Reliability analysis for chain topology wireless sensor networks with multiple-sending transmission scheme

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Abstract
Reliability analysis is a key problem in wireless sensor networks (WSNs). The primary contribution of this paper is an in-depth study of the reliability of a chain topology wireless sensor network with multiple-sending scheme. We study the wireless link reliability for the fading channels. The node energy availability for the source and relay nodes is investigated in terms of the limited node energy. The instantaneous network reliability and the mean time to failure are derived. Finally, the initial node energy allocation scheme is proposed to balance the lifetime of each sensor node, thus reducing the total energy consumption. The simulation results substantiate the correctness of the theoretical results.

Keywords: Chain topology; WSN; Link reliability; Energy availability; Energy-saving

1 Introduction
A wireless sensor network (WSN) consists of a large number of low-cost sensor nodes [1] distributed in large geographic area. It senses the interested events, generates packets, and transmits packets to the sink node or the access point via wireless communication. Each node is equipped with sensing, communication, computational, and energy supply modules. Considering the size and the cost, sensor nodes are devices with limited resources, particularly communication capability and battery energy [2]. However, due to the superiority in monitoring the spatial phenomena, the wireless sensor network is adopted in many applications, such as military applications, environment monitoring, biological detection, and smart home.

Generally, wireless sensor nodes may often be deployed in a harsh and inhospitable physical environment [3]. Therefore, the packet loss rate in wireless sensor network is much higher than other networks due to the influence of the environment, energy depletion, and hardware failure. Nevertheless, many safety-critical applications are proposed recently, for example, structural health monitoring [4], clinical monitoring [5], etc. The missing of urgent packets in these applications may cause severe property loss and casualties which are often unacceptable [6]. Hence, the reliable transmission is essential for applications of wireless sensor network. In order to guarantee the practicability of applications, how to measure the reliability of the wireless sensor network is an important issue which motivates us to investigate the reliability analysis for such networks.

Recently, the reliability analysis has drawn significant attention for wireless network. There exist many intensive studies about reliability analysis for traditional wireless communication network. Chen [7] evaluated the end-to-end expected reliability and its corresponding mean time to failure (MTTF) in different wireless communication schemes for wireless CORBA networks. Cook [8] discussed the two-terminal reliability analysis using the random waypoint mobility model for a mobile ad hoc wireless network. Snow [9] analyzed the reliability, availability, and survivability for a typical cellular or personal communication service network. Bai [10] analyzed the reliability of DSRC wireless communication for vehicle safety applications through experiment based on real-world experimental data. Egeland [11] analyzed the k-terminal reliability and network availability for planned...
and random wireless mesh networks. However, the reliability analysis for wireless sensor network is quite different due to the non-repairable sensor node and limited battery energy. It is reported in [12] that the energy constraint is the main factor preventing from the full exploitation of wireless sensor network technology. So far, there are a few research works about reliability analysis for wireless sensor network. In [13], the author first conducted actual experiments to characterize link reliability measures in an actual sensor network setting and then investigated how link-level re-transmission and multi-path routing might improve the reliability of wireless sensor network. However, the influence of environment to link reliability between two neighboring node was not involved. Cheng [14] proposed the high energy first clustering algorithm to address network lifetime predictability by the worst case energy consumption analysis. But the designed reference model cannot illuminate the relations between the data transmission and energy consumption. Wang [15] studied the reliability for an event-driven wireless sensor network which only the source node is able to sense events and generate packets.

In this paper, we analyze the reliability of chain topology wireless sensor networks where the source and relay nodes can sense events and generate packets using the multiple-sending scheme. Considering the impact from the channel fading and node failure caused by the battery energy depletion, the wireless link reliability and node energy availability are analyzed, respectively. Then, the instantaneous network reliability and the MTTF of WSNs are derived. In order to reduce the total energy consumption, the energy-saving initial node energy allocation scheme is proposed to balance the lifetime of sensor nodes.

The remainder of the paper is organized as follows: The next section derives the expression of wireless link reliability with a composite channel model which is represented as a mixture of the path loss and the shadow fading. In Section 3, the node energy availability for the source and relay nodes, the instantaneous network reliability, and the MTTF of WSNs are discussed. Furthermore, the allocation of initial energy for each node is investigated for energy saving. Numerical results are presented in Section 4. The conclusions are drawn in Section 5.

2 Wireless link reliability

We focus on a chain topology wireless sensor network shown in Figure 1 which includes a source node, \( N \) relay nodes, and a sink node. The source and relay nodes generate packets by sensing events and transmit the locally generated packets to the next sensor node via the wireless channel. Relay nodes receive packets from the previous sensor node and retransmit them. Except for the sink node, all other nodes are powered by batteries. There exist two transmission schemes to improve the reliability between neighboring nodes in recent research papers: the multiple-sending-based transmission scheme and the acknowledgement-based transmission scheme. The multiple-sending-based transmission scheme can eliminate the ACK control mechanism and reduce the latency of the data delivery as described in [3]; it has similar performance with the acknowledgement-based scheme when the environment of network is not good. We adopt the multiple-sending-based scheme in this paper. Let the source node and the sink node be node 0 and node \( N + 1 \), respectively. The relay node \( n \) \( (n = 1, 2, \ldots, N) \) are called node \( n \).

Since all packets are transmitted over the wireless channel, packets are not always able to be successfully received due to the channel fading. The success probability of one transmission from node \( n \) to node \( n + 1 \) is defined in [15]:

\[
p_{\text{succ}}^{n} = \Phi(\sqrt{2\varphi^{n}}),
\]

(1)

where the function \( \Phi(x) \) is the cumulative distribution function of the standard normal distribution which is given as

\[
\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \exp \left( -\frac{t^2}{2} \right) dt,
\]

(2)

and the parameter \( \varphi^{n} \) is expressed as

\[
\varphi^{n} = \frac{1}{\sqrt{2\sigma}} \left( \left[ P_{\text{tran}}^{n}\right]_{\text{dB}} + [G]_{\text{dB}} - \eta \left[ \frac{d_n}{d_{\text{ref}}} \right]_{\text{dB}} \right.
\]

\[
- \left( [n_0]_{\text{dB}} - [\gamma_{th}]_{\text{dB}} \right),
\]

(3)

where the function \( [x]_{\text{dB}} \) is the dB value of \( x \) which denotes for \( 10\log_{10} x \), \( \sigma \) is the variance of shadow fading, \( P_{\text{tran}}^{n} \) is the transmission power of node \( n \), \( G \) is a dimensionless constant which indicates the line-of-sight (LOS) path loss between node \( n \) and node \( n + 1 \), \( \eta \) is the path loss exponent for wireless channel which is decided by the environment, \( d_n \) is the distance between node \( n \) and node \( n + 1 \), \( d_{\text{ref}} \) is the far-field reference distance depending on the antenna characteristics and the average channel attenuation, \( n_0 \) is the background noise power at the receiver, and \( \gamma_{th} \) is the threshold value of signal-to-noise ratio (SNR).

