AN UNCERTAIN PROGRAMMING MODEL FOR SINGLE
MACHINE SCHEDULING PROBLEM WITH BATCH DELIVERY

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(Communicated by Changzhi Wu)

Abstract. A single machine scheduling problem with batch delivery is studied in this paper. The objective is to minimize the total cost which comprises earliness penalties, tardiness penalties, holding and transportation costs. An integer programming model is proposed and two dominance properties are obtained. However, sometimes due to the lack of historical data, the worker evaluates the processing time of a job according to his past experience. A pessimistic value model of the single machine scheduling problem with batch delivery under an uncertain environment is presented. Since the objective function is non-monotonic with respect to uncertain variables, a hybrid algorithm based on uncertain simulation and a genetic algorithm (GA) is designed to solve the model. In addition, two dominance properties under the uncertain environment are also obtained. Finally, computational experiments are presented to illustrate the modeling idea and the effectiveness of the algorithm.

1. Introduction. Scheduling problems including both earliness and tardiness penalties are getting more and more attention due to the importance of just-in-time (JIT) scheduling. In the past few decades, scheduling problems with batch delivery, which play a significant role in supply chain management have received a lot of attention. Mason and Anderson \[25\] considered a scheduling problem with batch delivery of minimizing the weighted flow time and many dominance properties of the feasible solution are derived. A branch-and-bound procedure and a simpler depth-first search was proposed for solving this problem. A single machine sequencing problem with batch delivery was analyzed by Zdrzalka \[47\] and the objective was to minimize the delivery time of all jobs. Three efficient approximation algorithms were described and analyzed. Cheng et al.\[3\] investigated the single machine scheduling with batch delivery and the objective was to minimize the sum of delivery costs and job earliness penalties. They developed polynomial time algorithms for different cases. Hall et al.\[9\] investigated a variety of scheduling problems which a job is dispatched to a customer at the earliest fixed delivery date. They showed that such an algorithm is unlikely to exist for different scenarios. Hall and Potts \[10\] considered many batch delivery scheduling problems in an arborescent supply chain. Several dynamic programming algorithms were provided. Hall and Potts \[11\] considered several scheduling problems where deliveries

2010 Mathematics Subject Classification. Primary: 90B99; Secondary: 90C10.

Key words and phrases. Single machine scheduling, batch delivery, uncertain environment, genetic algorithm, uncertain simulation.

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are made in batches and each batch was delivered to a customer in a single shipment. They proposed several efficient algorithms. Yang [40] studied some single machine scheduling problems with generalized batch delivery dates and earliness penalties. They showed that these problems can be solved in polynomial time if all processing times are equal and an $O(n\log(n))$ algorithm for the weighted earliness case was given. Qi [28] addressed a new scheduling model for a firm with an option of outsourcing. They discussed four objective functions and solved them by dynamic programming algorithms. Wang and Cheng [36] studied a logistics scheduling problem and the objective was to minimize the sum of work-in-process inventory and transport costs. Some polynomial-time algorithms to solve several special cases were developed. Wang and Cheng [37] proposed a scheduling model which contains production scheduling, material supply, and product delivery and some heuristics for some special cases were proposed. Soukhal et al. [32] considered some two-machine flow shop scheduling problems with transportation constraints and analyzed complexity of some cases. Stecke and Zhao [33] studied various scenarios of the production and transportation problem with a commit-to-delivery mode of business. An efficient polynomial time heuristic algorithm was provided for the NP-hard problem. They also provided models and analyzed other scenarios where shipping cost accounts for customer locations and quantity discounts. Steiner and Zhang [34] investigated a supply chain scheduling problem in which jobs have to be scheduled on a single machine and delivered to customers in batches. They presented a pseudo-polynomial algorithm for a restricted case and made a parametric analysis for its performance ratio. Cakici et al. [1] presented a model of minimizing the total weighted tardiness and total distribution costs in an integrated production and distribution environment. Different heuristics based on GA were developed. Fu et al. [6] considered a production and delivery scheduling problem and developed two polynomial-time approximation schemes for single delivery case and multiple delivery times case, respectively. A single-machine scheduling with batch delivery and controllable processing times was studied by Yin et al. [42]. Some properties of the optimal schedule were obtained and an $O(n^5)$ dynamic programming algorithms was proposed for the problem. Some other types of single-machine scheduling problems with batch delivery were also studied by Yin et al. [43, 44, 45, 46]. They conducted theoretical analysis of various problems and many effective dynamic programming algorithms were proposed.

