Statistical Characteristics of the Received Signal for Stochastic Surface Models

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1 Surface model

Reflected signal has some variations in structure, which appear due to scattering features of the surface. In mapping, for example, it is possible to separate useful signals (reflected from the surface) and interference signals (scattered from hydrometeors, clouds, and atmosphere turbulence). And receiver noise, atmospherics, discretization errors may be considered as an equivalent to the interference signals.

The accurate relationship between the received signal and the surface parameters may be derived by means of electromagnetic equations for the selected surface model. But in common case the solution of such equation is very difficult, especially for the real, complicated surfaces.

Moreover, in real condition the reflected signal is always fluctuating due to aspect angle changing, surface vibration, beam jitter, unstable SAR motion and so on. These facts allow using stochastic surface models instead of functional-determined ones. These stochastic models are characterised by the fluctuations of the scattered electromagnetic field.

Let’s write input signal as follows:

\[ u(t) = S_0 |F(\hat{r}, \hat{\lambda}(\hat{r}))| S(t, \hat{r})| + n(t) \]

where \( S_0 \) denotes signal reflected from the surface; \( F(\hat{r}, \hat{\lambda}(\hat{r})) \) denotes the complex scattering function of the element \( \hat{r} \) with electrophysical parameters \( \hat{\lambda}(\hat{r}) \); \( S(t, \hat{r}) \) denotes the point target response in case \( F(\hat{r}, \hat{\lambda}(\hat{r})) = 1 \); \( n(t) \) denotes additive noise, gaussian with zero mean and constant noise power spectral density \( N_0/2 \).

With stochastic surface models, the coefficient \( F(\hat{r}, \hat{\lambda}(\hat{r})) \) is a random, non-stationary in space. Adequate definition of this process is very difficult, so usually the real scene is represented by some simplified models. One of these models

\textbf{KEY WORDS} SAR; surface model; statistical characteristics; estimation

\textbf{ABSTRACT} This paper describes the stochastic model of the scattered electromagnetic field. Unlike common-used functional-determined models the proposed is characterised by amplitude/phase fluctuation of the received signal.

This paper derives the statistical characteristic of the input signal and describes algorithm for its estimation in post-processing and real-time processing modes. Achieved characteristics allow the mapping and estimation of the surface models more accurate, moreover, such processing increase space resolution of synthetic aperture radar.
is formed by assuming the surface as a set of
elementary (point) scatters with random dispositions.
So, the mapping of this complex function gives un-
stable results. The correct mapping may be
achieved by using statistical characteristics of
$F[\hat{r},\hat{\lambda}(\hat{r})]$, one of which is a radar cross section
$\sigma[\hat{r},\hat{\lambda}(\hat{r})]$.

The signal reflected from $i$-th point reflector
(with space coordinates $\hat{r}$) is given as:

$$S(t,\hat{r}) = G(t,\hat{r})K(t,\hat{r})S_0[t-t_3(t,\hat{r})] \cdot$$

$$\exp{j\omega_0[t-t_3(t,\hat{r})]}$$

(1)

where $G(t,\hat{r})$ denotes the antenna pattern; $K(t,$
$\hat{r})$ denotes the signal aberration in atmosphere;
$S_0(t) = S_0(t)\exp{j\theta(t)}$ denotes the complex
envelope of the signal; $\theta(t)$ denotes the phase
modulation; $t_3$ denotes the delay time.

With the simplified surface model, the received
signal will be the addition of point signals $S_{ni}(t)$
with noise $n(t)$, i.e.,

$$u(t) = \Re \sum_{n} S_{ni}(t) + n(t).$$

Even if scattering probability of each point is ran-
dom with unknown probability density function, the
received signal will be gaussian. Experimental data
shows that the received signal can be accurately
approximated by gaussian process, as shown in
Reference [1].

