Softly Broken $N = 1$ Supersymmetric QCD

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Abstract

We study softly broken $N = 1$ supersymmetric QCD with the
gauge group $SU(N_c)$ and $N_f$ flavours of quarks for $N_f > N_c + 1$.
We investigate the phase structure of its dual theory adding generic
soft supersymmetry breaking terms, i.e. soft scalar masses, trilinear
coupling terms of scalar fields and gaugino masses. It is found that
the trilinear coupling terms play an important role in determining the
potential minima. Also we compare softly broken original and dual
theories in the broken phase.

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1 Introduction

In the last few years, the understanding of strongly coupled supersymmetric (SUSY) Yang-Mills theories has made important progress. A number of exact results have been obtained [1], because holomorphy and global symmetries including $R$-symmetry restrict the possible structure of the theory [2]. In particular, the old Montonen-Olive duality conjecture [3] has been gained new insights, although this duality was shown to possibly exist only in $N = 4$ SUSY Yang-Mills theory [4]. Seiberg and Witten have found the exact low-energy effective action of $N = 2$ SUSY gauge field theory and its vacuum structure through duality [5]. Furthermore, Seiberg’s duality was found to be realized in the infrared region of $N = 1$ SUSY QCD with the $SU(N_c)$ gauge group and $N_f$ flavours of quarks for the case $N_f > N_c + 1$ [6], which actually includes QCD in the real world, i.e. $(N_c, N_f) = (3, 6)$. This conjecture is strongly supported by the ’t Hooft anomaly matching condition [7]. Since duality can relate a strongly coupled theory to a weakly coupled one, thus this has provided with a possible way to explore the non-perturbative aspects of the strongly coupled theory.

It is important to extend such analyses to non-SUSY cases. However, it is very difficult to discuss strongly coupled non-SUSY Yang-Mills theories directly, since we cannot use holomorphy or $R$-symmetry in a non-SUSY theory. Thus, for the study of a non-SUSY theory it is of much interest to consider $N = 1$ SUSY theory with soft SUSY breaking terms. Actually in Ref.[8-10] $N = 1$ SUSY QCD with soft scalar masses as well as with gaugino masses was discussed and interesting results were obtained [9].

If all the symmetries allow it, then other soft SUSY breaking terms, i.e. trilinear ($A$-terms) and bilinear ($B$-terms) interactions of scalar fields in general also appear in the low energy effective theory, e.g. from the viewpoint of supergravity or dynamical SUSY breaking. These terms are very important to determine the potential minima. For instance, in the minimal supersymmetric standard model, the successful electroweak symmetry breaking can not be realized without the $B$-term of two Higgs fields [12] and large values of $A$-terms lead to charge and/or colour breaking vacua [13].

When the superpotential $W$ includes the term $\hat{q} T \hat{q}$, the scalar potential in general also has the corresponding $A$-term $q T \bar{q}$, where $q (T, \bar{q})$ denotes the

*Softly broken $N = 2$ SUSY QCD has been studied in [11].
scalar component corresponding to the supermultiplet $\hat{q}$ ($\hat{T}, \hat{q}$). The above superpotential corresponds to the dual to $N = 1$ SUSY QCD theory with the gauge group $SU(N_c)$ and $N_f$ flavours of quark supermultiplets $\hat{Q}$ and $\hat{\bar{Q}}$ for $N_f > N_c + 1$ [6]. In this case $\hat{q}$ and $\hat{\bar{q}}$ are the dual quark superfields and $\hat{T}$ is the meson superfield. In this paper we study such a dual theory with soft masses and $A$-terms. We investigate the phase structure of this theory taking the soft parameters as free parameters. If the fundamental theory for SUSY breaking is specified, e.g. originated from certain type of supergravity model, superstring theory [14] or dynamical SUSY breaking [15], these soft parameters can be written in terms of more fundamental quantities such as $F$-term condensations. However, here it is more instructive to take the soft parameters as free parameters in order to understand the generic phase structure of the theory. We also compare softly broken dual theory with softly broken original one.

