Entropic force and its fluctuation in Euclidian Quantum Gravity

Yue Zhao\textsuperscript{1,*}

\textsuperscript{1}NHETC and Department of Physics and Astronomy, Rutgers University, Piscataway, NJ 08854–8019, USA

(Dated: August 17, 2010)

Abstract

In this paper, we study the idea about gravity as entropic force proposed by Verlinde. By interpreting Euclidean gravity in the language of thermodynamic quantities on holographic screen, we find the gravitational force can be calculated from the change of entropy on the screen. We show normal gravity calculation can be reinterpreted in the language of thermodynamic variables. We also study the fluctuation of the force and find the fluctuation acting on the point-like particle can never be larger than the expectation value of the force. For a black hole in AdS space, by gauge/gravity duality, the fluctuation may be interpreted as arising from thermal fluctuation in the boundary description. And for a black hole in flat space, the ratio between fluctuation and force goes to a constant $\frac{T}{m}$ at infinity.

Keywords: entropic force, gauge/gravity duality, black hole

*Electronic address: zhaoyue@physics.rutgers.edu
I. INTRODUCTION

The thermodynamics of black hole has been studied for several decades since the discovery of Hawking radiation \[2\]. It reveals a deep connection between the structure and dynamics of space-time and laws of thermodynamics. And more recently, the work by Jacobson shows an explicit derivation from laws of thermodynamics to Einstein equation \[4\]. The attempts to explain gravity as an emergent phenomena is based on the holographic principle \[5, 6\]. And AdS/CFT correspondence provides strong support and explicit examples on how thermodynamics of space-time can be related to thermodynamics of the dual system living on holographic screen \[7–10\].

Recently, Verlinde \[1\] proposed a conjecture that the origin of gravity can be interpreted as entropy changing on the holographic screen, which can be explicitly expressed as

\[
F \Delta x = T \Delta S
\]

(I.1)

There are several assumptions required to realize his idea. Firstly, one imposes how entropy on elements of holographic screen changes when one moves the particle \(\Delta x\)

\[
\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x
\]

(I.2)

where the distance between particle and screen element should be smaller than the Compton wavelength of the particle. And this will imply the Unruh-like relation between temperature and acceleration

\[
k_B T = \frac{1}{2\pi} \frac{ha}{c}
\]

(I.3)

Then to relate energy and temperature of the system, one imposes the equipartition of energy, see also \[11\]

\[
E = \frac{1}{2} N k_B T
\]

(I.4)

where \(N\) is the number of degree of freedom on the holographic screen as

\[
N = \frac{Ac^3}{G\hbar}
\]

(I.5)

With identifying the total energy \(E\) of the system living on holographic screen with the bulk energy inside the screen, one can derive the Newton gravitational force and Einstein equation.
There has been many papers following this conjecture[14–25], and some other papers apply this idea into cosmology[26–29], especially, [30] studies possible fluctuation seeds induced by thermal fluctuation on holographic screen during inflation.

(I.1) is the equation showing the most important point of the whole idea, one interprets the origin of gravitational force as the change of entropy on the holographic screen. However, to derive the correct expression for gravitational force or equation, one needs to impose several other conditions. How far can those imposed conditions go? In what circumstance do those conditions fail to work? Is the interpretation of gravity always correct or just a coincidence in some particular cases?

In section 2, we apply some basic concepts and relations obtained from AdS/CFT to study the entropic interpretation of gravity, leaving those potentially unsafe conditions along, one finds that only by imposing (I.1), one can obtain the correct expression of gravitational force for generic static spherically symmetric metric background. The derivation in this section is an analog to the calculation in polymer molecule system, which is used to motive entropic force in Verlinde’s paper.

In section 3, we consider our method to derive entropic force in more details, and we find it has a good interpretation from gravitational side alone, without correspondence between gravity and field theory. The point of this section is following: if one interprets the derivation from gravity side along, the whole calculation is just generalized Euler-Lagrange equation. However, if one gives thermodynamic interpretation ( as quantities on holographic screen ) to each step during derivation as we did in section 2, one will automatically draw the conclusion that gravity as entropic force. Together the analysis in section 2, it provides a solid base for entropic force interpretation. Moreover, since our derivation can be done in a quite generic metric background, it could be a clue to find more explicit connections and understanding on thermodynamics of two systems.

