SPIN POLARISABILITIES OF THE NUCLEON AT NLO IN THE CHIRAL EXPANSION

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We present a calculation of the fourth-order (NLO) contribution to spin-dependent Compton scattering in heavy-baryon chiral perturbation theory, and we give results for the four spin polarisabilities. No low-energy constants, except for the anomalous magnetic moments of the nucleon, enter at this order. The NLO contributions are as large or larger than the LO pieces, making comparison with experimental determinations questionable. We address the issue of whether one-particle reducible graphs in the heavy baryon theory contribute to the polarisabilities.

1 Introduction

The usual notation for the Compton scattering amplitude in the Breit frame is, for incoming real photons of energy $\omega$ and momentum $\mathbf{q}$ to outgoing real photons of the same energy and momentum $\mathbf{q}'$,

\[
T = \epsilon'^{\mu} \Theta_{\mu\nu} \epsilon^\nu \\
= \epsilon' \cdot \epsilon A_1(\omega, \theta) + \epsilon \cdot \hat{\mathbf{q}} \epsilon' \cdot \hat{\mathbf{q}} A_2(\omega, \theta) \\
+ i\sigma \cdot (\epsilon' \times \epsilon) A_3(\omega, \theta) + i\sigma \cdot (\hat{\mathbf{q}}' \times \hat{\mathbf{q}}) \epsilon' \cdot \epsilon A_4(\omega, \theta) \\
+ \left( i\sigma \cdot (\epsilon' \times \hat{\mathbf{q}}) \epsilon \cdot \hat{\mathbf{q}}' - i\sigma \cdot (\epsilon \times \hat{\mathbf{q}}') \epsilon' \cdot \hat{\mathbf{q}} \right) A_5(\omega, \theta) \\
+ \left( i\sigma \cdot (\epsilon' \times \hat{\mathbf{q}}') \epsilon \cdot \hat{\mathbf{q}} - i\sigma \cdot (\epsilon \times \hat{\mathbf{q}}) \epsilon' \cdot \hat{\mathbf{q}} \right) A_6(\omega, \theta),
\]

(1)

where hats indicate unit vectors. By crossing symmetry the functions $A_i$ are even in $\omega$ for $i = 1, 2$ and odd for $i = 3 - 6$. The leading pieces in an expansion in powers of $\omega$ are given by low-energy theorems, and the next terms contain the electric and magnetic polarisabilities $\alpha$ and $\beta$ and the spin polarisabilities $\gamma_i$:

\[
A_1(\omega, \theta) = -\frac{Q^2}{m_N} + 4\pi(\alpha + \cos \theta \beta)\omega^2 + \mathcal{O}(\omega^4) \\
A_2(\omega, \theta) = -4\pi\beta\omega^2 + \mathcal{O}(\omega^4)
\]
\begin{align*}
A_3(\omega, \theta) &= \frac{e^2 \omega}{2m_N^2} \left( Q(Q + 2\kappa) - (Q + \kappa)^2 \cos \theta \right) + 4\pi \omega^3 (\gamma_1 + \gamma_5 \cos \theta) + \mathcal{O}(\omega^5) \\
A_4(\omega, \theta) &= -\frac{e^2 \omega}{2m_N^2} (Q + \kappa)^2 + 4\pi \omega^3 \gamma_2 + \mathcal{O}(\omega^5) \\
A_5(\omega, \theta) &= \frac{e^2 \omega}{2m_N^2} (Q + \kappa)^2 + 4\pi \omega^3 \gamma_4 + \mathcal{O}(\omega^5) \\
A_6(\omega, \theta) &= \frac{e^2 \omega}{2m_N^2} Q(Q + \kappa) + 4\pi \omega^3 \gamma_3 + \mathcal{O}(\omega^5)
\end{align*}

where the charge of nucleon is $Q = (1 + \tau_3)/2$ and its anomalous magnetic moment is $\kappa = (\kappa_s + \kappa_v \tau_3)/2$. Only four of the spin polarisabilities are independent since three are related by $\gamma_5 + \gamma_2 + 2\gamma_4 = 0$. The polarisabilities are isospin dependent.

Compton scattering from the nucleon has recently been the subject of much work, both experimental and theoretical. The unpolarised polarisabilities have been well known for a number of years now, at least for the neutron, but it is only very recently that determinations of the spin polarisabilities have been extracted from fixed-$t$ dispersion analyses of photoproduction data. The forward spin polarisability $\gamma_0 = \gamma_1 + \gamma_5$ has a longer history, with determinations that are in the range of recent values, namely $-0.6$ to $-1.5 \times 10^{-4}$ fm$^4$ for the proton.

