Regression-Based Correction and I-PSO-Based Optimization of HMCVT’s Speed Regulating Characteristics for Agricultural Machinery

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Abstract: To improve the speed regulating characteristics of continuously variable transmission for agricultural machinery, in order to meet the engineering and technical requirements of precision agriculture and intelligent agriculture, the paper researches and proposes a method combining the analysis of speed regulating characteristics, regression-based correction, and the improved particle swarm optimization (I-PSO) algorithm. First, the paper analyzes the degree of deviation between the linearization degree and the theoretical value of the speed regulating characteristics of the variable-pump constant-motor system of agricultural machinery according to the measurement results of the bench test. Next, the paper corrects the speed regulating characteristics and compares the regression results based on four models. Finally, the paper proposes a design method for the expected speed regulating characteristics of agricultural machinery and it completes the optimization of speed regulating characteristics and the matching of transmission parameters with the I-PSO algorithm. Results indicate that the speed regulating characteristics of the variable-pump constant-motor system show high linearization (with a coefficient of determination of 0.9775). The theoretical and measured values of the speed regulating characteristics have a certain deviation (with a coefficient of determination of 0.8934). Therefore, correcting the speed regulating characteristics of the variable-pump constant-motor system is highly necessary. In addition, the second reciprocal function model proposed has the highest correction precision (with a coefficient of determination of 0.9978). The I-PSO algorithm is applicable to the design and application of hydro-mechanical continuously variable transmission (HMCVT) for agricultural machinery. The new method proposed can improve the HMCVT’s speed regulating characteristics efficiently and quickly. It also ensures that the speed regulating characteristics are highly consistent with the expected design characteristics (with a mean error of 1.73%). Thus, the research offers a theoretical direction and design basis for the research and development of continuously variable transmission units in agricultural machinery.

Keywords: agricultural machinery; HMCVT; correction of characteristics; I-PSO algorithm; parameter match

1. Introduction

There is a wide variety of agriculture and forestry machinery. The tractor is one of the most important pieces of agricultural working machinery applied in the fields of agriculture and forestry [1–3]. Moreover, the agriculture working machinery also includes the grain harvester combine, the cotton picker, and so on. The forestry working machinery includes the skidder, the forest fire truck, the harvesting–cultivating combination machine, and so on [4]. The agriculture and forestry machinery generally works in the conditions of severe environments and variable loads [5–8]. Running reliably at the required speed is one of the most basic performance requirements of agriculture and forestry machinery. It requires a wider range of speed changes.
For vehicles (mainly fuel engine vehicles [9,10] and electric vehicles [11–13]), when the engine is determined, the transmission system plays an important role in changing the speed. Agricultural machinery and vehicles have a certain degree of commonality. Tractors are also non-road vehicles. Therefore, the method of analogy can be used to study and analyze the power transmission system. The HMCVT, as a power dividing (composed of the mechanical power and the hydraulic power), continuously variable transmission unit [14], is applied to agricultural and forestry machinery such as tractors [15–17].

The HMCVT receives the power transmitted from the engine and then outputs the power after changing the speed according to the transmission characteristics. Then, the power goes through other transmission systems (with the fixed transmission ratio) and wheels [18–20]. Finally, the running speed of the agriculture and forestry machinery is formed. Therefore, to ensure that the agriculture and forestry machinery runs reliably at the required speed, it is necessary to design the speed regulating characteristics properly. However, designing the speed regulating characteristics of a HMCVT properly requires the correct mastery of the speed regulating characteristics. Meanwhile, the proper design of the speed regulating characteristics is the premise of using the continuously transmission unit.

Recently, a few scholars have conducted studies on the analysis and optimization design of the HMCVT’s speed regulating characteristics.

In the studies of the HMCVT’s speed regulating characteristic analysis, most scholars use theoretical analysis based on the mechanical transmission principle to derive the relational expression of the HMCVT’s speed regulating characteristics (i.e., the relational expression between the displacement ratio of the pump–motor system and the HMCVT’s ratio). Xu et al. [21] gave a relational expression between the HMCVT speed ratio and variable-pump constant-motor displacement ratio based on a theoretical analysis. The research pointed out that the relational model could offer a theoretical basis for the transmission scheme determination, parameter matching and performance analysis of the HMCVT. Later, Xu et al. [22] pointed out the requirement of the HMCVT’s steady section shift to the working section transmission ratio. Sung et al. [23] researched the speed regulating characteristics of 12 HMCVTs using the network analysis method. Yu et al. [24], after obtaining a theoretical analysis model of speed regulating characteristics, analyzed the linearization degree of the speed regulating characteristics and the change range of the HMCVT’s output speed, researched using a prototype test. Li et al. [25] analyzed the relationship between the transmission ratio and displacement ratio using a theoretical model of output speed built in the environment of Matlab. Meanwhile, the research analyzed the hydraulic–mechanical mixed section and the purely hydraulic section separately and pointed out the continuity of the speed regulating characteristics. Xia and Sun [26] derived the change relational expression of speed regulating characteristics based on the working principle of continuously variable transmission. Then, on this basis, they analyzed the working advantages of the HMCVT.

