G^3 M^2} \right]^{1/(4 - 3k_p)},
\]

where \( M \) is the mass of the core interior to the point at which the density is \( \rho \). In general, only a fraction \( \epsilon_{\text{sd}} \) of this mass accretes onto the star-disk system, \( \dot{m}_{\text{sd}} = \epsilon_{\text{sd}} M \), since some of the infalling mass may be blown out by feedback processes from the star and some of the material that reaches the disk may be diverted to the outflow. The instantaneous mass of the star is related to \( m_* \), by \( m_* = m_s + m_d = (1 + f_d) m_s \), with \( f_d \) being the ratio of the disk to stellar mass. For the high accretion rates generally considered in this paper, and when most accretion is via a disk, the rate limiting step for accretion will be viscous processes and gravitational torques in the disk. In the absence of magnetic fields, we assume these will only become efficient once the disks become relatively massive (\( f_d \simeq 1/3 \)) and self-gravitating (Adams, Ruden, & Shu 1989; Shu et al. 1990; Gammie 2001), and so we set this as our fiducial value for \( f_d \). Thus we also have \( \dot{m}_s = (1 + f_d)^{-1} \dot{m}_{\text{sd}} \). Once a core of a given mass, \( M \), has collapsed to form a star-disk system of \( m_{\text{sd}} = \epsilon_{\text{sd}} M \) and star mass \( m_* = (1 + f_d)^{-1} \epsilon_{\text{sd}} M \), the disk mass may continue to accrete to the star, although probably not with complete efficiency. In contemporary star formation, the main driver for protostellar outflows is believed to be a hydromagnetic wind (Konigl & Pudritz 2000; Shu et al. 2000). In this paper we assume that in primordial star formation magnetic fields are negligible and therefore that protostellar outflows are negligible as well, so that \( \epsilon_{\text{sd}} = 1 \); however, we shall keep \( \epsilon_{\text{sd}} \)
in the equations for generality. (Tan & Blackman (2003) consider the possibility of magnetic field generation by a disk dynamo, and the influence of the resulting outflow on $t_{sd}$.) Using equation (3) in equation (1), we then find

$$\dot{m}_{sd} = \frac{8\phi_1 \epsilon_{sd}}{\sqrt{3}} \left[ \frac{(3 - k_p)k_3^3K^3}{2(2\pi)^{3/2}G^{3/2} \rho_{sd}^{-1}} \right]^{2(4 - 3\gamma_p)j} M^j,$$

(4)

where

$$j \equiv 3 \left( \frac{1 - \gamma_p}{4 - 3\gamma_p} \right).$$

(5)

For $m_{sd} \propto M$ (actually, we are assuming $m_{sd} = M$), this corresponds to a time dependence $\dot{m}_{sd} \propto t^{j/(1 - j)}$. Thus, whereas contemporary star formation accelerates ($\gamma_p < 1$ so that $j/[1 - j] > 0$—McLaughlin & Pudritz 1997; McKee & Tan 2002, 2003), primordial star formation decelerates ($4/3 > \gamma_p > 1$ so that $j/[1 - j] < 0$) (Omukai & Nishi 1998).

Omukai & Nishi (1998) have shown that the central regions of a collapsing primordial gas cloud are characterized by $\gamma \simeq 1.09$, or equivalently that $k_p \simeq 2.2$, so that $\dot{m}_{sd} \propto M^{-0.37}$. The dependence of $\dot{m}_{sd}$ on $M$ is quite sensitive to $\gamma_p$. Ripamonti et al. (2002) followed the growth of the protostar up to about 0.5$M_{\odot}$, and their result of $m_{sd} \propto M^{-0.522}$, i.e. $j = -0.522$, implies an effective value of $\gamma_p = 1.144$. We shall adopt an intermediate value $\gamma_p = 1.1$ for the entire collapsing cloud. With this value we have $k_p = 20/9$ and $k_3 = 22/9$.

The normalization of the accretion rate then depends on two parameters: $K$ and $\phi_*$. The calculations by Abel et al. and by Bromm et al. show that before the gas can collapse it passes through a stage when its density is about $10^4$ cm$^{-3}$, which is the density at which H$_2$ is thermalized, and a temperature of about 200 K, which is the minimum temperature to which H$_2$ can cool the gas. However, the core can also be supported by turbulent motions: in the simulation of ABN it is found that $\epsilon_{sd} = 20/9$ and $k_3 = 22/9$.

The parameter $\phi_*$ depends on the type of collapse solution, which depends on the initial condition: Collapse from a singular isothermal sphere leads to the Shu (1977) solution for inside-out collapse with $\phi_* = 0.663$, whereas analogous collapse from a singular polytropic sphere with $\gamma_p = 1.1$ gives $\phi_* \simeq 0.55$ (McKee & Tan 2003). Collapse from a static uniform isothermal sphere gives the Larson (1969) - Penston (1969) solution: at the point of protostellar core formation the gas envelope is contracting at about three times the sound speed. Hunter (1977) has described the family of isothermal solutions that span this interval. In particular his model (11b) has collapse velocities of about $v_{sd} \simeq c_{th}/3$ for the gas at the time of protostar formation. Compared to the Shu solution (eq. 4), the accretion rate is about a factor 2.6 larger. Assuming the same increase applies to the $\gamma_p \simeq 1.1$ case gives (from eq. 4)

$$\dot{m}_{sd} = 0.026\epsilon_{sd}K^{15/7} \left( \frac{M}{M_{\odot}} \right)^{-3/7} M_{\odot} \text{yr}^{-1},$$

(8)

$$= 0.026\epsilon_{sd}K^{15/7} \left( \frac{m_*}{M_{\odot}} \right)^{-3/7} M_{\odot} \text{yr}^{-1}. $$

(9)

Note that the stellar accretion rate and the mass flow rate through the inner accretion disk is a factor $(1 + f_d)^{-1} \rightarrow 3/4$ smaller than this. The accretion rate declines in time as $t^{-3/10}$. The time required to build up a given stellar mass is

$$t_* = 27(1 + f_d)^{10/7} \epsilon_{sd}^{-10/7}K^{15/7} \left( \frac{m_*}{M_{\odot}} \right)^{10/7} \text{yr}. $$

(10)

The lifetimes, $t_{life}$, of very massive stars have been estimated to be $\sim 2$ Myr (Schaerer 2002). The condition $t_* < t_{life}$ implies that a maximum stellar mass of about 2000$M_{\odot}$ could be accumulated in the absence of any feedback processes. On these scales the effect of dark matter becomes important, so that the formation timescale for a given baryonic mass becomes shorter, raising the above mass limit by a modest factor.

The accretion rate for this model is shown in Figure 1, along with the rates predicted by Ripamonti et al. (2002) and Omukai & Nishi (1998). Omukai & Nishi assumed that the collapse would follow the Larson-Penston solution, which is appropriate for an initially uniform cloud. However, the calculations of Abel et al. (2000, 2002) show that the protostellar core is highly centrally concentrated and has only subsonic infall motions. This is to be expected, since it is evolving quasistatically due to the slow radiation losses from the trace amounts of molecular hydrogen present in the primordial gas. In order to fit their observed accretion rates with an analytic model, ON were forced to adopt an artificially small value of the entropy parameter $K$, because of their use of the Larson-Penston solution. Equation (9) is a more accurate description of how the normalization of the accretion rate depends on the properties of the pre-stellar core.
Fig. 1.— Mass accretion rate onto the protostar and disk as a function of the current collapsed mass, $m_{\text{sd}}$ (we assume $\epsilon_{\text{sd}} = 1$, so that $m_{\text{sd}} = M$). Solid line fiducial model (with $K^\prime = 1$) from eq. (9). Dotted line from Omukai & Nishi (1998). Dashed line is analytic result from Ripamonti et al. (2002). Long-dashed line is the constant accretion rate used in protostellar evolution models of Stahler et al. (1986) and Omukai & Palla (2001). Dot-dashed line is the settling inflow rate at the final stage of the simulation of ABN, now as a function of the enclosed mass. Note that the decline of this rate at small masses is due to the lack of the full set of high density cooling processes in the simulation of ABN.

3.1. The Mass Scale of Primordial Star Formation

The mass of the protostellar cloud can be inferred from equation (3):

$$M = 543K^{53/2} \left(\frac{10^4 \text{ cm}^{-3}}{n_{\text{H}}}\right)^{7/20} M_\odot,$$

where $M$ is the mass of gas at densities greater than $n_{\text{H}}$ under the assumption that most of the mass is polytropic. This mass is somewhat less than the maximum possible mass that can be gravitationally stable for these values of $\gamma_p$ and $n_{\text{H}}$, as is generally true for polytropes (see Fig. 6 in McKee & Holliman 1999).

