NATURE OF THE BACKGROUND ULTRAVIOLET RADIATION FIELD AT HIGH REDSHIFTS

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ABSTRACT

We have tried to determine the flux of the ultraviolet background radiation field from the column density ratios of various ions in several absorption systems observed in the spectra of QSOs. We find that in most cases the flux is considerably higher than what has been estimated to be contributed by the AGNs. The excess flux could originate locally in hot stars. In a few cases we have been able to show that such galactic flux can only contribute a part of the total required flux. The results suggest that the background gets a significant contribution from an unseen QSO population.

Key Words: quasars: absorption lines — diffuse radiation

I. INTRODUCTION

The shape and the intensity of the intergalactic UV background radiation are crucial factors in determining the ionization balance of the intergalactic medium and therefore influence the structure formation in the universe. Knowledge of this radiation field is thus necessary for the understanding of the early universe. AGNs are believed to be the major contributors to this background, though a significant contribution from star forming galaxies can not be ruled out. Detailed calculations of the propagation of AGN like ionizing radiation through intergalactic space, taking into account the absorption and reradiation by the galactic and intergalactic material, have been carried out by Haardt and Madau (1996, hereafter HM96). They have determined the frequency and redshift dependence of the background. Observationally, the intensity of the radiation has been determined in recent years, by studying the proximity effect in the Lyman alpha forest of the absorption lines in the spectra of QSOs (Bajtlik, Duncan & Ostriker 1988). This analysis is insensitive to the shape of the radiation (Bechtold, 1994 and Das and Khare, 1997). Values of the intensity of the background at the Lyman limit, J_{\nu,LL}, obtained by Bechtold (1994) and Cooke et al (1996) are considerably higher than the value expected from the distribution of visible
QSOs. Several sources of uncertainty in the value of the flux obtained by the proximity effect analysis have been considered by various authors (Bechtold, 1994, 1995, Srianand and Khare, 1995, Das and Khare, 1997). It has also been suggested (Fall & Pei 1993) that the actual number of QSOs may be larger than their observed number and that several QSOs may be rendered invisible due to dust extinction in the intervening absorbers. It is possible that the background radiation gets a significant contribution from star forming galaxies (Madau & Shull 1996, Giroux & Shapiro 1996, Khare and Ikeuchi, 1998). Here we try to obtain an independent estimate of the background flux by studying the ionization state of the QSO absorption systems for which an estimate of the particle density is available from the observations of the fine structure excited lines of C II. Wherever possible, we also try to estimate the contribution of the galactic flux to the total ionizing flux for the systems.

In section II we present our analysis, the results are discussed in section III.

II. DATA AND ANALYSIS

Several absorption systems reported in the literature, have the absorption lines of C II, C II*, C IV, H I, Si II, Si IV etc. Particle density in the absorber can be obtained from the column densities of C II and C II*. The column density ratio of C II to C IV is a good indicator of the ionization parameter as it changes rapidly with change in the ionization parameter, \( \Gamma = \frac{\Phi}{c n_H} \), where \( \Phi \) is the flux of the ionizing radiation i.e. the number of photons cm\(^{-2}\) s\(^{-1}\), \( n_H \) is the particle density and \( c \) is the velocity of light. This ratio is insensitive to the particle density and the abundance of carbon, but is very sensitive to the neutral hydrogen column density, \( N_{HI} \), in the absorber, specially for \( N_{HI} > 10^{17} \) cm\(^{-2}\) (Bergeron and Stasinska 1986). This particular column density ratio is also very sensitive to the shape of the ionizing radiation. This is shown in Fig.1 which shows the ratios for neutral hydrogen column densities of \( 10^{17} \) cm\(^{-2}\) and \( 10^{19} \) cm\(^{-2}\) for (i) spectral shape as given by HM96 for a redshift of 2.5, (ii) galactic spectra as taken from Bruzal (1983) and (III) a combination of both the above with the AGN flux taken to be same as the actual value given by HM96 for
These results have been obtained from the code ‘CLOUDY’, kindly supplied to us by Prof. Ferland. The galactic spectra produces much smaller values of the ratio compared to the AGN or power law spectra. This is due to the fact that the galactic spectra has much smaller number of photons having sufficient energy to produce C IV. Thus it is necessary to know the shape of the ionizing radiation to determine the ionization parameter from the C II to C IV ratio. In order to get information about the intensity and the shape of the radiation field we can make use of the column density ratio of other ions. Si II to Si IV ratio is an useful ratio for this purpose. In Fig. 2 we have plotted the ratio of column densities of Si II to Si IV for the three cases listed above. This ratio is not very sensitive to the shape and can be used with the C II to C IV ratio to constrain the shape as well as the intensity of the ionizing radiation. Fe II to Fe III and Al II to Al III ratios can also be used for this purpose. These ratios are also plotted in Fig. 2. We have, therefore, selected from the literature, absorption systems for which the column densities of H I, C II and C II*, or limits on their values, have been determined and for which additional ions like the C IV, Si II, Si IV, Fe II, Fe III etc are also available. The details are given in Table 1.