In recent years, some scholars began to study scheduling problems under random environments. For example, Wu and Zhou [38] studied a single machine scheduling with random due dates. Three highly accurate and efficient heuristic policies were proposed. Framinan and Gonzalez [5] investigated expected makespan of a permutation flow shop scheduling problem with stochastic processing time and they proposed a procedure with a variable number of iterations. Karimi et al. [18] considered a flexible job-shop scheduling problem with job transportation time and formulated the problem by two mixed integer linear programming models. An imperialist competitive algorithm mixed with a simulated annealing-based local search was proposed and the computational results showed that the proposed algorithm outperforms two other existing competitive algorithms. Hao et al. [15] considered a job shop scheduling problem with stochastic processing time. An effective algorithm for minimizing the expected average makespan within a reasonable amount of calculation time was proposed.
However, it’s not reasonable to use probability theory to solve all the scheduling problems. It is well known that the estimated probability distribution is close enough to the long-run cumulative frequency. However, in many cases, no samples are available to estimate the probability distribution. For example, a lot of papers assumed that the processing time is a random variable. A worker can only estimated the processing time based on his past experience. Therefore, the processing time of a job has something to do with the experience of a worker. Due to the absence of data for these variables, we have to invite some domain experts to evaluate the belief degree that each event will happen. In order to deal with the involved human uncertainty, uncertainty theory was founded by Liu [21] in 2007 and refined it in 2010 [22]. Uncertainty theory is a branch of axiomatic mathematics for modeling human uncertainty, which has been deeply developed in many fields such as uncertain programming, uncertain risk analysis, and uncertain uncertain calculus. Liu and Yao [23] proposed an uncertain multilevel programming method for modeling uncertain decentralized planning problem with multiple decision makers in a hierarchical system. Besides, a GA was employed to solve the model. As an illustration, the uncertain multilevel programming model was applied to a product control problem. Ning et al. [26] studied a parallel-machine scheduling problem with uncertain processing time and release date. They proposed an uncertain multiple objectives programming model and an effective GA for the problem. Ke et al. [19] studied a hybrid multilevel programming problem with uncertain random parameters based on expected value dependent-chance. They proposed an approach integrates with uncertain random simulation, Nash equilibrium searching approach and GA. Han et al. [14] used a numerical method to solve the uncertain maximum flow problem. They used the 99-method to obtain the uncertainty distribution and the expected value of the maximum flow of uncertain network. Gao [7] gave the uncertainty distribution of the shortest path length. Gao et al. [8] studied the frequency service network design problem in a railway freight transportation system and they proposed a budget-constrained model and a possibility-constrained model for freight transportation system. A linear-quadratic control problem for discrete time switched systems under uncertainty was studied by Yan et al. [39] and an effective two-step pruning scheme was proposed. Cui and Sheng [4] proposed an expected constrained programming model for an uncertain solid transportation problem. This model can be transformed to deterministic form according to inverse uncertainty distribution method. Jiang [16] considered an uncertain chance constrained programming model for empty container allocation problem. Shen and Zhu [30] studied an uncertain two-stage supply chain scheduling problem. A pessimistic value model was established at the scheduling stage and a batch delivery method at the transportation stage was introduced. Chen and Zhu [2] considered a linear quadratic optimal control with process state inequality constraints in discrete-time uncertain systems and a necessary condition for the existence of optimal state feedback control was proposed. Rong [29] proposed two new models of economic order quantity for inventory problem with uncertain cost. Li and Peng [20] presented a new approach to risk comparison in an uncertain environment. Yao [41] presented an uncertain integral of a matrix of uncertain processes with respect to multidimensional Liu process [24].

The batch delivery system decreases the delivery costs while increasing the earliness/tardiness penalties and inventory costs. This paper attempts to establish a balance between these two factors while keeping the smallest possible sum by proper
scheduling. A single machine scheduling problem with job delivery is considered in this paper. It is an extension of [13] by taking into consideration due date windows and uncertainties. Some hypotheses in [13] are limitations: continuous batch processing, non-delay scheduling, all parameters are deterministic. Due to the order adjustment, continuous batch processing may be interrupted. Sometimes, forming a delay scheduling may be beneficial to the objective function. In reality, due to the uncertainty of human and environment, these factors must be taken into account. Precise processing times are difficult to obtain due to many uncertainties such as machine breakdown, raw material differences, tool wear, external environment, etc. For example, in the process of wood cutting, due to the different varieties of timber, poor machine adjustment and no properly installed cutters, workers will estimate the processing time according to the actual situation. Firstly, an integer programming model is proposed in this paper. To speed up the convergence rate of the heuristic, two dominance properties of the optimal solution are proposed. Secondly, a pessimistic value model under an uncertain environment is proposed. Since the objective function is non-monotonic, the pessimistic value model can’t be converted into its deterministic form by the inverse distribution method [22]. The problem is solved successfully by means of uncertain simulation. In addition, two dominance properties under an uncertain environment are obtained.

The rest of the paper is organized as follows. In Section 2, the problem is described and two dominance properties of the feasible solution are proposed. In Section 3, some basic definitions about uncertainty theory are introduced. In Section 4, the problem is formulated under an uncertain environment. In Section 5, the proposed solution method is described. Section 6 gives some numerical examples.

2. Problem formulation. To establish the integer programming model, the problem is described at first. There are $n$ jobs that have to be processed on a machine. The machine is always available and only can process a job at one time. Preemption is not allowed. All of the jobs are available at the begin. These are $K$ customers and customer $l$ has $n_l$ jobs, $n = \sum_l n_l$. Each job has a processing time $p_i$ and a due date window $[d_{si}^l, d_{ti}^l]$. If the job $i$ is shipped to the customer before $d_{si}^l$, an earliness penalty is generated. If the job $i$ is shipped to the customer after $d_{ti}^l$, a tardiness penalty is generated. If the job has not been sent after the process is finished, it will generate a holding penalty. The unit earliness penalty cost, the unit tardiness penalty cost and the unit holding cost are denoted by $\alpha_i$, $\beta_i$ and $h_i$, respectively. The transportation cost of each batch of customer $l$ is denoted by $D_l$. Assume that there is no capacity limit in each batch delivery. Because a truck can carry at least a job at one shipment. It is reasonable that the number of jobs may equal to the number of transportation batch. The delivery time of a batch is equal to the completion time of the last job in the batch. The goal is to minimize the sum of earliness, tardiness, holding and transportation costs. It is known that the classic single-machine scheduling problem is NP-hard in the ordinary sense. Then the problem in this paper is significantly more complicated and so it is strongly NP-hard too.

Before the problem be formulated, some notations are introduced in Table 1.

Decision variables:

$x_{ij}$ : 1, if job $i$ is processed on machine at the $j$th position,
0, otherwise. $i = 1, 2, \ldots, n; j = 1, 2, \ldots$
Table 1. List of notations.

| notations | definitions |
|-----------|-------------|
| \(i = 1, 2, \ldots, n\) | the index of job |
| \(j = 1, 2, \ldots\) | the index of position |
| \(b = 1, 2, \ldots\) | the index of batch |
| \(l = 1, 2, \ldots, K\) | the index of customer |
| \(n_l\) | the number of jobs of customer \(l\), \(l = 1, 2, \ldots, K\) |
| \(p_i\) | processing time of job \(i\), \(i = 1, 2, \ldots, n\) |
| \([d_{is}^l, d_{it}^l]\) | due window of job \(i\), \(i = 1, 2, \ldots, n\) |
| \(\alpha_i\) | unit earliness penalty cost of job \(i\), \(i = 1, 2, \ldots, n\) |
| \(\beta_i\) | unit tardiness penalty cost of job \(i\), \(i = 1, 2, \ldots, n\) |
| \(h_i\) | unit holding cost of job \(i\), \(i = 1, 2, \ldots, n\) |
| \(D_l\) | transportation cost of customer \(l\), \(l = 1, 2, \ldots, K\) |
| \(C_i\) | completion time of job \(i\), \(i = 1, 2, \ldots, n\) |
| \(T_j\) | completion time of the \(j\)th job on machine, \(j = 1, 2, \ldots\) |
| \(R_i\) | delivery date of job \(i\), \(i = 1, 2, \ldots, n\) |
| \(R'_{lb}\) | delivery date of the \(b\)th batch of customer \(l\), \(l = 1, 2, \ldots, K; b = 1, 2, \ldots\) |

