Article

Convective Heat and Mass Transfer in Third-Grade Fluid with Darcy–Forchheimer Relation in the Presence of Thermal-Diffusion and Diffusion-Thermo Effects over an Exponentially Inclined Stretching Sheet Surrounded by a Porous Medium: A CFD Study

Amir Abbas 1,*, Ramsha Shafqat 1, Mdi Begum Jeelani 2,⁎ and Nadiyah Hussain Alharthi 2,⁎

1 Department of Mathematics and Statistics, Faculty of Science, The University of Lahore, Sargodha-Campus, Sargodha 40100, Pakistan; ramshawarriach@gmail.com
2 Department of Mathematics and Statistics, College of Science, Imam Mohammad Ibn Saud Islamic University, Riyadh 13314, Saudi Arabia; nhalharthi@imamu.edu.sa
* Correspondence: cfdamirabbas4693@gmail.com (A.A.); mbshaikh@imamu.edu.sa (M.B.J.)

Abstract: The current study aims to investigate the thermal-diffusion and diffusion-thermo effects on heat and mass transfer in third-grade fluid with Darcy–Forchheimer relation impact over an exponentially inclined stretching sheet embedded in a porous medium. The proposed mechanism in terms non-linear and coupled partial differential equations is reduced to set of ordinary differential equations by employing an appropriate similarity variable formulation. The reduced form of equations is solved by using the MATLAB built-in numerical solver bvp4c. The numerical results for unknown physical properties such as velocity profile, temperature field, and mass concentration along with their gradients such as the skin friction, the rate of heat transfer, and the rate of mass transfer at angle of inclination \( \alpha = \pi/6 \) are obtained under the impact of material parameters that appear in the flow model. The solutions are displayed in forms of graphs as well as tables and are discussed with physical reasoning. From the demonstration of the graphical results, it is inferred that thermal-diffusion parameter \( Sr \) velocity, temperature, and concentration profiles are augmented. For the increasing magnitude of the diffusion-thermo parameter \( D_f \) the fluid velocity and fluid temperature rise but the opposite trend in mass concentration is noted. The current results are compared with the available results in the existing literature, and there is good agreement between them that shows the validation of the present study.

Keywords: thermal-diffusion; diffusion-thermo; third-grade fluid; Darcy–Forchheimer relation; stretching sheet; porous medium

1. Introduction

The complicated physical behavior of non-Newtonian fluids, as well as their widespread use in industrial, medical, and military applications, has attracted the attention of researchers. Non-Newtonian fluids have been studied using a variety of models. These models can be divided into three categories of fluids: (i) grade fluids, (ii) Maxwell fluids, and (iii) integral fluids. The study of grade fluids has received a lot of interest in recent years. The third-grade fluid model is one of the most comprehensive fluid models, displaying all shear thinning and shear thickening fluid properties. Researchers paid attention to such fluid models due to much application in engineering and industry such as in food processing, papermaking, and lubricating, etc.

Sahoo and Do [1] numerically examined the mechanism of heat transfer in third-grade fluid flow along the surface of a linear stretching sheet with the effects of slip condition. Javanmard et al. [2] focused their attention on the process of fully developed flow of...
third-grade fluid in a pipe under the impact of applied magnetic field and convective boundary conditions. The boundary layer flow of fluid of grade three is discussed in detail by Pakdemirli [3]. The process of heat transfer in fluid of third grade over a linear stretching sheet with partial slip condition is explored by Sahoo [4]. Jawanmard et al. [5] investigated the heat transfer analysis of third-grade fluid in two coaxial pipes with a radius under the influence of magnetic field. Zhang et al. [6] considered the effects of heat generation/absorption, thermal radiation, magnetic field, and Newtonian conditions on third-grade nanofluid flow over a slender stretching sheet. The effects of chemical reaction, heat generation/absorption, and magnetic field on the third-grade nanofluid flow over a non-linear stretching sheet with convective boundary conditions. Imtiaz et al. [9] examined the impact of Cattaneo–Christov heat flux along with chemical reaction influence on third-grade fluid. Hayat et al. [10] discussed the model of third-grade fluid flow along the surface of a rotating disk with the combined impact of activation energy and non-linear chemical reaction with nanomaterial.

In the above paragraphs, the studies on the non-Newtonian third-grade fluids were documented. Now, we highlight the work concentrating the Darcy–Forchheimer relation in porous medium within Newtonian and non-Newtonian fluids. The importance of flows saturating porous space in engineering and commercial applications such as oil reservoirs, resin transfer models, porous insulation, packed beds, geothermal energy, fossil fuel beds, and nuclear waste disposal has aroused academics’ interest. Permeable media with Darcy’s relation, in which the pressure gradient and volume average velocity are directly related, have received a lot of attention in the literature. Forchheimer [11] incorporated the factor of square velocity in the expression of Darcy’s velocity to investigate the features of inertia and boundary. Muskat and Wyckoff [12] named this the Forchheimer term that always holds for problems regarding high Reynolds numbers. Physically, higher filtration flow rates can deliver quadratic drag for permeable space in the velocity expression [13]. The Darcy–Forchheimer model coupled with natural, forced, and mixed heat transfer in non-Newtonian power law fluid has attracted the attention of Shenoy [14]. The effects of generalized Fourier’s law coupled with three-dimensional nanofluidic convective flows under the impact of Hall current with Darcy–Forchheimer law effects have been studied by Raja et al. [15]. Pan [16] used the mixed element method to solve the Darcy–Forchheimer model. Ramzan et al. [17] proposed the model of Williamson nanofluid equipped with generalized Fourier’s and Fick’s laws with Darcy–Forchheimer relation influence in stratified medium. Mahdi et al. [18] examined the convective heat transfer in nanofluid flow saturated with porous medium.

