Field-controlled randomness of colloidal paths on a magnetic bubble lattice

C Jungnickel\(^1\), Z Khattari\(^2\), T H Johansen\(^3\) and Th M Fischer\(^1,4\)

\(^1\) Institut für Experimentalphysik V, Universität Bayreuth, 95440 Bayreuth, Germany
\(^2\) Department of Physics, Hashemite University, Zarqa, Jordan
\(^3\) Department of Physics, The University of Oslo, Oslo, Norway
E-mail: christiane.jungnickel@uni-bayreuth.de, zkhattari@hu.edu.jo
and thomas.fischer@uni-bayreuth.de

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**Abstract.** Paramagnetic colloidal particles move in the potential energy landscape of a magnetically modulated bubble lattice of a magnetic garnet film. The modulation causes the energy minima to alternate between positions above the centres of the bubbles and interstitial positions. The particles deterministically follow the time-dependent positions of the energy minima until the minima become unstable in one or several directions and allow the particles to hop to a new minimum. We control the time delay between instabilities of the minima in alternative directions by the angle of the external magnetic field with the crystallographic directions of the bubble lattice. When the time delay is large, the particles deterministically hop to the next minimum along the direction that becomes unstable first. When the time delay is short, diffusion of the particle in the marginal potential randomizes the choice of the hopping directions or the choice of the transport network. Gradual changes of the external field direction from 0° to 30° lead to a continuous crossover from a deterministic to a fully stochastic path of the colloids.

\(^4\) Author to whom any correspondence should be addressed.
1. Introduction

The application of external fields periodically varying in both space and time can be used to probe transport properties near critical points. They have been used in superconductors to understand the dynamic transition between a vortex liquid and a vortex glass phase [1, 2]. Stochastic resonance of activation processes in classical metastable systems in the presence of periodic driving has been shown to exhibit interesting transport phenomena [3]. External drive has also been used for the dispersion-free transport of particles [4, 5]. In these systems, the particles are forced to constantly relax towards the instantaneous local equilibrium position and if the field is strong, then such fields may wipe out any randomness occurring due to weaker thermal fluctuations. Of particular interest are fields that vary in more than one spatial dimension. In these systems, a series of locked transport states occur as the angle at which the particles are driven with respect to the periodicity of the field is varied [6]. In the locked states, the particles move along a high-symmetry direction of the lattice even when this is not the same as the direction of the drive. The locking produces sharp transitions in the transport direction response of the particles as a function of the driving angle that acts as the control parameter of the transition.

The deterministic nature of such transport processes is in sharp contrast to the motion of particles in one-dimensional (1D) ratchet potentials [7]–[9], where the strength of time-dependent periodic potentials only overcomes the thermal energy for some fraction of the modulation period. The transport in thermal ratchet potentials in contrast to deterministic ratchets is therefore accompanied by a dispersion of the speed of transport. The combination of thermal ratchets with deterministic spatiotemporal potentials further enriches the variety of transport phenomena. The transport in such systems can be deterministic in one but random in a second parameter. Here we report on a periodic 2D and time-dependent potential that is free from dispersion of the speed of the particles but where the direction of the motion or the transport network is dispersed. Thermal effects smear out either the sharp transition between the high-symmetry transport directions or the sharp transitions between alternative transport networks and we may continuously vary from a deterministic to a fully randomized motion by varying the driving angle. The external control of the paths of the particles might be useful for directing molecular cargo attached to the particles in a laboratory on a chip device.

2. The experimental setup

The realization of our system consists of a magnetic garnet film magnetized normal to the film of saturation magnetization $M_S = 10^4$ A m$^{-1}$ showing a hexagonal magnetic bubble array pattern...
Figure 1. Scheme of a magnetic bubble lattice of lattice spacing $d$ and bubble size $R$. The external magnetic field is applied at an angle $\vartheta$ with the film normal and an azimuth angle $\varphi$ with the (10) direction. Three unit cells showing the location of the bubbles, the two types of interstitials and the two types of networks $B I$ (green) and $B \bar{I}$ (red) are shown on the left. Paramagnetic particles are placed above the garnet film that is covered with water.

