Destabilising Divergences in the NMSSM

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Abstract

The problem of destabilising divergences is discussed for singlet extensions of the MSSM. It is shown that models which possess either gauged-$R$ symmetry or target space duality at the Planck scale are able to circumvent this problem whilst avoiding cosmological domain walls.
1 Introduction

There has lately been some interest in the problem of how to accommodate an extra
gauge singlet field into the minimal supersymmetry standard model (MSSM). This is the
simplest extension which is consistent with a lightest higgs boson whose mass exceeds
the upper bound found in the MSSM [1]. Previously it was thought that, by acquiring a
vacuum expectation value of $O(M_W)$, such a singlet could also provide a simple solution to
a fine-tuning problem in the MSSM, the so-called ‘$\mu$–problem’ [2, 3]. Because of difficulties
with cosmology (specifically the appearance of domain walls) this now no longer appears
to be the case [4, 5]. In fact, it was shown in ref. [5] that models with singlets are likely
to require symmetries in addition to those in the MSSM if they are to avoid problems
with either domain walls or fine-tuning. In this respect models with gauge singlets are
singularly less efficient at solving fine-tuning problems. However since they allow for more
complicated higgs phenomenology, it is still worth pursuing them. This paper concentrates
on the task of building an MSSM extended by a singlet, which avoids reintroducing the
hierarchy problem, fine-tuning, and domain walls.

Let us take as our starting point a low-energy effective theory which includes all the
fields of the MSSM, plus one additional singlet $N$. The superpotential is assumed to be
the standard MSSM Yukawa couplings plus the higgs interaction

$$W_{\text{higgs}} = \mu H_1 H_2 + \mu' N^2 + \lambda N H_1 H_2 - \frac{k}{3} N^3,$$

and the soft supersymmetry breaking terms are taken to be of the form

$$V_{\text{soft higgs}} = B \mu h_1 h_2 + B' \mu' n^2 + \lambda A_\lambda h_1 h_2 - \frac{k}{3} A_k n^3 + \text{h.c.}$$

$$+ m_1^2 |h_1|^2 + m_2^2 |h_2|^2 + m_N^2 |n|^2,$$

where throughout scalar components will be denoted by lower case letters. For the moment let us put aside the question of how the $\mu$ and $\mu'$ terms get to be so small (i.e.
$O(M_W)$ instead of $O(M_{Pl})$), and return to it later. From a low-energy point of view the
only requirement is that the additional singlet should significantly alter the higgs mass
spectrum. This means that $\lambda \neq 0$. There are four possibilities which can arise:

If all the other operators are absent, then in the low energy phenomenology there is
an apparent (anomalous) global $\tilde{U}(1)$ symmetry (orthogonal to the hypercharge), which
leads to a massless goldstone boson. Generally one expects significant complication to be
required in order that axion bounds are satisfied.

There are two cases which lead to a discrete symmetry. These are $\mu = 0, k = 0$ which leads to a $Z_2$ symmetry, and $\mu = 0, \mu' = 0$ which leads to a $Z_3$ symmetry.

The latter is usually referred to as the next-to-minimal supersymmetric standard model
(NMSSM) [3, 4], and has been the main focus of work on singlet extensions of the MSSM.

Thus the second possibility is that there is an exact discrete symmetry, and thus a domain
wall problem associated with the existence of degenerate vacua after the electroweak phase
transition. Weak scale walls cause severe cosmological problems (for example their density falls as $T^2$ whereas that of radiation falls as $T^4$ so they eventually dominate and cause
power law inflation) [3]. This is not true however, if the discrete symmetry is embedded
in a broken gauge symmetry. In this case the degenerate vacua are connected by a
gauge transformation in the full theory \[8\]. After the electroweak phase transition, one expects a network of domain walls bounded by cosmic strings to form and then collapse \[8\]. As discussed in ref.\[9\] bounds from primordial nucleosynthesis (essentially on the reheat temperature after inflation) require that the potential be very flat. In addition this mechanism depends rather strongly on the cosmology, and so models with discrete symmetry (such as the NMSSM) remain questionable.

The third possibility is that the discrete symmetry is broken \[10\] by gravitationally suppressed interactions \[7, 11\]. This was the case considered and rejected in ref.\[5\]. Here the very slight non-degeneracy in the vacua, causes the true vacuum to dominate once the typical curvature scale of the domain wall structure becomes large enough. However one must ensure that the domain walls disappear before the onset of nucleosynthesis and this means that the gravitationally suppressed terms must be of order five. It was shown in ref.\[5\] that, no matter how complicated the full theory (i.e. including gravity), there is no symmetry which can allow one of these terms, whilst forbidding the operator \(\nu N\), where \(\nu\) is an effective coupling. Furthermore, any such operator large enough to make the domain walls disappear before nucleosynthesis generates these terms at one loop anyway (with magnitude \(\sim M_W^2 M_{Pl}^2 N\)), even if they are set to zero initially. This constitutes a reintroduction of the hierarchy problem as emphasised in ref.\[12\] and as will be clarified in the following section.

The final case which is the subject of this paper, is when there is no discrete symmetry at the weak scale (exact or apparent). This is true when either \(\mu \neq 0\) or both \(\mu' \neq 0\) and \(k \neq 0\). It is well known that (as in the previous case) this type of model can lead to dangerous divergences due to the existence of tadpole diagrams. Such divergences have the potential to destroy the gauge hierarchy unless they are either fine-tuned away, or removed by some higher symmetry. In the next section the problem is quantified for the model in eq.(1), and the dangerous diagrams identified. It is also shown that normal gauge symmetries are not able to forbid these diagrams, and that they are therefore not a good candidate for the higher symmetry in question. Then in sections 2 and 3, it is shown that models which possess gauged-\(R\) symmetry and target space duality respectively, can avoid such problems. (For the reasons discussed in ref.\[13\], gauged \(R\)-symmetry \[13, 14\] might be favoured over global, although the arguments presented will apply to either case.)

