Gauge invariance of nonlinear Landau damping rate of Bose excitations in quark-gluon plasma

Yu.A.Markov * and M.A.Markova

Institute of System Dynamics
and Control Theory Siberian Branch
of Academy of Sciences of Russia,
P.O. Box 1233, 664033 Irkutsk, Russia

Abstract

On the basis of the approximate dynamical equations describing the behavior of quark-gluon plasma (QGP) in the semiclassical limit and Yang-Mills equation, the kinetic equation for longitudinal waves (plasmons) is obtained. With the Ward identities the gauge invariance of obtained nonlinear Landau damping rate is proved. The physical mechanisms defining nonlinear scattering of a plasmon by QGP particles are analyzed. The problem on a connection of nonlinear Landau damping rate of longitudinal oscillations with damping rate, obtained in the framework of hard thermal loops approximation, is considered. It is shown that the gauge-dependent part of nonlinear Landau damping rate for the plasmons with zero momentum vanishes on mass-shell.

*e-mail:markov@icc.ru
1. INTRODUCTION

In recent 15-20 years, a theoretical investigations of properties of quark-gluon plasma has been of great interest. It is connected with intensive looking for a QGP in the experiments with collision of ultrarelativistic heavy ions and application concerning the physics of the early universe.

Two methods to study of the nonequilibrium phenomena in a QGP are used: method of temperature Green functions and kinetic approach. Significant progress has been achieved in the development of the first method. The effective perturbative theory was constructed in the papers [1] by Pisarski and Braaten, Frenkel and Taylor on the basis of the resummation of so-called hard thermal loops (HTL’s), and the problem on the sign and gauge dependence of the damping rate of the long wavelength excitations in QGP was solved. Independent solution of "the plasmon puzzle" problem was received by Kobes, Kunstatter, and Rebhan [2]. The damping rate for heavy quarks interacting with light thermal quarks and gluons, was found in [3], for soft plasmon and plasmino - in [4, 5]. The damping rate for energetic fermions and bosons was derived by Lebedev and Smilga [6] with extension of the program of resummation outlined in Ref. [1], by means of inclusion of higher order effects in a hard propagator. The alternative derivation of damping rate for energetic fermions, based on introduction of an infrared cutoff was deduced by Burgess, Marini, and Rebhan [7]. The progress of the thermal QCD makes possible a new look at the existing of kinetic theory of QGP, developed by Heinz, Winter, Elze, Vasak and Gyullasy, Mrówczyński and others [8], and it has given impetus to its further development.

In spite of the fact that the language and methods of these approaches are very different, there are close similarities between HTL approach and transport theory. Originally the kinetic theory was used by Silin to derive HTL in the photon’s self-energy [9]. HTL’s in the quark and gluon self-energies can be computed similarly. Moreover, Kelly, Liu, Lucchesi and Manuel [10] have shown that the generating functional of HTL’s (with an arbitrary number of soft external bosonic legs) can be derived from the classical kinetic theory of QGP. This points to the classical nature of the hard thermal effects. For hard excitations, the damping rate has been computed by Heiselberg and Pethick [11] from a Boltzmann - like equation, with the collision terms included.

A further step in development of kinetic theory was made by Blaizot and Iancu [12]. In contrast to the early papers on transport theory of QGP [8], these authors use from outset the ideas developed in thermal QCD in deriving of the kinetic equations. The equations obtained by them on the basis of a truncation of the Schwinger-Dyson hierarchy isolate consistently the dominant terms in the coupling constant \( g \) in a set of equations, which describe the response of plasma to weak and slowly varying disturbances, and encompass all HTL’s (concerning recent investigations in this area, see also papers [13-16]). However, here it should be noted that if the influence of the average fermionic field is neglected, then the expression for current induced by soft gauge fields, obtained in [12] (and the nonlinear equation of motion, connected with it) fully coincide with the corresponding
expression obtained in [10] from usual classical kinetic theory on the basis of consistent expansion of distribution function in powers of the coupling constant. This somewhat justifies use of the (semi)classical kinetic equations found in [8], in spite of the fact that intermediate approximation schemes in which these equations were derived, mix leading and nonleading contributions with respect to the powers of $g$ and so, are not entirely consistent.

Such close interlacing of two methods of investigation of nonequilibrium phenomena in QGP leads to the question: can we calculate the damping rate of soft bosonic modes corresponding to the hard thermal result [4], remaining in the framework of classical (semiclassical) kinetic theory only? Xiaofei and Jiarong [17] were the first to put this question. Because of obtained results, they have given a positive answer.

As was shown by Heinz and Siemens [18], in linear approximation the Landau damping is absent in QGP. In fact, the only mechanism, with that one can connect the damping following from the kinetic theory with one from HTL approach, is so-called nonlinear Landau damping. It bounds up with the nonlinear effects of waves interaction and particles in QGP. The multiple time-scale method which has proved successful in study of the nonlinear properties of electromagnetic (Abelian) plasma [19], was used in [17] for determination of this association. By means of this method the nonlinear shift of the mass-shell of the longitudinal modes in the temporal gauge has been obtained by Xiaofei and Jiarong. Its imaginary part defines required nonlinear Landau damping rate. Further, the limiting expression of the derived damping rate for $k = 0$-mode was obtained, and numerical computations for approximate estimate were performed. The value derived by this means is in close agreement with similar numerical one obtained by Braaten and Pisarski [4] in the framework of effective perturbative theory.

However, under close examination of above-mentioned paper we found certain mistakes in computations, which were of both principle and nonprinciple character. As it was shown in our early paper [20], the elimination of these inaccuracies finally leads not only to a numerical modification of the limiting value of nonlinear Landau damping rate obtained in [17], but what is more important, it changes the sign of obtained expression. This points to some prematurity of the statements in [17] on obtained connection between nonlinear Landau damping rate and damping rate from the HTL-approach.

In this paper, that is further development the ideas outlined in Ref. [20], we consider the above problem, using the approach based on obtaining of kinetic equation for waves in quark-gluon plasma developed by Kadomtsev, Silin, Tsytovich and others [21] in connection with ordinary plasma. We have shown that nonlinear Landau damping rate $\gamma_l(k)$ (which even is not of fixed sign for arbitrary value of $k$) for longitudinal waves in QGP defines two various processes: the effective spectral pumping of energy from short to long waves (with complete conservation of excitations energy), and properly nonlinear dissipation of plasma waves energy in the medium. The main conclusion of this work is that there is a need to compare the piece of $\gamma_l(k)$ that corresponds to nonlinear dissipation of waves energy and is a positive for any value of a wave vector $k$ and, in particular for
\( k = 0 \) - mode, with damping rate of soft boson modes from HTL-approximation.

The outline of the paper is as follows. In Sec. 2 we derive a system of self-consistent equations in the covariant gauge for regular (coherent) and random parts of both the distribution functions of QGP particles and gauge field. In Sec. 3 the first order approximation of the colour current is considered and the correlation function of random oscillations is introduced. In Sec. 4 we discuss the consistency with gauge symmetry of used approximation scheme. In sec. 5 the second and the third orders approximation of the colour current are studied, and the terms leading in the coupling constant are separated. In Sec. 6 the kinetic equation for longitudinal waves in QGP is derived. In Sec. 7 the nonlinear Landau damping rate is rewritten in the term of HTL-amplitudes. In Sec. 8 by the effective Ward identities, the gauge invariance of obtained damping rate is proved. In Sec. 9 the physical mechanisms defining the nonlinear scattering of waves by plasma particles are considered. In Sec. 10 association of the nonlinear Landau damping rate with damping rate, obtained on the basis of hard thermal loops approximation is discussed. In Conclusion possible ways of further development of the scrutinized theory are discussed.

2. THE INITIAL EQUATIONS. THE RANDOM-PHASE APPROXIMATION

We use metric \( g^{\mu\nu} = diag(1, -1, -1, -1) \) and choose units such that \( c = k_B = 1 \). The gauge field potentials are \( N_c \times N_c \)-matrices in a color space defined by \( A_{\mu} = A_{\mu}^a t^a \) with \( N_c^2 - 1 \) hermitian generators of \( SU(N_c) \) group in the fundamental representation. The field strength tensor \( F_{\mu\nu} = F_{\mu\nu}^a t^a \) with

\[
F_{\mu\nu}^a = \partial_\mu A_{\nu}^a - \partial_\nu A_{\mu}^a + gf^{abc} A_{\mu}^b A_{\nu}^c
\]

obeys the Yang-Mills (YM) equation in a covariant gauge

\[
\partial_\mu F^{\mu\nu}(X) - ig[A_{\mu}(X), F^{\mu\nu}(X)] - \xi^{-1} \partial_{\nu} A_{\mu}(X) = -j_{\nu}(X),
\]

where \( \xi \) is a gauge parameter. \( j_{\nu} \) is the colour current

\[
j_{\nu} = gt^a \int d^4y \rho^\nu'[Sp t^a (f_q - f_{\bar{q}}) + Tr(T^a f_q)],
\]

where \( T^a \) are hermitian generators of \( SU(N_c) \) in the adjoint representation \( ((T^a)^{bc} = -if^{abc}, Tr(T^a T^b) = N_c \delta^{ab}) \). We denote the trace over color indices in adjoint representation as \( Tr \). Distribution functions of quarks \( f_q \), antiquarks \( f_{\bar{q}} \), and gluons \( f_g \) satisfy the dynamical equations which in the semiclassical limit (neglecting spin effects) are \( [8] \)

\[
p^{\mu}D_{\mu} f_{q,\bar{q}} = \frac{1}{2} gp^{\mu} \{ F_{\mu\nu}, \frac{\partial f_{q,\bar{q}}}{\partial p_{\nu}} \} = 0,
\]

4
\[ p^\mu \tilde{D}_\mu f_g + \frac{1}{2} gp^\mu \{ F_{\mu\nu}, \frac{\partial f_g}{\partial p_\nu} \} = 0, \]

where \( D_\mu \) and \( \tilde{D}_\mu \) are covariant derivatives which act as

\[ D_\mu = \partial_\mu - ig [ A_\mu(X), \cdot ], \]
\[ \tilde{D}_\mu = \partial_\mu - ig [ A_\mu(X), \cdot ], \]

\([,]\) denotes commutator, \(\{,\}\) denotes anticommutator, and \( A_\mu, F_{\mu\nu} \) are defined as \( A_\mu = A^a_\mu T^a, F_{\mu\nu} = F^a_{\mu\nu} T^a \). Upper sign in the first equation (2.4) refers to quarks and lower one - to antiquarks.

We begin with consideration of dynamical equations (2.4). The distribution functions \( f_q, \bar{f}_q \) and \( f_g \) can be decomposed into two parts: regular and random ones, where latter are generated by spontaneous fluctuations in the plasma

\[ f_s = f_s^R + f_s^T, \ s = q, \bar{q}, g, \]

so that

\[ \langle f_s \rangle = f_s^R, \ \langle f_s^T \rangle = 0. \]

Here, angular brackets \( \langle \cdot \rangle \) indicate a statistical ensemble of averaging. The initial values of parameters which characterize the collective degree of a plasma freedom is such statistical ensemble. For almost linear collective motion to be considered below it may be initial values of oscillation phases.

Further we set

\[ A_\mu = A_\mu^R + A_\mu^T, \ \langle A_\mu^T \rangle = 0, \]

by definition. The regular (background) part of the field \( A_\mu^R \) will be considered equal to zero. The condition for which the last assumption holds, will be closer considered in Sec. 4.

Averaging the equation (2.4) over statistical ensemble, in view of (2.5)-(2.7), we obtain the equations for the regular parts of the distribution functions \( f^R_{q,q} \) and \( f^R_g \)

\[ p^\mu \partial_\mu f^R_{q,q} = igp^\mu \langle [ A^T_\mu, f^T_{q,q} ] \rangle = \frac{1}{2} gp^\mu \langle \{( F^T_{\mu\nu})_{L}, \frac{\partial f^T_{q,q}}{\partial p_\nu} \} \rangle \mp \frac{1}{2} gp^\mu \langle \{( F^T_{\mu\nu})_{NL}, \frac{\partial f^R_{q,q}}{\partial p_\nu} \} \rangle \mp \]

\[ + \frac{1}{2} gp^\mu \langle \{( F^T_{\mu\nu})_{NL}, \frac{\partial f^R_{q,q}}{\partial p_\nu} \} \rangle, \]

\[ p^\mu \partial_\mu f^R_g = igp^\mu \langle [ A^T_\mu, f^T_g ] \rangle - \frac{1}{2} gp^\mu \langle \{( F^T_{\mu\nu})_{L}, \frac{\partial f^T_g}{\partial p_\nu} \} \rangle - \frac{1}{2} gp^\mu \langle \{( F^T_{\mu\nu})_{NL}, \frac{\partial f^R_g}{\partial p_\nu} \} \rangle - \]

\[ - \frac{1}{2} gp^\mu \langle \{( F^T_{\mu\nu})_{NL}, \frac{\partial f^T_g}{\partial p_\nu} \} \rangle. \]
Here, indices "L" and "NL" denote the linear and nonlinear parts with respect to field $A^a_\mu$ of the strength tensor (2.1).

Subtracting (2.8) from (2.4), we define the equations for $f^T_{q\bar{q}}$ and $f^T_g$

$$p^\mu \partial_\mu f^T_{q\bar{q}} = ig^\mu ([A^T_\mu, f^T_{q\bar{q}}] - ([A^T_\mu, f^T_{q\bar{q}}]) + \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_L, \frac{\partial f^R_{q\bar{q}}}{\partial p_\nu}\} +$$

$$+ \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_L, \frac{\partial f^T_{q\bar{q}}}{\partial p_\nu}\} - \{(F^T_{\mu\nu})_L, \frac{\partial f^T_{q\bar{q}}}{\partial p_\nu}\} + \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_{NL} - (F^T_{\mu\nu})_{NL} \}, \frac{\partial f^T_{q\bar{q}}}{\partial p_\nu}\} +$$

$$+ \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_{NL}, \frac{\partial f^T_{q\bar{q}}}{\partial p_\nu}\} - \{(F^T_{\mu\nu})_{NL}, \frac{\partial f^T_{q\bar{q}}}{\partial p_\nu}\})\} (2.9)$$

$$p^\mu \partial_\mu f^T_g = ig^\mu ([A^T_\mu, f^T_g] - ([A^T_\mu, f^T_g]) - \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_L, \frac{\partial f^R_g}{\partial p_\nu}\} -$$

$$- \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_L, \frac{\partial f^T_g}{\partial p_\nu}\} - \{(F^T_{\mu\nu})_L, \frac{\partial f^T_g}{\partial p_\nu}\} - \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_{NL} - (F^T_{\mu\nu})_{NL} \}, \frac{\partial f^T_g}{\partial p_\nu}\} -$$

$$- \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_{NL}, \frac{\partial f^T_g}{\partial p_\nu}\} - \{(F^T_{\mu\nu})_{NL}, \frac{\partial f^T_g}{\partial p_\nu}\})\} (2.9)$$

The system of equations (2.8) and (2.9) is suitable for investigation of nonequilibrium processes in QGP such that the excitation energy of waves is small quantity in relation to the total energy of particles. In this case it can be used expansion in powers of oscillations amplitude of the random functions $f^T_s$

$$f^T_s = \sum_{n=1}^{\infty} f^T_{s(n)}, \ s = q, \bar{q}, g, \ (10)$$

where $f^T_{s(n)}$ collects the contributions of the $n$-th power in $A^T_\mu$. Substituting expansion (2.10) into (2.9), and collecting terms of the same order in $A^T_\mu$, we derive the system of equations

$$p^\mu \partial_\mu f^{T(1)}_{q\bar{q}} = \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_L, \frac{\partial f^R_{q\bar{q}}}{\partial p_\nu}\}, \ (11)$$

$$p^\mu \partial_\mu f^{T(2)}_{q\bar{q}} = ig^\mu ([A^T_\mu, f^{T(1)}_{q\bar{q}}] - ([A^T_\mu, f^{T(1)}_{q\bar{q}}]) +$$

$$+ \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_L, \frac{\partial f^{T(1)}_{q\bar{q}}}{\partial p_\nu}\} - \{(F^T_{\mu\nu})_L, \frac{\partial f^{T(1)}_{q\bar{q}}}{\partial p_\nu}\} + \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_{NL} - (F^T_{\mu\nu})_{NL} \}, \frac{\partial f^{T(1)}_{q\bar{q}}}{\partial p_\nu}\} +$$

$$+ \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_{NL}, \frac{\partial f^{T(1)}_{q\bar{q}}}{\partial p_\nu}\} - \{(F^T_{\mu\nu})_{NL}, \frac{\partial f^{T(1)}_{q\bar{q}}}{\partial p_\nu}\})\} \ (12)$$

$$p^\mu \partial_\mu f^{T(3)}_{q\bar{q}} = ig^\mu ([A^T_\mu, f^{T(2)}_{q\bar{q}}] - ([A^T_\mu, f^{T(2)}_{q\bar{q}}]) + \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_L, \frac{\partial f^{T(2)}_{q\bar{q}}}{\partial p_\nu}\} -$$

$$- \{(F^T_{\mu\nu})_L, \frac{\partial f^{T(2)}_{q\bar{q}}}{\partial p_\nu}\} + \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_{NL} - (F^T_{\mu\nu})_{NL} \}, \frac{\partial f^{T(1)}_{q\bar{q}}}{\partial p_\nu}\} -$$

$$- \frac{1}{2} g^\mu \{(F^T_{\mu\nu})_{NL}, \frac{\partial f^{T(1)}_{q\bar{q}}}{\partial p_\nu}\} - \{(F^T_{\mu\nu})_{NL}, \frac{\partial f^{T(1)}_{q\bar{q}}}{\partial p_\nu}\})\} etc.
The nonlinear colour current is expressed as

\[ j_\mu = j_\mu^{\text{R}} + j_\mu^{\text{T}}, \quad \langle j_\mu \rangle = j_\mu^{\text{R}}, \quad j_\mu^{\text{T}} = \sum_{n=1}^{\infty} j_\mu^{(n)}, \quad (2.14) \]

where

\[ j_\mu^{(n)} = g t^a \int d^4 p p_\mu [\text{Sp} t^a (f_\mu^{(n)} - f_\mu^{\text{R}}) + \text{Tr} (T^a f_\mu^{(n)})]. \quad (2.15) \]

Now we turn to the Yang-Mills equation (2.2), connecting the gauge field with the colour current. Averaging Eq. (2.2) and subtracting the average d equation from (2.2) in view of Eqs. (2.7) and (2.14), we find (for \( A_R^{\mu} = 0 \))

\[ \partial_{\mu} (F_{\mu \nu}^{\text{T}}) - \xi^{-1} \partial^{\nu} \partial_{\mu} A_{\mu}^{\text{T}} + j_\nu^{\text{T}(1)} = -\left( j_\nu^{\text{T}(N_L)} - \langle j_\nu^{\text{T}(N_L)} \rangle \right) + ig \partial_{\mu} ([A_{\mu}^{\text{T}}, A_{\nu}^{\text{T}}] - \langle [A_{\mu}^{\text{T}}, A_{\nu}^{\text{T}}] \rangle) + g^2 ([A_{\mu}^{\text{T}}, [A_{\mu}^{\text{T}}, A_{\nu}^{\text{T}}]] - \langle [A_{\mu}^{\text{T}}, [A_{\mu}^{\text{T}}, A_{\nu}^{\text{T}}]] \rangle). \quad (2.16) \]

Here, in the l.h.s. we collect all linear terms with respect to \( A_{\mu}^{\text{T}} \) and we denote: \( j_\nu^{\text{T}(N_L)} \equiv j_\nu^{\text{T}(2)} + j_\nu^{\text{T}(3)} + \ldots \). To account for nonlinear interaction between waves and particles in QGP (in first non-vanishing approximation over the energy of waves), it is sufficiently to restrict the consideration to third order in powers of \( A_{\mu}^{\text{T}} \) in expansion (2.10).

