Yukawa Textures, New Physics and Nondecoupling

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Abstract

We point out that New Physics can play an important rôle in rescuing some of the Yukawa texture zero ansätze which would otherwise be eliminated by the recent, more precise measurements of $V_{CKM}$. As an example, a detailed analysis of a four texture zero ansatz is presented, showing how the presence of an isosinglet vector-like quark which mixes with standard quarks, can render viable this Yukawa texture. The crucial point is the nondecoupling of the effects of the isosinglet quark, even for arbitrary large values of its mass.

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1 Introduction

The increasingly higher precision in the determination of the elements of the fermion mixing matrices, both in the quark and lepton sectors is clearly one of the most significant recent developments in particle physics and provides a great challenge to flavour models.

In most of the attempts at understanding the observed pattern of fermion masses and mixing, one assumes the existence of family symmetries either abelian or non-abelian, leading to special flavour structures in the Yukawa matrices, often involving texture zeros and/or a Froggatt-Nielsen type power structure of the matrix elements, in terms of a small expansion parameter. In the search for the allowed texture zeros, one may take a bottom-up approach, where one uses the input data on fermion masses and mixing to derive the Yukawa textures which are allowed by experiment. Some years ago, Ramond, Roberts and Ross (RRR) [2], in a pioneering work, followed this bottom-up approach and made a systematic search for allowed quark Yukawa structures. Assuming symmetric or Hermitian Yukawa matrices and using the experimental data available at the time, RRR found a total of five possible solutions in a survey of all six and five texture-zero ansätze. Meanwhile, with the impressive improvement in the experimental determination of the $V_{CKM}$ matrix, all the texture-zero structures found in [2] have great difficulty in reproducing the data. One of the greatest challenges to these models arises from the precise determination of the rephasing invariant angle $\beta \equiv \arg (-V_{cd}^* V_{td}^* V_{tb})$. Indeed, it has been pointed out [3] that in a large class of texture zero models which include all those considered by RRR, one cannot have a sufficiently large value of $\sin(2\beta)$, to conform to the present experimental value $\sin(2\beta) = 0.687 \pm 0.032$ [4].

Another important constraint arises from the experimental value of $B_d^0 - \bar{B}_d^0$ mixing, combined with the recent measurement of $B_s^0 - \bar{B}_s^0$ mixing by D0 [5] and CDF [6], which leads to the extraction of the ratio $|V_{td}|/|V_{ts}|$ with relatively small errors. Very recently, the rephasing invariant phase $\gamma \equiv \arg (-V_{ud}^* V_{ub}^* V_{cd}^* V_{cb})$ has been measured by Belle [7], [8], [9], and BaBar [10] leading to the value $\gamma = (63^{+15}_{-12})^0$ [4]. In spite of the large experimental errors, the measurement of $\gamma$ is of crucial importance due to the fact that its extraction from input data is essentially not affected by the presence of New Physics (NP) contributions to $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings.

In the study of the impact of NP on the test of Yukawa textures, one has to specify what are the assumptions on the nature of NP. In most of the NP scenarios considered in the literature, one usually assumes that NP does not contribute significantly to the tree level decays of strange and B-mesons. This implies that NP does not affect the extraction of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and $\gamma$, from experimental data. With these four inputs, one can reconstruct the full $V_{CKM}$ matrix and in particular the reference unitarity triangle [11], [12], [13]. However, we may assume that there may be significant contributions from NP to $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixing, thus affecting the extraction of $|V_{td}|$, $|V_{ts}|$ as well as $\beta$ from experiment. Apart from these effects, we may assume in these scenarios that NP decouples from low energy physics. This
assumption holds in a large class of models beyond the SM and in particular in most of the supersymmetric extensions of the Standard Model (SM). Let us now consider the implications for the tests of Yukawa textures. If a particular texture predicts a value of $\beta$ and/or of $|V_{td}|$ in disagreement with experiment, it may be rescued by the presence of NP contributions to $B_d^0-\bar{B}_d^0$ mixing. However, this class of NP cannot rescue a texture which predicts a value $|V_{ub}|/|V_{cb}|$ and/or $\gamma$ in disagreement with experiment. This question is specially relevant, due to the fact that some of the most attractive Yukawa textures do have difficulty in conforming to the measured values of $|V_{ub}|/|V_{cb}|$ and $\gamma$.