\(y_{ilb} : 1\), if job \(i\) is delivered in the \(b\) batch of customer \(l\),

\(0\), otherwise. \(i = 1, 2, \ldots, n; l = 1, 2, \ldots, K; b = 1, 2, \ldots\)

Objective function:

\[
f = \sum_{i=1}^{n} \alpha_i \cdot \max\{0, d_{is}^l - R_i\} + \sum_{i=1}^{n} \beta_i \cdot \max\{0, R_i - d_{it}^l\} + \sum_{i=1}^{n} h_i \cdot (R_i - C_i) + \sum_{l=1}^{K} \sum_{b=1}^{n_l} y_{ilb} D_l.
\]  

(1)

The objective function includes the sum of earliness costs, tardiness costs, holding costs and delivery costs. These variables in (1) have the following relationship.

\[
C_i = \sum_{j=1}^{n} x_{ij} T_j, \ i = 1, 2, \ldots, n,
\]  

(2)

\[
T_j = T_{j-1} + \sum_{i=1}^{n} x_{ij} p_i, \ j = 1, 2, \ldots,
\]  

(3)

\[
R'_{lb} = \max_{1 \leq i \leq n} \{y_{ilb} C_i\}, \ l = 1, 2, \ldots, K; b = 1, 2, \ldots,
\]  

(4)

\[
R_i = \sum_{l=1}^{K} \sum_{b=1}^{n_l} y_{ilb} R'_{lb}, \ i = 1, 2, \ldots, n.
\]  

(5)

Then the problem can be formulated as the following integer programming mathematical model:

\[
\min_{x_{ij}, y_{ilb}} f
\]  

subject to

\[
\sum_{i=1}^{n} x_{ij} = 1, \ j = 1, 2, \ldots, n,
\]  

(7)
\[ \sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n, \quad (8) \]

\[ \sum_{i=1}^{n} y_{ilb} = 1, \quad l = 1, 2, \ldots, K; \quad b = 1, 2, \ldots, (9) \]

\[ \sum_{b=1}^{K} y_{ilb} = 1, \quad i = 1, 2, \ldots, n; \quad b = 1, 2, \ldots, (10) \]

\[ \sum_{l=1}^{n} y_{ilb} = 1, \quad i = 1, 2, \ldots, n; \quad l = 1, 2, \ldots, K. \quad (11) \]

The model is to minimize \( f \) subject to some constraints. Eq. (7) ensure that each position on the machine can only process one job. Eq. (8) ensure that each job can only be processed one position on the machine. Eq. (9) ensure that one job can only belong to a batch. Eq. (10) ensure that one job can only belong to a customer. Eq. (11) ensure that one batch can only belong to a customer.

GAs can be used to solve the model. However, the convergence of GA is slow. Dominance properties based on interchanging the position of jobs which can provide necessary conditions for optimal solution. Dominance properties can get better solutions before implementing the GA. It implies that dominance properties can help GA to converge faster. In order to accelerate the convergence speed, dominance properties are used to search the optimal solution locally.

To get the dominance properties, we try to exchange the order of two job and then two dominance properties for the optimal schedule are derived.

**Property 1.** A feasible schedule has the following structure: job \( j \) is processed after \( i \) in a feasible schedule, and the two jobs belong to the same batch \( b \), and there is no job belong to batch \( b \) between the two jobs. If \( h_i - h_j \geq \frac{h_ip_i - hjp_j}{G - A} \), the two jobs should be interchanged.

**Proof.** A feasible schedule satisfying the condition is denoted by \( \pi_1 \). Construct a new feasible schedule \( \pi_2 \) in which the two jobs are interchanged. Suppose that \( \tilde{R}_b \) is the delivery date of the batch. Obviously, the delivery dates of the two jobs remain unchanged. Then the penalty cost for each job has not changed.

The start processing time instant of job \( i \) and the completion time instant of job \( j \) in \( \pi_1 \) are denoted by \( A \) and \( G \), respectively. It is easy to see that positions of \( A \) and \( G \) remain unchanged in \( \pi_2 \). The holding costs of job \( i \) in \( \pi_1 \) and job \( j \) in \( \pi_2 \) are denoted by \( F(\pi_1) \) and \( F(\pi_2) \), respectively.

Then according to the definition of holding costs, we have

\[
F(\pi_1) = h_i \cdot (\tilde{R}_b - A - p_i) + h_j \cdot (\tilde{R}_b - G),
\]

\[
F(\pi_2) = h_i \cdot (\tilde{R}_b - G) + h_j \cdot (\tilde{R}_b - A - p_j).
\]

It is easy to see that \( F(\pi_1) - F(\pi_2) = h_ip_i - hjp_j > 0 \). Thus, \( \pi_2 \) dominates \( \pi_1 \).

The property is proved.

**Property 2.** A feasible schedule has the following structure: job \( j \) is processed after \( i \), and the two jobs belong to the same batch \( b \), and there are some jobs belong to batch \( b \) between the two jobs. The job set belong to batch \( b \) between job \( i \) and \( j \) is denoted by \( \{h_m\}, m \neq i, j \). If \( p_i > p_j \) and \( (h_i - h_j)(G - A) > h_ip_i - hjp_j + \sum h_m(p_i - p_j), \) the two jobs should be interchanged. If \( p_i = p_j \) and \( h_i > h_j \), the two jobs should be
interchanged. If \( p_i < p_j \) and \((h_i - h_j)(G - A) > h_i p_i - h_j p_j - \sum h_m(p_j - p_i), \) the two jobs should be interchanged.

