Abstract

Communication is a cornerstone of social interactions, be it with human or artificial intelligence (AI). Yet it can be harmful, depending on the honesty of the exchanged information. To study this, an agent based sociological simulation framework is presented, the reputation game. This illustrates the impact of different communication strategies on the agents’ reputation. The game focuses on the trustworthiness of the participating agents, their honesty as perceived by others. In the game, each agent exchanges statements with the others about their own and each other’s honesty, which lets their judgments evolve. Various sender and receiver strategies are studied, like sycophant, egocentricity, pathological lying, and aggressiveness for senders as well as awareness and lack thereof for receivers. Minimalist malicious strategies are identified, like being manipulative, dominant, or destructive, which significantly increase reputation at others’ costs. Phenomena such as echo chambers, self-deception, deception symbiosis, clique formation, freezing of group opinions emerge from the dynamics. This indicates that the reputation game can be studied for complex group phenomena, to test behavioral hypothesis, and to analyze AI influenced social media. With refined rules it may help to understand social interactions, and to safeguard the design of non-abusive AI systems.

Key words: sociophysics; information theory; dynamical systems; computational psychology; artificial intelligence

1 Introduction

Communications are essential to human interactions. They allow to exchange information and often shape group interactions. With the raise of social media, which are guided and influenced by artificial intelligence (AI) systems, the need to understand the vulnerabilities and shortcomings of social communication increases. Presently, AI systems shape social interactions by influencing which message reaches which participant, or by actively participating in the communications while pretending to be human. Even though communications can be extremely complex, their dynamics may exhibit identifiable patterns that could be the result of a conscious or unconscious strategy to achieve certain goals, which can be malicious. Determining the key elements that inform these strategies might help to counteract them. While these elements are known to psychologists, isolating their impact on individuals is challenging because of the complex nature of humans and their environment.

Here, we explore the hypothesis that the need to gain, maintain, or boost one’s reputation is a central element to many human communication strategies and may lead to some identifiable patterns in their communication dynamics. These patterns can be amplified or exploited by AI systems managing communications or participating in them. The need for reputation exists since reputation is a catalyst and currency for social interactions. In order to test this hypothesis and explore its consequences we develop an agent based simulation, which will be referred to as the reputation game in the following. The reputation game can be regarded as an extension of socio-physical simulations of gossip or rumor dynamics and trust networks, which aims at a more detailed cognitive and psychological modeling of each individual agent.

The game simulates the opinions that agents have about other agents and how these opinions may change through communications with others. In this game, the agents are virtual entities that are intended to mimic certain aspects of human beings. All of them will strive for a high reputation, and use strategies to reach this goal. Here, reputation refers to how honest an agent is perceived by the others. A key element of this game is the need for agents to form an opinion about the peers’ honesty and this is achieved by exchanging messages with other agents. Since each message contains some information, the agents use probabilistic logic and some rudimentary rules to determine whether the information received is considered as trustworthy and need to be memorized by the agents in the game.

1 All messages potentially affect the payoff of the game, as they...
Not all agents will interpret a given message in a similar way as their beliefs on the honesty of others will differ; this makes the game highly complex and its outcome non-trivial. The beliefs that agents maintain about each other are stored in a simple cognitive model, which is based on information theoretical principles (probabilistic logic [22] and optimal belief approximation [23]), but has limited capabilities. For efficient lie construction and detection, agents also maintain guesses on the beliefs and intentions of the other agents; i.e. they possess a rudimentary theory of mind [24].

Our motivation for developing the reputation game is to provide a theoretical framework to test and describe the elements of communication strategies that could affect the self-esteem and reputation of agents. How people share their information depends on their personality [25]. Personality traits are not necessarily obvious, but they can be inferred by observing behavior patterns. AI systems that manage social media communications with the aim to keep the attention of participants [26] most likely will discover and exploit such patterns. The game presented here should ultimately allow to test the link between emerging communication patterns, cognitive states of the participating agents, social situations, and the impact on idealized model characters in a straightforward fashion. It further allows to investigate communication dynamics of social groups, to test the origin of strategies observed in the real world, to understand the vulnerability of humans to deception and manipulations, and to help the development of methods to identify and counteract malicious communications. The latter is becoming increasingly important with the rise of AI based communication systems, which have opened the door for large-scale malicious communication attacks on human minds as well as on other AI systems. Eventually, as we refine the rules, the reputation game might help to study the impact of such attacks and to develop counter-measures by enabling simulations that surpass what can be achieved in a laboratory.

The game as it stands is built to reproduce some exaggerated behaviors that show some resemblance with humans. It is designed as a proof of concept, which – after refinement and input from psychologists – can become a tool to validate the impact of certain personality traits on others and group dynamics. It may also permit to study the impact of AI communication on group sociology. The game is built on principles of information theory, which should be universal to any functional cognitive systems. Thus, some of the effects it exhibits can be expected to capture mechanisms that show resemblance to well-known real-world sociological, cognitive, and psychological phenomena and therefore could help identifying new ones. Our work takes inspiration from work in socio-physical simulations, computational cognition and computational psychology [7, 9, 11, 17, 19, 27] but the use of information theory [22, 23, 30, 31] enables us to consider more complex phenomena.

The paper is organized as follows: The principles and the idea of the reputation game and related approaches are discussed in Sect. 2. The basic concepts of the game, an overview of the agent’s interactions, and the rules of the game are specified in Sect. 3. The used principles of information representation and processing are discussed in detail in Sect. 4. The different receiver and communication strategies used by agents are described in Sects. 5 and 6 respectively. Simulation runs of the reputation game in various configurations are discussed in Sect. 7. We discuss our main findings in Sect. 8 and conclude in Sect. 9. The discussion is structured such that technical and mathematical descriptions (Sec. 3-6) are largely kept separate from conceptual and sociological discussions (Sec. 2, 7, 8) and 9). This allows selective reading of the work, but implies some redundancy in the presentation. An overview on the used mathematical symbols can be found in Tab. 1 and summaries of the different receiver and communication strategies of agents in Tab. 2 and Tab. 3 respectively.

2 Principles

2.1 Players and their strategies

The idea of the reputation game is to provide a model that allows to study the impact of different communication strategies on the evolution of various agent’s reputation. Our model is kept as simple as possible to highlight the set of rules that are essential to certain types of strategies. However it is also complex enough to mimic phenomena occurring in the real world.

The game contains at least two agents. Each can send messages and receive them. All use both a number of communication and a number of receiver strategies. These strategies specify each agent’s personality. In this initial work, we will choose the personalities such that their performance can be studied in isolation. Strategies to send messages can be malicious (e.g. manipulative, destructive, aggressive etc.) or ordinary, if they are devoid of bad intentions. Agents interpret the messages they receive according to their level of psychological awareness or intelligence. For example a naive agent will not have the ability to determine if a message is honest or dishonest. Agents can be deaf, naive, uncritical, ordinary, strategic, anti-strategic, flattering, egocentric, aggressive, shameless, smart, deceptive, clever, manipulative, dominant and destructive. This is fixed at the beginning of the game upon definition of the agents in the game.

The environment for each agent playing the game is defined by the set of other agents. This environment is noisy with a noise level that depends on the agents’ characters (aka the used strategies) and moods (their intrinsic parameters). Agents communicate with each other in binary conversations, and – in order to strip the model from unessen-
tial complexity – there is only one single type of conversation topic, namely the honesty of a third agent, or that of one of the conversation partners. The statements agents make can be honest or dishonest. The choice is made randomly, according to agent specific, fixed properties, namely the agents’ honesties. Thus, the agents have to figure out how honest everybody is from the unreliable statements they get and some sparse clues. The only reliable information they get are their self-observations – they are aware whether they speak honestly or dishonestly – and accidental signs of other agents that can give their lies away. In particular, we account for the possibility that an agent may be “blushing” when they lie. This is emulated in the game by introducing a probability to “blush”. In the remainder of the paper, the value for this probability will be set to 10% for most agents.

Even though some communications may be deceptive, they nearly always contain valuable information and, as such, help determining whether agents are trustworthy or not. For example, an agent may recognize that a message is in strong contradiction with their own knowledge, which then adds information about the speaker, for example. Whether a diverging opinion in a message is recognized as valuable information or as a sign of a deception attempt depends among other things on the opinion that the receiver has about the honesty of the speaker. We call this opinion the reputation of the speaker with the receiver in the following. The self-reputation of an agent will be called the self-esteem of that agent.

What each agent thinks about the honesty of other agents and about themselves is described by separate probability distribution functions. If the game contains three agents, then there should be nine such probability functions. These probability functions depend on parameters whose values enable to describe whether an agent is trustworthy or not. After receiving an information, an agent will update these parameters to reflect the new information they got. The result of this opinion update will depend on their previous opinion of the sender and whether they consider the message trustworthy. The apparent honesty of the speaker plays a central role in this information update, as it partly determines how much their message is believed. The influences of opinions on the update of other opinions couple to the beliefs of the agents in a complicated way and eventually leads to emergent behaviors. Indeed, the messages of a more reputed agent, i.e. an agent perceived by the others to be more honest, will have a larger impact than messages from a less reputed one. However, more dishonest agents have more opportunities to manipulate others’ beliefs into a direction that is favorable to them. Here the word “favorable” means achieving a higher reputation. When lying, agents can promote agents, who seem to talk more positively about them, or try to reduce the reputation of those, who seem to make statements that are more harmful to their own reputation. In order to keep track of whom to support and whom to marginalize, each agent maintains a friend and an enemy list, which are updated by the agents whenever they hear a statement about themselves. Depending on how favorable this statement is in comparison to other agents’ statements, the speaking agent becomes a friend or an enemy to the receiver.

When agents lie, they try to undermine the receiver’s ability to detect the lie. In addition to the speakers reputation, lie detection is largely based on the similarity of the expressed opinion to the receiver’s own belief, thus liars try to send a slightly modified version of this belief back to their victim. To do this, they need to maintain an idea of what the victim believes on the different topics.

The interactions of agents allow for various strategies to boost their own reputation. Ordinary agents pick conversation partners and topics randomly and uniformly from the set of agents, while strategic agents target highest reputed agents as communication partners and egocentric agents prefer to speak about themselves. The reputation game allows to study how successful different strategies are in promoting their users’ reputation and which impact these strategies have on the social group, in terms of the networks defined by the reputations with each other and the relationships between agents.

2.2 Related approaches

The reputation game shows how a set of dynamical rules based on information or probability theory can lead to phenomena that resemble known sociological effects. There exists a rich literature on probabilistic modeling of trust and reputation in a sociological and economical context [e.g. 11, 16, 18] and on the evolution and maintenance of cooperation [e.g. 32, 33]. As mentioned already, this work can be regarded as an extension of these works into the direction of modeling the individual cognition and psychology in more detail. Our work uses ideas and principles from sociophysical simulations [7, 9, 34], computational cognition [29] and references therein, computational psychology [27, 28, 35], information theory [22, 23, 30, 31, 36], and complex adaptive system research [37].

Agent based simulations are frequently used in studies of gossip or rumor dynamics [e.g. 7, 10]. The reputation game takes inspiration from these works, but might differ from many of such simulations because our model is more complex. This complexity is required to capture more subtle aspects of human communication that are needed in the battle of deception and counter-deception. The price to pay for this complexity is a larger number of modeling choices than what might be typical for a socio-physical model [see for example trust networks in 17, 19]. We motivate our modeling choices, but these will need to be questioned, revised, and improved in future research to provide a more realistic setting.

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3Human reputation and self-esteem are certainly a more complex phenomena.

4As agents have unspecified genders we use the singular they and them to refer to them individually.
models try to capture the inner workings of minds, often from a bottom up approach by simulating low level brain functionalities like individual neurons and by shaping them in a way that intelligent behavior follows [e.g. 38]. Here, instead a top down approach is chosen, in which a certain amount of rationality of the agents’ minds is postulated (in the spirit of Bayesian models of cognition, 39, 40). This amount is mostly limited by the finite computational resources available.

Here, rationality mimics Bayesian reasoning or probabilistic logic [22, 31] and follows information theory [23, 30, 36]. Bayesian frameworks are used in many works on computational psychology and cognition [e.g. see 27] as well as in information theory [41] and references therein. Human minds are far from being perfect probabilistic logical systems [42], and the relevance of cognitive limitations w.r.t. perfect reasoning has been recognized for realistic psychological modeling [43]. The necessity for shortcuts and approximate logic in mental operations lies partly in the need for quick response times, but also in the vast resources required by perfect probabilistic logic, which assigns a probability value to each conceivable state of the world. Also our agents are not maintaining probabilities for each conceivable possibility, nor do they remember all past events. Instead, they only memorize compressed information. Information theory specifies how this is done optimally [23]. The limited information available to their reasoning is likely to make agents more vulnerable to manipulative strategies.

The interactions of the agents in the game result in a complex adaptive system [37]. Agents are strongly interacting, adapt to their social environment by learning about the honesty of other agents, and try to shape this environment to their advantage by raising or lowering the reputation of other agents that seem to be beneficial or harmful to them, respectively. Such hedging of other agents’ reputation is also called attribution of credit in complex adaptive system research [37]. Here, we focus on the impact of strategies on group dynamics and the emerging phenomena that they create. We do not study the origin of the strategies themselves. Some of the behavioral patterns that we investigate are inspired from the Dark Triad personalities of the Machiavellian, narcissistic, and sociopathic type [e.g. 44, 49]. For example, Babiak et al. [48] write about psychopaths:

Specifically, their game plans involved manipulating communication networks to enhance their own reputation, to disparage others, and to create conflicts and rivalries among organization members, thereby keeping them from sharing information that might uncover the deceit.

The reputation game allows to assess how successful such strategies are, and thereby might help to explain why such strategies have evolved in the real world in the first place.

3 The reputation game

3.1 Basic elements

In the game, a set $A$ of $n$ agents communicate together in sequential conversations. A conversation is defined as two agents exchanging statements. The conversation initiating agent $a \in A$ chooses a conversation partner $b \in A \setminus \{a\}$ and a conversation topic $c \in A$. Then $a$ and $b$ exchange statements about the reputation of $c$, denoted by $a \rightarrow b (a$ speaks to $b$ about $c)$ and $a \leftarrow b (b$ speaks to $a$ about $c)$ for the individual communications, as well as by $a \leftrightarrow b$ for the conversation (of $a$ and $b$ about $c$). Finally both $a$ and $b$ update the reputation of $a$, $b$, and $c$ according to their experiences and interpretations thereof. Agent $c$ could be a third agent, $a$, or $b$. A game round is a sequence of $n$ conversations, in which each agent initiates one conversation. The game ends after a predefined number of rounds. A single conversation and a conversation round are depicted in Fig. 1 and 2, respectively. The goal of each agent is to eventually obtain a reputation as high as possible.

The belief an agent $a$ maintains about some other agent $c$’s honesty $x_c \in [0, 1]$ is given by a parametrized, one-dimensional probability density distribution (PDF $\mathcal{P}(x_c | I_{ac})$, where $I_{ac} = (\mu_{ac}, \lambda_{ac}) \in \mathbb{R}^2$ is a tuple of parameters, which store the knowledge of $a$ on $x_c$.

In what follows, we choose $\mathcal{P}(x_c | I_{ac})$ to be a Beta distribution, as it is also used in related works [e.g. 15] and is a natural choice, as shown in Sect. 4.2. When $\mu_{ab}$ and $\lambda_{ab}$ are natural numbers they can be interpreted as being respectively the number of honest and dishonest statements that $a$ believes $b$ has made. We allow, however, both parameters to take values in the continuous interval $(-1, 10^6)$ (chosen for numerical reasons). The two parameters of the distribution allow to express an assumed mean honesty $\bar{x}_{ac}$, which we identify with $c$’s honesty according to $a$ (aka $c$’s reputation with $a$), as well as the uncertainty $\sigma_{ac}$, around that mean, which expresses how sure $a$ is about $c$’s reputation in terms of a standard deviation. The message from $a$ to $b$ consists of the topic $c$ and $a$’s belief encoding parameters $I_{ac}$ in case $a$ was honest, or a distorted version thereof, in case $a$ lied. Thus, agents own, maintain, and exchange beliefs in form of probabilities.

Similarly, agent $a$ will form their own views of the honesty of agent $b$ they communicate with. This is embodied by the set of parameters $I_{ba}$ for each $b \in A$. We will refer to $a$’s belief state as $I_a = (I_{ab})_{b \in A}$. If not mentioned otherwise, belief states do not contain information at the beginning of the game. But this changes in the course of the game, as agents’ belief states evolve with time in accordance with their experiences.

We denote probabilities with $P$ and PDFs with $\mathcal{P}$. They are related via integration: $P(x \in [x_1, x_2]| I) = \int_{x_1}^{x_2} dx \mathcal{P}(x| I)$. Note that probabilities take values in $P \in [0, 1]$, whereas PDFs in $\mathcal{P} \in \mathbb{R}_{\geq 0}^+ = [0, \infty]$. Bayes’ theorem applies to both, so that a strict discrimination between those is not always necessary. We therefore use the word “probability” for both, probabilities and PDFs.
The moves an agent $a$ can make in the game are to choose a conversation partner $b$ and a topic $c$, as well as to decide to lie. By default, these choices are made randomly. For example, whether an agent $a$ communicates honestly is chosen randomly according to agent $a$’s honesty parameter $x_a \in [0, 1]$. This specifies the frequency with which $a$ is honest and therefore $x_a \equiv P(a \text{ honest} | x_a)$. Other choices might be guided by the agent’s strategy. For example, we will define strategic agents, who preferentially pick highly reputed agents as conversation partners.

### 3.2 Information handling

When an agent $b$ receives a message from agent $a$ about agent $c$, agent $b$ has to judge how reliable the message is. If the message appears honest, the information contained in the message should be used to update $b$’s belief about $c$. The fact that $a$ was honest is also recorded by $b$. If the message appears to be a lie, $b$ should discard the message’s content and only record a lie for $a$. The problem is that $b$ rarely knows whether a message is honest or not, and can at best assign a probability to these possibilities. As a consequence, the PDF describing the correct posterior knowledge that an agent should have after receiving a message is a superposition of these two possible updates.

Furthermore, the PDFs, with which agent $b$ describes the honesty of speaker $a$ and of topic $c$, become entangled, as $b$ needs to recognize whether $a$ sent genuine information about $c$. The functional form of this potentially bi-modal, two dimensional, and potentially entangled PDF $\mathcal{P}(x_a, x_c | d, I_b)$, with $d$ the data obtained by $b$ and $I_b$ the prior knowledge of $b$, cannot be precisely captured by the functional form of the one dimensional PDFs $\mathcal{P}(x_a | I'_a)$ and $\mathcal{P}(x_c | I'_{bc})$ agent $b$ uses to store the updated knowledge $I'_b$. These only allow for product belief states of the form $\mathcal{P}(x_a, x_c | d, I_b) = \mathcal{P}(x_a | I'_a) \mathcal{P}(x_c | I'_{bc})$, which cannot express entanglements. Thus, information gets lost in an update, and agent $b$ should choose the new parameters $I'_b$ such that as much information as possible is kept from $\mathcal{P}(x_a, x_c | d, I_b)$.

