Squeezed cooling of mechanical motion beyond the resolved-sideband limit

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Abstract – Cavity optomechanics provides a unique platform for controlling micromechanical systems by means of optical fields that cross the classical-quantum boundary to achieve solid foundations for quantum technologies. Currently, optomechanical resonators have become promising candidates for the development of precisely controlled nano-motors, ultrasensitive sensors and robust quantum information processors. For all these applications, a crucial requirement is to cool the mechanical resonators down to their quantum ground states. In this paper, we present a novel cooling scheme to further cool a micromechanical resonator via the noise squeezing effect. One quadrature in such a resonator can be squeezed to induce enhanced fluctuations in the other, “heated” quadrature, which can then be used to cool the mechanical motion via conventional optomechanical coupling. Our theoretical analysis and numerical calculations demonstrate that this squeeze-and-cool mechanism offers a quick technique for deeply cooling a macroscopic mechanical resonator to an unprecedented temperature region below the zero-point fluctuations.

Introduction. – Cavity optomechanics [1] concerns a strong interaction between optical fields and mechanical oscillators that are derived from the mechanical effects of photons. However, the mechanical effect of radiation pressure on a macroscopic object is extremely weak and always suffering from large noises [2]. High-Q optical cavities can resonantly enhance this optical force by the confinement of high-intensity light in a very small volume, such as in a Fabry-Pérot cavity which consists of a heavy fixed mirror and a light movable mirror (see fig. 1) [1]. Since the photon is reflected multiple times (e.g., 106 [3]) between two cavity mirrors before it decays, a coherent cavity field with low fluctuations builds up, resulting in a large optical force on the movable mirror and making it vibrating at a frequency from kHz to GHz [4]. In recent decades, the field of cavity optomechanics has witnessed rapid growth, and the related research has become increasingly important for both fundamental physics and applied technology [5].

Cooling massive mechanical resonators to the quantum ground states is a prominent achievement of cavity optomechanics [6–8], and an average phonon occupation of 0.20 ± 0.02 has recently been achieved [9]. A vibration mode in the ground state with a noise below the standard quantum limit is a fundamental requirement for various optomechanical applications, such as reliable nano-motors, high-precision sensors and robust quantum processors. However, the quantum uncertainty principle prevents complete halting of the resonator’s motion to access a temperature beyond the quantum fluctuations. In an attempt to achieve deeper cooling towards the quantum limit [10], the typical mechanism based on dynamical backaction loses efficiency due to the quantum backaction limit, and various improved cooling schemes within the resolved-sideband limit have been proposed [11–14]. Although a recent cooling experiment using squeezed light has achieved cooling below the quantum backaction limit [15], all sideband cooling techniques are eventually suffering from the single-phonon scattering balance [9], which makes ground-state cooling below one-phonon level difficult for a low cooling efficiency.

In this paper, we present an alternative technique for a deep cooling via direct squeezing. The basic idea of this technique is very similar to magnetic refrigeration. The confinement of magnetic dipoles in one direction by an
external magnetic field will drive fluctuations in the spatial degrees of freedom into the momentum channel, thereby effectively improving the temperature of the refrigerant. This thermal squeezing effect can be essentially understood in terms of an oscillator entering a tighter potential with a more confined position, causing its momentum to increase. Thus, the problem becomes one of cooling a hotter refrigerant, which is easier than cooling a cold one. When the confinement is finally removed, the motion fluctuations will return back to the spatial degrees of freedom, and the overall temperature will decrease. Similarly, in the quantum domain, when one quadrature of a mechanical resonator is squeezed, the other conjugate quadrature will be “heated” by increased quantum fluctuations due to the quantum uncertainty principle. For a given optomechanical oscillator, the quantum fluctuations in the momentum quadrature can be amplified by quantum squeezing on the position quadrature. Then, the “hotter” quadrature, with its amplified motion fluctuations, can be directly coupled to a blue-detuned cavity mode for conventional optomechanical cooling.