The heat transfer caused by concentration (mass) gradient is called the diffusion-thermo effect (Dufour effect). On the other hand, mass transfer caused by the temperature gradient is called the thermal-diffusion (Soret) effect. Research on these topics has been the focus of researchers due to significant application in different areas of science. Kafoussias and Williams [19] highlighted the effects of thermal-diffusion and diffusion thermo on free, forced and mixed convection flow and mass transfer with variable viscosity. Abd-El Aziz [20] determined the numerical solutions of thermal-diffusion and diffusion-thermo effects on magnetohydrodynamic convective three-dimensional flows over a porous plate with radiation effects. Hayat et al. [21] investigated the axially symmetric flow of second-grade fluid with thermal-diffusion and diffusion effects. Srinivas et al. [22] studied the process of fluid flow between two expanding and contracting walls in the presence of thermal-diffusion and diffusion-thermo effects. Afify [23] documented the study accomplishing the magnetohydrodynamic fluid flow and heat transfer with the combined effects of suction/injection, thermal-diffusion and diffusion-thermo effects over a stretching surface. The combined effects of variables viscosity, suction/injection, thermal-diffusion and diffusion-thermo effects over an accelerating surface have been considered by Seddeek [24].
The thermal-diffusion and diffusion-thermo effects on Erying–Powell nanofluid fluid flow with gyrotactic microorganisms was examined by Eldabe et al. [25].

In the above paragraphs, the literature was confined to examination of the non-Newtonian fluids, with different flow features and fluid characteristics on different geometries. Studies based on the stretching sheet are highlighted in this paragraph. The study of laminar boundary layer flow over a stretching sheet has gained a lot of attention in the past because of its applications in industries such as materials made by extrusion, the boundary layer along a liquid film in condensation, and heat-treated materials travelling between a feed roll and a wind-up roll or on a conveyor belt that pose the characteristics of a moving continuous surface. The numerical study of the different processes with different flow over the exponentially stretching sheet is explored in [26–29]. Kumar et al. [30] investigated heat and mass transportation on exponentially angled stretching sheets imbedded in porous media using Soret, Dufour, magnetic field, slip effects, Joule heating, and chemical reaction. Magyari and Keller [31] examined heat and mass transfer in the boundary layers over an exponentially stretching continuous sheet. Different studies considering the Newtonian and non-Newtonian fluids with diverse flow features over different geometries have been presented in [32–38]. The study of their grade fluid over an exponentially stretching is discussed in [39,40].

Being motivated by the above-mentioned physical significance of the third-grade fluid, Darcy–Forchheimer relation, porous medium, thermal-diffusion and diffusion-thermo effects in the existing literature, the gap in the study of combined effects of thermal-diffusion and diffusion-thermo with Darcy–Forchheimer relation impact on heat and mass transfer in third-grade fluid over an exponentially inclined stretching sheet embedded in a porous medium has been filled. The mathematical model of the boundary layer flow of third-grade fluid and heat transfer is developed and solved with an appropriate method. The whole model, along with its solution procedure and solutions of the physical properties of interest, is demonstrated in the following sections.

2. Mathematical Modeling

Consider steady, incompressible two-dimensional and viscous boundary layer flow the third-grade fluid with the Darcy–Forchheimer relation with thermal-diffusion and diffusion-thermo effects over an inclined exponentially stretching sheet embedded in a porous medium. The velocity components \((u, v)\) along \((x, y)\) are aligned and the schematic diagram for flow configuration is shown in Figure 1. By following the equation in [1,30], the governing equations are given as below:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
u \frac{\partial u}{\partial x} + \nu \frac{\partial v}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + \frac{a_1}{\rho} \left( \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial u}{\partial x} \frac{\partial^2 u}{\partial y^2} + \frac{3}{\partial u} \frac{\partial^2 u}{\partial y^2} + \nu \frac{\partial^2 u}{\partial y^2} \right) + 2 \frac{a_2}{\beta} \frac{\partial u}{\partial y} \frac{\partial^2 v}{\partial y^2}
+ 6 \frac{a_3}{\beta} \left( \frac{\partial u}{\partial y} \right)^2 + 2 \frac{\partial^2 T}{\partial y^2} + g \beta_T (T - T_{\infty}) \cos \alpha + g \beta_C (C - C_{\infty}) \cos \alpha - \frac{\nu u}{\kappa} - Fu^2
\]

\[\frac{\partial T}{\partial x} + \frac{\partial T}{\partial y} = \alpha_m \frac{\partial^2 T}{\partial y^2} + \frac{D_m k_T}{C_C \rho} \frac{\partial^2 T}{\partial y^2}
\]

\[\frac{\partial C}{\partial x} + \frac{\partial C}{\partial y} = \frac{D_m k_T}{\rho} \frac{\partial^2 C}{\partial y^2} + \frac{D_m k_T}{T_m} \frac{\partial^2 T}{\partial y^2}
\]
Subject to the boundary conditions are

\[ u = U_w, \quad v = 0, \quad T = T_w(x), \quad C = C_w(x) \quad \text{at} \quad y = 0 \]
\[ u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as} \quad y \to \infty. \]  (5)

Equation (1) represents the equation of continuity, Equation (2) represents boundary layer form of momentum equation for third-grade fluid, Equation (3) represents the energy equation compatible for the current flow features, Equation (4) represents the mass concentration, and Equation (5) represents the boundary conditions imposed on the current phenomenon. In Equation (5), the flow is induced due to exponential stretching of the sheet and is fixed at an angle of inclination \( \alpha = \pi / 6 \). The sheet is heated at temperature \( T_w \) and exact the surface mass concentration is \( C_w \). Away from the surface the velocity approaches zero and temperature of the fluid and concentration in the fluid approach free stream temperature and mass concentration. Here, \( U_w = U_0 e^{x/3}, \quad T_w = T_{\infty} + C_{\omega} e^{x/3}, \quad C_w = C_{\infty} + C_{\omega} e^{x/3} \), are the stretching velocity, wall temperature, and wall concentration, respectively. The symbols \( U_w, T_w, \) and \( C_w \) are reference velocity, temperature, and concentration, respectively. Here, \((u, v)\) are the velocity components along \((x, y)\) directions, respectively. The notations \((\alpha_1, \alpha_2, \beta_3), K_0, \) \( F = \frac{C_p}{\sqrt{K_0}} \) are material moduli, permeability of the porous medium, coefficient of inertia with \( C_p \) the drag coefficient, respectively. The designations \( \rho, C_p, \mu, \nu, \alpha_m, k, D_m, T_m C_s, \) and \( k_T \) are the density of the fluid, specific heat at constant pressure, dynamic viscosity, kinematic viscosity, thermal diffusivity, mass diffusivity, thermal conductivity, means fluid temperature, concentration susceptibility, and thermal-diffusion ratio, respectively. The symbols \((T, C, \ T_\infty, C_\infty)\) are the temperature and concentration within the boundary layer and in the free stream region, respectively.

