Evaluation of Two-Hinged Wings

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Two types of wing geometries imitating wings of birds in gliding flight are analyzed with the vortex lattice method in terms of lift-drag ratio and root bending moment while varying the joint positions and joint angles under the constant lift coefficient and constant flight velocity conditions. One geometry is, what is sometimes called, a gull wing which has a dihedral inner wing and horizontal outer wing, named “DH wing.” On the contrary, the other geometry is a drooped wing which has a horizontal inner wing and anhedral outer wing, named “HA wing.” The lift-drag ratio of the HA wing becomes larger when the joint of the anhedral outer wing is closer to the wing tip with a larger anhedral angle, while the root bending moment of the HA wing becomes smaller when the joint of the anhedral outer wing is closer to the wing root with a larger anhedral angle. In contrast, the DH wing has no combination of a joint position and joint angle to improve the lift-drag ratio, and it is found that the HA wing effectively contributes to the reduction of the root bending moment and the lift-drag ratio compared with the DH wing.

Key Words: Aerodynamics, Birds, Root Bending Moment, Wings and Airfoil Sections

Nomenclature

- \( AR \): aspect ratio, [-]
- \( b \): span, m
- \( C_D \): induced drag coefficient, [-]
- \( C_L \): lift coefficient, [-]
- \( C_N \): normal force coefficient, [-]
- \( C_{RBM} \): root bending moment coefficient, [-]
- \( c \): chord, m
- \( D \): drag, N
- \( e \): span-efficiency factor, [-]
- \( L \): lift, N
- \( LBD \): lift-drag ratio compared with a planar wing, [-]
- \( M \): moment, Nm
- \( M_{RBM} \): root bending moment, Nm
- \( N \): number of panels
- \( n \): normal vector on a panel of a wing, [-]
- \( RBM \): root bending moment compared with a planar wing, [-]
- \( S \): area, \( \text{m}^2 \)
- \( V \): flight velocity, \( \text{m/s} \)
- \( w \): normal component of downwash, \( \text{m/s} \)
- \( \Gamma \): circulation, \( \text{m}^2/\text{s} \)
- \( \eta \): length from root to joint as a fraction of semispan, [-]
- \( \theta \): dihedral or anhedral angle, deg

Subscripts

- \( \text{ahd} \): anhedral
- \( \text{DH} \): wing with a dihedral inner portion and horizontal outer portion
- \( \text{dhd} \): dihedral
- \( e \): effective

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1. Introduction

The minimum induced drag on a planar wing is well known to be achieved by an elliptic planform,1) which means there is no margins to improve for a planar wing from the view point of the induced drag or span-efficiency factor. However, the span-efficiency factor of a cambered-span wing in nonplanar wings was theoretically demonstrated to be improved by 50% compared with an elliptical wing in planar wings.2) Jones3) paid attention to root bending moment on a wing with maximizing the span-efficiency factor of a planar wing. In Ref. 4, root bending moments on the wings with drooped outer portion and other wing geometries were experimentally and numerically analyzed under the common span length. When the root bending moment on the wing with a drooping outboard was nearly equal to that of the planar wing, the span-efficiency factor of the former was greater than that of the latter. Andrews et al.5) investigated the span-efficiency factor of a wing model of a gull under the various physical limitations. The steady aerodynamic performances in gliding flights of static wing geometries in the flapping cycle were analyzed. They concluded that the maximum span-efficiency factor was obtained when the wing was slightly drooped under a condition that the physical limitations were fulfilled.

From another perspective, it was revealed that the drooped
wing contributed to the static pitch stability. In the research, the pitching moments around the wings’ centers of gravity about the four types of biologically inspired wings were measured in a wind tunnel test. A negative slope of the pitching moment coefficient was obtained in the drooped wing. Furthermore, Jones analyzed the optimum condition of a distribution of a downwash under a fixed total lift and root bending moment for a non-planar wing which has variable arc length and arc angle.

In this paper, a wing geometry with a horizontal inner portion and drooped outer portion is modeled as a two-hinged wing and investigated while varying the joint position and joint angle. In addition to the drooped wing, a wing with a dihedral inner portion and horizontal outer portion is also investigated. This paper focuses on the lift-drag ratio and the root bending moment of the two types of wing geometries. They are computed with the vortex lattice method, and their effects on the flight performance are discussed herein.

