Analytic Amplitudes for Hadronic Forward Scattering and the Heisenberg $\ln^2 s$ Behaviour of Total Cross Sections

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Abstract

The $\ln^2 s$ behaviour of total cross sections, first obtained by Heisenberg 50 years ago, receives now increased interest both on phenomenological and theoretical levels. We present a modification of the Heisenberg’s model in connection with the presence of glueballs and we show that it leads to a realistic description of all existing hadron total cross-sections data, in agreement with the COMPETE analysis.

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1 The COMPETE analysis of forward data

Analytic parametrizations of forward (\(t = 0\)) hadron scattering amplitudes is a well-established domain in strong interactions.

However, in the past, the phenomenology of forward scattering had quite a high degree of arbitrariness: i) An excessive focus on \(pp\) and \(\bar{p}p\) scattering; ii) Important physical constraints are often mixed with less general or even ad-hoc properties; iii) The cut-off in energy, defining the region of applicability of the high-energy models, differs from one author to the other; iv) The set of data considered by different authors is sometimes not the same; v) No rigorous connection is made between the number of parameters and the number of data points; vi) No attention was paid to the necessity of the stability of parameter values; vii) The experiments were performed in the past in quite a chaotic way: huge gaps are sometimes present between low-energy and high-energy domains or inside the high-energy domain itself.

The COMPETE (COMputerized Models and Parameter Evaluation for Theory and Experiment) project tries to cure as much as possible the above discussed arbitrariness. The \(\chi^2/dof\) criterium is not able, by itself, to cure the above difficulties: new indicators have to be defined. Once these indicators are defined\([1]\), an appropriate sum of their numerical values is proposed in order to establish the rank of the model under study: the highest the numerical value of the rank the better the model under consideration.

The final aim of the COMPETE project is to provide our community with a periodic cross assessments of data and models via computer-readable files on the Web\([2]\).

We consider the following exemplar cases of the imaginary part of the scattering amplitudes:

\[
ImF^{ab} = s\sigma_{ab} = P_1^{ab} + P_2^{ab} + R_+^{ab} \pm R_-^{ab} \tag{1}
\]

where:
- the \(\pm\) sign in formula \(1\) corresponds to antiparticle (resp. particle) - particle scattering amplitude.
- \(R_\pm\) signify the effective secondary-Reggeon (\((f, a_2), (\rho, \omega)\)) contributions to the even (odd)-under-crossing amplitude

\[
R_\pm(s) = Y_\pm \left( \frac{s}{s_1} \right)^{\alpha_\pm}, \tag{2}
\]

where \(Y\) is a constant residue, \(\alpha\) - the reggeon intercept and \(s_1\) - a scale factor fixed at 1 GeV\(^2\);
- \(P_1(s)\) is the contribution of the Pomeron Regge pole

\[
P_1^{ab}(s) = C_1^{ab} \left( \frac{s}{s_1} \right)^{\alpha_{P_1}}, \tag{3}
\]

\(\alpha_{P_1}\) is the Pomeron intercept \(\alpha_{P_1} = 1\), and \(C_1^{ab}\) are constant residues.
- \(P_2^{ab}(s)\) is the second component of the Pomeron corresponding to three different \(J\)-plane singularities:

a) a Regge simple - pole contribution

\[
P_2^{ab}(s) = C_2^{ab} \left( \frac{s}{s_1} \right)^{\alpha_{P_2}}, \tag{4}
\]

with \(\alpha_{P_2} = 1 + \epsilon, \epsilon > 0\), and \(C_2^{ab}\) const. ;
b) a Regge double-pole contribution

\[ P_{2ab}^{ab}(s) = s \left[ A^{ab} + B^{ab} \ln \left( \frac{s}{s_1} \right) \right], \]  

(5)

with \( A^{ab} \) and \( B^{ab} \) const.;

c) a Regge triple-pole contribution

\[ P_{2ab}^{ab}(s) = s \left[ A^{ab} + B^{ab} \ln^2 \left( \frac{s}{s_0} \right) \right], \]  

(6)

where \( A^{ab} \) and \( B^{ab} \) are constants and \( s_0 \) is an arbitrary scale factor.