In section 4, since the force can be interpreted as a quantity in theromodynamics, we go further step to study its thermal fluctuation. We find the thermal fluctuation \( F^2 - \overline{F^2} \) is always positive. And for a point-like particle, the fluctuation is never larger than the real force. The metrics of black hole in asymptotic flat and AdS space are taken as examples in this section.
II. ENTROPIC FORCE IN GAUGE/GRAVITY DUALITY

One of the main ideas in Verlinde’s paper is to interpret the origin of gravity as the change of the entropy in the dual theory on holographic plane. Gauge/gravity correspondence provides us a natural and very-well understood place to test this idea. The change of entropy in field theory on the holographic screen of the bulk space should give the correct expression for gravitational force experienced by particles in bulk space.

In gauge/gravity duality, one has the identification between Euclidian action of the field theory and the Euclidian action from gravity. And this will be our starting point to see how change of entropy causes gravity.

Firstly, we consider the simplest case, one point-like particle moving in a fixed geometric background with black hole. And we need to assume that the gravity set-up has a dual theory on field theory side. The temperature of black hole in gravity side should be identified as the temperature of the field theory.

Suppose there is an external force acting on the point particle holding it fixed. From the boundary point of view, the black hole plays the role of heat bath, the point particle should be interpreted as a perturbation away from equilibrium in the bath. If there is no external force acting on the particle, the particle will fall into black hole, which is analog to the process that the perturbation is erased and gets equilibrium with heat bath. When there is force holding particle fixed outside horizon, this corresponds to an effective force keeping perturbation away from thermal equilibrium with the bath.

Now let us applying the similar analysis used in section 2 in Verlinde’s paper [1] for polymer molecule system. We choose to use micro-canonical ensemble. Thus, the entropy of the whole system can be written as $S(E + F_{\text{ext}} r_0, r_0)$, where $E$ is the total energy of black hole and particle (heat bath and molecule chain). Since in micro-canonical ensemble one has

$$\frac{d}{dr_0} S(E + F_{\text{ext}} r_0, r_0) = 0 \quad \text{(II.6)}$$

which implies

$$\frac{F_{\text{ext}}}{T} - \frac{\partial S}{\partial r_0} = 0 \quad \text{(II.7)}$$
The next thing to do is to find the expression for entropy from our gravitational set-up. To achieve that, one recalls that in gauge/gravity duality, the Euclidean action calculated from gravity is identified as entropy of the dual field theory if one treats the system as micro-canonical ensemble. That provides a solid way to calculate entropy in (II.6). The Euclidean action of our gravitational set-up can be written as

\[ I = I_{\text{Grav}} + I_{\text{part}} + I_{\text{int}} \] (II.8)

where \( I_{\text{Grav}} \) is the Euclidean Einstein-Hilbert action of background metric. \( I_{\text{part}} \) is the contribution from the static particle outside black hole. And \( I_{\text{int}} \) is from the interaction which keeps particle fixed. One can easily see that, in micro-canonical ensemble, \( I_{\text{int}} \) will contribute the first term in (II.7), and \( (I_{\text{Grav}} + I_{\text{part}}) \) contribute to the second term. So to calculate the expression for external force, one needs to do derivative on \( (I_{BG} + I_{\text{part}}) \) respect to \( r_0 \).

The action of a point particle in general geometric background is proportional to the proper mass times the integral of its proper time

\[ I_{\text{part}} = im \int d\lambda \] (II.9)

We want to consider the particle fixed at some point. So one can write the coordinates of the particle as \( X^\mu = \{t, r_0, \theta_0, \phi_0\} \).

Since we need the Euclidian action, so we take \( \lambda \to i\tau \). And one has to give the correct range where Euclidian proper time \( \tau \) runs.

