We propose using the lightest CP-even Higgs boson in the minimal supersymmetric standard model (MSSM) to probe the stop sector. Unlike measuring stop masses in production/decay processes which requires knowledge of masses and mixing angles of other superparticles, our strategy depends little on supersymmetric parameters other than those in the stop sector in a large region of the parameter space. We show that measurements of the Higgs mass and the production rate in the gluon fusion channel, the dominant channel at the LHC, allow for determination of two parameters in the stop mass-squared matrix, including the off-diagonal mixing term. This proposal is very effective when stops are light and their mixing is large, which coincides with the region where the electroweak symmetry breaking is minimally fine tuned. We also argue that a lightest CP-even Higgs mass in the upper range of allowed values and a production rate significantly smaller than the rate predicted in the standard model would be difficult to reconcile within the MSSM, except in extreme corners of the parameter space.
I. INTRODUCTION

Supersymmetry (SUSY) is usually considered the leading candidate for physics beyond the standard model. Among many virtues of SUSY, perhaps the most prominent ones are the stabilization of the electroweak scale up to very high energies such as the grand unification scale and the possibility of radiatively driven electroweak symmetry breaking (EWSB). However, neither the Higgs boson nor any superpartners have been found in collider experiments so far, and it is discomforting to realize that majority of natural parameter space of MSSM has been ruled out by current experimental limits on the Higgs mass \[1\], leaving us with the parameter space where EWSB is achieved with fine tuning of soft SUSY breaking parameters at a few percent level. \(^1\)

The EWSB and the mass of the Higgs boson in the MSSM are tightly connected with the stop sector: stop mass squared parameters, \(m_{\tilde{t}_L}^2\) and \(m_{\tilde{t}_R}^2\), and the mixing, \(X_t = A_t - \mu / \tan \beta\), where \(A_t\) is the top soft trilinear coupling, \(\mu\) is the supersymmetric Higgs mass and \(\tan \beta = v_u/v_d\) is the ratio of the vacuum expectation values of up-type and down-type Higgs bosons. These parameters enter the calculation of physical stop masses, \(m_{\tilde{t}_1}\) and \(m_{\tilde{t}_2}\), which is what we measure in experiments. Information about the mixing is not given from mass eigenstates. The splitting between \(m_{\tilde{t}_1}\) and \(m_{\tilde{t}_2}\) can originate either from the difference between \(m_{\tilde{t}_L}^2\) and \(m_{\tilde{t}_R}^2\) or from large mixing. However, the mixing in the stop sector is crucial for the Higgs boson mass. In the MSSM the mass of the lightest CP-even Higgs boson is bounded at the tree-level by the Z boson mass, \(m_h \leq m_Z |\cos 2\beta|\). In order to lift the Higgs mass above the LEP limit \(m_h \geq 114\) GeV, radiative corrections from stops are required to be large, which then implies either stop masses heavier than about 900 GeV for moderate mixing, or large stop mixing for fairly light stop masses. Indeed the region of large mixing, \(X_t/m_{\tilde{t}_L,R} \simeq \pm 2\), and stop masses \(m_{\tilde{t}_L} \simeq m_{\tilde{t}_R} \simeq 300\) GeV minimizes the fine tuning of EWSB while satisfying the limit on the Higgs mass. There has been some effort to realize the large mixing scenario in models, see e.g. Refs. \[3, 4, 5, 6\], in order to address the naturalness issue of the MSSM. It goes without saying that determining parameters of the stop sector precisely in collider experiments will be of great importance for understanding the EWSB, the Higgs mass and the internal consistency of the MSSM. \(^2\)

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\(^1\) For recent discussion of fine tuning in EWSB see e.g. Refs. \[2, 3, 4, 5, 6\].

\(^2\) It is important to note that the discussion in this paragraph is specific to the MSSM. In models with more
So how does one measure stop masses and the mixing angle? This is a simple question without a simple answer. In the MSSM with R-parity the lightest supersymmetric particle (LSP) is stable. In a large class of models the lightest neutralino is (or can be) the LSP and a good candidate for dark matter. In collider experiments the lightest neutralino (being stable and electrically neutral) will escape direct detection and result in events with missing transverse energy ($E_T$). Due to R-parity superparticles need to be pair-produced and they eventually cascade-decay into the LSP plus standard model particles. Thus a typical event for the production and decay of superparticles is multi-jet and multi-lepton with large missing $E_T$. In the end the stop, if produced, is never directly observed in collider detectors. Any reconstruction of masses and mixing angle in the production/decay processes has to rely on the visible decay product and missing $E_T$.

At the Large Hadron Collider (LHC) various reasons complicate the measurement of masses and mixing angle in the production/decay process. First, the large missing $E_T$ makes event-by-event reconstruction of masses impossible; one has to resort to measuring kinematic endpoints and edges of invariant mass distributions of final particles. The position of such endpoints and edges is sensitive to masses of all particles involved in the decay chain, including the LSP which escapes detection. Second, at hadron colliders it is the partons inside the proton that collide with each other and the center-of-mass energy is not a known quantity. Thus there is no kinematic constraint to impose in the longitudinal direction of the collider. Third, because of long decay chains of SUSY particles there are usually many jets and leptons in the final state, leading to large combinatoric factors. Previous studies showed that in the end it is quite a complicated and elaborate analysis to extract mass parameters in the production/decay processes, and the outcome crucially depends on knowing the mass and nature of other particles in the decay chain such as charginos and neutralinos. For stops, there is an added layer of complexity because decays of stops sometimes involve top quarks, which require extra efforts to identify.