Since the multiple-sending transmission scheme is adopted, the packet will be transmitted \( K \) times to improve the successful probability of transmission between node \( n \) and node \( n + 1 \). Therefore, the wireless link reliability between node \( n \) and node \( n + 1 \) can be derived as

\[
\rho^{L}_{n} = 1 - \left( 1 - p_{\text{succ}}^{n} \right)^{K} = 1 - \left( 1 - \Phi(\sqrt{2\varphi^{n}}) \right)^{K},
\]

\( \forall n \in \{0, 1, \ldots, N\} \).

(4)
Assuming that the wireless link reliability measures for the source node and relay nodes are independent, we can calculate the link reliability $R_n^L$ according to the path from the node $n$ to the sink node. For the source node, packets need to be transmitted through all relay nodes until they reach the sink node. As a result, the wireless link reliability for the source node can be given by

$$R_n^L = \prod_{k=n}^{N} r_k^L = \prod_{n=0}^{N} \left(1 - \left[1 - \Phi\left(\sqrt{2\phi_k}\right)\right]^K\right). \tag{5}$$

For relay nodes, the locally generated packets are transmitted to the next sensor node and independent of previous sensor nodes. Thus, the wireless link reliability for node $n$ ($n \in \{1, 2, \ldots, N\}$) can be calculated by

$$R_n^L = \prod_{k=n}^{N} r_k^L = \prod_{k=n}^{N} \left(1 - \left[1 - \Phi\left(\sqrt{2\phi_k}\right)\right]^K\right) \forall n \in \{1, 2, \ldots, N\}. \tag{6}$$

### 3 Node energy availability

Due to the limited battery energy, the source node and relay nodes will lose their function when the energy of the battery is less than the threshold energy. So as to analyze the node energy availability, we divide the energy consumption into the following three aspects: energy for sensing events, energy for receiving packets, and energy for transmitting packets. In order to save the energy and prolong the lifetime of the WSN, sensor nodes have two operating modes. One is the active mode which is energy-consuming. In this mode, the sensor node is transmitting or receiving packets. The other is the sleep mode which is energy-saving. In this mode, the sensor node is waiting for the arrival of the next event with negligible energy. Generally speaking, each node has its own duty cycle when sensing events. Hence, the energy consumption for sensing events for node $n$ at time $t$ is given by

$$E_n^{\text{sens}}(t) = \tau_n P_n^{\text{sens}} t \forall n \in \{1, 2, \ldots, N\}, \tag{7}$$

where $\tau_n$ is the duty cycle of node $n$, $P_n^{\text{sens}}$ is the power needed to sense events in unit time for node $n$. When a sensor node senses an event and generates a packet, the node switches to the active mode and transmits the packet to the next node. The energy consumption for transmitting one packet can be defined as

$$E_n^{\text{trans}} = \left(\frac{P_n^{\text{elec}} + P_n^{\text{trans}} KL}{r}\right) \forall n \in \{0, 1, \ldots, N\}, \tag{8}$$

where $P_n^{\text{elec}}$ is the power consumption to operate the wireless communication module, $P_n^{\text{trans}}$ is the power used to transmit the amplifier, $L$ is the packet length in bit, and $r$ is the data transmission rate in bit per second. When the previous node transmits a packet, the next node is activated from the sleep mode and receives the packet. Because the amplifier does not need to be operated when receiving packets, the energy consumption for receiving one packet can be expressed as

$$E_n^{\text{receive}} = \frac{P_n^{\text{elec}} KL}{r} \forall n \in \{0, 1, \ldots, N\}. \tag{9}$$

Since both the source node and relay nodes can sense events and generate packets, we assume that $M_n(t)$ indicates the number of packets generated by node $n$ ($n \in \{0, 1, \ldots, N\}$) during $[0, t]$ which satisfies a non-homogeneous Poisson process with intensity function $\lambda_n(t)$. The probability density function of $M_n(t)$ with mean $\Lambda_n(t)$ is defined as

$$\Pr\{M_n(t) = k\} = \frac{(\Lambda_n(t))^k}{k!} \exp(-\Lambda_n(t)), k = 1, 2, \ldots. \tag{10}$$

Specially, if $\lambda_n(t)$ does not change over time, which means $M_n(t)$ satisfies the homogeneous Poisson process, the probability density function of $M_n(t)$ can be rewritten as

$$\Pr\{M_n(t) = k\} = \frac{(\lambda_n t)^k}{k!} \exp(-\lambda_n t), k = 1, 2, \ldots. \tag{12}$$
Based on the energy consumption for sensing events, transmitting and receiving one packet for each node, the node energy availability for the source node and relay nodes will be investigated, respectively. To facilitate the following discussion, we assume that $E_n^{\text{init}}$ denotes the initial energy of node $n$, $E_n^{\text{res}}$ stands for the residual energy of node $n$, and $E_{\text{th}}$ indicates the threshold energy.

