Chiral Restoration and the Scalar and Vector Correlations in Hot and Dense Matter
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Abstract

First, it is pointed out that hadron/nuclear physics based on QCD should be regarded as “condensed matter physics” of the QCD vacuum. We indicate that phase shift analyses which respect chiral symmetry (ChS), analyticity and crossing symmetry of the scattering amplitude show the $\sigma$ meson pole in the $s$-channel in the low mass region as well as the $\rho$ meson pole in the $t$-channel in the $\pi$-$\pi$ scattering in the scalar channel. We review recent developments in exploring possible precursory phenomena of partial restoration of ChS in nuclear medium by examining the spectral function in the scalar and the vector channels. We emphasize that the wave function renormalization of the pion in the medium plays an essential role to induce the decrease of the pion decay constant as the order parameter of chiral transition. An emphasis is also put on the importance to examine the scalar and vector channels simultaneously for exploring the possible restoration of chiral symmetry.

1 Introduction

The basic observation on which the whole discussions in this report are based is that the dynamical breaking of chiral symmetry (ChS) is a phase transition of the QCD vacuum with an order parameter $\langle \bar{q}q \rangle$. This is a reflection of the complicated structure of QCD vacuum, which actually makes hadron/nuclear physics based on QCD tricky:

(1) The QCD Lagrangian is not written in terms of hadron fields but in terms of quark- and gluon-fields from which hadrons are composed, and the quarks and gluons are colored objects which can not exist in the asymptotic states. The low-lying elementary excitations on top of the non-perturbative QCD vacuum are composite and colorless particles, which we call hadrons.

(2) Symmetries possessed by the QCD Lagrangian, such as the chiral $\text{SU}(3)_L \times \text{SU}(3)_R$ symmetry in the massless limit of the first three quarks, and the color gauge symmetry, are not manifest in our every-day world. This complication is owing to the fact that the true QCD vacuum is completely different from the perturbative one and is actually realized through the phase transitions, i.e., the confinement-deconfinement and the chiral transitions. The notion of such a complicated vacuum structure, i.e., the collective nature of the vacuum and the elementary particles was first introduced by Nambu[1], in analogy with the physics of superconductivity[2].

(3) Some phenomenological rules extracted on the hadron dynamics such as the vector-meson dominance and the Okubo-Zweig-Iizuka rule might be related with some fundamental properties of the QCD vacuum. One may notice that the so called $U_A(1)$ anomaly which is responsible to make $\eta'$ as heavy as 960 MeV also characterizes the non-perturbative QCD vacuum[3].

(4) Thus one recognizes that hadron/nuclear physics as sub-atomic physics based on QCD should be a study of the nature of QCD vacuum; i.e., hadron/nuclear
physics is a combination of the condensed matter physics of the QCD vacuum and the atomic physics as played with the constituent quark-gluon model where the vacuum structure is taken for granted.

In the present report, focusing on the chiral transition in hot and/or dense hadronic matter, I will discuss some characteristic changes in the scalar and vector correlations associated with the (partial) restoration of ChS in the hadronic medium; the major part of this report is based on some previous ones.

2 Restoration of Chiral Symmetry as a Phase Transition of QCD vacuum

The lattice simulations show that the true QCD vacuum is realized through the chiral transition. At finite $T$, the lattice simulations also show that ChS is restored at $T = T_c \sim 150-175$ MeV, depending the number of the active light flavors. A heuristic argument based on a Hellman-Feynman theorem can tell us that the chiral condensate decreases at finite density $\rho_B$ as well as at finite $T$.

