Characterization of short necklace states in the logarithmic transmission spectra of localized systems

Liang Chen\textsuperscript{1,2} and Xunya Jiang\textsuperscript{1,3,4}

\textsuperscript{1} State Key Laboratory of Functional Materials for Informatics, Shanghai Institute of Microsystem and Information Technology, CAS, Shanghai 200050, People’s Republic of China
\textsuperscript{2} Graduate School of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
\textsuperscript{3} Department of Illuminating Engineering and Light Sources, School of Information Science and Engineering, Fudan University, Shanghai, People’s Republic of China
\textsuperscript{4} Engineering Research Center of Advanced Lighting Technology, Fudan University, Ministry of Education, Shanghai, People’s Republic of China

E-mail: jiangxunya@fudan.edu.cn

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Abstract

High transmission plateaus exist widely in the logarithmic transmission spectra of localized systems. Their physical origins are short chains of coupled localized states embedded inside the localized system, which are dubbed as ‘short necklace states’. In this work, we define the essential quantities and then, based on these quantities, we investigate the properties of the short necklace states statistically and quantitatively. Two different approaches are utilized and their results agree very well. In the first approach, the typical plateau-width and the typical order of short necklace states are obtained from the correlation function of the logarithmic transmission. In the second approach, we investigate the statistical distribution of the peak/plateau-width measured in the logarithmic transmission spectra. A novel distribution is found, which can be exactly fitted by the summation of two Gaussian distributions. These two distributions are the results of sharp peaks of localized states and the high plateaus of short necklace states. The center of the second distribution also tells us the typical plateau-width of short necklace states. With increasing system length, the scaling property of the typical plateau-width is very special since it hardly decreases. The methods and quantities defined in this work can be widely used in Anderson localization studies.

(Some figures may appear in colour only in the online journal)

1. Introduction

The most extraordinary phenomenon of wave transport in random media is Anderson localization [1], which is possessed by various systems, such as electronic, photonic and acoustic systems [2]. In an Anderson localized system the ensemble average of the logarithmic transmission $\langle \ln T \rangle$, not the transmission $\langle T \rangle$, is additive with the system length $L$ [3, 4]. Naturally, the localization length can be defined as $\xi = -2L/\langle \ln T \rangle$. In strongly localized systems ($L \gg \xi$), the transmission is generally small, but also shows large fluctuations [4]. These extremely large $T$ values will dominate the transmission of localized systems. In recent years, study of the physical origins of these large $T$ values has yielded abundant results to aid in understanding the transport phenomena in random systems and the Anderson localization [5–25].

Previously, it was pointed out by Azbel et al [5] that the resonant transport through the localized states lying near the system center can contribute very large $T$ values, of order unity. Although such a mechanism can generate a high resonant transmission peak [5, 6], its contribution to the transmission is vanishingly small in a strongly localized system because the resonant peak is
Degenerate localized states are required to be evenly exponentially sharp \( \sim e^{-L/\xi} \) \[5, 6, 9\]. Later, Pendry \[7\] and Tartakovskii \[8\] independently predicted a kind of quasi-extended state, called the necklace state (NS). The NS is formed through the coupling between nearly degenerate localized states which are evenly distributed in the system. Although the spatial overlaps between the localized states are small, because of the degenerate coupling, the NS contributes very wide transmission ‘mini-bands’, which can dominate the transmission of strongly localized systems. NSs have been demonstrated experimentally in photonic systems \[10, 11\]. Their fundamental character and statistical consequences for the transmission are widely studied \[12, 14–17\]. Recently, it was demonstrated that the NSs have an evident contribution to the short-time transport of the wave package \[19–22\] and the dynamics of fluctuations of localized waves \[23\]. More profoundly, the number of NSs can increase dramatically on approaching the Anderson transition point, which strongly supports a mode coupling induced quantum percolation scenario for the Anderson localization–delocalization transition \[24, 25\].

