Abstract

We show that the boundary state description of a Dp-brane is strictly related to the corresponding classical solution of the low-energy string effective action. By projecting the boundary state on the massless states of the closed string we obtain the tension, the R-R charge and the large distance behavior of the classical solution. We discuss both the case of a single D-brane and that of bound states of two D-branes. We also show that in the R-R sector the boundary state, written in a picture which treats asymmetrically the left and right components, directly yields the R-R gauge potentials.
1 Introduction

Classical solutions of various low-energy string actions carrying a non-vanishing charge with respect to some \((p + 1)\)-form field have been intensively studied in the last few years. The simplest of them are discussed in detail in Ref. [1] where one can also find the references to the original papers. The classical solutions having a non-vanishing electric or magnetic charge under the Neveu-Schwarz–Neveu-Schwarz (NS-NS) 2-form correspond, respectively, to the fundamental string and the solitonic 5-brane, or, in the dual formulation, to the solitonic string and the fundamental 5-brane. On the contrary, the classical solutions with a non-vanishing charge under the various \((p + 1)\)-forms of the Ramond-Ramond (R-R) sector do not have any relation to the perturbative closed string or its solitons. In fact, as it has been recognized by Polchinski [2], these solutions which are required by various string dualities, correspond to membranes on which open strings can end with Dirichlet boundary conditions in the transverse directions and the usual Neumann boundary conditions in the longitudinal directions. For this reason they are called Dirichlet branes (D-branes). The properties of D-branes\(^1\) can be discussed either by studying their interaction with open strings\(^2\) or, more efficiently, by introducing the so-called boundary state\(^2\).

The boundary state is a BRST invariant state written in terms of the closed string oscillators which basically identifies the left and the right sector of the closed strings attached to it. It contains the couplings of the D-brane with all the states of the closed string spectrum and, in particular, as we show in Sections 2 and 4, it can be used to compute the coupling with massless states, providing an independent calculation of the D-brane tension and R-R charge\(^2\).

In this paper we show that the boundary state can also be used to efficiently extract target-space information, such as the value of background fields corresponding to the classical description of a D-brane. We show that the boundary state formalism greatly simplifies the calculation of the long range fields produced by a flat membrane presented in Refs. [5, 12] where the string scattering from a D-brane was studied in detail and then, in the low energy limit, the various amplitudes were factorized in a pure tree graph and in a source term generated by the presence of the D-brane. However, this factorization can be performed once for all at string level and, indeed, it can be seen as the very definition of the boundary state\(^2\). Then the long-distance behavior of the various fields present in the low energy string action can be deduced by studying the emission of massless string states from the boundary state. This shows that there is a very direct connection between the classical D-brane solution and the corresponding boundary state describing its quantum properties. While for the NS-NS sector the calculations in the case of superstring are very similar to the ones performed for the bosonic string, in the R-R sector we see that the expression for the boundary state has a particularly simple expression.

\(^1\)For a review of their properties see Ref. [3].
\(^2\)For a pre-D-brane discussion of the boundary state see Refs. [10, 11].
if it is written in an unusual picture which treats asymmetrically the left and the right sector. The same analysis can be performed also in the case of D-brane bound states, again finding complete agreement with the classical solutions.

The paper is organized as follows. In Section 2, in order to avoid all the technicalities of the superstring, which at first sight may obscure the otherwise very clear physical picture, we concentrate on the bosonic string. In this theory we write the boundary state for a Dp-brane and show how to extract from it the brane tension that appears in the effective description provided by the Born-Infeld action. Then, after a short discussion of the classical D-brane solution, we derive its large-distance behavior and compare it with what we get from the boundary state by studying the emission of the graviton, dilaton and antisymmetric Kalb-Ramond field. As expected, we find complete agreement between the two approaches provided the space-time dimension is ten. This forces us to consider the superstring case. In Section 3 we write the boundary state for the NS-NS sector of the superstring theory and show a complete agreement between the large-distance behavior of the classical solution and that obtained from the boundary state. In Section 4 we write the boundary state for the R-R sector of superstring and show that, when it is saturated with states in the corresponding asymmetric picture, there is complete agreement between the long-distance behavior of the classical solution and the boundary state results, also in the case of the $(p+1)$-form potential. Finally, in Section 5 we turn to D-brane bound states with two non-vanishing $p$-form fields whose dimensions differ by two. They can be described by a boundary state containing a non trivial background gauge field. Again we find a complete agreement between the large-distance behavior obtained from the boundary state and that following from the low energy solutions described in Ref. [13]. Some technical details as well as our conventions and notations for spinors in the R-R sector are contained in the Appendix.

2 D-brane tension and large-distance behavior

A Dp-brane can be conveniently described in terms of a string world-sheet with a boundary on which $d - p - 1$ coordinates satisfy Dirichlet boundary conditions, $d$ being the space-time dimension. In this framework a very useful object is the boundary state $|B\rangle$ which is a BRST invariant state of the closed string that inserts a boundary on the world-sheet and enforces on it the appropriate boundary conditions. It was originally introduced in order to factorize open string loop diagrams in terms of the closed string states $|\Omega\rangle$, and its explicit form can be obtained either by solving overlap equations between the left and right fields induced by the presence of a boundary $|\Gamma\rangle$, or by factorizing amplitudes of closed strings emitted from a disk $|\Delta\rangle$.

In order to see in the most transparent way how the boundary state describes a
Dp-brane, we begin with the bosonic string and, for the sake of simplicity, understand the ghost degrees of freedom. The boundary state for a D-brane located at $y$ is

$$|B\rangle = \mathcal{N} \delta^{(d_\perp)}(q - y) \exp \left[ - \sum_{n=1}^{\infty} a_n^{\dagger} S_{\mu\nu} \tilde{a}_n^{\dagger} \right] |0; k = 0\rangle,$$  

(2.1)

where the $\delta$-function is only over the $d_\perp = d - p - 1$ directions transverse to the brane, and $a_n$ ($\tilde{a}_n$) are the left (right) moving modes of the string coordinates. The matrix

$$S_{\mu\nu} = (\eta_{\alpha\beta}, -\delta_{ij}) = 2 (\eta_{\alpha\beta}, 0) - \eta_{\mu\nu}$$  

(2.2)

is obtained from the metric $\eta_{\mu\nu}$ by performing a T-duality transformation on the last $d_\perp$ directions to enforce the Dirichlet boundary conditions appropriate for a Dp-brane. The normalization constant $\mathcal{N}$ was derived in Ref. [8] from the factorization of amplitudes of closed strings emitted from a disk in the case where each target-space direction was compactified on a circle of radius $R$. In the decompactification limit $R \to \infty$, the boundary state normalization can be written as

$$\mathcal{N} = \frac{4}{\alpha'} \sqrt{\frac{W \alpha'}{8\pi V}} \left( \frac{2\pi \sqrt{2\alpha'}}{2\alpha'} \right)^{-d/2} (2\pi R)^{d_\perp},$$  