To find the range of \( \tau \), one firstly recalls how temperature relates to the metric. Suppose one has a static spherically symmetric metric

\[ ds^2 = -g_{tt}(r)dt^2 + g_{rr}(r)dr^2 + \ldots \] (II.10)

Here \( g_{tt} \) and \( g_{rr} \) are only functions of \( r \), and \( \ldots \) just the normal angular parts of the metric. After taking \( t \to it \), one finds \( t \) has to have period as

\[ \beta = \frac{1}{T} = \frac{4\pi}{\sqrt{g'_{tt}(r)(g_{rr}(r))'|_{r=r_h}}} \]

where we set \( k_B \) to 1, \( T \) is identified as temperature of the system, and \( r_h \) is the position of horizon.
This is the period of coordinate time $t$, and it differs with particle’s proper time $\tau$ by a factor of $\sqrt{g_{tt}}$. Thus one has

$$I_{\text{part}} = -m \frac{\sqrt{g_{tt}}}{T} \quad (\text{II.11})$$

Here we neglect the back reaction from the particle to background metric. This is corresponding to treat the heat bath, where the perturbation sits, infinitely large, and the small variation of the perturbation is infinitesimal respect to whole system. Then $I_{\text{Grav}}$ actually has no dependence on the position of particle. Now one can study how entropy changes respect to $r_0$

$$\Delta S = \frac{\partial I_{\text{part}}}{\partial r_0} \Delta r_0 = -m \frac{\partial r_0 g_{tt}}{2 \sqrt{g_{tt}T}} \Delta r_0 \quad (\text{II.12})$$

Now applying Verlinde’s conjecture, taking the origin of gravitational force as the change of entropy in dual theory, one has, in a covariant form,

$$F^a = T \nabla_a S \quad (\text{II.13})$$

Before going into calculation, several points needs to be clear. The definitions of work and force need to be clarified. There are two sets of coordinates describing the system, so one needs to choose the coordinates very carefully.

To motivate our choices of coordinates, let us briefly review the definition of work and force in special relativity. In special relativity, one also has two sets of coordinates, the proper $(\tau, \vec{l})$ and observer $(t, \vec{x})$ coordinates. One defines work as

$$W = \int_{x_0}^{x_1} \vec{F} \cdot d\vec{x} \quad (\text{II.14})$$

and force is defined as

$$F_i = \frac{dp_i}{dt} = \sqrt{g_{tt}} \frac{d}{d\tau} \left( m_0 \frac{dx^i}{d\tau} \right) \quad (\text{II.15})$$

Notice according to definition, force showed up here is not a 4-vector, and the upper subscript $i$ on $F$ is coming from $x^i$. Now one can write the generalized definition for work in a general background metric.

With background metric, one also has two sets of coordinates, one set is used to describe the field theory side $(\tau, \vec{l})$ which should be interpreted as proper coordinates, another set is used in gravity theory side $(t, \vec{x})$, which is analog to observer coordinates in special relativity. Thus one can write down the definition of work in general background metric as

$$W = \int_{x_0}^{x_1} F_i dx^i = \int_{x_0}^{x_1} g_{ij} F^i dx^j \quad (\text{II.16})$$
And in our case, we only care the radial component, thus we have

$$\Delta W = F_r \Delta r = g_{rr} F^r \Delta r = g_{rr} \sqrt{g_{tt}} \frac{d}{d\tau} (m_0 \frac{dr}{d\tau}) \Delta r \quad (II.17)$$

Now we can plug (II.12) and (II.17) into (II.13) and find the expression of the force as

$$m_0 \frac{d^2 r}{d\tau^2} = -m_0 \frac{1}{2} g_{tt} g_{rr} \partial_{r_0} g_{tt} \quad (II.18)$$

Since the particle is static, the external force should be balanced by gravitational force. And from (II.7), this force emerges from the changing of entropy of the dual system on holographic screen. If the entropic force interpretation is correct, one should get the same answer as one gets from geodesic equation. From the $r$ component of geodesic equations, one has

$$\frac{d^2 r}{d\tau^2} + \frac{1}{2} \Gamma_{\mu \nu}^{r} \frac{dX^\mu}{d\tau} \frac{dX^\nu}{d\tau} = F_{ext} \quad (II.19)$$

$F_{ext}$, again, is the external force keeping the particle fixed at $r_0$. Since the particle is not moving on radial direction, and the external force is balanced by gravitational force, one gets

$$F_{Grav} = m \frac{d^2 r}{d\tau^2} = -F_{ext} = -m \frac{1}{2} g_{tt} g_{rr} \partial_{r_0} g_{tt} \quad (II.20)$$

which is exactly the same expression as the gravitational force calculated from entropic force interpretation using the thermodynamic language on holographic screen.

