The \( \sigma \) and \( f_0(980) \) from \( K\pi 4 \oplus \pi\pi \) scatterings data

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We systematically reconsider, within an improved “analytic K-matrix model”, the extraction of the \( \sigma \equiv f_0(600) \) and \( f_0(980) \) masses, widths and hadronic couplings using new \( K\pi\pi \equiv K \rightarrow \pi\pi e^+e^- \) data on \( \pi\pi \) phase shift below 390 MeV and different sets of \( \pi\pi \rightarrow \pi\pi/K\bar{K} \) scatterings data from 400 MeV to 1.4 GeV. Our results are summarized in Tables 1, 2 and 5. In units of MeV, the complex poles are: \( M_\sigma = 452(12) \) – \( i \) 260(15) and \( M_f = 981(34) \) – \( i \) 18(11), which are comparable with some recent high-precision determinations and with PDG values. Besides some other results, we find: \( |g_{\pi\pi\pi\pi}|/|g_{\pi\pi\pi\pi}| = 0.37(6) \) which confirms a sizeable \( g_{\pi\pi\pi\pi} \) coupling found earlier, and which disfavours a large \( \pi\pi \) molecule or four-quark component of the \( \sigma \), while its broad \( \pi\pi \) width (relative to the one of the \( \rho \)-meson) cannot be explained within a \( q\bar{q} \) scenario. The narrow \( \pi\pi \) width of the \( f_0(980) \) and the large value: \( |g_{\pi\pi\pi\pi}|/|g_{\pi\pi\pi\pi}| = 2.59(134) \), excludes its pure \( (i\bar{u}u + \bar{d}d\bar{d}) \) content. A significant gluonium component eventually mixed with \( q\bar{q} \) appears to be necessary for evading the previous difficulties.

1. Introduction

Understanding the nature of scalar mesons in terms of quark and gluon constituents is a long standing puzzle in QCD \(^1\). The problem here is that some states are very broad (\( \sigma \) and \( \kappa \) \(^3\) if confirmed) mesons) and others are close to an inelastic threshold (\( f_0(980) \), \( \eta(958) \)) which disfavours a large \( \pi\pi \) molecule or four-quark component of the \( \sigma \), and which cannot be reproduced by a set of Feynman diagrams, including resonance (bare) couplings to \( \pi\pi/\bar{K}K \) and (in the original model \(^5\)) 4-point \( \pi\pi/\bar{K}K \) interaction vertices which we shall omit for simplicity in \(^4\) and here. A subclass of bubble pion loop diagrams including resonance poles in the \( s \)-channel are resummed (unitarized Born). In this letter, we discuss the approach for the case of \( \pi\pi \rightarrow \pi\pi/\bar{K}K \) in the \( s \)-channel. In the present analysis, the introduction of a real analytic form factor shape function, which takes explicitly into account left-handed cut singularities for the strong interaction amplitude, allows a more flexible parametrisation of the \( \pi\pi \rightarrow \pi\pi/\bar{K}K \) data. In our low energy approach, it can be conveniently approximated by:

\[
 f_P(s) = \frac{s - s_{AP}}{s + \sigma_{DP}}, \quad P = \pi, K,
\]

which multiplies the scalar meson couplings to \( \pi\pi/\bar{K}K \). In this form, the shape function allows for an Adler zero at \( s = s_{AP} \) and a pole at \( \sigma_{DP} > 0 \) simulating the left hand cut.

1 channel \( \oplus \) 1 “bare” resonance

\(^3\)Some applications of the model have been discussed in \(^5\).
Let’s first illustrate the method in this simple case. The unitary $PP$ amplitude is then written as:

$$T_{PP}(s) = \frac{G_{PP}(s)}{s - s - G_{PP}(s)} = \frac{G_{PP}(s)}{D_P(s)},$$

(2)

where $T_{PP} = e^{i\delta_P} \sin \delta_P / \rho_P(s)$ with $\rho_P(s) = (1 - 4m_P^2/s)^{1/2}$, $G_P = g_P^2$ and $\rho_P(s)$ are the bare coupling squared and:

$$\text{Im } D_P = \text{Im } (-G_P \tilde{f}_P) = -(\theta P P) G_P f_P,$$

(3)

with: $(\theta P P)(s) = 0$ below and $(\theta P P)(s) = \rho_P(s)$ above threshold $s = 4m_P^2$. The “physical” couplings are defined from the residues, with the normalization:

$$g_P^2 = g_{PP}^2/(16\pi).$$

(4)

The amplitude near the pole $s_0$ where $D_P(s_0) = 0$ and $D_P(s) \approx D_P(s_0)(s - s_0)$ is:

$$T_{PP}(s) \sim \frac{g_P^2}{s_0 - s}; \quad g_P^2 = \frac{G_{PP}(s_0)}{-D_P(s_0)}.$$