The chiral condensate at $T \neq 0$ is given by

$$\langle\bar{q}q\rangle_T = \frac{1}{Z} \text{Tr} \left[ \bar{q}_i q_i \, e^{-(H_{QCD} - \mu N)/T} \right] = \frac{\partial \omega(T)}{\partial m_i} \equiv \langle\bar{q}_i q_i\rangle_0 + \delta\langle\bar{q}_i q_i\rangle_T,$$

(2.1)

where $Z$ is the QCD partition function, $\omega$ is the free energy density and $m_i$ is the current quark mass ($i = u, d, s, \ldots$). For the free pion gas, the modification due to finite $T \neq 0$ is given by

$$\delta\langle\bar{q}_i q_i\rangle_T = \sum_p n_\pi(p)\langle\pi(p)|\bar{q}_i q_i|\pi(p)\rangle,$$

(2.2)

with $\langle\pi(p)|\bar{q}_i q_i|\pi(p)\rangle = m_\pi\langle\bar{q}_i q_i\rangle_\pi/E_p$, being the pion matrix element of the scalar charge $\bar{q}_i q_i$ due to the $i$ quark. The non-covariant matrix element $\langle\bar{q}_i q_i\rangle_\pi$ is given by the Hellman-Feynman theorem, again; $\langle\bar{q}_i q_i\rangle_\pi = \partial m_\pi/\partial m_i$. At small temperature, it can be shown that the pion matrix element of the scalar charge $\langle\bar{q}_i q_i\rangle_\pi \simeq 6.25 > 0$ with the use of Gell-Mann-Oakes-Renner relation, $f_\pi^2 m_\pi^2 = -(m_u + m_d)/2 \cdot (\bar{u}u + \bar{d}d)$. Thus, one may expect that $\langle\bar{q}_i q_i\rangle_T$ decreases in the absolute value; restoration of ChS at finite temperature. More systematic calculations using a nonlinear chiral Lagrangian confirm the above result.

As for the the degenerate nucleon system $|N\rangle$, one may start from the formula

$$\langle N|\bar{q}q|N\rangle = \frac{\partial\langle N|H_{QCD}|N\rangle}{\partial m_q},$$

(2.3)

where the expectation value of QCD Hamiltonian may be evaluated to be

$$\langle N|H_{QCD}|N\rangle = \varepsilon_{vac} + \rho_B[M_N + B(\rho_B)].$$

Here, $\varepsilon_{vac}, M_N$ and $B(\rho_B)$ denote the vacuum energy, the nucleon mass and the nuclear binding energy per particle, respectively. Thus one ends up with

$$\frac{\langle\bar{q}q\rangle}{\langle\bar{q}q\rangle_0} = 1 - \frac{\rho_B}{f_\pi^2 m_\pi^2} \left( \Sigma_{\pi N} + \tilde{m} \frac{d}{dm} B(\rho_B) \right),$$

(2.4)

where $\Sigma_{\pi N} = (m_u + m_d)/2 \cdot \langle N|\bar{u}u + \bar{d}d|N\rangle$ denotes the $\pi$-N sigma term with $\tilde{m} = (m_u + m_d)/2$; the semi-empirical value of $\Sigma_{\pi N}$ is known to be $(40 - 50)$ MeV.
Notice that the correction term with finite $\rho_B$ is positive and gives a reduction of almost 35% of $\langle \bar{q}q \rangle$ already at the normal nuclear matter density $\rho_0 = 0.17 \text{fm}^{-3}$. We notice that the physical origin of this reduction is common in finite-$T$ and -$\rho_B$ cases: The scalar probe $\bar{q}_i q_i$ hits either the vacuum or a particle $h$ present in the system at $T \neq 0$ and/or $\rho_B \neq 0$; in the latter case, $\bar{q}_i q_i$ picks up a positive contribution to the chiral condensate because of the positive scalar charge $\langle h | \bar{q}q | h \rangle > 0$ of the particle.

From the above estimate, one may consider that the central region of heavy nuclei could be dense enough to cause a partial restoration of ChS, realizing some characteristic phenomena of the chiral restoration in nuclear medium; some of them may be observed by experiments in the laboratories on Earth [3, 12].

It is a well-known fact in many-body or statistical physics that if a phase transition is of second order or weak first order, there may exist specific collective excitations called soft modes [14]; they actually correspond to the quantum fluctuations of the order parameter. In the case of chiral transition, there are two kinds of fluctuations; those of the phase and the modulus of the chiral condensate. The former is the Nambu-Goldstone boson, i.e., the pion, while the latter the $\sigma$ with the quantum numbers $I = 0$ and $J^{PC} = 0^{++}$. Some effective models [26] of QCD and the argument based on the “mended symmetry” of Weinberg [13] predict the $\sigma$ mass below or equal to the $\rho$ meson mass.

