Dynamic Effect of Rudder Rotary Response on Ship Manoeuvrability

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Abstract. Simulation mathematical model of manoeuvrability such as zig-zag and turning circle test under different rudder and speed is built and the rudder rotary response effect on ship maneuvrability parameters is analyzed extensively. In the paper, MMG (maneuvering model group) model is adopted to set up the 3 degrees of freedom governing equations of ships, in which the rudder rotary response is approximated by the assumption of uniform rotary speed. A sea-going vessel of 10 thousand deadweight tonnages is selected as the numerical simulation target to investigate the rudder rotary response effect on ship maneuvrability parameters, such as the zig-zag and turning circle test. Some meaningful conclusions are concluded through comparative analysis, the conclusions indicate that rudder rotary response has remarkable effect on ship maneuvrability, and it is a significant factor which needs to be taken into consideration for actual ships.

1. Introduction
Ship maneuvrability is a basic navigation performance. With the development of modern shipping industry and the increase of navigation density, the research scope of ship maneuvrability is gradually expanded. On the whole, ship maneuvrability is not only dependent on the hydrodynamic performance of the ship itself, but also closely related to the maneuvring motion response of each maneuvring mechanism, such as rudder and the host speed control response effect, which has varying degrees of influence on the conventional slewing, stopping and zig-zag of ships[1]. However, for a long time, the research on maneuvrability often focuses on the hydrodynamic of the hull itself, while the research on the influence of the motion response of the control mechanism has hardly been studied in the reference [2-7]. In this paper, the influence of the rudder rotary response on the maneuvring performance is studied, and a ship maneuvring motion model is established to simplify it appropriately. Meanwhile, numerical simulation method is used to analyze its influence on the operation performance such as zig-zag and turning circle, in order to give some qualitative or quantitative conclusions and some results with practical reference value.
2. Ship movement control equation

At present, this paper mainly focuses on the maneuvering movement in the constant velocity region and considers that the ship is in the infinite deep and wide water area, without wind and wave flow, and the ship moves with three degrees of freedom. As a preliminary study on the influence of rudder rotary response, this paper ignores the transient rudder rotary response of rudder engine and uses the average rudder rotary velocity to approximate the whole rudder rotary process. At the same time, according to the purpose of this paper, the change of main engine speed in the process of ship maneuvering is ignored.

2.1. Coordinate system

Two coordinate systems, the inertial coordinate system \((\xi\eta\zeta)\) and the hull coordinate system \((xyz)\), are introduced to describe the ship's maneuvering motion. The inertial coordinate system is used to describe the position and status of a ship with its origin on the surface of the water and its \(\zeta\) axis vertically downward. Hull coordinate system is used to solve the motion control equation and describe the angular velocity of the ship. The origin is located in the center of the ship, with axis x pointing to the bow, axis y pointing to the starboard side and vertical axis z pointing to the bottom. Heading Angle \(\psi\) is the Angle of heading clockwise off the \(\xi\) axis, and the sitting mark of the ship center in the inertial coordinate system is \((x_0, y_0, z_0)\), which is shown in the following formula.

\[
\begin{bmatrix}
 x \\
 y \\
 z
\end{bmatrix} =
\begin{bmatrix}
 \cos\psi & \sin\psi & 0 \\
 -\sin\psi & \cos\psi & 0 \\
 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
 \xi \\
 \eta \\
 \zeta
\end{bmatrix}
\]

(1)

The two coordinate systems are correlated with each other through transformation matrix, while for the transformation of such parameters as position, the coordinate value of the origin of the hull motion coordinate system under the inertial system \((x_0, y_0, z_0)\) should be added on the basis of it, which is shown in the following figure.