We can estimate the mass of the molecular core in this cloud by equating the formation time for $\text{H}_2$ to the dynamical time, which we take to be $t_{\star}$. Rapid $\text{H}_2$ formation can occur only at high densities via a three-body process. The rate constant, $k_{3b}$, for $\text{H} + \text{H} + \text{H} \rightarrow \text{H}_2 + \text{H}$ used in the simulation of ABN was $k_{3b} = 1.3 \times 10^{-32} (T/300 \text{ K})^{-0.38} \text{ cm}^6 \text{ s}^{-1}$ for $T < 300 \text{ K}$ (Orel 1987) and $k_{3b} = 1.3 \times 10^{-32} (T/300 \text{ K})^{-1} \text{ cm}^6 \text{ s}^{-1}$ for $T > 300 \text{ K}$ (Palla et al. 1983). The characteristic time for the gas to become molecular is then

$$t_{\text{H}_2} = \frac{1}{2k_{3b}n_{\text{H}}} = 1.22 \times 10^{16} K^\prime \left(\frac{10^4 \text{ cm}^{-3}}{n_{\text{H}}}\right)^{1.9} \text{ yr}.$$

Equating this to the star formation time $t_{\star}$ (which may be inferred from equations 10 and 11 with $m_{\star} = \epsilon_{\text{sd}} M/(1 + f_d)$), we find that the gas becomes molecular at a density $n_{\text{H}} = 2.8 \times 10^{13} K^{53/7} \text{ cm}^{-3}$, corresponding to a mass $m_{\text{sd}} = 1.34\epsilon_{\text{sd}} K^{53/4} M_\odot$. ABN’s results show that the gas becomes molecular at $T \sim 600 \text{ K}$, so it has a slightly different entropy parameter ($K^\prime \simeq 0.4$) than the gas at $n_{\text{H}} = 10^4 \text{ cm}^{-3}$. We conclude that the central $0.4M_\odot$ of the initial gas cloud is molecular, which is close to the result in the final timestep of ABN’s simulation.

4. Rotating Infall

The collapsing clouds formed in the simulations of ABN and BCL are rotating. When the initial angular momentum is very high, then the structure may form a rotationally-supported disk on quite large scales $\sim 10^{3} M_\odot$ (BCL). It then fragments into protostellar cores with masses $\sim 100 M_\odot$. For more typical cosmological initial conditions, ABN find that the first “molecular cloud” that forms in their simulation does not fragment. It is rotating, but never (on the scales resolved $\sim 20 \text{ AU}$) fast enough to be completely supported by rotation. Their figure 4b shows that the radial mass-weighted rotational velocities are about half of Keplerian for a broad range of the enclosed mass. These rotational velocities are therefore also about the same magnitude as the sound speed.
We assume the gas conserves angular momentum once it passes through the sonic point of the collapsing flow (Ulrich 1976; Cassen & Moosman 1981; Terebey, Shu, & Cassen 1984). For the Hunter settling solution the sonic point starts moving outwards from the center at about the sound speed right after the protostar forms ($t = 0$). Note that this phase was not simulated by ABN. We parameterize the rotation of the cloud material that crosses the sonic point by the parameter

\[ f_{\text{Kep}} = \frac{v_{\text{circ}}}{v_{\text{Kep}}} = \frac{v_{\text{circ}}}{(GM_{sp}/r_{sp})^{1/2}}, \]

where $v_{\text{circ}}$ is the circular velocity (i.e., normal to the radius) and $M_{sp}$ is the mass interior to the sonic point. Averaging over spherical shells, ABN found $f_{\text{Kep}} \approx 0.5$, independent of $r$, and we adopt this as a fiducial value. Note that to reach this state there must be transport of angular momentum as the core is forming, and this is thought to be due to the turbulent motions in the gas.

The mass-radius relation of the equilibrium core is

\[ M = \left[ \frac{4\pi}{3 - \kappa_p} \right]^{1 - \gamma_p} \left( \frac{k_p K}{G} \right)^{3 - \gamma_p} r^{4 - 3\gamma_p} \]

\[ \rightarrow 980 \left( \frac{r}{\text{pc}} \right)^{7/9} K^{10/9} M_\odot. \]

from equation (3) together with the relation $M = 4\pi r^3\rho/(3 - k_p)$. The mass inside the sonic point is made up of the core of mass $m_0c_{th}^2 t/G$ (with $m_0 = 2.577$ for the isothermal settling solution vs. $m_0 = 0.975$ for the Shu solution) and the infall envelope, which has a mass $\approx c_{th}^2 t/G$ for both solutions (see Fig. 3 in Hunter 1977; most of the mass is near $\zeta \sim 1$). Thus in the isothermal case the mass within the sonic point radius of the settling solution is about $(2.577 + 1)/(0.975 + 1) \approx 1.8$ times greater than the equilibrium state. We assume a similar increase applies to the $\gamma_p \approx 1.1$ case. The above analysis assumes a quasi-spherical core, which is able to collapse with 100% efficiency ($\epsilon_{sd} = 1$). If the protostar generates a bipolar outflow that sweeps up material, then the mass inside a given $r_{sp}$ will be smaller by a factor of order $\epsilon_{sd}$.

The outer radius, $r_{d}$, of the accretion disk that forms when angular momentum is conserved is given by

\[ v_{Kep}^2 = \frac{Gm_{sd}}{r_{d}}, \]

where $v_{Kep}$ is the Keplerian velocity and $m_{sd} = (1 + f_{\text{disk}}) m_*$ is the mass interior to $r_d$. Conservation of angular momentum from the sonic point, $r_{sp}$, to $r_d$ implies

\[ r_d = f_{\text{Kep}}^2 \left( \frac{M_{sp}}{m_{sd}} \right) r_{sp} \]

\[ \rightarrow 3.44 \left( \frac{f_{\text{Kep}}}{0.5} \right)^2 \left( \frac{M}{M_\odot} \right)^{9/7} K^{1-10/7} \text{ AU}, \]

where in the numerical evaluation we have assumed $M_{sp} = m_{sd}$ and used equation (14), boosted by a factor 1.8, to replace $r_{sp}$ (this implicitly assumes that $\epsilon_{sd} \approx 1$).

Equation (17) defines the outer size of the disk. Material falls on to this disk at all radii $r_* < r < r_d$ (Figure 2). There is also some direct accretion to the star of material that had very little initial angular momentum. We follow Ulrich (1976) in describing the density distribution of this freely-falling and rotating accretion envelope. Note that Ulrich and those who followed assumed that the rotation was initially uniform, so applying this solution to a turbulent, differentially rotating cloud is necessarily approximate. The fact that the characteristic disk scale is generally much greater than the stellar radius (which is always less than a few hundred solar radii, §7) implies that most accretion proceeds via a disk. Provided that $r_d \geq r_*$, the mass accreting directly onto the disk is

\[ \dot{m}_{\text{disk, direct}} = m_{sd} \left( 1 - \frac{r_*}{r_d} \right)^{1/2}, \]

whereas that accreting directly onto the star is

\[ \dot{m}_{*, \text{ direct}} = m_{sd} \left[ 1 - \left( 1 - \frac{r_*}{r_d} \right)^{1/2} \right] \]

(Adams & Shu 1986). Once the disk is large compared to the star, direct accretion onto the star accounts for only a fraction $r_*/2r_d$ of the total.

The properties of protostellar disks supplied by inflow with the geometry shown in Figure 2 have been considered by Cassen & Moosman (1981). The impact of gas streamlines with the disk provides some viscosity to enable accretion through the disk, although this is not important for $r \ll r_d$. However, if the disk becomes sufficiently massive we expect the gravitational turbulent viscosity of clumps in a massive disk (Gammie 2001) or large scale ($m = 1$ mode) instabilities (Adams et al. 1989; Shu et al. 1990) to be the most important processes for driving inflow. Dynamo amplification of seed magnetic field (Tan & Blackman 2003) may lead to the development of magneto-rotational instability (MRI) (Balbus &
Hawley 1991), which can provide a source of viscosity. Fragmentation of a gravitationally unstable disk may occur if the local thermal time of the disk becomes less than about half the orbital time (Gammie 2001), but this does not seem to occur in these disks (Tan & Blackman 2003). The structure of the inner accretion disk is considered in §6.

One consequence of the geometry of rotating infall for initial protostellar core formation is that the optically thick stages of the collapse (with respect to both molecular line and continuum cooling, see §2) are reached at somewhat higher masses than in the case of spherical accretion. Since material can dissipate some of its energy in the disk, it has a smaller bulk kinetic energy when it reaches the star. It also has a higher temperature because of the dissipation of this energy, which has implications for the temperature of the post-accretion shock relaxation region and the protostellar evolution (see below). By changing the size and temperature of the photosphere around the star, disk accretion tends to create a hotter radiation field, which has an important effect on the feedback processes that occur (§8; Paper II). The density and a simple model of the optical depth near the star are shown in figure 3.

