I. INTRODUCTION

Numerous studies have been conducted to better understand the dynamic behavior of financial markets. Most of the studies mainly focus on the statistical properties of the independent events occurring at discrete time steps. In recent years, the waiting time between consecutive events has drawn much attention, for instance, the intertrade duration between two consecutive trades [1–4], the duration time that the price or volatility keeps below or above its initial value [5–7], and the waiting time that the price return first exceeds a predefined level [8–13].

To the best of our knowledge, the concepts raised above have been carefully studied, and have multifractal nature, and the power-law distribution has been presently applied to many other fields, e.g., climatic cataclysm, and has been widely studied by many scientists. It offers an inspiration that the analysis of the independent events occurring at discrete time steps. In recent years, the recurrence interval analysis has been conducted to study the relationship between the trading volumes and the price returns, and growing evidence shows that large price movements might be driven by large trading volumes, described well by the price impact function [18–20]. Some of these studies have revealed that the trading volumes and the magnitude of the price returns have universal properties, such as the fat-tailed distribution and the long-term memory effect [21–25]. The recurrence interval analysis method has been presently applied to many other fields, e.g., tur-
bulence \[44\] and networks \[45\] \[46\]. In this paper, we attempt to study the recurrence intervals between large trading volumes, and test if recurrence intervals of the trading volumes show power-law distribution and memory effects similar to those of the price returns.

The paper is organized as follows. In Section 2, we introduce the data sets analyzed and the investigated variables. Sections 3 and 4 study the probability distribution and the memory effects of the recurrence intervals respectively. In Section 5, we study the relationship between trading volumes and price returns based on the recurrence interval analysis. Section 6 gives the conclusion.

II. DATA SETS

We analyze the 1-min intraday data of 20 liquid stocks actively traded on the Shanghai Stock Exchange and the Shenzhen Stock Exchange from January 2000 to May 2009, the Shanghai Stock Exchange Composite Index (SSEC) and the Shenzhen Stock Exchange Composite Index (SZCI) from January 2003 to April 2009. The data are retrieved from a database developed by GTA Information Technology Co., Ltd, see [http://www.gtadata.com/]. Since the sampling time is 1 minute, the total number of data points is about 340000 for the two Chinese indices and 500000 for individual stocks. These 20 stocks are actively traded stocks representative in a variety of industry sectors. Each stock is uniquely identified with a stock code which is a unique 6-digit number. A stock with the code initiated with 60 is traded on the Shanghai Stock Exchange, while a stock with the code initiated with 00 is listed on the Shenzhen Stock Exchange.

The 1-min trading volumes of the Chinese stock markets exhibit a U-shaped intraday pattern, like many Western stock markets \[20\] \[22\] \[23\] \[25\]. The intraday pattern is defined as

\[
A(s) = \frac{\sum_{i=1}^{N} V_i(s)}{N},
\]

which is the average volume at a specific minute \(s\) of the trading day averaged over all \(N\) trading days and \(V_i(s)\) is the trading volume at time \(s\) of day \(i\). The intraday pattern is illustrated as Fig. 1. The Shanghai Stock Exchange and Shenzhen Stock Exchange open at 9:30 a.m., and close at 15:00 p.m., during which there exists a midday break between 11:30 a.m. and 13:00 p.m.. Similar to many Western stock markets, the average volume of the Chinese stock markets displays sharp peaks close to the opening and closing times. In addition, the average volume of the Chinese stock market surges soon after midday break which may due to the information aggregation and the cumulative orders submitted during the midday break. Taking a more careful look at the curves for the two Chinese indices, one observes that the average volume shows a relatively sharp jump at around 10:30 a.m. This phenomenon can be explained by the trading rule that for the stocks which behaved abnormally the Stock Exchange will suspend trading in relevant securities. Until the day the parties with disclosure obligation make relevant announcements, the stocks involved will resume trading at 10:30 a.m. on that day. All these factors will affect the measurement of the trading volumes, and will consequently lead to periodic behavior in the distribution of recurrence intervals between large trading volumes.

\[
A(s) = \frac{\sum_{i=1}^{N} V_i(s)}{N},
\]

FIG. 1: (Color online) Interval intraday patterns of trading volumes for SSEC, SZCI and four representative stocks.

