Numerical model of a passive microsystem detecting and saving independent acceleration shocks

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Abstract. This paper describes a numerical model for a passive microsystem to detect and save acceleration shocks. For the first time a system is presented, where more than one or two acceleration events can be detected and saved and where the latching position is not influenced by the seismic mass position before the acceleration event. Therefore the latching position indicates the maximal external acceleration amplitude acted on the microsystem. A numerical model is used for prediction of the dynamic system behavior. Utilizing the model a parameter study was performed to determine optimal parameters for clear correlation between ratcheting position and its acceleration threshold. A mass of the ratcheting parts in the same range as the seismic mass enable a clear result in a range of up to 500 g and five latching positions.

1. Introduction
Shocks can lead to damages within e.g. the logistics chain. Also for supervision of product reliability and preventive maintenance, a detection of off-limit conditions is of interest [1]. To measure the maximum acceleration in this case, a passive microsensor is adequate. A passive sensor measures an environmental event without requiring electrical energy at that time. A non-electrical status change is utilized to store this event for an arbitrary long time. If the stored value is required the status can be read out e.g. by RFID.

The majority of passive acceleration sensors uses a spring-guided mass to detect the acceleration events [2-10]. A ratcheting mechanism stores the acceleration threshold for an arbitrary long time [2-5, 8-10]. The known systems contain one or two stable positions of the seismic mass [2,4-5,7-9] for detecting and storing one or two thresholds. In [10], a system is presented which features 20 ratcheting positions. The tooth shape secures the seismic mass position so that an acceleration event directed in the opposite direction of the sensitive direction of the microsystem leads not to a change of the seismic mass position. However, the acceleration thresholds for the next ratcheting position depends on the acceleration event itself as well as on the ratcheting position before the acceleration event. For most applications, it is important that the stored maximum value is independent from previous, smaller shock events.

The goal of this paper is a mathematical model for a system where the ratcheting position is independent from the ratcheting position and thus previous shock events before. So, every latching position can be correlated with a discrete acceleration amplitude range. The concept is derived to the idea in [10]. The parameters are optimized with respect to the system parameter clearness, which mean, that the seismic mass position does not influence the seismic mass position of the next acceleration event. The clearness of the latching position of the seismic mass in a passive microsystem is described by the first time.
2. Description of the mathematical model

The schematic system design is shown in figure 1. A seismic mass is coupled to the system frame by four guiding springs. They allow a movement of the seismic mass in the sensitive direction. Two stationary ratcheting parts are spring-guided too. The ratcheting mechanism is realized by numerous teeth between the three parts. The movement direction of the ratcheting parts is orthogonal to the seismic mass movement.

![Figure 1. Basic system design of the passive shock sensor](image)

In case of a sufficiently large acceleration amplitude, the seismic mass collides with the two corresponding ratcheting parts that are deflected due to this. If the seismic mass passes the first tooth, the measured acceleration was bigger than the first threshold. The tooth shape inhibits the backwards directed movement of the seismic mass. The number of passed teeth depends on the acceleration amplitude, only. The ratcheting parts are symmetrically arranged to avoid a seismic mass rotation around the z-axis during collision.

![Figure 2. Sequence of latching, forces are only showed in first section.](image)

For more than 2 latching positions where each latching position can be correlated to a discrete acceleration amplitude range, a more complex mathematical model is required. Here, a mass-spring-damp-model is used to predict the dynamic seismic behavior. The model used in [10] is therefore expanded for the aim of distinct acceleration values. The model allows the calculation of the seismic mass movement and the movement of corresponding ratcheting parts. In contrast to this, in [10] only the movement of the seismic mass has been taken into account. The complete motion sequence is divided into 5 different steps that are shown in figure 2.
In section 1, the seismic mass movement is described by the following equation in positive x-direction:

\[
m_{\text{SM}} \ddot{x}_{\text{SM}} = m_{\text{SM}} a(t) - c_{\text{SM},x} x_{\text{SM}} - k_{\text{SM}} \dot{x}_{\text{SM}} - \frac{F_{\text{a},x}}{|F_{\text{a},x}|} + \frac{F_{\text{v},\text{SM}}}{|F_{\text{v},\text{SM}}|} - \frac{F_{\text{D},\text{SM}}}{|F_{\text{D},\text{SM}}|}
\]

In this equation, \( m_{\text{SM}} \) is the mass of the seismic mass, \( a(t) \) is the external acceleration acting on the seismic mass, \( c_{\text{SM},x} \) is the damping coefficient, \( k_{\text{SM}} \) is the spring constant of the guiding springs of the seismic mass, \( F_{\text{a},x} \) is the force from the external acceleration event acting on the seismic mass in sensitive direction, \( F_{\text{v},\text{SM}} \) is the spring force described by the spring rate of the guiding springs of the seismic mass \( c_{\text{SM},x} \), \( F_{\text{D},\text{SM}} \) is the damping force with the damping factor \( k_{\text{SM}} \). The accelerations course is assumed to be ideally half sinusoidal-like in [10, 11] and as used in manufacturers’ instructions [12, 13].

