Abstract

The $Sp(2)$-gauge fixing of $N = 1$ super-Yang-Mills theory is considered here. We thereby apply the triplectic scheme, where two classes of gauge-fixing bosons are introduced. The first one depends only on the gauge field, whereas the second boson depends on this gauge field and also on a pair of Majorana fermions. In this sense, we build up the BRST extended (BRST plus antiBRST) algebras for the model, for which the nilpotency relations, $s_1^2 = s_2^2 = s_1s_2 + s_2s_1 = 0$, hold.

PACS: 11.15 , 03.70
1 Introduction

In globally supersymmetric gauge theories, gauge choices may be adopted that break supersymmetry; that is the case, for example, of the Wess-Zumino gauge choice. This sort of breaking is not spontaneous. Indeed, supersymmetry becomes nonmanifest and, in some cases, it may jeopardize the quantization program, for nonphysical states may appear in the spectrum. Such an issue may be very systematically treated by means of the Becchi-Rouet-Stora-Tyutin- (BRST-) extended quantization procedure by Batalin and Marnelius, also referred to as the triplectic scheme \([1, 2, 3]\).

The triplectic scheme is a general covariant \(Sp(2)\)-symmetric Lagrangian gauge theory quantization procedure that follows the general procedure of the field-antifield or Batalin-Vilkovisky (BV) method\([4, 5]\); however, it relies on with the additional requirement of an extended BRST\([6, 7, 8]\) (BRST plus anti-BRST) invariance, rather than just BRST symmetry. In the usual BV quantization, the BRST invariance is translated into the so-called master equation. At zero-loop order, this equation is well-defined and its solution, together with the appropriate requirements corresponding to the gauge fixing, leads to the construction of the complete structure of ghosts, antighosts, ghosts for ghosts, etc\([9, 10]\). At higher orders in \(\hbar\), one needs however to introduce some regularization procedure in order to give a well-defined meaning to the mathematical objects involved in the formal master equation. Anomalies and Wess Zumino terms may in this way be calculated at one-loop order\([11, 12]\).

In the triplectic quantization, the extended BRST invariance is expressed through a set of two master equations that correspond to the requirements of BRST and anti-BRST invariances, respectively. As in the standard BV case, both equations formally admit a loop expansion. One then expects that anomalies and Wess Zumino terms should show up at one-loop order, as long as one is able to introduce appropriate regularization schemes. These features are not manifest in the recently discussed case of the Yang Mills theory presented in ref\([13]\).

Our proposal in this paper is to discuss the gauge fixing of \(N = 1\)-SYM model by considering the background-field procedure. Firstly, we obtain the complete structure for the BRST extended symmetry (BRST plus anti-BRST). This algebraic structure will come to help us in the development of two classes of gauge-fixing bosons. The first class boson has a dependence only on the gauge field, \(A_\mu\), but the second class boson is a function of \(A_\mu\) and the \(\lambda\) and \(\bar{\lambda}\) Majorana fermions. In both cases, we obtain the gauge-fixed action for the model. We shall also show how to fix the gauge by means of canonical transformations by considering the method of
2 Review

The field-antifield formalism for quantization of general dynamical systems is the most powerful method to treat gauge models. In this way, we start by considering some gauge theory, and enlarge the original field content, $\phi^i$, adding up all the usual gauge-fixing structure: ghosts, antighosts and auxiliary fields associated to the original gauge symmetries. The resulting set will be labeled by $\phi^A$. Then, we associate to each of these fields five new quantities, introducing the sets: $\bar{\phi}^A$, $\phi^*_A$, $\phi^A$, $\pi^1_A$ and $\pi^2_A$. The Grassmannian parities of these fields are: $\epsilon(\phi^A) = \epsilon(\bar{\phi}^A) \equiv \epsilon_A$, $\epsilon(\phi^{*A}) = \epsilon(\pi^{1A}) = \epsilon_A + 1$. The ideas of the extended BRST quantization in the antifield context previously discussed in [15, 16, 17] are brought into a completely anticanonical setting. The extended BRST invariance of the generating functional, defined on this $6n$-dimensional space, is equivalent to the fact that the quantum action, $W$, is a solution of the two master equations:

$$\frac{1}{2} (W, W)^a + V^a W = i\hbar \Delta^a W,$$

(1)

where the indices $a = 1, 2$ correspond respectively to BRST and anti-BRST invariances and the extended form of the antibrackets, triangle and $V$ operators read as below:

$$(F, G)^a \equiv \frac{\delta^r F}{\delta \phi^A} \frac{\delta^l G}{\delta \phi^A} + \frac{\delta^r F}{\delta \phi^{*A}} \frac{\delta^l G}{\delta \phi^{*A}} - \frac{\delta^r F}{\delta \pi^{1A}} \frac{\delta^l G}{\delta \pi^{1A}} - \frac{\delta^r F}{\delta \pi^{2A}} \frac{\delta^l G}{\delta \pi^{2A}}$$

(2)

$$\Delta^a \equiv (-1)^{\epsilon_A} \frac{\delta^l}{\delta \phi^{*A}} \frac{\delta^l}{\delta \phi^{*a}} + (-1)^{\epsilon_A} \frac{\delta^l}{\delta \pi^{1A}} \frac{\delta^l}{\delta \pi^{1a}}$$

(3)

$$V^a = \frac{1}{2} \epsilon^{ab} \left( \phi^{*b} \frac{\delta^r}{\delta \phi^A} - (-1)^{\epsilon_A} \pi^{ab} \frac{\delta^r}{\delta \phi^A} \right).$$

(4)

Here and in the sequel, unless explicitly indicated, we shall be adopting the convention of summing up over repeated indices.

The field-antifield functional integral is defined including also an extra action functional, $X$, with a gauge-fixing status:

$$Z = \int [D\phi][D\phi^*][D\pi][D\bar{\phi}][D\lambda] \exp\{\frac{i}{\hbar}(W + X)\};$$

(5)
this functional is required to satisfy the following master equation

\[ \frac{1}{2}(X, X)^a - V^a X = i\hbar \Delta^a X. \] (6)

Another way of gauge fixing is by means of the canonical transformations rather than including the functional \( X \), was proposed in [13]. In this work, we subsequently apply this method for the gauge fixing of an \((N = 1)\) SYM theory.

For a gauge theory with closed and irreducible algebra, corresponding to a classical action \( S_0[\phi^i] \), a solution for the zero-loop order action, \( S \), is:

\[ S = S_0 + \phi^a s_a \phi^A + \frac{1}{2} \tilde{\phi} A s_2 s_1 \phi^A + \frac{1}{2} \epsilon^{ab} \phi^a \lambda^b \] (7)

where the \( s_a \) represents BRST \((a = 1)\) and anti-BRST \((a = 2)\) transformations of the fields (in other words, for theories with closed algebra, the standard BRST extended algebra associated with the gauge theory). In this article, we shall not be dealing with the generalized BRST transformations of the triplectic formalism[1, 2, 3], but just with standard transformations that do not involve the antifields.

3 Extended BRST Symmetry for \( N = 1 \) - SYM

Traditionally, Lagrangians invariant under supersymmetry and a local gauge symmetry also exhibit spin-\( \frac{1}{2} \) fermions and scalar fields. Here, we are interested in a Lagrangian invariant under the smallest degree of supersymmetry, namely \( N = 1 \), whose multiplet displays the gauge boson \((A_\mu)\) and its physical Majorana fermion partner \((\lambda)\), the gaugino. Let us consider the classical action for this model:

\[ S_0 = -\frac{1}{g^2} 2tr \int d^4x(\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + i\bar{\lambda} \sigma^{\mu} \tilde{D}_\mu \lambda), \] (8)

where \( g \) is the gauge coupling, and we adopt:

\[ D_\mu \cdot = \partial_\mu \cdot - i[A_\mu, \cdot] \]

