Two-dimensional Dirac fermions with random axial-vector potential

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A Dirac fermion model with random axial-vector potential is proposed. At a special strength of randomness, the symmetry of the action is enhanced, which is due to the gauge symmetry à la Nishimori. Some exact scaling exponents of single-particle Green functions are computed. The relationship with the XY gauge glass model is discussed.

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Random Dirac fermions have attracted much current interest in condensed matter physics. They have actually intimate relationship with integer quantum Hall (IQH) transitions [1–3], dirty d-wave superconductors [4–6], etc. Critical theory of IQH transitions is believed to be a strong coupling fixed point of the Dirac fermion with random mass, vector and scalar potentials [1], though the fixed point is still missing. Near the zero energy quasi-random mass, vector and scalar potentials [1], the strong coupling fixed point of the Dirac fermion with random axial-vector potential [1] is given by [1] (the class of superconductors with broken spin rotation and time reversal symmetry) in Zirnbauer’s classification [6]. Although the enhancement of the symmetry on the Nishimori-line has already been discussed in [7], Gruzberg et. al. have explicitly shown that the symmetry is enhanced to O(2n + 1) due to the gauge symmetry [8]. This class is not included in Zirnbauer’s classification.

Among these developments, the Dirac fermion with random vector potential only is one of simplest but nontrivial models, still providing hot topics. It gives multifractal scaling exponents of local composite operators [9–11], exact zero-energy wave function for any realization of disorder [12–15], replica symmetry breaking [16], etc.

In this paper, we study a Dirac fermion in two dimensions including random axial-vector potential. We show that the symmetry of the model is enhanced at a strong disorder strength, which is analogous to the Nishimori-line of statistical models. Similarly to the conventional model with random vector potential, this model can be solved exactly, giving us some exact scaling dimensions. It turns out that this model corresponds to the spin wave model for the XY gauge glass model.

The system we will study is

$$\mathcal{H}_A = \bar{\psi} h_A \psi, \quad h_A = \gamma_\mu (-i \partial_\mu - \sqrt{g} \gamma_5 A_\mu),$$

(1)

where $\gamma_1, \gamma_2, \gamma_5$ denote the three Pauli matrices, and $A_\mu$ is a random axial-vector potential. Being defined in 2D Euclidean space, this Hamiltonian is non-hermitian due to $\gamma_5$ coupled with the vector potential. Therefore, the model (1) may have a relationship with Hatao-Nelson model [17]. However, ensemble-average yields the square of the antihermitean term, and the Lagrangian becomes hermitian. It should also be mentioned that without mass, the Hamiltonian has chiral symmetry $\gamma_5 \gamma_A + h_A \gamma_5 = 0$, telling that the model belongs to the class AIII in Zirnbauer’s classification.

In the previous work [18], the same model but with usual vector potential, $h_V = \gamma_\mu (-i \partial_\mu - \sqrt{g} A_\mu)$ was studied. In the present case, the coupling with $\gamma_5$ may imply that the right- and left-moving fermions have opposite charges. In what follows, the present model (1) is sometimes referred to as A-model, whereas the conventional one as V-model. The probability distribution of the axial-vector potential is assumed to be of Gaussian type

$$P[A] = \frac{1}{N_A} e^{-\frac{1}{2} \int d^2 x A^2_\mu},$$

(2)

where the normalization factor is given by $N_A = \int D A e^{-\frac{1}{2} \int d^2 x A^2_\mu}$. At a given Matsubara frequency $\omega$, the partition function of the model with quenched disorder is given by

$$Z = \int D \psi \bar{D} \psi e^{-\int d^2 x \bar{\psi} (h_A + i\omega) \psi}.$$  

(3)

The Lagrangian is invariant under the following chiral gauge transformation,

$$\psi \rightarrow e^{i \sqrt{g} \theta(x) \gamma_5} \psi, \quad \bar{\psi} \rightarrow \bar{\psi} e^{i \sqrt{g} \theta(x) \gamma_5}, \quad A_\mu \rightarrow A_\mu + \partial_\mu \theta(x).$$

(4)

It is well-known that this transformation gives rise to a nontrivial Jacobian, but as we shall see later, it is irrelevant to our problem. The $\omega$-term serves as a symmetry-breaking term.

