SOLAR WIND ACCELERATION BY THE DISSIPATION OF ALFVÉN WAVES

A. A. GALEEV AND A. M. SADOVSKI
Space Research Institute of Russian Academy of Sciences
117810, Profsoyuznaya 84/32, GSP-7, Moskow, Russia

Abstract. Axford and McKenzie [1992] suggested that the energy released in impulsive reconnection events generates high frequency Alfvén waves. The kinetic equation for spectral energy density of waves is derived in the random phase approximation. Solving this equation we find the wave spectrum with the power law “-1” in the low frequency range which is matched to the spectrum above the spectral brake with the power low “-1.6.” The heating rate of solar wind protons due to the dissipation of Alfvén waves is obtained.

1. Introduction

The solar wind which is formed by the gas-dynamic expansion of solar corona into the interplanetary space can be divided into two states: the fast (> 700 km/s) and slow (≈ 400 km/s) streams. The fast solar wind originates in the polar coronal holes, regions of the solar corona with relatively low electron temperature $T \approx 10^6$ K, reduced plasma density $n = 10^8$ cm$^{-3}$ and magnetic field at the base of corona $B_0 \approx 10$ G [1], [2]. Variations of the velocity of this fast solar wind streams are insignificant (700–800 km/s). The slow solar wind is associated with transient openings of closed field regions in the corona. The streams of slow wind are limited by the range ±13° near the equator. The transition zone between this regions has the width ±13° – ±20° on the north and south latitudes in the solar minimum [3]. The radiation balance analysis in the near equatorial coronal holes [4] shows that to accelerate solar wind it is necessary to dissipate the energy flux of radiationless origin $\approx (5 \pm 1) \cdot 10^5$ erg/cm$^2$·s within the distance of one-
two solar radii above the Sun’s surface in order to maintain the observed temperature $1.5 \cdot 10^5$ K of slowly rising expanding gas.

Axford and McKenzie [1] suggested that the energy source, necessary to accelerate fast solar wind, is in the regions of strong magnetic field that define the boundaries of chromospheric supergranules. If the magnetic field is not strictly unipolar in this regions, for example, the closed loops of magnetic field lines are existing at the base of the corona, then the necessary energy is released in the processes of impulsive reconnection of magnetic field lines on the characteristic spatial scale of network activity $l = 100$ km. Such processes of impulsive reconnection are accompanied by the generation of Alfvén and fast magnetosonic waves. Using the parameters of plasma at the coronal base described above we can find the Alfvén wave velocity $V_A = B/\sqrt{4\pi n m_i} = 2 \cdot 10^8$ cm/s, the ion’s thermal velocity $V_{Ti} = 2 \cdot 10^7$ cm/s, and the characteristic time of magnetic field lines reconnection [5]:

$$
\tau_R = \frac{\pi v_A}{4l \ln \text{Re}_m} = \frac{\pi \cdot 2 \cdot 10^8}{4 \cdot 10^7 \text{ cm} \cdot \ln 3.5 \cdot 10^{11}} \sim 0.6 \text{ s},
$$

where: $\text{Re}_m = 4\pi n e^2 v_A l/m_e c^2 = 3.5 \cdot 10^{11}$ is the magnetic Reinoold’s number, $\nu_e = 20n/(T^0)^3/2 = 2.3 \text{ s}^{-1}$ is the collision frequency. It is evident that the characteristic period of Alfvén waves is of the order of obtained reconnection time. The total energy flux necessary to accelerate fast solar wind is estimated as $8 \cdot 10^5 \text{ erg/cm}^2 \cdot \text{s}$ [1] and as in the case of the near equatorial coronal hole the major part of the energy of generated waves should be dissipated within one-two solar radii above the solar surface.

