Unification of Gauge and Yukawa Couplings without Symmetry *

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Abstract

A natural gradual extension of the idea of Grand Unification is to attempt to relate the gauge and Yukawa couplings; Gauge-Yukawa Unification (GYU). However, within the framework of renormalizable field theories, there exists no realistic symmetry that leads to a GYU. Here we propose an approach to GYU which is based on the principle of the reduction of couplings and finiteness in supersymmetric Grand Unified Theories. We elucidate how the observed top-bottom mass hierarchy can be explained in terms of supersymmetric GYU by considering an example of the $SU(5)$ Finite Unified Theory. It is expected that, when more accurate measurements of the top and bottom quark masses are available, it will be possible to discriminate among the various GYU models.

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1 Introduction

The traditional way to reduce the independent parameters of a theory is the introduction of a symmetry. Grand Unified Theories (GUTs) \cite{1, 2, 3} are representative examples of such attempts. For instance, the minimal $SU(5)$ reduces the gauge couplings of the Standard Model (SM) by one and gives us a testable prediction for one of them. In fact, LEP data \cite{4, 5} seem to suggest that the $N = 1$ global supersymmetry \cite{6, 8} should be required in addition to make the prediction viable. GUTs also relate Yukawa couplings among themselves, which can lead to predictions for the parameters of the SM. The prediction of the ratio $m_\tau/m_b$ \cite{7} in the minimal $SU(5)$ was an example of a successful reduction of the independent parameters of this sector. On the other hand, requiring more symmetry (e.g., $SO(10)$, $E_6$, $E_7$, $E_8$) does not necessarily lead to more predictions for the SM parameters, due to the presence of new degrees of freedom, various ways and channels of breaking the symmetry, etc. An extreme case from this point of view are superstrings, which have huge symmetries, but no real predictions for the SM parameters.

In a series of papers \cite{9}–\cite{14}, we have proposed that a natural gradual extension of the GUT ideas, which preserves their successes and enhances the predictions, is to attempt to relate the gauge and Yukawa couplings, or in other words, to achieve Gauge-Yukawa Unification (GYU). Searching for a symmetry that could provide such a unification, one is led to introduce a symmetry that relates fields with different spins, i.e., supersymmetry and in particular $N = 2$ supersymmetry \cite{15}. Unfortunately, $N = 2$ supersymmetric theories have serious phenomenological problems due to light mirror fermions. Needless to say that there exists GYU in superstrings, too \cite{16, 17}.

In the following we would like to emphasize an alternative way to achieve unification of couplings, which is based on the fact that within the framework of a renormalizable field theory, one can find renormalization group (RG) invariant relations among parameters, that can improve the calculability and the predictive power of a theory. In our recent studies \cite{9}–\cite{14}, we have considered the GYU which is based on the principles of reduction of couplings \cite{18}–\cite{33} and finiteness \cite{34}–\cite{43}. These principles, which are formulated in
perturbation theory, are not explicit symmetry principles, although they might imply symmetries. The former principle is based on the existence of RG invariant relations among couplings, which preserve perturbative renormalizability. Similarly, the latter one is based on the fact that it is possible to find RG invariant relations among couplings that keep finiteness in perturbation theory, even to all orders. Applying these principles one can relate the gauge and Yukawa couplings without introducing necessarily a symmetry, nevertheless improving the predictive power of a model. Concerning recent related studies, we would like to emphasize that our approach to GYU for asymptotically non-free theories [11, 12] covers work done by other authors [14], though the underlying idea might be different.

In the next section we begin by illustrating the idea of reduction of couplings, and in section 3 we consider a Finite Unified Theory (FUT) based on $SU(5)$–one of the successful Gauge-Yukawa Unified theories–which, moreover, is attracting a renewed interest because of duality in supersymmetric field theories [15, 16].

## 2 Reduction of couplings

To illustrate the idea of the reduction of couplings, we consider a theory containing two scalar fields $\phi_I$, $I = 1, 2$. The renormalizable Lagrangian, which has two parities $\phi_I \rightarrow -\phi_I$, is given by

$$\mathcal{L} = \frac{1}{2} \sum_{I=1,2} \left( \partial_\mu \phi_I \partial^\mu \phi_I - m_I^2 \phi_I^2 \right) - \frac{g_1}{4!} \phi_1^4 - \frac{g_0}{4} \phi_1^2 \phi_2^2 - \frac{g_2}{4!} \phi_2^4. \tag{1}$$

The theory defined by this Lagrangian has originally three dimensionless couplings $g_i$, $i = 0, 1, 2$ and two dimensionful parameters $m_1$ and $m_2$, and we would like to consider the reduction in these numbers.

