The Usage of Isoguarantees for Currency Portfolio and Integrated Asset and Liability Management

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This article suggests using the idea of isoguarantees or the management of complicated financial processes and other processes depending upon them or in some way joining them together, emphasizing the importance of guaranty and reliability management. Financial risk investigations let us talk about commensurability of the amount of most of the financial indicators and the risk.

There are some conceptions quite uncommon in financial literature, which, however, are necessary needed in order to analyze reliability of expected results using methods mentioned in the article. One of those conceptions is isoguarantee, which is a line joining effectiveness (profit, profitability, incomes etc) indicators of the same guarantee in the possibilities - risk plane.

Using ideas and techniques of Markowitz or modern portfolio and adequate with stochasticity of profit possibilities portfolio, integrated assets and liabilities portfolio is being offered, which helps to put in order the possibilities of analyzed process or subject expansion according to its levels of guarantees. Two academic examples are shown as an illustration for ideas analyzed: formation of currency portfolio and formation and management of integrated assets and liabilities portfolio. And so the article is divided into two parts.

Although the methods of analytic solution search are sometimes used in the analysis of cases, still the main method for analyzing complicated cases remains imitative technologies that can be understood as an interaction of computer counting and imitative modeling. Obtained results are shown in the geometric form. It is also necessary to admit that computer resources sometimes were not sufficient to present continuous processes in visually adequate discrete manner.

To analyze extremely complicated situations huge resources of computer power and velocity are needed, but taking into account the fact that innovations outrun demand in this area, imitation technologies should be recognized as the most perspective mean of analysis of complicated quantitative cases helping to solve any analytical problem successfully.

Keywords: isoguarantees, imitative technologies, adequate portfolio, risk, integral asset liability management portfolio.

Introduction

Globalization processes equalize the opportunities and differences of most financial markets in such a way that the same financial instruments begin to acquire set the same prices in all markets. Although, the influence of globalization on risk variety and scale is not known
yet [Held D., 1999; Becker S., 2001], it is clear, that globalization, while decreasing some risks, creates the new ones. Globalization is especially dangerous in financial markets where globalization processes can cause the global financial catastrophe. Such circumstances make us emphasize the guaranties or reliability of expected results in financial management problems.

Although, you will not find so called determined financial processes, still most of the methods, even the new ones refer to common deterministic schemes of thinking [Oguzcoy C. B., 1997].

Preparation for management in case of risk and uncertainty starts with putting in order the relevant information to assure optimal solutions, as the future of process can be evaluated only as a specter of possibilities and only one of them will come true in the future.

In this article the problem of information arrangement and preparation for decision making and management under the risk is analyzed in two stages. First, we are trying to apply investment profitability portfolio adequate with stochastic nature of profit possibilities [Rutkauskas A. V., 2000; Rutauskas A. V., 2003] to simplified currency management system, and to form the opportunities of portfolio (taking into account its guaranties) using forecast of macro environment. In the second stage, we are trying to expand the opportunities of integrated assets and liabilities portfolio, in order to put portfolio into integrated assets and liabilities scheme. We are doing so to seek for the further goal - to classify opportunities of integrated assets and liabilities strategies, taking into account their reliability. Both stages are illustrated with examples.

1. Conception of isoguarantees for currency portfolio management

1.1. Presumptions for problem appearance

Nowadays, currency portfolio management problems become urgent not only for subjects serving currency markets or fisc but also for different companies. At globalisation and market integration conditions, not only multinational, but also national corporations must dispose different currencies and other financial assets of different countries. For all corporations – for multinational and national – it is very important to formulate rationally multi-exchange portfolio of assets and liabilities. Undeniable, that corporation's ability to choose and manage currency portfolio properly is the problem of priority that corporation financial managers deal with and managing such kind of portfolio, problems of different aspects arise [Korhonen A., 1987; Oguzcoy C. B., 1997].

Here we will analyse the problem and procedure of forming and choosing currency portfolio, by solving problems in a described situation. Suppose, one of departments of large corporation, existing in one of the countries in the Euro zone, dispose of 1mln. EUR capital, which could be invested for one-year period. Meanwhile, other department after one year will have to borrow 1mln. EUR, to fulfil its international operations. A large commercial bank, working in Euro market, agrees to accept deposits in currencies and suggests more favourable interests for deposits in currencies, then interests for deposits in Euro in this period. The bank also pledges to return accumulated sum in wanted currency, taking into consideration currency exchange rates at that moment.
Currency interest rates offered by banks per year, and future currency exchange rate after one year, could be evaluated only as random variables which, possible probabilities values are shown in the table 1.

What kind of portfolio of corporations' deposits in currencies should be, which could maximize accumulated sum $S$, measured in Euro, corporations' usefulness function $U = u(S, \sigma_S)$. Here $\sigma_S$ - average standard deviation, index of risk.

Banks suggested average interest rate for different currency deposits and forecasted currency rate changes in one year table 1:

| Currency | Spot exchange in EUR | Forecasted interest rate for year | Forecasted changes of exchange rate in EUR |
|----------|----------------------|----------------------------------|--------------------------------------------|
| USD      | 0.8959               | N(0.07; 0.001)                   | N(-0.01; 0.001)                            |
| CHF      | 0.7321               | N(0.06; 0.0014)                  | N(-0.015; 0.0075)                          |
| GBP      | 1.4505               | N(0.09; 0.0012)                  | N(+0.05; 0.008)                            |
| RUB      | 0.1305               | N(0.04; 0.01)                    | N(-0.25; 0.009)                            |

Remark. In the table $N(\ )$ are Gaussian distribution functions.

