Theory of the Normal State of Cuprate Superconducting Materials

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Abstract

We have proposed a model Hamiltonian, which describes a simple physical picture that the holes with single occupation constraint introduced by doping move in the antiferromagnetic background of the copper spins, to describe the normal state of the cuprate superconducting materials, and used the renormalization group method to calculate its anomalous magnetic and transport properties. The anomalous magnetic behavior of the normal state is controlled by both the copper spin and the spin part of the doping hole residing on the O sites. The physical resistivity is determined by both the quasiparticle-spin-fluctuation and the quasiparticle-gauge-fluctuation scatterings and the Hall coefficient is determined by the parity-odd gauge interaction deriving from the nature of the hard-core boson which describes the charge part of the doping holes.

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Since the discovery of the cuprate superconducting materials\textsuperscript{[1]} there has still considerable controversy over the choice of the appropriate microscopic Hamiltonian. Although there have appeared a lot of models, for example, the one-band effective Hubbard model\textsuperscript{[2]}, t-J model\textsuperscript{[3]}, three-band Hubbard model\textsuperscript{[4]}, phenomenological marginal Fermi liquid\textsuperscript{[5]}, nearly antiferromagnetic Fermi liquid\textsuperscript{[6]} and so on, to try to describe the normal and superconducting states of the cuprate superconducting materials, it is generally agreed now that Anderson’s starting point\textsuperscript{[2]}, namely, strongly on site Coulomb interactions among a partially filled band of Cu 3d level, is correct. The controversial point is that one is how to treat doping holes residing on O 2p level. At zero doping, it is generally agreed that the insulating ”parent” phases of the cuprate materials are charge-transfer insulators\textsuperscript{[7]} and can be described by quantum antiferromagnetic Heisenberg model. Substantial progress has been achieved in understanding of the Heisenberg limit, both theoretically and experimentally\textsuperscript{[8–10]}. Under a finite doping range, Zhang and Rice\textsuperscript{[3]} showed that the three-band Hubbard model can be reduced into a single band effective Hamiltonian—the t-J model under the case of spin singlet phase that hybridization strongly binds a hole on each square of O atoms to the central Cu\textsuperscript{2+} ions in a similar way as a hole in the single band effective Hamiltonian, then two holes feel a strong repulsion against residing on the same square. In fact, in this representation, the doping hole residing on the O site
only contributes its charge degree and its spin degree is completely confined to con-
struct the spin singlet with the hole on the Cu site. It is not clear to what extent it
is valid as one uses this representation to study the effect of the doping. The gauge
theory of t-J model[11–13] gives a good description of the transport properties of the
normal state of the cuprate superconducting materials, but it is difficult to give a
reasonable explanation to its anomalous magnetic behavior which is shown by the
nuclear magnetic resonance (NMR) and other experiments.

Recently, it has been shown by Sokol and Pines[14] using purely scaling considera-
tions, that the experimental data of Refs.[15-18] on the NMR spin lattice relaxation
rate $T_1$ and spin echo decay rate $T_{2G}$ in the cuprates imply a quantum critical(QC),
$z = 1$ ($z$ a dynamical exponential), behavior over a broad doping range, and in low
temperature a quantum disorder(QD), $z = 1$, behavior. The crossover to the $z = 2$
regime, occurs only in the fully doped materials. This unusual magnetic behavior
reminds one that even in a broad doping range, the magnetic behavior of the system
is still determined by the critical point there appearing in the zero doping limit. The
unusual physical properties of the normal state may derive from its strongly antifer-
romagnetic correlation behavior. The doping will destroy the long range antiferro-
magnetic correlation, but the system still remains the short range antiferromagnetic
correlation behavior even in the superconducting state, which induces the system
shifting from renormalized classical (RC) regime to QC or QD regimes.

It is clear that the remarkably anomalous behavior of the normal state of the cuprates is mainly on the following aspects in a wide doping range: a. It shows an antiferromagnetic correlation behavior. However, the NMR spin-lattice relaxation rates on the Cu site and the O site in the Cu-O plane have completely different temperature dependence, the relaxation rate \((T_1T)^{-1}\) on the Cu site obeys a Curie-Weiss law in the higher temperature region, while the relaxation rate \((17T_1T)^{-1}\) on the O site has a linear temperature behavior in the higher temperature region. b. It has a linear temperature dependence of the resistivity in a narrow optimal doping range, but for the underdoping range, only in the higher temperature region one has this relation. c. Its Hall coefficient is inversely proportional to temperature and the carrier number is of order doping density. d. It shows a Drude behavior in far-infrared region and a non-Drude behavior in mid-infrared region. e. In the underdoping range, there exists a gap (or a pseudogap) in the spin excitation spectrum\(^{[19]}\). To explain these properties of the normal state but not only one among them, there needs more effort both on the theory and experiment. In Ref.20, in contrast to the t-J model, we loosed the spin degree of the doping hole on the O site and considered that it has a Kondo-type interaction with the hole on the Cu site. In fact, in some strongly interacting region, these two models should give the same physical behavior of the
cuprates. Using this model, we can betterly explain some anomalous magnetic and transport properties of the normal state of the cuprates. In this paper, we follow this line to study the normal state more detail.

The common property of the cuprate materials is that they have one or more Cu-O planes which determine the anomalous behavior of the system in the normal and superconducting states. In the low temperature region, the carriers of the system are the doping holes residing on the O sites. It means that if we want to study the anomalous properties of the system, we must consider the dynamics of the doping holes, which maybe is a key point to understand these anomalous behavior. In fact, the real physics we must consider is that there exists two kinds of the holes, ones reside on the Cu sites and others on the O sites called the doping holes. However, the appearence of the mid-infrared region means that there exists a strongly coupling between the doping holes and the holes on the Cu sites, but it is not strongly enough to eliminate the spin degree of the doping hole and enforce one to construct a spin singlet with the hole on the Cu site at all time. While the doping hole will drastically influence the magnetic behavior what the holes on the Cu sites have.

