Stability analysis of large slenderness ratio horizontal hydraulic cylinder

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Abstract. An improved two sections pressure bar method (ITSPBM) was presented to analyze the stability of the large slenderness ratio horizontal hydraulic cylinder. The friction moments, the bearing reactions at the two hinge joints as well as the clearances between the piston and the inner wall of the cylinder were taken into consideration. The result shows that the stability safety factor is 6.20. Meanwhile, in the finite element model of the horizontal hydraulic cylinder, the nonlinear friction force and clearances were involved. The result reveals that the stability safety factor is 9.24. Through comparing the results of the ITSPBM, the traditional two sections pressure bar method (TTSPBM), the finite element method (FEM) and the method in NB/T 35020-2013, it suggests that the stability of the large slenderness ratio horizontal hydraulic cylinder meet the actual engineering requirements and the friction moments at the two hinge joints can extremely enhance the stability.

1 Introduction

Under the condition of large axial load, the stability problem of large slenderness ratio horizontal cylinder is critical for the safety and reliability of the whole structure design. As for the large slenderness ratio horizontal hydraulic cylinder, it is simplified as equal section mechanical model in engineering design to analyze the stability according to the Euler formula. In this algorithm, the hydraulic cylinder and the piston rod are fixedly connected. The clearance between the piston rod and the guide sleeve, the friction moments at the two hinge joints and the bearing reactions are not considered. The calculation results cannot fully reflect the deflection and buckling characteristics of the large slenderness ratio horizontal hydraulic cylinder. GUO Ying-long et al [1] put forward two sections pressure bar mechanical model and the calculation formula of the buckling critical load is derived taking into account the reactions between the piston rod and the hydraulic cylinder and the stiffness of the hydraulic cylinder. Considering the initial deflection of the hydraulic cylinder and the piston rod, self-weight, and the friction moments at the two hinge joints, P J Gamez -Montero et al [2-3] derived the calculation formula of the buckling critical load and verified the variation law of the buckling critical load with the initial deflection. The results of the tests show that the influence of the friction moments cannot be ignored when the clearance between the pivot and sleeve is enough small. Under the actual working conditions of the hydraulic cylinder system, the axial pressure exerted by the oil in the hydraulic cylinder applies on both ends of the piston rod. Based on the mechanical model, WU Ting-ping [4] considered the bearing reactions existing at both ends of the hydraulic cylinder and calculated the stability of the
hydraulic cylinder. S Baragetti [5] established the numerical model of the buckling critical load, considering initial deflection of the hydraulic cylinder, the stiffness of the wear rings between the piston and the inside walls of the cylinder, the piston rod and the guide sleeve, as well as the friction moments at the two hinge joints. The results show that the stability of the hydraulic cylinder and the piston rod can be improved apparently with the increase of the stiffness of wearing pieces. The nonlinear stability analysis of three steps pressure bar was carried out by I G Raftoyiannis [6] using the finite element analysis software ALGOR on the basis of theoretical analysis. The results show that the initial defect obviously cuts down the ultimate bearing capacity of bars. Considering the initial geometric defect and nonlinear material characteristics, D Yang [7] used ABAQUS to analyze the partial and global buckling load. E Narvydas [8] established TRD (tube and rod diameter) finite element model taking into account the material nonlinearity and geometry nonlinearity. P V Binh [9] analyzed the stability of the thin-walled structure under complex loading condition through the energy method and evaluate the accuracy of the results with the finite element method.

In present study, the slenderness ratio of the horizontal hydraulic cylinder is set to 111.0. An improved two sections pressure bar method was established, considering the friction moments, the bearing reactions at the two hinge joints of the large slenderness ratio horizontal hydraulic cylinder as well as the clearances between the piston and the inner wall of the cylinder. The corresponding transcendental equations for calculating the buckling critical load was derived. The stability safety factor of the large slenderness ratio horizontal hydraulic cylinder was calculated by the ITSPBM, the TTSPBM, the FEM and the method in NB/T 35020-2013 respectively and the calculation results were compared.