3. Solution Methodology

In this section, we elaborate the entire solution procedure to solve the governing Equations (1)–(4) along with boundary conditions (5).
3.1. Similarity Formulation

The Equations (1)–(5) are non-linear and coupled partial differential equations. First, we reduce these equations into a set of ordinary differential equations by using the following similarity variables used by [30].

\[ u = U_o e^{\xi}, \quad v = \sqrt{\frac{\nu}{2L}}(f(\eta) + \eta f'(\eta)) e^{\xi}, \quad T = T_o + C_0 \theta e^{\xi}, \quad C = C_o + C_0 \phi e^{\xi}, \]

\[ \eta = \sqrt{\frac{L}{2
u}}y e^{\xi/L}. \] (6)

By utilizing the above-mentioned similarity variable in Equation (6), Equation (1) is satisfied automatically, and the Equations (2)–(4) with boundary condition Equation (5) takes the following form.

\[ f'''' + f'ff'' - 2f''^2 + 2Ri(\theta + N\phi) \cos \alpha + K \left( 6f'''' - 2\eta f'''' - 9f''^2 - L \left( 3f''^2 + \eta f'' f''' \right) + 3\beta Rf''^2 f''' - K* f' - M f' - Fr f''^2 \right) = 0 \] (7)

\[ f''\theta - f\theta' = \frac{1}{Pr} \theta'' + Df\phi'' \] (8)

\[ f''\phi - f\phi' = \frac{1}{Sc} \phi'' + Sr\theta'' \] (9)

Boundary conditions

\[ f = 0, \quad f' = 1, \quad \theta(0) = 1, \quad \phi(0) = 1 \quad \text{as} \quad \eta \to 0 \]

\[ f' \to 1, \quad f'' \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty. \] (10)

Here, the parameters \( Sr = \frac{D_o \delta r T_o}{\nu L}, \quad D_f = \frac{D_o \delta r C_o}{\nu L}, \quad Ri = \frac{G\alpha}{K^*}, \quad N = \frac{\beta - C_o}{Pr}, \quad K = \frac{a_1 \nu L e e}{4 \nu}, \quad L = \frac{a_2 \nu L e e}{L^2}, \quad \beta = \frac{3 \beta^*}{K^*}, \quad K^* = \frac{2\nu L}{K_\alpha}, \quad Fr = \frac{C_o \nu L e e}{\sqrt{\nu}}, \quad Pr = \frac{T_o}{T_o}, \quad Sc = \frac{T_m}{T_m}, \) are thermal-diffusion parameter, diffusion-thermo parameter, Richardson number, buoyancy ratio parameter, the non-dimensional viscoelastic parameter, cross-viscous parameter, the third-grade fluid parameter, permeability parameter, local inertial coefficient, Prandtl number, and Schmidt number, respectively. Prime notation denotes differentiation w.r.t to the similarity variable \( \eta. \)

Here, \( f'(\eta) \) denotes dimensionless velocity, \( \theta(\eta) \) designates the dimensionless temperature, and \( \phi(\eta) \) denotes the mass concentration.

3.2. Solution Technique

It is not possible to find an exact solution for the highly non-linear coupled ordinary differential equation. The numerical solutions of Equations (7)–(9) along with boundary conditions (10) for different values of the governing parameters are obtained, namely, Richardson number \( Ri, \) buoyancy ratio parameter \( N, \) third-grade fluid parameter \( \beta, \) viscoelastic parameter \( K, \) cross-viscous parameter \( L, \) permeability parameter \( K^*, \) local inertial coefficient \( Fr, \) thermal-diffusion parameter \( Sr, \) diffusion-thermo parameter \( D_f, \) Prandtl number \( Pr, \) and Schmidt number \( Sc. \) The numerical solutions of the proposed model are solved by utilizing MALAB built-in Numerical Solver bvp4c. In the computation, \( \eta \to 10.0 \) is taken and the axis according to the clear figure visibility. Numerical Solver bvp4c is a finite difference code that implements the three-stage Lobattoo formula. This is a collocation formula and the collocation polynomial is a \( C^1 \)-continuous solution that has fourth-order accuracy uniformly in the interval of integration. Mesh selection and error control are based on the residual of the continuous solution. The collocation technique uses a mesh of the points to divide the interval of integration into subintervals. The solver determines a numerical solution by solving a global system of algebraic equations resulting from the boundary conditions and the collocation condition imposed on all the subintervals.

The solver estimates the error of the numerical solution in each subinterval. If the solution does
not satisfy the tolerance criteria, the solver adapts the mesh and repeats the process. There is a need to provide the points of the initial mesh, as well as an initial approximation of the solution at the mesh points. The Equations (7)–(10) are converted into a system of first order ordinary differential equations and then are put into the bvp4c numerical solver code for the final solutions. We set

\[ f = y(1), f' = y(2), f'' = y(3), f''' = y(4), \theta = y(5), \theta' = y(6), \phi = y(7), \phi' = y(8), \]

\[ f^{(iv)} = yy1 = \left(1/K \times y(1)\right) \times (2 \times Ri \times \left(N \times y(7) + y(5)\right)) \times \cos \alpha + y(4) + y(1) \times y(3) - 2 \times y(2)^2 + K \times \left(6 \times y(2) \times y(4) - 2 \times \eta \times y(3) \times y(4) - 9 \times y(3)^2\right) - L \times (3 \times y(3)^2 + \eta \times y(2) \times y(4)) + 3 \times \beta \times Re \times y(3)^2 \times y(4) \]

\[ \theta'' = yy2 = \left(Pr \times (y(2) \times y(5) - y(1) \times y(6)) - D \times Sc \times (y(2) \times y(7) - y(1) \times y(8))\right) / (1 + D \times Sr) \]

\[ \phi'' = Sc \times (y(2) \times y(7) - y(1) \times y(8)) - Sr \times (Pr \times (y(2) \times y(5) - y(1) \times y(6)) - D \times Sc \times (y(2) \times y(7) - y(1) \times y(8))) / (1 + D \times Sr) \]

