Dissipation-driven quantum phase transitions in a Tomonaga-Luttinger liquid electrostatically coupled to a metallic gate

M. A. Cazalilla, F. Sol, and F. Guinea

Introduction: In the study of properties of quantum wires (and other mesoscopic systems), proximity to metallic gates is frequently regarded as a source of static screening of the interactions in the wire. Within a classical electrostatic picture, which is valid at long distances from the gate, this arises because electrons in the wire interact, not only amongst themselves, but also with their image charges in the gate. Thus the Coulomb potential becomes a more rapidly decaying dipole-dipole potential. Moreover, metallic environments (e.g. gates) are also a source of dephasing and dissipation (for experimental research on related topics, see [1]), which arise because the electrons in the wire can exchange energy and momentum with the low-energy electromagnetic modes of the gate. This effect is described by the dissipative part of the screened potential and, as we show below, it can lead to backscattering (i.e. a scattering process where one or several electrons reverse their direction of motion) in a one-dimensional (1D) quantum wire. We find that, for an arbitrarily small coupling to the gate, the backscattering can drive a quantum phase transition provided that the interactions between the electrons in the wire are sufficiently repulsive.

The effects of dissipation on quantum phase transitions have attracted much attention [2, 3, 4, 5]. We discuss here transitions induced by dissipation, as in [6, 7, 8]. Some aspects of this work are also related to previous research on 1D systems coupled to environments [9, 10]. In particular, the system studied here can be considered a microscopic realization of the model studied in [11]. A discussion of the relations between our findings and this work will be given below.

The model: The electrostatic coupling of the wire to a metallic gate is described by

$$H_{gw} = \int d\mathbf{r} d\mathbf{r}' v_c(\mathbf{r} - \mathbf{r}') \rho_w(\mathbf{r}) \rho_g(\mathbf{r}')$$

where $v_c(\mathbf{r}) = e^2/4\pi\epsilon |\mathbf{r}|$ is the (statically screened) Coulomb potential, where $e < 0$ is the electron charge and $\epsilon$ the dielectric constant of the insulating medium located between the gate and the wire. The operators $\rho_g(\mathbf{r})$ and $\rho_w(\mathbf{r})$ describe the density fluctuations in the gate and the wire, respectively. For a 1D quantum wire, $\rho_w(\mathbf{r}) \approx \rho_w(x)\delta(y)\delta(z - z_0)$, where $z_0 > w > 0$ is the distance measured from the surface of the gate (see Fig. 1) and $w$ is the width of the wire. Following [12, 21], we integrate out the density modes of the metallic gate, and obtain the following effective action for the 1D wire ($L$ is the length of the wire and $T = \beta^{-1}$ is the temperature):

$$S_{\text{diss}} = \frac{1}{2\hbar} \int_0^L dxdx' \int_0^{\hbar/\beta} d\tau d\tau' \rho_w(x, \tau) \times V_{\text{scr}}(x - x', z_0, \tau - \tau') \rho_w(x', \tau'),$$

where we have assumed a translationally invariant flat gate so that the screened interaction $V_{\text{scr}}(x, x', \tau) = -\langle V\mathbf{r} - \mathbf{r}'\rangle_g/\hbar$, depends only on $\mathbf{R} - \mathbf{R}'$ ($\mathbf{r} = (x, y, z)$); $V(\mathbf{r}, \tau) = \int d\mathbf{r} v_c(\mathbf{r} - \mathbf{r}') \rho_g(\mathbf{r}')$ and $\langle \ldots \rangle_g$ denotes average over the gate’s degrees of freedom. To obtain an effective description of the effect of the gate on the low-temperature and low-frequency properties of the 1D wire, we first employ bosonization [21, 22, 23]. In the absence of the gate, the wire [24] is a Tomonaga-Luttinger liquid, described by the action [21, 22, 23]:

$$S_{\text{0}}[\phi] = \frac{1}{2\pi g} \int dx d\tau \left[ \frac{1}{v} (\partial_\tau \phi)^2 + v (\partial_x \phi)^2 \right].$$
where $\phi(x, \tau)$ is a plasmon field that varies slowly on the scale of lattice constant, $a$, and $v = v_F/g$ is the phase velocity of the plasmons, being $v_F$ the bare Fermi velocity. Treating the ions in the wire as a positive uniform background of the same density $\rho_0$ as the electrons, we focus on the density fluctuations about $\rho_0$. In bosonized form, $\rho_\omega(x, \tau) = \frac{1}{2} \partial_\omega \phi(x, \tau) + \frac{1}{2 \pi} \sum_{m \neq 0} e^{i m (k_F x + \phi(x, \tau))}$. Each of the terms describes the low-energy density fluctuations of momentum $q \approx 2mk_F$, where $m$ is an integer and $k_F$ the Fermi momentum. Replacing $\rho_\omega(x, \tau) \rightarrow \frac{1}{2} \partial_\omega \phi(x, \tau)$ in $\rho$ yields a term that describes forward ($q \approx 0$) scattering between electrons. Such a term leads to the (static) screening of the Coulomb interaction described in the introduction, and its dissipative part yields a contribution to the action of the form $\int dq d\omega |q| q^2 f(q) |\phi(q, \omega)|^2$. We find at small $q$ that $f(q) \sim \ln(1/q)$ for a semi-infinite 3D diffusive gate or a granular gate. Thus, the forward scattering term is irrelevant in the renormalization-group (RG) sense. For a 2D diffusive gate, $f(q) \sim q^{-1}$ and the term is marginal in the RG sense but it can be shown that it does not modify the power-law correlations of the system at zero-temperature. Moreover, in the limit of strongly repulsive interactions which interests us here its effect is small. We therefore focus on the (backscattering) terms with $q \approx 2k_F$, which describe Friedel oscillations. In the limit of very repulsive interactions, the electrons in 1D wire are a Tomonaga-Luttinger liquid (TLL) that is close to a Wigner-crystal state and density correlations are dominated by this oscillating term. Thus, $S_{\text{diss}}[\phi] = -\frac{\eta}{2 \pi} \int_0^L dx \int_0^{\hbar^2} d\tau d\tau' K(\tau - \tau') \times \cos 2p [\phi(x, \tau) - \phi(x, \tau')]$ for a 2D gate, $f(q) \sim q^{-1}$ and the term is marginal in the RG sense but it can be shown that it does not modify the power-law correlations of the system at zero-temperature. Moreover, in the limit of strongly repulsive interactions which interests us here its effect is small. We therefore focus on the (backscattering) terms with $q \approx 2k_F$, which describe Friedel oscillations. In the limit of very repulsive interactions, the electrons in 1D wire are a Tomonaga-Luttinger liquid (TLL) that is close to a Wigner-crystal state and density correlations are dominated by this oscillating term. Thus, $S_{\text{diss}}[\phi] = -\frac{\eta}{\pi} \int_0^L dx \int_0^{\hbar^2} d\tau d\tau' K(\tau - \tau') \times \cos 2p [\phi(x, \tau) - \phi(x, \tau')]$, where $\eta \propto a^{-2} S(Q_p, z_0)$, $S(q, z_0)$ being the Fourier transform of the spatial dependence of the dissipative part of $V_{\text{sc}}, (x, z_0, \tau)$ of $W_{\text{sc}}(x, z_0) \delta(\tau) + K(\tau) S(x, z_0)$ at low energies. The static part, $W_{\text{sc}}(x, z_0)$, has the effect of screening the interactions in the wire, and therefore leads to an effective dependence of $g$ on $z_0$: as $z_0 \rightarrow 0$, the more screened the Coulomb interactions are, and therefore $g \rightarrow 1$. The dissipative kernel $K(\tau) = (\pi/\hbar^2)^{1+s} |\sin(\pi/\hbar^2)|^{-1-s}$, for $\omega \ll \omega_c$, with $\omega_c = \min\{E_F/h, \omega_g(\tau_c)\}$ being the bandwidth of the wire and $\omega_g(\tau_c)$ the characteristic response frequency of the gate electrons. We have generalized the model to consider general dissipative environments (characterized by $s$, a metallic gate corresponding to ohmic dissipation, $s = 1$) as well as generic backscattering processes for $q \approx Q_p = 2p^2 k_F$. Spinless electrons correspond to $p = 1$, spin-1/2 electrons to $p = \sqrt{2}$ (provided that $q \leq \frac{1}{2}$), and nanotubes to $p = 2$ (provided that $q \leq \frac{1}{2}$).