1.2. The peculiarities of solution of the problem

Many of methods devoted to the solution of the main portfolio problems seem not very sophisticated [Markowitz H. M., 1990; Tobin J., 1965] though the way to this simplicity was quite difficult. On the other hand simplicity of the problem was a consequence of simplification of the reality [Giokas D., 1991; Chopra V. K., 2001]. First of all it concerns mean value – the main component of the diptychs mean value – standard deviation. An investor knows that the perspective should presume a set of possibilities so he (she) is concerned to know their means. Moreover, an investor considers that possibilities are not symmetrically distributed along mean value. So mean value (average) is not the most expected value. Thus, if a standard deviation is not a deviation of the most expected value it cannot be used as more objective measure of the risk.

As a result of these facts an investor ought to suspect that presumption about utility function depending only on mean value and standard deviation is not reasonable, though maximization of utility function is an adequate presumption.

1.3. The role of isoguarantees in portfolio decisions

What motivates investor to make one or another decision? It seems that modern investment theory gives an undoubtedly right answer: an investor intends to maximize his average income under the accepted level of risk or intends to minimize risk while willing to have the chosen level of average profit. When an investor uses mean value - standard deviation ideology then these criteria give satisfactory results [Hensel C. R., 2001; Rutkauskas A. V. 2003]. But an investor wants to encounter all possibilities which could be presumed by the future situation and wants to estimate the reliability of each of them and first of all reliability of the most expected of them. In this case mean – standard deviation portfolio model couldn't be used as instrument of decision-making.

Further more formalized criterion will be discussed. Let's presume that the main goal of
an investor is to maximize the guarantee for profitability to the level not below (more or equal) the desirable level or – is maximization of the upper bound of profit with the chosen guarantee. Evidently, such kind of criterion is more understandable among many investors. Of course in this case it is necessary to cover the meaning of some used categories. Guarantee (fr. Guarantie – guarantee) is a probability of the event that investor's profit will exceed the given level. Isoguarantee is a line on possibilities – risk plane joining points with an equal guarantee.

What is the sense of isoguarantee in the statistical terms? In this case isoguarantee is a line on the possibilities – risk plane joining quintiles of equal level in one line. So, the term isoguarantees in investment adequate term in statistics is isoquintiles. But similar term – isoquant is already properly used in physics, economics etc.

If it is reasonable to realise that an entire set of profit possibilities exists for each level of risk, then one could find a quintile of every level of confidence. So coincidence of isoguarantees conception and isoquintiles conception in statistics is very useful for portfolio investigation.

1.4. Practical usage of isoguarantees in portfolio decision

Further let us consider the case from the 1 table. Firstly let's analyse possibilities of 1 EUR to be invested in 4 currencies: USD, CHF, GBP and RUR. Geometric body of possibilities of mean value – standard deviation portfolio is given in the picture 1a Efficiency line of these possibilities is illustrated in the picture 1b and optimal solution for the investor is illustrated in picture 1c (point S).

But an investor couldn't be finally satisfied only with information about possibilities mean value under different levels of risk couldn't be finally satisfied only with inform
tion about possibilities of mean value under different levels of risk if only evaluation of guarantee of the possibility's are not clear. That's why further discussion will be concentrated on all possibilities - risk portfolio model (see fig. 2). Fig. 2a provides a sight on the "bunch of all quartiles" portfolio set efficiency lines (fig. 2b) of which indicate efficiency zone almost entirely, because zero and fourth quartiles are replaced by 0.005 and 0.995 quintiles correspondingly. Now, once again ought to be remembered that these quartiles - standard deviation portfolios efficiency lines also are the isoguarantees for investment portfolio of respective level of confidence.

What kind of role could perform isoguarantees for the investor in the portfolio decision-making? For his aspiration - to join the points situated on possibilities - risk plane with the same guarantee level - isoguarantee could be as a constructive and evident criterion for portfolio selection. If entirely the whole isoguarantee probability that the meaning of the portfolio would be less than the changeable (increasing) meaning of the portfolio the investor should select the highest profitability. In this case the right points of isoguarantees should be selected portfolios of which are described by such values: \((w_1 = , w_2 = , w_3 = , w_4 = )\). It is evident that higher level of profit with equal level of confidence not certainly express higher utility. Therefore more general cases should be analysed.

1.5. The selected case study

In this chapter more adequate then earlier portfolio management case will be discussed and imitative technologies will be usual more rationally for the solution of declared problems. The bank seeks for the stabilization of its activity and suggests for the investors not to change very drastically of their currency portfolio. The will preserve its privileges if changes per year in amount of each currently will not exceed 33 percent of the amount. For the manager of the portfolio which now consist at equal amounts of 1 mln. units of all cur-
rencies now it is necessary to define optimal solution according to financial restrictions given in table 1 as well as the requirement at the bank. The results of the solution are illustrated in fig 3.

The purpose of this paragraph is to analyse all possibilities usable by the portfolio idea with the aspiration to rank them by different level of profitability (abscise) with selected levels of risk (ordinate) and confidence (coordinate). Such arrangement of information would be helpful for selection and even could help to develop the utility concept. The information arranged in such a way would be enough not only for the selection of the solution according to sufficient complicated utility functions but also could serve for their improvement. By the way for the investors, especially non-professionals, more advantage information is indicated in the aggregate survival function (survival surface – 3b fig.), which is determinately obtained from the distribution (density) functions, however has logically well-arranged definition of investment portfolio guarantee system. The value on the possibilities – risk plane are the same here and in the fig. 3a., but on the coordinate we have the guarantees (possibilities) that the value of the portfolio would be lower than the possibility level on the coordinate under the corresponding levels of risk (abscissa). There can clearly be seen the crossing between uniform possibility degree and the survival function surface and, in this turn presumed and found how isoguarantees look like.

