Four-fermion interactions and sgoldstino masses in models with a superlight gravitino

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Abstract

We discuss the rôle of the effective interactions among four matter fermions in supersymmetric models with a very light gravitino. We show that, from a field-theoretical viewpoint, no model-independent bound on the gravitino mass can be derived from such interactions. Making use of a naturalness criterion, however, we are able to derive some interesting but not very stringent bounds, complementary to those obtained from the direct production of supersymmetric particles. We also show that, generically, masses for the spin-0 partners of the goldstino (sgoldstinos) of the order of the gravitino mass and much smaller than squark and slepton masses do not obey a naturalness criterion.
1. In the study of realistic supersymmetric extensions of the Standard Model (for reviews and references, see e.g. [1]), the old subject [2, 3] of the phenomenological implications of a very light gravitino was recently revamped in a series of papers [4, 5, 6, 7, 8].

It is well known that, if the gravitino is light (say, $eV < \sim m_{3/2} < \sim keV$), then the effective interactions of its goldstino components with the fields of the Minimal Supersymmetric Standard Model (MSSM) play an important phenomenological rôle. Pair-production of MSSM R-odd particles (sparticles) at colliders is still controlled by the renormalizable MSSM couplings, but each of these particles can decay via its effective coupling with the corresponding ordinary particle and the goldstino. For a given sparticle mass, and apart from mixing effects, the latter coupling is entirely controlled by the gravitino mass $m_{3/2}$ or, equivalently, by the supersymmetry-breaking scale $F = \sqrt{3} m_{3/2} M_P$, where $M_P \equiv (8\pi G_N)^{-1/2} \simeq 2.4 \times 10^{18}$ GeV is the Planck mass [1].

If the gravitino is very light, say $m_{3/2} \ll eV$, then its effective interactions with the MSSM fields are even stronger, and additional phenomenological implications must be taken into account. For example, diagrams involving goldstino exchange can be important for the pair-production of MSSM sparticles. Also, the gravitino can be produced in association with an MSSM sparticle, such as a sfermion or a gaugino. Finally, pair-production of gravitinos can be considered, tagged by a single photon or a single jet. By combining the phenomenological analyses of all these processes, an absolute lower bound on the gravitino mass can be established. A first estimate of this bound can be obtained [3] by considering the last class of processes, in a situation where the MSSM sparticles are sufficiently heavy to escape detection. With this method, the present lower bound on the gravitino mass can be estimated to be $m_{3/2} \gtrsim 10^{-5}$ eV, corresponding to $\sqrt{F} \gtrsim G_F^{-1/2} \sim 300$ GeV. An important feature of this limit is its model-independence, since, apart from some controllable ambiguity [4], the goldstino effective interactions in the low-energy limit depend only on $m_{3/2} \leftrightarrow \sqrt{F}$.

The case of a very light gravitino is naturally associated with the existence of some new dynamics at a scale very close to the electroweak one, responsible for the breaking of supersymmetry, the generation of supersymmetry-breaking masses for the MSSM sparticles and the scalar partners of the goldstino (sgoldstinos), and also the non-renormalizable four-fermion effective interactions involving four gravitinos, or two gravitinos and two ordinary fermions. This unknown dynamics may also generate effective four-fermion interactions involving ordinary fermions only, which are significantly constrained by the Tevatron data [10] (we are concerned here with flavour-conserving interactions, since the flavour-changing ones can be naturally suppressed by suitable flavour symmetries). We may then ask if the study of these interactions can lead to indirect, model-independent bounds on $m_{3/2} \leftrightarrow \sqrt{F}$, comparable with the bounds coming from direct production processes. This is the first question that will be addressed in the present paper.

The second question to be addressed here concerns the class of supersymmetric models

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1 We consider here, for simplicity, the case of pure $F$-breaking, with $F$ real and positive.
where the sgoldstinos have masses much smaller than the MSSM sparticles: we are going to study the stability of such a situation with respect to quantum corrections.