3 Low-energy QCD and the $\sigma$ meson

3.1 The $\sigma$ and chiral symmetry, unitarity, analyticity and crossing symmetry in the $\pi \pi$ scattering amplitudes

The elusiveness of the $\sigma$ meson comes from the fact that it strongly couples to two pions to acquire a large width $\Gamma \sim m_\sigma$. After the establishment of the chiral perturbation theory [15] for describing low energy hadron phenomena, one of the most important problems has been to describe resonances in a consistent way with ChS [16]. The central issue there is to incorporate the fundamental properties of the scattering amplitude such as unitarity, analyticity and the crossing symmetry together with ChS. In this way, the recent cautious phase shift analyses of the $\pi\pi$ scattering have come to claim a pole identified with the $\sigma$ in the $s$ channel together with the $\rho$ meson pole in the $t$ channel [7, 17, 18]. The $\sigma$ pole has the real part $\text{Re} m_\sigma = 500-600$ MeV and the imaginary part $\text{Im} m_\sigma \simeq \text{Re} m_\sigma$ [18]. Afterwards, it has been also found that the $\sigma$ pole gives a significant contribution in the decay processes of heavy particles involving a charm and $\tau$ leptons; as $D \to \pi\pi\pi$, $J/\psi \to \sigma \omega \to 2\pi\omega$ and $\tau \to a_1\nu \to \sigma\pi\nu \to 3\pi\nu$ [19, 20, 21].

A summary of the locations of the $\sigma$ pole in the complex energy plane may be found in [22].

To see the significance in establishing the $\sigma$ pole of incorporating ChS, analyticity and crossing symmetry as well as unitarity, we notice that the same phase shift can be well reproduced with a unitarized scattering amplitude lacking the $\sigma$ pole but including the $\rho$ meson pole in the $t$-channel [23, 24]. One may naturally wonder whether the existence of the $\sigma$ pole in the $s$ channel is real or not. It was apparent that the urgent problem is to incorporate crossing symmetry in the scattering amplitudes consistently with ChS. Then Igi and Hikasa [25] constructed
the invariant amplitude for the $\pi$-$\pi$ scattering using the $N/D$ method so that it satisfies the ChS low energy theorem, analyticity, unitarity and especially (approximate) crossing symmetry. They calculated two cases with and without the scalar pole degenerated with the $\rho$ meson, the existence of which was taken for granted. What they found is that the $\rho$ only scenario can account only about a half of the observed phase shift, while the degenerate $\rho$-$\sigma$ scenario gives a reasonable agreement with the data. In the phenomenological approaches like those given in [23], it was unclear how the fundamental properties of the scattering matrix are respected, such as ChS and crossing symmetry.

### 3.2 The $\sigma$ as a collective $q$-$\bar{q}$ mode on top of the QCD vacuum

In fact, the interpretation of the $\sigma$ has a long history of controversies [18]: First of all, the conventional constituent quark model has difficulties to describe the low-lying $\sigma$ as a $q$-$\bar{q}$ meson; the lowest scalar $q$-$\bar{q}$ meson should be a $P$ wave ($^3P_0$) state, which is in turn usually heavier than 1 GeV. Then there are various proposals for interpretation of the low-mass $\sigma$, commonly denying a pre-existing $q$-$\bar{q}$ state. However, it should be emphasized that the $\sigma$ as the quantum fluctuation of the chiral order parameter must be a collective state composed of many $q$-$\bar{q}$ states as the pion is [1, 26]; notice that the pion can not be understood within the conventional constituent quark model.

In [6], it is argued that the linear realization of ChS as given in the NJL-like dynamical models, especially incorporating the vector terms is consistent with the chiral perturbation theory in the sense that they reproduce the low-energy constants $L_i$ and $H_i$ in the nonlinear chiral Lagrangian [27].

