The Kinematic and Dynamic Analysis of a Bionic Manipulator

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Abstract. Bionic manipulators can help people with disabilities to improve their quality of life, they thus became a focus researched by many scholars. In this paper, the kinematic and dynamic analysis of a bionic manipulator was conducted. Firstly, the kinematic model of the bionic manipulator was developed. The relationships between input and out variables were obtained. Then, the dynamic model of the bionic manipulator was established by using the Lagrangian method. The dynamic parameters including the angular displacement, angular velocity and angular acceleration under the action of torque were analyzed. The results indicated that the angular displacements of the three angles are increasing with time. With the increasing of time, the angular velocity first increased and then decreased. The angular accelerations are oscillation with an increase in time. The work in this paper lays the groundwork for future applications of bionic manipulators.

1. Introduction

In the past few decades, the number of disabled people has increased due to the outbreak of war and traffic accidents. According to the second national sample survey on disabled persons [1], there are 70.5 million households, accounting for 17.80% of the total number of households, with disabled persons in China. Among them, there are 8.76 million households with two or more disabled persons, accounting for 12.43 percent of all households with disabled persons. The population of households with disabled persons accounts for 19.98 percent of the national total population. Moreover, many disabled persons among disabilities are hand amputees and the quality of their life has been greatly affected. It follows that the research on bionic manipulators has become a hot point recently.

The bionic manipulators have been researched by lots of scholars. A novel method based of electromyography was proposed to adjust the grip strength of the prosthetic hand automatically [2-4]. Afterwards, the rope-driven under-actuated manipulator was studied and the dynamic model was developed by [5]. It was obtained that the proposed manipulator can grasp objects stably. High-density electromyography (EMG) signals were employed in [6] to characterize the accuracy of recognizing the discharges of motor units during natural movement and their correlation with finger kinematics was then researched. The dynamics should be considered during the design of the bionic manipulators. Many models including pulley combination model and connecting rod model were proposed by [7-9]. Their dynamic characteristics were studied. Li [10] researched a single finger by using the principle of virtual work and obtained relationship between the driving torques, the internal forces of the coupling rod spring and the gripping forces. Moreover, when analyzing the dynamics of the bionic manipulators, the fingers are usually regarded as light rods. The weight of the fingers is thus ignored to simplify the
calculation procedures. The angular displacement, angular velocity and angular acceleration of a bionic manipulator were analyzed by modeling the bionic manipulator as hinges and linkage mechanisms [11-14].

The main contribution of this article is to obtain the relationship between the input and output parameters of one finger of the bionic manipulator and studied the dynamics of the finger considering its weight.

2. Mechanism description

As illustrated in figure 1, a human palm is composed of carpal bones, metacarpal bones and phalanx bones. Inspired by the composition of a human hand, a bionic manipulator composed of rigid rods and revolute joints is proposed, shown in Figure 2.

From figure 2, it can be seen that the bionic manipulator is composed of finger prostheses, a palm of hand, fishing lines, spring plates and motors. The fingers can generate rotational motions driven by the motors through the fishing lines, shown in figure 3. The spring plates are used to restore the original shape when the finger relaxes. The motor used to close the thumb is denoted by motor 1 while the motor used to rotate the thumb is denoted by motor 2. Moreover, the closure of the index finger is controlled by motor 3 while the closure of the middle finger, ring finger and pinkie is controlled by motor 4.

From figure 2, it can be seen that the index finger has three DOFs (degrees of freedom). It follows that three motors should be used to make the index finger move accurately. In the follows sections, the kinematics and dynamics of the index finger are discussed. Firstly, the reference frame should be established, which is shown in figure 4. In figure 4, the fixed reference frame is locate at the base of the index finger, with its Z axis parallel to the rotational axis of the base revolute joint.
3. Direct kinematic analysis

According to the composition of the index finger, it can be considered as a serial mechanism of three degrees of freedom. The motion sketch of the index finger is thus shown in figure 5. From figure 5, it can be seen that the index finger is composed of three rigid rods $OA$, $AB$ and $BC$. $C_1$, $C_2$ and $C_3$ are the centroid of the rigid rods $OA$, $AB$ and $BC$ respectively. The angle between the $X$ axis and the rigid rod $OA$ is defined as $\theta_1$ while the angle between the rigid rod $AB$ and the line joining nodes $O$ and $A$ is defined as $\theta_2$. Moreover, the angle between the rigid rod $BC$ and the line joining nodes $A$ and $B$ is defined as $\theta_3$. The distance between nodes $O$ and $C_1$ is $d_1$ while the distance between nodes $A$ and $C_2$ is $d_2$. Furthermore, the distance between nodes $B$ and $C_3$ is $d_3$. The lengths of rigid rods $OA$, $AB$ and $BC$ are $l_1$, $l_2$ and $l_3$ respectively.