In this paper we propose an approach, complementary to studying the production/decay processes of stops, that does not require prior knowledge of masses and mixing angles of other superparticles. The proposal is to use properties of the lightest CP-even Higgs boson complicated Higgs sector, the mass of the Higgs boson can receive additional contributions or the 114 GeV limit on the Higgs mass might not apply due to modified Higgs decays. See e.g. Ref. for related discussion.
in the MSSM to extract parameters in the stop sector. At the LHC the Higgs boson is produced dominantly through the gluon fusion process $gg \rightarrow h$ and subsequently decays into other standard model particles. By measuring the invariant mass of the decay products it is possible to determine the Higgs mass precisely. As it turns out in the MSSM both the gluon fusion production rate and the Higgs mass are sensitive only to parameters in the stop sector and not to masses and mixing of other superpartners. The only exception to this is the large $\tan\beta$ region where contributions from sbottom sector to both the Higgs mass and the gluon fusion rate can be significant. Furthermore, we will demonstrate that, if the variables are chosen appropriately, the Higgs mass and the gluon fusion rate depend on only two out of the three parameters in the mass matrix; the dependence on the third parameter is negligible in a significant region of the parameter space. Therefore with two measurements in the Higgs sector we are able to extract two parameters in the stop mass-squared matrix, including the mixing term $X_t$.

There is also an interesting possibility that the two measurements (the Higgs mass and the gluon production rate) would point to mutually inconsistent values of stop masses and mixing, even after taking into account the current (large) estimates of experimental and theoretical errors. This is the case for a large Higgs mass $m_h \gtrsim 130$ GeV and a significantly reduced production rate in the gluon fusion channel. Even though, taken separately, these two measurements are perfectly allowed in the MSSM, we will argue that the combined scenario is very difficult to reconcile except in some extreme (insane) corners of the parameter space. Finally, in every SUSY breaking scenario in which $m_{\tilde{t}_L}^2$ and $m_{\tilde{t}_R}^2$ are related to each other in any specific way, and, in addition, parameters in the sbottom sector are related to parameters in the stop sector, our procedure can be used to fix the parameters of the model, or it could possibly rule out the scenario if the measured value of the Higgs mass and the gluon production rate are impossible to satisfy for any choice of parameters.

II. THE STOP SECTOR IN MSSM

The stop mass-squared matrix in the MSSM in the flavor basis ($\tilde{t}_L, \tilde{t}_R$) is given by

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{\tilde{t}_L}^2 + m_{\tilde{t}}^2 + D_L^t & m_t X_t \\ m_t X_t & m_{\tilde{t}_R}^2 + m_{\tilde{t}}^2 + D_R^t \end{pmatrix},$$

(1)
where

\[ D'_L = \left( \frac{1}{2} - \frac{2}{3}s_w^2 \right) m_Z^2 \cos 2\beta, \]  
(2)

\[ D'_R = \frac{2}{3}s_w^2 m_Z^2 \cos 2\beta, \]  
(3)

\[ X_t = A_t - \frac{\mu}{\tan \beta}. \]  
(4)

In the above, \( s_w \) is the sine of Weinberg angle. From Eq. (1) we see that there are four free parameters in the stop mass matrix: \( \tan \beta \) (through the dependence on \( \cos 2\beta \)), \( m_{\tilde{t}_L}^2 \), \( m_{\tilde{t}_R}^2 \), and \( X_t \). Nevertheless, the dependence on \( \tan \beta \) is rather weak because \( m_Z^2 \ll m_t^2 \).

Furthermore, the mass of the lightest CP-even Higgs boson in MSSM is insensitive to \( \tan \beta \) once \( \tan \beta \gtrsim 10 \), in which case \( \cos 2\beta \sim 1 \). In this way neither the Higgs mass nor the stop mass-squared matrix is dependent on \( \tan \beta \). On the other hand, the off-diagonal mixing in the sbottom mass matrix,

\[ m_b X_b = m_b (A_b - \mu \tan \beta), \]  
(5)

becomes substantial when \( \tan \beta \sim m_t/m_b \) and the supersymmetric Higgs mass \( \mu \) is large simultaneously. In this situation the sbottom contribution to both the Higgs mass and the production rate in the gluon fusion channel could be significant \([9, 10]\). Therefore the region of parameter space we would like to focus on in this paper is:

\begin{itemize}
  \item \( 10 \lesssim \tan \beta \lesssim m_t/m_b \),
  \item \( |m_b \mu \tan \beta| \lesssim m_{\tilde{b}_L}^2, m_{\tilde{b}_R}^2 \),
\end{itemize}

for which our strategy will not depend on SUSY parameters other than those in the stop sector. In this case the stop mass matrix is controlled by three parameters \( m_{\tilde{t}_L}^2 \), \( m_{\tilde{t}_R}^2 \), and \( X_t \). In addition we will be interested in the so-called “decoupling limit” \([11]\), in which the lightest CP-even Higgs, \( h \), is standard model-like in that its couplings to quarks and leptons approach the standard model values, and all other Higgs bosons in the MSSM are much heavier than \( h \) and roughly degenerate.