### 3.3 Possible roles of the $\sigma$ in hadron phenomenology

If the $\sigma$ meson with a low mass has been identified, many experimental facts which otherwise are mysterious might be nicely accounted for in a simple way [3, 28]. The phenomena which have a possible relevance to the $\sigma$ include: (1) $\Delta I = 1/2$ rule in the kaon decay [29], (2) the intermediate-range attraction in nuclear force [30], (3) $\pi$-$N$ sigma term [31, 32] and so on. They are related to the collective nature of the $\sigma$, which causes an enhancement of the matrix elements involving the $\sigma$ in the intermediate state.

### 4 Partial chiral restoration and the $\sigma$ meson in hadronic matter

As remarked above, it still remains uncertain whether the $\sigma$ pole reported really corresponds to the quantum fluctuation of the chiral order parameter or only a $\pi$-$\pi$ molecule generated dynamically without the pre-existing $\sigma$. As was first shown in [33], if the $\sigma$ is really associated with the fluctuation of the chiral order parameter, one can expect that the $\sigma$ pole moves toward the origin in the complex energy plane in the chiral limit and the $\sigma$ may become a sharp resonance as ChS is restored.
at high temperature and/or density; the $\sigma$ can be a soft mode\cite{14} of the chiral restoration; see also \cite{34}.

One must, however, notice that a hadron put in a heavy nucleus may dissociate into complicated excitations to lose its identity. Then the most proper quantity to observe is the response function or spectral function in the channel with the same quantum number as the hadron has. When the coupling of the hadron with the environment is relatively small, then there may remain a peak with a small width in the spectral function, corresponding to the hadron, thereby it may be meaningful to tell the mass and the width of the hadron in the medium.

How about at finite density? The lattice QCD is unfortunately still premature to give any reliable results for the system at finite chemical potential. If numerical experiment may not be relied on, one can listen to Nature directly. Some years ago\cite{35, 28}, the present author proposed several nuclear experiments including one using electro-magnetic probes to create the scalar mode in nuclei, thereby obtain a clearer evidence of the existence of the $\sigma$ meson and also examine the possible restoration of chiral symmetry in nuclear medium. It was also mentioned that to avoid the huge amount of two pions from the $\rho$ meson, detecting neutral pions through four $\gamma$’s may be convenient.

Hatsuda, Shimizu and the present author (HKS)\cite{36} showed that the spectral enhancement near the $2m_{\pi}$ threshold takes place in association with partial restoration of ChS at finite baryon density. The calculation is a simple extension of the finite $T$ case done by Chiku and Hatsuda\cite{37}: Chiku and Hatsuda performed a model calculation of the spectral functions in the pion and the $\sigma$ meson channel at finite $T$ using a linear $\sigma$ model. They found that the spectral function in the $\sigma$ channel shows an enhancement near two-$m_{\pi}$ threshold as ChS is restored as $T$ goes high.

In \cite{36}, HKS started from the following linear sigma model:

$$L = \frac{1}{4}\text{Tr}[\partial\!\!\!\!\!\!\!\!\partial M\partial M^\dagger - \mu^2 MM^\dagger - \frac{2\lambda}{4!}(MM^\dagger)^2 - h(M + M^\dagger)] + \bar{\psi}(i\not\partial - gM_5)\psi + \cdots,$$

where $M = \sigma + i\vec{\tau} \cdot \vec{\pi}, M_5 = \sigma + i\gamma_5\vec{\tau} \cdot \vec{\pi}, \psi$ is the nucleon field, and Tr denotes the trace for the flavor index.

The spectral function in the scalar channel is obtained from the propagator. The $\sigma$-meson propagator at rest in the medium reads $D_{\sigma}^{-1}(\omega) = \omega^2 - m_{\sigma}^2 - \Sigma_{\sigma}(\omega; \rho_B)$, where $m_{\sigma}$ is the mass of the $\sigma$ in the tree-level, and $\Sigma_{\sigma}(\omega; \rho_B)$ represents the loop corrections in the vacuum as well as in the medium. The corresponding spectral function is given by $\rho_{\sigma}(\omega) = -\frac{1}{\pi}\text{Im}D_{\sigma}(\omega)$.

