THE FORMATION OF GALAXY DISKS

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ABSTRACT

Galaxy disk formation must incorporate the multiphase nature of the interstellar medium. The resulting two-phase structure is generated and maintained by gravitational instability and supernova energy input, which yield a source of turbulent viscosity that is able to effectively compete in the protodisk phase with early angular momentum loss of the baryonic component via dynamical friction in the dark halo. Provided that star formation occurs on the viscous drag time-scale, this mechanism provides a means of accounting for disk sizes and radial profiles. The star formation feedback is self-regulated by turbulent gas pressure-limited percolation of the supernova remnant-heated hot phase, but can run away in gas-rich protodisks to generate compact starbursts. A simple analytic model is derived for a Schmidt-like global star formation law in terms of the cold gas volume density.

Subject headings: galaxies:formation–galaxies: ISM–galaxies: kinematics and dynamics–galaxies: stellar content

1. Introduction

An outstanding problem in understanding the formation of galaxy disks is the origin of the characteristic disk scale length. In the absence of a theory that determines this parameter, one can place little credence in simulations of numerical or semi-analytical galaxy formation. Yet until recently, the elements of a robust theory for disk scales appeared to be in place.

Tidal torques between neighbouring density fluctuations generate an initial dimensionless angular momentum \( \lambda_i \equiv J/E|G^{-1}M^{-5/2} \approx 0.05 \), with a broad dispersion \( \Delta \lambda_i \sim \lambda_i \). Baryons cool and contract once dark halos virialize, conserving specific angular momentum to form a centrifugally supported disk when \( \lambda \sim 1 \). The final disk radius is \( r_d \sim \lambda_i r_h \), where \( r_h \) is the halo radius. This yields a disk scale length of several kpc for an \( L_\ast \) galaxy, as observed (Fall and Efstathiou 1980).
However with the advent of numerical hydrodynamical simulations with adequate resolution, it has become apparent that angular momentum is effectively transferred outwards as dense baryon clumps sink into the central regions of the forming galaxy. Clumpiness in the dark halo also tidally perturbs the disk and aggravates the angular momentum transfer problem. The result of the simulations is that the disk sizes are invariably too small (Steinmetz and Navarro 1999), by a factor $\sim 5$.

Two schemes have been proposed in order to halt excessive angular momentum transfer. Photoionization of the intergalactic medium destroys small-scale substructure (Navarro and Steinmetz 1997). These structures are the building blocks of large-scale structure and a source of clumpiness during massive halo formation. However the CDM structure is unaffected and there is little effect on the formation of massive galaxies. Another problem related to that of angular momentum transfer in that it involves baryonic dissipation is that of overcooling. This results in premature exhaustion of the gas supply. A solution designed to tackle both of these problems appeals to feedback from star formation via supernovae. Provided gas cooling is delayed to $z \sim 1$, one can form disks of the appropriate size and mass (Weil, Eke and Efstathiou 1998).

However this success comes at a price. Late disk formation implies strong evolution in disk size, luminosity and surface brightness at $z \sim 1$. Large disks are observed to $z \sim 1$, and there is little evidence for any evolution in scale length and surface brightness, other than passive stellar population ageing (Lilly et al. 1998). Disks are observed to $z \sim 1$ with a Tully-Fisher relation that is essentially indistinguishable from that of present-day disks, apart from passive evolution. Not all analyses agree: Bouwens and Silk (2000) find that up to 2 mag of surface brightness evolution is required at $z \sim 1$ in an analysis of a much larger data set (Simard et al. 1999). However the very existence of substantial numbers of disk galaxies at $z \sim 1$ means that early heating of protodisks and consequent suppression of star formation to $z \sim 1$ appears to be an unacceptable resolution of the angular momentum transfer problem.