To this end, we first compute one-loop diagrams in $4 - 2\epsilon$ dimensions and employ the minimal subtraction (MS) scheme for renormalization. One finds in this order

$$g_0^{(0)} = \mu^{2\epsilon} \left[ g_0 + \frac{1}{\epsilon} \frac{1}{16\pi^2} \left( g_0^2 + \frac{1}{4} g_1 g_0 + \frac{1}{4} g_2 g_0 \right) \right], \tag{2}$$
\[ g_i^{(0)} = \mu^{2i} \left[ g_i + \frac{1}{\epsilon} \frac{1}{16\pi^2} \left( \frac{3}{4} \right) (g_0^2 + g_i^2) \right] (i = 1, 2), \quad (3) \]

\[ (m_1^{(0)})^2 = m_2^2 + \left[ \frac{1}{\epsilon} \frac{1}{16\pi^2} \left( \frac{1}{2} \right) (g_1 m_1^2 + g_0 m_2^2) \right] , \quad (4) \]

\[ (m_2^{(0)})^2 = m_2 + \left[ \frac{1}{\epsilon} \frac{1}{16\pi^2} \left( \frac{1}{2} \right) (g_2 m_2^2 + g_0 m_1^2) \right] , \quad (5) \]

where \( \mu \) is the 't Hooft renormalization scale, and \( g^{(0)} \)'s and \( m^{(0)} \)'s stand for the bare couplings and masses. To maintain renormalizability of the theory, it is usually assumed that these five parameters are independent. There may be, however, exceptional situations. Obviously, in the presence of the \( O(2) \) symmetry, we have \( m_1 = m_2 \) and \( g_1 = g_2 = 3g_0 \) so that only one dimensionless and one massive parameter are independent. This is true to all orders in perturbation theory, because the \( O(2) \) symmetry is anomaly-free in the present case.

Are there other possibilities? To answer this question at one-loop order, we assume that

\[ g_i = \rho_i g_0 , \quad (i = 1, 2) , \quad m_1 = \epsilon m_2 , \quad (6) \]

and insert them into the renormalization eqs. (2)–(5). One finds that (under the assumption that \( m_1^2 , m_2^2 > 0 \)) there are two solutions that are consistent with the one-loop renormalizability:

\[ \rho_1 = \rho_2 = 3 \text{ or } \rho_1 = \rho_2 = 1 \quad (7) \]

with \( \epsilon = 1 \). The first one is the symmetric one, but the second one is associated with no obvious symmetry. So, the second one might be an artifact of one-loop order and could disappear if one goes to higher orders. It is remarkable that one can check at one-loop order already, whether the second possibility of reducing the number of parameters persists in higher orders. We will see it in a moment.

The reduction of couplings was originally formulated for a massless theory on the basis of the Callan-Symanzik equation. The extension to theories with massive parameters is not straightforward if one wants to keep the generality and the rigor on the same level as for the massless case; one has to fulfill a set of requirements coming from the
renormalization group equations, the Callan-Symanzik equations, etc., along with the normalization conditions imposed on irreducible Green’s functions. There have been some progresses in this direction [47]. Here we would like to present the idea of the reduction of dimensionless couplings. As we have done in the example above, we assume, to make the method transparent, that the MS scheme has been employed so that all the RG functions such as β functions depend only on dimensionless couplings. Then we would like to investigate whether a solution like eq. (7), which is not a consequence of a symmetry, persists to higher orders in perturbation theory.

To be general, we consider a massless renormalizable theory which contain a set of \((N+1)\) dimensionless couplings. The renormalized irreducible Green’s function in the MS scheme satisfies the RG equation

\[
0 = \left[ \frac{\mu}{\partial \mu} + \beta_i \frac{\partial}{\partial g_i} + \Phi_I \gamma_{IJ} \frac{\delta}{\delta \Phi_J} \right] \Gamma(\Phi, g_0, g_1, \ldots, g_N, \mu),
\]

where \(\Phi\) stands for a set of fields, \(\beta\)’s for the \(\beta\) functions and \(\gamma\) for the \(\gamma\) functions. We then ask ourselves whether the reduction of parameters, i.e.,

\[
g_i = g_i(g), \quad (i = 1, \ldots, N), \quad g \equiv g_0
\]

is consistent with the RG equation

\[
0 = \left[ \frac{\mu}{\partial \mu} + \beta_g \frac{\partial}{\partial g} + \Phi_I \gamma_{IJ} \frac{\delta}{\delta \Phi_J} \right] \Gamma(\Phi, g, g_1(g), \ldots, \mu),
\]

where \(g\) is called the primary coupling. We find that the following set of equations has to be satisfied:

\[
\beta_g = \beta_0, \quad \beta_g \frac{dg_i}{dg} = \beta_i, \quad (i \neq 0),
\]

which are called the reduction equations [22].

