ΛCDM model with a scalar perturbation vs. preferred direction of the universe

Xin Li, Hai-Nan Lin, Sai Wang and Zhe Chang

Institute of High Energy Physics,
Theoretical Physics Center for Science Facilities,
Chinese Academy of Sciences,
100049 Beijing, China

Abstract

We present a scalar perturbation for the ΛCDM model, which breaks the isotropic symmetry of the universe. Based on the Union2 data, the least-$\chi^2$ fit of the scalar perturbed ΛCDM model shows that the universe has a preferred direction $(l, b) = (287^\circ \pm 25^\circ, 11^\circ \pm 22^\circ)$. The magnitude of scalar perturbation is about $-2.3 \times 10^{-5}$. The scalar perturbation for the ΛCDM model implies a peculiar velocity, which is perpendicular to the radial direction. We show that the maximum peculiar velocities at redshift $z = 0.15$ and $z = 0.015$ equal to $73 \pm 28 \text{km} \cdot \text{s}^{-1}$ and $1099 \pm 427 \text{km} \cdot \text{s}^{-1}$, respectively. They are compatible with the constraints on peculiar velocity given by Planck Collaboration.

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I. INTRODUCTION

The standard cosmological model, i.e., the ΛCDM model \[1, 2\] has been well established. It is consistent with several precise astronomical observations that involve Wilkinson Microwave Anisotropy Probe (WMAP) \[3\], Planck satellite \[4\], Supernovae Cosmology Project \[5\]. One of the most important and basic assumptions of the ΛCDM model states that the universe is homogeneous and isotropic on large scales. However, such a principle faces challenges \[6\]. The Union2 SnIa data hint that the universe has a preferred direction \((l, b) = (309°, 18°)\) in galactic coordinate system \[7\]. Toward this direction, the universe has the maximum expansion velocity. Astronomical observations \[8\] found that the dipole moment of the peculiar velocity field on the direction \((l, b) = (287° \pm 9°, 8° \pm 6°)\) in the scale of \(50h^{-1}\)Mpc has a magnitude \(407 \pm 81\) km \(\cdot\) s\(^{-1}\). This peculiar velocity is much larger than the value \(110\) km \(\cdot\) s\(^{-1}\) given by WMAP5 \[9\]. The recent released data of Planck Collaboration show deviations from isotropy with a level of significance \((\sim 3\sigma)\) \[10\]. Planck Collaboration confirms asymmetry of the power spectrums between two preferred opposite hemispheres. These facts hint that the universe may have a preferred direction.

Many models have been proposed to resolve the asymmetric anomaly of the astronomical observations. An incomplete and succinct list includes: an imperfect fluid dark energy \[11\], local void scenario \[12, 13\], noncommutative spacetime effect \[14\], anisotropic curvature in cosmology \[15\], and Finsler gravity scenario \[16\].

In this paper, we present a scalar perturbation for the flat Friedmann-Robertson-Walker (FRW) metric \[17\]. Based on the Union2 data, the least-\(\chi^2\) fit of the scalar perturbed ΛCDM model shows that the universe has a preferred direction. In the scalar perturbed ΛCDM model, the universe could be treated as a perfect fluid approximately. In comoving frame, however, the fluid has a small velocity \(v\). It could be regarded as the peculiar velocity of the universe. The data of Planck Collaboration gives severe constraints on the peculiar velocity \[18\]. For the bulk flow of Local Group, it should be less than \(254\) km \(\cdot\) s\(^{-1}\). For bulk flow of galaxy clusters at \(z = 0.15\), it should be less than \(800\) km \(\cdot\) s\(^{-1}\).

The paper is organized as follows. In Sec. II, we present a scalar perturbation for the FRW metric. Explicit relation between luminosity and redshift is obtained. In Sec. III, we show a least-\(\chi^2\) fit of the scalar perturbed ΛCDM model to the Union2 SnIa data. The preferred direction is found \((l, b) = (287° \pm 25°, 11° \pm 22°)\). The magnitude of the scalar
perturbation is at the scale of $10^{-5}$. This perturbation implies a peculiar velocity with value $73 \pm 28 \text{km} \cdot \text{s}^{-1}$ at $z = 0.15$, and $1099 \pm 427 \text{km} \cdot \text{s}^{-1}$ at $z = 0.015$. The conclusions and remarks are given in Sec. IV.

