Question for $SU(5) \times SU(5)$ string unification

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ABSTRACT: The heterotic string theory can embeds the crossed $SU(5) \times SU(5)$ gauge group. Here we investigate the string unification in this framework and the concerning problems. We show generically that only a very constrained parameter space is allowed for new particles, mostly due to the gauge coupling constant $\alpha^{-1}$. One possible but unfavorable solution is given by the introduction of three fermion generations of $SU(5)_L$-adjoint representation. Only the low-scale decompositions of $SU(5)_L$ with vanishing hypercharge, $\Sigma_3 \sim (1,3)_0$ and $\Sigma_8 \sim (8,1)_0$, of fermionic and bosonic types can be included to circumvent the problem. The triplets must live in TeV region and could be accessible at colliders. We also show that non-supersymmetric scenario is exclusively compatible with the introduction of additional color-$SU(2)_L$-triplet field while supersymmetry is solely possible at high-energy scale. All these intermediated thresholds are easily incorporated into the called Adjoint $SU(5)$ schemes.

KEYWORDS: Grand unified theories, string model, $SU(5) \times SU(5)$, new physics

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1. Introduction

The unification idea, mainly in context of SU(5) group \[ SU(5) \], is still as an interesting alternative for the physics beyond the standard model (SM) \[ SM \]. Many models have been proposed in this framework in recent decades trying to explain the incorporation of SM and the phenomenology discovered beyond this theory. Unfortunately, the Minimal Supersymmetric Standard Model (MSSM) \[ MSSM \], where the best fit for the running gauge couplings is obtained, was thought to be not working correctly, predicting a proton less stable than the observed by the SuperKamiokande bound \[ SuperKamiokande \]. On the other hand, recent few works have been showed that appropriate Yukawa choices can induces a proton consistently with this stability bound \[ stability bound \].

While electroweak and strong interactions can have their couplings easily unified around \[ 10^{16} \text{GeV} \], the quest on the unification theory of four couplings is still alive. In spite of this, the superstring theory has emerged as the most promising candidate for quantum theory of all known interactions \[ superstring \].

As the first requisite for any Grand Unified Theory (GUT) or Supersymmetric GUT (SUSY GUT), superstring theory must contain the SM in its low-energy effective theory \[ SM \]. GUTs and SUSY GUTs are particularly favorable in the wide range of options because they are a truly unified theory with one gauge coupling in the theory at low-energy scale \[ low-energy scale \]. Indeed, it has been shown that low-energy physics can be embedded in the heterotic string profile \[ heterotic string \], exhibiting many appealing properties that we see today \[ appealing properties \] and the incorporation of some currently known SUSY GUTs could be suitable in this scheme. Another requisite is the solution of some known problems. For example, a consequence of string theory is that the strong CP problem can be solved by axions \[ axions \]. Another example is that the theory elegantly can explain the doublet-triplet-splitting problem \[ doublet-triplet-splitting problem \].

On the other hand, some specific choices for the string embody are more attractive than others options \[ more attractive choices \]. Thus, the question is changed from what is the simplest GUT model to what is the most natural pattern based on \[ E_8 \times E_8 \] weakly heterotic string \[ E_8 \times E_8 \]. Here we are considering a natural answer to this question in a SU(5) \[ SU(5) \] gauge group \[ SU(5) \]. In addition, this model has many attractive features, e.g., as the generalized seesaw \[ generalized seesaw \] and it do not needs the introduction of group singlets. In this scheme, it has been showed in some previous studies that the doublet-triplet-splitting problem could be solved with the inclusion of discrete symmetries even in string scenario \[ discrete symmetries \] or in a pure GUT profile \[ pure GUT profile \]. Yet, some phenomenological implications to LHC in generic SU(5)-based gauge group have been analyzed recently \[ phenomenological implications \] and the interesting in LHC searches in this sense are increasing. These searches could be naturally extended to the SU(5) \[ SU(5) \] \( \times \) SU(5) \[ SU(5) \] theory with some complementary analysis.

Towards to the running of gauge couplings constants, in string unification the relation of gauge with gravitational couplings is given at tree level by the following \[ relation \].

\[
\alpha_{\text{string}} = \frac{2G_N}{\alpha'} = k_i \alpha_i,
\]  

(1.1)
where $\alpha_{\text{string}} = g^2_{\text{string}}/4\pi$ is the string-scale unification coupling constant, $G_N$ is the Newton constant, $\alpha'$ is the Regge slope, $\alpha_i = g^2_i/4\pi$ are the gauge couplings, with $i$ running over the gauge groups, $U(1)_Y$, $SU(2)_L$, $SU(3)_C$, being denoted by $i = y, w, s^1$, respectively. $k_i$ are the so-called affine or \textit{Kac-Moody} levels at the group factors $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$ are realized in the four-dimensional string. Each of the multiplications in Eq.(1.1) is renamed as a new coupling, $k_y\alpha_y = \alpha_1$, $k_w\alpha_w = \alpha_2$, $k_s\alpha_s = \alpha_3$, which could be indeed unifiable at $\Lambda$ scale. For the canonical case, where $SU(5)$, $SO(10)$, $E_6$, $[SU(3)]^3 \times Z_3$, $SO(18)$, $E_8$, $SU(15)$, $SU(16)$ and $SU(8) \times SU(8)$ are subjects [11], [12] [13], these levels are given by $k_y = 5/3$, $k_w = 1$ and $k_s = 1$, but they are different for other group choices such as $SU(5) \times SU(5)$, $[SU(6)]^3 \times Z_3$, Pati-Salam models, etc. Indeed, these levels have the power of constrain the Renormalization Group Equations (RGEs) for a new class of particles [13]. In this sense, the stringy nature of the unification in each theory is due to the new equation that relates the gauge coupling constants with string scale, $\Lambda$,

$$\alpha_{\text{string}} = \frac{1}{4\pi} \left( \frac{\Lambda}{\Lambda_s} \right)^2, \quad (1.2)$$

which is an additional bond to the RGEs. In this way, the parameter space for the string scale by the unification coupling could be quite constrained. Thus, instead of these two independent parameters, in string theory it is reflected in only one parameter of freedom, that could be a parametrization among $\Lambda$ and $\alpha_{\text{string}}$.