### 3.1 Source node energy availability

For the source node, the energy consumption includes the energy for sensing events and transmitting packets. The energy dissipation for receiving packets is not included because the source node does not receive any packets from other nodes. Therefore, the residual energy of the source node at time $t$ is as follows:

$$
E_0^{\text{res}}(t) = E_0^{\text{init}} - E_0^{\text{sens}}(t) - E_0^{\text{tran}} + M_0(t) = E_0^{\text{init}} - \tau_0 E_0^{\text{sens}} t - \frac{(p_{\text{elec}} + p_{\text{tran}}^{\text{elec}}) KLM_0(t)}{r}.
$$

(13)

To guarantee the proper function of the source node, the residual energy needs to be higher than the threshold energy. Assume that the symbol $A_0$ denotes the state that the source node is energy available at time $t$. Hence, the energy availability of the source node at time $t$ can be obtained as

$$
\Pr\{A_0\} = \Pr\{E_0^{\text{res}}(t) \geq E_{\text{th}}\} = \Pr\left\{M_0(t) \leq \frac{(E_0^{\text{init}} - \tau_0 E_0^{\text{sens}} t - E_{\text{th}}) r}{(p_{\text{elec}} + p_{\text{tran}}^{\text{elec}}) K L}ight\}.
$$

(14)

According to Equation 10, $M_n(t)$ satisfies the non-homogeneous Poisson process. The probability such as the source node is energy available can be calculated as

$$
\Pr\{A_0\} = \sum_{k=0}^{M_0} \frac{(A_0(t))^k}{k!} \exp(-A_0(t)) \cdot M_0 \geq 0 , \; M_0 < 0.
$$

(15)

$M_0$ is an integer and can be defined as

$$
M_0 = \left\lceil \frac{(E_0^{\text{init}} - \tau_0 E_0^{\text{sens}} t - E_{\text{th}}) r}{(p_{\text{elec}} + p_{\text{tran}}^{\text{elec}}) K L} \right\rceil
$$

(16)

where $\lceil x \rceil$ is the largest integer lesser than or equal to $x$. According to the cumulative distribution function (CDF) of Poisson distribution defined in [16], Equation 15 can be further modified as

$$
\Pr\{A_0\} = \sum_{m=0}^{\text{max}} \frac{1}{m!} \Gamma(M_0 + 1, A_0(t)) \cdot M_0 \geq 0 , \; M_0 < 0
$$

(17)

where $\Gamma(\mu, x) = \int_x^\infty e^{-t} t^{\mu-1} dt$ is the upper incomplete gamma function.

### 3.2 Relay node energy availability

For relay nodes, the energy consumption includes the energy for receiving and retransmitting packets of previous nodes and the energy for sensing events and transmitting the locally generated packets. Thus, the residual energy of the node $n (n = 1, 2, \ldots, N)$ is as follows:

$$
E_n^{\text{res}}(t) = E_n^{\text{init}} - E_n^{\text{sens}}(t) - (E_n^{\text{elec}} + E_n^{\text{tran}}) \cdot D_n(t) - E_n^{\text{elec}} \cdot M_n(t) + (p_{\text{elec}} + p_{\text{tran}}^{\text{elec}}) KLD_n(t)
$$

(13)

$$
= E_n^{\text{init}} - E_n^{\text{sens}}(t) - (E_n^{\text{elec}} + E_n^{\text{tran}}) \cdot D_n(t)
$$

$$
- (p_{\text{elec}} + p_{\text{tran}}^{\text{elec}}) KLD_n(t)
$$

(18)

where $D_n(t)$ denotes the number of packets which are successfully received at node $n$ during $[0, t]$. Similar to the analysis of the source node energy availability, we assume the symbol $A_n$ indicates the state that the relay node $n (n = 1, 2, \ldots, N)$ is energy available at time $t$. Thus, the relay node energy availability at time $t$ can be obtained as

$$
\Pr\{A_n\} = \Pr\{E_n^{\text{res}}(t) \geq E_{\text{th}}\} = \Pr\left\{\left(\frac{(p_{\text{elec}} + p_{\text{tran}}^{\text{elec}}) KLD_n(t)}{r} + (p_{\text{elec}} + p_{\text{tran}}^{\text{elec}}) KLM_n(t)\right) \leq (E_n^{\text{init}} - E_n^{\text{sens}}(t) - E_{\text{th}}) r\right\}.
$$

(19)

From Equation 19, in order to obtain the relay node energy availability, $D_n(t)$ should be measured firstly. Since all the nodes preceding node $n$ generate and transmit packets to node $n$, we assume $D_{m,n}(t)$ denotes the number of packets which are generated by node $m$ and received by node $n$ during $[0, t]$. Apparently, $D_n(t)$ can be expressed as

$$
D_n(t) = \sum_{m=0}^{n-1} D_{m,n}(t),
$$

(20)

where the probability distribution of $D_{m,n}(t)$ can be derived as (the detailed derivation can be found in Appendix 1)

$$
\Pr\{D_{m,n}(t) = i|A_m A_{m+1} \cdots A_{n-1}\} = \left(\frac{R_{m,n}^{\text{L}} \cdot \Lambda_m(t)}{i!}\right)^i \exp\left(-\frac{R_{n,n}^{\text{L}} \cdot \Lambda_n(t)}{i!}\right),
$$

(21)

where $R_{m,n}^{\text{L}}$ is defined as

$$
R_{m,n}^{\text{L}} = \prod_{j=m}^{n-1} r_j.
$$

(22)
From Equation 21, we can find out that $D_{m,n}(t)$ satisfies a non-homogeneous Poisson process with mean $R_{m,n}^L \cdot \Lambda_m(t)$ which can be expressed as

$$D_{m,n}(t) \sim \text{Poisson} \left( R_{m,n}^L \cdot \Lambda_m(t) \right).$$

(23)

Substituting Equation 23 into Equation 20 and using the sums of Poisson distribution in [17], we can obtain

$$D_n(t) \sim \text{Poisson} \left( \sum_{m=0}^{n-1} (R_{m,n}^L \cdot \Lambda_m(t)) \right),$$

(24)

and the probability density function of $D_n(t)$ can be written as

$$\Pr(D_n(t) = i | A_0 A_1 \cdots A_{n-1}) = \frac{\left( \sum_{m=0}^{n-1} (R_{m,n}^L \cdot \Lambda_m(t)) \right)^i}{i!} \exp \left( - \sum_{m=0}^{n-1} (R_{m,n}^L \cdot \Lambda_m(t)) \right),$$

(25)

Substituting Equations 25 and 10 into Equation 19, the probability density function of $D_n(t)$ can be measured as (the detailed derivation is presented in Appendix 2):

$$\Pr(A_n | A_0 A_1 \cdots A_{n-1}) = \sum_{k=0}^{M_n} \frac{(\Lambda_n(t))^k}{k!} \exp(-\Lambda_n(t)) \times \frac{1}{M_{n,k}!} \times \Gamma \left( M_{n,k} + 1, \sum_{m=0}^{n-1} (R_{m,n}^L \cdot \Lambda_m(t)) \right),$$

