Applying a Langevin description of the linear sigma model we investigate four different scenarios for the evolution of a disoriented chiral condensate: annealing or quench with initial conditions governed by effective ‘light’ or physical mass pions. We present pion number distributions estimated from the zero mode (i.e. \( k = 0 \)-field) component. The best DCC signal is expected for the quench scenario with initial conditions centered around \( \langle \sigma \rangle \approx 0 \) as would be the case of effective light ‘pions’ close to the phase transition. Our investigations support the idea of looking for DCC formation in individual events.

1 Disoriented Chiral Condensate

The idea of so called disoriented chiral condensate (DCC) first appeared in a work of Anselm but it was made widely known due to Bjorken and Krzywicki. This idea has been carried over to the field of high energy heavy ion collisions as one of the most interesting suggestions of exotic phenomena as it might give direct evidence for the chiral phase transition expected to occur at high energy densities. Since then many works appeared on various aspects of DCC formation in heavy-ion collisions (for a review see and references listed). Usually these considerations assume an initial state at high temperature in which the chiral symmetry is restored by vanishing collective fields (or equivalently by vanishing quark condensates). Independent thermal fluctuations in each isospin direction of the \( O(4) \) sigma-model are present. This configuration sits on the top of the barrier of the potential energy at zero temperature, so a sudden cooling of the system supposedly brings it into an unstable state. This picture is referred to as the quenched approximation. The spontaneous growth and subsequent decay of these configurations would give rise to large collective fluctuations in the number of produced neutral pions.

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compared to charged pions, and thus could provide a mechanism explaining a family of peculiar cosmic ray events, the Centauros. A deeper reason for these strong fluctuations lies in the fact that all pions are assumed to sit in the same momentum state and the overall wavefunction can carry no isospin.

The proposed quench scenario, however, assumes that the effective potential governing the evolution of the long wavelength modes immediately turns to the classical one at zero temperature. This is a very drastic assumption as the soft classical modes completely decouple from the residual thermal fluctuations at the chiral phase transition temperature in an ad hoc manner.

An alternative, the annealing scenario was suggested by Gavin and Müller. They used the one-loop effective potential instead of the classical one including thermal fluctuations. For only moderate expansion in one or more dimension this view did not lead to a prediction of a huge DCC signal. The smallness of the order parameter at the beginning of the DCC formation has also been criticized: if the soft field remains in thermal contact with the fluctuations giving rise to the one-loop potential, then one also has to allow for thermal fluctuations in the initial conditions. Quenched initial conditions within the linear sigma model are statistically unlikely.

The likeliness of an instability leading potentially to a DCC event during the evolution with a continuous contact with the heat-bath of thermal pions was investigated by us in [11]. In the present contribution we will first review the theoretical ideas behind a classical Langevin description of the soft fields employed in [11] and then shortly discuss our model for simulating the evolution of the order parameter and the soft pion fields. We then state some new numerical results of the simulation on the evolution of a coherent pionic field considered by different scenarios.

Our picture of a possible DCC formation in high energy heavy-ion collisions is as follows:

- in the first stage of the collision ($\tau = 0 \ldots 0.5$ fm/c) a parton gas is formed with a temperature well over the chiral restoration point ($T \gg T_c$),
- in the following ($\tau = 1 \ldots 2$ fm/c) the temperature is around critical ($T \approx T_c$) and small chirally disoriented domains (bubbles) of collective pionic fields start to form together with a thermalized background of (quasi-)pions,
- at the time ($\tau = 2 \ldots 10$ fm/c) the temperature drops and the fireball expands rapidly, at most one bigger DCC domain survives this phase in the heat bath of thermal pions,
- finally ($\tau \geq 10$ fm/c) the fireball breaks off and — in some events — many soft pions are emitted with isospin distribution characteristic to DCC.
2 Description of soft modes within a thermal environment

One of the recent topics in quantum field theory at finite temperature or near thermal equilibrium concerns the evolution and behavior of the long wavelength modes. These modes often lie entirely in the non-perturbative regime. Therefore solutions of the classical field equations in Minkowski space have been widely used in recent years to describe long-distance properties of quantum fields that require a non-perturbative analysis. The distinction between soft and hard modes separates the energy and momentum scales as \( gT \ll k_c \ll T \), allowing for a perturbative resummation of certain interactions. Soft modes \( (k < k_c) \) are non-perturbative, but - this is the hope - can be treated (semi)classically. Hard modes \( (k > k_c) \) are treated perturbatively. A justification of the classical treatment of the long-distance dynamics of weakly coupled bosonic quantum fields at high temperature is based on the observation that the average thermal amplitude of low-momentum modes is large and approaches the classical equipartition limit. The classical field equations should therefore provide a good approximation for the dynamics of such highly occupied modes. However, the thermodynamics of a classical field is only defined if an ultraviolet cut-off \( k_c \) is imposed on the momentum \( p \) such as a finite lattice spacing \( a \). Many thermodynamical properties of the classical field depend strongly on the value of the cut-off parameter \( k_c \) and diverge in the continuum limit \( (k_c \to \infty) \). In a correct semi-classical treatment of the soft modes the hard modes thus cannot be neglected, but it should incorporate their influence in a consistent way.