In the research field of HMCVT design, most scholars have used the optimization algorithm for the design and matching adjustment of parameters. Volpe et al. [27] used the difference evolution algorithm, the simulated annealing algorithm, and the simplex method for the optimization design of two types of power-dividing continuously variable transmission. Macor and Rossetti [28,29] set parameters, such as the gear ratio and the planetary gear transmission ratio, as the variables of optimization design, and used the PSO algorithm for the multi-object optimization design. Moreover, the research pointed out that the HMCVT’s optimization design is complicated and has strongly nonlinear characteristics. Zhang et al. [30] used the non-dominated sorting genetic algorithm for the optimization of the planetary row and transmission ratio. Some scholars first determined the form of HMCVT transmission scheme (an equal-ratio or equal-difference transmission scheme), and then determined the values of transmission parameters according to the empirical method and formulas. For instance, Ni et al. [31] designed a four-stage HMCVT with the common transmission ratios of sections as 1.88, and verified the continuity of output speed. Zhang et al. [32] and He et al. [33] designed the HMCVT’s parameters with the equal-ratio and equal-difference principles, respectively. Moreover, Liu et al. [34] used
the theoretical analysis formulas of kinematics and mechanics combined with the Newton–Raphson method for parameter matching. Cheng et al. [35] performed a mechanism analysis of the HMCVT’s speed regulating characteristics, composed of the multi-planetary-row compound transmission, and designed a non-equal-ratio-transmission HMCVT using the improved genetic algorithm. The research matched the required range of tractor working speed with the change range of each HMCVT section transmission ratio. This helped to improve the flexibility of the HMCVT’s speed regulating characteristics design.

To sum up, using the theoretical relational expression of speed regulating characteristics to design the HMCVT is common. However, the method depends on the precision of the model of the speed regulating characteristics. The HMCVT is composed of the mechanical system (generally composed of the fixed-shaft gear pair and the planetary gear train) and the hydraulic system (generally composed of the pump and the motor). In the process of transmission, it can be considered that the actual transmission characteristics are consistent with the theoretical transmission characteristics of the mechanical system. However, the hydraulic system is greatly affected by the environment, thus causing a certain deviation between the actual and theoretical transmission characteristics. Therefore, only when the transmission characteristics of the mechanical and hydraulic systems are both correct, the HMCVT’s speed regulating characteristics have certain precision. In particular, it is difficult to improve the precision of the HMCVT’s speed regulating characteristics model in the research, development, and design stage. Generally, the comparative verification of the speed regulating characteristics can only be done after the HMCVT is made, thus resulting in increasing research and development costs and duration. In addition, current studies have a serious deficiency in this respect.

To solve the problem described above, the paper proposes a regression-based correction method and an I-PSO-based optimization method for the HMCVT’s speed regulating characteristics. The paper mainly researches the following three aspects. Firstly, the paper describes a test of the speed regulating characteristics of the hydraulic system using the test bench of a variable-pump constant-motor system. Then, based on the measurement results of the test, the paper analyzes the linearization degree of the measured output speed and the deviation degree between the measured and theoretical values. The method of least squares is used for the regression analysis of the speed regulating characteristics of the variable-pump constant-motor system. The regression model is considered as the correction model of the HMCVT hydraulic system’s speed regulating characteristics. Secondly, according to agricultural machinery’s working speed requirement, engine characteristics, and the corrected model of the HMCVT’s speed regulating characteristics, the paper uses the I-PSO algorithm for the optimization design of the speed regulating characteristics and matching of transmission parameters. Thirdly, using the coefficient of determination $R^2$ and the mean absolute percentage error MAPE as evaluation indexes, the paper compares before and after optimization and before and after correction of the speed regulating characteristics. Comparison results show that the corrected model of the speed regulating characteristics of the hydraulic system has high precision; the speed regulating characteristics based on the I-PSO algorithm are highly consistent with the expected characteristics and the matching of the transmission design parameters has a good result. According to the method proposed, it is only necessary to test the output speed of the pump–motor system chosen to complete the optimization design of the speed regulating characteristics and the matching of the transmission parameters of the HMCVT effectively in the research, development, and design stage.