From the current experimental data, we think that the property of the normal state of the cuprates is determined by two different regions, one is the central region of the first Brillouin zone of the copper spins, another is its corner region, i.e., near
the regions $Q = (\pm \frac{\pi}{a}, \pm \frac{\pi}{a})$ which mainly reflects the antiferromagnetic behavior of the system. We study the following model Hamiltonian\cite{20}

$$H = -t \sum_{<ij>} \hat{c}^+_i \hat{c}_j + V_0 \sum_i \hat{S}_i \cdot \hat{s}_i + J \sum_{<ij>} \hat{S}_i \cdot \hat{S}_j$$  \hspace{1cm} (1)$$

where $\hat{s}_i = \frac{1}{2} \hat{c}^+_i \hat{\sigma}_{\alpha\beta} \hat{c}_i \hat{\beta}$, $\hat{c}_i = (1 - n_{i-\alpha}) c_i$, $c_i (c^+_i)$ is the hole operator which derives from the doping (i.e., the doping hole operator). $\hat{S}_i$ is the spin operator at the $i$ site which represents the copper spin. Here we adopt an effective square lattice for O which identifies the Cu lattice, the doping holes have a single occupation constraint because of the strong repulsive interaction among the doping holes (we think that in the heavily doping range this constraint is invalid, it is valid only in the underdoping and optimal doping region). For more reality, we should consider the doping hole moving in the O sites, but we take a mapping in phase space, the hopping term can be written as $\sum_p \epsilon_p \hat{c}^+_p \hat{c}_p$, $\epsilon_p = -4t' \cos(\frac{1}{2}ap_x) \cos(\frac{1}{2}ap_y)$. In the long wavelength limit, its behavior is the same as that of the first term in (1). Second term describes the Kondo interaction between the copper and doping hole spins which derives from the hybridization between the holes on the Cu sites and the holes on the O sites. Last term describes the antiferromagnetic interaction among the copper spins which is valid for the doping and undoping cases. This model Hamiltonian can be seen as an effective Hamiltonian deriving from the three bands Hubbard model where the
hybridization between the hole on the $Cu$ site and the hole on the $O$ site induces the exchange (Kondo) interaction, here we think that the holes on the $Cu$ sites are localized, the doping holes on the $O$ sites are hopping in an effective lattice which identifies the $Cu$ sites. This model Hamiltonian describe a very simple physical picture that the doping holes with single occupation condition move in an copper spin background. In the Kondo regime, the copper and doping hole spins bind into a Kondo singlet, which is similar to the Zhang and Rice’s singlet, then it has an effective hopping on the different sites. We think that the model Hamiltonian describes the same physical properties as the t-J model in some energy scale, but it can give the NMR data better than the t-J model, and here the degrees of the doping holes is appearantly written out. It is noted that this model is different from the Kondo lattice model because of the single occupation condition of the doping holes and different from the t-J model because of the appearing of the freedom degrees of the doping holes.

In fact, this model Hamiltonian is an effevtive Hamiltonian of the three-band Hubbard model in the strongly on site Coulomb interaction limit. Because of the strongly on site Coulomb interaction among the holes on the Cu sites, the copper spin approaches to localization. The doping holes have strongly hybridization with the holes on the Cu sites, which will drastically affect the magnetic behavior of the
copper spins. In addition to the strongly on site Coulomb interaction among the
doping holes, this hybridization enforces the charge and spin degrees of the doping
holes to separate each other. The Kondo-type coupling between its spin part and
the copper spin will determine the magnetic behavior of the system, its charge part
will determine the transport behavior of the system. This characteristic property
of the cuprate superconducting materials derives from the strongly on site Coulomb
interaction and the strong hybridization between the doping holes and the holes on the
Cu sites. But it will disappear as one heavily dopes the parent insulators. Because of
the doping density increased, the influence of the strongly on site Coulomb interaction
becomes weak and the short-range antiferromagnetic phase of the system disappears.
In some cases, the heavily doping cuprate superconducting materials should show
some characteristic property of the heavy fermion system if the dimensionality is
not important. On the other hand, if we do not consider the single occupation
constraint of the doping holes, i.e., the doping holes construct a doping conduct band,
in the antiferromagnetic phase of the copper spin, we can easily explain the magnetic
behavior of the normal state\textsuperscript{[21]}, but we cannot give a reasonable explanation to the
transport behavior of the normal state.

We adopt the common method to deal with the single occupation condition by
introducing slave boson: $\tilde{c}_{i\sigma} = p_{i\sigma}^{+} f_{i\sigma} = b_{i} f_{i\sigma}$, $p_{i}^{+} p_{i} + f_{i\sigma}^{+} f_{i\sigma} = 1$ (or $b_{i}^{+} b_{i} = f_{i\sigma}^{+} f_{i\sigma}$).
\( b_i (= p_i^+) \) is a hard-core boson operator which describes the charge degree of the hole, \( f_{i\sigma} \) is a fermion operator which describes its spin degree. In the representation of the hole, the Hamiltonian in (1) can be written as

\[
H = -t \sum_{<ij>} (\eta^{*}_{ij} f^+_{i\sigma} f_{j\sigma} + \chi_{ij} b^+_{i} b_{j}) + V_0 \sum_{i} b^+_{i} b_{i} \hat{S}_i \cdot \hat{s}_i \\
+ J \sum_{<ij>} \hat{S}_i \cdot \hat{S}_j + t \sum_{<ij>} \eta^{*}_{ij} \chi_{ij} + \sum_{i} \lambda_i (b^+_{i} b_{i} - f^+_{i\sigma} f_{i\sigma})
\]  

Here we introduce two Hubbard-Stratonovich fields \( \eta^{*}_{ij} \) and \( \chi_{ij} \) to decouple the hard-core boson and fermion, \( \lambda_i \) is a Lagrangian multiplier which ensures the single occupation condition. To treat the hard-core nature of the bosons effectively, we make the following transformation which transforms the hard-core bosons into the fermions with a vortex tube carrying one flux quantum attached to each\textsuperscript{[22,23]}

\[
b^+_i = h^+_i \exp[-i \sum_{j \neq i} \theta_{ij} n_j], \quad n_j = h^+_j h_j
\]

where the operators \( h^+_i, h_i \) obey Fermi statistics and \( \theta_{ij} \) is the angle between the direction from site \( i \) to site \( j \) and some fixed direction, the x axis for example. We think that in the low temperature, low energy and long wavelength limits, the hard-core nature of the bosons is important, we must consider its effect.

In the spin and fermion coherent state representations, for spin part, we take the antiferromagnetic Néel order as its background, because although the doping
destroys the long range antiferromagnetic order, the short range antiferromagnetic order is still remained. The reason of taking this approximation is that at zero doping, the magnetic behavior of the system is controlled by a quantum critical point which is determined by the coupling constant and temperature parameters$^8$. The doping will drastically influence the magnetic behavior of the system, but it does not change the characteristic property of the quantum critical point which still determines the magnetic behavior of the doping system. Because of the interlayer very weak interaction, there exists a long range antiferromagnetic Néel order at low temperature for the parent insulators. Even in the fully (optimal) doping range, the doping system still has the short-range antiferromagnetic order both in the normal state and in the superconducting state, so the antiferromagnetic behavior is one of the remarkable character properties of the cuprate materials. Therefore it is reasonable that we take this approximation as a starting point to study the magnetic behavior of the cuprate materials. But in the heavily doping range, this starting point will be wrong if the short-range antiferromagnetic order disappears. For the doping hole part, we take the following approximations, i.e., only consider the effect of the phase fluctuation

$$
\eta_{ij} = <\eta>e^{iA_{ij}}, \quad \chi_{ij} = <\chi>e^{iA_{ij}}
$$

$$
\lambda_i = \lambda + iA_0(i), \quad A_{ij} = (r_i - r_j) \cdot A\left(\frac{r_i - r_j}{2}\right)
$$

