Effects of size and surrounding medium on whispering-gallery-mode lasers in Er$^{3+}$-doped silica microspheres

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Abstract

This work reports the experimental fabrication and characterization of the Er$^{3+}$-doped silica microspheres (μS) and numerical investigation of the effects of size and surrounding medium on the optical properties of whispering-gallery-mode (WGM) lasers. The heat melting method of two discharge electrodes was used to produce the Er$^{3+}$-doped silica μSs of diameters up to several tens of micrometers. The 125-μm diameter single-mode optical fiber was tapered with a cone angle formed by chemical etching in hydrofluoric acid (HF) solutions. It was used to produce the μSs and couple the pumped laser into μS surface as well was coupled out the lasing emission. The WGM lasers at telecom regime of ~1520–1570 nm were characterized in both clockwise (CW) and counterclockwise (CCW) propagation directions. By adjusting the coupling gap between the tapered optical fiber and the μS surface, the selectivity of the multi- or single-emitted modes of the μS laser was achieved. We performed finite-difference time-domain (FDTD) simulations to examine the size dependence and analyze the effect of the surrounding medium’s refractive index on the optical characteristics, such as emission wavelength, intensity, as well as the shape of WGM lasing emission. The facile approach and quantitative investigation of this work has attracted much attention from researchers in the micro-photonic field and may be useful in many applications from tunable single-mode lasing sensing to optical micro-devices.

Keywords Microsphere · Whispering-gallery-mode · Single-mode · Lasers
1 Introduction

In the last few decades, optical micro-resonators have attracted a growing interest due to their special characteristics and various promising applications for quantum electrodynamics, quantum information processing, nonlinear optics, data storages, chemical or biosensing, lasers, and filters (Berman 1994; Chang and Campillo 1996; Chang et al. 2018; Oraevsky 2002; Vahala 2003; Liu et al. 2009; Righini and Soria 2016; Zhang et al. 2017; Kfir et al. 2020; Nguyen et al. 2019). Dielectric microspheres (μS), in which lights can be guided in the shape of whispering-gallery-modes (WGMs), can provide ultrahigh quality (Q) factors and small modal volumes (V) (Oraevsky 2002; Vahala 2003; Liu et al. 2009; Righini and Soria 2016; Zhang et al. 2017; Kfir et al. 2020; Nguyen et al. 2019). When doped with rare-earth ions, μSs can exhibit a lasing emission which acts as both gain media and resonators for emitted lasers. These lasing emissions were characterized and shown to have very narrow emission linewidths. The achieved Q-factors can be up to $10^{10}$, indicating very long light storage times, achieving lasing actions at low threshold pump powers (Nguyen et al. 2019; Lissillour et al. 2001; Fürst et al. 2010; Pham et al. 2013; Wu et al. 2014; Toncelli et al. 2017; Fang et al. 2017). In recent work (Nguyen et al. 2019), Er$^{3+}$-doped silica μSs with diameters of ~25–40 μm were successfully fabricated and WGM lasing emissions of single- or the multi-emitted modes in the wavelength (λ) range of 1520–1570 nm were quantitatively analyzed. The integration of these μSs with silicon-on-insulator (SOI) slotted photonic crystal (PhC) waveguides was proposed and modelled using 3D-FDTD simulations. For μS diameter of ~30 μm, the integrated system deals with the challenge of confining light inside the μS with refractive index ~1.44 when surrounded by a higher refractive indexed medium, i.e. effective refractive index of the conducted SOI PhC (~2.95 at λ ~ 1550 nm). The μS was wrapped with an air ring ~1.0 μm, which acts as a low refractive index environment of air to prevent the WGMs inside the μS from leaking out, following the total internal reflection phenomenon. Although the proposed compact structure proved successful; it requires further research to be processed for systematic understanding and applications of ultrahigh sensitivity sensors, lasing sources, and towards the advanced quantum communications. Although several studies reported the influence of surrounding medium’s refractive index on the WGMs and lasing emissions of rare-earth ions doped silica μS; they have not provided details of the influence the refractive index has on the lights guided in the form of WGMs, as well as the intensities and linewidths of lasing emissions (Righini and Soria 2016; Berneschi et al. 2020; Brice et al. 2020; Cai et al. 2020a, 2020b; Zhang et al. 2020).

In term of the relationship between the WGM resonances and the μS diameters, a dimensionless size parameter has been introduced as $x = 2\pi.R/\lambda = k.R$ (1), where $R$ is the radius of the μS and $k$ indicates the wavenumber. When the radius of the μS is much larger than the resonant wavelength ($R \gg \lambda$ or $x \gg 1$) and the light ray is at the glancing incidence of the surface of μS, the condition to have WGM resonance is equal to the optical path length. The optical path length should correspond to an integer number ($l$) of the wavelength, called the angular number as discussed in Sect. 2. In order to remain in phase $l$ must be almost equivalent to the perimeter of the μS: $2\pi.N.R = l.\lambda$ (2), where $N$ is the refractive index of the material of the μS (Little et al. 1999; Righini et al. 2011; Yu et al. 2014; Sun et al. 2017). To (1) fit much larger wavelength of WGM lasing emissions and (2) obtain the smaller integrated size scale ~100×100 μm² for minimizing the loss of the photonic integrated circuits (PICs), the Er$^{3+}$-doped silica μS with controlled diameter of ~25–40 μm was presented and discussed in our previous work (Nguyen et al. 2019). The
other studies of integration systems between the $\mu$S and different platforms have been continuously studied.