The fine structure excited level of C II is primarily populated by collisions with electrons for the absorption systems considered here, which are either the Lyman limit or the damped Lyman alpha systems, while the collisions with H I and also the collisional deexcitations by electrons can be ignored (Bahcall and Wolfe, 1968, Morris et al 1986). As we are considering the absorption systems at high redshifts, the excitation of the finestructure level by the absorption of the cosmic microwave background photons may be important. In order to check this we calculated the excitation temperatures of C II* from the observed column densities. These are much higher than the corresponding temperatures of the microwave background at the redshifts of the absorbers as can be seen from Table 2. We have therefore, ignored the excitation by the microwave background photons and have assumed the electron densities (assumed to be equal to the proton density) to be given by $n_e = 21 \left[ \frac{N_{C II^*}}{N_{C II}} \right]$ (Morris et al 1986). The hydrogen density obtained by adding the neutral hydrogen density to the electron density, corrected for the electrons coming from He II and He III, for each system
is also given in Table 2. For a few of the systems, only an upper limit on C II*/C II was available. For these systems only an upper limit on the particle density could be obtained. For these systems we have assumed a lower limit on particle density to be 0.045 cm$^{-2}$, which is smaller than all the lower limits to the particle densities obtained for the systems considered here and is also considerably lower than the mean interstellar particle density. For each absorption system we have constructed a number of photoionization models for the observed neutral hydrogen column density and different spectral shapes. The AGN spectral shape at the redshift of the absorption system is taken from HM96. For some of the systems only the total neutral hydrogen column density is available, while C II* has been observed in some particular velocity component of the absorption line. For such systems we have assumed the neutral hydrogen column density in individual components to be in the same ratio as the Si II column densities, as the ionization potential of Si II is close to that of H I. Details of the models and comparison of their predictions with the observations are discussed below for individual absorption systems. The results are given in Table 2. Note that all the absorption systems are sufficiently far away from the respective QSOs (relative velocity is greater than 15000 km s$^{-1}$) and can be considered to be intervening (however, see Richards et al, 1999) so that the radiation of the parent QSO can be ignored. In the following analysis we have only used the ratios of the column densities of different ions of the same element. Our conclusions are, therefore, independent of the assumed values of chemical abundances.

A. $z=1.7765$ system towards Q1331+170

This QSO has been observed by Kulkarni et al (1996). However, for this system C IV column density has not been reported and so we could not constrain the spectral shape. Errors on column densities have not been reported by the authors and we assumed errors of 25% in the column densities. Neutral hydrogen column density in the component showing C II* has been obtained from the total H I column density (Green et al 1995) by assuming
the H I column densities to be in the same ratio as the Si II column densities. Analysis of the Si lines assuming HM96 spectral shape for \( z = 2 \) yields 
\[ -2.4 > \Gamma > -2.6 \]

giving 
\[ 3.2 \times 10^7 \leq \Phi \leq 1.4 \times 10^8. \]

