Bayesian Inference for Predicting Rock Mass Rating Ahead of Tunnel-Face Excavation Integrating Multi-Source Information

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Abstract. The rock mass rating (RMR14) system can effectively evaluate the quality of the rock mass surrounding a tunnel, which combines the characteristics of the rock mass strength and the discontinuity. However, it is difficult to infer the RMR14 value accurately due to the data often being limited, as well as there being extensive data sources of geological exploration. The rock mass classification systems, which are known as empirical methods, are still used in current engineering design and construction. In this paper, a quantitative prediction method of the RMR14 value on tunnel excavation is proposed that is based on the Bayesian network theory. A variety of empirical formulas between the RMR14 with the geological strength index and the basic quality method are fused in the Bayesian network fusion framework; MATLAB is used to generate 500 random samples, and the conditional probability table of the Bayesian network is generated based on the expected maximum algorithm. The uncertainty of the RMR value prediction is quantified by the RMR14 Progressive Prediction Probability Model. The proposed method is applied to a rock tunnel project, i.e., the Laoying tunnel in Yunnan province, China. The results show that this method is able to provide a reasonable probability inference of the RMR14.

Key words: Rock mass prediction, Bayesian network, multi-sources data integration, RMR Progressive Prediction Probability Model

1. Introduction

As the environmental medium of tunnel engineering, the quality of the surrounding rock plays an important role in the safety of the project. In the quantification of the quality of the surrounding rock and the geological conditions of the tunnel, many evaluation methods have previously been proposed at home and abroad. For example, the rock quality designation (RQD) classification, the Q system classification, the geological mechanics RMR14 classification (Celada B , Tardaguila I, 2014), the geological strength index (GSI) classification (Hoek and Brown,1997.), and the engineering rock classification basic quality (BQ) method. These are the commonly used rock classification systems for rock tunnels and are habitually used to select the appropriate excavation methods and rock support systems.
However, limited by the observation cost in practical engineering, the information obtained from the tunnel excavation site is usually insufficient for accurate classification of the rock mass quality. For a complex engineering medium, there are four types of uncertainties in a rock mass classification system: inherent variability due to complex geological conditions, measurement errors during in situ and laboratory investigations, model uncertainty in various estimation models, and statistical uncertainty when calculating statistical parameters (Aladejare and Wang, 2017; Yuan et al., 2020). In describing the uncertainty of the rock mass parameters, the probability method is usually introduced to correct the classification of the surrounding rock (Aladejare and Wang, 2019; Asem and Gardoni, 2019). The commonly used classification methods of the surrounding rock, such as GSI and the RMR, are reliant on engineering experience and reasonable judgment. Therefore, the result of the surrounding rock classification is extremely subjective. The Bayesian method is widely used in the probability classification of the surrounding rock because it is able to integrate engineering experience and new tunnel excavation data (Aladejare and Idris, 2020; Feng and Jimenez, 2015; Feng and Jimenez, 2014; Santos et al., 2018). Wenmin Yao (2020) used the Bayesian sequence updating method to predict and analyze the GSI by integrating the RMR, the Q classification, and the RMi value. Zhang (2017) processed the RMR14 classification of the surrounding rock of a mountain tunnel based on the Mamdani fuzzy inference, successfully applying it to rock tunnel engineering.

From the method of obtaining rock mass information and the characteristics of the linear extension of tunnel engineering, it can be seen that the acquisition of rock mass information is accumulative over time. The exposure degree of the rock mass is different during different stages of tunnel engineering, and the acquisition degree of the parameters required for different surrounding rock classification systems also differs. Sometimes, it is essential to infer the other surrounding rock classification results based on some known surrounding rock classification results. Therefore, to make full use of limited data, it is necessary to study the surrounding rock rating method under the condition of incomplete information. At present, the research on rock mass classification methods generally focuses on the condition of complete rock information; there are only a few research studies on the rock mass classification under the condition of incomplete information. In particular, tunnel engineering has the characteristics of gradual exposure of the surrounding rock’s geology with tunneling, and there are few studies on rock mass uncertainty classification under this condition. Focusing on the RMR14 classification method, this paper proposes a probability rock mass classification model that can integrate multi-source data based on Bayesian network theory.