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu]. \] (9)

The \( N = 1 \) SUSY algebra reads as below:

\[ \delta^{SUSY} A_\mu = i(\bar{\xi} \sigma_\mu \lambda - \bar{\lambda} \sigma_\mu \xi) \]
\[
\delta^{SUSY} \lambda = \sigma^{\mu\nu} \xi F_{\mu\nu}
\]
\[
\delta^{SUSY} \bar{\lambda} = -\bar{\xi} \sigma^{\mu\nu} F_{\mu\nu}
\] (10)

An appropriate form to write down the BRST transformations may be found in [R], and a BRST extended (BRST \(s_1\) plus anti-BRST \(s_2\)) version satisfying the extended nilpotency,

\[
s_1^2 = s_2^2 = s_1 s_2 + s_2 s_1 = 0,
\] (11)

may be written as

\[
s_1 A_\mu = iD_\mu c_1 \\
s_1 \lambda = -[c_1, \lambda] \\
s_1 \bar{\lambda} = -[c_1, \bar{\lambda}] \\
s_1 c_1 = -\frac{1}{2}[c_1, c_1] \\
s_1 c_2 = -B \\
s_1 B = 0;
\]

\[
s_2 A_\mu = iD_\mu c_2 \\
s_2 \lambda = -[c_2, \lambda] \\
s_2 \bar{\lambda} = -[c_2, \bar{\lambda}] \\
s_2 c_1 = B - [c_1, c_2] \\
s_2 c_2 = -\frac{1}{2}[c_2, c_2] \\
s_2 B = [B, c_2];
\] (12)

we take here \(c_1 = c_1^a T^a\), \(c_2 = c_2^a T^a\) and \(B = B^a T^a\) for ghosts, antighosts and auxiliary fields. Also, we adopt the normalization condition \(Tr(T^a T^b) = \frac{1}{2} \delta^{ab}\).

An important point to consider now is the general form of the gauge-fixing action in the triplectic scheme [1]. The method consists in the construction of a non-degenerate (gauge-fixed) action that belongs to the same cohomological class as the classical action; consequently, it describes the same physical observable [9]. The gauge fixing with the BRST extended invariance has been analyzed in many papers [6, 8, 13, 14, 15, 17]. A most general form of the BRST extended gauge-fixed action has been analyzed in Hamiltonian framework, in [1]. In this paper, we claim that the gauge-fixed action does not have the general form \(S_{GF} = S_0 + s_2 s_1 \chi\). We
will however be interested just in the $Sp(2)$-symmetric case described in Section 2, for which this results hold.

Let us consider the triplectic functional of (5) with a suitable gauge-fixing functional, $X$, solution to the eq.(6). After integrating over $\bar{\phi}^A, \phi^*_A, \pi^a_A$, the conclusive result will be the exponential of $\frac{1}{\hbar}$ times a (non-degenerated) gauge-fixed action of the form,

$$S_{GF} = S_0[\phi^i] + s_2s_1\chi.[\Phi^A].$$

(13)

This action is trivially BRST-extended invariant. The precise relation between the bosonic functional, $\chi$, and the triplectic gauge fixing action, $X$, of (6) is not relevant for our purposes here. The relevant question in our study is to obtain the gauge-fixed action for the $N = 1$ supersymmetric Yang-Mills model by using this framework of the background. In this way, some questions may appear: How to construct the gauge fixed-boson? What is its functional dependence?

The heart of our analysis is that, for the gauge-fixing action (13), the SUSY charge density, $J^0$, is

$$J^0 = J^0_{Naive} + BRST \text{ extended exact},$$

(14)

if

$$\delta^{SUSY}s_2s_1\chi = s_2s_1\delta\chi,$$

(15)

where $\delta$ is a supersymmetry transformation and $s_2s_1$ is the BRST-extended piece. A recent paper, [18], shows how to gauge-fix this model in the BRST framework; for this purpose, the authors use the well-known fermion, $\Psi = c_2\partial_\mu A^\mu$.

