Wave Physics of a Graphene Lattice Emulated in a Ripple Tank

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Using the example of graphene, we have extended the classic ripple tank experiment to illustrate the behavior of waves in periodic lattices. A loudspeaker driving air through a periodically perforated plexiglass plate onto the tank’s water surface creates wave patterns that are in agreement with numerical simulations and are explained in terms of solid state theory. From an educational point of view, the experiment provides an illustrative example of the concepts of reciprocal space and symmetry.

At public outreach events as well as in schools or undergraduate lectures, the ripple tank is widely used to illustrate wave phenomena. However, a standard ripple tank’s one or two dappers cannot create periodic wave patterns. This challenged us to modify the setup in a way that provides multiple periodic sources. Employing a loudspeaker and a plexiglass plate with hexagonally arranged holes we were able to realize wave patterns in the ripple tank, that are reminiscent of the microscopic images of artificial graphene. Two characteristic interference patterns are depicted in Figs. 1(a) and (b). We find that the strongest interference patterns are not obtained when the excitation wavelength is equal to the nearest neighbor separation $a_0$. Instead, the symmetry of the lattice favors wavelengths about 10 % shorter than $a_0$. The discrepancy hints at the non-trivial symmetry properties of the hexagonal lattice and is explained by means of solid state theory, testifying to the applicability of our ripple tank for educational purposes.

The paper is organized as follows: section I describes the technical details of the setup, in section II and III we measure its physical properties, and in section IV the favorable excitation wavelengths are extracted from the discrete Fourier transform of a finite hexagonal lattice. We close with summary and outlook.

I. SETUP

Figure 2 gives an overview of the modified ripple tank setup. A loudspeaker is used as a precise and inexpensive source of excitation. It is mounted face-down inside a plexiglass hood. An aluminium rack holds both the hood and an interchangeable source plate 295 $\times$ 295 $\times$ 4.2 mm$^3$ in size. The source plate seen in Fig. 2 is patterned with a hexagonal lattice of holes. Directly underneath the source plate we place a commercially available ripple tank (Leybold, No. 401 501, with see-through water trough, tilted mirror and observation screen). The distance between source plate and water surface is only a few mm. A white LED mounted in front of the center of the loudspeaker illuminates the ripple tank surface and can be used as stroboscope to observe a wave pattern at a fixed phase. To that end, the LED is typically powered for 1 % of time of each wave cycle, thus allowing the observation of the wave during a $4^\circ$ phase interval.

Both the stroboscope’s and the loudspeaker’s frequencies, relative phase-shift, wave form and amplitude are controlled with a LabView program on a standard PC via an USB Audio Adapter (LogiLink, UA0078). Between PC and loudspeaker or stroboscope, amplifiers are used to scale the signals appropriately. The loudspeaker requires between 0.3 and 3V AC input voltage, the LED 4V DC. The loudspeaker receives a sinusoidal input signal, the stroboscope’s waveform is set to rectangular.

In combination with the loudspeaker, the hole pattern...
of the source plate thus creates a controlled spatial pattern of point sources of sinusoidal-in-time airblasts directed downward onto the surface of the water. Since ripple crests/valleys act on the illuminating light as converging/diverging lenses, the resulting wave pattern on the water surface becomes visible as bright and dark areas projected onto the observation screen.

II. SINGLE SOURCE

We performed a series of single source experiments to determine the dispersion relation of water waves in our setup. The water height in the tank was kept constant at \( h = 1.8 \) cm. The wavelength lies between 0.5 cm and 2.5 cm. Since wavelength and water height are of the same order of magnitude, the dispersion relation for ripples is to a good approximation given by

\[
f(\lambda) = \frac{1}{\lambda} \sqrt{\frac{2\pi S}{\rho \lambda} + \frac{g \lambda}{2\pi}}
\]

where \( f \) is the frequency of the ripple, \( \lambda \) its wavelength, \( \rho \) the density of the water (1 g/cm\(^3\)), \( S \) the surface tension (73 g/s\(^2\)), and \( g \) the absolute value of the gravitational acceleration (981 cm/s\(^2\)).

For single source experiments, we fabricated a source plate with a single hole 2 mm in diameter. The frequency was swept from 5 to 40 Hz. With a digital camera, we took pictures of the observation screen. The radial difference between two adjacent bright circles in Fig. 3(a) corresponds to the wavelength. Fig. 3(b) shows our measured wavelengths in comparison to Eq. 1. Fitting the data we extract a measurement uncertainty of \( \sigma = 1 \) Hz for each data point. Thus the frequency of the water waves generated by our setup can be controlled with 1 Hz precision.

III. TWO SOURCES

Using a plate with two holes, we can emulate a double slit experiment. We choose this well-known geometry to compare measurement with simulation as well as study the effect the finite hole diameter has on the pattern.

