Slavnov-Taylor1.0: A Mathematica package for computation in BRST formalism

Marco Picariello\textsuperscript{a,b} Emilio Torrente-Lujan\textsuperscript{b,c}

\textsuperscript{a}INFN sezione di Milano, Via Celoria 16, I-20133 Milano, Italy
\textsuperscript{b}Departamento de Fisica, Grupo de Fisica Teorica, Universidad de Murcia, Murcia, Spain
\textsuperscript{c}CERN-TH, CH-1211 Geneve 23, Switzerland

Abstract

\textbf{Slavnov-Taylor} is a Mathematica package which allows us to perform automatic symbolic computation in BRST formalism. This article serves as a self-contained guide to prospective users, and indicates the conventions and approximations used.

\textit{Key words:} Slavnov-Taylor, BRST, Mathematica

1 Program summary

\textit{Title of program: Slavnov-Taylor}
\textit{Available at:} http://pcpicariello.mi.infn.it/ST/
\textit{Programming Language:} Mathematica 4.0
\textit{Platform:} Any platform supporting Mathematica 4.0
\textit{Computers tested on:} Pentium PC
\textit{Operating systems under which the program has been tested:} Linux
\textit{Memory required to execute:} Minimal: 1.254.784 bytes, Standard: 1.281.248 bytes
\textit{No. of bytes in distributed program, including test data, etc:} 35.636 bytes
\textit{Keywords:} Slavnov-Taylor, BRST, Mathematica
\textit{Nature of physical problem:} Symbolic computation in the Slavnov-Taylor formalism for gauge theories in 4-Dimensional space-time based on a semi-simple compact Lie group for a general BRS transformations.
\textit{Restrictions on the complexity of the problem:} Only matter in the adjoint is allowed.
\textit{Typical running time:} less than one second
2 Introduction

Symbolic computation in the Slavnov-Taylor (ST) formalism is very useful to study properties of a field theory such as quantum stability. The main computation problem is to find the most general counter-term compatible with the ST identity, and then absorb this counter-term by a field and coupling constant redefinition. This can be a not easy task if the BRS transformation is not nilpotent. It is therefore desirable to construct a calculational tool which may perform the computation automatically. In this article we present a tool \( \text{Slavnov-Taylor} \) which partially solves this problem. \text{Slavnov-Taylor} allows us to perform generic computation of BRST variation and construct generic polynomials of a given dimension and ghost number. It can be used to check hand computation for generic ST operator which may not be nilpotent. Moreover the symbolic manipulation allows us to perform computation in general gauges.

We applied \text{Slavnov-Taylor} to verify the correctness of the computation used to prove the quantum stability of the Curci-Ferrari gauge in a gluon-ghost condensation scenario, where the BRS is not nilpotent.

2.1 The package

\text{Slavnov-Taylor} has been written as a Mathematica package. The interface with the user is given by the Mathematica application, which can be run either in on-line command or with the front-end feature.

The algorithm is not optimized, because the running time is not an important point due to the fact that people will need to run it few times (or even once) for a given model.

The imposition of the ST identity turns out to be equivalent to the solution of a linear problem. \text{Slavnov-Taylor} is a tool whose output could be used for constructing such linear system.

It may also be used as help for more theoretical studies such as the stability of gauge theory when the ST is not nilpotent.

2.2 Aims and Contents

The main aims of this article are to provide a manual for the use of \text{Slavnov-Taylor}, to describe the limit of application and the notation used (to
allow for user generalization).

The rest of this paper proceeds as follows:

- The relevant parameters are presented in sec. 3.
- The approximations employed are noted in sec. 4. The algorithm of the calculation is also outlined.
- A description of how to use the package is given in sec. 5, including information on the input. More technical informations related to running and extending the package are placed in appendices.
- Sample output from one run is displayed and explained in appendix A.
- Appendix B gives a more technical example of the package.
- Finally, in appendix C, a description of the relevant modules and objects and their relation to each other is presented.

3 The creation of a model

In this section, we introduce the objects and parameters in the Slavnov-Taylor conventions. Translation to the actual name used in the Slavnov-Taylor package are shown in Appendix C.