(4)
It is well known that the Hamiltonian (2) can be transformed into the following action

\[ S[a, \psi_h] = \frac{1}{2g} \int_0^\beta d\tau \int d^2x [(\nabla \tilde{\Omega})^2 + \frac{1}{c^2} (\partial_\tau \tilde{\Omega})^2] \]

\[ + \int_0^\beta d\tau \int d^2x \{ \psi_f^{\ast} (\partial_\tau - \mu_F - i e A_0) \psi_f \]

\[ + \psi_h^{\ast} (\partial_\tau - \mu_B + i e A_0 + i e a_0) \psi_h \]

\[ - \frac{1}{2 m_F} \psi_f^{\ast} (\nabla - i e A)^2 \psi_f - \frac{1}{2 m_B} \psi_h^{\ast} (\nabla + i e a + i e A)^2 \psi_h \}

\[ + \sum_n \int \frac{d^2q}{(2\pi)^2} u \hat{s}(q, \omega_n) \cdot \hat{\Omega}(Q - q, -\omega_n) \]

\[ + \frac{\alpha}{4i\pi} \int_0^\beta d\tau \int d^2x \varepsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \]

where \( \alpha = \frac{e^2}{2l+1}, l = 0, 1, 2, \ldots \); \( Q = (\pm \frac{\pi}{a}, \pm \frac{\pi}{a}) \), \( u = V_0 S \delta, g = \frac{1}{JS^2}, c^2 = 8a^2 J^2 S^2, \)

\( m_F = \frac{1}{\langle \eta \rangle}, m_B = \frac{1}{\langle \chi \rangle}, \delta \) is the doping density, \( \hat{\Omega}(x_i) = \eta_i S_i / S, \eta_i = \pm \), is the staggered spin field, \( \psi_f, \psi_h \) the fermion fields, \( A_\mu \) the gauge field which derives from the phase fluctuation of the Hubbard-Stratonovich fields \( \eta_{ij} \) and \( \chi_{ij} \), \( a_\mu \) the Chern-Simons gauge field which derives from the nature of the hard-core boson. If we integrate out the gauge field \( a_\mu \), it is equivalent to take the following transformation in the action

\[ \int Da_\mu D\psi_h e^{-S[a, \psi_h]} = \int D\phi e^{-S[0, \phi]} \]

where \( \phi \) is the hard-core boson field.

We can easily see that the physical picture described by the action in (5) is more clear, the spin and charge degrees of the doping hole are separated, but there
exists an interaction between them via the gauge field. The spin part of the doping hole has an interaction with the staggered spin field, which mainly describes the magnetic behavior of the normal state. The transport properties of the normal state is dominated by both the charge part of the doping hole and the quasiparticle-spin-fluctuation scattering. The spin part of the system is mainly controlled by the corner region of the first Brillouin zone \( q \sim Q \), but the charge part of the system is mainly controlled by its central region \( q \sim 0 \). We think that the spin part of the system does not drastically affect its charge part and will not change its transport properties at the normal state if the quasiparticle-spin-fluctuation scattering is not dominant, because the staggered spin field has less influence on the hard-core boson field and gauge field. But in the superconducting state, the spin part does drastically affect the charge part. The antiferromagnetic spin fluctuation tends to make the fermions pair and destroy the gauge invariance, but the gauge field is strongly pair breaking, and will in general suppress the pairing transition temperature significantly. The competition of the antiferromagnetic fluctuation and gauge field fluctuation will determine the transition temperature of the superconducting state. This problem will be addressed in a separate paper. Here we only consider the physical properties of the normal state.

Generally, at the normal state, we can integrate out the fermion field \( \psi_{f\sigma} \) and
obtain an effective action including the staggered spin field, hard-core boson field and gauge field. Because of the gauge invariance, the staggered spin field does not directly interact with the gauge field, they only affect each other by through the fermion field. After integrating out the fermion field, we have the following effective action

$$S_{\text{eff.}} = S_{\text{eff.}}^s + S_{\text{eff.}}^c.$$  \hspace{1cm} (7)

$$S_{\text{eff.}}^s = \beta \sum_n \int \frac{d^2q}{(2\pi)^2} \{ \frac{1}{2g} (q^2 + \frac{1}{c^2} \omega_n^2) |\hat{\Omega}|^2 - F(Q - q) \frac{\omega_n}{\omega_{AF}} |\hat{\Omega}|^2 \}.$$  \hspace{1cm} (8)

$$S_{\text{eff.}}^c = \int_0^\beta \int d\tau \int d^2x \{ \psi_h^* (\partial_\tau - \mu_B + ieA_0) \psi_h 
- \frac{1}{2m_B} \psi_h^* (\nabla + ieA + ieA_0)^2 \psi_h \}
+ \sum_{\omega_n, q} [\chi_F q^2 + \frac{\gamma_1 |\omega_n|}{q}] \cdot [\delta_{ij} - \frac{q_i q_j}{q^2}] A_i(q, i\omega_n) A_j(-q, -i\omega_n)
+ \frac{\alpha}{4i\pi} \int_0^\beta \int d^2x \varepsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda$$  \hspace{1cm} (9)

where, $\omega_{AF} = \frac{2v_F}{aN(E_F)} \frac{1}{a^2}$, $\gamma_1 = \frac{\pi}{v_F}$, $N(E_F)$ is the density of the state at the Fermi surface, $\chi_F$ and $v_F$ are the diamagnetic susceptibility and the Fermi wave velocity, the factor $F(Q - q)$ can be written in the one-loop approximation $F(Q - q) \sim \frac{\pi}{a|Q-q|} \sim 1$ for $q << 1$. We have omitted term $A_0(q, i\omega)\tilde{D}_{00}(q, i\omega)\tilde{D}_0(-q, -i\omega)$, because of longitudinal screening effect, in the limit of the low energy and long wavelength, $\tilde{D}_{00}(q, i\omega)$ takes constant value, $\tilde{D}_{00}(q, i\omega) \rightarrow \text{const. as } \omega, q \rightarrow 0$. We see that the
spin and charge parts are completely separated (here omiting the high order term). The higher terms may induce the interaction between the spin and charge parts, but they are the irrelevant terms in the low energy limit. This is true because the Fermi surface is stable which is not destroyed by the gauge fluctuation due to the nature of the hard-core bosons, one can safely integrate out the fermion field.