The dependence of the friction coefficient $\eta$ on the gate-wire distance $z_0$ can be obtained for various models of the gate: $\eta \approx a^{-2} (\pi Q_p \sigma_{2D})^{-1} L_0(2Qz_0)$ for a 2D gate, and $\eta \approx a^{-2} (\pi \sigma_{3D})^{-1} K_0(2Qz_0)$ where $\sigma_{2D}$ and $\sigma_{3D}$ are gate conductivities measured in units of $e^2/h$, and $L_0(x) = K_0(x)$, $K_0(x)$ being the modified Bessel function of the 2nd kind. Thus, the values of $\eta$ and $g$ can be tuned either by bringing the wire closer to the gate or, alternatively if the wire is connected to leads, by charging the gate to vary the chemical potential (and therefore the density $\rho_0 \propto k_F \propto Q_p$) of the wire. In deriving Eq. (1), we have assumed that the wire is away from half-filling. The analysis of the half-filled case is more involved and will be reported elsewhere [24].

**Weak coupling RG analysis:** To assess the stability of the TLL when perturbed by $S_{\text{diss}}[\phi]$ we have studied the RG flow of the above model. Assuming that the dimensionless coupling $\alpha = (v_F c)(\tau_c)^{1-s} \eta$ is small, we perturbatively integrate high-frequency density fluctuations to lowest order in $\alpha$, and obtain the following RG equations:

$$\frac{d \alpha}{d \ell} = 4p^2 g \alpha$$

$$\frac{d g}{d \ell} = -4p^2 g^2 \alpha - 4p^2 g^2 \alpha$$

where $\ell = \ln(\omega_c/T)$. These equations describe a Kosterlitz-Thouless-like transition around a quantum critical point where $\alpha = \alpha^* = 0$ and $g = g^* = (2-s)/2p^2$ ($g^* = 1/2p^2$ for the ohmic case). At the critical point correlations decay as power-laws with universal exponents determined by $g^*$. E.g. density correlations at $2k_F$ decay as $(x^2 + v^2 \tau^2)^{-s'}$, which implies that the dynamical exponent $z = 1$. It also interesting to point out that dissipation does not drive any phase transition for $s > 2$. Furthermore, for an infinite-range dissipative kernel ($s = -1$) and $p = 1$, [11, 12] reduce to the equations derived by Voit and Schulz [20] and Giamarchi and Schulz [21] for spinless electrons in the presence of phonons and disorder, respectively.

In the phase where $\alpha$ flows towards strong coupling, the system is characterized by a length scale which diverges as $\xi_1 \approx k_F^{-1} e^{-\pi/[(2-s)p^2\omega_c(\sigma_{3D})]}$ with the distance $\alpha - \alpha_c$ to the transition, and behaves as $\xi_1 \approx k_F^{-1} [\alpha(0)]^{2p^2(2-s')}$ far from it. On the side where $\alpha$ scales down to zero, the system is a TLL with infinite conductivity at zero temperature (frequency). However, at finite temperature (frequency) the (optical) conductivity is finite a behaves as a power-law: $\sigma(T) \sim \frac{1}{T^{2-\mu}} \omega^{\frac{\mu}{2}} \omega^{\frac{\mu}{2}}$, and $\sigma(\omega > 0) \sim \alpha \omega^{1-s}$, with $\omega > 0$ and $\alpha$ (the transition), where the exponent $\mu = (2p^2 g + s + 1)$. In the phase where $S_{\text{diss}}[\phi]$ is relevant, an illustrative but rather crude estimate of the conductivity, hopefully valid in the large $\eta$ limit, can be obtained by expanding the cosine in Eq. (1) and keeping the quadratic terms in $\phi(x, \tau)$ only. Using the Kubo formula [22],