# 2 The Dangerous Diagrams

In order to demonstrate which are the dangerous diagrams associated with the model of eq.(1), it is convenient to use the formalism of \(N = 1\) supergravity \[13\]. In this section the formalism will be described, and some specific examples given. Using standard power counting rules, some general observations will then be made about the divergent diagrams.

For completeness, let us first summarize the perturbation theory calculation of the offending, divergent diagrams \[13, 14\]. The lagrangian of \(N = 1\) supergravity depends only on the Kähler function,

\[
G = K(z^i, \bar{z}^i) + \ln |\hat{W}(z^i)|^2 \tag{3}
\]

where \(z^i\) is used to denote a generic chiral superfield (visible or hidden), and \(\bar{z}^i = \bar{z}^i\).
Although the holomorphic function $\hat{W}$ is referred to as the superpotential, it does not necessarily correspond to the superpotential in the low energy (i.e. softly-broken, global supersymmetry) approximation. This point will be important later; hence the hat on this superpotential. The function $K = K^\dagger$ is the Kähler potential. When supersymmetry is spontaneously broken, divergent diagrams are most efficiently calculated using the augmented perturbation theory rules described in ref. [12] which are as follows. The breaking of supersymmetry is embodied in $\theta$ and $\theta'$ dependent, classical VEVs for the chiral compensator, $\phi$, and Kähler potential which take the form

$$\phi \sim 1 + \frac{M_S^2}{M_{Pl}} \theta^2$$

$$e^{-K/3M_{Pl}^2} \sim 1 + \frac{M_S^2}{M_{Pl}} \theta^2 + \frac{M_S^2}{M_{Pl}^2} \theta^2 \theta'^2,$$

(4)

where $M_S$ is the scale of supersymmetry breaking in the hidden sector, of order $M_S^2 \sim M_W M_{Pl}$. (The precise forms, which are not important here, may be found in ref. [12].) Generally, in addition to renormalisable terms, the Kähler potential and superpotential are expected to contain an infinite number of non-renormalisable terms suppressed by powers of $M_{Pl}$. There are therefore two types of vertex which can appear in diagrams; those coming from the dimension-3, $\hat{W}$ operators of the form

$$\phi^3 \hat{W}_{ij...},$$

(5)

and those coming from dimension-2, $K$ operators, of the form

$$\phi \bar{\phi} (-3e^{-K/3M_{Pl}^2})_{ij...},$$

(6)

for a vertex with $z^i, z^j, \bar{z}^k, z^l...$ exiting. Here the indices $ijkl...$ denote covariant differentiation (with respect to Kähler transformations), so that

$$D_i \hat{W} = e^{-K/3M_{Pl}^2} \partial_i e^{K/M_{Pl}^2}$$

$$\hat{W}_{ij} = D_j \hat{W}_i - \Gamma^k_{ij} \hat{W}_k$$

(7)

where $\Gamma^k_{ij}$ is the connection of the Kähler manifold described by the metric $\partial_i \partial_j K$. In order to calculate the divergent diagrams, one may now use global superspace perturbation rules. In particular, using the standard definitions for $D_\alpha$ and $\overline{D}^{\dot{\alpha}}$ operators [13], a $K$-vertex with $m$ chiral legs and $n$ antichiral legs throws $m$ of the $-\overline{D}^2/4$ and $n$ of the $-\overline{D}^2/4$ operators onto the surrounding propagators. On the other hand a chiral vertex with $n$ chiral legs throws only $n - 1$ of the $-\overline{D}^2/4$ operators onto the surrounding propagators and similarly for antichiral with $-\overline{D}^2/4$ operators (the difference being due to the conversion of integrations to full superspace ones). The propagators are as follows [12],

$$\langle z^i \bar{z}^j \rangle = K^{ij} P_i \frac{e^{K(\theta, \bar{\theta})/3}}{\phi(\theta) \phi(\bar{\theta})} \delta^4(x - x') \delta^4(\theta - \theta')$$

$$\langle \bar{z}^i z^j \rangle = K^{ij} P_i \frac{e^{K(\theta, \bar{\theta})/3}}{\phi(\theta) \phi(\bar{\theta})} \delta^4(x - x') \delta^4(\theta - \theta'),$$

(8)
where $P_1$ and $P_2$ are the chiral and anti-chiral projection operators

\[
P_1 = \frac{D^2 D^2}{16 \Box}, \quad \quad P_2 = \frac{D^2 D^2}{16 \Box},
\]

and where

\[
\delta^4(\theta - \theta') = (\theta - \theta')^2 (\bar{\theta} - \bar{\theta})^2.
\]

Since we are only interested in determining the leading divergences, it is quite sufficient to use the massless approximation here.

