Gravitational and Dilatonic Radiations from the Parallel Dressed-Dynamical Unstable Dp-Branes

Hamidreza Daniali and Davoud Kamani

Department of Physics, Amirkabir University of Technology (Tehran Polytechnic)
P.O.Box: 15875-4413, Tehran, Iran
e-mails: hrdl@aut.ac.ir, kamani@aut.ac.ir

Abstract

In the context of the bosonic string theory, we shall extract the general radiation amplitude of a massless closed string from the interaction of two parallel unstable Dp-branes. The branes are non-stationary and have been dressed by background fields. The foregoing amplitude will be rewritten for the massless state radiation from the branes with the large distance. Besides, we shall study the gravitational and dilatonic radiations from this configuration.

PACS numbers: 11.25.-w; 11.25.Uv

Keywords: Boundary state; Background fields; Brane dynamics; amplitude; Gravitational radiation; Dilatonic radiation.
1 Introduction

Some significant physical results in string theory have been obtained by investigating the D-branes and their interactions \[1, 2\]. The boundary state formalism \[3]-[16]\ is an adequate technique for calculating the interaction of the branes in the closed string channel. The D-branes in the presence of the nonzero background fields and dynamics possess some appealing properties. Therefore, by adding various background fields and dynamics a general boundary state, corresponding to a Dp-brane, can be obtained. The interaction of such branes has been widely studied via the boundary state formalism \[17]-[28]\.

The Dp-branes can radiate closed strings in a wide range of configurations. One of the most significant setups is the production of closed strings from a single unstable Dp-brane \[29, 30, 31]\. The closed string radiation from a single unstable brane in the presence of the various background fields has been also studied \[32, 33]\. Besides, the supersymmetric form of the closed string radiation was investigated \[34]\. The closed string radiation from the interacting D-branes is another setup for the production of particles in the closed string spectrum \[35, 36]\. We shall apply the latter case for a generalized configuration of the interacting branes.

The background fields and dynamics of the branes motivated us to study the effects of these variables on the radiation of a massless closed string from the interacting branes. Thus, in this paper, in the context of the bosonic string theory, we study a massless closed string radiation via the interaction of two parallel unstable Dp-branes. The branes have tangential dynamics and they have been dressed by the quadratic tachyonic fields, the Kalb-Ramond field and the $U(1)$ gauge fields in a specific gauge. The boundary state formalism will be used to compute the radiation amplitude. Hence, by entering an appropriate vertex operator in the worldsheet of the mediated closed string between the branes, we obtain a radiated closed string. We shall accurately utilize the eikonal approximation, which ignores the recoil of the branes. At first, we compute the amplitude for radiating a general massless closed string. Afterward, we focus on the graviton and dilaton emission from large distance branes. We shall see that the resultant particle is radiated from one of the branes (a Bremsstrahlung-like process) or from the middle area.
between the branes. Note that the interaction between two branes, with a large distance, entirely occurs through the exchange of the massless closed strings.

This paper is organized as follows. In Sec. 2, we shall introduce the boundary state, which is associated with a dressed-dynamical unstable Dp-brane. In Sec. 3, the radiation amplitude of a generic massless closed string, created from the interaction of two parallel Dp-branes, will be computed. This amplitude will be deformed to describe radiation from the large distance branes. In Sec. 4, the emission of the graviton and dilaton from the large distance branes, with a specific set of the background fields and dynamics, will be calculated. Sec. 5 is devoted to the conclusions.

2 Dressed-dynamical unstable Dp-brane: the boundary state

In order to compute the boundary state, corresponding to a dressed-dynamical unstable Dp-brane, one should start with the following string sigma-model action

\[
S_\sigma = - \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \left( \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X^\nu + \varepsilon^{ab} B_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \right) + \frac{1}{2\pi\alpha'} \int_{\partial\Sigma} d\sigma \left( A_\alpha \partial_\sigma X^\alpha + \omega_{\alpha\beta} \mathcal{J}_\tau^{\alpha\beta} + \mathcal{T}(X^\alpha) \right),
\]

where \( \Sigma \) is the worldsheet of the closed string and \( \partial\Sigma \) represents its boundary. The spacetime indices, i.e., \( \mu, \nu \in \{0, 1, \cdots, 25\} \) are split into the indices for the worldvolume directions \( \alpha, \beta \in \{0, 1, \cdots, p\} \), and the ones for the perpendicular directions \( i, j \in \{p+1, \cdots, 25\} \). In addition, the worldsheet directions are labeled by \( a, b \in \{0, 1\} \). We employ a constant Kalb-Ramond field \( B_{\mu\nu} \), a \( U(1) \) internal gauge field \( A_\alpha = -\frac{1}{2} F_{\alpha\beta} X^\beta \) with the constant field strength \( F_{\alpha\beta} \), and a quadratic open-string tachyonic field \( \mathcal{T} = \frac{1}{2} U_{\alpha\beta} X^\alpha X^\beta \), where the matrix \( U_{\alpha\beta} \) is symmetric and constant. The antisymmetric tensor \( \omega_{\alpha\beta} \) will be used for the angular velocity (angular momentum density). Hence, the explicit form of the dynamics term is given by \( \omega_{\alpha\beta} \mathcal{J}_\tau^{\alpha\beta} = 2\omega_{\alpha\beta} X^\alpha \partial_\tau X^\beta \). We should note that in the presence of the background fields on the worldvolume of the brane, the Lorentz symmetry has been prominently broken. Hence, the brane tangential dynamics along its worldvolume directions is meaningful.
By applying the flat spacetime and flat worldsheet, and also by setting the variation of the action to zero, the following boundary state equations are conveniently extracted

