Abstract

This paper studies the attitude control of a satellite in three-axis by thrusters. The mathematical model of attitude dynamics and kinematics of the satellite is represented as a switched system with sub-systems. Each sub-system is defined according to on/off thrusters state. A training method based on dynamic programming is utilized which can find the appropriate switching between sub-systems such that a cost function is optimized. Furthermore to extend the solution for a specific domain of interest neural network is used for approximating the cost function with basis functions. The training method is offline and it finds the optimal weights of basis functions which can be used to find optimal switching. It is shown that the proposed method can execute a maneuver in fixed final time and bring the attitude to final desired condition. Moreover, the proposed method is robust against uncertainties in system modeling. Finally, it is shown that the control scheme can be used to design low cost attitude control unit for microsatellite or as a backup unit.

1. Introduction

Attitude control and determination (ACD) unit plays a vital role in satellite mission. A satellite must take precise measurements of its position and orientation. ACD measures the orientation and generates command signals to point the satellite to desired direction. Various actuators can be used for orientation maneuver such as thrusters, momentum wheels, control moment gyros, and magnetic torquers. Among these, thrusters are commonly used due to their advantages such as capability of producing wide range of force magnitude, reliability, and long life-time [4, 5, 12, 13, 21]. Also, based on number of thrusters and their location on satellite, attitude can be controlled in one, two or three axis [5, 14, 15, 19, 21]. Hence the number of thrusters and their configuration is important in attitude control and it is well documented [3, 10, 17, 18]. For example in [18], attitude control in three-axis with different number of thrusters has been discussed and in [3], configuration of four thrusters was suggested which was inspired by European student earth orbiter (ESEO) team for a microsatellite.

So far, numerous methods have been developed for thruster control, e.g. see [4, 5, 18, 19, 21]. Because of the discrete on/off thruster operation, devising a control scheme is challenging. In this sense, switching moment (from on to off and vice versa) is directly related to many factors such as fuel consumption, accuracy, maneuver time and etc. [6, 16, 20, 22]. Therefore, an ideal control scheme is the one that can accomplish the maneuver in desirable time with minimum error while consume the least amount of fuel.

Usually the attitude maneuver time needs to be specified and fixed. One suitable approach for a fixed final time control problem is dynamic programming (DP) [9, 11]. Even though this technique tends to be computationally demanding as the system dimension increases due to the curse of dimensionality [11], it has the privilege of being used as an offline method. Dynamic programming can find an optimal decision, e.g. on/off switching time, and it is simple enough to be used in learning algorithms to expand the optimal solution withing a desirable domain of interest. Here, a neural network (NN) trains the solution, off line, using DP to generate an online optimal control [2, 8].

In this research, dynamics of satellite is defined as a switched system based on thrusters on/off condition, using a table consists of all the possible cases. A case is defined as one possible combination of the thrusters on/off condition. Using N thrusters, there are 2^N cases. Then DP is utilized to find an optimal switching between subsystems such that a cost function is optimized. Finally, NN as an intelligent approach employed to approximate the cost function to generalize the solution for every point of system’s domain.

The outline of this paper is as follows: In section two, satellite attitude dynamics is explained; In sections three and four; optimal switching between autonomous subsystems and NN scheme are explained and the application of using DP and NN in attitude control is explained in section five; In section six, results from simulation of the system with proposed method are shown and Finally section seven offers some conclusions and future work topics.

2. Satellite Attitude Kinematics and Dynamics

In this section, a brief review of the attitude kinematics and dynamics equation of motion using Euler angles is shown [3]. It is assumed that the satellite is non-spinning and rigid, and it is orbiting the earth in a circular orbit. Also four thrusters are the only active actuators during attitude maneuver. Four thrusters is the minimum number of thrusters needed for satellite three-axis attitude control [3, 18].

Thrust vector of \( \kappa \)-th thruster, \( \mathbf{F}_\kappa \), \( \kappa \in \{1, 2, 3, 4\} \), in body frame \((XYZ)_{B}\) is depicted in Fig. 1 where \( \alpha_\kappa \) and \( \beta_\kappa \) are elevation and azimuth angles, respectively.
The thrust vector in body frame can be written as:

\[ \mathbf{F}_k = F_k \begin{bmatrix} \cos(\alpha_k) \cos(\beta_k), \sin(\alpha_k), \cos(\alpha_k) \sin(\beta_k) \end{bmatrix}^T \] (1)

