Prediction of construction bases frozen soil temperature development under intense heating

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Abstract. The purpose of the article is to assess development of the temperature field of the bases for foundations of horizontal burner devices (HBDs), considering seasonal changes in temperature and operation of horizontal flares (HFs). Predictive analysis is based on the solution of a quasilinear system of thermoelasticity equations, which takes into account the mutual influence of mechanical and temperature fields. An iterative algorithm is proposed that allows, at each time step, to obtain separately the solution of the heat conduction and elasticity equations while maintaining their physical coupling. The results of thermal insulation calculation of a flare pit for a gas well cluster are presented.

1. Introduction
The history of the boundaries of permafrost formations (PMF), which occupy a significant territory of Russia, is associated both with climatic factors and human activities in the development of these territories. In particular, the exploitation of natural deposits may be accompanied by thawing of the surrounding permafrost and lead to technological disasters caused by unacceptable settlement of the earth surface [1,2].

An urgent task in the design of construction projects in a PMF zone is to preserve the bases soil in the frozen state. A mathematical model used in the design should take into account the structural heterogeneity of the environment, the dependence of the thermomechanical modules of the environment on possible phase transitions, changes in the seasonal temperatures of the ambient air and solar radiation, possible heat sources. A finite element model and its thermomechanical fundamentals, as well as the main approaches of the authors of the article to solving such problems are described in [3], which is the basis for a set of applied programs that has been developed for solving thermoelasticity problems in application to soil mechanics. In [4], the influence of the temperature and strain fields coupling on the sizes of the thawing region of PMF and settlement of construction bases was considered.

2. Problem statement
A soil body will be simulated with a construction (flare pit) located on it, a layered two-phase environment occupying volume $V$, each layer of which is homogeneous and isotropic. This environment is characterized by the behavior of water-saturated soil, one phase of which is thawed, and the other is frozen. The case of small deformations of a continuous medium will be considered. Let the radius vector of soil points be denoted by $\vec{x}$, and the increment of temperature be denoted by $T$ with respect to its initial distribution $T_0(\vec{x})$. 
The system of thermoelasticity equations, consisting of the Duhamel-Neumann and thermal conductivity equations, can be written as follows [5]:

\[
\begin{align*}
\frac{\partial}{\partial x_i} \left( \frac{c_{ijkl}}{\lambda_{ij}} \frac{\partial u_j}{\partial x_i} \right) &= \rho F_i + \alpha_i K \frac{\partial T}{\partial x_i}, \\
\frac{\partial}{\partial x_i} \left( \lambda_{ij} \frac{\partial T}{\partial x_i} \right) + q_v &= \alpha_i KT_0 \frac{\partial}{\partial t} \left( \frac{\partial u_i}{\partial x_i} \right) + \rho c \frac{\partial T}{\partial t}, \quad i, j = 1, 2, 3,
\end{align*}
\]

(1)

where \( c_{ijkl} \) denotes the tensor of soil elasticity moduli, \( \alpha_i(\bar{x},T) \) is the coefficient of cubical thermal expansion of soil, \( \lambda_{ij} = \lambda(\bar{x},T) \delta_{ij} \) is the soil thermal conductivity tensor, \( q_v(\bar{x}) \) is the power of internal heat sources per soil volume unit, \( \rho(\bar{x},T) \) is the soil consistency, \( c(\bar{x},T) \) is the soil specific heat. Convective transport in a continuous medium will be neglected because of the small velocities of the medium points. Therefore, in the equations (1), the total time derivative \( \frac{\partial}{\partial t} \) is replaced by the partial.

