Optimized Base-Station Cache Allocation for Cloud Radio Access Network with Multicast Backhaul

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Abstract—The performance of cloud radio access network (C-RAN) is limited by the finite capacities of the backhaul links connecting the centralized processor (CP) with the base-stations (BSs), especially when the backhaul is implemented in a wireless medium. This paper proposes the use of wireless multicast together with BS caching, where the BSs pre-store contents of popular files, to augment the backhaul of C-RAN. For a downlink C-RAN consisting of a single cluster of BSs and wireless backhaul, this paper studies the optimal cache size allocation strategy among the BSs and the optimal multicast beamforming transmission strategy at the CP such that the user’s requested messages are delivered from the CP to the BSs in the most efficient way. We first state a multicast backhaul rate expression based on a joint cache-channel coding scheme, which implies that larger cache sizes should be allocated to the BSs with weaker channels. We then formulate a two-timescale joint cache size allocation and beamforming design problem, where the cache is optimized offline based on the long-term channel statistical information, while the beamformer is designed during the file delivery phase based on the instantaneous channel state information. By leveraging the sample approximation method and the alternating direction method of multipliers (ADMM), we develop efficient algorithms for optimizing cache size allocation among the BSs, and quantify how much more cache should be allocated to the weaker BSs. We further consider the case with multiple files having different popularities and show that it is in general not optimal to entirely cache the most popular files first. Numerical results show considerable performance improvement of the optimized cache size allocation scheme over the uniform allocation and other heuristic schemes.

Index Terms—Alternating direction method of multipliers (ADMM), base-station (BS) caching, cloud radio access network (C-RAN), data-sharing strategy, multicasting, wireless backhaul

I. INTRODUCTION

Cloud radio access network (C-RAN) has been recognized as one of the enabling technologies to meet the ever-increasing demand for higher data rates for the next generation (5G) wireless networks [2]–[4]. In C-RAN, the base-stations (BSs) are connected to a centralized processor (CP) through high-speed fronthaul/backhaul links, which provide opportunities for cooperation among the BSs for inter-cell interference cancellation. The performance of C-RAN depends crucially on the capacity of the fronthaul/backhaul links. The objective of this paper is to explore the benefit of utilizing caching at the BSs to augment the fronthaul/backhaul links.

There are two fundamentally different fronthauling strategies that enable the cooperation of the BSs in C-RAN. In the data-sharing strategy [5]–[8], the CP directly shares the user’s messages with a cluster of BSs, which subsequently serve the user through cooperative beamforming. In the compression strategy [5], [9], the CP performs the beamforming operation and sends the compressed version of the analog beamformed signal to the BSs. The relative advantage of the data-sharing strategy versus the compression strategy depends highly on the fronthaul/backhaul channel capacity [10], [11]. In general, the compression strategy outperforms data-sharing when the fronthaul/backhaul capacity is moderately high, in part because the data-sharing strategy relies on the backhaul to carry each user’s data multiple times to multiple cooperating BSs. Thus, the finite backhaul capacity significantly limits the BS cooperation size.

The capacity limitation in fronthaul/backhaul is especially pertinent for small-cell deployment where high-speed fiber optic connections from the CP to the BSs may not be available and wireless backhauling may be the most feasible engineering option. The purpose of this paper is to point out that under this scenario, the data-sharing strategy has a distinct edge in that it can take advantage of: (i) the ability of the CP to multicast user messages wirelessly to multiple BSs at the same time; and (ii) the ability of the BSs to cache user messages to further alleviate the backhaul requirement. Note that the multicast opportunity in the wireless backhaul and the caching opportunity at the BSs are only available to facilitate the data-sharing strategy in C-RAN, but not the compression strategy, as the latter involves sending analog compressed beamformed signals from the CP to the BSs, which are different for different BSs and are also constantly changing according to the channel conditions, so are impossible to cache.

This paper considers a downlink C-RAN in which the CP utilizes multiple antennas to multicast user messages to a single cluster of BSs using the data-sharing strategy, while the BSs pre-store fractions of popular contents during the off-peak time and request the rest of the files from the CP using coded delivery via the noisy wireless backhaul channel. Given a total cache constraint, we investigate the optimal cache size allocation strategy across the BSs and the optimal multicast
beamforming transmission strategy at the CP so that the file requests can be delivered most efficiently from the CP to the BSs. It is important to emphasize that the optimizations of the BS cache size allocation and the beamforming strategy at the CP occur in different timescales. While the beamformer can dynamically adapt to the instantaneous channel realization, the cache size is optimized only at the cache allocation phase and can only adapt to the long-term statistics of the backhaul channel.

This paper proposes a sample approximation approach to solve the above two-timescale optimization problem. The optimal cache size allocation considers the long-term channel statistics in allocating larger cache sizes to the BSs with weaker backhaul channels, while accounting for the potential effect of beamforming. It also considers the difference in file popularities in caching larger portions of more popular files.

A. Related Work

While caching has been extensively used at the edge of Internet, the idea of coded caching that takes advantage of multicast opportunity has recently attracted extensive research interests due to the pioneering work of [12], which uses the network coding method to simultaneously deliver multiple files through a common noiseless channel to multiple receivers, each caching different parts of the files. This paper studies a different scenario in which the same content is requested by multiple receivers (BSs), hence no network coding is needed and the coded multicasting in this paper refers to channel coding across multiple wireless backhaul channels with different channel conditions between the CP transmitter and the BS receivers.

C-RAN with BS caching has been previously considered in [13]–[16], but most previous works assume fixed cache allocation among the BSs. More specifically, [13] and [14] examine how BS caching helps in reducing both backhaul capacity consumption and BS power consumption under given users’ quality-of-service constraints; [13] studies how BS caching changes the way that backhaul is utilized and proposes a similar scheme as in [17] that combines the data-sharing strategy and the compression strategy to improve the spectral efficiency of the downlink C-RAN. This paper differs from the above works in focusing on how to optimally allocate the cache sizes among the BSs and design multicast beamformers at the CP to improve the efficiency of sharing user’s requested files via the wireless backhaul channel.

Previous works on caching strategy optimization rely on the assumptions of either simplified networks [18], [19] or Poisson distributed networks [20], [22] that are reasonable in a network with a large number of BSs and users, and focus on analyzing how BS caching helps in improving the performance of the BS-to-user layer. This paper instead considers a C-RAN with a single cluster of BSs and investigates how BS caching improves the efficiency of file delivery between the cloud and the BSs layer.

This paper is motivated by [23] which shows from an information-theoretical perspective the advantage of allocating different cache sizes to different BSs depending on their channel conditions. In addition, [23] proposes a joint cache-channel coding scheme that optimally utilizes the caches at the BSs in a broadcast erasure channel, which is further generalized to the degraded broadcast channel in [24]. We take the findings in [23] one step further by considering the effect of multiple-antenna beamforming in a downlink C-RAN backhaul network. We also extend [23] to the case of multiple files with different popularities and demonstrate that the optimal caching strategy also highly depends on the file popularities.

B. Main Contributions

This paper considers the joint optimization of BS cache size allocation and multicast beamformer at the CP in two timescales for a downlink C-RAN with a single BS cluster under the data-sharing strategy. The main contributions of this paper are summarized as follows:

- **Problem Formulation**: We derive a new multicast backhaul rate expression with BS caching based on the joint cache-channel coding scheme of [23]. We then formulate two new cache size allocation problems of minimizing the expected file downloading time and maximizing the expected file downloading rate subject to the total cache size constraint. The cache size allocation is optimized offline and is fixed during the file delivery phase, while the transmit beamformers are adapted to the real-time channel realization.

- **Algorithms**: We propose efficient algorithms for solving the formulated cache size allocation problems. More specifically, to deal with the intractability of taking expectation over the channel realizations in the objective functions, we approximate the expectation via sampling [25]. Note that the sample size generally needs to be large in order to guarantee the approximation accuracy. We further propose to solve the sample approximation problem using the successive linear approximation technique and the alternating direction method of multipliers (ADMM) algorithm [26], which decomposes the potentially large-scale problem (due to the large sample size) into many small-scale problems on each sample.