2. Methodology

2.1. Vortex lattice method

The vortex lattice method was used for the analysis. The wing geometry was discretized into a number of panels in spanwise, and the horseshoe vortices and collocation points were distributed on the quarter-chord line and 3/4-chord line, respectively. After solving the distribution of circulation on the wing, aerodynamic properties are known. The wing lift coefficient can be calculated as follows:

\[ C_L = \frac{2}{S_{\text{ref}}} \sum_{i=1}^{N} \left( \frac{\Gamma_i}{V_i} \right) \cos \theta_i \cdot d_{s_i} \]  

where \( d_{s_i} \) is the width of the \( i \)th panel and \( S_{\text{ref}} \) is the projected area of a wing. The induced drag coefficient on the wing can be computed as:

\[ C_D = \frac{2}{S_{\text{ref}}} \sum_{i=1}^{N} \left( \frac{w_i}{V_i} \right) \frac{\Gamma_i}{V_i} \cdot d_{s_i} \]  

(2)

The suction force at the leading edge, whose direction is nearly opposite to the induced drag is ignored in the present research. In the calculation, the angle of attack was adjusted so that \( C_L \) is close to 0.5. The span-efficiency factor is defined as:

\[ e = \frac{AR_e}{AR_{\text{ref}}} \]  

where the effective aspect ratio \( AR_e \) is expressed as \( C_L^2 / \pi C_{D_e} \), and the reference aspect ratio \( AR_{\text{ref}} \) is expressed as \( b_{\text{ref}}^2 / S_{\text{ref}} \). The root bending moment coefficient is expressed as:

\[ C_{\text{RBM}} = \frac{1}{S_{\text{ref}} b_{\text{ref}}} \sum_{i=1}^{N} r_i \times \left( \frac{\Gamma_i}{V_i} \right) d_{s_i} n_i \cdot e_x \]  

(4)

where \( r_i \) is the vector from the wing root to the middle point of the \( i \)th chord, and \( e_x \) is the basis vector in the direction of the \( x \)-axis, shown in Fig. 1, for the right wing.

In the previous equations, the reference span and area are the length or area projected on the \( xy \) plane, respectively. The root bending moment coefficient is calculated for a right-hand side of a wing. A root bending moment due to the wing weight acts in the opposite direction of the root bending moment due to the lift. The former was not considered, because the weight is expected to be much smaller than the total weight of an airplane.

Figure 1 portrays two types of wing geometry. One wing geometry has a horizontal inner wing and anhedral outer wing, named “HA wing.” The other geometry has a dihedral inner wing and horizontal outer wing, named “DH wing.” In the present research, the aerodynamic characteristics of the HA and DH wings are compared with those of the planar wing, which is the wing in the HA with \( \theta_{\text{dhd}} = 0^\circ \) and DH wing with \( \theta_{\text{dhd}} = 0^\circ \), respectively. The following two parameters are used for comparison. The first one is a lift-drag ratio of a wing. In the present research, only the induced drag on a wing is considered for the total drag. And the lift is equal to the weight and it is constant for all conditions. Then, the ratio of the lift-drag ratio between the HA/DH wing and the planar wing is given as:

\[ LBD = \frac{(C_L)_{\text{planar}}}{(C_L)_{\text{HA/DH}}} \]  

\[ = \frac{(D_i)_{\text{planar}}}{(D_i)_{\text{HA/DH}}} \]  

\[ = \frac{(C_L)_{\text{planar}}}{(C_L)_{\text{HA/DH}}} \frac{e}{(e)_{\text{planar}}} \left( \frac{b_{\text{ref}}}{b_{\text{wet}}} \right) \]  

where “+” denotes the condition (A) or (B) as stated below. The other one is a root bending moment of a wing. The ratio of the root bending moment between the HA/DH wing and the planar wing is given as:

