About Geometric Scaling and Small-$x$ Evolution at RHIC and LHC

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We show that the whole range of RHIC data for hadron production in $d$-$Au$ collisions is compatible with geometric scaling. To establish the scaling violations expected from small-$x$ evolution a larger kinematic range in transverse momentum and rapidity would be needed. We point out that the fall-off of the $p_t$ distribution of produced hadrons at large $p_t$ is a sensitive probe of small-$x$ evolution especially at the LHC.

It is clearly observed that the small-$x$ DIS data show the property of geometric scaling (GS) [2], i.e. the cross section depends on the combination $Q^2/Q_s^2(x)$ only, where $Q_s(x)$ is referred to as the saturation scale. On the other hand, geometric scaling is a feature of the asymptotic solutions of nonlinear evolution equations, such as the BK equation [3]. Hence, geometric scaling is seen as a strong indication for small-$x$ gluon saturation.

A phenomenological study of DIS data using a model for the dipole cross section was performed by Golec-Biernat and Wüsthoff (GBW) [4]. They found that the inclusive HERA data at low $x \ll 0.01$ could be described well by a dipole cross section

$$N_{GBW}(r_t, x) = 1 - \exp \left( -\frac{1}{4} r_t^2 Q_s^2(x) \right).$$

The $x$-dependence of the saturation scale is given by

$$Q_s(x) = Q_0 \left( \frac{x_0}{x} \right)^{\lambda/2}, \quad Q_0 = 1 \text{ GeV}$$

where the parameters $x_0 \simeq 3 \times 10^{-4}$ and $\lambda \simeq 0.3$ are fitted to the small-$x$ data. The amplitude depends on $x$ and $r_t$ (the transverse size of the dipole) only through the combination $r_t^2 Q_s^2(x)$. In DIS, this directly results in a scaling cross section. Hence, in the dipole picture, GS is equivalent to a dependence of the amplitude on $r_t^2 Q_s^2(x)$ only.

Despite that GS is expected already sub-asymptotically in a growing range around $Q_s$ [5], the values of $x$ probed at recent colliders may be not small enough to expect GS and it may be more important to test violations of GS to establish small-$x$ evolution. In order to investigate GS violations in the RHIC data, in [6, 7] a phenomenological model has been put forward. We will refer to this model as the DHJ model. It offers a good description of the $p_t$ distribution of hadrons produced in $d$-$Au$ collisions at RHIC in the forward region.

According to Refs. [6, 7] the cross section of hadron production in high-energy nucleon-nucleus collisions can be described in terms of the dipole scattering amplitude,

$$\frac{d N_h}{d y_h \, d^2 p_t} = \frac{K(y_h)}{(2\pi)^2} \int_{x_F}^{1} d x_1 \, \frac{x_1}{x_F} \left[ \sum_q f_{q/p}(x_1, p_t^2) \, N_F \left( \frac{x_1}{x_F} p_t, x_2 \right) \, D_{h/q} \left( \frac{x_F}{x_1}, p_t^2 \right) + f_{g/p}(x_1, p_t^2) \, N_A \left( \frac{x_1}{x_F} p_t, x_2 \right) \, D_{h/g} \left( \frac{x_F}{x_1}, p_t^2 \right) \right].$$

*As it turned out the study of Ref. [7] contained an artificial upper limit on the $x_1$ integration to exclude large $x_2$. Without this cut, the larger $p_t$ data for $y_h = 0, 1$ are in fact not well-described by the DHJ model.

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generally, the \( \text{BK} \) equation. The parameterization that we adopted reads

\[ Q_0 \approx 2 \text{GeV}^2 \text{.} \]

The momentum fraction of the target partons equals \( x_2 = x_1 \exp(-2y_h) \). We can to good approximation neglect finite mass effects, i.e. we equate the pseudorapidity \( \eta \) and the rapidity \( y_h \) and use \( x_F = \sqrt{p_t^2 + m^2} \exp(\eta) \approx p_t / \sqrt{s} \exp(y_h) \). Finally, there is an overall \( y_h \)-dependent \( K \)-factor that effectively accounts for NLO corrections. It should be noted that the scaling properties of the dipole scattering amplitude are not directly visible in the hadron production data, due to its convolution with the parton distribution and fragmentation functions.

The dipole scattering amplitude of the DHJ model is given by [6,7]:

\[
N_A(q_t, x_2) = \int d^2r_t \ e^{i \vec{q}_t \cdot \vec{r}_t} \left[ 1 - \exp \left( -\frac{1}{4} (r_t^2 Q_s^2(x_2))^{\gamma(q, x_2)} \right) \right]. \tag{4}
\]

Note that \( \gamma \) is a function of \( q_t \) rather than \( r_t \). This allows one to compute the Fourier transform more easily. The corresponding expression \( N_F \) for quarks is obtained from \( N_A \) by the replacement \( (r_t^2 Q_s^2)^\gamma \rightarrow ((C_F/C_A)r_t^2 Q_s^2)^\gamma \), with \( C_F/C_A = 4/9 \). The exponent \( \gamma \) is usually referred to as the “anomalous dimension”, although the connection between \( N_{A/F} \) and the gluon distribution inside the nucleus cannot always be made.