The angles of constructive and destructive interference can be read from Fig. 3(c). Starting with the solution to the wave equation of a 2D time-harmonic source for a radially propagating wave

\[
\psi(r, k, \omega, t) = \frac{1}{2\pi \sqrt{2}} (H_0^2[kr]e^{i\omega t} + H_0^2[-kr]e^{-i\omega t})
\]

with \( H_0^2 \) the zero order Hankel function of the second kind, \( k \) the absolute value of the wave vector, \( r \) the radial distance from the source, \( \omega \) the radial frequency, and \( t \)
the time, we performed a Mathematica-based simulation of the 2D wave pattern.

The result of these calculations is shown in Fig. 3(d). To estimate the effects introduced by the finite size of the holes, we compared simulated wave patterns that took the finite diameter of the holes into account to those where an infinitesimally small hole diameter was assumed. We define the maximum effect of the finite hole diameter on the wave pattern as the maximum of the absolute value of the difference of those images. We found that this maximum effect corresponds to the ratio of source radius to separation between the two holes. This relation is also supported by our experimental observations during a series of variation of hole size and separation between the holes $a_0$. Since $a_0 = 1\text{ cm}$ and a hole diameter of 2 cm fulfill our requirements of accuracy and are at the same time convenient to realize experimentally, we used them for all subsequent source plates.

IV. HEXAGONAL LATTICE

The symmetry properties of the hexagonal lattice can be inferred by performing the Fourier transform of the wave pattern. We exploit that this transform can be expressed as the product of the Fourier transform of a single time harmonic point source and that of the lattice. We perform a discrete Fourier transform of a finite lattice and compare the results to key parameters known from the Fourier transform of an infinite lattice, which are deduced from the pseudopotential method for a crystal with a two-atom basis.

We created a 42 point lattice with $a_0 = 1\text{ cm}$ for the discrete transform. Figure 4(a) pictures this lattice in real space. Figure 4(b) shows the squared modulus of its discrete Fourier transform. As expected from the pseudopotential method we find a pattern with two classes of discrete Fourier transform. Figure 4(a) pictures this lattice in real space. Most importantly, it is not the correspondence of the 2D wave pattern.

![Figure 4](image)

FIG. 4. (a): Hexagonal pattern with 42 holes: nearest-neighbor distance $a_0$ is 1 cm. Dashed black lines indicate the unit cell of the lattice. (b): 2D discrete Fourier transform of the pattern depicted in a. Dark spots correspond to high amplitude. Purple arrows indicate the six lowest reciprocal lattice vectors, the black arrow indicates the next highest reciprocal lattice vector. (c): 2D discrete Fourier transform of a single time harmonic source with wavelength $\lambda = \sqrt{3}/2\text{ cm}$. (d): 2D discrete Fourier transform of 42 time harmonic point sources with wavelengths of $\lambda = \sqrt{3}/2\text{ cm}$.

This analysis illustrates that the resulting wave pattern in real space is, at the given wave length, essentially a superposition of six plane wave states. In general, the excitation wavelength, which determines the radius of the circle in Fig. 4(c), therefore serves as a tool to select the finite set of plane wave states that superimpose in real space. Most importantly, it is not the correspondence between the nearest-neighbor separation and the wave length in real space that gives rise to strong interference patterns, as one could naively think. Instead, the wavelength of strongest interference $\lambda = \sqrt{3}/2a_0$ corresponds to half the side length of the unit cell of the graphene lattice. The graphene lattice is built in real space by a triangular Bravais lattice with a two-atom basis. The basis vectors of the triangular lattice are given by

$$a_1 = a_0 \sqrt{3} \left(1/2, \sqrt{3}/2 \right), \quad a_2 = a_0 \sqrt{3} \left(-1/2, \sqrt{3}/2 \right).$$

To conclude, by showing the strongest interference
patterns when excited at those wavelengths that correspond to the symmetry of the triangular Bravais lattice, the modified ripple tank experiment illustrates that the unit cell of graphene is not its first Brillouin zone, the hexagon, but an equilateral rhombus, as indicated by the dashed black lines in Fig. 4(a).

V. SUMMARY AND OUTLOOK

With a modification that provides periodic sources, we have extended the classic ripple tank experiment to illustrate symmetry properties of the graphene lattice. The experimentally observed patterns can be reproduced closely by numerical simulations. A next step could be to introduce phase shifts between the individual point sources. However, this requires a completely new excitation technique. A more promising application for the present technique is to mimic electron waves in cavities. In this case, the apparatus could not only be used to make current science accessible to students or the general public, but might also prove to be a useful modelling tool for microstructured geometries that are costly and time-consuming to fabricate.

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