On the structure of the $\pi\pi$ invariant mass spectra of the $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ decays

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We perform a model-independent analysis for recently reported data of the $\pi^+\pi^-$ invariant mass spectra in the $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ decays and point out that there does exist a broad peak below 0.6 GeV in the data of the $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ decay, which is analogous to that in the $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$ decay. With the data of $\Upsilon(4S)$ decays, we further test our model developed for studying the puzzle in the $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$ decay. The result shows that with such a model, all the $\pi^+\pi^-$ invariant mass spectra of $\Upsilon(4S)$ decays can be described. We also predict the $\cos TH_{\pi}$ distributions of $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ decays, which can be used to justify our model prediction.

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$\Upsilon(4S)$ is the first bottomonium state above the $B\bar{B}$ threshold. The branching fraction for $\Upsilon(4S) \to B\bar{B}$ is larger than 96% [1]. Recently, the $\pi\pi$ invariant mass spectrum of the $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ decay was measured by the Belle Collaboration [2,3] and the BaBar Collaboration [4], respectively, and the decay of $\Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^-$ was reported by the BaBar Collaboration [4]. Their results with rather large error bars are shown in Fig. 3. The branching ratios of decay modes reported by different collaborations are listed in Table I. Although the branching ratio in Ref. [2] seems inconsistent with the newly reported one in Ref. [3], it is compatible with those in Refs. [4,5].

An interesting phenomenon in the the $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$ decay is that the $\pi^+\pi^-$ invariant
TABLE I: The branching ratios of the decay modes $\Upsilon(4S) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$. 

|                  | $B(\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-)(10^{-4})$ | $B(\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-)(10^{-4})$ |
|------------------|---------------------------------------------------------------|---------------------------------------------------------------|
| CLEO [5]         | $< 1.2$                                                       | $< 3.9$                                                       |
| BaBar [4]        | $0.90 \pm 0.15$                                              | $1.29 \pm 0.32$                                              |
| Belle [2]        | $1.1 \pm 0.2 \pm 0.4$                                        |                                                               |
| Belle [3]        | $1.8 \pm 0.3 \pm 0.2$                                        |                                                               |

mass spectrum has a double peak structure, namely a broad peak shows up below 0.6 GeV [6]. In past years, much attention has been attracted to such a phenomenon [7, 8, 9, 10, 11, 12, 13]. Now, the reported $\pi^+\pi^-$ invariant mass spectrum of the $\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ decay also exhibits a distinguishable double peak structure, but in the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay, the lower structure seems not evident [2, 4, 14]. Based on this observation, Vogel suspected that the dipion transitions between two $\Upsilon(nS)$ states with $\Delta n = 2$ are special [15].

In order to explain the double peak structure and the angular distribution in the $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay, we proposed an additional sequential process in our model. In such a process, an intermediate $b\bar{b}qq$ state with $J^P = 1^+$ and $I = 1$ (called $X$) is suggested. By including this sequential process and considering the $S$ wave $\pi\pi$ final state interaction (FSI), we well-described the $\Upsilon(nS) \rightarrow \Upsilon(mS)\pi^+\pi^- (n = 2, 3, m = 1, 2$ and $n > m$) data available in a systematical way [7]. Obviously, the newly reported data will provide us an opportunity to justify our model and check Vogel's assertion.

In this paper, we perform a model-independent analysis for all the $\pi^+\pi^-$ invariant mass spectrum data in the $\Upsilon(4S) \rightarrow \Upsilon(1S, 2S)\pi^+\pi^-$ transitions. Although the Belle Collaboration claimed that the data in Ref. [2] should be superseded by those in Ref. [3], we still use all these data to justify our model proposed in Ref. [7], and consequently, predict the angular distribution for further confirmation in future.

Before processing detailed calculations, we analyze the $\pi^+\pi^-$ invariant mass spectra of the $\Upsilon$ $\pi\pi$ transitions, qualitatively.

