Spherical Collapse Model And Dark Energy (II)

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ABSTRACT

This is a second paper of a series of two. In this paper, we directly correct the problem pointed out in the first paper of this series, dark energy does not cluster on the scale of galaxy clusters, but the current describing the flowing of dark energies outside the clusters is ignored in almost all the previous papers. We set up and solve a first order differential equation which describes the evolution of the clusters in a back ground universe containing dark energies. From the solution we extract the key parameters of the model and find them depending on the equation of state coefficients of dark energies rather non-trivially. We then apply the results in Press-Scheter theory and calculate the number density of galaxy clusters and its evolutions, we find the observable quantities are strongly affected by the equation of state coefficients of dark energies.

Subject headings: top-hat spherical collapse model, galaxy clusters formation, dark energy

1. Introduction

In a previous paper (Zeng & Gao 2005), we point out that, in most of the existing literatures about top-hat spherical collapse model of galaxy clusters formation, (Barrow & Saich 1991; Eke, Cole & Frenk 1996; Wang & Steinhardt 1998; Lokas & Hoffman 2001; Percival, Miller & Peacock 2000; Weinber & Kamionkowski 2002), the dark energy is assumed not to cluster on the scale of galaxy clusters. But the current which describes the flowing of dark energy outside the clusters is usually ignored, so the discussions in these literatures are not self-consistent. In that paper, by assuming that dark energy clusters synchronously with ordinary matters so that the dark energy current flowing outside the clusters does not exist at all, we make our discussions self-consistent. However, by so doing, we do not correct the problem in the existing literatures directly. We only from the contrary indicate that the effects of the dark energy current may be important.

The purpose of this paper is to directly consider the effects of such a current. Just as we state in that paper, when we add such a current in the energy-momentum tensor, Einstein equation becomes complicated and the metric of the cluster-inside space-time cannot be factorized as usual. So not only Friedman equation, but also Raycharduri equation does not follow as thought by the author of (Wang & Steinhardt 1998) and (Weinber & Kamionkowski 2002).

Using energy conservation law, we set up a first order differential equation to describe the evolution of the radius of a cluster and solved it in the subsection 2.1. We then give numerical as well as formal analytical solutions for this equation. In subsection 2.2, we extract the key parameters of this model which will be used in the application of Press-Scheter theory. In subsection 3.1 we derived theoretical formulae to calculate the number-density v.s. temperature of galaxy clusters using Press-Scheter theory. In subsection 3.2, we provide numerical results for the number density of galaxy clusters and its evolutions and study the effects of dark energy equation of coefficients on this two quantities. We end the paper with the main conclusions and some discussions.

2. Spherical Collapse Model in QCDM Cosmologies

2.1. The Basic Equation and Its Solution

There is a simple way (Weinberg 1972) to derive a Friedmann-like equation to describe the evolution of the radius of the clusters. Consider a test particle
It is worth noting that, besides the physical radius of the clusters, the symbol \( a_p \) appearing in eq(1) can also be understood as the scale factor of the clusters-inside space-time. But that in eq(2) it can only be understood as the radius of the clusters, it cannot be understood as the scale factor of the cluster-inside space-time. This is because, when dark energy is not assumed to move synchronously with ordinary matters on the galaxy clusters scale, the metric function \( U(t, r) \) and \( V(t, r) \) in the ansatz (Weinberg 1972) \( ds^2 = -dt^2 + U(t, r)dr^2 + V(t, r)(d\theta^2 + \sin^2 \theta d\phi^2) \) cannot be factorized \( U(t, r) = a^2(t)f(r) \) and \( V(t, r) = a^2(t)r^2 \) uniformly inside the clusters (Zeng & Gao 2005).

Combining eq(2) with the background universe Friedman equation:

\[
\dot{a}^2 = \frac{8\pi G}{3}(\rho_{mb}a^2 + \rho_{Qb}a^2) \tag{3}
\]

we get

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{\rho_{mc,ta}a^2 + \rho_{Qb}(\frac{3}{2}a^2 - \frac{1}{2}a_p^2) - k}{\rho_{mb}a^2 + \rho_{Qb}a^2} \tag{4}
\]

using notations

\[
x = \frac{a}{a_p}; \ y = \frac{a_p}{a_{p,ta}}; \\
\zeta = \frac{\rho_{mc,ta}}{\rho_{mb,ta}}; \ \nu = 1 - \frac{\Omega_{mb,ta}}{\Omega_{mb,ta}}
\]