Although there numerous papers have reported the experimental and numerical quantitative capability of rare-earth ions doped silica $\mu$S in generating optical signals or the numerical investigation of the WGM characteristics of the $\mu$S resonators (Nguyen et al. 2019; Lissillour et al. 2001; Fürst et al. 2010; Pham et al. 2013; Wu et al. 2014; Toncelli et al. 2017; Fang et al. 2017; Little et al. 1999; Righini et al. 2011; Yu et al. 2014; Sun et al. 2017; Mescia et al. 2012); only few works have comprehensively performed experimental, characterization, numerical simulation as well as application in optical devices of telecom-like spectroscopy, optical switches, dense wavelength division multiplexing (DWDM) communications, and lab-on-a-chip sensors (Schliesser et al. 2005; Del’Haye, P., Schliesser, A., Arcizet, O., Wilken, T., Holzwarth, R., Kippenberg, T.J. 2007; Savchenkov et al. 2008). However, there was a drawback using narrow-linewidth emission for spectroscopy and sensors, owing to the low power per emission line and the difficulty of resolving features that are smaller than the emitted mode spacing (Del’Haye et al. 2009). In addition, the vibration of the coupling gap, which makes the loss and the instability of the laser and signal coupled into and out the $\mu$S, respectively (Nguyen et al. 2019; Gorodetsky and Ilchenko 1999; Ma et al. 2007; Sprenger et al. 2009; Watkins et al. 2012); hinders the practical application of these optical devices. Accurate numerical simulations are required to predict device performance, design, and rectification as well as to obtain unexpected properties that may be exploited in the fabrication of the complex optical systems. In this paper, the heat melting method of two discharge electrodes was used to fabricate the $\text{Er}^{3+}$-doped silica $\mu$S with controlled diameters of ~30–80 $\mu$m. Using a 125 $\mu$m diameter single-mode optical fiber, the tapered fiber tips of several micrometers were formed by chemical etching in hydrofluoric acid (HF) solutions and used to produce the $\mu$S as well as couple the pumped laser onto the $\mu$S surface and the lasing emission. The WGM lasing emission wavelength in the range of ~1520–1570 nm was characterized in the clockwise (CW) and counterclockwise (CCW) propagation directions. The selectivity of multi- or single-emitted modes of the $\mu$S laser was achieved by adjusting the coupling gap between the collection tapered fiber and the $\mu$S surface. The FDTD simulations (Lumerical FDTD Solutions xxxx; Taflove 1995; Haus 1984) calculated the size dependence and analyzed the refractive index of surrounding medium on the optical characteristics, such as emission wavelength, and intensity of WGM lasers in $\text{Er}^{3+}$-doped silica $\mu$S. The achieved results have shown that the microscopic size of ~30 $\mu$m is suitable for sensing applications that detect small changes in the surrounding medium. The shape of the WGM laser still exists and changes little with a refractive index of less than 1.20, but the shape of the WGM laser changes a lot when the refractive index is greater than 1.20. The facile approach and investigation of this work has attracted much attention from researchers in micro-photonic field and in applications from tunable single-mode lasing sensing to optical micro-devices.

2 Fabrication, characterization, and simulation details

2.1 Fabrication

Several methods have been used to fabricated the WGM micro-resonators, such as the melting rare-earth doped optical fiber tips using fusion techniques, the microwave plasma torch fusion, the electric tube furnace, the sol–gel or glass coating of silica $\mu$Ss and by
rare-earth ion implantation with different shapes (μSs, microdisks, toroids, etc.), made of various glasses (silica, telluride, phosphate, …), and various dopants (Er, Yb, Nd, Tm, …) (Chang and Campillo 1996; Righini and Soria 2016). In this paper, we developed the electrical discharge method to produce the μSs at the end of Er\(^{3+}\)-doped silica tapered fibers. The silica fibers with Er\(^{3+}\) concentration of ~2000 ppm were etched in hydrofluoric acid (HF) solutions at 25% v/v for 120 min and then 3% v/v for 45 min in order to dissolve the cladding to the naked core parts as illustrated in Fig. 1a. The achieved diameters of the tapered fiber tips were ~1.7–6.1 μm as depicted in Fig. 1b, c. The same procedures were used for the single-mode collection tapered fibers.

