Focus Point Gauge Mediation without a Severe Fine-tuning

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Abstract

We consider focus point gauge mediation within the framework of the next-to-minimal supersymmetric standard model, which substantially reduces the degree of fine-tuning for the electroweak symmetry breaking. The milder fine-tuning is realized by a messenger field in the adjoint representation of $SU(5)$ gauge group with $SU(3)_c$ octet being heavy. Our model has a simple ultraviolet completion. The fine-tuning measure $\Delta$ can be as small as 40-50 without any contradiction with LHC constraints.

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1 Introduction

Supersymmetric (SUSY) extensions of the standard model (SM) are the most attractive models beyond the SM, since they not only explain the observed mass of the Higgs boson \(^1\) naturally, but also provide us with a consistent framework of the unification of all the known gauge coupling constants at the scale around \(10^{16}\) GeV, called the grand unified theory (GUT) scale. However, there is a serious problem in the SUSY SM, that is, we have too large flavor changing neutral currents (FCNCs) \(^2\). There have been observed, so far, only two solutions to this problem with generic Kähler potential. One is gauge mediation \(^3,5\) and the other high scale SUSY with the gravitino mass \(\gtrsim 100-1000\) TeV \(^6,14\). Furthermore, the former does not have a serious cosmological problem so called “Polonyi Problem” \(^15\).

The purpose of this paper is to discuss the fine-tuning problem for the electroweak symmetry breaking (EWSB) scale in gauge mediation models. We first show that minimal gauge mediation in the minimal SUSY SM (MSSM) already needs a severe fine-tuning as \(\Delta \gtrsim 1500\) at the present, where \(\Delta\) shows the sensitivity of the \(Z\)-boson mass scale to fundamental mass parameters \(^16,17\). This is because we need large stop masses to explain the observed Higgs boson mass of 125 GeV \(^18,22\). Therefore, we introduce a singlet chiral multiplet to the MSSM (NMSSM) to lower the stop masses while keeping the Higgs boson mass. However, we find that a fine-tuning of \(\Delta \gtrsim 300\) is still required. Finally, we invoke focus point gauge mediation proposed by Fukuda et al. \(^23\) (see also \(^24,26\) for earlier attempts) some time ago and show that we do not need such a severe fine-tuning.\(^4\) In fact, we find a wide parameter region with \(\Delta \simeq 40-50\) in our model. We discuss predictions and testability of the model in conclusion.

2 Minimal gauge mediation in MSSM and the fine-tuning problem

In this section, we show that the minimal gauge mediation model in the MSSM requires a very high degree of fine-tuning due to large masses for colored SUSY particles. The superpotential for \(\mathbf{5}\) and \(\overline{\mathbf{5}}\) messengers is given by

\[
W = \lambda_I^L Z \Psi_I^L \Psi_I^L + \lambda_D^I Z \Psi_D^I \Psi_D^I,
\]

where \(I = 1 \ldots N_5\), \(\Psi_I^L\) and \(\Psi_D^I\) are \(SU(2)_L\) doublet and \(SU(3)_c\) triplet messengers, respectively, and \(Z\) is a SUSY breaking field which has non-vanishing vacuum expectation values (VEVs): \(Z = M + F_Z \theta^2\). Since \(\Psi_I^L\) and \(\Psi_D^I\) consists of a complete multiplet of \(SU(5)\) GUT gauge group, \(\overline{\mathbf{5}}, \lambda_I^L = \lambda_D^I\) at the GUT scale is expected. By assuming all the couplings are of the same order, we define the messenger scale as \(M_{\text{mess}} = \lambda_5 M\), where \(\lambda_5\) is a coupling \(\lambda_I^L\) or \(\lambda_D^I\).

After integrating out the messenger fields, we obtain the soft SUSY breaking masses for the MSSM particles as

\[
M_t = \frac{g_2^2}{16\pi^2}(N_5 \Lambda), \quad M_{\tilde{t}} = \frac{g_2^2}{16\pi^2}(N_5 \Lambda), \quad M_{\tilde{b}} = \frac{g_3^2}{16\pi^2}(N_5 \Lambda),
\]

\(^1\)Pure gravity mediation \(^10,13,14\) which belongs to high scale SUSY does not have the Polonyi field and hence it is free from the cosmological problems.

