On cosmic rotation*

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Abstract

We overview our recent studies of cosmological models with expansion and global rotation. Problems of the early rotating models are discussed, and the class of new viable cosmologies is described in detail. Particular attention is paid to the observational effects of the cosmic rotation.

1. INTRODUCTION

This paper is dedicated to the memory of Professor Dmitri D. Ivanenko who was deeply interested in the problem of universal rotation and made essential contributions to this subject. For the first time attention to cosmological models with rotation was drawn in 1946 by George Gamov [1] (although we should mention also the earlier work of Lanczos [2]). Soon after this, K. Gödel [3] had suggested to describe cosmic rotation with the help of a spacetime metric of the form

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\[ ds^2 = a^2(dt^2 - 2e^x dt dy + \frac{1}{2} e^{2x} dy^2 - dx^2 - dz^2). \]  

(1.1)

Matter in this model is dust with the energy density \( \varepsilon \), and the cosmological constant \( \Lambda \) is nontrivial and negative (i.e. its sign is opposite to that introduced by Einstein). The angular velocity \( \omega \) of the cosmic rotation in (1.1) is given by \( \omega^2 = \frac{1}{2\omega^2} = 4\pi G \varepsilon = -\Lambda \). For many years this model became a theoretical “laboratory” for the study of rotating cosmologies. As compared to the Gödel’s world, the model suggested earlier by Lanczos appears to be less physical in that it describes a universe as a rigidly rotating dust cylinder of infinite radius. Dust density in this solution (later rederived by van Stockum [4]) diverges at radial infinity.

Using the Gödel model one can clearly understand the idea of the cosmic rotation of matter in the universe [3]: let us consider a particle with the initial velocity (in the comoving coordinates) \( \left\{ \dot{t} = 1, \dot{x} = \beta, \dot{y} = 0, \dot{z} = 0 \right\} \) which starts moving from \( \left\{ t = x = y = z = 0 \right\} \). Straightforward analysis shows that such a particle deviates from the initial \( x \)-axis direction in the rotating metric.

We have no intention of giving a complete review of all the cosmological models with rotation. Instead we present here our understanding of the main problems of the cosmic rotation, and explain how, in our opinion, these problems can be solved.

\section*{2. PROBLEMS OF COSMIC ROTATION}

It is worthwhile to notice that along with the constant deep interest to the rotating cosmologies, historical development revealed several problems which were considered by the majority of relativists as the arguments against the models with nontrivial cosmic rotation. It seems useful to list them here. In the next sections we will demonstrate that it is possible to solve all these problems within the framework of wide class of viable rotating cosmological models.
A. First problem: causality

Gödel himself proved the existence of closed time-like curves in the metric (1.1). This was immediately recognized as an unphysical property because it violates the causal structure of space-time. Considerable efforts were thus focused on deriving completely causal rotating cosmologies. In his last work devoted to rotation, Gödel without proof mentions the possibility of positive solution of the causality problem [6]. First explicit solutions were reported later [7,8]. Maitra [7] formulated a simple criterium for the existence of closed time-like curves in rotating metrics.

B. Second problem: expansion

Apparent expansion of the universe is usually related to the fact of the red shift in the spectra of distant galaxies, and thus all the standard cosmological models are necessarily non-stationary. However, it was immediately noticed that it is impossible to combine pure rotation and expansion in a solution of the general relativity field equations for a simple physical matter source. Some solutions are known [9–20] which describe certain stages of the universe’s evolution, but the complete cosmological scenario for a rotating world was not available.

C. Third problem: microwave background radiation

Discovery of the microwave background radiation has revealed the remarkable fact that its temperature distribution is isotropic to a very high degree. This fact was for a long time considered as a serious argument for isotropic cosmological models and was used for obtaining estimates on the possible anisotropies which could take place on the early stages of the universe’s evolution. In particular, homogeneous anisotropic rotating cosmologies were analysed in [21–23], and strong upper limits on the value of the cosmic rotation were
obtained. Plainly speaking, these numerical estimates did not leave any chance for rotation to be a significant factor in cosmology.