3.1 Field content

The gauge theories in 4-Dimensional space-time considered in the Slavnov-Taylor package are based on a semi-simple compact Lie group. In a general gauge model the gauge fields are Lie algebra valued and the belong to the adjoint representation:

\[ A_\mu(x) = A_\mu^a(x)\tau_a \]

where the matrices \( \tau_a \) are the generators of the group and obey

\[ [\tau_a, \tau_b] = i f_{abc} \tau_c, \quad Tr \tau_a \tau_b = \delta_{ab} \]

Matter fields can be included if they are in the adjoint representation (which is for example the case of the gaugino for pure Yang-Mills theory in Supersymmetry).

The action is constructed as the most general gauge invariant and power-counting renormalizable action. We define the curvature:

\[ F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i [A_\mu, A_\nu] \]
and the invariant is given by:

\[ S_{\text{inv}} = -\frac{1}{4g^2} \int d^4x \ F_{\mu\nu} F^{\mu\nu} \]

The operator \( \cdot \) is the trace over the gauge group index.

The gauge fixing is done in the BRS way: one introduce the Faddeev-Popov ghost and antighost fields \( c(x)^a \) and \( \bar{c}(x)^a \), and the Lautrup-Nakanishi field \( b(x) \). The latter normally play the role of a Lagrange multiplier for the gauge condition. These three fields are Lie-algebra valued: \( c(x) = c^a(x)\tau^a \), etc. They are used to write the gauge fixing term, needed to extend to quantum level the theory.

Renormalization requires the introduction of an external source coupled to the BRS variation of the fields. This is done by adding to the action the terms

\[ S_{\text{ext}} = \int d^4x \sum_{\Phi} X[\Phi].(s\Phi) \]

where the sum is over the fields which have a non linear BRS transformation. We used the Slavnov-Taylor notation \( X[\Phi] \) as the external field coupled to the non linear BRS variation of the \( \Phi \) field.

### 3.2 BRST transformation

The BRS transformation are defined as:

\[
\begin{align*}
    sA_\mu &= \partial_\mu c + iA_\mu \wedge c \\
    sc &= \frac{1}{2} c \wedge c \\
    s\bar{c} &= b \\
    sb &= 0
\end{align*}
\]

where the \( \wedge \) operator is the external product and is defined on two general quantities Lie algebra valued \( \chi \) and \( \eta \) as:

\[
(\chi \wedge \eta)^a = f^{abc}_d \chi^b \eta^c
\]

BRST invariance can thus be expressed by a functional identity, the ST identity:

\[ S(S) = 0 \]

where the nonlinear ST operator is given, for any functional \( F \) by:

\[ S(F) = \int d^4x \sum_{\Phi} \frac{\delta F}{\delta X[\Phi]} \frac{\delta F}{\delta \phi} + s\phi \frac{\delta F}{\delta \phi} \]
where the sum over $\Phi$ is a over the fields with non linear BRS transformation, while the sum over $\phi$ is over the fields with linear BRS transformation.

In case the BRS is nilpotent the gauge fixing action is given by an $s$ variation of an integrated local polynomial:

$$S_{gf} = s \int d^4x \ P(fields)$$

otherwise it is a ST invariant integrated local polynomial.

ST formalism allows us to study stability and unitarity of the model, but these properties are not investigated by our package. These points are reserved for a future version of the Slavnov-Taylor package.

### 3.2.1 Fields Properties

The field properties used by the Slavnov-Taylor package are:

#### 3.2.1.1 Lorentz properties:

The boson gauge fields $A_\mu^a(x)$ have a Lorentz index, while the $b^a(x), c(x)$ and $\bar{c}(x)$ have no Lorentz index (they are scalar).

#### 3.2.1.2 Statistic:

The boson gauge fields $A_\mu^a(x)$ and the Lautrup-Nakanishi fields $b^a(x)$ are commuting fields, while the components of $c(x)$ and $\bar{c}(x)$ are anti-commuting.

#### 3.2.1.3 Canonical dimension:

The boson gauge fields $A_\mu^a(x)$, the ghosts $c^a(x)$ and the antighost $\bar{c}^a(x)$ have canonical dimension one, while the fields $b^a(x)$ have canonical dimension two.

#### 3.2.1.4 Ghost number:

The boson gauge fields $A_\mu^a(x)$ and the Lautrup-Nakanishi fields $b^a(x)$ are fields with ghost number zero, while the components of $c(x)$ have ghost number one and the components of $\bar{c}(x)$ have ghost number minus one.

