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Susceptible-Infected-Susceptible Epidemic Discrete Dynamic System Based on Tsallis Entropy

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Abstract: This investigation deals with a discrete dynamic system of susceptible-infected-susceptible epidemic (SISE) using the Tsallis entropy. We investigate the positive and maximal solutions of the system. Stability and equilibrium are studied. Moreover, based on the Tsallis entropy, we shall formulate a new design for the basic reproductive ratio. Finally, we apply the results on live data regarding COVID-19.

Keywords: Tsallis entropy; discrete dynamic system; equilibrium point; COVID-19; fractional calculus

1. Introduction

Discrete dynamic systems of SISE were extensively discussed for a long historical period, that successfully described the procedure in disease diffusion (see [1]). A decay ago, in the year 1927, the traditional SISE was offered [2]. After that, there established an enormous number of periodicals on SISE [3–5]. In overall, SISEs are considered to be homogeneously combined, which indicates that susceptible persons are infected with the same information. Nevertheless, there are various systems of populations in human culture [6], and the joining between persons is not identical. The stability and convergence of the systems are studied by using the basic reproductive ratio. This ratio is given in different formula based on the system and the situation of the solution. In our discussion, we suggest new formal of this ratio based on the entropy concept. For COVID-19, the researchers established a suitable ratio called the case fatality rate (CFR).

In [7], the researchers studied SISE at level-liberated networks; it designates that under the suitable parameters, there is probably a threshold at which the disease will persevere. In view of [7], some inoculation approaches are investigated, which additional develop the mechanisms of SISE on networks [8–10]. Obviously, the classes of difference equations have numerous practices in SISE system [11–13]. In reality, a positive interval, and discrete simulations often give information about disease [14]. On the other hand, a difference equation is the discretion of the continuous model [15,16], which indicates it practical to respond to the approximation method. Particularly, discrete simulations show a more complex dynamical conduct than the conforming continuous representations [17–20].

Under these compensations in attention, the state of the discrete SISE system of networks is about excessive investigation care. From the above-mentioned details, we shall deal with a discrete-time SISE system involving Tsallis entropy, which will be important work. We apply the results to live data regarding COVID-19.
2. The SISE Dynamical System Involves Tsallis Entropy

In traditional statistical methods, the entropy function formerly presented by Rudolf Clausius is construed as statistical entropy utilizing probability theory. The statistical entropy view was introduced in 19th century with the work of physicist Ludwig Boltzmann. This entropy was generalized by Tsallis as follows [21]: Consider a discrete set of probabilities \( \{\rho_j\} \) satisfying the condition \( \sum_j \rho_j = 1 \), and \( \alpha \) any real number, the Tsallis entropy is formulated by the terms

\[
\Upsilon_\alpha(\rho_j) = \frac{1}{\alpha - 1} \left( 1 - \sum_j \rho_j^\alpha \right), \quad \alpha \neq 1,
\]

where \( \alpha \) is a real parameter which is known as the entropy-index. One of the most important property of Tsallis entropy is that it has a maximum value determining when each micro-state is equiprobable \( \rho_j = 1/\Phi \) for all \( j \) and then we get

\[
\Upsilon_{\alpha}^{\text{max}} = \frac{1 - \Phi^{1-\alpha}}{\alpha - 1}, \quad \alpha \neq 1.
\]

If \( \alpha > 1 \) then \( \Upsilon_{\alpha}^{\text{max}} \to 1/(\alpha - 1) \) and if \( \alpha \to \infty \) then \( \Upsilon_{\alpha}^{\text{max}} \to 0 \) (see [22]).

Machado [23,24] presented novel formulas for entropy inspired by using the behavior of fractional calculus. The results of the generalized fractional entropy are examined both in usual probability distributions and data series. Moreover, by using the quantum deformed calculus, Hasan et al. [25] introduced a generalized q-entropy.

Numerous issues rule the transmissibility of the infection from the affected to the unaffected. In addition, disease dynamical systems can be investigated at altered rules: the single distinct, small collections of people, and among whole people. Different representations are selected given by the complexity of available data. In their contemporary avatar, computers that generate the numbers and distribution designs of infections simulate systems (see [26–29]).

