Reduced Phase Space of the first order
Einstein Gravity on $\mathbb{R} \times T^2$

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Abstract

Chern-Simons formulation of the 2+1 dimensional Einstein gravity with negative cosmological constant is investigated when the spacetime has the topology $\mathbb{R} \times T^2$. The physical phase space is shown to be a direct product of two sub-phase spaces having symplectic structures with opposite signs. Each sub-phase space is found to be a non-Hausdorff manifold plus a set of measure zero. Geometrical interpretation of each point in the phase space is also given. A prescription to quantize this phase space is proposed.
Since the first order formalism of 2+1 dimensional Einstein gravity was shown to be equivalent to the Chern-Simons gauge theories with noncompact gauge groups\[1\][2], many works have appeared on this “Chern-Simons gravity” (CSG). Particularly in the case where the spacetime topology is $\mathbb{R} \times T^2$ and the cosmological constant vanishes, various aspects of the CSG including its geometrical interpretation and the structure of its phase space seem to have been elucidated \[3\] [4] [5] [6] [7].

As for the case with nonvanishing cosmological constant, except a series of works on the holonomy algebra which are made by Nelson and Regge \[8\] [9], relatively few people deal with this case\[10\]. In the previous paper\[11\], we have shown that the physical phase space in the negative cosmological constant case has nine sectors when the spacetime has the topology $\mathbb{R} \times T^2$ and one of these sectors is in 1 to 2 correspondence with the ADM phase space. However we knew little about the remaining eight sectors. In this paper, we will give the topological and symplectic structures to this phase space as a whole. We will find that this phase space is not equipped with a cotangent bundle structure. We will give geometrical interpretations to each of the nine sectors. We will also discuss its quantization.

Before going into the subject, we make some remarks. We use the notation in the previous paper\[11\]. In giving the geometrical meaning to each sector of the phase space, we will use the construction of spacetimes proposed by Witten\[1\]. Namely we identify the spacetime $M$ with a quotient space $\mathcal{F}/G$, where $\mathcal{F}$ is a subspace of the anti-de Sitter space $AdS^3$ and $G$ is a subgroup of $SO(2, 2)$ which is specified by a point on the physical phase space. Since we want to associate a spacetime with every point on the phase space, we will tacitly take the universal covering of $\mathcal{F}$ if $G$ involves rotations. It therefore seems to be natural to use as the gauge group the universal covering group $\tilde{SO}(2, 2)$ of $SO(2, 2)$. Thus a $2\pi$-rotation is no longer equivalent to the identity. For simplicity of the analysis, we will use its subgroup $\tilde{SO}_0(2, 2)$ which is connected with the identity. The prescription for reducing the phase space used in ref.\[11\] still remains valid also in this case (though with a few exceptions), since the conjugation transformation by an element of $\tilde{SO}(2, 2)$ induces the adjoint representation of the $SO(2, 2)$ Lie algebra, whose group structure is $SO(2, 2)$.

First we look into the topological structure of the physical phase space. We start with
the reduced action of the CSG with the negative cosmological constant \( \Lambda = -\frac{1}{L^2} \) on \( \mathbb{R} \times \Sigma \)

\[
I_W = \frac{L}{2} \int \eta_{ab} \left\{ -(2) A^{(+)a} \wedge (2) \dot{A}^{(+)b} + (2) A^{(-)a} \wedge (2) \dot{A}^{(-)b} \right\},
\]

(1)

We have used (the pullback into \( \Sigma \) of) the (anti-)self-dual \( SO(2,2) \) connection

\[
(2) A^{(\pm)a} \equiv \left( \frac{1}{2} e^a_{bc} \omega^{bc}_i \pm \frac{1}{L} e^a_i \right) dx^i,
\]

(2)

where \( e^a_\mu \) and \( \omega^{ab}_\mu \) are a triad and a spin connection respectively. Note that \( (2) A^{(\pm)a} \) is flat since we have solved the constraint. We see from the action (1) that the physical phase space \( \mathcal{M} \) is expressed as a direct product:

\[
\mathcal{M} = \mathcal{M}^{(+)} \times \mathcal{M}^{(-)},
\]

where \( \mathcal{M}^{(\pm)} \) is the sub-phase space which is the moduli space of flat (anti-)self-dual \( SO(2,2) \) connections modulo gauge transformations. We can also realize that \( \mathcal{M}^{(+)} \) and \( \mathcal{M}^{(-)} \) have symplectic structures with opposite signs.

