Granular transport in a horizontally vibrated sawtooth channel

Shahin Mobarakabadi,1 Ehsan Nedaaee Oskoee,1,∗ Matthias Schröter,2 and Mehdi Habibi1

1Institute for Advanced Studies in Basic Sciences, Gava Zang, Zanjan 45195-159, Iran
2Max Planck Institute for Dynamics and Self-Organization (MPIDS), 37077 Göttingen, Germany

(Dated: January 29, 2014)

We present a new mode of transport of spherical particles in a horizontally vibrated channel with sawtooth shaped side walls. The underlying driving mechanism is based on an interplay of directional energy injection transformed by the sidewall collisions and density dependent interparticle collisions. Experiments and matching numerics show that the average particle velocity reaches a maximum at 60 % of the maximal filling density. Introducing a spatial phase shift between the channel boundaries increases the transport velocity by an order of magnitude.

PACS numbers: 45.70.-n, 45.70.Mg, 83.80.Fg, 87.15.hj

I. INTRODUCTION

Brownian motors extract useful work from a noisy environment by means of a broken spatial symmetry [1, 2]. The concept of Brownian motors is of great importance in cell biology and nanotechnology, however their underlying principle is not limited to thermal noise and can therefore be also implemented in macroscopic setups. One example for such an athermal, macroscopic, and noisy environment are granular gases [3].

Due to the dissipative collisions between particles, granular gases require constant external driving which is in most cases provided by shaking the container. Work can then e.g. be extracted by means of a rotational ratchet, where the symmetry is broken by different coatings on the two sides of each vane [4]. Or probe particles with an asymmetric shape can be set into translational [5, 6] or rotational [7] motion.

Another class of granular Brownian motors converts the random motion of the particles into a directed flow. This can either be implemented by breaking the symmetry of the driving [8–10], or by breaking the spatial symmetry of the container boundaries [11–18]. All setups in the latter group include a sawtooth shaped base plate which is shaken vertically to drive the granular gas. Their phenomenology includes height-dependent flow directions [11, 13, 16], segregation of binary mixtures [14, 15], and rotational motion of the circular base [17].

In this paper we decouple the direction of driving from gravity: our system is a horizontally vibrated channel where sawtooth shaped side walls break the symmetry. This geometry allows us to control the average particle density, a feature which can not be realized with vertically shaken cells with sawtooth floor. We observe unidirectional transport with a non-monotone density dependence. In section 2 we discuss the experimental setup and results. In section 3 we introduce and validate our numerical simulations which are then used in section 4 to gain insight into the transport mechanisms.

II. EXPERIMENTAL RESULTS

The experimental setup consists of two sawtooth corridors which are connected by two small pentagonal cells at each end of the corridors, as shown in Fig. 1a. Each corridor consists of five triangular cells; both floor and walls are made from plexiglass. The channels are mounted horizontally and shaken in X direction (c.f. fig. 1b) using an off center pulley which is driven by an AC motor. The oscillations amplitude is 7.5 ± 0.5 mm, the driving frequency 5 Hz.

The channel is filled with steel spheres of radius 6 mm. The maximum number of spheres per triangular cell is 21, this value defines the maximal area fraction \( \phi_c \). Ten percent of the beads have been colored in order to use them

*Electronic address: nedaaee@iasbs.ac.ir

FIG. 1: (Color online) Sawtooth channel. Top view (a), and dimensions (b) of the experimental setup and visualization of the setup used in the simulations (c). Particles are transported in the negative Y-direction in panel (b).
as tracers, this has been achieved by oxidizing them in a fire for one minute. We checked with a simple bouncing experiment that their restitution coefficient was within the error bar of untreated steel spheres. The average speed of the beads is then determined by measuring the rate at which tracers enter a given cell; the sample time is chosen so that the average tracer has completed at least two full cycles.