This completes our review of the perturbation theory rules. Now let us consider the NMSSM, in which the renormalisable part of k"ahler potential has the canonical form,

\[
K = \frac{z_i z_j}{\delta^4} + K_{\text{non-renorm}}
\]

and the superpotential is of the following form;

\[
\hat{W}_{\text{higgs}} = \lambda N H_1 H_2 - \frac{k}{3} N^3 + \hat{W}_{\text{non-renorm}}.
\]

The extra terms, which represent possible higher order, non-renormalisable operators, are the terms which we are going to examine. As a warm-up exercise, consider the case where there are no non-renormalisable operators in $K$, and only a single non-renormalisable coupling in the superpotential of the form

\[
\hat{W}_{\text{non-renorm}} = \frac{\lambda'}{M_{\text{Pl}}} (H_1 H_2)^2.
\]

One may hope that by adding such a coupling it is possible to remove the domain walls which would otherwise form due to the global $Z_3$ symmetry apparent in the renormalisable part of eq.(11). However, as discussed in ref.3, there is no sufficiently large, non-renormalisable operator that can be added to the superpotential, which does not destabilise the gauged hierarchy. Here ‘sufficiently large’ means that the cosmological walls must disappear before the onset of primordial nucleosynthesis for which one requires $\lambda' \gtrsim 10^{-7}$.

For the operator in question, this is due to the 3-loop diagram in fig.(1), which gives rise to a contribution to the effective action of the form,

\[
\delta S = -\frac{k' \lambda^2}{M_{\text{Pl}}} \int d^4 x_1 \ldots d^4 x_4 d^4 \theta_1 \ldots d^4 \theta_4 N(x_1, \theta_1) \frac{\phi(\theta_1)}{\phi(\theta_4)} e^{K_{(12)}^2 / 3 e^{K_{(13)}^2 / 3 e^{2K_{(42)}^2 / 3 e^{2K_{(43)}^2 / 3} (14)}} \times \left( \frac{D_1^2 \delta_{12}}{4 \Box_1} \right) \left( \frac{D_2^2 \delta_{24}}{4 \Box_2} \right) \left( \frac{D_4^2 \delta_{43}}{4 \Box_4} \right) \left( \frac{D_2^2 D_4^2 \delta_{24}}{16 \Box_2} \right) \left( \frac{D_4^2 D_4^2 \delta_{43}}{16 \Box_4} \right),
\]

where $\delta_{ij} = \delta^4(x_i - x_j) \delta^4(\theta_i - \theta_j)$, and here $K_{(ij)} = K(\theta_i, \bar{\theta}_j)$.
One can evaluate this expression by integrating by parts to expose factors of $\delta^4(\theta_i - \theta_j)$ and thus eliminating $\theta$ integrals in the standard manner. Acting on the $\phi$ or $e^{K/3}$ factors always reduces the degree of divergence as is obvious from eqn.(11). Factors of $D^2\mathcal{D}^2$ may be removed using the identities,

$$
D^2\mathcal{D}^2 D^2 = 16 \Box D^2
$$

$$
\mathcal{D}^2 D^2 \mathcal{D}^2 = 16 \Box \mathcal{D}^2
$$

$$
16 = \int d^4\theta_2 \delta^4(\theta_2 - \theta_1) D^2\mathcal{D}^2 \delta^4(\theta_2 - \theta_1)
$$

$$
16 = \int d^4\theta_2 \delta^4(\theta_2 - \theta_1) D^2 \mathcal{D}^2 \delta^4(\theta_2 - \theta_1).
$$

The integral is reduced to a single integral over $\theta_1$ of the form,

$$
\delta S = \frac{-2k\lambda^2}{M_{Pl}} \int d^4x_1 \ldots d^4x_4 d^4\theta_1 N(x_1, \theta_1)e^{2K_{(11)}} \left( \frac{\delta^4x_{31}}{\Box_3} \right) \left( \frac{\delta^4x_{43}}{\Box_4} \right)^2 \left( \frac{\delta^4x_{24}}{\Box_2} \right)^2 \delta^4x_{12},
$$

where $\delta^4x_{ij} = \delta^4(x_i - x_j)$. Converting the delta functions to momentum space, one finds a contribution to the effective action of

$$
\delta S = -2k\lambda^2 \int d^4x_1 d^4\theta_1 N(x_1, \theta_1)e^{2K_{(11)}} I_3,
$$

in which $I_3$ is the quadratically divergent 3-loop integral,

$$
I_3 = \int \frac{d^4k_1 \, d^4k_2 \, d^4k_3}{(2\pi)^4 (2\pi)^4 (2\pi)^4} \frac{1}{k_1^2 k_2^2 k_3^2 (k_1 - k_2)^2 (k_1 - k_3)^2} = \mathcal{O}(M_{Pl}^2/(16\pi^2)^3),
$$

where the integral has been regularised with a cut-off of order $M_P$. Inserting the $\theta$ dependent VEVs of eqn.(11) into the above, results in terms in the effective potential of the form

$$
\delta V \approx \frac{2k\lambda^2}{(16\pi^2)^3} \left( (n + n^*) M_{Pl} M_W^2 + (F_N + F_N^*) M_{Pl} M_W \right)
$$

in which $I_3$ is the quadratically divergent 3-loop integral.
which clearly destabilises the hierarchy unless $\lambda'$ is sufficiently small, so small in fact that it is unable to remove the cosmological domain walls before the onset of nucleosynthesis. The non-renormalisable term in eq. (13), is (to leading order in $M_{Pl}^{-1}$) equivalent to adding instead the term

$$K_{\text{non-renorm}} = -\frac{\lambda'}{\lambda} \left( \frac{N^\dagger H_1 H_2 + \text{h.c.}}{M_{Pl}} \right) - \frac{k\lambda'}{\lambda^2} \left( \frac{N^\dagger H_1 H_1^\dagger + \text{h.c.}}{M_{Pl}} \right),$$

in the Kähler potential. This may be seen by making the redefinitions

$$N \rightarrow N - \frac{\lambda' N H_1}{\lambda M_{Pl}},$$

$$H_1 \rightarrow H_1 - \frac{\lambda' k N H_1}{\lambda^2 M_{Pl}}.$$  \hspace{1cm} (21)

This provides a useful check of the perturbation theory rules. The divergent diagrams in the redefined model are of the form shown in fig. (2), where black vertices are chiral and white ones come from the $K_{\text{non-renorm}}$ terms in the Kähler potential.

