Charm- and Bottom- Baryons: A Variational Approach Using Heavy Quark Symmetry

C. Albertus*, J. E. Amaro*, E. Hernández† and J. Nieves*

*Dpto. de Física Moderna, U. Granada, Spain.
†Grupo de Física Nuclear, Facultad de Ciencias, U. Salamanca, Spain.

Abstract. We evaluate masses of bottom and charmed baryons using several non-relativistic quark potentials which parameters have been adjusted to the meson spectra. Heavy Quark Symmetry leads to important simplifications of the three body problem, which turns out to be easily solved by a simple variational ansatz. The wave functions obtained can be readily used to compute further observables as mass densities or form factors. The quark-quark potentials explored so far, show an overall good agreement with the experimental masses.

INTRODUCTION

The non-relativistic constituent quark model (NRCQM), using QCD-inspired potentials, has proved to be an excellent tool to predict properties of hadrons.

In the case of baryons including one heavy flavour (c, b) and two light ones (u, d, s), it is possible to take advantage of yet another property of QCD: Heavy Quark Symmetry (HQS) Ref. [1, 2]. In the limit in which the mass of the heavy quark is infinity, the quantum numbers of the light degrees of freedom are well defined always. Furthermore, in this limit, the masses of the baryons depend only on the quark content and on the light-light quantum numbers of the baryon. All of this is a clear simplification for solving the three body problem. Thus, for bottom- and charm-baryons we can consider the quantum numbers of the two light quark system to be fixed, and neglect corrections terms in the wave function that scale as \( \mathcal{O}(\Lambda_{QCD}/m_{c,b}) \).

The aim of this work is to determine masses and other properties like mass densities and electromagnetic form factors for baryons containing a heavy quark and two light ones. This study includes all baryons compiled in Table 1 and some more details will be given elsewhere [3].

Table 1. Baryons considered in this work. The information enclosed in the different columns is strangeness, spin-parity, isospin, spin-parity of the light degrees of freedom and quark content. The spin-parity of the light quarks, fifth and eleventh columns, in some cases are determined thanks to HQS.

| Baryon | (S) | \( J^P \) | (I) | \( s^0 \) | Quark content | Baryon | (S) | \( J^P \) | (I) | \( s^0 \) | Quark content |
|--------|-----|---------|-----|---------|---------------|--------|-----|---------|-----|---------|---------------|
| \( \Lambda_{c,b} \) | (0) | \( \frac{1}{2}^+ \) | (0) | 0^+ | \( (u,d)_{c,b} \) | \( \Xi_{c,b} \) | (-1) | \( \frac{1}{2}^+ \) | (1) | 1^+ | \( (u,s)_{c,b} \) |
| \( \Sigma_{c,b} \) | (0) | \( \frac{3}{2}^+ \) | (1) | 1^+ | \( (u,u)_{c,b} \) | \( \Xi^*_{c,b} \) | (-2) | \( \frac{3}{2}^+ \) | (0) | 1^+ | \( (s,s)_{c,b} \) |
| \( \Sigma^*_{c,b} \) | (0) | \( \frac{1}{2}^+ \) | (1) | 1^+ | \( (u,u)_{c,b} \) | \( \Omega_{c,b} \) | (-2) | \( \frac{3}{2}^+ \) | (0) | 1^+ | \( (s,s)_{c,b} \) |
| \( \Xi_{c,b} \) | (-1) | \( \frac{1}{2}^+ \) | (1) | 0^+ | \( (u,s)_{c,b} \) | \( \Omega^*_{c,b} \) | (-2) | \( \frac{3}{2}^+ \) | (0) | 1^+ | \( (s,s)_{c,b} \) |
THE THREE-BODY PROBLEM