- **Engineering Insight**: We quantify how much cache should be allocated among the BSs in a practical C-RAN setup, and show that, as compared to the uniform and proportional cache size allocation schemes, the proposed scheme allocates aggressively larger cache sizes to the files with higher popularities, and for each file the proposed scheme allocates aggressively larger cache sizes to the BSs with weaker backhaul channels.

C. Paper Organization and Notations

The remainder of this paper is organized as follows. Section II introduces the considered system model for C-RAN. We derive the backhaul multicast rate with BS caching in Section III and state the problem setup in Section IV. Sections V and VI focus on the proposed cache size allocation schemes for the single file case and the multiple files case,
respectively. Simulation results are provided in Section VII. Conclusions are drawn in Section VIII.

Throughout this paper, lower-case letters (e.g. \( x \)) and lower-case bold letters (e.g. \( \mathbf{x} \)) denote scalars and column vectors, respectively. We use \( \mathbb{C} \) to denote the complex domain. The transpose and conjugate transpose of a vector are denoted as \( (\cdot)^T \) and \( (\cdot)^H \), respectively. The expectation of a random variable is denoted as \( \mathbb{E} [\cdot] \). Calligraphy letters are used to denote sets.

II. SYSTEM MODEL

Consider a downlink C-RAN model in Fig. 1 consisting of BSs connected to a cloud-based CP through shared wireless backhaul. The cloud employs a data-sharing strategy which delivers each user’s intended message to a predefined cluster of BSs and the BS cluster subsequently serves the user through cooperative beamforming. The capacities of the backhaul is a significant limiting factor to the performance of the C-RAN [5], [8]. To alleviate the backhaul requirement, this paper considers the scenario where each BS is equipped with a local cache of size \( C_l \) that can pre-store some contents of the file. This paper addresses two questions:

- Given fixed local cache sizes and fixed cached contents, at a fast timescale, how should the transmit beamforming strategy be designed as function of the instantaneous realization of the wireless channel in order to most efficiently delivery a common user message to all the BSs?
- At a slow timescale, how should the contents be cached and how should the cache sizes be allocated across the BSs so that the expected delivery performance across many channel realizations is optimized?

The answers to the above two questions would be trivial if the cache size at each BS is large enough to store the entire file library in the network, in which case no backhaul transmission is needed. This paper considers a more realistic scenario where the network operator has a fixed budget to deploy only a limited amount of total cache size \( C \). Because of the limited cache size, each BS can only cache a subset of the files. In the next section, we define the file delivery performance in the backhaul link in terms of both the delivery rate and the downloading time, which are expressed as functions of cache sizes at the BSs.

III. BROADCAST CHANNEL WITH RECEIVER CACHING

In this section, we investigate the optimal caching strategy for the backhaul network with given cache size at each BS. We then formulate the two-stage joint cache and beamforming design problem considered in this paper in the next section.

A. Separate Cache-Channel Coding

Without BS caching, the downlink C-RAN wireless backhaul network with a single cluster of BSs as shown in Fig. 1 can be modeled as a broadcast channel (BC) with common message only, whose capacity is given as

\[
R_0 \leq I(\mathbf{x}; y_l), \forall l \in L,
\]

where \( R_0 \) denotes the multicast rate, \( I(\mathbf{x}; y_l) \) is the mutual information between the transmit signal \( \mathbf{x} \) at the CP and the received signal \( y_l \) at BS \( l \). It can be seen from (2) that the common information rate is limited by the worst channel across the BSs.

To deal with the channel disparity issue in (2), this paper considers the use of BS caching to smooth out the difference
in channel quality across the BSs. Assuming that BS \( l \) has cache size \( C_\ell \) bits, filled up by caching the first \( C_\ell \) bits of a file with a total size of \( F \) bits, a simple caching strategy is to let the CP deliver only the rest \( F - C_\ell \) bits of the file to BS \( l \). However, since the BSs are served through multicasting, the CP has to send the maximum of the rest of the requested file, i.e., \( \max_i \{F - C_i\} \), to make sure that the BS with least cache size can get the entire file. Assuming that the channel coherent block is large enough so that the file can be completely downloaded within one coherent block, then the amount of time needed to finish the file downloading is

\[
T_0 = \frac{\max_i \{F - C_i\}}{\min_l \{I(x; y_l)\}},
\]

and the effective file downloading rate is

\[
D_0 = \frac{F}{T_0} = \frac{\min_l \{I(x; y_l)\}}{\max_i \{1 - C_i/F\}}.
\]

As we can see from (3) or (4), with this naive caching strategy, it is optimal to allocate the cache size uniformly among the BSs, i.e., \( C_l = C/L, \forall l \in \mathcal{L} \).

### B. Joint Cache-Channel Coding

It is possible to significantly improve the naive separate cache-channel coding strategy by considering cached content as side information for the broadcast channel. The achievable rate of this strategy, named as joint cache-channel coding in \([23]\), can be characterized as below.

**Lemma 1 (\([23]\))**: Consider a BC with common message, if receiver \( l \in \mathcal{L} \) caches \( \alpha_l \) (\( 0 \leq \alpha_l \leq 1 \)) fraction of the message, then the multicast common message rate \( R \) is achievable if and only if the following set of inequalities are satisfied:

\[
R(1 - \alpha_l) \leq I(x; y_l), \forall l \in \mathcal{L},
\]

where \( x \) is the input and \( y_l \)'s are the output of the broadcast channel.

**Proof**: We outline an information-theoretic proof as follows. Consider that a message \( w \) is chosen uniformly from the index set \( \{1, 2, \ldots, W\} \) and is to be transmitted to a set of receivers \( \mathcal{L} \) over \( n \) channel uses at a rate of \( R = \frac{\log W}{n} \) bits per channel use. A codebook \( C \) of size \( 2^{nR} \times n \) is first generated by drawing all symbols \( x_i(j), i = 1, 2, \ldots, n, \) and \( j = 1, 2, \ldots, 2^{nR}, \) independently and identically according to the channel input distribution, where each row of \( C \) corresponds to a codeword. To send the message \( w \), the \( w \)-th row of \( C \), denoted as \( X^n(w) = [x_1(w) \ x_2(w) \ \ldots \ x_n(w)] \), is transmitted over the channel. Note that the codebook \( C \) is revealed to both the transmitter and the \( L \) receivers. After receiving \( Y^n_l \), the receiver \( l \) tries to decode the index \( w \) by looking for a codeword in the codebook \( C \) that is jointly typical with \( Y^n_l \) and the cached content.

Suppose that each receiver \( l \) caches a fraction of the message—specifically caches the first \( \alpha_l \log W \) bits of \( w \). Then, receiver \( l \) only needs to search among those codewords whose indices start with the same \( \alpha_l \log W \) bits as the cached bits. Since there exist a total number of \( 2^{nR-\alpha_l \log W} \) such codewords, by the packing lemma \([28]\), receiver \( l \) would be able to find the correct codeword with diminishing error probability in the limit \( n \to \infty \) as long as the inequality \( nR - \alpha_l \log W \leq nI(x; y_l) \) is satisfied, or equivalently \( R - \alpha_l R \leq I(x; y_l) \), where \( x \) is the input channel symbol. This inequality needs to be satisfied by all \( l \in \mathcal{L} \) to ensure that the common message is recovered by all the receivers, which leads to the proof of the achievability of (5) in Lemma 1 For the proof of converse, we refer to \([23]\) for the details.

In the setup of this paper, given cache size allocation \( C_\ell \) and the file size \( F \), each BS \( l \) can cache \( C_l/F \) fraction of the file. Hence, by Lemma 1 the file delivery rate \( D_c \), with the joint cache-channel coding strategy can be formulated as

\[
D_c = \min_l \left\{ \frac{I(x; y_l)}{1 - C_l/F} \right\},
\]

and the downloading time \( T_c \) can be written as

\[
T_c = \frac{F}{D_c} = \max_l \left\{ \frac{F - C_l}{I(x; y_l)} \right\}.
\]

Clearly, the above file downloading time and delivery rate are strictly better than the ones in (3) and (4) except when all \( I(x; y_l) \) are equal to each other. Instead of allocating the cache size \( C_\ell \) uniformly, (6) and (7) suggest that it is advantageous to allocate more cache to the BSs with weaker channels to achieve an overall higher multicast rate or shorter downloading time. The difficulty, however, lies in the fact that in practice the channel condition changes over time while the cache size allocations among the BSs can only be optimized ahead of time at the cache deployment phase. In the next section, we formulate a two-stage optimization problem that jointly optimizes the cache size allocation strategy based on the long-term channel statistics and the beamforming strategy based on the short-term channel realization.