2. Materials and Methods

2.1. Working Principle of Tractor HMCVT Researched

Figure 1 shows the transmission scheme of the tractor HMCVT researched in the paper. The continuously variable transmission unit has eight gear pairs (suppose that their transmission ratios are $i_1$, $i_2$, $i_3$, $i_4$, $i_5$, $i_6$, $i_7$, and $i_8$, respectively), two planetary rows $P_1$ and $P_2$ (suppose that the parameters of planetary rows are $k_1$ and $k_2$, respectively), three wet clutches $C_0$, $C_1$, and $C_2$, and one variable-pump constant-motor system.
2. Materials and Methods

2.1. Working Principle of Tractor HMCVT Researched

When a tractor runs normally and works (corresponding to the HMCVT hydraulic–mechanical-power mixed working mode), the overall power of the transmission system is output from the engine and then input into the variable-pump constant-motor system (the hydraulic power flow) and the planetary gear train (the mechanical power flow). The divided power converges through the planetary gear train and then is output through P1’s gear ring or P2’s planetary carrier. When a tractor starts up (corresponding to the purely hydraulic working mode of HMCVT), the overall power of the transmission system is output from the engine and then input into the variable-pump constant-motor system directly, and drives gear pair \( i_8 \) directly to offer the power to the output end. The core part of the continuously variable unit (i.e., the power confluence mechanism) of the tractor is the Simpson planetary gear transmission mechanism and has three working sections (including the purely hydraulic working section \( H_0 \) and the hydraulic–mechanical mixed working sections \( HM_1 \) and \( HM_2 \)). Figure 1 also shows the power output routes of working sections, in which sections \( H_0 \), \( HM_1 \), and \( HM_2 \) are shown in blue, green, and red, respectively.

The calculation formulas for the speeds of the confluence mechanism planetary carrier, gear ring, and sun gear are as follows [36,37]:

\[
k_1 n_r = (1 + k_1) n_c - n_s
\]

in which \( n_r \) is the output speed of the gear ring; \( n_c \) is the input speed of the planetary carrier, and \( n_s \) is the input speed of the sun gear.

Theoretically, the transmission relationship between the variable pump and constant motor is as follows [38]:

\[
n_m = n_p \varepsilon
\]

in which \( n_m \) is the speed of the constant motor; \( n_p \) is the working speed of the variable pump, and \( \varepsilon \) is the displacement ratio of the variable-pump constant-motor system.

Combining Equation (1) with Equation (2), we then obtain the relational expression of the 3-section HMCVT’s speed regulating characteristics (i.e., the relational expression between the displacement ratio of the pump–motor system and the transmission ratio of HMCVT) as follows [39]:

\[
i_{H_0} = \frac{-i_1 i_2 i_8}{\varepsilon}
\]

\[
i_{HM_1} = \frac{i_1 i_2 i_3 i_4 i_5 i_6 k_1}{\varepsilon i_3 i_4 i_5 + i_1 i_2 (1 + k_1)}
\]

\[
i_{HM_2} = \frac{i_1 i_2 i_3 i_4 i_5 (1 + k_2)}{i_1 i_2 k_2 - \varepsilon i_3 i_4 i_5}
\]
2.2. Build the ‘Variable-Pump and Constant-Motor’ Test Bench of HMCVT for Tractor

Figure 2 shows the variable-pump constant-motor test bench built for the HMCVT for a tractor.

![Test bench of ‘variable-pump and constant-motor’ system. Note:](image)

The test bench uses the speed torque sensors of the ZJ-2000A model and ZJ-5000A model of Lanling Jiangsu, China. Table 1 gives the ranges of speed and torque.

**Table 1. Related parameters of motor.**

| Model    | Measurement Range               |
|----------|---------------------------------|
| ZJ-2000A | Rated Torque: 2000 N · m         |
|          | Working Speed: 0–3000 r /min     |
| ZJ-5000A | Rated Torque: 4000 N · m         |
|          | Working Speed: 0–5000 r /min     |

The test bench uses the variable pump of the Linde HPV-02 model with 55 cm³/rev displacement and 75 kW continuous working power, and the constant motor of the Linde HMF-02 model with 55 cm³/rev displacement and 93 kW continuous working power.

2.3. Test of Speed Regulating Characteristics for ‘Variable-Pump and Constant-Motor’ System

The test aims to measure the speed regulating characteristics of the variable-pump constant-motor system in the real environment, and offers measured data to the studies on the linearization degree of the motor output speed, the deviation degree between the measured value and theoretical value of output speed, and the correction of the speed regulating characteristics based on regression.

The test requires us to fix the variable pump’s input speed and record the constant motor’s output speed by changing displacement ratio \( \varepsilon \).