A new approach is required. I have previously proposed (Silk 1997) the following model for disk formation. Feedback from supernovae results in a two-phase interstellar medium, the porosity of which controls feedback on the supply and compression of cold gas and the ensuing star formation rate. Gas-rich disk gravitational instabilities drive cloud-cloud collisions that provide an effective turbulent viscosity which is responsible for the formation and contraction of the disk. Viscous galactic disk models have a long history, commencing with the original proposal by Silk and Norman (1981), the definitive study by Lin and Pringle (1987a) that derived exponential stellar surface density profiles, and the chemical evolution studies of Clarke (1989,1991). In the cosmological context of cold
dark matter halos, turbulent viscosity has been incorporated into disk models by Firmani
et al. (1996). I show here that the viscous drag time-scale is of the same order as the
dynamical friction time-scale that controls the loss of angular momentum from baryons to
dark matter (Section 2). I apply porosity feedback to derive a Schmidt-like star formation
law that depends on cold gas volume density (Section 3), and I examine outflows and radial
flows (Section 4). The requirement that the star formation time-scale is on the order of
the viscous time-scale suffices to give an exponential disk with an appropriately large-scale
length and that moreover forms at a relatively early stage (Section 5).

2. Angular momentum problem resolved

The loss of baryonic angular momentum during disk formation is primarily due to the
dynamical friction exerted by the inner dark halo on the baryonic clumps. The time-scale
for dynamical friction to act is approximately \( (M/M_{\text{cloud}}) \) dynamical times, or

\[
t_{df} \approx \left( \frac{r}{V} \right) \left( \frac{r}{l_t} \right)^3 \approx \frac{r}{V} \left( \frac{V}{\sigma_g} \right)^3
\]

where the first equality made use of the tidal radius \( l_t \) defined by

\[\rho/\bar{\rho} = (l_t/l)^3\]

for a cloud of density \( \rho \) and size \( l \) in a medium of mean density \( \bar{\rho} \). The second equality
replaces the disk circular velocity \( V \) by \( \Omega r \) and \( \sigma_g \) by \( \Omega l_t \), the latter being the cloud velocity
dispersion induced by gravitational instabilities (Gammie, Ostriker and Jog 1991).

To compare \( t_{df} \) with the viscous drag time-scale, defined by

\[t_\nu = r^2/\nu,\]

I compare three alternate definitions of viscosity. In a non-self-gravitating cold accretion
disk, the \( \alpha \)-prescription (Shakura and Sunyaev 1973) is commonly used:

\[\nu_1 = \alpha H c_s \quad \alpha \sim 1,\]

where \( H \) is the scale height and \( c_s \) is the sound velocity. This prescription is lacking
in fundamental motivation. A recent proposal (Duschl, Strittmatter and Biermann
2000) appeals to turbulence experiments which suggest that a limiting Reynolds number
\( R_{\text{crit}} \sim 10^2 - 10^3 \) demarcates the onset of turbulence, and yields the ansatz

\[\nu_2 = \beta r V, \quad \beta \approx R_{\text{crit}}^{-1}.\]
This expression should be applicable to self-gravitating systems and to spheroidal as well as disk geometries. Finally, non-axisymmetric gravitational instabilities of a cold disk provide an effective turbulent viscosity that transfers angular momentum (Lin and Pringle 1987b). This results in a characteristic instability scale ($l_i$) and turbulent velocity ($\sigma_g \approx \Omega l_i$), motivating the hypothesis that

$$\nu_3 = \gamma l_i \sigma_g, \quad \gamma \sim 1.$$  

The corresponding time-scales for viscous drag are

$$t_{\nu_1} = \frac{1}{\alpha} \left( \frac{r}{V} \right) \left( \frac{\sigma_g}{c_s} \right) \left( \frac{V}{\sigma_g} \right)^3; \quad t_{\nu_2} = \beta^{-1} \left( \frac{r}{V} \right); \quad t_{\nu_3} = \frac{1}{\gamma} r \left( \frac{V}{\sigma_g} \right)^2.$$  

In all three prescriptions for viscosity, the viscous time is of order the dynamical friction time; and indeed $t_{\nu_3}$ (and even $t_{\nu_2}$) can be substantially less. It is likely that only $t_{\nu_2}$ and $t_{\nu_3}$ are relevant to self-gravitating disk formation, and I conclude that viscosity in the protodisk can operate more effectively than dynamical friction.

