Asymptotically exact solution of a local copper-oxide model

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We present an asymptotically exact solution of a local copper-oxide model abstracted from the multi-band models. The phase diagram is obtained through the renormalization-group analysis of the partition function. In the strong coupling regime, we find an exactly solved line, which crosses the quantum critical point of the mixed valence regime separating two different Fermi-liquid (FL) phases. At this critical point, a many-particle resonance is formed near the chemical potential, and a marginal-FL spectrum can be derived for the spin and charge susceptibilities.

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One of the most challenging theoretical issues has been raised by the experimental observations on copper-oxide based metals: What is the appropriate description of the normal state which does not fit into the Fermi-liquid (FL) phenomenology. In an attempt to find the unifying features in the diverse observed anomalies, a phenomenological marginal-FL (MFL) spectrum for the spin and charge fluctuations was proposed by Varma et al. [1].

The essential point is that the frequency dependence of the susceptibilities is singular, while the momentum dependence is assumed smooth. It is expected that the study of the related impurity models would shed some light on the basic physics of the lattice models, in the same sense as the Anderson model vs the single band Hubbard model.

Recently, Varma et al. [2,3] have explored the local non-FL properties of a generalized Anderson model, equivalent to a single-impurity version of one of the multi-band models proposed to describe the physics of the copper-oxide compounds [4,5]. In fact, such a model was first studied by Haldane in the late 70s on Anderson’s suggestion [6]. They realized that the usual Anderson model should be supplemented by screening channels to saturate the Friedel sum rule, which is crucial for the understanding of the mixed valence physics.

The recent Wilson’s renormalization-group (RG) study [2] and analytical arguments [3] have provided a microscopic scenario for the MFL phenomenology within this generalized Anderson model.

In this Letter, we derive explicitly physical properties of this local copper-oxide model through exactly solving the Hamiltonian in the asymptotic limit. We first separate the model into ”hybridizing” and ”screening” parts. The first part is the usual Anderson model plus X-ray edge-like (XRE) scattering terms, which were neglected in all previous studies [2,3,6]. This part reduces to a hybridization process and potential scatterings, which can be merged into an effective hybridization via a canonical transformation, giving rise to a local FL behavior. The second part is the multi-channel XRE scattering terms, and can be reduced to a single spinless channel, exhibiting the Anderson catastrophe. These XRE singularities can also be transformed into the effective hybridization, so the physics of the full model results from a competition between the above two different physical factors in the
hybridization. Then we apply the RG theory [7] to derive flow equations from which a phase diagram is found. There exist two different FL phases: a Kondo phase and a local free FL phase separated by a mixed valence phase, displaying a local MFL behavior. Finally, from the partition function and the effective Hamiltonian, we take the strong coupling limit, and an exactly solved line (analogue of the Toulouse limit in the Kondo problem) can be found, which crosses the quantum critical point of the mixed valence phase where a many-particle resonance is formed between the localized impurity and the conduction electrons. Properties of the above three phases, especially, the mixed valence phase controlled by the critical point can be exactly derived.

The local copper-oxide model is specified by the Hamiltonian [2]

$$ H = H_h + H_s, $$

$$ H_h = \sum_{k,\sigma} \epsilon_k C_{k,\sigma,0}^\dagger C_{k,\sigma,0} + \epsilon_d n_d + U n_d,\uparrow n_d,\downarrow + \frac{t}{\sqrt{N}} \sum_{k,\sigma} (C_{k,\sigma,0}^\dagger d_\sigma + h.c.) $$

$$ + \sum_{k,k',\sigma,\sigma'} \frac{V_0}{N} C_{k,\sigma,0}^\dagger C_{k',\sigma,0} (n_d,\sigma - \frac{1}{2}) + \sum_{k,k',\sigma,\sigma'} \frac{V_{l,0}}{N} C_{k,\sigma,0}^\dagger C_{k',\sigma,0} (n_d,\sigma - \frac{1}{2}), $$