The shape of the index finger can be controlled by varying the angles $\theta_1$, $\theta_2$ and $\theta_3$, driven by motors, which are chosen as the input variables of the index finger. The position of node $C$, expressed by its Cartesian coordinates $(x, y$ and $z)$ is chosen as the output of the index finger.

![Figure 5. Motion sketch of the index finger](image)

Let $X_1 = [x_1, y_1, z_1]^T$, $X_2 = [x_2, y_2, z_2]^T$ and $X_3 = [x_3, y_3, z_3]^T$ denote the coordinates of points $C_1$, $C_2$ and $C_3$ respectively. These coordinates can be written as

$$X_1 = [x_1, y_1, z_1]^T = [d_1 \cos \theta_1, d_1 \sin \theta_1, 0]^T$$

$$X_2 = [x_2, y_2, z_2]^T = [l_1 \cos \theta_2 + d_2, l_1 \sin \theta_2 + d_2 \sin \theta_2, 0]^T$$

$$X_3 = [x_3, y_3, z_3]^T = [l_1 \cos \theta_3 + l_2 \cos \theta_2 + d_3, l_1 \sin \theta_3 + l_2 \sin \theta_2 + d_3 \sin \theta_2, 0]^T$$

The direct kinematic analysis of the index finger corresponds to the computation of position of the node $C$ for given angles $\theta_1$, $\theta_2$ and $\theta_3$. From figure 5, the coordinates of node $C$ can be easily computed, which take the following form

$$X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} l_1 \cos \theta_1 + l_1 \cos \theta_2 + l_1 \cos \theta_3 \\ l_1 \sin \theta_1 + l_2 \sin \theta_2 + l_3 \sin \theta_3 \\ 0 \end{bmatrix}$$

From equation (4), it can be seen the movement of node $C$ is constrained to the $XY$ plane. For a set of given input variables, the output variables can be easily computed.

4. Dynamic analysis

The Lagrangian approach is employed in this work to derive an appropriate dynamic model of the index finger. Afterwards, the kinetic and potential energy should be firstly computed.

4.1. Kinetic energy

According to equations (1)-(3), the velocity of centroid of the rigid rods $OA$, $AB$ and $BC$ can be written as
\[ v_1 = \left[ \dot{x}_1^2 + \dot{y}_1^2 \right]^{\frac{1}{2}} = \left[ d_1 \dot{\theta}_1 \right]^{\frac{1}{2}} \]  

\[ v_2 = \left[ \dot{x}_2^2 + \dot{y}_2^2 \right]^{\frac{1}{2}} = \left[ d_2 \cos(\theta_1 + \theta_2) \dot{\theta}_1 + \dot{\theta}_2 + l, \cos \theta_1 \cdot \dot{\theta}_1 \right]^{\frac{1}{2}} + \left[ d_2 \sin(\theta_1 + \theta_2) \dot{\theta}_1 + \dot{\theta}_2 + l, \sin \theta_1 \cdot \dot{\theta}_1 \right]^{\frac{1}{2}} \]  

\[ v_3 = \left[ \dot{x}_3^2 + \dot{y}_3^2 \right]^{\frac{1}{2}} = \left[ \left( l, \cos \theta_1 \cdot \dot{\theta}_1 + l_2 \cos(\theta_1 + \theta_2) \cdot (\dot{\theta}_1 + \dot{\theta}_2) + d_3 \cos \sum_{i=1}^{m} \theta_i \cdot \sum_{i=1}^{m} \dot{\theta}_i \right)^2 \right. \]  

\[ \left. + \left( l, \sin \theta_1 \cdot \dot{\theta}_1 + l_2 \sin(\theta_1 + \theta_2) \cdot (\dot{\theta}_1 + \dot{\theta}_2) + d_3 \sin \sum_{i=1}^{m} \theta_i \cdot \sum_{i=1}^{m} \dot{\theta}_i \right)^2 \right]^{\frac{1}{2}} \]  

(5)

(6)

(7)

Since the kinetic energy of a rigid rod contains the translational kinetic energy corresponding to the rod’s translations and the rotational kinetic energy corresponding to the rod’s rotations, the kinetic energy of the index finger can thus be computed as

\[ E_k = \sum_{i=1}^{m} \sum_{j=1}^{n} \left( m_i v^2_i + J_{oi} \dot{\theta}_i^2 \right) \]  

(8)

where \( J_{o1}, J_{o2} \) and \( J_{o3} \) are the moments of inertia of the rods \( OA, AB \) and \( BC \) with respect to their centers.