eq(4) becomes:

\[
\left( \frac{dy}{dx} \right)^2 = \frac{\zeta y^{-1} + \nu x^{-3(1+w)}(\frac{3}{2}\zeta^{-\frac{3}{2}}x^2 - \frac{1}{2}y^2) - k a_{p,ta}^{-2}}{x^{-1} + \nu x^{-1-3w}} \tag{6}
\]

where, as a result of \( \left[ \frac{dx}{dt} \right]_{x=1} = 0 \),

\[
ka_{p,ta}^{-2} = \zeta + \nu \left( \frac{3}{2}\zeta^{-\frac{3}{2}} - \frac{1}{2} \right) \tag{7}
\]

Eq(6) is a first order non-linear ordinary differential equation which contains a characteristic parameter \( \zeta \) but satisfies a two-boundary condition:

\[
y_{x=0} = 0; \ y_{x=1} = 1 \tag{8}
\]

This kind of equation can be easily solved by the numerical method described in (Press et al 1992), chapter 17. We display our numerical results in FIG.2. Which is also depicted in the figure is the results of \( \zeta(w, \Omega_{mb,ta}) \) when dark energy is assumed to move...
synchronously with ordinary matters both on Hubble scale and galaxy cluster scales. The actual fact should lie between this two extreme cases. But the result of (Wang & Steinhardt 1998) does not lie between this two extreme cases, so it cannot be thought as an approximation of the actual facts.

Eq(6) can also be solved analytically. Note
\[
\frac{y}{x} = \frac{a_p}{a_{p,ta}} = \frac{a_p}{a} \frac{a_{ta}}{a_{p,ta}} = \frac{a_p}{a} \zeta^{\frac{1}{4}} \delta^{\frac{1}{4}}
\]
so
\[
\frac{y}{x} \rightarrow 0 = \zeta^{\frac{1}{4}} \cdot 1\]

Let
\[
y = \zeta^{\frac{1}{4}} x \left[ 1 - \alpha(a_{ta}x) + \beta(a_{ta}x)^2 + \ldots \right]
\]
substituting eq(11) into (6) and equating the two sides of the resulting equation order by order in x, we can get
\[
\alpha = \frac{1}{5 a_{ta}} \left[ \zeta^{\frac{1}{4}} + \nu \left( \frac{3}{2} \zeta^{-\frac{1}{4}} - \frac{1}{2} \zeta^{-\frac{3}{4}} \right) \right]
\]
and a similar expression for \( \beta, \ldots \) etc. In this paper we only need to know \( \alpha \). Some people may argue that if progressional solution of eq(6) is not of the form as we wrote in eq(11) or does not exist at all, then our ansaltz eq(11) will be invalid. We would like to point out that if such things occur, then when eq(11) is substituted into eq(6) we can not get a self-consistent equation with the two sides equated order by order in x.

2.2. Extracting The Key Parameters of Press-Schecter Theory from Spherical Collapse Model

In the ideal model, if there is an over-dense region in a flat background universe, then at very early times, this region will expand as the background universe expands; but because this region’s over-dense, its expanding rate will decrease and stop doing so at some middle times; then it starts to shrink because of self-gravitating, the final fate of this over-dense region is a singular point. But in practice, when this region shrinks to some degree, the pressures originate from the random moving of particles inside the over-dense region will balance the self-gravitating and the system will enter the virialization period. In theoretical studies, it is usually assumed that the virialization point is coincident with the collapse point of the ideal model on the time axis.

According to Press-Schecter theory, if an over-dense region is to be virialized at some time \( a_c \), its density-contrast should be no less than \( \delta_c(w, \Omega_m, a_c) \).

\[
\delta_c = \left[ \frac{\rho_{mc}(a)}{\rho_{mb}(a)} - 1 \right] \frac{1}{D_1(a)} \quad a \rightarrow 0
\]

where \( \rho_{mc} \) and \( \rho_{mb} \) are the matter densities of cluster and background respectively, while \( D_1(a) \) is the growth function of linear perturbation theory (Dodelson 2003),

\[
D_1(a) = \frac{5 \Omega_{m0} H_0^2}{2}(a) \int_0^a da' [a' H(a')]^{-3}
\]

