Scattering processes in antiprotonic hydrogen - hydrogen atom collisions

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The elastic scattering, Stark transitions and Coulomb deexcitation of excited antiprotonic hydrogen atom in collisions with hydrogenic atom have been studied in the framework of the fully quantum-mechanical close-coupling method for the first time. The total cross sections \( \sigma_{l n}(E) \) and averaged on the initial angular momentum cross sections \( \sigma_{l n s}(E) \) have been calculated for the initial states of \((pp)_n\) atom with the principal quantum number \( n = 3 - 14 \) and at the relative energies \( E = 0.95 - 50 \text{ eV} \). The energy shifts of the \( ns \) states due to the strong interaction and relativistic effects are taken into account. Some of our results are compared with the semiclassical calculations.

I. INTRODUCTION

The slowing down and Coulomb capture of the negative particle \( M^- \) in hydrogen media lead to the formation of the \( M^- \)-molecular complex the decay of which results in the exotic atom formation in highly excited states with the principal quantum number \( n \). Where is the reduced mass of the exotic atom. Their initial population and kinetic energy distribution of the exotic atom are determined by the selection of different decay modes of this complex. The further destiny of the exotic atom depends on the kinetics of the processes occurring in the deexcitation cascade. The experimental data are mainly appropriate to the last stage of the atom cascade, such as X-ray yields and the products of the weak or strong interaction of the exotic particle in the low angular momentum states with hydrogen isotopes.

Hadronic hydrogen atoms are of special interest among exotic atoms because they have the simplest structure and are the probe in the investigations of the various aspects of both the exotic atom physics and the elementary hadron-nucleon interactions at zero energy. In particular, in order to analyze the precision spectroscopy experimental data [1] the kinetic energy distribution must be taken into account. The velocity of the exotic hydrogen atom plays an important role due to the effect of the Stark transitions on the X-rays yields and the Doppler broadening of the L-lines owing to the preceding Coulomb deexcitation transitions. The energy release in the last process leads to an acceleration of colliding partners. So the reliable theoretical backgrounds on the processes both in low-lying and in highly excited states are required for the detailed and proper analysis of these data.

In this paper we present the first step toward quantum-mechanical treatment of non-reactive scattering processes of the excited antiprotonic hydrogen atom in collisions with the hydrogenic atom in the ground state:

- elastic scattering

\[
(ax)_{h1} + (be)_{ls} \to (ax)_{h1} + (be)_{ls} \tag{1}
\]

- Stark mixing

\[
(ax)_{h1} + (be)_{ls} \to (ax)_{h1} + (be)_{ls} \tag{2}
\]

- Coulomb deexcitation

\[
(ax)_{h1} + (be)_{ls} \to (ax)_{h1} + (be)_{ls} \tag{3}
\]

Here \((a;b) = (pp);t\) are hydrogen isotopes and \(x = K\); \(pp\); \((n;1)\) and \(ls\) are the principal and orbital quantum numbers of the exotic and hydrogenic atom, respectively. The processes (1) - (2) decelerate and accelerate while the Coulomb deexcitation (3) accelerates the exotic atom and in uncencing their quantum number and energy distributions during the cascade. The last process has attracted particular attention.

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especially after the "hot" p atom s with the kinetic energy up to 200 eV were found experimentally. Due to the similarity of the general features of the exotic atom s the Coulomb deexcitation process must be also taken into account for the other exotic atom s.

Starting from the classical paper by Leon and Bethe [3], Stark transitions have been treated in the semi-classical straight-line-trajectory approximation (see [4] and references therein). The first fully quantum-mechanical treatment of the processes (1) - (2) based on the adiabatic description was given in [5]-[8]. Recently [2,10] the elastic scattering and Stark transitions (for n = 2 - 5) have also been studied in a close-coupling approach treating the interaction of the exotic hydrogen atom with the hydrogenic one in the dipole approximation with electron screening taken into account by the model. As for higher exotic atom states (n > 5), the semi-classical approach [10] is used for the description of these processes.

As concerning the acceleration process (3) in the muonic and hadronic hydrogen atom s, the parametrization based on the calculations in the semi-classical model [11] (see also [13]) is used for the low-lying states (n = 3 - 7) and the results of the semi-classical Monte Carlo model [12,13] are used for higher exotic atom states in the cascade calculations.