Boundary conditions

\[ y_o(1) = 0, y_o(2) = 1, y_o(5) = 1, y_o(7) = 1, yin f(2) = 0, yin f(3) = 0, yin f(5) = 0, yin f(7) = 0 \]

The numerical solutions for the unknown material properties such as velocity field \( f'(\eta) \), temperature field \( \theta(\eta) \), and mass concentration \( \phi(\eta) \) are calculated at an angle of inclination \( \alpha = \frac{\pi}{6} \), presented in graphical form and discussed with physical reasoning. The gradients of the quantities outlined above, such as the skin friction \( f''(0) \), the heat transfer rate \( \theta'(0) \), the mass transfer rate \( \phi'(0) \) exactly at the surface of the geometry are determined and tabulated. The forthcoming section is presents the analysis with a detailed discussion of the graphed and tabulated physical quantities of interest.

4. Results and Discussion

In the current section, the physical behavior of the material properties of interest is discussed for elevating the pertinent parameters that appear in the flow equations. The values of \( \eta_{\infty} \), the numerical infinity values, are kept large enough and are retained as \( \eta_{\infty} = 10.0 \). Actually, this value is dependent on the physical parameters of the phenomenon and its value \( \eta_{\infty} = 10.0 \) is adequate to simulate \( \eta = \infty \) for all cases presented in Figures 2–19. Under this condition, it was possible to explore the numerical influence of Richardson number \( Ri \), buoyancy ratio parameter \( N \), viscoelastic parameter \( K \), cross-viscous parameter \( L \), third-grade fluid parameter \( \beta \), permeability parameter \( K^* \), local inertial coefficient \( Fr \), thermo-diffusion parameter \( Sr \), diffusion-thermo parameter \( D_t \), Prandtl number \( Pr \), and Schmidt number \( Sc \) on velocity field \( f'(\eta) \), temperature field \( \theta(\eta) \), and mass concentration \( \phi(\eta) \).
Processes 2022, 10, x FOR PEER REVIEW 7 of 21

geometry are determined and tabulated. The forthcoming section is presents the analysis with a detailed discussion of the graphed and tabulated physical quantities of interest.

4. Results and Discussion

In the current section, the physical behavior of the material properties of interest is discussed for elevating the pertinent parameters that appear in the flow equations. The values of $\eta_\infty$, the numerical infinity values, are kept large enough and are retained as $\eta_\infty = 10.0$. Actually, this value is dependent on the physical parameters of the phenomenon and its value $\eta_\infty = 10.0$ is adequate to simulate $\eta = \infty$ for all cases presented in Figures 2–19. Under this condition, it was possible to explore the numerical influence of Richardson number $Ri$, buoyancy ratio parameter $N$, viscoelastic parameter $K$, cross-viscous parameter $L$, third-grade fluid parameter $\beta$, permeability parameter $K^*$, local inertial coefficient $F_S$, thermo-diffusion parameter $S_S$, diffusion-thermo parameter $D_f$, Prandtl number $P_S$, and Schmidt number $S_c$ on velocity field $f'(\eta)$, temperature field $\theta(\eta)$, and mass concentration $\phi(\eta)$.

Figure 2. Consequences of $\beta$ on $f'(\eta)$.

Figure 3. Consequences of $\beta$ on $\theta(\eta)$.
Figure 3. Consequences of $\beta$ on $\theta(\eta)$.

Figure 4. Consequences of $\beta$ on $\phi(\eta)$.

Figure 5. Consequences of $K^*$ on $f'(\eta)$.
Figure 5. Consequences of $K^*$ on $f'(\eta)$.

Figure 6. Consequences of $K^*$ on $\theta(\eta)$.

Figure 7. Consequences of $K^*$ on $\phi(\eta)$.
Figure 7. Consequences of $K_*$ on $\phi(\eta)$.

Figure 8. Consequences of $F_S$ on $f'(\eta)$.

Figure 9. Consequences of $F_S$ on $\theta(\eta)$.
Figure 9. Consequences of $F_S$ on $\theta(\eta)$.

Figure 10. Consequences of $F_r$ on $\phi(\eta)$.

Figure 11. Consequences of $Pr$ on $f'(\eta)$.
Figure 11. Consequences of $Pr$ on $f'(\eta)$.

Figure 12. Consequences of $Pr$ on $\theta(\eta)$.

Figure 13. Consequences of $Pr$ on $\phi(\eta)$. 

\[ \text{Parameters: } Ri = 5.0, N = 0.5, K = 0.1, L = 0.1, \alpha = \pi/6, K^* = 8.0, \beta = 0.1, Fr = 5.0, Sc = 1.0, D_f = 0.1, Sr = 0.1 \]
Figure 14. Consequences of $D_f$ on $f'(\eta)$.

Figure 15. Consequences of $D_f$ on $\theta(\eta)$. 

$\text{Ri} = 5.0, \ N = 0.5, \ K = 0.1, \ L = 0.1, \ \alpha = \pi/6, \ K^* = 8.0, \ \beta = 0.1, \ Fr = 5.0, \ Sc = 1.0, \ Pr = 2.0, \ Sr = 0.1$
Figure 15. Consequences of $D_f$ on $\theta(\eta)$.

Figure 16. Consequences of $D_f$ on $\phi(\eta)$.