(5)

The real part of $D_P$ is obtained from a dispersion relation with subtraction at $s = 0$ and one obtains:

$$\tilde{f}_P(s) = \frac{2}{\pi} \left[ h_P(s) - h_P(0) \right].$$

(6)

$h_P(s) = f_P(s)\tilde{L}_{s1}(s) - (\sigma_{NP}/(s + \sigma_{DP}))\tilde{L}_{s1}(s)$, $\sigma_{NP}$ is the residue of $f_P(s)$ at $s = -\sigma_{DP}$ and: $\tilde{L}_{s1}(s) = \left[(s - 4m_P^2)/m_P^2\right] \tilde{L}_1(s, m_P^2)$ with $\tilde{L}_1$ from [55].

2 channels + 2 “bare” resonances

The generalization to this case is conceptually straightforward though cumbersome. Let us consider 2-body channels coupled to 2 “bare” resonances labelled $a$ and $b$, with bare masses $sRa$ and $sRb$.

– Let $f_{\pi a}(s)$, $f_{\pi b}(s)$, $f_{Ka}(s)$, $f_{Kb}(s)$ be four shape functions, real analytic in the s-plane, with left cut, and $\tilde{f}_{\pi a}(s)$, $\tilde{f}_{\pi b}(s)$, $\tilde{f}_{Ka}(s)$, $\tilde{f}_{Kb}(s)$, $\tilde{f}_{Kab}(s)$, $\tilde{f}_{Kab}(s)$, six functions, real analytic in the s-plane, with right cut.

Their imaginary parts on the cut for $s \geq 4m_\pi^2$ are:

$$\text{Im } f_{\pi a}(s + i\epsilon) = (\theta_P P) f_{\pi a}(s),$$

$$\text{Im } \tilde{f}_{\pi b}(s + i\epsilon) = (\theta_P P) f_{\pi b}(s),$$

$$\text{Im } \tilde{f}_{\pi a}(s + i\epsilon) = (\theta_P P) f_{\pi a}(s),$$

and analogous for the 2nd $K$ channel.

– Let’s define the bare inverse propagators:

$$D_a(s) = (sRa - s); \quad D_b(s) = (sRb - s),$$

(8)

and the “bare” couplings $g_{\pi a}, g_{\pi b}, g_{Ka}, g_{Kb}$ of the resonances to the channels, through the pure 1-resonance inverse propagators:

$$D_a(s) = D_a(s - g_{\pi a}^2 \tilde{f}_{\pi a}(s) - g_{Ka}^2 \tilde{f}_{Ka}(s));$$

$$D_b(s) = D_b(s - g_{\pi b}^2 \tilde{f}_{\pi b}(s) - g_{Kb}^2 \tilde{f}_{Kb}(s)).$$

(9)

– Let’s define the full denominator function $D(s)$, analytic in the s-plane, with right cut $s \geq 4m_\pi^2$:

$$D(s) = D_a D_b - (g_{\pi a} g_{\pi b} \tilde{f}_{\pi a} \tilde{f}_{\pi b} + g_{Ka} g_{Kb} \tilde{f}_{Ka} \tilde{f}_{Kb})^2,$$

$$= D_a D_b - D_a \left( g_{\pi a}^2 \tilde{f}_{\pi a} + g_{Ka}^2 \tilde{f}_{Ka} \right) - D_b \left( g_{\pi b}^2 \tilde{f}_{\pi b} + g_{Kb}^2 \tilde{f}_{Kb} \right) - \left( g_{\pi a} g_{\pi b} \tilde{f}_{\pi a} \tilde{f}_{\pi b} + g_{Ka} g_{Kb} \tilde{f}_{Ka} \tilde{f}_{Kb} \right)^2,$$

(10)

and the partial propagators

$$P_{aa} = \frac{D_a}{D}, \quad P_{bb} = \frac{D_b}{D},$$

$$P_{ab} = \frac{1}{D} (g_{\pi a} g_{\pi b} \tilde{f}_{\pi a} + g_{Ka} g_{Kb} \tilde{f}_{Ka}).$$

(11)

– Then

$$T_{\pi \pi} = \frac{g_{\pi a}^2 f_{\pi a}}{D} P_{aa} + 2g_{\pi a} g_{\pi b} f_{\pi a} f_{\pi b} P_{ab} + \frac{g_{\pi b}^2 f_{\pi b}^2}{D} P_{bb},$$

$$T_{KK} = T_{\pi \pi} : \pi \rightarrow K,$$

$$T_{K \pi} = g_{Ka} g_{Ka} f_{Ka} P_{aa} + g_{Ka} g_{Kb} f_{Ka} f_{Kb} + g_{Ka} g_{Kb} f_{Ka} f_{Kb} (P_{ab} + P_{bb}).$$

(12)

is a set of unitary elastic amplitudes.