This paper is organized as follows. In section 2 we give a brief review of $N = 1$ SUSY QCD and its Seiberg’s dual theory. We add the generic soft SUSY breaking terms to the dual theory and study its phase structure. In section 3 we compare this phase structure of the dual theory with softly broken original $N = 1$ SUSY QCD theory. Section 4 is devoted to conclusions and discussions.

2 Softly broken dual to $N = 1$ SUSY QCD theory

At first we review briefly $N = 1$ SUSY QCD and its dual theory [6]. Here we consider $N = 1$ SUSY QCD with the gauge group $SU(N_c)$ and $N_f$ flavours of quark supermultiplets $\hat{Q}^i$ and $\hat{\bar{Q}}_i$ ($i = 1, \cdots, N_f$), where $\hat{Q}$ and $\hat{\bar{Q}}$ transforms under $N_c$ and $\bar{N}_c$ representations of $SU(N_c)$. Hereafter the colour indices are omitted. This theory has a vanishing superpotential and has the global symmetry

$$SU(N_f)_Q \times SU(N_f)_{\bar{Q}} \times U(1)_B \times U(1)_R.$$  \hspace{1cm} (1)

†Application of coupling reduction theory [16] to the soft SUSY breaking terms is another type of interesting approach to fix the relations among soft SUSY breaking terms [17].
Quark superfields \( \hat{Q} \) and \( \hat{\bar{Q}} \) transform as the multiplets \((N_f, 0, 1)\) and \((0, N_f, 1)\) of the global symmetry \( SU(N_f)_Q \times SU(N_f)_{\bar{Q}} \times U(1)_B \), respectively.

Its dual theory has the gauge group \( SU(\tilde{N}_c) \), where \( \tilde{N}_c \equiv N_f - N_c \), and contains \( N_f \) flavours of dual quark supermultiplets \( \hat{q}_i \) and \( \hat{\bar{q}}_i \) and singlet superfields \( \hat{T}_{ij} \), which correspond to meson supermultiplets in the original theory. This dual theory has the same global symmetry as the original one and the superfields \( \hat{q}_i, \hat{\bar{q}}_i \) and \( \hat{T}_{ij} \) transform as \((\bar{N}_f, 0, N_c/\tilde{N}_c)\), \((0, N_f, -N_c/\tilde{N}_c)\) and \((N_f, \bar{N}_f, 0)\) under the global symmetry \( SU(N_f)_q \times SU(N_f)_{\bar{Q}} \times U(1)_B \), respectively. The dual theory has the superpotential,

\[
W = \hat{q}_i \hat{T}_{ij} \hat{\bar{q}}_j. \tag{2}
\]

This dual pair has the same anomaly structure for the global symmetries, i.e. the 't Hooft anomaly matching condition \[7\] is satisfied. The 't Hooft anomaly matching condition plays basic role in the probe of dual pairs, i.e. massless fermions and their global symmetries are important.

Now let us consider the non-SUSY case. Here we break \( N = 1 \) SUSY softly. We add the following soft SUSY breaking terms to the dual theory:

\[
\mathcal{L}_{SB} = -m_q^2 \text{tr}|q|^2 - m_{\bar{q}}^2 \text{tr}|\bar{q}|^2 - m_T^2 \text{tr}|T|^2 + (h q_i T_j \bar{q}^j + h.c.). \tag{3}
\]

Also the gaugino mass terms are added. Here soft scalar mass terms and the \( A \)-term are flavour-independent. Note that these terms are all the possible soft terms to be added and they do not break any global symmetry except \( R \)-symmetry. For the kinetic term, we assume the canonical form with normalization factors \( k_q \) and \( k_T \) for \( q, \bar{q} \) and \( T \). Then we write the following scalar potential:

\[
V(q, \bar{q}, T) = \frac{1}{k_T} \text{tr}(qq^\dagger \bar{q}^\dagger \bar{q}) + \frac{1}{k_q} \text{tr}(qTT^\dagger q^\dagger + q^\dagger T^\dagger Tq) \\
+ \frac{\tilde{g}^2}{2} (\text{tr}q^\dagger \tilde{T} q - \text{tr}q^\dagger T q)^2 + m_q^2 \text{tr}q^\dagger q + m_{\bar{q}}^2 \text{tr}\bar{q}^\dagger \bar{q}^\dagger \\
+ m_T^2 \text{tr}T^\dagger T - (h q_i T_j \bar{q}^j + h.c.), \tag{4}
\]

