Bifurcation analysis of the influence of generator excitation forms on system voltage stability

Wenqian Song¹,²*, Hao Liang¹, Lin Jia³, Xue Xia¹, Guangtao Zhang¹, Feng Zhao³, Lili Liu², Qibang Tan²
¹Electric Power Research Institute of State Grid Jibei Electric Power Company Limited, Beijing, 100045, China
²Department of Electric Engineering, North China Electric Power University, Baoding, Hebei Province, 071003, China
³North China Branch of State Grid Corporation of China, Beijing, 100010, China
*swq_hbdldx@ncepu.edu.cn
*Corresponding author’s e-mail: 18300604413@163.com

Abstract. Based on a typical power system model, this paper simulates the scenario of the increase of active power output of a proximal generator around certain bus node and applies bifurcation theory to study the influence of different excitation forms on system voltage stability after reaching generator excitation saturation. With numerical bifurcation analysis software MATCONT, the simulation analysis results show that the smaller the excitation output upper limit value is, the earlier the bus voltage reaches the Hopf bifurcation with the increase of active power output. In the case of self-shunt excitation, the output of the excitation upper limit value is constrained by the generator terminal voltage. Therefore, the bifurcation point in this case occurs earlier than that in the case of separate excitation.

1. Introduction
In order to make more rational use of energy and solve the problem of unbalanced power production and consumption, China's new energy power generation and UHV AC/DC transmission projects have developed rapidly. The increase of new energy generation results in the decrease of the ratio of conventional hydro-power and thermal-power units, which weakens the support ability of synchronous generators to the reactive power and thus deteriorates the stability of power grid. In addition, the operation of UHV AC-DC transmission projects greatly improves the transmission capacity of electric energy, but it will correspondingly reduce the ratio of running synchronous generator of the receiving-end grid. Especially in the case of line failure, the receiving-end grid is prone to voltage instability or even voltage collapse due to insufficient reactive power support.

The generator excitation control system is one of the most effective and economical control measures to maintain the system voltage stability, which can provide sufficient dynamic reactive power support after disturbance and improve the system voltage recovery process after failure [1-2]. In recent years, researchers have studied and explored the voltage stability of power system related to excitation system. Literature [3] made a qualitative analysis of the influence of excitation control system on stability by simulating the three-phase short-circuit fault of excitation control system of single-machine infinite bus system. Literature [4] demonstrated the influence of excitation system...
mathematical model and parameters on dynamic stability analysis results of power grid. Due to the
diversity of power supply forms and the complexity of power grid operation mode, the excitation
system is easy to approach or trigger its operation and control limits in case of power grid or unit
failure. Therefore, the output limits should be considered and paid attention to when studying the
excitation system's action mechanism on voltage stability. Literature [5] analyzed the influence of
auxiliary excitation control functions such as overexcited restriction on reactive power characteristics
of DC near-area generators and bus voltage of converter stations. Literature [6] studied the influence
of generator excitation upper limit and PSS circuit on the oscillation instability and chaos phenomenon
of power system.

As an aspect of system stability, voltage stability has strong nonlinear characteristics. Bifurcation
theory, as a basic method for analyzing and studying the structural stability of nonlinear systems, has
been widely applied in the study of power system stability in recent years [7-9]. Literature [10]
conducted bifurcation analysis on a typical power system model and studied various bifurcation
behaviors caused by excitation limitation. A smooth function was used to simulate the limiting effect
of the limiter, but the error was unavoidable. Literature [11] showed the influence of different
excitation upper limit value on the system voltage stability under the circumstance of excitation
saturation, this kind of stability study either adopted ideal excitation model or simply considered the
excitation output upper limit value as a fixed value. However, in the actual operation of the units, the
self-shunt excitation method is more and more widely used than the separate excitation method, due to
the advantages of simple structure, less investment and maintenance work as well as quick response.
Especially in recent years, most large-capacity hydro-power and thermal-power units above 300MW
in production adopt the self-shunt excitation method. Compared with the separate excitation system
whose excitation output upper limit value is a fixed value, the excitation output upper limit of the self-
shunt system is restricted by the generator’s terminal voltage, which will have a prominent impact on
system stability in the case of system failure. The literature has little discussion on the influence of the
two excitation forms of separate excitation and self-shunt excitation on the stability of power system.

