Theory of charmless hadronic $B$-decays

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Abstract

I summarize results and performance of the dynamical theory of charmless hadronic $B$ decays, based on QCD factorization in the heavy quark limit. On the theoretical side, a number of NNLO ($\alpha_s^2$) amplitudes are now available, all showing a well-behaved perturbative expansion. The large observed branching fraction in $B^0 \to \pi^0 \pi^0$ remains a challenge, implying either a large inverse moment of the $B$-meson wave function or a sizable power correction (or unexpected new physics). $B$-factory/Belle2 analyses of $B \to \gamma \ell \nu$ may shed light on this. On the other hand, the new Belle and LHCb measurements of $A_{\CP}(\pi^+ \pi^-)$ bring the experimental result closer to QCDF predictions, similar to what is found in $B \to \pi K$ decays, while $S_{\pi\pi}$ gives a competitive $\gamma$-determination. I remind the reader that for vector-vector final states, a theoretical treatment of the full set of helicity amplitudes has existed and met with some success since the $B$-factory era, and applies equally to LHCb measurements of e.g. $B_s \to \phi \phi$. 

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Table 1: Topological amplitudes. First row: notation prevailing in the data-driven/$SU(3)$-based literature. Second row: notation used in the context of QCD factorization (originating from naive factorization). Further rows: scaling in powers of the Cabibbo angle $\lambda$, in $1/N_c$, and in $\Lambda_{QCD}/m_b$. Some multiply suppressed amplitudes are omitted.

| traditional name | T | C | $P_{ut}$ | $P_{et}$ | $P_{EW}$ | $P_{EW}^C$ | (P$_{et}$) | PA | E | A |
|------------------|---|---|---------|---------|---------|-----------|-----------|----|---|---|
| Cabibbo ($b \to d$) | all amplitudes are $O(\lambda^3)$ |
| Cabibbo ($b \to s$) | $\lambda^4$ | $\lambda^4$ | $\lambda^4$ | $\lambda^2$ | $\lambda^2$ | $\lambda^2$ | $\lambda^2$ | $\lambda^4$ | $\lambda^4$ |
| $1/N$ | $1$ | $1/N$ | $1/N$ | $1$ | $1/N$ | $1/N$ | $1/N$ | $1$ |
| $\Lambda/m_b$ | $1$ | $1$ | $1$ | $1$ | $1$ | $\Lambda/m_b$ | $\Lambda/m_b$ | $\Lambda/m_b$ | $\Lambda/m_b$ |

1 Introduction

Charmless hadronic $B$-decays have been at the center of experimental and theoretical interest for many years. The reasons are threefold: (i) there are a large number of measurements, with over 100 final states when considering light pseudoscalars and vectors alone; (ii) they are sensitive both to CKM elements and, being short-distance-dominated rare processes, to possible new heavy particles; (iii) they are conceptually interesting as they involve an intricate interplay of the three different Standard-Model (SM) interactions and the hierarchy of energy scales $M_W$, $m_b$, $\Lambda_{QCD}$.

Concretely, any weak $B$-decay into two charmless hadrons has an amplitude

$$A(\bar{B} \to M_1 M_2) = e^{-i\gamma} |V_{ud}V_{ub}| T_{M_1M_2} + |V_{cd}V_{cb}| P_{M_1M_2} + A_{NP},$$

where $D = d$ or $s$, the “tree” $T_{M_1M_2}$ and “penguin” $P_{M_1M_2}$ are CP-even “strong” amplitudes, comprising hadronic matrix elements of the weak Hamiltonian $H = \sum_i C_i Q_i$, and $A_{NP}$ denotes a possible beyond-SM (BSM) contribution. Extracting CKM information or identifying a BSM contribution from the data requires knowledge about the strong amplitudes $T_{M_1M_2}$ and $P_{M_1M_2}$.

