Application of P- median method in logistics node location

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Abstract. In view of the logistics node location problems, according to build logistics nodes needed to meet the timeliness and economic cost minimization requirement, combined with the specific case, write the corresponding algorithm program, based on P-value model which is constructed and improve the plan of site selection of logistics company division static P-value model, the plan of site selection of dynamic for selection of logistics node provides an effective analysis method and the decision basis, to solve the problem for a class of the actual location.

1. Previous literature
With the rapid development of e-commerce and the Internet, online shopping has gradually become a shopping method that consumers are happy to choose, followed by the rapid development of the logistics industry. Modern logistics industry is developing rapidly all over the world. New ideas and operation methods lead to a large number of theoretical and applied research problems, which also promote a series of research work. Disciplines theoretical achievements of application in the field of logistics also gradually developed, in which operational research is used to solve the problem of optimization of logistics field type of commonly used method. These methods can be divided into quantitative analysis and comprehensive evaluation system to two categories. In particular, there are mixed integer nonlinear programming, double layer planning, analytic hierarchy process, data envelopment analysis and other methods [1].

2. Problem analysis of P- median method

2.1. Meaning of P- median
The P- median problem was originally proposed by the Hakimi. The model is to find a suitable location for P facilities and assign each request point to a specific facility under a set of requirements and candidate facility locations for a given number and location, thus minimizing the transportation cost between the plant and the request point. P- median problem is applicable to the location of problems such as express outlets and schools [2]. The advantage of the p-median method is that it can calculate the logistics site selection scheme with the best economic benefit and the lowest cost. The disadvantage is that the model has certain limitations. Each demand point can only correspond to one alternative logistics center but not multiple logistics centers.

2.2. Hub positioning
Hub facility point (hub node) is an important central node in the same level of facility node in a certain region, which plays a certain role of transit and circulation, and usually has a large storage space and information control capacity. Other common nodes (non-hub nodes) are only responsible for the
distribution service, and the flow of goods between them needs to be carried out through hub facility points. Hub positioning problem, that is, in a given number and location of distribution facility node set, n hub nodes are selected to meet the flow needs of all nodes in the set. At the same time, each customer is assigned to a specific node to meet the needs of the customer, so as to minimize the transportation and circulation costs of the distribution system.

2.3. Location problem of logistics distribution node
Most enterprises need to decide the location of two or more facilities, and they are not isolated from each other, so the solution of the problem becomes relatively complex. According to the characteristics of the region to be selected, the problem of site selection can be divided into continuous site selection and discrete site selection. According to the objective function, the continuous site selection is carried out in a certain area, and the representative methods of the continuous site selection are the cross median method and the center of gravity method. Discrete site selection is to select the desired number of addresses from several alternative addresses so that the objective function can be optimized. The models commonly used for discrete site selection are maximum coverage model, set coverage model and p-median model. In the continuous site selection, the address selected by the cross median method may not be the optimal address, and the center of gravity method can only realize the location of a single logistics node. The results of continuous site selection may not meet the actual requirements and can be used as a reference in the actual site selection problem. In this paper, the discrete site selection method is used to establish the p-median model, and the heuristic algorithm is used to quickly solve the model.

3. Establishment and implementation of P- median model

3.1. Basic assumptions
(1) each demand area and the alternative logistics center are treated as geometric points, and the demand and location of each demand point and the location of the alternative logistics center are known; (2) the distance between the demand point and the logistics center is calculated according to the linear distance; (3) the freight from the demand point to the logistics center is directly proportional to the distance between the two points; (4) the demand of each demand point can be met by the logistics center.

3.2. Establishment of the model
In order to simplify the model, the sum of the product of the weight of the demand point and the distance from the demand point to the logistics center can be used as the objective function to be optimized as follows:

\[
\min f = \sum_{i \in I} \sum_{j \in J} w_i \sqrt{x^2 - 2xx_j + x_j^2 + y^2 - 2yy_j + y_j^2}
\]

The constraint conditions obtained according to the site selection requirements are:

\[
\sum_{j \in J} y_{ij} = 1 \quad \forall i \in I \quad \tag{2}
\]

\[
\sum_{i \in I} x_j = p \quad \tag{3}
\]

\[
y_{ij} \leq x_j \quad \forall i \in I, \forall j \in J \quad \tag{4}
\]

\[
y_{ij} \leq x_j \quad \forall i \in I, j \in J \quad \tag{5}
\]

I is the set of demand points, \( I = \{1, 2, \ldots, n\} \); \( w_i \) is the weight of the fourth demand point (for example, the quantity demanded); \( J \) is the collection of alternative logistics centers, \( J = \{1, 2, \ldots, m\} \); \( p \) is the number of logistics centers to be established and \( p < m \); When the demand point \( i \) is satisfied, the logistics center, it is 1; otherwise, it is 0. The second formula means that each demand point can only be assigned to one logistics center. The meaning of \( x_j \) is when the logistics center is established at the point \( j \), it is 1; otherwise, it is 0. The third formula indicates that the
number of logistics centers selected is $p$. Formula 4 ensures that the demand point is not assigned to an unselected logistics center\(^6\).