In this paper we will show that there is a class of NP which can solve the above conflict and thus render viable those Yukawa textures. At this stage, it is worth recalling that in most proposals for family symmetries which could shed some light on the flavour puzzle, there is the underlying assumption that, even if the family symmetry is embedded in a grand-unified theory, the heavy particles decouple and do not affect low energy physics, in particular the masses and mixing observable at low energies. In this paper, we will consider a scenario with heavy fermions where the decoupling does not occur and in particular heavy fermions do affect the effective standard fermion mass matrices at low energies. We will show that the influence of heavy vector-like quarks is such that they may render viable some of the texture zero structures which would otherwise be eliminated by the more precise data presently available on the $V_{CKM}$ matrix. The embedding of a family structure into a larger framework where heavy fermions are included, has the following interesting feature. Let us consider a flavour model based on $SU(3)_c \times SU(2)_L \times U(1) \times F$ where $F$ denotes a family symmetry responsible for the presence of a set of texture zeros in the three by three quark mass matrices $M_u$ and $M_d$. This symmetry can be trivially embedded into a larger framework with isosinglet vector-like heavy quarks $Q$ by assuming that the SM fields keep their transformation properties under $F$ and allowing for $F$ to be softly broken by $SU(3)_c \times SU(2)_L \times U(1)$ invariant mass terms connecting $Q$ to standard quarks. The striking feature of the example we will consider, with one singlet down-type vector-like quark, is that its effect in the low energy standard fermion masses and mixing can be sizeable even in the limit where heavy quark masses are very large and deviations from unitarity of the $V_{CKM}$ matrix are arbitrarily suppressed.

The paper is organized as follows. In the next section, we analyse in detail a four texture zero Hermitian ansatz and illustrate its difficulties in accommodating the present data on $V_{CKM}$. In section 3, we present an example of nondecoupling of NP and analyse in an analytical qualitative way how the presence of vector-like quarks can render the four texture zero ansatz compatible with our present knowledge on $V_{CKM}$. In section 4, we provide an explicit example which is solved numerically, confirming our analysis of section 3. Finally, we present our conclusions in section 5.
2 A Four Texture Zero Hermitian Ansatz

Several Hermitian ansätze with texture zeros have been studied in the literature. These ansätze lead in general to predictions which usually consist of simple relations for the mixing angles expressed in terms of quark mass ratios. It is worth emphasizing that Hermiticity is as important as the existence of texture zeros, in order to obtain predictive ansätze. Indeed, it has been shown [14] that if one drops the requirement of Hermiticity, most of the texture-zero ansätze can be obtained, starting from arbitrary quark mass matrices $M_u, M_d$, by simply making weak-basis transformations. This shows that without the requirement of Hermiticity those texture zeros have no physical implications.

For definiteness, we consider a specially interesting four zero ansatz which has been analysed in detail in the literature [15], [16]. The quark mass matrices $M_u, M_d$ are assumed to have the form:

$$M_u = \lambda_u K_u^+ \begin{bmatrix} 0 & a_u & 0 \\ a_u & b_u & c_u \\ 0 & c_u & 1 - b_u \end{bmatrix} K_u \quad ; \quad M_d = \lambda_d \begin{bmatrix} 0 & a_d & 0 \\ a_d & b_d & c_d \\ 0 & c_d & 1 - b_d \end{bmatrix}$$ (1)

where $K_u = diag(e^{i\phi_1}, 1, e^{i\phi_3})$ and all other parameters are real.

It is clear from Eq. (1) that the trace of each matrix was factored out so that one has, by construction:

$$\text{Tr}(M_u) = \lambda_u \quad ; \quad \text{Tr}(M_d) = \lambda_d$$ (2)

The convention of phases adopted in $K_u$ corresponds to the factoring out of all phases in $M_u$ and $M_d$ and the elimination of the maximum number of non-physical ones. It is clear that no non-factorizable phases remain in this ansatz due to the existence of one zero off-diagonal entry in both $M_u$ and $M_d$. It was shown in Ref. [3] that the absence of nonfactorizable phases leads to important restrictions on $\sin 2\beta$.