### III. STOPS AND THE GLUON FUSION PRODUCTION

At hadron colliders the dominant production mechanism for the Higgs boson is the gluon fusion production \([12, 13, 14]\). The contributing Feynman diagram is shown in Fig. 1.
which it is the top quark running in the loop. The gluon fusion production, being a loop induced process, is very sensitive to new physics, especially to any new colored particle which couples to the Higgs significantly. In the MSSM there is only one such particle, the stop, whereas all the other colored superparticles have much smaller coupling to the lightest CP-even Higgs due to small Yukawa couplings. Therefore in the MSSM the gluon fusion production rate of the lightest CP-even Higgs boson probes the stop sector and is insensitive to other parts of the spectrum.\textsuperscript{3}

Obviously the gluon fusion production rate is directly proportional to the decay rate of $h \rightarrow gg$, for which the stop contribution at one-loop level has been computed \cite{8}. The analytic expression, including the top quark contribution, is

$$\Gamma(h \rightarrow gg) = \frac{G_F \alpha_s^2 m_h^3}{36\sqrt{2}\pi^3} \left| \frac{3}{4} A^h_1(\tau_i) + \sum_{i=1,2} \frac{3 g_{h\tilde{t}_i\tilde{t}_i}}{4 m_{\tilde{t}_i}^2} A^h_0(\tau_i) \right|^2,$$

where $\tau_i = m_h^2/(4m_{\tilde{t}_i}^2)$ and the form factors are

$$A^h_1(\tau) = \frac{2}{\tau^2} [\tau + (\tau - 1)f(\tau)],$$

$$A^h_0(\tau) = -\frac{1}{\tau^2} [\tau - f(\tau)],$$

$$f(\tau) = \begin{cases} \arcsin^2 \sqrt{\tau} & \tau \leq 1 \\ -\frac{1}{4} \left[ \log \frac{1 + \sqrt{1 - \tau^{-1}}}{1 - \sqrt{1 - \tau^{-1}}} - i\pi \right] & \tau > 1 \end{cases}.$$  

Furthermore, $g_{h\tilde{t}_i\tilde{t}_i}$ is the coupling of the lightest CP-even Higgs boson to stop mass eigen-

![Diagram of Gluon fusion production of the Higgs boson in the standard model.](image)

\textsuperscript{3} The exception is, as commented earlier, the contribution from the sbottom sector for very large $\tan \beta \sim m_t/m_b$ and small sbottom masses \cite{10}.
states, normalized to $2(\sqrt{2}G_F)^{1/2}$,

$$g_{h\tilde{t}_1\tilde{t}_1} = m_Z^2 \cos 2\beta \left( \frac{1}{2} \cos^2 \theta_t - \frac{2}{3} v_w \cos 2\theta_t \right) + m_t^2 - \frac{1}{2} m_t X_t \sin 2\theta_t,$$

$$g_{h\tilde{t}_2\tilde{t}_2} = m_Z^2 \cos 2\beta \left( \frac{1}{2} \sin^2 \theta_t + \frac{2}{3} v_w \cos 2\theta_t \right) + m_t^2 + \frac{1}{2} m_t X_t \sin 2\theta_t,$$

where $\theta_t$ is the mixing angle between the flavor basis and mass eigenbasis,

$$\sin 2\theta_t = \frac{2m_t X_t}{m_{t_1}^2 - m_{t_2}^2}, \quad \cos 2\theta_t = \frac{m_{t_1}^2 + D'_t - m_{\tilde{t}_1}^2 - D'_L}{m_{t_1}^2 - m_{t_2}^2},$$

such that

$$\begin{pmatrix} \cos \theta_t - \sin \theta_t \\ \sin \theta_t - \cos \theta_t \end{pmatrix} M_t^2 \begin{pmatrix} \cos \theta_t, \sin \theta_t \\ -\sin \theta_t, \cos \theta_t \end{pmatrix} = \begin{pmatrix} m_{t_1}^2, 0 \\ 0, m_{t_2}^2 \end{pmatrix}.$$

The form factors $A_0^\mu(\tau)$ and $A_2^\mu(\tau)$ approach 4/3 and 1/3, respectively, for $\tau = m_h/(4m_t^2) \to 0$. For $m_h \sim 120$ GeV, $m_t = 172$ GeV, and $m_{\tilde{t}} \sim 200$ GeV, one can check that $\tau \to 0$ is a good approximation for the form factors.

The $\tau_i \to 0$ limit is equivalent to approximating the one-loop diagram in Fig. 1 by a dimension-five local operator $(h/v)G^a_{\mu\nu}G^{a\mu\nu}$ whose coefficient has long been known to be related to the QCD one-loop beta function [15, 16]. If we turn on a background Higgs field $h$ and consider the squark threshold effect for the running of one-loop beta function of QCD, neglecting other contributions for now, we get

$$-\frac{1}{4g^2(\mu_{\tau})}G^a_{\mu\nu}G^{a\mu\nu} = -\frac{1}{4} \left( \frac{1}{g^2(\Lambda)} - \frac{b_3^{UV}}{16\pi^2} \log \frac{\Lambda^2}{\mu_t^2} - \sum_{i=1,2} \frac{b_3^{(0)}}{16\pi^2} \log \frac{m_{\tilde{t}_i}^2(h)}{\mu_t^2} - \cdots \right) G^a_{\mu\nu}G^{a\mu\nu}$$

$$= -\frac{1}{4} \left( \frac{b_3^{(0)}}{16\pi^2} \log \frac{\det M_t^2(h)}{\mu_t^2} - \cdots \right) G^a_{\mu\nu}G^{a\mu\nu},$$