2. To keep the discussion as simple as possible, we consider an $N = 1$ globally supersymmetric model containing only two chiral superfields, $Y \equiv (y, \psi_y, F_y)$ and $Z \equiv (z, \psi_z, F_z)$. Despite its simplicity, this model should reproduce all the relevant aspects of the realistic case: the $Y$ multiplet will mimic the rôle of the matter superfields of the MSSM (in the limit of massless quarks and leptons), whereas the $Z$ multiplet will contain the goldstino and the (complex) sgoldstino. The most general effective Lagrangian with the above field content is determined, up to higher-derivative terms, by a superpotential $w$ and by a Kähler potential $K$. Here we choose:

$$w = \Lambda_S^2 Z, \quad (1)$$

$$K = Y Y + Z Z - \frac{Y^2 Y^2}{4 \Lambda_{yy}^2} - \frac{Y Y Z Z}{\Lambda_{yz}^2} - \frac{Z^2 Z^2}{4 \Lambda_{zz}^2} + \frac{Y^3 Y}{9 \Lambda_{yy}^4} + \frac{Y^2 Y^2 Z Z}{4 \Lambda_{yyz}^4} + \frac{Y Y Z Z^2}{4 \Lambda_{yzz}^4} + \frac{Z^3 Z}{9 \Lambda_{zzz}^4} + \ldots, \quad (2)$$

where $(\Lambda_S, \Lambda_{yy}, \Lambda_{yz}, \Lambda_{zz}, \Lambda_{yyz}, \Lambda_{yzz}, \Lambda_{zzz})$ are all parameters with the dimension of a mass, to be taken for now as independent, and the dots stand for higher-order terms in a power-expansion in the $Y$ and $Z$ fields. Notice that the Kähler potential (2) is the most general one compatible with a global $U(1)_Y \times U(1)_R$ symmetry, preserved by the superpotential (1). We recall that the appearance of non-canonical terms in $K$ implies that the model under consideration is an effective theory, valid up to some energy cutoff $\Lambda_0$ (see the discussion below). Whilst it is not restrictive to choose $\Lambda_S$ real and positive, the signs in front of the higher-dimensional operators in $K$ are purely conventional. In the conventions of eq. (2), it is crucial to have positive $\Lambda_{zz}^2$ and $\Lambda_{yz}^2$ to obtain a stable vacuum, whereas all the remaining parameters in $K$ can have either sign.

It is straightforward to derive the component Lagrangian corresponding to the chosen $w$ and $K$. We give here, for illustration, some of the lowest-order non-derivative terms. The expansion of the scalar potential around the origin is

$$V = \Lambda_S^4 + \frac{\Lambda_S^4}{\Lambda_{yz}^2} y \bar{y} + \frac{\Lambda_S^4}{\Lambda_{zz}^2} z \bar{z} + \ldots = F^2 + m_y^2 y \bar{y} + m_z^2 z \bar{z} + \ldots, \quad (3)$$

thus $V$ has a local minimum for

$$\langle y \rangle = \langle z \rangle = 0, \quad \langle F_y \rangle = 0, \quad \langle F_z \rangle = \Lambda_S^2. \quad (4)$$

Supersymmetry is spontaneously broken, with vacuum energy $\langle V \rangle \equiv F^2 = \Lambda_S^4$, and the global symmetry remains unbroken. Notice that the Kähler metric is canonical at the
minimum, so that the fields are automatically normalized. The matter sfermion $y$ and the
sgoldstino $z$ have masses

$$m_y^2 = \frac{\Lambda_y^4}{\Lambda_{yz}^2}, \quad m_z^2 = \frac{\Lambda_z^4}{\Lambda_{zz}^2}. \quad (5)$$