Let us now consider the case where \( \Sigma \approx T^2 \). We will denote two generators of \( \pi_1(T^2) \) by \( \alpha \) and \( \beta \). We know that each sub-phase spaces \( \mathcal{M}^{(\pm)} \) consists of three subsectors \( \mathcal{M}^{(\pm)}_S \), \( \mathcal{M}^{(\pm)}_N \), and \( \mathcal{M}^{(\pm)}_T \) (plus a set \( \mathcal{M}^{(\pm)}_0 \) with measure zero). The \( \mathcal{M}^{(\pm)}_S \) is parametrized by

\[
S^{(\pm)}[\alpha] = \exp(\lambda_2 \alpha_{\pm}), \quad S^{(\pm)}[\beta] = \exp(\lambda_2 \beta_{\pm}),
\]

(3)

with \( (\alpha_{\pm}, \beta_{\pm}) \in (\mathbb{R}^2 \setminus \{(0,0)\})/\mathbb{Z}_2 \). Its symplectic structure is given by

\[
\mp L d\alpha_{\pm} \wedge d\beta_{\pm}.
\]

(4)

\( \mathcal{M}^{(\pm)}_0 \) is the union of a point \( \{e^\pm\} \equiv \{ S^{(\pm)}[\alpha] = S^{(\pm)}[\beta] = 1 \} \) and four countable sets of 1-parameter families:

\[
\mathcal{M}^{(\pm)}_{n,\pm} = \bigcup_{n \in \mathbb{Z} \setminus \{0\}} \mathcal{M}^{(\pm)}_{n,\pm}, \quad \mathcal{M}^{(\pm)}_{n,\pm} = \left\{ S^{(\pm)}[\alpha] = \exp(2n\pi \lambda_0), \quad S^{(\pm)}[\beta] = \exp(\pm \lambda_0 + b \lambda_2) \mid b \geq 1 \right\},
\]

\[
\mathcal{M}^{(\pm)}_{m,\pm} = \bigcup_{m \in \mathbb{Z} \setminus \{0\}} \mathcal{M}^{(\pm)}_{m,\pm}, \quad \mathcal{M}^{(\pm)}_{m,\pm} = \left\{ S^{(\pm)}[\alpha] = \exp(\pm \lambda_0 + a \lambda_2), \quad S^{(\pm)}[\beta] = \exp(2m\pi \lambda_0) \mid a \geq 1 \right\}.
\]

\( \lambda_0 \) denotes (the universal covering version of) the pseudo-Pauli matrices.

\( \mathbb{Z}_2 \) in the denominator is generated by the internal inversion: \( (\alpha_{\pm}, \beta_{\pm}) \rightarrow -(\alpha_{\pm}, \beta_{\pm}) \).
Parametrization of the $\mathcal{M}_N^{(\pm)}$ is

$$S^{(\pm)}[\alpha] = \exp\{(\lambda_0 \pm \lambda_2) \cos \theta_\pm\}, \quad S^{(\pm)}[\beta] = \exp\{(\lambda_0 \pm \lambda_2) \sin \theta_\pm\}, \quad (5)$$

with $\theta_\pm + 2\pi$ being identified with $\theta_\pm$. This sector by itself does not have a symplectic structure. The $\mathcal{M}_T^{(\pm)}$ is expressed by the following parametrization

$$S^{(\pm)}[\alpha] = \exp(\lambda_0 \rho_\pm), \quad S^{(\pm)}[\beta] = \exp(\lambda_0 \sigma_\pm), \quad (6)$$

with $(\rho_\pm, \sigma_\pm) \in \mathbb{R}^2 \setminus \{(0,0)\}$. Its symplectic structure is:

$$\pm L d\rho_\pm \wedge d\sigma_\pm. \quad (7)$$

We would like to provide a construction in which these three subsectors $\mathcal{M}_S^{(\pm)}$, $\mathcal{M}_N^{(\pm)}$ and $\mathcal{M}_T^{(\pm)}$ appear in one parametrization. It turns out that this unification can be done as in the $\Lambda = 0$ case. For this purpose we first consider two commuting $\overline{PSL}(2, \mathbb{R})$ holonomies in the following form:

$$S^{(\pm)}[\alpha] = \exp\left[\cos \theta_\pm \left\{\left(r_\pm + \sqrt{r_\pm^2 + 1}\right)^{1/2} \lambda_0 \pm \left(-r_\pm + \sqrt{r_\pm^2 + 1}\right)^{1/2} \lambda_2\right\}\right],$$

$$S^{(\pm)}[\beta] = \exp\left[\sin \theta_\pm \left\{\left(r_\pm + \sqrt{r_\pm^2 + 1}\right)^{1/2} \lambda_0 \pm \left(-r_\pm + \sqrt{r_\pm^2 + 1}\right)^{1/2} \lambda_2\right\}\right]. \quad (8)$$

The above holonomies with $r_\pm < 0$, $r_\pm = 0$ and $r_\pm > 0$ give parametrization of $\mathcal{M}_S$, $\mathcal{M}_N$ and $\mathcal{M}_T$ respectively. Relations between these new parameters $(r_\pm, \theta_\pm)$ and the old ones $(\alpha_\pm, \beta_\pm)$ for $\mathcal{M}_S^{(\pm)}$ and $(\rho_\pm, \sigma_\pm)$ for $\mathcal{M}_T^{(\pm)}$ are obtained by performing on (8) the conjugation using $\exp(\mp \lambda_1 \Phi_\pm)$ with $\Phi_\pm = \frac{1}{2} \ln\{|r_\pm|/(\sqrt{r_\pm^2 + 1} + 1)\}$:

$$(\alpha_\pm, \beta_\pm) = \pm \sqrt{-2r_\pm(\cos \theta_\pm, \sin \theta_\pm)} \quad for \quad r_\pm < 0,$$

$$(\rho_\pm, \sigma_\pm) = \sqrt{2r_\pm(\cos \theta_\pm, \sin \theta_\pm)} \quad for \quad r_\pm > 0. \quad (9)$$

Using the new parametrization, symplectic structures (4), (7) are expressed by the unified form:

$$\pm L d\rho_\pm \wedge d\theta_\pm. \quad (11)$$

Besides, vanishing of the symplectic structure in $\mathcal{M}_N^{(\pm)}$ can be explained by the fact that $r_\pm$ is a constant (i.e. zero) in this subsector.

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4This parametrization is different from that in [1]. In fact the former includes the latter as a special case with $\theta_\pm \in (-\pi/2, \pi/2)$. 

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In summary, we give the topological structure of
\[ \mathcal{M}'(\pm) \equiv \mathcal{M}(\pm) \setminus \mathcal{M}_0(\pm) = \mathcal{M}_S(\pm) \cup \mathcal{M}_N(\pm) \cup \mathcal{M}_T(\pm). \]

We should notice that the period of the parameter \( \theta_\pm \) is \( \pi \) for \( r_\pm < 0 \) and \( 2\pi \) for \( r_\pm \geq 0 \).

The \( \mathcal{M}'(\pm) \) defined above therefore turns out to be a non-Hausdorff manifold constructed by gluing together a punctured cone (the \( \mathcal{M}_S(\pm) \)) and a punctured plane (the \( \mathcal{M}_T(\pm) \)) at the puncture in the one to two fashion. The circle which serves as the glue is provided by the \( \mathcal{M}_N(\pm) \). The structure near the puncture precisely coincides with that of the base space of cotangent bundle structure of the phase space in the case without cosmological constant [6]. In the case with negative cosmological constant, however, the phase space \( \mathcal{M} \) does not have a cotangent bundle structure even after the removal of the set involving \( \mathcal{M}_0(\pm) \).

The phase space is represented by the direct product of two non-Hausdorff manifolds plus the set of measure zero.