In this paper we study this issue even without introduction of complete scalar structure since the parameter space is set univocally by low-energy multiplets content in non-SUSY and in SUSY cases, by SM and MSSM, respectively. This paper has been organized as follows: in Sec. 2 we introduce the canonical unification and the new analysis that is suitable for the string unification; in Sec. 3 the $SU(5)_L \times SU(5)_R$ theory is introduced; in Sec. 4 we address the problems concerning the coupling constants at one-loop and two-loop levels for standard and SUSY theories; in Sec. 5 we discuss the inclusion of the adjoint $SU(5)_L$ subgroup in non-SUSY and in SUSY theories; in Sec. 6 some phenomenological implications are presented; finally the conclusions and the summary are subject of Sec. 7.

2. Canonical unification

2.1 Standard Model

2.1.1 One-loop analysis

The renormalizability of the theory is induced by a mass-dimensional scale parameter, $\mu$, which is a response of the Green functions of the theory in each energy scale. The change in $\mu$ induces a new adjustment in the coupling constant, mass and vertex renormalization of the theory, being governed by the RGEs. The coefficients in these equations are associated with the finite shift generated by a increase in this scale parameter for each of this three constants of the theory [14], [15], [16]. Concerning to the gauge couplings constants, the

\[ ^1 \text{In this notation } y \text{ stands for hypercharge, while } w \text{ stands for weak and } s \text{ stands for strong} \]
evolution of these parameters at one-loop level is governed by the following RGEs:

\[ \alpha_i^{-1}(\mu) = \alpha_i^{-1}(M_Z) - \frac{b_i^z}{2\pi} \log \frac{\mu}{M_Z}, \]

(2.1)

where \( \alpha_i^{-1} = \alpha_i^{-1}(M_Z) \) and the one-loop \( \beta \)-function coefficients \( b_i^z \) at \( M_Z \) scale are given by

\[ b_i^z = \frac{1}{3} \sum_R [s(R)N_i(R)] - \frac{11}{3} C_2(G_i) \]

(2.2)

with \( s(R) \) definite for the particle representations as

\[ s(R) = \begin{cases} 
1 \text{ complex scalar} \\ 
2 \text{ chiral fermions} \\ 
4 \text{ vector-like fermions}
\end{cases} \]

(2.3)

The \( C_2(G_i) \) is the Casimir group invariant of adjoint representation of Group \( G_i \) (equal to \( n \) for SU(\( n \)) and null for U(1)). The functions \( N_i(R) \) encode the group structure contributions as follows,

\[ N_i(R) = T_i(R) \prod_{j \neq i} d_j(R), \]

(2.4)

where \( d_j(R) \) is the dimension of the representation concerning the invariant subgroup, \( G_i \), and \( T_i(R) \) is the Dynkin index, which, in our convention, is 1/2 for the fundamental representations of SU(\( n \)) groups and \( Y^2 \) for the U(1)_{Y} group. We use the convention where such hypercharge of singlets coincides with its electrical charges, \( Y = Q - T_3L \).

For a theory beyond SM one can consider the contribution of \( N \) new thresholds that can include anything, as new fermions, scalars or other varieties and it is essentially content dependent. Thus, at one-loop level the \( \beta \)-functions comes out as

\[ \alpha_i^{-1} = k_i \alpha_i^{-1}_{\text{string}} + \frac{b_i^z}{2\pi} \log \frac{\Lambda}{M_Z} + \frac{\Delta b_i^1}{2\pi} \log \frac{\Lambda}{M_1} + \frac{\Delta b_i^2}{2\pi} \log \frac{\Lambda}{M_2} + \cdots + \frac{\Delta b_i^N}{2\pi} \log \frac{\Lambda}{M_N}. \]

(2.5)

Explicitly, for the SM we have

\[ b_y^z = \frac{20}{9} N_g + \frac{n_H}{6}, \]

(2.6)

\[ b_w^z = \frac{4}{3} N_g + \frac{n_H}{6} - \frac{22}{3}, \]

(2.7)

\[ b_s^z = \frac{4}{3} N_g - 11, \]

(2.8)

where \( N_g \) is the number of fermionic generations and \( n_H \) is the number of Higgs SU(2)_{L} doublets at \( M_Z \) scale.

Furthermore, at one-loop level, unless we couple new extra gauge bosons, only positive \( \Delta b_j \)'s are allowed \([17]\). Since additional gauge bosons at intermediate scales usually induces the fast proton decay, we will not consider this possibility. As a result we can estimate
an over contribution for the string unification without specifying the particle content that are needed, but after this estimative we introduce the necessary threshold(s) with the properties found in the analysis. Algebraically we can rewrite Eq. (2.1) in a more general form at string level for $N$ additional scales above electroweak scale. We simply have to consider an effective contribution to the SM, given by three new functions, $f_i$, for each gauge coupling constant,

$$\alpha_{iZ}^{-1} = k_i \alpha_{\text{string}}^{-1} + \frac{b_i^z}{2\pi} \log \frac{\Lambda}{M_Z} + f_i, \quad (2.9)$$

with $b_i^z = (41/6, -19/6, -7)$ being the SM $\beta$-function coefficients at one-loop level and the functions $f_i$ are the positive contributions which are necessary to reach the string unification at scale $\Lambda$. We notice that the quantities at $M_Z$ scale are experimental constants, then our parameter freedom is only in the couplings at string scale, the own string scale and in the functions $f_i$ since the freedom of couplings at new thresholds are inside these latter parameters. However, these first two quantities are put together to form the stringy character of theory. Thus the string scale is restricted by $\alpha_{\text{string}}$ in Eq. (1.2) which is transferred to $f_i$ functions. Then, a simplification could be obtained choosing the following parametrization

$$\Lambda = z \Lambda_s, \quad (2.10)$$

and $z$ we call stringy intensity, a parameter which is contained in the interval $1 \geq z > 0$ for perturbative theory. The perfect string unification occurs at $z = 1$ and, therefore, when $\Lambda = \Lambda_s$ and $\alpha_{\text{string}}^{-1} = 4\pi$. For values of stringy intensity below 1 the string scale is lower but $\alpha_{\text{string}}^{-1}$ increases very fast from the lower bound at $4\pi$. Considering the central values of electroweak constants [48],

$$\alpha^{-1} = 127.916 \pm 0.015, \quad (2.11)$$

$$\sin^2 \theta_W = 0.23116 \pm 0.00013, \quad (2.12)$$

$$\alpha_s = 0.1184 \pm 0.0007, \quad (2.13)$$

where the scale $\Lambda_s$ is given by

$$\Lambda_s = e^{(1-\gamma)/2} \frac{3^{-3/4}}{4\pi} M_P \approx 5.27 \times 10^{17}\text{GeV}, \quad (2.14)$$

where $\gamma \approx 0.577$ is the Euler constant.

