Explicit analysis of nonlinearities in time-interleaved ADC

Wentao Wei\(^a\), Peng Ye, Yu Zhao, Kuojun Yang, Jian Gao, and Wuhuang Huang

School of Automation Engineering, University of Electronic Science and Technology of China,
No. 2006, Xiyuan Ave, West Hi-Tech Zone, Chengdu, 611731, China
\(^a\)wentaowei0228@gmail.com

Abstract: TIADC (Time-Interleaved ADC) architecture suffers from errors introduced by mismatches among the interleaved channels, which degrade performance of TIADC significantly. In this paper, a behavioral model for TIADC based on Wiener model is proposed to describe the nonlinearities in TIADC system. The time-domain and frequency-domain representations of the model are derived. Besides, the discrete-time equivalent model is proposed by transforming the hybrid TIADC model to a purely discrete time system, which is useful for the analysis and implementation of subsequent calibration method. What’s more, numerous studies have investigated compensation methods for discrete-time Wiener model. The methods in these papers can be modified easily for calibration of TIADC using the model proposed in this paper. The experiment results show the nonlinearities of TIADC in practice are consistent with the model proposed in this paper and simulation results indicates the validity of the proposed discrete-time equivalent model.

Keywords: nonlinearities, TIADC, behavioral model, Wiener model, discrete-time equivalent model

Classification: Circuits and modules for electronic instrumentation

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1 Introduction

Time-interleaving is an effective way to meet the fast increasing requirements for high speed and high resolution application scenarios, e.g., communication systems and measurement instruments [1, 2]. ADCs using time interleaving architecture can achieve high sampling rate beyond a certain process technology limit, which makes demand for high resolution and high sampling rate satisfied simultaneously [3].

However, mismatches among sub-ADCs lead to additional errors and the effective resolution of TIADC (Time-Interleaved ADC) degrades considerably [4, 5]. Offset, gain, time, bandwidth and frequency mismatches are investigated and discussed extensively in the last three decades. Numerous calibration method have been proposed to compensate for these mismatches [6, 7, 8, 9].
Recent research have shown the interest on nonlinear mismatches in TIADC since the effect of nonlinear behavior is becoming increasingly apparent as the input frequency increases [10, 11, 12, 13]. These nonlinearities are mainly caused by switch-induced charge injection errors and input signal dependence of the switch on-resistance [14]. The nonlinear behavior causes additional spurious tones in the TIADC output. In many realistic applications, weak signals are accompanied by strong signals. The spurious tones produced by strong signals can interfere with the weak signals, thus these mismatch errors must be compensated.

So far, however, there has been little discussion about the mathematical models of the nonlinearity mismatches in TIADC. In [13], authors model nonlinearity mismatch using hybrid filter banks and show nonlinearity mismatch is a generalization of offset, gain and time mismatch. In [10], the authors build static and dynamic nonlinear models of TIADC. In these models, the TIADC output is expressed as the sum of original signal and error signal, which is useful for further calibration methods.

This paper develops a mathematical model based on Wiener model to investigate the nonlinearities in TIADC. Wiener model is a widely used mathematical tools in nonlinear signal processing, and a considerable amount of literature has been published on Wiener model. This paper combines Wiener model with TIADC, and derive the time-domain and frequency-domain representations of the nonlinear model. We show that offset, gain, time, bandwidth and frequency mismatch are just a special case of the proposed model. The location of $p$th order nonlinear mismatches with sinusoidal input is also given and results show higher order nonlinearities superpose onto the same frequency bins as lower order nonlinearities. Besides, we propose the discrete-time equivalent model of TIADC by transforming hybrid TIADC system to a purely discrete time system, which is useful for analysis and implementation of subsequent calibration method. What’s more, numerous studies have investigated the compensation methods for discrete-time Wiener model. The methods in these papers can be modified easily for calibration of TIADC using the model proposed in this paper.

The rest of this paper is organized as follows. Section 2 propose the system model based on Wiener model. In Section 3, the discrete-time equivalent model is derived. Section 4 gives the implementation of acquisition system using TIADC. Simulation and experimental results are given in Section 5. Finally, Section 6 concludes the paper.

## 2 Modeling of TIADC using Wiener model

An $M$ channel time-interleaved architecture is illustrated in Fig. 1. Each sub-ADC has the same sampling rate $f_s/M$ but with different clock phase. With interleaving, the overall system has a sampling rate of $f_s$, which enables the time-interleaved ADC to operate at $M$ times higher sampling rate than sub-ADC.