We introduce the following assumption. Eqs. (2.8) represent the kinetic equations for averaged distribution functions. The correlation functions in the r.h.s. of these equations have meaning of the collision terms due to particle-wave interaction and describe the influence of plasma waves to a background state. Recently research of similar equations has attracted detailed attention [16], since on scale large wavelengths of collective excitations (\( \lambda \sim 1/g^2 T \)) they lead to Bödeker’s effective theory [13]. We suppose that a characteristic time of nonlinear relaxation of the oscillations is small quantity as compared with a time of relaxation of the distribution particles \( f_\mu^{\text{R}} \). Therefore we neglect by change of regular part of the distribution functions with space and time, assuming that these functions are specified and describe the global equilibrium in QGP

\[ f_{q,\bar{q}}^{R} \equiv f_{q,\bar{q}}^{0} = \frac{2N_f \theta(p_0)}{(2\pi)^3} \frac{1}{e(pu)/T + \mu + 1}, \quad f_{g}^{R} \equiv f_{g}^{0} = \frac{2 \theta(p_0)}{(2\pi)^3} \frac{1}{e(pu)/T - 1}, \quad (2.17) \]

where \( N_f \) - being the number of flavours for massless quarks, \( u_\mu \) is the four-velocity of the plasma at temperature \( T \), and \( \mu \) is the quark chemical potential.

3. THE LINEAR APPROXIMATION. THE CORRELATION FUNCTION OF THE RANDOM OSCILLATIONS

We will now come to the derivation of kinetic equation for waves. The initial equation is Eq. (2.16). The l.h.s. of Eq. (2.16) contains a linear approximation of the colour
current, explicit form of which is easily defined from Eq. (2.11). We prefer to work in momentum space; the corresponding equations are obtained by using

\[ A^T_\mu(x) = \int d^4 k A^T_\mu(k) e^{-ikx}, \]

and similar translations for \( f^T_{q,q}, f^T_g \). The result of Fourier transformation for Eq. (2.11) is

\[ f^{T(1)}_{q,q}(k,p) = \mp g \frac{\chi^\nu\lambda(k,p)}{pk + ip_0\epsilon} \frac{\partial f^0_{q,q}}{\partial p^\lambda} A^T_\nu(k), \quad f^{T(1)}_g(k,p) = -g \frac{\chi^\nu\lambda(k,p)}{pk + ip_0\epsilon} \frac{\partial f^0_g}{\partial p^\lambda} A^T_\nu(k), \] (3.1)

\( \epsilon \to +0. \)

Here \( \chi^{\nu\lambda}(k,p) = (pk) g^{\nu\lambda} - p^k g^{\nu\lambda} \). Substituting (3.1) into (2.15) (more precisely, in Fourier transformation of (2.15), we define a well-known form [8, 22] current approximation which is linear with respect to a gauge field

\[ j^{T(1)}(k) = \Pi^{\mu\nu}(k) A^T_\nu(k), \] (3.2)

where

\[ \Pi^{\mu\nu}(k) = g^2 \int d^4 p \frac{\mu^\nu(p^\mu(k)\partial_p) - (kp)\partial_p^\nu) N_{eq}}{pk + ip_0\epsilon} \]

is the high temperature polarization tensor, and \( N_{eq} = \frac{1}{2}(f^0_q + f^0_g) + N_c f^0_g \).

Further we rewrite Eq. (2.16) in the momentum space. Taking into account (3.2), we obtain

\[
\begin{align*}
& [k^2 g^{\mu\nu} - (1 + \xi^{-1}) k^\mu k^\nu - \Pi^{\mu\nu}(k)] A^{Tb}_\nu(k) = j^{Tb\mu}_N(k) - \langle j^{Tb\mu}_N(k) \rangle + \\
& + f^{bde} \int S^{(I)\mu\nu\lambda}_k(k_1,k_2) A^{Tc}_\nu(k_1) A^{Td}_\lambda(k_2) - \langle A^{Tc}_\nu(k_1) A^{Td}_\lambda(k_2) \rangle \delta(k - k_1 - k_2) dk_1 dk_2 + \\
& + f^{bde} f^{fde} \int \sum_{k_1,k_2,k_3} S^{(I)\mu\nu\sigma}_k(k_1,k_2,k_3) A^{Tc}_\nu(k_1) A^{Td}_\lambda(k_2) A^{Te}_\sigma(k_3) - \langle A^{Tc}_\nu(k_1) A^{Td}_\lambda(k_2) A^{Te}_\sigma(k_3) \rangle \\
& \quad \delta(k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3,
\end{align*}
\] (3.3)

where

\[ S^{(I)\mu\nu\lambda}_k(k_1,k_2) = -ig(k^\nu g^{\mu\lambda} + k^\nu g^{\mu\lambda} - k^\mu g^{\nu\lambda}), \quad S^{(I)\mu\nu\sigma}_k(k_1,k_2,k_3) = g^2 g^{\nu\lambda} g^{\mu\sigma}. \] (3.4)

Let us multiply Eq. (3.3) by the complex conjugate amplitude \( A^{T^*a}_\mu(k') \) and average it

\[
\begin{align*}
& [k^2 g^{\mu\nu} - (1 + \xi^{-1}) k^\mu k^\nu - \Pi^{\mu\nu}(k)] \langle A^{T^*a}_\mu(k') A^{Tb}_\nu(k) \rangle = \langle A^{T^*a}_\mu(k') j^{Tb\mu}_N(k) \rangle + \\
& + f^{bde} \int S^{(I)\mu\nu\lambda}_k(k_1,k_2) \langle A^{T^*a}_\mu(k') A^{Tc}_\nu(k_1) A^{Td}_\lambda(k_2) \rangle \delta(k - k_1 - k_2) dk_1 dk_2 + \\
& + f^{bde} f^{fde} \int \sum_{k_1,k_2,k_3} S^{(I)\mu\nu\sigma}_k(k_1,k_2,k_3) \langle A^{T^*a}_\mu(k') A^{Tc}_\nu(k_1) A^{Td}_\lambda(k_2) A^{Te}_\sigma(k_3) \rangle \delta(k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3.
\end{align*}
\] (3.5)

We introduce the correlation function of the random oscillations

\[ I^{ab}_{\mu\nu}(k',k) = \langle A^{T^*a}_\mu(k') A^{Tb}_\nu(k) \rangle. \] (3.6)
In conditions of the stationary and homogeneous of QGP, when the correlation function (3.6) in the coordinate representation depends on the difference of coordinates and time \( \Delta X = X' - X \) only, we have

\[
I^{ab}_{\mu \nu}(k', k) = I^{ab}_{\mu \nu}(k') \delta(k' - k).
\] (3.7)

By the effects of the nonlinear interaction of waves and particles, the state of QGP becomes weakly inhomogeneous and weakly nonstationary. The medium nongomogeneity and nonstationary lead to a delta-function broadening, and \( I^{ab}_{\mu \nu} \) depends on both arguments.

Let us introduce \( I^{ab}_{\mu \nu}(k', k) = I^{ab}_{\mu \nu}(k, \Delta k) \), \( \Delta k = k' - k \) with \( | \Delta k/k | \ll 1 \) and insert the correlation function in the Wigner form

\[
I^{ab}_{\mu \nu}(k, x) = \int I^{ab}_{\mu \nu}(k, \Delta k) e^{-i \Delta k x} d\Delta k,
\]
slowly depending on \( x \). In Eq. (3.5) we change \( k \leftrightarrow k' \), \( a \leftrightarrow b \), complex conjugate and subtract obtained equation from Eq. (3.5), beforehand expanding of the polarization tenzor into Hermitian and anti-Hermitian parts

\[
\Pi^{\nu \sigma}(k) = \Pi^{H \nu \sigma}(k) + \Pi^{A \nu \sigma}(k),
\]

\[
\Pi^{H \nu \sigma}(k) = \Pi^{* H \nu \sigma}(k),
\]

\[
\Pi^{A \nu \sigma}(k) = -\Pi^{* A \nu \sigma}(k).
\]

We assume that anti-Hermitian part of \( \Pi^A \) is small in comparison with \( \Pi^H \) and it is a value of the same smallness order, as the nonlinear terms in the r.h.s.. Therefore it can be suggested that \( \Pi^{A \nu \sigma}(k) \simeq \Pi^{A \nu \sigma}(k') \), and the term with \( \Pi^A \) can be rearranged to the r.h.s.. The remaining terms in the l.h.s. we expanded in a series in powers of \( \Delta k \) to first smallness order. This corresponds to \( \text{gradient expansion} \) procedure usually used in derivation of kinetic equations \([8, 12, 14]\). Multiplying obtained equation by \( e^{-i \Delta k x} \) and integrating over \( \Delta k \) with regard to

\[
\int \Delta k \lambda I^{ab}_{\mu \nu}(k, \Delta k) e^{-i \Delta k x} d\Delta k = i \frac{\partial I^{ab}_{\mu \nu}(k, x)}{\partial x^\lambda},
\]
we obtain finally

\[
\frac{\partial}{\partial k^\lambda}[k^2 g^{\mu \nu} - (1 + \xi^{-1}) k^\mu k^\nu - \Pi^{H \mu \nu}(k)] \frac{\partial I^{ab}_{\mu \nu}}{\partial x^\lambda} = 2i \Pi^{A \mu \nu} I^{ab}_{\mu \nu} -
\]

\[
-i \int dk' \{ (A^{T_s a}(k') j^{T b \mu}(k)) - (A^{T b \mu}(k) j^{T b \mu}(k')) \} =
\]

\[
-i \left\{ \int_{bcde} dk' dk_1 dk_2 S^{(I)}_{k', k_1, k_2} (A^{T^a \mu}(k') A^{T^c \nu}(k_1) A^{T^d \lambda}(k_2)) \delta(k - k_1 - k_2) -
\]

\[
-i \left\{ \int_{bcde} dk' dk_1 dk_2 S^{(I) \mu \lambda}_{k', k_1, k_2} (A^{T^a \mu}(k) A^{T^c \nu}(k_1) A^{T^d \lambda}(k_2)) \delta(k' - k_1 - k_2) \right\} - (3.8)
\]

\[
-i \left\{ \int_{bcde} \sum_{k, k_1, k_2, k_3} (A^{T^a \mu}(k') A^{T^c \nu}(k_1) A^{T^d \lambda}(k_2) A^{T^e \sigma}(k_3)) \delta(k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3 \right\} -
\]
\[-f^{ac} f^{de} \int \Sigma_{k', k_1, k_2, k_3} \langle A^{T^d}_\mu(k_1) A^{T^c}_\nu(k_2) A^{T^e}_\lambda(k_3) \rangle \delta(k' - k_1 - k_2 - k_3) dk_1 dk_2 dk_3 \},
\]
where \( j_{NL}^{T \mu}(k) = j^{T(2) \mu}(k) + j^{T(3) \mu}(k) \).

We make several remarks relative to obtained Eq. (3.8). The term with \( \Pi^A \) introducing in the r.h.s. of Eq. (3.8) corresponds to the linear Landau damping. However, as was shown by Heinz and Siemens [18], linear Landau damping for waves in QGP is absent and hence this term vanishes. Further, the terms with \( \Sigma(I) \) can be omitted also. These terms will be enter into kinetic equation for plasmons (Sec. 5) with \( \text{Im} \Sigma(I) \) that vanishes by reality of the function \( \Sigma(I) \).

4. CONSISTENCY WITH GAUGE SYMMETRY

In this Section we shall discuss the consistency of approximation scheme which we use with requirements of the non-Abelian gauge symmetry.

The initial dynamical equations (2.4) and Yang-Mills equation (2.2) (without the gauge-fixing condition) transform covariantly under local transformations

\[ \tilde{A}_\mu(X) = h(X)(A_\mu(X) + \frac{i}{g} \partial_\mu)h^\dagger(X), \quad h(X) = \exp(i\theta^a(X)t^a), \]

with parameter \( \theta^a(X) \). We also have transformations of the quark, antiquark and gluon distribution functions [8]

\[ \tilde{f}_{q, \bar{q}}(p, X) = h(X)f_{q, \bar{q}}(p, X)h^\dagger(X), \quad \tilde{f}_g(p, X) = H(X)f_g(p, X)H^\dagger(X), \]

where \( H^{ab}(X) = \text{Sp}[t^a h(X)t^b h^\dagger(X)] \).

As known (see, e.g. [16]), after the splitting of (2.5), (2.7) the resulting equations left two symmetries, the background gauge symmetry,

\[ \tilde{A}^R_\mu(X) = h(X)(A^R_\mu(X) + \frac{i}{g} \partial_\mu)h^\dagger(X), \quad \tilde{A}^T_\mu(X) = h(X)A^T_\mu(X)h^\dagger(X), \quad (4.1) \]

and the fluctuation gauge symmetry,

\[ \tilde{A}^R_\mu(X) = 0, \quad \tilde{A}^T_\mu(X) = h(X)(A^R_\mu(X) + \tilde{A}^T_\mu(X) + \frac{i}{g} \partial_\mu)h^\dagger(X), \quad (4.2) \]

The condition which we impose on regular part of a gauge field \( A^R_\mu \) and requirement that the statistical average of the fluctuation vanishes \( \langle A^T_\mu \rangle = 0 \), break both of types symmetry (4.1) and (4.2). Thus in the case of a gauge transformation (4.1) we obtain \( \tilde{A}^R_\mu \neq 0 \), and in the case of (4.2) we come to noninvariance of the constraint \( \langle A^T_\mu \rangle = 0 \). Moreover, introduced correlation function (3.6) also have explicitly a gauge noncovariant
character. This leads to the fact that calculations in the preceding Sections are gauge noncovariant, and therefore their value is doubted.

Nevertheless, there is the special case, when preceding (and following) conclusions are justified. This is the case of a colourless fluctuation, for which \( I_{\mu\nu}^{ab}(k, x) = \delta^{ab} I_{\mu\nu}(k, x) \). We can obtain a gauge invariant equation for \( I_{\mu\nu}(k, x) \) only in this restriction, in spite of the fact that the intermediate calculations spoil non-Abelian gauge symmetry of the initial equations (2.2)-(2.4).

In principle, we shall be able to maintain explicit background gauge symmetry (4.1) at each step of our calculations, as it has been done, for example, by Blaizot and Iancu [14] for derivation of the Boltzmann equation describing the relaxation of ultrasoft (\( \lambda \sim 1/g^2 T \)) colour excitations. First of all we assume that \( A_R^\mu \neq 0 \). Then as the gauge-fixing condition for the random field \( A_R^\mu \), we choose the background field gauge

\[
\mathcal{D}_\mu^R(X) A_R^\mu(X) = 0, \quad \mathcal{D}_\mu^R(X) \equiv \partial_\mu - ig A_R^\mu(X),
\]

which is manifestly covariant with respect to the gauge transformations of the background gauge field \( A_R^\mu(X) \). We lastly define a gauge covariant Wigner function as in Refs. [8, 14]

\[
\hat{I}_{\mu\nu}^{ab}(k, x) = \int \hat{I}_{\mu\nu}^{ab}(s, x) e^{iks} ds, \ s = X_1 - X_2, \ x = \frac{1}{2}(X_1 + X_2),
\]

where

\[
\hat{I}_{\mu\nu}^{ab}(s, x) \equiv U^{aa'}(x, x + \frac{s}{2}) I^{a'b'}(x + \frac{s}{2}, x - \frac{s}{2}) U^{b'b}(x - \frac{s}{2}, x),
\]

instead of the usual Wigner function \( I_{\mu\nu}^{ab}(k, x) \), whose "poor" transformation properties follow from initial definition \( I_{\mu\nu}^{ab}(X_1, X_2) = \langle A_T^{a}(X_1) A^{b}(X_2) \rangle \). The function \( U(x, y) \) is the non-Abelian parallel transporter

\[
U(x, y) = \text{P} \exp \left\{ -ig \int_{\gamma} dz^\mu A_R^\mu(z) \right\}.
\]

The path \( \gamma \) is the straight line joining \( x \) and \( y \).

The derivation of the kinetic equation for plasmons in this approach becomes quite cumbersome and non-trivial. For example, in the l.h.s. of equations for random parts of distributions (2.11)-(2.13), the covariant derivative \( \mathcal{D}_\mu^R \) will be used instead of the ordinary one \( \partial_\mu \). Besides, we cannot suppose that regular parts of distributions functions are specified and equal to equilibrium Fermi-Dirac and Bose-Einstein distributions (2.17). It is necessary also take into account their change using kinetic equations (2.8) with the collision terms in the r.h.s. of (2.8), which describe the reaction of the soft fluctuation on the background distributions. The correlators in the r.h.s. of Eq. (2.8) can be expressed in terms of the function \( I_{\mu\nu}^{ab} \) and the distributions of hard particles \( f_s^R(p, X), s = q, \bar{q}, g \), only.

However, if we restrict our consideration to study of colourless excitations and replace the distribution functions of hard particles by their equilibrium values (2.17), then it
leads to effective vanishing of terms explicitly contained the mean field $A^R_\mu$. This follows e.g., from the analysis of derivation of Boltzmann equation by Blaizot and Iancu [14]. Therefore the most simple way of derivation of kinetic equations for soft colourless QGP excitations is assume $A^R_\mu = 0$ and use prime gauge noncovariant correlator (3.6). In this case the background field gauge (4.3) is reduced to a covariant one. The kinetic equation obtained by this means is gauge invariant if all contributions at the leading order in $g$ to the probability of nonlinear wave scattering by hard thermal particles are taken into account (see also discussion in Conclusion).

In fact, the requirement of nontriviality of a colour structure of Wigner function $I^{ab}_{\mu\nu}(k, x)$ leads to necessity of existence of nonvanishing mean field $A^R_\mu$ and/or the external colour current and conversely, the existence of mean field in QGP results in colour structure $I^{ab}_{\mu\nu}(k, x)$, which is different from the identity.

5. THE SECOND AND THIRD APPROXIMATIONS OF THE COLOUR CURRENT

Now we concerned with computation of the nonlinear corrections to the current in the r.h.s. of basic Eq. (3.8).