We assume that phase transitions in the soil occur at a known constant temperature \( T = T^* \). In environments with phase transitions, it is necessary to set the condition on the temperature and its gradient at the phase boundary \( S(\bar{x},t) \). This boundary divides the region \( V \) into two subregions: \( V^o = \{ (x,y,z) \in V, T(x,y,z) > T^* \} \), occupied with thawed soil, and \( V^f = \{ (x,y,z) \in V, T(x,y,z) < T^* \} \), occupied with frozen soil. A similar notation will be used for thermomechanical parameters in each phase. The equation system coefficients are discontinuous functions of coordinates:

\[
K, G, \alpha_v, \lambda_{ij}, q_v, \rho, c = \begin{cases} K^0, G^0, \alpha^0_v, \lambda^0_{ij}, q^0_v, \rho^0, c^0, \bar{x} \in V^0, \\
K^*, G^*, \alpha^*_v, \lambda^*_ij, q^*_v, \rho^*, c^*, \bar{x} \in V^*.
\end{cases}
\]

(2)

The border \( S(\bar{x},t) \) is movable and its position at any time \( t \) is not known in advance. Let's write the conditions at the boundary \( S \) of the phase transition. It assumes the continuity of the temperature field [5]:

\[
[T] = T^* - T^* = 0, \quad \bar{x} \in S.
\]

(3)

The square brackets denote the jump in the function during the crossing of the phase boundary. This condition is homogeneous. In addition to it, it is necessary to set the condition for the heat flux at the boundary. Therefore, it is natural to assume discontinuity of the heat flux at the phase boundary:

\[
\lambda_{ij} \frac{\partial T}{\partial x_j} n_i = -L \dot{S}_s, \quad \bar{x} \in S,
\]

(4)

where \( n_i \) is the component of the unit normal to the boundary \( S \), \( L \) is the phase transition enthalpy, \( \dot{S}_s \) is the normal component of the velocity of the phase transition boundary. The problem of solving the heat equations of (1) in the region \( V \) with discontinuous coefficients from (2), the conjugation boundary conditions (3) and (4) at the phase boundary \( S \) and the initial and boundary conditions at the outer boundary \( \Sigma \) of region \( V \) is a classical Stefan problem. Following [5], we include the inhomogeneous condition at the phase boundary (4) in the heat equation. We take into account the concentrated heat capacity \( c_s \) as an additional term in the heat equation from system (1):

\[
\frac{\partial}{\partial x_i} \left( \lambda_{ij} \frac{\partial T}{\partial x_j} \right) + q_v - \delta_s L \dot{S}_s = \alpha_i KT_0 \frac{\partial}{\partial t} \left( \frac{\partial u_i}{\partial x_i} \right) + \rho c \frac{\partial T}{\partial t}, \quad i, j = 1, 2, 3.
\]

(5)

where \( \delta_s \) denotes a surface Dirac delta function. It is determined in a way that for any function \( A(\bar{x}) \) the following is true:

\[
\int_V \delta_s A(\bar{x})dV = \int_S A(\bar{x})d\Sigma.
\]
At the points of the phase transition boundary, we introduce a natural orthonormal coordinate system \((x', y', z')\) so that the boundary equation \(S\) takes the form: \(x' = x_0\). For boundary points motion velocity we get: \(\dot{S}_i = dx'/dt\). The boundary condition for temperature in new coordinates takes the form: \(T(x_0', y', z') = T^*\). Whence the expression follows:

\[
\delta_i \dot{S}_i = \delta(x' - x_0) \frac{dx'}{dt} = \delta(T - T^*) \frac{dT}{dt}.
\]  

(6)

Neglecting the convective component in the total time derivative in the heat equation from (1), we get it in the form:

\[
\frac{\partial}{\partial x_i} \left( \lambda_{ij} \frac{\partial T}{\partial x_j} \right) + q_v = \left( \rho c + L \delta(T - T^*) \right) \frac{\partial T}{\partial t}.
\]  

(7)

In equation (7), the phase transition boundary is not present clearly. In this case, taking account of the phase transition heat can be defined by the effective heat capacity \(c_{eff}\):

\[
c_{eff} = c + \rho^{-1}L \delta(T - T^*).
\]  

(8)