### IV. TWO-STAGE CACHING AND BEAMFORMING DESIGN

We are now ready to formulate the two-stage joint cache size allocation and beamforming design problem. At a slow timescale, cache size allocation is done at the cache deployment phase, so they can only adapt to the channel statistics. At a fast timescale, the beamforming vector can be designed to adapt to each channel realization during the file delivery phase.

First, we fix cache size allocation and content placement and focus on the beamforming design in the fast timescale. Assuming that the BS uses a single-datastream multicast beamforming strategy for the multiple-antenna BC \([1]\), the transmit signal is given by \( x = w^s \), where \( w \in \mathbb{C}^{M \times 1} \) is the beamformer vector and \( s \in \mathbb{C} \) is the user message, which can be assumed to be complex Gaussian distributed \( \mathcal{CN}(0, 1) \). Then, the mutual information in the previous section becomes

\[
I(x; y_l) = \log \left( 1 + \frac{\text{Tr}(\mathbf{H}_l \mathbf{W})}{\sigma^2} \right),
\]

where \( \mathbf{H}_l = \mathbf{h}_l \mathbf{h}_l^H \), and \( \mathbf{W} = \mathbf{w} \mathbf{w}^H \) is the beamforming covariance matrix of the transmit signal \( x \) restricted within the constraint set

\[
\mathbf{W} = \{ \mathbf{W} \geq 0 | \text{Tr} \left( \mathbf{W} \right) \leq P, \text{rank} \left( \mathbf{W} \right) = 1 \}.
\]

\[1\] In a similar vein, a related problem formulation of using secondary backhaul links to compensate for channel disparity is investigated in \([22]\).
with \( P \) being the transmit power budget at the CP.

The above set is nonconvex due to the rank-one constraint.

To obtain a numerical solution, a common practice is to drop the rank-one constraint to enable convex optimization, then to recover a feasible rank-one beamformer from the resulting solution [29]. While the solution so obtained is not necessarily global optimum, this strategy often works very well in practice, when compared to the globally optimal branch-and-bound algorithm [30].

Under fixed channel realization \( \mathbf{H}_l \) and cache size \( C_l \), the optimal beamformer design problem, after dropping the rank-one constraint, can now be formulated in terms of maximizing the delivery rate (or equivalently minimizing the downloading time):  

\[
\begin{align*}
\text{maximize} & \quad D_c \\
\text{subject to} & \quad \text{Tr}(\mathbf{W}) \leq P, \quad \mathbf{W} \succeq 0,
\end{align*}
\]

which can be reformulated as the following convex optimization problem:

\[
\begin{align*}
\text{maximize} & \quad \xi \\
\text{subject to} & \quad \log \left( 1 + \frac{\text{Tr}(\mathbf{H}_l \mathbf{W})}{\sigma^2} \right) \geq \xi(F - C_l), \quad l \in \mathcal{L}, \\
& \quad \text{Tr}(\mathbf{W}) \leq P, \quad \mathbf{W} \succeq 0.
\end{align*}
\]

This problem can be solved efficiently using standard optimization toolbox such as CVX [31]. To obtain a rank-one multicasting beamforming vector afterwards, we can adopt a strategy of using the eigenvector corresponding to the largest eigenvalue of solution \( \mathbf{W}^* \). The simulation section of this paper later examines the performance loss due to such a relaxation of the rank-one constraint.

Next, we consider the allocation of cache sizes in the slow timescale. The challenge is now to find the optimal allocation \( C_l \) that minimizes the expected file downloading time or maximizes the expected file downloading rate over the channel distribution. Intuitively, the role of caching at the BSs is to even out the channel capacity disparity in the CP-to-BS links so as to improve the multicasting rate, which is the minimum capacity across the BSs. At the fast timescale, transmit beamforming already does so to some extent. BS caching aims to further improve the minimum. The challenge here is to optimize the cache size allocation, which is done in the slow timescale, while accounting for the effect of beamforming, which is done in the fast timescale as a function of the instantaneous channel. We note that the caching strategy outlined in Lemma 1 is universal in the sense that it depends only on \( C_l \) and not on \( \mathbf{H}_l \). In the next two sections, we devise efficient algorithms that optimize the cache size allocation at the BSs based on the long-term channel statistics using a sample approximation technique.

V. CACHE ALLOCATION OPTIMIZATION ACROSS THE BSs

In this section, we formulate the cache size allocation problem for delivering a single file case of fixed size \( F \) bits in order to illustrate a sample approximation technique that allows us to quantify how much cache should be allocated to the BSs with different average channel strengths. The multiple files case is treated in Section VI.

A. Minimizing Expected Downloading Time

For given cache size \( C_l \), the optimal file downloading time [7] can be written as

\[
T_c^* = \min \max \{ \frac{F - C_l}{\log \left( 1 + \frac{\text{Tr}(\mathbf{H}_l \mathbf{W})}{\sigma^2} \right)} \}.
\]

Note that the file downloading time has also been considered in [16], [32] as the objective function. Differently, in this paper, we take the expectation of \( T_c^* \) over the channel distribution and aim to find an optimal cache size allocation that minimizes the long-term expected file downloading time. The cache optimization problem under a total cache size constraint \( C \) across the BSs is formulated as:

\[
\begin{align*}
\text{minimize} & \quad \mathbb{E}_{\{\mathbf{H}_l\}} [T_c^*] \\
\text{subject to} & \quad \sum_{l \in \mathcal{L}} C_l \leq C, \quad 0 \leq C_l \leq F, \quad l \in \mathcal{L}.
\end{align*}
\]

Finding a closed-form expression for the objective function in (13a) is difficult. This paper proposes to replace the objective function in (13a) with its sample approximation [25] and to reformulate the problem as:

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{N} \sum_{n=1}^{N} \max_l \left\{ \frac{F - C_l}{\log \left( 1 + \frac{\text{Tr}(\mathbf{H}_l^n \mathbf{W}^n)}{\sigma^2} \right)} \right\} \\
\text{subject to} & \quad \sum_l C_l \leq C, \quad 0 \leq C_l \leq F, \quad l \in \mathcal{L}, \\
& \quad \text{Tr}(\mathbf{W}^n) \leq P, \quad \mathbf{W}^n \succeq 0, \quad n \in \mathcal{N},
\end{align*}
\]

where \( N \) is the sample size, \( \mathcal{N} := \{1, 2, \ldots, N\} \), \( \{\mathbf{H}_l^n\}_{n \in \mathcal{N}} \) are the channel samples drawn according to the distribution of \( \mathbf{H}_l \), and \( \mathbf{W}^n \) is the beamforming covariance matrix adapted to the samples \( \{\mathbf{H}_l^n\}_{l \in \mathcal{L}} \). Note that we do not assume any specific channel distribution here. In fact, the above sample approximation scheme works for any general channel distribution. Furthermore, even if in practice when the channel distribution is unknown, we can still use the historical channel realizations as the channel samples, as long as they are sampled from the same distribution.

Problem (14) is still not easy to solve mainly due to the following two reasons. First, the objective function of problem (14) is nonsmooth and nonconvex, albeit all of its constraints are convex. Second, the sample size \( N \) generally needs to be sufficiently large such that the sample average is a good approximation to the original expected downloading time, leading to a high complexity for solving problem (14) directly. In the following, we first reformulate problem (14) as a smooth problem and linearize the nonconvex term, then leverage the ADMM approach to decouple the problem into \( N \)
low-complexity convex subproblems to improve the efficiency of solving the problem.

