The low-density spin susceptibility and effective mass of mobile electrons in Si inversion layers

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We studied the Shubnikov-de Haas (SdH) oscillations in high-mobility Si-MOS samples over a wide range of carrier densities \( n \simeq (1 - 50) \times 10^{11} \text{cm}^{-2} \), which includes the vicinity of the apparent metal-insulator transition in two dimensions (2D MIT). Using a novel technique of measuring the SdH oscillations in superimposed and independently controlled parallel and perpendicular magnetic fields, we determined the spin susceptibility \( \chi^s \), the effective mass \( m^* \), and the \( g^s \)-factor for mobile electrons. These quantities increase gradually with decreasing density; near the 2D MIT, we observed enhancement of \( \chi^s \) by a factor of \( \sim 4.7 \).

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Many two-dimensional (2D) systems exhibit an apparent metal-insulator transition (MIT) at low temperatures as the electron density \( n \) is decreased below a critical density \( n_c \) (for reviews see, e.g., Refs. \[1\]-\[3\]). The phenomena of the MIT and ‘metallic’ conductivity in 2D attract a great deal of interest, because it addresses a fundamental problem of the ground state of strongly correlated electron systems. The strength of electron-electron interactions is characterized by the ratio \( r_s \) of the Coulomb interaction energy to the Fermi energy \( \epsilon_F \). The 2D MIT is observed in Si MOSFETs at \( n_c \sim 1 \times 10^{11} \text{cm}^{-2} \), which corresponds to \( r_s \sim 8 \) (for 2D electrons in (100)-Si, \( r_s = 2.63 \sqrt{10^{11} \text{cm}^{-2}/n} \)).

In the theory of electron liquid, the electron effective mass \( m^* \), the \( g^s \)-factor, and the spin susceptibility \( \chi^s \propto g^s m^* \) are renormalized depending on \( r_s \). Though the quantitative theoretical results \[2\]-\[6\] vary considerably, all of them suggest enhancement of \( \chi^s \), \( m^* \) and \( g^s \) with \( r_s \). Earlier experiments \[8\]-\[12\] have shown growth of \( m^* \) and \( g^s m^* \) at relatively small \( r_s \) values, pointing to a ferromagnetic type of interactions in the explored range \( 1 \lesssim r_s < 6.5 \). Potentially, strong interactions might drive an electron system towards ferromagnetic instability \[1\]. Moreover, it has been suggested that the ‘metallic’ behavior in 2D is accompanied by a tendency to a ferromagnetic instability \[12\]. Thus, in relation to the still open question of the origin of the 2D MIT, direct measurements of these quantities in the dilute regime near the 2D MIT are crucial.

In this Letter, we report the direct measurements of \( \chi^s \), \( m^* \), and hence \( g^s \) over a wide range of carrier densities \( 1 \leq r_s \leq 8.4 \), which extends for the first time down to and across the 2D MIT. The data were obtained by a novel technique of measuring the interference pattern of Shubnikov-de Haas (SdH) oscillations in \textit{crossed magnetic fields}. The conventional technique of measuring \( g^s m^* \) in tilted magnetic fields \[8\]-\[11\] is not applicable when the Zeeman energy is greater than the cyclotron energy \[13\]. The crossed field technique removes this restriction and allows us to extend measurements over the wider range of \( r_s \). We find that for small \( r_s \), the \( g^s m^* \) values increase slowly in agreement with the earlier data by Fang and Stiles \[8\] and Okamoto et al. \[10\]. For larger values of \( r_s \), \( g^s m^* \) grows faster than it might be extrapolated from the earlier data. At our highest value of \( r_s \approx 8.4 \), the measured \( g^s m^* \) is greater by a factor of 4.7 than that at low \( r_s \).

Our resistivity measurements were performed by ac (13Hz) technique at the bath temperatures \( 0.05 - 1.6 \text{K} \) on six (100) Si-MOS samples selected from four wafers: Si12 (peak mobility \( \mu \simeq 3.4 \text{m}^2/\text{Vs} \) at \( T = 0.3 \text{K} \)), Si22 (\( \mu \simeq 3.2 \text{m}^2/\text{Vs} \)), Si57 (\( \mu \simeq 2.4 \text{m}^2/\text{Vs} \)), Si6-14/5, Si6-14/10, and Si6-14/18 (\( \mu \simeq 2.4 \text{m}^2/\text{Vs} \) for the latter three samples) \[14\]. The gate oxide thickness was \( 190 \pm 20 \text{nm} \) for all the samples; the 2D channel was oriented along [011] for sample Si3-10, for all other samples - along [010]. The crossed magnetic field system consists of two magnets, whose fields can be varied independently. A split coil, producing the field \( B_\perp \leq 1.5 T \) normal to the plane of the 2D layer, is positioned inside the main solenoid, which creates the in-plane field \( B_\parallel \) up to 8T. The electron density was determined from the period of SdH oscillations (Figs. 1 and 3).