Thus, using the canonical levels and the RGE coefficients in the Appendix A, the functions $f_i$ must have the necessary contributions showed in Fig. I. We can easily see that we have a wide space for all three parameters. Furthermore, these functions, besides the very different origin, have the same order in every stringy intensity. For the values of $z$ near the perfect string unification, greater as $z \simeq 90\%$, these three functions have the impressive approximate values, as showed in Fig 2. This means that it can be adjusted to string implementation naturally since neither of parameters is much favored or disfavored by the introduction of new content.
Figure 1: String Standard Model running at one-loops level with canonical SU(5) unification with allowed range of stringy intensity \( z \).

### 2.1.2 Two-loop analysis

The one-loop analysis of running of gauge couplings is a very good approximation. But the two-loop level analysis provides an extremely precise quantification of \( f_i \) functions \[44, 45, 46, 49\]. At this level we cannot find analytical solutions to the functions \( f_i \), however we can solve numerically the RGEs and then evaluate that functions with these numerical solutions for the RGEs starting at electroweak scale. Thus, now we have to solve the following equations at electroweak scale running to unification scale, \( \Lambda \),

\[
\frac{\partial \alpha_i^{-1}(t)}{\partial t} + \frac{1}{2\pi} \left( b^z_i + \frac{1}{4\pi} b^x_{ij} \alpha_j(t) \right) = 0, \tag{2.15}
\]

where \( b^z_{ij} \) are the \( \beta \)-function coefficients of RGE at two-loop level explicitly given by

\[
b_{ij} = \begin{pmatrix}
0 & 0 & 0 \\
0 & -\frac{136}{3} & 0 \\
0 & 0 & -102
\end{pmatrix} + N_g \begin{pmatrix}
\frac{95}{27} & \frac{1}{3} & \frac{44}{9} \\
\frac{1}{3} & \frac{49}{3} & 4 \\
\frac{11}{18} & \frac{3}{2} & \frac{76}{3}
\end{pmatrix} + n_H \begin{pmatrix}
\frac{1}{2} & \frac{3}{2} & 0 \\
\frac{1}{2} & \frac{15}{2} & 0 \\
0 & 0 & 0
\end{pmatrix}, \tag{2.16}
\]

in a SM extension with \( N_g \) generations and \( n_H \) Higgs doublets. To SM \( b^z_{ij} = b_{ij} \) with \( N_g = 3 \) and \( n_H = 1 \). On the other hand, the string constraint imposes that

\[
\alpha_i^{-1}(\Lambda) - f_i = \frac{4\pi k_i}{z^2}, \tag{2.17}
\]

where we insert the functions \( f_i \) in the same way as in the Eq.(2.9) and these functions are still positive. Now the unification is not a simple GUT scale but the string scale. With the introduction of the second loop correction to RGEs we can obtain the comparison with one-loop level, as showed in Fig. 2, in the region where \( z \geq 90\% \).
We can conclude that the canonical unification could be finely adjusted to include the string unification. Besides the string theory could be implemented in such non-SUSY scheme \cite{50} this is rather unnatural since string theory includes natively the supersymmetry. In this sense, we show in the next subsection the SUSY scenario.

2.2 Minimal Supersymmetric Standard Model

2.2.1 One-loop analysis

The supersymmetrization of SM is a well-motivated extension and could be tested in the collision experiments, for example, at LHC \cite{51, 52}. The minimal version, the Minimal Supersymmetric Standard Model (MSSM) changes from SM the RGE coefficients at $M_s$ scale. In this context the string theory is included more naturally \cite{53, 54}. At one loop level the MSSM with $N_g$ generations and $n_H$ Higgs doublets has the $\beta$-coefficients\footnote{Here we denote the SUSY $\beta$-coefficients by a bar over these quantities.} given by

\begin{align}
\bar{b}_y &= \frac{10}{3} N_g + \frac{n_H}{2}, \\
\bar{b}_w &= 2N_g + \frac{n_H}{2} - 6, \\
\bar{b}_s &= 2N_g - 9,
\end{align}

and these reads as $\bar{b}^Z = (11, 1, -3)$ for the MSSM with $N_g = 3$ and $n_H = 2$ at $M_Z$ scale. The new RGEs at one-loop level is then given by the new $f_i$’s functions showed in Fig. 3.

We notice that the SUSY results are more favored than SM since the inclusion of only few particles can reach the string unification scale due to function be somewhat in the
Figure 3: String Minimal Supersymmetric Standard Model running at one-loop (thick lines) and two-loop (thin lines) level with canonical SU(5) unification with allowed range of string intensity (z).

same interval but approximately three times smaller than SM $f_i$ functions. In this case the functions are limited by positivity and no result can be found for stringy intensity less than $\simeq 75\%$ with $M_s = 1$ TeV. Here we can see that the $f_y$ function is the most restrictive contribution to canonical MSSM since it covers only a small range in stringy intensity. Notice that even with introduction of gauge bosons, the parameter space is still constrained because, at one-loop level, only positive $f_y$ function is allowed due to the positivity of hypercharges in every threshold.