The quantum noise squeezing effect and its applications have been found three decades ago [17,18], and noise squeezing on optomechanical resonators has also been achieved using various schemes [19–23]. Because parametric amplification is a robust squeezing technique applied in many quantum and classical systems [24–27], we prefer parametric driving as a means of creating mechanical squeezing due to its low noise adding [28], and theoretically study the cooling scheme for an optomechanical oscillator [29]. Our theoretical analysis shows that this squeezed resonator can be effectively cooled down to a low temperature that is limited only by the degree of squeezing. With repeated cycles of squeezing and cooling, the resonator can be deeply cooled to a temperature region that is inaccessible for conventional cooling methods without noise squeezing.

Model. — The main schematic of our system, which consists of a high- Q cavity and a movable mirror, is shown in fig. 1. The mechanical mirror, with a suspension frequency of ωm and an effective mass of m_eff, is separately controlled by parametric driving on the spring constant k(t) at a double frequency of ωm and with a driving shift of δ_m. Thus, the Hamiltonian of the driven parametric oscillator (DPO) is [24]

\[ \hat{H}_{DPO} = \frac{\hbar^2}{2m_{eff}} + \frac{1}{2}[k_0 - k_\delta \sin(\omega_d t + 2\theta)] \hat{\hat{a}}^2, \]  

(1)

where the free spring constant is k_0 = m_eff ω_m^2, the driving frequency is ω_d = 2(ω_m − δ_m), the driving amplitude is k_δ, and θ is the driving-induced phase shift. For a classical DPO, strong squeezing of thermomechanical noise has been experimentally demonstrated [19]. To highlight the quantum squeezing effect, the original Hamiltonian \( \hat{H}_{DPO} \) in a frame rotating at a frequency of ω_m − δ_m can be written as [24]

\[ \hat{H}'_{DPO} = \hbar \delta_m \hat{b}^\dagger \hat{b} + \frac{\hbar}{2} \left( \xi^* \hat{b}_{\hat{a}}^2 - \xi \hat{b}_{\hat{a}}^2 \right), \]  

(2)

where \( \hat{b} (\hat{b}^\dagger) \) is the phonon annihilation (creation) operator, with an effective mechanical frequency \( \delta_m \). The second term of \( \hat{H}'_{DPO} \) takes the exact form of a squeezing operator, which can generate quantum noise squeezing on the DPO. The squeeze parameter is \( \xi = re^{-2i\theta} \), where the squeeze factor is \( r = \omega_m k_r / 4 k_0 \) and the phase shift \( \theta \) adjusts the squeezing direction of the coupled quadrature [30]. Therefore, the whole system depicted in fig. 1 can be described by the following Hamiltonian:

\[ \hat{H} = \hat{H}'_{DPO} + \hbar \delta_c \hat{a}^\dagger \hat{a} - \hbar g \hat{a}^\dagger \hat{a} (\hat{b}^\dagger + \hat{b}) + i \hbar (\eta \hat{a}^\dagger - \eta^* \hat{a}). \]  

(3)

The second term of eq. (3) represents a cavity mode \( \hat{a} \) with a detuning \( \delta_c = \omega_{cav} - \omega_L \), where \( \omega_{cav} \) and \( \omega_L \) are the frequencies of the cavity mode and the pumping laser, respectively. The third term describes the optomechanical coupling between the cavity mode \( \hat{a} \) and the position quadrature of the mechanical mode \( \hat{b} \), which has a coupling strength \( g \) and is maintained by a phase-shift feedback of the parametric driving [2]. The last term represents the laser pumping with a strength \( \eta \) (see SM).