Notice that the two masses are controlled by two independent parameters. In particular, a hierarchical relation between them could be arranged, at the classical level, by suitably choosing those parameters. Similarly, the non-derivative part of the Lagrangian bilinear in the fermion fields reads

$$\mathcal{L}_{2f} = -\frac{\Lambda_y^2}{\Lambda_{yz}} (\bar{\psi}_y \psi_z \bar{y} + \text{h.c.}) - \frac{1}{2\Lambda_{zz}^2} (\bar{\psi}_z \psi_z z + \text{h.c.}) + \ldots$$

$$= -\frac{m_y^2}{F} (\bar{\psi}_y \psi_z \bar{y} + \text{h.c.}) - \frac{1}{2\Lambda_{zz}^2} (\bar{\psi}_z \psi_z z + \text{h.c.}) + \ldots \quad (6)$$

We remark that there is no fermion mass term, as expected from the facts that $\psi_z$ is the goldstino and that a mass for the matter fermion $\psi_y$ would break the global $U(1)_Y$. Finally, the effective four-fermion interactions are:

$$\mathcal{L}_{4f} = -\frac{1}{4\Lambda_{zz}^2} \bar{\psi}_z \psi_z \bar{\psi}_z \psi_z - \frac{1}{\Lambda_{yz}^2} \bar{\psi}_y \psi_z \bar{\psi}_y \psi_y - \frac{1}{4\Lambda_{yy}^2} \bar{\psi}_y \psi_y \bar{\psi}_y \psi_y + \ldots$$

$$= -\frac{m_y^2}{4F^2} \bar{\psi}_z \psi_z \bar{\psi}_z \psi_z - \frac{m_z^2}{F^2} \bar{\psi}_y \psi_z \bar{\psi}_y \psi_y - \frac{1}{4\Lambda_{yy}^2} \bar{\psi}_y \psi_y \bar{\psi}_y \psi_y + \ldots \quad (7)$$

The important fact to notice is that, whilst the coefficients of the Yukawa interactions and of the four-fermion interactions involving at least two goldstinos can be reexpressed in terms of the supersymmetry-breaking scale $F$ and the supersymmetry-breaking masses $(m_y^2, m_z^2)$, the coefficient of the four-fermion interaction involving only matter fermions is controlled by an independent mass parameter, $\Lambda_{yy}$. At the classical level, then, the possibility of a suppression of the latter coefficient with respect to the former ones is perfectly consistent. Only the knowledge of the underlying dynamics could allow us to say more on the relative size of the different mass parameters appearing in eqs. (1) and (2).

3. Even if it is mathematically and phenomenologically consistent to assume that $\Lambda_{yy} \gg \Lambda_{yz}, \Lambda_{zz}$, no obvious symmetry seems to be recovered in the limit $\Lambda_{yy} \to \infty$. Similarly, we may consistently assume that $\Lambda_{zz} \gg \Lambda_{yz}$, corresponding to $m_z^2 \ll m_y^2$, but again no obvious symmetry is recovered in the limit $\Lambda_{zz} \to \infty$. We may then ask how natural such situations are. To answer this question, we shall now compute the most divergent contributions to the one-loop effective action, and use them to estimate a naturalness bound on the relative size of the mass scales controlling the different physical observables of the model.

Thanks to supersymmetry, quartic divergences are absent, and the most divergent contribution to the one-loop effective action is the quadratically divergent one. We should
warn the reader that, if the cutoff scale $\Lambda_0$ is not very large, also the logarithmically divergent and finite contributions may be numerically important. However, our simplifying choice of considering only the quadratic divergences will be sufficient for a qualitative discussion of the naturalness bounds. The quadratically divergent contributions to the one-loop effective action are summarized by the following renormalization of the Kähler potential \[11\]

$$\Delta_Q K = \frac{\Lambda_0^2}{16\pi^2} (\log \det K_{mn}) ,$$  \hspace{1cm} (8)

where $\Lambda_0$ is an ultraviolet cutoff in momentum space and $K_{mn}$ is the (field-dependent) Kähler metric. Expanding in powers of the fields, we can write the uncorrected superpotential $w$ and the corrected Kähler potential $K_Q = K + \Delta_Q K$ in the same functional form as in eqs. (1) and (2),