This conclusion would not resolve, and indeed would even aggravate, the angular momentum problem, since viscous drag also removes angular momentum from the gas clouds, were it not for the fact that the star formation time $t_*$ likely to be comparable to the viscous time. This is plausible because the radial gas flow $\dot{M}_g = 2\pi \Sigma_g \nu$ is regulated by viscosity, and the gas flow controls the gas supply available for star formation at any given radius. Strong support for this conjecture comes from the model of a self-gravitating viscous disk, in which Lin and Pringle (1987a) find that $t_\nu \approx t_*$ results in an exponential surface density profile. Hence I conclude that star formation develops before substantial angular momentum loss is induced by dynamical friction of infalling clouds against the dark matter, and that the resulting disk scale-length is therefore of order $\lambda_i r_h$, where $r_h$ is the initial halo radius ($\sim 100$ kpc) at overdensity 200 (the virialization radius) and $\lambda_i \approx 0.05$ is the dimensionless halo angular momentum parameter induced by gravitational torques in the early universe. Moreover the resulting stellar disk surface density profile is exponential. Of course, numerical simulations are needed to verify these conjectures and should eventually be feasible.

3. **Global star formation rate: the second parameter**

It is usually assumed that the star formation rate in a self-gravitating gas-rich disk is regulated by gravitational instability according to whether the Toomre parameter

$$Q_g \approx \frac{\delta K \sigma_g}{\pi G \Sigma_g}$$
is smaller or larger than unity, for instability or stability respectively. Here, \( \kappa \) is the epicyclic frequency, \( \sigma_g \) is the gas-velocity dispersion, \( \Sigma_g \) is the gas surface density, and \( \delta \) is a parameter, of order unity for a gas-rich disk, that allows for the stellar contribution to the self-gravity of the gas component of the disk. H\( \alpha \), HI and CO mapping of disks shows that \( Q_g \sim 1 \) in the star-forming regions of disk galaxies, suggesting that star formation is self-regulated by gravitational instability (Kennicutt 1989). This is plausible, since disk instabilities heat the disk and raise \( Q_g \) via tidal acceleration of clouds, whereas gas cloud collisional dissipation decreases the gas velocity dispersion, decreasing \( Q_g \). Star formation occurs in the clouds that form by gravitational instability, grow in mass by coalescence, and become unstable to fragmentation. \( Q_g \) self-regulation predicts that there is a threshold to star formation. Star formation ceases when the gas surface density drops below a critical value

\[
\Sigma_{\text{crit}} \approx \delta \kappa \sigma_g / \pi G.
\]

Now \( \kappa \approx 2^{12} \Omega \) for a flat rotation curve, \( \Omega \approx 10^{-15} \text{ s}^{-1} \) and \( \sigma_g \sim 10 \text{ km s}^{-1} \) for the Milky Way, as a typical example, so that \( \Sigma_{\text{crit}} \approx 10 M_\odot \text{ pc}^{-2} \). Clouds of giant HII regions, both in our galaxy and in nearby spirals, support this star formation threshold (Kennicutt 1998). However H\( \alpha \) mapping (Ferguson et al. 1998) has shown that substantial star formation occurs at \( \Sigma < \Sigma_{\text{crit}} \). To properly account for star formation, a second parameter is evidently needed that supplements or modifies the \( Q_g \) criterion.

A promising direction for such a modification would involve the dependence of the star formation rate on gas volume density, rather than surface density. Disk flaring develops in the outer disk, and the resulting density decrease at fixed \( \Sigma_g \) would be a logical inhibitor of star formation that occurs near, but independently of, the radius at which \( \Sigma_g \) drops below \( \Sigma_{\text{crit}} \). Indeed, it is entirely possible that the physics that controls disk flaring, namely the transition between the dominance of disk and halo self-gravity, is ultimately responsible for determining the transition between the \( Q_g < 1 \) (inner disk) and \( Q_g > 1 \) (outer disk) regimes. I develop here a physical criterion for controlling the star formation rate via feedback that depends primarily on the gas density, rather than the gas surface density. Supernovae control the energy and momentum input that heats and stirs the interstellar medium. The rate of supernovae is controlled by the star formation rate, and therefore provides a natural means of self-regulating star formation. The essence of this self-regulation can be described by a two-phase model of the interstellar medium, consisting of the hot gas directly heated by supernova remnants and the cold gas that is compressed by supernova blast waves into dense shells, dense cold clumps of which also survive the passage of the supernova blast waves.