(26)

where $M_n$ and $M_n'$ are non-negative integers described as

$$M_n = \left[ \frac{E_n^{\text{init}} - \tau_n^{\text{pens}} t - E_{th}}{\rho_{\text{elec}}^n + \rho_{\text{tran}}^n} KL \right], \forall n \in \{1, 2, \ldots, N\},$$

(27)

$$M_{n,k}' = \left[ \frac{E_n^{\text{init}} - \tau_n^{\text{pens}} t - E_{th}}{\rho_{\text{elec}}^n + \rho_{\text{tran}}^n} KL - k KL (\rho_{\text{elec}}^n + \rho_{\text{tran}}^n) \right],$$

(28)

where $M_n$ and $M_n'$ are non-negative integers described as

$$M_n = \left[ \frac{E_n^{\text{init}} - \tau_n^{\text{pens}} t - E_{th}}{\rho_{\text{elec}}^n + \rho_{\text{tran}}^n} KL \right], \forall n \in \{1, 2, \ldots, N\},$$

(27)

$$M_{n,k}' = \left[ \frac{E_n^{\text{init}} - \tau_n^{\text{pens}} t - E_{th}}{\rho_{\text{elec}}^n + \rho_{\text{tran}}^n} KL - k KL (\rho_{\text{elec}}^n + \rho_{\text{tran}}^n) \right],$$

(28)

If $M_n < 0$ or $M_n' < 0$, according to Equations 25 and 10, the energy availability of the relay node $n$ satisfies

$$\Pr \{ A_n | A_0 A_1 \cdots A_{n-1} \} = 0.$$  

(29)

### 3.3 Instantaneous network reliability

Considering the wireless link reliability and node energy availability, the instantaneous network reliability is defined as:

$$R_{\text{sys}} = \Pr \{ A_0 A_1 A_2 \cdots A_N \} = \Pr \{ A_0 \} \prod_{n=1}^{N} \Pr \{ A_n | A_0 A_1 \cdots A_{n-1} \}. $$

(30)

Based on the law of total probability [18] and Equations 15 and 26, the instantaneous network reliability can be rewritten as

$$R_{\text{sys}}(t) = \frac{1}{M_0} \Gamma(M_0 + 1, \Lambda(t)) \times$$

$$\prod_{n=1}^{N} \left\{ \sum_{k=0}^{M_n} \frac{(\Lambda_n(t))^k}{k!} \exp(-\Lambda_n(t)) \times \frac{1}{M_{n,k}!} \Gamma \left( M_{n,k} + 1, \sum_{m=0}^{n-1} (R_{m,n}^L \cdot \Lambda_m(t)) \right) \right\}.$$  

(31)

### 3.4 MTTF

The MTTF is the most widely used parameter to measure the system reliability. In our paper, it is assumed that the failed system cannot be repaired since wireless sensor nodes may often be deployed in a harsh and inhospitable physical environment. Thus, we use the MTTF to measure the average time when any sensor node fails to work. In this system, the MTTF of WSN can be obtained by

$$\text{MTTF} = \int_{0}^{\infty} R_{\text{sys}}(t) \, dt$$

(32)

### 3.5 Energy-saving initial node energy allocation

According to Equation 25, the probability density function of $D_n(t)$ are quite different, especially when the wireless link reliability is high. It means that the expected numbers of packets successfully received at node $n$ during $[0, t]$ are different, thus causing unbalanced energy consumptions between nodes. However, the MTTF of the system is determined by the node with the shortest lifetime. Therefore, the energy-saving initial node energy allocation scheme is proposed to balance the lifetime of each node and save energy of the WSN.

From Equation 13, the expected value of energy consumption for the source node at time $t$ can be obtained as

$$E \left[ E^{\text{cons}}_0(t) \right] = t_0 \rho_{\text{pens}}^n + \frac{1}{\rho_{\text{elec}}^n + \rho_{\text{tran}}^n} \left\{ \frac{E^{\text{init}}}{\rho_{\text{elec}}^n + \rho_{\text{tran}}^n} + \frac{E_{th}}{\rho_{\text{elec}}^n + \rho_{\text{tran}}^n} \right\}$$

(33)
To guarantee the proper function of the source node at time $t$, the initial energy of the source node need to satisfy $E_{0}^{\text{init}} - E_{\text{th}} \geq E_{0}^{\text{cons}}$. Generally speaking, the initial energy of the node is much larger than the threshold energy ($E_{0}^{\text{init}} >> E_{\text{th}}$). As a result, the expected value of initial energy for the source node at time $t$ can be expressed as

$$E[\hat{E}_{n}^{\text{init}} (t)] \approx E[\hat{E}_{n}^{\text{cons}} (t)] = r_{0}P_{n}^{\text{sens}}t + \left( \frac{P_{\text{elec}}^{n} + P_{\text{tran}}^{n}}{r} \right) KL \Lambda_{0} (t).$$

(S4)

Specially, if $\Lambda_{0} (t)$ does not change over time, Equation 34 can be transformed to

$$E[\hat{E}_{n}^{\text{init}} (t)] \approx \left[ r_{0}P_{n}^{\text{sens}}t + \left( \frac{P_{\text{elec}}^{n} + P_{\text{tran}}^{n}}{r} \right) KL \right] \Lambda_{0} (t).$$

(S5)

Similar to the source node, from Equation 18, the expected value of energy consumption for the relay node $n$ at time $t$ can be obtained as

$$E[\hat{E}_{n}^{\text{cons}} (t)] = \tau_{n}P_{n}^{\text{sens}}t + \left( \frac{P_{\text{elec}}^{n} + P_{\text{tran}}^{n}}{r} \right) KL \Lambda_{n} (t) + \sum_{m=0}^{n-1} \left( \frac{P_{\text{elec}}^{n} + P_{\text{tran}}^{n}}{r} KL \right) R_{m,n}^{L} \Lambda_{m} (t).$$

(S6)