$$ H_s = \sum_{k,\sigma,l>0} \epsilon_k C_{k,\sigma,l}^\dagger C_{k,\sigma,l} + \sum_{k,k',\sigma,l>0} \frac{V_l}{N} C_{k,\sigma,l}^\dagger C_{k',\sigma,l} (n_d - \frac{1}{2}), \tag{1} $$

where $n_d = \sum_\sigma n_{d,\sigma}$, $N$ is the number of the lattice sites, and we have separated the Hamiltonian into hybridizing and screening parts and distinguished parallel-spin and opposite-spin XRE scatterings in the hybridizing channel. The localized impurity hybridizes only with channel $l = 0$. The chemical potential $\mu$ is set to zero, and we are interested in the case when the local impurity level $\epsilon_d$ is close to zero. The spinless version of this model, i.e. the multi-channel resonant level model, has been solved exactly [8,9], displaying a FL vs non-FL transition as the interaction parameters are varied. In fact, $H_h$ is the usual Anderson model plus XRE potential scatterings, while $H_s$ is the multi-channel XRE Hamiltonian. It is more convenient to take an infinite $U$ limit as in the usual treatments for the Anderson model (the low-energy physics is kept). Thus, we add a local constraint for the local impurity $n_d \leq 1$.

First, we use abelian bosonization to handle the XRE singularities of the screening channels, which reduce to a one-dimensional problem with only one Fermi point for each
channel and the dispersion is linearized with a cutoff $k_D$ [10]. Since the spin degrees of freedom of the screening channel electrons are trivially involved, they can be separated from $H_s$. Moreover, we can assume $V_l = V_s$ for all $l > 0$ without loss of generality, so the channel index can be dropped. Thus, all the screening channels are described by a single spinless channel [3]. The resulting form is

$$H^b_s = \sum_{k > 0} \frac{k}{\rho} a^\dagger_k a_k + \sum_{k > 0} \bar{V}_s \sqrt{\frac{k}{N}} (a^\dagger_k + a_k) (n_d - \frac{1}{2}),$$

with $\bar{V}_s \equiv \sqrt{2N_s} V_s$ and $N_s$ is the number of the screening channels. $a^\dagger_k$ and $a_k$ are the bosonic operators describing the charge degrees of freedom of the screening channel and $\rho = (\hbar v_F)^{-1}$ is the density of states at the Fermi point. Employing the inverse bosonization, we can transform bosons back to fermions: $H^f_s = \sum_k \epsilon_k s^\dagger_k s_k + \frac{\bar{V}_s}{N} \sum_{k, k'} s^\dagger_k s_{k'} (n_d - \frac{1}{2})$. The Hamiltonian (4) can be diagonalized through a canonical transformation [8]: $U = \exp \{ \sum_{k > 0} \frac{\delta_s}{\sqrt{N}} (a_k - a^\dagger_k) (n_d - \frac{1}{2}) \}$, where $\delta_s \approx \pi \rho \bar{V}_s$ is the phase shift generated by the XRE scattering of all screening channels at the impurity.

For the hybridizing channel, employing another canonical transformation [8,11]: $S = \exp \{ \frac{\delta_0}{\sqrt{N}} (b_{k, \sigma} - b^\dagger_{k, \sigma}) (n_{d, \sigma} - \frac{1}{2}) \} \exp \{ \sum_{k > 0, \sigma} \frac{\delta'_0}{\sqrt{N}} (b_{k, \sigma} - b^\dagger_{k, \sigma}) (n_{d, \sigma} - \frac{1}{2}) \}$, where $\delta_0 \approx \pi \rho V_0$ and $\delta'_0 \approx \pi \rho V'_0$ are the phase shifts induced by the parallel-spin ($\sigma$) and opposite-spin ($\bar{\sigma}$) XRE scattering of the hybridizing channel, respectively, we merge XRE scatterings into the hybridization and obtain