The schematic of the electrical discharge equipment that was used to produce the μSs is shown in Fig. 2a. It includes three main parts: controllable voltage source, electrodes, and 3D micro-precision stage. The discharge system driving voltage is controlled by a DC power supply. The potential difference between the electrodes must be larger than the breakdown voltage for a given electrode spacing and influence the heating of the fiber tip. The E-field strength and melting time are the dominant parameters that affect the size and surface roughness of the μSs. The first arc’s strength and duration, i.e., the one that forms the μS, were ~300 V/mm and ~750 ms, respectively. For the subsequent arcs, the arc strength remained unchanged, but the duration time was reduced to 100–200 ms. A detailed procedure of the heat melting method used in this work has been reported previously (Nguyen et al. 2019; Ji et al. 2011; Huang et al. 2013). In this work, the Er\(^{3+}\)-doped silica μSs with diameters of ~30–80 μm have been controlled as shown in Fig. 3 in order to (1) fit much larger wavelength of WGM lasing emissions (λ ~ 1520–1570 nm) and (2) obtain the photonic integrated circuit at size scale ~100 × 100 μm\(^2\) for minimizing the loss.

2.2 Characterization

The sketch of in-house setup for characterizing the WGM lasers of μS is illustrated in Fig. 2b. Our experiment focuses on the optical emission at L-band from Er\(^{3+}\)-doped silica μSs. The laser diode with output power ~180 mW at λ = 1470 nm (SLA5653-QD-71/CV1) was polarized at TE mode for excitation of the Er\(^{3+}\) ions. The pumped laser and the emitted lasers to/from the μS were guided by different single-mode optical tapered fibers, to

![Fig. 1](image_url) **Fig. 1** a SEM images representing the tapered fiber tips. The fiber taper has a smooth shape and its diameter gradually reduces to ~1.7—6.1 μm b, c, where it forms an almost sphere
ensure that the pumped light does not change the effective pumped power that contributes to lasing. This technique, which show simple, reproducible, and reversible procedure, has allowed for flexibility in controlling the coupling gap between the collection fiber and

Fig. 2 Schematic illustrations of the a electrical discharge equipment to produce the μS at the end of Er3+-doped glass silica tapered fiber; b tapered fiber coupled μS optical measurement system and c three lowest energy-level diagram of Er3+ ion

Fig. 3 SEM images of the Er3+-doped silica glass μSs with diameters of ~30–80 μm. a 29.7 μm, b 38.5 μm, c 60.1 μm, and d 79.8 μm
μS surface, when the position of excited fiber is fixed. It allows not only changing from multi- to single-emitted modes of the collection fiber, but also-more importantly—the ability chooses a desired single-emitted mode among several. The spectral characteristics of the lasing emissions were characterized by the optical spectrum analyzer (OSA)-Advantest Q8384 with 0.01 nm resolution. The coupling gap was adjusted by a 3D micro-precision stage (PZT-AE0203D08F) with the accuracy of 0.10 μm. Two propagation directions were measured (CW and CCW).

In the measured configuration depicted in Fig. 2c, pumped laser at λ = 1470 nm takes the excited state \(^4\)I\(_{13/2}\) from the ground state \(^4\)I\(_{15/2}\). Then the stimulated emission has been obtained in the wavelength range of ~1520–1570 nm. The quantitative of rare earth interaction, mechanism of “lasing action” and laser output power versus incident pumped power characteristics have been reported previously (Mescia et al. 2012; Jung et al. 2010; Subramaniam 2015; Pham et al. 2005) and are not discussed in detail here.

2.3 Theoretical approach and simulation

As in the well-known WGM resonators, light is confined in a band around of a great circle of the μS and defined as the caustic region comprised between the outer and the inner μS, to which the propagating and bouncing rays are tangent. Equation (2) clearly shows that changes to either the radius or refractive index of the material of the μS will change the resonance of the optical mode. The resonant wavelength changes is given (Righini and Soria 2016):

\[
\frac{Δλ}{λ} = \frac{ΔR}{R} = \frac{ΔN_s}{N_s}
\]  

Equation (3) indicates that any small change in the surrounding medium’s refractive index of \(N_s\) or in the radius \(R\) of the μS must be balanced by a small change in the resonant wavelength.

Several analytical models were used to realize the WGMs of the μS (Little et al. 1999; Schiller 1993; Spillane et al. 2003). A simplified 2D model for understanding the physical and optical properties of the 3D optically coupled μS systems was considered. Schiller’s approximation was used to predict the resonant characteristics of the μS due to its high accuracy and straightforward calculation procedure (Schiller 1993). It is well known that the optical WGM is formed by repeated total internal reflections of light, which is constrained on the μS surface. Three mode numbers used, are described as: angular (\(l\)), azimuthal (\(m\)), and radial (\(n\)). The angular number, \(l\), indicates the number of modal wavelengths that fit into the circumference of the equatorial plane of the μS; the radial number, \(n\), presents the number of intensity maxima along the radial direction. Therefore, \(2l\) is the number of the field maxima in the angular variation around the μS equator. The azimuthal mode number, \(m\), indicates the field variation in the polar direction, with the number of intensity maxima along this direction being equal to \(l—|m|+1\). In practice, we are interested in exciting low-order WGMs with small \(n\) and with \(m ≈ l\) for maximizing the resonant advantages of the μS. These modes have the optical field distributions near the μS surface, which are approximated to the equator and the electric fields are compressed into the smallest modal volume.

