Quintessential perturbations during scaling regime

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The scalar field with an exponential potential allows a scaling solution where the density of the field follows the density of the dominating fluid. Such a scaling regime is often used as an important ingredient in many models of quintessence. We analyse evolution of perturbations while the background follows the scaling. As the results, the perturbed scalar field also scales with the perturbed fluid, and the perturbations accompany the adiabatic as well as the isocurvature mode between the fluid and the field.

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Although the prime motivation of reducing the fine tuning problem has not been quite successful, we notice a important ingredient in many models of quintessence. We analyse evolution of perturbations while perturbed scalar field also scales with the perturbed fluid, and the perturbations accompany the adiabatic as well as the isocurvature mode between the fluid and the field.

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variables indicate the gauge-invariant notation. For our convention of the gauge-invariant combinations see §3 in [10].

Under our gauge condition eq. (A8) gives \( \alpha = -\frac{w}{1 + w} \delta r \), thus for a pressureless fluid \( \nu_f = 0 \) gauge implies \( \alpha = 0 \) which is the synchronous gauge condition; the opposite is not necessarily true. In a more realistic situation we should consider a system with a radiation \((r)\) a pressureless matter \((c)\) and a scalar field \((\phi)\). Many of the previous works prefered to use the synchronous gauge. Equation (A8) gives \((ac\nu) = k\alpha\), thus, \( \nu_c \equiv 0 \) implies the synchronous gauge condition; by further ignoring \( \nu_c \), which otherwise will give the gauge mode, the synchronous gauge becomes effectively the same as the \( \nu_c = 0 \) gauge. From eqs. (A7,A8) we can derive the set of equations for the three component system under the \( \nu_c = 0 \) gauge. In the radiation-less limit such equations will produce correctly \( w = 0 \) limit of eqs. (4) - this is the case for \( \delta_c \sim \delta_c \). Without the pressureless fluid, however, such three-component equations will not produce \( w = \frac{1}{3} \) limit of eqs. (4); the difference is due to the different gauge conditions used. Since the \( \nu_c = 0 \) gauge is based on the pressureless matter, without or negligible amount of the pressureless matter (as in the radiation dominated era, RDE) we could encounter situations where the gauge condition is not suitable for handling the problem; the situation is not incorrect as long as we have a tiny fraction of the pressureless matter in the RDE, but later we will notice a situation (in the literature) where it (the synchronous gauge) causes more troublesome analytic handling compared with analyses based on our present gauge condition.

**Scaling background:** It is known that with a potential \( V = V_0 e^{-k\phi} \) the scalar field approaches an attractor solution of the dominant component \( \tilde{\phi} \). We set \( 8\pi G = 1 \).

In a flat background without \( \Lambda \), if the energy density of dominant component \( (f) \) behaves as \( \mu_f \propto a^{-3(1+w)} \), eqs. (A4-A6) allow [37,21,30]:

\[
\mu_\phi = \frac{2}{1-w} \frac{V}{1+w}, \quad \Omega_\phi = \frac{3(1+w)}{\lambda^2}, \quad \delta \phi' = \frac{3(1+w) a'}{\lambda} \frac{\delta \phi}{a} = \frac{6(1+w)}{1+3w} \frac{1}{\lambda} \eta, \quad (4)
\]

and \( \Omega_f = 1 - \Omega_\phi \); a prime indicates a time derivative based on \( \eta \). With this we can show \( p/\mu = w = p_\phi/\mu_\phi \) as well. In the following we analyse the perturbations for the cases of a radiation dominated era, a matter dominated era, and an era with general \( w \).

**Case with \( w = \frac{1}{3} \):** When the dominant component is the radiation, eqs. (4) give

\[
\delta r' + \left[ \frac{k^2}{3} - \left( 1 - \frac{8}{3a} \right) \frac{2}{\eta^2} \right] \delta r = \frac{16}{3} \left( \frac{2}{\eta} \delta \phi' + \frac{1}{\eta^2} \delta \phi \right), \quad (5)
\]

\[
\delta \phi'' + \frac{2}{\eta} \delta \phi' + \left( k^2 + \frac{4}{\eta^2} \right) \delta \phi = \frac{2}{\lambda} \left( \frac{1}{\eta} \delta r' - \frac{1}{\eta^2} \delta r \right). \quad (6)
\]

In the large-scale limit, ignoring \( k^2 \) terms, we have solutions \( \delta_r \propto \delta \phi \propto \eta^q \) where

\[
q = 2, \quad -1, \quad \frac{1}{2} (1 \pm \sqrt{1 - 16\Omega_r}). \quad (7)
\]