As the dominant mechanism of dissipation of Alfvén waves generated by the processes of impulsive reconnection of magnetic field lines, we will consider the induced scattering of Alfvén waves by plasma ions [6]. For the sake of simplicity as in the paper [6] we will limit ourselves by the case of the circularly polarized Alfvén waves propagating along the radial magnetic field $B_0 = B_0(r)z$ from the polar coronal holes with the wave vector $k = (0,0,k)$, where $r$ is the radial distance from the center of the Sun. With the help of the above cited parameters of plasma and magnetic field at the base of the corona, we fix the following ordering of the ion cyclotron $\omega_{ci}$, Alfvén $\omega_k$ and ion Doppler $kv_{Ti}$ frequencies:

$$
\omega_{ci} \gg \omega_k \gg kV_{Ti},
$$

where $\omega_{ci} = eB_0(r)/m_i c, \omega_k = kV_A$. Under this conditions plasma is magnetized and the resonant condition describing the interaction of Alfvén waves with the different polarization propagating in the opposite directions along the magnetic field lines can be written as:

$$
\omega_k + \omega_{k'} - (k + k')v_z = 0,
$$
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where \( \omega_k = kV_A > 0, \omega_{k'} = -k'V_A < 0, k, k' > 0 \) and \( v_z \) is the velocity along the magnetic field. Here in the process of induced scattering the quantum of higher frequency is exchanged for that of lower frequency. As a result of this some fraction of wave energy is transferred to plasma ions.

In the next section we will derive the kinetic equation for the interacting waves in the random phase approximation. In this derivation we limit ourselves by the case \( T_i \gg T_e \) [7], [8], that is valid for the solar corona. Then in the section 3 we will describe the evolution of the Alfvén waves spectrum in the processes of their propagation and show that the wave energy flux necessary to accelerate the fast solar wind can be dissipated at the heights of 1-2 solar radii above the solar surface that is required by the Withbroe [4] analysis.

2. Derivation of kinetic equation in the random phase approximation

The kinetic equation for the particle distribution function \( f_j \) in the absence of collisions takes the form [9], [10]:

\[
\left( \frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z} + \omega_\theta \frac{\partial}{\partial \theta} \right) f_j = \frac{i e_j}{2 m_j c} B^\pm_k e^{\mp i \theta} \left[ \left( \frac{\omega_k}{k} - v_z \right) \frac{\partial}{\partial v_\perp} + \frac{\partial}{\partial v_z} v_\perp \mp i \left( \frac{\omega_k}{k} - v_z \right) \frac{\partial}{\partial v_\perp} \right] f_j \exp \left[ -i \left( \omega_k t - k z \right) \right],
\]

(4)

where: \( v_\perp \) and \( \theta \) are the velocity and the azimuthal angle of the cyclotron rotation of particles, \( B^+_k \) and \( B^-_k \) are the waves amplitudes with the left and the right polarizations propagating away from Sun and towards the Sun respectively.

Following perturbation theory we solve the kinetic equation (4) by finding the distribution function of plasma particles as power series in the wave amplitudes:

\[
f_j = f_{j0} + f_{j1} + f_{j2} + f_{j3}, \tag{5}
\]

where \( f_{j0} \) is the isotropic maxwellian distribution of particles of \( j \) species in the zero approximation and \( f_{jn} \) is the particle distribution function on the \( n \)-th step of iteration. Using the iteration procedure for solving the kinetic equation (4) we obtain the corresponding Fourier’s components of the particle distribution functions for the first two steps of iteration:

\[
f^+_1 = -\frac{e_j \omega_k}{2 m_j k c} \omega_k - k v_z - \omega_{ij} \partial f_{j0} / \partial v_\perp B^+_k e^{-i \theta} \exp \left[ -i \left( \omega_k t - k z \right) \right], \tag{6}
\]

\[
f^-_1 = \frac{e_j \omega_{k'}}{2 m_j k' c} \omega_{k'} - k' v_z + \omega_{ij} \partial f_{j0} / \partial v_\perp B^-_{k'} e^{i \theta} \exp \left[ -i \left( \omega_{k'} t - k' z \right) \right], \tag{7}
\]
\[ \frac{f_{j2}}{2} = -\frac{\omega_{cj}}{2} V_A \left[ 1 + \frac{(k + k') V_A}{2\omega_{cj}} + \frac{(k^2 + k'^2) V_A^2}{2\omega_{cj}^2} + \frac{(k - k') v_z}{2\omega_{cj}} \right. \\
\left. + \frac{(k'^2 - k^2) V_A v_z}{\omega_{cj}^2} \frac{\partial f_{j0}}{\partial v_z} \omega_k + \omega_{k'} - (k + k') v_z + i0 \right] \]