5. ACCRETION DIRECTLY ONTO THE PROTOSTAR

Gas that falls directly onto the surface of the protostar goes through an accretion shock, which we identify with the surface of the protostar. The accretion shock has a complex structure: the shock front, in which the gas temperature jumps to a high value; the post-shock relaxation layer, in which the gas cools by emitting radiation; and the radiative precursor, in which the gas upstream from the shock front absorbs and re-radiates the emission from the post-shock relaxation layer (McKee & Hollenbach 1980). The gas and radiation are thus far from LTE inside the accretion shock. Label the surface just outside the accretion shock front by “1”, and that just inside the postshock relaxation layer by “2”. Stahler et al. (1980) have shown that the Rosseland mean opacity of the gas between \( r_1 \) and \( r_2 \) is small, so that the radiation pressure is the same at both points (\( P_{\text{rad},1} = P_{\text{rad},2} \)). Since the radiation emitted by the postshock relaxation layer is approximately the same in the upstream and downstream directions, it follows that \( J/P_{\text{rad}} \) for the radiation emitted in this layer is the same on both sides of this layer; because the layer is optically thin, the same is true for radiation emitted outside the layer and therefore \( J_1 \approx J_2 \).

The luminosity in the outer layers of the protostar varies as

\[
L = L_0 + \dot{m}_a w, \tag{20}
\]

where

\[
w \equiv \frac{5kT}{2\mu} + \epsilon_1 - \frac{1}{2}(1 - f_k)v_{H}^2 \tag{21}
\]

(eq. A11). Here \( \mu \) is the mean mass per particle at \( r \) and \( f_k \equiv v^2/v_{H}^2 \). For accretion onto the star, \( f_k1 = 1 \) since the unshocked gas is in free-fall collapse and \( f_k2 \approx 0 \) since the shocked gas is nearly at rest. There is thus a jump in luminosity.
across the shock,
\[ L_1 = L_2 + \frac{1}{2} f k_1 v_{\text{rs}}^2 + \frac{5}{2} \left( \frac{k T_1}{\mu_1} - \frac{k T_2}{\mu_2} \right) + \epsilon_{\text{I1}} - \epsilon_{\text{I2}}. \] (22)

For free-fall collapse, \( L \) decreases beyond \( r_1 \) as \( \epsilon_{\text{th}} \) and \( \epsilon_I \) decrease.

### 5.1. Optically Thin Accretion

To evaluate the conditions at the accretion shock, we must distinguish between the cases in which the accretion flow is optically thick or thin to photospheric photons; we assume that the flow is opaque to the energetic photons emitted by the shocked gas (cf., Stahler et al. 1980). Let the hot gas behind the shock front emit a flux of energetic photons \( F_x \) upstream and the same flux downstream. The value of \( F_x \) can be inferred from equation (22), \( 4 \pi r_2^2 \cdot 2F_x = L_1 - L_2 \).

These fluxes are reprocessed into less energetic photons in the radiative precursor and the postshock relaxation layer, respectively. In each case, half the photons go upstream and half downstream. (Actually, Calvet & Gullbring 1998 have shown that the downstream flux from the radiative precursor can exceed the upstream flux due to line opacity, but we ignore this complication here.) As a result, a net flux \( F_x \) of reprocessed photons enters the postshock gas at \( r_2 \). These photons are absorbed and re-emitted by the shocked gas so that the net flux is zero.

In this section, we are assuming that the accretion flow is transparent. We further assume that the radiation emitted by the hot gas in the postshock relaxation layer and by the gas in the radiative precursor is significantly more energetic than that emitted by the photosphere, and that correspondingly the opacity for the shock photons is significantly greater than the photospheric opacity (this approximation is marginal for the reprocessed photons emitted by the radiative precursor.) First consider the case in which the flux from the interior is negligible \( (F_{\text{int}} \ll F_x) \). The gas at \( r_2 \) must then be able to radiate the flux \( F_x \) into space, so that \( \sigma T_{\text{eff},2}^4 = F_x \). In this case the gas inside \( r_2 \) is isothermal, so this is also the effective temperature \( T_{\text{eff},2} \). Including the effect of \( F_{\text{int}} \) increases the effective temperature of the gas behind the postshock relaxation layer to
\[ \sigma T_{\text{eff},2}^4 = F_{\text{int}} + F_x. \] (23)

In the Eddington approximation the contribution of the interior flux to \( \sigma T^4 \) is reduced by a factor 2 at the surface, since the radiation then occupies only half the available solid angle. As a result, we have
\[ \sigma T_2^4 = \frac{1}{2} F_{\text{int}} + F_x. \] (24)
Stahler et al. (1980) evaluated the temperature inside the postshock relaxation layer, at a point at which the X-rays had been emitted but not yet reprocessed, and they obtained a factor 2 in front of $F_*$ in this equation (corrected by Stahler 1988). The calculations of Calvet & Gullbring (1998—see below) show that the temperature structure of the shocked gas is actually more complicated than these simple analytic models imply, however. The effective temperature at $r_1$ includes the contribution of the reprocessed flux $F_i$ emitted upstream, so that

$$\sigma T_{\text{eff},1}^4 = F_{\text{int}} + 2F_i. \quad (25)$$

Calvet & Gullbring (1998) carried out both analytic and numerical models of the accretion shock. In their analytic calculation, they divided the postshock relaxation layer into two parts, the region in which the energetic photons are emitted and that in which they are absorbed, and they included the latter region in their calculation. With a flux $F_i$ incident on the absorbing gas, they found

$$\sigma T^4 = \frac{3}{4} F_{\text{int}} \left( \tau + \frac{2}{3} \right) + \frac{1}{4} F_i \left[ 2 + \frac{3}{q} + \left( \frac{q - 3}{q} \right) e^{-q\tau} \right], \quad (26)$$

where $q$ is the ratio of the opacity of the energetic photons to the photospheric photons. The first term is the standard Eddington approximation for the temperature structure of a plane parallel atmosphere carrying a flux $F_{\text{int}}$. If $q \gg 1$, as we have assumed, then the contribution of the shock radiation to the effective temperature is

$$\frac{1}{4} F_i \left( 2 + q e^{-q\tau} \right).$$

At $\tau = 0$ (which is inside the shock in our terminology), $T^4$ can be much larger than $F_i$. Only the reprocessed photons penetrate into the region $r \leq r_2$. Since an approximately equal flux of reprocessed photons enters this region from the radiative precursor, the solution in this region is equivalent to having $F_i = 2F_p$. Since the energetic photons have been absorbed at $r_2$ ($q\tau \gg 1$) but not photospheric photons ($\tau \ll 1$), it follows that $\sigma T^4_p = \frac{1}{2}(F_{\text{int}} + F_i) = \frac{1}{2}F_{\text{int}} + F_x$, in agreement with our result in equation (24).

Calvet & Gullbring’s (1998) solution shows that the shock is thin only if $q \gg 1$, as we have assumed. In this case, the gas emitting the reprocessed photons is hotter than the photosphere by a factor $\sim q^{1/4}$. As a result, the radiation outside the photosphere does not have a blackbody spectrum; correspondingly, the analytic model for the temperature distribution is approximate. However, their numerical calculations show that equation (23) for the effective temperature behind the shock is accurate to within a few percent. The gas between the photosphere and the shock is generally warmer than that at the photosphere due to the heating by reprocessed photons.

How does the heating and ionization of the gas ahead of the shock affect the emitted luminosity? The luminosity that emerges from the protostar can be inferred from equation (A11)

$$L_0 = L_2 - \dot m_* \left( \frac{5kT_2}{2\mu_2} + \epsilon_{12} - \frac{1}{2} \mu_2 \right), \quad (27)$$

where we have assumed that $L_0$ is measured at a large enough distance from the star that $T_0$ and $\epsilon_0$ are both negligible. To approximately allow for the non-blackbody nature of the spectrum of the radiation near the shock, we evaluate the ionization at the effective temperature behind the shock, $\epsilon_{12} = \epsilon_l(T_{\text{eff},2})$.

### 5.2. Optically Thick Accretion

When the accretion flow is opaque, we determine the temperature just behind the accretion shock, $T_2$, from the radiation momentum equation (Mihalas & Weibel-Mihalas 1999, eqs. 97.2 and 97.3),

$$\nabla \cdot \mathbf{P}_{\text{rad}}' = -\frac{\kappa F'}{c}, \quad (28)$$

where $\mathbf{P}_{\text{rad}}'$ is the radiation pressure tensor and $F'$ is the flux, both measured in the comoving frame, and where $\kappa$ is the Rosseland mean opacity. The comoving flux is related to that in the protostar frame by $F' = F - v'(u'_{\text{rad}} + P'_{\text{rad}})$ (Mihalas & Weibel-Mihalas 1999, eq. 91.17). Hence, for isotropic radiation that is in LTE ($P'_{\text{rad}} = \alpha_RT^4/3$) we have

$$\frac{\partial T}{\partial r} = -\kappa \left( \frac{3F}{\epsilon a_RT^3} - \frac{vT}{c} \right). \quad (29)$$

Note that $v < 0$ for accretion flow, so the advection term $\kappa vT/c$ increases the magnitude of the temperature gradient. To simplify the integration, we approximate the luminosity as a constant, $L \approx \frac{1}{4}(L_1 + L_p)$, where $L_p$ is the photospheric luminosity. The flux is then

$$F \simeq \frac{L_1 + L_p}{8\pi r_p^3}. \quad (30)$$

To solve for $T_2$, we first guess a value of $L_p$, which determines the photospheric temperature $T_p$ in terms of the unknown radius of the photosphere, $r_p$ through $T_p = (L_p/4\pi\sigma r_p^2)^{1/4}$. We solve for the temperature structure and thus the total optical depth of the radiative precursor from the photosphere to the accretion shock, using the opacities of Rogers & Iglesias (1992) and Iglesias & Rogers (1996) for $T > 7000$ K and those of Lenzuni, Chernoff, & Salpeter (1991) for
that the use of a spatially constant "although the dispersion in quoted results from numerical studies is about two orders of magnitude. It must be stressed

where

$T < 7000 \text{K}$. This is carried out at an angle of $\pi/3$ from the pole, so that the optical depth is a reasonable approximation for the mean value from the stellar surface. According to equation (20), $L_p$ and $L_2$ are related by

$$L_p = L_2 + \frac{1}{2} \dot{m}_* v_{h, *}^2 - \frac{5}{2} \dot{m}_* \left( \frac{kT_2}{\mu_2} - \frac{kT_p}{\mu_p} \right) - \dot{m}_* (\epsilon_{P2} - \epsilon_{Pp}).$$

(31)

This value of $L_p$ is used to determine $T_p$ and the cycle is repeated until convergence is achieved.