To reduce the numerical computation burdens, we will use the fitting formulae provided by (Ma et al 1999). We check that when \(-1.7 < w < -0.4 \) and \(0.1 < \Omega_m < 0.7\), the formulae provided by this work is accurate to \( 2\% \). To relate the mass of a galaxy cluster with its characteristic X-ray temperature, the ratio of cluster/background matter densities at the virialization point is another very important parameter,

\[
\Delta_c(w, \Omega_m, a_c) = \frac{\rho_{mc,c}}{\rho_{mb,c}}
\]

It can be shown that \( D_1(a) \rightarrow a \). Using eq(11), we have
\[
\left[ \frac{a_p}{a} \right]_{a \rightarrow 0} = (1 - \alpha \cdot a)
\]
Substituting eq(16) into eq(13) and using the fact that
In the above four equations, \(U_{mm,c}\) and \(U_{QQ,c}\) denote the matter-matter, \(\rho_{mc,ta}\) and \(\rho_{QQ,ta}\) the Quintessence-Quintessence gravitation potentials respectively. The subscripts \(_c\) and \(_{ta}\) indicate that quantities carrying them should take values at the collapse point, the system virializes fully, we can get the following relations:

\[
\delta_c(w, \Omega_{m0}, a_c) = \frac{3}{5a_{ta}} \left[ \zeta^2 + \nu \left( \frac{3}{2} - \frac{4}{5} - \frac{1}{2} \zeta^{-2} \right) \right] D_1(a_c) \tag{17}
\]

To calculate the second factor of the above equations' right-most part, we can use energy conserving condition and virial theorem. If Quintessence clusters synchronously with ordinary matters, by assuming that at the collapse point, the system virializes fully, we can get the following relations:

\[
E_{\text{kinetic},c} = -\frac{1}{2} U_{mm,c} + U_{mQ,c} - \frac{1}{2} U_{QQ,c} \tag{19}
\]

\[
\frac{1}{2} U_{mm,c} + 2U_{mQ,c} + \frac{1}{2} U_{QQ,c} = U_{mm,ta} + U_{mQ,ta} + U_{QQ,ta} \tag{20}
\]

just as we do in (Zeng & Gao 2005). Now, since Quintessence is assumed not to cluster on the scale of galaxy clusters at all, we can only write down the following relations:

\[
E_{\text{kinetic},c} = -\frac{1}{2} U_{mm,c} + U_{mQ,c} \tag{21}
\]

\[
\frac{1}{2} U_{mm,c} + 2U_{mQ,c} = U_{mm,ta} + U_{mQ,ta} \tag{22}
\]

in the above four equations, \(U_{mm}\), \(U_{mQ}\) and \(U_{QQ}\) denote the mass conserving condition, \(\rho_{mc,ta} = \rho_{mc,ta}\left[\frac{a_{ta}}{a_c}\right]^{-3(1+w)}\), we can change eq(24) into the following form:

\[
\frac{a_{p,ta}}{a_{p,c}} \left[ 1 + 4\xi \left( \frac{5}{2} \frac{a_{p,ta}}{a_{p,c}} - \frac{1}{2} \frac{\rho_{Qb,ta}}{\rho_{mc,ta}} \right) \right]^{-3(1+w)} = \left[ 2 + 2\left( \frac{5}{2} \zeta^2 - \frac{1}{2} \nu \zeta^{-1} \right) \right] \tag{25}
\]

where \(\xi = \frac{w}{\zeta} \left[ \frac{3}{a_c} \right]^{3(1+w)}\). Eq(25) can be solved analytically, \(\frac{a_{p,ta}}{a_{p,c}} = \frac{1}{3} \left[ -\beta + \frac{2}{\gamma(\beta, \alpha)} + \gamma(\beta, \alpha) \right] \),

\[
\alpha = 2\nu \left( \frac{a_{ta}}{a_c} \right)^{(1+w)} \gamma(\beta, \alpha) = 10\nu \zeta \left( \frac{a_{ta}}{a_c} \right)^{(1+w)} - \left[ 2 + (5\xi - 1) \right] \nu \zeta^{-1} \gamma = \left[ -2\beta^3 + 27\alpha + \sqrt{27(-4\beta^3\alpha + 27\alpha^2)} \right]^{1/2} \tag{26}
\]
Using the fact that $t_c = 2t_{ta}$ and background Friedman equation $(\frac{a}{a_c})^2 \propto (\rho_m + \rho_q \delta)$ we can set up an integration equation

$$
\int_0^{a_c} da' \sqrt{\frac{a'}{1 + \nu_0 a'^{w-3}}} = 2 \int_0^{a_{ta}} da' \sqrt{\frac{a'}{1 + \nu_0 a'^{w-3}}}
$$

(27)

where $\nu_0 = \frac{1 - \Omega_{m0}}{\Omega_{m0}}$. Solve eq(27) numerically, we can get the relation $a_{ta}$ v.s. $a_c$. Substituting eq(26) and $\frac{a_{ta}}{a_c}$ solved from eq(27) into eq(18), we will finally get the quantity $\Delta_c$.