\[
\left([\eta_{\alpha\beta} + 4\omega_{\alpha\beta}] \partial_\tau X^\beta + \mathcal{F}_{\alpha\beta}\partial_\sigma X^\beta + U_{\alpha\beta}X^\beta\right)_{\tau=0} \ket{\mathcal{B}} = 0,
\]

\[
\delta X^i|_{\tau=0}\ket{\mathcal{B}} = 0,
\]

(2.2)

where we have defined the total field strength as \(\mathcal{F}_{\alpha\beta} = F_{\alpha\beta} - B_{\alpha\beta}\).

By utilizing the solution of the equation of motion into the boundary state equation, we can rewrite Eqs. (2.2) in terms of the string oscillators. Besides, the solution of the total boundary state equations can be represented as follows

\[
\ket{\mathcal{B}}^{(\text{tot})} = T_p \ket{\mathcal{B}}^{(\text{osc})} \otimes \ket{\mathcal{B}}^{(0)} \otimes \ket{\mathcal{B}}^{(\text{gh})},
\]

(2.3)

where \(T_p\) is the Dp-brane tension, and

\[
\ket{\mathcal{B}}^{(\text{osc})} = \prod_{n=1}^{\infty} [\det Q_{(\mu)}]^{-1}\exp \left[ -\sum_{m=1}^{\infty} \frac{1}{m} (\alpha_m^{\mu\nu}S_{(m)\mu\nu}^\alpha \tilde{c}_{-m}^\nu) \right] \ket{0}_\alpha \otimes \ket{0}_{\tilde{\alpha}},
\]

(2.4)

\[
\ket{\mathcal{B}}^{(0)} = \prod_i \delta(x^i - y^i)|p^i = 0\rangle \int_{-\infty}^{\infty} \prod_{\alpha=0}^p \left\{ dp^\alpha \exp \left[ i\alpha' \left( (U^{-1}A)_{\alpha\beta}p^\beta \right) \right] \right\},
\]

(2.5)

\[
\ket{\mathcal{B}}^{(\text{gh})} = \exp \left[ \sum_{m=1}^{\infty} \left( c_{-m} \tilde{b}_{-m} - b_{-m} \tilde{c}_{-m} \right) \right] \frac{c_0 + \tilde{c}_0}{2} |q = 1\rangle \otimes |\tilde{q} = 1\rangle.
\]

(2.6)

The coherent state method has been applied to calculate the first equation. The last state exhibits the conformal ghosts contribution to the total boundary state. In addition, we have defined the following matrices

\[
Q_{(m)\alpha\beta} = A_{\alpha\beta} - F_{\alpha\beta} + \frac{i}{2m} U_{\alpha\beta},
\]

(2.7)

\[
S_{(m)\mu\nu} = (\Delta_{(m)\alpha\beta} , \delta_{ij}),
\]

(2.8)

\[
\Delta_{(m)\alpha\beta} = (Q^{-1}_{(m)}N_{(m)})_{\alpha\beta},
\]

(2.9)

\[
N_{(m)\alpha\beta} = A_{\alpha\beta} + F_{\alpha\beta} - \frac{i}{2m} U_{\alpha\beta},
\]

(2.10)

\[
A_{\alpha\beta} = \eta_{\alpha\beta} + 4\omega_{\alpha\beta}.
\]

(2.11)

It should be mentioned that the normalization prefactor in Eq. (2.4) comes from the disk partition function.
3 General formulation for a massless state radiation

In this section, we concentrate on the radiation of a massless closed string from the interaction of two dressed-dynamical unstable D\(p\)-branes. To achieve a generalized result, distinct fields and different dynamics are applied to each of the branes. Thus, the fields and dynamics of the first and second branes are labeled by the subscripts (1) and (2), respectively.

