Topological phase transitions and Weyl semimetal phases in chiral photonic metamaterials

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Abstract
Recently, topologically nontrivial phases in chiral metamaterials have been proposed. However, a comprehensive description of topological phase diagrams and transitions in chiral metamaterials has not been presented. In this work, we demonstrate several forms of topological phase transitions and study the existence of edge states in different phases. In the local/lossless chiral media system, the topological phase transitions are associated with Weyl points. Along with the transitions, the edge state and Fermi arc exhibit a series of changes. When the nonlocal effect is introduced, the system shows phase transition between type-I/II Weyl semimetal phase and trivial phase. Moreover, the dissipative system also undergoes topological phase transitions owing to the annihilation of the topological charges. Our work could be helpful for the application of topological concepts and rich the topological wave physics in metamaterials.

1. Introduction

Topology is a branch of mathematics. It describes the invariance under continuous deformation [1–5]. The idea of topological phases in photonics stems from the exciting development of solid materials, which have fundamentally changed our understanding of the phase of matter [6–16]. The novel topological phases have been proposed and studied in photonic systems, including topological insulators [17], topological semimetals [18], and non-abelian topological charges [19]. The topological semimetals are gapless in bulk structures, and their nontrivial topological properties are characterized by band degeneracy [20]. The development of topological semimetals reveals fascinating phenomena such as Dirac degeneracies [21–24], triple degenerate points [25–28], Weyl points [29–37], and nodal line (surface) [38–40]. Particularly, the Weyl point is regarded as a chiral singularity in three-dimensional momentum space [36]. It is a monopole Berry flux which is like the source or drain of magnetic flux in real space [1]. An important feature of the Weyl point is the existence of the Fermi arc connecting Weyl points at different positions [41].

The chiral metamaterials possess time-reversal symmetry but do not have spatial inversion symmetry owing to the chiral coupling between the magnetic field and electric field [42]. Different from homogeneous materials without time-reversal symmetry, chiral metamaterials do not require magneto-optical effects. Generally, the magneto-optical effects need a large static magnetic field [41]. It may be challenging to demonstrate experimentally. Recently, topological insulators [43], Weyl semimetals [44], and multiplexing and dynamically routing [45] have been proposed and studied in chiral metamaterials. However, there are very few investigations about topological phase transitions in chiral metamaterials. On the other hand, the bulk-edge correspondence is a widely used principle in topological wave physics [46]. According to the bulk-edge correspondence, the edge states can span the common band gap at the interface between two materials with different topologies [1]. The number of unidirectional edge modes is determined according to the topological properties of bulk states. Moreover, the topological structure of a
band gap is characterized by its band gap Chern number. It is defined as \( C_{\text{gap}} = \sum_i C_i \), i.e., the sum of Chern numbers of all bulk states below the band gap [46]. Specifically, the bulk-edge correspondence has been established in the gyroelectric media [47] and magnetized cold plasmas [48].

In the presented study, a comprehensive description of the topological phase diagrams and topological phase transitions in chiral metamaterials is given. The topological phase transitions are associated with Weyl points in the local/lossless chiral media system. We clarify that the Fermi arcs exist not only on the vacuum–chiral media interface but also on the chiral–chiral media interfaces. The type-I/II Weyl points and topological phase transitions are investigated when the nonlocal effect is considered. The Weyl exceptional rings and topological phase transitions are further shown in the dissipative chiral metamaterials. The dissipative system undergoes topological phase transitions owing to the annihilation of the topological charges.

2. Topological phase transitions in the local/lossless chiral metamaterials

2.1. Band structures

For the chiral metamaterials, the relative permittivity, permeability, and chirality tensors are given by \( \varepsilon = \text{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3) \), \( \mu = \text{diag}(\mu_1, \mu_2, \mu_3) \), where \( \varepsilon_2 = 1 - \omega_p^2/\omega^2 \), \( \omega_p \) represents the plasma frequency, and \( I \) is the identity matrix. The chiral effect exists in many natural materials, and the development of metamaterials enables us to synthesize strong chiral media [49–51]. A metallic helix structure along the \( x \) direction can be used to break the inversion symmetry and form a kind of chiral metamaterial [44]. Moreover, the metal wire along the \( z \) direction is used to produce the plasma longitudinal mode along the \( z \) direction [45].

