Investigating students’ failure in fractional concept construction

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Abstract. Failure is a failure to achieve goals. This failure occurs because a larger scheme integrates the schemes in mind that are related to the problem at hand. These schemes are integrated so that they are interconnected to form new structures. This new scheme structure is used to interpret the problems at hand. This research is a qualitative research done to trace student's failure which happened in fractional concept construction. Subjects in this study as many as 2 students selected from 15 students with the consideration of these students meet the criteria that have been set into two groups that fail in solving the problem. Both groups, namely group 1 is a search group for the failure of students of S1 subject and group 2 is a search group for the failure of students of S2 subject.

1. Introduction

Fractions studied by students in elementary school are part of a rational number that can be written in the form of a/b with a and b being integers and b not equal to zero [3]. Fractions can be given various meanings according to the context in which the concept is used. While the concept is defined as an abstract idea that can be used to classify a set of objects [6], [12] states in order to understand the meaning of fractions one must “pay attention to the mathematical theory of embedded fractions, applied, and referential mapping between their theory and situation” (p 54).

According to [17] the fraction model assigned to elementary students is the area or area model, the measurement model or the length and the set model. Freudenthal [7] sees fractions as a phenomenological source of rational numbers. Fractions are interpreted into five major subconstructs: part of the whole or set, division, ratio, operator, and measurement [2, 4, 11, 16]. Meanwhile, according to Kennedy [8] fractions have 3 meanings that will be used in this study, as follows:

(1) Fractions show a unit partitioned into parts of the same size.

Fractions can be interpreted as part of something intact. Unity broken into smaller parts produces fractions of the same size [2].

(2) Fractions show a set that is partitioned into sub-subsets of equal size.

If a set of objects are grouped into equal parts, then the situation is clearly related to the division.

(3) Fractions show the ratio (ratio).

The relationship between a pair of numbers is often expressed as a comparison.
According to [13], elementary students are in concrete operational period only able to recognize concrete facts logically, but still cannot think abstractly. In the concrete operation phase, this experience must be concrete [14,15]. This means that the object of action to be applied must be concrete, i.e., it can be perceived by the student's senses [14,15]. This concrete experience gives students constructive meaning of knowledge.

The success of constructing fractional concepts by students is evident from the work they do in the absence of errors in the process and perform the steps in correctly solving the problem on the outcome. However, not all the fractional concept constructs performed by students succeed, there are times when the fractional concept construction is done fail. The failure may be wrong to use the strategy, ignore the error in the process of obtaining the solution, assessing the wrong solution is right, wrong in doing so causing deadlock or wrong in changing the problem so that it is not in accordance with the concept structure.

Failure according to Big Indonesian Dictionary is the failure to reach the goal. So it can be interpreted in the failure of student construction on fractional concept is the student's failure to do a series of thinking process to solve the problem of fractions in writing, because the concept of the student is different from the scientific concept.

Examples of student construction failures in Arnon's research, et al [1] which illustrates the interiorization of false action in constructing fractions as a set that is partitioned into sub-subsets of equal size.

In Avi interviews of images created representing are as follows:

![Figure 1. The work of Avi](image)

Researcher (Arnon) : How many fifths do we see in your picture?
Avi : Ah, so in every circle there are five, and if we combine all fifteen.
Researcher : So three fifths or five fifteen?
Avi : this (pointed pictures), five more, er, three, lim ... three more.
Researcher : Explain again, why are you doing here three circles?
Avi : because each has, e because it is written in a numerator three. 

Avi's action (action) is wrong in two ways: first it misrepresents the numerator by thinking it can be represented by three circles, the second of which divide each of the three circles into five equal parts of 15 parts. None of these steps of action are true.

If we see from the mistakes Avi made above it can be felt Avi uses the wrong strategy in working on the problem, ignoring the error in the process of obtaining the solution, assessing the wrong solution is correct and changing the problem so that it is not in accordance with the concept structure.

Another example of the failure of the students in the first article researcher Kurniawan, et al [9] so that the need for limited intervention provided by teachers to students.

The form of failure can be seen from the meaning of fractions by kennedy are:

a. Fractions show a unit partitioned into parts of the same size

determine the fraction as a congruent part of the whole based on blue or pink student answers
b. Fractions show a set that is partitioned into sub-subsets of the same size.

specify fractions as part of a set that is congruent based on orange, black or blue student answer

c. Fraction show the ratio (ratio).

determine the fraction as the ratio of the whole based on orange color with chocolate student answers

The three answers of 3 students in determining the fractions of the given problem fail in obtaining a solution that is inconsistent with the fractional concept structure, so it needs to be traced more thoroughly, why it fails, what causes failure, and describes the failure process to find the solution of the solution. From this preliminary research, the researcher tried to trace the failure of students in constructing the concept of fractions by using different problem from the research.

