Study on the Equivalent Pressure Coefficient of the Viscosity Model Shaped by Ultra-thin Injection Molding

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Abstract. In ultra-thin injection molding, with the increase of pressure, the nonlinear relationship between pressure and viscosity becomes more and more obvious. The pressure dependence of Cross-WLF viscosity model can no longer reflect the actual production. So, this paper based on the free volume theory, expounds how the pressure affects the melt viscosity in the Cross-WLF viscosity model, and based on the variations of the specific volume in the PVT melt curve, demonstrates that the pressure dependence of melt viscosity is significantly nonlinear in the ultra-thin high-pressure injection molding, which is inconsistent with the traditional linearity. In order to reflect the actual production better, this paper defines the equivalent pressure coefficient of the viscosity model and justifies it in the production of the ultra-thin light guide plate.

1. Introduction

The Polymer Rheology defines that the viscosity of high polymer melt increases with the rise of pressure, otherwise it decreases in normal cases. In regular injection molding, the injection pressure is generally below 100Mpa, thus, most researches tend to focus on the pressure dependence of melt viscosity under low pressure. However, in ultra-thin injection molding where the injection pressure is up to 300Mpa, the injection is speedy and the product precision is demanding, the pressure dependence of melt viscosity will have a significant impact on the molding process. Given its uniqueness, relevant literature is rarely seen.

Westover\cite{1} discovers that the rise of pressure enables the viscosity of polystyrene to increase by more than 100 times provided that the shearing rate is constant. Son Y\cite{2} find that the viscosity of polystyrene is increased by 134 times on the condition that the temperature stays at 196\textdegree C and the pressure boosts from 0 to 126Mpa while the viscosity of polystyrene is only increased by 14 times when the temperature stands at 149\textdegree C and the pressure changes by the same degree. In light of the relations between the fractional free volume and the temperature, Kadijk S E \cite{3} comprehensively explains the pressure dependence of the viscosity by using zero shear viscosity and shear thinning practice in Arrhenius-WLF Equation and Carreau Equation respectively. Fatma Dinc\cite{4} employs free volume theory and develops a new viscosity model. According to this model, zero shear viscosity, constant shear stress viscosity, and constant shear rate viscosity are all in a linear relationship with the fractional free volume, in which the viscosity declines as the fractional free volume increases. P. Van Puyvelde\cite{5} controls the capillary in the fashion of back pressure and improves the pressure’s impact on the viscosity by way of correcting the pressure at the inlet. Gu Zhengwei\cite{6} argues that since zero shear viscosity \( \eta_0 \) is obtained in low pressure (below 50Mpa) environment, the pressure’s impact on
the viscosity must be included if the pressure of injection molding is high. As a result, he corrects the reference point of zero shear viscosity in the Cross Model and verifies how important the pressure dependence of melt viscosity is to the simulation analysis of thin-wall items.

In numerous literatures have been discussed the pressure’s impact on the viscosity within the low-pressure scope. As for high-speed ultra thin molding technology, it is commonly recognized that the pressure dependence of the viscosity is crucial to the molding process. But subject to the limitation of laboratory conditions, few literatures make reference.

2. pressure dependence of viscosity in Cross-WLF seven-parameter model

The Cross-WLF seven-parameter model[7] is

$$\eta(\dot{\gamma}, T, P) = \frac{\eta_0(T, P)}{1 + (\eta_0 / \tau^*)^{1-n}}$$

where

$$\eta_0(T, P) = D_1 \exp\left(-\frac{A_1(T - T^*)}{A_3 + (T - T^*)}\right), \quad T^* = D_2 + D_4 \cdot P, \quad A_2 = \tilde{A}_2 + D_3 \cdot P$$

n is a non-Newtonian index; \(\tau^*\), a material constant; \(D_1\), the shear stress prior to the shear thinning of melt; \(D_2\), the glass transition temperature under low pressure; \(D_3\), a coefficient affecting the viscosity and pressure; \(T^*\), the temperature at which the melt transforms into glass; \(A1, \tilde{A}_2\) factors related to temperature.

The pressure’s impact on viscosity in the model is determined by the product of \(D_3 \times P\). The recommended values of several plastic \(D_3\) are shown in Table 1. The value of \(D_3\) is small. When the pressure of injection molding is low, the product of \(D_3 \times P\) would also be small and have unnoticeable impact on viscosity. When the pressure of injection molding is high, the product of \(D_3 \times P\) would be large. Therefore, in the ultra thin high-pressure injection molding, the value of \(D_3\) is significant to the molding process.

| Materials | \(D_3\) (K/Pa) |
|-----------|--------------|
| PC        | \(1.9 \times 10^{-7}\) |
| PMMA      | \(1.7 \times 10^{-7}\) |
| PS        | \(2.3 \times 10^{-7}\) |

3. Free Volume Theory and Pressure Coefficient of the Cross-WLF Viscosity Model

Fox and Flory[8] believe that the total volume \(V\) of liquid or solid has two components: one is \(V_0\) the volume occupied by molecules; the other is \(V_f\) the free volume, that is,

$$V = V_0 + V_f$$

The free volume is scattered in the matter in the form of "cavity". The high polymer molecule chain adjusts its conformation through the rotation and displacement of the free volume. As the segment is frozen and the free volume is also frozen and kept at a fixed value, the free volume of high polymer in glassy state starts defrosting and swelling when the temperature rises up to the transition point of glassy state, as shown in Figure 1.