Now we separately study the spin and charge parts. We can use the renormalization group method to study the spin part\[^{[8,24]}\]. From the action (8), we see that there exist three regimes, $z = 1$ (a dynamic exponential) regime, the square term of frequency $\omega$ is dominant over its linear term; $z = 2$ regime, the linear term is dominant; and the crossover regime from $z = 1$ to $z = 2$. The $z = 1$ and $z = 2$ regimes are controlled by different quantum critical points, the $z = 1$ regime is controlled by the quantum critical point of the undoping parent insulators, the $z = 2$ regime is controlled by a new quantum critical point induced by the overdamping spin wave effect which derives from the coupling between the copper spins and the doping holes. The crossover regime is a border of these two regimes. This classification depends upon the assumption that the short-range antiferromagnetic order is still robust under the doping. If the short-range antiferromagnetic order of the system disappears, one cannot use the action (8) to describe the system. There exists a character energy $\omega_s$, as $\omega_{AF} \gg \omega_s$, the overdamping effect of the spin spectrum is less important, the
magnetic behavior of the system is determined by the $z = 1$ regime, as $\omega_{AF} \ll \omega_s$, the overdamping effect is dominant, the magnetic behavior of the system is determined by the $z = 2$ regime.

We use the renormalization group methods developed in Refs.[8,24] to study the behavior of the effective action in (8). Here we adopt the symbols in Ref.8, $g_0 = h c \Lambda g, t_0 = k_B T g, \Lambda$ is a cutoff of the wave vector. For intermediate doping, the last term in (8) is a small quantity which can be treated perturbatively, the frequency $\omega$ has the scaling transformation $\omega' = \omega e^l$ which corresponds to the $z=1$ regime. In order to get the low energy behavior of the system, we can integrate out the high energy parts which will induce the effective coupling constants depending upon the renormalization parameter $l$. In one-loop approximation, we can get the following renormalization group equations of the coupling constants

\[
\frac{dt}{dl} = \frac{gt}{4\pi \sqrt{1 - a^2 g^2}} \frac{1}{\sinh(\frac{g}{7} \sqrt{1 - a^2 g^2}) \cosh(\frac{g}{7} \sqrt{1 - a^2 g^2}) - \cos(\frac{g a^2}{7})} \\
\frac{da}{dl} = \left[2 - \frac{g}{4\pi \sqrt{1 - a^2 g^2}} \frac{1}{\sinh(\frac{g}{7} \sqrt{1 - a^2 g^2}) \cosh(\frac{g}{7} \sqrt{1 - a^2 g^2}) - \cos(\frac{g a^2}{7})}\right] a \\
\frac{d}{dl} \left(\frac{g}{l}ight) = -\frac{g}{t}
\]

where $a_0 = \frac{1}{\omega_{AF}}$. If we assume the density of state at the Fermi surface $N(E_F) \propto m_F$, the quantity $\omega_{AF}$ takes the form $\omega_{AF} \propto \frac{1}{N^2(E_F) u^2}$. For simplicity, we can omit the dependence of the first equation on $a$, and only keep the linear term in the second
equation in (10). These approximations will be enough to give the correct behavior of the characteristic frequency $\omega_{AF}$ and correlation length $\xi$ in different regimes (see below). With the above approximations, we have

$$
\begin{align*}
t(l) &= \frac{t_0}{1 + \frac{t_0}{2\pi} \ln \frac{\sinh\left(\frac{1}{2} x_0 e^{-l}\right)}{\sinh\left(\frac{1}{2} x_0\right)}} \\
g(l) &= \frac{g_0 e^{-l}}{1 + \frac{t_0}{2\pi} \ln \frac{\sinh\left(\frac{1}{2} x_0 e^{-l}\right)}{\sinh\left(\frac{1}{2} x_0\right)}} \\
a(l) &= a_0 e^{2i} \frac{t_0}{t(l)}
\end{align*}
$$

(11)

where $x_0 = \frac{a}{t_0}$. In the QC regime, we have ($t(\hat{l}) = 2\pi$)

$$
\begin{align*}
\xi &= a e^{\hat{l}} \sim \frac{1}{T} \\
a(\hat{l}) &= a_0 \left(\frac{\xi}{a}\right)^2 \frac{k_B T}{2\pi \rho_0^0}
\end{align*}
$$

(12)

In the QD regime, we have ($g(\hat{l}) = 8\pi$)

$$
\begin{align*}
\xi &= \text{const.} \\
a(\hat{l}) &= \frac{1}{8\pi} a_0 g_0 \left(\frac{\xi}{a}\right)
\end{align*}
$$

(13)

Here we see that the correlation length $\xi$ given in (12) and (13) is independent of the doping density $\delta$. If one solves equations (10) without using the above approximations, one can find that the correlation length $\xi$ will decrease with increasing the doping density $\delta$ because of the quantity $a_0$ depending upon the doping density $\delta$. In
the present case, the RC regime will disappear, because in the intermediate doping range, the system enters the QC or QD regime.

On the other hand, for fully doping the last term in (8) is more important than the second term, we can not treat it as a perturbative term, the frequency \( \omega \) has the scaling transformation \( \omega' = \omega e^{2t} \), corresponding to the \( z = 2 \) regime, which means that the system undergoes the crossover regime from the \( z = 1 \) to the \( z = 2 \) regimes due to the doping. In the \( z = 2 \) regime, the second term in (8) and high order terms of the frequency are irrelevant under the scaling transformation, we will omit these terms. Using the above methods, in one-loop approximation we have

\[
\frac{dt}{dl} = \frac{gt}{4\pi} \cot(g \frac{\bar{\omega}}{2l})
\]

\[
\frac{dg}{dl}(-\frac{g}{t}) = -2 \frac{g}{t}
\]

(14)

here \( \bar{\omega} = \frac{\omega_{AF}}{g_0} \) will be treated as a renormalization invariance. We can easily solve the equations (14) and get following expressions

\[
t(l) = \frac{t_0}{1 + \frac{t_0}{4\pi\bar{\omega}} \ln \frac{\sin(\frac{1}{2}x_0\bar{\omega}e^{-2l})}{\sin(\frac{1}{2}x_0\bar{\omega})}}
\]

\[
g(l) = \frac{g_0e^{-2l}}{1 + \frac{t_0}{4\pi\bar{\omega}} \ln \frac{\sin(\frac{1}{2}x_0\bar{\omega}e^{-2l})}{\sin(\frac{1}{2}x_0\bar{\omega})}}
\]

(15)

In the QC regime, we have \( t(l) = 2\pi \)

\[
\xi'^2 \sim \frac{1}{\bar{\omega}T}
\]

(16)
In the QD regime, we have \( (g(l) = 8\pi) \)

\[
\xi' \sim \text{const.}
\]

We see that there exists qualitatively difference between the \( z = 1 \) and \( z = 2 \) regimes. In the \( z = 1 \) regime, the correlation length \( \xi \) is inversely proportional to the temperature \( T \), and the characteristic energy \( \omega_{AF} \) is renormalized. However, in the \( z = 2 \) regime, the square of the correlation length is inversely proportional to the temperature, \( \xi'^2 \sim \frac{1}{T} \), because the characteristic energy \( \omega_{AF} \) is very less than the character energy \( \omega_s \), it is not renormalized. In fact, equation (10) and (14) is obtained in the different critical regions, they are valid only near their fixed points correspondence with the \( z = 1 \) and \( z = 2 \) regions, respectively. We can intuitively understand these problem in this way, in the low doping, the density of state \( N(E_F) \) is very small, then the quantity \( \omega_{AF} \) is very larger than the character energy \( \omega_s \), the linear term of \( \omega_n \) is less important than the quadratic term of \( \omega_n \), the system is in the \( z = 1 \) region. In the large doping, the quanity \( \omega_{AF} \) is very less than the character energy \( \omega_s \), the linear term of \( \omega_n \) is more important than the quadratic term, the system is going into the \( z = 2 \) region.