We consider all the existing forward data for \( pp, \bar{p}p, \pi p, Kp, \gamma \gamma \) and \( \Sigma p \) scatterings. The number of data points is: 904, 742, 648, 569, 498, 453, 397, 329 when the cut-off in energy is 3, 4, 5, 6, 7, 8, 9, 10 GeV respectively. A large number of variants were studied and classified. All definitions and numerical details can be found in Ref. 1.

The 2-component Pomeron classes of models are RRPE, RRPL and RRPL2, where by RR we denote the two effective secondary-reggeon contributions, by P - the contribution of the Pomeron Regge-pole located at \( J = 1 \), by E - the contribution of the Pomeron Regge-pole located at \( J = 1 + \epsilon \), by L - the contribution of the component of the Pomeron, located at \( J = 1 \) (double pole), and by L2 - the contribution of the component of the Pomeron located at \( J = 1 \) (triple pole). We also studied the 1-component Pomeron classes of models RRE, RRL and RRL2.

The highest rank are get by the RRPL2u models (see Table 1 and Figure 1), corresponding to the \( \ln^2 s \) behaviour of total cross sections first proposed by Heisenberg 50 years ago.

The \( u \) index denotes the universality property: the coupling \( B \) of the \( \ln^2 s \) term is the same in all hadron-hadron scatterings and \( s_0 \) is the same in all reactions. We note that the coupling \( B \) is remarkably stable for the different models: 0.3157 mb (RRPL2u(19)), 0.3152 mb (RRPL2u(21)), 0.3117 mb (RRPL2u(15)), etc. This reinforces the validity of the universality property.

We note also that the familiar RRE Donachie-Landshoff model is rejected at the 98% C.L. when models which achieve a \( \chi^2/dof \) less than 1 for \( \sqrt{s} \geq 5 \text{ GeV} \) are considered.

The predictions of the best RRPL2u model, adjusted for \( \sqrt{s} \geq 5 \text{ GeV} \), are given in Tables 2-4.

The uncertainties on total cross sections, including the systematic errors due to contradictory data points from FNAL (the CDF and E710/E811 experiments, respectively), can reach 1.9% at RHIC, 3.1% at the Tevatron, and 4.8% at the LHC, whereas those on the \( \rho \) parameter are respectively 5.4%, 5.2%, and 5.4%. The global picture emerging from fits to all data on forward observables supports the CDF data and disfavors the E710/E811 data at \( \sqrt{s} = 1.8 \text{ TeV} \).

Any significant deviation from the predictions based on model RRPL2u will lead to a re-evaluation of the hierarchy of models and presumably change the preferred parametrisation to another one. A deviation from the “allowed region” would be an indication that strong interactions demand a generalization of the analytic models discussed so far, e.g. by adding Odderon terms, or new Pomeron terms, as suggested by QCD.
2 A generalization of the Heisenberg model for the total cross-section

In his remarkable paper of 1952, Heisenberg investigated production of mesons as a problem of shock waves. One of his results was that the total cross section increases like the square of the logarithm of the centre-of-mass energy. It is noteworthy that this result coincides with very recent calculations based on AdS/CFT dual string-gravity theory \[5\] or on the Colour Glass Condensate Approach \[6\] and, of course, saturates the Froissart-Martin bound \[7\]. In contradistinction to the latter case however the coefficient of the \(\ln^2\) term is an estimate at finite energies and not an asymptotic bound as the one obtained by Lukaszuk and Martin \[8\].

We have shown that \[9\] by modifications of the original model of Heisenberg motivated by the enormous progress of knowledge in the 50 years that passed thence, the model yields some general and even some quantitative results which describe the data very well.

The considerations of Heisenberg concerning the total cross section are essentially geometrical ones, but the crucial ingredient is that the energy density and not the hadronic density is the essential quantity to be taken into account.

Proton-proton collisions are considered in the centre-of-mass system and the energy \(\sqrt{s}\) is supposed to be high enough that Lorentz contraction allows to view the nucleons as discs.

