Cold Dark Matter Hypotheses in the MSSM

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We perform a Bayesian model selection analysis in the the R-parity conserving MSSM to compare two different assumptions: whether the lightest neutralinos make all or only part of the cold dark matter. This corresponds to either imposing full WMAP relic density limits or just its upper bound for constraining the MSSM parameters. We consider several realisations of the MSSM, namely, three GUT-scale SUSY breaking scenarios with a handful of parameters corresponding to the CMSSM, anomaly mediation and the large volume string scenarios as well as the weak-scale 25-parameter phenomenological MSSM (pMSSM). The results give a data-based quantitative evidence for a multicomponent cold dark matter. The pMSSM posterior samples indicate that the choice of imposing full WMAP limits or just its upper bound affects mostly the gaugino-higgsino content of the neutralino and, against naive expectations, essentially not any other sector.

Astrophysical observations [1] indicate that more than 20% of the energy density of the universe is made of neutral and weakly interacting non-baryonic ‘dark matter’ (DM) (for reviews, see Refs. [2–4]). Neutrinos are known to make up a minor component of the total DM [5]. The bulk of the DM has to be cold in order to be consistent with structure formation in the universe. A cold DM (CDM) candidate should be massive and stable on cosmic time-scales. The neutralino lightest supersymmetric particle (LSP) in the minimal supersymmetric standard model (MSSM) with conserved R-parity is an excellent CDM candidate. Other CDM candidates from models beyond the standard model include axions/axinos, gravitinos, Kaluza-Klein gravitons and stable moduli fields.

In most of the MSSM literature the neutralino has been considered to be the only thermal relic contribution to the DM energy density. However this assumption may be too strong and unnecessary. In fact it is very difficult, i.e. requires much fine-tuning, to obtain the observed value of the CDM relic density with the above mentioned assumption. Typically the neutralino relic density either radically over-closes or under-closes the universe. For example bino-like neutralinos yield relic densities between 1 to 4 orders of magnitude more than the WMAP fit value, and it is known that in the minimal anomaly mediated symmetry breaking (mAMSB), which maps to a subset of the MSSM parameters space, neutralino co-annihilations are so important that the predicted relic densities are generically far below the cosmological and astrophysical fit value [6] to the extent that an extra CDM component or some other modifications is unavoidable [7]. Mixed axion-axino [8] and/or stable moduli [9] could make up additional components to the neutralino CDM. In Refs. [10–12] multicomponent DM models were constructed to explain its indirect detection observations.

The assumption that CDM are solely made of neutralinos and the alternative case where additional non-neutralino components are allowed are mutually exclusive hypotheses that could be compared against each other in light of the currently available indirect collider and astrophysical data. One could ask the question of which of the two alternative hypotheses has more support from current data relative to the other [71]. Answers to questions of this sort, which can be updated as new data become available, can be obtained via the Bayesian technique for model selection.

Bayesian model selection techniques have been successfully applied in cosmology and astrophysics, see for instance Ref. [13] and references therein, and recently in particle physics phenomenology as well. For example the sign of the MSSM Higgs doublets mixing parameter, $\mu$, is not yet fixed by any observation and is an important quantity in determining possible SUSY contribution to the muon anomalous magnetic moment, $\delta a_{\mu}$ [14]. In Ref. [15, 16] Bayesian technique were employed to select between the $\text{sign}(\mu) > 0$ and $\text{sign}(\mu) < 0$ branches of the constrained minimal supersymmetric model (CMSSM) and of the pMSSM. Another example which is conceptually different is that of Bayesian comparisons between different models as illustrated in Ref. [17] where various models for SUSY breaking mediation were compared with one another in light of current indirect collider and WMAP data.

Bayesian approach to MSSM and DM phenomenology for the LHC have been used by various groups; see for instance [18–21] and references therein. In this article we compare the two CDM hypotheses in the context of the pMSSM [16, 22, 23] and GUT-scale models for SUSY breaking mediations, using Bayes’ theorem and current data, to

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make statement about which of the hypotheses receive better support from the data. In the first hypothesis, \( \mathcal{H}_0 \), the WMAP inferred CDM data is imposed on the pMSSM and other GUT-scale models parameters in a way compatible with the assumption that the whole CDM in the universe are solely made of neutralinos. For the second hypothesis, \( \mathcal{H}_1 \), where a multicomponent CDM is assumed, pMSSM and other GUT-scale models points that give relic densities lower than WMAP central point are not penalised so that non-neutralino CDM components are allowed for. The way in which the WMAP CDM data is employed for the analysis is summarised in Fig. 1(a).

The remaining part of the article is organised as follows. We first give a brief review of the Bayesian model selection technique followed by a description of how it is applied for the pMSSM and other GUT-scale models for the two CDM hypotheses. Then we compare and explain some differences between the sampled points, particularly in the gaugino and neutralino sectors, from this analysis and the previous corresponding pMSSM analysis considered in Ref. [16]. Finally we explain the implications of the hypotheses for direct DM detection experiments before ending the article with summary and conclusions.