6. PROPERTIES OF THE INNER ACCRETION DISK

Since much of the accretion onto the star goes through an accretion disk, it is necessary to determine the properties of this disk, particularly near the star where the disk emission can have significant feedback effects. Our objective in this section is to estimate the luminosity and approximate spectrum emitted from the inner region of the accretion disk. The luminosity from the inner part of the accretion disk can be comparable to that produced at the boundary layer. The structure of the accretion disk, and in particular its scale-height, is also of interest because it affects the size of the region where the accretion luminosity from the boundary layer is released, and thus the spectrum. The disk may be the site of dynamo action that can amplify weak seed magnetic fields and perhaps generate a hydromagnetic bipolar outflow, a common occurrence in contemporary star formation. For the primordial case this question has been addressed by Tan & Blackman (2003), where it is concluded that strong large-scale magnetic fields are likely to be produced, particularly by the time the protostar is massive and contracting to the main sequence. Even in the absence of magnetic fields the disk may generate a radiatively driven outflow (Paper II), which depends somewhat on the structure of the disk. Finally, the disk may act to shadow gas in the equatorial plane from the brunt of the stellar radiative feedback, thus influencing the efficiency of star formation.

In order for the star to continue accreting, it must keep its rotation rate slower than the break-up rate. This is most likely achieved either by the action of magnetic fields coupled to the larger scale disk or an outflow, or by the excitation of density waves in the disk by the star (if it is rotating so quickly that it starts to assume a non-axisymmetric structure). We have assumed that these processes occur; furthermore, we assume that they are sufficiently effective that they maintain the overall rotation of the star to be much less than break-up, so that the rotational energy is negligible.

To calculate the radial structure of the inner accretion disk at any given point in the evolution of the protostar, we assume that it is fed smoothly at a rate given by equation (9) and use the standard theory of steady, thin, viscous accretion disks, with a spatially constant viscosity parameter, $\alpha_{ss}$ (Shakura & Sunyaev 1973; Frank, King, & Raine 1992). The viscosity is assumed to be a function of the total pressure, which is the sum of gas and radiation pressures. We have evaluated two cases with $\alpha_{ss} = 0.01, 0.3$ but regard the $\alpha_{ss} = 0.01$ case as the most appropriate for the very inner regions that are relevant here (see Tan & Blackman 2003 for a discussion on these choices and for the full results from both cases). This is a typical value for viscosity generated by the magneto-rotational instability (MRI) (Balbus & Hawley 1991, 1998), although the dispersion in quoted results from numerical studies is about two orders of magnitude. It must be stressed that the use of a spatially constant “$\alpha_{ss}$” viscosity model is motivated by its theoretical simplicity and convenience, and results of this modeling should be viewed as only an approximate guide to reality.

The inner scale of the disk is set by $r_*$, which we estimate in §7 below. We ignore the effects of energy injection from the star, deferring this to paper II. We again use the opacities of zero metallicity gas of Rogers & Iglesias (1992) and Iglesias & Rogers (1996) for $T \gtrsim 6000 \text{K}$ and of Lenzuni et al. (1991) for $T \lesssim 6000 \text{K}$ (however, here the interest is only in the higher temperature regime). The method of solution assumes that the central disk temperature, $T_c$, is much greater than the surface temperature, $T_{\text{eff}}$, which is a good assumption for the inner regions that we consider, except in the early stages.

The high accretion rates and large sizes of primordial protostars cause the ionization energy to have an important effect on the disk structure, particularly during the earlier stages of the evolution. The standard energy equation ignoring ionization is (e.g. Frank et al. 1995)

$$\frac{4\pi T^4}{3T} = \frac{3Gm_\ast \dot{m}_*}{8\pi r^3} \left[ 1 - \left( \frac{r_*}{r} \right)^{1/2} \right],$$

(32)

where $T$ is the optical depth through the disk with midplane temperature $T_c$. The term on the right hand side of eq. (32) is the viscous dissipation per unit disk surface area. At a given radial location in the disk there is also a rate of energy absorbed per unit disk face area by the ionization of the accreting gas. The full energy equation is therefore

$$\frac{4\pi T_c^4}{3T} = \frac{3Gm_\ast \dot{m}_*}{8\pi r^3} \left[ 1 - \left( \frac{r_*}{r} \right)^{1/2} \right] + \frac{\dot{m}_* \Delta \epsilon_I}{2\pi r} dr,$$

(33)

where $\Delta \epsilon_I$ is the internal energy due to ionization and dissociation per unit gas mass (see Appendix A.1). Note that $d\epsilon_I/dr < 0$ in general.

To solve the structure of the disk we divide it into a large number of radial zones, and start at the outer edge with an assumed ionization state that is almost completely neutral. We then solve the modified disk structure equations to find the temperature, density, ionization state of H and He (from the Saha equation), etc, and then advance the solution inwards given the new ionization state. Figure 4 shows three examples of disk structure for $\alpha_{ss} = 0.01$, with
At these masses the stellar sizes are taken to be $r_*=100, 300, 4R_\odot$ and the accretion rates (eq. 9) are $\dot{m}_*=(17, 6.4, 2.4) \times 10^{-3} M_\odot \, \text{yr}^{-1}$, respectively.

As we shall see below, at early times the large accretion rates in primordial protostars lead to large rates of energy absorption by ionization and dissociation (eq. 37), and this can regulate the midplane temperature to about $10^4 \, \text{K}$ in the early stages. The effect of including and not including the ionization energy is shown in the temperature plots of Figure 4.

Our disk model breaks down when the predicted surface temperature is very cool ($T < 5000 \, \text{K}$), which occurs for low stellar masses. This break down occurs because in our thin disk model we approximate the opacity by its value at the midplane, whereas in fact the opacity increases rapidly with temperature for $T < 10^4 \, \text{K}$. For example, the $1 \, M_\odot$ model in Figure 4 suggests that the surface temperature of the inner disk might be as low as a few hundred degrees because most of the energy has gone into ionizing the accreting material. In this model, the central temperature of the disk is about $10^4 \, \text{K}$, so that (1) the opacity is high and (2) the gas is partially ionized. Had we resolved the vertical structure of the disk, then the average opacity and hence the central temperature would have been lower; with less ionization, the effective temperature would have been higher. However, since we are most interested in the radiative feedback from the star and disk at higher masses, we do not attempt to correct this limitation of the thin disk model.

**Fig. 4.—** Protostellar disk structure for models with $\alpha_{ss}=0.01$ and $m_*=1, 10, 100 M_\odot$, for which $r_*=100, 300, 4R_\odot$ (see §7) and $\dot{m}_*=(17, 6.4, 2.4) \times 10^{-3} M_\odot \, \text{yr}^{-1}$ (see §3), respectively. From top to bottom the panels show (1) surface density, $\Sigma$; (2) ratio of scale-height to radius, $h/r$, and ratio of gas pressure to total pressure, $\beta$; (3) ionization fractions of H, He$^+$, He$^{2+}$; and (4) mid-plane temperature, $T_c$, and photospheric temperature, $T_{\text{phot}}$ (the dotted lines show results for when the ionization energy is neglected).
7. PROTOSTELLAR EVOLUTION TO THE MAIN SEQUENCE

The basic process we wish to model is the evolution of the protostar's radius as it grows in mass by accretion of material from an accretion disk or directly from the infalling envelope. Stahler et al. (1986) and Omukai & Palla (2001) considered a detailed model for the evolution of a protostar with spherically symmetric accretion and a constant accretion rate of $4.4 \times 10^{-3}M_\odot$ yr$^{-1}$. Like these authors, we also define the protostellar radius to be the location of the accretion shock. This size is important for determining the accretion luminosity of the protostar and it sets a lower limit for the photospheric radius of the protostar, which controls the spectrum of emitted radiation.