where the third term is the \( D \)-term and \( \tilde{g} \) denotes the gauge coupling constant of the dual theory.
We assume $h$ is real. The minimum of potential can be obtained along the following diagonal direction [9],

\[ q = \begin{pmatrix} q^{(1)} & 0 \\ 0 & q^{(2)} & \cdots \\ & 0 & \cdots & q^{(\tilde{N}_c)} \end{pmatrix}, \]

\[ \tilde{q} = \begin{pmatrix} \tilde{q}^{(1)} & 0 \\ 0 & \tilde{q}^{(2)} & \cdots \\ & 0 & \cdots & \tilde{q}^{(\tilde{N}_c)} \end{pmatrix}, \]

\[ T = \begin{pmatrix} T^{(1)} & 0 \\ 0 & T^{(2)} & \cdots \\ & 0 & \cdots & T^{(\tilde{N}_c)} \end{pmatrix}, \]

where all the entries, $q^{(i)}$, $\tilde{q}^{(i)}$ and $T^{(i)}$, can be made real. In this case the scalar potential is written as

\[ V(q, \tilde{q}, T) = \frac{1}{kq} \sum_{i=1}^{\tilde{N}_c} q^{2(i)} + \frac{\tilde{g}^2}{4N_c} \sum_{i<j}^{\tilde{N}_c} (q^{2(i)} - \tilde{q}^{2(i)} - q^{2(j)} + \tilde{q}^{2(j)})^2 + \frac{m_q^2}{kq} \sum_{i=1}^{\tilde{N}_c} q^{2(i)} + m_q^2 \sum_{i=1}^{\tilde{N}_c} T^{2(i)} + \frac{1}{kq} \sum_{i=1}^{\tilde{N}_c} T^{2(i)} q^{2(i)} + \tilde{q}^{2(i)} - 2h \sum_{i=1}^{\tilde{N}_c} q^{(i)} T^{(i)} \tilde{q}^{(i)}. \]  

Let us study the minimum of the potential (8). For fixed values of $q^{(i)}$ and $\tilde{q}^{(i)}$, this potential is unbounded from below along $T \to \infty$, if $m_T^2 + (q^{(i)} + \tilde{q}^{(i)})/kq < 0$, which corresponds to $m_T^2 < 0$ in the limit $q^{(i)} = \tilde{q}^{(i)} = 0$.

The stationary condition, $\partial V/\partial T^{(i)} = 0$, requires

\[ T^{(i)\text{min}} = \frac{hq^{(i)} q^{(i)}}{m_T^2 + (q^{(i)} + \tilde{q}^{(i)})/kq}. \]
Using this, we write the scalar potential as

\[ V(q, \bar{q}, T_{min}) = \frac{1}{kT} \sum_{i=1}^{\tilde{N}_c} q_i^2 q_i^2 + \frac{\hat{g}^2}{4 \tilde{N}_c} \sum_{i<j} (q_i^2 - \bar{q}_j^2 - q_j^2 + \bar{q}_i^2)^2 \]

\[ + m_q^2 \sum_{i=1}^{\tilde{N}_c} q_i^2 + m_{\bar{q}}^2 \sum_{i=1}^{\tilde{N}_c} \bar{q}_i^2 \]

\[ - \frac{\tilde{N}_c}{m_T} \frac{h^2 q_i^2 \bar{q}_i^2}{(q_i + \bar{q}_i)/k_q}. \]  

(10)

Quartic terms appear in the first and second terms as well as the last term. The potential minimum corresponds to the direction along which some of these quartic terms vanish. Note that if the first term vanishes, the last term also vanishes.