The main aim of this paper is to obtain the cross sections of the processes (1)-(3) for the excited antiprotonic atom beginning from the low collision energies in the framework of the fully quantum-mechanical approach. For this purpose the unified treatment of the elastic scattering, Stark transitions and Coulomb deexcitation within the close-coupling method has been used. This approach has been recently applied for the study of the di-essential and total cross sections of the elastic scattering, Stark transitions and Coulomb deexcitation in the collisions of the excited muonic [14] and pionic [15] hydrogen atom s with the hydrogen ones. In the following Section we briefly describe the close-coupling formalism. The results of the close-coupling calculations concerning the total cross sections of the processes (1)-(3) are presented and discussed in Section III. Finally, summary and concluding remarks are given in Section IV.

II. CLOSE-COUPLING APPROACH

A. Total wave function of the binary system in terms of basis states

The total wave function \( \psi(r;R) \) of the four-body system (ap + be) satisfies the time-independent Schrödinger equation with the Hamiltonian which after separating the center of mass motion can be written as

\[
\hat{H} = \frac{1}{2m} \nabla^2_R + h_p(\tau) + h_e(\tau) + V(r;\tau;R) \tag{4}
\]

\( m \) is the reduced mass of the system. Here we use the set of Jacobi coordinates \( (R;\tau) \):

\[
R = R_H \quad R_{pa} = r_p \quad r_e = r_{pa}; \quad \tau = r_e \quad r_{be};
\]

where \( r_p, r_b, r_e, r_{be} \) are the radius-vectors of the nucleon, muon and electron in the lab-system and \( R_H, R_{pa} \) are the center of mass radius-vectors of the hydrogenic and exotic atom \( s \), respectively. The Hamiltonians \( h_p \) and \( h_e \) of the free exotic and hydrogen atom \( s \), respectively, satisfy the Schrödinger equations

\[
h_{n'm'}(\tau) = \lambda_{n'm'}(\tau);
\]

\[
h_e'_{n'e}(\tau) = \lambda_{n'e}(\tau);
\] \hspace{1cm} \tag{5}

where \( n_{m'}(\tau) \) and \( ' n_e(\tau) \) are the hydrogen-like wave functions of the exotic atom and hydrogen atom bound states, \( \lambda_{n'm'} \) and \( \lambda_{n'e} \) are the corresponding eigenvalues. In the present study \( \lambda_{n'm'} \) includes beyond the standard non-relativistic two-body Coulomb problem the energy shifts due to the strong interaction, vacuum polarization and finite size. It is worthwhile noting that in order to treat the hadronic ns states as non-asymptotic states in the scattering problem we take into consideration only the real part of the complex strong interaction energy shift.

The interaction potential

\[
V(r;\tau;R) = V_{ab} + V_{b} + V_{ae} + V_{e}
\] \hspace{1cm} \tag{7}

includes the two-body Coulomb interactions between the particles from two colliding subsystems:

\[
V_{ab} = \frac{1}{l_{ab}} = \mathbf{j} + e\mathbf{r}_j \quad V_{pb} = \frac{1}{l_{pb}} = \mathbf{j} + e\mathbf{r}_j ;
\]

\[
V_{pe} = \frac{1}{l_{pe}} = \mathbf{j} + e\mathbf{r}_j \quad V_{ae} = \frac{1}{l_{ae}} = \mathbf{j} + e\mathbf{r}_j ;
\] \hspace{1cm} \tag{8}

\[
V = \mathbf{j} + e\mathbf{r}_j ;
\] \hspace{1cm} \tag{9}
where the following notations are used:

\[ m_p = (m_p + m_a); \quad m_a = (m_p + m_a); \quad e = m_e = (m_e + m_b); \quad e = m_b = (m_e + m_b); \quad (10) \]

\[ (m_a, m_b, m_p, e) \text{ are the masses of hydrogen isotopes, antiproton and electron, respectively.} \]

\[ \text{Atom units} (a.u.) = e = m_e = m_b = m_e + m_b = 1 \text{ will be used throughout the paper unless otherwise stated.} \]

In this paper, as well as in the previous studies [11, 12–15], we assume that the state of the target electron is xed during the collision. The electron excitations can be taken into account in a straightforward manner. In a space-fixed coordinate frame we built the basis states from the eigenvectors of the operators \( h_0; h_p; J^2; J^z; J \text{ and the total parity} \) with the corresponding eigenvalues \( {}_{ls}^n L; \lambda(l+1); L(L+1); J(J+1); M \) and \( (1)^{\pm L} \), respectively:

\[ \Psi_{s;nlL;JM} = \sum_{\ell} \frac{1}{4} R_{1\ell}(R) R_{n1}(R) Y_{\ell M}^J (\ell R); (12) \]

where

\[ Y_{\ell M}^J (\ell R) = \frac{X}{R} \text{ isom L JM iY}_{\ell M}^J (\ell R); (13) \]

Here the orbital angular mom entum \( L \) of \( {}_{a;nl} \) is coupled with the orbital mom entum \( L \) of the relative motion to give the total angular mom entum \( L \). The explicit form of the radial hydrogen-like wave functions \( R_{n1}(R) \) will be given below.