Even though NSs can contribute very large transmission, their formation has rigorous conditions, i.e., the nearly degenerate localized states are required to be evenly distributed inside the system \[7, 14, 16, 17\]. In general, localized states with similar frequencies are not so well distributed in a specific random configuration. In such cases, ideal NSs \[14, 15\] crossing the whole configuration are not formed. However, eventually, those nearly degenerate states that are also spatially close to each other couple together, forming short chains of coupled localized states embedded inside the configuration. Since these coupled localized states have similar properties to the NS, we call them short necklace states (SNSs) \[26\]. The SNS is also manifested as a high transmission plateau which has significant transmission contribution. In transmission spectra the SNS is difficult to distinguish from isolated localized states since the valley between coupled peaks seems to be very low. However, it can be clearly identified from the logarithmic transmission spectra, for instance, see the coupled peaks in figure 1. Because of the strong coupling between the neighboring localized states inside the SNS, on top of the plateau there are sharp peaks and valleys, which correspond to non-smooth changes of the transmission phase between the coupled peaks \[14\].

In contrast to the NSs, the SNSs have the special properties \[26\] that (1) SNSs exist widely in every random configuration while NSs only occur once in millions of random configurations with large length \( L \gg \xi \); (2) the plateau-width of an SNS only depends on the coupling strength between the neighboring localized states and is insensitive to the system length. With these properties, SNSs are superior qualitatively compared with NSs. However, the way in which to quantitatively characterize the statistical properties and scaling behavior of SNSs is still an unexplored topic. Moreover, some essential quantities in SNS study, such as the ‘half-widths’ of SNS plateaus in \( \ln T \) spectra, need to be defined since there is no obvious physical quantity in previous studies that can directly describe SNSs.

In this paper, we study the statistical manifestation of an SNS using physical quantities that are defined in the logarithmic transmission spectra. To characterize an SNS, we find two different approaches, whose results can be compared with each other. The first approach is based on the correlation functions \( C_{\ln T} \) (defined in the \( \ln T \) spectra) and \( C_t \) (defined in the transmission coefficient \( t \) spectra). From \( C_{\ln T} \), we show that there is a typical frequency correlation range of logarithmic transmission, which is explained as the typical SNS plateau-width. From this frequency range and compared with the results of \( C_t \), we can find the most probable order of the SNS. The second approach is from direct measurement of the peak/plateau-width in \( \ln T \) spectra. We find that the statistical distribution of the peak/plateau-width is quite abnormal and can be fitted very well by the summation of two Gaussian distribution functions, where the primary Gaussian center gives the typical width of the resonant peaks of the localized states while the secondary one gives exactly the typical width of the SNS plateaus. Excellent agreement is found between the two approaches. The dependence of the properties of SNSs on the scaling parameter \( L/\xi \) is also studied. We find that the plateau-width of SNS hardly depends on \( L/\xi \), while the peak-width of localized states decays exponentially with \( L/\xi \), indicating that SNSs have more significant transmission contributions in longer systems. The essential quantities defined in this work also provide new means for further quantitative study of the statistical properties of the transmission of Anderson localized systems.

The rest of this paper is organized as follows. In section 2, we introduce our model and the basic properties of the \( \ln T \) spectra. In section 3, we present our numerical results and theoretical analysis of three correlation functions: the correlation of the transmission \( C_T \), the correlation of the logarithmic transmission \( C_{\ln T} \) and the field correlation \( C_t \). We show that the half-width of \( C_{\ln T} \) gives the typical
plateau-width of an SNS. The most probable order of an SNS can be obtained by comparing \( C_{\ln T} \) and \( C_t \). In section 4, we study the statistical distribution of the peak/plateau-width (defined in the \( \ln T \) spectra). The probability distributions are calculated from very large numbers of samples and can be fitted very well by the summation of two Gaussian distributions, where the first Gaussian center gives the typical width of the resonant peaks of localized states and the second one gives the typical width of the SNS plateau. In section 5, we discuss the system length dependence of the SNS. Finally, a summary of this work is given in section 6.