(figure 1) with a lattice constant of $d = 13.5 \mu m$. Paramagnetic colloids (dynabeads) of diameter $2a = 2.8 \mu m$ and magnetic susceptibility $\chi = 0.17$ with a carboxylate functionalization are dispersed in an aqueous solution placed above the magnetic garnet film. The magnetic garnet film was covered with polystyrene sulfonate (PSS), a negatively charged polyelectrolyte. The PSS repels the negatively charged particles and therefore stops the sedimentation of the beads a few nanometres prior to contact. The gravitational and electrostatic potential arising from this functionalization is rather stiff such that the particles are strongly confined in the direction normal to the film. In this way, one avoids adhesion of the beads to the garnet film surface and the beads remain mobile in the plane parallel to the garnet film.

The regular bubble array leads to a heterogeneous magnetic field in the region of the sedimented colloids above the garnet film. The magnetic field of a single bubble exhibits a logarithmic singularity right at the domain wall if the domain wall is infinitely sharp. For finite width it still exhibits a maximum close to the wall that becomes indistinguishable from the logarithmic singularity as one moves away from the wall a distance greater than the domain width. The colloidal particles sense the magnetic field at an elevation of roughly a bead radius above the garnet film. The heterogeneities of the magnetic field close to the wall fall off as one moves away from the film. At the elevation of the beads, details of the domain wall structure have already decayed but the smeared out heterogeneities from the bubbles and the continuous phase of opposite magnetization remain.

An external magnetic field normal to the film pointing along or in the opposite direction of the bubble magnetization will strengthen or weaken the total field above the bubbles. The paramagnetic beads are attracted towards the strongest magnetic field at the elevation determined by the balance of gravitational and electrostatic interactions. They will prefer a position above the magnetic bubbles for an external field parallel to the bubble magnetization. An antiparallel field causes them to jump to an interstitial position between three bubbles. An alternating homogeneous external magnetic field hence leads to consecutive jumps from...
a bubble position towards an interstitial position and vice versa. There are two triplets of interstitial positions \( I \) and \( \bar{I} \) near a bubble. Each interstitial position of one of the triplets corresponds to three equivalent positions separated by a lattice vector. There are three bubbles next to each interstitial position, and connecting the different interstitial positions with its neighbouring bubbles creates two different connected networks (the \( BI \) and the \( B\bar{I} \) network). The symmetry of the system gives the particle six equivalent choices of direction when jumping from a bubble towards an interstitial in the first half-cycle of the modulation. Three of these choices belong to pathways on the first network and the other half to those on the second network. From the interstitials there are only three equivalent choices when hopping towards a bubble in the second half-cycle of the modulation since only the network serving these interstitials can be used.

3. External path selection

Superposition of an external field modulation parallel to the film breaks the symmetry and this may be used to bias the hopping into a desired direction and to bias the use of the two networks. Here we study the effect of the orientation of this parallel field on the statistic or deterministic nature of the hopping pathway. We will show that thermal diffusion occurring during the short time where a choice between competing directions has to be made enables us to continuously vary between a deterministic and a random choice between the competing alternatives, resulting in a tuneable dispersion of hopping directions and a tuneable dispersion on the networks at a dispersion-free speed of the particles. The control parameter for these dispersions is the angle of the symmetry breaking in plane modulation orientation with respect to the magnetic bubble lattice.

We describe the geometry of our bubble lattice within a Cartesian coordinate system with the \( x \)-axis aligned with one of the bubble lattice crystallographic axes (the 10 direction) in the plane of the garnet film, the \( y \)-axis along the \([-21] \) direction in the film and the \( z \)-axis normal to the film. The lattice vectors are thus given by \( \mathbf{a}_1 = d \mathbf{e}_x \) and \( \mathbf{a}_2 = \frac{d}{2} \mathbf{e}_x + \frac{\sqrt{3}}{2} \mathbf{e}_y \). We apply a magnetic external field modulation of the form \( \mathbf{H}^\text{ext}(t) = \mathbf{H} \sin \omega t [\sin \vartheta (\cos \varphi \mathbf{e}_x + \sin \varphi \mathbf{e}_y) + \cos \vartheta \mathbf{e}_z] \). Two sets of parameters were used in the experiment. The field amplitude was \( \mathbf{H} = 810 \) A m\(^{-1}\), the tilt of the field was \( \vartheta = 12^\circ \) and the angular frequency was \( \omega = 2\pi \nu = 3.76(18.9) \) s\(^{-1}\), respectively.