Here we give definitions and set terminology that would allow for a self-contained discussion. Bayesian inference studies are centered around two quantities: model evidence and parameters posterior probability distributions. These are defined as follows. Consider a hypothesis (or model with all its assumptions) \( \mathcal{H} \) with parameters \( \mathbf{m} \). The a priori assumed form of values which the model parameters can take is encoded in the prior probability distribution of the model parameters given that the hypothesis is true, \( p(\mathbf{m}|\mathcal{H}) \). The posterior distribution of the parameters in light of the data considered and in the context of the hypothesis is represented by \( p(\mathbf{m}|\mathbf{d},\mathcal{H}) \). The data, \( \mathbf{d} \), is a set of experimentally determined values for certain quantities (observables) which the model can predict. The support or evidence which the model obtains from the data is defined as the probability density of observing the data set given that the hypothesis is true and represented by \( Z \equiv p(\mathbf{d}|\mathcal{H}) \).

The evidence is calculated as

\[
Z = \int p(\mathbf{d}|\mathbf{m},\mathcal{H}) p(\mathbf{m}|\mathcal{H}) \, d\mathbf{m} \tag{1}
\]

where the integral is \( N \)-dimensional, with \( N \) the dimension of the set of parameters \( \mathbf{m} \). \( p(\mathbf{d}|\mathbf{m},\mathcal{H}) \) is called the likelihood, it is the probability of obtaining the data set \( \mathbf{d} \) from certain model parameters \( \mathbf{m} \) and is a function of \( \chi^2 \) for the analyses presented in this article. Eq. (1) is obtained directly from Bayes’ theorem

\[
p(\mathbf{m}|\mathbf{d},\mathcal{H}) = \frac{p(\mathbf{d}|\mathbf{m},\mathcal{H}) p(\mathbf{m}|\mathcal{H})}{p(\mathbf{d}|\mathcal{H})} . \tag{2}
\]

In the reign of searches for new physics [24] from upcoming LHC and other experimental data, Bayesian inference analysis would play an important role in selecting between various models based on the data. For such LHC−1 problem and the Bayesian selection example presented in this article, evaluating the evidence is more important compared to computing parameters posterior probability distributions. In general one would not bother about finding a model posterior probability distribution if, to begin with, it is known that its evidence is poor. However, evaluating the evidence is computationally very expensive especially for multi-dimensional parameter spaces. In fact until the work in Ref. [23], the magnitude of the evidence were mostly disregarded and only the model posterior probability distributions were computed. For evaluating the evidence we use the MultiNest algorithm [26, 27] which implements nested sampling technique [25] and is very efficient in dealing with complex and multi-modal parameter spaces.

In order to select between two hypotheses \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) one needs to compare their respective posterior probabilities given the data set \( \mathbf{d} \) to be used for the comparison. The posterior probabilities are compared via Bayes’ theorem as

\[
\frac{p(\mathcal{H}_1|\mathbf{d})}{p(\mathcal{H}_0|\mathbf{d})} = \frac{p(\mathbf{d}|\mathcal{H}_1)p(\mathcal{H}_1)}{p(\mathbf{d}|\mathcal{H}_0)p(\mathcal{H}_0)} = \frac{Z_1}{Z_0} \frac{p(\mathcal{H}_1)}{p(\mathcal{H}_0)} , \tag{3}
\]

where \( p(\mathcal{H}_1)/p(\mathcal{H}_0) \) is the ratio of prior probabilities for the two models usually set to unity with the assumption that the two hypothesis are a priori equally likely. \( Z_1/Z_0 > 1 \) would mean that the data favours model \( \mathcal{H}_1 \) relative to \( \mathcal{H}_0 \) and vice versa if the ratio is less than one. Given various evidences to be compared beside one another, Jeffreys’ scale [28], which gives a calibrated spectrum of significance for the relative strength between the evidences,

\[
\Delta \log_e Z = \log_e \left[ \frac{p(\mathcal{H}_1|\mathbf{d})}{p(\mathcal{H}_0|\mathbf{d})} \right] = \log_e \left[ \frac{Z_1}{Z_0} \right] , \tag{4}
\]

is used. The Jeffreys’ scale convention we employ is shown in Tab. I. Values in the column labeled “Probability” are calculated as follows. Given that \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) are mutually exclusive and a priori equally likely then

\[
p(\mathcal{H}_0|\mathbf{d}) + p(\mathcal{H}_1|\mathbf{d}) = 1 \text{ and } p(\mathcal{H}_0) = p(\mathcal{H}_1) \tag{5}
\]
so that the posterior probabilities are given by
\[ p(\mathcal{H}_0|d) = \frac{Z_0}{Z_0 + Z_1} \quad \text{and} \quad p(\mathcal{H}_1|d) = \frac{Z_1}{Z_0 + Z_1}. \] (6)

Next, we are going to apply the selection technique presented above to assess the mutually exclusive CDM hypotheses using current indirect collider and WMAP data.