Interaction takes place only in the overlap region and the crucial assumption is made that a reaction can only occur if the energy density is high enough in order to create at least a meson pair.

The result of Heisenberg is

\[
\sigma = \frac{\pi}{m^2} \ln^2 \frac{\sqrt{s}}{k_0}.
\]  

(7)

We see that implicitly the assumption has been made that if a meson production is energetically possible, it will happen (black disk). Of course, Heisenberg was taking the pion mass for the meson mass. For the energy of the produced mesons he deduced, in his dynamical considerations, assuming interactions of maximal strength, that the energy \(k_0\) (for two produced mesons) increases only slowly with energy, at any rate not by a power of \(s\). Therefore the asymptotically leading term in the cross section is \((\pi/4m^2)\ln^2 s\), the coefficient \(\pi/4m^2\) being 1/4 of the Lukaszuk-Martin bound. The argument can be extended easily to hadron-hadron scattering in general, and therefore we have the result that the coefficient of the \(\ln^2 s\) term is universal for all hadron reactions.

There are two obvious necessary modifications of the Heisenberg model:

1) If we want to apply it to all kind of hadrons, we have to take care of the different hadron sizes, since in the above treatment all sizes are equal to \(1/m\).

2) We have to take into account that direct pion exchange, though being the exchange with the lightest particle, is not relevant at high energies. This is due to the fact that exchanged gluons have spin 1 and pions spin 0. Therefore already in Born approximation gluon exchange dominates at high energies. In Regge theory this is manifested by the fact that intercept of the pion is much lower than that of the Pomeron. For the mass we rather insert a mass \(M\) in the range of the glueball
mass instead of the pion mass \( m \), since we believe that the high-energy behavior is dominated by gluon exchange.

We then obtain

\[
\sigma = \frac{\pi}{4M^2} \ln^2 s \\
+ \frac{\pi}{M} \ln s \left\{ (R_1 + R_2) + \frac{1}{M} \ln \frac{\alpha}{k_0} \right\} \\
+ \pi(R_1 + R_2)^2 + \frac{\pi}{M^2} \ln^2 \frac{\alpha}{k_0} \\
+ \frac{2\pi}{M}(R_1 + R_2) \ln \frac{\alpha}{k_0},
\]

where \( 0 \leq \alpha \leq 1 \).

We see that the leading \( \ln^2 s \) term is still universal, but now dominated rather by a glueball than by the pion mass. Since \( R_1 \) and \( R_2 \) are supposed to be of the size of the electromagnetic radii, the second term in Eq. (8) will dominate over the \( \frac{\pi}{M^2} \) term except at high energies, \( s \gg k_0^2/\alpha^2 \).

In order to perform a rough numerical estimate, we may insert for the glueball mass a value between 1.4 and 1.7 GeV, yielding

\[
\frac{\pi}{4M^2} = 0.11 - 0.16 \text{ mb.}
\]

For \( R_1 \) and \( R_2 \) we may insert the electromagnetic radii. In contrast to Heisenberg, we insert for \( k_0 \) the minimal energy of two produced particles. Since production seems to occur in clusters with mass around 1.3 GeV \( [10] \), we can put \( k_0 = 2.6 \) GeV. The value of \( \alpha \) (\( 0 \leq \alpha \leq 1 \)) might be process dependent. For very small objects ("onia") it might be very small.

In the past, application of the Heisenberg model to the global analyses of the forward hadronic data were performed in [11], but the universality of the leading term was not discussed there. This universality was treated by Gershtein and Logunov [12], who made the assumption, as in the present paper, that the growth of the hadron-hadron total cross-sections is related to resonance production of glueballs.

The COMPETE value of \( B \) (see the previous section) corresponds to a mass \( M \) of 1 GeV, a bit small for a glueball, but not unreasonable given the crude approximations. \( Z^{HH} \) (where \( Z^{HH} = C_1^{HH} + A^{HH} \)) are in the right order of magnitude of \( R^2 \).