2. Research Methods

This research reveals how the process of fractional concept construction by students in problem solving failure. So as to get the picture of the process of failure of fractional concept construction. According to Creswel [5], this kind of research belongs to qualitative research. The subjects in this study are 2 students of SD N Blimbing 3 Malang class V and VI selected from 15 students with consideration of two students meet the criteria that have been set into two groups that fail in solving the problem. The problem used in this study is “Nadia cuts a square ABCD into small squares (like a red square) there are 4 pieces, large squares (like a blue square) there are 2 pieces and there are two rectangles. If the number 1 is expressed by the square area of the ABCD and the red square is the square unit that makes up the square ABCD. While the rectangle is a cut of two square units. Write down the pieces on the red, blue and red and blue combinations of rectangles, circles and equilateral triangles?”

The given problem is done with think aloud. Then the students are interviewed according to the flow of the students. Then the two students are grouped into two groups, namely: group 1 (the search for the failure of the students of S1) and group 2 (the search for the failure of the students of S2)

3. Results and Discussion

Search for student failure of subject S1

Subject S1 is a student who has high ability in his class, visible from the test scores, the value of raport and the researchers’ communication with classroom teachers and the results of observations and research conducted by researchers at school.

From the results of observations of researchers on the results of work and subject S1 interviews indicate that the subject S1 experiencing confusion that resulted in not able to connect the fractional scheme on the square ABCD and rectangle to the fractional scheme on the circle [10]. Here are the work and interviews taken to show the fractions on the red part.
Figure 2. The work of subject S1 work in the presentation of red denominations

Figure 2 provides information on failures made by subject S1. After confirming the fraction on the red part in the square ABCD, subject S1 succeeded in presenting a fraction of the worth of square ABCD to rectangle and equilateral triangle. While the representation of fractions from square ABCD to circle images is not well constructed. The subject S1 knows that the number of pieces in the circle is as much as the one on the square ABCD, rectangles and equilateral triangles, but is unable to give justification to the cut shape on the circle. (the integrated problem scheme is greater than the initial scheme)

When observing and starting cutting parts of the circle the subject S1 realizes there is something wrong with the pieces he did on the circle image or not the same shape of the cuts. S1 subject tries to think, move the ruler, move the pen and measure the circle. Then he pours it into a circle image of the results that are in his mind but still wrong. This is because the subject S1 can not connect the scheme that belongs to the new scheme on the fraction in the circle. Footage of interview result with subject S1.

R : What do you do for fractions on the red part in square ABCD
S1 : I am trying to equate part of this ABCD square. (pointing to a red square)
R : Then can you explain where the fraction of 1/16 can be found?
S1 : I divide this square ABCD into the same part as the red part.
   And the result is 16, the red one is a part of 16 parts.
   (while pointing and cutting into square ABCD)

then

R : You serve the fragment (pointing to 1/16) here (pointing to the rectangle) and you shaft one part here. Right?
S1 : yes ....
R : You also do on the circle. Why do you hesitate and are looking for a form for presentation, can you explain!
S1 : I am trying to make this (pointing around the circle), like this (pointing rectangle) or this (pointing square ABCD).
R : Owh so ..... Why are pieces like this that you do?
S1 : Confused me how to share (with a smile)

This dialog illustrates the confusion experienced by the subject S1 when deciding which piece will be
done in the circle image, whereas in rectangular and equilateral triangles can be done correctly. This
confusion occurs because the subject S1 tries to equate (shape) the shape of the piece on a circle with
a square ABCD and a rectangle. Subject S1 realized there are 4 ways to cut the circle into equal
parts. How to cut on the circle, namely: pieces in a simple way, pieces using percent, pieces using
degrees and pieces with similarity of form (similarity). Of the four ways that the subject S1 using
pieces with the same shape, because it is considered easier.

Subject S1 tries to continue to solve the next problem. This is evident from the work and interview
footage conducted to show fractions on the blue.

Figure 3. The work of subject S1 in the presentation of a blue fraction

Figure 3 still seen subject S1 failure. Although already trying to find other ways. In the fractional
representation of the square ABCD to the same triangular image of the subject S1 performs the cut on
the equilateral triangle into two equal parts, then the result of the piece of the right part is moved to
the left side so that the shape becomes rectangle. The rectangle is cut into parts of the same shape so
that the number is the same as the square ABCD. While in the presentation of fractions from square
ABCD to circle images are not constructed again properly [10].

