Non-Gaussianity and Baryonic Isocurvature Fluctuations in the Curvaton Scenario

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Abstract

We discuss non-Gaussianity and baryonic isocurvature fluctuations in the curvaton scenario, assuming that the baryon asymmetry of the universe originates only from the decay products of the inflaton. When large non-Gaussianity is realized in such a scenario, non-vanishing baryonic isocurvature fluctuations can also be generated unless the baryogenesis occurs after the decay of the curvaton. We calculate the non-linearity parameter $f_{\text{NL}}$ and the baryonic isocurvature fluctuations, taking account of the primordial fluctuations of both the inflaton and the curvaton. We show that, although current constraints on isocurvature fluctuations are severe, the non-linearity parameter can be large as $f_{\text{NL}} \sim O(10 - 100)$ without conflicting with the constraints.
1 Introduction

Current cosmological observations are now very precise to give us much information about the early universe. In particular, from observations of the cosmic microwave background (CMB) such as WMAP \cite{1, 2}, we can probe the physics of the early universe through the nature of primordial fluctuations, which are usually characterized by the amplitude of the fluctuations and its scale dependence as well as by a possible contribution of gravity waves. Recently in addition to these quantities, non-Gaussianity has been attracting much attention since it provides information about different aspects of the physics of the early universe. Non-Gaussianity is usually quantified by the so-called non-linearity parameter $f_{\text{NL}}$ and the recent constraint on this quantity from WMAP5 is \cite{1}:

$$-9 < f_{\text{NL}} < 111 \quad (95 \% \text{ C.L}).$$  \hspace{1cm} (1)

Although the data is still consistent with Gaussian fluctuation, which corresponds to $f_{\text{NL}} = 0$, the central value is away from zero as $f_{\text{NL}} \sim 50$. If future observations confirm such non-zero large value of $f_{\text{NL}}$, it will give very important implications to the scenario of the early universe; a simple model of inflation would be excluded since fluctuations from the inflaton are almost Gaussian. In such a case, some mechanism is needed to generate large non-Gaussian fluctuations.

Importantly, the simple inflationary scenario is not the only possibility to generate the density fluctuations. In particular, from the viewpoint of particle physics, there may exist a scalar field other than the inflaton, i.e., so-called the curvaton \cite{3, 4, 5}, which acquires primordial fluctuation and generate the present density fluctuations although it is a sub-dominant component during inflation. With the curvaton, large non-Gaussianity can also be generated \cite{6, 7} and the curvaton scenario seems attractive in the light of constructing a successful model of generating large non-Gaussianity.

In order to generate large non-Gaussianity with the curvaton, it is necessary that the energy density of the curvaton at the time of its decay should be much smaller than that of the dominant component of the universe (which is expected to be from the inflaton). Density fluctuations generated in such a scenario are (almost) scale-invariant, and hence can be consistent with the observations if the fluctuations of all the components are adiabatic. However, in such a scenario, there exist entropy fluctuations between components from the inflaton and those from the curvaton. In order to have vanishing isocurvature fluctuations, it is necessary to generate baryon asymmetry and cold dark matter (CDM) at low-temperature universe after the decay of the curvaton. Although there are several possibilities that the dark-matter density is determined at a relatively low temperature, such as axion dark matter, the lightest superparticle dark matter and so on, baryogenesis

\begin{itemize}
\item \#1Here $f_{\text{NL}}$ represents the so-called “local type” non-Gaussianity. In this letter, we only consider non-Gaussianity of this type.
\item \#2Some other models generating large non-Gaussianity have also been discussed, such as the modulated reheating scenario \cite{8}.
\end{itemize}
at a low temperature may be challenging. Indeed, for some of the scenarios of baryogenesis, like the thermal \cite{9} and non-thermal \cite{10} leptogenesis, a relatively high cosmic temperature is required\footnote{However, the electroweak baryogenesis \cite{11} may be a possibility. Another possible model is Affleck-Dine baryogenesis \cite{12} and the issues of non-Gaussianity in the model is discussed in \cite{13}.}. This fact indicates that, to realize large non-Gaussianity, it may be necessary to generate the baryon asymmetry of the universe before the decay of the curvaton. Since the size of baryonic isocurvature fluctuations is now severely constrained by observations, such constraints are important in considering baryogenesis in the curvaton scenario generating large non-Gaussianity. In particular, in the simplest curvaton scenario where the cosmic density fluctuations are totally from the primordial fluctuation of the curvaton, large non-Gaussianity cannot be generated without conflicting with the constraints from the baryonic isocurvature fluctuations if the baryon asymmetry is solely from the decay product of the inflaton. This argument excludes some of the scenarios of baryogenesis which require high cosmic temperature in the curvaton scenario.

However, in the mixed fluctuation scenario where fluctuations from the inflaton and the curvaton both contribute to cosmic density fluctuations \cite{14, 15}, the situation changes; even if the baryon number of the universe originates only from the decay products of the inflaton, the amplitude of isocurvature fluctuations relative to adiabatic ones can be suppressed. Thus, in such a case, large non-Gaussianity may be generated without conflicting with the constraint on the isocurvature fluctuations.

