Radial acceleration relation and dissipative dark matter

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Observations indicate that ordinary matter, the baryons, influence the structural properties of dark matter on galactic scales. One such indication is the radial acceleration relation, which is a tight correlation between the measured gravitational acceleration and that expected from the baryons. We show here that the dark matter density profile that has been motivated by dissipative dark matter models, including mirror dark matter, can reproduce this radial acceleration relation.

Introduction

The matter content of the Universe can be inferred from the cosmic microwave background and large scale structure studies to be dominated by a nonbaryonic dark matter component [1]. Dark matter has also been inferred to exist on galaxy scales, with rotation curve measurements providing the most detailed information about the structural properties of dark matter on small scales, see e.g. [2–8]. From such studies evidence that the dark matter structural properties are influenced by ordinary matter, baryons, is in abundance. The first such indication was the Tully-Fisher relation [9], which relates galaxy luminosity with the asymptotic value of the rotational velocity. Another such indication is the radial acceleration relation, which is a tight correlation between the measured gravitational acceleration and that expected from the baryons [10] (for earlier closely related work see [11]). See also e.g. [12–15] for other related studies and discussions of the baryonic influence of the structural properties of dark matter in galaxies.

The radial acceleration relation is a one-parameter formula which summarizes many of the empirical correlations that rotation curve measurements show with the baryon content. Such a relation was in fact initially motivated by modified Newtonian dynamics (MOND) [16], which was originally posed as an alternative to dark matter. While the radial acceleration relation arises quite naturally within MOND it might seem unclear how such a connection with baryons could arise in dark matter theories. There is however a kind of dark matter which requires nontrivial interactions with baryons, called dissipative dark matter [17–23]. Further, we shall show here that the dark matter density profile, that has been motivated by dissipative dark matter models, can reproduce the radial acceleration relation.

Given that the radial acceleration relation was originally introduced in MOND, let us briefly digress to indicate how it arises in that framework. MOND postulates that the acceleration $g$ of a test mass in a galaxy gravitational field is fully determined by the Newtonian acceleration expected at the same location due to the gravitational field strength $g_{\text{bar}}$ created by the baryonic matter of the galaxy. In spherical or axial symmetric case, the relation has the form

$$g = g_{\text{bar}} \nu \left( \frac{g_{\text{bar}}}{a_0} \right),$$

(1)

where $a_0$ is a new fundamental constant and the interpolating function $\nu(y)$ satisfies $\nu(y) = 1$ in the $y = g_{\text{bar}}/a_0 \gg 1$ limit of large accelerations (Newtonian limit). The behaviour of the interpolating function in the opposite limit $y \ll 1$ of small accelerations (deep-MOND regime) can be uniquely specified if the gravitational dynamics becomes scale
invariant [24, 25]. Under the scale transformation \((t, \vec{r}) \rightarrow \Lambda (t, \vec{r})\), the acceleration \(g\) and the gravitational field strength \(g_{\text{bar}}\) scale as follows:

\[
g = \left| \frac{d^2 \vec{r}}{dt^2} \right| \rightarrow \frac{1}{\Lambda} g, \quad g_{\text{bar}} \propto \frac{1}{r^2} \rightarrow \frac{1}{\Lambda^2} g_{\text{bar}}. \tag{2}
\]

Therefore, in order to have scale invariant dynamics in the limit \(y \ll 1\), the interpolating function \(\nu(y)\) must satisfy

\[
\frac{1}{\Lambda} \nu \left( \frac{y}{\Lambda^2} \right) = \nu(y), \quad y \ll 1. \tag{3}
\]

The solution of this functional equation is \(\nu(y) = k/\sqrt{y}\) with some constant \(k\). It is clear that this dimensionless constant \(k\) can be absorbed in the definition of the critical acceleration \(a_0\), so that we can set \(k = 1\) without loss of generality.

The scale invariant gravitational dynamics at low accelerations is perhaps the most important aspect of MOND, which (typically) does not depend significantly on the specific form of the interpolating function \(\nu(y)\). A simple choice for the interpolating function is [26]:

\[

\nu(y) = \frac{1}{1 - e^{-\sqrt{y}}}. \tag{4}
\]

The relation Eq. (1) with this functional choice for the interpolating function was analyzed in the recent comprehensive study of [10] which involved 153 rotationally supported galaxies from the SPARC data base [27]. Within observational uncertainties this relation was found to hold in rotationally supported galaxies. The value for \(a_0\) was estimated to be: 

\[
a_0 = (1.20 \pm 0.02 \pm 0.24) \times 10^{-10} \text{ m s}^{-2}. \]

The results of [10] have already generated considerable interest and attempts to explain them, e.g. [28–30].