\( \times \frac{B_k^* B_{k'}^*}{B_0^2} \exp \left[ -i (\omega_k + \omega_{k'}) t + i (k + k') z \right]. \)

Here we have neglected the small contribution related to the differentiation of nonresonant denominator on \( v_z \) in the equation (8).

Perturbation of the charge density related to the particle distribution function of the second order \( f_{j2} \) excites the longitudinal wave with the potential \( \phi_{k+k'} \), that can be found from the Poisson equation:

\[ (k + k')^2 \phi_{k+k'} = 4\pi \sum_j \epsilon_j \int d^3v f_{j2}, \]

where \( \epsilon_{k+k'} \) is the dielectric permeability of the plasma:

\[ \epsilon_{k+k'} = 1 + \sum_j \frac{\omega_{pj}^2}{(k + k')^2} \int d^3v \frac{(k + k') \partial f_{j0}}{\omega_k + \omega_{k'} - (k + k') v_z + i0}. \]

With the help of the operator of particle interaction with the longitudinal waves in a form:

\[ f_{j\phi} = -\frac{e_j}{m_j \omega_k + \omega_{k'} - (k + k') v_z + i0} \frac{\partial f_{j0}}{\partial v_z} e^{-i(\omega_k + \omega_{k'}) t + i(k + k') z} \]

we find on the next iteration step the particle distribution function of the third order for the waves propagating from Sun:

\[ f_{j3}^{-\ast} = -\frac{e_j}{2m_j} \int \frac{dk'}{2\pi} \left[ \frac{\omega_{k'} - v_z}{\omega_k - k' v_z - \omega_{cj} + i0} \left( \frac{\partial}{\partial v_{\perp}} + \frac{\partial}{\partial v_z} \right) f_{j2} + f_{j\phi} \right] \]

\( \times B_k^{-*} e^{-i\theta} \exp \left[ i(\omega_k t - k' z) \right]. \)

The kinetic equations for the waves propagating from Sun and towards the Sun respectively can be described in terms of Maxwell equations for the Fourier’s components of the wave amplitudes:

\[ B_k^\pm = \frac{4\pi}{kc} \sum_j e_j \int dV v_{\perp} e^{\pm i\theta} \left( f_{j1j}^\pm + f_{j3j}^\mp \right), \]

Here we take into account that the second order current, associated with the distribution functions \( f_{j2} \) and \( f_{j\phi} \), does not contribute to (13).
In the linear approximation the dispersion relation (13) can be written as [11]:

\[
\left\{ 1 - \sum_j \frac{\omega^2_{pj} \omega}{2k^2c^2} \int d^3v \frac{v_\perp \partial f_{j0}/\partial v_\perp}{\omega_k - kv_z - \omega_{cj} + i0} \right\} B_k^+ = \epsilon_{k1}^+ B_k^+ = 0. \tag{14}
\]

Taking the frequencies ordering (2) we obtain from the equation (14) the well-known dispersion relation for the Alfvén waves:

\[
\epsilon_{k1}^+ \approx 1 - \frac{\omega^2_k}{k^2v_A^2} = 0 \tag{15}
\]

In the order to obtain the time dependent evolution of Alfvén waves we multiply the equation (14) by \(B_k^+(k, \omega)\exp[i(\tilde{\omega} - \omega)t]\) and integrate it by \(d\tilde{\omega}d\omega\). Using the relation \(\epsilon_{k1}^+ = \epsilon_{k1}^+ + i\epsilon_{k1}^{''}\), we obtain:

\[
\int \int d\tilde{\omega}d\omega \epsilon_{k1}^+ B_k^+(k, \omega)B_k^{+\ast}(k, \tilde{\omega})e^{i(\tilde{\omega} - \omega)t} = \frac{i}{2} \frac{\partial \epsilon_{k1}^+'(\omega_k)}{\partial \omega_k} \frac{d|B_k^+|^2}{dt} + i\epsilon_{k1}^{''}(\omega_k)|B_k^+|^2, \tag{16}
\]

where the time dependent wave amplitude is defined as:

\[
B_k^+(t) = \int d\omega B_k^+(k, \omega)e^{-i(\omega - \omega_k)t}. \tag{17}
\]