Let us study the direction where

\[ q_i = q, \quad \bar{q}_i = 0. \]  

(11)

In this direction each quartic term disappears and \( T_{(i)min} \) also vanish. In this case we have the scalar potential as \( V = \tilde{N}_c m_q^2 q^2 \). Thus, the scalar potential is unbounded from below along the direction \( q \to \infty \), if \( m_q^2 < 0 \). If \( m_q^2 > 0 \), the scalar potential has the minimum \( V = 0 \) at \( q = 0 \). For the direction with \( q_i = 0 \) and \( \bar{q}_i = \bar{q} \), we have similar results, i.e. the potential is unbounded from below if \( m_{\bar{q}}^2 < 0 \).

The second term in (8), i.e. the \( D \)-term, vanishes along the following direction:

\[ q_i = \bar{q}_i = X_i. \]  

(12)

Along this direction, the scalar potential (8) is written as

\[ V(X, T) = \sum_{i=1}^{\tilde{N}_c} \left[ \frac{1}{kT} X_i^4 + (m_q^2 + m_{\bar{q}}^2) X_i^2 + m_T^2 T_{(i)}^2 \right] \]

\[ + \frac{2}{k_q} T_{(i)}^2 X_i^2 - 2h T_{(i)} X_i^2. \]  

(13)

Note that the \( i \)-th elements, i.e. \( X_i \) and \( T_{(i)} \), are decoupled from the \( j \)-th elements (\( i \neq j \)). The stationary condition, \( \partial V/\partial X_i = 0 \), requires

\[ \frac{2}{kT} X_i^3 + [(m_q^2 + m_{\bar{q}}^2) + \frac{2}{k_q} T_{(i)}^2 - 2h T_{(i)}] X_i = 0. \]  

(14)
If this equation as well as eq.(9) has a solution except \( X_i = 0 \), this point then corresponds to the potential minimum which has a lower energy than \( V = 0 \) given at the origin \( X_i = T(i) = 0 \). Recall that if \( X_i = 0, T(i) \) always vanishes owing to eq.(9). One of the conditions leading to the broken phase, i.e. \( X_i \neq 0 \), is obtained from

\[
f(T(i)) \equiv \frac{2}{k_q} T(i)^2 - 2h T(i) + m_q^2 + m_{\bar{q}}^2 \leq 0. \tag{15}
\]

Otherwise, we always have the unbroken phase, i.e. \( X_i = T(i) = 0 \). The above inequality is satisfied for the values of \( T(i) \)

\[
\frac{h - \sqrt{h^2 - 2(m_q^2 + m_{\bar{q}}^2)/k_q}}{2/k_q} \leq T(i) \leq \frac{h + \sqrt{h^2 - 2(m_q^2 + m_{\bar{q}}^2)/k_q}}{2/k_q}, \tag{16}
\]

if

\[
h^2 \geq \frac{2}{k_q}(m_q^2 + m_{\bar{q}}^2). \tag{17}
\]

The inequality (17) is one of conditions on soft SUSY breaking parameters to realize the broken phase. Furthermore, both of stationary conditions, \( \partial V/T(i) = \partial V/X_i = 0 \), should be satisfied. That leads to the following trilinear equation for \( T(i) \) through the use of eqs. (9) and (14),

\[
g(T(i)) \equiv \left( \frac{2}{k_q} T(i) - h \right) f(T(i)) - \frac{2}{k_T} m_T^2 T(i) = 0. \tag{18}
\]

If this equation has a solution in the region given by (16), the broken phase is realized. If the inequality (17) is satisfied and \( m_T^2 \) is not negative, the function \( g(T(i)) \) has always a local maximum point.