Figure 17. Consequences of $Sr$ on $f'(\eta)$.
4.1. Influence of Flow Parameters on Velocity Profile

Figure 18. Consequences of $Sr$ on $\theta(\eta)$.

Figure 19. Consequences of $Sr$ on $\phi(\eta)$. 

$\theta(\eta)$ vs $\eta$

$\phi(\eta)$ vs $\eta$
4.1. Influence of Flow Parameters on Velocity Profile $f'(\eta)$, Temperature Profile $\theta(\eta)$, and Mass Concentration $\phi(\eta)$

Figure 2 shows the consequences of third-grade parameter $\beta$ on $f'(\eta)$ when the rest of the parameters are kept at their fixed values. Figure 2 illustrates that as $\beta$ is enhanced, there is intensification in velocity of the fluid. It is due to fact that as $\beta$ rises, basically reference velocity is maximized and viscosity of the fluid is reduced. Figure 3 shows the results for temperature profile $\theta(\eta)$ for different values of $\beta$. It is viewed that, owing to the increase in magnitude of $\beta$, the temperature field declines. The physical behavior of the concentration profile $\phi(\eta)$ for different values of $\beta$ is displayed in Figure 4. It is seen from the graphical result that mass concentration decreases as $\beta$ is enhanced. Figure 5 depicts the influence of permeability parameter $K^*$ on velocity of the fluid. Figure 5 highlights that as $K^*$ is raised, velocity of the fluid decreases. When $K^*$ increases, viscosity of the fluid essentially increases and porosity of the medium decreases, due to which the fluid velocity slows down. The numerical results for the temperature profile against increasing values of $K^*$ are shown in Figure 6. The graph reveals that as $K^*$ is raised, temperature of the fluid rises rapidly. The concentration profile behavior versus $K^*$ is presented in Figure 7. It is concluded that as $K^*$ is enlarged, the concentration profile boosts with reasonable difference. Figure 8 shows the evaluation of the impact of local inertial coefficient $Fr$ on velocity. The graph for velocity of the fluid in Figure 8 shows that as $Fr$ is intensified, a reduction in $f'(\eta)$ is noted. Due to increase in drag coefficient and reduction in porosity of the medium, velocity declines. The physical behavior of the temperature profile $\theta$ for different values of the local inertial coefficient $Fr$ is illustrated in Figure 9. It is seen that as $Fr$ is raised, temperature field increases. The numerical solutions for the mass concentration $\phi$ are plotted in Figure 10. It is noted that as $Fr$ is enhanced, the mass concentration is strengthened. It is due to the reason that as $Fr$ is enlarged, the inertial coefficient is essentially expanded, and the porosity coefficient is reduced which causes the velocity to slow down and compels $\theta$ and $\phi$ to boost up their values.

The effect of Prandtl number $Pr$ on $f'(\eta)$ is demonstrated in Figure 11. The plot in Figure 11 indicates that the decline in the value of $f'(\eta)$ is seen as $Pr$ is raised. The highest magnitude for $f'$ is obtained for $Pr = 0.61$ and the bottom value is secured for $Pr = 10.0$. Due to the increase in $Pr$, viscosity of the fluid is enhanced which stops the fluid from moving fast. Figure 12 shows the graphical solutions for temperature profiles. Figure 12 highlights that a downfall in temperature is noted, owing to the rise in the values of Prandtl number $Pr$. Figure 13 presents the numerical results for mass concentration for diverse values of $Pr$. Results reveal that mass concentration increases for increasing magnitude of $Pr$. Very interestingly, the behavior of the velocity and temperature profiles follows the science of the Prandtl number, where, as $Pr$ increases, the viscous force increases causing the decline in velocity field. As $Pr$ is enhanced, the heat transfer due the thermal diffusion is reduced; hence, temperature of the fluid flow domain reduces due to low thermal conductance of the fluid. The increase in the $Pr$ is due to enhancement of the viscous force and reduction in the thermal conductance of the fluid which slows down the velocity and temperature profiles. The effect of diffusion-thermo parameter $D_f$ on the velocity profile is shown in Figure 14. Figure 14 indicates that as $D_f$ is augmented, the velocity is strengthened. Figure 15 displays the solutions for temperature distribution for several values of $D_f$. It is revealed that temperature of the fluid is intensified as $D_f$ is maximized. The mass concentration is attenuated in Figure 16. From a physical point view, it is shown that as $D_f$ is increased more heat flux occurs due to increasing temperature gradient that supports the enhancement in the temperature. As energy is absorbed by the fluid this causes the velocity to grow and increasing concentration gradients correspond to decreasing concentration in the boundary layer. Figure 17 illustrates the control of thermal-diffusion parameter $Sr$ on $f'(\eta)$. It is concluding that as $Sr$ is increased velocity increases. Figure 18 portrays the value of temperature against different values of $Sr$. It is observed that as $Sr$ rises, temperature of the fluid increases. Figure 19 demonstrates the mass concentration behavior for increasing values of $Sr$. It is noted that as $Sr$ is increased, the mass concentration increases visibly. Due
to the increase in $Sr$, more mass flux is observed due to which the velocity and temperature of the fluid increase and mass concentration increases as well.