![Fig. 1. DH and HA wing geometries.](image-url)
The root bending moment between the HA and DH wings, their contributions to the total lift coefficient, is determined only by the wing geometry and independent of the angle of attack. This is because the distribution of the lift along the span depends only on the wing geometry in the case that the total lift is fixed, which means $C_{RBM}/(C_{RBM})_{planar}$ is independent of the angle of attack in the calculation. The comparisons are evaluated under the following two conditions. (A) The lift coefficient $C_L$ among the HA and DH wings and the planar wing is given by:

$$LBD_{(A)} = \frac{e}{(e)_{planar}} \cdot \frac{b_{ref}}{b_{wet}}$$

(7)

The ratio of the root bending moment between the HA/DH wing and the planar wing is given by:

$$RBM_{(A)} = \frac{C_{RBM}}{(C_{RBM})_{planar}} \cdot \frac{b_{ref}}{b_{wet}}$$

(8)

2.1.2. Condition (B)

The ratios of the lift-drag ratio and the root bending moment between the HA/DH wing and the planar wing are given by:

$$LBD_{(B)} = \frac{e}{(e)_{planar}} \cdot \left( \frac{b_{ref}}{b_{wet}} \right)^2$$

(9)

$$RBM_{(B)} = \frac{C_{RBM}}{(C_{RBM})_{planar}} \cdot \left( \frac{b_{ref}}{b_{wet}} \right)^2$$

(10)

Equations (9) and (10) were derived by substituting Eq. (11) into Eqs. (5) and (6), respectively.

$$\frac{1}{2} \rho V^2 S_{ref} C_L = \left( \frac{1}{2} \rho V^2 S_{wet} C_L \right)_{planar}$$

(11)

Eqs. (7) and (9) mean that $LBD$ is determined only by the wing geometry and independent of the angle of attack, similarly to $RBM$. This is because the span-efficiency factor $e$ and $b_{ref}/b_{wet}$ are not dependent of the angle of attack but also dependent of the wing geometry. Then, both of $LBD$ and $RBM$ are determined by the geometry. The indices expressed in Eqs. (7)–(10) do not depend on the sign of $\theta_{abd}$ and $\theta_{abd}$ because the boundary condition in the vortex lattice method is independent of the sign, although $\theta_{abd}$ and $\theta_{abd}$ are set to be positive in the following analysis.

### 2.2. Computation model

Two simple models for gliding-bird wings, the HA and DH wings portrayed in Fig. 1, were investigated. Both computational models have joints at the shoulders and wrists, which are called double-hinge wings. The wing inner/outer portions are assumed to be a rectangular geometry without twist angles or changes in airfoil along the span. This is because the effect of the difference of the wing geometry from the planner wing on the aerodynamic performance is focused in the present analysis.

The parameters in these geometries are the anhedral angle, $\theta_{abd}$, on the outer wing of the HA wing, the dihedral angle, $\theta_{abd}$, on the inner wing of the DH wing, and the joint to semi-span ratio, $\eta$, on both wings. The joint to semispan ratio is defined as:

$$\eta = \frac{b_s}{b_{wet}}$$

(12)

The study cases on the HA wing with $\theta_{abd}$ from $0–80^\circ$ and $\eta$ of $0.0–1.0$ and on the DH wing with $\theta_{abd}$ from $0–80^\circ$ and $\eta$ of $0.0–1.0$ were investigated.

Some fixed parameters are imposed on this analysis for a reasonable comparison. The fixed parameters are summarized in Table 1. In this paper, a wing lift $L$ is assumed to be constant, then a product of a squared flight velocity $V^2$, projected area $S_{ref}$, and lift coefficient $C_L$ is constant as well. That is, a change in the projected area results in a change in the flight velocity under the condition (A) or a change in the lift coefficient under the condition (B).

The total wetted wing span and wing lift are common for the HA and DH wings. The case that the inner/outer wings have a lift coefficient larger than 1, which cannot be ordinarily observed, should be excluded. Then, the constraints for $\eta$, which will be indicated by Eqs. (16) and (17), should be satisfied.