Interaction between two D-branes, in the closed string channel, is elaborated by exchanging a closed string between two boundary states. The geometry of the worldsheet of the exchanged closed string is a cylinder. Let \(\tau\) denote the coordinate along the length of the cylinder, \(0 \leq \tau \leq t\), and \(\sigma\) as the periodic coordinate, i.e. \(0 \leq \sigma \leq \pi\). The radiation of a closed string is described by inserting an appropriate vertex operator, corresponding to the radiated string, into the interaction amplitude. From the mathematical point of view, one should calculate the following amplitude

\[
A = \int_0^\infty dt \int_0^t d\tau \int_0^\pi d\sigma \langle B_1 | e^{-tH} \mathcal{V}(\tau, \sigma) | B_2 \rangle^{(\text{tot})},
\]

(3.1)

where \(H\) is the closed string Hamiltonian, which comprises the matter and ghost parts. Besides, \(\mathcal{V}(\tau, \sigma)\) represents a vertex operator, associated with an arbitrary massless string. Let us exert the complex coordinate \(z = \sigma + i\tau\) and the derivative \(\partial = \partial_z\). Thus, the vertex operator is given by

\[
\mathcal{V}(z, \bar{z}) = \epsilon_{\mu\nu} \partial X^\mu \bar{\partial} X^\nu e^{ip \cdot X},
\]

(3.2)

where the momentum of the radiated closed string is \(p^\mu\) (with \(p^\mu p_\mu = 0\)), and its polarization tensor is \(\epsilon_{\mu\nu}\).

By substituting the vertex operator (3.2) into the amplitude (3.1), we find

\[
\mathcal{A} = \frac{T_p^2}{4} (2\pi)^{26} \epsilon_{\mu\nu} \int_0^\infty dt \int_0^t d\tau \int_0^\pi d\sigma \mathcal{D}(y_1, y_2) e^{At} \int_{-\infty}^{+\infty} \prod_{\alpha=0}^p dk \bar{\mathcal{D}}(k, k')
\times e^{-t\alpha' k^2} e^{-\alpha' (k^2 - k^2)} Z^{(\text{osc}, g)}(e^{ip \cdot X_{\text{osc}}}) \left\{ \langle \partial X^\mu \bar{\partial} X^\nu \rangle_{\text{osc}} - \langle \partial X^\mu p \cdot X \rangle_{\text{osc}} \langle \bar{\partial} X^\nu p \cdot X \rangle_{\text{osc}} \right.
- \left. \alpha' k^\mu \langle \bar{\partial} X^\nu \rangle_{\text{osc}} + \alpha' k^\nu \langle \partial X^\mu p \cdot X \rangle_{\text{osc}} - \alpha'^2 k^\mu k^\nu \right\},
\]

(3.3)
where the following quantities originate from the zero-mode part of the amplitude

\[
D(y_1, y_2) \equiv \prod_{i=p+1}^{25} \delta(x^i - y_1^i) \delta(x^i - y_2^i) \delta(p^i),
\]

\[
D(k, k') \equiv \prod_{\alpha=0}^{p} \delta(p^\alpha + k'^\alpha - k^\alpha)
\times \exp \left[ -i\alpha' \left( \sum_{\alpha=0}^{p} \left[ (U_1^{-1}A_1)_{\alpha\alpha} (k^\alpha)^2 - (U_2^{-1}A_2)_{\alpha\alpha} (k'^\alpha)^2 \right] + 2 \sum_{\beta \neq \alpha} \left[ (U_1^{-1}A_1)_{\alpha\beta} k^\alpha k'^\beta - (U_2^{-1}A_2)_{\alpha\beta} k'^\alpha k^\beta \right] \right) \right].
\]

Besides, the oscillating part of the partition function \(Z^{(\text{osc}, g)}\) and the correlators in Eq. (3.3) possess the following definitions

\[
Z^{(\text{osc}, g)} = \langle B_1 | e^{-tH(\text{osc})} | B_2 \rangle^{(\text{osc})} (g) \langle B_1 | e^{-tH(\text{g})} | B_2 \rangle^{(\text{g})},
\]

\[
\langle O(\sigma, \tau) \rangle^{\text{osc}} \equiv \frac{\langle B_1 | e^{-tH(\text{osc})} O(\sigma, \tau) | B_2 \rangle^{(\text{osc})}}{\langle B_1 | e^{-tH(\text{osc})} | B_2 \rangle^{(\text{osc})}}.
\]

According to Eq. (3.3), \(O(\sigma, \tau)\) takes four values.

The first and second integrals in Eq. (3.3) guarantee that all points of the worldsheet cylinder have been considered. Let us introduce another proper time, i.e. \(t' = t - \tau\). It enables us to modify the integrations as

\[
\int_0^\infty dt \int_0^t d\tau = \int_0^\infty d\tau \int_0^\infty dt'.
\]

Note that \(t' (\tau)\) denotes the proper time of the closed string, which is radiated from the brane with the boundary state \(|B_1\rangle (|B_2\rangle)\). This means that \(t' = 0\) and \(t' = t\) (or \(\tau = 0\)) correspond to the radiation from the first brane and the second one, respectively. When the radiation occurs from the middle points between the branes, we have \(\tau, t' > 0\).