If the body frame is located in the center of mass (CM) of satellite, the torque about CM of the satellite is

\[ \mathbf{M}_k = \mathbf{r}_k \times \mathbf{F}_k \text{, where } \mathbf{r}_k = \begin{bmatrix} r_{xk}, r_{yk}, r_{zk} \end{bmatrix}^T \text{ is the vector distance of the } k \text{th thruster from CM.} \]

\[ \mathbf{M}_k = F_k \begin{bmatrix} r_{xk} \cos(\alpha_k) \sin(\beta_k) - r_{zk} \sin(\alpha_k) \\
 r_{zk} \cos(\alpha_k) \cos(\beta_k) - r_{xk} \cos(\alpha_k) \sin(\beta_k) \\
 r_{xk} \sin(\alpha_k) - r_{yk} \cos(\alpha_k) \cos(\beta_k) \end{bmatrix} \] (2)

The net torque of all thrusters (\( \mathbf{M}_T^B \)), can be written as:

\[ \mathbf{M}_T^B = \begin{bmatrix} M_x, M_y, M_z \end{bmatrix}^T = \sum_{k=1}^{4} \mathbf{M}_k \] (3)

The attitude dynamics is given by:

\[ \dot{\mathbf{M}}_T^B = \mathbf{I} \frac{d(\omega^B)}{dt} + \omega^B \times \omega^B \] (4)

Where \( \mathbf{I} = \text{diag} \{I_X, I_Y, I_Z\} \) is the inertia matrix, and \( I_j, j \in \{X, Y, Z\} \) is the moment of inertia about each body frame axes, and \( \omega^B = [\omega_x, \omega_y, \omega_z]^T \) is the body angular velocity vector with respect to inertial frame. After some manipulation and simplification, Eqs. 3 and 4 can be written as:

\[ \begin{bmatrix} \dot{\omega}_x \\
 \dot{\omega}_y \\
 \dot{\omega}_z \end{bmatrix} = \begin{bmatrix} \dot{I}_x \omega_x \omega_z \\
 \dot{I}_y \omega_y \omega_z \\
 \dot{I}_z \omega_z \omega_y \end{bmatrix} + \begin{bmatrix} M_x / I_x \\
 M_y / I_y \\
 M_z / I_z \end{bmatrix} \] (5)

Where \( \dot{I}_x = (I_y - I_z) / I_x, \dot{I}_y = (I_z - I_x) / I_y \) and \( \dot{I}_z = (I_x - I_y) / I_z \). In Eqn. 5, the torque can be obtained and substituted from Eqn. 3.

The attitude kinematics equations, Eqn. 6, are based on small Euler angles which are defined as the rotational angles about body axes as roll(\( \phi \)), pitch(\( \theta \)) and yaw(\( \psi \)) about the \( X_B, Y_B \) and \( Z_B \), respectively [5,18]. In Eqn. 6, \( \omega = (\mu_s / R)^{1/2} \) is the angular velocity of satellite in a circular orbit where \( \mu_s \) is the standard gravitational parameter and \( R \) is the distance from orbit to center of earth [18].

\[ \begin{aligned}
\dot{\phi} &= \omega_x + \psi \omega_y \\
\dot{\theta} &= \omega_y + \phi \omega_z \\
\dot{\psi} &= \omega_z - \phi \omega_y
\end{aligned} \] (6)

Using Eqn. 5 and 6, the linearized state space representation of attitude kinematics and dynamics can be written as Eqn. 7. It is worth mentioning that, due to using only four thrusters, there is coupling in torque applied to each axis which cause coupling in three-axis model.

\[ \begin{bmatrix} \dot{\phi} \\
 \dot{\theta} \\
 \dot{\psi} \end{bmatrix} = \begin{bmatrix} \omega_x + \psi \omega_y \\
 \omega_y + \phi \omega_z \\
 \omega_z - \phi \omega_y \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\
 0 & 0 & 0 \\
 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \phi \\
 \theta \\
 \psi \end{bmatrix} + \begin{bmatrix} M_x \\
 M_y \\
 M_z \end{bmatrix} \] (7)

The dynamics and kinematics equations in Eqn. 7 are based on small Euler angles. However, if the ACD only uses thrusters, it is not possible to achieve linear velocity augmentation in the desired body directions [18]. Here, the goal is to drive the satellite from the initial position/velocity to the final desired states (\( x_d \)) in a fixed time (\( t_f \)). Since it is not possible to drive the states to exact desired values, an invariant set, \( S \), is defined in Eqn. 8. When \( t \to t_f \), the final states values (\( x(t_f) \)) are within a defined boundary (\( \delta \)) of final desired value.

\[ S = \{ x(t) \ | \ | x(t_f) - x^d || \leq \delta \} \] (8)

As mentioned before, there is coupling in applied torque. This essentially makes the analysis of attitude hard and therefore designing a control scheme is challenging. The main idea is to find the appropriate input, i.e. \( \mathbf{M}_T^B \) to execute a maneuver. Finding the input is directly related to find the on/off situation of each thruster. That is, when a thruster should be turned on or off. So the challenge is basically finding the times of on/off for each thruster. In this research, intelligent control is utilized to find a solution for aforementioned problem. The solution is based on converting the system’s dynamics and kinematics model to switched form where DP can be used to find an optimal switching. The idea is explained in the following.