**Proof.** The definitions of \( \pi_1, \pi_2 \) and \( R' \) are the same as property 1.

If \( p_i > p_j \), we have

\[
F(\pi_1) = h_i \cdot (R' - A - p_i) + h_j \cdot (R' - G) + U, \\
F(\pi_2) = h_i \cdot (R' - G) + h_j \cdot (R' - A - p_j) + U + \sum h_m(p_i - p_j).
\]

where \( U \) represents the total holding cost of jobs belong to batch \( b \) between the two jobs. Compare \( F(\pi_1) \) with \( F(\pi_2) \), we have \( F(\pi_1) - F(\pi_2) = (h_i - h_j)(G - A) - h_i p_i + h_j p_j. \) Because \((h_i - h_j)(G - A) > h_i p_i - h_j p_j + \sum h_m(p_i - p_j), \) we have \( F(\pi_1) - F(\pi_2) > 0. \) Then job \( i \) and \( j \) should be interchanged.

If \( p_i = p_j \), we have

\[
F(\pi_1) = h_i \cdot (R' - A - p_i) + h_j \cdot (R' - G) + U, \\
F(\pi_2) = h_i \cdot (R' - G) + h_j \cdot (R' - A - p_j) + U.
\]

Compare \( F(\pi_1) \) with \( F(\pi_2) \), we have \( F(\pi_1) - F(\pi_2) = (h_i - h_j)(G - A) - h_i p_i + h_j p_j. \) Because \((h_i - h_j)(G - A) > h_i p_i - h_j p_j + \sum h_m(p_j - p_i), \) we have \( F(\pi_1) - F(\pi_2) > 0. \) Then job \( i \) and \( j \) should be interchanged.

If \( p_i < p_j \), we have

\[
F(\pi_1) = h_i \cdot (R' - A - p_i) + h_j \cdot (R' - G) + U, \\
F(\pi_2) = h_i \cdot (R' - G) + h_j \cdot (R' - A - p_j) + U - \sum h_m \cdot (p_j - p_i).
\]

Compare \( F(\pi_1) \) with \( F(\pi_2) \), we have \( F(\pi_1) - F(\pi_2) = (h_i - h_j)(G - A) - h_i p_i + h_j p_j + \sum h_m(p_i - p_j). \) Because \((h_i - h_j)(G - A) > h_i p_i - h_j p_j + \sum h_m(p_j - p_i), \) we have \( F(\pi_1) - F(\pi_2) > 0. \) Then job \( i \) and \( j \) should be interchanged.

The property is proved. \( \square \)

3. **Problem under an uncertain environment.** To describe an uncertain variable which refers to human uncertainty, Liu [24] established the uncertainty theory and has been developed well up to now. Next some concepts in uncertainty theory will be introduced.

Let \( \Gamma \) be a nonempty set, \( \mathcal{L} \) is a \( \sigma \)-algebra over \( \Gamma \), and each element \( \Lambda \) in \( \mathcal{L} \) is called an event. A set function \( \mathcal{M} \) from \( \mathcal{L} \) to \([0, 1]\) is called an uncertain measure if it satisfies normality axiom, duality axiom, subadditivity axiom and product axiom [21, 24].

An uncertain variable is a measurable function \( \xi \) from an uncertainty space \((\Gamma, \mathcal{L}, \mathcal{M})\) to the set \( R \) of real numbers, i.e., for any Borel set \( B \) of real numbers, the set \( \{ \xi \in B \} = \{ \gamma \in \Gamma | \xi(\gamma) \in B \} \) is an event. The uncertain distribution \( \Phi \) of an uncertain variable \( \xi \) is defined by \( \Phi(x) = \mathcal{M}\{\xi \leq x\} \) for any real number \( x \). The uncertain variables \( \xi_1, \xi_2, \cdots, \xi_m \) are said to be independence (Liu [24]) if

\[
\mathcal{M}\left(\bigcap_{i=1}^{m} (\xi_i \in B_i)\right) = \min_{1 \leq i \leq m} \mathcal{M}\{\xi_i \in B_i\}
\]

for any Borel sets \( B_1, B_2, \cdots, B_n \) of real numbers.
Example 1. An uncertain variable $\xi$ is called normal if it has a normal uncertainty distribution

$$\Phi(x) = \left(1 + \exp\left(\frac{\pi(e - x)}{\sqrt{3}\sigma}\right)\right)^{-1}, \quad x \in \mathbb{R}$$

where $e$ and $\sigma$ are real numbers with $\sigma > 0$. A normal uncertain variable $\xi$ is denoted by $\xi \sim N(e, \sigma)$.

Given an increasing function $\Phi(x)$, Peng and Iwamura [27] introduced an uncertainty space $(\mathbb{R}, \mathcal{B}, \mathcal{M})$ as follows. Let $\mathcal{B}$ be the Borel algebra over $\mathbb{R}$. Let $\mathcal{C}$ be the collection of all intervals of the form $(-\infty, a]$, $(b, +\infty)$, $\emptyset$ and $\mathbb{R}$. The uncertain measure $\mathcal{M}$ is provided in such a way: first,

$$\mathcal{M}\{(-\infty, a]\} = \Phi(a), \quad \mathcal{M}\{(b, +\infty)\} = 1 - \Phi(b), \quad \mathcal{M}\{\emptyset\} = 0, \quad \mathcal{M}\{\mathbb{R}\} = 1.$$  

Second, for any $B \in \mathcal{B}$, there exists a sequence $\{A_i\}$ in $\mathcal{C}$ such that

$$B \subset \bigcup_{i=1}^{\infty} A_i.$$ 

Thus

$$\mathcal{M}\{B\} = \begin{cases} \inf_{B \subset \bigcup_{i=1}^{\infty} A_i} \sum_{i=1}^{\infty} \mathcal{M}\{A_i\}, & \text{if } \inf_{B \subset \bigcup_{i=1}^{\infty} A_i} \sum_{i=1}^{\infty} \mathcal{M}\{A_i\} < 0.5 \\ 1 - \inf_{B \subset \bigcup_{i=1}^{\infty} A_i} \sum_{i=1}^{\infty} \mathcal{M}\{A_i\}, & \text{if } \inf_{B \subset \bigcup_{i=1}^{\infty} A_i} \sum_{i=1}^{\infty} \mathcal{M}\{A_i\} < 0.5 \\ 0.5, & \text{otherwise.} \end{cases} \quad (12)$$

The uncertain variable defined by $\xi(\gamma) = \gamma$ from the uncertainty space $(\mathbb{R}, \mathcal{B}, \mathcal{M})$ to $\mathbb{R}$ has the uncertainty distribution $\Phi$.