Figure 2 shows a trajectory of a particle for a tilt azimuth of \( \varphi = 2^\circ \) and for \( \varphi = 88^\circ \). Both trajectories follow along a path passing from a bubble domain towards an interstitial position back to a bubble domain and so forth. However, the trajectory for \( \varphi = 2^\circ \) is completely deterministic with the hopping during the next period following the same path shifted by the lattice vector \( \mathbf{a}_1 \). The position of the bead after \( n \) periods \( /n/ \approx 2\pi /\omega \) can be described simply by \( \mathbf{x}(t + n/\nu) = \mathbf{x}(t) + n \mathbf{a}_1 \). The trajectory for \( \varphi = 88^\circ \), on the other hand, is a random path with the first hop from the bubble to the interstitial always occurring in the same \((-12) \) direction and the second part of the hopping from the interstitial position to the next bubble either occurring along the (11) or the \((-21) \) direction. We may describe the possible trajectories of the particles by measuring the probabilities of hopping in the various directions.

These probabilities are \( p_{11}^{B\rightarrow I} \), \( p_{12}^{B\rightarrow I} \), \( p_{21}^{B\rightarrow I} \), \( p_{11}^{B\rightarrow I} \), \( p_{12}^{B\rightarrow I} \), \( p_{21}^{B\rightarrow I} \), \( p_{11}^{I\rightarrow B} \), \( p_{12}^{I\rightarrow B} \), \( p_{21}^{I\rightarrow B} \) for the hops from the bubble towards an interstitial position of type \( I \) or \( \bar{I} \) and \( p_{11}^{I\rightarrow B} \), \( p_{12}^{I\rightarrow B} \), \( p_{21}^{I\rightarrow B} \), \( p_{11}^{I\rightarrow B} \), \( p_{12}^{I\rightarrow B} \), \( p_{21}^{I\rightarrow B} \) for the hops from the interstitials of type \( I \) and \( \bar{I} \) to the bubbles. We have
Figure 2. Two typical trajectories of paramagnetic beads above the bubble lattice. (a) The external field (blue) is tilted with an azimuth $2^\circ$ off the $(10)$ direction and the trajectory follows a deterministic path but randomly using the $B I$ (green) and $B \bar{I}$ network (red). The random choice of the network is made in the first half-cycle of the modulation. (b) The external field (blue) is tilted with an azimuth $2^\circ$ off the $(-12)$ direction and the trajectory follows a random path along the $B \bar{I}$ network (red). Directions from the interstitials towards the bubbles that would have been alternative choices during the second half-cycle of the modulation are indicated as yellow arrows. The sector accessible with this mode of transportation is shaded in grey.

measured those probabilities as a function of the tilt azimuth $\varphi$ by counting the number of hops along the different directions for a fixed value of $\varphi$. Typical numbers of counts for fixed $\varphi$ were $N \approx 100$ events. In figure 3, we show those measured probabilities plotted as a function of $\varphi$.

Figure 3(a) shows the probabilities of hopping from a bubble to an interstitial during the first half-cycle. For all the directions of the external magnetic field that are significantly closer to one of the interstitial directions the hopping is deterministically oriented towards this closest interstitial. Only for external fields oriented in a direction between two interstitials is there a choice for the paramagnetic beads to hop to either of the two different types of interstitials. The random choice in the first half-cycle allows the particles to use either network; however, it does not result in a dispersion of the global direction of transport, because the consecutive probabilities of hopping from both interstitials towards the next bubble in the second half-cycle (figures 3(b) and (c)) are both deterministic for this external field orientation and the paramagnetic particles are forwarded to the same bubble. The alternative trajectories that move along different networks rejoin after one period. The transport mode randomly accesses both networks but there is no dispersion of the global transport direction.

