General Superfield Quantization Method.

II. General Superfield Theory of Fields: Hamiltonian Formalism

A.A. Reshetnyak

Department of Mathematics, Seversk State Technological Institute, Seversk, 636036, Russia

Abstract

In the framework of started in Ref.[1] construction procedure of the general superfield quantization method for gauge theories in Lagrangian formalism the rules for Hamiltonian formulation of general superfield theory of fields (GSTF) are introduced and are on the whole considered.

Mathematical means developed in [1] for Lagrangian formulation of GSTF are extended to use in Hamiltonian one. Hamiltonization for Lagrangian formulation of GSTF via Legendre transform of superfunction $S_L(A(\theta),\hat{A}(\theta),\theta)$ with respect to $\hat{A}(\theta)$ is considered. As result on the space $T^*_{odd}M_{cl} \times \{\theta\}$ parametrized by classical superfields $A^i(\theta)$, superantifields $A^*_i(\theta)$ and odd Grassmann variable $\theta$ the superfunction $S_H(A(\theta),A^*(\theta),\theta)$ is defined. Being equivalent to different types of Euler-Lagrange equations the distinct Hamiltonian systems are investigated. Translations along $\theta$ for superfunctions on $T^*_{odd}M_{cl} \times \{\theta\}$ being associated with these systems are studied. Various types of antibrackets and differential operators acting on $C^k(T^*_{odd}M_{cl} \times \{\theta\})$ are considered.

Component (on $\theta$) formulation for GSTF quantities and operations is produced. Analogy between ordinary Hamiltonian classical mechanics and GSTF in Hamiltonian formulation is proposed. Realization of the GSTF general scheme is demonstrated on 6 models.

PACS codes: 03.50.-z, 11.10.Ef, 11.15.-q

Keywords: Lagrangian quantization, Gauge theory, Hamiltonization, Superfields.

Los Alamos database number: hep-th/0303262

I Introduction

The problem of construction of the general superfield quantization method (GSQM) for general gauge theories in Lagrangian formalism, in which it is possible to establish a superfield realization for BRST symmetry type transformations [2] and which includes the BV quantization method [3] on component (on $\theta$) level, can not be adequately and noncontradictory decided without the preliminary creation of GSTF. The latter contains the all information on ways
of the superfield formulation for concrete models of GSTF. Construction of the Lagrangian formulation for GSTF, as the 1st stage in creating of GSQM, was realized in paper [1]. In that work both preliminary historical excursus on the ways of development of the quantization methods for gauge theories on the BRST symmetry basis and motivations for origin of the GSQM construction problem were considered. In this connection, nevertheless it is appropriate to note the papers [4] where one had been made one from the first indications on the Grassmann variable $\theta$ ($\theta^2 = 0$) role for generalization of the differential equations concept, being interpreted as the odd time, and Refs.[5] describing the possibility to use that variable for BV method formulation. By the following stage after Lagrangian formulation for GSTF [1] being by composite part of GSQM in the Lagrangian formalism (in the usual sense) it appears the creation of the Hamiltonian formulation for GSTF that represents the investigation subject of this paper.

In Sec.II having a preparatory character the possibility for determination of the Euler-Lagrange equations integrals is considered together with introduction of special constraints system for one of integrals in the Lagrangian formulation for GSTF. Moreover a sequence of algebraic and analytic questions of the mathematical means from Ref.[1] are transferred with some modifications onto superalgebra of superfunctions $C^k(T_{\text{odd}}(T_{\text{odd}} M_{\text{cl}}) \times \{\theta\}), k \leq \infty$. Creating of the Hamiltonian formulation for GSTF on a basis of realization of the Legendre transform of superfunction $S_L(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta) \equiv S_L(\theta) \in C^k(T_{\text{odd}} M_{\text{cl}} \times \{\theta\})$ with respect to superfields $\mathcal{A}(\theta)$ and construction of $S_H(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta) \equiv S_H(\theta) \in C^k(T_{\text{odd}}^* M_{\text{cl}} \times \{\theta\})$ together with obtaining of dynamical equations in $T_{\text{odd}}^* M_{\text{cl}}$ essentially reflecting the properties of $S_H(\theta)$ are realized in Sec.III.

Sec.IV is devoted to detailed study of the properties and relations among different Lagrangian and Hamiltonian systems of ordinary differential equations (ODE) with respect to differentiation on $\theta$ describing an arbitrary model of GSTF with various modifications.

Important role of the 2nd order odd (with respect to $\varepsilon_P$ and $\varepsilon_{\text{gradings}}$) operator $\Delta^\text{cl}(\theta)$ acting on $C^k(T_{\text{odd}}^* M_{\text{cl}} \times \{\theta\})$ and nontrivial master equations in $T_{\text{odd}}^* M_{\text{cl}} \times \{\theta\}$ together with translation transformations on $\theta$ along integral curves of different Hamiltonian systems (and their some modifications) for arbitrary superfunctions given on $T_{\text{odd}}^* M_{\text{cl}} \times \{\theta\}$ are described in Sec.V. In connection with these transformations the key role of solvability conditions fulfilment for the Hamiltonian systems is demonstrated here as well. Systematic investigation of the gauge theories in the Hamiltonian formulation for GSTF including the solvable Hamiltonian systems together with introduction of detailed concepts on constraints, representations on the gauge theories, gauge transformations of general and special types are carried out in Sec.VI.

Sec.VII is devoted to extension of the superalgebra $\mathcal{A}_{\text{cl}}$ [1] of the 1st order differential operators acting on the superfunctions from $C^k(T_{\text{odd}} M_{\text{cl}} \times \{\theta\})$ to superalgebra $\mathcal{B}_{\text{cl}}$ of the 1st and 2nd orders ones defined on $C^k(T_{\text{odd}}^* M_{\text{cl}} \times \{\theta\})$. The different types of antibrackets being generated by the 2nd order odd operators are studied here as well. The concept on transformation of operators from $\mathcal{B}_{\text{cl}}$ is introduced.

Component (on $\theta$) formulation for quantities and relations of the Hamiltonian formulation of GSTF is considered in Sec.VIII.

General statements of Secs.II–VIII are demonstrated in Sec.IX on the example of 6 models, whose superfield Lagrangian formulation had been proposed in paper [1]. As it had been noted in that work the almost all from mentioned models appear by initial ones to constructing of the interacting superfield (on $\theta$) Yang-Mills type models in realizing of the gauge principle [6].

The final propositions and analogy for Hamiltonian formulation of GSTF with ordinary classical mechanics in the standard (even) Hamiltonian formalism conclude the work in Sec.X.
All assumptions, in the framework of which the paper is made, in fact had been pointed out in the introduction of work [1]. In the paper unless otherwise stated it is used the system of notations suggested in Ref.[1].

II Special Properties of the Lagrangian Formulation for GSTF. Superalgebra $C^k(T_{odd}(T_{odd}^*M_{cl}) \times \{\theta\})$

II.1 Integrals of Lagrangian System

E.Noether’s theorem [7] analog applied to translation along $\theta$, being one from the coordinates of the superspace $M = \{(z^a, \theta)\}$ [1], on a constant parameter $\mu \in \Lambda_1(\theta)$ being considered in question as continuous transformation of global symmetry for integrand in

$$ Z[\mathcal{A}] = \int d\theta S_L(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta) $$

permits one to find the integral of motion for Euler-Lagrange equations

$$ \frac{\delta Z[\mathcal{A}]}{\delta \mathcal{A}^i(\theta)} = \left( \frac{\partial}{\partial \mathcal{A}^i(\theta)} - (-1)^{\varepsilon_i} \frac{d}{d\theta} \frac{\partial}{\partial \mathcal{A}^j(\theta)} \right) S_L(\theta) \equiv \mathcal{L}_i^l(\theta)S_L(\theta) = 0, \; i = 1, \ldots, n, $$

or one for equivalent system $2n$ ODE of the 2nd order with respect to differentiation on $\theta$

$$ \mathcal{A}^i(\theta) \frac{\partial^2 S_L(\theta)}{\partial \mathcal{A}^j(\theta) \partial \mathcal{A}^l(\theta)} = 0, $$

$$ \Theta_i(\mathcal{A}(\theta), \dot{\mathcal{A}}(\theta), \theta) \equiv \frac{\partial S_L(\theta)}{\partial \mathcal{A}^l(\theta)} - (-1)^{\varepsilon_i} \left( \frac{\partial}{\partial \theta} + \dot{U}_+(\theta) \right) \frac{\partial S_L(\theta)}{\partial \mathcal{A}^l(\theta)} = 0, $$

in the following form

$$ S_E(\mathcal{A}(\theta), \dot{\mathcal{A}}(\theta), \theta) = \mathcal{A}^i(\theta) \frac{\partial S_L(\theta)}{\partial \mathcal{A}^i(\theta)} - S_L(\theta), $$

$$ \frac{dS_E(\theta)}{d\theta} \big|_{\mathcal{L}_i^l(\theta)S_L(\theta)=0} = - \frac{\partial S_L(\theta)}{\partial \theta} - 2 \mathcal{A}^i(\theta) \frac{\partial S_L(\theta)}{\partial \mathcal{A}^i(\theta)} = 0. $$

The fulfillment of relation (2.4b) is the nontrivial fact. In particular, in absence of the explicit dependence of $S_L(\theta)$ on $\theta$ (that corresponds to above-mentioned symmetry transformation)

$$ \tilde{P}_1(\theta)S_L(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta) = 0 \iff \frac{\partial S_L(\theta)}{\partial \theta} = 0, $$

Eq.(2.4b) is reduced to

$$ \mathcal{A}^l(\theta) \frac{\partial S_L(\theta)}{\partial \mathcal{A}^l(\theta)} \big|_{\mathcal{L}_i^l(\theta)S_L(\theta)=0} = 0. $$

By the sufficient condition for solvability of the last equation is the fulfillment of the following system of equations on arbitrary solution $\mathcal{A}^i(\theta)$ for system (2.3)

$$ \mathcal{A}^A_{i}(\theta)_{|\mathcal{A}(\theta)} = 0, \; a = (A_1, A_2), \; A_1 = 1, \ldots, n_1, \; n_1 = (n_{1+}, n_{1-}), $$

$$ \frac{\partial S_L(\theta)}{\partial \mathcal{A}^{A_2}(\theta)_{|\mathcal{A}(\theta)}} = 0, \; A_2 = n_1 + 1, \ldots, n. $$
Indices $A_1, A_2$, generally speaking, will be able to destroy the locality and covariance of the corresponding superfunctions, being by elements of representation $T_J \equiv T \text{ space, with respect to index } j \text{ corresponding to the restriction } T_{ij} \text{ of the superfield (on } \theta \text{) representation } T \text{ of supergroup } J = J \times P \text{ onto subsupergroup } J \text{ [1]. By one from the possible choices for system (2.7a,b) parametrized by discrete value } n_1 \text{ it appears the setting } n_1 = 0 \ (a = i) \ \ \ (2.8) .

Thus in order to the superfunction $S_E(\theta)$ should appear by integral for Eqs.(2.3) in fulfilling of Eq.(2.5) it is necessary to reduce the system (2.3) to one of the 2nd order on $\theta$ 3n ODE

$$
\mathcal{A}'(\theta) \frac{\partial^2 S_L(\mathcal{A}(\theta), \mathcal{A}(\theta))}{\partial \mathcal{A}'(\theta) \partial \mathcal{A}'(\theta)} = 0 ,
$$

$$
\Theta_i(\mathcal{A}(\theta), \mathcal{A}(\theta)) = 0 ,
$$

$$
\mathcal{A}^{A_1}(\theta) = \Theta_i(\theta)\lambda_1^{A_1i}(\mathcal{A}(\theta), \mathcal{A}(\theta)), \varepsilon(\lambda_1^{A_1i}) = \varepsilon_{A_1} + \varepsilon_i + 1 ,
$$

$$
\frac{\partial S_L(\mathcal{A}(\theta), \mathcal{A}(\theta))}{\partial \mathcal{A}^{A_2}(\theta)} = \Theta_j(\theta)\lambda_2^{A_2j}(\mathcal{A}(\theta), \mathcal{A}(\theta)), \varepsilon(\lambda_2^{A_2j}) = \varepsilon_{A_2} + \varepsilon_j ,
$$
called further the extended Lagrangian system (ELS). Subsystem (2.9c,d) appears by additional constraints to differential constraints in Lagrangian formalism (DCLF) (2.9b) [1]. For $n_1 = 0$, $\lambda_j(\mathcal{A}(\theta), \mathcal{A}(\theta)) = 0$ and absence of explicit dependence upon $\theta$ for $S_L(\theta)$ the constraints (2.8) coincide with DCLF corresponding to the GSTF model which is the natural system from corollary 2.2 of theorem 2 of the Ref.[1]. The presence of arbitrary superfunctions $\lambda_1^{A_1i}(\theta), \lambda_2^{A_2j}(\theta) \in C^k(T_{odd}M_{cl})$ in Eqs.(2.9c,d) reflects the fact of the Eqs.(2.7) fulfilment on the solutions for LS (2.3).

In general case the equation (2.4b) is always fulfilled without additional constraints (2.7) for following choice of explicit dependence of $S_L(\theta)$ upon $\theta$

$$
\tilde{P}_1(\theta)S_L(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta) = -\theta \left( 2\mathcal{A}'(\theta) \frac{\partial S_L(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta)}{\partial \mathcal{A}'(\theta)} \right) ,
$$

where as in (2.5) the quantity $\tilde{P}_1(\theta)$ is one from the system of projectors $\{\tilde{P}_a(\theta), U(\theta)\}, a = 0, 1$ acting on $C^k(T_{odd}M_{cl} \times \{\theta\})$ [1]. The consistency of Eq.(2.4b) and its solution (2.10) follows from the fact that for the system of the 1st order with respect to $\theta$ n ODE of the form

$$
h^i(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta) = 0, \ h^i(\theta) \in C^k(T_{odd}M_{cl} \times \{\theta\})
$$
it is necessary to fulfill the solvability conditions [1] which have the form of the 1st or 2nd orders on $\theta$ system of 2n ODE

$$
\frac{d}{d\theta} h^i(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta) = 0 .
$$

Equivalently, in place of the 1st subsystem in Eqs.(2.12) the following system can be written (no longer in the superfield form with respect to superfield representation $T$ [1])

$$
h^i(P_0\mathcal{A}(\theta), \mathcal{A}(\theta), 0) = 0 .
$$
It follows from (2.20) the superfunctions $S$ the superfunction $S$ together with (2.17) and identical fulfillment of Eq.(2.4b) the following equality is valid

$$
\frac{\partial S_L(\mathcal{A}(\theta), \hat{\mathcal{A}}(\theta))}{\partial A^i(\theta)} = \Theta_j(\mathcal{A}(\theta), \hat{\mathcal{A}}(\theta)) K^j(\mathcal{A}(\theta), \hat{\mathcal{A}}(\theta)) = \left(\mathcal{L}^i_j(\theta) S_L(\theta)\right) K^j(\theta). \quad (2.14)
$$

Eq.(2.14) imposes definite restrictions on $\lambda_1^{A_1\theta}(\theta)$, $\lambda_2^{A_2\theta}(\theta)$ in (2.9c,d)

$$
K^j(\mathcal{A}(\theta), \hat{\mathcal{A}}(\theta)) = \lambda_1^{A_1\theta}(\theta, \hat{\mathcal{A}}(\theta)) \frac{\partial S_L(\theta)}{\partial A^{\lambda_1}(\theta)} + \lambda_2^{A_2\theta}(\theta, \hat{\mathcal{A}}(\theta)) A^{A_2}(\theta). \quad (2.15)
$$

Consider two possible limiting cases of the Eq.(2.14) fulfillment. Namely, having put the constraints (2.9c) for $n_1 = n$ and $\lambda_1^{A_1\theta}(\theta) = 0$ satisfying to equations

$$
\hat{A}^i(\theta) = 0, \quad (2.16a)
$$

obtain their solution [1] in the whole $T_{odd} \mathcal{M}_{cl}$

$$
\mathcal{A}'(\theta) = P_0(\theta) \mathcal{A}'(\theta) = \mathcal{A}'(0). \quad (2.16b)
$$

It means the superfunction $S_L(\theta)$ does not depend on $\hat{A}^i(\theta)$ and has the form

$$
S_L(\mathcal{A}(\theta), \hat{\mathcal{A}}(\theta)) \equiv S_L(\mathcal{A}(\theta)) = S_L(\mathcal{A}(0)). \quad (2.17)
$$

Together with (2.17) and identical fulfillment of Eq.(2.4b) the following equality is valid

$$
S_E(\mathcal{A}(\theta)) = -S_L(\mathcal{A}(\theta)). \quad (2.18)
$$

Another variant is derived in fulfilling of the condition (2.5) and of constraints (2.9c,d) for $n_1 = 0$ for any configuration of $\mathcal{A}'(\theta) \in \mathcal{M}_{cl}$. This case is realized by means of identities

$$
S_L(\theta) = 0. \quad (2.19)
$$

An arbitrary differential consequence of the last relation being obtained in calculating of the $l$ order derivative ($l = 0, 1, \ldots$) of Eq.(2.19) with respect to $\mathcal{A}^i(\theta)$ means the independence of the superfunction $S_L(\theta)$ on superfields $\mathcal{A}^i(\theta)$ in the whole $T_{odd} \mathcal{M}_{cl}$

$$
S_L(\mathcal{A}(\theta), \hat{\mathcal{A}}(\theta)) = S_L(\mathcal{A}(\theta)) = S_L(\mathcal{A}(\theta)). \quad (2.20)
$$

It follows from (2.20) the superfunctions $\frac{\partial S_L(\mathcal{A}(\theta))}{\partial \mathcal{A}^i(\theta)}$ appear not only by integrals of motion for system (2.3) but are constant for any value of $\mathcal{A}^i(\theta)$ by virtue of $\hat{\mathcal{A}}^i(\theta) \equiv 0$. The last fact can be interpreted as well with help of Noether’s theorem. Namely, the invariance of $S_L(\theta)$ with respect to symmetry transformation, being by translation of the superfield $\mathcal{A}^i(\theta) \rightarrow \mathcal{A}^i(\theta) = \mathcal{A}^i(\theta) + C^i(\theta)$ on an arbitrary constant superfield $C^i(\theta)$ ($\varepsilon(C^i) = \varepsilon_i$), corresponds to existence of integrals for Eqs.(2.3), i.e. to conservation of the ”momenta” $\frac{\partial S_L(\mathcal{A}(\theta))}{\partial \mathcal{A}^i(\theta)}$. The condition (2.5) have been made use for the last conclusion as well.
Remark: The definition of the symmetry transformations, Noether’s Theorem formulation and its proof in the context of GSQM are the separate problems and are eliminated outside the paper’s scope.

The independence of the "momenta" upon $\theta$ implies together with (2.19) the identical fulfilment of Eqs. (2.3) meaning formally that almost any superfields $A^i(\theta)$ are the solutions for LS (2.3) and under condition

$$\text{rank} S_L''(\theta) = n, \quad S_L''(\theta) \equiv \left( \frac{\partial S_L'(\theta)}{\partial \theta} \right) \equiv \left( \frac{\partial \tilde{A}^i(\theta)}{\partial \tilde{A}^i(\theta)} \right) = 0,$$  \hspace{1cm} (2.21)

exactly the any $A^i(\theta)$ are the same.

From these two cases of Eq. (2.6) fulfilment for any superfield $A^i(\theta)$ it follows that by the sufficient condition for its resolution is the existence of the solution for system (2.7) for arbitrary configuration $A^i(\theta)$ and $n_1$. The latter implies the identical vanishing of $\lambda_1^{A^i}(\theta), \lambda_2^{A^i}(\theta)$ in (2.9c,d) together with constraints (2.7) themselves, i.e. their fulfilment for any $A^i(\theta) \in \mathcal{M}_d$.

Thus, the validity of Eq. (2.6) for any $A^i(\theta)$ imposes the strong restrictions on the structure of the superfunction $S_L(\theta)$. Indicate in this case the necessary condition on the structure of $S_E(A(\theta), \hat{\theta}(\theta))$ (2.4a) and therefore for $S_L(\theta)$

$$\text{rank} \left\| \frac{\partial_r}{\partial \tilde{\Gamma}^p(\theta)} \frac{\partial \hat{S}_E(\theta)}{\partial \tilde{\Gamma}^q(\theta)} \right\|_{\partial \tilde{S}_E(\theta)} = 0, \quad p, q = 1, \ldots, 2n, \quad \tilde{\Gamma}^p(\theta) = \left( A^i(\theta), \frac{\partial S_L(\theta)}{\partial \tilde{A}^i(\theta)} \right).$$ \hspace{1cm} (2.22)

Except for Noether’s integral for LS the another (non Noetherian) one exists of the form

$$V_E(A(\theta), \hat{\theta}(\theta), \theta) = \hat{\theta}(\theta) \cdot \frac{\partial S_L(\theta)}{\partial \hat{\theta}(\theta)} + S_L(\theta), \quad \frac{dV_E(\theta)}{d\theta} \bigg|_{\dot{\theta}^i(\theta)S_L(\theta)} = 0,$$ \hspace{1cm} (2.23)

that is equivalent to realization of the only condition (2.5) without relationships (2.7), (2.22).

Consider the problem of reduction of the 2nd order on $\theta$ system of $2n$ ODE (2.3) to the normal form (NF), i.e. to the system of solvable ODE with respect to higher derivatives: superfields $\hat{A}^i(\theta)$ (formally). It is possible in $T_{odd} \mathcal{M}_d \times \{\theta\}$ if and only if the condition (2.21) on the rank of supermatrix $K(\theta)$ is fulfilled almost everywhere in $T_{odd} \mathcal{M}_d$. The comparison of the conditions (2.21) with (2.22) leads to very strong restriction on the structure of $S_L(\theta)$, namely to its independence upon superfields $A^i(\theta)$. Therefore for realizing of the Eq. (2.6) on the given stage of study of the classical theory properties confine ourselves by conditions (2.5)–(2.7) instead of (2.22).

Having suggested the realization of (2.21) the formulated problem can be solved by means of Legendre transform of $S_L(\theta)$ with respect to superfields $\hat{A}^i(\theta)$. To this end, introduce the additional to $A^i(\theta)$, $\hat{A}^i(\theta)$ superfields of $\bar{J}$ (Lorentz) type $\bar{J}^i(\theta)$ as the superfunctions over $\Lambda_{D|N_c+1}(z^a, \theta; \mathbf{K})$. The last set is the Berezin superalgebra over number field $\mathbf{K}$ ($\mathbf{R}$ or $\mathbf{C}$) being by Grassmann algebra with $(D + N_c)$ even with respect to $\varepsilon_P$ grading (from latter only $D$ are even with respect to $\varepsilon_J$ grading and other $N_c$ are odd with respect to $\varepsilon_J$ one) generating elements $z^a$ and with one odd with respect to $\varepsilon_P$ grading one $\theta$ [1]. $A^i(\theta)$ are transformed with respect to supergroup $J$ superfield representation $T^*$ which is connected by a special form with representation $^1T$ (conjugate to $T$ with respect to a some bilinear form). Grading

\[^1\text{as one had already been noted in Ref. [1] the separate work would be devoted to detailed study of (ir)reducible supergroup $J$ superfield representations}\]
properties of those superfields, called further as superantifields, and their derivatives on \( \theta \):
\[
\mathcal{A}^i_\theta(\theta) \equiv \frac{d^{A^i_\theta}}{d\theta} \text{ are written as follows}
\]
\[
(\varepsilon_p, \varepsilon_j, \varepsilon) \mathcal{A}^i_\theta(\theta) = (\varepsilon_p(\mathcal{A}^i_\theta(\theta))) + 1, \varepsilon_j(\mathcal{A}^i_\theta(\theta)), \varepsilon(\mathcal{A}^i_\theta(\theta)) + 1 = (1, \varepsilon, \varepsilon, + 1). \tag{2.24}
\]

In accordance with terminology of the work [1] the superantifields \( \mathcal{A}^i_\theta(\theta), \mathcal{A}^i_\theta(\theta) \) belong to \( \mathcal{A}_{D|Nc+1}(z^a, \theta; \mathbf{K}) \), \( \mathbf{K} = (\mathbf{R} \text{ or } \mathbf{C}) \) being by superalgebra of the superfunctions, to be transformed with respect to supergroup \( J \) superfield representations and defined on \( \Lambda_{D|Nc+1}(z^a, \theta; \mathbf{K}) \).

### II.2 Elements of Algebra and Analysis on \( T_{odd}(T_{odd}^* \mathcal{M}_{cl}) \times \{ \theta \} \)

The transformation laws for superantifields \( \mathcal{A}^i_\theta(\theta), \mathcal{A}^i_\theta(\theta) \) with respect to action of the supergroup \( J = (\mathcal{M} \odot \mathcal{J}_A) \times \mathbf{P}[1] \) (ir)reducible superfield representation \( T^* \) operators have the form
\[
\mathcal{A}^i_\theta(\theta) \mapsto \mathcal{A}^i_\theta(\theta) = (T^*(e, \bar{g}) \mathcal{A}^i_\theta)(\theta), \ e \in \mathcal{M}, \ \bar{g} \in \mathcal{J}_A, \tag{2.25a}
\]
\[
\mathcal{A}^i_\theta(\theta) \mapsto \mathcal{A}^i_\theta(\theta) = (T^*(e, \bar{g}) \mathcal{A}^i_\theta)(\theta), \tag{2.25b}
\]
\[
\mathcal{A}^i_\theta(\theta) \mapsto \mathcal{A}^i_\theta(\theta) = (T^*(e, \bar{g}) \mathcal{A}^i_\theta)(T(h^1(\mu))\theta) = (T^*(e, \bar{g}) \mathcal{A}^i_\theta)(\theta - \mu), \tag{2.26a}
\]
\[
\mathcal{A}^i_\theta(\theta) \mapsto \mathcal{A}^i_\theta(\theta) = (T^*(e, \bar{g}) \mathcal{A}^i_\theta)(\theta), \ h(\mu) \in \mathcal{P}, \ \mu \in \mathcal{I}_A(\theta). \tag{2.26b}
\]

and realize the finite-dimensional (2.25) and infinite-dimensional (2.26) superfield representations respectively.

Under permutation of two and more elements from \( \Lambda_{D|Nc+1}(z^a, \theta; \mathbf{K}) \) the sign ("+" or "-"") arises being dictated by their \( \varepsilon(!) \) grading.

Starting from supermanifolds \( \mathcal{M}_{cl} \) and \( T_{odd} \mathcal{M}_{cl} \) let us formally construct the following supermanifolds \( T_{odd}^* \mathcal{M}_{cl}, T_{odd}^* \mathcal{M}_{cl} \times \{ \theta \}, T_{odd}(T_{odd}^* \mathcal{M}_{cl}) \) and \( T_{odd}(T_{odd}^* \mathcal{M}_{cl}) \times \{ \theta \} \) parametrized by local coordinates \( \Gamma^p(\theta) \equiv (\mathcal{A}^i(\theta), \mathcal{A}^i(\theta)), (\Gamma^p(\theta), \theta), (\Gamma^p(\theta), \tilde{\Gamma}^p(\theta)) \equiv (\mathcal{A}^i(\theta), \mathcal{A}^i(\theta))) \) and \( (\Gamma^p(\theta), \tilde{\Gamma}^p(\theta), \theta), p = 1, \ldots, 2n \) respectively with \( \mathcal{A}^i(\theta) \in \mathcal{M}_{cl} \).

In complete analogy with Ref.[1] define the superalgebras of superfunctions \( \mathbf{K}[[T_{odd}(T_{odd}^* \mathcal{M}_{cl}) \times \{ \theta \}] \supset \mathbf{K}[[T_{odd}^* \mathcal{M}_{cl} \times \{ \theta \}]] \) determined on \( T_{odd}(T_{odd}^* \mathcal{M}_{cl}) \times \{ \theta \}, T_{odd}^* \mathcal{M}_{cl} \times \{ \theta \} \) respectively and being by formal power series with respect to generating elements \( \Gamma^p(\theta), \tilde{\Gamma}^p(\theta), \theta \) and \( \Gamma^p(\theta), \tilde{\Gamma}^p(\theta), \theta \) correspondingly \( (\mathbf{K} = \mathbf{R} \text{ or } \mathbf{C}) \). In particular, these sets contain superalgebras of superfunctions \( \mathbf{K}[[T_{odd}(T_{odd}^* \mathcal{M}_{cl}) \times \{ \theta \}] \supset \mathbf{K}[[T_{odd}^* \mathcal{M}_{cl} \times \{ \theta \}]] \) respectively appearing by finite polynomials with respect to \( \Gamma^p(\theta), \tilde{\Gamma}^p(\theta) \). For any superfunction \( \mathcal{F}(\theta) \equiv \mathcal{F}(\Gamma(\theta), \tilde{\Gamma}(\theta), \theta) \in \mathbf{K}[[T_{odd}(T_{odd}^* \mathcal{M}_{cl}) \times \{ \theta \}] \) the transformation laws are valid, in acting of representation \( \tilde{T} \) constructed from \( T, T^* \) on the indicated superalgebra, following from (2.25), (2.26) respectively
\[
\mathcal{F} \left( \Gamma^p(\theta), \frac{d\Gamma^p(\theta)}{d\theta}, \theta \right) = \mathcal{F} \left( (T \oplus T^*)(e, \bar{g}) \Gamma)(\theta), (T \oplus T^*)(e, \bar{g}) \tilde{\Gamma})(\theta), \theta \right), \tag{2.27}
\]
\[
\mathcal{F} \left( \Gamma^p(\theta), \tilde{\Gamma}(\theta), \theta \right) = \mathcal{F} \left( (T \oplus T^*)(e, \bar{g}) \Gamma)(\theta - \mu), (T \oplus T^*)(e, \bar{g}) \tilde{\Gamma})(\theta), \theta - \mu \right). \tag{2.28}
\]

By definition \( \mathcal{F}(\theta) \) is expanded in the formal power series (in finite sum for polynomials corresponding to local superfunctions) in powers of \( \Gamma^p(\theta), \tilde{\Gamma}^p(\theta) \)
\[
\mathcal{F}(\mathcal{A}(\theta), \mathcal{A}(\theta), \mathcal{A}^*(\theta), \mathcal{A}^*(\theta), \theta) = \sum_{i=0}^{\infty} \frac{1}{i!} \mathcal{A}^i_{(i)}(\theta)\mathcal{F}^{(i)}(\theta, \mathcal{A}(\theta), \mathcal{A}(\theta), \mathcal{A}^*(\theta), \theta) =
\]
\[
\sum_{m,l=0}^{\infty} \frac{1}{m!l!} \vec{\partial} (\vec{\partial})_l \vec{\partial} (\vec{\partial})_m \vec{\partial} (\vec{\partial})_m (A(\theta), \vec{A}(\theta), \vec{\theta}) \tag{2.29}
\]

where the notations are introduced

\[
\vec{A}^*_l (\theta) \equiv \Pi_{p=1}^l \vec{\partial}^*_p (\theta) \quad \vec{F} (\theta), \vec{\partial} (\theta) \equiv \vec{F} (\theta), \vec{\partial} (\theta), \theta 
\]

\[
\vec{A}^*_m (\theta) \equiv \Pi_{p=1}^m \vec{A}^*_p (\theta) \quad \vec{F}^{(m)} (\theta), \vec{\partial} (\theta) \equiv \vec{F}^{(m)} (\theta), \vec{\partial} (\theta), \theta 
\]

Coefficients of decomposition in (2.29) appear themselves by superfuntions. Moreover they are expanded in power series with respect to \(A^i(\theta), \vec{A}^* (\theta)\) as well and \(\vec{F}^{(m)} (\theta), \vec{\partial} (\theta, \theta) \in K[[T_{odd}M_{cl} \times \{\theta\}]] [1]\). In addition the coefficients possess the following properties of generalized symmetry, for instance, for \(\vec{F}^{(m)} (\theta), \vec{\partial} (\theta, \theta)\)

\[
\vec{F}^{(m)} (\theta), \vec{\partial} (\theta) = (-1)^{(s+1)(s-1)} \vec{F}^{(m)} (\theta), \vec{\partial} (\theta, \theta) \equiv (-1)^{(s-1)(s+1)} \vec{F}^{(m)} (\theta), \vec{\partial} (\theta, \theta), s = \overline{2m}, r = \overline{2l} \tag{2.31}
\]

Introducing the operations of differentiation with respect to superfuns \(\Gamma^p (\theta), \vec{\partial}^p (\theta)\) on \(K[[T_{odd}T_{odd}M_{cl} \times \{\theta\}]]\) one can convert the last superalgebra in one of the \(k\)-times differentiable superfuns \(C^k(T_{odd}T_{odd}M_{cl} \times \{\theta\}) \equiv D_{cl}^k, k \leq \infty\). In equipping of \(K[[T_{odd}T_{odd}M_{cl} \times \{\theta\}]]\) by a some norm one can convert this set into functional space regarding the series in (2.29) by convergent with respect to above norm and operations of differentiation on \(\Gamma^p (\theta), \vec{\partial}^p (\theta)\) by commutative with sign of sum.

