Measurement of the distance to the central stars of Nebulae by using Expansion methods with Alladin Sky Atlas

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Abstract. The usual methods of distance determination in Astronomy parallax and Spectroscopic with Expansion Methods are seldom applicable to Nebulae. In this work determination of the distances to individual Nebulae are calculated and discussed. The distances of Nebulae to the Earth are calculated. The accuracy of the distance is tested by using Aladin sky Atlas, and comparing Nebulae properties were derived from these distance made with statistical distance determination. The results showed that angular Expansions may occur in a part of the nebulae that is moving at a velocity different than the observed velocity. Also the results of the comparison of our spectroscopic distances with the trigonometric distances is that the spectroscopic distances are 55% larger. Since using trigonometric parallaxes with large relative measurement errors can introduce systematic errors, we carried out a Monte Carlo simulation of the biases introduced by selection effects and measurement errors. It turns out that a difference between both distance scales of the observed size is expected for the present day data if the underlying distance scales are identical.

1. Introduction

Assumption of Zero-Age Main Sequence Stars Spectroscopic parallax distance estimates involve comparison of (de-reddened) apparent magnitudes with absolute magnitudes, which are assumed based on stellar evolutionary status, to derive a distance modulus. Several early works reported distances ranging from 185 to 2000 pc (falling off the range of distance versus publication year. In the first modern set of distance estimates to the Orion Nebula region, derived a distance modulus (DM) of 8.5 magnitudes by comparing absolute magnitudes [1] along the main sequence with the de-reddened apparent magnitudes of B stars in the cluster. His method would be repeated by numerous authors in later studies[1,2] re-calculated the total absorption towards the three brightest stars in the Trapezium, used a different absolute magnitude relation[2] and derived a much smaller distance modulus, 7.38 magnitudes [1,2].

Eight methods are considered as independent ways of determining the distance. Some of the methods are reliable than others, but most have a limited application. Each of the methods , their expected accuracy and some results for the more reliable methods will be discussed in turn [3].

(1) Trigonometric parallax. Unfortunately, none of the nebulae are close enough to allow distances to be reliably determined. A possible exception is NGC 7293 for which parallax of 0.020+..006 has been given, corresponding to a distance of 50 pc. This value is very uncertain.

(2) Spectroscopic parallax. Some planetary nebulae are excited by stars which have binary companions. If the spectral type and luminosity class of the companion (which is presumably a ‘normal’ star since it has a ‘normal’ spectrum) can be measured, a spectroscopic distance for the system can be determined this method may have a wide application, since it is estimated that at
least 10% of the exciting stars are binaries. The method may be applied both to visual binaries (where both stars are seen separately) and to spectroscopic binaries where the ‘normal’ star dominates the spectrum. Only a very limited number of such cases have been well studied. These nebulae are all nearby. This is consistent with the fact that the nebulae all have large angular diameters.

(3) Expansion distances. Radial velocity measurements of the nebulae show a splitting of emission lines. This is interpreted as an expansion of the nebulae, with velocities typically of 20 km s\(^{-1}\). This expansion velocity may be compared with angular expansions derived from studying the location of knots, filaments, edges, and other features seen on both old and new photographs [3].

2. Basic Consideration and Equations

2.1 Distance modulus:

The distance modulus \( \mu = m - M \) is the difference between the apparent magnitude \( m \) and the absolute magnitude \( M \) of an astronomical object. It is related to the distance \( d \) or \( D \) in parsecs by [3,4]:

\[
\log_{10}(d) = 1 + \frac{\mu}{5} 
\]

\[
\mu = 5 \log_{10}(d) - 5 
\]

............ (1)

\[
m - M = 5\log_{10}(D(pc)) - 5 
\]

............. (2)

Where: \( m \) = Apparent magnitude; \( M \) = Absolute magnitude; \( D \) = the distance in parsecs.

This definition is convenient because the observed brightness of a light source is related to its distance by the inverse square law (a source twice as far away appears one quarter as bright) and because brightness’s are usually expressed not directly, but in magnitudes.

Absolute magnitude \( M \) is defined as the apparent magnitude of an object when seen at a distance of 10 parsecs. Suppose a light source has luminosity \( L(d) \) when observed from a distance of \( d \) parsecs [4].