A consequence of the universal \( \ln^2 s \) term is that at asymptotic energies all hadron cross sections become equal. At finite but high energies the pion and kaon proton cross sections are therefore expected to rise somewhat faster than the nucleon-nucleon cross sections. This seems indeed to be indicated by the data.

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Figure 1: Model RRPL2u(19).
Table 1: Ranking of the 23 models having nonzero area of applicability. The number between parenthesis (Model Code column) denotes the number of free parameters.

| Model Code       | Rank |
|------------------|------|
| RRPL2_u(19)      | 234  |
| RRP_n_f L2_u(21) | 221  |
| (RR_c)^d PL2_u(15)| 216  |
| RRL_n_f(19)      | 214  |
| (RR)^d P_n_f L2_u(19)| 206  |
| R^p L^p|R_c(12)    | 194  |
| (RR)^d PL2_u(17) | 189  |
| (RR_c)^d P^p L2_u(14)| 185  |
| (RR)^d P_n_f L2(20)| 182  |
| (RR)^d P^p L2_u(16)| 180  |
| RR_n L^p(15)    | 170  |
| RRL^p(17)       | 164  |
| RR_c L^p(15)    | 159  |
| RRPL(21)        | 155  |
| RR{PL2}^p(18)   | 155  |
| R^p L^p|R(14)     | 154  |
| RRL^p(17)       | 153  |
| RR{PL2}(20)     | 152  |
| R^p L^p|R_n(12) | 170  |
| RRPE_u(19)      | 146  |
| RR_c PL(19)     | 144  |
| RRL2(18)        | 143  |
| RRL2(18)        | 142  |

Table 2: Predictions for $\sigma_{tot}$ and $\rho$, for $\bar{p}p$ (at $\sqrt{s} = 1960$ GeV) and for $pp$ (all other energies). The central values and statistical errors correspond to the preferred model RRPL2_u.

| $\sqrt{s}$ (GeV) | $\sigma$ (mb) | $\rho$    |
|------------------|---------------|-----------|
| 100              | 46.37 ± 0.06  | 0.1058 ± 0.0012 |
| 200              | 51.76 ± 0.12  | 0.1275 ± 0.0015 |
| 300              | 55.50 ± 0.17  | 0.1352 ± 0.0016 |
| 400              | 58.41 ± 0.21  | 0.1391 ± 0.0017 |
| 500              | 60.82 ± 0.25  | 0.1413 ± 0.0017 |
| 600              | 62.87 ± 0.28  | 0.1416 ± 0.0018 |
| 1960             | 78.27 ± 0.55  | 0.1450 ± 0.0018 |
| 10000            | 105.1 ± 1.1   | 0.1382 ± 0.0016 |
| 12000            | 108.5 ± 1.2   | 0.1371 ± 0.0015 |
| 14000            | 111.5 ± 1.2   | 0.1361 ± 0.0015 |
Table 3: Predictions for $\sigma_{\text{tot}}$ for $\gamma p \rightarrow \text{hadrons}$ for cosmic-ray photons. The central values and the statistical errors are as in Table 2.

| $p_{\text{lab}}^\gamma$ (GeV) | $\sigma$ (mb) |
|-------------------------------|--------------|
| $0.5 \cdot 10^6$              | 0.243 ± 0.009|
| $1.0 \cdot 10^6$              | 0.262 ± 0.010|
| $0.5 \cdot 10^7$              | 0.311 ± 0.014|
| $1.0 \cdot 10^7$              | 0.333 ± 0.016|
| $1.0 \cdot 10^8$              | 0.418 ± 0.022|
| $1.0 \cdot 10^9$              | 0.516 ± 0.029|

Table 4: Predictions for $\sigma_{\text{tot}}$ for $\gamma \gamma \rightarrow \text{hadrons}$. The central values and the statistical errors are as in Table 2.

| $\sqrt{s}$ (GeV) | $\sigma$ ($\mu$ b) |
|------------------|-------------------|
| 200              | 0.546 ± 0.027     |
| 300              | 0.610 ± 0.035     |
| 400              | 0.659 ± 0.042     |
| 500              | 0.700 ± 0.047     |
| 1000             | 0.840 ± 0.067     |