FLIPPING $SU(5)$ OUT OF TROUBLE

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ABSTRACT

Minimal supersymmetric $SU(5)$ GUTs are being squeezed by the recent values of $\alpha_s$, $\sin^2 \theta_W$, the lower limit on the lifetime for $p \to \bar{\nu}K$ decay, and other experimental data. We show how the minimal flipped $SU(5)$ GUT survives these perils, accommodating the experimental values of $\alpha_s$ and $\sin^2 \theta_W$ and other constraints, while yielding a $p \to e/\mu^+\pi^0$ lifetime beyond the present experimental limit but potentially accessible to a further round of experiments. We exemplify our analysis using a set of benchmark supersymmetric scenarios proposed recently in a constrained MSSM framework.

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One of the key pieces of circumstantial evidence in favour of grand unification has long been the consistency of the gauge couplings measured at low energies with a common value at some very high energy scale, once renormalization effects are taken into account. This consistency is significantly improved when light supersymmetric particles are included in the renormalization-group running, in which case the agreement improves to the per-mille level [1].

However, this circumstantial evidence is not universally accepted as convincing. For example, it has recently been suggested that the logarithmic unification of the gauge couplings is as fortuitous as the apparent similarity in the sizes of the sun and moon [2]. Alternatively, it has been argued that the unification scale could be as low as 1 TeV, either as a result of power-law running of the effective gauge couplings in theories with more than four dimensions [3], or in theories with many copies of the $SU(3) \times SU(2) \times U(1)$ gauge group in four dimensions [4].

For some time now, detailed calculations have served to emphasize [5] how much fine tuning is needed in models with power-law running to reproduce the effortless success of supersymmetric grand unification with logarithmic running of the gauge couplings. Moreover, data from particle physics and cosmology provide independent hints for low-energy supersymmetry. Precision electroweak data favour quite strongly a low-mass Higgs boson [6], as required in the minimal supersymmetric extension of the Standard Model (MSSM) [7], and the lightest supersymmetric particle is a perfect candidate [8] for the cold dark matter thought by astrophysicists to infest the Universe. Many studies have shown that these and other low-energy data - such those on $b \rightarrow s\gamma$ decay [9] and $g_\mu - 2$ [10] - are completely consistent with low-energy supersymmetry, and a number of benchmark supersymmetric scenarios have been proposed [11].

Issues arise, however, when one considers specific supersymmetric grand unified theories. One is the exact value of $\sin^2 \theta_W$, which acquires important corrections from threshold effects at the electroweak scale, associated with the spectrum of MSSM particles [12, 13], and at the grand unification scale, associated with the spectrum of GUT supermultiplets [12, 14]. Precision measurements indicate a small deviation of $\sin^2 \theta_W$ even from the value predicted in a minimal supersymmetric $SU(5)$ GUT, assuming the range of $\alpha_s(M_Z)$ now indicated by experiment [15].

The second issue is the lifetime of the proton. Minimal supersymmetric $SU(5)$ avoids the catastrophically rapid $p \rightarrow e^+\pi^0$ decay that scuppered non-supersymmetric $SU(5)$. However, supersymmetric $SU(5)$ predicts $p \rightarrow \bar{\nu}K^+$ decay through $d = 5$ operators at a rate that may be too fast [16] to satisfy the presently available lower limit on the lifetime for this decay [17, 18]. The latter requires the $SU(5)$ colour-triplet Higgs particles to weigh $> 7.6 \times 10^{16}$ GeV, whereas conventional $SU(5)$ unification for $\alpha_s(M_Z) = 0.1185 \pm 0.002$, $\sin^2 \theta_W = 0.23117 \pm 0.00016$ and $\alpha_{em}(M_Z) = 1/(127.943 \pm 0.027)$ [18] would impose the upper limit of $3.6 \times 10^{15}$ GeV at the 90% confidence level [16]. This problem becomes particularly acute if the sparticle spectrum is relatively light, as would be indicated if the present experimental and theoretical central values of $g_\mu - 2$ [10] remain unchanged as the errors are reduced.