Generally, for the \( z = 1 \) regime, according to these resolutions we can write the
following expression of the spin susceptibility in the low energy limit

\[ \chi_1(q, \omega) = \frac{\chi_0}{\xi^{-2} + q^2 - \frac{1}{\xi} \omega^2 - iF(Q - q)\omega_{AF}} \]  

(18)

where, \( \frac{1}{\omega_{AF}} = \frac{4\pi a^2}{k_BT\omega_{AF}(l)\xi^2} + \frac{1}{\omega_R} \), \( \xi \sim \frac{1}{T} \), for QC regime; \( \frac{1}{\omega_{AF}} = \frac{16\pi a}{\omega_{AF}(l)\xi} + \frac{1}{\omega_R(T)} \), \( \xi \sim const. \), for QD regime. \( \omega_R \) and \( \bar{\omega}_R \) derive from the high order quantum fluctuation without the doping\[^{[10]}\], in QD regime, \( \omega_R \) is very large, as \( T \to 0, \bar{\omega}_R(T) \to \infty \); In QC regime, \( \omega_R \sim \lambda T(\xi)^2 \), \( \lambda \) is a constant. \( \omega_{AF}(l) \) is the renormalized quantity of \( \omega_{AF} \).

We see that the image term is consist of two parts, one is deriving from the effect of undoping and another is induced by doping. For the \( z = 2 \) regime, we can write a general expression of the spin susceptibility in the low energy limit

\[ \chi_2(q, \omega) = \frac{\chi'_0}{\xi'^{-2} + q^2 - iF(Q - q)\omega_{AF}} \]  

(19)

where, \( \xi'^2 \sim \frac{1}{\lambda} \), in QC regime; \( \xi' \sim const. \), in QD regime. We need some explanation for the quantities \( \chi_0 \) and \( \chi'_0 \). In the QD regimes of the \( z = 1 \) and \( z = 2 \) regimes, because of the correlation length taking constant values, there will appear a gap \( \Delta_0(\Delta'_0) \) in the spin spectrum, the spin wave excitation will be suppressed, so the quantities \( \chi_0 \) and \( \chi'_0 \) should be exponentially decaying functions as decreasing the temperature in the QD regimes. In the QC regimes, the spin wave excitation energy is very larger than the energy gap(s) \( \Delta_0(\Delta'_0) \), so one can take the quantities \( \chi_0 \) and \( \chi'_0 \) as constants. These two expressions of the spin susceptibility are valid in the coner
region and mainly describe the physical properties of the normal state controlled by the corner region (i.e., \( Q = (\pm \frac{\pi}{a}, \pm \frac{\pi}{a}) \)) of the first Brillouin zone.

Using above spin susceptibilities, we can calculate the NMR spin lattice relaxation rate \( T_1 \) and spin echo rate \( T_{2G} \) at Cu sites which are completely determined by the real and imaginary parts of the spin susceptibility, respectively. In the QD regime we have

\[
\begin{align*}
\frac{1}{T_1 T} &\propto \begin{cases} 
\chi_0, & z=1 \\
\chi', & z=2 
\end{cases} \\
\frac{1}{T_{2G}} &\propto \begin{cases} 
\chi_0, & z=1 \\
\chi', & z=2 
\end{cases}
\end{align*}
\]

if the hyperfine coupling constants for both the relaxation rate and the spin echo rate have the similar momentum dependence. Similarly, in the QC regime we have

\[
\begin{align*}
\frac{1}{T_1 T} &\propto \begin{cases} 
\frac{1}{\omega_{AF}(j)T} + \frac{1}{\omega_R T}, & z=1 \\
\frac{1}{\omega_{AF} T}, & z=2 
\end{cases} \\
\frac{1}{T_2} &\propto \begin{cases} 
\frac{1}{T}, & z=1 \\
\frac{1}{\omega_{AF} T^{1/2}}, & z=2 
\end{cases}
\end{align*}
\]

(21)

We see that the NMR relaxation rate \( \frac{1}{T_1 T} \) increases and reaches a top point and then decreases as the temperature \( T \) increases.
Now we consider the NMR spin-lattice relaxation rate on O site which is mainly
determined by the $q \sim 0$ region, $(^{17}T_1T)^{-1} \propto \chi(q = 0)$, $\chi(q = 0)$ being a static
spin susceptibility. If we take the relation for the static spin susceptibility \cite{25} $\chi(q = 0) \propto N_R(E_F)$, $N_R(E_F)$ being a renormalized density of state, we can explain the
temperature dependence of the static spin susceptibility and the relaxation rate on
O site. In equation (5), the spin coupling term is
\[
\frac{1}{2} u \sum_n \int \frac{d^2q}{(2\pi)^2} \frac{d^2q'}{(2\pi)^2} d\omega \psi_{\sigma_\alpha}^* (q, \omega) \hat{\sigma}_{\alpha\beta} \psi_{\beta} (q + q', \omega + \omega_n) \cdot \hat{\Omega} (Q - q', -\omega_n)
\]
If we adopt the method in Ref.26 to make the renormalization group transformation
for the fermion field $\psi_{f\sigma}$, we can obtain the relation $u(l) \propto ue^{2l}$, according to the
equation (10), $a(l) \propto (N(E_F)u)^2(l) \propto a_0 e^{2l}$, so we have the relations $N(E_F, l) \propto
N(E_F) e^{-l}$, $N_R(E_F) = N(E_F, \hat{l}) \propto N(E_F)/\xi$. If the system is in the $z = 1$ QC regime,
$\xi \propto \frac{1}{T}$, we have the relation
\[
(^{17}T_1T)^{-1} \propto \chi(q = 0) \propto T
\]
which is reasonable at least in the $q \sim 0$ regime. If the system is in the $z = 2$
QC regime, using the same method as above, we can have the relation $\chi(q = 0) \propto
\sqrt{T}$. However, in the $z = 2$ QC regime, the doping holes can construct a big Fermi
surface, quasiparticle-hole pairing excitation heavily damps the spin wave spectrum.
If we assume that the main process of the quasiparticle-hole pairing excitation is the quasiparticle-hole pair \((q, Q + q)\), the spin coupling term in (5) can be written as

\[
\frac{1}{2} u \sum_n \int \frac{d^2q}{(2\pi)^2} \frac{d^2q'}{(2\pi)^2} d\omega \psi^*_\alpha(q, \omega) \hat{\sigma}_{\alpha\beta} \psi_{\beta}(Q + q + q', \omega + \omega_n) \cdot \hat{\Omega}(-q', -\omega_n)
\]

Using the same method as above, we find the coupling constant \(u(l) = u e^l\), so we have the relation

\[
\left(\frac{1}{17}T_1 T \right)^{-1} \propto \chi(q = 0) \sim \text{const.}
\]

which is correct at zeroth order approximation, the higher orders contribute a small quantity. However, in Ref. 27, the authors showed that the higher order term can give a correction \(-\alpha T\), \(\alpha \ll 1\), to the spin susceptibility \(\chi(q = 0)\) in (23) for a large Fermi surface.