At first we define $f^{T(2)}_{q\bar{q}}$. We carry out the Fourier transformation of Eq. (2.12) and substituting the obtained $f^{T(1)}_{q\bar{q}}$ from (3.1) into derived expression, we find

\[
f^{T(2)}_{q\bar{q}} = g^2 \left[ t^b, t^c \right] p^\nu p^\lambda \int \frac{(k_2 \partial_\rho f^0_{q\bar{q}})}{pk_2 + ip_0\epsilon} \left( A^b_\nu(k_1)A^c_\lambda(k_2) - \langle A^b_\nu(k_1)A^c_\lambda(k_2) \rangle \right) \delta(k - k_1 - k_2)dk_1dk_2 + \]
\[+ \frac{\mu^2}{2} \left\{ t^b, t^c \right\} \int \chi^{\nu\lambda}(k_1, p) \frac{\partial}{\partial p^\lambda} \left( \chi^{\sigma\rho}(k_2, p) \frac{\partial f^0_{q\bar{q}}}{p_2 + ip_0\epsilon} \right) \left( A^b_\nu(k_1)A^c_\lambda(k_2) - \langle A^b_\nu(k_1)A^c_\lambda(k_2) \rangle \right) \delta(k - k_1 - k_2)dk_1dk_2.
\]

From here on the suffix "T" for a gauge field is omitted. The expression for $f^{T(2)}_q(k, p)$ is obtained from (5.1) by choosing upper sign and replacements $f^0_q \rightarrow f^0_q, t^a \rightarrow T^a$. Substituting obtained expressions $f^{T(2)}_s, s = q, \bar{q}, g$ into (2.15) (for $n = 2$), we find required current correction

\[
j^{T(2)\mu}(k) = -ig^3 f^{abc} \int d^4p \frac{p^\mu p^\nu p^\lambda}{pk + ip_0\epsilon} \frac{k_2\partial_\rho N_{eq}}{pk_2 + ip_0\epsilon} \left( A^b_\nu(k_1)A^c_\lambda(k_2) - \langle A^b_\nu(k_1)A^c_\lambda(k_2) \rangle \right) \delta(k - k_1 - k_2)dk_1dk_2 + \]
\[+ \frac{g^3}{4} f^{abc} \int d^4p \frac{p^\mu \chi^{\nu\lambda}(k_1, p)}{pk + ip_0\epsilon} \frac{\partial}{\partial p^\lambda} \left( \chi^{\sigma\rho}(k_2, p) \frac{\partial f^0_q}{p_2 + ip_0\epsilon} \right) \left( A^b_\nu(k_1)A^c_\lambda(k_2) - \langle A^b_\nu(k_1)A^c_\lambda(k_2) \rangle \right) \delta(k - k_1 - k_2)dk_1dk_2.
\]
The contribution of gluons to the expression with symmetric structure constant \( d^{abc} \) here drops out. This is connected with the fact that in calculation of trace of anti-commutators we have: \( \text{Sp} t^a \{ t^b, t^c \} = \frac{1}{2} d^{abc} \) - for quarks and antiquarks, and \( \text{Tr} T^a \{ T^b, T^c \} = 0 \) - for gluons. The symmetry of contributions can be restored if we note that besides usual gluon current \( j^g(x) = g t^a \int d^4p \, p^\mu \text{Tr}(T^a f_g(x, p)) \), the dynamical equation for gluons admits a covariant conserving quantity

\[
\zeta g t^a \int d^4p \, \text{Tr}(\mathcal{P}^a f_g(x, p)),
\]

where \( (\mathcal{P}^a)^{bc} = d^{abc} \) and \( \zeta \) is a certain arbitrary constant. The covariant continuity of (5.3) is evident from the identity: \( [\mathcal{P}^a, T^b] = i f^{abc} T^c \). On addition of (5.3) to (2.3) we have contributions to the nonlinear current corrections only. Adding (5.3) to the second current iteration (2.15) and taking into account the equality

\[
\text{Sp} \mathcal{P}^a \{ T^b, T^c \} = N_c d^{abc},
\]

we derive more general expression for \( j^{T(2)} \), instead of (5.2)

\[
j^{T(2)\mu}(k) = \int S_{k,k_1,k_2}^{abc\mu\nu}(A^a_\nu(k_1) A^a_\nu(k_2) - \langle A^a_\nu(k_1) A^a_\nu(k_2) \rangle) \delta(k-k_1-k_2) dk_1 dk_2,
\]

where \( S_{k,k_1,k_2}^{abc\mu\nu} = f^{abc} S_{k,k_1,k_2}^{(III)\mu\nu} + d^{abc} S_{k,k_1,k_2}^{(III)\mu\nu} \),

\[
S_{k,k_1,k_2}^{(III)\mu\nu} = -ig^3 \int d^4p \frac{p^\mu p' \partial \zeta N_{eq}}{p k + i p_\rho \epsilon} \frac{(k_2 \partial p_{\rho N_{eq}})}{p k_2 + i p_\rho \epsilon},
\]

\[
S_{k,k_1,k_2}^{(III)\mu\nu} = \frac{g^3}{2} \int d^4p \frac{p^\mu \chi^{\nu}(k_1, p) \partial \lambda^\mu}{p k + i p_\rho \epsilon} \frac{(k_2 \partial p_{\rho \chi^{\nu}})}{p k_2 + i p_\rho \epsilon},
\]

\[
\tilde{N}_{eq} = \frac{1}{2} (f_q^0 - \bar{f}_q^0) + \zeta N_{eq} f_g^0.
\]

The tensor structure of \( S_{k,k_1,k_2}^{(III)\mu\nu} \) exactly coincides with appropriate expression obtained in calculation of \( j^{T(2)\mu} \) in Abelian plasma [21], and hence piece of current with \( d^{abc} \) has a meaning of Abelian part of the colour current \( j^{T(2)\mu} \). The term with \( S_{k,k_1,k_2}^{(III)\mu\nu} \) is purely non-Abelian, i.e. it has no Abelian counterpart.

Let us estimate orders of \( S^{(II)} \) and \( S^{(III)} \). Following usual terminology [1], we call an energy or a momentum "soft" when it is of order \( gT \), and "hard" when it is of order \( T \). We will be considered, as in Ref. [12], that collective excitations carrying soft momenta, i.e. \( k \sim gT \), and plasma particles have the typical hard energies: \( p \sim T \). In a coordinate representation the first of conditions denotes that oscillation amplitude \( A^a_\mu \) and distribution functions of hard particles \( f_s(X, p), s = q, \bar{q}, g \) change on the scale \( X \sim 1/gT \). On this basis, we have the following estimate for \( S^{(II)} \)

\[
S_{k,k_1,k_2}^{(II)\mu\nu} \sim g^2 T.
\]
Here, we considered that by virtue of the definitions (2.17) \( N_{eq} \sim 1/T^2 \).

In expression (5.6) the integral of energy with gluon distribution function is infrared divergent. In a similar manner \([17]\), we regulate it by introducing an electric mass cutoff of order \( gT \), and only take the leading term in \( g \). Then it can be found that in (5.6), the part related to the gluon distribution function is of order \( g^0T\ln g \) and the other part related to the quark and antiquark distribution functions is of order \( g^3T \), i.e.

\[
S_{k,k_1,k_2}^{(III)\mu\nu\lambda} \sim g^2T(g\ln g) + g^3T.
\]

Hence, \( S^{(III)} \) which is purely non-Abelian, is of lower order in the coupling constant than \( S^{(III)} \), which has an Abelian counterpart. This fact was first seen in Ref. [17].

The expression for a colour current in third order in field is defined by means of reasoning similar previous ones. Performing the Fourier transformation of equation (2.13), taking into account (3.1), (5.1) and equalities

\[
\text{Tr} \{\{T^a, T^b\}\{T^d, T^e\} \} = N_c d^{abc} d^{cde} + 4\delta^{ab}\delta^{cd} + 2\delta^{ad}\delta^{ce} + 2\delta^{ae}\delta^{bd},
\]

we find the required form of \( j^{T(3)\mu\lambda}(k) \)

\[
j^{T(3)\mu\lambda}(k) = \int \sum_{k,k_1,k_2,k_3} \delta^{ab}\delta^{cd} (A^b_{\mu}(k_3) A^d_{\lambda}(k_1) A^e_{\sigma}(k_2) - A^b_{\mu}(k_3) A^d_{\lambda}(k_1) A^e_{\sigma}(k_2)) - \\
- \langle A^b_{\nu}(k_3) A^d_{\lambda}(k_1) A^e_{\sigma}(k_2) \rangle \delta(k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3 + \\
+ d^{abc} d^{cde} \int \sum_{k,k_1,k_2,k_3} A^b_{\mu}(k_3) A^d_{\lambda}(k_1) A^e_{\sigma}(k_2) - \langle A^b_{\nu}(k_3) A^d_{\lambda}(k_1) A^e_{\sigma}(k_2) \rangle \\
\delta(k - k_1 - k_2 - k_3) dk_1 dk_2 dk_3.
\]

Here,

\[
\Sigma^{abde\mu\nu\lambda\sigma}_{k,k_1,k_2,k_3} = f^{abc} f^{cde} \Sigma^{(II)\mu\lambda\sigma}_{k,k_1,k_2,k_3} + f^{abc} d^{cde} \Sigma^{(III)\mu\nu\lambda\sigma}_{k,k_1,k_2,k_3} + \delta^{ab}\delta^{de} \Sigma^{(IV)\mu\lambda\sigma}_{k,k_1,k_2,k_3} + d^{abc} f^{cde} \Sigma^{(V)\mu\lambda\sigma}_{k,k_1,k_2,k_3} + \\
+ d^{abc} d^{cde} \Sigma^{(VI)\mu\nu\lambda\sigma}_{k,k_1,k_2,k_3} + \delta^{ab}\delta^{de} + \delta^{ad}\delta^{ce} + \delta^{ae}\delta^{bd} \Sigma^{(V)\mu\lambda\sigma}_{k,k_1,k_2,k_3} + \delta^{ab}\delta^{de} \Sigma^{(VI)\mu\lambda\sigma}_{k,k_1,k_2,k_3}.
\]

\[
\Sigma^{(II)\mu\lambda\sigma}_{k,k_1,k_2,k_3} = -g^4 \int d^4p \frac{p^\mu p^\nu p^\lambda p^\sigma}{pk + ip_0\epsilon} \frac{1}{p(k_1 + k_2) + ip_0\epsilon} \frac{1}{p(k_2 + ip_0\epsilon)},
\]

\[
\Sigma^{(IV)\mu\nu\lambda\sigma}_{k,k_1,k_2,k_3} = -\frac{g^4}{2N_c} \int d^4p \frac{p^\mu \chi_{\nu\tau}^\rho(k_3, p)}{pk + ip_0\epsilon} \frac{\partial}{\partial p^\tau} \left( \frac{\chi^\lambda_{\nu\tau}(k_1, p)}{p(k_1 + k_2) + ip_0\epsilon} \frac{\partial}{\partial p^\lambda} \left( \frac{\chi^\rho_{\nu\tau}(k_2, p)}{p(k_2 + ip_0\epsilon)} \frac{\partial}{\partial p^\rho} \right) \right),
\]

The expression for \( \Sigma^{(III)} \) is obtained from (5.11) by exception of quark and antiquark contributions. The availability of the term with \( \Sigma^{(III)} \) is reflection of more complicated
colour structure of the gluon kinetic equation in comparison with quark and antiquark equations, that manifests here, in appearance of additional terms in (5.8) as compared with

\[ \text{Sp}(\{t^a, t^b\}\{t^d, t^e\}) = \frac{1}{2} d^{abc} d^{cde} + \frac{1}{N_c} \delta^{ab} \delta^{de}. \]

The terms with \( \Sigma^{(III)}, \Sigma^{(V)} \) and \( R \) are defined as the interference of Abelian and non-Abelian contributions. For colourless fluctuations of QGP, which we study in a given paper, the correlation function (3.6) is proportional to the identity. This leads to the absence of the interference of Abelian and non-Abelian contributions. For this reason their explicit form is not given here.

At the end of this Section we estimate the order of \( \Sigma^{(I)} \) and \( \Sigma^{(IV)} \). It follows from the expression (5.1) that

\[ \Sigma_{k,k_1,k_2,k_3}^{(II)} \sim g^2. \] (5.12)

Cutting off, as in the previous Section, integration limit for the gluon distribution function, we find

\[ \Sigma^{(IV)} \sim \Sigma^{(VI)} \sim g^3 + g^4, \quad \Sigma^{(VII)} \sim g^3. \]

By this means, purely non-Abelian contribution of \( \Sigma^{(II)} \) is of lower order in the coupling constant than Abelian - \( \Sigma^{(IV)}, \Sigma^{(VI)} \) and \( \Sigma^{(VII)} \).

6. THE KINETIC EQUATION FOR LONGITUDINAL WAVES

Now we turn to initial equation for waves (3.8). We substitute obtained nonlinear corrections of induced current by field (5.4) and (5.9) into this equation.

Because of a nonlinear wave interaction the phase correlation effects take plays. By virtue of their smallness, fourth-order correlators can be approximately divided into product of the correlation functions \( \langle A^*(k')A(k) \rangle \). For third-order correlation functions this decomposition vanishes, and it should be considered a weak correlation of phases fields. For this purpose we use the nonlinear equation of a field (3.3), taking into account only the terms of the second order in \( A \), in the r.h.s. of (3.3)

\[ [k^2 g^{\mu\nu} - (1 + \xi^{-1}) k^\mu k^\nu - \Pi^{\mu\nu}(k)] A_\nu^a(k) = \]

\[ = f^{abc} \int S_{k,k_1,k_2}^{\mu\nu\lambda}(A_\nu^b(k_1) A_\lambda^c(k_2) - \langle A_\nu^b(k_1) A_\lambda^c(k_2) \rangle) \delta(k - k_1 - k_2) dk_1 dk_2. \] (6.1)

The approximate solution of this equation is in the form

\[ A_\mu^a(k) = A_\mu^{(0)}(k) - D_{\mu\nu}(k) f^{abc} \int S_{k,k_1,k_2}^{\nu\lambda\sigma}(A_\nu^{(0)b}(k_1) A_\lambda^{(0)c}(k_2) - \langle A_\nu^{(0)b}(k_1) A_\lambda^{(0)c}(k_2) \rangle) \delta(k - k_1 - k_2) dk_1 dk_2, \] (6.2)
where \( S_{k,k_1,k_2}^{\mu\lambda} \equiv S_{k,k_1,k_2}^{(I)\mu\lambda} + S_{k,k_1,k_2}^{(II)\mu\lambda} \) and \( A_\mu^{(0)}(k) \) is a solution of homogeneous Eq. (6.1) corresponding free fields, and

\[
D_{\mu\nu}(k) = -[k^2 g_{\mu\nu} - (1 + \xi^{-1}) k_\mu k_\nu - \Pi_{\mu\nu}(k)]^{-1}
\]

represents the medium modified (retarded) gluon propagator.

Now we substitute (6.2) into third-order correlation functions entering to Eq. (3.8). Because \( A_\mu^{(0)} \) represents amplitudes of entirely uncorrelated waves, the correlation function with three \( A_\mu^{(0)} \) drops out. In this case every term in \( \langle A_\mu^{*a}(k') A_\nu^{*c}(k_1) A_\sigma^{d}(k_2) \rangle \) and \( \langle A_\mu^{*a}(k) A_\nu^{*c}(k_1) A_\sigma^{d}(k_2) \rangle \) should be defined more exactly. In the correlation functions of four amplitudes, within the accepted accuracy, it can not make distinctions between the fields \( A \) and \( A_\mu^{(0)} \).

Finally Eq. (3.8) becomes

\[
\frac{\partial}{\partial k_\lambda} [k^2 g_{\mu\nu} - (1 + \xi^{-1}) k_\mu k_\nu - \Pi_{\mu\nu}(k)] \frac{\partial I_{ab}^{\mu\nu}}{\partial x_\lambda} =
\]

\[
= -i \int dk'dk_1 dk_2 dk_3 \left[ f^{be} f^{de} \delta(k - k_1 - k_2 - k_3) \sum_{k,k_1,k_2,k_3} \left( \langle A_\mu^{*a}(k') A_\nu^{*c}(k_3) A_\sigma^{d}(k_1) A_\sigma^{d}(k_2) \rangle - 
\right.
\]

\[
- \langle A_\mu^{*a}(k') A_\nu^{*c}(k_3) \rangle \langle A_\sigma^{d}(k_1) A_\sigma^{d}(k_2) \rangle - 
\]

\[
- f^{ac} f^{de} \delta(k' - k_1 - k_2 - k_3) \sum_{k',k_1,k_2,k_3} \left( \langle A_\mu^{*a}(k) A_\nu^{*c}(k_3) A_\sigma^{d}(k_1) A_\sigma^{d}(k_2) \rangle - 
\right.
\]

\[
- \langle A_\mu^{*a}(k) A_\nu^{*c}(k_3) \rangle \langle A_\sigma^{d}(k_1) A_\sigma^{d}(k_2) \rangle \right) +
\]

\[
+i f^{bd} f^{ae} \int dk' \int dk_1 dk_2 dk_3 \left( \langle A_\mu^{*a}(k') \rangle - D_{\rho\sigma}(k) \right) S_{k,k_1,k_2}^{\rho\sigma} \langle A_\mu^{*a}(k_1) A_\sigma^{d}(k_2) \rangle
\]

\[
A_\sigma^{d}(k_1) A_\sigma^{d}(k_2) \rangle \langle A_\mu^{*a}(k_1) A_\sigma^{d}(k_2) \rangle \delta(k - k_1 - k_2) \delta(k' - k_1 - k_2') \right).
\]

Here, we keep the terms of leading order in \( g \) only and set

\[
\tilde{\Sigma}_{k,k_1,k_2,k_3}^{\mu\nu\lambda\sigma} \equiv \sum_{k,k_1,k_2,k_3} \left( S_{k,k_1,k_2}^{\mu\nu\rho} + S_{k,k_1,k_2}^{\mu\rho\nu} \right) D_{\rho\sigma}(k_1 + k_2) S_{k_1,k_2}^{\sigma\lambda\rho}(k_1 + k_2) S_{k_1,k_2,k_3}^{\mu\sigma\lambda\rho}.
\]

It follows from the definition (6.3) that the propagator is of the order \( \sim 1/g^2 T^2 \). Taking into account (3.4), (5.7) and (5.12), we see that all terms in the r.h.s. of (6.4) are of the same order. This explains the fact that in the expansion of the current (2.14) the following term - \( j^{T(3)} \) should be retained in addition to the first nonlinear correction \( j^{T(2)} \): it leads to the effects of the same order of magnitude.

Let us make the correlation decoupling of the fourth-order correlators in the r.h.s. of Eq. (6.4) in the terms of the pair ones by the rule

\[
\langle A(k_1) A(k_2) A(k_3) A(k_4) \rangle = \langle A(k_1) A(k_2) \rangle \langle A(k_3) A(k_4) \rangle + \langle A(k_1) A(k_3) \rangle \langle A(k_2) A(k_4) \rangle +
\]

\[
+ \langle A(k_1) A(k_4) \rangle \langle A(k_2) A(k_3) \rangle.
\]
Taking into account that the spectral densities in the r.h.s. of Eq. (6.4) can be considered as stationary and homogeneous those, i.e. having the form (3.7), and setting $I_{\mu\nu}^{ab} = \delta^{ab}I_{\mu\nu}$, we find instead of Eq. (6.4)

\[
\frac{\partial}{\partial k_\lambda} \left[ k^2 g^{\mu\nu} - (1 + \xi^{-1})k^\mu k^\nu - \Pi^{H\mu\nu}(k) \right] \frac{\partial I_{\mu\nu}}{\partial x^\lambda} = 2N_c \int dk_1 \text{Im}(\tilde{\Sigma}^{\mu\nu\sigma\lambda}_{k,k,k_1,-k_1} - \tilde{\Sigma}^{\mu\nu\sigma\lambda}_{k,k,k_1,k_1})I_{\mu\lambda}(k)I_{\nu\sigma}(k_1) + \text{N}_c\text{Im}(D_{\mu\rho}(k)) \int dk_1 dk_2 (S_{k,k_1}^{\mu\nu} - S_{k_1,k_2}^{\mu\nu}) (S_{k,k_1}^{\nu\sigma\lambda} - S_{k,k_2}^{\nu\sigma\lambda}) I_{\mu\lambda}(k_1)I_{\nu\sigma}(k_2)\delta(k-k_1-k_2).
\]

As it is known [23, 24], in global equilibrium QGP the oscillations of three types can be extended: the longitudinal, transverse and nonphysical (4-D longitudinal) ones. In this connection we define the Wigner function $I_{\mu\nu}(k,x) = I_{\mu\nu}$ in the form of expansion

\[
I_{\mu\nu} = P_{\mu\nu}I_k^t + Q_{\mu\nu}I_k^l + \xi D_{\mu\nu}I_k^\xi, I_k^{(t,l,n)}(k) = \frac{\partial I_k}{\partial x^{(t,l,n)}}(k,x).
\]

The Lorentz matrices in (6.7) are members of the basis [24, 25]

\[
P_{\mu\nu}(k) = g_{\mu\nu} - D_{\mu\nu}(k) - Q_{\mu\nu}(k), Q_{\mu\nu}(k) = \frac{\bar{u}_\mu(k)\bar{u}_\nu(k)}{\bar{u}^2(k)}, C_{\mu\nu}(k) = -\frac{(\bar{u}_\mu(k)k_\nu + \bar{u}_\nu(k)k_\mu)}{\sqrt{-2k^2u^2(k)}},
\]

\[
D_{\mu\nu} = k_\mu k_\nu/k^2, \bar{u}_\mu(k) = k^2u_\mu - k_\mu(ku).
\]

The effective gluon propagator (6.3) can be written in more convenient form

\[
\mathcal{D}_{\mu\nu}(k) = -P_{\mu\nu}(k)\Delta'(k) - Q_{\mu\nu}(k)\Delta^0(k) + \xi D_{\mu\nu}(k)\Delta^0(k),
\]

where $\Delta'(k) = 1/(k^2 - \Pi'^{\mu\nu}(k))$, $\Pi' = \frac{1}{\pi}I^{H\mu\nu}P_{\mu\nu}$, $\Pi' = \Pi^{H\mu\nu}Q_{\mu\nu}$; $\Delta^0(k) = 1/((\omega + i\epsilon)^2 - k^2)$. The shift $i\epsilon$ is introduced in $\Delta^0(k)$ to provide the right analytical properties. At finite temperature, the velocity of plasma introduces a preferred direction in space-time which breaks manifest Lorentz invariance. Let us assume that we are in the rest frame of the heat bath, so that $u_\mu = (1,0,0,0)$.