Restoring the contributions of the nonlinear currents on the right-hand side of the equation (13) we find the kinetic equation for the weakly interacting waves propagating from the Sun:

\[
\frac{d|B_k^+|^2}{dt} = Im \sum_j \frac{\omega^2_{pj} \omega}{k^2c^2} \int \frac{dk'}{2\pi} \int d^3v \frac{\omega_{k'}}{\omega_k - kv_z - \omega_{cj} + i0} \times \left( f_{j2}^+ + f_{j0}^+ \right) B_k^{+\ast} B_{k'}^{+\ast} e^{i(\omega_k - \omega_{k'})t - i(k - k')z}, \tag{18}
\]

To obtain the above equation in such form we integrate by parts the expressions for nonlinear currents to get rid of the derivatives over \(v_z\) and \(v_\perp\) and neglect small corrections related to the differentiation of the nonresonant denominator by \(v_z\).

Expanding the expressions for the nonresonant denominator and for the particle distribution functions \(f_{j2}\) and \(f_{j0}\) as a power series in \(\omega_k/\omega_{cj}\) and \(\omega_{k'}/\omega_{cj}\), \(kv_z\) and \(k'v_z\) to the second order we find the equation for waves propagating from the Sun in the form:
\[
\frac{d}{dt}|B_k^+|^2 = -Im \sum_j \frac{\omega_{pj}^2 V_A^2}{2c^2} \int \frac{dk'}{2\pi} \int d^3v \frac{\omega_k}{k} \left( \frac{\omega_{k'}}{k'} - v_z \right) \left( 1 + \frac{kV_A}{\omega_{cj}} - \frac{kv_z}{\omega_{cj}} \right) \times \left\{ 1 + A_j - \frac{\sum_{j'} \omega_{pj'}^2}{\omega_k^2 + \omega_{k'}^2 - (k + k')v_z + i0} \left[ \frac{\partial f_{j0}}{\partial v_z} (1 + A_{j'}) \frac{\partial f_{j0}}{\partial v_z} \right] \right\} \times \left[ \frac{\omega_k}{\omega_k + \omega_{k'} - (k + k')v_z + i0} \frac{|B_k^+|^2 |B_k'|^2}{B_0^2} \right],
\]

(19)

where

\[
A_j = \frac{(k + k')V_A}{2\omega_{cj}} + \frac{(k^2 + k'^2)V_A^2}{2\omega_{cj}^2} + \frac{(k - k')v_z}{2\omega_{cj}} + \frac{(k'^2 - k^2)V_A v_z}{\omega_{cj}^2},
\]

and \(A_{j'}\) and differ from \(A_j\) only by the index \(j'\).

Considering the limit \(T_e \ll T_i\) we obtain in the last term in figure brackets that the electron contribution to integrals dominates that for ions:

\[
\sum_{j'} \int d^3v \frac{\partial f_{j0}}{\partial v_z} (1 + A_{j'}) \frac{\partial f_{j0}}{\partial v_z} \approx -\frac{\omega_{pe}^2}{k + k'} \int d^3v \frac{\partial f_{j0}}{v_z \partial v_z} \gg \omega_{pe}^2 \frac{(k + k')v_z}{v^2_{Ti}}.
\]

(20)

As the result the first term in the brackets cancels with the last one approximately equal \(-1\). Taking the integral by parts over \(v_z\) and then over \(dk'\) we reduce equation (19) to the equation obtained by Livshits and Tsytovich [7], [8]:

\[
\frac{d}{dt}|B_k^+|^2 = -\omega_k \frac{|B_k^+|^2 k}{B_0^2} \frac{\partial}{\partial k} |B_k^-|^2 k.
\]