Now we apply the Wick’s rotation \(\tau \to -i\tau\). Using the explicit forms of the oscillating part and ghost part of the boundary state, i.e. Eqs. (2.4) and (2.6), we obtain the following results

\[
Z^{(\text{osc}, g)} = \prod_{n=1}^{\infty} \det \left[ Q^\dagger_{(n)1} Q_{(n)2} \right]^{-1} \prod_{n=1}^{\infty} \frac{(1 - q^{2n})^2}{\det \left( 1 - \mathcal{S}_{(n)}^{(1)} \mathcal{S}_{(n)}^{(2)} q^{2n} \right)},
\]
\[ \langle e^{i p \cdot X_{\text{osc}}} \rangle = \exp \left\{ \frac{\alpha'}{2} p_{\mu} p_{\nu} \sum_{n=1}^{\infty} \frac{1}{n} \left( S_{(n)}^{(1)\mu\eta} S_{(n)}^{(2)\nu_{\eta}} \text{Tr} \left[ \ln \left( 1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n} \right) \right] \right) 
\right. 
- S_{(n)}^{(2)\mu\nu} T_{t'}^{(n)} - S_{(n)}^{(1)\mu\nu} T_{\tau}^{(n)} \right\}, \]

where \( q = e^{-2(\tau + t')} \), and

\[ T_{(t',\tau)}^{(n)} = \text{Tr} \left[ \ln \left( 1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4(t',\tau)} \right) - \left( 1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2(n-1)} e^{-4(t',\tau)} \right)^{-1} \right]. \]

The exponential correlator was calculated by using the Cumulant expansion in the eikonal approximation. For the one-derivative correlators we obtain the following expression

\[ \langle \partial X^\mu \partial X^\nu \rangle_{\text{osc}} = \langle \partial X^\mu \bar{\partial} X^\nu \rangle_{\text{osc}} = -i\alpha' \sum_{n=1}^{\infty} \left\{ \eta^{\mu\nu} \text{Tr} \left( \frac{1 + S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}} \right) \right. 
- S_{(n)}^{(1)\mu\eta} S_{(n)}^{(2)\nu_{\eta}} \text{Tr} \left( \frac{S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}}{1 - S_{(n)}^{(1)\dagger} S_{(n)}^{(2)} q^{2n}} \right) \right. 
- \frac{1}{4} \left( S_{(n)}^{(2)\mu\nu} \partial_{t'} T_{t'}^{(n)} - S_{(n)}^{(1)\mu\nu} \partial_{\tau} T_{\tau}^{(n)} \right) \left. \right\}. \]

According to the equation \( \langle X^\mu \partial \partial X^\nu \rangle = 0 \), the correlator \( \langle \partial X^\mu \bar{\partial} X^\nu \rangle_{\text{osc}} \) in Eq. (3.3) can be obtained by computing the derivative of Eq. (3.12).

In fact, the integrand of the amplitude (3.3) is independent of the worldsheet coordinate \( \sigma \) (see Eqs. (3.4)-(3.6) and (3.9)-(3.12)). Therefore, the integration over the coordinate \( \sigma \) is trivial, i.e. its effect is given by multiplying the amplitude by the factor \( \pi \).

### 3.1 Radiation from the branes with large distance

We are interested in the radiation of the closed strings from the branes whose positions are far from each other. In the large distance limit (LDL), the main contribution to the interaction obviously comes from the exchange of the massless closed strings. Since the large distance of the branes corresponds to the long enough time, the limit \( t \to \infty \) should be exerted on the oscillating portion of the amplitude, i.e. on Eqs. (3.4)-(3.12). Therefore, for the distant branes, the quantity \( Z^{(\text{osc},g)} \) and the correlators in the amplitude (3.3) take
the following features

\[ Z^{(\text{osc,g})}_{\text{LDL}} = \prod_{n=1}^{\infty} \det \left[ Q^{(n)1}_{(n)1} Q^{(n)2}_{(n)2} \right]^{-1}, \quad (3.13) \]

\[ \langle e^{i p \cdot X_{\text{osc}}} \rangle_{\text{LDL}} = \exp \left\{ -\alpha' \left( S^{(2)\mu\nu}_{(1)} T^{(1)}_{\nu'} + S^{(1)\mu\nu}_{(1)} T^{(1)}_{\tau} \right) \right\}, \quad (3.14) \]

\[ \langle \partial X^\mu X^\nu \rangle_{\text{osc}}|_{\text{LDL}} = -i\alpha' \left\{ 13 \eta^{\mu\nu} + \frac{1}{4} \left( S^{(2)\mu\nu}_{(1)} \partial_{\nu} T^{(1)}_{\nu'} + S^{(1)\mu\nu}_{(1)} \partial_{\tau} T^{(1)}_{\tau} \right) \right\}. \quad (3.15) \]

Besides, the derivative of Eq. (3.15) with respect to \( \bar{z} \) may be calculated to obtain the two-derivative correlator \( \langle \partial X^\mu \partial X^\nu \rangle_{\text{osc}}|_{\text{LDL}} \).