– The inelasticity $\eta$ is related to the amplitudes or S-matrix as:

$$\eta e^{2i\delta_P} = S_{PP} \equiv 1 + 2i\rho_P T_{PP}, \quad P \equiv \pi, K,$$

$$\sqrt{1 - \eta^2 e^{2i\delta_K}} \equiv -i S_{KK} = 2\sqrt{\rho_P K} T_{\pi \pi},$$

(13)

where the sum of pion and kaon phase shifts is:

$$\delta_{\pi K} = \delta_{\pi} + \delta_{K}.$$ (14)

– In the following, we shall work in the minimal case with one shape function:

$$f_{\pi a}(s) = f_{\pi b}(s) = f_{Ka}(s) = f_{Kb}(s),$$

(15)

where:

$$\sigma_D \equiv \sigma_{D\pi} = \sigma_{DK}; \quad s_A \equiv s_{A\pi} = s_{AK}.$$ (16)

3. Phenomenology of elastic $\pi \pi \rightarrow \pi \pi$ scattering

Data input

The only data input used in this process is the pion phase shift $\delta_{\pi}$ well measured experimentally. We shall use the new precise data from NA48/2 on $Ke^+\pi^- \rightarrow \pi^0\pi^0$ for the $\pi\pi$-phase shift below 390 MeV [19] and use from 400 to 900 MeV the CERN-Munich [39] and Hyams et al. [51] $\pi\pi$-phase from $\pi \pi \rightarrow \pi \pi$ which agree each others above 400 MeV. These data are shown in Fig. [1].

0 “bare” resonance $\equiv \lambda \phi^4$ model

Let’s first fit the elastic $\pi\pi$ data by using a $\lambda \phi^4$ model without any “bare” resonance. In this old version of the model [55], one can introduce the shape function $f_P$ [37]:

$$T_{PP} = \frac{\Lambda f_P(s)}{1 - \Lambda f_P(s)}; \quad f_P(s) = \frac{s - s_{AP}}{(s + \sigma_{D1})(s + \sigma_{D2})},$$

(17)

where $\sigma_{D1} = \sigma_{D\pi}$ and:

$$\tilde{f}_2(s) = \frac{2}{\pi} \left[ h_2(s) - h_2(0) \right].$$

(18)

with:

$$h_P(s) = f_P(s)\tilde{L}_{s1}(s) - \sum_{i=1,2} \frac{\sigma_{N_i}}{s + \sigma_{D_i}} \tilde{L}_{s1}(s),$$

(19)
we look for a minimum of $\chi^2$ over the interval $[4, 5]$ for the fit. Here, this is obtained for $\sqrt{s} = 0.7$ GeV; red (continuous) line for 1 “bare” resonance and $\chi^2_{\text{min}}$ reached at $\sqrt{s} = 0.7$ GeV; green (dashed) line for 2 “bare” resonances and $\chi^2_{\text{min}}$ reached at $\sqrt{s} = 0.75$ GeV.

$\sigma_{N1}, \sigma_{N2}$ in Eq. (17) are the residues of $f^i(s)$ at $\sigma_{D1} \equiv \sigma_{D\pi}, \sigma_{D2}$. In fitting the “bare” parameters, we look for a minimum of $\chi^2 \equiv \chi^2_{\text{min}}$ by varying the range of the interval $[4m^2_{\pi}, \infty)$ inside which we perform the fit. Here, this is obtained for $\sqrt{s} = 0.7$ GeV where: $\chi^2_{\text{min}}/ndf = 12.04/14 = 0.86$. The fitted values of the “bare” parameters and the resulting values of the physical pole parameters are given in Table 1. The quoted errors of the “bare” parameters come from the fit program MINUIT. The errors induced by each of these “bare” parameters on the physical poles can be added (as currently done) linearly or quadratically $^6$. These results indicate that, though not accurate, this original version of the model gives a reasonable value of the physical parameters.