Figure 2 shows the average drift speed as a function of the renormalized area fraction \((\phi/\phi_c)\) where \(\phi\) is the number of spheres per cell. Three different regimes can be distinguished. For low area fractions \((0.005 < \phi/\phi_c < 0.08)\) the particles move in negative Y direction (i.e. downward in fig. 13) with a velocity that is independent of \(\phi\). As here are on average less than 2 beads in a cell, particle-particle collisions are rare. We refer to this range as the dilute regime.

For intermediate area fractions \((0.08 < \phi/\phi_c < 0.6)\) the average speed increases monotonically. Fitting a line to the experimental data in figure 2 shows that \(\langle V_y \rangle \sim (\phi/\phi_c)^{0.6(2.05)}\) in the intermediate regime. This effect delineates the cooperative regime.

For even higher densities \((\phi/\phi_c \geq 0.6)\) the drift speed decreases sharply. As the system approaches the fluid-solid transition density, we refer to this as the jamming regime.

III. SIMULATION METHOD

To elucidate the transport mechanism we have performed molecular dynamics simulations. The normal force between two grains \(i\) and \(j\), \(F_{n,i,j}\), is given by

\[
F_{n,i,j} = f(\frac{\delta_{i,j}}{d})(k_n\delta_{i,j}\mathbf{n}_{i,j} - \gamma_n m_{eff} v_{n,i,j}).
\]

Here \(v_{n,i,j}\) is the normal component of the relative velocity between the grains (evaluated at the contact point), \(\mathbf{n}_{i,j}\) is the unit vector corresponding to the distance \(r_{i,j}\) between the two grain centers. \(d\) is the particle diameter, \(m_{eff}\) the effective mass \(m_im_j/(m_i + m_j)\), and \(\delta_{i,j} = d - r_{i,j}\) is the normal compression at the contact. \(f(x)\) equals \(\sqrt{x}\) which models Hertzian contacts. The density of the grains is set to 7.87 g/cm³. The numerical values of the elastic constant \(k_n\) and the viscoelastic constant \(\gamma_n\) are listed in table I. They have been determined by numerically matching the experimental results of a steel sphere bouncing of a steel plate (respectively a plexiglass plate for the particle wall collisions described below).

We do neglect the viscoelastic tangential interaction between grains, this assures that particles do not experience any vertical forces and remain therefore in contact with the bottom surface of the channel. The agreement shown below between our simulations and experiments justifies this decision by hindsight.

The same equations are used to model the interaction between grains and the channel walls, the coefficients have an added subscript \(w\). Viscoelastic tangential forces between the grains and side walls are neglected too. The mass of the container is set to be infinity, consequently \(m_{eff} = m\).

For the evaluation of the tangential force \(f\) exerted by the channel bottom on a particle we follow a method proposed by Kondic [10] which distinguishes between sliding and rolling contacts. In a first step the contact is assumed to be rolling with \(f = ma\). Taking the rotation into account, the acceleration \(\mathbf{a}\) of the sphere can be computed as \(\mathbf{a} = 2/7\mathbf{a}_s\) with \(\mathbf{a}_s\) being the acceleration of the surface. If the no-sliding condition \(|f| \leq f_{max} = \mu_s mg\) (\(m_s\) is the static coefficient of friction) is satisfied, this is the final result.

Otherwise, the sliding contact has to be evaluated using the dynamic friction coefficient \(\mu_d\):

\[
f = -\mu_d mg \frac{v_w}{|v_w|}
\]

where \(v_w\) is the velocity of the contact point relative to the substrate. The values of \(\mu\) used are listed in table I; they are only justified by their ability to reproduce the experimental results.

![Graph](image_url)
FIG. 3: Modified boundary geometries. Channel a) interpolates between a sawtooth and a triangular shape by increasing the angle $\alpha$. $\beta$ is kept constant at 72 degree as in figure 1. Channel b) introduces a relative displacement $d$ of the two sidewalls.