When \(T < T_g\), the total volume of high polymer is[9]-[10]:

$$V_g = V_f + V_0 + \left(\frac{dV}{dT}\right)_g T_g$$

where \(V_f\) free volume under glassy state

\(V_0\)-occupied volume of glassy high polymer under absolute zero
Similarly, when $T < T_g$, the volume of high polymer is:

$$V_r = V_g + \left( \frac{dV}{dT} \right)_r (T - T_g)$$

The free volume under the rubbery state when the temperature is $T$

$$\left( V_f \right)_r = V_f + (T - T_g) \left[ \left( \frac{dV}{dT} \right)_r - \left( \frac{dV}{dT} \right)_g \right]$$

(4)

(5)

Where the difference between the rubbery state and glassy state $\left( \frac{dV}{dT} \right)_r - \left( \frac{dV}{dT} \right)_g$ is the expansion rate of free volume above $T_g$ the expansion coefficients of high polymer above or below $T_g$ are respectively as follows:

$$a_r = \frac{1}{V_g} \left( \frac{dV}{dT} \right)_r, a_g = \frac{1}{V_g} \left( \frac{dV}{dT} \right)_g$$

(6)

The expansion coefficient of free volume adjacent to $T_g$ is the difference of expansion efficient of high polymer above and below $T_g$,

$$a_f = a_r - a_g$$

(7)

The fractional free volume at a temperature above the glass transition temperature is expressed:

$$f_T = f_g + a_f (T - T_g) \quad (T \geq T_g)$$

(8)

Where $f_g$ is the factional free volume of glassy high polymer.

According to free volume theory, the very reason the viscosity of melt decreases is that the temperature rises above the glass transition temperature and the fractional free volume expands.[11]

The pressure dependence of high polymer viscosity can be summarized as follows: under pressure, the free volume within macromolecule materials dwindles and the molecule chain becomes less active, making the glass transition temperature rise up. Suppose the high polymer has State 1 ($P_1, T$) and State 2 ($P_2, T$), let isothermal compression be the equal of increasing the glass transition temperature of polymer, here comes:

$$\alpha_f (T_{g2} - T_{g1}) = K_f (P_2 - P_1)$$

(9)

$K_f$ is the isothermal compression coefficient of free volume as a result

$$\alpha_f (T_2 - T_1) = K_f (P_2 - P_1) \quad \frac{dT_g}{dP} = \frac{K_f}{\alpha_f} = D_3$$

(10)

(11)

The equation 11 indicates that in the Cross-WLF viscosity model, $D_3$ is regarded as a constant because $K_f$ is thought to be irrelevant with $P$. But actually it is not true. For example, as shown in the Figure 2, Panlite L-1225L(PC) which is an exhibition of the relationship between the specific volume and the pressure and temperature, there is a non-linearity between the pressure and the specific volume when the temperature is constant and this non-linearity intensifies as the pressure stays at high level in particular. Thus, when the melt is in compression, $K_f$ is not a constant value. As the pressure rises, the value of $K_f$ is on the decline, so is the value of $D_3$. 


4. Dₙ, the Equivalent Pressure Coefficient of the Cross-WLF Viscosity Model

In order to linearly represent the melt compressibility so that the linear representation of the pressure’s impact on viscosity is realized in the Cross-WLF viscosity model, the equivalent compression gradient (shown in dotted line) of melt volume is selected in this paper to linearly calculate the amount of melt compression. Its value selection principle is to make the area of S₁ and S₂ in the figure equal. The objective is to compensate the positive and negative deviation of the compressed volume under high pressure and low pressure and appropriately reflect the actual compression amount of melt. The relationship between the compression amount of specific volume of Panlite L-1225L (PC) and the pressure is shown in Figure 3.

To master the compression law of free volume, macromolecules and free volume in melt are seen as two different series elastomers of elasticity modulus. These two elasticity modulus are all the function of the pressure P, that is kₚ(P) and kₙ(P), as shown in Figure 4.

Suppose the elasticity modulus of two elastomers are in direct proportion when the melt is under pressure, namely,

\[ k_f(P) \propto k_n(P) \quad \text{and} \quad k_f(P) = \xi k_n(P) \tag{12} \]

then comes

\[ k_f(P) + \xi k_n(P) = \bar{k}(P) \tag{13} \]
\[ k_f(P)\left(1 + \frac{1}{\xi_f}\right) = \bar{k}(P) \] \hspace{1cm} (14)

\[ k_f(P) = \left(\frac{\xi_f}{1 + \xi_f}\right)\bar{k}(P) \] \hspace{1cm} (15)

Based on the study on the free volume, macromolecule expansibility and compressibility in literature, \( \xi_f \) is valued at 1.68\cite{12].