In this letter, we discuss non-Gaussianity and baryonic isocurvature fluctuations in the curvaton scenario where only the decay products of the inflaton are responsible for the baryogenesis. We pay particular attention to the question how large $f_{NL}$ can be while imposing the isocurvature constraints. We will see that $f_{NL} \sim \mathcal{O}(10-100)$ can be realized even if severe constraints on isocurvature fluctuations are imposed.

2 Scenario and Formalism

We first describe the scenario we consider in this letter and summarize formulae to calculate density fluctuations and its non-Gaussianity.

Here, we consider the curvaton scenario where fluctuations from the inflaton also contribute to the total curvature fluctuations along with that from the curvaton. In addition, we assume that only the decay products of the inflaton (not those of curvaton) are responsible for the baryon asymmetry of the universe; the possibilities include a baryogenesis in the thermal bath before the decay of the curvaton and that in association with the decay of the inflaton. We also assume that the density of CDM is determined after the decay of the curvaton so that there is no CDM isocurvature fluctuations. We can easily apply our results to the case where CDM originates only from the decay products of the inflaton.

In our study, we adopt the following form for the scalar-field potential:

$$V(\phi, \sigma) = V(\phi) + \frac{1}{2}m_\sigma^2\sigma^2,$$

(2)
where $\phi$ and $\sigma$ represent the inflaton and curvaton fields, respectively, $V(\phi)$ is the potential for the inflaton, and $m_{\sigma}$ is the mass of the curvaton. (We assume a quadratic potential for the curvaton.) In general, the curvature fluctuations from the inflaton depend on the inflaton potential. However, for the purpose of the following discussion, we only need to specify the value of the slow-roll parameter $\epsilon$, which is defined as

$$\epsilon = \frac{1}{2} M_{\text{pl}}^2 \left( \frac{V_{\phi}}{V} \right)^2,$$

where $V_{\phi} \equiv \partial V / \partial \phi$, and $M_{\text{pl}} \simeq 2.4 \times 10^{18}$ GeV is the reduced Planck scale. We also assume that the mass of the curvaton is small and the curvaton acquires primordial fluctuation during the inflation. (We denote the initial amplitude of the curvaton as $\sigma^*$; hereafter, the subscript "*" is for quantities at the time of the horizon exit.)

After the inflation, the inflaton begins to oscillate around the minimum of its potential and then decays. We denote the decay rate of the inflaton as $\Gamma_{\phi}$ and define the reheating temperature $T_R$ as

$$T_R = \left( \frac{10}{g_{\text{SM}} \pi^2 M_{\text{pl}}^2} \Gamma_{\phi}^2 \right)^{1/4},$$

where we use $g_{\text{SM}} = 106.75$ as the effective number of the massless degrees of freedom. As the universe expands, the expansion rate of the universe $H$ becomes comparable to $m_{\sigma}$ and the curvaton starts to oscillate. Then, when the expansion rate becomes comparable to the decay rate of $\sigma$, which is denoted as $\Gamma_{\sigma}$ (and is related to the lifetime of the curvaton as $\tau_{\sigma} = \Gamma_{\sigma}^{-1}$), the curvaton decays. The start of the oscillation and the decay may occur before or after the reheating due to the inflaton decay, depending on the values of $m_{\sigma}$ and $\Gamma_{\sigma}$.

Assuming that the potential of the inflaton is well approximated by a quadratic one around its minimum, its energy density behaves as that of matter for the period of the inflaton oscillation. Then, denoting the energy densities of radiation components from the decays of $\phi$ and $\sigma$ as $\rho_{\gamma_{\phi}}$ and $\rho_{\gamma_{\sigma}}$, respectively, evolutions of these variables (as well as those of the energy density of the inflaton field and the curvaton amplitude) are governed by

$$\dot{\rho}_{\gamma_{\phi}} + 4H \rho_{\gamma_{\phi}} = \Gamma_{\phi} \rho_{\phi},$$

$$\dot{\rho}_{\gamma_{\sigma}} + 4H \rho_{\gamma_{\sigma}} = \Gamma_{\sigma} \sigma^2;$$

$$\dot{\rho}_{\phi} + 3H \rho_{\phi} = -\Gamma_{\phi} \rho_{\phi},$$

$$\dot{\sigma} + (3H + \Gamma_{\sigma}) \dot{\sigma} + m_{\sigma}^2 \sigma = 0,$$

where the dot represents derivative with respect to the cosmic time and $\rho_i$ indicates the background energy density of the component $i$. Notice that, when $H \ll m_{\sigma}$, Eqs. (6) and (7) are
can be well approximated by
\[ \dot{\rho}_\gamma + 4H\rho_\gamma = \Gamma_\sigma \rho_\sigma, \]
\[ \dot{\rho}_\sigma + 3H\rho_\sigma = -\Gamma_\sigma \rho_\sigma, \]
respectively, where \( \rho_\sigma \) is the energy density of the curvaton field. In addition, it should be noted that the total radiation energy density is given by
\[ \rho_\gamma = \rho_{\gamma_\phi} + \rho_{\gamma_\sigma}. \]