**Figure 3.** All possibilities – risk portfolio: 3a – density functions; 3b – survival functions; 3c – isoguarantees; 3d - numerical table of sales PSB probability distributions
1.6. Imitative technologies as instrument of financial modelling and decision-making.

Imitation is the act or an instance of imitating or being imitated. Imitative technologies means the connection of analytical sense and computer power for description and investigation a situation, when pure analytical methods hardly could be exploited. Analytical form of the isoguarantees it is the case when solution of the problem without means of imitative technologies is practically ineffective.

2. Integrated assets and liabilities portfolio formation and management

2.1. Introduction to the problem of integral asset and liability management

This part of the article discusses the conception and techniques of integrated assets and liabilities portfolio management (IALPM) development problems as intersection of problems arising in development of new management perspective – integrated asset-liability management (IALM) perspective and in development of integrated assets and liabilities portfolio (IALP) or investment portfolio adequate for stochasticity of investment profit possibilities.

The predominant organization management perspective that in manufacturing and trading entities was titled as systemic and as functional in finance called for an organization to be structured into line of functional units the decisions (management) of which are coordinated by a corporate plan based on a forecasts of macroeconomics environmental and individual indicators [Holmer M., 2001].

It is very important to emphasize that the forecasts were not treated as sets of possibilities i.e. possible outcomes with its probabilities and moreover behavioural decisions were not oriented to reliability management [Spronk J., Rutkauskas A. V., 2002].

In the year 1970 John Galbraight named the last of XX century as an age of risk and uncertainty. The beginning of the XXI century also corresponds with this denomination. Among financial intermediaries this perspective was nominated as integrated asset-liability management (IALM) perspective. This perspective call for an organization to be structured into integrated units that include all the functional activity related to a line of business and call for business units to make decisions using risk-adjusted or hedged profitability. IALM is based on computerised decision models “that represent both the assets and liabilities associated with the business line, characterize the uncertainty of the future environment, and produce strategies for structuring the assets and liabilities of business line in ways that are profitable across a range of alternative future environments” [Holmer M. R., 2001]. Because of complex volatility of future there is no alternative to IALM for financial intermediaries.

2.2. IALM perspective: implementation results, challenges and problems

The analysis of development of IALM perspective would testify the premises that, first, an evolution of management perspectives is an innovative management response to business problems and, second, the individual success of new management conception must be supported by an adequate technique. Really very often new management perspective evolves through the piecemeal implementation of new management techniques introduced to solve
concrete problems arising under the older management perspective.

The subject of the process to investigate in the paper will be financial intermediaries as well as personal finance where IALM already has gained its right to be used conceptually and practically. Often is supposed that management for financial intermediary is nothing more than definition of correct structure of assets and liabilities. It seems to be the truth if one could define this structure under stochastical behaviour of main assets' and liabilities' properties.

Financial intermediaries sell their liabilities, which become assets for savers or other intermediaries. A liability's scheduled cash flow ought be seen as contingent in the sense that it depends on the occurrence of certain future events. Liabilities with contingent cash flows are inherently risky and buyer will pay for an intermediary's liability taking into account risk of the cash flows.

Financial intermediaries usually use the proceeds of the liability's sale plus equity capital to buy assets, which are the liabilities of investors or other intermediaries. The cash flow of these assets are contingent and are used to pay the liability's scheduled cash flow. Any asset cash flow remaining after the payment of the liability's cash flow is profit for the financial intermediary. Since most financial intermediaries issue liabilities with contingent cash flow schedules the future profitability of the intermediary is quite uncertain. That's why the basic management objective for intermediary is formulated as "to sell liabilities and to buy assets in a way that the net cash flow or profit is both substantial relative to equity and consistent across the range of contingent events that effect future asset and liability cash flow [Holmer M. R., 2001].

Thus stochasticity is a characteristic feature for intermediary's profitability as well as for the income of assets and for expenditure of liabilities. However, the nature of the stochasticity is different in both cases. And indeed, if assumption the type of variability of assets profitability immanent is the tendency: the higher the expected value of profit the higher is the variance [Kouwenberg R., 2001; Rutkauskas A. V., 2000].

For the tendency of liability's expenditure (negative cash flow), the concept that higher guaranteed loan require higher expenditure and vice versa seems could be correct. Then the differences in the tendency of relations between cash flow variance (s) and expectation (EXP) of assets and liabilities are presented in fig. 4.

Regardless of conceptual sophisticate about the character of the cash flow i.e. about stochasticity of its nature positive cash flow from assets and negative – for expenses on liabilities altogether must comply with some request:

1. The cash flow from the assets must be sufficient to pay the scheduled cash flow

\[ EXP \]

![Picture 4. The differences in the tendency of relation between cash flow variance (s) and expected value (EXP) for assets (semi plane A) and liabilities (semi plane L.)](image-url)
and taxes and difference between positive cash flow and negative cash flow and taxes must be sufficient for expected profit, also is treated as stochastical;
✓ Positive and negative cash flow must be interbalanced in every time interval.

Taking into account the above mentioned requirements and keeping in mind the stochastical nature of the cash flow it becomes obvious how complicated IALM techniques are needed. At that time every one can understand the importance of integrated assets and liabilities portfolio (IALP) to develop IALM for realization strategic goals: intermediary value maximization at the chosen time or other purpose. Certainly it is necessary to observe that presumption about stochasticity of cash flows bear the analogous presumption about stochasticity of intermediary's value. In this case the adequate concept and technique are needed for value management.

