Noncommutative gauge theory with the symmetry $U(n \otimes m)^*$ and standard-like model with fractional charges

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Abstract

U($n \otimes m$)$^*$ gauge field theory on noncommutative spacetime is formulated and the standard-like model with the symmetry U($3_c \otimes 2 \otimes 1_Y$)$^*$ is reconstructed based on it. On the noncommutative spacetime, the representation that fields belong to is fundamental, adjoint or bi-fundamental. For this reason, one had to construct the standard model by use of bi-fundamental representations. However, we can reconstruct the standard-like model with only fundamental and adjoint representations and without using bi-fundamental representations. It is well known that the charge of fermion is 0 or $\pm 1$ in the U(1) gauge theory on noncommutative spacetime. Thus, there may be no room to incorporate the noncommutative U(1) gauge theory into the standard model because the quarks have fractional charges. However, it is shown in this article that there is the noncommutative gauge theory with arbitrary charges which symmetry is for example U($3 \otimes 1$)$^*$. This type of gauge theory emerges from the spontaneous breakdown of the noncommutative U(4)$^*$ gauge theory in which the gauge field contains the $0$ component $A_{\mu}^0(x, \theta)$. The standard-like model in this paper also has fermion fields with fractional charges. Thus, the noncommutative gauge theory with fractional U(1) charges can not exist alone, but it must coexist with noncommutative nonabelian gauge theory.

1 Introduction

In the past several years, field theories on the noncommutative(NC) spacetime have been extensively studied from many different aspects. The motivation comes from the string theory which makes obvious that end points of the open strings trapped on the D-brane in the presence of two form B-field background turn out to be noncommutative [1] and then the noncommutative supersymmetric gauge theories appear as the low energy effective theory of such D-brane [2], [3].

After the completion of standard model in particle physics more than decades ago, there have been several occasions that indicated the experimental deviations from the standard model. However, such deviations ultimately shrank to nothing and the correctness of the standard model has been confirmed. Thus, many advanced theories beyond standard model must reduce to the standard model in their characteristic limits. If not, such theory is branded not to be the qualified theory beyond standard model. This is the case also in the NC field theory. However, there are many difficulties in gauge theories on NC spacetime because of the noncommutativity of spacetime. One of the difficulty [7, 8, 9] is that in order to respect the gauge invariance the field must belong to the fundamental, adjoint or bi-fundamental representation of the gauge group on NC spacetime. In the standard model, three symmetries such as color, weak isospin and hypercharge are there. Thus, quarks have to be bi-fundamental in the symmetry U($3_c \times U(2)_L \times U(1)_Y$) as in [10, 11] which tried to construct the standard model on NC spacetime.

In this article, U($n \otimes m$)$^*$ gauge field theory on noncommutative spacetime is formulated. U($n \otimes m$)$^*$ gauge group is the resultant gauge group from the spontaneous symmetry breakdown of U($N$)$^*$ ($N = n+m$). It reduces to U($n \otimes m$) on the commutative spacetime which is not U($N$) but isomorphic to SU($n$)$\otimes$ SU($m$)$\otimes$ U(1). The standard-like model with the symmetry U($3_c \otimes 2 \otimes 1_Y$)$^*$ is reconstructed without using the bi-fundamental representation of fields. It should be noted that the symmetry U($3_c \otimes 2 \otimes 1_Y$)$^*$ is different from U($3_c \times U(2)_L \times U(1)_Y$)$^*$ as explained in the fourth section.

There are other difficulties in NC gauge theory. Hayakawa [4] indicated that the matter field must have charge 0 or $\pm 1$ in U(1) NC gauge theory in order to keep the gauge invariance of the theory. Matsubara [5] and Armoni [6] also indicated that U(N) gauge theory has the consistency in calculations.
of gluon propagator and three gluons vertex to one loop order, whereas SU(N) gauge theory is not consistent. These problems make it very difficult to reproduce the standard model in the framework of NC gauge theory. The indication made by Hayakawa [14] is serious since quarks in standard model have fractional charges. There must be the story other than Hayakawa’s indication if the noncommutativity on the spacetime is somewhat true in nature. We can overcome this difficulty by considering nonabelian U(n ⊗ m)* gauge theory [13] which results from the spontaneous symmetry breakdown of U(N)* gauge theory. We conclude that gauge theory that has the matter fields with fractional charges can not exist alone, but it must coexist with NC nonabelian gauge theory.

