Seeking the Ground State of String Theory

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The greatest obstacle to developing a string phenomenology is our lack of understanding of the ground state. We explain why the dynamics which determines this state is not likely to be accessible to any systematic approximation. We note that the racetrack scheme, often cited as a counterexample, suffers from similar difficulties. We stress that the weakness of the gauge couplings, the gauge hierarchy, and coupling unification suggest that it may be possible to extract some information in a systematic approximation. We review the ideas of Kahler stabilization, an attempt to reconcile these facts. We consider whether the system is likely to sit at extremes of the moduli space, as in recent proposals for a low string scale. Finally we discuss the idea of Maximally Enhanced Symmetry, a hypothesis which is technically natural, compatible with basic facts about cosmology, and potentially predictive.

§1. Introduction

In thinking of Yukawa’s great work, one cannot help but consider the importance of asking the right questions. It is not merely that Yukawa predicted the existence of a particular particle, but that the questions which he asked, and the answer which he provided, remain fruitful to this day. In thinking about the fundamental interactions, we would like to pose and answer similar, qualitative questions. This has been the spirit of this conference. It would be presumptuous to suppose that the questions which I will pose here will be of such importance, or the answers so significant. It is likely, as we will see, that it is premature to ask these questions. It is also possible that the questions I pose here will someday seem inappropriate, in much the way we no longer see the computation of the total cross section as an important problem in strong interactions. Still, I hope that the questions which I will phrase here will be helpful in confronting some of the issues which we face in thinking about the formulation of string theory and its connection with nature.

Up until now, our approach to string theory has suffered from a certain schizophrenia. At weak coupling, we have a beautiful picture, containing gravity, gauge interactions, chirality, generations, calculable interactions, and other features which we view as crucial to any fundamental theory. But there are also problems with this picture. First, there are many vacua. These vacua carry both discrete and continuous labels. The discrete labels, especially the number of supersymmetries, are extremely useful in gaining control over the theory; they can be used to severely constrain, for example, the effective lagrangian for the moduli, the fields whose expectation values correspond to the continuous labels. The greater the number of supersymmetries, the stronger the constraints and the more powerful the statements we can make. This power is the origin of virtually all of the recent developments connected with duality. Unfortunately, one of the easiest statements to make about ground states with more than four supersymmetries (\(N > 1\) in four-dimensional counting) is that they are ground states, perturbatively and non-perturbatively.
For the interesting cases with $N \leq 1$ supersymmetry, there is no such simple argument, in general. Generically, perturbative ground states with four or less supersymmetries are unstable. Assuming that string theory describes nature, duality is not likely to be of much help in establishing the properties of the non-supersymmetric state which corresponds to the world we observe. Duality generally relates very strong coupling in one theory to weak coupling in another theory. Yet a simple argument shows that any stable ground state of the theory is either degenerate with other ground states or lies at a point in the moduli space where no weakly coupled description is possible.\(^1\) While this argument was originally formulated with the heterotic string in mind, it applies to any weak coupling description. The point is simply that at weak coupling, any potential for the modulus which describes the coupling necessarily goes to zero. Recent developments in duality are thus not directly useful for addressing the problem of vacuum stability.\(^2\) If a very strong coupling region of one theory corresponds to a weakly coupled description of another, then it suffers from the same problem. This suggests that in seeking the phenomenologically interesting ground state(s) of the theory, we need to look for regions of the moduli space which admit no weak coupling description.\(^*\) David Gross has dubbed this the “Principal of Minimal Calculability (PMC)”.

Still, nature exhibits weak coupling and seems to exhibit perturbative unification. Surely, these are important clues. One of the themes of this lecture will be an effort to reconcile these seemingly contradictory facts. A related fact, which also figures heavily in this talk, is that nature exhibits hierarchies. From the perspective of string theory, the issue is: how does a theory with no parameters generate large (small) pure numbers. We will confront these issues quite directly in this discussion. We will see that the answers traditionally given by particle physicists provide only limited comfort. For example, we often say that hierarchies can be understood through the slow running of couplings, or through the natural appearance in field theories of expressions involving $e^{-8\pi^2/g^2}$. We say that it is natural that $g$ should be of order 1, and therefore the exponential can be extremely small. But, especially in light of duality, a more natural condition is that $g^2/4\pi$ should be 1. Then the exponential should not be thought of as small at all. To my knowledge, there are only three proposals in the literature to deal with this conundrum, and I will review them here. The first of these is known as the “racetrack” scheme.\(^3\) The idea is that one has several gaugino condensates (or similar sources of moduli superpotentials). It is assumed that the superpotential is a sum of the superpotentials generated by gluino condensation in each sector, i.e., it has the form

$$W = \sum e^{-8\pi^2/g_i^2}.$$  

Generically, any ground states one finds in such a picture will suffer from the problem described above, that $e^{-8\pi^2/g^2}$ is not small, since the various terms in the sum must be comparable. It has been argued, however, that for some choices of low energy gauge groups, large hierarchies can result if there are terms in the sum with large

\(^*) The notion of a moduli space, in this context, is a bit fuzzy; we have in mind the classical moduli space; later, we will introduce the notion of an approximate moduli space.
β-functions. This proposal has been most persuasively developed by Kaplunovsky and Louis. We will see that while this is a logical possibility, it cannot be studied in a systematic approximation, and it seems unlikely that minima with the desired properties exist. The second proposal is known as Kahler stabilization, and will be briefly reviewed below. We will spend most of our time, however, on a third proposal, that of “Maximally Enhanced Symmetry”. This proposal, as we will see, has problems of its own, but if we hypothesize that it is correct, it makes definite predictions for low energy physics, and also suggests an approach to developing a real string phenomenology.