The SISE system is formulated with \( N \) patrons and all the patrons are separated into \( n \) groups by their joints (junctions) \( j(j = 1, 2, ..., n) \). Consequently, it has \( N = \sum_{j=1}^n N_j \), where \( N_j \) represents the total number of the patron with position \( j \). It is considered that every patron has two positions, the first position is infected \( (I) \) and the second position is the susceptible \( (S) \). The susceptible patron may be infected with transmission ratio \( \tau \), and the infected patron may improve to a susceptible patron with repossession ratio \( \varrho \). Hence, we obtain the equation

\[
N_j(t) = S_j(t) + I_j(t)
\]

and the discrete system

\[
\begin{align*}
S_j(t+1) &= S_j(t) \left( 1 - \tau/T \ Y_\alpha(I_j(t)) \right) + \varrho T I_j(t) \\
I_j(t+1) &= I_j(t) \left( 1 - \varrho T \right) + \tau/T S_j(t) Y_\alpha(I_j(t)),
\end{align*}
\]

\( (0 \leq S_j(0) \leq N_j, \ 0 \leq I_j(0) \leq N_j) \)

where \( Y_\alpha(I_j(t)) \) is the Tsallis entropy introduced by the probability that any given connect points to an infected node and \( T \) indicates the time-step measure. It is a value indicating out that scheme (1) is recognized by employing the forward Euler pattern to the continuous SISE system and the equilibrium points (in discrete system they are equal to the fixed points) of structure (1) are similar as for the continuous equivalent. By letting

\[
S(t) = \sum_{j=1}^n S_j(t), \quad \text{and} \quad I(t) = \sum_{j=1}^n I_j(t),
\]
system (1) becomes
\[ S(t + 1) = S(t) \left( 1 - \tau T \, Y_a(I(t)) \right) + \varrho T \, I(t) \]
\[ I(t + 1) = I(t) \left( 1 - \varrho T \right) + \tau T \, S(t) \, Y_a(I(t)), \]
\( (0 \leq S(0) \leq N, \ 0 \leq I(0) \leq N, \ t = 0, 1, 2, \ldots) \).

Approximate (2) to entropy system, we have
\[ S(t + 1) = S(t) - \tau T \, \Lambda(Y_a(I(t)), Y_a(S(t))) + \varrho T \, I(t) \]
\[ I(t + 1) = I(t) \left( 1 - \varrho T \right) + \tau T \, \Lambda(Y_a(I(t)), Y_a(S(t))), \]
where \( \Lambda(Y_a(I(t)), Y_a(S(t))) = Y_a(I(t)) \times Y_a(S(t)) \).

By the maximum value of the Tsallis entropy, System (3) implies that
\[ I(1) = I(0) \left( 1 - \varrho T \right) + \tau T \, \Lambda(Y_a(I(0)), Y_a(S(0))) \]
\[ \leq N(1) + \tau T \, Y_a^{\text{max}} I \times Y_a^{\text{max}} S \]
\[ \leq N(1) + \frac{\tau T}{(a - 1)^2}. \]

By letting \( a \to \infty \), we have \( \lim_{a \to \infty} \frac{\tau T}{(a - 1)^2} = 0 \) for all fixed parameters \( \tau \) and \( T \). Thus, \( I(1) \) is bounded by \( N \). Moreover, since \( I(0) = N \) with \( \varrho T I \) then this yields that for \( a \to \infty \), the initial solution becomes \( I(0) = N \) and consequently the step one of solution becomes \( I(1) = N(1 - \varrho T) \geq 0 \), which leads to the non-negative solution \( I \). Hence, by induction, one can prove that \( 0 \leq I(t) \leq N \) for \( t = 0, 1, 2, \ldots \). By the above construction together with the initial condition \( S(0) = 0 \), we confirm that \( S(t) \leq N \) for all \( t = 0, 1, 2, \ldots \). Furthermore, since \( \varrho T > 0 \) then \( S(t) \to \varrho TI(t) \geq 0 \). We indicate that System (3) has a bounded non-negative solution. \( \square \)
3. Stability of SISE System

