Model of local optimal control for technological modes in electric power associations

A G Ponomarev

Institute of Computer Science and Technology, Peter the Great St. Petersburg Polytechnic University, 29 Polytechnicheskaya, St.Petersburg, 195251, Russia

E-mail: alexey.g.ponomarev@gmail.com

Abstract: The task of controlling the technological modes of electric power associations is solved in the form of a problem of locally optimal control under constraints. A nonlinear conditional optimization operator is proposed for the solution. Mathematical models of the energy system, technological requirements, control tasks are formulated and analytical methods are proposed.

1. Model of electric power association (EPA)
We consider linearized models of electric power systems in the form of vector-matrix equations of the control object. An example of a system of equations of electromechanical processes with controllers can be represented as follows [1-3]:

\[
\dot{\varphi}_i = \omega_i, \\
\dot{\omega}_i = -\frac{1}{T_i^{(M)}} \sum_{j=1}^{n} \rho_{ij} (\varphi_i - \varphi_j) - \frac{R_i}{T_i^{(M)}} \omega_i + \frac{1}{T_i^{(M)}} p_i - \frac{1}{T_i^{(M)}} \mu_i, \\
\dot{p}_i = -\frac{1}{T_i^{(S)}} p_i + \frac{1}{T_i^{(S)}} q_i, \\
\dot{q}_i = -\frac{k_i}{T_i^{(C2)}} \omega_i - \frac{1}{T_i^{(C2)}} q_i + \frac{1}{T_i^{(C2)}} \sigma_i, \\
\dot{\sigma}_i = -\frac{1}{T_i^{(C2)}} \sigma_i + \frac{1}{T_i^{(C2)}} u_i.
\]

The first two equations describe the dynamics of the rotation frequency of the rotor of the th generator (equivalent unit for the i-th thermal electric power system area). Variables: \( \varphi_i \) - the deviation of the absolute angle of the i-th rotor from the base angle; \( \omega_i \) - the deviation of i-th frequency from the set-point, \( p_i \) - the deviation of power transmitted to the i-th generator; \( \mu_i \) - unplanned power load. Parameters: \( T_i^{(M)} \) - reduced mechanical inertia constant, \( R_i \) - rotor damping constant; \( \rho_{ij} \) - specific sync momentum (power) between i-th and j-th equivalent generating units. In
this case $S_{ij} = \sum_{j=1}^{n} \rho_{ij}(\varphi_i - \varphi_j)$ represents the tie-line power flow between the $i$-th and $j$-th EPA areas.

The third equation relates the power $p_i$ and the heat carrier (steam) power $q_i$. Here $T_i^{(S)}$ is the steam-power equation time constant. The fourth equation corresponds to the primary governor, where $\sigma_i$ is the secondary governor signal, $T_i^{(C1)}$ is the time constant of the primary turbine governor, and $k_{\omega}$ is the gain of the turbine speed primary governor. The last equation describes the secondary turbine governor, $u_i$ is the controlling signal, $T_i^{(C2)}$ is the time constant of the secondary turbine governor.

We suppose that in one step of discrete time the turbine governors provide a quasi steady state for variables, thus setting the derivatives in the equation (1) to zero we get $p_i = k_{\omega} \sigma_i + u_i$.

By specifying the state vector of the whole EPA and the vectors of control signals and loads in the following form [4,5]:

$$X = [\varphi_1, ..., \varphi_n, \omega_1, ..., \omega_n]^T, \quad U = [u_1, ..., u_n], \quad M = [\mu_1, ..., \mu_n]^T$$

we can obtain the matrix-vector representation of the control object in continuous or discrete time. Based on the differential equations, discrete-time equations of the object can be obtained in the form [4]:

$$X(k+1) = HX(k) + F_U U(k) - F_M M, \quad Y(k) = CX(k), \quad X(0) = X_0$$

where the vector $X(k)$ characterizing the state of the system corresponds to the state vector in the form of (2), and the vector of the output coordinates $Y(k)$ is the vector of frequencies and active power flows in the EPA tie-lines.

Similar models can be obtained for the EPA with the hydraulic units or with the other structure of the unit control system if the requirements of linearity of regulators are met.

