An Approach for Analyzing the Dynamic Strength Increment of Concrete Material

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Abstract: The Split Hopkinson Pressure Bar (SHPB) is usually used to get the dynamic compressive strength of concrete materials at strain-rate between $10^1$ and $10^3$ s$^{-1}$. It is widely accepted that the main influencing factors of dynamic strength include strain-rate effect and inertial effect in SHPB tests, which result in obvious scattered phenomenon existing in the test data. However, many empirical formulae of dynamic increase factor were directly obtained from the scattered test data to describe the material properties, which has been overestimated strain-rate effect for design analysis. In present paper, numerical simulations with a rate-independent material model are used to study this pseudo-strain-rate sensitive phenomenon because it is very difficult to quantify these influences by conducting laboratory tests. Then a unified approach for processing the scattered test data of normal concrete is available especially for mid-high strain rate. Moreover, the quantitative relations among the strain-rate effect, inertial effect and calibration test data have been obtained with the help of the corresponding numerical simulations, and semi-empirical formulae are also proposed to describe the inertial effect and strain-rate effect.

1. Introduction

The inertial effect and strain-rate effect on the dynamic strength of diversified concrete materials, such as mortar, concrete and geomaterial etc, have become non-ignorable factors in both the material constitutive model in numerical analysis and the design of the civil engineering. Actually the great attention has been paid to investigate the dynamic compressive strength of concrete material with help of Split Hopkinson Pressure Bar (SHPB) [1,2]. It is widely accepted that the dynamic strength of concrete material can be defined as the ratio of dynamic to static strength, i.e., the dynamic increase factor (DIF) [3]. A lot of DIF formulae based on laboratory test data are available [2-15]. These empirical formulae obtained directly by fitting test data may overestimate the dynamic compressive strength of concrete material, because the inertial effect has significant contributions to its dynamic strength increment at mid-high strain rate.

The objective of the present study is to propose a new approach for processing the scattered test data of normal concrete at mid-high strain rate. The numerical SHPB tests are performed to study inertia-induced strength increment of concrete materials by virtue of the rate-independent material model. The dynamic strength increment due to inertial effect will be quantified according to numerical results, and then the strength increment due to strain-rate effect only will be obtained from the relations between test data and inertial effect. The present studies are aimed at proposing a method to study the dynamic strength increment of concrete material.
2. Empirical formulae

Although DIFs of concrete material obtained from different SHPB tests seem to have similar increasing trend with the increase of strain rate, obvious scattered phenomenon of test data can be observed in the different experiments [16]. However, many empirical formulae obtained directly by fitting test data are listed in Table 1, which maybe overestimate the dynamic compressive strength of concrete material. For this matter, it is necessary to quantify the inertial effect and material strain-rate effect contributing to the dynamic compressive strength increment.

| Year | Formula | Notes |
|------|---------|-------|
| 2003 | \[ \text{DIF} = \frac{F_m}{W_r} - 1 + 1 \] | [10] |
| 2005 | \[ \text{DIF} = N_m(a_i) \] | [11] |
| 2007 | \[ \text{DIF} = 0.2583(\log \dot{e})^2 - 0.5076(\log \dot{e}) + 1.021 \] | [12] |
| 2013 | \[ \text{DIF} = 0.03438(\log \dot{e} + 3) + 1 \] | [13] |
| 2014 | \[ \text{DIF} = 1.274(\log \dot{e})^2 - 3.443(\log \dot{e}) + 3.976 \] | [14] |
| 2017 | \[ \text{DIF} = 1 + \frac{F_m}{1 + \exp(-\phi(\log \dot{e} - \log \dot{e}_c))} \] | [15] |

3. Theoretical formulae and numerical results

In this section, the relationship of the test data, strain-rate effect and inertial effect will be derived first and then a rate-independent material model [17] is used to carry out simulations of SHPB tests. Thus, the numerically obtained dynamic strength increment is only caused by inertial effect.