Our work uses a dielectric μS which has the refractive index and diameter of 1.44 and \(2R\) in the medium of refractive index of \(N_s\), respectively. Due to its individual nature, a cubic region with dimension of larger than \(2R \times 2R \times 2R\) was used for calculation of the
whole structure. The problem of the light interacting with such structure was described by Maxwell’s equations. In this work, the simulations were carried out using commercial FDTD software package LUMERICAL (Lumerical FDTD Solutions xxxx; Taflove 1995; Haus 1984). The excitation of TE polarized point-source electric dipoles in the vicinity of the µS surface have been taken for the WGM-µS lasers. To avoid unexpected noise waves reflected from the computational domain walls, the perfectly matched layers (PML) were set at surrounding boundaries. An intensity monitor that collects the emission pattern from the µS was placed 1.0 µm behind the surface.

3 Results and discussion

To couple light into the µS efficiently, the evanescent wave mode of the tapered fiber must match the WGM of the µS. When the Er³⁺-doped silica µS is adequately coupled with the tapered fiber, multi-mode emission can be achieved; in which a single-mode emission can be observed (Nguyen et al. 2019; Pham et al. 2013). In the measurement, single- and multi-mode emissions were measured and the orientations of the WGM orbits have been determined by the collected tapered fiber positions. As shown in Fig. 4a, the µS with the diameter of ~29.7 µm and coupling gap of 1.5 ± 0.1 µm, produced multi-mode lasing emissions with spacing of ~2.5–4.3 nm and side mode suppression ratios (SMSR) of ~14.1–27.1 dB. As it is shown, these emissions have the CW sense of circulation (red curve). It means that there is strong coupling and high contrast in the CW configuration, which is referred to as

![Fig. 4](image)

*Fig. 4* WGM emission spectra of the Er³⁺-doped silica glass µS with diameters of 29.7 µm (*a, b*) and 79.8 µm (*c, d*). The coupling gap between the tapered fiber and the µS surface changes from *a* 1.5 ± 0.1 µm to *b* 0.7 ± 0.1 µm, 0.5 ± 0.1 µm, and 0.3 ± 0.1 µm; *c* 1.5 ± 0.1 µm and 1.2 ± 0.1 µm to *d* 0.3 ± 0.1 µm. The CW and CCW configurations are indicated in Fig. 2b
the in-phase matched coupling (Righini et al. 2011; Dong et al. 2008). For example, the lasing emission at \( \sim 1547.31 \) nm: (1) the optical powers are \(-30.0 \) dBm and \(-49.2 \) dBm and (2) the SMSRs are 27.1 dB and 17.6 dB for CW and CCW geometries, respectively. The optical characteristics of the multi-emitted modes from two propagation directions of CW and CCW geometries for \( \mu S \) diameter of 29.7 \( \mu m \) are summarized in Table 1.

To explore the characteristics of the single WGMs, when the coupling gap \( g \) is smaller than \( \sim 0.7 \) \( \mu m \), we have collected its output power by precise adjustment of the coupling gap. Due to the vibration of the fiber tip caused from the dipole-field interaction effect and the force of the field in the tangential direction as it approaches the \( \mu S \) surface, it is not an effective way to make the coupling gap less than 0.15 \( \mu m \) (Nguyen et al. 2019; Pham et al. 2013; Treussart et al. 1994). In the presented scheme, the diameter of \( \mu S \) and pumped power remained as those indicated in Fig. 4a, while Fig. 4b shows the single-mode spectra extracted from the CW configuration for several coupling gaps of \( 0.7 \pm 0.1 \) \( \mu m \) (black curve), \( 0.5 \pm 0.1 \) \( \mu m \) (red curve), and \( 0.3 \pm 0.1 \) \( \mu m \) (blue curve). As the collection tapered fiber approaches the \( \mu S \) surface, higher output power and SMSR could be achieved. As it is shown, the output power and SMSR are \(-24.7 \) dBm and \(18.0 \) dB for the coupling gap of \( 0.7 \pm 0.1 \) \( \mu m \), while they could be as high as \(-11.0 \) dBm and 29.0 dB for the coupling gap of \( 0.3 \pm 0.1 \) \( \mu m \), respectively. This technique has good reproducibility in practice and extracts most of the WGMs that can oscillate in the \( \mu S \) with a suitable adjustment of the coupling gap. It is important to note that the comprehension of the results is not as simple as mentioned in several published works (Pham et al. 2013; Righini et al. 2011; Gorodetsky and Ilchenko 1999).