The bare quantities are given by

\[
\Phi_I^{(0)} = \mu^{k_{i\ell}} Z_I^{\phi \phi^* J}(g) \Phi_J, \quad g_i^{(0)} = \mu^{k_{i\ell}} Z_i^{\phi \phi J}(g) g_j(g).
\]

The renormalization constants above are those which are first computed in the original theory and then rewritten by means of eq. (9), and the \(k\)’s are introduced to match the
dimension in \((4 - 2\epsilon)\) dimensions. Therefore, the requirements for the reduced theory to be perturbative renormalizable means that the functions \(g_i(g)\) should have a power series expansion in the primary coupling \(g\). That is,

\[
g_i(g) = g \sum_{n=0}^{\infty} \rho_i^{(n)} g^n \quad (i \neq 0).
\] (13)

Recalling ourselves that \(\beta\)'s and \(\gamma\)'s are also a power series and assuming that the expansion coefficients with \(n \leq n_0\) are determined, we insert the power series ansatz (13) into the reduction equations (11). One finds that to obtain the \((n_0 + 1)\)th order coefficients, we have to solve a linear system of equations with \(N\) unknown quantities, where its coefficients are given by the lowest order quantities in the reduction procedure. This is the reason why one can investigate at the lowest order, whether the linear system in \((n_0 + 1)\)th order can be uniquely solved.

For our example of a \(\phi^4\) theory, one finds

\[
\beta_0 = \mu \frac{dg_0}{d\mu} = \frac{1}{16\pi^2} (4g_0^2 + g_1g_0 + g_2g_0) + \ldots ,
\] (14)

\[
\beta_i = \mu \frac{dg_i}{d\mu} = \frac{3}{16\pi^2} (g_0^2 + g_i^2) + \ldots , \quad (i = 1, 2),
\] (15)

where \(\ldots\) indicates higher order terms. The power series ansatz for the present case takes the form

\[
g_i(g) = g \left( \rho_i^{(0)} + \sum_{n=1}^{\infty} \rho_i^{(n)} g^n \right), \quad (i = 1, 2),
\] (16)

where

\[
\rho_1^{(0)} = \rho_2^{(0)} = 3 \text{ or } 1.
\] (17)

As described above, we insert them into the corresponding reduction equations and assume that \(\rho_i^{(n)}\) with \(n \leq n_0\) are determined already. Collecting terms of \(O(g^{n_0+3})\), we find that

\[
\begin{pmatrix}
(n_0 + 2)(4 + \rho_1^{(0)} + \rho_2^{(0)}) - 5\rho_1^{(0)} & \rho_1^{(0)} \\
\rho_2^{(0)} & (n_0 + 2)(4 + \rho_1^{(0)} + \rho_2^{(0)}) - 5\rho_2^{(0)}
\end{pmatrix}
\begin{pmatrix}
\rho_1^{(n_0+1)} \\
\rho_2^{(n_0+1)}
\end{pmatrix} = \text{known quantities}.
\] (18)
Since the matrix on the l.h. side of eq. (18) is regular, we conclude that $\rho_i^{(n+1)}$ can be uniquely determined. That is, the power series (13) exists uniquely.

Moreover, it is possible [22] to find a reparametrization of couplings in such a way that $\rho_i^{(n)}$ for all $n > 0$ exactly vanish. In fact, this theory corresponds to [21]

$$L = \sum_{I=+, -} \left( \frac{1}{2} \partial_\mu \phi_I \partial^\mu \phi_I - \frac{g_0}{6} \phi_I^4 \right), \quad \phi_{+(-)} = \frac{1}{\sqrt{2}}(\phi_1 + (-)\phi_2).$$

3 Finite Unified Model Based on SU(5)

As a realistic example for the reduction of couplings, we consider a Finite Unified Model Based on SU(5). From the classification of theories with vanishing one-loop $\beta$ function for the gauge coupling [33], one can see that using SU(5) as gauge group there exist only two candidate models which can accommodate three fermion generations. These models contain the chiral supermultiplets $\mathbf{5}$, $\overline{\mathbf{5}}$, $\mathbf{10}$, $\overline{\mathbf{10}}$, $\mathbf{24}$ with the multiplicities $(6, 9, 4, 1, 0)$ and $(4, 7, 3, 0, 1)$, respectively. Only the second one contains a $24$-plet which can be used for spontaneous symmetry breaking (SSB) of SU(5) down to $SU(3) \times SU(2) \times U(1)$. (For the first model one has to incorporate another way, such as the Wilson flux breaking to achieve the desired SSB of SU(5).) Therefore, we would like to concentrate only on the second model.