A global dispersion of the transport direction occurs when the external magnetic field points from a bubble to an interstitial. During the first half-cycle the particle deterministically hops from a bubble to the interstitial that lies in just this direction. The following hop from interstitial to a bubble during the second half-cycle, however, leaves the particle two choices that results in trajectories moving along different bubbles. The ensemble of possible trajectories lies
Figure 3. Polar plot of the hopping probabilities of the individual hops of the particles (a) from bubble to interstitial, (b) from interstitial of type \( I \) towards a bubble and (c) from interstitial of type \( \bar{I} \) towards a bubble, plotted as a function of the tilt azimuth \( \phi \) of the external magnetic field.

in a 60° sector (shaded grey in figure 2) between the two crystallographic directions enclosing the direction of the external field. Figure 4 shows the probabilities of hopping directions from one bubble to a neighbouring bubble during a full period of the field modulation. These probabilities result from the probabilities of the individual hops occurring during each half-cycle by considering the hopping via the two interstitials that are common neighbours to the start and target bubbles.

\[
P_{10}^{B ightarrow B} = p_{11}^{B ightarrow I} p_{2-1}^{I ightarrow B} + p_{2-1}^{B ightarrow \bar{I}} p_{11}^{\bar{I} ightarrow B},
\]

\[
P_{01}^{B ightarrow B} = p_{12}^{B ightarrow I} p_{11}^{I ightarrow B} + p_{11}^{B ightarrow \bar{I}} p_{12}^{\bar{I} ightarrow B},
\]

\[
P_{11}^{B ightarrow B} = p_{21}^{B ightarrow I} p_{12}^{I ightarrow B} + p_{12}^{B ightarrow \bar{I}} p_{21}^{\bar{I} ightarrow B},
\]

\[
P_{10}^{B ightarrow B} = p_{11}^{B ightarrow I} p_{12}^{I ightarrow B} + p_{21}^{B ightarrow \bar{I}} p_{12}^{\bar{I} ightarrow B},
\]

\[
P_{0-1}^{B ightarrow B} = p_{1-2}^{B ightarrow I} p_{1-1}^{I ightarrow B} + p_{1-1}^{B ightarrow \bar{I}} p_{1-2}^{\bar{I} ightarrow B},
\]

\[
P_{1-1}^{B ightarrow B} = p_{2-1}^{B ightarrow I} p_{1-2}^{I ightarrow B} + p_{1-2}^{B ightarrow \bar{I}} p_{2-1}^{\bar{I} ightarrow B}.
\]
The hopping during one period is deterministic if the field is not pointing in a direction just between two crystallographic directions. There are, hence, two types of degeneracies occurring in the transport. For $\phi = 2n\pi/6$ the external magnetic field is oriented along a crystallographic axis. There is network dispersion. The global transport direction is fixed, but the particles have the choice to use either the $BI$ or the $B\bar{I}$ network for the transport. For $\phi = (2n + 1)\pi/6$ the external magnetic field is oriented between two crystallographic axes. There is transport direction dispersion. The particles are confined to use only one of the networks, but have a choice of direction. For tilt azimuth directions pointing in a non-degenerate direction $\phi \neq n\pi/6$, there is neither a choice of transport direction nor a choice of network.

4. Dynamic broadening

We performed experiments with modulation frequencies of $f = 0.6$ and 3 Hz. Figure 5 shows the hopping probabilities $p_{2\rightarrow 1}^B$ on the $BI$ network during the second half-cycle in the transition region $\phi \approx 30^\circ$. Both frequencies give essentially the same results; however, the azimuthal width $\Delta \phi$ of the transition region is slightly larger for higher modulation frequencies, $\Delta \phi (3 \text{ Hz}) \approx 20^\circ$, than for the lower frequencies, $\Delta \phi (0.6 \text{ Hz}) \approx 13^\circ$.