Isospin symmetry may be used to group together classes of decays (such as all $B \to K\pi$ decays), which may involve up to two tree amplitudes (colour-allowed and colour-suppressed), a QCD-penguin amplitude, electroweak penguin amplitudes, and so on. These “topological” amplitudes (first row of Table 1) can be visualized as weak-interaction diagrams. Existing theoretical treatments either eliminate the strong amplitudes by simultaneously considering $b \to d$ and $b \to s$ transitions and invoking $SU(3)_F$ symmetries [1, 2] (at the expense of a smaller number of BSM-sensitive observables), or attempt to calculate some strong amplitudes, or some combination thereof. Of the computational approaches, those that achieve some degree of model-independence are based on an expansion in $\Lambda/m_b$: QCD factorization (QCDF) [3, 4, 5] and its effective-field theory formulation in SCET [6, 7, 8], and the still more
ambitious, but also more model-dependent, pQCD approach \cite{9,10}.

2 Status of QCD factorization

Structure The QCD factorization approach \cite{3,4} is based on the collinear factorization of the hadronic matrix elements $\langle M_1 M_2 | Q_i | B \rangle$ in the heavy-quark limit. To leading power in $\Lambda/m_b$, the collinear and soft dynamics is contained in $B \to$ light hadron form factors and light-cone distribution amplitudes (LCDA) for the initial- and final-state mesons, convoluted with perturbative hard-scattering kernels. Schematically,

$$F_{BM_1} f_{M_2} a = F_{BM_1} f_{M_2} \int t^I(u) \phi_{M_2}(u) du + f_B f_{M_1} f_{M_2} \int t^{II}(u, v) \phi_{M_2}(v) \phi_{M_1}(u) du dv,$$

where now $a$ denotes any of the amplitudes in Table \ref{tab:1}. $F_{BM_i}$ is a form factor and $f_B, f_{M_i}, f_{M_2}$ are decay constants.\footnote{By convention, a product of a form factor and a decay constant is factored out of the $a_i / \alpha_i$ amplitudes, but not in \cite{1} (nor in most SU(3)-based analyses). Accordingly the normalization conventions differ between the first and second rows of Table \ref{tab:1}.} The kernels $t^I$ and $t^{II}$ are hard-scattering kernels and are perturbatively calculable as power series in $\alpha_s$. The structure \cite{2} holds at the leading power in $\Lambda/m_b$ and is unambiguous, free from scale or scheme dependences, etc. Some important terms at the first subleading power also factorize, but others do not (an attempt to factorize them leads to endpoint-divergent convolutions).

The hard-scattering kernels are computed by considering appropriate partonic states with the quantum numbers of the initial and final mesons, consisting of soft partons for the $B$ meson and collinear ones for the two final-state mesons (Figure \ref{fig:1}). The structure \cite{2} emerges most transparently within soft-collinear effective theory (SCET), whereby the hard kernels become Wilson coefficients and the form factors and light-cone distribution amplitudes become matrix elements of operators

\begin{equation}
\begin{aligned}
\text{Figure 1: Matching calculation for the hard-scattering kernels. The red lines are hard (virtuality $\sim m_b^2$), the green lines hard-collinear (virtuality $\sim m_b \Lambda$). Gluon exchanges of lower virtualities are reproduced by the effective-theory matrix elements, which, for hadronic states, define form factors and light-cone distribution amplitudes.}
\end{aligned}
\end{equation}
in the effective theory; the full equivalence (up to a change of operator basis) to the original, diagram-based formula has been demonstrated in [8, 11]. Any differences in practice arise from (independent) approximations at the phenomenological stage, such as neglect of higher orders or the treatment of power corrections. (In this context see also [12].)

At the leading power, only two-particle partonic states need to be considered. The $B$-meson lines can either annihilate via the weak Hamiltonian (these terms can be shown to be power-suppressed), or the valence light quark in the $B$ meson can form a spectator line with one of the lines representing $M_1$. The “hard-spectator-scattering” terms on the second line of (2) involve all diagrams that include a hard gluon exchange involving the spectator line (green lines in Figure 1). However, there is a second leading-power contribution from the end-point region where the spectator quark enters $M_1$ as a soft parton. These diagrams are responsible for the first line in (2). As a consequence, at zeroth order in $\alpha_s$ and in the heavy-quark limit, “naive factorization” is obtained.

The pQCD approach [9, 10] aims to also factorize the form factors, introducing some new conceptual issues and parametric dependences. We refer to the original literature for more details.