### 3.3. Model implementation

From the process of model establishment, it can be seen that to implement the p-median model, it is necessary to select the appropriate logistics center location and assign each demand point to the selected logistics center. The algorithms for solving the p-median model include precise algorithm and heuristic algorithm. The precise algorithm is not suitable for large scale problems, and the greedy heuristic algorithm is commonly used. The specific steps of the algorithm are as follows\(^7\):

1. **Step 1:** set the cycle variables $k = m$, select all the alternative logistics centers, and assign each demand point to the alternative logistics center nearest to the demand point (hereinafter referred to as the alternative point);
2. **Step 2:** fetch $j \in J$, reassign the assigned requirement points to other alternative points according to the latest principle, and calculate the target function value under the new assignment, denoted as $C_j$;
3. **Step 3:** when the set is selected, select such, let $C_{j_0} = \min_{j \in J} \{C_j\}$, and $J = J - j_0$, $k = k - 1$;
4. **Step 4:** if $k = p$, the algorithm stops; otherwise, Step 2 is returned.

### 4. Logistics node optimization scheme based on P-median method

SDJM is a modern logistics enterprise focusing on road LCL, urban distribution, supply chain logistics services. The company plans to build a transfer center in the east of Jinan. SDJM logistics center is located between the government of Dongjia Town, Jinan City and the government of Guodian Town, near Jinan International Airport Lingang Economic Development Zone, Jigang District and Jinan High-tech Development Zone, near Guotang Road to the west, Yushan Avenue to the south, about 4 kilometers from the entrance of Guodian Expressway, 18.3 kilometers from Jinan Remote Wall International Airport, about 20 kilometers from the city center (Quancheng Road), convenient transportation conditions and unique location advantages. In order to simplify the model, 11 typical supermarket stores in Jinan were selected, and external goods were accepted by the established transit center, and then goods were delivered to each store by trucks with small load.

#### 4.1. Assumptions

SDJM logistics co., LTD. ’s Jinan transfer center location problem is to select a certain number of store addresses from the address set of a given 11 stores as alternative points to expand into the transfer center, so as to establish a series of distribution areas and realize the minimum total logistics cost of the distribution system formed by the transfer center to each store. In order to facilitate the establishment of mathematical model, the following assumptions are made:

1. Transport distance and unit transport price between stores are known;
2. The daily demand of each store is certain;
3. The warehouse in the transit center has no capacity constraints.

The freight table of each store is shown in table 1, and the daily demand of each store is shown in table 2.
Table 1. Transportation costs between stores

|   | 0  | 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9  | 10 |
|---|----|----|----|----|----|----|----|----|----|----|----|
| 0 | 0  |    |    |    |    |    |    |    |    |    |    |
| 1 | 51 | 0  |    |    |    |    |    |    |    |    |    |
| 2 | 27 | 37 | 0  |    |    |    |    |    |    |    |    |
| 3 | 16 | 26 | 23 | 0  |    |    |    |    |    |    |    |
| 4 | 44 | 53 | 63 | 41 | 23 | 0  |    |    |    |    |    |
| 5 | 18 | 56 | 43 | 31 | 29 | 22 | 0  |    |    |    |    |
| 6 | 42 | 96 | 74 | 69 | 69 | 50 | 16 | 0  |    |    |    |
| 7 | 48 | 96 | 74 | 69 | 69 | 50 | 16 | 0  |    |    |    |
| 8 | 48 | 96 | 74 | 69 | 69 | 50 | 16 | 0  |    |    |    |
| 9 | 48 | 96 | 74 | 69 | 69 | 50 | 16 | 0  |    |    |    |
|10 | 48 | 96 | 74 | 69 | 69 | 50 | 16 | 0  |    |    |    |

Table 2. Daily demand of goods in each store

| Stores make up, | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------------|---|---|---|---|---|---|---|---|---|---|----|
| Demand for     | 5 | 4 | 3.5| 2 | 4.5| 4 | 3 | 3.5| 2.5| 4  | 3  |

4.2. Center of gravity method for site selection

The principle of the center of gravity method is to regard the resource points and demand points of the logistics system as the object system distributed in a plane. The material flow of each resource point and demand point can be regarded as the weight of the object respectively. The center of gravity of the object system will be the optimal location of the logistics center.