To guarantee the normal operation of node $n$ at time $t$, the initial energy of the relay node $n$ needs to satisfy $E_{n}^{\text{init}} - E_{\text{th}} \geq E_{n}^{\text{cons}}$. Thus, the expected value of the initial energy for the relay node $n$ can be calculated as

$$E[\hat{E}_{n}^{\text{init}} (t)] \approx E[\hat{E}_{n}^{\text{cons}} (t)] = \tau_{n}P_{n}^{\text{sens}}t + \left( \frac{P_{\text{elec}}^{n} + P_{\text{tran}}^{n}}{r} \right) KL \Lambda_{n} (t) + \sum_{m=0}^{n-1} \left( \frac{P_{\text{elec}}^{n} + P_{\text{tran}}^{n}}{r} KL \right) R_{m,n}^{L} \Lambda_{m} (t).$$

(S7)

Specially, if $\Lambda_{n} (t)$ does not change over time, Equation 3 can be rewritten as

$$E[\hat{E}_{n}^{\text{init}} (t)] \approx \left[ \tau_{n}P_{n}^{\text{sens}} + \left( \frac{P_{\text{elec}}^{n} + P_{\text{tran}}^{n}}{r} KL \right) \right] \Lambda_{n} (t) + \sum_{m=0}^{n-1} \left( \frac{P_{\text{elec}}^{n} + P_{\text{tran}}^{n}}{r} KL \right) R_{m,n}^{L} \Lambda_{m} (t).$$

(S38)

According to Equations 35 and 38, it can be easily observed that the energy consumption for each node per unit of time is independent of $t$ when $\Lambda (t)$ does not change over time. Therefore, the ratio of the initial node energy between the source and relay nodes is constant when $M_{n} (t)$ satisfies the homogeneous Poisson process.

4 Numerical results

In this section, we will evaluate the effectiveness of the derived expressions of wireless link reliability for each node, the instantaneous network reliability, and MTTB by using Monte Carlo (MC) simulations. In our simulation, we consider a chain topology WSN where $N$ relay nodes are uniformly placed in a $D$-meter-long linear region. Due to the considered scenario, the source node and each relay node can generate packets independently by sensing events and transmit packets to the next sensor node until the sink node. For the sake of simplicity, the intensity function for NHPP does not change over time and some parameters are assumed to be the same, i.e., $E_{0}^{\text{init}} = E_{n}^{\text{init}}, p_{\text{elec}}^{n} = p_{\text{elec}}, p_{\text{tran}}^{n} = p_{\text{tran}}, p_{\text{sens}}^{n} = p_{\text{sens}}, \lambda_{n} = \lambda$.

Table 1 Main simulation parameters

| Parameters                  | Symbol | Value       |
|-----------------------------|--------|-------------|
| Transmit times for one packet | $K$   | 1           |
| Length of the linear region | $D$   | 180 m       |
| Threshold value of SNR      | $\gamma_{\text{th}}$ | 6 dB        |
| Variance of shadow fading   | $\sigma$ | 8 dB        |
| Path loss exponent          | $\eta$ | 3.71        |
| LOS path loss at $d_{\text{ref}}$ | $G$ | $-31.54$ |
| Power of background noise   | $n_{0}$ | $4 \times 10^{-14}$ W |
| Power required by sensing event per second | $p_{\text{sens}}$ | 0.1 mW |
| Power dissipation to operate the wireless communication module | $p_{\text{elec}}$ | 0.1 mW |
| The initial energy of each node | $E_{0}^{\text{init}}$ | 0.05 J |
| The threshold energy of each node | $E_{\text{th}}$ | 0 J        |
| Duty cycle                  | $r$   | 0.01        |
| Packet length               | $l$   | $2.5 \times 10^{5}$ bit/s |
| Transmission rate           | $r$   | 5,000 bits  |
| Intensity function for NHPP | $\lambda (t)$ | 1           |
and $\tau_n = \tau$ for $n = 0, 1, 2, \ldots, N$. In addition, the main simulation parameters are listed in Table 1.

Figure 2 depicts the wireless link reliability for the source and relay nodes versus transmit power when the number of relay nodes is 3. Obviously, with the increase of transmit power, the received SNR will increase, and thus the wireless link reliability for each node will increase. Furthermore, the results in Figure 2 show that the wireless link reliability for each sensor node is quite different when the transmit power is low. It is because the nodes farther away from the sink node need more hops to reach the sink node. Hence, we adopt the multiple-sending scheme to mitigate the impact on wireless link reliability from multi-hops.

Figure 3 illustrates the relationship between the instantaneous network reliability and time with different numbers of relay nodes when $R_{L0} = 0.7$. Since the wireless link reliability of the source node is lowest which is described in Figure 2, we adopt the wireless link reliability of the source node ($R_{L0}$) to guarantee the wireless link reliability
of the network in the following analysis. It can be seen that the instantaneous network reliability is one at the beginning. The reason is that the energy of each sensor node is enough to maintain the normal work of the network at the initial stage. With the passage of time, the residual energy of each node is getting less to lower the probability of normal operation. As a result, the instantaneous network reliability falls sharply to 0. Accordingly, the instantaneous network reliability decreases with the increase of time. Moreover, it can be observed that the instantaneous network reliability with less relay nodes falls firstly since fewer relay nodes may result in the higher transmit power of each node required to get the same wireless link reliability [15]. Therefore, the instantaneous network reliability increases with the number of relay nodes.

Figure 4 illustrates the instantaneous network reliability from another different perspective. The figure shows the instantaneous network reliability versus time with different wireless link reliability when the number of different relay nodes.

Figure 5 MTTF versus wireless link reliability with different $N$. 
relay nodes $N = 4$. The instantaneous network reliability decreases with the increase of time which has been explained in the above paragraph. Simultaneously, it can be noted that the higher wireless link reliability may lead to a lower instantaneous network reliability. The reason is that each node needs to consume more energy to keep a higher wireless link reliability as shown in Figure 2. Furthermore, from Figures 3 and 4, simulation results match well with the theoretical results which validate our derivation.

Figure 5 plots the MTTF of the WSN versus wireless link reliability with different numbers of relay nodes. Since higher wireless link reliability needs higher transmit power which is shown in Figure 2 and the initial energy of each sensor node is fixed. The MTTF decreases with the increase of wireless link reliability. In addition, the instantaneous network reliability increases with the number of relay nodes as shown in Figure 3 leading to greater value of MTTF. Moreover, the value of MTTF tends to 0 when the wireless link reliability tends to 1. That is because the transmit power tends to be infinity in order to guarantee that the wireless link reliability is equal to 1. Furthermore, the simulation results verify the accuracy of theoretical derivations.

