Impact of the Atmospheric Refraction on the Precise Astrometry with Adaptive Optics in Infrared

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Abstract

We study the impact of the atmospheric differential chromatic refraction on the measurements and precision of relative astrometry. Specifically, we address the problem of measuring the separations of close pairs of binary stars with adaptive optics in the J and K bands.

We investigate the influence of weather conditions, zenithal distance, star’s spectral type and observing wavelength on the astrometric precision and determine the accuracy of these parameters that is necessary to detect exoplanets with existing and planned large ground based telescopes with adaptive optics facilities. The analytical formulae for simple monochromatic refraction and a full approach, as well as moderately simplified procedure, are used to compute refraction corrections under a variety of observing conditions.

It is shown that the atmospheric refraction must be taken into account in astrometric studies but the full procedure is not necessary in many cases. Requirements for achieving a certain astrometric precision are specified.

Key words: atmospheric effects, astrometry, stars: binaries, instrumentation: adaptive optics

PACS: 41.85.Gy, 95.10.Jk, 95.75.Qr, 95.85.Jq

1 Introduction

Recently, the atmospheric refraction (AR) has been subject of several studies about its impact on the observations (Roe, 2002), the determination of AR

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at various wavelength (eg. Kunz et al., 2005) or its theoretical models (eg. Garfinkel, 1967; Yatsenko, 1995). The simplest yet quite precise model of the atmosphere and the refraction assumes spherical symmetry of the Earth and the dependence on local weather conditions (eg. Green, 1985). The refraction decreases the real zenithal distance of an object $z_t$. The refraction angle $R = z_t - z_a$, where $z_a$ denotes the apparent observed zenithal distance, is highly dependent on many factors such as the observing wavelength, air pressure and temperature, humidity and $z_t$ itself. In infrared it usually reaches tenths of arcseconds. For relative astrometry, it means that AR changes the apparent separation of two stars at two different zenithal distances by $R_{21} = |R_2 - R_1|$ along the direction to the zenith. The smaller the difference between the zenithal distances of stars, the smaller $R_{21}$. Nevertheless, even for separations of several arcseconds, AR’s contribution to the apparent separation can be larger than the precision of an astrometric measurement. Clearly, the impact of AR on the relative astrometry of close pairs deserves to be studied carefully.

Modern adaptive optics (AO) systems allow us to obtain diffraction-limited images of stars. Sharp and well sampled images are the key to achieving μas-precision which in case of close binaries means an ability to detect massive planets around their components. A precision below 100 μas was already achieved with the 8-m VLT (Neuhäuser et al., 2006) and 200-in Hale telescope (Cameron et al., 2008). Future large and extremely large telescopes like the Thirty-Meter Telescope, Giant Magellan Telescope or the European Extremely Large Telescope, equipped with a new-generation extreme AO (ExAO) should reach a level of astrometric precision of 10 μas or better. Such a precision is sufficient to astrometrically detect the movement of a 1 $M_\odot$ star 10 pc away around a common mass-center with a body of $\sim 0.16$ Jupiter masses on a 1 AU orbit. As we will demonstrate, the relative astrometry at this level of precision will require very accurate knowledge of meteorological conditions near the telescope.

2 Modeling AR

2.1 Refractive index $n$

It is not straightforward to derive an analytic formula for a refractive index $n$. A relatively simple but useful approximation is given by Roe (2002) who uses and corrects Shubert & Walterscheid (2000):

1 See Equation 4.1 in Helminiak & Konacki (2008)
\[ n(\lambda, p, T, p_w) = 1 + \left[ 64.328 + \frac{2948.1}{146 - \lambda^2} + \frac{255.4}{471 - \lambda^2} \right] \frac{p_T}{p_s} 10^{-6} - 43.49 \left[ 1 - \frac{0.007956}{\lambda^2} \right] \frac{p_w}{p_s} 10^{-6} \]  

(1)

where the observing wavelength, \( \lambda \), is given in \( \mu m \), \( p \), \( T \) and \( p_w \) are the pressure \([hPa]\), temperature \([K]\) and partial pressure of water vapor \([hPa]\) respectively. The symbols with the index \( s \) refer to the standard values of air pressure \((1013.25 \ hPa)\) and temperature \((288.15 \ K)\).