Suppose, for a while, that \( h > 0 \). Note that the values of \( g(T(i)) \) at the boundaries of the region (16) are negative if \( m_T^2 > 0 \). Thus, it is the condition for the broken phase that the local maximum point of \( g(T(i)) \) should be within the region (16) and at that point the value of \( g(T(i)) \) should not be negative. That leads to the following condition

\[
\left[ \frac{1}{3} h^2 + \frac{2m_T^2}{3k_T} - \frac{4}{3} m_{av}^2 \right]^3 - \left( \frac{m_{av}^2 h}{k_T} \right)^2 \geq 0, \tag{19}
\]

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where $m_{av}^2 = (m_q^2 + m_{\bar{q}}^2)/(2k_q)$. Similarly we can calculate the case with $h < 0$. The above condition (19) is available for both cases, $h > 0$ and $h < 0$. Here we define the ratio $\rho$,

$$\rho \equiv \frac{m_T^2}{m_{av}^2 k_T}.$$  \hfill (20)

For $\rho \geq 1$, the condition (19) implies

$$\frac{h^2}{m_{av}^2} \geq A_1(\rho), \quad A_2(\rho) \geq \frac{h^2}{m_{av}^2} \geq A_3(\rho),$$  \hfill (21)

where the points, $h^2 = A_i(\rho)m_{av}^2$ for $i = 1, 2, 3$, are the boundary points for the inequalities (19) to become equalities: $A_1(\rho) > A_2(\rho) > A_3(\rho)$. Fig. 1 shows $A_1(\rho)$ and $A_2(\rho)$ as a function of $\rho$, while $A_3(\rho)$ is always negative. For $\rho < 1$, only the first inequality in (21) is meaningful. Note that $A_1(\rho) > 4$, for any value of $\rho$. Then we obtain the broken phase in the soft parameter region satisfying the conditions (17) and (19). In the region with $A_2(\rho) < 4$, the broken phase is realized only for $h^2 \geq A_1(\rho)m_{av}^2$. On the other hand, in the region with $A_2(\rho) > 4$ the broken phase is realized for $A_2(\rho)m_{av}^2 \geq h^2 \geq 4m_{av}^2$, as well as for $h^2 \geq A_1(\rho)m_{av}^2$. The scalar potential also has the unbounded-from-below directions for negative soft scalar mass squared, e.g. for $m_q^2$, $m_{\bar{q}}^2$ and $m_T^2$. Otherwise, we have the unbroken phase. Figs. 2 and 3 show this phase structure, as an example for $\rho = 1$ and 20, respectively.
Fig. 1: $A_1(\rho)$ and $A_2(\rho)$ in (21) as functions of $\rho$ defined in (20).

Fig. 2: The phase structure for $\rho = 1$. UFB denotes the unbounded-from-below direction.
In the unbroken phase none of local or global symmetries is broken except the $R$-symmetry. All the dual quarks and singlet fermions $\chi_T$ are massless, although their scalar partners become massive due to the soft mass terms. In the broken phase the dual gauge symmetry is broken completely and $\tilde{N}_c$ flavours of dual quarks become massive. Only $N_c (= N_f - \tilde{N}_c)$ flavour of dual quarks and $N_c \times N_c$ singlet fermions $\chi_T$ remain massless. They have the global symmetry $SU(N_c)_q \times SU(N_c)_{\bar{q}} \times U(1)_B$. All the scalar fields become massive. This breaking pattern is similar to the one discussed in Ref. [9], but slightly different. In Ref. [9], the flavour symmetry is broken by hand, i.e. by nondegenerate soft scalar masses, while some of them are taken to be imaginary. That leads to the same type of gauge symmetry breakdown. However, in that model the fields $T$ do not develop their vacuum expectation values. On the other hand, in our model spontaneous symmetry breaking can occur without the breaking of flavour symmetry by hand even for positive values of soft scalar mass squared. In addition, the fields $T$ also develop their vacuum expectation values in the broken phase of our model.