Regarding that \(D_{cl}^k \) and \(C^k \times \{\theta\} \equiv C^k(T_{odd}T_{odd}M_{cl} \times \{\theta\}) \subset D_{cl}^k \) are supplied by above-mentioned structure of norm and by corresponding convergence of series (2.29) for arbitrary \(\vec{F} (\theta) \in D_{cl}^k \) we assume to be valid the following expansion in the functional Taylor’s series in powers of \(\delta \Gamma^p (\theta) - \Gamma^p (\theta)\), \(\delta \vec{\partial}^p (\theta) = \vec{\partial}^p (\theta) - \vec{\partial}^p (\theta)\) in a some neighbourhood (possibly in the whole \(T_{odd}T_{odd}M_{cl} \times \{\theta\}\)) of \(\Gamma^p (\theta), \vec{\partial}^p (\theta)\) (write this fact only for \(\vec{A}^*_p (\theta), \vec{\partial}^* (\theta)\)

\[
\vec{F} (\Gamma (\theta), \vec{\partial} (\theta), \theta) = \sum_{m,l=0}^{\infty} \frac{1}{m!l!} \delta \vec{A}^*_l (\theta) \delta \vec{A}^*_m (\theta) \vec{F} (\theta) \left( A (\theta), \vec{A}^* (\theta), \vec{\partial}^* (\theta), \vec{\partial}^* (\theta), \theta \right) \equiv \sum_{m,l=0}^{\infty} \frac{1}{m!l!} \delta \vec{A}^*_l (\theta) \delta \vec{A}^*_m (\theta) \left( \prod_{p=0}^{m-1} \partial_{A^*_p (\theta)} \prod_{q=0}^{l-1} \partial_{\vec{A}^*_q (\theta)} \vec{F} (\theta) \left( A (\theta), \vec{A}^* (\theta), \vec{\partial}^* (\theta), \vec{\partial}^* (\theta), \theta \right) \right) \tag{2.32}
\]

For coefficient’s superfuns \(\vec{F}^{(m)} (\theta), \vec{\partial} (\theta, \theta) \) the properties (2.31) and expansion in Taylor’s series in powers of \(\delta A^i (\theta), \delta \vec{A}^* (\theta)\) hold [1]. Notations of the form (2.30) have been made use in (2.32) and the left partial superfuns derivatives with respect to \(\vec{A}^*_p (\theta), \vec{\partial}^*_p (\theta)\) for fixed \(\theta\) were introduced nontrivially acting on the \(\vec{F} (\theta) \in D_{cl}^k \) only at coinciding \(\theta\). Their nonzero action on \(\vec{A}^*_p (\theta)\) and \(\vec{\partial}^*_p (\theta)\) reads as follows

\[
\frac{\partial \vec{A}^*_p (\theta)}{\partial \vec{A}^*_p (\theta)} = \delta^*_p, \quad \frac{\partial \vec{\partial}^*_p (\theta)}{\partial \vec{\partial}^*_p (\theta)} = \delta^*_p \tag{2.33}
\]
At last, according to Ref. [1] one can use the combination of decompositions (2.29) and (2.32) regarding, for instance, $\mathcal{F}(\theta)$ is expanded with respect to $\mathcal{A}^*_i(\theta), \tilde{\mathcal{F}}^p(\theta)$ as a polynomial and in Taylor’s series in powers of $\delta \mathcal{A}^i(\theta)$ in a neighbourhood of $\mathcal{A}^*_b(\theta)$. It should be noted that for local superfunctions the series (2.32) (or mentioned combination of decompositions (2.29), (2.32)) pass into finite sum. The action of projector’s systems $P_a(\theta), a = 0, 1$ and $\{P_b(\theta), U(\theta), V(\theta)\}$, $b = 0, 1$ are naturally extended onto $D^k_{cl}$ decomposing the last set in direct sum

$$D^k_{cl} = C^k(P_0(T_{odd}(T^*_a\mathcal{M}_{cl}))) \oplus C^k(P_1(T_{odd}(T^*_a\mathcal{M}_{cl}))) \oplus C^k(P_0(T_{odd}(T^*_a\mathcal{M}_{cl}))) \times \{\theta\} \equiv 0.0D^k_{cl} \oplus 1.0D^k_{cl} \oplus 0.1D^k_{cl}. \quad (2.34)$$

By invariant subsuperspaces in $D^k_{cl}$ with respect to action of projectors $\{\tilde{P}_a(\theta), U(\theta), V(\theta)\}$ are the following ones

$$\tilde{P}_0(\theta)D^k_{cl} = 0.0D^k_{cl}, \quad \tilde{P}_1(\theta)D^k_{cl} = 0.1D^k_{cl}, \quad (U + V)(\theta)D^k_{cl} = 1.0D^k_{cl}, \quad (2.35)$$

from which the only $0.0D^k_{cl}$ is the nontrivial subsuperalgebra, whereas $1.0D^k_{cl}$ and $0.1D^k_{cl}$ are nilpotent ideals in $D^k_{cl}$. To decompose $1.0D^k_{cl}$ into a direct sum of subsuperalgebras it is necessary to make use in explicit form the superfield-superantifield structure of the coordinates $\Gamma^p(\theta) = (\mathcal{A}^i(\theta), \mathcal{A}^*_i(\theta))$ while in decomposition (2.35) that polarization was not taken into account. Preliminarily indicate the corresponding definite gradings for superfields $\Gamma^p(\theta), \tilde{\Gamma}^p(\theta)$ in ignoring of the superfield-superantifield structure

$$(\varepsilon_P, \varepsilon_J, \varepsilon)\Gamma^p(\theta) = (\varepsilon_P(\tilde{\Gamma}^p(\theta)) + 1, \varepsilon_J(\tilde{\Gamma}^p(\theta)), \varepsilon(\tilde{\Gamma}^p(\theta)) + 1) = (\varepsilon_P(\Gamma^p), \varepsilon_J(\Gamma^p), \varepsilon_P). \quad (2.36)$$

Projector $\mathcal{W}(\theta)$ with respect to coordinates $\Gamma^p(\theta)$

$$\mathcal{W}(\theta) = (U + V)(\theta) \quad (2.37)$$

is given to be covariant, and $1.0D^k_{cl}$ with respect to action of projectors $\{\tilde{P}_a(\theta), \mathcal{W}(\theta)\}$ are not reduced further. Choosing the coordinates on $T^*_a\mathcal{M}_{cl}$ in the form $(\mathcal{A}^i(\theta), \mathcal{A}^*_i(\theta))$ we get

$$1.0D^k_{cl} = 1.0.0D^k_{cl} \oplus 0.1.0D^k_{cl} \equiv U(\theta)D^k_{cl} \oplus V(\theta)D^k_{cl}. \quad (2.38)$$

System of even, with respect to $\varepsilon_P, \varepsilon_J, \varepsilon$ gradings, projectors $\{\tilde{P}_a(\theta), U(\theta), V(\theta)\}$ or $\{\tilde{P}_a(\theta), \mathcal{W}(\theta)\}$ are described by relations being analogous to one $\{\tilde{P}_a(\theta), U(\theta)\}$ in Ref. [1] with multiplication table and completeness relation respectively

| $\tilde{P}_a$ | $U$ | $V$ | $\mathcal{W}$ |
|--------------|-----|-----|-------------|
| $P_b$        | $\delta_{ab}P_b$ | 0   | 0           |
| $U$          | 0   | $U$ | $0$         |
| $V$          | 0   | 0   | $V$         |
| $\mathcal{W}$ | 0   | $U$ | $V$         |

Systems $P_a(\theta)$ and $\{\tilde{P}_a(\theta), \mathcal{W}(\theta)\}$ on $T^*_a\mathcal{M}_{cl}$ are connected by means of relations

$$P_0(\theta) = \tilde{P}_0(\theta), \quad \tilde{P}_1(\theta) + \mathcal{W}(\theta) = P_1(\theta). \quad (2.40)$$

Projectors $\tilde{P}_1(\theta), U(\theta), V(\theta)$ are derivations on $D^k_{cl}$, whereas for $\tilde{P}_0(\theta), P_0(\theta)$ the following rule in acting on the product of any $\mathcal{F}(\theta), \mathcal{J}(\theta) \in D^k_{cl}$ is valid

$$D_0(\theta)(\mathcal{F}(\theta) \cdot \mathcal{J}(\theta)) = (D_0(\theta)\mathcal{F}(\theta))(D_0(\theta)\mathcal{J}(\theta)), \quad D_0 \in \{P_0, \tilde{P}_0\}. \quad (2.41)$$

$$\mathcal{W}(\theta) = (U + V)(\theta) \quad (2.37)$$

is given to be covariant, and $1.0D^k_{cl}$ with respect to action of projectors $\{\tilde{P}_a(\theta), \mathcal{W}(\theta)\}$ are not reduced further. Choosing the coordinates on $T^*_a\mathcal{M}_{cl}$ in the form $(\mathcal{A}^i(\theta), \mathcal{A}^*_i(\theta))$ we get

$$1.0D^k_{cl} = 1.0.0D^k_{cl} \oplus 0.1.0D^k_{cl} \equiv U(\theta)D^k_{cl} \oplus V(\theta)D^k_{cl}. \quad (2.38)$$

System of even, with respect to $\varepsilon_P, \varepsilon_J, \varepsilon$ gradings, projectors $\{\tilde{P}_a(\theta), U(\theta), V(\theta)\}$ or $\{\tilde{P}_a(\theta), \mathcal{W}(\theta)\}$ are described by relations being analogous to one $\{\tilde{P}_a(\theta), U(\theta)\}$ in Ref. [1] with multiplication table and completeness relation respectively

| $\tilde{P}_a$ | $U$ | $V$ | $\mathcal{W}$ |
|--------------|-----|-----|-------------|
| $P_b$        | $\delta_{ab}P_b$ | 0   | 0           |
| $U$          | 0   | $U$ | $0$         |
| $V$          | 0   | 0   | $V$         |
| $\mathcal{W}$ | 0   | $U$ | $V$         |

Systems $P_a(\theta)$ and $\{\tilde{P}_a(\theta), \mathcal{W}(\theta)\}$ on $T^*_a\mathcal{M}_{cl}$ are connected by means of relations

$$P_0(\theta) = \tilde{P}_0(\theta), \quad \tilde{P}_1(\theta) + \mathcal{W}(\theta) = P_1(\theta). \quad (2.40)$$

Projectors $\tilde{P}_1(\theta), U(\theta), V(\theta)$ are derivations on $D^k_{cl}$, whereas for $\tilde{P}_0(\theta), P_0(\theta)$ the following rule in acting on the product of any $\mathcal{F}(\theta), \mathcal{J}(\theta) \in D^k_{cl}$ is valid

$$D_0(\theta)(\mathcal{F}(\theta) \cdot \mathcal{J}(\theta)) = (D_0(\theta)\mathcal{F}(\theta))(D_0(\theta)\mathcal{J}(\theta)), \quad D_0 \in \{P_0, \tilde{P}_0\}. \quad (2.41)$$
Any $\mathcal{F}(\theta) \in D_{cl}^k$ is decomposed onto the component functions (not being by elements from a superspace of superfield representation $T$)

$$
\mathcal{F}(\Gamma(\theta), \tilde{\Gamma}(\theta), \theta) = \tilde{P}_0(\theta)\mathcal{F}(\theta) + U(\theta)\mathcal{F}(\theta) + V(\theta)\mathcal{F}(\theta) + \tilde{P}_1(\theta)\mathcal{F}(\theta) \equiv \mathcal{F}(P_0(\Gamma(\theta), \tilde{\Gamma}(\theta), 0) + P_1(\theta)\mathcal{F}(\mathcal{A}(\theta), P_0\mathcal{A}^*(\theta, \tilde{\Gamma}(\theta), 0) + P_1(\theta)\mathcal{F}(P_0\mathcal{A}(\theta), \mathcal{A}^*(\theta, \tilde{\Gamma}(\theta), 0) + \mathcal{F}(P_0(\Gamma(\theta), \tilde{\Gamma}(\theta), \theta)
$$

(2.42)

from subsuperalgebras $0,0D^k_{cl}, 1,0D^k_{cl}, 0,1D^k_{cl}, 0,1D^k_{cl}$ respectively.

The matrix realization of elements from $D_{cl}^k$ (as the vector superspace elements) in the form of column-vector from 4 components

$$
\mathcal{F}(\theta) \mapsto (\tilde{P}_0\mathcal{F}, U\mathcal{F}, V\mathcal{F}, \tilde{P}_1\mathcal{F}) \equiv (\mathcal{F}^{0,0}, \mathcal{F}^{1,0}, \mathcal{F}^{0,1}, \mathcal{F}^{0,1})^T,
$$

(2.43)

permits to represent the projectors in the form of $4 \times 4$ matrices

$$
\tilde{P}_a(\theta) = \text{diag}(\delta_{a0}, 0, 0, \delta_{a1}), \quad U(\theta) = \text{diag}(0, 1, 0, 0), \quad V(\theta) = \text{diag}(0, 0, 1, 0).
$$

(2.44)

The analytic representation for $\mathcal{F}(\theta)$ by means of relations (2.32), (2.42) leads to validity of the projectors realization as the 1st order differential operators under their action on $D_{cl}^k$

$$
\tilde{P}_0(\theta) = 1 - \theta \frac{\partial}{\partial \theta}, \quad U(\theta) = P_1(\theta)A^i(\theta) \frac{\partial_i}{\partial A^i(\theta)} - V(\theta) = P_1(\theta)A^i(\theta) \frac{\partial_i}{\partial A^i(\theta)} + \tilde{P}_1(\theta) = \theta \frac{\partial}{\partial \theta}, \quad W(\theta) = P_1(\theta)\Gamma^p(\theta) \frac{\partial_i}{\partial \Gamma^p(\theta)^2} + \frac{\partial_i}{\partial \Gamma^p(\theta)} = \left( \frac{\partial_i}{\partial A^i(\theta)}, \frac{\partial_i}{\partial A^i(\theta)} \right).
$$

(2.45)

The left partial superfield derivative with respect to superfield $\Gamma^p(\theta)$ for fixed $\theta$ is introduced in (2.45). The connection between derivatives $\frac{d}{d\theta}$ and $\frac{\partial}{\partial \theta}$ under their action on the elements from $D_{cl}^k$ is established by the formula (being by continuation of the corresponding one given on $C^k(T_{odd}\mathcal{M}_{cl} \times \{\theta\})$ [1])

$$
\frac{d}{d\theta} = \frac{\partial}{\partial \theta} + \Gamma^p(\theta)P_0(\theta)\frac{\partial_i}{\partial \Gamma^p(\theta)} \equiv \frac{\partial}{\partial \theta} + P_0(\theta)\tilde{\omega}(\theta), \quad \tilde{\omega}(\theta) = \left[ \frac{d}{d\theta}, W(\theta) \right]_s.
$$

(2.46)

Class $C_{FH,cl}$ of regular (analytic) over $\mathbb{K}$ superfunctionals on $T_{odd}(T_{odd}\mathcal{M}_{cl}) \times \{\theta\}$ as continuation of the class $C_F$ of superfunctionals on $T_{odd}\mathcal{M}_{cl} \times \{\theta\}$ [1] is defined by the formula

$$
F_{H,cl}[\Gamma] \equiv F_{H,cl}[\mathcal{A}, \mathcal{A}^*] = \int d\theta \mathcal{F}(\Gamma(\theta), \tilde{\Gamma}(\theta), \theta) \equiv \frac{d}{d\theta} \mathcal{F}(\theta), \quad F_{H,cl}[\Gamma] \in C_{FH,cl}, \mathcal{F}(\theta) \in D_{cl}^k.
$$

(2.47)

It is evident that $F_{H,cl}$ is given with accuracy up to $P_0(\theta)\mathcal{F}(\theta)$ component part of $\mathcal{F}(\theta)$. Since the operator $\frac{d}{d\theta}$ does not lead out $\mathcal{F}(\theta)$ from $D_{cl}^k$, then $F_{H,cl}[\Gamma]$ belongs to $D_{cl}^k$. Besides $F_{H,cl}[\Gamma]$ is a scalar with respect to action of $\tilde{T}_r$ operators, being by restriction of representation $\tilde{T}$ operators onto $P$, if $\mathcal{F}(\theta)$ is transformed by the rules (2.27) or (2.28).

An analog of the basic lemma of variational calculus [1] is valid in this case, from which it follows the connection for variational superfield derivatives of $F_{H}[\Gamma]$ with respect to $\Gamma^p(\theta)$ with partial superfield ones of its density $\mathcal{F}(\theta) \in D_{cl}^k$ with respect to $\Gamma^p(\theta)$, $\tilde{\Gamma}^p(\theta)$. For instance,

$$
2P_1(\theta)\Gamma^p(\theta) = (P_1(\theta)A^i(\theta), P_1(\theta)A^i(\theta)) \text{ is understood as the indivisible object for partial differentiation with respect to } \theta: \frac{\partial}{\partial \theta}P_1(\theta)\Gamma^p(\theta) = 0, \quad \text{but } \frac{\partial}{\partial \theta}(\theta\Gamma^p(\theta)) = \tilde{\Gamma}^p(\theta).
$$
obtain for the left and right derivatives with respect to superantifields the formulae respectively

\[
\frac{\delta_l F_H[\Gamma]}{\delta \dot{A}_i^s(\theta)} = \frac{\partial_l \mathcal{F}(\theta)}{\partial \dot{A}_i^s(\theta)} + (-1)^{\varepsilon_i} \frac{d}{d\theta} \frac{\partial_r \mathcal{F}(\theta)}{\partial \dot{A}_i^s(\theta)} \equiv \mathcal{L}_i^s(\theta) \mathcal{F}(\theta) ,
\]

\[
\frac{\delta_r F_H[\Gamma]}{\delta \dot{A}_i^s(\theta)} = \left[ \frac{\partial_r \mathcal{F}(\theta)}{\partial \dot{A}_i^s(\theta)} + (-1)^{\varepsilon_i} \frac{d}{d\theta} \left( \frac{\partial_r \mathcal{F}(\theta)}{\delta \dot{A}_i^s(\theta)} \right) \right] \mathcal{F}(\theta) \equiv \mathcal{L}_i^s(\theta) \mathcal{F}(\theta) .
\]

The connection of above left and right derivatives is established by the relationship

\[
\frac{\delta_l F_H[\Gamma]}{\delta \dot{A}_i^s(\theta)} = (-1)^{(\varepsilon_i+1)} \varepsilon_r F_H[\Gamma] \frac{\delta_r F_H[\Gamma]}{\delta \dot{A}_i^s(\theta)} .
\]

Superfield derivatives have the following table of Grassmann parities according to (2.24), (2.36)

| Function | Parity of Argument | Parity of Value | Parity of Term | Parity of Term | Parity of Term |
|----------|--------------------|----------------|--------------|--------------|--------------|
| \varepsilon_p | \varepsilon_i | \varepsilon_i | \varepsilon_j | \varepsilon_j | \varepsilon_j |

The nth superfield variational derivative with respect to superantifields \( \dot{A}_i^s(\theta_1), \ldots, \dot{A}_i^s(\theta_k) \) of superfunctional \( F_H[\Gamma] \) is expressed in terms of partial superfield ones with respect to \( \dot{A}_i^s(\theta), \ldots, \dot{A}_i^s(\theta), \dot{A}_i^s(\theta), \ldots, \dot{A}_i^s(\theta) \) of its density \( \mathcal{F}(\theta) \) by the formula

\[
\left( \prod_{l=0}^{k-1} \frac{\delta_l \mathcal{F}(\theta)}{\delta \dot{A}_{k-l}^s(\theta_{k-l})} \right) F_H[\Gamma] \equiv \frac{\delta^k F_H[\Gamma]}{\delta \dot{A}_i^s(\theta_1) \ldots \delta \dot{A}_i^s(\theta_k)} \equiv F^{(k)}_H[\Gamma, \theta_k] = \left( \prod_{l=0}^{k-2} \mathcal{L}_i^{s_{k-l}}(\theta_k) \delta(\theta_k - \theta_{k-l-1}) \right) \mathcal{L}_i^{s_{k-1}}(\theta_k) \mathcal{F}(\Gamma(\theta_k), \tilde{\Gamma}(\theta_k), \theta_k) ,
\]

\[
\delta(\theta' - \theta) = \theta' - \theta , \int d\theta' \delta(\theta' - \theta) y(\theta') = y(\theta) , \tilde{\theta}_k \equiv \theta_1, \ldots, \theta_k .
\]

By means of the last relation the superfield variational derivative with respect to \( \dot{A}_i^s(\theta) \) of arbitrary superfunction \( \mathcal{F}(\Gamma(\theta'), \tilde{\Gamma}(\theta'), \theta') \) is defined for necessarily(!) not coinciding values of \( \theta \) and \( \theta' \). For its calculation from arbitrary \( \mathcal{F}(\Gamma(\theta'), \tilde{\Gamma}(\theta'), \theta'; \tilde{\theta}_k) \in \mathcal{D}_{\theta_1} \times \{ \tilde{\theta}_k \} \) it is sufficient to know the values of the following derivatives arising from (2.33), (2.48), (2.51a)

\[
\frac{\delta_l \dot{A}_i^s(\theta')}{\delta \dot{A}_i^s(\theta)} = (-1)^{\varepsilon_j} \delta_j(\theta - \theta') , \frac{\delta_l \left( \frac{d \dot{A}_i^s(\theta')}{d\theta'} \right)}{\delta \dot{A}_i^s(\theta)} = \delta_j^s .
\]

Remark: Further all derivatives with respect to \( \dot{A}_i^s(\theta), \dot{A}_i^s(\theta) \) are considered (under omission) by the left and the right ones with respect to these superantifields are labeled by the sign "r". 
III Basic Statements of the Hamiltonian Formulation for GSTF

Define on $T_{\text{odd}}(T_{\text{odd}}^* M) \times \{\theta\}$ the superfunction $S_{H,L} \in D^k_{cl}, \; k \leq \infty$ by the relation

$$S_{H,L}(\theta) = S_{H,L}(A(\theta), \dot{A}(\theta), A^*(\theta), \theta) = \dot{A}^*(\theta) - S_L(A(\theta), \dot{A}(\theta), \theta) = \text{extr} S_{H,L}(\theta, \dot{A}(\theta), A^*(\theta), \theta), \quad (3.1a)$$

Having considered the problem on extremum for $S_{H,L}(\theta)$ with respect to superfields $\dot{A}^i(\theta)$ we arrive to standard Legendre transform of $S_L(\theta)$ with respect to $\dot{A}^i(\theta)$ being defined by the relationships

$$\dot{A}^*_i(\theta) = \frac{\partial S_L(A(\theta), \dot{A}(\theta), \theta)}{\partial \dot{A}^i(\theta)}, \quad S_H(A(\theta), A^*(\theta), \theta) = \text{extr} S_{H,L}(A(\theta), \dot{A}(\theta), A^*(\theta), \theta), \quad (3.2)$$

expressing by virtue of (2.21) $\dot{A}^i(\theta)$ in terms of generalized ”momenta” – superantifields $A^*_i(\theta)$: $\dot{A}^i(\theta) = \dot{A}^i(A(\theta), A^*(\theta), \theta)$. It follows from expressions (3.1), (3.2) the consequences for Legendre transform

$$S_H(A(\theta), A^*(\theta), \theta) = \dot{A}^i(A(\theta), A^*(\theta), \theta)A^*_i(\theta) - S_L(A(\theta), \dot{A}(\theta), A^*(\theta), \theta), \quad (3.3)$$

$$\frac{d_r A^*_i(\theta)}{d\theta} = \frac{\partial S_H(A(\theta), A^*(\theta), \theta)}{\partial A^*_i(\theta)}, \quad \frac{d_r A^i(\theta)}{d\theta} = \frac{\partial S_H(A(\theta), A^*(\theta), \theta)}{\partial A^i(\theta)}, \quad \Theta_{\epsilon_1}(A(\theta), \dot{A}(\theta), \theta)_{|\dot{A}(\theta)=\dot{A}(A(\theta), A^*(\theta), \theta)}^{\theta^H} \equiv \Theta^H_i(A(\theta), A^*(\theta), \theta) = 0 \quad (3.5c)$$

Definition: Call the system of the 1st order on $\theta$ $3n$ ODE, $2n$ from which are in NF

$$\frac{d_r A^*_i(\theta)}{d\theta} = (-1)^{\epsilon_{i+1}} \frac{\partial S_H(A(\theta), A^*(\theta), \theta)}{\partial A^*_i(\theta)}, \quad (3.5a)$$

$$\frac{d_r A^i(\theta)}{d\theta} = \frac{\partial S_H(A(\theta), A^*(\theta), \theta)}{\partial A^i(\theta)}, \quad (3.5b)$$

the generalized Hamiltonian system (GHS) being defined by superfunction $S_H(\theta)$ (3.3) and the subsystem of $2n$ ODE (3.5a,b) is called the Hamiltonian system (HS).

DCLF (2.3b) expressed in terms of $T_{\text{odd}}^* M_{cl}$ coordinates in Eqs.(3.5c) are according to definition the system of $n$ algebraic (i.e. of the 0 order with respect to operators of differentiation on $\theta$) equations on $2n$ unknowns $\Gamma^p(\theta)$ for any values of $\theta$. Call the subsystem (3.5c) the generalized constraints in Hamiltonian formalism (GCHF). In the same way just as DCLF for Euler-Lagrange equations restrict the formulation of Cauchy problem in $T_{\text{odd}}^* M_{cl} \times \{\theta\}$ [1] the GCHF have the analogous meaning with respect to HS (3.5a,b).

Under fulfilment of the condition

$$\deg_{A^*_i(\theta)} \Theta^H_i(\Gamma(\theta), \theta) = 0 \quad (3.6)$$

let us call $\Theta^H_i(A(\theta), \theta)$ the holonomic constraints in Hamiltonian formalism (HCHF).

The integer-valued functions $\deg_{A^*_i(\theta)}$ written in (3.6) appear according to Ref.[1] by functions of degree on $D^k_{cl}$ with respect to generating elements $A^*_i(\theta)$. In addition we assume the
following functions given on $D^k_{cl}$

$$
\min \deg_{C(\theta)}, \deg_{C(\theta)}: D^k_{cl} \to \mathbb{N}_0,
$$

$$
C(\theta) \in \{ \mathcal{A}(\theta), \tilde{\mathcal{A}}(\theta), \mathcal{A}^*(\theta), \tilde{\mathcal{A}}^*(\theta), \Gamma(\theta), \tilde{\Gamma}(\theta), \mathcal{A}(\theta)\tilde{\mathcal{A}}(\theta), \mathcal{A}^*(\theta)\tilde{\mathcal{A}}^*(\theta), \Gamma(\theta)\tilde{\Gamma}(\theta), \ldots \} \quad (3.7)
$$

and called the least degree and degree of corresponding superfunctions with respect to $C(\theta)$.

To be solvable GHS (3.5) must satisfy to solvability conditions of the type (2.12) written additionally to GHS. Therefore the solvable GHS is equivalent to the system of the 1st order on $\theta$ 5n ODE consisting, in addition to (3.5), of the subsystem

$$
\frac{d_r}{d\theta} \left( \frac{\partial S_H(\theta)}{\partial \mathcal{A}^*(\theta)} \right) = 0, \quad \frac{d_r}{d\theta} \left( \frac{\partial S_H(\theta)}{\partial \tilde{\mathcal{A}}^*(\theta)} (-1)^{\varepsilon_1+1} \right) = 0 ,
$$

being valid by virtue of identity $\mathcal{A}^*(\theta) = \tilde{\mathcal{A}}^*(\theta) = 0$.

At last note that $S_H(\theta)$ (3.3) is a scalar with respect to action of the superfield representation $\tilde{T}_{|j}$ operators and is transformed with respect to $\tilde{T}_{|p}$ ones by the rule (2.26), (2.28)

$$
S_H \left( \mathcal{A}^*(\theta), \mathcal{A}''(\theta), \theta \right) = S_H \left( T(h^{-1}(\mu))\mathcal{A}(\theta), T(h^{-1}(\mu))\mathcal{A}^*(\theta), T(h^{-1}(\mu))\theta \right) =
$$

$$
S_H \left( \mathcal{A}(\theta - \mu), \mathcal{A}^*(\theta - \mu), \theta - \mu \right) \equiv S_H(\theta') , \quad (3.9)
$$

so that

$$
\delta S_H(\theta) = S_H(\theta') - S_H(\theta) = -\mu \frac{dS_H(\theta)}{d\theta} = -\mu \left[ \frac{\partial}{\partial \theta} + P_0(\theta)\tilde{W}(\theta) \right] S_H(\theta) . \quad (3.10)
$$

**Statement 3.1** (equivalence of GHS (3.5) and LS (2.3))

GHS (3.5) is equivalent to system of the 2nd order on $\theta$ 2n ODE (2.3) under following admissible formulation of the initial conditions [1] (Cauchy problem) for GHS in $T_{odd}^{*r}\mathcal{M}_{cl} \times \{ \theta \}$ for $\theta = 0$

determining the integral curve $\mathcal{T}^p(\theta)$ for GHS

$$
P_0(\theta)\mathcal{A}^*_i(\theta) = \mathcal{A}^*_i(\theta)_{|\theta=0} = \left( P_0(\theta) \frac{\partial S_L(\theta)}{\partial \mathcal{A}^*(\theta)} \right)_{|\{ \mathcal{A}^*(\theta) = \mathcal{A}^*_i(0), \tilde{\mathcal{A}}^*(\theta) = \tilde{\mathcal{A}}^*_i(0) \}} \equiv \tilde{\mathcal{A}}^*_i(0) , \quad (3.11a)
$$

$$
P_0(\theta)\mathcal{A}^i(\theta) = \mathcal{A}^i(\theta)_{|\theta=0} = \mathcal{A}^i(0) , \quad (3.11b)
$$

$$
\Theta^H_i(\Gamma(\theta), \theta) = 0 . \quad (3.11c)
$$

In this case the Cauchy problem for LS (2.3) in $T_{odd}^{*r}\mathcal{M}_{cl} \times \{ \theta \}$ means the integral curve for Eqs.(2.3) $\mathcal{A}^i(\theta)$ in $T_{odd}^{*r}\mathcal{M}_{cl} \times \{ \theta \}$ for $\theta = 0$ goes through point $\left( \mathcal{A}^i(0), \tilde{\mathcal{A}}^i(0) \right)$ [1]

$$
\mathcal{A}^i(\theta)_{|\theta=0} = \mathcal{A}^i(0), \quad \mathcal{A}^i(\theta)_{|\theta=0} = \tilde{\mathcal{A}}^i(0) \quad (3.12)
$$

and must satisfy to DCLF: $\Theta^H_i(\mathcal{A}^i(\theta), \tilde{\mathcal{A}}^i(\theta), \theta) = 0$.

Proof of the Statement 3.1 will be developed later in fulfilling of the condition (2.21).

**Remarks:**

1) The question on formulation of the Cauchy problem is more complicated for field theory since both GHS (3.5) and LS (2.3) with respect to indices of supergroup $J$ representation have the nontrivial (hidden in this work in the condensed notations) functional structure;
(2) system (3.5) can be obtained from problem on conditional extremum for superfunctional

$$Z_H[\Gamma] \equiv Z_H[A, A^*] = \int d\theta \left( \dot{A}^i(\theta) A^*_i(\theta) - S_H(A(\theta), A^*(\theta), \theta) \right), \quad Z_H[\Gamma] \in C_{FH,cl}$$

(3.13)

in fulfilling of GCHF (3.5c), or equivalently from problem on unconditional extremum for the extended superfunctional

$$Z_H^{(1)}[\Gamma, D] = \int d\theta \left( \dot{A}^i(\theta) A^*_i(\theta) - S_H^{(1)}(\Gamma(\theta), D(\theta), \theta) \right), \quad (3.14a)$$

$$S_H^{(1)}(\Gamma(\theta), D(\theta), \theta) = S_H(\Gamma(\theta), \theta) - D^i(\theta) \delta_i^H(\Gamma(\theta), \theta) , \quad (3.14b)$$

in setting of the additional superfields $D^i(\theta)$ to be equal to 0 after variation. Superfields $D^i(\theta)$ play the standard role of Lagrange multipliers for GCHF (3.5c) extending $T_{odd}(T_{odd}M_{cl})$, being transformed with respect to representation $T$ and having the same $\varepsilon_p, \varepsilon_j, \varepsilon$ gradings as for $A^i(\theta)$ $(\varepsilon_p, \varepsilon_p, \varepsilon_j) D^i(\theta) = (0, \varepsilon_i, \varepsilon_i))$.

Superfunctional $S_H(\theta)$ is the expression of the integral (2.4a) for Eqs.(2.3) in terms of coordinates on $T_{odd}M_{cl} \times \{\theta\}$

$$S_H(\Gamma(\theta), \theta) = S_E(A(\theta), \dot{A}(\theta), \theta)_{\dot{A}(\theta) = \dot{A}(\theta), A^*(\theta), \theta} . \quad (3.15)$$

The translation of the superfield $\Gamma^p(\theta)$ along $\theta$ on a constant parameter $\mu \in \Lambda_1(\theta)$

$$\delta \mu \Gamma^p(\theta) = \Gamma^p(\theta + \mu) - \Gamma^p(\theta) = \mu \frac{d \Gamma^p(\theta)}{d\theta} \mu$$

(3.16)

for $S_H(\theta)$ has the form (3.10) (further denote $(-\delta S_H(\theta))$ in (3.10) as $\delta \mu S_H(\theta)$). Expression (3.10) on the solution $\bar{\Gamma}^p(\theta)$ of GHS (3.5) is given by the formulae taking representation (2.46) for $\bar{\nabla}(\theta)$ into account

$$\delta \mu S_H(\theta)_{\bar{\nabla}(\theta)} = \mu \left[ \frac{\partial S_H(\bar{\nabla}(\theta), \theta)}{\partial \theta} - P_0(\theta) \left( \frac{\partial S_H(\bar{\nabla}(\theta), \theta)}{\partial \Gamma^p(\theta)} \right) \right] =$$

$$= \mu \left[ \frac{\partial S_H(\bar{\nabla}(\theta), \theta)}{\partial \theta} - P_0(\theta) \left( \frac{\partial S_H(\bar{\nabla}(\theta), \theta)}{\partial \Gamma^p(\theta)} \right) \right] , \quad (3.17)$$

being true on the solutions for HS (3.5a,b) as well.