2. The model

We study 1D random stacks composed of binary-dielectric layers (called A and B) with thicknesses \( d_A = d_B = 500 \) nm and with refractive indices \( n_A = 1.0 \) and \( n_B = 3.0 \times (1 + W_\gamma) \), where \( \gamma \) is a random number uniformly distributed in \([-0.5, 0.5]\) and \( W \) gives the randomness strength. This periodic on average model is an optical counterpart of the Anderson model of electronic systems and is widely used for localization studies [12, 14, 15, 19, 25]. The transmission coefficients of optical waves are calculated by the standard transfer matrix method [25]. Without randomness, \( W = 0 \), the system exhibits a pass band in the frequency range (89.13, 124.1 THz). Our study will focus on the range (100, 103.2 THz), where the localization length is almost a constant for a certain randomness \( W (0.2 \lesssim W \lesssim 0.6) \) [25]. The localization length is calculated by \( \xi = -2L/\langle \ln T \rangle \). We can see they have similar average heights, being approximately \(-2L/\xi = -20\). After being averaged over a large number of configurations, the \( \ln T \) spectrum becomes a flat line exactly falling on the mean value \( \langle \ln T \rangle = -20 \). It is a natural result according to the single parameter scaling theorem that \( \ln T \) follows a Gaussian distribution with mean value \(-2L/\xi\). An interesting phenomenon in the \( \ln T \) spectra is that there are some clear plateau structures, such as the one marked by arrows in figure 1(a).

Figure 1(a) shows two typical logarithmic transmission spectra with the same \( L/\xi = 10 \) but with different localization lengths, \( \xi = 215.6a \) (black thin) and \( \xi = 34.14a \) (red bold). We can see they have similar average heights, being approximately \(-2L/\xi = -20\). After being averaged over a large number of configurations, the \( \ln T \) spectrum becomes a flat line exactly falling on the mean value \( \langle \ln T \rangle = -20 \). It is a natural result according to the single parameter scaling theorem that \( \ln T \) follows a Gaussian distribution with mean value \(-2L/\xi\). An interesting phenomenon in the \( \ln T \) spectra is that there are some clear plateau structures, such as the one marked by arrows in figure 1(a).

From the wave intensity distributions one can find that these plateaus are formed by the SNSs (i.e., from the degenerate coupling between some spatially close localized states) embedded inside the configuration [26]. Since the peaks of the localized states in the spectra are extremely sharp \((\sim e^{-L/\xi})\), the fluctuation of \( \ln T \) is dominated by these plateaus. Moreover, these SNSs may be essential for an understanding of the Anderson transition phenomenon [26]. After observing a large number of \( \ln T \) spectra for fixed \( \xi \) and \( L/\xi \), we find that the plateaus appear with some intuitive regularities: (1) most plateaus have similar frequency widths for a configuration; (2) the numbers of peaks on each plateau are similar. For example, for most plateaus of the \( \xi = 34.14a \) system shown in figure 1, the plateau-width is about 0.3–0.4 THz and the number of peaks is about 3–4. In the following we will focus on the statistical properties of these SNSs and try to characterize them quantitatively.

To quantitatively study the SNS, we need to find proper physical quantities. Figure 1 shows that the plateaus of the SNSs can hardly be distinguished from the localized states in a \( T \) spectrum (see figure 1(b)), but can be clearly identified by the plateaus in the \( \ln T \) spectra (figure 1(a)). Therefore, we try to define physical quantities in the \( \ln T \) spectra for the SNS study. In the following, we will define and study the correlation function of the \( \ln T \) spectra in our first approach. Then, we will define the \( \ln T \) peak/plateau-width \( \Gamma \) and study the statistical distribution of \( \Gamma \) in our second approach.

3. Correlation functions

According to the standard definition of mathematics, we define the correlation functions of \( T \) and \( \ln T \) in the frequency domain as

\[
C_T(\Delta \omega) = \frac{\text{Cov}(T_\omega, T_{\omega+\Delta \omega})}{\sigma(T_\omega)\sigma(T_{\omega+\Delta \omega})}
\]

(1)

\[
C_{\ln T}(\Delta \omega) = \frac{\text{Cov}(\ln T_\omega, \ln T_{\omega+\Delta \omega})}{\sigma(\ln T_\omega)\sigma(\ln T_{\omega+\Delta \omega})}
\]

(2)

where \( T \) is the transmission coefficient, \( \sigma \) is the standard deviation, \( \sigma(x) = \sqrt{(x-\langle x \rangle)^2} \), and Cov is the covariance, \( \text{Cov}(x, y) = \langle (x-\langle x \rangle)(y-\langle y \rangle) \rangle \). Physically, the meaning of \( C_T \) is the frequency correlation of the transmitted wave intensity. Its Fourier transformation corresponds to the dynamical response at the outgoing interface, which has been intensively studied in weakly localized systems [27]. If two frequencies are on the same transmission peak, \( C_T \) is close to unity. Otherwise \( C_T \) will be close to zero. Hence, the half-width of \( C_T \) is usually used to characterize the line-width of localized states [27]. Similarly, in the \( \ln T \) spectra, since the \( \ln T \) of two frequencies on the same plateau are much larger than the mean value \( \langle \ln T \rangle \) and contribute a significantly large \( C_{\ln T} \), we expect \( C_{\ln T} \) to be close to unity when \( \Delta \omega \) is smaller than a typical plateau-width and to decay to zero when \( \Delta \omega \) is larger than a typical plateau-width.