In this paper, we use Wiener model to describe the nonlinear dynamic property of TIADC. Wiener model is a dynamic nonlinear model which is composed of a dynamic linear system followed by a static (memoryless) nonlinearity, as shown in Fig. 2. The input-output relationship of the Wiener model is given by
\[ y(t) = \sum_{p=0}^{\infty} a_p \left[ \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau \right]^p \]  

where \( x(t) \) and \( y(t) \) are input and output respectively, \( h(t) \) is the impulse response of the linear system, and \( a_p \) denotes the parameters of the \( p \)th order nonlinearity.

An \( M \) channel TIADC using Wiener model is shown in Fig. 3. The output of \( m \)th channel is given as

\[ y_m(t) = \sum_{k=-\infty}^{\infty} \sum_{p=0}^{\infty} a_m \left[ \int_{-\infty}^{\infty} h_m(\tau)x(t-\tau)d\tau \right]^p \cdot s(t) \]  

with

\[ s(t) = \delta(t - mT_s - kMT_s) \]  

where \( T_s \) is the sampling rate of the overall TIADC system and \( h_m(\tau) \) denotes impulse response of \( m \)th channel \((m = 0, 1, \ldots, M - 1)\). The output of the TIADC is expressed as...
\[ y(t) = \sum_{m=0}^{M-1} y_m(t) \]  

(4)

Poisson summation formula is given as

\[ \sum_{n=-\infty}^{\infty} \delta(t - nT) = \frac{1}{T} \sum_{k=-\infty}^{\infty} e^{j2\pi kT} \]  

(5)

By replacing \( t \) with \( t - \frac{mT_s}{T} \) and \( T \) with \( MT_s \), (5) can be expressed as

\[ \sum_{k=-\infty}^{\infty} \delta(t - mT_s - kMT_s) = \frac{1}{MT_s} \sum_{k=-\infty}^{\infty} e^{j2\pi k(t - mT_s)/MT_s} \]  

(6)

Thus, output of TIADC can be reformulated as

\[ y(t) = \sum_{m=0}^{M-1} y_m(t) \]

\[ = \frac{1}{MT_s} \sum_{m=0}^{M-1} \sum_{k=-\infty}^{\infty} \sum_{p=0}^{\infty} a_{mp} \left[ \int_{-\infty}^{\infty} h_m(t)\chi(t - \tau)d\tau \right]^p e^{j2\pi k(t - mT_s)/MT_s} \]  

(7)

The frequency domain expression of TIADC output can be obtained by taking fourier transform of (7), which is given by

\[ Y(j\Omega) = \frac{1}{MT_s} \sum_{m=0}^{M-1} \sum_{k=-\infty}^{\infty} \sum_{p=0}^{\infty} a_{mp} \left[ H_m\left( j\left( \Omega - k\frac{2\pi}{MT_s} \right) \right) \chi\left( j\left( \Omega - k\frac{2\pi}{MT_s} \right) \right) \right]^p \cdot e^{j2\pi km/M} \]  

(8)

where \((*p)\) denotes the \( p \)-fold convolution [13].

By separating the zero and first order terms of (8), the relationship between proposed model and mismatch errors mentioned before can be clearly seen. (8) can be reformulated as

\[ Y(j\Omega) = \frac{1}{MT_s} \sum_{m=0}^{M-1} \sum_{k=-\infty}^{\infty} a_{m0} \delta \left( j\left( \Omega - k\frac{2\pi}{MT_s} \right) \right) e^{j2\pi km/M} \]

\[ + \frac{1}{MT_s} \sum_{m=0}^{M-1} \sum_{k=-\infty}^{\infty} a_{m1} H_m\left( j\left( \Omega - k\frac{2\pi}{MT_s} \right) \right) \chi\left( j\left( \Omega - k\frac{2\pi}{MT_s} \right) \right) e^{j2\pi km/M} \]

\[ + \frac{1}{MT_s} \sum_{m=0}^{M-1} \sum_{k=-\infty}^{\infty} \sum_{p=2}^{\infty} a_{mp} \left[ H_m\left( j\left( \Omega - k\frac{2\pi}{MT_s} \right) \right) \chi\left( j\left( \Omega - k\frac{2\pi}{MT_s} \right) \right) \right]^p \cdot e^{j2\pi km/M} \]  

(9)

As stated before, offset, gain, time, bandwidth and frequency mismatches are common mismatch errors of TIADC [4, 7]. It is easy to see \( a_{m0} \) is the offset of \( m \)th channel and \( H_m(j\Omega) \) is the frequency response of the \( m \)th channel. Gain, time, bandwidth mismatch errors are just a special case of frequency mismatches [7, 8]. The remaining terms are higher order effects introduced by nonlinear behavior. Compared with model in [4] and [7], the proposed model is a more general model, which can deal with both linear and nonlinear effects in TIADC.
3 Discrete-time equivalent model of TIADC

In section 2, we have modeled the TIADC based on Wiener model. The reason why we propose this model is that there are numerous literature focusing on compensating nonlinearity in Wiener model. These methods can be modified to compensate for nonlinearities in TIADC. However, there is a difficulty in making use of these methods because a fundamental difference exists between TIADC problem and general nonlinearity problem.