Vanishing of the expression (3.17) means by virtue of (2.4b), (3.4) the fulfillment on the solutions for GHS (3.5) of the following equation

$$\frac{\partial S_H(\bar{\nabla}(\theta), \theta)}{\partial \theta} + P_0(\theta) \left( \frac{\partial S_H(\bar{\nabla}(\theta), \theta)}{\partial \Gamma^p(\theta)} \right) = 0 . \quad (3.18)$$

Eq.(3.18) under validity of the condition (2.5) with regard for (3.4) is reduced to the system

$$\bar{P}_i(\theta) S_H(\Gamma(\theta), \theta) = 0 \iff S_H(\Gamma(\theta), \theta) = S_H(\Gamma(\theta), 0) \equiv S_H(\Gamma(\theta)) , \quad (3.19a)$$

$$\left( S_H(\bar{\nabla}(\theta)), S_H(\bar{\nabla}(\theta)) \right)_\theta = 0 . \quad (3.19b)$$

By sufficient condition for Eq.(3.19b) resolution is in force of (2.7) the fulfillment on the solutions for GHS of the following system

$$\frac{\partial S_H(\Gamma(\theta))}{\partial A_{A_1}(\theta)}_{\bar{\nabla}(\theta)} = 0, \quad \frac{\partial S_H(\Gamma(\theta))}{\partial A_{A_2}(\theta)}_{\bar{\nabla}(\theta)} = 0, \quad A_1 = 1, \ldots, n_1, \quad A_2 = n_1 + 1, \ldots, n . \quad (3.20)$$
With join of the superfields \((-A^*_A(\theta), A^{A_2}(\theta))\) into uniform superfield \(\varphi^a(\theta)\) (being transformed with respect to a some subrepresentation \(T_\varphi\) from \(T \oplus T^*\) now without definite \(\varepsilon_p\) parity the system (3.20) will be written in the form

\[
\frac{\partial S_H(\Gamma(\theta))}{\partial \varphi^a(\theta)}|_{\bar{\Gamma}(\theta)} = 0, \quad \Gamma^p(\theta) = (\varphi^a(\theta), \varphi^*_a(\theta)) = ((-A^*_A(\theta), A^{A_2}(\theta)), (A^{A_1}(\theta), A^*_A(\theta))).
\] (3.21)

System (3.21) according to (2.8) and (3.4) takes the form for \(n_1 = 0\)

\[
\frac{\partial S_H(\Gamma(\theta))}{\partial A^i(\theta)}|_{\bar{\Gamma}(\theta)} = 0.
\] (3.22)

The designation for odd Poisson bracket (antibracket) \((,)_\theta\) as a bilinear differential mapping on \(D^k_{cl}\) is introduced in the relations (3.17) – (3.19) having the form in coordinates \(A^i(\theta), A^*_a(\theta)\) and \(\Gamma^p(\theta)\) for arbitrary \(F(\theta), J(\theta) \in D^k_{cl}\)

\[
(F(\theta), J(\theta)) = \begin{pmatrix}
\frac{\partial F(\theta)}{\partial A^i(\theta)} & \frac{\partial J(\theta)}{\partial A^*_a(\theta)} \\
\frac{\partial F(\theta)}{\partial A^*_a(\theta)} & -\frac{1}{(\varepsilon_p + \varepsilon_q + 1)} \frac{\partial J(\theta)}{\partial A^i(\theta)}
\end{pmatrix} = \begin{pmatrix}
\omega^{pq}(\theta) & \omega^{pq}(\theta) \\
\omega^{pq}(\theta) & -\frac{1}{(\varepsilon_p + \varepsilon_q + 1)} \omega^{pq}(\theta)
\end{pmatrix},
\] (3.23)

where

\[
\omega^{pq}(\theta) = (\Gamma^p(\theta), \Gamma^q(\theta))_\theta = P_0(\theta)\omega^{pq}(\theta), \quad \omega^{pq}(\theta) = -(\varepsilon_p + \varepsilon_q + 1)\omega^{pq}(\theta), \quad \|\omega^{pq}(\theta)\| = \begin{pmatrix} 0_n & 1_n \\ -1_n & 0_n \end{pmatrix}, \quad (\varepsilon_p, \varepsilon_J, \varepsilon)\omega^{pq}(\theta) = (\varepsilon_p(\Gamma^p) + \varepsilon_p(\Gamma^q) + 1, \varepsilon_J(\Gamma^p) + \varepsilon_J(\Gamma^q), \varepsilon_p + \varepsilon_q + 1).
\] (3.24)

In the 3rd equality in (3.23) it is assumed that \(\mathcal{F}(\theta), \mathcal{J}(\theta)\) are to be homogeneous with respect to \(\varepsilon\) grading. Properties of \(\varepsilon_p, \varepsilon_J, \varepsilon\) parities for antibracket read as follows

\[
(\varepsilon_p, \varepsilon_J, \varepsilon)(\mathcal{F}(\theta), \mathcal{J}(\theta))_\theta = (\varepsilon_p(\mathcal{F}) + \varepsilon_p(\mathcal{J}) + 1, \varepsilon_J(\mathcal{F}) + \varepsilon_J(\mathcal{J}), \varepsilon(\mathcal{F}) + \varepsilon(\mathcal{J}) + 1).
\] (3.25)

Under equivalent parametrization for \(T^*_{odd}M_{cl}\) by coordinates \(\Gamma^p(\theta) = (\varphi^a(\theta), \varphi^*_a(\theta))\) (3.21) the antibracket (3.23) takes the form

\[
(F(\theta), J(\theta))^{(\varphi, \varphi^*)}_\theta = \begin{pmatrix}
\frac{\partial F(\theta)}{\partial \varphi^a(\theta)} & \frac{\partial J(\theta)}{\partial \varphi^*_a(\theta)} \\
\frac{\partial F(\theta)}{\partial \varphi^*_a(\theta)} & -\frac{1}{(\varepsilon_p + \varepsilon_q + 1)} \frac{\partial J(\theta)}{\partial \varphi^a(\theta)}
\end{pmatrix} = \begin{pmatrix} \varepsilon_p(\varphi^a, \varphi^*_a) = \begin{cases} (1, 0), & a = 1, \ldots, n_1 \\ (0, 1), & a = n_1 + 1, \ldots, n \end{cases} \end{pmatrix}.
\] (3.26a, 3.26b)

Index \((\varphi, \varphi^*)\) on the antibracket (3.26a) denotes the local coordinates with respect to which one is calculated. By omission we suppose that \((F(\theta), J(\theta))_\theta\) is calculated in the initial coordinates \((A^i(\theta), A^*_a(\theta))\). Antibracket satisfies to standard properties of an generalized antisymmetry, Leibnitz rule and Jacobi identity for arbitrary \(\mathcal{F}(\theta), \mathcal{J}(\theta), K(\theta) \in D^k_{cl}\) with definite \(\varepsilon\) parity

\[
(F(\theta), J(\theta))_\theta = \varepsilon(\varepsilon_p + \varepsilon_q + 1) (J(\theta), F(\theta)),
\] (3.27a)

\[
(F(\theta) \cdot J(\theta), K(\theta))_\theta = F(\theta)(J(\theta), K(\theta))_\theta + (-1)^{\varepsilon(\varepsilon_p + \varepsilon_q + 1)} (F(\theta), K(\theta))_\theta J(\theta) + ((F(\theta), J(\theta))_\theta) K(\theta) - \varepsilon_p(\varepsilon_p + \varepsilon_q + 1) + \text{cycl. perm.}(F(\theta), J(\theta), K(\theta))_\theta = 0.
\] (3.27b, 3.27c)

Summarizing the developed investigation of the properties for \(S_H(\Gamma(\theta), \theta)\) let us formulate the results in the form of
Statement 3.2
In order to the superfunction $S_H(\Gamma(\theta))$ not explicitly depending upon $\theta$ would be by integral for GHS (3.5) it is sufficient to fulfill of the following system of equations (being algebraic with respect to $\theta$ on solutions for GHS)

$$(\varphi^*_a(\theta), S_H(\Gamma(\theta)))_{\theta|\Gamma(\theta)} = 0^3.$$ 

(3.28)

Remarks:
1) System (3.28) or (3.21) considered for any values of superfields $\Gamma^p(\theta)$ may be interpreted as a consequence from Noether’s theorem of the fact that translation transformation on a constant superfield $d^n(\theta)$: $\varphi^a(\theta) \rightarrow \varphi^a(\theta) + d^n(\theta) \langle \varepsilon(d^n) = \varepsilon(\varphi^a) \rangle$ is a symmetry transformation for $S_H(\Gamma(\theta))$ and therefore in this case the relation is true

$$S_H(\Gamma(\theta)) = S_H(\varphi^a(\theta)).$$

(3.29)

Then the set of superantifields $\varphi^*_a(\theta)$ is conserved on solutions for GHS

$$\varphi^*_a(\theta)|_{\Gamma(\theta)} = 0;$$

(3.30)

2) Statement 3.2 in fact permits one to claim that $S_H(\Gamma(\theta))$ appears by the integral for only Hamiltonian part (3.5c) of the GHS under conditions (3.28).

Thus $S_H(\Gamma(\theta))$ in fulfilling of (3.19a) is the integral of the 1st order on $\theta$ system 4n ODE

$$\frac{d}{d\theta} \Gamma^p(\theta) = \omega^{pq}(\theta) \frac{\partial S_H(\Gamma(\theta))}{\partial \Gamma^q(\theta)}, \quad p, q = 1, \ldots, 2n,$$

(3.31a)

$$\Theta^H(\Gamma(\theta)) = 0,$$

(3.31b)

$$\chi^H_a(\Gamma(\theta)) \equiv (\varphi^*_a(\theta), S_H(\Gamma(\theta)))_{\theta} = \left(\frac{d}{d\theta} \Gamma^p(\theta) - (\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta}\right) \lambda_{pa}(\Gamma(\theta)),$$

(3.31c)

with arbitrary superfunctions $\lambda_{pa}(\Gamma(\theta)) \in C^k \equiv C^k(T^*_{odd}M_{cl})$ satisfying to the property

$$\varepsilon(\lambda_{pa}(\Gamma(\theta))) = \varepsilon_p + 1 + \varepsilon(\varphi^a), \quad a = (A_1, A_2) = \overline{1, n}.$$ 

(3.32)

Definition: Systems (3.31) and (3.31a,c) are called the extended generalized Hamiltonian system (EGHS) and the extended HS (EHS) respectively. Subsystem (3.31c) we will call the special constraints in Hamiltonian formalism (SCHF) and their analog (2.7) as the special constraints in Lagrangian formalism (SCLF). In fulfilling of the conditions

$$\deg_{\mathcal{A}^*(\theta)} \chi^H_a(\Gamma(\theta)) = 0, \quad \deg_{\mathcal{A}^*_{\bar{\theta}}(\theta)} \left(\chi^H_a(\Gamma(\theta))|_{\mathcal{A}^*_{\bar{\theta}}(\theta)} = \frac{\partial S_H(\Gamma(\theta))}{\partial \mathcal{A}^*_{\bar{\theta}}(\theta)}\right) \equiv \deg_{\mathcal{A}^*_{\bar{\theta}}(\theta)} \chi_a(\mathcal{A}(\theta), \mathcal{A}(\theta)) = 0,$$

(3.33)

call SCHF, SCLF the holonomic SCHF (HSCHF), holonomic SCLF (HSCLF) respectively.

From relations (2.3b) for $\Theta_{\bar{\theta}}(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta)$ with regard of (3.4) it follows the explicit representation for GCHS (3.5c) in terms of $\Gamma(\theta)$ and $S_H(\Gamma(\theta))$

$$\Theta^H_{\bar{\theta}}(\Gamma(\theta), \theta) = (\mathcal{A}^*_\theta(\theta), S_H(\Gamma(\theta)))_{\theta} + (-1)^{\varepsilon_1} \left(S^{-1}_H(\Gamma(\theta))\right)_{ik}(\theta) \left[\frac{\partial}{\partial \theta}(\mathcal{A}^k(\theta), S_H(\Gamma(\theta)))_{\theta}(-1)^{\varepsilon_k} + \right.$$

$$\left.((\mathcal{A}^k(\theta), (\mathcal{A}^*_\theta(\theta), S_H(\Gamma(\theta)))_{\theta}) \left[ (\mathcal{A}^k(\theta), \mathcal{A}^*_\theta(\theta), S_H(\Gamma(\theta)))_{\theta} - \frac{d_{\theta}}{d\theta} \mathcal{A}^k(\theta) \right] (-1)^{\varepsilon_k} - \right.$$

$$\left.(-1)^{\varepsilon_1} (\mathcal{A}^k(\theta), (\mathcal{A}^*_\theta(\theta), S_H(\Gamma(\theta)))_{\theta} \mathcal{A}^k(\theta), S_H(\Gamma(\theta)))_{\theta} \right].$$

(3.34)

Note: the proof of the fact that condition (3.28) is necessary for realization of Eq.(3.19b) appears by more technically complicated problem.
In obtaining of (3.34) the formulae following from calculation rules for composite derivatives and from Legendre transform (3.2) have been made use in addition to above-mentioned ones

\[
\frac{\partial F(A(\theta), A^*(A(\theta), \hat{A}(\theta), \theta), \theta)}{\partial \hat{A}^i(\theta)} = \frac{\partial A_k^i(\theta)}{\partial \hat{A}^i(\theta)} \frac{\partial F(\Gamma(\theta), \theta)}{\partial A_k^i(\theta)}, \quad \forall F(\theta) \in C^{k*} \times \{\theta\},
\]

(3.35a)

\[
\frac{\partial A_k^i(\theta)}{\partial \hat{A}^i(\theta)} = (S_L^m)_{ik}(\theta), \quad (S_H^m)^k_j(\theta) = \frac{\partial}{\partial A_k^i(\theta)} \frac{\partial S_H(\Gamma(\theta), \theta)}{\partial A_j^i(\theta)},
\]

(3.35b)

\[
(S_L^m)_{ik}(\theta), (\hat{A}(\theta), \theta)(S_H^m)^k_j(\theta)(-1)^{\epsilon_j+1} = \delta_{ij}, \quad (S_H^{-1})_{ik}(\theta)(S_H^m)^k_j(\theta) = \delta_{ij},
\]

(3.35c)

\[
(S_H^m)_{ik}(\Gamma(\theta), \theta) = (-1)^{\epsilon_j+1}(S_L^m)_{ik}(\theta), \quad (\hat{A}(\theta), \theta)(S_H^{-1})_{ik}(\theta) = (-1)^{\epsilon_j+1}(\epsilon_k+1)(S_H^m)_{ik}(\theta),
\]

(3.35d)

\[
\delta_F(\theta - \theta, \hat{A}(\theta), \hat{A}(\theta), (\hat{A}(\theta), \hat{A}(\theta), \theta), \theta) \frac{\partial}{\partial \hat{A}^i(\theta)} = \hat{A} \left( \frac{\delta(\theta - \theta)F(\theta, \hat{A}(\theta), \hat{A}(\theta), \theta, \theta)}{\partial \hat{A}^i(\theta)} \right)
\]

(3.35e)

\[
\frac{d}{d\theta_1} \left((S_H^{-1})_{ik}(\theta) \frac{\partial \delta(\theta_1 - \theta)F(\Gamma(\theta_1), \theta_1)}{\partial A_k^i(\theta_1)} \right) = \frac{\partial \delta(\theta_1 - \theta)F(\Gamma(\theta_1), \theta_1)}{\partial \hat{A}^i(\theta_1)} - \frac{\partial \delta(\theta_1 - \theta)F(\Gamma(\theta_1), \theta_1)}{\partial \hat{A}^i(\theta_1)}
\]

(3.35f)

One can show the SCHF (3.31c) under definite choice for \(\lambda_{pq}(\theta)\) are equivalent to the SCLF (2.9c,d) out of solutions for LS (2.3) as well. Analyzing a set of solutions for SCHF (SCLF) simultaneously with other Eqs.(3.31) (2.9) one can pass by means of addition to the subsystem (3.31c), (2.9c,d), in general, a nonlinear combination of Eqs.(3.31a) (2.9b) to equivalent EGHS (3.31) (ELS(2.9)) in which the superfunctions \(\lambda_{pq}(\theta)\) \((\lambda_{1,2}^1(\theta), \lambda_{2A,4}^1(\theta))\) vanish.

Described transformation of EGHS (3.31) (ELS(2.9)) changing the SCHF (3.31c) (SCLF (2.9c,d)) themselves, being considered without other Eqs.(3.31a,b) (2.9a,b), does not change a set of solutions for EGS (ELS) on the whole. Under various choice of the SCHF, SCHF being realized by changing of parameter \(n_1\) and under special structure of the superfunctions \(S_L(\theta), S_H(\theta)\) it is possible to achieve that the Eqs.(2.6), (3.19b) written in the whole spaces \(T_{odd} M_{cl}, T_{odd} M_{cl}^c\) would be by double zeros of solutions for LS (2.3), GHS (3.5) respectively

\[
\hat{A}^i(\theta) \frac{\partial S_L(\theta)}{\partial \hat{A}^i(\theta)} = \left( \frac{\partial S_L(\theta)}{\partial \hat{A}^i(\theta)} \hat{A}^i(\theta) + \hat{A}^j(\theta) \hat{A}^k(\theta) \right) = \left( L_j^i(\theta) S_L(\theta) \right) \left( L_k^i(\theta) S_L(\theta) \right) d^{jk}(\theta),
\]

(3.36a)

\[
(S_H(\Gamma(\theta)), S_H(\Gamma(\theta)))_{\phi^{\omega}} = -2 \frac{\partial S_H(\phi^{\omega})}{\partial \phi^{\omega}} \chi_H^H(\Gamma(\theta)) = \left[ \frac{d_1 \Gamma^\phi(\theta)}{d\theta} - (\Gamma^p(\theta), S_H(\theta))_\theta \right] \frac{d \Gamma^\phi(\theta)}{d\theta} - (\Gamma^q(\theta), S_H(\theta))_\theta \right] d_{pq}(\Gamma(\theta)),
\]

(3.36b)

with arbitrary superfunctions \(d_{pq}(\Gamma(\theta)) \in C^{k*}\) and \(d^{jk}(\theta) \in C^k(T_{odd} M_{cl}), (p, q = \overline{1, 2n}; j, k = \overline{1, n})\) possessing by the properties

\[
\varepsilon(d_{pq}) = \varepsilon_p + \varepsilon_q + 1, \quad \varepsilon(d^{jk}) = \varepsilon_j + \varepsilon_k + 1, \quad d_{pq} = (-1)^{(\varepsilon_p+1)(\varepsilon_q+1)} d_{qp}, d^{jk} = (-1)^{\varepsilon_j \varepsilon_k} d^{kj},
\]

(3.37)
The fact, that the left-hand side of expressions (3.36) are double zeros for solutions of the systems (2.3), (3.5) respectively, means the fulfilment on their solutions of the equalities

\[
\frac{\partial}{\partial \bar{A}^i(\theta)} \left( \frac{\partial}{\partial A^i(\theta)} \left( \bar{A}^i(\theta) \frac{\partial S_{L}(\theta)}{\partial \bar{A}^i(\theta)} \right) \right)_{|L_{ij}(\theta)S_{L}(\theta)=0} = \frac{\partial}{\partial A^i(\theta)} \left( \frac{\partial}{\partial \bar{A}^i(\theta)} \left( \bar{A}^i(\theta) \frac{\partial S_{L}(\theta)}{\partial \bar{A}^i(\theta)} \right) \right)_{|L_{ij}(\theta)S_{L}(\theta)=0} = 0 , \quad (3.38)
\]

\[
\frac{\partial}{\partial \bar{\Gamma}^p(\theta)} (S_{H}(\Gamma(\theta)), S_{H}(\Gamma(\theta)))_{\bar{\Gamma}(\theta)} = 0 . \quad (3.39)
\]

Note that under definite choice for superfunctions \( d_{pq}(\theta), d^{kj}(\theta) \) the equations (3.36) themselves will become by equivalent to each other (in sense of change of variables under Legendre transform) out of solutions for GHS and LS as well.

Being easily obtained from definition of Legendre transform (3.2) the following relation taking Eqs.(3.4), LS (2.3) in the form (2.2) and formulae (3.5c) into account

\[
\Theta^H_i(\Gamma(\theta), \theta) = - \left( \frac{d_{s}A^i(\theta)}{d\theta} + \frac{\partial T_{S}(\theta)}{\partial A^i(\theta)} \right) \quad (3.40)
\]

establishes the coincidence of GCHF (3.5c), therefore of DCLF (2.3b) as well, with Eqs.(3.5a) from HS. On the solutions for the other half of equations from HS (3.5b), in fulfilling of the condition (3.19a), the GCHF (3.34) are defined only in \( T_{odd}^* \mathcal{M}_{cl} \) and have the more simple form

\[
\Theta^H_i(\Gamma(\theta)) = (A^i(\theta), S_{H}(\theta))_{\theta} + (-1)^{e_{i}+e_{j}}(S_{H}^{-1})_{ik}(\theta)(A^k(\theta), (A^j(\theta), S_{H}(\theta))_{\theta}) \times \frac{(A^i(\theta), S_{H}(\theta))_{\theta}}{\theta} . \quad (3.41)
\]

Relationship (3.40) leads to validity of

**Statement 3.3** (on equivalence of GHS and HS)

GHS (3.5) is equivalent to its proper subsystem – HS (3.5a,b).

Representation (3.34) is very useful for proof of implication from GHS (3.5) to LS (2.3) in the Statement 3.1. To this end, it is sufficient to differentiate with respect to \( \theta \) the subsystem (3.5b), next to multiply the obtained expression on the supermatrix elements \( (S_{H}^n)_{ij}(\theta) \) taking the formulae (3.35b,c,d) into account, having got from the left in derived differential consequence for (3.5b) the Eq.(2.3a) in terms of coordinates on \( T_{odd}^* \mathcal{M}_{cl} \) and \( S_{L}(\theta) \).

But from the right with use of the property for Legendre transform to be involutory, relations (3.34) and Eqs.(3.5a,b) themselves the system of DCLF \( \Theta_i(A(\theta), \hat{A}(\theta), \theta) \) will be obtained, which vanishes by virtue of identity (3.5c) and (3.40)

\[
\hat{A}^i(\theta)(S_{H}^n)_{ij}(A(\theta), \hat{A}(\theta), \theta) = \frac{d}{d\theta} \left( \frac{\partial S_{H}(\Gamma(\theta), \theta)}{\partial \hat{A}^i(\theta)} \right) (S_{H}^{-1})_{ij}(\Gamma(\theta), \theta)_{|A^i(\theta) = A^i(A(\theta), \hat{A}(\theta), \theta)} \cdot \quad (3.42a)
\]

\[
\frac{d}{d\theta} \left( \frac{\partial S_{H}(\Gamma(\theta), \theta)}{\partial \hat{A}^i(\theta)} \right) (S_{H}^{-1})_{ij}(\Gamma(\theta), \theta)_{|A^i(\theta) = A^i(A(\theta), \hat{A}(\theta), \theta)} = \Theta^H_j(A(\theta), \hat{A}(\theta), \theta, \theta)(-1)^{e_{i}+e_{j}} \equiv \Theta^H_j(A(\theta), \hat{A}(\theta), \theta, \theta)(-1)^{e_{i}+e_{j}} = 0 . \quad (3.42b)
\]

Thus one can take for granted the Statement 3.1 entirely (inverse implication was developed by direct construction of GHS (3.5) from LS (2.3)). Combining the results of Statements 3.1, 3.3 we arrive to conclusion on equivalence of HS (3.5a,b) and LS (2.3). It means in setting of Cauchy problem for HS the identical fulfilment of the Eqs.(3.11c). In what follows we will be able to ignore the Eqs.(3.11) in this question.

Relations (3.42) mean the solvability conditions for subsystem (3.5b), written in the 1st subsystem in (3.8), are identically realized on solutions for HS (3.5a) in force of the formula
(3.42). GCHF (3.5c) are the differential consequence of (3.5b) coinciding with (3.5a). But solvability of subsystem (3.5a) written in the 2nd subsystem in (3.8) and being equivalent to solvability of (2.3b), that with allowance made for (2.12) is given by the equations

$$\frac{d}{d\theta} \left( \frac{\partial S_L(\theta)}{\partial \mathcal{A}_i(\theta)} \right) = 0,$$

(3.43)

being additional to (2.3), is more complicated problem. From equivalence of HS (3.5a,b) and LS (2.3), Statements 3.1, 3.3 it follows the easily provable validity of the

**Statement 3.4** (on equivalence of EGHS and EHS, of EGHS and ELS)

a) EGHS (3.31) is equivalent to its proper subsystem EHS (3.31a,c);

b) EGHS (3.31) is equivalent to ELS in setting of Cauchy problem for EGHS in $T^\ast_{\text{odd}} \mathcal{M}_{cl} \times \{\theta\}$ for $\theta = 0$ in the form (3.11) and integral curve $\bar{A}_i(\theta)$ must satisfy to the equations

$$\chi_a^H(\bar{\Gamma}(\theta)) = 0, \ a = (A_1, A_2) = 1, \ldots, n.$$  

(3.44)

Cauchy problem for system (2.9) in the Lagrangian formalism in this case is formulated in $T^\ast_{\text{odd}} \mathcal{M}_{cl} \times \{\theta\}$ and corresponding integral curve $\bar{A}_i(\theta)$ satisfies to DCLF and SCLF

$$\Theta_i \left( \bar{A}(\theta), \bar{A}(\theta) \right) = 0, \ \chi_a \left( \bar{A}(\theta), \bar{A}(\theta) \right) = 0.$$  

(3.45)

From the last statement it follows the subsystem (3.43) is formally the solvability conditions for EGHS (3.31) which on the whole in terms of antibracket have the form

$$\frac{d_c}{d\theta} \left( \omega^{pq}(\theta) \frac{\partial_i S_H(\Gamma(\theta))}{\partial \Gamma^p(\theta)} \right) = ((\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta}, S_H(\Gamma(\theta)))_{\theta} = 0,$$

(3.46a)

$$\frac{d_c}{d\theta} \left( \lambda_a^H(\Gamma(\theta)) - \left( \frac{d_c \Gamma^p(\theta)}{d\theta} - (\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta} \right) \lambda_{pa}(\Gamma(\theta)) \right) = 0.$$  

(3.46b)

In its turn, the subsystem (3.46b) is equivalent to the following one on solutions for (3.31)

$$((\varphi^a(\theta), S_H(\Gamma(\theta)))_{\theta}, S_H(\Gamma(\theta)))_{\theta} - ((\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta}, S_H(\Gamma(\theta)))_{\theta} \lambda_{pa}(\Gamma(\theta)) (-1)^{\varepsilon_p + \varepsilon(\varphi^a)}$$

$$- \left[ \frac{d_c \Gamma^p(\theta)}{d\theta} - (\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta} \right] \lambda_{pa}(\Gamma(\theta)), S_H(\Gamma(\theta)))_{\theta} = 0.$$  

(3.47)

From explicit form of Eqs.(3.46a), (3.47) it is obvious that on the solutions for EGHS and on any solution for (3.46a) it follows the identical fulfilment of Eqs.(3.46b). In obtaining of the relations (3.46), (3.47) the identity $\bar{\Gamma}^p(\theta) \equiv 0$ with following formula have been made use

$$\frac{d_c \mathcal{F}(\Gamma(\theta), \theta)}{d\theta} = \left( \frac{\partial_i \mathcal{F}(\Gamma(\theta), \theta)}{\partial \theta} + \left( \mathcal{F}(\Gamma(\theta), \theta), S_H(\Gamma(\theta)))_{\theta} \right) \bar{\Gamma}(\theta) \right)$$  

(3.48)

being valid $\forall \mathcal{F}(\theta) \in C^{k*} \times \{\theta\}$ (and even from $D^b_{\wedge}$). In (3.48) through $\bar{\Gamma}(\theta)$ an integral curve of HS (3.31a) is denoted as in (3.17) taking statement 3.3 into consideration. The fact, that projector $P_0(\theta)$ in (3.48) and, therefore in (3.46), (3.47) was omitted in comparison with (3.18), for now means the tending to obtain, namely, the such expression that is provided by solvability of the relations (3.46a).

**Remarks:**
1) Statements 3.3, 3.4 have been obtained with use of not only Legendre transform but with regard for validity of LS (2.3). One can give rise to doubt the equivalence of GHS and HS, for instance, for following structure of superfunction $S_L(\theta)$ satisfying to (2.5)

$$S_L(\mathcal{A}(\theta), \dot{\mathcal{A}}(\theta)) = T(\dot{\mathcal{A}}(\theta)) - S(\mathcal{A}(\theta)).$$

(3.49)

In that case the relationships are valid

$$\frac{d}{d\theta} \left( \frac{\partial S_L(\theta)}{\partial \dot{\mathcal{A}}^i(\theta)} \right) = \frac{d}{d\theta} \left( \frac{\partial T(\dot{\mathcal{A}}(\theta))}{\partial \dot{\mathcal{A}}^i(\theta)} \right) \equiv 0, \quad \Theta^H_1(A(\theta)) = \frac{\partial S_L(\theta)}{\partial \mathcal{A}^i(\theta)} = -\frac{\partial S(A(\theta))}{\partial \mathcal{A}^i(\theta)} = 0, \quad (3.50)$$

and GHS has the form

$$\frac{d}{d\theta} \mathcal{A}^i(\theta) = -\frac{\partial S_H(A(\theta), \mathcal{A}^*(\theta))}{\partial \mathcal{A}_i^*(\theta)} = \frac{\partial T(\dot{\mathcal{A}}^*(\theta))}{\partial \mathcal{A}_i^*(\theta)}, \quad \frac{d}{d\theta} \dot{\mathcal{A}}^i(\theta) = -\frac{\partial S(A(\theta))}{\partial \mathcal{A}^i(\theta)} \quad (3.51a)$$

$$\Theta^H_1(A(\theta)) = 0. \quad (3.51b)$$

In that example the HS is the subsystem (3.51a) which has its 1st subsystem to be necessarily considered by solvable and then the equivalence of GHS and HS is obvious;

2) if we will ignore the dynamical Eqs.(2.3b) in obtaining of HS (3.51a) by means of Legendre transform (3.3), (3.4) then the 2nd subsystem in (3.51a) has the form

$$\mathcal{A}^*_i(\theta) = 0. \quad (3.52)$$

HS being defined in question by the 1st subsystem in (3.51a) and by Eqs.(3.52) will not be equivalent to HS (3.51a), therefore to GHS (3.51) and to one’s own GHS!

**Statement 3.5** (indication for solvability of EHS (ELS))

In order that EHS (3.31a,c), ELS (2.9) were solvable it is sufficient that corresponding master equations (3.19b), (2.6) would be by double zeros for solutions of HS (3.31a), LS (2.9a,b) respectively.

**Proof:** 1) Hamiltonian formulation.

In force of remark after relation (3.47) it is sufficient to verify in validity of subsystem (3.46a). Reduce the right-hand side of (3.46a) to more convenient form for further analysis with allowance made for Jacobi identity for antibracket (3.27c). Jacobi identity with superfunctions $X(\theta) \equiv X(\Gamma(\theta), \mathcal{A}(\theta), S_H(\Gamma(\theta)), S_H(\Gamma(\theta)))$ leads to expression

$$(X(\theta), S_H(\Gamma(\theta)))_\theta, S_H(\Gamma(\theta)))_\theta = \frac{1}{2}(X(\theta), (S_H(\Gamma(\theta)), S_H(\Gamma(\theta))))_\theta. \quad (3.53)$$

Choosing as $X(\theta)$ the coordinates $\Gamma^p(\theta)$ obtain the equivalent subsystem for (3.46a)

$$\frac{1}{2}(\Gamma^p(\theta), (S_H(\Gamma(\theta)), S_H(\Gamma(\theta))))_\theta = 0, \quad (3.54)$$

which, by virtue of nondegeneracy of the supermatrix $\|\omega^{pq}(\theta)\|$ (as the ordinary matrix (!)) (3.24), is equivalent to the system (3.39). Having used by hypothesis of the Statement on the structure for master equation (3.19b), therefore having the form (3.36b), we verify in validity of the Statement for EHS.

2) Lagrangian formulation.

The solvability conditions for ELS are the system consisting of Eqs.(2.3b), (4.33) and

$$\frac{d}{d\theta} \left( \chi_a(\mathcal{A}(\theta), \dot{\mathcal{A}}(\theta)) - \Theta_j(\mathcal{A}(\theta), \dot{\mathcal{A}}(\theta))d^a_j(\mathcal{A}(\theta), \dot{\mathcal{A}}(\theta)) \right)_{|\mathcal{A}(\theta)} = 0. \quad (3.55a)$$
or equivalently for nonvanishing summands on the integral curve $\mathcal{A}(\theta)$ for LS (2.3)

$$
\frac{d}{d\theta} \left( \chi_a(\mathcal{A}(\theta), \mathcal{A}(\theta)) \right)_{|\mathcal{A}(\theta)} - \frac{d}{d\theta} \left( \frac{\partial S_L(\theta)}{\partial \mathcal{A}^i(\theta)} \right) d^\alpha_a(\mathcal{A}(\theta), \mathcal{A}(\theta))_{|\mathcal{A}(\theta)} = 0 .
$$

(3.55b)

The structure of constraints $\chi_a(\theta)$ (2.7) and representation (3.55b) permit one to state that for solvability of the system (3.43), (3.55a) it is sufficient the solvability only for subsystem (3.43) which is equivalently given by the expression (without $P_0(\theta)$ projector)

$$
\mathcal{A}^i(\theta) \frac{\partial}{\partial \mathcal{A}^i(\theta)} \frac{\partial S_L(\mathcal{A}(\theta), \mathcal{A}(\theta))}{\partial \mathcal{A}^i(\theta)} = 0 .
$$

(3.56)

Representation (3.56) for solvability conditions (3.43) is equivalent to Eqs.(3.38). Having used by hypothesis of the statement on the Lagrangian master equation (2.6), therefore having the form (3.36a) we verify in validity of the statement in this case as well.

Statement 3.5 permits to make more deeper inference on a solvability and on the solvability conditions (3.46a) for EHS themselves, on ones (3.43) for ELS and etc. Namely, the calculation of the $k$th ($k = 1, 2, \ldots$) derivative with respect to $\theta$ of expression (3.46a) on the integral curve $\Gamma(\theta)$ value for HS leads to the expression taking Eq.(3.48) into account

$$
\frac{d^k}{d\theta^k}(\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta|\Gamma(\theta)} = (\ldots((\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta}, S_H(\Gamma(\theta)))_{\theta}, \ldots, S_H(\Gamma(\theta)))_{\theta} = 0 .
$$

(3.57)

The identical vanishing of the left-hand side of (3.57) implies the vanishing (without $P_0(\theta)$ projector) in the superfield form the right-hand one as well for any $k = 1, 2, \ldots$. That fact is really provided in force of Statement 3.5 and Jacobi identity of the type (3.35) with superfunctions $(\ldots((\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta}, S_H(\Gamma(\theta)))_{\theta}, \ldots, S_H(\Gamma(\theta)))_{\theta}$, $S_H(\Gamma(\theta))$

$$
(\ldots((\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta}, S_H(\Gamma(\theta)))_{\theta}, \ldots, S_H(\Gamma(\theta)))_{\theta} = \frac{1}{2}((\ldots((\Gamma^p(\theta), S_H(\Gamma(\theta)))_{\theta}, S_H(\Gamma(\theta)))_{\theta}, \ldots, S_H(\Gamma(\theta)))_{\theta}, (S_H(\Gamma(\theta)), S_H(\Gamma(\theta)))_{\theta})_{\theta} .
$$

(3.58)

The analogous statement applied to ELS has the trivial form being written by the formula

$$
\frac{d^k}{d\theta^k} \left( \frac{\partial S_L(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta)}{\partial \mathcal{A}^i(\theta)} \right)_{|\mathcal{A}(\theta)} = \left( \frac{\partial}{\partial \theta} + \frac{\partial}{\partial \theta} \right)^k \frac{\partial S_L(\mathcal{A}(\theta), \mathcal{A}(\theta), \theta)}{\partial \mathcal{A}^i(\theta)}_{|\mathcal{A}(\theta)} = 0 ,
$$

(3.59)

by virtue of nilpotency of the operators $\frac{\partial}{\partial \theta}$, $\frac{\partial}{\partial \theta}$ [1] and Eq.(2.5).

Finally, having considered the 2nd derivative on $\theta$ of an arbitrary $\mathcal{F}(\theta) \in D_{\mathcal{A}}^k$ calculated on the solution $\Gamma(\theta)$ for HS in scope of Statement 3.5 validity obtain from (3.48) the formula

$$
\frac{d^2}{d\theta^2} \mathcal{F}(\theta)_{\theta|\Gamma(\theta)} = ((\mathcal{F}(\theta), S_H(\theta))_{\theta}, S_H(\theta))_{\theta|\Gamma(\theta)} .
$$

(3.60)

If the left-hand side of (3.60) vanishes identically, then vanishing of the right-hand one is provided by Jacobi identity in the form (3.35) for $X(\theta) = \mathcal{F}(\theta)$. Analogously to proof of the expression (3.57) vanishing one can show the same for any $\mathcal{F}(\theta) \in D_{\mathcal{A}}^k$. Thus, it is proved the
technical, but very important for formulae writing,

**Statement 3.6**
In fulfilling of the solvability conditions (3.46a) for EHS (3.31a,c) the formula for translation of arbitrary superfunction $\mathcal{F}(\theta) \in D_{cl}^k$ with respect to variable $\theta$ on a constant parameter $\mu \in \Lambda_1(\theta)$ along solution $\hat{\Gamma}(\theta)$ for HS (3.31a) is written without $P_0(\theta)$ projector in front of antibracket

$$\delta_\mu \mathcal{F}(\Gamma(\theta), \hat{\Gamma}(\theta), \theta)_{\Gamma(\theta)} = \frac{d_r \mathcal{F}(\theta)}{d\theta} |\Gamma(\theta)|^\mu = \left( \frac{\partial_r \mathcal{F}(\theta)}{\partial \theta} + (\mathcal{F}(\theta), S_H(\Gamma(\theta)))_\theta \right) |\Gamma(\theta)|^\mu. \quad (3.61)$$

In obtaining of (3.61) the conditions (3.19) and following derivation rule of antibracket have been made use being valid for any $\mathcal{F}(\theta), \mathcal{J}(\theta) \in D_{cl}^k$

$$\frac{\partial_r (\mathcal{F}(\theta), \mathcal{J}(\theta))_\theta}{\partial \theta} = (\mathcal{F}(\theta), \frac{\partial_r \mathcal{J}(\theta)}{\partial \theta})_\theta + (-1)^{\varepsilon(\mathcal{J})+1} (\frac{\partial_r \mathcal{F}(\theta)}{\partial \theta}, \mathcal{J}(\theta))_\theta. \quad (3.62)$$

**Remark:** In Statement 3.5 (later in 3.6 too) in fact it is considered as the integral curve for EHS (ELS) not only the integral curve $\Gamma(\theta)$ for HS (3.31a) ($\hat{A}(\theta)$ for LS (2.3)) but such $\Gamma(\theta),$ ($\hat{A}(\theta)$) which satisfies to SCHF (3.31c) (SCLF (2.9c,d)) and is denoted further as

$$\hat{\Gamma}^p(\theta) = \Gamma^p(\theta) \cap \{ \chi_a^H(\Gamma(\theta)) = 0 \} \text{ \((\hat{A}^p(\theta) = \hat{A}^p(\theta) \cap \{ \chi_a(\hat{A}(\theta), \hat{A}(\theta)) = 0 \})\).} \quad (3.63)$$

**IV Properties of the Differential Systems for Lagrangian and Hamiltonian Formulations of GSTF**

The combined analysis of Statements 3.1, 3.3, 3.4 leads to the result deserving of the exceptional consideration.