First, drop the constant $1/N$ in (14a) and introduce the auxiliary variable $\{\xi^n\}$, and reformulate problem (14) as

$$\begin{align*}
\text{minimize} & \quad \sum_{n=1}^{N} \frac{1}{\xi^n} \\
\text{subject to} & \quad \log\left(1 + \frac{\text{Tr}(H^n W^n)}{\sigma^2}\right) \geq \xi^n (F - C_l), l \in \mathcal{L}, \quad n \in \mathcal{N},
\end{align*}$$

(15a) and (15b)

The above problem (15) is smooth but still nonconvex due to constraint (15b). To deal with this nonconvex constraint, we approximate the nonconvex term $\xi^n (F - C_l)$ in (15b) by its first-order Taylor expansion at some appropriate point $(\bar{\xi}^n, \bar{C}_l)$, i.e.,

$$\xi^n (F - C_l) \approx \bar{\xi}^n (F - \bar{C}_l) + [F - \bar{C}_l, -\bar{\xi}^n]^T \bar{C}_l - \bar{\xi}^n, \quad l \in \mathcal{L}, \quad n \in \mathcal{N},$$

(16)

Based on (17), an iterative first-order approximation is proposed in Algorithm 1 for solving problem (15). More specifically, let $\{\xi^n(t), C_l(t)\}$ be the iterates at the $t$-th iteration, the algorithm solves

$$\begin{align*}
\text{minimize} & \quad \sum_{n=1}^{N} \frac{1}{\xi^n} \\
\text{subject to} & \quad \log\left(1 + \frac{\text{Tr}(H^n W^n)}{\sigma^2}\right) \geq \xi^n(t) (F - C_l(t)) \quad (18a) \\
& \quad \sum_{n=1}^{N} \xi^n(t) \geq \tau \quad (18b) \\
& \quad |\xi^n(t) - \xi^n(t)| \leq r(t), \quad n \in \mathcal{N}, \quad l \in \mathcal{L}, \quad (18c) \\
& \quad |C_l - C_l(t)| \leq r(t), \quad l \in \mathcal{L}, \quad (18d)
\end{align*}$$

(14b) and (14c)

with fixed $\{\xi^n(t), C_l(t)\}$, where (18c) and (18d) are the trust region constraints $33$, within which we trust that the linear approximation in (18b) is accurate, and $r(t)$ is the trust region radius at the $t$-th iteration, which is chosen in a way such that the following condition is satisfied:

$$\begin{align*}
\sum_{n=1}^{N} \frac{1}{\xi^n} - \sum_{n=1}^{N} \max_{l} \left\{ \frac{F - C_{l}^*(t)}{\log\left(1 + \frac{\text{Tr}(H^n W^n(t))}{\sigma^2}\right)} \right\} & \geq \tau, \\
\sum_{n=1}^{N} \frac{1}{\xi^n} - \sum_{n=1}^{N} \frac{1}{\xi^n} & \geq \tau
\end{align*}$$

(19)

where $W^n(t)$, $C_l^*(t)$, $\xi^n(t)$ are solutions to problem (18) and $r(t)$ is a constant. Notice that the numerator in (19) is the actual reduction in the objective of problem (14) and the denominator is the predicted reduction. The condition in (19) basically says that the trust region radius is accepted only if the ratio of the actual reduction and the predicted reduction is greater than or equal to a constant, in which case problem (18) is a good approximation of the original problem (14).

Algorithm 1 Optimized Cache Allocation with Single File

**Initialization:** Initialize $C_l(1) = C_l / L$, $l \in \mathcal{L}$, and $\xi^n(1)$ as the solution to problem (15) with $C_l = C_{l}^*(1)$; set $t = 1$;

**Repeat:**

1) Initialize the trust region radius $r(t) = 1$;

**Repeat:**

a) Use the ADMM approach in Appendix B to solve problem (18);

b) Update $r(t) = r(t)/2$;

**Until** condition (19) is satisfied.

2) Update $\{\xi^n(t+1), C_l(t+1)\}$ according to (20) and (21), respectively;

3) Set $t = t + 1$;

**Until** convergence

For the initial point, we can set $C_l(1)$ to be $C_l / L$ for all $l \in \mathcal{L}$, then problem (15) can be decoupled into $N$ convex optimization subproblems to solve for $\xi^n(1)$ for all $n \in \mathcal{N}$.

It remains to solve problem (18). Note that problem (18) is a convex program but with a potentially large number of variables due to the large sample size. We propose an ADMM approach $26$ to solve problem (18), which decouples the high-dimensional problem into $N$ decoupled small-dimensional subproblems. The details of solving problem (15) using the ADMM approach can be found in Appendix B. It can be shown that the ADMM approach is guaranteed to converge to the global optimum solution of the convex optimization problem (18).

Once problem (13) is solved using Algorithm 1 we fix the obtained optimized cache size allocation and evaluate its effectiveness under a different set of independently generated channels and calculate the file downloading time$3$ for each channel by solving the convex problem (11).

Algorithm 1 is guaranteed to converge to a stationary point of the optimization problem (14). In the rest of this section, we prove the convergence of Algorithm 1. First, we define the stationary point of problem (14) as in (34).

**Definition 1:** Consider a more general problem

$$\min_{x \in X} F(x)$$

(22)

$^3$Note that the optimal downloading time for each given channel is the inverse of the optimal objective value of problem (11).
where $\mathcal{X}$ is the feasible set and $F(x)$ is defined as

$$F(x) := \frac{1}{N} \sum_{n=1}^{N} \max_i \{ f_{nl}(x) \}. \quad (23)$$

Here, $\{ f_{nl}(x) \}$ is a set of continuously differentiable functions. Given any feasible point $\bar{x}$, define

$$\Phi(\bar{x}) = \max_{\{(d) \leq 1, \bar{x} + d \in \mathcal{X}\}} \left\{ F(\bar{x}) - \frac{1}{N} \sum_{n=1}^{N} \max_i \{ f_{nl}(\bar{x}) + \nabla f_{nl}(\bar{x})^T d \} \right\}. \quad (24)$$

A point $\bar{x} \in \mathcal{X}$ is called a stationary point of problem (22) if $\Phi(\bar{x}) = 0$.

Two remarks on the above definition of the stationary point are in order. First, it is simple to see that $\Phi(\bar{x})$ is always nonnegative as $d = 0$ is a feasible point of (24). If $\Phi(\bar{x}) = 0$, it means that there does not exist any feasible and decreasing direction at point $\bar{x}$ in the first-order approximation sense. Second, problem (14) is in the form of problem (22) if we set $x = \{C_l, W^n\}$, $f_{nl}(x) = \log \left( 1 + \frac{\ln(W^n)}{\sigma} \right)$, and $\mathcal{X}$ to be the feasible set of problem (14), which is convex and bounded.

Based on the above stationary point definition, we now state the convergence result of Algorithm 1 in the following theorem.

**Theorem 1**: Algorithm 1 is guaranteed to converge. Any accumulation point of the sequence generated by Algorithm 1 is a stationary point of problem (14), or equivalently problem (15).

**Proof**: Algorithm 1 is a special case of the general nonsmooth trust region algorithm discussed in [35], Chapter 11, which can be proved to converge to a stationary point of the general problem (22). For completeness of this paper, we provide a proof outline in Appendix A.

### B. Maximizing Expected Downloading Rate

In this subsection, we consider maximizing the expected file downloading rate as the objective function to optimize the BS cache size allocation, which can be formulated as

$$\begin{align*}
\text{maximize } & \mathbb{E}_{H_l} \left[ \frac{F}{T^*_l} \right] \\
\text{subject to } & \sum_{l \in \mathcal{L}} C_l \leq C, \quad 0 \leq C_l \leq F, \quad l \in \mathcal{L},
\end{align*} \quad (25a)$$

where $T^*_l$ is the optimal file downloading time defined in (12) under given channel realization and cache size allocation. Note that the expected value of the inverse of a random variable $X$, $\mathbb{E} \left[ \frac{1}{X} \right]$, is in general different from the inverse of the expected value of $X$, $\frac{1}{\mathbb{E}[X]}$. Thus, the cache size allocation obtained from solving problem (25) is different from the one obtained from solving problem (13).

