Chiral Lagrangian with Heavy Quark-Diquark Symmetry

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(Dated: February 2, 2008)

Abstract

We construct a chiral Lagrangian for doubly heavy baryons and heavy mesons that is invariant under heavy quark-diquark symmetry at leading order and includes the leading $O(1/m_Q)$ symmetry violating operators. The theory is used to predict the electromagnetic decay width of the $J = \frac{3}{2}$ member of the ground state doubly heavy baryon doublet. Numerical estimates are provided for doubly charm baryons. We also calculate chiral corrections to doubly heavy baryon masses and strong decay widths of low lying excited doubly heavy baryons.

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Heavy quark-diquark symmetry relates mesons with a single heavy quark to antibaryons with two heavy antiquarks. Savage and Wise \[1\] argued that quark-diquark symmetry was realized in the heavy quark limit of Quantum Chromodynamics (QCD) and studied this symmetry using the methods of Heavy Quark Effective Theory (HQET) \[2\]. Recently, Refs. \[3, 4\] derived effective Lagrangians for heavy diquarks within the framework of Non-Relativistic QCD (NRQCD) \[3, 4\]. These papers obtain a prediction for the hyperfine splitting of the ground state doubly heavy baryons in terms of the ground state heavy meson hyperfine splitting\(^1\). Heavy quark-diquark symmetry also relates other properties of heavy mesons and doubly heavy baryons. A useful tool for studying low energy strong and electromagnetic interactions of heavy hadrons is heavy hadron chiral perturbation theory (HH\(\chi\)PT) \[8, 9, 10\]. This theory has heavy hadrons, Goldstone bosons, and photons as its elementary degrees of freedom and incorporates the approximate chiral and heavy quark symmetries of QCD. In this paper we derive a version of HH\(\chi\)PT that includes doubly heavy baryons and incorporates heavy quark-diquark symmetry. The theory is used to calculate chiral corrections to doubly heavy baryon masses and to obtain model-independent predictions for the electromagnetic decay of the \(J = \frac{3}{2}\) member of the ground state doubly heavy baryon doublet. Our formulae are applicable to either doubly bottom or doubly charm baryons, and we give numerical estimates for the case of doubly charm baryons. We also discuss the low lying excited doubly heavy baryons, show how these states can be included in the effective theory, and calculate their strong decay widths.

Motivation for this work comes from the SELEX experiment’s recent observation of states which have been tentatively interpreted as doubly charm baryons \[11, 12, 13\], and also the COMPASS experiment, which in its second phase run in 2006 hopes to observe doubly charm baryons \[14\]. Many aspects of the SELEX states are difficult to understand. States observed by SELEX include the \(\Xi_{cc}^+(3520)\), which decays weakly into \(\Lambda_c^+\pi^+K^-\) \[11\], as well as \(pD^+K^-\) \[13\], the \(\Xi^{++}_{cc}(3460)\), which decays weakly into \(\Lambda_c^+K^-\pi^+\pi^+\) \[12\], and a broader state, \(\Xi^{++}_{cc}(3780)\), also seen to decay into \(\Lambda_c^+K^-\pi^+\pi^+\) \[12\]. The ground states of the \(\Xi_{cc}^+\) and \(\Xi^{++}_{cc}\) are related by isospin symmetry and therefore should differ in mass by only a few MeV, so the observed difference of 60 MeV seems implausible. On the other hand, an

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\(^1\) The formula for the hyperfine splittings in Ref. \[1\] differs from the correct formula in Refs. \[3, 4\] by a factor of 2.
unpublished talk \[15\] and conference proceedings \[16\] present evidence for additional states, \(\Xi_{cc}^{+}(3443)\) and \(\Xi_{cc}^{++}(3541)\). If these states exist the isospin splittings are closer to theoretical expectations, but still quite large. The difference between the mass of the \(\Xi_{cc}^{+}(3520)\) and \(\Xi_{cc}^{+}(3443)\) is 77 MeV, and the splitting between \(\Xi_{cc}^{++}(3541)\) and \(\Xi_{cc}^{++}(3460)\) is 81 MeV. These splittings agree remarkably well with calculations of the doubly charm hyperfine splittings in quenched lattice QCD \[17, 18, 19\] and are within \(\sim 25\) MeV of the heavy quark-diquark symmetry prediction obtained in Refs. \[3, 4\], an acceptable discrepancy given the expected \(O(\Lambda_{QCD}/m_c)\) corrections. However, interpretation of the \(\Xi_{cc}^{+}(3520)\) as the \(J = \frac{3}{2}\) member of the ground state doublet is impossible to reconcile with the fact that the \(\Xi_{cc}^{+}(3520)\) is observed to decay weakly because if the \(\Xi_{cc}^{+}(3520)\) is not the ground state of the \(ccd\) system it should decay electromagnetically. There are also discrepancies between the weak decay lifetimes predicted by HQET \[20, 21, 22\] (\(\sim 100\) fs) and the observed lifetimes (\(< 33\) fs) \[11, 13\]. Production cross sections are also poorly understood within perturbative QCD \[23\]. However, the SELEX states are observed in the forward region, \(\langle x_F \rangle \sim 0.3\), where nonperturbative production mechanisms such as intrinsic charm \[24, 25, 26\] or parton recombination \[27, 28\] may be important.

Even if there is difficulty interpreting the SELEX data, doubly charm baryons must exist and are expected to have masses of approximately 3.5 GeV, where the SELEX states are. In light of existing and future experimental efforts to observe doubly charm baryons, it is desirable to have model independent predictions for other properties besides the relation for the hyperfine splittings derived in Refs. \[1, 3, 4\]. Therefore it is important to develop theoretical tools for analyzing the properties of doubly heavy baryons systematically.

