Emergence of agent swarm migration and vortex formation through inelastic collisions

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Abstract. Biologically inspired models of self-propelled interacting agents display a wide variety of collective motion such as swarm migration and vortex formation. In these models, active interactions among agents are typically included such as velocity alignment and cohesive and repulsive forces that represent agents’ short- and long-range ‘sensing’ capabilities of their environment. Here, we show that similar collective behaviors can emerge in a minimal model of isotropic agents solely due to a passive mechanism—inelastic collisions among agents. The model dynamics shows a gradual velocity correlation build-up into the collective motion state. The model displays a discontinuous transition of collective motion with respect to noise and exhibits several collective motion types such as vortex formation, swarm migration and also complex spatio-temporal group motion. This model can be regarded as a hybrid model, connecting granular materials and agent-based models.
1. Introduction

The agent-based studies of collective motion of animals such as flocks of birds, schools of fish [1], colonies of insects [2, 3] and also vortex motion in bacterial colonies [4, 5] is a rapidly developing field of research [6], with results also applied to human crowds and even vehicular traffic flow [7]. In these models, animals are commonly modeled by agents utilizing active interactions such as velocity correlation, cohesive and repulsive forces. In the minimal model of Vicsek et al. [8], collective motion emerges in a group of agents where each agent aligns its velocity according to the average velocity direction of the agents surrounding it. It was shown that this active interaction produces collective migration—all agents moving in the same randomly chosen direction. Biologically oriented models apply more specific active mechanisms to reproduce the actual behavior observed in various specific groups of animals or colonies [2, 3, 5, 9, 10]. It should be emphasized that in these models the mechanism that leads to the collective motion represents implicit ‘sensing’ and information-processing capabilities of the agents enabling them to respond to the average velocity vector and the average agent density over a certain distance. While the inclusion of such active agent mechanisms is well justified from the biological perspective, actual interaction mechanisms can vary widely even for the simplest microorganisms such as motile bacteria: from chemotaxis [11] to hydrodynamic alignment [5, 12]. One of the fundamental questions arising in this context is what are the minimal interactions required and how crucial are active mechanisms in the emergence of agent collective motion.

This question also applies to the observed emergence of collective motion in various systems of inanimate objects. In particular, collective behavior of the ‘self-propelled particles’ (SPPs) has also been investigated experimentally and theoretically in the context of granular materials [13]–[15]. Here, inanimate grains are propelled mechanically by a vibrating base of a container. In this case, the particles become self-propelled due to either shape anisotropy (rods) [16] or anisotropic mass distribution (asymmetric dumbbells). Moreover, under certain conditions, the combination of the rod’s vertical tilt and a vibrating bottom results in directed mechanical motion in the direction of the rod’s tilt [15]; hence, an equivalent self-propulsion force is attained. While the connection between these very different systems and groups of animals might appear questionable at first, there seems to be a connection embedded in the underlying physics: there is a resemblance between the collective motion of elongated...
swimming bacteria [12, 17] and grains of rice being vibrated on a vertical shaker [13, 18]. In both cases, the emergence of dynamical structures much larger than the particle (or bacteria) length scale has been reported. Clearly, the two systems are essentially different as is simply reflected by the fact that the bacteria are truly self-propelled and self-organized, while the inanimate system requires an external driving force and imposed constraints. However, a comparison between such remote living and azoic systems might provide some valuable insights into the behavior of living systems.

The study presented here is motivated by a simple question: what are the minimal passive (or physical) mechanisms that can also lead to the emergence of collective behavior and whether such mechanisms might also be employed by living systems. To be able to deduce from experimental observations what is the nature of the underlying mechanisms, it is important to determine whether there are qualitative differences between collective motions generated by passive versus active mechanisms in terms of the onset of motion and its statistical properties.

