Has the QCD Critical Point been Signaled by Observations at RHIC?

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The shear viscosity to entropy ratio ($\eta/s$) is estimated for the hot and dense QCD matter created in Au+Au collisions at RHIC ($\sqrt{s_{NN}} = 200$ GeV). A very low value is found $\eta/s \sim 0.1$, which is close to the conjectured lower bound ($1/4\pi$). It is argued that such a low value is indicative of thermodynamic trajectories for the decaying matter which lie close to the QCD critical end point.

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A primary objective for studying ultra-relativistic heavy ion collisions is to map out the accessible regions of the Quantum Chromo Dynamics (QCD) phase diagram. The existence of a critical point – an end point of the first order chiral transition in this diagram – has attracted considerable attention since it was first proposed \cite{1, 2}. Several theoretical techniques have been exploited to locate this critical end point (CEP) in the plane of temperature vs baryon chemical potential ($T, \mu_B$) i.e. the QCD phase diagram \cite{3, 4, 5, 6, 7}. The properties of the CEP have also been sought via model calculations and the universality hypothesis \cite{8, 9, 10}. Nonetheless, experimental verification of the CEP constitutes a major current scientific challenge.

The study of heavy ion collisions to search for the CEP was proposed several years ago \cite{11}. Recently, a resurgence of experimental interest has centered around the prospects for successful experimental searches at Brookhaven’s Relativistic Heavy Ion Collider (RHIC) \cite{12, 13} and CERN’s Super Proton Synchrotron (SPS) \cite{14, 15}. Indeed, a recent theoretical investigation involving two flavors of light dynamical quarks \cite{16} gives $T_c/T_{co} \sim 0.95$ and $\mu_B^2/T_{co} \sim 1.1$, where $T_{co} \sim 170$ MeV is the temperature for a cross-over to the quark gluon plasma (QGP) at zero baryon chemical potential and $T_c$ and $\mu_B^2$ are the temperature and the baryon chemical potential (respectively) at the critical end point. It is widely believed that these values for $T_c$ and $\mu_B$ place the CEP in the range of observability for future energy scans at both RHIC and the SPS \cite{13}.

In recent hydrodynamical calculations \cite{17, 18}, an interesting aspect of the CEP has been reported which could have important consequences for its detection. That is, it acts as an attractor for thermodynamic trajectories of evolving hot and dense QCD matter \cite{11}. This aspect of the CEP is illustrated in Fig. 1 where isentropic trajectories in the ($T, \mu_B$) plane are shown for a broad range of values for the entropy per baryon $s/n_B$. Trajectories for the range $s/n_B \sim 100 – 50$, are clearly focused toward the CEP. Such a focusing is not observed for isentropic trajectories in similar model calculations which do not take explicit account of the CEP in the equation of state (EOS). Instead, trajectories are merely shifted on the phase coexistence line \cite{17}. Given this aspect of the CEP, it is worthwhile to investigate whether or not the decay dynamics of the high energy density QCD matter created in heavy ion RHIC collisions show any signals for the CEP.

Here, we argue that estimates for the ratio of viscosity to entropy density ($\eta/s$) made from RHIC measurements \cite{19, 20, 21}, indicate decay trajectories which are close to the CEP. This implies that the prospects for constraining the location of the CEP in the ($T, \mu_B$) plane are very good. Possibly, such constraints can be best achieved via a study of excitation functions with relatively large steps in collision energy prior to focusing on a series of fine steps.
There is strong experimental evidence for the creation of locally equilibrated nuclear matter at unprecedented energy densities, in heavy ion collisions at RHIC [22, 23, 24, 25, 26, 27, 28, 29]. Jet suppression studies indicate that the constituents of this matter interact with unexpected strength and this matter is almost opaque to high energy partons [30, 31].

Elliptic flow measurements [32, 33] validate the predictions of perfect fluid hydrodynamics for the scaling of the elliptic flow coefficient $v_2$ with eccentricity $\varepsilon$, system size and transverse kinetic energy $KE_T$ [34, 35, 36]; they also indicate the predictions of valence quark number ($n_q$) scaling [37, 38, 39], suggesting that quark-like degrees of freedom are pertinent when elliptic flow develops [32, 33]. The result of such scaling is illustrated in Fig. 2 where we plot $v_2/n_q\varepsilon$ vs $KE_T/n_q$: it shows that the relatively “complicated” dependence of $v_2$ on centrality, transverse momentum, particle type and quark number can be scaled to a single function [32, 33].