Note that, for acquiring Eq. (3.14) we assumed that the momentum of the radiated closed string and the setup parameters satisfy the relation

\[ p_{\mu} p_{\nu} \sum_{n=2}^{\infty} \left( S^{(1)*}_{(n)1} + S^{(2)}_{(n)2} \right)^{\mu\nu} = 0. \quad (3.16) \]

This condition clarifies that for the given setup parameters, the components of the momentum of the radiated closed string are not independent, i.e. two of them are specified in terms of the others.

Substitute the foregoing correlators into Eq. (3.3), the amplitude for the massless string radiation from the interaction of the branes in the large distance limit will be obtained. Since this form of the amplitude is very long we do not explicitly write it.

### 3.2 Radiation in the presence of the condition \( \Delta^{(1)\dagger}_{(1)} \Delta^{(2)}_{(1)} = 1 \)

From now on, for simplification, we impose the following condition on the setup parameters \( \Delta^{(1)\dagger}_{(1)} \Delta^{(2)}_{(1)} = 1 \). Thus, for the large distance limit, the two-derivative correlator can be written in the form

\[ \langle \partial X^\mu \partial X^\nu \rangle_{\text{osc}}|_{\text{LDL}} = -\langle \partial X^\mu X^\nu \rangle_{\text{osc}}|_{\text{LDL}} \left[ \langle p \cdot \partial X \rangle_{\text{osc}}|_{\text{LDL}} + \frac{i\alpha'}{2} (k^2 - k'^2) \right]. \quad (3.17) \]
Substitute this correlator into Eq. (3.3), the amplitude for the closed string radiation from the large distance branes takes the feature

\[
A(0)|_{\text{LDL}} = -\frac{\pi}{16} T_p^2 \alpha'^2 (2\pi)^{26} \int_{-\infty}^{0} \int_{\infty}^{+ \infty} \prod_{\alpha=0}^{p} dk^\alpha dk'^{\alpha} \mathcal{D}(k, k') e^{-\alpha' (t' k^2 + \tau k'^2)} \int_{0}^{\infty} dt' D(y_1, y_2) e^{A(t' + \tau)} \int_{-\infty}^{+ \infty} \int_{-\infty}^{\infty} \prod_{n=1}^{\infty} \det \left[ Q^2_{(n)} \right]^{-1} \langle e^{ip \cdot X_{\text{osc}}}' \rangle' \times \left\{ A(\partial_t \mathcal{T}^r_{1})^2 + B(\partial_\tau \mathcal{T}^r_{\tau})^2 + C \partial_\tau \mathcal{T}^r_{1} + D \partial_\tau \mathcal{T}^r_{\tau} + E \right\},
\]

where $\mathcal{T}^r_{(n)}$ and $\langle e^{ip \cdot X_{\text{osc}}}' \rangle'$ indicate Eqs. (3.11) and (3.14) in the presence of the equation $\Delta_{(1)}^{(1)} \Delta_{(1)}^{(1)} = 1$. With the help of integration by part we obtain the following equations

\[
\partial_\tau \mathcal{T}^r_{1} = -\frac{2(\alpha' k^2 - 4)}{\alpha' k_{\mu}p_{\nu} S^1_{(1)}^{\mu \nu}},
\]

\[
\partial_t \mathcal{T}^r_{1} = -\frac{2(\alpha' k'^2 - 4)}{\alpha' k'_{\mu}p_{\nu} S^2_{(1)}^{\mu \nu}}.
\]

In fact, Eq. (3.18) formally describes any massless string radiation. Each of these radiations possesses its own quantities $A, B, C, D$ and $E$. In the next section the explicit forms of these variables for the graviton and dilaton radiations will be written.

4 Graviton and dilaton radiations

In this section, we shall focus on the graviton and dilaton emission from the interacting distant branes. Precisely, the amplitude (3.18) will be rewritten for the graviton and dilaton radiations, i.e., the explicit forms of the variables $A, B, C, D$ and $E$ will be written.

4.1 The amplitude for the graviton radiation

The graviton’s polarization tensor is symmetric and traceless, i.e., $\epsilon^{(gr)}_{\mu \nu} = \epsilon^{(gr)}_{\nu \mu}$ and $\epsilon^{(gr)}_{\mu} = 0$. In addition, the polarization tensor should satisfy $p_{\mu} \epsilon^{(gr)}_{\mu \nu} = 0$. Performing
some heavy calculations, for the graviton radiation we obtain