Our purpose is to obtain a bosonic functional of $A^\mu, \lambda$ and $\bar{\lambda}$ fields. We can easily show that SUSY and BRST extended symmetries commute with these quantities, by considering the algebras (3) and (12), where

$$\delta^{SUSY}s_1s_2A_\mu = i[B, \bar{\xi}\sigma_\mu\lambda - \bar{\lambda}\sigma_\mu\xi] + [[\bar{\xi}\sigma_\mu\lambda - \bar{\lambda}\sigma_\mu\xi, c_1], c_2]$$

$$\delta^{SUSY}s_1s_2\lambda = -\sigma^{\mu\nu}\xi[F_{\mu\nu}, B] + \sigma^{\mu\nu}\xi[F_{\mu\nu}, c_1]c_2 + \sigma^{\mu\nu}c_2[F_{\mu\nu}, c_1]$$

$$\delta^{SUSY}s_1s_2\bar{\lambda} = \bar{\xi}\sigma^{\mu\nu}[F_{\mu\nu}, B] - \bar{\xi}\sigma^{\mu\nu}c_2[F_{\mu\nu}, c_1] - \xi\sigma^{\mu\nu}[F_{\mu\nu}, c_1]c_2.$$  

(16)

These identities allow us to write a gauge-fixing boson in two different cases.

(I) Let us start the first and particular case for the boson that involves only the field $A_\mu$: 

$$\chi = -\frac{1}{2}A_\mu A^\mu$$

(17)
In the second and most general case, the boson is a functional of the $A_\mu$, $\lambda$, and $\bar{\lambda}$ fields:

$$\chi = -\frac{1}{2} A_\mu A^\mu - \frac{1}{2} \lambda \bar{\lambda}$$  

(18)

In both cases, we find the usual gauge-fixed action for this model:

$$s_1 s_2 \chi = tr(\partial_\mu BA^\mu + i \partial_\mu c_2 D^\mu c_1).$$  

(19)

Another interesting way to get the gauge-fixed action is to perform canonical transformations in the triplectic space [13]. For each of the antibrackets of (2), we introduce a generator of transformations $F_a [\phi^A, \bar{\phi}^A, \phi^{*a}, \pi^a]$ and write down the set of transformations:

$$\phi^{A'} = \frac{\delta F_a}{\delta \phi^A},$$

$$\phi^{*a} = \frac{\delta F_a}{\delta \phi^A},$$

$$\bar{\phi}^{A'} = \frac{\delta F_a}{\delta \pi^A},$$

$$\pi^a = \frac{\delta F_a}{\delta \phi^A},$$

(20)

where their general form is

$$F_a = 1_a + f_a$$

(21)

with

$$1_a = \phi^A \phi^{*a} + \bar{\phi}^{A'} \pi^a$$

$$f_1 = g_1[\phi, \bar{\phi}] + g_3^A[\phi, \bar{\phi}]\pi_A^1 + g_4^A[\phi, \bar{\phi}]\phi^{*1}$$

$$f_2 = g_2[\phi, \bar{\phi}] + g_3^A[\phi, \bar{\phi}]\pi_A^2 + g_4^A[\phi, \bar{\phi}]\phi^{*2}.$$  

(22)

In this approach, the fundamental condition for the canonical transformation reproduces the gauge-fixing corresponding to some boson $\chi$, after we express the result in terms of the transformed fields and impose the condition that $\bar{\phi}^{A'}, \phi^{*a}$ and $\pi^a'$ are set to zero:

$$\frac{\delta f'_a}{\delta \phi^A} s_a \phi^A + \frac{1}{2} g_3^a s_2 s_1 \phi^A - \frac{1}{2} \epsilon^{ab} \frac{\delta f'_a}{\delta \phi^A} \frac{\delta f'_b}{\delta \phi^A} = s_2 s_1 \chi.$$  