Subject S1 can not take advantage of the schemes that are owned (construction is not intact) on the
fractions in the circle. When observing and starting cutting parts of a circle the subject S1 is still
aware there is something wrong with the pieces he did on the circle’s image or not the same shape of
the cuts.

Here’s a snippet of the interview.

R : You present the fraction (pointing to 4/16) here (pointing to the rectangle) and you shove one
here.
S1 : yes (heheh)
R : Well, this one (pointing to the circle) also you still hesitate ......?
S1 : Hehehe
R : But still you keep doing like this, why not change?
S1 : Still confused how to share it (hehehe)

Subject S1 still fails in constructing the similarity (similarity) on the shape of a loop cut. This happens
because the subject has not maximized the scheme it has.

Then subject S1 continues to resolve the last problem. This is evident from the work and interview
footage conducted to show the fractions on the combined parts of red and blue.
Figure 4 still shows the subject S1 has failed because the scheme that has been in can not be utilized (construction is not intact). This is seen in the presentation of the combined fractions of the square ABCD to the circle image by the subject S1 by trying to change the shape of the cut on the circle image. The piece is formed into 3/4 of the top and 1/4 of the bottom. The bottom of the circle is placed on the right and left side above 3/4 of the circle.

Here's a snippet of the interview.

R : You present a fraction (pointing to 5/16) here (pointing to the rectangle). Well here you start to change it. Can you explain, why you change.
S1 : I just realized, if this (pointed circle) can be cut and placed here (the bottom piece of the circle)
R : Is this piece (pointing to the bottom piece of the circle) the same as the part you put this on?
S1 : Yes (hehe) .... I confuse this one (pointing to the bottom piece of the circle) not the same as this one (pointing to the combined pieces of the top of the circle)
R : Well just keep on doing it, why?
S1 : Yes, how do I change it like this (his expression still shows confusion)

Behavior of the stages of the subject S1 on the fractions of red, blue and combined red and blue, so the occurrence of failure in constructing the concept of fractions (fractions as equal parts of the whole) can be seen in the following figure.

Figure 5. Failure on fractional concept observed on the subject of S1
Figure 5 gives an overview of the stages of the subject S1, beginning from the stage of understanding, observing, measuring, cutting, moving pieces, shading to belief (by writing how to find answers) of the given problem.

At the stage of the process of determining how to cut the circle, subject S1 knows there are 4 ways (simple, percent, similarity and degree). The subject then uses the similarity of the square ABCD as the ultimate image which is most easily done by cutting resulting in another image to be the same shape as the square ABCD. So the subject S1 is not able to prune the cut shape on the circle that has the same fraction as the fraction in square ABCD.

**Search for student failure of subject S2**

The subject of S2 is a student who has a moderate ability in his class and netted based on observations and communication of researchers with classroom teachers. Based on the observation of the researcher at the time of the subject S2 of study failed in presenting the concept of fracture (fraction as equal parts of whole) and the concept of fractions worth of two different images.

Here are the work and interviews taken to show the fractions on the red part.

![Figure 6. The work of the subject of S2 in the presentation of red denominations](image)

The failure performed by the subject S2 of Figure 5 is seen in the unsuccessful S2 in presenting the equivalent fractions in equilateral triangle images. The representation of a fraction of the value of the square ABCD to an equilateral triangle image is done by cutting the sides of the equilateral triangle into triangular, square and rectangular shapes. This happens because the subject of S2 has an existing schema integrated greater than the problem scheme that is being encountered [10]. So the construction that happened failed, while the presentation of a fraction of the value of square ABCD to rectangle and circle successfully constructed well. The subject of S2 can not connect the scheme owned by the new scheme to fractions in equilateral triangles. When observing and initiating the cutting of equilateral triangle parts the subject S2 realizes that the cuts are not of the same shape. Here's a snippet of interviews with the subject of S2.

R : What do you do for fractions on the red part in square ABCD
S2 : I cut this part of square ABCD into the same part as the red part. (pointing to a red square)
R : can you explain where fractions of 1/16 can be found?
S2 : I divide this square ABCD into the same part as the red part. the result is 16, its red 1. So 1/16 (while pointing and cutting into square ABCD)
then

R : You present the shards (pointing to 1/16) here (pointing to rectangles and circles) and you shove one here. Right?
S2 : right sir.
R : how to draw this equilateral triangle?
S2 : I tried to cut it out so like this (pointed around the first equilateral triangle) not the same
shape, then I draw again like this (pointing to the equilateral equilateral triangle), keep me shading one part. (thinking)
R : Owh so ..... Are you sure of this piece?  
S2 : Confused (hehe), not sure sir.