3. Optimal Switching in Switched System Using Dynamic Programming

In this section, a DP based method is introduced which finds the switching between sub-systems of a system such that a cost function is optimized. For the remainder of the paper this is called optimal switching.

**Definition:** A switched system is described by a collection of indexed differential (difference) equations as Eqn. 9, each equation called a sub-system:

\[ \dot{z}(t) = f_s(z(t)), z(0) = Z_0 \] (9)
Where \( z \in \mathbb{R}^m \) is the state vector and \( i \in \{1, 2, ..., n\} \) indicates the sub-system. At each time only one sub-system is active. The objective is to find the sequence of switching between subsystems so cost function of Eqn. 10 is optimized \([9, 23]\) where convex functions \( \hat{\Psi}(t_f) : \mathbb{R}^m \to \mathbb{R} \) and \( Q(z(t)) : \mathbb{R}^m \to \mathbb{R} \) correspond to the cost at the end and during the time period, respectively.

\[
J(z(t)) = \hat{\Psi}(t_f) + \int_{t_0}^{t_f} \dot{Q}(z(t))dt \tag{10}
\]

Eqn. 9 is in continuous-time framework. However, since DP works in discrete time framework, Eqs. 9 and 10 can be discretized with a very small time step \( \delta_t \) and can be written according to Eqn. 11 and 12, respectively.

\[
z_{k+1} = F_i(z_k), \quad z(0) = z_0 \tag{11}
\]

\[
J(z_k) = \Psi(z_N) + \sum_{k=1}^{N-1} Q(z_k) \tag{12}
\]

In the discrete-time domain, \( k \) is the index of time instant, \( N \) denotes the final time such that \( t_f = N\delta_t \), \( \Psi \) is the cost function of final states, i.e. \( z_N \), and \( Q \) is the cost function of states at each time instant. The system switches between sub-systems to move from the initial states to the desired final states such that the cost function is minimized. The optimal switching (policy) can be obtained from Bellman’s principle of optimality.

According to Bellman principle, “[a]n optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision” \([11]\). Defining \( J_k \) as the cost function at each time instant, from Bellman principle, the following results can be obtained directly.

\[
J_N(z_N) = \Psi(z_N) \tag{13a}
\]

\[
J_k(z_k) = Q(z_k) + J_{k+1}(z_{k+1}) \tag{13b}
\]

According to Bellman principle, in Eqn. 13b optimizing \( J_k(z_k) \) is equivalent to optimizing \( J_{k+1}(z_{k+1}) \) \([11]\). The interpretation of Eqn. 13b for the system defined in Eqn. 11 is as the optimal switching at each instant that optimize the cost function. In other words, no matter what the active sub-system is, the best switching to another sub-system is found by optimizing the cost. If the system is at time \( k \), the cost for system is \( J_k(z_k) \), according to Eqn. 13b, minimizing \( J_k(z_k) \) is equal to minimizing \( J_{k+1}(z_{k+1}) \). The concept can be defined by Eqn. 14, where \( \chi = \{1, 2, ..., n\} \) and \( i_k(z_k)^* \) is the optimal switching obtained from comparing costs of all subsystems, i.e. \( J_{k+1}(F_i(z_k)) \), and choosing the sub-system with minimum cost \([9]\).

\[
i_k(z_k)^* = \arg\min_{i \in \chi} \{J_{k+1}(F_i(z_k))\} \tag{14}
\]

If an autonomous system according to Eqn. 11 at some initial condition exists, then optimal switching can be obtained as follows: For \( k = 0 \) the cost, \( Q(z_0) \), can be calculated then \( z_1 \) can be obtained from Eqn. 11 using the optimal sub-system that can be determined from Eqn. 14 by comparing value of \( J_1(F_i(z_0)) \). This process can be advanced till final time.

The sequence of the optimal sub-systems indicates the optimal switching. However, the process is only valid for the initial condition that the process started with and if the initial condition changes, the process should be repeated. So it is necessary to have a solution which covers any initial condition within desired domain. Such solution can be obtained using NN technique which trains the system for whole range of domain. In the following section, an algorithm is explained to train the system.