Typical traces of the longitudinal resistivity \( \rho_{xx} \) as a function of \( B_\perp \) are shown in Fig. 1. Due to the high electron mobility, oscillations were detectable down to 0.25T and a large number of oscillations enabled us to extract the fitting parameters \( g^s m^* \) and \( m^* \) with a high accuracy. The oscillatory component \( \delta \rho_{xx} \) was obtained by subtracting the monotonic ‘background’ magnetoresistance (MR) from the \( \rho_{xx}(B_\perp) \) dependence (the background MR is more pronounced for lower \( n \), compare Figs. 1 and 3).

The theoretical expression for the oscillatory compo-
Differential of the magnetoresistance is as follows [13]:

$$\frac{\delta \rho_{xx}}{\rho_0} = \sum_s A_s \cos(\pi s(\frac{h \pi n}{e B_\parallel} - 1))Z_s,$$

(1)

where

$$A_s = 4 \exp(-2\pi^2 \frac{k_B T D}{h \omega_c}) \frac{2 \pi^2 \sqrt{s} k_B T}{\sinh(2\pi \sqrt{s} k_B T/h \omega_c)}.$$

(2)

Here $\rho_0 = \rho_{xx}(B_\parallel = 0)$, $\omega_c = e B_\perp/m^*\nu_c$ is the cyclotron frequency, $m^*$ is the dimensionless effective mass, $\nu_c$ is the free electron mass, and $T_D$ is the Dingle temperature. We take the valley degeneracy $g_e$ as observed as a function of $B_\parallel$ and two values of $n$: $B_\parallel \sim g^* m^*$: observation of several nodes enables us to determine this quantity with a high accuracy. The positions of the nodes on the $B_\perp$ axis (and, thus, the value of $g^* m^*$) are $T$-independent at $T < 1K$ within the experimental accuracy $\sim 2\%$. We have observed a non-monotonic dependence of $g^* m^*$ on $B_\parallel$, which will be discussed elsewhere [17]. This dependence is more pronounced near the 2D MIT, where $g^* m^*$ varies with $B_\parallel$ by $\sim 15\%$. The data discussed below have been obtained in the fields sufficiently low to ignore the $g^* m^*(B)$ dependence. For example, to ensure the linear regime, at high densities ($n \sim 10^{12} \text{cm}^{-2}$) we have used $B_\parallel \approx \pm 0.8 T$, which corresponded to the spin polarization $P \sim 1\%$; at low densities ($n \sim 10^{11} \text{cm}^{-2}$), the applied $B_\parallel \approx 0.1 - 0.2 T$ corresponded to $P \sim 5 - 10\%$.

Comparison between the measured and calculated dependences $\delta \rho_{xx}/\rho_0$ versus $B_\perp$, both normalized by the amplitude of the first harmonic $A_1$, is shown in Fig. 2 for two carrier densities. The normalization assigns equal weights to all oscillations. We analyzed SdH oscillations over the low-field range $B_\perp \leq 1T$; this limitation arises from the assumption in Eq. (1) that $h \omega_c \ll \epsilon_F$ and $\delta \rho_{xx}/\rho_0 \ll 1$. The latter condition also allows us to neglect the inter-level interaction which is known to enhance $g^*$ in stronger fields [5].

The amplitude of SdH oscillations at small $B_\perp$ can be significantly enhanced by applying $B_\parallel$ (see Fig. 3), which

![FIG. 1. Shubnikov-de Haas oscillations for $n = 10.6 \times 10^{11} \text{cm}^{-2}$ (sample Si6-14/10) at $T = 0.35K$ and two values of $B_\parallel$.](image)

![FIG. 2. Examples of data fitting with Eq. (1): (a) $m^* = 0.22, T_\sigma = 0.55K, g^* = 2.63$; (b) $m^* = 0.275, T_\sigma = 0.57K, g^* = 3.35$. The data for sample Si6-14/10 are shown as the solid lines, the fits (with parameters shown) as the dashed lines. Both are normalized by $A_1(B_\parallel)$.](image)
is another advantage of the cross-field technique. Indeed, for low $B_\parallel$ and $n$, the electron energy spectrum is complicated by crossing of levels corresponding to different spins/valleys. By applying $B_\parallel$, we can control the energy separation between the levels, and enhance the amplitude of low-$B$ oscillations (see Fig. 3). We have verified that application of $B_\parallel$ (up to the spin polarization $\sim 20\%$) does not affect the extracted $m^*$ values (within $10\%$ accuracy), provided the sample remains in the ‘metallic’ regime (the insets to Fig. 3 show that the values of $m^*$ measured at $B_\parallel = 0$ and $3.36$ T do coincide).

Fitting of the data provides us with two combinations of parameters: $g^*m^*$ and $(T + T_D)m^*$. The measured values of $g^*m^*$, as well as $m^*$ which are discussed below, were similar for different samples. Figure 4a shows that for small $r_s$, our $g^*m^*$ values agree with the earlier data by Fang and Stiles [8] and Okamoto et al. [10]. For $r_s \geq 6$, $g^*m^*$ increases with $r_s$ faster than it might be expected from extrapolation of the earlier results [10]. The MIT occurs at the critical $r_s$-value ranging from $r_c = 7.9$ to 8.8 for different samples; in particular, $r_c \approx 8.23$ for samples Si6-14/5,10,18 which have been studied down to the MIT.