Fig. 1 Coordinate systems and definitions of ship maneuverability

2.2. Ship movement control equation

Based on the MMG modeling method, the ship's three degrees of freedom control equation can be obtained without considering the ship's pitching heave and rolling motion, shown in formula (2).

\[
\begin{align*}
(m + m_\gamma)\dot{u} - (m + m_\gamma)v\dot{r} - mx_\gamma \dot{r}^2 &= X_H + X_R + X_P \\
(m + m_\gamma)\dot{v} + (m + m_\gamma)ur + mx_\gamma \dot{r} &= Y_H + Y_R \\
(I_z + J_z)\dot{r} + mx_\gamma (\dot{\psi} + ur) &= N_H + N_R
\end{align*}
\]

(2)

where, \(m\) and \(I_z\) are the mass and moment of inertia of the ship; For the longitudinal, transverse and heading, \(m_x, m_y, J_z\) are mass of the ship when moving in water; \((u,v,r)\) is the ship's longitudinal, lateral and head turning speed, and the point above it represents the derivative of time, that is, motion acceleration; \(x_\gamma\) is longitudinal center of gravity of ship; \((X,Y,N)\) are the longitudinal forces, transverse forces and head turning moments acting on the hull; The subscripts \(H, R\) and \(P\) represent the forces exerted by the hull, rudder, and propeller respectively. They can be respectively:

- Hull hydrodynamics: The hydrodynamics of the hull adopts the Guidao model, which is shown in formular(3).
\[ \begin{align*} 
X_H &= R(u) + X_v v^2 + X_r r^2 \\
Y_H &= Y_v v + Y_r r + Y_{v,v} v + Y_{r,r} r + (Y_{v,v} v + Y_{v,r} r) v \\
N_H &= N_v v + N_r r + N_{v,v} v + N_{v,r} r + (N_{v,v} v + N_{v,r} r) r 
\end{align*} \] (3)

In formula (3), \( R(u) \) is ship resistance, \( X_{v,r}, X_{v,v} \) and \( X_{r,r} \) are hull longitudinal hydrodynamic coefficients, \( Y_v, Y_r, N_v \) and \( N_r \) are linear hydrodynamic coefficients, \( Y_{v,v}, Y_{r,r}, Y_{v,r}, Y_{v,v}, N_{v,v}, N_{r,r}, N_{v,r} \) and \( N_{v,v} \) are non-linear hydrodynamic coefficients; The empirical determination method of hydrodynamic coefficients can refer to the literature[1,6,8].

- **Calculation of rudder force**: rudder force can be expressed as the following equation.

\[ \begin{align*} 
X_h &= -(1-t_p)F_h \sin \delta \\
Y_h &= -(1+a_h)F_h \cos \delta \\
N_h &= -(x_h + a_h \sin \delta)F_h \cos \delta 
\end{align*} \] (4)

where, \( t_p \) is thrust deduction coefficient, \( a_h \) is transverse force induction coefficient, \( x_h \) is the ordinate of the rudder, \( F_h \) is the normal hydrodynamic force of rudder. In addition, Fangcun model is used for calculation in this paper. According to the purpose of this paper, the asymmetrical effect of rudder force is ignored.

- **Calculation of propeller force**: only longitudinal thrust is considered for propeller.

\[ X_p = (1-t_p)\rho n^2 D^4 K_T J \] (5)

where, \( t_p \) is thrust deduction, \( \rho \) is water density, \( n \) is propeller speed, \( D \) is propeller diameter, \( K_T \) is its thrust coefficient, \( J \) is the speed coefficient of the rear oar, as \( J = u (1-w_p)/nD \).

In order to determine the position and attitude of the ship in the inertial coordinate system, the following motion auxiliary equation is needed. Namely, the moving speed and heading Angle of the ship under the inertial system can be expressed in the following formula.

\[ \begin{bmatrix} \dot{\xi} \\
\dot{\eta} \\
\dot{\psi} \end{bmatrix} = \begin{bmatrix} \cos \psi & -\sin \psi & 0 \\
\sin \psi & \cos \psi & 0 \\
0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\
v \\
r \end{bmatrix} \] (6)

2.3. Simulation of rudder rotary

There are many factors affecting the rudder rotary response of the actual ship rudder engine, so it is difficult to make a strict mathematical statement. In this aspect, the first order inertial response plus maximum rudder speed is usually used for simulation, or the second order dynamic response plus maximum rudder speed acceleration response time is used for simulation. In this paper, as a preliminary study on the rudder rotary response, the transient response of rudder rotary is ignored in view of the small transition time. It is considered that the whole process of rudder rotary is uniform rotation and the motion of rudder Angle can be expressed as,

\[ \dot{\delta} = \omega \] (7)

where, \( \delta \) is rudder angle, \( \omega \) is the average rudder speed.