$$H_{\text{eff}} = \sum_{k > 0, \sigma} \frac{k}{\rho} b^\dagger_{k, \sigma} b_{k, \sigma} + \epsilon'_d n_d + t \sum_{\sigma} (\Delta \psi^\dagger_{\sigma} d_{\sigma} + \text{h.c.}) + \sum_{k > 0} \frac{k}{\rho} a^\dagger_k a_k,$$

with $\psi_{\sigma} \equiv \sqrt{k_D} \exp \{ \sum_{k > 0} \frac{1 - \delta_0}{\sqrt{N}} (b_{k, \sigma} - b^\dagger_{k, \sigma}) \} \exp \{ - \sum_{k > 0} \frac{\delta'_0}{\sqrt{N}} (b_{k, \sigma} - b^\dagger_{k, \sigma}) \}$, $\Delta \equiv \exp \{ \sum_{k > 0} \frac{\delta_{\bar{\sigma}}}{\sqrt{N}} (a_k - a^\dagger_k) \}$, $\epsilon'_d \equiv \epsilon_d - \rho V_0 V'_0$, and $n_d \leq 1$. A fermionic form is obtained by inverse bosonization

$$\bar{H}_{\text{eff}} = \sum_{k, \sigma} \epsilon_k C^\dagger_{k, \sigma} C_{k, \sigma} + \epsilon'_d n_d + t \sum_{\sigma} (\Delta \psi^\dagger_{\sigma} d_{\sigma} + \text{h.c.}) + \sum_{k} \epsilon_k s^\dagger_k s_k,$$

where $\psi_{\sigma} = (k_D)^{\frac{\delta_0}{2\pi}} (C_{\sigma})^{\frac{1 - \delta_0}{2\pi}} (C_{\bar{\sigma}})^{-\frac{\delta'_0}{2\pi}}$, $C_{\sigma} = \frac{1}{\sqrt{N}} \sum_k C_{k, \sigma}$, $\Delta = (k_D)^{-\frac{\delta_0}{2\pi}} (s_0)^{\frac{\delta_0}{2\pi}}$, $s_0 = \frac{1}{\sqrt{N}} \sum_k s_k$, and $n_d \leq 1$. 

4
Next, we derive the partition function, dividing $\tilde{H}_{eff}$ as $\tilde{H}_{eff} = \tilde{H}_0 + \tilde{H}_I$ with $\tilde{H}_0$ and $\tilde{H}_I$ as the free and the hybridization parts of (4), respectively. Paralleling all the strategies of previous studies [6-8], we write the partition function in terms of a sum over histories of the impurity. Each history is a sequence of transitions between the three local $d$ states $|\alpha\rangle = |0\rangle$, and $|\sigma\rangle$, $\sigma = \uparrow, \downarrow$. The transitions take place at the imaginary time $0 < \tau_1 < ... < \tau_n < \beta = \frac{1}{\tau}$; along the Feynman trajectory the local state is at $|\sigma_{i+1}\rangle$ from $\tau_i$ to $\tau_{i+1}$ ($i=1$ to $n$). The partition function of (4) is now given by

$$Z = \sum_{n=0}^{\infty} \sum_{\{\sigma, \sigma_1, \ldots, \sigma_{n+1}=\sigma\}} \sum_{\{\alpha=0, \sigma\}} (\Gamma)^n \int_0^\beta \frac{d\tau_{2n}}{\tau} \int_0^{\tau_{2n}-\tau} \frac{d\tau_{2n-1}}{\tau} \cdots \int_0^{\tau_{2}-\tau} \frac{d\tau_1}{\tau} \Pi_I g_{\sigma_i, \sigma_{i-1}} \exp\{E_d^a \sum_i (-1)^i \tau_i + \sum_{i>j} (-1)^{i+j}[(1 - \frac{\delta\alpha_i}{\pi})^2 \delta\sigma_i, \sigma_j + (\frac{\delta\alpha_i}{\pi})^2 \delta\sigma_i, \sigma_j + (\frac{\delta\alpha_i}{\pi})^2] \ln |\frac{\tau_i - \tau_j}{\tau}|\}, \quad (5)$$

where the bare hybridization strengths are defined as $\Gamma = \rho t^2$, while the cutoff factor $\tau = \frac{\rho}{\kappa_D}$.