(2.3)

where $W$ is the normalization of the open string momentum eigenstates

$$W = (2\pi R)^{p+1} \left( \frac{2\pi \alpha'}{R} \right)^{d_\perp},$$  

(2.4)

while $V$ is the space-time volume $V = (2\pi R)^d$. Note however, that the factorization procedure naturally leads to a boundary state containing the closed string propagator

$$D_a = \frac{\alpha'}{4\pi} \int_{|z|\leq 1} \frac{d^2z}{|z|^2} z^{L_0-a} \bar{z}^{\bar{L}_0-a}.$$  

(2.5)

with intercept $a = 1$. Thus the expression derived in Ref. [8] is slightly different from Eq. (2.1), since the latter does not contain the closed string propagator (2.5). For future use, it is convenient to rewrite the normalization factor $\mathcal{N}$ in the following form

$$\mathcal{N}' = \sqrt{\frac{2}{\alpha' \pi}} \left( \frac{2\pi \sqrt{2\alpha'}}{\alpha'} \right)^{-d/2} \left( \frac{2\pi \sqrt{\alpha'}}{\alpha'} \right)^{d_\perp}.$$  

(2.6)

The first two factors correspond to the normalization in the pure Neumann case, while the last one provides a factor of $(2\pi \sqrt{\alpha'})$ for each transverse component of the $\delta$-function present in the boundary state in Eq. (2.1). This observation will be used later on when we construct the boundary state for a non localized D-brane.

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$^3$Throughout this paper, the indices $\alpha$ and $\beta$ denote the $p + 1$ longitudinal directions of the Dp-brane, while the indices $i, j$ denote the transverse directions.
Using Eq. (2.6), we can rewrite the boundary state $|B\rangle$ as

$$|B\rangle = \frac{\hat{T}_p}{2} \delta^{(d-1)}(q - y) \exp \left[ -\sum_{n=1}^{\infty} a_n^{\mu*} S_{\mu\nu} \tilde{a}_{\nu n}^{*} \right] |0; k = 0\rangle \quad ,$$  

(2.7)

where

$$\hat{T}_p = \frac{\sqrt{\pi}}{2(d-10)/4} \left( 4\pi^2 \alpha' \right)^{(d-2p-4)/4} \quad .$$  

(2.8)

If the matrix element $\langle B|D_{a=1}|B\rangle$ is computed, one obtains the same result derived by Polchinski [2] using the open string formalism for the interaction between two D-branes. Thus, $\hat{T}_p$ can be identified with the tension of the D$p$-brane. That this identification is correct can also be seen in a more direct way. In fact, the boundary state (2.7) is the generator of the interaction amplitudes of any closed string state with a D-brane. These amplitudes can be simply computed by saturating $|B\rangle$ with the corresponding normalized closed string states. In particular, the coupling between a level 1 state of momentum $k$ and a D-brane is given by

$$A^{\mu\nu} = \langle 0; k|a_1^\mu \tilde{a}_1^\nu|B\rangle = -\frac{\hat{T}_p}{2} V_{p+1} S^{\mu\nu} \quad ,$$  

(2.9)

where $V_{p+1}$ is the world-volume of the D-brane. The explicit expression of the emission amplitude for a graviton and a dilaton can then be obtained by saturating $A^{\mu\nu}$ with the polarization tensors

$$\epsilon^{(h)}_{\mu\nu} = \epsilon^{(h)}_{\nu\mu} \quad , \quad \epsilon^{(h)}_{\mu\nu} \eta^{\mu\nu} = \epsilon^{(h)}_{\mu\nu} k^{\mu} = 0 \quad ,$$

$$\epsilon^{(\phi)}_{\mu\nu} = \frac{1}{\sqrt{d-2}} [\eta_{\mu\nu} - k_\mu \ell_\nu - k_\nu \ell_\mu] \quad , \quad k \cdot \ell = 1, \quad \ell^2 = 0 \quad ;$$  

(2.10)

thus, using Eq. (2.2) and the fact that the outgoing state has non-vanishing momentum only along the transverse directions, one obtains

$$A_{grav} = A^{\mu\nu} \epsilon^{(h)}_{\mu\nu} = -\hat{T}_p V_{p+1} \eta^{\alpha\beta} \epsilon^{(h)}_{\alpha\beta} \quad ,$$

$$A_{dil} = A^{\mu\nu} \epsilon^{(\phi)}_{\mu\nu} = \hat{T}_p V_{p+1} \frac{d - 2p - 4}{2\sqrt{d-2}} \quad .$$  

(2.11)

Since $A^{\mu\nu}$ in Eq. (2.9) is symmetric, one can immediately deduce that the coupling of the brane with the antisymmetric Kalb-Ramond tensor $B_{\mu\nu}$ is vanishing. On the other hand, the interaction between the massless fields and a D-brane is described by the Born-Infeld action

$$S_{BI} = -\frac{T_2}{\kappa} \int d^{p+1} \xi \ e^{-\phi} \sqrt{-\det [G_{\alpha\beta} + B_{\alpha\beta} + 2\pi \alpha' F_{\alpha\beta}]} \quad .$$  

(2.12)

We normalize each component of the momentum eigenstates such that $\langle k|k\rangle = 2\pi \delta(k - k')$ and take $(2\pi)^d \delta^{(d)}(0) = V$.

$^5\kappa$ is related to Newton’s constant by $\kappa^2 = 8\pi G_N$. 

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where $T_p$ is, by definition, the D-brane tension and $F$ is the field strength of an external gauge field. Notice that the action (2.12) is written using the string metric $G$, and should be considered together with the gravitational bulk action in the string frame

$$S_{\text{bulk}} = \frac{1}{2\kappa^2} \int d^d x \sqrt{-\hat{G}} \ e^{-2\phi} \left[ R(\hat{G}) + 4(\nabla \phi)^2 - \frac{1}{12}(dB)^2 \right]. \quad (2.13)$$

This action, however, leads to a mixed propagator between the dilaton and the graviton. To remove this mixed term, one has to go to the Einstein frame and then rescale the dilaton field so that its kinetic term is correctly normalized. This can be done by writing

$$G_{\mu\nu} = e^{4\phi/(d-2)} \eta_{\mu\nu}, \quad \phi = \kappa \frac{\sqrt{d-2}}{2} \chi. \quad (2.14)$$

Using these definitions in Eq. (2.12) and putting for simplicity $B = F = 0$, we get

$$S_{\text{BI}} = -\frac{T_p}{\kappa} \int d^{d+1} \xi \ e^{-\kappa \chi(d-2p-4)/(2\sqrt{d-2})} \sqrt{-\det [g_{\alpha\beta}]} \ . \quad (2.15)$$

If we consider the Dp-brane in the static gauge where $x^\alpha \equiv \xi^\alpha$, and expand the metric around the Minkowski background $g_{\mu\nu} = \eta_{\mu\nu} + 2\kappa h_{\mu\nu}$, it is not difficult to derive that the couplings of the D-brane with the graviton and the dilaton are

$$A_{\text{grav}} = -T_p V_{p+1} \eta^{\alpha\beta} \epsilon_{\alpha\beta}^{(h)} \ , \quad A_{\text{dil}} = T_p V_{p+1} \frac{d-2p-4}{2\sqrt{d-2}}. \quad (2.16)$$

Comparing Eqs. (2.11) and (2.16), we deduce that $T_p = \hat{T}_p$ in agreement with the result obtained by Polchinski [2]. Notice that our calculation is shorter than his because we know the precise normalization of the boundary state and do not need to insert the coupling just derived into a tree diagram with the propagator of the massless states in order to compare with the string calculation.