**Dissipative dark matter model**

The observed correlations of rotation curve shapes with baryon content in galaxies, for which the radial acceleration relation is an indicator, is an interesting challenge for dark matter theories. This seems to be particularly relevant to the often considered case of dark matter consisting of collisionless (or weakly interacting) particles. In particular, the correlations are observed to hold in gas-rich dwarf irregular galaxies which are dark matter dominated at all radii. It is hard to envisage how baryons, which are gravitationally insignificant, could have such a large influence on the structural properties of the dark matter, if gravity were the only interaction with ordinary matter.

Dissipative dark matter is a specific kind of dark matter candidate that actually requires nontrivial interactions with baryons for a consistent picture of dark matter in rotationally supported galaxies [17–23]. Such generic model involves considering a dark sector consisting of dark matter particles coupled with massless dark photons [20]. A theoretically constrained case is mirror dark matter [31] which involves a duplicate set of particles and forces with exactly the same fundamental parameters (particle masses and coupling constants) as the standard particles and forces. In other words, we are envisaging dark matter with either similar or exactly the same particle properties to ordinary matter.

Such dark matter is dissipative. We are interested in the parameter space where dissipation plays an important role, that is, in the absence of any heat source the cooling rate is sufficiently strong for the galactic dark matter to collapse into a disk on timescales much smaller than the Hubble time. However, in the presence of heating a dissipative dark matter halo could, in principle, exist in an approximately spherical pressure supported dark plasma. This is the scenario that we consider.\(^1\)

In such models the only viable heat source identified is ordinary type-II supernovae, which can become an energy source for the dark sector if small kinetic mixing interaction exists. The kinetic mixing interaction [34, 35]

\[

\mathcal{L}_{\text{int}} = -\frac{\epsilon}{2} F^{\mu\nu} F'_{\mu\nu}, \tag{5}
\]

\(^1\) The only other scenario involving dissipative dark matter that we are aware of is the “double disk” dark matter model of [32, 33]. In that model, dissipative dark matter is assumed to be only a subcomponent of the total nonbaryonic dark matter sector. No heat source is envisaged in that model so that the dissipative dark matter collapses forming a dark disk.
facilitates halo heating by enabling ordinary supernovae to be a source of these ‘dark photons’ \cite{36,37}. These dark photons can transport a large fraction of a supernova’s core collapse energy to the halo (potentially up to \( \sim 10^{53} \) erg per supernova for kinetic mixing of strength \( \epsilon \sim 10^{-9} \)). The supernovae generated dark photons propagate out into the halo where they can eventually be absorbed via some interaction process, with dark photoionization being the mechanism in the specific models studied \cite{18,21,23}.

Such strongly interacting dark matter can be modelled as a fluid governed by Euler’s equations. The halo is dynamic and is able to expand and contract in response to the heating and cooling processes. It is envisaged that for a sufficiently isolated galaxy its halo can evolve into a steady state configuration which is in hydrostatic equilibrium, and heating and cooling rates are locally balanced. These conditions determine the temperature and density profiles of the dark matter halo around such disk galaxies. More generally, for interacting disk galaxies undergoing perturbations or other non-equilibrium conditions (such as rapidly changing star formation rate as in starburst galaxies) the dark matter density and temperature profiles would be time dependent and would require the full solution of Euler’s equations.

Insight into the steady-state solution, applicable to an isolated disk galaxy with a stable star formation rate, arises from the following simple argument \cite{20,21}. The (supernova sourced) heating rate at a particular location \( r \) in the halo is proportional to the product of the dark matter density and dark photon energy flux at that point: \( \Gamma_{\text{heat}}(r) \propto n(r)F_{\gamma D}(r) \). The halo is dissipative and cools via dark bremsstrahlung (and potentially other processes), which means that the cooling rate at the position \( r \) is proportional to the square of dark matter density: \( \Gamma_{\text{cool}}(r) \propto n(r)^2 \).