Following reference [42], the chiral media system is modeled as an extended eigenvalue equation

\[
\hat{H} |\Psi\rangle = \omega \hat{A} |\Psi\rangle, \quad |\Psi\rangle = \left[ \begin{array}{c} \mathbf{E}_x \sqrt{\mu_0/\varepsilon_0} \mathbf{H}, \mathbf{J}/\omega_0 \varepsilon_0 \end{array} \right]^T,
\]

where \( \varepsilon_0 \) and \( \mu_0 \) are the permittivity and permeability of free space, \( \mathbf{E}, \mathbf{H}, \) and \( \mathbf{J} \) are the electric field vector, magnetic field vector, and current density vector. The \( 9 \times 9 \) tensors \( \hat{H} \) and \( \hat{A} \) in equation (1) are given by

\[
\hat{H} = \begin{pmatrix} 0 & -c \gamma & 0 \\ c \gamma & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{A} = \begin{pmatrix} \gamma_1 & i \gamma_2/c & 0 \\ -i \gamma_2/c & I & 0 \\ 0 & 0 & I \end{pmatrix},
\]

where \( \gamma \) is the skew-symmetric tensor of the normalized wave vector \( \mathbf{k}, \gamma = \text{diag}(0, 0, \omega_p) \), \( c \) represents the speed of light in vacuum, and \( \gamma_i = \text{diag}(\varepsilon_1, \varepsilon_2, 1) \). Here and in the following sections, all the wave vectors are normalized to \( k_p (k_p = \omega_p/c) \), and the eigenfrequency is normalized by \( \omega_p \).

We first analyze the non-chiral anisotropic homogeneous materials where \( \gamma = 0 \). For each \( k_z \), there are nine solutions for the eigenfrequencies \( \omega_n \) in equation (1), where \( \omega_{-n} = -\omega_n \) and \( n = -4, -3, \ldots, 3, 4 \) represents the indices for the eigenmodes. The corresponding eigenvectors are denoted by \( |\mathbf{p}_n\rangle \). Note that \( \omega_{-1} = \omega_{-2} = \omega = 0 \) are the zero frequency eigenmodes. The non-chiral system supports three propagation modes along the \( z \) direction, one is longitudinal and the other two are transverse, as shown in figure 1(a). The two transverse modes (\( \omega = ck_z/\sqrt{\varepsilon_1} \)) are degenerate (figure 1(a)). In particular, the electric fields of these two transverse modes are in the \( x \)–\( y \) plane, so they are only affected by the dielectric constant in the transverse direction (\( \varepsilon_2 \)). In figure 1(a), the longitudinal mode and two transverse modes intersect at \( k_z = k' = \omega_p\sqrt{\varepsilon_1}/c \). Therefore, the band structures possess three degenerate points at \( k = (0, 0, \pm k') \).

Moreover, the chirality of the triple degenerate point is zero because the system preserves both time-reversal and inversion symmetries. The dispersion relation of the triple degenerate point (the black square in figure 1(a)) in the \( k_z = k_p \) plane at \( k_z = k' \) is shown in figure 1(c). There is a flat band in the center, and the three bands are linearly dispersive in all directions.

When nonzero chirality (\( \gamma \neq 0 \)) is introduced into the system, the triple degenerate point is split into two Weyl points (figure 1(b)). The Weyl points are highlighted by the green and purple spots. The green and purple dots represent the Weyl points with positive and negative chirality, respectively. The band structure around one of the Weyl points (the green spot in figure 1(b)) in the \( k_z = k_p \) plane at \( k_z = k' \) is shown in figure 1(d). The dispersion is linear in all directions, indicating that the band cross point is a Weyl point.