In our analysis, we use the \( \delta N \) formalism \cite{16} to calculate the perturbations. Then, perturbation variables are obtained by evaluating the number of e-folds from some time during inflation to the time after the curvaton decay as a function of model parameters such as \( \Gamma_\phi, \Gamma_\sigma, m_\sigma \) and \( \sigma_* \). Here, let us briefly summarize the resultant formulae of the density fluctuations and non-linearity parameter \( f_{\text{NL}} \).

With the \( \delta N \) formalism, if there is no isocurvature fluctuation, the curvature fluctuation originating from scalar-field fluctuations is given by
\[ \zeta^{(\text{adi})} = N_a \delta \varphi_*^a + \frac{1}{2} N_{ab} \delta \varphi_*^a \delta \varphi_*^b + \cdots, \]
where \( N \) is the number of e-folds, \( \delta \varphi_*^a \) is the primordial fluctuation of the scalar field \( \varphi^a \), and
\[ N_a \equiv \frac{\partial N}{\partial \varphi^a}, \quad N_{ab} \equiv \frac{\partial^2 N}{\partial \varphi^a \partial \varphi^b}, \]
with \( \varphi^a = \phi \) and \( \sigma \) in our case.

In the following, we consider the scenario where fluctuations of the inflaton and the curvaton are both responsible for cosmic density fluctuations. For simplicity, we assume that these fields are uncorrelated. Then, the curvature perturbations originating from these scalar fields can be calculated separately. The curvature fluctuation from the inflaton, which we denote \( \zeta_{\phi}^{(\text{adi})} \), is given by
\[ \zeta_{\phi}^{(\text{adi})} \simeq \frac{1}{\sqrt{2\epsilon M_{\text{pl}}}} \delta \phi_* + \frac{1}{2} \left( 1 - \frac{\eta}{2\epsilon} \right) \delta \phi_*^2, \]
where \( \eta \equiv M_{\text{pl}}^2 V_{\phi\phi}/V \), and we have used the slow-roll approximation. (We neglect terms of the order of \( \delta \phi_*^3 \) which are irrelevant for our discussion.) In addition, the curvaton contribution to \( \zeta^{(\text{adi})} \), which we denote \( \zeta_{\sigma}^{(\text{adi})} \), is expressed as
\[ \zeta_{\sigma}^{(\text{adi})} = N_\sigma \delta \sigma_* + \frac{1}{2} N_{\sigma\sigma} \delta \sigma_*^2. \]

We solve the set of equations Eqs. (5) – (8) with parameters \( \Gamma_\phi, \Gamma_\sigma, m_\sigma \) and \( \sigma_* \) being fixed, then we determine \( N_\sigma \) and \( N_{\sigma\sigma} \) to obtain \( \zeta_{\sigma}^{(\text{adi})} \). For the important case where
\[ \frac{\rho_{\gamma\sigma}}{\rho_{\gamma}} \ll 1 \text{ (and } m_{\sigma} \ll \Gamma_{\phi}) \], in which large non-Gaussianity can be generated in the curvaton scenario, the relation \[ 4N(\sigma_{s}) = \left[ \frac{\rho_{\gamma\sigma}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \] holds and the \( \sigma_{s} \) dependence of \( N \) is well approximated as \cite{15}

\[
N(\sigma_{s}) \simeq \frac{1}{3 \sqrt{2\pi}} \frac{\Gamma^{2}(5/4)}{M_{\text{pl}}^{2}} \frac{\sigma_{s}^{2}}{\sqrt{\Gamma_{\sigma}/m_{\sigma}}} : \text{ for } \left[ \frac{\rho_{\gamma\sigma}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \ll 1, \tag{16}
\]

while, for the case where the curvaton eventually dominates the universe, \( \left[ \frac{\rho_{\gamma\sigma}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \simeq 1 \), it is given by

\[
N(\sigma_{s}) \simeq \frac{2}{3} \ln \sigma_{s} : \text{ for } \left[ \frac{\rho_{\gamma\sigma}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \simeq 1. \tag{17}
\]

Now, we discuss the entropy fluctuation between baryon and radiation, which is given up to the second order by

\[
S_{b\gamma} \equiv \frac{\delta \rho_{b}}{\rho_{b}} - \frac{1}{2} \left( \frac{\delta \rho_{b}}{\rho_{b}} \right)^{2} - \frac{3}{4} \left[ \frac{\delta \rho_{\gamma}}{\rho_{\gamma}} - \frac{1}{2} \left( \frac{\delta \rho_{\gamma}}{\rho_{\gamma}} \right)^{2} \right]. \tag{18}
\]