We use the principle of minimal information loss for choosing the parameters in $I'_b$. Information loss can be quantified using the Kullback-Leibler (KL) divergence [36], which measures the information difference between original and approximate PDF. This choice of the information measure rests on a solid mathematical proof, which states, that in the absence of any other criteria, the KL is the only consistent choice to quantify how optimal a belief update is [23]. The KL based principle of minimal information loss has also proven to be extremely useful in many areas,

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In human psychology, however, additional criteria might be relevant that can lead to deviations from a pure KL based data compression. Recognizing liars might be more essential than differentiating between mostly honest people. Consequently, a positive-negative asymmetry of diagnosticity of information seems to be used by human minds when deciding how to store morality related information (e.g. honesty-dishonesty) [53]. In this first incarnation of the reputation game, we ignore such subtleties.
like information field theory\cite{54, 56} and information field dynamics\cite{57, 60}.

Some of the information will inevitably get lost during the belief update of an agent due to the limited flexibility of the parametric form and the product structure of belief states. This makes them vulnerable to rumors, misinformation, and self-deception, which in turn can be exploited by special communication strategies of deceptive agents.

### 3.3 Belief update

In the following, we explain the update due to the initial communication \( a \xrightarrow{c} b \). The update due to the response \( a \xleftarrow{c} b \) is analogous.

First, the speaker \( a \) updates their self-image according to whether \( a \) was honest or lied in the conversation, i.e. agent \( a \) increases \( \mu_{aa} \) by one if the message was honest, otherwise \( \lambda_{aa} \) is increased by one, as explained in Sect. \( \ref{belief-update} \).

The information agent \( b \) uses for the update is the overall communication setting \( a \xrightarrow{c} b \), the messages exchanged \( J(t) \), the blushing observation \( o_t \), and \( b \)'s assessments of \( x_a \) and \( x_c \). We call the tuple \( d_t = (a \xrightarrow{c} b, J(t), o_t) \) the data of the communication at time \( t \) and \( \mathcal{P}(x_a, x_c | I_b) = \mathcal{P}(x_a | I_b) \mathcal{P}(x_c | I_b) \) the prior of the update. The update of agent \( b \) proceeds in three stages:

1. First, \( b \) constructs the joint posterior probability function \( \mathcal{P}(x_a, x_c | d_t, I_b, A_b) \propto \mathcal{P}(d_t | x_a, x_c, I_b, A_b) \mathcal{P}(x_a, x_c | I_b) \). This expression contains the likelihood \( \mathcal{P}(d_t | x_a, x_c, I_b, A_b) \) to obtain the message \( d_t \). The functional form of this likelihood depends on agent \( b \)'s receiver strategy (see Tab. \( \ref{receiver-strategies} \)).

   A receiver strategy is the background information that determines the form of the likelihood \( b \) is using, given \( x_a, x_c, I_b \) and additional auxiliary information \( A_b \). This auxiliary information is dynamical and is used by \( b \) for orientation. It comprises of \( \kappa_b \), the level of surprise marking for \( b \) the border between typical lies and honest statements, agent \( b \)'s guesses for agent \( a \)'s beliefs and intentions w.r.t. \( c, I_{bac} \) and \( I_{bac} \), respectively, and other quantities.

2. This joint posterior is then approximated by the parametric form used to store beliefs, \( \mathcal{P}(x_a, x_c | I_{ba}', I_{bc}') = \mathcal{P}(x_a | I_{ba}') \mathcal{P}(x_c | I_{bc}') = \text{Beta}(x_a|\mu_{ba}', \lambda_{ba}') \text{Beta}(x_c|\mu_{bc}', \lambda_{bc}') \), by choosing values of \( I_{ba}' = (\mu_{ba}', \lambda_{ba}') \) and \( I_{bc}' = (\mu_{bc}', \lambda_{bc}') \), which then become the new belief parameters at time \( t + 1 \).

   The principle of least information loss is used to compress data, but information is inevitably lost in this step, since (i) the parametric form of the beta function is not able to represent all posterior structures and (ii) the entanglement of the variable \( x_a \) and \( x_c \) due to the received information cannot be represented by the product structure of the belief representation.

3. Finally, the auxiliary variables in \( A_b \) are updated. How this is done in detail is explained in Sect. \( \ref{belief-update} \).

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**Figure 2:** In a game round, each agent is initiator of one conversation (arrow with small agent icon directly next to it), but not necessarily addressed as a conversation partner (arrow without agent icon directly next to it). The opinion expressed by agent red to blue about agent green can affect green’s self-esteem, as it might partly propagate to green via blue. Whether the statement by blue on green was more positive or negative decides whether green regards blue as a friend, and therefore eventually how green speaks about blue to red. Thus, how red speaks about green can affect later on what red is made to believe about blue. This illustrates the high level of entanglement of the agent’s interactions.

### 3.4 Strategies

A **communication strategy** consists of a set of rules about whom to pick as a conversation partner and as a topic, as well as rules that guide the decisions on how to lie. For example, a malicious communication strategy could be to expose a victim to propaganda in form of massive self-appraisal. This can lead to a nearly complete conversion of the victim to the position expressed in the propaganda, as we show later on (Sect. \( \ref{self-appraisal} \)).

A communication \( a \xrightarrow{c} b \) received by agent \( b \) may provide relevant information on agent \( c \) if the information is trustworthy, but also on the speaker \( a \)'s honesty and intention. A receiver might judge the honesty of a message on the basis of various signs of deception. How an agent analyzes a message is the agent’s **receiver strategy**. All, but naive agents, use the reputation of the speaker with them as one of the indicators that gives weight to the message. This makes more reputed agents automatically more influential and being influential is what we regard as the agents’
ultimate goal.

Being influential in the reputation game means to be able to efficiently shift the opinions of others for one’s own advantage. This requires the ability to lie as well as a good reputation to get the lies accepted. We measure the general reputation of an agent as the agent’s average reputation with the others. Any boost of an agent’s reputation above the agent’s objective honesty is a measure of influence.

Since agents do not know their objective honesty a priori, they have to learn this from self-observations and feedback of the other agents. The resulting self-esteem is an important variable as well, as it is the basis of the communicated self-picture of an agent in case of honest communications. Obtaining a high self-esteem might therefore be a secondary goal of agents, as this permits self-appraisal without the risks involved in lying.

3.5 Rules of the game

The protocol of the reputation game consists of the following steps. Some of the mathematical and technical details mentioned here will be introduced in more detail in the remainder of the article.

1. A set of labeled agents \( A = \{ \text{red, black, cyan, yellow, blue, ...} \} \) participates in the game. Each agent \( a \in A \) has a number of static properties \((x_a, \text{the set of used strategies, ...})\) specifying the agent’s communication strategy and a set of dynamical variables \((I_a, I_b, K_a, \kappa_a, \ldots)\), being the parameters of the agent’s world model.

2. Time \( t \) is measured in communication events, which happen sequentially.

3. The central property of each agent \( a \) is the agent’s frequency to be honest, \( x_a = P(a \text{ is honest}\mid x_a) \). Other properties determine other aspects of the agent’s communication and receiving strategies.

4. The belief of agent \( a \) regarding the honesty of agent \( b \) is encoded in the parametric probability distribution \( P(x_b\mid I_{ab}) = \text{Beta}(x_b\mid \mu_{ab}, \lambda_{ab}) \), where \( I_{ab} = (\mu_{ab}, \lambda_{ab}) \) is the tuple of dynamical variables parameterizing \( b \)'s belief and Beta is the beta distribution. The joint belief state of an agent regarding the honesty of all other agents is set to the direct product of the single agent beliefs, \( P(x_I\mid I_a) = \prod_{b \in A} P(x_b\mid I_{ab}) \), with \( x = (x_b)_{b \in A} \) and \( I_a = (I_{ab})_{b \in A} \). This implies that agents are unable to keep track of entangled information of the sort “only one of \( b \) and \( c \) can be honest, not both”. Such an knowledge state would actually be appropriate in case the two agents \( b \) and \( c \) accuse each other to be liars.

5. A conversation \( a \xrightarrow{c} b \) is an exchange of statements between two agents \( a \) and \( b \) about agent \( c \), who is the topic of the conversation. The conversation starts at time \( t \) with the conversation initiator \( a \) choosing another agent \( b \in A \setminus \{a\} \) (excluding themselves to avoid a soliloquy without information exchange), and \( c \in A \) out of the set of all agents. Then \( a \) composes and transmits a statement \( J(t) \) about \( c \) to \( b \), which we also refer to as \( J = J(t) = J_{a\rightarrow b}(t) \) to clarify that message \( J \) is associated with the communication \( a \rightarrow b \). The initial communication is followed by a reciprocal message \( J(t + 1) \) from \( b \) to \( a \) about \( c \), denoted by \( a \xrightarrow{c} b \). The full conversation is denoted by \( a \xrightarrow{c} b \). Only after the statements are exchanged, the agents update their beliefs. By choosing conversation partner and topic, the initiating agent \( a \) basically requests \( b \) to make a statement on \( c \) (which could as well be \( a \) or \( b \)). How agents make these choices depends on their communication strategy (see Tab. 3). Agents that are initiating conversations about themselves, for example, will get to know who are their friends and enemies. See Fig. 1 for an illustration of a conversation.

6. The game is played in a number of \( N_{\text{rounds}} \) rounds. In each round, each agent initiates exactly one conversation with another agent, which consists of two communications and subsequent belief updates. The game ends after \( N_{\text{rounds}} \) rounds at time \( t_{\text{end}} = 2N_{\text{rounds}}|A| \). See Fig. 2 for an illustration of a round of conversations.

7. The format of the messages is that of the internal belief representation. For an honest communication \( a \rightarrow b \) at time \( t \) we therefore have the message \( J_{a\rightarrow b}(t) = I_{ac}(t) \).

8. Whether an agent \( a \) lies in a given conversation is usually decided by chance, with the agent specific frequency \( x_a \). When lying, all, except one category of agents called shameless agents, risk to accidentally reveal to their communication partner the fact that they are lying. The probability of being caught lying is \( f_b = 0.1 \). Here \( b \) stands for “blushing”, to mimic the fact that agents can give away the act of lying. This gives other agents some direct information about one’s honesty. We denote the observation of the blushing status of the speaker at time \( t \) as \( o_t \). Note, that agent \( b \) can also become convinced that \( a \) was honest. This happens when \( a \) makes a confession, a disadvantageous self-statement (without blushing).

9. After a conversation about agent \( c \), both communicating agents \( a \) and \( b \) update their beliefs in response to the information perceived about all involved agents, i.e. \( a, b, \) and \( c \) as explained before. With this slightly delayed update for the initial receiver \( b \), a communicated opinion \( J_{a\rightarrow b} \) of the conversation initiator \( a \) is not directly mirrored back to \( a \) in \( b \)'s response \( J_{b\rightarrow a} \). Side effects on other agents \( A\setminus\{a,b,c\} \) need not to be taken care of in the update due to the independent product structure of the belief representation, as detailed later in Sect. 4.3.2

10. After the game is over, the performance of the agents is
judged with respect to a number of performance metrics, such as the average reputation of an agent, which is an average of the other agents’ posterior means on the agent’s honesty, the frequency of obtaining a top reputation, and the like.

The motivations for the choices made by these rules and further details will be explained later on in depth.

4 Information representation

We first introduce probabilistic reasoning, before discussing the agent’s belief representation and updating in Sect. 4.2 and 4.3 respectively. The optimal data compression is introduced in Sect. 4.4.

4.1 Probabilistic reasoning

Agents need to maintain a picture of their social environment, to know who is honest and who is not. Since they do not have direct access to the intrinsic honesty parameters of any other agent, nor even to their own, they need to deduce these values from the information they get. This information, however, is incomplete, noisy, and often biased, with a noise level that depends on the evolving social atmosphere. Therefore, agents have to cope with significant amounts of uncertainty.

Bayesian probabilities are ideal for logical reasoning under uncertainty. Thereby, probabilities are regarded as a device that keeps book of the plausibility of different possibilities given some information $I$. Assigning a probability value $P(E|I) \in [0,1]$ to a possibility or an event $E$ therefore is not necessarily expressing how often $E$ happens on average, i.e. its frequency, but expresses the strength of the belief in $E$ being the case. If, however, an event $E$ has a frequency $f$, then the event’s probability equals this frequency if the latter is known, $P(E|I, f) = f$ with $I = "f \text{ is the frequency of } E"$.

Probabilities are subjective, in the sense that different probability values are assigned by agents with different knowledge. They are objective, in the sense that given the same knowledge state, any ideal mind should assign the same probability values. We use this in the following by only labeling the belief state $I_a$ of an agent $a$ on some quantity $x$, but not explicitly the induced probability $P(x|I_a)$ used by this agent. Any other agent $b$ with an identical belief state $I_b = I_a$ would assign exactly the same probability to $x$, $P(x|I_b) = P(x|I_a)$.

If there are a number of imperfectly known continuous quantities $x_1, \ldots, x_n$ then the PDF $P(x|I)$, with $x = (x_1, \ldots, x_n)$, expresses their joint probability density. The probability (density) of individual quantities is obtained from this by marginalization over the other parameters,

$$P(x_i|I) = \int dx_1 \ldots dx_{i-1} dx_{i+1} \ldots dx_n P(x|I). \quad (1)$$

In case the quantities are independent, the joint probability factorizes into marginal ones,

$$P(x|I, \text{independence}) = \prod_{i} P(x_i|I). \quad (2)$$

Often, probabilities do not factorize, $P(x|I) \neq \prod_i P(x_i|I)$. This expresses the entanglement between quantities, like that certain combination of two variables are particularly probable. Complicated entanglements can arise in the setting of the reputation game, since agents make statements about the trustworthiness of each other that are only believed in case they appear trustworthy themselves.

Here, $x_a$ will denote the honesty of agent $a$, with $x_a = 0$ stating that agent $a$ always lies and $x_a = 1$ that $a$ is always honest. These honesty values are denoted by the tuple $\bar{x} = (x_i)_{i \in A}$ with $A$ the set of agents. Any agent $b$ will maintain a belief state $I_b$ about these honesty values in form of the PDF $P(\bar{x}|I_b)$.

This is updated when new data $d$ becomes available according to Bayes’ theorem,

$$P(\bar{x}|d, I_b) = \frac{P(d|\bar{x}, I_b) P(\bar{x}|I_b)}{\int d\bar{x} P(d|\bar{x}, I_b) P(\bar{x}|I_b)} = \frac{P(d|\bar{x}, I_b)}{P(d|I_b)}. \quad (3)$$

Here, $P(d|\bar{x}, I_b)$ is the likelihood, the probability to have obtained the data $d$ given $\bar{x}$ and $I_b$. $P(\bar{x}|d, I_b)$ is the posterior, the probability for $\bar{x}$ given $d$ and $I_b$. The latter PDF is the knowledge about $\bar{x}$ updated by the data.

4.2 Belief representation

Ideally, after receiving new data $d$, agent $b$ would update the knowledge by just memorizing it, i.e. $I_b \to I'_b = (d, I_b)$, and use all recorded statements and Bayes’ theorem to construct their current beliefs. However, this would be computational expensive, as then all reasoning has to be repeated over and over again whenever new information arrives or an action has to be chosen. Therefore, our agents will follow the design of many cognitive systems, which only store and update some compressed information. This will be the tuple $I_b = (I_{bi})_{i \in A}$ consisting of $n$ parameter tuples $I_{bi}$ that describe agent $b$’s honesty impression of agent $i$, as well as some auxiliary information $A_b$. As we will not use probabilistic updates for the auxiliary information to limit the complexity of the simulation, we will omit $A_b$ in our equations in the following. Thus, we write $P(d|\bar{x}, I_b)$ instead of the more accurate $P(d|\bar{x}, I_b, A_b)$.

We will assume that agents do not store information on parameter entanglements, but simply keep track of the individual marginal probabilities about the honesty of each other agent and themselves. The knowledge of agent $b$ about the honesty of all agents is then given by the direct product of individual marginal probabilities,

$$P(\bar{x}|I_b) = \prod_{i \in A} P(x_i|I_{bi}). \quad (4)$$
| variable or symbol | ref. | range | meaning |
|-------------------|------|-------|---------|
| \( P(A|B), P(x|y) \) | 4.1  | \([0, 1], \mathbb{R}_0^+\) | probability of \( A \) given \( B \), PDF of \( x \) given \( y \) |
| \( n \) | 3.1  | \( \mathbb{N} \) | number of agents |
| \( \mathcal{A} \) | 3.1  | \{red, cyan, ...\} | set of \( n \) named agents |
| \( a, b, c, i \) | 3.1  | \( \mathcal{A} \) | some agents |
| \( a, b, c \) | 3.5  | \( \mathcal{A} \) | usually sender, receiver, and topic of a communication |
| \( x_i \) | 3.5  | \([0, 1]^n\) | indexed set of honesty of all agents |
| \( x = (x_i)_{i \in \mathcal{A}} \) | 3.5  | \( \mathbb{R}_0^+ \) | beta distribution |
| \( \text{Beta}(\alpha, \beta) \) | 3.5  | \beta, gamma function | |
| \( \psi(\alpha) = d\ln\Gamma(\alpha)/d\alpha \) | 4.5  | \( \mathbb{R}_0^+ \) | digamma function |
| \( \bar{t} = (\mu, \lambda) \) | 4.4  | \(-1, \infty)^2\) | stored belief about honesty of an agent |
| \( \bar{t}' \) | 3.2  | \(-1, \infty)^2\) | some other belief, not necessarily in the format of \( \bar{t} \) |
| \( \bar{t}'' = (\mu', \lambda' \) | 4.4  | \(-1, \infty)^2\) | encoding of \( \bar{t}' \) into storage format |
| \( \mu \) | 3.5  | \(-1, \infty\) | number of honest statements counted for an agent |
| \( \lambda \) | 3.5  | \(-1, \infty\) | number of lies counted for an agent |
| \( I_0 \) | 4.2  | \(-1, \infty)^2\) | prior information, here \( I_0 = (0, 0) \) |
| \( I_{ab} \) | 3.1  | \(-1, \infty)^2\) | belief of agent \( a \) on honesty of agent \( b \) |
| \( I_a = (I_{ai})_{i \in \mathcal{A}} \) | 3.1  | \(-1, \infty)^2n\) | beliefs of \( a \) on honesty of all agents |
| \( I_{abc} = (\mu_{abc}, \lambda_{abc}) \) | 3.5  | \(-1, \infty)^2\) | \( a \)'s assumption about belief of \( b \) about \( c \) |
| \( K \) | 3.5  | \(-1, \infty)^2\) | \( a \)'s assumption about \( b \)'s intention for \( c \) |
| \( \overline{D}_{KL}(x|\bar{t}) \) | 3.3  | \\(\mathbb{R}_0^+\) | Kullback-Leibler divergence \( \overline{D}_{KL}(P(x|\bar{t})||P(x|\bar{t}')) \) |
| \( \overline{D}_{KL}(x|\bar{t}) \) | 3.3  | \(-1, \infty)^2\) | expected \( x \) given information \( \bar{t} \) |
| \( J = J_{a \sim b}(t) = (\mu_j, \lambda_j) \) | 3.3  | \(-1, \infty)^2\) | message in communication \( a \leftrightarrow b \) at time \( t \) |
| \( \Delta J = J_{a \sim b} - I_{abc} \) | 3.3  | \(\mathbb{R}^2\) | apparent novel information in \( J_{a \sim b} \) on \( c \) |
| \( h = \text{honest} \) | 4.2  | \{true, false\} | whether message was honest, meaning \( J_{a \sim b} = I_{ac} \) |
| \( -h = \text{lie} \) | 4.2  | \{true, false\} | whether message was a lie, meaning \( J_{a \sim b} \neq I_{ac} \) |
| \( \text{state} \) | 5.1  | \{\(h, -h\}\} | state of a message |
| \( b = \text{blush} \) | 5.1  | \{true, false\} | whether speaker blushed because of lying |
| \( o = \text{a.o} \) | 5.1  | \{\(b, -b\}\} | blushing observation of comm. \( J \), \( b = \text{blush} \) |
| \( f \) | 5.3  | 0.1 | frequency of blushing while lying |
| \( d = (a \leftrightarrow b, J, o) \) | 4.4  | \(\mathbb{A}^3(-1, \infty)^2\{b, -b\}\) | data: communication, message, blushing observation |
| \((\mu, \lambda)^+ := \begin{cases} (\mu, \lambda) & \text{if } \mu, \lambda \geq 0 \\ I_0 & \text{else} \end{cases} \) | 4.4  | \(-1, \infty)^2\) | ensures convex PDFs, reduces confusing updates |
| \( y_J \) | 4.4  | \([0, 1]\) | probability of received message being honest |
| \( K_J \) | 5.1  | \(\mathbb{R}^+\) | amount of new information in message \( J \) if honest |
| \( K_b \) | 5.1  | \(\mathbb{R}^+\) | last ten non-zero \( K_J \)s encountered by agent \( b \) |
| \( S_J = K_J/K_b \) | 5.1  | \(\mathbb{R}^+\) | relative surprise of message \( J \) for agent \( b \) |
| \( R(d) \) | 4.7  | \(\mathbb{R}^+\) | ratio of likelihoods for \( J \) lie and for \( J \) honest |
| \( I^+ = I^{+}(1, 0) \) | 4.3  | \(-1, \infty)^2\) | belief \( I \) on speaker, updated for being honest |
| \( I^\circ = I^{+}(0, 1) \) | 4.3  | \(-1, \infty)^2\) | belief \( I \) on speaker, updated for being dishonest |
| const, const' ... | 36   | \(\mathbb{R}\) | irrelevant constants |