2.3. The Main Equations and Constrains of the IALP

Integrated asset and liability management system (IALMS) has its development and implementation for large financial institutions: commercial banks [Giokas D., 1991; Oguzcyö C. B., 1997], insurance companies [Carino D. R., 2001; Dert c., 2001; Merton R. C., 2001], etc. as well as for individual finance [Berger A. J., 2001].

This paragraph describes IALP as one chain of IALMS when it is used for strategic planning. We present the core of IALP’s principle models system that could be used for financial management of intermediaries as well as for management of personal finance. The IALP helps to reinforce IALMS as strategic decision-making system. The main decision points over which IALP is integrated into IALMS are: where and how much to invest where and how much to borrow how to use leverage, how to maximize corporation or individual wealth at each stage, how to make adequate decision under the risk and uncertainty.

On the other hand, IALP is used for financial forecasting system (FFS) when the main financial statements: balance sheet, income statement, cash flow statement etc. are being generated [Rutkauskas A. V., 2000]. Thus IALP could be helpful to coordinate IALM and FFS. Because IALP techniques are based entirely on stochastic modelling the principles of integrated risk management (IRM) could be implemented into IALMS. So IALP could be helpful to overcome uncertainty and complexity of many financial management problems.

Picture 5 represents the mechanic IALP and FFS, IRM and IALMS operation model, where IALP works as a bearing, which puts together forecasting and planning, risk and uncertainty, assets and liabilities and guaranties dynamics of financial subject.
The distinctiveness of IALP is its conformity with entirely stochastic system i.e. the system where the set of possibilities and the guaranties of these possibilities are regarded. In order to understand easier and for practice of implementation IALMS, FFS and IRM as dynamic instruments are used in the discrete form. So the IALP is also presented in discrete form as mathematical model where the time period T also is divided by time moments: \( t_0, t_1, \ldots, t_n \) into static intervals or stages \( [t_0, t_1), [t_1, t_2), \ldots, [t_{n-1}, t_n] \).

For convenience of exposition we suppose that the stage coincide with one-year period. The IALP switches on at start of each forecasting (planning) stage rendering changes to the asset and liability position, evaluating the results over the coming stage. Rebalancing assets and liabilities at times between reviewed points is not allowed. The mechanism of rebalancing depends on planning strategy is that stage-by-stage strategy or we have an integrated by stages (over time) strategy. In the case we use the stage-by-stage strategy. This situation simplifies exposition of portfolio techniques.

In the case we will present IALP as decision instrument on separate of all time period \( T \) though the behaviour of the managed system reacts on the issue of this decision and, vice versa, the objectives of the system cause the objectives and constraints of decision on separate stage. Different behaviour of the system means different changes in balance sheet, different incomes, and cash flow etc. statement results.

In our turn we will present only core changes that happen throughout one stage as static ring of all chain: changes in wealth, structure of asset and liability as well as changes in microenvironment (price, risk etc.).

We will define the relevant sets, accounting and decision variables, inputs, identities and governing equations for the IALP. However, before doing so we should note that for analytical convenience change in amount of every asset and liability will be treated as consisting of two non-intersecting components. First component appears as a result of rebalancing of already existing amount of assets and liabilities. The total amount of asset and liability doesn't change in this component. The second component is an increasement in every kind of asset and liability as a result of increase of total amount. Consequently those components are called: first – changes as result of rebalancing, second – newly introduced growth.

Let us define the following sets:

- \( t_1, t_2, \ldots, t_n \) – discrete times at which the IALP will be rebalanced.
- \( T \) – time horizon (period) consisting of \( n \) intervals, \( [t_0, t_1), [t_1, t_2), \ldots, [t_{n-1}, t_n] \), \( t_k = T \).
- \( i \) – asset categories, \( i = 1, 2, \ldots, n \).
- \( j \) – liability categories, \( j = 1, 2, \ldots, m \).
- \( a_i \) – amount of \( i \) asset in money at time \( t \).
- \( l_j \) – amount of \( j \) liability at time \( t \).

Define the following decision and accounting variables:

- \( I_i^{\text{th}} \) – growth index of \( i \) asset in result of rebalancing at the moment \( t \).
- \( I_i^{\text{th}} = (1 + \lambda_i^a) \), where \( \lambda_i^a \) – change rate of \( i \) asset at the moment \( t \).
- \( I_j^{\text{th}} \) – growth index of \( j \) liability in result of rebalancing at the moment \( t \).
\[ I_{i,t} = (1 + \lambda_{i,t}^j), \quad \text{where } \lambda_{i,t}^j - \text{change rate of } j \text{ liability at the moment } t. \]

\[ \lambda_{i,a}^a - \text{fraction of assets newly invested in asset category } i \text{ at time } t_0 \text{ or at the beginning of interval } (t, t+1); \sum \lambda_{i,a} = 1, \lambda_{i,a} \geq 0. \]

\[ \overline{\lambda}_{i,a}^a - \text{fraction of amount of } \text{"old" assets in newly invested } i \text{ asset.} \]

\[ \overline{\lambda}_{i,a}^a = \lambda_{i,0}^a \times \lambda_{i,a}, \quad \text{where } l_n - \text{growth rate of asset at time } t. \]

\[ a_i = \text{amount of } i \text{ assets after rebalancing and introducing new at moment } t. \]
\[ a_i = a_i (1 + \lambda_{i,t}^j + \lambda_{i,a}) \]

