Mathematical modeling of ice melt during irradiation by selective source of thermal radiation

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Abstract. A numerical simulation of the melting of an ice layer on a vertical substrate heated from a radiation source in the form of a halogen lamp in the single-phase formulation of the Stefan problem was carried out. Ice is presented in the form of a pure, non-scattering, selectively absorbing material with two spectral bands of bulk absorption. The computational model takes into account the selective nature of the radiation source. Analysis of the calculation results shows the dominant effect of the incident radiation on the formation of the flux density field of the resulting radiation in the medium. Comparison of the results shows a satisfactory agreement with experimental data.

1. Introduction.
Due to the development of the Arctic and the observed climate changes, the theoretical forecast of the state of glaciers and ice fields seems to be relevant and important. Simulation of ice melting is necessary to understand natural processes, as well as to ensure the safety of building structures, equipment and people in northern latitudes. Krass and Merzlikin [1] presented the state of early studies of thermophysical processes in snow and ice cover and proposed physical models of a new class of problems in which snow and ice are considered as light-scattering media characterized by volumetric absorption and scattering of radiation. The current state of theoretical and numerical modeling in the snow-ice stratum under solar irradiation can be judged from recent publications [2] and [3]. Simulation of heating and subsequent melting of ice is based on the Stefan problem for a translucent medium. Numerical and experimental studies of heat transfer in semitransparent media with allowance for thermal radiation using two and three phase formulations of the Stefan problem are described in [4–6]. Mathematical modeling of radiation-conductive heat exchange using the single-phase formulation of the Stefan problem is considered in [7, 8], but there are very few experimental works that could be used to validate a computational model in the literature. Seki et al. [9] performed a computational-experimental investigation, where an ice layer on a vertical opaque substrate was studied in a climate chamber at a constant temperature of 0 °C under the influence of radiation of two types of lamps (halogen and nichrome filament). Thus, radiation melting of ice was simulated under conditions of an external short-wave and long-wave radiation flux. In the mathematical model of the process, the authors neglected the presence of a water film on the surface and calculation was performed in a single-phase formulation of the Stefan problem. The authors compared the melting and heating rates of the non-irradiated ice surface and obtained a satisfactory agreement between the experimental and numerical results. In this case, the effect of short-wave radiation on the appearance of significant surface roughness of ice was first shown.
In the computational part, the authors used fitting parameters and direct integration according to the Bouguer law to account for radiation.

In this paper, the task is to validate the previously proposed computational model, including its radiation part [8], which takes into account the volume selective absorption of radiation by a layer of ice and a selective radiation source. Validation of the computational model is carried out by comparison with the experimental data presented in [9].

2. Statement of the problem and methods of solution

Figure 1 shows the geometrical scheme of the problem in which a layer of clear and non-scattering ice is adhered to the substrate with thickness $L_0$ and is located in a climate chamber with constant temperature $T_\infty$. The right surface of the ice layer is illuminated by a lamp with a filament temperature of 3200 K and constant incident flux $E_\nu^* = 1162.22 \text{ W/m}^2$. The radiation range of this lamp falls mostly on the part of the spectrum up to 1.2 μm, therefore it is necessary to take into account the selectivity of the radiation source. To do this, using the function of radiation of the first kind [10], at a given filament temperature and range of wavelengths, we calculate the fractions of radiation flux $i_f$ per one or another part of the spectrum.

The ice surfaces are optically gray, they diffusely absorb, reflect and transmit radiation, so that, $A_i + R_i + D_i = 1$, where $A_i$, $R_i$, $D_i$ are the absorption, reflective and transmittance hemispherical abilities of the ice surface respectively ($i = 1$, 2). The left surface of the substrate is maintained at constant temperature $T_{\text{sub}} = 256.15 \text{ K}$, which coincides with the initial temperature $T(x, 0)$, the temperature inside the chamber is $T_\infty = 273.15 \text{ K}$.