Direct measurements of the polarised cross-section at MAMI have been used to obtain a value of $-0.8 \times 10^{-4}$ fm$^4$, as reported by Pedroni at this conference. No direct measurements of polarised Compton scattering have yet been attempted. However the backwards spin polarisability $\gamma_\pi = \gamma_1 - \gamma_5$ has recently been extracted from unpolarised Compton scattering from the proton. The LEGS group\textsuperscript{6} obtained $-27 \times 10^{-4}$ fm$^4$, far from the previously accepted value of $-37 \times 10^{-4}$ fm$^4$, which is dominated by $t$-channel pion exchange. In contrast results presented by Wissmann at this conference give a value extracted from TAPS data which is compatible with the old value.

2 Polarisabilities in HBCPT

The non-spin polarisabilities have previously been determined to NLO (fourth order) in heavy baryon chiral perturbation theory (HBCPT). The values are in good agreement with experiment, with the NLO contribution (where LEC’s enter) being small compared to the LO part (which comes from pion-nucleon loops). The spin polarisabilities have also been calculated\textsuperscript{7} at lowest order the value $\gamma_0 = \alpha_{em} g_A^2 / (24\pi^2 f^2 Nc^2 m_N^2) = 4.51$ is obtained for both proton and
neutron, where the entire contribution comes from $\pi N$ loops. The effect of the $\Delta$ enters in counter-terms at fifth order in standard HBCPT, and has been estimated to be so large as to change the sign. The calculation has also been done in an extension of HBCPT with an explicit $\Delta$ by Hemmert et al. They find that the principal effect is from the $\Delta$ pole, which contributes $-2.4$, with the effect of $\pi\Delta$ loops being small, $-0.2$. Clearly the next most important contribution is likely to be the fourth-order $\pi N$ piece, and this is the result which is presented here. Two other groups have also presented fourth order calculations of the spin polarisabilities recently: Ji et al. calculated $\gamma_0$ and obtained an expression in complete agreement with ours. Gellas et al. have also calculated all four polarisabilities. Their calculations agree with ours, but we disagree on what constitutes the polarisabilities; we will say more about this later.

In HBCPT the fixed terms in the amplitudes $A_3$ to $A_6$ are reproduced at leading (third) order, by the combination of the Born terms and the seagull diagram. The same terms are produced entirely from Born graphs in the relativistic theory, but integrating out the antinucleons generates a seagull term in the third-order Lagrangian which has a fixed coefficient. This illustrates a point which we will come back to, namely that one cannot determine by inspection which graphs in HBCPT are one-particle reducible. The loop diagrams of Fig. 1 have contributions of order $\omega$ which cancel and so do not affect the LET, while the $\omega^3$ terms give the polarisabilities at this order.

At NLO, the diagrams which contribute are given in Fig. 2. In the Breit frame, only diagrams 2a-h contribute, and there can be no seagulls at this order. It follows that there are no undetermined low-energy constants in the final amplitude. When the amplitudes are Taylor expanded, there are contributions at order $\omega$ and $\omega^3$. The former do not violate the LETs, however. The third-order contributions to the LETs actually involve the bare values of $\kappa$ which enter in the second-order Lagrangian. However $\kappa_v$ has a pion loop contribution at the next order: $\delta\kappa_v = -g_A^4 m_\pi M_N / 4\pi f^2$. This then contributes to the fourth-order Compton scattering amplitude. Reproducing these terms is one check on our calculations. The order $\omega^3$ pieces give the polarisabil-
ties. The requirement $\gamma_5 + \gamma_2 + 2\gamma_4 = 0$ is satisfied, which provides another non-trivial check on the results.

The loop contributions to the polarisabilities to NLO are

$$\begin{align*}
\gamma_1 &= \frac{\alpha em g_4^2}{24\pi^2 f^2\pi m^2} \left[ 1 - \frac{\pi m}{8M_N} (8 + 5\tau_3) \right] \\
\gamma_2 &= \frac{\alpha em g_4^2}{48\pi^2 f^2\pi m^2} \left[ 1 - \frac{\pi m}{4M_N} (8 + \kappa_v + 3(1 + \kappa_s)\tau_3) \right] \\
\gamma_3 &= \frac{\alpha em g_4^2}{96\pi^2 f^2\pi m^2} \left[ 1 - \frac{\pi m}{4M_N} (6 + \tau_3) \right] \\
\gamma_4 &= \frac{\alpha em g_4^2}{96\pi^2 f^2\pi m^2} \left[ -1 + \frac{\pi m}{4M_N} (15 + 4\kappa_v + 4(1 + \kappa_s)\tau_3) \right] \\
\gamma_0 &= \frac{\alpha em g_4^2}{24\pi^2 f^2\pi m^2} \left[ 1 - \frac{\pi m}{8M_N} (15 + 3\kappa_v + (6 + \kappa_s)\tau_3) \right]
\end{align*}$$