\(^2\)With a specific form of the Kähler potential, such as a sequestered Kähler potential, the too large FCNCs are also avoided. In this case, focus point gaugino mediation \(^27,29\) is considered, which ameliorates the fine-tuning of the EWSB scale. The reduction of fine-tuning is a consequence of non-universal gaugino masses at the GUT scale (see for example Refs. \(^30,40\)).
Figure 1: The contours of $\Delta$ (black solid) and $m_h$ (red dashed), where $m_h$ is shown in units of GeV. We take $M_{\text{mess}} = 1500$ TeV and $N_5 = 1$ ($M_{\text{mess}} = 700$ TeV and $N_5 = 4$) in the left (right) panel. Here, $\alpha_s(m_Z) = 0.1181$ and $m_t(\text{pole}) = 173.34$ GeV.

and

\[
\begin{align*}
    m_Q^2 &\simeq \frac{N_5}{256\pi^4} \left[ \frac{8}{3} g_1^4 + \frac{3}{2} g_2^4 + \frac{6}{5} g_3^4 \left( \frac{1}{6} \right)^2 \right] \Lambda^2, \\
    m_U^2 &\simeq \frac{N_5}{256\pi^4} \left[ \frac{8}{3} g_3^4 + \frac{2}{3} \left( \frac{2}{3} \right)^2 \right] \Lambda^2, \\
    m_D^2 &\simeq \frac{N_5}{256\pi^4} \left[ \frac{8}{3} g_3^4 + \frac{6}{5} g_4^4 \left( \frac{1}{3} \right)^2 \right] \Lambda^2, \\
    m_L^2 &\simeq \frac{N_5}{256\pi^4} \left[ \frac{3}{2} g_2^4 + \frac{6}{5} g_4^4 \left( \frac{1}{2} \right)^2 \right] \Lambda^2, \\
    m_E^2 &\simeq \frac{N_5}{256\pi^4} \left[ 6 \frac{g_4^4}{5} \right] \Lambda^2, \\
    m_{H_u}^2 = m_{H_d}^2 &= m_L^2, \\
\end{align*}
\]

where $M_b$, $M_{\tilde{g}}$, and $M_{\tilde{t}}$ are the bino, wino and gluino masses, respectively; $m_Q$, $m_U$ and $m_D$ are squark masses; $m_L$ and $m_E$ are slepton masses; $m_{H_u}^2$ and $m_{H_d}^2$ are soft masses for the up-type and down-type Higgs, respectively; $g_1$, $g_2$ and $g_3$ are gauge coupling constants of $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$. Here, $\Lambda = F_Z/M$, which controls the overall mass scale.

The fine-tuning is estimated using the following measure \cite{16,17}:

\[
\Delta = \max \left| \frac{\partial \ln m_Z}{\partial \ln a_i} \right|, \quad a_i = \{ |\mu|, |M|, |F_Z|, |B_\mu(M_{\text{mess}})| \},
\]

where $\mu$ is a higgsino mass and $B_\mu(M_{\text{mess}})$ is a Higgs $B$-term at the messenger scale, which is defined as $V \supset B_\mu H_u H_d + \text{h.c.}$. We consider $a_i$ is a fundamental mass parameter, and check the sensitivity of the $Z$-boson mass $m_Z$ to $a_i$.

In Fig. 1 we show contours of the Higgs boson mass $m_h$ and $\Delta$ on $m_{\text{gino}}$-$\tan \beta$ plane, where $m_h$ is shown in units of GeV. Here, $m_{\text{gino}}$ is a physical gluino mass and $\tan \beta$ is a ratio of the Higgs VEVs, $\langle H_u^0 \rangle / \langle H_d^0 \rangle$. SUSY mass spectra and $m_h$ are calculated using SPheno-4.0.3 \cite{41}.
In the left (right) panel, we take $M_{\text{mess}} = 1500 \text{ TeV}$ and $N_5 = 1$ ($M_{\text{mess}} = 700 \text{ TeV}$ and $N_5 = 4$). In the case of $N_5 = 4$, the fine-tuning is slightly milder than the case of $N_5 = 1$ for fixed $m_h$, as the messenger scale can be lower. However, even in this case, the fine-tuning of $\Delta \gtrsim 1500$ is required to explain the Higgs boson mass of 125 GeV. The stop mass scale, $\sqrt{m_{\tilde{Q}3}m_{\tilde{U}3}}$, for $m_h = 125 \text{ GeV}$ is around 10 TeV in both cases. Note that $\Delta$ is dominated by the sensitivity to $\mu$-parameter.