$$w = \hat{\Lambda}^2_S \hat{Z} ,$$  \hspace{1cm} (9)

$$K_Q = \hat{Y} \hat{Y} + \hat{Z} \hat{Z} - \frac{\hat{Y}^2 \hat{Y}^2}{4\hat{A}_{yy}} - \frac{\hat{Y} \hat{Z} \hat{Z}}{\hat{A}_{yz}} - \frac{\hat{Z}^2 \hat{Z}^2}{4\hat{A}_{zz}} + \ldots ,$$  \hspace{1cm} (10)

in terms of renormalized fields and parameters\[1]\n
\[1\hat{Y} = \left[ 1 - \frac{1}{2} \frac{\Lambda_0^2}{16\pi^2} \left( \frac{1}{\hat{A}_{yy}^2} + \frac{1}{\hat{A}_{yz}^2} \right) \right] Y ,\]

(11)

\[1\hat{Z} = \left[ 1 - \frac{1}{2} \frac{\Lambda_0^2}{16\pi^2} \left( \frac{1}{\hat{A}_{yz}^2} + \frac{1}{\hat{A}_{zz}^2} \right) \right] Z ,\]

(12)

\[1\hat{\Lambda}^2_S = \left[ 1 + \frac{1}{2} \frac{\Lambda_0^2}{16\pi^2} \left( \frac{1}{\hat{A}_{yy}^2} + \frac{1}{\hat{A}_{zz}^2} \right) \right] \Lambda^2_S ,\]

(13)

\[1\frac{1}{\hat{A}_{yy}^2} = \frac{1}{\Lambda_{yy}^2} + \frac{\Lambda_0^4}{16\pi^2} \left( \frac{4}{\Lambda_{yy}^4} + \frac{2}{\Lambda_{yy}^2 \Lambda_{yz}^2} + \frac{2}{\Lambda_{yz}^2 \Lambda_{zz}^2} - \frac{4}{\Lambda_{yyy}^2} - \frac{1}{\Lambda_{yyzz}} \right) ,\]

(14)

\[1\frac{1}{\hat{A}_{yz}^2} = \frac{1}{\Lambda_{yz}^2} + \frac{\Lambda_0^4}{16\pi^2} \left( \frac{3}{\Lambda_{yy}^4} + \frac{2}{\Lambda_{yy}^2 \Lambda_{yz}^2} + \frac{2}{\Lambda_{yz}^2 \Lambda_{zz}^2} - \frac{1}{\Lambda_{yyy}^2} - \frac{1}{\Lambda_{yyzz}} \right) ,\]

(15)

\[1\frac{1}{\hat{A}_{zz}^2} = \frac{1}{\Lambda_{zz}^2} + \frac{\Lambda_0^4}{16\pi^2} \left( \frac{4}{\Lambda_{yy}^4} + \frac{2}{\Lambda_{yy}^2 \Lambda_{yz}^2} + \frac{2}{\Lambda_{yz}^2 \Lambda_{zz}^2} - \frac{4}{\Lambda_{yyy}^2} - \frac{1}{\Lambda_{yyzz}} \right) ,\]

(16)

The previous results, obtained from the general formula of eq. (8), have a simple diagrammatic interpretation. We consider here, for illustration, the effective interaction involving four matter fermions, whose quadratic renormalization is given in eq. (14). The (component-field) one-loop diagrams contributing to eq. (14) are shown in Fig. 1, where the dots denote crossed diagrams in (a) and (b), and diagrams with self-energy insertions on different lines in (d). The contribution proportional to $1/\Lambda_{yy}^4$ comes from the $\psi_y$-loops\[2\]

\[2\]Since we have shown the expansion of $K$ up to the sixth order in the fields, for consistency we have shown the one of $K_Q$ up to the fourth order.
in (a) and the $y$-loops in (b), (c), (d); the one proportional to $1/(\Lambda_{yy}^2\Lambda_{yz}^2)$ from the $z$-loops in (d); the one proportional to $1/\Lambda_{yz}^4$ from the $\psi_z$-loop in (a) and the $z$-loop in (b); the one proportional to $1/\Lambda_{yy}^4$ from the $y$-loop in (c); the one proportional to $1/\Lambda_{yzz}^4$ from the $z$-loop in (c). The interaction vertices originate from the couplings of eq. (7), including those