To simplify the situation, we neglect the intergenerational mixing among the lepton and quark supermultiplets and consider the following SU(5) invariant cubic superpotential for the (second) model:

$$W = \sum_{i=1}^{3} \left[ \frac{1}{2} g_i^u 10, 10, H_i + \sqrt{2} g_i^d 10, \overline{5}, \overline{T}_i \right] + \sum_{\alpha=1}^{4} g_\alpha^f H_\alpha 24 \overline{T}_\alpha + \frac{g_\lambda}{3} (24)^3,$$

where the $10_i$'s and $\overline{5}_i$'s are the usual three generations, and the four $(\mathbf{5} + \overline{\mathbf{5}})$ Higgses are denoted by $H_\alpha$, $\overline{T}_\alpha$. The superpotential is not the most general one, but by virtue of the non-renormalization theorem, this does not contradict the philosophy of the coupling unification by the reduction method. (A RG invariant fine tuning is a solution of the
Given the superpotential \( W \), we can compute the \( \beta \) functions of the model. We denote the gauge coupling by \( g \) (with the vanishing one-loop \( \beta \) function), and our normalization of the \( \beta \) functions is as usual, i.e., \( dg_i/d \ln \mu = \beta_i^{(1)}/16\pi^2 + O(g^5) \), where \( \mu \) is the renormalization scale. We find:

\[
\begin{align*}
\beta_g^{(1)} &= 0, \\
\beta_u^{(1)} &= \frac{1}{16\pi^2} \left[ -\frac{96}{5} g^2 + 9 (g_i^u)^2 + \frac{24}{5} (g_i^f)^2 + 4 (g_i^d)^2 \right] g_i^u, \\
\beta_d^{(1)} &= \frac{1}{16\pi^2} \left[ -\frac{84}{5} g^2 + 3 (g_i^u)^2 + \frac{24}{5} (g_i^f)^2 + 10 (g_i^d)^2 \right] g_i^d, \\
\beta^{(1)} &= \frac{1}{16\pi^2} \left[ -30 g^2 + \frac{63}{5} (g^\lambda)^2 + 3 \sum_{\alpha=1}^4 (g_\alpha^f)^2 \right] g^\lambda, \\
\beta_f^{(1)} &= \frac{1}{16\pi^2} \left[ -\frac{98}{5} g^2 + 4 (g_i^d)^2 \delta_{i\alpha} + \frac{48}{5} (g_\alpha^f)^2 + \sum_{\beta=1}^4 (g_\beta)^2 + \frac{21}{5} (g^\lambda)^2 \right] g_\alpha^f.
\end{align*}
\]

We then regard the gauge coupling \( g \) as the primary coupling and solve the reduction equations (11) with the power series ansatz. One finds that the power series,

\[
\begin{align*}
(g_i^u)^2 &= \frac{8}{5} g^2 + \ldots, \\
(g_i^d)^2 &= \frac{6}{5} g^2 + \ldots, \\
(g^\lambda)^2 &= \frac{15}{7} g^2 + \ldots, \\
(g_4^f)^2 &= g^2, \\
(g_\alpha^f)^2 &= 0 + \ldots \quad (\alpha = 1, 2, 3),
\end{align*}
\]

exists uniquely, where \( \ldots \) indicates higher order terms and all the other couplings have to vanish. As we have done in the previous section, we can easily verify that the higher order terms can be uniquely computed. Consequently, all the one-loop \( \beta \) functions of the theory vanish. Moreover, all the one-loop anomalous dimensions for the chiral supermultiplets,

