Photoionization of Rydberg States by Ultrashort Wavelet Pulses

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Abstract. Photoionization of Rydberg atomic state by the ultra-short wavelet pulses (USWP) is calculated in the frame of perturbation theory with the use of Kramers ionization cross-section. Two simplest types of USWP are considered, namely, cosine and sine pulses. Analytical expressions containing scaling functions for photoionization probability are derived for above-mentioned USWP.

1. Introduction
Atoms in highly excited (Rydberg) states are important nanoobjects both from theoretical point of view and for different applications in nanophysics particularly in nanophotonics. Rapid development of ultra-short pulse generation technology makes it urgent to investigate the peculiarities of the USWP-Rydberg atom interaction.

2. General formulas for radiation absorption probability
The present work is devoted to theoretical description of atomic Rydberg states photoionization by USWP within the frame work or perturbation theory. We consider two types of USWP cosine- and sine ones. Fourier transforms of these pulses are given by the following expressions:

\[ E_{\cos}(\omega) = 2 \sqrt{\frac{2}{3}} \sqrt{\pi} E_0 \omega^2 \tau^3 \exp(-\omega^2 \tau^2 / 2), \]
\[ E_{\sin}(\omega) = 2 i \sqrt{\pi} E_0 \omega \tau^2 \exp(-\omega^2 \tau^2 / 2), \]

here \( E_0 \) is the amplitude of electric field strength in the pulses and \( \tau \) is pulse duration parameter.

For the calculation of Rydberg state photoionization probability during all time of USWP action we use the formula obtained in paper [1]

\[ W_{ph} = \frac{c}{4\pi^3} \int_{\omega_1}^{\omega_2} \sigma_{ph}(\omega') \left| \frac{E(\omega')}{\hbar \omega'} \right|^2 d\omega' \]

here \( \sigma_{ph}(\omega) \) is photoionization cross section for which we use Kramers formula:

\[ \sigma_{ph}(\omega) = \frac{8\pi}{3\sqrt{3}} \frac{Z_{eff}^2}{n^2 c \omega^3}. \]
Here $n$ is principal quantum number of Rydberg state and $Z_{eff}$ is effective charge of atomic core. Using equalities (1), (3) – (4) we obtain the following expression for photoionization probability of Rydberg states by cosine pulse ($I_{nZ_{eff}} = Z_{eff}^2 / 2 n^2$ is ionization potential of Rydberg state):

$$W_n^{(\cos)}(\tau) = \frac{2^8}{9 \sqrt{3}} \left[ \frac{E_0}{E_{sc}} \right]^2 f_c(\tilde{\tau} = \tau I_{nZ_{eff}}),$$  

$$E_{sc}(n, Z_{eff}) = \frac{Z_{eff}^3}{n^{3/2}},$$

$$f_c(\tilde{\tau}) = \tilde{\tau}^5 \text{erfc}(\tilde{\tau}).$$

Here $f_c(\tilde{\tau})$ is scaling function of dimensionless time $\tilde{\tau} = \tau I_{nZ_{eff}}$, $\text{erfc}(x)$ is complimentary error function ($\text{erfc}(x) = 1 - \text{erf}(x)$).

For sine pulse we have

$$W_n^{(\sin)}(\tau) = \frac{2^8}{3 \sqrt{3}} \left[ \frac{E_0}{E_{sc}} \right]^2 f_s(\bar{\tau} = \tau I_{nZ_{eff}}),$$

here

$$f_s(\bar{\tau}) = \bar{\tau}^5 \left[ \exp(-\frac{\bar{\tau}^2}{\pi \bar{\tau}}) - \text{erfc}(\bar{\tau}) \right].$$

It follows from the definitions of scaling functions (7), (9) that they have maxima at

$$\tilde{\tau}_{\text{max}}^{(c)} = 1.457, \quad \tilde{\tau}_{\text{max}}^{(s)} = 1.175$$

and corresponding maximum values:

$$f_{c, \text{max}} \approx 0.258, \quad f_{s, \text{max}} \approx 0.054.$$  

3. Results and conclusions

Plots of function (7) and (9) are shown in figure 1.