5. Theoretical description

Theoretically we may describe the hopping by using the equation

$$- f \eta a \ddot{x} - \nabla V(x(t), t) + \zeta(t) = 0,$$

where $\eta = 10^{-3} \text{ N s m}^{-2}$ denotes the viscosity of water, $f = 20$ is the friction coefficient of a bead near the surface of the garnet film [10],

$$V(x, t) = -\chi \frac{4\pi}{3} a^3 \mu_0 H^2(x, t)$$
Figure 5. The hopping probabilities in the second cycle for two different modulation frequencies plotted in the transition region around $\phi \approx 30^\circ$. The lines are least square fits with the equation $p = (1 - \tanh [(\phi - \phi_c)/\Delta \phi])/2$ and serve as guides for the eye. The data for $\nu = 3$ Hz exhibit a somewhat broader transition region than the data for $\nu = 0.6$ Hz.

is the magnetic potential energy of the bead in the magnetic field with $\mu_0 = 4\pi \times 10^{-7}$ N A$^{-2}$ being the vacuum permeability and $\zeta(t)$ is a thermal random force satisfying the fluctuation dissipation theorem

$$\langle \zeta(t_1)\zeta(t_2) \rangle = 2f \eta k_B T \delta(t_1 - t_2)\mathbf{I},$$

with $k_B$ being Boltzmann’s constant, $T = 300$ K the temperature, $\delta(t)$ the Dirac delta function and $\mathbf{I}$ the unit tensor in 2D. The magnetic energy in (2) is much larger than the thermal energy and for low frequencies of the magnetic field modulation, the relaxation time into the magnetic minimum energy position is generically much shorter than the modulation period. In those situations, we may simplify equation (1) to

$$\nabla V(x_s(t), t) = 0$$

for most of the time. Hence, the particle will adiabatically move within the energy minimum $x_s(t)$ defined by the superposition of the heterogeneous field of the garnet film and the external field. This confinement of the particle to the energy minimum renders the dynamics completely deterministic. Moreover, a periodic modulation of the field will lead to a periodic closed path the stationary point of the potential energy traces with time. Indeed, if the amplitude of the magnetic field is small, no transport of particles is observed and the particles just follow the closed trace of the minimum.

A new situation arises if the nature of the stationary point $x_s(t)$ defined by equation (4) changes from a minimum to a saddle point. This change requires the amplitude of the external field to surmount a threshold. The critical time $t_c$ at which one reaches this threshold is defined by

$$\nabla V(x_c(t_c), t_c) = 0 \quad \text{and} \quad \xi_c \cdot \nabla V(x_c(t_c), t_c) = 0.$$
At the time $t_c$ the minimum changes to a saddle point and one may lower the energy of the particle by a displacement in either the $+\xi_c$ or the $-\xi_c$ direction depending on the sign of $(\xi_c \cdot \nabla)^3 V$. The confinement of the particle to the minimum does not exist any longer. Near the saddle point and for a short time around the critical time $t_c$, we may approximate equation (1) along the $+\xi_c$ direction by the 1D equation

$$-f\eta a\dot{x} - \partial_t \partial_\xi V \big|_{x(t_c), t_c} (t - t_c) + \xi(t) = 0.$$  \hspace{1cm} (6)

The solution of this equation is straightforward and using the fluctuation dissipation theorem (3), we find the mean square displacement of the particle from the saddle point as a function of the time $\Delta t = t - t_c$ to be given by

$$\langle x^2(t) \rangle = \frac{1}{4} \left( \frac{\partial_t \partial_\xi V \big|_{x(t_c), t_c}}{\eta f a} \right)^2 \Delta t^4 + 2k_B T \eta f a \Delta t.$$  \hspace{1cm} (7)

The mean square displacement crosses over from a diffusive motion ($\Delta t < \tau_D$) towards an accelerated motion $\Delta t > \tau_D$ at a crossover time

$$\tau_D = \left( \frac{8\eta f a k_B T}{\left( \partial_t \partial_\xi V \big|_{x(t_c), t_c} \right)^2} \right)^{1/3}. \hspace{1cm} (8)$$