We note that, if $w \leq -1$, then for very small $\Omega_{m0}$, if a cluster is to virialize too later, then at the virialization point, its radius will be larger than that of the turn around time, i.e. $\frac{a_{ta}}{a_c} < 1$, see FIG.3. Physically this means that, after the "turn around" point, instead of collapsing, the cluster experience a period of expansion to reach virialization status. Mathematically this only means that the turn around point is a local minimum instead of a local maximum of the cluster radius and has no problem in principle. This may be new structure which has not been discovered previously, we call it "phantom hole" and leave the detailed discussion of this structure for future works.

Fig. 3.— For too small $\Omega_{m0}$, if a cluster is to virialize too later and $w \leq -1$, then at the virialization point, its radius will be larger than that at the turn around time, i.e. $\frac{a_{ta}}{a_c} < 1$.

The formation of "phantom hole" will make the kinetic energy of the matter-system in it less than 0, please see eq(30), which may be a serious problem.

We provide numerical results for $\delta_c(w, \Omega_{m0}, a_c)$ and $\Delta_c(w, \Omega_{m0}, a_c)$ in FIG.4-5. About this two figure, what we would like point out is that, as $a_c \rightarrow 0$, $\delta_c \rightarrow 1.686$ and $\Delta_c \rightarrow 178$ whatever $w$ and $\Omega_{m0}$ is. Physically, this is because, the earlier an over-dense region collapse, the more the background universe is like a totally matter dominated one. While in a totally matter dominated universe, the fact that $\delta_c = 1.686$ and $\Delta_c = 178$ can be proved analytically.

Fig. 4.— $\delta_c$’s dependence on $w$, $\Omega_{m0}$ and $a_c$. If $a_c \rightarrow 0$, $\delta_c \rightarrow 1.686$ asymptotically whatever $w$ and $\Omega_{m0}$ is.

Fig. 5.— $\Delta_c$’s dependence on $w$, $\Omega_{m0}$ and $a_c$. As $a_c \rightarrow 0$, $\Delta_c \rightarrow 178$ asymptotically.
3. The Number Density of Galaxy Clusters and Its Evolutions

3.1. Theoretical Formulaes

According to Press-Schechter theory, the comoving number density of clusters which have collapsed (i.e., virialized) at certain red-shift $z$ and have masses in the range $M \sim M + dM$ could be calculated (Dodelson 2003):

$$n(M, z)dM = -\sqrt{2/\pi} \frac{\rho_{mb} \delta_c}{M} \frac{d\sigma}{dR} \exp\left[-\frac{\delta_c^2}{2\sigma^2}\right] dM$$

(28)

where: $\rho_{mb}$ is the matter density of background universe; the factor $\frac{d\sigma}{dR}$ denotes the average number density of clusters with mass $M = \frac{4\pi}{3}R^3$ $\rho_{mb}$; $\delta_c$ is given by eq(13); and

$$\sigma^2(R, z) = \sigma_8^2 \left[ \int k^{n_s+2} T^2(k) W^2(k \cdot R) dk \right] D^2([1 + z] - 1)$$

$$\sigma_8^2 \left[ \int k^{n_s+2} T^2(k) W^2(k \cdot R) dk \right] D^2(1)$$

(29)

In this paper we use the notations of (Dodelson 2003) where $n_s$ is the primordial power spectrum index, $T(k)$ is the BBKS transfer function, $W(k \cdot R)$ is the top-hat window function.