![Figure 1](image.png)

Figure 1. Solid curve – cosine USWP, dotted curve – sine USWP, abscissa is dimensionless time.
Figure 2 shows the photoionization probability of Rydberg state with given principal number \( n = 10, \ Z_{\text{eff}} = 1 \) by cosine- and sine wavelet pulses as function of pulse duration for electric field amplitude \( E_0 = 10^{-6} \) at.u. Note that such small value of electric field enable one to neglect the mixing of Rydberg states due to Stark effect.

![Figure 2](image)

**Figure 2.** Photoionization of Rydberg state \( (n = 10, Z_{\text{eff}} = 1) \) by cosine (solid curve) and sine (dotted curve) wavelet pulses.

Let us consider the photoionization of Rydberg state with given principal \( n \) and orbital \( l \) quantum numbers. Corresponding cross section is given by the following expression

\[
\sigma_{nl}(\omega) = \frac{32\pi}{3\sqrt{3}} \frac{(l+1/2)Z_{\text{eff}}^2}{n^3 c \omega^2} \exp\left(-\frac{2\omega(l+1/2)^3}{3Z_{\text{eff}}^2}\right).
\]  
(12)

This formula can be obtained in the frame of quasi-classical approach as it shown in [3], [4].

Substituting formulas (1), (12) in expression (3) after simple algebra we have:

\[
W_{nl}^{(\text{con})}(\tau, \omega) = \frac{2^9}{9\sqrt{3}\pi} \left(\frac{E_0}{E_{\text{sc}}}ight)^2 f_{cl}(\bar{\tau}, \zeta),
\]  
(13)

here
\[ f_{cl}(\tilde{\tau}, \zeta) = \tilde{\tau}^4 \left\{ \exp\left(-\tilde{\tau}^2 - 2\zeta\right) - \sqrt{\pi} \left(\zeta/\tilde{\tau}\right) \exp\left(\zeta^2/\tilde{\tau}^2\right) \text{erfc}\left[\zeta + \zeta/\tilde{\tau}\right] \right\} \] (14)

is appropriate scaling function for cosine-pulse and parameter \( \zeta \) is given by equality:

\[ \zeta = \frac{(l + 0.5)^3}{6n^2}. \] (15)

Analogously for photoionization probability of Rydberg state \( nl \) by sine-pulse (2) one can find

\[ W_{nl}^{(\sin)}(\tau, \omega) = \frac{2^9}{3\sqrt{3\pi}} \left(\frac{E_0}{E_{nc}}\right)^2 f_{sl}(\tilde{\tau}, \zeta), \] (16)

here

\[ f_{sl}(\tilde{\tau}, \zeta) = \tilde{\tau}^4 \int_{-\infty}^{\infty} \exp\left(-2\zeta w - w^2\tilde{\tau}^2\right) \frac{dw}{w} \] (17)

is scaling function for Rydberg state (with given principal and orbital quantum numbers) ionization by sine wavelet pulse.

Photoionization probabilities of Rydberg states with given principal quantum number \( n = 10 \) and different orbital numbers \( l \) by cosine wavelet pulse with amplitude \( E_0 = 10^{-6} \) at.u. as a function of pulse duration are presented in figure 4

![Figure 4](image)

**Figure 4.** Photoionization probabilities of Rydberg states \( n = 10 \) by cosine wavelet pulse as a function of pulse duration for different \( l \) numbers: solid line \(- l = 1 \), dotted line \(- l = 3 \), dashed line \(- l = 6 \)

One can see from this figure that the photoionization probability dependence upon orbital quantum number has non-monotonic character.
One can see from figures 2-5 that photoionization probability dependencies upon wavelet pulses duration are the curves with maxima and go to zero for long pulses. The last conclusion is a specific feature of photoionization by wavelet pulses without carrier frequency. So in the considered situation we have no transition to the monochromatic limit as in the case of atom photoionization by Gaussian pulses [5] when the probability of the process is linear function of pulse duration for sufficiently long pulse.

Thus in the framework of perturbation theory and quasi-classical approach we obtain general expressions for total photoionization probability of Rydberg states by cosine and sine wavelet pulses for two cases, namely, for (I) Rydberg state with given principal quantum number \( n \) and for (II) Rydberg state with given principal \( n \) and orbital \( l \) quantum numbers. Obtained expressions describe the photoionization probability as a function of wavelet pulse duration via scaling functions in universal manner and can be used for simple description of considered process.

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References

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Figure 5. Just the same as in figure 4 for sine wavelet pulse.