Analytical and Numerical Design Analysis of Concentric Tube Heat Exchangers – A Review

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Abstract. This paper considers an analytical and a numerical approach in the design of a concentric tube heat exchanger. Sensible heat transfer is considered in the analysis and the heat exchanger is developed for actual operating conditions in a chemical plant. The heat exchanger is a concentric tube heat exchanger where hot oil exchanges heat with hot water. Hot oil is in the inner pipe and the heating medium, hot water, is in the outer pipe (annular side) of the heat exchanger. An analytical model employing effectiveness-number of transfer units (ɛ-NTU) approach and log mean temperature difference (LMTD) approach were employed in the design of the concentric tube heat exchanger. In the design process, performance charts were developed for concentric tube heat exchanger. Performance charts describe the performance of the heat exchanger in terms of crucial dimensionless parameters. Performance charts help to select the right number of transfer units (NTU) for the given heat exchanger. Both parallel and counter flow configurations were considered for the design analysis. Likewise, a numerical model was also considered in the design of the heat exchanger. The results from the analysis are presented and compared. From the results it can be seen that both numerical and analytical approaches produce the exact same results. The designer certainly has the flexibility to choose an appropriate design methodology based on the available inputs and requirements.

1. Introduction
Heat exchangers (HX) are used in all process and manufacturing industries worldwide. Heat exchanger equipment transfers heat from a hot fluid to a cold fluid. Herein both fluids are separated by means of a solid wall. Typically, most heat exchangers are two-fluid heat exchangers though three-fluid heat exchangers are becoming popular. Heat exchangers can be classified in terms of flow and construction. In terms of flow, heat exchangers can be classified as parallel flow, counter flow and cross flow. In terms of construction, they can be classified as shell and tube, concentric tube and finned tube heat exchangers. The choice of a heat exchanger for a given application is dependent on the application itself, available resources, space, existing connections in the field, etc. Whatever may be the choice of a heat exchanger, it is very essential that a heat exchanger be designed such that it delivers the required heat transfer while occupying less space, being light weight, and yet be priced competitively.

In this project, a concentric tube heat exchanger is designed for heating hot oil. Hot oil is used for a certain process heating application and hot water is the heating medium. The heat exchanger will be designed such that the hot oil shall flow through the inner pipe and hot water through the outer pipe (annular side) of the heat exchanger.

A conventional ε-NTU approach, LMTD approach and a numerical approach are employed in design the concentric tube heat exchanger. Both parallel and counter flow configurations are considered in the
design analysis. Herein, performance charts were developed to aid the design process. Performance charts describe the variation of heat exchanger effectiveness with respect to capacity rate ratio and number of transfer units (NTU). Since all parameters in performance charts are expressed in dimensionless basis, the developed performance charts are applicable for any system of units, inlet fluid temperatures, fluid flow rates, materials of construction and size. Each design approach has unique characteristics and these aspects are discussed in the subsequent sections of the paper. There are numerous references available in the literature pertaining to heat exchanger performance modelling, and only the most pertinent are discussed. Kays and London [1] and Rohsenow [2] described both the logarithmic mean temperature difference and the methods in order to size and predict the performance of a heat exchanger. Domingos [3] presented a general method of calculating overall performance and intermediate temperatures of complex crossflow heat exchangers using the concept of effectiveness and a local energy balance. Pignotti and Shah [4] and Shah and Pignotti [5] discussed the tools developed previously (such as Domingos’ method, the Pignotti chain rule, etc.,) to determine the relationship for highly complex heat exchanger flow arrangements. As compared with the present investigation, these studies pertained to quite different geometries such as cross flow and shell and tube heat exchangers. Furthermore, they did not address the design of optimal heat exchangers, which achieve the required task at the lowest cost while satisfying imposed constraints. Mott and Mills [6] and Genic et al. [7] are among those researchers who described optimization analysis based primarily on minimizing energy costs related to pumping of a fluid. Kovarik [8] described a technique to optimize a cross flow heat exchanger. The objective function which was employed included cost factors related to the heat exchanger size and pumping power, as well as the required heat transfer rate. Similarly Rao et al. [9] and Caputo et al. [10] proposed methods to minimize capital and operating costs for shell-and-tube heat exchangers while satisfying the required heat transfer duty. Silaipillayarputhur and Idem [11] considered a numerical approach in the design and optimization of a double pipe heat exchanger. Therein, the heat exchanger was optimized such that the capital cost and operation cost of the heat exchanger were minimized.