4.2. Potential energy

The potential energy of the index finger includes the gravitational potential energy of the rigid rods and the elastic potential energy of the torsional springs, which are used to restore the finger’s original position without the control of the motor. The potential energy of the index finger takes the following form

\[ E_p = \sum_{i=1}^{m} E_{pi} \]  

(9)

where

\[ E_{p1} = k \left( \frac{\pi \theta_1}{180} \right)^2 + d_1 g m_1 \sin \theta_1 \]  

(10)

\[ E_{p2} = g m_2 \left[ l_1 \sin \theta_1 + d_1 \sin(\theta_1 + \theta_2) \right] + k \left( \frac{\pi \theta_1}{180} \right)^2 \]  

(11)

4.3. Equations of motion

The equations of motion of the index finger are developed here using the Lagrangian method, namely

\[ \tau = d \frac{\partial E_k}{\partial \dot{\theta}} - \frac{\partial E_k}{\partial \theta} + \frac{\partial E_p}{\partial \dot{\theta}} \]  

(12)

where \( \theta = [\theta_1, \theta_2, \theta_3]^T \). By substituting equations (8) and (9) into equation (12), the dynamic model of the index finger can be obtained as

\[ M \ddot{\theta} + H \dot{\theta}^2 + K \theta_t + G = F \]  

(13)

where

\[ G = \left[ d_1 g m_1 \cos \sum_{i=1}^{m} \theta_i + (d_2 g m_2 + g l_m) \cos \sum_{i=1}^{m} \theta_i + (d_1 g m_1 + g l_1 m_1) \cos \theta_1 + k \left( \frac{\pi}{180} \right)^2 \theta_1 \right. \]  

\[ \left. + (d_2 g m_2 + g l_m) \cos \sum_{i=1}^{m} \theta_i + (d_2 g m_2 + g l_m) \cos \sum_{i=1}^{m} \theta_i + k \left( \frac{\pi}{180} \right)^2 \theta_1 \right. \]  

\[ \left. + d_3 g m_1 \cos \sum_{i=1}^{m} \theta_i + k \left( \frac{\pi}{180} \right)^2 \theta_1 \right] \]  

\[ F = [\tau_1, \tau_2, \tau_3]^T, \theta_v = \left[ \dot{\theta}_1 \dot{\theta}_2 \dot{\theta}_3 \dot{\theta}_3 \right]^T \]  

(14)

(15)
Furthermore, $M$, $H$ and $K$ are all $3 \times 3$ matrices whose elements are defined as follows (e.g. $M_{ij}$ is the element located on the $i$th line and the $j$th column of $M$, etc.):

\[
M_{11} = J_0 + d_1^2 m_1 + d_2^2 m_2 + d_3^2 m_3 + l_1^2 m_1 + l_2^2 m_2 + 2 d_1 l_1 m_2 + 2 d_1 l_1 m \cos \theta_1 + 2 d_1 l_1 m \cos \theta_2 + 2 d_1 l_1 m \cos (\theta_2 + \theta_3)
\]

\[
M_{12} = l_2^2 m_1 + d_1^2 m_2 + d_2^2 m_1 + 2 d_1 l_2 m_2 + 2 l_2 l_1 m \cos \theta_2 + 2 d_1 l_1 m \cos \theta_3 + 2 d_1 l_1 m \cos (\theta_2 + \theta_3)
\]

\[
M_{13} = d_1^2 m_1 + d_1 l_1 m \cos \theta_1 + d_1 l_1 m \cos (\theta_2 + \theta_3)
\]

\[
M_{21} = d_2^2 m_2 + d_2^2 m_2 + l_2^2 m_2 + d_2 l_2 m_2 + l_2 l_2 m_2 \cos \theta_2 + 2 d_2 l_1 m_2 \cos \theta_2 + d_2 l_2 m_2 \cos \theta_2 + d_2 l_2 m_2 \cos (\theta_2 + \theta_3)
\]

\[
M_{22} = J_0 + d_2^2 m_2 + d_3^2 m_2 + l_2^2 m_2 + 2 d_2 l_1 m_3 \cos \theta_3 + 2 d_2 l_2 m_3 \cos \theta_3
\]

\[
M_{23} = d_2^2 m_2 + d_1 l_1 m \cos \theta_1
\]

\[
M_{31} = d_3^2 m_3 + d_3^2 m_3 \cos \theta_1 + d_1 l_1 m \cos \theta_2 + d_1 l_1 m \cos (\theta_2 + \theta_3)
\]

\[
M_{32} = d_3^2 m_3 + d_1 l_1 m \cos \theta_1
\]