where $b_3^{(0)} = 1/6$ [17]. Expanding $\det M_t^2(h)$ in the presence of the background Higgs field $h$ with respect to $\langle h \rangle = v/\sqrt{2}$, one immediately obtains the dimension-five operator $(h/v)G^a_{\mu\nu}G^{a\mu\nu}$, whose coefficient is essentially determined by the quantity

$$v \frac{\partial}{\partial h} \log \det M_t^2(h) \bigg|_{h=v/\sqrt{2}}.$$

In fact, it is straightforward to verify in Eq. (8) that in the limit $\tau_i \to 0$ the stop contribution to the decay width $\Gamma(h \to gg)$ is controlled by

$$\sum_{i=1,2} \frac{g_{h\tilde{t}_i\tilde{t}_i}}{m_{t_i}^2} = \frac{m_{t_i}^2 g_{h\tilde{t}_2\tilde{t}_2} + m_{t_i}^2 g_{h\tilde{t}_1\tilde{t}_1}}{\det M_t^2},$$

$$= \frac{1}{2} v \frac{\partial}{\partial h} \log \det M_t^2(h) \bigg|_{h=v/\sqrt{2}}.$$
FIG. 2: Plot of $R_g$ as a function of $r$ for $m_{\tilde{t}}^2 = 500$ GeV, $\tan \beta = 10$ (green/gray), $\tan \beta = 30$ (red/dark gray), $\tan \beta = 50$ (blue/black). The solid lines are for other SUSY masses fixed to 400 GeV. For comparison, the (green/gray) dashed lines are for other SUSY masses fixed to 800 GeV and $\tan \beta = 10$. The three clusters of lines correspond to $X_t/m_{\tilde{t}} = 0, -1, -2$ as indicated in the plot.

If we further drop the subleading contribution proportional to $m_Z^2$ in $g_{h\tilde{t}_i\tilde{t}_j}$, then Eq. (16) becomes

$$m_{\tilde{t}}^2(m_{\tilde{t}_i}^2 + m_{\tilde{t}_j}^2) + 2m_{\tilde{t}}^2X_t^2 \over \det M_{\tilde{t}}^2.$$ (17)

Defining variables

$$m_{\tilde{t}}^2 = {m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2 \over 2}, \quad r = {m_{\tilde{t}_L}^2 - m_{\tilde{t}_R}^2 \over m_{\tilde{t}_L}^2 + m_{\tilde{t}_R}^2},$$ (18)

we see that Eq. (17) depends mostly on $X_t$ and $m_{\tilde{t}}^2$, and weakly on $r$ which only appears in the denominator. In Fig. 2 we demonstrate that $R_g$, the ratio of the full gluon fusion rate in the MSSM over the rate in the standard model, varies little for $|r| \lesssim 0.4$. The value of $r = 0.4$ for $m_{\tilde{t}} = 500$ GeV corresponds to $m_{\tilde{t}_L} \sim 590$ GeV and $m_{\tilde{t}_R} \sim 390$ GeV. Most SUSY breaking scenarios generate comparable $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$, and since the renormalization group running of stop masses is dominated by the gluino mass, the contribution of which is identical for both $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$, the weak scale values of both masses remain close to each other. For example, all the SPS benchmark scenarios for supersymmetry in Ref. [18] have stop mass splittings that fall within $|r| \lesssim 0.4$. Throughout this paper we use the publicly available code FeynHiggs2.5 [19] to obtain numerical results presented in plots. The set of relevant input
parameters we use throughout this study is the top quark pole mass at $m_t = 172.5$ GeV, bottom quark pole mass at $m_b = 4.7$ GeV, and the pseudo-scalar Higgs mass at $m_A = 400$ GeV. In Fig. 2 we plot the production rate for $\tan\beta = 10, 30, 50$ (although these three cases are plotted with different color/shades they are hard to distinguish because of the negligible dependence on $\tan\beta$) and two different common masses of all other superpartners, 400 GeV (solid) and 800 GeV (dashed, only for $\tan\beta = 10$). The three clusters of lines correspond to $X_t/m_{\tilde{t}} = 0, -1, -2$ as indicated in the plot. It is clear that the dependence on $\tan\beta$ and masses of other superpartners is very small.

It is worth commenting that FeynHiggs computes the approximate Higgs production cross-sections using extrapolation of the standard model production rate [20]. Higher-order corrections such as the next-to-leading order QCD corrections might be important in determining the Higgs production rate in the MSSM and should be included in future analysis. However, the important observation relevant for our proposal is that the change in the gluon fusion production rate is largely a constant shift [21] and does not introduce a significant dependence on other SUSY parameters such as the gaugino mass $m_{1/2}$. In order to demonstrate our method we find it sufficient to use the approximation of FeynHiggs.

IV. STOPS AND THE HIGGS MASS

In the Higgs sector of the MSSM there are two Higgs doublets, $H_u$ and $H_d$, coupling to the up-type and down-type quarks respectively. After electroweak symmetry breaking three components are eaten Goldstone bosons and give mass to the electroweak gauge bosons through the Higgs mechanism. The remaining physical states are two CP-even neutral Higgs bosons, $h$ (the lighter one) and $H$ (the heavier one), one CP-odd neutral Higgs boson $A$, and the charged Higgses $H^\pm$. In the MSSM supersymmetry imposes very tight constraints on the Higgs potential at tree-level, in particular, the scalar quartic couplings are completely fixed by $SU(2)_w \times U(1)_Y$ gauge couplings. As a result there are two free parameters in the MSSM Higgs sector, usually taken to be $\tan\beta$ and $m_A$, and one can derive hierarchical relations for masses of different Higgs bosons [8]. Among them the most important one is perhaps the upper bound on the mass of the lightest Higgs boson $h$,

$$m_h \leq m_Z |\cos 2\beta| \leq m_Z = 91.2 \text{ GeV}, \quad (19)$$
which is clearly below the LEP bound \( m_h \geq 114 \text{ GeV} \).