In a recent paper\(^{12}\) it was shown how to construct an effective semi-classical action for describing not only the classical behavior of the long wavelength modes below some given cutoff \( k_c \), but taking into account also perturbatively the interaction among the soft and hard modes. Also for studying non-equilibrium dynamics the separation of soft and hard contributions is of importance. The resulting effective action\(^{12,13}\). \( S_{\text{eff}}[\phi] \) turns out to be complex, leading to a stochastic equation of motion for the soft modes. If the hard modes are already in thermal equilibrium then the evolution of the soft modes is described by a set of generalized Langevin equation - the equation of motion corresponding to the above complex effective action.

We briefly sketch the above following\(^{12}\) by considering a scalar field with interaction \( \mathcal{L}_{\text{int}} = \frac{g^2}{4!} \phi^4 \). The splitting of the Fourier-components, \( \phi(p,t) = \phi(p < k_c,t) + \phi(p > k_c,t) \), leads to the following interaction part in the action

\[
S_{\text{int}}[\phi, \varphi] = -\int_{t_0}^{t} d^4x \left( \frac{g^2}{4!} \phi^4 + \frac{g^2}{3!} \left( \phi^3 \varphi + \frac{3}{2} \phi^2 \varphi^2 + \phi \varphi^3 \right) \right). \tag{1}
\]
By integrating out the ‘influence’ of the hard modes on the two-loop level, one notices that Feynman graphs contributing at order $O(g^4)$ to the effective action contain imaginary contributions. The stochastic equation of motion has the general shape

$$\Box \phi + \bar{m}^2 \phi + \frac{g^2}{3!} \phi^3 + \frac{3}{(2N-1)!} \phi^{N-1} (\text{Re} \Gamma_{2N}) \phi^N = \sum_{N=1}^{3} \phi^{N-1} \xi_N. \quad (2)$$

Here $\Gamma_{2N}$ denotes the effective contribution with $2N$ soft legs, $\bar{m}^2$ the re-summed Hartree-Fock self energy (cactus graphs) and $\xi_N$ are associated noise variables with a correlation $\langle \xi_N \xi_N' \rangle = \text{Im} \Gamma_{2N}$. When the characteristic time scale in the heat-bath of hard modes is short compared to the evolution of the soft order parameter fields, then the Markovian limit has a form

$$\frac{\partial^2 \phi}{\partial t^2} + (k^2 + \bar{m}_{\text{eff}}^2) \phi + \left( \frac{\bar{g}_{\text{eff}}^2}{6} \right) \phi^3 + \mu_{\text{eff}} \phi^5 + \sum_{N=1,2,3} \eta^{(N)} \cdot \phi^N \approx \sum_{N=1}^{3} \xi_N \phi^{N-1}. \quad (3)$$

This leads to the familiar form of plasmon damping ('$\eta$') and to momentum dependent mass corrections. The hard modes act as an environmental heat bath. They also guarantee that the soft modes become, on average, thermally populated with the same temperature as the heat bath. The noise terms are related to the plasmon damping in the Markovian approximation via the simple fluctuation dissipation relation

$$\langle \xi(t) \xi(t') \rangle = 2TV \eta \delta(t-t') \quad (4)$$

in the semiclassical (high temperature) limit.

3 Langevin description of linear sigma model

For the description of the evolution of collective pion and sigma fields in the $O(4)$-model we utilize the following simplified equations of motion (taking $k_c \to 0$) for the order parameters $\Phi_a = \frac{1}{V} \int d^3x \phi_a(x,t)$ in a volume $V$:

$$\ddot{\Phi}_0 + \left( \frac{D}{\tau} + \eta \right) \dot{\Phi}_0 + m_\pi^2 \Phi_0 = f_\pi m_\pi^2 + \xi_0,$$
$$\ddot{\Phi}_i + \left( \frac{D}{\tau} + \eta \right) \dot{\Phi}_i + m_\sigma^2 \Phi_i = \xi_i, \quad (5)$$
with $\Phi_0 = \sigma$ and $\Phi_i = (\pi_1, \pi_2, \pi_3)$ being the chiral meson fields and

$$m_T^2 = \lambda \left( \Phi_0^2 + \sum_i \Phi_i^2 + 1 \over 2 T^2 - f_\pi^2 \right) + m^2_\pi,$$

$$m_L^2 = m_T^2 + 2 \lambda \left( \Phi_0^2 + \sum_i \Phi_i^2 \right),$$

the effective masses. These coupled Langevin equations resemble in its structure a Ginzburg-Landau description of phase transition.