\[
H_{12} = (d_1 l_1 m_2 - l_1 l_1 m_3) \sin \theta_2 - d_1 l_1 m_2 \sin (\theta_2 + \theta_3)
\]

\[
H_{13} = -d_1 l_1 m_3 \sin \theta_2 - d_1 l_1 m_3 \sin (\theta_2 + \theta_3)
\]

\[
H_{21} = (d_2 l_2 m_2 + d_2 l_2 m_2) \sin \theta_2 + d_2 l_2 m_2 \sin (\theta_2 + \theta_3),
\]

\[
H_{23} = -d_2 l_2 m_2 \sin \theta_2, H_{22} = H_{32} = H_{33} = 0
\]

\[
H_{31} = d_3 l_3 m_3 \sin \theta_2 + d_3 l_3 m_3 \sin (\theta_2 + \theta_3),
\]

\[
H_{32} = d_3 l_3 m_2 \sin \theta_2
\]

\[
K_{11} = (-2d_1 l_1 m_2 - 2l_1 l_1 m_3) \sin \theta_2 - 2d_1 l_1 m_2 \sin (\theta_2 + \theta_3),
\]

\[
K_{12} = -2d_1 l_1 m_2 \sin \theta_2 - 2d_1 l_1 m_3 \sin (\theta_2 + \theta_3)
\]

\[
K_{13} = -2d_1 l_1 m_3 \sin \theta_2 - 2d_1 l_1 m_3 \sin (\theta_2 + \theta_3)
\]

\[
K_{21} = -2d_2 l_2 m_2 \sin \theta_2, K_{22} = -2d_2 l_2 m_2 \sin \theta_2
\]

\[
K_{23} = 2d_2 l_2 m_2 \sin \theta_3, K_{21} = K_{32} = K_{33} = 0
\]

5. Numerical simulations
The dynamic model of the index finger is established in equation (13). By using equation (13), the motions of the index finger can be obtained for the given torques $\tau_1$, $\tau_2$ and $\tau_3$. In this section, the numerical simulations of the finger’s motions are conducted. Moreover, the coefficients of equation (13) are listed in table 1.

| Parameter | Value | Unit | Parameter | Value | Unit |
|-----------|-------|------|-----------|-------|------|
| $l_1$     | 1.5   | m    | $m_1$     | 0.1   | kg   |
| $l_2$     | 1.5   | m    | $m_2$     | 0.1   | kg   |
| $l_3$     | 1.5   | m    | $m_3$     | 0.1   | kg   |
| $d_1$     | 1     | m    | $J_{01}$  | 1     | kg·m²|
| $d_2$     | 1     | m    | $J_{02}$  | 1     | kg·m²|
| $d_3$     | 1     | m    | $J_{03}$  | 1     | kg·m²|
| $k$       | 1     | N/m  | $J_0$     | 1     | kg·m²|
The driven torques are chosen as \( \tau_1 = \tau_2 = \tau_3 = 10 \text{ Nm} \) while the initial values of \( \theta_1, \theta_2 \) and \( \theta_3 \) are chosen as \( \theta_{10} = \theta_{20} = \theta_{30} = 30^\circ \). By employing the Runge-Kutta method, the motion laws of the index finger can be obtained, which are shown in figures 6-11.

From figures 6-11, it can be seen that the angular displacement \( \theta_1 \) increases with time. The angular velocity \( \dot{\theta}_1 \) oscillates gradually and the average of \( \dot{\theta}_1 \) increases with time. Moreover, the angular displacement \( \theta_2 \) increases in a parabolic trend at 0-8s and remains oscillating after \( t = 8s \). The angular velocity \( \dot{\theta}_2 \) first increases and then oscillates with an increase in time. The angular displacement \( \theta_3 \) increases slowly during 0-4s, and shows an oscillating trend during 4-8s. Then, the increasing trend is more obvious after \( t = 8s \). The angular velocity \( \dot{\theta}_3 \) presents an upward trend during 0-4s. Afterwards, it oscillates when \( 4s \leq t \leq 8s \). However, when \( t > 8s \), it presents the upward trend again. These motion laws lay a foundation for the controller design of the index finger.

6. Conclusion
In this work, the kinematic and dynamic analysis of the index finger has been conducted. By employing the Runge-Kutta method, the motion of the index finger is simulated. The results indicate that the angular displacements of the three angles are all increasing with an increase in time for the given torques. The angular velocities are all oscillating with the increasing of time. These motion laws should be considered during the design of the index finger. The reason for the angular velocities
present an oscillating trend is that the damping factors such as friction between moving pairs and friction of the motor are ignored in this study. Furthermore, the influence of the damping factors on the dynamics of the index finger will be researched in the future work.

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