\[
\begin{align*}
\gamma_{10}^{(1)} &= \frac{1}{16\pi^2} \left[ -\frac{36}{5} g^2 + 3 (g_i^u)^2 + 2 (g_i^d)^2 \right], \\
\gamma_5^{(1)} &= \frac{1}{16\pi^2} \left[ -\frac{24}{5} g^2 + 4 (g_i^d)^2 \right], \\
\gamma_{H_\alpha}^{(1)} &= \frac{1}{16\pi^2} \left[ -24 g^2 + 3 (g_i^u)^2 \delta_{i\alpha} + \frac{24}{5} (g_\alpha^f)^2 \right], \\
\gamma_{\mu_\alpha}^{(1)} &= \frac{1}{16\pi^2} \left[ -24 g^2 + 4 (g_i^d)^2 \delta_{i\alpha} + \frac{24}{5} (g_\alpha^f)^2 \right], \\
\gamma_{24}^{(1)} &= \frac{1}{16\pi^2} \left[ -\frac{10}{5} g^2 + \sum_{\alpha=1}^4 (g_\alpha^f)^2 + \frac{21}{5} (g^\lambda)^2 \right],
\end{align*}
\]

\(^1\)In the case at hand, however, one can find a discrete symmetry that can be imposed on the most general cubic superpotential to arrive at the non-intergenerational mixing \(^2\).
also vanish in the reduced system. A very interesting result is that these conditions are necessary and sufficient for finiteness at the two-loop level \[34\].

A natural question is what happens in higher loops. Interestingly, there exists a powerful theorem \[40\] which provides the necessary and sufficient conditions for finiteness to all loops. The theorem makes heavy use of the non-renormalization property of the supercurrent anomaly \[41\]. In fact, the finiteness theorem can be formulated in terms of one-loop quantities, and it states for supersymmetry gauge theories, the necessary and sufficient conditions for $\beta_g$ and $\gamma$’s to vanish to all orders are \[40\]:

(a) The validity of the one-loop finiteness conditions, i.e., $\beta_g^{(1)} = \gamma^{(1)} s = 0$.

(b) The reduction equation (11) admit a unique power series solution.

Since the solution (22) can be extended to a unique power series in $g$, the reduced theory (which has a single coupling $g$) has $\beta$ and $\gamma$ functions vanishing to all orders. In this way, the Gauge-Yukawa Unification is achieved.

In most of the previous studies of the present model \[37, 38\], however, the complete reduction of the Yukawa couplings, which is necessary for all-order-finiteness, was ignored. They have used the freedom offered by the degeneracy in the one- and two-loop approximations in order to make specific ansätze that could lead to phenomenologically acceptable predictions. In the above model, we found a diagonal solution for the Yukawa couplings, with each family coupled to a different Higgs. However, we may use the fact that mass terms do not influence the RG functions in a certain class of renormalization schemes, and introduce appropriate mass terms that permit us to perform a rotation in the Higgs sector such that only one pair of Higgs doublets, coupled to the third family, remains light and acquires a non-vanishing VEV \[38\]. Note that the effective coupling of the Higgs doublets to the first family after the rotation is very small avoiding in this way a potential problem with the proton lifetime \[48\]. Thus, effectively, we have at low energies the Minimal Supersymmetric Standard Model (MSSM) with only one pair of

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\[9\]

There is an alternative way to find finite theories, which has been found in connection to duality in supersymmetric theories \[10\].
Higgs doublets satisfying the boundary conditions at $M_{\text{GUT}}$

\[ g_i^2 = \frac{8}{3}g^2 + O(g^4) , \quad g_b^2 = g_\tau^2 = \frac{6}{5}g^2 + O(g^4) , \tag{23} \]

where $g_i \ (i = t, b, \tau)$ are the top, bottom and tau Yukawa couplings of the MSSM, and the other Yukawa couplings should be regarded as free.

Adding soft breaking terms (which are supposed not to influence the $\beta$ functions beyond $M_{\text{GUT}}$), we can obtain supersymmetry breaking. The conditions on the soft breaking terms to preserve one-loop finiteness have been given already some time ago \[36\]. Recently, the same problem in higher orders has been addressed \[42\]. It is an open problem whether there exists a suitable set of conditions on the soft terms for all-loop finiteness.

4 Predictions of Low Energy Parameters

Since the $SU(5)$ symmetry is spontaneously broken below $M_{\text{GUT}}$, the finiteness conditions do not restrict the renormalization property at low energies, and all it remains is a boundary condition on the gauge and Yukawa couplings at $M_{\text{GUT}}$, i.e., eq. (23). So we examine the evolution of these couplings according to their renormalization group equations at two-loop with this boundary condition.