with extra scalars, and from derivative couplings involving two fermions and two scalars. A similar diagrammatic interpretation holds for the quadratically divergent corrections to the other four-fermion interactions and to the scalar masses. Notice that the renormalized scalar masses can be directly obtained from the above formulae as $\hat{m}_y^2 = \hat{\Lambda}_S^4/\hat{\Lambda}_{yz}^2$ and $\hat{m}_z^2 = \hat{\Lambda}_S^4/\hat{\Lambda}_{zz}^2$. We have independently checked this result via explicit evaluation of the relevant self-energy diagrams.

Since all the quadratic divergences can be reabsorbed in a redefinition of fields and parameters, all the predictions obtained from $K_Q$ will be identical in form to the predictions originally obtained from $K$. From the technical point of view, then, a possible suppression of the four-fermion interactions not involving the goldstinos remains viable also at the quantum level, and the same is true for a possible suppression of $m_y^2$ with respect to $m_z^2$. On the other hand, we may want to take more seriously the physical meaning of the cut-off scale $\Lambda_0$, and to ask how much suppression can be considered natural in the two cases.

In order to proceed, we should first make a statement about the plausible values that can be assigned to the cutoff $\Lambda_0$ in the two cases of interest. We first address the question

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{diagrams.png}
\caption{Quadratically divergent diagrams contributing to the $\psi_y\psi_y\bar{\psi}_y\bar{\psi}_y$ amplitude.}
\end{figure}
of four-fermion interactions, assuming for simplicity that \( \Lambda_{yz} = \Lambda_{zz} \equiv \Lambda \), corresponding to \( m_y^2 = m_z^2 \equiv m^2 = F^2/\Lambda^2 \), and that \( \Lambda_{yy}, \Lambda_{yyy}, \Lambda_{yzz}, \Lambda_{yyz}, \Lambda_{zzz} \geq \Lambda \). Then a fair estimate is

\[
m^2 \lesssim \Lambda^2 \lesssim 16\pi\Lambda^2, \tag{17}\]

where the lower bound is obvious, and the upper bound is an estimate of the energy scale at which perturbative unitarity is violated by the most dangerous four-fermion scattering amplitudes, proportional to \( E^2/\Lambda^2 \). Incidentally, notice that the interval in eq. (17) shrinks to a point when the bound \( m^2 \lesssim \sqrt{16\pi F} \) is saturated [the latter bound corresponds to the requirement that the spin-0 fields have a particle interpretation, \( \Gamma(y \rightarrow \psi_y\psi_z) = 2\Gamma(z \rightarrow \psi_z\psi_z) \lesssim m \)].

We can now see from eqs. (13)–(17) that, under the previous assumptions, there is no naturalness problem for the supersymmetry-breaking scale and for the coefficients of the four-fermion amplitudes involving the goldstinos, since they receive at most relative corrections of order one. Instead, if we assume \( \Lambda_{yy} \gg \Lambda \) there is a potential problem for the four-fermion amplitudes not involving the goldstinos, controlled by \( \Lambda_{yy}^2 \).

To begin with, assume that also the scale parameters associated with the sixth-order terms of \( K \) are much larger than \( \Lambda \). Then the natural values of \( \Lambda_{yy} \) are those satisfying the bound

\[
\frac{1}{\Lambda_{yy}^2} \gtrsim \frac{\Lambda_0^2}{8\pi^2\Lambda^4}. \tag{18}\]

For the two extreme choices of the cutoff scale in (17), the bound (18) translates into

\[
\frac{1}{\Lambda_{yy}^2} \gtrsim \frac{m^6}{8\pi^2 F^4} \tag{19}\]

in the least restrictive case, and into

\[
\frac{1}{\Lambda_{yy}^2} \gtrsim \frac{2m^2}{\pi F^2} \tag{20}\]

in the most restrictive one. We shall comment later on the phenomenological implications of such inequalities.