I have argued that global feedback operates via the porosity of a two-phase interstellar medium (Silk 1997). I now further extend these arguments by deriving an expression for
the star formation rate that does not explicitly depend on \( Q \). The porosity is related to the hot gas fraction by \( P = -\ln(1 - f) \), where \( f \) is the volume filling factor of hot gas. For global feedback, the hot gas volume fraction must be significant, say \( f > 0.1 \). If the gas density or pressure were sufficiently high, supernova feedback would be restricted to such small volumes that there would be no global effect. A two-phase interstellar medium is adopted in order to describe the interaction of cold and hot gas. The cold gas is atomic and molecular gas, and initially comprises the entire disk. Once stars form and die, a steady state is reached in which bubbles of hot gas driven by supernovae sweep out and envelop much of the cold gas. The hot gas mass fraction is determined by cold cloud evaporation and by cooling of the hot gas. The diffuse hot gas erodes cold clouds and leaks out of the disk via chimneys and fountains. I will estimate below the rate at which gas ejection occurs. The porosity is a convenient and calculable parameter that defines the filling factor of the hot phase, and hence its ability to diminish the effectiveness of star formation by eroding the cold gas supply.

Supernova remnants deposit energy and, ultimately, momentum into the interstellar gas at a rate \( \dot{\rho}_* v_{SN} \) where \( v_{SN} \) is the specific momentum injected by supernovae per unit star formation rate \( \dot{\rho}_* \) and is given by

\[
v_{SN} = E_{SN} / v_c m_{SN} = 500 E_{51}^{13/14} n_g^{-1/7} m_{250} \zeta_g^{-3/14} \text{km s}^{-1}
\]

with \( v_c = 413 E_{51}^{1/4} n_g^{1/7} \zeta_g^{3/14} \) being the velocity at which the remnant enters the momentum conserving regime, \( E_{51} \equiv E_{SN} / 10^{51} \text{ergs} \) the supernova energy (taken to be \( 10^{51} \) ergs), and \( \zeta_g \) the metallicity relative to solar of the ambient gas (Cioffi, Mckee and Bertchinger 1988). Also \( m_{SN} \) is the mean mass required in forming stars in order to produce a supernova. For SNII, one simply assumes an initial mass function (IMF) with all stars of mass above \( 8 \, M_\odot \) becoming supernovae, so that \( m_{SN} \approx 250 \, M_\odot \) for a Miller-Scalo IMF. One can crudely double the inferred rate for an estimate of the rate of Types I and II supernovae after the first \( 10^8 \) years have elapsed (to allow sufficient time for SNIa to form).

Porosity is defined to be the product of the supernova remnant 4-volume at maximum extent, when halted by ambient gas pressure, and the rate of bubble production. The porosity is a measure of the fraction of volume \( f \) occupied by the hot phase (\( T \sim 10^6 \text{K} \)) associated with the interiors of supernova remnants. By definition \( f = 1 - e^{-P} \), and the porosity \( P = (\dot{\rho}_* / m_{SN}) \left( \frac{4}{3} \pi R_a^3 t_a \right) \) where \( \dot{\rho}_* \) is the star formation rate, \( m_{SN} \) is the mass in stars formed per supernova, and \( R_a \) is the radius of the supernova remnant at time \( t_a \) when halted by the ambient (turbulent) gas pressure \( p_g \). One finds that the porosity

\[
P \propto \dot{\rho}_* p_g^{-1.36} \rho_g^{-0.11}
\]

is extremely sensitive to the interstellar pressure. This provides the motivation for the
feedback prescription. By incorporating an analytic fit to the evolution of a spherically symmetric supernova-driven shell (Cioffi et al. 1988), one can write

\[ P = G^{-1/2} \sigma_f^{2.72} p_g^{1.36} \rho_g^{-0.11} \dot{\rho}_* , \]  

(2)

where \( \dot{\rho}_* \) is the star formation rate, \( p_g \) is the ambient gas pressure, both thermal and turbulent, and \( \sigma_f \) is a fiducial velocity dispersion that is proportional to \( E_{SN}^{1.27} m_{SN}^{-1} \zeta_g^{-0.2} \) and may be taken to be 22 km s\(^{-1}\) for \( E_{SN} = 10^{51} \) erg, \( m_{SN} = 250 \, M_\odot \) and \( \zeta_g = 1 \). Note that at large pressure, \( P \ll 1 \), and porosity is primarily controlled by the ambient pressure, which I take to be dominated by turbulence: \( p_g = \rho_g \sigma_g^2 \).