### 4. Neural Network Scheme

In order to be able to evaluate Eqn. 14 for all \( z_k \in \mathbb{R}^m \), the cost function should be well defined in the domain. Assuming the cost function is bounded and smooth in the domain, based on universal approximation theorem in NN, it can be written as the sum of infinite basis functions, Eqn. 15, where \( \sigma_j \) are independent basis functions with weight \( w_j \) \([7]\).

\[
J_k(z_k) = \sum_{j=1}^{\infty} w_k(j)\sigma_j(z_k) \tag{15}
\]

With \( F \) basis functions, the cost function or cost to go, \( J_k(z_k) \), can be estimated accurately with Eqn. 16, where \( W_{\chi} = [w_{\chi}(1), w_{\chi}(2), ..., w_{\chi}(F)]^T \) is a vector of unknown weights and \( \Sigma(z_k) = [\sigma_1(z_k), \sigma_2(z_k), ..., \sigma_F(z_k)]^T \) is a vector of smooth basis functions \([7]\).

\[
J_k(z_k) \approx W_{\chi}^T \Sigma(z_k) \tag{16}
\]

The choice of basis functions, called neurons, is usually a natural choice guided by engineering experience and intuition \([7]\). In this study polynomial functions are chosen as neurons and it is assumed that the cost function is accurately estimated with selected neurons.

To substitute \( J_k(z_k) \) with Eqn. 16, we need to determine \( W_{\chi} \). This can be done by training the system in a specific domain of interest. A training algorithm is introduced in next section.

#### 4.1. Training algorithm based on NN and DP

To train Eqn. 16 and obtain weight vector, an offline algorithm can be utilized. Once the weights are determined, optimal switching can be obtained using Eqn. 14. Here we use an algorithm which is suggested in \([8]\) to find weights. In this algorithm, recurrence Eqn. 13b is used for training and obtaining weights. For fixed final time, the method is a backward fashion from final time to initial time, i.e. from \( k = N \) to \( k = 0 \). In this method, a batch least square method is utilized to determine weights. To use the least square method, several sample states should be picked from domain.
of interest which is defined as $\Upsilon \in \mathbb{R}^m$. This sampling can be done either randomly or using a mesh grid in $\Upsilon$. The training algorithm is outlined in the following:

**Training algorithm steps:**

1) Create $P \in \mathbb{R}^m$ vectors of sampling points in $\Upsilon$ where $P$ is an arbitrary big integer number indicated number of sampling.
2) Set $k = N$ and train network weights to obtain $W_N$:
   
   $$W_N^T \Sigma(z_N) = \Psi(z_N)$$

3) Set $k = k - 1$ and find the optimal subsystem:
   
   $$i_k(z_k)^* = \text{arg min} \{W_{k+1}^T \Sigma(F_i(z_k))\}$$

4) Train the network weights $W_k$:
   
   $$W_k^T \Sigma(z_k) = \bar{Q}(z_k) + W_{k+1}^T \Sigma(F_{i_k}^*(z_k))$$

5) Go to step 3 and repeat steps 3 and 4 until $k = 0$.

Note that the algorithm is an offline training method and uses batch least square.

4.2. Training algorithm implementation

For offline training implementation, we use the batch least square method which is easy to program. Note that at each time step $k$, there is a vector of weights, i.e. $W(k) \in \mathbb{R}^N$. If there are $N$ time steps, then we can define a matrix $\mathcal{M} \in \mathbb{R}^{P \times N}$ where $k$th column is assigned for weights at $k$th time step. In first step of implementation, we create a matrix as

$$M_{m \times p} = [M_1 | M_2 | ... | M_P] \tag{17}$$

where $M_i \in \mathbb{R}^m$, $i = \{1, 2, ..., P\}$ is a vector with random arrays within $\Upsilon$, $P$ should be chosen big enough so the sampling covers all the domain. This is the implementation of first step in training algorithm.