The 1-loop divergent contributions were shown by Jain in ref. [12] to cancel unless the trilinear terms couple directly to hidden sector fields. This result can easily be recovered here, since the diagram gives

$$\delta S = \frac{M_{Pl}}{2(16\pi^2)} \int d^4x_1 d^4\theta_1 K_{N H_1 H_1} K^{H_1 H_1} N(x_1, \theta_1) + \text{h.c.}$$

where we have approximated

$$\int \frac{d^4k_1}{(2\pi)^4} \frac{1}{k_1^2} = O(M_{Pl}^2/(16\pi^2)).$$

Without any direct coupling between $H_1$ and a hidden sector field, the VEVs of eq. (11) do not appear, and the diagram does not give dangerous terms. The 2-loop contributions are easily found to cancel amongst themselves. With a little effort the remaining divergences can also be shown to cancel except the single (Mercedes) diagram of fig. (3).
The contribution of this diagram to the effective action is,

$$\delta S = \frac{-k\lambda \chi^2}{M_{Pl}} \int d^4x_1 \ldots d^4x_5 d^4\theta_1 \ldots d^4\theta_5 N(x_1, \theta_1) \frac{\phi(\theta_1)}{\phi(\theta_5)}$$

$$\times e^{K(12)/3} e^{K(13)/3} e^{K(42)/3} e^{K(43)/3} e^{K(45)/3} e^{K(52)/3} e^{K(53)/3} \left( \frac{D_1^2 \delta_{12}}{4 \Box_1} \right) \left( \frac{D_2^2 \delta_{25}}{16 \Box_2} \right)$$

$$\times \left( \frac{D_4^2 \delta_{53}}{4 \Box_4} \right) \left( \frac{D_3^2 \delta_{31}}{4 \Box_3} \right) \left( \frac{D_2^2 D_3^2 \delta_{24}}{16 \Box_2} \right) \left( \frac{D_4^2 D_4^2 \delta_{43}}{16 \Box_4} \right) \left( \frac{D_4^2 \delta_{54}}{4 \Box_5} \right).$$

(24)

By integrating by parts with $D_4^2$, $D_5^2$, and $D_5^2$, and using the rules in eqn.(13), the last factor becomes simply $\delta_{54}$. The $\langle 45 \rangle$ propagator effectively collapses and the integral over $(x_5, \theta_5)$ results in eqn.(14) as required. (Again, when evaluating the leading divergences, one may ignore $D^2$ operators acting on $\phi$ and $e^{K/3}$.)

Having gained some confidence in calculation of divergences, we can now go on to systematically consider the other operators which may appear in $\tilde{W}$ or $K$. In order to determine exactly which ones are dangerous, let us first restrict our attention to operators in $W_{\text{non-renorm}}$. Obviously the degree of fine-tuning decreases with higher order since each loop gives a factor $\Lambda^2/(16\pi^2)$ where $\Lambda$ is a cut-off, and involves more Yukawa couplings. It therefore seems reasonable to disregard contributions which are higher than six-loop since they are unable to destabilise the hierarchy. Up to and including six loop, the following operators are potentially dangerous if they appear in the superpotential (multiplied by any function of hidden sector fields), since one can write down a tadpole diagram using them (together with the trilinear operators of the NMSSM);
The corresponding tadpole diagrams for each operator are shown in fig.(4a-h). (Figure (4c) is the diagram which was evaluated above.) Notice that, since the leading divergences involve chiral or antichiral vertices only, an operator must break the $Z_3$ symmetry in $\hat{W}$ in order for it to be dangerous (so that for example $N^2(H_1H_2)^2$ does not destabilise the hierarchy). The first two operators are the exception in this list, since one cannot say with certainty whether or not their contributions to the effective potential will be dangerous. This depends on how the couplings $\mu$ or $\mu'$ are generated. Specifically, the diagram in fig.(4a) generates logarithmically divergent terms of the form

$$\delta V = \frac{\log \Lambda^2}{32\pi^2} \int d^4\theta F^\dagger \log F_i \frac{\lambda^i N^+}{M_{Pl}} \frac{m\phi^m}{m\phi^m} F_i \sim \left(\frac{M_{Pl}}{M_W}\right)^{1/m} M_W F_N^\dagger. \tag{25}$$

These are the divergent terms which lead to logarithmic running of the soft-breaking scalar masses. However, if there is a $\mu$-term produced directly in the superpotential from some product of hidden sector fields ($\mu = \Phi^m / M_{Pl}^{m-1}$ for example), the contribution above includes

$$\frac{\log \Lambda^2}{32\pi^2} \int d^4\theta \mu(\Phi) \lambda^i N^+ = \frac{\log \Lambda^2}{32\pi^2} \lambda^i F_N^\dagger \frac{m\phi^m}{m\phi^m} F_i \sim \left(\frac{M_{Pl}}{M_W}\right)^{1/m} M_W F_N^\dagger. \tag{26}$$

where $\Phi$ is a hidden sector field, one can assume that $F_\Phi \sim M_W M_{Pl}$, and that also $\langle|\phi|^m\rangle \sim M_WM_{Pl}^{m-1}$ in order to get $\mu \sim M_W$. This leads to a value of $F_N \gg M_W$ unless $m$ is extremely large, destabilising the gauge hierarchy. If $\mu$ is generated in the visible sector on the other hand, it may be possible to avoid this conclusion. In this sense such terms have the same status as the trilinear couplings in the Kähler potential which were discussed above.