$$\sigma(\omega) = \frac{i D}{\omega + i/\tau_d}$$
where $D = g e^2 v / h \pi$ is the Drude weight and $\tau_d^{-1} = 4\pi^2 q v n$. However, for $\omega \gg \tau_d^{-1}$ (but $\omega \ll \omega_c$), we expect a crossover to a power-law such that $\text{Re } \sigma(\omega) \sim \alpha \omega^{d-4}$.

**Self-consistent Harmonic Approximation (SCHA):** A better approximation than expanding the cosine in \( \xi \) can be obtained by using the SCHA, which approximates $S_{\text{diss}}$ by a quadratic term \[ 30 \] \( -\frac{1}{2} \int dx \, dv \int d\tau \, d\tau' \, \Sigma(\tau - \tau') \left| \psi(x, \tau) - \psi(x, \tau') \right|^2 \). Assuming that $\Sigma(\tau) \sim \eta / (\pi \tau^2)$ at long times ($s = 1$), and optimizing the free energy, we find that $\eta \sim (\nu \tau_c)^{-1} \left| \eta / (\nu \tau_c) \right| \frac{1}{b^s}$. Note that $\eta \rightarrow 0$ as $g \rightarrow g^* = \frac{1}{2\nu}$ thus signaling a transition and in agreement with the previous RG analysis. Away from the transition ($g < g^*$), the SCHA yields a diffusive plasma propagator, $G^{-1}(q, \omega) \approx \eta |\omega| + v q^2 / (\pi g)$ at small $\omega$, signaling the breakdown of the TLL. Moreover, $\Phi(x, \tau) = \langle e^{2i \nu \phi(x, \tau)} e^{-2i \nu \phi(0, 0)} \rangle_{\text{SCHA}} \approx N_0^2 [1 + C_0 (\eta x, \eta \nu \tau / g)]$, where $N_0 = N_0 (\eta \nu \tau_c) = 0$ is non-universal and $C_0 (\eta x, \eta \nu \tau / g) \sim (\eta x)^{-1}$ whilst $C_0 (0, \eta \nu \tau / g) \sim (\eta \nu \tau / g)^{-1/2}$.

**Large $N$ approximation:** Further insight into the properties of the model can be obtained by means of the large $N$ approach, which captures many of the properties of dissipative quantum rotor models \[ 24, 31, 32, 33 \]. Let $n(x, \tau) = (\cos 2 \rho \phi(x, \tau), \sin 2 \rho \phi(x, \tau))$, so that the action $S = S_0 + S_{\text{diss}}$ becomes:

\[
S[n, \lambda] = \frac{1}{2} \int \frac{dq \, d\omega}{(2\pi)^3} \, G_0^{-1}(q, \omega) |n(q, \omega)|^2 + \frac{i}{2} \int dx \, dv \, \lambda(x, \tau) \left[ n^2(x, \tau) - 1 \right],
\]

where $G_0^{-1}(q, \omega) = \eta |\omega| + \kappa_p (|\omega / v|^2 + q^2)$, $\kappa_p = 4p^2 v^2 / (\pi g)$, and $\lambda(x, \tau)$ is a Lagrange multiplier ensuring that $n(x, \tau) = 1$. After generalizing the symmetry of the model from $O(2)$ to $O(N)$, the field $n$ is integrated out. In the large $N$ limit, the path-integral is dominated by a saddle point at $\lambda(x, \tau) = -i\kappa_p \xi^{-2}$, which obeys:

\[
N \int \frac{dq \, d\omega}{(2\pi)^3} \, G(q, \omega) |\lambda| = -i\kappa_p \xi^{-2} = 1,
\]