§2. Fixing the moduli

2.1. Generalities

String theory is a theory without free parameters, but the role of parameters is played by the moduli. Even without any detailed understanding of string dynamics, there are only a few logical possibilities for the fate of the moduli:

- The moduli are not fixed. Perturbatively and non-perturbatively, their potential vanishes. This is the case for $N > 1$ supersymmetry in four dimensions and for supersymmetry in $D > 4$. In such cases, the supersymmetry prevents one from writing any potential for the moduli in the low energy effective theory at all.
- The moduli are unstable to runaway to $\infty$. Consider, for example, the modulus which describes the string coupling. One expects that any potential which is generated either perturbatively or non-perturbatively will vanish as the coupling tends to zero. One can imagine loopholes to this argument, but it is certainly true of all known cases. A similar argument applies to moduli which describe, for example, compactification radii, at least in cases with an approximate, low energy supersymmetry. As the radii become large, the theory becomes a theory in more than four dimensions with supersymmetry, and in such theories the energy necessarily vanishes. Again, one can conceive of loopholes to this argument, but it holds for all known cases.
- The moduli are fixed. Necessarily this occurs for couplings of order one. This follows from our discussion above: if any weak coupling approximation is valid, the potential cannot have a minimum. A similar argument holds for the moduli which describe compactification (at least, as discussed above, in cases with low energy supersymmetry). Of course, we can hope for accidents. Ratios of scales might be small, couplings in effective lagrangians might be small, simply by accident. But the underlying theory is still likely to be strongly coupled. Without some further assumptions, one has little hope of predicting anything under these circumstances. As we will discuss below, supersymmetry, holomorphy, and other symmetries may allow one to make some predictions.

String duality is an exciting development, but it does not help directly with this problem. Indeed, these arguments imply that we expect realistic ground states of string theory to lie where no weak coupling description is valid (this is the PMC
How, then, can we imagine developing any phenomenology? How can we reconcile these remarks with the facts that the gauge couplings we observe in nature are small and seem to be unified? In this lecture, I will review what has been said about these questions, and offer some speculations. I will focus on the third possibility in our list above, that the moduli are fixed and in some sense of order one. I will focus principally on the questions: why is $\alpha \ll 1$, and why is $\frac{m_W}{m_P} \ll 1$. I will not offer much insight into the questions: what is the origin of the fermion mass hierarchy, and, most importantly, why is the cosmological constant so small. It will be clear from later discussion that our lack of an answer to this last question is an extremely serious limitation, and quite possibly an indication that all of this discussion is premature.

Before turning to general theoretical issues, I would like to mention one more possible clue and constraint. This is the cosmological moduli problem. Most proposals for supersymmetry breaking in string theory postulate that the moduli develop a shallow potential, typically with a minimum at some more or less random value. In that case, the early universe has no reason to start out close to the ground state and the system generically stores too much energy. The conventional solution to this problem is to suppose that when these moduli decay, they heat the universe above nucleosynthesis temperatures, and produce the observed baryon asymmetry at the same time. This requires that the moduli be quite heavy. An alternative possibility is that all of the moduli are charged under symmetries at the minimum, so that the values of these fields in the early universe can naturally coincide with those at the present time. Prior to the recent developments in string duality, it was not possible to say much about this possibility. Now it is easy to construct examples of this phenomenon. This will be the focus of the latter part of this talk.

It is worthwhile to first review the conventional particle physics wisdom about small numbers. Usually we say that the gauge couplings ($g_s$, $g$, $g'$) are numbers of order 1. Thus it is natural that $\alpha_{\text{gut}} \sim \frac{1}{30}$, for example. This also provides a way of understanding hierarchies, since non-perturbative effects in weakly coupled field theories typically behave as $e^{-\frac{8\pi^2}{g^2}}$. But this begs the question: why is it $g$ which is $O(1)$, and not $\alpha$. In the case of electric-magnetic duality, for example, $\alpha = 1$ at the self-dual point. We will confront this issue shortly. About small Yukawa couplings, there are a few ideas. Most assume that there are approximate symmetries, broken by the expectation values of fields. In string theory, it has long been noted that some Yukawa couplings vanish exponentially with compactification radius, and this could be a source of Yukawa hierarchies. A version of this idea suggested by brane physics has recently been studied by Refs. 10 and 11). About the cosmological constant, there is no real conventional wisdom, and I will not have anything to add today. Recently, Kachru and Silverstein have exhibited non-supersymmetric models in which the cosmological constant vanishes, and Harvey has noted that in some string theories the cosmological constant may be exponentially small. Witten earlier made an interesting proposal motivated by string dualities. But none of these ideas is as yet complete.
It is also useful to recall the conventional approaches to string phenomenology. Essentially all string phenomenology ignores the problem of moduli and simply assumes that the moduli are fixed to some convenient values. The couplings are invariably assumed weak. The rationale is that, after all, weakly coupled strings look much like nature. Prior to 1995, virtually all string model building started with the assumption that only the heterotic string was phenomenologically viable. In this theory, the unified coupling is related to the string coupling and the compactification volume through
\[ \alpha_{\text{gut}} \approx \frac{g_s^2}{V}. \]  
(2)

Here \( g_s \) is the dimensionless string coupling and \( V \) is the compactification volume in units of the string tension. Requiring that \( g_s \leq 1 \), so that a weak coupling string description should be valid, yields that \( V \sim 1 \), while \( M_s \sim M_{\text{gut}} \sim M_p. \)\(^{15,16}\) The requirement that \( g_s < 1 \), however, was always artificial, a reflection of our wishful thinking that the theory should be weakly coupled. Our post 1995 understanding of duality permits many new possibilities. Perhaps the simplest of these is the proposal by Witten,\(^ {17}\) following the realization of Horava and Witten\(^ {18}\) that the strongly coupled heterotic string theory is in fact an eleven-dimensional theory with two walls in the eleventh dimension. Taking the values of the gauge coupling and unification scale at face value, this could be the appropriate description of the theory. It leads to a picture in which the world is approximately five dimensional; six dimensions are compactified, say, on a Calabi-Yau space with radius of order one in eleven-dimensional Planck units, while one dimension is significantly larger. More radical possibilities\(^ {20}-23\) have been proposed recently, and will be discussed further below.