This dialogue illustrates the confusion and uncertainty of the subject of S2 in determining the shape of the cut made on an equilateral triangle image. This confusion occurs because the subject S2 can not cut out the same cuts on an equilateral triangle as it does on the square ABCD, rectangles and circles. The subject of S2 goes on to the next issue. This is evident from the work and interview footage conducted to show fractions on the blue.

![Figure 7. The work of the subject S2 in the presentation of a blue fraction](image)

Figure 7 still shows the subject S2 to failure. Although it has tried to bypass the other way on an equilateral triangle image, either an equilateral triangle image is provided as well as illustrated by the subject S2. It is seen that the subject S2 did not succeed in cutting the same triangular image with the same shape. The subject of S2 can not take advantage of the schemes they have (old schemes not yet activated) on fractions in equilateral triangles.

Here’s a snippet of the interview.
R : You present a denomination (pointing to 4/16) here (pointing to the rectangle and circle) and you shove one here.  
S2 : Yes, Sir.  
R : how is this (pointing to an equilateral triangle) are you still confused?  
S2 : right sir. I cut it here (pointing to the first equilateral triangle image) not sure yet, keep me drawing again its triangle. Then I cut also still not sure ..... hehe  
R : But still you keep doing like this, why not change?  
S2 : Still confused how to share it sir (hehehe)

The subject of S2 still fails to construct similarity on the equilateral triangle pieces. This happens because the subject has not maximized the scheme it has.

S2 subject tries to continue to solve the next problem. This is evident from the work and interview footage conducted to show the fractions on the combined parts of red and blue.

![Figure 8. The work of the subject of S2 in the presentation of the combined fractions of red and blue](image)
Figure 8 still visible subject of S2 failure. The subject of S2 does not have a scheme to solve the problem because it has not been in the activation of schemes owned on fractions in equilateral triangles.

Here’s a snippet of the interview.

R : You present a fraction (pointing to 5/16) here (pointing to the rectangle and circle). Well here you start to change it (pointing to an equilateral triangle). Can you explain, why you change.
S2 : I believe it looks like this (pointing an equilateral triangle)
R : Does this snippet (pointing to the same piece of triangle) is appropriate?
S2 : if the number is the same Mr. .... but the shape is not the same
R : So what?
S2 : Yes, just like this sir (his expression still shows confusion)

The behavior of the stages performed by the subject S2 on the combined fractions of red and blue so that the failure to construct the concept of fractions (fractions as equal parts of the whole) can be seen in the following figure.

Figure 9 gives an overview of the stages of the subject of S2, beginning with the stage of understanding, observing, cutting, thinking, counting, shading to belief (by writing how to find answers) of the given problem.

At the stage of the process of determining how to cut an equilateral triangle, the subject of S2 knows there are 3 ways (divided into two equal parts, patterned pieces, and pieces inside there are triangles). Then the subject uses similarity in the previous image (eg in the square ABCD and the equilateral triangle (on the issue)) as part of the resulting image on the cutting of the equilateral triangle which is cut into two equal parts and the piece inside is the triangle.

The subject of S2 fails in constructing the similarity of the equilateral triangle pieces. This happens because the Subject S2 can not connect the schemes that belong to the new scheme on fractions in equilateral triangles and the subject S2 has not yet maximized the scheme it has.

So the subject of S2 is incapable of imaging the cut shape on the equilateral triangle having the same fraction as the fraction in the square ABCD.
4. Conclusion

Search of student failure in fractional concept construction divided into two groups, that is group 1 is group of student failure search of subject S1 and group 2 is group of student failure search subject S2.

Group Search 1 is marked with subject of S1
a) integrated problem scheme greater than the initial scheme (owned scheme)

b) can not connect the scheme that belongs to the new scheme in the fractional scheme on the square ABCD to the fractional scheme on the circle.

c) realize there are 4 ways to cut the circle into equal parts, namely: cut in a simple way, cut with percent, cut with degree and cut with similarity. Of the four ways that the subject S1 using pieces with the same shape, because it is considered easier.

d) the schemes that have been owned can not be utilized (construction is not intact)

Group Search 2 is marked with subject of S2
a) the initial schema integrated is greater than the problem scheme

b) Unable to maximize the scheme, so as not to be able to imitate the cut shape on the equilateral triangle having the same fraction as the fraction in square ABCD.

c) know there are 3 ways to cut the triangle equally, ie a. divided into two equal parts, b. patterned pieces, and c. pieces inside there is a triangle.

d) has no scheme to solve the problem because it has not been in activation of schemes owned on fractions in equilateral triangles.

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