2. Nomenclature

| Symbol | Name |
|--------|------|
| $A_o$  | Heat exchanger surface area (m$^2$) |
| $C_{min}$ | Minimum capacity rate fluid (W/K) |
| $C_{max}$ | Maximum capacity rate fluid (W/K) |
| $C_r$  | Capacity rate ratio (dimensionless) |
| $C_p$  | Specific heat (J/kg·K) |
| $D_H$  | Tube hydraulic diameter (m) |
| $d_{in}$ | Tube inside diameter (m) |
| $d_{out}$ | Tube outside diameter (m) |
| $h_i$  | Internal heat transfer coefficient (W/m$^2$·K) |
| $h_o$  | External heat transfer coefficient (W/m$^2$·K) |
| $\dot{m}$ | Mass flow rate (kg/s) |
3. Analytical Approach

This project considers the design of a concentric tube heat exchanger wherein the hot oil is in the inner pipe of the heat exchanger and hot water is in the outer pipe (annular side) of the heat exchanger. The hot oil is used for a certain heating application in the process plant. It is proposed to pre-heat the oil using the available “waste” hot water in the facility. It is desired to raise the temperature of 400 lbm/hr of hot oil from 90°F to 100°F by employing hot water that is available at 5000 lbm/hr at 150°F. The hot oil employed in the study is a heat transfer fluid, rated for open systems, having a viscosity grade of ISO 46. The thermophysical properties of the fluids employed in the heat exchanger are described in Tables 1 and 2 respectively.
Table 1. Hot Oil Thermal Properties.

| $\rho$ (kg/m$^3$) | $m$ (kg/s) | $C_p$ (J/kg-°C) | $k$ (W/m-°C) | $\mu$ (N-s/m$^2$) |
|------------------|-----------|----------------|-------------|------------------|
| 761              | 0.0504    | 2558           | 0.133       | 0.00117          |

Table 2. Hot Water Thermal Properties.

| $\rho$ (kg/m$^3$) | $m$ (kg/s) | $C_p$ (J/kg-°C) | $k$ (W/m-°C) | $\mu$ (N-s/m$^2$) |
|------------------|-----------|----------------|-------------|------------------|
| 1000             | 0.63      | 4180           | 0.65613     | 0.000446         |

The design constraints for the project are described in Table 3. The design constraints are heat transfer duty, heat exchanger diameter, and length. An additional design constraint, allowable pressure drop, will be considered while employing the numerical model. Employing this constraint in the conventional $\varepsilon$-NTU approach and LMTD approach will make the design process a little tedious. Therefore, in practical applications, this constraint is examined in terms of flow velocity. In real life applications, to keep the pressure drop within reasonable limits, it is a common accepted practice to keep the velocity of flow in the pipe to be around 3 m/s.

Table 3. Design Constraints.

| Design Constraints |
|--------------------|
| Parameter | Condition |
| Discharge oil temperature | 37°C |
| Max. heat exchanger external diameter | 75 mm (2.5 in NPS) |
| Max. heat exchanger length | 1.2 m (4 ft) |
| Allowable pressure drop, oil side* | 500 Pa |

* - for Numerical design analysis

The assumptions employed in the analysis are that the heat exchanger is operating at steady state, the properties of the fluids and the heat exchanger wall remains constant, and the heat exchanger is operating adiabatically.

The rate of heat transfer or the heat transfer duty, being one of the design constraint can be determined as follows [12]

$$Q = \dot{m}c_p \Delta T$$  \hspace{1cm} (1)

Here, the term $\Delta T$ refers to the absolute temperature difference between the fluid at inlet and at exit of the heat exchanger. Assuming the concentric tube heat exchanger to be well insulated it can be assumed that the rate of heat gained by the hot oil will be equal to rate of heat lost by the hot water such that
\[ Q = Q_{\text{oil}} = Q_{\text{hotwater}} \quad (2) \]

The average specific heat and the mass flow rate of both the fluids rates are known. The required discharge temperature of the hot oil is known as well. Therefore, the rate of heat transfer and the discharge temperature of hot water can be readily determined by employing Equations (1) and (2).