Table 1: Used variables and symbols, the Sec. or Eq. of their definition, their ranges, and meanings.
The functional form of the belief about the honesty of a single agent $c$, $\mathcal{P}(x_c|I_a)$, should be derived here from the case where agent $c$ makes unambiguous observations, namely the self-observation of their own actions. To investigate this, let us first concentrate on the case agent $a$ communicates honestly, the message is in the state “honest”. This happens with the frequency $x_a$. The update of the self-belief state $I_{aa}$ of agent $a$ should then be according to Eq. 3

$$\mathcal{P}(x_a|I_{aa}, h) = \frac{P(h|x_a, I_{aa}) \mathcal{P}(x_a|I_{aa})}{P(h|I_{aa})} \propto x_a \mathcal{P}(x_a|I_{aa}),$$

since $P(h|x_a, I_{aa}) = x_a$. Thus, whenever agent $a$ communicates honestly, the probability expressing the self-perception should be multiplied with $x_a$ and then normalized.

Now, let us investigate the case of agent $a$ lying, the message state is “lie” = $-h$, which happens with frequency $1 - x_a$. Then we have

$$\mathcal{P}(x_a|I_{aa}, -h) = \frac{P(-h|x_a, I_{aa}) \mathcal{P}(x_a|I_{aa})}{P(-h|I_{aa})} \propto (1 - x_a) \mathcal{P}(x_a|I_{aa}),$$

since $P(-h|x_a, I_{aa}) = 1 - x_a$. Thus, whenever lying, the self-perception probability should be multiplied with $(1 - x_a)$.

It is therefore reasonable to represent the self-perception via numbers of honest and dishonest statements, $\mu_{aa}$ and $\lambda_{aa}$, respectively. The corresponding probability is then

$$\mathcal{P}(x_a|I_{aa}) = \frac{(\mu_{aa} + \lambda_{aa} + 1)!}{\mu_{aa}! \lambda_{aa}!} x_a^{\mu_{aa}} (1 - x_a)^{\lambda_{aa}},$$

with $I_{aa} = (\mu_{aa}, \lambda_{aa})$. Here, it is assumed that the prior distribution in absence of further information is flat, $\mathcal{P}(x_a|I_0) = 1$ with $I_0 = (0, 0)$.

We adopt this functional form for the honesty information representation for all agents. We drop agent indices for a moment and the requirement of integer parameters $\mu$ and $\lambda$ by allowing $\mu, \lambda \in (-1, 10^6]$ in the following, where the lower limit ensures proper (integrable) PDFs and the upper limit numerical stability. With this, the corresponding probability generalizes to

$$\mathcal{P}(x|I) := \frac{x^\mu (1 - x)^\lambda}{\mathcal{B}(\mu + 1, \lambda + 1)} = \text{Beta}(x|\mu + 1, \lambda + 1),$$

with $I = (\mu, \lambda)$,

$$\mathcal{B}(\alpha, \beta) := \int_0^1 dx x^{\alpha-1}(1 - x)^{\beta-1} = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha + \beta)}$$

being the beta function, and

$$\text{Beta}(x|\alpha, \beta) := \frac{x^{\alpha-1}(1 - x)^{\beta-1}}{\mathcal{B}(\alpha, \beta)}$$

the beta distribution. This provides a bit more flexibility compared to the case of $\alpha, \beta \in \mathbb{N}$ to express small information gains, which is needed in case the obtained data contains ambiguous information. Such probabilities $\mathcal{P}(x|I)$ for a number of belief states for an agent’s honesty are shown in Fig. [3].

We note that $\mathcal{P}(x|I)$ defined this way has a mean and variance of

$$\overline{x}_I := \langle x |x|I \rangle = \int_0^1 dx x \mathcal{P}(x|I) = \frac{\mu + 1}{\mu + \lambda + 2},$$

$$\sigma^2_I := \langle (x - \overline{x}_I)^2 |x|I \rangle = \frac{\overline{x}_I (1 - \overline{x}_I)}{\mu + \lambda + 3},$$

and denote with $\overline{x}_{ab} := \overline{x}_{I_{ab}}$ the reputation $b$ has in the eyes of $a$. 

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Figure 3: PDFs corresponding to belief states about an agent’s honesty $x$ as given by $I = (0, 0)$, $(1, 0)$, $(2, 0)$, $(9, 1)$, $(26, 4)$, $(70, 15)$ as marked by solid lines, and $(-0.99, -0.99)$ as marked by the dashed line. The peaks of the PDFs increase in this order. The case shown with the dashed line corresponds to believing the respective agent to be either a very frequent liar or to be very honest, but not an occasional liar. Left: Linear scale. Right: Logarithmic scale.
4.3 Belief update

When receiving a statement \( J = J_{a → b}(t) = (\mu_J, \lambda_J)_{a → b}(t) \) from agent \( a \) at time \( t \), agent \( b \) assesses the reliability of the statement by assigning the probability

\[
y_J = P(h|d, I_b)
\]

to the possibility that \( a \) communicated honestly depending on \( b \)'s receiver strategy as will be discussed in Sect. 5. This assignment is based on the prior belief \( I_{ba} \) on \( a \)'s honesty (and some auxiliary information \( A_b \)) and the data \( d = d(t) = (a → b, J, o)(t) \), which consist of the message \( J \) and the observation \( o \) whether agent \( a \) blushed or not (accidentally revealed a lie or not).

Untrustworthy message

If a communication \( J \) appears completely untrustworthy to the receiver \( b \) they will set \( y_J = 0 \) and ignore the statement made about the conversation topic. However, \( b \)'s opinion about the speaker \( a \) will be updated. Agent \( b \)'s posterior about \( a \) will change according to

\[
\mathcal{P}(x_a|h, I_{ba}) = \frac{\mathcal{P}(h|x_a, I_{ba}) \mathcal{P}(x_a|I_{ba})}{\mathcal{P}(h|I_{ba})} \propto x_a^{\mu_{ba}} (1 - x_a)^{1 + \lambda_{ba}},
\]

since \( \mathcal{P}(h|x_a, I_{ba}) = 1 - x_a \) irrespective of \( I_{ba} \). This new belief state is represented by increasing \( b \)'s lie-counter \( \lambda_{ba} \) for \( a \) by one,

\[
I_{ba}(t) \rightarrow I_{ba}(t + 1) = (\mu_{ba}(t), \lambda_{ba}(t) + 1) = I_{ba}^0(t).
\]

All other beliefs of agent \( b \) stay unchanged, \( I_{bi}(t + 1) = I_{bi}(t) \) for all \( i \in A \backslash \{a\} \). For later usage we introduced the notation \( I^0 := I + (0, 1) \) for a belief state \( I \) updated by one lie. Similarly, \( I^\oplus := I + (1, 0) \) should denote a belief state \( I \) updated by one observed honest statement.

Trustful update

If, however, agent \( b \) is convinced that the statement received from \( a \) is honest, then agent \( b \) assigns \( y_J = 1 \). Let's assume that \( a \) does not make a statement about themselves, \( a \neq c \). If agent \( b \) believes the statement is honest (\( b \) thinks \( “J_{a → b}(t) = I_{ac}(t)” \)) then \( b \) only needs to identify the new information in it. For spotting the news in an expressed opinion, the receiver needs to know the opinion of the speaker at the time of their last conversation. Agents maintain guesses on each other’s previous beliefs for this purpose. The guess of agent \( b \) at time \( t \) what \( a \) believed about \( c \) at their last conversation is denoted as \( I_{bac}(t) = I_{bac} = (\mu_{bac}, \lambda_{bac}) \). How this is maintained is explained later in Sect. 5.3. The new information in an honest statement of \( a \) on \( c \) is then

\[
\Delta J = \Delta J(t) = (\Delta \mu, \Delta \lambda)(t) = J_{a → b}(t) - I_{bac}(t).
\]

If all accounting was correct, \( \Delta \mu, \Delta \lambda \geq 0 \) should be the case. If this is not the case, something went wrong and agent \( b \) better assumes not to have received any new information, as expressed in \( \Delta J \rightarrow I_0 = (0, 0) \). We denote this by

\[
\Delta J^+ = (\Delta \mu^+, \Delta \lambda^+) := \begin{cases} 
(\Delta \mu, \Delta \lambda) & \text{if } \Delta \mu, \Delta \lambda \geq 0 \\
I_0 & \text{else}.
\end{cases}
\]

Agent \( b \) therefore realizes that agent \( a \) is reporting agent \( c \) to have made \( \Delta \mu^+ \) new honest and \( \Delta \lambda^+ \) new dishonest statements since they spoke last about \( c \). The belief update on \( c \) should then be[9]

\[
\mathcal{P}(x_c|\Delta J^+, h, I_b) = \frac{\mathcal{P}(\Delta J^+|x_c, I_{bc}) \mathcal{P}(x_c|I_{bc})}{\mathcal{P}(\Delta J^+|I_{bc})} \propto x_c^{\mu_{bc} + \Delta \mu^+} (1 - x_c)^{\lambda_{bc} + \Delta \lambda^+}.
\]

This can be represented by agent \( b \) just increasing the counts for assumed honest and dishonest statements of \( c \), i.e.,

\[
I_{bc}(t) \rightarrow I_{bc}(t + 1) = I_{bc}(t) + \Delta J^+(t).
\]

Such a trustful update is illustrated in Fig. 3.

Since agent \( b \) assumes that agent \( a \) has said the truth, \( b \) registers

\[
I_{ba}(t) \rightarrow I_{ba}(t + 1) = (\mu_{ba}(t) + 1, \lambda_{ba}(t)) = I_{ba}^\oplus(t).
\]

All other beliefs of agent \( b \) stay unchanged, \( I_{bi}(t+1) = I_{bi}(t) \) for all \( i \in A \backslash \{a,c\} \).

Finally, we need to deal with the case that agent \( a \) made a self-statement that agent \( b \) regards as absolutely honest. Then, the two above update rules for \( c \) and \( a \) just need to be merged into a single one for \( a \),

\[
I_{ba}(t) \rightarrow I_{ba}(t + 1) = I_{ba}^\oplus(t) + \Delta J^+(t)
\]

and \( I_{bi}(t + 1) = I_{bi}(t) \) for all \( i \in A \backslash \{a\} \).

Skeptical update

The two cases of updates discussed above lead to joint belief states on \( a \) and \( c \) for agent \( b \) that again are represented by product states without any entanglement,

\[
\mathcal{P}(x_a, x_c|d(t), I_b(t)) = \mathcal{P}(x_a|I_{ba}(t + 1)) \mathcal{P}(x_c|I_{bc}(t + 1)).
\]

When agent \( b \) is unsure whether \( a \) was honest or lied, the resulting belief state should be a superposition of the state

\[9\text{To limit the complexity of the simulation, our agents ignore the possibility that other agents might be misinformed.} \]
after an assumed honest communication and a perceived lie. The former is given by Eq. 23 (Eq. 24 for \( a \neq c \) or Eq. 25 for \( a = c \)) and the latter by Eq. 18. The superimposed states should have weights according to their probabilities. Thus, \( y_J = P(h|d, I_0) \) is the weight of the honest message state and \( 1 - y_J = P(\neg h|d, I_0) \) the weight of the dishonest message state.

Let us first assume that \( a \neq c \). We then have

\[
\begin{align*}
\mathcal{P}(x_a, x_c|d, I_b) &= y_J \mathcal{P}(x_a|I^\beta_{ba}) \mathcal{P}(x_c|\Delta J^+, h, I_{bc}) \\
&\quad + (1 - y_J) \mathcal{P}(x_a|I^\beta_{ba}) \mathcal{P}(x_c|I_{bc}) \\
&= y_J \text{Beta}(x_a|I^\beta_{ba}) \text{Beta}(x_c|I_{ba} + \Delta J^+) \\
&\quad + (1 - y_J) \text{Beta}(x_a|I^\beta_{ba}) \text{Beta}(x_c|I_{bc}).
\end{align*}
\]

We note that this is not a direct product of marginal distributions any more used in the agent’s memories since \( b \)-s knowledge on the honesty of \( a \) and \( c \) got entangled.

When \( a \) speaks about themselves, we have \( c = a \) and assign\(^{10}\)

\[
\begin{align*}
\mathcal{P}(x_a|d, I_b) &= y_J \text{Beta}(x_a|I^\beta_{ba} + \Delta J^+) + (1 - y_J) \text{Beta}(x_a|I^\beta_{ba}).
\end{align*}
\]  

In general, this is also not in the format used by agent \( b \) to memorize beliefs, \( \mathcal{P}(x_a|I_{ba}(t + 1)) = \text{Beta}(x_a|I_{ba}(t + 1)) \).

Since the cases of a certainly honest and a certainly dishonest message are enclosed in Eqs. 28 and 29 by setting \( y_J = 1 \) and \( y_J = 0 \), respectively, we only have to consider skeptical updates in the following.

\(^{10}\) Actually, since a self-statement is self-referential with respect to its truth value, a logically fully consistent update would require to solve an implicit self-consistent relation. This can be seen for example in case agent \( a \) makes the statement to be a notorious liar, which if true is contradicted by just have made an honest confession. We, as well as our agents, do not invest mental energy in such philosophical calculations, but just use the pragmatic Eq. 29.

4.4 Optimal belief approximation

Usually the honesty of a message is unclear to the receiver \( b \). In this case, the belief state \( \mathcal{P}(x|I') \) with \( I' = I'(t) := (d(t), I_b(t)) \) as given by Eq. 28 is a superposition of the two belief states that would arise if the message is known to be honest and to be dishonest. In order to cast \( \mathcal{P}(x|I') \) into the functional form of \( \mathcal{P}(x|I) \) a new \( I'' = I_b(t + 1) \) has to be found that captures as much as possible the information of \( I' \). The information loss in this approximation of \( I' \) by \( I'' \) is measured by the Kullback-Leibler (KL) divergence

\[
\text{KL}_{E}(I', I'') := D_{KL}(\mathcal{P}(x|I')||\mathcal{P}(x|I'')) (30)
\]

\[
:= \int dE \mathcal{P}(x|I') \ln \left( \frac{\mathcal{P}(x|I')}{\mathcal{P}(x|I'')} \right) (31)
\]

in units of nits (\( = 1.44 \) bits) \(^{23}\). Thus, \( \text{KL}_{E}(I', I'') \) should be minimized with respect to \( I'' \), the parameters of the approximate belief state \(^{11}\). These then form the next information state \( I_b(t + 1) = I'' \) of \( b \).

Since the update concerns only the knowledge about agents \( a \) and \( c \), the sender and topic of a message, only the beliefs about those need updating. Side effects do not occur here as agents do not track entanglements. Learning that \( a \)'s honesty is different from what \( b \) has previously assumed is not letting \( b \) reevaluate \( a \)'s past statements as \( b \) neither memorizes those precisely, nor the entanglements these imply.

Thus, the relevant KL for agent \( b \)'s belief update after receiving information \( d \) from \( a \) about \( c \) is \( \text{KL}_{E}(x_a, x_c)(I', I'') \) with \( \mathcal{P}(x_a, x_c|I') = \mathcal{P}(x_a, x_c|d, I_{ba}, I_{bc}) \) being the accurate, potentially entangled belief state and

\[
\begin{align*}
\mathcal{P}(x_a, x_c|I'') &= \mathcal{P}(x_a|I^\beta_{ba}) \mathcal{P}(x_c|I^\beta_{ba}) \quad (32)
\end{align*}
\]

\[
\begin{align*}
&= \text{Beta}(x_a|I^\beta_{ba}) \text{Beta}(x_c|I^\beta_{ba}) (33)
\end{align*}
\]

\(^{11}\) Note that we regard the KL here to be a function of the information sets \( I' \) and \( I'' \) on the quantity \( \frac{\mathcal{P}(x)}{\mathcal{P}(x)} \) in contrast to the standard convention to define \( D_{KL} \) as a functional of the PDFs implied by these.
being the simplified state over the relevant subspace of $x_a$ and $x_c$ that will be memorized. As the latter is a direct product of one dimensional PDFs, it turns out that it is sufficient to perform only two one dimensional updates based on the two marginals

$$P(x_a|I') = \int_0^1 dx_c P(x_a, x_c|I')$$ and (34)
$$P(x_c|I') = \int_0^1 dx_a P(x_a, x_c|I').$$ (35)

This is because the two dimensional KL$(x_a, x_c)$ of the joint update on agents $a$ and $c$ separates into two one dimensional KLs for the marginal distributions of $x_a$ and $x_c$.

$$\text{KL}(x_a, x_c)(I', I'') = \int dx_a \int dx_c P(x_a, x_c|I') \ln \frac{P(x_a, x_c|I')}{P(x_a, x_c|I''')}$$

$$= \int dx_a \int dx_c P(x_a, x_c|I') \times$$

$$[- \ln P(x_a|I''') - \ln P(x_c|I''')] + \text{const}$$

$$= - \int dx_a P(x_a|I') \ln P(x_a|I''')$$

$$- \int dx_c P(x_c|I') \ln P(x_c|I''') + \text{const}$$

$$= \text{KL}_{x_a}(I', I''') + \text{KL}_{x_c}(I', I''') + \text{const},$$ (36)

and these can be minimized individually with respect to $I'''$ and $I''''$. Constant terms w.r.t. $I''''$ are denoted const and const'.