II. SCALAR PERTURBATION FOR FRW METRIC

The FRW metric describes the homogeneous and isotropic universe. In order to describe the deviation from isotropy, we try to add a scalar perturbation for the FRW metric. The scalar perturbed FRW metric is of the form

$$ds^2 = (1 - 2\phi(\vec{x}))dt^2 - a^2(t)(1 + 2\phi(\vec{x}))\delta_{ij}dx^i dx^j. \quad (1)$$

It should be noticed that the scalar perturbation field $\phi(\vec{x})$ is time-independent. And the scalar perturbation can be interpreted as a sort of space-dependent spatial curvature. By setting the scale factor $a(t) = 1$, one can find that the spatial Ricci tensor of metric (1) is of the form

$$R_{ij} = -\delta_{ij}\phi_{,k,k}. \quad (2)$$

The nonvanishing components of Einstein tensor for the metric (1) are given as

$$G_0^0 = 3(1 + 2\phi)H^2 - 2a^{-2}\phi_{,i,i}, \quad (3)$$

$$G_i^j = \delta_{ij}(1 + 2\phi) \left(H^2 + 2\frac{\dot{a}}{a}\right), \quad (4)$$

$$G_j^0 = -2H\phi_{,j}, \quad (5)$$

where the commas denote the derivatives with respect to $x^i$, the dot denotes the derivatives with respect to cosmic time $t$ and $H \equiv \frac{\dot{a}}{a}$. The scalar perturbation breaks homogeneity and isotropy of the universe. Since $\phi$ is a perturbation, the cosmic inventory could be treated as a perfect fluid approximately. In comoving frame, however, the fluid has a perturbed velocity $v$. The energy-momentum tensor is given by

$$T^{\mu\nu} = (\rho + p)U^{\mu}U^{\nu} - pg^{\mu\nu}, \quad (6)$$

where $\rho$ and $p$ are the energy density and pressure density of the fluid, respectively. Here, we set $U^{\mu}$ as $U^0 = 1, \frac{U^i}{U^0} \equiv v^i$, to first order in $v$. In this paper, we just investigate low redshift region of the universe, where the universe is dominated by matter and dark energy.
Thus, the nonvanishing components of energy-momentum tensor are given as
\begin{align}
T^0_0 &= \rho_m + \rho_{de}, \\
T^0_i &= \rho_m v_i, \\
T^j_i &= \delta^j_i \rho_{de},
\end{align}
(7)  
where \( \rho_m \) and \( \rho_{de} \) denote the energy density of matter and dark energy, respectively. Then, the Einstein field equation \( G^\mu_\nu = 8\pi G T^\mu_\nu \) gives three independent equations
\begin{align}
(1 + 2\phi)H^2 - \frac{2a^{-2}}{3}\phi_{,i,i} &= \frac{8\pi G}{3}(\rho_m + \rho_{de}), \\
(1 + 2\phi)(H^2 + 2\frac{\ddot{a}}{a}) &= 8\pi G \rho_{de}, \\
H\phi_{,j} &= -4\pi G \rho_m v_j.
\end{align}
(10)  
(11)  
(12)  
The energy-momentum conservation equation reads
\begin{equation}
\frac{\partial T^\mu_\nu}{\partial x^\mu} + \Gamma^\mu_\alpha\nu T^\alpha_\nu - \Gamma^\nu_\alpha T^\mu_\alpha = 0,
\end{equation}
(13)  
where \( \Gamma^\mu_\alpha\nu \) is the Christoffel symbol. Then, following the theory of general relativity, we obtain the specific form of energy-momentum conservation equation for matter and dark energy in the perturbed FRW universe (11). It is as follows:
\begin{align}
\frac{\partial \rho_m}{\partial t} + 3H\rho_m + \frac{\partial \rho_m v^i}{\partial x^i} &= 0, \\
\frac{\partial \rho_m v_i}{\partial t} + 3H\rho_m v_i - \phi_{,i}\rho_m &= 0, \\
\frac{\partial \rho_{de}}{\partial t} &= 0, \\
\frac{\partial \rho_{de}}{\partial x^i} &= 0.
\end{align}
(14)  
(15)  
(16)  
(17)  
The equations (16) and (17) show that the energy density of dark energy remaining constant in our model. By making use of the field equation (12), we find from equation (14) that
\begin{equation}
\frac{\partial (\rho_m a^3)}{\partial t} = -aH \frac{\phi_{,i,i}}{4\pi G}.
\end{equation}
(19)  
The solution of equation (19) reads
\begin{equation}
\rho_m a^3 = -\frac{\phi_{,i,i}}{4\pi G} (a - 1) + \rho_{m0},
\end{equation}
(20)
where \( \rho_{m0} \) denotes the energy density of matter at present. We have already used the initial condition that the present energy density of matter is constant to deduce continuity equation (20).