In this section, we aim to study the stability of SISE (1). By substituting $I_j(t) = N_j(t) - S_j(t)$ in the first equation of System (1), we have

$$S_j(t+1) = S_j(t) \left(1 - \tau_j \mathcal{T} \right) Y_a(I_j(t)) + \varphi \mathcal{T} (N_j - S_j(t))$$

$$I_j(t+1) = I_j(t) \left(1 - \varphi \mathcal{T} \right) + \tau_j \mathcal{T} S_j(t) Y_a(I_j(t)),$$

which is equivalent to the following system

$$S_j(t+1) = S_j(t) \left(1 - \varphi \mathcal{T} - \tau_j \mathcal{T} \right) Y_a(I_j(t)) + \varphi \mathcal{T} N_j(t)$$

$$I_j(t+1) = I_j(t) \left(1 - \varphi \mathcal{T} \right) + \tau_j \mathcal{T} S_j(t) Y_a(I_j(t)).$$

The disease free equilibrium of SISE (6) can be computed by the following construction

$$\Xi_0 = (S_1(0), S_2(0), ..., S_n(0), I_1(0), ..., I_n(0)) = (N_1, ..., N_n, 0, ..., 0).$$

By employing the linearization matrix method [19] on the system (6) at the point $\Xi_0$, we obtain

$$\Psi = \begin{pmatrix} \psi + \sigma & 0 \\ -\psi & \sigma \end{pmatrix}_{2n \times 2n},$$

where $\psi$ (the vector of new infections) and $\sigma$ (the vector of all other transitions including disease-connected deaths) are non-negative such that $\psi + \sigma$ is irreducible and $\Psi$ indicates the Jacobi matrix at $\Xi_0$ (we assume that this point is unique). Note that

$$\psi = \begin{pmatrix} \tau T N_1 Y_{a}^{\text{max}} & ... & \tau T n N_1 Y_{a}^{\text{max}} \\ \tau T n n N_1 Y_{a}^{\text{max}} & ... & \tau T N_n Y_{a}^{\text{max}} \end{pmatrix}_{n \times n},$$

and

$$\sigma = \begin{pmatrix} 1 - \varphi \mathcal{T} & 0 & ... & 0 \\ 0 & ... & 0 & 1 - \varphi \mathcal{T} \end{pmatrix}_{n \times n}.$$
The Basic Reproductive Ratio

The basic reproductive ratio ($\lambda_0$) can be explained as the predictable ratio of cases openly produced by one case in a resident where all persons are subject to infection. Mathematically, it is known as the spectral radius of the matrix $\psi(I_d - \sigma)^{-1}$ (the largest absolute number of the eigenvalues).

There are other different definitions and formulas can describe the situation properly. This ratio plays an important role to achieve the stability. It has been shown in many studies if $\lambda_0 > 1$ then we indicate an unstable situation and if $\lambda_0 < 1$ then the situation is asymptotically stable, while the case $\lambda_0 = 1$ indicates the stability, but not being asymptotic [19]. Recently, for COVID-19, researchers suggested the case fatality rate (CFR, the aim is to reduce this ratio) [32]

$$CFR(t) = \frac{D(t)}{I(t)} \times P(t), \quad t = 0, 1, 2, \ldots$$

where $D$ indicates the number of dying people. For example, if the number $D = 10$ and $I = 500$, then the ratio is 2%. Simultaneously, if it is recorded that there are 500 susceptible persons then

$$CFR_N(t) = \frac{D(t)}{I(t) + S(t)} = \frac{10}{1000} = 1\%.$$  

The idea of the probability of the survival function is not suitable for COVID-19. Therefore, based on our SISE system, we suggest to use the maximum entropy $\Upsilon_{\alpha}^{max}(I)$ as follows:

$$\lambda_0 = \frac{I(t) \times P(I)}{N(t)} = \frac{500 \times 0.98}{1000} = 49\%.$$  

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$$\lambda_0 = \frac{I(t) \times P(I)}{N(t)} = \frac{500 \times 0.98}{1000} = 49\%.$$  

Hence, the SISE system is stable, while for $\alpha < 1$, the system is unstable. For example, when $\alpha = 0.5$ and the probability $P(I) = 0.1$ this implies that $\Phi = \frac{1}{10^{0.5}} = 10$, we get $\Upsilon_{\alpha}^{max}(I) = \frac{1-10^{0.5}}{0.5} = 4.324$; which leads $\lambda_0(0.5) = 4.324 \times 0.5 = 2.162 > 1$.