### 2.2.1. HA wing geometry constraint

When the HA wing geometry consists of inner and outer wings, their contributions to the total lift coefficient can be estimated as follows:

$$C_{L_{diff}} = \eta C_{L_i} + (1 - \eta) C_{L_v}$$

$$\equiv \eta C_{N_i} + (1 - \eta) C_{N_v} \cos \theta_{abd}$$

(13)

The right-hand side in this equation can be expressed with the normal force coefficients, $C_{N_i}$ and $C_{N_v}$. Then, the maxi-

| Table 1. Fixed parameters for the analysis. |
|-------------------------------------------|
| **Fixed properties**                     | **Values** |
| Wetted aspect ratio, $A_{Rwet}$, [-]      | 10         |
| Wetted semispan, $b_{wet}/2 = b_s + b_v$, m | 5.0        |
| Chord, $c$, m                             | 1.0        |
| Number of panels for the entire wing, $N_{panel}$, [-] | 100        |
| Wetted area, $S_{wet}$, m²               | 10         |
| Twist along span, deg                     | 0.0        |
| Zero-lift angle along span, deg           | 0.0        |

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maximum value of $C_{L,HA}$ can be expressed by using the maximum values of $C_{N,}$ and $C_{N,}$ as:

$$C_{L,HA,max} = \eta C_{N,max} + (1 - \eta)C_{N,max} \cos \theta_{ahd}$$  \hfill (14)

Because the wing lift coefficient is required to be less than the maximum wing lift coefficient,

$$C_{L,HA} < C_{L,HA,max} = \eta C_{N,max} + (1 - \eta)C_{N,max} \cos \theta_{ahd}$$  \hfill (15)

Assuming that $C_{N,max} = C_{N,max} = C_{L,max}$ in Eq. (15),

$$\cos \theta_{ahd} > \frac{C_{L,HA}}{C_{L,max}} - \eta$$  \hfill (16)

### 2.2.2. DH wing geometry constraint

Similarly, the following relation on the DH wing can be obtained as follows:

$$\cos \theta_{ahd} > \frac{C_{L,DH}}{C_{L,max}} + (\eta - 1)$$  \hfill (17)

Equations (16) and (17) show the constraints for $\eta$ under given values of $\theta_{ahd}$ and $\theta_{ahd}$. In this analysis, the $C_{L,ext}/C_{L,max}$ and the $C_{L,ext}/C_{L,max}$ are set to 0.50 (e.g. $C_{L,ext} = C_{L,ext} = 0.5$ and $C_{L,max} = 1.0$). Note that the curves expressed as $LBD$ and $RBM$ defined in Eqs. (7)-(10) in all figures shown below are independent of the lift coefficient, as stated above. However, the region of $\eta$, which is determined by Eqs. (16) and (17) depends on the ratio of the lift coefficient to the maximum lift coefficient.

### 3. Results

#### 3.1. HA wing

Initially, plots of $LBD$ under the conditions of (A) and (B) on the HA wing compared with the planar wing are shown in Figs. 2 and 3, respectively. The lines which do not satisfy Eq. (16) are portrayed in light gray, while the lines which satisfy are portrayed in black.

In Fig. 2, the configuration for maximizing $LBD$ under the condition that the lift coefficient $C_{L}$ is identical is obtained when $\eta = 0.8$ and $\theta_{ahd} = 80^\circ$ and the value of $LBD$ is improved compared with that of the planar wing by 2.5%. This geometry is close a geometry of a wing with winglets. The value of $LBD$ is monotonically deteriorated as $\eta$ decreases and $\theta_{ahd}$ increases, except the configuration with $\eta$ of 0.6–1.0. Figure 3 portrays $LBD$ under the condition that the flight velocity $V_{\infty}$ is identical, and shows there is not any HA configuration which overcomes the planar wing, which is different from the characteristics seen in Fig. 2. The differences are due to the difference between Eqs. (7) and (9), that is the squared $b_{ref}/b_{wet}$.

Secondly, $RBM$ on the HA under the conditions of (A) and (B) wing compared with the planar wing are plotted in Figs. 4 and 5 with the same color code as in Figs. 2 and 3. Figures 4 and 5 show that $RBM$ under the conditions of (A) and (B) improves for any combination of $\eta$ and $\theta_{ahd}$. $RBM$ decreases with the decrease of $\eta$ and the increase of $\theta_{ahd}$ except the configuration with a small $\eta$. The decrease of $RBM$ under the condition (B) is larger than that under the condition (A). Figures 2 and 3 show that the disadvantage of $LBD$ obtained in the condition (B) is larger than that in the condition (A) due to the multipler $b_{ref}/b_{wet}$ as stated before. On the other hand, the advantage of $RBM$ in the condition (B) is larger than that in the condition (A) because of the same reason.