For calculating these single agent marginal KLs, $\text{KL}_{x_a}(I', I''')$ and $\text{KL}_{x_c}(I', I''')$, we need expressions for the marginal updates on speaker and topic, $P(x_a|I')$ and $P(x_c|I')$ as given by Eqs. 34 and 35. The involved integrals can be calculated analytically and the results for the different cases unify and generalize to a single expression of the marginal update for any agent $i \in \mathcal{A}$.

$$P(x_i|I') = y_J \text{Beta}(x_i|I_{bi}^n) + (1-y_J) \text{Beta}(x_i|I_{bi}^h),$$ (37)

with

$$I_{bi}^n := I_{bi} + (1,0)_i \text{ speaker} + \Delta J_{i \text{ topic}},$$ (38)

$$I_{bi}^h := I_{bi} + (0,1)_i \text{ speaker},$$ (39)

$$I_{i \text{ condition}} := \begin{cases} I & \text{condition is true} \\ 0 & \text{condition is false} \end{cases}$$ (40)

an information that only is taken into account in case the condition is true. Eq. 37 is valid for all agents $i \in \mathcal{A}$, including the topic $c$, the speaker $a$, the receiver $b$, or anybody else. In case $i \not\in \{a,c\}$, Eq. 37 states that for agent $i$ the initial belief is to be kept, $P(x_i|I') = \text{Beta}(x_i|I_{bi}) = P(x_i|I_b)$, as no information about $i$ was revealed.

The single agent’s marginal KLs are then

$$\text{KL}_{x_a}(I', I'') = y_J \text{KL}_{x_a}(I_{bi}^n, I_{bi}^n) + (1-y_J) \text{KL}_{x_a}(I_{bi}^h, I_{bi}^h),$$ (41)

$$\text{KL}_{x}(I', I'') = (\mu - \mu'') \psi(\mu+1) - \psi(\mu+\lambda+2) + (\lambda - \lambda'') \psi(\lambda+1) - \psi(\mu+\lambda+2) + \ln B(\mu+1, \lambda'') - B(\mu+1, \lambda''),$$ (42)

$$\psi(\alpha) = \frac{d \ln \Gamma(\alpha)}{d \alpha}$$ (43)

the digamma function. These KLs, $\text{KL}_{x_a}$ for speaker $a$ and $\text{KL}_{x_c}$ for speaker $c$, are especially needed for $\psi(\alpha)$. The expression in Eq. 42 can be derived using the expectation

Figure 5: Belief updates of an agent $b$ listening for the first time to the self-appraisal of agent $a$. The initial belief state $I_{ba} = (7,5)$ (black solid line) changes after receiving the message $J_{ga,b} = (23,1)$ (black dotted line) by an amount that depends on whether agent $b$ trusts the message fully ($y_c = 1 \Rightarrow I'' = (31,6)$, blue), with the sender's reputation ($y_J = \pi_{ab} = 0.57 \Rightarrow I'' = (4.0,1.2)$, orange) or only a little ($y_J = 0.1 \Rightarrow I'' = (4.6,3.3)$, green). For these cases, the correct posteriors are shown with dashed lines and the memorized PDFs as solid lines in the corresponding colors. The perceived honesty of the message is included in the updates shown in color, but not in the naive update (black dashed line). Left: Linear scale. Right: Logarithmic scale.
5 Receiver strategies

In the reputation game, a speaker tries to construct effective lies when deceiving. An effective lie should on the one hand be as big as possible (as measured in bits) to pursue the speaker’s agenda, and on the other hand sufficiently small to go unnoticed by the receiver. These are opposite requirements and the optimal scale depends on the lie detection abilities of the receiver. It can therefore be assumed that lie construction and detection strategies should be the result of an antagonistic co-evolution. Here, we follow some imagined first steps of such an evolution by first constructing some basic lie detection strategies in Sect. 5.1, then introduce an adapted lie construction strategy in Sect. 5.2 and finally a smart lie detection strategy adapted to this in Sect. 5.3. Finally, we explain how the auxiliary variables used by agents in lie construction and detection are maintained in Sect. 5.4. An overview on the different receiver strategies is given in Tab. 2.

5.1 Basic lie detection

A lie detection strategy of an agent is a recipe for how to choose the weight \( y_J := \mathcal{P}(h|d) \) of a message \( J \) in a communication \( a \to b \). For example, naive agents always assign value \( \langle x \ln x \rangle_{\text{Beta}(x|\alpha, \beta)} = \frac{\alpha}{\alpha + \beta} [\psi(\alpha + 1) - \psi(\alpha + \beta + 1)] \)[2]. \( y_J = 1 \) irrespectively of the data. This is obviously a poor strategy. It already is problematic in case of non-deceptive agents \(^{13} \) for the strong echo chamber effect it allows, which leads to a too rapid convergence of premature opinions.

The message weight \( y_J \) should best be assigned according to Bayes theorem, yielding

\[
y_J = \frac{\mathcal{P}(d|h) \mathcal{P}(h)}{\mathcal{P}(d)} \tag{44}
\]

\[
y_J = \frac{\mathcal{P}(d|h) \mathcal{P}(h)}{\mathcal{P}(d) \mathcal{T}_{ba}} = \frac{\mathcal{P}(d|h) \mathcal{T}_{ba}}{\mathcal{P}(d|h) \mathcal{T}_{ba} + \mathcal{P}(d|\neg h) (1 - \mathcal{T}_{ba})}, \tag{45}
\]

\[
\mathcal{R}(d) = \frac{\mathcal{P}(d|h)}{\mathcal{P}(d)} \tag{47}
\]

the likelihood ratio, \( d = (a \xrightarrow{\neg} b, J, a) \) the data available to \( b \), and

\[
\mathcal{P}(h) = \int dx_a \mathcal{P}(x_a|I_{ba}) = \langle x_a \rangle_{\langle x_a | I_{ba} \rangle} = \mathcal{T}_{ba} \tag{48}
\]

the prior probability that \( b \) assigns to \( a \) for being honest. Thus, a receiver strategy is fully specified as soon as the likelihoods \( \mathcal{P}(d|h) \) and \( \mathcal{P}(d|\neg h) \) are given or even just their lie-to-honest likelihood ratio \( \mathcal{R}(d) = \mathcal{P}(d|\neg h)/\mathcal{P}(d|h) \).

The reputation of a speaker has a strong influence on whether their potentially outrageous statements will be believed or not. If we set \( y_J = 1/2 \) to investigate which statements are at the margin to being trustworthy and solve Eq. 46 for the likelihood ratio

\[
\mathcal{R} = \frac{y_J - 1}{\mathcal{T}_{ba} - 1} \bigg|_{y_J = 1/2} = \frac{\mathcal{T}_{ba}}{1 - \mathcal{T}_{ba}} \tag{49}
\]

\( ^{13} \)Non-deceptive agents would even communicate honest statements when they should lie according to their lie-frequencies 1 - \( x_a \).

\( ^{2} \)Beta function.
we see that three agents with reputations \( \tau_{ba} = 0.1, 0.5, \) and 0.9 reach \( y_f = \frac{1}{2} \) for \( \mathcal{R} = \frac{1}{n}, 1, \) or 9, respectively. Thus, the lie-to-honest likelihood ratio of a statement can be 81 times larger for the most reputed agent (\( \tau_{ba} = 0.9 \)) compared to that of a statement by the least reputed of those agents (\( \tau_{ba} = 0.1 \)) before it is perceived as only half trustworthy. Statements of reputed agents are much more trusted.

We assume in the following that these likelihoods are given by independent probabilities for a number of data features \( f_j(d) \) with \( j \) labeling the different features. Thus, for the honesty state \( \in \{h, \neg h\} \) we have

\[
\mathcal{P}(d|\text{state}) = \prod_j \mathcal{P}(f_j(d)|\text{state}).
\]

The features used in basic lie detection are naive trust, speaker reputation, blushing, confessions, and message surprise. Smart lie detection will additionally use expectation matching. These features will be explained in the following. The assumption of their independence is not entirely realistic; however, our aim is to set up a reasonably functioning lie detection, but not necessarily the best possible. The independence assumption permits to write

\[
\mathcal{R}(d) = \prod_j \mathcal{R}_j(f_j(d)) = \prod_j \frac{\mathcal{P}(f_j(d)|\neg h)}{\mathcal{P}(f_j(d)|h)}. 
\]

To calculate the likelihood ratio, naive agents use only naive trust, uncritical agents use additionally the speaker reputation and blushing, critical agents use further confessions and surprise information, whereas smart agents exploit expectation matching in addition to the former features, which is whether a message looks more like what the speaker seems to believe, or what the speaker apparently wants them to believe. We also introduce deaf agents, who only use blushing information to learn about others, as an illustrative reference. Deaf agents are also uncritical, as they do not inspect the message content, neither for deciding about its honesty, not for updating their beliefs.

### Naive trust

Naive agents always trust the speaker and set \( y_f = 1 \). This implies that for them

\[
\mathcal{R}(d) = \mathcal{R}_{\text{naive}}(d) = 0
\]

or \( \mathcal{P}(d|\neg h) = 0 \), meaning that they assume a lie would never have reached them.

### Speaker reputation

Let us first inspect the case that no feature beyond the message existence is used at all, and that this existence does not imply any information on the honesty, \( \mathcal{P}(d|h) = \mathcal{P}(d|\neg h) \). Therefore, \( \mathcal{R}(d) = 1 \) and \( y_f = \tau_{ba} \). Thus, without inspecting the message data agent \( b \) assigns the prior average belief on the honesty of \( a \) to the message being honest. This already provides some amount of defense against liars, since if identified as such, they get their messages down weighted.

### Blushing

The blushing feature \( f_b(d) = a \in \{\text{blush}, \text{no blush}\} =: \{b, \neg b\} \) has the likelihood

\[
\mathcal{P}(b|\neg h) = f_b, \quad \mathcal{P}(b|h) = 0, \text{ and } \mathcal{P}(\neg b|\text{state}) = 1 - \mathcal{P}(b|\text{state}).
\]

Therefore, \( \mathcal{R}(b) = \infty \) and \( \mathcal{R}(\neg b) = 1 - f_b \). Thus, the uncritical agent assigns

\[
\mathcal{R}_{\text{uncritical}}(d) = \mathcal{R}_b(a) = \frac{1 - f_b}{\mathcal{P}(\neg b|a)} = \left\{ \begin{array}{ll} \infty & \text{b} \\ 1 - f_b & \text{\neg b} \end{array} \right.,
\]

where \( \mathcal{P}(\neg b|a) = \mathcal{P}(a = \neg b|a) \in \{0, 1\} \) is the logical theta function that is unity in case of no blushing, and otherwise zero. The uncritical agent, who uses only blushing information, assigns \( y_f = 0 \) in case the speaker blushes, otherwise \( y_f = \tau_{ba} / [(1 - f_b) + f_b \tau_{ba}]^{-1} = \tau_{ba} [0.9 + 0.1 \tau_{ba}]^{-1} \approx \tau_{ba} \), since \( f_b = 0.1 \). The small enhancement of \( y_f \) w.r.t. \( \tau_{ba} \) is due to the weak indication of honesty implied by non-blushing, see Eq. 46 with \( \mathcal{R}(d) = \mathcal{R}_b(\neg b) = 1 - f_b = 0.9 \) inserted.
Confession

As agents rather overstate their honesty than underestimate it, a self-statement of a currently non-blushing agent $a$ that indicates an honesty $\tau_{ja\sim a,b}^{\sim}$ below the agent’s reputation $\tau_{ba}$ to $b$ must be an honest confession. Whether a confession is present is given by

$$f_c(d) := c := (\tau_{ja\sim a,b}^{\sim} < \tau_{ba}) \in \{\text{true}, \text{false}\} \quad (57)$$

and we have $P(c|h) = 0$, such that

$$R_c(c, \neg b) = \frac{P(c \land \neg b|\neg h)}{P(c \land \neg b|h)} = 0 \quad (58)$$

and therefore $y_J = 1$ if a confession is present. The absence of a confession does not bear much information, as it could be caused by a lie or by agent $b$ being misinformed about the true honesty of $a$. The former has a probability of $1 - x_a$, but the probability of the latter is hard to estimate accurately. Thus, it is safer to set the likelihood ratio for all other cases to be uninformative,

$$R_c(\neg(c \land \neg b)) = 1, \quad (59)$$

than to risk to get misleading hints. We collect all these cases in

$$R_c(J,o) = P(\neg c \lor b|J,o), \quad (60)$$

again using the probability notation to express a logical theta function.

Message surprise

Critical agents use in addition to the blushing and confession information the cognitive dissonance the message generates if taken for true, which we associate to the surprise of a message $J_{a\sim a}^{\sim}$, with respect to their own beliefs. This surprise is $s_J = KL_{x_a}(J_{a\sim a}^{\sim}, I_{bc})$, the number of nits a plain adaption of the message would cause in $b$’s mind. This gets compared to an agent specific and learned reference scale $\kappa_b$ to form the normalized surprise data feature

$$f_b(d) := S_J := \frac{s_J}{\kappa_b} \quad (61)$$

Here, we make the ad-hoc assumption that critical agents implicitly assume the distributions of $S_J \in \mathbb{R}_0^+$ to be

$$P(S_J|h) := e^{-S_J} \quad (62)$$

$$P(S_J|\neg h) := \frac{S_J^2}{2} e^{-S_J} \quad (63)$$

such that for them

$$R_{\text{critical}}(d) = \frac{R_a(S_J) R_c(J,o) R_b(o)}{P(\neg c \lor b|J,o) \frac{1 - f_b}{P(\neg b|o)}} \quad (64)$$

$$= \frac{S_J^2}{2} P(\neg c \lor b|J,o) \frac{1 - f_b}{P(\neg b|o)}. \quad (65)$$

This means critical agents assume the surprises of honest messages to be distributed exponentially, with a clear peak at zero surprise, whereas that of lies to have a typical surprise $s_J$ of at least $\sqrt{2} \kappa_b$, the surprise level above which the lies should dominate. The assumed surprise likelihood functions are depicted in Fig. 7. They allow for a more critical discrimination of lies from honest statements than blushing, confession, and the speaker’s reputation alone. Tuning their auxiliary parameters $\kappa_i$ allows agents to adapt the absolute surprise distribution functions $P(s_J|\text{state})$ to the social situation they find themselves in. This will be detailed later in Sect. 5.4.

5.2 Lie construction

With the basic receiver strategies to detect lies in place, the question can be addressed how agents should construct their lies.

Lies towards naive and uncritical agents can be arbitrarily big, as these do not inspect the messages closely. Thus,
these agents are very vulnerable to propaganda.\footnote{In the current version of the reputation game, this would not be fully exploited by malicious agents, as agents do not infer the character of other agents except for the level of their honesty.} Lies towards critical as well as smart agents need to balance the push for the speaker’s agenda, favoring larger lies, and the risk to get caught, which increases with the size of the lie (where the size is measured by the receiver in units of bits).

As a statement towards a critical or smart agent gets judged on the basis of how much it diverges from the receiver’s own opinion, it better stays close to this opinion and deviates only so little in the desired direction that it can go unnoticed. In order not to be too predictable in this, lies are designed such that their surprises approximately match the assumed surprise distribution of honest statements, Eq. 62.

A liar $a$ proceeds in the following way when talking to an enemy or a friend. The agent takes $I_{abc}$, the assumed belief of the recipient $b$ on the topic $c$, decides on the direction $D \in \{(1,0),(0,1)\}$ of the bias to be applied according to whether $c$ is a friend to $a$ or an enemy, respectively. The lie will be constructed as

$$J_{a \rightarrow b} = I_{abc} + \alpha D,$$

with $\alpha \in \mathbb{R}_0^+$ such that the receiver is expected to experience only a certain surprise by the lie. This is achieved by drawing randomly a target normalized surprise $S_J \leftarrow \mathcal{P}(S_J|\|h\|)$, multiplying it with $\kappa_a f_{\text{caution}}$, where $\kappa_a$ is agent $a$’s substitute for $\kappa_b$ unknown by $a$ and $f_{\text{caution}} = 0.3$ is a caution factor to compensate for the mistake thereby done, and choose $\alpha$ via a numerical line search such that

$$\text{KL}_{x,c}(I_{abc} + \alpha D, I_{abc}) = \kappa_a f_{\text{caution}} S_J. \quad (66)$$

For an agent being a topic, who is neither a friend or an enemy, a white lie is used by setting $J_{a \rightarrow b} = I_{abc}$. White lies do not necessarily bias the recipient’s opinion on $c$, however they let the speaker appear honest without revealing the speaker’s true opinion.

5.3 Smart lie detection

A more efficient, smart lie detection takes into account the way lies are constructed. The agent’s lies are constructed as biased copies of what the speaker $a$ thinks the receiver $b$ believes on the topic. This opens the possibility for a smart agent $b$ to discriminate messages by matching them up against expected honest and dishonest statements of the speaker. For this $b$ needs an idea of what $a$ believes on topic $c$, denoted as $I_{bac}$ (agent $b$’s guess for $a$’s belief on $c$), as well as an idea of what $a$ wants $b$ to believe on that topic, denoted as $\tilde{I}_{bac}$ (b’s guess for what $a$ wants $b$ to think about $c$). Which of those matches better to the message $J_{a \rightarrow b}$ is then an indicator of the message’s honesty. The data features used by smart agents are the message surprises w.r.t. $I_{bac}$, and $I_{abc}$, $s_h := \text{KL}_{x,c}(J_{a \rightarrow b}, I_{bac})$ and $s_{-h} := \text{KL}_{x,c}(J_{a \rightarrow b}, \tilde{I}_{bac})$, respectively. The corresponding normalized surprises $S_{\text{state}} := s_{\text{state}}/\kappa_b$ (with state $\in \{h, -h\}$) are again assumed to be zero peaked exponential distributions,

$$\mathcal{P}(S_{\text{state}}|\text{state}) := e^{-S_{\text{state}}}, \quad (68)$$

with the lie detection scale parameter $\kappa_b$. This specifies the distribution of $S_h$ in case $h$, as well as of $S_{-h}$ in case $-h$. The distribution of $S_h$ in case $-h$ and that of $S_{-h}$ in case $h$ are not needed in detail, we only assume them to be identical,

$$\mathcal{P}(S_{h}|\text{-h}) = \mathcal{P}(S_{-h}|h). \quad (69)$$

Furthermore, we assume these two features to be independent of each other, so that their lie-to-honest likelihood ratio becomes

$$\mathcal{R}_{\text{em}}(S_h, S_{-h}) := \frac{\mathcal{P}(S_h, S_{-h}|\text{-h})}{\mathcal{P}(S_h, S_{-h}|h)} \quad (70)$$

$$= \frac{\mathcal{P}(S_h|-h)}{\mathcal{P}(S_h|\text{h})} \quad (71)$$

$$= e^{S_h-S_{-h}}. \quad (72)$$

For the smart lie detection, this likelihood ratio is just multiplied to the likelihood ratio critical agents use:

$$\mathcal{R}_{\text{smart}}(d) = \mathcal{R}_{\text{em}}(S_h, S_{-h}) \mathcal{R}_c(J_o) \mathcal{R}_b(o) \mathcal{R}_r(b) \quad (73)$$

$$= e^{S_h-S_{-h}} \frac{S^2_b}{2 \mathcal{P}(\text{-c} \vee b)|J_o} \frac{1-f_h}{\mathcal{P}(b|o)} \quad (74)$$

Special deception strategies, which circumvent or even exploit smart lie detection, can be imagined as well. These are beyond the scope of this work. The above strategies are sufficient to illustrate what kind of strategies might be used by real humans. Note that we do not claim that the ones chosen here are exhaustive.