The simplest way to avoid these potential pitfalls is to flip $SU(5)$ [19, 20]. As is well known, flipped $SU(5)$ offers the possibility of decoupling somewhat the scales at which the Standard Model $SU(3), SU(2)$ and $U(1)$ factors are unified. This would allow the strength of the $U(1)$ gauge to become smaller than in minimal supersymmetric $SU(5)$, for the same
value of $\alpha_s(M_Z)$ \[13\]. Moreover, in addition to having a longer $p \to e/\mu^+\pi^0$ lifetime than non-supersymmetric $SU(5)$, flipped $SU(5)$ also suppresses the $d = 5$ operators that are dangerous in minimal supersymmetric $SU(5)$, by virtue of its economical missing-partner mechanism \[19\].

In this paper, we re-analyze the issues of $\sin^2 \theta_W$ and proton decay in flipped $SU(5)$ \[13\], in view of the most recent precise measurements of $\alpha_s(M_Z)$ and $\sin^2 \theta_W$, and the latest limits on supersymmetric particles. We study these issues in the MSSM, constraining the soft supersymmetry-breaking gaugino masses $m_{1/2}$ and scalar masses $m_0$ to be universal at the GUT scale (CMSSM), making both a general analysis in the $(m_{1/2}, m_0)$ plane and also more detailed specific analyses of benchmark CMSSM parameter choices that respect all the available experimental constraints \[11\]. We find that the $p \to e/\mu^+\pi^0$ decay lifetime exceeds the present experimental lower limit \[17\], with a significant likelihood that it may be accessible to the next round of experiments \[21\]. We recall the ambiguities and characteristic ratios of proton decay modes in flipped $SU(5)$.

We first recall the lowest-order expression for $\alpha_s(M_Z)$ in conventional $SU(5)$ GUTs, namely

$$\alpha_s(M_Z) = \frac{7}{5} \frac{\alpha}{\sin^2 \theta_W - 1}. \quad (1)$$

The present central experimental value of $\alpha_s(M_Z) = 0.118$ is obtained if one takes $\sin^2 \theta_W = 0.231$ and $\alpha^{-1} = 128$, indicating the supersymmetric grand unification is in the right ballpark. However, at the next order, one should include two-loop corrections $\delta_{2\text{loop}}$ as well as electroweak and GUT threshold corrections, that we denote by $\delta_{\text{light}}$ and $\delta_{\text{heavy}}$. Their effects can be included by making the following substitution in (1) \[12\]:

$$\sin^2 \theta_W \to \sin^2 \theta_W - \delta_{2\text{loop}} - \delta_{\text{light}} - \delta_{\text{heavy}}, \quad (2)$$

where $\delta_{2\text{loop}} \approx 0.0030$, whereas $\delta_{\text{light}}$ and $\delta_{\text{heavy}}$ can have either sign. If one neglects $\delta_{\text{light}}$ and $\delta_{\text{heavy}}$, the conventional $SU(5)$ prediction increases to $\alpha_s(M_Z) \approx 0.130$ \[13\]. A value of $\alpha_s(M_Z)$ within one standard deviation of the present central experimental value requires $\delta_{\text{light}}$ and/or $\delta_{\text{heavy}}$ to be non-negligible, so that the combination $(\delta_{2\text{loop}} + \delta_{\text{light}} + \delta_{\text{heavy}})$ is suppressed. However, in large regions of parameter space $\delta_{\text{light}} > 0$, which does not help. Moreover, in conventional $SU(5)$, as was pointed out in \[12\], a compensatory value of $\delta_{\text{heavy}}$ is difficult to reconcile with proton decay constraints. This problem is exacerbated by the most recent lower limit on $\tau(p \to \bar{\nu}K^+)$ \[17\].