The topological property of the chiral metamaterial can be characterized by the nonzero Berry flux. Mathematically, the Berry flux \( \Phi \) can be described as the surface integral of the Berry curvature:

\[
\Phi = \int \Omega \cdot d\mathbf{s} = \int \nabla \times (\mathbf{U}(k)) \cdot \mathbf{V}_k |\mathbf{U}(k)| \cdot d\mathbf{s}, \quad \text{where} \quad \mathbf{U}(k) = [\mathbf{E}, \mathbf{H}]^T \text{is the eigenpolarization state of metamaterial [43].}
\]

Particularly, the Berry curvature of the chiral metamaterial is calculated at every
Figure 1. Band structures, triple degenerate point, and Weyl points. (a) Three-dimensional band structures of the non-chiral metamaterials ($\gamma = 0$). TDP: triple degenerate point. (b) Band diagrams and Weyl points in the chiral metamaterials ($\gamma = 0.8$). WP: Weyl points. (c) The band structure of the triply degenerate point (the black square) in (a) at $k_z = k_t$. (d) The band structure around one of the Weyl points in the chiral metamaterials (the green spot) in (b) at $k_z = k'$. (e) Equi-frequency surfaces and Berry curvatures plot when shifted frequency $\delta \omega = -0.01 \omega_p$ ($\omega = 0.99 \omega_p$) in (b). (f) Location of the triple degenerate point in (a) and Weyl points in (b) in three-dimensional momentum space. (g) The dispersion relations $\omega(k_z)$ at fixed $k_y$ of the chiral metamaterials. $\varepsilon_t = 2$ and $\omega_p = c = 1$ for all the plots.

point $(k_y, k_z)$ on the two-dimensional equal frequency surface [52], as illustrated in figure 1(e). In momentum space, Chern number can be regarded as the number of monopoles (Weyl points) of Berry flux [1]. Therefore, the Chern number of each equal frequency surface in figure 1(e) can be obtained by $C = \Phi/2\pi$.

Around the Weyl points, we calculate the Berry curvature on an equal frequency surface ($\omega = 0.99 \omega_p$) and depict the results in figure 1(e). Black and red arrows denote the inward and outward Berry curvature, respectively. The magnitude of the Berry curvature decreases rapidly from the center on the equal frequency surfaces. The Berry flux on hyperbolic equal frequency surfaces is outward, so it can be integrated into a nonzero Chern number of $+1$. It corresponds to the Weyl point with positive chirality. On the other hand, the Chern number is $-2$ for the central equal frequency surface since it contains two Weyl points with negative chirality, i.e., the Berry flux is inward. The topological conservation behavior of the Chern numbers is illustrated in figure 1(f). Weyl points with opposite chirality are generated from a triple degenerate point. To better illustrate the Weyl points between branches, the dispersion relations $\omega(k_z)$ at different values of $k_y$ are given in figure 1(g). The Weyl points appear only where $k_y = 0$, which is a highly symmetric point in momentum space.

The location of the Weyl points in momentum space can be expressed by

$$k^\pm = \omega_p \sqrt{\left(\mp \gamma + \sqrt{\varepsilon_t}\right)} \sqrt{\varepsilon_t / c}.$$