One intriguing possibility is that the standard model lives on a brane. In this case, the gauge couplings are independent of the compactification volume, and one can, again, consider the possibility that the compact space is large. In all of these pictures, one must still ask why some dimensions are large.

Finally, what is the conventional wisdom about fixing the moduli? There are a number of approaches:

- Ignore the issue.
- Assume that the moduli are stabilized at strong coupling, and that the smallness of the observed couplings is an accident. This could be well be the answer, and is in line with the PMC enunciated earlier. But it is very disappointing, and leaves us without an explanation of why string theory gets even qualitative things right.
- Racetrack models:\(^3\) In theories with two or more gaugino condensates, it has been argued that there is a superpotential for the moduli of the form
\[ W = \sum C_a(T)e^{-\frac{\pi}{b_a}a(T)}. \]  
(3)

Then one can have, for example, isolated supersymmetric minima with fixed, large values of the moduli, provided that some of the \( b_a \) are large and nearly equal. Supersymmetry can then be broken at a lower scale.
• Kahler stabilization: In this picture, holomorphic quantities, such as $e^{-8\pi^2/g^2}$ and the holomorphic gauge couplings are small. However, non-holomorphic quantities, such as the Kahler potential, are assumed to receive large corrections, and to be responsible for the stabilization of the moduli.

• Large topological charges: The authors of Ref. 24) argue that large topological charges could stabilize compactification radii at large values. One could imagine that similar effects stabilize other moduli. The question would then be why these charges take such values.

• Maximally enhanced symmetry: It is natural to postulate that the ground state lies at a point where all of the moduli transform under unbroken symmetries. This postulate makes some definite predictions. Whether there really exist string vacua with such symmetries, and simultaneously with small effective gauge couplings, is an open question.

We do not have much to add to the first two items. In the next section, we will explain why the multiple gaugino condensate idea cannot be studied systematically. The following section then explains the basic ideas of Kahler stabilization. The final section is devoted to the hypothesis of maximally enhanced symmetries.

2.2. Racetrack models

Racetrack models have been offered to explain how string vacua might arise at perturbatively weak couplings. The idea is that the superpotential of the theory is a sum of terms of the form of Eq. (3). In fact, as has been explained in Ref. 3), it is quite natural that effects associated with low energy dynamics be larger than stringy non-perturbative dynamics. Noting that low energy gauge groups in string theory can be extremely large, the authors of Ref. 3) argue that if two groups have very large $\beta$-functions, some of the moduli can be fixed, without breaking supersymmetry.

There are several difficulties with this picture. All are related to the problem that there is no small parameter to justify the approximations; i.e., there is nothing like the $N$ of the large $N$ expansion. In this version of the mechanism, for example, $x = e^{-8\pi^2/b}$ should be of order one. But this means that the scale of the low energy groups is of the order of the fundamental scale, so the low energy analysis is not really consistent. Alternatively, higher order operators complicate the analysis. As a result, it is difficult to determine whether a vacuum state even exists. Other versions of the scheme suffer from similar difficulties. One can obtain smaller $x$ at the price of fine tuning, but it is still difficult to obtain a small cosmological constant. In all of these versions, the Kahler potential is not calculable; this is a particular important issue in versions of the scheme in which supersymmetry is broken by the condensates. It would seem that one should have simply hypothesized the desired result: the coupling is fixed in a way that the gauge coupling is small. One has no control over the final answer. These issues will be explored in a future publication.

2.3. Kahler stabilization

Quite generally, if the superpotential is responsible for stabilizing the moduli, it is unlikely that the effective couplings can be weak, in the sense that $e^{-8\pi^2/g^2}$ is
small. This follows simply from holomorphy. Consider, for example, weakly coupled string theory. We expect that the superpotential is roughly of the form

\[ W = e^{-S} + be^{-2S} + \cdots. \] (4)

If the Kahler potential is not significantly modified from its tree level form, then at any minimum of the potential, \( e^{-S} \sim e^{-2S} \sim 1 \). Thus the coupling is strong and there is no hierarchy.\(^*\)

We can be more precise if we exploit holomorphy and the discrete axion shift symmetry. These restrict the couplings of \( S \) to the gauge fields to

\[ \left( \frac{1}{2\pi^2 S} + e^{-S} + \cdots \right) W^2, \] (5)

while the superpotential has the form \(^4\)

\[ W = e^{-S} + e^{-2S} + \cdots. \] (6)

The Kahler potential, on the other hand, is not restricted by holomorphy. It is known that string perturbation theory is not as convergent as field theory perturbation theory. Assume, then, that there are large corrections even for \( \alpha \sim \frac{1}{30} \). The full potential is given, in terms of \( W \) and \( K \) by (for simplicity, considering only the field \( S \))

\[ V = e^{-K} \left[ \frac{\partial W}{\partial S} + \frac{\partial K}{\partial S} W \right]^2 \left( \frac{\partial^2 K}{\partial S \partial S^*} \right)^{-1} - 3|W|^2 \]. \] (7)

Kahler stabilization is the suggestion that this potential has its minimum when \( e^{-S} \) is small due to the structure of \( K \). One can certainly postulate forms for \( K \) which yield a local minimum of the potential for such values of \( S \), with vanishing cosmological constant (if one allows sufficient fine tuning). This approach is predictive. Because \( e^{-S} \) is small, it predicts coupling constant unification and that the superpotential is not significantly altered from its weak coupling form. It also predicts that there is approximate, low energy supersymmetry. On the other hand, explaining, say, squark degeneracy requires additional inputs. While squark masses, for example, are sometimes degenerate in the weak coupling limit, our basic assumption is that the Kahler potential is very different from its weak coupling form. So one needs to postulate, say, approximate flavor symmetries.