The solution of the problem is carried out in two stages. At the first stage, radiation-conductive heat exchange is considered until the right surface of the ice layer reaches the phase transition temperature $T_f$. At the second stage, the Stefan problem with a fixed value of the right boundary temperature $T(L(t), t) = T_f$ is considered, and a film of water flowing under the influence of gravity is supposed to appear on the irradiated surface.

We assume that the temperature of the film $T_{\text{film}}$ is higher than the phase transition temperature of ice and, thus, the boundary condition on the irradiated surface takes into account its own radiation and convective heat transfer from the film. In [9] the presence of thin films of melt water was noted, but it was neglected in calculations. The position of the phase boundaries $L(t)$ is determined from the solution of the boundary value problem.

The energy conservation equations for ice with the temperature $T(x, t)$ is written as:

$$c_p \rho \frac{\partial T(x, t)}{\partial t} = \frac{\partial}{\partial x} \left( \lambda \frac{\partial T(x, t)}{\partial x} - E_{\nu}(x, t) \right), \quad 0 < x < L(t) \, .$$

(1)
Here $c_p$ is the heat capacity at constant pressure, $\rho$ is the density, $\lambda$ is the thermal conductivity, $E_r(x,t) = E_r^+(x,t) - E_r^-(x,t)$ is the flux density of the resulting radiation.

The boundary conditions for equation (1) at the first stage of the process are written as follows:

$$\frac{\partial T}{\partial x} = 0 \text{ at } x = 0,$$

$$\lambda \frac{\partial T}{\partial x} - h(T_n - T) - |E_{ex,2}| = 0 \text{ at } x = L_n.$$ (2)

where $|E_{ex,2}| = A_e \left(E_r^+(x,t) + E_r^-(x,t)\right) - \epsilon_e \sigma T^4(x,t)$.

It is assumed that the surface of the substrate on the left boundary is maintained at constant temperature $T_{sub}$ and there is no heat flow from the substrate to the ice. The right boundary is exposed to radiation from a radiation source, and cooling associated with convection is also taken into account.

Equations (1) and (2) are supplemented by the initial condition: $(x,0)\rightarrow T_{x,0} = T_{sub}$.

At the second stage of the process, the temperature of the surface of the right boundary, when $x = L(t)$, is fixed: $(x,t)\rightarrow T_{f}(x,t) = T_f$. The boundary condition (2) is transformed into the Stefan condition with allowance for a thin film of water formed on the surface. We assume that the water film is isothermal, and the temperature difference over its thickness is negligible:

$$\lambda \frac{\partial T}{\partial x} + h(T_{f} - T) - |E_{ex,2}| = \rho_\gamma \frac{dT}{dt}.$$ (3)

Where $|E_{ex,2}|$ has the following form:

$|E_{ex,2}| = A_e \left(E_r^+(x,t) + E_r^-(x,t)\right) - \epsilon_e \sigma(T_f^4(x,t) - T_{f}^4)$ at $x = L(t)$.

Here, $T_f = 273.15$ K is the melting temperature of water, $T_{f} = 277.15$ K is the temperature of the water film, $\gamma$ is the latent heat of the phase transition. In condition (3), heat transfer from the outer surface of the water film is taken into account, in (4), the natural radiation of the film and the right surface is considered.

The assumption of the presence of a thin film of water on the ice surface does not contradict the single-phase approximation of the Stefan problem, since radiation does not absorb radiation in the film itself and it acts only as an additional boundary condition on the interfacial surface with constant values. The thermal problem is solved only in the thickness of ice on a vertical substrate.

The dimensionless densities of radiation fluxes $\Phi_j = E_j^+/\left(\sigma T_j^4\right)$, $\Phi_\gamma = \sum (\Phi_j^+ - \Phi_j^-)$ included in equations (1) – (4), are determined by solving the radiation transfer equation in a flat layer of the radiating and absorbing medium with a known temperature distribution over the layer, $j$ is the number of the spectral band [2, 4].