Although it is possible to take quenched average of the partition function without resort to the replica
trick, we will first apply it in order to explain the enhancement of the symmetry at a special coupling constant. Integration over disorder for replicated partition function $Z^n$ yields an effective Lagrangian, $\mathcal{L}_A = \bar{\psi}^a (\gamma_\mu \partial_\mu + \bar{\imath} \omega) \psi^a - \bar{\frac{1}{2}} \left( \bar{\psi}^a \gamma_5 \bar{\psi}^a \right)^2$, where $a = 1, \ldots, n$. Due to the identity $\gamma_\mu \gamma_5 = -\bar{\imath} \epsilon_{\mu\nu} \gamma^\nu$, we have $-\frac{1}{2} \left( \bar{\psi}^a \gamma_5 \gamma_\mu \psi^a \right)^2 = \frac{1}{2} \left( \bar{\psi}^a \gamma_\mu \gamma^5 \psi^a \right)^2$. Compared with the effective Lagrangian of the V-model, $\mathcal{L}_V = \bar{\psi}^a (\gamma_\mu \partial_\mu + \bar{\imath} \omega) \psi^a - \frac{1}{2} \left( \bar{\psi}^a \gamma_\mu \gamma^5 \psi^a \right)^2$, the present A-model turns out to be equivalent to the V-model but with negative coupling constant $g$. Namely, the partition function $Z_A^n(g)$ of the A-model with the coupling constant $g > 0$ is equivalent to that of the V-model $Z_V^n(-g)$. Therefore, some properties, e.g., the scaling dimensions of some local operators for the A-model can be obtained directly from the V-model by the use of the formal replacement $g \to -g$, as we shall see momentarily.

However, the A-model has some peculiar properties. One of them is the enhancement of the symmetry at a special coupling constant $g = \pi$. The Lagrangian in Eq. (8) has global $U(n) \times U(n)$ symmetry for any $g$ when $\omega = 0$. Explicitly, it is readily seen that for any realization of the axial-vector potential it is invariant under the transformation $\psi \to (U_1 P_+ + U_2 P_-) \psi$ and $\bar{\psi} \to \bar{\psi} (\bar{U}^\dagger P_+ + \bar{U}^\dagger P_-)$, where $P_\pm = (1 \pm \gamma_5) / 2$ and $U_i$ is a $n \times n$ unitary matrix which acts on the replica space. Since the Lagrangian is invariant under local chiral gauge transformations, we have

$$Z^n = \int D\psi D\bar{\psi} D\bar{A} e^{-\frac{1}{2} \int d^2 x A^2 \epsilon e^{-\frac{1}{2} \int d^2 x (A_\mu - \partial_\mu \theta)^2 e^{-\int d^2 x \mathcal{L}}},$$

(5)

where $\mathcal{L} = -\bar{\psi}^a \gamma_\mu (i \partial_\mu + \sqrt{g} \gamma_5 A_\mu) \psi^a$, $V$ is the “volume” of the gauge space $V = \int D\theta$, and $\Gamma(\theta)$ is a Jacobian due to chiral gauge transformation [4]. This equation is based on the same argument as Nishimori applied to the random-bond Ising model [1]. Since the Jacobian is multiplied by a factor $n$, it should vanish in the replica limit $n \to 0$. This is also justified below in the direct computation of the exact scaling dimensions of the single-particle Green functions. Now using the fermionization of the gauge variable via fermion-boson correspondence in 2D,

$$\frac{1}{2} \left( \partial_\mu \theta \right)^2 \leftrightarrow \bar{\psi}^a \gamma_\mu \partial_\mu \psi^a, \quad \frac{1}{\sqrt{\pi}} \epsilon_{\mu\nu} \partial_\nu \theta \leftrightarrow \bar{\psi}^a \gamma_\mu \psi^a,$$