Here, \( \delta \rho_{i} \) denotes the energy-density fluctuation of the component \( i \) on the uniform density slicing. We evaluate \( S_{b\gamma} \) when \( \tau_{\sigma} \ll t \ll t_{\text{eq}} \) with \( t_{\text{eq}} \) being the time of the radiation-matter equality. In the present scenario, there exist two sources of radiation: one originating from the inflaton and the other from the curvaton. We treat them separately and write \( \delta \rho_{\gamma}/\rho_{\gamma} \) as

\[
\frac{\delta \rho_{\gamma}}{\rho_{\gamma}} = \frac{\delta \rho_{\gamma\phi} + \delta \rho_{\gamma\sigma}}{\rho_{\gamma\phi} + \rho_{\gamma\sigma}}. \tag{19}
\]

Since the baryon asymmetry and \( \gamma_{\phi} \) has the same source (i.e., the inflaton), the adiabatic relation holds between these two components. Furthermore, as will be discussed later, large non-Gaussianity can be generated when \( \left[ \frac{\rho_{\gamma\sigma}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \ll 1 \), thus we concentrate on such a case. Neglecting the terms which are second or higher order in \( \left[ \frac{\rho_{\gamma\sigma}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \), and using the adiabatic relation between the baryon and \( \gamma_{\phi} \), we obtain

\[
S_{b\gamma} \simeq - \frac{3}{4} \left[ \frac{\rho_{\gamma\phi}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \frac{\delta \rho_{\gamma\phi}}{\rho_{\gamma\phi}}, \tag{20}
\]

where terms of the order of \( (\delta \rho_{\gamma\phi}/\rho_{\gamma}\phi)^{2} \) vanish. (Here and hereafter, it should be understood that \( \left[ \frac{\rho_{\gamma\phi}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \) is equal to \( 4N(\sigma_{s}) \), and is proportional to \( \sigma_{s}^{2} \).) Notice that, compared to \( \delta \rho_{\gamma\phi}/\rho_{\gamma}\phi \), \( \delta \rho_{\gamma\sigma}/\rho_{\gamma\sigma} \) is of the order of \( \left[ \frac{\rho_{\gamma\sigma}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \) and hence its contribution is irrelevant in the present discussion.

When \( \left[ \frac{\rho_{\gamma\sigma}}{\rho_{\gamma}} \right]_{t \gg \tau_{\sigma}} \ll 1 \), the cosmic expansion is solely determined by radiation from the inflaton, and we can neglect the effect of \( \gamma_{\phi} \) on the background evolution. Then, \( \rho_{\gamma\sigma} \) is proportional to \( \sigma_{s}^{2} \) and

\[
\frac{\delta \rho_{\gamma\sigma}}{\rho_{\gamma\sigma}} = 2 \left( \frac{\delta \sigma_{s}}{\sigma_{s}} + \frac{1}{2} \frac{\delta \sigma_{s}^{2}}{\sigma_{s}^{2}} \right). \tag{21}
\]
which results in
\[ S_{b\gamma} = -\frac{3}{2} \left[ \frac{\rho_{\gamma}}{\rho_{\gamma}} \right]_{t >> T_{\gamma}} \left( \frac{\delta \sigma_s}{\sigma_s} + \frac{1}{2} \frac{\delta \sigma_s^2}{\sigma_s^2} \right). \]
(22)

In discussing the non-Gaussianity in the present framework, it should be noted that the isocurvature fluctuations also generate curvature perturbation. Indeed, using the relation \( 3 \zeta_{(iso)} = S_{m\gamma} \) (with \( S_{m\gamma} \) being the entropy fluctuation between the total matter and radiation), which holds in the matter-dominated universe, we obtain the isocurvature contribution to the curvature fluctuation as
\[ \zeta_{(iso)} = -\frac{1}{2} \frac{\Omega_b}{\Omega_m} \left[ \frac{\rho_{\gamma}}{\rho_{\gamma}} \right]_{t >> T_{\gamma}} \left( \frac{\delta \sigma_s}{\sigma_s} + \frac{1}{2} \frac{\delta \sigma_s^2}{\sigma_s^2} \right). \]
(23)

where \( \Omega_b \) and \( \Omega_m \) are density parameters of the total matter and baryon, respectively. In our numerical analysis, we use \( \Omega_b/\Omega_m \approx 0.17 \) [1].

Since the fluctuations of the inflaton and the curvaton are assumed to be uncorrelated, the amplitude of the total curvature fluctuation \( \zeta \) in matter dominated epoch is given by
\[ \zeta = \zeta_\phi + \zeta_\sigma, \]
(24)

where \( \zeta_\phi = \zeta_{(adi)} \) and \( \zeta_\sigma = \zeta_{(adi)} + \zeta_{(iso)} \). From the observations of the cosmic density fluctuations, the size of \( \zeta \) is constrained. In our study, we determine the amplitudes of the primordial scalar-field fluctuations so that \( \zeta \) becomes consistent with the observed value.