As in previous works of the authors, the radiation transfer is calculated using a simple and fairly accurate modified mean flux method [2, 4, 6]. According to this approach, the integral-differential equation of radiation transfer is reduced to a system of two nonlinear differential equations for a flat layer of a semitransparent absorbing medium. The differential analogue of the transport equation for hemisphericaler flows is represented as [2, 4]:

$$\frac{d}{d\tau_j} \left(\Phi_j^+ - \Phi_j^-\right) + \left(m_j^+ \Phi_j^+ - m_j^- \Phi_j^-\right) = n_j^2 \Phi_0,$$

$$\frac{d}{d\tau_j} \left(m_j^+ \Phi_j^+ - m_j^- \Phi_j^-\right) + \left(\Phi_j^+ - \Phi_j^-\right) = 0.$$ (5)

The boundary conditions for the system of equations (9) in dimensionless variables are written as [2]:
In (6) a selective radiation source is taken into account.

Here \( \Phi_{0,j} = n^2 B_\nu \left( \frac{\alpha}{4 \sigma n T_j^*} \right) \) is dimensionless flux density of equilibrium radiation, \( B_\nu \) is Planck function, \( n \) is refractive index of ice, \( n^* \) is refractive index of the environment, \( \tau = \alpha L(t) \) is spectral optical thickness of the layer at the moment of time \( t \). The values of coefficients \( m^\pm, l^\pm \) are determined from the recurrence relation obtained using the formal solution of the radiation transfer equation, where \( j \) is the number of the spectral band \([2, 4]\). Layer I refers to ice, layer II refers to the external space (Fig. 1).

The solution of the boundary value problem is reduced to determining the temperatures \((T_x, T_\theta)\) and flux densities of the resulting radiation \( \Phi_\nu(\xi, \eta) \) in the region \( G = \{0 \leq \xi \leq 1; 0 \leq \eta \leq \eta_1\} \), which is a flat layer of clean, absorbing, radiating and non-scattering ice. The position of the phase transition front \( s(\eta) \) varies from 1 to 0. The boundary value problem (1) - (4) is solved by a finite difference method, the nonlinear system of implicit difference equations is solved by a sweep method and iterations. In solving the radiation problem iterations are used, when the boundary value problem (5) – (6) is solved by the matrix factorization method at each step. The rapid convergence of this method of solution allows us to obtain fairly accurate results.

### 3. Result analysis

Below is an analysis of the results of numerical modeling of vertically positioned clear, non-scattering ice with the following physical parameters: initial ice thickness \( L_0 = 0.045 \) m; the temperature of the left boundary of the substrate and the initial temperature of the substrate and ice is \( T_{\text{sub}} = 253.15 \) K, the temperature of the atmosphere inside the chamber is maintained at a constant value of \( T_\infty = 273.15 \) K, equal to the melting ice temperature \( T_f \), constant density of the incident radiation flux is \( E_\nu = 1162.22 \) W/m². The following thermophysical properties of ice are taken: thermal conductivity \( \lambda = 1.9 \) W/(m K); thermal diffusivity \( \alpha = 9.3 \cdot 10^{-7} \) m²/s; latent heat of the phase transition \( \gamma = 335 \) kJ/kg. Optical parameters are as follows: refractive index of ice \( n = 1.31 \), refractive index of air \( n^* = 1 \); reflection coefficients \( R_1 = 0.5 \) and \( R_2 = 0.063 \); boundary emissivity \( \varepsilon_1 = 1 - R_1 \). The spectral characteristics are presented in Table 1.

In solving the problem, the heat transfer coefficient and the degree of blackness on the right irradiated surface are varied. Heat transfer at the stage of ice heating is \( h = 15.16 \) W/m², at the phase transition stage \( h = 8 \) W/m². The emissivity of the irradiated border at the first stage is \( \varepsilon_2 = 0.97 \) [1]. At the phase transition stage, the right boundary emissivity \( \varepsilon_2 \) is assumed to be 0.47, and the reflection coefficient \( R_2 = 1 - \varepsilon_2 \). This parameter is obtained in the course of numerical experiments and corresponds to the appearance of highly rough surfaces when ice is irradiated with a halogen radiation source [9] and, thus, to an increase in its reflectivity.