**Definition:** We denote the model of GSTF on $\mathcal{M}_{cl}$ given on $T_{odd} \mathcal{M}_{cl}$ ($T_{odd}^* \mathcal{M}_{cl}$) with superfunction $S_L(\theta)$ ($S_H(\theta)$), with corresponding dynamic system LS, ELS [GHS, HS, EGHS, EHS] by the triple of quantities ($T_{odd} \mathcal{M}_{cl}, S_L(\theta), A) [(T_{odd}^* \mathcal{M}_{cl}, S_H(\theta), B)],$ where $A \in \{ \text{LS, ELS} \}$ ($B \in \{ \text{GHS, HS, EGHS, EHS} \}$).

**Statement 4.1** (diagram of equivalent formulations for GSTF model on $\mathcal{M}_{cl}$)

The following commutative diagram is valid in formulating of the GSTF model

$$\begin{array}{c c}
(T_{odd} \mathcal{M}_{cl}, S_L(\theta), \text{LS}) & \rightarrow & (T_{odd} \mathcal{M}_{cl}, S_L(\theta), \text{ELS}) \\
\downarrow & & \downarrow \\
(T_{odd}^* \mathcal{M}_{cl}, S_H(\theta), \text{GHS}) & \rightarrow & (T_{odd}^* \mathcal{M}_{cl}, S_H(\theta), \text{EGHS}) \\
\downarrow & & \downarrow \\
(T_{odd}^* \mathcal{M}_{cl}, S_H(\theta), \text{HS}) & \rightarrow & (T_{odd}^* \mathcal{M}_{cl}, S_H(\theta), \text{EHS})
\end{array} \quad (4.1)$$

whose vertical arrows formally denote the equivalence relation (i.e. presence, for instance, of special isomorphic mappings between $T_{odd} \mathcal{M}_{cl}$ and $\mathcal{T}_{odd}^* \mathcal{M}_{cl}, S_L(\theta)$ and $S_H(\theta)$, LS and GHS (HS)), and horizontal ones are mappings of the restriction of the left column by means of SCLF in the 1st row or by SCHF in the 2nd and 3rd rows.

**Remarks:**
1) Conducting of a detailed proof is the separate problem and requires except for established correlation for $S_L(\theta)$ and $S_H(\theta)$ by Legendre transform a more exact description for $T_{odd} \mathcal{M}_{cl}, T_{odd}^* \mathcal{M}_{cl}$ from geometric viewpoint;
2) the right column is described by $S_L(\theta), S_H(\theta)$ not explicitly depending on $\theta;$
3) if the master equations (2.6), (3.19b) have the form (3.36) then it follows from Statement 3.5 the solvability of the corresponding dynamical systems of equations from the right column.

ELS (2.9) arises from variational problem on conditional extremum for superfunctional (2.1) in fulfilling of Eqs.(2.9c,d) for SCLF. On the other hand, ELS follows from variational problem on unconditional extremum for superfunctional

$$Z(1)[A, \lambda] = \int d\theta S_L(1)(A(\theta), \dot{A}(\theta), \lambda(\theta))$$

$$S_L(A(\theta), \dot{A}(\theta)) + \lambda^a(\theta) \left( \chi_a(A(\theta), \dot{A}(\theta)) - \Theta_i(A(\theta), \dot{A}(\theta)) \lambda^i_a(A(\theta), \dot{A}(\theta)) \right),$$

with vanishing, after calculation of the 1st order variation for $Z(1)[A, \lambda]$, of the superfields $\lambda^a(\theta)$ extending the $T_{odd} A_d \times \{\theta\}$ and being by Lagrange multipliers to SCLF (2.9c,d). $\lambda^a(\theta)$ are transformed on a some $J$ subrepresentation from $T \oplus T^*$ and possess by the gradings

$$(\varepsilon_p, \varepsilon_j, \varepsilon) \lambda^a(\theta) = (\varepsilon_p(\chi_a(\theta)), \varepsilon_j(\chi_a(\theta)), \varepsilon(\chi_a(\theta))), a = (A_1, A_2).$$

**Statement 4.2**

1) Superfunctions $S_L(A(\theta), \dot{A}(\theta))$, $S_L(1)(A(\theta), \dot{A}(\theta), \lambda(\theta))$ at absence of the explicit dependence upon $\theta$ appear by the integrals of solvable ELS (2.9);

2) On the integral curve $A'(\theta)$ for solvable ELS the superfunctionals $Z[A]$ and $Z(1)[A, \lambda]$ achieve their critical values for arbitrary configurations of $\lambda^a(\theta)$

$$Z[\dot{A}] = Z(1)[\dot{A}, \lambda] = 0.$$  

**Proof:** 1) Expressions

$$\tilde{S}_L(A(\theta), \dot{A}(\theta)) = A'(\theta) \frac{\partial S_L(\theta)}{\partial A'(\theta)} = 0,$$

$$S_L(1)(A(\theta), \dot{A}(\theta), \lambda(\theta)) = \left( \tilde{S}_L(A(\theta), \dot{A}(\theta)) + \lambda^a(\theta) \chi_a(\theta) + \lambda^a(\theta) \dot{\chi}_a(\theta)(-1)^{c(\chi_a)} \right) = 0$$

taking the equations $\Theta_i(\theta) = \dot{\Theta}_i(\theta) = 0$ into account prove the 1st part of the Statement.

2) Identical formulae for the critical values of $Z[A]$, $Z(1)[A, \lambda]

$$Z[\dot{A}] = \tilde{S}_L(A(\theta), \dot{A}(\theta)) = \tilde{S}_L(1)(A(\theta), \dot{A}(\theta), \lambda(\theta)), Z(1)[\dot{A}, \lambda] = \tilde{S}_L(1)(A(\theta), \dot{A}(\theta), \lambda(\theta)),$$

arising from (2.1), (4.2) with allowance made for (4.5) prove the formula (4.4) as well.

From Statement 4.1 it follows the possibility to formulate the proposition for Hamiltonian formulation being analogous to Statement 4.2. To this end indicate that EHS can be obtained from variational problem on conditional extremum for superfunctional $Z_H[\Gamma]$ (3.13) under condition (3.31c) or on conditional one for $Z_H(1)[\Gamma, D]$ (3.14a) in fulfilling of SCHF (3.31c) (after variation with respect to $\Gamma^p(\theta)$ it is necessary to put $D'(\theta) = 0$), or on unconditional one for the extended by superfields $\lambda^a(\theta)$ superfunctional

$$Z_H(2)[\Gamma, \lambda] = \int d\theta (\tilde{A}'(\theta) A'_i(\theta) - S_H(2)(\Gamma, \lambda(\theta))),$$

$$S_H(2)(\Gamma(\theta), \lambda(\theta)) = S_H(\Gamma(\theta)) - \lambda^a(\theta) \left[ \chi^H(\Gamma(\theta)) - \left( \frac{d\Gamma^p(\theta)}{d\theta} -(\Gamma^p(\theta), S_H(\theta))_\theta \right) \lambda^a(\theta) \right].$$
Superfields $\lambda^a(\theta)$ in (4.7) can be chosen by the same as ones in (4.2). In order to obtain EHS (3.31a,c) it is necessary to put $\lambda^a(\theta) = 0$ after variation in $\delta_1 Z^{(2)}_H$.

**Statement 4.3**

1) Superfunctions

$$S_L(\mathcal{A}(\theta), \mathcal{B}(\theta)) |_{\mathcal{A}(\theta) = \mathcal{B}(\Gamma(\theta))} = \mathcal{B}^*(\theta)\mathcal{A}^*(\theta) - S_H(\Gamma(\theta)),$$  \hspace{1cm} (4.8a)

$$\dot{S}_L(\mathcal{A}(\theta), \mathcal{B}(\theta), D(\theta)) |_{\mathcal{A}(\theta) = \mathcal{B}(\Gamma(\theta))} = \dot{\mathcal{B}}^*(\theta)\mathcal{A}^*(\theta) - S_H^{(1)}(\Gamma(\theta), D(\theta)),$$  \hspace{1cm} (4.8b)

$$S_L^{(1)}(\mathcal{A}(\theta), \mathcal{B}(\theta), \lambda(\theta)) |_{\mathcal{A}(\theta) = \mathcal{B}(\Gamma(\theta))} = \dot{\mathcal{B}}^*(\theta)\mathcal{A}^*(\theta) - S_H^{(2)}(\Gamma(\theta), \lambda(\theta))$$  \hspace{1cm} (4.8c)

without explicit dependence upon $\theta$.

2) On the integral curve for solvable EGHS $\tilde{\Gamma}^H(\theta)$ for arbitrary $D^i(\theta)$, $\lambda^a(\theta)$ the superfunctionals $Z^{(k)}_H$, $k = 0, 1, 2$ ($Z^{(0)}_H \equiv Z_H[\Gamma]$) achieve their critical values

$$Z_H[\tilde{\Gamma}] = Z^{(1)}_H[\tilde{\Gamma}, D] = Z^{(2)}_H[\tilde{\Gamma}, \lambda] = 0.$$ \hspace{1cm} (4.9)

**Proof:**

1) Relations

$$\frac{d}{d\theta} (\mathcal{B}^*(\theta)\mathcal{A}^*(\theta)) |_{\mathcal{A}(\Gamma(\theta))} = \mathcal{B}^*(\theta)\mathcal{A}^*(\theta),$$ \hspace{1cm} (4.10a)

$$\dot{S}_H(\Gamma(\theta)) |_{\mathcal{A}(\Gamma(\theta))} = \frac{d}{d\theta} (S_H^{(1)}(\Gamma(\theta)) + \frac{1}{2} (S_H^{(2)}(\Gamma(\theta)), \lambda(\theta))) = 0,$$ \hspace{1cm} (4.10b)

$$\frac{d}{d\theta} (D^i(\theta)\Theta^H_i(\Gamma(\theta))) |_{\mathcal{A}(\Gamma(\theta))} = \frac{d}{d\theta} (D^i(\theta)\Theta^H_i(\Gamma(\theta))) = 0,$$ \hspace{1cm} (4.10c)

$$\frac{d}{d\theta} \left( \lambda^a(\theta) \left[ \chi^H_a(\Gamma(\theta)) - \frac{d}{d\theta} \left( \Gamma^a(\theta), S_H(\Gamma(\theta)) \right) \right] \right) |_{\mathcal{A}(\Gamma(\theta))} = 0$$ \hspace{1cm} (4.10d)

being obtained with the help of solvability conditions for EGHS, representation (3.36b), formula (3.53) and Statement 3.6 prove the 1st part of the Statement.

2) The validity of the 2nd one follows from formulae (4.10) and

$$Z_H[\tilde{\Gamma}] = S_L(\mathcal{A}(\theta), \mathcal{B}(\Gamma(\theta))) |_{\mathcal{A}(\Gamma(\theta))}, \hspace{0.5cm} Z_H^{(1)}[\tilde{\Gamma}, D] = S_L^{(1)}(\mathcal{A}(\theta), \mathcal{B}(\Gamma(\theta))) |_{\mathcal{A}(\Gamma(\theta))},$$ \hspace{1cm} (4.11)

$$Z_H^{(2)}[\tilde{\Gamma}, \lambda] = S_L^{(2)}(\mathcal{A}(\theta), \mathcal{B}(\Gamma(\theta)), \lambda(\theta)) |_{\mathcal{A}(\Gamma(\theta))},$$

**Corollary:** From Statement 4.3 it follows the $S_H^{(k)}(\theta), k = 0, 1, 2$ ($S_H^{(0)}(\theta) = S_H(\Gamma(\theta))$) appear by the integrals for EHS as well.

Representation (3.31a,c) for EHS implies for $\lambda_{pa}(\Gamma(\theta)) = 0$ the following consequence for EHS under parametrization of $T_{odd}^* \mathcal{M}_{cl}$ by the coordinates (3.21)

$$\frac{d_r \varphi^a(\theta)}{d\theta} = (\varphi^a(\theta), S_H(\Gamma(\theta)))_\theta, \hspace{0.5cm} \frac{d_r \varphi^a(\theta)}{d\theta} = 0,$$ \hspace{1cm} (4.12a)

$$\chi^H_a(\Gamma(\theta)) = 0.$$ \hspace{1cm} (4.12b)

Assume now the superfunction $S_H(\theta)$ depends explicitly upon $\theta$. Then its translation along integral curve $\Gamma^H(\theta)$ for Hamiltonian subsystem (4.12a) has the form

$$\delta_\mu S_H(\Gamma(\theta), \theta) |_{\mathcal{A}(\Gamma(\theta))} = \left[ \frac{\partial S_H(\Gamma(\theta), \theta)}{\partial \theta} + P_0(\theta) \frac{\partial S_H(\theta, \varphi^a(\theta))}{\partial \varphi^a(\theta)} \right] \mu$$

$$= \left[ \frac{\partial S_H(\Gamma(\theta), \theta)}{\partial \theta} + \frac{1}{2} P_0(\theta) (S_H(\Gamma(\theta), \theta), S_H(\Gamma(\theta), \theta))_\theta \right] \mu,$$ \hspace{1cm} (4.13)
where in view of absence of the solvability conditions fulfilment for \((4.12a)\) without use of SCHF \((4.12b)\) one can not apply the Statement 3.6.

Result \((4.13)\) is differed from one \((3.17)\). The requirement of invariance for \(S_H(\Gamma(\theta), \theta)\) under such translations leads to the equation taking place on solutions \(\tilde{\Gamma}^p(\theta)\) for Eqs.\((4.12a)\)

\[
\frac{\partial_r S_H(\tilde{\Gamma}(\theta), \theta)}{\partial \theta} + \frac{1}{2} P_0(\theta)(S_H(\tilde{\Gamma}(\theta), \theta), S_H(\tilde{\Gamma}(\theta), \theta))_{\theta} = 0 .
\] (4.14)

A solution for \((4.14)\) will be equivalent to one for \((3.18)\) only in fulfilling of conditions \((3.19)\). Moreover the transformations \(\delta_{\mu_1}(\delta_{\mu_2} F(\theta))_{|\tilde{\Gamma}(\theta)} \neq 0 .\) (4.15)

At the same time the restriction of those transformations by SCHF \((4.12b)\), so that the master equation \((3.36b)\) holds, makes them by coinciding with \(\delta_{\mu_1} F(\theta)_{|\tilde{\Gamma}(\theta)}\) (see \((3.61), (3.63)\)).

V Operator \(\Delta^cl(\theta)\), Master Equation, Translations on \(\theta\) along Integral Curves for HS

V.1 Operator \(\Delta^cl(\theta)\)

For an arbitrary superfunction \(F(\theta) \in D^k_{cl}\) the expression for supercommutator is valid

\[
\left[ \frac{\partial_r}{\partial \Gamma^p(\theta)}, \frac{d_r}{d \theta} \right] \mathcal{F}(\theta) = 0 .
\] (5.1)

That formula is not in general true in calculating of mentioned supercommutator in the direction of integral curve \(\Gamma(\theta)\) for Hamiltonian part of EHS \((3.31a)\) which has the form

\[
\left[ \frac{\partial_r}{\partial \Gamma^p(\theta)}, \frac{d_r}{d \theta} \right] \mathcal{F}(\theta)_{|\Gamma(\theta)} = (-1)^{\epsilon_q}(\mathcal{F}(\theta), (\Gamma^q(\theta), S_H(\Gamma(\theta)))_{\theta})_{\omega_{qp}(\theta)_{|\Gamma(\theta)}} .
\] (5.2)

In deriving of \((5.2)\) it was used the formulae \((3.61), (3.62)\) and

\[
\frac{\partial_r \mathcal{F}(\theta)}{\partial \Gamma^p(\theta)} \equiv (\mathcal{F}(\theta), \Gamma^q(\theta))_{\theta} \omega_{qp}(\theta), \quad \frac{\partial \Gamma^p(\theta)}{\partial \theta} = 0 ,
\] (5.3a)

\[
(\epsilon_p, \epsilon_q, \epsilon) \omega_{qp}(\theta) = (\epsilon_p (\Gamma^p) + \epsilon_q (\Gamma^q) + 1, \epsilon_q (\Gamma^q) + 1, \epsilon_q + \epsilon_p + 1), \quad \omega_{qp}(\theta) = P_0(\theta) \omega_{qp}(\theta) ,
\]

\[
\omega_{qp}(\theta) = (-1)^{\epsilon_q \epsilon_p} \omega_{pq}(\theta), \quad \omega^{dq}(\theta) \omega_{qp}(\theta) = \delta^d_p, \quad ||\omega_{qp}(\theta)|| = \left\| \begin{array}{cc} 0_n & -1_n \\ 1_n & 0_n \end{array} \right\| , \quad d, p, q = \frac{1}{2} n \] (5.3b)

together with Jacobi identity \((3.27c)\) for \(\mathcal{F}(\theta), \Gamma^q(\theta), S_H(\Gamma(\theta))\).

Remark: The fulfilment of Eqs.\((3.18)\) without formal writing of \(P_0(\theta)\) projector leads to the contradictory relation

\[
0 = \frac{d^2_r \mathcal{F}(\theta)}{d \theta^2} \bigg|_{\Gamma(\theta)} = \left( \mathcal{F}(\theta), \frac{\partial_r S_H(\theta)}{\partial \theta} + \frac{1}{2} (S_H(\theta), S_H(\theta))_{\theta} \right)_{\theta|\Gamma(\theta)} \neq 0 .
\] (5.4)

The contradiction is taken off in fulfilling of Eqs.\((3.19)\) and \((3.36b)\).
The requirement of renewal of supercommutator value in the form (5.1) on solutions for Hamiltonian part of EGHS (3.31a) by calculating of the supercommutator on the coordinates \( \Gamma^q(\theta) \) of \( T^*_\text{odd} \mathcal{M}_\text{cl} \) with consequent summation under condition \( p = q \) uniquely leads to relation

\[
\left[ \frac{\partial}{\partial \Gamma^p(\theta)}, \frac{d_r}{d\theta} \right] \Gamma^p(\theta)|_{\Gamma(\theta)} = (-1)^{\varepsilon_q} \omega_{qp}(\theta)(\Gamma^p(\theta), (\Gamma^q(\theta), S_H(\theta))_\theta)|_{\Gamma(\theta)} = 2\Delta^{cl}(\theta)S_H(\Gamma(\theta), \theta)|_{\Gamma(\theta)} = 0 .
\] (5.5)

In (5.5) it was introduced the definition of the 2nd order odd (with respect to \( \varepsilon_p \) and \( \varepsilon \) gradings, but not on \( \varepsilon_f \) one) differential operator

\[
\Delta^{cl}(\theta) = \frac{1}{2} (-1)^{\varepsilon_q} \omega_{qp}(\theta)(\Gamma^p(\theta), (\Gamma^q(\theta), \theta))_\theta \equiv (-1)^{\varepsilon_q} \frac{1}{2} \frac{\partial}{\partial \Gamma^p(\theta)} \left( \omega_{qp}(\theta) \frac{\partial}{\partial \Gamma^p(\theta)} \right),
\] (5.6)

which in Darboux coordinates \( (\mathcal{A}^i(\theta), \mathcal{A}^*_i(\theta)) \) and \( (\varphi^a(\theta), \varphi^*_a(\theta)) \) has the form

\[
\Delta^{cl}(\theta) = (-1)^{\varepsilon} \frac{\partial}{\partial \mathcal{A}^i(\theta)} \frac{\partial}{\partial \mathcal{A}^*_i(\theta)} = (-1)^{\varepsilon(\varphi^a)} \frac{\partial}{\partial \varphi^a(\theta)} \frac{\partial}{\partial \varphi^*_a(\theta)} .
\] (5.7)

The geometric interpretation of relation (5.5) is as follows: antisymplectic divergence of tangent vector \( \hat{\Gamma}^p(\theta) \) in any point of the integral curve \( \Gamma^p(\theta) \) for HS (3.5a,b) is equal to doubled value of the result of operator \( \Delta^{cl}(\theta) \) action on \( S_H(\Gamma(\theta), \theta) \) out of dependence from Eqs.(3.19), (3.31c). Vanishing of the above divergence is equivalent to validity of the equation

\[
\Delta^{cl}(\theta)S_H(\Gamma(\theta), \theta)|_{\Gamma(\theta)} = 0 .
\] (5.8)

**Statement 5.1**

In calculating of the supercommutator (5.5) along integral curve \( \hat{\Gamma}^p(\theta) \) (3.63) for EHS (3.31a,c) the equation (5.8) at absence of the explicit dependence upon \( \theta \) for \( S_H(\theta) \) is the differential consequence for SCHF (3.31c) with vanishing \( \lambda_{pa}(\Gamma(\theta)) \).

**Proof:** Using of the fact that action of \( \Delta^{cl}(\theta) \) on \( S_H(\Gamma(\theta)) \) under comparison of the formulae (5.7), (3.31c) takes the form in question

\[
\Delta^{cl}(\theta)S_H(\Gamma(\theta))|_{\Gamma(\theta)} = (-1)^{\varepsilon(\varphi^a)} \frac{\partial}{\partial \varphi^a(\theta)} \frac{\partial}{\partial \varphi^*_a(\theta)}|_{\Gamma(\theta)} = 0 ,
\] (5.9)

we arrive directly to validity of the Statement. □

**Remarks:**

1) The relation to be analogous to (5.5) in the case of symplectic geometry and usual Hamiltonian equations is identically fulfilled;

2) the conservation of supercommutator value (5.5) on the solutions \( \hat{\Gamma}^p(\theta) \) for EHS in terms of Darboux coordinates \( \mathcal{A}^i(\theta), \mathcal{A}^*_i(\theta) \) leads to consequence for (5.5)

\[
\left[ \frac{\partial}{\partial \mathcal{A}^i(\theta)}, \frac{d_r}{d\theta} \right] \mathcal{A}^i(\theta)|_{\Gamma(\theta)} = \Delta^{cl}(\theta)S_H(\Gamma(\theta))|_{\Gamma(\theta)} = 0 ,
\] (5.10a)

\[
\left[ \frac{\partial}{\partial \mathcal{A}^*_i(\theta)}, \frac{d_r}{d\theta} \right] \mathcal{A}^*_i(\theta)|_{\Gamma(\theta)} = \Delta^{cl}(\theta)S_H(\Gamma(\theta))|_{\Gamma(\theta)} = 0 .
\] (5.10b)

The requirement of Eqs.(3.19b) fulfilment for any configuration \( \Gamma^p(\theta) \) irrespective to EHS (3.31a,c) appears by stronger condition on the structure of \( S_H(\Gamma(\theta)) \)

\[
(S_H(\Gamma(\theta)), S_H(\Gamma(\theta)))_\theta = 0, \forall \Gamma^p(\theta) \in T^*_\text{odd} \mathcal{M}_\text{cl} .
\] (5.11)
On the Lagrangian formalism language the situation considered in Sec.II beginning with Eqs.(2.16a) up to (2.22) inclusively corresponds to this equation.

**Statement 5.2** (solvability criterion for Eq.(5.11))

For existence of a solution for Eq.(5.11) it is necessary and sufficient to realize the following representation \( \forall \Gamma^p(\theta) \in T^*_{\text{odd}} \mathcal{M}_{cl} \)

\[
\frac{\partial_t S_H(\Gamma(\theta))}{\partial \Gamma^p(\theta)} = \left( \frac{\partial_t S_H(\Gamma(\theta))}{\partial \varphi^a(\theta)} \right) \lambda^{ba} (\Gamma(\theta)) \frac{\partial_t S_H(\Gamma(\theta))}{\partial \varphi^b(\theta)} \), \quad p = 1, 2n , \quad (5.12a)
\]

\[
\varepsilon(\lambda^{ba}) = \varepsilon(\varphi^b) + \varepsilon(\varphi^a) + 1, \quad \lambda^{ba} = -(1)^{\varepsilon(\varphi^b)\varepsilon(\varphi^a)} \lambda^{ab}, \quad a, b = (A_1, A_2), (B_1, B_2) = \overline{1, n} \ (5.12b)
\]

with superfunctions \( \lambda^{ab}(\theta) \in C^{k\times d} \).

**Proof 1** The sufficiency by a certain way follows from representation (3.26) for antibracket and the equality respectively

\[
(S_H(\theta), S_H(\theta))^{(\varphi, \varphi^*)}_\theta = 2 \frac{\partial_t S_H(\theta)}{\partial \varphi^a(\theta)} \frac{\partial_t S_H(\theta)}{\partial \varphi^b(\theta)} \), \quad \lambda^{ab}(\theta) \frac{\partial_t S_H(\theta)}{\partial \varphi^b(\theta)} \ . \quad (5.13)
\]

2) Let us represent the antibracket in the invariant coordinate-free form

\[
(F(\theta), J(\theta))_\theta = \omega_\theta(\text{osgrad}F(\theta), \text{osgrad}J(\theta)), \quad \forall F(\theta), J(\theta) \in C^{k\times} \times \{\theta\} , \quad (5.14)
\]

where \( \omega_\theta \) is the nondegenerate closed odd differential 2-form on \( T^*_{\text{odd}} \mathcal{M}_{cl} \). That form directly inherits all above properties from ones for antibracket (3.25), (3.27). The explicit form for antibracket (3.25) in local coordinates \( \Gamma^p(\theta), (A^i(\theta), A^*_i(\theta)) \) on \( T^*_{\text{odd}} \mathcal{M}_{cl} \) or in coordinates \( (\varphi^a(\theta), \varphi^*_a(\theta)) \) of the type (3.26a) permits one to present \( \omega_\theta \) by the formulae

\[
\omega_\theta = \frac{1}{2} \omega_{qp}(\theta) d\Gamma^p(\theta) \wedge d\Gamma^q(\theta) = dA^i(\theta) \wedge dA^*_i(\theta) = d\varphi^a(\theta) \wedge d\varphi^*_a(\theta) , \quad (5.15)
\]

where " \( \wedge \) " is the sign of external multiplication. Grassmann parities \( \varepsilon_p, \varepsilon_j, \varepsilon \) for elements \( d\Gamma^p(\theta), dA^i(\theta), dA^*_i(\theta), d\varphi^a(\theta), d\varphi^*_a(\theta) \) are defined to be coinciding with \( \varepsilon_p, \varepsilon_j, \varepsilon \) gradings of the coordinates \( \Gamma^p(\theta), A^i(\theta), A^*_i(\theta), \varphi^a(\theta), \varphi^*_a(\theta) \) respectively.

Symbol \( \text{osgrad}F(\theta) \) in (5.14) denotes by definition the odd (with respect to \( \varepsilon_p, \varepsilon \) but not \( \varepsilon_j \) skew-symmetric (antisymplectic) gradient of superfunction \( F(\theta) \). Obtain in coordinates \( \Gamma^p(\theta) \) on \( T^*_{\text{odd}} \mathcal{M}_{cl} \) the expression for coordinates of \( \text{osgrad}F(\theta) \) as the vector field on \( T^*_{\text{odd}} \mathcal{M}_{cl} \) in the basis \( \{\frac{\partial}{\partial \Gamma^p(\theta)}\} \) with regard for (3.24)

\[
(\text{osgrad}F(\theta))^p = \frac{\partial_t F(\theta)}{\partial \Gamma^q(\theta)} \omega_{qp}(\theta) \ . \quad (5.16)
\]

In every point of \( T^*_{\text{odd}} \mathcal{M}_{cl} \) a vector fields subset turning the form \( \omega_\theta \) into zero forms the isotropic subsuperspace \( L_\theta \) of the maximal dimension \( (k, n-k) \), where \( k \) and \( n-k \) are dimensions of even and odd (with respect to \( \varepsilon \) grading) isotropic subsuperspaces respectively. For the case of maximal dimension \( L_\theta \) is called by the Lagrangian superspace.

From Eq.(5.11) and relation (5.14) it follows that \( \text{osgrad}S_H(\theta) \in L_\theta \) in any point from \( T^*_{\text{odd}} \mathcal{M}_{cl} \). In its turn, it means that in basis \( \{\frac{\partial}{\partial \Gamma^p(\theta)}\} \) in an arbitrary point from \( T^*_{\text{odd}} \mathcal{M}_{cl} \), with coordinates \( \Gamma^p(\theta) \), among coordinates of the vector \( \text{osgrad}S_H(\theta) \) at most \( n \) are linearly independent. By virtue of supermatrix \( ||\omega_{qp}(\theta)|| \) nondegeneracy in (5.16) the last conclusion is

\(^4\)representation \( \Gamma^p(\theta) = (\varphi^a(\theta), \varphi^*_a(\theta)) \) being used in (5.12) can be distinguished from adopted one in (3.21) by value of parameter \( n_1 \). It is main that antibracket in terms of \( (\varphi^a(\theta), \varphi^*_a(\theta)) \) has the same form as in (3.26)
valid for column-vector $\frac{\partial S_H(\theta)}{\partial \Gamma^p(\theta)}$ as well, i.e. the representation (5.12a) with $\lambda^{ab}(\Gamma(\theta))$ possessing by properties (5.12b) follows from (5.11) and (5.13). □

**Remark:** Coordinates $(\mathcal{A}^*(\theta), \mathcal{A}_1^*(\theta))$ and $(\varphi^a(\theta), \varphi_a^*(\theta))$ of the same point from $T_{odd}^*\mathcal{M}_{cl}$ defined in (3.21) are equivalent in the sense that they are connected by anticanonical transformation.

**Statement 5.3** (indication for solvability of Eq.(5.11))

For fulfilment of Eq.(5.11) it is necessary to be valid the following condition on solutions for equations $\frac{\partial S_H(\theta)}{\partial \Gamma^p(\theta)} = 0$ for a some parametrization of $\Gamma^p(\theta) = (\varphi^a(\theta), \varphi_a^*(\theta))$

$$\text{rank} \left| \frac{\partial_r}{\partial \Gamma^p(\theta)} \frac{\partial_l S_H(\Gamma(\theta))}{\partial \Gamma^q(\theta)} \right| = \text{rank} \left| \frac{\partial_r}{\partial \varphi^a(\theta)} \frac{\partial_l S_H(\Gamma(\theta))}{\partial \varphi^b(\theta)} \right| = l \leq n, \ l = (l_+, l_-). \ (5.17)$$

The proof follows from the results of Statement 5.2, namely from representation (5.12). □

Note that Lagrangian subsuperspace $L_\theta$ is defined ambiguously. Relation (5.17) itself is the consequence for formula (2.22) taking account of (2.4a), (3.3), (3.4). The comparison of the condition (5.17) with one (2.21) in terms of $S_H(\Gamma(\theta))$ and coordinates $(\mathcal{A}^*(\theta), \mathcal{A}_1^*(\theta))$ on $T_{odd}^*\mathcal{M}_{cl}$, with regard for (3.35d), in the form

$$\text{rank} K^*(\theta) = \text{rank} \| (S_H')^\mu(\Gamma(\theta)) \| = n, \ (5.18)$$

in fact leads to independence of $S_H(\Gamma(\theta))$ upon $\mathcal{A}^*(\theta)$ that is very strong restriction on a structure of the model. Ignoring of the condition (5.18) after Legendre transform of $S_L(\theta)$ realization results in completely different dynamics.

**Statement 5.4**

$S_H(\Gamma(\theta))$ satisfying to (5.11) is the integral for HS (3.5a,b) constructed with respect to given $S_H(\Gamma(\theta))$.

Really the translation of $S_H(\Gamma(\theta))$ with respect to $\theta$ along integral curve $\tilde{\Gamma}^p(\theta)$ of the pointed HS on $\mu \in 1\Lambda_1(\theta)$ is given by the formula

$$\delta_\mu S_H(\Gamma(\theta))|_{\tilde{\Gamma}(\theta)} = (S_H(\tilde{\Gamma}(\theta)), S_H(\tilde{\Gamma}(\theta)))_{a\mu} = 0. \ (5.19)$$

This fact provides the solvability of corresponding HS (3.5a,b) in the sense of the type (2.12) relations as well and validity of Statement 3.6 in question.

**Remark:** Without fulfilment of equation (5.11) HS itself does not satisfy to solvability conditions (i.e. is unsolvable), and therefore the Statement 3.6 is not valid for such HS in the sense of the Eqs.(3.8) nonfulfilment. The removal of that contradiction is provided by restriction of the right-hand sides of HS (3.5a,b) until their $P_0(\theta)$ components [1]. But the latter is the explicit violation of the superfield form of equations and appears by the obstacle for further interpretation of HS role in the quantization method itself.

**Statement 5.5**

There exists a parametrization of $T_{odd}^*\mathcal{M}_{cl}$ by Darboux coordinates $(\varphi^a(\theta), \varphi_a^*(\theta))$ that in fulfilling of Eq.(5.11) the identities are valid

$$-\frac{\partial_l S_H(\Gamma(\theta))}{\partial \varphi^a(\theta)} = (\varphi_a^*(\theta), S_H(\Gamma(\theta)))_{\theta}^{(\varphi, \varphi^*)} = 0, \ \forall \Gamma^p(\theta) \in T_{odd}^*\mathcal{M}_{cl}. \ (5.20)$$

HS in this coordinates has the representation

$$\frac{d_r \varphi^a(\theta)}{d\theta} = (\varphi^a(\theta), S_H(\theta))_{\theta}^{(\varphi, \varphi^*)}, \ \frac{d_r \varphi_a^*(\theta)}{d\theta} = 0. \ (5.21)$$
From Statements 5.2, 5.3 in fact it follows the validity of the last Statement for $\lambda^{ab}(\Gamma(\theta)) = 0$, but more detailed proof is not carried out here.

Relations (5.20) represented through antibracket are conserved under anticanonical transformations together with form of the operator $\Delta^{cl}(\theta)$ action on an arbitrary superfunction from $C^{k*}$. Then the following equation holds in any local coordinates on $T^*_d\mathcal{M}_{cl}$ (connected with each other via anticanonical transformations)

$$\Delta^{cl}(\theta)S_H(\Gamma(\theta)) = 0, \forall \Gamma^p(\theta) \in T^*_d\mathcal{M}_{cl} \tag{5.22}$$

defining the further limitation for $S_H(\Gamma(\theta))$. Really, having acted on Eqs.(5.20) by $\frac{\partial}{\partial \phi_i(\theta)}$ obtain the realization of the result (5.22). Thus, Eq.(5.22) appears by the consequence of Eq.(5.11)! Equation (5.22) permits among them the same interpretation as it was made for relation (5.5).

