Dark matter and dark energy from quark bag model

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Abstract. We calculate the present expansion of our Universe endowed with relict colored objects – quarks and gluons – that survived hadronization either as isolated islands of quark-gluon ”nuggets” or spread uniformly in the Universe. In the first scenario, the QNs can play the role of dark matter. In the second scenario, we demonstrate that uniform colored objects can play the role of dark energy providing the late-time accelerating expansion of the Universe.
1 Introduction

Over two decades ago [1, 2] (see also [3]) the accelerated expansion of the early Universe was derived from a quark bag model with the proper equations of state (EoS). It was called tepid [1, 2] or little [3] inflation, in view of its moderate scales, compared to the better known earlier inflation. However, occurring at a later time (when the temperature $\sim 200$ MeV) and smearing a lot of the earlier effects, it may have important consequence for the observable Universe.

The derivation was based on a quark-gluon bag EoS completing the Friedmann equations. Our Universe was cooling down along the "hot" (i.e. quark-gluon) branch of the EoS until it reached the point/area of the transition to the confined hadron phase. In most of the papers along these lines (see [1] and references therein) the process of inflation terminates by a phase transition (hadronization) to the state of colorless objects. The details of the phase transition, depending on unknown confining forces, are poorly known and leave much room for speculations.

In the present paper, we consider the possibility that a small fraction of colored objects – quarks and gluons – escaped hadronization. They may survive as islands of colored particles, called quark-gluon nuggets (for brevity sometimes also quark nuggets (QNs)). This possibility was first considered by E. Witten [4] and scrutinized further in [5–7]. In his paper [4], E. Witten discusses the possibility that QNs can survive even at zero temperature and pressure. If so, the "hot" quark-gluon phase in the form of QNs may affect the present expansion of the Universe. Indeed, our investigation shows that nuggets can contribute to dark matter provided that their interaction with ordinary matter is weak.

The size distribution of QNs was calculated in [8, 9]. The authors found that a large number of stable QNs exists in the present Universe. They also claimed that QNs could be a viable candidate for cosmological dark matter. The survival probability of these QNs, i.e. the question whether the primordial QNs can be stable on a cosmological time scale, is a key issue, and it was studied by a number of our predecessors. In particular, the authors of [10], using the chromoelectric flux tube model, have demonstrated that the QNs will survive against baryon evaporation if the baryon number of the quark matter inside the nuggets...
is larger than $10^{42}$ which is a rather conservative estimate. A scenario where the Universe would be closed with QNs with the baryon number density window $10^{39.40} \leq N \leq 10^{49}$ or, in other words, the proverbial cosmological dark matter, containing 90% or more of all matter in the Universe, is made of QNs, was considered in the paper [11]. The special role of the strange quark matter in the phase transition, both in the context of the early Universe and in compact stars, was discussed in [12]. A relativistic model for strange quark stars was proposed in [13] (see also [14] for a different approach to get compact quark objects). Quark matter is believed to exist at the center of neutron stars [15], in strange stars [16] and as small pieces of strange matter [17]. The latter can result in ultra-high energy cosmic rays [18, 19]. The search (in lunar soil and with an Earth orbiting magnetic spectrometer) for cosmic ray strangelets may be the most direct way of testing the stable strange matter hypothesis.

Another possibility is that a very small (to be specified!) fraction of colored objects – quarks and gluons – survived after the phase transition in the form of a perfect fluid uniformly spread within the colorless hadronic medium. This picture is physically less motivated than the nugget model. We suppose that this fluid has the same thermodynamical properties as the quark-gluon plasma (QGP). Therefore, we call it as a QGP-like perfect fluid. Nevertheless, it is of interest to investigate cosmological consequences of such assumption. In our paper, we demonstrate that such fluid can provide an alternative (with respect to the cosmological constant) explanation to the late-time accelerating expansion of the Universe. It is worth mentioning that quark-gluon plasma as dark matter in the halo of galaxies was investigated in the paper [20]. The authors arrived at a very interesting conclusion that flatness of the rotational curves can be explained due to the presence of quark matter in halos (because of additional attraction from such matter). In our paper, we spread this quark matter over the whole Universe and demonstrate that it can result in the late time acceleration of the Universe. Analogously, the authors of [20] also claimed that quark-gluon plasma on the global level behaves like dark energy.

The paper is structured as follows. In Sec. 2, we briefly remind the quark bag equations of state. In Sec. 3 and 4, we consider the influence of nuggets and QGP-like perfect fluid on the late-time expansion of the Universe. The main results are briefly summarized in concluding Sec. 5.

