Building a model for calculating stress under dynamic loading of metal rings by the magnetic pulse method

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Abstract. Many recent studies have focused on specifics of behavior of materials under dynamic loading as it is considerably different from cases with quasi-static loading. This paper considers the method of magnetic pulse deformation of metal rings, which is a continuation of the authors’ research. [1]. The methods used in the experiment, which are described in this paper, allow to carry out laboratory studies into material deformation at a speed reaching the values of around $10^7$ c$^{-1}$. This paper is dedicated to building a mathematical model for determining the stress in the test samples under dynamic loading of materials by the magnetic pulse method.

1. The magnetic pulse method

The electrical loading diagram by the magnetic pulse method is shown in Figure 1. The metal ring sample is coaxially placed on the inductance coil. The repulsive force occurring between the solenoid and the ring causes deformation of the latter.

![Diagram of the unit implementing sinusoidal electromagnetic loading](image)

**Figure 1.** The diagram of the unit implementing sinusoidal electromagnetic loading with a period of 5.5-7 ms.

Where $R_{ch}$ is charging resistance; $AT$ is autotransformer; $REC$ is rectifier; $C$ is condenser; $S$ is spark gap; $L$ is coil (coreless solenoid); Sample is a metal ring; $RC$ is Rogowski coil; $PD$ is photodiode; $OSC$ is oscillograph.
In the course of the research [2] it was found out that oscillations occur in the stress profile (Figure 2). Their origin is attributed to the fact that under harmonic loading of the ring samples during the transition from quasi-static to dynamic deformation the oscillatory nature of the circumferential stress profile maintains. Under high-speed dynamic loading [3] it is already a relatively smooth function.

![Figure 2. Experimentally determined stress profiles. a) copper sample, b) aluminium sample.](image)

It was established that this type of profile cannot be described as part of fragile fracture; for this reason, it is necessary to consider the processes related to viscous fracture. Consequently, the mathematical model for describing the behavior of the material will be presented from the point of view of dislocation dynamics.

2. Stress calculation model

Previously, [4] the thin ring motion equation was as follows:

$$\rho \left( \frac{1}{2R} \left( \frac{dR}{dt} \right)^2 + \frac{d^2R}{dt^2} \right) + \frac{\sigma}{R} = \frac{q(t)}{h},$$

where $R$ is ring radius; $\rho$ is ring material density, $q(t)$ is loading impacting the inner surface of the ring; $\sigma$ stress occurring in the sample, $h$ is thickness of the sample in question.

Radial loading on the inner surface of the ring sample was measured in the course of the experiment by way of using the piezoelectric sensor, as shown in the diagram (Figure 3).

![Figure 3. Pressure measurement diagram.](image)

Equation (1) was used to obtain differential equation to calculate circumferential stress $\sigma(t)$ in linear approximation using Hooke’s law. However, this model did not properly reflect the behavior of the material. In this paper, the ring motion equation (1) is supplemented with Sokolovsky-Malvern equation, where the difference between the speed of the stress change and deformation is different from the zero function, which, in its turn, is proportionate to shift plastic deformation:
\[ \frac{\partial \sigma}{\partial t} - \rho c^2 \frac{\partial \varepsilon}{\partial t} = g(\sigma, \varepsilon), \] (2)

where \( \varepsilon \) is deformation, \( c \) is speed sound, \( t \) is time, \( g(\sigma, \varepsilon) \) is relaxation function.

According to the equation [5] relaxation function can be presented as:

\[ g(\sigma, \varepsilon) = -\frac{8}{3} \mu b N_m(\tau, \gamma)v_d(\tau, \gamma), \] (3)

where \( \mu \) is dislocation shift magnitude, \( b \) is the magnitude of the Burgers vector, \( N_m(\tau, \gamma) \) is mobile dislocation density, \( v_d(\tau, \gamma) \) is their speed, \( \gamma \) is dislocation plastic deformation, \( \tau \) dislocation stress.