**V.2 Translations on $\theta$ along Integral Curves for the Hamiltonian Systems**

Let us classify some properties of translations with respect to $\theta$ on $\mu \in \Lambda_1(\theta)$ of an arbitrary $\mathcal{F}(\Gamma(\theta), \theta) \in C^{k*} \times \{\theta\}$ along integral curves $\Gamma(\theta)$, $\tilde{\Gamma}(\theta), \hat{\Gamma}(\theta), \check{\Gamma}(\theta)$ of the corresponding differential systems of equations: HS (3.5a,b) (or equivalently (3.31a)), EHS (3.31a,c) with master equation (3.36b), HS given by (4.12a) and HS described in Statement 5.4 respectively

$$\delta_{\mu}\mathcal{F}(\theta)_{\Gamma(\theta)} = \bar{s}(\Gamma(\theta))\mathcal{F}(\Gamma(\theta), \theta)\mu = \left[ \frac{\partial_{\mu}\mathcal{F}(\Gamma(\theta), \theta)}{\partial_{\theta}} + (\mathcal{F}(\Gamma(\theta), \theta), S_H(\Gamma(\theta)))_\theta \right] \mu, \tag{5.23a}$$

$$\delta_{\mu}\mathcal{F}(\theta)_{\hat{\Gamma}(\theta)} = \hat{s}(\hat{\Gamma}(\theta))\mathcal{F}(\hat{\Gamma}(\theta), \theta)\mu = \left[ \frac{\partial_{\mu}\mathcal{F}(\hat{\Gamma}(\theta), \theta)}{\partial_{\theta}} + (\mathcal{F}(\hat{\Gamma}(\theta), \theta), S_H(\hat{\Gamma}(\theta)))_\theta \right] \mu, \tag{5.23b}$$

$$\delta_{\mu}\mathcal{F}(\theta)_{\check{\Gamma}(\theta)} = \check{s}(\check{\Gamma}(\theta))\mathcal{F}(\check{\Gamma}(\theta), \theta)\mu = \left[ \frac{\partial_{\mu}\mathcal{F}(\check{\Gamma}(\theta), \theta)}{\partial_{\theta}} + (\mathcal{F}(\check{\Gamma}(\theta), \theta), S_H(\check{\Gamma}(\theta)))_\theta \right] \mu. \tag{5.23c}$$

$$\delta_{\mu}\mathcal{F}(\theta)_{\tilde{\Gamma}(\theta)} = \tilde{s}(\tilde{\Gamma}(\theta))\mathcal{F}(\tilde{\Gamma}(\theta), \theta)\mu = \left[ \frac{\partial_{\mu}\mathcal{F}(\tilde{\Gamma}(\theta), \theta)}{\partial_{\theta}} + (\mathcal{F}(\tilde{\Gamma}(\theta), \theta), S_H(\tilde{\Gamma}(\theta)))_\theta \right] \mu. \tag{5.23d}$$

Call the differential operators of the 1st order $\bar{s}(\theta)$, $\hat{s}(\theta)$, $\check{s}(\theta)$, $\tilde{s}(\theta)$ by the generators of translations on $\theta$ along corresponding integral curve.

Formally formulae (5.23c,d) point to coincidence with respect to the form with corresponding transformations (5.23b,a) but it is not in general the case. The action of the generators on $S_H(\Gamma(\theta))$ is equal respectively

$$\bar{s}(\theta)S_H(\Gamma(\theta)) = (S_H(\Gamma(\theta)), S_H(\Gamma(\theta)))_\theta \neq 0, \tag{5.24a}$$

$$\hat{s}(\theta)S_H(\hat{\Gamma}(\theta)) = (S_H(\hat{\Gamma}(\theta)), S_H(\hat{\Gamma}(\theta)))_\theta = 0, \tag{5.24b}$$

$$\check{s}(\theta)S_H(\check{\Gamma}(\theta)) = \frac{1}{2}(S_H(\check{\Gamma}(\theta)), S_H(\check{\Gamma}(\theta)))_\theta \neq 0, \tag{5.24c}$$

$$\tilde{s}(\theta)S_H(\tilde{\Gamma}(\theta)) = (S_H(\tilde{\Gamma}(\theta)), S_H(\tilde{\Gamma}(\theta)))_\theta = 0. \tag{5.24d}$$

Additional relation (4.12b) applied to $\tilde{s}(\theta)$ in its definition permits to make conclusion that

$$\tilde{s}(\tilde{\Gamma}(\theta))|_{\lambda_H^\theta(\tilde{\Gamma}(\theta))=0} = \tilde{s}(\tilde{\Gamma}(\theta)) = \frac{1}{2}(S_H(\tilde{\Gamma}(\theta)), S_H(\tilde{\Gamma}(\theta)))_\theta. \tag{5.25}$$
It is not difficult to establish the properties of possession by nilpotency for the transformations (5.23), that is equivalently reformulated for their generators

\[
\tilde{s}^2(\theta) = -\frac{1}{2} \left( (S_H(\Gamma(\theta)), S_H(\Gamma(\theta)))_\theta, \right)_\theta \neq 0 , \quad (5.26a)
\]

\[
\hat{s}^2(\theta) = -\frac{1}{2} \left( (S_H(\hat{\Gamma}(\theta)), S_H(\hat{\Gamma}(\theta)))_\theta, \right)_\theta = 0 , \quad (5.26b)
\]

\[
\hat{s}^2(\theta) = \frac{\partial \hat{\varphi}}{\partial \varphi_b(\theta)} \frac{\partial S_H(\hat{\Gamma}(\theta))}{\partial \varphi_a(\theta)} \frac{\partial S_H(\hat{\Gamma}(\theta))}{\partial \varphi^*_a(\theta)} \neq 0 , \quad (5.26c)
\]

\[
\hat{s}^2(\theta) = -\frac{1}{2} \left( (S_H(\hat{\Gamma}(\theta)), S_H(\hat{\Gamma}(\theta)))_\theta, \right)_\theta = 0 . \quad (5.26d)
\]

Let us note that in this case in spite of equality (5.25) validity the relations hold

\[
\tilde{s}^2(\tilde{\Gamma}(\theta)) \neq \hat{s}^2(\tilde{\Gamma}(\theta))|_{\chi_{\tilde{\Gamma}}(\theta) = 0}, \quad \hat{s}^2(\tilde{\Gamma}(\theta))|_{\chi_{\tilde{\Gamma}}(\theta) = 0} \neq 0 . \quad (5.27)
\]

In obtaining of the formulae (5.26) the antibracket’s properties (3.25), (3.27), master equation (5.11) for (5.26d), Eq.(3.19b) in the form (3.36b) and its differential consequence (3.39) for (5.26b) (see remark after (3.62)) have been made use.

In terms of operators \( \hat{s}(\theta), \tilde{s}(\theta) \) we have proved the

**Statement 5.6**

The nilpotency of operators \( \hat{s}(\theta), \tilde{s}(\theta) \) appears by sufficient condition for the type (2.12) solvability respectively for EHS (3.31a,c) and HS of the form (3.31a), built with respect to \( S_H(\Gamma(\theta)) \) being by a solution for master equation (5.11).

The generators in (5.23a,b,d) \( \pi(\theta), \tilde{s}(\theta), \hat{s}(\theta) \) given through only the operator \( \frac{\partial}{\partial \theta} \) and antibracket do not depend on a concrete choice of the coordinates \( \Gamma_p(\theta) \) on \( T_{\text{odd}}^{*} M_{\text{cl}} \) connected with each other via anticanonical transformations. Therefore, the translation transformation with respect to \( \theta \) along integral curve \( \hat{\Gamma}(\theta) \) of HS (3.31a) given, equivalently, by the Statement 5.5. (Eqs.(5.21)) together with Eq.(5.20) is defined by a generator \( s_\varphi(\theta) \) coinciding with \( \tilde{s}(\theta) \). Properties (5.24d), (5.26d) are valid for \( s_\varphi(\theta) \) as well.

**Remark:** Under attainment of the maximal value for rank in the expression (5.17) the corresponding Lagrangian subsupermanifold \( M_{\text{cl}}^{T} \subset T_{\text{odd}}^{*} M_{\text{cl}} \) in terms of coordinates \( \varphi^a(\theta), \varphi^*_a(\theta) \) is defined one-valued in the framework of the Statement 5.5 and is parametrized only by \( \varphi^*_a(\theta) \). On the other hand, the projection of solution \( \hat{\Gamma}(\theta) \) for HS of the type (3.31a), in fulfilling of (5.11), on subsystem of the form (5.21) without realization of the identity (5.20) leads to unsolvable HS. That fact means the supermanifold \( M_{\text{cl}}^{T} \) is not already parametrized by superfields \( \varphi^*_a(\theta) \) (value of parameter \( n_1 \) in (3.21) characterizing index \( a \) is now differred from formulated in Statement 5.5). Corresponding \( \tilde{s}_\varphi(\theta) \) being by the translation generator on \( \theta \) along projection for given \( \hat{\Gamma}(\theta) \) on the above-described type (5.21) subsystem is defined by the formula

\[
\tilde{s}_\varphi(\theta) = \frac{\partial \mathcal{F}(\theta)}{\partial \theta} + \frac{\partial \mathcal{F}(\theta)}{\partial \varphi_b(\theta)} \frac{\partial S_H(\tilde{\varphi}(\theta), \varphi^*_b(\theta))}{\partial \varphi^*_a(\theta)} , \quad \mathcal{F}(\theta) \in D_{\text{cl}}^k , \quad (5.28)
\]

has \( S_H(\hat{\Gamma}(\theta)) \) as an eigenfunction with zero eigenvalue, but is not nilpotent

\[
\tilde{s}_\varphi(\theta) S_H(\tilde{\varphi}(\theta), \varphi^*_b(\theta)) = \frac{1}{2} (S_H(\theta), S_H(\theta))_{\theta(\tilde{\varphi}(\theta), \varphi^*_b(\theta))} = 0, \quad \tilde{s}^2(\theta) = \tilde{s}^2(\theta) = \tilde{s}^2(\theta) = 0 . \quad (5.29)
\]

It is the same operator \( \tilde{s}_\varphi(\theta) \) for \( n_1 = 0 \) (\( \varphi^a(\theta) \equiv A^i(\theta) \)) is considered in the BV quantization method (without auxiliary, for instance, ghost-antighost type superfields and for \( \theta = 0 \)).
HS (3.5a,b) with $S_{\mathcal{H}}(\Gamma(\theta))$ satisfying to (5.11) corresponds in the Lagrangian formulation to solvable LS (2.3) with $S_L(\mathcal{A}(\theta),\mathcal{A}(\theta))$ satisfying to Eqs.(2.5), (2.6) for any $\mathcal{A}(\theta) \in \mathcal{M}_{cl}$

$$
\dot{\mathcal{A}}(\theta) \frac{\partial_t S_L(\theta)}{\partial \dot{\mathcal{A}}(\theta)} = 0, \quad \forall \mathcal{A}(\theta) \in \mathcal{M}_{cl}.
$$

(5.30)

Condition (5.18) for $S_{\mathcal{H}}(\Gamma(\theta))$, meaning for $S_L(\theta)$ being one-valued constructed via Legendre transform of the given $S_{\mathcal{H}}(\Gamma(\theta))$ with respect to $\mathcal{A}(\theta)$ in view of the supermatrix $K(\theta)$ (2.21) nondegeneracy, leads to identity

$$
\frac{\partial_t S_L(\theta,\mathcal{A}(\theta))}{\partial \mathcal{A}(\theta)} = 0, \quad \forall \mathcal{A}(\theta) \in \mathcal{M}_{cl}.
$$

(5.31)

Formula (5.31) means the independence of $S_{\mathcal{H}}(\theta)$ upon $\mathcal{A}(\theta)$ that leads to the trivial dynamics on $\theta$. The latter implies that LS (2.3) is restricted to proper trivial subsystem (2.3a). Note that any superfields $\mathcal{A}(\theta)$ satisfy to that subsystem and in fulfilling of (2.5) the corresponding superfunctional $Z[\mathcal{A}]$ (2.1) vanishes identically.

Eq.(5.22) for $S_{\mathcal{H}}(\theta)$ taking into account of (5.30), (5.31) and (3.35) means the realization of the equivalent equation for $S_L(\mathcal{A}(\theta))$

$$
-(\mathcal{L}) \frac{\partial_t S_L(\mathcal{A}(\theta),\mathcal{A}(\theta))}{\partial \mathcal{A}(\theta)} = 0 .
$$

(5.32)

VI  Detailed Investigation of the Hamiltonian Formulation of GSTF

In connection with addition to LS (2.3) the SCLF $\chi_{\infty}(\mathcal{A}(\theta),\mathcal{A}(\theta))$ (2.7) it is necessary for Lagrangian formulation of GSTF to modify the system of initial postulates formulated in [1]. SCLF $\chi_{\infty}(\theta)$ can be linearly (functionally) dependent themselves. Taking into account of the SCLF structure the assumptions have the form being consistent with ones in Ref.[1].

1. There exists a configuration $(\mathcal{A}_0(\theta),\mathcal{A}_1(\theta)) \in T_{odd} \mathcal{M}_{cl}$ that

$$
\Theta(\mathcal{A}(\theta),\mathcal{A}(\theta)) = \chi_{\infty}(\mathcal{A}(\theta),\mathcal{A}(\theta)) |_{(\mathcal{A}_0(\theta),\mathcal{A}_1(\theta))} = 0 ;
$$

(6.1)

2. There exists, at least locally, a smooth supersurface $1\Sigma \subset \mathcal{M}_{cl} \left[ (\mathcal{A}_0(\theta),\mathcal{A}_1(\theta)) \in T_{odd} 1\Sigma \right]$

$$
dim 1\Sigma = 1m = (1m_+,1m_-), \quad \dim T_{odd} 1\Sigma = (1m_+ + 1m_-) + (1m_+ + 1m_-),
$$

(6.2a)

$$
\chi_{\infty}(\mathcal{A}(\theta),\mathcal{A}(\theta)) |_{T_{odd} 1\Sigma} = 0 ,
$$

(6.2b)

the such that the following condition almost everywhere on $T_{odd} 1\Sigma$ is fulfilled

$$
\text{rank} \left| \frac{\delta_t \chi_{\infty}(\mathcal{A}(\theta),\mathcal{A}(\theta))}{\delta \mathcal{A}(\theta_1)} \right|_{T_{odd} 1\Sigma} = n - 1m .
$$

(6.3)

Remarks:
1) Assumption 2 from Ref.[1] on the rank of supermatrix of the 2nd superfield variational derivatives with respect to $A^i(\theta)$ and $A^i(\theta_1)$ of superfunctional $Z[A]$ \[ or \frac{\delta \Theta_i(A^i(\theta_1)/A^i(\theta))}{\delta A^i(\theta_1)} \] remains valid without changes;

2) the rule for calculation of the type (6.3) supermatrices ranks had been pointed out in [1];

3) because of SCLF $\chi_\alpha(\theta)$ and DCLF $\Theta_i(\theta)$ are not in general case (functionally) independent from each other, then in addition to assumption 2 of the work [1] and $2_M$ (6.2), (6.3) it is necessary to introduce the value for rank of $2n \times 2n$ supermatrix

$$\text{rank} \left[ \frac{\partial (\Theta_i(\theta), \chi_\alpha(\theta))}{\partial (A^i(\theta), A^k(\theta))} \right]_{T_{odd}(\Sigma^n \Sigma)} = (6.4)$$

where $\Sigma$ is the smooth local supersurface described in [1];

4) the smooth supersurfaces $^1\Sigma$ and $\Sigma$ are defined at least locally in a some neighbourhood of the corresponding configuration $A^i_0(\theta)$ and $A^i_0(\theta)$ below.

In consequence of SCLF $\chi_\alpha(\theta)$ dependence for $^1\Sigma$ $m \neq 0$ it follows by analogy with Theorem 2 from the Ref.[1] a possibility to realize an equivalent set of SCLF with one’s own differential operators $\hat{\cal R}_{\beta_1}^a(A(\theta), \hat{\cal A}(\theta), \theta; \theta') \equiv \hat{\cal R}_{\beta_1}^a(\theta; \theta')$, $a = (A_1, A_2) = 1, \ldots, 2m$ leading to identities among $\chi_\alpha(\theta)$ [1]. The joint investigation of $\chi_\alpha(\theta)$ and $\Theta_i(\theta)$, based on the relation (6.4), together with corresponding operators $\hat{\cal R}_{\beta_1}^a(\theta; \theta')$, $\alpha = 1, \ldots, m$ [1] and $\hat{\cal R}_{\beta_1}^a(\theta; \theta')$ is not developed here.

Analysis of GCHF $\Theta_i^H(\Gamma(\theta))$, SCHF $\chi_\alpha^H(\Gamma(\theta))$ independence under condition (3.19a) one can separately and jointly develop in $T^*_{odd}M_{cl}$ directly. To this end it is necessary to reformulate the assumptions 1–3 suggested in [1] in terms of $Z_H[\Gamma]$, $Z_H^{(1)}[\Gamma, D]$, $Z_H^{(2)}[\Gamma, \lambda]$, $\check{S}_H^{(k)}(\theta)$, $k = 0, 1, 2$. $1_H$. There exists a configuration $\Gamma_0^p(\theta) = (A_i^0(\theta), A^*_0(\theta)) \in T^*_{odd}M_{cl}$ that

$$\Theta_i^H(\Gamma_0(\theta), \theta) = 0 ; \hspace{1cm} (6.5)$$

$2_H$. A smooth local supersurface $\Sigma \subset M_{cl}$ : $\Gamma_0^p(\theta) \in T^*_{odd}\Sigma$ exists the such that

$$\Theta_i^H(\Gamma(\theta), \theta)|_{T^*_{odd}\Sigma} = 0 . \hspace{1cm} (6.6)$$

A separation of index $\iota$ [1] exists

$$\iota = (A, \alpha), \ A = 1, \ldots, n - m, \ \alpha = n - m + 1, \ldots, n, \ m = (m_+, m_-), \hspace{1cm} (6.7a)$$

that the following condition with regard of (3.35f) almost everywhere on $\Sigma$ holds

$$\text{rank} \left[ \frac{\delta_l Z_H^{(1)}[\Gamma, D]}{\delta A^i(\theta_1) \delta D^{(i)}(\theta)} \right]_{\Sigma} = \text{rank} \left[ \frac{\delta_l Z_H^{(1)}[\Gamma, D]}{\delta A^i(\theta_1) \delta D^{(i)}(\theta)} \right]_{\Sigma} = n - m ; \hspace{1cm} (6.7b)$$

$3_H$. $\text{rank}K^*P(\theta)|_{T^*_{odd}V} \equiv \text{rank} \left[ (S_H^{(n)}(\Gamma(\theta), \theta))|_{T^*_{odd}V} = n, \ \Sigma \subset V \subset M_{cl} , \hspace{1cm} (6.8)$$

where $V$ is a some neighbourhood of $\Sigma$ described, for instance, in [1].

$1_{HM}$. A configuration $\Gamma_0^p(\theta) \in T^*_{odd}M_{cl}$ exists that

$$\Theta_i^H(\Gamma_0(\theta)) = \chi_\alpha^H(\Gamma_0(\theta)) = 0 ; \hspace{1cm} (6.9)$$
$2_{HM}$. A smooth local supersurface $^1\Sigma \subset \mathcal{M}_cl : \tilde{\Gamma}_0^p(\theta) \in T_{odd}^{*}1\Sigma$ exists the such that together with (6.2a) being valid for $T_{odd}^{*}1\Sigma$ the expression is true

$$\chi_a^H(\Gamma(\theta))|_{T_{odd}^{*}1\Sigma} = 0 \tag{6.10},$$

and almost everywhere on $^1\Sigma$ the relation holds

$$\text{rank} \left\| \frac{\delta \chi_a^H(\Gamma(\theta))}{\delta \hat{\mathcal{A}}(\theta_1)} \right\|_{T_{odd}^{*}1\Sigma} = \text{rank} \left\| \frac{\delta \chi_a^H(\Gamma(\theta))}{\delta \hat{\mathcal{A}}(\theta_1)} \frac{\delta Z_H^{(2)}[\Gamma, \lambda]}{\delta \lambda^a(\theta)} \right\|_{T_{odd}^{*}1\Sigma} = n - 1m \tag{6.11}.$$

Adapting remark 3) after (6.3) to the case of the Hamiltonian formulation of GSTF note that by analog of the formula (6.4) under a simultaneous analysis of $\Theta_i^H(\theta)$ and $\chi_a^H(\theta)$ in addition to assumptions $2_H$ and $2_{HM}$ it appears the rank value of $2n \times 2n$ supermatrix

$$\text{rank} \left\| \frac{\partial (\Theta_i^H(\theta), \chi_a^H(\theta))}{\partial (\mathcal{A}^i(\theta), \hat{\mathcal{A}}^i_k(\theta))} \right\|_{T_{odd}^{*}1\Sigma} \tag{6.12}.$$

Assumptions $1_H, 2_H$ are sufficient in separate investigating of GCHF $\Theta_i^H(\theta)$ structure without SCHF and $2_{HM}$ in investigating of $\chi_a^H(\theta)$.

In the framework of conditions $1_H - 3_H$ the theorem being almost analogous to Theorem 2 in Ref.[1] is formally valid.

**Theorem 1** (on reduction of GCHF to equivalent system in generalized normal form (GNF))

A nondegenerate parametrization for superfields $\Gamma^p(\theta)$ exists of the form

$$\Gamma^p(\theta) = (\mathcal{A}^i(\theta), \hat{\mathcal{A}}^i_k(\theta)) = (\delta^p(\theta), \delta^k_p(\theta); \beta^p_1(\theta), \beta^p_2(\theta); \xi^a(\theta), \xi^a_\alpha(\theta)) \equiv \langle \phi^A(\theta), \phi^*_A(\theta); \xi^\alpha(\theta), \xi^\alpha_\alpha(\theta) \rangle, \ i = (a, \alpha), \ [\xi^\alpha] = m \tag{6.13}.$$

that the system of $n$ GCHF $\Theta_i^H(\Gamma(\theta), \theta)$ (3.5c) being by algebraic with respect to derivatives on $\theta$ is equivalent to one of independent algebraic equations in GHF

$$\delta^p_\alpha(\theta) = \phi^\alpha(\delta(\theta), \xi(\theta), \xi^\alpha(\theta), \theta), \ \beta^p_\alpha(\theta) = \kappa^\alpha(\delta(\theta), \xi(\theta), \theta). \tag{6.14}.$$

Superfields $\xi^\alpha(\theta)$ and their ”momenta” $\xi^\alpha_\alpha(\theta)$ are arbitrary and the number of them ($\xi^\alpha(\theta)$) coincides with number of algebraic (on $\theta$) identities among GCHF

$$\int d\theta \frac{\delta Z_H^{(1)}[\Gamma, D]}{\delta D^\nu(\theta)} \delta^{H_a}(\Gamma(\theta), \theta; \theta') = 0, \ (\varepsilon_P, \varepsilon_J, \varepsilon) \delta^{H_a}(\theta; \theta') = (1, \varepsilon_i + \varepsilon_\alpha, \varepsilon_i + \varepsilon_\alpha + 1), \tag{6.15}$$

with a) local and b) linear independent operators $\delta^{H_a}(\Gamma(\theta), \theta; \theta') \equiv \delta^{H_a}(\theta; \theta')$:

a) $$\delta^{H_a}(\theta; \theta') = \sum_{k=0}^{1} \left( \frac{d}{d\theta} \right)^k \delta(\theta - \theta'), \ \delta^{H_a}(\Gamma(\theta), \theta), \ \tag{6.16}.$$

b) functional equation

$$\int d\theta' \delta^{H_a}(\theta; \theta') u^a(\Gamma(\theta'), \theta') = 0, \ u^a(\theta) \in C^{k_a} \times \{\theta\} \tag{6.17}$$

has the unique vanishing solution.
Remarks:
1) Operators \( \hat{R}^i_{H\alpha}(\theta;\theta') \) can be chosen in a such way that from (3.5c), (2.21) and Ref.[1] it follows the equalities
\[
\hat{R}^i_{H\alpha}(A(\theta),\hat{A}(\theta),\theta;\theta') \big|_{A=A(\Gamma(\theta),\theta)} = \hat{R}^i_{H\alpha}(\theta;\theta'), \quad \hat{R}^i_{H\alpha}(A(\theta),\hat{A}(\theta),\theta) \big|_{A=A(\Gamma(\theta),\theta)} = \hat{R}^i_{H\alpha}(\theta); \quad (6.18)
\]
relation (3.40) permits to conclude on convention of Theorem 1 because the subsystem (3.5a) of the 1st order with respect to derivatives on \( \theta \) differential equations is in N on now. However there are the forcible arguments to consider Theorem 1 (see corollary 1.2);
2) the writing of GCHF (3.5c) in equivalent form (6.14) leads to necessity of the type (2.12) solvability conditions fulfilment for Eqs.(6.14). Realization of those conditions with necessity results in absence of explicit dependence on \( \theta \) for \( \Theta^H_1(\theta) \) and therefore the Eq.(3.19a) holds;
3) representation of equivalent GCHF in the form (6.14) is possible, in particular, under assumption that in realizing of Theorem 2 in Ref.[1] additionally to postulate 3_H the following condition is valid
\[
\text{rank} \left| \frac{\partial^2 S_L(\theta)}{\partial \bar{\delta}^1(\theta) \partial \bar{\delta}^2(\theta)} \right|_{T_{odd}V} = \text{rank} \left| \frac{\partial^2 S_H(\theta)}{\partial \bar{\delta}^1(\theta) \partial \bar{\delta}^2(\theta)} \right|_{T_{odd}V}; \quad (6.19)
\]
permitting to present \( \bar{\delta}^1(\theta) \) as the superfunctions being essentially dependent upon \( \bar{\delta}^1(\theta) \).

The proof of Theorem 1 is out of the paper's scope. Let us indicate a some main corollaries (being by analogs of the corollaries for Theorem 2 in Ref.[1]).

Corollary 1.1
HCHF \( \Theta^H_1(\theta) \equiv \Theta_1(A(\theta),\theta) \) in fulfilling of the conditions
\[
\text{rank} \left| \frac{\partial \Theta^H_1(\theta)}{\partial A^i(\theta)} \right|_{\Sigma} = n - m \quad (6.20)
\]
and under nondegenerate parametrization \( A^i(\theta) = (\varphi^A(\theta),\xi^a(\theta)) \) are equivalent to system of algebraically independent in a sense of differentiation on \( \theta \) HCHF
\[
\Theta^H_1(\varphi(\theta),\xi(\theta),\theta) = 0. \quad (6.21)
\]
The number of \( [\xi^a(\theta)] \) coincides with one of the algebraic on \( \theta \) identities among \( \Theta^H_1(\theta) \) with linearly independent operators \( \mathcal{R}^i_{H\alpha}(A(\theta),\theta) \in C^k(\mathcal{M}_{cl} \times \{\theta\}) \)
\[
\Theta^H_1(\theta) \mathcal{R}^i_{H\alpha}(A(\theta),\theta) = 0, \quad \mathcal{R}^i_{H\alpha}(A(\theta),\theta) = \mathcal{R}^i_{h\alpha}(A(\theta),\theta), \quad (6.22)
\]
where \( \mathcal{R}^i_{h\alpha}(A(\theta),\theta) \) are the operators satisfying to conditions of the corollary 2.1 from Ref.[1].

Corollary 1.2
For a model of GSTF in the Lagrangian formulation representing the almost natural system (see Corollary 2.2 of Ref.[1]) with \( S_L(\theta) \in C^k(T_{odd}\mathcal{M}_{cl} \times \{\theta\}) \) and HCLF \( \Theta_1(A(\theta),\theta) \)
\[
S_L(A(\theta),\hat{A}(\theta),\theta) = T(A(\theta),\hat{A}(\theta)) - S(A(\theta),\theta), \quad \min \deg_A S(\theta) = 2, \quad (6.23a)
\]
\[
T(A(\theta),\hat{A}(\theta)) = T_1(\hat{A}(\theta)) + \hat{A}^i(\theta)T_\alpha(A(\theta)), \quad T_\alpha(A(\theta)) = g_{\alpha}(\theta)A^i(\theta), \quad (6.23b)
\]
\[
g_{\alpha}(\theta) = (-1)^{\varepsilon_\alpha} g_{\alpha}(\theta), \quad g_{\alpha}(\theta) = P_{0}(\theta)g_{\alpha}(\theta), \quad \min \deg A^i(\theta) T_1(\theta) = 2, \quad (6.23c)
\]
\[
T_1(\hat{A}(\theta)) = \frac{1}{2} \hat{A}^i(\theta) \kappa_{\alpha}(\hat{A}(\theta))A^i(\theta)(1)^{\varepsilon_\alpha}, \quad \kappa_{\alpha}(\hat{A}(\theta)) = (-1)^{\varepsilon_\alpha+1}(\varepsilon_\alpha+1)\kappa_{\alpha}(\hat{A}(\theta)), \quad (6.23c)
\]
\[
\Theta_1(A(\theta),\theta) = -S_{\hat{A}^i}(A(\theta),\theta)(-1)^{\varepsilon_\alpha} = 0, \quad (6.23d)
\]
the superfunction $S_H(\Gamma(\theta), \theta)$ and HCHF $\Theta_i^H(\theta)$ are given by sequence of relationships

\begin{align}
S_H(\Gamma(\theta), \theta) &= S_{H_0}(\Gamma(\theta), \theta) + S_{H_{\text{int}}}(\Gamma(\theta), \theta), \\
S_{H_0}(\Gamma(\theta), \theta) &= T_0(\hat{\mathcal{A}}^*(\theta)) + S_0(\mathcal{A}(\theta), \theta), \quad \text{deg}_{\hat{\mathcal{A}}^*(\theta)} T_0(\theta) = \text{deg}_{\mathcal{A}(\theta)} S_0(\theta) = 2, \\
T_0(\hat{\mathcal{A}}^*(\theta)) &= \frac{1}{2} \left( \kappa_0^{-1} \right)^{\gamma_j}(\theta) \hat{\mathcal{A}}_j^*(\theta) \hat{\mathcal{A}}_k^*(\theta)(-1)^{\varepsilon_i}, \quad \hat{\mathcal{A}}^*_j(\theta) = \mathcal{A}^*_j(\theta) - T_1(\mathcal{A}(\theta)), \\
\hat{\mathcal{A}}^*_j(\theta) &= \frac{\partial T_1(\hat{\mathcal{A}}(\theta))}{\partial \hat{\mathcal{A}}^*(\theta)} = \left( \kappa_{ij}(\theta) \hat{\mathcal{A}}^*_j(\theta) - \frac{1}{2} \frac{\partial \kappa_{ij}(\theta)}{\partial \hat{\mathcal{A}}^*(\theta)} \hat{\mathcal{A}}^*_j(\theta) \hat{\mathcal{A}}^*_k(\theta)(-1)^{\varepsilon_k} \right) (-1)^{\varepsilon_j}, \\
\kappa_{ij}(\hat{\mathcal{A}}(\theta)) &= \kappa_{ij}(\hat{\mathcal{A}}(\theta)) + \kappa_{ij}(\hat{\mathcal{A}}(\theta)), \quad \text{min deg}_{\hat{\mathcal{A}}^*(\theta)} \kappa_{ij}(\theta) \geq 1, \quad \text{deg}_{\mathcal{A}(\theta)} \kappa_{ij}(\theta) = 0, \\
\Sigma_i^H(\theta) &= -S_{\nu_j}(\mathcal{A}(\theta), \theta)(-1)^{\varepsilon_i} = 0.
\end{align}

Equation (6.24d) is resolvable with respect to $\hat{\mathcal{A}}^*(\theta)$

\begin{align}
\hat{\mathcal{A}}^*_i(\theta) &= h^{\nu_j}(\hat{\mathcal{A}}^*(\theta)) \hat{\mathcal{A}}^*_j(\hat{\mathcal{A}}^*(\theta))(-1)^{\varepsilon_i}, \quad h^{\nu_j}(\hat{\mathcal{A}}^*(\theta)) = (\kappa_0^{-1})^{\nu_j}(\theta) + h^{\nu_j}(\hat{\mathcal{A}}^*(\theta)), \\
\text{min deg}_{\hat{\mathcal{A}}^*(\theta)} h^{\nu_j} &\geq 1, \quad h^{\nu_j}(\hat{\mathcal{A}}^*(\theta)) = P_0(\theta) h^{\nu_j}(\hat{\mathcal{A}}^*(\theta)), \quad (\kappa_0^{-1})^{\nu_j}(\theta) = -(-1)^{\varepsilon_j}(\kappa_0^{-1})^{\nu_j}(\theta).
\end{align}

In (6.24) $S_{H_0}(\theta), T_0(\theta)$ are quadratic parts with respect to extended superantifields $\hat{\mathcal{A}}^*_j(\theta)$ and $\mathcal{A}^*_j(\theta)$, whereas $S_{H_{\text{int}}}(\theta)$ is at least cubic on $\mathcal{A}^*_j(\theta), \hat{\mathcal{A}}^*_j(\theta)$. Condition (6.7b) and identities with linearly independent operators $\mathcal{R}_{H_0}^j(\mathcal{A}(\theta), \theta)$ are given by the relations respectively

\begin{align}
\text{rank} \| S_{\nu_j}(\mathcal{A}(\theta), \theta) \|_{\Sigma} = n - m, \quad S_{\nu_j}(\mathcal{A}(\theta), \theta) \mathcal{R}_{H_0}^j(\mathcal{A}(\theta), \theta) = 0.
\end{align}

**Remark**: If HCHF are resolvable then the explicit dependence upon $\theta$ does not take place for all superfunctions in Corollaries 1.1, 1.2.

**Corollary 1.3**
If for GCHF (3.5c) (HCHF $\Theta_i^H(\theta)$) the conditions almost everywhere in any neighbourhood $U \subset \mathcal{M}_{cl}$ are realized:

\begin{align}
\text{rank} \| \delta_{\hat{\mathcal{A}}^*(\theta_1)} Z_{\mathcal{H}}^{(1)}(\Gamma, D) \|_{T_{\text{od}} U} = n, \quad \left( \text{rank} \| \Theta_i^H(\theta) \|_{U} = n \right),
\end{align}

then all $\Theta_i^H(\theta)$ ($\Theta_i^H(\theta)$) appear by linearly independent and already are in GHF.

**Corollary 1.4**
Under a particular choice of $S_H(\Gamma(\theta), \theta)$ in Corollary 1.2 being not explicitly dependent upon $\theta$ and for $T_1(\mathcal{A}(\theta)) = 0$ (natural system) in the form

\begin{align}
S_H(\Gamma(\theta)) = T(\mathcal{A}^*(\theta)) + S(\mathcal{A}(\theta)),
\end{align}

the Hamiltonian subsystem in GHS (3.51) obtained via only Legendre transform (3.2) without regard for dynamical equations of HCLF (2.3b) is represented by means of the system consisting of the 1st subsystem in (3.51a) and identities (3.52)

\begin{align}
\frac{d\mathcal{A}^*(\theta)}{d\theta} = \frac{\partial S_H(\Gamma(\theta))}{\partial \mathcal{A}^*(\theta)} = \frac{\partial T(\mathcal{A}^*(\theta))}{\partial \mathcal{A}^*(\theta)}, \quad \frac{d\mathcal{A}^*_j(\theta)}{d\theta} = 0.
\end{align}

a) The above system (6.28) satisfies to solvability condition, but $S_H(\Gamma(\theta))$ is not its integral;
b) By adding to that system the HCHF $\overline{\Theta}_i^H(\theta)$ the solvability conditions for given HS is conserved and $S_H(\Gamma(\theta))$ appears now by integral for obtained GHS to be equivalent to LS (2.3). However the HCHF themselves do not appear by solvable in view of nonfulfillment of Eqs.(3.39). Corresponding integral curve according to Existence and Uniqueness Theorems for solution of the 1st order on $\theta$ ODE [1] has the form

\[
\mathcal{A}_i^\prime(\theta) = \mathcal{A}_i^0(0) + \frac{\partial T(A^\ast(\theta))}{\partial A^\ast_i(\theta)} \theta, \quad \mathcal{A}_i^\prime(\theta) = \mathcal{A}_i^0(0), \quad (6.29a)
\]

\[
\Theta_i^H(\mathcal{A}(\theta)) = (-1)^{\varepsilon_{i+1}} S_\tau(\mathcal{A}(\theta)) = 0. \quad (6.29b)
\]

The fulfilment of (6.29b) reflects the requirement of compatibility of $\mathcal{A}_i(\theta)$ with HCHF. Eqs.(6.29b) themselves for all $\theta$ already are the Euler-Lagrange equations for $S(\mathcal{A}(\theta))$ in an usual sense (as in the classical field theory).