In second step, we define a zero matrix $\Sigma \in \mathbb{R}^{P \times F}$ and a vector $\Psi \in \mathbb{R}^{P}$. The following iteration algorithm can be used to complete second step:

1) For $j = 1, ..., P$ repeat steps 2 and 3:

2) Evaluate $\Sigma(M_j)$ and substitute its value in $\Sigma$ such that:
   
   $$\Sigma(j,:) := \sigma(M_j)$$

3) Evaluate $\Psi(M_j)$ and substitute its value in $\Psi$ such that:
   
   $$\Psi(j,1) := \Psi(M_j)$$

Where in the iteration, $\Sigma(j,:)$ means all columns of $j$th row and $\Psi(j,1)$ means $j$th row of the vector. Filling all elements with iteration, the gains can be obtained using Eqn. 18. The result of Eqn. 18 is the last column of $\mathcal{M}$:

$$W_N = (\Sigma^T \Sigma)^{-1} \Sigma^T \Psi \tag{18}$$

Third and forth steps can be performed, using one iterative algorithm. First we create $\Sigma$ and $\Psi$ as mentioned before, then following iteration is used:

1) For $j = 1, ..., P$ repeat steps 2, 3 and 4:

2) Evaluate $\Sigma(M_j)$ and substitute its value in $\Sigma$ such that:
   
   $$\Sigma(j,:) := \sigma(M_j)$$

3) Evaluate $W_{k+1}^T \Sigma(F_i(M_j))$ for all $n$ subsystems, compare and choose the subsystem which has minimum value.

4) Calculate $Q(M_j) + W_{k+1}^T \Sigma(F_i^*(M_j))$ and substitute its value in $\Psi$ such that:
   
   $$\Psi(j,1) := \Psi(M_j)$$

By using Eqn. 18, $W_k$ will be obtained and it will be placed in $k$th column of $\mathcal{M}$.

After training the system offline, we can use $\mathcal{M}$ for online implementation. One problem with the offline training algorithm is the switching frequency. The training does not take into account the physical limitations of system. In fact, in the training there is no procedure to consider switching frequency and if the switching between sub-systems is high, it may cause damage to the system. To reduce the possible high switching frequency, a remedy called minimum dwell time is suggested in [9]. That is, during a minimum specific time of $\Delta_t$, there is no switching. Considering dwell time is important in online implementation. In the next section, it is explained how the introduced method can be used for attitude control problem.

5. Switched Form of Attitude Control

In sections three and four, utilization of DP and NN for a switching system was explained. Although the attitude problem does not have a switched form, it has discontinuous input. That is, based on thrusters on/off, only specific torque level is applied to the system. This is the key to convert satellite attitude dynamics, Eqn. 7, to switched form of Eqn. 9. It is trivial that with four thrusters, $T_n \in \mathbb{R}^4$, there are sixteen different combinations of thrusters that are fired. All possible combinations of thrusters can be written according to Table 1.

The attitude dynamics originally is in the form of Eqn. 19. The control input, $U$, is a function of torque applied by four thrusters.

$$\dot{x} = f(x, u) \tag{19}$$

By converting Eqn. 19 to format of Eqn. 20, the optimal switching between subsystems can be found. In Eqn. 20, for each sub-system, the fired thrusters can be found from Table 1. For example, for $i = 1$, according to the table only $T_1$ is fired, so Eqn. 3 is determined with only $T_1$ on and therefore the dynamics of sub-system one can be calculated by replacing Eqn. 3 (with $T_1$ on) in Eqn. 7.

$$\dot{x} = f_i(x) \quad i = \{1, 2, ..., 16\} \tag{20}$$

Finally, an operational consideration for attitude control is the thruster pulse duration ($P_i$). As mentioned in the last section, to avoid high frequency
switching, a minimum dwell time is necessary. Interestingly, the definition of dwell time matches with $P_d$. That is the dwell time can be set equal to $P_d$ so the switching frequency is guaranteed to be more than the time is need for a thruster to turn on and off.

6. Simulation

In this section, the proposed control solution is applied to a microsatellite. The first step for offline training is defining basis functions. The choice of basis functions is often based on sight and experience in engineering [7]. Here, the basis function is chosen as:

$$\Sigma(x_k) = \phi^a \omega_x^b \phi^c \omega_y^d \psi^e \omega_z^f$$  \hspace{1cm} (21)$$

where $a + b + c + d + e + f \leq 5$, and all powers are real integer numbers. Based on this choice 210 basis functions have been created with weights vector $W_k = [w_1(k), w_2(k), ..., w_{210}(k)]^T$.

The cost function is chosen as Eqn. 22 where $Q$ and $Q_f$ are positive definite diagonal matrices.

$$J = x_f^T Q_f x_f + \int_{t_0}^{t_f} x(t)^T Q x(t) dt$$ \hspace{1cm} (22)$$

The satellite is assumed to be in an circular orbit in the altitude of 500 km and the maneuver time, $t_f$, is five seconds. The training domain is $\pm 5^\circ$ for Euler angles and $\pm 5$ deg/sec for angular velocities. The Satellite’s thrusters installation parameters are defined in Table 2.