where $G^{-1}(q, \omega; \lambda_0) = G_0^{-1}(q, \omega) + i\lambda_0$. For a single ohmic quantum rotor, only solutions of \[ 41 \] with $\xi > 0$ exist \[ 32, 33, 34 \]. However, the spatial coupling of the rotors described by the $q^2$ term of the action, allows for solutions with $\xi = 0$. Thus we find $\xi^{-1} \sim (\eta - \eta_c)$, which implies that the critical exponent $\nu = 1$ and $z = 2$, in the large $N$ limit \[ 32 \] (more accurate estimates, $\nu \approx 0.689(6)$ and $z = 1.97(3)$, have been reported in \[ 10 \]). Critical correlations $\Phi(x, \tau) = \langle n(x, \tau) \cdot n(0, 0) \rangle_{N \rightarrow \infty} = NC_0 (\eta x, \eta \nu \tau / g)$, with $C_0 (x, \tau)$ defined as above (a more correct form for $N = 2$ can be found in \[ 10 \]). Away from criticality, $\Phi(x, 0) \sim e^{-|x|/\xi} / [\xi^{-1} |x|]$ and $\Phi(0, \tau) \sim \xi^2 g^2 / (\nu \tau)^2$ in the phase with $\xi \neq 0$. For $\eta > \eta_c$, we set $n_1(x, \tau) = N_0$ and integrate out the remaining $N - 1$ components of $n$, and proceed as above. Assuming that the phase is ordered, i.e. $N_0 \neq 0$, implies that $\xi^{-1} = 0$. The correlation function $\Phi(x, \tau)$ takes the same asymptotic form as that found using the SCHA.

**Phase diagram:** The simplest flow and phase diagram compatible with all above results is shown in Fig. 2. We find three phases: i) The TLL phase, where the coupling to the gate flows towards zero, ii) An ordered, gapless, dissipative phase, which has diffusive plasmons, and iii) A disordered phase with a finite spatial correlation length and density correlations at $g \approx Q_p$ decaying as $\tau^{-2}$. Just like for the single dissipative quantum rotor, we expect this result, obtained in the large $N$ limit, to remain valid for $N = 2$. The form of the spatial correlations indicates that the system can be regarded as consisting of independent superconducting “puddles” of size $\sim \xi$, each puddle behaving as a single ohmic quantum rotor. In the TLL phase, an analysis of the leading irrelevant operators shows that $\Phi(0, \tau) \sim \tau^{-2}$, at least. These results are in agreement with Griffiths’ theorem \[ 34 \].

We believe the model considered here to be equivalent, within the bosonization approach and for $\alpha \sim 1$, to the dissipative 2D XY model studied in \[ 10 \]. The two dissipative phases found here correspond to those reported in \[ 10 \]. However, we note that on the line $\alpha = 0$ (an for $\alpha$ small too) the two models differ: whereas the model of \[ 10 \] undergoes a Kosterlitz-Thouless transition for large $g$ and $\alpha = 0$, and therefore, it is in a disordered (plasma) phase, our model does not undergo a transition and instead exhibits a line of fixed points for $\alpha = 0$ (the TLL phase). A detailed comparison of the two models will be published elsewhere.

**Dual model:** A model dual to the one discussed above can be realized if one considers an 1D metal of spin-$\frac{1}{2}$ fermions with attractive interactions (i.e. a Luther-
the lead fermions and use that field $L$, amplitude, and $\Delta$ metal, respectively. The field $\theta(x)$ is dual to the density field $\phi(x)$. Assuming that $t_J$ is small, we integrate out the lead fermions and use that $\langle \Delta L(x, \tau) \Delta L(0, 0) \rangle_L \approx |\mathcal{G}_L^0(x, \tau)|^2 e^{-|x|/\ell_p}$, where $\ell_p$ is the mean-free path and $\mathcal{G}_L^0(x, \tau) = [2\pi (u LF \tau - i x)]^{-1}$ the single-particle Green's function of the lead electrons. Taking into account that $\theta(x, \tau)$ varies slowly on the scale of the correlation-length $\xi_s = \hbar c/\Delta_s$ $\gg \ell_p \gg k_{LF}^{-1}$, where $k_{LF}$ is the Fermi momentum of the lead, we obtain

