Full Field of View Super-Resolution Imaging via Two Static Masks

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Abstract. The usage of two static gratings for obtaining super resolved imaging dates back to the work by Bachl and Lukosz in 1967. However, in that approach, a severe reduction in the field of view was the necessary condition for improving the resolution. In this paper we present two approaches that are also based upon two static gratings but without the need to sacrifice in the field of view. The key idea for not paying with the field of view is performed in two ways: First, by using white light illumination that averages the ghost images obtained outside the region of interest since the positions of those images are wavelength dependent. Second, by using two random functions for the encoding and the decoding of the spatial information instead of using the spatially periodic gratings.

1. Introduction

Super resolution is an interesting as well as practical field in which one exceeds the diffraction as well as the geometrical limits of the imaging system and image smaller spatial features [1-3]. This is obtained by paying with various other domains such as time [4], field of view [5-7], code [8,9], polarization [10] and wavelengths [11]. The operation principle is basically the same and it includes an encoding of the small spatial features, which can not pass through the imaging system, in the domain in which the payment is done and then transmitting them through the imager and decoding them back into the space domain.

In this paper we use two static masks for the encoding/decoding [6,7]. However, instead of a grating we propose two approaches that allow obtaining super resolved imaging without reduction in the field of view: first by using of two random masks rather than gratings and second by using two gratings but illuminating with white light. The payment in both approaches is as follows: in the random masks approach it is related to the fact that the masks need to have much smaller features than those we aim to resolve in the object. In addition we pay in contrast of the resulted output image similarly to what happens in code division multiplexing approach [9]. In the white light illumination
idea we need the object to be gray level object. Note also that the approach with the two random masks, by moving one of the masks we can refocus our imaging system and obtain an auto focusing mechanism.

2. Mathematical Derivation

2.1. Random masks

The schematic sketch of the proposed setup is seen in Fig. 1(a). We will now analyze it mathematically and prove that indeed a super resolved imaging is obtained. In addition we will show the required trade offs that are needed to be paid in order to obtain the desired outcome. For simplicity we will perform 1-D analysis. The extension to 2-D is straightforward.

The field distribution after free space propagation of $z_1$ equals to:

$$g_1(x) = \int G(\mu) \exp(\pi i \lambda z_1 \mu^2) \exp(2\pi i x \mu) d\mu$$

(1)

where $G(\mu)$ is the Fourier transform of the field distribution. We multiply this distribution by the random encoding mask equals to $m(x)$ and the obtained product equals to:

$$\int \left[ \int M(\mu - \mu_i) G(\mu) \exp(\pi i \lambda z_1 \mu^2) d\mu \right] \exp(2\pi i x \mu) d\mu$$

(2)

While $M(\mu)$ is the Fourier transform of the mask. Now we perform back free space propagation of $-z_1$ to obtain the field distribution in the input plane while the effect of the encoding mask is included:

$$\int \left[ \int M(\mu - \mu_i) G(\mu) \exp(\pi i \lambda z_1 \mu^2) d\mu \right] \exp(-\pi i \lambda z_1 \mu^2) \exp(2\pi i x \mu) d\mu$$

(3)

we progress now to the aperture plane by performing a Fourier transform, which after mathematical simplification equals to:

$$\int M \left( \frac{\mu_x}{\lambda F} - \mu_i \right) G(\mu) \exp(\pi i \lambda z_1 \mu^2) d\mu \exp\left( -\pi i \lambda z_1 \left( \frac{\mu_x}{\lambda F} \right)^2 \right)$$

(4)

If we multiply by the aperture (we assume a rect function for the aperture) and after additional optical Fourier transform we reach the output plane (yet without taking into account the second decoding mask):

$$\int \left[ \int M \left( \frac{\mu_x}{\lambda F} - \mu_i \right) G(\mu) \exp(\pi i \lambda z_1 \mu^2) d\mu \right] \exp\left( -\pi i \lambda z_1 \left( \frac{\mu_x}{\lambda F} \right)^2 \right) \text{rect}\left( \frac{\mu_x}{\Delta \mu_z / \lambda F} \right) \exp\left( -\pi i \lambda z_1 \frac{\mu_x}{\lambda F} \right) d\mu$$

(5)

We change first the variables into $v = \frac{\mu_x}{\lambda F}$:

$$\int \left[ \int M (v - \mu_i) G(\mu) \exp(\pi i \lambda z_1 \mu^2) d\mu \right] \exp(-\pi i \lambda z_1 v^2) \text{rect}\left( \frac{v}{\Delta \mu_z / \lambda F} \right) \exp(-2\pi i v x) dv$$

(6)

Now we need to have a free space propagation of $z_2$ in order to reach the random decoding mask. To do that we use the angular spectrum approach for computing the free space propagation, i.e. we multiply the spectrum by the chirp phase factor:
\[
\int \left[ \int M(v - \mu) \exp(\pi i \lambda z, \mu^2) d\mu \right] \exp\left( \frac{2\pi i v x}{\Delta \mu / \lambda F} \right) e^{-(2\pi ivx)d\nu} 
\]
(7)