The acceptable final error, $\delta$, is 0.1 $^\circ$ for Euler angles and 0.1 deg/sec for angular velocities. It is assumed that the pulse duration ($P_d$) of each thruster is 30ms.

After off line training, the weights are obtained as shown in Fig. 2. Using these weights in online simulation, the Euler angles and angular velocities are as shown in Fig. 3 and 4 respectively.

It is clear that the final value of Euler angles and angular velocities is not zero. This is because of on/off nature of thrusters. However, the error at end of maneuver time is reasonably small so it is recommended to use another actuator, e.g. momentum wheel, to compensate the error. As it was stated before in section two, because of using only four thrusters, it is not possible to achieve linear velocity augmentation. This can be confirmed from Fig. 4. However, by using more thrusters, linear velocity augmentation is possible. The proposed method is capable of being adapted to any number of thrusters. So by defining a new table of all possible combinations of thrusters, the same procedure can be repeated to achieve linear velocity augmentation.

Figure 5 shows the active sub-system at each time. Since the dwell time is equal to $P_d = 30ms$, the minimum active time of a sub-system is 30ms. In Fig. 6 on/off time history of each thruster is shown with binary states (1:on, 0:off). It is important to notice that after five seconds, the control with thrusters is out of control loop and another actuator can be utilized which applies less torque than thrusters to bring the states to exact final value.

The plot in Fig. 6 is particularly important because it suggests a firing pattern for each thruster. That is, starting from an arbitrary initial condition, there is a firing pattern for each thruster that by following the pattern, the maneuver can be accomplished as desired. A possible application of firing pattern is introducing a low cost attitude control system for small satellites. Since in designing a satellite, budget and space is very important, having a ready to command attitude control system, with minimum required equipment, is beneficial. The possible system does not need any sensor for states feedback which can save money and space in satellite design. The firing pattern just needs the initial condition and based on that, it can command the thrusters in open loop fash-

---

Table 1. Different combination of thrusters

| Subsystem ($f_k$) | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  | 16  |
|-------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $T_1$             | ON  | OFF | OFF | OFF | ON  | OFF | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  |
| $T_2$             | OFF | ON  | OFF | OFF | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  |
| $T_3$             | OFF | OFF | ON  | OFF | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  |
| $T_4$             | OFF | OFF | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  | OFF | ON  |

Table 2. Satellite’s thrusters installation parameters. The vectors are defined in body frame and the configuration is symmetric.

| Parameter [unit] | Value                           |
|------------------|---------------------------------|
| $r_1$ [m]        | (0.5, 0.05, -0.7)               |
| $r_2$ [m]        | (0.5, -0.05, -0.7)              |
| $r_3$ [m]        | (-0.5, 0.05, -0.7)              |
| $r_4$ [m]        | (-0.5, -0.05, -0.7)             |
| $F_x, F_y, F_z$ [N] | 0.5                            |
| $I_x, I_y, I_z$ [kg.m$^2$] | 4.35, 4.33, 3.66            |
| $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ [deg] | -135, 135, -45, 45            |
| $\beta_1$ [deg]  | 0                              |

Fig. 2. Attitude dynamics and kinematics training weights ($W_k$) for $t_f = 5$. There are totally 210 lines, each carries the weight of a basis function. $\delta$ for this training is 0.001 sec.
ion. Since it does not need sensor during the operation, less power is consumed for a maneuver. Such a system can also be introduced as a backup control unit.

![Fig. 3. Euler angles propagation from initial condition \( [\phi, \theta, \psi] = [1, 0, -2.5] \text{deg} \). The final value of all angles is within acceptable error.](image)

![Fig. 4. Angular velocities propagation from initial condition \( [\omega_x, \omega_y, \omega_z] = [-0.5, -1, 1.5] \text{deg/sec} \).](image)

![Fig. 5. Active sub-system at each time with \( P_d = 30 \text{ms} \).](image)

6.1. System with Modeling Errors

In this part, robustness of the method against measurement errors in system is analyzed. Let’s assume there is difference between the value of torque level in offline training, i.e. measured value, and its actual values in online simulation. Analyzing the error in applied-torque is important because a disturbance torque can be modeled as the error between the actual and measured torque level [1]. So if the system is robust against such errors, then it can handle disturbances. If the system has been trained offline with some values different than actual values for torque, then the following relation is effective:

\[
M'_\kappa = M_\kappa (1 + \gamma_\kappa), \quad \kappa = 1, 2, 3, 4 \tag{23}
\]

Where \( M_\kappa \) is the measured torque which has been used for offline training, \( M'_\kappa \) is actual values of torque and, \( \gamma_\kappa \) represents the modeling error factor. The error in torque is a result of error in either measured thrust level \( (F_\kappa) \) or thruster installation position and orientation, i.e. \( \alpha_\kappa, \beta_\kappa \) and \( r_\kappa \).