The measurements of lasing emissions of the other \( \mu S \) diameters were also considered and exhibited the same tendencies. As shown in Fig. 4c, WGM emission spectra extracted from Er\(^{3+}\)-doped silica \( \mu S \) with 79.8 \( \mu m \) diameter in the CW geometry for the coupling gaps \( g = 1.5 \) \( \mu m \) (red curve) and 1.2 \( \mu m \) (blue curve). As it is shown, the multi-mode lasing emissions for two coupling gaps have been observed. The number of lasing emissions was reduced for a narrower coupling gap. Table 2 shows the characteristics of the multi-mode emissions from CW geometry for \( \mu S \) diameters of 79.8 \( \mu m \) for coupling gaps of 1.5 \( \mu m \) and 1.2 \( \mu m \). When the coupling gap \( g \) was smaller than \( \sim 0.7 \) \( \mu m \), the single-mode lasing

| Table 1 Characteristics of the multi-mode lasing emissions from CW and CCW configurations for \( \mu S \) diameter of 29.7 \( \mu m \) |
|---------------------------------|---|---|---|---|---|---|---|
| **CW geometry**                |   |   |   |   |   |   |   |
| Emitted wavelength, \( \lambda \) (nm) | 1540.66 | 1544.10 | 1547.31 | 1550.84 | 1554.20 | 1556.68 | 1560.93 |
| Power (dBm)                     | -32.7 | -35.5 | -30.0 | -33.5 | -31.0 | -39.1 | -43 |
| FWHM (nm)                       | 0.07 | 0.08 | 0.03 | 0.07 | 0.07 | 0.05 | 0.04 |
| SMSR (dB)                       | 24.6 | 22.4 | 27.1 | 22.9 | 25.8 | 18.5 | 14.1 |
| **CCW geometry**               |   |   |   |   |   |   |   |
| Emitted wavelength, \( \lambda \) (nm) |   |   |   |   |   | 1544.91 |   |
| Power (dBm)                     |   | -58.8 | -49.2 | -51.8 | -62.9 | -56.3 |   |
| FWHM (nm)                       |   | 0.07 | 0.07 | 0.05 | 0.06 | 0.06 |   |
| SMSR (dB)                       |   | 9.0 | 17.6 | 14.6 | 4.5 | 10.4 |   |

*FWHM full-width at half-maximum, SMSR side mode suppression ratio*
emission was also achieved. The single-mode emitted laser of the Er$^{3+}$-doped μS with diameter of 79.8 μm for the CW geometry and coupling gap of 0.3±0.1 μm is shown in Fig. 4d. As it is shown, the optical output power and SMSR could be as high as -10.2 dBm and 30.0 dB, respectively.

For visualizing the modes and fields, the 3D-FDTD method combined with PML were implemented to simulate the radiation patterns of WGMs from μS (Lumerical FDTD Solutions xxxx; Taflove 1995; Haus 1984). As mentioned above, the electric-dipole emitters located near the surface and oriented along the radials of the μS, produced the ~20 fs sinusoidal pulses centered at emission wavelengths. Figure 5a–c show the WGM $H$-field distributions for diameters 29.7 μm, 38.5 μm, and 79.8 μm; their refractive index of 1.44 in air, at the wavelengths of 1550.84 nm, 1550.92 nm, and 1551.03 nm, respectively. The pair of mode parameters for WGMs of ($l$ = 66, $n$ = 4) in Fig. 5a, ($l$ = 85, $n$ = 4) in Fig. 5b, and ($l$ = 177, $n$ = 6) in Fig. 5c were determined by counting the number of $H$-field intensity maxima. Figure 5d demonstrates the $E$-field intensity corresponding to the WGM-μS for 29.7 μm diameter, where the light is well confined in the equatorial plane.

To confirm the FDTD simulations, the modes and fields of the WGMs of μS have been realized by the approximate expressions described by Schiller (Schiller 1993). The spherical mode is described conventionally in terms of three integer numbers $l$, $m$, and $n$. The value $l – m + 1$ is the number of field maxima in the polar direction, normal to the equatorial plane and between two poles. The value of $n$ counts the number of field maxima in the radial direction. At the equatorial plane, $m = l$, the “fundamental mode”, the resonant wavelength is determined by the value of $l$ and $n$. The values of $l$ that correspond to various $n$ for the μS with diameters of 29.7 μm, 38.5 μm, 60.1 μm, and 79.8 μm; refractive indices $N = 1.44$ and $N_c = 1.0$ in the wavelength of interest were calculated. For the considered diameters here, the dependence of resonant wavelength on ($l$, $n$) for TE mode can be expressed as,

\[
N \lambda_1^l = l + \frac{1}{2} - \zeta_n \sqrt{\frac{l + 0.5}{2}} - \frac{q}{\sqrt{q^2 - 1}} + \frac{3 \zeta_n^2}{2^{3/3} 10^{l + 0.5}} + \frac{1}{2^{1/3} \sqrt{(q^2 - 1)^3 \sqrt{(l + 0.5)^2}}} \tag{3}
\]
where $\lambda_n^l$ is resonant wavelength, which is well identified with the pair $(l, n)$; $\zeta_n$ denotes the $n^{th}$ zero of the Airy function $Ai(\zeta_n)$, the field intensity distribution or the total number of field strength spikes, either positive or negative, or in the radial direction and $q = N/N_S$ (the ratio of the refractive index of $\mu S$ material and the surrounding medium). Normally for a given WGM, the $n$ number is a small integer. In our research, Eq. (3) was used to calculate the $l$ parameter using the resonant wavelength $\lambda$ when a given $n$ is known. The calculated $(l, n)$ pairs are listed in Table 3. A more intuitive way to determine the pairs $(l, n)$ is from the simulation in which the field maxima could be seen in artistic pictures and those