We use the same idea as in the previous subsection to solve problem (25). First, we replace the objective function (25a) with its sample approximation and reformulate the problem as

$$\begin{align*}
\text{maximize } & \sum_{n=1}^{N} \xi^n \\
\text{subject to } & (14b), (14c), \text{ and } (15b),
\end{align*} \quad (26a)$$

in which we have dropped the constants $N$ and $F$ from the objective function. Then, we replace the nonconvex term in constraint (15b) by its linear approximation (17) and solve problem (26) via optimizing a sequence of linearly approximated problems similar to problem (13). The approximated problem at each iteration is solved via an ADMM approach similar to the one described in Appendix B with the only difference being that the first term in (37) needs to be replaced by $-\bar{\xi}^n$.

Same as in the previous subsection, once the optimized cache size allocation is obtained from solving problem (26), we evaluate its effectiveness on different sets of channels and solve the multicast rate by optimizing problem (11).

### VI. Cache Allocation Optimization Across Files

In this section, we consider the cache size allocation problem for the general case with multiple files having different popularities. Due to the minimal difference between the downloading rate and the downloading time as described in the previous section, we only focus on minimizing the expected file downloading time as the objective function in this section.

We assume that each file $k$ of equal size $F$ bits is requested from the user with given probability $p_k, k \in K := \{1, 2, \ldots, K\}$, $\sum_k p_k = 1$, and that BS $l$ caches $C_{lk}/F$ fraction of file $k$ with a total cache size constraint given by $\sum_{l,k} C_{lk} \leq C$. Given that file $k$ is requested, according to Lemma 1 the optimal downloading time for file $k$, denoted as $T^*_k$, can be written as

$$T^*_k = \min_{W_k \in \mathcal{W}} \max_l \left\{ \frac{F - C_{lk}}{\log \left( 1 + \frac{\ln(W_k)}{\sigma} \right)} \right\}. \quad (27)$$

Different from the downloading time (12) in the single file case, the above optimal downloading time $T^*_k$ is a random variable depending on not only the channel realization but also the index of the requested file. We take the expected value of $T^*_k$ on both the channel realization $H_l$ and the file index $k$ as the objective function and formulate the multi-file cache size allocation problem as

$$\begin{align*}
\text{minimize } & \sum_k p_k \mathbb{E}_{H_l} \left[ T^*_k \right] \\
\text{subject to } & \sum_{l,k} C_{lk} \leq C, \quad 0 \leq C_{lk} \leq F, \quad l \in \mathcal{L}, \quad k \in \mathcal{K}.
\end{align*} \quad (28a)$$

Although intuitively the more popular file should be allocated larger cache size, the question of how much cache should be allocated to each file is nontrivial. In particular, it is in

---

3 The optimal multicast rate for given channel is the optimal objective value of problem (11).
general not true that one should allocate the most popular file in its entirety first, then the second most popular file, etc., until the cache size is exhausted. This is because the gain in term of the objective function of the optimization problem (28) due to allocating progressively more cache size to one file diminishes as more cache is allocated. At some point, it is better to allocate some cache to the less popular files, even when the most popular file has not been entirely cached. The optimal allocation needs to be found by solving problem (28).

To solve problem (28), we use the same sample approximation idea as in the single file case. With an additional set of auxiliary variables \( \xi^n_k \), problem (28) after the sample approximation can be formulated as

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{1}{t_k} \\
\text{subject to} & \quad \log \left( 1 + \frac{\text{Tr}(H^n_k W^n_k)}{\sigma^2} \right) \geq \xi^n_k (F - C_k), \\
& \quad l \in \mathcal{L}, \ n \in \mathcal{N}, \ k \in \mathcal{K}, \\
& \quad \sum_{l,k} C_{lk} \leq C, \ 0 \leq C_{lk} \leq F, \\
& \quad l \in \mathcal{L}, \ k \in \mathcal{K}, \\
& \quad \text{Tr}(W^n_k) \leq P, \ W^n_k \succeq 0, \\
& \quad n \in \mathcal{N}, \ k \in \mathcal{K}.
\end{align*}
\]

Problem (29) is then solved in an iterative fashion. At each iteration the nonconvex term on the right hand side of (29b) is replaced by its first-order approximation and the resulting convex problem to be solved at the \( t \)-th iteration is given by:

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{1}{t_k} \\
\text{subject to} & \quad \log \left( 1 + \frac{\text{Tr}(H^n_k W^n_k)}{\sigma^2} \right) \geq \xi^n_k (t) (F - C_k) \\
& \quad + (F - C_{lk}(t)) (\xi^n_k(t) - \xi^n_k(t)), \\
& \quad l \in \mathcal{L}, \ n \in \mathcal{N}, \ k \in \mathcal{K}, \\
& \quad |\xi^n_k(t) - \xi^n_k(t)| \leq r(t), \ n \in \mathcal{N}, \ k \in \mathcal{K}, \\
& \quad |C_{lk} - C_{lk}(t)| \leq r(t), \ l \in \mathcal{L}, \ k \in \mathcal{K}, \\
\end{align*}
\]

\[\xi^n_k(t + 1) = \min_{l \in \mathcal{L}} \left\{ \log \left( 1 + \frac{\text{Tr}(H^n_k W^n_{k}(t))}{\sigma^2} \right) F - C_{lk}^*(t) \right\}, \]

\[n \in \mathcal{N}, \ k \in \mathcal{K}, \]

\[C_{lk}(t + 1) = C_{lk}^*(t), \ l \in \mathcal{L}, \ k \in \mathcal{K},\]

where \( \xi^n_k(t) \) and \( C_{lk}(t) \) are fixed parameters obtained from the previous iteration and are updated for the next iteration according to

\[\left\{ \begin{array}{l}
\xi^n_k(t + 1) = \min_{l \in \mathcal{L}} \left\{ \log \left( 1 + \frac{\text{Tr}(H^n_k W^n_{k}(t))}{\sigma^2} \right) F - C_{lk}^*(t) \right\}, \\
\end{array} \right.
\]

\[n \in \mathcal{N}, \ k \in \mathcal{K}, \]

\[C_{lk}(t + 1) = C_{lk}^*(t), \ l \in \mathcal{L}, \ k \in \mathcal{K},\]

where \( W^n_{k}(t) \) and \( C_{lk}^*(t) \) are solutions to problem (30).

Similar to (19), the trust region radius \( r(t) \) in (30a) and (30d) is picked to satisfy the following condition:

\[
\sum_{k=1}^{K} \sum_{n=1}^{N} \frac{1}{\xi^n_k(t)} - \max_l \left\{ \log \left( 1 + \frac{\text{Tr}(H^n_k W^n_{k}(t))}{\sigma^2} \right) F - C_{lk}^*(t) \right\} \geq \tau
\]

for some constant \( \tau \in (0, 1) \).

Note that problem (30) can also be solved by using an ADMM approach as explained in Appendix C which decouples problem (30) into \( NK \) subproblems and each subproblem corresponds to a pair of sample channel and file request. The overall proposed algorithm for solving the cache size allocation problem with multiple files is summarized in Algorithm 2.

Once the cache size allocation for the multiple files case is optimized through Algorithm 2, we calculate the downloading time for file \( k \) by solving problem (27) with fixed \( C_{lk} \), which can be formulated as a convex optimization problem similar to (11). We then compute the average downloading time by averaging under different sets of channel realizations.

\[\text{Algorithm 2 Optimized Cache Allocation with Multiple Files} \]

\[\text{Initialization: Initialize } C_{lk}(1) = C/LK, \ l \in \mathcal{L}, \ k \in \mathcal{K} \text{ and } \xi^n_k(1) \text{ as the solution to problem (29) with } C_{lk} = C_{lk}(1); \]

\[\text{Repeat:} \]

1) Initialize the trust region radius \( r(t) = 1 \);

2) Update \( r(t) = r(t)/2 \);

\[\text{Until condition (33) is satisfied.} \]

3) Set \( t = t + 1; \)

\[\text{Until convergence} \]

Note that problem (30) can also be solved by using an ADMM approach as explained in Appendix C which decouples problem (30) into \( NK \) subproblems and each subproblem corresponds to a pair of sample channel and file request. The overall proposed algorithm for solving the cache size allocation problem with multiple files is summarized in Algorithm 2.