Notice that $k^+$ will disappear when $\gamma \geq \sqrt{\varepsilon_t}$. Generally, the formation of Weyl points needs to break spatial inversion symmetry or time-reversal symmetry of the system. Both types of symmetry breaking can generate Weyl points, but they are different in essence [45]. Specifically, according to the Nielsen–Ninomiya no-go theorem [53], the minimum numbers of Weyl points in the system that breaks the symmetry of spatial inversion and time-reversal are four and two, respectively. The chiral metamaterials have time-reversal symmetry but do not possess spatial inversion symmetry owing to the chiral coupling between the electric field and magnetic field [42]. Therefore, the minimum number of Weyl points in the chiral
metamaterial is four according to the Nielsen–Ninomiya no-go theorem. In figure 1(f), there are indeed four Weyl points in metamaterial when the chiral coupling term $\gamma$ is less than $\sqrt{\varepsilon_t}$. In this case, the minimum number of Weyl points in the chiral metamaterial satisfies the Nielsen–Ninomiya no-go theorem. However, only two Weyl points appear in the metamaterials when $\gamma > \sqrt{\varepsilon_t}$ because the Weyl point with smaller value of $|k_z|$ has a nonphysical solution (equation (3)). Obviously, the minimum number of Weyl points violates the Nielsen–Ninomiya no-go theorem. Moreover, the large chiral coupling term $\gamma > \sqrt{\varepsilon_t}$ is challenging to realize experimentally [43]. Thus, considering the validity (Nielsen–Ninomiya no-go theorem) and practicability of our scheme, we only analyze the case of $\gamma < \sqrt{\varepsilon_t}$ in the present study.

2.2. Topological phase diagram and topological phase transitions

We now construct the topological phase diagram of the chiral metamaterials. In photonics, the topology of material system is defined on the dispersion band in momentum space. The topological invariant of the two-dimensional dispersion band is the Chern number which is a quantity characterizing the quantization collective behavior of the wave function on the band [1]. The Chern number of each band can be changed only when the different bands cross [48]. In the chiral metamaterials, the band crossing can only occur at the points $(k_x, k_y, k_z) = (0, 0, \pm k_{\pm})$ when $\gamma \neq 0$ and $k_z \neq 0$, as shown in figure 1(b). Thus, the locations of $k_z = \pm k_{\pm}$ define the boundaries between different regions on the topology. Figure 2(a) shows the surfaces $\gamma = 0$ and $k_z = \pm k_{\pm}$ in the $(\varepsilon_t, \gamma, k_z)$ space. The parameter space is divided into several different regions. The cross-section of the three-dimensional surfaces ($\varepsilon_t = 2, \gamma > 0, k_z > 0$) is illustrated in figure 2(b). The Roman numerals I–III express the three different phases in the two-dimensional cross-section. Notably, the blue line ($k = k_{-}$) and red line ($k = k_{+}$) intersect when the chirality equals zero ($\gamma = 0$) to form a triple degenerate point.

The topological phase transitions can happen when the state of the system passes through the Weyl point, e.g., along the black dash line in figure 2(b). For example, phase I changes to phase II when the parameter $k_z$ crosses the boundary of $k_z = k_{+}$. We plot the band structures of the bulk metamaterial in figures 2(c)–(g) which correspond to the ‘a’–’c’ points in the phase diagram in figure 2(b). Along with the phase transition from ‘a’ to ‘c’, the second (blue) and third (cyan) bands intersect at $k_z = \pm k_{\pm}$ and their Chern numbers change from $\pm 1$ to $0$, as shown in figures 2(c)–(e). This change is consistent with the fact that the Weyl point at $k_z = \pm k_{\pm}$ possesses topological charge of 1 (figure 2(d)). Similar behavior can be observed between the first (orange) and second (blue) bands when the parameter $k_z$ crosses the boundary of

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**Figure 2.** Topological phase diagrams and topological phase transitions of the chiral metamaterials ($\omega_p = c = 1$).

(a) Three-dimensional phase diagrams of metamaterials in the $(\varepsilon_t, \gamma, k_z)$ space. (b) Two-dimensional cross-sections of (a) at $\varepsilon_t = 2$. (c)–(g) The 2D band structures and Chern numbers are shown at the colored spots (marked by ‘a’–‘e’) in (b). The parameters are set to $\gamma = 0$ and $k_z = 0$. In (c), $k_z = 0.7$ in (d), $k_z = 1.5$ in (e), $k_z = 1.76957$ in (f), and $k_z = 2$ in (g), respectively.
$k_z = k^- \text{ and transforms from phase II to phase III, as illustrated in figures 2(e)–(g). Notably, the gap Chern number } \mathcal{C}_\text{gap in figures 2(e) and (g) is changed due to the exchange of band Chern numbers on both sides of the band gap. Moreover, in terms of band gap topology (gap Chern number is zero), phase III (metamaterial) is the same as the vacuum state.}