(23)
For the present case, we may to consider two possibilities

(I)

\begin{align*}
g_1 & = g_3 = g_4 = 0 \\
g_2 & = s_1 \chi_i \\
f_2 & = s_1 \chi_i
\end{align*}

or

(II)

\begin{align*}
g_2 & = g_3 = g_4 = 0 \\
g_1 & = -s_2 \chi_i \\
f_1 & = -s_2 \chi_i
\end{align*}

where $i = 1, 2$ refers to the two classes of bosons \([17]\) and \([18]\). In both cases, we get the gauge-fixed action, whenever we perform the corresponding canonical transformation in the fields and then set all the primed antifields to zero.

4 Concluding Remarks

We have shown that it is possible to obtain the gauge-fixed action for an $N = 1$ -- Supersymmetric Yang-Mills model with BRST extended invariance. To carry out such a programme, we consider two paths for the gauge-fixing. In the first one, we adopt the triplectic scheme where, for a particular solution to the gauge-fixing action $X$, two kinds of bosons are considered; in both cases, we obtain the correct form of the gauge-fixed action. In the second way, we perform the gauge-fixing process with the help of canonical transformations by means of a particular choice for the generator of the transformations.
5 Acknowledgments

The authors would like to express their deepest gratitude to J. A. Helaïel-Neto for a careful reading and suggestions on our original manuscript. CNPq-Brazil is acknowledged for the invaluable financial help.
References

[1] I. A. Batalin and R. Marnelius, Phys. Lett. B350 (1995) 44.
[2] I. A. Batalin, R. Marnelius and A. M. Semikhtov, Nucl. Phys. B446 (1995) 249.
[3] I. Batalin and R. Marnelius, Nucl. Phys. B465 (1996) 521.
[4] I. A. Batalin and G. A. Vilkovisky, Phys. Lett. B102 (1981) 27.
[5] I. A. Batalin and G. A. Vilkovisky, Phys. Rev. D28 (1983) 2567.
[6] P. Grégoire and M. Henneaux, Phys. Lett. B277 (1992) 459.
[7] P. Grégoire and M. Henneaux, Comm. Math. Phys. 157 (1993) 279.
[8] P. Grégoire and M. Henneaux, J. Phys. A 26 (1993) 6073.
[9] M. Henneaux and C. Teitelboim, “Quantization of Gauge Systems”, Princeton University Press, 1992, Princeton, New Jersey.
[10] M. Henneaux, “Lectures on the Antifield-BRST Formalism for Gauge Theories”, Nucl. Phys. B - Proc. Suppl. 18A (1990) 47.
[11] W. Troost, P. van Nieuwenhuizen and A. Van Proeyen, Nucl. Phys. B333 (1990) 727.
[12] F. De Jonghe, “The Batalin-Vilkovisky Lagrangian Quantization scheme with applications to the study of anomalies in gauge theories”, Ph.D. thesis K.U. Leuven, hep-th 9403143.
[13] E.M.C. Abreu, N.R.F. Braga and C.F.L. Godinho, Nucl. Phys. B524(1998)779.
[14] Nelson R. F. Braga and Cresus F. L. Godinho Phys. Rev. D61 (2000)125003
[15] I. A. Batalin, P. M. Lavrov and I. V. Tyutin, J. Math. Phys. 31 (1990) 1487.
[16] I. A. Batalin, P. M. Lavrov and I. V. Tyutin, J. Math. Phys.32 (1991) 532.
[17] I. A. Batalin, P. M. Lavrov and I. V. Tyutin, J. Math. Phys.32 (1991) 2513.
[18] Kazuo Fujikawa and Kazumi Okuyama, Nucl.Phys. B521(1998)401