(21)

In the case of stationary expansion of plasma from the solar corona along approximately radial magnetic field lines this equation takes the form:

\[
\left[ u(r) + V_A(r) \right] \frac{\partial}{\partial r} |B_k^+|^2 = -kV_A \frac{|B_k^+|^2 k}{B_0^2(r)} \frac{\partial}{\partial k} |B_k^-|^2 k.
\]

(22)

3. Evolution of Alfvén wave spectrum in the processes of their propagation

Evolution of Alfvén wave amplitudes in the processes of their propagation into the interplanetary space we calculate theoretically assuming that (a)
the interplanetary magnetic field $B_0(r)$ and the solar wind velocity $u$ are radial and spherically symmetric, (b) the Alfvén waves are circularly polarized, (c) wave amplitudes are small $|B_k^+(r)|^2k \ll B_0^2(r)$.

Integrating the equation for the spectral energy density of waves

$$[u(r) + V_A(r)] \frac{\partial |B_k^+(r)|^2k}{4\pi} = -\kappa k^2 V_A(r) \frac{|B_k^+(r)|^2k}{B_0^2(r)} \frac{\partial |B_k^+(r)|^2k}{4\pi},$$  

we find the general solution [12]:

$$|B_k^+(r)|^2k = \Phi \left[ \kappa I \int_{R_\odot}^r \frac{V_A(r)dr}{u + V_A(r)} + \frac{1}{k(R_\odot)} - \frac{1}{k} \right].$$  

Here: $\Phi$ is an arbitrary function; $V_A(r) = V_A(R_\odot)R_\odot/r; k(R_\odot)V_A(R_\odot)/2\pi = 1.7$ Hz; $R_\odot = 6.96 \cdot 10^{10}$ cm; $\kappa = |B_k^-(r)|^2/|B_k^+(r)|^2$ is the ratio of the spectral energy density of Alfvén waves, propagating to the Sun and from the Sun respectively, that is the constant because both waves have the same spectral index $\alpha = -1$ and radial profiles. The ratio of the wave energy density to the energy of the magnetic field $I = |B_k^+(r)|^2k/B_0^2(r)$ happens to be the constant too due to the identical radial profiles of the energy density of Alfvén waves and the energy of magnetic field and very weak dependence of the wave energy density $|B_k^+(r)|^2k$ on wave vector $k$ due to the very small left-hand side of the equation (23). Numerical value for $I$ can be found by equating the energy flux of Alfvén waves generated at the base of the corona to the energy flux of the radiationless nature with the value $8 \cdot 10^5$ erg/cm$^2$·s, necessary for the acceleration of the fast solar wind from polar coronal holes:

$$V_A(r) \frac{|B_k^+|^2k}{4\pi} = 8 \cdot 10^5 \text{ erg/cm}^2\cdot\text{s}. \quad (25)$$

As the result we find that: $I = 5 \cdot 10^{-4}$.

In the process of wave propagation to the interplanetary space the position of the spectral break, separating spectral domains with the different slopes, is shifted to the low frequencies [13]. From the general solution (24) we construct the particular solution:

$$\frac{1}{k_{br}} = \kappa I \int_{R_\odot}^r \frac{V_A(r)dr}{u + V_A(r)} + \frac{1}{k(R_\odot)}$$  

$^1$In the works [7, 8], authors were interested in stationary spectra of Alfvén waves when the left-hand side of the equation has been zero, in the other words $\partial/\partial t = 0$. That’s why they didn’t call any attention that the sign in the right-hand side of the equation should be mines not plus.
We can find the exact position of the spectral break frequency as a function of radial distance by dividing both sides of this equation by the Alfvén wave velocity:

\[
    f_{br}(r) = \left[ 0.6 + 2\pi\kappa I \int_{R_{\odot}}^{r} \frac{V_A(r)dr}{u + V_A(r)} \right]^{-1} \frac{1}{R_{\odot}}. \tag{27}
\]