The pMSSM \cite{16, 22, 23, 29} has unconstrained MSSM parameters defined at the weak-scale. In order to suppress sources of unobserved CP-violation and FCNC the parameters are set to be real with diagonal sfermion mass parameters, \( m_f \), and trilinear scalar couplings. In addition, first and second generation mass parameters are set to be degenerate. The soft SUSY breaking parameters that remain, out of the initially more than 100 parameters, make the pMSSM parameters set
\[ m = \{\tan \beta, m_{H_1}^2, m_{H_2}^2; M_{1,2,3}; m_{1,2,3,4,5}^{3rdgen\ f_{j,2,3,4,5}}, m_{1,2,3,4,5}^{1/2ndgen\ f_{j,2,3,4,5}}; A_{t,b,c,\mu}=0\} \] (7)
where the notations are as in Ref. \cite{16} and the integers 1, 2, 3, 4, 5 label third generation and degenerate first and second generation of the sfermion masses there.

Cosmological and astrophysical fits to the standard cosmological constant plus CDM model imply a relic density \( \Omega_{\text{CDM}} h^2 = 0.1143 \pm 0.0034 \) \cite{17}.

On the other hand, for \( \mathcal{H}_1 \) only the upper half of the above constraint, Eq. (9), is imposed such that predicted relic densities lower than the central value are not penalised as shown in Fig. \ref{fig:1}(a).

Apart from the CDM relic we also impose precision electroweak and B-physics indirect collider data for checking the neutralino CDM hypotheses. Various predictions, \( \mathcal{O} \), for the observables were obtained via SOFTSUSY2.0.17 \cite{31} for producing the MSSM spectrum; micrOMEGAs2.2 \cite{32} for computing the neutralino CDM relic density, the branching ratio \( BR(B_s \rightarrow \mu^+\mu^-) \) and the anomalous magnetic moment of the muon \( (g-2)_\mu \); SuperIso2.0 \cite{33} for predicting the Isospin asymmetry in the decays \( B \rightarrow K^*\gamma \) and \( BR(B \rightarrow s\gamma) \); and susyPOPE \cite{34, 35} for computing precision observables that include the W-boson mass \( m_W \), the effective leptonic mixing angle variable \( \sin^2 \theta_{\text{eff}}^{\ell\ell} \), and the total Z-boson decay width, \( \Gamma_Z \), at two loops in the dominant MSSM parameters.

The experimentally determined central values, \( \mu_i \), and their corresponding uncertainties, \( \sigma_i \), of the observables
\[
\mathcal{O} = \{m_W, \sin^2 \theta_{\text{eff}}^{\ell\ell}, \Gamma_Z, \delta_{\text{eff}} \mu, R_U, A_{tb}^{0.1}, A^t = A_e, R_{b,c}, A_{tb,c}, A_{b,c}, BR(B \rightarrow X_s\gamma), BR(B_s \rightarrow \mu^+\mu^-), \Delta_{3\tau}, R_{BR(B_s \rightarrow \tau\nu)}, R_{\Delta M_{B_s}}, \Omega_{CDM} h^2\}
\] (10)
make the data set (same as those in Ref. \cite{16}) for the analysis
\[
d = \{\mu_i, \sigma_i\}
\] (11)
where \( i = 1, \ldots, 19 \) label the individual observables. The central values and errors of the observables are summarised in Tab. \ref{tab:1}. We assume that the observables are independent to form combined likelihoods

| \( \Delta \log_e Z \) | Probability | Remark |
|---------------------|-------------|--------|
| < 1.0               | < 0.750     | Inconclusive |
| 1.0                 | 0.750       | Weak Evidence |
| 2.5                 | 0.923       | Moderate Evidence |
| 5.0                 | 0.993       | Strong Evidence |

TABLE I: Jeffreys’ scale for the interpretation of relative evidences. \( \Delta \log_e Z > 5 \), e.g. 10, would indicate an irrefutable relative evidence.
neutralino relic densities are centred around a point about two orders of magnitude less than the WMAP central value except Table II for a quantitative measure. present and discuss the results of the samplings in the paragraphs that follow. represents the predicted value of neutralino CDM relic density; $H$ is the likelihood function for $H$ lightest neutralinos (solid line) or where a multicomponent CDM is allowed (dotted line). The posterior distributions show that generically the most preferred neutralino relic densities are centred around a point about two orders of magnitude less than the WMAP central value except when the single neutralino CDM scenario is imposed. This indicates that the single neutralino hypothesis is disfavoured. See Table II for a quantitative measure.