2.2 Spectroscopic distances

Spectroscopic distances can be derived from the model atmosphere analyses described in [5] and offer an approach to the PN distance scale independent from the properties of the nebulae. After the stellar mass is estimated from a comparison with evolutionary models [5] the distance can be calculated in a straightforward way from effective temperature, gravity, and the reddened apparent magnitude of the stars [5].

1. The stellar radius is computed from surface gravity and mass.
2. The surface flux in the V band is calculated from model atmospheres, with the appropriate parameters (thus bypassing the uncertain bolometric corrections),
3. and finally the absolute magnitude \( M_V \) is derived and compared with the red end \( V_0 \) magnitude of the star.

4. This procedure can be condensed into one formula [6]:
\[ D = 0.111 \frac{\sqrt{M \times F_V}}{g} 10^{0.4v_0} \]  

\[ \text{Where: } M = \text{the solar mass in } M_\odot; \]
\[ g = \text{is the surface gravity in } \text{cm/s}^2; \]
\[ F_V = \text{is the model atmosphere flux in } 10^6 \text{erg cm}^{-2} \text{s}^{-1} \text{Å}^{-1}. \]

2.3 Expansion distances

Distances for PNe can also be determined by the parallax expansion method as is usually done for resolved novae. As a nebula expands its rate of motion appears as a proper motion measurable in arc seconds per year. Spectra obtained at the same period of time give the expansion velocity directly in kilometers per second. If the expansion is uniform then the proper motion corresponds to the same velocity. The geometry of the expansion allows to find the distance D to the nebula as[7]:

\[ D_{\text{pc}} = \frac{V}{211} \frac{\text{km/sec}}{\text{mas/yr}} \]  

\[ \text{Where } V = \text{is the radial velocity; } \Theta = \text{is the proper motion.} \]

This method has been applied both at optical and radio wavelengths to determine distances to PNe [7].

3. Determine the physical size of the PN using the small-angle formula

Explains the derivation of the small-angle formula [8]

\[ l = \frac{r \Theta}{206265} \]  

\[ \text{Where } l : \text{is the physical size, } r : \text{is the distance (note that } l \text{ and } r \text{ must be expressed in like units } \text{Mpc for galaxies) as shown in figure 1. } \Theta : \text{Angular size of nebulae in arcs units} \]

\[ 206265 = \text{is the number of arc seconds in a radian [7,8] } \]

4. Aladin(atlas sky)

Aladin is an interactive sky atlas developed and maintained by the Centre de Donnie’s astronomies’ DE Strasbourg (CDS) for the identification of astronomical sources through visual analysis of reference sky images. Aladin allows the user to visualize digitized images of any part of the sky, to superimpose entries from the CDS astronomical catalogues and tables, and to interactively access related data and information from various data servers [7,8].

In this case we use Aladin in the undergraduate mode (developed in the frame work of the EuroVO-AIDA European project) [8,9]

edit -> user preferences -> profile ->
undergraduate. Restart Aladin in order to validate the new Configuration [9,10] Open the server selector window as shown in figure 2:

File -> Load astronomical image

-> Aladin image server [8,11]

![Server selector window for Aladin Sky Atlas-v9.0](image)

Figure 2. Server selector window for Aladin Sky Atlas-v9.0 [8]

5. The Results

In this work the spectroscopic distances were calculated by using the Equation (1) and compared with observed distance as shown in table 1:

Table 1. Represents the values of Spectroscopic distances of nebulae with extinction E(B-V).

| NP       | SPECTRAL TYPE | mv    | Mv   | E(B-V) | Observed D(pc) | The calculated D(pc) |
|----------|---------------|-------|------|--------|----------------|-----------------------|
| NGC 245  | K0 V          | 14.3  | 5.9  | 0.01   | 470            | 478                   |
| NGC 1514 | A0 III        | 9.42  | -0.2 | 0.045  | 400            | 470                   |
| NGC 2346 | A2 V          | 11.12 | 1.4  | 0.22   | 640            | 690                   |
| NGC 3132 | AO V          | 10.06 | 0.7  | 0.07   | 670            | 744                   |
| A CO 35  | G8 III-IV     | 9.63  | 1.9  | 0      | 360            | 346                   |
According to the relation (1) had been showed between (D) and (E(B-V)) , as along as the distance increased the Extinction E(B-V) increased too, leads to conclude that when the nebulae has largest observed distance (D) the value of E(B-V) stron and that will increase as shown in figure 3,A. The values of the calculated D is less than the values in observed D . the relation between Calculated D and extinction E(B-V) shows a big range as(400-700) in the first interval . The distance increases the extinction increase too, as shown in figure 3,B.