Shubert & Walterscheid (2000) computed values of \( n \) for the range of wavelengths from 0.2 to 10 \( \mu m \). This regime contains many regions, where the presence of atmospheric CO\(_2\) and water vapor lines cause fluctuations in the refractive index (Mathar, 2004). These so called resonances make the dependence of \( n(\lambda) \) not so simple as Shubert & Walterscheid (2000) claim. It is especially important for the K band which ”redder” side is strongly influenced by a resonance with water at \( \sim 2.6 \mu m \).

This particular resonance is also not included in the model proposed by Ciddor (1996, with further supplements) which now is considered as the state-of-the-art and is recommended for geological and astronomical research. This model is based on a revised equation for the density of moist air (with CO\(_2\)), known as the BIPM 1981/91 equation (Davis, 1992) and assumes that the atmosphere is a mixture of ”dry air”, containing a variable amount of carbon-dioxide and water vapor. The entire recipe for calculating the refractive index is rather complicated (Appendix B in: Ciddor, 1996). The validity of Ciddor’s model extends from 0.3 to 1.7 \( \mu m \) and from 100 to 1400 \( hPa \). It means that we need to extrapolate it to the K band \((\sim 2.2 \mu m)\) without any warranty of validity, but it covers lower air pressures typical for high-altitude observatories.

The last model we considered was presented by Mathar (2004). It is based on the calculations of a complex-valued dielectric function \( \varepsilon \) (where \( n = \Re(\sqrt{\varepsilon}) \)) as the response of a superposition of independent molecular oscillators whose strengths were derived from the HITRAN database (Rothman et al., 1998). Almost 60,000 H\(_2\)O and CO\(_2\) lines between 0.44 and 25 \( \mu m \) were incorporated to ensure that the influence of the resonances. For the results obtained outside the resonances, the following smooth polynomial was fitted (Mathar, 2007):

\[ n - 1 = \sum_{i=0,5} c_i(T, p, H)(\nu - \nu_{ref})^i \]  

(2)

2 In Ciddor (1996) named the ”phase” index, not the ”group” one
\[ c_i(T, p, H) = c_{iref} \]
\[ + c_{iT}(1/T - 1/T_{ref}) + c_{iTT}(1/T - 1/T_{ref})^2 \]
\[ + c_{iH}(H - H_{ref}) + c_{iHH}(H - H_{ref})^2 \]
\[ + c_{ip}(p - p_{ref}) + c_{ipp}(p - p_{ref})^2 \]
\[ + c_{iTH}(1/T - 1/T_{ref})(H - H_{ref}) \]
\[ + c_{iTp}(1/T - 1/T_{ref})(p - p_{ref}) \]
\[ + c_{iHp}(H - H_{ref})(p - p_{ref}), \]

where \( H \) denotes the relative humidity (in %), \( \nu = 1/\lambda \) is the wavenumber and reference values for \( T, p \) and \( H \) are set to 290.65 \( K \), 75,000 \( Pa \) and 10 % respectively. Values of all \( c \) coefficients and \( \nu_{ref} \) are dependent on the wavelength range and for 1.3 to 2.5 \( \mu m \) are given in Table I in Mathar (2007). This range also limits the validity of the fit.

In order to compare the three models above, we plot in Figure 1 the refractive index as a function of wavelength. For all the cases the conditions are \( p = 1013.25 \) \( hPa \), \( T = 288.15 \) \( K \), 50% of relative humidity and 375 ppm (particles per million) of \( \text{CO}_2 \) (not present in Roe’s model). The transmission curves of \( J, H \) and \( K \) filters of the Palomar High Angular Resolution Camera (PHARO Hayward et al., 2001) are overplotted.