### 3 Minimal gauge mediation in NMSSM

In the MSSM, the very high degree of fine-tuning is required since the stops need to be heavy as $\sim 10 \text{ TeV}$ to explain the observed Higgs boson mass. Thus, we introduce a singlet chiral multiplet $S$ to the MSSM, lowering the stop masses. In this section, only one pair of $5$ and $\bar{5}$ messengers is introduced, i.e. $N_5 = 1$. This is because a stau becomes the next-to-lightest SUSY particle for larger $N_5$, which is quite severely constrained by LHC experiments [43]. Here, we assume the stau is long-lived with the gravitino heavier than $O(10) \text{ keV}$.

The relevant superpotential and scalar potential in the NMSSM are written as

$$W = \lambda S H_u H_d + \xi_F S + \frac{1}{2} \mu' S^2,$$

and

$$V_{\text{soft}} = m_S^2 |S|^2 + (A_\lambda \lambda S H_u H_d + \xi_S S + \frac{1}{2} m'^2_S S^2 + h.c.),$$

respectively. The mass parameters in the superpotential are $|\xi_F|^{1/2} \sim \mu' \sim O(1) \text{ TeV}$, and we assume $S^4$ term is suppressed by a symmetry. In order to realize the correct EWSB, non-zero soft SUSY breaking mass parameters in the scalar potential are required. In our setup, the correct EWSB is realized by the tadpole, $\xi_S \sim O(\text{TeV}^3)$. We take $m_S^2 = 10^6 \text{ GeV}^2$ and $A_\lambda = m'^2_S = 0$ at the messenger scale. An explicit example model inducing these soft SUSY breaking parameters is shown in Appendix A.

In this setup, the fine-tuning is estimated using

$$\Delta = \max \left| \frac{\partial \ln m_Z}{\partial \ln a'_i} \right|, \quad a'_i = \{ |\mu'|, |\xi_F|, |\xi_S(M_{\text{mess}})|, |m_S^2(M_{\text{mess}})|, M, |F_Z| \} ,$$

where $a'_i$ is considered to be a fundamental mass parameter in this setup.

We show contours of $\Delta$ and $m_h$ in Fig. 2 on $m_{\text{gluino}}$-$\mu'$ plane. SUSY mass spectra and $m_h$ are estimated using NMSSMTools 5.2.0 [45] [46]. The coupling $\lambda$ is defined at the stop mass scale while $\mu'$ is given at the messenger scale. In the framework of the NMSSM, the Higgs boson mass of 125 GeV is easily explained thanks to the $F$-term contribution from $\lambda S H_u H_d$ with $\mu'$ of $O(1) \text{ TeV}$; therefore, the minimum of $\Delta$ is not determined by $m_h$ but current LHC constrains on gluino and squark masses [47]. By considering these constraints, $\Delta$ around 300 is required in this framework.

### 4 Focus point gauge mediation in NMSSM

The fine-tuning of the EWSB scale is further relaxed when we adopt focus point gauge mediation [23], which is realized in $SU(5) \times U(3)_H$ product group unification (PGU) [48] [49]. In $SU(5) \times U(3)_H$ PGU, the doublet-triplet splitting problem in GUTs is elegantly solved.