This is considerably higher than the value of HM96 flux at the redshift of the absorption system. It is, however, possible that the excess flux comes from the galaxies. We explored this possibility by constructing photoionization models with the shape as well as the intensity of the background as given by HM96 at the redshift of the absorber, the rest coming from the galaxies. For these models the limits become 
\[ -2.7 > \Gamma > -2.9 \]

giving 
\[ 1.5 \times 10^7 \leq \Phi_G \leq 6.9 \times 10^7, \Phi_G \text{ being the galactic flux in cm}^{-2} \text{ s}^{-1}. \]

B. \( z = 2.279 \) system towards Q2348-14

This QSO has been observed by Pettini et al (1994). An upper limit on the column density ratio of Si II to Si IV gives, for HM96 spectral shape for \( z = 2.5 \), a lower limit of 
\[ -2.2 \]

for log \( \Gamma \). However, as only an upper limit on the particle density could be obtained, this can not be converted to a limit on the flux. Assuming the minimum value of the particle density to be 
\[ 0.045 \text{ cm}^{-3}, \]

which is smaller than the observed upper limit by 1.2 dex, requires the flux to be larger than 
\[ 8.5 \times 10^6. \]

The column density ratio of Al II to Al III gives 
\[ -1.3 \]

\[ > \log \Gamma > -1.4, \]

giving 
\[ 5.3 \times 10^7 < \Phi < 1.2 \times 10^9. \]

Taking the actual value of the HM96 flux and assuming the rest of the contribution from the galactic sources requires 
\[ -1.4 > \log \Gamma > -1.5, \]

giving the galactic flux to be between 
\[ 4.1 \times 10^7 \text{ and } 9.7 \times 10^8. \]

C. \( z = 2.638 \) system towards PKS 2126-158

This system has been observed by Giallongo et al (1993) and has 7 components spread over a velocity width of 270 km s\(^{-1}\). A neutral hydrogen column density for the whole system has been obtained by Young et al (1979) to be 
\[ 1.1 \times 10^{19} \text{ cm}^{-2}. \]

We have assumed the neutral hydrogen column density in individual components to be in the same ratio as the Si II column densities. C II* is observed in two of the components. These are considered below.
(i) $z=2.6376$: We obtain $\log N_{\text{HI}} = 17.92$. As Si IV lines are not observed we take a 3 $\sigma$ upper limit on equivalent width, which translates to an upper limit of $10^{13.23}$ cm$^{-2}$ on the column density of Si IV, assuming a velocity dispersion parameter of 24.2 km s$^{-1}$, same as that for C IV. Assuming AGN shape, we get $-3.1 > \log \Gamma > -3.3$, giving $1.6 \times 10^6 \leq \Phi \leq 1.4 \times 10^8$. Taking the AGN flux to be that given by HM96 for $z=2.5$, and additional flux from the galaxies, the observed column density ratios can not be explained and a minimum flux of $9.4 \times 10^5$ from the AGN is required, needing a total of $-2.9 > \log \Gamma > -3.1$, giving $1.6 \times 10^6 \leq \Phi_G \leq 2.3 \times 10^8$. Note that the Al II to Al III ratio can not, however be explained by the same range of $\Gamma$ for any shape and requires $\log \Gamma \leq -3.3$.

(ii) $z=2.6364$. We obtain $\log N_{\text{HI}} = 16.4$. As Si II and Si IV lines have not been detected for this system, we could not obtain any constraints on the relative contribution from galaxy to the radiation flux. Taking all of the radiation to be AGN type, we obtained limits on the ionization parameter to be $-2.7 \geq \log \Gamma \geq -3.3$ so that the flux lies between $1.05 \times 10^6$ and $6.97 \times 10^8$. Taking the actual value of the AGN flux (HM96 value at $z=2.5$), the rest coming from galaxy, requires $\log \Gamma \geq -1.0$, giving $\Phi_G \geq 1.2 \times 10^{10}$.