It has already been demonstrated that the next three operators will lead to dangerous divergences and must be forbidden. Not all of the remaining operators are dangerous however. Consider for instance adding a dimension-7 operator to the superpotential;

$$\hat{W}_{\text{non-renorm}} = \frac{\lambda'}{M_{Pl}^4} N^7. \tag{27}$$

In this case the (Garfield) diagram of fig.(4e) looks potentially dangerous, since it also appears to be a divergent tadpole contribution. Its contribution to the effective action is

$$\delta S = \frac{k^2\lambda'}{18 M_{Pl}^4} \int d^4x_1 d^4x_2 d^4x_3 d^4\theta_1 d^4\theta_2 d^4\theta_3 N(x_1, \theta_1) \frac{1}{\phi(\theta_1)^3} e^{K_{(12)}} e^{K_{(13)}} \times \left(\frac{D_{\theta_2}^2 D_{\theta_3}^2 \delta_2}{16 \Box_2}\right)^2 \left(\frac{D_{\theta_1}^2 D_{\theta_3}^2 \delta_3}{16 \Box_3}\right)^2 \left(\frac{-D_{\theta_1}^2 \delta_1}{4 \Box_3}\right). \tag{28}$$

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1I would like to thank G. G. Ross for pointing this out.
Again by integrating by parts with $\overline{D}_2^2$ and $\overline{D}_3^2$ one can extract the leading term, but this time, one is forced to act at least once upon the $e^K$ factors, because in total there is an odd number of $D^2$ and $\overline{D}^2$ operators. The result is

$$\delta S = \frac{k^2 \lambda'}{18 M_{Pl}^4} \int d^4 x_1 d^4 \theta_1 N(x_1, \theta_1) \frac{1}{\phi(\theta_1)^3} \left( -\frac{D^2}{4} e^{2K_{(11)}} \right) I_4,$$

in which $I_4$ is the quartically divergent 4-loop integral,

$$I_4 = \int \frac{d^4 k_1}{(2\pi)^4} \frac{d^4 k_2}{(2\pi)^4} \frac{d^4 k_3}{(2\pi)^4} \frac{d^4 k_4}{(2\pi)^4} \frac{1}{k_1^2 k_2^2 k_3^2 k_4^2 (k_1 - k_2)^2 (k_3 - k_4)^2} = \mathcal{O}(M_{Pl}^4/(16\pi^2)^4).$$

The final contribution to the effective potential is not harmful to the gauge hierarchy;

$$\delta V \approx -\frac{k^2 \lambda'}{9(16\pi^2)^4} \left( (F_N + F_N^*) M_W^2 + (n + n^*) M_W^2 \right).$$

This is clearly the case whenever the total number of $D^2$ and $\overline{D}^2$ operators is odd. This fact leads one quite easily to the chief result of this section, which is that, for the model

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**Figure 4:** Tadpole diagrams for non-renormalisable operators in $\hat{W}$ up to 6-loop.
of eqn.(12), any extra odd-dimension operators in \( \hat{W} \) or even-dimension operators in \( K \) are not harmful to the gauge hierarchy.

This may be deduced by first generalising the supergraph, power counting rules. Let there be \( V_d \) superpotential vertices of dimension \( d + 3 \) (that is of the form \( z^{d+3}/M_{\text{Pl}}^d \)), and \( U_d \) Kähler potential vertices of dimension \( d + 2 \) (of the form \( z^{d+2}/M_{\text{Pl}}^d \)). To the divergence, a propagator counts as \( 1/p^2 \), a \( V_d \) vertex as \( p^{d+2} \) (from the \( D^2 \) factors on its legs), a \( U_d \) vertex as \( p^{d+2} \), and each loop variable as \( p^2 \). In addition each external chiral leg removes a \( D^2 \) operator of the vertex, effectively contributing \( 1/p \). Hence the total degree of divergence is

\[
D = 2L - 2P - E_c + \sum_d V_d(d + 2) + \sum_d U_d(d + 2),
\]

(32)

where \( L \) is the number of loops, \( P \) is the number of propagators, and \( E_c \) is the number of external chiral legs. There are two useful relations; the first is

\[
2P + E_c = \sum_d V_d(d + 3) + \sum_d U_d(d + 2),
\]

(33)

the right hand side being simply the number of external legs when there are no propagators; the second arises from counting the internal momentum variables, one of which is removed by each vertex delta function,

\[
P - L = \sum_d V_d + \sum_d U_d - 1.
\]

(34)

Substituting these gives the following value for the divergence

\[
D = 2 - E_c + \sum_d V_d + \sum_d U_d.
\]

(35)

The actual contribution to the effective potential is therefore of the form

\[
\delta V \sim \frac{\Lambda^{2-E_c+\sum_d V_d+\sum_d U_d}}{M_{\text{Pl}}^{\sum_d V_d+\sum_d U_d}} \sim M_{\text{Pl}}^{2-E_c}.
\]

(36)

This is the result of ref.\([13, 12]\), which says that in \( N = 1 \) supergravity, apart from a quadratic vacuum term, the only divergent contribution to the effective potential is linear in fields (\( E_c = 1 \)). Now consider the total number, \( N_{D^2} \), of \( D^2 \) and \( D^2 \) operators. There are \( d + 2 \) from every vertex, \(-1\) from every external chiral line, and \( 2 \) on every propagator, giving

\[
N_{D^2} = 2P - E_c + \sum_d V_d(d + 2) + \sum_d U_d(d + 2)
\]