In this paper, the bifurcation theory is applied to analyze the voltage stability of a typical power
system model, and the emphasis is on the influence of two saturation forms of excitation system,
namely the separate excitation (the output upper limit value is fixed) and the self-shunt excitation (the
output upper limit value is constrained by the terminal voltage) on the voltage stability of the system.
The results show that the lower the upper limit value, the earlier the system voltage approaches the
saddle junction bifurcation point and Hopf bifurcation point. Besides, the bifurcation point in this case
of shunt-excitation occurs earlier than that in the case of separate excitation, because the self-shunt
excitation system has its upper limit value constrained by the terminal voltage and thus has weaker
ability of voltage support.

2. System model
This paper uses a typical three-node power system model composed of an infinite bus, a load bus and
a generator, as shown in Figure 1. Specific parameters can be obtained from literature [12].

![Figure 1. The model of the three-node power system.](image)

2.1 Generator model

2.1.1 Rotor equations. The motion equations of generator rotor side are as follows:
The transient process of the excitation winding is defined as follows:

\[
\dot{E}_q = \frac{-E' + (x_q - x_q')i_q + E_{jd}}{T_{d0}}
\]

(3)

Ignoring the q-axis damping winding, its value can be determined by (4):

\[
E_d' = -(x_q - x_q')i_q
\]

(4)

The mechanical input power of the generator can be expressed as:

\[
P_e = E'_q i_q + E'_d i_d + (x_q' - x_q')i_d i_q
\]

(5)

### 2.1.2. Stator equations.

Neglecting the stator resistance and its transient process, the stator side can be specified by the following equations:

\[
\dot{E}_q' + x_q' i_d = u_q
\]

(6)

\[
E_d' - x_q' i_q = u_d
\]

(7)

### 2.2. Excitation system model

The generator excitation system is described by a high gain single time constant automatic voltage regulator and limiter. Without considering excitation saturation, the separate excitation mode model and the self-shunt excitation mode model are as shown in Figure 2. When the output of the excitation system does not reach the upper limit value, the mathematical model can be described by the following equation:

\[
\dot{E}_{jd} = \frac{-E_{jd} + K_A(u_{ref} - u_i)}{T_A}
\]

(8)

When the output of excitation system reaches saturation under the separate excitation mode, the mathematical model can be described by equation (9):

\[
E_{jd} = E_{jd}^{max}
\]

(9)

where \(E_{jd}^{max}\) is excitation output upper limit value.

When the output of excitation system reaches saturation under the self-shunt excitation mode, the mathematical model can be described by equation (10):

\[
E_{jd} = u_i U_{rmax} - K_s i_{jd}
\]

(10)
where $U_{R\text{max}}$ is output upper limit value, $K_c$ is reversing pressure drop, $i_{id}$ is field current. $E_{fd}^{\text{max}}, U_{R\text{max}}$ and $K_c$ can be obtained through field tests.