Pattern formation in granular systems (see [19] for an extensive review), and in particular the similarities in dynamics of anisotropic grains and elongated bacteria, makes it plausible that there might be simple passive or physical mechanisms that affect the coherent motion in both the inanimate granular systems and the biologically inspired agent models. Here, to identify possible passive mechanisms, we start from a model that contains no explicit active mechanisms—the agents do not possess any sensing capabilities and they have no information regarding their surroundings. As opposed to previous granular models, our model is perfectly isotropic: the agents and the interactions do not have any distinguishable orientation or direction. We find that even in such a minimal model of self-propelled agents collective motion can be generated by a simple passive mechanism of inelastic collisions. We show that this mechanism can display many of the basic collective behaviors generated by the active mechanisms in the biologically inspired agent systems: onset of coherent motion, large-scale vortices, etc.

2. The model

The self-propelled isotropic agents are represented by round smooth inelastically colliding disks of unit mass \( m = 1 \) moving on a two-dimensional (2D) frictionless flat surface. This passive interaction mechanism is described by the spring-dashpot model [20, 21] in which an agent’s normal and tangent accelerations during collision with another agent are given by (see figure 1):

\[
\begin{align*}
\ddot{x}_n &= -K_{nd} \cdot r_{ab}^\alpha \cdot r_{nab} - K_{nr} \cdot r_{ab}^\beta, \\
\ddot{x}_t &= -\min(|\mu \cdot \dot{x}_n|, \dot{x}_t \cdot K_{td}).
\end{align*}
\]  

The normal repulsion force is modeled by a spring pulling the agents away from each other with a spring constant \( K_{nr} \), and depending \((\beta = 1.5)\) on the overlap between the two colliding agents \( r_{ab} \). A friction force proportional to the relative velocity \((\alpha = 0.5)\) reduces kinetic energy with constant \( K_{nd} \). In the tangent direction, a friction force is also applied, and it is limited to be no larger than the normal force multiplied by a factor \( \mu \). Usually, \( K_{nd} = K_{td} \), and it is convenient to express the degree of dissipation in each interaction by the normal restitution coefficient: \( r \), the ratio of pre- and post-collision velocity magnitudes in the normal direction \((r = 1\) is a totally elastic collision and \( r = 0\) is a totally inelastic collision).
Each agent also possesses a self-propulsion force that compensates for the energy lost in the course of collisions and prevents the system from immanent freezing:

\[
\dot{x}_{sp} = \begin{cases} 
0 & \frac{|\dot{x}|}{|\dot{x}|} \geq 1 \\
-\frac{c}{\dot{x}} \cdot \hat{x} & \frac{|\dot{x}|}{|\dot{x}|} \leq 1
\end{cases}
\]

(2)

The self-propulsion force is applied in the direction of the current agent velocity, and its magnitude \(c\) is set to 1. An agent traveling at a speed greater than unity loses its self-propulsion.

Simulations of the model show that this simple system shows many modes of observed coherent motion, while utilizing only simple inelastic collisions as an interaction between agents.

3. Vortex formation

When reflecting boundary conditions are imposed, the ensemble of agents exhibit a range of collective behaviors: from the formation of vortices to ordered migration (traveling back and forth between system boundaries) and also random chaotic-like motion of subgroups. Examples of vortex formation are shown in figure 2 for both circular and square boundaries. In the circular system, the aggregate quickly organizes into a steady vortex with zero density in the center. Angular velocity distribution is similar to that of a rigid rotating disk. Since boundaries are perfectly reflecting, the velocity perpendicular to a wall is suppressed in the course of collisions with other agents on the way back from the wall. As a consequence, the velocity parallel to the wall survives and becomes a natural attractor for the dynamics. The circular system boundaries conform to parallel velocity perfectly, and a tight vortex is formed. Square geometries also lead to vortex formation, though the corners cause some disturbances and the vortex becomes more loosely packed.