Here ”dots” denote the time derivative with respect to proper time $\tau$, $\eta$ the effective damping coefficient and $D/\tau$ is the Raleigh-damping corresponding to a $D$-dimensional scaling flow of the fireball. The later assumed (rapid) expansion and cooling results in time dependent temperature and volume according to

$$\frac{\dot{T}}{T} + \frac{D}{3\tau} = 0, \quad \frac{\dot{V}}{V} - \frac{D}{\tau} = 0.$$

For the friction coefficient $\eta$ of the $\sigma$ and pion field we take the on-shell plasmon damping rate for standard $\phi^4$-theory generalized to the O(4)-model:

$$\eta = 2 \gamma_{pl} = \frac{9}{16\pi^3} \lambda^2 T^2 \frac{m}{f_{Sp}} (1 - e^{-f_\pi}) ,$$

where $f_{Sp}(x) = - \int_1^x dt \frac{\ln t}{t}$ defines the Spence function. We treat the dissipation term in a first step as Markovian (mainly because of numerical implementation). Furthermore we take $m = m_\pi = 140$ MeV for specifying the damping coefficient $\eta$ in the following. (We remark that both rather crude approximations can be relaxed.) In the semiclassical Markovian approximation the noise is effectively white and at two-loop level it is Gaussian and thus follows a similar relation like (4).

Of course, semi-classical Langevin equations may not hold for a strongly interacting theory as for highly non trivial dispersion relations the frequencies of the long wavelength modes are not necessarily much smaller than the temperature. Still, when the soft modes become tremendously populated one can argue that the long wavelength modes being coherently amplified behave classically. Aside from a theoretical justification one can regard the Langevin equation as a practical tool to study the effect of thermalization on a sub-system, to sample a large set of possible trajectories in the evolution, and to address also the question of all thermodynamically possible initial configurations in a systematic manner.
The ‘Brownian’ motion of the soft field configuration leads to equipartition of the energy at constant temperature. In fig. 1 we show the effective transversal masses $m_T$ of the pion modes and $m_L$ of the $\sigma$ mode as a function of the temperature obtained with solving eqs. (5) for a static equilibrium system (i.e. $D = 0$) at fixed temperature $T$ and sufficiently large volume $V$. The masses shown are thus taken as an ensemble average of the different realizations of the Langevin scheme. For large volumes the fluctuations in the obtained masses are of the order $1/V$ and thus small. For the situation that the vacuum pion mass is assumed to be zero (no explicit symmetry breaking) one can realize from fig. 1 the situation for a true second order phase transition occurring at the transition temperature $T = T_c = \sqrt{2f_\pi^2 - 2m^2_\pi}/\lambda \approx 125$ MeV. On the other hand for the physical situation of a nonvanishing pion mass of $m_\pi = 140$ MeV the ‘phase transition’ resembles more the form of a smooth crossover. In this case, at $T \approx T_c$, the $\sigma$-field still possesses a nonvanishing value of $\langle \sigma(T \approx T_c) \rangle \approx f_\pi/2 \approx \sigma_{vac}/2$. 

Figure 1: The order parameter $\sigma$, the transverse (eff. 'pion') mass $m_T$ and the longitudinal (eff. 'sigma') mass as a function of temperature $T$ for an equilibrium situation for physical vacuum pion mass and for vanishing pion mass.
4 Stochastic DCC

In ref. 11 we had studied the average and statistical properties of individual solutions of the above Langevin equations with the emphasis on such periods of the time evolution when the transverse mass $m_T$ becomes imaginary and therefore an exponential growth of unstable fluctuations in the collective fields might be expected. We had found that in different realistic initial volumes individual events lead to sometimes significant growth of fluctuations.

\[
\begin{align*}
\text{Figure 2: The pion number distribution in four different DCC scenarios.}
\end{align*}
\]

Here we want to show now the distribution in the number of produced long wavelength pions $N_\pi$ out of the evolving chiral pion order fields within the DCC domain $V(\tau)$. Within the semiclassical interpretation these can approximately be written as

\[
N_\pi^{\text{zeromode}} \approx \frac{1}{2} m_\pi \left( \vec{\pi}^2 + \frac{1}{m_\pi^2} \vec{\pi}^2 \right) V(\tau). \tag{10}
\]

As a rough estimate one can think of the quench energy density $\lambda^4 f_\pi^4$ being transformed into low momentum pions yielding $N_\pi/V \approx \frac{1}{2} \frac{f_\pi^4}{m_\pi^2} \approx 0.3 \text{ fm}^{-3}$. 