Then, for the xed values of \( J; M; \) the exact solution of the Schrödinger equation

\[ (\varepsilon - H)_{nL}^J (\ell; R) = 0; \quad (14) \]

is expanded as follows

\[ \sum_{\ell} \frac{1}{4} R_{1\ell}(R) R_{n1}(R) Y_{\ell M}^J (\ell R); (15) \]

where the \( G_{nL}^J (R) \) are the radial functions of the relative mom entum and the sum is restricted by the \( (n; L) \) values to satisfy the total mom entum conservation. This expansion leads to the coupled radial scattering equations

\[ \frac{d^2 G}{dR^2} + \frac{k_n^2}{R^2} G_{nL}^J (R) = 2m \frac{X}{n^{2J}n^{2J_m} (R) G_{nL}^J (R)}; (16) \]

where \( k_n^2 = 2m (\varepsilon_{cm} + n l_m - n l) \) specifies the channel wave num bers; \( \varepsilon_{cm} \) and \( n l_m \) are relative mom entum energy and exotic atom bound energy in the entrance channel, respectively.

The radial functions \( G_{nL}^J (R) \) satisfy the usual plane-wave boundary conditions at \( R = 0 \)

\[ G_{nL}^J (0) = 0; \quad (R^{L+1}) \quad \text{at asymptotic distances} \quad (R = 1) \quad (17) \]

and at asymptotic distances \( (R = 1) \)

\[ G_{nL}^J (R) \quad \text{if} \quad n^{2J}; n^{2J_m} (R) = 2m (n l_m L L -1) \quad S_{nL}^J (n l_m ! n l_m ! L L) e^{i(k_x R L = 2)} \quad (18) \]

where \( k_x, k_y \) are the wave num bers of initial and final channels and \( S_{nL}^J (n l_m ! n l_m ! L L) \) is the scattering matrix in the total angular mom entum representation. Here and below the indices of the entrance channel and target electron state are omitted for brevity.
B. Potentialm atri

Here we present the derivation of the exact matri of the interaction potentials involved in the close-coupling calculations. The interaction potentialm atri \( W_{n^P L^m n^Q L'^m} \) coupling the asymptotic initial (nL;J) and nal (n^P L^m ;J) channels is de
ned by

\[
W_{n^P L^m n^Q L'^m} (R) = \frac{1}{4} \int dr d^2 R^2 R_{ij} (r) R_{n1} (\ ) R_{n1} (\ )
\]

\[
Y^m_{LM} (\ ; \ ) V (r; \ ; R) \ Y^m_{L'M'} (\ ; \ ) (\ ; \ )
\]

(19)

where the radial hydrogen-like wave functions are given explicitly by

\[
R_{n1} (\ ) = N_{n1} \frac{2}{na} \ exp (\ n a) \ S_q (n; l) \frac{2}{na} q
\]

(20)

(a is the Bohr' radius of the exotic atom \( \text{in a.u.} \) with

\[
N_{n1} = \frac{2}{na} \ \frac{\exp (-2a)}{n!} \ \frac{1}{q!} \ \frac{n^2}{2n} \ l^2 + 1
\]

and

\[
S_q (n; l) = (\ )^q \ \frac{1}{q!} \ \frac{n^2}{2n} \ l^2 + 1
\]

(22)

Averaging \( V (r; \ ; R) \) over 1s-state of hydrogen atom leads to

\[
V (R; \ ) = \frac{1}{4} \ \int dr R^2 (r) V (r; \ ; R) = \exp (-2a) \ \frac{1}{e} \ \frac{1}{\exp (-2a)} \ \frac{1}{e} \ \frac{1}{\exp (-2a)} \ U (r; \ ) U (r; \ ) g
\]

(23)

Then we use the transform ation

\[
U (r; \ ) = (1 + \ \frac{1}{R + \ j}) \ \exp (\ \frac{2a}{R + \ j}) \ \lim_{x \to 0} \ \frac{1}{2x} \ \frac{e^{2a} \ b + 1}{R + \ j}
\]