According to Equation (2), the change range of the absolute value of the displacement ratio is 0–1. Therefore, to cover the change range of displacement ratio \( \varepsilon \) completely, the test chooses displacement ratios of 0.2, 0.25, 0.3, 0.375, 0.5, 0.625, 0.75, and 1 (in total, 8 groups of tests), and considers the average of stable constant motor output speeds in each group of tests as the output speed in the current working condition. In addition, the research records the input speed data of the variable pump to analyze and determine whether the volatility of the input speed is reasonable. The analysis aims to alleviate the influence of the input speed volatility on the system’s output speed.
2.4. The Analysis Method for Speed Regulating Characteristics of ‘Variable-Pump and Constant-Motor’ System

The paper calculates and obtains $\pi_p$ (the average input speed of variable pump) and $\sigma_p$ (the standard deviation) to determine whether the volatility of the input speed is reasonable. The calculation formulas of average input speed $\pi_p$ and standard deviation $\sigma_p$ are as follows:

$$
\pi_p = \frac{1}{N_p} \sum_{i=1}^{N_p} n_{pi}
$$

(6)

$$
\sigma_p = \sqrt{\frac{1}{N_p} \sum_{i=1}^{N_p} (n_{pi} - \pi_p)^2}
$$

(7)

in which $N_p$ is the total number of measured data and $n_{pi}$ is the value of the $i$th measured datum.

The higher the linearization degree of the HMCVT’s speed regulating characteristics, the better the adjustability of the tractor’s driving speed [24]. According to the expression of the speed regulating characteristics of the HMCVT used, i.e., Equations (3)–(5), the HMCVT’s output speed and displacement ratio $\epsilon$ have a linear variation relationship theoretically. The gear system transmission is reliable, and its theoretical and actual transmission characteristics are basically consistent. Therefore, the linearization degree of the variable-pump constant-motor system’s speed regulating characteristics plays a decisive role in the linearization degree of the HMCVT’s speed regulating characteristics.

The paper proposes to perform linear fitting to the motor output speed based on the least squares method and using $R^2$, the coefficient of determination of linear fitting, as the determination basis of the linearization degree. The following is the calculation formula of $R^2$:

$$
R^2 = 1 - \frac{\sum_{i=1}^{N} [y_{measured} - y_{ideal}]^2}{\sum_{i=1}^{N} [y_{ideal} - (\sum_{i=1}^{N} y_{ideal}) / N]^2}
$$

(8)

in which $y_{ideal}$ is the theoretical value, $y_{measured}$ is the measured value of the bench test, and $N$ is the total number of data to be fitted.

The paper uses the coefficient of determination $R^2$ to evaluate the degree of consistency between the measured and theoretical output speeds of the pump–motor system.

2.5. The Regression-Based Correction Method of HMCVT’s Speed Regulating Characteristics for Tractor

According to Equation (2), the relational expression of $i_{pm}$, the transmission ratio of the variable-pump constant-motor system (i.e., $n_p/n_m$, the ratio of the input speed of pump and the output speed of motor), and the displacement ratio is as follows:

$$
i_{pm} = \frac{1}{\epsilon}
$$

(9)

According to the variation trend of measured data, the research uses the linear and nonlinear least squares methods for the regression analysis of the speed regulating characteristics of the variable-pump constant-motor system. The polynomial regression [40,41] has good generalization ability and strong applicability in the field of engineering technology. In addition, according to Equations (2) and (9), the research adds items to the numerator and the denominator based on the original theoretical model to correct the model. To sum up, the paper uses four models in the regression analysis, including the 2-order polynomial model, the 3-order polynomial model, and two $\epsilon$ reciprocal function models. The four models’ forms are as follows.

The 2-order polynomial model of speed regulating characteristics:

$$
i_{pm} = a_1\epsilon^2 + a_2\epsilon + a_3
$$

(10)
The 3-order polynomial model of speed regulating characteristics:

\[ i_{pm} = a_1 \varepsilon^3 + a_2 \varepsilon^2 + a_3 \varepsilon + a_4 \]  
(11)

The first reciprocal function model of speed regulating characteristics:

\[ i_{pm} = a_1 / (\varepsilon + a_2) \]  
(12)

The second reciprocal function model of speed regulating characteristics:

\[ i_{pm} = (a_1 \varepsilon + a_2) / (\varepsilon + a_3) \]  
(13)

The paper uses the coefficient of determination \( R^2 \) to evaluate and determine the final variation expression of the speed regulating characteristics of the variable-pump constant-motor system, and substitutes the final form of \( i_{pm} \) into Equations (3)–(5) to obtain the corrected change law of the speed regulating characteristics of the tractor HMCVT. Figure 3 shows the technical route of the correction method proposed.