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On the other hand one might also estimate that the chiral order field ‘circles’ around with \( \langle \vec{\pi}^2 + \frac{1}{m_\pi^2} \vec{\pi}^2 \rangle \leq f_\pi^2 \) resulting in \( N_\pi / V \approx f_\pi^2 / (2m_\pi) \approx 0.08 \, fm^{-3} \).

For the total pion number the crucial question is then how large has the evolving volume \( V(\tau) \) of the DCC domain increased when the pion oscillations have emerged after the roll down period.

In the following we present results for four different DCC scenarios:

- **Normal** or annealing scenario: The system is preheated at \( T = T_c \) for sufficiently long time in order to cover a complete set of possible initial thermal conditions using its own dynamics and then switch over to a D-dimensional scaling expansion.

- **Quench** scenario: As for **Normal**, however, after preheating and thus during the expansion the term \( \lambda / 2T^2 \) of the potential in the equations of motion is being omitted in order to mimique an abrupt occurrence of the zero temperature vacuum potential.

- **\( m_\pi^{eff} \approx 0 \)** scenario: As for **Normal**, however, the preheating at \( T = T_c \) is carried out with \( m_\pi = 0 \) (comp. fig. 1) in order to generate initial conditions which are centered around the origin \( \sigma = \vec{\pi} = 0 \).

- **Quench** and **\( m_\pi^{eff} \approx 0 \)** scenario: Like the previous, however, the \( \lambda / 2T^2 \) term of the potential in the equations of motion is being omitted.

One might argue that the use of the initial conditions prepared within the \( m_\pi^{eff} \approx 0 \) scenario is inconsistent within the linear sigma model with a physical pion mass. From fig. 1 one notices that then the phase transition resembles a smooth crossover. From lattice calculations, however, one knows that the chiral transitions happens much sharper within a very narrow window close at \( T = T_c \). This means that it might very well be that the order parameter will fluctuate around zero slightly above the critical temperature.

In fig. 2 we depict the distribution of produced pions within \( 10^4 \) events within the four different DCC scenarios. As parameters we had taken \( D = 3 \), \( \tau_0 = 7 \, fm/c \) and \( V(\tau_0) = 100 \, fm^3 \). A comparison of annealing and quench scenarios, both with finite and vanishing pion mass (for generating the initial conditions) reveals that the most productive DCC events would lead to a few (6-8 in annealing scenario with finite pion mass), to a moderate number (20 - 40 in annealing scenario with massless pions and quench with massive pions) or to about 120 pions (in quench scenario with initial conditions generated by massless pions), respectively. The final distribution does not follow a usual Poissonian distribution. Instead, fluctuations with a large number of produced pions are still likely with some finite probability! In principle, an ensemble averaged description of potential DCC formation carried out within the mean
field approximation, as presented in the various literature, cannot account for such fluctuations and thus has to fail at some point. Also the isospin ratio signal in the latter three scenarios is close to that of expected for a DCC event. These findings support the idea of looking for DCC formation in individual events.

In fig. 3 we depict the individual trajectories of the best candidates. Especially for the first scenario one recognizes the fact that the initial conditions for the order parameters are centered around $\sigma \approx f_\pi/2$ (comp. with fig. 1) and do not allow for initial fluctuations in the backward hemisphere with $\sigma < 0$. In any case, all these trajectories clearly illustrate that the collective pion field in the late stage of the evolution within the scaling expansion oscillates around the ‘chiral’ circle $\sigma^2 + \bar{\sigma}^2 \approx f_\pi^2$ ($f_\pi = 93$ MeV) as expected for DCC formation.
5 Summary and Conclusion

In summary we want to remark that within the presented Langevin description one can simulate, on an event by event analysis, the possible evolution of various DCC scenarios in a rather transparent form. It has been demonstrated that although the average evolution of the order parameter field is damped, there are individual events which show pionic instabilities and appreciable pion yields. One can repeat the presented calculations for different sets of parameters (the expansion dimension $D$, the scaling parameter of initial time $\tau_0$ and the initial volume $V(\tau_0)$). In addition, one can also start to describe the evolution with initial conditions generated at much higher initial temperatures $T \gg T_c$ as carried out by Randrup. We will leave this for future work.

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