We now prove that the boundary state contains another fundamental information about the D-brane, namely the long-distance behavior of the massless fields of the D-brane solution of the string effective action. Before showing this, we recall that an electric Dp-brane is a solution of the field equations derived from

$$S = \frac{1}{2\kappa^2} \int d^d x \sqrt{-\hat{g}} \left[ \hat{R}(\hat{g}) - \frac{\hat{\gamma}}{2} (\nabla \hat{\phi})^2 - \frac{1}{2(n+1)!} e^{-\hat{a}\hat{\phi}} \left( \hat{d}C(n) \right)^2 \right], \quad (2.17)$$

where $n = p + 1$ and $\hat{\gamma} = 2/(d-2)$, which corresponds to the ansatz

$$d\hat{s}^2 = [H(x)]^{2a} (\eta_{\alpha\beta} dy^\alpha dy^\beta) + [H(x)]^{2b} (\delta_{ij} dx^i dx^j) \ , \quad (2.18)$$

for the metric $\hat{g}$, and

$$e^{-\hat{\phi}(x)} = [H(x)]^7, \quad \hat{C}_{01...p}(x) = \pm \sqrt{2\sigma[H(x)]^{-1}} \ , \quad (2.19)$$
for the dilaton $\hat{\phi}$ and for the $(p+1)$-form potential $\hat{C}$ respectively. (The two signs in $\hat{C}$ correspond to the brane and the anti-brane case.) If the parameters are chosen as

$$a = \frac{-d-p-3}{2(d-2)}, \quad b = \frac{p+1}{2(d-2)}, \quad \tau = \frac{\hat{a}}{2}, \quad \sigma = \frac{1}{2},$$

(2.20)

with $\hat{a}$ obeying the equation

$$(p+1)(d-p-3) + \hat{a}^2 = 2(d-2),$$

(2.21)

then the function $H(x)$ satisfies the flat space Laplace equation. An extremal $p$-brane solution is constructed by introducing in the right-hand side a $\delta$-function source term in the transverse directions. If we restrict ourselves to the simplest case of just one $p$-brane, we can write

$$H(x) = 1 + 2\kappa T_p G(x),$$

(2.22)

where

$$G(x) = \begin{cases} \frac{1}{2\pi} \log |x| & p = d-3, \\ \left[(d-p-3)|x|^{(d-p-3)}\Omega_{d-p-2}\right]^{-1} & p < d-3, \end{cases}$$

(2.23)

with $\Omega_q = 2\pi^{(q+1)/2}/\Gamma\left((q+1)/2\right)$ being the area of a unit $q$-dimensional sphere $S_q$. Looking at the behavior for $|x| \to \infty$ of the fields in Eqs. (2.18) and (2.19), we see that they tend in general to non-vanishing background values. At large distance, the fluctuations around these background values (which we denote by $\hat{h}_{\mu\nu}, \hat{\varphi}$ and $\hat{A}_{01...p}$ respectively) correspond to the exchange of the massless states: this is precisely what is encoded in the boundary state. To see this explicitly, it is first convenient to rescale the fields according to (see e.g. Eq. (4.8))

$$h_{\mu\nu} = \frac{1}{2\kappa} \hat{h}_{\mu\nu}, \quad \varphi = \frac{1}{\kappa \sqrt{d-2}} \hat{\varphi}, \quad A_{01...p} = \frac{\sqrt{2}}{\kappa} \hat{A}_{01...p},$$

(2.24)

and then make the Fourier transform

$$\int dt \ d^p y \ d^{d-2} x \ e^{ik_{d-2}x} G(x) = \frac{V_{p+1}}{k_{d-2}^2}. \quad (2.25)$$

The transformation (2.24) is determined by requiring that, in term of the new fields, the action (2.17) becomes the one usually obtained from the string calculations in the low energy limit. Thus, to make easier the comparison with the string results, we rewrite the classical solutions (2.18) and (2.19) as

$$h_{\mu\nu}(k) = 2T_p \frac{V_{p+1}}{k_{d-2}^2} \ \text{diag}(-a, a ... a, b ... b),$$

(2.26)

and

$$\varphi(k) = -\frac{\hat{a}}{\sqrt{d-2}} T_p \frac{V_{p+1}}{k_{d-2}^2}, \quad A_{01...p}(k) = \mp 2\sqrt{2} T_p \frac{V_{p+1}}{k_{d-2}^2}. \quad (2.27)$$
The classical solution has a mass per unit \( p \)-volume, \( M_p \), and an electric charge with respect to the R-R field, \( \mu_p \), given respectively by

\[
M_p = \frac{T_p}{\kappa}, \quad \mu_p = \pm \sqrt{2} T_p. \tag{2.28}
\]

The fact that the coefficient of the Coulomb-like behavior for the gauge potential in Eq. (2.27) is not exactly equal to \( \mu_p \) is due to the fact that the field \( A \) of Eq. (2.24) is not "canonically" normalized. The "canonically" normalized field is indeed \( A_{\text{can}} = A/2 \), and in terms of it the coefficient of the Coulomb-like term is exactly equal to \( \mu_p \).

Now we will show that it is possible to derive the long-distance behavior of the \( p \)-brane solution directly from the boundary state, without making any reference to field theory. In fact, a classical solution of the field equations can be seen as a source for particles interacting with external quantum states. The probability amplitude for this interaction to occur is proportional to the Fourier transform of the classical field and is related to the emission vertex of the various states. In our case, such a vertex is, as we have just shown, the boundary state. However, it should be borne in mind that the string description cannot be really continued off shell; thus we can derive only the most singular terms in \( k_\perp \) which correspond to the next-to-leading terms in the solutions (2.18) and (2.19). These can be obtained by projecting the boundary state onto the level-one states with the closed string propagator (2.5) inserted in between. Simple algebra shows that such a projection is

\[
J_{\mu\nu}(k) \equiv \langle 0;k|a_1^\mu a_1^\nu D_{a=1}|B \rangle = -\frac{T_p}{2} \frac{V_{p+1}}{k_{\perp}^2} S_{\mu\nu}, \tag{2.29}
\]