(37)

in total. In order for a diagram to be harmful, this number must be even, and hence when \( E_c = 1 \),

\[
\sum_d V_d d + \sum_d U_d d = \text{odd}.
\]

(38)

This can only be satisfied if there is at least one vertex which has an odd \( d \), thus proving the statement above. (Substituting eq.(33) shows that this also means the total number of chiral and antichiral vertices is even.)
The relatively restrictive constraint that the superpotential be a holomorphic function means that there are now only 13 dangerous operators in \( \hat{W} \). The Kähler potential is restricted only by the condition, \( K = K^\dagger \) however. Apart from the trilinear operators (which as we have seen above only destabilise the gauge hierarchy if they directly couple visible and hidden sector fields), there is a much larger number of higher dimension operators which must be forbidden here. For example the operator,

\[
K_{\text{non-renorm}} = \lambda' N^\dagger N(H_1 H_2)
\]  

leads to the diagram in Fig.(5), whose contribution to the effective action is

\[
\delta S \approx -\frac{M_P k \lambda'}{18(16\pi^2)^4} \int d^4x_1 d^4\theta_1 N(x_1, \theta_1) \frac{\phi(\theta_1)}{\phi(\bar{\theta}_1)} e^{5K_{(11)/3}},
\]

which again gives \( n \) a VEV of \( O(10^{11} \text{ GeV}) \). Clearly any odd-dimension operator which breaks the \( Z_3 \) symmetry of eq.(13) may appear in \( \hat{K} \) and will destroy the gauge hierarchy if it does so.

Hence a particularly attractive way to ensure a model with singlets which is natural, is to devise a symmetry which forbids odd-dimension terms in \( K \), and even-dimension terms in \( \hat{W} \). This is the approach taken in the next two sections. (A possible alternative which will not be considered here is to include an extra symmetry in the visible sector, which ensures these couplings are always suppressed by some field whose VEV is extremely small.)

To finish this section, let us recapitulate the arguments of ref.[3] which make it clear that such a symmetry cannot be a normal gauge symmetry. For simplicity, take this to be a \( U(1)_X \) symmetry (the extension to non-abelian cases is trivial), and let the \( Z_3 \) symmetry be broken by a \( H_1 H_2 \) or \( N^2 \) term in \( K \). Such couplings provide naturally small \( \mu \sim M_W \) or \( \mu' \sim M_W \) in the effective low energy global superpotential \( W \). The other effective couplings at the weak scale are in general arbitrary functions of hidden sector
fields which carry charge under the new $U(1)_X$ which shall be referred to collectively as $\Phi$ (with $\xi = \Phi/M_{Pl}$). It is simple to see that one cannot use this symmetry to forbid terms linear in $N$. If $\mu(\xi) \neq 0$ then $\mu(\xi)$ must have the same charge as $\lambda(\xi)N$ and therefore $(\mu(\xi))^\dagger \lambda(\xi)N$ is uncharged. If both $\mu' \neq 0$ and $k \neq 0$ then $\mu'(\xi)$ must have the same charge as $k(\xi)N$ and therefore $(\mu'(\xi))^\dagger k(\xi)N$ is uncharged. Once such a linear operator has been constructed, it is of course trivial to construct all the other dangerous operators.

One should bear in mind that if one sets these couplings to zero by hand in the first place, they remain small to higher order in perturbation theory. So this is merely a fine-tuning problem. One might also argue that the nature of this fine-tuning problem is different from that of the $\mu$-problem, since in the latter the coupling has to be very small, whereas here the couplings may just happen to be absent (as for example are superpotential mass terms in string theory). However, the extremely large number of dangerous operators makes this fine tuning problem a particularly serious one. In the next two sections, two examples are presented which are able to avoid this problem.

## 3 Models with $R$-symmetry

The reason that it has not been possible to forbid divergent tadpole diagrams in the models that have been discussed here and in ref.[5], is that the Kähler potential and superpotential have the same charges (i.e. zero). There are however two available symmetries in which the Kähler and superpotentials transform differently. These may accommodate singlet extensions to the MSSM simply and without fine-tuning.

The first is gauged $U(1)_R$-symmetry [13, 14]. In this case the Kähler potential has zero $R$-charge, but the superpotential has $R$-charge 2. This means that the standard renormalisable NMSSM higgs superpotential,

$$\hat{W}_{\text{higgs}} = \lambda N H_1 H_2 - \frac{k}{3} N^3,$$

has the correct $R$-charge if $R(N) = 2/3$ and $R(H_1) + R(H_2) = 4/3$. So consider the Kähler potential

$$\mathcal{G} = y_i y^i + \Phi \overline{\Phi} + \left( \frac{\alpha}{M_{Pl}^2} \Phi H_1 H_2 + \frac{\alpha'}{M_{Pl}^2} \Phi N^2 + \text{h.c.} \right) + \log |\hat{W} + g(\Phi)|^2,$$

where $y_i$ are the visible sector fields and where $\Phi$ represents a hidden sector field with superpotential $g(\Phi)$ which acquires a VEV of $O(M_{Pl})$. (It may represent arbitrary functions of hidden sector fields in what follows). This next-to-minimal choice of Kähler potential is the one proposed in ref.[3] which leads to naturally small $\mu$ and $\mu'$ couplings in the low energy (global supersymmetry) approximation $\hat{W}$. Specifically, the terms which arise in the scalar potential are [3, 4]

