Pixellated Lenses and Estimates of $H_0$ from Time-delay Quasars

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Abstract. The largest source of uncertainty in the determination of $H_0$ from a multiply-imaged QSO system is the unknown mass distribution in the lensing galaxy. Parametric models severely restrict the shape of the galaxy thereby underestimating errors and biasing the best estimate of $H_0$. We present a method that explores the whole of the model space allowed by the image observables and a few general properties of galaxies. We describe blind tests of the method and then apply it to PG1115+080 and B1608+656 which yield that $H_0$ is between 45 and 80 km s$^{-1}$ Mpc$^{-1}$ at 90% confidence level, with the best estimate of 60 km s$^{-1}$ Mpc$^{-1}$.

1. Introduction

It has been 35 years since Refsdal (1964) proposed an elegant method to derive the Hubble constant, $H_0$, from a multiply-imaged QSO system. Until a few years ago the main obstacle to implementing the method was the lack of sufficiently accurate data on the image positions and time delays between images. As the precision of the observational measurements improves the errors in $H_0$ become dominated by the uncertainties in the galaxy mass distribution. These errors are very hard to quantify using parametric shape(s) for the galaxy lens model; the derived errors will tend to be underestimated as was noted by Bernstein and Fischer (1999) who constructed many types of parametric models for Q0957+561: ‘The bounds on $H_0$ are strongly dependent on our assumptions about a “reasonable” galaxy profile.’ In this contribution we develop and apply a non-parametric method for modeling lensing galaxies and thus estimating $H_0$ with the errorbars derived entirely from the uncertainties in the galaxy mass map. The method was initially applied to reconstructing mass maps of lensing galaxies (Saha & Williams 1997); in this volume, Saha et al. show how it can be extended to recover mass distribution in galaxy clusters.

2. The Method

We start by tiling the lens plane with $\sim 25^2$ independent mass pixels each $\sim 0.1''$ on the side. Then we pixellate the lens equation and the arrival time surface.
Figure 1. Probability distribution of $h$ derived using four synthetic lens systems. Dashed line: inversion symmetry applied to all 4 galaxies, dotted line: inversion symmetry constraint not used, solid line: inversion symmetry used where modeler deemed it appropriate.

The image observables, which we take to be exact, enter as primary modeling constraints. The secondary constraints pertain to the main lensing galaxy:

1. mass pixel values, $\kappa_n$ must be non-negative;
2. location of the galaxy center is assumed to be coincident with that of the optical/IR image;
3. the gradient of the projected mass density of the galaxy must point away from galaxy center to within a tolerance angle of $\pm 45^\circ$;
4. inversion symmetry, i.e. a galaxy must look the same if rotated by $180^\circ$ (applied only if galaxy optical/IR image looks symmetric);
5. logarithmic projected density gradient in the image region, $\frac{d \log \kappa}{d \log r} = \text{ind}(r)$ should not be any shallower that -0.5;
6. external shear, i.e. influence of mass other than the main lensing galaxy is restricted to be constant across the image region, i.e. it is represented by adding a term $\frac{1}{2} \gamma_1 (\theta_1^2 - \theta_2^2) + \gamma_2 (\theta_1 \theta_2)$ to the lensing potential.

After both primary and secondary constraints have been applied, we are left with a large number of viable galaxy models. We then generate a fair sample of the remaining model space by random walking through it until we accumulate 100 mass maps. Each of these galaxies reproduce the image properties exactly and each looks reasonably like a real galaxy. The distribution of the 100 corresponding $H_0$’s is therefore the derived probability distribution $p(h)$. The width of the distribution indicates the uncertainty arising from our lack of sufficient knowledge about the lensing galaxy. (See Williams & Saha 1999 for details.)

3. Blind tests

First we apply the method to a synthetic sample of lenses. One of us constructed 4 sets of QSO-galaxy four-image lenses, some including external shear, and con-
veyed the image position and time delay ratio information only to the other one of us, the modeler. In each case the modeler decided, based on the appearance of the reconstructed galaxy mass maps whether inversion symmetry was appropriate or not. The modeler was told the correct answer for $h$, which is 0.025 in our synthetic universe, after the final $p(h)$ distributions were obtained.

Distributions from the four synthetic lenses were multiplied together to yield the combined estimated $p(h)$ shown in Figure 1 as the solid line. The dashed histogram is for the case where inversion symmetry was imposed on all four galaxies, and the dotted histogram represents the case where inversion symmetry was not applied in any of the systems. All three resultant distributions recover $h$ fairly well, with the 90% of the models contained within 10% of the true $h = 0.025$. However the distributions are not the same; the most probable values are different by $\sim 10\%$. This illustrates how a relatively minor feature in modeling constraints, namely inclusion or exclusion of inversion symmetry, can make a considerable difference in the estimated $h$ value when the goal is to achieve precision of better than 10%. Based on this observation we conclude that the assumed galaxy shape in parametric reconstructions plays a major role in determining the outcome of $H_0$.

4. PG1115+080 and B1608+656

We now apply the method to PG1115 and B1608, both of which have accurate image position and time delay measurements (Schechter et al. 1997, Barkana 1997, Myers et al. 1995, Fassnacht et al. 1999). Figure 2 shows an ensemble average of 100 reconstructed mass maps for PG1115. Note that the density contours are smooth and roughly elliptical. Figure 3 is a plot of the double logarithmic slope of the projected density profile in the vicinity of the images vs. the derived estimate for $H_0$ for each of the 100 reconstructed galaxies. Since
Figure 3. The slope of the projected density profile in the image region vs. the derived $H_0$ for each of the 100 reconstructed galaxies for the case of PG1115. Isothermal slope is -1.

Figure 4. Combined probability distribution based on the lensing data from PG1115 and B1608. The anticorrelation is well defined and is understood in terms of the arrival time surface, it can potentially be used as an additional modeling constraint. The combined $p(h)$ distribution based on the lensing data of PG1115 and B1608 is presented in Figure 4. The median is at about 60 km s$^{-1}$ Mpc$^{-1}$, and the 90% confidence range extends from 45 to 80 km s$^{-1}$ Mpc$^{-1}$.

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