2.3. Load model
The Walfve comprehensive load model is adopted, which describes the dynamic behaviour of induction motors under large disturbance. Its model is as follows:

$$P = P_{ld} + P_0 + p_1 \dot{\delta}_1 + p_2 u_i + p_3 u_i$$  \hspace{1cm} (11)

$$Q = Q_{ld} + Q_0 + q_1 \dot{\delta}_1 + q_2 u_i + q_3 u_i^2$$  \hspace{1cm} (12)

2.4. Network model
Using the notation in Figure 1, the network equations under the unified DQ0 coordinate can be obtained as:

$$u_i + \frac{i_i}{Y_i} = u_i$$  \hspace{1cm} (13)

$$u_i = (u_q + j_u_d)e^{i\delta}$$  \hspace{1cm} (14)

$$i_i = (i_q + j_i_d)e^{i\delta}$$  \hspace{1cm} (15)

$$Y_i = Y_i \angle \phi_i$$  \hspace{1cm} (16)

From (13)-(16), we can write

$$i_q \cos \phi_i + i_d \sin \phi_i = Y_i u_q - Y_i u_i \cos(\delta_i - \delta)$$  \hspace{1cm} (17)

$$i_q \cos \phi_i - i_d \sin \phi_i = Y_i u_d - Y_i u_i \sin(\delta_i - \delta)$$  \hspace{1cm} (18)

2.5. System comprehensive model
By substituting equations (4), (6) and (7) into equations (17) and (18), we can get:

$$\begin{bmatrix} i_q \\ i_d \end{bmatrix} = \begin{bmatrix} \cos \phi_i & \sin \phi_i - Y_i x_{ld} \\ -\sin \phi_i + Y_i x_q & \cos \phi_i \end{bmatrix} \begin{bmatrix} Y_i E_q - Y_i u_i \cos(\delta_i - \delta) \\ -Y_i u_i \sin(\delta_i - \delta) \end{bmatrix}$$  \hspace{1cm} (19)

The power balance equation at bus 2 is:

$$P = u_i u_i Y_1 \cos(\delta_1 - \theta - \phi_1) - u_i^2 Y_1 \cos \phi_1 + E_{u_1} Y_1 \cos(\delta_1 - \phi_2) - u_i^2 Y_2 \cos \phi_2$$  \hspace{1cm} (20)

$$Q = u_i u_i Y_1 \sin(\delta_1 - \theta - \phi_1) + u_i^2 Y_1 \sin \phi_1 + E_{u_1} Y_1 \sin(\delta_1 - \phi_2) + u_i^2 Y_2 \sin \phi_2$$  \hspace{1cm} (21)

After linking the system network equation with each component model, a comprehensive model describing the system can be obtained. Its general form is as follows:

$$\dot{x} = f(x, \lambda)$$  \hspace{1cm} (22)

where $x = [\delta, \omega, E_{d}, E_{d} \delta, u_i]^T$ and $\lambda$ is bifurcation parameter. If the saturation part of excitation is considered, from the perspective of mathematical model, at this time $E_{d}$ is no longer a differential variable, while other differential equations remain unchanged. Thus, it can be seen that the dimension of the differential equations is reduced by one dimension. In this case, the state variables are:

$$x = [\delta, \omega, E_{d}, \delta, u_i]^T$$
3. Bifurcation analysis

In this paper, the numerical bifurcation analysis software MATCONT is adopted for the bifurcation analysis of the system. After the output of the excitation system reaches the upper limit value, the difference of the dynamic behaviour of the system under the two modes of separate excitation and self-shunt excitation is compared. By adjusting the mechanical input power $P_m$, the voltage characteristic of load point is simulated as the active power of peripheral units increases.

3.1. Voltage bifurcation analysis of load points without considering excitation saturation

In all analyses of this article, $P_m$, $u_l$ and $E_{fd}$ are per unit values as well as solid and dotted lines in the figure indicate stable and unstable equilibrium points respectively. The saddle point bifurcation and Hopf bifurcation are abbreviated as SNB and HB respectively. Without considering excitation saturation, bifurcation analysis is carried out on the system. The excitation voltage and load point voltage output are shown in Figure 3 and Figure 4. Point 1 in the figures is the initial stable equilibrium point, and the bifurcation parameter values corresponding to the bifurcation point are shown in Table 1.