\[
A = \epsilon_{\alpha\beta}^{(gr)} \left\{ p_\lambda p_\eta \left[ \Delta^{(2)\alpha\lambda}_{(1)} \Delta^{(2)\beta\eta}_{(1)} - \Delta^{(2)\alpha\beta}_{(1)} \Delta^{(2)\lambda\eta}_{(1)} \right] + p_\perp^2 \Delta^{(2)\alpha\beta}_{(1)} \right. \\
+ \left. 2p^\alpha p_\lambda \Delta^{(2)\beta\lambda}_{(1)} + p^\alpha p^\beta \right\} + \epsilon_{\alpha}^{(gr)} \left( p_\perp^2 + p_\lambda p_\eta \Delta^{(2)\lambda\eta}_{(1)} \right), \tag{4.1}
\]

\[
B = \epsilon_{\alpha\beta}^{(gr)} \left\{ p_\lambda p_\eta \left[ \Delta^{(1)\dagger\alpha\lambda}_{(1)} \Delta^{(1)\dagger\beta\eta}_{(1)} - \Delta^{(1)\dagger\alpha\beta}_{(1)} \Delta^{(1)\dagger\lambda\eta}_{(1)} \right] + p_\perp^2 \Delta^{(1)\dagger\alpha\beta}_{(1)} \right. \\
+ \left. 2p^\alpha p_\lambda \Delta^{(1)\dagger\beta\lambda}_{(1)} + p^\alpha p^\beta \right\} + \epsilon_{\alpha}^{(gr)} \left( p_\perp^2 + p_\lambda p_\eta \Delta^{(1)\dagger\lambda\eta}_{(1)} \right), \tag{4.2}
\]

\[
C = \epsilon_{\alpha\beta}^{(gr)} \left\{ \frac{1}{2} (k^2 - k'^2) \Delta^{(2)\alpha\beta}_{(1)} - k^\alpha p_\lambda \Delta^{(2)\beta\lambda}_{(1)} - k^\beta p_\lambda \Delta^{(2)\alpha\lambda}_{(1)} \right. \\
+ \left. 2p^\alpha p^\beta \right\} - \frac{1}{2} \epsilon_{\alpha}^{(gr)} k^2 k'^2, \tag{4.3}
\]

\[
D = -\epsilon_{\alpha\beta}^{(gr)} \left\{ \frac{1}{2} (k^2 - k'^2) \Delta^{(1)\dagger\alpha\beta}_{(1)} - k^\alpha p_\lambda \Delta^{(1)\dagger\beta\lambda}_{(1)} - k^\beta p_\lambda \Delta^{(1)\dagger\alpha\lambda}_{(1)} \right. \\
+ \left. 2p^\alpha p^\beta \right\} + \frac{1}{2} \epsilon_{\alpha}^{(gr)} k^2 k'^2, \tag{4.4}
\]

\[
E = \epsilon_{\alpha\beta}^{(gr)} k^\alpha k^\beta. \tag{4.5}
\]

Note that all indices represent the worldvolume directions, i.e., they belong to the set \(\{0, 1, \cdots, p\}\).

### 4.2 The amplitude for the dilaton radiation

In contrast to the graviton’s and Kalb-Ramond’s polarization tensors, we have the explicit form of the dilaton polarization tensor. In the 26-dimensional spacetime, it possesses the following form

\[
\epsilon_{\mu\nu}^\phi = \frac{1}{\sqrt{24}} \left( \eta_{\mu\nu} - p_\mu l_\nu - p_\nu l_\mu \right), \quad p \cdot l = 1, \quad l^2 = 0. \tag{4.6}
\]
Thus, the radiation amplitude for the dilaton is given by Eq. (3.18), accompanied by the following variables

\[
A = \frac{1}{\sqrt{24}} p_\xi p_\gamma \left\{ S^{(2)\mu\xi}_\mu S^{(2)}_\gamma - S^{(2)\mu}_\mu S^{(2)\xi}_\gamma \right\},
\]

(4.7)

\[
B = \frac{1}{\sqrt{24}} p_\xi p_\gamma \left\{ S^{(1)\mu\xi}_\mu S^{(1)\gamma}_\gamma - S^{(1)\mu\xi}_\mu S^{(1)\gamma}_\gamma \right\},
\]

(4.8)

\[
C = -\frac{1}{\sqrt{24}} \left\{ 2k^\mu p_\nu S^{(2)}_\nu - \frac{1}{2} (k^2 - k'^2) S^{(2)\mu}_\mu - S^{(2)\mu\nu}_\nu \left[p_\mu p_\nu (195 + 2l \cdot k) + 2l^\mu p_\nu (k^2 - k'^2) \right] \right\},
\]

(4.9)

\[
D = \frac{1}{\sqrt{24}} \left\{ 2k^\mu p_\nu S^{(1)\nu}_\mu - \frac{1}{2} (k^2 - k'^2) S^{(1)\mu\nu}_\nu - S^{(1)\mu\nu}_\nu \left[p_\mu p_\nu (195 + 2l \cdot k) + 2l^\mu p_\nu (k^2 - k'^2) \right] \right\},
\]

(4.10)

\[
E = \frac{1}{\sqrt{24}} \left[ k^2 - 2 (l \cdot k)(p \cdot k) - 28 k \cdot p - 111 (k^2 - k'^2) \right].
\]

(4.11)

Note that all indices in these parameters are the spacetime indices. They belong to the set \(\{0, 1, \ldots, 25\}\).