Because the entire system is subjected to fluctuations originating from both the external reservoirs and the internal quantum dynamics, the full motion of the system

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1The relation between energy fluctuation (ΔE) and temperature (T) of a classical system in equilibrium with a bath can be expressed, according to the fluctuation-dissipation theorem, as ΔE = μkB_T, where μ is the mobility for the energy damping to the bath (e.g., the energy variance in a canonical ensemble satisfies ΔE ≈ k_B T). However, for a well-isolated quantum system, the relation is ambiguous because of the variety of temperature definitions beyond that with respect to a bath. Nevertheless, the fluctuation-dissipation theorem is assumed to be valid in the quantum regime [16], and thus, we use the word “heated” in the sense of an effective temperature T_eff related to the energy fluctuations of a nonequilibrium system.

2See the Supplemental Material Supplementarymaterial.pdf (SM) for the details of the calculations of the theoretical model, the stability analysis, the critical squeeze factor and the phonon spectrum in the strong-coupling regime.
can be described by the quantum Langevin equations of \( \hat{H} \), which contain noise terms for both the optical mode (\( \hat{a}_{in} \)) and the mechanical mode (\( \hat{b}_{in} \)) as follows:

\[
\frac{d\hat{a}}{dt} = -i\Delta_c + \frac{\kappa}{2} \hat{a} + ig\hat{b}(\hat{b} + \hat{b}^\dagger) + \eta + \sqrt{\kappa}\hat{a}_{in}, \tag{4}
\]

\[
\frac{d\hat{b}}{dt} = -(i\Delta_m + \Gamma_m \frac{\kappa}{2}) \hat{b} + ig\hat{a}\hat{a}^\dagger - \xi\hat{b}^\dagger + \sqrt{\Gamma_m}\hat{b}_{in}, \tag{5}
\]

where \( \kappa \) is the total decay rate of the cavity mode and \( \Gamma_m \) is the damping rate of the mechanical mode (see SM). The noise operators \( \hat{a}_{in} \) and \( \hat{b}_{in} \) represent the corresponding vacuum fluctuations, which have the following statistical properties: \( \langle \hat{a}_{in}(t)\hat{a}_{in}^\dagger(t') \rangle = (n_{th}^c + 1)\delta(t - t') \), \( \langle \hat{b}_{in}(t)\hat{b}_{in}^\dagger(t') \rangle = (n_{th}^m + 1)\delta(t - t') \), and \( \langle \hat{b}^\dagger(t)\hat{b}(t') \rangle = n_{th}^m\delta(t - t') \). Here, we assume that the cavity field contains a number of thermal photons equal to \( n_{th}^c = [\exp(h\omega_{vac}/k_BT) - 1]^{-1} \) and that the number of thermal phonons is given by \( n_{th}^m = [\exp(h\omega_m/k_BT) - 1]^{-1} \) [31].

In the weak-pumping regime after cryogenic precooing, all dynamical quantities modulate around their classical equilibrium states as follows: \( \hat{a} = a_s + \delta a, \hat{a}^\dagger = a_s^* + \delta a^\dagger, \hat{b} = b_s + \delta b, \) and \( \hat{b}^\dagger = b_s^* + \delta b^\dagger \). Thus, the equilibrium position of the resonator can be determined using the following implicit equation:

\[
x_s = 2g\eta^2(\delta_m + r \sin 2\theta) \left[ [(\kappa/2)^2 + (\delta_c - g x_s)^2]/(\Gamma_m/2)^2 + \delta_m^2 - r^2 \right]. \tag{6}
\]

where \( x_s = b_s + b_s^* \). In fig. 2, a squeezing-enhanced sensitive displacement of \( x_s \) near the critical squeezing point \( r \sim r_c \equiv \sqrt{\delta_m^2 + G_m^2/4} \) is identified (vertical dashed line), and it implies an efficient method of phonon cooling when \( r \sim r_c \) (see SM). The nonlinear static responses of \( x_s \) with respect to the pumping detuning \( \delta_c \) also sensitively depend on \( r \) (upper inset) and \( \theta \) (lower inset). To ensure reliable performance, the stability requirement of the DPO imposes an upper bound on the squeezing degree of \( r \) such that \( r < r_c \) for steady-state parametric squeezing [26]. Therefore, the instability beyond the squeezing bound should be overcome during the squeezed cooling process. There are two ways to fulfill this purposes. One is to use a red-detuned cooling laser to induce a positive damping rate \( \Gamma_{opt} \) on the DPO (through optical spring effects), which can stabilize the steady-state squeezing over a large squeezing region [32]. The other reliable technique is to introduce quantum feedback control [33–35] over the dynamics of the DPO that extends beyond its steady-state performance (e.g., locking onto a position-squeezed self-sustained oscillation) and can also work in the deep squeezing regime [24].