In a program area, there is a resource point and a demand point, and the demand or resource of each point, their respective coordinates are \( n \ w_j (j = 1, 2, 3, \ldots, n) (x_j, y_j) (j = 1, 2, 3, \ldots, n) \)

Firstly, it is planned to set up a logistics center in the region. The coordinate of the logistics center is set as \((x_d, y_d)\), and the freight rate from the logistics center to the resource point or demand point is \(a_j\).

According to the method of finding the center of mass of the object in the plane, we can get:

\[
\begin{align*}
\bar{x} &= \frac{\sum_{j=1}^{n} a_j w_j x_j}{\sum_{j=1}^{n} a_j w_j} \quad (6) \\
\bar{y} &= \frac{\sum_{j=1}^{n} a_j w_j y_j}{\sum_{j=1}^{n} a_j w_j} \quad (7)
\end{align*}
\]

The coordinate chart of 11 stores is shown in Figure 1.

![Figure 1. Coordinates of 11 stores](https://example.com/figure1.png)

The final coordinate of the transit center is \((7.1, 6.1)\).

The target points obtained by the barycenter method do not necessarily accord with a certain store, and several candidate transit centers are determined within a certain range around the geographical coordinates of the above calculation results to form the candidate transit center set. And then using the
P- median method, two candidate stores are selected from the transit center, and finally identified as the target store for expansion into the transit center.

4.3. P-median model
Establishment of the mode
According to the above analysis, the following model is established:

Objective function:
\[ \min \sum_{i \in N} \sum_{j \in M} c_{ij}y_{ij} \] (8)

The constraint conditions are as follows:
Each store can only set up one transit center
\[ \sum_{j \in M} y_{ij} = 1 \quad i \in N \] (9)
The total number of transit centers established is two
\[ \sum_{j \in M} x_j = 2 \] (10)
Stores that cannot be expanded as transit centers do not allocate demand points
\[ y_{ij} \leq x_j \quad i \in N, j \in M \] (11)
\[ x_j \in \{0,1\} \quad j \in M \] (12)
\[ y_{ij} \in \{0,1\} \quad j \in M, i \in N \] (13)

Type:
\[ N = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}, \text{11 demand points in the system;} \]
\[ M = \{0,6,7\}, \text{3 alternative points of transit center;} \]
\[ d_i -- \text{the requirements of the first demand point; } i \]
\[ c_{ij} -- \text{the unit transportation cost from the demand point to the transfer center; } i j \]
\[ x_j \text{Establish a transit center at the point.} \]
\[ 0, 1 \quad j \in M, i \in N \]

Model solution
So that the first store is assigned to the smallest candidate. \( k = 4i c_{ij} \)
First assignment: \( A = (a_0, a_1, \ldots, a_{10}) = (0, 0, 0, 0, 6, 6, 6, 7, 7, 7, 7) \)
Total cost of transportation: \( TC = \sum_{i=0}^{10} c_i a_i d_i = 934 \)

Delete candidate points 0, 6 and 7 respectively for analysis, and calculate the increment of each:
If the candidate point 0 is deleted, then \( (\ldots a_0, a_1, \ldots) =, = 1649.5, \text{the increment is} \)
\( 715.5; a_{10}(7, 6, 7, 6, 6, 6, 7, 7, 7, 7, 7)TC \)
If the candidate point 6 is deleted, then \( (\ldots a_0, a_1, \ldots) =, = 1060.5, \text{the increment is} \)
\( 126.5; a_{10}(0, 0, 0, 0, 0, 0, 7, 7, 7, 7)TC \)
If candidate point 7 is deleted, then \( (\ldots a_0, a_1, \ldots) =, = 1086, \text{increment is} \)
\( 152; a_{10}(0, 0, 0, 0, 6, 6, 6, 0, 6, 6)TC \)

Therefore, the increase generated by the removal of candidate point 6 is minimal, so the candidate location to be removed is candidate point 6.

At this point, \( k=2 \), the calculation ends, and the result is that the extension of the transfer center at the candidate location 0,7, and the total transportation cost is 1060.5.

5. Conclusion
As the core part of the logistics system, the location of logistics nodes is becoming more and more complex. Therefore, how to optimize the layout of logistics nodes in a more scientific and reasonable way and reduce the operation cost of the entire logistics system has become an urgent problem to be solved in China. In this paper, the p-median and its improved model are applied to the comprehensive decision-making problem of logistics node site selection scheme, and a p-median model is established.
to solve a class of site selection problem, so as to realize the rapid solution of the model and minimize the freight in the logistics center site selection problem.

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