where the momentum \( k \) has non-vanishing components only in the transverse directions. If we decompose \( J_{\mu\nu}(k) \) into its irreducible components, we obtain a traceless symmetric tensor \( h_{\mu\nu} \), a scalar \( \phi \) and an antisymmetric tensor \( B_{\mu\nu} \), which we can identify, respectively, with the long-distance fluctuations of the graviton, the dilaton and the Kalb-Ramond field in the presence of a D-brane. In particular, using Eqs. (2.2), (2.10), we have

\[
h_{\mu\nu}(k) \equiv J_{\mu\nu}(k) - \frac{J(k) \cdot \epsilon(\phi)}{\eta \cdot \epsilon(\phi)} \eta_{\mu\nu} = 2T_p \frac{V_{p+1}}{k_{\perp}^2} \text{diag}(-a,a\ldots b\ldots b), \tag{2.30}
\]

\[
\varphi(k) \equiv J(k) \cdot \epsilon(\phi) = T_p \frac{V_{p+1}}{k_{\perp}^2} \frac{d - 2p - 4}{2\sqrt{d - 2}}, \tag{2.31}
\]

\[
B_{\mu\nu}(k) \equiv \frac{1}{2} (J_{\mu\nu}(k) - J_{\nu\mu}(k)) = 0, \tag{2.32}
\]

where \( a \) and \( b \) are given in Eq. (2.20).

Comparing with Eqs. (2.26) and (2.27), we find perfect agreement for the metric and the Kalb-Ramond field, which indeed was absent in the \( p \)-brane solution; for
the dilaton instead the agreement occurs only if $d = 10$. This strongly suggests to consider the superstring theory. As a matter of fact, a comparison between the $p$-brane solution of the action (2.17) and a string calculation, does make sense only in the superstring case where the graviton, dilaton and Kalb-Ramond field come from the NS-NS sector and the antisymmetric gauge potentials like $A_{\mu_1 \ldots \mu_n}$ from the R-R sector. Nonetheless, the bosonic case we have considered in this section already tells us what are the distinctive features of the boundary state and how the long-distance behavior of the massless fields is encoded in it. In the next Sections we will consider in detail the superstring case and get a complete mapping between the D$p$-brane solutions and the boundary state calculations.

3 The NS-NS sector of superstring

In the superstring the boundary state is similar to that of Eq. (2.7) but contains an additional part depending on the oscillators of the fermionic coordinates. In the Neveu-Schwarz sector, the orbital part of the GSO projected boundary state is

$$|B\rangle_{NS} = \frac{T_p}{2} \delta^{(d_\perp)} (q-y) \exp \left[ - \sum_{n=1}^{\infty} \alpha_n^{\mu \dagger} \mathcal{S}_{\mu\nu} \tilde{\alpha}_n^{\nu \dagger} \right] \sin \left[ \sum_{r=1/2}^{\infty} b_r^{\mu \dagger} \mathcal{S}_{\mu\nu} \tilde{b}_r^{\nu \dagger} \right] |0; k = 0\rangle ,$$

(3.1)

where $b_r^{\mu}$ ($\tilde{b}_r^{\mu}$) are the left (right) moving modes of the NS fermionic string coordinates $\psi^{\mu}$. The normalization constant $T_p$ is given by Eq. (2.8) with $d = 10$. However, to have a non-ambiguous description, it is necessary to specify the superghost charge or, equivalently, to select a picture for the boundary state. In our case, the simplest choice is to write

$$|\hat{B}\rangle_{NS} = |B\rangle_{NS} |\Omega_{NS}\rangle ,$$

(3.2)

where $|\Omega_{NS}\rangle$ is the superghost vacuum with charge $(-1, -1)$. Of course, in principle one is allowed to change this choice by applying the usual picture changing operator, but this modifies also the structure of the orbital part (3.1).

As we have seen in the previous section, in order to compare with the classical solution we project the boundary state onto the massless states. Of course, the latter have to be chosen in the picture dual to that of the boundary state in order to soak up the superghost anomaly on the disk. In NS-NS case these states are produced by the vertex operator

$$V^{NS-NS}(k; z, \bar{z}) = \epsilon_{\mu\nu} : \mathcal{V}_{-1}^{\mu}(k/2; z) \mathcal{V}_{-1}^{\nu}(k/2; \bar{z}) : ,$$

(3.3)

6Here and in the following, we explicitly write only the superghost vacuum and understand the full ghost and superghost contributions to the boundary state, since these do not play any significant role for our present purposes.
with
\[ V_{-1}^\mu(k; z) = e^{-\phi(z)} c(z) \psi^\mu(z) e^{ik \cdot X(z)} . \] (3.4)

The antiholomorphic part, \( \tilde{V}_{-1}^\nu \), is also given by Eq. (3.4) but with the left-moving fields replaced by the corresponding right-moving ones \( (X^\mu(z) \to \tilde{X}^\mu(\bar{z}), \text{ etc. etc.)} \), and the polarization tensor \( \epsilon_{\mu\nu} \) is as in Eq. (2.10).

Thus, the projection one has to consider is
\[ J_{\mu\nu}(k) \equiv \lim_{z, \bar{z} \to \infty} \langle 0; 0 | : V_{-1}^\mu(k/2; z) \tilde{V}_{-1}^\nu(k/2; \bar{z}) : | D_{a=0} \rangle_{NS} . \] (3.5)

Computing the ghost and superghost contributions, one obtains
\[ J_{\mu\nu}(k) = \langle 0; k | b_{1/2}^\mu b_{1/2}^\nu D_{a=4} | B \rangle_{NS} = -\frac{T_p}{2} \frac{V_{p+1}}{k^2} S_{\mu\nu} , \] (3.6)

which is the same result of the bosonic string calculation, Eq. (2.29).

The irreducible components of \( J_{\mu\nu}(k) \) give the long-distance behavior of the graviton, dilaton and Kalb-Ramond fields as in Eqs. (2.30)–(2.32). Since now \( d = 10 \), this result is in complete agreement with the \( p \)-brane solution in Eqs. (2.26) and (2.27).

4 The R-R sector of superstring

The massless spectrum in the R-R sector of type II string theories consists of a collection of antisymmetric tensor fields \( A_{\mu_1...\mu_n} \) playing the role of gauge potentials. In the type IIA theory, these R-R fields are forms of odd degree \( (n = 1, 3, ...) \), while in the type IIB theory they are forms of even degree \( (n = 0, 2, ...) \) [19]. In the BRST invariant formalism, the emission of a R-R field from a closed string is described by a vertex operator which usually is taken to be
\[ V^{R-R}(k; z, \bar{z}) = \mathcal{F}_{\alpha\beta} : V_{-1/2}^\alpha(k/2; z) \tilde{V}_{-1/2}^\beta(k/2; \bar{z}) : \] (4.1)

for type IIA, and
\[ V^{R-R}(k; z, \bar{z}) = \mathcal{F}_{\dot{\alpha}\dot{\beta}} : V_{-1/2}^{\dot{\alpha}}(k/2; z) \tilde{V}_{-1/2}^{\dot{\beta}}(k/2; \bar{z}) : \] (4.2)

for type IIB. Here we have adopted the conventions of Ref. [18] and denoted by \( \alpha (\dot{\alpha}) \) the sixteen-dimensional indices of a chiral (antichiral) Majorana-Weyl spinor of \( SO(1,9) \). The holomorphic component of the type IIA vertex operators is
\[ V_{-1/2}^\alpha(k; z) = e^{-\phi(z)/2} c(z) S^\alpha(z) e^{ik \cdot X(z)} , \] (4.3)

[7] Here and in the following we use the notations of Ref. [13]: \( X^\mu(z) \) and \( \psi^\mu(z) \) are the bosonic and fermionic string coordinates, \( b(z) \) and \( c(z) \) are the ghost fields, while \( \phi(z), \xi(z) \) and \( \eta(z) \) are related to the superghost fields.