(24)

which allows us to apply the additional theorem for the spherical Bessel functions [13].

\[
\exp (\ \frac{1}{R + \ j}) = \frac{4}{\pi R_1 f_1} X \\ (1)^2 Y_{\ell} (R_1 Y_{\ell} (f_1)
\]

\[
K_{\ell+1} (R_1) L_{\ell+1} (f_1) \ + \ (R_1) K_{\ell+1} (R_1) L_{\ell+1} (f_1)
\]

(25)

\( (f_1) \) and \( K_{\ell} (x) \) are the modified spherical Bessel functions of the irst and third kind. Furtherm one, by substituting the Eqs. (20)–(25) into Eq. (19) we can integrate over the angular variables \( R; \ ). Finally, applying the angular m on etum algebra and integrating over , we obtain:

\[
W_{n^P L^m n^Q L'^m} (R) = \ (1)^2 W_{\ell} (r; \ ; e; n^P L^m ; j) \ W_{\ell} (r; \ ; e; n^Q L'^m ; j)
\]

(26)
\( (m \text{ is the maximum value of the allowed multipoles}). \) Here the next notations are used:

\[
W (\ell; s; n_l m_\ell | \ell) = N_{n_l m_\ell} \frac{n^X \ell^1}{m_\ell = 0} \frac{2n^0}{n + n^0} \frac{n^X \ell^1 m^1}{m_\ell = 0} \frac{S_\ell}{n + n^0} \frac{2n^0}{n + n^0}
\]

\[
H_\ell (x)J^{\ell m}_1 (x; (n;n^0; s; )); + f_\ell (x)J^{\ell m}_3 (x; (n;n^0; s; )); + f_\ell (x)J^{\ell m}_4 (x; (n;n^0; s; )); = H_\ell (x)J^{\ell m}_1 (x; (n;n^0; s; )); + f_\ell (x)J^{\ell m}_3 (x; (n;n^0; s; )); + f_\ell (x)J^{\ell m}_4 (x; (n;n^0; s; ));
\]

where \( x = 2R = s + 1 + \frac{1}{L} + m_1 + m_2, \frac{1}{L} = 2L + 1; \)

\[
N_{n_l m_\ell} = \frac{1}{n + n^0} \frac{2n^0}{n + n^0} \frac{2n^0}{n + n^0} \frac{n^{l+1} \ell}{(n + 1)!(n + 1)!(n^0 + 1)!(n^0 + 1)!
\]

\[
(n;n^0; s; ) = \frac{2nn^0}{n + n^0};
\]

\[
H_\ell (x) = (1, 2t)h_\ell (x) + xh_{\ell + 1} (x);
\]

\[
J^{\ell m}_1 (x; ) = (1, 2t)f_\ell (x) + xf_{\ell + 1} (x);
\]

The functions \( h_\ell (x) \) and \( f_\ell (x) \) are given by

\[
h_\ell (x) = \frac{2}{x} K_{\ell + 1; \ell + 1} (x)
\]

and

\[
f_\ell (x) = \frac{2}{x} I_{\ell + 1; \ell + 1} (x)
\]

The radial integrals \( J^{\ell m}_1 (x; ) \) are defined as follows:

\[
J^{\ell m}_1 (x; ) = \int_0^\infty y^{x-2} e^y f_\ell (y) dy;
\]

\[
J^{\ell m}_2 (x; ) = J^{\ell + 1 m + 1}_1 (x; );
\]

\[
J^{\ell m}_3 (x; ) = \int_x^\infty y^{x-2} e^y h_\ell (y) dy;
\]

\[
J^{\ell m}_4 (x; ) = J^{\ell + 1 m + 1}_3 (x; )
\]

and calculated analytically using the power series for the modified Bessel functions.

C. Cross sections

The transition amplitude from the initial state \( \ell m > \) to the final state \( \ell' m' > \) of the exotic atom can be defined by

\[
f (\ell m ! \ell' m' k_\ell k_{\ell'} i = \frac{1}{k_\ell k_{\ell'}} X^{\ell,X_{\ell'} L \ell m \ell' m} \frac{1}{k_\ell k_{\ell'}} X^{\ell,X_{\ell'} L \ell m \ell' m} \frac{1}{k_\ell k_{\ell'}} X^{\ell,X_{\ell'} L \ell m \ell' m} \frac{1}{k_\ell k_{\ell'}} X^{\ell,X_{\ell'} L \ell m \ell' m}
\]

\[
Y_{\ell L} (\ell L) Y_{\ell' m'} (\ell' L') T (n L ! n' L');
\]