5.4 Auxiliary parameters update

We now summarize how all the auxiliary parameters are maintained. After receiving the communication $J = J_{a \rightarrow b}$ (and eventually having responded) agent $b$ performs updates of the following parameters: Lists of friends $F_b$ and enemies $E_b$, guesses for agent $a$’s belief on and intention for $c$, $I_{abc}$ and $I_{abc}$, respectively, as well as the reference surprise scale $\kappa_b$.

**Friends and enemies**

Agent $b$ updates the list of friends $F_b$ and that of enemies $E_b$, where $F_b = \{b\}$ and $E_b = \{\}$, initially. An agent in none of these lists is regarded by $b$ as being neutral to $b$.

In case agent $a$ made a statement $J_{a \rightarrow b}$ about $b$ to $b$, agent $b$ memorizes how much respect $r_{ab} := \text{KL}_{x,c}(J_{a \rightarrow b}, I_{abc})$ agent $a$ thereby expresses for $b$, where we define respect as the by a communication stated honesty of an agent. Then the
median \( \hat{r}_b = \text{median}\{(r'_{ab})_{i \in A} | a, b\} \) of the memorized respect values of all other agents is calculated and compared to this updated one. If \( r'_{ab} > \hat{r}_b \) agent \( a \) is added to the set \( F_b \) of \( b \)'s friends and removed from \( E_b \), the list of \( b \)'s enemies (if listed there). If \( r'_{ab} < \hat{r}_a \) agent \( a \) will be added to the enemy list and removed from the friend list. In case \( r'_{ab} = \hat{r}_b \), these lists stay as they are.

In summary, an agent \( a \) is regarded as a friend by \( b \) whenever \( a \)'s last statement about \( b \) to \( b \) was more positive than the median of other agents' last statements at that point in time and \( a \) is regarded as an enemy, if this was less positive.

### Theory of mind

Agents maintain an image of the opinions of the other agents, a rudimentary theory of mind. Agent \( b \) does not have direct access to the beliefs of agent \( a \), but only to the received message \( J = J_{\text{a} \rightarrow \text{b}} \). This message has to be analyzed to determine \( a \)'s beliefs.

Agent \( b \) extracts from the message what \( a \) seems to believe on topic \( c \), whenever \( a \) seems to be honest, and stores this as \( I_{bac} \) (agent \( b \)'s best guess for \( I_{ac} \)). Similarly, agent \( b \) can also determine the intention of \( a \) when \( a \) is lying. In that case the message contains what \( a \) wants \( b \) to believe about \( c \). This intention is stored by \( b \) as \( \bar{I}_{bac} \) (agent \( b \)'s best guess for by lie distortions modified \( I_{abc} \)).

The updates for \( I_{bac} \) and \( \bar{I}_{bac} \) are done by blending the message \( (J) \) into the present value of these variables with a weight according to how much the message seems to be honest (weight \( y_J \)) or dishonest (weight \( 1 - y_J \)), respectively:

\[
\begin{align*}
I_{bac}(t) & \rightarrow I_{bac}(t+1) = y_J J + (1 - y_J)I_{bac}(t) \quad (75) \\
\bar{I}_{bac}(t) & \rightarrow \bar{I}_{bac}(t+1) = (1 - y_J)J + y_J\bar{I}_{bac}(t) \quad (76)
\end{align*}
\]

Similar update rules are used for agent based simulations on trust networks [34]. These updates should provide guesses of \( b \) for \( I_{ac} \) and \( I_{abc} \) (modified by the bias of the lie), respectively. The corresponding guesses, \( I_{bac} \) and \( \bar{I}_{bac} \), become accurate whenever the speaker \( a \) reveals to be honest \((y_J = 1 \Rightarrow I_{bac} = J = I_{ac}) \) or to be lying \((y_J = 0 \Rightarrow \bar{I}_{bac} = J = I_{abc}) \), respectively. Hopefully for \( b \), these guesses should stay reasonably accurate at other times.

### Typical surprises

Agent \( b \)'s lie detection relies on the surprise reference scale \( \kappa_b \). This determines the assumed PDFs for message surprises \( s_I = KL_{x_J}(J,F) \) from various reference points \( I = (I_{bc},I_{bac},I_{abc}) \). No static value can be assigned to \( \kappa_b \), as the surprise PDFs \( P(s_{\text{h}} | x) \) and \( P(s_{\text{h}} | x) \) differ in different social situations and usually also evolve as a function of time. A simple heuristic is used to update \( \kappa_b \).

Initially, we set \( \kappa_b = 1 \). For the update of \( \kappa_b \), it will be used that given the assumed surprise distributions for honest and dishonest statements, Eqs. (62) and (63) respectively, and given that half of the statements are a priori expected to be honest and half to be dishonest (as implied by \( P(x_{\text{h}} | I_b) = 1 \)), the median value for message surprises \( s_J \) (with respect to \( I_{bc} \)) should be located at \( \sqrt{\frac{\pi}{2}} \). This is the expected median of the assumed surprise distribution and marks the expected transition from mostly honest to mostly dishonest statements. Thus, agent \( b \) just maintains a tuple \( K_b \) with the \( N_b \) last non-zero surprises received and sets

\[
\kappa_b = \frac{\text{median}(K_b)}{\sqrt{\pi}} \quad (77)
\]

whenever a new message arrives. The size of \( N_b \) determines how quickly or slowly agent \( b \) adapts to a changing social atmosphere, and is set to \( N_b = 10 \) in our simulations. We initialize \( K_b \) with \( (\sqrt{\pi}, \ldots, \sqrt{\pi}) \) to be consistent with the initial \( \kappa_b = 1 \).

### 6 Communication Strategies

A communication strategy of an agent \( a \) is a set of rules (or frequencies) that specify how to select the conversation partner \( b \), which topic \( c \) to choose, how frequently to lie, and in which way. We will use the names of strategies also as adjectives for agents, meaning that an aggressive agent always uses an aggressive communication strategy. Many strategies can be combined, for example a clever agent is smart and deceptive.

The basic reference communication strategy is that of an ordinary agent, and all other basic strategies are described in terms of their differences to this in Sect. 6.1. For special agents\(^{\text{15}} \) agents that use a combination of various basic strategies, the reference will be the clever agent as discussed in Sect. 6.2. An overview of the different communication and receiver strategies is given by Tab. 3.

#### 6.1 Basic strategies

The ordinary agent \( a \) picks the communication partner \( b \) randomly and uniformly from all other agents, \( b \leftarrow A \setminus \{a\} \), the topic \( c \) randomly and uniformly from all agents, \( c \leftarrow A \), communicates honestly with the frequency \( x_a \), promotes friends and demotes enemies when lying, and uses a critical receiver strategy.

The strategic agent \( a \), however, picks communication partners according to their reputation, by setting

\[
P(a \leftarrow b | a \text{ strategic}) = \frac{x_{ab}}{\sum_{bc \in A \setminus \{a\}} x_{ab}} (1 - \delta_{ab}), \quad (78)
\]

By concentrating communications on presumably reputed, if not even really honest agents, the strategic agent’s opinions, if adapted by \( b \), might propagate more efficiently into

\(^{15}\text{The fictional special agent 007}, for example, exhibits strategies that resemble the here introduced manipulative and destructive strategies, which are both special in our nomenclature.}
third agents. This is because the communicated opinion benefits from the reputed agents being more influential and the higher frequency with which honest agents express their true beliefs. Strategic agents therefore target optimal multipliers for their communications. Being strategic will be part of the dominant and the destructive strategies.

We call an agent preferring low reputed agents as communication partners an anti-strategic\(^\text{\footnote{An anti-strategic agent is also “strategic” in the original sense of the word, similar to an anti-particle, which actually is a particle, or an anti-correlation, which is also a correlation.}}\) agent:

$$P(a \rightleftharpoons b|a \rightleftharpoons a) = \frac{(1 - \bar{\pi}_{ab})}{\sum_{b \in A \setminus \{a\}} (1 - \bar{\pi}_{ab})} (1 - \delta_{ab}). \quad (79)$$

Being anti-strategic may pay off for flattering agents, who always lie positively when their conversation partner is the topic, and pick the conversation partner as topic whenever they have the opportunity to initiate a conversation,

$$P(a \rightleftharpoons b|a \rightleftharpoons a, \text{a flattering}) = \delta_{bc}. \quad (80)$$

Flattering agents should be efficient in befriending others. Being an agent b’s friend pays off for a whenever b lies about a. Thus, flattering agents are best advised to be anti-strategic as well, in order to ensure the friendship of the most frequent liars they can identify. Being flattering and anti-strategic will therefore be part of the manipulative strategy.

Egocentric agents prefer to speak about themselves. In half of the cases in which they initiate a conversation, they directly pick themselves as topic, in the other half, they pick randomly and uniformly from the set of all agents $A$, as

$$P(a \rightleftharpoons b|a \rightleftharpoons a, \text{a egocentric}) = \frac{1}{2} \left( \delta_{ac} + \frac{1}{n} \right). \quad (81)$$

Thus, they present themselves in more than half of the conversations they initiate. Egocentric agents can benefit from being strategic, as this should increase their reach. For this reason, being also egocentric in addition to being strategic will be part of the dominant strategy.

Aggressive agents only speak about enemies when initiating a conversation,

$$P(a \rightleftharpoons b|a \rightleftharpoons a, \text{a aggressive}) = \frac{\delta_{ce} \in E_n}{|E_n|}, \quad (82)$$

and neither praise friends nor themselves. The aggressive agent’s destructiveness w.r.t. other agents’ reputations can unfold best if the agent is also strategic and therefore the destructive agent will be both, aggressive and strategic.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
agent $a$ & $P(a \rightleftharpoons b|a \rightleftharpoons a)$ & $P(a \rightleftharpoons b|a \rightleftharpoons a)$ & $P(h|a \rightleftharpoons b)$ & $P(b|h)$ & deception & receiver \\
& $\propto (1 - \delta_{ab}) \times$ & & \vspace{1pt} & \vspace{1pt} & strategy & strategy \\
\hline
deaf & 1 & $\frac{1}{n}$ & $x_a$ & $f_b$ & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & deaf \\
naive & 1 & $\frac{1}{n}$ & $x_a$ & $f_b$ & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & naive \\
uncritical & 1 & $\frac{1}{n}$ & $x_a$ & $f_b$ & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & critical \\
ordinary & 1 & $\frac{1}{n}$ & $x_a$ & $f_b$ & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & uncritical \\
strategy & $\pi_{ab}$ & $\frac{1}{n}$ & $x_a$ & $f_b$ & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & critical \\
anti-strategy & $\{1 - \pi_{ab}\}$ & $\frac{1}{n}$ & $x_a$ & $f_b$ & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & critical \\
f \text{flattering} & 1 & $\frac{1}{2} \left( \delta_{ac} + \frac{1}{n} \right)$ & $x_a (1 - \delta_{bc})$ & $f_b$ & a, b, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & critical \\
eglect & 1 & $\frac{1}{n}$ if $E_n = \{\}$ \vspace{1pt} & $x_a$ & $f_b$ & a, fr.$\odot$; en.$\downarrow$; n.$\odot$ & critical \\
shameless & 1 & $\frac{1}{n}$ & $x_a$ & 0 & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & critical \\
smart & 1 & $\frac{1}{n}$ & $x_a$ & $f_b$ & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & smart \\
deceptive & 1 & $\frac{1}{n}$ & $x_a$ & 0 & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & critical \\
clever & 1 & $\frac{1}{n}$ & $x_a$ & 0 & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & smart \\
manipulative & $\{1 - \pi_{ab}\}$ & $\delta_{bc}$ & 0 & $f_b$ & a, b, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & smart \\
manipulative & $\pi_{ab}$ & $\frac{1}{2} \left( \delta_{ac} + \frac{1}{n} \right)$ & 0 & $f_b$ & a, fr.$\uparrow$; en.$\downarrow$; n.$\odot$ & smart \\
manipulative & $\pi_{ab}$ & $\frac{1}{2} \left( \delta_{ac} + \frac{1}{n} \right)$ & 0 & 0 & a, fr.$\odot$; en.$\downarrow$; n.$\odot$ & smart \\
\hline
\end{tabular}
\caption{Summary of agents’ communication strategies, which determine how an agent $a$ picks (if being initiator of a conversation $a \rightleftharpoons b$) the partner $b$, topic $c$, whether (in any communication $a \rightleftharpoons b$) $a$ is honest ($h$) or lies ($\neg h$), whether $a$ bluses ($b$), how $a$ lies about $c$, about a friend ($\text{fr.} \in F_a$), about an enemy ($\text{en.} \in E_n$), and about a neutral agent ($n. \in A \setminus \{F_a \cup E_n \cup a\}$), and how $a$ receives messages. “$\uparrow$” means that the testimony in a lie is biased positively (with $(\delta_{\mu}, \delta_{\lambda}) = J_{a \rightarrow b} - I_{abc} \propto (1, 0)$) and “$\downarrow$” means negatively ($(\delta_{\mu}, \delta_{\lambda}) \propto (0, 1)$) w.r.t. to $I_{abc}$, the by-$a$-assumed opinion of $b$ about $c$. “$\odot$” means white lies, in which $a$ tries to tell $b$ exactly what $b$ believes about $c$, $J_{a \rightarrow b} = I_{abc}$. Differences in behavior w.r.t. an ordinary agent are marked in blue.}
A 

shameless agent a lies without blushing, which gives a clear advantage if lying frequently and we will assume thedestructive agent also to be shameless.

Finally, a (fully) 
decceptive agent a lies without exception, \( x_a = 0 \), and therefore does not risk to make any confession or to be caught lying due to contradictions between expressed true beliefs and lies. All special agents introduced in the following, the clever, the manipulative, the dominant, and the destructive agent, will be fully deceptive. In the typical simulation presented later on, one agent, namely agent red, will be fully deceptive as well as smart, and uses further special strategies.

The impact of most basic communication strategies on the agent’s reputation is modest if used alone. They unfold their special power if used in matching combinations.

6.2 Special strategies

A clever agent is smart and deceptive and is the reference point for the special agents, which are all smart and deceptive as well. Smartness permits the agent to understand the beliefs and intentions of other agents better, allowing for more precisely placed lies.

The manipulative agent is clever, flattering, and anti-strategic. This should enable the agent to efficiently identify and befriended other dishonest agents, who more frequently praise their friends and therefore the befriended manipulative agent than other agents. Manipulative agents should thereby become popular and influential. Their presence in a social group is expected to lift the self-esteem of the group members, owning to flattering. This lift should be stronger for less reputed agents, as those are preferentially targeted for conversations. These are often the more dishonest agents, which then, as a friend of the manipulative agent, hopefully give positively biased testimonies about this agent. As a consequence, we expect manipulative agents to frequently establish mutual friendship.

The dominant agent is clever, egocentric, and strategic. The agent’s communications are targeting the most reputed agents to praise themselves. If successful, these will most efficiently propagate a positive image of the dominant agent to others as well as mirroring this image back to the dominant agent themselves. The latter effect might efficiently boost the self-esteem of the dominant agent. Dominant agents will be best informed about their own reputation, by making themselves the conversation topic. This will, however, couple their self-esteem more to their reputation compared to other special agents. This will also provide them with a more accurate friend and enemy classification, as they see how other agents talk about them. This classification will not be accurate, however, in case they are interacting with manipulative agents, as the latter speak differently about a topic depending on whether the topic is also their conversation partner or not. Nevertheless, dominant agents are expected to be drawn towards manipulative agents w.r.t. to their communication partner choice and friendship, whenever the manipulative agent manages to become reputed.

Finally, the destructive agent is clever, deceptive, aggressive, and shameless. By also targeting reputed agents for communications, the agent’s disrespectful propaganda about the agent’s enemies can unfold best. Since destructive agents are shameless, they are not risking to blush while lying. This might compensate for the lack of direct self-promotion of the destructive agent. Their lack of self- and friend-promotion lowers the surprise variance the receivers experience compared to just deceptive or clever agents, which helps the destructive agent to appear more honest. The presence of a destructive agent in a social group is expected to lower the reputation and self-esteem values of the other agents significantly due to that agent’s tendency to concentrate conversation topics on enemies, about which the agent talks disrespectfully.

To summarize, we have defined a number of communication and receiver strategies and can now see how agents equipped with different sets of such strategies interact.

7 Simulations

We can now discuss our reputation game simulations. All agents’ initial beliefs and assumptions on honesty and reputations of other agents are set to be non-informative, \( I_{ab}(0) = I_{abe}(0) = I_{abe}(0) = I_0 = (0,0) \), if not specified differently. In the displayed simulation runs, individual random sequences for the different processes like choosing conversation partners, topics, whether to lie, how strong to lie and the like are kept identical between the simulations. The intrinsic honesty of agents is also kept identical, with \( \mathbf{x} = (x_{\text{red}}, x_{\text{cyan}}, x_{\text{black}}) = (0.27, 0.80, 0.97) \), if not specified differently. Differences in dynamics therefore only arise here because of the different strategies used by agents in the different simulation runs, as these specify how the random number sequences are used in detail. This should help to highlight the effects of the strategies, and to facilitate their comparison. However, differences in the performance of individual strategies observed this way are only indicative. The dynamics is chaotic and thus firm conclusions about the efficiency of strategies can only be drawn from a sufficiently large statistical ensemble of simulation runs with varying random number sequences, which we discuss in what follows.

First, we present a few reputation game simulations to motivate the complexity of the agents’ receiver strategies (Sect. 7.1). Sect. 7.2 presents simulations of propaganda situations without random elements. These give insights into the cognitive models assumed in this work and explain the need for their complexity. Sect. 7.3 illustrates the effects of basic and special communication strategies with individual simulations. Sect. 7.4 presents statistical results based on one hundred simulation runs per setup. These allow to measure the effects of the different deceptive strategies quantitatively, and provide statistics of the resulting repu-
7.1 Receiver strategies

First, we investigate different receiver strategies of agents in order to investigate their performance. The sender strategy is that of ordinary agents in what follows. An overview on the different receiver strategies is given in Tab. 2.