Now after propagating a free space distance of \(z_2\) we will multiply by the decoding random mask \(m^*(x)\) and the expression that we obtain is a convolution in the Fourier domain:

\[
\int \left[ \int M(v_i - \mu_i) \exp(\pi i \lambda z, \mu^2) d\mu \right] \exp\left( \frac{\nu}{\Delta \mu / \lambda F} \right) M^*(-v_i + v) \exp(2\pi ivx)d\nu d\nu
\]
(8)

Now we need to do additional free space propagation of \(-z_2\), which means another Fourier multiplying by the chirp factor and inverse Fourier:

\[
\int \left[ \int M(v_i - \mu_i) \exp(\pi i \lambda z, \mu^2) d\mu \right] \exp\left( \frac{\nu}{\Delta \mu / \lambda F} \right) M^*(-v_i + v) \exp(2\pi ivx)d\nu d\nu
\]
(9)

This is the field distribution in the output plane. Note that the masks of encoding and decoding are random and therefore uncorrelated, therefore we can write

\[
\int f(\nu) \cdot M(\nu) M^*(\nu - v_i)d\nu = \delta(v_i)
\]
(10)

For any general function \(f(\nu)\). Since we are talking about fields, M can be complex and non hermitic. We rewrite Eq. 9 as:

\[
\int G(\nu_i) \exp(\pi i \lambda z, \nu^2) \exp(-\pi i \lambda z, \nu^2) \exp(2\pi ivx) \cdot \left[ \int \exp(\pi i \lambda z, \nu_i^2) \exp\left( \frac{\nu_i}{\Delta \mu / \lambda F} \right) M(v_i - \mu_i) M^*(-v_i + v_i)d\nu \right] d\nu d\nu
\]
(11)

and use the assumption of Eq. 10, yielding:

\[
\int G(\nu_i) \exp(\pi i \lambda (z_i - z_2), \nu_i^2) \exp(2\pi ivx)d\nu
\]
(12)

For intensity, in the spatially coherent case we may write:

\[
I(x) = \int G(\nu_i) \exp(\pi i \lambda (z_i - z_2), \nu_i^2) \exp(2\pi ivx)d\nu
\]
(13)

For \(z_i = z_2\) one obtains super resolution since the field of the output equals to the fill resolution object's field \(g(x)\). An interesting application for the proposed setup can be filtering. By choosing \(z_1 - z_2\) not being zero we actually apply filtering over the input object.

Time multiplexing super resolution is capable of assisting in increasing the depth of focus of an imaging system. Since the setup of Refs. [12] generates synthetic OTF which equals to the replications of the original OTF of the imaging system, then if the spectral distance between the replications is smaller than the half the width of the original OTF then despite of the fact that the replicated OTF is narrowed due to the defocusing, after the replications a wider synthetic OTF is generated that allows imaging of high spatial frequencies:

\[
S(\mu) = \sum A_n B_n \text{trian} \left( \frac{\mu - n \mu_0}{\Delta \mu} \right)
\]
(14)

where \(S(\mu)\) is the synthetic OTF, \(\text{trian}\) is the triangle function represents the original OTF, \(A_n\) and \(B_n\) are the Fourier coefficients of the encoding and the decoding gratings respectively, \(\mu_0\) is the distance between replications and \(\Delta \mu\) is half the width of the original OTF.

However, it is clear that such a setup is problematic in respect to the projection of the encoding gratings as well as to the energetic efficiency of the image [12]. The conventional auto focusing involves movement of the lenses in the imaging module. In this paper we see a very elegant solution to the focusing topic. By varying the difference \(z_1 - z_2\) one may actually create an auto focus imaging system where in order to focus we just shift one of the random masks, i.e. change the difference \(z_1 - z_2\).

Note that for the assumption of Eq. 10 the Fourier of the encoding/decoding mask \(M\) must contain a lot of features which means that \(m(x)\) should be large in the spatial domain, at least as large as \(g(x)\).
and definitely much larger than the point spread function of the imaging system before super resolution (within the width of the aperture which was a \textit{rect} in our case, the function \(M\) should have as many features as possible). In addition, the spectral width of the coding/decoding mask i.e. the width of \(M\) should be as large as the synthetic aperture we aim to generate in the super resolution process. This in a way resembles CDMA coding where also orthogonality is required in order to separate mixed bits. In our case the resolution of \(m(x)\), i.e. its smallest feature should at least as small as the smallest feature in \(g(x)\) that we want to see divided by the super resolution factor. This is our payment for the super resolution improvement in addition to the energy and contrast loses.

