Castillo, Federico; Liu, Fu

On the Todd class of the permutohedral variety. (English) Zbl 1467.52022

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Summary: In the special case of braid fans, we give a combinatorial formula for the Berline-Vergne's construction for an Euler-Maclaurin type formula that computes the number of lattice points in polytopes. Our formula is obtained by computing a symmetric expression for the Todd class of the permutohedral variety. By showing that this formula does not always have positive values, we prove that the Todd class of the permutohedral variety $X_d$ is not effective for $d \geq 24$.

Additionally, we prove that the linear coefficient in the Ehrhart polynomial of any lattice generalized permutohedron is positive.

MSC:

52B20 Lattice polytopes in convex geometry (including relations with commutative algebra and algebraic geometry)
14M25 Toric varieties, Newton polyhedra, Okounkov bodies

Keywords:

Ehrhart polynomials; generalized permutohedra; Berline-Vergne construction

Software:

SageMath

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