Savage and Wise \[1\] wrote down a version of heavy quark effective theory (HQET) which includes diquarks as elementary degrees of freedom and derived a formula relating heavy meson and doubly heavy baryon hyperfine splittings. HQET only separates the scales \(\Lambda_{QCD}\) and \(m_Q\), where \(m_Q\) is the heavy quark mass. The dynamics of a bound state of two heavy quarks is characterized by additional scales \(m_Qv\) and \(m_Qv^2\), where \(v\) is the typical velocity of the heavy quarks in the bound state. The correct effective theory for hadrons with two heavy quarks is Non-Relativistic QCD (NRQCD) \[5\], which properly accounts for the scales \(m_Qv\) and \(m_Qv^2\). Analysis of heavy diquarks within the framework of NRQCD was recently performed in Refs. \[3, 4\]. These papers derived Lagrangians for diquark fields starting from NRQCD and obtained the correct heavy quark symmetry prediction for the hyperfine
splittings of the doubly heavy baryons. For simplicity, we will consider only one flavor of heavy quark. The lowest mass diquark will consist of two heavy antiquarks in an orbital $S$-wave in the $3$ representation of color $SU(3)$. Then Fermi statistics demands that they have total spin one. In the rest frame of the heavy quark and lowest mass diquark, the Lagrangian to $O(1/m_Q)$ is

$$\mathcal{L} = h^\dagger \left( iD_0 - \frac{\vec{D}^2}{2m_Q} \right) h + \vec{V}^\dagger \cdot \left( iD_0 + \delta - \frac{\vec{D}^2}{m_Q} \right) \vec{V} + \frac{g_s}{2m_Q} h^\dagger \vec{\sigma} \cdot \vec{B}^a \frac{\lambda_a}{2} h + \frac{ig_s}{2m_Q} \vec{V}^\dagger \cdot \vec{B}^a \frac{\lambda_a}{2} \times \vec{V}. \quad (1)$$

Here $h$ is the heavy quark field, $\vec{V}$ is the field for the diquarks, the $\lambda^a/2$ are the $SU(3)$ color generators, $\text{Tr}[\lambda^a \lambda^b] = 2 \delta^{ab}$, $D_0$ and $\vec{D}$ are the time and spatial components of the gauge covariant derivative, respectively, $\vec{B}^a$ is the chromomagnetic field, and $m_Q$ is the heavy quark mass. The term proportional to $\delta$ is the residual mass of the diquark. The heavy antiquarks in the diquark experience an attractive force and therefore the mass of the diquark is not $2m_Q$ but $2m_Q - \delta$, where $\delta$ is the binding energy. This residual mass can be removed by rephasing the diquark fields. Physically, this corresponds to measuring diquark energies relative to the mass of the diquark, rather than $2m_Q$. Once this is done the Lagrangian, at lowest order in $1/m_Q$, is invariant under a $U(5)$ symmetry which permutes the two spin states of the heavy quark and the three spin states of the heavy antiquark. The $U(5)$ symmetry is broken by the $O(1/m_Q)$ kinetic energy and chromomagnetic couplings of the heavy quark and diquark. The latter terms are responsible for the hyperfine splittings.

The ground state doublet of heavy mesons is usually represented in $\text{HH}_\chi\text{PT}$ as a $4 \times 4$ matrix transforming covariantly under Lorentz transformations, and transforming as a doublet under $SU(2)$ heavy quark spin symmetry,

$$H_v = \left( \frac{1 + \gamma_5}{2} \right) (P^{*\mu}_{\nu} \gamma_{\mu} - \gamma_5 P_{\nu}). \quad (2)$$

Here $P^{*\mu}_{\nu}$ is the $J^P = 1^-$ vector heavy meson field which obeys the constraint $v_\mu P^{*\mu}_{\nu} = 0$, where $v^\mu$ is the four-velocity of the heavy meson. $P_{\nu}$ is the $J^P = 0^-$ pseudoscalar heavy meson field. The superfield $H_v$ obeys the constraints $\gamma_5 H_v = -H_v \gamma_5 = H_v$, so $H_v$ only has four independent degrees of freedom. These can be collected in a $2 \times 2$ matrix. For example,
in the heavy meson rest frame where \( \nu^\mu = (1, 0, 0, 0) \),

\[
H_v = \begin{pmatrix}
0 & -\vec{P}_v \cdot \vec{\sigma} & -P_v \\
-\vec{P}_v \cdot \vec{\sigma} & 0 & 0 \\
-P_v & 0 & 0
\end{pmatrix},
\]

(3)

where we have used the Bjorken-Drell conventions for \( \gamma_\mu \) and \( \gamma_5 \). For a process such as the weak decay \( B \rightarrow D \ell \nu \), in which the initial and final heavy hadrons have different four-velocities, the covariant representation of fields is needed to determine heavy quark symmetry constraints on heavy hadron form-factors. However, for studying low energy strong and electromagnetic interactions in which the heavy meson four-velocity is conserved (up to \( O(\Lambda_{QCD}/m_Q) \) corrections), it is also possible to work in the heavy meson rest frame and use \( 2 \times 2 \) matrix fields. This makes some calculations simpler and we find it easiest to formulate the extension of HH\( \chi \)PT with \( U(5) \) quark-diquark symmetry in this frame. We define the heavy meson field in our theory to be

\[
H_a = \vec{P}_{a}^* \cdot \vec{\sigma} + P_a,
\]

(4)

where \( a \) is an \( SU(3) \) flavor anti-fundamental index and the \( \vec{\sigma} \) are the Pauli matrices. Since we have chosen to work in the heavy meson rest frame, Lorentz covariance is lost and the symmetries of the theory are rotational invariance, \( SU(2) \) heavy quark spin symmetry, parity, time reversal and \( SU_L(3) \times SU_R(3) \) chiral symmetry. Under these symmetries the field \( H_a \) transforms as

- rotations \( H'_a = U H_a U^\dagger \)
- heavy quark spin \( H'_a = S H_a \)
- parity \( H'_a = -H_a \)
- time reversal \( H'_a = -\sigma_2 H_a^* \sigma_2 \)
- \( SU_L(3) \times SU_R(3) \) \( H'_a = H_b V^\dagger_{ba} \).

(5)

Here \( U \) and \( S \) are \( 2 \times 2 \) rotation matrices and \( V^\dagger_{ba} \) is an \( SU(3) \) matrix which gives the standard nonlinear realization of \( SU_L(3) \times SU_R(3) \) chiral symmetry. In the two component notation the HH\( \chi \)PT Lagrangian is:

\[
\mathcal{L} = \text{Tr}[H^I_{a}(iD_0)_{ba}H_b] - g \text{Tr}[H^I_{a}H_b \vec{\sigma} \cdot \vec{A}_{ba}] + \frac{\Delta H}{4} \text{Tr}[H^I_{a} \sigma^I H_b \sigma^I].
\]

(6)

The last term breaks heavy quark spin symmetry and \( \Delta H \) is the hyperfine splitting of the heavy mesons. The time component of the covariant chiral derivative is \( (D_0)_{ba} \), \( \vec{A}_{ba} \) is the
spatial part of the axial vector field, and $g$ is the heavy meson axial coupling. Our definitions for the chiral covariant derivative, the axial current, and the Lagrangian for the Goldstone boson fields are the same as Ref. 29.