**Status of the perturbative kernels**

Over the last 7 years a number of substantial works have pushed the precision from NLO ($\alpha_s$) order (at which nontrivial factorization of infrared physics first occurs) to NNLO ($\alpha_s^2$), beginning with spectator scattering for the leading-power tree [8, 13, 14] and (QCD and QED) penguin [11, 15] amplitudes, followed by the form factor terms for the trees [16, 17, 18]. In all cases, a well-behaved perturbation series is obtained, and the structure of (2) holds, i.e. infrared physics factorizes as expected. While no formal all-orders proof of factorization has been published, in view of the high complexity and intricate cancellations observed, this should be considered strong evidence that factorization holds at higher orders. The missing piece at NNLO at the leading power is the two-loop form-factor correction to the penguin amplitudes.

**Power corrections**

In phenomenological applications certain terms that are formally $\Lambda/m_b$-suppressed cannot be neglected. First, there is the so-called “scalar-penguin” amplitude $a_6^\alpha$. This forms part of the QCD penguin amplitude $\alpha_6$ and includes the hadronic matrix element of the $(V - A) \times (V + A)$ QCD penguin operator $Q_6$, as well as “charming-penguin” loop contributions. (The superscript refers to the CKM structure $V_{cb}V_{cD}^\ast$ in (1).) For some of the final states that involve at least one pseudoscalar, this contribution is “chirally enhanced” by a large normalisation factor. E.g. for $M_2$ a pion, the normalisation $r_\pi^\alpha = 2 m_\pi^2/(m_b(m_u + m_d))$ is formally

\footnote{A partial calculation exists, comprising the chromomagnetic-operator contribution [14].}
power-suppressed but numerically greater than one. Fortunately the scalar penguin factorizes. At the moment, the NNLO contributions, which might be phenomenologically relevant (see below), are not known. A second, likely important power correction is the annihilation amplitude $\beta_3$. (The possible importance of such a term, which involves a large colour factor, was first pointed out in the pQCD framework [9].) Note that “annihilation” refers to the way the external lines are contracted with the weak Hamiltonian in the matching onto SCET, and not to a topological annihilation amplitude: Like $a_6^c$, $\beta_3^c$ forms part of the topological (QCD-)penguin amplitude, which decomposes in the heavy-quark limit as

$$P_c \propto \alpha_4^c + \beta_3^c \equiv a_4^c \pm r_\chi M_2 a_6^c + \beta_3^c.$$  

(3)

The sign in front of $a_6^c$ depends on the spin of the final-state meson $M_1$. $\beta_3^c$ (and other annihilation amplitudes) do not factorize and need to be modelled, usually according to the parameterization given in [5]. Thirdly, there are contributions to the topological amplitudes which arise at the level of higher-twist LCDA’s for $M_1$ or multi-particle LCDA’s for the $B$ meson, which do not factorize. These terms are (by convention) included in the amplitudes $a_{1,2,4}^{u,c}$; in the SCET formulation, they involve power-suppressed parts of SCET matrix elements. $a_4^c$ also includes power-suppressed “charming penguin” corrections, for which however no enhancement mechanism has been identified.

**Electroweak amplitudes, singlets, vector-vector final states, etc** The electroweak penguin operators in the weak Hamiltonian, together with QED effects, generate further topological amplitudes which factorize at the leading power. If one or both of the final-state mesons are $SU(3)_F$ singlets, some extra amplitudes appear. We refer to the original literature, specifically [20] (see also [21]), for a comprehensive discussion. If both final-state particles are vector mesons, the number of amplitudes triples. In this case, only the helicity-0 ("longitudinal") amplitudes factorize. The other helicity amplitudes are power-suppressed, but are numerically not negligible [22].