Theorem 1. [48] (i) Let $\xi$ be a common uncertain variable with the continuous distribution $\Phi(x)$ and $f(x)$ a Borel function. Then the distribution of the uncertain variable $f(\xi)$ is

$$\Psi(x) = \mathcal{M}\{f(\xi) \leq x\} = \begin{cases} \inf_{\{f(\xi) \leq x\} \subset \bigcup_{i=1}^{\infty} A_i} \sum_{i=1}^{\infty} \mathcal{M}\{A_i\}, & \text{if } \inf_{\{f(\xi) \leq x\} \subset \bigcup_{i=1}^{\infty} A_i} \sum_{i=1}^{\infty} \mathcal{M}\{A_i\} < 0.5 \\ 1 - \inf_{\{f(\xi) > x\} \subset \bigcup_{i=1}^{\infty} A_i} \sum_{i=1}^{\infty} \mathcal{M}\{A_i\}, & \text{if } \inf_{\{f(\xi) > x\} \subset \bigcup_{i=1}^{\infty} A_i} \sum_{i=1}^{\infty} \mathcal{M}\{A_i\} < 0.5 \\ 0.5, & \text{otherwise.} \end{cases}$$

(ii) Let $f : \mathbb{R}^n \to \mathbb{R}$ be a Borel function, and $\xi = (\xi_1, \xi_2, \cdots, \xi_n)$ be a common uncertain vector. Then the distribution of the uncertain variable $f(\xi)$ is

$$\Psi(x) = \mathcal{M}\{f(\xi_1, \xi_2, \cdots, \xi_n) \leq x\} = \mathcal{M}\{(\xi_1, \xi_2, \cdots, \xi_n) \in f^{-1}(-\infty, x)\}$$

$$= \begin{cases} \sup_{A_1 \times A_2 \times \cdots \times A_n \subset \Lambda} \min_{1 \leq k \leq n} \mathcal{M}_k\{A_k\}, & \text{if } \sup_{A_1 \times A_2 \times \cdots \times A_n \subset \Lambda} \min_{1 \leq k \leq n} \mathcal{M}_k\{A_k\} > 0.5, \\ 1 - \sup_{A_1 \times A_2 \times \cdots \times A_n \subset \Lambda} \min_{1 \leq k \leq n} \mathcal{M}_k\{A_k\}, & \text{if } \sup_{A_1 \times A_2 \times \cdots \times A_n \subset \Lambda} \min_{1 \leq k \leq n} \mathcal{M}_k\{A_k\} > 0.5, \\ 0.5, & \text{otherwise.} \end{cases}$$

where $\Lambda = f^{-1}(-\infty, x)$, and each $\mathcal{M}_k\{A_k\}$ is derived from (12).
\textbf{Definition 1.} [21] Let $\xi$ be an uncertain variable, and $\alpha \in (0, 1)$. Then 
\[ \xi_{\text{sup}}(\alpha) = \sup\{r \mid M\{\xi \geq r\} \geq \alpha\} \]
is called the $\alpha$-optimistic value to $\xi$, and 
\[ \xi_{\text{inf}}(\alpha) = \inf\{r \mid M\{\xi \leq r\} \geq \alpha\} \]
is called the $\alpha$-pessimistic value to $\xi$.

\textbf{Definition 2.} [21] An uncertain distribution $\Phi(x)$ is said to be regular if its inverse function $\Phi^{-1}(x)$ exists and is unique for each $\alpha \in (0, 1)$. Then the inverse function $\Phi^{-1}$ is called the inverse uncertainty distribution of $\xi$.

\textbf{Example 2.} The inverse uncertainty distribution of normal uncertain variable $\mathcal{N}(\mu, \sigma)$ is 
\[ \Phi^{-1}(\alpha) = \mu + \frac{\sigma}{\sqrt{\pi}} \ln\frac{\alpha}{1-\alpha}. \]

\textbf{Theorem 2.} [21] Assume the constraint function $g(x, \xi_1, \xi_2, \ldots, \xi_n)$ is strictly increasing with respect to $\xi_1, \xi_2, \cdots, \xi_k$ and strictly decreasing with respect to $\xi_{k+1}, \xi_{k+2}, \cdots, \xi_n$. If $\xi_1, \xi_2, \cdots, \xi_n$ are independent uncertain variables with uncertainty distributions $\Phi_1, \Phi_2, \cdots, \Phi_n$, respectively, then the chance constraint 
\[ M\{g(x, \xi_1, \xi_2, \cdots, \xi_n) \leq 0\} \geq \alpha \]
holds if and only if 
\[ g(x, \Phi_1^{-1}(\alpha), \cdots, \Phi_k^{-1}(\alpha), \Phi_{k+1}^{-1}(1-\alpha), \cdots, \Phi_n^{-1}(1-\alpha)) \leq 0. \]

However, in reality many parameters will be affected by some emergencies. Hence, indeterminacy should be taken into account by the decision maker. The processing time of a job is often considered as a deterministic variable. In fact, it has a great relationship with the worker who evaluates approximate time based on his past experience and current situation. To deal with these empirical data, uncertain variables are introduced into the integer programming model. The uncertain processing time of job $i$ is denoted by $\xi_i$.