### 4 Calculation

In fig 4 we show the algorithm used to perform the calculation. Due to the fact that user can define the field content he needs and their BRS transformation, he can easily modify the model. For example to study pure Supersymmetric Yang-Mills theory, he needs to define gaugino fields and the Supersymmetric
and the general polynomial of given dimension and ghost number

User obtains ST transformations of a general integrated polynomial

and the general polynomial of given dimension and ghost number

Fig. 1. Algorithm used to calculate ST transformations of general quantities in a given model. Each step (represented by a box) is detailed in the text.

BRS transformations. In the same way the generalized BRS transformations introduced in (3) can be treated. Moreover theories with non nilpotent BRS transformation can be taken into account with no problem (for example on-shell formulation of Supersymmetric theory (4), or gauge theory in presence of condensates (5)).

5 Running Slavnov-Taylor

The program ST.nb is included in the Slavnov-Taylor distribution. This program is can be run by Mathematica. The program contains:

- the definition of the fields $a, c, \bar{c}, b$ and their BRS transformation;
- the invariant action and the gauge fixing Curci-Ferrari action as an $s$ variation of a polynomial;
- The definition of the full action as the sum of the invariant action, the gauge-fixing action and the external action.

The program gives as output:

- the external action $S_{ext}$ create by the program;
- the list of the field defined by the user;
- the list of the external field defined by the program;
- the explicit form of the full action;
• the dimension of the full action;
• the ghost number of the full action;
• the values of $c \frac{\delta S_{tot}}{\delta \phi}$
• the $s$ variation of the full action (which is zero);
• the most general polynomial of dimension three and ghost number $-1$;
• the most general polynomial of dimension two and ghost number 0;
• the $s$ variation of $a_\mu a_\mu$;
• the $s$ variation of the $s$ variation of $a_\mu a_\mu$;

5.1 Description of the main procedures and global parameters

Setting the global parameter $\$QUIET$ to a non-zero value gives additional information on field creations.
The properties of the fields content of the model are encoded in objects and must be provided. The same for the BRS transformations. We give here a description of the main procedures that user can use.

• DefineField
  • Parameters:
    Name is the name of the field;
    Indices is a list which determines the Lorentz properties of the field (an empty list define a scalar);
    Type determines the statistic of the field, Type=$\$BOSE$ define a bosonic (commuting) field; Type=$\$FERMI$ define a fermionic (anti-commuting) field;
    Dimension is the dimension of the field;
    Gh define the ghost number of the field.
  • Object created:
    Field
  • Set quantities:
    Stat the statistic of the field: $+/−1$ for commuting/anti-commuting field;
    Dim the canonical dimension of the field
    Gh the ghost number of the field
  • Global variables modified:
    appends to the variable $\$FieldList$ the name of the field

• SetBRST
  • Parameters:
    Name The field name of which we are defining the BRS transformation;
    brs The BRS transformation of the field.
  • Object created:
    $\$ExtField$ (if the BRS transformation is not linear in the fields);
Set quantities:
- \( s \) is set to be the BRS transformation;
- BRST is set to TRUE;

Global variable modified:
- for non linear BRS transformations it adds to \( \text{Sext} \) the right quantity

Module called:
- for non linear BRS transformations it call the module \( \text{DefineExtField} \)
  (which appends to the variable \( \$\text{ExtFieldList} \) the name of the external field, and works in a similar way than \( \text{DefineField} \) module)

Create
- Parameters:
  - \textbf{Dimension} the canonical dimension of the integrated polynomial we are looking for
  - \textbf{Gh} the ghost number of the integrated polynomial we are looking for

An object \( \text{Field} \) can be removed by the module \( \text{UndefineField} \). An object \( \text{ExtField} \) can be deleted by the module \( \text{UndefineExtField} \).