The proportionality coefficients depend on dissipative particle physics and do not need to be specified for the present discussion.\(^4\) Assuming now for simplicity an approximately isothermal halo\(^5\), the dark matter density can be obtained directly from the presumed dynamically driven local balancing of the heating and cooling rates, \( \Gamma_{\text{heat}}(r) = \Gamma_{\text{cool}}(r) \), so that:

\[
n(r) \propto F_{\gamma D}(r).
\]

As already mentioned, the timescale of this halo evolution is assumed to be much smaller than the current age of the Universe. Given that the dark photon energy flux is presumed to originate from ordinary supernovae, it can be straightforwardly calculated by summing up all the sources in the disk. Define a spherical co-ordinate system with the origin at the center of the galaxy and with the baryonic disk described in terms of \( \tilde{r}, \tilde{\phi} \) at \( \theta = \pi/2 \). If supernovae occur at an average rate per unit area of \( \Sigma_{SN}(\tilde{r}, \tilde{\phi}) \), then the flux of dark photons in a particular wavelength range from a disk area element: \( \tilde{r}d\tilde{\phi}d\tilde{r} \) is proportional to:

\[
dF_{\gamma D} \propto e^{-\tau} \frac{\Sigma_{SN}(\tilde{r}, \tilde{\phi})}{4\pi d^2} \tilde{r}d\tilde{\phi}d\tilde{r}.
\]

where \( \tau \) is the wavelength dependent optical depth and \( d \) is the distance between the point \( r \) and the disk element defined by \( \tilde{r}, \tilde{\phi} \). For an optically thin halo, this leads to the relatively simple formula for the dark matter density profile:\(^6\)

\[
\rho(r, \theta, \phi) = \lambda \int d\tilde{\phi} \int d\tilde{r} \frac{\Sigma_{SN}(\tilde{r}, \tilde{\phi})}{4\pi[r^2 + \tilde{r}^2 - 2r\tilde{r}\sin \theta \cos(\tilde{\phi} - \phi)]}.
\]

---

\(^2\) If halo heating is indeed due to ordinary supernovae then at early times, prior to the onset of star formation, the dissipative dark matter might have been able to cool to form a dark disk composed of dark stars made almost entirely of dark matter. In the particular case of mirror dark matter such ‘mirror stars’ can evolve extremely rapidly \cite{33} and thereby potentially source the heavy mirror nuclei (mirror metals) needed for the halo to absorb the dark photon radiation from ordinary supernovae. The remnant dark stars themselves are assumed to be a subdominant mass component of the halo at the present time.

\(^3\) Dwarf spheroidal and elliptical galaxies are devoid of baryonic gas and have negligible active star formation. Ordinary supernovae cannot provide a viable heat source, and thus a very different picture of the dark matter halo in these galaxy types is envisaged. In these galaxies the dark matter halo might have cooled and collapsed into compact objects, ‘dark stars’ in the case of mirror dark matter.

\(^4\) The emitted dark photon radiation from a local volume can only be an effective cooling agent if the halo is optically thin. In specific models including mirror dark matter, the halo may in fact be optically thick for a range of dark photon wavelengths. In which case we can consider as an approximation only the optically thin cooling component, as the optically thick component will be reabsorbed and to first order can be neglected.

\(^5\) The halo could not be exactly isothermal as this would be incompatible with the hydrostatic equilibrium condition. Thus, corrections to the results following from the simple assumptions adopted here are in fact inevitable.

\(^6\) As briefly noted already, the halo may be optically thick for a range of wavelengths: \( \lambda_L \lesssim \lambda_{\gamma D} \lesssim \lambda_H \) (the precise values of \( \lambda_L, \lambda_H \) are model/parameter dependent and are uncertain). Nevertheless, the optically thin dark photons can dominate the energy transport if the energy spectrum of dark photons originating in the region around ordinary supernova peaks at a wavelength below the low-wavelength cutoff scale, \( \lambda_L \).
The proportionality coefficient $\lambda$ in Eq. (8) is a combination of many quantities. In specific dissipative dark matter models it depends on fundamental parameters related to the dark photoionization cross section and radiative cooling cross sections. It also depends on other quantities including the supernovae dark photon energy spectrum, and halo properties: ionization state and composition. In general, we also expect $\lambda$ to depend on halo temperature and, hence, also on galaxy properties (mass, luminosity etc) and therefore it is not expected to be a universal constant but can have some (possibly weak) scaling between galaxies. Furthermore, $\lambda$ may not even be a constant within a galaxy but have dependence on spatial coordinates. We assume that such variation is subleading. Some work justifying this assumption has been done \cite{18, 19}, and more work is in progress \cite{30} on this important issue for these kinds of dark matter models. If these assumptions are indeed valid, then dissipative dark matter can be extremely predictive as far as galaxy dynamics is concerned.