1) The isoscalar character of bottomonia limits the isospin of the dipion system to be $I = 0$. Because both $\Upsilon(4S)$ and $\Upsilon(1S)$ are the vector state, the $\pi^+\pi^-$ system favors the $S$ wave than the higher ones. A mass difference of 1.12 GeV between $\Upsilon(4S)$ and $\Upsilon(1S)$ gives the upper limit of the physical $M_{\pi^+\pi^-}$ region. In this region, the well-established isoscalar-scalar meson of $f_0(980)$ would couple to $\pi\pi$ strongly, which causes its width of about 40-100 MeV [1]. Therefore, there might be
a narrow peak or dip in the $\pi^+\pi^-$ invariant mass spectrum of the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay. However, if the bin in the experimental data is wide, as shown in all the reported data sets, the narrow state of $f_0(980)$ might not show up explicitly.

2) In comparison with the measured invariant mass spectrum, namely the differential width, the squared decay amplitude ($|M|^2$) reflects the decay dynamics more directly. Among low partial waves between two pions, only $S$- and $D$-waves can contribute to the decay amplitude. Because the contribution from the $D$-wave is suppressed by a factor of $2/45$ due to the integration of the 2nd order Legendre polynomial, the squared decay amplitude can be approximated by the invariant mass spectrum divided by the integrated phase space. The $\pi^+\pi^-$ invariant mass spectra of the $\Upsilon(3S,4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decays without the phase space factor are shown in Fig. 1. For comparison, we normalize the magnitudes around $m_{\pi^+\pi^-} = 0.41$ GeV in all the data sets from Refs. [2, 3, 4] for the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay and from Ref. [6] for the $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay to the values close to each other. From this figure, one sees that the tendency of the data in the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay, especially in the region lower than 0.65 GeV, is analogous to that in the $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay. This indicates that in the $\pi\pi$ invariant mass spectrum of the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay, a broad peak does exist in the lower $M_{\pi\pi}$ region, which implies that the above mentioned $\Delta n = 2$ rule might not be true.

FIG. 1: The $\pi^+\pi^-$ invariant mass spectra of the $\Upsilon(3S,4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decays without the phase space factor.

Now, we use our proposed model [7] to study $\Upsilon(4S) \rightarrow \Upsilon(1S,2S)\pi^+\pi^-$ decays. The decay mechanism of $\Upsilon(nS) \rightarrow \Upsilon(mS)\pi^+\pi^-$ is shown in Fig. 2, where (a) and (c) represent the contact
diagram and the diagram in tree level with $X$, respectively, and (b) and (d) denote the corresponding diagrams with the $S$-wave $\pi\pi$ FSI.

FIG. 2: The decay mechanism of $\Upsilon(nS) \rightarrow \Upsilon(mS)\pi^+\pi^-$. Solid external lines represent $\Upsilon(nS)$, dashed lines represent pions, and solid intermediate lines represent $X$.

By incorporating appropriately the chiral expansion with the heavy quark expansion to the lowest order, the amplitude of the contact term in the heavy quarkonium $\pi^+\pi^-$ transition can be written as \[ V_0 = -\frac{4}{f_\pi^2} \left[ (g_1 p_1 \cdot p_2 + g_2 p_1^0 p_2^0 + g_3 m_\pi^2) \epsilon^* \cdot \epsilon' + g_4 (p_{1\mu} p_{2\nu} + p_{1\nu} p_{2\mu}) \epsilon^* \epsilon' \right], \] (1)
where $f_\pi = 92.4$ MeV is the pion decay constant, $g_i$ ($i = 1, 2, 3, 4$) are coupling constants, $p_1$ and $p_2$ represent the 4-momenta of $\pi^+$ and $\pi^-$, respectively, $p_{i0}$ ($i = 1, 2$) denote the energies of $\pi^\pm$ in the lab frame, and $\epsilon$ and $\epsilon'$ describe the polarization vectors of the heavy quarkonia, respectively.