**D. Fourth problem: observations**

The last but not least problem is the lack of direct observational evidence for the cosmic rotation. Attempting at its experimental discovery one should study possible systematic irregularities in angular (ideally, over the whole celestial sphere) distributions of visible physical properties of sources located at cosmological distances. Unfortunately, although a lot of data is already potentially accumulated in various astrophysical catalogues, no complete analysis was made in search of the global cosmic rotation. Partially this was explained by the insufficient theoretical study of observational cosmology with rotation. To our knowledge, till the recent time there were only few theoretical predictions concerning the possible manifestations of the cosmic rotation, mainly these were the above mentioned estimates of MBR anisotropies [21–23] (also of the X-ray background anisotropies [24]) and the number counts tests analyses [2,6,25–27]. Few purely empirical studies (without constructing general relativistic models) of the angular distributions of astrophysical data are available which interpreted the observed systematic irregularities as the possible effects of rotation, [28–32].

**3. CLASS OF SHEAR-FREE COSMOLOGICAL MODELS WITH ROTATION AND EXPANSION**

In [33,34] we considered a wide class of viable cosmological models with expansion and rotation. Let us describe it here briefly. Denoting \( x^0 = t \) as the cosmological time and \( x^i, i = 1, 2, 3 \) as three spatial coordinates, we write the space-time interval in the form

\[
d s^2 = d t^2 - 2 R n_i d x^i d t - R^2 \gamma_{ij} d x^i d x^j,
\]

where \( R = R(t) \) is the scale factor, and
\[ n_i = \nu_a e_i^{(a)}, \quad \gamma_{ij} = \beta_{ab} e_i^{(a)} e_j^{(b)}. \quad (3.2) \]

Here \((a, b = 1, 2, 3) \nu_a, \beta_{ab}\) are constant coefficients, while \[ e^{(a)} = e_i^{(a)}(x) dx^i \quad (3.3) \]
are the invariant 1-forms with respect to the action of a three-parameter group of motion which is admitted by the space-time \((3.1)\). We assume that this group acts simply-transitively on the spatial \((t = \text{const})\) hypersurfaces. It is well known that there exist 9 types of such manifolds, classified according to the Killing vectors \(\xi^{(a)}\) and their commutators \([\xi^{(a)}, \xi^{(b)}] = C^{c}_{ab} \xi^{(c)}\). Invariant forms \((3.3)\) solve the Lie equations \(L_{\xi^{(b)}} e^{(a)} = 0\) for each Bianchi type, so that models \((3.1)\) are spatially homogeneous.

Kinematical characteristics of \((3.1)\) are as follows: volume expansion is
\[ \dot{\vartheta} = 3 \frac{\dot{R}}{R}, \quad (3.4) \]
nontrivial components of vorticity tensor are
\[ \omega_{ij} = -\frac{R}{2} \hat{C}^{k}_{ij} n_k, \quad (3.5) \]
and shear tensor is trivial,
\[ \sigma_{\mu\nu} = 0. \quad (3.6) \]

Hereafter the dot (\(\dot{}\)) denotes derivative with respect to the cosmological time coordinate \(t\). Tensor \(\hat{C}^{k}_{ij} = e^{k}_{(a)}(\partial_i e^{(a)}_j - \partial_j e^{(a)}_i)\) is the anholonomity object for the triad \((3.3)\); for I-VII Bianchi types values of its components numerically coincide with the corresponding structure constants \(C^a_{bc}\). The list of explicit expressions for \(\xi^{(a)}, e^{(a)}, C^a_{bc}, \hat{C}^{k}_{ij}\) for any Bianchi type is given in \([33]\).

We choose the constant matrix \(\beta_{ab}\) in \((3.2)\) to be positive definite. This important condition generalises results of Maitra \([7]\), and ensures the absence of closed time-like curves.

One can immediately see that space-times \((3.1)\) admit, besides tree Killing vector fields \(\xi^{(a)}\), a nontrivial conformal Killing vector.
\[ \xi_{\text{conf}} = R \partial_t. \]  

(3.7)

All models in the class (3.1) have a number of common remarkable properties:

- Space-time manifolds are spatially homogeneous and completely causal.
- MBR is totally isotropic for any moment of \( t \).
- Rotation (3.3) does not produce parallax effects.

These properties were proved in [33,34], and here we only remark that causality is provided by the positivity of \( \beta_{ab} \) (one can immediately write the original Gödel metric (1.1) in the form (3.1) and check that its \( \beta_{ab} \) matrix is not positive definite), while isotropy of MBR and absence of parallax effects are related to the existence of the conformal Killing vector (3.7). Hence, the most strong limits on the cosmic rotation, obtained earlier from the study of MBR [21–23] and of the parallaxes in rotating world [36,37], are not true for this class of cosmologies.