2. Basic Information of Rock Mass Quality Classification

2.1. Relationship between rock mass classification methods

The mechanical strength parameters of rock mass are affected by the discontinuity of the rock mass, and the acquisition of parameters is always the central and difficult point in the study of rock mass engineering mechanics. Generally, the GSI classification method considers the distribution characteristics and the structural surface conditions of the rock mass, including the structural surface roughness, the weathering grade, and the filling properties. The GSI is conventionally estimated using a graphical method from the standard GSI chart. The GSI classification can be used directly with the Hoek–Brown failure criterion or the generalized Zhang-Zhu criteria failure criterion, which can provide the rock strength parameters for the stability analysis of jointed rock masses. However, the GSI classification index does not consider the strength information of the rock blocks.

The rock mass basic quality (BQ) classification method determines the grade of the surrounding rock by consideration of the integrity of the rock mass and the strength of the intact rock, and it is corrected by the distribution characteristics of the groundwater. The formula for the BQ classification is

\[ BQ = 100 + 3R_c + 250K_v \]  \hspace{1cm} (1)

where \( R_c \) is the saturated uniaxial compressive strength of the rock and \( K_v \) is the integrity coefficient of the rock mass.

Celada (2014) modified the RMR rock mass classification method by the addition of factors such as the excavation method, the stress characteristics of the excavation face, and the rock collapse
resistance index, obtaining an RMR14 surrounding rock classification method that can describe the quality of the tunnel rock mass more accurately and comprehensively. The RMR14 classification index mainly includes the rock mass strength, the rock mass integrity, the structural plane characteristics, the joint filling properties, and the groundwater conditions. Its calculation formula is as follows:

\[ RMR14 = (RMR + F_0)F_eF_s \] (2)

where RMR is the basic rock mass quality score without consideration of the excavation factors, \( F_0 \) is the influence coefficient of the joint strike, \( F_e \) is the influence coefficient of the tunnel excavation mode, and \( F_s \) is the correction coefficient of the stress–strain characteristics. The expression of RMR is as follows:

\[ RMR = R_1 + R_2 + R_3 + R_4 + R_5 \] (3)

where \( R_1, R_2, R_3, R_4 \), and \( R_5 \) are the uniaxial compressive strength of rock (UCS), the RQD, the most influential discontinuity spacing, the discontinuity state score, and the groundwater state score, respectively. Due to the difference and the number of factors considered in different rock mass classification methods, there are also differences in obtaining the corresponding indexes. Table 1 compares the GSI, the BQ, and the RMR14 surrounding rock classification indexes.

| Joint spacing | Joint set number | Trace length | Joint filling property | UCS | Groundwater | Tunnel excavation method |
|---------------|-----------------|--------------|-----------------------|-----|-------------|--------------------------|
| GSI           | √               | √            | √                     |     |             |                          |
| BQ            | √               | √            | √                     |     | √           |                          |
| RMR14         | √               | √            | √                     |     | √           | √                        |

As can be seen from the table above, the RMR14 classification method represents the surrounding rock characteristics in the most complete way but requires the greatest number of parameters and is the most difficult to measure accurately (Chen et al., 2017). Tunnel engineering has the characteristics of progressive acquisition of the surrounding rock information. In the case of insufficient rock mass information on site, the more parameters that are required, the more difficult the evaluation will be when the information pertaining to the excavation faces is insufficient. For example, the GSI evaluation can be obtained without the groundwater and rock mass strength measurement, but the evaluation results of the BQ and the RMR14 cannot be evaluated if only approximate measurement of the excavation face is carried out. The BQ value can reflect the joint spacing of the rock mass and the state of the groundwater, and the GSI index can supplement the characteristics of joint filling. Therefore, the BQ and the GSI, which are relatively easy to calculate, are utilized to predict the RMR value. In this paper, a progressive probability evaluation model of rock mass classification was established based on the Bayesian network, and the results of the GSI and the BQ rock mass classifications were combined to predict the RMR14.