![Fig. 5](image)

Fig. 5 Magnetic field distributions in the WGM plane for the wavelengths ($\lambda$) and $\mu S$'s diameters ($\phi$): a $\lambda \sim 1554.20$ nm and $\phi = 29.7$ $\mu$m; b $\lambda \sim 1554.70$ nm and $\phi = 38.5$ $\mu$m; c $\lambda \sim 1555.20$ nm and $\phi = 79.8$ $\mu$m. The E-field magnitude of the $\mu S$ with diameter of 29.7 $\mu$m for the wavelength of $\sim 1554.20$ nm is shown in (d).

| No. | $\mu S$ diameter (µm) | Schiller approximation | FDTD simulation |
|-----|------------------------|------------------------|-----------------|
| 1   | 29.7                   | (67, 4)                | (66, 4)         |
| 2   | 38.5                   | (86, 4)                | (85, 4)         |
| 3   | 60.1                   | (135, 5)               | (133, 5)        |
| 4   | 79.8                   | (175, 6)               | (177, 6)        |
parameters could be counted along the circumference and diameter of the μS. These values were also listed in Table 3 and are in good agreement. However, the simulation resolution and the analytical approximation condition will cause some discrepancies.

WGM-μS have been used extensively for temperature, humidity, and stress sensing. The working principle of WGM-μS sensor is that the emission wavelength or intensity of the output signal varies with the change in external environment. Here we consider the influence of the surrounding medium on the optical characteristics of the WGM-μS, such as the wavelength and intensity of lasing emission. As mentioned above, there is no influence of changing refractive index on the pumped light that couples to Er$^{3+}$-doped silica μS surface. Figure 6 illustrates the dependences of the wavelength and intensity of the emitted laser on the surrounding medium’s refractive index. For the given emission mode, as the ambient refractive index increases, the displacement wavelength shall be towards longer wavelengths and the emission intensity decreases. For example, when the ambient refractive index increases from air to 1.20, the emission wavelength changes from 1554.20 to 1581.0 nm and from 1554.47 to 1565.61 nm (the variation in wavelength is 26.8 nm and 11.14 nm for 0.2 refractive index units), corresponding to the μS diameters of 29.7 μm and 79.8 μm, as shown in Fig. 6a, b, respectively. In addition, the emission intensity also decreases as the refractive index of the medium increases. As Fig. 6 depicts, the wavelength and emission intensity of lasing emission are quite sensitive to the surrounding environment for the small μS size. Therefore, the microscopic size of ~30 μm is suitable for sensing applications that detect small changes in the surrounding medium.

In order to see the influence of the surrounding medium on the shape and intensity of the WGM lasing emission, the field distributions in the equatorial plane were simulated as shown in Figs. 7 and 8. Figure 7 shows the field profiles in the equatorial plane for μS diameter of 29.7 μm immersed in different refractive index media: (a) air ($\lambda \sim 1554.20$ nm); (b) $N_S = 1.04$ ($\lambda \sim 1557.28$ nm); (c) $N_S = 1.05$ ($\lambda \sim 1558.18$ nm); (d) $N_S = 1.10$ ($\lambda \sim 1563.81$ nm). The shape of WGM exists and changes little with refractive indices less than 1.05, but they change a lot when the refractive index is greater than 1.05 although the emission intensity is insignificantly low.

Same to Figs. 7 and 8 shows the distribution of magnetic fields in the equatorial plane for μS diameter of 79.8 μm in different refractive index media: (a) air ($\lambda \sim 1555.20$ nm); (b) $N_S = 1.10$ ($\lambda \sim 1558.40$ nm); (c) $N_S = 1.20$ ($\lambda \sim 1565.61$ nm); (d) $N_S = 1.30$ ($\lambda \sim 1571.70$ nm). The shape of the WGM still exists and changes little with

![Fig. 6](image)

Fig. 6 Wavelength and real part of electric field at resonances of various refractive indices of surrounding media for the μS with diameters of a 29.7 μm and b 79.8 μm
a refractive index of less than 1.20, but the shape of the WGM changes a lot when the refractive index is greater than 1.20.

The Er$^{3+}$-doped silica $\mu$Ss in air surrounding medium achieved ultrahigh $Q$-factors and finesses because they can trap light via near-total internal reflection using ultrasmooth silica interface. When the refractive index of the surrounding medium increases, it makes the radiation loss increases and lower $Q$-factor has been achieved. The decayed emission intensities and deformed shapes of the WGM-$\mu$S lasers for increasing the ambient refractive indices have been achieved. When the refractive index exceeds 1.20, the emission intensity significant deforms due to the low $Q$-factor of the Er$^{3+}$-doped silica $\mu$S as shown in Figs. 7d, 8d.