1 “bare” resonance

This analysis has been done in $^{47}$ using the CGL parametrization based on Roy equations with constraints from chiral symmetry $^{58}$. In the following, we shall use instead the new precise data from NA48/2 on $K\pi$ for the $\pi\pi$-phase below 390 MeV $^{10}$ and use from 400 to 900 MeV the CERN-Munich $^{50}$ and Hyams et al. $^{51}$ $\pi\pi$-phase from $\pi\pi \rightarrow \pi\pi$ which agree each others above 400 MeV. We extract the “bare” parameters from these data:

– In the first step, we leave all “bare” parameters free and find a minimum $\chi^2$: $\chi^2_{\text{min}}/ndf = 9.43/17 = 0.55$ for $\sqrt{s} = 0.75$ GeV. The fitted value of the Adler zero is:

$$s_{A\pi} = 0.0394(92) \text{ GeV}^2,$$

(20)

Table 1

| Output | 0 res. | 1 res. | 2 res. | Average |
|--------|--------|--------|--------|---------|
| $s_A$  | 0.009(6) | fixed | fixed |
| $\sigma_{D\pi}$ | 6.2(3.2) | 1.41(7) | 1.78(10) |
| $\sigma_{D2}$ | 7.6(4.5) | 7.6(4.5) | 7.6(4.5) |
| $s_{R\alpha}$ | - | 1.94(9) | 26.97(1.54) |
| $\Lambda$ | 108(34) | - | - |
| $g_{\pi a}$ | - | 2.54(8) | 10.42(30) |
| $s_{R\beta}$ | - | 0.61(31) | - |
| $g_{\pi b}$ | - | -0.39(8) | - |
| $\chi^2_{\text{min}}$/ndf | 12.04/14 = 0.86 | 11.73/15 = 0.78 | 12.71/16 = 0.79 |
| $M_\pi$ | 468(181) | 456(19) | 448(18) | 452(13) |
| $\Gamma_\pi/2$ | 261(211) | 265(18) | 260(19) | 259(16) |
| $|g_{\sigma\pi\pi}|$ | 2.58(13.1) | 2.72(16) | 2.58(14) | 2.64(10) |

Table 2

| Processes | $M_\pi - i\Gamma_\pi/2$ | Refs. |
|-----------|------------------------|-------|
| This work | $K\pi \rightarrow \pi\pi$ | $452(13) - i 259(16)$ |
| $K\pi \rightarrow \pi\pi/K\bar{K}$ | $448(43) - i 266(43)$ |
| Average | $452(12) - i 260(15)$ |
| Others | $\pi\pi \rightarrow \pi\pi/\pi\pi$ | $441^{+16}_{-8}$ - $i 272^{+9}_{-15}$ |
| $\pi\pi \rightarrow \pi\pi/K\bar{K}$ | $461 \pm 15 - i (255 \pm 16)$ |
| $J/\psi \rightarrow \omega\pi\pi$ | $541 \pm 39 - i (222 \pm 42)$ |
| $D^+ \rightarrow \pi^+\pi^-\pi^0$ | $478 \pm 29 - i (162 \pm 46)$ |

which is relatively bad compared with the theoretical expectation$^5$:

$$s_{A\pi} = \frac{m_{\pi}^2}{2} = 0.0094 \text{ GeV}^2.$$

(21)

– Then in the second step, we fix the Adler zero at the value in Eq. (21) and deduce the results in Table 1. The fit is shown in Fig. 1. These “bare” parameters lead to the physical poles in Table 1 which we consider as improvements of the previous results in $^{47}$. This result is comparable in size and errors with the precise determinations from recent analyses of the analogous $\pi\pi \rightarrow \pi\pi/K\bar{K}$ scatterings data using different ap-

$^5$Fixing the Adler value at the one in Eq. (21) does not bring any improvements here.

$^6$Notice that if we have added linearly the errors induced by each “bare” parameters by taking into account their signs, we would have obtained about 2 times more accurate predictions. For a more conservative error, we shall take here and in the following the quadratic sum of these errors.
proaches (Roy equations ⊕ chiral symmetry constraints [58]), Roy equations ⊕ control of the high-energy behaviour of the amplitude [59] (Table 2) which have been obtained before the last K e4 NA48/2 precise data [49].

2 “bare” resonances
We repeat the previous analysis by working instead with 2 “bare” resonances. We fix the Adler zero at the value in Eq. 21 and fit the other “bare” parameters. We obtain the results quoted in Table 1 for a $\chi^2_{min}/ndf=12.71/16=0.794$ at $\sqrt{s} = 0.75$ GeV. The fit is shown in Fig. 1.

Comments and final results from $\pi\pi \to \pi\pi$
From previous studies, we conclude that:
- The results from different forms of the model in Table 1 are very stable. The final results from elastic $\pi\pi \to \pi\pi$ are the average of the ones from 0, 1 and 2 “bare” input resonances quoted in this Table 1, which are:

\[
M_{\sigma}[\text{MeV}] = 452(13) - i 259(16) , \\
|g_{\pi\pi\pi\pi}| = 2.64(10) \text{ GeV} .
\]

- The results, from the 0 “bare” resonance or $\lambda\delta^4$ model show that the existence of the $\sigma$ pole is not an artifact of the “bare” resonance entering in the parametrization of the $\pi\pi$ amplitude $T_{PP}$.