This article consists of 5 sections. In second section, the NC nonabelian gauge theory with the symmetry U(n ⊗ m)* is proposed. In third section, the spontaneous symmetry breakdown of SU(4)* gauge theory is discussed in order to show that quarks has the fractional B-charges whereas the lepton has charge −1. In fourth section, the standard-like model with the symmetry U(3 ⊗ 2 ⊗ 1y)* is proposed to show that fields with color as well as flavor quantum numbers can be expressed in terms of the fundamental representation and also fields with fractional charges are incorporated in the gauge field on NC spacetime. The last section is devoted to discussions and conclusions.

2 Nonabelian gauge theory on noncommutative spacetime

Let us first consider the nonabelian gauge theory on the NC spacetime with the symmetry U(n + m)* given by the Lagrangian

$$L = -\frac{1}{2} \text{Tr} [F_{\mu\nu}(x) * F^{\mu\nu}(x)] + \bar{\Psi}(x) * \left\{ i\gamma^\mu (\partial_\mu - igA_\mu(x)) - m \right\} * \Psi(x) + \text{Tr} \left[ (D^\mu \varphi(x))^\dagger * D_\mu \varphi(x) + m^2 \varphi(x)^\dagger * \varphi(x) - \lambda (\varphi(x)^\dagger * \varphi(x))^2 \right],$$

(2.1)

where we omit the gauge fixing and FP ghost terms. The Moyal star product of functions \(f(x)\) and \(g(x)\) is defined as

$$f(x) * g(x) = e^{i\theta^{\mu\nu} \partial_\mu \partial_\nu f(x_1) g(x_2)} \bigg|_{x_1=x_2=x},$$

(2.2)

where \(\theta^{\mu\nu}\) is constant with two subscripts to characterize the noncommutativity of spacetime. The noncommutative parameter \(\theta^{\mu\nu}\) is usually seemed to be a constant not to transform corresponding to Lorentz transformation. \(\Psi(x)\) is the fermion field with the fundamental representation. The quantity

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) - ig [A_\mu(x), A_\nu(x)]_*$$

(2.3)

is the field strength of gauge field with the configuration

$$A_\mu(x) = \sum_{a=0}^{n^2-1} \sum_{b=0}^{m^2-1} A_{\mu}^{ab}(x) T^a \otimes \tilde{T}^b.$$  

(2.4)

and the field \(\varphi(x)\) is the Higgs boson belonging to the adjoint or fundamental representation of U(n) ⊗ U(m) which covariant derivative is

$$D^\mu \varphi(x) = \partial^\mu \varphi(x) - ig [A^\mu(x), \varphi(x)]_*,$$

(2.5)

or

$$D^\mu \varphi(x) = \partial^\mu \varphi(x) - ig A^\mu(x) * \varphi(x).$$

(2.6)

The gauge transformations of fields in (2.1) are defined as

$$A^a_\mu(x) = U(x, \theta) * A_\mu(x) * U^{-1}(x, \theta) + \frac{i}{g} U(x, \theta) * \partial_\mu U^{-1}(x, \theta),$$

$$\Psi^a(x) = U(x, \theta) * \Psi(x),$$

$$\varphi^a(x) = U(x, \theta) * \varphi(x) * U^{-1}(x, \theta)$$

(2.7)

where the gauge transformation function \(U(x, \theta)\) is written as

$$U(x, \theta) = e^{i\alpha(x, \theta)*} = \sum_{n=0}^{\infty} \frac{i^n}{n!} \alpha(x, \theta) * \alpha(x, \theta) * \alpha(x, \theta) * \cdots * \alpha(x, \theta)$$

(2.8)
in terms of the Lie algebra valued function

\[ \alpha(x, \theta) = \sum_{a=0}^{n^2-1} \sum_{b=0}^{m^2-1} \alpha^{ab}(x, \theta) T^a \otimes \hat{T}^b \]  

(2.9)

with the condition that

\[ \alpha^{ab}(x, \theta) = f^a(x, \theta) * g^b(x, \theta). \]  

(2.10)