**2.3. Bulk-edge correspondence and Fermi arc**

Due to the difference of the Chern numbers at both sides of the gap, an edge state is supposed to connect the upper and lower bands. For a vacuum-metamaterial structure, the edge state can be obtained by matching the boundary conditions of Maxwell’s equations along the open boundary direction ($x$ direction) \[54\]. We depict the band diagram of the edge states for phases I, II, and III chiral metamaterials in figures 3(a)–(c), respectively. Different from electronic materials, the vacuum state is not an insulator of photons and there are photonic bands in the vacuum. For the bulk-edge correspondence, the vacuum effect should be considered if it directly contacts with the surface of chiral metamaterials. In figures 3(a) and (b), for phases I and II, edge states can span the common band gap (gray region) between the vacuum and metamaterial. However, for phase III, no edge state exists in the common band gap in figure 3(c). The number of edge states is equal to the gap Chern number in figures 3(a)–(c). It is revealed that the bulk-edge correspondence works well for the chiral metamaterials.

Figures 3(d)–(f) represent the time snapshots (COMSOL Multiphysics) of the electric field ($E_z$). In particular, the $k_z$ of the electric dipole source is set to a fixed value and is placed at the interface of vacuum and chiral media to excite the edge waves. In addition, the excitation frequency of the dipole source is located in the common band gap region (figures 3(a)–(c)). In these cases, the regions $x > 0$ and $x < 0$ are the vacuum and chiral media, respectively. In figures 3(d) and (e), the edge modes are located in the common band gap region of metamaterials and vacuum, so the edge waves can move along the positive $y$-direction and is robust to the triangular defect. It should be noted that only the edge modes in the common band gap are localized at the boundary, as shown by the gray regions in figures 3(a) and (b). Otherwise, it will leak to the vacuum or chiral media band. On the other hand, when the $x < 0$ region is replaced by phase III chiral media, the electric dipole source is unable to excite the edge wave because the vacuum-metamaterial interface does not support edge mode, as displayed in figure 3(f). Moreover, there is no energy leaking to the vacuum side, because no vacuum band exists in the common band gap region, as shown in figures 3(a)–(c). The simulation results in figures 3(d)–(f) are consistent with the bulk-edge correspondence (figures 3(a)–(c)).

To further understand the edge states in different phases, the band structures at $k_y = 0$ with two different configurations are shown in figure 4. In topological semimetals, these edge states are called Fermi arcs \[30\]. Firstly, when the chiral media are contacted with vacuum (figure 4(a)), the Fermi arc is located in region of $|k_x| < k^- \text{ which corresponds to phases I and II in the phase diagram in figure 2(b). When } \omega = \omega_p \text{ (Weyl degeneracy frequency), the Weyl points of metamaterial can be connected by the Fermi arcs.}
parameters are the two chiral media are the same. Figure 4(f). The Weyl points are connected by the Fermi arcs when the frequencies are equal to 0. The nonlocal parameter can be used as an additional dimension to elucidate the topological phase transitions when the nonlocal effect is considered in the chiral media system. The nonlocal effect in the chiral–chiral media interfaces.

Next, we move to the double-deck chiral media configuration as illustrated in inset of figure 4(e). The parameters are $\varepsilon_l = 2$, $\omega_{p1} = \omega_{p2} = 1$, and $\gamma = 0.8$ for the chiral media 1 and $\varepsilon_l = 1$, $\omega_{p1} = 0.5\omega_{p2} = 0.5$, and $\gamma = 0.5$ for the chiral media 2. The positions of the Weyl points are different in the two chiral media. The Fermi arc can connect two Weyl points with the same chirality. Similar to the twisted magnetized plasma configuration in reference [55], the Weyl point at $k^{\pm 2}$ ($x < 0$) can be viewed as a pseudo Weyl point with opposite chirality corresponding to the Weyl point at $k^{1 \pm 2}$ ($x > 0$) on the other side of the boundary.