The presence of several zeros in this Hermitian ansatz renders the analytical diagonalization of the mass matrices quite simple. The column vectors of the unitary matrices $U^u, U^d$ which diagonalize $M_u, M_d$ can be determined, in each case, via the vector product of the first and third rows, with the inclusion of the mass eigenvalues. Each one of the three columns can be expressed as:

$$(-m_i', a, 0) \times (0, c, 1 - b - m_i') \quad \frac{1}{N_i}$$ (3)

where we have omitted the $u, d$, sub-indices, $N_i$ is a normalization factor and $m_i'$ denotes the $i$th mass eigenvalue divided by the sum of the three mass eigenvalues. The mass eigenvalues do not depend on $K_u$ and thus both in the up and down quark sectors, the four parameters $a, b, c, \lambda$ can be expressed in terms of quark masses, leaving only one free parameter in each sector, which we may chose to be $b_{(u,d)}$. This allows one to write each of the unitary matrices $U^u, U^d$ in terms of quark mass ratios, a single free parameter and the phases $\phi_1, \phi_3$ which have been factored out.
In this way one obtains the well known texture zero relations [17], [2], [18], [16] valid to leading order:

\[
\left| \frac{V_{ub}}{V_{cb}} \right| = \sqrt{\frac{m_u}{m_c}} \quad \left| \frac{V_{td}}{V_{ts}} \right| = \sqrt{\frac{m_d}{m_s}} \quad \left| V_{us} \right| = \sqrt{\frac{m_d}{m_s}} e^{i\phi_1} - \sqrt{\frac{m_u}{m_c}}
\]  

(4)

which are verified by a wide class of models [17]. Furthermore, as pointed out in the introduction, it has been shown [3] that texture zero ansätze with no non-factorizable phases, such as this one, cannot reach values of \(\sin(2\beta)\) as high as the present central value [4]:

\[\sin(2\beta) = 0.687 \pm 0.032 \]  

(5)

It was already pointed out in Ref. [16] that the relation obtained for \(\left| \frac{V_{ub}}{V_{cb}} \right|\) was problematic, and strongly disfavoured this ansatz, due to the smallness of the ratio \(\sqrt{\frac{m_u}{m_c}}\). At present the constraint has become even more severe, since the new experimental average for \(\left| V_{ub} \right|\) went up significantly. The current experimental values for these two \(V_{CKM}\) entries are [4]:

\[\left| V_{ub} \right| = (4.31 \pm 0.30) \times 10^{-3} \quad \left| V_{cb} \right| = (41.6 \pm 0.6) \times 10^{-3}\]  

(6)

whilst the values of the quark mass matrices are taken as [4]:

\[m_u = 1.5 - 3.0 \text{ (Mev)}, \quad m_c = 1250 \pm 0.090 \text{ (Mev)}, \quad m_t \simeq 300 \text{ (Gev)}\]
\[m_d = 3 - 7 \text{ (Mev)}, \quad m_s = 95 \pm 25 \text{ (Mev)}, \quad m_b = 4.2 \pm 0.07 \text{ (Gev)},\]  

(7)

\[m_u/m_d = 0.3 - 0.7 \quad m_s/m_d = 17 - 22\]

The new theoretically clean and significantly improved constraint on \(\left| \frac{V_{td}}{V_{ts}} \right|\) is [4]:

\[\left| \frac{V_{td}}{V_{ts}} \right| = 0.208^{+0.008}_{-0.006}\]  

(8)

Taking into account the small experimental error, this results deviates significantly from the value predicted by the ansatz, \(\left| \frac{V_{td}}{V_{ts}} \right| = \sqrt{\frac{m_d}{m_s}}\) which leads to the range:

\[0.213 < \left| \frac{V_{td}}{V_{ts}} \right| < 0.243\]  

(9)

where we took into account the experimental constraint on \(m_s/m_d\) given above.