It was shown in Refs. \[11, 13\] that the $g_4$-term in Eq. (1) can be ignored in the viewpoint of QCD multipole expansion. Then, Eq. (1) can be reduced to

\[ V_0 = -\frac{4}{f_\pi^2} (g_1 p_1 \cdot p_2 + g_2 p_1^0 p_2^0 + g_3 m_\pi^2) \epsilon^* \cdot \epsilon'. \] (2)

Note that part of $D$-wave components still exist in the $g_2$ term \[7\]

\[ p_{1\mu} p_{2\nu}^\mu = \frac{1}{1-\beta^2} \left( \left( p_1^{0\nu} - \frac{\beta^2 p_1^{\nu}}{3} \right) P_0(\cos \theta_\pi^\nu) - 2\beta^2 p_1^{\nu} p_2(\cos \theta_\pi^\nu) \right), \] (3)

where $\beta$ is the velocity of the $\pi\pi$ system in the rest frame of the initial particle, $p_{1\mu}^* = (p_{1\mu}^0, p_1^\nu)$ and $p_{2\mu}^* = (p_{2\mu}^0, p_2^\nu)$ represent the four-momenta of $\pi^+$ and $\pi^-$ in the c.m. frame of the $\pi\pi$ system, respectively. $P_0(\cos \theta_\pi^\nu) = 1$ and $P_2(\cos \theta_\pi^\nu) = (\cos^2 \theta_\pi^\nu - 1/3)/2$ are the Legendre functions of the 0-th order and the 2-nd order, respectively.

In our model, we stressed that an additional sequential process $\Upsilon(nS) \rightarrow \pi X \rightarrow \Upsilon(mS)\pi\pi$ should also contribute to the decay. Taking a simple $S$-wave coupling for the vertex $\Upsilon(nS) \rightarrow \pi X$, the decay amplitude of the sequential process in Fig. 2 (c) can be written as

\[ V_{X}^{tree} = g_n m \epsilon_\mu^\nu \epsilon'^\nu \left( \frac{-g^\mu\nu + p_X^\mu p_X^\nu/m_X^2}{p_X^2 - m_X^2 + i m_X \Gamma_X} + \frac{-g^\mu\nu + p_{X^-}^\mu p_{X^-}^\nu/m_X^2}{p_{X^-}^2 - m_X^2 + i m_X \Gamma_X} \right), \] (4)
where \( g_{nm} \) is the effective coupling constant among \( \Upsilon(mS), \Upsilon(nS), \pi^+ \) and \( \pi^- \) via an intermediate state \( X \), and \( p_{X \pm} \) denote the momenta of the charged intermediate states of \( X^\pm \), respectively. In Ref. \[7\], we showed that the \( X \) state is necessary in getting a global fit to all the \( \Upsilon(nS) \to \Upsilon(mS)\pi^+\pi^- (n = 2, 3, m = 1, 2, n > m) \) data. Especially in the \( \Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^- \) decay, the contribution of \( X \) is very important for both the dipion invariant mass spectrum and the angular distribution. The predicted mass and width of the \( X \) state are \( m_X = 10.05 \text{ GeV} \) and \( \Gamma_X = 0.688 \text{ GeV} \), respectively \[7\]. It should be mentioned that the predicted \( X \) state is supported by later theoretical calculations with the constituent quark model \[17\] and the QCD sum rules \[18\], respectively.