Figure 6 MTTF versus wireless link reliability with different initial node energy schemes when $N = 3$.

Figure 7 Total energy of sensor nodes versus wireless link reliability with different initial node energy schemes when $N = 3$. 
Figures 6 and 7 verify the effectiveness of the energy-saving initial node energy allocation scheme. Figure 6 illustrates the simulation results about the MTTF of WSN versus wireless link reliability with different initial node energy allocation schemes when the number of relay node \( N = 3 \). In the simulation, we find out that the last relay node is always the first one to stop working properly. Therefore, we calculate the initial energy of each node according to the lifetime of node 3.

As shown in Figure 6, the MTTF of WSN which adopts the energy-saving initial node energy allocation scheme is almost the same as that using uniform initial node energy. Figure 7 shows how much energy we save with different wireless link reliability. We assume the symbol \( E_{\text{init}} \) indicates the total initial energy of sensor nodes including the source and relay nodes. With the increase of wireless link reliability, more packets will be successfully received at the next sensor node. Hence, the differences of \( D_n(t) \) are growing which lead to more unbalanced energy consumptions among nodes. Consequently, the \( E_{\text{total}} \) decreases with the increase of the wireless link reliability.

5 Conclusions

When the safety-critical applications are introduced in WSNs, a significant challenge is that the missing of the urgent information or the node failure owing to the energy depletion will bring serious casualties and property loss. In this paper, we have studied the reliability of chain-topology WSNs using multiple-sending scheme from the aspects of the wireless link reliability and the node energy availability. The impact from the channel fading caused by the influence of environment is discussed. Based on the relationship between the wireless link reliability and the energy consumption, the node energy availability, the instantaneous network reliability, and MTTF are analyzed. Furthermore, due to the unbalanced load for each sensor node, the energy-saving initial node energy allocation scheme is presented to reduce the energy consumption of the concerned network. The simulation results have confirmed the results obtained analytically. Also, the results are useful for designing a WSN with good network performance.

Notation

\[ N \] number of relay nodes
\[ G \] line-of-sight path loss between node \( n \) and node \( n + 1 \)
\[ \eta \] path loss exponent for the wireless channel
\[ d_n \] distance between node \( n \) and node \( n + 1 \)
\[ d_{\text{ref}} \] far-field reference distance
\[ n_0 \] background noise power
\[ \gamma_{\text{th}} \] threshold value of SNR
\[ \sigma \] variance of shadowing fading
\[ K \] transmit times for one packet
\[ L \] packet length in bit
\[ r \] data transmission rate in bit per second
\[ \tau_n \] duty cycle of node \( n \)
\[ p_{\text{sen}}^n \] power required by sensing events per second for node \( n \)
\[ p_{\text{elec}}^n \] power dissipation to run the transmitter circuitry for node \( n \)
\[ p_{\text{tran}}^n \] transmit power of node \( n \)
\[ L_{\text{init}}^n \] total energy of node \( n \)
\[ L_{\text{sen}}^n (t) \] energy consumption for sensing events for node \( n \) at time \( t \)
\[ L_{\text{tran}}^n \] energy consumption for transmitting one packet for node \( n \)
\[ L_{\text{rec}}^n \] energy consumption for receiving one packet for node \( n \)
\[ E_{\text{cons}}^n (t) \] energy consumption for node \( n \) at time \( t \)
\[ E_{\text{ch}} \] threshold energy of sensor nodes
\[ E_{\text{total}} \] total initial energy of all sensor nodes except the sink node
\[ R_{\text{sys}} (t) \] instantaneous network reliability at time \( t \)
\[ \Pr \{X\} \] probability of \( X \)
\[ \Pr \{X|Y\} \] conditional probability of \( X \) given \( Y \)
\[ \lfloor x \rfloor \] largest integer less than \( x \)
\[ 10 \log_{10} x \]

Appendices

Appendix 1

Derivation of probability distribution of random variable \( D_{mn} (t) \)

Proposition 1. If \( X \sim B(n, p) \) and conditioning on \( X \), \( Y \sim B(X, q) \), then \( Y \) is a simple binomial variable with distribution \( Y \sim B(n, pq) \).

Proof. Based on the law of total probability [18], the probability distribution of random variable \( Y \) can be calculated by

\[
\Pr \{ Y = y \} = \sum_{X=0}^{n} Pr \{ Y = y|X = x \} Pr \{ X = x \} = \sum_{x=0}^{n} \binom{n}{x} q^x (1 - q)^{n-x} \Pr \{ X = x \} = (pq)^y \sum_{w=0}^{n-y} \binom{n-y}{w} (p (1 - q))^w (1 - p)^{(n-y) - w}.
\]

(39)
Since
\[
\binom{w+y}{y} \binom{n}{y} = \binom{n+y}{y} \binom{n-y}{w},
\]
(40)
Substituting Equation 40 into Equation 5 and according to the binomial theorem, the probability distribution of random variable \(Y\) can be further modified as
\[
Pr \{Y = y\} = \binom{n}{y} \left(pq\right)^y \left(1 - pq\right)^{n-y}.
\]
(41)
Proposition 1 is proved.

In this appendix, the probability distribution of random variable \(D_{m,i}\) will be deduced.