Progression from the point representation to the
continuous surface model may be attained by the
integration of elementary scatters of infinitesimal
sizes $d\hat{r}$ as follows:

$$u(t) = \Re \int_{\hat{r}} \hat{S}(t,\hat{r}) d\hat{r}$$

(2)

Finally, from Eqs. (1) and (2) the received signal
can be expressed as:

$$u(t) = \Re \int_{\hat{r}} F[\hat{r},\hat{\lambda}(\hat{r})] \cdot$$

$$K(t,\hat{r})S_0[t-t_3(t,\hat{r})] \cdot$$

$$\exp{j\omega_0[t-t_3(t,\hat{r})]} \cdot d\hat{r} + n(t)$$

(3)

2 The statistical characteristics of
the reflected electromagnetic
field

As mentioned above, the input signal is gaussian.
For complete definition of such processes it is nec-
sessary to determine first and second statistical mo-
ments (mean and correlation function). Usually the
mean of the received signal is equal to zero
$\langle u(t) \rangle = 0$, so the main characteristic of the in-
put signal is a correlation function $R(t_1,t_2) =$$

$$R(t_1,t_2) = \left[ \Re \int_{\hat{r}} F[\hat{r},\hat{\lambda}(\hat{r})]S(t_1,\hat{r}) d\hat{r} +$$

$$n(t_1) \cdot \Re \int_{\hat{r}} F[\hat{r},\hat{\lambda}(\hat{r})]S(t_2,\hat{r}) d\hat{r} + n(t_2) \right].$$

After some transformations (random processes
$F[\hat{r},\hat{\lambda}(\hat{r})]$ and $n(t)$ are statistically indepen-
dent, $\langle n(t) \rangle = 0$):

$$R(t_1,t_2) = \frac{1}{2} \Re \int_{\hat{r}} F[\hat{r},\hat{\lambda}(\hat{r})] \cdot$$

$$F[\hat{r}',\hat{\lambda}(\hat{r}')] S(t_1,\hat{r}) S(t_2,\hat{r}') d\hat{r} d\hat{r}' +$$

$$\frac{N_0}{2} \delta(t_1 - t_2).$$

(4)

Let's consider random process $F[\hat{r},\hat{\lambda}(\hat{r})]$ as a
delta-correlated in space domain:

$$\langle F[\hat{r}_1,\hat{\lambda}(\hat{r}_1)] F[\hat{r}_2,\hat{\lambda}(\hat{r}_2)] \rangle \equiv$$

$$\sigma[\hat{r}_1,\hat{\lambda}(\hat{r}_1)] \delta(\hat{r}_1 - \hat{r}_2).$$

(5)

where $\sigma[\hat{r},\hat{\lambda}(\hat{r})]$ denotes the radar cross-sec-
tion (RCS); $\delta(\hat{r}_1 - \hat{r}_2)$ denotes the delta-function.

From Eqs. (4) and (5), it is easy to obtain:

$$R(t_1,t_2) = \frac{1}{2} \Re \int_{\hat{r}} \sigma[\hat{r},\hat{\lambda}(\hat{r})] S(t_1,\hat{r}) \cdot$$

$$\int_{\hat{r}} S(t_2,\hat{r}') d\hat{r}' + \frac{N_0}{2} \delta(t_1 - t_2).$$

(6)

Correlation function (CF) width is restricted by
double synthesis interval, and as usual this function
decreases while $t_1 - t_2$ increases. Moreover, it is
asymmetric and periodical.

In Fig. 1, $T_c$ denotes time of synthesis; $T_p$ de-
notes pulse repetition interval, $T_p$ denotes
pulsewidth.

"Signal" part of the CF $R_c(t_1,t_2) = \int_{\hat{r}} \sigma[\hat{r},$
$\hat{\lambda}(\hat{r})] \frac{1}{2} \Re \int S(t_1,\hat{r}) S(t_2,\hat{r}') d\hat{r}'$ may be consid-
ered as coefficients of series
\[
\sigma[\hat{r},\hat{\lambda}(\hat{r})] = \sum_{n=1}^N \frac{1}{2} \Re \int S(t_1,\hat{r}) S(t_2,\hat{r}') d\hat{r}'
\]
So, the estimation of the CF gives possibility of the RCS estimation.