Then the stress equation \( \sigma \) can be presented as:

\[ \frac{\rho R}{E} \left[ \frac{d^2 \sigma}{dt^2} + \frac{1}{2E} \left( \frac{d\sigma}{dt} \right)^2 - \frac{1}{2E} \frac{d\sigma}{dt} g(\sigma, \varepsilon) + \frac{1}{2E} g^2(\sigma, \varepsilon) + \frac{dg(\sigma, \varepsilon)}{dt} \right] + \frac{\sigma}{R} = q(t) \frac{h}{\tau}, \] (4)

where \( E \) is Young’s modulus.

Let’s consider three most characteristic correlations between the speed of mobile dislocations and their density as functions of stress and deformation.

In case of small deformations, there is a linear correlation between the density of mobile dislocations and shift plastic deformation:

\[ N_m = N_0 + \alpha \gamma, \] (5)

where \( \alpha \) is coefficient of dislocation multiplication, \( \gamma \) is shift plastic deformation, \( N_0 \) is initial dislocation density, \( N_m \) is density of mobile dislocations.

In this case we talk about viscous drag on dislocations, i.e. correlation between the speed of dislocations and shift stress is linear:

\[ v_d = \frac{(\tau - \tau_0)b}{B}, \] (6)

where \( \tau \) is shift stress, \( \tau_0 \) is characteristic stress, \( b \) is value of the Burgers vector, \( B \) is coefficient of drag on dislocations, \( v_d \) is speed of dislocations.

In case when part of dislocations becomes sessile, i.e. when some part of the total density of dislocations remains immobile, Gilman’s form of the dislocation multiplication law can be used [5]:

\[ N_m = (N_0 + \alpha \gamma) \exp \left( -\frac{H \gamma}{\tau} \right), \] (7)

where \( H \) is coefficient of material hardening.

The speed of dislocations for this case can be determined as follows:

\[ v_d = v_{dm} \exp \left( -\frac{\tau_0}{\tau} \right), \] (8)

as \( v_{dm} \) is the maximum possible value of the speed of dislocations; \( \tau_0 \) is characteristic drag strain.

Under dynamic loading, it is not so much the value of the full plastic deformation as its speed that influences material hardening. Therefore, the dislocation multiplication law can be as follows:

\[ N_m = (N_0 + \alpha \gamma) \exp \left( -\frac{H_1 \gamma'}{\tau_0} \right), \] (9)

where \( \gamma' \) is the speed of dislocation plastic deformation, while \( H_1 \) can be defined on the basis of the following equation:

\[ H_1 = H \tau \phi, \] (10)
where $\tau_f$ is elastic precursor front duration.

In this case, equation (8) can be used for calculating the speed of dislocation motion.

Let’s consider a case of loading using the loading of a thin aluminum ring as an example. We are going to use numerical solution of the equation (4), where relaxation function $g(\sigma, \varepsilon)$ from equation (3) is represented by using equations (8) and (9).

Calculation results are shown in Figure 4. From Figure 4 it is obvious that equation (4) solution using the dislocation multiplication law in the above-described form (9) demonstrates the oscillatory nature of the stress profile. Such profile behavior can be attributed to the fact that dislocation motion contributes to lowering the stress which, in its turn, slows them down. Growing stresses during dislocation locking result to their subsequent acceleration.

The calculated stress profile (Figure 4) unambiguously reflects the experimentally measured profile (Figure 2). Thus, on the basis of the review of the results of many experiments and experimental modifications, which we developed for stress, deformation and fracture of thin metal ring samples in a wide range of deformation speed using the magnetic pulse method, mathematical models for calculating circumferential stress suggested earlier have been improved. The developed model is presented from the point of view of the dynamics of dislocations. Three most characteristic cases of correlation of the speed of mobile dislocations and their density with stress and deformation were considered. The developed model for the most complex transitional stress mode from quasi-static to dynamic unambiguously describes experimentally measured stress profiles where the oscillatory nature of stress still maintains.

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References

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