In the second section, the seismic mass collides with the stationary ratcheting parts which can be described with the conservations of momentum and energy. Hence, the velocity of the seismic mass after collision is described by:

\[
\dot{x}_{\text{SM},2} = \frac{\frac{m_{\text{SM}}}{2} m_{\text{RP}} (1 + \cotan \alpha) \pm \sqrt{\left(\frac{m_{\text{SM}}}{2} m_{\text{RP}} \cotan \alpha + m_{\text{SM}}\right)^2 + \left(2 m_{\text{SM}} m_{\text{RP}} + 2 m_{\text{SM}}^2 + \frac{m_{\text{SM}}}{\cotan \alpha}\right) y_{\text{RP}}^2}}{2 \left(\frac{m_{\text{SM}}}{2} + \frac{m_{\text{SM}}}{m_{\text{RP}}} (1 + \cotan \alpha)\right)}
\]

In comparison to [10], the velocity of the ratcheting parts is calculated, too. However, for the first collision the velocity of the corresponding ratcheting parts is zero. The spring rate of the ratcheting parts in x-direction is estimated to be about 300 N/m, hence it can be assumed, that the ratcheting parts do not move in x-direction. External acceleration on the ratcheting parts and the seismic mass in y- or z-directions are not noticed in this model. For applications like logistic monitoring or product reliability is just one acceleration direction dominant.

Dependent on the seismic mass velocity at the moment of collision, the seismic mass and the ratcheting parts move either coupled or decoupled in section 3. In case of a coupled movement, Coulomb friction occurs between the tooth flanks and has to be added to the differential equation of the seismic mass (1):

\[
m_{\text{SM}} \ddot{x}_{\text{SM}} = m_{\text{SM}} a(t) - c_{\text{SM},x} x_{\text{SM}} - k_{\text{SM}} \dot{x}_{\text{SM}} - 2 \tan \alpha \left(\frac{\sin \alpha + \mu \cos \alpha}{\cos \alpha - \mu \sin \alpha}\right) (m_{\text{RP}} \ddot{x}_{\text{SM}} + k_{\text{RP}} \dot{x}_{\text{SM}} + c_{\text{RP}} x_{\text{SM}})
\]

If the seismic mass and the corresponding ratcheting parts have different velocities after the collision, they are moving decoupled (1) in section 3. Consequently, the movement of the ratcheting parts has to be considered in the model in order to predict the moment and position of the next collision. Their movement can be described using a differential equation:

\[
m_{\text{RP}} \ddot{y}_{\text{RP}} = m_{\text{RP}} a(t) - c_{\text{RP}} y_{\text{RP}} - k_{\text{RP}} \ddot{y}_{\text{RP}}
\]

After a travel of about 5 μm in y-direction (additional to the tooth height) the ratcheting parts reach a mechanical stop. The stop is assumed to have an infinite mass. Hence, the velocity of the ratcheting parts becomes zero and they are pulled back by the spring force afterwards. The movement range towards the seismic mass is restricted by these stops as well.

In section 4 the seismic mass movement is calculated with the speed of the seismic mass after collision as initial conditions. If the ratcheting parts move decoupled from the seismic mass, the movement of the seismic mass is described by the same equation (1) from section 1. For the next collision there are two interesting facts to calculate: The velocity and movement direction of the seismic mass as well as the position of the next collision. This position is calculated considering the movement of the seismic mass and the ratcheting parts.

If the acceleration force is smaller than the retractive force caused by the seismic mass guiding springs, the seismic mass is slowed down, pulled backwards and stopped on the next foregoing tooth flank which represents the end of the ratcheting sequence (section 5).

The dynamic performance of the system is solved numerically by using the event function of the ODE45 solver in Matlab. Every section of the movement can be detected and the latching can be considered within the model.
### 3. System design

Based on this mathematic model, a system is designed that features 5 latching positions with a unique performance of *clearness*. That means every latching position can be dedicated to a specific acceleration range and does not depend on the seismic mass position prior to the next higher acceleration event. To achieve this behavior, the ratcheting parts have to be deflected suchlike that the ratcheting parts pass exactly one tooth.

A parameter study was executed to determine the influence of key system parameters on the ratcheting behavior. Key parameters are:

- the spring rate of the guiding springs for the seismic mass $c_{SM}$,
- the spring rate of the guiding springs for the ratcheting parts $c_{RP}$,
- the mass of the ratcheting parts $m_{RP}$,
- the ratcheting period $b_z$,
- the ratio of the tooth length to the tooth height $eta$.