Noting that the concavity of the fit curve in Fig. 1 around the $\rho$-meson mass region has raised some doubts on the data of the phase shift $\delta_\pi$ [62], we have redone the fit by assuming that the data increases linearly from the $Ke4$ one. Using 1 or 2 resonances, we still find, in this extreme case, a $\sigma$ pole:

\[
M_{\sigma}[\text{MeV}] \simeq 413 - i 300 , \\
(23)
\]

where a similar value has been obtained earlier [62].

This result may indicate that the existence and the dynamics of the $\sigma$ is mainly due to the low-energy behaviour of the $\pi\pi$ phase shift $\delta_\pi$ data, which are accurately determined from $Ke4$ by NA48/2 [49].

4. Phenomenology of inelastic $\pi\pi \to \pi\pi/KK$

2 “bare” resonances ⊕ 2 channels parameters
In so doing, we take in Table 3 three representatives sets of $\pi\pi \to \pi\pi/KK$ data in the existing literature:

| Input Set 1 | Set 2 | Set 3 |
|------------|-------|-------|
| $\delta_\pi$ | 4950 [51] | 4960 [51] | 4960 [51] |
| $\eta$ | 50 | 51 | 52 |
| $\delta_{\pi K}$ | 53 | 52 | 52 |

See also Table 2 for some other determinations.

Figure 2. a) Fit of the $\pi\pi$ phase $\delta_\pi$ versus $\sqrt{s}$. Set 1 (blue: dotted); Set 2 (green: dashed); Set 3 (red: continuous); b) Fit of the inelasticity $\eta$; c) Fit of the sum of $\pi$ and $K$ phase $\delta_{\pi K}$.

- The choice for $\delta_\pi$ which we have used in the analysis of elastic $\pi\pi \to \pi\pi$ scattering is unique and comes from the new $Ke4$ data of NA48/2 [49] below 390 MeV and from $\pi\pi \to \pi\pi/KK$ data above 400 MeV measured by CERN-Munich [50] and Hyams et al. [51] [see Figs. 1 and 2 a)].

- For the inelasticity $\eta$, different data exhibits a minimum $\eta_{min}$ just above the $KK$ threshold. CERN-Munich and Hyams et al. give the smallest value: $\eta_{min} \approx 0.4$, while Cohen et al. [52] provide the largest one: $\eta_{min} \approx 0.7$ [see Fig. 2 b)].

- For the sum of $\pi$ and $K$ phase $\delta_{\pi K}$, we use the one from Cohen et al and from Etkin-Martin [53], which represent the two extreme cases [see Figs. 2 c)]. With these choices, we expect to span all possible re-
regions of the space of parameters, and then to extract results which do not only come from a single experiment. We have not used the data of Kaminski et al. [54] due to the large errors, which, however, agree within the errors with the other data sets used here.

Table 4

Values in GeV\(^4\) (\(d = 1, 2\)) of the bare parameters of the K-matrix model for 2 channels \(\oplus 2\) bare resonances from \(K\bar{K} \oplus \pi\pi \to \pi\pi/\bar{K}K\) scatterings. The fit has been performed until \(\sqrt{s} \simeq 1.225 \sim 1.250\) GeV, where the \(\chi^2/\text{ndf}\) is minimal (see Fig. 3). The correlated errors come from the fitting procedure using the program MINUIT.

| Output | Set 1 | Set 2 | Set 3 |
|--------|-------|-------|-------|
| \(s_A\) | 0.016 \(\pm 0.004\) | 0.013 \(\pm 0.006\) | 0.010 \(\pm 0.006\) |
| \(\sigma_D\) | 0.740 \(\pm 0.097\) | 0.909 \(\pm 0.201\) | 1.116 \(\pm 0.262\) |
| \(s_{R_e}\) | 4.112 \(\pm 0.499\) | 2.230 \(\pm 0.271\) | 2.447 \(\pm 0.298\) |
| \(g_{\pi}\) | -0.557 \(\mp 0.177\) | 0.846 \(\mp 0.391\) | 0.997 \(\mp 0.516\) |
| \(g_{K\bar{K}}\) | 3.191 \(\pm 0.499\) | 1.458 \(\pm 0.262\) | 1.684 \(\pm 0.363\) |
| \(s_{R_b}\) | 1.291 \(\pm 0.062\) | 1.187 \(\pm 0.094\) | 1.354 \(\pm 0.149\) |
| \(g_{\pi b}\) | -1.562 \(\mp 0.117\) | -1.527 \(\mp 0.134\) | -1.756 \(\mp 0.183\) |
| \(g_{K\bar{K} b}\) | 0.748 \(\pm 0.062\) | 0.999 \(\pm 0.149\) | 1.159 \(\pm 0.261\) |

\[
\chi^2_{\text{min}}/\text{ndf} = \frac{70.6}{27} = 0.914, \quad \frac{48.8}{64} = 0.759, \quad \frac{44.3}{58} = 0.763
\]