From above, we conclude that Theorem 1 can be extended to include the stability as follows:

**Theorem 2.** Consider the entropy discreet system of SISE (3). If the conditions

$$0 < \varrho T < 1, \quad \alpha > 1$$

hold. Then every solution of (3) is bounded non-negative and stable satisfying the basic reproductive ratio

$$\lambda_0(\alpha) = \frac{I(t) \times \Upsilon_{\alpha}^{max}(I)}{N(t)}, \quad t = 0, 1, 2, \ldots$$

The survival function (or it called reliability function) is a function that offers the probability that a patient, scheme, or other thing of concern will survive further than any indicated time and it
is one of the techniques to define and show survival data. It states as the probability that a subject survives longer than time $t$. The distribution of survival times may be approximated well by a function such as the exponential distribution. Numerous distributions are usually utilized in survival analysis, containing the exponential, Tsallis entropy, gamma, normal and log-logistic. These distributions are formulated by parameters. The entropy optimization principle (includes the maximum entropy) converts it from a measure of information into an implement of statistics conclusively [34]. Since the higher maximum entropy goes to Tsallis entropy (see [35]), then it is a confidence to employ this fact to define CFR.

4. Applications

In this section, we use live data to examine our theoretical results, especially the stability of the SISE system by using $(\lambda)_a$. Table 1 shows the data from the first infected countries until the end of May. The rate of death is given by using $\text{CFR}_N(t)$. The basic reproduction ratio is evaluated by using $(\lambda_0)_a$ for some $\alpha > 1$. We use the conditions of Theorem 2 to get non-negative bounded and stable solution. One can consider the following system

$$
S(t+1) = aS(t)(1-S(t)) \times \Upsilon_{\alpha}(I)
$$

$$
I(t+1) = bI(t)(1-I(t)) \times \Upsilon_{\alpha}(I),
$$

(10)

where $a := \tau T$, $b := \varphi T$. Let $a = b = 0.4$, initial condition $(S_0, I_0) = (0.1, 0.1)$ and $\alpha = 2$, we have a stable limit cycle for the system of period one (see Figure 1). The red line shows the values of each considered case.

### Table 1. Data of COVID-19 until end of May ($\alpha = 2, 3, 4$).

| Country Name | Total ($N$) | Infected Number ($I$) | Death | $\text{CFR}_N$ | $(\lambda)_2$ | $(\lambda)_3$ | $(\lambda)_4$ |
|--------------|-------------|-----------------------|-------|---------------|---------------|---------------|---------------|
| USA          | 1,837,170   | 599,867               | 106,195 | 15%           | 0.326         | 0.163         | 0.097         |
| Brazil       | 514,992     | 279,096               | 29,341 | 12%           | 0.541         | 0.275         | 0.162         |
| Russia       | 405,843     | 171,883               | 4693   | 1%            | 0.423         | 0.211         | 0.127         |
| Spain        | 286,509     | 196,958               | 27,127 | 12%           | 0.687         | 0.343         | 0.206         |

**Example 1.** Consider the following system

$$
S(t+1) = aS(t)(1-S(t)) \times \Upsilon_{\alpha}(I)
$$

$$
I(t+1) = bI(t)(1-I(t)) \times \Upsilon_{\alpha}(I),
$$

(10)

where $a := \tau T$, $b := \varphi T$. Let $a = b = 0.4$, initial condition $(S_0, I_0) = (0.1, 0.1)$ and $\alpha = 2$, we have a stable limit cycle for the system of period one (see Figure 1). The red line shows the values of each considered case.