Now one can easily verify that $\text{Im}\Sigma_{\sigma} \propto \theta(\omega - 2m_{\pi})\sqrt{1 - \frac{4m_{\pi}^2}{\omega^2}}$ near the two-pion threshold in the one-loop order. On the other hand, the pole mass $m_{\sigma}^*$ in the medium is defined by $\text{Re}D_{\sigma}^{-1}(\omega = m_{\sigma}^*) = 0$. Partial restoration of ChS implies that $m_{\pi}^*$ approaches to $m_{\pi}$. Thus one sees that there should exist a density $\rho_c$ at which $\text{Re}D_{\sigma}^{-1}(\omega = 2m_{\pi})$ vanishes even before the complete restoration of ChS where $\sigma$-$\pi$ degeneracy gets realized: $\text{Re}D_{\sigma}^{-1}(\omega = 2m_{\pi}) = [\omega^2 - m_{\pi}^2 - \text{Re}\Sigma_{\sigma}(\omega = 2m_{\pi})] = 0$. At this point, the spectral function is solely given in terms of the imaginary part of the
Figure 1: The spectral function $\rho_\sigma(\omega)$ (the upper panel) and $\text{Re} D_\sigma^{-1}(\omega)$ (the lower panel) calculated with a linear sigma model. $\Phi(\rho) \equiv \langle \sigma \rangle / \sigma_0$ measures the rate of the partial restoration of the ChS at the baryonic density $\rho$.  

$$\rho_\sigma(\omega \simeq 2m_\pi) = -\frac{1}{\pi \text{Im}\Sigma_\sigma} \propto \frac{\theta(\omega - 2m_\pi)}{\sqrt{1 - \frac{4m_\pi^2}{\omega^2}}},$$

which clearly shows the near-threshold enhancement of the spectral function. This should be a general phenomenon to be realized in association with partial restoration of ChS.

In fact, the effects of the meson-loop as well as the baryon density was treated as a perturbation to the vacuum quantities in the above. Therefore, our loop-expansion should be valid only at relatively low densities. When we parameterize the chiral condensate in nuclear matter $\langle \sigma \rangle$ as $\langle \sigma \rangle = \sigma_0 \Phi(\rho)$, one may take the linear density approximation for small density; $\Phi(\rho) = 1 - C \rho / \rho_0$, with $C = (g_s / \sigma_0 m_\pi^2) \rho_0$.

The spectral function $\rho_\sigma(\omega)$ together with $\text{Re} D_\sigma^{-1}(\omega)$ calculated with a linear sigma model are shown in Fig.1 with the bare mass $m_\sigma = 550$ MeV: The characteristic enhancements of the spectral function is seen just above the $2m_\pi$. It is also to be noted that even well before $m_\sigma^*$ is close to $m_\pi$, when ChS is restored, a large enhancement of the spectral function is seen near $2m_\pi$ threshold.

5 Chiral restoration in nonlinear realization; role of the wave function renormalization

The near-threshold enhancement obtained above is based on the linear representation of the ChS, where the $\sigma$ degree of freedom is explicit from the outset. Actually, some related works use the non-linear realization of ChS supported by the development of the chiral perturbation theory; a unitarized chiral perturbation theory is...
intended to be applied to nuclear medium, which is a far from simple task. Nevertheless it would be intriguing to see how the possible restoration of ChS is implemented in the nonlinear realization of ChS where the $\sigma$ degrees of freedom is absent.

Jido et al showed that the nonlinear realization of the chiral symmetry can also give rise to a near $2m_\pi$ enhancement of the spectral function in nuclear medium as shown in Fig.2. The enhancement of the cross section is found due to the wave function renormalization of the pion in nuclear medium which causes the decrease of the pion decay constant $f_\pi^*$. Then (4.1) is cast into the following form

$$L = \frac{1}{2}[(\partial S)^2 - m_\sigma^2 S^2] - \frac{\lambda}{6} S^3 - \frac{\lambda}{4!} S^4$$

$$+ \frac{\langle \sigma \rangle + S}{4} \text{Tr}[\partial U\partial U^\dagger] + \frac{\langle \sigma \rangle + S}{4} h \text{Tr}[U^\dagger + U]$$