The capacity rate ratio for the given arrangement may be given as [12]

\[ C_r = \frac{C_{\text{min}}}{C_{\text{max}}} = \frac{\dot{m}c_p}{\dot{m}c_p} \quad (3) \]

Here, \( C_{\text{min}} \) corresponds to the capacity rate of minimum capacity rate fluid, which is hot oil and \( C_{\text{max}} \) corresponds to the capacity rate of maximum capacity rate fluid, which is hot water. The values for \( C_{\text{min}} \) and \( C_{\text{max}} \) can be readily computed as the pertinent information are available. The heat exchanger effectiveness may then be determined by employing the following equation [12]

\[ \varepsilon = \frac{Q}{Q_{\text{max}}} = \frac{Q}{C_{\text{min}}[T_{\text{Hotwater,inlet}} - T_{\text{oil,inlet}}]} \quad (4) \]

Here, the rate of heat transfer \( Q \) is given by Equation (1). For a concentric tube heat exchanger subjected to parallel flow, the heat exchanger effectiveness may also be given as [12]

\[ \varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r} \quad (5) \]

Likewise, for a concentric tube heat exchanger subjected to counter flow, the heat exchanger effectiveness may also be given as [12]

\[ \varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 + C_r \exp[-NTU(1 - C_r)]} \quad (6) \]

Number of transfer units (NTU) is a dimensionless parameter that is widely used by process engineers and heat exchanger designers. NTU is a physically significant dimensionless parameter as it encompasses material characteristics, fluid characteristics, flow characteristics, heat exchanger size, fouling, etc. Using Equation (5) and (6), for a range of NTU varying between 0.1 and 10, and capacity rate ratios varying between 0 and 1 the heat exchanger effectiveness can be readily plotted and performance charts can be developed for the concentric tube heat exchanger. The following figures 1 and 2 describe the performance charts for concentric tube heat exchanger. Using the performance charts, the required NTU for the heat transfer duty can be determined.
Figure 1. Performance of concentric tube heat exchanger subjected to parallel flow.

Figure 2. Performance of concentric tube heat exchanger subjected to counter flow.

NTU is a dimensionless parameter that accounts for material characteristics, flow characteristics, size, construction, fouling, etc. Therefore, NTU is a physically significant dimensionless parameter that is used by engineers during the design phase of the heat exchanger. Likewise, it must be recognized that higher the NTU, the more would the area, material, size, weight and cost. In keeping up with the competition, it is important to design the equipment such that it is compact, less weight, low cost and yet deliver the required heat transfer. Performance charts described in Figures 1 and 2 shall help in optimizing the heat exchanger during the development phase. In addition, it is a common belief that increasing the surface area enhances the rate of heat transfer. However, this is true only until a threshold limit and increasing the surface area (i.e., NTU) beyond that limit, adds unnecessary weight.
and cost. Figures 1 and 2 is certainly helpful in choosing the right surface area (i.e., NTU) for the given application.

The determination of internal and external heat transfer coefficients are described extensively in [11] and [12] and therefore they are not discussed herein.

Considering the fouling resistances, and conduction resistances, the overall heat transfer coefficient may be given as [12]

$$\frac{1}{U_o} = \frac{1}{h_i} + R_f + \frac{\frac{r_{in,\text{out}}}{k_{\text{piping,\text{in}}}}} \ln \left( \frac{r_{in,\text{out}}}{r_{in,\text{in}}} \right) + \frac{r_{in,\text{out}}}{r_{in,\text{in}}} R_s + \frac{r_{in,\text{out}}}{r_{in,\text{in}}} \frac{1}{h_i}$$

(7)

Recall that the required NTU for the heat exchanger was determined using performance charts. The NTU of a heat exchanger can also be described as [12]

$$\text{NTU} = \frac{U_o A_i}{C_{\min}}$$

(8)

Hence, from Equation (8), the required surface area of the heat exchanger can be readily determined. The surface area of the heat exchanger can be described as

$$A_s = \pi d_{in,\text{out}} L$$

(9)

From Equation (9) the required length for the concentric tube heat exchanger can be determined.

If log mean temperature difference (LMTD) approach were to be used, the rate of heat transfer may be described as [12]

$$Q = U_o A_o \Delta T_{in}$$

(10)

The computation of log mean temperature difference $\Delta T_{in}$ for parallel and counter flows are described in detail in [12] and therefore not included herein.

For LMTD approach, the required heat transfer surface area is determined by employing Equation (10) and the required length of the concentric tube heat exchanger can be determined by employing Equation (9).