Further derivation of kinetic equation for longitudinal oscillations involves the same type of manipulations as in the theory of electromagnetic plasma, so we can afford to be sketchy.

Now we omit nonlinear terms and anti-Hermitian part of the polarization tensor in Eq. (3.5). Further substituting the function $\delta^{ab}Q_{\mu\nu}(k)I_k^t\delta(k' - k)$ instead of $I_{\mu\nu}^{ab}(k',k)$, we lead to the equation

\[
\Re (\varepsilon^t(k)) I_k^t = 0.
\]

Here, we use relation

\[
\Delta^{-1}(k) = k^2\varepsilon^t(k),
\]

\[\text{(6.9)}\]
where
\[
\varepsilon^l(k) = 1 + \frac{3\omega_p^2}{k^2}[1 - F(\frac{\omega}{|k|})], \quad F(x) = \frac{x}{2}[\ln \frac{1 + x}{1 - x} - i\pi(1 - |x|)]
\]
is the longitudinal colour electric permeability and \(\omega_p^2 = \frac{1}{18} g^2 T^2 (N_f + 2N_c)\) is a plasma frequency. The solution of this equation has the structure
\[
I_k^l = I_k^l \delta(\omega - \omega_k^l) + I_{-k}^l \delta(\omega + \omega_k^l), \quad \omega_k^l > 0,
\]
where \(I_k^l\) is a certain function of a wave vector \(k\) and \(\omega_k^l \equiv \omega^l(k)\) is a frequency of the longitudinal eigenwaves in QGP.

The equation describing the variation of spectral density of longitudinal oscillations is obtained from Eq. (6.6) by replacement: \(I_{\mu\nu} \rightarrow Q_{\mu\nu}(k)I_k^l\), where \(I_k^l\) is defined by (6.10). \(\delta\)-functions in (6.10) enable us to remove integration over frequency and thus we have instead of Eq. (6.6)
\[
\left(\frac{h^2}{k^2} \frac{\partial \text{Re} \varepsilon^l(k)}{\partial \omega^\lambda} \right)_{\omega = \omega_k^l} \frac{\partial I_k^l}{\partial x^\lambda} = 2N_c I_k^l \int dk_1 I_{k_1}^l \left(\text{Im} \left[\hat{\Sigma}_{\mu\nu\lambda\sigma} - \hat{\Sigma}_{\mu\nu\sigma\lambda} \right] \right)_{\omega = \omega_k^l, \omega_1 = \omega_k^l} +
\]
\[
+ \frac{N_c}{2} \int_0^\infty d\omega \int dk_1 dk_2 I_{k_1}^l I_{k_2}^l \left(G_{k_1,k_2} + G_{-k_1,k_2} + G_{k_1,-k_2} + G_{-k_1,-k_2}\right)_{\omega = \omega_k^l, \omega_1 = \omega_k^l, \omega_2 = \omega_k^l},
\]
where
\[
G_{k_1,k_2} = \text{Im}(\mathcal{D}_{\rho\alpha}(k)) \left(S_{\rho\mu\nu} - S_{\rho\mu\nu, \kappa_2} \right) \left(S_{\kappa_1,\kappa_2}^* - S_{\kappa_1,\kappa_2}^*, \kappa_1 \right) Q_{\mu\lambda}(k_1) Q_{\nu\sigma}(k_2) \delta(k - k_1 - k_2).
\]

Let us consider the terms in the r.h.s. of Eq. (6.11). The integral with the function \(G_{k_1,k_2}\) is different from zero if the conservation laws are obeyed
\[
k = k_1 + k_2,
\]
\[
\omega_k^l = \omega_{k_1}^l + \omega_{k_2}^l.
\]
These conservation laws describe a decay of one longitudinal wave into two longitudinal waves. However for a dispersion law of the longitudinal oscillations in QGP, these resonance equations have no solutions, i.e. this nonlinear process is forbidden. Therefore the integral with \(G_{k_1,k_2}\) vanishes. Remaining integrals with \(G\)-functions differ from \(G_{k_1,k_2}\) in that some of the interacting waves are not radiated but absorbed. They also vanish.

The expression
\[
\left(\hat{\Sigma}_{\mu\nu\lambda\sigma} - \hat{\Sigma}_{\mu\nu\sigma\lambda} \right) Q_{\mu\lambda}(k_1) Q_{\nu\sigma}(k_1) \mid_{\omega = \omega_k^l, \omega_1 = \omega_k^l},
\]
contains the factors
\[
1/(pk + ip_0\epsilon), 1/(pk_1 + ip_0\epsilon), 1/(p(k - k_1) + ip_0\epsilon).
\]
Imaginary parts of first two factors should be setting equal to zero, because they are connected with linear Landau damping of longitudinal waves (which is absent in QGP), and therefore, the imaginary part of the expression \((6.12)\) will be defined as

\[
\operatorname{Im} \left. \frac{1}{p(k - k_1) + ip_0 \epsilon} \right|_{\omega = \omega'_k, \omega_1 = \omega'_{k_1}} = -\frac{i \pi}{p_0} \delta(\omega'_k - \omega'_{k_1} - \mathbf{v}(k - k_1)).
\]

It follows that nonlinear term in the r.h.s. of \((6.11)\) with the function \((6.12)\) is different from zero if the conservation law is obeyed

\[
\omega'_k - \omega'_{k_1} - \mathbf{v}(k - k_1) = 0.
\]

This conservation law describes the process of scattering of plasmon by the hard thermal particle in QGP.

Let us consider in more detail the term in \((6.12)\) (see definition \((6.5)\) with propagator \(D_{\rho \alpha}(k - k_1)\). By expansion \((6.8)\) this propagator represents the nonlinear interaction of longitudinal waves with longitudinal ones through three types of intermediate oscillations: the transverse, longitudinal and nonphysical oscillations depending on a gauge parameter. The term with

\[
\left( P_{\rho \alpha}(k - k_1) \Delta^l(k - k_1) \right)_{\omega = \omega'_k, \omega_1 = \omega'_{k_1}}
\]

in general, describes two fundamentally different nonlinear processes:

1. if \((\omega'_k - \omega'_{k_1}, \mathbf{k} - \mathbf{k}_1)\) is a solution of the dispersion equation \(\Delta^l(k - k_1) = 0\), then this term describes the fusion process of two longitudinal oscillations in transverse eigenwave;

2. otherwise, it defines the process of nonlinear scattering of longitudinal waves in longitudinal those through the transverse virtual oscillation (for a virtual wave in distinction to the eigenwave, a frequency \(\omega\) and a wave vector \(\mathbf{k}\) are not connected with each other by the dispersion dependence \(\omega \neq \omega(\mathbf{k})\)).

The equality \(\Delta^l(k - k_1)|_{\omega = \omega'_k, \omega_1 = \omega'_{k_1}} = 0\) does not hold for longitudinal oscillations, as we see above and therefore, the term

\[
\left( Q_{\rho \alpha}(k - k_1) \Delta^l(k - k_1) \right)_{\omega = \omega'_k, \omega_1 = \omega'_{k_1}}
\]

defines only the process of scattering of longitudinal waves in longitudinal those through the longitudinal virtual oscillation.

The contribution of nonphysical intermediate oscillations

\[
\left( \xi D_{\rho \alpha}(k - k_1) \Delta^0(k - k_1) \right)_{\omega = \omega'_k, \omega_1 = \omega'_{k_1}}
\]
will be considered in Sec. 12.

The remaining terms in the Eq. (6.11) with \( \Sigma \) are distinguished from above considered ones by a sign of \( k_1 \), and describe the processes of simultaneous radiation or absorption by particles of two plasmons.

Summing the preceding and going to the function

\[
W^l_k = - \left( \omega_k^2 \frac{\partial \text{Re} \varepsilon^l(k)}{\partial \omega} \right)_{\omega = \omega_k^l} I^l_k,
\]

having the physical meaning of spectral density of longitudinal oscillations energy, we find from (6.11) the required kinetic equation for longitudinal waves in QGP

\[
\frac{\partial W^l_k}{\partial t} + \mathbf{V}_k^l \frac{\partial W^l_k}{\partial \mathbf{x}} = - \hat{\gamma} \{ \left( \frac{W^l_k}{\omega_k^l} \right) \} W^l_k,
\]

(6.14)

where

\[
\mathbf{V}_k^l = \frac{\partial \omega_k^l}{\partial k} = - \left[ \left( \frac{\partial \text{Re} \varepsilon^l(k)}{\partial \mathbf{k}} \right) \left( \frac{\partial \text{Re} \varepsilon^l(k)}{\partial \omega} \right) \right]_{\omega = \omega_k^l}
\]

is the group velocity of longitudinal oscillations and

\[
\hat{\gamma} \{ \left( \frac{W^l_k}{\omega_k^l} \right) \} \equiv \gamma^l(k) = 2N_c \int d\mathbf{k}_1 \left( \frac{W^l_{k_1}}{\omega_{k_1}} \right) \left[ \frac{1}{k^2 k_1^2} \left( \frac{\partial \text{Re} \varepsilon^l(k)}{\partial \omega} \right)^{-1} \left( \frac{\partial \text{Re} \varepsilon^l(k_1)}{\partial \omega_1} \right)^{-1} \right.
\]

\[
\left. \left( \frac{d}{dt} \left( \frac{W^l_{k_1}}{\omega_{k_1}} \right) \right) \right]_{\omega = \omega_k^l, \omega_1 = \omega_{k_1}^l}
\]

(6.15)

presents the damping rate caused by nonlinear effects and being the linear functional of spectral density of energy.

One can write (6.14) in more convenient form if the spectral density of number of longitudinal oscillations is represented as

\[
N^l_k = \frac{W^l_k}{\omega_k^l}.
\]

It fulfils the role of distribution function of plasmons number. Then instead of (6.14) we have

\[
\frac{dN^l_k}{dt} = \frac{\partial N^l_k}{\partial t} + \mathbf{V}_k^l \frac{\partial N^l_k}{\partial \mathbf{x}} = - \hat{\gamma} \{ N^l_k \} N^l_k.
\]

(6.16)

7. HTL-AMPLITUDES. WARD IDENTITIES

Before proceeding to a question on a gauge dependence of obtained nonlinear Landau damping rate (6.15), we rewrite derived expression in the terms of HTL-amplitudes [1,
follows. We present HTL-corrections to the bare four-gluon vertex \([1, 12]\), that is a sum of the bare three-gluon vertex [1, 12]. This makes possible to extend a procedure of a gauge invariance proof of the damping rate for QGP collective excitations in quantum theory [1] to our case.

We present the integration measure \(d^4p\) as \(dp_0 |p|^2 dp |d\Omega\), where \(d\Omega\) is the angular measure. Using the definition of equilibrium distributions (2.17) (for \(\mu = \text{0}\)) and taking into account\(\int_{-\infty}^{+\infty} |p|^2 dp |\int_{-\infty}^{+\infty} p_0 dp_0 \frac{dN_{eq}(p_0)}{dp_0} = -\frac{3}{4\pi} \left(\frac{\omega_{pl}}{g}\right)^2\),

we perform the integral over \(dp_0\) and the radial integral over \(|p|\) in the expressions for \(S^{(II)}\)-function (5.5) and \(\Sigma^{(II)}\)-function (5.10). Further we rewrite the expression (6.15) as follows

\[
\gamma^i(k) = -2g^2 N_c \int d\mathbf{k}_1 \left(\frac{W_{\mathbf{k}_1}^{\alpha}}{\omega_{\mathbf{k}_1}}\right) \left[\frac{1}{k^2 k_1^2} \left(\frac{\partial \text{Re} \, \varepsilon^j(k)}{\partial \omega} \right)^{-1} \left(\frac{\partial \text{Re} \, \varepsilon^j(k)}{\partial \omega_1} \right)^{-1}\right]_{\text{on-shell}} \text{Im} \, \tilde{T}(\mathbf{k}, \mathbf{k}_1),
\]

where

\[
\tilde{T}(\mathbf{k}, \mathbf{k}_1) \equiv \delta \Gamma^{\mu\nu\lambda\sigma}(k, k_1, -k, -k_1) - \ast \Gamma^{\mu\nu\rho}(k, -k_1, -k + k_1) \ast \Gamma^{\alpha\lambda\sigma}(k_1 - k, -k, k_1) - \ast \Gamma^{\mu\nu\rho}(k, k_1, -k - k_1) \ast \Gamma^{\alpha\lambda\sigma}(k + k_1, -k, -k_1) \ast Q_{\mu\lambda}(k) Q_{\nu\sigma}(k_1)|_{\omega = \omega_k, \omega_1 = \omega_{k_1}}
\]

and

\[
\delta \Gamma^{\mu\nu\lambda\sigma}(k, k_1, k_2, k_3) = 3 \omega_{pl}^2 \int \frac{d\Omega}{4\pi} \frac{v^\mu v^\nu v^\lambda v^\sigma}{vk + i\epsilon} \left[\frac{1}{v(k + k_1) + i\epsilon} \left(\frac{\omega_2}{vk_2 - i\epsilon} - \frac{\omega_3}{vk_3 - i\epsilon}\right) - \frac{1}{v(k + k_3) + i\epsilon} \left(\frac{\omega_1}{vk_1 - i\epsilon} - \frac{\omega_2}{vk_2 - i\epsilon}\right)\right], \quad (v^\mu = (1, \mathbf{v}))
\]

present HTL-corrections to the bare four-gluon vertex \([1, 12]\).

\[
\ast \Gamma^{\mu\nu\rho}(k, k_1, k_2) \equiv \Gamma^{\mu\nu\rho}(k, k_1, k_2) + \delta \Gamma^{\mu\nu\rho}(k, k_1, k_2)
\]

is the effective three-gluon vertex \([1, 12]\), that is a sum of the bare three-gluon vertex

\[
\Gamma^{\mu\nu\rho}(k, k_1, k_2) = g^{\mu\nu}(k - k_1)^\rho + g^{\nu\rho}(k_1 - k_2)^\mu + g^{\rho\mu}(k_2 - k)^\nu
\]

and corresponding HTL-correction

\[
\delta \Gamma^{\mu\nu\rho}(k, k_1, k_2) = 3 \omega_{pl}^2 \int \frac{d\Omega}{4\pi} \frac{v^\mu v^\nu v^\rho}{vk + i\epsilon} \left(\frac{\omega_2}{vk_2 - i\epsilon} - \frac{\omega_1}{vk_1 - i\epsilon}\right).
\]

The polarization tensor in these notations takes the form

\[
\Pi^{\mu\nu}(k) = 3 \omega_{pl}^2 \left(\frac{v^\mu v^\nu - \omega}{4\pi} \frac{d\Omega}{vk + i\epsilon}\right).
\]
For writing the expression (7.1), for example, in the temporal gauge $A_0^a = 0$, it is sufficiently to replace the projection operators $Q_{\mu\lambda}(k)$ and $Q_{\nu\sigma}(k_1)$ by

$$Q_{\mu\lambda}(k) = \frac{\sqrt{-2k^2u^2}}{k^2(ku)} C_{\mu\lambda}(k) + \frac{\bar{u}^2(k)}{k^2(ku)^2} D_{\mu\lambda}(k) = \frac{\bar{u}_\mu(k)\bar{u}_\lambda(k)}{\bar{u}^2(k)},$$

$$\bar{u}_\mu(k) \equiv \frac{k^2}{(ku)}(k\mu - u_\mu(ku)),$$

(similarly for $Q_{\nu\sigma}(k_1)$), and the propagator (6.8) by

$$\tilde{D}_{\rho\alpha}(k) = D_{\rho\alpha}(k) - \left(\frac{\sqrt{-2k^2u^2}}{k^2(ku)} C_{\rho\alpha}(k) + \frac{\bar{u}^2(k)}{k^2(ku)^2} D_{\rho\alpha}(k)\right)\Delta^l(k) - \xi D_{\rho\alpha}(k)\Delta^0(k) - \xi_0 \frac{k^2}{(ku)^2} D_{\rho\alpha}(k),$$

(7.7)

where $\xi_0$ is a gauge parameter in the temporal gauge.

To establish the gauge invariance of nonlinear Landau damping rate $\gamma^l(k)$ there is a need to show that expression $\text{Im} \tilde{T}(k, k_1)$, where function $\tilde{T}(k, k_1)$ defined by (7.2) in a covariant gauge equals to similar expression in the temporal gauge.

We prove a gauge invariance for more general expression that in a covariant gauge has the form

$$^{*}\tilde{\Gamma}(k, -k_2, k_1, -k_3) \equiv \{^{*}\Gamma^{\mu\sigma\lambda\nu}(k, -k_2, k_1, -k_3) - \tilde{D}_{\rho\alpha}(-k_1 + k_2) ^{*}\Gamma^{\mu\rho\nu}(k, -k_3, k_1 - k_2) ^{*}\Gamma^{\alpha\lambda\sigma}(-k_1 + k_2, k_1, -k_2) - \tilde{D}_{\rho\alpha}(-k_1 + k_3) ^{*}\Gamma^{\mu\rho\sigma}(k, -k_2, k_1 - k_3) ^{*}\Gamma^{\alpha\lambda\nu}(-k_1 + k_3, k_1, -k_3)\}\bar{u}_\mu(k)\bar{u}_\lambda(k_1)\bar{u}_\nu(k_3)\bar{u}_\sigma(k_2)|_{\text{on-shell}}.$$

Here, $k + k_1 = k_2 + k_3$. The effective vertex $^{*}\Gamma^{\mu\sigma\lambda\nu}$ are formed by adding the hard thermal loop (7.3) to the bare four-gluon vertex

$$\Gamma^{\mu\sigma\lambda\nu} = 2g^{\mu\sigma}g^{\lambda\nu} - g^{\mu\nu}g^{\sigma\lambda} - g^{\mu\lambda}g^{\nu\sigma}.$$

The association of the expression (7.2) with (7.8) is given by

$$\text{Im} \tilde{T}(k, k_1) = \frac{1}{\bar{u}^2(k)\bar{u}^2(k_1)} \text{Im} ^{*}\tilde{\Gamma}(k, -k_2, k_1, -k_3)|_{k_1 = -k, k_2 = -k_1, k_3 = k_1}.$$

As will be shown in a separate publication [26], the expression (7.8) is associated with probability of plasmon-plasmon scattering.