Next we construct a spacetime from each point in the phase space. Henceforth we will use \((x, y)\) as periodic coordinates on \( T^2 \) with period 1. Identification conditions are therefore obvious. We only consider the subspace \( \mathcal{M}' \equiv \mathcal{M}'(+) \times \mathcal{M}'(-) \) with nonzero measure, which consists of the nine sectors. We will denote these sectors as \( \mathcal{M}(\psi, \Phi) \equiv \mathcal{M}_\psi(+) \times \mathcal{M}_\psi(-) \) (\( \psi, \Phi = S, N, T \)).

As an illustration we review the spacetime construction from \( \mathcal{M}_{(S,S)} \) [11]. The simplest connection which gives the holonomies (3) is
\[ A(\pm) \equiv \lambda_\alpha A_\mu^{(\pm)\alpha} dx^\mu = \lambda_2 d \varphi_\pm, \quad (\varphi_\pm \equiv \alpha_\pm x + \beta_\pm y). \quad (12) \]

By performing the time-dependent gauge transformation \( g^{(\pm)} = e^{\mp \lambda_0 t} \) and by extracting the triad part, we can construct the spacetime metric
\[ L^{-2} ds^2 = -dt^2 + \cos^2 t d\left( \frac{\varphi_+ - \varphi_-}{2} \right)^2 + \sin^2 t d\left( \frac{\varphi_+ + \varphi_-}{2} \right)^2. \quad (13) \]

Parametrization of the \( AdS^3 \) which reproduces this metric is:
\[ (T, X, Y, Z) = L(\sin t \cosh \frac{\varphi_+ + \varphi_-}{2}, \sin t \sinh \frac{\varphi_+ + \varphi_-}{2}, \cos t \sinh \frac{\varphi_+ - \varphi_-}{2}, \cos t \cosh \frac{\varphi_+ - \varphi_-}{2}). \quad (14) \]

We should remark that the periodicity condition for the above parametrization is expressed by the identification under two \( \tilde{SO}(2,2) \) transformations given by the holonomies
Witten’s construction therefore seems to be equivalent to the standard construction explained above.

Indeed, it turns out that these two alternative constructions give the same spacetime also to the remaining eight sectors. We will omit the detail of its derivation and give only parametrization in the $AdS^3$ which represent the spacetime constructed from a point in each sectors.\footnote{\label{footnote1}We always consider that $T^2 - X^2 - Y^2 + Z^2 = L^2$ holds, and assume that the universal covering is taken if necessary. The metric is obtained by substituting the parametrization into the pseudo-Minkowski metric:}

\[ds^2 = -dT^2 + dX^2 + dY^2 - dZ^2.\]

\footnote{\label{footnote2}We define the following new coordinates on $T^2$:}

\[\eta_{\pm} \equiv x \cos \theta_{\pm} + y \sin \theta_{\pm}, \quad r_{\pm} \equiv \rho_{\pm} x + \sigma_{\pm} y.\]
As for the other three sectors $\mathcal{M}_{(S,T)}$, $\mathcal{M}_{(N,S)}$ and $\mathcal{M}_{(N,T)}$, the following holds generically. The metric obtained from a point in $\mathcal{M}_{(\Phi,\Psi)}$ ($\Phi \neq \Psi$) can be made into the same form as the one obtained from $\mathcal{M}_{(\Psi,\Phi)}$ with the subscripts $\pm$ replaced by $\mp$. On the other hand, the triad and the parametrization in the former are respectively obtained by reversing the orientation of the triad and by replacing $Z$ with $-Z$ in the latter. Taking these facts into account, we can say that the universe obtained from a point in $\mathcal{M}_{(\Phi,\Psi)}$ is the “mirror image” of that in $\mathcal{M}_{(\Psi,\Phi)}$.

The eight sectors except $\mathcal{M}_{(S,S)}$ give spacetimes in which each torus $T^2$ is timelike, so they do not correspond to the ordinary ADM formalism. Though timelike closed curves contained in these tori are forbidden if we require the causality, there seems to be no reason to suppress them in the quantum gravity, which is the quantum theory of spacetime. So we can consider that each point in $\mathcal{M}' \setminus \mathcal{M}_{(S,S)}$ gives such an “exotic” spacetime [6].

