Testing a Topology Conserving Gauge Action in Lattice QCD

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We study lattice QCD with a gauge action, which suppresses small plaquette values. Thus the MC history is confined to a single topological sector over a significant time, while other observables are decorrelated. This enables the accumulation of statistics with a specific topological charge, which is needed for simulations of QCD in the \( \epsilon \)-regime. The same action may also be useful for simulations with dynamical quarks. The update is performed with a local HMC algorithm.

1. MOTIVATION

Our goal is to explore the applicability of lattice gauge actions for QCD, which are designed such that small plaquette values are strongly suppressed. If such an action can be identified, we expect the following virtues:

- an acceleration of dynamical fermion simulations
- control over the topological charge

In particular we hope for applications in the \( \epsilon \)-regime of QCD [1]. In that regime, the pion Compton wave length clearly exceeds the box length, \( m_\pi^{-1} \gg L \), which is an unphysical situation. However, simulations in a such small volume may provide physically significant information, since the low energy constants of chiral perturbation theory take the same values as in a large volume. Hence the hope is to evaluate them without the requirement of a large lattice.

However, this implies several conditions for the lattice formulation: the lattice fermions should keep track of the chiral symmetry and give access to very light pions. Moreover, the topological charge \( Q_{\text{top}} \) needs a sound definition, since the measurements are performed at distinct \( |Q_{\text{top}}| \) [2].

Both of these requirements are provided by Ginsparg-Wilson fermions: they have an exact, lattice modified chiral symmetry [3], and the fermionic index defines \( Q_{\text{top}} \) [4]. Their simulation is now possible, at least quenched [5]. Recent results were obtained for the Dirac spectrum [6] and for meson correlation functions [7].

In particular the Neuberger Dirac operator [8] can be applied. Its index — and hence the topological sector — cannot change as long as each plaquette variable \( U_P \) obeys the constraint [9] [10]

\[
S_P := 1 - \frac{1}{3} \text{Re} \text{Tr}(U_P) < \epsilon \simeq 1/20.49 ,
\]

where \( \beta S_P \) is the plaquette action.

Therefore \( Q_{\text{top}} \) is fixed under continuous deformations of the gauge configuration if we use the plaquette action \( \beta S_\alpha \), with

\[
S_\alpha(U_P) = \left\{ \begin{array}{ll}
\frac{S_P(U_P)}{[1-S_P(U_P)/\epsilon]^{\alpha}} & S_P(U_P) < \epsilon \\
+\infty & \text{otherwise}
\end{array} \right.
\]

for \( \alpha > 0 \). This lattice gauge action (for \( \alpha = 1 \)) was introduced by M. Lüscher for conceptual purposes [11], and applied by Fukaya and Onogi in Schwinger model simulations [13].

The use of such a lattice gauge action has the advantages that the continuum property of stable topologies is reproduced, and that tedious computations of the index can be saved. Finally it allows for the cumulation of statistics in a specific topological sector; first experience in the \( \epsilon \)-regime shows that in particular \( |Q_{\text{top}}| = 1, 2 \) are useful (the sector \( Q_{\text{top}} = 0 \) suffers from strong fluctuations, and at \( |Q_{\text{top}}| > 2 \) quenched chiral perturbation theory [12] fails in volumes with box length \( L \lesssim 1.5 \) fm).

On the other hand, an obvious problem is that due to the constraint implemented in \( S_\alpha(U_P) \) the physical lattice spacing tends to be very small.
Figure 1. Above: ratio between the standard plaquette action $S_P$ and the modified plaquette action $S_\alpha$. Below: ratio between forces in configuration space, which drive the local HMC algorithm.

For practical simulations we hope that $\varepsilon$ values clearly above the theoretical bound still suppress topological transitions sufficiently. For instance, in the Schwinger model $\varepsilon = 1$ already stabilized $Q_{\text{top}}$ over hundreds of configurations \cite{13}.

2. A LOCAL HYBRID MONTE CARLO ALGORITHM

Since gauge actions of the type (1) are non-linear in the link variables, the heat bath algorithm cannot be applied. Instead we use a local HMC algorithm \cite{14}. Compared to the Wilson action, the force is just changed by an extra factor on each plaquette,

$$F_\alpha = \frac{\delta S_\alpha(U_P)}{\delta U_{x,\mu}} = \frac{\delta S_P(U_P)}{\delta U_{x,\mu}} \cdot \frac{1 + \frac{\alpha-1}{\varepsilon} S_P}{(1 - S_P/\varepsilon)^{\alpha+1}}.$$ 

The new plaquette action $S_\alpha$ and the force $F_\alpha$ are illustrated as functions of the standard plaquette action $S_P$ in Fig. 1.

3. RESULTS

We set $\alpha = 1$ and as a first experiment we searched for the line of a constant plaquette variable $\langle S_P \rangle$ on a $4^4$ lattice, as $\beta$ and the action parameter $\varepsilon$ are varied. The result is shown in Fig. 2. As we decrease $\varepsilon$, very small values of $\beta$ are needed to keep the plaquette constant.

Table 1

| $1/\varepsilon$ | $\beta$ | $r_0/a$ | $\beta_W$ | $\tau_{\text{cool}}$ | $\tau_{\text{plaq}}$ |
|----------------|---------|---------|-----------|-----------------------|----------------------|
| 0              | 6.18    | 7.14(3) | 6.18      | 1.17                  | 7.27                 |
| 1.25           | 0.8     | 7.0(1)  | 6.17      | 5.76                  | 1.11                 |
| 1.52           | 0.3     | 7.3(4)  | 6.19      | 21.04                 | 0.84                 |

As a more serious approach to identify a line of a constant physical scale, we now proceed to a $16^4$ lattice and measure $r_0/a$ (at $r_0 = 0.5$ fm) following the standard procedure, see e.g. Ref. \cite{15}. Finite size effects are expected to be on the percent level for our results presented in Table 1 (though we only indicate the statistical error).

We also performed first tests of the topological stability. In order to arrive at a quick first impression this was done with cooling and searching for the first plateau of the action (which is not sensitive to the sign of $Q_{\text{top}}$). The histories for $|Q_{\text{cool}}|$ — ignoring instanton/anti-instanton cancellations — are shown in Fig. 3.

We recognize a clear progress in view of the topological stability as $\varepsilon$ decreases along a line of approximately constant physics. This can also be quantified by the autocorrelation in these histories, as the fifth column in Table 1 shows.

On the other hand, the last column in Table 1 shows that the decorrelation with respect to the plaquette values becomes even better as $\varepsilon$ decreases (at a fixed scale).
4. CONCLUSIONS

"Topology conserving gauge actions" could be highly profitable in QCD simulations. The suppression of small plaquette values may speed up the simulations with dynamical quarks. A stable $Q_{\text{top}}$ is useful in particular in the $\varepsilon$-regime.

We are exploring the applicability of such actions, in view of the physical scale and the topological stability. This is an ongoing project. At present we have first promising candidates for suitable parameters which provide topological stability to some extent, while keeping the physical lattice spacing at a reasonable value.

Future tests will involve index measurements to verify the topological stability. At last we mention that the gauge action (I) has the problem that once a plaquette violates the constraint, a force in the wrong direction sets in. Hence such configurations had to be rejected. This happens more frequently as $\varepsilon$ is decreased. We now want to test variants of the action (I), where the denominator is replaced by a quadratic or exponential factor in order to avoid this problem, and also in view of the caveat pointed out in Ref [16].

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