**Figure 1.** Bifurcation diagrams of System (10) with 100 iterations, when $a = b = 0.4$ and $\alpha = 2$ (left column, with initial condition $S_0 = I_0 = 0.1$). The middle column is for $a = b = 0.7$ and initial condition $S_0 = I_0 = 0.2$ The right column indicates the case $a = b = 0.9$ under the initial condition $S_0 = I_0 = 0.1$. All cases indicate a stable limit cycle of period one. The red line indicates the values of each case.
Example 2. Consider the following system

\[
\begin{align*}
S_i(t + 1) &= -a S_i(t)^2 \times Y_{\alpha_i}^{\text{max}}(I) + I_i(t) + 1, \\
I_i(t + 1) &= b S_i(t) \times Y_{\alpha_i}^{\text{max}}(I),
\end{align*}
\]  

(11)

where \( a := \tau T, b = \varrho T \) and \( \alpha = 2 \) with \((0.2, 0.6)\) as initial point. In all figures, the red line indicates the values of each considered case as follows:

- For \( a = 0.9, b = 0.3 \), the system has a limit cycle with period 4, while for \( a = 0.5, b = 0.3 \) the system has a limit cycle with period 2.
- For \( a = 0.1, b = 0.3 \), it has no limit cycle (see Figure 2). The positive fixed point of the third case is \( \varphi = 0.3 = (\lambda)_2 \) in the USA’s situation. While there are two positive fixed points (equilibrium point in the difference equation) for the first case, \( \varphi_1 = 0.7 = (\lambda)_2 \) for Spain and \( \varphi_2 = (\lambda)_3 \) for Russia.
- Also, for the initial condition \((0,0)\) and the case \( a = 0.9, b = 0.5 \), we get two positive fixed points \( \varphi_1 = 0.8 \) and \( \varphi_2 = 0.4 = (\lambda)_2 \) for Russia.
- Figure 3 represents the bifurcation behavior of the system (9) with the initial condition \((0,0), a = 0.1, b = 0.5 \) (left), \( a = 0.4, b = 0.3 \) (middle) and \( a = 0.1, b = 0.5 \) (right). In the second case, we have a limit cycle of period 2 and two positive fixed points \( \varphi_1 = 0.9 \) and \( \varphi_2 = 0.27 = (\lambda)_3 \) for Brazil.
- The last case \( a = 0.7, b = 0.2 \), we have two positive fixed points \( \varphi_1 = 0.7 = (\lambda)_2 \) (Spain) and \( \varphi_2 = 0.16 = (\lambda)_4 \) (Brazil).

Figure 2. Bifurcation diagrams of System (11), when \( a = 0.9, b = 0.3 \) (left), \( a = 0.5, b = 0.3 \) (middle) and \( a = 0.1, b = 0.3 \) (right). The value of the \( \alpha = 2 \). Note that \( x = S(t) \).

Note that the simplest situation is the case where there is no recuperation rate. This leads to an SI-like model, so that the pathogen infects all individuals on the long run. The simple continuous SI model has the logistic function as a solution and its discretized version is the logistic map, which presents the traditional bifurcation diagram as stable solutions.
Figure 3. Bifurcation diagrams of System (11), when $a = 0.1$, $b = 0.5$ (left), $a = 0.4$, $b = 0.3$ (middle, with a limit cycle of period 2) and $a = 0.7$, $b = 0.2$ (right, with a limit cycle of period 2). The red line represents the values of each considered case.

5. Conclusions

The correct balance between short- and long-term data loading in the world of big data has the following strategies:

- An applied perception based on the main usages for the data;
- The data constructions wanted for analysis;
- The relevancy of the data over time;
- Development with little organization is essential.

We prepared a new formula of the basic reproductive ratio $(\lambda_0)_\alpha$, which is defined by the Tsallis entropy. The formula is useful for the stability of SISE system involving the Tsallis entropy. It is related to long time data (by taking in account the above stratifies, it may modify for short time data). We applied the suggested system by using live data regarding COVID-19.

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