The shaking of the container is modeled by a sinusoidal excitation with an oscillation frequency $\omega$ and an amplitude $x_0$. As the simulations are performed in the referential frame of the container, a force $-\omega^2x_0 \cos(\omega t)$ is added to each particles center of mass. The code is written in C++ using a 5th order predictor-corrector algorithm for numerical integration of the equations of motion. The time step increment ($\Delta t$) is set to $10^{-5}$ s in all simulations; the simulation time is chosen so that the average particle has performed four full cycles which requires at maximum $3.6 \times 10^7$ time steps.

Figure 2 shows that the results of the simulation are in good agreement with the experiment. Therefore we will use our simulations to gain insight into the driving mechanism in the three regimes.

IV. TRANSPORT MECHANISM

Understanding the transport mechanism depicted in figure 2 requires answers to the following three questions:

(A) Why are the particles moving unidirectionally at all? This is best discussed in the dilute regime.

(B) Why does the drift velocity increase in the cooperative regime?

(C) Why does the drift velocity go down at even higher densities?

The dilute regime is specifically apt to study the underlying transport mechanism as it allows us to ignore inter-particle collisions. First, it should be pointed out that dissipative collisions are not a necessary condition to have a finite drift velocity; the mechanism is therefore different from fluxes seen in granular gases going through beveled pores [20]. This has been tested by short simulations where $\gamma_n$ and $\gamma_{nw}$ have been set to zero.

Secondly, like in other ratchet systems the symmetry breaking of the boundary is a necessary condition to obtain a finite $\langle v_y \rangle$. If we increase in our simulations the angle $\alpha$ shown in Figure 3a, we change the channel profile from a sawtooth shape to a triangular one. This goes together with a reduction of the average drift velocity towards zero as shown in figure 4.

We therefore conjecture the following driving mechanism: spheres gain kinetic energy in x-direction due to the frictional contacts with the horizontally vibrating base. This kinetic energy is then converted by the sidewall collisions into a directed motion in negative y-direction. To substantiate this conjecture, we have computed the distribution $P$ of particle direction with respect to the Y-axis $\gamma$, measured after the particles collided with the sidewall (c.f. Fig. 1b). As it can be seen in figure 5 the majority of post collisional velocities point in negative y-direction which corresponds to $\gamma$ values larger than 90 degree. For $\phi/\phi_c = 0.047$ more than 60% of post collision vectors fall into this range. Our system seems therefore in the dilute limit amenable for the analysis presented in [16], which would explain why for our comparatively elastic steel particles and large ratchet angles no reversal of the transport direction was observed.
B. Cooperative regime

For values of $\phi/\phi_c > 0.08$, the average drift velocity increases proportional to $(\phi/\phi_c)^{0.6}$ as shown in figure 2. However, the post sidewall collision angle distribution $P(\gamma)$ even becomes more balanced, at $\phi/\phi_c = 0.476$ only 58% of the vectors point in negative y-direction. Therefore the mechanism behind the increase of $\langle v_y \rangle$ has to be something different. We hypothesize that the increased density keeps the particles longer in close proximity to the narrow passage between cells.

There are two indications for the importance of the narrow passage area. The first one can be seen in figure 6 which shows that $\langle v_y \rangle$ has (at all densities) a local maximum when the shaking amplitude $x_0$ is approximately half of the width of the passage. It is also interesting to note that only in the in the cooperative regime the average drift speed increases systematically with the amplitude $x_0$.

The second sign that improved overcoming of the narrow passage is the mechanism behind the cooperative regime can be found when studying a channel where the two sidewalls are shifted by a length $d$ against each other as in figure 3b. Figure 7 clearly demonstrates that the distinction between dilute and cooperative regime vanishes when the ratio $d/L$ goes to 0.5 i.e. the narrow passages are taken out of the system. The accompanying increase of the average drift speed by one order of magnitude is also significant from the perspective of potential applications.

C. Jamming regime

The strong decrease in drift velocity for $\phi/\phi_c > 0.6$ can be explained back to two mechanisms. First the effective redirection of the kinetic energy at the sidewalls stops. As can be seen in figure 5 at $\phi/\phi_c = 0.857$ only 49% of the post collisional velocities have $\gamma$ values larger than 90 degree.