With the experimental data curve of PC PVT, based on equation 6
\[ a_f = \Delta a = a_{s} - a_{r} \] \hspace{1cm} (16)
the free volume expansibility of PC is:
\[ a_f = 6.2 \times 10^{-4} - 1.9 \times 10^{-4} = 4.3 \times 10^{-4} \] \hspace{1cm} (17)
with equation 10, the equivalent pressure impact coefficient is:
\[ \bar{D}_3 = \frac{k_f(P)}{a_f} = \frac{\left(\frac{\xi_f}{1 + \xi_f}\right)\bar{k}(P)}{a_f} \] \hspace{1cm} (18)

With equation 18, the equivalent compression coefficient of the viscosity of Panlite L-1225L is shown in Figure 5.

The parameters of Cross-WLF viscosity model of the material Panlite L-1225L (from moldflow2012) are shown in table 2. The comparison of viscosity curves is shown in Figure 6 on the conditions that the pressure-affected coefficient \( D_3 \) takes the recommended value \( 1.9\times10^{-7} \) and equivalent value \( \bar{D}_3 \). The pressure in the Figure 6 is valued at 50 Mpa, 150 Mpa, and 300 Mpa. The Figure 6 shows that the impact the equivalent pressure-affected coefficient \( D_3 \) exerts on the viscosity is less than the recommended value \( 1.9\times10^{-7} \). Therefore, it is more appropriate to use equivalent pressure-affected coefficient to make calculations and actual production in the ultra-thin high pressure injection molding.

**Table 2** parameters of Cross-WLF viscosity model of Panlite L-1225L

| parameters | \( n \) | \( \tau^* \) | \( D_3 \) | \( D_2 \) | \( D_3 \) | \( A_1 \) | \( \bar{A}_2 \) |
|------------|---|---|---|---|---|---|---|
| values     | 0.17 | 7.8e005 | 2.9e010 | 417 | 1.9e-007 | 25.9 | 61.2 |

**Figure 5** the equivalent compression coefficient pressure of the viscosity of Panlite L-1225L

**Figure 6** The different effects of equivalent coefficients and recommended values
5. Verification of the Equivalent Pressure Coefficient

Injection molding machine is TOYO SI-80IV. Test process parameters are shown in Table 3. There is a sample of 0.4mm and 3in ultra-thin light guide plate for mobile phone manufactured by Dongguan Yidong Electronics Co., Ltd for Motorola company. Its dimension is shown in Figure 7. The injection molding machine is TOYO SI-V series full-automatic injection molding machine. The materials of test-piece are GF8002 (PC) and the mold for test is shown in Figure 8.

| materials      | Melt temperature | mold temperature | injection speed | screw diameter | maintained pressure | Time of maintaining | Mold-locking force |
|----------------|------------------|------------------|-----------------|----------------|---------------------|---------------------|-------------------|
| Panlite L-1225L| 310°C            | 110°C            | 500mm·s⁻¹       | 20 mm          | 60%                 | 0.5 s               | 70 Tom            |

Figure 7 Geometric dimensions of samples  

Figure 8 The mold for test

As shown in Fig 9(a), short injections are conducted on test piece as per the process parameters in Table 3 under the conditions that the injection pressure is set at 150Mpa, 200Mpa, 250Mpa, and 300Mpa respectively. Then as shown in Fig. 9(b), simulation is made in accordance with process parameters when the equivalent pressure-affecting coefficient $D_3$ is respectively set at 1.623e-007, 1.540-007, 1.496e-007, and 1.410e-007 selected based on the injection pressure. Finally, the results of simulation are shown in Fig.9 (c) when the pressure-affecting coefficient $D_3$ is set 1.9e-007 and the process parameters are kept unchanged.

The compassion between the short injection results and simulation results in Fig. 9 shows that the results of filled area of cavity simulated through using equivalent pressure-affecting coefficient $D_3$ are extremely close to the results of short injection, particularly, under the high-pressure injection state.

(a) Short injected samples
The above two experiments have demonstrated that the pressure dependence of viscosity model in ultra-thin injection molding must be considered because the injection pressure is forceful. It has been spotted from the practice that the pressure-affecting coefficient of Cross-WLF model in ultra-thin injection molding is largely deviated from the audit results under high pressure molding state. In light of the equivalent pressure-affecting coefficient proposed in this paper, the shortcomings of the Cross-WLF model in ultra-thin simulation have been satisfactorily addressed.

6. Summary
This paper concludes that the expansion ratio and compression ratio of free volume in melt are not proportional. The law of pressure’s impact on viscosity cannot be scientifically reflected by equalizing the pressure’s impact on viscosity with the linear increase of glassy transition temperature of the melt. To compensate the shortcomings of viscosity model in ultra-thin injection molding, on the basis of the free volume theory, the melt compression under high or low pressure, the compression ratio of appropriate and constant specific volume is selected under specific pressure to accurately reflect the non-linear changes of the specific volume as the pressure changes. On such basis, the concept of equivalent pressure-affecting coefficient is developed to rationally reflect objective reality. Through the comparison between value analysis and practice, the simulated results of equivalent pressure-affecting coefficient fit relatively well to the production practice.

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