We are now ready to generalize the Lagrangian to incorporate the doubly heavy baryons and the $U(5)$ quark-diquark symmetry. The field $H_a$ transforms like the tensor product of a heavy quark spinor and a light antiquark spinor. (This is how representations of heavy hadron fields were constructed in Ref. 30.) Writing the field with explicit indices, $(H_a)_{\alpha\beta}$, the index $\alpha$ corresponds to the spinor index of the heavy quark and the index $\beta$ is that of the light antiquark spinor. In the theory with quark-diquark symmetry, the heavy quark spinor is replaced with a five-component field, the first two components corresponding to the two heavy quark spin states and the last three components corresponding to the three spin states of the diquark:

$$Q_\mu = \begin{pmatrix} h_\alpha \\ V_i \end{pmatrix}. \quad (7)$$

In terms of $Q_\mu$ the kinetic terms of the Lagrangian in Eq.(1) are

$$\mathcal{L} = Q_\mu^\dagger iD_0 Q_\mu. \quad (8)$$

The fields in $\chi$PT with heavy quark-diquark symmetry transform as tensor products of the five component field $Q_\mu$ and a two-component light antiquark spinor. Thus, the $2 \times 2$ matrix field $H_a$ is promoted to a $5 \times 2$ matrix field

$$H_{a,\alpha\beta} \rightarrow \mathcal{H}_{a,\mu\beta} = H_{a,\alpha\beta} + T_{a,i\beta}. \quad (9)$$

Here the index $\mu$ takes on values between 1 and 5, $\alpha, \beta = 1$ or 2, and $i = 3, 4, \text{ or } 5$. The doubly heavy baryon fields are contained in $T_{a,i\beta}$. Under the symmetries of the theory $\mathcal{H}_a$ transforms as

- rotations: $\mathcal{H}'_a = R \mathcal{H}_a U^\dagger$
- heavy quark spin: $\mathcal{H}'_a = S \mathcal{H}_a$
- parity: $\mathcal{H}'_a = -\mathcal{H}_a$
- time reversal: $\mathcal{H}'_a = -\Sigma_2 \mathcal{H}_a^* \sigma_2$
- $SU_L(3) \times SU_R(3)$: $\mathcal{H}'_a = \mathcal{H}_b V_{ba}^\dagger. \quad (10)$
The matrix $S$ is now an element of $U(5)$ and $R$ is a $5 \times 5$ reducible rotation matrix

$$R_{\mu\nu} = \begin{pmatrix} U_{\alpha\beta} & 0 \\ 0 & R_{ij} \end{pmatrix},$$

(11)

where $U_{\alpha\beta}$ is an $SU(2)$ rotation matrix and $R_{ij}$ is an orthogonal $3 \times 3$ rotation matrix related to $U$ by $U^\dagger \sigma_i U = R_{ij} \sigma_j$. The $5 \times 5$ matrix appearing in the time reversal transformation is

$$(\Sigma_2)_{\mu\nu} = \begin{pmatrix} (\sigma_2)_{\alpha\beta} & 0 \\ 0 & \delta_{ij} \end{pmatrix}.$$ 

(12)

Under rotations the field $T_{a,i\beta}$ transforms as

$$T'_{a,i\beta} = R_{ij} T_{a,j\gamma} U^\dagger \gamma\beta.$$ 

$T_{a,i\beta}$ can be further decomposed into its spin-$3/2$ and spin-$1/2$ components,

$$T_{a,i\beta} = \sqrt{2} \left( \Xi^*_a i\beta + \frac{1}{\sqrt{3}} \Xi_{a,\gamma} i\beta \right),$$

(13)

where $\Xi^*_a i\beta$ and $\Xi_{a,\gamma}$ are the spin-$3/2$ and spin-$1/2$ fields, respectively. The factor of $\sqrt{2}$ is a convention that ensures that the kinetic terms of the doubly heavy baryon fields have the same normalization as the heavy meson fields. The field $\Xi^*_a i\beta$ obeys the constraint $\Xi^*_a i\beta \sigma_{\beta\gamma} = 0$.

The $U(5)$ invariant generalizations of the first two terms of Eq. (6) are simply obtained by making the replacement $H_a \rightarrow H_a$. To determine the proper generalization of the $U(5)$ breaking term we note that the chromomagnetic couplings in Eq. (11) can be written as

$$\frac{g_s}{2m_Q} Q_{\mu} \Sigma^i_{\mu\nu} \cdot \bar{B}_a \lambda^a \frac{\lambda^a}{2} Q_{\nu},$$

(14)

where the $\Sigma_{\mu\nu}$ are the $5 \times 5$ matrices

$$\Sigma_{\mu\nu} = \begin{pmatrix} \bar{\sigma}_{\alpha\beta} & 0 \\ 0 & T_{jk} \end{pmatrix},$$

(15)

and $(T^i)_{jk} = -i \epsilon_{ijk}$. It is now obvious that the correct generalization of Eq. (11) is

$$\mathcal{L} = \text{Tr}[\mathcal{H}_a^i (iD_0)_{ba} \mathcal{H}_b] - g \text{Tr}[\mathcal{H}_a^i \mathcal{H}_b \bar{\sigma} \cdot \bar{A}_{ba}] + \frac{\Delta_H}{4} \text{Tr}[\mathcal{H}_a^i \Sigma^i \mathcal{H}_a \sigma^i]$$

$$+ \text{Tr}[T^i_a (iD_0)_{ba} T_b] - g \text{Tr}[T^i_a T_b \bar{\sigma} \cdot \bar{A}_{ba}] + \frac{\Delta_H}{4} \text{Tr}[T^i_a T^i T_a \sigma^i].$$

(16)
The last line of Eq. (16) contains the terms relevant for doubly heavy baryons. Heavy quark-diquark symmetry relates the couplings in the doubly heavy baryon sector to the heavy meson sector. The propagator for the spin-$\frac{1}{2}$ doubly heavy baryon is

$$\frac{i\delta_{ab}\delta_{\alpha\beta}}{2(k_0 + \Delta_H/2 + i\epsilon)},$$

while the propagator for the spin-$\frac{3}{2}$ doubly heavy baryon is

$$\frac{i\delta_{ab}\mathcal{P}_{i\alpha,j\beta}}{2(k_0 - \Delta_H/4 + i\epsilon)} = \frac{i\delta_{ab}(\delta_{ij}\delta_{\alpha\beta} - \frac{1}{3}(\sigma^i\sigma^j)_{\alpha\beta})}{2(k_0 - \Delta_H/4 + i\epsilon)}.$$

The projection operator $\mathcal{P}_{i\alpha,j\beta}$ satisfies $\sigma^i_{\gamma\alpha}\mathcal{P}_{i\alpha,j\beta} = \mathcal{P}_{i\alpha,j\beta}\sigma^j_{\beta\gamma} = 0$. Comparison of the poles of the propagators shows that the hyperfine splitting for the doubly heavy baryons is $\frac{3}{4}\Delta_H$, reproducing the heavy quark-diquark symmetry prediction

$$m_{\Xi^*} - m_{\Xi} = \frac{3}{4}(m_{P^*} - m_P), \quad (17)$$

obtained in Refs. [3, 4].