In Ref. \[7\], it was also shown that the \( S \)-wave \( \pi\pi \) FSI is very important to reproduce the double peak structure in the \( \pi^+\pi^- \) invariant mass spectrum of the \( \Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^- \) decay. Such an importance has further been presented in the study of the heavy quarkonium chromo-polarizability \[19\] that describes the interaction of heavy quarkonia with soft gluons \[20\]. The \( S \)-wave \( \pi\pi \) FSI can properly be treated by the so-called coupled-channel chiral unitary approach \[21\]. In this approach, the \( \pi\pi \) \( S \)-wave phase shifts can be well-described with barely one parameter, 3-momentum cut-off \( q_{\text{max}} = 1.03 \text{ GeV} \), and the low lying scalar mesons \( (\sigma, f_0(980), a_0(980) \) and \( \kappa \) can dynamically be generated with reasonable masses and widths \[21, 22\]. In the \( S \)-wave isoscalar sector, the \( \pi\pi \) and \( K\bar{K} \) channels are taken into account (for detailed information, refer to Ref. \[21\]). By using the phase convention \( |\pi^+\rangle = -|1,1\rangle \) and the normalization \( \langle \pi^+\pi^-|\pi^+\pi^-\rangle = \langle \pi^-\pi^+|\pi^-\pi^+\rangle = \langle \pi^0\pi^0|\pi^0\pi^0\rangle = 2 \) (considering the fact that \( \pi^+, \pi^- \) and \( \pi^0 \) are in the same isospin multiplet), the full amplitude of the \( \pi^+\pi^- + \pi^-\pi^+ + \pi^0\pi^0 \to \pi^+\pi^- \) process can be denoted by \( 2t_{I=0}^{I=0}\pi\pi,\pi\pi \) with \( t_{I=0}^{I=0}\pi\pi,\pi\pi \) being the full coupled-channel amplitude of the \( I = 0 \) \( S \)-wave \( \pi\pi \to \pi\pi \) process. In the \( \Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^- \) decay, the large phase space allows the \( K\bar{K} \) channel to be on shell. Thus, the full amplitude of the \( K^+K^- + K^0\bar{K}^0 \to \pi^+\pi^- \) process can similarly be described by \( 2t_{KK,\pi\pi}^{I=0}/\sqrt{3} \) with the phase convention \( |K^0\rangle = |1/2,1/2\rangle \) and the amplitude \( t_{KK,\pi\pi}^{I=0} \) which denotes the full coupled-channel amplitude of the \( I = 0 \) \( S \)-wave \( K\bar{K} \to \pi\pi \) process. To factorize the FSI from the contact term in Fig. \[2\] (b), the on-shell approximation is adopted as usual. The off-shell effects can be absorbed into phenomenological coupling constants. In this approximation, both the \( \pi\pi \) and \( K\bar{K} \) loops shown in Fig. \[2\] (b) are allowed in the \( \Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^- \) decay, but only the \( \pi\pi \) loop is permitted in the \( \Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^- \) decay. Then, the total transition amplitude for \( \Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^- \) can be given by

\[
t = V_0 + V_{0S} \cdot G_{11} \cdot 2t_{\pi\pi,\pi\pi}^{I=0} + V_{0S}(m_K) \cdot G_{22} \cdot \frac{2}{\sqrt{3}}t_{KK,\pi\pi}^{I=0} + V_{\text{tree}}^{\pi\pi} + g_{ij} \epsilon^\nu \epsilon^\rho G^{\mu\nu}_{\pi\pi} \cdot t_{\pi\pi,\pi\pi}^{I=0},
\]  

(5)
and the transition amplitude for $\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ by

$$
t = V_0 + V_{0S} \cdot G_{11} \cdot 2t^{I=0}_{\pi\pi,\pi\pi} + V_X^{free} + g_{12}\epsilon_\mu\epsilon_\nu G_X^{\mu\nu} \cdot 2t^{I=0}_{\pi\pi,\pi\pi},
$$

where $V_{0S}$ is the $S$-wave projection of $V_0$, and $V_{0S}(m_K)$ is the one for the process with two kaons. $G_{ii}$ and $G_X^{\mu\nu}$ are the two-meson loop integral ($i = 1$ for the $\pi\pi$ loop, and $i = 2$ for the $K\bar{K}$ loop) and the three-meson loop integral, respectively.

$$
G_{ii} = i \int \frac{d^4q}{(2\pi)^4} \frac{1}{q^2 - m_i^2 + i\varepsilon} \frac{1}{(p' - p - q)^2 - m_i^2 + i\varepsilon},
$$

$$
G_X^{\mu\nu} = i \int \frac{d^4q}{(2\pi)^4} \frac{-g^{\mu\nu} + p'_X p'_X/m_X^2}{p'_X - m_X^2 + i\varepsilon} \frac{1}{q^2 - m_\pi^2 + i\varepsilon} \frac{1}{(p' - p - q)^2 - m_\pi^2 + i\varepsilon},
$$

where $p'$ and $p$ represent the momenta of $\Upsilon(4S)$ and $\Upsilon(1S)$, respectively. The loops are calculated with a cut-off momentum $q_{max} = 1.03$ GeV which was used in explaining the data of the $\pi\pi$ $S$-wave scattering [21]. Thus, the FSI of $\pi\pi$ used here is consistent with the interaction of $\pi\pi$ employed in fitting the $\pi\pi$ scattering data. It should be mentioned that in calculating the transition amplitude for the diagram in Fig. 2(d), we do not consider the intermediate $X_{s}(Bb\bar{s}\bar{s})$ state, the strange cousin of $X$, due to its negligible contribution (refer to Ref. [7]). In this way, no additional parameter is introduced in the calculation.

