Transport properties of rippled graphene

Maciej Zwierzycki

Institute of Molecular Physics, Polish Academy of Sciences, Smoluchowskiego 17, 60-179 Poznań, Poland

E-mail: maciej.zwierzycki@ifmpan.poznan.pl

Received 12 November 2013, revised 20 January 2014
Accepted for publication 30 January 2014
Published 11 March 2014

Abstract
The exceptionally high mobility of carriers in graphene is one of its defining characteristics, especially in view of potential applications. Therefore it is of both practical and fundamental importance to understand the mechanisms responsible for limiting the values of the mobility. The aim of the paper is to study theoretically one such mechanism, i.e. scattering on ripples. The transport properties of rippled graphene are studied using the single-band tight-binding model. Both the bond-length variation, corresponding to the vector potential in the effective mass picture, and the fluctuating scalar potential are included in the formalism. The samples are modeled as self-similar surfaces characterized by the roughness exponent, with values ranging from those typical for graphene on SiO$_2$ to those seen for suspended samples. The range of calculated resistivities and mobilities overlaps with those from experiments. The results presented in this paper support the notion of rippling as one of the important factors limiting the mobility of carriers in graphene.

Keywords: graphene, transport, mobility, ripples

(Some figures may appear in colour only in the online journal)

1. Introduction
The enormous interest in graphene, a two-dimensional (2D) honeycomb lattice of carbon atoms discovered in 2004 [1], has both fundamental and practical reasons. The electronic properties of graphene are related to the peculiar nature of its band states in the vicinity of the Fermi level. These states possess a linear dispersion and are described by an effective Hamiltonian formally identical to that of relativistic massless particles (Dirac fermions), albeit with a velocity 300 times lower than the speed of light [2]. This leads to a range of interesting phenomena like finite minimal conductivity, the Klein paradox (i.e. tunneling with no attenuation) and the anomalous quantum Hall effect which can be observed even at room temperatures (see e.g. [3] for a review). The practical interest stems mostly from the fact that carriers in graphene, whose number and character (electrons or holes) can be modified by the field effect, possess exceptionally high mobility. The values range from of the order of $10^4–15000$ cm$^2$ V$^{-1}$ s$^{-1}$ for exfoliated graphene on SiO$_2$ substrate [2, 4, 5] to over $100000$ cm$^2$ V$^{-1}$ s$^{-1}$ for suspended samples [6–9], making graphene a strong candidate for use in ultra-fast electronic devices [10–14].

The precise mechanism limiting the mobility of carriers in graphene is still a matter of debate. The values and temperature dependence of mobilities measured in suspended samples (in the doped regime) seem to support the phonon scattering model [6]. The situation is less clear for samples on substrates, where little temperature dependence is observed. An often suggested source of scattering in the latter case is charged impurities, possibly located in the substrate [15]. However, an experiment designed specifically to test this scenario failed to produce supporting results [16]. Another possibility is scattering induced by ripples [17], which has been observed in both suspended [18] and exfoliated samples [19–22]. Theoretical studies indicate that ripples can be caused by direct interaction with the substrate [22], thermodynamical instability inherent to 2D systems [23, 24], functionalization e.g. by OH groups [25] or doping [26]. The picture which emerges is that of graphene as a self-similar rippled membrane.

One way to characterize such structure is via a height-correlation function $(h(r) - h(0))^2$ which exhibits $r^{2H}$ scaling for small values of $r$ and eventually saturates at twice...
the standard deviation of the height, \( \text{std}(h) = \sqrt{\langle h^2 \rangle} \) (we assume \( \bar{h} = 0 \)). The scaling exponent \( H \) is related to the fractal dimension of the rippled surface and assumes values ranging from 0.5 for exfoliated [20] to 0.7–1.0 for suspended [23, 24, 27] samples. The larger values of \( H \) in the latter case indicate locally smoother structures. The lateral size of surface features can be characterized by the correlation length \( \xi \), defined as the rollover point of the height-correlation function\(^1\), varying between several and a few tens of nanometers [24, 20, 22]. Finally, the size of out-of-plane fluctuations can be described using the standard deviation of the height, \( \text{std}(h) \), usually equal to a few Ångströms.