We know that the $\mathcal{M}_{(S,S)}$ is in 1 to 2 correspondence with the ADM formalism [[1]]. The expressions of the ADM variables (complex modulus $m$, conjugate momentum $p$ and Hamiltonian $H$) in terms of new parameters ($r_{\pm}, \theta_{\pm}$) are as follows:

$$m = \frac{e^{it} \sin \theta_+ \sqrt{-2r_+} + e^{-it} \sin \theta_- \sqrt{-2r_-}}{e^{it} \cos \theta_+ \sqrt{-2r_+} + e^{-it} \cos \theta_- \sqrt{-2r_-}},$$  \hspace{1cm} (20)$$

$$p = \frac{-iL}{4 \sin t \cos t} \left( e^{-it} \cos \theta_+ \sqrt{-2r_+} + e^{it} \cos \theta_- \sqrt{-2r_-} \right)^2,$$  \hspace{1cm} (21)$$

$$H = \frac{-L}{\sin t \cos t} \sin(\theta_+ - \theta_-) \sqrt{r_+ + r_-},$$  \hspace{1cm} (22)$$

which are essentially the same as those given in ref.[9]. These new parameters are related with the parameters ($\alpha, \beta, u, v$) in [[1]] by an ordinary canonical transformation

$$2L(vd\alpha - ud\beta) = L(r_+ d\theta_+ - r_- d\theta_-) + dV,$$

$$V(\alpha, \beta, \theta_+, \theta_-) = -2L \frac{(\alpha \sin \theta_- - \beta \cos \theta_-)(\alpha \sin \theta_+ - \beta \cos \theta_+)}{\sin(\theta_+ - \theta_-)}. \hspace{1cm} (23)$$

However, the canonical transformation from the ADM variables to the new parameters is singular in the sense that it does not contain the generating function:

$$\text{Re}(\bar{p}dm) - Hdt = L(r_+ d\theta_+ - r_- d\theta_-). \hspace{1cm} (24)$$

We conjecture that this singular nature is related to the fact that $r_{\pm}$ and therefore $p$ cannot be expressed in terms of $(m, \bar{m}, \theta_+, \theta_-)$ alone.
We should note that the spacetimes we have given are not the unique ones constructed from the points in $M'$. It is because the gauge group $SO(2,2)$ (or $\tilde{SO}_0(2,2)$) is in fact larger than the direct product of the 2+1 dimensional local Lorentz group and the group of diffeomorphisms $[6][7]$. For illustration, we consider the $M_{(S,S)}$. By choosing time dependent gauge transformations other than that giving the spacetime (13), we can construct various spacetimes. There are for example Louko-Marolf-type universe $[6]$ in which the evolution of the torus is roughly:

$$T \times S \rightarrow S \times S \rightarrow S \times T,$$

and Unruh-Newbury-type universe in which the rough evolution of the torus is:

$$S \times T \rightarrow S \times S \rightarrow T \times S \rightarrow S \times S \rightarrow S \times T,$$

with an intermediate singularity in $T \times S$. Though these spacetimes coincide with one another in the region where the ADM is well-defined ($T > |X|, Z > |Y|$), their behaviors in the other region vary considerably by the choice of gauge. There seems to be no criterion for choosing the most relevant gauge.

Finally we consider how to quantize the subspace $M'$ of the physical phase space. Since $M'$ does not have a cotangent bundle structure, we cannot naively perform the quantization where quantum states are represented by functions of coordinates. We have to invoke geometric quantization scheme $[12]$ such as Kahler quantization $[1]$. To many physicists these methods are not familiar. By a trick we provide the $M'$ with a cotangent bundle structure.