[8] We understand that in ten dimensions a form of degree \( n \) is equivalent to a form of degree \( 8 - n \), since their field strengths are dual to each other.
where $S^\alpha(z)$ is the spin field [13]. The antiholomorphic component is given by analogous expressions in terms of the right-moving fields. The bispinor $F_{\alpha\dot{\beta}}$, which plays the same role of the polarizations $\epsilon_{\mu\nu}$ in the NS-NS vertices, is

$$F_{\alpha\dot{\beta}} = \frac{i}{16(n+1)!} \left( C^{\mu_1...\mu_{n+1}} \right)_{\alpha\dot{\beta}} F_{\mu_1...\mu_{n+1}} , \quad (4.4)$$

where $\Gamma^{\mu_1...\mu_{n+1}}$ is the antisymmetrized product of $(n+1)$ $\Gamma$-matrices and $C$ is the charge conjugation matrix (see the Appendix for our conventions and notations). Similar equations hold also for type IIB. The BRST invariance of the vertices (4.1) and (4.2) requires $k^2 = 0$, $d^* F = 0$ and $dF = 0$, so that $F$ must be identified with a field strength (i.e. $F = dA$).

The vertices of Eqs. (4.1) and (4.2) have the same superghost charge in the left and right sectors, like the NS-NS vertices (3.3), and are the ones that have been usually considered in the literature for the calculation of correlation functions. However, this is not the only existing possibility. For example, one could choose to work with R-R vertices that carry a total superghost charge $-2$ (which is the right amount to saturate the superghost anomaly on a disk [1]) and that can be directly projected on a boundary state as we did with the NS-NS vertices in the previous subsection. R-R vertices of this kind can be easily constructed. For example, in the type IIA theory, one can use the vertex operator

$$W_{-R}^{R-R}(k; z, \bar{z}) = A_{\dot{\alpha}\dot{\beta}} : V^\dot{\beta}_{-3/2}(k/2; z) \tilde{V}^\dot{\alpha}_{-1/2}(k/2; \bar{z}) : \quad (4.5)$$

with

$$A_{\dot{\alpha}\dot{\beta}} = \frac{1}{4\sqrt{2n!}} \left( C^{\mu_1...\mu_n} \right)_{\dot{\alpha}\dot{\beta}} A_{\mu_1...\mu_n} \quad (n \text{ odd}) \quad (4.6)$$

and

$$V^\dot{\beta}_{-3/2}(k; z) = e^{-3\phi(z)/2} c(z) S^\dot{\beta}(z) e^{ik \cdot X(z)} . \quad (4.7)$$

In this case the BRST invariance implies $k^2 = 0$, $d^* A = 0$ and $F = dA$, so that $A$ must be identified with a gauge potential. Of course the vertices (4.1) and (4.2) are equivalent; this can be checked directly either by showing that $W_{-R}^{R-R}(k; z, \bar{z})$ and $V_{R}^{R-R}(k; z, \bar{z})$ are related to each other by a picture-changing operation in the left sector, or by showing that they produce the same correlation functions. In this respect it is worth pointing out that even if $W_{-R}^{R-R}(k; z, \bar{z})$ contains the bare potential $A$, its correlation functions depend only on the field strength $F$; moreover, the second line of Eq. (4.5) does not give any contribution inside expectation values, but it is necessary for the BRST invariance of the whole vertex. Notice that $W_{-R}^{R-R}(k; z, \bar{z})$
has the correct GSO projection for a vertex of the type IIA theory; in fact even if
the left and right spinor indices are in the same spin representation, the different
superghost charges give the left and right parts a different $F$-parity. Of course a
similar construction can be done also for the type IIB with obvious changes in the
spinor indices.

It is possible to check that the scattering amplitudes among the R-R vertex
operators (4.1), (4.2), (4.5) and the NS-NS ones (3.3) are reproduced, in the field
theory limit, by the following action

$$S' = \int d^{10}x \sqrt{-g} \left[ -\frac{1}{8(n+1)!} \frac{e^{\frac{3\kappa_\phi}{2}}}{\sqrt{g}} \left(dA_n)^2 \right] \right].$$

(4.8)

By comparing this action with that in Eq. (2.17), used to study the classical solution,
one can easily explain the rescalings in Eq. (2.24).

Let us now consider the interaction of the massless R-R fields with a D-brane,
and introduce the boundary state for the R-R sector which can be used with the
asymmetric vertex operators (4.5). As explained in the Appendix, this boundary
state, after GSO projection, is

$$|\tilde{B}\rangle_R = \pm \frac{T_p}{2} \delta^{d_-}(q - y) \exp \left[ -\sum_{n=1}^\infty d_n^{\mu\dagger} S_{\mu\nu} \tilde{d}_n^{\nu\dagger} \right] |0; k = 0\rangle$$

(4.9)

with $d_n$ ($\tilde{d}_n$) being the left (right) moving modes of the fermionic coordinates and

$$|\Omega_R\rangle^{(1)} = \begin{cases} M_{\alpha\beta} |\alpha\rangle_{-1/2} |\beta\rangle_{-3/2} & \text{for IIA (p even)} \\ M_{\dot{\alpha}\dot{\beta}} |\dot{\alpha}\rangle_{-1/2} |\dot{\beta}\rangle_{-3/2} & \text{for IIB (p odd)} \end{cases}$$

(4.10)

and

$$|\Omega_R\rangle^{(2)} = \begin{cases} M_{\dot{\alpha}\dot{\beta}} |\dot{\alpha}\rangle_{-1/2} |\dot{\beta}\rangle_{-3/2} & \text{for IIA} \\ M_{\alpha\beta} |\alpha\rangle_{-1/2} |\beta\rangle_{-3/2} & \text{for IIB} \end{cases}$$

(4.11)

where we have introduced the notation

$$|\alpha\rangle_{\ell} \equiv \lim_{z \to 0} : S^\alpha(z) e^{\ell\phi(z)} : |0\rangle.$$

(4.12)

For a Dp-brane the explicit expression for the matrix $M$ to be used in Eqs. (4.10)
and (4.11) is

$$M_{AB} \equiv \begin{pmatrix} M_{\alpha\beta} & M_{\alpha\dot{\beta}} \\ M_{\dot{\alpha}\beta} & M_{\dot{\alpha}\dot{\beta}} \end{pmatrix} = (CT^0 \Gamma^1 \cdots \Gamma^p)^{AB}.$$

(4.13)

We first consider the type IIA theory and compute the interaction between the
R-R massless potential and a D-brane, following the same procedure presented in
Section 4 for the bosonic case. Saturating $|\hat{B}⟩_R$ with the corresponding state (4.5), namely
\[
\lim_{z,\bar{z}→∞}⟨ 0 | W^{R-R} (k; z, \bar{z}) |\hat{B}⟩_R = \mp \sqrt{2} T_p V_{p+1} A_{0...p} ,
\]
we can reconstruct the Wess-Zumino term of the D-brane effective action
\[
S_{WZ} = -\mu_p \int A ,
\]
and derive again Eq. (2.28).