Non-Gaussianity of fluctuations is usually quantified with higher order statistics such as bispectrum. Here, we consider the bispectrum of \( \zeta \):
\[ \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{\vec{k}_3} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2 + \vec{k}_3) B_{\zeta}(k_1, k_2, k_3). \]
(25)

Then, \( B_{\zeta}(k_1, k_2, k_3) \) is obtained as
\[ B_{\zeta}(k_1, k_2, k_3) = \left( \tilde{N}_{\sigma}^2 + \tilde{N}_{\sigma\sigma}^2 \Delta_{\sigma\sigma}^2 \ln k_{\min} L \right) \tilde{N}_{\sigma\sigma} \left[ P_{\delta\sigma}(k_1) P_{\delta\sigma}(k_2) + (2 \text{ perms.}) \right], \]
(26)

where
\[ \tilde{N}_{\sigma} = N_{\sigma} - \frac{1}{2} \frac{\Omega_b}{\Omega_m} \left[ \frac{\rho_{\gamma}}{\rho_{\gamma}} \right]_{t >> T_{\gamma}} \sigma_s^{-1}, \]
(27)
\[ \tilde{N}_{\sigma\sigma} = N_{\sigma\sigma} - \frac{1}{2} \frac{\Omega_b}{\Omega_m} \left[ \frac{\rho_{\gamma}}{\rho_{\gamma}} \right]_{t >> T_{\gamma}} \sigma_s^{-2}. \]
(28)

Here, \( P_X(k) \) denotes the power spectrum of the variable \( X \) defined as
\[ \langle X_{\vec{k}_1} X_{\vec{k}_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_X(k_1), \]
(29)
and is related to $\Delta^2_X$ as
\[ P_X(k) = \frac{2\pi^2}{k^3} \Delta^2_X. \] (30)

Assuming that the curvaton fluctuation is due to the quantum fluctuation during inflation, we obtain
\[ \Delta^2_{\delta \sigma} = \left( \frac{H_*}{2\pi} \right)^2, \] (31)

where $H_*$ is the expansion rate during inflation and we assume $\Delta^2_{\delta \sigma}$ to be scale-invariant.

In deriving Eq. (26), following [17, 18], we have regularized the infrared divergence by introducing the infrared cutoff parameter $L^{-1}$ and $k_{\text{min}} = \min(k_1, k_2, k_3)$. (We have neglected an $O(1)$ coefficient in front of $\ln k_{\text{min}} L$.) In order to discuss the implication to the cosmological observations, both $k_{\text{min}}$ and $L^{-1}$ are taken to be the cosmological scale. Since the scale dependence from $\ln k_{\text{min}} L$ is rather weak, we approximate $\ln k_{\text{min}} L = 1$ in our numerical analysis.

As mentioned in the introduction, the non-linearity parameter $f_{\text{NL}}$ is often used to characterize non-Gaussianity of fluctuations, which is given by
\[ B_\zeta(k_1, k_2, k_3) = \frac{6}{5} f_{\text{NL}} [P_\zeta(k_1)P_\zeta(k_2) + (2 \text{ perms.})]. \] (32)

Here $P_\zeta(k)$ is the the power spectrum of $\zeta$ defined as
\[ \langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \rangle = (2\pi)^3 \delta(\vec{k}_1 + \vec{k}_2) P_\zeta(k_1). \] (33)

In the present scenario, $P_\zeta(k)$ becomes
\[ P_\zeta(k) = \frac{2\pi^2}{k^3} \Delta^2_{\phi} = \frac{2\pi^2}{k^3} \left[ \frac{1}{2\epsilon M^2_{\text{pl}}} \Delta^2_{\phi} + \left( \tilde{N}_{\sigma}^2 + \tilde{N}_{\sigma\sigma}^2 \Delta^2_{\delta \sigma} \ln kL \right) \Delta^2_{\delta \sigma} \right]. \] (34)

Then, the non-linearity parameter $f_{\text{NL}}$ is given by
\[ \frac{6}{5} f_{\text{NL}} = \frac{(2\epsilon - \eta) + 4\epsilon^2 M^4_{\text{pl}}(\tilde{N}_{\sigma}^2 + \tilde{N}_{\sigma\sigma}^2 \Delta^2_{\delta \sigma} \ln k_{\text{min}} L) \tilde{N}_{\sigma\sigma}}{\left[ 1 + 2\epsilon M^2_{\text{pl}}(\tilde{N}_{\sigma}^2 + \tilde{N}_{\sigma\sigma}^2 \Delta^2_{\delta \sigma} \ln k_{\text{min}} L) \right]^2}, \] (35)