In the low temperature region, the system enters into the QD regime, there will appear the energy gap \(\Delta_0\) in the antiferromagnetic quantum fluctuation excitation spectrum in the \(q \sim Q\) region. However, the opening energy gap also drastically influences the static spin susceptibility \(\chi(q = 0)\) and the NMR relaxation rate at \(O\) sites. We think that the relation \(\frac{1}{17}T_1 T \propto \chi(q = 0)\) can be approximately extended to the QD regime and the static spin susceptibility \(\chi(q = 0)\) is an exponentially decaying function as decreasing the temperature if the energy gap in the spin excitation
spectrum extends over the whole Fermi surface. These results are in good agreement with the current experimental data which shows that the model Hamiltonian in (1) can be betterly used to describe the magnetic behavior of the normal state of the cuprates. We think that the model Hamiltonian captures the key point of the cuprate superconducting materials that the holes induced by the doping has a strongly magnetic correlation with the copper spin, this property is responsible for the anomalously magnetic behavior of the cuprate superconducting materials.

At the normal state, the transport property of the system is mainly determined by the effective action (9) and the fermion $\psi_{f\sigma}$. First we consider the contribution from the fermion $\psi_{f\sigma}$. Using the expression given in Ref.6, we study the behavior of the resistivity produced by the quasiparticle-spin-fluctuation scattering

$$\rho_{\psi}(T) \propto \frac{1}{T} \int d\omega d^2q \frac{\omega e^{\omega/T}}{(e^{\omega/T} - 1)^2} \chi''(q, \omega) \propto T \int_0^{\infty} dx \frac{xe^x}{(e^x - 1)^2} \frac{T \xi_i^2}{\omega_i[1 - \alpha_i(\xi_i^2 T^2 x^2 / c^2)]}$$

(24)

where $\alpha_i = 0$ for $z = 2$ or $1$ for $z = 1$. $\xi_i = \xi'$ for $z = 2$ or $\xi$ for $z = 1$, $\omega_i = \bar{\omega}_{AF}$ for $z = 2$ or $\omega_{AF}^R$ for $z = 1$. We see that, in the QC ($z = 1$ or $2$) regime, the resistivity varies linearly with the temperature $\rho_{\psi}(T) \propto T$, and its slope depends upon the doping density. However, in the QD regime, the resistivity can be nearly
written as \( \rho(T) \propto T^\alpha \), i.e.,

\[
\rho_\psi(T) \propto \begin{cases} 
T, & T > T^* \\
T^\alpha, & T < T^* 
\end{cases}
\] (25)

where \( \alpha \sim 2 \), for \( T \ll T^* \), \( T^* \) is a characteristic temperature indicating the system going from the \((z = 1 \text{ or } 2)\) QC regime into the QD regime. While as the system goes from the \( z = 2 \) QC regime into the \( z = 1 \) QC regime, the resistivity still varies linearly with the temperature, but its slope will be changed. The resistivity produced by the quasiparticle-gauge-fluctuation scattering is\(^{[12]} \rho'_F(T) \propto T^{4/3} \). The physical resistivity should be

\[
\rho(T) = \rho_\psi(T) + \bar{\rho}(T)
\] (26)

where \( \bar{\rho}(T) = \rho_F(T) + \rho_B(T) \sim \rho_B(T) \), \( \rho_B(T) \) is the contribution of the hard-core boson.

However, so far there is not an effective method to deal with the hard-core boson, although one often meets it in the literature. Here we follow the method in Ref.20 to deal with the hard-core boson. In a higher energy range, the hard-core boson shows the behavior of a boson; In a lower energy range, it effectively shows the nature of a fermion. So there exists a character energy \( \omega_c \), as \( \omega \gg \omega_c \), we can treat it as a boson, as \( \omega \ll \omega_c \), we should treat it as a fermion. For the former case, it has been
extensively studied by many authors[11–13]. In this case, the action (9) can be written as

\[
S_{\text{eff.}}^c = \int_0^\beta d\tau \int d^2x \phi^*[\partial_\tau + ieA_0 + \frac{1}{2m}(\nabla + ieA)^2]\phi \\
+ \sum_{\omega_n,q} \left( \chi_F q^2 + \frac{\gamma_1 |\omega_n|}{q} \right) (\delta_{ij} - \frac{q_i q_j}{q^2}) A_i(q, i\omega_n) A_j(-q, -i\omega_n)
\]

where \( \phi \) is the boson field. It shows the resistivity \( \rho_B(T) \) has a linear temperature dependence and the Hall coefficient is inversely proportional to the doping density (holon density), but it cannot explain the temperature dependence of the Hall coefficient which we think that it derives from the nature of the hard-core boson.

To determine the low energy and long wavelength behavior of the gauge field, we integrate out the fermion field \( \psi_h \) in the action (9) and obtain an effective action

\[
S_{\text{eff.}} = \sum_{\omega_n,q} \left( \chi_F q^2 + \frac{\gamma_1 |\omega_n|}{q} \right) (\delta_{ij} - \frac{q_i q_j}{q^2}) A_i(q, i\omega_n) A_j(-q, -i\omega_n) \\
+ \sum_{\omega_n,q} \left( \chi q^2 + \frac{\gamma_2 |\omega_n|}{q} \right) (\delta_{ij} - \frac{q_i q_j}{q^2}) a_i(q, i\omega_n) a_j(-q, -i\omega_n) \\
+ \sum_{\omega_n,q} a_0(q, i\omega_n) D_{00}(q, i\omega_n) a_0(-q, -i\omega_n) \\
- \frac{\alpha}{4\pi} \sum_{\omega_n,q} \epsilon_{\mu\nu\lambda} (a - A)_\mu q_\nu (a - A)_\lambda
\]

where \( q_\mu = (q_i, \omega_n) \), we have taken the Coulomb gauge \( \nabla \cdot a = \nabla \cdot A = 0 \). If there is not longitudinal screening effect for the CS gauge field, \( D_{00}(q, i\omega) \) will take the value \( D_{00}(q, i\omega) \sim q^2 \); If there is longitudinal screening effect, \( D_{00}(q, i\omega) \) would take the constant value in the limit of low energy and long wavelength, \( D_{00}(q, i\omega) \sim \text{const.} \).
The longitudinal screening effect of the CS gauge fields will drastically influence the low energy behavior of the system. First, we consider the latter case, there existing the longitudinal screening effect of the CS gauge field. Because of $D_{00}(q, i\omega)$ taking constant value, the CS term in (28) cannot provide a gap to the transverse parts of the gauge field and CS gauge field. So we can use the method in Ref.24 and take following scaling transformations

$$q \rightarrow sq, \quad \omega_n \rightarrow s^3\omega_n, \quad s \rightarrow 0$$