The most direct mechanism by which rippling modifies the electronic structure of graphene is the alteration of bond lengths (and consequently hopping integrals). Interestingly, in the effective mass approximation the resulting modifications to the Hamiltonian have the form of a random gauge field [28–30], whose presence may be responsible for the suppression of weak localization observed in some graphene samples [31]. Furthermore, the presence of ripples can lead to fluctuations of the charge density and, in particular, the formation of electron and hole puddles in globally neutral samples. Gibertini et al [32] calculated the deformation potential for geometries obtained from Monte Carlo simulations [23, 27] and found it and the local charge density to vary on a length scale of 1–2 nm with no apparent correlation with the height variation\(^2\). Similar length scales were also obtained in the \textit{ab initio} calculations of [34]. Kim and Castro [25] and Gazit [35] on the other hand obtained a close correlation between the charge fluctuations and the local curvature of the graphene sheet. In this case the effective scalar potential varies on length scales comparable to the coherence length. In all cases the amplitudes of the scalar potential variation were on the scale of tens of meV. On the experimental side, electron and hole puddles were observed in scanning tunneling microscopy (STM) studies. However, the interpretation of their origin is a matter of ongoing discussion [36, 37].

While qualitative estimates of scattering rates associated with ripples were presented in [17], little in the way of explicit calculations of transport properties appears in the literature to date. Simple one-dimensional models were studied in [38, 39]. More sophisticated structural models were used in [40, 41] but the effect of rippling was found to be modest compared to that of scattering on charged impurities. However, in both cases the modeling of the ripples was tailored to the specific case of relatively smooth structures [23], characterized by \( H = 1 \). The aim of the present contribution is to study the transport properties for the structures over the whole range of \( H \) and \( \text{std}(h) \) values. In the following paragraphs I will outline the details of the method of calculations and then present the results for both ballistic and diffusive regimes. On the basis of these I argue that the scattering on ripples has measurable impact on transport properties and leads in fact to resistance and mobility values compatible with experimental data.

\(^{1}\) The height-correlation function typically behaves approximately like \( \langle (h(r) - \bar{h})^2 \rangle \sim 2 \langle h^2 \rangle (1 - e^{-r^2/\xi^2}) \).

\(^{2}\) Longer length scales were obtained by the same group [33] when their formalism was applied to the experimental structure of [21].

2. The method

The calculations were performed using a single-band tight-binding model with hopping restricted to nearest neighbors only. The effect of rippling was included in the Hamiltonian by modification of the hopping constants. A simple scaling formula was adopted to this end:

\[
 t(r) = t_0 (a_0/r)^{-1/3} \tag{1}
\]

where \( r \) is the interatomic distance, \( a_0 = 1.42 \text{ Å} \) is the equilibrium lattice constant and \( t_0 = t(a_0) \) equals \(-2.8 \text{ eV} \). The choice of scaling formula is further discussed in appendix A.

The scattering geometry was employed with a system consisting of a rippled region sandwiched between flat-graphene leads. The leads were assumed to be heavily doped with the Fermi energy located 1 eV over the neutrality point. For such systems, coefficients of transmission, \( t_{\nu\nu} \), between the incoming state \( \nu \) in the left electrode and outgoing state \( \mu \) in the right electrode were calculated using the wavefunction matching method [42, 43]. These were subsequently used in the Landauer–Büttiker formula for the conductance:

\[
 G_{\text{LB}} = \frac{e^2}{h} \sum_{\mu\nu} |t_{\mu\nu}|^2. \tag{2}
\]

The self-similar landscape of the scattering region was generated using the diamond–square algorithm [44], described in appendix B, as multiples of 20 nm \( \times \) 20 nm non-repeating tiles. This yields the correlation length of \( \xi \approx 10 \text{ nm} \). The exponent \( H \) and \( \text{std}(h) \) were adjustable parameters. Two examples of generated surfaces are shown in figure 1. The honeycomb lattice of graphene was mapped onto the resulting surface with no adjustments to the in-plane positions of the carbon atoms.