### 4 Conclusions

In conclusion, the Er$^{3+}$-doped silica $\mu$S with diameters of ~ 30–80 $\mu$m were fabricated by using an electrical discharge method and their WGM lasing emissions at the telecom regime were quantitatively analyzed. In the measurement, the coupling gap between the
collection tapered fiber and the μS surface resulted in the single- or the multi-emitted modes. The FDTD simulations were performed to examine the size dependence and analyze the refractive index of surrounding medium on the optical characteristics, such as emission wavelength and intensity of WGM lasers in Er$^{3+}$-doped silica μS, which justifies the observation of the lasing emission modes. The demonstrated approach of this work may be beneficial in applications from tunable single-mode lasing and sensing to micro-devices.

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**Declarations**

**Conflict of interest** The authors declare no conflicts of interest.
References

Berman, P.R.: Cavity Quantum Electrodynamics. Academic Press, New York (1994)

Berneschi, S., Bettazzi, F., Giannetti, A., Baldini, F., Conti, G.N., Pelli, S., Palchetti, I.: Optical whispering gallery mode resonators for label-free detection of water contaminants. Trends Anal. Chem. 126, 115856 (2020)

Brice, I., Grundsteins, K., Atvars, A., Alnis, J., Viter, R., Ramanavicius, A.: Whispering gallery mode resonator and glucose oxidase based glucose biosensor. Sens. Actuators B: Chem. 318, 128004 (2020)

Cai, L., Pan, J., Zhao, Y., Wang, J., Xiao, S.: Whispering gallery mode optical microresonators: Structures and sensing applications. Phys. Status Solidi A 217, 1900825 (2020a)

Cai, L., Pan, J., Hu, S.: Overview of the coupling methods used in whispering gallery mode resonator systems for sensing. Opt. Lasers Eng. 127, 105968 (2020b).

Chang, R.K., Campillo, A.J.: Optical Processes in Microcavities, Advanced Series in Applied Physics, vol. 3. World Scientific, Singapore (1996)

Chang, P., Li, X., Huang, L., Gao, F., Zhang, W., Bo, F., Zhang, G., Xu, J.: Fast light in the generation configuration of stimulated Brillouin scattering based on high-Q micro-cavities. Opt. Express 26(12), 15377–15383 (2018)

Del’Haye, P., Schliesser, A., Arcizet, O., Wilken, T., Holzwarth, R., Kippenberg, T.J.: Optical frequency comb generation from a monolithic microresonator. Nature 450, 1214–1217 (2007)

Del’Haye, P., Arcizet, O., Gorodetsky, M.L., Holzwarth, R., Kippenberg, T.J.: Frequency comb assisted diode laser spectroscopy for measurement of microcavity dispersion. Nat. Photon. 3, 529–533 (2009)

Dong, C., Xiao, Y., Yang, Y., Han, Z., Guo, G., Yang, L.: Directly mapping whispering gallery modes in a microsphere through modal coupling and directional emission. Chin. Opt. Lett. 6, 300 (2008)

Fang, Z., Chormaic, S.N., Wang, S., Wang, X., Yu, J., Jiang, Y., Qiu, J., Wang, P.: Bismuth-doped glass microsphere lasers. Photon. Res. 5(6), 740–744 (2017)

Fürst, J.U., Strekalov, D.V., Elser, D., Lassen, M., Andersen, U.L., Marquardt, C., Leuchs, G.: Naturally phase-matched second-harmonic generation in a whispering-gallery-mode resonator. Phys. Rev. Lett. 104, 153901 (2010)

Gorodetsky, M.L., Ichenko, V.S.: Optical microsphere resonators: optimal coupling to high-Q whispering-gallery modes. J. Opt. Soc. Am. B 16(1), 147–154 (1999)

Haus, H.A.: Waves and Fields in Optoelectronics. Prentice-Hall, Englewood Cliffs (1984)

Huang, Y.T., Guo, C.L., Huang, Y., Zhang, P.J.: Ytterbium-doped silica microsphere laser. Appl. Mecha. Mater. 278–280, 1063–1067 (2013)

Ji, H., Chua, J., Hsu, H.-Y., Wedding, A.B.: Development of a fabrication process for the manufacturing of a microspherical probe for coordinate measuring machine applications. J. Mecha. Design 133(11), 111003 (2011)

Jung, M., Shin, J.H., Jhon, Y.M., Lee, J.H.: A theoretical and experimental investigation into pair-induced quenching in bismuth oxide-based Erbium-doped fiber amplifiers. J. Opt. Soc. Korea 14(4), 298–304 (2010)

Kfir, O., Lorencço-Martins, H., Storeck, G., Sivis, M., Harvey, T.R., Kippenberg, T.J., Feist, A., Ropers, C.: Controlling free electrons with optical whispering-gallery modes. Nature 582, 46–50 (2020)

Lissillour, F., Messager, D., Stéphan, G., Féron, P.: Whispering-gallery-mode laser at 1.56μm excited by a fiber taper. Opt. Lett. 26(14), 1051–1053 (2001)