### 3 Phenomenology

Generically, QCDF implies that the predictions of naive factorization hold up to corrections of order $\alpha_s$ or $\Lambda/m_b$, in particular (i) direct CP asymmetries are typically suppressed, (ii) the corrections $\Delta S_f$ to the sine-coefficients in time-dependent CP asymmetries, due to sub-leading amplitudes, are given by naive factorization up to small corrections. Moreover, pure-annihilation modes are power-suppressed. All of this matches with the available data, with some well-known qualifications.
The topological tree amplitudes are known to NNLO. The colour-allowed and colour-suppressed trees $a_1$ and $a_2$ evaluate to

$$a_1 = 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050}) i, \quad a_2 = 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078}) i$$

for a $B \to \pi\pi$ decay, where all errors have been combined in quadrature. We see that the colour-allowed tree amplitude carries a very small uncertainty and a tiny strong phase (and is very close to the naive-factorisation result $C_1 + C_2/3 = 1.009$). On the other hand, the colour-suppressed tree amplitude carries a large uncertainty. The detailed anatomy of the NNLO central value including twist-3 power corrections is

$$a_2 = \left\{ [0.220]_{\text{LO}} + [-0.179 - 0.077i]_{\text{NLO}} + [-0.031 - 0.050i]_{\text{NNLO}} \right\}_{\text{FF}}$$

$$+ \left[ \frac{r_{sp}}{0.445} \right] \left\{ [0.114]_{\text{NLO}} + [0.049 + 0.051i]_{\text{NNLO}} + [0.067]_{\text{tw3}} \right\}_{\text{spec}}.$$

Here, the two lines correspond to the two lines of (2). There is strong destructive interference between the LO term and the higher-order form-factor corrections, which amplifies the relative importance of spectator scattering and of power corrections. In addition, the form-factor and spectator-scattering corrections interfere destructively with each other. The spectator scattering suffers moreover from an uncertain normalization $r_{sp} = (9f_{M1}f_B)/(m_B\lambda_B f_F^{\pi}(0))$. This crucially depends on the first inverse moment $1/\lambda_B$ of the relevant $B$-meson LCDA, which is poorly known. Experimental data on $BR(B \to \pi^0\pi^0)$ suggest a very large magnitude of $a_2$; this may point to a small value of $\lambda_B$, or to an underestimate of the twist-3 spectator-scattering power correction. Further $B$-factory measurements of $B \to \gamma\ell\nu$, which is very sensitive to $\lambda_B$, may help with resolving this; see [23] in this context. Measurements with $\pi^0\rho^0$ and $\rho^0\rho^0$ show less drastic discrepancies; see also [25]. The remaining amplitudes (QCD and EW penguins) are free from such strong cancellations, and consequently carry more modest uncertainties. They are currently known only to NLO.

If the aim is to use QCDF to search for BSM effects, then its phenomenological usefulness must be first judged against data on BSM-insensitive observables. This has been done for modes dominated by tree and QCD penguin amplitudes in the SM, based on the argument that these are in turn dominated by SM $W$-boson exchange at tree-level or within a charm loop. This class includes all $B \to \pi\pi$ observables as well as the direct CP asymmetry $A_{CP}(\bar{B} \to \pi^+K^-)$, and related observables obtained by replacing one of the final-state mesons by a vector ($\pi \to \rho$ or $K \to K^*$). The case of $BR(\bar{B} \to \pi^0\pi^0)$ has already been discussed. Figure 2 (left) shows a complex penguin-to-tree ratio that can be extracted from the time-dependent CP-asymmetry in $\bar{B} \to \pi^+\pi^-$, given the CKM angle $\gamma$. The ellipses correspond to $1\sigma$ experimental errors, the cross denotes the QCDF prediction with errors combined in quadrature, and the blue square corresponds to the parameter set “G” [8] which accommodates...
Figure 2: Penguin-to-tree ratios: fits to data compared to QCDF predictions (points with error bars). Left panel: $P_{\pi^+\pi^-}/T_{\pi^+\pi^-}$. The three ovals correspond to (from right to left) $\gamma = 60^\circ, \gamma = 70^\circ, \gamma = 80^\circ$. Middle panel: $\hat{\alpha}_4^c(\pi K)/(a_1(\pi\pi) + a_2(\pi\pi))$, right panel: $\hat{\alpha}_4^c(\pi K^*)/(a_1(\pi\pi) + a_2(\pi\pi))$. See text for details.