The light propagation satisfies \( ds = 0 \), which gives

\[
\frac{dt}{a(t)} = (1 + 2\phi(\vec{x}))\delta_{ij}dx^i dx^j .
\] (21)

The right-hand side of the equation (21) is time-independent. During a very short time, the location of a galaxy is unchanged. Then, we get

\[
\int_{t_1}^{t_0} \frac{dt}{a(t)} = \int_{t_1+\delta t_1}^{t_0+\delta t_0} \frac{dt}{a(t)} .
\] (22)

Thus, the redshift \( z \) of galaxy satisfies

\[
1 + z = \frac{1}{a},
\] (23)

where we have set the scale factor \( a(t) \) to be 1 at present.

A particular form of \( \phi(\vec{x}) \) is necessary for deriving relation between luminosity distance and redshift. The form of \( \phi(\vec{x}) \) is determined by the perturbed energy-momentum tensor. However, the information of the perturbed energy-momentum tensor is unknown. On the contrary, we choose a specific form of \( \phi \) to determine the perturbed energy-momentum tensor. It is given as

\[
\phi = A \cos \theta ,
\] (24)

where \( A \) is a dimensionless parameter and \( \theta \) is the angle between \( \vec{r} \) and \( z \)-axis. By making use of (24), we reduce the equation (10) to

\[
\frac{da}{dt} = H_0 a(1 - A \cos \theta) \sqrt{\Omega_{m0} a^{-3} + 1 - \Omega_{m0} - \frac{4A \cos \theta}{3r^2 H_0^2 a^{-3}}} ,
\] (25)

where \( H_0 \) is Hubble constant and \( \Omega_{m0} \equiv 8\pi G \rho_{m0}/(3 H_0^2) \) is the energy density parameter for matter at present.

Combining the equations (21), (23), (24), (25) and (20), and using the definition of luminosity distance [17], we obtain the relation between luminosity distance and redshift

\[
H_0 d_L = (1 + z) \int_0^z \frac{(1 - A \cos \theta) dx}{\sqrt{\Omega_{m0}(1 + x)^3 + 1 - \Omega_{m0} - \frac{4A \cos \theta(1+x)^3}{3H_0^2 d_L^2}}} ,
\] (26)

where \( d_{L0} \equiv (1 + z) \int_0^z \frac{dx}{H_0 \sqrt{\Omega_{m0}(1+x)^3 + 1 - \Omega_{m0}}} .
\)
III. NUMERICAL RESULTS

Our numerical studies are based on the Union2 SnIa data \[20\]. Our goal is to find whether the universe has a preferred direction or not. We perform a least-$\chi^2$ fit to the Union2 SnIa data

$$\chi^2 \equiv \sum_{i=1}^{557} \frac{(\mu_{th} - \mu_{obs})^2}{\sigma_\mu^2},$$

(27)

where $\mu_{th}$ is theoretical distance modulus given by

$$\mu_{th} = 5 \log_{10} \frac{d_L}{\text{Mpc}} + 25.$$  

(28)

$\mu_{obs}$ and $\sigma_\mu$, given by the Union2 SnIa data, denote the observed values of the distance modulus and the measurement errors, respectively. The least-$\chi^2$ fit of the $\Lambda$CDM model gives $\Omega_m = 0.27 \pm 0.02$ and $H_0 = 70.00 \pm 0.35 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$. Before using our model to fit the Union2 SnIa data, we fix the values of $\Omega_m$ and $H_0$ as their mean values. Such an approach is valid for the scalar perturbed model, since it is just a perturbation for the $\Lambda$CDM model. Then, the least-$\chi^2$ fit of the formula (26) gives $A = (-2.34 \pm 0.91) \times 10^{-5}$ and $(l, b) = (287^\circ \pm 25^\circ, 11^\circ \pm 22^\circ)$. The preferred direction is plotted as point G of Fig.1.

The preferred directions given by other models are plotted in Fig.1 for contrast. Kogut et al. \[21\] got $(l, b) = (276^\circ \pm 3^\circ, 30^\circ \pm 3^\circ)$ is shown as point A, Antoniou et al. \[7\] got $(l, b) = (309^{\circ \pm 23^\circ}, 18^\circ \pm 11^\circ)$ is shown as point B, Cai and Tuo \[22\] got $(l, b) = (314^{\circ \pm 20^\circ}, 28^\circ \pm 11^\circ)$ is shown as point C, Kalus et al. \[23\] got $(l, b) = (325^\circ, -19^\circ)$ is shown as point D, Cai et al. \[24\] got $(l, b) = (306^\circ, -13^\circ)$ is shown as point E, Chang et al. \[16\] got $(l, b) = (304^\circ \pm 43^\circ, -27^\circ \pm 13^\circ)$ is shown as point F. Within a level of significance (1$\sigma$), it is shown in Fig.2 that our results are consistent with the one of Kogut et al. \[21\], Antoniou et al. \[7\] and Cai et al. \[22\].