Similar expression (7.8) in the temporal gauge is obtained with replacements $\bar{u}_\mu(k) \rightarrow \tilde{u}_\mu(k)$ and the propagator (6.8) by (7.7).

The gauge invariance proof is based on using the identities, analogous the effective Ward identities in hot gauge theories [1]. It can be shown that the following equalities hold

$$k_\mu ^{*}\Gamma^{\mu\nu\lambda\sigma}(k, k_1, k_2, k_3) = ^{*}\Gamma^{\nu\lambda\sigma}(k_1, k_2, k + k_3) - ^{*}\Gamma^{\nu\lambda\sigma}(k + k_1, k_2, k_3),$$

22
\[ k_{1\nu} \ast \Gamma^{\mu\nu\lambda\sigma}(k, k_1, k_2, k_3) = \ast \Gamma^{\mu\lambda\sigma}(k + k_1, k_2, k_3) - \ast \Gamma^{\mu\lambda\sigma}(k_1 + k_2, k_3), \quad (7.9) \]

(similar contractions with \( k_{2\lambda}, k_{3\sigma} \)),

\[ k_\mu \ast \Gamma^{\mu\nu\rho}(k, k_1, k_2) = D^{-1\nu\rho}(-k_1) - D^{-1\nu\rho}(-k_2), \quad (7.10) \]

(similar contractions with \( k_{1\nu}, k_{2\rho} \)).

Here \( D^{-1\nu\rho}(k) = P^{\mu\nu}(k)\Delta^{-1\mu}(k) + Q^{\mu\nu}(k)\Delta^{-1\mu}(k) \) is the inverse propagator for which the following useful relation holds

\[ D_{\rho\alpha} D^{-1\alpha\lambda}(k) = \delta_{\rho}^\lambda - \frac{k_\rho k_\lambda}{k^2}. \quad (7.11) \]

8. THE GAUGE INVARIANCE

Our proof of gauge invariance is reduced to contraction of the projectors \( Q \) and \( \tilde{Q} \) with resummed three- and four-gluon vertices and the use of the Ward identities (7.9), (7.10). At the beginning we consider the term with a gauge parameter for the propagator in a covariant gauge.

By using the Ward identities (7.10), we have

\[ \xi D_{\rho\alpha}(k_1 - k_2)\Delta^0(k_1 - k_2) \ast \Gamma^{\mu\nu\rho}(k_1, -k_3, k_1 - k_2) \ast \Gamma^{\alpha\lambda\sigma}(-k_1 + k_2, k_1, -k_2) = \]
\[ = \xi(\Delta^0(k_1 - k_2))^2(D^{-1\mu\sigma}(k) - D^{-1\mu\sigma}(k_3))(D^{-1\lambda\nu}(k) - D^{-1\lambda\nu}(-k_1)). \quad (8.1) \]

Further this expression is contracted with \( \bar{u}_\mu(k)\bar{u}_\nu(k_3) \). It is easily shown that it vanish whether because \( D^{-1\mu\nu}(k) \) is transverse, or by the definition of the mass-shell condition, i.e.

\[ k_\mu D^{-1\mu\nu}(k) = 0, \quad D^{-1\nu\rho}(k) |_{\omega = \omega^I_k} = 0. \quad (8.2) \]

Similar statement holds in the temporal gauge also.

The gauge-dependent parts in the above calculation drops out \( \gamma^I(k) \), since they are multiplied by the mass-shell factor. These factors are proportional to \( (\omega - \omega^I_k) \). However, in a quantum case Baier, Kunstatter, and Schiff [27] observed that naive calculation in covariant gauge appears to violate this consideration. Mass-shell factor is multiplied by the integral involving a power infrared divergence which is cutoff exactly on the scale \( (\omega - \omega^I_k) \sim g^2T \). By this means the gauge-dependent part yields a finite contribution to the gluon damping rate. This problem is considered for damping rate of Fermi-excitations in QGP, also [28, 5, 29].

It can be shown that in our case, the integral preceding the mass-shell conditions, diverges for lower limit also and thus the similar problem is arised: does (8.1) yield a
finite, the gauge-dependent contribution to the nonlinear Landau damping rate? In Sec. 12 we provide the answer to the question.

Now we consider the remaining terms in (7.8). We calculate the contraction with effective four-gluon vertex \(\ast\Gamma_4\). Slightly cumbersome, but not complicated computations by using the effective Ward identities (7.9), (7.10) and relations (7.11), (8.2) lead to the following expression

\[
\ast\Gamma^{\mu\nu\lambda\sigma}(k, -k_2, k_1, -k_3)\bar{u}_\mu(k)\bar{u}_\lambda(k_1)\bar{u}_\nu(k_3)\bar{u}_\sigma(k_2)|_{\text{on-shell}} = \\
k^2k_1^2k_2^2k_3^2 \ast\Gamma^{0000}(k, -k_2, k_1, -k_3) + \Xi(k, -k_2, k_1, -k_3),
\]

where

\[
\Xi(k, -k_2, k_1, -k_3) = \left\{(k_1^2k_2^2[k\omega k_3^2 + \omega k_1^2]) \ast\Gamma^{0000}(k - k_3, k_1, -k_2) - \\
-\omega\omega_3k_1^2k_2^2k_3\ast\Gamma^{0000}(k - k_3, k_1, -k_2) + (k \leftrightarrow k_3) + (k_1 \leftrightarrow k, k_2 \leftrightarrow k_3)\right\} + \\
+\left\{(\omega k_2^2 + \omega k_3^2)(\omega k_1^2 + \omega k_2^2)\mathcal{D}^{-1000}(-k_1 + k_2) - \omega_1\omega_2(\omega k_1^2 + \omega k_2^2)k_3\mathcal{D}^{-1000}(-k_1 + k_2) - \\
-\omega_2(\omega k_1^2 + \omega k_2^2)k_3\mathcal{D}^{-1000}(-k_1 + k_2) + \omega_1\omega_2\omega_3 k_3\mathcal{D}^{-1000}(-k_1 + k_2)\right\}.
\]

Further, we calculate the contractions with the terms containing \(\ast\Gamma_3\) in (7.8). Here, we are led to the expression

\[
\{\mathcal{D}_{\rho\sigma}(-k_1 + k_2) \ast\Gamma^{\mu\rho\sigma}(k, -k_3, k_1, -k_2) \ast\Gamma^{\alpha\lambda\sigma}(-k_1 + k_2, k_1, -k_2) + (k_2 \leftrightarrow k_3)\}\bar{u}_\mu(k)\bar{u}_\lambda(k_1) \\
\bar{u}_\nu(k_3)\bar{u}_\sigma(k_2)|_{\text{on-shell}} = \\
k^2k_1^2k_2^2k_3^2 \{\mathcal{D}_{\rho\sigma}(-k_1 + k_2) \ast\Gamma^{0000}(k, -k_3, k_1, -k_2) - \\
-\mathcal{D}_{\rho\sigma}(k - k_3) \ast\Gamma^{0000}(k, -k_3, k_1, -k_2)\} + \Xi(k, -k_2, k_1, -k_3).
\]

Note that the function \(\Xi\) in the r.h.s. of (8.4) is identical to that in (8.3). Subtracting (8.4) from (8.3), we arrive at desired expression

\[
\ast\Gamma(k, -k_2, k_1, -k_3) = k^2k_1^2k_2^2k_3^2 \{ \ast\Gamma^{0000}(k, -k_2, k_1, -k_3) - \mathcal{D}_{\rho\sigma}(k - k_3) \ast\Gamma^{0000}(k, -k_3, k_1, -k_2) - \\
-\mathcal{D}_{\rho\sigma}(k - k_3) \ast\Gamma^{0000}(k, -k_3, k_1, -k_2)\} + \Xi(k, -k_2, k_1, -k_3).
\]

Now we consider the structure of \(\ast\Gamma\) in the temporal gauge. For this purpose we replace \(\bar{u}_\mu\) by \(\bar{u}_\mu\) in (7.8) and the propagator in the covariant gauge by the propagator in the temporal gauge (7.7). The contraction with effective four-gluon vertex leads to

\[
\ast\Gamma^{\mu\nu\lambda\sigma}(k, -k_2, k_1, -k_3)\bar{u}_\mu(k)\bar{u}_\lambda(k_1)\bar{u}_\nu(k_3)\bar{u}_\sigma(k_2)|_{\text{on-shell}} = \\
k^2k_1^2k_2^2k_3^2 \ast\Gamma^{0000}(k, -k_2, k_1, -k_3) + \tilde{\Xi}(k, -k_2, k_1, -k_3),
\]

where

\[
\tilde{\Xi}(k, -k_2, k_1, -k_3) = \left\{(\omega_1\omega_2(\omega + \omega_3) \ast\Gamma^{0000}(k - k_3, k_1, -k_2) - \\
-\omega_1\omega_2k_3\ast\Gamma^{0000}(k - k_3, k_1, -k_2)\right\} + (k \leftrightarrow k_3) + (k_1 \leftrightarrow k) + (k_1 \leftrightarrow k, k_2 \leftrightarrow k_3) + \\
\}

24
\begin{align*}
&+\{(\omega_1 + \omega_2)(\omega + \omega_3)D^{-100}(-k_1 + k_2) - (\omega + \omega_3)k_{1\lambda}D^{-10\lambda}(-k_1 + k_2) - \\
&-(\omega_1 + \omega_2)k_{3\nu}D^{-10\nu}(-k_1 + k_2) + k_{1\lambda}k_{3\nu}D^{-1\nu\lambda}(-k_1 + k_2)) + (k_2 \leftrightarrow k_3)\}.
\end{align*}

Contracting with the terms containing $\Gamma_3$, yields

\begin{align*}
\{\tilde{D}_{\rho\alpha}(-k_1 + k_2)^*\Gamma^{\mu\nu}(k,-k_3, k_1 - k_2)^*\Gamma^{0\lambda\sigma}(-k_1 + k_2, k_1, -k_2) + (k_2 \leftrightarrow k_3)\}	ilde{u}_\mu(k)\tilde{u}_\lambda(k_1)
\end{align*}

\begin{equation}
\bigg|_{on-shell} = \tag{8.7}
\end{equation}

\begin{align*}
&= k^2 k_1^2 k_2^2 k_3^2 \{\tilde{D}_{\rho\alpha}(-k_1 + k_2)^*\Gamma^{00\nu}(k,-k_3, k_1 - k_2)^*\Gamma^{00\lambda}(-k_1 + k_2, k_1, -k_2) + (k_2 \leftrightarrow k_3) + \\
&+\Xi(k,-k_2,k_1,-k_3)\}
\end{align*}

Subtracting (8.7) from (8.6), we are led to similar expression (8.5). Thus, we have shown that at least in a class of the covariant and temporal gauges, the nonlinear Landau damping rate (7.1) (exactly, its piece, independent of a gauge parameter) is a gauge-invariant.

9. THE PHYSICAL MECHANISM OF NONLINEAR SCATTERING OF WAVES

In Sec. 7 the expression of the nonlinear Landau damping rate $\gamma^l(k)$ for Bose excitations in a quark-gluon plasma was obtained. Now we transform it to the form, in which each term in $\gamma^l(k)$ has direct physical relevance. The first transformation of this type has been proposed by Tsytovich for Abelian plasma [21]. In [20] it taken into account the contribution with longitudinal virtual wave only. In order to satisfy gauge invariance, we extend the transformation of similar type to the case with the transverse virtual wave. Since the nonlinear Landau damping rate is independent of the choice of gauge, we choose the temporal gauge for simplicity.

As was mentioned in Sec. 6, the expression for $\gamma^l(k)$ contains the contributions of two different processes. The first is associated with absorption of a plasmon by QGP particles with frequency $\omega$ and a wave vector $k$ with its consequent radiation with frequency $\omega_1$ and a wave vector $k_1$. It defined by the second term in the r.h.s. of (7.2). The frequency and wave vectors of a incident plasmon and a recoil plasmon satisfy the conservation law (6.13). The second process represents simultaneous radiation (or absorption) of two plasmons with frequency $\omega, \omega_1$ and wave vectors $k, k_1$ satisfying the conservation law

\begin{equation}
\omega_k^l + \omega_{k_1}^l - v(k + k_1) = 0, \tag{9.1}
\end{equation}

and it defined by third term in (7.2). In contrast to previous scattering process, this process not conserves the plasmons number and its contribution are not important to the order of interest. The first term in (7.2), associated with HTL-correction to four-gluon vertex, contains both processes. Futher, we have taken into account the terms with (6.13) and we drop the terms which contain a $\delta$-function of (9.1).
Now we consider the expression with \( \delta \Gamma_4 \). With regard to above-mentioned and by the definition (7.3), the contribution of a given term to \( \gamma_l(k) \) can be represented as

\[
3(\omega pl/g)^2 \int \left( \frac{\omega_k}{\omega_{k1}} \right) w_v^c(k, k_1) \left( \frac{W^I_{k1}}{\omega_{k1}} \right) \frac{d\Omega}{4\pi} dk_1,
\]

where

\[
w_v^c(k, k_1) = 2\pi N_c \left( \frac{\partial \text{Re} \beta(k)}{\partial \omega} \right)^{-1} \left( \frac{\partial \text{Re} \beta(k_1)}{\partial \omega_1} \right)^{-1} \delta(\omega_k - \omega_{k1} - v(k - k_1)) |\mathcal{M}^c(k, k_1)|^2,
\]

(9.2)

To clear up the physical origin of contribution (9.2), it is convenient to compare it with corresponding contribution in the theory of electromagnetic plasma. In this case as shown in [21] this contribution describes the normal Thomson scattering of a wave by particles: a wave with the original frequency \( \omega_k \) is set in oscillatory particle motion of plasma and oscillating particle radiates a wave with modified frequency \( \omega_{k1} \). The corresponding function \( w_v^c(k, k_1) \) presents the probability of Thomson scattering. As it was shown above in quark-gluon plasma for a soft long-wavelength excitations all Abelian contributions is at most \( g \ln g \) times the non-Abelian ones and the basic scattering mechanism here, is essentially another (our discussion is schematic, the details of a computation displayed in Appendix A).

To show this mechanism we use the classical pattern of QGP description [8], in which the particles states are characterized besides position and momentum, by the color vector (non-Abelian charge) \( Q = (Q^a), a = 1, \ldots, N_c^2 - 1 \) also. As was shown by Heinz [8] there is an close connection between the classical kinetic equations and semiclassical ones (2.4). Therefore in this case the use of classical notions is justified.

Let the field acting on a colour particle in QGP represents a bundle of longitudinal plane waves

\[
\tilde{A}_\mu^a(x) = -\int [(\omega^2/k^2)Q_{\mu
u}(k)A_{k\nu}^a] e^{ikx - i\omega_k t} dk.
\]

(9.5)

The particle motion in this wave field is described by the system of equations

\[
m \frac{d^2 x^\mu}{d\tau^2} = g Q^a \tilde{F}_{a\mu\nu} dx^\nu/
\]

\[
\frac{dQ^a}{d\tau} = -g F_{abc} dx^\mu \tilde{A}_\mu^b Q^c.
\]

(9.6), (9.7)

Here, \( \tau \) is a proper time of a particle. Eq. (9.7) is familiar Wong equation [30]. The system (9.6), (9.7) is solved by the approximation scheme method - the \textit{weak field expansion}. A zeroth approximation describes uniform restlinear motion, and the next one - constrained
charge oscillations in the field (9.5). With a knowledge of the motion law of a charge, the radiation intensity by it longitudinal waves can be defined. In this case Eq. (9.6) defines the Abelian contribution to radiation, whereas (9.7) - non-Abelian one and interference of these two contributions equals zero. The scattering probability computed by this means, based on Eq. (9.7) is coincident with obtained above (9.3).

In this manner the contribution of (9.2) to \( \gamma^l(k) \) are caused by not the spatially oscillations of a colour particle, as it occurs in electromagnetic plasma, but it induced by a precession of a colour vector \( Q \) of a particle in field of a longitudinal wave (9.5) \( (Q^a Q^a = \text{const} \text{ by Eq. (9.7)}. \)

Let us consider now more complicated terms in (7.1) associated with *\( \Gamma \)-functions. By the definition of three-gluon vertex the following equality is obeyed

\[
* \Gamma^{\mu\nu\rho}(k, -k_1, -k_2) = -* \Gamma^{\rho\mu\nu}(k_2, -k, k_1),
\]

(hereafter, \( k_2 \equiv k - k_1 \)). Using this relation, the contribution of the term with longitudinal virtual wave to \( \gamma^l(k) \) can be presented as

\[
-2g^2 N_c \int \mathrm{d}k_1 \left( \frac{W^l_{k_1}}{\omega^l_{k_1}} \right) \frac{1}{k^2 k_1^2 k_2^2} \left( \frac{\partial \text{Re} \varepsilon^l(k)}{\partial \omega} \right)^{-1}_{\omega=\omega^l_{k_1}} \left( \frac{\partial \text{Re} \varepsilon^l(k_1)}{\partial \omega_1} \right)^{-1}_{\omega_1=\omega^l_{k_1}} \left[ \frac{1}{(\omega_1 \omega_2)^2} \text{Im}(k^2_2 \Delta^l(k_2)(* \Gamma^{ijl}(k, -k_1, -k_2) k^i_1 k^j_1 k^l_2)^2) \right]_{\omega=\omega^l_{k_1}, \omega_1=\omega^l_{k_1}}.
\]

Further we use the relation

\[
\text{Im}(k^2_2 \Delta^l(k_2)(* \Gamma^{ijl}(k, -k_1, -k_2) k^i_1 k^j_1 k^l_2)^2) = -\text{Im}(k^2_2 \Delta^l(k_2))^{-1}|k^2_2 \Delta^l(k_2)(* \Gamma^{ijl}(k, -k_1, -k_2) k^i_1 k^j_1 k^l_2)|^2 + 2(\text{Im} * \Gamma^{ijl}(k, -k_1, -k_2) k^i_1 k^j_1 k^l_2) \text{Re}(k^2_2 \Delta^l(k_2)(* \Gamma^{ijl}(k, -k_1, -k_2) k^i_1 k^j_1 k^l_2)).
\]

Here, we drop the momentum dependence on vertices. Taking into consideration the equality

\[
\text{Im}(k^2_2 \Delta^l(k_2))^{-1} = 3\pi \omega^2 pl \omega_2 \int \frac{\mathrm{d}\Omega}{4\pi} \delta(v k_2),
\]

one can write contribution of the first term in the r.h.s. of (9.8) to the nonlinear damping rate in the form

\[
3(\omega pl/g)^2 \int (\omega^l_k - \omega^l_{k_1}) w^l_v(k, k_1) \left( \frac{W^l_{k_1}}{\omega^l_{k_1}} \right) \frac{\mathrm{d}\Omega}{4\pi} \mathrm{d}k_1,
\]

where

\[
w^l_v(k, k_1) =
2\pi N_c \left( \frac{\partial \text{Re} \varepsilon^l(k)}{\partial \omega} \right)^{-1}_{\omega=\omega^l_k} \left( \frac{\partial \text{Re} \varepsilon^l(k_1)}{\partial \omega_1} \right)^{-1}_{\omega_1=\omega^l_{k_1}} \delta(\omega^l_k - \omega^l_{k_1} - v(k-k_1)) |M^l_v(k, k_1)|^2.
\]
\[ \mathcal{M}^\parallel(k, k_1) = \frac{g^2}{|k| |k_1|} \frac{(k_2 \cdot \mathbf{v})}{k_2^2} \left( \frac{k_2^2 \Delta^l(k_2)}{\omega_1 \omega_2^2} \delta^{ijl}(k, -k_1, -k_2) k^j_1 k^j_2 \right)_{\omega = \omega_k, \omega_1 = \omega_{k_1}}. \] (9.9)

HTL-correction \( \delta \Gamma_3 \) enters in the amplitude (9.9) only. Here the contribution of bare three-gluon vertex is dropped out, since by the definition (7.5), \( \Gamma^{ijl}(k, -k_1, -k_2) k^j_1 k^j_2 \equiv 0 \).