Therefore, one usually resorts to large radiative corrections from superpartners with significant coupling to the Higgs boson to raise \( m_h \). This is why \( m_h \) is most sensitive to the parameters in the stop sector, and not to masses of other superparticles. \(^4\) For simplicity if we assume \( m_{\tilde{t}_R} \simeq m_{\tilde{t}_L} = m_{\tilde{t}} \), the one-loop correction to \( m_h \) is approximately given as

\[
\Delta m_h^2 \simeq \frac{3G_F}{\sqrt{2}\pi^2} m_t^4 \left\{ \log \frac{m^2_{\tilde{t}}}{m^2_{\tilde{t}}} + \frac{X^2}{m^2_{\tilde{t}}} \left( 1 - \frac{X^2}{12m^2_{\tilde{t}}} \right) \right\},
\]

which grows logarithmically with the stop mass \( m_{\tilde{t}} \). On the other hand, the up-type Higgs mass squared parameter increases quadratically with \( m_{\tilde{t}} \),

\[
\Delta m_{H_u}^2 \simeq -\frac{3}{8\pi^2} m_{\tilde{t}}^2 \log \frac{\Lambda^2}{m^2_{\tilde{t}}},
\]

It is the logarithmic versus quadratic dependence on the stop mass that dictates the fine-tuning in the MSSM. For \( m_{\tilde{t}_R} \simeq m_{\tilde{t}_L} \) the stop masses need to be very large, \( \mathcal{O}(1 \text{ TeV}) \), to evade the LEP limit on the Higgs mass, which leads to large \( (\mathcal{O}(m^2_{\tilde{t}}/m^2_t) \lesssim 1\%) \) fine-tuning in electroweak symmetry breaking. On the other hand, the stop masses could be significantly below 1 TeV if there is large mixing in the stop sector, in which case the fine tuning can be reduced to the level of 5\%. The Higgs mass is maximized for \( |X_t/m_{\tilde{t}}| \sim 2 \) and with this mixing light stops, \( m_{\tilde{t}_R} \simeq m_{\tilde{t}_L} \simeq 300 \text{ GeV} \), are sufficient to push the Higgs mass above the LEP limit.

From the discussion above, we see that for \( \tan \beta \gtrsim 10 \) the tree level contribution to the Higgs mass is saturated and the residual dependence of the Higgs mass on \( \tan \beta \) is very weak (\( \tan \beta \) does not enter the leading one loop correction). Furthermore, in spite of Eq. (20) being derived for \( m_{\tilde{t}_R} \simeq m_{\tilde{t}_L} \) and the Higgs mass in general being dependent on all three parameters in the stop sector: \( m_{\tilde{t}} \), \( X_t \) and \( r \), the dependence on \( r \) is very weak for quite large deviations of \( m_{\tilde{t}_R} \) and \( m_{\tilde{t}_L} \) from the average value. In Fig. 3 we plot the sensitivity of the Higgs mass to \( r \) for three different values of \( \tan \beta \) (distinguished by colors/shades) and two different common masses of all other superpartners, 400 GeV (solid) and 800 GeV (dashed, only for \( \tan \beta = 10 \)). The three clusters of lines correspond to \( X_t/m_{\tilde{t}} = 0, -1, -2 \) as indicated in the plot. Again we see that \( m_h \) is not much dependent of \( r \), \( \tan \beta \), and masses of other superpartners in the region of the parameter space we are considering.

\(^4\) The exception is again the sbottom sector for very large \( \tan \beta \) and large \( \mu \).
FIG. 3: Plot of the Higgs boson mass as a function of $r$ for $m_{\tilde{t}}^2 = 500$ GeV, $\tan \beta = 10$ (green/gray), $\tan \beta = 30$ (red/dark gray), $\tan \beta = 50$ (blue/black). The solid lines are for other SUSY masses fixed to 400 GeV. For comparison, the (green/gray) dashed lines are for other SUSY masses fixed to 800 GeV and $\tan \beta = 10$. The three clusters of lines correspond to $X_{\tilde{t}}/m_{\tilde{t}} = 0, -1, -2$ as indicated in the plot.

V. RESULTS

In this section we present our results, concentrating on the observables $m_h$ and $R_g \equiv \Gamma_{g}^{\text{MSSM}}/\Gamma_{g}^{\text{SM}}$ which is the ratio of the Higgs production rate in the gluon fusion channel in the MSSM and in the standard model. Contours of constant $m_h$ and $R_g$ are plotted in the $m_{\tilde{t}} - X_{\tilde{t}}/m_{\tilde{t}}$ plane, as shown in Fig. 3.