To avoid the effect of this periodic oscillation, the intraday pattern is removed by

\[
V'(t) = \frac{V(t)}{A(s)}.
\]

Then the volatility is normalized by dividing its standard deviation

\[
v(t) = \frac{V'(t)}{\sqrt{\langle V'(t)^2 \rangle - \langle V'(t) \rangle^2}}^{1/2}.
\]

III. PROBABILITY DISTRIBUTION OF RECURRENT INTERVALS BETWEEN LARGE TRADING VOLUMES

A. Empirical distribution

It is well accepted that there exists a close relationship between price returns and trading volumes. Recent empirical studies on the recurrence intervals between price
returns have shown that the tail of the recurrence interval distribution follows a power-law scaling. Here we study the probability distribution function (PDF) \( P_q(\tau) \) of the recurrence intervals between trading volumes, and see if the power-law scaling behavior maintains.

In Fig. 2 the scaled PDFs \( P_q(\tau) \) are plotted as a function of the scaled recurrence interval \( \tau / \langle \tau \rangle \) for various values of \( q = 2, 3, 4, 5 \) for the two Chinese indices and four representative stocks. For small values of \( \tau / \langle \tau \rangle \), \( P_q(\tau) \) diverges for different \( q \) values, which may partially due to the discreteness of recurrence intervals. However, one can also observe that for large values of \( \tau / \langle \tau \rangle \) the curves for different \( q \) values almost collapse onto a single curve and show nice linear behavior in the double logarithmic plot. Hence, we can conclude that the tails of the PDFs of the recurrence intervals between trading volumes follow a scaling behavior as

\[
P_q(\tau) = \frac{1}{\langle \tau \rangle} f(\tau / \langle \tau \rangle).
\]

This indicates that the scaled PDF for large values of the scaled interval is independent of the threshold \( q \).

![FIG. 2: (Color online) Empirical probability distributions of scaled recurrence intervals for different thresholds \( q = 2, 3, 4, 5 \) for SSEC, SZCI and four representative stocks. The solid curves are the fitted functions \( cx^{-\delta} \) with parameters listed in Table I.](image)

**B. Determining the scaling function**

The linear behavior of the scaled PDF \( P_q(\tau / \langle \tau \rangle) \) for large values of \( \tau / \langle \tau \rangle \) in the double logarithmic plot suggests that the scaling function may follow a power law as

\[
f(\tau / \langle \tau \rangle) = f(x) = cx^{-\delta}, \quad x \geq x_{\min}
\]

for \( x \) larger than a lower bound \( x_{\min} \). We aggregate the interval samples above \( x_{\min} \) for different values of \( q = 2, 3, 4, 5 \), and fit them using a common power-law function presented in Eq. (5).

Recently, an efficient method of fitting power-law distributions based on the Kolmogorov-Smirnov (KS) statistic is proposed by Clauset, Shalizi and Newman [47]. We use this method with the purpose of making a relatively accurate estimation of the parameters in Eq. (5). The KS statistic is defined as

\[
KS = \max_{x \geq x_{\min}} (|F - F_{PL}|)^{0.3},
\]

where \( F \) is the cumulative distribution function (CDF) of the empirical data and \( F_{PL} \) is the CDF of the power-law fit. To make the empirical PDF and the best power-law fit as similar as possible, we determine the estimate of \( x_{\min} \) by minimizing the KS statistic, then we estimate the values of the other two parameters \( c \) and \( \delta \) using the maximum likelihood method. The estimated parameters \( \hat{x}_{\min}, \hat{\delta}, \hat{c} \) and the resultant KS statistic are depicted in Table I and the power-law fits are correspondingly illustrated in Fig. 2. For the two indices, \( x_{\min} \) shows small values close to 1.4 indicating a scaling region over almost three orders of magnitude with power-law exponents \( \delta = 1.74 \pm 0.04 \). Among 20 stocks, 16 stocks have \( \hat{x}_{\min} < 9.0 \) indicating a scaling region over more than two orders of magnitude with power-law exponents \( \delta = 2.0 \pm 0.3 \). The rest of stocks (600009, 600058, 600104 and 000063) have \( \hat{x}_{\min} > 9.0 \), and show scaling regions spanning almost two orders of magnitude with exponents larger than 2.0.