(6)

we have

$$Z^n = \frac{1}{V} \int D\psi D\bar{\psi} D\bar{A} e^{-\frac{1}{2} \int d^2 x A^2 \epsilon e^{-\frac{1}{2} \int d^2 x \mathcal{L}}},$$

(7)

where the Lagrangian now includes an additional 0th fermion $\mathcal{L} = -\bar{\psi}^a \gamma_\mu (i \partial_\mu + \bar{\imath} \omega) \psi^a - \sqrt{g} (\bar{\psi}^a \gamma_5 \psi^a)^2 + \sqrt{g} (\bar{\psi}^a \gamma_5 \psi^a)^2 A_\mu$. The Greek superscript $\alpha$ runs from 0 to $n$, whereas $a$ from 1 to $n$. When $g = \pi$ (and $\omega = 0$), the ensemble-averaged Lagrangian can be written in a symmetric way together with the 0th fermion as $\mathcal{L} = -\bar{\psi}^a \gamma_\mu (i \partial_\mu + \bar{\imath} \omega) \psi^a + \frac{1}{2} \left( \bar{\psi}^a \gamma_5 \psi^a \right)^2$, telling that the symmetry is enhanced to $U(n+1) \times U(n+1)$. This enhancement of the symmetry for the Dirac fermion with random axial-vector potential is quite analogous to that for the Majorana or Dirac fermion with random mass (the random-bond Ising model). Interestingly, such an enhancement does not occur in the case with conventional vector potential.

What happens at this special coupling constant? For the conventional model $h_V$, the exact scaling properties have been obtained by various methods [3, 4]. Since the present Hamiltonian $h_A$ is nonhermitian, we exactly solve the model by taking the quenched average directly, without using the replica trick or the SUSY technique. One of most interesting properties in disordered systems is the multifractality of the scaling exponents. The Dirac fermion with random vector potential is a typical example having such exponents. By the use of the single-particle Green function,

$$G_{ij}(x, y, i\omega) = \langle \psi_i(x) \bar{\psi}_j(y) \rangle,$$

(8)

where

$$\langle O \rangle = \frac{1}{Z} \int D\psi D\bar{\psi} O e^{-\int d^2 x \mathcal{L}},$$

(9)

ensemble-averaged density of state (DOS) and a field-theoretical analogue of the inverse participation ratios (IPR) $P^{(k)}(E) = \frac{\sum_n |\Psi_n(x)|^2 \delta(E - E_n) / \sum_n |\Psi_n(x)|^2 \delta(E - E_n)}{C_k}$ can be computed as [21]

$$\rho(E) = -\frac{1}{\pi} \lim_{\omega \to 0} \text{Im} \text{tr} G(x, x, i\omega - E),$$

$$P^{(k)}(E) \rho(E) = \frac{1}{C_k} \lim_{\omega \to 0} \omega^{k-1} \text{Im} \text{tr} G(x, x, E - i\omega)^k,$$

(10)

where $\Psi_n(x)$ is a two component spinor wave function of $n$th eigenstate, $\text{tr}$ is a trace for spinor indices, and $C_k = \pi(2k - 3)!!/(2n - 2)!!$. It is useful to introduce the chiral basis, $\psi_{\pm} = P_{\pm} \psi$ and $\psi_{\mp} = \bar{\psi} P_{\mp}$, which was actually used by Mudry et. al. [2]. In what follows, we will take similar notations to this reference. Since $\psi \bar{\psi} = \psi_{\mp} \psi_{\pm} + \psi_{\pm}^\dagger \psi_{\mp}^\dagger$, the $k$th power of the Green function can be written as

$$[G(x, x, E - i\omega)]^k \propto \prod_{a=1}^k \left[ \psi_{\mp}^a \psi_{\pm}^a(x) + \psi_{\pm}^a \psi_{\mp}^a(x) \right].$$

(11)

Here we have introduced species $a$ to calculate the $k$th power of the Green function, although we will not use the replica trick. Expanding the product, we have various operators with various scaling dimensions. Most dominant operators depends on whether $k$ is even or odd. Define
then an example of most dominant operator is $O^{(k)} = O^{(k)}$ for even $k$ and $O^{(k)} = O^{(k)}_a \psi^k_a \psi^k_a$ for odd $k$.