In order to be able to follow the evolution of a protostar for an arbitrary accretion rate, we use the analytic approach developed by Nakano et al. (1995; 2000). A more rigorous derivation of their result, with a slight correction, is given in the Appendix. By equating the time derivative of the energy contained in the protostar, the structure of which is taken to be a polytrope, to the rate at which energy is added, we obtain

$$\frac{d\ln r_*}{d\ln m_*} = 2 + \frac{d\ln \beta}{d\ln m_*} - \frac{4}{a_\beta \epsilon_{\text{eff},*}} \left[ \frac{1}{2} \epsilon_{\text{I}_{*\lambda}} + \epsilon_{\text{I}_{\mu}} - \left( \frac{5kT_\lambda}{2\mu_2} + \epsilon_{\text{I}_{\mu}} \right) + \frac{1}{m_*} \left( L_\lambda - \bar{E}_{\text{nuc}} \right) \right], \quad (34)$$

which is equation (A13). Here $\beta$ is the mean value of the ratio of gas pressure to total pressure in the protostar; $a_\beta$ is a number of order unity that is proportional to the gravitational energy; $\epsilon_{\text{I}_{\mu}}$ is the energy per unit mass required to fully ionize the gas; and $\bar{E}_{\text{nuc}}$ is the rate at which nuclear reactions release energy.

We now consider the evaluation of each of the terms in this equation. First, the term $d\ln \beta/d\ln m_*$ becomes significant only when radiation pressure becomes important, which occurs only at relatively high masses. The appropriate polytropic index in this regime is $n = 3$, and in this case the mean value of $\beta$ is the same as the central one. We calculate $\beta$ assuming a standard Eddington stellar structure model with $n = 3$ applies to the protostar.

Next consider the temperature behind the accretion shock. For accretion directly onto the star, the value of $T_2$ in the optically thin case is given by equation (24). As remarked in §5.1, we allow for the non-blackbody nature of the radiation field there by using the Saha equation to evaluate $\epsilon_{\text{I}_{\mu}}$ at $T_{\text{eff},2}$. When the accretion is optically thick, we integrate through the radiative precursor to find $T_2$, as described in §5.2. In the optically thick case, $\epsilon_{\text{I}_{\mu}}$ is evaluated at $T_2$.

Disk accretion presents a more complicated problem. Gas reaches the surface of the star at a temperature $T_3$ that we have estimated from standard thin disk theory (§6). However, as this gas joins the star, it will spread over the stellar surface, where it can lose heat. Integrating the radiative transfer equation through the disk shows that the central temperature is related to the effective temperature by $T_3 \propto T^{1/4}$. In this regime, $T_3 \propto 1/A$, where $A$ is the area normal to the direction of propagation of the radiation; similarly, $T_{\text{eff}} \propto 1/A^{1/4}$, so that altogether $T \propto 1/A^{1/2}$. For the disk, we identify the $A$ as the area of the star covered by the disk, $4\pi r_0^2 \cos \theta_0$, where $\theta_0$ is the angle from the axis at which the disk intersects the star. We therefore adopt the approximation that for disk accretion, $T_3 = T_3(r_0)(\cos \theta_0)^{1/2}$. In this paper, we estimate the height of the disk as 1.5 times the scaleheight; a more accurate estimate will be given in Paper II.

To evaluate the luminosity $L_2$, which is effectively the luminosity that can be transported radiatively, we follow Nakano et al. (1995) and approximate it as being equal to the value of the ZAMS luminosity (Schaerer 2002) of a star of the same mass. However, this is not a very good approximation at lower stellar masses, $m_* \lesssim 10 M_\odot$, when, because of its expanded size, the protostar's internal luminosity is much smaller than the ZAMS value (Stahler et al. 1986). In this regime we evaluate $L_2$ from analytic fits to the results of Omukai & Palla (2003, fig. 5), so that $L_2 \simeq 0$ for $m_* \lesssim 7 M_\odot$, then rises as $L_2 / L_0 \simeq 390(m_*/M_\odot)^{2} - 2700 m_*/M_\odot$, which is used until $L_2 \simeq L_{\text{ZAMS}}$. The idea behind this approach of specifying $L_2(m_*)$ is that the amount of radiative luminosity that a star can carry is determined by its structure. In a main sequence star, the rate of energy generation adjusts to give this luminosity; in a rapidly accreting protostar, the rate of energy generation, both nuclear and gravitational, can differ substantially from that which can be transported, and this difference can cause the star to either swell or shrink. The evolution during the early stages is not particularly sensitive to this approximation because the accretion luminosity is much greater than the internal luminosity.

The rate of nuclear energy generation, $\bar{E}_{\text{nuc}}$, is determined by both deuterium burning and the nuclear reactions that occur on the main sequence. We do not explicitly include the latter, but note that they prevent $r_*$ from shrinking below its main sequence value. Deuterium burning sets in when the central temperature $T_\epsilon \simeq 10^8$ K. The central temperature is

$$T_\epsilon = \beta_\text{gas} a_T \frac{\mu m_1}{k} \frac{G m_*}{r_*} \quad \text{(35)}$$

where $\mu$ is the mean particle weight in the ionized interior and $a_T = 0.54, 0.84$ for $n = 1.5, 3$ (Chandrasekhar 1939). For $m_* \lesssim 10 M_\odot$, which is the regime where D-burning can be relatively important, $\beta_\text{gas}$ and $\beta_\epsilon$ are close to unity. The reaction rate rises sharply with temperature so that, while there is fuel available, this process can act as a thermostat and maintain an almost constant $T_\epsilon$ (Stahler 1988). If the protostar reaches this central temperature at relatively low masses (as in contemporary star formation), then the luminosity produced by D burning can be large compared to that which can be transported radiatively through the star. The star becomes convective, which allows freshly accreted deuterium to be brought to the center for burning (we take $D/H = 2.2 \times 10^{-5}$; Pettini & Bowen 2001). The rate of D-burning can be calculated in this case using the model of Nakano et al. (2000) (see also McKee & Tan 2003). However, for realistic accretion rates (i.e. from cores with $K \sim 1$), the protostar only reaches the D-burning temperature at relatively high masses ($m_* \sim 10 M_\odot$) so that the star remains radiatively stable, D is quickly depleted from the stellar center, and the protostar can continue to evolve to hotter central temperatures, little affected by this
nuclear energy generation. In this case we set $\dot{E}_{\text{nuc}} = 10^4 L_\odot$, independent of stellar mass, which approximates the more detailed calculations of Omukai & Palla (2003). Eventually the central temperature becomes hot enough to allow support via hydrogen burning. Some H–burning reactions start generating significant luminosity when $T_c \gtrsim 2 \times 10^7$ K (we set $\dot{E}_{\text{nuc}} = 10^5 L_\odot$ at this point, Omukai & Palla 2003). However, full support of the star only occurs once $T_c \sim 10^8$ K, which is hotter than in the contemporary case, leading to smaller main sequence radii (e.g. Schaerer 2002). In this way the protostar settles on the zero age main sequence, where it may continue to grow in mass if the accretion is on-going.

Finally, the evolution of the polytropic index of the star needs to be accounted for. We take $n$ to be constant during specific phases of the evolution, but allow for certain transitions as follows. By comparison to the results of Stahler et al. (1986), we find that an initial value of $a_g = 1.1$ (corresponding to $n = 2.3$) is a reasonable description of the initial structure, while the star is younger than its Kelvin-Helmholz time. If the central temperature becomes hot enough for D-burning and $\dot{E}_{\text{nuc}} \gtrsim L_2$, so that the star becomes convective, then $n = 1.5 (a_g = 0.86)$ (this case is not realized in our models with $K' = 1$). In this case, the star eventually becomes radiative ($n = 3$ and $a_g = 1.5$). The star also relaxes to this structure in the absence of convective D-burning, after the age of the star becomes greater than the current Kelvin-Helmholz time (this is the case realized in all the models presented in this paper). The expansion of the protostar after a Kelvin-Helmholz time reflects the fact that it is relaxing to a globally more compact state, which releases gravitational energy and causes the outer layers of the star to expand. From comparison with the results of Omukai and Palla (2001) we set this expansion factor to be three. For protostars that evolve from a convective D-core burning state to a radiative state including D-shell burning, there is an analogous expansion of the outer layers, by about a factor of two (Palla & Stahler 1991).

We take our initial condition to be a protostar of mass $M_\ast = 0.3M_\odot$ and radius $r_\ast = 30 R_\odot$, which is a reasonable extrapolation of the numerical results of Ripamonti et al. (2002), who formed a core with a mass of 0.04$M_\odot$ and a size of $10^{12}$ cm $\approx 14 R_\odot$ at the end of the core formation phase of their simulation (see also Omukai & Nishi 1998). In any case, since the gas in front of the accretion shock is optically thick at early times, the subsequent evolution is insensitive to the initial condition.