We have performed a power-law fit for the empirical interval distributions, and given a relatively accurate estimation of the parameters. Furthermore, a goodness-of-fit test should be adopted to examine the goodness of the power-law fit. Based upon the KS statistic, we test the hypothesis that the empirical PDFs could be fitted well by their common power-law fit. To do this, a bootstrap method is adopted following Refs. [14,18]. One thousand synthetic data sets are randomly generated from the best power-law fit of the empirical PDFs. For each synthetic data set, a KS statistic is obtained as

\[
KS_{\text{sim}} = \max_{x \geq x_{\min}} (|F_{\text{sim}} - F_{\text{sim,PL}}|)^{0.3},
\]

where \( F_{\text{sim}} \) is the CDF of the synthetic data and \( F_{\text{sim,PL}} \) is the CDF of the power-law fit for synthetic data. The KS statistic for the empirical data is defined by Eq. (5). The \( p \)-value is defined as the frequency that \( KS_{\text{sim}} > KS \), which is regarded as the probability that the power-law fit is coincident with the empirical PDF.

A variant of the KS statistic, known as the KSW statistic [18], is also used to perform the goodness-of-fit
TABLE I: Estimated parameters for the power-law fit of empirical PDFs, and goodness-of-fit tests based on KS, KSW, and CvM statistics for two Chinese indices and 20 stocks.

| Code | $\tau_{\text{min}}$ | $\alpha$ | KS | $p_{\text{KS}}$ | KSW * | $p_{\text{KSW}}$ | $W^2$ |
|------|------------------|--------|-----|----------------|-------|----------------|-----|
| SSEC | 1.46             | 1.76   | 0.0257 |
| SZCI | 1.40             | 1.71   | 0.0285 | 0.121          | 0.016 | 0.17 |
| 600000 | 5.12 | 2.04   | 0.0383 | 0.003 | 0.26 |
| 600009 | 20.27 | 2.75   | 0.0328 | 0.035 | 0.07 |
| 600016 | 1.82 | 2.08   | 0.0322 | 0.032 | 0.14 |
| 600058 | 15.06 | 2.17   | 0.0196 | 0.092 | 0.02 |
| 600060 | 1.85 | 1.92   | 0.0347 | 0.0004 | 1.65 |
| 600073 | 4.53 | 2.03   | 0.0191 | 0.158 | 0.09 |
| 600088 | 2.95 | 1.99   | 0.0381 | 0.008 | 1.42 |
| 600098 | 2.57 | 1.93   | 0.0312 | 0.055 | 0.35 |
| 600100 | 7.54 | 1.97   | 0.0316 | 0.176 | 0.145 |
| 600104 | 9.24 | 2.59   | 0.15   | 0.0265 | 0.19 | 0.17 |
| 600001* | 2.73 | 1.94   | 0.0311 | 0.029 | 0.72 |
| 600002* | 2.33 | 2.00   | 0.0197 | 0.171 | 0.27 |
| 600021† | 0.72 | 1.75   | 0.0315 | 0.019 | 0.61 |
| 600027* | 0.78 | 1.79   | 0.0348 | 0.0 | 2.93 |
| 600029 | 2.31 | 2.12   | 0.0229 | 0.508 | 0.220 |
| 600031 | 2.31 | 1.89   | 0.0202 | 0.320 | 0.064 |
| 600039 | 2.37 | 2.03   | 0.0133 | 0.733 | 0.111 |
| 600060 | 3.71 | 1.96   | 0.0146 | 0.816 | 0.274 |
| 600063 | 37.88 | 3.27  | 0.0258 | 0.997 | 0.311 |
| 600066* | 0.56 | 1.72   | 0.0305 | 0.0 | 0.157 |

The KSW statistic for the empirical data is defined as

$$KSW = \max_{\tau \geq \tau_{\text{min}}} \left( \frac{|F - F_{\text{PL}}|}{\sqrt{F_{\text{PL}} - (1 - F_{\text{PL}})}} \right). \quad (8)$$

The KSW statistic with this definition is more sensitive on the edges of the cumulative distributions. In the bootstrap process, 1000 synthetic data sets are generated from the best power-law fit, and therefore the KSW statistic for the synthetic data is calculated by

$$KSW_{\text{sim}} = \max_{\tau \geq \tau_{\text{min}}} \left( \frac{|F_{\text{sim}} - F_{\text{PL, sim}}|}{\sqrt{F_{\text{PL, sim}} - (1 - F_{\text{sim, PL}})}} \right). \quad (9)$$

Similar to the p-value for KS statistic, the p-value for the KSW statistic could be obtained by calculating the frequency that $KSW_{\text{sim}} > KSW$.