In the chiral basis the Lagrangian is converted into

$$
\mathcal{L} = \bar{\psi}^a (2i\partial \!\!\!\!/ - \sqrt{g} A_+) \psi^a_+ + \bar{\psi}^a (2i\partial \!\!\!\!/ + \sqrt{g} A_-) \psi^a_-, \tag{13}
$$

where $z = x + iy$, $\bar{z} = x - iy$, $A_+ = A_x \pm i A_y$, and $a = 1, 2, \ldots, k$. The different sign for $A_+$ tells that the right- and left-movers have opposite charges, as mentioned previously. In 2D the gauge fields can be decomposed into two independent components $A_\mu = \epsilon_{\mu \nu} \partial_\nu \eta + \partial_\mu \xi$, or in the chiral basis, $A_+ = -2i \partial \!\!\!\!/ (\eta + i \xi)$, and $A_- = 2i \partial \!\!\!\!/ (\eta - i \xi)$. The probability distribution $A$ is written now in terms of these fields as

$$
P[\xi, \eta] = \frac{1}{N_{E} N_{\eta}} e^{-\frac{1}{2} \int d^2 \!\!\!\!/ [i(\partial \!\!\!\!/ \xi) (\partial \!\!\!\!/ \eta)]^2}, \tag{14}
$$

where $N_{E} N_{\eta}$ is a normalization factor $N_{E} = \int d \xi e^{-\frac{1}{2} \int d^2 \!\!\!\!/ (\partial \!\!\!\!/ \xi)^2}$ and similar for $N_{\eta}$. Via the gauge transformation $\psi^a_\pm \rightarrow e^{i \gamma^a \xi(\partial \!\!\!\!/ \xi)} \psi^a_\pm$ and $\bar{\psi}^a_\pm \rightarrow \bar{\psi}^a_\pm e^{-i \gamma^a \xi(\partial \!\!\!\!/ \xi)}$, the axial-vector potential completely disappears in the Lagrangian,

$$
\mathcal{L} \rightarrow \bar{\psi}^a_\pm (2i \partial \!\!\!\!/ A_\pm) \psi^a_\pm + \bar{\psi}^a_\pm (2i \partial \!\!\!\!/ A_-) \psi^a_-. \tag{15}
$$

The chiral gauge transformation yields a nontrivial Jacobian $D\psi/D\bar{\psi} \rightarrow D\psi/D\bar{\psi} e^{i\gamma(\xi)}$. However, this factor appears in the denominator $Z$ as well as in the numerator in Eq. (4), and hence cancels out. Finally, $O^{(k)}$ is gauge-invariant while $O^{(k)}_a$ is not. Actually, they obey the transformation laws

$$
O^{(k)} \rightarrow O^{(k)}_c, \quad O^{(k)}_a \rightarrow e^{-2i \sqrt{g} \xi} O^{(k)}_a. \tag{16}
$$

By the use of the free action (13) after the gauge transformation, the correlation function of $O^{(k)}_a$ can be evaluated as follows. The two point correlators of the free fermi fields are $\langle \psi^a_+ (z) \psi^{a'}_+ (0) \rangle \sim \delta^{aa'} \bar{z}$ and $\langle \psi^a_-(z) \psi^{a'}_-(0) \rangle \sim \delta^{aa'} \bar{z}^{-1}$. Therefore, we have

$$
\langle O_{c} (x) O^{(1)}_c (0) \rangle = |x|^{-2k}, \quad \langle O_{a} (x) O^{(1)}_a (0) \rangle = e^{-2i \sqrt{g} \xi} \bar{z}^{2i \sqrt{g} \xi} |x|^{-2k}. \tag{17}
$$