2.2. Empirical formulas for RMR14

Many empirical methods of rock mass classification have been developed based on tunnel engineering-related case studies by various researchers. Table 2 shows some empirical formulas for the RMR with the GSI and BQ values.

| Rock mass classification | ID | Empirical formulas | References       |
|--------------------------|----|--------------------|------------------|
| GSI                      | 1  | \( GSI = RMR - 5 \) | Hoek et al. (2000)|
|                          | 2  | \( GSI = \frac{RMR}{1.36} - 4.34 \) | Hoek et al. (1997)|
|                          | 3  | \( GSI = 6 \exp \left( \frac{RMR}{20} \right) \) | Singh et al. (2013)|
3. The concept and constitution of Bayesian networks

The Bayesian network (Li et al., 2017) is also called the belief network. The topology of the Bayesian network is a directed acyclic graph (DAG). The nodes of the DAG in a Bayesian network represent random variables that can be observable variables, hidden variables, or unknown parameters. Variables or propositions that are believed to be causally related are connected by arrows, i.e., the arrows of the two linked nodes represent the fact that the two random variables are causally related or unconditionally independent. If two nodes are connected by a single arrow, indicating that one node is the parent and the other node is the child, the two nodes produce a conditional probability value. For example, assuming that node E directly affects node H, the directed arc from node E to node H is established by an arrow pointing to node H, and the weights are represented by conditional probability P(H|E).

\[
G = (I, E)
\]

where G denotes a DAG, I represents the collection of all nodes in the graph, and E represents the collection of directed connection segments.

\[
X = (X_i)i \in I
\]

The random variable is represented by a node i in its DAG. If the joint probability of node X can be expressed as

\[
p(x) = \prod_{i} P(x_i | x_{pa(i)})i \in I
\]

then X is called a Bayesian network relative to a DAG, G, where pa (i) denotes the parent of a node.

The composition of a Bayesian network mainly comprises two parts: (1) an acyclic graph composed of directed line segments between variables and connecting variables. Directed line segments point from the parent nodes to the child nodes, representing the mutual relationship between the variables. (2) The conditional probability table (CPT) of the relationship between the variables and their related parent nodes. The CPT expresses the strong and weak relationships between the parent–child nodes. The task of a Bayesian network is data analysis and probabilistic reasoning, to obtain some data or knowledge as the basis of the evidence variables and to use the prior probability of the evidence variables to evaluate the posterior probability of the variables containing uncertain information.

4. Bayesian inference for predicting the RMR

4.1. Bayesian network probabilistic model and related parameters

The GSI, BQ values and the RMR were selected as the basic nodes of the Bayesian network. To determine the CPT, 500 equivalent samples were generated by random sampling with MATLAB based on the empirical model in Table 2. The RMR was assumed to be normally distributed. The RMR sample range could be as follows: \( RMR_{\text{max}} = 100 \), \( RMR_{\text{min}} = 0 \). The mean value of the RMR could be taken as \( \mu_{RMR} = 50 \). Based on the 6\( \sigma \) principle, the standard deviation of the RMR could be taken as \( \sigma_{RMR} = \frac{190 - 0}{6} = 16.67 \). Five hundred equivalent samples were generated based on this distribution. Sample distribution probability models of the GSI, the BQ and the RMR are shown in Figure 1.
According to the probability distribution model of each parameter and the evaluation criteria of each parameter in the Rock Mass Description Method (ISRM, 1978), Table 3 shows the parameters and the corresponding interval division in the Bayesian network.