The goodness-of-fit tests based on the KS and KSW statistics are carried out for the two Chinese indices and 20 individual stocks, and the resultant p-values are depicted in Table I. Consider the significance level of 1%, if the p-value is larger than 0.01, we can conclude that the stock passes the test, and consequently the null hypothesis that the empirical PDFs of the relevant stock for different q values could be fitted well by their common power-law fit is accepted. Among the 20 stocks, four stocks (600060, 600088, 0000027, 0000066) marked with * fail in the test using both KS and KSW statistics, and three stocks (000001, 000002, 000021) marked with † fail in the test using KS or KSW statistic. In other words, 13 stocks pass the test using both KS and KSW statistics, and 16 stocks pass the test using at least one of KS and KSW statistics. Based upon this fact, we may roughly conclude that for most of the stocks the tails of empirical PDFs follow scaling behavior and the scaling function could be approximated by a power law. Similar results are obtained for the two Chinese indices composed of individual stocks traded on relevant stock exchanges: SSEC passes the goodness-of-fit test using both KS and KSW statistics, and SZCI passes the goodness-of-fit test using KS statistic.

We also use other variation of the goodness-of-fit test on place of the KS statistic. The Crémér-von Mises (CvM) statistic is another common used statistic, which is defined as

$$W^2 = N \int_{-\infty}^{\infty} (F - F_{\text{PL}})^2 dF_{\text{PL}}, \quad (10)$$

where $F$ and $F_{\text{PL}}$ are the CDFs of the empirical data and its power-law fit respectively, and $N$ is the number of scaled interval samples $x = \tau/\langle \tau \rangle$. For a sequence of scaled interval samples $x_1, x_2, \ldots, x_N$ arranged in ascending order, the computational form of $W^2$ statistic is given by

$$W^2 = \frac{1}{12N} + \sum_{i=1}^{N} \left( x_i - \frac{2i - 1}{2N} \right)^2. \quad (11)$$

In Table II $W^2$ for the two Chinese indices and 20 individual stocks is also depicted. For a stock which has a $W^2$ smaller than the critical value 0.743, it successfully passes the goodness-of-fit test under the significance level of 1%. From the table, only four stocks (600060, 600088, 0000027, 0000066) show $W^2$ larger than the critical value, thus fail in the CvM test. In general, the CvM statistic offers results very similar to those of the KS statistic, and for the stock which has a large p-value close to 1.0 the $W^2$ statistic of the relevant stock is significantly smaller than the critical value.

IV. MEMORY EFFECTS IN RECURRENCE INTERVALS BETWEEN TRADING VOLUMES

A. Conditional PDF and mean conditional recurrence interval

Besides the investigation of the probability distribution, the computation of the temporal correlation offers another important way of understanding the statistical properties of the recurrence intervals between trading volumes. Previous studies have detected the presence of memory effects in the recurrence intervals between price returns [31, 32, 33]. We thus conjecture similar memory effects may also exist in the recurrence intervals between trading volumes.

To verify our conjecture, we first calculate the conditional PDF of the recurrence intervals between trading
volumes. The conditional PDF computes the probability of finding a recurrence interval $\tau$ conditioned on the preceding interval $\tau_0$. The conditional PDF is calculated for a bin of $\tau_0$ to get better statistics. We rank the whole sequence of the recurrence intervals in an ascending order, and partition it to four bins with equal size. Fig. 3 plots the scaled conditional PDF $P_q(\tau|\tau_0)(\tau)$ as a function of the scaled recurrence interval $\tau/\langle \tau \rangle$ with $\tau_0$ in the largest and smallest bins for different $q$-values for SSEC, SZCI and four representative stocks. The curves for different $q$-values with $\tau_0$ in the largest and smallest bins approximately collapse on to two separate solid curves as illustrated in Fig. 3. For small $\tau/\langle \tau \rangle$, $P_q(\tau|\tau_0)(\tau)$ with $\tau_0$ in the smallest bin is larger than that with $\tau_0$ in the largest bin, while $P_q(\tau|\tau_0)(\tau)$ with $\tau_0$ in the largest bin shows values larger than that with $\tau_0$ in the smallest bin when $\tau/\langle \tau \rangle$ is large. This means that small $\tau_0$ tends to be followed by small $\tau$ and large $\tau_0$ tends to be followed by large $\tau$, and therefore provides an evidence of the short-term memory.