The heat equation in the form of (8), which allows one to take into account conditions (3) and (4) at the phase boundary, leads to the so-called generalized formulation of the Stefan problem. Thus, further on the system of thermoelasticity equations will be used in the form:

\[
\begin{align*}
\frac{\partial}{\partial x_i} \left( C_{ijkl} \frac{\partial u_j}{\partial x_i} \right) &= \rho F_i + \alpha_v K \frac{\partial T}{\partial x_i} \\
\frac{\partial}{\partial x_i} \left( \lambda_{ij} \frac{\partial T}{\partial x_j} \right) + q_v &= \alpha_v KT_0 \frac{\partial}{\partial t} \left( \frac{\partial u_j}{\partial x_i} \right) + \rho c_{eff} \frac{\partial T}{\partial t}, \quad i, j = 1, 2, 3.
\end{align*}
\]  

(9)

The \(eff\) index in the designation of the effective heat capacity will be omitted. The numerical implementation of the Stefan problem in a generalized formulation is called the shock-capturing method, which was first proposed in the works of A. Samarsky [5] and B. Budak [6].

The parts of the outer boundary \(E\) of the region \(V\), on which the boundary conditions of the first kind (Dirichlet) and the third kind (Newton-Richmann) for temperature will be set, are denoted by \(\Sigma^F\) and \(\Sigma^3\), respectively.

For element \(\Sigma^s\) heat exchange with the environment is defined as follows:

\[
\alpha q^o = \beta(T - T_{av}) - (\lambda \cdot \text{grad}T) \cdot \hat{n}.
\]  

(10)

where \(\alpha q^o\) is the solar radiation vector flux across the boundary, \(\beta(T - T_{av})\) is the boundary heat exchange with air.

On the element of the boundary \(\Sigma^T\) we assume the temperature \(T^0(t)\) to be defined:

\[
T(t) = T^0.
\]  

(11)

The standard boundary conditions for displacements and the initial conditions complete the problem statement:

\[
\bar{u}(\bar{x}, t) = 0, \quad \bar{x} \in \Sigma^H, \quad \left( C_{ijkl} \frac{\partial u_j}{\partial x_i} - \alpha_v KT \delta_{ij} \right) \bar{n} \cdot \bar{e}_i = S^o, \quad \bar{x} \in \Sigma^\sigma.
\]  

(12)

Thus, the model of quasilinear thermoelasticity with phase transitions is described by equations (9) with boundary and initial conditions (10) - (12). Accounting for solar energy in the boundary condition (10) is a difficult task, since the value of the parameter \(\alpha\) depends on hard-to-determine factors, such as vegetation and snow cover, the fraction of
long-wave radiation reflected by the atmosphere towards the Earth surface, which are generally
unknown in a particular geographical area [7,8].

The parameters included in the quasilinear boundary condition (10) will be determined using an
iterative regularizing process, assuming that for a given geographical point we know the values that
are schematically presented in Tables 1 and 2. The distribution of the initial temperature in the soil
\( T_0(x_0, x_0, x_i, J^n) \) is taken based on the data of the exploratory thermal well at a certain moment in time
\( t^n \) at the points of the well \( (x_i, x_0, x_i) \) from the calculation region with a vertical coordinate variable
along the well. Solving iteratively the problem of minimizing the regularizing functional with a certain
initial value of the parameter \( \alpha \) and matching the annual periodic temperature \( T_0 \) with the given
accuracy, we select the correct coefficient \( \alpha \) from this numerical experiment [9,10].