Now we turn to the R-R part of the solutions (2.19) and insert a closed string propagator in the boundary state projection
\[
J^{\dot{\alpha} \dot{\beta}} (k) \equiv \lim_{z,\bar{z}→∞} ⟨ 0 | : V^{\dot{\alpha} - 3/2}_- (k/2; z) \bar{V}^{\dot{\beta} - 1/2}_- (k/2; \bar{z}) : D_{a=0} |\hat{B}⟩_R
\]
\[
= \mp \frac{T_p V_{p+1}}{2 k_\perp^2} (C^{-1} MC^{-1})^{\dot{\alpha} \dot{\beta}} ,
\]
(4.16)
Then, we decompose $J^{\dot{\alpha} \dot{\beta}}$ into irreducible components to obtain the gauge potentials. Since Eq. (4.6) instructs us to identify the trace of the bispinor with the potential $A$ times a factor of $2\sqrt{2}$, we have
\[
A_{\mu_1...\mu_n} (k) = \frac{1}{2\sqrt{2}} \text{Tr} (J (k) C \Gamma_{\mu_1} \cdots \Gamma_{\mu_n}) .
\]
(4.17)
Using Eq. (4.16), we see that the potentials above are zero unless $n = p + 1$, in which case we get
\[
A_{\mu_1...\mu_{p+1}} (k) = \mp 2\sqrt{2} T_p \frac{V_{p+1}}{k_\perp^2} \epsilon^{(v)}_{\mu_1...\mu_{p+1}} ,
\]
(4.18)
where $\epsilon^{(v)}$ is the completely antisymmetric tensor of the world-volume. This result is in perfect agreement with the classical solution in Eq. (2.27). The vanishing of all other potentials is consistent with the fact that a D$p$-brane is charged only under the $(p + 1)$ gauge field of the R-R sector.

Similar calculations also hold for type IIB theory and, also in this case, we correctly reproduce the long-distance behavior of the classical solution in agreement with the results of Ref. [12].

5 Delocalized D-brane bound states

We now show that, using the same procedure described in the previous sections, it is possible to derive the long-distance behavior of a broader class of $p$-branes solutions; in particular we examine those background field configurations that present two non
trivial R-R potentials whose dimension differs by two. These solutions are described in detail in Ref. [13], where they are explicitly constructed from those that present only one form field potential using the T-duality symmetry between type IIA and IIB theories. It is useful to outline here this approach since it can be directly applied at a string level, obtaining a generalization of the boundary state presented in the previous sections. For simplicity, we again use the boundary state of the bosonic string given in Eq. (2.1). The same procedure can then be generalized in a straightforward way to the superstring.

The first step of the procedure presented in Ref. [13] is to delocalize the \( p \)-brane solution removing its dependence also on the \((p + 1)\)th spatial direction; from the world-sheet point of view this operation simply consists in fixing to zero the momentum \( k\) along a Dirichlet direction so that, in the final result, the square of transverse momentum does not involve \( k\), that is \( k^2_{\perp} = \sum_{i=p+2}^{d} k_i k^i\). The boundary state corresponding to this delocalized solution is again given by the same expression as in Eq. (2.1) with, however, two important modifications. One is the absence of the \( \delta \)-function along the delocalized direction \((p + 1)\) and the other is a different normalization: in fact, according to Eq. (2.6), one \( \delta \)-function less implies also a factor of \((2\pi\sqrt{\alpha'})\) less, and this corresponds to substitute \( T_p \) with \( T_{p+1} \) in the normalization. In conclusion the boundary state corresponding to a \( D_p \)-brane delocalized in the \((p + 1)\)th direction is given by

\[
|B\rangle = \frac{T_{p+1}}{2} \delta^{(d')} (q - y) \exp \left[ -\sum_{n=1}^{\infty} a_n^{\mu+n} S_{\mu\nu} a_n^{\nu+n} \right] |0; k = 0\rangle ,
\]

where \( d' \) means that the \( \delta \)-function in the \((p + 1)\)th direction is absent.

Now, in the plane defined by the delocalized Dirichlet direction and by the \( p \)th Neumann direction, we perform first a rotation with angle \( \phi \) and then a T-duality transformation along the \((p + 1)\)th transverse coordinate axis. These operations do not affect the structure of the boundary state (5.1) but simply that of matrix \( S_{\mu\nu} \). In fact, under the rotation \( S_{\mu\nu} \) behaves as a tensor, while the T-duality changes the sign of the component of the \((p + 1)\)th column, yielding

\[
S_{\mu\nu} = \begin{cases} 
\eta_{\mu\nu} & \text{if } \mu = 0, \ldots, p - 1 , \\
-\delta_{\mu\nu} & \text{if } \mu = p + 2, \ldots, d - 1 , \\
(\cos 2\phi & -\sin 2\phi) \\
(\sin 2\phi & \cos 2\phi) 
\end{cases} \quad \text{in the plane } p, p + 1 .
\]

The rotation can be performed with the same procedure used in Ref. [4] for boosting the boundary state. There is, however, a very important difference with respect to the boost case. Unlike in the case discussed in Ref. [4] in this case there is no \( \delta \)-function in the transverse direction in which we perform the rotation and therefore the rotation acts trivially on the zero modes. As a result the rotation in a delocalized \( D_p \)-brane does not generate any “Born-Infeld” factor as instead happens in the localized case.
With this new matrix appearing in the exponent the boundary state satisfies new overlap equations in the plane where we performed the rotation
\[ (\partial_\tau X^p + i \tan \phi \partial_\sigma X^{p+1}) |_{\tau=0} B) = 0 , \]
\[ (\partial_\tau X^{p+1} - i \tan \phi \partial_\sigma X^p) |_{\tau=0} B) = 0 . \]

Those are the same overlap equations satisfied by a boundary state in presence of a constant background field \( F = B + 2\pi \alpha' F \), where \( B \) is the Kalb-Ramond potential and \( F \) is the external gauge field strength:
\[ (\partial_\tau X^\mu - i F^{\mu\nu} \partial_\sigma X^\nu) |_{\tau=0} B) = 0 , \]
where \( \mu, \nu = p \) or \( p + 1 \). Comparing Eqs. (5.3) and (5.4) we get that
\[ F_{(p+1)p} = -F_p^{(p+1)} = \tan \phi . \]

The normalization of the boundary state of a delocalized \( p \)-brane differs, however, from the one with an external field because of the lack, in the case of a delocalized \( p \)-brane, of the Born-Infeld factor present instead in the boundary state with an external field.