Meanwhile, we also study the field correlation function,

\[
C_t(\Delta \omega) = \frac{\langle t_\omega(t_\omega+\Delta \omega) + t_\omega^*t_{\omega+\Delta \omega} \rangle}{\langle T_\omega \rangle + \langle T_{\omega+\Delta \omega} \rangle}
\]

(3)

where \( t(\omega) \) is the complex transmission coefficient and \( T = \text{tr}^* \). The complex amplitude of the incident wave is chosen to be \( E_\text{in}(x = 0) = 1 \) so that at the outgoing interface \( t = E(x = L) = |E|e^{i\phi} \), where \( E \) is the complex electronic field and \( \phi \) is its phase. Similarly to \( C_T \), the physical meaning of \( C_t \) is simply the frequency correlation of the transmitted field, i.e., the field correlation \( C_t \) is the intensity correlation.

The definition of \( C_t \) is the same as that in [7], and it can be negative valued, depending on the averaged phase difference \( \Delta \phi \) between \( \omega \) and \( \omega + \Delta \omega \). Generally, when \( \Delta \omega \) crosses a resonant peak, the phase of \( r \) jumps by \( \pi \) [7, 25]. Hence, the phase difference between \( t(\omega) \) and \( t(\omega + \Delta \omega) \) is approximately \( \Delta \phi = \phi(\omega) - \phi(\omega + \Delta \omega) \approx \pi \). Then, \( t_\omega t_{\omega+\Delta \omega}^* + t_{\omega+\Delta \omega}^*t_\omega = |E_\omega|^2 |E_{\omega+\Delta \omega}|^2 \cos(\Delta \phi) \) becomes
negative so that $C_T$ is negative. When $\Delta \omega$ reaches a value that typically contains two peaks (plus one average distance between two peaks), $C_T$ will be positive since $\Delta \Phi \approx 2\pi$, and so forth. Hence, when increasing $\Delta \omega$, we expect $C_T(\Delta \omega)$ to oscillate between positive and negative. Each time $C_T(\Delta \omega)$ changes its sign, the average number of resonant peaks in $\Delta \omega$ increases by one. Therefore, $C_T(\Delta \omega)$ provides a way to check the average resonant peak number in a certain frequency range.

In our calculations we set the original frequency point to $\omega = 100$ THz without loss of generality. The correlation functions calculated from a very large number of configurations for different $\xi$ are shown in figure 2(a), where the ratio $L/\xi$ is fixed at 10. The solid, dashed and dotted curves respectively represent $C_\text{S}$, $C_T$ and $C_{\text{inT}}$. The red curves marked by crosses correspond to the smaller $\xi$ system and the black curves marked by solid dots correspond to the larger $\xi$ system. In figure 2(b) the transverse axis is shown on a logarithmic scale.

Let us first discuss $C_T$. Figure 2(a) shows that $C_T$ is a very singular function at the origin, $\Delta \omega \rightarrow 0$. The line-width $\Delta \omega$ of localized states, corresponding to $C_T(\Delta \omega) = 0.5$, is larger in the smaller $\xi$ systems. This is consistent with the observation in figure 1 that the line-width of resonant peaks in smaller $\xi$ systems is larger. Detailed data show that the half-width of the larger $\xi$ system (black solid curve) is about 0.382 GHz and that of the smaller $\xi$ system (red solid curve) is 2.483 GHz. This is also quantitatively consistent with the observation from the T spectra.