The system of variational equations of the initial-boundary value problem (9) - (12) for trial
functions \( \bar{h}, q \) takes the following form:

\[
\begin{align*}
\int V \left( \text{Grad} \cdot \bar{h} \right) : \mathcal{C} : \text{Grad} \bar{h} dV + \int \bar{h} \cdot \alpha \cdot K \text{grad} T dV = \\
\int \bar{h} \cdot \rho \bar{F} dV + \int \bar{h} \cdot (\bar{S} + \alpha \cdot KTn) d\Sigma,
\end{align*}
\]

\[
\begin{align*}
\int \text{grad} q \cdot \bar{q} \cdot \text{grad} \bar{T} dV + \int q (\alpha \cdot KTq \text{div} \bar{u} + \rho cT) dV - \int \bar{q} \cdot q dV = \\
- \int q (\alpha q - \beta (T - T_n)) d\Sigma.
\end{align*}
\]

To obtain its numerical solution for \( \bar{h} = \overline{u}, q = T \), spatial discretization was performed using the
finite element method, and the implicit finite difference Euler scheme was applied for time
\( t^{n+1} = t^n + \Delta t \) [11,12]. For visual reference the system linearized in such a way (13) is written in the
block-matrix form:

\[
\begin{pmatrix}
\frac{\partial}{\partial x_1} \left( C_{ij} \frac{\partial}{\partial x_j} \right) - \frac{\partial}{\partial x_1} \left( \alpha_i \cdot K \cdot \delta_j \left( \cdot \right) \right) \\
\frac{-T_0}{\Delta t} \cdot \frac{\partial}{\partial x_1} \left( \alpha_i \cdot K \cdot \delta_j \left( \cdot \right) \right) \\
\frac{\bar{h}_i}{\Delta t} \\
\frac{\bar{u}_i}{\Delta t} \\
\frac{-\rho F^n}{\Delta t} \\
\frac{-T_0}{\Delta t} \cdot \frac{\partial}{\partial x_j} \left( \alpha_i \cdot K \cdot \delta_j \left( \cdot \right) \right) - \frac{\rho c^n}{\Delta t} T^n - \bar{q}_i^n
\end{pmatrix}
\begin{pmatrix}
\bar{u}_i^{n+1} \\
T^{n+1}
\end{pmatrix}
= \begin{pmatrix}
\bar{f}_1 \\
\bar{f}_2
\end{pmatrix}
\]

The shorter form of the system (14) is as follows:

\[
[H] \begin{bmatrix} \vec{X}^n \end{bmatrix} = \begin{bmatrix} \vec{F} \end{bmatrix}, \quad n = 1, 2, ..., \text{ where:}
\]

\[
[H] = \begin{bmatrix}
H_{11} & H_{12} \\
\frac{1}{\Delta t} H_{21} & H_{22}
\end{bmatrix}; \quad \begin{bmatrix} \vec{X}^n \end{bmatrix} = \begin{bmatrix} \bar{h}_i^n \end{bmatrix} \text{; } \begin{bmatrix} \vec{F} \end{bmatrix} = \begin{bmatrix} \bar{F} \end{bmatrix} \text{; } \begin{bmatrix} \vec{f}^1 \end{bmatrix} \text{; } \begin{bmatrix} \vec{f}^2 \end{bmatrix}
\]

Occurrence of the factor \( \sqrt{\Delta t} \) for the element \( H_{21} \) of the matrix block in (15) leads to the fact that
its solution is equivalent to finding the saddle point of some functional, and therefore the system itself
is called a saddle system. In case of a numerical solution of the saddle system, a problem arises related
to the stability of its inversion procedure. In addition to satisfying the strong ellipticity condition of the
submatrix \( H_{11} \), the fulfillment of the Ladyzhenskaya-Babuska-Brezzi condition (LBB) must be
required which imposes a restriction on the time step value \( \Delta t \), which must be greater than a certain
minimum allowed value. The LBB condition is burdensome; therefore, the system will be inverted using the two-layer stationary iterative method with a special preconditioner $[H_0]$:

$$
[H_0]\left\{\frac{X^{n,m} - X^{n,m-1}}{\tau}\right\} + [H]\left\{X^{n,m}\right\} - \{F\} = 0, \quad m = 1, 2, \ldots,
$$

$$
[H_0] = \begin{bmatrix} \tau H_{11} & 0 \\ 0 & H_{22} - \frac{1}{\Delta t} E \end{bmatrix}
$$

(16)