$$S_{\text{diss}}[\theta] = \frac{\eta J}{\pi} \int dx d\tau d\tau' \cos[\theta(x, \tau) - \theta(x, \tau')]/|\tau - \tau'|^2,$$

for $|\tau - \tau'| > \tau_c$, where $\tau_c = \tau_p = \ell_p/u LF$ and $\eta J = \nu L \ell p (t J / \hbar)^2 / \hbar \nu L$ being the density of states of the lead. The RG flow for this term can be obtained from Eqs. (10) by replacing $g \rightarrow g^{-1}$ and $\alpha \rightarrow \alpha_s = (\eta J \ell / g_{\text{diss}})$. However, in a consistent treatment [21], the lead must be also treated as a source of dissipation for the density fluctuations.

Conclusions: We have analyzed a 1D metallic system with a few channels coupled to a metallic gate. In the absence of this coupling, the system is a Tomonaga-Luttinger liquid (TLL). We have shown that this phase is stable if the interactions in the wire are attractive or weakly repulsive, and the coupling is small. For sufficient repulsion, the coupling to the gate induces a phase transition to a gapless phase characterized by diffusive charge excitations, in contrast to the acoustical plasmons found in one dimensional conductors. For large compressibility, $v/g \rightarrow 0$, the coupling to the gate can induce a phase with a finite spatial correlation length, and ohmic correlations in the temporal direction.

Although for $g < g^*$ an arbitrarily small coupling to a gate destabilizes the TLL, in practice finite temperature $T > 0$ or finite length $L$ of the wire will cut-off the RG flow before the system can exhibit the full properties of the ordered (or disordered) phase described above. Thus, at finite $T$ and $L$, it is important to find the optimal conditions for the coupling to the gate to lead to measurable effects. The dimensionless quantity that measures the strength of the coupling to the gate is, e.g., for a 2D gate, $\alpha \approx g(k GF / k F)^2 e^{-4 \pi^2 k F z_0} / \sigma_{2D}$, where $k GF$ is the Fermi momentum of the gate electrons. Therefore, the gate should be a high resistance metal, and the Fermi wavelength of the wire needs to be small compared to the distance to the gate. Note, however, that close to the gate, the interactions within the wire strongly screened and $g \rightarrow 1$, and the TLL is stable. One way around this problem would be to use a granular gate, which provides ohmic dissipation but not metallic screening of the interactions for $q \approx 0$. It should be also mentioned that we have also neglected the possibility that disorder destabilizes the TLL before the effects of the gate are significant. These constraints suggest that a reasonable system where some of the phases discussed here can be observable is a weakly doped clean nanotube, where $k F z_0$ can be made small, coupled to a metallic gate with a short elastic mean free path or to a granular gate.

We thank T. Giamarchi, A. Millis, A. Muramatsu, and P. Werner for useful conversations. This work has been supported by Gipuzkoako Foru Aldundia (MAC), MEC (Spain) under Grants No FIS2004-06490-C03-00 (MAC), MAT2002-0495-C02-01 (FG) and FIS2004-05120 (FS), and R. Arcecs Foundation (FS).