![FIG. 3: (Color online) Scaled conditional PDF $P_q(\tau|\tau_0)(\tau)$ of scaled return interval $\tau/\langle \tau \rangle$ with $\tau_0$ in the largest 1/4 subset (open symbols) and the smallest 1/4 subset (filled symbols) for SSEC, SZCI and 4 representative stocks. The solid lines are guidelines for looking.](image)

To detect the short-term memory of the recurrence intervals, we can also calculate the mean conditional recurrence interval $\langle \tau|\tau_0 \rangle$ conditioned on the preceding interval $\tau_0$. In Fig. 4 the scaled mean conditional recurrence interval $\langle \tau|\tau_0 \rangle/\langle \tau \rangle$ is plotted with respect to the scaled preceding interval $\tau_0/\langle \tau \rangle$ for the two Chinese indices and four representative stocks. Though $\langle \tau|\tau_0 \rangle/\langle \tau \rangle$ strongly fluctuates in the whole region of $\tau_0/\langle \tau \rangle$, it approximately shows a monotonically increasing tendency as the increase of $\tau_0/\langle \tau \rangle$. This indicates that for small (large) preceding interval $\tau_0$ the mean value of the following interval is also small (large), and this further confirms the short-term memory revealed in the conditional PDF.

![FIG. 4: (Color online) Scaled mean conditional return interval $\langle \tau|\tau_0 \rangle/\langle \tau \rangle$ as a function of scaled return interval $\tau_0/\langle \tau \rangle$ for SSEC, SZCI and four representative stocks.](image)

B. Detrended fluctuation analysis

To further investigate the long-term memory of the recurrence intervals between trading volumes, we use the detrended fluctuation analysis (DFA) method, known as a common method of measuring the temporal correlation of a time series [52–56]. The DFA method computes the detrended fluctuation function $F(l)$ of the time series within a window of $l$ data points after removing a linear trend, and $F(l)$ is expected to follow a scaling form

$$F(l) \sim l^{\alpha}. \quad (12)$$

The scaling exponent $\alpha$ contains the information of temporal correlation between the time series: if $\alpha = 0.5$ the time series are uncorrelated, and if $0.5 < \alpha < 1.0$ the time series are long-term correlated. By calculating the exponent $\alpha$ of the recurrence intervals, we can investigate the temporal correlation between them.

![FIG. 5: (a) and (b) plot $F(l)$ of the recurrence intervals for the two Chinese indices. Different symbols represent $F(l)$ for different values of threshold $q$. One observes that](image)
for both two indices $F(l)$ for different $q$ values approximately obey a scaling form, and similar scaling behaviors are observed for the 20 individual stocks. By calculating the slope of its best linear fit in the double logarithmic plot, we can obtain the estimate of $\alpha$. In Fig. 5 (c), the estimate values of $\alpha$ for the two Chinese indices and 20 stocks are depicted. It is clear that the values of $\alpha$ for all the indices and stocks are significantly larger than 0.5, which indicates there exists a long-term memory in the recurrence intervals. We further assume that the long-term memory of the trading volumes may arise from the long-term memory of the recurrence intervals. To verify this we also calculate $F(l)$ of the recurrence intervals between shuffled trading volumes in which the long-term memory is artificially eliminated by shuffling. The exponent $\alpha$ of the recurrence intervals between shuffled trading volumes for the two Chinese indices and 20 stocks shows values very close to 0.5, as illustrated in Fig. 5 (c). This may provide an evidence to substantiate the assumption that the long-term memory of the trading volumes leads to the long-term memory of the recurrence between them.

FIG. 5: (Color online) Detrended fluctuation function $F(l)$ of return intervals for SSEC and SZCI. The curves are vertically shifted for clarity. Exponents $\alpha$ of trading volumes (black circles), recurrence intervals (open symbols) and shuffled recurrence intervals (filled symbols) for two Chinese indices and 20 stocks.

V. DEPENDENCE OF TRADING VOLUMES ON PRICE RETURNS

A. Correlation between recurrence intervals of trading volumes and their preceding price returns

Empirical studies have shown that the trading volumes are highly correlated with the price returns, that large price returns are usually accompanied by large trading volumes. We attempt to address the question how the price returns affect the trading volumes based on the return interval analysis. Suppose $v(t)$ and $v(t + \tau)$ are two consecutive trading volumes exceeding threshold $q$ and $r(t)$ is the preceding return of recurrence interval $\tau$ as illustrated in Fig. 6 the correlation function is defined as

$$C(|r|, \tau) = \frac{\langle |r(t)| \tau \rangle - \langle |r(t)| \rangle \langle \tau \rangle}{\sqrt{\langle |r(t)|^2 \rangle - \langle |r(t)| \rangle^2 \langle \tau^2 \rangle - \langle \tau \rangle^2}} \quad (13)$$

It measures the correlation between the magnitude of the price return $|r(t)|$ and the volume recurrence interval $\tau$ immediately after it. Here $\langle \cdots \rangle$ is the average over time $t$.