\[ \lambda_{i,t}^j - \text{fraction of newly borrowed liabilities in liability category } j \text{ at time } t_0 \text{ or at the beginning of interval } (t, t+1); \]

\[ \overline{\lambda}_{i,t}^j - \text{fraction of amount of } \text{"old" liabilities in newly borrowed } j \text{ liability.} \]

\[ \overline{\lambda}_{i,t}^j = \lambda_{i,t}^j \times \lambda_{i,a}, \quad \text{where } l_{t+1} - \text{growth rate liability at moment } t. \]

\[ P_{t+1}^i - \text{index of } i \text{ asset's price change in year } (t+1); \]

\[ a_{i,t+1}^i = \text{amount of } i \text{ asset in a result } t+1 \text{ of changes at moment } t \text{ and price change } \]
\[ a_{i,t+1}^i = a_i P_{t+1}^i. \]

\[ \Delta_{i,t+1}^a - \text{change in amount of } i \text{ asset in the year } (t+1) \text{ because of price and increase in amount at moment } t. \]
\[ \Delta_{i,t+1}^a = a_{i,t+1}^i - a_i. \]

\[ a_{i,t+1}^i = a_i (1 + l_{i,t+1} + \lambda_{i,t+1}^a) \times P_{t+1}^i. \]

\[ a_{i,t+1}^i = a_i (1 + l_{i,t+1} + \lambda_{i,t+1}^a) \times (1 + \Delta P_{t+1}^i), \quad \text{where } \]
\[ \Delta P_{t+1}^i - \text{price rate of } i \text{ asset.} \]

\[ \Delta_{t+1,w}^i - \text{changes in wealth generated by } i \text{ asset. It consist of changes in amount of asset } i \text{ because of price changes by adding interest earned by asset } i \text{ and substracting some transaction cost if some amount asset was sold.} \]

\[ \Delta_{t+1,w}^i = a_i (1 + l_{i,t+1} + \lambda_{i,t+1}^a) \times (\Delta P_{t+1}^i + + e_{t+1}^i) - C_{t+1}^i, \quad \text{where } \]
\[ \Delta P_{t+1}^i - \text{price change rate on } i \text{ asset at year } (t+1); \]
\[ e_{t+1}^i - \text{interest rate on } i \text{ asset at year } (t+1); \]
\[ C_{t+1}^i - \text{transaction cost of } i \text{ asset sold at time } t. \]
\[ C_{t+1}^i = \begin{cases} a_i \left( \lambda_{t+1}^i + \lambda_{i,0}^a \right) & \text{if } \lambda_{t+1}^i + \lambda_{i,0}^a < 0, \\ 0 & \text{if } \lambda_{t+1}^i + \lambda_{i,0}^a \geq 0 \end{cases} \]

where

\[ C_{t}^i - \text{transaction cost rate per unit of asset sold;} \]

\[ l_{t+1}^j = \text{amount of } j \text{ liability at time } t+1; \]

\[ l_{t+1}^j = l_j (1 + l_{t+1}^j + \lambda_{t+1}^j). \]

\[ \Delta_{t+1}^j - \text{amount of expenditure on } j \text{ liability in } (t+1) \text{ year. These expenditures consist of debt service on } j \text{ liability in } (t+1) \text{ year plus transaction cost } C_{t+1}^j \text{ of } j \text{ liabilities sold (changed) at the moment.} \]
\[ \Delta_{t+1}^j = l_j^i (1 + l_{t+1}^j + \lambda_{t+1}^j), \quad \text{O}_{t+1}^j + C_{t+1}^j. \]
C_t^j = \begin{cases} l_t (\lambda_t^j + \lambda_{t,n}^j), & \text{if } \lambda_t^j + \lambda_{t,n}^j < 0, \\ 0, & \text{if } \lambda_t^j + \lambda_{t,n}^j \geq 0, \end{cases}

where

O_t^{\text{eb}} - payment rate per unit of liability j,

C_t^{\text{eb}} - transaction rate per unit of j liability sold at t.

\Delta_{t+1}^{\text{net}} - increase in total net amount of wealth in year (t+1),

\Delta_{t+1}^{\text{net}} = \left( \sum_i \Delta_i^{a_i} - \sum_j \Delta_j^{b_j} \right).

Now let's define inputs and identities of the IALP:

d_{i,t+1} - probability distribution function of i asset price index at the moment (t+1).

d_{i,t+1} - probability distribution function of interest rate on asset i in year (t+1).

d_{t+1}^{b_j} - probability distribution function of payment rate on j liability in year (t+1).

l_{t+1} - growth rate of assets and liabilities in year (t+1).

\sum_i a_i = \sum_j l_i - the main identities the beginning and at the end of each time interval.

Management mechanisms of IALP:

U_{t+1}^j \left( \Delta_{t+1}^{\text{net}} \right) \{S\} - utility function defining subject utility from amount of net wealth generated by exploitation of all assets and liabilities.

\{S\} - symbolize stochasticity of \Delta_{t+1}^{\text{net}}.

Strategy:

\max_{A_{t+1}^{\text{eb}}} \left( \sum_i \Delta_i^{a_i,\text{w}} - \sum_j \Delta_j^{b_j} \right) \Rightarrow \text{optimum}.

Optimum means maximum utility from chosen possibility taking into account guarantee of this possibility.

2.4. Some remarks on IALP criteria adequacy to subject IALM strategy and possibilities of numerical solution

It is difficult to deny that one couldn't maximise the utility of any development strategy throughout chosen time horizon T without knowing how to maximise utility of changes on each time interval (stage).