Radial acceleration relation in dissipative dark matter model

In order to explore the radial acceleration relation in the context of dissipative dark matter, consider a generic disk galaxy with the supernova distribution approximated as an axisymmetric thin Freeman disk \cite{40}:

$$\Sigma_{SN}(r) = R_{SN} \frac{e^{-r/r_D}}{2\pi r_D^2},$$

(9)

where $R_{SN}$ is the type-II supernova rate in the galaxy under consideration and $r_D$ is the disk scale length.

For an actual galaxy this exponential distribution could only be a rough approximation; a better approximation would be to use the measured UV surface brightness profile of the galaxy under consideration \cite{22}. Anyway, with the axisymmetric distribution given in Eq. (9), the dark matter density in Eq. (8) depends only on $r$ and $\theta$, and the dark matter contribution to the gravitational acceleration at a point in the plane of the disk can be straightforwardly obtained from Newton’s law of gravity:

$$g_{\text{dark}}(r) = G_N \int d\tilde{\phi} \int d\cos \tilde{\theta} \int d\tilde{r} \tilde{r}_D^2 \frac{\rho(\tilde{r}, \tilde{\theta}) \cos \omega}{d^2}.$$  

(10)

Here, $d^2 \equiv r^2 + \tilde{r}_D^2 - 2r \tilde{r} \sin \tilde{\theta} \cos \tilde{\phi}$. $\cos \omega \equiv (r - \tilde{r} \sin \tilde{\theta} \cos \tilde{\phi})/d$ and $G_N$ is Newton’s constant. The baryonic contribution to the gravitational acceleration will also be needed, and for an axisymmetric thin disk of stellar mass $m$ and surface density $\Sigma(r) = m e^{-r/r_D}/(2\pi r_D^2)$ it is described by an equation of the form: Eq. (11)

The dark matter density function motivated by dissipative dark matter, Eq. (8), is relatively simple and depends on only one parameter if the supernovae distribution is known. In the axisymmetric case with the exponential $\Sigma_{SN}$ profile it is possible to do the $\phi$-integration analytically. The result is that the density takes the form of a Laplace transformation:

$$\rho(r, \theta) = \frac{\lambda R_{SN}}{4\pi r_D^2} \int_0^\infty \frac{t e^{-rt/r_D}}{\sqrt{1 + t^2 + 2t^2 \cos 2\theta}} dt.$$  

(11)

From this expression, the radial dependence of the density for a fixed angular direction $\theta \neq \pi/2$ can be shown to satisfy:

$$\rho(r) = \begin{cases} \frac{\lambda R_{SN}}{4\pi r_D^2} \log(r/r_D) & \text{for } r \ll r_D \\ \frac{\lambda R_{SN}}{4\pi r_D^2} & \text{for } r \gg r_D \end{cases}$$  

(12)

The transition region occurs roughly when $r/r_D \approx 1.5$. The density has a log divergence as $\theta \to \pi/2$ which would be regulated by considering a disk of finite thickness. The gravitational acceleration, however, is finite and thus a thin disk is suitable for the current purposes. Despite its angular dependence, such a density profile results in rotation curves similar to that from a spherically symmetric cored isothermal profile: $\rho = \rho_0 r_D^2/(r^2 + r_D^2)$ with $r_0 \approx 1.4 r_D$.