2 Equations of state in the quark-gluon bag model

We first briefly remind the quark bag equation of state, a simple model of quark confinement. For vanishing chemical potential, $\mu = 0$, it is a system of two equations

$$p_q(T) = A_q T^4 - B ,$$  \hspace{1cm} (2.1)

$$p_h(T) = A_h T^4 .$$  \hspace{1cm} (2.2)

The first line corresponds to the "hot" phase of deconfined quarks and gluons, and the second one relates to confined particles, i.e. hadrons. A system of strongly interacting particles, made of free quarks and gluons, is cooling down and meets the "cold" phase transforming in colorless hadrons. The coefficients are defined by the degrees of freedom and are equal to: $A_q \approx 1.75$, $A_h \approx 0.33$, $B = (A_q - A_h) T_c^4$ and $T_c \approx 200$ MeV.
Knowing the pressure, \( p(T) \), for \( \mu = 0 \), one can easily calculate the remaining thermodynamical quantities, e.g., for the energy density we have

\[
\varepsilon(T) = T \frac{dp}{dT} - p .
\]  

(2.3)

The above EoS is not unique. There is a number of interesting modifications [1, 2, 21–25]. First such modification was considered by C. Källmann [21], who introduced a temperature-dependent bag “constant”, namely, by replacing in the first line of the EoS, Eq. (2.1), \( B \rightarrow \bar{B}(T) = \bar{B}T \), where \( \bar{B} = (A_q - A_h)T_c^3 \). This modification has immediate consequences, namely, by producing a minimum in the “hot” line of the EoS, corresponding to metastable deeply supercooled states of the deconfined strongly interacting matter. Also, it drives inflation of the Universe, as shown in [1, 2]. A detailed discussion of the above EoS and their consequences, both for the heavy ion collisions and the early Universe, can be found in the review paper [22].

Since the idea of the present paper is that a small fraction of deconfined quarks and gluons survives to present days, we shall be interested in the “hot” branch of the bag EoS. As we mentioned above, there is a number of different modifications of Eq. (2.1). For our present purposes, however, two simple representatives will be sufficient. They are the Källmann modified model (which we call Model I):

\[
p_q(T) = A_q T^4 - \bar{B}T \equiv \bar{A}_1 T + \bar{A}_4 T^4 ,
\]  

(2.4)

and the original model (Model II) described by Eq. (2.1):

\[
p_q(T) = A_q T^4 - B \equiv A_0 + \bar{A}_4 T^4 .
\]  

(2.5)

It is worth noting that in these equations, we measure temperature in energetic units, i.e. in \( \text{erg} \) or \( \text{MeV} \) (\( \text{1MeV} \approx 0.1602 \times 10^{-5} \text{erg} \)). Then, pressure is measured in \( \text{erg}^4 \) or \( \text{MeV}^4 \).

### 3 Quark nuggets

As we wrote above, there is a possibility that after a phase transition from quark gluon plasma (QGP) to hadronic matter, a part of QGP was preserved in the form of quark gluon nuggets [4–7]. They are isolated “islands” of QGP in a sea of a new hadronic phase. Now, we want to investigate cosmological consequences of this assumption. Obviously, for considered models, a cosmological scenario strongly depends on thermodynamical properties of QGP. We focus on two possible Eqs. (2.4) and (2.5). With the help of standard thermodynamical Eq. (2.3) we get the expressions for the energy density:

\[
\varepsilon = 3 \bar{A}_4 T^4
\]  

(3.1)

and

\[
\varepsilon = -\bar{A}_0 + 3 \bar{A}_4 T^4
\]  

(3.2)

for Model I and Model II, respectively. Eqs. (2.4), (2.5), (3.1) and (3.2) describe the pressure and energy density inside of the nuggets. The total pressure and energy density of all nuggets

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1 Usually, the dimension of pressure is \( \text{erg}/\text{cm}^3 \). It is not difficult to get the relation \( \text{1MeV}^4 \approx 2.09 \times 10^{26} \text{erg}/\text{cm}^3 \). However, to transform to the usual units, it is more convenient to redefine the coefficients as follows: \( \bar{A}_i \rightarrow \bar{A}_i = \bar{A}_i/[(\text{MeV}^4)^i \text{L}_\text{Pl}^3], i = 0, 1, 4 \), where \( \text{M}_{\text{Pl}} \approx 2.177 \times 10^{-5} \text{g} \) is the Planck mass and \( \text{L}_{\text{Pl}} \approx 1.616 \times 10^{-33} \text{cm} \) is the Planck length.
in the Universe can be calculated as follows. Let us take, e.g., Model I with Eq. (2.4). Then, for total pressure of nuggets we get
\[ P = \frac{\sum_i p_{qi} v_i}{V} = \frac{A_1 T + A_4 T^4}{a^3}, \]
where \( p_{qi} \) is the pressure of the i-th nugget with the volume \( v_i \) and \( V \propto a^3 \) is the total volume of the Universe (\( a \) is the scale factor of the Friedmann-Robertson-Walker metric). We consider the case where all nuggets have the same pressure (2.4) and their volumes are either constant or only slightly varying with time. The total volume of nuggets \( \sum_i v_i \) is included in the coefficients \( A_1 \) and \( A_4 \) (i.e. \( A_1, A_4 \) have dimension \( \tilde{A}_1, \tilde{A}_4 \times \text{cm}^3 \), so, taking into account the footnote 1, \( A_1 \) is dimensionless and \( A_4 \) has the dimension \( \text{erg}^{-3} \)). Therefore,
\[ \frac{A_1}{A_4} = \frac{\tilde{A}_1}{\tilde{A}_4} = -0.8114 \ T_c^3. \]

Similarly, from Eq. (3.1), for the energy density of all nuggets we get:
\[ \mathcal{E} = \frac{3A_4 T^4}{a^3}. \]
The same procedure holds for the Model II. Let us consider two models separately.