2. EPA transient processes control

When an unplanned load $M$ occurs in the system, a transient process begins, which is regulated by the controls $U$. After completion of transient processes a new quasi-steady state mode is established, and the resulting state vector $X^*$ remains constant under the action of the resulting control $U^*$:

$$X^* = HX^* + F_{U} U^* - F_{M} M, \quad Y^* = CX^*.$$ (4)

From this point the task of the external control is to calculate control signals $u(k)$ to reduce output vector $Y(k)$ to set values. Thus the current deviation of the state vector from the established steady state is described by the linear equation in discrete time:

$$x(k+1) = Hx(k) + F_{U} u(k)$$ (5)

According to the well-known load-frequency control principle (LFC) [2,6] control signals are to be calculated according to the unplanned load $M$, which in turn can be estimated based on area control error (ACE)

$$\lambda_i(k) = (\omega_i(k) - \omega_{0}) (k_{\omega} + R_i) + \sum_{j} (S_{ij}(k) - S_{ij})$$ (6)

where $\omega_{0}$ is the frequency set-point and $S_{ij}$ are the set-points of power flows.
Let the controls be formed according to the law \( \sum_i u_i(k) = K \lambda(k) \), where \( \lambda(k) = \sum_{i} \lambda_i(k) \), which can be written as \( Iu(k) = K \lambda(k) \), \( I = (1, ..1) \).

3. Technological requirements for the EPA modes.
During the transitional process it is necessary to fulfill the technological requirements of validity and optimality to the modes of electric power associations.

The validity requirements are restrictions on the current values of controls and the active power flows in th EPA lines caused by them at the next time point.

\[
U(k) \in [U_{\text{min}}, U_{\text{max}}], \quad Y(k+1) \in [Y_{\text{min}}, Y_{\text{max}}]
\]

(7)

where do restrictions for the control signals follow:

\[
u(k) \in [u_{\text{min}}, u_{\text{max}}], \quad Cx(k+1) \in [y_{\text{min}}, y_{\text{max}}]
\]

(8)

The requirements of optimality consist in minimizing the quadratic quality-functional which characterizes the degree of deviation of frequencies, power flows and controls from the set-points:

\[
J(k) = (y(k+1) - y_0)^T Q_1 (y(k+1) - y_0) + (u(k+1) - u_0)^T Q_2 (u(k+1) - u_0)
\]

(9)

where \( Q_1, Q_2 \) are diagonal matrices of the weight factors.

Thus, the task of discrete local optimal control of the modes of EPA can be formulated as follows: to calculate at the each point of discrete time \( k \) the required controls \( u(k) \), providing compensation of total unplanned load and satisfying the validity and optimality requirements:

\[
u(k) = \arg \min \{ J(k) : Iu(k) = K \lambda(k); Cx(k+1) \in [y_{\text{min}}, y_{\text{max}}]; u(k) \in [u_{\text{min}}, u_{\text{max}}] \}
\]

(10)

4. Calculation of the local optimal controls.
To solve the problem we propose the following approach. We introduce the vector of extended state coordinates in the form of

\[
z(k) = \begin{pmatrix} y(k+1) \\ u(k) \end{pmatrix}
\]

(11)

then the EPA dynamics equation for deviations from the quasi-static mode can be rewritten as

\[
y(k+1) - CF_u u(k) = CHx(k)
\]

(12)

and taking into account the power balance requirements \( Iu(k) = K \lambda(k) \), the extended coordinate vector requirements taking the form

\[
Az(k) = \begin{pmatrix} E & -CF_u \\ 0 & I \end{pmatrix} \begin{pmatrix} y(k+1) \\ u(k) \end{pmatrix} = \begin{pmatrix} CHx(k) \\ K \lambda(k) \end{pmatrix} = b(k)
\]

(13)

where \( E \) is identity matrix.