$$+ \mathcal{L}_{\pi N}^{(1)} - gSNN. \quad (5.1)$$

Here $\mathcal{L}_{\pi N}^{(1)} = \tilde{N}(i\not{\partial} + i\not{\gamma}_5 - m_\pi^*)N$, and $t(v_{\mu}, a_{\mu}) = (\xi \partial_{\mu} \xi^\dagger \pm \xi^\dagger \partial_{\mu} \xi)/2$, with $m_N^* = g\langle \sigma \rangle$. In this representation, the in-medium $\pi\pi$ scattering amplitude reads in the tree level

$$A(s) = \frac{s - m_\pi^2}{(\langle \sigma \rangle)^2} - \frac{(s - m_\pi^2)^2}{\langle \sigma \rangle^2} \frac{1}{s - m_\sigma^2}. \quad (5.2)$$

The first term in (5.2) comes from the contact $4\pi$ coupling generated by the expansion of the second line in (5.1) with the coefficient proportional to $1/\langle \sigma \rangle^2$. On the other hand, the second term in (5.2) is from the contribution of the scalar meson $S$ in the $s$-channel. Fig.2 shows a unitarized in-medium $\pi\pi$ cross section solely with the first term in (5.2), i.e., the non-linear chiral Lagrangian. One can see a clear enhancement of the cross section near the threshold or a softening, associated with restoration of ChS.

Although there is no explicit $\sigma$-degrees of freedom in this heavy $S$ approximation, there arises a decrease of the pion decay constant $f_\pi^*$ in nuclear medium. Precisely speaking, the heavy $S$ limit is defined as $\lambda$ and hence $m_\sigma^*$ go infinity with $g/\lambda$ and
\[ \langle \sigma \rangle_0 = f_\pi \text{ fixed.} \] Then integrating out the S field in this limit, one has the following low-energy Lagrangian

\[
\mathcal{L} = \left( \frac{f_\pi^2}{4} - \frac{gf_\pi}{2m_\sigma^2} \bar{N}N \right) \left( \text{Tr}[\partial U \partial U^\dagger] - \frac{h}{f_\pi} \text{Tr}[U^\dagger + U] \right) + \mathcal{L}_{\pi N}^{(1)} + \cdots , \tag{5.3}
\]

where all the constants take their vacuum values: \( f_\pi = \langle \sigma \rangle_0, m_\sigma^2 = \lambda \langle \sigma \rangle_0^2 / 3 + m_\pi^2 \), and \( m_N = g\langle \sigma \rangle_0 \). Note that \( gf_\pi / 2m_\sigma^2 \) in front of \( \bar{N}N \) approaches to a finite value \( 3g/2\lambda f_\pi \) in the heavy limit, thus it cannot be neglected. In (5.3), \( \cdots \) denotes other higher dimensional operators which are not relevant for the discussion below.

It should be remarked that the near-threshold enhancement is caused by the decrease of \( f_\pi^* \) owing to the following new vertex:

\[
\mathcal{L}_{\text{new}} = - \frac{3g}{2\lambda f_\pi} \bar{N}N \text{Tr}[\partial U \partial U^\dagger]. \tag{5.4}
\]

In Fig.3, \( 4\pi-N-N \) vertex generated by \( \mathcal{L}_{\text{new}} \) is shown as an example. In the uniform nuclear matter, \( \bar{N}N \) in eq.(5.4) may be replaced by \( \rho; \bar{N}N \to \rho \).

Then the coefficient of the first term of eq.(5.3) is written as \( f_\pi^2 / 4 \cdot (1 - 2g/f_\pi m_\sigma^2 \cdot \bar{N}N) \), which shows that the properly normalized pion field in nuclear matter must be

\[
\phi^* = (\phi / f_\pi) \cdot f_\pi^*. \tag{5.5}
\]

with \( f_\pi^* = f_\pi(1 - g\rho / f_\pi m_\sigma^2) \). The in-medium \( \pi\pi \) scattering with this normalization exactly reproduces the first term in (5.2) as it should be. It is important that the wave function renormalization of the pion field in the medium implies a reduction of the vacuum condensate

\[
f_\pi = \langle \sigma \rangle_0 \to f_\pi^* = \langle \sigma \rangle . \tag{5.6}
\]

The essential role of the wave-function renormalization of the pion field on the partial restoration of ChS is confirmed rigourously in the chiral perturbation theory \[40\] and has been also revealed in accounting for the anomalous repulsion seen in the deeply bound pionic nuclei\[41\].