If to start from a priority of superfunctionals $Z[\mathcal{A}]$ (2.1), $Z_H[\Gamma]$ (3.13), then they are defined with accuracy up to $P_0(\theta)$ component of one’s densities. This ambiguity leads to possibility to satisfy to the nondegeneracy conditions for $K(\theta)$ (2.21) and therefore for $K^*(\theta)$ (6.8) by means of addition of the corresponding $P_0(\theta) F(\theta)$. However above described operation can destroy a explicit superfield form for integrands ($S_L(\theta), S_H(\theta)$) in the expressions for $Z[\mathcal{A}], Z_H[\Gamma]$. If it is necessary to hold the structure of densities $S_L(\theta), S_H(\theta)$ by fixed (that is more natural from viewpoint of physics), then $Z[\mathcal{A}]$ and $Z_H[\Gamma]$ must not possess of the such above-mentioned ambiguity in one’s definitions and hypothesis (2.21) is not fulfilled obstructing to possibility for the Hamiltonization (3.2) realization. It means that assumption $3_H$ (6.8) does not take place. In this case a corresponding problem of the theory investigation in Hamiltonian formulation for GSTF is more complicated one and requires the combined study of DCLF (2.3b) and constraints arising on the 1st stage of Dirac-Bergmann algorithm [8]. The last problem appears by separate subject of investigation just as a study of the SCHF $\chi_a^H(\theta)$ independence problem both separately and simultaneously with GCHF $\Theta_i^H(\theta)$ in a presence of the assumptions $1_{HM}, 2_{HM}, 3_H$ and condition (6.12).

Led in Secs.II–VI investigation permits to introduce the terminology being analogous to one for the Lagrangian formulation of GSTF [1].

**Definitions:**

1) The model of superfield theory of fields (mechanics) being given by superfunction $S_H(\theta) \in \mathcal{O}^{k*} \times \{\theta\}$, $k \leq \infty$ (or by superfunctional $Z_H[\Gamma] \in C_{FH,cl}$) under condition (3.5c) (or by $Z_H^{(1)}[\Gamma, D]$) satisfying to assumptions $1_{H-3_H}$ (6.5)–(6.8) is called the gauge theory of general type (GThGT) of superfields $\Gamma^p(\theta)$ with nondegenerate $K^*(\theta)$ and in realizing of the 1st condition in (6.26) for GCHF the nondegenerate theory of general type (ThGT).

2) Under additional fulfilment of the Corollary 1.1 conditions (6.20) on HCHF $\overline{\Theta}_i^H(\theta)$ and $m > 0$ let us call the model by the gauge theory of special type (GThST) of superfields $\Gamma^p(\theta)$ with nondegenerate $K^*(\theta)$. In realizing for HCHF of the 2nd relation in (6.26) we will call the model by the nondegenerate theory of special type (ThST).

3) Call the formulation of GThGT, GThST with $S_H(\theta)$ (or $Z_H[\Gamma]$) by the Hamiltonian formalism for description of GThGT, GThST, or equivalently the Hamiltonian formalism (formulation) for GSTF.

**Remark:** At presence of SCHF $\chi_a^H(\Gamma(\theta))$ in the Hamiltonian formalism for the model description we will say that the GThGT (GThST) is given with SCHF.

4) GThGT (GThST) with or without SCHF, for which the $S_H(\Gamma(\theta), \theta)$ possibly satisfies to Eqs.(3.19), (3.39) on an integral curve of the corresponding system (GHS, HS, EGHS, EHS), we will regard by belonging to the I class of gauge field theory models. The same definition...
we relate to GThGT (GThST) with SCLF $\chi_c(A(\theta), \hat{A}(\theta))$ or without them, for which the Eqs. (2.5), (2.6), (3.38) on an integral curve of the corresponding system (LS, ELS) in the Lagrangian formalism for GSTF are possibly valid.

5) GThGT (GThST) for which the $S_L(\Gamma(\theta))$ satisfies to master equation (5.11) let us relate to gauge field theories of the II class. The same we will say on GThGT (GThST) in the Lagrangian formalism with $S_L(\theta)$ satisfying to (5.30).

A set of GThGT (GThST) is not divided onto models belonging only to the I class or only to the II one. As the example, consider in Hamiltonian formalism the GT hST belonging to the I class with SCHF $\chi_c^H(\Gamma(\theta))$ for $n_1 = n$

$$\frac{\partial S_H(\theta)}{\partial A_i^*(\theta)} = 0. \quad (6.30)$$

Given SCHF corresponds to SCLF $\chi_c(\hat{A}(\theta))$ defined in (2.7a). The latter means that $S_H(\Gamma(\theta))$ satisfying to (3.15), (2.18) may contain only potential term on solutions for (6.30)

$$S_H(\Gamma(\theta)) = -S(A(\theta)), \quad (6.31)$$

and therefore the model belongs on solutions for SCHF (6.30) to the II class theories.

As for the generators of gauge transformations of general type (special type) GGTGT (GGTST) in Lagrangian formalism [1] the identities (6.15) for GThGT and (6.22) for GThST with operators $\mathcal{R}_{\theta_H}(\theta; \theta')$, $\mathcal{R}_{\theta_H}(A(\theta), \theta)$ respectively, whose sets are complete and linearly independent, i.e. are the bases in corresponding linear spaces $Q(Z_H^{(1)}) = \text{Ker}\left\{\delta Z_H^{(1)}[R, D]\right\}$, $Q(S_H) = \text{Ker}\left\{\mathcal{R}_{\theta_H}(\theta)ight\}$, make possible the following interpretation for quantities $\mathcal{R}_{\theta_H}(\theta; \theta')$, $\mathcal{R}_{\theta_H}(\theta)$.

**Definitions:**

1) Any $\mathcal{R}_{\theta_H}(\Gamma(\theta), \theta) \in C^{k*} \times \{\theta\}$, $\mathcal{R}_{\theta_H}(A(\theta), \theta) \in C^k(\mathcal{M}_{cl} \times \{\theta\})$ satisfying to identities

$$\int d\theta \frac{\delta Z_H^{(1)}[\Gamma, D]}{\delta D^i(\theta)} \mathcal{R}_{\theta_H}(\Gamma(\theta), \theta) = 0, \quad (\varepsilon_p, \varepsilon_j, \varepsilon) \mathcal{R}_{\theta_H}(\theta) = (0, \varepsilon_i, \varepsilon_i), \quad (6.32a)$$

$$\Theta_{\theta_H}^H(A(\theta), \theta) \mathcal{R}_{\theta_H}(A(\theta), \theta) = 0, \quad (\varepsilon_p, \varepsilon_j, \varepsilon) \mathcal{R}_{\theta_H}(\theta) = (0, \varepsilon_i, \varepsilon_i), \quad (6.32b)$$

let us call by the GGTGT and GGTST respectively in the Hamiltonian formalism;

2) Call superfunctions $\hat{\tau}_H(\Gamma(\theta), \theta)$, $\tau_H(A(\theta), \theta)$ given by the formulae

$$\hat{\tau}_H(\Gamma(\theta), \theta) = \int d\theta \frac{\delta Z_H^{(1)}[\Gamma, D]}{\delta D^i(\theta')} \hat{E}_{\theta_H}^i(\Gamma(\theta), \theta; \theta'), \quad (\varepsilon_p, \varepsilon_j, \varepsilon) \hat{\tau}_H(\theta) = (0, \varepsilon_i, \varepsilon_i), \quad (6.33a)$$

$$\tau_H(A(\theta), \theta) = \Theta_{\theta_H}^H(A(\theta), \theta) \hat{E}_{\theta_H}^i(A(\theta), \theta), \quad (\varepsilon_p, \varepsilon_j, \varepsilon) \tau_H(\theta) = (0, \varepsilon_i, \varepsilon_i), \quad (6.33b)$$

the trivial GGTGT, GGTST respectively. Properties for superfunctions $\hat{E}_{\theta_H}^i(\theta; \theta')$, $E_{\theta_H}^i(\theta)$ are the same as for the analogous superfunctions in Lagrangian formalism for GSTF [1].

Any GGTGT can be represented respectively in the form

$$\mathcal{R}_{\theta_H}(\Gamma(\theta), \theta) = \int d\theta' \hat{R}_{\theta_H}(\Gamma(\theta), \theta; \theta') \hat{\xi}_c^a(\Gamma(\theta), \theta'), \quad \hat{\xi}_c^a(\theta) \in C^{k*} \times \{\theta\}, \quad (6.34)$$

$$\mathcal{R}_{\theta_H}(\Gamma(\theta), \theta) = R_{\theta_H}(A(\theta), \theta) \xi^{(1)}_0(A(\theta), \theta) + \tau_H(\Gamma(\theta), \theta), \quad \xi^{(1)}_0(\theta) \in C^k(\mathcal{M}_{cl} \times \{\theta\}), \quad (6.35)$$

$$(\varepsilon_p, \varepsilon_j, \varepsilon) \xi^{(1)}_0(\theta) = (\varepsilon_p, \varepsilon_j, \varepsilon) \xi^{(1)}_0(\theta) = (0, \varepsilon_\alpha, \varepsilon_\alpha).$$

These formulae convert $Q(Z_H^{(1)})$ in an affine $C^{k*} \times \{\theta\}$-module and $Q(S_H)$ in an affine $C^k(\mathcal{M}_{cl} \times \{\theta\})$-module.
The so-called equivalence transformations (affine transformations of modules \(Q\)) are valid for the basis elements \(\mathcal{R}^n_{HA}(\theta; \theta')\), \(\mathcal{R}^i_{HA}(\theta)\) respectively

\[
\mathcal{R}^n_{HA}(\theta; \theta') = \int d\theta_1 \left[ \hat{\mathcal{R}}^i_{HB}(\theta; \theta_1) \zeta_{\alpha_1}^\beta(\Gamma(\theta_1), \theta_1; \theta') + \frac{\delta_{\beta}^\gamma Z_{\theta \theta'}^1(\Gamma, D)}{\delta D^i(\theta_1)} \hat{E}^i_{HA}(\Gamma(\theta), \theta, \theta_1; \theta') \right],
\]

\[
\mathcal{R}^i_{HA}(\theta) = \mathcal{R}^i_{HA}(\mathcal{A}(\theta), \theta) \xi_{\alpha 0}(\mathcal{A}(\theta), \theta) + \Theta^H_j(\mathcal{A}(\theta), \theta) E_{HA}^j(\mathcal{A}(\theta), \theta).
\]

Properties of the superfunctions \(\hat{E}_{\theta}^1(\theta, \theta_1; \theta')\), \(E_{HA}^j(\theta, \theta')\), \(\zeta_{\alpha 0}(\theta, \theta')\) completely coincide with analogous ones for corresponding superfunctions in Lagrangian formalism [1]. Relationships (6.15) for GTThGT are interpreted as a consequence of invariance for superfunctional \(Z_{\theta}^1(\Gamma, D)\) with respect to following transformations of \(\Gamma^p(\theta), D^i(\theta)\) in the infinitesimal form

\[
(\Gamma^p(\theta), D^i(\theta)) \rightarrow (\Gamma^p(\theta), D^i(\theta)) = (\Gamma^p(\theta), D^i(\theta) + \delta D^i(\theta)) : \delta D^i(\theta) = \int d\theta' \hat{\mathcal{R}}^i_{HA}(\theta; \theta') \xi^\alpha(\theta')
\]

with arbitrary superfields \(\xi^\alpha(\theta)\) \((\varepsilon_P, \varepsilon_J, \varepsilon)\xi^\alpha(\theta) = (0, \varepsilon, \varepsilon)\) defined on \(\Lambda_{D|Nc+1}(z^a, \theta; \mathbf{K})\).

Really the sequence of the formulae holds

\[
Z_{\theta}^1(\Gamma', D') = Z_{\theta}^1(\Gamma, D) + \int d\theta d\theta' \Theta^H_i(\Gamma(\theta), \theta) \hat{\mathcal{R}}^i_{HA}(\theta; \theta') \xi^\alpha(\theta') = Z_{\theta}^1(\Gamma, D).
\]

Identities (6.22) for GTThST, in general, can not be interpreted as a consequence of invariance for \(S_{\theta}(\mathcal{A}(\theta), \theta)\) with respect to the infinitesimal transformations

\[
\mathcal{A}'(\theta) \rightarrow \mathcal{A}^\alpha(\theta) = \mathcal{A}'(\theta) + \delta \mathcal{A}'(\theta) : \delta \mathcal{A}'(\theta) = \mathcal{R}^i_{HA}(\mathcal{A}(\theta), \theta) \xi^\alpha(\theta)
\]

with arbitrary superfields \(\xi^\alpha_0(\theta)\) having the same Grassmann gradings as for \(\xi^\alpha(\theta)\) in (6.37). However for superfunction \(S(\mathcal{A}(\theta), \theta)\) in Corollaries 1.2, 1.4 (in the latter \(S(\theta), \hat{\mathcal{R}}^i_{HA}(\theta)\) do not depend on \(\theta\) explicitly) a property of its invariance holds just as in Lagrangian formalism [1].

Formally one can call the considered infinitesimal transformations written in the form (6.37) the GTGT for \(Z_{\theta}^1(\Gamma)\) and the GTST for \(S(\mathcal{A}(\theta), \theta)\), if latters are of the form (6.39).

The concluding remarks from Sec.VI of the paper [1] on irreducible and reducible GTThGT, GTThST and on investigations of corresponding gauge algebras of GTThGT, GTThST are valid for the Hamiltonian formalism of GSTF as well.

VII Extension of Superalgebra \(\mathcal{A}_{cl}\) up to \(\mathcal{B}_{cl}\)

Let us continue the action of special involution \(*\) [1] from \(\hat{\Lambda}_{D|Nc+1}(z^a, \theta; \mathbf{K})\) and \(C^k(T_{odd}\mathcal{M}_{cl} \times \{\theta\})\) onto \(D^k_{cl}\) by means of the relations

\[
(\Gamma^p(\theta))^* = \Gamma^p(\theta) = \Gamma^p(\theta), \quad \left(\hat{\Gamma}^p(\theta)\right)^* = \hat{\Gamma}^p(\theta),
\]

where \(\Gamma^p(\theta)\) are the superfields being \(*\)-conjugate to superfields \(\Gamma^p(\theta)\) with components

\[
\Gamma^p(\theta) = P_0(\theta)\Gamma^p(\theta) - P_1(\theta)\Gamma^p(\theta), \quad P_1(\theta)\Gamma^p(\theta) = -P_1(\theta)\Gamma^p(\theta).
\]

The subspace of superfields being invariant with respect to \(*\) is formed by the superfields

\[
P_0(\theta)\Gamma^p(\theta) = \frac{1}{2} \left(\Gamma^p(\theta) + \Gamma^p(\theta)\right), \quad \hat{\Gamma}^p(\theta).
\]
Superfields $Γ^p(θ)$ are transformed with respect to the supergroup $J$ superfield representation $\mathcal{T}$ being by $*$-conjugate to representation $T$: $(\mathcal{T}_J = T_J, \mathcal{T}_P = (T_P)^{-1})$. The restrictions of involution onto subsuperalgebras $0^0C^k(T^*_{odd}M_{cl} \times \{θ\}), 0^0D^k_{cl}$ appear by the identity mappings.

The continuation of the superalgebra $\mathcal{A}_{cl}$ [1] of the 1st order differential operators acting on $C^k(T_{odd}M_{cl} \times \{θ\})$ being by linear span of $\{U_a(θ), U_+(θ), \hat{U}_a(θ), \hat{U}_+(θ), a = 0, 1\}$ to superalgebra $\mathcal{B}_{cl}$ of the 1st and 2nd orders ones acting on $D^k_{cl}$ through elements of the basis

$$\mathcal{B}^b_{cl} = \left\{U_a(θ), \hat{U}_a(θ), V_a(θ), \hat{V}_a(θ) = \left[\frac{d}{dθ}, V_a(θ)\right]_s, \Delta_{ab}(θ), a, b = 0, 1\right\}.$$  \hspace{1cm} (7.4)

is realized with help of tensor product $⊗_θ$ introduced in [1] and written, for instance, for

$$U_a(θ) = (P_1(θ)A^i(θ))⊗_θ\frac{∂_t}{∂P_a(θ)A^i(θ)} =$$

$$(0, P_1(θ)A^i(θ))⊗_θ\left(\frac{∂_t}{∂P_0(θ)A^i(θ)}δ_{a0}, \frac{∂_t}{∂P_1(θ)A^i(θ)}δ_{a1}\right)^T \equiv P_1(θ)A^i(θ)\frac{∂_t}{∂P_a(θ)A^i(θ)}, \hspace{1cm} (7.5a)$$

$$\hat{U}_a(θ) = \hat{A}^i(θ)⊗_θ\frac{∂_t}{∂P_a(θ)A^i(θ)} =$$

$$\left(\hat{A}^i(θ), 0\right)⊗_θ\left(\frac{∂_t}{∂P_0(θ)A^i(θ)}δ_{a0}, \frac{∂_t}{∂P_1(θ)A^i(θ)}δ_{a1}\right)^T \equiv \hat{A}^i(θ)\frac{∂_t}{∂P_a(θ)A^i(θ)}.$$ \hspace{1cm} (7.5b)

For other operators (7.4) write compactly with allowance made for structure of formulae (7.5)

$$V_a(θ) = P_1(θ)A^*_i(θ)\frac{∂}{∂P_a(θ)A^*_i(θ)}; \hspace{0.5cm} \hat{V}_a(θ) = \hat{A}^*_i(θ)\frac{∂}{∂P_a(θ)A^*_i(θ)}, \hspace{1cm} (7.6a)$$

$$\Delta_{ab}(θ) = \frac{∂_t}{∂P_a(θ)A^i(θ)}\frac{∂}{∂P_b(θ)A^*_i(θ)}(-1)^{ε_p} = \frac{1}{2}\frac{∂_t}{∂P_a(θ)Γ^p(θ)}\frac{∂}{∂P_b(θ)Γ^p(θ)}(-1)^{ε_p}. \hspace{1cm} (7.6b)$$

Note that $Δ_{11}(θ) ≡ 0$ on $D^k_{cl}$. Properties of gradings for operators (7.4) are given by the table

| $U_a(θ)$ | $\hat{U}_a(θ)$ | $V_a(θ)$ | $\hat{V}_a(θ)$ | $Δ_{ab}(θ)$ |
|----------|----------------|----------|----------------|------------|
| $ε_p$    | 0              | 1        | 0              | 1          |
| $ε_J$    | 0              | 0        | 0              | 0          |
| $ε$      | 0              | 1        | 0              | 1          |
|          |                |          |                |            | (7.7)      |

In terms of the coordinates $Γ^p(θ)$ the operators $U_a(θ), V_a(θ)$ and $\hat{U}_a(θ), \hat{V}_a(θ)$ are combined into the expressions respectively

$$\mathcal{W}_a(θ) = U_a(θ) + V_a(θ) = P_1(θ)Γ^p(θ)\frac{∂_t}{∂P_a(θ)Γ^p(θ)}, \hspace{1cm} (7.8a)$$

$$\hat{\mathcal{W}}_a(θ) = \hat{U}_a(θ) + \hat{V}_a(θ) = \hat{Γ}^p(θ)\frac{∂_t}{∂P_a(θ)Γ^p(θ)}. \hspace{1cm} (7.8b)$$

By means of the basis elements (7.5), (7.6), (7.8) one can write the superfield (at least with respect to operators of differentiation) elements of $\mathcal{B}_{cl}$ not being contained in $\mathcal{A}_{cl}$

$$V_+(θ) = V_0(θ) + V_1(θ) = P_1(θ)A^*_i(θ)\frac{∂}{∂A^*_i(θ)}.$$ \hspace{1cm} (7.9a)
\[ V_- = V_0(\theta) - V_1(\theta) = P_1(\theta)A^*_p(\theta) \frac{\partial}{\partial A^*_p(\theta)} = -(V_+(\theta))^* , \quad (7.9b) \]
\[ \hat{V}_+ = \hat{A}^*_p(\theta) \frac{\partial}{\partial A^*_p(\theta)} \quad \hat{V}_- = \hat{A}^*_p(\theta) \frac{\partial}{\partial A^*_p(\theta)} = (\hat{V}_+)^* , \quad (7.9c) \]
\[ \Delta_{+\theta}(\theta) = \sum_{a,b} \Delta_{ab} = (-1)^{\varepsilon_a} \frac{\partial}{\partial A^a(\theta)} \frac{\partial}{\partial A^*_b(\theta)} , \quad (7.10a) \]
\[ \Delta_{-\theta}(\theta) = \sum_{a,b} (-1)^{\varepsilon_a} \frac{\partial}{\partial P_a(\theta)A^{*}(\theta)} \frac{\partial}{\partial P_b(\theta)A^*_p(\theta)} = (-1)^{\varepsilon_a} \frac{\partial}{\partial A^a(\theta)} \frac{\partial}{\partial A^*_b(\theta)} , \quad (7.10b) \]
\[ \Delta_{-\theta}(\theta) = \sum_{a,b} (-1)^{\varepsilon_a} \frac{\partial}{\partial P_a(\theta)A^{*}(\theta)} \frac{\partial}{\partial P_b(\theta)A^*_p(\theta)} = (-1)^{\varepsilon_a} \frac{\partial}{\partial A^a(\theta)} \frac{\partial}{\partial A^*_b(\theta)} = (\Delta_{+\theta}(\theta))^* , \quad (7.10c) \]
\[ \Delta_{\theta}(\theta) = \sum_{a,b} (-1)^{\varepsilon_a} \frac{\partial}{\partial P_a(\theta)A^{*}(\theta)} \frac{\partial}{\partial P_b(\theta)A^*_p(\theta)} = (-1)^{\varepsilon_a} \frac{\partial}{\partial A^a(\theta)} \frac{\partial}{\partial A^*_b(\theta)} = (\Delta_{+\theta}(\theta))^* . \quad (7.10d) \]

By writing of (7.9), (7.10) it is taken into consideration according to Ref.[1] that by relationships
\[
\frac{\partial}{\partial P_0(\theta)A^{*}(\theta)} = \frac{\partial}{\partial P_0(\theta)A^*_p(\theta)} - \frac{\partial}{\partial P_1(\theta)A^*_p(\theta)} = \left( \frac{\partial}{\partial A^*_p(\theta)} \right)^* , \quad (7.11) \]

involution * is extended onto \( \mathcal{B}_{cl} \) as well.

The superfield elements built from operators (7.8) have the form in coordinates \( \Gamma^p(\theta) \)
\[
\mathcal{W}_+(\theta) = \mathcal{W}_0(\theta) + \mathcal{W}_1(\theta) = P_1(\theta)\Gamma^p(\theta) \frac{\partial}{\partial \Gamma^p(\theta)} , \quad (7.12a) \]
\[
\mathcal{W}_-(\theta) = \mathcal{W}_0(\theta) - \mathcal{W}_1(\theta) = P_1(\theta)\Gamma^p(\theta) \frac{\partial}{\partial \Gamma^p(\theta)} = -(\mathcal{W}_+(\theta))^* , \quad (7.12b) \]
\[
\hat{\mathcal{W}}_+(\theta) = \hat{\mathcal{W}}_0(\theta) \frac{\partial}{\partial \Gamma^p(\theta)} , \quad \hat{\mathcal{W}}_-(\theta) = \hat{\mathcal{W}}_0(\theta) \frac{\partial}{\partial \Gamma^p(\theta)} = (\hat{\mathcal{W}}_+(\theta))^* , \quad (7.12c) \]
\[
\frac{\partial}{\partial \Gamma^p(\theta)} = \frac{\partial}{\partial P_0(\theta)\Gamma^p(\theta)} - \frac{\partial}{\partial P_1(\theta)\Gamma^p(\theta)} = \left( \frac{\partial}{\partial \Gamma^p(\theta)} \right)^* . \quad (7.13) \]

The operators \( U_1(\theta), \hat{U}_0(\theta), V_1(\theta), \hat{V}_0(\theta), \mathcal{W}_1(\theta), \hat{\mathcal{W}}_0(\theta), \Delta_{00}(\theta), \Delta_{11}(\theta) \) are invariant with respect to *, but the only \( \hat{U}_\pm(\theta), \hat{V}_\pm(\theta), \hat{\mathcal{W}}_\pm(\theta), \Delta_{\pm\pm}(\theta), \Delta_{\pm\mp}(\theta) \) are defined in the superfield form with respect to \( \tilde{T}, \tilde{T} \) representations. Operators \( V_+(\theta), \mathcal{W}_+(\theta), U_+(\theta) \) appear by the realizations of projectors \( V(\theta), \mathcal{W}(\theta), U(\theta) \) on \( D_{cl}^k \) in the relation (2.37).

All algebraic properties for \( \mathcal{A}_{cl} \) [1] under the composition of operators are literally transferred onto subsuperalgebra \( \mathcal{B}_{cl}^i \) of the 1st order operators in \( \mathcal{B}_{cl} \). The additional properties for analogous ones from [1] have the form

1) \[ V^2_a = \delta_{a1} V_a \quad V^2_\pm = \pm V_\pm \quad V_+ V_- = V_- \quad V_- V_+ = -V_+ \] 2) \[ [V_a, V_b]_- = \varepsilon_{ab} V_0 \quad [V_+, V_-] = 2V_0 \quad \varepsilon_{ab} = -\varepsilon_{ba}, \varepsilon_{10} = 1, a, b = 0, 1, \]
\[ [V_+, V_-] = -2V_1 \quad [V_+, V_a]_- = (-1)^a V_0 \quad [V_-, V_a]_- = -V_0 \] 3) \[ \hat{V}^2_i = 0 \quad \hat{V}_i \hat{V}_j \]
4) \[ [\bar{V}_0, V_i]_\theta = 0 , \quad [\bar{V}_1, V_i]_\theta = \bar{V}_i , \quad [\bar{V}_\pm, V_i]_\theta = \pm \bar{V}_i , \quad (7.14d) \]

5) \[ [U_i, V_j]_\theta = [U_i, V_j] = [\bar{U}_i, V_j] = [\bar{U}_i, V_j]_\theta = 0 . \quad (7.14e) \]

For subset $\bar{B}_ct$ with basis $\{U_\alpha(\theta), V_\alpha(\theta)\}$ the remarks being analogous for $\bar{A}_ct$ [1] are valid.

For the 2nd order operators let us point out only the following formulae

\[
(\Delta^2(\theta))^2 = \left[ \Delta^2(\theta), \bar{W}_+(\theta) \right]_+ = \left[ \Delta^2(\theta), \bar{U}_+(\theta) \right]_+ = \left[ \Delta^2(\theta), \bar{V}_+(\theta) \right]_+ = \Delta^2_{ab}(\theta) = 0 , \quad (7.15a)
\]

\[
\left[ \Delta^2(\theta), W_+(\theta) \right]_+ = \Delta_{1+}(\theta) + \Delta_{+1}(\theta) . \quad (7.15b)
\]

A deviation of $\Delta_{ab}(\theta)$ in order to be derivation in acting on the product of arbitrary $\mathcal{F}(\theta)$, $\mathcal{J}(\theta) \in D^k$ yields the definition for antibrackets $\Delta_{ab}(\theta)(\mathcal{F}(\theta) \cdot \mathcal{J}(\theta)) = (\Delta_{ab}(\theta)\mathcal{F}(\theta))\mathcal{J}(\theta) + (-1)^{\varepsilon(\mathcal{F})}\mathcal{F}(\theta)\Delta_{ab}(\theta)\mathcal{J}(\theta) + \ldots$

\[
(\mathcal{F}(\theta), \mathcal{J}(\theta))_{ab} = \frac{\partial \mathcal{F}(\theta)}{\partial P_a(\theta) A^i(\theta)} \frac{\partial \mathcal{J}(\theta)}{\partial P_b(\theta) A^i(\theta)} - (-1)^{(\varepsilon(\mathcal{F})+1)(\varepsilon(\mathcal{J})+1)} (\mathcal{F} \mapsto \mathcal{J}) \quad (7.17)
\]

which have the gradings as in (3.25). Antibracket $(\ , )_{11}$ is not obtained by means of (7.16) from operator $\Delta_{11}(\theta) \equiv 0$. It follows from (7.17) the definitions for antibrackets in the superfield form

\[
(\mathcal{F}(\theta), \mathcal{J}(\theta))_{++} = \sum_{a,b} (\mathcal{F}(\theta), \mathcal{J}(\theta))_{ab} = (\mathcal{F}(\theta), \mathcal{J}(\theta))_\theta , \quad (7.18a)
\]

\[
(\mathcal{F}(\theta), \mathcal{J}(\theta))_{+-} = \sum_{a,b} \left( \frac{\partial \mathcal{F}(\theta)}{\partial P_a(\theta) A^i(\theta)} \frac{\partial \mathcal{J}(\theta)}{\partial P_b(\theta) A^i(\theta)} - (-1)^{(\varepsilon(\mathcal{F})+1)(\varepsilon(\mathcal{J})+1)} (\mathcal{F} \mapsto \mathcal{J}) \right) , \quad (7.18b)
\]

\[
(\mathcal{F}(\theta), \mathcal{J}(\theta))_{-+} = \sum_{a,b} \left( \frac{\partial \mathcal{F}(\theta)}{\partial P_a(\theta) A^i(\theta)} \frac{\partial \mathcal{J}(\theta)}{\partial P_b(\theta) A^i(\theta)} - (-1)^{(\varepsilon(\mathcal{F})+1)(\varepsilon(\mathcal{J})+1)} (\mathcal{F} \mapsto \mathcal{J}) \right) , \quad (7.18c)
\]

\[
(\mathcal{F}(\theta), \mathcal{J}(\theta))_{--} = \sum_{a,b} \left( \frac{\partial \mathcal{F}(\theta)}{\partial P_a(\theta) A^i(\theta)} \frac{\partial \mathcal{J}(\theta)}{\partial P_b(\theta) A^i(\theta)} - (-1)^{(\varepsilon(\mathcal{F})+1)(\varepsilon(\mathcal{J})+1)} (\mathcal{F} \mapsto \mathcal{J}) \right) . \quad (7.18d)
\]

Antibrackets (7.17), (7.18) satisfy to standard properties for odd Poisson bracket (3.25), (3.27). In particular, Jacobi identity for corresponding antibrackets can be obtained from action of squared (formally) operators $\Delta_{ab}(\theta)$, $\Delta_{\pm\pm}(\theta)$, $\Delta_{\pm\mp}(\theta)$ on the product of 3 arbitrary superfunsions from $D^k$.

Operators $\bar{U}_i(\theta)$, $\bar{V}_i(\theta)$, $\Delta_{ij}(\theta)$, $i, j \in \{0, 1, +, -\}$ differentiate the antibrackets $(\ , )_{ij}$ with corresponding (coinciding for $\Delta_{ij}(\theta)$ and $(\ , )_{ij}$) indices by Leibnitz rule. For example, for any from operators $B(\theta) \in \{\bar{U}_i(\theta), \bar{V}_i(\theta), \Delta^{ij}(\theta)\}$ and for $(\ , )_\theta$ the following relation holds

\[
B(\theta)(\mathcal{F}(\theta), \mathcal{J}(\theta))_\theta = (B(\theta)\mathcal{F}(\theta), \mathcal{J}(\theta))_\theta + (-1)^{(\varepsilon(\mathcal{F})+1)}(\mathcal{F}(\theta), B(\theta)\mathcal{J}(\theta))_\theta . \quad (7.19)
\]

In fact the relationships (7.16)–(7.18), (7.6), (7.10), (5.6), (5.7) make having the same rights a construction of antisymplectic differential geometry on $T^*_{\text{odd}}\mathcal{M}_c$ both starting from the operators $\Delta_{ij}(\theta)$, $i, j \in \{0, 1, +, -\}$, next obtaining the antibrackets $(\ , )_{ij}$ and vice versa starting from the antibrackets(!) (with exception of $(\ , )_{11}$ and $\Delta_{11}(\theta) \equiv 0$).
With help of antibrackets one can define a so-called transformation of the operators from $\mathcal{B}_cl$. Let us confine ourselves by the case of operators $B(\theta) \in \{ \hat{U}_+ (\theta), \hat{V}_+ (\theta), \Delta^{cl}(\theta) \}$ having required the fulfillment of the transformation rule

$$B(\theta) \mapsto B'(\theta) = B(\theta) + (\mathcal{F}(\theta), B(\theta))_\theta \equiv B(\theta) + \text{ad}_{\mathcal{F}(\theta)}, (\varepsilon_p, \varepsilon_j, \varepsilon)\mathcal{F}(\theta) = (0, 0, 0) \, . \quad (7.20)$$

The condition of nilpotency conservation for $B'(\theta)$ with allowance made for antibracket’s properties (3.27) leads to the equation on a superfunction $\mathcal{F}(\theta) \in D^k_{cl}$

$$B(\theta)\mathcal{F}(\theta) + \frac{1}{2} (\mathcal{F}(\theta), \mathcal{F}(\theta))_\theta = f(\hat{\mathcal{G}}(\theta), \theta), (\varepsilon_p, \varepsilon_j, \varepsilon)f(\theta) = (1, 0, 1), f(\theta)|_{T_{odd}^* \mathcal{M}_cl} = 0 \, . \quad (7.21)$$

Transformation (7.20) permits to achieve of the vanishing for one(!) from $B(\theta)$ with help of a special choice for $\mathcal{F}(\theta)$ satisfying to (7.21). So putting successively

$$\mathcal{F}_1(\theta) = \hat{\mathcal{A}}'(\theta) \mathcal{A}^*_\theta(\theta), \quad \mathcal{F}_2(\theta) = -\hat{\mathcal{A}}'_\theta(\theta) \mathcal{A}'(\theta) \, , \quad (7.22)$$

we obtain respectively

$$\hat{U}'_+(\theta) = 0 \iff \hat{U}_+(\theta) = -\text{ad}_{\mathcal{F}_1(\theta)}, \quad \hat{V}'_+(\theta) = 0 \iff \hat{V}_+(\theta) = -\text{ad}_{\mathcal{F}_2(\theta)} \, . \quad (7.23)$$

Choosing as $\mathcal{F}(\theta)$ in (7.20) the sum of $\mathcal{F}_1(\theta)$ and $\mathcal{F}_2(\theta)$ from (7.22) we derive according to (7.20), (7.22), (7.23) the formulae

$$\hat{U}'_+(\theta) = -\hat{V}_+(\theta), \quad \hat{V}'_+(\theta) = -\hat{U}_+(\theta) \, . \quad (7.24)$$

For arbitrary $\mathcal{F}_1(\theta), \mathcal{F}_2(\theta) \in D^k_{cl}$ a requirement of the properties (7.14e) conservation taking account of antibracket’s ones (3.27) and (7.19) leads to necessity of the compatibility condition fulfilment

$$\hat{V}_+(\theta)\mathcal{F}_1(\theta) + \hat{U}_+(\theta)\mathcal{F}_2(\theta) + (\mathcal{F}_1(\theta), \mathcal{F}_2(\theta))_\theta = f_1(\hat{\mathcal{G}}(\theta), \theta) \, ,$$

$$(\varepsilon_p, \varepsilon_j, \varepsilon)f_1(\theta) = (1, 0, 1), f_1(\theta)|_{T_{odd}^* \mathcal{M}_cl} = 0 \quad (7.25)$$

which is the additional relation to Eqs.(7.21) for $B(\theta) \in \{ \hat{U}_+(\theta), \hat{V}_+(\theta) \}$.