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[9] Here the term dual “quark” has not to be taken literally since the dual gauge group is completely broken. Thus, these dual quarks are nothing but singlet fermions.
3 Softly broken dual pair

In this section we consider the relation between softly broken original SUSY QCD and dual theories. In the original theory the soft scalar mass terms as well as gaugino mass terms are all we can add as soft SUSY breaking terms, i.e.

$$L_{SB} = -m_Q^2 |Q|^2 - m_{\bar{Q}}^2 |\bar{Q}|^2.$$  \hspace{1cm} (22)

Let us discuss the unbroken phase \( q = \bar{q} = T = 0 \), where one of the conditions (17) and (19) is not satisfied. In this case the structure of massless fermions and global symmetries except gauginos and \( R \)-symmetry is not changed compared with the SUSY limit. Thus, this case leads to the same anomaly structure for the unbroken global symmetry \( SU(N_f) \times SU(N_f) \times U(1)_B \) as the SUSY limit. On the other hand, we have the unbroken phase \( Q = \bar{Q} = 0 \) for \( m_Q^2 > 0 \) and \( m_{\bar{Q}}^2 > 0 \). In this case no local or global symmetry is broken except the \( R \)-symmetry, which is broken by gaugino mass terms. Moreover, all the quarks remain massless. Thus the anomaly structure is the same as for the SUSY limit, e.g.

$$SU(N_f)^3 \text{ and } SU(N_f)^2 U(1)_B.$$  \hspace{1cm} (23)

Therefore, this dual pair has the same anomaly structure in the unbroken phase even in the presence of soft SUSY breaking terms. That seems to imply the presence of Seiberg’s duality in this phase even after SUSY breaking with the \( A \)-terms. This observation has been already made in Ref. [8], although the \( A \)-terms were not included in the discussions.

Let us extend the above consideration to the broken phase and notice that large symmetry breaking takes place in the broken phase discussed in the previous section. Here we simplify the issue and consider the following model. When adding the soft SUSY breaking terms, we break the flavour symmetry \( SU(\tilde{N}_c)_q \times SU(\tilde{N}_c)_{\bar{q}} \) into \( SU(\tilde{N}_c - 1)_q \times U(1)_q \times SU(\tilde{N}_c - 1)_{\bar{q}} \times U(1)_{\bar{q}} \). Then we assume the first flavour has soft scalar masses, \( m_{q1} \) and \( m_{\bar{q}1} \), different from the others, \( m_q \) and \( m_{\bar{q}} \). Recall that the \( i \)-th flavour is decoupled from the other flavours in all the conditions and equations to realize the broken phase. Here we assume that only the soft scalar masses of the first flavour, \( m_{q1} \) and \( m_{\bar{q}1} \), satisfy the breaking conditions, (17) and (19). In this

\[\text{This scenario where only one flavour is different from the others, could be conceivable in the same way as the top quark is much heavier than the rest in the real world.}\]
case only the vacuum expectation values $X_1$ and $T_{(1)}$ are developed. That leads to the gauge symmetry breaking,

$$SU(\tilde{N}_c) \to SU(\tilde{N}_c - 1) \ (= SU(N_f - 1 - N_c)).$$  \hfill (24)

Furthermore, $(N_f - 1)$ flavours of dual quarks and $(N_f - 1) \times (N_f - 1)$ singlet fermions $\chi_T$ remain massless. These massless fermions have the global symmetry $SU(N_f - 1)_q \times SU(N_f - 1)_{\tilde{q}} \times U(1)_{B'}$. Massless dual quarks, $\psi_q$ and $\psi_{\tilde{q}}$, and singlet fermions $\chi_T$ transform as $(\tilde{N}_f, 0, N_c/(\tilde{N}_c - 1))$, $(0, N_f, -N_c/(\tilde{N}_c - 1))$ and $(N_f, \tilde{N}_f, 0)$ under this global symmetry, respectively. All the scalar fields become massive. This structure of massless fermions obviously corresponds to the SUSY model with $SU(N_f - 1 - N_c)$ gauge group and $(N_f - 1)$ flavours of quarks. This SUSY model is dual to SUSY QCD theory with $SU(N_c)$ gauge group and $(N_f - 1)$ flavours of quarks.