$$V_{\text{scalar}} = W_i W^i + m^2 y_i y^i + m \left[ y^i W_i + (A - 3)\hat{W} + (B - 2)m \mu H_1 H_2 + (B - 2)m \mu' N^2 + \text{h.c.} \right],$$

where $W$ are the trilinear terms of the superpotential $\hat{W}$, rescaled according to

$$\hat{W} = \langle \exp (\Phi \overline{\Phi}/2M_{Pl}^2) \rangle \hat{W}.$$
Here $W$ is the new low energy superpotential including the $\mu$ and $\mu'$ terms,

$$W = \hat{W} + \mu H_1 H_2 + \mu' N^2,$$

(45)

and $m$ is the gravitino mass

$$m = \langle \exp (\Phi / 2M_{Pl}^2)g^{(2)} \rangle,$$

(46)

where $g^{(2)}$ are the quadratic terms in $g$, and where the VEV of $g^{(2)} = M_S^2 / M_{Pl}$ is set by hand such that $M_s \sim 10^{11}$ GeV. Applying the constraint of vanishing cosmological constant, one finds that the universal trilinear scalar coupling, $A = \sqrt{3} \langle \Phi / M_{Pl} \rangle$, and that the bilinear couplings are given by,

$$B = \frac{(2A - 3)/(A - 3)}{|\mu| = \frac{m\alpha(A - 3)}{\sqrt{3}}},$$

$$|\mu'| = \frac{m\alpha'(A - 3)}{\sqrt{3}}.$$  

(47)

All dimensionful parameters at low energy are of order $M_W$.

Invariance of the Kähler potential requires that $R(\Phi) = -4/3$. It is easy to see that with this set of $R$-charges there can never be odd-dimension operators in $K$, or even-dimension ones in $\hat{W}$. Indeed the operators which can appear in the superpotential can be written as,

$$\hat{O}_c = \frac{\Phi^c y^{(d+3)}}{M_{Pl}^2 M_{Pl}^2},$$

(48)

where $y$ stands for any of the visible sector fields. In order to have $R$-charge 2, they must satisfy

$$\frac{2(d + 3)}{3} - \frac{4c}{3} = 2$$

(49)

or $d = 2c$. Hence only odd-dimension operators are allowed in $\hat{W}$. The operators which can appear in the Kähler potential are of the form

$$\hat{O}_{abc} = \frac{(\Phi \Phi)^b \Phi^c}{M_{Pl}^2 M_{Pl}^2} (y y^\dagger)^a y^{(d+2-2a)} \frac{M_{Pl}^d}{M_{Pl}^d},$$

(50)

where negative $c$ can be taken to represent powers of $\Phi$. The condition $R = 0$ becomes,

$$d = 2(a + c - 1),$$

(51)

so that only even-dimension operators may appear in $K$ as required. In a fully viable model, one would also have to take account of anomalies in the $R$ symmetry which can usually be cancelled if there are enough hidden sector singlets $\mathbf{13}$. This will not be considered here.
4 Models with Duality Symmetry

The second symmetry one can use to forbid terms linear in $N$ is target space duality in a string effective action. Generally, these have flat directions, some of which correspond to moduli determining the size and shape of the compactified space. Furthermore these moduli have discrete duality symmetries, which at certain points of enhanced symmetry become continuous gauge symmetries [16].

In Calabi-Yau models, abelian orbifolds and fermionic strings the moduli include three Kähler class moduli ($T$-type) which are always present, plus the possible deformations of the complex structure ($U$-type), all of which are gauge singlets. Additionally there will generally be complex Wilson line fields [17, 18]. When the latter acquire a vacuum expectation value they result in the breaking of gauge symmetries. There has been continued interest in string effective actions since they may induce the higgs $\mu$-term [3, 18, 19, 20], be able to explain the Yukawa structure [21, 22], and be able to explain the smallness of the cosmological constant in a no-scale fashion [21, 23]. Since the main objective here is simply to find a route to a viable low energy model with visible higgs singlets, these questions will only be partially addressed.

Typically the moduli and matter fields describe a space whose local structure is given by a direct product of $SU(n, m)/SU(n) \times SU(m)$ and $SO(n, m)/SO(n) \times SO(m)$ factors [17, 18]. As an example consider the Kähler potential derived in refs. [18], which at the tree level is of the form

$$K = - \log(S + \overline{S}) - \log[(T + \overline{T})(U + \overline{U}) - \frac{1}{2}(\Phi_1 + \overline{\Phi}_2)(\Phi_2 + \overline{\Phi}_1)] + \ldots$$

The $S$ superfield is the dilaton/axion chiral multiplet, and the ellipsis stands for terms involving the matter fields. The fields $\Phi_1$ and $\Phi_2$ are two Wilson line moduli. As in ref. [3, 18, 19, 24], let us identify these fields with the neutral components of the higgs doublets in order to provide a $\mu$-term. Problems such as how the dilaton acquires a VEV, or the eventual mechanism which seeds supersymmetry breaking will not be addressed here.