To relate the mass of a cluster with its characteristic X-ray temperature, consider a virialized spherical over-dense region in the background universe containing dark energies. If dark energies cluster synchronously with ordinary matters, just as we assumed in (Zeng & Gao 2005), then we have

$$E_{\text{kinetic, vir}} = \left[-\frac{1}{2} U_{mn} + U_{mQ} - \frac{1}{2} U_{QQ} \right]_{\text{vir}}$$

i.e.

$$(\rho_{mc,c} + \rho_{Qc,c})\bar{V}^2_{\text{vir}} = \frac{4\pi G}{3}\left[\sigma^2(\rho_{mc} - \rho_{Qc})^2\right]_{\text{vir}}$$

where $\bar{V}^2_{\text{vir}}$ is the mean square velocity of particles in the cluster when the system is fully virialized and $\alpha_p$ is the scale factor of the cluster. However, if dark energies do not cluster on the scale of galaxy clusters at all, then what we can get should be

$$\rho_{mc,c}\bar{V}^2_{\text{vir}} = \frac{4\pi G}{3}\sigma^2\left[\sigma^2 - \frac{5}{2} \frac{\alpha_{p,c}}{\alpha_{q,c}} \frac{\Delta_c}{\varDelta} \right]_{\text{vir}}$$

So

$$\bar{V}^2_{\text{vir}} = \frac{3}{5}(GMH)^{\frac{1}{2}}\left(\frac{\Delta_c}{\varDelta} \right)^{\frac{1}{2}}$$

$$\times \left[1 - \frac{5}{2} \frac{\Delta_c}{\varDelta} - \frac{1}{2} \frac{\Omega_{Qb,c}}{\Omega_{mb,c}} \right]$$

(30)

where $a_p$ should be understood as the radius of clusters in units of Hubble length $H^{-1}$, since in this case we cannot define a scale factor globally in the clusters and we normalize $a_p$ as $a_p \approx a$ when $a \rightarrow 0$. Using relation:

$$k_B T = \frac{\mu m_p V^2_{\text{vir}}}{\beta}$$

(31)

where $k_B$ is the Boltzmann constant, $m_p$ is the mass of proton, while $\mu m_p$ is the average mass of particles, $\beta$ is the ratio of kinetic energy to temperature. The composition $\frac{m_p}{\mu m_p}$ has physical meaning of energy transformation efficiency from thermal dynamic form to x-ray form. Substituting eq(30) into eq(31) we get the following mass-temperature relation:

$$M = \frac{1}{GH(z)} \left[ \frac{5 \beta k_B T}{\mu m_p} \frac{1}{f(z)} \right]^{\frac{1}{2}}$$

or

$$R = \left[ \frac{2GM}{H^2} \right]^{\frac{1}{2}} = \frac{1}{H(z)} \left[ \frac{5 \cdot 2 \frac{\beta k_B T}{\mu m_p}}{f(z)} \right]^{\frac{1}{2}}$$

(32)

with

$$H(z) = H_0[\Omega_{m0} a_c^{-3} + (1 - \Omega_{m0} a_c^{-3(1+w)})]^\frac{1}{2}$$

$$f(z) = \left(\frac{\Delta_c}{\varDelta} \right)\left[1 - \frac{5}{2} \frac{\Delta_c}{\varDelta} - \frac{1}{2} \frac{\Omega_{Qb,c}}{\Omega_{mb,c}} \right]$$

(33)

and $\Delta_c$ given by eq(18) and $z = a_c^{-1} - 1$.

Just as (Wang & Steinhardt 1998) pointed out, since the mass-temperature relation is red-shift dependent, simply substituting eq(32) into eq(28) cannot give us correct number density of clusters in a given temperature range today. Instead, we should first find out the virialization rate and multiply it by the mass-temperature relation then integrate over red-shift

$$n(T, z)dT =$$

$$-\frac{1}{\sqrt{2\pi}} \int_z^\infty \frac{\rho_{tot} \frac{d\sigma}{d\Delta_c} \frac{d\sigma}{d\Delta_c} \frac{\delta_c^2}{\varDelta^2} - 1)\exp[-\frac{\delta_c^2}{2\varDelta^2}] d\Delta_c dT$$

(34)
From eqs(34), (32) and (29) we can see that besides \( \frac{\sigma}{\mu} \), \( n(T, z) \) will also depend on the cosmological parameters \( w, \Omega_m, h, n_s \) and the normalization \( \sigma_8 \) of the cosmic density fluctuations. In principle, if we can measure the number density \( v.s. \) temperature relation precisely enough, by numerical fittings, we can determine all these parameters simultaneously from observations. However, in practice, because of parameter degeneracy and measure errors, we can only determine some of them or their combinations partly.

Now let us return to the strange phenomenon displayed in FIG.2 and 3, i.e. \( \zeta < 1 \) and \( \frac{\sigma}{\mu, p} < 1 \) respectively. We have explained that these two things take place when \( w < -1 \) and \( \Omega_{mb, ta} \) or \( \Omega_m \) takes too small values. It can be checked that when \( \frac{\sigma}{\mu, p} < 1 \), we must have \( \zeta < 1 \) and the kinetic energy of the virialized matter system will be less than zero, see eq(30). We think such a “cluster” can not emit X-rays.