\textbf{Definition 3.} A feasible solution $(x_{ij}^*, y_{ilb}^*)$ is called the possibility-constrained optimal solution if for any feasible solution $(x_{ij}, y_{ilb})$, solution $(x_{ij}^*, y_{ilb}^*)$ satisfies 
\[ \min \{ \mathcal{M}\{f(x_{ij}^*, y_{ilb}^*) \leq \mathcal{T}\} \geq \alpha \} \leq \min \{ \mathcal{M}\{f(x_{ij}, y_{ilb}) \leq \mathcal{T}\} \geq \alpha \}, \]
where $\alpha$ is a predetermined confidence level.

The decision maker must determine a cost $\mathcal{T}$ such that there exists a solution $(x_{ij}^*, y_{ilb}^*)$ satisfying $\mathcal{M}\{f(x_{ij}, y_{ilb}) \leq \mathcal{T}\} \geq \alpha$. For example, let $\alpha = 0.95$, the decision maker has to determine a cost $\mathcal{T}$ and then chooses a solution $(x_{ij}, y_{ilb})$ which satisfying $\mathcal{M}\{f(x_{ij}, y_{ilb}) \leq \mathcal{T}\} \geq 0.95$. It means that if the decision maker chooses the solution $(x_{ij}, y_{ilb})$ $(x_{ij}, y_{ilb})$, the total cost will be lower than $\mathcal{T}$ at least $95\%$. Then the problem can be formulated as the following pessimistic value model according to Definition 3:

\[ \min \min_{x_{ij}, y_{ilb}} \mathcal{T} \text{ subject to } \mathcal{M}\{f \leq \mathcal{T}\} \geq \alpha, \] (13)
\[ \sum_{i=1}^{n} x_{ij} = 1, \quad j = 1, 2, \ldots, n, \]
\[ \sum_{j=1}^{n} x_{ij} = 1, \quad i = 1, 2, \ldots, n, \]
\[ \sum_{i=1}^{n} y_{ib} = 1, \quad l = 1, 2, \ldots, K; b = 1, 2, \ldots, \]
\[ \sum_{l=1}^{K} y_{ib} = 1, \quad i = 1, 2, \ldots, n; b = 1, 2, \ldots, \]
\[ \sum_{b=1}^{n} y_{ib} = 1, \quad i = 1, 2, \ldots, n; l = 1, 2, \ldots, K. \]

The model is to minimize the \( \alpha \)-pessimistic to \( f \). Constraint (14) ensures that the total cost will be less than or equal to \( \mathcal{F} \) under confidence level \( \alpha \).

Note that the objective function \( f \) is a non-monotonic function with respect to \( \xi_i \) for \( i = 1, 2, \ldots, n \), so constraint (14) can’t be transformed to a deterministic form by using the inverse uncertainty distribution method [22] while the deterministic form of objective function can be obtained by the uncertain simulation [48].

Before introducing the hybrid algorithm, two dominance properties under the uncertain environment which can accelerate the convergence speed of the hybrid algorithm are obtained.

**Theorem 3.** A feasible solution has the following structure: job \( j \) is processed after \( i \), and the two jobs belong to the same batch \( b \). There is no job belong to batch \( b \) between the two jobs. The uncertain distribution of \( \xi_i \) and \( \xi_j \) are denoted by \( \Phi_{\xi_i} \) and \( \Phi_{\xi_j} \), respectively. If \( h_j \Phi_{\xi_j}^{-1}(\alpha) - h_i \Phi_{\xi_i}^{-1}(1-\alpha) \leq (G-A)(h_i-h_j) \), the position of the two jobs should be interchanged. \( \alpha \) is a preset confidence level.

**Proof.** Because the processing time is an uncertain variable, the relationship of uncertain variables can be compared under a certain confidence level \( \alpha \). According to the relationship derived from property 1, we have \( \mathcal{M}\{h_j \xi_j - h_i \xi_i < (G-A)(h_i-h_j)\} \geq \alpha \). Then according to Theorem 2, we have \( h_j \Phi_{\xi_j}^{-1}(\alpha) - h_i \Phi_{\xi_i}^{-1}(1-\alpha) \leq (G-A)(h_i-h_j) \).

**Theorem 4.** A feasible solution has the following structure: job \( j \) is processed after \( i \), and the two jobs belong to the same batch \( b \), and there are some jobs belong to batch \( b \) between the two jobs. The job set belong to batch \( b \) between job \( i \) and \( j \) is denoted by \( \{h_m\}, m \neq i, j \). If \( \Phi_{\xi_i}^{-1}(\alpha) < \Phi_{\xi_j}^{-1}(1-\alpha) \) and \( (h_i + \sum h_m)\Phi_{\xi_i}^{-1}(\alpha) - (h_j + \sum h_m)\Phi_{\xi_j}^{-1}(1-\alpha) < (h_i-h_j)(G-A), \) then the position of the two jobs should be interchanged. If \( \Phi_{\xi_i} = \Phi_{\xi_j} \) and \( h_i > h_j, \) the position of the two jobs should be interchanged. If \( \Phi_{\xi_i}^{-1}(\alpha) < \Phi_{\xi_j}^{-1}(1-\alpha) \) and \( (h_i + \sum h_m)\Phi_{\xi_i}^{-1}(\alpha) - (h_j + \sum h_m)\Phi_{\xi_j}^{-1}(1-\alpha) < (h_i-h_j)(G-A), \) the position of the two jobs should be interchanged. \( \alpha \) is a preset confidence level.

**Proof.** Because the processing time is an uncertain variable, the relationship of uncertain variables can be compared under a certain confidence level \( \alpha \). According to the relationship derived from property 2, we have \( \mathcal{M}\{\xi_j < \xi_i\} \geq \alpha \) and \( \mathcal{M}\{(h_i + \sum h_m)\xi_i - (h_j + \sum h_m)\xi_j < (h_i-h_j)(G-A)\} \geq \alpha \). Then according to Theorem 2,
we have \(\Phi^{-1}_j(\alpha) < \Phi^{-1}_j(1 - \alpha)\) and 
\((h_1 + \sum h_m)\Phi^{-1}_j(\alpha) - (h_j + \sum h_m)\Phi^{-1}_j(1 - \alpha) < (h_i - h_j)(G - A)\). The following two conclusions can be obtained in a similar way. \(\Box\)

4. Hybrid heuristic algorithm. Because objective function \(f\) is a non-monotone function, inequality (14) can’t be transformed to a deterministic form by the inverse uncertain distribution method [22]. The objective function can be transformed to a deterministic form effectively by the uncertain simulation [48]. The uncertain simulation is introduced as follows.