Note that these do not depend on $\xi$ and therefore the change of the scaling dimension due to disorder is involved with $\xi$ only. The average-over $\xi$ is easily taken and the scaling dimension of $O^{(k)}_c$ finally reads $\Delta_k = k$ for even $k$ whereas $\Delta_k = k + \frac{5}{2}$ for odd $k$, and this is itself nothing but the dominant scaling dimension of the $k$th power of the single-particle Green function $(trG)^k$. Some comments may be in order. In the conventional V-model the random field $\eta$ plays a role in the change of the dimensions due to disorder. In the present case, the role of $\eta$ and $\xi$ is exchanged by $\gamma_\xi$ in Eq. (11), and $\xi$ causes the change of the scaling dimensions. Furthermore, the field $\eta$ gives in general negative scaling dimensions for the V-model, whereas $\xi$ gives positive dimensions for the present model.

Using the dominant scaling dimensions of the Green functions obtained so far, we can calculate the scaling properties of the DOS and IPR. Since $\omega$ couples with $\bar{\psi} \psi$ in the Lagrangian, the dimension of $\omega$ reads $z = 1 - \frac{4}{2}$. Therefore, we expect $\rho(\omega) \sim \omega^{(2-\tau_\omega)/2}$. This is just the same formula for the V-model but with negative $g$. It is also readily seen that the random axial-vector potential is a marginal perturbation and the theory moves along a critical line. However, starting from the free fermion point, the A-model moves, as $g$ increses, to the opposite direction to which the V-model moves. Therefore, the line on which the A-model lies is not reached by the conventional random vector potential model. In this sense, the present model is complementary to the full critical line of $U(1) \times U(1)$ symmetry. The dimension of $\rho$ is an increasing function of $g$, so that the DOS is suppressed around the zero energy if the random axial-vector potential is switched on. At the special point $g = \pi$ where the symmetry is enhanced, the exponent becomes infinity. Therefore, it is likely that the theory is well-defined only for $g < \pi$, and the enhancement of the symmetry is a signal that the theory reached at a singularity.

The scaling behavior of the IPR is obtained in a similar way: $P^{(k)} \sim \omega^{5/2} z^k$, where $\tau_\omega = -2$ for even $k$ and $\tau_\omega = 0$ for odd $k$. As stressed by Mudry et al. [3], the IPR defined by Eq. (11) is not necessarily coincides with that of the original definition $P^{(k)} = |\Psi_n(x)|/N_\xi^{|\omega|}$, where the normalization $N_\xi$ is defined by $N_\xi^2 = \int d^2 x |\Psi_0(x)|^2$. Actually, the zero energy wave function can be obtained exactly as

$$
\Psi_0 (x) = e^{i(\eta(x) - \xi(x)) \gamma_\eta \phi_0}, \tag{18}
$$

for the present A-model, where $\phi_0$ is a constant spinor, while $\Psi_0 (x) = e^{i(\eta(x) - \xi(x)) \gamma_\eta \phi_0}$ for the conventional V-model. The IPR of the latter wave function has been studied by various methods, e.g., replica method [1], SUSY method [2], the method using the equivalence to the random energy model [4], etc. They have all given the same scaling exponent for small $g$. Eq. (18) for the present model also gives $|\Psi_0 (x)|^2 \propto e^{2\eta(x)}$. Since the present A-model has the same probability distribution as the conventional V-model has, we expect that the A-model should have the same scaling dimension $\tau_k = (2 - \frac{5}{2} k)(k - 1)$ as the V-model has, which is quite different from the conjectured $\tau_k^* \rightarrow k$ based on [1].