Little, B.E., Laine, J.-P., Haus, H.A.: Analytic theory of coupling from tapered fibers and half-blocks into microsphere resonators. J. Lightwave Technol. 17(4), 704–715 (1999)

Liu, J., Ngo, Q.M., Park, K.H., Kim, S., Ahn, Y.H., Park, J.Y., Koh, K.H., Lee, S.: Optical waveguide and cavity effects on whispering-gallery mode resonances in a ZnO nanonail. Appl. Phys. Lett. 95, 221105 (2009)

Lumerical FDTD Solutions.: Lumerical Solutions, Inc. https://www.lumerical.com/

Pham, V.H., Bui, H., Pham, T.S., Nguyen, T.A., Nguyen, T.V., Le, H.T., Bui, T.N., Nguyen, V.P., Coisson, R.: Control of whispering-gallery-mode spectrum from erbium-doped silica microsphere lasers. J. Opt. Soc. Am. B 30(6), 1586–1589 (2013)
Pham, V.H., Chu, T.T.H., Hoang, Q.H.: Long-band emission of microsphere lasers based on erbium-doped sol-gel silica-alumina glasses. Appl. Phys. Lett. 87, 161110 (2005).
Righini, G.C., Soria, S.: Biosensing by WGM microspherical resonators. Sensors 16, 905 (2016)
Righini, G., Dumeige, Y., Férón, P., Ferrari, M., Conti, G.N., Ristic, D., Soria, S.: Whispering gallery mode microresonators: Fundamentals and applications. Rivista Del Nuovo Cimento 34(7), 435–488 (2011)
Savchenkov, A.A., Matsko, A.B., Ilchenko, V.S., Solomatine, I., Seidel, D., Maleki, L.: Tunable optical frequency comb with a crystalline whispering gallery mode resonator. Phys. Rev. Lett. 101, 093902 (2008)
Schiller, S.: Asymptotic expansion of morphological resonance frequencies in Mie scattering. Appl. Opt. 32(12), 2181–2185 (1993)
Schliesser, A., Brehm, M., Keilmann, F., van der Weide, D.W.: Frequency-comb infrared spectrometer for rapid, remote chemical sensing. Opt. Exp. 13(22), 9029–9038 (2005)
Spillane, S.M., Kippenberg, T.J., Painter, O.J., Vahala, K.J.: Ideality in a fiber-taper-coupled microresonator system for application to cavity quantum electrodynamics. Phys. Rev. Lett. 91, 043902 (2003)
Sprenger, B., Schwefel, H.G.L., Wang, L.J.: Whispering-gallery-mode-resonator-stabilized narrow-linewidth fiber loop laser. Opt. Lett. 34(21), 3370–3372 (2009)
Subramaniam, T.K.: Erbium doped fiber lasers for long distance communication using network of fiber optics. Am. J. Opt. Photon. 3(3), 34–37 (2015)
Sun, H., Chen, X., Wang, H., Lu, Q., Yang, H., Xie, S., Wu, X.: Fabrication of lasing whispering gallery mode microresonators by controllable injection method. IEEE Photon. J. 9(3), 1503006 (2017)
Taflove, A.: Computational Electrodynamics. Artech House, Boston (1995)
Toncelli, A., Capuj, N.E., Garrido, B., Sledzinska, M., Sotomayor-Torres, C.M., Tredicucci, A., Navarro-Urrios, D.: Mechanical oscillations in lasing microspheres. J. Appl. Phys. 122, 053101 (2017).
Treussart, F., Hare, J., Collot, L., Lefèvre, V., Weiss, D.S., Sandoghdar, V., Raimond, J.M., Haroche, S.: Quantized atom-field force at the surface of a microsphere. Opt. Lett. 19(20), 1651–1653 (1994)
Vahala, K.J.: Optical microcavities. Nature 424, 839–846 (2003)
Watkins, A, Ward, J., Chormaic, S.N.: Thermooptical tuning of whispering gallery modes in erbium: ytterbium doped glass microspheres to arbitrary probe wavelengths. Jpn. J. Appl. Phys. 51(5R), 052501 (2012)
Wu, T., Huang, Y., Huang, J., Huang, Y., Zhang, P., Ma, J.: Laser oscillation of Yb3+:Er3+ co-doped phosphosilicate microsphere. Appl. Opt. 53(21), 4747–4751 (2014)
Yu, H., Huang, Q., Zhao, J.: Fabrication of an optical fiber micro-sphere with a diameter of several tens of micrometers. Materials 7, 4878–4895 (2014)
Zhang, C., Cocking, A., Freeman, E., Liu, Z., Tadigadapa, S.: On-chip glass microspherical shell whispering gallery mode resonators. Sci. Rep. 7, 14965 (2017)
Zhang, Y.-N., Zhu, N., Zhou, T., Zheng, Y., Shum, P.P.: Research on fabrication and sensing properties of fiber-coupled whispering gallery mode microsphere resonator. IEEE Sens. J. 20(2), 833–841 (2020)

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