Feedback is regulated by the porosity parameter \( P \). Previously, the star formation rate had no explicit feedback. Consider the possibility of strong feedback. In this case, the filling factor of hot gas is of order fifty percent, which is the requirement for strong feedback. Rewriting (2) as

\[ \dot{\rho}_* \approx G^{1/2} \rho_g^{3/2} \left( \frac{\sigma_g}{\sigma_f} \right)^{2.7} P , \]  

(3)

one explicitly incorporates feedback into the star formation rate. The natural limit in which feedback self-regulates has \( P \sim f \sim 1/2 \). If \( P \ll 1 \), feedback is irrelevant: if \( P \gg 1 \), star formation would be completely suppressed. It seems likely, though hard to prove rigorously, that \( f \sim 1/2 \) corresponds to self-regulation of disk star formation. Certainly, \( f \sim 0.2 - 0.5 \) prevails in the local interstellar medium (Slavin and Cox 1993; Ferriere 1995). In this case, the star formation rate is independent of \( Q \), and is controlled by ambient pressure. The turbulent pressure is fed by disk gravitational instabilities, and implicitly depends on \( Q \). The derived global star formation law has a normalisation, equivalent to star formation efficiency, that depends explicitly on cloud velocity dispersion. The star formation rate and efficiency are enhanced by stirring: this might be expected near a bar or as a consequence of a merger.

Another limiting case is \( P \sim f \ll 1 \). Here porosity is small and feedback is unimportant. This might be relevant in a dense gaseous protodisk, and would suggest the possibility of runaway star formation.

4. Feedback and outflow

Supernovae are responsible for the feedback from star formation that renders star formation inefficient. The star formation efficiency \( \varepsilon \) may be defined explicitly by writing the star formation rate as

\[ \dot{\rho}_* = \varepsilon \Omega \rho_g , \]
where $\varepsilon$ is the fraction of gas converted into star per dynamical time. Adapting the porosity model for feedback, one now can write the porosity as

$$P \approx \left(\frac{\rho}{\rho_g}\right)^{\frac{1}{2}} \left(\frac{\sigma_f}{\sigma_g}\right)^{2.7} \varepsilon,$$

where $\rho$ is the total (gas plus star) density in the disk or protodisk. High star formation efficiency drives high porosity and significant feedback.

The momentum input from supernovae is dissipated via cloud-cloud collisions and outflow from the disk. In a steady state, the momentum input rate $\dot{\rho}_s v_{SN}$ must balance the cloud collisional dissipational rate $\rho_g \sigma_g^2 t^{-1}$ and the momentum carried out in outflows $f p_g H^{-1}$, where $p_g$ is the turbulent pressure $\rho_g \sigma_g^2$ of the two-phase interstellar medium, $H \equiv \sigma_g^2 / 2 \pi G \Sigma$ is the disk gas scale height, and $\Sigma$ is the surface mass density. At a given disk radius, there is also viscosity-driven inflow, $M_g = 2 \pi \Sigma_g \nu$, which provides a gas supply for star formation. Ignoring the outflow and infall contributions in the momentum budget, one has

$$\varepsilon \Omega \mu_g v_{SN} = \mu_g \sigma_g \Omega,$$

so that $\varepsilon = 0.02 (\sigma_g / 10 \text{ kms}^{-1}) (500 \text{ kms}^{-1} / v_{SN})$. This suggests that supernova feedback can indeed yield the required low efficiency of star formation.

For a galaxy such as the Milky Way, the global star formation efficiency is expected to be around 2 percent, both as inferred from the global values of gas mass ($\sim 6 \times 10^9 M_\odot$) and star formation rate ($\sim 3 M_\odot \text{yr}^{-1}$) after allowance for gas return from evolving stars (the returned fraction $\sim 0.5$ for a Miller-Scalo IMF) over a galactic dynamical time and as more directly inferred from studies of HII region radio luminosities summed over molecular cloud masses (e.g. Williams and McKee 1997).