In the following, we shall use:

\[
m_K \equiv \frac{1}{2} (m_K + m_{K^0}) = 495.65 \text{ MeV} .
\]

Letting all “bare” parameters free, we study in Fig. 3 using the fitting program MINUIT, the variation of \(\chi^2/\text{ndf}\) versus \(\sqrt{s}\) until 1.4 GeV where the data are available. In the fitting procedure, we have chosen the same initial conditions for the 3 sets, where a good convergence with a good \(\chi^2/\text{ndf}\) of the solutions has been obtained for Set 2 and Set 3. A minimum value \(\chi^2_{\text{min}}/\text{ndf}\) is reached for \(\sqrt{s} \simeq (1.225 - 1.250)\) GeV at which we extract the optimal outputs given in Table 4. At each corresponding value of \(\chi^2_{\text{min}}/\text{ndf}\), the fits for different sets of data are shown in Fig. 2. All three sets give good values of \(\chi^2_{\text{min}}/\text{ndf}\) less than one.

### Poles from 2 “bare” resonances \(\oplus 2\) channels

We use the results of the “bare” parameters in Table 4 obtained at \(\chi^2_{\text{min}}/\text{ndf}\) for deducing the ones of the complex poles in Table 5. The errors on the physical poles are induced by the ones of the “bare” parameters in Table 4 and have been added quadratically. The iteration of solutions from Set 1 has only a local minimum in \(\chi^2\) such that, in order to be more conservative, we have multiplied by a factor 2 the related uncertainties of the results coming from the fit. The last column gives the mean value from the three different determinations. We have taken (as is usual in the literature) the weighted average, where the corresponding error is more weighted by the most accurate predictions.\(^9\)

- For the \(\sigma\), we obtain the average of the complex pole mass and width given in Table 5:

\[
M_{\sigma} \text{[MeV]} = 448(43) - i 266(43) .
\]

This result is in perfect agreement with the mean value from elastic \(\pi\pi \to \pi\pi\) scattering in Eq. 22 and comparable in size and errors with the ones in Table 2. Averaging the two predictions in Eqs. 22 and 23, we deduce our final value:

\[
M_{\sigma} \text{[MeV]} = 452(12) - i 260(15) .
\]

Averaging the result in Table 5 and Eq. 22 we deduce:

\[
|g_{\sigma\pi\pi}| \approx 2.65(10) \text{ GeV}, \quad r_{\sigma\pi\pi K} = \frac{|g_{\sigma K+K-}|}{|g_{\sigma\pi\pi}|} = 0.37(6) ,
\]

\(^9\)Alternatively, we can take, as a final value, the most accurate prediction, which leads about the same result.
which improves and confirms our previous rough findings in [48] and which is comparable with some other determinations in Table 6 from [48] 10. the sizeable coupling of the \( \sigma \) to \( \bar{K}K \) disfavours the usual \( \pi\pi \) molecule and four-quark assignment of the \( \sigma \), where this coupling is expected to be negligible.

For the \( f_0(980) \), we obtain the mean value:

\[
M_f[\text{MeV}] = 981(34) - i 18(11) ,
\]

which is comparable with the PDG range of values [64]:

\[
M_f[\text{MeV}] = 980(10) - i(20 \sim 50) .
\]

From Table 5 we also find:

\[
|g_{f\pi^+\pi^-}| = 1.12(31) \text{ GeV},
\]

\[
r_{f\pi}K = \frac{|g_{fK^+K^-}|}{|g_{f\pi^+\pi^-}|} = 2.59(1.34) ,
\]

in agreement with the determinations in the existing literature (see Table 6). The large value of this ratio of coupling and the relative narrowness of the \( f_0(980) \) width (compared to e.g. the \( \rho \)-meson) does not favour the pure (\( u\bar{u} + d\bar{d} \)) content of the \( f_0(980) \) where \( r_{fK} \) is expected to be about 1/2 and the width of about 120 MeV [62]. This feature has been used as an indication of the four-quark nature of the \( f_0(980) \) (see e.g. [63]) or alternatively of its large gluonium component via a maximal mixing with a \( q\bar{q} \) state (see e.g. [50]).