2 An improved two sections pressure bar method

2.1 Establishment of mechanical model

The primary parameters of the large slenderness ratio horizontal hydraulic cylinder: the distance between two hinges \( L_z = 11210 \text{ mm} \), the length of the piston rod \( L_d = 10241 \text{ mm} \), the diameter \( d = 380 \text{ mm} \), the external diameter of the cylinder \( D_1 = 770 \text{ mm} \), the internal diameter of the cylinder \( D = 640 \text{ mm} \), the length of outer left hinge joint of the cylinder \( L_0 = 7190 \text{ mm} \). The slenderness ratio can be calculated to be 111.0 approximately. The cylinder outside the hinge joint and self-weight were not considered in the analysis for convenience.

![Figure 1. Mechanical model of large slenderness ratio horizontal hydraulic cylinder.](image)

The mechanical model of the hydraulic cylinder is shown in Fig.1. \( L_1 = L_z - L_d \), \( L_2 = L_z - L_1 \), \( \delta \) is the maximum deflection of the hydraulic cylinder. \( \mu \) is the radius clearance between the piston and the inside wall of the cylinder. The axial pressure \( P \) exerted on the piston rod is not on the same horizontal line as axial reaction of the right hinge joint when the deflections of the hydraulic cylinder occur. In order to maintain the balance, bearing reactions \( R_1 \) and \( R_2 \) at the two hinge joints of the large slenderness ratio horizontal hydraulic cylinder are produced respectively. \( f_1 \) is the friction coefficient of the hinge joint at the cylinder. \( f_2 \) is the friction coefficient of the hinge joint at the piston rod. \( M_1 \) is the friction moment at the hinge joint of the cylinder and \( M_2 \) is the friction moment at the hinge joint of the piston rod.

The friction moments at two hinge joints of the large slenderness ratio horizontal hydraulic cylinder are much less than the buckling critical axial pressure. Consequently, the influences of the friction moments were not considered in the calculation of the bearing reactions \( R_1 \) and \( R_2 \). \( R_1 \) and \( R_2 \) were
indicated as:

\[ R = R_1 = R_2 = \frac{P\delta}{(L_1 + L_2)} \]  

\( I_1 \) and \( I_2 \) are the moment of inertia of the cylinder and the piston rod, respectively. Differential equations of calculation for deflection were established in two parts. Equation (2) and (3) solution can be obtained as:

\[ EI_1 \psi_1'' = M_1 - Rx_1, \quad 0 \leq x_1 \leq L_1, \]
\[ EI_2 \psi_2'' + Py_2 = M_2 + Rx_2, \quad 0 \leq x_2 \leq L_2 \]

General solutions of equation (2) and (3):

\[ y_1 = \frac{M_1}{2EI_1} x_1^2 - \frac{x_1^3}{6EI_1} + A_1 x_1 + B_1, \quad 0 \leq x_1 \leq L_1, \]
\[ y_2 = A_2 \sin(k_2x_2) + B_2 \cos(k_2x_2) + \frac{M_2}{k_2^2EI_2} x_2^2 + \frac{M_1}{k_2^2EI_1}, \quad 0 \leq x_2 \leq L_2 \]

The boundary conditions were set as follows:

\[ x_1 = 0, y_1 = 0 \]  
\[ x_2 = 0, y_2 = 0 \]  
\[ x_1 = L_1, x_2 = L_2 \]  
\[ y_1 = y_2 = \delta, y_1' = -y_2' \]  

The boundary conditions (6) was added into the equations (4)(5). Then, the model that results in:

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
L_1 & 1 & 0 & 0 & -\frac{k_2^2L_1^3}{6(L_1 + L_2)} - 1 & \frac{L_1^2}{2EI_1} & 0 \\
0 & 0 & \sin(k_2L_2) & \cos(k_2L_2) & \frac{k_2^2L_1}{k_2^2L_1 + L_2} - 1 & 0 & 1 \\
1 & 0 & k_2\cos(k_2L_2) & -k_2\sin(k_2L_2) & \left(\frac{1}{L_1 + L_2}(1 - \frac{k_2^2L_1^2}{2})\right) & \frac{L_1}{EI_1} & 0 \\
\end{bmatrix}
\begin{bmatrix}
A_1 \\
B_1 \\
A_2 \\
B_2 \\
\delta \\
M_1 \\
M_2 \\
\end{bmatrix} =
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
\end{bmatrix}
\]