The effective "magnetic field" reflects the differences of the local state energies: $E_d^0 = -e'_d$, $E_d^\sigma = e'_d$. The fugacity $(y_{\sigma_i, \sigma_{i+1}}^{\alpha} \sqrt{\Gamma})$ is the amplitude associated with a transition from $|\sigma_i\rangle$ to $|\sigma_{i+1}\rangle$, with $y_{\sigma_{2i}, \sigma_{2i+1}}^{\sigma} = \delta_{\sigma_{2i}, \sigma_{2i+1}}$, $y_{\sigma_{2i+1}, \sigma_{2i}}^{0} = 1; y_{\sigma_{2i+1}, \sigma_{2i}}^{\sigma} = 1; y_{\sigma_{2i+1}, \sigma_{2i+1}}^{0} = \delta_{\sigma_{2i+1}, \sigma_{2i}}$. The long-range logarithmic interaction between the flipping events arises from the reaction of the conduction electron bath towards the transition between the local states. The local disturbance on the bath involves two factors: the absorption or emission of the local conduction electrons and the change in the local potential that the conduction electrons experience [8]. Both kinds of disturbance are incorporated in the effective "charge" factor, i.e. the coefficient of the logarithmic function of (5).

The partition function (4) could be obtained directly from the model Hamiltonian (1) without bosonization, using the famous fermion techniques [12] in certain asymptotic limit which, we believe, is also valid here. This alternative derivation would allow us to rectify the phase shifts obtained by the bosonization treatments to the exact expressions: $\delta_s = 2\tan^{-1}(\frac{2}{3}\rho \bar{V}_s)$, $\delta_0 = 2\tan^{-1}(\frac{2}{3}\rho V_0)$, and $\delta'_0 = 2\tan^{-1}(\frac{2}{3}\rho V'_0)$ so that the corresponding effective Hamiltonian (4) might be used beyond the range of validity for the bosonization method, especially in the following strong coupling limit where the renormalized parameters
of the model recover their bare values.

To set up the RG flow equations, we can directly employ the scaling theory proposed by Anderson et al. [7] in the Coulomb gas representation. The RG equations describe the flow behavior as the bandwidth is reduced. They are given by

\[
\frac{d\Gamma}{d\ln\tau} = -\gamma\Gamma, \quad \frac{d\epsilon_d}{d\ln\tau} \approx \frac{\Gamma}{\pi}, \quad \frac{d\gamma}{d\ln\tau} \approx -\frac{(\gamma + 1)^2}{\Gamma},
\]

with \(\gamma \equiv -\frac{2\delta_0}{\pi} + (\frac{\delta_0}{\pi})^2 + (\frac{\delta_0}{\pi})^2\) describing the total interaction strength between the conduction electrons and the local impurity, which should be positive in our case. These equations were derived by assuming \(\Gamma\tau \leq 1\), a rare gas of spin-flips. In the zeroth order, we can construct two invariants (\(\Gamma^*\tau^* = 1\)):

- \(\Gamma^* = \Gamma(\tau_0)^{1+\gamma}\)
- \(\epsilon_d^* = \epsilon_d + \frac{\Gamma}{\gamma\pi}[1 - (\Gamma\tau_0)^{1/\gamma}]\)

where \(\epsilon_d\), \(\Gamma\), and \(\tau_0\) are initial (bare) parameters. In terms of these scaling invariants, the running resonance width and impurity level are written as \(\Gamma(\tau) = (\Gamma^*)^{1-\gamma}(\tau)^{-\gamma}\) and \(\epsilon_d(\tau) = \epsilon_d^* + \frac{\Gamma}{\gamma\pi}[1 - (\Gamma^*)^{-\gamma}]\). Obviously, for the case \(\tau = 0\), the two expressions become \(\Gamma(\tau) = \Gamma^*\) and \(\epsilon_d(\tau) = \epsilon_d^* + \frac{\Gamma}{\gamma\pi}\ln(\Gamma^*\tau)\), which exactly recovers the Haldane’s RG results for the standard Anderson model [7]. Moreover, a complete phase diagram can be determined by comparing the invariants \(\epsilon_d^*\) and \(\Gamma^*\). In the plane \(\Gamma^* - \gamma\) (Fig.1), there are three phases corresponding to different impurity-occupancies:

- Single-occupancy regime \((\epsilon_d^* \ll -\Gamma^*)\) where a singlet state is formed and the Kondo effect shows up;
- Zero-occupancy regime \((\epsilon_d^* \gg \Gamma^*)\) where the model corresponds to a local free FL phase, and the mixed valence regime \((|\epsilon_d^*| \leq \Gamma^*)\) where \(<n_d>\) fluctuates between 0 and 1 phases. The mixed valence regime separates the \(<n_d> = 1\) and the \(<n_d> = 0\) phases with crossover lines \(\Gamma^* \approx \frac{\Gamma}{1-\gamma\pi}\) and \(\Gamma^* \approx \frac{\Gamma}{1+\gamma\pi}\), respectively. Although parameter \(\gamma\) is not renormalized in the zeroth order of \(\Gamma\tau\), in its first order it is renormalized to smaller values as \(\tau\) increases. On the other hand, from \(\Gamma^* \approx -\gamma\pi\epsilon_d^* + \Gamma\), \(\Gamma^*\) increases when \(\epsilon_d^* > 0\) and decreases, while it decreases if \(\epsilon_d^* < 0\) and decreases in absolute value. Hence the flow directions indicated in Fig.1. In the end, the renormalized \(\gamma\) tends to zero as \(\epsilon_d^* \rightarrow 0\), \(\Gamma^* \rightarrow \Gamma\). However, when \(\Gamma(\tau)\tau \approx 1\), the RG equations are not correct quantitatively.

In addition to this RG analysis, a strong coupling limit can be extracted independently.
from the partition function (5) and the effective Hamiltonian (4). As follows from (4), when opposite-spin XRE scattering in the hybridizing channel renormalizes to zero while the XRE scatterings of the parallel-spin and the screening channel reach their respective unitary limit, i.e. \( \delta_0 = \delta_s = \pi \) and \( \delta_0' = 0 \) or \( \gamma = 0 \), the phase shifts due to hybridization and parallel-spin XRE scattering in the hybridizing channel compensate each other, thus the hybridizing electrons become completely free. In such a strong coupling limit, the effective Hamiltonian becomes

\[
H_T = \sum_{k,\sigma} \epsilon_k C_{k,\sigma}^\dagger C_{k,\sigma} + \epsilon_d n_d + t \sum_{\sigma} (s_0^\dagger d_{\sigma}^\dagger + h.c.) + \sum_k \epsilon_k s_k^\dagger s_k, \tag{7}
\]

with constraint \( n_d \leq 1 \) or \( n_d + s_0^\dagger s_0 = 1 \), which reflects the Friedel sum rule in this limit. It is obvious that the partition function derived from the Hamiltonian (7) is exactly the same as (5) for \( \delta_0 = \delta_s = \pi \) and \( \delta_0' = 0 \), i.e. \( \gamma = 0 \). In this sense, the strong coupling limit found here is somehow analogous to the Toulouse limit of the Kondo problem [13], although the actual physics involved is quite different. The most essential difference is that the unitary limit has been actually reached in our case. The vanishing of opposite-spin XRE scattering in the hybridizing channel is exactly what is required by the infinite U limit, because any finite hybridization between opposite-spin hybridizing electrons and the local impurity (to compensate XRE scattering) will contradict the single occupancy constraint.