### 3.1 Model I

Here, the pressure and energy density of all nuggets are given by the above formulae (3.3) and (3.5), respectively. In these formulae, temperature is a function of the scale factor \( a \): \( T = T(a) \). Let us specify this dependence. From the energy conservation equation
\[ d(\mathcal{E} a^3) + P d(a^3) = 0 \]
we can easily get
\[ T = \left( \frac{(C/a)^{3/4} - A_1}{A_4} \right)^{1/3}. \]
As we mentioned above, we consider the model where the coefficients \( A_1 < 0 \) and \( A_4 > 0 \). In Eq. (3.7), \( C \geq 0 \) is the constant of integration which is defined by the temperature \( T_0 \) and scale factor \( a_0 \) at the present time:
\[ C = (A_1 + A_4 T_0^3)^{4/3} a_0 = A_4^{4/3} \left( -0.8114 T_c^3 + T_0^3 \right)^{4/3} a_0. \]
The temperature \( T \) tends to the constant value when the scale factor approaches infinity:
\[ T \to T_\infty = \left( \frac{-A_1}{A_4} \right)^{1/3} = 0.9327 T_c \quad \text{for} \quad a \to \infty, \]
and the pressure goes asymptotically to zero: \( P \to 0 \). On the other hand, for \( C \equiv 0 \), the temperature is constant all the time \( T \equiv T_\infty \), and nuggets behave as a matter with zero pressure \( P = 0 \). It is worth noting that in this model the temperature of the QNs at present time is not arbitrary low, rather it is close to the critical temperature \( T_c \) of the phase transition (see also Eq. (3.29) for the Model II below).
We consider our Universe starting from the moment when we can drop the radiation. It is well known that the radiation dominated (RD) stage is much shorter than the matter dominated (MD) stage. Hence, the neglect of the RD stage does not affect much the estimate of the lifetime of the Universe. Starting from the MD stage, the first Friedmann equation for our model reads

\[ 3H^2 + K = \kappa\varepsilon_{0}^{\text{mat}} \left( \frac{a_0}{a} \right)^3 + \Lambda, \quad (3.10) \]

where we take into account the cosmological constant \( \Lambda \) and the \((\text{usual + dark})\) matter with the present value of the energy density \( \varepsilon_{0}^{\text{mat}} \). In \((3.10)\), \( H = a'/a = (da/d\eta)/a \), \( \kappa = 8\pi G_N/c^4 \), \( G_N \) is the gravitational constant and \( K = \pm 1,0 \) is the spatial curvature. The conformal time \( \eta \) is connected with the synchronous time \( t \) via \(ad\eta = cdt \). Taking into account Eqs. \((3.5)\) and \((3.7)\), we get for the Hubble parameter \( H = (1/a)da/dt = (c/a^2)da/d\eta \) the following expression:

\[ H^2 = H_0^2 \left\{ \left[ \beta \left( \frac{a_0}{a} \right)^3 + \gamma \left( \frac{a_0}{a} \right)^{9/4} + \Omega_M \left( \frac{a_0}{a} \right)^3 + \Omega_{\Lambda} + \Omega_K \left( \frac{a_0}{a} \right)^2 \right] \right\}, \quad (3.11) \]

where the cosmological parameters are

\[ \Omega_M = \frac{c^2}{3H_0^2\kappa\varepsilon_{0}^{\text{mat}}}, \quad \Omega_\Lambda = \frac{c^2}{3H_0^2\Lambda}, \quad \Omega_K = -K \left( \frac{c}{a_0H_0} \right)^2. \quad (3.12) \]

and we introduce the dimensionless parameters

\[ \beta = \left( \frac{C}{a_0} \right)^{3/4} \frac{1}{A_1^{1/4}} \left( \frac{\kappa c^2}{a_0^2H_0^2} \right)^{3/4}, \quad \gamma = -A_1 \frac{A_1}{A_4^{1/4}} \left( \frac{\kappa c^2}{a_0^2H_0^2} \right)^{3/4}. \quad (3.13) \]

From the second Friedmann equation

\[ \frac{2\dot{H} + H^2 + K}{a^2} = -\kappa P + \Lambda \quad (3.14) \]

after some obvious algebra we obtain the deceleration parameter

\[ -q = \frac{1}{aH^2} \frac{d^2a}{dt^2} = \left( \frac{H_0}{H} \right)^2 \left\{ \frac{\gamma}{2} \left[ \beta \left( \frac{a_0}{a} \right)^{9/4} + \gamma \left( \frac{a_0}{a} \right)^9 \right]^{1/3} \right. \]

\[ - \left[ \beta \left( \frac{a_0}{a} \right)^3 + \gamma \left( \frac{a_0}{a} \right)^{9/4} \right]^{4/3} - \frac{\Omega M}{2} \left( \frac{a_0}{a} \right)^3 + \Omega_{\Lambda} \left\}. \quad (3.15) \]