In this picture, other moduli are also fixed by the form of \( K \). Just as one does not expect the gauge couplings to be extremely small, one does not expect large hierarchies of compactification radii. This follows from the fact that as the radii become large, the theory is effectively a supersymmetric theory in a higher dimension, where one cannot write down a potential for the moduli. In other words, the potentials for

\(^*\) One might hope to get around this by supposing that, say, considering, as in the racetrack schemes, two terms, \( ae^{-\alpha S} + be^{-\beta S} \), and hoping to find a minimum where \( e^{\alpha - \beta} = b/a \), and \( \alpha - \beta \) is small, while \( b/a \) is also small. This is similar to the failed racetrack schemes described above, but perhaps occurs in some other context.
the moduli which describe the size of the internal dimension necessarily vanish as the size tends to infinity.

One does not expect, in such a picture, particularly large hierarchies of compactification scales and $M_p$. Of course, one has provided no explanation of the cosmological constant puzzle.

§3. New insights from duality

3.1. Horava-Witten: A large eleventh dimension

Duality has opened up new ways to think about these problems. One puzzle in the early days of superstring compactifications was reconciling the observed values of the unified scale and couplings with weakly coupled string theory. In light of our understanding of duality, it is reasonable, following Horava and Witten\(^{18,17}\) to suppose that string theory is described by the heterotic string theory in a strongly coupled regime. In this regime, the theory looks eleven dimensional with two walls in the 11’th dimension separated by a distance $R_{11}$.\(^{17,26}\) Calling $M_{11}$ the eleven-dimensional Planck scale, and $V$ the compactification volume of, say, some six-dimensional Calabi-Yau manifold, one finds

$$R_{11}^3 = \frac{\alpha_{\text{gut}}^3 V}{512 \pi^4 G_N^2}, \quad (8)$$

$$M_{11} = R_{11}^{-1} \left( 2(4\pi)^{-2/3} \alpha_{\text{GUT}} \right)^{-1/6}. \quad (9)$$

In these equations, $G_N$ is the ordinary Newton constant. Plugging in the observed values for the unification scale ($R_{11}^{-1}$) and the unified coupling constant, one finds

$$M_{11} R_{11} = 72, \quad M_{11} R \approx 2. \quad (10)$$

In this picture, then, the eleven-dimensional Planck scale is close to the unification scale, while $R_{11}$ is significantly larger. This viewpoint has other interesting consequences. For example, it ameliorates the cosmological axion problem of string theory.\(^{26,27}\) On the other hand, it is still hard to understand the stabilization of the moduli. For large $R_{11}$, the bulk theory is approximately five dimensional. Supersymmetry in five dimensions forbids a potential, so the potential must tend to zero as $R_{11} \rightarrow \infty$. This can be made precise, by using the five-dimensional supersymmetry to restrict the form of $K$ and $W$, and one finds that the potential does tend rapidly to zero, consistent with this heuristic argument.\(^{26,19}\) Thus one expects that, if there is a stable minimum of the potential, it occurs when the various radii are of order $M_{11}$. The problem of explaining why this ratio is of order 70 seems similar to the problem of understanding why the gauge coupling is of order $1/30$. Again, one needs something like Kahler stabilization of the moduli.

3.2. A more radical proposal: String theory at the TeV scale

All of the previous discussion has been based on the idea that, string theory being a theory without parameters, all dimensionless couplings and ratios of scales
should be numbers of order one (with the exception of the supersymmetry breaking scale, which is understood as the exponential of a number of order one). Indeed, we have seen that in the supersymmetric case, one can prove this. Recently, various authors have proposed that perhaps the string scale lies at another familiar scale in physics, the scale of weak interaction symmetry breaking.\textsuperscript{20,21} Such ideas, in fact, had been considered in the past, but had not been taken too seriously because such a possibility corresponds to enormous string coupling, in the case of the heterotic string. Newton’s coupling is so small, or the Planck scale so large, in such a picture, because the internal space has a very large volume. For example, if one compactifies the eleven-dimensional theory, one has

\[ G_N = \frac{\text{TeV}^9}{V^{(7)}}. \tag{11} \]

In particular, if all of the dimensions are of comparable size, then \( r \), the radius of the compact space, satisfies \( r \sim \text{MeV}^{-1} \), while if, for example, two dimensions are large, and the others are of order the fundamental scale, \( r \sim \text{mm} \). Most of the proposals of this type assume that the fields of the standard model live on a brane. Gravity looks 4 + \( n \) dimensional on scales small compared to \( r \), where \( n \) is the number of large compact dimensions. Exciting new phenomenological possibilities exist: long range forces (in the mm case, and possibly in others, as we will discuss below), production of large numbers of Kaluza-Klein states, and production of stringy excitations.