D. $z=2.6522$ system towards Q2231-00

This Lyman limit system has been analysed in details by Prochaska (1999). We have reanalysed this system taking the shape of the background to be that given by HM96 for $z=2.5$. We determined the column density ratios for various ions for $\log N_{\text{HI}}$ of 19.12. The Fe II to Fe III and Si II to Si IV column density ratios constrain the ionization parameter to between -2.4 to -2.55 resulting in a background flux between $3.9 \times 10^6$ and $8.5 \times 10^6$. This is considerably higher than the value of $8.59 \times 10^5$ for HM96 for $z=2.5$. Models with the shape as well as the intensity of the background as given by HM96 for $z=2.5$, the rest of the flux coming from galaxy fail to yield a result as the column density ratios of Si and Fe can not simultaneously be produced by a single value (range) of ionization parameter. By gradually increasing the value of the flux contributed by the AGN background above the
HM96 value, we find that a minimum AGN flux of $3.3 \times 10^6$ was needed to explain the ion ratios, for which we get $-2.4 > \log \Gamma > -2.6$, requiring $\Phi_G$ to be between $6.7 \times 10^5$ and $5.2 \times 10^6$. Thus the minimum of AGN type flux required is more than a factor of 3.8 larger than that obtained by HM96.

**E. z=2.844 system towards HS1946+7658**

This system has been observed and analysed by Fan and Tytler (1994). Cloudy models with HM96 spectral shape at $z=3$ give $-2.3 > \log \Gamma > -2.8$, giving $10^8 \leq \Phi \leq 10^9$. A minimum flux of $8.4 \times 10^7$ of the HM96 type is needed, with $-1.9 \geq \log \Gamma \geq -2.6$, giving $1.9 \times 10^8 \leq \Phi_G \leq 3.9 \times 10^9$. Thus almost all of the flux is being contributed by galaxy. Note that the flux for $z=3$ of HM96 is $7.8 \times 10^5$.

**F. z=3.025 system towards Q0347-38**

This QSO has been observed by Prochaska and Wolfe (1999). Only a lower limit is available for the C II to C IV ratio, so that the galactic fraction of the flux could not be constrained. AGN shape for $z=3.0$ for the radiation gives $-2.6 > \log \Gamma > -2.8$, giving $2.1 \times 10^6 \leq \Phi \leq 9.8 \times 10^7$, for the assumed minimum value of the particle density. Taking the actual value of the HM96 flux for $z=3$ and assuming the rest of the contribution from the galactic sources requires $-2.5 \geq \log \Gamma \geq -2.7$, giving $1.9 \times 10^6 \leq \Phi_G \leq 1.2 \times 10^8$, for the assumed minimum value of $n_H$.

**G. z=3.6617 system towards Q2212-1626**

This QSO has been observed by Lu et al (1996). Only a lower limit is available for the C II to C IV ratio, so that the galactic fraction of the flux could not be constrained. AGN shape for $z=3.5$ for the radiation gives $-2.5 \geq \log \Gamma \geq -2.7$, giving $3.4 \times 10^6 \leq \Phi \leq 1.1 \times 10^8$ for the assumed minimum value of $n_H$. Taking the actual value of the HM96 flux at $z=3.5$ and
assuming the rest of the contribution from the galactic sources requires \(-2.4 > \log \Gamma > -2.5\),
giving \(3.8 \times 10^6 \leq \Phi_G \leq 1.4 \times 10^8\), for the assumed minimum value of \(n_H\). Note that the
value of flux for HM96 for \(z=3.5\) is \(5.0 \times 10^5\).