![Figure 3. Excitation voltage bifurcation diagram.](image1)

![Figure 4. Load voltage bifurcation diagram.](image2)

It can be seen from Figure 3 and Figure 4 that the balance point curve of the power system has undergone four bifurcations, corresponding to points 2, 3, 4 (Hopf) and 5 (SNB) in the figure respectively. Before the second point, the equilibrium points are stable. As $P_m$ gradually increases, when passing the second point, the equilibrium point becomes unstable. With the further increase of $P_m$, the unstable equilibrium point disappears at the third point, and the system returns to a stable state. This stable characteristic will be maintained until $P_m$ passes through the fourth point; at this moment, the system equilibrium point will become unstable again. At the fifth point, the saddle node bifurcation occurred in the system does not change the tendency of system instability.

![Table 1. Bifurcation parameters without considering the excitation saturation link.](image3)

| number | type | $P_m$ | $u_l$ | $E_{fd}$ |
|--------|------|------|------|--------|
| 2      | Hopf | 0.587| 1.004| 2.048  |
| 3      | Hopf | 1.088| 0.981| 2.696  |
| 4      | Hopf | 1.161| 0.975| 2.821  |
| 5      | SNB  | 1.922| 0.752| 5.341  |

3.2. Voltage bifurcation analysis of load point under different peak values of separate excitation mode after reaching excitation saturation

With considering the excitation saturation link along with the separate excitation mode, the bifurcation analysis under different excitation peak values is shown in Figure 5. The bifurcation types and bifurcation parameters are summarized in Table 2. Curves 1, 2, and 3 are the bifurcation curves when the excitation peak voltage is 2.75p.u., 3p.u. and 3.25p.u. and points1, 2 and 3 are the initial saturation points of these three cases.
Figure 5. Bifurcation analysis of different excitation upper limit values in separate excitation.

It can be seen that before the saddle node bifurcation points, the system is unstable due to the occurrence of Hopf bifurcation. As the upper limited value decreases, the Hopf bifurcation points and the saddle node bifurcation points are both occur earlier. When the excitation peak voltage is 2.75p.u., the $P_m$ corresponding to the first Hopf point is 1.270p.u. and the $P_m$ corresponding to SNB point is 1.299p.u.; When the excitation peak voltage is 3p.u., the $P_m$ corresponding to the first Hopf point is 1.334p.u. and the $P_m$ corresponding to SNB point is 1.386p.u.; When the excitation peak voltage is 3.25p.u., the $P_m$ corresponding to the first Hopf point is 1.395p.u. and the $P_m$ corresponding to SNB point is 1.467p.u.. As smaller the output upper limit value of the excitation system, it is less possible to provide stronger excitation support and the less helpful to the system voltage stability.

Table 2. Mechanical power at the bifurcation point of saturated separate excitation mode.

| excitation peak voltage | 2.75 | 3    | 3.25 |
|-------------------------|------|------|------|
| first Hopf              | 1.270| 1.334| 1.395|
| SNB                     | 1.299| 1.386| 1.467|
| second Hopf             | 1.226| 1.343| 1.437|

3.3. Voltage bifurcation Analysis of load point under separate excitation and self-shunt excitation after reaching excitation saturation

Considering the limitation of excitation saturation, bifurcation analysis on the system with separate excitation and self-shunt excitation are conducted, which is shown in Figure 6. The two methods adopt the same initial excitation saturation point marked as 1-point. Its excitation upper limit value voltage $E_{max}$ is 2.118p.u., commutator coefficient $K_c$ is 0.08 and output upper limit value voltage $U_{Rmax}$ is 2.1p.u. Curve 1 and curve 2 respectively represent the simulation results of voltage bifurcation of load bus with self-shunt excitation and separate excitation.
It can be observed from Figure 6 that the saddle node bifurcation points of load bus voltage arise in advance after considering the excitation saturation link, where system loses its stability. The saddle node bifurcation point of self-shunt excitation mode appears at 2-point where \( P_m \) is 0.681p.u. and saddle node bifurcation point emerges at 3-point where \( P_m \) is 1.046p.u. in self-shunt excitation mode. It appears earlier in self-shunt excitation than that in separate excitation mode, which leads to the system loses stability earlier. This is because the output peak value of separately excited excitation system is not subject to generator terminal voltage and hence it can better provide reactive voltage support, which is conducive to system voltage stability. In the self-shunt excitation mode, as the increase of \( P_m \), the generator terminal voltage decreases. Consequently, the excitation output peak value is reduced and the bifurcation point appears earlier.