In some calculations a fluctuating scalar potential \( V_{s}(r) \), included via the on-site elements of the tight-binding Hamiltonian, was taken into account. In contrast to the case for [35, 32, 33, 25, 34], where the calculated local potentials were explicitly dependent on the geometry, a simplified approach was adopted here. The potentials were generated independently of the geometry, using the same diamond–square algorithm but with the Fermi energy located 1-2 eV below the neutrality point. For such systems, coefficients of transport, \( t_{\nu\nu} \), between the incoming state \( \nu \) in the left electrode and outgoing state \( \mu \) in the right electrode were calculated using the wavefunction matching method [42, 43]. These were subsequently used in the Landauer–Büttiker formula for the conductance:

\[
 G_{\text{LB}} = \frac{e^2}{h} \sum_{\mu\nu} |t_{\mu\nu}|^2. \tag{2}
\]

The self-similar landscape of the scattering region was generated using the diamond–square algorithm [44], described in appendix B, as multiples of 20 nm \( \times \) 20 nm non-repeating tiles. This yields the correlation length of \( \xi \approx 10 \text{ nm} \). The exponent \( H \) and \( \text{std}(h) \) were adjustable parameters. Two examples of generated surfaces are shown in figure 1. The honeycomb lattice of graphene was mapped onto the resulting surface with no adjustments to the in-plane positions of the carbon atoms.

In some calculations a fluctuating scalar potential \( V_{s}(r) \), included via the on-site elements of the tight-binding Hamiltonian, was taken into account. In contrast to the case for [35, 32, 33, 25, 34], where the calculated local potentials were explicitly dependent on the geometry, a simplified approach was adopted here. The potentials were generated independently of the geometry, using the same diamond–square algorithm but with the Fermi energy located 1-2 eV below the neutrality point. For such systems, coefficients of transport, \( t_{\nu\nu} \), between the incoming state \( \nu \) in the left electrode and outgoing state \( \mu \) in the right electrode were calculated using the wavefunction matching method [42, 43]. These were subsequently used in the Landauer–Büttiker formula for the conductance:

\[
 G_{\text{LB}} = \frac{e^2}{h} \sum_{\mu\nu} |t_{\mu\nu}|^2. \tag{2}
\]

The self-similar landscape of the scattering region was generated using the diamond–square algorithm [44], described in appendix B, as multiples of 20 nm \( \times \) 20 nm non-repeating tiles. This yields the correlation length of \( \xi \approx 10 \text{ nm} \). The exponent \( H \) and \( \text{std}(h) \) were adjustable parameters. Two examples of generated surfaces are shown in figure 1. The honeycomb lattice of graphene was mapped onto the resulting surface with no adjustments to the in-plane positions of the carbon atoms.

In some calculations a fluctuating scalar potential \( V_{s}(r) \), included via the on-site elements of the tight-binding Hamiltonian, was taken into account. In contrast to the case for [35, 32, 33, 25, 34], where the calculated local potentials were explicitly dependent on the geometry, a simplified approach was adopted here. The potentials were generated independently of the geometry, using the same diamond–square algorithm but with the Fermi energy located 1-2 eV below the neutrality point. For such systems, coefficients of transport, \( t_{\nu\nu} \), between the incoming state \( \nu \) in the left electrode and outgoing state \( \mu \) in the right electrode were calculated using the wavefunction matching method [42, 43]. These were subsequently used in the Landauer–Büttiker formula for the conductance:

\[
 G_{\text{LB}} = \frac{e^2}{h} \sum_{\mu\nu} |t_{\mu\nu}|^2. \tag{2}
\]
The value of $3$ reduced in the latter case. This correlates with the structures clearly in figure 3(a). Comparison of results obtained for $\sigma$ functional dependence of $\sigma$ effect increases for higher values of std ($\approx \frac{4e^2}{\hbar^2}$) in flat graphene), as a function of std($h$). The descent of the curves gets slower with increasing $H$; however, even for $H = 1$, the effect is non-negligible. The dependence of the conductivity on the standard deviation of the scalar potential, std($V_s$), is presented in the inset of figure 4 for $H = 0.5$ and std($h$) = 2 Å. One sees that while both types of potential lead to decrease in the conductivity, the ‘long-wavelength’ type B has a significantly stronger effect.