At the present time \( t_0 \), Eqs. \((3.11)\) and \((3.15)\) read

\[ 1 = (\beta + \gamma)^{4/3} + \Omega_M + \Omega_\Lambda + \Omega_K, \quad (3.16) \]

\[ -q_0 = \frac{\gamma}{2} (\beta + \gamma)^{1/3} - (\beta + \gamma)^{4/3} - \frac{\Omega_M}{2} + \Omega_\Lambda. \quad (3.17) \]

Additionally, we obtain from \((3.11)\) the differential equation

\[ \frac{\ddot{a}}{a} = \frac{\dot{a}a}{\sqrt{(\beta + \gamma\ddot{a}^{3/4})^{1/3} + \Omega_M\dot{\ddot{a}} + \Omega_\Lambda\dot{\ddot{a}}^4 + \Omega_K\dot{\ddot{a}}^2}}, \quad (3.18) \]
where we introduce the dimensionless quantities

\[ \tilde{a} = \frac{a}{a_0}, \quad \tilde{t} = H_0 t. \]  

(3.19)

Therefore, the age of the Universe \( \tilde{t}_0 \) is defined by the equality

\[ -\tilde{t}_0 = \int_1^0 \frac{\tilde{a} d\tilde{a}}{\sqrt{\left(\beta + \gamma \tilde{a}^{3/4}\right)^{4/3} + \Omega_M \tilde{a} + \Omega_\Lambda \tilde{a}^4 \Omega_K \tilde{a}^2}}. \]  

(3.20)

Now, we consider the case of the flat space \( K = 0 \rightarrow \Omega_K = 0 \). Then, Eq. (3.16) reads

\[ 1 = (\beta + \gamma)^{4/3} + \Omega_M + \Omega_\Lambda. \]  

(3.21)

Eq. (3.17) demonstrates that accelerated expansion of the Universe at the present time (i.e. \( -q_0 > 0 \)) can be ensured by the first and the last terms on the right side of this equation. It is tempting to explain the acceleration only at the expense of the first term, i.e. due to the presence of QNs when the cosmological constant is absent. However, simple analysis of Eqs. (3.17) and (3.21) in the case \( \Omega_\Lambda = 0 \) shows that the acceleration \( -q_0 > 0 \) is achieved only for \( \beta < 0 \) that contradicts our model. The inclusion of the negative curvature \( \Omega_K > 0 \) does not affect this conclusion due to the smallness of \( \Omega_K \).

Nevertheless, QNs can contribute to the dark matter if they weekly interact with usual baryon matter and light, or they can explain the problem of missing baryons [26, 27] if their interaction with usual matter is not negligible. As we have mentioned above, nuggets behave as matter either asymptotically when \( a \rightarrow \infty \) or for all time in the case \( C = 0 \rightarrow \beta = 0 \). In the latter case we can exactly restore the ΛCDM model so long as Eqs. (3.17) and (3.21) take the usual form for this model:

\[ 1 = \Omega_{M,\text{total}} + \Omega_\Lambda \]  

(3.22)

and

\[ -q_0 = -\frac{1}{2} \Omega_{M,\text{total}} + \Omega_\Lambda \Rightarrow \Omega_\Lambda = \frac{1}{3} \left( \Omega_M + 2q_0 \right), \]  

(3.23)

where \( \Omega_{M,\text{total}} \equiv \gamma^{4/3} + \Omega_M \). Let \( \Omega_M \) correspond to just the visible matter. According to observations, \( \Omega_M \approx 0.04 \). Then, we can easily restore the parameters of the ΛCDM model. For example, taking the deceleration parameter \( q_0 \approx -0.595 \), as in the ΛCDM model [28, 29], we get \( \Omega_\Lambda \approx 0.73 \) and \( \gamma \approx 0.33 \rightarrow \gamma^{4/3} \approx 0.23 \). Therefore, \( \Omega_{M,\text{total}} \approx 0.27 \). For the age of the Universe, we get from (3.18) (where we should put \( \beta = 0, \Omega_K = 0 \)) \( \tilde{t}_0 \approx 1 \Rightarrow t_0 \approx H_0^{-1} \sim 13.7 \times 10^9 \text{yr} \). Hence, weekly interacting QNs may be candidates for dark matter.