Assuming that the acceleration of fast solar wind is terminated at \( r = 3R_{\odot} \) [16] we find that \( f_{br}(3R_{\odot}) \approx 0.28 \text{ Hz} \). At this stage most of the heating has been deposited within \((1 - 2)R_{\odot}\) above the surface as required by Withbroe [4] analysis. We can simplify this expression at the distance \( r > 10R_{\odot} \) assuming that the velocity of the fast solar wind \( u = 7.5 \cdot 10^7 \text{ cm/s} \gg V_A(r) \). As a result we reduce the expression (27) to the form:

\[
    f_{br}(r) = \left[ 0.6 + \pi\kappa \ln \frac{r}{R_{\odot}} \right]^{-1} \frac{1}{R_{\odot}}. \tag{28}
\]

Taking the numerical value of the spectral break \( 6 \cdot 10^{-2} \text{ Hz} \) at the distance \( r = 60R_{\odot} \) [13], separating the domain of Alfvén wave turbulence with the spectral indexes \( \alpha = -1 \) and domain with \( \alpha = -1.6 \) and equating it to the theoretical value obtained from the equation (28) we find the ratio of the spectral energy density of Alfvén waves propagating to the Sun and from the Sun respectively that is equal to \( \kappa = 0.17 \). This means that the intensity of waves propagating to the Sun is six times lower than those for the waves propagating from the Sun.

Experimental observations show that above the spectral break a wave spectrum with a slope near \( -1.6 \) [14] is established:

\[
    |B_k^+(r)|^2 = |B_{k_{br}}^+(r)|^2 k_{br}^{1.6}/k^{1.6}. \tag{29}
\]

The equation for the spectral energy density of waves in this case should inherit the structure of the equation (23) with the continuous transition across the spectral break:

\[
    [u(r) + V_A(r)] \frac{\partial}{\partial r} \left| B_k^+(r) \right|^2 k^{1.6} = -\kappa k^{1.6} V_A(r) k_{br}^{-0.2} \left| B_k^+(r) \right|^2 k_{br}^{-1.6} \frac{\partial}{\partial k} \left| B_k^+(r) \right|^2 k_{br}^{1.6}, \tag{30}
\]

Equation for characteristics of (30) take a form:

\[
    \frac{1}{k^{1.6}} \frac{dk}{dr} = \frac{\kappa I \left[ 2\pi f_{br}(r) \right]^{0.4} V_A(r)^{0.6}}{u + V_A(r)}, \tag{31}
\]
Figure 1. The characteristics of the equation (30).

where \( I = |B_{k_{br}}^+(r)|^2 k_{br} / B_0^2(r) = 5 \cdot 10^{-4}, \ u \gg V_A(r). \) Beyond the 187.5R⊙ (0.87 astronomical units — AU) the collisional hydromagnetic turbulence spectrum of Kraichnan [15] with the spectral index \( \alpha = -1.5 \) [13] replace the collisionless spectra with the spectral indexes \( \alpha = -1 \) and \( \alpha = -1.6. \)

Integrating (31) and transforming the result to the frequency dependence on the radial distance we obtain:

\[
    f = \left[ C \left( \frac{r}{R_\odot} \right)^{0.6} - \left( \frac{1}{f_{br}(r)} \right)^{0.6} \right]^{-1.6},
\]

where \( C \) is an arbitrary constant.

Figure 1 shows the characteristics of the equation (30) between the curves of the spectral break frequency \( f_{br} \) and the ion cyclotron frequency
\[ f_{ci} = 1.6 \cdot 10^4 \left( \frac{R_\odot}{r} \right)^2, \] where the wave energy is totally absorbed in the ion cyclotron resonance.