**Table 3. Parameters and corresponding interval division**

| Index   | Parameter  | Range            |
|---------|------------|------------------|
| GSI     | state      | [Very poor] [0,20] Poor (20,40] Fair (40,60] Good (60,80] Very Good (80,100] |
| BQ      | state      | Class Five (0,250] Class Four (250,350] Class Three (350,450] Class Two (450,550] Class One (550,880] |
| RMR14   | state      | Very Bad [0,20] Bad (20,40] Normal (40,60] Good (60,80] Very Good (80,100] |

Statistical analysis was conducted on the rock mass classification indexes, and the distribution probability model of the rock mass classification indexes and the constructed Bayesian network are shown in Figure 2.

**Figure 1. Equivalent samples probability distribution**

**Figure 2. RMR Progressive Prediction Probability Model**

Based on the quality information of rock mass samples obtained by sampling, the Netica (Norsys Software Corporation) software was used to input the sample data into the network model; the expectation maximization algorithm was used to obtain the CPT values of the nodes through parameter learning to obtain the probability model of the progressive evaluation of the rock mass classification. Through this process, some experience of the rock mass classification could be solidified, providing support for the engineering rock mass classification. When the number of samples obtained from the field monitoring increased, the actual rock mass classification data could also be updated to the basic database of the tunnel surrounding rock classification cases so as to realize the updating of probability model parameters for the progressive evaluation of the rock mass classification.
The probability classification of the rock mass under given conditions can be obtained based on the probabilistic model of the progressive evaluation of the rock mass classification. Suppose, for example, that the [BQ] values can be obtained from data acquired at the excavation face, when P (BQ = Class Three), i.e., when [BQ] is known to be Class Three, based on the progressive grading model of rock mass, the probability grading of the RMR14 evaluation is P (RMR14 = Very_Bad) = 0.112; P (RMR14 = Bad) = 0.126; P (RMR14 = Normal) = 0.414; P (RMR14 = Good) = 0.236 and P (RMR14 = Very_Good) = 0.236. Therefore, it could be judged that the rock mass quality at this time was most likely to be Grade III. Similarly, when the GSI was known to be the “Good” rating, the probability rating of the RMR14 was also known to be P (RMR14 = Very_Bad) = 0.0833; P (RMR14 = Bad) = 0.0833; P (RMR14 = Normal) = 0.0833; P (RMR14 = Good) = 0.667 and P (RMR14 = Very_Good) = 0.0833, it could be judged that the rock mass quality was most likely to be Grade IV.

As the information pertaining to the rock mass became more complete, the probabilistic model based on the progressive evaluation of the rock mass classification could be used to judge the quality and classification of the rock mass more accurately.

4.2. Case study

To demonstrate the application of the Bayesian inference method for RMR14 prediction for tunnel engineering, the Laoying tunnel is selected as a case study. The Laoying tunnel is a rock tunnel located in Baoshan City, Yunnan Province, China. The length of the tunnel is approximately 11.5 km, and the maximum buried depth is approximately 1265 m. The surrounding rock mainly comprises granite and limestone, and there are five faults. The study area is in the interval between K10 + 559 and K11 + 708. The geological information is sampled from the excavation faces in 15 discrete positions. The measured BQ, GSI, and RMR14 values are shown in Table 4.