3.1 Derivation of the material DIF

The compressive dynamic increase factor \( \text{DIF} \) obtained from tests can be expressed as[18]

\[ \text{DIF}_c = \text{DIF}_{\dot{e}_c} + R_e - 1 \]

where \( \text{DIF}_{\dot{e}_c} \) and \( R_e \) are the strain-rate effect and inertial effect, respectively.

There is an assumption for normal concrete that the inertial effect is the strain rate and specimen size dependent, while the strain-rate effect is intrinsic property of material [18]. Hence, all test results...
of normal concrete can be summed up in a unified volume of specimen and the unified $DIF_{c\phi}$ can be calibrated by

$$DIF_{c\phi} = DIF_{c\phi} + R_{c\phi} - 1$$

where $R_{c\phi}$ is obtained from the simulations of standard volume specimen.

Then substituting the Eq. (1) into Eq. (2), it can be obtained

$$DIF_{c\phi} = DIF_{c\phi} + R_{c\phi} - R_c$$

In order to quantify the inertial effect, a formula is proposed to describe the $R_c$, which is similar to the empirical formulae used in Ref.[13], i.e.

$$R_c = 1 + \frac{F_m - 1}{\lambda_c} \left\{ \tanh \left( \left( \log(\dot{\varepsilon}) - W_c \right) S \right) - B_{c\phi} \right\}$$

where $\dot{\varepsilon}$ is the dimensionless strain rate, normalized with respect to a reference strain rate $\dot{\varepsilon}_0 = 1.0 \text{ s}^{-1}$; $B_{c\phi} = \tanh \left( \left( \log(\dot{\varepsilon}_0) / \dot{\varepsilon}_0 \right) - W_c \right) S$ is a static reference point in which $\dot{\varepsilon}_0 = 10^{-7} \text{ s}^{-1}$ is a quasi-static strain rate; $\lambda_c = 1 - B_{c\phi}$; $W_c$ and $S$ are the horizontal coordinate value and the tangent slope of the inflection point of the curve, respectively; $F_m$ denotes the ultimate value of $R_c$ when the strain rate goes to infinity.

As for material strain-rate effect, the compressive dynamic increase factor can be described more suitable as

$$DIF_{c\phi} = 1 + \frac{F_m - 1}{\lambda_c} \left\{ \atan \left( \left( \log(\dot{\varepsilon}) - W_c \right) S \right) - B_{c\phi} \right\}$$

where $B_{c\phi} = \atan \left( \left( \log(\dot{\varepsilon}_0) / \dot{\varepsilon}_0 \right) - W_c \right) S$, $\lambda_c = \pi/2 - B_{c\phi}$ and the other parameters are the same as Eq. (4).

### 3.2 Numerically obtained inertial effect

The static strength of 45.6MPa as a ‘standard concrete’ is adopted in present paper and the various parameters used in material model are the same as Ref.[19]. The results will be obtained from the simulations by virtue of three-wave method formula proposed by [20].

The inertia-induced $R_c$ obtained from numerical simulations are shown in Figure 1(a). Hereon, in order to obtain the $R_{c\phi}$ of the standard volume specimen, $\Phi 64\times 32$mm (diameter$\times$length) cylindrical specimen is assumed as a benchmark. The numerical results obtained from different specimen size can be unified to the standard volume by using Eq. (4) as shown in Figure 1(b) and the corresponding parameters are listed in Table 2.

![Figure 1](image-url)
Table 2. Various values of Eq. (4) for different $R_c$

| $DIF$ | $W_i$ | $S$ | $F_u$ |
|-------|-------|-----|-------|
| $R_{0a}$ | 3.44  | 1.2 | $F_u\text{=}10$ |
| $R_c$   | 3.44  | 1.2 | $F_u\text{=}0(V_i/V_o)^\alpha$ |

* $V_i$ and $V_o$ are the arbitrary and standard volume of specimen, respectively. $F_{0u}$ is ultimate value of the $R_{0a}$ with the standard volume specimen, when $V_i=V_o$, $F_m=F_{0u}$ and $R_c=R_{0a}$. In other cases, $F_m=F_{0u}\times(V_i/V_o)^\alpha$, in which the $\alpha=0.35$ is a fitting constant.