At first sight, this idea seems outrageous, but in fact it is quite difficult to definitively rule out.\textsuperscript{28} There are several obvious problems to worry about:

- **Proton decay:** proton decay must be highly suppressed; if the relevant scale is of order a TeV, then operators up to very high dimension must be forbidden. This can be arranged, however, by assuming, for example, that there is a discrete symmetry which is a large subgroup of baryon number.
- **Other types of flavor violation:** These can be suppressed if one assumes that the theory has a large flavor symmetry, broken, perhaps, on distant walls.\textsuperscript{10,11} Still, these processes constrain the scale to be greater than \( 5 - 10 \) TeV.\textsuperscript{7}
- **Production of Kaluza-Klein modes:** In this picture, typical Kaluza-Klein modes of the graviton couple with gravitational strength. However, there are a huge number of such modes, so one needs to worry about processes in which one produces these modes and they carry off energy. The lower limits on the string scale arising from these types of considerations are of order a few TeV.
- **Astrophysical constraints:** here one needs to worry about production of these particles in red giants. The problem is most severe in the case of two compact dimensions. Here one obtains limits in the 30 TeV range if \( n = 2 \);\textsuperscript{28} recently it has been argued that the limit is 50 TeV.\textsuperscript{29} This means that it will be difficult to observe the associated change in Newton’s law, and is certainly problematic from the perspective of the hierarchy problem.
- **Cosmology:** Even for general \( n \) this is more problematic. One has in these theories a serious moduli problem, for example. The authors of Ref. 28 argue that, if the scale is not too low, provided the universe was in the correct ground state shortly before nucleosynthesis, production of Kaluza-Klein modes will not
spoil this. As we will describe later, it is hard to imagine how to establish such an initial condition. One can contemplate, for example, several stages of inflation, but one is still left with a severe moduli problem.\textsuperscript{30}

While some of these issues may make one uncomfortable with the idea of a low string scale, it is clear that these considerations alone do not rule out the possibility. The laboratory and astrophysical constraints at best place the lower limit on the scale at 10 TeV, and the cosmological constraints, while potentially more severe, require assumptions about aspects of early universe physics about which we do not have direct evidence.

Still, one can ask: is there any physics which suggests a low string scale. The literature on this problem refers to the hierarchy problem. Indeed, if the scale is close to the weak scale, then Higgs scalars with mass of this order are natural. However, if the scale is 10 TeV, then this is less clear. We have argued that the true ground state of string theory should be strongly coupled. But in this case, supersymmetry being absent, one expects any fundamental scalars to have masses of order the scale, i.e., of order 10 TeV. So one has a fine tuning to at least one part in several hundred, or perhaps even worse. In weakly coupled string theory, this might be acceptable. At string tree level, one often finds particles which are massless for no symmetry reason. Loop corrections to the mass might be in an acceptable range. However, as we have argued, it is not likely that there is such a weak coupling parameter.

So there is already a potential hierarchy problem. A more severe problem arises when we ask: how might we stabilize the radius at such a large value? After all, we have argued that most dimensionless ratios in the theory should be of order 1. There would seem to be two possible explanations for such large numbers. One is that some modulus (not associated with the large dimensions) takes an extreme value, and some lagrangian parameter relevant to fixing the size of the compact space is exponentially small in this modulus. It is not easy to see how this would work in practice, and in any case fixing this modulus would represent one more mystery. An alternative possibility has been explored in Ref. \textsuperscript{24}, following earlier suggestions of Sundrum;\textsuperscript{31} in this scenario, the large dimensions are connected with the large value of some topological charge. The problem of large radii is then replaced by the question of why this topological charge is so large.

The problem of stabilization has been discussed in Ref. \textsuperscript{24} and further in Ref. 6). In order to discuss stabilization of the moduli, it is crucial to make some assumptions about the way in which the cosmological constant is cancelled. One possibility is that, independent of the value of the cosmological constant in the effective low energy field theory, say at energy scales slightly below the radius $r^{-1}$, the large distance cosmological constant vanishes, for some unknown reason. In this case, the values of the bulk cosmological constant and the cosmological constant of the brane theory are independent. We expect the brane cosmological constant to be of order $M^4$. In any large radius scenario, on the other hand, the bulk cosmological constant must be many orders of magnitude smaller than the value expected from dimensional analysis, $M^{4+n}$. Indeed, the bulk cosmological constant makes a contribution to the masses of the Kaluza-Klein states of order $A_b/M^{n+2}$. This mass is greater than $1/R$
unless $\Lambda_b < 1/(r^2 M^2) M^{4+n}$. This is an additional fine tuning which must be explained. Moreover, the actual small value of the number requires that there be some modulus besides the radial dilaton which takes some extreme value. The authors of Ref. 24) argue that such a value of $\Lambda_b$ is at least plausible in the case that the bulk theory is supersymmetric. But in that case, one can show that the bulk cosmological constant vanishes; a different analysis, described in Ref. 6), is required. I will return to this case below. Finally, as noted in Ref. 24), the required topological charges are very large.

An approach which yields a more plausible picture is to assume that the cosmological constant must already nearly vanish in the theory at scales below $r^{-1}$. More precisely, one assumes that, on account of some unknown mechanism, the cosmological constant in the bulk adjusts to cancel the contribution of the brane, of possible higher curvature terms, and of any topological charges in the internal dimensions. The brane contribution is expected to be of order $M^4$. If one assumes that the internal space is flat, then the radial dilaton mass is of order $mm^{-1}$, independent of the number of internal dimensions. This is interesting, but, as explained in Ref. 6), a flat internal space requires far more fine tuning than a curved one. Various possibilities arise in the case of curved manifolds. If the bulk is not supersymmetric, the required topological charges are still rather large, and generically the radial dilaton has mass of order $1/R$. If the bulk theory is supersymmetric, the story is more complicated, and depends sensitively on the values of $n$ and the nature of the stabilizing charges. One turns out to require $n > 4$, and even then, one typically finds that there are contributions to the masses of the Kaluza-Klein states larger than $1/r$. If one is willing to suppose that supersymmetry is hierarchically broken on the branes, one can find an example with rather small charges and a radial dilaton light enough to affect Cavendish experiments. This case comes closest to realizing a solution of the hierarchy without very large parameters, and with no more mystery than the usual one of understanding the smallness of the observed cosmological constant. The methods of Ref. 6) can also be applied to the possibility that the radii are large due to some extreme values of other moduli. Finally, in the case $n = 2$ (and certain suitable generalizations) there may be additional possibilities. If one has bulk supersymmetry, the potential for $r$ is a function of $\ln(r)$. If this function has a minimum for a value of $\ln(r) \sim 40$, this would give rise to a large radius. Moreover, because the fields in the bulk vary logarithmically, and some inevitably couple to $F\tilde{F}$, such a picture might account for the smallness of the gauge couplings.* This picture of stabilization has much in common with the idea of Kahler stabilization discussed earlier.