The intrinsic hamiltonian that describes the dynamics of the baryon is given by

\[ H = -\frac{\vec{\nabla}_1^2}{2\mu_1} - \frac{\vec{\nabla}_2^2}{2\mu_2} + \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_h} + V_{l,h}(\vec{r}_1, \text{spin}) + V_{l,h}(\vec{r}_2, \text{spin}) + V_{l,l_2}(\vec{r}_1, \vec{r}_2, \text{spin}), \]  

(1)

where \( \vec{r}_i \) is the position of the \( i \)-th light quark with respect to the heavy one, \( m_h \) stands for the mass of the heavy quark, while \( \mu_i \) accounts the reduced mass of the heavy and the \( i \)-th light quark system, \( V_{l,h} \) and \( V_{l,l_2} \) are the light–heavy and light–light interaction potentials, and the words \( \text{spin} \) stands for possible spin dependence of the potentials. Note the presence of the Hughes-Eckart term \( \vec{\nabla}_1 \cdot \vec{\nabla}_2/m_h \) that results from the separation of the CM movement.

The potentials used in this work are the one proposed by R.K. Bhaduri et al. in Ref. [4], and the set of potentials proposed by B. Silvestre-Brac and C. Semay that can be found in Ref. [5]. The parameters of those potentials have been adjusted in the meson sector. For their use in the \( qq \) sector they have to be adequately transformed. We use the prescription \( V_{qq}^{ij} = \frac{1}{2} V_{q\bar{q}}^{ij} \) that assumes a \( \lambda_i \cdot \lambda_j \) color dependence in all terms of the potential [5].

To solve the three-body problem one can use Faddeev equations [5]. This is a non-trivial task from the computational point of view, and leads to wave functions that are difficult to use in other contexts. Here we propose an extremely simple variational scheme. As it is usual, we assume an antisymmetric wave-function for the color degrees of freedom and the spin-flavour wave function is determined by the quantum numbers specified in Table 1. Finally for the spatial wave function, we propose the ansatz

\[ \psi(r_1, r_2, r_{12}) = NF(r_{12}) \phi_1(r_1) \phi_2(r_2) \]  

(2)

where \( N \) is a normalization factor, \( \phi_i(r_i) \) is the ground state wave function for the \( V_{l,h}^{qq} \) potential, and \( F(r_{12}) \) is a Jastrow correlation function in the relative distance of the two light quarks \( r_{12} \). For \( F \) we take

\[ F(r_{12}) = \left(1 - e^{-c_1 r_{12}}\right) \sum_{j=2}^{4} a_j e^{-b_j^2 (r_{12} -d_j)^2} \]  

(3)

where the term \( e^{-c_1 r_{12}} \) would be excluded in those cases where the potential \( V_{l,h}^{qq} \) does not show a repulsive hard core at the origin. Taking into account that the color wave function is antisymmetric we use symmetrized wave functions in the spin-isospin and orbital degrees of freedom of the two light quarks.

This variational scheme shows clear resemblances to that successfully used in the study of double \( \Lambda \) hypernuclei [6].

1 In this hamiltonian the motion of the center of mass (CM) of the baryon has been taken out.
2 We have assumed that the relative orbital angular momentum between the light quarks is zero. Thus the spatial wave function can only depend on \( r_1, r_2 \) and \( r_{12} \)
PRELIMINARY RESULTS

Our results for the masses obtained with the AL1 potential of Ref. [5] are given in Tables 2 and 3. We find good agreement with experimental data [7], when available, and with previous results from lattice [8] and Faddeev calculations [5].

In Table 4 we give the mass radii obtained also for the AL1 potential. Our results agree with the ones obtained in Ref. [5] using a Faddeev approach. Conclusions are similar when the potential of [4] or potentials AL2, AP1 or AP2 of Ref. [5] are used.