The Ciddor’s model, considered here as the reference one, produces values significantly higher by about \( 5 \times 10^{-8} \) than Roe’s over the whole range. It may be due to the fact that Roe’s model does not include \( \text{CO}_2 \). Nevertheless, the Mathar’s model is in excellent agreement with Roe’s up to \( \sim 2.1 \mu m \) where the previously mentioned resonance with water plays a significant role. The majority of the \( J \) band is out of Mathar’s model validity range so the curve was extrapolated. As it was shown in Mathar (2007), this model exceeds the measurements of the refractive index of moist air by an almost constant value of \( 4 \times 10^{-8} \) except for the resonance region where the empirical data is smaller by \( 6 \times 10^{-8} \).

For the remaining calculations, we decided to reject the Ciddor’s model as the harder to do, and use the two others: Roe’s model as the simple one and Mathar’s as the one which includes resonances, thus the more accurate one. Their accuracy is well enough for this application.

Let us note that in order to make the influence of the refraction more predictable in general, it is better to observe in the infrared. For \( \lambda \sim 0.5 \mu m \), the refraction index is a much steeper function of wavelength than even for \( J \) band (1.25\( \mu m \)) while in the \( K \) band (2.2\( \mu m \)) can be considered as almost constant (see Fig. 2 in: Roe, 2002). Obviously, also for the adaptive optics purposes it
is better to operate in the longer wavelengths. We have also found it interesting to explore the accuracy of a simplified model in the presence of the water vapor resonance. Thus only the K band was chosen for further calculations.

2.2 Refraction angle and relative astrometry

Deriving the relation between the refractive index \( n(\lambda, p, T, p_w) \) (or \( H \) instead of \( p_w \)) and the refraction angle \( R(n, z_t) \) is not straightforward either. The relation for the refraction angle of a monochromatic beam of light, \( R_{\text{mon}} \), proposed by Shubert & Walterscheid (2000) and Roe (2002) is:

\[
R_{\text{mon}} \, [\text{as}] \equiv z_t - z_a \simeq 206265 \left( \frac{n^2 - 1}{2n^2} \right) \tan z_t, \tag{4}
\]

A more sophisticated approach is presented by Stone (1996) where \( R_{\text{mon}} \) in the visible (VIS) depends on \( \tan^2 z_t \) and the non-spherical shape of the Earth is taken into account. Stone (1996) also presents a simple way to compute the mean refraction \( R_m \) by weighting the individual refractions \( R_{\text{mon}}(\lambda) \) with the apparent stellar flux at the wavelength \( \lambda \) and by averaging across the bandpass:

\[
R_m = \frac{\int_0^\infty S(\lambda)E(\lambda)A(\lambda)L(\lambda)F(\lambda)D(\lambda)R_{\text{mon}}(\lambda)d\lambda}{\int_0^\infty S(\lambda)E(\lambda)A(\lambda)L(\lambda)F(\lambda)D(\lambda)d\lambda}, \tag{5}
\]

where \( S(\lambda) \) is the spectral energy distribution of a star, \( E(\lambda) \) – the transmittance of interstellar dust, \( A(\lambda) \) – transmission of the atmosphere at a given airmass, \( L(\lambda) \) – the transmission of the telescope optics, \( F(\lambda) \) – the filter transmission and \( D(\lambda) \) – the quantum efficiency of the detector. This averaging again favors the IR, where the method by Stone approaches the one by Roe.

Figure 2 depicts the influence of AR on relative astrometry of binaries or other close pairs. AR changes the separation between the objects by a value of \( R_{21} = |R_{m1} - R_{m2}| \), along the direction to the Zenith. From the geometry of the effect, the following relation can be derived:

\[
\rho^2 = \rho'_2 + R_{21}^2 + 2\rho R_{21} \cos(\theta - \psi), \tag{6}
\]

where \( \rho', \rho \) are respectively the true and the apparent separations, \( \theta' \) and \( \theta \) are respectively the true and the apparent position angles of the second star, both measured from the vector pointing to the North counter-clockwise to the
position vector of the star B relatively to A, and $\psi$ is the paralactic angle (between the North and Zenith).

One has also keep in mind that during a single observation, the zenithal distance of the system changes with time, thus, in general, also the relative observed separation. Most of the Adaptive Optics systems guide in VIS, while observations are in IR. Dependence of $R$ on $z_t$ and $\lambda$ will lead to a drift of the star’s image across the CCD chip during one exposure. This fact puts some limits on the exposure times and was investigated by [Rod 2002] for the Keck II telescope. If the exposure time is too long, not only the measurement of position but also the refraction correction is more uncertain.