### 4.3 Some physical properties

Here, by performing the integration over the proper times, we rewrite the radiation amplitude (3.18). Assume that the momentum of the radiated string is small. Afterward, by expanding the exponential, we obtain

\[
\int_0^\infty dt \int_0^\infty dt' e^{-(\alpha' k^2 - 4)} e^{-\tau(\alpha' k'^2 - 4)} \langle \epsilon p \cdot X_{osc} \rangle' \approx \frac{1}{(\alpha' k^2 - 4)(\alpha' k'^2 - 4)}. \tag{4.12}
\]

Now apply a constant shift to the momenta, i.e., \(k^\alpha = K^\alpha - u^\alpha\) and \(k'^\alpha = K'^\alpha - u^\alpha\). Besides, let the worldvolume vector \(u^\alpha\) satisfy the following conditions

\[
k \cdot u = k' \cdot u = 0, \quad u^2 = -\frac{4}{\alpha'}.
\]

(4.13)
Adding all these together, we obtain the final version of the amplitude as in the following

\[
\mathcal{A}^{(0)}_{\text{LDL}} = \frac{\pi}{4} (2\pi)^{26} T_p^2 D(y_1, y_2) \prod_{n=1}^{\infty} \left[ \det \left( Q_{(n)1} Q_{(n)2} \right) \right]^{-1} \\
\times \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \prod_{\gamma=0}^{P} dK^\gamma dK'^\gamma \mathcal{D}(K, K') \left( \frac{\Gamma}{K^2} - \frac{\Theta}{K'^2 K^{2'} + \Upsilon} \right), \tag{4.14}
\]

where \( \Gamma, \Upsilon \) and \( \Theta \) are given by

\[
\Gamma = \frac{1}{2\alpha' p_\mu p_\nu S^{(2)\mu\nu}_{(1)}} \left( D' - \frac{BK'^2}{2\alpha' p_\mu p_\nu S^{(2)\mu\nu}_{(1)}} \right), \\
\Upsilon = -\frac{1}{2\alpha' p_\mu p_\nu S^{(1)\mu\nu}_{(1)}} \left( C' - \frac{AK'^2}{2\alpha' p_\mu p_\nu S^{(1)\mu\nu}_{(1)}} \right), \\
\Theta = E', \tag{4.15}
\]

where \((D', C', E') \equiv (D, C, E)\mid_{k' \rightarrow K' \rightarrow -u}.\) The amplitude (4.14) elaborates the graviton or dilaton radiation from the interaction of two parallel dressed-dynamical unstable Dp-branes in the large distance. Note that we applied only one vertex operator, hence, we do not have simultaneous radiation of the graviton and dilaton. However, the three terms in this amplitude clarify that the graviton and or dilaton can be radiated in the three distinct physical processes. The squared momenta in the denominators, i.e. \(K^2\) and \(K'^2\), correspond to the propagators of the radiated strings by the D-branes. The quantities \( \Gamma \) and \( \Upsilon \) correspond to the residue of a single-pole process. That is, a massless state is emitted by one of the branes and is absorbed by the other one, then after traveling as an excited state, it re-decays by emitting the final massless string. The \( \Theta \)-term exhibits a double-pole process, in which the radiation occurs between the branes, from the mediated closed string. The mediated string, which is exchanged between the branes, is responsible for the interaction of the branes.

5 Conclusions

We calculated a general amplitude for radiating any massless closed string via the interaction of two parallel unstable Dp-branes. The branes have tangential dynamics, and they have been dressed by the Kalb-Ramond field, a quadratic tachyonic field and
a $U(1)$ gauge potential in a particular gauge. The presence of the background fields and dynamical parameters drastically dedicated a generalized form to the amplitude. These parameters enable us to adjust the value of the radiation amplitude to any desirable value.

We deformed the foregoing general amplitude to extract any massless string radiation from the branes with the large distance. Then, the amplitude for the graviton and dilaton radiations from the branes with the large distance were explicitly computed. By evaluating the proper-time integrals in the eikonal approximation, it was revealed that three distinct radiation processes can occur. That is, the gravitational or dilatonic radiation can take place from the first brane, from the second brane and from the middle region between the branes.

References

[1] J. Polchinski, “String Theory”, (Cambridge University Press, Cambridge, 1998), Volumes I and II; C. V. Johnson, “D-Branes”, (Cambridge University Press, Cambridge, 2003).

[2] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724-4727.

[3] C. G. Callan and I. R. Klebanov, Nucl. Phys. B 465 (1996) 473.

[4] M.B. Green and P. Wai, Nucl. Phys. B 431 (1994) 131.