![Figure 3. a) ideal clear system behavior; b) unclear system behavior](image)

The seismic mass and the spring rate of its guiding springs show a similar influence on the system behavior. Due to the easy adjustability of the spring rate, the seismic mass is set as constant (0.3 mg). Parameters are successively varied in a reasonable technology range whereby the other parameters are constant. Subsequently the system performance is analyzed with 5 threshold-accelerations. As shown in fig. 3, clear system behavior comprises an equal distance between these 5 acceleration thresholds. An unclear system behavior is shown in figure 3 b). To calculate the clearness factor, two criteria are defined. The first criterion of distinctness is defined:

\[
\text{clearness}_{\text{distinctness}} = e_1 = \frac{a_{0.5} - a_{3.4}}{a_{3.4} - a_4 - 1} \tag{5}
\]

$a_5$ is the desired acceleration amplitude for latching position 5. $a_{3.4}$ means the minimal necessary acceleration to reach the latching position 4 from the latching position in initial state 3. This is the main factor. The optimal value would be 1 as shown in figure 3 a). In case of an unclear behavior the value becomes negative or larger than 2.

The second criterion using the gradient of the threshold-acceleration on latching position 5 is calculated with the equation:

\[
\text{clearness}_{\text{rise}} = e_2 = \frac{a_{4.5} - a_{0.5}}{a_{0.5}} \tag{6}
\]

The optimum for this criterion is 0. Both criteria are influencing each other as shown figure 3 b).

The influences of the investigated parameters are shown in figure 4 a). The influence was determined by taking the standard deviation from varied parameters in a reasonable range of technology.
Figure 4. a) Influence of the parameter on the clearness; b) Influence of the mass of the corresponding ratcheting parts with $m_{SM} = 0.3$ mg, $c_{SMx} = 2$ N/m, $c_{RP} = 30$ N/m, $b_Z = 200$ µm, $\alpha = 30^\circ$, $a_Z = 35$ µm

An increase of the parameter $c_{SMx}$ causes an increase of all acceleration thresholds but has just a little influence on the clearness. The spring rate of the guidance of the ratcheting part $c_{PR}$ has nearly no influence on the system performance due to the uncoupled movement for the majority of acceleration events. The mass of the ratcheting parts has great influence on the system behavior (shown in figure 4 b)). The greater the mass of the ratcheting parts, the more energy of the seismic mass is transferred to the ratcheting parts. In figure 4 b) the optimum of the ratcheting parts is shown. The influence of the latching period is similar to the influence of the parameter $c_{SMx}$. The parameter latching period, which contains the travel of the seismic mass and the parameter $c_{SMx}$ are linear components in the spring force. So, the mass of the ratcheting part has the biggest influence of the parameter clearness.

For verifying the presented model demonstrator systems will be fabricated, which can detect accelerations until 500 g (shown in figure 5). The system parameters are summarized in table 1. The clearness factor distinctness is 1.1, which is close to the optimum 1 and the gradient of threshold-acceleration is -0.2 %, close to the optimum 0. The gradient of the other threshold-acceleration is about 10 %. The first threshold-acceleration is by 73 g.

Figure 5. Clearness of shock sensor with the parameters: $m_{SM} = 0.75$ mg, $c_{SMx} = 7.37$ N/m, $m_{RP} = 0.6$ mg, $c_{RP} = 12.5$ N/m, $b_Z = 100$ µm, $a_Z = 35$ µm

| Parameter                                      | Symbol | Unit  | Assumed Value |
|-----------------------------------------------|--------|-------|---------------|
| Movement range (ratcheting period)            | $x_{max}$ | µm    | 500 (100)     |
| Number of ratcheting positions                | $n$    |       | 5             |
| Spring rate of the guiding springs for the seismic mass | $c_{SMx}$ | N/m | 7.4; 30 -7. |
| Seismic mass                                   | $m_{SM}$ | mg    | 0.75          |
| Spring rate of the guiding springs for the ratcheting part | $c_{RP}$ | N/m | 12.5         |
| Mass ratcheting part                           | $m_{RP}$ | mg    | 0.6           |
| Tooth shape                                    | $a_Z$, $h_Z$, $\alpha$ | µm or ° | 52.5; 30; 30 |
By having an optimum by other maximum accelerations, the ratio of the seismic mass and the mass of the ratcheting parts change. While by 500 g maximum acceleration the ratio is about 1:0.8, the ratio by 5000 g is about 1:0.2.

4. Conclusions and further work
A numerical model for a passive acceleration threshold detector and the influences of the system parameters on its behaviour were shown. The parameters are optimized to have a distinct measurement. That means, that every latching position a range of acceleration can be collated. Of course a lower seismic mass and a higher stiffness of the guiding spring of the seismic mass leads to a system which senses higher accelerations. The spring rate of the guiding springs of the ratcheting parts have very low effect on the behaviour of the seismic mass. The strongest influence of the parameter clearness has the mass of the ratcheting parts, because the energy of the seismic mass is transferred to the ratcheting parts by impact. A mass of the ratcheting parts in the same range as the seismic mass enable a clear result in a range of up to 500 g and five latching positions.

For verifying the presented model the next step is the fabrication of a demonstrator system with the shown parameters from table 1. In an adequate measurement setup the behavior of the seismic mass will be reviewed.

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