Below $M_{\text{GUT}}$ their evolution is assumed to be governed by the MSSM. We further assume a unique threshold $M_{\text{SUSY}}$ for all superpartners of the MSSM so that below $M_{\text{SUSY}}$ the SM is the correct effective theory. We recall that $\tan \beta$ is usually determined in the Higgs sector, which however strongly depends on the supersymmetry breaking terms. Here we avoid this by using the tau mass $M_\tau$ as input $^3$. That is, assuming that

\[ M_Z \ll M_t \ll M_{\text{SUSY}} , \tag{24} \]

we require the matching condition at $M_{\text{SUSY}}$ \[19\],

\[ \alpha_t^{\text{SM}} = \alpha_t \sin^2 \beta , \quad \alpha_b^{\text{SM}} = \alpha_b \cos^2 \beta , \quad \alpha_\tau^{\text{SM}} = \alpha_\tau \cos^2 \beta , \]

$^3$This means that we partly fix the Higgs sector indirectly.
\[ \alpha_\lambda = \frac{1}{4} \left( \frac{3}{5} \alpha_1 + \alpha_2 \right) \cos^2 2\beta , \quad (25) \]

to be satisfied \[ \] where \( \alpha_i^{\text{SM}} (i = t, b, \tau) \) are the SM Yukawa couplings and \( \alpha_\lambda \) is the Higgs coupling. This is our definition of \( \tan \beta \), and eq. (25) fixes \( \tan \beta \), because with a given set of the input parameters \[52\],

\[ M_\tau = 1.777 \text{ GeV} , \quad M_Z = 91.188 \text{ GeV} , \quad (26) \]

with \[53\]

\[ \begin{align*}
\alpha_{\text{EM}}^{-1}(M_Z) & = 127.9 + \frac{8}{9\pi} \log \frac{M_t}{M_Z} , \\
\sin^2 \theta_W(M_Z) & = 0.2319 - 3.03 \times 10^{-5} T - 8.4 \times 10^{-8} T^2 , \\
T & = M_t/[\text{GeV}] - 165 ,
\end{align*} \quad (27) \]

the matching condition (25) and the GYU boundary condition at \( M_{\text{GUT}} \) (23) can be satisfied only for a specific value of \( \tan \beta \). Here \( M_\tau, M_t, M_Z \) are pole masses, and the couplings are defined in the \( \overline{\text{MS}} \) scheme with six flavors. The translation from a Yukawa coupling into the corresponding mass follows according to

\[ m_i = \frac{1}{\sqrt{2}} g_i(\mu) v(\mu) , \quad i = t, b, \tau \quad \text{with} \quad v(M_Z) = 246.22 \text{ GeV} , \quad (28) \]

where \( m_i(\mu) \)'s are the running masses satisfying the respective evolution equation at two-loop order. The pole masses can be calculated from the running ones of course. For the top mass, we use \[49, 50\]

\[ M_t = m_t(M_t) \left[ 1 + \frac{4}{3} \frac{\alpha_3(M_t)}{\pi} + 10.95 \left( \frac{\alpha_3(M_t)}{\pi} \right)^2 + k_t \frac{\alpha_t(M_t)}{\pi} \right] , \quad (29) \]

where \( k_t \simeq -0.3 \) for the range of parameters we are concerned with in this paper \[50\]. Note that both sides of eq. (29) contains \( M_t \) so that \( M_t \) is defined only implicitly. Therefore, its determination requires an iteration method. As for the tau and bottom masses, we

\[\text{[4]There are MSSM threshold corrections to this matching condition [50, 51], which will be discussed later.}\]
assume that $m_\tau(\mu)$ and $m_b(\mu)$ for $\mu \leq M_Z$ satisfy the evolution equation governed by the $SU(3)_C \times U(1)_{EM}$ theory with five flavors and use

$$M_b = m_b(M_b) \left[ 1 + \frac{4}{3} \frac{\alpha_{3(5f)}(M_b)}{\pi} + 12.4 \left( \frac{\alpha_{3(5f)}(M_b)}{\pi} \right)^2 \right],$$

$$M_\tau = m_\tau(M_\tau) \left[ 1 + \frac{\alpha_{EM(5f)}(M_\tau)}{\pi} \right],$$

(30)

where the experimental value of $m_b(M_b)$ is $(4.1 - 4.5)$ GeV [32]. The couplings with five flavors entered in eq. (30) $\alpha_{3(5f)}$ and $\alpha_{EM(5f)}$ are related to $\alpha_3$ and $\alpha_{EM}$ by

$$\alpha_{3(5f)}^{-1}(M_Z) = \alpha_{3}^{-1}(M_Z) - \frac{1}{3\pi} \ln \frac{M_t}{M_Z},$$

$$\alpha_{EM(5f)}^{-1}(M_Z) = \alpha_{EM}^{-1}(M_Z) - \frac{8}{9\pi} \ln \frac{M_t}{M_Z}.$$

(31)

Using the input values given in eqs. (26) and (27), we find

$$m_\tau(M_\tau) = 1.771 \text{ GeV}, m_\tau(M_Z) = 1.746 \text{ GeV}, \alpha_{EM(5f)}^{-1}(M_\tau) = 133.7,$$

(32)

and from eq. (28) we obtain

$$\alpha_{\tau}^{SM}(M_Z) = \frac{g_\tau^2}{4\pi} = 8.005 \times 10^{-6},$$

(33)

which we use as an input parameter instead of $M_\tau$.