#5 Since there exists the contribution from isocurvature fluctuations in this scenario, $f_{\text{NL}}$ here is not the same as the one for the case only with adiabatic perturbations. However, as shown in Eq. (23), the isocurvature contribution is suppressed by the factor $\Omega_b / \Omega_m$. In fact, because of the difference between the transfer function for the adiabatic contribution and that for the isocurvature one, the bispectrum is enhanced at the Sachs-Wolfe plateau. In the present case, the enhancement factor for the corresponding term is roughly estimated to be $\sim 2^{2/3}$ for large-scale fluctuations [19]. Even if the second terms in Eqs. (27) and (28) are multiplied by this factor, the contribution from the isocurvature fluctuations is still sub-dominant. For the details of non-Gaussianity from isocurvature fluctuations, see [19].
where we have used the relation $\Delta^2_\delta = \Delta^2_\sigma$. Since we are interested in the case where the non-linearity parameter becomes large, we neglect $(2\epsilon - \eta)$ in the numerator of (35) in the following analysis. We can find approximated formulae of $\tilde{N}_\sigma$ and $\tilde{N}_{\sigma\sigma}$ for the most important case of $[\rho_{\gamma\sigma}/\rho_\gamma]_{t\gg \tau_\sigma} \ll 1$, for which $f_{NL} \gg 1$ may be realized. In such a case, as we have mentioned, the relation $4N(\sigma_*) = [\rho_{\gamma\sigma}/\rho_\gamma]_{t\gg \tau_\sigma}$ holds and hence
\[
\tilde{N}_\sigma \simeq \left(1 - \frac{\Omega_b}{\Omega_m}\right) N_\sigma, \quad \tilde{N}_{\sigma\sigma} \simeq \left(1 - \frac{\Omega_b}{\Omega_m}\right) N_{\sigma\sigma} : \text{for} \ [\rho_{\gamma\sigma}/\rho_\gamma]_{t\gg \tau_\sigma} \ll 1. \tag{36}
\]

For the sake of the following arguments, we also calculate the power spectrum of the isocurvature perturbations. With Eq. (22), we obtain
\[
\Delta^2_{S_{b\gamma}} = \frac{9}{4} \left[\frac{\rho_{\gamma\sigma}}{\rho_\gamma}\right]^2_{t\gg \tau_\sigma} \left(\frac{1}{\sigma^2_*} + \frac{1}{\sigma^4_*} \Delta^2_{\delta\sigma} \ln k_{\min}L\right) \Delta^2_{\delta\sigma}. \tag{37}
\]

Before showing the numerical results, we briefly consider the pure curvaton case where all the cosmic density fluctuations are only from the curvaton. In such a case, we obtain
\[
\frac{\Delta_{S_{b\gamma}}}{\Delta_\zeta} \simeq \frac{N_{\sigma\sigma}}{N^2_\sigma} \left[\frac{\rho_{\gamma\sigma}}{\rho_\gamma}\right]_{t\gg \tau_\sigma}, \tag{38}
\]
and $\Delta_{S_{b\gamma}}$ and $\Delta_\zeta$ are of the same order irrespective of $\sigma_*$. (See, for example, Eqs. (16) and (17).) In this case, the entropy fluctuation is too large to be consistent with the observations $[5, 6, 20, 21]$. Thus when the baryon number is produced at high temperature, it is difficult to have large non-Gaussianity in the simplest curvaton paradigm without conflicting with the isocurvature constraint. As we will see in the following, the situation changes in the mixed fluctuation scenario. In particular, when the curvature perturbation mainly comes from the inflaton fluctuation and $[\rho_{\gamma\sigma}/\rho_\gamma]_{t\gg \tau_\sigma} \ll 1$, large non-Gaussianity becomes possible without conflicting with the isocurvature constraint.

3 Numerical Results

Now, we show our numerical results. In our analysis, we numerically solve Eqs. (5) – (8) and calculate the number of e-folds as a function of $\sigma_*$. Then, we calculate $f_{NL}$ and $S_{b\gamma}$ for various values of the model parameters. In the following, we take $m_\sigma = 100$ GeV.

First, we show how large the baryonic isocurvature fluctuations can be. In Fig. 1, we show contours of constant $\Delta_{S_{b\gamma}}/\Delta_\zeta$ on the $\sigma_*$ vs. $\Gamma_\sigma/m_\sigma$ plane. Here, we take $T_R = 10^{10}$ GeV, and $\epsilon = 10^{-2}$ $\#6$. When $\tilde{N}_\sigma^2 > \tilde{N}_{\sigma\sigma}^2 \Delta^2_{\delta\sigma} \ln k_{\min}L$, one can see that the baryonic isocurvature fluctuations are suppressed as $\sigma_*$ becomes smaller. This fact can be easily