**Figure 3.** The technical route of correction method proposed for speed regulating characteristics.

2.6. The I-PSO-Based Optimization Design Method of HMCVT’s Speed Regulating Characteristics for Tractor

Figure 4 shows the original speed regulating characteristics of the HMCVT researched. The HMCVT of this model has three sections (H\(_0\), HM\(_1\), and HM\(_2\)), of which the speed
regulating characteristics intersect mutually. Moreover, there are two synchronous section-shift transmission ratios. The design aims to ensure the continuity of the transmission ratio of the HMCVT.

![Figure 4](image-url)

**Figure 4.** The original transmission characteristics of HMCVT researched.

To make the HMCVT system easier to control, the two synchronous section-shift transmission ratios should be distributed in the marginal area with the displacement ratio \( \varepsilon \) of \(-1~1\) as much as possible because, in this case, the displacement ratio \( \varepsilon \) has a wider range of change. If the actually required displacement ratio \( \varepsilon \) has a small change range, the requirement to control precision is high. For instance, suppose that the designed HMCVT transmission ratio \( i_{cvt} \) has a change range of \( 1~11 \); then, if displacement ratio \( \varepsilon \) changes in the range of \(-1~1\), \( \varepsilon \) changing by 0.1 corresponds to \( i_{cvt} \) changing by 0.5, on average; if displacement ratio \( \varepsilon \) changes in the range of \(-0.5~0.5\), \( \varepsilon \) changing by 0.05 corresponds to \( i_{cvt} \) changing by 0.5 on average. This indicates that the wider displacement ratio \( \varepsilon \)'s available range is, the more applicable it is to practical engineering.

The paper sets the tire radius of the tractor as 0.9 m and the other transmission system's transmission ratio as 9. Figure 5 shows the external characteristics of the diesel engine used.

![Figure 5](image-url)

**Figure 5.** External characteristic curve of diesel engine used.

In Figure 5, the values of torque in the positions marked with the left and right red lines (i.e., the speeds of the diesel engine are 1085 and 1992 r/min, respectively) correspond to 90% of the maximum torque of the diesel engine. The design in the paper uses Wang's analysis of the research results of Resch and Renius for reference [42]. In the whole life cycle of the tractor, the proportion of time for which the tractor works in the speed section of 4~20 km/h is approximately 76~93% (in which the proportion of time in 4~12 km/h is 61~68% and the proportion of time in 12~20 km/h is approximately 15~25%).
Therefore, the paper designs section HM$_1$ as the farmland working section of the tractor and section HM$_2$ as the driving and transportation section of the tractor. The design can also avoid the frequent switch problem of the wet clutch. To sum up, the transmission ratios of the sections of the tractor HMCVT are shown in Table 2.

### Table 2. Transmission ratios of HMCVT’s sections for tractor.

| Section | Vehicle Speed (km/h) | Transmission Ratio       |
|---------|----------------------|--------------------------|
| HM$_1$  | 4~20                 | 4.34~14.14               |
| HM$_2$  | 20~40 and above      | 2.17 and below~4.34      |

Section H$_0$ is the starting section of the tractor, so, in the design, it should have a synchronous section-shift transmission ratio with section HM$_1$, and the minimum vehicle speed corresponding to the synchronous section-shift transmission ratio should be equal to or greater than 4 km/h.

Moreover, when the displacement ratio $\epsilon = 0$, in section HM$_1$, only the gear system is transmitting the power, so the power transmission efficiency is the maximum in this case. To ensure that section HM$_1$ fully plays its role in the case where the displacement ratio $\epsilon = 0$, the paper considers the speed of the diesel engine corresponding to 90% of maximum torque as the critical speed for calculation, and calculates the range of vehicle speed corresponding to the critical speed. Meanwhile, we further calculate the proportion of the vehicle speed range in 4~12 km/h, and take the HMCVT transmission ratio corresponding to 80% of the maximum proportion as the designed value of the expected transmission ratio of section HM$_1$ in the case where displacement ratio $\epsilon = 0$. The corresponding calculation formula (engine output speed through the HMCVT and other transmission systems to cause the tire to produce translation speed) of the tractor’s speed and the HMCVT transmission ratio is as follows (0.377 is the coefficient used for unit conversion):

$$u_d = \frac{0.377 \, r_d \, n_e}{i_0 \, i_{cvt}}$$

(14)

in which $u_d$ is the running speed of the tractor, $r_d$ is the radius of the wheel, $n_e$ is the working speed of the engine, and $i_0$ is the overall transmission ratio of the other transmission system.

According to the calculation result, when displacement ratio $\epsilon = 0$, the value range of $i_{cvt}$, the HMCVT transmission ratio corresponding to section HM$_1$, is 5.37~7.82.