\footnote{For the Freeman disk the gravitational acceleration in the equatorial plane ($\theta = \pi/2$) is known analytically \cite{49}: $g_B(x) = [G_N m_{bary}/r_D^2] (x/2)[I_0(x/2)K_0(x/2) - I_1(x/2)K_1(x/2)]$, where $I_0, I_1$ are modified Bessel functions of the first kind, $K_0, K_1$ are modified Bessel functions of the second kind, and $x \equiv r/r_D$.}
At low $r \lesssim r_D$ the approximately flat density profile [Eq. (12)] implies a linearly rising rotation curve:

$$v(r) \simeq r \sqrt{2\pi G N \lambda \Sigma_{SN}(0)/3}.$$ (13)

Such linearly rising rotation curves are seen in dwarf disk galaxies, where dark matter generally dominates over ordinary matter even at low radii [4, 8] (for recent studies see e.g. [6]). In particular note that the inner rotation curve slope satisfies: $dv/dr \propto \sqrt{\Sigma_{SN}(0)}$, that is, the rotation curve slope is expected to scale with the square root of the central surface brightness (in the UV band), which is in fact in agreement with observations [14] (see also [41, 42] for related discussions). Considering next the $\rho \propto 1/r^2$ behaviour at large radii, $r \gg r_D$, Newton’s laws imply that the rotation curve has a flat (i.e. radially independent) asymptotic rotational velocity, $v_{\text{asym}} = \sqrt{G N \lambda R_{SN}}$, in agreement with long standing observations [2, 3].

The baryonic Tully Fisher relation $(m_{\text{bar}} \propto v_{\text{asym}}^4)$ [12] will require the approximating scaling: $\lambda R_{SN} \propto \sqrt{m_{\text{bar}}}$. Core-collapse supernovae have been observed in the local Universe to occur with a rate that roughly matches this scaling [43], which means that an approximately constant $\lambda$ (or modestly varying $\lambda$) would suffice to reproduce the baryonic Tully Fisher relation. Actually, this kind of dark matter model motivates a somewhat different form to the Tully Fisher-type scaling. Instead of converting $\lambda R_{SN}$ into $m_{\text{bar}}$, which itself can only be obtained indirectly as observations measure light not mass, one can use the luminosity in the UV band $[L_{UV}]$ as a proxy for the current star formation rate. Thus we expect $R_{SN} \propto L_{UV}$, and hence $v_{\text{asym}} = \sqrt{G N \lambda R_{SN}}$ suggests that: $\lambda L_{UV} \propto v_{\text{asym}}^2$. One can further motivate a $\lambda \propto 1/v_{\text{asym}}$ scaling in simple dissipative dark matter models if bremsstrahlung effectively dominates the cooling leading to the rough scaling: $L_{UV} \propto v_{\text{asym}}^3$ [22].

Another interesting feature of the density profile Eq. (5) is that it leads to gravitational accelerations that have scale invariance in a similar sense as MOND. That is, under the scale transformation $(t, \vec{r}) \rightarrow \Lambda (t, \vec{r})$, $r_D \rightarrow \Lambda r_D$ (with $G, R_{SN}, \lambda$ unchanged) the density scales: $\rho \rightarrow \rho / \Lambda^2$, and $g_{\text{dark}} \rightarrow g_{\text{dark}} / \Lambda$. A consequence of this is that the dark halo contribution to the rotational velocity is scale invariant, this means that it will depend only on the dimensionless ratio $r/r_D$ rather than $r$ and $r_D$ separately. Such scale invariant dynamics is supported by observations, see e.g. [15] for a recent discussion. This scale invariance feature, along with the baryonic Tully Fisher scaling ($\lambda R_{SN} \propto \sqrt{m_{\text{bar}}}$), imply that the rotation curves have a characteristic form. The dark matter rotational velocity function $v(r)$ for a particular value of $r_D$, $m_{\text{bar}}$ can be mapped onto any other $r_p$, $m_{\text{bar}}$ value.

The characteristic form for $v(r)$ can be conveniently specified by considering the velocity ratio (where, following [14], the dimensionless parameter is taken to be $r/r_{\text{opt}}$ where the optical radius $r_{\text{opt}} \simeq 3.2 r_D$; although any other scaling with $r_D$ could be used):

$$R(r/r_{\text{opt}}) \equiv \frac{v(r/r_{\text{opt}})}{v(r = r_{\text{opt}})}.$$ (14)