The ensemble of gauge function is extended gauge group denoted by U(n ⊗ m)*. The star commutator between two Lie algebra valued functions is calculated as

\[ [\alpha(x, \theta), \beta(x, \theta)]_* = \sum_{a=c=0}^{n^2-1} \sum_{b,d=0}^{m^2-1} \left( \alpha^{ab}(x, \theta) * \beta^{cd}(x, \theta) T^a T^c \otimes \hat{T}^b \hat{T}^d - \beta^{cd}(x, \theta) * \alpha^{ab}(x, \theta) T^c T^a \otimes \hat{T}^d \hat{T}^b \right). \]  

(2.11)

Thus, the enveloping Lie algebra closes within itself for the star commutator (2.11). It should be noted that

\[ \lim_{\theta \to 0} U(x, \theta) = \exp \left[ i \sum_{a=0}^{(n^2-1)} f^a(x) T^a \right] \otimes \exp \left[ i \sum_{b=0}^{(m^2-1)} g^b(x) \hat{T}^b \right] \in SU(n) \otimes SU(m) \otimes SU(1), \]  

(2.12)

which indicates that the extended group U(n ⊗ m)* reduces to nonabelian group SU(n) ⊗ SU(m) ⊗ SU(1) when \( \theta^{\mu \nu} \) approaches to 0. However, we consider this limit in classical level, not in quantum level.

Under the gauge transformation in (2.7) the field strength \( F_{\mu \nu}(x) \) and the covariant derivative of \( D_\mu \varphi(x) \) are transformed covariantly

\[ F_{\mu \nu}^g(x) = U(x, \theta) * F_{\mu \nu}(x) * U^{-1}(x, \theta), \]  

(2.13)

\[ (D_\mu \varphi(x))^g = U(x, \theta) * D_\mu \varphi(x) * U^{-1}(x, \theta). \]  

(2.14)

Then, the gauge field term in (2.11) is transformed as in

\[ \text{Tr} \left[ F_{\mu \nu}^g(x) * F^{g \mu \nu}(x) \right] = \text{Tr} \left[ U(x, \theta) * F_{\mu \nu}(x) * F^{\mu \nu}(x) * U^{-1}(x, \theta) \right] \]  

(2.15)

which shows the gauge term itself is not gauge invariant because of the Moyal product but the action is invariant thanks to the rule

\[ \int d^4x \, f(x) * g(x) = \int d^4x \, g(x) * f(x). \]  

(2.16)

We call this situation pre-invariance to gauge transformation. That is, the gauge boson term in Lagrangian is pre-invariant. That is the case for the Higgs boson term in (2.11) because of (2.14). On the other hand, the fermion term in Eq. (2.11) is invariant under gauge transformations (2.7).

Here, we take \( N = 4 \) without loss of generality in order to define the more general gauge theory on NC spacetime. As stated in the next section, we obtain the U(3 ⊗ 1)* gauge theory resulting from the spontaneous symmetry breakdown of U(4)* gauge theory. However, regardless of the spontaneous breakdown, we can consider the U(3 ⊗ 1)* gauge theory as the gauge theory with the generators \( \lambda^a \) which is \( 4 \times 4 \) matrix constructed from the Gell-Mann matrix \( \lambda^a \) by adding 0 components to fourth line and column

\[ \lambda^a = \begin{pmatrix} \lambda^a & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \]  

(2.17)

\( \lambda^{15} = \frac{\lambda_3}{2\sqrt{3}} \text{Diag}(1, 1, 1, -3) \) and \( 4 \times 4 \) unit matrix \( \lambda^0 \).

It should be noted that the U(3 ⊗ 1)* gauge theory is different with the product gauge theory with the symmetry U(3) × U(1)* because of the spacetime noncommutativity. We can also define the more general gauge theory with the symmetry such as U(3 ⊗ 2 ⊗ 1)* consisting of the 16 × 16 matrix generators, which is used to reconstruct the standard-like model in the fourth section. It should be noticed that the noncommutative gauge group U(3 ⊗ 2 ⊗ 1)* becomes U(3) × U(2) × U(1)* except for the overall U(1)) in the commutative limit though U(N)* would not resolve into SU(N) × U(1) in the quantum level.
3  The spontaneous breakdown of U(4)\ast gauge theory