[5] C. Bachas, Phys. Lett. B 374 (1996) 37.

[6] M. Li, Nucl. Phys. B 460 (1996) 351.

[7] M.B. Green and M. Gutperle, Nucl. Phys. B 476 (1996) 484.

[8] M. Frau, A. Liccardo and R. Musto, Nucl. Phys. B 602 (2001) 39.

[9] F. Hussain, R. Iengo and C. Nunez, Nucl. Phys. B 497 (1997) 205.

[10] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, Nucl. Phys. B 507 (1997) 259.
[11] C. G. Callan, C. Lovelace, C. R. Nappi and S. A. Yost, Nucl. Phys. B 288 (1987) 525; Nucl. Phys. B 308 (1988) 221.

[12] O. Bergman, M. Gaberdiel and G. Lifschytz, Nucl. Phys. B 509 (1998) 194.

[13] S. Gukov, I. R. Klebanov and A. M. Polyakov, Phys. Lett. B 423 (1998) 64.

[14] M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda and R. Marotta, Nucl. Phys. B 621 (2002) 157.

[15] P. Di Vecchia, A. Liccardo, R. Marotta and F. Pezzella, Int. J. Mod. Phys. A 20 (2005) 4699-4796.

[16] J. Polchinski, “TASI lectures on D-branes”, arXiv:hep-th/9611050.

J. Polchinski, S. Chaudhuri and C. V. Johnson, “Notes on D-branes”, arXiv:hep-th/9602052.

[17] C. Bachas, “(Half) a lecture on D-branes,” arXiv:hep-th/9701019 [hep-th].

[18] P. Di Vecchia and A. Liccardo, “D-branes in string theory. II.”, arXiv:hep-th/9912275 [hep-th].

[19] M. Billo, D. Cangemi and P. Di Vecchia, Phys. Lett. B 400 (1997) 63.

[20] L. Girardello, C. Piccioni and M. Porrati, Mod. Phys. Lett. A 19 (2004) 2059-2068.

[21] J. X. Lu, Nucl. Phys. B 934 (2018) 39-79.

[22] F. Hussain, R. Iengo, C. A. Nunez and C. A. Scrucca, Phys. Lett. B 409 (1997) 101-108.

[23] M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, Phys. Lett. B 400 (1997) 52.

[24] H. Arfaei and D. Kamani, Phys. Lett. B 452 (1999) 54-60, arXiv:hep-th/9909167.

D. Kamani, Phys. Lett. B 487 (2000) 187-191, arXiv:hep-th/0010019; E. Maghsoodi and D. Kamani, Nucl. Phys. B 922 (2017) 280-292, arXiv:1707.08383 [hep-th]; F. Safarzadeh-Maleki and D. Kamani, Phys. Rev. D 90 (2014) 107902,
[25] D. Kamani, Eur. Phys. J. C 26 (2002) 285-291, arXiv:hep-th/0008020; S. Teymour-tashlou and D. Kamani, Eur. Phys. J. C 81 (2021) 761, arXiv:2108.10164 [hep-th]; D. Kamani, Mod. Phys. Lett. A 17 (2002) 237-243, arXiv:hep-th/0107184.

[26] F. Safarzadeh-Maleki and D. Kamani, Phys. Rev. D 89 (2014) 026006, arXiv:1312.5489 [hep-th].

[27] H. Arfaei and D. Kamani, Nucl. Phys. B 561 (1999) 57-76, arXiv:hep-th/9911146; D. Kamani, Nucl. Phys. B 601 (2001) 149-168, arXiv:hep-th/0104089; H. Daniali and D. Kamani, Nucl. Phys. B 975 (2022) 115683, arXiv:2202.09347 [hep-th].

[28] D. Kamani, Ann. Phys. 354 (2015) 394-400, arXiv:1501.02453 [hep-th]; H. Arfaei and D. Kamani, Phys. Lett. B 475 (2000) 39-45, arXiv:hep-th/9909079; D. Kamani, Mod. Phys. Lett. A 15 (2000) 1655-1664, arXiv:hep-th/9910043.

[29] N. D. Lambert, H. Liu and J. M. Maldacena, JHEP 03, 014 (2007).

[30] B. Chen, M. Li and F. L. Lin, JHEP 11, 050 (2002).

[31] I. R. Klebanov, J. M. Maldacena and N. Seiberg, JHEP 07, 045 (2003).

[32] K. Nagami, JHEP 01, 005 (2004).

[33] S. J. Rey and S. Sugimoto, Phys. Rev. D 67, 086008 (2003).

[34] J. Shelton, JHEP 01, 037 (2005).

[35] F. Hussain, R. Iengo, C. A. Nunez and C. A. Scrucca, Nucl. Phys. B 517 (1998) 92-124; AIP Conf. Proc. 415 (1998) 421, arXiv:hep-th/9711021.

[36] J. D. Blum, Phys. Rev. D 68, 086003 (2003).