Using the equations described in this section a MATLAB model was developed to design the concentric tube heat exchanger.

4. Numerical Approach
This section of the document describes the governing equations required to design the heat exchanger using numerical approach. Only the hot oil side is considered in the analysis, as heating up of hot oil is the primary focus in the project. The dimensions of the inner pipe and the flow velocity of oil are determined such that the required heat transfer duty is satisfied for a given mass flow rate and for an allowable pressure drop. From Equation (1), for the given conditions, the temperature difference in the hot water between the inlet and discharge was found to be negligible. This means that the hot water temperature remains approximately a constant while flowing through the concentric tube heat
exchanger. Hence, it is reasonable to assume that the heat exchanger wall is at constant temperature during the steady state operation of the heat exchanger.

Considering the heat exchanger to operate adiabatically, an overall energy balance yields the heat transferred from the tube wall to the hot oil \[11, 12\]

\[ Q = \dot{m}_{\text{hottub}} c_{p,\text{hottub}} (T_{\text{hottub},\text{out}} - T_{\text{hottub},\text{in}}) = \pi D_{H,\text{inner}} L h \Delta T_{m} \]  

(11)

The log-mean temperature difference is the mean temperature difference between the tube wall and the hot oil \[11, 12\]

\[ \Delta T_{m} = \left( T_{w} - T_{\text{hottub},\text{Lin}} \right) - \left( T_{w} - T_{\text{hottub},\text{out}} \right) \]  

\[ \ell \ln \left( T_{w} - T_{\text{hottub},\text{Lin}} \right) \]  

\[ \ell \ln \left( T_{w} - T_{\text{hottub},\text{out}} \right) \]  

(12)

The empirical Dittus-Boelter correlation for turbulent convection heating can be given as \[11, 12\]

\[ \text{Nu} = \frac{h_{D,H,\text{inner}}}{k_{\text{hottub}}} = 0.023 \text{Re}^{0.8} \text{Pr}^{0.4} \]  

(13)

The Darcy friction factor represents the dimensionless pressure loss per unit length in a pipe and can be expressed in terms of the mean oil velocity \[11, 12, 16\]

\[ f = \frac{\Delta P/L}{\frac{1}{2} \rho_{\text{hottub}} V_{\text{hottub}}^{2} / D_{H,\text{inner}}} \]  

(14)

An expression from Haaland \[11, 13, 15, 16\] provides an explicit relation between the friction factor, pipe relative roughness, and the Reynolds number

\[ \frac{1}{\sqrt{f}} = -1.8 \log \left( \left( \frac{e_{i}}{D_{H,\text{inner}}} \right)^{1.11} + \frac{6.9}{\text{Re}} \right) \]  

(15)

The Reynolds number may be defined as \[11, 12\]

\[ \text{Re} = \frac{\rho_{\text{hottub}} V_{\text{hottub}} D_{H,\text{inner}}}{\mu_{\text{hottub}}} \]  

(16)

The mean oil velocity in the pipe is determined from the continuity equation \[11, 16\]

\[ V_{\text{hottub}} = \frac{4 \dot{m}_{\text{hottub}}}{\pi D_{H,\text{inner}}^{2}} \]  

(17)

Upon rearranging, the energy conservation can be expressed in dimensionless as follows \[11\]
\[ f_1 = \frac{m_{\text{hot oil}} c_{\text{p, hot oil}}}{k_{\text{hot oil}} L} - 0.023 \pi \left( \frac{\rho_{\text{hot oil}} V_{\text{hot oil}} D_{H,\text{inner}}}{\mu_{\text{hot oil}}} \right)^{0.8} \left( \frac{\mu_{\text{hot oil}} c_{\text{p, hot oil}}}{k_{\text{hot oil}}} \right)^{0.4} \frac{1}{\ln \left( \frac{T_w - T_{\text{hot oil, in}}} {T_w - T_{\text{hot oil, out}}} \right)} = 0 \]  

Upon rearranging Equations (14) and (15), the following dimensionless equation can be obtained \[11\]

\[ f_2 = \sqrt[4]{\frac{\rho_{\text{hot oil}} V_{\text{hot oil}}^2 L}{2 \Delta p D_{H,\text{inner}}^2}} + 1.8 \log \left[ \frac{\left( \frac{\varepsilon_1}{D_{H,\text{inner}}} \right)}{3.7} \right]^{1.11} + 6.9 \frac{\mu_{\text{hot oil}}}{\rho_{\text{hot oil}} V_{\text{hot oil}} D_{H,\text{inner}}} = 0 \]  