### 3.2 Model II

Quark nuggets for the Model II are defined by the thermodynamical functions (2.5) and (3.2). Similar to Eqs. (3.3) and (3.5), the total pressure and energy density of all nuggets in the Universe are

\[ P = \frac{A_0 + A_4 T^4}{a^3}, \]  

(3.24)

\[ \mathcal{E} = \frac{-A_0 + 3A_4 T^4}{a^3}, \]  

(3.25)
where
\[ \frac{A_0}{A_4} = \frac{\bar{A}_0}{\bar{A}_4} = -0.8114 \ T_c^4. \] (3.26)

In this model, \( A_0 < 0 \) and \( A_4 > 0 \). Taking into account the footnote 1, we may conclude that the coefficients \( A_0 \) and \( A_4 \) have dimensions erg and erg\(^{-3}\), respectively. For the thermodynamical functions (3.24) and (3.25), energy conservation Eq. (3.6) gives the following dependence of the temperature on the scale factor:
\[ T = \left( \frac{(\bar{C}/a) - A_0}{A_4} \right)^{1/4}, \] (3.27)

where \( \bar{C} \geq 0 \) is the constant of integration which is defined by the temperature \( T_0 \) and the scale factor \( a_0 \) at the present time:
\[ \bar{C} = (A_0 + A_4 T_0^4) a_0 = A_4 (-0.8114 T_c^4 + T_0^4) a_0. \] (3.28)

Similar to the previous case, the temperature \( T \) tends to the constant value when the scale factor approaches infinity:
\[ T \to T_\infty = \left( -\frac{A_0}{A_4} \right)^{1/4} = 0.9491 T_c \text{ for } a \to \infty, \] (3.29)
and the pressure goes asymptotically to zero: \( P \to 0 \). On the other hand, for \( \bar{C} \equiv 0 \), the temperature is constant all the time \( T \equiv T_\infty \), and QNs behave as a matter with zero pressure \( P = 0 \).

In this model, the pressure and energy density have simple dependence on the scale factor:
\[ P(a) = \frac{\bar{C}}{a^4}, \] (3.30)
\[ E(a) = 3 \frac{\bar{C}}{a^4} - 4 \frac{A_0}{a^3}. \] (3.31)

Formally, such perfect fluid can be considered as a mixture of radiation and matter. However, for ordinary radiation \( T \sim 1/a \).

From the first Friedmann equation (3.10), we obtain the expression for the Hubble parameter:
\[ H^2 = H_0^2 \left\{ \beta \left( \frac{a_0}{a} \right)^4 + \gamma \left( \frac{a_0}{a} \right)^3 + \Omega_M \left( \frac{a_0}{a} \right)^3 + \Omega_\Lambda + \Omega_K \left( \frac{a_0}{a} \right)^2 \right\}, \] (3.32)

where the cosmological parameters are defined in (3.12) and the dimensionless parameters \( \beta \) and \( \gamma \) are
\[ \beta = \frac{\bar{C}}{a_0} \left( \frac{\kappa c^2}{a_0^3 H_0^2} \right), \quad \gamma = -4 \frac{A_0}{3} \left( \frac{\kappa c^2}{a_0^3 H_0^2} \right). \] (3.33)

Therefore, the age of the Universe \( \tilde{t}_0 \) is defined by the equality
\[ -\tilde{t}_0 = \int_0^1 \frac{\bar{a} d\bar{a}}{\sqrt{\beta + \gamma \bar{a} + \Omega_M \bar{a}^4 + \Omega_\Lambda \bar{a}^4 + \Omega_K \bar{a}^2}}. \] (3.34)
The second Friedmann equation (3.14) results in the deceleration parameter
\[ -q = \frac{1}{aH^2} \frac{d^2a}{dt^2} = \left( \frac{H_0}{H} \right)^2 \left\{ -\beta \left( \frac{a_0}{a} \right)^4 - \frac{1}{2} \gamma \left( \frac{a_0}{a} \right)^3 - \frac{1}{2} \Omega_M \left( \frac{a_0}{a} \right)^3 + \Omega_\Lambda \right\}. \] (3.35)

At the present time \( t_0 \), Eqs. (3.32) and (3.35) read
\[ 1 = \beta + \gamma + \Omega_M + \Omega_\Lambda + \Omega_K, \] (3.36)
\[ -q_0 = -\beta - \frac{1}{2} \gamma + \Omega_\Lambda - \frac{1}{2} \Omega_M. \] (3.37)

It can be easily seen that similar to the previous model we also reproduce the standard \( \Lambda \)CDM model in the case of the flat space \( \Omega_K = 0 \) and \( \bar{C} = 0 \rightarrow \beta = 0 \). The only difference is that in Eqs. (3.22) and (3.23) \( \Omega_{M,\text{total}} = \gamma + \Omega_M \). For example, if we take \( \Omega_M \approx 0.04 \) and \( q_0 \approx -0.595 \), then we get \( \Omega_\Lambda \approx 0.73 \) and \( \gamma \approx 0.23 \rightarrow \Omega_{M,\text{total}} \approx 0.27 \) as in the \( \Lambda \)CDM model. For these parameters, the age of the Universe is \( t_0 \approx 1 \Rightarrow t_0 \approx H_0^{-1} \sim 13.7 \times 10^9 \text{ yr} \). Hence, we again arrive at the conclusion that weekly interacting QNs may be candidates for dark matter.