The non-dimensional circulation distributions with $\theta_{ahd}$ of 0°, 40°, 60°, 70°, and 80° and $\eta$ of 0.35 are illustrated in Fig. 6. The inflection points correspond to the joints, whose values of $y/(1/2h_{ref})_{mid}$ are 0.350, 0.413, 0.519, 0.612, and 0.721, respectively. Their values approach 1.0 as the increase of $\theta_{ahd}$, because the $b_{ref}$ defined in Fig. 1 becomes smaller. This tendency can be obvious from the following equation.
\[ y \left( \frac{1}{2} h_{\text{ref}} \right)_{\text{jet}} = \frac{\eta - \frac{1}{2} b_{\text{wet}}}{\{\eta + (1 - \eta) \cos \theta_{\text{anh}}\} \frac{1}{2} b_{\text{wet}}} \]

\[ = \frac{\eta}{\eta + (1 - \eta) \cos \theta_{\text{anh}}} \]

Note that the wetted span \( b_{\text{wet}} \) is fixed to be 10 m. In Fig. 6, the non-dimensional circulation is a value divided by \( C_\text{L} \). The obtained curves are independent of the wing lift coefficient because \( \Gamma \) is proportional to \( C_\text{L} \). The distribution of circulation for \( \theta_{\text{anh}} \) of 0° in Fig. 6 corresponds to that of the planar wing. The circulation on the inner wing increases and conversely, that on the outer wing decreases, as the anhedral angle increases. This bell-shaped distribution of circulation leads to the smaller root bending moments, shown in Figs. 4 and 5.

3.2. DH wing

Subsequently, plots of \( LBD \) under the conditions of (A) and (B) on the DH wing are shown in Figs. 7 and 8, respectiv-
transformation. The lines which do not satisfy Eq. (17) are portrayed in light gray, while the lines which satisfy the equation are portrayed in black.

The DH wing configuration does not improve LBD under the conditions of (A) and (B), though the combination of $\eta$ and $\theta_{dhd}$ to improve the LBD of the HA wing configuration under the condition (A) was found as shown in Fig. 2.

Finally, RBM on the DH wing under the conditions of (A) and (B) are plotted in Figs. 9 and 10 with the same color code as in Figs. 7 and 8.

Figure 9 shows that RBM under a given $\theta_{dhd}$ under the condition (A) improves for any combination of a joint position and joint angle, but this improvement is less than that seen in Fig. 4 for the HA wing. RBM decreases with the increase of $\theta_{dhd}$ and it decreases with the increase of $\eta$ except for wings having a large $\eta$ under condition (A). Although the evaluated LBD and RBM on the DH wing are inferior to those on the HA wings, the DH wing with the positive $\theta_{dhd}$ could be more feasible from the viewpoint of the lateral/ directional stability.

In the above analysis, two kinds of indices, LBD and RBM, were evaluated with wide ranges of $\theta_{dhd}$, $\theta_{fl}$ and $\eta$ under two different conditions. The focused configurations on the DH/HA wings in the above discussions, such as the configurations depicted in Figs. 2, 4 and 9 have distinctive characteristics with respect to LBD and RBM, then the results in this paper can be utilized as one of the charts for a non-planar wing design.

4. Conclusions

The lift-drag ratio and the root bending moment on the HA wing with a horizontal inner portion and drooped outer portion and DH wing with a dihedral inner portion and horizontal outer portion were investigated under the constant lift coefficient and constant flight velocity conditions while varying the joint positions and joint angles with the vortex lattice method. The HA wing has two distinctive configurations. One configuration with a drooped short outer wing, which is close to a wing with winglets, improves a little the lift-drag ratio under the condition that lift coefficient is constant compared with a planar wing. The other configuration with a highly drooped outer wing realizes a reduction of the root bending moment under the constant lift coefficient condition and the constant flight velocity condition compared with the planar wing, though it deteriorates the lift-drag ratio.

There is no configuration of the DH wing to improve the lift-drag ratio under both of the conditions, while any combination of a joint position and joint angle of the DH wing realizes a reduction of the root bending moment. The DH wing derives the smaller reduction of the root bending moment compared with the HA wing.

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