We can also include operators which mediate electromagnetic decays. The Lagrangian for electromagnetic decays of the heavy mesons in the two-component notation is [31]

$$\mathcal{L} = \frac{e\beta}{2}\text{Tr}[H\dagger H\bar{\sigma} \cdot \vec{B} Q_{ab}] + \frac{e}{2m_Q}Q'\text{Tr}[H\dagger\bar{\sigma} \cdot \vec{B} H_a], \quad (18)$$

where $Q_{ab} = \text{diag}(2/3, -1/3, -1/3)$ is the light quark charge matrix, $\beta$ is the parameter introduced in Ref. [31], and $Q'$ is the heavy quark charge. For charm, $Q' = 2/3$. The first term is the magnetic moment coupling of the light degrees of freedom and the second term is the magnetic moment coupling of the heavy quark. Both terms are needed to understand the observed electromagnetic branching fractions of the $D^{*+}$ and $D^{*0}$ [31]. The magnetic couplings of the heavy quark and diquark are

$$\mathcal{L}_{em} = \frac{e}{2m_Q}Q' h^\dagger \bar{\sigma} \cdot \vec{B} h - \frac{ie}{m_Q}Q' \vec{V}^\dagger \cdot \vec{B} \times \vec{V} \quad (19)$$

where the $\vec{\Sigma}_{\mu\nu}$ are the $5 \times 5$ matrices

$$\vec{\Sigma}_{\mu\nu} = \begin{pmatrix} \bar{\sigma}_{\alpha\beta} & 0 \\ 0 & -2\vec{T}_{jk} \end{pmatrix}.$$ (20)
The magnetic coupling of the diquark has the opposite sign as that of the heavy quark because it is composed of two heavy antiquarks. The coefficient of the chromomagnetic coupling of the diquark in Eqs. (11) is a factor of 2 smaller than the coefficient of the electromagnetic coupling of the diquark in Eq. (19) due to a color factor. The magnetic couplings in the HHχPT Lagrangian for heavy mesons and doubly heavy baryons are

\[ L = \frac{e\beta}{2} \text{Tr}[H^\dagger_a H_b \vec{\sigma} \cdot \vec{B} Q_{ab}] + \frac{e}{2m_Q} Q'\text{Tr}[H^\dagger_a \vec{Y}' \cdot \vec{B} H_b]. \] (21)

The part of this Lagrangian involving the doubly heavy baryon fields is

\[ L = \frac{e\beta}{2} \text{Tr}[T^\dagger_a T_b \vec{\sigma} \cdot \vec{B} Q_{ab}] - \frac{e}{m_Q} Q'\text{Tr}[T^\dagger_a \vec{T} \cdot \vec{B} T_b]. \] (22)

This can be used to obtain the following tree level predictions for the electromagnetic decay widths:

\[ \Gamma[P^* \rightarrow P \gamma] = \frac{\alpha}{3} \left( \beta Q_{aa} + \frac{Q'}{m_Q} \right)^2 \frac{m_P}{m_{P^*}} E_\gamma^3 \]
\[ \Gamma[\Xi^* \rightarrow \Xi \gamma] = \frac{4\alpha}{9} \left( \beta Q_{aa} - \frac{Q'}{m_Q} \right)^2 \frac{m_{\Xi}}{m_{\Xi^*}} E_\gamma^3. \] (23)

Here \( E_\gamma \) is the photon energy. These results could also be obtained in the quark model, with the parameter \( \beta = 1/m_q \), where \( m_q \) is the light constituent quark mass. The effective theory allows one to improve upon this approximation by including corrections from loops with light Goldstone bosons, which give \( O(\sqrt{m_q}) \) corrections to the decay rates (31). If these loop corrections are evaluated in an approximation where heavy hadron mass differences are neglected, the correction to the above formulae can be incorporated by making the following replacements (31)

\[ \beta Q_{11} \rightarrow \frac{2}{3} \beta - \frac{g^2 m_K}{4\pi f_K^2} - \frac{g^2 m_\pi}{4\pi f_\pi^2} \]
\[ \beta Q_{22} \rightarrow -\frac{1}{3} \beta + \frac{g^2 m_\pi}{4\pi f_\pi^2} \]
\[ \beta Q_{33} \rightarrow -\frac{1}{3} \beta + \frac{g^2 m_K}{4\pi f_K^2}. \] (24)

For charm mesons, hyperfine splittings are \( \approx 140 \text{ MeV} \) and the \( SU(3) \) splitting is \( \approx 100 \text{ MeV} \), while for bottom mesons the hyperfine splittings are \( \approx 45 \text{ MeV} \) and \( SU(3) \) splitting is \( \approx 90 \text{ MeV} \). The approximation of neglecting heavy hadron mass differences and keeping Goldstone boson masses is reasonable for kaon loops but not for loops with pions.
the largest $O(\sqrt{m_q})$ corrections come from loops with kaons. When data on double heavy
baryon electromagnetic decays is available, more accurate calculations along the lines of
Ref. [29] should be performed. In this paper, we will use Eqs. (23) and (24) to obtain
estimates of doubly charm baryon electromagnetic decay widths.