Summarizing, cosmological models with rotation and expansion (3.1) solve the first three problems of cosmic rotation. This class of metrics is rich enough, as it contains all kinds of worlds: open and closed with different topologies.

4. Gödel-type cosmological model

To the end of this paper we will consider now the natural non-stationary generalization of the original Gödel metric (1.1) which has drawn considerable attention in the literature. This generalized model is described by the interval

\[ ds^2 = dt^2 - 2\sqrt{\sigma}R(t)e^{mx}dt dy - R^2(t)(dx^2 + ke^{2mx}dy^2 + dz^2), \]

where we denoted \( x^1 = x, x^2 = y, x^3 = z \), and \( m, \sigma, k > 0 \) are constant parameters. The condition \( k > 0 \) guarantees the absence of closed time-like curves. The metric (1.1) is usually called the Gödel-type model with rotation and expansion. Coordinate \( z \) gives the direction of the global rotation, the magnitude of which
\[ \omega = \sqrt{\frac{1}{2} \omega_{\mu \nu} \omega^{\mu \nu}} = \frac{m}{2R} \sqrt{\frac{\sigma}{k+\sigma}} \]  \hspace{1cm} (4.2)

decreases in expanding world.

The three Killing vector fields are

\[ \xi_{(1)} = \frac{1}{m} \partial_x - y \partial_y, \quad \xi_{(2)} = \partial_y, \quad \xi_{(3)} = \partial_z. \]  \hspace{1cm} (4.3)

These satisfy commutation relations

\[ [\xi_{(1)}, \xi_{(2)}] = \xi_{(2)}, \quad [\xi_{(1)}, \xi_{(3)}] = [\xi_{(2)}, \xi_{(3)}] = 0, \]  \hspace{1cm} (4.4)

showing that the model (4.1) belongs to the Bianchi type III.

From the point of view of the Petrov classification, one can verify that the Gödel–type model is of the type \( D \).

It is convenient to choose at any point of the space-time (4.1) a local orthonormal (Lorentz) tetrad \( h_\mu^a \) so that, as usual, \( g_{\mu \nu} = h_\mu^a h_\nu^b \eta_{ab} \) with \( \eta_{ab} = \text{diag}(+1, -1, -1, -1) \) the standard Minkowski metric. This choice is not unique, and we will use the gauge in which

\[ \hat{h}_0^0 = 1, \quad \hat{h}_2^2 = -R\sqrt{\sigma} e^{mx}, \quad h_1^1 = h_3^3 = R, \quad \hat{h}_2^2 = R e^{mx} \sqrt{k+\sigma}. \]  \hspace{1cm} (4.5)

Hereafter a caret denotes tetrad indices; Latin alphabet is used for the local Lorentz frames, \( a, b, ... = 0, 1, 2, 3 \).

A. Dynamical realizations

Several dynamical realizations (i.e. construction of exact cosmological solutions for the gravitational field equations) of the Gödel–type metric (4.1) are known. In the Einstein’s general relativity theory such models were described in [38–40], with different matter sources. Rotation, spin and torsion are closely interrelated in the Poincaré gauge theory of gravity [41–45], and hence it is quite natural to study cosmologies with rotation within the gauge gravity framework. General preliminary analysis of the separate stages of the universe’s
evolution was made in our works \cite{16,18}, while in \cite{19,20} complete cosmological scenarios are considered.

Here we shall mention a model \cite{50} in which the gravitational field dynamics is determined by the minimal quadratic Poincaré gauge model. This is perhaps the closest extension of the Einstein’s general relativity theory. The matter source is represented by the Weyssenhoff spinning fluid and the magnetic field. In the analysis of the evolution of the scale factor $R(t)$ it is convenient to distinguish several qualitatively different stages in the history of the universe. First stage is the shortest one and it describes a bounce at $t = 0$. There is no initial singularity due to the dominating spin contribution. Matter is characterized by the approximate equation of state $p \approx \varepsilon$ at this stage. Next comes the second stage when the scale factor increases like $\sqrt{t}$, while the equation of state is of the radiation type, $p \approx \varepsilon/3$. This expansion lasts until the size of the metagalaxy approaches $\approx 10^{27}$ cm. After this the “modern” stage starts with the effectively dust equation of state $p_0 \approx 0$. Scale factor still increases, but the deceleration of expansion takes place. The final stage depends on the value of the cosmological term, and either the future evolution enters the eternal de Sitter type expansion, or expansion ends and a contraction phase starts. The details of this complete scenario \cite{50} depend on the values of the coupling constants which determine the structure of the gravitational Lagrangian. The principal difficulty of this dynamical realization, in our opinion, is presented by the magnitude of magnetic field which at the “modern” stage should be close to the upper limits established for the global magnetic field from astrophysical observations.