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[3] The gravitino lighter than about 10 keV is strongly constrained by the Lyman-α forest data [44], if it is a dominant component of the dark matter.
To realize focus point gauge mediation, a messenger superfield in the adjoint 24 representation of $SU(5)$ is introduced. The messenger field of $SU(3)_c$ octet in the 24 representation becomes much heavier than others, due to a Dirac mass with another $SU(3)_c$ octet chiral field, which belongs to the adjoint representation of $SU(3)_H(\subset U(3)_H)$ before $SU(5) \times U(3)_H$ is broken to the SM gauge group. Then, the low-energy Lagrangian in the messenger sector is given by

$$W = \lambda_X ZX \bar{X} + \lambda_3 Z \text{Tr}(\Sigma_3^2),$$

(8)

where $X$, $\bar{X}$ and $\Sigma_3$ correspond to $(3, 2)$, $(\bar{3}, 2)$ and $(1, 3)$ of the $SU(3)_c \times SU(2)_L$ gauge group, and $U(1)_Y$ charges of $X$ and $\bar{X}$ are -5/6 and 5/6, respectively. Note that the gauge coupling unification is still maintained with contributions from the $SU(3)_c$ octet fields [23].

After integrating out the messenger fields, we obtain

$$M_b = \frac{g_1^2}{16\pi^2}(5\Lambda), \quad M_{\tilde{w}} = \frac{g_2^2}{16\pi^2}(5\Lambda), \quad M_{\tilde{\beta}} = \frac{g_3^2}{16\pi^2}(2\Lambda),$$

(9)

and

$$m_Q^2 \simeq \frac{1}{256\pi^4} \left[ \frac{8}{3} g_1^4(2\Lambda^2) + \frac{3}{2} g_2^4(5\Lambda^2) + \frac{6}{5} g_1^4(5\Lambda^2) \frac{1}{6^2} \right]$$

$$m_{\tilde{U}}^2 \simeq \frac{1}{256\pi^4} \left[ \frac{8}{3} g_1^4(2\Lambda^2) + \frac{6}{5} g_1^4(5\Lambda^2) \left( \frac{2}{3} \right)^2 \right]$$

$$m_D^2 \simeq \frac{1}{256\pi^4} \left[ \frac{8}{3} g_1^4(2\Lambda^2) + \frac{6}{5} g_1^4(5\Lambda^2) \frac{1}{3^2} \right]$$

$$m_L^2 \simeq \frac{1}{256\pi^4} \left[ \frac{3}{2} g_1^4(5\Lambda^2) + \frac{6}{5} g_1^4(5\Lambda^2) \frac{1}{2^2} \right]$$

$$m_E^2 \simeq \frac{1}{256\pi^4} \left[ \frac{6}{5} g_1^4(5\Lambda^2) \right]$$

$$m_{H_u}^2 = m_{H_d}^2 = m_L^2.$$  

(10)
Figure 3: The contours of $\Delta$ (black solid) and $m_h$ (red dashed), where $m_h$ is shown in units of GeV. The parameters are same as in Fig. 2.

Notice that $SU(2)_L$ contributions to the gaugino and sfermion masses correspond to the effective messenger number of five, while $SU(3)_c$ contributions correspond to the effective messenger number of two. These larger $SU(2)_L$ contributions allow the small fine-tuning. Since the focus point leading to the small fine-tuning is fixed by the representations of $SU(5)$ and $U(3)_H$, the scenario is highly predictive and robust. As in the previous section, we set $m^2_S = 10^6$ GeV$^2$ and $A_\lambda = m'_S = 0$.

In Fig. 3, we show contours of $\Delta$ (cf. Eq. (7)) and $m_h$ in focus point gauge mediation within the framework of the NMSSM. As in the previous case, SUSY mass spectra and $m_h$ are computed using NMSSMTools with modifications to incorporate focus point gauge mediation. One can see that $\Delta$ is reduced to 40-50 in this framework for the gluino mass of 2.4-2.6 TeV.

Finally, some examples of mass spectra are shown in Table 1. For all the benchmark points, the higgsino is the next-lightest SUSY particle, which is assumed to be stable in the collider time scale. Due to the small tan $\beta$, smuon and stau masses are almost degenerate with selectron masses.

5 Conclusions

We have considered focus point gauge mediation in the NMSSM and shown that the fine-tuning $\Delta$ is significantly reduced compared to the minimal gauge mediation in the MSSM. It has been found that there exists a wide range of parameter with $\Delta \simeq 40$-50, where the minimum value of $\Delta$ is determined by the LHC constraints on the gluino and squark masses. The EWSB is correctly explained by the SUSY breaking tadpole term in the scalar potential for the singlet Higgs. An explicit example model generating the tadpole is shown in Appendix A.