Currently $\Gamma[D^{*+}]$ is measured to be $96 \pm 22$ keV, while there is only an upper limit for
$\Gamma[D^{*0}]$. The branching ratios for the $D^{*+}$ decays are $\text{Br}[D^{*+} \to D^{0}\pi^+] = 67.7 \pm 0.5\%$, $\text{Br}[D^{*+} \to D^{+}\pi^0] = 30.7 \pm 0.5\%$ and $\text{Br}[D^{*+} \to D^+\gamma] = 1.6 \pm 0.4\%$. The branching ratios
for $D^{*0}$ decays are $\text{Br}[D^{*0} \to D^{0}\pi^0] = 61.9 \pm 2.9\%$ and $\text{Br}[D^{*0} \to D^{0}\gamma] = 38.1 \pm 2.9\%$. Isospin symmetry can be used to relate the strong partial width of the $D^{*0}$ to the known strong partial width of the $D^{*+}$. Then the measured branching fractions of the $D^{*0}$ can be used to obtain the partial electromagnetic width of the $D^{*0}$. We find

$$\Gamma[D^{*0} \to D^{0}\gamma] = 26.1 \pm 6.0\text{ keV}$$
$$\Gamma[D^{*+} \to D^+\gamma] = 1.54 \pm 0.35\text{ keV},$$

(25)

where the error is dominated by the uncertainty in $\Gamma[D^{*+}]$. $\Gamma[D^{*+} \to D^+\gamma]$ is suppressed because of a partial cancellation between the magnetic moments of the light degrees of freedom and the charm quark. Using the partial widths in Eq. (25) and the formulae in Eqs. (23) and (24), we obtain predictions for doubly charm baryon electromagnetic decays in Table I.

In our calculations of the doubly charm baryon decay widths the factor $m_{\Xi}/m_{\Xi^*}$ in Eq. (23) has been set equal to one. For the expected masses and hyperfine splittings of the doubly charm baryons, this factor changes the predictions for the widths by less than 3%. The fits are labeled in the left hand column of Table I

In the fits labeled QM we have not included the $O(\sqrt{m_q})$ corrections in Eq. (24). Therefore, these predictions for the doubly charm baryon electromagnetic decays are the same as what would be obtained in the quark model. The values of the parameters $\beta$ and $m_c$ for each fit are shown along with the predictions for the electromagnetic decay widths. In QM 1, we have treated $\beta$ and $m_c$ as free parameters and fit these to the central values in Eq. (25). In QM 2 we have set $m_c = 1500$ MeV and performed a least squared fit to $\beta$. In the fits labeled $\chi$PT, we have included the leading $O(\sqrt{m_q})$ chiral corrections in Eq. (24). We have used $f_\pi = 130$ MeV, $f_K = 159$ MeV, and $g = 0.6$ which is extracted from a tree level fit to the $D^{*+}$ width. In $\chi$PT 1, we fixed $\beta$ and $m_c$ to reproduce the central values in Eq. (25). In $\chi$PT 2, we set
m_c = 1500 MeV and performed a least squares fit to $\beta$. There are several sources of error in the calculation. We expect 30% theoretical errors due to heavy quark symmetry breaking effects, 30% errors due to higher order $SU(3)$ breaking effects, and 25% uncertainty from the experimentally measured value of $\Gamma[\Xi^{*+}]$ leading to at least 50% error in the predictions in Table I.

Chiral perturbation theory and the nonrelativistic quark model give similar size estimates for the $\Xi^{*+}_{cc}$ electromagnetic decay widths which are expected to be $\sim 2-3$ keV if the hyperfine splitting is 80 MeV. The electromagnetic decay should completely dominate any possible weak decay, even if the weak decay rates are an order of magnitude greater than calculated in Refs. [20, 21, 22]. The quark model predicts $\Gamma[\Xi^{*++}]$ slightly greater than $\Gamma[\Xi^{*+}]$. This is in contrast with the charm meson sector where the magnetic moment of the light degrees of freedom and the magnetic moment of the charm quark add constructively to give a large $\Gamma[D^{*0} \to D^{0}\gamma]$ and destructively to give a small $\Gamma[D^{*+} \to D^{+}\gamma]$. In the doubly heavy baryon sector, the relative sign of the magnetic moments is reversed, and both decay rates are approximately the same. In fact from Eq. (23), we can see that for $\beta = 4/m_c$ the two rates are exactly equal in the quark model. Fits to the $D^*$ electromagnetic decays yield values of $\beta$ and $m_c$ that are close to this point in parameter space. Including the $O(\sqrt{m_q})$ corrections from chiral perturbation theory, the most important effect is the kaon loop correction whose contribution to the $\Xi^{*+}_{cc}$ decay has opposite sign as the contribution
from $\beta$ at tree level, therefore suppressing the $\Xi^{*+}_{cc}$ decay relative to $\Xi^{*+}_{cc}$.

The theory can also be used to compute chiral corrections to doubly heavy baryon masses. The one loop corrections to the hadron masses are

$$
\delta m_{\Xi^*} = \sum_{i,b} \mathcal{C}_{iab} \frac{g^2}{16 \pi^2 f_i^2} \left( \frac{5}{9} K(m_{\Xi^*_b} - m_{\Xi^*_a}, m_i, \mu) + \frac{4}{9} K(m_{\Xi^*_b} - m_{\Xi^*_a}, m_i, \mu) \right),
$$

$$
\delta m_{\Xi^0} = \sum_{i,b} \mathcal{C}_{iab} \frac{g^2}{16 \pi^2 f_i^2} \left( \frac{1}{9} K(m_{\Xi^0_b} - m_{\Xi^0_a}, m_i, \mu) + \frac{8}{9} K(m_{\Xi^0_b} - m_{\Xi^0_a}, m_i, \mu) \right),
$$

$$
\delta m_{H^+} = \sum_{i,b} \mathcal{C}_{iab} \frac{g^2}{16 \pi^2 f_i^2} K(m_{H^+_b} - m_{H^+_a}, m_i, \mu),
$$

$$
\delta m_{H^0} = \sum_{i,b} \mathcal{C}_{iab} \frac{g^2}{16 \pi^2 f_i^2} \left( \frac{1}{3} K(m_{H^0_b} - m_{H^0_a}, m_i, \mu) + \frac{2}{3} K(m_{H^0_b} - m_{H^0_a}, m_i, \mu) \right). \quad (26)
$$