According to Equations (3)–(5), the HMCVT’s speed regulating characteristics have a nonlinearity characteristic and many characteristic parameters (i.e., many transmission parameters to be designed). We classify the transmission parameters of the HMCVT researched and obtain the following seven transmission parameters to be designed: $i_1$ and $i_2$; $i_3$, $i_4$, and $i_5$; $i_6$, $i_7$; $i_8$; $k_1$; and $k_2$. If we use the enumeration method to match the transmission parameters and optimize the speed regulating characteristics (suppose that the precision of the transmission parameter of the mechanical part is 0.01), it is necessary to carry out $35^5 + 150^2 = 5.25 \times 10^{12}$ times of calculation and matching processes (the range of gear transmission ratio is 0.5~4 and the value range of planetary row’s characteristic parameter is 2.5~4). In addition, there are constraint conditions for the transmission parameters in the matching process. Meanwhile, a great amount of work is required if replanning the HMCVT’s speed regulating characteristics to match the working requirements of other agricultural machinery. The reasons above cause the difficulty in optimizing the speed regulating characteristics using the enumeration method. The heuristic intelligent optimization algorithm has obvious advantages in performance optimization, parameter matching, and identification, and has been used to accurately solve a series of complex engineering problems [43–46].

Therefore, the paper proposes using the I-PSO algorithm for the optimization of the speed regulating characteristics of the tractor HMCVT and the matching of the transmission param-
eters. As for the I-PSO algorithm used, the paper uses the I-PSO algorithm process [47,48], proposed in previous research and verified for engineering applications, for reference.

The optimization of the speed regulating characteristics of the HMCVT is a multi-object optimization problem. The objective function \( \text{fitness} \) of the I-PSO algorithm proposed and used in the research is as follows:

\[
\text{fitness} = |i_{HM_1}(\varepsilon = -1) - 14.14| + |i_{HM_1}(\varepsilon = 1) - 4.34| + |i_{HM_2}(\varepsilon = 1) - 4.34| \quad (15)
\]

in which \( i_{HM_1}(\varepsilon = -1) \) and \( i_{HM_1}(\varepsilon = 1) \) are the transmission ratios of the HMCVT working in section HM_1 with the displacement ratios of -1 and 1, respectively, and \( i_{HM_2}(\varepsilon = 1) \) is the transmission ratio of the HMCVT working in section HM_2 with the displacement ratio of 1.

According to Equation (15), the paper considers the sum of three sub-objective functions with equal weight as the overall objective function. Each term of a sub-objective function considers the absolute value of error between the design value and expected value as the calculation formula. Meanwhile, the constraint conditions of parameter matching are as follows:

\[
i_{HM_2}(\varepsilon = -1) \leq 2.17 \quad (16)
\]

\[
5.37 \leq i_{HM_1}(\varepsilon = 0) \leq 7.82 \quad (17)
\]

\[
i_{H_0}(\varepsilon = c) = i_{HM_1}(\varepsilon = c) \geq 8.48 \quad (18)
\]

in which \( i_{H_0}(\varepsilon = c) \) is the transmission ratio of the HMCVT working in stage H_0 when the displacement ratio is equal to \( c \).

To sum up, Figure 6 shows the optimization process of the speed regulating characteristics based on the I-PSO algorithm proposed.

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**Figure 6.** Flow diagram of optimization for speed regulating characteristics based on the I-PSO algorithm.
2.7. The Comparative Analysis Method of Speed Regulating Characteristics

To compare the optimization results of the speed regulating characteristics better, the paper uses three comparative analysis methods. First, we calculate the errors of $i_{HM_1}(\varepsilon = -1)$, $i_{HM_1}(\varepsilon = 1)$, and $i_{HM_2}(\varepsilon = 1)$ of the speed regulation of the tractor HMCVT after optimization with their expected values to verify the optimization result of the speed regulating characteristics. Meanwhile, we check whether the transmission ratios of $i_{HM_2}(\varepsilon = -1)$, $i_{HM_1}(\varepsilon = 0)$ and the intersection position of section $H_0$ and section $HM_1$ are in the ranges specified by Equations (16)–(18).

Next, we compare and analyze the difference in speed regulating characteristics before and after optimization by calculating and generating figures. Finally, based on the optimization results, we compare and analyze the difference in the speed regulating characteristics before and after correction by calculating and generating figures.

The research uses the coefficient of determination $R^2$ and the mean absolute percentage error $MAPE$ for the analysis and evaluation of different types of speed regulating characteristics. See Equation (8) for the calculation formula of the coefficient of determination $R^2$.