The expected suppression of flow due to the shear viscosity, as predicted by weak-coupling transport calculations is also noted. In fact, only a rather small viscosity, $\eta/s \lesssim 0.1$, could be accommodated by the data in early estimates [19]. This value of $\eta/s$, which is much lower than that obtained from weak coupling QCD [40] or hadronic computations [41], has been interpreted as evidence that the quark gluon plasma (QGP) created in the early phase of RHIC collisions is more strongly coupled than expected [28, 42]. An alternative interpretation is that the low value for $\eta/s$ could result from an anomalous viscosity $\eta_A$, arising from turbulent color magnetic and electric fields dynamically generated in the expanding quark-gluon plasma [43]. That is, $\eta^{-1} = \eta_A^{-1} + \eta_C^{-1}$ and $\eta_A$ dominates over the collisional viscosity $\eta_C$.

The ratio $\eta/s$ cannot be arbitrarily small because quantum mechanics limits the size of cross sections via unitarity. A conjectured lower bound for $\eta/s$ is $1/(4\pi)$, reached in the strong coupling limit of certain gauge theories [44]. It has even been speculated that this lower bound holds for all substances [44]. The temperature dependence of $\eta/s$ also provides invaluable insights. This is illustrated in Fig. 3 where we have plotted $\eta/s$ vs $(T - T_c)/T_c$ for molecular, atomic and nuclear matter. The data for He, N$_2$ and H$_2$O were obtained for their respective critical pressure. They are taken from Ref. [45] and the references therein. The calculated results shown for the meson-gas (for $T < T_c$) are obtained from chiral perturbation theory with free cross sections [46]. Those for the QGP (ie. for $T > T_c$) are from lattice QCD simulations [47]. The value $T_c \sim 170$ MeV is taken from lattice QCD calculations [48].

Figure 3 illustrates the observation that for atomic and molecular substances, the ratio $\eta/s$ exhibits a minimum of comparable depth for isobars passing in the vicinity of the liquid-gas critical point [44, 45, 46]. When an isobar passes through the critical point (as shown in Fig. 3), the minimum forms a cusp at $T_c$: when it passes below the critical point, the minimum is found at a temperature below $T_c$ (liquid side) but is accompanied by a discontinuous change across the phase transition. For an isobar passing above the critical point, a less pronounced minimum is found at a value slightly above $T_c$. The value $\eta/s$ is smallest in the vicinity of $T_c$ because this corresponds to the most difficult condition for the transport of momentum [45].

Given the observations for atomic and molecular substances, one expects a broad range of trajectories in the $(T, n_B)$ or $(T, \mu_B)$ plane for nuclear matter, to show $\eta/s$ minima with a possible cusp at the critical point. The exact location of this point (in the $(T, \mu_B)$ plane) is of course not known, and only coarse estimates of where it might lie are available. The open triangles in Fig. 3 show calculated values for $\eta/s$ along the $\mu_B = 0$, $n_B = 0$ trajectory. For $T < T_c$, the $\eta/s$ values for the meson-gas show an increase for decreasing values of $T$. For $T$ greater than $T_c$, the lattice results [47] indicate an increase of $\eta/s$ with $T$, albeit with large error bars. These trends suggest a minimum for $\eta/s$.

The open hexagons in Fig. 3 show estimates for $\eta/s$ constrained by experimental data; they are for a trajectory which starts along $n_B = n_0$. The $\eta$ values are extracted from momentum transport data obtained in intermediate-energy (IE) collisions [49, 50, 51]; values for $s$ were obtained from composite yields measured in the same energy and mass region. The filled hexagon in
shown a recent estimate for $\eta/s$ with an estimated systematic error of $+0.05$ primarily due to the uncertainty in $\lambda_f$. This value for $\eta/s$ is in good agreement with the experimentally based estimates of Gyulassy and Shuryak [42, 55]. However, it is in stark contrast to the predictions of perturbative QCD [19, 40].