A Sample Output

We now present the output for a single example calculation. When you load the package you obtain some information on the version in use. We load the gauge theory model with usually BRS transformation which is defined in the \texttt{ST.nb} file. Once the model is defined, we can perform the computation. We run the commands

\[
\text{In}[1]:= \text{\textless \textless PackageST.m; }
\text{In}[2]:= \text{\textless \textless ST.nb; }
\text{In}[3]:= \\
\text{s[a[\mu].a[\mu]]} \\
\text{s[\%]}
\]

The output obtained is

\[
\text{Out}[3]= -2 a \cdot d \cdot c \\
\text{Out}[4]= 0
\]

The first output is the consequence of the application of the ST operator to quantity \( a^a_Ta \ast a^b_Tb \). The second output is the consequence of the application of the ST operator twice to the same quantity. In this case, due to the nilpotency of the ST operator, the result is zero.
Table B.1
Detailing arguments in order for module used to define a model.

| Name       | Arguments                                      |
|------------|------------------------------------------------|
| DefineField| Name, Lorentz indices, Statistic, Dimension, Ghost Number |
| SetBRST    | Name, BRST transformation                      |

B Sample Program

We now present the sample program from which it is possible to run Slavnov-Taylor in a simple fashion. The most important features of the modules and objects are described in appendix C. The sample program has the form displayed in figure B.

After an initial introductory print-out, the fields $a, b, c, \bar{c}$ are defined and their BRST transformation are set. For these, the same notation as appendix 5 is used.

The user has created the field content of his model. Now is time to define the classical action and to fix the gauge (for example the Curci-Ferrari gauge): The first results that can be easily obtained are the external action $S_{\text{ext}}$, the list of defined fields and external fields. Moreover the explicit form of the full action is obtained, and the dimension and ghost number of a general integrated polynomial (in the example $S_{\text{tot}}$). To show the use of the ST operator $s$, an application of the $Dr$ operator is shown and the variation of the full action (which turn out to be zero) is reported.

The use of the module Create is shown and the most general polynomials of dimension 3 and ghost number -1, and dimension 3 and ghost number zero are reported.

Finally a simple example showing the nilpotency of the $s$ operator is reported.

C Modules and objects

In this section we stretch the most important features of the modules and objects. In tab. C.1 we list the modules with their parameter with a short description, in tab. C.2 and tab. C.3 we give a list of the operators with their arguments and description.
### Table C.1
List of the modules with their parameter and description

| Name         | Arguments                     | Output |
|--------------|-------------------------------|--------|
| Gh           | Pol                           | Ghost number of Pol |
| UndefineField| name, Lorentz                 | Delete a Field |
| DefineField  | name, Lorentz, type, dim, gh  | Create a Field |
| UndefineExtField | name, Lorentz              | Delete an ExternalField |
| DefineExtField| name, Lorentz, type, dim, gh  | Create an ExternalField |
| SetBRST      | name, BRS                     | Associate a BRS variation to a Field |
| Create       | dimax, gh                     | Generate the most general integrated polynomial of the fields with dimension less or equal to dimax and ghost number gh |

### Table C.2
List of the operators with their arguments and description

| Name   | Arguments                     | Output |
|--------|-------------------------------|--------|
| Stat   | Mon                           | +1 if Mon is bosonic, -1 if it’s fermionic |
| Dim    | Pol                           | The dimension of Pol |
| Dr     | Pol, name                     | The functional derivative of Pol with respect to field name |
| s      | Pol                           | BRST variation of Pol |

### Table C.3
List of the overloaded operators with their arguments and description

| Name            | Arguments  | Output |
|-----------------|------------|--------|
| NonCommutativeMultiply | Pol1, Pol2   | Take into account the non commutativity of Lie algebra valued polynomial |
| Wedge           | Pol1, Pol2  | External product between Pol1 and Pol2 |
| Dot             | Pol1, Pol2  | Scalar product between Pol1 and Pol2 |

### Acknowledgments

We would like to thank Prof. R. Ferrari for enlightening conversations on the subject. One of us (M.P.) would like to thank Prof. S.P. Sorella for useful discussion during the preparation of this paper. One of us (E.T) would like to thank the hospitality of CERN-TH division, the department of physics of University of Milan, and financial support of INFN-CICYT grant.
References

[1] O. Piguet and S. P. Sorella, *Algebraic Renormalization*, Monograph series **m28**, Springer Verlag, 1995

[2] D. Dudal et al., Phys.Lett. B569 (2003) 57-66 [arXiv:hep-th/0306116].

[3] P. Federbush, [arXiv:hep-th/9906245].

[4] N. Maggiore, Contemp. Math. **219** (1998) 141.
   N. Maggiore, O. Piguet and S. Wolf, Nucl. Phys. B **458** (1996) 403 [Erratum-ibid. B **469** (1996) 513] [arXiv:hep-th/9507045].