2.4. Numerical computation method

Equation (2,6) together constitutes the ship’s three degrees of freedom governing equation, the final motion parameters of the ship are \( (\xi, \eta, \psi) \) and \( (u,v,r) \). In order to reduce the error and ensure the accuracy, the fourth-order Runge-Kutta method is used to solve the governing equations in time domain.

3. Simulation calculation and analysis

The research object is a conventional 10,000-ton single-propeller single-rudder ship. The initial position coordinates of the ship are \((0,0)\) m, and the seawater density is 1025 kg/m³. The main parameters are as follows,

The main scale is 126 (length between perpendiculars) \( \times 20.2 \times 7.6 \) m, displacement is 13700 t, square coefficient \( C_b = 0.691 \), waterline surface coefficient \( C_w = 0.815 \).
Rudder area is 17.04m, aspect ratio is 1.56, wake \( w_R = 0.24 \), thrust reduction \( t_R =0.21 \), rectification coefficient \( \gamma =0.409 \).

Propeller diameter \( D = 4.8 \) m, pitch 3.864 m, disc ratio 0.61, wake \( w_P = 0.28 \) in direct flight, thrust reduction \( t_P = 0.226 \).

The ship resistance is approximately equal to within the speed range considered in this paper, \( R(u) = 1.62u + 4.085u^2 + 0.00438u^{4.715} (KN) \).

The propeller thrust performance curve is \( K_T(J) = 0.3476-0.253J-0.229J^2 + 0.0731J^3 \).

In order to analyze the influence of rudder rotary response on specific operating performance parameters, two kinds of conventional steering motions, namely turning circle and zig-zag, are considered. Ship speed selected two speeds: full speed \((v= 7.96 \) m/s, \( n=136 \) rpm\), half speed \(v= 4.88 \) m/s, \( n=80 \)). Reference rudder turning speed \( \omega_0 = (65/28) \) deg/s, as the reference rudder speed for comparison in this paper, it corresponds to the rudder rotary speed in the sea ship specification.

3.1. Turning circle test simulation result

Three rudder angles are considered for the turning circle, which are 10,20,30 deg respectively, 5 rudder rotary speeds, \( \omega/\omega_0=[0.5,1,2,0.4,0,\infty] \), \( \infty \) represents step steering, and the rudder rotary time is 0. After calculation, table 1 gives the parameters of steering performance at different rudder rotary speeds and different rudder angles. Due to space limitations, the trajectory of the turning circle under different conditions is only given when the rudder angle is 30 degrees, as shown in Figure 2.

The results show that the relationship between rudder rotary speed and turning circle steering performance parameters is as follows: with the increase of rudder rotary speed (the rudder rotary time decreases),

- The transverse distance and tactical diameter vary slightly and are almost negligible.
- The longitudinal distance is obviously varied, and it is opposite to the rudder rotary response. The faster the rudder rotary response, the smaller the longitudinal distance. The variation is positively related to the speed and rudder Angle, such as 30-degree rudder Angle. The step rudder rotary at half speed is 6% smaller than the longitudinal distance of the reference rudder, and at full speed it is 10% smaller.
- The turning circle time decreases correspondingly, and the change has little relation with the change of speed and has a positive correspondence with the turning circle rudder angle.