(29)

We see that the dynamic exponent is $z = 3$. We must notice that these (and below) scaling transformations are taken in the $q \sim 0$ (not $q \sim Q$) region of the first Brillouin zone. To keep the quadratic term invariance, the gauge field and CS gauge field would take following scaling transformations

$$a_0 \rightarrow s^{-5/2}a_0, \quad a_i \rightarrow s^{-7/2}a_i$$

$$A_0 \rightarrow s^{-5/2}A_0, \quad A_i \rightarrow s^{-7/2}A_i$$

(30)

Under these transformations, all higher order interaction vertex functions $\Gamma^{(n)}A^n$ and $\Gamma'(n)a^n, n \geq 3$, are irrelevant. Let us see the CS term, the terms $\sum \epsilon_{ij}\omega_n a_i(a-A)_j$ and $\sum \epsilon_{ij}\omega_n A_i(a-A)_j$ are irrelevant because of their scaling dimension being one, the terms $\sum \epsilon_{ij}q_i a_0(a-A)_j$ and $\sum \epsilon_{ij}q_i A_0(a-A)_j$ are marginal because of their scaling dimension being zero. We see that in $z = 3$ regime, the coupling constant $e$ and
statistical parameter $\alpha$ are exactly marginal. The physical property of the system is controlled by following effective action at the quantum critical point $e(s) = e(0)$ and $\alpha(s) = \alpha(0)$

$$S_{\text{eff.}}^c = \sum_{\omega_n, q} \left( \chi_F q^2 + \frac{\gamma_1 |\omega_n|}{q} \right) \left( \delta_{ij} - \frac{q_i q_j}{q^2} \right) A_i(q, i\omega_n) A_j(-q, -i\omega_n)$$

$$+ \sum_{\omega_n, q} \left( \chi q^2 + \frac{\gamma_2 |\omega_n|}{q} \right) \left( \delta_{ij} - \frac{q_i q_j}{q^2} \right) a_i(q, i\omega_n) a_j(-q, -i\omega_n)$$

$$+ \sum_{\omega_n, q} a_0(q, i\omega_n) D_{00}(q, i\omega_n) a_0(-q, -i\omega_n)$$

$$- \frac{\alpha}{4\pi} \sum_{\omega_n, q} \epsilon_{ij} q_i (a - A)_0(q, i\omega_n) (a - A)_j(-q, -i\omega_n)$$

We see that the parity-odd term $\epsilon_{ij} q_i a_0 a_j$ survives. This term will determine the anomalous behavior of the Hall coefficient (see below).

Using the gauge propagators given in (31), by a simple calculation, the imaginary parts of the fermion self-energy and its Green function can be written as, respectively:

$$\Sigma^\prime_{c}(k_F, \omega) \sim \omega^{2/3}, \quad G^\prime_{c}(k_F, \omega) \sim \omega^{-2/3}$$

We see that at the quantum critical point, the spectral density $G^\prime_{c}$ has a power law divergence, removing all remnant characters of the quasiparticle and destroying the Fermi liquid behavior.

If we consider that there is not longitudinal screening effect for the CS gauge
field, i.e., \( D_{00}(q, i\omega) \sim q^2 \), we can obtain another conclusion. We think that this case only takes place in a very low energy range, although we cannot give an exact character energy. In this case, the CS term will provide an energy gap \( \Delta \) to the transverse parts of the gauge field and CS gauge field, which will drastically influence the physical property of the system. The effective action (28) can be written as

\[
S_{\text{eff.}} = \sum_{q, \omega_n} (\chi_F q^2 + \frac{\gamma_1 |\omega_n|}{q} + \Delta)(\delta_{ij} - \frac{q_i q_j}{q^2}) A_i(q, i\omega_n) A_j(-q, -i\omega_n)
+ \sum_{q, \omega_n} (\chi q^2 + \frac{\gamma_2 |\omega_n|}{q} + \Delta)(\delta_{ij} - \frac{q_i q_j}{q^2}) a_i(q, i\omega_n) a_j(-q, -i\omega_n)
- \frac{\alpha}{4\pi} \sum_{q, \omega_n} \epsilon_{\mu\nu\lambda} A_\mu(q, i\omega_n) q_\nu A_\lambda(-q, -i\omega_n)
\]

where in the mean field theory (RPA) approximation\[23\], \( \Delta \sim \frac{\delta}{m} \), \( \delta \) is the holon density (doping density). As \( \Delta \ll \chi q_0^2 \), or \( \chi_F q_0' \), \( q_0 = (\gamma_2 \omega_0 / \chi)^{1/3} \), \( q_0' = (\gamma_1 \omega_0 / \chi_F)^{1/3} \), \( \omega_0 \) is some characteristic energy in which the overdamped mode is dominant, we can omit the gap \( \Delta \) although it is relevant as taking the scaling transformations (29), and obtain the same conclusion as that there existing longitudinal screening effect of the CS gauge field. As the gap \( \Delta \) is dominant, we must take following scaling transformations

\[
q \rightarrow sq, \quad \omega_n \rightarrow s\omega_n
\]

\[
a_i \rightarrow s^{-3/2} a_i, \quad A_i \rightarrow s^{-3/2} A_i, \quad s \rightarrow 0.
\]

which keeps the quadratic terms but the \( q^2 \) terms in (33) invariance. We see that
the dynamic exponent $z = 1$. However, under these transformations, the CS and $q^2$ terms are irrelevant, $\alpha(s) \to 0, \chi_F(s)(\chi(s)) \to 0$, as $s \to 0$. At this quantum critical point, the effective action (33) can be written as

$$S_{\text{eff}}^c = \sum_{q,\omega_n} \left( \frac{\gamma_1|\omega_n|}{q} + \Delta \right) \left( \delta_{ij} - \frac{q_i q_j}{q^2} \right) A_i(q, i\omega_n) A_j(-q, -i\omega_n)$$

$$+ \sum_{q,\omega_n} \left( \frac{\gamma_2|\omega_n|}{q} + \Delta \right) \left( \delta_{ij} - \frac{q_i q_j}{q^2} \right) a_i(q, i\omega_n) a_j(-q, -i\omega_n)$$

By using these gauge propagators, we can obtain the imaginary part of the fermion self-energy

$$\Sigma^\prime_c(k_F, \omega) \sim \omega^2$$

We see that at this quantum critical point, the system has the Fermi liquid behavior.