To estimate the fraction of Alfvén wave energy transferred to the protons of solar wind plasma in the process of induced scattering of waves we use, following Tu [17], the equation for the magnetic moment of protons. Doing this we take into account that due to the smallness of the left-hand side of the equation (30) the product \( |B_k^+|^2 k^{1.6} \) has a very weak dependence on the wave vector \( k \). Therefore the right-hand side of (30) also depends weakly on \( k \) and we can rewrite it in form:

\[ [u(r) + V_A(r)] \frac{\partial |B_k^+|^2}{\partial r} = - \frac{\partial}{\partial k} 0.25\kappa f_{br}(r) I^2 B_0^2(r) \equiv - \frac{\partial}{\partial k} \Pi(r), \quad (33) \]

where \( \Pi(r) \) is the volumetric energy flux.

The equation for the magnetic moment take a form:

\[ \frac{u}{d} \ln \left( \frac{T_\perp}{B} \right) = \frac{\Pi(r)}{n(r) k_B T_\perp}, \quad (34) \]

where \( T_\perp \) is the perpendicular proton temperature in Kelvin, \( k_B \) Boltzmann’s constant, \( n(r) = 10^8 \left( \frac{R_\odot}{r} \right)^2 \) cm\(^{-3}\) density of plasma.

We integrate the equation (34) from the three solar radii assuming that the acceleration of solar wind is already achieved. As a result we have:

\[ \frac{T_\perp}{B_0} - \frac{T_\perp}{B_0} = \int_{3R_\odot}^r \frac{\Pi(r)}{n(r) k_B u B_0(r)} dr = 0.144 \ln \left[ 0.6 + \pi \kappa \ln \frac{r}{R_\odot} \right], \quad (35) \]

Non-adiabatic heating of the protons related to the presence of the volumetric energy flux \( \Pi(r) \) reach only 0.16 K/nT in the range of radial distances [3\( R_\odot \), 200\( R_\odot \)]. Therefore the source of energy for the acceleration of fast solar wind from the polar coronal holes must now be found in the solar corona itself [18].

Below the spectral brake frequency \( f_{br}(r) \) we have the spectrum of Livshits-Tsytovich [7], [8] with the spectral index \( \alpha = -1 \). However this spectrum doesn’t extend from \( f_{br} \) to the whole low frequency domain but truncated by the WKB solution.

Solving the stationary equation for Alfvén waves [19], [20], [21]:

\[ \nabla \left( 3u(r) \pm 2V_A(r) \right) \frac{|B_k^+(r)|^2 k}{8\pi} - u(r) \nabla \frac{|B_k^+(r)|^2 k}{8\pi} = 0, \quad (36) \]

the equation for the wave amplitude was found in the form:

\[ \left[ 1 \pm \frac{V_A(r)}{u} \right]^2 n(r)^{-3/2} |B_k^+(r)|^2 k = \text{const.} \quad (37) \]
At the distances $r > 10R_\odot$ we can neglect the small parameter $V_A(r)/u \ll 1$ and rewrite it in the form defining the reduction of the wave amplitudes and their spectral index $\alpha = -2/3$ [13]:

$$\text{const} \cdot n(r) = \left( \left| B^+_k(r) \right|^2 k \right)^{2/3}. \quad (38)$$

Besides that, knowing from observations that the spectral break curve $f_{br}$ and the WKB solution are merging at the distance 0.87 AU (187.5 $R_\odot$) we are able to find the form and position of WKB solution curve from the equation (38) in the form (fig. 1):

$$f_{WKB}(r) = 1.6 \cdot 10^{-3} \cdot \left( \frac{r}{187.5R_\odot} \right)^{2/3}. \quad (39)$$

4. Conclusion

Using the collisionless kinetic equation for interacting Alfvén waves in the random phase approximation we have followed analytically the evolution of these waves in the process of their propagation from the polar solar corona. As a result of such evolution the boundary of waves with the spectral index $\alpha = -1$ is shifted towards low frequencies due to induced scattering of waves by ions, which increase energy of ions by the recoil effect. However, this spectrum does not extend over the whole low frequency range but is truncated by WKB solution.

Above the frequency $f_{br}$ the spectrum with the spectral index $\alpha = -1.6$ is obtained. Let us note that both in the polar solar corona and in the interplanetary space the approximation of collisionless plasma is valid at least up to the distance 0.87 AU.

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