Next we briefly describe how the value of \(\gamma\), \(\gamma \equiv \arg (-V_{ud}V_{ub}^{\ast}V_{cd}^{\ast}V_{cb})\) for this ansatz can be derived analytically. It is clear from Eq. (1) that in our parametrization \(\arg V_{ud} = \phi_1\), to an excellent approximation. The phase \(\phi_1\) is fixed by the experimental value of \(\left| V_{us} \right|\), given by Eq. (4). Using the central values for the quark mass ratios:

\[\sqrt{\frac{m_d}{m_s}} = \sqrt{\frac{1}{20}} = 0.224; \quad \sqrt{\frac{m_u}{m_c}} = 0.042\]  

(10)

together with [4] \(\left| V_{us} \right| = 0.2257 \pm 0.0021\), one obtains

\[\phi_1 = -87^\circ\]  

(11)
Now, to leading order, one has $V_{ub}/V_{cb} = -\sqrt{\frac{m_u}{m_c}}$, implying $\arg(V_{ub}^* V_{cb}) \simeq \pi$ and one obtains in good approximation:

$$\gamma \simeq \arg(V_{ud} V_{cd}^*)$$

(12)

Using the fact that in leading order in our parametrization

$$V_{cd} = -\sqrt{\frac{m_d}{m_s}} + \sqrt{\frac{m_u}{m_c}} e^{i\phi_1}$$

(13)

and taking central values for the quark mass ratios, one obtains

$$\arg(V_{cd}^*) = 169^\circ$$

(14)

Which finally leads to:

$$\gamma = \arg V_{ud} + \arg V_{cd}^* = (-87^\circ + 169^\circ) = 82^\circ$$

(15)

It is clear that in the framework of this ansatz, the value of $\gamma$ is very constrained, even allowing for one $\sigma$ deviations from central values of the experimental parameters. At present, the current experimental value $[4] \gamma = (63^{+15}_{-12})^\circ$ has large errors and therefore it is not possible to exclude the ansatz only on the grounds of the $\gamma$ constraint. However, it is clear that the ansatz tends to give values for $\gamma$ larger than the central experimental value.

Concerning $\beta$, it was shown in a previous work $[3]$ that only $\arg(V_{cd})$ contributes significantly, so that in this framework we have:

$$\beta \simeq -\arg V_{cd} \simeq 180^\circ - 169^\circ = 11^\circ$$

(16)

In summary, the four texture zero ansatz of Eq. (1) has serious difficulties in accommodating the recent, more precise, experimental data on $V_{CKM}$. It is useful to separate these difficulties of the ansatz in two classes:

(i) The ansatz predicts too small a value for $\beta$ and too large a value for $|V_{td}|/|V_{ts}|$.

(ii) The ansatz predicts too small a value for $|V_{ub}|/|V_{cb}|$ and too large a value for $\gamma$

The important point we wish to emphasize is that, while difficulties of class (i) can be avoided by assuming New Physics (NP) contributions to $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$ mixings, those of class (ii) remain a challenge to the ansatz even in the presence of NP contributions to mixing. It is useful to parametrize NP contributions to mixing in the following way:

$$M_{12}^{(q)} = (M_{12}^{(q)})^{SM} r_q^2 e^{-2i\phi_q} \quad q = d, s$$

(17)
The SM corresponds to \( r_q = 1, \phi_q = 0 \). In the presence of NP, instead of \( \beta \) one measures \( \beta - \phi_d \) and \( \Delta M_{B_q} = (\Delta M_{B_q})^{SM}_q r_q^2 \). It is clear that even a small contribution of \( \phi_d \) (i.e. \( \phi_d = -11^\circ \)) together with a small deviation of \( r_d/r_s \) from unity can rescue the ansatz from discrepancies of class (i). On the contrary, the extraction of \( |V_{ub}|/|V_{cb}| \) and \( \gamma \) from experiment is unaffected by the presence of NP in the mixing. At this stage it should be noted that in most of the extensions of the SM, including the supersymmetric ones, there are NP contributions to the mixing [19] – [25].

An alternative way of checking that this interesting ansatz is in conflict with experiment is through the use of the following exact unitarity relation [26]:

\[
\frac{\sin \beta}{\sin(\gamma + \beta)} = \frac{|V_{ub}| |V_{ud}|}{|V_{cb}| |V_{cd}|}
\]  

as well as another unitarity relation, that holds to an excellent approximation [26]:

\[
\frac{\sin \gamma}{\sin(\gamma + \beta)} \simeq \frac{|V_{td}| / |V_{ts}|}{|V_{us}|} 
\]

Replacing in Eq. (18) the values obtained in this ansatz for \( |V_{ud}| \simeq 1, |V_{cd}| \) and the ratio \( |V_{ub}| / |V_{cb}| \) we obtain the prediction \( \sin \beta / \sin(\gamma + \beta) \simeq 0.19 \). This is to be compared to the value computed with experimental central values \( \sin \beta / \sin(\gamma + \beta) \) \text{exp} \simeq 0.37. Likewise for the relation given by Eq. (19), where in this case the ansatz predicts \( \sin \gamma / \sin(\gamma + \beta) \simeq 0.99 \), while the value computed with the experimental central values is 0.89. We have also used the unitarity relations of Eqs. (18), (19) to verify the validity of our approximate analytical evaluation of the elements of \( V^{CKM}_\mathbb{C} \), predicted by the ansatz.