| Processes | \( |g_{\pi^+\pi^-}| \) | \( r_{\pi^+\pi^-} \pi \) | \( r_{f\pi^+\pi^-} \) | Models |
|-----------|----------------|----------------|----------------|---------|
| \( \pi\pi \to \pi\pi/K\bar{K} \) | 2.65(10) 0.37(6) 1.17(26) 2.6(1.3) | 17 55 |
| Others | 2.03(3) 0.65(18) 0.97(6) 1.7(2) | 2.5 0.62 1.6 1.20 | 54 |
| \( \psi \to \phi \pi\pi/K\bar{K} \) | 0.67 | 2.35 1.80 | 67 |
| Average | 2.4 0.6 1.5 1.8 |

**Table 6**

Modulus of the \( \pi^+\pi^- \) and \( K^+K^- \) complex couplings in GeV of the \( \sigma \) and of \( f_0(980) \) from S- and K-matrix models for \( \pi\pi \to \pi\pi/K\bar{K} \) scatterings compared with the ones from \( \phi \) and \( J/\psi \) decays. \( r_{f\pi}K \equiv |g_{S\pi^+\pi^-}|/|g_{f\pi^+\pi^-}|: S \equiv \sigma, f \). As (intuitively) expected, the previous results for the \( \sigma \) parameters are approximately reproduced:

\[
M_\sigma[\text{MeV}] \approx 377 - i 195 ,
\]

and

\[
|g_{f\pi^+\pi^-}| \approx 2.13 \text{ GeV}, \quad r_{\sigma\pi\pi}K \approx 0.42 ,
\]

while the \( f_0 \) mass is pushed far away from the \( \bar{K}K \) threshold:

\[
M_f[\text{GeV}] \approx 3.8 + i 1.7 .
\]

Due to the bad quality of \( \chi^2/\text{ndf} \), the result from this version of the model will not be retained.

### 5. On-shell mass, width and couplings of the \( \sigma \)

Due to the large width of the \( \sigma \), a direct comparison of the previous results with the ones obtained from QSSR or some other theoretical predictions in the real axis is questionable. For better comparing the results obtained in the complex plane with the theoretical predictions obtained in the real axis, it is more appropriate to introduce like in [17] the on-shell meson [68] masses and hadronic widths, where the amplitude is purely imaginary at the phase 90°:

\[
\text{Re}D((M_\sigma^{\text{os}})^2) = 0 \implies M_\sigma^{\text{os}} \approx 0.9 \text{ GeV} .
\]

In the same way as for the mass, one can also define an “on-shell width” [17] from Eqs. (3) and (5) evaluated at \( s = (M_\sigma^{\text{os}})^2 \):

\[
M_\sigma^{\text{os}} \Gamma_\sigma^{\text{os}} \approx \frac{\text{Im} \frac{D}{D'}}{\text{Re} \frac{D}{D'}} \implies \Gamma_\sigma^{\text{os}} \sigma \pi \pi^+ \pi^- \approx 0.7 \text{ GeV} ,
\]

which are comparable with the Breit-Wigner mass and width [51]:

\[
M_{BW} \approx \Gamma_{BW} \approx 1 \text{ GeV} .
\]

These values lead to the on-shell coupling:

\[
|g_{\pi^+\pi^-}|^{\text{os}} \approx 6 \text{ GeV} .
\]

### 6. Comparison with QSSR \( \oplus \) LET predictions

One on hand, the corresponding on-shell (or Breit-Wigner) mass and coupling of the \( \sigma \) are comparable in size with the predictions from combined QSSR \( \oplus \) LET analysis [40][42] for a glueball with a large OZI violation for its coupling to \( \pi\pi \) and \( K\bar{K} \):[11]

\[
M_\sigma \approx 1 \text{ GeV} , \quad |g_{\sigma\pi^+\pi^-}| \approx |g_{\sigma K^+K^-}| \approx 5 \text{ GeV} ,
\]

implying:

\[
\Gamma_{\sigma \pi^+\pi^-} = \frac{|g_{\sigma\pi^+\pi^-}|^2}{16\pi M_\sigma^2} \sqrt{1 - \frac{4m_\pi^2}{M_\sigma^2}} \approx 0.7 \text{ GeV} .
\]

The existence of the \( \sigma \) is necessary for a consistency between the subtracted and unsubtracted QSSR [32],