Once the cache size allocation for the multiple files case is optimized through Algorithm 2, we calculate the downloading time for file \( k \) by solving problem (27) with fixed \( C_{lk} \), which can be formulated as a convex optimization problem similar to (11). We then compute the average downloading time by averaging under different sets of channel realizations.

\[\text{VII. SIMULATION RESULTS} \]

This section evaluates the performance of our proposed caching schemes through simulations. Consider a downlink C-RAN model with \( L = 5 \) BSs randomly placed on the half plane below the CP with the relative distances between the CP and the 5 BSs shown in Fig. 2. We generate 1000 sets of channel realizations from the CP to the BSs according to \( \text{H}_t = \text{K}_t^{1/2} \nu_t \), where \( \text{K}_t \) models the correlation between the CP transmit antennas to BS \( l \) and is generated mainly according to the angle-of-arrival and the antenna pattern, as described in [36], with the path-loss component modeled as \( \sigma^2 = 128.1 + 37.6 \log_{10}(d) \) dB and \( d \) is the distance between the cloud and the BS in kilometers; \( \nu_t \) is a Gaussian random vector with each element independently and identically distributed as \( \mathcal{N}(0, 1) \). The first \( N = 100 \) sets of channel realizations are generated.
used in the sample approximation to optimize the cache allocation while the rest 900 are used to evaluate the performance under the obtained cache size allocation. The details of the simulation parameters are listed in Table I.

### A. Cache Allocation for BSs with Varying Channel Strengths

In this subsection, we evaluate the performances of the proposed schemes for caching a single file across multiple BSs with different channel strengths as discussed in Section V. We compare the optimized cache size allocations obtained from minimizing the expected file downloading time \( T \) and maximizing the expected file downloading rate in (25) with the following set of schemes:

- **No Cache**: Cache sizes \( C_l = 0 \) for all BSs;
- **Uniform Cache Allocation**: Cache sizes among the BSs are uniformly distributed as \( C_l = C/L \), which serves as a baseline scheme;
- **Proportional Cache Allocation**: Cache sizes among the BSs are proportionally allocated such that \( (F - C_l) / \log\left(1 + \frac{P_t\mu(K)}{L\sigma^2}\right) \) for all \( l \) are equalized, if possible, which serves as another baseline scheme;
- **Lower/Upper Bound**: Cache sizes among the BSs are dynamically and optimally allocated by solving problem (11) for each channel realization by treating \( \{C_l\} \) as the optimization variables, which is impractical in reality but serves as a lower bound for minimizing the expected file downloading time and an upper bound for maximizing the expected file downloading rate;
- **Rank-One Multicast Beamformer**: Cache sizes among the BSs are the same as the optimized caching schemes, but the multicast beamformer is restricted to be rank-one and is set to be the eigenvector corresponding to the largest eigenvalue of the optimized beamforming matrix \( \mathbf{W}_n \) in each test sample channel.

In Fig. 3, we compare the allocated BS cache sizes between the proposed schemes trained on the first 100 channels and the baseline schemes under normalized file size \( F = 100 \) and total cache size \( C = 100 \). As we can see, both of the proposed caching schemes are more aggressive in allocating larger cache sizes to the weaker BS 3 as compared to the uniform and proportional caching schemes. We then evaluate the performances of different cache size allocation schemes on the rest 900 sample channels and report the file downloading time and downloading rate (or spectral efficiency) in Table II and III, respectively, under two different settings of total cache size \( C = 100 \) and \( C = 200 \), normalized with respect to file size \( F = 100 \). As we can see, the proposed caching scheme improves over the uniform and proportional caching schemes by 10% – 15% on average, but the gains are more significant for the 90th-percentile downloading time and the 10th-percentile downloading rate, which are around 20% – 27% and 26% – 36%, respectively.

We note here that without caching, the average and 90th-percentile file downloading time are 11.45 ms/Mb and 14.76 ms/Mb, respectively, in this setting. The average and 10th-percentile file downloading rate are 4.63 bps/Hz and 3.39 bps/Hz. Thus, the optimized BS caching schemes with \( C = 100 \) and \( C = 200 \) (normalized with respect to \( F = 100 \)) improve the average downloading time by about 33% and 50% respectively, and improve the average downloading rate.

### TABLE I

| Parameters                        | Values       |
|-----------------------------------|--------------|
| Number of BSs                     | 5            |
| Backhaul channel bandwidth        | 20 MHz       |
| Number of antennas at CP         | 10           |
| Number of antennas at each BS    | 1            |
| Maximum transmit power \( P \) at CP | 40 Watts   |
| Antenna gain                      | 17 dBi       |
| Background noise                  | -150 dBm/Hz  |
| Path loss from CP to BS          | 128.1 + 37.6 \log_{10}(d) |
| Rayleigh small scale fading       | 0 dB         |
| Normalized file size              | 100          |
| Training sample size \( N \)      | 100          |
| Test sample size                  | 900          |

**Fig. 2.** A downlink C-RAN setup with 5 BSs. The distances from the CP to the 5 BSs are (398, 278, 473, 286, 267) meters, respectively.

**Fig. 3.** Cache allocation for different schemes under total cache size \( C = 100 \), normalized with respect to file size \( F = 100 \).
by about 43% and 91% respectively.

In Figs. 4 and 5 we compare the cumulative distribution functions (CDFs) of the downloading time and the downloading rates evaluated on the 900 test channels with different caching schemes. Similar to what we have seen in Tables II and III, the proposed caching scheme shows significant gains in improving the high downloading time regime and the low downloading rate regime. This is due to the fact that BSs farther away from the cloud are more aggressively allocated larger amount of cache under the optimized scheme. Second, the rank-one beamformer derived from the general-rank covariance matrix does not degrade the performance much at all. Hence, we only focus on the performance of the proposed caching schemes without the rank-1 constraint on the covariance matrix in the next subsection for the multiple files case.

B. Cache Allocation for Files of Varying Popularities

In this subsection, we present simulation results for the caching schemes with multiple files having different popularities and focus on the expected file downloading time as the performance metric. We first consider only two files with different pairs of request probabilities \((p_1, p_2)\) listed on
the first row of Table IV where each column denotes the cache size allocation among the 5 BSs under the specific file popularity given in the first row and each cell gives the cache size allocation between the two files within each BS. The cache sizes in each column add up to the total cache size $C = 100$, normalized with respect to file size $F = 100$.

From Table IV we see that for each column with given file popularity, the weakest BS 3 always gets the most cache size as in the single file case shown in Fig. 3. Moreover, as the difference between the popularities of the two files increases across the columns, more cache is allocated to the first file. For example, the proposed caching scheme decides to allocate all the cache to only the more popular file 1 when $(p_1, p_2) = (0.9, 0.1)$.

In Fig. 6 we compare the average file downloading time between the optimized cache scheme and the following baseline schemes:

- **No Cache**: Cache size $C_{lk} = 0$ for all BSs and files;
- **Uniform Cache Allocation**: Cache size for file $k$ at each BS $l$ is set to be as $C_{lk} = C/LK$ for all $k$ and $l$;
- **Proportional Cache Allocation**: We first set the total cache size allocated for file $k$ as $p_k C$, then distribute $p_k C$ among the BSs according to the rule described in the Proportional Cache Allocation scheme in Section VII-A.
- **Caching the Most Popular File**: We cache the most popular file in its entirety first, then the second most popular file, etc. When a file cannot be cache entirely, we distribute the remaining cache among the BSs according to the Proportional Cache Allocation scheme described in Section VII-A.