SO(10) grand unified theory (GUT) including its supersymmetric version is most promising model in particle physics since it can incorporate the 15 existing fermions in addition to the right-handed neutrino and has possibilities to explain so many phenomenological puzzles. Pati-Salam symmetry SU(3) × SU(4) × SU(2) \ast \ast \ast is one of the intermediate symmetry of the spontaneous breakdown of SO(10) GUT. This symmetry spontaneously breaks down to the left-right symmetric gauge model with the symmetry SU(3) × SU(2) \ast \ast \ast \ast SU(2) \ast \ast R \ast \ast \ast \ast U(1) \ast \ast. In this stage, the spontaneous breakdown SU(4) \to SU(3) \ast \ast \ast \ast U(1) \ast \ast \ast occurs. B \ast \ast \ast \ast \ast \ast \ast \ast \ast charge of fermions is given by

\[ Q_B = \begin{pmatrix} q^a \\ q^b \\ l \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} q^a \\ q^b \\ l \end{pmatrix}. \] (3.1)

As an example, we pick up this process in order to investigate whether fermions have B \ast \ast \ast \ast \ast \ast \ast \ast \ast charge in the noncommutative version of SU(4) gauge theory.

The gauge boson in SU(4)\ast gauge theory is expressed in terms of 16 component gauge bosons by

\[ A_\mu(x) = \sum_{a=0}^{15} A^a_\mu(x) T^a \]

\[ = \frac{1}{2} \begin{pmatrix} A_{\mu}^{11} & G_{\mu}^1 & G_{\mu}^2 & X_{\mu}^1 \\ \bar{G}_{\mu}^1 & A_{\mu}^{22} & G_{\mu}^3 & X_{\mu}^2 \\ \bar{G}_{\mu}^2 & \bar{G}_{\mu}^3 & A_{\mu}^{33} & X_{\mu}^3 \\ \bar{X}_{\mu}^1 & \bar{X}_{\mu}^2 & \bar{X}_{\mu}^3 & A_{\mu}^{44} \end{pmatrix}. \] (3.2)

where

\[ \begin{align*}
A_{\mu}^{11} &= \frac{1}{\sqrt{2}} A_{\mu}^0 + A_{\mu}^3 + \frac{1}{\sqrt{3}} A_{\mu}^8 + \frac{1}{\sqrt{6}} A_{\mu}^{15}, \\
A_{\mu}^{22} &= \frac{1}{\sqrt{2}} A_{\mu}^0 - A_{\mu}^3 + \frac{1}{\sqrt{3}} A_{\mu}^8 + \frac{1}{\sqrt{6}} A_{\mu}^{15}, \\
A_{\mu}^{33} &= \frac{1}{\sqrt{2}} A_{\mu}^0 - \frac{2}{\sqrt{3}} A_{\mu}^8 + \frac{1}{\sqrt{6}} A_{\mu}^{15}, \\
A_{\mu}^{44} &= \frac{1}{\sqrt{2}} A_{\mu}^0 - \frac{3}{\sqrt{6}} A_{\mu}^{15}, \\
G_{\mu}^1 &= A_{\mu}^1 - i A_{\mu}^2, \\
G_{\mu}^2 &= A_{\mu}^4 - i A_{\mu}^5, \\
G_{\mu}^3 &= A_{\mu}^6 - i A_{\mu}^7, \\
X_{\mu}^1 &= A_{\mu}^9 - i A_{\mu}^{10}, \\
X_{\mu}^2 &= A_{\mu}^{11} - i A_{\mu}^{12}, \\
X_{\mu}^3 &= A_{\mu}^{13} - i A_{\mu}^{14}. \end{align*} \] (3.3)

The gauge field \( A_\mu(x) \) contains 8 color gluons, 6 gauge bosons causing proton decay, one extra boson \( A_{\mu}^{15}(x) \), and one 0 component boson \( A_{\mu}^0(x) \) dependent on other bosons. Here, we denote \( A_{\mu}^{15}(x) \) by \( B_\mu(x) \) and call it B-field.