Given a model for the stabilization of the radial dilaton, the question of early universe cosmology is also brought into focus. Perhaps the most serious issue is production of bulk modes. As noted in Ref. 28), if the temperature on the brane is higher than some temperature, $T_o$, then the bulk modes are overpopulated at nucleosynthesis. In the case $n = 2$, this temperature is only a few MeV. This already

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* I thank Nima Arkani-Hamed for discussions of this possibility. Elaborations on these ideas will appear elsewhere. The special role of $n = 2$ has been discussed in Ref. 32).
seems to be a fine tuning. Moreover, in this case, there may be other, very efficient, mechanisms for production of bulk modes, as will be described below. For any dimension, there is also a potential, very severe, moduli problem. In particular, if there is a period of inflation, it would seem that the bulk cosmological constant, and hence the location of the minimum, would be modified. The authors of Ref. 28) deal with this problem by supposing that the inflaton lives on a brane. A specific proposal along these lines was made in Ref. 33). This model, however, illustrates several generic problems: it is hard to obtain a reasonable fluctuation spectrum, and it is hard to understand why the reheating temperature is not of order \( M. \)

Other potential problems abound.\(^{34),35),6}\) Cosmology in higher dimensions opens many new and interesting possibilities, but the difficulties look formidable.

All of this leads to the following conclusion: given the current state of our understanding, the view that the compactification scales should be small is, at best, prejudice. There is no decisive theoretical argument that the fundamental scale of string theory could not be a few TeV. It is important to keep this possibility in mind, both in thinking about experiments and in developing string phenomenology. On the other hand, the issues raised above, and the absence of any compelling argument in favor of large dimensions, in my view, provide some reinforcement for the earlier prejudice.

§4. Maximal symmetry

We now turn to another approach to the problem of moduli,\(^5\) inspired, in part, by recent developments in duality, and in part by the cosmological moduli problem.\(^7\)

Imagine the classical moduli space of some string compactification. If there are points in this moduli space where all of the moduli are charged under unbroken symmetries, then:

- Such points are automatically stationary points of the full effective action.
- Because of their high degree of symmetry, it is natural for the early universe to start out at such a point.

From studies of duality, we know that there are many such points of “Maximal Symmetry”. Probably the simplest example of this phenomenon is provided by the IIB theory in 10 dimensions. This has a well-known \( SL(2,Z) \) symmetry, under which

\[
\tau = \frac{i}{g} + a
\]

transforms as

\[
\tau \rightarrow -\frac{1}{\tau}, \quad \tau \rightarrow \tau + 1.
\]

With \( a = 0 \), the first transformation has a self-dual point, a particular value of the coupling at which the \( Z_2 \) symmetry is “restored”. At this point, the dilaton transforms.

Upon compactification, one can construct many more examples of this phenomenon, with varying amounts of supersymmetry. For example, one can consider
toroidal compactification of the IIB theory, with special radii and angles for the torus; Gepner compactifications of the Type II theory, and many others. One interesting example in four dimensions is provided by toroidal compactification of the heterotic string. If one takes the torus to be a product of circles, each at the appropriate $SU(2)$ point, then all of the moduli are charged under the $SU(2)$ except the dilaton. But this theory also has an $SL(2, Z)$ symmetry; at the self-dual point, $S$ transforms. Presumably this phenomenon occurs also in theories with $N = 1$ supersymmetry; this is currently under study.

As we stated, such points are automatically stationary points of the effective action and thus candidate minima. Moreover, it is natural for the early universe to favor states with high degrees of symmetry, so this proposal solves the cosmological moduli problem. However, there are some obvious objections to this possibility. In particular, one expects that, generically, $\alpha = \mathcal{O}(1)$ at these points. This follows from our discussion in the previous section regarding holomorphy. It also is true for some of our particular examples. In the case of electric-magnetic duality (the heterotic example above), the Dirac quantization condition shows that the gauge coupling is indeed of order 1 at the self-dual point. In the case of four supersymmetries ($N = 1$ in four-dimensional counting), the situation might be better. First, as in the discussion of the previous section, Kahler potential effects might be relevant in the symmetry restoration phenomenon. A toy example of this was provided in Ref. 5). It would be of interest to survey examples of the enhanced symmetry phenomenon to determine if the couplings are ever small.

In any case, for the rest of this section, we will adopt a set of hypotheses and explore their consequences. In particular, we will assume that, at some high scale, $M$, one has

- Maximally enhanced symmetry: all of the moduli transform under symmetries.
- Approximate $N = 1$ supersymmetry.
- Small, unified gauge couplings. In particular, $e^{-S}$ is an extremely small number.

Before going further, we should note that gaugino condensation cannot play a role in supersymmetry breaking at such points. This is because linear couplings of the moduli to the gauge fields, $M W^2_\alpha$, are forbidden by the symmetries. This means that we require supersymmetry breaking at relatively low energies.