With stochastic surface model, the estimation of
the optimal parameter is given in Reference [2]:
The estimation of the statistical characteristics

With an ensemble of $N$ independent realisations (snapshots) estimation is given by:

$$\hat{R}(t_1, t_2) = \frac{1}{N} \sum_{i=1}^{N} u_i(t_1) u_i(t_2)$$

(8)

The mean of this estimation is:

$$\langle \hat{R}(t_1, t_2) \rangle = \frac{1}{N} \sum_{i=1}^{N} \langle u_i(t_1) u_i(t_2) \rangle = \langle R(t_1, t_2) \rangle$$

The dispersion is:

$$D[\hat{R}(t_1, t_2)] = \langle [\hat{R}(t_1, t_2) - R(t_1, t_2)]^2 \rangle = \frac{1}{N^2} \sum_{i=1}^{N} \langle [\hat{R}_i(t_1, t_2) - R(t_1, t_2)]^2 \rangle$$

(9)

As we can see from the above equation that any finite values of the CF dispersion $D[\hat{R}(t_1, t_2)]$ and CF probability function density give consistent estimate of the CF if $N \to \infty$.

But often it is necessary to determine correlation function even if we have only one realisation—signal itself, in this case the one-assessment (for example in one-look SAR) is more difficult because of impossibly averaging. The estimating methods and characteristics are derived below.

3.1 Averaging over adjacent intervals

It is possible to estimate CF as an averaged value over adjacent intervals. Errors in this case depend on pulse repetition time, SAR velocity, RCS behaviour. Estimation error is given by:

$$\Delta R(t_1, t_2) \equiv \frac{1}{2} \text{Re} \int_{-\Delta}^{\Delta} d\tilde{r} \tilde{S}(t_1, \tilde{r}) S(t_2, \tilde{r}) d\tilde{r}$$

(10)
where $\Delta D$ is a space "offsetting" interval. Geometrical relations for averaging over 3 intervals are shown in Fig. 2.

The bias of this estimation is:
\[
\Delta R(t_1,t_2) = \frac{1}{2} \text{Re} \left[ \int_{\mathcal{D}} \delta(t_1,\tilde{\lambda}(\tilde{\tau})) S(t_1,\tilde{\tau}) S(t_2,\tilde{\tau}) d\tilde{\tau} \right]
\]

The dispersion is:
\[
D[R(t_1,t_2)] = \frac{1}{N^2} \sum_{i=1}^{N} [R_i(t_1,t_2) - R(t_1,t_2)]^2
\]
\[
R(t_1,t_2)^2 = \frac{[\Delta R(t_1,t_2)]^2}{N}
\]

3.2 Assuming input process as a periodically pseudo-stationary

If we consider input process as a pseudo-stationary in the one period of the CF, the estimation may be written as follows:
\[
R(t_1,t_2) = R(\tau; t_1, t_2) = \frac{1}{T_u} \int_{0}^{T_p} u(t_1; t_2) u(t_1; t_2 - \tau) dt_2 \quad (11)
\]

The bias is determined by simplifying input process to stationary. The dispersion of this estimation is:
\[
D[R(\tau; t_1, t_2)] = \frac{1}{(T_p)^2} \int_{0}^{T_p} \int_{0}^{T_p} \left[ u(t_3; t_1, t_2) u(t_4; t_1, t_2) - \tau; t_1, t_2 \right] dt_3 dt_4 - R^2(\tau; t_1, t_2)
\]

For the gaussian input process
\[
< u(t_3; t_1, t_2) u(t_4; t_1, t_2) > = R^2(\tau; t_1, t_2) + R^2(t_4 - t_3; t_1, t_2) + R(t_4 - t_3; t_1, t_2) R(t_4 - t_3 + \tau; t_1, t_2)
\]

Finally, the dispersion is given by:
\[
D[R(\tau; t_1, t_2)] = \frac{2}{T_p} \left[ 1 - \frac{\tau}{T_u} \right] \cdot \left[ R^2(z) + R(z - \tau) R(z + \tau) \right] dz
\]

The maximal value of the dispersion will be, if $\tau = 0$, equal to the dispersion of the estimation.

3.3 Surface statistical characteristic estimation

As we can see from Eq. (6), a kind of the possible CF estimation is an RCS $R_0[\hat{r},\tilde{\lambda}(\hat{r})]$ measuring. With a stochastic surface model it is useful to assume RCS stable in some space interval and achieve results by averaging in some space interval. On the one hand the bigger interval gives smaller dispersion of the estimation, but on the other hand the smaller interval gives higher resolution.

Assume an estimation of the RCS, as:
\[
\hat{R}_0[\hat{r}_0,\tilde{\lambda}(\hat{r})] = \text{Re} \left[ \tilde{\lambda}(\hat{r}) \right]
\]

This equation is an optimal output of the functional-determined model.