data on $B \to \pi\pi$ and $B \to \pi K$ well. The fitted imaginary part is weakly sensitive to $\gamma$ and opposite in sign from expectations. The discrepancy is less striking than in the past, however, due to the significantly reduced value of $|C_{\pi\pi}|$ reported by Belle at this conference [27, 28], and the small LHCb result [26]. Note also that the theory prediction may still receive an important charm-loop correction (proportional to the large Wilson coefficient $C_1$). In spectator scattering, such contributions first enter at order $\alpha_s^2$ and are not known for $a_6$ yet. The fitted real part is in agreement with theory for $\gamma \sim 60^\circ - 70^\circ$. Turning this around, one can say that QCDF allows for a $\sim (5 - 7)^\circ$-precision determination of $\gamma$ from $B \to \pi^+\pi^-$ alone. The middle and right panels in Fig. 2 show another penguin-to-tree ratio that can be extracted from $B \to \pi\pi$ and $B \to \pi K$ (middle) and $B \to \pi K^*$ data (see [20] for details). The fitted value corresponds to the intersection of the left wedge and the circle. The blue, onion-like shape delimits an estimate of the complex annihilation contribution $\beta_3^c$. In the case of $\pi K$, one observes a reasonable comparison between theory and data within errors. This is quite a nontrivial check – the fitted result could a priori lie anywhere in the part of the complex plane depicted (or even outside). The discrepancy (mainly) in the imaginary part simply reflects that there is no precise prediction for the direct CP asymmetry $A_{CP}(\pi^+K^-)$, and is very similar (involving $\hat{\alpha}_4^c$) to what is seen for $B \to \pi^+\pi^-$. Finally, in the case of $\pi K^*$ one has agreement between QCDF and data within errors. The magnitude of the penguin amplitude $\hat{\alpha}_4^c$ is significantly smaller than in the $\pi K$ case. This can be understood based on the structure in [3]: In the case of a vector $M_2 = \overline{K}^*$, the scalar-penguin contribution $a_6^c$ is no longer chirally enhanced, and effectively absent. This is a characteristic prediction of the heavy-quark limit; I am not aware of any alternative theoretical explanation.
It is also worth noting that several dedicated studies of vector-vector final states exist, notably [22, 29] dating from the $B$-factory era. As said above, two out of three helicity (or equivalently transversity) amplitudes are power-suppressed, but as pointed out in [22], in the case of the penguin-amplitudes this suppression is effective only for the positive-helicity amplitude (in a $B$ decay). Modelling power corrections in the usual way, in the case of $B \rightarrow \phi K^*$, the full angular analyses performed by Belle and Babar can be reproduced by QCDF [29]. Much of the theory carries over to $B_s \rightarrow \phi \phi$, under study at LHCb [30]. The suppression of, for example, certain triple-product asymmetries is a simple consequence of (and depends on!) the survival of the suppression of the positive-helicity amplitude in the presence of QCD effects. For a comprehensive study focusing on the (calculable) longitudinal-polarization observables, see [31], which also contains references to earlier work on vector-vector final states in the heavy-quark expansion.

Finally, a large number of studies of BSM effects employing QCD factorization exist, but it is not possible to do justice to them in the limited space here. However, the discussion here should convey that there are observables which remain NP-sensitive, after uncertainties on strong amplitudes are taken into account.

4 Conclusion

The QCD factorization approach provides a consistent and unambiguous framework for computing hadronic $B$ decay amplitudes, including strong phases, in the heavy-quark limit. The leading-power amplitudes are partly available at NNLO, and further calculations by several groups are underway. The same is true for the factorizable “scalar-penguin” power corrections. All results so far point to a well-behaved perturbation expansion and no violations of factorization have been found nor are any expected. At the phenomenological level, upon modelling non-factorizable power corrections with a reasonable model one obtains a consistent description of most data within the Standard Model, within uncertainties. This includes many nontrivial predictions. The main tension is with the $B \rightarrow \pi^0\pi^0$ mode, where new-physics contributions are generally not expected. This could be explained through an underestimated power correction or a large inverse moment of the $B$-meson LCDA. Future experimental data may help. The framework extends to vector-vector final states and can accommodate $B$-factory data on $B \rightarrow \phi K^*$, and can make predictions for LHCb measurements of the angular distribution in $B(B_s) \rightarrow VV$.

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