2.2. Polychromatic illumination

Returning again to the setup of Fig. 1(a) where the two random masks are replaced with two identical gratings. Following the mathematical analysis given in Ref. 2, we proved that for \(z_1 = -z_2\) super resolution can be obtained while the super resolved reconstruction is obtained within the field of view limited by:

\[
x_{m,n} = \lambda z \nu_0 (m + n)
\]

where \(\nu_0\) is the basic frequency of the two gratings and \(m, n\) are integers. The meaning is that the super resolved image is obtained only for \(m = pn\) and all the replicas that do not fulfill this condition (crossed terms with \(n \neq pm\)) will appear at the spatial positions described by Eq. 14.

By using white light illumination will cause the undesired term to appear each at different locations (since according to Eq. 14 the locations of the undesired orders is proportional to the wavelength \(\lambda\) and thus all will be averaged into an uniform background affecting the resulted image only by reducing its dynamic range or its signal to noise ratio (SNR) in the detector. The averaging over the wavelengths reduces eventually the SNR of the information.

3. Results

3.1. Random masks

For the system simulations we have assumed the configuration of Fig. 1(a). In Fig. 2 we present the obtained results. In Fig. 2(a) we show the high resolution reference. In Fig. 2(b) we see the low resolution reference as it is seen after the spatial blurring due to the high F number of the imaging lens. After applying the proposed approach on the imaging quality of Fig. 2(b) and after adding the random masks, the obtained result is seen in Fig. 2(c).

![Figure 2](image)

\textbf{Figure 2.} Simulation results: (a). The high resolution reference. (b). The low resolution image as it is seen with the low resolution imager. (c). The result obtained after adding the random masks to the imaging quality of 2(b).

One may see that the reconstructed image is very similar to the original high resolution reference of Fig. 2(a). The resolution improvement factor in this case was of more than a factor of 4 in every axis.
3.2. Polychromatic illumination
To demonstrate the presented approach, the optical setup shown in Fig. 1(b) was used. The experimental setup included two imaging modules. The magnification of the first imaging system was selected to be 7.5x. The second imaging module magnifies the first image plane into output plane and its magnification can be selected according to our benefit. As first imaging system we used a long working distance infinity corrected Mitutoyo microscope lens with 0.14NA. A zoom photographic objective is used as second imaging system (acting as a tube lens). This lens should not have the restriction of having a fixed magnification.

White light illumination is provided by a halogen lamp source and a 3CCD color video camera (SONY Model DXC-950P) captures the final images. The halogen lamp has relatively uniform spectrum in the visible range and therefore the assumption for the spectral uniformity as done in the mathematical analysis of Section 2 is valid.

Two precision Ronchi ruling slides were used as diffraction gratings in the experiment. The period of both G1 and G2 gratings was $p_1=600$ lp/mm and $p_2=80$ lp/mm, respectively (due to the ratio of magnifications between the two parts of our setup, the second grating could be a low frequency grating). The period of the first grating is selected depending on the NA of the microscope lens that was used as first imaging system. To achieve a resolution gain factor close to 2, the diffraction angle for a central wavelength of the broadband spectral light used as illumination must be nearly twice the angle defined by the NA of the objective. This means that a period of around 500 lp/mm is suitable for such a resolution improvement. Once the first grating is selected, one can do both: fixing the magnification of the microscope objective and properly selecting the G2 grating, or the opposite. In our case, a ratio of 7.5 was defined by the periods of both diffraction gratings and this will be the magnification that will be aimed for the microscope lens.

We have used a negative high resolution USAF test target for 1D experiments. Figure 3(a) depicts the full field of view image when the presented approach for a magnification of the second imaging system close to 1. One can see that as the ghost images are wavelength sensitive, they are averaged in the background (which means that there is no limitation on the field of view). On the other hand, the proper combination of diffraction orders between both gratings compensates their chromatic dispersion and reinforces the white light super resolved image. In Fig. 3(b) we show the classical Bachl and Lukosz monochromatic experiment by simply placing an interference filter (515 nm main wavelength) before the input plane.

![Figure 3](image)

**Figure 3.** Experimental results: (a) The full field of view superresolved image obtained using the presented approach, and (b) The full field of view image with monochromatic illumination.

One may see as the ghost images are not averaged the final resolution is limited by the distance between the replicated diffraction orders. In this case, a reduction in the field of view is needed to allow super resolution over the region of interest.

In Fig. 4 one may see the central part of the resolution target where a magnification of close to 7x is chosen for the tube lens system. One may see that indeed the resolution of the vertical lines...
(Group9, Element 2 corresponding with 575 lp/mm) is much higher than the resolution of the horizontal lines (Group 8, Element 4 corresponding with 362 lp/mm). Therefore, the experiment has demonstrated resolution improvement by a factor of almost 2.

Figure 4. Experimental results: High-resolution region of interest. (a) Reference image obtained without the gratings (b). White squares mark the resolution limit with and without our approach.

4. Conclusions
In this paper we have presented a new direction for super resolution. It involves two static masks. The outcome is a super resolved image in which one does not pay neither in time nor in field of view. The payment is done in the spatial resolution which is required from the encoding/decoding masks as well as the dynamic range of the detector.

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