So far we have studied some peculiar properties of the Dirac fermion with random axial-vector potential. As already mentioned, the Hamiltonian is nonhermitian, because it is defined in the Euclidean space. Then, is this model unrealistic? To address the question, we next examine the bosonized form of the A-model, as has been
done for the V-model by Bernard [3]. By the use of the correspondence [1], the Hamiltonian (1) is converted into
\[ H = \frac{1}{2} (\partial_\mu \phi)^2 + \sqrt{g} \partial_\mu \phi A_\mu. \]  
(19)

This is a low-temperature effective Hamiltonian of the XY gauge glass model [1-4] whose Hamiltonian is defined on a 2D square lattice,
\[ -\beta H = K \sum_{(i,j)} \cos(\phi_i - \phi_j + \chi_{ij}), \]  
(20)

where $\chi_{ij}$ is a random gauge variable with a probability distribution
\[ P[\chi] \propto e^{K_p \sum_{(i,j)} \cos \chi_{ij}}. \]  
(21)

Provided that $K$ and $K_p$ are large, we actually reach the Hamiltonian (19) with $g = \frac{e^2}{\sqrt{K_p}}$ and the probability distribution (21) as well by expanding the cosine terms up to second order, defining $\phi_i - \phi_j \sim a_0 \partial_i \phi(x)$ and $\chi_{ij} \sim a_0 A_\mu(x)$ with the lattice constant $a_0$, and rescaling the fields $\phi \rightarrow \phi/\sqrt{K}$ and $A_\mu \rightarrow A_\mu/\sqrt{K_p}$. This is the so-called spin-wave approximation. The spin-wave Hamiltonian (19) is expected to be unstable against vortex excitations if $K$ or $K_p$ becomes small.

On the other hand, it has been shown that the model [20] with (21) has a Nishimori-line $K_p = K$, on which some exact results can be obtained in a similar way as the random-bond Ising model [1-2]. The equivalence of the condition $K_p = K$ to $g = \pi$ implies that the symmetry enhancement of the Dirac fermion with random axial-vector potential reflects the gauge symmetry on the Nishimori-line of the gauge glass model. As to the symmetry of the model, it is difficult to read its enhancement from the spin-wave Hamiltonian (19). Even if we apply the replica method, the symmetry always remains $U(1)$. This reminds us of the case of the random-bond Ising model: We only come across the continuous $O(n)$ symmetry when the model has been described by the Majorana fermion via the Jordan-Wigner transformation. In the present case, the enhanced symmetry also becomes manifest only in the fermion description. However, the Lagrangian discussed so far is just for the spin-wave part of the gauge glass model and it is quite necessary to include vortices to fully describe the lattice model (20). Then, such theories should be well-defined field theories with manifest enhanced symmetry on the Nishimori-line. It is an interesting issue to derive and study such field theories.

Finally, let us discuss the symmetry for the dual theory of the gauge glass model (23) [24]. The periodicity of the Hamiltonian can be described by $e^{K \cos(\phi_i - \phi_j + \chi_{ij})} \sim \sum_{m=-\infty}^{\infty} e^{K e^{\gamma \left( \phi_i - \phi_j + \chi_{ij} \right) - 2\pi m}}$. By the use of the Poison summation formula (24), we reach the following Hamiltonian,
\[ H_d = \frac{1}{K} (\partial_\mu \theta)^2 + \epsilon_\mu A_\mu \partial_\mu \theta - 2y \cos 2\pi \theta, \]  
where $\theta$ is a dual field and $y$ is a fugacity. This is the sine-Gordon model coupled with random vector potential $A_\mu$.

Therefore, it turns out that there is a duality between the random axial-vector potential and conventional vector potential. The former model yields a symmetry enhancement whereas the latter does not. Recently, the replica symmetry-breaking of the V-model has been suggested [7,8], which might be understood via this duality relation.

In summary, we have studied a Dirac fermion with random axial-vector potential. This model has an enhanced symmetry at a special strength of randomness, which reminds us of the Nishimori-line of the statistical models. Indeed, the model is equivalent to the spin wave Hamiltonian of the XY gauge glass model. It turns out that this model moves along a critical line for increasing $g$ up to $\pi$, and some exact scaling exponents have been obtained.

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