In the derivation, starting from micro-canonical ensemble, $F_a = T \nabla_a S$ is the only formula we use, it does not depend on other assumptions, such as equipartition of energy. So this is a very safe check on the idea about the entropic origin of gravity. And one can claim that gravity always points to the direction to increase the system entropy, which leads the phenomena that gravity is always attractive.

### III. UNDERSTANDING ENTROPIC FORCE FROM GRAVITY SIDE ALONE

In previous section we have seen how entropic force interpretation in dual field theory gives the correct expression for gravity in bulk. In this section, we will consider our derivation again but interpret only from gravity side alone, without any knowledge about dual description on holographic screen.

The way we work in last section is firstly to write down the total action, then identify it with the entropy in field theory side. Taking the expression for black hole temperature, and applying (I.1), one can derive the expression for gravity.
Now let us firstly forget about the field theory interpretation, and only consider the point-like particle in background metric. Instead of writing the Polyakov point particle action, we write it in Nambu-Goto form as

\[ I_{\text{part}} = im \int d\lambda \sqrt{\dot{X}_\mu \dot{X}^\mu} \]  

(III.21)

For a particle fixed at some point, after gauge fixing \( \lambda \) as particle’s proper time, the action just reduces to Polyakov point particle action.

Again, since the particle is in the heat bath by black hole, the Euclidian time direction, after taking \( \lambda \to i\tau \), should be periodical.

Following the standard procedure in classical mechanics, consider a system whose action integration range is coordinate dependent

\[ S = \int_0^{\tau} \! d\tau (L_{\text{part}}[q, \dot{q}] + L_{\text{int}}[q]) \]  

(III.22)

here \( q \) is the canonical coordinate and \( \dot{q} = dq/d\tau \), \( L_{\text{int}} \) is the interaction term acting on the particle to keep it fixed, and it will give the term of \( F_{\text{ext}} \) in equation of motion. E.O.M. is given by functional derivative of \( q \) and \( \dot{q} \). Thus one gets

\[ \delta S = f(q) \frac{\partial L}{\partial q} - f(q) \frac{d}{d\tau} \frac{\partial L}{\partial \dot{q}} + \frac{\partial f(q)}{\partial q} L[q, \dot{q}] = 0 \]  

(III.23)

In our case,

\[ L_{\text{part}} = m \sqrt{\dot{X}_\mu \dot{X}^\mu} \]  

(III.24)

and \( q \) is taken as \( r \). Thus,

\[ \frac{d}{d\tau} \frac{\partial L}{\partial \dot{r}} = g_{rr} \frac{d^2 r}{d\tau^2} \]  

(III.25)

Taking (III.25) into (III.23), and notice in our case \( f(q) = \sqrt{g_{tt}} \), one can again generate (II.20), which is what we expect.

We can see that, the same calculation procedure, from gravity side point of view without knowledge anything about dual description, is just solving the generalized Euler-Lagrange equation. The point here is that the normal gravity calculation can be reinterpreted in the language of thermodynamic variables on the holographic screen, where gravity can be treated as change of entropy from dual theory point of view. The concept of gravity as entropic force just comes out naturally if the correspondence between gravity and field theory is set up and interpreted correctly.
IV. THERMAL FLUCTUATION

In the previous sections, we checked the entropic interpretation of gravitational force in the language of thermodynamics variables on holographic screen. And since we interpret the force as a thermodynamic quantity, we can push this idea one more step to study the fluctuation of gravitational force. In our set-up, $F_{\text{Grav}}$ and $r_0$ are analog to pressure and volume in dual field theory side. And since the particle is held by external force and the system is in equilibrium, one is allowed to use normal thermodynamics analysis to do calculation. Since we are interested in the fluctuation of the force acting on the particle, i.e. the fluctuation of the gravitational force, we can treat $(I_{\text{Grav}} + I_{\text{part}})$ as the partition function of the system. One can consistently find that the first derivative of partition function respect to $r_0$ does give the correct expression for external force, which is the standard way to calculate pressure in normal thermodynamic analogy.