As has been pointed out in Ref. [7], to describe all the bottomonium dipion transition data, including the newly reported data, self-consistently, the $g_2$ value is fixed to be $g_2/g_1 = -0.23$. Then, the data of the $\pi^+\pi^-$ invariant mass spectra in the $\Upsilon(4S) \rightarrow \Upsilon(1S,2S)\pi^+\pi^-$ decays [2, 3, 4] are fitted by three parameters, $g_1$, $g_3/g_1$ and $g_{nm}$, where $m = 1$ and $m = 2$ represent the final states of $\Upsilon(1S)$ and $\Upsilon(2S)$, respectively. Namely, in terms of the function minimization and error analysis package (MINUIT) [23], these parameters can be determined by a least square fit. The resultant parameters with relevant errors are listed in Table III. In order to demonstrate the necessity of the suggested intermediate state $X$, the data fitting without $X$ is also performed, and the relevant parameters are presented in Table III too. Moreover, the central values of the experimental branching widths, $\Gamma(\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-) = 2.2$ keV [2], 3.7 keV [3], 1.8 keV [4] and $\Gamma(\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-) = 2.7$ keV [4], are used in determining the physical values of coupling constants as well. It should be noted that due to $g_{nm} = g_{nX}g_{mX}$, the values of $g_{nm}$ with different $n$ and $m$ are not fully independent. Using the values of $g_{41}$ and $g_{42}$ given in Table III and the value of $g_{31}$ given in Ref. [7], one can deduce $g_{32} = 7.36$ GeV$^2$ which is different from the value of $-0.00418$ GeV$^2$ given in Ref. [7]. Fortunately, because the contribution from the sequential process is not
important in the $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ decay, such a change does not affect the description of the data.

The calculated $\pi^+\pi^-$ invariant mass spectra of the $\Upsilon(4S) \rightarrow \Upsilon(1S,2S)\pi^+\pi^-$ decays are plotted in Fig. 3 where the solid and dashed curves denote the results with and without $X$, respectively. In the case with $X$, a clear bump exists in the low-energy region of the $\pi^+\pi^-$ invariant mass spectra in both the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ and the $\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ decays, while in the case without $X$, no such a structure appears in the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay. It seems that the newly reported $\Upsilon(4S)\pi\pi$ transition data support our model in which the sequential process with $X$ is influential. Moreover, in the $\pi^+\pi^-$ invariant mass spectrum of the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay, our model gives a narrow structure at the place just below 1 GeV. In addition, in the $\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ decay, the calculated results for Fig. 3(d) show a better description of the data if the sequential process with $X$ is also considered. These results can be understood by analyzing the contributions of different terms. For simplicity, in the case of $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$, we only focus our attention on the data set in Ref. [3]. The contributions from different terms are plotted in Fig. 4 where the solid curve denotes the total result, and the dashed, dotted and dash-dotted curves represent the contributions from the terms without $X$, with $X$ only, and the interference term, respectively. It is seen that the narrow peak close to 1 GeV comes from the $S$-wave coupled-channel $\pi\pi$ FSI. Recalling that the isoscalar-scalar meson $f_0(980)$ can be generated dynamically in the coupled-channel chiral unitary approach [21], one can attribute the origin of the narrow peak to the effect of $f_0(980)$. However, due to the limited resolving power of the present data, this narrow peak might not be observed. Higher statistical data should be called. If this peak still cannot be found in the future higher statistical data, one should not be surprised, because it might be canceled by other stuff. An analogue is that in studying $J/\psi \rightarrow \omega\pi^+\pi^-$ decay in the coupled-channel chiral unitary approach [26], due to the contribution of the intermediate state $b_1(1235)$, the peak of $f_0(980)$ cannot be observed [24, 25]. Similar to the situation in the $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay [7], the contribution from the sequential process plays a dominant role in the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay. But, in the $\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-$ decay, the dominant contribution comes from the terms without $X$. Furthermore, in both $\Upsilon(4S)$ decays, the contribution from the interference term is important. Anyway, in order to justify the heavy quarkonium $\pi\pi$ transition model further, the data with higher statistics should be requested.