Let us focus first on the contours of constant $R_g$, observing that $R_g \gtrsim 1$ when the mixing in the stop sector is small $|X_{\tilde{t}}/m_{\tilde{t}}| \lesssim 1.6$ regardless of $m_{\tilde{t}}$. Moreover, for small mixing $R_g$ increases as $m_{\tilde{t}}$ decreases, since lighter stops give more significant contributions to the production rate. On the other hand, in the region where stops are light $m_{\tilde{t}} \sim \mathcal{O}(500 \text{ GeV})$ and mixing is large $|X_{\tilde{t}}/m_{\tilde{t}}| \sim 2$, we see $R_g \lesssim 1$. The fact that the Higgs production in the gluon fusion channel in the MSSM could be smaller than in the standard model for large mixing in the stop sector has previously been observed in Ref. [10]. It is interesting to note that $R_g$ alone seems to give a good sense of the magnitude of $X_{\tilde{t}}/m_{\tilde{t}}$: $R_g \gtrsim 1$ if the mixing is small and $R_g \lesssim 1$ if the mixing is large.

For contours of constant Higgs mass, the story is similar to what has been said repeatedly
in the literature. If there is no mixing in the stop sector, the stop mass $m_{\tilde{t}}$ needs to be close to 1 TeV in order to have a Higgs mass above the LEP bound of 114 GeV. The Higgs mass starts increasing when one turns on the mixing and eventually reaches a maximum value for $|X_t/m_{\tilde{t}}| \sim 2$. In the region of large mixing light stops, $m_{\tilde{t}} \simeq 300$ GeV, are still allowed by $m_h \geq 114$ GeV.

When we consider both kinds of contours together, there are several observations to be made. First consider the region of small mixing. In this region contours of constant $m_h$ and $R_g$ run somewhat parallel to each other vertically, implying a very loose constraint on $m_{\tilde{t}}$, the overall stop mass scale, unless the gluon production rate can be measured precisely in experiments. Furthermore, the region where $R_g \gtrsim 1$ corresponds to the region where EWSB is more fine-tuned. Once we move into the region where $R_g \lesssim 1$, contours of constant $R_g$ run at large angles with contours of constant $m_h$, which means it is possible to determine both $m_{\tilde{t}}$ and $X_t/m_{\tilde{t}}$ fairly well even if there is a large uncertainty in $R_g$. This is because in this region $R_g$ is quite sensitive to $m_{\tilde{t}}$ and (especially) $X_t/m_{\tilde{t}}$, and decreases rapidly with increasing mixing and decreasing stop masses. Therefore measurements of $m_h$ and $R_g$ will allow for a fairly accurate determination of $m_{\tilde{t}}$ and $X_t/m_{\tilde{t}}$ in the region of large mixing and light stops. All these measurements involve properties of the Higgs boson and can be done without prior knowledge of other masses and mixing angles in the MSSM spectrum. As
demonstrated in previous sections, these results are not sensitive to other parameters, and choosing different values of $\tan \beta$, $\mu$ or masses of other superpartners would only negligibly change results presented in Fig. 4.

In Fig. 4 we have also zoomed in on the region of large mixing with negative $X_t$ and small stop mass, since this is the region of particular interest: both $m_h$ and $R_g$ vary rapidly. This is also the least fine-tuned region of the MSSM. From the zoomed in plot one can see that, for example, if the experimental central values are $m_h = 118$ GeV and $R_g = 0.7$, then the corresponding central values for $m_{\tilde{t}}$ and $X_t/m_{\tilde{t}}$ will be 380 GeV and $-2$, respectively. Of course we should not forget that there is another solution for roughly the same $m_{\tilde{t}}$ but positive $X_t/m_{\tilde{t}}$. We also mention in passing that all the constant $R_g$ contours re-appear in the dense region near the bottom-left corner where $m_{\tilde{t}} \sim 300$ GeV and $X_t/m_{\tilde{t}} \sim -2.2$. They re-appear because in this region the lightest stop is extremely light, $\sim 120$ GeV, for which the stop contribution in the gluon fusion rate, Eq. (6), completely overwhelms the standard model contribution. Therefore the production rate at first decreases all the way to zero, when stop contribution reaches a critical value and become equal and opposite to the top contribution, and then starts growing when stop contribution becomes more negative than the critical value. In this region $R_g$ is a very rapidly growing function for decreasing stop mass. The end result is a region with very densely populated contours in the very bottom-left corner in Fig. 4.

At this point we would like to comment on various theoretical and experimental uncertainties one might encounter in implementing our strategy. For the lightest CP-even Higgs boson $h$, if it is observed in the golden channel $h \rightarrow ZZ \rightarrow \ell^+\ell^-\ell^+\ell^-$ or the silver mode $h \rightarrow \gamma\gamma$, then its mass can be measured with an accuracy of $\Delta m_h / m_h \sim 0.2\%$ at the LHC [22]. For $m_h \sim 120$ GeV, this gives an uncertainty of only 250 MeV! Unfortunately, the theoretical uncertainty in computing $m_h$ within MSSM is still quite large in comparison. In the MSSM the full one-loop and dominant two-loop corrections to $m_h$ have been calculated, however, results from two different renormalization schemes differ by about $2-3$ GeV [8]. The difference can be seen as a rough estimate of the magnitude of the unknown higher order corrections. On the other hand, the situation with uncertainties in the partial width $\Gamma(gg \rightarrow h)$, and hence $R_g$, is less optimistic. The reason is two-fold. First the production

\footnote{In fact, in this region $m_h < 114$ GeV which is ruled out by the LEP limit.}
rate of $gg \rightarrow h$ is not directly measurable in experiments since the Higgs can only be seen through its decay products. Instead, what can be measured directly is the cross-section times the branching ratio such as $\sigma(gg \rightarrow h) \times \text{Br}(h \rightarrow 2\gamma)$. By combining measurements on Higgs production and decay in different channels it is possible to extract individual partial decay width, and at the LHC with a 200 fb$^{-1}$ luminosity the error is expected to be $\Delta \Gamma_g/\Gamma_g \sim 30\%$ when including systematic errors of approximately 20% from higher order QCD corrections [23, 24, 25].