5. NULL GEODESICS IN THE GÖDEL-TYPE MODEL

Practically all the information about the structure of the universe and about the properties of astrophysical objects is obtained by an observer in the form of different kinds of electromagnetic radiation. Thus, in order to be able to make theoretical predictions and compare them with observations, it is necessary to know the structure of null geodesics in
the cosmological model with rotation. All the models from the class (3.1) have three Killing vectors and one conformal Killing vector field. Hence the null geodesics equations

\[ k^\mu \nabla_\mu k^\nu = 0, \quad k^\mu k_\mu = 0 \]  \hspace{1cm} (5.1)

(where \( k^\mu = \frac{dx^\mu}{ds} \) is the tangent vector to a curve \( x^\mu(s) \) with an affine parameter \( s \)) have four first integrals,

\[ q_0 = \xi_{\text{conf}}^\mu k_\mu, \quad q_a = -\xi_s^\mu k_\mu, \quad a = 1, 2, 3. \]  \hspace{1cm} (5.2)

Solving (5.2) with respect to \( k^\mu \), one obtains a system of ordinary first order nonlinear equations which can be straightforwardly integrated. Complete solution of the null geodesics equations in the Gödel–type model is given in \cite{51,33}, and here we present only short description of null geodesics in (4.1).

To begin with, let us define convenient parameterization of null geodesics. Without loosing generality (using the spatial homogeneity) we assume that an observer is located at the space-time point \( P = (t = t_0, x = 0, y = 0, z = 0) \). Now, arbitrary geodesics which passes through \( P \) is naturally determined by its initial direction in the local Lorentz frame of observer at this point. In the tetrad (4.3) we may put

\[ k_a^\mu = (h_a^\mu k^\mu)_P = (1, \sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta), \]  \hspace{1cm} (5.3)

where \( \theta, \phi \) are standard spherical angles parameterizing the celestial sphere of an observer.

Then from (3.7), (4.3), (5.3) and (5.2) one finds the values of the integration constants

\[ q_0 = R_0, \]
\[ q_1 = \frac{R_0}{m} \sin \theta \cos \phi, \]
\[ q_2 = R_0(\sqrt{\sigma} + \sqrt{k + \sigma} \sin \theta \sin \phi), \]  \hspace{1cm} (5.4)
\[ q_3 = R_0 \cos \theta. \]

Generic null geodesics, initial directions of which satisfy \( (\sin \theta \sin \phi + \sqrt{\sigma}) \neq 0 \), are then described by
\[ e^{-mx} = \frac{\sqrt{\sigma + \sqrt{k + \sigma}} \sin \theta \sin \Phi}{\sqrt{\sigma + \sqrt{k + \sigma}} \sin \theta \sin \phi}, \quad (5.5) \]

\[ y = \frac{\sin \theta (\cos \Phi - \cos \phi)}{m(\sqrt{\sigma + \sqrt{k + \sigma}} \sin \theta \sin \phi)}, \quad (5.6) \]

\[ z = \left( \frac{k + \sigma}{k} \right) \cos \theta \left[ \int_{t_0}^{t} \frac{dt'}{R(t')} + \sqrt{\frac{\sigma}{k + \sigma}} \left( \frac{\Phi - \phi}{m} \right) \right], \quad (5.7) \]

where the function \( \Phi(t) \) satisfies the differential equation

\[ \frac{d\Phi}{dt} = -m \left( \frac{\sqrt{\frac{\sigma}{k + \sigma}} + \sin \theta \sin \Phi}{1 + \sqrt{\frac{\sigma}{k + \sigma}} \sin \theta \sin \Phi} \right), \quad (5.8) \]

with initial condition \( \Phi(t_0) = \phi \).

For a detailed discussion of rays which lie on the initial cone \( \sin \theta \sin \phi + \sqrt{\frac{\sigma}{k + \sigma}} = 0 \) and different subcases of (5.5)-(5.8) see [33].