The discrete-time output of TIADC is obtained by taking uniform samples of (7) with sample interval $T_s$, which is given by

$$y(n) = \sum_{m=0}^{M-1} y_m(nT_s)$$

$$= \frac{1}{MT_s} \sum_{m=0}^{M-1} \sum_{k=0}^{M-1} \sum_{p=0}^{\infty} a_{mp} \left[ \int_{-\infty}^{\infty} h_m(t)x(nT_s - \tau) d\tau \right]^p e^{j2\pi k(n-m)/M}$$

(10)

In TIADC system, input signal $x(t)$ is a continuous-time signal while the output of the TIADC $y[n]$ is a discrete-time signal. Besides, as can be seen from (10), the time delay $\tau$ is also a continuous variable. Thus, the model is a hybrid system which is more difficult to handle than purely continuous-time or discrete-time systems.

In this section, we transform the hybrid system to a purely discrete-time system. In the discrete time system, only the discrete points are defined, while the interval between two points are not defined. Thus, if the time delay $\tau$ is not an integer multiples of the sampling period $T_s$, the continuous-time system model cannot be transformed to discrete-time system model directly. Therefore, we need to derive an equivalent discrete-time model to represent the TIADC output. We will get started from frequency domain representation to derive the equivalent discrete-time model.

If we assume input signal $x(t)$ is a band-limited signal, i.e.

$$X(j\Omega) = 0, \quad |\Omega| \geq B; \quad B \leq \frac{\pi}{T}$$

(11)

then we have

$$X(e^{j\omega}) = \frac{1}{T_s} X(j\Omega), \quad \omega \in [-\pi, \pi]$$

(12)

where

$$\omega = \Omega T_s$$

(13)

Discrete-time fourier transform of $y[n]$ can be obtained from (8),

$$Y(e^{j\omega}) = \frac{1}{M} \sum_{m=0}^{M-1} \sum_{k=0}^{M-1} \sum_{p=0}^{\infty} a_{mp} [H_m(e^{j(\omega-k\Omega)/T}) X(e^{j(\omega-k\Omega)/T})]^p$$

$$\times e^{j2\pi km/M}$$

(14)

where

$$H_m(e^{j\omega}) = H_m\left(j \frac{\omega}{T}\right), \quad \omega \in [-\pi, \pi]$$

(15)

By transforming (14) to time-domain, we can get
where \( \tilde{h}_m(n) \) is the discrete-time equivalent system’s impulse response and satisfies

\[
\tilde{h}_m(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_m(e^{j\omega})e^{j\omega n} d\omega.
\]

(17)

The reason why we call it equivalent model is \( \tilde{h}_m(n) \) is not equal to \( h_m(nT_s) \). To make it clear, we use an example to explain the above statement. Supposing frequency response of channel \( m \) is \( H_m(e^{j\omega}) = e^{-j\omega d T_s} \), where \( d \) is not an integer number. Then, based on inverse continuous-time fourier transform, \( h_m(t) = \delta(t - d T_s) \). According to [15], for noninteger values of \( d \), the discrete-time impulse response of system is an infinitely long, shifted and sampled version of the sinc function. Thus the interval of \( \tilde{h}_m(n) \) is expanded to \([-\infty, \infty]\) and \( \tilde{h}_m(n) \) is not just a sampled output of \( h_m(t) \).

In practice, we can only handle system with finite memory length and finite order. Thus, we use the discrete-time equivalent model to represent the input-output relationship of TIADC, which is given by

\[
y(n) = \sum_{m=0}^{M-1} y_m(n) = \frac{1}{M} \sum_{m=0}^{M-1} \sum_{k=0}^{M-1} \sum_{p=0}^{P} a_{mp} \left[ \sum_{l=-\infty}^{\infty} \tilde{h}_m(l)x(n-l) \right]^p e^{j2\pi k(n+m)/M}
\]

(16)

The discrete-time equivalent model is useful for the analysis and implementation of subsequent calibration method by making it a purely discrete time system. The block diagram of discrete-time equivalent model is shown in Fig. 4.

\[\text{Fig. 4. Block diagram of discrete-time equivalent model.}\]

The reason why we can build the equivalent model of TIADC is based on two cornerstones. One is the assumption of fading memory, which is described clearly in [16] and is a very useful assumption in practical situation. The other is Stone-Weierstrass theorem. According to Stone-Weierstrass theorem, every continuous functions defined on a closed interval \([a, b]\) can be uniformly approximated as
closely as desired by a polynomial function. Thus, the differences between real system and discrete-time equivalent model can be made arbitrarily small.