Secondly, figure 8 shows that the average collision duration doubles in the jamming regime. For Hertzian sin-
FIG. 8: The average collision duration doubles in the jamming regime, this hints towards the beginning formation of force-chains. These numerical data are measured in the standard setup (figure 1c) with an oscillations amplitude of 7.5 mm and a driving frequency of 5 Hz.

FIG. 8: The average collision duration doubles in the jamming regime, this hints towards the beginning formation of force-chains. These numerical data are measured in the standard setup (figure 1c) with an oscillations amplitude of 7.5 mm and a driving frequency of 5 Hz.

V. CONCLUSIONS

We have demonstrated experimentally and numerically a new method of granular transport in a horizontally vibrated channel with sawtooth shaped boundaries. The average drift speed depends on the filling fraction, increasing for intermediate densities and decreasing again when approaching jamming. The underlying driving mechanism was found to be the redirection of the injected kinetic energy in the particle sidewall collisions. The reason for the drift speed increase at intermediate densities seems to be the improved flux through the narrow passages between the tips of the saw-teeth; removing them by introducing a phase shift between the two boundaries resulted in a more than tenfold increase in drift velocity. At high densities the average contact duration increases which indicates that the energy injection through the shaking bottom plate becomes significantly less efficient. However, more work will be needed for establishing a microscopic theory of this transport mechanism.

ACKNOWLEDGMENTS

We thank Y. Rahmani for his help in preparing the experimental setup and H. Fazli for helpful discussions. Support from the IASBS Research Council (Grant No. G2012IASBS103 and G2012IASBS106) is acknowledged.

[1] P. Hänggi and F. Marchesoni, Reviews of Modern Physics 81, 387 (2009).
[2] P. Reimann, Physics Reports 361, 57 (2002).
[3] N. V. Brilliantov and T. Pöschel, Kinetic Theory of Granular Gases (Oxford University Press, 2004).
[4] P. Eshuis, K. van der Weele, D. Lohse, and D. van der Meer, Physical Review Letters 104, 248001 (2010).
[5] B. Cleuren and C. Van den Broeck, Europhysics Letters 77, 50003 (2007).
[6] G. Costantini, U. Marini Bettolo Marconi, and A. Puglisi, Physical Review E 75, 061124 (2007).
[7] R. Balzan, F. Dalton, V. Loreto, A. Petri, and G. Ponthuale, Physical Review E 83, 031310 (2011).
[8] J. A. Gallas, H. J. Herrmann, and S. Sokolowski, Journal de Physique II 2, 1389 (1992).
[9] R. Grochowski, P. Walzel, M. Rouijaa, C. A. Kruelle, and I. Rehberg, Applied Physics Letters 84, 1019 (2004).
[10] J. Blaschke and J. Vollmer, Physical Review E 87, 040201 (2013).
[11] I. Derényi, P. Tegzes, and T. Vicsek, Chaos 8, 657 (1998).
[12] Z. Farkas, P. Tegzes, A. Vukics, and T. Vicsek, Physical Review E 60, 7022 (1999).
[13] M. Levanon and D. C. Rapaport, Physical Review E 64, 011304 (2001).
[14] Z. Farkas, F. Szalai, D. E. Wolf, and T. Vicsek, Physical Review E 65, 022301 (2002).
[15] J. F. Wambaugh, C. Reichhardt, and C. J. Olson, Physical Review E 65, 031308 (2002).
[16] A. Bae, W. Morgado, J. Veerman, and G. Vasconcelos, Physica A 342, 22 (2004).
[17] M. Heckel, P. Müller, T. Pöschel, and J. A. C. Gallas, Physical Review E 86, 061310 (2012).
[18] K. Chen, Y. C. Chou, and K. To, Physical Review E 87, 012711 (2013).
[19] L. Kondic, Phys. Rev. E 60, 751 (1999).
[20] R. S. Shaw, N. Packard, M. Schröter, and H. L. Swinney, PNAS 104, 9580 (2007).
[21] T. Schwager and T. Pöschel, Physical Review E 57, 650 (1998).