6. OBSERVATIONS IN ROTATING COSMOLOGIES

The qualitative picture of specific rotational effects which could be observed in the Gödel–type model (4.1) is in fact independent of the dynamical behavior of the scale factor \( R(t) \). To some extent the same is true also for the quantitative estimates, especially if one uses the Kristian-Sachs formalism [52,53] in which all the physical and geometrical observable quantities are expressed in terms of power series in the affine parameter \( s \) or the red shift \( Z \). In this case the description of observable effects on not too large (although cosmological) scales involves only the modern values (i.e. calculated at the moment of observation \( t = t_0 \)) of the scale factor \( R_0 = R(t_0) \), Hubble parameter \( H_0 = (\dot{R}/R)_P \), rotation value \( \omega_0 = \omega(t_0) \), deceleration parameter \( q_0 = -(\ddot{R}R^2/\dot{R}^2)_P \), etc.

In this section we will discuss possible observational manifestations of the cosmic rotation in the Gödel–type universe. Estimates for the value of vorticity and for the direction of rotation axis can be find from the recent astrophysical data, see sects. 6.2 and 6.3.
A. Classical cosmological tests

Classical cosmological tests, such as apparent magnitude – red shift \((m - Z)\), number counts – red shift \((N - Z)\), angular size – red shift relations, and some other, reveal specific dependence of astrophysical observables on the angular coordinates \((\theta, \phi)\) in a rotating world. Thus a careful analysis of the angular variations of empirical data over the whole celestial sphere is necessary.

The knowledge of null geodesics enables one to obtain the explicit form of the area distance \(r\) between an observer at a point \(P\) and any source \(S\), which is a crucial step in deriving classical cosmological tests. The area distance is defined \([54, 52, 53]\) by

\[
dA_S = r^2 d\Omega_P,
\]

where \(dA_S\) is the intrinsic area of the source which subtends the solid angle \(d\Omega_P\) at \(P\) when observer looks at a source \(S\). In general, \(r\) is thus a function of direction of observation, that is \(r = r(\theta, \phi)\). (Besides this, \(r\) of course depends on the value of the affine parameter \(s\), or equivalently on the moment of \(t\) at which source radiates a ray detected by an observer at \(t_0\)).

Using (5.5)-(5.8), one can find for the area distance along the axis of rotation the following exact result

\[
r^2(t; \theta = 0) = \frac{\sin^2 \left( \int_{t_0}^{t} dt' \omega(t') \right)}{\omega^2(t)}.
\]

In general, rotational effects are always maximal in the directions close to the \(z\) axis, and quite remarkably observations in the direction of rotation can be described by simple and clear formulas. As for an arbitrary direction, exact formulas become very complicated and it is much more convenient to replace them by the Kristian-Sachs expansions. Recall that the red shift \(Z\), which reflects the dependence of frequency on the motion of a source and observer, is defined by

\[
1 + Z = \frac{(k^\mu u_\mu)_S}{(k^\mu u_\mu)_P}.
\]
where \( u^\mu \) is the four–velocity of matter in the universe. In the Kristian-Sachs approach this exact relation is replaced by the expansion \([52,53]\)

\[
1 + Z = 1 + rK^\mu K^\nu (\nabla_\mu u_\nu)_P + \frac{1}{2} r^2 K^\mu K^\nu K^\lambda (\nabla_\mu \nabla_\nu u_\lambda)_P + ..., \tag{6.4}
\]

where

\[
K^\mu = \left( \frac{k^\mu}{k^\nu u_\nu} \right)_P.
\]

One can invert (6.4) and use the resulting expansions in the calculations of observable effects in rotating cosmologies. Now all the angular dependent rotational contributions are contained in the coefficients of these expansions.

For the Gödel–type cosmology \([4.1]\) the classical \((m - Z)\) and \((N - Z)\) relations read as follows.

**Apparent magnitude \(m\) vs. red shift \(Z\):**

\[
m = M - 5 \log_{10} H_0 + 5 \log_{10} Z + \frac{5}{2} (\log_{10} e)(1 - q_0)Z +
-5 \log_{10} \left( \sqrt{\frac{\sigma}{k + \sigma}} \sin \theta \sin \phi \right) +
-\frac{5}{2} (\log_{10} e) \frac{\omega_0 \sin \theta \cos \phi \left( \sqrt{\frac{\sigma}{k + \sigma}} + \sin \theta \sin \phi \right)}{H_0 \left( 1 + \sqrt{\frac{\sigma}{k + \sigma}} \sin \theta \sin \phi \right)^2} Z + O(Z^2). \tag{6.5}
\]