In Fig. 6 we fix the number of files to be $K = 4$ and generate the file popularity according to the Zipf distribution given by $p_k = \frac{k^{-\alpha}}{\sum_{i=1}^{K} i^{-\alpha}}$, $\forall$ $k$, with different settings of $\alpha$. As the Zipf distribution exponent $\alpha$ increases, the difference among the file popularities also increases. As we can see from Fig. 6 the average downloading time for all schemes, except for the uniform caching scheme, decreases as $\alpha$ increases. This is because in uniform cache allocation the cache size is the same for all files, hence the downloading time is the same no matter which file is requested. In contrast, all other three schemes tend to allocate more cache to the more popular files. In particular, the proposed caching scheme converges to the scheme of caching the most popular file when $\alpha = 1.5$, while it consistently outperforms the proportional caching scheme.

From Fig. 6 we conclude that first, the uniform cache size allocation scheme performs poorly when the files have different popularities and especially when the difference is large. Second, it is advantageous to allocate larger cache size to the more popular file, however, it is not trivial to decide how much more cache is needed for the more popular file.

Our proposed caching scheme provides a better cache size allocation solution as compared to the heuristic proportional caching scheme and the most popular file caching scheme.

**TABLE IV**

| File Popularity $(p_1, p_2)$ | $(0.5, 0.5)$ | $(0.6, 0.4)$ | $(0.7, 0.3)$ | $(0.8, 0.2)$ | $(0.9, 0.1)$ |
|-----------------------------|-------------|-------------|-------------|-------------|-------------|
| BS1 $(8.2, 8.2)$           | $(13.2, 2.1)$ | $(16.8, 0)$ | $(20.2, 0)$ | $(22.2, 0)$ |
| BS2 $(0.0, 0)$             | $(0.0, 0)$   | $(0.0, 0)$  | $(3.6, 0)$  | $(7.1, 0)$  |
| BS3 $(11.8, 11.8)$         | $(48.2, 36.9)$ | $(53.6, 27)$ | $(60.8, 10.9)$ | $(68.8, 0)$ |
| BS4 $(0.0, 0)$             | $(0.0, 0)$   | $(2.0, 0)$  | $(7.5, 0)$  | $(10.1, 0)$ |
| BS5 $(0.0, 0)$             | $(0.0, 0)$   | $(0.0, 0)$  | $(1.8, 0)$  |             |
| Total $(60, 50)$            | $(61.4, 38.6)$ | $(73.27)$   | $(89.1, 10.9)$ | $(100, 0)$   |

**Fig. 6.** Average downloading time for different Zipf file distributions under the same number of files $K = 4$ and total cache size $C = 400$, normalized with respect to file size $F = 100$.

**VIII. Conclusion**

This paper points out that caching can be used to even out the channel disparity in a multicast scenario. We study the optimal BS cache size allocation problem in the downlink C-RAN with wireless backhaul to illustrate the advantage of multicast and caching for the data-sharing strategy. We first derive the optimal multicast rate with BS caching, then formulate the cache size optimization problem under two objective functions, minimizing the expected file downloading time and maximizing the expected file downloading rate, subject to the total cache size constraint. By leveraging the sample approximation method and ADMM, we propose efficient cache size allocation algorithms that considerably outperform the heuristic schemes.

**Appendix A**

**Proof of Theorem 1**

We use the notations introduced in Definition 1 in the following convergence proof. First of all, it is simple to show that the objective sequence $\{F(x(t))\}$ generated by Algorithm 1 monotonically decreases and is lower bounded by zero.
Second, by using the continuously differentiable property of the function \( f_{\eta_1}(x) \), it can be shown that there always exists a trust region radius \( r(t) \) such that the condition (19) is satisfied and that \( r(t) \) is lower bounded by some constant \( r > 0 \), i.e., \( r(t) \geq r > 0 \), for all \( t \). Moreover, since the generated sequence \( \{x(t)\} \) lies in the bounded set \( \mathcal{X} \), there must exist an accumulation point. Without loss of generality, let \( \bar{x} \) denote an accumulation point of some convergent subsequence indexed by \( T \). Finally, we show \( \Phi(\bar{x}) = 0 \) by contradiction: Suppose that \( \bar{x} \) is not a stationary point, i.e., \( \Phi(\bar{x}) = \delta > 0 \), then there exists a subsequence of \( \{x(t)\}_{t \in T} \) that is sufficiently close to \( \bar{x} \) such that

\[
\frac{1}{N} \sum_{n=1}^{N} \frac{1}{\xi_n(t)} - \frac{1}{N} \sum_{n=1}^{N} \frac{1}{\xi_n(s)} \geq r \Phi(x(t)) \geq \frac{r \delta}{2}, \tag{34}
\]

where the first inequality is due to \([38]\) Lemma 2.1 (iv). Combining (34) with (19) and (20), we get

\[
F(x(t)) - F(x(t + 1)) \geq \frac{\tau r \delta}{2} > 0,
\]

which further implies that \( F(x(t)) \to -\infty \) as \( t \to +\infty \) in \( T \). This contradicts the fact that the sequence \( \{F(x(t))\} \) is bounded below by zero. The proof is completed.

**APPENDIX B**

**The ADMM Approach to Solve Problem (38)**

To apply the ADMM approach to solve problem (38), we first introduce a set of so-called consensus constraints \( C_l^n = C_l, \ l \in L, n \in N \), and reformulate problem (38) as

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{\xi_n} \sum_{n=1}^{N} \xi_n \\
\text{subject to} & \quad \log \left( 1 + \frac{\text{Tr}(H^n W^n)}{\sigma^2} \right) \geq \xi_n(t) (F - C_l^n) \\
& \quad + (F - C_l(t)) (\xi_n - \xi_n(t)) \quad \forall l \in L, n \in N, \tag{35a} \\
& \quad C_l^n = C_l, \ l \in L, n \in N, \tag{35b} \\
& \quad |\xi_n - \xi_n(t)| \leq r(t), \ n \in N, \tag{35c} \\
& \quad |C_l - C_l(t)| \leq r(t), \ l \in L, \tag{35d}
\end{align*}
\]

where we replace the variable \( C_l \) in (38) with the newly introduced variable \( C_l^n \) in (35b). We form the partial augmented Lagrangian of problem (35) by moving the constraint (35c) to the objective function (35a) as follows:

\[
\mathcal{L}_\rho(\xi^n, W^n, C_l^n, C_l; \lambda_l^n) = \frac{1}{\xi_n} + \sum_{l=1}^{L} \sum_{n=1}^{N} \left[ \lambda_l^n (C_l^n - C_l) + \frac{\rho}{2} (C_l^n - C_l)^2 \right], \tag{36}
\]

where \( \lambda_l^n \) is the Lagrange multiplier corresponding to the constraint \( C_l = C_l^n \) and \( \rho > 0 \) is the penalty parameter.

The idea of using the ADMM approach to solve (35) is to sequentially update the primal variables via minimizing the augmented Lagrangian (36), followed by an update of the Lagrange multiplier. Particularly, at iteration \( j + 1 \), the ADMM algorithm updates the variables according to the following three steps:

**Step 1** Fix \( \{C_l, \lambda_l^n\}_j \) obtained from iteration \( j \), update \( \{\xi^n, W^n, C_l^n\}_j \) for iteration \( j + 1 \) as the solution to the following problem

\[
\begin{align*}
\text{minimize} & \quad \mathcal{L}_\rho(\xi^n, W^n, C_l^n, C_l; \lambda_l^n) \\
\text{subject to} & \quad (35b), (35d) \text{ and (14c)}.
\end{align*}
\]

**Step 2** Fix \( \{\xi^n, W^n, C_l^n\}_{j+1} \) obtained from Step 1, update \( \{C_l\}_j \) for iteration \( j + 1 \) as the solution to the following problem

\[
\begin{align*}
\text{minimize} & \quad \mathcal{L}_\rho(\xi^n, W^n, C_l^n, C_l; \lambda_l^n) \\
\text{subject to} & \quad (35b), (14b).
\end{align*}
\]

**Step 3** Fix \( \{C_l^n\}_{j+1} \) and \( \{C_l\}_{j+1} \) obtained from Steps 1 and 2 respectively, update the Lagrange multiplier as:

\[
\lambda_l^{j+1} = \lambda_l^n + \rho (C_l^{j+1} - C_l^{j+1}).
\]