### 4 QGP-like perfect fluid

Above, we considered a scenario where QNs form after the phase transition the "isolated islands" in the sea of baryon matter. In this section we suppose a less physically motivated model where a part of quark gluon plasma survived after the phase transition in the form of the homogeneously and isotropically distributed perfect fluid. This perfect fluid has the thermodynamical functions of the form (2.4), (3.1) or (2.5), (3.2). We do not know what part of QGP survived (see however some estimate at the very end of this section). So, we demand only that the ratio between coefficients \( \bar{A}_i \) was preserved. Maybe, it is more correct to speak about some unknown perfect fluid with the thermodynamical functions motivated by the QGP. Therefore, we call this fluid as a QGP-like perfect fluid. We are going to investigate the cosmological consequences of such proposal.

In general, we can consider the pressure of the form
\[ p(T) = \hat{A}_0 + \hat{A}_1 T + \hat{A}_2 T^2 + \hat{A}_3 T^3 + \hat{A}_4 T^4, \] (4.1)
which, via Eq. (2.3), results in the energy density
\[ \varepsilon(T) = -\hat{A}_0 + \hat{A}_2 T^2 + 2\hat{A}_3 T^3 + 3\hat{A}_4 T^4. \] (4.2)

Eq. (3.6) leads to the differential equation
\[ \frac{da}{a} = -\frac{\left( 2\hat{A}_2 + 6\hat{A}_3 T + 12\hat{A}_4 T^2 \right)}{3 \left( \hat{A}_1 + 2\hat{A}_2 T + 3\hat{A}_3 T^2 + 4\hat{A}_4 T^3 \right)} dT. \] (4.3)

The solution of this equation enables to determine the dependence \( T = T(a) \). Unfortunately, there is no solution of (4.3) in elementary functions. Therefore, we consider two particular models by analogy with the previous section.
4.1 Model I

First, we consider the case \( \hat{A}_0 = 0, \hat{A}_2 = 0, \hat{A}_3 = 0 \). As we mentioned above, the coefficients \( \hat{A}_1 \) and \( \hat{A}_4 \) satisfy the condition similar to (3.4):

\[
\frac{\hat{A}_1}{\hat{A}_4} = -0.8114 T_c^3. \tag{4.4}
\]

Therefore, we consider the case \( \hat{A}_1 < 0 \) and \( \hat{A}_4 > 0 \). Following the footnote 1, the coefficients \( \hat{A}_1 \) and \( \hat{A}_4 \) have dimensions \( \text{cm}^{-3} \) and \( \text{erg}^{-3} \text{cm}^{-3} \), respectively.

Integrating (4.3), we get

\[
T = \left( \frac{\hat{C}^3 - \hat{A}_1 a^3}{4 \hat{A}_4 a^3} \right)^{1/3} \Rightarrow \varepsilon(T) = 3 \hat{A}_4 \left( \frac{\hat{C}^3 - \hat{A}_1 a^3}{4 \hat{A}_4 a^3} \right)^{4/3}, \tag{4.5}
\]

where \( \hat{C} \geq 0 \) is the dimensionless constant of integration which is defined by the temperature \( T_0 \) and the scale factor \( a_0 \) at the present time:

\[
\hat{C} = \left( \hat{A}_1 + 4 \hat{A}_4 T_0^3 \right)^{1/3} a_0 = \hat{A}_4^{1/3} (-0.8114 T_c^3 + 4 T_0^3)^{1/3} a_0. \tag{4.6}
\]

The difference between the first equation in (4.5) and Eq. (3.7) is due to the prefactor \( 1/a^3 \) in (3.3) and (3.5). The temperature \( T \) tends to the constant value when the scale factor approaches infinity

\[
T \to T_\infty = \left( \frac{-\hat{A}_1}{4 \hat{A}_4} \right)^{1/3} = 0.5876 T_c \quad \text{for} \quad a \to \infty. \tag{4.7}
\]

It can be easily verified that, in the limit \( a \to \infty \), the energy density \( \varepsilon \to (3/4) [\hat{A}_1^4 / (4 \hat{A}_4)]^{1/3} \) and the pressure \( p \to - (3/4) [\hat{A}_1^4 / (4 \hat{A}_4)]^{1/3} \), i.e. the perfect fluid has asymptotically the vacuum-like equation of state \( p = -\varepsilon \). If \( \hat{C} \equiv 0 \), then the perfect fluid has this equation of state for all time.

From the first Friedmann equation (3.10), we get the Hubble parameter

\[
H^2 = H_0^2 \left\{ \left[ \beta \left( \frac{a_0}{a} \right)^3 + \gamma \right]^{4/3} + \Omega_M \left( \frac{a_0}{a} \right)^3 + \Omega_\Lambda + \Omega_K \left( \frac{a_0}{a} \right)^2 \right\}, \tag{4.8}
\]

where

\[
\beta = \frac{\hat{C}^3}{4 (a_0^3 \hat{A}_4)^{1/4}} \left( \frac{\kappa c^2}{a_0^3 H_0^3} \right)^{3/4}, \quad \gamma = -\frac{\hat{A}_1 a_0^3}{4 (a_0^3 \hat{A}_4)^{1/4}} \left( \frac{\kappa c^2}{a_0^3 H_0^3} \right)^{3/4}. \tag{4.9}
\]