**Number of sources \(N\) vs. red shift \(S\):**

\[
\frac{dN}{d\Omega} = \frac{n_0 Z^3}{3H_0^3 \left( 1 + \sqrt{\frac{\sigma}{k + \sigma}} \sin \theta \sin \phi \right)^3} \left[ 1 - \frac{3}{2} (1 + q_0)Z - 
-3 \frac{\omega_0 \sin \theta \cos \phi \left( \sqrt{\frac{\sigma}{k + \sigma}} + \sin \theta \sin \phi \right)}{H_0 \left( 1 + \sqrt{\frac{\sigma}{k + \sigma}} \sin \theta \sin \phi \right)^2} Z + O(Z^2) \right]. \tag{6.6}
\]

In \(6.5)-(6.6)\) \(M = -\frac{5}{2} \log_{10} L_S\) is the absolute magnitude of a source with an intrinsic luminosity \(L_S\) and \(n_0\) is the modern value of number density of sources \(n = n(t)\) (as usual, \(6.6\) is derived under the assumption of the absence of source evolution).

The \((N - Z)\) relation describes the number of sources observed in a solid angle \(d\Omega\) up to the value \(Z\) of red shift. One can estimate the global difference of the number of sources visible in two hemispheres of the sky, \(N^+, N^-\), by integrating \(6.6\). The result is
It seems worthwhile to draw attention to the absence of a correction proportional to \( Z \) in (6.7). It is difficult to make a comparison of our results with [25–27] as they study stationary rotating models in which there is no red shift.

For some time classical cosmological tests were carefully carried out for standard models, but later it was recognized that evolution of physical properties of sources often dominates over geometrical effects. However, specific angular irregularities predicted in rotating cosmologies, (6.5), (6.6)-(6.7), may revive the importance of the classical tests.

**B. Periodic structure of the universe**

Recent analysis of the large–scale distribution of galaxies [55] has revealed an apparently periodic structure of the number of sources as a function of the red shift. Cosmic rotation may give a natural explanation of this fact [56]. The crucial point is the structure of null geodesics in the Gödel–type model: explicit solutions (5.5)-(5.7) demonstrate a helicoidal behavior of rays in directions close to the rotation axis. This yields a periodicity of the area distance as a function of red shift, and hence the visible distribution of sources turn out to be also approximately periodical in \( Z \).

This effect is most transparent for the direction of rays straight along the axis of rotation. The area distance is then given by (6.2). In order to be able to make some quantitative estimates, let us assume the polynomial law for the scale factor,

\[
R(t) = R_0 \left( \frac{t - t_\infty}{t_0 - t_\infty} \right)^b,
\]

which is naturally arising in a number of cosmological scenarios \((0 < b < 1)\). Then it is straightforward to derive (analogously to (6.6)) the distribution of number of sources per red shift per solid angle,

\[
\frac{dN}{d\Omega dZ} = \frac{n_0}{\omega_0^2 H_0 (1 + Z)^{1/b}} \sin^2 \left( \frac{b\omega_0}{(1 - b)H_0} \left[ (1 + Z)^{(b-1)/b} - 1 \right] \right).
\]

(6.9)
This shows that the apparent distribution of visible sources is an oscillating function of red shift, with slowly decreasing amplitude. Similar generalized formula can be obtained for arbitrary directions, so that (6.9) is modified by additional angular dependence of the magnitude of successive extrema of distribution function.

Observational data [55] give for the distance between maxima the value $128h^{-1}\text{Mps}$ (where $H_0 = 100h\text{km sec}^{-1}\text{Mps}^{-1}$). From this one can estimate the rotation velocity which is necessary to produce such a periodicity effect,

$$\omega_0 \approx 74H_0. \quad (6.10)$$

This result does not depend on $b$.

C. Polarization effect

Cosmic rotation affects polarization of radiation which propagates in (6.1), and this produces a new observable effect which has been already reported in the literature by Birch [28,29]. In the geometrical optics approximation, polarization is described by a space-like vector $f^\mu$ which is orthogonal to the wave vector, $f_\mu k^\mu = 0$, and is parallelly transported along the light ray,

$$k^\mu \nabla_\mu f^\nu = 0. \quad (6.11)$$

Study of (6.11) reveals that the cosmic rotation forces a polarization vector to change its orientation during propagation along the null geodesics. It is clear that this conclusion has physical meaning only when one defines a frame at any point of the ray with respect to which polarization rotates. Let us describe how this can be achieved.