4.2. Influence of the Materials Parameters on $f''(0), \theta'(0), \text{and} \phi'(0)$

Tables 1 and 2 show the comparison of the present numerical results for the rate of heat transfer and the skin friction with the previously published results. From the close observation of the numerical solutions, both the previously documented and current results, we conclude that there is good agreement between them which indicates the validation of the present study. Tables 3 and 4 show the numerical results of the skin friction, rate of heat transfer, and rate of mass transfer under the influence of Richardson number $Ri$ and Schmidt number $Sc$, respectively. Table 3 displays the numerical solutions of $f''(0), \theta'(0) \text{ and } \phi'(0)$ under the effects of $Ri$. Results highlight that the skin friction reduces but rate of heat transfer and rate of mass transfer increase gradually. Table 4 depicts the physical effects of Schmidt number on $f''(0), \theta'(0) \text{ and } \phi'(0)$. The solution indicates that as $Sc$ is augmented, the $f''(0)$ and $\theta'(0)$ increase but the reverse trend is noted in $\phi'(0)$. The results presented in Tables 1–3 were calculated at the surface.

| Table 1. Comparison of the $-\theta'(0)$ values for several values of Prandtl number for Newtonian fluid when $Ri = 0, N = 0, Sc = 0, \beta = 0, L = 0, K = 0, K^* = 0, Fr = 0, Sr = 0, D_f = 0$. |
|---|---|---|
| Pr | Magyari and Keller [31] | Present Study |
| 1.0 | 0.9547 | 0.9551 |
| 3 | 1.8691 | 1.8121 |
| 5 | 2.5001 | 2.5577 |
| 10 | 3.6604 | 3.6868 |

| Table 2. Comparison of the $f''(0)$ values for Newtonian fluid when $Ri = 0, N = 0, Sc = 0, \beta = 0, L = 0, K = 0, K^* = 0, Fr = 0, Sr = 0, D_f = 0, Pr = 0$. |
|---|---|---|
| Mukhopadhyay et al. [39] | Hayat et al. [40] | Present Study |
| $-1.2818$ | $-1.2818$ | $-1.2819$ |

| Table 3. Consequences of $Ri$ on (a) $f''(0)$ (b) $\theta'(0)$ (c) $\phi'(0)$ when $N = 0.5, \beta = 0.1, K = 0.1, L = 0.1, \alpha = \pi/6, K^* = 8.0, Fr = 5.0, Pr = 2.0, Sc = 1.0, D_f = 0.1, Sr = 0.1$. |
|---|---|---|
| $Ri$ | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
| 0.1 | 2.08746 | 1.16064 | 0.62817 |
| 3.0 | 1.67881 | 1.31154 | 0.76902 |
| 7.0 | 1.17803 | 1.42548 | 0.86850 |
| 10.0 | 0.83114 | 1.48769 | 0.92125 |

| Table 4. Consequences of $Sc$ on (a) $f''(0)$ (b) $\theta'(0)$ (c) $\phi'(0)$ when $Ri = 5.0; N = 0.5, \beta = 0.1, K = 0.1, L = 0.1, \alpha = \pi/6, K^* = 8.0, Fr = 5.0, Pr = 2.0, D_f = 0.1, Sr = 0.1$. |
|---|---|---|
| $Sc$ | $f''(0)$ | $-\theta'(0)$ | $-\phi'(0)$ |
| 1.0 | 1.42189 | 1.37481 | 0.82479 |
| 3.0 | 1.47903 | 1.27574 | 1.73512 |
| 7.0 | 1.53384 | 1.16031 | 2.88998 |
| 10.0 | 1.55589 | 1.09747 | 3.54354 |
5. Conclusions

The current study was devoted to investigating the thermal-diffusion and diffusion-thermo effects on heat and mass transfer in third-grade fluid with the impact of the Darcy–Forchheimer relation over an exponentially inclined stretching sheet embedded in a porous medium. The proposed phenomenon was given a mathematical form in terms of nonlinear and coupled partial differential equations and then reduced to an ordinary differential equation by utilizing an appropriate similarity transformation. For the numerical solutions of the governing flow problem, a MATLAB built-in numerical solver bvp4c was used. The numerical results of the material properties are portrayed in graphs and tables. The main outcomes are summarized as below:

- The velocity profile \( f' \) increases as \( Sr, D_f, \) and \( \beta \) increase and reductions in \( K^*, Fr, \) and \( Pr \) are enlarged.
- The temperature field \( \theta \) is increased as \( K^*, Fr, D_f, \) and \( Sr \) are augmented but the reverse behavior is viewed when increasing the values of \( \beta \) and \( Pr. \)
- The concentration profile \( \phi \) is increased as \( K^*, Fr, \) and \( Sr \) are augmented but attenuated as \( \beta, Pr, \) and \( D_f \) are elevated.
- The skin friction \( f''(0) \) is increased owing to the rise in values of \( Sc \) and the opposite trend is noted with the increasing values of \( Ri. \)
- The rate of heat transfer \( \theta'(0) \) is augmented as \( Ri \) rises but reduces with increasing magnitudes of \( Sc. \)
- The mass transfer rate \( \phi'(0) \) increases as \( Ri \) and \( Sc \) are augmented.
- The tabular results for \( f''(0), \theta'(0) \) and \( \phi'(0) \) are computed exactly at the surface.
- All the numerical results at the inclined exponentially stretching plate fixed at the angle of inclination of \( \alpha = \pi/6 \) were computed.
- All the numerical results presented in graphs in Figures 2–19 satisfied the given boundary conditions asymptotically; therefore, the numerical results given in tabular form are accurate.
- The current results were compared with the available results in the existing literature for the special case, and there was good agreement between them showing the validation of the present study.
- In future, the study will be extended to third-grade nanofluid and hybrid nanofluid with the inclusion of different flow features and physical effects of the different fluid characteristics over the exponentially inclined stretching sheet embedded in a porous medium with different flow conditions.

Author Contributions: Conceptualization, A.A. and R.S.; methodology, R.S.; software, A.A.; validation, M.B.J., N.A. and A.A.; formal analysis, N.A.; investigation, A.A.; resources, M.B.J.; data curation, N.A.; writing—original draft preparation, A.A.; writing—review and editing, A.A.; visualization, A.A.; supervision, M.B.J.; project administration, M.B.J.; funding acquisition, M.B.J. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by [Imam Mohammad Ibn Saud Islamic University] grant number [RG-21-09-13].

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Acknowledgments: (Research Group) The authors extend their appreciation to the Deanship of Scientific Research at Imam Mohammad Ibn Saud Islamic University for funding this work through Research Group no. RG-21-09-13.