First we consider the symmetry under large diffeomorphisms, in particular the inversion:

$$I : (\alpha, \beta) \rightarrow - (\alpha, \beta) \quad \text{or} \quad (x, y) \rightarrow -(x, y),$$

which induces the following simultaneous transformations:

$$I : (\theta, r) \rightarrow (\theta + \pi, r).$$

$M'$ does not have a cotangent bundle structure even after imposing this symmetry. If we adopt one of the following artificial prescription, however, the resulting phase space $M'$

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7The notation $T \times S$ for example denotes the torus with $(T, X)$- and $(Z, Y)$-directions being respectively timelike and spacelike.
acquires a cotangent bundle structure $\mathcal{M}^\circ = T_\ast B$ with the base space $B$:

i) We impose the inversion symmetry independently on $\mathcal{M}^\prime (+)$ and on $\mathcal{M}^\prime (−)$. Then $\mathcal{M}^\circ = T_\ast T^2$. The coordinates $(\theta_+, \theta_-)$ which parametrize the base space $T^2$ have periods $\pi$.

ii) We assume that $(\theta_+, \theta_- + \pi)$ can be distinguished from $(\theta_+, \theta_-)$ even when either $r_+$ or $r_-$ is negative. This involves the assumption that the $\mathcal{M}(S,S)$ is not in 1 to 2 correspondence but equivalent with the ADM phase space. The base space $B$ then becomes a torus $T^2$ different from that in i).

If we use one of these cotangent bundle structures, we can construct a representation where the quantum states are functions of $(\theta_+, \theta_-)$. In the quantum theory the canonical variables $(\theta_\pm, r_\pm)$ are promoted to the basic operators which satisfy the canonical commutation relations derived from the symplectic structure (II):

$$\left[\hat{\theta}_\pm, \hat{r}_\pm\right] = \pm i \frac{1}{L}, \quad \text{zero otherwise.}$$

(27)

It is probable that the action of $\hat{\theta}_\pm$ on the wavefunction $\chi$ is given by multiplication

$$\hat{\theta}_\pm \chi(\theta_+, \theta_-) = \theta_\pm \cdot \chi(\theta_+, \theta_-) .$$

(28)

To determine the action of $\hat{r}_\pm$, however, we have to know the integration measure or the inner product, the determination of which in turn requires the knowledge of transformation properties of the wavefunction and of the basic operators under the modular group $\Gamma = PSL(2,\mathbb{Z})$. Transformations of the classical variables under the two elementary modular transformations:

$$S : (\alpha, \beta) \rightarrow (−\beta, \alpha), \quad T : (\alpha, \beta) \rightarrow (\alpha + \beta, \beta),$$

prove to be given by the following simultaneous transformations

$$S : (\theta_\pm, r_\pm) \rightarrow (\theta_\pm + \frac{\pi}{2}, r_\pm),$$

$$T : (\theta_\pm, r_\pm) \rightarrow \left(\frac{1}{2\pi} \ln \left\{\frac{e^{i\theta_\pm} + \sin \theta_\pm}{e^{-i\theta_\pm} + \sin \theta_\pm}\right\}, (1 + \sin 2\theta_\pm + \sin^2 \theta_\pm) r_\pm\right).$$

(29)

We can show that these transformations preserve the symplectic structure of $\mathcal{M}'$ and the cotangent bundle structure of $\mathcal{M}^\circ$. We therefore expect that under the assumption made above a consistent quantum theory can be defined on the “fundamental region” $B/\Gamma$.

Since the quantum theory constructed as above is defined on $\mathcal{M}^\circ$ which is larger than the ADM phase space, we may find a process which is not expected by quantizing the ADM
formalism. In our quantum theory, momentum eigenstates would play an important role because each sector is identified by the signature of \((r_+, r_-)\). Whether our quantization prescription works well, however, depends on the validity of the two assumptions. First we have assumed that the relevant gauge group is the universal covering \(\widetilde{SO}(2, 2)\) of the anti-de Sitter group. Second we have used the trick to give the cotangent bundle structure to the phase space. If either of these two turns out to be physically irrelevant, such a quantization prescription cannot be used and we are obliged to take the geometric quantization methods.

Complete formulation of our quantum theory \([14]\) includes a definition of the measure and the quantum version of the modular transformation \([29]\), its quantum relation to the ADM formalism \([13][11]\), and its relations to the quantum theory of refs. \([8][9]\) and to the geometric quantization \([12]\).

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