H. \(z=4.0803\) system towards Q2237-0608

This QSO has been observed by Lu et al (1996). Only a lower limit is available for the
C II to C IV ratio, so that the galactic fraction of the flux could not be constrained. AGN
shape for \(z=4\) for the radiation gives \(-2.6 > \log \Gamma > -2.7\), giving \(2.6 \times 10^6 \leq \Phi \leq 2.1 \times 10^7\)
for the assumed minimum value of \(n_H\). Taking the actual value of the HM96 flux at \(z=4\) and
assuming the rest of the contribution from the galactic sources requires \(-2.4 \geq \log \Gamma > -2.5\),
giving \(4.0 \times 10^6 \leq \Phi_G \leq 3.3 \times 10^7\), for the assumed minimum value of \(n_H\). Note that the
value of flux for HM96 for \(z=4\) is \(2 \times 10^5\).

III. DISCUSSION

For 5 of the systems we could derive the range of flux values assuming the radiation to
be AGN type. All of these are higher than the corresponding HM96 values by minimum
factors ranging from 1.2 to 158. Note that we have taken into account the uncertainties in
the column densities of all the ions which is the reason for obtaining large ranges for the
flux values. For three of these systems we could obtain a minimum value for the flux of the
AGN background. These values are 1.1, 3.8 and 98.8 times higher than the HM96 values at
the appropriate redshifts. For these systems a large flux is needed from galaxies. For 4 other
systems a lower limit to the flux could only be obtained with an assumption of the lower
limit on the particle density to be \(0.045 \text{ cm}^{-3}\), which is about half of the mean interstellar
value of the particle density and which indicates that the actual C II column densities are
higher than the observed lower limits by 0.64 to 1.37 dex. This is a reasonable lower limit
as the systems being considered are Lyman limit or damped Lyman alpha systems and also
as this value is considerably lower than the range of density values for systems for which
the values could be obtained from the observations. For these systems, the required values of flux are higher than the HM96 values by minimum factors of 2.7 to 62. On the other hand, assuming the AGN flux to be that given by HM96, and assuming the rest of the required flux to be of local, galactic origin, very high galactic flux is required. For most of the systems, this high flux requires the absorption systems to be present within 100 parsecs of typical O stars. The typical radius of the Stromgren spheres of these stars is of the same order, indicating that the absorption systems are inside the H II regions. Such conclusions have earlier been rejected on the basis of statistical arguments about the properties of the absorption systems (Srianand and Khare, 1994). The flux could come from QSOs which happen to lie close to the lines of sight at redshifts similar to the redshifts of the absorption systems. We have searched the catalogues for presence of any such QSOs near the line of sight to Q2231-00. However, no QSO is found to lie closer than 1000 Mpc to the line of sight within the required redshift range. The high values of the flux indicated by our analysis for almost all the systems, may be interpreted to indicate the presence of an unseen population of dust extinct QSOs.

Note that in all our analysis we have assumed that all the ions producing absorption in a given velocity range in an absorption system are physically located in the same region (cloud). This may not be always valid. Kirkman and Tytler (1999) and Churchill and Charlton (1999) have found evidence for ions with the same velocity structure in their absorption lines belonging to a given redshift system, arising in physically different gaseous components. If C IV ions are from a more widely distributed component, then, the C II/C IV column density ratio in the region of interest will be smaller and may require lower values of the flux.

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Figure Captions

**Figure 1.** Column density ratios of C II to C IV as a function of the ionization parameter, for different shapes of the background radiation spectrum as explained in the text. The solid lines are for $N_{\text{HI}}=10^{17}$ cm$^{-2}$ and dashed lines are for $N_{\text{HI}}=10^{19}$ cm$^{-2}$.

**Figure 2.** Column density ratios of Si II to Si IV, Al II to Al III and Fe II to Fe III. for the three shapes of the background radiation spectrum as explained in the text, for $N_{\text{HI}}=10^{18}$ cm$^{-2}$. Solid, dotted and dashed lines are for galactic spectra, AGN (HM96) spectra and AGN together with the galactic spectra respectively.
TABLE I. Observational data for the absorption systems