The vacuum expectation value of Higgs boson \( \varphi(x) \) takes the form

\[ < \varphi(x) > = v \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -3 \end{pmatrix}, \] (3.6)

which yields the gauge boson mass term

\[ \left[ -i g |A_\mu(x), < \varphi(x) > \right|^2 = 8 g^2 v^2 \left( X_{\mu}^1 \bar{X}_{\mu}^1 + X_{\mu}^2 \bar{X}_{\mu}^2 + X_{\mu}^3 \bar{X}_{\mu}^3 \right). \] (3.7)

Equation (3.7) shows that 6 proton decay causing gauge bosons acquire mass, so that symmetries relating to Lie algebra \( T^a (a = 1, 2, \cdots, 8, 15) \) keep unbroken. Let us consider the gauge transformation specified
by
\[ U^{cb}(x, \theta) = e^{i\alpha(x, \theta)\star} = \exp i \left\{ \sum_{a=0}^{8} T^a \alpha^a(x, \theta) * + T^{15} \alpha^{15}(x, \theta) * \right\}. \] (3.8)

Under this gauge transformation, the part of the gauge field \( A_\mu(x) \) in (2.4)
\[ A^{cb}_\mu(x) = \sum_{a=0}^{8} A^a_\mu(x) T^a + B_\mu(x) T^{15} \] (3.9)
and fermion field \( \Psi(x) \) and Higgs field transform in the similar way as in (2.7). Thus, it is easily shown that the Lagrangian (2.1) is still pre-invariant after the spontaneous breakdown resulting from (3.6). This indicates that color symmetry yielding the strong interaction and \( B \)-symmetry due to the generator \( T^{15} \) remain unbroken. In the commutative field theory, this breakdown is written as
\[ SU(4) \rightarrow SU(3) \times U(1). \] (3.10)

However, we can’t do it in the same way because
\[ U^{cb}(x, \theta) \neq \exp i \left\{ \sum_{a=1}^{8} \alpha^a(x, \theta) \star T^a \right\} * \exp i \left\{ \alpha^{15}(x, \theta) \star T^{15} \right\} \] (3.11)
owing to the Moyal product. Thus, in the NC field theory, we should write the spontaneous breakdown explained so far as
\[ U(4) * \rightarrow U(3 \otimes 1) * . \] (3.12)

Interaction terms between fermion and \( B \)-gauge field extracted from the fermion term in (2.4) is given by
\[ I_D = \bar{\Psi}(x) * \left\{ \gamma^\mu(gB_\mu(x) T^{15}) \right\} * \Psi(x) \]
\[ = \frac{3}{2\sqrt{6}} g \bar{\Psi}(x) \star \gamma^\mu \left( \begin{array}{cccc} \frac{1}{3} & 0 & 0 & 0 \\ 0 & \frac{1}{3} & 0 & 0 \\ 0 & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 0 & -1 \end{array} \right) B_\mu(x) * \Psi(x) \] (3.13)

Then, if we define \( B \)-charge operator \( Q_B \) and \( B \)-charge \( e_B \)
\[ Q_B = \frac{2\sqrt{6}}{3} T^{15}, \quad e_B = \frac{3}{2\sqrt{6}} g \] (3.14)
(3.1) is reproduced.

We considered the spontaneous breakdown of \( U(4) * \) gauge symmetry down to \( U(3 \otimes 1) * \) symmetry. Thus, charges of fermions are limited as shown in (3.1). However, apart from such construction, we can consider such a case that if the Lagrangian is pre-invariant under the gauge transformation function \( U^s(x, \theta) \) given by
\[ U^s(x, \theta) = e^{i\alpha(x, \theta)\star} = \exp i \left\{ \sum_{a=0}^{8} T^a \alpha^a(x, \theta) * + Q \beta(x, \theta)* \right\} \] (3.15)
where
\[ Q = \left( \begin{array}{cccc} e & 0 & 0 & 0 \\ 0 & e & 0 & 0 \\ 0 & 0 & e & 0 \\ 0 & 0 & 0 & e' \end{array} \right) \] (3.16)
with arbitrary constants \( e \) and \( e' \), fermions may have arbitrary charges. This is because Interaction terms between fermion and \( B \)-gauge is given by
\[ I_D = \bar{\Psi}(x) * \left\{ \gamma^\mu(gB_\mu(x)Q) \right\} * \Psi(x). \] (3.17)
If there is only $U(1)_	heta$ gauge symmetry, the gauge transformation of gauge field $A_\mu(x) = Q B_\mu(x)$ given by

$$U^\nu(x, \theta) = \exp i \{ Q \beta(x, \theta) \}.$$  

leads to the inconsistency as indicated by Hayakawa [4]. But, in our case, there are two kinds of symmetry and therefore, the gauge transformation of gauge field $A_\mu(x) = \sum_{a=0}^{8} T^a A^a_\mu(x, \theta) + Q B_\mu(x)$ given by (3.15) has nothing to do with any contradiction because of $A^0_\mu(x, \theta)$ existence.