| Speed (m/s) | Rudder (deg) | Rudder ratio (deg/s) | Transfer (m) | Advance (m) | Tactical diameter (m) | Heading 90 time (s) | Heading 180 time (s) |
|------------|-------------|----------------------|--------------|-------------|----------------------|--------------------|--------------------|
| 7.96       | 10.0        | 0.5                  | 646.6        | 879.4       | 1362.1               | 161.5              | 308.5              |
|            |             | 1                    | 647.0        | 862.8       | 1361.9               | 159.5              | 306.5              |
|            |             | 2                    | 643.7        | 854.1       | 1361.9               | 158.0              | 305.5              |
|            |             | 4                    | 643.9        | 849.9       | 1361.9               | 157.5              | 305.0              |
|            |             | \( \infty \)         | 644.1        | 845.8       | 1361.9               | 157.0              | 304.5              |
| 4.88       | 10.0        | 0.5                  | 653.9        | 881.6       | 1379.5               | 265.0              | 510.5              |
|            |             | 1                    | 654.3        | 871.3       | 1379.5               | 263.0              | 508.5              |
|            |             | 2                    | 654.5        | 866.1       | 1379.5               | 262.0              | 507.5              |
|            |             | 4                    | 654.6        | 863.5       | 1379.5               | 261.5              | 507.0              |
|            |             | \( \infty \)         | 654.7        | 861.0       | 1379.5               | 261.0              | 506.5              |
| 7.96       | 20.0        | 0.5                  | 381.9        | 611.0       | 813.9                | 110.5              | 208.0              |
|            |             | 1                    | 379.9        | 579.2       | 812.8                | 106.5              | 204.5              |
|            |             | 2                    | 379.4        | 563.0       | 812.4                | 104.5              | 202.5              |
|            |             | 4                    | 379.3        | 554.8       | 812.3                | 103.5              | 201.5              |
3.2. Zig-zag test simulation result

In order to fully consider the impact of rudder rotary speed on zig-zag control, this section considers 2 speeds of full speed and half speed, 2 zig-zag steering rudder angles (\( \delta = 10/20° \)), and the rudder rotary speed is \( \omega/\omega_0 = [0.5, 1.0, 2.0, 4.0, \infty] \), a total of 5 rudder rotary speeds.

After calculation, the motion response under different steering speeds at full speed sailing of 10 degrees zig-zag steering is shown in Figure 2, the curve is the heading angle of the ship. Table 2 shows the specific influences of rudder rotary speed on the three performance parameters of zig-zag steering (the first overturning Angle \( \alpha_{z1} \), the second overturning Angle \( \alpha_{z2} \), and the maneuvering period \( T_z \)).

The results show that the rudder rotary response has a significant effect on the zig-zag steering with the increase of rudder rotary speed,

- The first and second overturning angles quickly decrease, and the change corresponds positively to the speed and rudder Angle. For example, at full speed of 20 degrees zig-zag steering, the two overshoot angles of the step steering are only 30% of the base rudder rotary speed.

- The maneuvering period of zig-zag steering is also reduced accordingly. Relative to the reference rudder rotary speed, the maneuvering period of step rudder rotary is reduced by about 10-20%, and the reduction is approximately proportional to the speed and the zig-zag steering rudder angle.

| Speed (m/s) | Rudder (deg) | Rudder ratio (deg/s) | \( z1 \) (°) | \( z2 \) (°) | \( T_z \) (t) |
|------------|-------------|----------------------|--------------|--------------|--------------|
| 7.96       | 10          | 0.5                  | 6.81         | 7.68         | 177.5        |
|            |             | 1                    | 4.47         | 5.19         | 156.5        |
|            |             | 2                    | 3.25         | 3.87         | 146          |
|            |             | 4                    | 2.63         | 3.21         | 141          |
|            |             | \( \infty \)         | 2.00         | 2.54         | 136          |
| 4.88       | 10          | 0.5                  | 5.02         | 5.81         | 270          |
|            |             | 1                    | 3.55         | 4.24         | 249          |
|            |             | 2                    | 2.81         | 3.43         | 238.5        |
|            |             | 4                    | 2.43         | 3.03         | 233          |
4. Conclusions

In this paper, the rudder rotary response effect on ship maneuvering performance is studied in detail, a three-degree-of-freedom ship motion mathematical model is established, and the process of steering is approximated to uniform motion for comparative analysis. Aiming at a typical 10,000-ton ship, the effects of different steering speeds on the ship's maneuvering performance under different conditions (speed, rudder angles) are studied through a series of simulation calculations. Through comparative analysis, some regularity conclusions are given. The results show that the rudder rotary speed, as a control parameter of maneuvering motion, has different degrees of influence on the maneuvering motion of the ship: the rudder rotary speed has obvious influence on the two overturning angles and the maneuvering period of the zig-zag maneuver with longitudinal rotation distance, and the magnitude of these influences is generally in a positive correspondence with the ship's speed and rudder Angle. Meanwhile, the effect of rudder rotary response on the tactical diameter of rotary pitch is very slight and can be ignored. The turning circle time decreases with the increase of rudder speed.

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