Deaf agents

We want to demonstrate the agent’s ability to learn from unbiased signals, like the agents’ self-observations and the blushing signals which are the only information sources available to deaf agents, as these do not hear the other’s messages. To this end, the top left panel in Fig. 8 shows the reputation dynamics of three deaf agents performing 300 conversation rounds. Agents learn relatively quickly and accurately their true honesty from their self-observations. Whenever they are honest, their self-esteem increases; when they lie, it decreases. The learning of the honesty of others is much more difficult, as it relies on the occasional blushing signals, which are visible as the sudden drops of the otherwise monotonically increasing reputation lines. Despite this difficulty, agents manage nevertheless to get the tendencies right. The discrepancies between correct honesty and their reputation seems to be consistent with the associated uncertainty estimates.

Despite the deaf agents not hearing each other, the patterns of their statements are instructive. These are shown in the top left panel of Fig. 9. This is a busy figure that we discuss briefly.

All honest statements from agent a to b about c are displayed as circles with the outer, middle, and inner color indicating the agent a, b, and c, respectively. These honest statements reflect the beliefs of the speaker a on topic c ($J_{a \rightarrow b} = I_{ac}$). For this reason, they are on top of the agents’ belief curves $x_{ac}(t)$, which are displayed as well. The main color of any of these lines is that of c and the dots on top are in the color of a. The circles on the unicolor lines are self-statements. Their densities reflect the intrinsic honesties of the speaking agent, with agent black making most frequently honest statements and agent red least frequently.

All lies ($J_{a \rightarrow b} = I_{abc} + \alpha D$) are displayed as triangles with the same color coding (speaker a specifying the outer, receiver b the intermediate, and topic c the inner color). The lies are mostly from agent red and fall into two categories: First, all lies about other agents (a ≠ c) are located at the horizontal line $x_f = 1/2$. The reason for this is that deaf agents do neither get friends nor enemies (as they do not listen to each other), and therefore make only white lies ($\alpha = 0 \Rightarrow J_{a \rightarrow b} = I_{abc}$). Since deaf agents can not update $I_{abc}$ (they don’t hear others’ opinions), their white lies are the initial value of this quantity, $J_{a \rightarrow b} = I_{abc} = I_0$, and therefore displayed at $x_f = 1/2$. The lies agents make about themselves, $J_{a \rightarrow a}$, are biased positively ($\alpha > 0$, $D = (1, 0) \Rightarrow x_{a \rightarrow a} > x_{I_{abc}} = x_{I_0} = 1/2$), and thus are found in the upper half of the diagram.

Uncritical agents

Agents should get much better estimates of each other’s honesty compared to the deaf agent scenario, if they exchange the information they collect. This is shown in the top right panel of Fig. 8, where uncritical agents, who listen to each other, perform the same set of conversations (as specified by $(a \rightleftharpoons b)(t)$) as the deaf agents did (top left panel), with also being honest or lying at exactly the same instances. What they say, however, differs from the deaf agents simulation, as agents now listen to each other and therefore their opinions and assumptions evolve differently to those of the previous run.

It is apparent that the agent’s guesses on each other’s honesty becomes much more accurate and definite. Actually, some overconfidence can be observed for the opinions on agent cyan, which have converged to a value significantly below the agent’s true honesty with a confidence that excludes the correct value. The self-esteem of cyan even follows this slightly incorrect value, despite cyan’s self-observation should inform cyan better. However, the opinions expressed by the others on cyan, in particular the ones of the most reputed agent black, seem to have a stronger pull. The collective development of the overconfident, but incorrect opinions on cyan is the result of an echo chamber: The initially more dispersed opinions of the different agents converge to a value that is partly decoupled from reality (cyan’s true honesty), and this value is largely determined by the group dynamics.

Inspecting the corresponding communication patterns in the top right panel of Fig. 9 shows for example the concentration of statements about agent cyan around cyan’s self-esteem. It is apparent that agent red regards cyan as a friend for most of the time, as agent red’s lies for cyan are typically above cyan’s self-esteem. Consequently, red regards black mostly as an enemy, as the lies about black are aiming for lowering black’s reputation and self-esteem. These lies by red, however, have little influence compared to the opinions expressed by cyan and in particular by black, due to the much higher reputations of black and cyan compared to red.

Investigating red’s reputation is also instructive. Initially it is high, as red’s early self-promoting lies fly. However, two confessions of red to cyan (at $t = 52$ and 100), who thereafter regards red as unreliable, and cyan’s repeated spreading of these news to black (cyan $\rightarrow$ black at $t = 57, 84, 93, \text{and } 150$), destroy red’s initially high reputation in an irreparable way.

Red would probably have overcome this resistance if red’s lies would simply have been much stronger. This is because the weight of a message, which an uncritical agent assigns, does not depend on how extreme the position of a message...
Figure 8: Reputation game simulations for three agents with the panels showing different receiver strategies: deaf agents (top left), uncritical agents (top right), ordinary agents, which have a critical receiver strategy (bottom left), and agent red being a smart agent (bottom right). All simulations are run with the same random number sequences, implying that the communication configurations (like $a \rightarrow b$) and message honesty states (honest or lie) exhibit exactly the same sequences. Differences are solely caused by differences in receiver strategies. This and other figures showing communication patterns intend to give an overview. To inspect details, we recommend to magnify their electronic, vector graphics versions. Text statements about certain precise communication events were not taken from these figures, but from the simulation log files. The self-esteem $x_{aa}$ of agent $a$ is shown as a thick solid line in the color of $a$, agent $b$’s reputation $x_{ab}$ in the eyes of agent $a$ is shown as a thin line, which carries the color of agent $b$ and has dots in the color of agent $a$ on it. One sigma uncertainties of self-esteem and reputations are displayed as transparently shaded areas in the color of agent $a$. The dashed lines show the actual honesty of the individual agents, the fraction of honest statements made, which is close to their intrinsic honesty of $\mathbf{x} = (x_{\text{red}}, x_{\text{cyan}}, x_{\text{black}}) = (0.27, 0.80, 0.97)$. The data points with bars at the right side display summary statistics of the full dynamics. The squares and their bars indicate the mean and variance of the reputation of the agent in the corresponding color. Similarly, the circles and bar indicate mean and variance of self-estems of agents.

is, but the shift of the receiver’s opinion does depend on this. Uncritical agents are therefore very prone to propaganda in form of exaggerated lies, as we show in Sect. 7.2

Critical agents

Ordinary agents use a critical receiver strategy, which is able to recognize exaggerated statements. A simulation run with such agents is shown in the bottom left panel of Fig. 8 again for the same sequence of communication decisions. Overall, the outcome of the simulation is similar to that of uncritical agents, in the sense that the final reputations and self-estems converge to values not too far from the correct honesty of the agents. However, at least two interesting differences to the uncritical agent simulation can be spotted here and should be discussed: The more volatile evolution of beliefs about cyan, with a significant gap between cyan’s self-esteem and cyan’s reputation (in the period 100 to 800) and the much later time in the critical simulation compared to the ordinary one ($t = 291$ instead of $t = 150$) at which agent black’s opinion about red joins that of cyan. Both are a consequence of critical agents being more reluctant to accept diverging opinions. This allows the self-esteem of cyan to evolve more decoupled from the lower opinions expressed by red and black, and makes black more skeptical about cyan’s reports on red’s dishonesty.
Smart agents

Smart agents have an even more sophisticated receiver strategy compared to critical agents. This should allow them to maintain a more accurate picture of the other agents’ beliefs, which improves their lie detection and lie construction. To illustrate this, the bottom right panels of Figs. 8 and 9 show a simulation run in which agents black and cyan still use critical receiver strategies (as in the bottom left panel), but red uses a smart strategy.

In this smart scenario (agent red being smart), the self-esteem and reputation of cyan do not show the strong growth that is visible in the critical scenario (agent red being only critical). The reason for this are the better targeted lies of red in the smart run, which undermine cyan’s and black’s lie detection more efficiently than in the critical run. This makes red’s lies more effective. As these lies mirror the other agents’ previously communicated beliefs, just in a slightly distorted manner, they counteract rapid evolution of these beliefs by pulling them back towards those previous values. Additionally, the echo chamber effect of group opinions converging to overconfident, but incorrect positions is strong, also due to the better targeted lies of red. Both, the retarding back-reaction and the opinion focusing effect of red’s more effective lies, effectively add inertia to cyan’s self-esteem and reputation, which keeps those from reaching the correct honesty value of cyan.  

7.2 Propaganda and resilience
Simulation setups

We claimed that agents with naive or uncritical strategies are very susceptible to exaggerated lies and that critical and smart receiver strategies provide some resilience against such lies. To demonstrate this, but also to illustrate the inner working of the cognitive model adapted, a number of propaganda situations are simulated. In those, all agents,
except agent red, who will be the propagandist, will be absolutely honest.

The basic propaganda situation is depicted on the left of Fig. 10. There, only agent red communicates to black, cyan, yellow, by repeatedly sending strong self-appraisal ($J_{\text{red} \rightarrow \cdot} = (10^3, 0)$) to them without blushing. Red’s initial reputation with them differs, being initially low with black ($x_{\text{black red}}(0) = 0$), medium with cyan ($x_{\text{cyan red}}(0) = 0.5$), and high with yellow ($x_{\text{yellow red}}(0) = 0.8$). These agents are isolated in this setup, as they only receive the propaganda, but can not exchange their positions among themselves.

This is changed in the setup with cross-communication among the propaganda receiving agents shown on the right of Fig. 10. There, every receiver communicates honestly the updated opinion on red to all other receivers after every exposure to the propaganda.

Simulations with the basic setup are shown in Fig. 11 for three uncritical agents (top left panel) and for three critical agents (top right) as receivers. Simulations with the cross-communication setup are run for three critical receivers (bottom left) and for one smart among two critical ones (bottom right). In all simulations, 75 propaganda rounds are performed.

**Isolated uncritical agents**

All uncritical, isolated agents rapidly adapt a high reputation for red under red’s propaganda (top left panel of Fig. 11), as the strong messages received are only slightly moderated by their initial limited respect for red. Although red’s reputation with them is steadily growing, it does not reach the position announced in the propaganda message of $\xi_{\text{red} \rightarrow \cdot} = 0.999$. This is caused by the receiver mechanism that tries to identify the novel part of a message, and disregards the part that already seems to be accounted for.

Naive agents would have fully adopted that latter position on the first exposure to the propaganda (not shown).

**Isolated critical agents**

Ordinary agents, which have critical minds, have much more resilience against propaganda, as can be seen in the top right panel of Fig. 11. Agents black and cyan, who are initially skeptical about red’s honesty, become immediately more skeptical under the exposure of the propaganda, as they perceive this as lies. This changes at $t = 15$, after 5 propaganda rounds, when red’s reputation with them starts to grow. What causes this change is that at this point in time, the large divergence of the propaganda messages from their own beliefs start to affect the scale $\kappa$, agents use to discriminate lies from honest statements, as this is based on the median of the last ten message surprises. Consequently, the mechanism to separate lies from honest statements starts to fail, which lets the propaganda appear slightly more trustworthy. Since the propaganda makes strong claims, it shifts – despite being still more dis-

\[18\text{This saturation effect can be read off from Eqs. 19 and 75 for a repeated message } J \text{ with } y_J \approx 1.\]
trusted than believed – black’s and cyan’s opinions on red upwards.

Agent black, who is initially most skeptical, is hit the strongest by this effect. Being initially most skeptical about red, black experiences the largest opinion divergence by the propaganda, and therefore the largest shift in $\kappa$black. This then makes black most vulnerable to propaganda.

We see that even critical agents, who are more wary, can be prone to propaganda. All their beliefs in red’s reliability increase and do this the more, the lower the initial trust was. At some moment, the novelty of the propaganda message wears off and red’s reputation stops to increase further. The propaganda statements are still received as mostly being lies, which thus makes the prestige of red finally disappear for each of the recipients again.

To summarize, the strategy to classify lies only according to the surprise they create works as long as the reference surprise value is not inflated. This quantity is determined empirically and increases to a too high value if there are many more lies than expected.

This enables the effect of propaganda. When an audience is exposed to a large amount of strong lies, it loses the confidence to recognize them as lies with certainty, which then - if these lies are sufficiently large - will pull the audience’s opinion in the desired direction. The statements that undermine the lie detection do not even need to be lies, they just need to strongly contradict the receiver’s belief system in order to have a general distorting effect on mental resistance.

**Cross-communicating critical agents**

A counter measure against attacks on the lie detection system can be the exposure to honest messages, or just messages with low surprise values. This can be achieved by honest and frequent exchanges with other honest agents. Such exchanges should help to a healthy lie detection sys-
A propaganda simulation with such honest cross-communication is shown in the bottom left panel of Fig. 11. As soon as the receiving agents cross-communicate their beliefs about red, the dynamics gets even more complicated. Although the receiving agents communicate honestly, they first have to build trust. This process exhibits a complex dynamic, which lets only agent cyan and yellow trusting each other in the end and distrusting red. Agent black, despite being initially very reputed, loses the trust of the other agents as well as black loses the trust in them.

This isolates black from the protecting effect of their communications, and lets black accept the propaganda even more than in the scenario without cross-communication. The reason for this is that diverging opinions of cyan and yellow about red harm black’s lie detection in addition to what the propaganda does to it. This was the basis of our comment above: Resistance to lies can also be destroyed by honest communications, if these contradict an agent’s belief system significantly.

Nevertheless, this simulation shows that honest cross-communication among recipients of propaganda can mitigate the propaganda’s impact to some degree.

Impact and resilience of a smart agent

The bottom right panel of Fig. 11 shows a simulation with a similar setup as before, but this time we assume agent black uses a smart receiver strategy. This means that black maintains and uses a set of guesses about the other agents’ beliefs (as stored in \( \tilde{I}_{black\ red\ red} \) for red) and intentions (as \( \tilde{I}_{black\ red\ red} \)) to detect lies. The smart receiver strategy allows black to identify red’s communications as propaganda, after a period of varying opinions about red, and to convince cyan and yellow also to distrust red in the end. Thus, the smart receiver strategy offers more resilience against exaggerated lies than the critical one.

We note that at the peak of red’s reputation with black, black has a bimodal belief state about red with \( \mu_{black\ red\ red} = (-0.39, -0.92) \). This is expressing that at that moment black is aware that either red is very honest or very dishonest, but certainly not anything in the middle between these extremes.

### 7.3 Communication strategies

We now discuss the impact of the basic and special communication strategies. The setup will be as in Sect. 7.1, but now agent red uses basic or special communication strategies. The intrinsic honesty of the three agents and the random sequences determining the course of simulation events will again be identical to what they were in the simulations shown in Figs. 8 and 9 for some of the runs. These are the runs with random number sequence No. 1 from our statistical set of one hundred simulations to be discussed later. For the special agents, we will also show runs with random sequence No. 2 to illustrate the variance in the dynamics with otherwise identical setup.

#### Basic communication strategies

Fig. 12 shows simulation runs, with the setup of Fig. 8 but here agent red is either strategic, egocentric, flattering, shameless, aggressive, or deceptive. The corresponding communication patterns can be found in Appendix 3. None of the basic strategies adapted by agent red appears to be efficient in boosting red’s reputation, except for the flattering and the deceptive strategies, which both let red lie more often.

The **strategic agent** red concentrates opinion exchanges on the most reputed agent black, and thereby manages to convince black that cyan is untrustworthy. This lets cyan, who actually is trustworthy, doubt black’s honesty as a reaction to black’s opinion on them. However, black’s and cyan’s reputations still stay well above that of red, as red’s reputation suffers from red’s occasional confessions.

The **egocentric agent** red speaks mostly about themselves. This has two visible consequences: Firstly, the others’ opinions on red converge faster, due to the larger number of confessions made by red. Secondly, the lies of red are not able to follow the development of the other agent’s opinions on other agents that well (see the horizontally aligned lies of red on cyan in the period \( t = 200 \) to 800 in the communication record displayed in Fig. 12).

The **flattering agent** red is somehow successful in obtaining an enhanced reputation. The key factor is that red is preferentially talking about others, thereby avoiding giving information about themselves away via confessions. This helps red to establish a slightly higher reputation than in the other scenarios discussed so far. The feedback to red by the other agents lets red’s self-esteem grow to this enlarged value until \( t = 1000 \). Thereafter, confessions by red are based on this enhanced value and do not let the other agents’ opinions fall below it. We witness here a successful and advantageous self-deception of an agent.

The **shameless agent** red lies without blushing, and therefore is more convincing. As a result red’s reputation grows slightly higher than in the ordinary agent scenario. Red’s reputation is held back by red’s confessions and the inertia the converging group opinion generates against the pull of red’s self-appraisal. We note, however, the significantly reduced reputations of cyan and black owing to the more convincing lies of a shameless agent red.

The **aggressive agent** red attacks preferentially cyan, who’s reputation and self-esteem suffer significantly from red’s vilification.

Finally, the **deceptive agent** red manages to get the highest reputation and self-esteem of red in all the scenarios discussed so far, since red does not make a single confession, and self-promotes with a high frequency.

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20 A negative \( \mu \) or \( \lambda \) creates an integrable singularity (for \( \mu, \lambda > -1 \)) in the belief distribution at \( x = 0 \) (complete dishonest) or \( x = 1 \) (complete honest), respectively.
Special communication strategies

Figs. 13 shows runs for agent red being clever (smart and deceptive), manipulative (clever, anti-strategic, and flattering), dominant (clever, strategic, and egocentric), and destructive (clever, strategic, aggressive, and shameless). On the left panels of Fig. 13 the random sequences are chosen as before (and like runs No. 1 of the statistics ensemble), whereas on the right panels different sequences (runs No. 2) were chosen. The latter was chosen to highlight that different dynamical regimes can appear in otherwise identical setups. The corresponding communication patterns can be found in Appendix B. For the dominant agent we display them also in Fig. 14 for a more detailed discussion.

Furthermore, Fig. 15 shows the evolution of the lie detection scale $\kappa_a$ for an instructive selection of simulation runs. A larger $\kappa_a$ of agent $a$ implies that this agent is used to receive messages that diverge more from the own opinions. This can make the agent blind for smaller lies.

The runs shown there with red being an ordinary agent shows that usually $\kappa_a$ varies on a logarithmic scale around unity, with a typical variance of one order of magnitude up or down.

The clever agent red performs slightly worse in terms of reputation than in the run where red is deceptive (see
Figure 13: As Fig. 12 but for agent red being deceptive (first row), manipulative (second row), dominant (third row), and destructive (fourth row). The left column shows simulations using random sequences No. 1 and the right using No. 2.
discussion before). The lie detection scale $\kappa_{\text{red}}$ of the clever agent red is significantly larger than that of the other two agents in the same run. However, thanks to being clever, red’s lies match the beliefs of the other agents better and these experience therefore reduced surprises compared to what they experience in the deceptive scenario. This will also be the case for many of the runs with the other special agents. The clever agent should serve here as a reference case, to compare the performance of other special agents.

Compared to the case when agent red is clever, the manipulative agent red is much more successful. As red is mostly flattering cyan, the latter gets a significant self-esteem boost in the simulation No. 1 (second row, left panel of Fig. 13), and partly also in No. 2 (second row, right panel).