Validity requirements for the technological mode can be written as

\[
z(k) \in [z_{\text{min}}, z_{\text{max}}], \quad z_{\text{min}} = \begin{pmatrix} y_{\text{min}} \\ u_{\text{min}} \end{pmatrix}, \quad z_{\text{max}} = \begin{pmatrix} y_{\text{max}} \\ u_{\text{max}} \end{pmatrix}
\]

(14)

and optimality requirement as
\[ J(k) = (z(k) - z_0)^T Q(z(k) - z_0) \rightarrow \min \] (15)

The task of the local optimal control is formulated as: to find

\[ z(k) = \arg \min \{ J(k) : Az(k) = b(k), z(k) \in [z_{\text{min}}, z_{\text{max}}] \} \] (16)

In the case of additional requirements for control the problem is formulated in a similar way with the necessary modification of the matrices \( A, b(k) \).

This problem is a minimization problem for a quadratic functional at the intersection of a linear manifold and parallelepiped. The following considerations are suggested for solving the problem.

The matrix \( Q \) is reduced to identity matrix by a suitable linear transformation of coordinates \( z = G \bar{z} \). As a result, the functional takes the form

\[ J(k) = \| z(k) - \bar{z} \|^2 \] (17)

where \( \| \cdot \| \) is Euclidean norm and the parallelepiped is approximated by a included sphere \( \| z(k) - \bar{z}_c \| \leq R \).

The task is to find the point \( \bar{z}(k) \) of Euclidean space closest to the point \( \bar{z}_0 \) and laying within the intersection of the linear manifold (hyperplane) \( \tilde{A}z(k) = b(k) \) and sphere \( \| z(k) - \bar{z}_c \| \leq R \), that is, to find the projection of the point \( \bar{z}_0 \) on the hyperplane-sphere intersection. The existence of non-empty intersection corresponds to the condition for the compatibility of technological requirements for the EPA mode.

Such a problem can be solved analytically using the finite-dimensional optimization projection operator as described in [7]:

\[ \bar{z}(k) = P(z_0, z_c, R, \tilde{A}, b(k)) \] (18)

This operator is non-linear and depends on vector \( b(k) \), which in turn depends on the current state vector \( x(k) \). Therefore, the predicted control vector \( u(k) = \Phi(x(k)) \) is obtained, where

\[ \Phi(x(k)) = TP(z_0, z_c, R, \tilde{A}, b(x(k)), \quad T = (0, E) \] (19)

5. Stability of the control process

The control system closed by feedback thus takes the form

\[ x(k + 1) = Hx(k) + B_U \Phi(x(k)) \] (20)

We suppose that \( x^* \) is a fixed point of the mapping, i.e. the root of the vector non-linear equation:

\[ x^* = Hx^* + B_U \Phi(x^*) \] (21)

and thus the current deviation of the state-vector from the fixed point follows as

\[ \Delta x(k + 1) = x(k + 1) - x^* = H(x(k) - x^*) + B_U (\Phi(x(k)) - \Phi(x^*)) = H \Delta x(k) + B_U (\Phi(x(k)) - \Phi(x^*)) \] (22)

and the following estimate is true for the norm of vector

\[ \| \Delta x(k + 1) \| \leq \| H \| \cdot \| \Delta x(k) \| + \| B_U \| L_\phi \| \Delta x(k) \| = \| H \| + L_\phi \| B_U \| \| \Delta x(k) \| \] (23)
where $L_\Phi$ is Lipshitz constant for the operator $\Phi$ which depends on K factor.

If condition $\|H\| + L_\Phi \|B_u\| < 1$ is fulfilled then requirement $\Delta x(k) \to 0$ is satisfied, i.e. closed control system ensured the Lyapunov asymptotic stability of the process [8,9].

If the object parameters $H, B_u$ are such that this condition is not fulfilled, the resulting controls can be formed on the basis of predicted controls according to the law $u(k) = \Gamma \Phi(x(k))$ where $\Gamma$ is the factor matrix, ensuring the condition of stability $\|H\| + L_\Phi \|\Gamma\| \|B_u\| < 1$.

6. Conclusion
In this paper we proposed a method for regulating the operating modes of electric power associations using analytical operators of finite-dimensional optimization. This method can be applied to control systems with simultaneous fulfillment of the requirements of validity (LFC control with tie-lines flow restrictions) and optimality.

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