The errors, in Euler angles and angular velocities, for change in orientation angles are shown in Figs. 7 and 8 respectively. In Fig. 7 the difference in Euler angles between the response without modeling error and two cases of modeling error, are depicted. In first case, there is a thruster installation error of \( \alpha_\kappa = 5^\circ \) and in second case the error is \( \beta_\kappa = 5^\circ \). For both cases, the difference in system response is small and it decreases at the end of maneuver. The errors prove that even though, a relatively large modeling error is applied to system, the response is reasonable and system does not become unstable. Of course, the system can tolerate modeling error to some extend and beyond a limit, the system response become unstable. The same behavior in angular velocities response can be seen in Fig. 8.

![Fig. 6. on/off time history of each thruster. (1: on, 0: off). At the beginning of the maneuver, the switching frequency is low and at the end of maneuver it is high.](image)

![Fig. 7. Difference in Euler angles between system without any error and system with error. (a): error in thruster installation of \( \alpha_\kappa = 5^\circ \). (b): error in thruster installation of \( \beta_\kappa = 5^\circ \). Depicted error, \( e = \| - \|^{*} \), is the difference between each angle in modeling without error and response with error which indicates with superscript \( * \).](image)
7. Conclusion and Future Work

The NN and DP successfully utilized to control the attitude of a satellite in three-axis. By offline training, a vector of basis functions weights can be constructed for time horizon of control. Starting from any initial condition within the training domain, attitude can be controlled with provided four thrusters. It is shown that it is possible to control the attitude with four thrusters using intelligent methods.

The proposed control scheme can be extended to control the attitude dynamics using any number of thrusters, e.g., six, eight, etc. The process would be the same except a new table of thrusters combinations should be defined similar to Table 1.

One possible application of this control method is introducing low cost attitude control unit without feedback for microsatellites or as a backup system for the main attitude control unit. For each initial condition, a firing pattern of each thruster can be suggested like Fig. 6. So starting from that initial condition, by executing the firing patterns and without any information about system orientation, satellite can be controlled. However a drawback and challenge is the amount of different firing patterns that need to be saved and stored.

It is shown that the method has promising performance for satellite attitude control with modeling error. The robustness of weights against uncertainties in physics of system, gives a safety margin in designing the satellite. This is particularly important for low cost projects of satellite design were accurate measurement in each parameter of satellite, e.g., $r_{xx}$, costs a lot. So having a controller that can work with estimated parameters is very important.

Other possible future works, includes real-time experiment demonstration. This study did not consider any real-time experiment as a contribution of the work. Furthermore, the effect of thrusters allocation can be studied which opens the question of: which is the best thrusters allocation for a satellite in terms of different factors such as fuel consumption accuracy, etc.

6.2. Pulse duration effect

Another important factor to study is the $P_d$ which is the minimum width of a thrust pulse. In reality, the pulse is not perfect and it is critical to make sure the method works for any $P_d$. The accuracy of method is validated for four different $P_d = 50, 100, 200, 300 \text{ms}$ as shown in Fig. 9. As it can be confirmed from figure for different $P_d$, the method works reasonably. However, by increasing $P_d$ there is more residual error at the end of maneuver. For $P_d$ more than $300 \text{ ms}$, the system becomes unstable. This can be explained by the fact that if the system stays in a sub-system for long time then it ignores the error till at some point, it is more than what the system can recover from it. However within the tested range of $P_d$, it shows promising results.

Fig. 8. Difference in angular velocities between system without any error and system with error. (a): error in thruster installation of $\alpha_c = 5^{\circ}$. (b): error in thruster installation of $\beta_c = 5^{\circ}$. Depicted error, $e = \left[\begin{array}{c} \alpha \\ \beta \\ \gamma \end{array}\right] - \left[\begin{array}{c} \alpha^* \\ \beta^* \\ \gamma^* \end{array}\right]$, is the difference between each angle in modeling without error and response with error which indicates with superscript $^*$.

AUTHORS
Amin Ghorbanpour∗ – Cleveland State University, 2121 Euclid Ave, Cleveland, OHIO, 44115, United States of America, e-mail: aghorbanspor@csuohio.edu.
Mohsen Sohrab – Tarbiat Modares University, Nasr, Jalal AleAhmad, Tehran, Iran, e-mail: mohsensohrab@gmail.com.