The CF estimation will be:
\[
\hat{R}_0(t_1,t_2) = \frac{1}{T_p} \int_{0}^{T_p} \left[ \hat{R}_0[\hat{r}_0,\tilde{\lambda}(\hat{r})] \cdot \text{Re}[S(t_1,\tilde{r})S(t_2,\tilde{r})] d\tilde{r} \right] \quad (12)
\]

Let $\xi(t_1,t_2,\hat{r}) = \frac{1}{2} \text{Re}[S(t_1,\tilde{r})S(t_2,\tilde{r})]$, so the mean is given by:
\[
< \hat{R}_0(t_1,t_2) > = \int_{0}^{\infty} < \hat{R}_0[\hat{r}_0,\tilde{\lambda}(\hat{r})] \cdot \xi(t_1,t_2,\hat{r}) > d\hat{r}
\]

In common case it is a biased estimate, and the bias is:

\[
B(R_0(t_1, t_2)) = \frac{1}{D} \mathbb{E} \left[ \frac{1}{D} \left| \mathbf{F} \left[ \tilde{r}_1, \tilde{\lambda}(\tilde{r}_1) \right] \right|^2 \right]
\]

The dispersion is:

\[
D(R_0(t_1, t_2)) = \int \mathbb{E} \left[ \left( R_0(t_1, t_2, \tilde{r}) - \mathbb{E}[R_0(t_1, t_2)] \right) \left( R_0(t_1, t_2, \tilde{r}') - \mathbb{E}[R_0(t_1, t_2)] \right) \right] d\tilde{r} d\tilde{r}'
\]

where \( R_0(t_1, t_2, \tilde{r}) \) is a correlation function of the scattering coefficient square fluctuations. Restriction of SAR resolution versus space-time harmonic \( \xi(t_1, t_2, \tilde{r}) \) brings up significant error in estimation from \( |t_1 - t_2| > T_1 \).

Accurate estimation may be achieved in recursive algorithm with Eq. (7), its structure is shown in Fig. 3.

**Fig. 3 Recurrent algorithm for CF estimation**

### 3.4 Real-time estimation

In real-time processing the CF estimation is difficult because of unknown behaviour RCS in the surface. A kind of estimation is given by:

\[
R_0(t_1, t_2) = A(t_1) \left\{ \frac{1}{N} \right. \mathbb{R} \left[ S(t_1, \tilde{r})S(t_2, \tilde{r}) \right] + \delta(t_1 - t_2) \left. \right\} \tag{13}
\]

where \( A(t_1) = \sigma^2(\tilde{r}) \) denotes the mean RCS of the scene at the moment \( t_1 \).

The mean is:

\[
B(R_0(t_1, t_2)) = \frac{1}{2} \mathbb{E} \left[ \left( \sigma^0(\tilde{r}, \tilde{\lambda}(\tilde{r})) - \sigma^0(\tilde{r}, \tilde{\lambda}(\tilde{r})) \right) \right]
\]

The dispersion is:

\[
D(R_0(t_1, t_2)) = \int \mathbb{E} \left[ \left( \sigma^0(\tilde{r}, \tilde{\lambda}(\tilde{r})) - \sigma^0(\tilde{r}, \tilde{\lambda}(\tilde{r})) \right) \left( \sigma^0(\tilde{r}, \tilde{\lambda}(\tilde{r})) - \sigma^0(\tilde{r}, \tilde{\lambda}(\tilde{r})) \right) \right] d\tilde{r} d\tilde{r}'
\]

where \( R_0(\tilde{r}, \tilde{r}) \) is a RCF correlation function.

More acceptable results may be achieved with average over adjacent intervals by using Eq. (13).

### 4 Modelling

Fig. 4 shows cross-section of the correlation function in coordinates \( t_1 - t_2 \). As mentioned above, generally this function is periodical, but for better interpretation its continuous analogue is shown in Fig. 4.

For higher reliability, results for each method are averaged over sets of surfaces (different \( \sigma^0(\tilde{r}) \)) and times \( (t_1, t_2) \).