#### Table 1. Spectral dependences of ice parameters.

| \( j \) | \( \nu_j \cdot 10^{14} \) Hz | \( \lambda_j, \mu m \) | \( \alpha_j, m^{-1} \) | \( f_{\theta j} \) | \( E_0^j, W/m^2 \) |
|---|---|---|---|---|---|
| 1 | 9.09 – 2.02 | 0.33 – 1.2 | 0.001 | 0.446 | 518.35 |
| 2 | 2.02 – 1.18 | 1.2 – \( \infty \) | 1 | 0.405 | 470.7 |
Figure 2. Calculated temperature field. 1 – process beginning of heating, 2 – melting beginning, 3 – end of melting.

Figure 3. The field of the flux density of the resulting radiation (solid lines – 1 stage, dashed lines – melting).

Figure 4. The ice melting rate (line – calculation, ● – data [9]).

Figure 5. Temperature dynamics of the left border of the ice (line – calculation, ● – data [9]).

Figure 2 shows the temperature field in the ice layer at the stages of heating and subsequent melting of ice. At the initial moment of heating (curves between 1 and 2), the temperature field in the volume of the medium is considerably non-uniform due to the high reflection coefficient on the left border. With a further increase in temperature, the temperature drop decreases. At the stage of phase transition (lines between 2 and 3), due to the presence of thawed water film, the temperature curves become almost linear, which indicates an equilibrium phase transition.

The flux density of the resulting radiation (RR) strongly depends on the optical properties of the medium. Figure 3 shows that volumetric thermal radiation is minimal at the left border (due to low temperature), and the role of incident radiation due to the reflection coefficient is high. On the right boundary, the role of volumetric thermal radiation increases, but does not exceed the incident radiation, since RR flux density is positive. At the stage of heating (solid lines between 1 and 2), there is a significant variation of values, a constant angle of inclination of the lines shows that the resulting radiation as the sample heats up has a constant value in volume. At the phase transition stage (dashed lines), the RR value on the left cold side is almost at one point, while on the right border there is a noticeably larger effect of the volumetric thermal radiation of the irradiated surface.
Figure 4 shows the change in ice thickness during melting in comparison with the experimental data of [9]. At the initial moment, the melting rate coincides with the experimental results; however, later, a constant variation is observed, not exceeding 3% of the experimental data. Figure 5 shows the temperature rise of the left side of ice with time. The agreement with the results of the experimental part of the above work is observed at the initial moment of time and upon reaching the steady state. A similar variation of values was observed when comparing the numerical results with the experimental ones carried out by the authors of [9]. This variation of values can be explained both by the experimental error and the physical phenomena unaccounted for in the model.

**Conclusion**

In this work, formation of the temperature field and flux density of the resulting radiation in the process of heating and melting a flat layer of ice upon irradiation with a short-wave radiation source in the laboratory has been studied numerically. A mathematical model of the phase transition in the single-phase approximation of the Stefan problem, which describes radiation-conductive heat transfer in the process of ice melting, is used [11]. A modified algorithm for the numerical calculation of radiation heat exchange is used, taking into account the selective nature of the medium and the radiation source. Indirectly, through a decrease in the emissivity and an increase in the reflection coefficient, the appearance of roughness on ice when irradiated with a halogen lamp is taken into account. The influence of radiation reflection at the boundaries and the role of volumetric thermal radiation on the process of heating and melting ice are analyzed. To validate the proposed mathematical model and the algorithm for numerical calculation, the melting rate of ice and the temperature rise on the left border were compared with experimental data [9]. A satisfactory agreement between the calculation results and experimental data is shown.

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