The quantity \( \mathcal{M}^\parallel \) can be interpreted as the scattering amplitude of a longitudinal wave by dressing ”cloud” of a particle. However, in contrast to the Abelian plasma here, the scattering is produced not by the oscillation of a screening cloud of color charge as a result of interaction with incident scattering wave, but as a consequence the fact that it induced by precession of color vectors of particles forming this cloud in the incident wave field \( \omega^l_k \). This process of transition scattering is a completely collective effect. For calculation of its probability \( w^\parallel(k, k_1) \) it is necessary to solve the kinetic equation describing a color charges motion in a screening cloud in the field that is equal to the sum of fields of incident wave (9.5) and a ”central” charge producing a screening cloud.

Now we consider the contribution with transverse virtual wave. Using the association of projectors

\[ P_{\rho \alpha}(k_2) = g_{\rho \alpha} - u_\rho u_\alpha - \frac{\omega^2}{k_2^2} Q_{\rho \alpha}(k_2), \]

we rewrite it in the form

\[ -2g^2 N_c \int \frac{dk_1}{(\omega^l_{k_1})^2} \left( \frac{1}{k^2 k_1^2} \left( \frac{\partial \text{Re} \varphi'(k)}{\partial \omega} \right)^{-1} \frac{\partial \text{Re} \varphi'(k_1)}{\partial \omega} \right)^{-1}_{\omega = \omega^l_k, \omega_1 = \omega^l_{k_1}} \]

\[ \frac{1}{(\omega_1)^2} \text{Im}\left( \Delta^l(k_2) \right) \right)^{-1} \frac{\partial \text{Re} \varphi'(k_1)}{\partial \omega} \right)^{-1}_{\omega = \omega^l_k, \omega_1 = \omega^l_{k_1}} \]

\[ \left[ \frac{1}{(\omega_1)^2} \text{Im}\left( \Delta^l(k_2) \right) \right]_{\omega = \omega^l_k, \omega_1 = \omega^l_{k_1}}. \] (9.10)

To simplify the expression, we expand \( \Gamma^{ijl} k^j_1 k^j_2 \) in two mutually orthogonal vectors: \( k_2 \) and \( \mathbf{n} k_2 \), where \( \mathbf{n} \equiv [kk_1] \)

\[ \Gamma^{ijl} k^j_1 k^j_2 = (\Gamma^{ijl} k^j_1 k^j_2) \frac{k^j_1}{k_2^2} + (\Gamma^{ijl} k^j_1 k^j_2) \frac{[n k_2]^l}{n^2 k_2^2}. \]

Substituting the last expression into (9.10), instead of the expression in square brackets, we define

\[ \left[ \frac{1}{(\omega_1)^2} \frac{1}{n^2 k_2^2} \text{Im}\left( \Delta^l(k_2) \right) \right]_{\omega = \omega^l_k, \omega_1 = \omega^l_{k_1}}. \]

We transform the imaginary part similar to (9.8)

\[ \text{Im}\left( \Delta^l(k_2) \right) (\Gamma^{ijl} k^j_1 [n k_2]^l)^2 = -\text{Im}\left( \Delta^{-l}(k_2) \right) \Delta^l(k_2) \Gamma^{ijl} k^j_1 [n k_2]^l)^2 + +2(\text{Im} \Gamma^{ijl} k^j_1 [n k_2]^l) \text{Re}\left( \Delta^l(k_2) \Gamma^{ijl} k^j_1 [n k_2]^l \right). \] (9.11)
If we take into account $\text{Im} \Delta^{-1}(k_2) = -(1/2) \text{Im} \Delta^{-1}(k_2)$, then the contribution of the first term in the r.h.s. of (9.11) to $\gamma^l(k)$ can be presented in the form

$$3(\omega_{pl}/g)^2 \int (\omega'_k - \omega'_{k_1}) w^+_v(k, k_1) \left( \frac{W^l_1}{\omega_{k_1}} \right) \frac{d\Omega}{4\pi} dk_1,$$

where probability $w^+_v(k, k_1)$ is obtained from $w^v(k, k_1)$ with replacement $\mathcal{M}^v(k, k_1)$ by

$$\mathcal{M}^v(k, k_1) = \frac{g^2}{|k||k_1|} \frac{\langle nk^2 \rangle v}{n^2 k^2_1} \left( \frac{\Delta^v(k_2)}{\omega_{k_1}} \ast \Gamma^v \right)^{ijl} k^i_1 k^j_1 [nk^2]^l \left( \omega_{k_1}, \omega_{k_1} \right). \quad (9.12)$$

In contrast to preceding case, $\mathcal{M}^v$ represents the sum of two contributions. The first one defined by bare three-gluon vertex is connected with self-interaction of a gauge field and has no analogy in Abelian plasma. The second contribution is associated with the scattering of wave by the screening cloud of the charges.

The remaining terms in the r.h.s. of (9.8) and (9.11) represent the interference of Thomson scattering by longitudinal and transverse virtual waves, respectively. It is easily to see this, having used

$$\text{Im}(\delta \Gamma^{ijl} k^i_1 k^j_1 k^l_1) = -3\pi \omega^2 \omega^2 \int \frac{d\Omega}{4\pi} \frac{(kv)(k_1 v)}{vk} \delta(vk_2),$$

$$\text{Im}(\ast \Gamma^{ijl} k^i_1 [nk^2]^l) = -3\pi \omega^2 \omega^2 \int \frac{d\Omega}{4\pi} \frac{(kv)(k_1 v)}{vk} \delta(vk_2).$$

The interference of the last two mechanisms of scattering is absent.

Thus, summing the preceding, instead of (7.1), (7.2) we have

$$\gamma^l(k) = 3(\omega_{pl}/g)^2 \int (\omega'_k - \omega'_{k_1}) Q(k, k_1) \left( \frac{W^l_1}{\omega_{k_1}} \right) dk_1, \quad (9.13)$$

where

$$Q(k, k_1) = 2\pi N_c \left( \frac{\partial \Re \epsilon^l(k)}{\partial \omega} \right)^{-1} \left( \frac{\partial \Re \epsilon^l(k_1)}{\partial \omega_1} \right)^{-1} \omega_{k_1} \omega_{k_1}, \quad (9.14)$$

$$\int \frac{d\Omega}{4\pi} \delta(\omega'_k - \omega'_{k_1} - v(k - k_1)) |\mathcal{M}^e(k, k_1) + \mathcal{M}^v(k, k_1) + \mathcal{M}^v(k, k_1)|^2.$$ 

The interference term in (9.14) between $\mathcal{M}^v$ and $\mathcal{M}^v$ vanishes by relation

$$k^i_1 [nk^2]^l \int \frac{d\Omega}{4\pi} v^i v^j \delta(vk_2) = 0.$$ 

It is convenient to interpret the terms intering to $\mathcal{M} \equiv \mathcal{M}^e + \mathcal{M}^v + \mathcal{M}^v$ using a quantum language. In this case the term $\mathcal{M}^v$ connected with the Thomson scattering can be represented as the Compton scattering of the quantum of the soft modes (plasmon).
by QGP termal particle. $\mathcal{M}^\parallel$ defines the scattering of a quantum oscillation through a longitudinal virtual wave with the propagator $\Delta^l(k_2)$, where a vertex of a three-wave interaction is induced by HTL-correction $\delta \Gamma_3$. $\mathcal{M}^\perp$ defines the quantum oscillation scattering by transverse virtual wave with propagator $\Delta^t(k_2)$. In this case $\mathcal{M}$ fulfils role of the total scattering amplitude.

At the end of this Section we note the principal distinction of obtained expression of the nonlinear Landau damping rate from the damping rate for hard particles [31]. In the last case the scattering amplitude involves the resummed gluon propagator in the electric and the magnetic channels only, i.e. $\Delta^l(k_2)$ and $\Delta^t(k_2)$ respectively. In our case, for small particle momenta ($|k| \leq gT$), the scattering amplitude takes into account the thermal masses of the particles and exchange contributions (vertex corrections). This leads to more complicated expression for $\mathcal{M}$ involving the effective vertices, in contrast to similar expression for the fast particles.

10. THE ASSOCIATION WITH HTL-APPROXIMATION

The kernel $Q(k,k_1)$ possesses two main properties. The following inequality results from definition (9.14)

$$Q(k,k_1) \geq 0. \quad (10.1)$$

Next from (9.4) it follows that $\mathcal{M}^c(k,k_1) = \mathcal{M}^{c*}(k_1,k)$. The correctness of this equality results from the conservation law (6.13). For $\mathcal{M}^\parallel$ and $\mathcal{M}^\perp$ we have the similar relations: $\mathcal{M}^{\parallel,\perp}(k,k_1) = (\mathcal{M}^{\perp,\parallel}(k_1,k))^*$. Their proof trivially follows from the definitions of $^*\Gamma_3$ and $\Delta^l,t$. The consequence of these equalities is a main property of a symmetry of kernel $Q(k,k_1)$ with respect to permutation of a wave vectors $k$ and $k_1$

$$Q(k,k_1) = Q(k_1,k). \quad (10.2)$$

From (9.13) follows that in the case of a global equilibrium plasma and by inequality (10.1), waves of a high frequencies are damped out, and a smaller ones are increased. In particular, in the limit of $|k| \to 0$ we obtain from (9.13)

$$\gamma^l(0) = 3(\omega_{pl}/g)^2 \int (\omega_{pl} - \omega_{k1}) Q(0,k_1) \left( \frac{W^l_{k1}}{\omega_{k1}} \right) dk_1 < 0,$$

i.e. $k = 0$-mode is not damped, as it was calculated in Ref. [17].

This clearly shows that nonlinear Landau damping rate (9.13) which is not of fixed sign, cannot be identified with gluon damping rate, calculated on the basis of resumming Braaten-Pisarski techniques. Actually $\gamma^l(k)$ defines two processes: the effective pumping of energy across the spectrum sideways of small wave numbers with conservation of excitation energy and properly nonlinear dissipation (damping) of longitudinal plasma waves.
by QGP particles, where the first process is crucial. To see this, we rewrite the kinetic
equation (6.14) as follows
\[
\frac{\partial W_k^l}{\partial t} = -3\left(\frac{\omega_{pl}}{g}\right)^2 \int (\omega_k^l - \omega_{k_1}^l) \omega_k^l Q(k, k_1) \left(\frac{W_k^l}{\omega_k^l} \right) \left(\frac{W_{k_1}^l}{\omega_{k_1}^l} \right) d\mathbf{k}_1. \tag{10.3}
\]
Hereafter, we have restricted ourseves to spatially-homogeneous case.

We perform replacement \(\omega_k^l = (\omega_k^l - \omega_{k_1}^l)/2 + (\omega_k^l + \omega_{k_1}^l)/2\), then last equation can be
presented as
\[
\frac{\partial W_k^l}{\partial t} = - \int Q_S(k, k_1) W_k^l W_{k_1}^l d\mathbf{k}_1 - \int Q_A(k, k_1) W_k^l W_{k_1}^l d\mathbf{k}_1, \tag{10.4}
\]
where we introduce symmetric and antisymmetric kernel, respectively
\[
Q_S(k, k_1) \equiv 3\left(\frac{\omega_{pl}}{g}\right)^2 \frac{(\omega_k^l - \omega_{k_1}^l)^2}{2\omega_k^l \omega_{k_1}^l} Q(k, k_1),
\]
\[
Q_A(k, k_1) \equiv 3\left(\frac{\omega_{pl}}{g}\right)^2 \frac{(\omega_k^l)^2 - (\omega_{k_1}^l)^2}{2\omega_k^l \omega_{k_1}^l} Q(k, k_1). \tag{10.5}
\]
We integrate Eq. (10.4) over d\(\mathbf{k}\) and introduce the total energy of longitudinal QGP
excitations: \(W_0^l = \int W_k^l d\mathbf{k}\). Then by the properties of kernels (10.5) we obtain
\[
\frac{\partial W_0^l}{\partial t} = - \int Q_S(k, k_1) W_k^l W_{k_1}^l d\mathbf{k} d\mathbf{k}_1 < 0,
\]
i.e. properly nonlinear dissipation of excitations is defined by symmetric part of a kernel
\(Q_S\) only. It is necessary to compare the parts of nonlinear Landau damping rate which
correspond to the nonlinear dissipation of waves only, with damping rate of boson modes
from HTL-approximation, namely
\[
\gamma_S^l(k) \equiv 3\left(\frac{\omega_{pl}}{g}\right)^2 \frac{(\omega_k^l - \omega_{k_1}^l)^2}{2\omega_k^l \omega_{k_1}^l} Q(k, k_1) W_{k_1}^l d\mathbf{k}_1. \tag{10.6}
\]
The function \(\gamma_S^l(k)\) is positive for any value of a wave vector \(k\) and particular, for \(|k| = 0\).

Antisymmetric part of a kernel is not associated with dissipative phenomenon and
defines the spectral pumping from short to long waves. It is easy to see, by considering
the model problem of interaction of two infinitely narrow packets with typical wave vectors
\(k_1, k_2\).

Let us introduce \(W_k^l\) as follows
\[
W_k^l(t) = W_1(t) \delta(k - k_1) + W_2(t) \delta(k - k_2), \quad |k_1| > |k_2|.
\]
Substituting the last expression into (10.4), we obtain the coupled nonlinear equations
\[
\frac{\partial W_1}{\partial t} = -(Q_S + Q_A) W_1 W_2, \quad W_1(t_0) = W_{10},
\]
\[ \frac{\partial W_2}{\partial t} = -(Q_S - Q_A)W_1W_2, \quad W_2(t_0) = W_{20}. \]

Here, \( Q_{S,A} \equiv Q_{S,A}(k_1, k_2) \). The general solution of this system has the form

\[ W_1(t) = -W_{10} C e^{-C(t-t_0)} \frac{e^{-C(t-t_0)}}{(Q_S + Q_A)W_{20} + (Q_S - Q_A)W_{10}}. \]

\[ W_2(t) = \frac{Q_S - Q_A}{Q_S + Q_A} W_1(t) - \frac{C}{Q_S + Q_A}, \quad C \equiv (Q_A - Q_S)W_{10} + (Q_A + Q_S)W_{20}. \]

For \(|k_1| > |k_2|\), by the definitions (10.5), \( C > 0 \) and therefore, in the limit for \( t \to \infty \) we have

\[ W_1(t) \to 0, \quad W_2(t) \to W_0 - \frac{2Q_S}{Q_S + Q_A} W_{10}. \]

Here \( W_0 = W_{10} + W_{20} \) is total initial energy of packets. Thus we see that as result of nonlinear interaction of two infinitely narrow packets the effective pumping of energy across the spectrum sideways of small wave numbers takes place.

In this case, as a result of the pumping, the part of excitations energy (proportionaled to \( Q_S \)) nonlinear absorbs by QGP particles. The absorption value converges to

\[ \Delta W = (\omega_{k_1}^l - \omega_{k_2}^l)(W_{10}/\omega_{k_1}^l). \]

The process of nonlinear scattering of plasmons by QGP particles only, not results in their relaxation in homogeneous isotropic plasma. In fact, by the kinetic equation (10.3) the general plasmons numbers conserves

\[ \frac{\partial N^l}{\partial t} = \frac{\partial}{\partial t} \int N^l_0 d\mathbf{k} = 0. \]

It follows that if for time \( t = t_0 \), \( N^l_0 \) plasmons are excited in a plasma, then excitation energy for any \( t \geq t_0 \) does not less than a value \( \omega_{n}N^l_0 \) by conservation law of plasmon numbers. In homogeneous plasma the total dissipation of longitudinal excitations energy is defind by slow processes of four-wave interaction.

**11. THE ESTIMATION OF \( \gamma_S^l(0) \)**

Now we present a complete calculation of \( \gamma_S^l(0) \) at zero momentum of an incident field. We start from representation of \( \gamma_S^l(0) \) in the form of (10.6) with kernel (9.14). We introduce the coordinate system in which axis 0Z is aligned with vector \( k_1 \); then the coordinates of vectors \( k \) and \( v \) equal \( k = (|k|, \alpha, \beta), \quad v = (1, \theta, \varphi), \) respectively. By \( \Phi \) we denote the angle between \( v \) and \( k \): \( \mathbf{v k} = |k| \cos \Phi \). The angle \( \Phi \) can be expressed as

\[ \cos \Phi = \sin \theta \sin \alpha \cos(\varphi - \beta) + \cos \theta \cos \alpha. \]
where
\[ v^l_{k_1} \equiv |\omega^l_{k_1}|, \quad k^2_1 \equiv (\omega^l_{k_1})^2 - |\omega^l_{k_1}|^2, \quad \rho^l_{k_1} \equiv (\omega^l_{k_1} - \omega_{pl})/|\omega^l_{k_1}| \geq 0, \quad d\Omega = \sin \theta d\theta d\varphi. \]

By using the definitions (9.4), (9.9) and (9.12), we find expressions \( \mathcal{M}^c(0, k_1), \mathcal{M}^\parallel(0, k_1) \) and \( \mathcal{M}^\perp(0, k_1) \). The first of them is defined more simply. In the limit of zero momentum
\[ \mathcal{M}^c(0, k_1) = \left( \frac{g}{\omega_{pl}} \right)^2 \cos \Phi \frac{\cos \theta}{\omega^l_{k_1}}. \]

Calculation of \( \mathcal{M}^\parallel(0, k_1) \) is more complicated. From (9.9) we obtain
\[ \mathcal{M}^\parallel(0, k_1) = \left( \frac{g}{\omega_{pl}} \right)^2 \omega^l_{k_1} \frac{1 - (\rho^l_{k_1})^2}{\omega^l_{k_1}} \lim_{|k| \to 0} \frac{1}{|k|} (\Delta^l(k_2) \delta \Gamma^{i jl}(k, -k_1, -k_2) k^i k^j_1 k^l_2). \] (11.3)

Using the definition of HTL-correction (7.6) to bare three-gluon vertex, after slightly cumbersome computations we define
\[ \lim_{|k| \to 0} \frac{1}{|k|} \delta \Gamma^{i jl}(k, -k_1, -k_2) k^i k^j_1 k^l_2 = - \cos \alpha \frac{k^2}{\omega_{pl}} \frac{(\rho^l_{k_1})^3}{1 - (\rho^l_{k_1})^2} \lim_{|k| \to 0} \Delta^{-1l}(k_2) + 3|k_1|^3 \cos \alpha \rho^l_{k_1} v^l_{k_1}. \]

In the last equality we use the definition of \( \Delta^{-1l}(k) \) as the function \( F(\omega/|k|) \) (6.9). Inserting (11.4) into (11.3), we reduce the scattering amplitude with longitudinal virtual oscillation to
\[ \mathcal{M}^\parallel(0, k_1) = - \left( \frac{g}{\omega_{pl}} \right)^2 (\rho^l_{k_1})^2 \cos \alpha + 3 \left( \frac{g}{\omega_{pl}} \right)^2 \omega_{pl} (1 - (\rho^l_{k_1})^2) \cos \alpha \lim_{|k| \to 0} \Delta^l(k_2). \] (11.5)