Let us first do some analysis on fluctuation in usual statistical system. Partition function of the system can be written as

$$Z = \sum_s e^{-\beta E_s} \tag{IV.26}$$

External force $Y$, by definition, is

$$Y = \frac{\partial E_s}{\partial y} \tag{IV.27}$$

$y$ is the extensive quantity corresponding to $Y$, so

$$Y = \sum_s \frac{\partial E_s}{\partial y} e^{-\beta(E_s)} \frac{1}{Z} \tag{IV.28}$$

Then the fluctuation of $Y$ can be derived as

$$Y^2 - \overline{Y^2} = \frac{1}{\beta^2} \frac{\partial^2}{\partial y^2} \ln Z + \frac{1}{\beta} \frac{\partial Y}{\partial y} \tag{IV.29}$$

If the system is adiabatic and in equilibrium, one has

$$\frac{\partial^2 E}{\partial y^2} = \frac{\partial Y}{\partial y} = \frac{\partial \overline{Y}}{\partial y} \tag{IV.30}$$

Now, come back to our analysis, we identify $Y$ with $F$ in (II.13), $y$ with $r_0$ and $\ln Z$ with Euclidian action $I$. Then we have

$$\overline{F^2} - \overline{F}^2 = \frac{1}{\beta^2} \frac{\partial^2}{\partial r_0^2} I + \frac{1}{\beta} \frac{\partial F}{\partial r_0} \tag{IV.31}$$
The set-up of our system is a point particle fixed at a point in some generic metric background, so it is an adiabatic system in equilibrium. Thus we can apply (IV.30).

After some derivations, one gets a simple formula

$$F_2^2 - F_1^2 = T m \frac{(\partial_{r_0} \sqrt{g_{tt}})^2}{\sqrt{g_{tt}}} = \frac{T}{m \sqrt{g_{tt}}} F^2$$

(IV.32)

One can see that (IV.32) is always positive. We will study (IV.32) in some more details in following sections.

A. Asymptotic AdS space with black hole

Since the duality between AdS space with Schwarzschild black hole and the CFT with finite temperature is one of the most understood examples in AdS/CFT, we take the explicit metric of that case into calculation. And also we take the near horizon limit of the metric. Thus we have, for example in AdS$_4$

$$g_{tt} = \left( \frac{r^2}{l^2} - \frac{2GM}{r} \right)$$

(IV.33)

which gives

$$\frac{F^2 - F_1^2}{F^2} = \frac{T}{m \sqrt{\frac{r^2}{l^2} - \frac{2GM}{r}}}$$

(IV.34)

Notice that the force we discuss here is not the conventional gravitational force experienced by particle in AdS space, i.e. $F_G = m \frac{d^2 x}{dr^2}$, they are proportional to each other by a coefficient as function of background metric at $r_0$.

The particle should be outside of black hole, so $r > \sqrt[3]{2G l^2 M}$, so ratio is always finite outside the black hole, it will blow up at black hole horizon and goes to zero when $r$ goes to infinity. Then one needs to find the point where the ratio becomes one, which implies the fluctuation is larger than the average value of gravitational force.

If the point where ratio becomes to one is close to black hole horizon, then there is nothing to worry about, since the particle will experience the heat radiation from black hole which naturally cause large fluctuation. The only dangerous case is that point is far away from black hole horizon, which is counter-intuitive and may potentially contradict with experiments. Let us talk about this possibility carefully.
Since we only care about the point where ratio equals to one is far away from horizon, we can take \( r^3 \gg 2GL^2M = r_{BH}^3 \), then (IV.34) approximately becomes

\[
\frac{F^2 - \overline{F}^2}{F^2/mr} = Tl
\]

(IV.35)

Using the expression for black hole temperature \( T \sim (GMl^2)^{1/3} \) and the radius of black hole \( R_{BH} \sim (GMl^2)^{1/3} \), one finds

\[
lm_0 \sim \frac{l}{\lambda} \sim \left( \frac{r_{BH}}{r_{\mathcal{O}(1)}} \right)
\]

(IV.36)

where \( r_{\mathcal{O}(1)} \) is the radial position where the ratio becomes order one, and \( \lambda \) is the Compton wavelength of the particle with mass \( m_0 \).