In Fig.1, \( \gamma = 0 \) is a strong coupling limit line of this local copper-oxide model. In (7), only charge of the local impurity \( \alpha \equiv \frac{1}{\sqrt{2}}(d_\uparrow + d_\downarrow) \) is coupled to the conduction electrons, while the spin \( \beta \equiv \frac{1}{2}(d_\uparrow - d_\downarrow) \) is decoupled except for the constraint. Thus, in this limit, the Hamiltonian is: \( \tilde{H}_T = \sum_k \epsilon_k s_k^\dagger s_k + \epsilon_d (n_{\alpha} + n_{\beta}) + \sqrt{2} t (s_0^\dagger \alpha^\dagger + h.c.) \), where \( n_{\alpha} + n_{\beta} \leq 1 \) and the hybridizing electrons do not show up explicitly. This Hamiltonian is essentially the same as Eq. (14) of [3]. Since \( \tilde{H}_T \) conserves \( n_{\beta} \), we can calculate physical quantities by taking the trace on its two subspaces \( n_{\beta} = 0, 1 \).

(i). When \( \epsilon_d \gg \epsilon_{dc} \), a critical value to be defined later, the \( n_{\alpha} = n_{\beta} = 0 \) state is favored in the low-energy regime. All charge fluctuation processes are frozen out and \( s_0^\dagger s_0 = 1 \) at the impurity site so that the Friedel sum rule is saturated. The hybridization strength \( \Gamma^* \)
should be renormalized to zero, and a local free FL behavior is thus displayed \[14\].

(ii). The opposite case $\epsilon_d \ll \epsilon_{dc}$ favors $n_\alpha + n_\beta = 1$ in the low-energy regime, and there are two possible configurations: $n_\alpha = 0, n_\beta = 1$; $n_\alpha = 1, n_\beta = 0$. All charge fluctuation processes are also frozen out but $s_0^\dagger s_0 = 0$. The hybridization strength $\Gamma^*$ should be renormalized to $+\infty$, and the system is scaled to Wilson’s strong coupling fixed point of the Kondo problem: a local FL behavior in its unitarity limit \[14\]. The symmetry of the ground state for the present case is different from that at $\epsilon_d \gg \epsilon_{dc}$, so we anticipate a quantum critical point at $\epsilon_d = \epsilon_{dc}$.

(iii). When $\epsilon_d \rightarrow \epsilon_{dc}$, the localized impurity fluctuates between zero- and single-occupancy $< n_\alpha + n_\beta > \rightarrow \frac{1}{2}$, and hybridizes with only part of the screening electron $< s_0^\dagger s_0 > \rightarrow \frac{1}{2}$, corresponding to the mixed valence phase. At the special point ($\gamma = 0$, $\Gamma^* = \Gamma$) along the strong coupling line, the above two different FL states are degenerate. This special point is just the quantum critical point controlling the physics of the whole mixed valence phase. An analogous quantum critical point was found in the two-channel or two-impurity Kondo problems \[15\]. At zero temperature, we find $\epsilon_{dc} \approx -\frac{3n\pi}{2}\Gamma$ and $< n_\alpha >\approx \frac{1}{2}$, $< n_\beta >\approx 0$. Thus, the local impurity level is close to the chemical potential. Using the phase-shift representation of the Friedel sum rule: $< n_d > + \frac{1}{\pi}\delta_h(\mu) + \frac{1}{\pi}\delta_s(\mu) = 1$, we easily obtain the phase shift of the screening electrons caused by the final hybridization is $\frac{\pi}{2}$ at the chemical potential. Since both hybridizing and screening electrons are involved and there is a constraint on the local impurity, the hybridization becomes a many-particle resonance, drawing some weight of the one-particle spectra from higher energies at the scale of the charge transfer gap in the insulating state. Such a many-particle resonance breaks down the Landau correspondence between the low-lying excitations of the interacting and non-interacting fermions \[3\]. At finite low temperatures, the impurity charge and longitudinal spin susceptibilities $\chi_{\sigma,\rho}$ can also be calculated using the relations $\sigma_z = (\alpha^\dagger\beta + \beta^\dagger\alpha)$, $\rho = (\alpha^\dagger\alpha + \beta^\dagger\beta - \frac{1}{4})$. It has been found that $\chi_{\sigma,\rho}$ are proportional to $\Gamma^{-1}ln(\frac{F}{T})$, as expected from the MFL phenomenology \[3\]. Thus, the MFL behavior controls the whole mixed valence regime.
In conclusion, we have presented an asymptotically exact solution of the local copper-oxide model including the charge fluctuations of the screening electrons. The physical picture of the breakdown of the FL behavior pointed out in Ref.[3] is basically correct. However, the justification of their physical arguments involves several unclear approximations. The crucial point is that in all the previous studies [2,3,6], the parallel-spin XRE scattering in the hybridizing channel was assumed to be zero, i.e. $V_0 = 0$. However, namely this assumption obscures the physical features of the model and makes the problem much more involved. Due to the emergence of new relevant variables in their theory, in principle, they can not reach the strong-coupling limit. Since our solution is based on both RG flow analysis and the strong coupling effective Hamiltonian, the MFL behavior should be a universal property of the mixed valence phase. Of course, whether a specific system is at the quantum critical point depends on a special combination of parameters. A more interesting question, whether the chemical potential of a real mixed valence system is pinned at the critical point, requires further studies.