$\#6$ In our following numerical analysis, we use $\epsilon = 10^{-2}$ and $10^{-10}$ for illustrational purposes. In fact, the value of $\epsilon = 10^{-2}$ corresponds to the case of quadratic chaotic inflation with $N_e = 50$, while $\epsilon \sim \mathcal{O}(10^{-10})$ is realized in some class of new inflation model.
understood as follows. When $\sigma_*$ is small enough, the curvature perturbation is dominated by the inflaton contribution $\zeta_\phi$ and hence is independent of $\sigma_*$. In addition, in such a case, the ratio $[\rho_{\gamma\sigma}/\rho_{\gamma}]_{T \gg \tau}$ is proportional to $\sigma_*^2$ and hence $\Delta S_{b\gamma}/\Delta \zeta$ becomes smaller as $\sigma_*$ decreases as far as the first term in the parenthesis of Eq. (37) dominates. On the contrary, for $N_2^\sigma \lesssim N_2^\sigma \Delta_\sigma^2 \ln k_{\text{min}} L$, the isocurvature perturbation $\Delta S_{b\gamma}$ is determined by the second term in Eq. (37). Then, $\Delta S_{b\gamma}/\Delta \zeta$ becomes insensitive to $\sigma_*$, as shown in the figure.

From current cosmological observations, baryonic isocurvature fluctuations are severely constrained; there is no sign of the isocurvature fluctuations in the observed angular power spectrum of the CMB, and $S_{b\gamma}$ is consistent with zero. In our analysis, we adopt the bounds on the baryonic isocurvature fluctuations obtained from the latest WMAP5 result. We classify the baryonic isocurvature fluctuations into correlated and uncorrelated parts as

\[ [S_{b\gamma}]_{\text{corr}} = -\Delta S_{b\gamma} \sin \delta, \quad [S_{b\gamma}]_{\text{uncorr}} = \Delta S_{b\gamma} \sqrt{1 - \sin^2 \delta}, \]

where $\sin \delta = \Delta \zeta_{\sigma}/\Delta \zeta$. Then, we adopt the following bounds on the ratios of these isocurvature modes to $\Delta \zeta$, reading off the 95 % C.L. constraints from the WMAP5 results \[1]\)

\[ \left[ \frac{S_{b\gamma}}{\Delta \zeta} \right]_{\text{corr}} > -0.31, \quad \left[ \frac{S_{b\gamma}}{\Delta \zeta} \right]_{\text{uncorr}} < 1.35. \]

Notice that, in the present scenario, $[S_{b\gamma}/\Delta \zeta]_{\text{corr}}$ is negative.\footnote{The bounds given in \cite{1} are separately obtained for the cases of totally correlated and totally uncorrelated modes. In Fig. 1 we shaded the regions excluded by the constraints (40) are shaded.}

Figure 1: Contours of constant $\Delta S_{b\gamma}/\Delta \zeta$ on $\sigma_*$ vs. $\Gamma_\sigma/m_\sigma$ plane for $T_R = 10^{10}$ GeV. For the slow-roll parameter, we take $\epsilon = 10^{-2}$. Regions excluded by the constraints (40) are shaded.
Figure 2: Contours of constant $f_{NL}$ on the $\sigma_*$ vs. $\Gamma_{\sigma}/m_\sigma$ plane for $T_R = 10^{10}$ GeV. ($f_{NL} = 1$, 10, and 100 from the top.) For the slow-roll parameter, we take $\epsilon = 10^{-2}$ (left) and $10^{-10}$ (right). Regions excluded by the constraints (40) are shaded.

region which is excluded by the above constraints; we found that the constraint on the correlated one is more stringent.

It should be noted that, in the allowed region, the curvature perturbation $\zeta$ is mainly from the inflaton fluctuation. Then, $\Delta^2_{\delta\sigma}$ and $\Delta^2_{\delta\phi}$ are related to $\Delta^2_{\zeta}$, which is known from the observation of the cosmic density fluctuations, as

$$\Delta^2_{\delta\sigma} = \Delta^2_{\delta\phi} = 2\epsilon M^2_{pl}\Delta^2_{\zeta}. \quad (41)$$

Neglecting the scale-dependence, we adopt $\Delta^2_{\zeta} = 2.457 \times 10^{-9}$ [1] to evaluate $f_{NL}$ given in Eq. (35).