4. Ordered migration and chaotic motion

Changing the system density and the aspect ratio of the system boundaries (i.e. from a circular to an elliptic system) results in different types of collective motion. For large enough aspect ratios,
there is a shift into a migration mode, with the entire aggregate moving back and forth along the longer axis (figure 3(a)), in what roughly resembles harmonic motion (figure 3(b)). When the aggregate collides with a wall at the end of its motion along the longer axis of the container, a density shock wave propagates back through the aggregate. This shock wave reverses the velocity of the aggregate faster than the velocity of the group.

For certain densities and aspect ratios, the system exhibits nontrivial spatio-temporal behavior of compact subgroups of agents depicted in figure 4(b). This state, which we shall refer to as ‘chaotic’, does not display a single system spanning the vortex (see figure 4(b) versus figure 4(a)), and does not show steady migration motion (as can be seen in average velocity and position plots in figures 4(c) versus figure 3(c)). This type of collective behavior appears to be a competition between vortex and migration motion modes, and does not converge into either of them.
5. Collective migration in periodic boundary conditions

To further investigate the properties of the collective motion and its connections with previous agent models, we considered a system with periodic boundary conditions: an agent wandering out of the system boundaries is repositioned at the opposite boundary with the same velocity. Systems of agents were simulated with various densities and friction coefficients. All systems gradually evolved into a state where all agents travel in the same direction (the collective migration state). The natural order parameter characterizing this state is the normalized average velocity:

$$\langle v \rangle = \frac{1}{N} \left| \sum_k \tilde{v}_k \right|,$$

where $v_k$ is the velocity vector of agent $k$, and $N$ is the number of agents in the system. When all the agents travel in the same direction at unit velocity, the order parameter is 1. The gradual increase of the order parameter in a simulation of a system with 1200 agents is shown in figure 5.
Figure 4. The reflecting boundary system exhibits a transition from vortex motion ((a) with $N = 1300$, $r = 0.85$, $D = 0.8$ and $AR = 1 : 3$) to chaotic behavior (b) with $N = 1000$, $r = 0.85$, $D = 0.5$ and $AR = 1 : 4$). The bottom panel (c) shows average position and velocity in the $x$-direction of the chaotic system in panel (b). A movie of panel (b) is available from stacks.iop.org/NJP/10/023036/mmedia.

The evolution towards steady state (figures 5(a)–(f)) starts with cooling, where random initial velocities are destroyed through collisions, and kinetic energy is decreased (figure 5(a)). Shortly thereafter, small localized structures appear (figure 5(b)). These consolidate into larger structures (figure 5(c)), and soon only a few competing structures remain (figure 5(d)). Kinetic energy is gradually increased as can be seen from the length of the position traces in figures 5(a)–(d). Convergence to steady state takes place on a larger timescale, with the system spanning structures slowly consolidating into a single directed collective motion with $\langle v \rangle = 1$ (figures 5(e)–(f)).

We propose that the results can be interpreted as if the inelastic interaction coupled with the self-propagation serves the equivalent role of the traditional velocity alignment force in the biologically inspired agent models (see the supplementary material for a more detailed explanation, available from stacks.iop.org/NJP/10/023036/mmedia). The fact that inelastic collisions create velocity correlation between colliding inanimate particles in the context of granular systems is well known, see e.g. [22], and this is the key ingredient of this model. The fundamentals of the present model can be outlined as follows (see figure 1): inelastic collisions diminish opposing (and therefore anti-correlated) velocities, thus enhancing post-collision velocity correlation. This is equivalent to an indirect velocity alignment force, with a kinetic energy ‘penalty’ for particles that collide with non-aligned velocities. The self-propulsion force then acts to regain the loss of kinetic energy after collisions and eliminates freezing of the system.
Figure 5. Time evolution of the system with periodic boundary conditions with $N = 1200$ agents, $d = 0.6$ and $r = 0.93$. The top panel shows the evolution of the order parameter $\langle v \rangle$—the normalized average velocity. Bottom panels show position traces of agents, during a small time window corresponding to the points (a)–(f) marked in the top panel. A movie of this figure is available from stacks.iop.org/NJP/10/023036/mmedia.