For simplicity, we introduce the replacements \( \delta \hat{a} \to \hat{a} \) and \( \delta \hat{b} \to \hat{b} \) to obtain an effective linearized Hamiltonian of the system:

\[
\hat{H}_{eff} = \hbar\Delta_c\hat{a}^\dagger\hat{a} + \hbar\delta_m\hat{b}^\dagger\hat{b} - \hbar g(a_s^*\hat{a} + a_s\hat{a}^\dagger)(\hat{b}^\dagger + \hat{b}) + \frac{\hbar}{2}(\xi\hat{b}^\dagger - \xi\hat{b}^\dagger), \tag{7}
\]

where \( \Delta_c \equiv \delta_c - g x_s \) is the position-shifted cavity detuning. Clearly, the last term of eq. (7) generates noise squeezing effect on the mechanical mode \( \hat{b} \) by means of the squeeze operator \( \hat{S}(\xi) = \exp\left(\frac{\xi}{2}\hat{b}^\dagger - \frac{\xi}{2}\hat{b}^\dagger\right) \), where \( \xi = re^{-2i\theta} \) determines the degree \( r \) and the angle \( \theta \) of the squeezing on the coupled quadratures [30].

**Phonon spectrum of the squeezed cooling and effective temperature.** – With the physical setup described above, our system can simultaneously apply mechanical squeezing to one quadrature and perform cooling on the other. Then, the final number of phonons in the mechanical resonator is determined by

\[
\tilde{n} = \langle \hat{b}^\dagger(t)\hat{b}(t) \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} S_n(\omega)d\omega, \tag{8}
\]

where \( S_n(\omega) \) is the phonon number spectrum (see SM), and it implies an effective cooling temperature \( T_{eff} \) given by

\[
T_{eff}(\tilde{n}) = \frac{\hbar\omega_m}{k_B \ln \left( \frac{1}{\tilde{n}} + 1 \right)}. \tag{9}
\]

Equation (9) is derived from the detailed balance expression between the Stokes and anti-Stokes processes [8,10,31], and \( \tilde{n} \) is equal to the area underneath the spectral curve \( S_n(\omega) \), implying that \( \lim_{\tilde{n} \to 0} T_{eff} \to 0 \).

As the effective temperature of the squeezed state can be directly measured by the phonon noise spectrum, in fig. 3, we present the calculated phonon number spectra.
S_\text{th}(\omega) of the DPO for different squeeze factors \( r \). The spectra exhibit a significant decrease in \( T_{\text{eff}} \) with an increasing squeeze factor \( r \). The inset figure reveals the critical squeeze factor \( r_c \) and the marked squeezing-based cooling that occurs when \( r > r_c \) (see SM). Here, we choose moderate parameters that can be easily realized in optomechanical systems [15]. A high-Q resonator with \( \Gamma_m/\omega_m = 0.001 \) is used to support the squeezing performance, and a precooling process is applied that achieves an average thermal occupation \( n^{th}_{\text{can}} = 10 \) to enhance the noise squeezing effect. The cavity mode, with an occupation \( n^{th}_{\text{can}} = 1 \), should be pumped by a far-red-detuned low-power laser source. These undemanding conditions for squeezing-based cooling can be easily fulfilled in a bad cavity (\( \kappa/\omega_m = 2 \)) beyond the resolved-sideband cooling limit and in the weak-coupling regime (\( g < \kappa \)). The cooling mechanism takes effect in this regime because the squeezed “heated” phonons with enhanced fluctuations are squeezed out and quickly taken away by the coupled photons leaking from the bad cavity. Because a larger squeeze factor \( r > r_c \) will drive the system into an unstable state, this technique should be dynamically controlled by a phase feedback to the parametric driving, and thus is limited by the squeeze factor \( r/\omega_m = k_r/4k_0 \), maintained by a relative modulation amplitude of DPO [34]. Nevertheless, a small squeeze factor of \( r \sim r_c \) still works because we can use successive squeezing-and-cooling cycles (cooling loops) to achieve lower and lower temperatures without a serious deterioration in cooling efficiency (see the lower three curves in fig. 3 indicated by the down arrow) [35].