### 3.2. Numerical Results, Effects of \( w \) on the Number Density of Galaxy Clusters and Its Evolutions

We display the effects of \( w \) on the number-density \( v.s. \) temperature of galaxy clusters in FIG.6 for four compositions of \( \sigma_8 \) and \( \frac{\sigma}{\mu} \). For any given parameter set \( \{ \sigma_8, \frac{\sigma}{\mu} \} \), if \( -1 < w < 0 \), then \( w \) affects the number-densities \( v.s. \) temperature of galaxy clusters exponentially. If \( w < -1 \), the effects are weak.

In FIG.7 we display the effects of \( w \) on the number-density \( v.s. \) red-shift of galaxy clusters whose mass is greater than \( 8h^{-1} \times 10^{14} M_{\odot} \) for two values of \( \sigma_8 \). From the figure we can easily see that \( w \) affects the number-density \( v.s. \) red-shift relation of galaxy clusters remarkably. When \( -1 < w < 0 \), the number density almost does not vary with time for \( \sigma_8 = 0.55 \), but for \( \sigma_8 = 0.85 \) case, the number density increases as we look back to the past. When \( w \) is less than \(-1\), the number density decreases at low red-shift, but increases at high red-shift, the turn around red-shift depends on \( w \). The less is \( w \), the higher red-shift the trend reverses. In FIG.8, we depict the same effects for the number density of galaxy clusters whose mass is greater than \( 1.5h^{-1} \times 10^{14} M_{\odot} \). From the figure, we can see for the smaller mass galaxy clusters, the effects of \( w \) on the number-density \( v.s. \) red-shift is more remarkable.

Comparing this fact with the observational results reported by (Bahcall, Fan & Cen 1997) and (Bahcall & Bode 2003), we can almost immediately say that \( w \) can not be greater than \(-1\). Of course, a reliable conclusion should be obtained by best fitting the current observational results with theoretical predictions. But if we want to fit the results of (Edge et al 1990; Henry & Arnaud 1991; Henry 1997) with our theoretical formulæ eq(34), we at least have four parameters \( \frac{\sigma}{\mu} \), \( w \), \( \sigma_8 \) and \( \Omega_m \) to determine. If we want to fit the results of (Bahcall & Bode 2003), we have to treat the problem of fitting data with errors in both coordinates. This two kinds of operations both cost time formidably. We leave them for future works.

Comparing the results in this paper with that of (Zeng & Gao 2005), we can see that the effects of \( w \) on the evolution of number density of galaxy clusters is even more strong under the assumption that dark energy does not cluster on the scale of galaxy clusters than the case where dark energy is assumed to cluster synchronously with ordinary matters. It’s easy to imagine that, the actual case should lie between this two extreme way, matter clusters and forms potential well, then dark energy falls into it and cannot climb up so clusters also. If we want to fit observation results into theoretical predictions to get reasonable constraints on \( w, \Omega_m, \sigma_8 \) and other cosmological parameters, theoretical formulæs in the both extreme cases are needed, and some kind of interpolation should be used.

Whatever the actual case is, according to the results found in this paper and its previous counterpart (Zeng & Gao 2005), we know that the effect of \( w \) on the number density of galaxy clusters is so strong that we think it should be possible to use this effect to measure \( w \).

### 4. Conclusions

We study the top-hat spherical collapse model of galaxy clusters formation in the flat QCDM or Phantom-CDM cosmologies under the assumption that Quintessence or Phantom does not cluster on this scale. We find that under this assumption, the key parameters of the model exhibit rather non-trivial and remarkable dependence on the equation of state coefficients \( w \) of Quintessence or Phantoms. We then applied the results in Press-Scheter theory and calculated the number density \( v.s. \) temperature function and the evolution of the number density of massive galaxy clusters and find that these two Quantities are both affected by \( w \) exponentially.