We now consider the transport properties of the system. We first assume that there exists an energy scale $\omega_c$ which will be used to characterize the nature of the hard-core bosons. If the energy $\omega < \omega_c$, the nature of the hard-core bosons is important, i.e., the fermionic character of the hard-core bosons is important. If the energy $\omega > \omega_c$, the nature of the hard-core bosons can be neglected, i.e., the bosonic character of the hard-core bosons is dominant. In the case, we must use the action (27) to determine the temperature dependence of the resistivity, it is well known$^{[12,13]}$ $\rho_B(T) \sim T$. In the case of $\omega < \omega_c$, if there exists the longitudinal screening effect of the CS gauge field, the CS term has less influence on the transverse parts of the gauge
field and CS gauge field, according to equation (32), the temperature dependence of the resistivity is \( \rho_B(T) \sim T^{4/3} \), the system has a non-Fermi liquid behavior. If there is not the longitudinal screening effect of the CS gauge field, the CS term has drastically influence on the transverse parts of the gauge field and CS gauge field and provides an energy gap to them, according to equation (36), the temperature dependence of the resistivity is \( \rho_B(T) \sim T^2 \), the system has a Fermi liquid behavior.

So we have the relation

\[
\rho_B(T) \propto \begin{cases} 
T, & T > \omega_c \\
T^{\alpha'}, & T < \omega_c 
\end{cases} \tag{37}
\]

where \( \alpha' = 4/3 \) or 2. In fact, we have three characteristic energies \( T^*, \omega_c \) and \( \tilde{T}^* \) to indicating the behavior of the system, here \( \tilde{T}^* \) is the characteristic energy indicating the crossover from \( z = 2 \) to \( z = 1 \) QC regimes for the spin part. Only under the condition \( T > max\{\omega_c, \tilde{T}^*\} \), the physical resistivity satisfies the relation \( \rho(T) \propto T \). However, in the little doping case, \( \tilde{T}^* \to \infty \), we can have the relation \( \rho(T) \propto T \) for \( T > max\{\omega_c, \tilde{T}^*\} \). From the equation (23) we see that the characteristic energy \( \tilde{T}^* \) corresponds to the temperature at which the static spin susceptibility tends to its maximal value as the system goes from the \( z = 1 \) QC regime to the \( z = 2 \) QC regime, so \( \tilde{T}^* \) should be similar as the characteristic temperature \( T_{\chi_{\max}} \) defined in Ref.[31,32], at which the spin susceptibility exhibits a broad peak. Generally, we
have the condition $\bar{T}^* < T_{\chi_{\text{max}}}$, because there may exist a transition region from $\chi(q = 0) \propto \sqrt{T}$ to $\chi(q = 0) \propto \text{const.}$ in the $z = 2$ QC regime which is dominant. The size of the transition region depends upon the shape of the Fermi surface of the doping holes. The experimental data in Ref.32 supports the above analysis for the $La_{2-\delta}Sr_{\delta}CuO_4$ sample in the doping range $0.09 < \delta < 0.16, \bar{T}^* \leq T_{\chi_{\text{max}}}$, if in this doping range the characteristic energy $\omega_c$ is less than $\bar{T}^*$.

We now consider the behavior of the temperature dependence of the Hall coefficient. We see that as there existing the longitudinal screening effect of the CS gauge field, the parity-odd gauge interaction has a little influence on the resistivity, but it drastically influences the Hall coefficient. An external magnetic field may have an activation effect on the nature of the hard-core bosons and turns on the parity-odd gauge interaction $D_{jo}(q, \omega) = <a_j a_0> = \sigma \varepsilon_{ij} q_i F(q^2, \omega), F(q^2, \omega) = <a_i a_i > < a_0 a_0 >$. This parity-odd gauge interaction will exist in a wide broad energy scale even to a higher energy in which the nature of the hard-core boson is less important because of the statistical parameter $\alpha$ being exactly marginal. If we take $F(q^2, \omega) = \frac{1}{\epsilon_0 q^2} \delta_{\omega,0}$, using the method in Ref.33 to calculate the Hall coefficient at lowest-order approximation, we get

$$R_H = R_H^* \frac{1}{1 + \beta T} + R_H^\infty$$

(38)
where $\beta = \frac{1}{8\pi^2\epsilon_0}(n_F - n_F^2), n_F = \{\exp(-\mu/k_B T) + 1\}^{-1}$ is the Fermi distribution function, the chemical potential $\mu$ is proportional to the doping density $\delta$. At the low doping and high temperature, the coefficient $\beta$ is nearly independent of temperature. $R_H^*$ is the Hall coefficient for the system without parity-odd gauge interaction, at low doping $R_H^* \propto \frac{1}{\delta}$, $R_H^\infty$ the Hall coefficient in the infinite temperature limit which can be identified$^{[34]}$ $R_H^\infty = R_0(\frac{\delta}{16} - \frac{1}{16} + \frac{3}{4})$, $R_0$ is a constant. We see that the anomalous temperature dependence of the Hall coefficient derives from the parity-odd gauge interaction which is the respondence of the nature of the hard-core bosons.

The expression of the Hall coefficient $R_H$ in (38) can better explain the current experimental data$^{[35,36]}$.

In summary, we have used a simple physical picture that the doping holes with single occupation constraint move in the antiferromagnetic background of the copper spins to describe the normal state of the cuprate superconducting materials by the traditional slave boson method. We have classified the first Brillouin zone of the copper spins into as two regions, one is the central region (near $q \sim 0$), which controls the behavior of the charge part of the doping holes and determines the transport property of the system; Another region is the corner region, i.e., near the regions $Q = (\pm \frac{\pi}{a}, \pm \frac{\pi}{a})$, which controls the behavior of the copper and doping spins and determines the magnetic property of the system. For the spin part, taking the

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long range antiferromagnetic Néel order of the copper spins as a background and using the renormalization method, we have given the spin susceptibilities of the system in the $z = 1$ and $z = 2$ regimes, and used it to calculate the NMR spin-lattice relaxation rate and spin echo rate. The results we have obtained are in good agreement with the current experimental data. For the charge part, we have extensively studied a hard-core boson system interacting via exchanging gauge bosons using the renormalization group method. As considering the longitudinal screening of the CS gauge field, the low energy and long wavelength behavior of the gauge fields is shown to be in the Gaussian universality class with a dynamical exponent $z=3$, and the parity-odd gauge interaction is exactly marginal. The anomalous transport properties of cuprate materials is controlled by the Hamiltonian at this quantum critical point $g(s) = g(0)$ and $\alpha(s) = \alpha(0)$, the system has a non-Fermi liquid behavior. As there is not the longitudinal screening effect of the CS gauge field, the system has a Fermi liquid behavior. We have showed that the physical resistivity is determined by both the quasiparticle-spin-fluctuation and quasiparticle-gauge-fluctuation scatterings, and the Hall coefficient is determined by the parity-odd gauge interaction which derives from the nature of the hard-core bosons.

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