Now we have lifted the whole construction of Ref. [13] at a string level. Thus, using the same procedure of previous sections, we can show that the new boundary state is the conformal description of those delocalized \( p \)-branes that present two non-vanishing R-R potentials.

The above construction of a delocalized boundary state for the bosonic string can be easily generalized to the NS-NS sector of the superstring as we have done in sect. 2. In particular, the boundary state for the NS-NS sector of a delocalized \( p \)-brane has exactly the same form as the one in Eq. (3.1) with the matrix \( S \) given in Eq. (5.2), with no \( \delta \)-function in the \((p+1)\)th transverse direction, and with \( T_p \) replaced by \( T_{p+1} \). Then, using Eq. (3.6), from this boundary state one can obtain the correct large distance behavior of the NS-NS fields of this type of solutions
\[ h_{\mu\nu}(k) = 2T_{p+1} \frac{V_{p+2}}{k_1^2} \text{diag}(-a, a, ..., a, c, b, ..., b) , \]
\[ \varphi(k) = -T_{p+1} \frac{V_{p+2} p - 3 + \cos 2\phi}{2\sqrt{2}} , \]
\[ B_{p(p+1)}(k) = -B_{(p+1)p}(k) = \frac{T_{p+1}}{2} \frac{V_{p+2}}{k_1^2} \sin 2\phi , \]
where \( a = (p+\cos 2\phi - 7)/16, c = (p-3(1+\cos 2\phi))/16, \) and \( b = (p+1+\cos 2\phi)/16 \). Remember that \( k_1^2 \) does not contain its transverse component along the \((p+1)\)th direction.

The boundary state of a delocalized \( p \)-brane for the R-R sector has the same form as the one in Eq. (4.9) suitably modified as we have done in the case of the NS-NS sector. In addition, in the R-R sector the rotation and the T-duality
transformation act also on the matrix $M$ present in the vacua of Eqs. (4.10) and (4.11). Since a rotation in the $(p, p + 1)$ plane acts on the $\Gamma$-matrices as follows

$$\left( \begin{array}{c} \Gamma^p \\ \Gamma^{p+1} \end{array} \right) \rightarrow \left( \begin{array}{cc} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{array} \right) \left( \begin{array}{c} \Gamma^p \\ \Gamma^{p+1} \end{array} \right)$$

(5.9)

and a T-duality transformation in the $(p + 1)$th direction amounts to multiply $M$ with $\Gamma^{p+1}$ from the right (see also Eq. (A.12)), then the matrix $M$ becomes

$$M = C \Gamma^0 \cdots \Gamma^{p-1} (\sin \phi + \cos \phi \, \Gamma^p \Gamma^{p+1}) .$$

(5.10)

Since the matrix $M$ now contains two terms with a different number of $\Gamma$-matrices, it is easy to realize that Eq. (4.16) yields two non-vanishing antisymmetric gauge potentials in the R-R sector, namely

$$A_{0\ldots p-1}(k) = \mp 2\sqrt{2} \, T_{p+1} \frac{V_{p+2}}{k_+^2} \sin \phi$$

(5.11)

$$A_{0\ldots p+1}(k) = \mp 2\sqrt{2} \, T_{p+1} \frac{V_{p+2}}{k_+^2} \cos \phi$$

(5.12)

The simultaneous appearance of a $(p - 1)$-form and a $(p + 1)$-form in the R-R sector indicates the presence of a D$(p - 1)$-brane and a D$(p + 1)$-brane forming a bound state, whose world-sheet realization is given by the rotated and T-dualized boundary state. As we have shown, this boundary state correctly reproduces the long-distance behavior of the delocalized D-brane classical solutions found in Ref. [13].

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**Appendix A**

Let $\gamma^i$ be the eight $16 \times 16$ $\gamma$-matrices of $SO(8)$. Starting from these matrices we can construct a chiral representation for the $32 \times 32$ $\Gamma$-matrices of $SO(1, 9)$, i.e.

$$\Gamma^i = \left( \begin{array}{cc} 0 & \gamma^i \\ \gamma^i & 0 \end{array} \right) = \sigma^1 \otimes \gamma^i$$

$$\Gamma^9 = \left( \begin{array}{cc} 0 & \gamma^1 \cdots \gamma^8 \\ \gamma^1 \cdots \gamma^8 & 0 \end{array} \right) = \sigma^1 \otimes \left( \gamma^1 \cdots \gamma^8 \right)$$

(5.1)

$$\Gamma^0 = \left( \begin{array}{cc} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{array} \right) = i \sigma^2 \otimes \mathbb{I}$$

\[\text{\footnote{Alternatively one can deduce the new form for the matrix } M \text{ by requiring it to satisfy Eq. (A.10) but with the new } S_{\mu\nu} \text{ given in Eq. (5.2).}}\]

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\[ \sigma^a \text{'s are the standard Pauli matrices. One can easily verify that these matrices satisfy } \{ \Gamma^\mu, \Gamma^\nu \} = 2\eta^{\mu\nu}. \text{ Other useful matrices are} \]

\[ \begin{align*}
\Gamma_{11} &= \Gamma^0 \cdots \Gamma^g = \left( \begin{array}{cc}
1 & 0 \\
0 & -1
\end{array} \right) = \sigma^3 \otimes 1
\end{align*} \]

\[ C = \left( \begin{array}{cc}
0 & -i
i & 0
\end{array} \right) = \sigma^2 \otimes 1, \]

where \( C \) is the charge conjugation matrix such that \( (\Gamma^\mu)^T = -C \Gamma^\mu C^{-1} \).

\[ \text{Let } A, B, \ldots \text{ be 32-dimensional indices for spinors in ten dimensions, and } |A\rangle|\bar{B}\rangle \text{ denote the vacuum of the Ramond fields } \psi^A(z) \text{ and } \tilde{\psi}^B(\bar{z}) \text{ with spinor indices } A \text{ and } B \text{ in the left and right sectors respectively, that is} \]

\[ |A\rangle|\bar{B}\rangle = \lim_{z, \bar{z} \to 0} S^A(z) \bar{S}^B(\bar{z}) |0\rangle \]

where \( S^A (\bar{S}^B) \) are the left (right) spin fields \( [15] \), and \( |0\rangle \) the Fock vacuum of the Ramond fields. The action of the Ramond oscillators \( d_\mu^A \) and \( \tilde{d}_\mu^A \) on the state \( |A\rangle|\bar{B}\rangle \) is given by

\[ d_\mu^A |A\rangle|\bar{B}\rangle = \tilde{d}_\mu^A |A\rangle|\bar{B}\rangle = 0 \]

if \( n > 0 \), and

\[ d_0^A |A\rangle|\bar{B}\rangle = \frac{1}{\sqrt{2}} (\Gamma^\mu)^A_C (\Gamma^\mu)^B_D |C\rangle|\bar{D}\rangle \]

\[ \tilde{d}_0^A |A\rangle|\bar{B}\rangle = \frac{1}{\sqrt{2}} (\Gamma_1)^A_C (\Gamma^\mu)^B_D |C\rangle|\bar{D}\rangle \]