The anomalous dimension of the DHJ model is parameterized as

\[
\gamma_{\text{DHJ}}(q_t, x_2) = \gamma_s + (1 - \gamma_s) \frac{|\log(q_t^2/Q_s^2(x_2))|}{\sqrt{y} + d \sqrt{y} + |\log(q_t^2/Q_s^2(x_2))|}, \tag{5}
\]

where \( y = \log 1/x_2 \) is minus the rapidity of the target parton. The saturation scale \( Q_s(x_2) \) and \( \lambda \) are taken from the GBW model [2]. Here \( Q_s \) includes a larger value of \( Q_0 \approx 1.63 \text{ GeV} \) to account for the size of the nucleus. The parameter \( d = 1.2 \) was fitted to the data. This choice of \( \gamma \) leads to a geometric scaling solution at \( q_t = Q_s \), where \( \gamma = \gamma_s = 0.628 \) and incorporates to a certain extent the violation expected from BFKL evolution for larger \( q_t \), see [5]. However, an analysis of the BK equation suggests that a smaller value of \( \gamma \approx 0.44 \) may be more appropriate at \( Q_s \) where the BFKL equation does not apply.

Not only a constant \( \gamma \) would lead to GS but also a \( \gamma \) that depends on \( q_t^2/Q_s^2 \) or \( r_t^2 Q_s^2 \) only. To check explicitly whether the RHIC data are compatible with GS, we propose a new scaling parameterization of \( \gamma \) that is similar in form to that of the DHJ model, but does not have the GS violating behavior nor the logarithmic rise expected from the BFKL (and more generally, BK) equation. The parameterization that we adopted reads

\[
\gamma_{\text{GS}}(w = q_t/Q_s) = \gamma_1 + (1 - \gamma_1) \frac{(w^a - 1)}{(w^a - 1) + b}. \tag{6}
\]

The two free parameters \( a \) and \( b \) will be fitted to the RHIC data. There are two major differences between the chosen parameterization \( \gamma_{\text{GS}} \) and \( \gamma_{\text{DHJ}} \). Firstly, \( \gamma_{\text{GS}} \) does not depend on the rapidity \( y \) explicitly. Therefore the resulting dipole scattering amplitude respects geometric scaling. Secondly, \( \gamma_{\text{GS}} \) approaches the large \( q_t \) limit of 1 much faster. This will lead to different large momentum slopes of the amplitude [4] and therefore to different predictions for the large \( p_t \) slope using Eq. [4]. For large \( w \) the exponential function can be expanded and the fall-off of the dipole scattering amplitude [4] is given by [9],

\[
N_A(q_t) \approx \begin{cases} \frac{Q_s^2}{q_t \log(q_t^2/Q_s^2)} & \text{for } \gamma \text{ of Eq. [5]} \\ \frac{Q_s^{2+a}}{q_t^{a+b}} & \text{for } \gamma \text{ of Eq. [6]} \end{cases} \tag{7}
\]

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Empirically, we find that the $p_t$ distribution falls off even faster. We note that the fall-off with $p_t$ is not determined by the size of the scaling violations. In order to observe such violations, one has to study both the $y_h$ and $p_t$ dependence over a significantly large range. Let us mention explicitly that our parameterization is not meant to replace other, physically better motivated models but to investigate in a general way which conclusion can really be drawn unambiguously from the RHIC data in the central and forward regions.

In Fig. 1a) we show our estimate for $dN_h/(dy_hdp_t)$ that follows from the integral in Eq. (3) by using $\gamma_{\text{GS}}(w)$ (9). All $p_t$ distributions of produced hadrons measured at RHIC in $d$-$Au$ collisions are well described. At the saturation scale we have chosen here for $\gamma_{\text{GS}}(w = 1) = \gamma_1$ the same value $\gamma_1 = 0.628$ as in the DHJ model. We also take the same parameterization of $Q_s(x)$. We obtain the best fit of the data for $a = 2.82$ and $b = 168$. As mentioned, this LO analysis requires the inclusion of a $K$-factor to account for NLO corrections, which are expected to become more relevant towards central rapidity. The $p_t$-independent $K$ factors we obtain for $y_h = 0, 1, 2, 2, 3, 2, 4$ are for our model equal to $K = 3.4, 2.9, 2.0, 1.6, 0.7$ and for the DHJ model $K = 4.3, 3.3, 2.3, 1.7, 0.7$. For further details of the calculation see [9].