(3)

Although the subleading pieces have a factor of $m_\pi/M_N$ compared with the leading piece, the numerical coefficients are often large. The anomalous magnetic moments are $\kappa_s = -0.12$ and $\kappa_v = 3.71$; with these values the numerical results for the polarisabilities to fourth order are

$$\begin{align*}
\gamma_1 &= \left[-21.3\right] + 4.5 - (2.1 + 1.3\tau_3) \\
\gamma_2 &= 2.3 - (3.1 + 0.7\tau_3) \\
\gamma_3 &= \left[10.7\right] + 1.1 - (0.8 + 0.1\tau_3) \\
\gamma_4 &= \left[-10.7\right] - 1.1 + (3.9 + 0.5\tau_3)
\end{align*}$$

Figure 2. Diagrams which contribute to spin-dependent forward Compton scattering in the $\epsilon \cdot \nu = 0$ gauge at NLO. The solid dots are vertices from $\mathcal{L}^{(2)}$. 

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\[ \gamma_0 = 4.5 - (6.9 + 1.5 \tau_3) \]
\[ \gamma_\pi = [-42.7] + 4.5 + (2.7 - 1.1 \tau_3) \]  

The term in square brackets, where it exists, is the third-order \(t\)-channel pion exchange contribution. (There is no fourth-order contribution.)

The NLO contributions are disappointingly large, and call the convergence of the expansion into question. While the fifth-order terms have also been estimated to be large\(^8\), this is due to physics beyond \(\pi N\) loops, namely the contribution of the \(\Delta\). Our results show that even in the absence of the \(\Delta\), convergence of HBCPT for the polarisabilities has not yet been reached.

3 Comments on the definition of polarisabilities

We now return to the difference between our results and those of the Jülich group, who give expressions for the polarisabilities which are analytically and numerically different from ours. The entire difference comes from the treatment of diagram 2g, which we include in the polarisabilities and they omit. The polarisabilities are not in fact usually defined as in Eq. 2, but as the first term in the expansion of the amplitudes after subtraction of the “Born terms”. This removes the LET terms, but, depending on the model used for the Born graphs, also some \(\omega\)-dependent terms. Gellas et al. argue that the contribution of 2g should also be removed by this subtraction.

There are two main objections to this definition. First, it is not model- and representation-dependent, as the one-particle reducible part of 2g beyond the LET piece involves an off-shell “formfunction” or “sideways formfactor”, and as stressed by Scherer,\(^{13}\) these cannot be unambiguously defined. Furthermore the procedure Gellas et al. have adopted does not respect Lorentz invariance. At this order there are terms that vanish in the Breit frame which are in fact generated by a lowest-order boost of the third-order (fully irreducible) loop amplitude. (In the centre-of-mass frame these show up as pieces with, apparently, the wrong crossing symmetry: they are even in \(\omega\) in amplitudes \(A_3\) to \(A_6\), and start at \(\omega^4\).) However the prescription of Gellas et al. discards the contribution of 2g to these pieces, violating the boost invariance of the resulting LO+NLO amplitude. (As Meißner explained in his talk, their prescription is to discard the part of 2g which has the form \(f(\omega)/\omega\), where \(f\) is analytic. In fact pieces like this also arise from other diagrams, notably 2f, while diagrams 2a-e, though apparently irreducible, contribute LET pieces. The distinction between reducible and irreducible in HBCPT is hidden, as mentioned earlier.)

The other objection to excluding so much from the definition of the po-
larisability is that, even if it is done consistently, it does not correspond to the definition used in the extraction from fixed-$t$ dispersion relations. There, the polarisabilities are related to the integral of the imaginary part of the amplitudes over the cut, where the amplitudes used have effectively been subtracted at the point where an intermediate nucleon would be on shell. This can at most change the spin polarisabilities by something of order $\alpha_{em} m_N^{-4}$, which is small numerically and is NNLO in HBCPT.

Thus the exclusion of $2g$ from the “structure constants” such as polarisabilities is neither a consistent definition, nor one that corresponds to dispersion relation determinations.

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