Let us consider now the corresponding original theory. If at the SUSY breaking scale, the flavour symmetry is broken in the same way as the one of the dual theory, $SU(N_f - 1)_q \times SU(N_f - 1)_{\tilde{q}}$, nothing would prevent the appearance of the following superpotential:

$$W = M_1 \hat{Q} \hat{\bar{Q}}.$$

Note that in this case the $B$-term, $-M_B^2 \hat{Q} \hat{\bar{Q}}$, can also appear as the soft terms in the lagrangian $L_{SB}$. Thus, the (mass)$^2$ matrix of the first flavour of squarks, $M_{11}^2$, is written as

$$M_{11}^2 = \begin{pmatrix} m_{Q1}^2 + M_1^2 & -M_B^2 \\ -M_B^2 & m_{\tilde{Q}1}^2 + M_1^2 \end{pmatrix}.$$ \hfill (26)

If $\det(M_{11}^2) > 0$, the potential minimum corresponds to $Q_1 = \bar{Q}_1 = 0$ and the gauge symmetry $SU(N_c)$ remains unbroken. In this case $(N_f - 1)$ flavours of quarks remain massless and these massless fermions have the global symmetry $SU(N_f - 1)_q \times SU(N_f - 1)_{\tilde{q}} \times U(1)_B$. All scalar fields become massive. This model has the same anomaly structure as the softly broken dual theory in the broken phase, e.g. for

$$SU(N_f - 1)^3 \quad \text{and} \quad SU(N_f - 1)^2 U(1)_B,$$

where $U(1)_B$ should be replaced by $U(1)_{B'}$ in the dual theory. That seems to suggest the presence of Seiberg’s duality after SUSY breaking even in the broken phase.
Let us discuss the case with \( \det(M^2_B) < 0 \) and \( m^2_{Q_1} + m^2_{\bar{Q}_1} + 2M^2_F > 2|M^2_B| \). In this case squarks \( Q^1 \) and \( \bar{Q}_1 \) develop their finite vacuum expectation values and the gauge symmetry is broken into \( SU(N_c - 1) \). Only the \((N_f - 1)\) flavours of quarks remain massless and they have the global symmetry \( SU(N_f - 1)_Q \times SU(N_f - 1)_{\bar{Q}} \times U(1)_B \). This case seems to correspond to the dual theory for the unbounded-from-below direction, i.e. \( m^2_{\bar{Q}_1} < 0 \), where the \( SU(\tilde{N}_c) \) gauge symmetry is unbroken and \((N_f - 1)\) flavours of dual quarks remain massless.

We note that such a dual theory along this specific direction has no stable vacuum within its own framework, and thus this unbounded-from-direction can not be described within the framework of the dual theory.

We have considered the case where only one flavour of squarks develop their vacuum expectation values. We can easily extend the above discussion to the case when more flavours of squarks develop their vacuum expectation values. Then we can obtain similar relations between softly broken original and dual theories in the broken phase.

Let us also give some comments on the unbounded-from-below directions for \( m^2_q < 0 \). In this case the scalar potential of the dual theory is unbounded from below and vacuum expectation values of \( q_{(i)} \) and the baryonic operator \( b = \prod q_{(i)} \) run away to infinity, \( q_{(i)} \rightarrow \infty \). This baryonic operator corresponds to the baryonic operator of \( Q_i \) in the original theory. Thus the unbounded-from-below direction for \( m^2_q < 0 \) in the dual theory corresponds to the unbounded-from-below direction for \( m^2_{\bar{Q}} < 0 \), where vacuum expectation values of \( Q_i \) and their baryonic operator go to infinity. We have the same situation for \( m^2_{\bar{Q}} < 0 \) and \( m^2_Q < 0 \).

4 Conclusions

We have studied the softly broken SUSY QCD taking into account the effects of \( A \)-terms. We have investigated the phase structure of the softly broken dual theory and have found that the \( A \)-terms play a basic role in the realization of the broken phase. Also we have found relations between softly broken dual pair even in the broken phase. These results should be useful in the understanding of QCD and confinement in the real world. Detailed quantitative results, including the mass spectra, will be discussed elsewhere.

Seiberg’s duality can be understood from the viewpoint of \( D \)-brane dynamics. It is interesting to study our results in non-SUSY cases also from
the viewpoint of $D$-brane dynamics.

One could discuss in a similar way the case with $N_f \leq N_c+1$ as treated in Ref.\[8\] but with generic soft breaking terms in order to investigate whether they play any specific role.

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