4. Conclusions
In this paper, a typical power system model is adopted to simulate the scenario of the increase of output active power around a bus node. The bifurcation analysis theory is applied to analyse and discuss the influence of different excitation forms on voltage stability after saturation of excitation system. The following conclusions are obtained: after considering the saturation link of excitation system, the bifurcation point of the system occur earlier under both excitation modes. The rate transmission limit is reduced, which makes the system more likely to lose stability. For the case of separately excited excitation mode, the lower the upper limit value of excitation system output is, the earlier the bifurcation point is, the less the unit can provide forced excitation support, consequently the less help to system voltage stability. Compared with the separate excitation mode, the excitation upper limit value output under the self-shunt excitation mode is constrained by the generator terminal voltage. Accordingly, the weaker the voltage support capacity is, the earlier the system reaches the bifurcation point.

Acknowledgements
This work is supported by the Science and Technology Programme of the State Grid Corporation titled “Research on the Source coordination mechanism of weak feeder of UHV AC/DC combined external transmission” (No.520101180050).

References
[1] Meng. C. (2018) Research on the influence of generator excitation limit functions on the stability of power system. Beijing: North China Electric Power University.
[2] Liu, Q. (2007) Power system stability and generator excitation control. China Electric Power Press, Beijing.
[3] Ding, Z.D., Liu, G.H. (2007) Research on the Influence of synchronous generator excitation on stability. Large Electric Machine and Hydraulic Turbine, (04):60-64.

[4] Shi, X. M., Wang, Z.H., Gui, G.L., and Dai, S.H. (2007) Research on the influence of the mathematical model and parameters of generator excitation system on the analysis results of grid dynamic stability. Relay, (21):22-27.

[5] Zhao, F., Wu, T., Xie, H., Dai, Q., and Liang, H. (2018) Study on effect of generator excitation auxiliary control on dynamic reactive power support capability of UHVDC transmission sending system. Power System Technology, 42 (7):2262-2269.

[6] Jia, H. J., Yu, Y.X., Wang, C.S. (2001) The chaotic and bifurcation phenomena considering power systems excitation limit and PSS. Automation of Electric Power Systems, 25 (1):11-14, 58.

[7] Tan, T.L., Zhang, Y., Zhong, Q. (2012) Multi-parameter bifurcation analysis of AC/DC power system. Electric Power Automation Equipment, 32 (2):23-28.

[8] Liu, S.F., Gao, J.F., Li, P. (2004) Multi-parameter bifurcation analysis of excitation system with the Walve aggregated load mode. Proceeding of the CSEE, 24(12):58-62.

[9] Zang, Y. (2009) Power system voltage stability research based on bifurcation theory. Beijing: North China Electric Power University.

[10] Wang, Q.H., Zhou, S.X. (2004) Bifurcation analysis with excitation limits in classic three-node power system. Journal of North China Electric Power University, 31 (1):5-10, 14.

[11] Li, G.Q., Zhang, H., Li, J., Wang, Y.W. and Zhang, P. (2015) Influence of excitation saturation element on power system voltage stability based on bifurcation theory. Electric Power Automation Equipment 35 (3):1-5, 46.

[12] Rajesh, K.G., Padiyar, K.R. (1999) Bifurcation analysis of a three node power system with detailed models. Electrical Power and Energy System, 1(21):375-393.