A summary of the basic features of the protostellar evolution is as follows. The mean density and central temperature ($T_c \sim 10^5$ K) of the initial state are relatively low compared to typical stellar values. However, as the mass increases via accretion, self-gravity compresses the star and raises the central temperature. The rate of this contraction is limited by the Kelvin-Helmholz time; indeed rapid accretion of material on shorter timescales swells the star. The star is typically able to reach a mass $\sim 10M_\odot$ by the point that its age is about equal to its instantaneous Kelvin-Helmholz time. It also reaches the D–burning temperature at about this time: too high a mass for D–burning to create a convective core. Structural rearrangement to a radiative ($n = 3$) core temporarily expands the outer layers of the star by factors of a few. However, the rapidly increasing internal luminosity causes fast contraction (see eq. 34) that is only halted once the star reaches the zero age main sequence, where the central temperature ($T_c \sim 10^8$ K) is hot enough for H–burning reactions to support the star. This occurs at stellar masses of about $100M_\odot$. In our numerical implementation of the above evolution we allow for accretion from both a disk and directly to a star, the relative proportions of which depend on $f_{\text{Kep}}$, the size of the star, and the mass that has collapsed.

Observe that because of the high accretion rates the total energy absorbed by dissociation and ionization (and the corresponding reduction in the protostar’s luminosity that escapes to infinity) can be quite large:

$$L_{I_{\text{max}}} \equiv \left(\frac{dE}{dt}\right)_{I_{\text{max}}} = \dot{m}_\ast \epsilon_{I_m}$$

$$= 2.65 \times 10^3 \left(\frac{\dot{m}_\ast}{10^{-5} M_\odot \text{ yr}^{-1}}\right) L_\odot. \quad (36)$$

This process helps to keep the radiated luminosity sub-Eddington during the early stages of protostellar evolution. In the later evolution of models of protostars accreting from realistic rotating cores, then equation (37) applies to the reduction in the accretion disk luminosity, as discussed in §6.

A comparison of our simple semi-analytic model with the spherical accretion test case of Stahler et al. (1986) and Omukai & Palla (2001) is shown in Figure 5a. As discussed above, the expansion factor of the luminosity wave has been adjusted to give agreement with the models. We then applied the same model to the accretion rate of equation (9), but still with spherical symmetry. We show two examples of rotation for the isentropic model: the fiducial $f_{\text{Kep}} = 0.5$ case and one with $f_{\text{Kep}}$ ten times smaller.

Rotation causes the infall envelope to become optically thin at much smaller masses. This causes the photospheric radius to be much closer to the stellar surface, leading to a hotter radiation field (§8). Also $f_k$ is reduced, because most material is processed through an accretion disk and so there is less bulk kinetic energy to advect, even if conditions are optically thick. However, the conditions at point “2”*, just behind the accretion shock are hotter in the disk case, so that overall more energy is advected into the star and its size is bigger.

8. RADIATION FROM THE PROTOSTAR

The total bolometric luminosity of the protostar is the sum of the radiation from the stellar surface, the boundary layer, and that from the inner accretion disk; we somewhat arbitrarily include emission out to a distance 10$r_\ast$ in the latter. Emission from further out in the disk is not significant for the feedback processes we consider in Paper II. The luminosity from the boundary layer is comparable to, but almost always somewhat greater than, the luminosity from the inner disk.
Fig. 5.— Protostellar (solid lines) and photospheric (dotted lines) radii as a function of protostellar mass. The dashed line in each plot is the zero age main sequence radius (Schaerer 2002). (a) Top left: Spherical isothermal accretion at a rate $4.4 \times 10^{-3} \, M_\odot \, yr^{-1}$ corresponding to $T = 1000 \, K$. The squares are the results of Stahler et al. (1986) and Palla & Omukai (2001). (b) Top right: Fiducial spherical isentropic accretion model. (c) Bottom left: isentropic case with small amount of rotation: $f_{\text{Kep}} = 0.05$. The abrupt decline of the photosphere between 30 and 40 $M_\odot$ reflects the development of an inner photosphere as the density declines (see Figure 3). (d) Bottom right: Fiducial isentropic case with $f_{\text{Kep}} = 0.5$.

At high stellar masses the internal luminosity of the star becomes dominant. Figure 6a shows the evolution of the total luminosity and its various sub-components for the fiducial $f_{\text{Kep}} = 0.5$, $K' = 1$, $\alpha_{\text{ss}} = 0.01$ case. The luminosites of the models with different rotational parameters are quite similar (Fig. 6b). The main differences are due to the different protostellar sizes (and thus accretion luminosities) and to the fact that our definition of the total luminosity only includes energy generated in the immediate vicinity of the star ($r < 10 \, r_*$) and not, for example, the outer accretion disk. This last point causes spherically-accreting protostars to have a higher total luminosity than disk-accreting ones, and is enough to make the difference between whether the star is sub- or super-Eddington at around $100 \, M_\odot$. This measure has been invoked in the spherical case as an important factor in determining the final mass of the star (Omukai & Palla 2003). While we regard the question of mass-limits due to feedback as being more complicated than a comparison of the protostar’s luminosity to the Eddington value (Tan & McKee 2003; Paper II), Figure 6b shows that if such criteria are to be used, then the dependence on the geometry of the accretion must be accounted for.

We calculate the spectra, and in particular the H–ionizing photon luminosity, $S$, of the various components as follows. When the accretion flow is optically thin, the luminosity from the stellar surface is $4\pi r^2 \sigma T_{\text{eff},1}^4$, where $T_{\text{eff},1}$ is given in equation (25). When the accretion flow is opaque, the effective temperature and radius of the photosphere are determined as described in §5.2. The assumption of a blackbody spectrum for the stellar spectrum is quite accurate for calculation of
fluxes of hydrogen-ionizing photons (Schaerer 2002), though not for photons that ionize He. Fortunately it is the feedback effects from H-ionization that are much more important (Paper II). The contribution from the boundary layer accretion luminosity is calculated assuming the radiation emerges with a blackbody spectrum from the upper and lower surfaces of an annulus that extends radially from the stellar surface by a distance equal to the height of the disk’s photosphere (estimated to be about 1.5 density scaleheights, $h$, from the disk model of §6; a more accurate treatment is given in Paper II). Finally the contribution from the inner accretion disk ($r < 10r_*$) is included, using the surface temperatures predicted by the radial disk model (§6).

The ionizing luminosities for different models are shown in Figure 7. The contribution from the protostellar photosphere is usually dominant. The differences between the low and high rotation parameter cases are very large, reflecting the different photospheric temperatures. These differences will prove crucial for determining mass limits to the star formation process from protostellar feedback (Paper II). Note that in the fiducial model the ionizing luminosity shows a very rapid increase from $m_* \simeq 20M_\odot$ to $m_* \simeq 40M_\odot$, reflecting the dramatic contraction of the star and the rapidly increasing internal luminosity. For these accretion rates ($K^* = 1$) the ionizing luminosity cannot increase much more quickly than this model, which applies to any rotating model in which the stellar photosphere coincides with its accretion surface. Thus without having completed the more detailed calculations of the effects of feedback on the accretion flow (Paper II), we expect that feedback processes will not become important until $m_*$ is at least $\sim 30M_\odot$, and this is an anticipated lower-limit to the mass of the first stars.

9. Conclusions

Recent numerical studies have indicated that the initial conditions for primordial star formation are dense, massive gas cores in approximate hydrostatic and virial equilibrium. These physical properties are set by the microphysics of H$_2$ cooling and not by the initial cosmological density perturbations. We have presented a theoretical model for star formation from these cores, basing our fiducial case on the core formed at the end of the simulation of Abel, Bryan, & Norman (2002). Following the philosophy of Stahler, Shu, & Taam’s (1980) study of low-mass, contemporary star formation, our approach has been to identify and isolate the various key stages that can be treated analytically and semi-analytically, and then combine them into a coherent picture of the whole formation process. This will be extended to include feedback processes in Paper II.

We have described the rate of collapse of the cores as a function of the entropy parameter of the gas, $K = P/\rho^p$, and the amount of mass that has already collapsed. The mass of gas in hydrostatic equilibrium inside the point at which $T \sim 300$ K, the minimum temperature due to H$_2$ cooling and $n \sim 10^4$ cm$^{-3}$, the critical density of H$_2$ rotational-vibrational line cooling, yields total core masses of about $10^3 M_\odot$. These cores collapse from the inside-out to form a protostar, which then grows rapidly in mass, with an accretion rate $\dot{m}_* \simeq 0.017(m_*/M_\odot)^{-3/7} M_\odot$ yr$^{-1}$.

We have developed a simplified method for modeling protostellar evolution and applied the appropriate accretion rate for primordial protostars. The method allows for the treatment of accretion of gas with angular momentum, so that part (most, in the typical case) of the accretion occurs via a disk. Using a realistic degree of rotation for the initial gas core, we find that conditions at the protostar rapidly become optically thin, in contrast to the spherical case. This means that the radiation field that the accretion envelope is exposed to is significantly hotter so that ionization and FUV radiation pressure feedback may become important at much smaller stellar masses than in the spherical case (Paper II).

In order to determine the emission spectrum of the protostar, we evaluated the spectrum of the radiation emitted from the stellar surface in both the optically thick and thin cases, and we included the emission from the inner accretion disk, including the boundary layer. These predictions will be used in Paper II to estimate the importance of various radiative feedback processes.