The following is the calculation formula of the mean absolute percentage error $MAPE$.

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \frac{y_{\text{design}} - y_{\text{ideal}}}{y_{\text{ideal}}} \right| \times 100\%$$

in which $y_{\text{ideal}}$ is the ideal expected value of the speed regulating characteristics of the HMCVT of the tractor, $y_{\text{design}}$ is the designed result of the speed regulating characteristics of the HMCVT of the tractor, and $n$ is the total number of data of the speed regulating characteristics.

3. Results and Discussion

3.1. The Results and Analysis of Speed Regulating Characteristics for ‘Variable-Pump and Constant-Motor’ System

Figures 7 and 8 show the test results of the speed regulating characteristics of the variable-pump constant-motor system. In the test, $\bar{\pi}_p$ (the mean working speed of variable pump) is 718.57 r/min and $\sigma_p$ (standard deviation) is 0.88 r/min. The results indicate that in the whole test process of the speed regulating characteristics, the variable pump’s working speed remains essentially stable, with low volatility. The variable pump’s working speed ultimately will not affect the constant motor’s output speed (i.e., it will not affect the speed regulating characteristics of the pump–motor system).

![Figure 7. Measured input speed of variable pump.](image-url)
In Figure 8, the output speed of the constant motor (showing the law of step rising) corresponds to the cases where the displacement ratio $\varepsilon$ is 0.2, 0.25, 0.3, 0.375, 0.5, 0.625, 0.75, and 1, respectively, from left to right. We use the least squares method for the linear fitting of the variation characteristics of displacement ratio $\varepsilon$, and obtain the following result:

$$n_m = 760.90\varepsilon + 30.79 \quad (20)$$

$R^2$, the coefficient of determination of linear fitting, is 0.9775. The result shows that the variable-pump constant-motor system researched has a high degree of linearization, consistent with the original transmission law of the variable-pump constant-motor system. Meanwhile, it indicates that the variation characteristics between the tractor’s speed and displacement ratio $\varepsilon$ also show a linear relationship in the adjustment of the system.

Figure 9 shows the comparison between the measured and theoretical values of the output speed of the variable-pump constant-motor system.

From Figure 9, we can see that the coefficient of determination $R^2$ between the measured and theoretical values is 0.8934. The result shows that the variation characteristics of the measured and theoretical values are basically consistent, but there is still a certain deviation. Therefore, correcting the model of the speed regulating characteristics of the variable-pump constant-motor system is particularly important.
3.2. The Results and Analysis of Regression-Based Correction of HMCVT’s Speed Regulating Characteristics

We use the least squares method for the regression analysis of Equations (10)–(13). Figure 10 shows the results.

Figure 10. Correction results of speed regulating characteristics of variable-pump constant-motor system: (a) 2-order polynomial correction model; (b) 3-order polynomial correction model; (c) the first type of reciprocal correction model; (d) the second type of reciprocal correction model.

Table 3 shows the values and coefficients of determination $R^2$ of the parameters of the four correction models.

| Type of Model                          | $a_1$    | $a_2$    | $a_3$    | $a_4$    | $R^2$  |
|---------------------------------------|----------|----------|----------|----------|--------|
| 2-order Polynomial Correction Model   | 8.1710   | -13.5400 | 6.4600   | -        | 0.9544 |
| 3-order Polynomial Correction Model   | -17.5100 | 39.7700  | -30.2700 | 8.9590   | 0.9848 |
| First Type of Reciprocal Correction Model | 0.8182  | -0.01756 | -        | -        | 0.9953 |
| Second Type of Reciprocal Correction Model | 0.2611  | 0.6309   | -0.04965 | -        | 0.9978 |

According to Figure 10 and Table 3, the second reciprocal correction model has the proper number of parameters and the highest coefficient of determination. Therefore, we choose the second reciprocal correction model as the model of the speed regulating characteristics of the variable-pump constant-motor system. The research also shows that the correction based on the original law model produces a good result.
Therefore, the modified models of the speed regulating characteristics of the HMCVT of the tractor are shown as follows and the variation law of characteristics is shown in Figure 11.

\[
i_{H0} = \frac{i_{12}i_{8}(0.2611\epsilon + 0.6309)}{(\epsilon - 0.04965)}
\]

\[
i_{HM1} = \frac{i_{12}i_{3}i_{4}i_{5}i_{k}k_{1}}{i_{3}i_{4}i_{5}(\epsilon - 0.04965)/(0.2611\epsilon + 0.6309) + i_{12}(1 + k_{1})}
\]

\[
i_{HM2} = \frac{i_{12}i_{3}i_{4}i_{5}i_{7}(1 + k_{2})}{i_{12}k_{2} - i_{3}i_{4}i_{5}(\epsilon - 0.04965)/(0.2611\epsilon + 0.6309)}
\]

Figure 11. The comparison of speed regulating characteristics of transmission parameters before and after optimization.