Here $m_i$ and $f_i$ are the mass and decay constant of the Goldstone boson in the one loop diagram and $\mathcal{C}_{iab}$ is a factor which comes from $SU(3)$ Clebsch-Gordan coefficients in the couplings. For loops with charged pions we have $\mathcal{C}_{i12}^{\pi^0} = \mathcal{C}_{i21}^{\pi^0} = 1$, for loops with neutral pions $\mathcal{C}_{i11}^{\pi^0} = \mathcal{C}_{i22}^{\pi^0} = \frac{1}{2}$, for loops with kaons $\mathcal{C}_{i11}^{K^0} = \mathcal{C}_{i22}^{K^0} = 1$ ($i=1$ or $2$), and for loops with $\eta$ mesons $\mathcal{C}_{i11}^{\eta} = \mathcal{C}_{i22}^{\eta} = \frac{1}{6}$ and $\mathcal{C}_{i33}^{\eta} = \frac{2}{3}$. The function $K(\delta, m, \mu)$ is related to the finite part of the integral

$$
\int i \frac{d^D l}{(2\pi)^D} \frac{\vec{l}^2}{l^2 - m^2} \frac{1}{l^0 - \delta + i\epsilon} = \frac{1}{(4\pi)^2} K(\delta, m, \mu), \quad (27)
$$

evaluated using dimensional regularization in the $\overline{MS}$ scheme. We find

$$
K(\delta, m, \mu) = (-2 \delta^3 + 3 m^2 \delta) \ln \left( \frac{m^2}{\mu^2} \right) + 2 \delta (\delta^2 - m^2) F \left( \frac{\delta}{m} \right) + 4 \delta^3 - 5 \delta m^2, \quad (28)
$$

where

$$
F(x) = 2 \sqrt{1 - x^2} \left[ \frac{x}{2} - \tan^{-1} \left( \frac{x}{\sqrt{1 - x^2}} \right) \right] \quad |x| < 1
$$

$$
= -2 \sqrt{x^2 - 1} \ln \left( x + \sqrt{x^2 - 1} \right) \quad |x| > 1,
$$

and $\mu$ is the renormalization scale. The $\mu$ dependence in the one loop calculation is cancelled by counterterms that have not been included.

We are interested in how the one loop corrections affect the leading order prediction for the hyperfine splittings. Unfortunately, it is impossible to give a reliable estimate without knowing the numerical value of the counterterms required to cancel the $\mu$ dependence in
the nonanalytic contribution. Furthermore, to compute the contribution from kaon loops, one must know the masses of doubly charm strange baryons which have not been observed. We will assume that the ground state doubly charm strange baryons are 100 MeV higher in mass than their nonstrange counterparts, similar to the $D$ meson system. We work in the isospin limit and use $g = 0.6$, $\Delta_H = 140$ MeV, $m_\pi = 137$ MeV, $m_K = 496$ MeV, $m_\eta = 548$ MeV and the experimental values of the pseudoscalar meson decay constants: $f_\pi = 130$ MeV, $f_K = 159$ MeV, and $f_\eta = 156$ MeV. The nonanalytic part of the one loop correction to the nonstrange doubly charm baryon hyperfine splitting is

$$\delta m_{\Xi^*_{cc}} - \delta m_{\Xi_{cc}} = \begin{cases} 
-7.0 \text{ MeV} & \mu = 500 \text{ MeV} \\
8.1 \text{ MeV} & \mu = 1000 \text{ MeV} \\
16.9 \text{ MeV} & \mu = 1500 \text{ MeV}
\end{cases} , \quad (29)$$

where we have shown our results for three values of $\mu$. For these choices of $\mu$ the nonanalytic part of the chiral correction varies between -7 MeV and +17 MeV. The nonanalytic part of the chiral correction to the doubly charm baryon hyperfine splitting is quite sensitive to the choice of $\mu$, and lies within 15% of the tree level prediction. We also calculate the correction to the hyperfine splitting relationship of Eq. (17) and find for the masses in the nonstrange sector

$$\delta m_{\Xi^*_{cc}} - \delta m_{\Xi_{cc}} - \frac{3}{4}(\delta m_{D^*} - \delta m_D) = \begin{cases} 
3.9 \text{ MeV} & \mu = 500 \text{ MeV} \\
5.3 \text{ MeV} & \mu = 1000 \text{ MeV} \\
6.1 \text{ MeV} & \mu = 1500 \text{ MeV}
\end{cases} . \quad (30)$$

The nonanalytic correction to the symmetry prediction is small ($< 10$ MeV) and relatively insensitive to the choice of $\mu$. Chiral perturbation theory predicts the nonanalytic dependence of the doubly heavy baryon masses on the light quark masses, and generalized to include the effects of quenching as well as other lattice artifacts, formulae such as those in Eq. (26) should be useful for chiral extrapolations of doubly heavy baryon masses and hyperfine splittings in lattice simulations.

Finally, we discuss excited doubly heavy baryons. There are two types of excitations in the doubly heavy baryon system: excitations of the light degrees of freedom and excitations of the diquark. Excitations of the first type are related to analogous excitations in the heavy meson sector by heavy quark-diquark symmetry. The lowest lying excited charm mesons are in a doublet of $J^P = 0^+$ and $1^+$ mesons with masses approximately 425 MeV above
the ground state in the nonstrange sector and 350 MeV above the ground state in the strange sector. In the nonstrange sector these states decay via S-wave pion emission and have widths in the range 250-350 MeV, while in the strange sector the strong decay is via $\pi^0$ emission which violates isospin, and therefore the states are very narrow with widths expected to be of order 10 keV. These states have light degrees of freedom with angular momentum and parity $j^p = \frac{1}{2}^+$.

The doubly charm baryons related to the even-parity excited charm mesons by quark-diquark symmetry are a doublet with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$. The excited charm mesons and doubly charm baryons can be incorporated into HH$\chi$PT with a $5 \times 2$ matrix field $S_{\mu\beta}$ which is like the field $H_{\mu\beta}$ except $S_{\mu\beta}$ has opposite parity. The excitation energies and strong decay widths of these excited doubly charm baryons should be similar to their counterparts in the charm meson sector. Since the excited $\Xi^{++}_{cc}(3780)$ state observed by SELEX is only 320 MeV above the $\Xi^{++}_{cc}(3460)$, the lowest mass $\Xi^{++}_{cc}$ candidate, and its width is considerably less than 300 MeV, it does not seem likely that this excited doubly charm baryon is related to the excited charm mesons by heavy quark-diquark symmetry.