Interestingly, the region around the neutrality point ($V_G \approx 0$) in figure 3 seems not to be affected by rippling. This is confirmed in figure 5 where the minimal conductance, $\sigma_{\text{min}} = \sigma(V_G = 0)$, is shown as a function of $H$ and std($h$). The main surface plot corresponds to the case with no additional scalar potential. We observe that $\sigma_{\text{min}}$ assumes values close to the universal value of $\frac{4e^2}{\hbar^2}$, predicted for flat graphene [45], over a large plateau and decreases only for std($h$) exceeding the $H$-dependent threshold value. Increasing the length of the scattering region ($L$) yields essentially the same plateau region (results not shown) with a steeper descent outside. Adding the type A scalar potential does not, for std($V_s$) $\lesssim 0.1$ eV, change the value of the minimal conductance but reduces the size of the plateau. Example results are shown in figure 5 using the red line for $H = 0.5$ and std($V_s$) = 0.05 eV. The type B potential, which varies laterally on the same length scale as structural corrugations, is capable of changing not only the stability but also the value of the minimal conductance as demonstrated by the blue curve in figure 5 (the same parameters apply). The observed increase of $\sigma_{\text{min}}$ in the presence of the slowly varying scalar potential can be qualitatively understood by noticing that in this case a carrier travels through a series of locally conductive regions (i.e. electron and hole puddles). For specific combinations of $H$, std($h$) and std($V_s$), $\sigma_{\text{min}}$ can achieve a value of $\frac{4e^2}{\hbar^2}$, often seen in exfoliated samples [2]. It should be noted, however, that such values are no longer universal in my calculations, i.e. in addition to depending on the above mentioned variables they also depend on the sample dimensions.

Following the usual convention, the results of figures 3 and 5 are presented as conductivities, $\sigma$. It should be understood, however, that these results characterize a specific

---

**Figure 1.** Two examples of surfaces generated using the diamond–square algorithm. The scaling exponent $H$ was set to 0.5 and 1.0 in panels (a) and (b) respectively. The standard deviation std($h$) = 3 Å was used in both cases.

**Figure 2.** Scalar potential generated using the diamond–square algorithm with $H_V = 1.0$ and std($V_s$) = 50 meV using tiles of 2 nm $\times$ 2 nm (type A).

**3. Results**

The calculated conductivity $\sigma = G_L B L^2$ (for $L = 100$ nm and $W = 20$ nm) is shown in figure 3 as a function of the distance between the Fermi level and the neutrality point in the scattering region. This quantity, $V_G$, would be controlled by the gate voltage in experiments. The convention used here is that positive values mean electron doping in the central part. It is immediately apparent that the conductance can be significantly lowered by the presence of the ripples. The effect increases for higher values of std($h$). Additionally the functional dependence of $\sigma$ can also change and deviate from $\sigma \sim V_G$, typical for the ballistic regime, as seen most clearly in figure 3(a). Comparison of results obtained for $H = 0.5$ and 1.0 reveals that the back-scattering is greatly reduced in the latter case. This correlates with the structures characterized by higher $H$ values being locally smoother (see figure 1). This point is further illustrated in figure 4 where the conductivities for three values of $H$ are shown, for a fixed value of $V_G = 0.1$ eV (corresponding to the electron density $n = 10^{12}$ cm$^{-2}$ in flat graphene), as a function of std($h$). The descent of the curves gets slower with increasing $H$; however, even for $H = 1$, the effect is non-negligible.