**VIII Component Formulation**

Continue in the framework of the Hamiltonian formulation for GSTF a programme, started in [1] for Lagrangian formalism, of the establishment of connection between superfield and component field quantities and relations of GSTF.

In fact all formulae of the analogous section in Ref.[1] are valid here as well under formal change of the form $\mathcal{A}^*(\theta) \to \Gamma^p(\theta) = (\mathcal{A}^*(\theta), \mathcal{A}_\theta^*(\theta))$ in the corresponding ones from [1]. So from (2.47) let us find the expression for densities of superfunctionals on $D^k_{cl}$ through latters themselves

$$\mathcal{F}(\Gamma(\theta), \hat{\mathcal{G}}(\theta), \theta) = P_0(\theta)\mathcal{F}(\theta) + \theta F_{H, cl}[\Gamma] \equiv \mathcal{F}(P_0\Gamma(\theta), \hat{\mathcal{G}}(\theta), 0) + \theta \mathcal{F}_{H, cl}[P_0\Gamma, \hat{\mathcal{G}}] \equiv$$

$$\mathcal{F}(\Gamma_0, \Gamma_1, 0) + \theta \mathcal{F}_{H, cl}[\Gamma_0, \Gamma_1] \, ,$$

$$\Gamma^p(\theta) = \Gamma^p_0 + \Gamma^p_1 \theta \equiv (\mathcal{A}^* + \lambda \theta, \mathcal{A}^*_\theta - \theta \mathcal{J}_\lambda) \, . \quad (8.1)$$
The formula
\[
\frac{\partial_r \mathcal{F}(\Gamma(\theta), \hat{\Gamma}(\Gamma(\theta), \theta))}{\partial \Gamma^p(\theta)} = \frac{\partial_r \mathcal{F}(\theta)}{\partial \Gamma^0(\theta) \Gamma^p(\theta)} + \frac{\partial_r \mathcal{F}(\theta)}{\partial \Gamma^1(\theta) \Gamma^p(\theta)} = \frac{\delta_r \mathcal{F}(\theta)}{\delta \Gamma^0(\theta)} + \frac{\delta_r \mathcal{F}(\theta)}{\delta \Gamma^1(\theta)}
\] (8.3)
establishes the connection of the partial superfield derivative with respect to superfield \( \Gamma^p(\theta) \) with component ones.

The component representation for supermatrix of the 2nd partial superfield derivatives of \( \mathcal{F}(\theta) \) with respect to \( \Gamma^p(\theta), \Gamma^q(\theta) \) is literally the same as in [1] under change of \( \mathcal{A}^i(\theta), \hat{\mathcal{A}}^i(\theta) \) on \( \Gamma^p(\theta), \Gamma^q(\theta) \) respectively.

From formulae
\[
\frac{\partial_r}{\partial \left( \frac{d_r \Gamma^p(\theta)}{d\theta} \right)} = \frac{\delta_r}{\delta \Gamma^1(\theta)} \left( -1 \right)^{\varepsilon_p+1} \equiv \left( \left( \frac{d_r \delta_r}{d\theta} \delta \lambda^i \right) \left( -1 \right)^{\varepsilon_i+1}, \left( \frac{d_r \delta_r}{d\theta} \delta \hat{\lambda}_i \right) \right),
\] (8.5)
where \( \left( \frac{d_r \delta_r}{d\theta} \right) \) are considered as a single differential object acting on \( D^k_{cl} \).

Connection for the right and left derivatives of the form (8.5) is yielded by the formulae
\[
\frac{\delta_r \mathcal{F}(\theta)}{\delta \Gamma^1(\theta)} = \left( -1 \right)^{\varepsilon_p+1} \delta_r P_0 \mathcal{F}(\theta) \delta \Gamma^0(\theta) + \frac{\partial_r \mathcal{F}_{H,cl}[\Gamma_0, \Gamma_1]}{\partial \Gamma^p(\theta)} \left( -1 \right)^{\varepsilon_{(H,cl)}} =
\] (8.6)

The relationship of the right variational superfield derivative with respect to \( \Gamma^p(\theta) \) and the component variational (in an usual sense) ones on \( \Gamma^0_0 \) and \( \Gamma^1_1 \) follows from (2.2), (2.48b), (8.1)

\[
\frac{\delta_r \mathcal{F}_{H,cl}[\Gamma]}{\delta \Gamma^p(\theta)} = \delta_r P_0 \mathcal{F}(\theta) \delta \Gamma^0(\theta) + \theta \frac{\partial_r \mathcal{F}_{H,cl}[\Gamma_0, \Gamma_1]}{\partial \Gamma^p(\theta)} + \frac{\partial_r \mathcal{F}_{H,cl}[\Gamma_0, \Gamma_1]}{\partial \Gamma^p(\theta)} \left( -1 \right)^{\varepsilon_{(H,cl)}} =
\]

\[
\frac{\partial_r P_0 \mathcal{F}(\theta)}{\partial \Gamma^p(\theta)} + \theta \frac{\partial_r \mathcal{F}_{H,cl}[\Gamma_0, \Gamma_1]}{\partial \Gamma^p(\theta)} + \frac{\partial_r \mathcal{F}_{H,cl}[\Gamma_0, \Gamma_1]}{\partial \Gamma^p(\theta)} \left( -1 \right)^{\varepsilon_{(H,cl)}} =
\]

\[
\frac{\partial_r P_0 \mathcal{F}(\theta)}{\partial \Gamma^p(\theta)} + \theta \frac{\partial_r \mathcal{F}_{H,cl}[\Gamma_0, \Gamma_1]}{\partial \Gamma^p(\theta)} + \frac{\partial_r \mathcal{F}_{H,cl}[\Gamma_0, \Gamma_1]}{\partial \Gamma^p(\theta)} \left( -1 \right)^{\varepsilon_{(H,cl)}} =
\]

(8.7)

From (8.7) it follows as in [1] the formulae in coordinates \( \Gamma^p(\theta) \) and (\( A^i(\theta), A^*_i(\theta) \)) respectively

\[
\frac{d_r \delta_r \mathcal{F}_{H,cl}[\Gamma]}{d\theta \delta \Gamma^p(\theta)} = \left( -1 \right)^{\varepsilon_p+1} \mathcal{F}_{H,cl}[\Gamma],
\]

(8.8)

\[
\left( \frac{d_r \delta_r}{d\theta} \delta A^i(\theta), \frac{d_r \delta_r}{d\theta} \delta A^*_i(\theta) \right) F_{H,cl}[\mathcal{A}, A^*] = \left( -1 \right)^{\varepsilon_i+1} \frac{\partial_r \mathcal{F}_{H,cl}[\Gamma_0, \Gamma_1]}{\partial \Gamma^p(\theta)} \delta_r \mathcal{F}_{H,cl}[\Gamma_0, \Gamma_1],
\]

(8.9)
Relationships being analogous to (8.8) for a superfunction $F(\theta)$ have the form
\[
\frac{d_r}{\delta \Gamma^p(\theta_1)} \mathcal{F}(\Gamma(\theta), \Phi(\theta), \theta) = \frac{d_r}{\delta \Gamma^p(\theta_1)} \left( \delta(\theta_1 - \theta) \frac{\partial_r}{\partial \Gamma^p(\theta_1)} \mathcal{F}(\Gamma(\theta_1), \Phi(\theta_1), \theta_1) \right) = \left( -1 \right)^{\epsilon(F)} \frac{\partial_r}{\partial \Gamma^p(\theta_1)} \mathcal{F}(\Gamma(\theta), \Phi(\theta), \theta) = \left( -1 \right)^{\epsilon(F)} \frac{\partial_r \mathcal{F}(\theta)}{\delta \Gamma^p_0}.
\] (8.10)

In obtaining (8.10) the identities have been made use
\[
P_0(\theta_1) \mathcal{F}(\theta_1) \equiv P_0(\theta) \mathcal{F}(\theta), \quad \frac{d}{d \theta} \mathcal{F}(\theta_1) \equiv \frac{d}{d \theta} \mathcal{F}(\theta), \quad \frac{d^2}{d \theta^2} \mathcal{F}(\theta_1) \equiv 0.
\] (8.11)

All above relations are sufficient in order to get the component expressions for operators from $\mathcal{B}_\text{cl}$ being additional to ones obtained in Ref.[1]. Let us point out the formulae only for basis operators $V_0(\theta), \bar{V}_0(\theta), \Delta_{00}(\theta), a, b = 0, 1$
\[
V_0(\theta) = -\theta J, \quad \bar{V}_0(\theta) = -\bar{J}, \quad \frac{\delta}{\delta \bar{A}^i} \bar{V}_1(\theta) = -J, \quad \frac{\delta}{\delta \bar{A}^i} V_1(\theta) = -\theta J, \quad \frac{\delta}{\delta \bar{A}^i} \bar{V}_1(\theta) = -\theta J, \quad \frac{\delta}{\delta \bar{A}^i} V_1(\theta) = -\bar{J},
\] (8.12)
\[
\Delta_{00}(\theta) = \frac{\delta_t}{\delta A^i} \delta(-\theta J), \quad \Delta_{01}(\theta) = \frac{\delta_t}{\delta A^i} \delta(-\bar{J}), \quad \Delta_{11}(\theta) = \frac{\delta_t}{\delta A^i} \delta(-\bar{J}).
\] (8.13)
\[
\Delta_{10}(\theta) = \left( \frac{d}{d \theta} \right) \left( \frac{d}{d \theta} \right) \delta(-\bar{J}^i) = \left( \frac{d}{d \theta} \right) \left( \frac{d}{d \theta} \right) \delta(-\bar{J}^i).
\] (8.14)

At last for the antibrackets (7.17) calculated on arbitrary $\mathcal{F}(\theta),\mathcal{J}(\theta)$ from $D_3^\text{cl}$ we have
\[
(\mathcal{F}(\theta), \mathcal{J}(\theta))_{00} = \frac{\delta \mathcal{F}(\theta)}{\delta A^i} \frac{\delta \mathcal{J}(\theta)}{\delta A^i} - \left( -1 \right)^{\epsilon(F)+1}(\epsilon(J)+1)(\mathcal{F} \leftrightarrow \mathcal{J}).
\] (8.15)

The antibrackets $(\mathcal{F}(\theta), \mathcal{J}(\theta))_{01}, (\mathcal{F}(\theta), \mathcal{J}(\theta))_{10}, (\mathcal{F}(\theta), \mathcal{J}(\theta))_{11}$ are yielded from (8.15) under corresponding changes for operators $\frac{\delta}{\delta \bar{A}^i}$ on $\delta(-\theta J)$, $\frac{\delta}{\delta \bar{A}^i}$ on $\delta(-\bar{J})$, and under their simultaneous change for antibracket $(\ , \ )_{11}$.

Formulae of this Section are sufficient in order to write all relations of the Hamiltonian formulation for GSTF in the component form.

IX Models in the Hamiltonian Formulation for GSTF

Continue investigation of the models from work [1] made in the Lagrangian formulation for the case of the Hamiltonian one.

IX.1 Models of Massive Complex Scalar Superfield

Begin from introduction of the superantifields\(^5\) $(\varphi_j(x, \theta))^* \in \Lambda_{40+1}(x^\mu, \theta; \mathbb{R}), j = 1, 2$ for real component superfields $\varphi_j(x, \theta)$ forming the complex scalar superfield $\varphi(x, \theta) \in \Lambda_{40+1}(x^\mu, \theta; \mathbb{C})$

\(^5\)in contrast to the designation of the complex conjugation $^*$ for superfields $\varphi^*(x, \theta)$ adopted in Ref.[1], in this section the sign " * " is reserved for notation of the only superantifields, whereas the complex conjugate quantity is denoted with bar: $\bar{g}(x, \theta)$ unlike of $\theta$-conjugate quantity $\tilde{g}(x, \theta)$ for $g(x, \theta)$ as in Sec.VII
defined on $\mathcal{M} = \mathbb{R}^{1,3} \times \tilde{P}$ [1]
\[
\varphi(x, \theta) = \varphi_1(x, \theta) + i\varphi_2(x, \theta) = \varphi(x) + \lambda(x) \theta .
\]

(9.1)

The noncontradictory join of the real superantifields $(\varphi_j(x, \theta))^*$ into complex ones $(\varphi(x, \theta))^*$, $(\overline{\varphi}(x, \theta))^*$ is realized by the formulae

\[
\begin{align*}
(\varphi(x, \theta))^* & = (\varphi_1(x, \theta))^* - i(\varphi_2(x, \theta))^* = (\varphi(x))^* - \theta J_\varphi(x), \quad (\varphi_j(x, \theta))^* = (\varphi_j(x))^* - \theta J_{\varphi_j}(x), \quad (9.2a) \\
(\overline{\varphi}(x, \theta))^* & = (\varphi_1(x, \theta))^* - i(\varphi_2(x, \theta))^* = (\varphi(x, \theta))^* + i(\varphi_2(x, \theta))^* , \quad (9.2b)
\end{align*}
\]

providing the fulfilment of the relationship with bar from the left-hand side meaning in question the complex conjugation

\[
(\overline{\varphi}(x, \theta))^* = (\overline{\varphi}(x, \theta))^* ,
\]

(9.3)

to be valid in this case in view of below following in (9.7a,b) relations.

The complex superantifields are the elements of complex irreducible massive Poincare group representation and possess by the following Grassmann gradings written, for instance, for $(\varphi(x, \theta))^*$ and its components with respect to $\theta$

\[
\begin{array}{cccc}
(\varphi(x, \theta))^* & J_{\varphi}(x) & (\varphi(x, \theta))^* & \theta J_{\varphi}(x) \\
\varepsilon_{\mathcal{P}} & 1 & 0 & 1 & 1 \\
\varepsilon_{\mathcal{P}} & 0 & 0 & 0 & 0 \\
\varepsilon & 1 & 0 & 1 & 1 ,
\end{array}
\]

(9.4)

in accordance with connection of spin with only statistic $\varepsilon_{\mathcal{P}}$.

Superantifields $(\varphi(x, \theta))^*$, $(\overline{\varphi}(x, \theta))^*$ are transformed with respect to $T^*_\mathcal{P}$ representation in the form

\[
\begin{align*}
\delta(\varphi(x, \theta))^* & = (\varphi(x))^* - (\varphi(x))^* = -\mu(\overline{\varphi}(x, \theta))^* = \mu J_{\varphi}(x) , \quad (9.5a) \\
\delta(\overline{\varphi}(x, \theta))^* & = (\overline{\varphi}(x))^* - (\overline{\varphi}(x))^* = -\mu(\overline{\varphi}(x, \theta))^* = \mu J_{\varphi}(x) . \quad (9.5b)
\end{align*}
\]

Superantifields $(\tilde{\varphi}(x, \theta))^*$, $(\tilde{\overline{\varphi}}(x, \theta))^*$ are the elements of mentioned Poincare group superfield representation and the scalars with respect to action of $T^*_\mathcal{P}$ operators (2.26b).

In view of the fact that supermatrix $K(\theta, x, y)$ of the 2nd derivatives of $S_L(\theta)$ with respect to $\tilde{\varphi}(x, \theta), \tilde{\overline{\varphi}}(y, \theta)$ can be chosen by nondegenerate [1], then it is possible to pass to the Hamiltonian formulation in $T^*_\mathcal{P} \mathcal{M}_{\mathcal{P}}$ parametrized by coordinates $\Gamma(x, \theta) = (\varphi(x, \theta), \overline{\varphi}(x, \theta), (\varphi(x, \theta))^*, (\overline{\varphi}(x, \theta))^*)$. Dimensions of the supermanifolds $\mathcal{M}_{\mathcal{P}}$ and $T^*_\mathcal{P} \mathcal{M}_{\mathcal{P}}$ with respect to $\varepsilon$ parity read as follows, regarding as the independent coordinates, for instance, the only $\varphi(x, \theta)$, $(\varphi(x, \theta))^*$

\[
\dim_{\mathcal{C}} \mathcal{M}_{\mathcal{P}} = (1, 0), \quad \dim_{\mathcal{C}} T^*_\mathcal{P} \mathcal{M}_{\mathcal{P}} = \dim_{\mathcal{C}} T^*_\mathcal{P} \mathcal{M}_{\mathcal{P}} = (1, 1) .
\]

(9.6)

Legendre transform (3.2), (3.3) is defined by the relations

\[
\begin{align*}
(\varphi(x, \theta))^* & = \partial_s S_L(\varphi(\theta), \overline{\varphi}(\theta), \tilde{\varphi}(\theta), \tilde{\overline{\varphi}}(\theta)) = \partial_s T(\tilde{\varphi}(\theta), \tilde{\overline{\varphi}}(\theta)) = -i\overline{\varphi}(x, \theta) , \quad (9.7a) \\
(\overline{\varphi}(x, \theta))^* & = \partial_s S_L(\overline{\varphi}(\theta), \overline{\varphi}(\theta), \tilde{\varphi}(\theta), \tilde{\overline{\varphi}}(\theta)) = \partial_s T(\tilde{\varphi}(\theta), \tilde{\overline{\varphi}}(\theta)) = -i\overline{\varphi}(x, \theta) , \quad (9.7b)
\end{align*}
\]

\[
S_H(\Gamma(\theta)) = T((\varphi(\theta))^*, (\overline{\varphi}(\theta))^*) + S_0(\varphi(\theta), \overline{\varphi}(\theta)), \quad T(\tilde{\varphi}(\varphi(\theta))^*, \tilde{\overline{\varphi}}((\varphi(\theta))^*)) = T((\varphi(\theta))^*, (\overline{\varphi}(\theta))^*) = \int d^4 x \frac{1}{l}(\varphi(x, \theta))^*(\overline{\varphi}(x, \theta))^* \equiv \int d^4 x L^*_\text{kin}(x, \theta) \equiv \int d^4 x L_{\text{kin}}(x, \theta) . \quad (9.7c)
\]

(9.7a-c)
Remind that this free GSTF model is the nondegenerate (and even nonsingular [5]) ThST. In the framework of Sec.VI terminology the model belongs to the II class in Lagrangian and Hamiltonian formulations and former one is given by superfunction $S_L(\theta)$ [1] defined on $T_{odd}M_{cl}$

$$S_L(\theta) \equiv S_L\left(\varphi(\theta), \overline{\varphi}(\theta), \tilde{\varphi}(\theta), \bar{\varphi}(\theta)\right) = T\left(\tilde{\varphi}(\theta), \bar{\varphi}(\theta)\right) - S_0(\varphi(\theta), \overline{\varphi}(\theta)),$$  

(9.8a)

$$T(\theta) \equiv T\left(\tilde{\varphi}(\theta), \bar{\varphi}(\theta)\right) = \int d^4x \frac{1}{i} \tilde{\varphi}(x, \theta) \bar{\varphi}(x, \theta) \equiv \int d^4x L_{kin}(x, \theta),$$  

(9.8b)

$$S_0(\theta) \equiv S_0(\varphi(\theta), \overline{\varphi}(\theta)) = \int d^4x (\partial{\varphi(\theta)} \partial{\overline{\varphi}(\theta)} - m^2 \varphi(\theta) \overline{\varphi}(\theta))(x, \theta) \equiv \int d^4x L_0(x, \theta),$$  

(9.8c)

and therefore the Hamiltonian formulation of model (9.7c) falls literally under conditions of Corollary 1.4 because of the Lagrangian one in this case had satisfied to conditions of Corollary 2.2 from the work [1].

Correspending GHS has the form (6.28), (3.51b)

$$\frac{d}{d\theta} \varphi(x, \theta) = \frac{\partial T(\theta)}{\partial(\varphi(x, \theta))} = \frac{1}{i} (\varphi(x, \theta))^*, \quad \frac{d}{d\theta} \overline{\varphi}(x, \theta) = \frac{\partial T(\theta)}{\partial(\overline{\varphi}(x, \theta))} = -\frac{1}{i} (\varphi(x, \theta))^*,$$  

(9.9a)

$$\frac{d}{d\theta} (\varphi(x, \theta))^* = 0, \quad \frac{d}{d\theta} (\overline{\varphi}(x, \theta))^* = 0,$$  

(9.9b)

$$\Theta^H_\varphi (\overline{\varphi}(x, \theta), \varphi(x, \theta)) = -\frac{\partial S_0(\theta)}{\partial \varphi(x, \theta)} = 0,$$  

(9.9c)

$$\Theta^H_\overline{\varphi} (\varphi(x, \theta), \overline{\varphi}(x, \theta)) = -\frac{\partial S_0(\theta)}{\partial \overline{\varphi}(x, \theta)} = 0.$$  

(9.9d)

The only half of them is independent, for instance, the equations for $\varphi(x, \theta)$, $\overline{\varphi}(x, \theta))^*$. HS (9.9a,b) is solvable. HCHF (9.9c,d) representing the superfield (on $\theta$) generalization of Klein-Gordon equation and coinciding with corresponding HCLF [1] do not appear by solvable. GHS (9.9) being equivalent to corresponding LS [1] has the superfunction $S_H(\theta)$ (9.7c) by one’s integral.

There are not of the type (6.22) identities among Eqs.(9.9c,d) although the only Eq.(9.9d) is independent.

Equation (2.6), in force of the fact that $S_H(\theta) = S_E(\theta) = T\left(\tilde{\varphi}(\theta), \bar{\varphi}(\theta)\right) + S_0(\theta)$ in terms of coordinates on $T_{odd}M_{cl}$, holds

$$\int d^4x \left[\tilde{\varphi}(x, \theta)(\Box + m^2)\varphi(x, \theta) + \bar{\varphi}(x, \theta)(\Box + m^2) \varphi(x, \theta)\right]_{\Theta^H_\varphi(x, \theta) = \Theta^H_\overline{\varphi}(x, \theta) = 0} = 0.$$  

(9.10)

Choosing SCLF and equivalent to them SCHF respectively in the form

$$\tilde{\varphi}(x, \theta)\big|_{\Theta^H_\varphi(x, \theta) = \Theta^H_\overline{\varphi}(x, \theta) = 0} = 0, \quad \bar{\varphi}(x, \theta)\big|_{\Theta^H_\varphi(x, \theta) = \Theta^H_\overline{\varphi}(x, \theta) = 0} = 0,$$  

(9.11)

$$\overline{\overline{\varphi}(x, \theta)}\big|_{\Theta^H_\varphi'(x, \theta) = \Theta^H_\overline{\varphi}'(x, \theta) = 0} = 0, \quad (\varphi(x, \theta))^*\big|_{\Theta^H_\varphi'(x, \theta) = \Theta^H_\overline{\varphi}'(x, \theta) = 0} = 0,$$  

(9.12)

we get the free massive complex scalar superfield model is described on their solutions by only $S_0(\varphi(\theta), \overline{\varphi}(\theta))$ and therefore belongs to the II class ThST with respect to terminology introduced in Sec.VI. Under choice of the SCLF, SCHF in the form (9.11), (9.12) the corresponding master equations (3.36a,b), in fact written in (9.10), appear by double zeros of the solutions for ELS and EGHS (9.9), (9.12). Therefore EGHS and corresponding ELS are solvable.
The antibracket $(\cdot, \cdot)_\theta (7.18a)$ given on $T_{odd}^*\mathcal{M}_{cl}$ has the form

$$(\mathcal{F}(\Gamma(\theta)),\mathcal{J}(\Gamma(\theta)))_\theta = \int d^4x \left[ \left( \frac{\partial \mathcal{F}(\Gamma(\theta))}{\partial \phi(x, \theta)} \frac{\partial \mathcal{J}(\Gamma(\theta))}{\partial (\phi(x, \theta))} + \frac{\partial \mathcal{F}(\Gamma(\theta))}{\partial \phi(x, \theta)} \frac{\partial \mathcal{J}(\Gamma(\theta))}{\partial (\phi(x, \theta))} \right) - (1)^{e(\mathcal{F})+1}(e(\mathcal{J})+1) (\mathcal{F} \leftrightarrow \mathcal{J}) \right], \mathcal{F}(\theta),\mathcal{J}(\theta) \in C^{k*}.$$  

(9.13)

Partial superfield derivatives with respect to superantifields $(\phi(x, \theta))^*, (\bar{\phi}(x, \theta))^*$ in terms of densities on $x^\mu$ have the same form that ones with respect to superfields $\phi(x, \theta), \bar{\phi}(x, \theta)$ in [1]. Operator $\Delta^{cl}(\theta)$ reads as follows on $T_{odd}^*\mathcal{M}_{cl}$

$$\Delta^{cl}(\theta) = \int d^4x \left[ \frac{\partial_t}{\partial \phi(x, \theta)} \frac{\partial}{\partial (\phi(x, \theta))^*} + \frac{\partial_t}{\partial \phi(x, \theta)} \frac{\partial}{\partial (\bar{\phi}(x, \theta))^*} \right].$$  

(9.14)

Eqs.(5.22) and therefore (5.8) hold trivially in question. The superfunctional $Z_H^{(1)}[\phi, \bar{\phi}, (\phi)^*, (\bar{\phi})^*, D, \bar{D}]$ (3.14a) with Lagrange complex scalar multipliers $D(x, \theta), \bar{D}(x, \theta)$ (having the same Grassmann parities as for $\phi(x, \theta), \bar{\phi}(x, \theta)$ [1]) leading on a basis of variational principle to the GHS of the type (3.51), being differed from (9.9) in the Eqs.(9.9b), has the form

$$Z_H^{(1)}[\Gamma, D, \bar{D}] = \int d\theta \int d^4x \left( \phi \phi + \phi \phi \right) (x, \theta) - S_H^{(1)}(\Gamma(\theta), D(\theta), \bar{D}(\theta)) \right], \ (9.15a)$$

$$S_H^{(1)}(\Gamma(\theta), D(\theta), \bar{D}(\theta)) = S_H(\Gamma(\theta)) - \int d^4x \left[ (\Box + m^2)\phi + D(\Box + m^2)\bar{\phi} \right] (x, \theta). \ (9.15b)$$

Because the GHS (9.9) and corresponding LS [1] (in fact being given by Eqs.(9.9c,d) and $\phi(\theta, \phi) = 0, \phi(\theta, \phi) = 0$) are the systems of differential superfield equations in partial derivatives of the 2nd order with respect to $x^\mu$ for GHS, LS and of the 1st (2nd) order on $\theta$ for GHS (LS), then as the independent initial conditions for LS one can choose

$$\left( \phi(\theta, \phi), \phi(\theta, \phi), \phi(\theta, \phi), \phi(\theta, \phi) \right)_{|\theta = \theta = 0} = \left( \phi_0, \phi_1, \phi_2, \phi_3 \right)(x^i), \ x^\mu = (x^0, x^i).$$

(9.16)

Then in correspondence with Statement 3.1 the Cauchy problem (9.16) both for GHS (9.9) and for GHS of the form (3.51) are set in $T_{odd}^*\mathcal{M}_{cl} \times \{\theta\}$ equivalently, with allowance made for (9.7a,b), by means of the independent relations

$$\left( \phi(\theta, \phi), \phi(\theta, \phi) \right)_{|\theta = \theta = 0} = \left( \phi_0, \phi_1 \right)(x^i),$$

$$\left( (\phi(\theta, \phi))^*, (\phi(\theta, \phi))^* \right)_{|\theta = \theta = 0} = (\bar{\phi}_0, \bar{\phi}_1)(x^i) \equiv \left( (\phi_0(x^i))^*, (\phi_1(x^i))^* \right).$$

(9.17)

The results of this subsection can be directly rewritten for the model with self-interaction described in Ref.[1]. To this end it is necessary only in formulae (9.7c), (9.8a,c), (9.9c,d), (9.10), (9.15) to make the changes of the form

$$S_H(\Gamma(\theta)) \rightarrow S_H M(\Gamma(\theta)) = T((\phi(\theta))^*, (\bar{\phi}(\theta))^*) + S_{0M}(\phi(\theta), \bar{\phi}(\theta)), \ (9.18a)$$

$$S_L(\theta) \rightarrow S_{LM}(\theta) = T(\theta) - S_{0M}(\theta),$$

$$S_0(\theta) \rightarrow S_{0M}(\theta) = S_0(\theta) - V(\phi(\theta), \bar{\phi}(\theta)), \ (9.18b)$$

$$V(\theta) \equiv V(\phi(\theta), \bar{\phi}(\theta)) = \int d^4x \left( \frac{\mu}{3} \bar{\phi}(\bar{\phi} + \phi) + \frac{\lambda}{2} (\bar{\phi}(\bar{\phi})^2 + \ldots) \right)(x, \theta),$$

$$\Theta^H_{\phi}(x, \theta) \rightarrow \Theta^H_{\phi}(x, \theta) = -\frac{\partial S_{0M}(\theta)}{\partial \phi(x, \theta)} = (\Box + m^2 + \frac{\mu}{3}(\bar{\phi} + 2\phi)(x, \theta) + \ldots) \frac{\partial S_{0M}(\theta)}{\partial \phi(x, \theta)}.$$
\begin{align}
\Theta^H_{\Psi}(x, \theta) & \quad \Rightarrow \quad \Theta^H_{\Psi_{\mathcal{M}}}(x, \theta) = \Theta^H_{\overline{\varphi}_{\mathcal{M}}}(x, \theta) = 0 , \\
\int d^n x \left[ \tilde{\varphi}(x, \theta) \Theta^H_{\varphi}(x, \theta) + \tilde{\varphi}(x, \theta) \Theta^H_{\overline{\varphi}_{\mathcal{M}}}(x, \theta) \right] |_{\Theta_{\varphi_{\mathcal{M}}}(x, \theta) = \Theta_{\overline{\varphi}_{\mathcal{M}}}(x, \theta) = 0} = 0 ; \\
Z^{(1)}_{\mathcal{HM}}[\Gamma, D, \overline{\mathcal{D}}] & = \int d\theta \left[ \int d^4 x \left( \tilde{\varphi}(\varphi^*) + \tilde{\varphi}(\overline{\varphi})^* \right)(x, \theta) - S^{(1)}_{\mathcal{HM}}(\Gamma(\theta), D(\theta), \overline{\mathcal{D}}(\theta)) \right] , \\
S^{(1)}_{\mathcal{HM}}(\Gamma(\theta), D(\theta), \overline{\mathcal{D}}(\theta)) & = S_{\mathcal{HM}}(\Gamma(\theta)) - \int d^4 x \left[ \overline{\Delta} \Theta^H_{\overline{\varphi}_{\mathcal{M}}} + D \Theta^H_{\overline{\varphi}_{\mathcal{M}}} \right](x, \theta) . \end{align}

The other characteristics, among them the classified ones, for free model are transferred onto self-interacting model taking the remarks made in Ref.[1] into account.

**IX.2 Models of Massive Spinor Superfield of Spin \( \frac{1}{2} \)**

Let us introduce the complex superantifields \( \Psi^*(x, \theta), \overline{\Psi}^*(x, \theta) \in \tilde{\mathcal{A}}_{4|0+1}(x, \theta ; \mathbb{C}) \) defined on \( \mathcal{M} = \mathbb{R}^{1,3} \times \tilde{\mathbb{P}} \) [1]

\[
\Psi^*(x, \theta) = (\psi^{*\alpha}(x, \theta), \chi^{*\dot{\alpha}}(x, \theta)), \quad \Psi^*(x, \theta) = \psi^*(x) + \psi^*(x)\theta = \psi^*(x) - \theta J(x) , \\
\overline{\Psi}^*(x, \theta) = (\overline{\chi}^\beta(x, \theta), \overline{\psi}^{*\dot{\beta}}(x, \theta))^T , \quad \overline{\Psi}^*(x, \theta) = \overline{\psi}^*(x) + \overline{\psi}^*(x)\theta = \overline{\psi}^*(x) - \theta J(x) ,
\]

which are the Dirac bispinors \( (\overline{\Psi}^*(x, \theta) = \Gamma^0(\Psi^*(x, \theta))^\dagger \equiv \overline{\Psi}^*(x, \theta) \) is Dirac conjugate to \( \Psi^*(x, \theta) \) as it follows from (9.24) below and, therefore, the elements of \( (\frac{1}{2}, 0) \oplus (0, \frac{1}{2}) \) reducible superfield (on \( \theta \) Lorentz group representation. The superantifields are given in (9.20) among them in two-component spinor formalism.

The Grassmann gradings for superantifields and their components it is sufficient to define, for instance, for \( \Psi^*(x, \theta) \) by the table

\[
\begin{array}{cccccc}
\varepsilon_P & | & \psi^*(x) & \psi^*_1(x) & J(x) & \Psi^*(x, \theta) & \psi^*_1(x, \theta) \\
\varepsilon_{\Pi} & | & 1 & 1 & 0 & 1 & 1 \\
\varepsilon & | & 0 & 1 & 1 & 0 & 0 \\
\end{array}
\]

that is corresponding to theorem on connection of spin with statistic \( \varepsilon_{\Pi} \) but not with \( \varepsilon \).

Superantifields \( \Psi^*(x, \theta), \overline{\Psi}^*(x, \theta) \) are transformed in a standard way (bispinors) with respect to restriction of representation \( T \) of the supergroup \( J \) onto \( \Pi(1,3)^\dagger \). As to \( T_{\mathcal{P}}^* \), then the superantifields are transformed according to (2.26a) in the form

\[
\delta \Psi^*(x, \theta) = \Psi^{*\dagger}(x, \theta) - \Psi^*(x, \theta) = -\mu \tilde{\psi}^*(x, \theta) = \mu \psi^*_1(x) = \mu J(x) , \\
\delta \overline{\Psi}^*(x, \theta) = \overline{\Psi}^{*\dagger}(x, \theta) - \overline{\Psi}^*(x, \theta) = -\mu \tilde{\overline{\psi}}^*(x, \theta) = \mu \overline{\psi}^*_1(x) = \mu \overline{J}(x) .
\]

Superantifields \( \tilde{\psi}^*(x, \theta), \tilde{\overline{\psi}}^*(x, \theta) \) are elements of mentioned Poincare group superfield representation as well and in addition are scalars with respect to action of \( T_{\mathcal{P}}^* \) operators (2.26b).