As it is well known, gravitational field affects the properties of an image of a source, such as shape, size and orientation [57,58]. Like the rotation of polarization vector, deformation and rotation of image depend on local coordinates and on the choice of an observer’s frame of reference. However, one can consider the combination of two problems, and this gives
rise to a truly observable effect which is coordinate and frame independent. Putting it in another way, one should calculate the influence of the cosmic rotation on the relative angle $\eta$ between the polarization vector and the direction of a major axis of an image. [This problem is discussed in the recent paper [60], but it is incorrect, in our opinion].

Most conveniently this can be done within the framework of the Newman-Penrose spin coefficient formalism. Namely, it is enough to construct a null frame $\{l, n, m, m\}$ is such a way that $l$ coincides with the wave vector $k$, and the rest of the vectors are covariantly constant along $l$. Then we can consider $m$ as a polarization vector, and thus deformation of an image of a source, calculated with respect to this frame $\{l, n, m, m\}$, gives at the end the observable relative angle $\eta$. Let us describe explicitly the null frame:

$$l = \frac{R_0}{R} \left[ \left\{ 1 + \sqrt{\frac{\sigma}{k + \sigma}} \sin \theta \sin \Phi \right\} \partial_t + \frac{1}{R} \sin \theta \cos \Phi \partial_x + \frac{e^{-mx}}{R \sqrt{k + \sigma}} \sin \theta \sin \Phi \partial_y + \frac{1}{R} \cos \theta \partial_z \right], \quad (6.12)$$

$$n = \frac{R}{R_0} \left( \frac{1}{1 + \cos \theta} \right) \left[ \partial_t - \frac{1}{R} \partial_z \right], \quad (6.13)$$

$$m = e^{i\Psi} \sqrt{2} \left[ \left\{ \sqrt{\frac{\sigma}{k + \sigma}} + i \left( \frac{\sin \theta}{1 + \cos \theta} \right) e^{-i\Phi} \right\} \partial_t + \frac{i}{R} \partial_x + \frac{e^{-mx}}{R \sqrt{k + \sigma}} \partial_y - \frac{i}{R} \left( \frac{\sin \theta}{1 + \cos \theta} \right) e^{-i\Phi} \partial_z \right], \quad (6.14)$$

$$m = e^{-i\Psi} \sqrt{2} \left[ \left\{ \sqrt{\frac{\sigma}{k + \sigma}} - i \left( \frac{\sin \theta}{1 + \cos \theta} \right) e^{i\Phi} \right\} \partial_t - \frac{i}{R} \partial_x + \frac{e^{-mx}}{R \sqrt{k + \sigma}} \partial_y + \frac{i}{R} \left( \frac{\sin \theta}{1 + \cos \theta} \right) e^{i\Phi} \partial_z \right]. \quad (6.15)$$

Here

$$\Psi(t, z) = z \frac{m}{2} \sqrt{\frac{\sigma}{k + \sigma}} + \Phi(t). \quad (6.16)$$

One should note that (6.12)-(6.15) is a smooth field of frames which cover all the space-time manifold. Direct computation proves that (6.12) is the null geodesics congruence with an
affine parameterization. We say that this congruence is oriented along the direction given by the spherical angles \((\theta, \phi)\) in the local Lorentz frame of an observer at \(P\), because a null geodesics (5.3)-(5.8) belongs to this congruence.

Direct computation of the spin coefficients (we are using definitions of [59], and denote spin coefficients by tildes in order to distinguish them from other quantities in this paper) gives

\[
\tilde{\varepsilon} = 0, \quad \tilde{\kappa} = 0, 
\]

\[
\tilde{\lambda} = 0, \quad \tilde{\nu} = 0, 
\]

\[
\tilde{\rho} = -\frac{R_0}{R^2} \left[ \hat{R} \left\{ 1 + \frac{\sigma}{k + \sigma} \sin \theta \sin \Phi \right\} + \frac{m}{2} \left( \frac{k}{k + \sigma} \right) \frac{\sin \theta \cos \Phi}{(1 + \frac{\sigma}{k + \sigma} \sin \theta \sin \Phi)} \right] + \frac{R_0 m}{R^2} \cos \theta \left( \sqrt{\frac{\sigma}{k + \sigma}} + \sin \theta \sin \Phi \right) \left( 1 + \sqrt{\frac{\sigma}{k + \sigma}} \sin \theta \sin \Phi \right), 
\]

\[
\tilde{\sigma} = \frac{R_0 m}{R^2} \left( \frac{k}{k + \sigma} \right) \frac{e^{2\psi}}{(1 + \sqrt{\frac{\sigma}{k + \sigma}} \sin \theta \sin \Phi)} \left( -\cos \Phi \left[ 2 \cos \theta - 1 - 2 \cos^2 \Phi (\cos \theta - 1) \right] + i \sin \Phi \left[ \cos \theta - 2 \cos^2 \Phi (\cos \theta - 1) \right] \right), 
\]