Conflicts of Interest: Authors declare no conflict of interest.
Nomenclature

| Nomenclature                        | Symbol     | Unit                      |
|-------------------------------------|------------|---------------------------|
| Specific heat at constant pressure  | \( C_p \)  | \( (Jkg^{-1}K^{-1}) \)   |
| Schmidt number                      | \( Sc \)   |                           |
| Thermal-diffusion parameter (Soret number) | \( Sr \) |                           |
| Viscoelastic parameter              | \( K \)    |                           |
| Velocity components in \( x \) and \( y \) directions | \( u \), \( \nu \) |                           |
| Wall temperature                    | \( T_w \)  |                           |
| Wall concentration                  | \( C_w \)  |                           |
| Dimensionless temperature           | \( \theta \) |                           |
| Dynamic mass concentration          | \( \phi \)  |                           |
| Dynamic viscosity                   | \( \mu \)   | \( (Pa.s) \)               |
| Electrical conductivity             | \( \sigma \) |                           |
| Fluid density                       | \( \rho \)  | \( (kgm^{-3}) \)           |
| Kinematic viscosity                 | \( \nu \)   | \( (m^2s^{-1}) \)          |
| Material moduli                     | \( \alpha_1, \alpha_2, \beta_3 \) |                   |
| Thermal diffusivity                 | \( \alpha \) | \( (ms^{-1}) \)            |
| Thermal-diffusion ratio             | \( k_T \)   |                           |
| Third-grade fluid parameter         | \( \beta \)  |                           |
| Thermal conductivity                | \( \chi \)   | \( (Wm^{-1}K^{-1}) \)      |
| Volumetric coefficient thermal expansion | \( \beta_T \) | \( (K^{-1}) \)            |
| Volumetric coefficient concentration expansion | \( \beta_C \) | \( (K^{-1}) \)            |
| Ambient conditions                  | \( w \)     |                           |