| Observable Constraint | Th. Source Ex. Source | Observable Constraint | Th. Source Ex. Source |
|------------------------|-----------------------|------------------------|-----------------------|
| $m_W$ [GeV]            | 80.399 ± 0.027        | $A_i = A_i^c$          | 0.1513 ± 0.0021       |
| $\Gamma_Z$ [GeV]       | 2.4952 ± 0.0025       | $A_b^b$                | 0.923 ± 0.020         |
| $\sin^2 \theta_{eff}$ | 0.2324 ± 0.0012       | $A_c^c$                | 0.670 ± 0.027         |
| $\delta a_\mu$         | (30.2 ± 9.0) × 10^{10} | $Br(B \rightarrow X_{\gamma}^\gamma)$ | (3.55 ± 0.42) × 10^{4} |
| $R^0_b$                | 20.767 ± 0.025        | $Br(B_s \rightarrow \mu^+ \mu^-)$ see Fig. 2 | 32, 51, 53 |
| $R^0_b$                | 0.21629 ± 0.00066     | $R_{\Delta M_{Bs}}$    | 0.85 ± 0.11           |
| $R^0_B$                | 0.1721 ± 0.0030       | $R_{Br(B_{s} \rightarrow \tau \nu)}$ | 1.26 ± 0.41 |
| $A^B_{FB}$             | 0.0992 ± 0.0016       | $\Delta_{0}$           | 0.0375 ± 0.029       |
| $A^B_{FB}$             | 0.0707 ± 0.035        | $\Omega_{CDM}h^2$      | 0.11 ± 0.02          |

TABLE II: Summary for the central values and errors for the observables. Theoretical uncertainties have been added in quadrature to the experimental uncertainties quoted.

$p(d|m, H_0) = \prod_{i=1}^{19} \exp \left[ -(O_i - \mu_i)^2 / 2\sigma_i^2 \right] / \sqrt{2\pi\sigma_i^2}$ and $p(d|m, H_1) = L(x) \prod_{i=1}^{18} \exp \left[ -(O_i - \mu_i)^2 / 2\sigma_i^2 \right] / \sqrt{2\pi\sigma_i^2}$ (12)

where for $H_i$ the index $i$ run over the different experimental observables (data) other than the CDM relic density; $x$ represents the predicted value of neutralino CDM relic density:

$L(x) = \begin{cases} 1 / (y + \sqrt{\pi s^2 / 2}), & \text{if } x < y \\ \exp \left[ -(x - y)^2 / 2s^2 \right] / (y + \sqrt{\pi s^2 / 2}), & \text{if } x \geq y \end{cases}$ (13)

$y = 0.1143$ is the WMAP central value quoted in Eq. (9) and $s = 0.02$ the inflated error. $L(x)$, shown in Fig. III(a), is the likelihood function for $H_0$ and $H_1$ respectively corresponding to the regions where $x < y$ and $x \geq y$. Next we present and discuss the results of the samplings in the paragraphs that follow.

The Bayesian evidence values from the samplings are summarised in Tab. III. Let us first comment on the case of the pMSSM. For the setting of Ref. 23, where only the linear prior was used, we obtain the following evidences: $\log_e \mathcal{Z}_0 = 29.902 ± 0.040$, $\log_e \mathcal{Z}_1 = 33.120 ± 0.028$ and difference $\Delta \log_e \mathcal{Z} = 3.218$. Wider prior parameters ranges and more B-physics observables were added for the new samplings for this article and for those in Ref. 16 relative to those
in Ref. [22]; as such the absolute evidence values differ significantly. However, here only the difference in the evidence values are important for the purpose of this paper. A significant difference in $\Delta \log_e Z$ is not expected if for the flat prior the constraints applied are as in Ref. [16] in place of those in Ref. [23] from which the flat prior result quoted in Tab. III is obtained [73]. The log prior results in the pMSSM [16] are $\log_e Z_0 = 65.043 \pm 0.042, \log_e Z_1 = 67.761 \pm 0.030$ with $\Delta \log_e Z = 2.718$. Translating the evidence ratios using Jeffreys’ scale, Tab. I implies that current indirect collider and cosmological data as employed here shows a significant evidence in support of multicomponent CDM hypothesis. The probability of an entirely neutralino-made CDM is around 0.04 or 0.06 respectively for the flat and log prior pMSSM. The pMSSM results imply an approximate prior independence since changing the log prior to a flat prior for the [16] fit should not lead to a large difference in $\Delta \log_e Z$.

In the second part of Tab. III the difference in the evidence values for the two hypotheses $H_0$ and $H_1$, is presented in the context of CMSSM, mAMSB and the LARGE volume scenarios (LVS) [65, 66]. Using the Jeffreys’ scale Tab. I we can extract the following information: the CMSSM results are prior dependent ranging from being weak to moderate evidence for the multicomponent CDM hypothesis. The LVS is approximately prior independent with a clear-cut indication of the presence of significant evidence against the solely neutralino CDM hypothesis, $H_0$. The mAMSB case is overwhelmingly prior independent and give an irrefutable evidence for multicomponent CDM hypothesis. The latter result can be considered as a check that proves that the Bayesian model selection technique is working since it is known [6] before hand that the mAMSB neutralino CDM cannot account for the WMAP fit, Eq.(9), and hence a multicomponent CDM is necessary in that scenario.