| QSO         | $z_{\text{abs}}$ | N$_{H I}$ cm$^{-2}$ | C$^\text{II}$/C$^\text{IV}$ | Si$^\text{III}$/Si$^\text{IV}$ | C$^\text{II}$/C$^\text{II}$ |
|-------------|------------------|----------------------|-----------------------------|----------------|----------------------|
| Q1331+170   | 1.7765           | 20.92a               | —                           | 7.71-18        | 0.035-0.060          |
| Q2348-14b   | 2.279            | 20.56± 0.075         | —                           | <1.458         | 0.002-0.046          |
| PKS 2126-158| 2.6364           | 16.401a              | 0.97-2.34                   | —              | 0.077-0.489          |
| PKS 2126-158c| 2.6376          | 17.92a               | 0.48-58.8                   | >6.6d          | 0.005-0.20           |
| Q2231-00e   | 2.652            | 19.12                | —                           | <1.2           | 0.002-0.003          |
| HS 1946+7658| 2.8443           | 20.23± 0.41          | 2.95-25.7                   | 0.32-7.07      | 0.012-2.570          |
| Q0347-38    | 3.025            | 20.80± 0.1           | >14.79                     | 10.59-18.5     | 0.002-0.028          |
| Q2212-1626  | 3.6617           | 20.20± 0.08          | >6.45                      | 1.99-2.51      | 0.002-0.046          |
| Q2237-0608  | 4.0803           | 20.52                | >10.984                     | 6.025-7.585    | 0.003-0.002          |

a: N$_{H I}$ in the components taken to be in the same ratio as N$_{Si II}$.

b: $\frac{Al^{II}}{Al^{III}}$ is 0.812±0.88 for this system.

c: $\frac{Al^{II}}{Al^{III}}$ is 6.918±41.68 for this system.

d: Taking 3σ upper limit to Si IV for this system.

e: $\frac{Fe^{II}}{Fe^{III}}$ is 0.338±0.461 for this system.
| \(z_{\text{abs}}\) | \(\frac{T_{\text{abs}}}{T_{\text{CMB}}}\) | \(n_H\ \text{cm}^{-3}\) | \(\Phi/\Phi_{\text{HM96}}\) | \(\Phi_{\text{HM96}}^{\text{min}}/\Phi_{\text{HM96}}\) | \(\Phi^a_G\) | \(d(\text{pc})^b\) |
|---|---|---|---|---|---|---|
| 1.7765 | 7.0-8.1 | 0.42-1.16 | 49.2-215.3 | 1.0c | 15.2-69.0 | 41.7-89.0 |
| 2.279 | 3.5-6.3 | 0.045d-0.82 | 62.3-1411 | 1.0c | 41.0-976.2 | 11-43.1 |
| 2.6364 | 6.6-15.3 | 0.07-11.66 | 1.23-811 | 1.0c | >12000 | <0.316 |
| 2.6376 | 3.7-9.5 | 0.11-6.28 | 1.88-176.1 | 1.1 | 1.6-236 | 22.5-273.9 |
| 2.652 | 3.3-3.3 | 0.047-0.072 | 4.58-10.0 | 3.8 | 0.67-5.2 | 151.0-423.0 |
| 2.8443 | 4.0-881.8 | 2.59-10.36 | 157.6-1997 | 98.8 | 194-3911 | 5.5-24.8 |
| 3.025 | 2.8-4.6 | 0.045d-1.31 | 2.73-126.4 | 1.0c | 1.9-122.0 | 31.1-110.1 |
| 3.6617 | 2.5-4.5 | 0.045d-1.23 | 6.78-234.4 | 1.0c | 3.76-145.6 | 25.4-120.2 |
| 4.0803 | 2.25-2.8 | 0.045d-0.28 | 13.45-106.5 | 1.0c | 4.06-33.87 | 59.54-108.4 |

\(a\) : In units of \(10^6\) cm\(^{-2}\) s\(^{-1}\)

\(b\) : Distance from O star.

\(c\) : \(\Phi_{\text{HM96}}^{\text{min}}\) is taken to be the actual value of \(\Phi_{\text{HM96}}\) at the redshift of the absorber in absence of necessary column densities.

\(d\) : Lower limit assumed to be 0.045 cm\(^{-3}\).