It should be understood, however, that these results characterize a specific
Conductivity $\sigma = G_{LB}$ of the scattering region with $L = 100$ nm and $W = 20$ nm calculated as a function of $V_G$, the distance between the Fermi level and the neutrality point in the scattering region. Different curves correspond to the different values of std$(h)$.

**Figure 3.**

Conductivity as a function of std$(h)$ for fixed $V_G = 0.1$ eV and different values of the scaling exponent $H$. The inset: conductivity as a function of the standard deviation of the scalar potential std$(V_s)$ for $H = 0.5$, $V_G = 0.1$ eV. The results for type A (solid line) and type B (dashed line) potentials are presented.

![Graph](image-a)

**Figure 4.**

Conductivity as a function of std$(h)$ for fixed $V_G = 0.1$ eV and different values of the scaling exponent $H$. The inset: conductivity as a function of the standard deviation of the scalar potential std$(V_s)$ for $H = 0.5$, std$(h) = 2$ Å and $V_G = 0.1$ eV. The results for type A (solid line) and type B (dashed line) potentials are presented.

![Graph](image-b)

**Figure 5.**

Minimal conductivity $\sigma_{\text{min}} = \sigma(V_G = 0)$ plotted as a function of $H$ and std$(h)$. The plateau values correspond to the universal value of $\sigma_{\text{min}} \approx 4e^2/h\pi$. No scalar potential was present for the main surface plot. The results, including a fluctuating scalar potential, are shown as red and blue lines for type A and type B potentials (with std$(V_s) = 0.05$ eV and $H = 0.5$) respectively.

![Graph](image-c)

Having calculated the resistivity we can now estimate the mobilities, $\mu$, corresponding to figure 6(b). Using $\rho^{-1} = e\mu$.
and assuming $n = 10^{12} \text{ cm}^{-2}$, appropriate for flat graphene with a chosen $V_G$, we obtain the values of $\mu$ shown in figure 7. The range of experimental values, measured on various substrates$^4$ and in suspended graphene, is also indicated in the figure. The STM studies reveal that exfoliated graphene on $\text{SiO}_2$ substrate [20] is characterized by $H \approx 0.5$ and std($h$) $\approx 2$–3 Å. These ranges of parameters correspond in my calculations to the mobility in the $\mu = 2000$–8000 cm$^2$ V$^{-1}$ s$^{-1}$ range, which is somewhat below experimental values, typically equal to 10000–15000 cm$^2$ V$^{-1}$ s$^{-1}$ $^2$ [2, 4, 5]. The values of the mobility are even more underestimated for the range of parameters corresponding to suspended graphene flakes, where calculations [23, 27, 24] predict $H \approx 0.7$–1.0 and std($h$) of several ångströms. This puts the calculated mobilities (see the corresponding curves in figure 7) in the 1000–10000 cm$^2$ V$^{-1}$ s$^{-1}$ range, whereas much higher values of over 100000 cm$^2$ V$^{-1}$ s$^{-1}$ were reported experimentally $^6$–$^9$.

One possible reason for overestimation of the scattering strength in my calculations may be the lack of structural relaxation in the model. It has been in fact demonstrated in the literature that relaxation of internal strain leads to a substantial reduction of the effect of rippling on the electronic structure of the sample $^{50}$.

4. Conclusions

In this paper I have studied the transport properties of rippled graphene using a structural model parameterized by the scaling exponent $H$ and the standard deviation std($h$) of the out-of-plane positions of carbon atoms. Both in ballistic and ohmic regimes, a substantial reduction of conductivities was already found for relatively modest amplitudes of corrugation. The range of resistivities and mobilities calculated in the ohmic

regime overlaps with experimental values. The mobility is however consistently underestimated when direct comparison is made, possibly because of the limitations of the structural model. The value of the minimal conductivity was found to be largely unchanged and equal to the universal value $\sigma_{\text{min}} = \frac{4e^2}{h\pi}$ when only the effect of changing the bond lengths was considered. The fluctuating scalar potential was found to decrease the conductance for doped samples and increase the minimal conductance for globally neutral samples. In conclusion, the results presented in this paper demonstrate that the scattering on ripples can lead to substantial reduction of the mobility of carriers in graphene. This indicates that carrier mobility can be improved by optimizing the geometry of the substrate, by reduction of either its characteristic scaling exponent $H$ or the standard deviation std($h$) of the height.