3 Dependence of AR on weather conditions and observing wavelength

We derived a simplified method which will be further called a semi-full approach. In order to compute the monochromatic refraction angle, the equations [1] and [3] were used (Roe’s model). The partial water vapor pressure was computed in the following way. The values of maximum water vapor pressure for a given temperature ($p_{w,max}(T)$) are presented in Table [1]. A 5-th order polynomial was fitted to this data, with $rms \simeq 0.051$, to derive a relation $p_w = H p_{w,max}(T)$ where $H$ is humidity. The following grid of parameters was used: $p_{[hPa]} = 613.25, 813.25, 1013.25 = p_s$; $H [\%] = 0, 50, 100$; $z_1 [\circ] = 0, 20, 40, 60$; $z_{21} \equiv z_2 - z_1 [\arcsec] = 1.5, 15$. For every point of this grid $R_{mon}$ was calculated for the temperatures from the range of 223.15 – 293.15 [K] every 1K. The range of weather conditions was chosen in order to simulate real conditions in many observatories starting from high-altitude ones where temperatures and air pressure values are low.

For the mean refraction, a moderate simplification of Stone’s (1996) method was used. $A(\lambda)$ and $E(\lambda)$ were computed using the equations 23–26 from [Stone 1996]. As an approximation of the spectral energy distribution $S(\lambda)$, we used spectra of black body in temperature of 7000 $K$, which corresponds to a F0 star. The transmission of a telescope optics and quantum efficiency of a detector were assumed to be constant across a given band. Instead of an exact filter transmission curve, we used a model of an ideal filter characterized by the central wavelength $\lambda_c$, total bandwidth $\Delta \lambda$ and constant transmission, which for modern filters is true down to a level of a few %. We applied the data for Palomar High Angular Resolution Camera (PHARO) for the K filter [Hayward et al., 2001]. As will be shown below, this semi-full approach is appropriate for relative astrometry.

Figure [3] shows $R_{12}$ computed with the semi-full approach for the K band. The
refraction correction is comparable to or bigger than 1 mas. Such a precision of relative astrometry is easily achievable from the ground. As the temperature and pressure dependence shows, the effect can also vary by a value higher than 1 mas. Obviously, the influence of the weather conditions decreases when the separation and zenithal distances are smaller. Less obvious is the fact that AR is more significant in low temperatures typical for high-altitude observatories. The correction $R_{21}$ is then about 30-40% higher, the function is slightly steeper and the air pressure has an impact as well. Thus, one may say that it is better to observe close pairs, high over the horizon, in low air pressure and high temperatures. However high temperatures are an issue in the K band for two reasons. Firstly, more turbulence is created, so the AO correction is less efficient, thus the precision of the astrometry is lower. Secondly, the thermal background becomes more significant and variable. This means more difficulties in measuring accurate positions of stars in an image.

The partial water pressure (or humidity) is not as important in the semi-full approach. Fig. 4 shows how the refraction correction changes with humidity. In the most extreme case (high $z$, $z_{21}$, air pressure and temperature), the scale of the change is smaller than 100 µas. This is a level of precision which can be achieved today (Helminiak & Konacki, 2008; Neuhäuser et al., 2006). However, for more probable temperatures, lower pressure and smaller zenithal distance and separation, the typical refraction correction is much smaller. What is interesting, $R_{21}$ decreases as the humidity rises, and the steepness of the function is constant. If humidity would not be included into calculations (the last factor in Equation 1 would be 0), no significant uncertainties would occur but we recommend to keep this factor in mind when calculating AR corrections.