This is not unexpected as the lowest lying excited doubly charm baryons are not excitations of the light degrees of freedom but rather states in which the diquark is excited. The lowest mass excited diquark is a P-wave excitation. Because of Fermi statistics the diquark is a heavy quark spin singlet. The diquark’s orbital angular momentum couples with the angular momentum of the light degrees of freedom to form baryons with $J^P = \frac{1}{2}^+$ and $J^P = \frac{3}{2}^+$, which we will refer to as $\Xi^P_{cc}$ and $\Xi^{P*}_{cc}$, respectively. The next lowest lying states are doubly heavy baryons with a radially excited diquark, which form a heavy quark doublet with $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ baryons, which we will refer to as $\Xi'_{cc}$ and $\Xi'^*_{cc}$, respectively. If the heavy antiquarks are sufficiently heavy that the force between them is approximately Coulombic, they interact via a potential which is 1/2 as strong as the potential between the quark and antiquark in a quarkonium bound state. Therefore we expect the excitation energies of the charm diquarks to be significantly smaller than the analogous excitation energies in charmonium. Quark model calculations of excited doubly charm baryons predict that the $\Xi^P_{cc}$ and $\Xi^{P*}_{cc}$ states are about 225 MeV above the $\Xi_{cc}$ and $\Xi^{*}_{cc}$, respectively, and that the heavy quark doublet containing $\Xi'_{cc}$ and $\Xi'^*_{cc}$ is about 300 MeV above the ground state doublet. These excitation energies are about 1/2 the corresponding excitation energies in the charmonium system: $m_{\Xi_c} - m_{J/\psi} = 430$ MeV and $m_{\psi'} - m_{J/\psi} = 590$.
MeV. The charm diquark excitation energies are less than the expected excitation energy of the light degrees of freedom and therefore the lowest lying excited doubly charm baryons have excited diquarks. Excitation energies of a diquark made from two bottom quarks are similar to the excitation energies of a diquark made from charm, so the same conclusion holds for doubly bottom baryons.

The doubly heavy baryons with P-wave excited diquarks decay to the ground state via S-wave pion emission. These decays violate heavy quark spin symmetry because the total spin of the diquark is changed in the transition. The Lagrangian for the excited $\Xi^P$ and $\Xi^{P^*}$ states, including kinetic terms, residual mass terms and terms which mediate the S-wave decays, is

$$\mathcal{L} = 2 (\Xi^P_a)^\dagger (i (D_0)_{ba} - \delta_P \delta_{ab}) \Xi^P_b + 2 (\Xi^{P^*}_a)^\dagger (i (D_0)_{ba} - \delta_{P^*} \delta_{ab}) \Xi^{P^*}_b$$

$$+ 2 \lambda_{1/2} (\Xi^+_a \Xi^P_b A_{ba}^0 + h.c.) + 2 \lambda_{3/2} (\Xi^{*+}_a \Xi^{P^*}_b A_{ba}^0 + h.c.) . \tag{31}$$

The strong decay widths of the P-wave excited nonstrange doubly charm baryons are

$$\Gamma[\Xi^{P^*}_{cc} \rightarrow \Xi^{*+}_{cc} \pi] = \frac{\lambda_{3/2}^2}{2\pi f^2} \left( \frac{1}{2} E_{\pi^0}^2 p_{\pi^0} + E_{\pi^+}^2 p_{\pi^+} \right) \frac{m_{\Xi^{*+}}}{m_{\Xi^{P^*}}} = \lambda_{3/2}^2 111 \text{ MeV} \,$$

$$\Gamma[\Xi^P_{cc} \rightarrow \Xi_{cc} \pi] = \frac{\lambda_{1/2}^2}{2\pi f^2} \left( \frac{1}{2} E_{\pi^0}^2 p_{\pi^0} + E_{\pi^+}^2 p_{\pi^+} \right) \frac{m_{\Xi}}{m_{\Xi^P}} = \lambda_{1/2}^2 111 \text{ MeV} \, .$$

To obtain numerical estimates, we have assumed the masses $m_{\Xi_{cc}} = 3440$ MeV, $m_{\Xi^{*}_{cc}} = 3520$ MeV, $m_{\Xi^P_{cc}} = 3665$ MeV and $m_{\Xi^{P^*}_{cc}} = 3745$ MeV, corresponding to a diquark excitation energy of 225 MeV. We sum over both charged and neutral pion decay modes. The coupling constants $\lambda_{1/2}$ and $\lambda_{3/2}$ are $O(\Lambda_{QCD}/m_Q)$ so we should expect this suppression makes $\lambda_{1/2}$ and $\lambda_{3/2} < 1$. Therefore these states could be quite narrow despite decaying via S-wave pion emission. The small widths are due to the small excitation energy which leaves little phase space for the decay. If the excitation energy is increased to 280 MeV, the widths are twice as large. Like the isospin violating decays of the $D_s^*$ mesons and the even-parity excited $D_s$ mesons, the excited doubly heavy strange baryons below the kaon threshold decay through a virtual $\eta$ which mixes into a $\pi^0$. Denoting the ground state doubly charm strange baryons as $\Omega^{(+)}_{cc}$ and the P-wave excited doubly charm strange baryons as $\Omega^{P(+)}_{cc}$ we obtain the following formulae for the isospin violating strong decay widths

$$\Gamma[\Omega^{P^*}_{cc} \rightarrow \Omega^{*+}_{cc} \pi^0] = \frac{\lambda_{3/2}^2}{2\pi f^2} \frac{2}{3} \theta^2 E_{\pi^0}^2 p_{\pi^0}$$

$$\Gamma[\Omega^P_{cc} \rightarrow \Omega_{cc} \pi^0] = \frac{\lambda_{1/2}^2}{2\pi f^2} \frac{2}{3} \theta^2 E_{\pi^0}^2 p_{\pi^0} \, .$$
Here $\theta = 0.01$ is the $\pi^0 - \eta$ mixing angle. We expect these widths to be in the range 1-5 keV, but without knowing the masses of the $\Omega_{cc}^{(*)}$ and $\Omega_{cc}^{P(*)}$ states or the couplings $\lambda_{1/2}$ and $\lambda_{3/2}$ we cannot make more precise predictions.

The $J^P = \frac{3}{2}^-$ and $J^P = \frac{1}{2}^-$ doubly heavy baryons with radially excited diquarks are members of a heavy quark doublet we will denote $T'_a$ whose definition in terms of component fields is identical to Eq. (13). The Lagrangian describing this field, including terms which mediate its decay to the ground state, is

$$\mathcal{L} = \text{Tr}[T'^i_a ((iD_0)_{ab} - \delta_{T'} \delta_{ab}) T'_b] - \bar{g} \text{Tr}[T'^i_a T'_b \vec{\sigma} \cdot \vec{A}_{ba}] + \frac{\Delta H}{4} \text{Tr}[T'^i_a T'^i_b \sigma] - \tilde{g} \left( \text{Tr}[T'^i_a T'_b \vec{\sigma} \cdot \vec{A}_{ba}] + \text{h.c.} \right).$$

(32)