4.1. The case of no moduli

Consider the possibility that there are no moduli. This has two immediate consequences. First, supersymmetry cannot be broken by high energy string effects, but must be broken by effects which are visible in the low energy theory. To understand this, note that any supersymmetry breaking effect must be describable in terms of a superpotential which is a function of the light fields, $W(\Phi)$. In order to obtain

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* In theories with more than four supersymmetries, the moduli space is generally exact quantum mechanically, i.e., there is no potential for the moduli. We have in mind with these remarks models with four or less (zero) supersymmetries.
supersymmetry breaking, one needs a linear term for some singlet field. But it is not plausible that there are such light fields at strong coupling.\textsuperscript{1)}

This observation, in turn, has the consequence that whatever breaks supersymmetry must be visible in the low energy dynamics. This is likely to mean that there is a supersymmetry breaking hidden sector. What is the scale? If there are no singlets, couplings of the form $\Phi W_{\alpha}^2$ cannot be the source of gaugino mass, as is usually assumed.\textsuperscript{2)} This suggests that supersymmetry breaking must be a low energy phenomenon, presumably mediated by gauge interactions. In other words, this general framework predicts something like low energy gauge mediation.

4.2. Maximal symmetry

Now consider the case that there are moduli, and that the minimum of the potential lies at a point of maximal symmetry. This turns out to be similar to the case of no moduli, in its low energy consequences. Supersymmetry breaking again must be a low energy phenomenon, since the symmetries forbid terms linear in the fields. Gaugino condensation and its generalizations are also forbidden, since couplings such as $\Phi W_{\alpha}^2$ do not respect the symmetries. So, just as in the case of no moduli, supersymmetry breaking must be a low energy phenomenon, presumably mediated by gauge interactions.

One question which we can ask in this framework is: how are the moduli stabilized? There are several rather natural possibilities. Perhaps the most interesting is the following. Suppose that all of the moduli are charged under standard model gauge symmetries. This is not such an outlandish suggestion. The MSSM has approximate flat directions in which all the gauge symmetry is broken. For example, there is a direction parameterized by $QQQL$, i.e.

\[
Q = \begin{pmatrix} v & 0 & 0 \\ 0 & v & 0 \\ 0 & 0 & v \end{pmatrix}, \quad L = (0, v), \tag{14}
\]

where the $Q$ field is written as a matrix in color and flavor. This flat direction has 12 parameters, or 12 candidate moduli. Note that this direction can be exactly flat even without string miracles; a discrete $R$ symmetry under which $Q$ and $L$ are neutral can insure this.

Now suppose supersymmetry is broken in a hidden sector, with the breaking communicated by gauge fields as in usual models of gauge mediation. Then the fields $Q$ and $L$ will receive positive masses-squared from low energy loop effects. This means that the potential has a minimum at the symmetric point.

\textsuperscript{1)} Here, we are assuming that some sort of conventional notion of naturalness holds in strongly coupled string theory, i.e., when one includes both perturbative and non-perturbative effects. Note we are also ignoring the effects of field-independent constants in the superpotential. This follows from our basic assumption that supersymmetry is not broken at the high energy scale. It can be enforced by an unbroken discrete $R$ symmetry.

\textsuperscript{2)} Several authors have noted recently that at one loop, effects associated with the Kahler anomaly can lead to gaugino masses. However, these effects tend to be quite small, and to require a large scale for supersymmetry breaking.\textsuperscript{38,37,39}
§5. The cosmological constant problem

Within each of these hypotheses, the cosmological constant problem remains a significant puzzle. Within the framework of Kahler stabilization, we had to suppose a fine tuning of the Kahler potential to obtain vanishing $\Lambda$. In the case of Kahler stabilization, we assumed that the supersymmetry was broken at the intermediate scale, and transmitted to light fields by effects suppressed by $1/M_p$. Examining Eq. (7), we see that the potential possesses terms of opposite signs. For intermediate scale supersymmetry breaking, these terms are of the same order of magnitude. If the Kahler potential has just the right form (i.e., if it is tuned in precisely the right way), these terms can cancel. In the case of maximal symmetry, however, we have argued that the breaking must be at a low scale. In this case, the contributions to the $3|W|^2$ term from the susy-breaking dynamics are suppressed by powers of the breaking scale over $M_p$. Something more is needed if the cosmological constant is to vanish at the level of the effective lagrangian. It is necessary that there be a large constant in the superpotential in order to cancel the contribution to the vacuum energy coming from low energy supersymmetry breaking. This constant could be generated by gluino condensation in a pure gauge theory. In the absence of moduli, such condensation does generate a constant in $W$. This constant would have to be just the right size to cancel the cosmological term from the other sectors. This is arguably a troubling feature of gauge mediation in general. Of course, given our total lack of understanding of the cosmological constant problem, perhaps this concern is misplaced.

§6. Conclusions

If string theory is ever to be directly tested, it is probably necessary to extract some general, qualitative prediction. One such prediction might be that there should be low energy supersymmetry, broken in some particular way. Another might be that the string scale is very low, so that there might be many new states at accessible energies. In this talk, we have explored some possibilities, but we do not have firm answers. Phenomenologically, a low string scale is not ruled out, though it may be hard to understand a scale less than about $6 - 10$ TeV. On theoretical grounds, however, this possibility seems unlikely. It requires that the minimum of the potential lie at a rather implausible extreme of the moduli space. It also requires a rather elaborate structure, and some number of fine tunings. Still, a real theory being absent, these arguments can at best be described as informed prejudice. The challenge for these ideas is to provide some compelling argument that the scale should, indeed, be at some particular, low value. Alternatively, we might be lucky and make the extraordinary discovery that Newtonian gravity is modified at short distances, or that phase space is more than four dimensional at TeV energies.