The transcendental equations for calculating the buckling critical load of the large slenderness ratio horizontal cylinder was obtained when the coefficient determinant of equation (7) is zero. \( M_1 = Pf_1d_1 \), \( M_2 = Pf_2d_2 \). \( d_1 \) and \( d_2 \) were substituted in equation (7) which are the diameters of two hinge joints respectively. \( \delta \) is the maximum deflection of the large slenderness ratio horizontal hydraulic cylinder.

\[
\delta = \frac{\sin(k_1\sqrt{f_1d_1}L_1)(L_1 + L_2)}{2L_1 + L_2}
\]

The relation between \( \delta \) and \( \mu \) can be obtained according to the geometrical connection condition.
\[ \delta - y(L_2) = \mu \]  

(9)

\( y(L_2) \) is the deflection of the large slenderness ratio horizontal cylinder under the buckling critical load without considering the clearance.

\[ y(L_2) = \sin(k_2 \sqrt{1 - f_2 d_2} L_2) \]  

(10)

\( \mu = 0.095 \text{mm} \) and the transcendental equations of the buckling critical load of the large slenderness ratio horizontal hydraulic cylinder is obtained according to (8) (9) (10).

\[ \sin(k_1 \sqrt{f_1 d_1} L_1) (L_1 + L_2) + (2L_1 + L_2) \sin(k_2 \sqrt{1 - f_2 d_2} L_2) - 0.095(2L_1 + L_2) = 0 \]  

(11)

2.2 Calculation of stability safety factor

The analytical solution cannot be obtained in equation (11) and it is difficult to converge in the solution of Newton iteration algorithm with two order convergence rate. An improved Newton iteration algorithm with seven order convergence rate was adopted to achieve effective convergence solutions. According to actual conditions, the friction coefficient \( f_1 \) is set to 0.015, \( f_2 \) is set to 0.1 as well as the radius clearance is 0.095 mm. The buckling critical load was solved to be 19838 kN and the stability safety factor can be obtain to be 6.20. The variation relationship between the friction coefficient \( f_2 \) at the hinge joint of the piston rod and \( \mu \) was shown in Table 1.

The buckling critical load decreases with the decrease of the friction coefficient \( f_2 \) or the increase of the clearance, because the friction moment at the hinge joint can reduce the initial deflection of the large slenderness ratio horizontal cylinder so as to enhance its overall stability. However, the initial deflection of the large slenderness ratio horizontal hydraulic cylinder is produced due to the existence of the clearance between the piston and the inside wall of the cylinder, which reduces the overall stability.

| Influence factor | Value | Result of stability safety coefficient |
|------------------|-------|----------------------------------------|
| Friction coefficient \( f_2 \) | 0.08  | 6.14                                   |
|                  | 0.06  | 6.08                                   |
|                  | 0.02  | 5.96                                   |
| Radial clearance \( \mu \) | 0.08  | 6.26                                   |
|                  | 0.06  | 6.34                                   |
|                  | 0.05  | 6.38                                   |

3 Finite element method to calculate stability safety factor

3.1 Relevant settings about finite element calculation

Figure 2. Schematic diagram of FEM calculation

The schematic diagram of finite element calculation is shown in Figure 2, the nonlinear contact was set at the clearance position in figure according to the actual clearances between the piston rod and the guide sleeve, the piston and the inner wall of the cylinder. The actual pin bearings at the two hinge joints of
the large slenderness ratio horizontal hydraulic cylinder were established in the finite element model. More specifically, the nonlinear friction contact was built at the fit between the pin and the pin-bushing in accordance with the theoretical friction coefficients so as to make the friction moments more similar with the actual engineering condition. Hence, the model in the finite element method is closer to the large slenderness ratio horizontal hydraulic cylinder actual engineering condition compared with the mechanical model in the ITSPBM.

3.2 Calculation of stability safety factor
The effect of the self-weight can be eliminated through repeated iterative computation in the FEM used to calculate the buckling critical load of the large slenderness ratio horizontal hydraulic cylinder. It was necessary to repeat calculation iterations in keeping the self-weight load unchanged in each iteration and adjust the axial load continuously until buckling load factor is equal to or close to 1.0 and the applied axial load at this moment is exactly the buckling critical load of the large slenderness ratio horizontal hydraulic cylinder.