By focusing on their reputation, the dominant agent red freezes the group opinion on red (third row, left panel of Fig. 13), preventing red to obtain a high reputation in about half of the simulations with red being dominant. This is accompanied by strongly reduced variances in opinions and therefore in $\kappa_i$ for every agent $i$ in such runs (see central panel of Fig. 15). The other half of the runs show much more volatility in red’s reputation with about one fifth of these runs leading to a top reputation and self-esteem for red. In the third row, right panel of Fig. 13 shows run No. 2 for the dominant agent red, which illustrates this latter case, and seems to be typical for this outcome. Before red’s dominance is established, a period of high opinion volatility and large uncertainty seems to be necessary. Red’s lies in this scenario are often on the extremes (see right panel of Fig. 14 for the period 600 to 1200), creating a social atmosphere that might be characterized as toxic, as any enemy of red is often blamed to be a complete liar. The reason for this is that red’s self-esteem does not manage to catch up with red’s inflated reputation due to red knowing their lies. Therefore, the many conversations of red about red lead to a high level of cognitive dissonance, which inflates $\kappa_{\text{red}}$ by two orders of magnitude above the usual $\kappa$ values (see bottom middle panel of Fig. 15). As $\kappa_{\text{red}}$ is also used by red in lie construction, red’s expressed opinions tend to be largely on the extreme, either very positive (about red and friends) or very negative (about enemies). Only after red’s self-esteem manages to become as high as red’s reputation, does $\kappa_{\text{red}}$ fall to a more normal level.

The destructive agent red manages to establish a high reputation in run No. 1, but not in run No. 2. In the latter red largely destroys cyan’s reputation during the initial period with a concentrated attack, though. Red’s surprise scale $\kappa_{\text{red}}$ in this run takes very extreme values, mostly due to the large difference between red’s and the others’ opinions about them. As a consequence, red speaks extremely negative about them, however, without being believed.

7.4 Statistics

Reputation statistics

In order to see how robust the observed impact of special agents on the individual runs are, an ensemble of one hundred simulations with differing random sequences was performed for each of the setups in which red is ordinary, deceptive, clever, manipulative, dominant, and destructive. All other configuration parameters are kept identical. Fig. 16 shows the time evolution of the ensemble mean and dispersion of the agent’s reputations and self-esteems for the different scenarios. Fig. 17 displays the correlations between the reputation of agent red (the least honest one) and that of agents cyan and black (the two more honest ones) for the different scenarios. Fig. 18 shows histograms of the agents’ reputations and their self-esteems occurring during the simulations for the different scenarios. We name these scenarios after the strategy agent red uses in them.

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21Fig. 17 shows a reconstructed density based on the simulation data points as given by the various time snapshots in the hundred simulations. The density is w.r.t. the plane spanned horizontally by black’s (or cyan’s) average reputation (averaged over the other two agents) and vertically by that of red. The observed data points were assumed to be drawn by a log-normal-Poisson process with log-density that exhibits a Matérn kernel correlation structure in each dimension separately. The log-density as well as the parameters of its Matérn kernel correlations were inferred from the data. Details of this information field theoretical reconstruction can be found in [63].
In the ordinary scenario, with ordinary agent red, reputation and self-esteem values of agents roughly reflect their honesty. Red is not able to significantly increase their reputation or self-esteem beyond red’s honesty ($x_{\text{red}} = 0.14$) in most of the runs, only in a few cases high values are reached (Fig. 18 shows a peak in the histogram of red’s reputation at 0.2, with a fat tail towards larger reputations up to 0.9). Agent cyan’s reputation ($\approx 0.7$) and self-esteem ($\approx 0.75$) are only slightly lower than they should be ($x_{\text{cyan}} = 0.80$). The largest disparity between reputation and honesty happens typically for agent black, who is too honest to defend their reputation (black’s reputation is on average at $\approx 0.65$, but shows large variance, whereas $x_{\text{black}} = 0.97$). Black’s reputation shows even a bimodal distribution, with a high reputation peak (at 0.925) shortly below black’s honesty ($x_{\text{black}} = 0.97$) and a low reputation peak at a much lower value (0.325). The reason for this low reputation peak is again black’s high honesty, which lets black more often express positions that are in contradiction to those of the other agents, letting black appear as untrustworthy. This can be regarded as the reputation game manifestation of the Cassandra syndrome: the most honest agent may appear less trustworthy than more dishonest agents. We note that agent cyan’s reputation, who is also mostly honest, is slightly bimodal as well.

In the scenario with the deceptive agent red, red reaches on average a significantly higher reputation (0.4) and self-esteem (0.26) than in the ordinary scenario (0.2 and 0.16, respectively). Red’s reputation distribution histogram shows now a broad plateau (from 0.05 to 0.3), a fat tail towards higher reputations (up to 0.95), and a distinct peak at highest reputations ($> 0.95$). This peak indicates that once accepted as being very honest, red can defend this position thanks to the higher influence a reputed agent has.

We note that cyan’s and black’s self-estees are higher than in the ordinary scenario and more focused on their intrinsic honesty values. Red’s more frequent lies in this scenario exhibit a stabilizing force to the other’s self-esteem. As lies mostly mirror the other’s beliefs, they can strengthen those beliefs if they are not too biased.

The scenario with the clever agent red looks nearly indistinguishable to the previous one, except for the self-esteem of agent red, which is now slightly higher (0.3) near the end of the simulation time. Being smart, red realizes that black and cyan are mostly honest when they speak about red’s honesty (which appears to them to be 0.4). Therefore, red’s self-opinion is more strongly drawn towards
Figure 16: Statistical summaries of 100 simulation runs with differing random sequences. Shown are the mean (lines) and dispersion (shaded areas) of reputations (thin lines) and self-esteems (thick lines) averaged over the 100 runs for the same moments. The colors code agents in the same way as in the other figures. The points and bars on the right indicate the mean and dispersion of the displayed temporal mean curves of reputations (via squares) and self-esteems (via circles). The bars do not take into account the dispersion of the individual runs (which is indicated by the shaded areas).

this value than in the previous scenario. The clever scenario will be the reference scenario to compare other special agent scenarios with, as agents are also clever in those.

The scenario with the manipulative agent red shows that the manipulative strategy is the most successful in allowing red to reach on average the highest reputation and self-esteem among all scenarios investigated. Both quantities show also the strongest rising trends at the end of the simulated period. Red’s chance of being regarded as very reputed (> 0.95) is nearly five times higher in the manipulative scenario compared to the clever one. Compared to the clever scenario, cyan and black’s reputations are lower and show more variance. Cyan’s reputation is now reaching lowest values nearly as frequently as black’s, thanks to cyan’s higher exposure to red’s confusing lies (red is anti-strategical here, thus mostly talking to cyan). It is noteworthy that the self-esteems of black and cyan are enhanced not only w.r.t. the clever scenario, but also w.r.t. black and
cyan’s intrinsic honesty. This is due to the flattering they get from red, which boosts their self-esteem. Although all agent’s reputations are generally higher here compared to the clever scenario, the number of cases in which red’s reputation surpasses the ones of the others is strongly increased (see Fig. 17).

The dominant agent red does not reach a higher average reputation than the clever agent red, but red’s reputation displays a larger dispersion in the dominant scenario than in the clever one or any of the others. Red’s chances to be regarded as very reputed (> 0.95) is the largest in the dominant scenario, being ten and two times higher than in the clever and manipulative scenarios, respectively. This top reputation chance of the dominant red is also slightly exceeding the value given by a uniform reputation distribution. Red’s self-esteem is higher on average by being dominant than being only clever, despite the lower average reputation of red in the dominant scenario. The higher frequency of conversations about red in the dominant scenario couple red’s self-esteem more strongly to red’s reputation. This effect outweighs the lower average reputation of red in this scenario. Being strategic, red targets predominantly black with self-promotion lies and thereby drives black’s opinion away from the other’s. As a consequence, black gets often confused and this lets black’s reputation reach lowest values (< 0.05) so frequently that black’s reputation distribution histogram exhibits a distinct peak there. Fig. 17 confirms this interpretation, with exhibiting the lowest
reputations for black for moments when red reaches highest reputation values.

In the scenario with the destructive agent red, red reaches on average a reputation significantly higher (0.45) than in the clever and dominant scenarios (0.4). However, the destructive red’s reputation exhibits a slowly declining temporal trend, whereas the ones of them being manipulative or dominant are increasing or constant, respectively. Destructive red’s reputation is uni-modal (with a broad peak centered on ≈ 0.5) and reaches neither the highest nor the lowest reputation values. Red’s self-esteem evolution is initially low but constantly raising during the further simulated period. Their self-esteem distribution function, however, peaks strongly at lowest values (< 0.05). This stronger detaching of red’s self-esteem from their reputation in the destructive scenario is caused by them avoiding themself as a topic; red mostly talks about enemies, not about red. The impact of red’s destructive strategy on red’s enemies is also clearly visible: Both other agents, black and cyan, experience now a high chance to be without any rep-

Figure 18: Frequency densities of agents (as indicated by color) to have a certain reputation (thin lines with shading below) or self-esteem (thick lines) based on the runs underlying also Fig. 16. An uniform distribution would appear as marked by the thin horizontal gray lines. The true honesty of an agent is marked as a vertical dashed line in the agent’s color.
The goal is to be highly deceptive, but still more reputed than other agents, the destructive strategy seems to be a choice as good as the manipulative one.

A comparison of red’s reputation histogram for the different strategies used is given by Fig. 19. This shows that among the strategies investigated here of deceptive agents, on average, the manipulative one seems to be the most successful, followed by the destructive one. If, however, success is defined as reaching the highest reputation values, the dominant strategy seems to be most favorable.

Fig. 19 also shows the reputation histogram results for runs with four or five agents. The three agents of the previous simulations were kept, just one or two additional agents are introduced there, who have a low honesty of $x_{\text{yellow}} = 0.31$ and $x_{\text{blue}} = 0.35$. These simulations can be regarded to be statistically independent of the simulation with three agents and w.r.t. each other for most practical purposes.

The corresponding reputation density plots for the four and five agent simulations are shown in Figs. 20 and 21. One sees that the reputations of these additional, mostly dishonest agents are correlated with that of red, and the correlation gets stronger the more dishonest agents are present. This indicates that there is some synergy between these least honest agents. There are two effects that can cause this. First, less honest agents are better in befriending each other. Second, there is a generic benefit for liars to draw from an atmosphere of general confusion that a larger number of dishonest agents creates. Their lies fly easier there.

These plots show further that the special strategies still pay off within larger groups, but with a reduced reputation gain compared to the three agent scenario. Now, the destructive agent red manages to reach higher average reputation values than by being manipulative or dominant. The latter are still more efficient in reaching the highest reputation values.

It seems safe to claim that these simulations show that the introduced special deceptive strategies are more successful than just being deceptive or clever. The details of which strategy is best with respect to the different success metrics might also depend on the precise composition of the social group. This was not varied much here, as we kept agent black very honest and agent cyan mostly honest in all runs. We leave the investigation of such effects to future research.

**Friendship statistics**

In the following, we want to investigate the friendship relational network of agents in the different setups. For the simulations with three agents, these are displayed in Fig. 22 and show that the most dishonest agent (red) manages to befriend best the others, in particular when being manipulative (bottom left). Red’s own friendship budget is nearly equally distributed among the other two agents, with a slight preference for cyan, who, also being a bit dishonest,
is slightly better in maintaining friendships than black.

The correlation of friendship and reputation relations can be studied in Fig. 23. For each of the hundred runs time-average $a \rightarrow b$ reputation relation values (with $a \rightarrow b$ meaning agent $b$’s reputation with $a$) and the time-fraction of $a \rightarrow b$ friendships (meaning agent $a$ regards $b$ as friend) were calculated and displayed. For visual clarity of the plot, the hundred points in the friendship-reputation plane of each $a \rightarrow b$ relation were converted into a density. Fig. 23 confirms the observation made with Fig. 22 that the most dishonest agents are preferentially regarded as friends. No distinct correlation between the friendship strengths and reputation values within the same $a \rightarrow b$ relation is seen, with two exceptions, the dominant and the destructive agents. The density distributions show different levels of dispersion in the friendship and reputation dimensions, but not much (linear) correlation between these variables.

The different strategies of agent red manifest themselves by clearly distinct friendship-reputation relation patterns. The ordinary agent red (top left panel of Fig. 23) manages to become both other agents’ preferred friend, at a moderate time averaged reputation of about 0.2. Becoming deceptive (top middle panel) increases red’s reputation to typically 0.4 without changing the friendship network much. The other agents’ reputations increase thereby also by a comparable margin. Red becoming clever (deceptive and smart, top right panel) lets the other agents’ reputations increase further on average, as red’s higher smartness now less often classifies them incorrectly as dishonest. The manipulative agent red (bottom left panel) manages to nearly monopolize black and cyan’s friendship, which turns them thereby into permanent mutual enemies. As the manipulative agent red interviews the others frequently about their self-images, red is well informed about their honesty. In contrast to this, the dominant agent red, who mostly speaks about red and less about others, therefore often incorrectly classifies black as less reliable (see distinct lower red contour in bottom middle panel). The dominant red’s own reputation can occasionally become very large, but usually stays below of that of the other two agents and that of the manipulative red agent. Finally, the destructive agent red (bottom right panel) creates the largest dispersion in other agents’ reputation and friendship values.

For the destructive agent red a clear correlation exists between the reputation and friendship red has for others,
7.5 Social atmospheres

The visual inspection of the belief state and communication dynamics in Figs. 8-9, Figs. 11-14, and App. B shows a variety of social atmospheres, ranging from frozen situations, in which opinions quickly converge to static values (e.g. dominant agent run shown on the left of Figs. 13-14), over adaptive regimes, in which individual and collective learning curves can be observed (e.g. ordinary agent run in Figs. 8-9), to very chaotic situations, in which the beliefs and expressed opinions change rapidly (e.g. dominant agent run shown on the right of Figs. 13-14). In order to classify these different regimes and to see how different strategies are related to those we introduce the a measure of social chaos in a run as

$$\text{chaos} := \langle (x_{ij}(t) - \overline{x}_{ij})^2 \rangle_{i,j \in A; t \in [0,T]}^{1/2}$$  \hspace{1cm} \text{(83)}$$

$$\overline{x}_{ij} := \langle x_{ij}(t) \rangle_{t \in [0,T]}.  \hspace{1cm} \text{(84)}$$

This characterizes the average volatility of all beliefs of a run.

Fig. 24 displays the relation of run averaged reputations of agents and this measure of social chaos in different scenarios (ordinary, deceptive, clever, manipulative, dominant, and destructive). All fully deceptive agents (all agents red except the ordinary agent red) seem to create and benefit from social chaos, as higher chaos values are reached and the average reputation of agent red correlates with this. The manipulative and dominant agents seem to benefit most strongly from chaos, whereas the destructive agent red shows the lowest level of a correlation between red’s reputation and the level of social chaos.

The reputation of the more honest agents black and cyan...
8 Discussion

8.1 The game and its players

The reputation game was introduced to simulate a number of communicating agents to study emerging social and psychological phenomena. It permits to investigate the vulnerability of individuals or groups to malicious communications. The rules of the game were designed to study a number of effects witnessed in group dynamics and can be summarized (and generalized) as follows:

A number of players exchange opinions on the other’s reputations (a partly shared reality) while aiming for orientation, reputation, and power.

The terms opinions, reputation, and power should be briefly explained in the game’s context. Here, the exchanged opinions are messages that can be honest or dishonest. Honesty is defined in the game as the frequency in which the players communicate their beliefs. Orientation, knowledge about the environment (or reality) \( \kappa_i \) is necessary to reach the other two goals, reputation and power. Reputation is defined as the beliefs of others about a player’s honesty. In the game, reputation is a prerequisite for power, which here is the ability to influence the environment, as only the statements of reputed players have a significant chance to impact other’s belief systems. Ultimately, reputation and power help to obtain other resources that are not modeled explicitly in the game. Although a high reputation can be reached by being honest, this typically does not imply a large empowerment, as is shown by the fact that the most honest agent often receives a low reputation in the presence of a deceptive agent. An honest player has little ability to steer others’ beliefs in comparison to a frequent liar. Thus, the most powerful players should be the ones that are least honest, but with a high reputation. The increase of their reputation with respect to their intrinsic honesty is therefore a good measure of power. Honest players might become reputed, but are rarely powerful.

At this level of abstraction, the reputation game can be played by humans as well as by computer programs. Here, we focus on agent based simulations, where agents are virtual entities that have certain traits of humans.

A number of decisions of agents in the game appear to be driven by chance, but this does not need to be the case. In principle agents could make decisions according to more sophisticated, deterministic calculations. However, using randomness for now permits to set up the game without having to discuss all principles behind decisions in detail. Nevertheless, a number of behavior strategies were introduced to understand their impact on the game.

These strategies were chosen to resemble deceptive strategies used by humans with various degrees. In particular, the manipulative, dominant, and destructive strategies introduced here resemble real world strategies that are used (neither necessarily nor exclusively) by members of the dark triad, Machiavellian, narcissistic, and sociopathic personalities.

8.2 The player’s minds

The agent’s information processing is designed to follow information theoretical principles, within some limits. The used cognitive model tries to follow the optimal Bayesian logic, however, agents are unable to memorize all fine details of the resulting high dimensional probability distributions. We believe that such a bound rationality model roughly captures how a human mind operates. Trying to maintain orientation in a complex and changing world requires to
follow information principles. These principles, however, demand computational resources beyond what is available to most finite physical systems, such as humans, our agents, or other AI systems. Thus, compromises in the accuracy to represent and process information are always necessary, and these could be the basis of some of the cognitive biases observed in real world psychology [e.g. 42, 65, 66] and AI.

The limitations of the agents’ knowledge representation, which is only a direct product of one dimensional, parametrized probability functions and not a multidimensional, non-parametric distribution as required by Bayesian logic, can be exploited by adversarial strategies of other agents. For example, a statement about some agent’s honesty that strongly disagrees with the receiver’s belief implies a bimodal posterior probability, with a peak associated with the possibility of an honest message and a second peak associated with the possibility of a lie. The relative height of these peaks depends on the clues the receiver got about the message honesty. However, this bimodal distribution cannot be stored in the agents’ belief representation and the information needs to be compressed into this form. As information is inevitably lost in this compression, the resulting reasoning of agents will be imperfect or irrational to a certain degree. This imperfection can be exploited by adversarial attacks, for example in form of large scale propaganda.

To decide whether a message is reliable, agents use a number of signs. Critical agents judge the trustworthiness of a message according to how much it fits their own beliefs or how surprising it is. The surprise of a message is measured in terms of the divergence of the belief resulting from accepting the message in comparison to the present belief. This divergence (or surprise) is measured in the number of bits that would be obtained by this update. The scale against which this surprise is compared to decide about the trustworthiness of messages needs to be learned and kept updated in a changing social environment. This adaptability, however, opens the door to manipulative attacks. Exposing an agent to a large number of strongly diverging opinions inflates this scale, thereby reduces the ability to detect lies, and thus makes manipulations easier. This is the principle of gas lighting communication patterns used by dark triad personalities. We simulated the case where agents are exposed to many messages that strongly diverge from their own beliefs and observed that even agents, which were initially getting more and more skeptical about the trustworthiness of the propaganda, converted eventually to the opinion expressed by the propaganda. The exposure to the propaganda let their reference surprise scale inflate, and thereby their lie detection break. Interestingly, the initially most skeptical agents convert most strongly to the propaganda position, since the propaganda causes the largest mental dissonance in skeptical minds.