As has been advertized previously \[13\], an alternative way to lower $\alpha_s(M_Z)$ is to flip $SU(5)$. In a flipped $SU(5)$ model, there is a first unification scale $M_{32}$ at which the $SU(3)$ and $SU(2)$ gauge couplings become equal, which is given to lowest order by \[24\],

$$\frac{1}{\alpha_2} - \frac{1}{\alpha_5} = \frac{\beta_2}{2\pi} \ln \frac{M_{32}}{M_Z}, \quad (3)$$

$$\frac{1}{\alpha_3} - \frac{1}{\alpha_5} = \frac{\beta_3}{2\pi} \ln \frac{M_{32}}{M_Z}, \quad (4)$$

\[1\] It is true, as pointed out recently \[22\], that this is not a problem if one allows arbitrary squark mixing patterns. However, such options must respect low-energy flavour-changing neutral-interaction limits \[23\], and are not possible in the CMSSM.
where $\alpha_2 = \alpha / \sin^2 \theta_W$, $\alpha_3 = \alpha_s(M_Z)$, and the one-loop beta function coefficients are $b_2 = +1$, $b_3 = -3$. The hypercharge gauge coupling $\alpha_Y = \frac{5}{3}(\alpha / \cos^2 \theta_W)$ has, in general, a lower value $\alpha_1'$ at the scale $M_{32}$:

$$\frac{1}{\alpha_Y} - \frac{1}{\alpha_1'} = \frac{b_Y}{2\pi} \ln \frac{M_{32}}{M_Z},$$

(5)

where $b_Y = 33/5$. Above the scale $M_{32}$, the gauge group is the full $SU(5) \times U(1)$, with the $U(1)$ gauge coupling $\alpha_1$ related to $\alpha_1'$ and the $SU(5)$ gauge coupling $\alpha_5$ as follows:

$$\frac{25}{\alpha_1'} = \frac{1}{\alpha_5} + \frac{24}{\alpha_1}. \quad (6)$$

The $SU(5)$ and $U(1)$ gauge couplings then become equal at some higher scale $M_{51}$. The maximum possible value of $M_{32}$, namely $M_{32}^{\text{max}}$, is obtained by substituting $\alpha_1' = \alpha_5(M_{32})$ into (5), and coincides with the unification scale in conventional $SU(5)$: $M_{32}^{\text{max}} = M_Z \times \exp((3 - 8 \sin^2 \theta_W)\pi/14\alpha_{em}(M_Z))$, where $M_Z = 91.1882 \pm 0.0022$ GeV [18]. In general, one has

$$\alpha_s(M_Z) = \frac{\frac{7}{3} \alpha}{5 \sin^2 \theta_W - 1 + \frac{25}{2\pi} \alpha \ln(M_{32}^{\text{max}}/M_{32})}, \quad (7)$$

and the flipped $SU(5)$ prediction for $\alpha_s(M_Z)$ is in general smaller than in minimal SU(5), for the same value of $\sin^2 \theta_W$. The next-to-leading order corrections to (7) are obtained by the substitution in (6). Numerically, an increase of $\sim 10\%$ in the denominator in (7), which would compensate for the decrease due to $\delta_{2\text{loop}}$, could be achieved simply by setting $M_{32} \approx \frac{1}{3} M_{32}^{\text{max}}$ in (6).

In order to understand the implications for $\tau(p \to e/\mu^+\pi^0)$ decay, we first calculate $M_{32}$, using (7) with $\sin^2 \theta_W$ replaced by $\sin^2 \theta_W - \delta_{2\text{loop}}$, leaving for later discussions of the possible effects of $\delta_{\text{light,heavy}}$. Fig. 8 exhibits the correlation between $M_{32}$ and $\alpha_s(M_Z)$ in flipped SU(5). The solid lines indicate the range of values of $M_{32}$ allowed for a given value of $\alpha_s(M_Z)$ (as given in the MC prescription), assuming the experimentally-allowed range $\sin^2 \theta_W^{\text{exp}} = 0.23117 \pm 0.00016$ [18], and making no allowance for either light or heavy thresholds. For the central experimental value $\alpha_s(M_Z) = 0.1185$, we see immediately that $M_{32}$ is significantly lower than its maximum value, which is $M_{32}^{\text{max}} = 20.3 \times 10^{15}$ GeV for our central values of $\alpha_s(M_Z)$ and $\sin^2 \theta_W$.