References

1. Sahoo, B.; Do, Y. Effects of slip on sheet-driven flow and heat transfer of a third grade fluid past a stretching sheet. *Int. Commun. Heat Mass Transf.* 2010, **37**, 1064–1071. [CrossRef]
2. Javanmard, M.; Taheri, M.H.; Ebrahimi, S.M. Heat transfer of third-grade fluid flow in a pipe under an externally applied magnetic field with convection on wall. *Appl. Rheol.* 2018, **28**, 56023. [CrossRef]
3. Pakdemirli, M. The boundary layer equations of third-grade fluids. *Int. J. Non-Linear Mech.* 1992, **27**, 785–793. [CrossRef]
4. Sahoo, B. Flow and heat transfer of a non-Newtonian fluid past a stretching sheet with partial slip. *Commun. Nonlinear Sci. Numer. Simul.* 2015, **20**, 602–615. [CrossRef]
5. Javanmard, M.; Taheri, M.H.; Askari, N.; Öztöp, H.F.; Abu-Hamdeh, N. Third-grade non-Newtonian fluid flow and heat transfer in two coaxial pipes with a variable radius ratio with magnetic field. *Int. J. Numer. Methods Heat Fluid Flow* 2020, **31**, 959–981. [CrossRef]
6. Zhang, L.; Bhatti, M.M.; Michaelides, E.E. Electro-magnetohydrodynamic flow and heat transfer of a third-grade fluid using a Darcy-Brinkman-Fourierheimer model. *Int. J. Numer. Methods Heat Fluid Flow* 2020, **31**, 2623–2639. [CrossRef]
7. Qayyum, S.; Hayat, T.; Alsaedi, A. Thermal radiation and heat generation/absorption aspects in third grade magneto-nanofluid over a slandering stretching sheet with Newtonian conditions. *Phys. B Condens. Matter* 2018, **537**, 139–149. [CrossRef]
8. Qayyum, S.; Hayat, T.; Alsaedi, A. Chemical reaction and heat generation/absorption aspects in MHD nonlinear convective flow of third grade nanofluid over a nonlinear stretching sheet with variable thickness. *Results Phys.* 2017, **7**, 2752–2761. [CrossRef]
9. Imtiaz, M.; Alsaedi, A.; Shafiq, A.; Hayat, T. Impact of chemical reaction on third grade fluid flow with Cattaneo-Christov heat flux. *J. Mol. Liq.* 2017, **229**, 501–507. [CrossRef]
10. Hayat, T.; Khan, S.A.; Khan, M.I.; Alsaedi, A. Impact of activation energy in nonlinear mixed convective chemically reactive flow of third grade nanomaterial by a rotating disk. *Int. J. Chem. React. Eng.* 2019, **17**, 20180170. [CrossRef]
11. Forchheimer, P. Wasserbewegung durch boden. *Z. Ver. Dtsch. Ing.* 1901, **45**, 1782–1788.
12. Muskat, M.; Wyckoff, R.D. The flow of homogeneous fluids through porous media. *Soil Sci.* **1938**, *46*, 169. [CrossRef]
13. Seddeek, M. Influence of viscous dissipation and thermophoresis on Darcy–Forchheimer mixed convection in a fluid saturated porous media. *J. Colloid Interface Sci.* **2006**, *293*, 137–142. [CrossRef][PubMed]
14. Shenoy, A.V. Darcy-Forchheimer natural, forced and mixed convection heat transfer in non-Newtonian power-law fluid-saturated porous media. *Transp. Porous Media* **1993**, *11*, 219–241. [CrossRef]
15. Raja, M.A.Z.; Khan, Z.; Zahra, S.; Chaudhary, N.I.; Khan, W.U.; He, Y.; Islam, S.; Shoaib, M. Cattaneo-christov heat flux model of 3D hall current involving biconvection nanofluidic flow with Darcy-Forchheimer law effect: Backpropagation neural networks approach. *Case Stud. Therm. Eng.* **2021**, *26*, 101168. [CrossRef]
16. Pan, H.; Rui, H. Mixed Element Method for Two-Dimensional Darcy-Forchheimer Model. *J. Sci. Comput.* **2012**, *52*, 563–587. [CrossRef]
17. Ramzan, M.; Gul, H.; Zahri, M. Darcy-Forchheimer 3D Williamson nanofluid flow with generalized Fourier and Fick’s laws in a stratified medium. *Bull. Pol. Acad. Sci. Tech. Sci.* **2020**, *68*, 327–335.
18. Mahdi, R.A.; Mohammed, H.A.; Munisamy, K.; Saeid, N. Review of convection heat transfer and fluid flow in porous media with nanofluid. *Renew. Sustain. Energy Rev.* **2015**, *41*, 715–734. [CrossRef]
19. Kafoussias, N.; Williams, E. Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscosity. *Int. J. Eng. Sci.* **1995**, *33*, 1369–1384. [CrossRef]
20. Abd El-Aziz, M. Thermal-diffusion and diffusion-thermo effects on combined heat and mass transfer by hydromagnetic three-dimensional free convection over a permeable stretching surface with radiation. *Phys. Lett. A* **2008**, *372*, 263–272. [CrossRef]
21. Hayat, T.; Nawaz, M.; Asghar, S.; Mesloub, S. Thermal-diffusion and diffusion-thermo effects on axisymmetric flow of a second grade fluid. *Int. J. Heat Mass Transf.* **2011**, *54*, 3031–3041. [CrossRef]
22. Srinivas, S.; Reddy, A.S.; Ramamohan, T. A study on thermal-diffusion and diffusion-thermo effects in a two-dimensional viscous flow between slowly expanding or contracting walls with weak permeability. *Int. J. Heat Mass Transf.* **2012**, *55*, 3008–3020. [CrossRef]
23. Affifi, A.A. Similarity solution in MHD: Effects of thermal diffusion and diffusion thermo on free convective heat and mass transfer over a stretching surface considering suction or injection. *Commun. Nonlinear Sci. Numer. Simul.* **2009**, *14*, 2202–2214. [CrossRef]
24. Seddeek, M.A. Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective mass flow and mass transfer over an accelerating surface with a heat source in the presence of suction and blowing in the case of variable viscosity. *Acta Mech.* **2004**, *172*, 83–94. [CrossRef]
25. Eldabe, N.T.; Rizkalla, R.R.; Abouzeid, M.; Ayad, V.M. Thermal diffusion and diffusion thermo effects of Eyring-Powell nanofluid flow with gyrotactic microorganisms through the boundary layer. *Heat Transf.—Asia Res.* **2020**, *49*, 383–405. [CrossRef]
26. Ishak, A. MHD boundary layer flow due to an exponentially stretching sheet with radiation effect. *Stains Malays.* **2011**, *40*, 391–395.
27. Sajid, M.; Hayat, T. Influence of thermal radiation on the boundary layer flow due to an exponentially stretching sheet. *Int. Commun. Heat Mass Transf.* **2008**, *35*, 347–356. [CrossRef]
28. Nadeem, S.; Haq, R.U.; Khan, Z.H. Heat transfer analysis of water-based nanofluid over an exponentially stretching sheet. *Alex. Eng. J.* **2014**, *53*, 219–224. [CrossRef]
29. Bidin, B.; Nazar, R. Numerical solution of the boundary layer flow over an exponentially stretching sheet with thermal radiation. *Eur. J. Sci. Res.* **2009**, *33*, 710–717.
30. Kumar, D.; Sinha, S.; Sharma, A.; Agrawal, P.; Dadheech, P.K. Numerical study of chemical reaction and heat transfer of MHD slip flow with Joule heating and Soret–Dufour effect over an exponentially stretching sheet. *Heat Transf.* **2022**, *51*, 1939–1963. [CrossRef]
31. Magyari, E.; Keller, B. Heat and mass transfer in the boundary layers on an exponentially stretching continuous surface. *J. Phys. D Appl. Phys.* **1999**, *32*, 577–585. [CrossRef]
32. Pandit, S.; Sharma, S. Sensitivity analysis of emerging parameters in the presence of thermal radiation on magnetohydrodynamic nanofluids via wavelets. *Eng. Comput.* **2021**, 1–10. [CrossRef]
33. Pandit, S.; Sharma, S. Wavelet strategy for flow and heat transfer in CNT-water based fluid with asymmetric variable rectangular porous channel. *Eng. Comput.* **2020**, 1–11. [CrossRef]
34. Mittal, R.; Pandit, S. Numerical simulation of unsteady squeezing nanofluid and heat flow between two parallel plates using wavelets. *Int. J. Therm. Sci.* **2017**, *118*, 410–422. [CrossRef]
35. Mittal, R.C.; Pandit, S. A Numerical Algorithm to Capture Spin Patterns of Fractional Bloch Nuclear Magnetic Resonance Flow Models. *J. Comput. Nonlinear Dyn.* **2019**, *89*, 799–808. [CrossRef]
36. Jiwari, R.; Kumar, V.; Singh, S. Lie group analysis, exact solutions and conservation laws to compressible isentropic Navier–Stokes equation. *Eng. Comput.* **2020**, 1–10. [CrossRef]
37. Khan, M.N.; Ahmad, S.; Ahammad, N.A.; Alqahtani, T.; Algarni, S. Numerical investigation of hybrid nanofluid with gyrotactic microorganism and multiple slip conditions through a porous rotating disk. *Waves Random Complex Media* **2022**, 1–16. [CrossRef]
38. Li, Y.-M.; Ullah, I.; Ahammad, N.A.; Ullah, I.; Muhammad, T.; Asiri, S.A. Approximation of unsteady squeezing flow through porous space with slip effect: DFM approach. *Waves Random Complex Media* **2022**, 1–15. [CrossRef]
39. Mukhopadhyay, S.; Moindal, I.C.; Hayat, T. MHD boundary layer flow of Casson fluid passing through an exponentially stretching permeable surface with thermal radiation. *Chin. Phys. B* **2014**, *23*, 104701. [CrossRef]
40. Hayat, T.; Khan, M.I.; Waqas, M.; Alsaedi, A.; Yasmeen, T. Diffusion of chemically reactive species in third grade fluid flow over an exponentially stretching sheet considering magnetic field effects. *Chin. J. Chem. Eng.*, 2017, 25, 257–263. [CrossRef]