Given the difficulties of this ansatz in conforming to the experimental data, one may wonder whether the ansatz may be “saved” if implemented in a larger framework. In the next section, we show that this is indeed the case. We describe a scenario where the presence of NP can fully rescue the four texture zero ansatz by embedding it into a minimal extension of the SM with one additional down vectorial isosinglet quark. This framework could result from a family symmetry of the Lagrangian leading to the existing texture zeros, which is softly broken by mass terms involving the additional heavy quark.

### 3 An example of nondecoupling

Let us consider a model with only one \( Q = -1/3 \) isosinglet vector-like quark. Vector-like quarks arise in a variety of extensions of the SM, in particular, within the framework of grand-unified theories based on \( E_6 \). Another motivation for introducing vector-like quarks arises if one requires spontaneous CP violation [27] – [30] in the context of supersymmetric extensions of the SM [31], [32]. Vector-like quarks are essential in order to generate a complex \( V^{CKM}_\mathbb{C} \) from vacuum phases [33].
It can be easily shown that, without loss of generality, one may choose a weak basis where $M_u$, the up quark mass matrix, is real and diagonal, and the down quark matrix $M_d$ can be cast in the form:

$$
M_d = \begin{pmatrix}
  m_d & | & 0 \\
  0 & | & 0 \\
  0 & | & 0 \\
  - & - & - \\
  M_D & | & H
\end{pmatrix}
$$

(20)

with $m_d$ a Hermitian $3 \times 3$ matrix, $M_D$ a $1 \times 3$ matrix and $H$ a single entry. The matrix $M_d$ is diagonalized by the usual bi-unitary transformation:

$$
U_L^\dagger M_d U_R = \begin{pmatrix}
  \overline{m} & 0 \\
  0 & \overline{M}
\end{pmatrix}
$$

(21)

where $\overline{m} = \text{diag} (m_d, m_s, m_b)$ and $\overline{M}$ is the heavy quark mass. One can write $U_L$ in block form,

$$
U_L = \begin{pmatrix}
  K & R \\
  S & T
\end{pmatrix}
$$

(22)

where $K$ is the usual $3 \times 3 V_{CKM}$ matrix. $U_L$ is the matrix that diagonalizes $M_d M_d^\dagger$, and the following relations can be readily derived in the limit $M_D, H >> O(m_d)$

$$
\overline{M}^2 \simeq (M_D M_d^\dagger + H^2) \equiv M^2
$$

$$
\overline{m}^2 \simeq K^\dagger m_{\text{eff}} m_{\text{eff}}^\dagger K
$$

(23)

(24)

with

$$
m_{\text{eff}} m_{\text{eff}}^\dagger \simeq m_d m_d^\dagger - \frac{(m_d M_D^\dagger M_D m_d^\dagger)}{M^2}
$$

(25)

Note that $K$ is the mixing matrix connecting standard quarks and has small deviations from unitarity given by $K^\dagger K = 1 - S^\dagger S$, with:

$$
S \simeq - \frac{M_D m_d^\dagger K}{M^2} \left(1 + \frac{\overline{m}^2}{M^2}\right)
$$

(26)

At this stage, it should be noted that the mass terms $M_D, H$ are $SU(2) \times U(1)$ invariant and thus they can be much larger than the electroweak scale. If one makes the natural assumption that $M_D M_D^\dagger$ and $H^2$ are of the same order of magnitude it is clear that in Eq. (25), the second term contributing to $m_{\text{eff}} m_{\text{eff}}^\dagger$, has a magnitude comparable to that of $m_d m_d^\dagger$. This is the crucial point which makes it possible to rescue the four texture ansatz considered in the previous section, through the introduction of a vector like isosinglet quark. Let us assume that there is a family symmetry which leads to the four texture zero ansatz, in the $3 \times 3$ quark mass matrices involving standard quarks. The $SU(2) \times U(1)$ invariant mass terms $M_D,$
\( H \) may break softly the family symmetry. It is clear that the presence of the second term contributing to \( m_{\text{eff}}m_{\text{eff}}^\dagger \) in Eq. (25), does affect the predictions of the ansatz, allowing for it to be in agreement with the present experimental data. In what follows, we explain how this is possible, first through a qualitative analysis and then in the next section through an exact numerical example.