[11]The expectation of a glueball chiral coupling to pair of Goldstone bosons [24] could not hold in this non-perturbative regime.
where the gluonium two-point correlator subtraction constant [35]:
\[\psi(0) \simeq -\frac{1}{16} \beta_1 \langle\alpha_s G^2\rangle\] (40)
plays a crucial role \((\beta_1 = -11/2 + n/3\) for n flavours), and where the value of the gluon condensate is:
\[\langle\alpha_s G^2\rangle = (6.8 \pm 1.3) \times 10^{-2} \text{ GeV}^4\] [71,72,73,74].
On the other hand, QSSR predicts for a \(S_2 \equiv 1/\sqrt{2}(\bar{u}u + d\bar{d})\) I=0 scalar meson [39]:
\[M_{S_2} \simeq 1 \text{ GeV}, \quad \Gamma_{S_2} \simeq 0.12 \text{ GeV},\] (41)
and
\[|g_{S_2\pi\pi-}| \simeq 2.5 \text{ GeV}, \quad \frac{|g_{S_2K^+K^-}|}{|g_{S_2\pi\pi-}|} = \frac{1}{2}.\] (42)

These results indicate that:
– The \(S_2\) is narrower and much higher in mass than the complex \(\sigma\) pole often identified with a \(\bar{q}q\) state in the current literature.
– The \(f_0(980)\) cannot be a pure \((\bar{u}u + d\bar{d})\) state due to the large ratio of its \(KK\) over its \(\pi\pi\) coupling \(r_{f_0K}\) [Eq. 40] and to its relative (compared to the \(\rho\)-meson) small \(\pi\pi\) width. It cannot be also a pure \(s\bar{s}\) or \(K\bar{K}\) molecule due to its non-negligible width into \(\pi\pi\).
– A large gluonium component eventually mixed with a \(\bar{q}q\) state in the \(\sigma\) and \(f_0(980)\) [25,20,22,32,47,48] can be advocated for evading these previous difficulties.
– Further phenomenological searches for gluonium have been proposed in the literature in \(\phi, J/\psi\) and \(T\) radiative decays [6,9,32], \(D, B\) semi-leptonic [75,9] and hadronic [22] decays.

7. Summary and conclusions
We have used new \(Ke4 \equiv K \rightarrow \pi\pi\nu\nu\) on \(\pi\pi\) phase shift below \(390 \text{ MeV} \oplus\) different \(\pi\pi \rightarrow \pi\pi/\bar{K}K\) scatterings data above \(400 \text{ MeV}\), for extracting the \(\sigma \equiv f_0(600)\) and \(f_0(980)\) masses, widths and hadronic couplings, within an improved “analytic \(K\)-matrix model”. Using a \(\lambda\phi^4\) version of the model, we have noticed from different analysis that the existence of the \(\sigma\) in the complex plane and having a mass of about \(452 \text{ MeV}\) is not an artifact of “bare” resonances used in the analytic \(K\)-matrix model.

We have also seen that our predictions are very stable versus the different forms (number of “bare” resonances) of the models. Our results are summarized in Table 3. The masses and widths [Table 2 and Eqs. 20] of the \(\sigma\) are comparable in size and errors with the most accurate determinations in the existing literature [58,59] (see also Table 2), while the ones of the \(f_0(980)\) in Eq. 25 are comparable with the PDG values [64]. The small uncertainties in our determinations can be mainly due to the new accurate data on \(Ke4\) from NA48/2.

The values of the couplings confirm and improve our previous results in [48] and are comparable with the ones from some other processes given in Table 6.
– The (unexpected) sizeable coupling of the \(\sigma\) to \(\bar{K}K\):

\[r_{\sigma\pi K} \simeq 0.37(6)\] [Eq. 27] is a strong indication against a pure \(\pi\pi\) molecule and four-quark substructure of the \(\sigma\), whilst its large width cannot be explained (using the QSSR results in a previous section) from a simple \((\bar{u}u + d\bar{d})\) assignment.
– The large value: \(r_{f_0K} \simeq 2.59(1.34)\) [Eq. 30] of the ratio of the \(f_0(980)\) couplings to \(\bar{K}K\) over the one to \(\pi\pi\) and of the \(f_0(980)\) relative narrow width compared e.g. with the one of the \(\rho\)-meson does not favour the pure \((\bar{u}u + d\bar{d})\) assignment of the \(f_0(980)\). In this scheme one would predict a ratio of coupling of about 1/2 and a width of about \(120 \text{ MeV}\) [Eq. 41].

The four-quark scenario can explain the large \(\bar{K}K\) coupling of the \(f_0(980)\) but it fails to explain the large coupling of the \(\sigma\) to \(\bar{K}K\).

The simple \(\bar{q}q\) scheme cannot explain the large \(\bar{K}K\) coupling and the narrowness of the \(f_0(980)\) as well as the broad width of the \(\sigma\).

A large gluonium component eventually mixed with a \(\bar{q}q\) state of the \(\sigma\) and \(f_0(980)\) can be advocated [48,20,22,32,47,48] for evading these above-mentioned difficulties.

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