Utility optimisation on each stage was understood as maximisation of net wealth growth taking into account riskness (volatility) of the growth. By the way changes in the amount of every asset can include three components:

- Changes in amount of assets because of rebalance between assets;
- Growth because of new acquisition of the assets;
- Changes in the asset value because of price changes.

The same scheme is used for liabilities.

One of paradoxes of stochastic approach to management is that manager or investigator has to realise that forecast is a wide spectrum of possibilities and reality will occur in single realization anyway. As consequence you have some losses because of expectation differs from occurrence or happening. So criteria for decisions ought to react to this objectivity fact seeking to minimise losses because of this non-coincidence. That's why understandings of expectation and guarantee are crucially important in decision-making and management under the risk.
Further, if \((i+1)\) year (or \(t_{i+1}\)) interval is the year of the investigation then IALP strategy is identical with modern investment portfolio and adequate portfolio maximisation problem. That is why the same technique as for mentioned portfolios could be used for analytical and numerical solution of the strategy. Let us demonstrate this concept and techniques on the numerical case.

Informative supply. Financial management whether it is a management of assets or liabilities, or risk management, in some sense it is the management of statistical relations between different variables of the system, or it is management regarding these relations and changing them in the needed way. Actually, the analysis of the system and numerical solution becomes more difficult when the statistical relations between different variables are complicated. We have to admit, that in most of the cases, the amount of information and historical data are not sufficient enough to evaluate all existing statistical relations in the system adequately.

That is why at the given situation we will cover only the analysis of the main statistical relations between exogenous variables. First, we will consider the existing statistical relations between interest rates earned by different categories of wealth and payments made at different liabilities, norms, which can be shown as correlation matrix \(C(A, L)\):

\[
C(A, L) = \begin{pmatrix}
C_{i_{a_{1}}, \ldots, i_{a_{s}}} & \ldots & C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}} & \ldots & C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}, i_{a_{1}}} \\
C_{i_{a_{1}}, \ldots, i_{a_{s}}} & \ldots & C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}} & \ldots & C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}, i_{a_{1}}} \\
C_{i_{a_{1}}, \ldots, i_{a_{s}}} & \ldots & C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}} & \ldots & C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}, i_{a_{1}}} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
C_{i_{a_{1}}, \ldots, i_{a_{s}}} & \ldots & C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}} & \ldots & C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}, i_{a_{1}}} 
\end{pmatrix}
\]

\(C_{i_{a_{1}}, \ldots, i_{a_{s}}}, \ldots, C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}}, \ldots, C_{i_{a_{1}}, \ldots, i_{a_{s}}, i_{a_{1}}, i_{a_{1}}} \) - is the correlation coefficient in year \(t\) between \(a\)-s category of wealth interest rate and payments according \(j\)-s category of liabilities norms.

We will also consider the existing statistical relations between price increase indexes of different wealth categories in year \((t+1)\), that can be shown as correlation matrix \(C(P)\):

\[
C(P) = \begin{pmatrix}
C_{p^{i_{1}}p^{j_{2}}} & C_{p^{i_{1}}p^{j_{3}}} \\
C_{p^{i_{1}}p^{j_{2}}} & C_{p^{i_{1}}p^{j_{3}}}
\end{pmatrix}
\]

2.5. The case

Suppose that at time \(t\) the subject possess 100 MEUR invested in four groups of assets as following: 40 MEUR in “1” asset, 20 MEUR in “2” asset, 30 MEUR in “3” asset and 10 MEUR in “4” asset. On the liability side there is following distribution: on “1” liability – 15 MEUR, on “2” liability – 30 MEUR, on “3” liability – 10 MEUR and on “4” liability – 45 MEUR. Equity capital is among declared liabilities.

Macroeconomic and marketing analysis let us make an assumption that the distribution of price increase possibilities of different assets categories will be such:

\(d_{i_{a_{1}}}^{p_{1}} - N (1,2; 0,05), \ d_{i_{a_{1}}}^{p_{2}} - N (1,3; 0,1), \ d_{i_{a_{1}}}^{p_{3}} - N (0,9; 0,04), \ d_{i_{a_{1}}}^{p_{4}} - N (1,0; 0,05)\).

The distribution of probability of interest rate possibilities in year \((t+1)\) would be:

\(d_{i_{a_{1}}}^{i_{1}} - N (0,05; 0,01), \ d_{i_{a_{1}}}^{i_{2}} - N (0,1; 0,02), \ d_{i_{a_{1}}}^{i_{3}} - N (0,1; 0,03), \ d_{i_{a_{1}}}^{i_{4}} - N (0,14; 0,03)\).

Adequately, the distribution of probability of liabilities payments norm would be:

\(d_{i_{a_{1}}}^{i_{1}} - N (-0,06; 0,01), \ d_{i_{a_{1}}}^{i_{2}} - N (-0,07; 0,01), \ d_{i_{a_{1}}}^{i_{3}} - N (-0,14; 0,01), \ d_{i_{a_{1}}}^{i_{4}} - N (-0,1; 0,02)\).
At that time the statistical relations between different variables would look like this:

$$
C(A; L) = \begin{bmatrix}
1 & 0.1 & -0.2 & 0 & 0.3 & 0.18 & 0 & 0 \\
1 & 0.15 & 0.1 & 0.25 & 0.12 & 0 & 0 \\
1 & 0 & 0 & 0.25 & 0 & 0 & 0 \\
1 & 0 & 0.2 & 0.1 & 0.3 & 0 & 0 \\
1 & -0.2 & 0 & 0 & 0 & 0.1 & 0 \\
1 & 0 & 0 & 1 & 0.05 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix},
$$

$C(P) = \begin{bmatrix}
1 & 0.1 & -0.2 & 0 \\
1 & -0.32 & 0.1 & 0 \\
1 & 0 & 0 & 1
\end{bmatrix}.$

Strategic goals for the subject on given stage are: amount existing asset to 110 MEUR by taking new loans on already existing liabilities and parallel rebalance existing structure of assets and liabilities in order to optimise the growth of net wealth in time period $(t, t+1)$.