We now explore the possible consequences of $\delta_{\text{light}}$ for $M_{32}$, following [12, 13]. We approximate the $\delta_{\text{light}}$ correction by

$$\delta_{\text{light}} = \frac{\alpha}{20\pi} \left[-3L(m_t) + \frac{28}{3} L(m_{\tilde{g}}) - \frac{32}{3} L(m_{\tilde{q}}) - L(m_h) - 4L(m_H) + \frac{5}{2} L(m_{\tilde{q}}) - 32L(m_{\tilde{\ell}_L}) + 2L(m_{\tilde{\ell}_R}) - \frac{35}{36} L(m_{\tilde{\ell}_L}) - \frac{19}{36} L(m_{\tilde{\ell}_L}) \right], \quad (8)$$

where $L(x) = \ln(x/M_Z)$. As already mentioned, we assume that the soft supersymmetry-breaking scalar masses $m_0$, gaugino masses $m_{1/2}$ and trilinear coefficients $A_0$ are universal at the GUT scale (CMSSM). We used ISASUGRA [24] to calculate the sparticle spectra in terms of...
The solid lines show the correlation between $M_{32}$ in flipped SU(5) and $\alpha_s(M_Z)$ in the $\overline{MS}$ prescription, assuming $\sin^2\theta_W^{\overline{MS}} = 0.23117 \pm 0.00016$, including $\delta_{\text{2loop}}$ but neglecting $\delta_{\text{light}}$ and $\delta_{\text{heavy}}$. The points indicate the changes in $\tau(p \to e/\mu^+\pi^0)$ found for $\alpha_s(M_Z) = 0.1185$ and the central value of $\sin^2\theta_W$ when including also the values of $\delta_{\text{light}}$ calculated for the CMSSM benchmark points.

Figure 1: The solid lines show the correlation between $M_{32}$ in flipped SU(5) and $\alpha_s(M_Z)$ in the $\overline{MS}$ prescription, assuming $\sin^2\theta_W^{\overline{MS}} = 0.23117 \pm 0.00016$, including $\delta_{\text{2loop}}$ but neglecting $\delta_{\text{light}}$ and $\delta_{\text{heavy}}$. The points indicate the changes in $\tau(p \to e/\mu^+\pi^0)$ found for $\alpha_s(M_Z) = 0.1185$ and the central value of $\sin^2\theta_W$ when including also the values of $\delta_{\text{light}}$ calculated for the CMSSM benchmark points.

These quantities, $\tan \beta$ and the sign of $\mu$, assuming $m_t = 175$ GeV $^2$. In evaluating (8), $m_\tilde{\chi} (m_H)$ ($m_{\tilde{q}}$) ($m_{\tilde{t}}$) were interpreted as the geometric means of the chargino and neutralino ($H, A, H^\pm$) ($\tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}$) ($\tilde{e}, \tilde{\mu}$) masses, respectively, and the mixings of $\tilde{\tau}, \tilde{b}$ and $\tilde{t}$ were all taken separately into account.