4 The standard-like model with the symmetry $U(3_c \otimes 2 \otimes 1_Y)_*$

We construct the standard-like model to show that fields with color as well as flavor quantum numbers can be expressed in terms of the fundamental representation and also fields with fractional charges are incorporated in the gauge field on NC spacetime.

We explain the gauge group $U(3_c \otimes 2 \otimes 1_Y)_*$ which generators are formed by the $8 \times 8$ matrices. The generators of color sector are

$$\Gamma^a = i^2 \otimes \lambda^a (a = 1, \cdots, 8)$$

(4.1)

where $1^4$ is the 4 dimensional unit matrix and $\lambda^a$ is given in (2.17). The generators of the weak isospin sector are written as

$$\Gamma^i = \tau^i \otimes 1^4$$

(4.2)

where $\tau^i (i = 1, 2, 3)$ is the Pauli matrix. The generator of the hypercharge is the 8 dimensional diagonal matrix

$$\Gamma^Y = \text{Diag} \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1 \right)$$

(4.3)

It should be noted that all these generators are 8 dimensional matrices and form closed algebra. The whole abelian parts are denoted by $\Gamma^0$ which are not explicitly written. With these generators explicitly written above, the group element of $U(3_c \otimes 2 \otimes 1_Y)_*$ is denoted as

$$g(x) = \text{Exp} \left\{ -\frac{i}{2} \left( \alpha^0(x) \Gamma^0 + \alpha^a(x) \Gamma^a + \alpha^i(x) \Gamma^i + \alpha^Y(x) \Gamma^Y \right) \right\}_*$$

(4.4)

It should be noted that $g(x)$ is not factored out into $2 \otimes 4 \otimes 1$ matrix owing to the spacetime noncommutativity.

The aggregate gauge field $A_\mu$ in terms of all gauge fields is described in the equation

$$A_\mu(x) = \frac{1}{2} \left( g_0 A^0_\mu(x) \Gamma^0 + g_c G^c_\mu(x) \Gamma^a + g A^i_\mu(x) \Gamma^i + g' B_\mu(x) \Gamma^Y \right),$$

(4.5)

where $g_0, g_c, g, g'$ are the coupling constants corresponding with each gauge field. Field strength $F_{\mu\nu}$ is defined as

$$F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + [A_\mu(x), A_\nu(x)]_*$$

$$= \frac{1}{2} \left( g_0 F^0_{\mu\nu}(x) \Gamma^0 + g_c G^c_{\mu\nu}(x) \Gamma^a + g F^i_{\mu\nu}(x) \Gamma^i + g' B_{\mu\nu}(x) \Gamma^Y \right)$$

(4.6)

$$+ \text{terms resulting from the noncommutativity of spacetime},$$

where

$$F^0_{\mu\nu}(x) = \partial_\mu A^0_\nu(x) - \partial_\nu A^0_\mu(x),$$

(4.7)

$$G^c_{\mu\nu}(x) = \partial_\mu G^c_\nu(x) - \partial_\nu G^c_\mu(x) + g_c f^{abc} G^b_\mu(x) \ast G^c_\nu(x),$$

(4.8)

$$F^i_{\mu\nu}(x) = \partial_\mu A^i_\nu(x) - \partial_\nu A^i_\mu(x) + g \epsilon^{ijk} A^j_\mu(x) \ast A^k_\nu(x),$$

(4.9)

$$B_{\mu\nu}(x) = \partial_\mu B_\nu(x) - \partial_\nu B_\mu(x).$$

(4.10)

The gauge field $A_\mu(x)$ transforms under the gauge transformation [45] by

$$A'_\mu(x) = g(x) \ast A_\mu(x) \ast g^{-1}(x) + ig(x) \ast \partial_\mu g^{-1}(x).$$

(4.11)
Then, the field strength $F_{\mu\nu}(x)$ is transformed covariantly

$$F_{\mu\nu}^\prime(x) = g(x) \ast F_{\mu\nu}(x) \ast g^{-1}(x).$$

We find the Yang-Mills Lagrangian $L_{YM}$

$$L_{YM} = \frac{1}{k^2} \text{Tr} \left[ F_{\mu\nu}(x) \ast F^{\mu\nu}(x) \right]$$

from which the relationships of coupling constants

$$g_c^2 = \frac{1}{4} k^2, \quad g^2 = \frac{1}{4} k^2, \quad g^\prime_2 = \frac{3}{8} k^2$$

are obtained. It should be noted that the integral of $L_{YM}$ over $x$ is gauge invariant.