The moduli space is given locally by

$$\mathcal{K}_0 = \frac{SU(1, 1)}{U(1)} \times \frac{SO(2, 4)}{SO(2) \times SO(4)},$$

which ensures the vanishing of the scalar potential at least at the tree level, provided that the $S$, $T$ and $U$ fields all participate in supersymmetry breaking (i.e. $G_S, G_T, G_U \neq 0$). In fact writing the Kähler function as

$$G = K(z_i, z^i) + \ln |\hat{W}(z_i)|^2,$$

the scalar potential becomes

$$\hat{V}_s = -e^G \left(3 - G_i G^\overline{j} G_{\overline{j}}\right) + \frac{g^2}{2} \text{Re}(G^i T^A j z_j)(G^k T^A t z_t),$$

where $G_i = \partial G/\partial z_i$, and $G^\overline{j} = (G_{\overline{j}i})^{-1}$. The dilaton contribution separates, and gives $G_S G^S G_S = 1$. To show that the remaining contribution is 2, it is simplest to define the vector

$$A^\alpha = a(t, u, h, \overline{h})$$

15
where the components are defined as $\alpha = (1 \ldots 4) \equiv (T, U, \Phi_1, \Phi_2)$, and $u = U + \mathcal{T}$, $t = T + \mathcal{T}$, $h = \Phi_1 + \Phi_2$. It is easy to show that

$$G_\alpha A^\alpha = -2a. \tag{57}$$

The vector $A^\alpha$ is designed so that $G_\beta A^\alpha$ is proportional to $G_\beta$, viz,

$$G_\beta A^\alpha = -aG_\beta. \tag{58}$$

Multiplying both sides by $G_\alpha G^\alpha G$ gives the desired result, i.e. that $G_\alpha G^\alpha G = 2$. Thus, if the VEVs of the matter fields are zero, the potential vanishes and is flat for all values of the moduli $T$ and $U$, along the direction $\langle |\Phi_1| \rangle = \langle |\Phi_2| \rangle = \rho_\phi$ (since this is the direction in which the $D$-terms vanish). The gravitino mass is therefore undetermined at tree level, being given by

$$m^2 = \langle e^G \rangle = \frac{|\hat{W}|^2}{s(ut - 2\rho_\phi^2)}. \tag{59}$$

In addition to the properties described above, there is an $O(2, 4, Z)$ duality corresponding to automorphisms of the compactification lattice [10, 18]. This constrains the possible form of the superpotential. The $PSL(2, Z)_T$ subgroup implies invariance under the transformations [10, 18],

$$
\begin{align*}
T &\rightarrow \frac{aT - ib}{icT + d} \\
U &\rightarrow U - \frac{ic}{2} \frac{\Phi_1 \Phi_2}{icT + d} \\
z_i &\rightarrow z_i (icT + d)^{n_i}.
\end{align*}
\tag{60}
$$

where $a, b, c, d \in Z$, $ad - bc = 1$, and where $z_i$ stands for general matter superfields with weight $n_i$ under the modular transformation above. The $\Phi_1$ and $\Phi_2$ fields have modular weight $-1$. It is easy to verify the invariance of the Kähler function under this transformation provided that

$$\hat{W} \rightarrow (icT + d)^{-1}\hat{W}. \tag{61}$$

The superpotential should be defined to be consistent with this requirement in addition to charge invariance, and this leads to a constraint on the modular weights of the Yukawa couplings and matter fields. (Anomalies occur here also, and must be cancelled in addition to the gauge anomalies. Again this is considered to be beyond the scope of the present paper.)

One may now easily find examples where this symmetry is able by itself, to forbid dangerous operators. Consider the NMSSM superpotential of eqn.(12). Identifying $\Phi_1$ and $\Phi_2$ with the higgs superfields $H_1$ and $H_2$ (in order to generate a $\mu H_1 H_2$ term in the low energy superpotential $W$) means that both of these fields have weight $-1$. Since the superpotential must transform as in eq.(61), the other weights must obey the following:

$$
\begin{align*}
3n_N + n_k &= -1 \\
n_N + n_\lambda &= +1.
\end{align*}
\tag{62}
$$

Since the Yukawa couplings are functions of the moduli, they too can carry weight under the transformation in eqn.(60).
One simple solution which forbids dangerous divergences is \( n_N = -1 \) and \( n_k = n_\lambda = +2 \). In this case it is obvious that (since the visible fields all have weight \(-1\)) even operators may be avoided in \( \hat{W} \). As for the Kähler potential, one expects the terms in \( K_{\text{non-renorm}} \) to be multiplied by powers of \((T+\bar{T})\). Thus terms in which the holomorphic and anti-holomorphic weights are the same may be allowed. Since all the weights are \(-1\), this can obviously only be achieved for operators which have an even number of fields.

There are clearly many ways in which one could devise similar models. A perhaps more obvious example would be models in which the superpotential transforms with weight \(-3\). There all the physical fields could be given weight \(-1\), with the couplings having weight 0. It is then clear that only trilinear couplings can exist in the superpotential, and only even-dimension terms can appear in the Kähler potential.

5 Conclusions

The problem of destabilising divergences in models which extend the MSSM with a singlet field has been discussed. In this paper the case where there is no discrete or global symmetry at the weak scale has been examined, and the dangerously divergent tadpole diagrams have been identified. In particular it was shown that half of the possible operators (i.e. those with odd-dimension in the superpotential \( \hat{W} \), or even-dimension in the Kähler potential) are perfectly harmless in the sense that they do not destroy the gauged hierarchy. Thus an attractive possibility for extending the higgs sector with a singlet is to generate the \( \mu \) term from couplings in the Kähler potential. Two examples were demonstrated in which all operators which are dangerous to the gauge hierarchy are forbidden. In order to achieve this, they had to incorporate either a gauged \( R \)-symmetry or a target space duality symmetry in the full theory including gravity. These models clearly satisfy all constraints from fine-tuning, primordial nucleosynthesis and cosmological domain walls. Since they have no discrete or continuous global symmetries in the weak scale effective theories, one expects all possible couplings (i.e. \( \mu H_1H_2, \mu N^2, \lambda NH_1H_2 \) and \( kN^3 \)) to be present. The phenomenological implications of these more general cases, have been discussed recently in ref.[24].

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