3.3. The Results and Analysis of I-PSO-Based Optimization for Speed Regulating Characteristics

Figure 12 shows the iteration evolution curve of the optimization of the speed regulating characteristics based on the I-PSO algorithm.

Figure 12. I-PSO-based iteration evolution curve.

According to Figure 12, for the strongly nonlinear problem, the I-PSO algorithm produces a good optimization result. From the perspective of the overall iteration process, the I-PSO algorithm’s optimization result can evolve continuously. It indicates that the I-PSO algorithm can avoid the problem of prematurity well.
After the optimization, \( i_{HM_i}(\varepsilon = -1) \) has an error of 1.08% compared with the expected value 14.14; \( i_{HM_i}(\varepsilon = 1) \) has an error of 2.32% compared with the expected value 4.3; \( i_{HM_i}(\varepsilon = 1) \) has an error of 1.79% compared with the expected value 4.34. This indicates that the optimization results of the speed regulating characteristics are consistent with the expected results.

After the optimization, \( i_{HM_1}(\varepsilon = 1) \) and \( i_{HM_2}(\varepsilon = 1) \) have an error of 0.51%; the transmission ratio characteristics of section \( H_0 \) and section \( HM_1 \) have an intersecting point when displacement ratio \( \varepsilon \) is \(-0.39\). This shows that after the optimization design, the tractor HMCVT’s speed regulating characteristics are continuous, without any point of discontinuity.

After the optimization, \( i_{HM_2}(\varepsilon = -1) \) = 0.95 < 2.17. The results show that the highest driving speed of the tractor after optimization meets the requirement of >40 km/h. Moreover, after the optimization design, \( i_{HM_1}(\varepsilon = 0) = 6.74 \), and the transmission ratio value in the intersection position of section \( H_0 \) and section \( HM_1 \) is 8.71. The optimization result completely meets the design requirement.

Figure 11 shows the comparison results of the speed regulating characteristics of the HMCVT of the tractor before and after optimization.

According to Figure 11, after the optimization, the position of the intersection point of section \( HM_2 \) and section \( HM_2 \) moves to the position with a larger displacement ratio and finally stops in the position with the displacement ratio of 1. This improves the available range of the displacement ratio, completely consistent with the design expectation.

Based on the optimization result, Figure 13 shows the comparison results of the speed regulating characteristics before and after correction.

![Figure 13. The comparison of speed regulating characteristics before and after correction.](image)

According to Figure 13, \( R^2 \), the coefficient of determination, of the speed regulating characteristics of section \( H_0 \) (choosing the transmission data with the displacement ratio of \(-1 \leq \varepsilon \leq -0.4\)), section \( HM_1 \) (choosing the transmission data with the displacement ratio of \(-1 \leq \varepsilon \leq -0.5\)), and section \( HM_2 \) (choosing the transmission data with the displacement ratio of \(0.5 \leq \varepsilon \leq 1\) and \(0.5 \leq \varepsilon < 1\)) is 0.5139, 0.6257, and 0.6351, respectively, before and after correction; \( MAPE \) is 13.69%, 6.95%, and 10.24%, respectively. Meanwhile, section \( HM_1 \) and section \( HM_2 \) have no intersection point before correction. This shows that the speed regulating characteristics of the HMCVT are not even continuous. To sum up, if we use the speed regulating characteristics in ideal conditions for optimization design, the design results have certain errors compared with the actual transmission characteristics.

4. Conclusions

The paper proposes an analysis method for the linearization degree and deviation degree of the speed regulating characteristics of a variable-pump constant-motor system.
The linear variation relationship between the actual output speed and displacement ratio of the tractor with the HMCVT is verified using the test bench results. Moreover, the research points out that the correction based on measured results is necessary for the speed regulating characteristics of the variable-pump constant-motor system.

The second reciprocal correction model proposed can be built easily (with only three parameters to be determined) and is highly consistent with the measured value of the speed regulating characteristics (the coefficient of determination is close to 1), so it should replace the original model of the speed regulating characteristics of the variable-pump constant-motor system. Moreover, the research shows that the correction of the original law model produces a good improvement result.

The method proposed for the optimization of the speed regulating characteristics and transmission parameter matching of agricultural machinery HMCVT is highly effective. Using the method combined with the I-PSO algorithm, the optimization design of the speed regulating characteristics and transmission parameter matching for the HMCVT can be completed effectively, merely by testing the output speed of the pump–motor system of the chosen model in the research, development, and design stage.

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