(38)
Here, $k_i$ and $k_f$ are the center of mass relative momenta in the initial and final channels; $\hat{k}_i$ and $\hat{k}_f$ are their unit vectors in the space-fixed system, respectively, and, finally, the transition matrix $T^J(nll! n^0l^0)$ used here is given by

$$T^J(nll! n^0l^0) = \sum_{n^0l^0} \frac{1}{2l+1} \frac{k_i}{k_f} X_{n^0l^0} \langle n^0l^0 | k_i | nll \rangle \langle nll | f \rangle.$$

(39)

In terms of the scattering amplitude (38) defined above, all the types of both the differential and total cross sections of the all processes under consideration for the transition from the initial (nll) state to the final (n'tl') state are defined as:

**Differential cross sections**

$$\frac{d nll! n^0l^0}{d\Omega} = \sum_{n^0l^0} \frac{1}{2l+1} \frac{k_i}{k_f} X_{n^0l^0} \langle n^0l^0 | k_i | nll \rangle \langle nll | f \rangle.$$ 

(40)

**Partial cross sections**

$$J_{nll! n^0l^0}(E) = \sum_{n^0l^0} \frac{1}{2l+1} \frac{k_i}{k_f} X_{n^0l^0} \langle n^0l^0 | k_i | nll \rangle \langle nll | f \rangle.$$ 

(41)

and the total cross sections for the nll! n^0l^0 transition are obtained by summing the corresponding partial cross sections over the total angular momentum $J$:

$$J_{nll! n^0l^0}(E) = \sum_J \sum_{n^0l^0} \frac{1}{2l+1} \frac{k_i}{k_f} X_{n^0l^0} \langle n^0l^0 | k_i | nll \rangle \langle nll | f \rangle.$$ 

(42)

Finally, the averaged over the initial orbital angular momentum entangled cross sections for the nll! n^0l^0 transitions are then defined over the partial and total cross sections with the statistical weight (2l+1)n^0l^0 in the case of the degenerated excited atom states and with the weight (2l+1)n^0l^0 in the case when the energy shift of the ns state is taken into account:

$$\langle nll! n^0l^0 | E \rangle = \sum_{n^0l^0} \frac{1}{2l+1} \frac{k_i}{k_f} X_{n^0l^0} \langle n^0l^0 | k_i | nll \rangle \langle nll | f \rangle.$$ 

(43)

and

$$\langle nll! n^0l^0 | E \rangle = \sum_{n^0l^0} \frac{1}{2l+1} \frac{k_i}{k_f} X_{n^0l^0} \langle n^0l^0 | k_i | nll \rangle \langle nll | f \rangle.$$ 

(44)

respectively.

### III. Results

The close-coupling method described in the previous Section has been used to obtain the total cross sections for the collisions of the pp atom states with hydrogen atom states. The present calculations had at least two goals: first, to apply the fully quantum-mechanical approach for the study of the processes (1) - (3) and, second, to clear the extent of the energy shifts of the ns states of the antiprotonic hydrogen atom on the cross sections of these processes.

The coupled differential equations (16) are solved numerically by the Numerov method with the standing-wave boundary conditions involving the real $K$-matrix. The corresponding $T$-matrix is obtained from the $K$-matrix using the matrix equation $T = 2K$ and $J K$. In the calculations both the exact interaction matrix and all the open channels with n^0n have been taken into account. The closed channels are not considered in the present study. The close-coupling calculations have been carried out for the relative collision energies $E_{cm}$ from 0.05 up to 50 eV and for the excited states with n = 3 - 14. At all energies the convergence of the partial wave expansion was achieved and all the cross sections were calculated with the accuracy better than 0.1%.

The results of the calculations are presented in Figures 1-4. In Fig. 1 we introduced the calculated total cross sections of the nll! n^0l^0 transitions for n = 8 at the kinetic energy $E_{cm} = 14$ eV both with and without the ns state energy shifts. The following measured world-average value [17] for the spin-averaged shift

$$\Delta = (721 \pm 14) eV$$
FIG. 1: The total cross sections $\sigma_{11}$ for the collisions of the pp atom in the $n = 8$ state with hydrogen atom at $E_{cm} = 1.4 \text{ eV}$. The dashed and dotted lines connect the points corresponding to the calculations both with and without taking into account the ns-state energy shifts, respectively. The dotted lines denote the results obtained without including the ns-states into the basis set.