At this stage the following comment is in order. It is well known that in models with isosinglet quarks there are Z mediated flavour changing neutral currents (ZFCNC) [35], [36], with strength proportional to deviations of \( 3 \times 3 \) unitarity of the \( V_{\text{CKM}} \) matrix. From Eq. (26) it is clear that deviations of unitarity are proportional to \( \frac{m_d^2}{M^2} \) [37], [34], and therefore are naturally suppressed. As a result, choosing both \( M \) and \( H \) much larger than \( m_d \) strongly suppresses ZFCNC. This in turn implies that for sufficiently large \( M \) the extraction of \( \beta \) and of \( |V_{ts}| / |V_{td}| \) are not significantly changed from the one based on SM physics.

Let us consider the following structure for \( M_d \), with the previous texture zero Hermitian ansatz embedded in the new four by four down mass matrix:

\[
\mathcal{M}_d = \begin{pmatrix}
M_d & 0 \\
- & - \\
M_D & -
\end{pmatrix}
= \begin{pmatrix}
0 & A & 0 & 0 \\
A & B & C & 0 \\
0 & C & D & 0 \\
0 & f & g & H
\end{pmatrix}
\tag{27}
\]

it is now possible to compute \( m_{\text{eff}}m_{\text{eff}}^\dagger \), to a good approximation, using Eq. (25). We are now interested in combining this larger ansatz for \( M_d \) with \( M_u \) following the previous pattern of Hermiticity and texture zeros.

It is known [38], [16] that the present experimental data can be well reproduced from the following Froggatt-Nielsen pattern for \( m_{\text{eff}} \):

\[
m_{\text{eff}} \sim m_b \begin{pmatrix}
0 & \varepsilon^3 & \varepsilon^4 \\
\varepsilon^3 & \varepsilon^2 & \varepsilon^1 \\
\varepsilon^4 & \varepsilon^2 & 1
\end{pmatrix},
\tag{28}
\]

with \( \varepsilon \simeq 0.2 \) together with a similar pattern for the up sector in terms of a smaller parameter \( \varepsilon \) with a value close to 0.06.

The required structure for \( m_{\text{eff}}m_{\text{eff}}^\dagger \) is:

\[
m_{\text{eff}}m_{\text{eff}}^\dagger \sim m_b^2 \begin{pmatrix}
\varepsilon^6 & \varepsilon^5 & \varepsilon^4 \\
\varepsilon^5 & \varepsilon^4 & \varepsilon^2 \\
\varepsilon^4 & \varepsilon^2 & 1
\end{pmatrix}
\tag{29}
\]

Using Eqs. (25), (27) it can be verified that, starting from a Froggatt-Nielsen pattern for \( M_d \) given by:

\[
|M_d| \sim m_b \begin{pmatrix}
0 & \varepsilon^3 & 0 \\
\varepsilon^3 & \varepsilon^2 & \varepsilon^2 \\
0 & \varepsilon^2 & 1
\end{pmatrix},
\tag{30}
\]
in the context of the previous four texture ansatz, implemented in the above extension of the SM, it is possible to obtain a matrix \( m_{\text{eff}} m_{\text{eff}}^\dagger \) with the structure given by Eq. (29), by assuming: \( \frac{\mu q}{H^2} \sim \bar{z} \), in Eq. (27).

This implies that the extension of the SM through the inclusion of one down vector-like isosinglet quark can save the ansatz discussed in the previous section. Since no additional quarks were introduced in the up sector, the matrix \( M_u \) remains unchanged. However, due to the different hierarchies of the quark masses in the up and down sector, the sensitivity of the \( V_{CKM} \) matrix is much higher to changes in the down sector than to changes in the up sector.

4 A numerical Example

In this section we give an explicit example which illustrates the above described framework.