Analytically the problem could be formulated in such a manner: define fractions of newly invested assets in $i$ category of asset $\alpha_{i,n}^A$ and growth index of $i$ asset in result of rebalancing existing up to the time $t$ assets – $I_{t+1}^A$ and fractions of newly borrowed liabilities in liability $j$ category $\alpha_{j,n}^L$ and growth index of $j$ liability in result of rebalancing existing up to the time liabilities – $I_{t+1}^L$ ($i = 1, 2, 3, 4; j = 1, 2, 3, 4$) in order to optimise subject utility from growth net wealth.

Of remember that interest rate on $I$ asset – $\epsilon_{t+1}^A$, payment rate per unit of liability $j - \alpha_{t+1}^L$ are stochastical variables and growth indexes of $i$ asset – $I_{t+1}^A$, and $j$ liability – $I_{t+1}^L$ in result of rebalancing existing up to time $I$ assets and liabilities are dependent on stochastical variables and event then the problem of optimisation is evidently the identical case of IALP and could be solved by the same techniques as classical (modern) investment portfolio or as adequate to the stochasticity of assets and liabilities portfolio [Rutkauskas A.V., 2000].

2.6. Interpretation of obtained results

Using imitation technologies [Rutkauskas A.V., 2000] we can define wealth increase possibilities of year $(t + 1)$ shown in graphic pictures.

Picture 6a shows all net assets increase possibilities of all categories of wealth, which are obtained while using the principle of portfolio formation. If we consider that Markowitz average-standard deviation portfolio is a standard, then we have the set of the analogues of quintiles: 0,001; 0,02; 0,04; ..., 0,98; 0,999 standard deviation portfolios (the conception of analogue will be explained better in the last paragraph). Picture 6b shows the effective lines of analogues of these portfolios that serve as isoguarantees in this case.

We can see isoguarantee of minimum (0,001), isoguarantee of maximum (0,995) and isoguarantees of all deciles in Picture 6b. Note that values of isoguarantees are almost entirely increasing as the dispersion increases so we have some remarks to make.

First, isoguarantee gives us information that with your chosen guaranty the net assets increase will be no smaller than the value of isoguarantee in a given level of risk, if you are not choosing from the set of possibilities, but from the effective lines only.

Second, to better understand net assets increase possibilities we have to use the spherical picture of survival functions family (6c picture), where we can find information about net assets possibilities, which are cho-
sen, below the level of isoguarantee. This can explain why the higher level of net assets, when guaranty is the same, not necessary means higher expected utility. Despite that, while trying to find the best structure of assets and liabilities, there were limited variation possibili-
ties for the variables causing them, because in other way we should also change $c_i$ coefficients parallely.

It is more difficult to interpret payments bound with expected possibility of liabilities service. Picture 7a shows portfolio analogues

![Picture 6. Possibility sets of asset portfolio analogues. (6a) - portfolio analogues of standard deviation 0.005; 0.25; 0.5; 0.75 and 0.995 levels of quintiles; (6b) - isoguarantees of corresponding level of portfolio analogue; (6c) - survival functions family of portfolio analogues](image)

![Picture 7a. Possibility sets of liabilities portfolio analogues. (7a) - portfolio analogues of standard deviation 0.005; 0.25; 0.5; 0.75 and 0.995 levels of quintiles; (7b) - isoguarantees of corresponding level of portfolio analogue; (7c) - survival functions family of portfolio analogues](image)
of payments possibilities for different levels of analogues. There is no doubt that analysts will be interested only in the points lying on the edge points of cost possible values. In case of Markowitz portfolio, this will be the envelop-

\[ \text{Possibility sets of asset and liability portfolio analogues. (8a) - portfolio analogues of standard deviation 0.005; 0.25; 0.5; 0.75 and 0.995 levels of quintiles; (8b) - isoguarantees of corresponding level of portfolio analogue; (8c) - survival functions family of portfolio analogues} \]

ment curve (Picture 7b). Survival functions analogue is shown in Picture 7c. We will not discuss the characteristics of the given pictures in detail here, because analysts and managers are interested in net assets increasement possibilities.

Picture 8 gives information about net assets increasement possibilities in year \((t+1)\). Picture 8a shows analogues of adequate investment portfolio for 0.001; 0.02; 0.04; ..., 0.98 and 0.995 level of quintiles. Picture 8b shows the spherical view of survival functions family generated by analogues of these portfolios. It gives good and relevant information about net assets increasement possibilities. We have the levels of possible risk on abscissa axis, assets increasement possibilities on coordinate axis, and the levels of guaranties on Z-axis. It is always easy to find a point or set of points that maximize the utility of a subject if the utility function is known. And finally, picture 8c shows the projection of the effective zone in the abscissa-coordinate system that visually gives information about utility dynamics possibilities in changing risk levels.