Acknowledgments

This work was supported by the Polish Ministry of Science and Higher Education as research project no. N N202 199239.
The author wishes to thank A Starikov for making his sparse-matrix computer programs available and S Krompiewski for many fruitful discussions.

**Appendix A. The scaling of the hopping constant**

A simple formula for the scaling of the hopping constant used throughout the paper

\[ t(r) = t_0 \left( \frac{r}{a_0} \right)^{-3} \]  

(A.1)

is a special case of a general \( t_{ij} \sim r^{-l-l' -1} \) scaling (with \( l = l' = 1 \) for the graphene \( \pi \)-band) motivated in turn by the scaling properties of the Hamiltonian of the linear muffin tin orbital (LMTO) [52] method. Other choices, frequently utilized in the literature, include a quadratic scaling suggested by Harrison [53]

\[ t(r) = t_0 \left( \frac{r}{a_0} \right)^{-2} \]  

(A.2)

and a more involved formula of Pereira et al [54]

\[ t(r) = t_0 e^{-3.37 \left( \frac{r}{a_0} - 1 \right)}. \]  

(A.3)

The performances of all three scaling formulas are compared in figure A.1 for both contracted (middle panel) and expanded (bottom panel) lattice constants. Additionally the results of tight-binding (TB) calculations were benchmarked against \textit{ab initio} full potential linearized augmented plane wave (FP LAPW) ones [55, 56]. From the results shown in figure A.1 it is clear that none of the formulas is consistently better than the others within the scope of the model employed in this paper. In the case of the contracted lattice constant, the best agreement between \textit{ab initio} and TB results is given by (A.2) with the other two being indistinguishable. However, the situation is reversed for the expanded lattice constant, where the \textit{ab initio} bands are best reproduced by either (A.3) (along the M–K direction) or (A.1) (along K–\( \Gamma \)) with (A.2) clearly the worst of the three. A generalized version of Harrison’s scaling suggested in [57] does not yield consistent improvement in the present case.

In the effective mass approximation, as used e.g. in [28–30], a linear scaling of the hopping constant is assumed and only the slope, usually parameterized as the logarithmic derivative

\[ \beta = - \frac{d \ln t}{d \ln r} \bigg|_{r=a_0}, \]

enters the theory. In this picture, (A.1)–(A.3) correspond to \( \beta \) equal to 3, 2 and 3.37 respectively. The theoretical and experimental estimates of \( \beta \) values found in the literature range between 1.1 and 3.6 [58–60].

**Appendix B. The diamond–square algorithm**

The diamond–square algorithm, first proposed in [44], is an iterative procedure for generating fractal landscapes with a chosen value of the scaling exponent \( H \). Consider a square with some predefined (possibly zero) corner values. The first iteration, shown in figure B.1, consists of two steps:

- **diamond**: the central value of the square is calculated as
  \[ h_5 = \sum_{i=1,...,4} h_i + \text{rand}(d) \]
  where \text{rand}(d) is a random variable with normal distribution\(^5\) centered at zero and with the standard deviation equal to \( d \);
- **square**: here one calculates the central values of the diamonds in an analogous fashion.

Cyclic boundary conditions can be used in the first iteration or alternatively one can skip the points outside of the original rectangle. Both steps are repeated the required number of times for squares and diamonds with dimensions

---

\(^5\) Alternatively a uniform distribution over \([-d, d]\) can be used.
The same is true for farther coordination zones, not shown in the plot. The peculiar, non-symmetric form of the peaks is an artifact of the model applied, namely the freezing of in-plane coordinates. A more natural looking shape could be generated e.g. by applying random three-dimensional shifts to each atom. Such a procedure does not seem to significantly alter the transport properties (results not shown here), provided that the amplitudes of random movements are kept small. I opted however not to include this step in my calculations, in order to reduce the number of arbitrary parameters in the model.