The uncertainty in the top quark mass has also an effect on calculation of both the Higgs mass and the gluon fusion production of the Higgs boson. At the LHC the mass of the top quark is expected to be measured with uncertainties of 1 GeV, dominated by systematic errors [26, 27]. An uncertainty at this level results in $\sim 0.5$ GeV difference in the calculated Higgs mass which is much smaller compared to the theoretical uncertainty. For the gluon fusion production the top quark mass uncertainty is negligible compared to the systematic error discussed above. Furthermore, a recent study suggests that a significantly better precision of the top quark mass measurement can be achieved using a sequence of effective field theories for the reconstruction of the top quark invariant mass distributions at collider experiments [28].

In the end, the uncertainty in $m_h$ is expected to be at the level of 2%, dominated completely by theoretical uncertainty, whereas the uncertainty in $R_g$ is much larger, at the level of 30%. However, we should stress that, even with a 30% uncertainty in $R_g$, in the region of large mixing and small stop mass it could still be useful to apply our strategy due to the fact that $R_g$ is very sensitive to $m_t$ and $X_t$ in this region. For example, even if the production rate is poorly measured to be in the region $0.7 \geq R_g \geq 0.3$, it is still possible to constrain the $m_\tilde{t} - X_t/m_\tilde{t}$ plane down to a small area by knowing $m_h$ with 2 GeV uncertainty, as can be seen from Fig. 4.

VI. EXTREME CORNERS OF THE MSSM

In this section we use our results to explore an interesting possibility that measurements of the Higgs mass and the production rate do not have overlapping contours in the $m_\tilde{t} - X_t/m_\tilde{t}$ plane. Both $m_h$ and $R_g$ is a measure on the overall stop mass scale $m_\tilde{t}$ and the mixing $X_t$. If $m_\tilde{t}$ and $X_t$ inferred separately from $m_h$ and $R_g$ are very far off, then it is a signal that the
region of parameter space we are considering,

\[ 10 \lesssim \tan \beta \lesssim \frac{m_t}{m_b}, \quad m_b |\mu \tan \beta| \lesssim m_{b_L}^2, \quad m_{b_R}^2, \quad \text{and} \quad |r| \lesssim 0.4, \quad (22) \]

is disfavored. In this case, we can further ask if it is possible to reconcile the differences in these two measurements by considering other parameter regions.

From Fig. 4 we see that the only situation in which contours from measurements of the Higgs mass and the production rate would not overlap (taking into account uncertainties discussed in the previous section) is when the Higgs is relatively heavy, \( m_h \gtrsim 130 \text{ GeV} \), and production rate very small, \( R_g \lesssim 0.6 \). The reason is that a Higgs mass around 130 GeV requires a high stop mass scale, \( m_{\tilde{t}} \gtrsim 1 \text{ TeV} \), whereas a small production rate prefers a low stop mass scale, \( m_{\tilde{t}} \lesssim 500 \text{ GeV} \).

In order to find a resolution in these two measurements, it is necessary to find ways to lower \( m_{\tilde{t}} \) while keeping \( m_h \) fixed at around 130 GeV, or increase \( m_{\tilde{t}} \) while maintaining a small \( R_g \) at roughly 0.6. Immediately we conclude that going to a smaller \( \tan \beta \) would not help because, in this case, the tree-level contribution to the Higgs mass is reduced and \( m_{\tilde{t}} \) has to be even higher in order to produce a larger radiative corrections to keep \( m_h \) large. This worsens the discrepancy.

An alternative is to have a large \( \tan \beta \sim \frac{m_t}{m_b} \) for which the sbottom contributions are important. Let us first discuss the effect of the sbottom sector on both the Higgs mass and the production rate. In the decoupling limit, a large \( \tan \beta \) is only a necessary but not a sufficient condition for the sbottom effects to be important for the Higgs mass; a sizable \( \mu \) term is also required. In this case the mixing in the sbottom sector can be very large which has a tendency to decrease the Higgs mass \[9\]. Obviously larger \( \mu \) term causes even larger \( X_b \) and therefore smaller Higgs mass. Moreover the Higgs mass is not an even function in \( X_b \rightarrow -X_b \) and hence not in \( \mu \rightarrow -\mu \) either.\[6\] Since the sbottom mass matrix and sbottom couplings to the lightest CP-even Higgs are very similar to those in the stop sector given in Sect. III with the corresponding electroweak charges, masses, and mixing term replaced by those for the sbottom, we expect that, in the same fashion as the stop, if the sbottom is light and mixing is large, it could decrease the production of the Higgs in the gluon fusion channel. The production rate does not depend explicitly on the sign of \( X_b \) but only implicitly through the Higgs mass \( m_h \).