Because of supermatrix \( K^{(1)}(\theta, x, y) \) of the 2nd derivatives of \( S^{(1)}_{\mathcal{L}}(\theta) \) with respect to \( \tilde{\psi}(x, \theta), \tilde{\overline{\psi}}(y, \theta) \) is the nondegenerate one [1], then it is possible according to Sec.III to pass to the Hamiltonian formulation in \( T_{\mathcal{odd}}^* \mathcal{M}_{cl} \) parametrized by coordinates \( \Gamma(x, \theta) = (\Psi(x, \theta), \overline{\Psi}(x, \theta), \Psi^*(x, \theta), \overline{\Psi}^*(x, \theta)) \). Dimensions of \( \mathcal{M}_{cl} \) and \( T_{\mathcal{odd}}^* \mathcal{M}_{cl} \) with respect to \( \varepsilon \) parity are the same

\[
\dim_{\mathbb{R}} \mathcal{M}_{cl} = (0, 8) , \quad \dim_{\mathbb{R}} T_{\mathcal{odd}}^* \mathcal{M}_{cl} = \dim_{\mathbb{R}} T_{\mathcal{odd}}^* \mathcal{M}_{cl} = (8, 8) .
\]
Legendre transform (3.2), (3.3) is given by the formulae

\[
\Psi^*(x, \theta) = \frac{\partial_t S_L(\Psi(\theta), \bar{\Psi}(\theta), \bar{\Psi}^*(\theta), \bar{\Psi}^T(\theta))}{\partial \bar{\Psi}^*(x, \theta)} + \frac{\partial_t T(\bar{\Psi}(\theta), \bar{\Psi}^T(\theta))}{\partial \bar{\Psi}^T(x, \theta)} = \bar{\Psi}(x, \theta), \quad (9.24a)
\]

\[
\bar{\Psi}^*(x, \theta) = \frac{\partial_t S_L(\bar{\Psi}(\theta), \bar{\Psi}^T(\theta), \bar{\Psi}(\theta), \bar{\Psi}^*(\theta))}{\partial \bar{\Psi}(x, \theta)} + \frac{\partial_t T(\bar{\Psi}(\theta), \bar{\Psi}^T(\theta))}{\partial \bar{\Psi}^T(x, \theta)} = \bar{\Psi}(x, \theta), \quad (9.24b)
\]

\[
S^{(1)}_H(\Gamma(\theta)) = T(\Psi^*(\theta), \bar{\Psi}^*(\theta)) = S_0(\Psi(\theta), \bar{\Psi}(\theta)) = T(\bar{\Psi}(\theta), \bar{\Psi}^T(\theta)) =
\int d^4x \Psi^*(x, \theta) \bar{\Psi}^*(x, \theta) \equiv \int d^4x \mathcal{L}^{(1)}_{\text{kin}}(x, \theta) = \int d^4x \mathcal{L}^{(1)}_{\text{kin}}(x, \theta). \quad (9.24c)
\]

That GSTF model is the nondegenerate ThST, belongs to the I class as in Lagrangian formalism as in Hamiltonian one and is given by means of superfunction \( S^{(1)}_H(\theta) \) defined on \( T_{\text{odd}} \mathcal{M}_c \) [1]

\[
S^{(1)}_L(\theta) \equiv S_L(\Psi(\theta), \bar{\Psi}(\theta), \bar{\Psi}(\theta), \bar{\Psi}^T(\theta)) = T(\bar{\Psi}(\theta), \bar{\Psi}^T(\theta)) - S_0(\Psi(\theta), \bar{\Psi}(\theta)), \quad (9.25a)
\]

\[
T^{(1)}(\theta) \equiv T(\bar{\Psi}(\theta), \bar{\Psi}^T(\theta)) = \int d^4x \bar{\Psi}(x, \theta) \bar{\Psi}^*(x, \theta) \equiv \int d^4x \mathcal{L}^{(1)}_{\text{kin}}(x, \theta), \quad (9.25b)
\]

\[
S^{(1)}_0(\theta) \equiv S_0(\Psi(\theta), \bar{\Psi}(\theta)) = \int d^4x \bar{\Psi}(x, \theta) (i\Gamma^\mu \partial_\mu - m) \Psi(x, \theta) \equiv \int d^4x \mathcal{L}^{(1)}_0(x, \theta). \quad (9.25c)
\]

Therefore the Hamiltonian formulation of the model (9.24c) satisfies to the conditions of Corollary 1.4. Relations (9.24a,b) establish the following correspondences in two-component spinor formalism

\[
\psi^\alpha(x, \theta) = \bar{\chi}^\alpha(x, \theta), \ x^\alpha(x, \theta) = \bar{\psi}^\alpha(x, \theta), \ \bar{x}^\alpha(x, \theta) = \bar{\psi}_\alpha(x, \theta), \ \bar{x}^\alpha(x, \theta) = \bar{\psi}_\alpha(x, \theta), \quad (9.26)
\]

so that the only half from them is independent.

Corresponding GHS has the form (6.28), (3.51b) and contains \( 3 \times 8 \) real component equations or equivalently 6 superfield spinor equations in four-component spinor formalism

\[
d_t \Psi(x, \theta) = \frac{\partial T^{(1)}(\theta)}{\partial \Psi^*(x, \theta)} = \bar{\Psi}^T(x, \theta), \quad \frac{d_t \Psi^*(x, \theta)}{d\theta} = \frac{\partial T^{(1)}(\theta)}{\partial \bar{\Psi}(x, \theta)} = \Psi(x, \theta), \quad (9.27a)
\]

\[
\frac{d_t \Psi^*(x, \theta)}{d\theta} = 0, \quad \frac{d_t \bar{\Psi}^*(x, \theta)}{d\theta} = 0, \quad (9.27b)
\]

\[
\Theta^H_\Psi(\bar{\Psi}(x, \theta), \partial_\mu \bar{\Psi}(x, \theta)) = - \frac{\partial_t S^{(1)}_0(\theta)}{\partial \bar{\Psi}(x, \theta)} = - (i\partial_\mu \bar{\Psi}(x, \theta) \Gamma^\mu + m \bar{\Psi}(x, \theta)) = 0, \quad (9.27c)
\]

\[
\Theta^H_\Psi(\Psi(x, \theta), \partial_\mu \Psi(x, \theta)) = - \frac{\partial_t S^{(1)}_0(\theta)}{\partial \Psi(x, \theta)} = - (i\Gamma^\mu \partial_\mu - m) \Psi(x, \theta) = 0. \quad (9.27d)
\]

The only half from them is independent, for example, the equations for \( \Psi(x, \theta), \ \Psi^*(x, \theta) \). HS (9.27a,b) is solvable. Superfunction \( S^{(1)}_H(\theta) \) (9.24c) is the integral of GHS (9.27) being equivalent to corresponding LS [1]. But HCHF (9.27c,d) representing themselves the superfield (on \( \theta \)) generalization of Dirac equation and coinciding with corresponding HCLF [1] do not satisfy to the solvability condition.

As one had been already mentioned there were not of the type (6.22) identities because of this model is not gauge although HCHF code 4 independent degrees of freedom among 8 component equations [1].
At last, the equation (2.6), in view of the fact that \( S_H^{(1)}(\theta) = S_E^{(1)}(\theta) = T(\dot{\Psi}(\theta), \overline{\Psi}(\theta)) + S_0^{(1)}(\theta) \) in terms of coordinates on \( T_{\text{odd}M_{cl}} \), is fulfilled and has the form

\[
- \int d^4x \left[ \overline{\Psi}(x, \theta)(i\Gamma^\mu \partial_\mu - m)\dot{\Psi}(x, \theta) + \overline{\Psi}(x, \theta)(i\Gamma^\mu \partial_\mu - m)\Psi(x, \theta) \right] \bigg|_{\Theta_\Psi(x, \theta) = 0, \Theta_{\overline{\Psi}}(x, \theta) = 0} = 0. \tag{9.28}
\]

Specifying SCLF and being equivalent to them SCHF respectively in the form

\[
\Psi(x, \theta)_{|\Theta_\Psi(x, \theta) = 0, \Theta_{\overline{\Psi}}(x, \theta) = 0} = 0, \quad \dot{\Psi}(x, \theta)_{|\Theta_\Psi(x, \theta) = 0, \Theta_{\overline{\Psi}}(x, \theta) = 0} = 0 ;
\tag{9.29}
\]

\[
\Psi^*(x, \theta)_{|\Theta_\Psi(x, \theta) = 0, \Theta_{\overline{\Psi}}(x, \theta) = 0} = 0, \quad \overline{\Psi}^*(x, \theta)_{|\Theta_\Psi(x, \theta) = 0, \Theta_{\overline{\Psi}}(x, \theta) = 0} = 0 , \tag{9.30}
\]

obtain that on their solutions according to (9.27a) the free massive spinor superfield model is described by only superfunction \( S_0(\Psi(\theta), \overline{\Psi}(\theta)) \) and therefore belongs to the II class ThST. Master equations (3.36a,b), in fact written in (9.28), appear by double zeros of the solutions for ELS and EGHS (9.27), (9.30) with SCLF, SCHF (9.29), (9.30). Therefore EGHS and corresponding ELS are solvable.

The antibracket \( (\ , \ )_\theta \) (7.18a), operator \( \Delta^\xi(\theta) \) are defined in the following way in coordinates \( \Gamma(x, \theta) \) on \( T^*_{\text{odd}M_{cl}} \)

\[
(F(\Gamma(\theta)), J(\Gamma(\theta)))_\theta = \int d^4x \left[ \left( \frac{\partial F(\Gamma(\theta))}{\partial \Psi(x, \theta)} \frac{\partial J(\Gamma(\theta))}{\partial \Psi^*(x, \theta)} - \frac{\partial F(\Gamma(\theta))}{\partial \Psi^*(x, \theta)} \frac{\partial J(\Gamma(\theta))}{\partial \Psi(x, \theta)} \right) - (1)^{(\varepsilon(F) + 1)(\varepsilon(J) + 1)}(F \leftrightarrow J) \right], \quad F(\theta), J(\theta) \in C^{k*} ,
\tag{9.31}
\]

\[
\Delta^\xi(\theta) = - \int d^4x \left[ \frac{\partial}{\partial \Psi(x, \theta)} \frac{\partial}{\partial \Psi^*(x, \theta)} + \frac{\partial}{\partial \Psi^*(x, \theta)} \frac{\partial}{\partial \Psi(x, \theta)} \right] . \tag{9.32}
\]

Applied in this case the Eq.(5.22) and therefore (5.8) are trivially valid. Superfunctional \( Z_H^{(1)}[\Psi, \overline{\Psi}, \Psi^*, \overline{\Psi}^*, D^{(1)}, D^{(1)}] \) (3.14a) with auxiliary bispinors \( D^{(1)}(x, \theta), D^{(1)}(x, \theta) \) (having the same \( \varepsilon_p, \varepsilon_n, \varepsilon \) gradings as for \( \Psi(x, \theta), \overline{\Psi}(x, \theta) \) [1]) from which it follows the GHS of the type (3.51), being differenced from (9.27) in the Eqs.(9.27b), has the form

\[
Z_H^{(1)}[\Gamma, D^{(1)}, D^{(1)}] = \int d\theta \left[ \int d^4x \left( \overline{\Psi} \Psi^* + \Psi^* \overline{\Psi} \right) (x, \theta) - S_H^{(1)}(\Gamma(\theta), D^{(1)}(\theta), D^{(1)}(\theta)) \right] , \tag{9.33a}
\]

\[
S_H^{(1)}(\Gamma(\theta), D^{(1)}(\theta), D^{(1)}(\theta)) = S_H^{(1)}(\Gamma(\theta)) + \int d^4x \left[ \overline{D}^{(1)}(x, \theta)(i\Gamma^\mu \partial_\mu - m)\Psi(x, \theta) + \overline{\Psi}(x, \theta)(i\Gamma^\mu \partial_\mu - m)D^{(1)}(x, \theta) \right]. \tag{9.33b}
\]

As far as the GHS (9.27) and corresponding LS [1] (in fact being given by formulae (9.27c,d) and \( \overline{\Psi}(x, \theta) = 0, \overline{\Psi^*}(x, \theta) = 0 \)) are the systems of differential superfield equations in partial derivatives of the 1st order with respect to \( x^\mu \) and of the 1st (2nd) order on \( \theta \) for GHS (LS) one can choose as the independent the following initial conditions for LS

\[
\Psi(x, \theta)_{|x^0 = \theta = 0} = \Psi(x^i), \quad \Psi^*(x, \theta)_{|x^0 = \theta = 0} = \Psi^*(x^i) . \tag{9.34}
\]

Then according to Statement 3.1 the Cauchy problem both for GHS (9.27) and for GHS of the form (3.51) are set in \( T^*_{\text{odd}M_{cl}} \times \{ \theta \} \) equivalently taking (9.24a,b) into account by means of the following independent relations

\[
\Psi(x, \theta)_{|x^0 = \theta = 0} = \Psi(x^i), \quad \Psi^*(x, \theta)_{|x^0 = \theta = 0} = \Psi^*(x^i) \equiv \Psi^*(x^i) . \tag{9.35}
\]
The generalization of the relationships from given subsection to the case of massive spinor superfield model with interaction [1] is produced sufficiently simple. To do this it is necessary to make the only following changes in formulae (9.24c), (9.25a,c), (9.27c,d), (9.28), (9.33)

respectively

\[
S_{H}^{(1)}(\Gamma(\theta)) \rightarrow S_{H}^{(1)}(\Gamma(\theta)) = T(\Psi^{*}(\theta), \overline{\Psi}(\theta)) + S_{0M}(\Psi(\theta), \overline{\Psi}(\theta)) ,
\]

\[
S_{L}^{(1)}(\theta) \rightarrow S_{L}^{(1)}(\theta) = T^{(1)}(\theta) - S_{0M}^{(1)}(\theta) ,
\]

\[
S_{0}^{(1)}(\theta) \rightarrow S_{0M}^{(1)}(\theta) = S_{0}^{(1)}(\theta) - V(\Psi(\theta), \overline{\Psi}(\theta)) ,
\]

\[
V^{(1)}(\theta) \equiv V^{(1)}(\Psi(\theta), \overline{\Psi}(\theta)) = \int d^{4}x \left[ \frac{\lambda_{1}}{2} (\overline{\Psi}\Psi)^{2} + \frac{\lambda_{2}}{2} (\overline{\Psi}\Gamma_{\mu}\Psi) \right](x, \theta) ,
\]

\[
\Theta_{\Psi}^{H}(x, \theta) \rightarrow \Theta_{\Psi}^{H}(x, \theta) = -\frac{\partial S_{0M}^{(1)}(\theta)}{\partial \Psi(x, \theta)} = -\left[ \partial_{\mu}\Gamma_{\mu}^{0} - m - \lambda_{1}(\overline{\Psi}\Psi)(x, \theta) - \lambda_{2}(\overline{\Psi}\Gamma_{\mu}\Psi)(x, \theta) \Gamma_{\mu}^{0} \right] \Psi(\theta, \overline{\Psi}(\theta), \partial_{\mu}\Psi(x, \theta) = 0 ,
\]

\[
\Theta_{\Psi}^{H}(x, \theta) \rightarrow \Theta_{\Psi}^{H}(x, \theta) = \left( \Theta_{\Psi}^{H}(x, \theta) \right)^{\dagger} \Gamma^{0} = 0 ,
\]

\[
\int d^{4}x \left[ \overline{\Psi} D_{\Psi}(\Psi, \overline{\Psi}, \partial_{\mu}) \overline{\Psi} + \overline{\Psi} D_{\Psi}(\Psi, \overline{\Psi}, \partial_{\mu}) \overline{\Psi} \right](x, \theta)_{[\Theta_{\Psi}^{H}(x, \theta) = 0, \Theta_{\Psi}^{H}(x, \theta) = 0]} = 0 ;
\]

\[
Z_{H}^{(1)}[\Gamma, D^{(1)}, \overline{D}^{(1)}] = \int d\theta \left[ \int d^{4}x \left( \overline{\Psi} D_{\Psi}(\Psi, \overline{\Psi}) \overline{\Psi} + \overline{\Psi} D_{\Psi}(\Psi, \overline{\Psi}) \overline{\Psi} \right)(x, \theta) - S_{0M}^{(1)}(\Gamma(\theta), D^{(1)}(\theta), \overline{D}^{(1)}(\theta)) \right] ,
\]

\[
S_{H}^{(1)}(\Gamma(\theta), D^{(1)}(\theta), \overline{D}^{(1)}(\theta)) = S_{H}^{(1)}(\Gamma(\theta)) - \int d^{4}x \left[ \overline{D}^{(1)} \Theta_{\Psi}^{H} + \overline{D} \Theta_{\Psi}^{H} + \Theta_{\Psi}^{H} D^{(1)}(\theta) \right](x, \theta) .
\]

Given model remains nondegenerate ThST taking corresponding remarks from Ref.[1] into account.

IX.3 Models of Free Vector Superfield

The real superantifields \( \mathcal{A}_{\mu}^{*(x, \theta)} \in \tilde{A}_{D}|(\theta+1)_{(x^{\mu}, \theta; \mathbb{R})} \) on \( \mathcal{M} = \mathbb{R}^{1,D-1} \times \hat{P} \) have the following expansion in powers of \( \theta \) and \( \varepsilon_{P}, \varepsilon_{\Pi}, \varepsilon \) gradings respectively

\[
\mathcal{A}_{\mu}^{*(x, \theta)} = A_{\mu}^{*(x)} + A_{1\mu}^{*(x)} \theta = A_{\mu}^{*(x)} - \theta J_{\mu}(x) , \quad \varepsilon = 1 \quad 0 \quad 0 \quad 1 \quad 1 . \quad (9.38)
\]

\( \mathcal{A}_{\mu}^{*(x, \theta)} \) are transformed with respect to massless (or massive for Proca model for \( D = 4 \) irreducible representation of group \( \tilde{J} = \Pi(1, D-1) \)) and have the usual transformation laws relative to that group. But with respect to \( T_{\hat{P}}^{I} \) the only \( \mathcal{A}_{\mu}^{*(x, \theta)} \) are nontrivially transformed according to (2.26a)

\[
\delta \mathcal{A}_{\mu}^{*(x, \theta)} = \mathcal{A}_{\mu}^{*(x)} - \mathcal{A}_{\mu}^{*(x)} = - \mu \mathcal{A}_{\mu}^{*(x)}(x, \theta) = - \mu J_{\mu}(x) .
\]

Let us remind that Lagrangian formulation of the massless model [1] had been given by the superfunction \( S_{L}(\mathcal{A}^{\mu}(\theta), \hat{A}^{\mu}(\theta)) \equiv S_{L}^{(2)}(\theta) \) on \( T_{\text{odd}} \mathcal{M}_{\text{cl}} = \{ (\mathcal{A}^{\mu}(x, \theta), \hat{A}^{\mu}(x, \theta)) \} \)

\[
S_{L}^{(2)}(\theta) = T(\hat{A}^{\mu}(\theta)) - S_{0}(\mathcal{A}(\theta)) ,
\]

\[
\mathcal{L}^{(2)}_{\text{kin}}(x, \theta) = \mathcal{L}^{(2)}_{\text{kin}}(\hat{A}^{\mu}(x, \theta)) = \frac{1}{2} \varepsilon_{\mu\nu} \mathcal{A}_{\nu}(x, \theta) \hat{A}^{\mu}(x, \theta) , \quad \varepsilon_{\mu\nu} = -\varepsilon_{\nu\mu} ,
\]

\[
S_{0}(\mathcal{A}^{\mu}(\theta)) = \int d^{D}x \mathcal{L}^{(2)}_{0}(\mathcal{A}^{\mu}(x, \theta), \partial_{\nu}\mathcal{A}^{\mu}(x, \theta)) \equiv \int d^{D}x \mathcal{L}^{(2)}_{0}(x, \theta) ,
\]

\[
\mathcal{L}^{(2)}_{0}(x, \theta) = -\frac{1}{4} F_{\mu\nu}(x, \theta) F^{\mu\nu}(x, \theta) , \quad F_{\mu\nu}(x, \theta) = \partial_{\nu} A_{\mu}(x, \theta) - \partial_{\mu} A_{\nu}(x, \theta) .
\]
whose form \((S_L^{(2)}(\theta))\) determined the GThST with the generators of gauge transformations

\[
R^\mu(x,y) = \partial^\mu \delta(x-y) .
\]  

(9.41)

For \(D = 2k + 1, k \in \mathbb{N}\) the supermatrix \(S_L^{(2)}(\theta,x,y) \equiv K^{(2)}(\theta,x,y)\) is always degenerate so that a standard Hamiltonization by the prescription of Sec.III is impossible. Choosing whose form \((\text{S})\) Legendre transform \((3.2), (3.3)\) according to \((9.40)\) is given by the formulae

\[
\text{appears by GThST belonging to the I class with GGTST (9.41) and HCHF having the form}
\]

\[
S_{\text{D}}(\text{to be nondegenerate for } D = 2k, k \in \mathbb{N} \text{ we conclude that the necessary condition for existence of Legendre transform for } S_L^{(2)}(\theta) \text{ with respect to } \hat{A}^\mu(x, \theta) \text{ is fulfilled.}}
\]

Dimensions of supermanifolds \(\mathcal{M}_{cl}\) and \(T_{odd}^*\mathcal{M}_{cl}\), in this example parametrized by superfields \(A^\mu(x, \theta)\) and \(\Gamma^p(x, \theta) = (A^\mu(x, \theta), A^*_{\mu}(x, \theta))\) respectively, with respect to \(\varepsilon\) grading are equal

\[
\text{dim}_R \mathcal{M}_{cl} = (4,0), \quad \text{dim}_R T_{odd}^* \mathcal{M}_{cl} = \text{dim}_R T_{odd} \mathcal{M}_{cl} = (4,4) .
\]

(9.42)

Legendre transform \((3.2), (3.3)\) according to \((9.40)\) is given by the formulae

\[
\hat{A}^*_\mu(x, \theta) = \frac{\partial S_L(\hat{A}^\nu(\theta), \hat{A}^*_{\nu}(\theta))}{\partial \hat{A}^\nu(x, \theta)} = \varepsilon_{\nu\mu} \hat{A}^\nu(x, \theta), \quad S_H(\Gamma^p(\theta)) = T(\hat{A}^*_{\mu}(\theta)) + S_0(\hat{A}^\mu(\theta)) ,
\]

(9.43a)

\[
T(\hat{A}^*_{\mu}(\theta)) = \int d^D x \frac{1}{2} (\varepsilon^{-1})^{\mu\nu} A^*_{\mu}(x, \theta) A^*_{\nu}(x, \theta), \quad (\varepsilon^{-1})^{\mu\nu} \varepsilon_{\nu\rho} = \delta_{\mu\rho} .
\]

(9.43b)

Therefore the Hamiltonian formulation of the model satisfies to conditions of Corollary 1.4 and appears by GThST belonging to the I class with GGTST \((9.41)\) and HCHF having the form

\[
\Theta^H_{\mu}(x, \theta) \equiv \Theta^H_{\mu}(A^\nu(x, \theta), \partial_{\nu} A^\nu(x, \theta)) = -\frac{\partial S_0(\hat{A}^\nu(\theta))}{\partial \hat{A}^\nu(x, \theta)} = -(\Box_{\mu\nu} - \partial_{\mu} \partial_{\nu}) A^\nu(x, \theta) = 0 .
\]

(9.44)

Corresponding GHS \((3.5)\) has the form \((6.28), (3.51b)\) in correspondence with Corollary 1.4

\[
\frac{d_r \hat{A}^\nu(x, \theta)}{d\theta}(\hat{A}^\nu(x, \theta), \hat{A}^*_{\nu}(x, \theta)) = (\varepsilon^{-1})^{\mu\nu} A^*_{\nu}(x, \theta), \quad \frac{d_r \hat{A}^*_\mu(x, \theta)}{d\theta} = 0, \quad \Theta^H_{\mu}(x, \theta) = 0 .
\]

(9.45)

GHS \((9.45)\) is equivalent to the corresponding LS \([1]\). Given GHS not being solvable, although the Hamiltonian subsystem in \((9.45)\) appears by solvable itself, has \(S_H(\Gamma^p(\theta))\) as one’s integral resulting in fulfilment of Eq.\((2.6)\) in this case in terms of coordinates on \(T_{odd}^* \mathcal{M}_{cl}\)

\[
\int d^D x \left[[(\Box_{\mu\nu} - \partial_{\mu} \partial_{\nu}) A^\nu(x, \theta)](\varepsilon^{-1})^{\mu\rho} A^*_\rho(x, \theta)\right]_{\Theta^H_{\mu}(x, \theta)=0} = 0 ,
\]

(9.46)

by virtue of the following expression

\[
S_H(\Gamma^p(\theta)) = S_E \left(A^\mu(x, \theta), \hat{A}^\mu(\theta)\right) |_{\hat{A}^\mu(x, \theta) = \hat{A}^\mu(A^*(x, \theta))} = T(\hat{A}^*_{\mu}(\theta)) + S_0(\hat{A}^\mu(\theta)) .
\]

Defining SCLF and SCHF, being equivalent to each other, in the form respectively

\[
\hat{A}^\mu(x, \theta)|_{\Theta^H_{\mu}(x, \theta)=0} = 0, \quad A^*_{\mu}(x, \theta)|_{\Theta^H_{\mu}(x, \theta)=0} = 0
\]

(9.47)

obtain that on their solutions the free massless vector superfield model is described by only \(S_0(\hat{A}^\mu(\theta)) \in \mathcal{M}_{cl}\) and, therefore belongs to the II class theory. Master equations of the form \((3.36a,b)\), in fact written in \((9.46)\), appear by double zeros for the solutions for EGHS \((9.45)\), \((9.47)\) and corresponding ELS. These systems therefore are solvable.
The antibracket (( , )θ) and operator Δcl(θ) have the form for any \( F(\Gamma(\theta)), J(\Gamma(\theta)) \in C^k \)

\[
(F(\Gamma(\theta)), J(\Gamma(\theta)))_{\theta} = \int d^Dx \left( \frac{\partial F(\Gamma(\theta))}{\partial A^\mu(x, \theta)} \frac{\partial J(\Gamma(\theta))}{\partial A^*_\mu(x, \theta)} - \frac{\partial F(\Gamma(\theta))}{\partial A^*_\mu(x, \theta)} \frac{\partial J(\Gamma(\theta))}{\partial A^\mu(x, \theta)} \right),
\]

(9.48a)

\[
\Delta^d(\theta) = \int d^Dx \frac{\partial}{\partial A^\mu(x, \theta)} \frac{\partial}{\partial A^*_\mu(x, \theta)}.
\]

(9.48b)

Eqs.(5.22) and hence Eqs.(5.8) trivially hold. GHS of the type (3.51), being differed from (9.45) in the 2nd subsystem, follows from variational problem for the superfunctional

\[
Z_H^{(1)}[A^\mu, \check{A}^\mu, D^\mu] = \int d\theta \left( \int d^Dx A^\mu(x, \theta) A^*_\mu(x, \theta) - S_H^{(1)}(\Gamma^\mu(\theta), D^\mu(\theta)) \right),
\]

(9.49a)

\[
S_H^{(1)}(\Gamma^\mu(\theta), D^\mu(\theta)) = S_H(\Gamma^\mu(\theta)) + \int d^Dx D^\mu(x, \theta) \left[ [\Box \eta_{\mu\nu} - \partial_\mu \partial_\nu] A^\nu(x, \theta) \right],
\]

\[
(\varepsilon_P, \varepsilon_\Pi, \varepsilon) D^\mu(x, \theta) = (0, 0, 0).
\]

(9.49b)

The setting of Cauchy problem, by virtue of functional dependence of HCHF (9.44), for GHS (9.45) and for corresponding LS [1] for \( D = 2k, k \in \mathbb{N} \) is nontrivial, and for \( D = 2k+1 \) is more complicated. It requires an additional study of HCLF and LS on the whole. GHS (9.45) for \( D = 2k \) itself and corresponding LS [1] (given with help of (9.44) and \( \varepsilon_{\mu\nu\theta}(x, \theta) = 0 \) for \( D \geq 2 \)) are the systems of differential superfield equations in partial derivatives of the 1st (2nd) order on \( \theta \) for GHS (LS) and of the 2nd order with respect to \( x^\mu \) for GHS and LS.

At last, \( \theta \)-superfield model of free massive real vector superfield described in the Lagrangian formulation for GSTF in [1] can be obtained in the Hamiltonian one under formal change of the corresponding superfunctions and differential operators in the relationships (9.40a,c), (9.43a), (9.44)–(9.46), (9.49) onto following ones

\[
S_L^{(2)}(\theta) \rightarrow S_L^{(2)}(\theta) = T(\check{A}^\mu(\theta)) - S_0m(A^\mu(\theta)),
\]

\[
S_H(\Gamma^\mu(\theta)) \rightarrow S_H(\Gamma^\mu(\theta)) = T(A^*_\mu(\theta)) + S_0m(A^\mu(\theta)),
\]

(9.50a)

\[
S_0m(A^\mu(\theta)) = \int d^4x L_0^{(2)}(x, \theta), \ L_0^{(2)}(x, \theta) = \left( -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m^2}{2} A_\mu A^\mu \right)(x, \theta),
\]

(9.50b)

\[
\Theta^H_\mu(x, \theta) \rightarrow \Theta^H_\mu(x, \theta) = -\left( \Box + m^2 \right) \eta_{\mu\nu} - \partial_\mu \partial_\nu A^\nu(x, \theta) = 0,
\]

(9.50c)

\[
- \int d^4x \Theta^H_\mu(x, \theta)(\varepsilon^{-1})^{\mu\nu} A^\nu(x, \theta)|_{\Theta^H_\mu(x, \theta) = 0} = 0;
\]

(9.50d)

\[
Z_{Hm}^{(1)}[A^\mu, \check{A}^\mu, D^\mu] = \int d\theta \left( \int d^Dx \check{A}^\mu(x, \theta) \check{A}^\mu(x, \theta) - S_{Hm}^{(1)}(\Gamma^\mu(\theta), D^\mu(\theta)) \right),
\]

(9.51a)

\[
S_{Hm}^{(1)}(\Gamma^\mu(\theta), D^\mu(\theta)) = S_{Hm}(\Gamma^\mu(\theta)) - \int d^DxD^\mu(x, \theta) \Theta^H_\mu(x, \theta).
\]

(9.51b)

As it had been noted in Ref.[1] there are not both differential identities among \( \Theta^H_\mu(x, \theta) \)
(9.50c) and therefore any gauge transformations. This model belongs to the class of nondegenerate ThST, being singular in correspondence with terminology from Ref.[8] in view of 2 second-class constraints presence.

## X Conclusion

The second composite part of GSQM for gauge theories in the Lagrangian formalism in a natural way continuing the ideas and elaborations of the Lagrangian formulation for GSTF
have been proposed in the work. On space $T^*_{\text{odd}} M_{\text{cl}} \times \{\theta\}$ the superfunction $S_H(\theta)$ is defined, equivalently just as $S_L(\theta)$ in Lagrangian formulation, coding all information about given GSTF model in Hamiltonian formalism. More complicated scheme of corresponding dynamical systems investigation on a basis of satisfaction of the solvability conditions has led to supplement of the additional special constraints aside from ones following from variational principle. This investigation trend has led to definite inverse connection, namely, to modification of Lagrangian formulation description for GSTF. In particular, it was expressed in introduction of concepts for theories of the I and II classes. GThGT and (or) GThST of the II class represent themselves the models of gauge theories with master equations being valid, in contrast to the theories of the I class, in the whole $T^*_{\text{odd}} M_{\text{cl}}, T^*_{\text{odd}} M_{\text{cl}}$ in correspondence with the kind of applied formalism.

Note that GThGT of the II class with $S_H(\Gamma(\theta)) \in C^k(T^*_{\text{odd}} M_{\text{cl}})$ has as consequence of the master equation the property of mentioned superfunction cancellation by means of the operator $\Delta_{\text{cl}}(\theta)$ action. Various constructions of the translation operations with respect to $\theta$ along integral curves for HS, GHS, EGHS, EHS together with their properties were examined.

Important problems of the study of the 1st and 2nd orders differential operators super-algebras and the component formulation for GSTF have been considered. Statements and conclusions for general formalism of the work have been demonstrated on the examples of free and self-interacting massive complex scalar superfield of spin 0 models, free and self-interacting massive spinor superfield of spin $\frac{1}{2}$ models in $D = 4$ and free massless and massive (Proca model) vector superfield ones in $D = 2k, k \in \mathbb{N}$. These examples appear by the starting models for construction of interacting superfield (on $\theta$) GThGT in Hamiltonian formulation such as $\theta$-superfield scalar or spinor or vector (for complex vector superfield in $D = 4$) electrodynamics on a basis of the gauge principle application [6] to models above. Note that nonlinear dynamical equations in fact written in HCHF for self-interacting models do not coincide with respect to form with each other for $P_0(\theta)$ and $P_1(\theta)$ components of the corresponding superfields in contrast to free models. For the latter examples the component on $\theta$ linear equations of motion have the same form and formally describe the same corresponding particles.

Natural algorithm to constructing from the models of usual relativistic field theory their superfield generalization in the form of natural system, described in the conclusion of Ref.[1] for Lagrangian formulation for GSTF, can be literally applied to analogous construction of their superfield generalization in Hamiltonian formalism for GSTF.

At last, let us continue the analogy [1] between quantities and relations from classical mechanics in its usual Hamiltonian formulation with even with respect to supergroup $P(\varepsilon_{\pi} \equiv 0)$
objects and ones from Hamiltonian formulation for GSTF.

| usual classical mechanics | Hamiltonian GSTF |
|---------------------------|------------------|
| 1. \( p_a(t) \) – generalized momenta \((\varepsilon, \varepsilon) p_a(t) = (\varepsilon, \varepsilon_a)\) | \( A^*_i(\theta) \) – superantifields |
| \( T^* M = \{(q^a, p_a)\} \) – phase space | \( T^*_\text{odd} M_{cl} = \{(A^*_i(\theta), A^*_i(\theta))\} \) – odd phase space |
| 2. \( p_a(t) = \frac{\partial L(t)}{\partial q^a(t)} \), \( H(p, q, t) = q^a p_a - L(t) \) – Hamiltonian | \( A^*_i(\theta) = \frac{\partial S_L(\theta)}{\partial A_i^*(\theta)} \), \( S_H(A(\theta), A^*(\theta), \theta) = \partial_{\theta} S_L(\theta) \) – superfunction of classical Hamiltonian action |
| 3. \( f(g(q, p, t), g(q, p, t)) \) – even Poisson bracket, \( f, g \in C^k(T^* M \times \{t\}) \) | \( (\mathcal{F}(\theta), \mathcal{J}(\theta))_{\theta} \) – antibracket, \( \mathcal{F}(\theta), \mathcal{J}(\theta) \in C^k(T^*_\text{odd} M_{cl} \times \{\theta\}) \) |
| 4. \( \frac{d}{dt} \{\Gamma^c(t), H(t)\} \) – Hamilton equations of motion | \( \frac{d}{dt} \Gamma^c(\theta) = (\mathcal{F}(\theta), S_H(\theta))_{\theta} \) – odd Hamilton equations of motion |
| 5. \( S_H[\Gamma] = \int dt \left( \dot{q}^a p_a - H(q, p, t) \right) \) – Hamiltonian action functional | \( Z_H[\Gamma] = \int d\theta (A^i(\theta) A^*_i(\theta) - S_H(\Gamma(\theta), \theta)) \) – odd Hamiltonian superfunctional |

Acknowledgments: I would like to thank Bogdan Mishchuk for discussions of some results of the present paper.

References

1. A.A. Reshetnyak, General Superfield Quantization Method. I. General Superfield Theory of Fields: Lagrangian Formalism, hep-th/0210207.

2. C. Becchi, A. Rouet and R. Stora, Phys. Lett. B 25 (1974) 344; Comm. Math. Phys. 42 (1975) 127; I.V. Tyutin, P.N. Lebedev Physical Inst. of the RF Academy of Sciences, preprint, No.39 (1975).

3. I.A. Batalin and G.A. Vilkovisky, Phys. Lett. B 102 (1981) 27; Phys. Rev. D 28 (1983) 2567.

4. V.N. Shander, Funct. analysis and its applications (on russian) 14 (1980) No.2 91; 17 (1983) No.1 89.

5. Ö.F. Dayi, Mod. Phys. Lett. A 4 (1989) 361; A 8 (1993) 811; ibid 2087; Int. J. Mod. Phys. A 11 (1996) 1.

6. C.N. Yang and R. Mills, Phys. Rev. 96 (1954) 191.

7. E. Noether, Nachr. König. Gesellsch. Wissen, Göttingen, Math-Phys. Klasse. (1918) 235, in English: Transport Theory and Stat. Phys. 1 (1971), 186.

8. D.M. Gitman and I.V. Tyutin, Quantization of Fields with Constraints (Springer-Verlag, Berlin and Heidelberg, 1990).