One may ask how the initial conditions and thus accretion rates of primordial star formation differ from those of contemporary star formation. There are several major differences: (1) the mass scale of cores is higher, being set by the microphysics of H$_2$ cooling, rather than CO and dust cooling; (2) the gas temperatures are higher, which follows from (1) and virial equilibrium, so that at the onset of dynamical collapse the quasi-equilibrium state is denser, the free-fall time shorter, and the accretion rate larger; (3) because the accretion rate is larger, primordial protostars are larger and reach the main sequence at higher masses; (4) correspondingly, support by deuterium burning is not particularly important, in contrast to protostars today; (5) thermal pressure is dominant in the initial gas cores, which is not true for contemporary massive cores (McKee & Tan 2002), primarily because metal-rich, contemporary cores are permeated by dynamically important magnetic fields and are able to radiate away most of their thermal energy, leaving themselves in a state of non-thermal pressure support that is supersonically turbulent; and (6) the temperature structure is such that the gas temperature decreases with radius, leading to accretion rates that decline with time, again opposite to the evolution of contemporary massive cores.

The contraction to the main sequence (after the star is approximately older than its Kelvin-Helmholz time), marks a major transition when radiative feedback from the protostar becomes much more important. Thus, even before a detailed analysis of these processes (Paper II), we anticipate that the masses of the first generation of stars must have been at least as great as $\sim 30M_\odot$.

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Fig. 6.— Top panel: Evolution of bolometric luminosities with protostellar mass for the fiducial isentropic model with rotation $f_{\text{Kep}} = 0.5$, $K' = 1$, and $\alpha_{\text{ss}} = 0.01$. Total radiated bolometric luminosity including contribution from inner accretion disk (solid line), stellar accretion luminosity including from direct spherical accretion and boundary layer accretion (dashed line), internal protostellar luminosity (long-dashed line), and accretion disk luminosity from $r < 10r_\ast$ (dotted line) are shown. The Eddington luminosity is shown by the dot-long-dashed line. The combination of a declining accretion rate, increasing stellar radius, and increasing stellar mass at later stages when the protostar is accreting on the main sequence, lead to approximately constant accretion luminosities. Note the total luminosity remains sub-Eddington over the entire evolution ($m_\ast \leq 1000M_\odot$). Bottom panel: Evolution of total bolometric luminosities for fiducial isentropic models with $K' = 1$ and $\alpha_{\text{ss}} = 0.01$, and with rotation $f_{\text{Kep}} = 0, 0.05, 0.5$ (dashed, dotted, solid lines, respectively). Note that the spherical accretion case ($f_{\text{Kep}} = 0$) exceeds the Eddington limit at $m_\ast \approx 100M_\odot$, but the more realistic disk accretion models are slightly sub-Eddington.

Formation Studies.

**APPENDIX**

A. THE RADIUS OF A PROTOSTAR

In this appendix, we first use the equations of radiation hydrodynamics to derive the rate at which a protostar accretes energy. Following Nakano et al. (1995; 2000), we then equate this to the rate of change of the energy of the protostar to obtain an equation that governs the evolution of the protostellar radius.

A.1. Energy Accretion

Let

$$\epsilon \equiv \frac{1}{2} v^2 + \epsilon_{\text{th}} + \epsilon_I \tag{A1}$$
be the total energy per gram of gas, where \( \epsilon_{\text{th}} = \frac{3}{2}kT/\mu \) is the thermal per unit mass and \( \epsilon_I \) is the internal energy per unit mass, including chemical binding energy. The zero point of \( \epsilon_I \) is determined by the boundary conditions. To relate this to the work of Nakano et al. (1995, 2000), we write \( \epsilon_I = \Psi_I/m_H \). If the gas is fully molecular, then since it takes 4.48 eV to dissociate \( H_2 \), 13.6 eV to ionize \( H \), and 79 eV to fully ionize \( He \), the maximum value of \( \Psi_I \) is \( \Psi_{Im} = 16.8 \) eV for a primordial gas with 0.079 He per H.

Conservation of energy, including radiation, is given by (see eqs. 94.15b and 96.17 in Mihalas & Weibel-Mihalas 1999)

\[
\frac{\partial}{\partial t} (\rho \epsilon + u_{\text{rad}}) + \nabla \cdot \left[ \rho \mathbf{v} \left( \epsilon + \frac{P_g}{\rho} \right) + \mathbf{F} \right] = \rho \mathbf{g} \cdot \mathbf{v} + \rho \dot{\epsilon}_{\text{nuc}},
\]

(A2)

where \( u_{\text{rad}} \) is the energy density of radiation, \( P_g \) is the gas pressure, \( \mathbf{F} \) is the flux of radiation measured in the frame of the protostar, and \( \rho \dot{\epsilon}_{\text{nuc}} \) is the rate of nuclear energy generation per unit volume. This equation is accurate to order \( v/c \). Now, the rate of work by the gravitational field can be written

\[
\rho \mathbf{g} \cdot \mathbf{v} = -\nabla \cdot (\rho \mathbf{v} \phi) - \frac{\partial}{\partial t} (\rho \phi) + \rho \frac{\partial \phi}{\partial t},
\]

(A3)

(Shu 1992), so that equation (A2) becomes

\[
\frac{\partial}{\partial t} [\rho(\epsilon + \phi) + u_{\text{rad}}] + \nabla \cdot \left[ \rho \mathbf{v} \left( \epsilon + \frac{P_g}{\rho} + \phi \right) + \mathbf{F} \right] = \rho \frac{\partial \phi}{\partial t} + \rho \dot{\epsilon}_{\text{nuc}}.
\]

(A4)
The rate of change of the energy of the protostar can be evaluated by integrating equation (A4) over the volume of the protostar. Let \( E_g \) be the total gas energy, etc. Then, since the gravitational energy satisfies
\[
\frac{dE_g}{dt} = \frac{1}{2} \int \frac{\partial}{\partial t} (\rho \phi) dV = \int \rho \frac{\partial}{\partial t} (\phi) dV,
\]
(Shu 1992), the total energy \( E = E_g + E_{\text{ke}} + E_r \) obeys
\[
\frac{dE}{dt} + \int dA \rho v \left( \epsilon + \frac{P_g}{\rho} + \phi \right) = L = E_{\text{nucl}},
\]
This equation is valid for an arbitrary mass distribution. The volume of integration must include all the self-gravitating mass, so it is valid only for \( r \gtrsim r_s \).

We now assume that the protostar is approximately spherical, so that for \( r \gtrsim r_s \), the potential is
\[
\phi = -\frac{Gm_s}{r} = -\frac{1}{2} v_{ff}^2,
\]
where \( v_{ff} \) is the free-fall velocity. We relate the velocity of the gas to the free-fall velocity by
\[
f_k = \frac{v^2}{v_{ff}^2},
\]
Inside the accretion shock, \( f_k \sim 0 \) since we assume that the star is not rapidly rotating; in the boundary layer of an accretion disk, \( f_k \simeq 1/2 \); and in free-fall collapse, \( f_k = 1 \). Denote the average over the surface of some quantity \( x \) by \( \langle x \rangle \), so that if the protostar accretes matter directly via free-fall collapse at a rate \( \dot{m}_s, \text{direct} \) and via disk accretion at a rate \( \dot{m}_s, \text{disk} \), then
\[
\langle x \rangle = \frac{1}{\dot{m}_s} (\dot{m}_s, \text{direct} x_{\text{direct}} + \dot{m}_s, \text{disk} x_{\text{disk}}).
\]
Equation (A6) for the energy then becomes
\[
\frac{dE}{dt} = \dot{m}_s \left[ \frac{5}{3} \epsilon_{\text{th}} + \epsilon_f - \frac{1}{2} (1 - f_k) v_{ff}^2 \right] - L + E_{\text{nucl}} \equiv \dot{m}_s \langle w \rangle - L + E_{\text{nucl}},
\]
where \( \dot{m}_s \) is positive for accretion and \( w \) is the enthalpy plus the potential energy per unit mass. Note that in contrast to Nakano et al. (1995, 2000), we do not include nuclear binding energy in our expression for the energy, and as a result the term \( E_{\text{nucl}} \) appears on the right-hand side of equation (A10).

Since the energy of the protostar is dominated by its interior, we are free to choose any surface layer at which to evaluate \( dE/dt \). Equivalently, we can evaluate equation (A4) for steady flow in a region in which \( \dot{\epsilon}_{\text{nucl}} = 0 \). Either way, we conclude that the luminosity is given by
\[
L = L_0 + \dot{m}_s \left[ \frac{5}{3} \epsilon_{\text{th}} + \epsilon_f - \frac{1}{2} (1 - f_k) v_{ff}^2 \right] = L_0 + \dot{m}_s \langle w \rangle,
\]
in terms of \( L_0 \), the emergent luminosity. This equation is similar to equation (96.18) of Mihalas & Weibel-Mihalas (1999), except that (1) we are focusing on the surface layers so that there is no nuclear energy generation and (2) we have not assumed spherically symmetric flow.