4 Implementation

To demonstrate validity of the proposed model, an acquisition system with sampling rate of 5GSPS is implemented based on a commercial TIADC EV8AQ160 of E2V company [17]. EV8AQ160 has four 8-bit ADC cores and each core has sampling rate of 1.25GSPS. By using the time-interleaved structure, the aggregate sampling rate can reach 5GSPS. The sampling clock phase of each core has difference of 90°.

Fig. 5 and Fig. 6 shows the simplified block diagram of the acquisition system and the photograph of the circuit board, respectively. The main modules of the acquisition system consist of TIADC, FPGA (Field Programmable Gate Array), DSP (digital signal processor), etc. The analog input signal passes through the analog cross point switch firstly, which is an analog input selector depending on the mode of ADC. Then the analog signals go through track-and-hold (T/H) stage and sampled by the corresponding ADC core. After that, the digital data are received, stored and processed by FPGA. In the implementation, ADC is configured as 1:2 DMUX and IDDR Transfer Mode to ensure the FPGA receives data accurately. The DSP is used for controlling, data processing and further implementation of non-linear calibration method.

Fig. 5. Simplified block diagram of 5GSPS acquisition system.

Fig. 6. Circuit board of the acquisition system.
In practical implementation, it is difficult for signal generator to generate clean single-tone sinusoid due to nonlinearity of the DAC (Digital-to-Analog Converter), which will influence the analysis and subsequent calibration of TIADC nonlinearity mismatch. Thus, the input signal should be filtered by analog filters before sending to the acquisition system.

5 Experiment and simulation results

In this section, we will show the validity of the proposed model by experiment and simulation results.

5.1 Experiment results

To evaluate proposed model, we use a sinusoidal signal as the input of the acquisition system. Assuming input signal is $A \cos(\Omega_{\text{in}} t)$. According to the derivation in section 2, there will be some additional spurious tones apart from original sinusoidal signal due to the distortion introduced by TIADC.

We can get the location of these spurious tones from frequency domain representation of (9). For offset mismatch, the spurious tones are located at $k\Omega_s/M (k = 0, \ldots, M - 1)$. For linear mismatches, the spurious tones appear at $k\Omega_s/M \pm \Omega_{\text{in}} (k = 0, \ldots, M - 1)$. For nonlinear mismatches, the situation becomes more complex, and we will illustrate it in detail.

According to Euler’s formula, the input can be represented as

$$A \cos(\Omega_{\text{in}} t) = \frac{A}{2} (e^{i\Omega_{\text{in}} t} + e^{-i\Omega_{\text{in}} t})$$

(19)

Since the Binomial theorem provides the solution to expand polynomial $(x + y)^n$, which are given by

$$(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k$$

(20)

Then the $p$th order nonlinearity of input signal is given by

$$\left[ \frac{A}{2} (e^{i\Omega_{\text{in}} t} + e^{-i\Omega_{\text{in}} t}) \right]^p = \left( \frac{A}{2} \right)^p \sum_{l=0}^{p} \binom{p}{l} e^{i(p-l)\Omega_{\text{in}} t} e^{-i(l)\Omega_{\text{in}} t}$$

$$= \left( \frac{A}{2} \right)^p \sum_{l=0}^{p} \binom{p}{l} e^{i(p-2l)\Omega_{\text{in}} t}$$

(21)

(21) shows that $p$th order nonlinearity generates tones not only at $\pm p\Omega_{\text{in}}$ but also at $\pm (p-2l)\Omega_{\text{in}} (l = 1, \ldots, \lfloor \frac{p}{2} \rfloor)$.

Thus, the spurious tones generated by $p$th order nonlinear mismatches are located at $k\Omega_s/M \pm (p-2l)\Omega_{\text{in}} (k = 0, \ldots, M - 1, l = 0, \ldots, \lfloor \frac{p}{2} \rfloor)$. Higher order nonlinearities superpose onto the same frequency bins as lower order nonlinearities which makes nonlinear mismatches more complex.