4 Requirements

The results obtained by the computations of the refractive corrections with the semi-full approach were compared to calculations based on full computations of all terms in Eq. 5 and with monochromatic refraction $R_{\text{mon}}$ itself. For the full approach the Mathar’s model of the refractive index was used, $A(\lambda)$ and $E(\lambda)$ were computed as previously, a black-body spectrum with $T_{\text{eff}} \sim 7000$ K was used for both stars’ $S(\lambda)$. The transmission curve of PHARO camera optics, as well as PHARO’s filters transmission curves (Fig. 1) and the detector’s quantum efficiency were adopted for $L(\lambda)$, $F(\lambda)$ and $D(\lambda)$ respectively. This data was kindly sent by Dr. Bernhard Brandl from Leiden Observatory. The telescope optics transmission curve unfortunately was unavailable, thus it was assumed to be constant.

In Figure 5 the differences between the monochromatic $R_{21,\text{mon}}$, semi-full $R_{21,\text{sf}}$
and full $R_{21,f}$ computations of the refraction corrections in the K band are shown. Humidity is set to 50%, air pressure to $p_s$. $z_1 - z_2$ is set to $10''$ and $z_1$ to $20^\circ$ (left panel) or $z_1 - z_2 = 15''$ and $z_1 = 60^\circ$ (right panel). They clearly show that the calculation of monochromatic AR only is good enough for achieving precision of single $\mu$as in many probable sets of weather and observing conditions. For today’s and future astrometric research in small fields in the infrared, one can compute only $R_{mon}$ set at $\lambda_c$ of a certain filter. Knowledge of all the transmission and quantum efficiency curves seems unnecessary, especially for the longer wavelengths.

Situation is rather more complicated when considering spectral types of stars. For stars with similar temperatures it does not matter if one uses full or semi-full approach. However when the effective temperatures differ, the differences between $R_{21,f}$ and $R_{21,sf}$ are not negligible if $\mu$as or even mas precision is required. Due to a weak wavelength dependence, these differences are lower in K than in J band.

The differences between the full and the semi-full approach in the K band are shown in Figure 6 for the same conditions as in Fig. 5 for three systems – O8 ($T_{eff} \simeq 37000$ K) + M1 ($T_{eff} \simeq 3700$ K), M1 + M7 ($T_{eff} \simeq 2700$ K) and F0 + F0 ($T_{eff} \simeq 7000$ K). Brighter (hotter) stars are closer to the zenith. At the higher zenithal distance, for the first binary, errors caused by the usage of the semi-full approach do not allow to go down to a precision of 100 $\mu$as except for the narrow range of temperatures around 250 K. Situation is only a bit better for the M-type binary. For the pair of identical stars, this level of precision is easily achievable with the semi-full approach.

For a comparison, we also show the results of calculations for the same pairs of stars but observed higher over the horizon ($z_1 = 20^\circ$) and with smaller separation ($z_{21} = 10$ as). For the new conditions, the situation in the K band improves (as expected) and even 10 $\mu$as precision is achievable in some conditions for every pair (especially around 250 K). It is also worthwhile mentioning that in the case of real astrometric measurements of O8 + M1 pair, an observer has to face several other sources of uncertainties caused by the large contrast of the observed stars. In the J band, the results would reveal a more complicated behavior. For cooler stars (or rather black-bodies) the maximum of the spectral energy distribution lies in the neighborhood of the J band. At 1.25 $\mu$m different types of stars have different shape of $S(\lambda)$, while at 2.2 $\mu$m the energy distribution looks more or less similar, no matter what the spectral type is. One must also remember that the refraction index $n$ is a steeper function of $\lambda$ in J than in K band. Thus it is better to calculate AR in the K than in J band or simply observe with a narrow-band filter.

This may be still not enough in the case of exoplanets. Their spectra are extremely different from the black-body. Possible various abundances of molecules
like water, methane, ozone, carbon dioxide and many others would make every planet’s spectrum unique. After direct imaging of a planet, which is said to be less difficult with ELTs, precise astrometry may be possible only after obtaining a high signal to noise spectrum in IR.