2.2.2 Two-loop analysis

In SM scenario we have seen that the challenging in $f_i$ function could be essential for the new particle content. For supersymmetry (in the $\overline{DR}$-scheme at $M_s$) the coefficients of RGEs at two-loop level are given by

$$
\overline{b}_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + N_g \begin{pmatrix} 190/27 & 2 & 88/9 \\ 3 & 14 & 8 \\ 9/3 & 3 & 68/3 \end{pmatrix} + n_H \begin{pmatrix} 1/7 & 3 & 0 \\ 7/2 & 7/2 & 0 \\ 0 & 0 & 0 \end{pmatrix},
$$

but there is no considerable variation from one-loop level as one can see in Fig. 3 (here none significant variation occurs even if SUSY scale is shifted from 1 TeV to electroweak scale at $M_Z$). At two-loop the functions decrease at most by a factor 1. Thus, with canonical Kać-Moody levels we can conclude that the MSSM is most plausible to string theory than the SM since it can be associated in a more natural way to the stringy character of the theory with only few thresholds. We will see in the next section that the same is not true in the non-canonical case of the SU(5)$_L \times$ SU(5)$_R$ gauge group.
3. SU(5)\(_L\) × SU(5)\(_R\) Model

We have seen that the canonical groups can be easily incorporated into string profile. The extension analysis to a non-canonical SU(5) × SU(5) theory with different Kač-Moody levels \[55, 56, 57, 58\] brings dramatically restrictive implications besides this small change from the canonical situation. For our purpose the weakly interacting heterotic string is a motivation for the use of crossed gauge group which could be a possible path to obtain the string group \(E_8 \times E_8\) near to \(\Lambda_s\). The gauge group \(SU(5)_L \times SU(5)_R\), specifically with the left-handed and right-handed components separated in each \(SU(5)\) invariant subgroup, should maintain the character of Georgi-Glashow in the left-handed sector but with a non-minimal modification. The non-SUSY \(SU(5) \times SU(5)\) has the following fermionic representations

\[
\psi_L = \begin{bmatrix}
D_1^c \\
D_2^c \\
D_3^c \\
e \\
-\nu
\end{bmatrix}_L \sim (5, 1), \quad \chi_L = \frac{1}{\sqrt{2}} \begin{bmatrix}
0 & U_3^c & -U_2^c & -u_1 & -d_1 \\
-U_3^c & 0 & U_1^c & -u_2 & -d_2 \\
U_2^c & -U_1^c & 0 & -u_3 & -d_3 \\
u_1 & u_2 & u_3 & 0 & -E^c \\
d_1 & d_2 & d_3 & E^c & 0
\end{bmatrix}_L \sim (10, 1) \quad (3.1)
\]

\[
\psi_R = \begin{bmatrix}
D_1^c \\
D_2^c \\
D_3^c \\
e \\
-\nu
\end{bmatrix}_R \sim (1, 5), \quad \chi_R = \frac{1}{\sqrt{2}} \begin{bmatrix}
0 & U_3^c & -U_2^c & -u_1 & -d_1 \\
-U_3^c & 0 & U_1^c & -u_2 & -d_2 \\
U_2^c & -U_1^c & 0 & -u_3 & -d_3 \\
u_1 & u_2 & u_3 & 0 & -E^c \\
d_1 & d_2 & d_3 & E^c & 0
\end{bmatrix}_R \sim (1, 10) \quad (3.2)
\]

where the first set of multiplets contains all left-handed components of SM and the second set contains all the right-handed SM ingredients. The introduction of SUSY do not introduces more fermion representations, but all the superfields should be written in the left-handed fashion as the MSSM, which could be made straight since we transfer the right-handed fields to conjugated representation in the Weyl representation instead of projections of complete Dirac multiplets \[59\]. The conventional fermions’ colors, flavors and electric charges are naturally indicated by notation. Here we are denoting the SM particles by lowercase letters while the capital letters indicates the vector-like particles that are naturally introduced in the theory to fulfill the multiplets. These particles introduces the main change in fermion sector. Then, we can notice that, beyond fermionic SM sector, we have only three types of particles, which are vector-like fermions, namely, \(U\), \(D\) and \(E\), apart from the natural right-handed neutrino, which is denoted by \(\nu_R\).

In this theory we have freedom to choose any route among the different possible symmetry-breaking paths due to a nontrivial potential that could be introduced once the scalar sector must be determined by the unification structure. Thus, we can adopt many symmetry-breaking patterns to electromagnetic theory as possible. We show explicitly
bellow two suitable samples \[30\]

\[
\begin{align*}
SU(5)_L & \times SU(5)_R \\
\downarrow & \Lambda \\
SU(3)_L & \times SU(2)_L \times U(1)_L \times SU(3)_R \times SU(2)_R \times U(1)_R \\
\downarrow & M_{LR} \\
SU(3)_{L+R} & \times SU(2)_L \times SU(2)_R \times U(1)_{L+R} \\
\downarrow & M_R \\
SU(3)_{L+R} & \times SU(2)_L \times U(1)_Y \\
\downarrow & M_W \\
SU(3)_{L+R} & \times U(1)_{EM}
\end{align*}
\] (3.3)

with the identification $SU(3)_{L+R} = SU(3)_C$, with both $SU(5)$ groups being broken simultaneously, and

\[
\begin{align*}
SU(5)_L & \times SU(5)_R \\
\downarrow & \Lambda \\
SU(3)_L & \times SU(2)_L \times U(1)_L \times SU(5)_R \\
\downarrow & v_R \\
SU(3)_{L+R} & \times SU(2)_L \times U(1)_Y \\
\downarrow & v_L \\
SU(3)_C & \times U(1)_{EM}
\end{align*}
\] (3.4)

with the $SU(5)_L$ being broken at higher energies than $SU(5)_R$.

Nevertheless, in approximation we can choose the simultaneous breaking of all intermediate patterns at string unification scale,

\[
\begin{align*}
SU(5)_L & \times SU(5)_R \\
\downarrow & \Lambda^{(all\ breakings)} \\
SU(3)_{L+R} & \times SU(2)_L \times U(1)_Y \\
\downarrow & M_W \\
SU(3)_C & \times U(1)_{EM}
\end{align*}
\] (3.5)

Thus, no symmetry breaking analysis is necessary neither complete scalar content is demanded in this context. A further analysis is required to determine the full setup and is beyond the scope of the investigation presented here. In this perspective, we shall run the RGEs starting only with the SM scalar at low-energy and for SUSY theory we include the additional MSSM scalar and all superpartners at $M_s$ scale.