In the double-deck configuration, the Fermi arc lies between phase II of chiral media 1 and phase III of chiral media 2. As shown in figures 2(e) and (g), phases II and III have gap Chern numbers $C_{\text{gap}} = 1$ and $C_{\text{gap}} = 0$, respectively. According to the bulk-edge correspondence, the Fermi arc can only exist in the region $k^{1 -} < |k_z| < k^{1 +}$ as shown in figure 4(e). There is no Fermi arc connecting the other two Weyl points in $k^{2 -} < |k_z| < k^{1 +}$ although both media is in phase I or II, because their gap Chern numbers are all $C_{\text{gap}} = 1$. It is also can be inferred that the Weyl points will coincide and the Fermi arc will disappear when the two chiral media are the same.

The equal frequency surfaces of the chiral media and Fermi arcs as a function of frequency are plotted in figure 4(f). The Weyl points are connected by the Fermi arcs when the frequencies are equal to $0.5\omega_{p2}$ and $\omega_{p2}$. At other frequencies (no Weyl point), the Fermi arcs connect the equal frequency surfaces of the chiral media. Simulation results demonstrate the topologically protected property of the Fermi arcs, as shown in figure 4(g). The edge waves conformally bend around the right-angled triangle and continue propagating along the negative $y$ direction without being scattered by the borders.

3. Type I/II Weyl points and topological phase transitions

The nonlocal parameter can be used as an additional dimension to elucidate the topological phase transitions when the nonlocal effect is considered in the chiral media system. The nonlocal effect in metamaterial means that the dielectric function is not only frequency dependent but also spatial dispersive.

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as shown in figure 4(b). The edge mode is prohibited if $|k_x| > k^{1 -}$ which means the Fermi arc does not exist in phase III region. To verify the topological protection property of the Fermi arc, a sharp defect is introduced into the interface, as illustrated in figures 4(c) and (d). The edge waves on the Fermi arc can stably bypass the defect. The edge waves can be excited by an electric dipole source is placed at the chiral–chiral media interfaces.

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Figure 5. Type-I/II Weyl points and topological phase transitions due to equi-frequency surface. (a) The purple/blue/black/yellow lines and green lines correspond to the longitudinal modes and transverse modes, respectively. The red and black dots represent Weyl points. (b) and (c) The band structures of the points ‘E’ and ‘F’ in the k_x−k_z plane at k_y = 0, respectively. (d) Each phase in the phase diagram is marked by the number and type of Weyl points. The parameters used are \( \varepsilon_t = 2, \omega_p = c = 1, \) and \( \gamma = 0.8 \) for the nonlocal chiral metamaterials.

The wave equation in metamaterials can be expressed as

\[
\left[ (\zeta + i\omega \tau) (\omega \tau)^{-1} (\zeta + i\omega \tau) + \omega \tau \right] \mathbf{E} = 0. \tag{4}
\]

Considering the specific forms of the relative permittivity, permeability, and chirality tensors in the nonlocal chiral metamaterials, equation (4) becomes

\[
\begin{pmatrix}
-\gamma^2 \omega^2 + k_z^2 + k_z^2 \\
\frac{k_x k_y}{\omega} + i\gamma k_z \\
-i\gamma k_y + \frac{k_y k_z}{\omega}
\end{pmatrix} + \omega \varepsilon_t 
\begin{pmatrix}
k_x k_y \\
k_x k_y + i\gamma k_z \\
-i\gamma k_y + \frac{k_y k_z}{\omega}
\end{pmatrix} = 0. \tag{5}
\]

Along the z \((k_x = k_y = 0)\) axis, for the nonlocal chiral metamaterials, the dispersions of transverse and longitudinal modes of metamaterials (the coefficient matrix of equation (5)) are given by

\[
-\gamma^2 \omega^2 \varepsilon_t + \left( k_z^2 - \omega^2 \varepsilon_t \right)^2 = 0, \tag{6}
\]

\[
1 - \omega_p^2 / \omega^2 + \alpha k_z^2 = 0. \tag{7}
\]

Equations (6) and (7) demonstrate that the presence of the nonlocal effect \(\alpha\) only affects the dispersion of the longitudinal mode (equation (7)).