The unknown parameters in (8) were constrained by requiring that electroweak symmetry breaking be triggered by radiative corrections, so that the correct overall electroweak scale and the ratio $\tan \beta$ of Higgs v.e.v.'s fix $|\mu|$ and $m_A$ in terms of $m_{1/2}$ and $m_0$. Before making a more general survey, we recall that a number of benchmark CMSSM scenarios have been proposed $^1$, which include these constraints and are consistent with all the experimental limits on sparticle masses, the LEP lower limit on $m_h$, the world-average value of $b \to s\gamma$ decay, the preferred range $0.1 < \Omega_\chi h^2 < 0.3$ of the supersymmetric relic density, and $g_\mu - 2$ within 2 $\sigma$ of the present experimental value. These points all have $A_0 = 0$, but otherwise span the possible ranges of $m_{1/2}, m_0, \tan \beta$ and feature both signs for $\mu$. Fig. $^1$ also shows the change in $M_{32}$ induced by the values of $\delta_{\text{light}}$ in these benchmark models, assuming a fixed value $\alpha_s(M_Z) = 0.1185$. In general, these benchmark models increase $M_{32}$ for any fixed value of $\alpha_s(M_Z)$ and $\sin^2\theta_W$. As $\alpha_s(M_Z)$ varies, the predicted value of $M_{32}$ in each model varies in the same way as indicated by the sloping lines. We recall that the estimated error in $\alpha_s(M_Z)$ is about 0.002, corresponding to an uncertainty in $M_{32}$ of the order of 20%, and hence a

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$^2$Heavy singlet neutrinos were not used in the renormalization-group equations.
corresponding uncertainty in the proton lifetime of a factor of about two. The error associated with the uncertainty in $\sin^2 \theta_W$ is somewhat smaller.\footnote{We note from Fig.\ 3 that \textit{there is no benchmark model} for which conventional $SU(5)$ grand unification is possible, with the measured values of $\alpha_s(M_Z)$ and $\sin^2 \theta_W$, unless one invokes GUT threshold effects.}

We now turn to the calculation of $\tau(p \to e/\mu^+\pi^0)$. We recall first that the form of the effective dimension-6 operator in flipped $SU(5)$ is different\footnote{Note the absence in the corresponding decay rate of the factor $(1 + (1 + |V_{ud}|^2)^2)$ found in conventional $SU(5)$, as recently re-emphasized in\cite{16}. This lengthens $\tau_p$ by $\approx 5$ in flipped $SU(5)$, an effect that is typically more than offset by the reduction in $M_{32}$.} from that in conventional $SU(5)$\cite{27, 28}:

$$\bar{\mathcal{L}}_{\Delta B\neq 0} = \frac{g_5^2}{2M_{32}^2} \left[ (e^{i\eta} \bar{d} e^{2i\eta_{11}} \gamma^\mu P_L d_j) (u_i \gamma_\mu P_L \nu_L) + h.c. \right. $$

$$+ \left. (e^{i\eta} \bar{d} e^{2i\eta_{21}} \cos \theta_c + \bar{s} e^{2i\eta_{21}} \sin \theta_c \gamma^\mu P_L u_j) (u_i \gamma_\mu P_L \ell_L) + h.c. \right]$$

where $\theta_c$ is the Cabibbo angle. Also appearing in (3) are two unknown but irrelevant CP-violating phases $\eta_{11,21}$ and lepton flavour eigenstates $\nu_L$ and $\ell_L$ that are related to mass eigenstates by unknown but relevant mixing matrices:

$$\nu_L = \nu_F U_\nu \quad , \quad \ell_L = \ell_F U_\ell.$$\hspace{1cm}(10)

Despite our ignorance of the mixing matrices (10), some characteristic flipped $SU(5)$ predictions can be made\cite{24}:

$$\Gamma(p \to e^+ \pi^0) = \frac{\cos^2 \theta_c}{2}|U_{e11}|^2 \Gamma(p \to \nu \pi^+) = \cos^2 \theta_c |U_{e11}|^2 \Gamma(n \to \bar{\nu} \pi^0)$$

$$\Gamma(n \to e^+ \pi^-) = 2\Gamma(p \to e^+ \pi^0) \quad , \quad \Gamma(n \to \mu^+ \pi^-) = 2\Gamma(p \to \mu^+ \pi^0)$$

$$\Gamma(p \to \mu^+ \pi^0) = \frac{\cos^2 \theta_c}{2}|U_{e12}|^2 \Gamma(p \to \nu \pi^+) = \cos^2 \theta_c |U_{e12}|^2 \Gamma(n \to \bar{\nu} \pi^0)$$\hspace{1cm}(11)