In any of patterns presented above we can directly see that the number of $SU(3)_C$ color theory is always two, but only one $SU(2)_L$ is contained into a unique $SU(5)_L$, while the factor for hypercharge cannot be immediately inferred but is dependent on the combinations inducing to the $U(1)_Y$ in the branch. This implies in the modification of the canonical levels. Mathematically, if $\alpha_i$ is the coupling constant of invariant subgroup $G_i$, the numerical factors can be seen as \[12\]

\[
\frac{1}{k_i} \equiv \frac{\alpha_i}{\alpha} = \frac{\text{Tr}T_i^2}{\text{Tr}T_i^2},
\] (3.6)
where $T$ is a generator of the subgroup $G_i$, properly normalized over a representation $R$ of $G$, and $T_i$ is the same generator but normalized over the representation of $G_i$ embedded in $R$, with the traces running over complete representations. Thus, the only possible values for $k_i \ (i = w, s)$ are integer numbers \[1, 2, 3\].

The $\sin^2 \theta_W$ is computed differently from the Georgi-Glashow theory. Here it depends substantially on the type of unification of different gauge couplings for each of SU(5) invariant subgroup. For the general case, when these two couplings, designed as $g_L$ and $g_R$ for left-handed and right-handed SU(5) groups, respectively, one can easily check that at unification scale, $\Lambda$,

\[
\sin^2 \theta_W(\Lambda) = \frac{1}{1 + k_y/k_w} = \frac{3}{8(1 + \alpha_L/\alpha_R)},
\]

(3.7)

where $\alpha_k \equiv g_k^2/(4\pi)$. Once $\sin^2 \theta_W(M_Z)$ for canonical groups increases with scale and it is always bigger than electroweak scale at unification scale, it was presumable that it always happens regardless of what is introduced at intermediate scale \[61, 62\]. However, one should only estimate the value of $\sin^2 \theta_W(\Lambda)$ at string scale considering all intermediate thresholds. In this aspect, we have recently shown that the introduction of $(15 + \overline{15}, 1) + (1, 15 + \overline{15})$ play the main role, opening the range of possibilities to the unification \[63\]. In this sense, the correspondence of two SU(5) groups with the same gauge group could be reached perfectly with $\sin^2 \theta_W(\Lambda) = 3/16$ and $\alpha_L = \alpha_R$ if the scalar content is adequately selected.

We have notice that the right-handed neutrino singlet has been introduced into the fermionic representation and we shall discuss it shortly. Without introduction of gauge singlets, this is the first non-canonical extension to SM. Once the suitable scalar representation is chosen, the neutrino can have their mass naturally generated. For instance, distinctly from the mirror symmetry \[64\], here with the introduction of $(\overline{5}, 5)$ scalar representation, the neutrino can get the usual Dirac mass contribution. On the other hand, with the introduction of $(1, 15)$ scalar representation the neutrinos get a Majorana mass contribution. Differently to the usual Georgi-Glashow theory, the introduction of $(1, 5) + h.c.$ does not induce the Dirac mass term inasmuch as a purely gauge singlet is no longer included. Apart from this arguments, the universal seesaw mechanism, which naturally works for all charged leptons and vector-like fermions \[65, 66\], can also be included for neutrino sector \[60\], but unlike to the charged sector, for neutrinos there is no vector-like fermion associated and the seesaw mechanism works likely a generalized type II mechanism \[65, 66, 67\]. There are a very wide range of varieties on the seesaw mechanism and this is not the main scope of this article and so further analysis is also required.

4. Problems with SU(5) $\times$ SU(5)

Back to string theory, now we shall present the context of SU(5) $\times$ SU(5) unification by the use of RGEs. Using the Eq.(3.6), the non-canonical levels are explicitly given by $k_y = 13/3$ (where $8/3$ from left-right symmetries plus $5/3$ from canonical SU(5)), $k_w = 1$ and $k_s = 2$ (for the two SU(3) factors in the whole gauge symmetry) as long as the $\beta$-coefficients are keep hold. This simple change straightly affects the unification which is
thoroughly modified. Hence from the canonical analysis at two-loop of Sec. 2 now we are going to parse the $f_i$'s functions as a dependence of string intensity with this modified Kač-Moody levels.

4.1 Standard $SU(5) \times SU(5)$

We have seen that the second loop leads a small variation which could be important in the final parameter arrangement. If we consider the Eqs. (2.17) and (2.15), we can estimate the rough contribution required to the $f_i$ functions of $SU(5) \times SU(5)$ crossed group. In the standard context, as showed in Fig. 4, we remark that this theory only reaches unification with stringy intensity above approximately 96% at two-loop level for the SM content at $M_Z$ scale. Only a very small shift is obtained from one-loop correction. We can see also that this scenario is very disfavored since a very substantial improvements to $f_w \simeq 33$ and to $f_s \simeq 22$ are required while a very small contribution is mandatory to $f_y$. In this standpoint the parameter $f_y$ is strongly constrained by the stringy intensity. This is a consequence of $U(1)$ decomposition inside the group structure, which is abundant inner the crossed group and brings about a level $k_y = 13/3$, more than twice of canonical $5/3$. We ought to emphasize that all modifications are absorbed into $f_i$ functions as a parameter redefinition. Thus, we should wisely introduce new particle content seeing that only small contributions to $f_y$ is toughly generated by reason of almost all particles have non-vanishing hypercharges. Consequently it is very difficult to reach the string unification since for this purpose at intermediate scales only particles without hypercharge, but not singlets of $SU(2)_L$ nor/neither $SU(3)_C$, must be started up whilst hypercharge being the most abundant quantum number in any theory. As a matter of fact, any $SU(n)$ is broken with $U(1)$ in the pattern as well as $SU(5) \times SU(5)$ theory. Thus, with the introduction of any new particle $f_y$ function can be entirely spoiled. Notwithstanding, we will show that a solution in the context of Adjoint $SU(5)$ with a three fermion family sector is possible although inappropriate in our particular opinion.

4.2 Supersymmetric $SU(5) \times SU(5)$

The SUSY version is also well motivate in this non-canonical context being an important framework for string unification. In this respect, without introduction of any particle content, but with the $M_s$ at TeV scale, we can also examine the effects of the non-canonical Kač-Moody levels.

The Fig. 5 shows that we cannot obtain string unification since the function $f_y$ is always negative for $M_s = 1$ TeV at two-loop level. Furthermore, the function $f_s$ is only positive for stringy intensity above $z \simeq 97\%$. Notice that not even with gauge boson contributions the function $f_y$ can be raised to positive regions since the gauge group $U(1)_Y$ do not receives negative improvement at one-loop level to $\beta$-coefficients. In this sense the study of the minimal energy for $M_s$ scale is pertinent, which will be shown in the next subsection.