Another property of squeezing-based cooling is that the cooling depends on the squeezing angle \( \theta \) when the system enters the strong-coupling regime of \( g \sim \kappa \) under a higher degree of squeezing (see error ellipse in the upper left inset of fig. 4). This dependence arises because the light mode couples to the asymmetric quadratures, whose squeezing degrees are modified by the angle \( \theta \). From the perspective of the squeezing picture, the Hamiltonian (7) will be \( \hat{S}(\xi)\hat{H}_{\text{eff}}\hat{S}(\xi)^\dagger \) and the third term of eq. (7), which describes the optical coupling of the mode \( \hat{a} \) to the mechanical quadrature of \( \hat{X} \) defined by \( \hat{b} = (\hat{X} + i\hat{P})/2 \), becomes \( (\cosh r - \cos 2\theta \sinh r)\hat{X} + (\sin 2\theta \sinh r)\hat{P} \). This expression clearly shows a \( \theta \) dependence of the coupling of the quadrature to the cavity mode. Figure 4 shows how the phonon spectrum changes with respect to the squeezing angle \( \theta \) when the squeeze factor \( r \) is fixed. The black curve is the reference phonon spectrum without squeezing (\( r = 0 \)), and the other curves represent the phonon spectra under a fixed squeeze factor \( r/\omega_m = 0.8 \) and with different squeezing angles. The results verify the \( \theta \) dependence of the cooling rate, and an optimal case arises when the cooling laser is coupled directly to the “hottest” quadrature with a maximum fluctuation. Both resonant peaks at \( \pm r_c/\omega_m \) appearing in fig. 4 (indicated by vertical dashed lines) are due to an increase in the coupling \( g \). The weak resonant peak at \( -r_c/\omega_m \) will be suppressed by the squeezing effect even in the strong-coupling regime. Moreover, phonon spectra within the resolved-sideband limit or in the strong-coupling regime still exhibit similar squeeze-induced enhancements of solid cooling (see SM).

Conclusion. – In summary, through direct numerical calculations based on an intuitive picture, we have revealed a deep cooling scheme for an optomechanical resonator in which the cooling performance is dramatically enhanced by the quantum squeezing effect induced by the
parametric driving. We demonstrated that by increasing the squeeze factor \( r \), we can effectively reduce the phonon number spectral curve, thereby extracting significant “heat” from the squeezed motion of a mechanical oscillator, to reach an effective temperature below the quantum shot noise. The idea proposed here is that one quadrature of the mechanical mode is “heated” by squeezing the other to improve the cooling capability by coupling the cooling laser directly to the “heated” quadrature, allowing the “hotter” phonons to be quickly taken away by the leakage of photons from a bad cavity. This method can be used to rapidly cool the mechanical motion down to its quantum ground state below the standard quantum limit. The resulting high cooling efficiency is of interest for many quantum applications, such as quick BEC evaporative cooling, quantum precision measurement or quantum sensors and rapid state initialization for quantum processing. We believe that this squeezing-based cooling scheme can serve as a universal technique for facilitating the development of quantum technology for use in macroscopic solid-state systems.

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