| Excavation faces | BQ value | BQ state | GSI value | GSI state | RMR14 value | RMR14 state |
|------------------|----------|----------|-----------|-----------|-------------|-------------|
| K10 + 559        | 467      | II       | 55        | Fair      | 64          | Good        |
| K10 + 630        | 454      | II       | 50        | Fair      | 70          | Good        |
| K10 + 680        | 315      | III      | 52        | Fair      | 58          | Normal      |
| K10 + 746        | 377      | III      | 54        | Fair      | 57          | Normal      |
| K10 + 865        | 385      | III      | 66        | Good      | 62          | Good        |
| K10 + 970        | 474      | II       | 47        | Fair      | 65          | Good        |
| K11 + 070        | 470      | II       | 52        | Fair      | 66          | Good        |
| K11 + 120        | 375      | III      | 38        | Poor      | 58          | Normal      |
| K11 + 211        | 490      | II       | 51        | Fair      | 74          | Good        |
| K11 + 302        | 460      | II       | 42        | Fair      | 68          | Good        |
| K11 + 386        | 477      | II       | 63        | Good      | 55          | Normal      |
| K11 + 462        | 390      | III      | 45        | Fair      | 62          | Good        |
| K11 + 541        | 410      | III      | 67        | Good      | 61          | Good        |
| K11 + 623        | 386      | III      | 65        | Good      | 67          | Good        |
| K11 + 708        | 473      | II       | 58        | Fair      | 67          | Good        |
Table 5 shows the results of the RMR14 Progressive Prediction Probability Model. For each measured position, the data sets with three degrees of integrity were employed in the prediction model to estimate the RMR14 values.

| Excavation faces | RMR14 measured state | BQ data prediction | GSI data prediction | GSI+BQ |
|------------------|----------------------|--------------------|---------------------|--------|
| K10 + 559        | Good                 | Good               | Normal*             | Good   |
| K10 + 630        | Good                 | Good               | Normal*             | Good   |
| K10 + 680        | Normal               | Normal             | Normal              | Normal |
| K10 + 746        | Normal               | Normal             | Normal              | Normal |
| K10 + 865        | Good                 | Normal*            | Good                | Good   |
| K10 + 970        | Good                 | Good               | Normal*             | Good   |
| K11 + 070        | Good                 | Good               | Normal*             | Good   |
| K11 + 120        | Normal               | Normal             | Bad*                | Normal |
| K11 + 211        | Good                 | Good               | Normal*             | Good   |
| K11 + 302        | Good                 | Good               | Normal*             | Good   |
| K11 + 386        | Normal               | Good*              | Good                | Good*  |
| K11 + 462        | Good                 | Normal*            | Normal*             | Normal*|
| K11 + 541        | Good                 | Normal*            | Good                | Good   |
| K11 + 623        | Good                 | Normal*            | Good                | Good   |
| K11 + 708        | Good                 | Good               | Normal*             | Good   |

* means that the actual classification deviates from the probability classification.

The results in Table 5 show that the rock mass quality evaluation of the 15 excavation faces selected by the Laoying tunnel and the results obtained by the probability classification model are relatively consistent with the measured results. When only the BQ data were included in the prediction, the prediction accuracy of the RMR reached 66.7%. When the data source was expanded to the BQ + GSI, the prediction accuracy of the RMR reached 86.7%. This indicates that the prediction accuracy of the RMR will be improved when more data sources are combined; it also verifies that the progressive probability classification model of rock mass proposed in this study has a high level of accuracy in the Laoying tunnel research section.

5. Conclusion

In this study, by comparing the index differences of related rock mass classification methods and combining the problems of the surrounding rock classification methods in the implementation process of tunnel engineering, it is proposed that the rock mass classification requires progressive evaluation. This paper establishes a progressive probability classification model of the RMR14 based on Bayesian network theory. The model combines the results of the GSI and the BQ to predict the RMR14. The model can obtain the probability results of the RMR14 classification under different data sources.

This paper collects the empirical formulas for the RMR14 with the GSI and the BQ, uses MATLAB to generate 500 sets of equivalent samples, and obtains the CPT of the model based on the expectation maximization method. In this study, 15 groups of basic rock mass classification information were obtained by investigating the construction site of Laoying tunnel in Yunnan Province. Through comparison with the actual measured data, the RMR progressive grading model proposed in this paper was found to have a high level of accuracy in the engineering example sections.

However, this is a statistical and probabilistic study that is limited to the available information used; the accuracy of the model in other tunnel engineering conditions also needs to be further verified. It is possible that this study may be extended and updated with data from other types of rock masses.

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