Let us represent the fermion field $\psi$ as a 8 dimensional vector with borrowing the names of existing leptons and quarks and then give the correct Dirac Lagrangian for the fermion sector in the standard-like model. Hereafter, the argument $x$ is often abbreviated if there is no confusion.

$$\psi = \begin{pmatrix}
  (u^r) \\
  u^g \\
  u^b \\
  \nu \\
  d^r \\
  d^g \\
  d^b \\
  e
\end{pmatrix}$$

with color indices $r$, $g$ and $b$. According to (4.11), the gauge transformation of fermion field $\psi$ in (4.15) is given by

$$\psi^\prime = g(x) \ast \psi.$$ 

With this configuration of fermion field, the Dirac Lagrangian is simply written as

$$L_D = i \bar{\psi} \ast \gamma^\mu (\partial_\mu - i A_\mu) \ast \psi.$$ 

It is evident that the Dirac Lagrangian $L_D$ is gauge invariant according to (4.11) and (4.16).

We consider a Higgs field $\Phi$ which belong to the adjoint representation in similar way as in (4.5). Gauge transformation of $\Phi$ is subject to

$$\Phi^\prime = g(x) \ast \Phi \ast g^{-1}(x).$$

Equation (4.18) yields the covariant derivative of Higgs field

$$D_\mu \Phi = \partial_\mu \Phi - i [A_\mu, \Phi],$$

from which we can construct the Higgs-gauge interaction term

$$L_D = \frac{1}{k^2} \text{Tr} \left( D_\mu \Phi \right) \dagger \ast (D^\mu \Phi)$$

with the normalization factor $k'$.

Yukawa interaction between fermion and Higgs fields is given as

$$L_Y = g_Y \bar{\psi} \ast \Phi \ast \psi$$

where $g_Y$ is the Yukawa coupling matrix. It should be noted that $L_Y$ in (4.24) is gauge invariant.

Spontaneous breakdown of gauge symmetry is caused by the vacuum expectation value of Higgs boson

$$< \Phi > = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \otimes 1^4 \mu$$
It is obvious that $[\Gamma^a, \langle \Phi \rangle] = 0$, and therefore, the color symmetry does not break spontaneously. Then, the gauge symmetry $U(3c \otimes 2 \otimes 1_Y)^*$ spontaneously breaks down to $U(3c \otimes 1_{em})^*$. Thus, gluon mass remains zero. In addition, since $[\Gamma^a, \langle \Phi \rangle = 0$, $[\Gamma^b, \langle \Phi \rangle = 0$ and $[\Gamma^c, \langle \Phi \rangle = 0$, gauge bosons with respect to these relations remain massless. However, charged gauge bosons acquire the masses due to the symmetry breakdown.

$$\text{Gauge boson mass}^2 = \text{Tr}[iA_\mu, \langle \Phi \rangle]^2 = -2g^2 \text{Tr} \left( \begin{array}{cc} 0^4 & -W^+ \mu \\ W^- \mu & 0^4 \end{array} \right)^2 \mu^2 = m^2 W^+ W^-,$$  \hspace{1cm} (4.23)

where

$$W^\pm_\mu = A^\pm_\mu \pm i B^\pm_\mu \sqrt{g^2 + g'^2}, \hspace{1cm} m^2 = 16g^2 \mu^2.$$  \hspace{1cm} (4.24)

Fermion mass term is

$$\bar{\psi} \langle \Phi \rangle \psi = g Y^\mu \bar{\psi} \left( \begin{array}{cc} 1^4 & 0^4 \\ 0^4 & (-1)^4 \end{array} \right) \psi$$

Thus,

$$\mathcal{L}_{fm} = m(\bar{u}^r u^r + \bar{u}^g u^g + \bar{u}^b u^b + \bar{\nu} \nu) + m(d^r d^r + d^g d^g + d^b d^b + \bar{e} e)$$  \hspace{1cm} (4.25)

where we performed the transformation $e^{i\pi/2} \gamma^5 \psi$ for $d^r, d^g, d^b$ and $e$ in order to make masses of these fermions have right sign. Fermion masses are all same including neutrino, so this model is really standard-like model.