The experimental and calculated $\eta/s$ values for nuclear matter shown in Fig. 3, give strong indication for a minimum in the vicinity of $T_c$. This minimum, achieved in RHIC collisions, is rather close to the conjectured lower bound of $\eta/s = 1/4\pi$. We therefore conclude that it is compatible with the minimum expected if the hot and dense QCD matter produced in RHIC collisions follow decay trajectories which are close to the CEP. As discussed earlier, such trajectories could be readily followed if the CEP acts as an attractor for thermodynamic trajectories for the decaying matter [17, 18]. Here, it is noteworthy that the system has to spend a considerable portion of its dynamics in the region of low $\eta/s$ for that low value to have a significant impact on the reaction dynamics. If the system gets momentarily, to a low $\eta/s$ and the rest of the dynamics takes place at high $\eta/s$, the momentary low value will have little impact. An optimal situation would be if the system stays at low $\eta/s$ and then very quickly freezes out thereafter. That is, significant collective motion develops prior to the systems evolution toward higher $\eta/s$ values.

In summary we have used the comprehensive overall scaling of elliptic flow data to constrain a rudimentary experimental estimate for $\eta/s$ in Au+Au collisions at RHIC [56]. A value close to the conjectured lower bound of $\eta/s = 1/4\pi$ is observed. We argue that such a low value for $\eta/s$ is suggestive of decay trajectories which lie close to the QCD critical end point, because it acts as an attractor for constant entropy phase space trajectories of evolving hot and dense QCD matter. This suggests that the prospects for a successful search for the first order QCD phase transition and the possibility for establishing reliable constraints (in the $(T, \mu_B)$ plane) for its associated critical end point are quite good. An initial measurement of excitation functions with coarse-grained collision energy steps may give sufficient information, or at least, give a better perspective on where to focus attention on fine steps.

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\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{(Color online) $\eta/s$ vs $(T-T_f)/T_f$ for several substances as indicated. The calculated values for the meson-gas have an associated error of $\sim 50\%$ [46]. The lattice QCD value $T_c = 170$ MeV [48] is assumed for nuclear matter. The lines are drawn to guide the eye.}
\end{figure}

\text{Fig. 3 shows a recent estimate for $\eta/s$ from high precision elliptic flow measurements in Au+Au collisions ($\sqrt{s_{NN}} = 200$ GeV) at RHIC [21]:}

$$\frac{\eta}{s} \sim T\lambda_f c_s,$$

where $T$ is the temperature, $\lambda_f$ is the mean free path and $c_s$ is the sound speed in the matter.

The temperature $T = 160$ MeV was employed in a blast wave model generalized to include viscous corrections [19]. We estimate the value $T = 165 \pm 3$ MeV via a hydrodynamically inspired fit to the universally scaled $v_2$ data shown in Fig. 2. The functional form used for the fit is $I_1(w)/I_0(w)$, where $w = KE_T/2T$ and $I_1(w)$ and $I_0(w)$ are Bessel functions [35]. For $c_s$, we use the recent observation that $v_2$ scales with eccentricity and across system size to constrain the estimate $c_s = 0.35 \pm 0.05$ [32, 33, 52]. The mean free path estimate $\lambda_f = 0.3 \pm 0.03 fm$ [53], is obtained from an on-shell transport model simulation of the gluon evolution (in space and time) in Au+Au collisions at 200 GeV [54]. This parton cascade model includes pQCD $2 \leftrightarrow 2$ and $2 \leftrightarrow 3$ scatterings. These values for $\lambda_f$, $T$ and $c_s$ lead to the estimate $\eta/s \sim 0.09 \pm 0.015$ with an estimated systematic error of $+0.1$ primarily due to the uncertainty in $\lambda_f$. This value for $\eta/s$ is in good agreement with the experimentally based estimates of Teaney and Gavin [19, 20] and the theoretical estimates of Gyulassy and Shuryak [42, 55]. However, it is in stark contrast to the predictions of perturbative QCD [19, 40].

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