Here the rotational velocity is that due to dark matter (i.e. obtained from $v^2/r = g_{\text{dark}}$). It is worth emphasizing that with the exponential supernovae distribution profile chosen, the velocity ratio Eq. (14) is predicted to be completely independent of the particular value of galaxy parameters: $r_D, m_{\text{bar}}, R_{SN}$ as well as the theory parameter $\lambda$. This parameter-free theoretical curve shown in Fig.1, can be compared to actual rotation curves of dwarf disk galaxies as they are typically dark matter dominated at all radii (summarized by synthetic rotation curves given in [14] and reproduced as the triangles in Fig.1). Also shown for comparison is the curve predicted by the radial acceleration relation in the deep-MOND regime where $v^2/r = \sqrt{a_0 g_{\text{bar}}}$. This relation depends on the total baryonic gravitational field $g_{\text{bar}}$ which we have derived for a gas dominated dwarf with stellar/gas fraction of $f_{\text{star}} = 0.2$, $f_{\text{gas}} = 0.8$. (Both the stellar and gas components are modelled with an exponential profile but the gaseous component is known from observations to be more radially extended which motivates a larger scale radius for that component: $r_{\text{gas}}^D = 3.0 r_D$, e.g. [44].)

In Fig.2 we give the radial acceleration relation predicted by the density profile Eq. (8) for some illustrative examples. The baryonic matter is modelled as a thin disk with both stellar and gas components each with an exponential profile (with $r_D^{\text{gas}} = 3.0 r_D$ as discussed above). Both the radial acceleration relation and the dark matter density motivated from dissipative dynamics have one free parameter, and we have normalized $\lambda R_{SN}$ such that it gives the same value for $g_{\text{obs}} = g_{\text{dark}} + g_{\text{bar}}$ as the $g$ obtained from the formula, Eqs. (11, 13) (with $a_0 = 1.20 \times 10^{-10}$ m s$^{-2}$) for the largest radii considered. The radial acceleration relation is in agreement with observations to within errors estimated to be 0.12 dex for the sample studied in [10].

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8 Some work has been done attempting to determine the scaling behaviour of $\lambda$ within the mirror dark matter context [13, 14]. Although some simplifying assumptions were made, that work indicates that $\lambda$ does indeed scale modestly, and $\lambda R_{SN}$ appears to have a scaling consistent with observations. Further work is underway which aims to provide a more rigorous check of this conclusion.
The triangles are the synthetic rotation curve obtained from dwarf disk galaxies. The parameters chosen were: $m_{\text{bar}} = 5.0 \times 10^8 m_\odot$, $r_D = 0.4$ kpc, $r_D = 0.6$ kpc, and $r_D = 1.5$ kpc, with stellar/gas mass fractions of $f_{\text{star}} = 0.2$, $f_{\text{gas}} = 0.8$ consistent with typical values for gas rich dwarfs.

The figures demonstrate that the density profile Eq. (8) motivated by dissipative dark matter leads to accelerations within galaxies consistent with the radial acceleration relation, which itself is known to be a good representation of the data. The differences are small and occur mainly at low radii $r < r_D$ where uncertainties are generally larger (especially in spirals). The reasons for this agreement are several. First, as already discussed, the concept of scale invariance can be used to explain the lack of significant $r_D$ variation between these curves. If they approximately agree for one value of $r_D$ then they should approximately agree for any other. Second, the basic velocity profiles in the dark matter dominated and deep-MONDian limits are very similar (Fig. 1). We don’t have any deep theoretical explanation for why this results, other than the basic velocity profile is a function of the baryonic distribution in each case. Finally, the agreement assumes $\Lambda R_{SN}$ values such that $g_{\text{obs}} = g_{\text{dark}} + g_{\text{bar}}$ is equal to the value of $g$ obtained from the radial acceleration relation [Eqs. (11), (14)], at large radii. This is approximately equivalent to having the scaling: $\Lambda R_{SN} \propto \sqrt{m_{\text{bar}}}$. As briefly mentioned already, the galactic supernova rate itself has an observed scaling consistent with $R_{SN} \propto \sqrt{m_{\text{bar}}}$, and thus a roughly constant or weak scaling of the model parameter $\lambda$ is required which appears to be possible in the specific models studied [18, 19, 39].