The output spectrum of TIADC with 790 MHz sinusoidal input signal is shown in Fig. 7. It can be seen that there are plenty of spurious tones in the output spectrum. The offset mismatch errors are located at 1250 MHz and 2500 MHz, which have the magnitude of $-48.77$ dBc and $-50.8$ dBc respectively. The linear mismatch errors are located at 460 MHz, 1710 MHz and 2040 MHz, which have the
magnitude of $-46.54 \text{ dBc}$, $-46.8 \text{ dBc}$ and $-42.69 \text{ dBc}$ respectively. Apart from these two mismatch errors extensively discussed by previous papers, there are still several spurious tones significantly higher than the noise floor which are caused by nonlinearity. The second and third nonlinear distortions are significant which have magnitude of $-50.87$ and $-47.51 \text{ dBc}$ respectively. The rest of spurious tones are all caused by nonlinear mismatch errors among sub-ADCs. For example, the spurious tones located at $920 \text{ MHz}$ and $130 \text{ MHz}$ are caused by second and third order nonlinear mismatches respectively.

As can be seen from the experiment results, the location of all spurious tones in the output spectrum are consistent with our proposed model, which shows the validity of the model. What’s more, from the experiment results, it can be clearly seen that nonlinear distortions are significant in TIADC thus should be eliminated.

5.2 Simulation results

Next, we performed the simulation in MATLAB on a two-channel TIADC system to verify the effectiveness of discrete-time equivalent model.

For numerical simulation with MATLAB, we assume channel frequency response as [10]

$$H_m(j\Omega) = \frac{e^{j\Omega T_{r_m}}}{1 + j\frac{\Omega}{\Omega_c}(1 + \Delta_m)} \tag{22}$$

where $\Omega_c$ is 3-dB cutoff frequency of first order filter, $r_m$ and $\Delta_m$ denote relative time offset and normalized frequency offset of mth channel respectively. We set $r_m = [0,0.02]$ and $\Delta_m = [0, -0.02]$. In the simulation, we introduce nonlinearity up to 4th order. The nonlinear parameters in (9) and (14) are set as $a_{m0} = [-0.02, 0.03]$, $a_{m1} = [1.02, 0.96]$, $a_{m2} = [0.02, -0.01]$, $a_{m3} = [0.02, 0.03]$ and $a_{m4} = [0.01, 0.005]$. The sampling rate of the TIADC system is set as 1 GHz.

Then we use the expression in (17) to obtain the discrete-time equivalent system’s impulse response. As discussed in section 4, we can only handle system with finite memory length. Thus, there is a trade-off between model complexity and accuracy.
We applied a sinusoidal input to the hybrid model and discrete-time equivalent model. The frequency of the input signal is 150 MHz, and the amplitude of it is set to 1. The output of hybrid model and discrete-time equivalent model are represented as $y_{hyb}(n)$ and $y_{dis}(n)$ respectively. The mean square error (MSE) between $y_{hyb}(n)$ and $y_{dis}(n)$ is defined by

$$MSE = \frac{1}{N} \sum_{i=1}^{n} (y_{hyb}(i) - y_{dis}(i))^2$$

where $N$ is the number of output used for calculation of MSE.

Table I shows the MSE versus order of discrete-time equivalent system’s impulse response $\tilde{h}_m(n)$. It can be seen that as order of $\tilde{h}_m(n)$ increases, the MSE decreases with it. In this example, the MSE is less than 0.05% when the order of $\tilde{h}_m(n)$ is beyond 4, which is sufficient for TIADC applications. Then, we can make use of the obtained discrete-time equivalent model for further calibration method.

### Table I. The MSE between hybrid model and discrete-time equivalent model versus order of $\tilde{h}_m(n)$

| Order of $\tilde{h}_m(n)$ | 0   | 1   | 2   | 3   | 4   |
|---------------------------|-----|-----|-----|-----|-----|
| MSE                       | 0.5462 | 0.0076 | 0.0037 | 0.0008 | 0.0005 |

6 Conclusion

In this paper, we develops a mathematical model based on Wiener model to represent nonlinearities in TIADC. We gives time-domain and frequency-domain representations of the nonlinear TIADC model. What’s more, we propose the discrete-time equivalent model of TIADC which is useful for analysis and implementation of subsequent calibration method. The experiment results show the nonlinearities of TIADC in practice are consistent with the model proposed in this paper and simulation results indicates the validity of the proposed discrete-time equivalent model. Besides, from the experiment results, it can be clearly seen that the nonlinear distortions are significant in TIADC, thus the future work of our study will be the calibration method based on the proposed discrete-time equivalent model.

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