Then, combining equations (6) and (7), the location of Weyl points (both type-I and type-II) can be defined as

\[
k_{z1} = \sqrt{-1 + \frac{1}{2\alpha} \left( 1 \pm \sqrt{1 + 4 \omega_p^2 \alpha^2 \gamma \sqrt{\varepsilon_t} + 4 \omega_p^2 \alpha^2 \varepsilon_t} \right)},
\]

\[
k_{z2} = \sqrt{-1 + \frac{1}{2\alpha} \left( 1 - \sqrt{1 + 4 \omega_p^2 \alpha^2 \gamma \sqrt{\varepsilon_t} + 4 \omega_p^2 \alpha^2 \varepsilon_t} \right)}. \tag{8}
\]

The nonlocal parameter \(\alpha\) can be controlled to produce negative or positive dispersion of the longitudinal mode, resulting in type-I or type-II Weyl points, respectively. We plot the dispersion of both transverse and longitudinal modes in figure 5(a) where \(\alpha\) is set to different values. When \(\alpha > 0\), the dispersion of the longitudinal modes all tilt downward, crossing the transverse modes twice and produce two type-I Weyl points. When \(\alpha < 0\), the dispersion of the longitudinal modes all tilt upward and produce more diverse distributions of Weyl point. The numbers of Weyl points for \(\alpha = -0.05, -0.22, \) and \(-0.45\)
Figure 6. Weyl exceptional rings and topological phase transitions in the dissipative chiral metamaterials. (a) Band structure (real eigenfrequency spectrum) for metamaterial with $k_x = 0$, $\gamma = 0.6$, and $\beta = 0.4\omega_p = 0.4$. EP: exceptional points. (b) Schematic of the Weyl points and Weyl exceptional rings in three-dimensional momentum space. Akin to Weyl points ($\gamma = 0.6$, $\beta = 0$), the Weyl exceptional rings ($\gamma = 0.6$, $\beta = 0.4\omega_p = 0.4$) with opposite topological charges exist in the dissipative chiral media system. The positive and negative topological charges are denoted by the green and purple color, respectively. (c) Phase diagram for the dissipative chiral media. The black line is the phase transition line. (d)–(f) Projection of the Weyl exceptional rings (mode degeneracy diagram) in the $k_y - k_z$ plane (real eigenfrequency spectrum) of metamaterial with $\gamma = 0.6$ and $\omega_p = c = 1$, corresponding to the red/cyan/green spots in (c), respectively.

are 4, 2, and 0, respectively. These Weyl points are called the type-II Weyl points. For the type-I and type-II Weyl points, we plot the band structures in figures 5(b) and (c), respectively. They all have conical dispersion structures around the Weyl points. However, the equal frequency surfaces are open near the type-II Weyl point (figure 5(c)). It is one of the notable differences between the type-I and type-II Weyl points [58].

A richer phase diagram of the nonlocal chiral media is shown in figure 5(d). The locations (black curves) of Weyl points are obtained by equation (8). The curves of Weyl points also divide the whole zone into three sections. However, the nonlocal chiral media system undergoes more complex phase transitions due to the different types and numbers of the Weyl points. There are always type-I Weyl semimetal phase when the nonlocal parameter $\alpha > 0$. The type-I Weyl semimetal phase changes to type-II Weyl semimetal phase when the nonlocal parameter changes from $\alpha > 0$ to $\alpha < 0$. Then, the type-II Weyl semimetal phase could further become trivial phase when all the Weyl points annihilate. The transition line as shown by the red dashed line in figure 5(d) could be expressed as $\alpha = 0$ and $\alpha = 1 / \left[4(\omega_p/c)^2\gamma\sqrt{\epsilon_t} - 4(\omega_p/c)^2\epsilon_t\right] = -0.288$, respectively.