Inserting the derived expression for star formation efficiency into the equation for the porosity, I find that

$$P \approx \left(\frac{\rho}{\rho_g}\right)^{\frac{1}{2}} \left(\frac{\sigma_f}{\sigma_g}\right)^{1.7} \sigma_f / v_{SN} = 0.5 \left(\frac{\rho / \rho_g}{\sigma_g / 0.1}\right)^{\frac{1}{2}} \left(\frac{\sigma_g}{10 \text{kms}^{-1}}\right)^{-1.7}.$$

The observed three-dimensional cloud velocity dispersion is 11 kms$^{-1}$ (for molecular clouds within 3 kpc of the sun) (Stark and Brand 1989). Thus the Milky Way interstellar medium has predicted porosity comparable to what is observed.

If the hot volume fraction is large, outflow from the disk is inevitable. The ratio of outflow to star formation rate per unit area of disk be written as $(f_s p_g H^{-1} V_{esc}^{-1}) (\varepsilon \Omega \rho_g)^{-1}$, where $V_{esc}$ is the escape velocity from the disk. Now $p_g = \pi G \Sigma_g \sigma_g = \frac{1}{2} \Sigma_g \Omega V$ and
\( \Sigma_g = 2 \rho_g H \), so that the outflow rate simplifies to

\[
\text{outflow rate} / \text{star formation rate} \approx \frac{f_s}{\varepsilon},
\]

where \( f_s \) is the surface area covered in hot bubbles.

This ratio will typically be, for say \( f \sim P \sim 0.3 \) and \( \varepsilon \sim 0.03 \), around 10, and is comparable to what is observed for both star-forming dwarfs and more luminous disk galaxies (Martin 1999). In general, the bulk of the gas cannot escape in a wind from normal galaxies, otherwise the gas reservoir would be seriously depleted. Known infall rates amount to at most 10 percent of the star formation rates. The gas must cool in the halo and fall back into the disk.

The viscosity prescription leads to a radial inflow of the gas. This amounts to \( v_r = \nu r^{-1} \), which can be rewritten as

\[
v_r = \gamma \sigma_g l_t r^{-1} = \gamma \sigma_g^2 / V,
\]

where I have used the expression for \( \sigma_g \) derived for gravitational instability-driven random motions. Hence one typically expects

\[
v_r \approx 0.5 \gamma \left( \frac{\sigma_g}{10 \text{km s}^{-1}} \right)^2 \left( \frac{200 \text{km s}^{-1}}{V} \right) \text{km s}^{-1}.
\]

The associated mass flux is

\[
\dot{M}_g = 2 \pi \nu \Sigma_g = 2 \pi \gamma l_t \sigma_g \Sigma_g.
\]

Again making use of \( \sigma_g = \Omega l_t \) and \( \Sigma = \Omega V / 2 \pi G \), the mass flux reduces to

\[
\dot{M}_g = \gamma \sigma_g^2 V G^{-1} (\Sigma_g / \Sigma) = \gamma (\sigma_g / V)^2 \Omega M (\Sigma_g / \Sigma).
\]

Comparing this with the global star formation rate \( \dot{M}_* = \varepsilon \Omega M_g \), with \( \varepsilon = \sigma_g / v_{SN} \), one obtains

\[
\dot{M}_g = \gamma (\sigma_g / V) (v_{SN} / V) \dot{M}_*,
\]

or

\[
\dot{M}_g / \dot{M}_* = \left( \frac{\gamma}{8} \right) \left( \frac{\sigma_g}{10 \text{km s}^{-1}} \right) \left( \frac{v_{SN}}{500 \text{km s}^{-1}} \right) \left( \frac{200 \text{km s}^{-1}}{V} \right)^2.
\]

I conclude that the viscous supply of gas may account for of order 30 percent of the net gas supply, after return, as is required to balance star formation. The associated radial flow is sufficient to modify chemical evolution in the solar neighbourhood and resolve the G dwarf problem (Clarke 1991), and to generate a disk metallicity gradient (Clarke 1989). However the most novel application is to the disk scale length, as I now demonstrate.
5. Disk properties

The disk size may be estimated by setting the viscous time-scale equal to the star formation timescale. This guarantees an exponential profile, and the characteristic length scale is obtained by setting \( t_\nu = t_{sfr} \), so that

\[
\rho = \frac{t_{sfr}}{2 \pi G \Sigma}.
\]