In the light of recent experimental evidence for near-maximal neutrino mixing, it is reasonable to think that (at least some of) the $e/\mu$ entries in $U_\ell$ are $O(1)$. In what follows, we assume that the lepton mixing factors $|U_{e11,12}|^2$ are indeed $O(1)$, and do not lead to large numerical suppressions of both the $p \to e/\mu^+ \pi^0$ decay rates. Note that there is no corresponding suppression of the $p \to \nu \pi^+$ and $n \to \bar{\nu} \pi^0$ decay rates, since all the neutrino flavours are summed over. However, without further information, we are unable to predict the ratio of $p \to e^+ X$ and $p \to \mu^+ X$ decay rates. Hereafter, wherever we refer to $p \to e^+ \pi^0$ decay, this mixing-angle ambiguity should be understood.

The $p \to e^+ \pi^0$ decay amplitude is proportional to the overall normalization of the proton wave function at the origin. The relevant matrix elements are $\alpha, \beta$, defined by

$$\langle 0| \epsilon_{ijk} (u^i d^j)_R u^k_L |p(k)\rangle \equiv \alpha u_L(k),$$

$$\langle 0| \epsilon_{ijk} (u^i d^j)_L u^k_R |p(k)\rangle \equiv \beta u_L(k).$$\hspace{1cm}(12) (13)
The reduced matrix elements $\alpha, \beta$ have recently been re-evaluated in a lattice approach \[29\], yielding values that are very similar and somewhat larger than had often been assumed previously, and therefore exacerbating the proton-stability problem for conventional supersymmetric $SU(5)$. Here, we use here the new central value $\alpha = \beta = 0.015$ GeV$^3$ for reference. The error quoted on this determination is below 10\%, corresponding to an uncertainty of less than 20\% in $\tau(p \to e^+\pi^0)$, which would be negligible compared with other uncertainties in our calculation. Thus, we have the following estimate, based on \[26, 16\] and references therein:

$$
\tau(p \to e^+\pi^0) = 3.8 \times 10^{35} \left( \frac{M_{32}}{10^{16}\text{GeV}} \right)^4 \left( \frac{\alpha_5(M_{32}^{\text{max}})}{\alpha_5(M_{32})} \right)^2 \left( \frac{0.015\text{GeV}^3}{\alpha} \right)^2 \gamma
$$

(14)

for use in the subsequent analysis, where we have absorbed reference values for $M_{32}$ and $\alpha_5(M_{32})$ as well as $\alpha$ and $\beta$, and $\alpha_5(M_{32}^{\text{max}})/\alpha_5(M_{32}) = 1 - (33/28)(\alpha_5(M_{32}^{\text{max}})/2\pi)\ln(M_{32}/M_{32}^{\text{max}})$.

We present a general view of flipped $SU(5)$ proton decay in the CMSSM in Fig. 2. The thick solid (blue) lines are contours of $\tau(p \to e^+\pi^0)$ for the indicated choices of $\tan \beta$ and the sign of $\mu$, which span (most of) the range of possibilities. Where applicable, we have indicated by (blue) crosses and labels the CMSSM benchmark points with the corresponding value of $\tan \beta$ and sign of $\mu$. Following \[31\], the dark (red) shaded regions in the bottom right-hand parts of each panel are excluded because the LSP is the lighter $\tilde{\chi}$: astrophysics excludes a charged LSP. The light (turquoise) shaded regions have LSP relic densities in the preferred range $0.1 < \Omega_\chi h^2 < 0.3$ for cold dark matter. The intermediate (green) shaded regions at low $m_{1/2}$ and large $m_0$ are those where electroweak symmetry breaking is no longer possible, and the horizontally-striped regions at low $m_0$ have tachyons. The dash-dotted (blue) line at small $(m_{1/2}, m_0)$ in panel (a) corresponds to $m_{\tilde{e}} = 100$ GeV. The near-vertical dashed (black) lines at small $m_{1/2}$ correspond to the LEP lower limit $m_{\tilde{\chi}}^\pm = 103.5$ GeV, and the dot-dashed (red) lines to LEP lower limit $m_h = 114$ GeV as calculated using the \texttt{FeynHiggs} code \[34\]. In each case, only larger values of $m_{1/2}$ are allowed, although there is uncertainty in the location of the $m_h$ line.$^4$