Note that from the evidence computations, the posterior samples of the models considered are obtained for free. For instance the posterior distributions of the predicted neutralino CDM for both hypotheses $H_0$ and $H_1$ are shown in Fig. I(b). The distribution for $H_1$ is centred at a value far less than the observed central value showing that addition non-neutralino CDM component(s) is(are) necessary in order to account for the WMAP data. Other analysis we performed from the posterior samples of the models are presented in the remaining parts of this article. We present and compare the difference between samples here for $H_1$ and from Ref. [10] for $H_0$. We address mostly the neutralino and chargino sectors which get most affected by the choice of $H_0$ or $H_1$ hypotheses.

The nature of the neutralino determines the processes by which it annihilates and/or co-annihilates into standard model particles and therefore the main determining factor for its relic density. The neutralino mass is given by

![Fig. 2: The chi-squared contour used for the B-physics observable $BR(B_s \to \mu^+ \mu^-)$.

| Models   | $\Delta \log_e Z_{\text{flat}}$ | $\Delta \log_e Z_{\text{log}}$ | $P(CDM = \tilde{\chi}_1^0)$ |
|----------|----------------------------------|---------------------------------|-----------------------------|
| pMSSM [22] | 3.2                             | -                              | 0.04                        |
| pMSSM [16] | -                               | 2.7                            | 0.06                        |
| CMSSM     | 1.4                             | 2.5                            | 0.18                        |
| mAMSB     | 14.1                            | 14.8                           | $6.8 \times 10^{-7}$        |
| LVS       | 3.6                             | 3.7                            | 0.02                        |

TABLE III: $\Delta \log_e Z$ values for the mutually exclusive CDM hypotheses in the pMSSM and some high scale models of SUSY breaking mediation. $P(CDM = \tilde{\chi}_1^0)$ represents the probabilities that the CDM are solely made of neutralinos. For the details of the comparisons between the CMSSM, mAMSB and the LARGE volume scenario (LVS) [65] models see Ref. [17].

The neutralino mass is given by
\[ \frac{1}{2} \psi^0 T M_N \psi^0 + H.c. \] where \( \psi^0 T = (\bar{\tilde{b}}, i \tilde{w}^3, \tilde{H}_1^0, \tilde{H}_2^0) \) and,

\[
M_N = \begin{pmatrix}
    M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta c_W \\
    0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\
    -m_Z c_\beta s_W & m_Z s_\beta c_W & 0 & -\mu \\
    m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0
\end{pmatrix},
\]

\( c_x = \cos x \) and \( s_x = \sin x \). The neutralino mass eigenstates are \( \tilde{\chi}_i^0 = N_{ij} \psi_j^0 \) where \( N \) is a unitary transformation that diagonalises \( M_N \). The neutralino mass eigenstate is a mixture of bino \( \tilde{b} \), wino \( \tilde{w}^3 \) and higgsinos \( \tilde{H}_{1,2} \)

\[ \tilde{\chi}_1^0 = N_{11} \tilde{b} + N_{12} \tilde{w}^3 + N_{13} \tilde{H}_1^0 + N_{14} \tilde{H}_2^0, \quad \sum_{i=1,2,3,4} (N_{1i})^2 = 1. \]

The coefficients \( N_{1i} \) with \( i = 1, 2, 3, 4 \), and the neutralino masses are complicated functions of the soft terms \( M_1, M_2, \mu, \text{sign}(\mu), \tan \beta \) etc (see e.g. \cite{67} and references therein) however the following statements approximate some relations between the \( N_{1i}s \) and the parameters. When \( M_1 \ll \text{min}(M_2, |\mu|) \), \( N_{11} \) dominates and the LSP is dominantly bino. Bino LSPs give relic densities too high, beyond the WMAP limits, in most regions of parameter space and therefore any pMSSM point in both \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) priors with such property would be excluded by the relic density constraint Eq.\( (\ref{14}) \). Next when \( M_2 < \text{min}(M_1, |\mu|) \), \( N_{12} \) dominates so the the neutralino is dominantly wino and is quasi mass degenerate with the lightest chargino. This leads to strong chargino co-annihilations to the extent that the relic density is typically much smaller than the WMAP constraint. The pMSSM parameter points that have that type of neutralino would be excluded in the \( \mathcal{H}_0 \) prior scenario but not in \( \mathcal{H}_1 \). For \( |\mu| < \text{min}(M_1, M_2) \), \( N_{13} \) and \( N_{14} \) are of order one and the LSP is dominantly higgsinos-like. In this case the neutralino efficiently annihilates into top and weak gauge boson pairs. In addition the \( \tilde{\chi}_1^0, \tilde{\chi}_2^0 \), and \( \tilde{\chi}_3^\pm \) are all approximately mass degenerate and higgsinos-like and hence there are more open channels for co-annihilations. Parameter points with this type of neutralino are also excluded in \( \mathcal{H}_0 \) prior.