4.1. Uncertain simulation. Let \(\xi = (\xi_1, \xi_2, \cdots, \xi_n)\) be an uncertain vector where \(\xi_i\) is an uncertain variable with continuous uncertainty distribution \(\Phi_i\) for \(i = 1, 2, \cdots, n\), and \(f: \mathbb{R}^n \rightarrow \mathbb{R}\) be a Borel function. The following algorithm is used to simulate the pessimistic value:

\[
    f_{\inf} = \inf\{r \mid M\{f(\xi) \leq r\} \geq \alpha\}
\]

where \(\alpha \in (0, 1)\) is a predetermined confidence level.

\textbf{Algorithm 1:} (Uncertain simulation for \(f_{\inf}\)) [48]:

\begin{itemize}
    \item \textbf{Step 1.} Randomly generate \(u_k = (\gamma^{(1)}_k, \gamma^{(2)}_k, \cdots, \gamma^{(n)}_k)\) with \(0 < \Phi_i(\gamma^{(i)}_k) < 1, i = 1, 2, \cdots, n, k = 1, 2, \cdots, m\).
    \item \textbf{Step 2.} Set \(a = f(u_1) \land f(u_2) \land \cdots \land f(u_m)\), \(b = f(u_1) \lor f(u_2) \lor \cdots \lor f(u_m)\).
    \item \textbf{Step 3.} Set \(r = (a + b)/2\).
    \item \textbf{Step 4.} If \(M\{f(\xi) \leq r\} \geq \alpha\), then \(b \leftarrow r\).
    \item \textbf{Step 5.} If \(M\{f(\xi) \leq r\} < \alpha\), then \(a \leftarrow r\).
    \item \textbf{Step 6.} Repeat the third to fifth steps until \(b - a < \epsilon\) for a sufficiently small number \(\epsilon\).
    \item \textbf{Step 7.} \(f_{\inf} = (a + b)/2\).
\end{itemize}

In the steps 4 and 5, the \(M\{f(\xi) \leq r\}\) can be obtained according to the Algorithm 2: (Uncertain simulation for \(L\)) in [48].

4.2. Genetic algorithm. Many approaches have been proposed for solving scheduling problem. GAs have been proved to be applicable to different types of scheduling problems. For instance, a hybrid heuristic combines with local search based on GA was proposed to minimize the sum of total earliness and tardiness in a single machine problem [12]. Computational results reflected the sizeable solution quality improvement induced by hybridization, and assessed the impact of each type of hybridization on the efficiency of the hybrid heuristic. Jolai et al.[17] developed a GA for a single machine problem of minimizing the number of tardy jobs and maximum earliness. Singh et al.[31] proposed three hybrid heuristics for a single machine scheduling problem with quadratic earliness and tardiness costs. Three improvement procedures were used and computational results showed that the hybrid approaches are very effective. Hamidinia et al.[13] used GA to solve a novel complex single machine scheduling problem and experiments showed that the proposed GA outperforms the traditional GA.

To obtain a compromised solution within a reasonable time, a hybrid heuristic based on GA is presented. To get a better initial solution and accelerate the convergence speed, Theorems 3 and 4 are used to optimize the initial solutions.

\begin{itemize}
    \item Solution representation:
\end{itemize}
The solution representation is a key step to set up a bridge between the solution and the chromosome. A matrix with 2 rows and \( N \) columns is used. Assume that the first row of the matrix is a permutation from 1 to \( N \). It indicates a processing sequence of these jobs. The second row of the matrix use a string of integers represent the batch index.

- Initialization:
  Initial solutions are generated randomly. Then better initial solutions are gotten by using Theorem 3 and 4.
- Fitness function:
  Use the objective function as a fitness function.
- Selection process:
  Select the chromosomes by spinning the roulette wheel.
- Crossover process:
  Two-point crossover method is used. Two chromosomes are randomly selected and a random number \( r_c \) is generated first. If \( r_c \) is smaller than \( P_c \), then crossover implements on this pair, else no crossover. Randomly assign two cutting points. Genes beyond the cutting points in parent 1 and parent 2 are directly duplicated to the offspring. The vacant positions in the offspring are duplicated from parent 2 and parent 1, respectively. An example of crossover operation is shown in Fig 1.
- Mutation process:
  Select two positions in a chromosome randomly and interchange their positions. In the mutation step, two chromosomes are randomly selected and a random number \( r_m \) is generated first. If \( r_m \) is smaller than \( P_m \), the mutation operation is executed, else no mutation. An example of mutation operation is shown in Fig 2.

- Termination:
  If arrive the maximum number of iterations, terminate; otherwise, circulate from the selection process.

  GA integrated with the uncertain simulation (Algorithm 1 and 2) can be used to solve the uncertain pessimistic value model.
Numerical experiment. Numerical experiments are conducted in order to assess the performance of the proposed mathematical model and hybrid algorithm. Since the objective function is a non-monotone function with respect to uncertain variables. The model can’t be transformed to a deterministic form by the inverse uncertain distribution method \[22\]. But the objective function can be dealt with by uncertain simulation \[48\]. Then the problem can be solved by a heuristic.

The performance of the proposed algorithm can be compared with the traditional GA which does not integrate with Theorem 3 and 4.