Inserting the star formation time-scale

\[
t_{sfr} = \frac{1}{\epsilon \Omega} \frac{\rho}{\rho_g} = \frac{1}{\Omega} \left( \frac{\rho}{\rho_g} \right) \left( \frac{v_{SN}}{500 \text{km s}^{-1}} \right) \left( \frac{\sigma_g}{10 \text{km s}^{-1}} \right) \left( \frac{50 M_\odot \text{pc}^{-2}}{\Sigma} \right) \left( \frac{\rho}{2 \rho_g} \right) \text{kpc},
\]

I infer that

\[
r_{d,\nu} = \gamma \left( \frac{v_{SN} \sigma_g}{2 \pi G \Sigma} \right) \left( \frac{\rho}{\rho_g} \right) = 2 \gamma \left( \frac{v_{SN}}{500 \text{km s}^{-1}} \right) \left( \frac{\sigma_g}{10 \text{km s}^{-1}} \right) \left( \frac{50 M_\odot \text{pc}^{-2}}{\Sigma} \right) \left( \frac{\rho}{2 \rho_g} \right) \text{kpc},
\]

comparable to the scale lengths of Milky Way-type disks.

It is interesting to compare this with the standard derivation of the final radius of a nonlinear cold dark matter density perturbation that collapses to approximate virialization at a spherically-averaged overdensity of 200. The radius at mean overdensity 200 is

\[
r_{200} \approx 0.01 \frac{V}{H(z) \Omega_0^{1/2}},
\]

where \( V \) is again the circular velocity, \( H(z) \) is the Hubble parameter, and \( \Omega_0 \) is the density parameter. The baryons dissipate and contract within the cold dark matter halo to a final disk radius, if angular momentum is conserved, of

\[
r_{d,\text{cdm}} = \lambda_i r_{200}
\]

In the viscous model, the disk radius can be written in terms of the virial radius defined by the self-gravitating baryonic component, \( r_v = V^2 (\pi G \Sigma)^{-1} \), so that

\[
r_{d,\nu} = \gamma \left( \frac{v_{SN} \sigma_g}{V^2} \right) \frac{\rho}{\rho_g} r_v.
\]

Allowing for the initial baryon fraction \( f_b \) (expected to be \( \Omega_b/\Omega_m \sim 0.05/0.3 \sim 1/6 \)), one finds that the gas first becomes self-gravitating at radius \( f_b r_{200} \). Hence the disk radius is comparable to that attained in the idealized cold dark model, with conservation of specific angular momentum of the baryons,

\[
\frac{r_{d,\nu}}{r_{d,\text{cdm}}} = \frac{f_b \gamma}{\lambda_i} 2 \left( \frac{v_{SN} \sigma_g}{V^2} \right) \left( \frac{\rho}{\rho_g} \right) \sim 1.
\]
The strong dependence of $r_v$ (or $r_{200}$) on $z$ leads to the predicted sensitivity of the Tully-Fisher relation to redshift (Mao, Mo and White 1998), in contradiction with observations (Voigt 1999). In the viscous model, this effect is diluted because of the proportionality of disk radius to gas velocity dispersion. I expect $\sigma_g$ to increase with redshift because the effect of mergers and infall will result in systematically more turbulent protodisks. Hence this should partially compensate the reduction of $r_d$ at earlier epochs arising from the explicit redshift dependence in the expression for $r_{200}$. Some reduction in $r_d$ is needed in order to account for the observed evolution in disk surface brightness at $z \sim 1$ (Bouwens and Silk 2000).

The Toomre parameter for disk instability may be rewritten as

$$Q_g \approx \frac{\delta \Omega \sigma_g}{\pi G \Sigma_g} = 2\delta \left( \frac{\Sigma}{\Sigma_g} \right) \left( \frac{\sigma_g}{V} \right),$$

where $\Sigma$ is the total disk surface density of stars plus gas. Initially $Q_g < 1$, so that the protodisk was unstable. At late times however, $Q_g$ increases as $\Sigma_g$ decreases, although $\sigma_g$ also decreases, so this increase in $Q_g$ is partially compensated by the decrease in gas turbulence, and the disk should remain at least marginally unstable, $Q_g \lesssim 1$.