An interesting question is whether the measured dependence $\chi^*(r_s)$ shown in Fig. 4a represents a critical behavior as might be expected if $\chi^*$ diverges at $n_c$. Analysis of the $B_\parallel$-induced magnetoresistance led the authors of Refs. [22,23] to the conclusion that there is a ferromagnetic instability at $n = n_c$. By forcing the critical dependence $\chi^* \propto (n/n_c - 1)^{-\alpha}$ to fit our data, we found that the correlated fitting parameters $n_c$ and $\alpha$ can be varied over a wide range, only weakly affecting the standard deviation. More importantly, the period of SdH oscillations shows that the electron states remain double spin degenerate across the 2D MIT down to at least $n = 0.98 \times 10^{11}$ cm$^{-2}$. The details of this analysis, which does not support the conclusion on spontaneous spin polarization and divergency of $\chi^*$ at $n = n_c$, are presented elsewhere [17].

![FIG. 3. Shubnikov-de Haas oscillations versus $B_\perp$ for sample Si6-14/10 ($n = 2.2 \times 10^{11}$ cm$^{-2}$) at $T = 0.4$; 0.5; 0.6; 0.7; 0.8K: (a) $B_\parallel = 0$ and (b) $B_\parallel = 3.36$ T. The insets show the temperature dependences of fitting parameters $(T + T_D)m^*$.](image)

![FIG. 4. Parameters $g^*m^*$, $m^*$, and $g^*$ for different samples as a function of $r_s$ (dots). The solid line in Fig. 4a shows the data by Okamoto et al. [10]. The solid and open dots in Figs. 4b and 4c correspond to two different methods of finding $m^*$ (see the text). The solid and dashed lines in Fig. 4b are polynomial fits for the two dependences $m^*(r_s)$. The values of $g^*$ shown in Fig. 4c were obtained by dividing the $g^*m^*$ data by the smooth approximations of the experimental dependences $m^*(r_s)$ shown in Fig. 4b.](image)

The second combination, $(T + T_D)m^*$, controls the amplitude of oscillations. In order to disentangle $T_D$ and $m^*$, we analyzed the temperature dependence of oscillations over the range $T = 0.3 – 1.6K$ (for some samples $0.4 – 0.8$ K) [19]. The conventional procedure of calculating the effective mass for low $r_s$ values ($\lesssim 5$), based on
the assumption that $T_D$ is $T$-independent, is illustrated by the insets in Fig. 3. In this small-$r_s$ range, our results are in a good agreement with the earlier data by Smith and Stiles [9], and by Pan et al. [11]. The assumption $T_D \neq f(T)$, however, becomes dubious near the MIT, where the resistance varies significantly over the studied temperature range; in this case, the two parameters $T_D$ and $m^*$ become progressively more correlated. The open dots in Fig. 4b were obtained by assuming that $T_D$ is $T$-independent over the whole explored range of $n$: $m^*$ increases with $r_s$, and the ratio $m^*/m_0$ becomes $\sim 2.5$ at $r_s = 8$ ($m_0 = 0.19$ is the band mass). As another limiting case, one can attribute the change in $R(T)$ solely to the temperature dependence of the short-range scattering and request $T_D$ to be proportional to $R(T)$. In the latter case, the extracted dependence $m^*(r_s)$ is weaker (the solid dots in Fig. 4b). To reduce the uncertainty of $m^*$ at large $r_s$, it is necessary to separate the effects of $T$-dependent scattering and ‘smearing’ of the Fermi distribution in the dependence $R(T)$; the adequate theory is currently unavailable.

Our data shows that the combination $(T + T_D)m^*$ is almost the same for electrons in both spin-up and spin-down subbands (e.g., for $n = 3.76 \times 10^{13}$ cm$^{-2}$ and $B_0 = 2.15$ T ($P = 20\%$), the $T_D$ values for ‘spin-up’ and ‘spin-down’ levels differ by $\leq 3\%$). This is demonstrated by the observed almost 100\% modulation of SdH oscillations (see, e.g., Fig. 2b). Thus, the carriers in the spin-up and spin-down subbands have nearly the same mobility; this imposes some constraints on theoretical models of electron transport in the 2D ‘metallic’ state.

In conclusion, using a novel cross-field technique for measuring SdH oscillations, we performed direct measurements of the spin susceptibility, effective mass, and $g$-factor of conducting electrons over a wide range of carrier densities. By studying the temperature dependence of SdH oscillations, we disentangled three unknown parameters: $g^*$, $m^*$, and $T_D$, and obtained their values in the limit of small magnetic fields. We found that both $g^* m^*$ and $m^*$ are almost independent of the temperature over the range $(0.2-1)$ K. At the 2D metal-insulator transition, we observed a finite value of the spin susceptibility, enhanced by a factor of $\sim 4.7$ with respect to the high-density limit.

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