Here, $\tau$ denotes the iteration parameter, $m$ denotes the iteration index, and $E$ denotes the unity matrix. Estimates for the convergence rate $v$ of this iterative method have been obtained:

$$
v = \frac{\gamma_1}{2}, \quad \gamma_1 = \min \left\{ 1, \frac{a}{c_2}, 1 + \rho c \right\}, \quad \gamma_2 = \max \left\{ 1, \frac{1}{c_1}, 1 + \rho c \right\}, \quad a \in (0, 1/2].
$$

(17)

In these expressions, as before, $\rho$ denotes the consistency, and $c$ denotes the specific heat of the soil. The constants $c_1$ and $c_2$ are determined based on the condition of positivity of the soil elasticity moduli:

$$
0 < \frac{c_1}{A} I \leq C \leq c_2 I, \quad I_{ijkl} = \frac{1}{2}(\delta_{ik} \delta_{jl} + \delta_{ij} \delta_{kl}),
$$

(18)

where $A$ is the Korn's constant. The condition for the iteration parameter $\tau$, at which the method converges, is as follows:

$$
\tau < \frac{2}{\gamma_2}.
$$

(19)

Thus, the convergence rate $v$ of the proposed iterative method (16) with the original preconditioner $[H_0]$ does not depend on the mesh steps, and therefore, not on the number of equations of the linear system to be solved, it does not depend on the spread of the thermal conductivity tensor in the soil layers, but it does depend on the layer spread of soil consistency and specific heat values. In addition, the proposed preconditioner allows to separate the Duhamel-Neumann and the thermal conductivity equations at each step of the linear system matrix inversion, since there is no need now to invert the entire block matrix $[H]$. To calculate the temperature in (16) at the $m$-th iteration, it suffices to invert the block $H_{12}$, which is, in fact, the Laplace operator. And for calculating the displacements at the same $m$-th iteration, it is enough to invert the block $H_{11}$ - an operator of elasticity theory. Therefore, the convergence rate of the iterative method does not depend on the parameter $1/\Delta t$, and there is no need to satisfy the LBB condition. Each of these blocks is inverted through the direct method using a solver from MKL program library.

The algorithm described above was implemented in the application software package [13]. An advantage of a finite element implementation over finite difference methods is its conservativeness and projectivity. These properties are important for the stability of the algorithm when solving large linear equation systems in regions of complex shape.

3. Numerical experiment

To simulate the heat trail of HF within the boundaries of a flare pit, the heat distribution on its surface was specified. Figure 1 presents the reference temperature isofields for HBD500-18 and the temperature field specified in the software package:
Points A, B and C indicate the places in which temperature values will be displayed vertically at different moments of time. Thus, it is necessary to additionally set the following boundary condition on the daylight surface $z = 0$:

$$T(x, y, z, t)|_{z=0} = T^0(x, y, 0, t)$$

(20)

where $T^0(x, y, 0, t)$ is the temperature of the flare pit surface at the moments of HF operation in accordance with fig. 1. At the other moments, the boundary condition (10) is fulfilled.

To describe the climate, the location of the survey objects, the data from tables 1 and 2 were used.

Table 1. Monthly and annual average air temperature, °C

|   | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | Year |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|------|
|   | 21.7| -22.4| -17.8| -13.5| -5.5| 2.0 | 7.3 | 7.0 | 3.7 | -4.5| -13.0| -18.0| -8.0 |
Table 2. The radiation balance of the underlying surface (kcal/cm²)

| Year | 01 | 02 | 03 | 04 | 05 | 06 | 07 | 08 | 09 | 10 | 11 | 12 |
|------|----|----|----|----|----|----|----|----|----|----|----|----|
|      | -1.3 | -1.0 | -0.9 | -0.2 | 1.1 | 5.5 | 8.9 | 4.3 | 1.2 | -0.7 | -1.2 | -1.2 | 14.5 |

Table 3 shows a proposal of vertical lithology of soil under a flare pit with dumping, consisting of plates made of heat-resistant concrete, dry sand, vermiculite, geomembrane, soil with layerwise ramming (sand):