To maintain $Q_g \sim 1$ over many dynamical times, as required for disks of spiral galaxies, the disk must remain gas-rich. Gas cooling is required to dissipate the dynamical instability heating. One can estimate that the cold gas controls the gravitational instability of the disk if $\mu_g/\sigma_g > \mu_*/\sigma_*$, and this condition requires a gas fraction of 30 percent or more, if the gas velocity dispersion $\sigma_g \sim 10$ km s$^{-1}$ and the stellar velocity dispersion $\sim 30$ km s$^{-1}$. This is indeed observed for the solar neighbourhood, where $\mu_g \approx 15$ M$_\odot$ pc$^{-2}$ and $\mu_* \approx 40$ M$_\odot$ pc$^{-2}$. The gas supply is prescribed by radial inflow in the viscous disk model, as is consistent with the gas reservoir inferred from observations of the high velocity clouds.

6. Discussion

In summary, I have argued that the justification for the viscous disk model comes from the fact that viscous and star formation time-scales are likely to be comparable,

$$\frac{t_{sfr}}{t_{\nu}} = \gamma \left( \frac{\rho}{\rho_g} \right) \left( \frac{\sigma_g}{V} \right) \left( \frac{v_{SN}}{\Omega_d r_d} \right),$$

where $\Omega_d$ is the rotation rate evaluated at the disk scale length. The baryons retain most of their initial angular momentum since the dynamical friction time-scale exceeds the viscous
time-scale,
\[ \frac{t_{df}}{\tau} = \gamma \left( \frac{V}{\sigma_g} \right). \]

This consequently justifies an exponential disk profile with scale length of order
\[ r_{d,\nu} \approx f_b (v_{SN} \sigma_g / V^2) r_{200}. \]
This scale coincidently happens to be of order \( \lambda_\nu r_{200} \), confirming that approximate angular momentum conservation is achieved, and disk sizes match the observed range.

The viscous disk model predicts a radial inflow rate of order 10% of the disk star formation rate. This will generate a gradient in the chemical abundances, and also provides a source of gas for bulge formation during the phase of early gas-rich disk evolution. The predicted ratio of bulge to disk stellar mass is of order \( \sigma_g v_{SN} / V^2 \), i.e. of order 10%. Late forming bulges by early viscous disk evolution would imply that bulges are more metal-rich than the disk, and that the age spread of bulge stars is comparable to that of the old disk.

Supernova feedback regulates the star formation rate. The dominant parameter is porosity, largely determined by the turbulent gas pressure \( p_g \). When \( p_g \) is in the range expected for typical disk galaxies, \( P \sim 1 \) and feedback is important. I assume that the feedback is negative: further work is needed to justify this. In the case of high turbulent pressure, expected in the aftermath of mergers, \( P \ll 1 \). The lack of feedback means that star formation can run away, only being limited by the available gas reservoir. In general, I find that supernova feedback results in a star formation rate
\[ \dot{\rho}_* \approx \Omega \rho_g (\sigma_g / v_{SN}). \]

Thus even in starbursts, a Schmidt-type relation is maintained. The predicted efficiency can be as high as 20 %, for merger-induced turbulence \( \sigma_g \sim 100 \text{ kms}^{-1} \) and a standard IMF, for which \( v_{SN} \approx 500 \text{ kms}^{-1} \).

A top-heavy IMF could substantially reduce \( v_{SN} \), and star formation efficiencies of order 50% would then be attainable. Such efficiencies may be needed in order to account for the luminosities measured in many ultraluminous infrared galaxies where the molecular gas masses are measured. A top-heavy stellar initial mass function might be required in protogalaxy mergers in order to reconcile the hypothesis that ellipticals form in such events with the observed paucity of young ellipticals at intermediate redshifts.

Gas density (and pressure) rather than surface density is the dominant gas parameter in porosity-regulated star formation. This means that disk flaring at the transition from disk self-gravity to halo gravity dominance, where the gas scale height increases due to the reduction in disk self-gravity, will tend to result in a reduction of the disk star formation rate. Disk gravitational instability, which controls the supply of massive molecular clouds, is
also quenched at approximately the same radius. This leads to a more complex dependence of star formation rate on gas surface density than suggested by the simple $Q_g$ threshold models that have hitherto been advocated.

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