It is well known that spatio-temporal noise acting on the agents plays a significant role in the dynamics of SPP systems [23]. Since the mechanism behind the velocity correlation in our model is inelastic collisions, it is important to check whether this mechanism is robust enough to mediate collective motion in the presence of noise. We therefore studied the case in which our model is augmented with a stochastic random force: in accordance with [8], this force randomly rotates the velocity of the agents at each time step. Simulating over a wide range of densities and noise amplitudes, we have found that the system was able to maintain the state of collective migration—high values of $\langle v \rangle$ (figure 6). For the large enough noise amplitudes, our system exhibits what appears to be a first-order phase transition, very similar to that described in [23].
Figure 6. Phase space of a periodic boundary system with noise. $N = 2500$, $r = 0.93$, no tangential friction. Left: order parameter versus noise at a given density. Right: order parameter with respect to density at given noise. The system exhibits apparent first-order phase transition.

6. Discussion

Our results show that a model, using simple inelastic collisions between the self-propelled agents, provides a wide range of self-organized collective behaviors: vortex formation, uniform migration and chaotic spatio-temporal motion. The model clearly demonstrates emergent collective motion of agents closely resembling systems with velocity correlation forces and agent-sensing capabilities, such as in [8, 23].

The underlying reason why coherent motion is attained in our model is that simple inelastic collisions between isotropic particles create an increase in velocity correlation each time two particles collide. The gradual build-up of correlations through myriads of collision events facilitates an effective velocity alignment force, without any active sensing or steering capability of the agents. Emergence of collective motion is insensitive to the exact values of the interaction parameters used for the simulation (i.e. $K_{nr}$, $K_{nd}$, $\alpha$, $\beta$ in (1)), and it is also robust enough to facilitate collective motion in the presence of noise, despite the delicate nature of the correlation mechanism. Agent models with active velocity alignment forces display a phase transition of the migration state with respect to noise magnitude or number density [8, 23]. While the order of the phase transition is still under debate for the case of agents with active interaction, our system appears to display a first-order phase transition.

Obviously, our model is a minimal one, since no cohesion forces were used, and finite density is maintained by the use of reflecting or periodic boundaries. This was done in order to simplify the model as much as possible and to ‘distil’ the minimal features needed for the emergence of collective motion. While implementation of cohesive forces is relatively straightforward, it remains for further investigation. It can be stated, however, that inelastic collisions are sufficient for the onset of collective motion.

Relating to ‘SPP’ models, our model also shows that there is no need, from the theoretical perspective, for interaction to be anisotropic, as was included in [13]–[15]. This point might only be of theoretical value, since isotropic SPPs might be very hard to come by in any experimental realization of inanimate systems.

Another model that is analogous to our model is studied in [24]. In this paper, pattern formation of microtubules is modeled by polar rods. Since the rods are not self-propelled, the order parameter for this system is the average rod orientation. Although the system is
anisotropic, the interaction rule is very similar: rods ‘collide’, and as a result the difference in their orientation is decreased by a finite amount. The system displays a rich phase space of disordered pattern, vortices and asters, analogous to disordered motion, vortices and migration motion in our system.

It can be argued that animals do indeed display collision avoidance behavior that can be modeled by velocity correlation forces. In this respect, the simplicity of this model might be considered as a bridge between a living system (with agents having complex sensing and steering capabilities) and an inanimate system of dissipative particles. As a closing remark, our system might also make itself more approachable to rigorous analysis.

7. Materials and methods

The model was studied using a so-called soft particles or a force-driven molecular dynamics simulation (the integration of Newton’s equations of motion in small time steps), with the number of agents, \( N \), typically around 1000–2000. Simulations were written in C++, and based on algorithms described in [25, 26].

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