In the above Step 1, the optimization problem is decoupled among the channel realizations and for each channel realization \( n \in N \) we solve the following subproblem:

\[
\begin{align*}
\text{minimize} & \quad \frac{1}{\xi_n} + \sum_{l \in L} \left[ \lambda_l^n (C_l^n - C_l) + \frac{\rho}{2} (C_l^n - C_l)^2 \right] \\
\text{subject to} & \quad \log \left( 1 + \frac{\text{Tr}(H^n W^n)}{\sigma^2} \right) \geq \xi_n(t) (F - C_l^n) \\
& \quad + (F - C_l(t)) (\xi_n - \xi_n(t)) \quad \forall l \in L, n \in N, \tag{37a} \\
& \quad C_l^n = C_l, \ l \in L, n \in N, \tag{37b} \\
& \quad |\xi_n - \xi_n(t)| \leq r(t), \ n \in N, \tag{37c} \\
& \quad |C_l - C_l(t)| \leq r(t), \ l \in L, \tag{37d}
\end{align*}
\]

where \( C_l \) and \( \lambda_l^n \) are fixed constants obtained from the previous iteration and set to be as \( C_l = C_l^n, \lambda_l^n = \lambda_l^{j+1} \). Note that problem (37) is a small-scale smooth convex problem and can be solved efficiently through the standard convex optimization tool like CVX [31]. The solutions to problem (37) are denoted as \( \{\xi^n, W^n, C_l^n\}_{j+1} \).
where \( a_l = \sum_{l=1}^{L} \phi_l^C + \lambda^C_n \rho_N \) is a constant with \( C^C_l = C^{n,j+1}_l \) obtained from Step 1 and \( \lambda^C_n = \lambda^{n,j} \) obtained from the previous iteration.

With the reformulated problem (39), it is easy to see that the optimal \( C_l \) admits a closed-form solution given by

\[
C_{l}^{j+1} = [a_l - \mu_i \bar{\theta}_i], \ i \in \mathcal{L},
\]

where

\[
\bar{\theta}_i = \max \{ C_i(t) - r(t), 0 \}, \ \bar{\theta}_i = \min \{ C_i(t) + r(t), F \},
\]

and \( \mu \) is the solution to

\[
\sum_{i=1}^{L} [a_i - \mu \bar{\theta}_i] = C
\]

conditioned on \( \sum_{i=1}^{L} a_i > C \); otherwise \( \mu = 0 \). The desired \( \mu \) can be found within \( \mathcal{O}(L \log_2(L)) \) operations.

In the above proposed ADMM algorithm, we introduce a set of auxiliary variables for problem (18), which is then optimized over two separate blocks of variables \( \{ \xi^n, W^n, C^n_l \} \) and \( \{ C_i \} \). In [26, Section 3.2] and [38, Proposition 15], the convergence guarantee of such a two-block ADMM algorithm is established based on two sufficient conditions: one that the objective function is closed, proper, and convex; the other is that the Lagrangian has at least one saddle point. It is simple to check that both of the conditions hold for the reformulated problem (39), which is equivalent to problem (18). Hence, the ADMM algorithm developed above converges to the global optimal solution of problem (18).

**APPENDIX C**

**The ADMM Approach to Solve Problem (30)**

Similar to problem (35), we first introduce a set of consensus constraints \( C^n_{lk} = C_{lk}, \ l \in \mathcal{L}, \ k \in \mathcal{K}, \ n \in \mathcal{N} \) for problem (30) and replace the variable \( C_{lk} \) in (30) with \( C^n_{lk} \). Then, the partial augmented Lagrangian of problem (30) can be written as

\[
\mathcal{L}_\rho (\xi^n, W^n, C^n, C_l ; \lambda^n_{lk}) = \sum_{k \in \mathcal{K}} \sum_{l \in \mathcal{L}} \sum_{n \in \mathcal{N}} \left[ \lambda^n_{lk} (C^n_{lk} - C_{lk}) + \frac{\rho_N}{\rho} (C^n_{lk} - C_{lk})^2 \right] + \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \frac{1}{\rho} \xi^n_{lk},
\]

where \( \lambda^n_{lk} \) is the Lagrange multiplier corresponding to the consensus constraint \( C^n_{lk} = C_{lk} \).

As in the three steps listed in Appendix B the first step at iteration \( j + 1 \) of the ADMM approach to solve problem (30) is to fix \( \{ \xi^n_{lk}, \lambda^n_{lk} \} \) as \( C_{lk} = C^n_{lk}, \lambda^n_{lk} = \lambda^{n,j} \) obtained from the \( j \)-th iteration and solve for \( \{ \xi^n_{lk}, W^n_{lk} \} \) by minimizing the Lagrangian (40), which is decoupled among each pair of sample channel realization and file index \( (n,k), n \in \mathcal{N}, k \in \mathcal{K} \). The subproblem to be solved in the first step is formulated as follows:

\[
\begin{align}
\text{minimize} \quad & \sum_{k \in \mathcal{K}} \frac{\rho_N}{\rho} (C^n_{lk} - C_{lk})^2 \\
\text{subject to} \quad & \log \left( \frac{1}{1 + \frac{\Tr (H^n_{lk} W^n_{lk})}{\sigma^2 \rho_N}} \right) \geq \xi^n_{lk}(t) (F - C^n_{lk}) \\
& \quad + (F - C_{lk}(t)) \left( \xi^n_{lk}(t) - \xi^n_{lk}(t) \right), \ l \in \mathcal{L}, \ k \in \mathcal{K}, \\
& \Tr (W^n_{lk}) \leq P, \ W^n_{lk} \succeq 0,
\end{align}
\]

The solutions to the above subproblem (41) are denoted as \( \{ \xi^n_{lk}, W^n_{lk}, C^n_{lk} \} \).

In the second step, variables \( C_{lk}, \ l \in \mathcal{L}, \ k \in \mathcal{K} \) are updated by minimizing the Lagrangian (40) under the total cache constraint with fixed \( C^n_{lk} = C^{n,j+1}_l \) obtained from solving problem (41) as well as fixed \( \lambda^n_{lk} = \lambda^{n,j} \) from the previous iteration. The subproblem in the second step can be formulated as

\[
\begin{align}
\text{minimize} \quad & \frac{1}{2} \sum_{l \in \mathcal{L}} \sum_{k \in \mathcal{K}} (C_{lk} - b_{lk})^2 \\
\text{subject to} \quad & \sum_{l \in \mathcal{L}} C_{lk} \leq C, \ 0 \leq C_{lk} \leq F, \ l \in \mathcal{L}, \ k \in \mathcal{K}, \\
& \quad |C_{lk} - C_{lk}(t)| \leq r(t), \ l \in \mathcal{L}, \ k \in \mathcal{K},
\end{align}
\]

where \( b_{lk} = \sum_{n \in \mathcal{N}} (C^{n,j+1}_l - \lambda^{n,j} \lambda^{n,j}) \), \( l \in \mathcal{L}, k \in \mathcal{K} \) are constants. The solution to problem (42) can be written as

\[
C_{lk}^{j+1} = [b_{lk} - v^b_{lk} \bar{\theta}_k], \ l \in \mathcal{L}, \ k \in \mathcal{K},
\]

where

\[
\bar{\theta}_k = \max \{ C_{lk}(t) - r(t), 0 \}, \ \bar{\theta}_k = \min \{ C_{lk}(t) + r(t), F \},
\]

and \( \nu \) is the solution to

\[
\sum_{l=1}^{L} \sum_{k=1}^{K} \sum_{n=1}^{N} |b_{lk} - v^b_{lk}| = C
\]

if \( \sum_{l=1}^{L} \sum_{k=1}^{K} b_{lk} > C \); otherwise \( \nu = 0 \). The desired \( \nu \) can be found within \( \mathcal{O}(LK \log_2(LK)) \) operations.

In the last step, we update the Lagrange multiplier \( \lambda^n_{lk} \) as

\[
\lambda^{n,j+1}_l := \lambda^{n,j}_l + \rho \left( C^n_{lk}^{j+1} - C_{lk}^{j+1} \right), \ \forall l, k, n.
\]

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