References

[1] Novoselov K S, Geim A K, Morozov S V, Jiang D, Zhang Y, Dubonos S V, Grigorieva I V and Firsov A A 2004 Science 306 666–9
[2] Novoselov K S, Geim A K, Morozov S V, Jiang D, Katsnelson M I, Grigorieva I V, Dubonos S V and Firsov A A 2005 Nature 438 197–200
[3] Castro Neto A H, Guinea F, Peres N M R, Novoselov K S and Geim A K 2009 Rev. Mod. Phys. 81 109–62
[4] Zhang Y B, Tan Y W, Stormer H L and Kim P 2005 Nature 438 201–4
[5] Chen J H, Jang C, Xiao S, Ishigami M and Fuhrer M S 2008 Nature Nanotechnol. 3 206–9
[6] Bolotin K I, Sikes K J, Hone J, Stormer H L and Kim P 2008 Phys. Rev. Lett. 101 096802
[7] Du X, Skachko I, Barker A and Andrei E Y 2008 Nature Nanotechnol. 3 491–5
[8] Bolotin K I, Sikes K J, Jiang Z, Fudenberg G, Hone J, Kim P and Stormer H L 2008 Solid State Commun. 146 351–5
[9] Elias D C et al 2011 Nature Phys. 7 701–4
[10] Lin Y M, Jenkins K A, Valdes-Garcia A, Small J P, Farmer D B and Avouris P 2009 Nano Lett. 9 422–6
[11] Liao L, Lin Y C, Bao M, Cheng R, Bai J, Liu Y, Qu Y, Wang K L, Huang Y and Duan X 2010 Nature 467 305–8
[12] Lin Y M et al 2011 Science 332 1294–7
[13] Schweizer F 2010 Nature Nanotechnol. 5 487–96
[14] Kim W, Pasanen P, Rikken J and Lipsanen H 2012 Nanotechnology 23 115201
[15] Adam S, Hwang E, Rossi E and Sarma S D 2009 Solid State Commun. 149 1072–9
[16] Ponomarenko L A, Yang R, Molihiinden T M, Katsnelson M I, Novoselov K S, Morozov S V, Zhukov A A, Schedin F, Hill E W and Geim A K 2009 Phys. Rev. Lett. 102 206603
[17] Katsnelson M and Geim A 2008 Phil. Trans. R. Soc. A 366 195–204
[18] Meyer J C, Geim A K, Katsnelson M I, Novoselov K S, Booth T J and Roth S 2007 Nature 446 60–3
[19] Stolyarova E, Rim K T, Ryu S, Maultzsch J, Kim P, Brus L E, Heinz T F, Hybertsen M S and Flynn G W 2007 Proc. Natl Acad. Sci. USA 104 9209–12
[20] Ishigami M, Chen J H, Cullen W G, Fuhrer M S and Williams E D 2007 Nano Lett. 7 1643–8
[21] Geringer V, Liebmann M, Echttermeyer T, Runte S, Schmidt M, Rickkamp R, Lemme M C and Morgenstern M 2009 Phys. Rev. Lett. 102 076102
[22] Cullen W G, Yamamoto M, Burson K M, Chen J H, Jang C, Li L, Fuhrer M S and Williams E D 2010 Phys. Rev. Lett. 105 215504
[23] Pasolino A, Los J H and Katsnelson M I 2007 Nature Mater. 6 858–61
[24] Abedpour N, Neek-Amal M, Asgari R, Shabhazi F, Nafari N and Tabar M R R 2007 Phys. Rev. B 76 195407
[25] Kim E A and Neto A H C 2008 Europhys. Lett. 84 5007
[26] Gazit D 2009 Phys. Rev. B 79 113411
[27] Los J H, Katsnelson M I, Yazeyev O V, Zakharchenko K V and Fasolino A 2009 Phys. Rev. B 80 121405
[28] Guinea F, Horovitz B and Le Doussal P 2008 Phys. Rev. B 77 205421
[29] Guinea F, Katsnelson M I and Vozmediano M A H 2008 Phys. Rev. B 77 205422