From (IV.36), if the equal ratio point is far away from black hole horizon, which means \( r_{\mathcal{O}(1)} \gg r_{BH} \), then it requires \( \lambda \gg l \). But we know that the particle with Compton wavelength larger then AdS radius cannot be well approximately described by a point-like particle. This is against our analysis using Polyakov point particle action. Thus, within a self-consistent analysis, one finds the fluctuation experienced by a point like particle can never dominate the real force, and this can be treated as a consistency check of our analysis.

Also here is another point needs to be clear. One requires the Compton wavelength of particle to be smaller than AdS radius to use point particle as approximation. This is not inconsistent with the treatment as neglecting the back-reaction from particle to background metric. To get a stable black hole in AdS space, one needs to take \( M_{BH} \gg \frac{1}{l} \). As long as the mass of particle is much smaller compared to black hole mass, our approximation is safe.

**B. Asymptotic flat space with black hole**

The result from (IV.34) is very interesting in the case of flat space with black hole, the ratio between fluctuation and the value of gravitational force goes to a constant!

\[
\frac{F^2 - \overline{F}^2}{F^2} = \frac{T}{m}
\]

(IV.37)

\( m \) again shows up in the dominator. However, just like the previous case, the ratio only becomes bigger than one for a particle whose Compton wavelength larger than \( R_{BH} \). And it is against our initial set-up.

We recover the units of quantities, we have

\[
\frac{F^2 - \overline{F}^2}{F^2} = \frac{k_B T}{mc^2}
\]

(IV.38)
We see that even when ratio goes to one in (IV.37), the fluctuation comparing with the real value of force is still extremely small. However this small number sheds light on an experimental verification on the idea of entropic force.

V. DISCUSSION

In this paper, we study Verlinde’s conjecture, gravitational force is induced by the change of entropy of dual field theory on holographic screen. Instead of applying other potentially dangerous assumptions, we take the idea from gauge/gravity duality, identifying the Euclidean action of gravity and field theory. We calculate the action of a point-like particle held fixed in a static metric background with black hole. Taking black hole’s temperature as the temperature of the dual system, we get the expression of the force induced by entropy change when we move the particle along radial direction. The entropic force derived this way agrees with the gravitational force in such background metric. Thus, this is a very safe check on the idea about gravity as emergent phenomena.

The key formula (I.1) is motivated from thermodynamics of the dual field theory. To give a more clear picture on how thermodynamics of gravity is related to that of the dual field theory, we study the gravity interpretation of (I.1). We did a similar calculation as entropic force in previous section, however we kept our interpretation in language of gravity side. We find $F_a = T \nabla_a S$ is just corresponding to a generalized Euler-Lagrange equation in gravity side. This gives an intuitive answer on why (I.1) gives the correct formula for gravitational force, and provide a solid base for entropic force interpretation. From this section, if one sticks on the language of thermodynamics on holographic screen to describe the system, one can naturally claim gravity is induced by the entropy change on the dual field theory.

After checking and giving a more explicit explanation on the entropic force, we take a further step to study the fluctuation of gravitational force in our formalism. We find the fluctuation in that metric is always positive. We take two widely used metrics, as examples. We find that, for a point-like particle, the fluctuation will never dominate the real value of gravitational force outside black hole. And for asymptotic flat metric with black hole, the ratio between fluctuation and force goes to a constant at infinity. This phenomena might provide a clue to experimentally test the concept of entropic force, or even holographic principle.
VI. ACKNOWLEDGEMENTS

I would like to thank S. Thomas for very helpful discussions and suggestion, and reading the draft. I also want to thank G. Pan for useful conversations. This research is supported by NHETC in Department of Physics and Astronomy at Rutgers University, and in part by DOE grant DE-FG02-96ER40949.