Using the central values of parameters in Table II, we calculate the $\cos\theta^*_\pi$ distributions for both $\Upsilon(4S)$ dipion transitions (for $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$, we use the parameter set determined from the data in Ref. [3]). The results are presented in Fig. 5 where the solid and dashed curves represent...
TABLE II: Resultant parameters in fitting the $\pi^+\pi^-$ invariant mass spectra of the $\Upsilon(4S) \to \Upsilon(1S,2S)\pi^+\pi^-$ decays.

| Decay | Data source | $g_1$ | $g_3/g_1$ | $g_{nm}$ (GeV$^2$) |
|-------|-------------|-------|-----------|-------------------|
| $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ | Belle [2] | $(6.00 \pm 1.43) \times 10^{-3}$ | $0.69 \pm 0.63$ | $5.28 \pm 1.20$ |
| | Belle [2] | $(6.51 \pm 1.18) \times 10^{-3}$ | $0.44 \pm 0.48$ | $0$ (fixed) |
| | Belle [3] | $(5.56 \pm 4.48) \times 10^{-3}$ | $0.94 \pm 1.74$ | $7.60 \pm 1.19$ |
| | Belle [3] | $(6.76 \pm 1.46) \times 10^{-3}$ | $1.04 \pm 0.52$ | $0$ (fixed) |
| | BaBar [4] | $(7.05 \pm 0.86) \times 10^{-3}$ | $-0.94 \pm 0.53$ | $3.70 \pm 1.02$ |
| | BaBar [4] | $(3.84 \pm 0.73) \times 10^{-3}$ | $1.65 \pm 0.70$ | $0$ (fixed) |
| $\Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^-$ | BaBar [4] | $0.15 \pm 0.01$ | $-3.37 \pm 0.17$ | $11.9 \pm 3.3$ |
| | BaBar [4] | $0.14 \pm 0.01$ | $-3.67 \pm 0.14$ | $0$ (fixed) |

FIG. 3: The $\pi^+\pi^-$ invariant mass spectra for the decays $\Upsilon(4S) \to \Upsilon(1S,2S)\pi^+\pi^-$. The solid and dashed curves denote the results with and without $X$, respectively.
FIG. 4: The contributions from different terms to the $\pi^+\pi^-$ invariant mass spectra. (a) $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$, (b) $\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-$. 

FIG. 5: Predictions of the $\cos\theta^*_\pi$ distributions. (a) $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$, (b) $\Upsilon(4S) \rightarrow \Upsilon(2S)\pi^+\pi^-$. The solid and dashed curves represent the results with and without the intermediate $X$ state, respectively. From the figure, one sees that the predicted $\cos\theta^*_\pi$ distribution of the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay is much flatter in the case without $X$ than in the case with $X$. Clearly, the angular distribution of the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay with $X$ exhibits a similar behavior shown in the $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay, because the sequential process plays dominant role in both decays. Therefore, we can expect that the future angular distribution data would be a criterion for further judgment.

Moreover, we would mention that in the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay, the numerical result
shows a negligible effect from the off-shell tensor resonance $f_2(1270)$.

In summary, we analyze recently reported data of the $\pi^+\pi^-$ invariant mass spectra of the $\Upsilon(4S) \rightarrow \Upsilon(1S,2S)\pi^+\pi^-$ decays in a model-independent way, and find that the behavior of the $\pi\pi$ invariant mass spectrum in the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay is an analogue of that in the $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay, namely there does exist a broad structure in the $\pi\pi$ mass region below 0.6 GeV. This disagrees with the presumed $\Delta n = 2$ rule in Ref. [15]. Then, we try to justify our model developed in studying the puzzle in the $\Upsilon(3S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay by using the newly reported data of $\Upsilon(4S)$ dipion decays. We find that with the additional contribution from the sequential process proposed in our model, one can describe the experimental data, namely the intermediate state $X$ plays an important role in the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay. We also predict the $\cos\theta^*_\pi$ distributions of $\Upsilon(4S) \rightarrow \Upsilon(1S,2S)\pi^+\pi^-$ decays. The result shows that the $\cos\theta^*_\pi$ distribution of the $\Upsilon(4S) \rightarrow \Upsilon(1S)\pi^+\pi^-$ decay in the without $X$ case is much flatter than that in the with $X$ case. Therefore, this observable can be employed to justify our model in the future.

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