4. Application

The derived formula $R_c$ has been verified in previous publication [18]. In this section, the $DIF_{0a}$ and $DIF_{c0}$ will be obtained by virtue of Eqs. (3) and (2), respectively.

4.1 Application in the SHPB experiment

The test data obtained from different specimen sizes can be summed up in a unified specimen size ($\Phi51\times51\text{mm}$) by virtue of Eq. (3), in which the $DIF_s$ are the known test data as shown in Figure 2(a), and then the unified $DIF_{c0}$ can be replotted in Figure 2(b).

![Figure 2](image_url)

Figure 2. The test data obtained from different compressive experiments: (a) the scattered data obtained from [2,6,16,18-24]; (b) the unified test data by virtue of Eq. (3).

Figure 2(b) shows the unified experimental results are derived from Figure 2(a) by conversion Eq. (3). After unified test data, the scattered phenomenon of the compressive DIFs has got obvious improvement especially for mid-high strain rate, which means that the unified approach for processing the compressive data of concrete is available. Therefore, this effective approach can improve the degree of dispersion of compressive test data obtained from different specimen size.

4.2 Further discussion

For further studies on the material strain-rate effect, the true compressive strain-rate effect can be easily obtained by Eq. (2) as shown in Figure 3, in which the solid symbols (the other symbols represent validation data) have been taken as a benchmark to calibrate the true strain-rate effect. The corresponding parameters of $DIF_{c0}$ used in Eq. (5) are listed in Table 3.
Figure 3. The material strain-rate effect by eliminating inertial effect from compressive test data

Table 3. Various values for true strain-rate effect

| DIF | Wc | S | Fm |
|-----|----|---|----|
| DIF | 1.8 | 1.2 | 1.8 |

Theoretically the specimen size and static strength used in simulations should be the same as that of experiment to remove the inertial effect. Hereon, the test data [16] that is very close to the 'standard concrete' has been used to obtain the true strain-rate effect. As can be seen from Figure 3, the red-green solid symbols represent the true strain-rate effect as calibration data obtained from Eq. (2) and all the other symbols represent validation data only. Although good agreements are obtained as shown in Figure 3, there still exists many uncertainties about the experimental/numerical results [3]. It is obvious that the concrete with different strength grade should have different strain-rate effect. Therefore, further studies are necessary for fully understanding the inherent material behavior of concrete at mid-high strain rate. Hereon, we only propose an approach to analyze the mechanism of the dynamic strength increment for concrete material.

5. Conclusions

The present numerical simulations using the rate-independent material model show that the inertial effect increases with the volume of specimen at the same strain rate. This effect leads to an apparent strength increment for concrete material when the strain rate is beyond $10^2 \text{s}^{-1}$, which coincides with the experimentally observed strain rate sensitivity in SHPB tests. Therefore, the experimentally observed strain-rate effect is pseudo-strain-rate effect, which is mainly caused by inertial effect. Based on the numerical results, the inertia-induced $R_c$ has been quantified into a simple mathematical formula and then a unified approach has been derived to improve the degree of dispersion of compressive test data. Finally, material true strain-rate effect of normal concrete has been separated from SHPB test data. It reveals that previous used DIF formulae obtained directly from test data overestimate the dynamic compressive strength of concrete material at strain rate from $10^1$ to $10^3 \text{s}^{-1}$, because the inertial effect has significant contributions to its dynamic strength. Hereon, great precaution should be paid when a rate-dependent material model of concrete material is used in both structural design and numerical simulations.

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