These provide the general solutions for \( \delta_r \) and \( \delta \phi \). Notice that the first two solutions are the well known growing and decaying modes of \( \delta_r \) in the absence of the field [35]. For the growing solution \( \delta_r \propto \eta^q \) we have

\[
\delta_r = \frac{5}{2} \delta \phi = 5 \lambda \delta \phi. \quad (8)
\]

We can show \( \delta \equiv \delta \mu/\mu = \Omega, \delta \phi + \Omega \phi \delta \phi = (1 - \frac{3}{\lambda} \Omega \phi) \delta r \), and \( \delta_r = \delta + 3H \delta \phi/\mu \) where \( \delta_r \equiv \delta + 3(1+w)(aH/k)v \). Thus, for the growing solution we have

\[
\delta_r = \left( 1 - \frac{2}{5} \Omega \phi \right) \delta r, \quad S_r = \frac{9}{20} \delta r, \quad (9)
\]

where [38]

\[
S_{ij} \equiv \frac{\delta \mu_i}{\mu_i - \delta \mu_j} - \frac{\delta \mu_j}{\mu_j + \delta \mu_j}. \quad (10)
\]

The adiabatic and the isocurvature perturbations are often characterized by \( \delta_r \) and \( S_{ij} \), respectively. From eqs. (A12,A13) we have

\[
\frac{k^2 - 3K}{a^2} \varphi_\chi = \frac{1}{2} \mu \delta_r, \quad (11)
\]

where \( \varphi_\chi \equiv \varphi - H \chi \); in the notation of [31,39] we have \( \varphi_\chi = \Phi_H = -\Psi \). Thus, vanishing \( \delta_r \) implies vanishing \( \varphi_\chi \), thus isocurvature in the multi-component situation. Therefore, eq. (4) shows that the growing mode of perturbation during the scaling regime accompanies the adiabatic (\( \delta_r \)) as well as the isocurvature (\( S_{i\phi} \)) modes; this is in contrast with the assumption made in [23], and the result in [34].

* The work in [31] was made in the zero-shear gauge which fixes the scalar-type shear of the normal hypersurface \( \chi \) equal to zero [38,40]. \( \delta_r \) in the zero-shear gauge is the same as \( \delta_f = \delta_f + 3(1+w)H \chi \). In our gauge condition we can show that \( H \chi = \chi \) is of the order \( \delta_f/(k\eta)^2 \). Thus, in the zero-shear gauge, the shear (thus metric) part dominates the density fluctuation in the large-scale limit. Although the authors of [31] concluded that they found no isocurvature mode during scaling, what they actually have shown was that to the order of perturbed potential \( \varphi_\chi \) the isocurvature mode vanishes. What we have shown is that we have an accompanied isocurvature mode which is of the order of density perturbation \( \delta_r = \frac{1}{2} (aH/k)^2 \varphi_\chi \). In order to derive our result in the zero-shear gauge one has to go to higher order terms in the large-scale expansion.
Results in [21] (see also [30,24]) also differ from ours in eqs. (13)–(14). We do not expect the results in eqs. (13)–(14) to be the same because the gauges used are different. The growing mode happens to coincide with ours in eq. (13), but notice that the relation between the growing solutions in eq. (8) differs; compare with results below eq. (44) in [21] and eq. (29) in [24]. However, since \( \delta_v \) and \( S_{\delta \phi} \) are gauge-invariant the relation between them

\[
\delta_v = \frac{20}{9} \left(1 - \frac{2}{5} \Omega_\phi \right) S_{\delta \phi},
\]

should be the same. We can derive the same solution using the fluid formulation of the perturbed field system; i.e., using eqs. (A7,A8,A10) for the field, see below eq. (44) in [21]. However, if we simply follow the analyses in [21,20,28] it is not easy to get our results. The works in [21,20,28] are based on the synchronous gauge which is effectively the comoving gauge based on the pressureless matter. Since the pressureless matter is subdominating in RDE we anticipate that the gauge condition is not the best one to suit the situation. Equation (8) is not presented previously, and somehow results in eqs. (A7,A8,A10) differ from ours in eq. (11) which are in errors.

In the small-scale limit we have two solutions \( \delta_v \propto \eta \) where \( q = \frac{1}{2} (-1 \pm \sqrt{1 + 2 \Omega_\phi} \), which lead to \( q = 2, -3 \) for vanishing \( \Omega_\phi \), [21]. This is the well known perturbation growth rate in the presence of unclustered component of matter derived in [21,34]. We can show that additional two solutions decay faster.