In the limit of infinite heavy quark mass, the light degrees of freedom in the radially excited doubly heavy baryons are in the same configuration as the ground state. Therefore, they are also related to the heavy meson ground state doublet by heavy quark-diquark symmetry. The axial coupling and hyperfine splitting of $T'_a$ are the same as $T_a$, as long as the spatial extent of the excited diquark, which is of order $1/(m_Q v)$, is much smaller than $1/\Lambda_{QCD}$. This is valid in the heavy quark limit, but could receive significant corrections in the charm sector. The last term in Eq. (32) mediates P-wave decays from the excited $J^P = \frac{3}{2}^-$ and $J^P = \frac{1}{2}^-$ doubly heavy baryons to the ground state. The partial decay widths are

$$\Gamma[\Xi'\to \Xi^* \pi] = C_{ab} \frac{5}{9} \frac{\tilde{g}^2}{2\pi f^2} \frac{m_{\Xi^*}}{m_{\Xi^0}} |p_\pi|^3$$
$$\Gamma[\Xi'^*\to \Xi_b \pi] = C_{ab} \frac{4}{9} \frac{\tilde{g}^2}{2\pi f^2} \frac{m_{\Xi^0}}{m_{\Xi^*}} |p_\pi|^3$$
$$\Gamma[\Xi'_a\to \Xi^* \pi] = C_{ab} \frac{8}{9} \frac{\tilde{g}^2}{2\pi f^2} \frac{m_{\Xi^*}}{m_{\Xi^0}} |p_\pi|^3$$
$$\Gamma[\Xi'_a\to \Xi_b \pi] = C_{ab} \frac{1}{9} \frac{\tilde{g}^2}{2\pi f^2} \frac{m_{\Xi^0}}{m_{\Xi^*}} |p_\pi|^3.$$  

(33)

Here $C_{ab}$ is an $SU(3)$ factor which is $1/2$ for decays involving $\pi^0$ and one for decays involving charged pions. The radially excited doubly heavy strange baryons should also be below the threshold for decays into kaons, and therefore should be quite narrow. The formulae in Eq. (33) can be used to obtain these decay widths as well. The isospin violating strong partial decay widths are obtained by using Eq. (33) with $C_{33} = \frac{2}{3}$ then multiplying by $\theta^2$. The expected widths of these states are of order 10 keV, but more precise estimates cannot be made until the masses of the states and the coupling $\tilde{g}$ are known. For the nonstrange doubly heavy baryons, in the limit of infinite heavy quark mass, we obtain

$$\Gamma[\Xi'] = \Gamma[\Xi'^*] = \frac{3\tilde{g}^2}{4\pi f^2} p_\pi^2 = 55 \text{ MeV} \left( \frac{\tilde{g}}{0.5} \right)^2 \left( \frac{p_\pi}{250 \text{ MeV}} \right)^3.$$  

(34)
for the total widths, and for the branching fractions we find

\[
\frac{\text{Br}[\Xi'^* \to \Xi^* \pi]}{\text{Br}[\Xi'^* \to \Xi \pi]} = \frac{5}{4} \quad \frac{\text{Br}[\Xi' \to \Xi^* \pi]}{\text{Br}[\Xi' \to \Xi \pi]} = 8. \tag{35}
\]

These relations receive large corrections due to phase space effects. Once the hyperfine splittings are taken into account the factors of \( p_\pi^3 \) will differ greatly for the four decays. To get a feeling for these effects in the doubly charm sector we choose \( m_{\Xi'_{cc}} = 3440 \) MeV, \( m_{\Xi'_{cc}} = 3520 \) MeV, \( m_{\Xi'_{cc}} = 3740 \) MeV, and \( m_{\Xi'_{cc}} = 3820 \) MeV, which corresponds to a diquark excitation energy of 300 MeV and hyperfine splittings of 80 MeV. We then find

\[
\Gamma[\Xi'_{cc}] = \tilde{g}^2 336 \text{ MeV} \quad \Gamma[\Xi'^{*}_{cc}] = \tilde{g}^2 78 \text{ MeV}
\]

\[
\frac{\Gamma[\Xi'^{*}_{cc} \to \Xi^*_{cc} \pi]}{\Gamma[\Xi'^{*}_{cc} \to \Xi_{cc} \pi]} = 0.56 \quad \frac{\Gamma[\Xi'^{*}_{cc} \to \Xi^*_{cc} \pi]}{\Gamma[\Xi'_{cc} \to \Xi_{cc} \pi]} = 2.3. \tag{36}
\]

Note that the \( \Xi'_{cc} \) unlike the \( \Xi'^{*}_{cc} \) strongly prefers to decay to \( \Xi^*_{cc} \) relative to \( \Xi_{cc} \) despite the phase space suppression. This may be useful for distinguishing \( \Xi'^{*}_{cc} \) and \( \Xi'_{cc} \) experimentally.

The SELEX \( \Xi^{++}_{cc}(3780) \) is broad relative to the other SELEX doubly charm candidates. Since it is 260 MeV heavier than the \( \Xi^+_{cc}(3520) \), it is a natural candidate for one of the low lying excited doubly charm baryons. Unfortunately, no measurement of the width exists and the pattern of decays is also hard to understand, since Ref. [12] states that 50% of the decays to \( \Lambda_c^+K^-\pi^+\pi^+ \) are through \( \Xi^+_{cc}(3520) \) \( \pi^+ \) while the other 50% are weak decays. More information on the quantum numbers of the \( \Xi^{++}_{cc}(3780) \) and the \( \Xi^+_{cc}(3520) \) are needed before we can determine which of the excited doubly charm baryons should be identified with the \( \Xi^{++}_{cc}(3780) \).

In this paper we have developed a generalization of HHχPT which incorporates heavy quark-diquark symmetry and includes the leading symmetry breaking corrections from the chromomagnetic couplings of the heavy quark and diquark. We also included electromagnetic interactions in the Lagrangian, and obtained an estimate of the width of the \( J = \frac{3}{2} \) member of the ground state doubly charm baryon doublet. The width of this state is completely dominated by electromagnetic decays. We showed how to include the lowest lying doubly charm baryons which are expected to be excitations of the doubly charm diquark rather than the light degrees of freedom. Strong decay widths of low lying excited states were calculated and the states are expected to be rather narrow because of limited phase space available for the decays. Of particular interest is the doubly charm strange sector where we
expect three pairs of excited baryons whose strong decay must violate isospin conservation because they are below the kaon decay threshold. These states will have narrow widths of 10 keV or less. Experimental efforts to observe the narrow doubly charm strange baryons would be of great interest.

**Acknowledgments**

This research is supported in part by DOE grants DE-FG02-96ER40945 and DE-AC05-84ER40150 and an Outstanding Junior Investigator Award.

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