2.7. Short comment on solution method

While solving the given problem, the idea of portfolio analogue has been used. In order to explain the essence of portfolio analogue we should remember the concept of investment portfolio: \(n\) investments \(a_i\) \((i = 1, 2, ..., n)\) portfolio is a set \(\{w_i, i = 1, n\}\) of any structural indicators \(w_i\) \((r = 1, 2, 0 \leq w_i \leq 1, \sum_{i=1}^{n} w_i = 1\). \(W = \sum_{i=1}^{n} w_i * a_i\) is called a value of portfolio. So we are analyzing the set of all possible portfolios and the set of all possible values of portfolios. Because \(a_i\) is random variable so \(w\) is
also a random variable. Markowitz or modern portfolio refers to analysis of interaction of average and standard deviation of these random measures and is based on using the characteristics of effective line EL. EL consists of points, obtained when choosing maximum from all possible portfolio values for each possible standard deviation value. Effective line is one of the best instruments for multicriteria analysis.

Not only average values are used in the case of adequate portfolio, but full distribution of portfolio values possibilities, or using the effective line all possible quintiles are being analyzed (e.g. quartiles, deciles, percentiles or their combinations). As a result, so-called effective zone is being formed instead of effective line (in case of Markowitz portfolio).

Portfolio analogue, solving ITIVS and other sophisticated problems, is a set \( \{x_i^p; i = 1, n\} \) of any ITIVS value of variables \( x_i (i = 1, 2, \ldots, n; a_i \leq x_i \leq b_i) \). The value of portfolio analogue is a value \( F \) of utility function, when \( x_i = x_i^p \):

\[
F = f(x_i^p, x_2^p, \ldots, x_n^p).
\]

In the case of adequate portfolio it is a function from the distribution of egzogenic variables possibilities. The example is survival functions family.

Having geometric view of survival functions family, i.e. the set of possible solutions, which is called restriction set in the mathematical forecasting problems, it becomes clear how to find a point or a set of points in the set of possible solutions when knowing the utility function (Pic. 8).

3. Conclusions

- Development of management theory and practices encounters two fully perceptible and mutually unobjectionable aspects with the intersection points very difficult to reveal. One of them is that the future of the process of many self-regulating and management objects cannot be defined determinately. The second is that in reality the development of the process will choose only one possibility. The adjusting process of these two aspects in portfolio management is burden by the fact that desirable states of portfolio results are defined by two one-aspected indicators: profitability and reliability. Consequently decision-making algorithms should encounter commensurable of these indicators.

- It is needed to consider every state of all kinds of quintile – risk portfolios for the creation of effective portfolio management algorithm. Its reliability should to be the inseparable characteristic of these states. Isoguarantees should serve for investor as easy understandable component of his (hers, its) decision-making criterion. Its capability to help for arrangement isoguaranteed states according level of another indicator is of incredible value in decision-making. Thought sense of isoguarantee used in portfolio decision coincides with the sense of term of isoquintile used in statistics and definition of its analytical expression usually is quite difficult and needs to use of imitative technologies.

- Integral asset and liability management becomes an independent perspective of financial process management, widely used in different financial institutions as well as in personal finances. Integral asset and liability portfolio, offered in the article, should be-
come compound element of asset and liability management perspective, helping to answer questions where and how much to borrow, where and how much to invest, and also helping to join forecasting and planning, and risk management systems together.

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IZOGARANTĖS KAIP PORTFELIO SPRENDIMŲ PAGRINDIMO PRIEMONĖS

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Santrauka

Šiame straipsnyje sudėtingoms finansinioms procesams ir juos sujungiančioms arba nuo jų priklausomoms reikšmėms valdyti siūloma pasitelkti izogarančių idėją, tuo pabrėžiant garantijos arba pasikliautinumo svarbą. Finansų rizikos tyrimai yra geras pagrindas kalbėti apie daugelio planuojamų finansinių indikatorių kiekybės ir rizikingumo subrendratinęms.

Straipsnyje vartojama keletas finansų literatūroje neįprastų terminų, kurie, deja, būtini nagrinėjant projekto rezultatų patikimumo pasirinktis metodais. Vienas iš jų terminų yra izogarantė, kuri suprantama kaip linija, jungianti vienodos garantijos efektyvumo (pelno, pelningumo, pajamų ir pan.) rodiklių ir šių rodiklių rizikingumą nusakančių rodiklių plokštumoje.

Remiantis Markowitzo arba moderniojo portfelio ir adekvataus pelno galimybės nevienarankščiškumui portfelio idėjomis ir technika siūlomas integralus turto ir įsipareigojimų portfelis, leidžiantis surašinti nagrinėjamo proceso ar subjekto plėtros galimybes pagal jų garantijų lygmenis. Nagrinėjami minčiai pailiuotai išnagrinėti du akademiniai pavyzdžiai: valiutų portfelio parengimas ir integruoto turto ir įsipareigojimų portfelio sudarymas ar valdymas. Dėl to straipsnis suskaidomas į dvi dalis.

Nors nagrinėjant situacijas nevengiama analitinio sprendimo ieškos metodų, tačiau pagrindine sudėtingų situacijų nagrinėjimo priemonė pasirinkta imitacinės technologijos, kurios suprantamos kaip kompiuterio skaičiuojamųjų galių ir imitacinio modeliavimo sąveiką išraiška. Gauti rezultatai paaiškinti geometrine forma. (Kartu reikia pripažinti, kad ne visuomet užteko kompiuterio resursų aproksimuoti tolydžiusios procesus vizualiai neprirkaištinais analogais.)

Ipač sudėtingoms situacijoms nagrinėti reikia dielių kompiuterinės galios ir greičių resursų, tačiau atsižvelgiant į tai, kad inovacijos šioje srityje stūkso poreikis, imitacinės technologijos reikia pripažinti perspektyviausiomis sudėtingų situacijų kiekvieno nagrinėjimo priemonėmis, leidžiantis sėkmingai spręsti visokius analitikai suvokiamus uždavinius.

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