We use Eq. (4.8) to find the age of the Universe:

\[
-\tilde{t}_0 = \int_1^0 \frac{\tilde{a} d\tilde{a}}{\sqrt{\left( \beta + \gamma \tilde{a}^3 \right)^{4/3} + \Omega_M \tilde{a} + \Omega_\Lambda \tilde{a}^4 + \Omega_K \tilde{a}^2}}. \tag{4.10}
\]
From the second Friedmann equation (3.14), the deceleration parameter is

$$-q = \left( \frac{H_0}{H} \right)^2 \left\{ 2\gamma \left[ \beta \left( \frac{a_0}{a} \right)^3 + \gamma \right]^{1/3} - \left[ \beta \left( \frac{a_0}{a} \right)^3 + \gamma \right]^{4/3} - \frac{1}{2} \frac{\Omega_M}{\Lambda} \left( \frac{a_0}{a} \right)^3 + \Omega_\Lambda \right\} \right. \} . (4.11)

At the present time $t_0$, Eqs. (4.8) and (4.10) read

$$1 = (\beta + \gamma)^{4/3} + \Omega_M + \Omega_\Lambda + \Omega_K , \quad \text{(4.12)}$$

$$-q_0 = 2\gamma (\beta + \gamma)^{1/3} - (\beta + \gamma)^{4/3} - \frac{1}{2} \Omega_M + \Omega_\Lambda. \quad \text{(4.13)}$$

The latter equation indicates that the acceleration of the Universe (i.e. $-q_0 > 0$) in this model can originate from the first and the last terms on the right-hand side of this equation. Up to now, the nature of the cosmological constant is still unclear. So, we try to explain the acceleration without it, i.e. we suppose that $\Omega_\Lambda = 0$. It is also well known that our Universe is very flat [28, 29]. Hence, we put $\Omega_K = 0$.

Let us consider two particular cases. The first one corresponds to the choice $\beta = 0$. In this case, Eqs. (4.12) and (4.13) take the form

$$1 = \Omega_M + \Omega_{\Lambda,\text{qgp}}, \quad \text{(4.14)}$$

$$-q_0 = -\frac{1}{2} \Omega_M + \Omega_{\Lambda,\text{qgp}}, \quad \text{(4.15)}$$

where $\Omega_{\Lambda,\text{qgp}} \equiv \gamma^{4/3}$. These equations reproduce exactly the standard ΛCDM model. Therefore, if we take $-q_0 \approx 0.595$ and $\Omega_M \approx 0.27$, then we get $\Omega_{\Lambda,\text{qgp}} \approx 0.73$ and for the age of the Universe $t_0 \approx 1 \Rightarrow t_0 \approx H_0^{-1} \sim 13.7 \times 10^9$ yr.

The second case with $\beta \neq 0$ is a bit more complicated, but also more interesting. Because the nature of dark matter is unclear and, according to the observations [28, 29], the visible matter has $\Omega_M \approx 0.04$, we shall take this value as a total contribution of matter. The other experimental restriction follows from the age of globular clusters which is $11 \div 16$ Gyr [30]. Therefore, the Universe cannot be younger. Now, we shall demonstrate that in our model we can satisfy this limitation if we suppose only $\Omega_M \approx 0.04$ and $-q_0 \approx 0.595$ (as in the ΛCDM model).

From Eqs. (4.12) and (4.13) (where $\Omega_\Lambda = \Omega_K = 0$), we express the parameters $\beta$ and $\gamma$ via $\Omega_M$ and $q_0$:

$$\beta = \frac{2 - 3\Omega_M + 2q_0}{4(1 - \Omega_M)^{1/4}}, \quad \gamma = \frac{2 - \Omega_M - 2q_0}{4(1 - \Omega_M)^{1/4}}. \quad \text{(4.16)}$$

Then, for $\Omega_M \approx 0.04$ and $-q_0 \approx 0.595$, we get $\beta \approx 0.174$ and $\gamma \approx 0.796$. For these values of $\beta$ and $\gamma$, the age of the Universe is $t_0 \approx 0.892 \Rightarrow t \approx 12.2$ yr, in agreement with the experimental data. Roughly speaking, the parameter $\gamma$ is responsible for the accelerated expansion of the Universe, and the parameter $\beta$ plays the role of dark substance.

### 4.2 Model II

Let us consider now the case $\hat{A}_1 = 0$, $\hat{A}_2 = 0$, $\hat{A}_3 = 0$. The coefficients $\hat{A}_0$ and $\hat{A}_4$ satisfy the condition similar to (3.26):

$$\frac{\hat{A}_0}{\hat{A}_4} = \frac{\bar{A}_0}{\bar{A}_4} = -0.8114 \ T_c^4 , \quad \text{(4.17)}$$

where $\hat{A}_0 < 0$ and $\hat{A}_4 > 0$, and they have dimensions erg × cm$^{-3}$ and erg$^{-3}$cm$^{-3}$, respectively.
Integrating (4.3), we get

\[ T = \frac{C}{a}, \quad (4.18) \]

where \( C \geq 0 \) is the constant of integration which is defined by the temperature \( T_0 \) and the scale factor \( a_0 \) at the present time: \( C = a_0 T_0 \). Therefore, the pressure and energy density depend on the scale factor as follows:

\[ \varepsilon = -A_0 + 3\hat{A}_4 \left( \frac{C}{a} \right)^4, \quad p = \hat{A}_0 + \hat{A}_4 \left( \frac{C}{a} \right)^4. \quad (4.19) \]