The influence of bandwidth on the refraction correction in the semi-full nomenclature is shown in Figure 7. Once again we used O8 + M1 pair observed 20° from zenith, separated by 10 mas (left) and observed 60° from zenith, separated by 15 mas (right). The air pressure was set to $p_s$ and 50% humidity was assumed. The plot shows the refraction correction $R_{21,sf}$ for: 1) a perfect filter with the central wavelength $\lambda_c = 2.196 \mu m$ and bandwidth $\Delta \lambda = 0.336 \mu m$ that corresponds to PHARO’s K filter (solid line) – ”normal”, 2) a ”wide” filter with the same central wavelength but 2 times wider bandwidth (dashed line), 3) and a ”narrow” filter with the same $\lambda_c$ but 2 times smaller bandwidth. When $\Delta \lambda$ for a given band reaches zero, $R_{21,sf}$ becomes $R_{21,mon}$. The correction is the smallest in ”wide” filter and the biggest in ”narrow” one, but only if the hotter star is higher over the horizon. It is exactly the opposite when M1 star is closer to the zenith. The differences between these two cases are the smallest in the ”narrow” case.

As seen in Figures 8, 9, and 10, the required precision of air pressure and temperature readings may vary with the zenithal distance and separation of stars. Especially the dependence on temperature in the full approach is interesting due to the influence of the resonances near the K band. For close pairs high over the horizon, any reasonable temperature and pressure can be set and mas precision is reachable. Note that in the semi-full approach even at 60 degrees from zenith, $p = p_s$, $R_{21}$ changes from 1.4125 to 1.0732 mas across a given temperature range which gives about 4.85 $\mu$as per 1 K. For the same case, at the constant temperature 223.15 K, the refraction correction changes from 0.8551 at 613.25 hPa to 1.4125 mas at $p_s$ which gives about 1.39 $\mu$as per 1 hPa. This allows us to achieve 1 mas actually without any knowledge of weather readings and 10 $\mu$as precision when conditions are known with an uncertainty of 1 unit. At the same time, it suggests that for other cases, when $R_{21}$ is bigger, a much higher than 1 unit precision of weather condition readings might be required to achieve 1 $\mu$as.

In Table 2 all the requirements for high-precision astrometry in the K band are collected. The maximum allowable errors of weather conditions readings are given and the need of knowing spectral type of stars (Sp.T.) and usage of the full approach (F.Ap.) is specified. The precision in humidity is not specified as a typical 1% precision will affect astrometry at a level comparable to 1 $\mu$as in the most extreme cases. For a given zenithal distance and $z_{21}$, the set of

\footnote{Note, that when sp. type is needed, the usage of monochromatic refraction only is forbidden}
requirements will lead to measurement errors at the level of given precision \( \sigma \) (in Table 2) or significantly smaller. As one can see, to reach the \( \mu \text{as} \) level precise readings of air temperature and pressure are needed in some cases. Those numbers can be compared to the accuracies of real measurements. For example the weather station at Paranal, provides the following accuracies of readings: 0.2 \( K \) in temperature, 0.1 \( hPa \) in pressure and 1\% in humidity. It simply means that 10 \( \mu \text{as} \) is not achievable there.

Unfortunately, milikelvin or higher variations of temperature as well as variations of the air pressure around the telescope’s dome are very probable. This means that achieving \( \mu \text{as} \)-measurements in wider fields can be impossible, at least by using this model of Earth’s atmosphere, even if weather instruments would be accurate enough. It refers not only to single-mirror telescopes but also to interferometers. Ground based astrometry might be limited to 1 or even 10 microarcsecond level by the impossibility to carry out proper AR calculations.

5 Summary

Undoubtedly, it is necessary to account for the atmospheric refraction to achieve precision of astrometric measurements at the level of miliarcseconds or better. In order to simplify the procedure it is better to observe in the infrared (the longer wavelength, the better) and with narrow-band filters. In cases of the most reasonable zenithal distances and observing conditions, the usage of the semi-full approach (without the transmission and quantum efficiency curves and with the black-body spectrum, Roe’s refraction model) allows to obtain precision well below 1 miliarcsecond. In many cases, also the monochromatic refraction is sufficient. Nevertheless, to compute AR properly one still has to know the air temperature and pressure rather well. It is also good to estimate stars’ spectral types when \( R_{\text{mon}} \) is not enough. In the semi-full approach (or any other) standard precision of weather readings (0.1 \( K \), 0.01 \( hPa \) and 1\% in humidity) is enough to reach \( \sigma \sim 10\mu\text{as} \) which is still mostly unreachable by todays facilities. Nevertheless, future ELT-s will probably be more accurate astrometrically but this will require more accurate weather readings and maybe a better model of the Earth’s atmosphere. Direct measurements of \( n \) in various conditions are usually carried out in laboratories or over long distances but inside the troposphere, so the light beam travels through a relatively stable environment, which is not the case in astronomy. Thus an improved atmosphere model, which may be required for reaching a 1 - 10 \( \mu \text{as} \) level of precision, would have to be calibrated using the measurements of stars’ positions taken from the space, which will be achievable only after launching \textit{Gaia} or \textit{SIM}. 
Acknowledgements