4.3 Small parameter space for $\alpha_1$

Once more we can show the contributions $f_i$ as functions of $z$, but now we choose the scale where the most restrictive function, $f_y$, matches to perfect unification at stringy
Figure 4: String Standard Model running at two-loop level with non-canonical $SU(5) \times SU(5)$ unification with allowed range of stringy intensity ($z$).

Figure 5: String Minimal Supersymmetric Standard Model running at two-loop level with non-canonical $SU(5) \times SU(5)$ unification with allowed range of stringy intensity ($z$).

Intensity $z = 1$. In this way we can evaluate the value for $M_s$ where no more contribution is needed to be added to $f_y$ function. We can see that, in this case, without introduction of any new content before $M_s$ scale, this scale is changed to $M_s \simeq 6.5 \times 10^{14}$ GeV at one-loop correction and to $M_s \simeq 1.3 \times 10^{15}$ GeV up to two-loop correction as can be seen in the Fig. 5.

The procedure employed above allowed us to replace the supersymmetry scale exactly to put $f_y = 0$ with perfect string scale. Now we need to introduce some new content with
vanishing hypercharge with enough contribution to \( f_w \approx 20 \) and \( f_s \approx 30 \). This inclusion is only possible with the introduction of scalar and fermion Adjoint sector below \( M_s \) scale. The fermion Adjoint contribution is necessary since purely scalar sector at very low-energy does not induce sufficient contribution even at \( M_Z \) scale. We discuss more on that in the next section.

5. Solution with Adjoint SU(5) subgroup

5.1 Model

Some years ago the study of adjoint fermionic content was proposed to correct the unification constraints by light fermionic triplets and correctly describe the neutrino mass with a mixed type I+III seesaw \(^{68}\). Indeed, many works and several applications have been considered in the unification direction of Adjoint SU(5)_L \(^{69, 70}\), as the seesaw for the neutrinos \(^{71}\) and also with the use of family symmetries with SUSY \(^{72}\). In this section we will briefly introduce this theory in an appropriate way and we show how this type of theory can help the string unification with significant modifications only in the \( f_w \) and \( f_s \) functions. The main modification in this scenario is due to the introduction of a fermionic adjoint multiplet in the representation (24, 1) of SU(5) \( \times \) SU(5), which belongs to SU(5)_L, at low energies. To avoid proton decay problems, only the vanishing hypercharge sector, namely \( \Sigma_3 \) and \( \Sigma_8 \), can be lowered below unification scale, or more generally speaking, at the minimal critical mass which induces proton decay. Further to the fermion sector, we can use also the scalar that can have similarly their \( \Sigma_3 \) and \( \Sigma_8 \) components below unification. The decrease of Adjoint components, specially both \( \Sigma_3 \)'s scalar and fermionic, are possible in many kinds of Adjoint SU(5) below TeV scale \(^{71}\) with a proton lifetime
Figure 7: Example of string unification without SUSY at two-loop level. The scale $M_1$ is where $\Sigma_3^F$'s, $\Sigma_3^S$ and $\Sigma_8^S$ are introduced, $M_2$ is where $\eta$ is included, the $M_3$ is the scale of $\Sigma_8^F$'s and $M_V$ is the scale of vector-like fermions.

in according to experiments while the neutral component of fermionic $\Sigma_3$ could also be a candidate to the Cold Dark Matter \[73\].

As we do not want any right-handed multiplet bellow unification scale, we only introduce the adjoint components in the left-handed sector. The modification in $\beta$-functions coefficients that modifies the running couplings for this new setup can be found in Appendix \[A\]. In order to induce adequate $f_w$ and $f_s$ contributions, in the non-SUSY as well as in SUSY case, we need three generations of adjoint sector. At one loop level the $f_y$ does not suffer any modification, but at two-loop level a small shift is required due to the interplay of the other two gauge couplings, $a_{1,2}^{-1}$.

5.2 Non-SUSY String Unification

We shall see that the non-supersymmetric case is slightly more restrictive than high-energy SUSY case. Indeed, even with all low scale contribution (at $M_Z$) of fermionic and scalar $\Sigma_3$ the function $f_w$ is still negative. As an attempt to save the unification character of the theory we need to introduce the $\eta \sim ((3, 3)_{-1/3}, 1)$ which comes from a possible scalar representation $H = (45, 1)$ and is the only possible contribution to fit the $f_w$ function to a perfect string scale at $z = 1$. This contribution to the RGEs can be seen in Appendix \[A\]. For completeness we have also run the RGEs starting from electroweak scale with an additional $SU(2)_L$ doublet, which is a component of $(45, 1)$. An example of string unification with adjoint content in non-SUSY scenario is showed in Fig. [ in the perspective of running of couplings. We can see that $\eta$ must be at very low scale while the vector-like contribution is needed at very-high scale essentially to fit the $f_y$ at perfect string scale.
Figure 8: Example of SUSY-string unification at two-loop level. The scale \( M_1 \) is where \( \Sigma_{F}^3 \)'s and \( \Sigma_{S}^3 \) are introduced, \( M_2 \) is where \( \Sigma_{F}^8 \)'s and \( \Sigma_{S}^8 \) are introduced while the high SUSY scale is denoted by \( M_s \).

5.3 High energy SUSY

We have seen in Sec. 4.2 that we can open the parameter space for SUSY if this symmetry is included at high-energy level. Some previous studies in the SM context have also been focused in this direction \[57, 58\] which is favored by string landscape \[74, 75, 76, 77\]. Now we are able to investigate this specific case with the introduction of adjoint components. The inclusion of adjoint super-multiplets (for both fermion and boson manifestation below \( M_s \)), with \( M_s \) scale at high-energy scale, provides the first approximation to relieve the theory.

The introduction of SUSY at high scale allows the string unification in the most promising setting. It can also accommodate the Adjoint theory \[78\]. The fermion masses can also be reconciled in a superstring scenario. For example, it was embodied into \( SO(10) \) model \[79\]. As an illustration we show in the Fig. 8 the string unification with stringy intensity \( z = 1 \), where the scalar and fermionic adjoint sectors are required.

In comparison to Fig. 8 we can notice that the role of \( M_s \) scale in the supersymmetric case is equivalent to the role played by vector-like fermions in Fig. 7, which diminishes the restrictive function \( f_y \) at high scales almost due to the interference of gauge coupling constants \( \alpha_{2,3} \).