Figure 3. A- Represents the relationship between the Observed distance and the extinction E(B-V) , B- Represents the Calculated distance in this work.

The angular Expansion distances were calculated by using the Equation 3 and compared with observed distances as shown in table 2, also the angular expansion were be calculated by using equation 4 for the adopted samples as estimated in table 3. The results compared with the observed values.

Table 2. Represents the values of the angular expansion rate and resultant distance.

| NP      | $\theta$'' (''/100years) | $\theta''$'' | V(KMS$^{-1}$) | Observed D(pc) | The calculated D(pc) |
|---------|--------------------------|--------------|--------------|----------------|---------------------|
| NGC246  | 1.4+0.5                  | 100          | 38           | 620            | 682.9               |
| NGC3242 | 0.83+0.25                | 15           | 30           | 760            | 767.5               |
| NGC3587 | 2.0+1.0                  | 99           | 41           | 430            | 453.4               |
| NGC6572 | 0.81+0.10                | 49           | 16           | 420            | 418.7               |
| NGC6720 | 0.9+0.1                  | 40           | 30           | 700            | 705.109             |
| NGC7009 | 0.75+0.3                 | 14           | 21           | 600            | 596.8               |
| NGC7662 | 1.0+0.6                  | 10           | 26           | 550            | 560.9               |
|         | 0.6+0.17                 | 10           | 26           | 900            | 917.4               |
| NGC2392 | 0.72+0.06                | 6            | 18           | 530            | 528.5               |
Where $V$: the radial velocity of the nebula in km/s unit.

Table 3. Represents the values of angular sizes and physical size with the observed and calculated distances for the adopted sample.

|       | $\Theta$ (arcsecond) | $L$ (Jy) | $D$ (pc) Observed | $D$ (pc) Calculated |
|-------|----------------------|----------|-------------------|---------------------|
| A30   | 63.5                 | 0.023    | 2020              | 1120                |
| A31   | 486.0                | 0.155    | 214               |                      |
| A33   | 134.0                | 0.014    | 1060              | 794                 |
| IC4637| 9.3                  | 0.132    | 1500              | 1525                |
| KI-14 | 23.0                 | 0.001    | 3500              | 3299                |
| KI-22 | 90.5                 | 0.011    | 1130              | 1033                |
| Mz3   | 12.7                 | 0.649    | 1440              | 916                 |
| NGC1535| 9.2                | 0.160    | 2310              | 1646                |
| NGC3132| 22.5              | 0.230    | 770               | 1250                |
| NGC7008| 14.0               | 0.217    | 870               | 1233                |
| SP3   | 17.8                 | 0.061    | 2380              | 1965                |
| A21   | 307.5                | 0.327    | 540               | 278                 |
| A35   | 368.0                | 0.255    | 230               | 255                 |
| NGC1514| 80.0               | 0.288    | 480               | 540                 |
| NGC6853| 165.0              | 1.324    | 380               | 310                 |
| NGC7296| 300.0              | 1.291    | 210               | 220                 |
| PHL932| 135.0                | 0.0100   | 310               | 540                 |

As the angular sizes and physical size increase the distance ($D$) decrease. Stretch a large $\Theta$ be a close distance as shown in figure 4 in a good agreements with the observed values for the angular size and distances as shown in the figure 5. The results showed the same behaviors and agree with observed values.

**Figure 4.** The angular size with calculated distances for the sample that adopted in table (3)
In this work the Aladin sky Atlas were be used to calculate the distance of the nebulae through Atlas Aladan program. Where some samples will be adopted for the nebulae and we introduced these samples in the program as the results show: To find the distance by using spectra method we will do the following steps in Aladin Atlas:
1- Choose the objects that we need to find the D(pc).
2- Determine the location related to our sample.
3- The simplest method to obtain the linear size of the nebula is to multiply the expansion velocity by the time passed since the explosion.
4- Need only to know the year the image has been taken. And calculate the time passed since the Explosion.