The preferred content-nature of the neutralinos, based on the data and CDM prior assumption considered, can be seen from the pMSSM posterior samples. We use a measure of the gauginos content \( Z_g = |N_{11}|^2 + |N_{22}|^2 \), so \( 1 - Z_g \) is approximately equal to unity or zero if the neutralino is mostly gaugino- or gaugino-like respectively. The posterior distributions of \( 1 - Z_g \) for both \( \mathcal{H}_0 \) (solid line) and \( \mathcal{H}_1 \) (dashed line) pMSSM priors are shown in Fig.\( (\ref{3}a) \). It shows that the neutralino is mostly higgsino-like for the \( \mathcal{H}_1 \) prior pMSSM. The distribution in the \( \mathcal{H}_0 \) prior is bimodal with dominantly gaugino-like neutralinos and a sub-dominant mixed gaugino-higgsino peak. The drastic difference between the posteriors resulting from the two hypotheses is due to the fact that the CDM constraints imposed reject points that predict relic densities much less than the observed central value in the case of \( \mathcal{H}_0 \). As result the posterior samples are dominantly gaugino-like but with sub-dominant mixed higgsino-gaugino neutralinos that allow moderate co-annihilations that keep the relic density within the \( \mathcal{H}_0 \) allowed range. \( \mathcal{H}_1 \) prior, on the other hand, allow and in fact prefer (as it turned out to be) channels that have very efficient co-annihilations which happen mostly for higgsino-like neutralinos.

The neutralino relic density is inversely proportional to the thermally averaged magnitude of its annihilation and co-annihilation reaction cross-sections at early universe time. The content-nature of the neutralino determines which of the early universe reactions dominate over others. For instance, a certain neutralino composition lead to an acceptable magnitude for the thermally averaged cross-sections (which is proportional to inverse of the relic density) with 8% of the cross-sections coming from direct \( \tilde{\chi}_i^0 \) annihilations into W- and Z-bosons, 63% from \( \tilde{\chi}_i^0 \tilde{\chi}_j^0 \) co-annihilations into quarks and 27% into leptons. Another example point has 46% and 48% of the thermally averaged cross-sections respectively coming from \( \tilde{\chi}_i^0 \tilde{u} \) and \( \tilde{\chi}_i^0 \tilde{c} \) co-annihilations. We keep record of this information for each of the pMSSM points visited during the nested sampling of the parameter space. The posterior distributions for the dominant annihilation and/or co-annihilation channels (or reactions) are shown in Fig.\( (\ref{3}b) \) for \( \mathcal{H}_0 \) where the dominant channels are co-annihilations with sleptons and in Fig.\( (\ref{3}c) \) for \( \mathcal{H}_1 \) where co-annihilations with charginos and annihilations via chargino exchange dominate. The parameters posterior probability distributions are approximately the same for both \( \mathcal{H}_0 \) and \( \mathcal{H}_1 \) pMSSM samples except in the neutralino and chargino sectors. This is mainly due to the difference in the properties of the neutralinos that lead to different co-annihilation processes which in turn control the consistency of the predicted relic densities with the imposed CDM constraints. The neutralino mass, chargino mass and the soft breaking parameters they depend on are shown in Fig.\( (\ref{4}) \). The neutralino mass and \( \tan \beta \) are not severely constrained by the relic density constraints and hence their posterior distributions do not drastically differ between the hypotheses. The other parameters and chargino masses, on the other hand, significantly differ. The reasons for this are explained below. There is also a wide difference in the posterior probability distributions of the SUSY breaking parameters \( M_1 \) and \( M_2 \) between the hypotheses since they determine the nature of the neutralino admixture which is tightly related to the relic density
FIG. 3: (a) Neutralino’s gaugino-higgsino admixtures for $\mathcal{H}_0$ (solid line) and for $\mathcal{H}_1$ (dashed line) log prior pMSSM CDM hypotheses. The neutralino is almost always purely higgsino (peak around zero) for the $\mathcal{H}_1$ hypothesis due to a single efficient dominant co-annihilation that leads to posteriors with much lower (than WMAP central value) relic densities. For the $\mathcal{H}_0$ hypothesis, however, there are mainly two dominant (co-)annihilation channels that keep the relic densities within the imposed WMAP limit. With $Z_0 = |N_{31}|^2 + |N_{32}|^2$, peaks to the right-hand side of the abscissa indicate higgsino domination. (b) $\mathcal{H}_0$ prior pMSSM posterior sample distribution for the early universe neutralino annihilation (solid line) and co-annihilations whose cross-sections dominate in determining the present neutralino relic density. Neutralino-sleptons co-annihilation is dominant in this scenario since the neutralinos are mostly gaugino-like as shown in plot (a) above. (c) Same as in (b) above but for $\mathcal{H}_1$ prior pMSSM posterior samples. Here chargino-neutralino co-annihilations dominate.

predictions via the preferred co-annihilation cross-sections. The $\mu$ parameter on the other hand is not as widely different since it is mostly controlled by the requirement of radiative electroweak symmetry breaking (EWSB) than by the CDM constraint.