Three numerical examples are considered. For the simple case, there are 10 jobs and 3 customers. For the mesoscale case, there are 50 jobs and 10 customers. For the large scale case, there are 100 jobs and 20 customers. Processing time of job \(i\) is assumed to be a normal uncertain variable: \(\xi_i \sim N(i, 1)\) for \(i = 1, 2, \cdots, 100\). Similar to \[35\], the centers of the due windows were uniformly and integrally distributed in \([(1 - T - RDD/2) \times \sum \xi_i, (1 - T + RDD/2) \times \sum \xi_i]\), where \(T\) is the tardiness factor and \(RDD\) is the relative range of the due windows. \(T\) and \(RDD\) take values randomly from 0.1, 0.2, 0.3 and 0.8, 1.0, 1.2, respectively. The sizes of the windows are uniformly distributed between 1 and \(\sum \xi_i/n\). The unit holding cost from a uniform distribution between 15 and 30. Unit delivery earliness penalty cost and tardiness penalty cost are generated in uniform distribution between 10 and 20. The transportation cost is chosen from a uniform distribution between 5 and 10. Confidence level \(\alpha = 0.8\).

HGA stands for GA integrate with Theorem 3 and 4. For each instance, GA and HGA are running five times, respectively. Population size of GA is 30. Crossover rate is 0.75. Mutation rate is 0.25. Maximum iteration number is 200. All instances are performed on a computer with an INTEL(R) i7 processor with a speed of 4.0 GHz and 16GB RAM. The following tables list values of objective function and the CPU time. “No” column lists running index of algorithm. “GA” column lists objective function values obtained by the traditional GA. “HGA” column lists objective function values obtained by the hybrid algorithm in Section 4.

| No | GA | HGA |
|----|----|-----|
|    | min | max | time(s) | min | max | time(s) |
| 1  | 3079 | 3361 | 356 | 3125 | 3349 | 301 |
| 2  | 2953 | 3415 | 338 | 2837 | 3437 | 312 |
| 3  | 2556 | 2889 | 313 | 2398 | 2735 | 263 |
| 4  | 3582 | 3764 | 359 | 3619 | 3923 | 329 |
| 5  | 2046 | 2358 | 271 | 2098 | 2491 | 254 |
Table 3. Results for mesoscale

| No | GA min | GA max | time(s) | HGA min | HGA max | time(s) |
|----|--------|--------|---------|---------|---------|---------|
| 1  | 8466   | 8893   | 1603    | 8501    | 8697    | 1354    |
| 2  | 9320   | 9671   | 1754    | 9455    | 9612    | 1340    |
| 3  | 8323   | 8569   | 1625    | 8361    | 8538    | 1385    |
| 4  | 8736   | 9028   | 1701    | 8798    | 9110    | 1313    |
| 5  | 7954   | 8077   | 1590    | 7839    | 8154    | 1397    |

Table 4. Results for large scale

| No | GA min | GA max | time(s) | HGA min | HGA max | time(s) |
|----|--------|--------|---------|---------|---------|---------|
| 1  | 21542  | 22634  | 8655    | 21696   | 21873   | 6320    |
| 2  | 19556  | 21391  | 8367    | 19374   | 20065   | 5966    |
| 3  | 21406  | 23767  | 8961    | 21250   | 22034   | 6257    |
| 4  | 21973  | 23891  | 9058    | 21630   | 22741   | 6299    |
| 5  | 20592  | 22378  | 8537    | 20063   | 20097   | 6035    |

For small size, GA and HGA can solve the small scale problem effectively. The two methods show little difference. For mesoscale case, the time of HGA needed is slightly less than GA. Results show that GA and HGA can be considered as a stable method for mesoscale case. For large size, the time of HGA needed is significantly less than GA. Results reflect that the two dominance properties have an important role in updating the initial solution as a local search method.

In addition, more numerical experiments are used to verify the effectiveness of HGA. The number of jobs is \( n = 20, 50, 100, 200 \). \( K = 5 \). The relative percentage error (RPE) is often used as an important tool for evaluating the merits of algorithms under small scale case. Its expression:

\[
RPE = \frac{C - C^*}{C^*} \times 100%.
\]

where \( C^* \) is optimum value and \( C \) is objective function value generated by a heuristic algorithm.

Table 5. Average relative percentage errors of GA and HGA

| \( n \) | GA     | HGA    |
|--------|--------|--------|
| 20     | 0.06266| 0.05713|
| 50     | 0.03316| 0.02685|
| 100    | 0.01295| 0.007372|
| 200    | 0.005723| 0.004993|
| Average| 0.028623| 0.024086|

Table 5 shows that (i) HGA is better than GA obviously at average relative error; (ii) the average relative error decreases with the increase of the number of jobs.

Moreover, some numerical experiments are used to test the sensitivity of the confidence level \( \alpha \). Assume that the numerical experiment are executed under the situation where \( n = 50 \) and \( K = 10 \). The results are shown in Fig.3. \( \alpha \in [0.1, 0.9] \)
with step sizes of 0.1. Fig. 3 indicates that the objective value is increasing with respect to $\alpha$.

![Figure 3: The sensitivity of the solution with respect to the confidence level](image)

**Figure 3.** The sensitivity of the solution with respect to the confidence level

6. **Conclusions.** In this paper, a single machine scheduling problem with batch delivery is considered. An integral programming model is proposed and two dominance properties are obtained. Later, a more reasonable pessimistic value model under an uncertain environment is proposed. The inverse distribution transformation method can’t be used owing to the objective function is a non-monotone function and the uncertain simulation [48] base on GA is used to obtain an approximate optimal solution. The proposed hybrid heuristic algorithm is evaluated by some numerical examples. For small and medium scale problems, GA and HGA have similar resultant values. But for large scale problem, the CPU time of HGA is shorter than GA obviously. Results show that GA integrates dominance properties can help for accelerating the convergence speed effectively. In addition, some numerical examples also show that HGA is superior to GA in the relative percentage error. The sensitivity test shows that the optimal solution of the pessimistic value model is increasing with respect to the confidence level.

Further research could focus on some more complex situations. For example, jobs are released at different times, the order is canceled, the machine failed, deterioration and learning effect. Moreover, it would be interesting to consider different criterions or more complicated workshop environment such as flow shop, job shop and open shop.

**Acknowledgments.** This work is supported by the National Natural Science Foundation of China (No.61673011).
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Received August 2017; 1st revision December 2017; 2nd revision January 2018.

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