Let us consider the following mass matrices in Gev units:

\[
\mathcal{M}_d = \begin{pmatrix}
0 & 0.0258 & 0 & 0 \\
0.0258 & 0.12 & 0.24 & 0 \\
0 & 0.24 & 4.97 & 0 \\
0 & 350 & 370 i & 500
\end{pmatrix} ; \quad M_u = K_u^\dagger \begin{pmatrix}
0 & 0.056 & 0 \\
0.056 & 1.3 & 2.8 \\
0 & 2.8 & 300
\end{pmatrix} K_u
\]

where \( K_u = \text{diag}(e^{i\phi_1}, 1, e^{i\phi_3}) \) with \( \phi_1 = -98.1^\circ \) and \( \phi_3 = 0.0^\circ \). It can be readily verified that \( \mathcal{M}_d \) and \( M_u \) lead to the following masses and mixing:

\[
(m_u, m_c, m_t) = (0.00246, 1.28, 300.0) \text{ in GeV;}
\]

\[
(m_d, m_s, m_b, M_4) = (0.0058, 0.0935, 4.3, 718.7) \text{ in GeV;}
\]

\[
V_{CKM}(3 \times 4) = \begin{pmatrix}
0.9743 & 0.2252 & 0.0036 & 0.000013 \\
0.2251 & 0.9735 & 0.0410 & 0.00016 \\
0.0084 & 0.0402 & 0.9991 & 0.0036
\end{pmatrix};
\]

The fourth column corresponds to matrix \( R \) in Eq. (22). An interesting feature of this example is the extreme smallness of the deviations from unitarity of the \( 3 \times 3 \) \( V_{CKM} \) matrix as confirmed by the smallness of all the entries in \( R \).

The corresponding values for \( \sin(2\beta) \) and \( \gamma \) are:

\[
\sin(2\beta) = 0.707, \quad \gamma = 66.1^\circ
\]

which are in good agreement with the present experimental bounds. The ratio \( |V_{ub}| / |V_{cb}| \) in this example is equal to 0.088 and therefore is significantly larger than \( \sqrt{m_u/m_c} \). The fact that \( |V_{ub}| \) in our example does not exactly agree with the new experimental constraint of Eq. (19) should not come as a surprise since in our analysis we are constrained by unitarity. It can be easily checked that the present experimental central values deviate from the unitarity relation of Eq. (18), although verifying
Eq. (19). Note that, as emphasized in Ref. [4] the new experimental average for $|V_{ub}|$ is somewhat above the range favoured by the measurement of $\sin(2\beta)$.

The fact that physics at a high energy scale can save a low energy texture that, by itself, was already ruled out by experiment, is perhaps unexpected. More surprising even is the fact that a similar effect could be obtained with a much heavier vectorial quark. It was shown in a recent paper [39] that down-type vectorial isosinglet quarks can also play an important rôle in generating sufficient CP violation in models with universal strength of Yukawa couplings [40],[41].

5 Conclusions

We have studied the impact of New Physics on tests of Yukawa texture zero ansätze, emphasizing that the greatest challenge for these textures arises from the measured values of $|V_{ub}| / |V_{cb}|$ and the rephasing invariant angle $\gamma$. This stems from the fact that while the presence of New Physics contributions to $B_d^0 - \bar{B}_d^0$ and/or $B_s^0 - \bar{B}_s^0$ mixings can solve eventual discrepancies in the predictions for $\beta$, $|V_{td}|$, $|V_{ts}|$, the extracted values of $|V_{ub}| / |V_{cb}|$ and $\gamma$ are unaffected by the presence of New Physics contributions to mixing. We then show that the presence of New Physics which does not decouple at low energies can save some of the most interesting ansätze which would otherwise be in conflict with experiment. We illustrate these effects through a specific four texture zero ansatz which is studied in the context of a minimal extension of the SM with an isosinglet vector-like heavy quark which mixes with the standard quarks. We show that the presence of the heavy quark is sufficient to render viable the ansatz which would otherwise be eliminated by the recent measurements of $|V_{ub}| / |V_{cb}|$ and $\gamma$. The crucial point is the nondecoupling of the effects of the heavy quark, even in the limit where its mass is arbitrarily large. It is clear that analogous considerations may in principle be applied to other texture zero ansätze which, without the input of New Physics would be ruled out by experiment.

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