[5] D. Dudal et al., JHEP **0306**, 003 (2003) [arXiv:hep-th/0305020].

D. Dudal, A. R. Fazio, V. E. Lemes, M. Picariello, M. S. Sarandy, S. P. Sorella and H. Verschelde, *Vacuum condensates of dimension two in pure Euclidean Yang-Mills*, [arXiv:hep-th/0304249].

D. Dudal, H. Verschelde, V. E. Lemes, M. S. Sarandy, S. P. Sorella and M. Picariello, Annals Phys. 308, 66-77 (2003) [arXiv:hep-th/0302168].

D. Dudal, V. E. Lemes, M. S. Sarandy, S. P. Sorella and M. Picariello, JHEP **0212**, 008 (2002) [arXiv:hep-th/0211007].

V. E. Lemes, M. S. Sarandy, S. P. Sorella, M. Picariello and A. R. Fazio, Mod. Phys. Lett. A **18**, 711 (2003) [arXiv:hep-th/0210036].
Automatic computation in the BRST formalism

- Definitions

- Reading Package

```mathematica
In[1]:= << PackageST.m;
Package ST
Version 1.0 by Marco Piccinello
```

- Definition of the fields

```mathematica
In[2]:= $Quiet = False;
```

```mathematica
In[3]:= DefineField[a__, b___, c___, d___];
In[4]:= DefineField[b__, c___, d___];
In[5]:= DefineField[c__, d___, c___];
In[6]:= Format[b___] = Ω;
In[7]:= Defined field a
In[8]:= Defined field b
In[9]:= Defined field c
In[10]:= Defined field d
```

- Definition of the transformation of the fields

```mathematica
In[11]:= SetField[a_, b___, c___];
In[12]:= SetField[b___, c___];
In[13]:= SetField[c___, d___];
In[14]:= Defined classical field X[c]
In[15]:= Defined classical field X[a]^
```

- Definitions of the action

- Definition of the constants

```mathematica
In[16]:= MyNumberOf[a_] = True;
In[17]:= MyNumberOf[b_] = True;
```

- Definition of the classical action

```mathematica
In[18]:= F[μ_, 3_] = d[μ, a[3]] - d[3, a[μ]] + a[μ] x a[3];
Sin = 1/q + 2 F[μ, V];
In[19]:= Sgf = a[barc] x d[μ, a[μ]] + (a/2) barc . (b - (1/2) barc x c)];
```

- The full action

```mathematica
In[20]:= Stot = Simplify[Expand[(Sin + Sgf + Sext)]];
```

- Results

- The action

```mathematica
In[17]:= (d[c] x X[a]) - c . (a x X[c]) + c . (c x X[c])
```

```mathematica
In[18]:= $FieldList
```

```mathematica
In[19]:= $ExtFieldList
```

```mathematica
In[20]:= X[c], X[a]^n
```

```mathematica
In[21]:= Stot
```

```mathematica
Out[20]= (d[c] x X[a]) - 2 (d[c] x X[a]) + (d[c] x X[a]) - (d[c] x X[a]) -
```

```mathematica
In[22]:= Dim[Stot]
```

```mathematica
Out[21]= 4
```

```mathematica
Out[22]= 0
```

- Dr operator and how to use it

```mathematica
In[23]:= c . Dr[Stot, b]
```

```mathematica
Out[23]= a b c + c . (d[c] x X[c]) - 1/2 a c . (c x c)
```

- Computations s[Stot]

```mathematica
In[24]:= Simplify[s[Stot] - a / c . Jacobi[barc, barc, c, c]]
```

```mathematica
Out[24]= 0
```

- General Counterterm

```mathematica
In[25]:= Create[3, -1]
```

```mathematica
Out[25]= k4 (d[mu]^2) . c + k5 a^mu . (d[mu] c) + k3 a^mu . X[a]^mu - k6 b . c + k2 c . X[c] + k7 c . c
```

```mathematica
In[26]:= Create[2, 0]
```

```mathematica
Out[26]= k9 a^mu . a^mu + k10 c c
```

- Example of the nilpotence of s

```mathematica
In[27]:= s[a[mu] . a[mu]]
```

```mathematica
Out[27]= - 2 a^mu . (d[mu] c)
```

```mathematica
Out[28]= 0
```