Now we consider the limit of the term with transverse virtual oscillation. From (9.12) it follows
\[ \mathcal{M}^\perp(0, k_1) = \left( \frac{g}{\omega_{pl}} \right)^2 \frac{\omega_{pl}}{\omega^l_{k_1}} \cos \Phi \frac{\cos \theta \cos \alpha}{k^2 \sin \alpha} \lim_{|k| \to 0} \frac{1}{|k|\nu} (\Delta^l(k_2) \delta \Gamma^{i jl}(k, -k_1, \nu) k^i k^j_1 k^l_2 |\nu k_2|^l). \] (11.6)

Using the definition of effective three-gluon vertex (7.4) and the relation
\[ (1 - (\rho^l_{k_1})^2)(1 - F(-\rho^l_{k_1})) = \frac{2}{3\omega^2_{pl}} \lim_{|k| \to 0} \Delta^{-1l}(k_2) + 1 + \frac{2k^2}{3\omega^2_{pl}} (1 - (\rho^l_{k_1})^2), \]
we obtain
\[
\lim_{|k| \to 0} \frac{1}{|\mathbf{k}|} \Gamma^{ijl}(k, -k_1, -k_2) k^i k^j k^l [\mathbf{n} \mathbf{k}_2]^l = 
\]
\[
= -\frac{3}{2} \frac{k_1^4}{\omega_{pl}} \sin \alpha (1 - v_{k_1}^l \rho_{k_1}^l) - \frac{k_2^2 \rho_{k_2}^l}{\omega_{pl}} \lim_{|k| \to 0} \Delta^{-1t}(k_2) \sin \alpha. \quad (11.7)
\]

Substitute (11.7) into (11.6) and take into account the relation (11.1), we obtained required limit
\[
\mathcal{M}^\perp(0, k_1) = -\left(\frac{g}{\omega_{pl}}\right)^2 \rho_{k_1}^l \sin \theta \sin \alpha \cos(\varphi - \beta) - 
\]
\[
-\frac{3}{2} \left(\frac{g}{\omega_{pl}}\right)^2 |k_1| (v_{k_1}^l)^{-1} - \rho_{k_1}^l \sin \theta \sin \alpha \cos(\varphi - \beta) \lim_{|k| \to 0} \Delta^t(k_2). \quad (11.8)
\]

The terms in the amplitude \(\mathcal{M}\) not containing \(\Delta^{l,t}\) combine to give
\[
\left(\frac{g}{\omega_{pl}}\right)^2 \frac{1}{\omega_{k_1}^l} \{\cos \Phi \cos \theta - (\rho_{k_1}^l)^2 \cos \alpha - \rho_{k_1}^l \sin \theta \sin \alpha \cos(\varphi - \beta)\}. \quad (11.9)
\]

By the \(\delta\)-function in (11.2), the expression in curly braces vanishes. Thus, all terms in \(\mathcal{M}\), not containing the factors \(\Delta^{l,t}\), are in relatively reduced in the limit of \(k = 0\)-mode.

The remaining terms, after substitution into (11.2) and integration over solid angle, yield
\[
Q(0, k_1) = \frac{9}{2} \pi g^4 N_c \frac{k_1^2 (1 - \rho_{k_1}^l)^2}{\omega_{pl}^2 (3\omega_{pl}^2 - k_1^2)} v_{k_1}^l \theta(1 - \rho_{k_1}^l) \{(1 - (\rho_{k_1}^l)^2) \cos^2 \alpha \lim_{|k| \to 0} (\Delta^t(k_2))^2 + 
\]
\[
+ \frac{k_1^2}{8\omega_{pl}^2} ((v_{k_1}^l)^{-1} - \rho_{k_1}^l)^2 \sin^2 \alpha \lim_{|k| \to 0} (\Delta^t(k_2))^2\}. \quad (11.9)
\]

We note that this expression is not dependent on angle \(\beta\). This enables us to represent the integration measure in the r.h.s. of (10.6) in the form
\[
\int d\mathbf{k}_1 = 2\pi \int_0^\infty k_1^2 d|\mathbf{k}_1| \int_1^{-1} d(cos \alpha).
\]

Futher we suppose that the excitations in QGP become isotropic over the directions of vector \(\mathbf{k}_1\) on a time scale which is much less than time scale of the nonlinear interaction. This enables us to consider the spectral density \(W_{k_1}^l\) as a function of \(|\mathbf{k}_1|\) only. Now we introduce the spectral function
\[
W_{|\mathbf{k}_1|}^l \equiv 4\pi k_1^2 W_{k_1}^l
\]

34
such that the integral $\int_0^\infty W_{|k_1|}^l d|k_1| = W^l$ is total energy of longitudinal oscillations in QGP. Substituting (11.9) into (10.6) (for $k = 0$) and performing the angular integration over $\alpha$, we obtain finally $\gamma^l_S(0)$

$$\gamma^l_S(0) \approx \int_0^{k_1^*} Q(|k_1|) W_{|k_1|}^l d|k_1|, \quad (11.10)$$

where a kernel $Q(|k_1|)$ has the form

$$Q(|k_1|) = \frac{9}{4} \pi g^2 N_c \frac{k_1^2 (1 - (\rho_{k_1}^l)^2)}{(3\omega_{pl}^2 - k_1^2)} |k_1| (\rho_{k_1}^l)^2 \theta (1 - \rho_{k_1}^l) \{(1 - (\rho_{k_1}^l)^2) \left| \lim_{|k| \to 0} (\Delta^l(k_2)) \right|^2 +$$

$$+ \frac{k_1^2}{4\omega_{pl}^2} ((v_{k_1}^l)^{-1} - \rho_{k_1}^l)^2 \left| \lim_{|k| \to 0} (\Delta^l(k_2)) \right|^2\}, \quad (11.11)$$

and the upper cutoff $|k_1^*|$ distinguishes between soft and hard momenta: $gT \ll |k_1^*| \ll T$. For crude estimation of $\gamma^l_S(0)$ let us expand the kernel $Q(|k_1|)$ in the momentum $|k_1|$, using the approximations

$$\omega_{k_1}^l \approx \omega_{pl}, \quad \rho_{k_1}^l \approx \frac{3|k_1|}{10\omega_{pl}}, \quad v_{k_1}^l \approx \frac{\omega_{pl}}{|k_1|}, \ldots.$$ 

Keeping the leading in $|k_1|$ term in the expansion (11.11), we obtain

$$Q(|k_1|) \approx \frac{9 \pi}{800} N_c g^2 \left( \frac{|k_1|}{\omega_{pl}^2} \right)^3. \quad (11.12)$$

The function $W_{|k_1|}^l$ is approximated by its equilibrium value [17]: $W_{|k_1|}^l \approx 4\pi T$ and therefore

$$W_{|k_1|}^l \approx 16\pi^2 k_1^2 T. \quad (11.13)$$

Substituting (11.12) and (11.13) into (11.10), and setting $|k_1^*| \sim gT$, we finally define

$$\gamma^l_S(0) \approx +1.04 N_c g^2 T. \quad (11.14)$$

In our case the coefficient of the $g^2 T$ has the same sign but is significantly large than corresponding one, calculated in [4].

12. THE $\gamma^l(0)$ DEPENDENCE ON A GAUGE PARAMETER

Now we estimate the contribution of a gauge dependence term of a propagator in covariant gauge (6.8) to the nonlinear Landau damping rate

$$\mathcal{D}_{\rho\alpha}^l(k_2) = \xi k_2\rho k_2\alpha/[(\omega_2 + i\epsilon)^2 - k_2^2]^2, \quad k_2 = k - k_1. \quad (12.1)$$
By the Ward identities (7.10), a gauge-dependent part of $\gamma'(k)$ can be represented as

$$
\gamma'_\xi(k) = -2\zeta g^2 N_c \text{Im} \int \frac{d\omega_{k_1}}{\omega_{k_1}} \left[ \left( \frac{\partial \text{Re} \varepsilon'(k)}{\partial \omega} \right) -1 \left( \frac{\partial \text{Re} \varepsilon'(k_1)}{\partial \omega_1} \right) -1 \right] 
$$

$$
\frac{(\omega\omega_{k_1}(kk_1) - k^2 k_1^2)^2}{k^2 k_1^2(k^2 k_1^2)^2} (\Delta^{-1}(k) - \Delta^{-1}(k_1))^2 \frac{1}{[(\omega_2 + i\epsilon)^2 - k_2^2]^2} \left[ \omega = \omega_{k_1} \omega_1 = \omega_{k_1} \right].
$$

(12.2)

If we take into account that $\Delta^{-1}(k)_{\omega = \omega_{k_1}} = \Delta^{-1}(k_1)_{\omega = \omega_{k_1}} \equiv 0$, then formally, a gauge-dependent part of nonlinear Landau damping rate vanishes, as it was mentioned in Sec. 8. However, we show that the integral in the r.h.s. of (12.2) develops on mass-shell poles. We consider, for example, the coefficient of $(\Delta^{-1}(k))^2$. In the limit $|k| \to 0$ this coefficient is equal to

$$
\int k_1^2 d|k_1| \left( \frac{W'_{k_1}}{\omega_{k_1}} \right) \left( \frac{\omega_{k_1}}{k_1^2} \right)^2 \left( \frac{\partial \text{Re} \varepsilon'(k_1)}{\partial \omega_1} \right) -1 \left( \frac{\partial \text{Re} \varepsilon'(k_1)}{\partial \omega_1} \right) -1 \frac{1}{\omega_1 = \omega_{k_1}} \text{Im} \left[ \frac{1}{[(\omega_{k_1} - \omega_{pd} - i\epsilon)^2 - k_1^2]^2} \right]
$$

(numerical factor is omitted). For lower limit, integrand expression (for $W'_{k_1} \simeq \text{const}$) is as follows

$$
\int \frac{d|k_1|}{|k_1|^2},
$$

i.e. the integral involves a power infrared divergence and is infinite at the pole. Thus, a gauge parameter in (12.2) is multiplied by $0 \times \infty$ uncertainty. We investigate this uncertainty, following by reasoning [27].

In order to evaluate the double poles in (12.1) we use the prescription

$$
\frac{1}{[(\omega_2 + i\epsilon)^2 - k_2^2]^2} = \lim_{m^2 \to 0} \frac{\partial}{\partial m^2} \frac{1}{[(\omega_2 + i\epsilon)^2 - k_2^2 - m^2]} = \begin{pmatrix} \frac{1}{\omega_2 - \sqrt{k_2^2 + m^2 + i\epsilon}} & \frac{1}{\omega_2 + \sqrt{k_2^2 + m^2 + i\epsilon}} \end{pmatrix}.
$$

(12.3)

By computing we first take $m^2 \to 0$ before the effective on mass-shell limit $\omega \to \omega_{pd}$.

We focus on the damping rate $\gamma'_\xi$ of excitation at rest (at vanishing three-momentum).

Performing the interesting limit $|k| \to 0$ in (12.2) and taking into account (12.3), we define

$$
\gamma'_\xi(0) = \frac{(2\pi)^2}{3} \zeta g^2 N_c \lim_{m^2 \to 0} \frac{1}{\omega_{pd}} \int_0^\infty \frac{k_1^2 d|k_1|}{\sqrt{k_1^2 + m^2}} \left( \frac{W'_{k_1}}{\omega_{k_1}} \right) \left( \frac{\partial \text{Re} \varepsilon'(k_1)}{\partial \omega_1} \right)_{\omega_1 = \omega_{k_1}} \left( \frac{\omega_{k_1}}{k_1^2} \right)^2
$$

$$
(\Delta^{-1}(\omega, 0) - \Delta^{-1}(\omega_{k_1}, k_1))^2 \{ \delta(\omega - \omega_{k_1} - \sqrt{k_1^2 + m^2}) - \delta(\omega - \omega_{k_1} + \sqrt{k_1^2 + m^2}) \}.
$$

(12.4)
Without restriction for the general case we choose \( \omega > \omega_{pl} + m \). Then we obtain from (12.4)

\[
\gamma_{l}^{\prime}(0) = \frac{2\pi^{2}}{3} \xi g^{2} N_{c} \frac{1}{\omega_{pl}} \lim_{m^{2} \to 0} \frac{\partial}{\partial m^{2}} \left[ \left( \frac{k_{10}}{\sqrt{k_{10}^{2} + m^{2}}} \right) \left( \frac{W_{k_{10}}}{\omega_{k_{10}}} \right) \left( \frac{\partial \text{Re} \varepsilon^{l}(k_{10})}{\partial \omega} \right)_{\omega = \omega_{k_{10}}}^{-1} \frac{(\omega_{k_{10}}^{l})^{2}}{(\omega_{k_{10}}^{l})^{2} - k_{10}^{2}} \right]
\]

\[
(\Delta_{-1}^{-1}(\omega, 0) - \Delta_{-1}^{-1}(\omega - \sqrt{k_{10}^{2} + m^{2}}, k_{10}))^{2} \left( \frac{\partial \sqrt{k_{10}^{2} + m^{2}}}{\partial k_{10}^{l}} + \left. \frac{\partial \omega_{k_{10}}^{l}}{\partial k_{10}^{l}} \right|_{k_{10} = k_{10}} \right)^{-1} \right], \quad (12.5)
\]

where \( k_{10} \) is a solution of equation

\[
\omega_{k_{10}}^{l} = \omega - \sqrt{k_{10}^{2} + m^{2}}.
\]

In order to perform the interesting limit \( m^{2} \to 0 \) and \( \omega \to \omega_{pl} \), we notice that the solution \( k_{10} \) vanishes for \( m^{2} = 0 \) as

\[
k_{10} \approx \omega - \omega_{pl} + O(k_{10}^{2}).
\]

Working out the derivative with respect to \( \partial/\partial m^{2} \) and taking \( m^{2} \to 0 \), we find that the most singular term in the limit \( k_{10} \to 0 \) comes from the derivative

\[
\frac{\partial}{\partial m^{2}} \left( \frac{k_{10}}{\sqrt{k_{10}^{2} + m^{2}}} \right) \to \frac{1}{2k_{10}^{2}}.
\]

Using the approximation \( W_{k_{10}}^{l} \approx 4\pi T \) and \( \Delta_{-1}^{-1}(\omega, 0) - \Delta_{-1}^{-1}(\omega - k_{10}, k_{10}) \approx 2\omega_{pl}k_{10} \), we finally obtain

\[
\gamma_{l}^{\prime}(0) \approx -2 \frac{2(2\pi)^{3}}{3} \xi g^{2} N_{c} T \left( \frac{\omega - \omega_{pl}}{\omega_{pl}} \right).
\]

(12.6)

From the last expression we notice that by going on mass-shell, the gauge-dependent part of the nonlinear Landau damping rate vanishes.

Excess factor \( (\omega - \omega_{pl}) \) of numerator (12.6) is arisen from the function

\[
\left( \frac{\partial \sqrt{k_{10}^{2} + m^{2}}}{\partial k_{10}^{l}} + \left. \frac{\partial \omega_{k_{10}}^{l}}{\partial k_{10}^{l}} \right|_{k_{10} = k_{10}} \right)^{-1}
\]

in the expression (12.5). In paper [27] this factor is compensated by singularity of statistic factor

\[
\lim_{k_{10} \to 0} \lim_{m^{2} \to 0} f_{g}(\sqrt{k_{10}^{2} + m^{2}}) \approx T \left( \frac{T}{\omega - \omega_{pl}} \right),
\]

associated with the spectral representation of bare propagator \( 1/(k^{2} + m^{2}) \). In our case this factor is absent.

One can lead to the result (12.6) in a different way. In derivation of the expression for \( \gamma^{l}(k) \) (6.15) we use representation of the spectral density \( I_{\omega,k}^{l} \) in the form (6.10). General speaking, this representation holds in the linear approximation only, when the time
correlation (dependence on \(\omega\)) is one-to-one correspondence with the spatial correlation (dependence on \(k\)). To include the effects of weakly nonstationary inhomogeneous plasma motions (when one-to-one correspondence between excitations frequency and its wave number fails), we replace the sharp \(\delta\)-function in the \(I_{\omega,k}^l\) by Breit-Wigner form \([6, 32, 29]\), with width \(\gamma^l\)

\[
I_{\omega,k}^l = \frac{1}{\pi} \left( I_k^l \frac{\gamma^l}{(\omega - \omega_k^l)^2 + (\gamma^l)^2} + I_{-k}^l \frac{\gamma^l}{(\omega + \omega_k^l)^2 + (\gamma^l)^2} \right),
\]

(12.7)

The parameter \(\gamma^l\) is interpreted as damping rate of boson mode of order \(g^2 T\). Note that in this case we are dealing with correlator \(\langle A_{a\mu}(X_1) A_{b\nu}(X_2) \rangle\) dependence on difference \(X_1 - X_2\).

Dependence on the midpoint \((X_1 + X_2)/2\) is accounted in the form of a dependence on a slow coordinate and a slow time in the l.h.s. of kinetic equation (6.14) by means of the operator \(\partial/\partial t + V^l_k \partial/\partial x\).

Using representation (12.7) and formula

\[
\frac{1}{[(\omega_1 - \omega_\text{pl} - i\epsilon)^2 - k_1^2]^2} = \frac{\partial}{\partial k_1^2} \frac{1}{2|k_1|} \left( \frac{1}{\omega_1 - \omega_\text{pl} - |k_1| - i\epsilon} - \frac{1}{\omega_1 - \omega_\text{pl} + |k_1| - i\epsilon} \right),
\]

in this case we derive, instead of (12.5)

\[
\gamma^l(0) = -\frac{\pi}{3} \xi g^2 N_c \frac{1}{\omega_\text{pl}} \int_0^\infty k_1^2 d|k_1| W_{k_1} \frac{\partial}{\partial k_1^2} \left[ \frac{1}{|k_1|} \int_{-\infty}^{+\infty} d\omega_1 \frac{\gamma^l}{(\omega_1 - \omega_\text{pl})^2 + (\gamma^l)^2} \right]
\]

\[
(\Delta^{-1\ell}(\omega, 0) - \Delta^{-1\ell}(\omega_1, 0))^2 \{ \delta(\omega_1 - \omega_\text{pl} - |k_1|) - \delta(\omega_1 - \omega_\text{pl} + |k_1|) \}.
\]

Perform the integration with respect to over \(d\omega_1\), and differentiation with respect to \(|k_1|^2\), we obtain expression

\[
\gamma^l(0) \simeq -4^2 \frac{(2\pi)^2}{3} \xi g^2 N_c T \left( \frac{\omega - \omega_\text{pl}}{\omega_\text{pl}} \right) \int_0^\infty k_1^2 d|k_1| \left| \frac{\gamma^l}{|k_1|^2 + (\gamma^l)^2} \right|^2.
\]

It follows the result (12.6).

13. CONCLUSION

Let us consider in more detail approximations scheme, which we use in this paper. In fact, here two types of the approximations are used. The first of them is connected with the employment of usual approach, developed in Abelian plasma, to QGP, i.e. the standard expansion of the current in powers of the oscillations amplitude and computation of interacting field in the form of a series of a perturbation theory in powers of a free field \(A^{(0)}\) (more precisely, in \(gA^{(0)}\)). However, in contrast to Abelian plasma, in our case even if
the first two nonlinear orders of the color current are taken into account, much more terms, defining the nonlinear scattering of waves are derived. Here, we use second approximation, connected with the notions going from the papers devoted to high-temperature QCD or more precisely, the set of rules for power counting these terms, developed by Blaizot and Iancu [12]. These rules enable us to singled out the leading terms in the coupling constant. These terms are purely non-Abelian in a full accordance with the conclusions of Ref. [12].