The scalar perturbation not only breaks the isotropy symmetry of the universe but also gives a peculiar velocity for the matter. By setting $\phi$ to be the form of (24), we find from (12) that

$$v \equiv \sqrt{|v_i v^i|} = a^4 \frac{2H|A| \sin \theta}{3rH_0^2 \Omega_m} = \frac{2H|A| \sin \theta}{3H_0^2 d_L \Omega_m} (1 + z)^{-3},$$

(29)

where we have used the relation $d_L = (1 + z)r$ to obtain the second equation. Substituting the value of $H_0d_L$ and $H$ (given by the $\Lambda$CDM model) into formula (29), and setting $\sin \theta = 1$ (the velocity $v$ is perpendicular to the preferred direction $(l, b) = (287^\circ \pm 25^\circ, 11^\circ \pm 22^\circ)$), we could obtain the value of peculiar velocity $v$ for a given redshift. At $z = 0.15$, we get...
FIG. 1: The direction of preferred axis in galactic coordinate. The point G denotes our result, namely, $(l, b) = (287° \pm 25°, 11° \pm 22°)$, which is obtained by fixing the parameters $\Omega_m = 0.27$ and $H_0 = 70.00$ and doing the least-$\chi^2$ to the Union2 data for formula (26). The results for preferred direction in other models are presented for contrast. Point A denotes the result of Kogut et al. [21], point B denotes the result of Antoniou et al. [7], point C denotes the result of Cai and Tuo [22], point D denotes the result of Kalus et al. [23], point E denotes the result of Cai et al. [24], point F denotes the result of Chang et al. [16].

$v|_{z=0.15} \simeq 73 \pm 28$km·s$^{-1}$. This peculiar velocity is compatible with the result of Planck Collaboration [18]. it gives a upper limit 800km·s$^{-1}$ for peculiar velocity at $z = 0.15$. It should be noticed that the peculiar velocity $v$ grows with time. We should check that if the peculiar velocity at the lowest redshift in the Union2 SnIa data is compatible with the result of Planck Collaboration [18]. The lowest redshift in the Union2 SnIa data is 0.015. At $z = 0.015$, we get $v|_{z=0.015} \simeq 1099 \pm 427$km·s$^{-1}$. Planck Collaboration [18] gives an upper limit on the bulk flow for Local Group, which equals to 254km·s$^{-1}$. Though, our result for peculiar velocity at $z = 0.015$ larger than 254km·s$^{-1}$, with a level of significance (1σ), it is still compatible with the upper limit 800km·s$^{-1}$ given by Planck Collaboration. And our result for peculiar velocity at $z = 0.015$ represents the upper limit of peculiar velocity in the scalar perturbed ΛCDM model.
IV. CONCLUSIONS AND REMARKS

We presented a scalar perturbation for the $\Lambda$CDM model that breaks the isotropic symmetry of the universe. Setting the scalar perturbation of the form $\phi = A \cos \theta$, we obtained a modified relation (26) between luminosity distance and redshift. The least-$\chi^2$ fit to the Union2 SNIa data showed that the universe has a preferred direction $(l, b) = (287^\circ \pm 25^\circ, 11^\circ \pm 22^\circ)$, which is close to the results of Kogut et al. and Antoniou et al. and Cai et al. [7, 21, 22]. Also, the least-$\chi^2$ fit to the Union2 SNIa data showed that the magnitude of scalar perturbation $A$ equals to $(-2.34 \pm 0.91) \times 10^{-5}$. The scalar perturbation has the same magnitude with the level of CMB anisotropy. The CMB anisotropy is a possible reason for the preferred direction of the universe.

The peculiar velocity was obtained directly from the Einstein equation (12). The numerical calculations showed that the peculiar $v|_{z=0.15} \simeq 73 \pm 28\text{km} \cdot \text{s}^{-1}$ and $v|_{z=0.015} \simeq 1099 \pm 427\text{km} \cdot \text{s}^{-1}$. They are compatible with the results of Planck Collaboration [18].
It should be noticed that the peculiar velocity we obtained is perpendicular to the radial direction.

Bianchi cosmology [25] has been studied for many years. It admits a set of anisotropic metrics such as Kasner metric [26]. The three dimensional space of Bianchi cosmology admits a set of Killing vectors $\xi^{(a)}_i$ which obey the following property

$$\left( \frac{\partial \xi^{(c)}_i}{\partial x^k} - \frac{\partial \xi^{(c)}_k}{\partial x^i} \right) \xi^i_{(a)} \xi^k_{(b)} = C^{c}_{ab},$$

(30)

where $C^{c}_{ab}$ is the structure constant of the symmetry group of the space. The scalar perturbation field $\phi(\vec{x})$ completely destroys the rotational symmetry of cosmic space. It means that no Killing vectors corresponding to the symmetry group of three dimensional cosmic space. Thus, there is no obvious relation between the Bianchi cosmology and our model.

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