Upon rearranging Equation (17), the following dimensionless equation can be obtained \[11\]

\[ f_3 = \frac{\rho_{\text{hot oil}} V_{\text{hot oil}} D_{H,\text{inner}}^2}{m_{\text{hot oil}}} - \frac{4}{\pi} = 0 \]  

The quantities \( f_1, f_2, \) and \( f_3 \) represent simultaneous nonlinear algebraic equations whose numerical roots are to be determined by employing techniques such as Newton Raphson method, Secant method, etc. MATLAB software has an inbuilt solver for solving simultaneous nonlinear algebraic equations. By solving the three simultaneous nonlinear equations, the three unknowns, namely, the pipe diameter, length and pipe velocity can be determined.

5. Results

Based on the analytical (\( \varepsilon \)-NTU and LMTD) approach and numerical approach as discussed in the sections 3 & 4 respectively, Matlab models were developed to design the concentric tube heat exchanger. The heat exchanger was subjected to the input conditions as described in section 3 and the design constraints as described in Table 3. The results from both models for parallel and counter flow concentric tube heat exchanger are described in Table 4.

| Table 4. Description of Concentric Tube Heat Exchanger. |
|--------------------------------------------------------|
| **Item** | **Inner Side** | **Annular Side** |
| Fluid | Hot Oil | Hot Water |
| Inlet Temp (°C) | 32 | 65 |
| Discharge Temp (°C) | 37 | 64.8 |
| Mass flow rate (kg/s) | 0.0504 | 0.63 |
| Heat Transfer (W) | 721 | 721 |
| Pressure drop, oil side (Pa)* | 250 | NA |
| Material | Carbon Steel, SMLS A53B | Carbon Steel, ERW A 53B |
| Diameter (in) - Analytical | 1/2 | 1 1/4 |
| Diameter (in) - Numerical | 1/2 | NA |
| Length (m) - (\( \varepsilon \)-NTU) approach, parallel flow | 0.81 | 0.81 |
| Length (m) - (\( \varepsilon \)-NTU) approach, counter flow | 0.81 | 0.81 |
| Length (m) - LMTD approach, parallel flow | 0.81 | 0.81 |
| Length (m) - LMTD approach, counter flow | 0.81 | 0.81 |
| Length (m) - Numerical approach | 0.81 | 0.81 |

* - for numerical design analysis
It can be clearly seen that both the analytical and numerical approach deliver the same results for the prescribed conditions. Though from performance charts, it can be clearly seen that the counterflow heat exchangers perform significantly better than the parallel flow heat exchangers, for the given application, there is absolutely no difference in the results between counter and parallel flow configurations. This is due to the prevailing uniform temperature of the hot fluid in the heat exchanger. The material selection for the heat exchanger was based on DuPont Engineering Standards as prescribed in [14].

6. Conclusions
This paper reviews the analytical and numerical approach in the design of a concentric tube heat exchanger. Therein, both parallel and counter flow configurations are analyzed. In the design process, performance charts were developed for concentric tube heat exchanger. Performance charts help the engineers to quickly determine the performance of the heat exchanger without performing tedious calculations. Likewise, performance charts help the engineers to determine the required NTU for the heat exchanger. NTU accounts for the physical characteristics of the heat exchanger, fluid and thermal properties, type of heat exchanger and fouling. Since NTU accounts for all the pertinent parameters, optimizing NTU is an essential item in the design of the heat exchanger. An over sized exchanger (having higher NTU) will certainly incur higher capital and operational costs and while an under sized heat exchanger (having lower NTU) shall certainly not deliver the required heat transfer. Performance charts developed in this paper will help the designers and engineers by providing this crucial information.

In addition, both analytical and numerical approaches are reviewed in this paper. Numerical approach accounts for tube side pressure drop and the heat exchanger can be designed such that this design criterion is satisfied. Analytical approach consisting of ε-NTU and LMTD methods do not explicitly satisfy this criterion. Likewise, employing the analytical approach can help to optimize the heat exchanger by choosing the right NTU for the given application. However, if the required discharge temperatures are unknown, it is quite tedious to model using LMTD method as the design process shall be iterative. Therefore, based on the available information and the required specifications, the designers can choose an appropriate method for the design of a concentric tube heat exchanger.

7. References
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