Case with general \( w \): Now, we consider the general situation with constant \( w \). In the large-scale limit we have solutions \( \delta_f \propto \delta \phi \propto \eta \) with

\[
q = 2, \quad \frac{3}{2} \left( -1 + \sqrt{1 - 8 \Omega_c} \right).
\]

(13)

The first two solutions are the well known growing and decaying modes of \( \delta_v \) in the absence of the field [21,24]. For the dominant solution \( \delta_c \propto \eta^2 \) we have

\[
\delta_c = -14 \delta \phi = \frac{7}{3} \lambda \delta \phi.
\]

(14)

We can show \( \delta = (1 - \frac{9}{14} \Omega_\phi) \delta_c \), thus,

\[
\delta_v = \left(1 - \frac{9}{14} \Omega_\phi \right) \delta_c, \quad S_{\delta \phi} = \frac{15}{14} \delta_c.
\]

(15)

These are the adiabatic and isocurvature fractions accompanied by the growing mode. For \( w = 0 \) our gauge condition coincides with the synchronous gauge. However, eq. (13) was not presented previously, and somehow results in eqs. (13,14) differ from the ones in [21]; compare with results below eq. (45) in [21] which are in errors.

In the large-scale limit we have two solutions \( \delta_c \propto \eta^q \)

\[
\quad \delta_c = \left(1 - \frac{9}{14} \Omega_\phi \right) \delta_c, \quad S_{\delta \phi} = \frac{15}{14} \delta_c.
\]

(15)

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These give the general solutions for \( \delta_f \) and \( \delta \phi \). The first two solutions are the well known growing and decaying modes of \( \delta_f \) in the absence of the field; \( \delta_f \propto a^{1+3w} \) and \( a^{-\frac{1}{2}(1-w)} \) [35,37]. For the dominant solution \( \delta_f \propto \eta \) we have

\[
\delta_f = \frac{2(7 + 9w)}{1 - 24w - 9w^2} \delta \phi = \frac{7 + 9w}{3(1 - w)} \lambda \delta \phi.
\]

(17)

Following the similar calculations as before we have

\[
\delta_v = \left[ 1 - \frac{9(1 - w^2)}{2(7 + 9w)} \Omega_\phi \right] \delta_f,
\]

\[
S_{\delta \phi} = \frac{3(1 - w)(5 + 3w)}{2(1 + w)(7 + 9w)} \delta_f,
\]

(18)

which show the adiabatic and isocurvature modes respectively accompanied by the growing mode of perturbation during the scaling regime.

It is convenient to properly normalize the coefficient with the well known conserved quantity in the large-scale (super-sound-horizon scale) limit: \( \varphi_v = \varphi - (aH/k)v \). For the growing solution we have \( \varphi_v = \frac{5 + 3w}{3 + 3w} \varphi_X \). Thus, using eq. (11) we have

\[
\varphi_v = \frac{2(5 + 3w)}{(1 + w)(1 + 3w)^2} \left[ 1 - \frac{9(1 - w^2)}{2(7 + 9w)} \Omega_\phi \right] \frac{1}{(k\eta)^2} \delta_f.
\]

(19)

In the large-scale limit we have \( \varphi_v = C(\chi) \) which will fix the normalizations of \( \delta_f \) and others in terms of \( C \).
Summary: We have considered a system of a field and a fluid with constant $\omega$. The system includes limits of a three component system ($r, c, \phi$) when either $r$ or $c$ is dominating over the other. We find that while the field scales with the fluid and the perturbed field also scales with the perturbed fluid, eqs. (A1-A7). The ordinary growing and decaying solutions of the fluid without the field remain valid for the coupled system, eqs. (A8-A13). The main results of our work are eqs. (A14-A15) which show that during the scaling the perturbations accompany both the adiabatic and the isocurvature modes. Considering the fact that a perturbed field accompanies nonvanishing entropic perturbation $\phi$, $v_\phi \equiv \delta p_\phi - c_s^2 \delta \mu_\phi$, it may not be a surprise to see the accompanied isocurvature mode in the multi-component system with the field.