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REFERENCES

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[1] C. M. Varma, P. B. Littlewood, S. Schmitt-Rink, E. Abrahams, and A. E. Ruckenstein, Phys. Rev. Lett. 63, 1996 (1989).

[2] I. Perakis, C. M. Varma, A. E. Ruckenstein, Phys. Rev. Lett. 70, 3467 (1993).

[3] C. Sire, C. M. Varma, A. E. Ruckenstein, and T. Giamarchi, submitted to PRL.

[4] C. M. Varma, E. Abrahams, and S. Schmitt-Rink, Solid State Commun. 62, 681 (1987).

[5] Q. Si and G. Kotliar, Phys. Rev. Lett. 70, 3143 (1993); Phys. Rev. B 48, 13881 (1993).

[6] F. D. M. Haldane, Phys. Rev. B 13, 281 (1976); ibid., 15, 2477 (1977); J. Phys. C 11, 5015 (1978); Ph. D. thesis, University of Cambridge, 1977 (unpublished).

[7] P. W. Anderson, G. Yuval, and D. R. Hamann, Phys. Rev. B 1, 4464 (1970); F. D. M. Haldane, Phys. Rev. Lett. 40, 416 (1978).

[8] G. M. Zhang, Z. B. Su, and L. Yu, Phys. Rev. B, March 15 issue (1994).

[9] T. Giamarchi, C. M. Varma, A. E. Ruckenstein, P. Nozières, Phys. Rev. Lett. 70, 3967 (1993).

[10] K. D. Schotte and U. Schotte, Phys. Rev. 182, 479 (1969).

[11] We are grateful to C. Sire, A. E. Ruckenstein and C. M. Varma for pointing out to us an error at this point in the first version of this paper.

[12] P. Nozières and C. de Dominicis, Phys. Rev. 178, 1097 (1969); P. W. Anderson and G. Yuval, Phys. Rev. Lett. 23, 89 (1969).

[13] G. Toulouse, C. R. Acad. Sc. Paris B 268, 1200 (1969).

[14] H. R. Krishna-murthy, J. W. Wilkins, and K. G. Wilson, Phys. Rev. B 21, 1044 (1980).
[15] I. Affleck and A. W. W. Ludwig, Phys. Rev. Lett. 67, 3160 (1991); ibid., 68, 1046 (1992); P. Nozières and A. Blandin, J. Phys. (Paris), 41, 193 (1980); V. J. Emery and S. Kivelson, Phys. Rev. B 46, 10812 (1992); B. Jones and C. M. Varma, Phys. Rev. B 40, 324 (1989).

Figure Caption

Fig. 1. The phase diagram of the local copper-oxide model. I, II, and III are single-occupancy phase, mixed valence phase, and zero-occupancy phase, respectively. The two thick lines correspond to the crossover lines, while the two slender lines with arrows roughly show up the flow directions and the horizontal one is $\epsilon^*_d = 0$ line.