The matching condition (25) suffers from the threshold corrections coming from the MSSM superpartners:

$$\alpha_i^{SM} \to \alpha_i^{SM}(1 + \Delta_i^{SUSY}), \; i = 1, 2, \ldots, \tau,$$

(34)

It was shown that these threshold effects to the gauge couplings can be effectively parametrized by just one energy scale [54]. Accordingly, we can identify our $M_{SUSY}$ with that defined in ref.[54]. This ensures that there are no further one-loop threshold corrections to $\alpha_3(M_Z)$ when we calculate it as a function of $\alpha_{EM}(M_Z)$ and $\sin^2 \theta_W(M_Z)$.

The same scale $M_{SUSY}$ does not describe threshold corrections to the Yukawa couplings, and they could cause large corrections to the fermion mass prediction [31, 51].

\footnote{It is possible to compute the MSSM correction to $M_t$ directly, i.e., without constructing an effective theory below $M_{SUSY}$. In this approach, too, large corrections have been reported [55]. In the present paper, evidently, we are following the effective theory approach as e.g. refs. [50, 51].}
For $m_b$, for instance, the correction can be as large as 50% for very large values of $\tan \beta$, especially in models with radiative gauge symmetry breaking and with supersymmetry softly broken by the universal breaking terms. As we will see later, the $SU(5)$-FUT model predicts (with these corrections suppressed) values for the bottom quark mass that are rather close to the experimentally allowed region so that there is room only for small corrections. Consequently, if we want to break the $SU(2) \times U(1)$ gauge symmetry radiatively, the model favors non-universal soft breaking terms. It is interesting to note that the consistency of the finiteness hypothesis is closely related to the fine structure of supersymmetry breaking and also to the Higgs sector, because these superpartner corrections to $m_b$ can be kept small for appropriate supersymmetric spectrum characterized by very heavy squarks and/or small $\mu_H$ describing the mixing of the two Higgs doublets in the superpotential.

To get an idea about the magnitude of the correction, we consider the case that all the superpartners have the same mass $M_{SUSY} = 500$ GeV with $M_{SUSY} >> \mu_H$ and $\tan \beta \gtrsim 50$. Using $\Delta$'s given in ref. [51], we find that the MSSM correction to the $M_t$ prediction is $\sim -1\%$ for this case. Comparing with the results of [51, 55], this may appear to be underestimated. Note, however, that there is a nontrivial interplay among the corrections between the $M_t$ and $M_b$ predictions for a given GYU boundary condition at $M_{GUT}$ and the fixed pole tau mass, which has not been taken into account in refs. [51, 55]. In the following discussion, therefore, we regard the MSSM threshold correction to the $M_t$ prediction as unknown and denote it by

$$\delta^{MSSM} M_t.$$ (35)

In table 1 we present the predictions of $M_t$ and $m_b(M_b)$ for various given values of $M_{SUSY}$.

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6The solution with small $\mu_H$ is favored by the experimental data and cosmological constraints [56]. The sign of this correction is determined by the relative sign of $\mu_H$ and the gluino mass parameter, $M_3$, and is correlated with the chargino exchange contribution to the $b \to s \gamma$ decay [50]. The later has the same sign as the Standard Model and the charged Higgs contributions when the supersymmetric corrections to $m_b$ are negative.
As we can see from the table, only negative MSSM corrections of at most $\sim 10\%$ to $m_b(M_b)$ are allowed ($m_b^{\text{exp}}(M_b) = (4.1 - 4.5) \text{ GeV}$), implying that FUT favors non-universal soft symmetry breaking terms as announced. The predicted $M_t$ values are well below the infrared value $[57]$, for instance, 194 GeV for $M_{\text{SUSY}} = 500 \text{ GeV}$, so that the $M_t$ prediction must be sensitive against the change of the boundary condition (23).