A.2. Equation for the Protostellar Radius

Following Nakano et al. (1995, 2000), we estimate the protostellar radius by equating equation (A10) for \( dE/dt \) to one calculated by evaluating the rate of change of the energy of the protostar,
\[
E = -\frac{a_\beta GM_s^2}{r_s} + m_s \epsilon_f.
\]
The first term in this equation represents the sum of the thermal energy of the gas, the energy of the radiation and the gravitational energy, and it follows directly from the virial theorem. Here \( \beta \) is the mean ratio of gas pressure to total pressure and \( a_\beta GM_s^2/r_s \) is the gravitational energy. We approximate the structure of the protostar as a polytrope of index \( \gamma_{ps} = 1 + 1/n \), so that \( a_\beta = 3/(5 - n) \) with \( n < 5 \). The second term in equation (A12) is the mean ionization energy of the material in the star. We shall assume that the star is approximately fully ionized \( (\epsilon_f \simeq \epsilon_{f,m}) \), which means that we cannot treat the earliest stages of protostellar evolution with our method. Differentiating equation (A12) and equating it to the result in equation (A10) evaluated at \( r_2 \) yields
\[
\frac{d \ln r_s}{d \ln m_s} = 2 + \frac{d \ln \beta}{d \ln m_s} - 4 \frac{a_\beta \gamma_{ps}}{a_\beta \gamma_{ps} + 1} \left[ \frac{1}{2} \epsilon_f \vert_{\text{ps}} + \epsilon_{f,m} + \frac{5kT_2}{2\mu_2} + \frac{1}{m_s} \left( L_2 - \dot{E}_{\text{nuc}} \right) \right],
\]
where \( \mu_2 \) is the mean mass per particle at \( r_2 \). This result is identical to that of Nakano et al. (2000), except that (1) equation (A13) has the enthalpy \( 5kT_2/2\mu_2 \) instead of the thermal energy \( 3kT_2/2\mu_2 \), (2) we allow the accretion flow to be optically thick, so that it is possible for \( T_2 \) to be much larger than the photospheric temperature, and (3) it explicitly allows for both direct accretion onto the star and for accretion via a disk.
REFERENCES

Abel, T., Bryan, G. L., & Norman, M. L. 2000, ApJ, 540, 39
Abel, T., Bryan, G. L., & Norman, M. L. 2002, Science, 295, 93
Adams, F. C., Ruden, S. P., & Shu, F. H. 1989, ApJ, 347, 959
Adams, F. C., & Shu, F. H. 1986, ApJ, 308, 836
Aoki, W., Norris, J. E., Ryan, S. G., Beers, T. C., & Ando, H. 2002, ApJ, 576, L141
Balbus, S. A., & Hawley, J. F. 1991, ApJ, 376, 214
Balbus, S. A., & Hawley, J. F. 1998, Rev. Mod. Phys., 70, 1
Baraffe, I., Heger, A., & Woosley, S. E. 2001, ApJ, 550, 890
Baumgarte, T. W., Kudritski, R. P., & Loeb, A. 2001, ApJ, 552, 464
Band, J., & Raine, D. 1995, Accretion Power in Structures (New York: Dover)
Christlieb, N., Bessell, M. S., Beers, T. C., Gustafsson, B., Korn, A., Barklem, P. S., Karlsson, T., Mizuno-Wiedner, M., & Rossil, S. 2002, Nature, 419, 904
Ciardi, B., Ferrara, A., Marri, S., & Raimondo, G. 2001, MNRAS, 324, 381
Frank, J., King, A., & Raine, D. 1995, Accretion Power in Astrophysics, 2nd Ed. (Cambridge: CUP)
Fryer, C. L., Woosley, S. E., & Heger, A. 2001, ApJ, 551, 372
Furlanetto, S., & Loeb, A. 2002, ApJ, 579, 1
Gammie, C. F. 2001, ApJ, 553, 174
Heger, A., & Woosley, S. E. 2002, ApJ, 567, 532
Hirase, T., Aizu, K., & Taketani, M. 1969, Progr. Theor. Phys., 32, 1
Hunter, C. 1977, ApJ, 218, 834
Hutchinson, J. B. 1976, ApJ, 205, 103
Iglesias, C. A., & Rogers, F. J. 1996, ApJ, 464, 943
Khop, A. et al. 2003, ApJ, in press, astro-ph/0302213
Königl, A., & Pudritz, R. E. 2000, in Protostars & Planets IV, eds. V. Mannings, A. P. Boss, & S. S. Russell (Tucson: The University of Arizona Press), 759
Larson, R. B. 1969, MNRAS, 145, 271
Lanzini, P., Chernoff, D. F., & Salpeter, E. E. 1991, ApJS, 76, 759
MacFadyen, A., & Woosley, S. E. 1999, ApJ, 524, 262
Madam, P. 2002, astro-ph/0210268
Madau, P. 1999, ApJ, 514, 648
Magliocchetti, M., Salvaterra, R., & Ferrara, A. 2003, MNRAS, submitted, (astro-ph/0304280)
Matzner, C. D. 2003, MNRAS, in press, (astro-ph/0203085)
McDowell, M. R. C. 1961, Observatory, 81, 240
McKee, C. F., & Hollenbach, D. J. 1980, ARA&A, 18, 219
McKee, C. F., & Holliman, J. H. 1999, ApJ, 522, 313
McKee, C. F., & Tan, J. C. 2002, Nature, 416, 59
McKee, C. F., & Tan, J. C. 2003, ApJ, 585, 850
McLaughlin, D. E., & Pudritz, R. E. 1997, ApJ, 476, 750
McLaughlin, D. E., & Pudritz, R. E. 1999, Foundations of Radiation Hydrodynamics (New York: Dover)
Mihalas, D., & Weibel-Mihalas, B. 1999, Foundations of Radiation Hydrodynamics (New York: Dover)
Nakano, T., Hasegawa, T., & Norman, C. 1995, ApJ, 450, 183
Nakano, T., Hasegawa, T., Morino, J.-I., & Yamashita, T. 2000, ApJ, 534, 576
Omukai, K., & Nishi, R. 1998, ApJ, 508, 141
Omukai, K., & Palla, F. 2001, ApJ, 561, L55
Omukai, K., & Palla, F. 2003, ApJ, 589, 677
Orel, A. E. 1987, J. Chem. Phys. 87, 314
Paczynksi, B. 1998, in Gamma-Ray Bursts: 4th Huntsville Symp., ed. C. A. Meegan, R. D. Price, & T. M. Koshyt (New York: AIP), 783
Palla, F., Salpeter, E. E., & Stahler, S. W. 1983, ApJ, 271, 632
Palla, F., & Stahler, S. W. 1991, ApJ, 375, 288
Penston, M. V., 1969, MNRAS, 144, 425
Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton: Princeton University Press)
Pettini, M., & Bowen, D. V. 2001, ApJ, 560, 41
Rees, M. J. 1984, ARA&A, 22, 471
Ripamonti, E., Haardt, F., Ferrara, A., & Colpi, M. 2002, MNRAS, 334, 401
Rogers, F. J., & Iglesias, C. A. 1992, ApJ, 401, 361
Salvaterra, R., & Ferrara, A. 2003, MNRAS, 339, 973
Schaerer, D. 2002, A&A, 382, 28
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337
Shu, F. H. 1977, ApJ, 214, 488
Shu, F. H. 1992, The Physics of Astrophysics, Vol. 2 (Mill Valley: Univ. Science Books)
Shu, F. H., Tremaine, S., Adams, F. C., & Ruden, S. P. 1990, ApJ, 361, 546
Shu, F. H., Najita, J., Shang, H., Li, Z.-H. 2000, in Protostars & Planets IV, eds. V. Mannings, A. P. Boss, & S. S. Russell (The University of Arizona Press), 789
Silk, J. 1977, ApJ, 211, 638
Spergel, D. N., et al. 2003, ApJ, in press, (astro-ph/0302209)
Stahler, S. W. 1988, ApJ, 332, 804
Stahler, S. W., & Taam, R. E. 1980, ApJ, 241, 637
Stahler, S. W., Palla, F., & Salpeter, E. E. 1986, ApJ, 302, 590
Tan, J. C., & Blackman, E. G. 2003, ApJ, submitted
Tan, J. C., Matzner, C. D., & McKee, C. F. 2001, ApJ, 551, 946
Tan, J. C., & McKee, C. F. 2003, in Emergence of Cosmic Structure, eds. S. Holt and C. Reynolds, (New York:AIP)
Terebey, S., Shu, F. H., & Cassen, P. 1984, ApJ, 286, 529
Tumlinson, J., & Shull, J. M. 2000, ApJ, 528, L65
Ulrich, R. K. 1976, ApJ, 210, 377
Woosley, S. E. 1993, ApJ, 405, 273
Yahil, A. 1983, ApJ, 265, 1047
Yoneyama, T. 1972, PASJ, 31, 505
Zeldovich, Ya. B., & Novikov, I. D. 1971, Relativistic Astrophysics, Vol. 1 (Chicago: Univ. Chicago Press)