Previous work [21, 22] along with the present results indicate that Eq. (8) provides a reasonably successful quantitative description of the physical properties of dark matter in disk galaxies. We emphasize again here that the dark matter density given by Eq. (8) could only be a rough approximation to the actual dark matter density as dictated by the dissipative dynamics. As briefly mentioned earlier, a more exact description is given by Euler’s equations of fluid dynamics. More work is needed to solve these fluid equations in both the general time-dependent case (applicable to e.g. star burst galaxies) as well as the simpler steady state solution applicable to isolated galaxies with stable star formation rates.
for illustrative examples. The exponential disk parameter $s$ chosen are $m_{\text{bar}} = 10^{12} m_\odot$ (dashed-dotted line), $r_D = 6.0$ kpc (thin dashed line) and $r_D = 12.0$ kpc (thick dashed line). In each case a stellar/gas mass fraction of $f_{\text{star}} = 0.8$, $f_{\text{gas}} = 0.2$ was used. Shown are accelerations obtained for $0.4 < r/r_D < 10$. The dotted line is the $g_{\text{obs}} = g_{\text{bar}}$ limit. b) Same as Fig. 2a, but with $m_{\text{bar}} = 10^{11} m_\odot$ and $r_D = 3.0$ kpc (dashed-dotted line), $r_D = 6.0$ kpc (thin dashed line) and $r_D = 12.0$ kpc (thick dashed line). c) Same as Fig. 2a, but with $m_{\text{bar}} = 10^{10} m_\odot$ and $r_D = 1.0$ kpc (dashed-dotted line), $r_D = 2.0$ kpc (thin dashed line) and $r_D = 5.0$ kpc (thick dashed line). d) Same as Fig. 2a, but with $m_{\text{bar}} = 5.0 \times 10^8 m_\odot$ and $r_D = 0.4$ kpc (dashed-dotted line), $r_D = 0.8$ kpc (thin dashed line) and $r_D = 1.5$ kpc (thick dashed line), with stellar/gas mass fraction of $f_{\text{star}} = 0.2$, $f_{\text{gas}} = 0.8$.

FIG. 2: a) Radial acceleration relation, Eqs. (11), (12) [solid line], compared with dissipative dark matter, $g_{\text{obs}} = g_{\text{dark}} + g_{\text{bar}}$, for illustrative examples. The exponential disk parameters chosen are $m_{\text{bar}} = 10^{12} m_\odot$ and $r_D = 3.0$ kpc (dashed-dotted line), $r_D = 6.0$ kpc (thin dashed line) and $r_D = 12.0$ kpc (thick dashed line). In each case a stellar/gas mass fraction of $f_{\text{star}} = 0.8$, $f_{\text{gas}} = 0.2$ was used. Shown are accelerations obtained for $0.4 < r/r_D < 10$. The dotted line is the $g_{\text{obs}} = g_{\text{bar}}$ limit. b) Same as Fig. 2a, but with $m_{\text{bar}} = 10^{11} m_\odot$ and $r_D = 3.0$ kpc (dashed-dotted line), $r_D = 6.0$ kpc (thin dashed line) and $r_D = 12.0$ kpc (thick dashed line). c) Same as Fig. 2a, but with $m_{\text{bar}} = 10^{10} m_\odot$ and $r_D = 1.0$ kpc (dashed-dotted line), $r_D = 2.0$ kpc (thin dashed line) and $r_D = 5.0$ kpc (thick dashed line). d) Same as Fig. 2a, but with $m_{\text{bar}} = 5.0 \times 10^8 m_\odot$ and $r_D = 0.4$ kpc (dashed-dotted line), $r_D = 0.8$ kpc (thin dashed line) and $r_D = 1.5$ kpc (thick dashed line), with stellar/gas mass fraction of $f_{\text{star}} = 0.2$, $f_{\text{gas}} = 0.8$.

### Conclusion

Observations have shown that the structural properties of dark matter and baryons in galaxies are deeply entwined. The radial acceleration relation, which can be viewed as a summary of much of the relevant information, gives a simple analytic form to the apparent collision. It is an approximate empirical law that dark matter theories need to explain if they are to describe nature. Dissipative dark matter models have the pertinent feature that they actually require baryons to influence dark matter properties as ordinary core-collapse supernovae appear to be the only viable heat source which can dynamically balance the radiative cooling of dark matter halos. Dissipative dark matter thereby motivates a particular dark matter density profile, Eq. (3), which, as we have shown here, approximately reproduces the empirical radial acceleration relation.
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