Hence, such perfect fluid can be formally considered as a mixture of vacuum and radiation. This conclusion is also confirmed by the form of the Friedmann equations for this model:

\[ H^2 = H_0^2 \left[ \beta \left( \frac{a_0}{a} \right)^4 + \gamma + \Omega_M \left( \frac{a_0}{a} \right)^3 + \Omega_\Lambda + \Omega_K \left( \frac{a_0}{a} \right)^2 \right], \quad (4.20) \]

\[ -q = \left( \frac{H_0}{H} \right)^2 \left[ \gamma - \beta \left( \frac{a_0}{a} \right)^4 - \frac{1}{2} \Omega_M \left( \frac{a_0}{a} \right)^3 + \Omega_\Lambda \right], \quad (4.21) \]

where

\[ \beta = \frac{\hat{A}_4 C}{a_0} \left( \frac{\kappa c^2}{a_0^3 H_0^2} \right), \quad \gamma = - \frac{\hat{A}_0 a_0^3}{3} \left( \frac{\kappa c^2}{a_0^3 H_0^2} \right). \quad (4.22) \]

For the age of the Universe we have

\[ -\tilde{t}_0 = \int_1^0 \frac{\tilde{a} d\tilde{a}}{\sqrt{\beta + \gamma \tilde{a}^4 + \Omega_M \tilde{a} + \Omega_\Lambda \tilde{a}^4 + \Omega_K \tilde{a}^2}}. \quad (4.23) \]

Obviously, the parameter \( \gamma \) plays the role of the cosmological constant. So, we may omit \( \Omega_\Lambda \) in above equations. According to the observations, we may also put \( \Omega_K = 0 \) because of its smallness. We restore exactly the \( \Lambda \)CDM model, e.g., with the choice \( \beta = 0 \) (then, \( \gamma \approx 0.73 \)). Therefore, in this model the cosmological constant arises due to QGP. Let us estimate the fraction of QGP which should remain after the phase transition to get the observable acceleration. It is clear that parameters \( \hat{A}_0 \) and \( \hat{A}_0 \) play the role of the vacuum energy density before and after the phase transition, respectively: \( \varepsilon_{V, in} = -\hat{A}_0, \varepsilon_{V, fin} = -\hat{A}_0 \). The initial vacuum energy density \( \varepsilon_{V, in} = -\hat{A}_0 \approx 1.42 T_4^4 \sim 2 \times 10^9 \text{MeV}^4 \) [22]. For \( \gamma \approx 0.73 \) from (4.22) we get \( \varepsilon_{V, fin} = -\hat{A}_0 \sim 6 \times 10^{-9} \text{erg} \times \text{cm}^{-3} \sim 3 \times 10^{-35} \text{MeV}^4 \). Thus, \( \varepsilon_{V, fin}/\varepsilon_{V, in} \sim 10^{-44} \).

5 Conclusions

Our paper was devoted to two great challenges of modern cosmology and high energy physics dubbed dark matter and dark energy. Up to now, there is no satisfactory explanation for both of them. In our paper, we proposed a possible solution to these problems. For this purpose, we considered the expansion of the present Universe, using the "hot", i.e. the quark-gluon branch of the bag EoS. Although we made reference to the role of this type of the EoS during the early universe, namely its inflation phase, here we postponed possible speculations about the continuous evolution of the universe, within the present formalism, from it early, quark-gluon stage to the present days, admitting only the possible continuity
in the existence in the present Universe of a small fraction of colored objects – quarks and gluons – which escaped hadronization. We considered two different scenarios.

In the first scenario, we supposed that the colored objects survived in the form of isolated islands, called quark-gluon nuggets, in a sea of a hadronic phase. In the second scenario, we assumed that a very small fraction of colored objects – quarks and gluons – survived after the phase transition in the form of a perfect fluid, called the QGP-like perfect fluid, uniformly spread within the colorless hadronic medium. Obviously, these cosmological scenarios are defined by EoS of quark gluon plasma (QGP). We focus on two possible Eqs. (2.4) and (2.5) dubbed Model I and Model II, respectively. We have shown that within considered scenarios, there are no fundamental differences in the obtained conclusions for these models.

For the nugget-scenario, we have shown that weekly interacting (with visible matter) QNs can play the role of dark matter for both of the models. In the case of QGP-like fluid, we have demonstrated that this fluid can play the role of dark energy providing the late-time accelerating expansion of the Universe for both of the models. Moreover, we defined that, to be in agreement with observations, only $10^{-44}$ part of the colored objects should survive after the phase transition. Therefore, the considered scenarios provide new possible ways of solving the problems of dark matter and dark energy.

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