For the number density \( v.s. \) temperature function of
Fig. 6.— Effects of $w$ on the number density v.s. temperature function of galaxy clusters when $z = 0$. The larger is $\sigma_8$ or $\beta$, the larger the function value will be. All four figures have $\Omega_m=0.27$, $h=0.71$ and $n_s=1.0$.

galaxy clusters, we find that it is an increasing function of $w$ and the dependence on $w$ is more strong in the range $-1 < w < 0$ than it is in the range $-\infty < w < -1$. While for the evolution of the number density of massive galaxy clusters, we find that when $w$ is less than $-1$, the number density decreases at low red-shift, but increases at high red-shift, the turn around red-shift depends on $w$. The smaller is the galaxy clusters’ mass, the stronger is the effect. According to the observational result, which says that the number density of massive galaxy clusters decreases as we look back to the past, we can qualitatively conclude that $w$ should not take values greater than $-1$ too much. It should take values less than $-1$.

The actual dark energy cluster property should lie between the two extreme cases discussed in this paper and that in (Zeng & Gao 2005). But whatever the fact is, our results here and that in (Zeng & Gao 2005) indicate that, $w$ affects the number density of galaxy clusters exponentially. So we think measuring the number density of galaxy clusters and its evolutions may be an effective way to determine $w$.

As discussions, we would like to state that, if the problem we pointed out in (Zeng & Gao 2005) is the fact, then we may have to accept that, we ignored a very important assumption in the usual ΛCDM cosmology. That is: the cosmic component denoted by $\Lambda$ and leading to the acceleration of the universe moves synchronously with ordinary matters. The reason is very clear, if $\Lambda$ does not move synchronously with ordinary matters on Hubble scales, then in our comoving reference frame build on the ordinary matters(such as supernovae), we should have a $\Lambda$ current flowing outside the hubble horizon. Once that occurs, we cannot define an all universe uniformly defined scale factor, so we have no Friedmann equation at all. In that case, our current explanation of the acceleration indicated
by the observation of supernovae will be very problematic.

If Λ component moves synchronously with ordinary matters, then we will have no reason to say that it is the vacuum energy of quantum field. On the contrary, some kinds of couplings between Λ (or dark energy) and the ordinary matters is a must be derivation.

Part of the numerical computations are performed on the parallel computers of the Inter-discipline Center of Theoretical Studies of ITP, CAS, Beijing, China.

Appendix. Comparing With Previous Works

A. Comparing The Basic Equations

Our results in this paper and its sibling one (Zeng & Gao 2005) are so different from the previous works that without a concrete comparison and explicit pointing out the problem in those works, few peoples will believe our conclusions.

To compare our basic equations with the basic equations used by (Wang & Steinhardt 1998), we can differentiate eq(2) and use energy conservation law to get,

\[ \frac{\dot{a}}{a} = -4\pi G \left[ (w + \frac{1}{3}) \rho Q a^3 \frac{\dot{a}}{a} - \frac{1}{2} \rho a^2 \dot{a} - \frac{1}{3} \rho \right] \]

In the right hand side of this equation, if we let a = a_p, i.e., assume that the dark energy moves synchronously with ordinary matters on the galaxy cluster scales, then it reduce to be the eq(A2) of (Wang & Steinhardt 1998). Note, in the right hand side of eq(35), a only appears in terms where dark energy is involved. But (Wang & Steinhardt 1998) does not use this assumption consistently, because when it combine its eq(A2) with the equation describing the dark energy’s evolution eq(A6), it uses the assumption that only on Hubble scales, dark energy moves synchronously with ordinary matters.

Although we have pointed out in (Zeng & Gao 2005), we still would like to point out that, as long as variable separation technique is used in solving Einstein equation, then whichever (Raychaudhuri or Friedmann) equation we choose to describe the evolution of the over-dense region, we will in fact have assumed that dark energy moves synchronously with ordinary matters in our over-dense region. Let us explain this point in more details. For an over-dense collapsing region, the most general metric describe its inside spacetime is

\[ ds^2 = -dt^2 + U(t, r)dr^2 + V(t, r)(d\theta^2 + \sin^2\theta d\phi^2), \]

Using Einstein equation \( G_{\mu\nu} = -8\pi GT_{\mu\nu} \), we can prove that, only when no energy current flowing outside the over-dense region, is the energy momentum tensor \( T_{\mu\nu} \) diagonal, and can the metric function \( U(t, r) \) and \( V(t, r) \) be factorized as

\[ U(t, r) = a_p^2(t)f(r), \quad V(t, r) = a_p^2(t)r^2. \]