FIG. 6: (Color online) Correlation function between recurrence intervals of trading volumes and their preceding returns for SSEC (filled black circles), SZCI (filled red squares), 10 stocks traded on Shanghai Stock Exchange (open black circles) and 10 stocks traded on Shenzhen Stock Exchange (open red squares).

Fig. 6 plots the correlation function $C(|r|, \tau)$ for the two Chinese indices and 20 stocks. For large $q$, $C(|r|, \tau)$ for the two Chinese indices and most of the individual stocks are significantly smaller than zero. The negative value of $C(|r|, \tau)$ indicates an anti-correlation between the volume recurrence interval and its preceding return. In other words, once a large positive or negative price change occurs, the following recurrence interval between large trading volumes tends to show small values, therefore large trading volumes are more likely to occur after
large price returns. To illustrate this, we randomly select a certain series of trading volumes initiated with an extremely large return $r(t=0) > 5$ for SSEC, and plot it in Fig. 4. The trading volume strongly fluctuates short after the large return for $t \leq 120$, and shows on average larger values than that after cooling down for a while for $t > 120$.

![FIG. 7: (Color online) Evolution of a certain series of normalized trading volumes conditioned on a large preceding return $r > 5$ for SSEC.](image)

**B. Comovement between trading volumes and price returns**

We have shown that there exists an anti-correlation between the recurrence intervals of trading volumes and their preceding price returns. The evolution curves of trading volumes and absolute price returns are plotted in Fig. 8, the observed comovement between them suggests that there might be some relationship between their recurrence intervals. Therefore, we further study the relationship between the recurrence intervals of both trading volumes and price returns. In doing so we calculate the probability that the trading volumes have the same recurrence intervals as the price returns as illustrated in Fig. 8. Suppose two consecutive trading volumes exceeding threshold $q$ occur at time $t$ and $t + \tau$, the probability is calculated as

$$P(\tau \mid |r| > Q) = \frac{N_{\tau, |r|>Q}}{N_{\tau}},$$  \hspace{1cm} (14)

where $N_{\tau, |r|>Q}$ is the number of $\tau$ conditioned on two consecutive absolute returns exceeding the threshold $Q$, i.e., $|r(t)| > Q$ and $|r(t+\tau)| > Q$, and $N_{\tau}$ is the total number of $\tau$.

In Fig. 8 $P(\tau \mid |r| > Q)$ is plotted as a function of the threshold $Q$ for the two Chinese indices and four representative stocks. In principle, $P(\tau \mid |r| > Q)$ equals one when $Q = 0$, and approximately follows a power-law decay for large scales of $Q$ as shown in the insets in Fig. 9. One observes that $P(\tau \mid |r| > Q)$ for large $q$ is obviously larger than that for small $q$, displaying a power-law tail with an exponent similar to that for small $q$. This indicates that the comovement between trading volumes and price returns is more pronounced for large thresholds $q$ and $Q$, which further confirms the close relationship between the trading volumes and the price returns.

**VI. CONCLUSION**

In this paper, we have introduced the recurrence interval analysis to the study of stock market trading volumes. The recurrence intervals between the trading volumes above a threshold $q$ for the two Chinese indices and 20 liquid Chinese stocks are carefully studied. Though the scaled PDFs of the recurrence intervals for different $q$ values diverge slightly for small intervals, they tend to show scaling behavior for large intervals. The goodness of fit tests are further performed to find out the specific form of the scaling function, and the results demonstrate that it could be approximated by a power law. The measurements of the conditional PDF, the mean conditional recurrence interval and the DFA method are carried out to detect the short-term and long-term memory of the recurrence intervals, and both memory effects are clearly observed.

The study of market impact of trading volumes is sup-
posed to be of great importance for both theoretical and practical reasons. Here, we tried to study the correlation between trading volumes and price returns using the recurrence interval analysis method. We calculated the correlation between the recurrence intervals of trading volumes and their proceeding price returns, and found that the trading volumes following large price returns are more likely to show large values, which may corresponds to the unstable state when after a market burst. The pronounced comovement between large trading volumes and large price returns further confirms our previous findings that large price returns are usually accompanied by large trading volumes. In fact, our study mainly focuses on the relationship between trading volumes and the magnitude of price returns. Further study concerning the dependence of trading volumes on price returns taking into account the effect of the signs of price returns needs to be proceeded, and this will certainly help to study the risk estimation in financial markets.

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