5- Determine the Angular size of the nebula:
In order to compute the angular size of the nebula we use the tool “dist” and track a vector from the centres of the nebula to the outermost visible part. The outer parts of the nebula are very faint, so we adjust the intensity distribution by increasing the contrast of the image. The centre of the nebula matches with the pulsar (marked on the image).

6- now have both the linear and the angular size of the nebula: by comparing them we can compute its distance from any object near it. In order to obtain the distance in parsec we have to express the sizes in the proper units: the linear size in parsec and the angular size in radians.

\[ 1 \text{ pc} = 3.085 \times 10^{16} \text{ m}, \quad 1 \text{ rad} = 206264'' \]

7- finally compute the distance to the nebulae by using the following relation [4]:

\[ d = R / r \text{ (pc)} \]

Where R is represents the radius of Nebula in parsec unit.

5.1 Linear size of the nebula.
The simplest method to obtain the linear size of the nebula is to multiply the expansion velocity by the time passed since the explosion. Observations performed several years apart reveal the slow expansion of the nebula, telling us the expansion is accelerating. In this tutorial we analyses an image of the Nebula as recent as possible and consider the expansion constant. This simplification gives us an approximated result, but enough accurate for our aims.

Start Aladin and switch to the undergraduate mode from the menu:
edit -> user preferences -> profile ->

![Figure 5](image-url)
undergraduate.
Restart Aladin in order to validate the new
configuration.
Open the server selector window:
File -> Load astronomical image
-> Aladin image server
In the field “target” enter “NGC246” and press “submit”. The list of available images appears (figure 6): look at the dates of the images and load in Aladin the most recent one (taken in 1994). We use this
the Skull (NGC246) Nebula image in the next step[11], when we will compute its projected angular
size.
Now we need only to know the year the image has been taken. We calculate the time passed since the
explosion (1785):
\[1\text{yr} = 3.154 \times 10^7 \text{s}.\]
\[(1994 - 1785) \text{yr} = 209 \text{yr} = 6.591 \times 10^{10} \text{s}.\]

**Figure 6.** The server selector window with the list of available image.

We multiply this time by the velocity and obtain the linear size (the radius) of the nebula:
\[R = 6.591 \times 10^{10} \text{s} \cdot 38 \text{ km/s} =
= 2.54 \times 10^{13} \text{ km}.\]

**5.2 Angular size of the nebula**
The modulus of the projected angular distance vector 7.6'. In the case of figure 7. it is
\[r = 4.103'. \] the angular size in radians:

\[1 \text{pc} = 3.085 \times 10^{16} \text{ m}\]
\[R = 2.54 \times 10^{13} \text{ km} = 2.54 \times 10^{16} \text{ m} = 0.8233 \text{ pc}.\]
Then
\[1 \text{ rad} = 206265''\]
\[r = 4.103' = 246.18'' = 1193 \times 10^{-3} \text{ rad}.\]
Finally compute the distance to the Crystal Ball Nebula.
\[d = R / r = 0.8233 \text{pc} / 1193 \cdot 10^{-3} = 690 \text{ pc}.\]
Figure 7. The distance vector for computing the angular size of the nebulae

In this sample, we calculated the distance from the nearest star through it is (UCAC2 27592058 (12.949, Star). The correct value of the distance to the Skull Nebula is about 620 pc, very close to the value obtained with our simple analysis. In particular, our value is slightly higher because of the approximations of our procedure.

6. Conclusions

The distances of planetary nebulae are discussed as derived from their angular expansions and radial expansion velocities. An assessment is given of distances derived by this method both at optical and radio wave lengths. The recent VLA radio data show promise in establishing a more accurate distance scale for planetary nebulae. Both the linear and the angular size of the nebula are comparing and computed their distance from us in parsec unit.

We have to express the sizes in the proper units: the linear size in parsec and the angular size in radians.

The distance calculated in with spectroscopic less distance in with expansion where the highest value in the spectroscopic is 879 pc. While the expansion is 917.4 pc, so note that the increase occurring in expansion.

7. References

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