[30] Vozmediano M, Katsnelson M and Guinea F 2010 Phys. Rev. B 80 125437
[31] Morozov S V, Novoselov K S, Katsnelson M I, Schedin F, Ponomarenko L A, Jiang D and Geim A K 2006 Phys. Rev. Lett. 97 016801
[32] Gibertini M, Tomadin A, Polini M, Fasolino A and Katsnelson M I 2010 Phys. Rev. B 81 125437
[33] Gibertini M, Tomadin A, Guinea F, Katsnelson M I and Polini M 2012 Phys. Rev. B 85 201405
[34] Partovi-Azar P, Nafari N and Tabar M R R 2011 Phys. Rev. B 83 165434
[35] Gazit D 2009 Phys. Rev. B 80 161406
[36] Deshpande A, Bao W, Miao F, Lau C N and LeRoy B J 2009 Phys. Rev. B 79 205411
[37] Zhang Y, Brar V W, Girir C, Zettl A and Crommie M F 2009 Nature Phys. 5 722–6
[38] Isacsson A, Jonsson L M, Kinaret J M and Jonson M 2008 Phys. Rev. B 77 035423
[39] Zwierzycki M 2012 Acta. Phys. Pol. A 121 1246–9
[40] Klos J W, Shylau A A, Zozoulenko I V, Xu H and Heinzl T 2009 Phys. Rev. B 80 245432
[41] Zhu W and Lv B 2013 Phys. Lett. A 377 1649–54
[42] Xia K, Zwierzycki M, Talanana M, Kelly P J and Bauer G E W 2006 Phys. Rev. B 73 064420
[43] Zwierzycki M et al 2008 Phys. Status Solidi b 245 623–40
[44] Fournier A, Fussell D and Carpenter L 1982 Commun. ACM 25 371–84
[45] Tworzydło J, Trauzettel B, Titov M, Rycerz A and Beenakker C W J 2006 Phys. Rev. Lett. 96 246802
[46] Sharvin Y V 1965 Zh. Eksp. Teor. Fiz. 48 984
[47] Sharvin Y V 1965 Sov. Phys.—JETP 21 655 (Engl. transl.)
[48] Datta S 1995 Electronic Transport in Mesoscopic Systems (Cambridge: Cambridge University Press)
[49] Xu P X and Xia K 2006 Phys. Rev. B 74 184206
[50] Dean C R et al 2010 Nature Nanotechnol. 5 722–6
[51] Wehling T O, Balatsky A V, Tsvetik A M, Katsnelson M I and Lichtenstein A I 2008 Europhys. Lett. 84 17003
[52] Berger C et al 2006 Science 312 1191–6
[53] Andersen O K, Jepsen O and Glötzl D 1985 Canonical description of the band structures of metals Highlights of Condensed Matter Theory International School of Physics ‘Enrico Fermi’ (Varenna, Italy) ed F Bassani, F Fumi and M P Tosi (Amsterdam: Elsevier) pp 59–176
[54] Harrison W A 2004 Elementary Electronic Structure (Singapore: World Scientific)
[55] Pereira V M, Castro Neto A H and Peres N M R 2009 Phys. Rev. B 80 045401
[56] Singh D 2006 Planewaves, Pseudopotentials, and the LAPW Method (New York: Springer)
[57] Blaha P, Schwarz K, Madsen G, Kvasnicka D and Luitz J 2001 WIEN2K: An Augmented Plane Wave + Local Orbitals Program for Calculating Crystal Properties (Austria: Techn. Universität Wien)
[58] Xu C H, Wang C Z, Chan C T and Ho K M 1992 J. Phys.: Condens. Matter 4 6047
[59] Suzuura H and Ando T 2002 Phys. Rev. B 65 235412
[60] Castro Neto A H and Guinea F 2007 Phys. Rev. B 75 045404
[61] Cappelluti E and Profa G 2012 Phys. Rev. B 85 205436