Table 2. Masses for the bottom- baryons considered. The spin-parity of the light degrees of freedom is shown in the second column. Results with our variational approach and with a Faddeev calculation from Ref. [5] are included. Lattice QCD [8] and experimental values [7], when available, are also given.

| B   | s^x content | M_{exp.} [MeV] | M_{Latt.} [MeV] | M_{Var} [MeV] | M_{Fad} [MeV] |
|-----|--------------|----------------|-----------------|---------------|--------------|
| Λ_b | 0^+ udb      | 5624 ± 9       | 5640 ± 60       | 5640          | 5638         |
| Σ_b | 1^+ llb      | 5770 ± 70      | 5846            | 5845          |              |
| Σ_b' | 1^+ llb    | 5780 ± 70      | 5877            |              |              |
| Σ_b'' | 0^+ lsb    | 5760 ± 60      | 5805            |              |              |
| Σ'_b | 1^+ lsb     | 5900 ± 70      | 5941            |              |              |
| Ξ_b | 1^+ lsb     | 5900 ± 80      | 5972            |              |              |
| Ξ_b' | 1^+ lsb    | 5990 ± 70      | 6034            |              |              |
| Ξ_b'' | 1^+ lsb    | 6000 ± 70      | 6065            |              |              |

Table 3. As in Table 2 for the charm sector.

| B   | s^x content | M_{exp.} [MeV] | M_{Latt.} [MeV] | M_{Var} [MeV] | M_{Fad} [MeV] |
|-----|--------------|----------------|-----------------|---------------|--------------|
| Λ_c | 0^+ udc      | 2285 ± 1       | 2270 ± 50       | 2291          | 2285         |
| Σ_c | 1^+ llc      | 2452 ± 1       | 2460 ± 80       | 2453          | 2455         |
| Σ_c' | 1^+ llc     | 2518 ± 2       | 2440 ± 70       | 2542          |              |
| Σ_c'' | 0^+ lsc    | 2469 ± 3       | 2410 ± 50       | 2476          | 2467         |
| Σ_c' | 1^+ lsc     | 2576 ± 2       | 2570 ± 80       | 2571          |              |
| Ξ_c | 1^+ lsc     | 2646 ± 2       | 2550 ± 80       | 2657          |              |
| Ξ_c' | 1^+ lsc    | 2698 ± 3       | 2680 ± 70       | 2677          | 2675         |
| Ξ_c'' | 1^+ lsc    | 2660 ± 80      |                |              |              |

Table 4. Results for mass radii using this variational here and those from the Faddeev calculation of Ref. [5].

| B   | \langle r^2 \rangle [fm^2] (Var) | \langle r^2 \rangle [fm^2] (Fad.) |
|-----|----------------------------------|----------------------------------|
| Λ_b | 0.045                            | 0.045                            |
| Σ_b | 0.054                            | 0.054                            |
| Σ_b' | 0.048                           | 0.048                            |
| Σ_b'' | 0.054                          | 0.054                            |
| Ξ_b | 0.095                            | 0.104                            |
| Σ_c | 0.117                            | 0.121                            |
| Ξ_c | 0.096                            | 0.104                            |
| Ξ_c' | 0.102                           | 0.108                            |

Finally in Figs. 1 and 2 we give our results for the charge density and electric form factor of the Λ_b and Ω_b^- baryons.
Figure 1. Charge density times $r^2$ for $Λ_b$ (solid) and $Ω_b^-$ (dashed).

Figure 2. Electric form factors for $Λ_b$ (solid) and $Ω_b$ (dot-dashed). The value at the origin is the charge of the baryon, 0 for $Λ_b$ and -1 for $Ω_b^-$. 
CONCLUDING REMARKS

The use of HQS simplifies considerably the solution of the three body problem in baryons with a heavy quark. Here we propose a method based on a simple variational approach that provides us with simple and portable wave functions that can be used in other contexts. Our results agree with previous ones obtained in the lattice or using a more complicate Faddeev approach. Calculations with potentials obtained from chiral quark models [9] and the study of the semileptonic decay of bottom baryons into charmed ones are under consideration.

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