Corresponding author

REFERENCES
[1] Brij N Agrawal and Hyochoong Bang. Robust closed-loop control design for spacecraft skew maneuver using thrusters. Journal of Guidance, Control, and Dynamics, 18(6):1336–1344, 1995.
[2] Richard E Bellman and Stuart E Dreyfus. Applied dynamic programming. Princeton university press, 2015.
[3] FR Blindheim. Attitude control of a micro satellite: three-axis attitude control of the eseo using reaction thrusters. Master of Technology Thesis, Høgskolen i Narvik, 2004.
[4] Vahid Bohlouri, Zeynab Khodamoradi, and Seyed Hamid Jalali-Naini. Spacecraft attitude control using model-based disturbance feedback control strategy. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 40(12):557, 2018.
[5] Arthur E Bryson Jr. Control of spacecraft and aircraft. Princeton university press, 2015.
[6] Runqi Chai, Al Savvaris, Antonios Tsourdos, Senchun Chai, and Yuanqing Xia. Optimal fuel consumption finite-thrust orbital hopping of aeroassisted spacecraft. Aerospace Science and Technology, 75:172–182, 2018.
[7] Tao Cheng, Frank L Lewis, and Murad Abu-Khalaf. A neural network solution for fixed-final time optimal control of nonlinear systems. Automatica, 43(3):482–490, 2007.
Fig. 9. Euler angles and angular velocities response for different values of $P_d$. (a,e): Euler angles and angular velocities for $P_d = 50\text{ms}$, (b,f): Euler angles and angular velocities for $P_d = 100\text{ms}$, (c,g): Euler angles and angular velocities for $P_d = 200\text{ms}$, (d,h): Euler angles and angular velocities for $P_d = 300\text{ms}$.

[8] Ali Heydari and Sivasubramanya N Balakrishnan. Optimal orbit transfer with on-off actuators using a closed form optimal switching scheme. In *Proceedings of the AIAA Guidance, Navigation, and Control Conference*, 2013.

[9] Ali Heydari and SN Balakrishnan. Optimal switching between autonomous subsystems. *Journal of the Franklin Institute*, 351(5):2675–2690, 2014.

[10] Jaehyun Jin, Bongkyu Park, Youngwoong Park, and Min-Jea Tahk. Attitude control of a satellite with redundant thrusters. *Aerospace science and technology*, 10(7):644–651, 2006.

[11] Donald E Kirk. *Optimal control theory: an introduction*. Courier Corporation, 2012.

[12] Christophe R Koppel, Francesco Di Matteo, Jose Moral, and Johan Steelant. Coupled simulation of the propulsion system and vehicle using the espss satellite library. *Progress in Flight Dynamics, Guidance, Navigation, and Control—Volume 10*, 10:179–198, 2018.

[13] Raymond Kristiansen and Per Johan Nicklasson. Satellite attitude control by quaternion-based backstepping. In *Proceedings of the 2005, American Control Conference, 2005.*, pages 907–912. IEEE, 2005.

[14] Krishna Dev Kumar, An-Min Zou, et al. A novel single thruster control strategy for spacecraft attitude stabilization. *Acta Astronautica*, 86:55–67, 2013.

[15] Wilfried Ley, Klaus Wittmann, and Willi Hallmann. *Handbook of space technology*, volume 22. John Wiley & Sons, 2009.

[16] Milad Pasand, Ali Hassani, and Mehrdad Ghorbani. A study of spacecraft reaction thruster configurations for attitude control system. *IEEE Aerospace and Electronic Systems Magazine*, 32(7):22–39, 2017.

[17] Pablo A Servidio and RS Sánchez Pena. Thruster design for position/attitude control of spacecraft. *IEEE Transactions on Aerospace and Electronic Systems*, 38(4):1172–1180, 2002.

[18] Marcel J Sidi. *Spacecraft dynamics and control: a practical engineering approach*, volume 7. Cambridge university press, 1997.

[19] Ashish Tewari. *Advanced control of aircraft, spacecraft and rockets*, volume 37. John Wiley & Sons, 2011.

[20] Min Wang and Yongchun Xie. Design of the optimal thruster combinations table for the real time control allocation of spacecraft thrusters. In *Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on*, pages 5063–5068. IEEE, 2009.

[21] James RWertz. *Spacecraft attitude determination and control*, volume 73. Springer Science & Business Media, 2012.

[22] Bong Wie. *Space vehicle dynamics and control*. Aiaa, 1998.
[23] Feng Zhu and Panos J Antsaklis. Optimal control of hybrid switched systems: A brief survey. *Discrete Event Dynamic Systems*, 25(3):345–364, 2015.