FIG. 2: The $l$-averaged cross sections of the elastic scattering for the collisions of the pp atom with the hydrogenic atom for the different values of the principal quantum number $n$. was used in the present calculations and the energy shifts of the ns states are dened by $1s = n^3$.

Contrary to the (p)–atom, the energy shifts of the ns states in (pp)–atom are repulsive, hence, the n1 ns transitions are closed below the corresponding threshold and, besides, according to the present study (e.g., see Fig. 1) the Stark transitions both from the ns-states and to the ns-state at the same collisional energy are strongly suppressed at the kinetic energies above the threshold. The similar e ect is also observed for the elastic np ! np transitions. The other transitions are practically unchangeable at xed energy. The analog of this e ect can be modeled by excluding ns-states from the basis states. At energy above a few thresholds the e ect is much weaker. Therefore, the strong interaction e ect in
the antiprotonic hydrogen (the similar effect must be also observed in the kaonic hydrogen atom) results in the essential and important difference of the Stark transition role in the absorption from the ns states in contrast to pionic hydrogen atoms. The influence of the strong interaction shift enhances for the lower states and becomes less pronounced for the highly excited states of the antiprotonic atom.

![Figure 3: The 1-averaged cross sections of the Stark transitions for the collisions of the pp atom with the hydrogenic one. The results of the calculations [10] in the semiclassical model are shown for n = 8 with triangles](image)

In Figures 2 and 3 the energy dependence of the 1-averaged total elastic and Stark cross sections are shown for the different principle quantum number values from n = 3 up to n = 12. Since the relative contribution of the ns state in the 1-averaged cross sections are small, the calculations both with and without the energy shift are practically indistinguishable at the energies above the corresponding threshold. So, in Figs. 2 and 3 the small energy region (below the corresponding threshold) corresponds to the calculations without the energy shift taken into account. In Fig. 3 the 1-averaged cross section for n = 8 are compared with the calculations in the framework of the semiclassical model [10]. As a whole a fair agreement is observed, but the semiclassical model results in a different energy dependence especially at low energy collision and gives smaller cross sections than those obtained in the present approach.

The dependence of the Coulomb deexcitation cross sections on energy obtained in the present study is illustrated in Fig. 4 for n = 8 and the different values of n = 1, 2 and 3. The special features of these cross sections are the following: the similar energy dependence but sharper than that of the elastic scattering and Stark transitions (see also in Fig. 4); the contribution of the transitions with n > 1 is comparable with the one for n = 1 and approximately equal about 50%. The effect of the ns state energy shift in the 1-averaged Coulomb deexcitation cross sections is small for the same reason as it was discussed above (due to small statistical weight of the ns-state). In Fig. 4 we also compare our results with those obtained in the semiclassical model (we use the parameterization suggested in [13] which gives a fair description of the Coulomb cross sections from [11]) for the n = 1 transition. The satisfactory agreement is observed, but this agreement is quite occasional and takes no place for other n values. The distribution over the final states n^0 is completely different from the semiclassical results [11] as it was illustrated in Fig. 4. The present calculations predict that n > 1 transitions give a substantial contribution to the Coulomb deexcitation of the highly excited antiprotonic hydrogen atom that is in agreement with our previous results for the muonic and pionic hydrogen [14,15] atoms.

**IV. CONCLUSION**

The unified treatment of the elastic scattering, Stark transitions and Coulomb deexcitation is presented in an ab initio quantum mechanical approach and for the first time the cross sections of these processes have
FIG. 4: The averaged cross sections of Coulomb deexcitation with \( n = 1; 2; 3 \) for the collisions of the pp atom (\( n = 8 \)) with the hydrogenic atom. The dashed line shows the used in cascade calculations \([13]\) for the transitions with \( n = 1 \) and based on the mass-scaling of the results \([11]\) for the muonic atom. The present calculations of the averaged elastic and Stark cross sections are shown for comparison.

been calculated for the highly excited antiprotonic hydrogen atom. The influence of the energy shifts of the ns states on these processes has been studied. We have found that strong interaction shifts in the antiprotonic hydrogen atom lead to substantial suppression of both \( n^0 \) \( l^6 \) 0 and \( n^1 \) \( l^6 \) 0 ns transitions. At the same time the cross sections of the elastic scattering and Stark transitions for the the states with \( l > 2 \) are practically unchanged. The present study is the first step to achieving a reliable theoretical input for realistic kinematic calculations.

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