We see in Fig. 2 that the ‘bulk’ regions of the parameter space preferred by astrophysics and cosmology, which occur at relatively small values of $(m_{1/2}, m_0)$, generally correspond to $\tau(p \to e^+\pi^0) \sim (1 - 2) \times 10^{35} \gamma$. However, these ‘bulk’ regions are generally disfavoured by the experimental lower limit on $m_h$ and/or by $b \to s\gamma$ decay. Larger values of $\tau(p \to e^+\pi^0)$ are found in the ‘tail’ regions of the cosmological parameter space, which occur at large $m_{1/2}$ where $\chi\ell$ coannihilation may be important, and at larger $m_{1/2}$ and $m_0$ where resonant direct-channel annihilation via the heavier Higgs bosons $A, H$ may be important.

We turn finally to the possible implications of the GUT threshold effect $\delta_{\text{heavy}}$ \[12, 14\]. A

\footnote{The horizontal spacing between points sampled was comparable to the thickness of these lines.}

\footnote{For fuller discussions of the implementations of these constraints with and without ISASUGRA, see \[1\], \[8\].}
Figure 2: The solid (blue) lines are contours of $\tau(p \rightarrow e/\mu^{+}\pi^{0})$ in the $(m_{1/2}, m_{0})$ plane for the CMSSM with (a) $\tan \beta = 10, \mu > 0$, (b) $\tan \beta = 10, \mu < 0$, (c) $\tan \beta = 35, \mu < 0$ and (d) $\tan \beta = 50, \mu > 0$. The (blue) crosses indicate the CMSSM benchmark points with the corresponding value of $\tan \beta$ and sign of $\mu$. Following [30], the dark (red) shaded regions are excluded because the LSP is charged, the light (turquoise) shaded regions have $0.1 < \Omega_{\chi}h^{2} < 0.3$, intermediate (green) shaded regions at low $m_{1/2}$ are excluded by $b \rightarrow s\gamma$, shaded (pink) regions at large $(m_{1/2}, m_{0})$ are consistent with $g_{\mu}-2$ at the 2-$\sigma$ level, and electroweak symmetry breaking is not possible in the hatched regions. The near-vertical dashed (black) lines correspond to the LEP lower limit $m_{\tilde{\chi}}^{\pm} = 103.5$ GeV, the dot-dashed (red) lines to $m_{h} = 114$ GeV as calculated using the FeynHiggs code [31], and the dotted (blue) lines at small $(m_{1/2}, m_{0})$ to $m_{\tilde{\tau}} = 100$ GeV.
As we have shown in this paper, it offers the possibility of lowering the prediction for partner mechanism. As in conventional supersymmetric $SU(5)$ neutrino modes would give novel insights into GUTs as well as mixing patterns. $SU(5)$ differ characteristically from those of conventional

$$\frac{M_{32}}{M_{32}^{\text{max}} \rightarrow \frac{M_{32}}{M_{32}^{\text{max}}} \ e^{-10\pi \delta_{\text{heavy}}/11\alpha}}.$$

We also recall that, in general, including $\delta_{\text{heavy}}$ leads to a re-scaling of the $M_{32}/M_{32}^{\text{max}}$.