[1] E. P. Verlinde, On the Origin of Gravity and the Laws of Newton. [arXiv:1001.0785] [hep-th].
[2] Hawking, S. W. (1974). "Black hole explosions?". Nature 248 (5443):30
[3] J. D. Bekenstein, Phys. Rev. D 7, 2333 (1973). J. M. Bardeen, B. Carter and S. W. Hawking, Commun. Math. Phys. 31, 161 (1973). S. W. Hawking, Commun. Math. Phys. 43, 199 (1975) [Erratum-ibid. 46, 206 (1976)]. P. C. W. Davies, J. Phys. A 8, 609 (1975). W. G. Unruh, Phys. Rev. D 14, 870 (1976).
[4] T. Jacobson, Phys. Rev. Lett. 75, 1260 (1995) [arXiv:gr-qc/9504004].
[5] G. ’t Hooft, arXiv:gr-qc/9310026
[6] L. Susskind, J. Math. Phys. 36, 6377 (1995) [arXiv:hep-th/9409089].
[7] J. M. Maldacena, Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/9711200].
[8] S. S. Gubser, I. R. Klebanov and A. M. Polyakov (1998). "Gauge theory correlators from non-critical string theory". Physics Letters B428: 105C114
[9] Edward Witten (1998). "Anti-de Sitter space and holography". Advances in Theoretical and Mathematical Physics 2: 253C291. http://arxiv.org/abs/hep-th/9802150.
[10] L. Susskind and E. Witten, “The holographic bound in anti-de Sitter space”. arXiv:hep-th/9802150.
[11] T. Padmanabhan, “Thermodynamical Aspects of Gravity: New insights”. arXiv:0911.5004 [gr-qc].
[12] L. Smolin, “Newtonian gravity in loop quantum gravity”. arXiv:1001.3668 [gr-qc].
[13] F. W. Shu and Y. Gong, “Equipartition of energy and the first law of thermodynamics at the apparent horizon”. arXiv:1001.3237 [gr-qc].
[14] C. Gao, “Modified Entropic Force”. arXiv:1001.4585 [hep-th].
[15] Y. Zhang, Y. g. Gong and Z. H. Zhu, “Modified gravity emerging from thermodynamics and holographic principle”. [arXiv:1001.4677] [hep-th].

[16] J. W. Lee, H. C. Kim and J. Lee, “Gravity from Quantum Information”. [arXiv:1001.5445] [hep-th].

[17] Y. S. Myung, “Entropic force in the presence of black hole”. [arXiv:1002.0871] [hep-th].

[18] J. Kowalski-Glikman, “A note on gravity, entropy, and BF topological field theory”. [arXiv:1002.1035] [hep-th].

[19] Y. X. Liu, Y. Q. Wang and S. W. Wei, “Temperature and Energy of 4-dimensional Black Holes from Entropic Force”. [arXiv:1002.1062] [hep-th].

[20] R. G. Cai, L. M. Cao and N. Ohta, “Notes on Entropy Force in General Spherically Symmetric Spacetimes”. [arXiv:1002.1136] [hep-th].

[21] F. Caravelli and L. Modesto, [arXiv:1001.4364] [gr-qc].

[22] Y. Tian and X. Wu, “Thermodynamics of Black Holes from Equipartition of Energy and Holography”, [arXiv:1002.1275] [hep-th].

[23] Y. S. Myung and Y. W. Kim, “Entropic force and entanglement system”, [arXiv:1002.2292] [hep-th].

[24] I. V. Vancea and M. A. Santos, “Entropic Force Law, Emergent Gravity and the Uncertainty Principle”, [arXiv:1002.2454] [hep-th].

[25] R. A. Konoplya, “Entropic force, holography and thermodynamics for static space-times”, [arXiv:1002.2818] [hep-th].

[26] R. G. Cai, L. M. Cao and N. Ohta, “Friedmann Equations from Entropic Force”. [arXiv:1001.3470] [hep-th].

[27] M. Li and Y. Wang, “Quantum UV/IR Relations and Holographic Dark Energy from Entropic Force”. [arXiv:1001.4466] [hep-th].

[28] S. W. Wei, Y. X. Liu and Y. Q. Wang, “Friedmann equation of FRW universe in deformed Horava-Lifshitz gravity from entropic force”. [arXiv:1001.5238] [hep-th].

[29] Y. Ling and J. P. Wu, “A note on entropic force and brane cosmology”. [arXiv:1001.5324] [hep-th].

[30] Y. Wang, “Towards a Holographic Description of Inflation and Generation of Fluctuations from Thermodynamics”. [arXiv:1001.4786] [hep-th].