The nature of the Higgs doublets mixing parameter, $\mu$, and mass parameters $m_{H_1,2}$ are controlled at leading order by the tree level requirement that

$$\frac{1}{2}m^2_Z = \frac{m^2_{H_1} - m^2_{H_2} \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2$$

at the EWSB scale. This reduces to $\frac{1}{2}m^2_Z \approx -m^2_{H_2} - \mu^2$ with moderate to high $\tan \beta$ values. That is for a fixed $Z$-boson mass, $m_{H_2}$ and $\mu^2$ would be roughly the same since renormalisation group (RG) running of the soft SUSY breaking parameters take $m_{H_2}$ to very large (relative to the gaugino mass parameters) negative values at the EWSB scale. This explains the shape of the diagrams for $\mu^2$ and $m^2_{H_2}$ parameters in Fig. 4. The $\mathcal{H}_1$ pMSSM $\mu^2$ posterior samples (shown in dashed line in the figure) are less than those in the $\mathcal{H}_0$ prior because in the former case the neutralinos are mostly higgsino-like compared to the dominantly gaugino-like neutralinos in the latter. The details of the neutralino gaugino-higgsino content or admixture is shown in Fig. 8 a).

The posterior distributions for the gaugino mass parameters $M_1$ and $M_2$ widely differs between the two CDM prior assumptions since these parameters control the nature of the neutralino gaugino-higgsino admixtures. We are going to address the $M_1$ and $M_2$ parameters structure for each of the priors, starting with $\mathcal{H}_1$. The scenario in the $\mathcal{H}_1$ prior is similar to the so-called focus point region in CMSSM where the $m_{H_2}$ RG running is independent of the gaugino and trilinear parameters, see e.g. [68]. As a result $M_1$ and $M_2$ would in principle remain unconstrained by the EWSB requirement. This indeed remains the case for the $M_1$ parameter as shown in the corresponding plot in Fig. 4. However $M_2$ is constrained from the chargino sector since neutralino-chargino co-annihilations requires $m_{\chi^+_1} \sim m_{\chi^0_1}$ which in turn imposes $M_2 \sim \mu^2$. The $M_2$ posterior distribution is similar to that of the $\mu$ parameter as shown in
When choosing between the two hypotheses $\mathcal{H}_0$ and $\mathcal{H}_1$. Except for $\tan \beta$, all numbers in the horizontal axis are in TeV units. All the dashed lines are for the prior hypothesis $\mathcal{H}_1$ where neutralinos form only part of multicomponent CDM. Solid lines are for the prior hypothesis $\mathcal{H}_0$ where neutralinos make up all of the CDM.

Fig. 4 (dashed lines). Now turning to the $\mathcal{H}_0$ prior: again moderate $\tan \beta$ implies $m_{H_0}^2 \sim \mu^2$ but here the neutralinos are mostly gauginos-like with $M_2^2 > \mu^2 > M_1^2$ and therefore different from the $\mathcal{H}_1$ focus point-like feature. $M_1$ is constrained to be small by co-annihilation scenarios requiring mass of the neutralino, controlled by $M_1 < M_2$, to be approximately degenerate with the sleptons it is co-annihilating with. In summary $M_2$ controls the mass of the mostly higgsino neutralinos via neutralino-chargino co-annihilations in $\mathcal{H}_1$ prior where the neutralinos make only part of the CDM energy of the universe. On the other hand if neutralinos were to make all the CDM then $M_1 < M_2$ controls its mass via co-annihilation with with sleptons.

We next turn to the chargino masses' posterior distributions. Co-annihilations with charginos constrain the chargino masses to be approximately equal to those of the neutralinos. This is more so in the case of the $\mathcal{H}_1$ prior (dashed lines in Fig. 4) than in $\mathcal{H}_0$ prior (solid lines) pMSSM since in the latter co-annihilations with sleptons dominate and hence the chargino masses can be much larger than the neutralino masses. The relation between $m_{\chi^\pm_1}$ and $m_{\chi^\pm_2}$ for each of the priors can be explained as follows. The masses are controlled by the $M_2$ and $\mu$ parameters

$$m^2_{\chi^\pm_1, \chi^\pm_2} = \frac{1}{2} \left[ |M_2|^2 + |\mu|^2 + 2m_W^2 \mp \sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2} \right]. \quad (17)$$

When $M_2^2 \sim \mu^2$, as is the case for the $\mathcal{H}_1$ prior pMSSM, then the following approximation holds true $(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 \sim 4|\mu M_2 - m_W^2 \sin 2\beta|^2$. This in turn would imply, from Eq. (17), that $m_{\chi^\pm_1} \sim m_{\chi^\pm_2}$ as can be seen in the corresponding plots in Fig. 4. In a similar manner when $M_2^2 \gg \mu^2$ as is the case for $\mathcal{H}_0$ prior pMSSM, then $\sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2} \sim M_2^2$. This way $m^2_{\chi^\pm_1} \sim \frac{1}{2} \mu^2$ and $m^2_{\chi^\pm_2} \sim M_2^2$ explain the structure of the corresponding $\mathcal{H}_0$ distributions in Fig. 4.