Firstly, \(D_{0,1}(t)\), which indicates the number of packets generated by the source node and received by node 1 during \([0,t]\), is considered. Apparently, \(D_{0,1}(t)\) is determined by the number of packets generated by the source node during \([0,t]\) and the wireless link reliability between the source node and the relay node 1. In this paper, we only concern the probability distribution of \(D_{0,1}(t)\) when the source node is available. Thus, the symbol \(D_{0,1}(t) | A_0\) is used to denote the \(D_{0,1}(t)\) when the source node is available. According to Equation 4, the wireless link reliability between the source node and node 1 is \(r_0^t\). For each packet transmitted from the source node, node 1 can either receive the packet or lose the packet. Therefore, with the condition of \(M_0(t) = k\), the process such that node 1 receives \(k\) packets is a \(k\) Bernoulli trial during \([0,t]\) with the probability of success \(r_0^t\). As a result, the random variable \(D_{0,1}(t) | A_0\) satisfies
\[
D_{0,1}(t) | A_0 \sim B \left(M_0(t) = k, r_0^t\right).
\]
(42)
The probability distribution of the above random variable can be given by
\[
Pr \{D_{0,1}(t) = i | A_0\} = \binom{k}{i} \left(r_0^t\right)^i \left(1 - r_0^t\right)^{k-i}, \quad i = 0, 1, \ldots, k,
\]
(43)
where \(\binom{k}{i}\) is defined as
\[
\binom{k}{i} = \frac{k!}{i!(k-i)!}.
\]
(44)
Based on the law of total probability \([18]\), Equations 10 and 43, the probability distribution of random variable \(D_{0,1}(t) | A_0\) can be calculated by
\[
Pr \{D_{0,1}(t) = i | A_0\} = \sum_{k=i}^{\infty} \binom{k}{i} \left(r_0^t\right)^i \left(1 - r_0^t\right)^{k-i} \times \frac{\left(A_0(t)\right)^k}{k!} \times \exp(-A_0(t)), \quad i = 0, 1, \ldots.
\]
(45)
According to Equation 44, \(\binom{k}{i} = 0\) when \(k < i\). By the variable substitution \(j = k - i\), Equation 45 can be rewritten as
\[
Pr \{D_{0,1}(t) = i | A_0\} = \frac{\left(r_0^t\right)^i \left(1 - r_0^t\right)^{-i} \exp(-A_0(t))}{i!} \times \sum_{j=0}^{\infty} \frac{\left(1 - r_0^t\right) \left(A_0(t)\right)^j}{j!}.
\]
(46)
Furthermore, the Taylor series \([19]\) of \(\exp\left((1 - r_0^t) \cdot A_0(t)\right)\) can be expressed as
\[
\exp\left((1 - r_0^t) \cdot A_0(t)\right) = \sum_{j=0}^{\infty} \frac{\left(1 - r_0^t\right) \cdot A_0(t)^j}{j!}.
\]
(47)
Substituting Equation 47 into Equation 45, the probability distribution of random variable \(D_{0,1}(t) | A_0\) can be given by
\[
Pr \{D_{0,1}(t) = i | A_0\} = \frac{\left(r_0^t \cdot A_0(t)\right)^i}{i!} \exp\left(-r_0^t \cdot A_0(t)\right), \quad i = 1, 2, \ldots.
\]
(48)
Apparently, the probability distribution of random variable \(D_{0,1}(t) | A_0\) satisfies the Poisson distribution with intensity function \(r_0^t \cdot A_0(t)\).

Similarly, for node 2, conditioning on \(D_{0,1}(t) | A_0 = i\), the random variable \(D_{0,2}(t) | A_0A_1\) also satisfies the binomial distribution with probability of success \(r_1^t\), namely
\[
D_{0,2}(t) | A_0A_1 \sim B \left(M_0(t) = k, r_1^t\right).
\]
(49)
According to Proposition 1, \(D_{0,2}(t) | A_0A_1\) can be transformed into a binomial variable with probability of success \(\prod_{j=0}^{i} r_j^t\) conditioning on \(M_0(t) = k\), namely
\[
D_{0,2}(t) | A_0A_1 \sim B \left(M_0(t) = k, \prod_{j=0}^{i} r_j^t\right).
\]
(50)
The probability distribution of \(D_{0,2}(t) | A_0A_1\) can be described as
\[
Pr \{D_{0,2}(t) = i | M_0(t) = k, A_0A_1\} = \binom{i}{j} \left(\prod_{j=0}^{i} r_j^t\right)^i \left(1 - \prod_{j=0}^{i} r_j^t\right)^{k-i}, \quad i = 0, 1, \ldots, k,
\]
(51)
Based on the law of total probability, the probability distribution of random variable $D_{0,2}(t)$ | $A_0A_1$ can be acquired as

$$\Pr \left\{ D_{0,2}(t) = i \mid A_0A_1 \right\} = \left( \prod_{j=0}^{i-1} r_j^I \cdot \Lambda_0(t) \right) \frac{1}{i!} \exp \left( - \prod_{j=0}^{i-1} r_j^I \cdot \Lambda_0(t) \right), \quad i = 0, 1, \ldots$$

(52)

Without loss of generality, for node $n$, the distribution of random variable $D_{m,n}(t) | A_mA_{m+1} \cdots A_{n-1}$ can be expressed as

$$D_{m,n}(t) | A_mA_{m+1} \cdots A_{n-1} \sim B \left( M_m(t) = k, \prod_{j=m}^{n-1} r_j^I \right).$$

(53)

The probability distribution of $D_{m,n}(t) | A_mA_{m+1} \cdots A_{n-1}$ can be obtained by

$$\Pr \left\{ D_{m,n}(t) = i \mid A_mA_{m+1} \cdots A_{n-1} \right\} = \left( \binom{k}{i} \left( \prod_{j=m}^{n-1} r_j^I \right) \left( 1 - \prod_{j=m}^{n-1} r_j^I \right)^{k-i} \right) \frac{1}{i!} \exp \left( - \prod_{j=m}^{n-1} r_j^I \cdot \Lambda_m(t) \right), \quad i = 0, 1, \ldots, k,$$

(54)

According to the law of total probability and Equations 10 and 54, we have

$$\Pr \left\{ D_{m,n}(t) = i \mid A_mA_{m+1} \cdots A_{n-1} \right\} = \left( \prod_{j=m}^{n-1} r_j^I \cdot \Lambda_m(t) \right) \frac{1}{i!} \exp \left( - \prod_{j=m}^{n-1} r_j^I \cdot \Lambda_m(t) \right),$$

(55)

which completes the derivation.

**Appendix 2**

**Derivation of energy availability for relay node $n$**

The derivation of energy availability for node $n$ $\Pr \{ A_n | A_0A_1 \cdots A_{n-1} \}$ is deduced in this appendix.