Another possibility is that also some of the scale parameters associated with the sixth-order terms of \( K \), in particular \( \Lambda_{yyy} \) and \( \Lambda_{yyz} \), are comparable in magnitude with \( \Lambda \). Then, due to the structure of eq. (14), there is the possibility of cancellations among the different contributions. Such cancellations may be accidental, in which case, beyond a given level of precision, we should check the contributions coming from the graphs with lower degree of divergence and from higher loops. We cannot exclude, however, possible cancellations of geometrical nature, related to the properties of the Kähler manifold. For example, if the only non-vanishing coefficients in (2) were those associated with \( \Lambda \) and \( \Lambda_{yyz} \), and the relation \( \Lambda^4 = 2\Lambda_{yyz}^4 \) held, then the correction to \( 1/\Lambda_{yy}^2 \) in (14) would vanish. More generally, we could look for manifolds with special properties. The simplest possibility that
comes to mind is to have an Einstein manifold, $R_i = kK_i$, with the hierarchy $\Lambda_{yy} \gg \Lambda$. If this were possible, the hierarchy would be automatically stable with respect to the correction of eq. (8). Unfortunately, it can be shown that for an Einstein manifold the relation $\Lambda_{yy} = \Lambda_{zz}$ must hold, so we should look for more subtle mechanisms.

We will now relax the assumption $\Lambda_{zz} = \Lambda_{yz}$ and see whether a possible hierarchy $\Lambda_{zz} \gg \Lambda_{yz}$, corresponding to $m_z^2 \ll m_y^2$, is stable or not. Assuming that none of the scale parameters in $K$ is smaller than $\Lambda_{yz}$, the range (17) of plausible cutoff values should now read $m_y^2 \lesssim \Lambda_{0y}^2 \lesssim 16\pi \Lambda_{yz}^2$. Naturalness questions can be addressed by looking again at eqs. (13)–(16). In particular, we can see that assuming $\Lambda_{zz} \gg \Lambda_{yz}$ does not generate a naturalness problem for the supersymmetry-breaking scale, but does imply a potential problem for the parameter $\Lambda_{zz}$ itself. Indeed, eq. (16) shows that, in that case, the quantum corrections proportional to $\Lambda_{0y}^2 / \Lambda_{yz}^4$ can be much larger than the tree-level value $1 / \Lambda_{yz}^2$, especially if we assign to the cutoff $\Lambda_{0y}^2$ the maximum (natural) value, of order $\Lambda_{yz}^2$. All this means that quantum corrections tend to spoil the assumed hierarchy $m_z^2 \ll m_y^2$, and drive $\hat{m}_z^2$ close to $\hat{m}_y^2$. From this point of view, for example, a situation with sparticle masses $m_y^2 \gg m_{3/2}^2$ and sgoldstino masses $m_z^2 \simeq m_{3/2}^2$ (hierarchy $\Lambda_{yz} \ll \Lambda_{zz} \simeq M_P$) does not appear natural.

A milder conclusion is reached if we assign to the cutoff $\Lambda_{0y}^2$ the minimum value, i.e. $m_y^2$. Then $m_z^2$ receives quantum corrections proportional to $m_y^6 / F^2$, which do not exceed $m_z^2$ itself provided $m_z F \gtrsim m_y^3$. In particular, a situation with $m_z^2 \simeq m_{3/2}^2$ would satisfy such a (milder) naturalness criterion provided $F^2 \gtrsim M_P m_y^3$, i.e. $m_{3/2} \gtrsim m_y^3 / M_P$. Finally, we recall that a tree-level hierarchy $m_z^2 \ll m_y^2$ could be maintained at the quantum level also if cancellations among different corrections took place in eq. (16), in analogy to what observed above when discussing eq. (14).