4. Weyl exceptional rings and topological phase transitions

In photonic systems, materials are usually accompanied by dissipation [59–63]. In this section, we will demonstrate that the present of the dissipation could greatly change the topological phase transitions. The dissipation parameter $\beta$ is involved in the chiral media by $\epsilon_z = 1 - \omega_p^2 / [\omega (\omega + i\beta)]$. In the dissipative chiral media, the $9 \times 9$ tensors $H$ in equation (2) becomes
\[
\hat{H} = \begin{pmatrix}
0 & -i\eta \\
\eta & 0 & 0 \\
-\eta & 0 & i\beta
\end{pmatrix},
\]

(9)

where \(\eta = \text{diag}(0, 0, \beta)\). In figure 6(a), the band structures for \(k_x = 0\) and dissipation parameter \(\beta = 0.4\omega_p\) are given. The real parts of the eigenfrequency are \(\omega_p \sqrt{1 - \beta^2/(4\omega_p^2)}\) in the dissipative chiral media. In the two-dimensional momentum space defined by \(k_y\) and \(k_z\), Weyl points on the \(k_z\) axis are divided into exceptional points and offset from the \(k_z\) axis as illustrated in figure 6(a). Each pair of exceptional points can be connected by the bulk Fermi arc [64, 65]. The exceptional points (two-dimensional momentum space) correspond to the Weyl exceptional rings in the three-dimensional momentum space. The distributions of the Weyl points (\(\beta = 0\)) and Weyl exceptional rings (\(\beta = 0.4\omega_p\)) in the three-dimensional momentum space are shown in figure 6(b). The Weyl exceptional rings are two slightly curved circular contours. Although the Weyl points transform into the Weyl exceptional rings in the presence of a dissipation perturbation, the Weyl points and the Weyl exceptional rings have the same topological charges [59, 66].

By adjusting the dissipation parameter \(\beta\) and chiral coupling term \(\gamma\), the position of Weyl exceptional rings in momentum space can move along the \(k_z\) direction, even merge or annihilate. In figure 6(c), the phase transition of the dissipative chiral media is shown in parameter space (\(\gamma, \beta\)). The transition from the Weyl semimetal phase to trivial phase of the chiral media can be realized by increasing \(\beta\). For example, we fix \(\gamma = 0.6\) and increase \(\beta\) from 1.1\(\omega_p\) to 1.7\(\omega_p\) along the black dashed line as shown in figure 6(c). When \(\beta = 1.1\omega_p\), the Weyl exceptional contours with opposite topological charges are well separated, as illustrated in figure 6(d). In this case, the dissipative chiral media correspond to the Weyl semimetal phase. While at the phase transition point (\(\beta = 1.5\omega_p\)), the Weyl exceptional contours coalesce (figure 6(e)). Further increasing the dissipation parameter \(\beta\), the chiral media change into the trivial phase (does not possess net topological charges) because the Weyl exceptional contours with opposite topological charges annihilate each other as shown in figure 6(f). A full transition line which separates the Weyl semimetal phase and trivial phase is depicted in figure 6(c).

5. Conclusion

In summary, the topological phase diagrams and topological phase transitions in chiral metamaterials are clarified. Several forms of topological phase transitions and the bulk-edge correspondence are studied. In the local/lossless chiral metamaterials, the topological phase transitions are associated with Weyl points. The Fermi arcs exist not only on the vacuum–chiral metamaterials interface but also on the chiral–chiral metamaterials interfaces. Moreover, the nonlocal or dissipation parameter appears as an additional dimension to explore the topological phase transitions in the nonlocal or dissipative chiral media system. Our work can broaden the possible applications of exotic topological phenomena in photonic metamaterial.

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Data availability statement

The data generated and/or analysed during the current study are not publicly available for legal/ethical reasons but are available from the corresponding author on reasonable request.

Conflict of interest

The authors declare no conflicts of interest.

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