Table 3. A proposal of vertical lithology of soil under a flare pit

| Layer name                                                                 | Layer thickness (mm) |
|---------------------------------------------------------------------------|----------------------|
| Plates of heat-resistant concrete BRPB35 И11 F400                          | 150                  |
| Tightened leveling layer of sand                                          | 300                  |
| Exfoliated vermiculite M150, state standard 12865-67, multifractional, packed in heat-resistant bags | 500                  |
| Impervious screen-layer of geomembrane «ГеоПласс» H-2L300(2)-5x40         | 2                    |
| Soil with layerwise ramming (sand)                                        | 2500                 |
| 120220 Hemic peat                                                         | 300                  |
| 140400а Very soft loam                                                    | 500                  |
| 141002а Frozen non-icy loam                                               | 1000                 |
| 141300а High-plastic loam                                                 | 700                  |
| 141200а Frozen loam                                                       | 7000                 |
| 141100а Frozen loam                                                       | 2000                 |
| 141000а Frozen loam                                                       | 3300                 |

The flare pit for the horizontal burner device HBD500-18 has the following dimensions:
- Pit length – 30m;
- Pit width – 5.5m;
- Pit dike height – 2m.

When using HBD500-18, the following is assumed:
- The estimated life of HBD is at least 30 years;
- HBD operating mode is periodic.

Maximum possible runtime:
- in the first year of field operation during commissioning, each well in the cluster takes 1 day for metering and research and 3 days for blowing-out;
- the maximum number of wells in the cluster is 4, thus in the first year, HBDs should work accordingly for (3+1)*4=16 days.

In subsequent operations, wells are blown once a year for 1 day, thus, the maximum possible operating time of HBDs after the first year of field operation is 4 days.

Numerical modeling includes the following steps:
- Simulating the annual (background) temperature cycle.
- Simulating the thermal effect of the flare system.

When simulating the operation of HBD, at least 10 calculations were performed and two HBD operation scenarios were considered: the start of operation in April (first scenario) and the start of operation in September (second scenario).

The computational region consists of two parallelepipeds measuring 80m*36m*14.5m and 30m*6m*3.45m. Time step: 1 day, simulation period: 5 years.
To simulate the thermal effect of the flare system, April was chosen as the coldest month, and September as the warmest.

4. Conclusion

Based on the phenomenological model of thermoelasticity with phase transitions, a numerically effective algorithm for solving the resulting system of iteratively coupled Duhamel-Neumann and thermal conductivity equations is proposed. The finite element program (FEP) created on the basis of the algorithm was used to calculate the heat-insulating dumping of the HBD flare pit. The dumping serves as artificial bases for buildings and constructions, prevents the technogenic impact on structurally unstable permafrost soils. It is also used in the relief provision and surface drainage of the site. Under natural conditions, the territory is subject to various cryogenic processes. The dumping reduces the thermal effect of constructions on natural soils, stabilizes the exfoliating process related to seasonal freezing-thawing of natural soils.

The numerical calculations showed that the first year of HBD operation, when the longest thermal effect on the dumping and soil (16 days) occurs, is the most significant in terms of impact, because after the first year, when HBD works only 4 days a year, the effect on dumping and soil is not so significant and winter temperatures can reduce the heat effects.

At the same time, other problems arise due to the fact that the heat flux of HBD, having penetrated through the thermal insulation, is poorly dispersed in the soil after the HBD shutdown, and freezing of the soil in winter is difficult due to such thermal insulation.

According to the computation results, the thickness of the insulating layer, consisting of plates made of heat-resistant concrete, dry sand, vermiculite, geomembrane and soil with layerwise ramming (sand), is sufficient to prevent heating of PMF.

In the first year of HBD operation, it is advisable not to carry out operation of HBD for 16 days during one month, but to draw up a special schedule for HBD operation during the first year.

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Figure 2. Finite element mesh
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