We also recall that, in general, including $\delta_{\text{heavy}}$ leads to a re-scaling of the $M_{32}/M_{32}^{\text{max}}$. We display in Fig. 3 the possible numerical effects of $\delta_{\text{heavy}}$ on $\tau(p \rightarrow e/\mu^+\pi^0)$ in the various benchmark scenarios, assuming the plausible ranges $-0.0016 < \delta_{\text{heavy}} < 0.0005$ [13]. The boundary between the different shadings for each strip corresponds to the case where $\delta_{\text{heavy}} = 0$. The left (red) parts of the strips show how much $\tau(p \rightarrow e^+\pi^0)$ could be reduced by a judicious choice of $\delta_{\text{heavy}}$, and the right (blue) parts of the strips show how much $\tau(p \rightarrow e^+\pi^0)$ could be increased. The inner bars correspond to the uncertainty in $\sin^2\theta_W$. On the optimistic side, we see that some models could yield $\tau(p \rightarrow e^+\pi^0) < 10^{35}$ y, and all models might have $\tau(p \rightarrow e^+\pi^0) < 5 \times 10^{35}$ y. However, on the pessimistic side, in no model can we exclude the possibility that $\tau_p(e^+\pi^0) > 10^{36}$ y.

We recall that a new generation of massive water-Čerenkov detectors weighing up to $10^6$ tonnes is being proposed [21], that may be sensitive to $\tau(p \rightarrow e^+\pi^0) < 10^{35}$ y. According to our calculations, such an experiment has a chance of detecting proton decay in flipped $SU(5)$, though nothing can of course be guaranteed. We recall that there is a mixing-angle ambiguity [14] in the final-state charged lepton, so any such next-generation detector should be equipped to detect $e^+$ and/or $\mu^+$ equally well. We also recall [24] that flipped $SU(5)$ makes predictions [12] for ratios of decay rates involving strange particles, neutrinos and charged leptons that differ characteristically from those of conventional $SU(5)$. Comparing the rates for $e^+, \mu^+$ and neutrino modes would give novel insights into GUTs as well as mixing patterns.

We conclude that flipped $SU(5)$ evades two of the pitfalls of conventional supersymmetric $SU(5)$. As we have shown in this paper, it offers the possibility of lowering the prediction for $\alpha_s(M_Z)$ for any given value of $\sin^2\theta_W$ and choice of sparticle spectrum. As for proton decay, we first recall that flipped $SU(5)$ suppresses $p \rightarrow \bar{\nu}K^+$ decay naturally via its economical missing-partner mechanism. As in conventional supersymmetric $SU(5)$, the lifetime for $p \rightarrow e/\mu^+\pi^0$ decay generally exceeds the present experimental lower limit. However, as we have shown in
Figure 3: For each of the CMSSM benchmark points, this plot shows, by the lighter outer bars, the range of $\tau(p \rightarrow e/\mu^+\pi^0)$ attained by varying $\delta_{\text{heavy}}$ over the range $-0.0016$ to $+0.0005$ [13]. The central boundary of the narrow inner bars (red, blue) corresponds to the effect of $\delta_{\text{light}}$ alone, with $\delta_{\text{heavy}} = 0$, while the narrow bars themselves represent uncertainty in $\sin^2\theta_W$. We see that heavy threshold effects could make $\tau(p \rightarrow e/\mu^+\pi^0)$ slightly shorter or considerably longer.

this paper, the flipped $SU(5)$ mechanism for reducing $\alpha_s(M_Z)$ reduces the scale $M_{32}$ at which colour $SU(3)$ and electroweak $SU(2)$ are unified, bringing $\tau(p \rightarrow e/\mu^+\pi^0)$ tantalizingly close to the prospective sensitivity of the next round of experiments. Proton decay has historically been an embarrassment for minimal $SU(5)$ GUTs, first in their non-supersymmetric guise and more recently in their minimal supersymmetric version. The answer may be to flip $SU(5)$ out of trouble.

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