Hence the particle has a typical time of the order of $\tau_D$ to randomly move in the saddle point, before the increasing slope of the potential forces it to follow the path of steepest descent towards the next minimum at the location $x_{s2}(t_{c1})$. The time needed for hopping from the saddle point to the next minimum is of the order of

$$\tau_{\text{hop}} = \frac{\eta f a s^2}{V(x_{c1}(t_{c1}), t_{c1}) - V(x_{s2}(t_{c1}), t_{c1})}. \hspace{1cm} (9)$$

Here, $s$ is the length of the path from $x_{c1}(t_{c1})$ to $x_{s2}(t_{c1})$, neglecting the shift of the second minimum from $x_{s2}(t_{c1})$ to $x_{s2}(t_{c1} + \tau_{\text{hop}})$.

We have shown that the motion of the particle can be divided into three sequences: an adiabatic motion in the local minimum, a random diffusion when the minimum converts to a saddle point and a hopping along the path of steepest descent. The first and the third sequence are sufficient to explain the occurrence of directed transport. If the modulation of the potential is performed in the right manner, one may arrange that each local minimum will develop into a saddle point with the unstable direction pointing in the desired transport direction. The random diffusion during the instability remains without consequence.

A new quality is added when the minimum becomes unstable with respect to two directions at the same time. Generically, such a simultaneous instability will not occur by chance but only if symmetry considerations require degeneracy in two directions. In our system, the potential becomes degenerate in two directions when the tilt azimuth of the external magnetic field is pointing in the direction between two optional minima. In such a situation, the particle may choose between the two directions and the probability for one direction is exactly $p = 1/2$. If the instability in one direction precedes the second direction, a deterministic description of the motion would predict the particle to follow whatever direction becomes unstable first. A deviation of the tilt azimuth from a symmetry direction in such a system would therefore lock the motion in the first unstable direction. This is the series of locked transport states predicted.
by Reichhardt and Reichhardt [6] as a function of the driving angle. The sharp transition from one to the other transport direction is however smeared out by the crossover time \( \tau_D \). Suppose the direction \( \xi_{c1} \) becomes unstable at the time \( t_{c1} \) as expressed in equation (5). We might find that the curvature of the same unstable point becomes unstable with respect to a second direction \( \xi_{c2} \) at the later time \( t_{c2} \)

\[
\xi_{c2} \cdot \nabla V(x_{c1}(t_{c1}), t_{c2}) = 0.
\]

A simple way of estimating the distribution of the particles in the different directions is to say that the particle randomly chooses a time \( t \) in the interval \( t_{c1} < t < t_{c1} + \tau_D \) to make a decision upon its future path. If this decision time \( t \) precedes \( t < t_{c2} \), it will follow the direction \( \xi_{c1} \). If the time falls between \( t_{c2} < t < t_{c1} + \tau_D \) it will go in either direction with the same probability. This leads to a probability of hopping in directions 1 and 2 given by

\[
P_{1/2} = \frac{1}{2} \pm \frac{t_{c2} - t_{c1}}{2\tau_D}.
\]

The time difference \( t_{c2} - t_{c1} \) linearly depends on the deviation of the tilt azimuth from the symmetry direction \( \phi - \phi_c \propto \omega(t_{c2} - t_{c1}) \) such that we expect a linear variation of the probability around the locking transition. Hence, the width \( \Delta \phi \) of the transition region is expected to scale as

\[
\Delta \phi \propto \omega \tau_D \propto \omega^{1/3}.
\]

The widths of the transition regions in figure 5 are consistent with such scaling.