In our scenario, the gluino dominantly decays into the higgsino, third generation quark and antiquark with the large top-Yukawa coupling. In this case, the LHC constraint on the gluino mass is about 2 TeV for decoupled squarks [50]. On the other hand, the lower-limit on the squark masses is more severe: the limit on the degenerated squark masses in the simplified model analysis is around 2.6 TeV for the gluino mass of 2.6 TeV [47]. Although this limit is not directly applicable to our model, the region with $\Delta < 50$ is expected to be tested in near future.
Table 1: Mass spectra in sample points. Here, $\mu_{\text{eff}} = \lambda \langle S \rangle$.

| Parameters               | Point I   | Point II  | Point III  |
|--------------------------|-----------|-----------|------------|
| $M_{\text{mess}} \,$(TeV) | 1000      | 500       | 1000       |
| $\Lambda \,$(TeV)        | 183       | 200       | 200        |
| $\lambda$                | 0.8       | 0.9       | 0.8        |
| $\mu'(M_{\text{mess}}) \,$(GeV) | 2600   | 4600      | 2200       |
| $\tan \beta$             | 4         | 4         | 4          |

| Particles | Mass (GeV) | Mass (GeV) | Mass (GeV) |
|-----------|------------|------------|------------|
| $\tilde{g}$   | 2600       | 2880       | 2830       |
| $\tilde{g}$   | 2700-3080  | 2960-3360  | 2930-3340  |
| $\tilde{t}_{1,2}$ | 2420, 2950 | 2690, 3230 | 2630, 3200 |
| $\tilde{\chi}_1^\pm$ | 597     | 486        | 596        |
| $\tilde{\chi}_2^\pm$ | 2360     | 2640       | 2580       |
| $\tilde{\chi}_1^0$   | 591       | 482        | 590        |
| $\tilde{\chi}_2^0$   | 605       | 492        | 604        |
| $\tilde{\chi}_3^0$   | 1280      | 1430       | 1400       |
| $\tilde{\chi}_4^0$   | 2340      | 2640       | 1990       |
| $\tilde{\chi}_5^0$   | 2360      | 4090       | 2580       |
| $\tilde{e}_{L,R}$    | 1560, 760 | 1680, 815  | 1700, 829  |
| $H^\pm$              | 1630      | 1680       | 1760       |
| $h_{\text{SM-like}}$ | 127.2     | 126.9      | 126.4      |
| $\mu_{\text{eff}} \,$(GeV) | 586     | 475        | 584        |
| $\Delta$             | 44        | 68         | 57         |

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A Generation of tadpole

To generate the tadpole term realizing the EWSB, we consider the following superpotential:

$$ W = \lambda'_{12} S\Psi_1 \bar{\Psi}_2 + \mu'_{12} \Psi_1 \bar{\Psi}_1 + \lambda_1 Z \Psi_1 \bar{\Psi}_1 + M_2 \Psi_2 \bar{\Psi}_2, $$

where $\langle Z \rangle = M + F_Z \theta^2$, $\mu'_{12} \sim 1 \text{TeV}$, and $\Psi_{1,2}$ is assumed to be charged under a gauged or global $SU(N)$. We take $\lambda_1 = 1$ and $M_2 = M$ for simplicity. After integrating out $\Psi_{1,2}$ and $\bar{\Psi}_{1,2}$, we obtain

$$ \xi_S \simeq N \frac{\lambda'_{12} F_Z^2}{192 \pi^2 M^2}, $$

and

$$ m_S^2 \simeq N \frac{\lambda'_{12} F_Z^2}{192 \pi^2 M^2} = \frac{\lambda'_{12} \xi_S}{\mu'_{12}}. $$

7
Other soft SUSY breaking terms can be suppressed as

\[ m_s^2/\mu' = 2A_\lambda \simeq N \frac{\lambda_{12}^2F_Z}{64\pi^2 M}, \]  

which are irrelevant to our discussion.

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