This scheme exposes an economical outline for SUSY string unification from particle sight. The low scale of fermionic \( \Sigma_3 \) is compatible with recent works in neutrino type I+III seesaw mechanisms for the simple Adjoint \( SU(5) \) \[69, 80, 81\].

5.4 Proton decay

The non-SUSY extension is restricted by the proton decay. Indeed, due to the intro-
duction of (45, 1)-scalar the proton could decay very fast. It can be seen in the Yukawa interaction of $y_{1ij} \bar{\psi}^T_{iL} C \chi^j_L H^*$ and $y_{2ij} \bar{\chi}^T_{iL} C \chi^j_L H$ which are allowed, where $y_k$ are the Yukawa couplings. Omitting the family index, these terms induce the proton decay through interactions like $q^T_{mL} y_1 \bar{C} \eta^m \ell$ and $\epsilon_{mnp} q^T_{mL} y_2 \bar{C} \eta^m n q^L$, respectively; here $\epsilon = i\sigma_2$ and $m, n, p$ are color indices, and $\eta^m$ is the colored scalar triplet belonging to (45, 1)-scalar with a relative low mass.

The situation is only ameliorated with introduction of vector-like fermions at unification scale, but without perfect string unification. It can be pictorially understood looking to Fig. 7. As the $\eta$ do not affect the coupling $\alpha_1$ very much, we can see the quasi-constant property of its angular coefficient in the Fig. 7. In fact, the $\eta$ was chosen for this propose and, therefore, extremely reduces $\alpha_2$ and $\alpha_3$ since the $\alpha_1$ just have almost the exact value for perfect string unification. Without vector-like fermions this occurs near $z = 1$, but with a new $\eta$ scale, $M_2$, rather above than in the Fig. 7 and $M_3$, for vector-like fermions, somewhat bellow than this latter analysis. We emphasize that any attempt to correct the proton decay problem have to take into account that only particles with small hypercharge must be introduced.

In contrast, the SUSY extension is not restricted by proton decay since the interaction with (45, 1) is not necessary anymore. In addition, other possible interactions, that are allowed in simple SU(5) context, even in the minimal supersymmetric SU(5)\cite{8}, are forbidden in the context of SU(5) $\times$ SU(5) gauge group, which does not mixes heavy vector-like quarks with light quarks at leading order\cite{61, 62}. Explicitly, the interactions allowed due to scalar sector ($\bar{5}, 1$) are $q^T_{mL} y'_1 \bar{C} H^c_m \ell$ and $\epsilon_{mnp} U^T_{mL} y'_2 \bar{C} H^c_n D^c_{pL}$, which the colored triplet $H^c_m$ belonging to ($\bar{5}, 1$) interacts with SM doublets but with vector-like SU(2)$_L$ singlet quarks. Chosen reasonable parameters it leads to proton lifetime consistent with experimental bound \cite{48}, similarly to the analysis performed in \cite{61, 62}. The supersymmetric model is in this respect phenomenologically safe, besides being unconventional with a three-family adjoint representation.

6. Phenomenological implications and discussion

Since we have shown that this theory is consistency in SUSY scheme, now is imperative a further discussion on phenomenological consequences than proton stability. As well as in the proton decay argumentation, here the R-parity is conserved due to interactions between SM SU(2)$_L$ doublets be only possible with vector-like SU(2)$_L$ singlets. More specifically, in SUSY GUTs the source of R-parity-breaking is due to interaction of $\bar{5}510$\cite{61, 62}. In the class of SU(5) $\times$ SU(5) models, R-parity-breaking is induced only by $q^T \ell D^c$ and $U^c D^c D^c$, where we are representing all Weyl spinors in the left-handed fashion, and this type of interactions do not lead to R-parity violation involving light fermions.

Contrary to the high energy symmetry proposed here, this type of theory could give some traces by the vector bosons which could have their mass at TeV scale and, if is accessible, could be observed for instance in LHC\cite{35}.

\footnote{The conditions for proton decay suppression with colored scalar triplet in the SU(5) context can be found in Refs. \cite{38, 37}.}
On the other hand, the interactions with heavy vector-like fermions can contribute to baryon asymmetry of the universe (BAU) since there is a plenty of CP-sources to this aim and this depends on the specific scalar choice as well as in the pure Adjoint SU(5), via the usual leptogenesis mechanism [82].

Concerning to generalized seesaw mechanism, this model can easily accommodate all low-mass spectrum of fermions with the vector-like fermions at very high scale, both in string case presented here and in GUT level [63]. For this propose it is required that scalar sector be explicitly introduced but the interaction among vector-like fermions and electroweak fermions is keep hold, which could provide a very elegant explanation to the low-energy scale of SM fermions.

7. Summary and Conclusions

We have seen that the string unification, compatible with weakly interacting heterotic string, is not possible in the context of SUSY SU(5) × SU(5) symmetry even at one- or two-loop level if the SUSY scale, $M_s$, is a low-energy symmetry. Nevertheless, the high-energy SUSY is still as a possible theory. Affortunately, these non-SUSY and SUSY SU(5) × SU(5) models are possible if the theory is enlarged to incorporate the called Adjoint SU(5)$_L$ model with three generations. However, some problems with proton decay in non-SUSY case can arise due to the introduction of color-triplet, $\eta$, of $(45, 1)$-scalar representation which is necessary to complete string unification with correct value for $\alpha_{\text{string}}$. For both implementations we need very light fermionic $\Sigma_3$ particles that can be tested in the LHC experiments. Besides the implementation of this solution, no other can emerge easily in the context analyzed above. The SUSY case do not have problems with proton decay and R-parity since the usual R-parity-breaking terms and L-nonconserving terms only interacts SM particles with very heavy vector-like particles, inducing a suppression in both violation contributions.

Thus, we conclude that the standard view of the weakly interacting string heterotic unification, with supersymmetry at TeV region, in the context of SU(5) × SU(5) symmetry, is almost excluded. This conclusion is legitimate in the scenario of perfect unification at same string scale for all four interactions. If we relax the condition of $z = 1$ the SUSY is still forbidden at low-energy due to very small parameter space for $f_\eta$ but can be lowered by only a small factor which do not affect any order of magnitude.