![Figure 1](image)

**Figure 1:** a) Transverse momentum distribution of produced hadrons in $d$-$Au$ collisions as measured at RHIC (symbols) for various rapidities $y_h$. To make the plot clearer, the data and the curves for $y_h = 0.1$ and 2.2 are multiplied with arbitrary factors, namely 16, 4 and 2, respectively.

b) Various fits of $\gamma_{\text{GS}}(w)$, which describe the RHIC data equally well. For comparison we show curves representing $\gamma_{\text{DHJ}}(w, y(w, y_h))$ at different rapidities $y_h$.

From this analysis we can conclude that a GS dipole amplitude is completely compatible with the data and therefore the conclusion that GS violations are observed at RHIC cannot be drawn. Of course, a scaling violating amplitude, i.e. a $\gamma$ that depends on $w$ and the rapidity $y$ explicitly, is not ruled out by the data either. What can be concluded further is that the logarithmic rise of $\gamma$ resulting from the BFKL evolution incorporated in the DHJ model is ruled out in the central region, see Fig. 1a). However, where the DHJ model starts to deviate from the data $x_2$ becomes larger than 0.01, although $Q_s$ then is still larger than in DIS at $x = 0.01$. If one were to exclude the central rapidity RHIC data in the model fit, one could also obtain a scaling model with a logarithmically rising, or even constant, $\gamma$.

To indicate how much $\gamma$ is constrained by the RHIC data, Fig. 1b) shows various $\gamma_{\text{GS}}(w)$'s that describe the available data equally well. They are all parameterized as in Eq. (6) with different $a$ and $b$ values. Furthermore, we added lines representing $\gamma_{\text{DHJ}}$ (5) for different rapidities. For this one needs to express the rapidity of the target parton $y$ in

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terms of \( w \) and \( y_h \), see [9] for details. It should be mentioned that the region below \( Q_s \), where the parameterization of \( \gamma_{\text{DHJ}} \) is not smooth, is hardly probed at RHIC. Clearly, \( \gamma \) is less well determined close to the saturation scale than in the dilute region. This is because the integrand entering the dipole scattering amplitude [4] is only weakly dependent on \( \gamma \) around the saturation scale \( r = 1/Q_s \). In addition, the forward data \((y_h = 3.2 \text{ and } 4)\) are essentially sensitive only to \( \gamma_1 \), since they probe the region where \( w \) is close to 1. Therefore, the rise of \( \gamma \) with \( w \) is effectively constrained only by the data for \( y_h = 0, 1 \). It is apparent from Fig. [1]b) that the DHJ model fails for larger \( w \), i.e. larger \( p_t \), in the central region but not in the forward region where the probed values of \( w \) are below 2.

Where the DHJ model curves deviate from the RHIC data, the probed \( x_2 \)-values are not very small. However, at LHC, due to the higher energies, the region of small \( x_2 \) extends to a much larger range of \( p_t \), so that the predictions of the DHJ model and the new one will be different even at small \( x_2 \). It has to be mentioned that the calculation of the \( p_t \) distribution using our scaling model should not be seen as a physically motivated prediction. However, a comparison of the estimate using \( \gamma_{\text{GS}} \) fitted to RHIC data with the estimate from the DHJ model allows drawing conclusions anyway. As we will explain, the estimates are so different that the LHC should be able to rule out one of these models at much smaller \( x \). Hence, the LHC can answer the question why the DHJ model fails in the central region at RHIC. Either because the probed \( x \)-values are not small enough or because some expectations from small-\( x \)-evolution has to be modified. For smaller \( p_t \) the predictions of the two models are expected to be comparable since only the region of small values of \( w \) is probed where \( \gamma_{\text{DHJ}} \) and \( \gamma_{\text{GS}} \) are similar, see Fig. [1]b). In the very forward region, i.e. \( y_h \approx 6 - 8 \), only this region is tested. However, there is quite a large range where the probed values of \( x_2 \) are small but the predictions are clearly different. One expects (see Fig. [1]b) differences in the central region between the two models for values of \( w \gtrsim 3 \), i.e. at RHIC for \( p_t \gtrsim 2.5 \text{ GeV} \). Since \( Q_s \) is larger at LHC such differences show up at larger momenta around 5 GeV. Even in the central region such momenta imply values of \( x_2 \) smaller than 0.001. Hence, a measurement of the slopes at moderate rapidities \( y_h \) at LHC would allow a discrimination between the two models in a region where small-\( x \) physics is expected to be applicable. The slower fall-off of the \( p_t \) distribution in the DHJ model is a direct consequence of the logarithmic rise of \( \gamma \) towards 1. Since such a behavior of \( \gamma \) is a generic signature of BFKL evolution, these measurements offer the possibility of testing whether such small-\( x \) evolution is actually relevant at present-day hadron colliders. Estimates of the \( p_t \) distributions for hadron production at LHC using both models can be found in [9].

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