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EQUATIONS OF FLUID-FIELD SYSTEM

We consider a system of a multiple ideal fluids and a field without direct mutual interactions among components; see [46] for more general situations. Our convention of the metric and the energy-momentum tensor is:

$$ds^2 = -a^2 (1 + 2\alpha) dt^2 - 2a^2 \beta_{\alpha\beta} d\eta d\xi^{\alpha},$$

$$+ a^2 \left[ g^{(3)}_{\beta\gamma} (1 + 2\varphi) + 2\gamma_{\alpha\beta} \right] dx^\alpha dx^\beta, \quad (A1)$$

$$T^0_0 = - (\bar{\mu} + \mu_\delta), \quad T^0_{\alpha} = - \frac{1}{k} (\mu + p_\delta) v_\alpha, \quad (A2)$$

$$T^\beta_\beta = (\bar{\rho} + \rho_\delta) \delta^\beta_\beta + \left( \frac{1}{k^2} \nabla_\beta \nabla_\beta + \frac{1}{3} \delta^\beta_\beta \right) \pi^{(s)}. \quad (A3)$$

$\beta$ and $\gamma$ always appear together as $\chi = a(\beta + a\dot{\gamma})$ which is spatially gauge-invariant. An overdot denotes a time derivative based on $t$ with $dt \equiv ad\eta$. $\bar{\mu} = \sum_i \mu_i$, $\delta \mu = \sum_i \delta \mu_i$, and similarly for $\bar{\rho}$, $\delta \rho$, $(\mu + p_\delta) v_\delta$ and $\pi^{(s)}$. The sum includes the field contribution which can be expressed in terms of fluid variables:

$$\mu_\phi = \frac{1}{2} \dot{\phi}^2 + V, \quad \delta \mu_\phi = \dot{\phi} \delta \dot{\phi} - \dot{\phi}^2 \alpha + V_{,\phi} \delta \phi, \quad (A4)$$

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V, \quad \delta p_\phi = \dot{\phi} \delta \dot{\phi} - \dot{\phi}^2 \alpha - V_{,\phi} \delta \phi, \quad (A5)$$

$$v_\phi = \frac{k}{a} \delta \phi, \quad \pi^{(s)}_\phi \equiv 0. \quad (A6)$$

The background evolution is governed by

$$H^2 = \frac{8\pi G}{3} \mu + \frac{\Lambda}{3} - \frac{K}{a^2}, \quad (A7)$$

$$\dot{\mu}_i + 3H (\mu_i + p_i) = 0, \quad (A8)$$

$$\dot{\phi} + 3H \phi + V_{,\phi} = 0. \quad (A9)$$

The energy-momentum conservation of individual fluid and the equation of motion of a scalar field give [10]

$$\delta \dot{\mu}_i + 3H (\delta \mu_i + \delta p_i) = (\mu_i + p_i) \left( \kappa - 3H \alpha - \frac{k}{a} \beta \right), \quad (A10)$$

$$\frac{a^2 (\mu_i + p_i) \nu_i}{a^2 (\mu_i + p_i)} = \frac{k}{a} \left( \alpha + \frac{\delta p_i}{\mu_i + p_i} \right), \quad (A11)$$

$$\delta \dot{\phi} + 3H \delta \phi + \frac{k^2}{a^2} \delta \phi + V_{,\phi} \delta \phi$$

$$= \dot{\phi} \left( \kappa + \alpha \right) + \left( 2\dot{\phi} + 3H \phi \right) \alpha, \quad (A12)$$

where $\delta_i \equiv \delta \mu_i/\mu_i$, $w_i \equiv p_i/\mu_i$, and $c_s^2 \equiv \rho_i/\mu_i$. Equations (A10-A14) remain valid for the scalar field with $i = \phi$. From eqs. (A13-A14), we can show

$$\delta p_\phi = \delta \mu_\phi + \frac{3aH}{k} (1 - c_s^2) (\mu_\phi + p_\phi) v_\phi. \quad (A15)$$

The metric parts are described by Einstein’s equations [17,18]:

$$\kappa = 3 (\dot{\phi} + H \alpha) + \frac{k^2}{a^2} \chi, \quad (A16)$$

$$- \frac{k^2 - 3K}{a^2} \varphi + H \kappa = - 8\pi G \delta \mu, \quad (A17)$$

$$\kappa - \frac{k^2 - 3K}{a^2} \chi = 12\pi G a \kappa (\mu + p) v, \quad (A18)$$

$$\chi + H \kappa - \alpha - \varphi = \frac{8\pi G}{k^2} a (\mu + p) v, \quad (A19)$$

$$\dot{\kappa} + 2H \kappa + \left( 3H - \frac{k^2}{a^2} \right) \alpha = 4\pi G \left( \delta \mu + 3 \delta p \right). \quad (A20)$$

These equations are in a gauge-ready form. That is, we have not chosen the temporal gauge condition so that we could use the freedom as an advantage in handling problems in diverse situations; the variables are all spatially gauge-invariant (17-18). More general equations with multiple fluids and fields with general interactions among them can be found in [10].

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