Some modification can occur if we include a different extended pattern with many symmetry breakings to the unified group SU(5) × SU(5). Another change is due to a non-perfect gauge group in the sense of the need of two distinct gauge couplings at string scale for each SU(5) group. The first case is more restrictive than analysis performed here, since new scalar content are inevitable and can induce particles with non-vanishing hypercharge. In the second case the interpretation of unification must be modified for the four forces in these two different gauge couplings or the running of gauge couplings must be performed to indistinguishable crossed superior gauge group, e.g., directly to E$_8$ × E$_8$.

The SUSY SU(5) × SU(5) with $M_s$ at high-scale enforces a very small parameter space for the string unification guided by the hypercharge contribution with Adjoint SU(5)$_L$.
invariant subgroup. The latter is extremely restrictive since any new hypercharged particle found in the LHC can discard these type of models in few years. Only low-energy SM is a plausible theory embedded in this group and the desert of particles could be justified in this framework. Then, our main conclusion is that, besides it is possible to find unification with $\text{SU}(5) \times \text{SU}(5)$ group, the string unification is a more elaborated issue and is very disadvantaged in the scenario of $\text{SU}(5) \times \text{SU}(5)$ gauge group, suffering of a strong fine-tuned particle content.

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A. Beta coefficients at one- and two-loop

Here we show the beta coefficients for the new content that can be introduced to correct the string unification. Inside the $\text{SU}(5)_L$ group the notation, given in the $(\text{SU}(3)_c, \text{SU}(2)_L)_Y$ structure, is the following:

$$\begin{align*}
\eta &= (3, 3)_{-1/3}, & U &= (3, 1)_{2/3} + (\overline{3}, 1)_{-2/3}, \\
\Sigma^S_3 &= \Sigma^F_3 = \Sigma_3 = (1, 3)_0, & D &= (3, 1)_{-1/3} + (\overline{3}, 1)_{1/3}, \\
\Sigma^S_8 &= \Sigma^F_8 = \Sigma_8 = (8, 1)_0, & E &= (1, 1)_{-1} + (1, 1)_1,
\end{align*}$$

(A.1)

and $F$ refers to Fermion’s while $S$ refers to Scalar’s.

A.1 Without supersymmetry

A.1.1 one-loop

The $\beta$-coefficients at one-loop level is given by:

$$\begin{align*}
\Delta b_y &= \frac{N_\eta}{3} + \frac{16}{9} N_U + \frac{4}{9} N_D + \frac{4}{3} N_E, \\
\Delta b_w &= 2 N_\eta + \frac{2}{3} N^S_3 + \frac{4}{3} N^F_3, \\
\Delta b_s &= \frac{N_\eta}{2} + 2 N^S_8 + 4 N^F_8 + \frac{2}{3} N_U + \frac{2}{3} N_D,
\end{align*}$$

(A.2) (A.3) (A.4)

where $N_i$ is the number of representations showed above.
A.1.2 two-loop

The $\beta$-coefficients at two-loop level is given by:

$$
\Delta b_{ij} = \left( \begin{array}{cccc}
\frac{4}{9} & 8 & \frac{16}{7} & \\
\frac{8}{27} & 56 & 32 & \\
\frac{8}{3} & 12 & 14 & \\
\frac{3}{2} & & & \\
\end{array} \right) N_\eta + \left( \begin{array}{cccc}
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
\end{array} \right) N^S_{\Sigma 3} + \left( \begin{array}{cccc}
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
\end{array} \right) N^S_{\Sigma 8} + \left( \begin{array}{cccc}
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
\end{array} \right) N^F_{\Sigma 3} \\
+ \left( \begin{array}{cccc}
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
\end{array} \right) N^F_{\Sigma 8} + \left( \begin{array}{cccc}
\frac{64}{9} & 0 & \frac{64}{9} & \\
0 & 0 & 0 & \\
\frac{4}{9} & 0 & \frac{16}{9} & \\
0 & 0 & 0 & \\
\end{array} \right) N_U + \left( \begin{array}{cccc}
\frac{64}{9} & 0 & \frac{64}{9} & \\
0 & 0 & 0 & \\
\frac{4}{9} & 0 & \frac{16}{9} & \\
0 & 0 & 0 & \\
\end{array} \right) N_D + \left( \begin{array}{cccc}
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
\end{array} \right) N_E.
\right) (A.5)
$$

A.2 With supersymmetry

A.2.1 one-loop

The $\beta$-coefficients at one-loop level is given by:

$$
\Delta b_y = N_\eta + \frac{8}{3} N_U + \frac{2}{3} N_D + 2 N_E,
$$

(A.6)

$$
\Delta b_w = 6 N_\eta + 2 N_{\Sigma 3},
$$

(A.7)

$$
\Delta b_s = \frac{3}{2} N_\eta + 3 N_{\Sigma 8} + N_U + N_D.
$$

(A.8)

A.2.2 two-loop

The $\beta$-coefficients at two-loop level is given by:

$$
\Delta b_{ij} = \left( \begin{array}{cccc}
\frac{4}{9} & 8 & \frac{16}{7} & \\
\frac{8}{27} & 56 & 32 & \\
\frac{8}{3} & 12 & 14 & \\
\frac{3}{2} & & & \\
\end{array} \right) N_\eta + \left( \begin{array}{cccc}
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
\end{array} \right) N^S_{\Sigma 3} + \left( \begin{array}{cccc}
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
\end{array} \right) N^S_{\Sigma 8} + \left( \begin{array}{cccc}
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
\end{array} \right) N^F_{\Sigma 3} \\
+ \left( \begin{array}{cccc}
\frac{128}{9} & 0 & \frac{128}{9} & \\
0 & 0 & 0 & \\
\frac{8}{9} & 0 & \frac{32}{9} & \\
0 & 0 & 0 & \\
\end{array} \right) N_U + \left( \begin{array}{cccc}
\frac{128}{9} & 0 & \frac{128}{9} & \\
0 & 0 & 0 & \\
\frac{8}{9} & 0 & \frac{32}{9} & \\
0 & 0 & 0 & \\
\end{array} \right) N_D + \left( \begin{array}{cccc}
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
0 & 0 & 0 & \\
\end{array} \right) N_E.
\right) (A.9)
$$

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