\[6\] Notice that our definition of \( \mu \) differs from that in \[9\] by a sign.
Now in order to produce a large effect in the production rate the sbottom has to be light, since its contribution decouples as $1/m_{b_1}^2$. On the other hand, the stop must be heavy to keep the Higgs mass large. At this point it is important to keep in mind that the soft-breaking masses for the left-handed sfermions are required to be the same by the $SU(2)_w$ gauge symmetry: $m_{t_L}^2 = m_{b_L}^2 = m_{q_3}^2$. Therefore there is a limited number of ways to keep at least one of the sbottoms light and at least one of the stops heavy. As an example, $m_h \sim 130$ GeV and $R_g \sim 0.6$ can be produced with the following choices of parameters (assuming large mixing in the stop sector that maximizes $m_h$):

(a) $\tan \beta \sim 50$, $m_{q_3} \sim m_{t_R} \sim 2000$ GeV, $m_{b_R} \sim 100$ GeV, and $\mu \sim -800$ GeV,\(^7\)

(b) $\tan \beta \sim 50$, $m_{q_3} \sim m_{b_R} \sim 300$ GeV, $m_{t_R} \sim 5000$ GeV, and $\mu \sim -250$ GeV, and small variations of these. As one can see, reconciling these two measurement in $m_h$ and $R_g$, by going outside of the choices of parameters we considered in Eq. (22), would require huge hierarchies in and between the stop and sbottom sectors. Such hierarchies are difficult to generate from a sensible UV model and we consider them rather extreme. The more plausible explanation of conflicting values of $m_h$ and $R_g$ would be contributions from physics beyond the MSSM.

VII. DISCUSSION AND CONCLUSIONS

In this paper we proposed using the Higgs boson as a probe of the stop sector. Our method relies on measurements of the Higgs mass as well as the production rate in the gluon fusion channel, the dominant production channel at the LHC. For $m_t/m_b \gtrsim \tan \beta \gtrsim 10$ and small $\mu$ term, our proposal is insensitive to other mass parameters in the MSSM and thus complementary to the conventional method of studying the production and decay processes of stops, which requires knowledge of masses and mixing angles in the chargino and neutralino sector.

In the stop mass-squared matrix, there are three free parameters $m_{t_L}^2$, $m_{t_R}^2$ and $X_t$ which (roughly) correspond to the two diagonal entries and the one off-diagonal entry. With only two measurements, the Higgs mass and the production rate in the gluon fusion channel, one might expect that a priori it is only possible to constrain the three parameters on a one-

\(^7\) In this case the lightest sbottom $b_1$ is slightly lighter than 100 GeV. However, if $b_1$ is mostly right-handed, which is the case here, the limit on its mass is very weak, much lower than 100 GeV.\(^{29}\)
dimensional surface. Nevertheless, we demonstrated that both measurements are sensitive to only two out of the three parameters in the mass matrix; there is a (almost) flat direction in the space of parameters. In the end, two measurements provide an access to, in terms of variables defined in Eq. (18), the overall stop mass scale $m_{\tilde{t}}^2$ and the mixing term $X_t$, as long as $|r| \leq 0.4$. It is worth pointing out that all the Snowmass benchmark scenarios for the MSSM have mass splittings satisfying $|r| \leq 0.4$. We also note that very often $r$ is calculable from a given UV model in which case it is not a free parameter and our procedure can be used to determine the stop sector of the model completely.

The proposed strategy is the least effective when the mixing in the stop sector is not large, for in this region contours of two different measurements run in parallel to each other. This happens when the Higgs is light and the production rate is close to the standard model value. On the other hand, our method is the most effective when stops are light and the mixing is large, in which case the allowed area in the $m_{\tilde{t}} - X_t/m_{\tilde{t}}$ plane is quite small. Because the production rate is very sensitive to $m_{\tilde{t}}$ and $X_t$ in this particular region, even with an uncertainty as large as 30% in the production rate, our proposal could be useful as discussed in the previous section.

As already emphasized, our proposal should be considered as complementary to methods of extracting stop mass parameters in direct production and decay processes. The point is to measure the same set of parameters in as many different ways as possible and see if there is a consistent set of numbers emerging. The computation presented in this study is at best exploratory in nature, since it does not include many of the recent higher order calculations of the Higgs production and decay rates. We only wish to demonstrate the feasibility of the proposal and identify regions of parameter space where the method is the most effective, in order to motivate and facilitate future studies.

We also considered the case when there is a discrepancy between the measurements of the Higgs mass and the production rate. This could happen if the lightest CP-even Higgs is heavy, $m_h \gtrsim 130$ GeV, and the production rate is significantly smaller than in the standard model. Even though it is possible to generate such a pattern in the MSSM, the required spectrum has large mass hierarchies in and between the stop and sbottom sectors which resides in extreme corners of the parameter space.

As a final comment, the effectiveness of our strategy is clearly limited by the possibly large uncertainty incurred in the measurement of the production rate in the gluon fusion
channel. We hope our proposal could serve as a strong motivation to make an effort to reduce the uncertainty in the gluon fusion production rate, either through a better experimental measurement or a more precise theoretical calculation. Before that goal is achieved, a better observable to consider is probably the event rate of $gg \rightarrow h \rightarrow \gamma\gamma$, which is directly measurable and has less uncertainty. However, the decay rate to two photons in the MSSM depends not just on stop masses but also on chargino masses. Thus if charginos are observed at the LHC and their masses are measured, then it could be useful to combine the measurement of $gg \rightarrow h \rightarrow \gamma\gamma$ with the Higgs mass, in the same fashion as described in this paper, to constrain the stop sector of the MSSM.

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