After several calculation iterations, the buckling critical load of the large slenderness ratio hydraulic cylinder is 92.0 MPa and the stability safety factor is 9.24 which satisfied the requirement of stability when the diameter of the piston rod is 380 mm.

4 Comparison of stability calculation method
According to the method in NB/T 35020-2013, the buckling critical load of the large slenderness ratio horizontal hydraulic cylinder is 18726 kN and the stability safety coefficient is 5.85. The buckling critical load of the large slenderness ratio horizontal hydraulic cylinder is 17430 kN the stability safety coefficient is 5.45 in accordance with the TTSPBM. The stability safety coefficients of the large slenderness ratio horizontal hydraulic cylinder obtained by different methods were listed in Table 2.

| Calculation method | Buckling critical load | Stability safety factor |
|--------------------|------------------------|------------------------|
| ITSPBM             | 19838 kN               | 6.20                   |
| TTSPBM             | 17430 kN               | 5.45                   |
| FEM                | 29569 kN               | 9.24                   |
| NB/T 35020-2013    | 18726 kN               | 5.85                   |

All of the stability safety factors satisfy the actual engineering requirement and were calculated by four different methods including the ITSPBM, the TTSPBM, the FEM and the method in NB/T 35020-2013, respectively. The difference between the ITSPBM and the TTSPBM, the method in NB/T 35020-2013 is 13.76% and 5.97%, respectively. Therefore, the differences indicate that the TTSPBM and the method in NB/T 35020-2013 are conservative relatively. The bearing reactions, the friction moments at the two hinge joints of the large slenderness ratio horizontal hydraulic cylinder as well as the clearance between the inner wall of the cylinder and the piston were involved in the ITSPBM. Hence, the combination of the friction moments at the two hinge points and the bending moments caused by bearing reactions at the two hinge joints reduce the initial deflection and strengthen the stability of the large slenderness ratio horizontal hydraulic cylinder.

Compared with the FEM, the calculation of the friction moments at the hinge joints of the large slenderness ratio horizontal hydraulic cylinder is more simple relatively in the ITSPBM. Besides, the nonlinear characters between the piston and the inner wall of the cylinder, the piston rod and the guide sleeve are neglected in the ITSPBM. As for the FEM, the friction moments at the two hinge joints of the large slenderness ratio horizontal hydraulic cylinder were simulated to be more similar with the actual engineering stress in the structure. Moreover, the nonlinear friction forces and clearances between the piston and the inner wall of the cylinder, the piston rod and the guide sleeve were covered in the FEM. Consequently, the result of the FEM is different greatly from other three methods. More specifically, the result of the FEM is 57.95% higher than that of the method in NB/T 35020-2013 (5.85 in comparison with 9.24). It comes into a conclusion that the friction moments at the two hinge joints
have a significant impact on the buckling critical load and extremely enhance the stability of large slenderness ratio horizontal hydraulic cylinder by comparing the results of the other three methods.

5 Conclusion
1) In this paper, the ITSPBM was used to calculate the stability of the large slenderness ratio horizontal hydraulic cylinder. The calculation results show that the stability of the large slenderness ratio horizontal hydraulic cylinder is enhanced by increasing the friction coefficient at the hinge joint of the piston rod and reducing the clearance between the piston and the inner wall of the cylinder.

2) All of the stability safety factors meet the actual engineering requirement. The calculation result of the ITSPBM is higher than that of the TTSPBM (6.20 in comparison with 5.45). Similarly, the result of the ITSPBM is higher than that of the NB/T 35020-2013 (6.20 in comparison with 5.85). The TTSPBM and the method in NB/T 35020-2013 are conservative relatively. Therefore, the ITSPBM and the FEM can be used for preliminary calculation and the method in NB/T 35020-2013 can be used for check subsequently in design.

3) Compared with the results of the ITSPBM, the TTSPBM, the FEM and the method in NB/T 35020-2013, the result of the FEM is much higher than other three methods, 57.95% higher than the method in NB/T 35020-2013. It is demonstrated that the friction moments at the two hinge joints have a greater impact on the buckling critical load and cannot be neglected. The ultimate load capacity of the large slenderness ratio horizontal hydraulic cylinder can be enhanced through increasing the friction coefficients at the two hinge joints in design.

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