Let us define photon field $A_\mu$ and the weak boson $Z_\mu$ as

$$A_\mu = \frac{g A^3_\mu + g B_\mu}{\sqrt{g^2 + g'^2}}, \hspace{1cm} Z_\mu = \frac{g A^3_\mu - g B_\mu}{\sqrt{g^2 + g'^2}}.$$  \hspace{1cm} (4.26)

Then, we can investigate the fermion electric charge which follows from (4.15), (4.17) and (4.26) and obtain the charge assignment

$$\left( \begin{array}{cccccc} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & -\frac{1}{3} \end{array} \right)$$  \hspace{1cm} (4.27)

for the fermion configuration (4.15).

The standard-like model on NC spacetime proposed in this section presents favorable aspects of the standard model. However, it contains several defaults such as the UV/IR mixing and extra U(1) gauge bosons which exist in all gauge theories on NC spacetime. We discuss these points in the following section.

5 Conclusions and Discussions

We have proposed nonabelian U$(n \otimes m)^*$ gauge theory constructed from the commutation relations of generators on NC spacetime. According to this noncommutative gauge theory, we considered the SU$(4)^*$ gauge theory which spontaneously breaks down to U$(3 \otimes 1)^*$ symmetry in order to obtain the gauge theory with fractional U(1) charges. It is shown that such NC gauge theory with fractional charges more than two can not exist alone, but it must coexist with NC nonabelian gauge theory. Then, we reconstructed the standard-like model based on the gauge group U$(3c \otimes 2 \otimes 1_Y)^*$ which shows that fields with color as well as flavor quantum numbers can be expressed in terms of the fundamental representation and also fields with fractional charges are incorporated in the gauge field on NC spacetime. It also shows favorable aspects of the standard model as well as the several defects discussed below.

Let us discuss the present situations of NC gauge theories including models proposed here and show several undesirable results with respect to the quantized version of NC gauge theory.

@1 Hayakawa [4] indicated that the charges of matter fields are restricted to 0 and ±.

@2 Fields can belong to the fundamental, bifundamental and adjoint representations of gauge groups [17, 18, 19].
The gauge groups are restricted to $U(N)^\ast$ group \[5, 6\].

Matusis, Susskind and Toumbas \[15\] found the unfamiliar IR/UV connection in NC gauge theory which involves non-analytic behavior in NC parameter $\theta$ making the limit $\theta \to 0$ singular.

Gauge anomalies cannot be cancelled in a chiral noncommutative theory, hence the anomaly free theory must be vector like \[14, 17, 20\].

In this paper, we constructed the standard-like model in order to overcome the problems (1) and (2). There is no problem of (5) since our standard-like model is vector like. However, the extra $U(1)$ gauge fields exist and our model suffers from (3) and (4) though we do not investigate the quantum effects of the $U(n + m)^\ast$ gauge theory. According to \[11\], there are the following observations with respect to (3) and (4). In the supersymmetric version of $U(N)^\ast$ gauge theory on NC spacetime \[15, 12, 13\], the UV/IR mixing occurs only for the $U(1)$ degree of freedom, which yields the decoupling from the remaining $SU(N)$ sector at the low energy. Thus, it looks like a safe commutative $SU(N)^\ast$ gauge theory at low energy. Armoni \[6\] observed the similar decoupling in the calculation of the one-loop gluon propagator in noncommutative QCD. Ruiz Ruiz \[16\] also showed that the defects stated above may be solved by considering the supersymmetric version of NC gauge theory. Thus, there are possibilities to be able to solve the defects stated above in the supersymmetric NC gauge theory.

The deviations from the standard model in particle physics have not yet been observed, and so any model beyond standard model must reduce to the standard model in its characteristic approximation. Then, according to the above indication, the supersymmetric gauge theory might overcome the defects stated above. With respect to the problem (5), we have to extend the model into the left-right symmetric gauge theory. This work will appear in future.

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