Now we discuss how large $f_{NL}$ can be in the parameter region consistent with the constraints [10]. In Fig. 2, taking $T_R = 10^{10}$ GeV, we show the contours of constant $f_{NL}$ for $\epsilon = 10^{-2}$ and $10^{-10}$. With such a choice of the reheating temperature, the curvaton field starts to oscillate after the reheating by the inflaton. If the curvaton begins to oscillate after the inflaton decay, the number of $e$-folds after the inflation depends only on the combination of $\Gamma_{\sigma}/m_\sigma$ once $\sigma_*$ is fixed. Thus, we show our results in the $\sigma_*$ vs. $\Gamma_{\sigma}/m_\sigma$ plane. (Notice that, for different values of $T_R$ and $m_\sigma$, the figure is almost unchanged as far as $m_\sigma \ll \Gamma_{\phi}$.) On the same figure, we shaded the region excluded by the constraints uncorrelated isocurvature fluctuations. Since the bounds for the case where the correlated and uncorrelated isocurvature fluctuations coexist are not available, we adopt the constraint [10] as reference values. In addition, since the constraint on the correlated isocurvature fluctuations are not shown for $|S_{b\gamma}/\Delta_\zeta|_{corr} < 0$ in [11], we adopt the constraint for the case of $|S_{b\gamma}/\Delta_\zeta|_{corr} > 0$ to derive the constraint [10], assuming that the bound on $|S_{b\gamma}/\Delta_\zeta|_{corr}$ does not significantly depend on the sign. For the validity of this assumption, see, for example, [22].
We can see that, even after imposing the isocurvature constraint, a very large value of $f_{\text{NL}}$ (i.e., $f_{\text{NL}} \sim \mathcal{O}(10^{-100})$) is possible with small enough $\sigma_*$; in such a case, even though the components related to the curvaton are always sub-dominant, $f_{\text{NL}}$ becomes large. (A possibility where a sub-dominant component generates large non-Gaussianity was first considered in [18].) In addition, when $\sigma_*$ becomes small enough, $f_{\text{NL}}$ becomes insensitive to $\sigma_*$. This is because, in such a parameter region, the second term in the first parenthesis of Eq. (26), which is independent of $\sigma_*$, dominates.

Next, let us consider the case where the curvaton decays before the reheating (i.e., during the inflaton oscillation). Even in such a case, large value of $f_{\text{NL}}$ can be obtained while satisfying the isocurvature constraints. To see this, in Fig. 3 we show the contours of constant $f_{\text{NL}}$ on $\sigma_*$ vs. $\Gamma_\sigma/m_\sigma$ plane for the case of $T_R = 10^6$ GeV. As one can see, in this case, $f_{\text{NL}} \sim \mathcal{O}(10^{-100})$ can be realized even if we impose the baryonic isocurvature constraints.

### 4 Implications for Scenarios of Baryogenesis

So far, we have seen that a large value of the non-linearity parameter of $f_{\text{NL}} \sim \mathcal{O}(10^{-100})$ can be realized without conflicting with the baryonic isocurvature constraints, assuming $T_R$, $\sigma_*$, and $\Gamma_\sigma$ as free parameters. If we fix the scenario of baryogenesis, however, it is often the case that a lower bound on $T_R$ is imposed to generate large enough baryonic asymmetry of the universe. So, finally, we discuss whether large $f_{\text{NL}}$ is possible for several scenarios of baryogenesis.

In order to realize the thermal leptogenesis [9], $T_R$ should be higher than $10^{9-10}$ GeV.
Then, for $m_\sigma = 100$ GeV and $\epsilon = 10^{-2}$, $\sigma_* \lesssim 10^{15}$ GeV and $\Gamma_\sigma \lesssim 10^{-7}$ GeV are required to realize $f_{NL} = 10$ without conflicting with the constraints (40). Notice that, even though $\Gamma_\sigma$ has to be much smaller than $m_\sigma$, the curvaton decay occurs when the cosmic temperature is $T \gtrsim 10^5$ GeV for $\Gamma_\sigma = 10^{-8}$ GeV, which is well before the start of the big-bang nucleosynthesis (BBN) [28] If $m_\sigma$ is larger, $f_{NL} \sim O(10)$ can be realized with a larger value of $\Gamma_\sigma$.

Even though the scenario of the thermal leptogenesis is attractive, it is incompatible with some class of supersymmetric model because, if $T_R \gtrsim 10^{9-10}$ GeV, overproduction of the gravitino may occur [25]. If a scenario of baryogenesis which works at a lower temperature is needed, one of the possibilities is the non-thermal leptogenesis in which right-handed neutrinos are directly produced by the decay of a scalar condensation (like the inflaton) [10]. In the non-thermal leptogenesis scenario, the lower bound on $T_R$ is reduced, and is given by $T_R \gtrsim 10^6$ GeV [26]. As one can see, even with $T_R \sim 10^6$ GeV, $f_{NL}$ can be as large as $O(10)$ (or larger) satisfying the baryonic isocurvature constraints. In this case, with $m_\sigma = 100$ GeV and $\epsilon = 10^{-2}$, $f_{NL} = 10$ requires $\sigma_* \lesssim 10^{14}$ GeV and $\Gamma_\sigma \lesssim 10^{-15}$ GeV, which corresponds to the decay temperature of the curvaton of $\sim O(10$ GeV).

In summary, even if the baryon asymmetry originates only from the decay products of the inflaton, a large non-Gaussianity of $f_{NL} \sim O(10 - 100)$ is possible in large class of scenarios of baryogenesis without conflicting with the observational constraints. In such a scenario, however, baryonic isocurvature fluctuations are inevitably generated and may be just below the observational bound. Thus, if a large value of $f_{NL}$ is confirmed in future observations, it is strongly encouraged to look for signals of the isocurvature fluctuations to test baryogenesis models in the curvaton scenario.

Note added: While we were finalizing the manuscript, Ref. [27] appeared on the arXiv, which has some overlap with our analysis.

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