Here it should be pointed out that the vertex eq.(5.4) which we have extracted from the linear sigma model has been known to be one of the next-to-leading order terms in the non-linear chiral Lagrangian in the heavy-baryon formalism \[39\]. Although roles of the new vertex have been extensively discussed in \[40\] in the context of chiral perturbation theory, the full non-perturbative incorporation of it, however, has not been done so far especially in calculations of the \( \pi\pi \) scattering amplitudes in nuclear matter starting from the non-linear chiral Lagrangian \[42\].

6 Concluding remarks

Interestingly enough, CHAOS collaboration \[43\] observed that the yield for \( M_A^{\pi^+\pi^-} \) near the \( 2m_\pi \) threshold increases dramatically with increasing \( A \); this experiment was motivated to explore the \( \pi-\pi \) correlations in nuclear medium \[44\]. They identified
that the $\pi^+\pi^-$ pairs in this range of $M_{\pi^+\pi^-}^A$ is in the $I = J = 0$ state. A similar experimental result which shows a softening of the spectral function in the $\sigma$ channel has been also obtained by TAPS group [45].

It is interesting that there are other possible experimental evidences for partial chiral restoration in nuclear matter than the chiral fluctuations in the sigma meson channel discussed so far. The deeply bound pionic atom has proved to be a good probe of the properties of the hadronic interaction deep inside of heavy nuclei. It has been suggested [46, 47] that the anomalous energy shift of the pionic atoms (pionic nuclei) owing to the strong interaction could be attributed to the decrease of the effective pion decay constant $f_\pi^*(\rho)$ at finite density $\rho$, which may imply that the ChS is partially restored deep inside of nuclei. It is interesting enough that the decrease of $f_\pi^*(\rho)$ is owing to the wave-function renormalization of the pion field in nuclei [41], as mentioned before. A KEK experiment also shows the softening of the spectral function in the vector ($\rho/\omega$) channel in heavy nuclei such as Cu [48], which might indicate also that restoration of ChS in nuclear medium as was suggested in [49].

Recently, Yokokawa et al [50] have applied the $N/D$ method a la Igi and Hikasa to hot and dense matter and examined the pole structures of the scattering matrices and the spectral functions in the $\sigma$ and the $\rho$ meson channels on the equal footing: The effect of chiral restoration is taken into account in the mean field level, which is tantamount to replacing $f_\pi$ by $f_\pi^*$. It was found that there exist two kinds of poles in the complex energy plane in both channels in the scattering matrix, one of which moves toward the origin in the chiral limit; this implies especially that the sigma meson which is elusive in the free space may appear as a rather sharp resonance in hot and/or dense medium where ChS is partially restored. This pole behavior was first suggested in [38] and shown in [8]. Yokokawa et al have also shown that the spectral functions in the both channels give rise to a softening in tandem as the ChS is restored.

More than a decade ago, the present author [51] examined characteristic behaviors of the baryon number susceptibility $\chi_B = \partial \rho / \partial \mu$ at finite temperature and density. It was shown that the $\chi_B$ is actually the correlation function in the vector channel (the longitudinal component) and coupled to the scalar susceptibility $\partial <\bar{q}q>/\partial m$ at finite chemical potential $\mu \neq 0$, and hence the latter can reflect the singular behavior of the former around the critical point of the chiral transition. It certainly be fruitful to examine the scalar and vector correlations in a combined way to explore possible critical phenomena of the chiral transition in hot and/or dense medium.

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1 In lack of time, I failed to present this part in the workshop.
Acknowledgements

The major part of this report is based on the work done in collaboration with T. Hatsuda, H. Shimizu, D. Jido and K. Yokokawa. I thank them for the collaboration and discussions. This work is supported by the Grants-in-Aids of the Japanese Ministry of Education, Science and Culture (No. 14540263).

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