K.G.H. would like to thank Prof. Maciej Konacki from the Nicolaus Copernicus Astronomical Center in Toruń for discussions and useful advices, Dr. Bernhard Brandl from the Leiden Observatory for sending the transmission and efficiency data of PHARO camera, Dr. Thomas Bertram from the Faculty of Mathematics and Natural Sciences of University of Cologne for valuable corrections, Prof. Andrzej A. Marsz from the Gdynia Maritime University for making available the data for Table 1, Dr. Marc Sarazin from ESO for the information on the VLT observatory weather station, and an anonymous referee for useful advices and priceless suggestions.

This work was supported by the Polish Ministry of Science and Higher Education through grants N203 005 32/0449 and 1P03D-021-29 and by the Foundation for Polish Science through a FOCUS grant.

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Fig. 1. The refractive index as a function of wavelength for the standard temperature, pressure and 50% relative humidity, overplotted with the filter transmission curves of the PHARO camera (thin solid line). Roe’s, Ciddor’s and Mathar’s models are plotted with thick solid, dashed and dot-dashed lines respectively. 100% transmission is at \((n - 1.00027) = 4 \times 10^{-6}\).
Fig. 2. Geometry of the refraction and its impact on the relative astrometry of a close pair of stars. N is the direction to the North, Z to the Zenith. The second star is observed at the point B (\(\rho, \theta\)), while its real location is at B'(\(\rho', \theta'\)), relatively to A.
Fig. 3. The refraction correction in the K band according to the semi-full approach. Each panel corresponds to a different zenithal distance of 0, 20, 40 and 60 degrees. The solid lines are for $z_2 - z_1 = 1$, dashed for 5 and dot-dashed for 15°. Every three lines correspond to $p = 613.25\, hPa$ (bottom of the three), $813.25\, hPa$ (middle) and $1013.25\, hPa = p_s$ (top).
Fig. 4. The impact of the humidity on the refraction correction (K band, semi-full approach). The solid line in the left panel refers to 0% humidity, dashed to 50% and dot-dashed to 100% humidity. The right panel shows the dependence of $|R_{21}|$ on humidity at $T = 293.15 \, K$. 

$z_1 = 60^\circ$
$z_{21} = 15 \, \text{as}$
$p = 1013.25 \, \text{hPa}$

$z_1 = 60^\circ$
$z_{21} = 15 \, \text{as}$
$p = 1013.25 \, \text{hPa}$
$T = 293.15 \, K$
Fig. 5. The differences between the monochromatic, $R_{21,mon}$, semi-full, $R_{21,sf}$, and full, $R_{21,f}$, computation of the refraction correction in the K band for F0 + F0 pair. $p = p_s$, $H = 50\%$ and $z_1 = 20^\circ$, $z_{21} = 10''$ (left) or $z_1 = 60^\circ$, $z_{21} = 15''$ (right). The difference $R_{21,sf} - R_{21,mon}$ is denoted with the solid line and $R_{21,f} - R_{21,mon}$ with the dashed line.
Fig. 6. The differences between the full and semi-full \( (R_{21,f} - R_{21,sf}) \), computation of the refraction correction in the K band for O8 + M1 (solid), M1 + M7 (dashed), and F0 + F0 (dot-dashed line). Observing conditions are given. \( p = p_s \) and \( H = 50\% \).
Fig. 7. The dependence of the refraction correction in semi-full approach $R_{21, sf}$ on the filter bandwidth for K band. Calculations are for O8 + M1 pair. Observing conditions are given, $p = p_s$ and $H = 50\%$. In both cases solid lines denote "normal" filter ($\lambda_c, \Delta \lambda$), dashed line - "wide" filter ($\lambda_c, 2\Delta \lambda$) and dot-dashed - "narrow" filter ($\lambda_c, \frac{1}{2} \Delta \lambda$).
Table 1
Maximum water vapor pressure as a function of the temperature.