In order to make agents more immune to propaganda we also introduced a smart receiver strategy, which compares a message with what the speaker seems to believe on a topic as well as what the speaker’s typical lies on a topic seem to be. These two reference points, but also the need to construct credible lies, require agents to maintain a mental representation of other’s belief systems, i.e. a rudimentary theory of mind. Here, we propose a simple description of the theory of mind updates, which is certainly ad-hoc and should be revised in future research. Smart agents, which are better in maintaining and using their theory of mind, are indeed more immune against propaganda and slightly better in discriminating lies from honest statements. Our special agents are all smart as well as deceptive (= pathological liars).

8.3 The player’s strategies

The basic strategies that agents can adopt are referred to as being strategic, anti-strategic, egocentric, flattering, aggressive, shameless, and deceptive. They can all be combined to form more complex, special strategies such as the clever (deceptive and smart), manipulative (clever, anti-strategic, and flattering), dominant (clever, strategic, and egocentric), and destructive (clever, strategic, aggressive, and shameless) strategies. The latter three are introduced to emulate communication patterns frequently associated with Machiavellian, narcissistic, and sociopathic personalities, respectively. The reputation game simulation permits to investigate the effectiveness of such communication patterns in achieving the goal of a high reputation and large power. Our simulations verify that such strategies are indeed effective to achieve such goals, at least in a statistical sense, not only in comparison to ordinary agents, but also if we compare to clever agents (= deceptive and smart).

The manipulative strategy most often leads to the highest relative reputation within small groups, the dominant strategy is able to reach the absolute highest reputation values most frequently, and the destructive strategy seems to become more efficient than the other two in larger groups (see Sect. 7.4). How many of these results can be transferred to real human communication settings will require more detailed investigations. However, our simulation can serve as a basis to study real world phenomena.

8.4 Emergent phenomena

The dynamic of the game is complex, stochastic, and chaotic. Nevertheless, emergent trends and patterns can be observed that resemble real world socio-psychological phenomena. Here, we list the ones we observed or strongly suspect to be present in the simulations.

The game setup is an echo-chamber, with only a few agents talking to each other, and who, thanks to the imperfect tracking of other agents’ information sources, do not realize when another agent’s apparent new information is in fact an echo of an earlier, own statement. Emergent echo-chambers in sub-sets of agents can also be observed in simulation runs (even though the size of the population
Figure 23: Distribution of reputation and friendship relations between pairs of agents in the hundred runs. For the reputation of agent $a$ with $b$ ($r_{ab}$, on the vertical axis) and for the time fraction $a$ is regarded as a friend by $b$ (on the horizontal axis) the density distribution color indicates agent $a$ and the color of the contour line, which is at 10% of the distribution’s peak value, indicates agent $b$. Displayed are the run-averaged friendship and reputation distributions. Thus, the cyan distribution with red contour expresses how cyan (agent $a$) is seen by red (agent $b$) on average within each of the hundred runs.

is below the size of real world echo chambers). The reputation network between agents defines who really listens to each other, where “really listening” is meant in the sense of accepting a received message as honest. The formation of an echo-chamber can for example clearly be observed in the simulation of cross-communicating ordinary agents under constant propaganda shown in Fig. 11 and discussed in Sect. 7.2. There, agents cyan and yellow form an echo-chamber, characterized by a growing mutual trust and converging opinions on red, a process in which agent black does not participate.

A similar, but slightly different sociological pattern is **clique formation**. Here, the bonding is through mutual friendship. Given the way friendship $f_{ab}$ is established in the game, by some agent talking more positively about some other agent, and how it manifests, by lying positively about a befriended agent, the occurrence of more stable friendship among more dishonest agents is to be expected, since these can maintain friendships better via their more frequent lies. We did not analyze the simulation data w.r.t. to the occurrence of cliques, but believe that they must be inevitable if the conditions are right, for example when several manipulative agents are present in a game, whose strategies make them perfectly compatible as friends. Since being anti-strategic, they most likely address each other more frequently, and when they talk with each other, they only exchange flattering statements about each other.

The occurrence of **group opinion building** is very manifest in most simulations and is discussed in the context of the smart agent in Sect. 7.1 of the shameless agent in Sect. 7.3 and the dominant agent in Sect. 7.3. There the phenomena of a **freeze-in of group opinions** was explicitly mentioned, which happens frequently thanks to the general echo-chamber setup of the reputation game.

The echo-chamber effect allows also for **self-deception**

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23Human friendship is certainly more complex.
of agents, which can be observed in many simulation runs. For example the run with the flattering among ordinary agents shown in Fig. 12 and discussed in Sect. 7.3 shows clearly self-deception of the mostly dishonest agent red. Despite the better direct information from the own self-observations, red’s final self-esteem follows red’s enhanced reputation (w.r.t. red’s honesty), despite the basis for this enhancement being red’s own lies.

The largest self-deceptions in the simulations can be found in some of the runs with dominant agents. There, the self-deception might be classified as an self-esteem boost via narcissistic supply. By preferring as conversation partners the most reputed agents and as topics the dominant agent themselves, dominant agents set up their environment in a way that makes efficient self-deception most likely. If the most reputed agent talks positively and frequently about the dominant agent to that agent, the dominant agent will start to believe in the echo of the own propaganda. The reputed agent has thereby become the supplier of the self-esteem boost. The transition from a realistic self-perception to the boosted self-esteem state can be seen in Fig. 13 (third row, right panel, e.g. shortly after time $t = 250, 400, \text{ and } 750$), and is a frequent phenomena for dominant agents as visible from the statistics presented in Fig. 18, Fig. 19, and Sect. 7.4.

Different social atmospheres can also be observed, for example by comparing the runs with dominant agents displayed in in the left and right panel of Fig. 13 and Fig. 14. In the first of the two displayed runs, the group opinions quickly freeze in, thanks to rapidly converging statements of the different agents. In the second run, the dynamics of the opinions is highly volatile and the expressed opinions scatter largely. Not only the opinions of the deceptive agent red, but also that of the other agents show large variance, even in case the latter are honest. This is because the focusing of group opinions is less efficient in such a situation. Fully deceptive agents create and most strongly benefit from chaotic social atmospheres, see Fig. 24 and Sect. 7.5.

This large diversity of opinions leads for all agents to an enlarged surprise reference scale for identifying lies. It also leads to larger lies, as the size of a lie is gauged against this
scale during lie construction. This again leads to an even larger scale, forming a run away effect. As a consequence, the agent’s critical lie detection breaks down and the propaganda of the dominant agent can pull opinions as strongly as if it would act on uncritical agents. See Fig. 11 for the reduced resilience of uncritical agents against propaganda and Fig. 15 (bottom middle and right panels) and Sect. 7.5 for the run away effect of the lie detection scale.

Thus, an attack on the lie detection system by exposing the victims to a large quantity of strong lies or just statements that create cognitive dissonances can be a successful strategy, in particular for strongly self-promoting agents. Such a strategy is gaslighting in which the victims are exposed to statements designed to confuse the victim’s belief system [e.g. 69 and references therein]. Gaslighting is a strategy often associated to narcissistic personalities. It is currently not explicitly implemented in the repertoire of strategies used by the dominant agent. Nevertheless, a variant of gaslighting seems to occur in the reputation game as a side product of the dominant agent’s strong focus on a single topic (the dominant agent) and a single conversation partner (the most reputed agent). If dominant agents become reputed, their self-esteem might stay low, for their many lies. This leads to a large cognitive dissonance for them, as in the frequent conversations they have about themselves, they are confronted with opinions that largely divergence from their self-picture. As a consequence, their reference scale for lies increases. Since they use this scale for lie construction and all their communications are lies, they express extreme opinions on any conversation topic. Since the extreme statements made by such a dominant agent with diverging reputation and self-image also affects the lie reference scales of other agents, the lies of those also become more extreme as well. A toxic social atmosphere can therefore result, which persists until the dominant agent’s self esteem and reputation agree, either on a high or on a low level. If the reputation and self-esteem of a dominant agent are both high, this agent has managed to manipulate the others into providing narcissistic supply, i.e. helping to maintain the inflated self-image of the dominant agent (see right panel of Fig. 14).

Too much cognitive dissonance, which agents experience if exposed to large scale propaganda, can lead to a breakdown of the mental defense against lies, as shown in Fig. 11. Working countermeasures that agents can take are honest and trustful exchanges with other propaganda victims and being smart in detecting lies. Both measures make agents more resilient against propaganda, as discussed in Sect. 7.2.

We also observed some form of Cassandra syndrome, in which the most honest agents experience the largest chance to get the lowest reputation and are unlikely to be believed anymore. The opinions expressed of an honest agent are bound to this agent’s beliefs and therefore do not follow as much an evolving group opinion as the opinions expressed by a dishonest agent, who targets other beliefs when lying. As a consequence, the expressed opinions of an honest agent might detach from the group position, which then lets the others perceive this agent as dishonest. These will then discard the opinions expressed by the most honest agent. Such a Cassandra syndrome situation can occur among ordinary agents, but becomes substantially more frequent when a dominant agent is present and manages to dominate the group. Interestingly, the Cassandra syndrome effect weakens with increasing levels of social chaos, probably due to the general loss of the other agents’ ability to discriminate between honest and dishonest messages.

Finally, we see a strong positive correlation of the reputations of the least honest agents. The mechanism generating this are the more easily maintained mutual friendships of dishonest agents, the general liar’s benefit from confusion, and the resulting inflation of the lie detection surprise scale in the presence of more other dishonest agents. This can lead to a deception symbiosis, in which the confusion created by a pathological liar makes it easier for other liars to plant their lies as well. This not only seems to hold for our agents. The negative impact of confusing, extreme messages on the ability of humans to discriminate correct and false statements is a documented psychological effect.

8.5 Future directions

The reputation game, as introduced here, is intended as a starting point for further developments and investigations. Probably most of its ingredients need to be revised and extended. Here, we want to discuss a few possible future directions.

Currently, the beliefs of agents about other agents and the auxiliary parameters used by agents for orientation are treated differently. The former in a probabilistic way and the latter more ad-hoc. This could be unified with a probabilistic treatment of any uncertain quantity.

Furthermore, the reasoning of agents and their theory of mind, the representation of other agents’ thinking, have disparate dynamics. In principle, this could be unified by agents just emulating other agents in their minds by using the same computational infrastructure for this, which they use for their own thinking. With such an architecture for the theory of mind, not only the description might become more natural, it might also be possible to simulate phenomena like hallucination as cross-talk between an emulated and the own personality of an agent.

The characters of agents are currently static, programmed strategies. Agents could be enabled to discover and learn strategies on their own, from trial and error, or by watching the actions of other agents. The level of randomness of their actions could also become an adjustable parameter. It would be interesting to see under which conditions for example the malicious strategies introduced here

24See triangles marking lies in the right panel of Fig. 14, which are either extremely positive (being at the top of the reputation range) or extremely negative (being at the bottom of the range) in the period 600 to 1200.
would develop on their own in an evolutionary scenario.

The language of agents can be enriched. More topics could be introduced, as aspects of an outer reality, or additional properties of agents. Also enabling agents to quote each other would be very interesting.

The mental representation agents used to memorize the learned can be made more complex. Real humans are, to some degree, able to remember an entanglement of statements. They can even remove information partly if it turns out that its source was deceptive. Agents could be provided with similar abilities.

Furthermore, the parameters of the cognitive model of the reputation game might be calibrated against real world data. Finally, the sizes of the simulated social networks need to be increased to mimic real social networks or even social media interactions. For simulation of the latter, the effects of attention steering AI systems should be included, in order to emulate their impact on society.

9 Conclusions

To conclude, we have introduced the reputation game as a socio-psychological simulation that is built on the premise that agents should process information according to simplified information theoretical principles. We showed that a large number of known sociological and psychological effects naturally seem to emerge from this premise. We believe that we have illustrated the usefulness of such simulations to understand human sociological behavior. Maybe, our work can help to mitigate the undesired effects that are discussed in the context of communications influenced by AI technologies.

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References

[1] David MJ Lazer et al. “The science of fake news.” In: Science 359.6380 (2018), pp. 1094–1096.
[2] Sinan Aral and Dean Eckles. “Protecting elections from social media manipulation.” In: Science 365.6456 (2019), pp. 858–861.
[3] Samantha Bradshaw and Philip Howard. “Troops, trolls and troublemakers: A global inventory of organized social media manipulation.” In: (2017).
[4] Samantha Bradshaw and Philip N Howard. “Challenging truth and trust: A global inventory of organized social media manipulation.” In: The Computational Propaganda Project 1 (2018).
[5] Liang Wu et al. “Misinformation in social media: definition, manipulation, and detection.” In: ACM SIGKDD Explorations Newsletter 21.2 (2019), pp. 80–90.
[6] Emilio Ferrara et al. “Characterizing social media manipulation in the 2020 US presidential election.” In: First Monday (2020).
[7] Guillaume Deffuant et al. “Mixing beliefs among interacting agents.” In: Advances in Complex Systems 3.01n04 (2000), pp. 87–98. DOI: 10.1142/S0219525900000078. URL: http://wwllisc.clermont.cemagref.fr/imagesproject/finalreport/mixbel.pdf
[8] Romualdo Pastor-Satorras et al. “Epidemic processes in complex networks.” In: Reviews of Modern Physics 87.3 (July 2015), pp. 925–979. DOI: 10.1103/RevModPhys.87.925 arXiv:1408.2701[physics.soc-ph]
[9] Andrea E. F. Clementi et al. “Rational Fair Consensus in the GOSSIP Model.” In: CoRR abs/1705.09566 (2017). arXiv:1705.09566 URL: http://arxiv.org/abs/1705.09566
[10] Daniel Geschke, Jan Lorenz, and Peter Holtz. “The triple-filter bubble: Using agent-based modelling to test a meta-theoretical framework for the emergence of filter bubbles and echo chambers.” In: British Journal of Social Psychology 58.1 (2019), pp. 129–149.
[11] Jean Tirole. “A theory of collective reputations (with applications to the persistence of corruption and to firm quality).” In: The Review of Economic Studies 63.1 (1996), pp. 1–22.
[12] Steven Tadelis. “What’s in a Name? Reputation as a Tradeable Asset.” In: American Economic Review 89.3 (1999), pp. 548–563.
[13] Rosaria Conte and Mario Paolucci. Reputation in artificial societies: Social beliefs for social order. Vol. 6. Springer Science & Business Media, 2002.
[14] Lik Mui et al. Ratings in Distributed Systems: A Bayesian Approach. 2001.
[15] Lik Mui, Mojdeh Mohtashemi, and Ari Halberstadt. “Notions of reputation in multi-agents systems: a review.” In: Proceedings of the first international joint conference on Autonomous agents and multiagent systems: part 1. 2002, pp. 280–287.
[16] Lik Mui. “Computational models of trust and reputation: Agents, evolutionary games, and social networks.” PhD thesis. Massachusetts Institute of Technology, 2002.
Furthermore, we use the machine learning package JAX \cite{jax} to implement and minimize the KLs with the second order schemes trust-exact and trust-nug \cite{trust-exact,trust-nug} in this sequence. In our experience, the former scheme seems to be more robust, and therefore provides the starting point for the latter scheme, which seems to be more accurate. Furthermore, we use the machine learning package jax \cite{jax} configured for 64 bit calculations to auto-differentiate the KLs to obtain their required Jacobians and Hessians as well as to speed up all KL-related computations via just-in-time compilation, which accelerates them substantially. Unfortunately, we found that the numerical results of the KL minimization do not exactly agree if executed on different computers. Since the game dynamics is chaotic, such tiny numerical differences can grow and result in differing game evolution in the later parts of some runs (visible to the eye typically after $t = 1000$ in some of the runs). We verified that the statistical results are not significantly affected by this. Furthermore, to ensure $\mu''', \lambda'' > -1$ in every optimization step the KL minimization is modified to the objective function

$$\begin{align*}
\text{KL}_x(I, (\gamma(\mu'''), \gamma(\lambda'')) + (\mu''' - \gamma(\mu''))^2 + (\lambda'' - \gamma(\lambda''))^2 &= \text{KL}_x(I, (\gamma(\mu'''), \gamma(\lambda''))) + \text{ReLU}(\mu''' - \gamma_0)^2 + \text{ReLU}(\lambda'' - \gamma_0)^2 \\
\text{ReLU}(x) &= \begin{cases} 0 & x < 0 \\
0 & x \geq 0 
\end{cases}
\end{align*}$$

with $\gamma(x) = \max(x, \gamma_0)$, $\gamma_0 = -1 + 10^{-10}$, and the rectified linear unit function. This way, the correct minimum is found as long it has coordinates $\mu''', \lambda'' \geq \gamma_0 > -1$. The additional terms gently push the calculations back to this boundary as soon it is violated. The upper limits of $\mu''', \lambda'' \leq 10^6$ are enforced after the minimization via re-scaling both variables by $10^6 / \max(\mu, \lambda)$ in case one of them exceeds this range.

## B Detailed Figures

We show here a number of figures that permit the inspection of further details of the simulation runs, but which are too crowded to be discussed in the main text.

Fig. 25 shows the communication patterns for the basic simulation runs, but which are too crowded to be discussed in the main text.

Fig. 26 shows the communication patterns for the basic simulation runs, but which are too crowded to be discussed in the main text.

Fig. 27 shows the run averaged relation of reputation and friendship between agents in the four and five agent simulations, respectively. Fig. 28 displays the relation between the run averaged reputation of an agent and the level of social chaos.

## A Numerical Details

Here, we detail how the KL minimization introduced in Sec. 4.4 is performed numerically. We use the Python package scipy \cite{scipy} to implement and minimize the KLs with the second order schemes trust-exact and trust-nug \cite{trust-exact,trust-nug} in this sequence. In our experience, the former scheme seems to be more robust, and therefore provides the starting point for the latter scheme, which seems to be more accurate. Furthermore, we use the machine learning package jax \cite{jax} configured for 64 bit calculations to auto-differentiate the KLs to obtain their required Jacobians and Hessians as well as to speed up all KL-related computations via just-in-time compilation, which accelerates them substantially. Unfortunately, we found that the numerical results of the KL minimization do not exactly agree if executed on different
Figure 25: Communication patterns as in Fig. 9 of the simulations of basic communication strategies with agent red being here strategic (top left), egocentric (top right), and flattering (middle left), shameless (middle right), aggressive (bottom left), and deceptive (fourth row). The used random sequences No. 1 are also identical to the simulations shown in Fig. 9.
Figure 26: As Fig. 25, just for agent red being clever (first row), manipulative (second row), dominant (third row), and destructive (fourth row). The left column shows simulations with the random sequences No. 1, and the right with No. 2.
Figure 27: Like Fig. 23, just for simulations with four (upper rows) and five (lower rows) agents.
Figure 28: Like Fig. 24 just for simulations with four (upper rows) and five (lower rows) agents.