It is easy to check that this action correctly reproduces the anticommutation properties of the \( d \)-oscillators, and in particular that \( \{ d_\mu^A, \tilde{d}_\nu^B \} = \{ d_\mu^A, d_\nu^B \} = \eta^{\mu\nu} \), and \( \{ d_0^A, \tilde{d}_0^B \} = 0 \).

We now use these definitions to derive the fermionic structure of the boundary state \( |B\rangle \) in the R-R sector of a Dp-brane. According to the general theory \( [3] \), \( |B\rangle \) has to satisfy the following overlap equations

\[ (d_\mu^n - i S_\mu^A \tilde{d}_\mu^n) |B\rangle = 0 \]

where \( S_\mu^B \) is the matrix defined in Eq. \( (2.2) \). For \( n \neq 0 \), Eq. \( (A.7) \) is easily solved by a Bogoliubov transformation; indeed, the non-zero mode part of \( |B\rangle \) is

\[ |B\rangle' = \exp \left[ i \sum_{n=1}^\infty d_\mu^n S_\mu^A \tilde{d}_\mu^n \right] |0\rangle \]
where, as usual, \(d_n^\dagger \equiv d_{-n}\). The zero-mode part \(|B\rangle^0\) requires more care due to the non-trivial action of the oscillators \(d_0^\dagger\) and \(d_0^\dagger\). If we write

\[
|B\rangle^0 = \mathcal{M}_{AB} |A\rangle |\bar{B}\rangle \tag{A.9}
\]

then, Eq. (A.7) for \(n = 0\) implies that the \(32 \times 32\) matrix \(\mathcal{M}\) has to satisfy the following equation

\[
(\Gamma^\mu)^T \mathcal{M} - i \mathcal{S}_\mu^\nu \Gamma_{11} \mathcal{M} \Gamma^\nu = 0 \tag{A.10}
\]

Using our previous definitions, one finds that a solution is

\[
\mathcal{M} = C \Gamma^0 \cdots \Gamma^p \frac{1 + i \Gamma_{11}}{1 + i} \tag{A.11}
\]

If one performs a T-duality transformation in one of the transverse directions of the D-brane, say for example in the \(i\)th direction, the boundary state retains its structure but one must change the signs in the \(i\)th column of \(\mathcal{S}_\mu^\nu\) and replace \(\mathcal{M}\) with the matrix

\[
\mathcal{M}' = i \mathcal{M} \Gamma^i \Gamma_{11} = C \Gamma^0 \cdots \Gamma^p \Gamma^i \frac{1 + i \Gamma_{11}}{1 + i} \tag{A.12}
\]

In our representation of \(\Gamma\)-matrices, it is natural to decompose the spinors in chiral and antichiral components \((A = (\alpha, \dot{\alpha})\) with sixteen-dimensional indices \(\alpha\) and \(\dot{\alpha}\) respectively. From Eqs. (A.1) and (A.2), it follows that for \(p\) even (i.e. in the type IIA theory), the matrix \(\mathcal{M}\) has non-vanishing entries only in the diagonal blocks, that is in the chiral-chiral sector and the antichiral-antichiral one, whereas for \(p\) odd (i.e. in the type IIB theory) \(\mathcal{M}\) is non-trivial only in the off-diagonal blocks, that is in the antichiral-chiral sector and in the chiral-antichiral one. Thus, in the sixteen-dimensional notation, Eq. (A.9) becomes

\[
|B\rangle^0 = |\Omega_R\rangle^{(1)} - i |\Omega_R\rangle^{(2)} \tag{A.13}
\]

where

\[
|\Omega_R\rangle^{(1)} = \begin{cases} 
M_{\alpha\beta} |\alpha\rangle_{-1/2} |\beta\rangle_{-3/2} & \text{for IIA}, \\
M_{\dot{\alpha}\dot{\beta}} |\dot{\alpha}\rangle_{-1/2} |\dot{\beta}\rangle_{-3/2} & \text{for IIB}
\end{cases} \tag{A.14}
\]

and

\[
|\Omega_R\rangle^{(2)} = \begin{cases} 
M_{\alpha\dot{\beta}} |\alpha\rangle_{-1/2} |\dot{\beta}\rangle_{-3/2} & \text{for IIA}, \\
M_{\dot{\alpha}\beta} |\dot{\alpha}\rangle_{-1/2} |\beta\rangle_{-3/2} & \text{for IIB}
\end{cases} \tag{A.15}
\]

In these equations we have defined

\[
M_{AB} \equiv \begin{pmatrix} 
M_{\alpha\beta} & M_{\dot{\alpha}\dot{\beta}} \\
M_{\dot{\alpha}\beta} & M_{\alpha\dot{\beta}}
\end{pmatrix} = \left( C \Gamma^0 \cdots \Gamma^p \right)_{AB} \tag{A.16}
\]

and introduced the appropriate superghost charge by treating asymmetrically the left and right sectors as we explained in Section 4. Then, the boundary state becomes

\[
|B\rangle = \exp \left[ i \sum_{n=1}^{\infty} d_n^{\dagger} S_{\mu\nu} \bar{d}_n^{\dagger} \right] |B\rangle^0 \tag{A.17}
\]
Finally, by applying on it the GSO projection, we obtain

\[ \langle \hat{B} \rangle_R = \frac{1 - (-1)^p(-1)^F}{2} \frac{1 + (-1)^\tilde{F}}{2} \langle B \rangle \]  

(A.18)

\[ = \cos \left[ \sum_{n=1}^{\infty} d_{n\mu}^\dagger S_{\mu\nu} \tilde{d}_{n\nu}^\dagger \right] |\Omega_R \rangle^{(1)} + \sin \left[ \sum_{n=1}^{\infty} d_{n\mu}^\dagger S_{\mu\nu} \tilde{d}_{n\nu}^\dagger \right] |\Omega_R \rangle^{(2)} \]

where \( F \) and \( \tilde{F} \) are the left and right F-parity operators defined in Ref. [18] which measure the fermion number, the chirality and the superghost charge. Note that the GSO projection in Eq. (A.18) is of type IIA for \( p \) even and of type IIB for \( p \) odd in accordance with the R-R charge that is carried by the Dp-brane.

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