[1] P. Cedraschi and M. Böttiker, Phys. Rev. B 63, 165312 (2001).
[2] F. Guinea, R. A. Jalabert, and F. Sols, Phys. Rev. B 70, 085310 (2004).
[3] F. Guinea, Phys. Rev. B 71, 045424 (2005).
[4] F. Marquardt and C. Bruder, Phys. Rev. B 68, 195305 (2003).
[5] J. Wei, S. Pereverzev, and M. E. Gershenson (2005), cond-mat/0508208.
[6] J. M. Wheatley, Phys. Rev. Lett. 67, 1181 (1991).
[7] A. Kapitulnik, N. Mason, S. A. Kivelson, and S. Chakravarty, Phys. Rev. B 63, 125322 (2001).
[8] G. Refael, E. Demler, Y. Oreg, and D. S. Fisher, Phys. Rev. B 68, 214515 (2003).
[9] M.-R. Li, K. Le Hur, and W. Hofstetter, Phys. Rev. Lett. 95, 086406 (2005).
[10] P. Werner, M. Troyer, and S. Sachdev, J. Phys. Soc. Jpn. Suppl. 74, 67 (2005).
[11] S. Sachdev, P. Werner, and M. Troyer, Phys. Rev. Lett. 92, 237003 (2004).
[12] P. Werner, K. Völker, M. Troyer, and S. Chakravarty, Phys. Rev. Lett. 94, 047201 (2005).
[13] A. H. Castro Neto, C. de C. Chamon, and C. Nayak, Phys. Rev. Lett. 79, 4629 (1997).
[14] A. H. Castro Neto, Phys. Rev. Lett. 78, 3931 (1997).
[15] F. Guinea, Phys. Rev. B 65, 205317 (2002).
[16] F. Guinea, Phys. Rev. B 67, 045103 (2003).
[17] F. Marquardt and D. S. Golubev, Phys. Rev. A 72, 022113 (2005).
[18] B. Horovitz and P. Le Doussal (2006), cond-mat/0602391.
[19] F. Guinea, Phys. Rev. Lett. 53, 1268 (1984).
[20] F. Sols and F. Guinea, Phys. Rev. B 36, 7775 (1987).
[21] F. D. M. Haldane, Phys. Rev. Lett. 46, 1840 (1981).
[22] A. O. Gogolin, A. A. Nersesyan, and A. M. Tsvelik, Bosonization and Strongly Interacting Systems (Cambridge University Press, Cambridge, 1998).
[23] T. Giamarchi, *Quantum Physics in One-dimension* (Oxford University Press, Oxford, 2004).
[24] M. A. Cazalilla, F. Sols, and F. Guinea, unpublished (2006).
[25] B. Gao, A. Komnik, R. Egger, D. C. Glattli, and A. Bachtold, Phys. Rev. Lett. 92, 216804 (2004).
[26] J. Voit and H. J. Schulz, Phys. Rev. B 36, 968 (1987).
[27] T. Giamarchi and H. J. Schulz, Phys. Rev. B 37, 325 (1988).
[28] E. Simánek, Phys. Lett. A 119, 477 (1987).
[29] S. Drewes, D. P. Arovas, and S. Renn, Phys. Rev. B 68, 165345 (2003).
[30] R. Citro, E. Orignac, and T. Giamarchi, Phys. Rev. B 72, 024434 (2005).
[31] S. Sachdev, A. V. Chubukov, and A. Sokol, Phys. Rev. B 51, 14874 (1995).
[32] S. R. Renn (1997), cond-mat/9708194.
[33] S. Pankov, S. Florens, A. Georges, G. Kotliar, and S. Sachdev, Phys. Rev. B 69, 054426 (2004).
[34] R. B. Griffiths, Journ. Math. Phys. 8, 478 (1967).
[35] We first consider the simpler case of spinless fermions and discuss further below how to extend our results to other systems such as spin-1/2 electrons and nanotubes.
[36] By construction[21] \( \phi(x, \tau) \) is an angle defined modulo \( \pi \).
[37] The dissipative part is \( \propto \omega \) to leading order. The first correction is \( A \omega^3 \), where \( A \) depends strongly on details of the gate response function and on \( z_0 \). To be able to compare it to the leading term, we use dimensional analysis and write \( A = \left( \omega_p^2 \right)^{-2} A \), where \( \omega_p^2 \approx \hbar v_G^2 / l_p \) for a diffusive gate with mean free path \( l_p \) and Fermi velocity \( v_G^2 \). For a clean gate \( \omega_c \approx \min\{\omega_p, E_F / \hbar\} \) being \( \omega_p \) the plasma frequency (3D gate) and \( E_F / \hbar \) the Fermi energy.