4. We have shown above that, if we do not invoke any naturalness criterion (the most appropriate attitude, in our opinion, when discussing model-independent bounds on $m_{3/2}$ and $F$), a suppression of the four-fermion operators not involving the goldstinos is completely self-consistent.

Nevertheless, it may be instructive to see if, when a naturalness criterion is adopted, interesting bounds on superlight-gravitino models can be obtained from the Tevatron bounds on effective four-fermion interactions involving ordinary fermions. For example, from an analysis of the dilepton mass spectrum, CDF has published bounds [10] on possible four-fermion interactions involving two quarks and two charged leptons. These bounds are expressed in terms of a compositeness scale, analogous (but not identical) to our $\Lambda_{yy}$, and depending on the Lorentz and flavour structure of the different operators. In the following, we shall denote by $\Lambda_{yy}^*$ the putative experimental lower bound on $\Lambda_{yy}$. When making numerical estimates, we shall use the reference value $\Lambda_{yy}^* = 1$ TeV, thus taking into account the CDF conventions for the normalization of the four-fermion operators. The Tevatron experiments should be also sensitive to the direct production of sfermion and
sgoldstino pairs. We shall denote by $m^*$ the putative lower bound on their masses, and use, when making numerical estimates, the reference value $m^* = 200$ GeV. Combining
the two types of searches, and using eqs. (19) and (20), we can derive the corresponding bounds on the scale of supersymmetry breaking:

$$\sqrt{F} \gtrsim 170 \text{ GeV} \left( \frac{m^*}{200 \text{ GeV}} \right)^{3/4} \left( \frac{\Lambda_{yy}^*}{1 \text{ TeV}} \right)^{1/4}$$

(21)

for the least restrictive choice of the cutoff scale, and

$$\sqrt{F} \gtrsim 400 \text{ GeV} \left( \frac{m^*}{200 \text{ GeV}} \right)^{1/2} \left( \frac{\Lambda_{yy}^*}{1 \text{ TeV}} \right)^{1/2}$$

(22)

for the most restrictive one. From eqs. (21) and (22) we see that the adoption of naturalness criteria on four-fermion (non-goldstino) interactions leads to bounds on $F$. These bounds are comparable with the more direct ones coming from tagged gravitino pair-production and from the pair production of sfermions and sgoldstinos. To say more, we should perform a detailed analysis, taking into account the dependences of the different signals on at least three independent parameters, e.g. $(m^2, F, \Lambda_{yy})$. At the level of the toy model, this would imply the combined study of several processes, such as $\psi_y \psi_y \rightarrow \psi_y \psi_y, \psi_y \psi_y \rightarrow \psi_y \psi_z, \psi_y \psi_y \rightarrow y y, \psi_y \psi_y \rightarrow z z, \ldots$ In a fully realistic model, there would be additional complications: the replacement of the $Y$ superfield with several superfields corresponding to left- and right-handed quarks and leptons; the introduction of gauge interactions, with additional processes and diagrams involving the gauginos coming into play. However, a detailed study of the interplay of the constraints coming from the different processes goes beyond the aim of the present paper.

We conclude by recalling our main results. On the one hand, we emphasized that four-fermion interactions not involving the goldstinos do not give direct model-independent bounds on $\sqrt{F}$ or $m_{3/2}$. On the other hand, the coefficients of such interactions can be indirectly related to $F$, after considering their renormalization properties and adopting some naturalness criterion. The latter viewpoint leads to bounds on $\sqrt{F}$ comparable and complementary to the direct, model-independent bounds. As for the sgoldstino mass $m_S^2$ (corresponding to $m_{S/2}, m_P^2$ in the more general case considered in the literature), we have shown that hierarchical situations with $m_S^2 \ll m_y^2$ (e.g. $m_S^2 \simeq m_{3/2}^2 \ll m_y^2$) are generically disfavoured by naturalness considerations, although the possibility of cancellations dictated by some symmetry of the underlying fundamental theory cannot be excluded.

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