We recall that if one includes the threshold effects of superheavy particles $[58, 59]$, the GUT scale $M_{\text{GUT}}$ at which $\alpha_1$ and $\alpha_2$ are supposed to meet is related to the mass of the superheavy $SU(3)_C$-triplet Higgs supermultiplets contained in $H_\alpha$ and $\overline{H}_\alpha$. These effects have therefore influence on the GYU boundary condition (23). The structure of the threshold effects in FUT is involved, but they are not arbitrary and probably determinable to a certain extent, because the mixing of the superheavy Higgses is strongly dictated by the fermion mass matrix of the MSSM. To bring these threshold effects under control is challenging. Here we assume that the magnitude of these effects is $\sim \pm 4 \text{ GeV}$ in $M_t$ (which is estimated by comparing the minimal GYU model based on $SU(5)$ $[14]$). We conclude $[14]$ that

$$M_t = (183 + \delta^{\text{MSSM}}M_t \pm 5) \text{ GeV} ,$$

(36)

where the finite corrections coming from the conversion from the dimensional reduction scheme to the ordinary $\overline{\text{MS}}$ in the gauge sector $[60]$ are included, and those in the Yukawa sector are included as an uncertainty of $\sim \pm 1 \text{ GeV}$. The MSSM threshold correction is denoted $\delta^{\text{MSSM}}M_t$ which has been discussed in the previous section. Comparing the $M_t$ prediction above with the experimental value $[64]$, $M_t = (175 \pm 9) \text{ GeV}$ $[14]$, we see it is consistent with the experimental data.
5 Conclusion

As a natural extension of the unification of gauge couplings provided by all GUTs and the unification of Yukawa couplings, we have introduced the idea of Gauge-Yukawa Unification. GYU is a functional relationship among the gauge and Yukawa couplings provided by some principle. In our studies GYU has been achieved by applying the principles of reduction of couplings and finiteness. The consequence of GYU is that in the lowest order in perturbation theory the gauge and Yukawa couplings above $M_{\text{GUT}}$ are related in the form

$$g_i = \kappa_i g_{\text{GUT}}, \; i = 1, 2, 3, e, \cdots, \tau, b, t,$$

where $g_i$ ($i = 1, \cdots, t$) stand for the gauge and Yukawa couplings, $g_{\text{GUT}}$ is the unified coupling, and we have neglected the Cabibbo-Kobayashi-Maskawa mixing of the quarks. So, Eq. (37) exhibits a boundary condition on the renormalization group evolution for the effective theory below $M_{\text{GUT}}$, which we have assumed to be the MSSM. As we have demonstrated in a number of publications [9, 10, 11, 12], especially in [14], there are various supersymmetric GUTs with GYU in the third generation that can predict the bottom and top quark masses in accordance with the experimental data. This means that the top-bottom hierarchy could be explained in these models, in a similar way as the hierarchy of the gauge couplings of the SM can be explained if one assumes the existence of a unifying gauge symmetry at $M_{\text{GUT}}$.

It is clear that the GYU scenario is the most predictive scheme as far as the mass of the top quark is concerned. It may be worth recalling the predictions for $m_t$ of ordinary GUTs, in particular of supersymmetric $SU(5)$ and $SO(10)$. The MSSM with $SU(5)$ Yukawa boundary unification allows $m_t$ to be anywhere in the interval between 100-200 GeV [31] for varying $\tan \beta$, which is now a free parameter. Similarly, the MSSM with $SO(10)$ Yukawa boundary conditions, i.e. $t - b - \tau$ Yukawa Unification gives $m_t$ in the interval 160-200 GeV [22, 30, 31, 33].

Clearly, to exclude or verify different GYU models, the experimental as well as theoretical uncertainties have to be further reduced. One of the largest theoretical uncertainties
for FUT, as we have seen, results from the not-yet-calculated threshold effects of the superheavy particles. Since the structure of the superheavy particles in FUT is basically fixed, it will be possible to bring these threshold effects under control, which will reduce the uncertainty of the $M_t$ prediction (5 GeV) to $\sim 2$ GeV. We have been regarding $\delta^{\text{MSSM}} M_t$ as unknown because we have no sufficient information on the superpartner spectra. Recently, however, it has been found that the principle of finiteness \cite{12} and also of reduction of couplings \cite{65} can be applied to dimensionfull parameters, e.g., soft breaking parameters, too. As a result, it becomes possible to predict the superpartner spectra to a certain extent and then to calculate $\delta^{\text{MSSM}} M_t$.

It will be very interesting to find out in the coming years, as the experimental accuracy of $m_t$ increases, if nature is kind enough to verify our conjectured Gauge-Yukawa Unification.

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