However we note that in [12] behaviour of a mean field \( \langle A^a_\mu(X) \rangle \) is investigated, which in our case vanishes. In the covariant derivative \( D_\mu \), containing only the random part of a gauge field in our approach, we suppose that \( \partial/\partial X^\mu \) and \( gA_\mu \) are of the different orders. In field strength tensor we distinguish the linear and nonlinear parts. Therefore the approximations scheme which have been carried out in this paper suffers from disadvantage of breaking non-Abelian gauge symmetry of a theory at each step of approximate calculation, as it was discussed in Sec. 4.

This is really true if we suppose that the magnitude of a soft random oscillations \( |A^a_\mu(X)| \) is of order \( T \) or \( |A^a_\mu(k)| \sim 1/g(gT)^3 \) in the Fourier representation. In this case, by using obtained expressions for terms in the expansion of a colour current (3.2), (5.4), (5.9) and estimations (5.7), (5.12), we have

\[
j^{T(1)}(k) \sim j^{T(2)}(k) \sim j^{T(3)}(k) \sim \ldots \sim \frac{1}{g^2 T},
\]
i.e. all terms in the expansion (2.14) are of the same order in magnitude and the problem of resummation of all the relevant contributions appears. Thus a gauge symmetry is recovered.

In this paper we have restricted ourselves to just a finite number of terms in the expansion (2.14). This impose more rigorous restriction on the magnitude of random oscillations: \( |A^a_\mu(X)| \sim gT \) \( (|A^a_\mu(k)| \sim 1/(gT)^3) \). In this case we have

\[
j^{T(1)}(k) \sim \frac{1}{gT}, \quad j^{T(2)}(k) \sim \frac{1}{T}, \quad j^{T(3)}(k) \sim \frac{g}{T}, \ldots ,
\]
each following term in the random current expansion is suppressed by more powers of \( g \) and use of the perturbation theory is justified. A gauge symmetry is restored when we take into account consistently all contributions at the leading order in \( g \) (Sec. 6) to the probability of the soft excitations scattering by the hard thermal particles. In this case we derive the expression for the nonlinear Landau damping rate (7.1), (7.2) which is closely allied in the form to corresponding gluon damping rate in HTL-approximation [4].

In Sec. 10 it was shown that the nonlinear interaction of longitudinal eigenwaves leads to effective pumping of energy across the spectrum sideways of small wave numbers. Consequence of this fact is the inequality \( \gamma'(0) < 0 \), i.e. \( k = 0 - \) mode is increased. The dissipation of the energy of plasma waves by QGP particles here, is not lead to total relaxation of a plasma excitations. In the scale of a small \(|k|\), effects described by
nonlinear terms in the expansion of the colour current of higher-order in the field, come into play. Consideration these effects involves suppression of increase of $k = 0$-mode.

The r.h.s. of obtained kinetic equation (6.16) in the regime $|k| \ll gT$ contains the part of possible processes of a plasmon scattering in QGP only, namely, the processes of a type

$$g^* + g \rightarrow g^* + g,$$
$$g^* + q(\bar{q}) \rightarrow g^* + q(\bar{q}),$$

where $g^*$ is plasmon collective excitations and $g, q, \bar{q}$ are excitations with characteristic momenta of order $T$. Diagrammatically this corresponds to graph with four external lines, where one of the incoming (outcoming) lines is a soft and the other is a hard. However it is clear that there are further contributions to the damping rate of a soft gluon, going without exchange of energy between a hard particles and waves. They are associated with the processes of nonlinear plasmon scattering by a soft excitations of QGP, i.e. with the processes of a type

$$g^* + g^* \rightarrow g^* + g^*,$$
$$g^* + q^*(\bar{q}^*) \rightarrow g^* + q^*(\bar{q}^*),$$

where $q^*, \bar{q}^*$ are plasmino collective excitations. The kinetic equation describing the process (13.1) and (13.2) is the equation of a purely Boltzmann type, i.e. collision term in the r.h.s. of this equation has standard Boltzmann structure, with on gain term and a loss term. The probability of these processes is defined by preceding methods from nonlinear current of a fourth order $j_T^{(4)}$, if the process of interaction iteration of higher-order in the field is taken into account.

As it was mentioned in Introduction, the Boltzmann equation was already used for definition of damping rate of the fast particles. In paper [11] (see also [14]) on the basis of Boltzmann equation, the damping rate for hard gluons in the leading logarithmic order has been computed. The value of obtained the damping rate is fully coincident with corresponding damping rate derived in quantum theory [6, 7]. The scattering matrix element appearing in collision term corresponds to elastic scattering of hard gluons in the resummed Born approximation. By Heiselberg and Pethick [11] was noted that the particle damping rate is considerably more difficult to calculate for small particle momenta, $|k| \leq gT$. Particles momentum becomes of the same order with momentum transfers. As in the case of scattering of plasmons by hard particles discussed above, vertex corrections should be taken into account also. It leads to more complicated expressions for probabilities of both plasmon-plasmon (13.1) and plasmon-plasmino (13.2) scattering in contrast to (9.14).

A detailed research of the process of (13.1) and its influence to relaxation of soft Bose excitations in QGP will be presented somewhere [26]. Here, we note only, that it is the process of higher-order in the field, which probability is defined with the help of three-gluon, four-gluon effective vertecies and effective propagator, as the probability of the process of nonlinear scattering of a plasmon by QGP particles, derived in this paper.
ACKNOWLEDGEMENTS

This work was supported by the Russian Foundation for Basic Research (Project No.97-02-16065).

APPENDIX A

We rewrite the system of equations (9.6), (9.7) in the coordinate representation. In the temporal gauge we have

\[
\frac{d^2 x}{dt^2} = \frac{g}{m} \sqrt{1-v^2} Q^a (E^a - v(E^a)),
\]

\[
\frac{dQ^a}{dt} = g f^{abc} (v A^b) Q^c, \tag{A1}
\]

where \(t\) is a coordinate time; \(v = dx/dt\), \(E^a = -\partial A^a/\partial t\), and

\[
A^a(x, t) = \int A^a_k e^{ikx-\omega_k t} dk, \quad A^a_k = \frac{k}{|k|} A^a_k. \tag{A2}
\]

The Eq. (9.6) corresponding to the component \(\mu = 0\), becomes identity.

If we neglect by the wave action on typical particle, then the colour charge motion will be constant and its colored vector will be fixed with initial condition: \(x_0(t) = v_0 t\), \(Q^a = Q^a_0\). To derive the oscillations of a color particle, excited by fields and being linear order in amplitude of field, it is necessary to neglect by variations of \(v\), \(Q^a\) in the r.h.s. of (A1) and set

\[
A^a(x, t) \simeq A^a(v_0 t, t) = \int A^a_k e^{-i(\omega_k - v_0)k x} dk, \text{ instead of (A2)}.
\]

In this approximation the solution of system (A1) has the form

\[
x(t) = v_0 t + \left( -\frac{g}{m} \right) Q^a_0 \sqrt{1-v_0^2} \int \frac{E^a_k - v_0(E^a_k)}{(\omega^2_k - v_0^2 k^2)} e^{-i(\omega_k - v_0)k t} dk \equiv x_0(t) + \Delta x(t),
\]

\[
Q^a(t) = Q^a_0 + ig f^{abc} Q^c_0 \int \frac{(v_0 A^b_k)}{\omega_k - v_0 k} e^{-i(\omega_k - v_0)k t} dk \equiv Q^a_0 + \Delta Q^a(t). \tag{A3}
\]

Using these expressions we derive the radiation intensity of oscillating colour charge.
It is equal to work of radiation field with charge in unit time

\[ \mathcal{W}^l = \int (E_Q^a J_Q^a) \, dx, \tag{A.4} \]

where \( E_Q^a \) is a field induced by a colour current \( j_Q^a \) of a charge \( Q^a \). The sign in the r.h.s. of (A4) corresponds to choose of a sign in front of current in the Yang-Mills equation (2.2).

We introduce \( E_Q^a \) and \( j_Q^a \) in the following form

\[ E_Q^a(x, t) = \int E_{k,\omega}^a e^{ikx - i\omega t} \, dk \, d\omega, \quad j_Q^a(x, t) = \int j_{k,\omega}^a e^{ikx - i\omega t} \, dk \, d\omega. \tag{A5} \]

The Fourier-component of a field \( E_{k,\omega}^a = k E_{k,\omega}^a/|k| \) is associated with \( j_{k,\omega}^a \) by Yang-Mills equation

\[ E_{k,\omega}^a = \frac{i}{\omega \varepsilon^I(\omega, k)} (k \cdot j_{k,\omega}^a). \tag{A6} \]

Substituting (A5) and (A6) into (A4) and taking into account reality of \( \mathcal{W}^l \), we obtain

\[ \mathcal{W}^l = \frac{(2\pi)^3}{2} \int i \left( \frac{1}{\omega \varepsilon^I(\omega, k)} - \frac{1}{\omega \varepsilon^I(\omega, k)'} \right) \frac{k^i k^j}{k^2} \langle j_{k,\omega}^{\alpha i} j_{k,\omega'}^{\beta j} \rangle e^{-i(\omega' - \omega)t} \, d\omega d\omega' dk. \tag{A7} \]

Here, by random character of a current phase, we replace the product \( j_{k,\omega}^{\alpha i} j_{k,\omega'}^{\beta j} \) by its averaged value.

The colour current formed by the constrained motion charge (A3) is presented as

\[ j_Q^a(x, t) = g v(t)^a(t) \delta(x - x(t)) = g v(t)^a(t) \int e^{ik(x - x(t))} \frac{dk}{(2\pi)^3} = j_k(t) e^{ikx} \, dk, \]

where we have in the linear approximation

\[ j_k(t) \equiv \frac{g}{(2\pi)^3} v(t)^a(t) e^{-ikx(t)} \approx \tag{A8} \]

\[ \simeq \frac{g}{(2\pi)^3} v_0 Q_0^a e^{-ikv_0 t} + \frac{g}{(2\pi)^3} \{ Q_0^a \Delta v(t) - i v_0 (k \cdot \Delta x(t)) \} + v_0 \Delta Q^a(t) \} e^{-ikv_0 t}, \]

and \( \Delta x(t), \Delta v(t), \Delta Q^a \) are defined by (A3).

The first term in the r.h.s. of (A8) is connected with linear Landau damping which is absent in QGP. Because of this fact this term is omitted. The term in braces, proportional to \( Q_0^a \) yields the Abelian contribution to radiation and is connected with usual spatial oscillation of a colour particle. The term with \( \Delta Q^a \) gives the non-Abelian part of radiation and is induced by precession of a colour vector of a particle in the field of incident wave. As it is easily to see the interference of these two contribution vanishes. Further we shall restrict our consideration to the second part of radiation which induced by non-Abelian part of a colour current

\[ (j_{k,\omega}^a)_{\text{non-Abelian}} = \int (j_k(t))_{\text{non-Abelian}} e^{ikx} \, dt. \]
\[
= \frac{ig^2}{(2\pi)^3} f^{abc} Q_0 \int \frac{v_0(v_0 A^b_{k'})}{\omega_{k'} - v_0 k'} \delta(\omega - \omega_{k'} + v_0(k' - k)) \, dk'.
\]

Substituting the last expression into \( (A7) \), taking into account

\[
\langle A^a_{k'} A^d_{k''} \rangle = I_{k'}^{l} \delta(k' - k'') \delta^{bd}
\]

and integrating over \( d\omega, d\omega' \) and \( dk'' \), we obtain with replacement of argument \( k' \rightarrow k_1 \)

\[
(W^l)_{\text{non-Abelian}} = -\frac{g^4 N_c q^2}{(2\pi)^3} \int \frac{dkdk_1}{k^2k_1^2} \text{Im} \left( \frac{1}{\omega_{k_1} \varepsilon^l(\tilde{\omega}_{k_1}, k)} \right) \frac{(k\nu)^2(k_1\nu)^2}{(\omega_{k_1} - v k_1)^2} I_{k_1}^{l}. \tag{A9}
\]

Here, \( q^2 \equiv Q_0^2 Q_0^a, \tilde{\omega}_{k_1}^l \equiv \omega_{k_1}^l - (k_1 - k)\nu \) and the suffix "0" of velocity is omitted.

For imaginary part of longitudinal permittivity we use approximation similar to [21]

\[
\text{Im} \frac{1}{\varepsilon^l(\tilde{\omega}_{k_1}, k)} \approx -\pi \left( \frac{\partial \text{Re} \varepsilon^l(k)}{\partial \omega} \right)^{-1} \omega = \omega_k \delta(\omega_k - \tilde{\omega}_{k_1}^l).
\]

Going from the spectral density \( I_{k_1}^{l} \) to density of longitudinal oscillations number \( N_{k_1}^{l} \) and setting the constant \( q^2 \) equal to \( 2/(2\pi)^3 \), we can cast radiation intensity expression \( (A9) \) into the final form

\[
(W^l)_{\text{non-Abelian}} = \int w_{\nu}^l(k, k_1) N_{k_1}^{l} \omega_{k_1}^{l} \frac{dkdk_1}{(2\pi)^6}, \tag{A10}
\]

where \( w_{\nu}^l(k, k_1) \) is defined by \( (9.3) \). Expression \( (A10) \) represents radiation intensity of isolated colour charge moving in a quark-gluon plasma in the direction \( \nu \) of a field of a longitudinal wave with frequency \( \omega_k^{l} \) and wave vector \( k \).
References

[1] E. Braaten and R.D. Pisarski, Nucl. Phys. B337, 569 (1990); ibid. B339, 310 (1990); J. Frenkel and J.C. Taylor, ibid. B334, 199 (1990)

[2] R. Kobes, G. Kunstatter, and A. Rebhan, Phys. Rev. Lett. 64, 2992 (1990); Nucl. Phys. B355, 1 (1991).

[3] R.D. Pisarski, Phys. Rev. Lett. 63, 1129 (1989)

[4] E. Braaten and R.D. Pisarski, Phys. Rev. D42, 2156 (1990)

[5] R. Kobes, G. Kunstatter, and K. Mak, Phys. Rev. D45, 4632 (1992); E. Braaten and R.D. Pisarski, ibid. D46, 1829 (1992)

[6] V.V. Lebedev and A.V. Smilga, Ann. Phys. (N.Y.) 202, 229 (1990); V.V. Lebedev and A.V. Smilga, Physica A181, 187 (1992)

[7] C.P. Burgess and A.L. Marini, Phys. Rev. D45, R17 (1992); A. Rebhan, ibid. D46, 482 (1992)

[8] U. Heinz, Phys. Rev. Lett. 51, 351 (1983); Ann. Phys. (N.Y.) 161, 48 (1985); 168, 148 (1986); J. Winter, J. Physique 45, C6-53 (1984); H.-Th. Elze, M. Gyulassy, and D. Vasak, Phys. Lett. B177, 402 (1986); Nucl. Phys. B276, 706 (1986); H.-Th. Elze and U. Heinz, Phys. Rep. 183, 81 (1989); St. Mrówczyński, Phys. Rev. D39, 1940 (1989); Yu.A. Markov and M.A. Markova, Teor. Math. Fiz. 111, 263 (1997) (Theor. Math. Phys. 111, 601 (1997))

[9] V.P. Silin, Zh. Eksp. Teor. Fiz. 38, 1577 (1960) [Sov. Phys. JETP 11, 1136 (1960)]

[10] P.F. Kelly, Q. Liu, C. Lucchesi, and C. Manuel, Phys. Rev. D50, 4209 (1994)

[11] H. Heiselberg and C.J. Pethick, Phys. Rev. D47, R769 (1993)

[12] J.P. Blaizot and E. Iancu, Nucl. Phys. B417, 608 (1994); ibid. B421, 565 (1994); ibid. B434, 662 (1995)

[13] D. Bödeker, Phys. Lett. B426, 351 (1998); preprint NBI-HE-99-04, hep-ph/9903478; preprint NBI-HE-99-13, hep-ph/9905239

[14] J.P. Blaizot and E. Iancu, preprint Saclay-T99/026, CERN-TH/99-71, hep-ph/9903389; preprint Saclay-T99/059, CERN-TH/99-172, hep-ph/9906485

[15] P. Huet and D.T. Son, Phys. Lett. B393, 94 (1997); P. Arnold, D.T. Son, and L.G. Yaffe, Phys. Rev. D59, 105020 (1999); ibid. D60, 025007 (1999)
[16] D.F. Litim and C. Manuel, Phys. Rev. Lett. 82, 4991 (1999); preprint ECM-UB-PF-99-12, CERN-TH-99-151, [hep-ph/9906210]; M.A. Valle Basagoiti, preprint U. Pais Vasco EHU-FT/9905, [hep-ph/9903462]

[17] Z. Xiaofei and L. Jiarong, Phys. Rev. C52, 964 (1995)

[18] U. Heinz and P.J. Siemens, Phys. Lett. B158, 11 (1985)

[19] A.G. Sitenko, *Fluctuations and Non-linear Wave Interactions in Plasma* (Naukova Dumka, Kiev, 1977) [English transl. publ. by Pergamon, Oxford, 1990]

[20] Yu.A. Markov and M.A. Markova, preprint ISDCT-99-2, [hep-ph/9902397], to be published by Trasp. Theory Stat. Phys.

[21] B.B. Kadomtsev, *Plasma Turbulence* (Academic Press, New York, 1965); V.V. Pustovalov and V.P. Silin, Proc. P.H. Lebedev Inst. 61, 42 (1972); V.N. Tsytovich, *Non-linear Effects in Plasma* (Nauka, Moscow, 1967) [English transl. publ. by Plenum, Oxford, 1970]; V.N. Tsytovich, *Theory of Turbulent Plasma* (Nauka, Moscow, 1971) [English transl. publ. by Plenum, Oxford, 1977]; V.N. Tsytovich, Phys. Rep. 178, 261 (1989)

[22] H.-Th. Elze, Z. Phys. C38, 211 (1988)

[23] O.K. Kalashnikov and V.V. Klimov, Yad. Fiz. 31, 1357 (1980) [Sov. J. Nucl. Phys. 31, 699 (1980)]; V.V. Klimov, Zh. Eksp. Teor. Fiz. 82, 336 (1982) [Sov. Phys. JETP 55, 199 (1982)]

[24] H.A. Weldon, Phys. Rev. D26, 1394 (1982)

[25] K. Kajantie and J. Kapusta, Ann. Phys. (N.Y.) 160, 477 (1985); U. Heinz, K. Kajantie, and T. Toimela, *ibid.* 176, 218 (1987)

[26] Yu.A. Markov and M.A. Markova, in preparation

[27] R. Baier, G. Kunstatter, and D. Schiff, Nucl. Phys. B388, 287 (1992)

[28] R. Baier, G. Kunstatter, and D. Schiff, Phys. Rev. D45, R4381 (1992); A. Rebhan, *ibid.* D46, 4779 (1992); H. Nakkagawa, A. Niégawa, and B. Pire, Phys. Lett. B294, 396 (1992); T. Altherr, E. Petitgirard, and T. del Río Gaztelurrutia, Phys. Rev. D47, 703 (1993)

[29] A.V. Smilga, Phys. At. Nuclei 57, 519 (1994)

[30] S.K. Wong, Nuovo Cim. A65, 689 (1970)

[31] G. Baym, H. Monien, C.J. Pethick, and D.G. Ravenhall, Phys. Rev. Lett. 64, 1867 (1990); J.P. Blaizot and E. Iancu, Phys. Rev. D55, 973 (1997)

[32] R.D. Pisarski, Phys. Rev. D47, 5589 (1993)