and we do not write other spin coefficients, because their values are irrelevant. Only one important step is to be done: spin coefficient \(\tilde{\pi} \neq 0\) in the frame (6.12)-(6.15), and we need to make an additional Lorentz transformation

\[
l \rightarrow l, \quad n \rightarrow n + a^* m + a m + a^* a \ l, \quad m \rightarrow m + a l, \quad \overline{m} \rightarrow \overline{m} + a^* l, 
\]

where the function \(a\) satisfies equation \(l^\mu \partial_\mu a + \bar{\pi}^* = 0\). This ensures that \(\bar{\pi} = 0\) in a new frame, while remarkably the transformation (6.21) does not change any of the spin coefficients (6.17)-(6.20). Thus one obtains finally the field of null frames \(\{l, n, m, \overline{m}\}\) with the required properties: \(l\) is the null geodesics congruence with affine parameterization, while \(n, m, \overline{m}\) are covariantly constant along \(l\). The latter is equivalent to \(\bar{\kappa} = \bar{\varepsilon} = \bar{\pi} = 0\).
As it is well known, deformation and rotation of an image along a null geodesics are described by the optical scalars

\[ \tilde{\vartheta} = -\text{Re} \tilde{\rho}, \quad \tilde{\omega} = \text{Im} \tilde{\rho}, \quad \tilde{\sigma}. \] (6.22)

We now assume, for definiteness, that the polarization vector \( f^\mu \) coincides with the vector \( m^\mu \) of the above null frame. Let the image of a source, as seen at the point corresponding to the value \( s = s_1 \) of the affine parameter, be an ellipse with the major axes \( a \) and minor axis \( b \). Then one can straightforwardly obtain for the angle of rotation of the major axis of the image at \( s_2 = s_1 + \delta s \),

\[ \delta \eta = -\tilde{\omega} \delta s - \frac{a^2 + b^2}{a^2 - b^2} \text{Im} \tilde{\sigma} \delta s. \] (6.23)

Integration along a ray gives finite angle of rotation.

It is worthwhile to notice that along the cosmic rotation axis the observer at \( P \) finds for the optical scalars

\[ \tilde{\vartheta}_P = H_0, \quad \tilde{\omega}_P = \omega_0, \quad \tilde{\sigma} = 0, \] (6.24)

thus the effect of rotation of the polarization vector in this direction is most explicit.

As for an arbitrary direction of observation, with the help of the Kristian–Sachs approach we finally find from (6.23)

\[ \eta = \omega_0 r \cos \theta + O(Z^2). \] (6.25)

This result is in good agreement with the observational data reported \[28\] on the dipole anisotropy of distribution of the difference between the position angles of elongation (the major axis) and polarization in a sample of 3CR radiosources. The estimate for the direction and the magnitude of cosmic rotation, obtained from the Birch’s data \[34,35\], read

\[ l^\circ = 295^\circ \pm 25^\circ, \quad b^\circ = 24^\circ \pm 20^\circ, \] (6.26)

\[ \omega_0 = (1.8 \pm 0.8)H_0. \] (6.27)
7. CONCLUSIONS

In our discussion of the properties of rotating cosmologies we have paid special attention to the Gödel-type model (4.1). However the main conclusions are true also for the whole class of metrics (3.1). A series of papers is now under preparation in which we make exact estimates for the cosmic rotation effects, and present also their dynamical realizations, for all nine Bianchi type rotating cosmologies. As one can notice, in some cases the above numerical estimates for the value of vorticity do not agree e.g., (6.10) and (6.27). One can only remark in this relation that too few empirical data were analysed until now, and further detailed discussion is required in order to make final estimates. It may also turn out that some of the above mentioned effects are explained after all by different physical (and geometrical) reasons, not related to the cosmic rotation. In the light of the modern COBE results [61–63] the purely rotating models (3.1) should be replaced by cosmologies with nontrivial shear. Preliminary analysis of such generalizations shows that rotating models can be made compatible with the COBE data without destroying the rest of the rotational effects (in particular, without essential modification of the polarization rotation formulas).

We believe that the cosmic rotation is an important physical effect which should find its final place in cosmology. In this paper we outlined one of the possible theoretical frameworks which can underlie our understanding of this phenomenon.

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