| Temper. [°C] | $p_{w,max}$ [hPa] | Temper. [°C] | $p_{w,max}$ [hPa] |
|--------------|-------------------|--------------|-------------------|
| 50           | 123.3             | 0            | 6.11              |
| 45           | 95.77             | -5           | 4.21              |
| 40           | 73.72             | -10          | 2.68              |
| 35           | 56.2              | -15          | 1.9               |
| 30           | 42.41             | -20          | 1.25              |
| 25           | 31.66             | -25          | 0.8               |
| 20           | 23.27             | -30          | 0.5               |
| 15           | 17.05             | -35          | 0.309             |
| 10           | 12.28             | -40          | 0.185             |
| 5            | 8.72              | -45          | 0.108              |
Table 2
The weather reading and other requirements for a relative astrometry with a given precision. The uncertainty in the humidity is assumed to be 1%. The symbols $dT$ and $dp$ refer to the errors in the temperature and pressure corresponding to an error in the refraction correction at least two times smaller than a given precision $\sigma$. The symbol "n–n" stands for not necessary — meaning that across a reasonable range of a parameter the variations in $R_{21}$ are at least 2 times smaller than a given astrometric precision.

| $\sigma \sim 1 \text{ mas}$: | dz [as] | $dT$ [K] | dp [hPa] | Sp.T. | F.Ap. | $dT$ [K] | dp [hPa] | Sp.T. | F.Ap. |
|-----------------------------|--------|--------|--------|------|------|--------|--------|------|------|
| $z = 0^\circ$               | 1      | n-n    | n-n    | no   | no   | n-n    | n-n    | no   | no   |
| $z = 20^\circ$              | 5      | 10     | 50     | no   | no   | 1      | 50     | no   | no   |
| $z = 40^\circ$              | 1      | n-n    | no     | no   | n-n  | 100    | n-n    | no   | no   |
| $z = 60^\circ$              | 5      | 10     | no     | no   | 10   | 50     | yes    | no   | no   |
| $z = 100 \mu\text{as}$:   | 1      | 1      | 10     | no   | no   | 10     | 10     | yes  | no   |
| $z = 20^\circ$              | 5      | 10     | no     | yes  | 5    | 10     | yes    | no   | no   |
| $z = 40^\circ$              | 15     | 1      | 5      | no   | no   | 0.5    | 5      | yes  | no   |
| $z = 60^\circ$              | 1      | 10     | 50     | no   | no   | 5      | 10     | yes  | no   |
| $\sigma \sim 10 \mu\text{as}$: | 1      | 1      | 10     | no   | no   | 1      | 10     | yes  | no   |
| $z = 20^\circ$              | 5      | 1      | no     | yes  | 0.5  | 1      | yes    | no   | no   |
| $z = 40^\circ$              | 15     | 0.1    | 1      | no   | no   | 0.1    | 0.5    | yes  | no   |
| $z = 60^\circ$              | 0.1    | 0.5    | yes    | no   | 0.05 | 0.1    | yes    | yes  | no   |
| $\sigma \sim 1 \mu\text{as}$: | 1      | 0.1    | 1      | no   | no   | 0.1    | 1      | yes  | no   |
| $z = 20^\circ$              | 5      | 0.1    | 0.1    | no   | no   | 0.1    | 0.1    | yes  | yes  |
| $z = 40^\circ$              | 15     | 0.01   | 0.1    | no   | yes  | 0.01   | 0.05   | yes  | yes  |
| $z = 60^\circ$              | 0.01   | 0.05   | yes    | yes  | 0.005| 0.01   | yes    | yes  | no   |