The magnetic field above the garnet film satisfies the static Maxwell equations \( \nabla \times \mathbf{H} = 0 \) and \( \nabla \cdot \mathbf{H} = 0 \) subject to the boundary condition \( H_z(x, y, z = +0) = M_s(x, y, z = -0) \) and \( H(x, y, z \to \infty) = H^{\text{ext}}(t) \). We may construct the solution from a superposition of the fundamental solution of a single bubble shifted by the lattice vector \( \mathbf{a}_{n,m} = n \mathbf{a}_1 + m \mathbf{a}_2 \)

\[
\mathbf{H} = H^{\text{ext}}(t) + \sum_{n,m} \mathbf{H}_{\text{bubble}}(x - \mathbf{a}_{n,m}, R),
\]

where

\[
H^\text{bubble}_{z}(\mathbf{x}, R) = \frac{M}{\pi} \left[ \frac{\left(\sqrt{\rho^2 + z^2} - R\right) \left(\sqrt{\rho^2 + z^2} - \rho\right)}{z \sqrt{(\rho + R)^2 + z^2}} \Pi \left(\frac{2\rho}{\rho + \sqrt{\rho^2 + z^2}}, \frac{4\rho R}{(\rho + R)^2 + z^2}\right) - \frac{\left(\sqrt{\rho^2 + z^2} + R\right) \left(\sqrt{\rho^2 + z^2} + \rho\right)}{z \sqrt{(\rho + R)^2 + z^2}} \Pi \left(\frac{2\rho}{\rho - \sqrt{\rho^2 + z^2}}, \frac{4\rho R}{(\rho + R)^2 + z^2}\right) \right],
\]

\[
H^\text{bubble}_{\rho}(\mathbf{x}, R) = \frac{M}{\pi} \sqrt{\frac{R}{\rho}} Q_{1/2}(\rho^2 + R^2 + z^2)
\]

is the field of one bubble of magnetization \( M \) in cylindrical coordinates, \( \rho = \sqrt{x^2 + y^2} \), \( \Pi \) is the complete elliptic integral of the third kind and \( Q \) is a Legendre function of the second kind. The sum in (13) extends over all lattice vectors. In the numerical treatment, we truncate the sum and treat the region excluded by the sum by a homogeneous mean magnetization using a muffin tin approximation [11].

In figure 6, we show a sequence of contour plots of the magnetic energy of a paramagnetic particle subject to an external field with azimuth \( \varphi = 210^\circ \) and \( \varphi = 205^\circ \) during a modulation.
Figure 6. Contour plots of the magnetostatic energy of a paramagnetic particle above the magnetic bubble lattice as a function of time $t$ (in units of the period $T$) for an external field with tilt azimuth $\varphi = 210^\circ$ and $\varphi = 205^\circ$ showing one modulation cycle. The times when hopping occurs (green arrows) are shown at higher resolution and the sequence is not spaced equally in time. Locations of the bubbles are marked in blue, interstitials $I$ in green and interstitials $\bar{I}$ in red. Differences in the energy landscape between the tilt azimuth $\varphi = 210^\circ$ and $\varphi = 205^\circ$ are significant only during the hop ($0.75 < t < 0.8$) from the interstitial $\bar{I}$ towards the bubble $B$. Details are explained in the text.

From the contour plot, we find a force rate

$$\partial_t \partial_x V |_{r_0(\varphi), t_c} = 0.4 \left( \frac{4\pi}{3d} \omega x a^3 \mu_0 M_s^2 \right) \approx 28 \text{ pN s}^{-1}$$
at $\omega = 2\pi v = 3.7$ s$^{-1}$, which corresponds to a crossover time of $\tau_D = 3.4$ ms (equation (8)). The delay between the instabilities in the $-21$ and $1-2$ directions is $t_2 - t_1 = \frac{0.01}{\varphi} \approx 2.6$ ms for $\varphi = 205^\circ$, which is of the same order as $\tau_D$. This confirms the explanation of the width of the transition region (equation (12)).

6. Conclusions

In summary, we have presented a 2D model system, where the deviation of the external driving field from a symmetry direction serves as a control parameter driving the transition from one transport direction to the next or a transition of the access of networks. A small control parameter in conjunction with a high frequency of the drive causes a time delay between two instabilities that is sufficiently small to allow a broadening of the transition via the random diffusion of the particles during that time. The azimuthal width of the transition therefore scales with the driving frequency to the power of one-third. In the adiabatic limit $\omega \to 0$, the transition becomes sharp. This allows us to tune the randomness of the transport process with the tilt azimuth of the driving field. Such tuning might be useful in the design of smart lab-on-a-chip devices.

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