$C_{\text{inT}}$ is a much smoother function than $C_T$ in the $\Delta \omega \rightarrow 0$ limit. Take the $\xi = 34.13a$ system (red curves) for example. $C_T$ rapidly falls to zero near $\ln(\Delta \omega) \approx 24$. $C_{\text{inT}}$ exhibits a plateau at the origin, then drops to 0.5 at $\ln(\Delta \omega) \approx 26.6$, i.e., its half-width is about 0.341 THz, which is much larger than that of $C_T$. This contrast reflects the different geometry properties of the $T$ and $\ln T$ spectra. In the $T$ spectrum, $C_T$ falls to zero typically when two frequencies are not on the same peak. Since the peaks of the localized states are exponentially sharp, $C_T$ decreases rapidly with increasing $\Delta \omega$. However, in the $\ln T$ spectrum, there exist many plateaus formed by SNSs, as shown in figure 1(a). These plateaus are usually higher than the average transmission background ($\ln T$). When two frequencies are on the same plateau but not the same peak, they will still contribute a significantly large value to $C_{\text{inT}}$. Hence, the half-width of $C_{\text{inT}}$, which is denoted as $\Omega_2$, in this paper, approximately gives the typical plateau-width of an SNS in the $\ln T$ spectrum.

With the help of $C_t$, we can find more detailed information on SNSs, such as the average SNS order, which is the average number of coupled localized states in one SNS. Similarly to $C_T$, $C_t$ also shows a sharp singularity at the origin and falls quickly with increasing $\Delta \omega$. Interestingly, $C_t$ shows some oscillations around $C_t = 0$ in the region where $C_T$ falls to nearly zero, see figure 2(c). As discussed earlier in this section, these oscillations can be understood from the $\pi$-phase jumps of the resonant peaks of the localized states. The different order minimal/maximal points of the $C_t$ correspond to the frequency ranges that can accommodate certain numbers of resonant peaks. We denote the $\Delta \omega$ at the first minimum of $C_t$ as $\Omega_1$, which is the typical frequency width of a resonant peak. We also denote $n$th order minimal/maximal points $\Omega_n$ as the average frequency range that can accommodate $n$ resonant peaks. Therefore, $C_t$ dictates the number of resonant peaks in a frequency range. With $\Omega_n$ in mind, we can see that the mean width of the SNS plateaus, obtained by $C_{\text{inT}}$, can accommodate about 3–4 resonant peaks, as indicated by the green arrows in figure 2(c).

Actually, this average number of resonant peaks in an SNS agrees very well with our direct observations of many spectra.

4. Peak/plateau-width statistics

To perform a test and verify the characteristics of the SNSs obtained from $C_{\text{inT}}$ and $C_t$, we next try our second approach, i.e., direct measurement of the peak/plateau-width in a large number of $\ln T$ spectra. Since the ensemble average of $\ln T$ spectra, $\langle \ln T \rangle$, is well defined, it is natural to define the peak/plateau-width $\Gamma$ as the frequency interval where $\ln T$ is always higher than $\langle \ln T \rangle$. From direct observation of the $\ln T$ spectra, one can find that $\langle \ln T \rangle$ crosses a lot of sharp peaks of localized states and fewer SNS plateaus. More precise statistical results should be obtained from a large number of realizations. We find that the statistical distribution of $\Gamma$ is extremely skewed. The reason is that in Anderson localized systems the peak-width of localized states is exponentially small and the width of coupled peaks also scales exponentially with the system length [5, 7, 25]. This is very similar to the probability distribution of the dimensionless conductance $g$, which is extremely skewed (nearly log-normal). An early
study [4] on Anderson localization has shown that it is better to study the additive quantity, ln, which is nearly Gaussian distributed. Similarly, instead of , we will study the ln distribution, which is likely Gaussian distributed.

We have measured in 10^5 spectra with high frequency precision that can distinguish each resonant peak in the L = 10^6 system. The probability distributions of ln for two different systems are shown in figure 3. Take the ξ = 34.13a system as an example. (All the following discussions also apply to the ξ = 215.6a system.) The probability distribution is roughly Gaussian-like, and a clear maximum is found at ln L ≈ 24.3 (ln ≈ 0.035 THz). This is exactly the ln(Ωt) at the first minimum of Ct shown in figure 2. As shown before, this value corresponds to the typical frequency range occupied by one single resonant peak. This result is coincident with our direct observation for the ln T spectra that the most probable peaks/plateaus cross by (ln T) are the single peaks. Suppose that the spectrum contains only sharp peaks and no plateaus. ln(Δω) should be nearly Gaussian distributed with a single maximum at the mean value. However, the real probability distribution of ln shows a strange shoulder at ln L ≈ 26.6 (ln ≈ 0.341 THz). Comparing with figure 2 we find that it exactly corresponds to the half-width of Cln T, Ωt, which is just the typical width of an SNS plateaux, referring to the physical meaning of Cln T. Actually, the measured probability distribution can be fitted very well by the summation of two Gaussian functions (dashed curves). The primary (left) Gaussian center corresponds to the typical width of resonant peaks of localized states, and the secondary (right, green) Gaussian center corresponds to the typical plateau-width of an SNS.