According to Equation 18, the node $n$ is energy available at time $t$ with appropriate values $D_n(t)$ and $M_n(t)$ which satisfy $E_n^{in} \geq E_{th}$. As a result, we will find out all possible combinations of $D_n(t)$ and $M_n(t)$. Obviously, when $D_n(t) = 0, M_n(t)$ may take the maximum value $M_n$ which can be expressed as

$$M_n = \left[ \frac{(E_n^{in} - \tau_n^{sens} t - E_{th}) r}{(p_{elec} + p_{tran}^{sens}) KL} \right], \forall n \in \{1, 2, \ldots, N\}.$$

(56)

Firstly, when $M_n(t) = 0$, the possible values of $D_n(t)$ is $\{0, 1, \ldots, M_{n,0} \}$ where $M_{n,0}$ can be calculated by

$$M_{n,0} = \left[ \frac{(E_n^{in} - \tau_n^{sens} t - E_{th}) r}{(p_{elec} + p_{tran}^{sens}) KL} \right].$$

(57)

Similarly, when $M_n(t) = 1$, the possible values of $D_n(t)$ is $\{0, 1, \ldots, M_{n,1} \}$ where $M_{n,1}$ can be calculated as

$$M_{n,1} = \left[ \frac{(E_n^{in} - \tau_n^{sens} t - E_{th}) r - KL(p_{elec} + p_{tran}^{sens})}{(2p_{elec} + p_{tran}^{sens}) KL} \right].$$

(58)

Without loss of generality, when $M_n(t) = k, k \leq M_n$, we have $D_n(t) \in \{0, 1, \ldots, M_{n,k} \}$ where $M_{n,k}$ can be calculated as

$$M_{n,k} = \left[ \frac{(E_n^{in} - \tau_n^{sens} t - E_{th}) r - kKL(p_{elec} + p_{tran}^{sens})}{(2p_{elec} + p_{tran}^{sens}) KL} \right],$$

(59)

According to the probability distribution function of $M_n(t)$ and $D_n(t)$ described in Equations 10 and 25, respectively, the energy availability of relay node $n$ can be given by

$$\Pr \{ A_n | A_0A_1 \cdots A_{n-1} \} = \sum_{k=0}^{M_n} \Pr \{ M_n(t) = k \} \sum_{i=0}^{M_{n,k}} \Pr \{ D_n(t) = i | A_0A_1 \cdots A_{n-1} \}$$

(60)

Based on the CDF of Poisson distribution described in Equation 17, the energy availability of relay node $n$ can be further modified into

$$\Pr \{ A_n | A_0A_1 \cdots A_{n-1} \} = \sum_{k=0}^{M_n} \frac{(\Lambda_n(t))^k}{k!} \exp(-\Lambda_n(t)) \times \frac{1}{M_{n,k}^{m_n}} \times \Gamma \left( M_{n,k} + 1, \sum_{m=0}^{n-1} (R_{m,n}^{sens} \cdot \Lambda_m(t)) \right).$$

This completes the derivation.

**Competing interests**

The authors declare that they have no competing interests.

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References

1. IF Akyildiz, W Su, Y Sankarasubramaniam, E Cayirci, A survey on sensor networks. IEEE Comm. Mag. 40(8), 102–114 (2002)

2. VC Gunjor, FC Lambert, A survey on communication networks for electric system automation. Comput. Network. 50(7), 877–897 (2006)

3. H Luo, H Tao, H Ma, SK Das, Data fusion with desired reliability in wireless sensor networks. IEEE Trans. Parallel Distr. Syst. 22(3), 501–513 (2011)

4. K Chintalapudi, T Fu, J Paek, N Kothari, S Rangwala, J Caffrey, R Govindan, E Johnson, S Masi. IEEE Internet Comput. 10(2), 26–34 (2006)

5. O Chipara, C Lu, TC Bailey, G-C Roman, Reliable clinical monitoring using wireless sensor networks: experiences in a step-down hospital unit, in Proceedings of the 8th ACM Conference on Embedded Networked Sensor Systems (ACM New York, 2010), pp. 155–168

6. Y Liu, Y Zhu, LM Ni, G Xue, A reliability-oriented transmission service in wireless sensor networks. IEEE Trans. Parallel Distr. Syst. 22(12), 2100–2107 (2011)

7. X Chen, MR Lyu, Reliability analysis for various communication schemes in wireless CORBA. IEEE Trans. Reliab. 54(2), 232–242 (2005)

8. JL Cook, JE Ramirez-Marquez, Two-terminal reliability analyses for a mobile ad hoc wireless network. Reliab Eng. Syst. Saf. 92(6), 821–829 (2007)

9. AP Snow, U Varshney, AD Malloy, Reliability and survivability of wireless and mobile networks. Computer. 33(7), 49–55 (2000)

10. F Bai, H Krishnan, Reliability analysis of dsr wireless communication for vehicle safety applications, in: Intelligent Transportation Systems Conference, 2006. ITSC’06. IEEE (IEEE Toronto, 2006), pp. 355–362

11. G Egeland, P Engelstad, The availability and reliability of wireless multi-hop networks with stochastic link failures. IEEE J. Sel. Area Comm. 27(7), 1132–1146 (2009)

12. T Kuro, RX Gao, S Sah, An adaptive sampling scheme for improved energy utilization in wireless sensor networks, in: Instrumentation and Measurement Technology Conference (I2MTC), 2010 IEEE (IEEE Austin, 2010), pp. 93–98

13. T Korkmaz, K Sarac, Characterizing link and path reliability in large-scale wireless sensor networks, in: 2010 IEEE 6th International Conference on Wireless and Mobile Computing, Networking and Communications (WiMob) (IEEE Niagara Falls, 2010), pp. 217–224

14. B-C Cheng, H-H Yeh, P-H Hsu, Schedulability analysis for hard network lifetime wireless sensor networks with high energy first clustering. IEEE Trans. Reliab. 60(3), 675–688 (2011)

15. J-B Wang, J-Y Wang, M Chen, X Zhao, S-B Si, L Cui, L-L Cao, R Xu, Reliability analysis for a data flow in event-driven wireless sensor networks using a multiple sending transmission approach. EURASIP J. Wireless Commun. Netw. 2013(1), 1–11 (2013)

16. FA Haight, Handbook of the Poisson Distribution. (Wiley, New York, 1967)

17. E Lehmann, Testing Statistical Hypotheses, 2nd edn. (Springer, New York, 1986)

18. S Kokoska, D Zwillinger, CRC Standard Probability and Statistics Tables and Formulas. (CRC Press, Florida, 2000)

19. M Abramowitz, IA Stegun, Handbook of Mathematical Functions: with Formulas, Graphs, and Mathematical Tables. (Courier Dover Publications, New York, 1970)