And only when the \( U(t, r) \) and \( V(t, r) \) function is factorized, can we have

\[ \frac{\dot{a}_p}{a_p} = -4\pi G \left[ \rho_{mc} + \rho_{Qc} + 3\rho_{Qc} \right], \]

in (WS98)'s notation

\[ \frac{\dot{R}}{R} = -4\pi G(pQ + \rho Q + \rho_{cluster}) \]

\[ \frac{\dot{a}_p}{a_p} + \frac{2}{3} \dot{a}_p^2 + 2 \frac{k}{a_p^2} = 4\pi G[\rho_{mc} + \rho_{Qc} - p_{Qc}] \]  

As a must be of eqs(38) and (39)

\[ \frac{\dot{a}_p^2}{a_p^2} + \frac{k}{a_p^2} = \frac{8\pi G}{3}(\rho_{mc} + \rho_{Qc}) \]

So Friedmann equation and Raychaudhuri equation must hold at the same time, or must not hold simultaneously. The statement of (Wang & Steinhardt 1998) and (Weinber & Kamionkowski 2002) that when dark energy does not cluster on the scale of galaxy clusters, Raychaudhuri equation can be used to describe the evolution of the over-dense region but Friedmann equation does not hold is an incorrect statement.

Let us say more explicitly, when dark energy does not cluster on the galaxy clusters, hence a dark energy exists which describe the flowing of dark energy outside the over-dense region, the basic equation which should be used to describe the evolution of the over-dense region is not the eq(A2) of (Wang & Steinhardt 1998), it should be our eq(2). Our eq(2) is not obtained by variable separation in solving Einstein equation. We obtained it by energy conservation, so we include the effects of the dark energy current on the evolution of the over-dense region.
B. Comparing The Numerical Results

Let us emphasize again that the problem in the existing works (Wang & Steinhardt 1998) and (Weinberg & Kamionkowski 2002) is, when writing down the basic equations describing the evolution of the over-dense region they assumed that dark energy moves synchronously with ordinary matters, please see eq(A2) of (Wang & Steinhardt 1998), but when writing down equations which will be used to describe the density of dark energies in the over-dense regions, please see eq(A6) of (Wang & Steinhardt 1998), a different assumption is made. That is, dark energy only moves synchronously with ordinary matters on Hubble scales.

Just as we pointed out in (Zeng & Gao 2005) and in the conclusion section of this paper. In realities, dark energy should have some degree of cluster behaviors on the galaxy clusters. We can imagine, ordinary matters cluster and form potential wells, when dark energy falls in and some degree of dark energy’s clustering will occur either. So the actual case of dark energy’s clustering phenomenon should lie between the following two extreme cases. The two extreme cases are, dark energy moves synchronously with ordinary matters on both Hubble scales and galaxy cluster scales or dark energy only moves synchronously with ordinary matters on Hubble scale but could not fall in the potential wells formed by the over-dense matter region at all.

We study the first extreme case in (Zeng & Gao 2005) and the second extreme case in this paper. A natural question is, can the results of (Wang & Steinhardt 1998) lie between our two extreme cases? If this is the case, then although inconsistent, the treatment of (Wang & Steinhardt 1998) can be thought as some kind of approximation of realities. We will see in the following that this is not the case. We compared the results of $\zeta$’s dependence on $w$ and $\Omega_{mb,ta}$ in FIG.9 and 10, FIG.9 is 3-dimensional, FIG.10 is 2-dimensional.

C. Some Comments On References

When this paper and its sibling one (Zeng & Gao 2005) are put on the e-preprint arXive and submitted to Astrophysical Journals, we are told that just recently, many authors have studied this problem in different depth. Such as (van de Bruck & Mota 2005), (Battye & Weller 2003), (Koivisto 2005), (Mota & van de Bruck 2004), (Nunes & Mota 2004) and (Manera & Mota 2005). We believe there must be more we do not know at this moment. We think we should first express our thanks to the referee and editors of APJ and the authors of these papers for their informing us of these works, we then also would like to point out that, almost all these works contain the problem we pointed out in this paper, i.e., when writing down the basic equations to describe the evolution of the over-dense regions, the assumption that dark energy moves synchronously with ordinary matters was made, but when the dark energy density was involved in the right hand side of the equation, a different assumption is made. That is, only on Hubble scales, dark energy moves synchronously with ordinary matters. Down left, dark energy is assumed to move synchronously with ordinary matters both on Hubble scales and on galaxy clusters scale. Down right, dark energy is only assumed moves synchronously with ordinary matters on Hubble scales, it is not assumed to move synchronously with ordinary matters on galaxy clusters scale.

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Fig. 10.— The same as FIG.9, but $\Omega_{mb,ta}$ is set to three special value 0.1, 0.3, 0.5.

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