In this picture:

- Trace 1: ensemble averaging, 50 realisations (Eq. (8));
- Trace 2: averaging over 9 adjacent intervals;
- Trace 3: assuming process as periodically pseudo-stationary (Eq. (11));
- Trace 4: surface statistical characteristics estimation (Eq. (12));
- Trace 5: recurrent surface statistical characteristics estimation (Fig. (3));
- Trace 6: real-time processing (Eq. (13));
- Trace 7: real-time processing with averaging over adjacent intervals.

As we see from this picture, real-time processing (Trace 7) gives results near to the ones obtained after the statistical characteristics estimation. As supposed, the statistical characteristics estimation gives CF errors when resolution of processing algorithm is less than frequency of the space harmonic \( \xi(t_1, t_2, \tilde{r}) \). For significant difference between \( t_1 \) and \( t_2 \) better results are given by the smoothed estimation of the RCS. Because of speckle-noise, whose space frequency may be near to the space frequency of \( \xi(t_1, t_2, \tilde{r}) \), the product of integration
proved throughout the entire image areas and that the brightness are improved only in the dark areas. The brightness difference between the light areas and the dark areas are reduced. However, both the brightness and the contrast are still lower in the dark areas than in the light areas, which means that the effects of non-uniform illumination still exist in the images after local filtering (also shown in Fig. 6). In addition, Fig. 3(c) shows more noises than those in the original image (Fig. 1(c)). This demonstrates that in the local window size (i.e. 11) should be greater in the low-density grains image than in the high-density grains image, which will consume more computing time.

![Image](a) the original moderate-density grains image  (b) after local filtering (c) after image subtracting (d) after enhanced stretching

**Fig. 6** Brightness curves vary with the x-locations

| Light areas | Moderate areas | Dark areas |
|-------------|----------------|------------|
| Original image | 205.92/46.90 | 132.97/41.53 | 69.47/40.41 |
| Local filtering algorithm | 166.01/63.80 | 131.14/50.78 | 97.91/50.68 |
| Image subtracting algorithm | 147.88/46.69 | 150.31/38.86 | 152.91/36.57 |
| Enhanced stretching algorithm | 163.94/96.83 | 156.19/88.79 | 174.80/105.83 |

As for the image subtracting algorithm (Fig. 4), it can be seen from Table 1 that the brightness in the dark areas are improved greatly, and are approximately equivalent all over the image areas. However, the contrast is reduced in the whole image areas, especially in the dark areas. Thus some of the dynamic range of the original image are lost (Fig. 6(c)). Though image subtracting algorithm is widely used in removing uneven illumination effects, it is not particularly suggested here.

As for the enhanced image stretching algorithm (Fig. 5), it can be seen from Table 1 that both the brightness and the contrast are improved in the dark areas. Result images in Fig. 5 show that with the method we can completely removes the effects of non-uniform illumination. However, there are more noises in Fig. 5(c) than in the original image (Fig. 1(c)), this is because the sampled points selected automatically to fit the brightness foreground might not be the grain pixels. A solution to this problem is to subdivide the image into fewer numbers of grids and select fewer sampled points.

### 5 Conclusion

This paper compares three methods to remove non-uniform illumination effects and three images were tested. The work shows that:

1) local filtering algorithm can only reduce the uneven illumination effects and might add noises to the low-density grains image;

2) image subtracting algorithm can remove uneven illumination effects on the brightness but cannot remove those on the contrast, and might lose some of the dynamic range of the original image;

3) the enhanced image stretching algorithm developed by the authors can balance the brightness and contrast all round the image areas and may be the most effective approach to non-uniform illumination effects approach to remove.

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in this case gives spikes.

Estimating error is necessary in further calculating the of the de-correlation operator \( W(t_1, t_2) \). Denote this error as \( \Delta R(t_1, t_2) \), so after calculation the bias appears in \( W(t_1, t_2) \). Generally, this means incomplete de-correlation and, decreased spatial resolution in comparison with accurate \( R(t_1, t_2) \) estimation.

5 Conclusion

This paper introduces stochastic models of the surface, which is characterised by fluctuation of the scattered electromagnetic field and input signal. Main characteristic of the input signal in this case is the correlation function. These methods may be used for post-processing and real-time processing. The results of modelling shows quite good reliability by the real-time algorithms in comparison with post-processing mode. This fact allows using such an algorithm in SAR for on-board processing and de-correlate input signal. De-correlation makes images statistically stable and increase the resolution of the image.

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