It could be that the usual arguments based on hierarchies for low energy supersymmetry are incorrect, and that there are good string ground states in which supersymmetry is badly broken. After all, our failure to understand the cosmological constant problem suggests that our ideas about naturalness and fine tuning are not
entirely correct. So it is hard, given the present state of our understanding, to argue persuasively that low energy supersymmetry is an outcome of string theory. But we have seen that the hypothesis of low energy supersymmetry, combined with maximally enhanced symmetry, makes some definite, qualitative predictions. We have argued that with these suppositions, supersymmetry must be broken at very low energy scales (perhaps a few orders of magnitude above the weak scale) with gauge interactions as the messengers of the breaking. This suggests, in fact, an approach to phenomenology which does not require complete control of strong dynamics. One might hope to study moduli spaces of $N = 1$ theories, and to determine the symmetry structure at their enhanced symmetry points. Some features, such as spectra and perhaps gauge couplings and some terms in the superpotential, might be restricted by symmetries and holomorphy.

It may well be that fundamental theory is entering an era where hypotheses will be tested principally by their self consistency, and by considering various gadanken experiments. But it would be disappointing if we did not have some picture of how string theory made contact with nature, and if this picture did not make some predictions. It is quite possible that none of the proposals for string dynamics described here are correct. The cosmological constant and the question of the smallness of the gauge couplings are serious challenges to the maximal symmetry hypothesis, in particular. But hopefully there is some approach which allows a qualitative — and perhaps somewhat quantitative — picture.

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References

1) M. Dine and N. Seiberg, Phys. Lett. 162B (1985), 299.
2) M. Dine and Y. Shirman, Phys. Lett. B377 (1996), 36; hep-th/9601175.
3) This idea has been presented most compellingly in V. Kaplunovsky and J. Louis, Phys. Lett. B417 (1998), 45; hep-th/9708049.
4) T. Banks and M. Dine, Phys. Rev. D50 (1994), 7454; hep-th/9406132.
5) M. Dine, Y. Nir and Y. Shadmi, Phys. Lett. B438 (1998), 61; hep-th/9806124.
6) T. Banks, M. Dine and A. Nelson, in preparation.
7) T. Banks, D. Kaplan and A. Nelson, Phys. Rev. D49 (1994), 779; hep-ph/9308292.
8) M. Dine, L. Randall and S. Thomas, Phys. Rev. Lett. 75 (1995), 398; hep-ph/9503303; Nucl. Phys. B458 (1996), 291.
9) E. Witten, Nucl. Phys. B268 (1986), 79.
10) N. Arkani-Hamed and S. Dimopoulos, hep-ph/9811353.
11) Z. Berezhiani and G. Dvali, hep-ph/9811378.
12) S. Kachru and E. Silverstein, Phys. Rev. Lett. 80 (1998), 4855; hep-th/9802183.
13) J. Harvey, Phys. Rev. D59 (1999), 026002; hep-th/9807213.
14) E. Witten, Mod. Phys. Lett. A10 (1995), 2153; hep-th/9506101.
15) V. S. Kaplunovsky, Phys. Rev. Lett. 55 (1985), 1036.
16) M. Dine and N. Seiberg, Phys. Rev. Lett. 55 (366), 1985.
17) E. Witten, Nucl. Phys. B471 (1996), 135; hep-th/9602070.
18) P. Horava and E. Witten, Nucl. Phys. B475 (1996), 94; hep-th/9603142.
19) A. Lukas, B. Ovrut, K. Stelle and S. Waldram, hep-th/9803235.
20) I. Antoniadis, Phys. Lett. B246 (1990), 377.
   J. Lykken, Phys. Rev. D54 (1996), 3693; hep-th/9603133.
21) N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Rev. Lett. B429 (1998), 263; hep-ph/9803315.
22) Some precursors of these ideas can be found in V. A. Rubakov and M. E. Shaposhnikov, Phys. Lett. 125B (1983), 136.
   G. Dvali and M. Shifman, Nucl. Phys. B504 (1996), 127; hep-th/9611213; Phys. Lett. B396 (1997), 64; hep-th/9612128.
23) Among the papers exploring the possibility of large dimensions are:
   K. R. Dienes, E. Dudas and T. Gherghetta, Phys. Lett. B436 (1998), 55; hep-ph/9803466.
   G. Shiue and S. H. Tye, Phys. Rev. D58 (1998), 106007; hep-th/9805157.
   P. C. Argyres, S. Dimopoulos and J. March-Russell, Phys. Lett. B441 (1998), 96; hep-th/9808138.
   L. E. Ibanez, C. Munoz and S. Rigolin, hep-ph/9812397.
   A. Pomarol and M. Quiros, Phys. Lett. B438 (1998), 255.
   T. E. Clark and S. T. Love, hep-th/9901103.
24) N. Arkani-Hamed, S. Dimopoulos and J. March-Russell, hep-th/9809124.
25) N. Seiberg, private communication.
26) T. Banks and M. Dine, Nucl. Phys. B479 (1996), 173; hep-th/9605136.
27) T. Banks and M. Dine, Nucl. Phys. B505 (1997), 445; hep-th/9608197.
28) N. Arkani-Hamed, S. Dimopoulos and G. Dvali, hep-ph/9807344.
29) S. Cullen and M. Perelstein, hep-ph/9903422.
30) N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and J. March-Russell, hep-ph/9903224.
31) R. Sundrum, hep-ph/9807348.
32) I. Antoniadis and C. Bachas, hep-th/9812093.
33) G. Dvali and S.-H. H. Tye, hep-ph/9812483.
34) K. Benakli and S. Davidson, hep-ph/9810280.
   M. Maggiore and A. Riotto, hep-th/9811089
35) N. Kaloper and A. Linde, hep-th/9811141.
36) D. H. Lyth, Phys. Lett. B448 (1999), 191; hep-ph/9810320.
37) L. Randall and R. Sundrum, hep-th/9810155.
38) G. Giudice, M. Luty and H. Murayama, hep-ph/9810442.
39) J. Bagger, private communication.