This is the first time that it has been seen directly from the statistical distribution that the SNS is clearly distinguished from other localized states. From the distribution, we can see that the logarithmic plateau-width of the SNS is Gaussian distributed and its mean value agrees excellently with the results of the correlation functions in section 3, as expected. Therefore, both Cln T and the probability distribution of ln provide proper physical values to characterize the SNSs.

5 Note that each spectrum covers a wide frequency range, where a lot of peaks and plateaus will be cut by (ln T), so that the ensemble of 10^5 spectra is considerably large, with about 10^3 sample peaks/plateaus.
an $L = 18'$ system the typical number of localized resonances on a short necklace state’s $\ln T$ plateau is about six. These typical orders can also be directly observed with the method described in section 3, i.e., by counting the number of minimal/maximal points within the frequency range $\Omega_q$. In our numerical simulations, it is shown that the number (order) of localized resonances increases nearly linearly with $L/\xi$. The different scaling behaviors of localized states and SNSs naturally result in a picture: with increasing $L$, the peaks of the localized states become more and more sharp (almost undetectable in a very large system), but the SNS plateau-width hardly decreases, so that the fluctuation of the $\ln T$ spectrum will be dominated by the SNSs in the large $L$ limit. The effect of SNSs on $\ln T$ fluctuation is discussed in detail in [26], where it is further argued that SNSs also affect the value of the localization length.

6. Discussion and summary

It is interesting to discuss the influence of an ideal NS on the results of this work. It is well known that although ideal NSs appear very rarely, they still dominate the transmission of Anderson localized systems [7]; this has been proved with the statistics of $T$ (integrated in the frequency domain) [10, 15, 25]. However, in the logarithmic transmission spectrum ($\ln T$), the root of the $\ln T$ plateau is enlarged and the difference in the $\ln T$ plateau-width between NSs and SNSs is small. The fluctuation of $\ln T$ (which directly relates to the fluctuation of the localization length) is dominated by the $\ln T$ plateaus, which are mainly contributed by the SNSs, because of their tremendous number compared to ideal NSs. In the systems studied in this paper, an ideal third-order NS appears only once in ~10000 samples, but SNSs exist in almost every random configuration, so that the statistical quantities defined in the $\ln T$ spectrum in this paper mainly characterize SNSs, and the influence of ideal NSs is negligible.

In summary, we have defined the basic quantities for SNS study and investigated the statistical properties of SNSs in strongly localized systems. We found two approaches to quantitatively study SNS properties. The first approach is based on the correlation functions and the second one is based on direct measurement of the peak/plateau-width in the logarithmic transmission spectra. In the first approach, we defined the correlation function $C_{\ln T}$ in the $\ln T$ spectra and showed that the typical width of an SNS plateau can be characterized by the half-width of $C_{\ln T}$. Moreover, with the help of $C_T$ (the correlation function of the transmission coefficient $t$), the most probable order of an SNS can be obtained. In the second approach, we defined the peak/plateau-width $\Gamma$ in the $\ln T$ spectra and studied the probability distribution of $\ln \Gamma$ directly measured from a large number of spectra. The probability distribution of $\ln \Gamma$ showed a novel shoulder and it could be fitted very well by the summation of two Gaussian distributions. We showed that the first one is from the distribution of localized state peaks and the second one is from the plateaus of SNSs. The center of the second distribution gives the average frequency width of the SNS plateaus, which agrees very well with the value obtained from the correlation function. With increasing system length $L$, the plateau-width of the SNSs decays very slowly, compared with the exponentially decaying peak-width of the localized states. Finally, we note that the methods we have used in our paper are not limited to the transport properties of localized systems. They may be used for the study of other statistical quantities that have similar properties to the transmission of localized systems.

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