Recapitulating, we have analysed the two mutually exclusive CDM hypotheses; whether it is solely made of neutralinos, $\mathcal{H}_0$, or whether a multicomponent scenario must be allowed, $\mathcal{H}_1$. We applied the Bayesian approach to inference and used the current indirect collider and cosmological data to select between these hypotheses within the context of the pMSSM where 20 phenomenologically viable MSSM parameters are simultaneously varied at the electroweak energy scale. The pMSSM setting is unbiased and free of theoretical assumptions and uncertainties from sources of SUSY breaking, mediation mechanisms and RG running. We also applied the Bayesian technique to GUT-scale defined models of SUSY mediation mechanisms. The results shown in Tab. III give a data-based evidence that a single neutralino CDM hypothesis is unrealistic and improbable. Based on the set of data employed for the analysis, the probability that neutralino make all the CDM for the pMSSM, mAMSB, CMSSM, and LVS are about 0.06 at most, 6.8 \times 10^{-7}, 0.18, and 0.02 respectively. The results in Tab. III and Jeffrey's scale (shown in Tab. I) for interpreting the evidence values shows that prior dependence of the results are strongest in the CMSSM, despite the fact that it has fewer number of parameters compared to the pMSSM. CMSSM has a weak-to-moderate evidence against $\mathcal{H}_0$, the solely neutralino made CDM hypothesis. The pMSSM and LVS results have moderate-to-sub-strong evidence against $\mathcal{H}_0$. LVS result is approximately prior independent. The mAMSB model have an exceptionally strong and prior independent evidence against $\mathcal{H}_0$. Based on these we therefore conclude that MSSM phenomenology studies should not neglect the mixed CDM possibility.
Let us, here, re-emphasise that prior dependence is a positive feature of Bayesian methods because it shows when the data and/or the physics under consideration are strong or well-defined enough to be free of ambiguous predictions and discriminations between alternative hypotheses. Prior (in)dependence is not a feature simply determined by the number of parameters in the sense that models with more parameters necessarily exhibit more prior dependence in their predictions. We hope that the results presented in this paper give good enough illustration of the mentioned observations. First, consider the mAMSB results in Tab. III. The results are prior independent because the physics is very well-pinned to the extent that even the not-that-strong indirect collider and cosmological data, Eq. (11), employed is good enough to give such exceptionally unambiguous results. Prior dependence can be completely absent if the data employed is strong enough. Secondly, the presence of relatively stronger prior dependence in the model with less number of parameters, CMSSM, compared to that with more parameters, the pMSSM, illustrates that prior dependence is not simply a feature directly derived from the number of model parameters but is only a feature indicating the strength of the data employed and/or the physics under consideration.

It is worth mentioning that it may seem that the analysis is not fair to the $\mathcal{H}_0$ prior pMSSM, where neutralinos are assumed to form all CDM. Not fair in the sense that any additional CDM candidate would be accompanied by its own parameters describing its mass and couplings. It would seem that these additional parameters would dilute the prior and in turn reduce the evidence for the scenario. We emphasise that this is necessarily not going to be the case because the additional DM parameters would make the model under consideration something completely different from the pMSSM. As such the comparison would then be between a pMSSM and a non-pMSSM. And that is not what we considered in this article. The comparison carried out is strictly between same pMSSMs with two distinct and mutually exclusive hypotheses: in the first, $\mathcal{H}_0$, the CDM is assumed to be solely made of the neutralino while in the second, $\mathcal{H}_1$, the neutralino forms only part of the CDM with the remaining to be accounted for by some other physics external and perpendicular to the pMSSM.

The posterior samples resulting from the analysis with $\mathcal{H}_0$, the assumption that the neutralinos make all CDM, and $\mathcal{H}_1$, where the neutralinos form only part of the CDM, the hypotheses differ drastically in the neutralino gaugino-higgsino content and neutralino/gauginos sector parameters. The dark matter relic density constraint in $\mathcal{H}_0$ dis-favours pMSSM points with higgsino-like or mixed gaugino-higgsino neutralinos that have efficient co-annihilation channels. However relaxing the constraint to $\mathcal{H}_1$ disfavoured pMSSM points become the preferred ones. Results in both this and previous pMSSM work indicate that, in light of the indirect collider and cosmological data considered, neutralino-chargino co-annihilations dominate at the early universe time before the neutralinos decouple from thermal equilibrium. It would be interesting to see what implications this feature would have for MSSM phenomenology at colliders.

Our results provide non-trivial quantitative evidence about the composition of DM in the MSSM, even after having introduced large number of parameters as in the pMSSM. We have assumed thermal dark matter and R-parity conserving MSSM. Modification of these assumptions or further extensions of the MSSM may modify our results.

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This question is different from attempts as in Ref. [70] seeking to estimate the effects, on preferred CMSSM regions, of changing the error bars on the (solely neutralino) CDM relic density constraint from WMAP. The WMAP value would not change the main results and conclusions of this work.

Here we use the five years WMAP values in order to properly compare with Ref. [16]. Updating this to the seven years changing the error bars on the (solely neutralino) CDM relic density constraint from WMAP.

The main difference between the pMSSM analysis in Ref. [23] and Ref. [16] is that the latter have additional set of electroweak observables from the code susyPOPE [34, 35] and have extended parameter ranges up to 4 TeV compared to 2 TeV for the former.