Non-equilibrium dynamics of Andreev states in the Kondo regime

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The transport properties of a quantum dot coupled to superconducting leads are analyzed. It is shown that the quasiparticle current in the Kondo regime is determined by the non-equilibrium dynamics of subgap states (Andreev states) under an applied voltage. The current at low bias is suppressed exponentially for decreasing Kondo temperature in agreement with recent experiments. We also predict novel interference effects due to multiple Landau-Zener transitions between Andreev states.

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Fig. 1: Schematic representation of the Andreev states dynamics in two different situations: a) quantum point contact and b) S-QD-S in the Kondo regime. The dashed lines correspond to the states for perfect normal transmission. The lower and upper striped regions represent the states in the continuous spectrum. The straight arrows indicate Landau-Zener transitions between AS’s, while the curved arrows represent inelastic transitions between the AS’s and the continuous spectrum (see text).

in which the current is due to Andreev processes whose order \( n \sim \text{int}(2\Delta/ev) \) becomes arbitrarily large as the voltage \( v \) tends to zero. The aim of this letter is to show that these properties can be understood in terms of the dynamics of the Andreev states under an applied bias voltage.

In order to give a qualitative explanation of this dynamics let us first analyze the behavior of the AS’s in different situations, as illustrated in Fig. 1. In the absence of a QD, a single channel S-S contact with normal transmission \( \tau \) exhibits AS’s located at \( \pm \Delta \sqrt{1 - \tau \sin^2 \phi / 2} \). where \( \phi \) is the phase drop across the contact. In equilibrium and at low temperatures only the lower state is occupied. Under a small bias voltage this state evolves adiabatically except for Landau-Zener (LZ) transitions to the upper state with probability \( p = \exp(-\pi \Delta r/ev) \),
where \( r = 1 - \tau \). This mechanism produces a net transfer of quasiparticles from the occupied to the empty states above the superconducting gap accounting for the low bias dc current in the system, as represented by the arrows in Fig. 1 a. As shown below, in a S-QD-S in the Kondo regime the situation is qualitatively different due to the fact that the AS’s detach from the continuous part of the spectrum (see Fig. 1 b). As a consequence the conversion of bound state electrons into free quasiparticles now requires an inelastic transition across the gap between the AS and the edge of the continuous spectrum. This feature is the main ingredient which determines the behavior of the current at low voltages. In particular we will show that there is an exponential suppression of the current as a function of the parameter \( T_K/\Delta \). The detachment of the AS’s from the continuous spectrum also gives rise to novel interference effects due to multiple LZ transitions between the bound states which could be observed directly as an oscillatory behavior of the dc current.

To describe a S-QD-S we start from the usual single level Anderson model, thus assuming a sufficiently large level separation. The model Hamiltonian is \( \mathbf{H} = \mathbf{H}_L + \mathbf{H}_R + \mathbf{H}_T + \mathbf{H}_D \), where \( \mathbf{H}_{L,R} \) describe the left and right leads as BCS superconductors, \( \mathbf{H}_T = \sum_{k,\sigma} \epsilon_{k,\sigma} \hat{c}_{k,\sigma}^\dagger \hat{d}_\sigma + \text{h.c.} \) is the term coupling the dot to the leads, \( \hat{d}_\sigma \) and \( \hat{c}_{k,\sigma}^\dagger \) being creation operators for electrons in the dot and in the leads respectively. \( \mathbf{H}_D = \sum_{\sigma} \epsilon_0 \hat{n}_\sigma + U \hat{n}_\uparrow \hat{n}_\downarrow \) is the uncoupled dot Hamiltonian characterized by the dot level position \( \epsilon_0 \) and the Coulomb interaction \( U \). It is assumed that the coupling of the dot to the leads in the normal state can be described by energy independent tunneling rates \( \Gamma_L = \Gamma_R = \Gamma \). As discussed in Ref. 10 the bias voltage can be introduced as a time-dependent phase factor in the hopping term.

A crucial point in the analysis of this model is how to deal with the electron correlation effects. In Ref. 8 a perturbative approach was used to study the zero-voltage case. It was found that the AS’s in the Kondo regime \( \langle T_K > \Delta \rangle \) have essentially the same phase dependence as in the non-interacting case but with a reduced amplitude. This is consistent with a Fermi liquid description of the normal state with renormalized parameters \( \epsilon_0^* \) and \( \Gamma^* \), where \( \Gamma^* = T_K/2 \) fixes the width of the Kondo resonance. Although different approximations differ in the way to relate these quantities to the bare parameters, the description in terms of the renormalized ones can be considered as universal 11. In the opposite limit \( (T_K < \Delta) \), the system is well described within the Hartree-Fock approximation 8 6.

In the present work we shall be concerned with the Kondo regime. At zero bias the AS’s are determined by the poles of the dot retarded Green function, which in the Nambu space and close to the Fermi energy adopts the form

\[
G_D^r(\omega) \approx \frac{\Gamma^*}{\Gamma} \left( \frac{\omega - \epsilon_0^* - 2\Gamma^* g^r}{2\Gamma^* f^r \cos \phi/2} \right)^{-1},
\]

where \( g^r = -i/\sqrt{\Delta^2 - (\omega + i0^+)^2} \) and \( f^r = \Delta/\sqrt{\Delta^2 - (\omega + i0^+)^2} \) correspond to the dimensionless Green functions of the uncoupled electrodes. In the electron-hole symmetric case \( (\epsilon_0^* = 0) \) the AS’s are then determined by the equation

\[
\omega_s \pm \Delta \cos \phi/2 + \omega_s \frac{\sqrt{\Delta^2 - \omega_s^2}}{2\Gamma^*} = 0.
\]

The solutions of Eq. 2 for \( \Gamma^* \gg \Delta \) and \( |\phi| \ll 1 \) are well approximated by \( \omega_s = \pm \Delta \cos \phi/2 \) with \( \Delta = \Delta[1 - 2(\Delta/2\Gamma^*)^2] \). Thus, as commented above, the AS’s for the S-QD-S are detached from the continuous spectrum for finite values of \( T_K/\Delta \), with a gap given by \( \Delta - \Delta \) (see Fig. 1 b). For decreasing \( T_K/\Delta \) the AS’s begin to deviate from this simple phase-dependence 8.

In a non-symmetric situation \( (\epsilon_0^* \neq 0) \) the two ballistic states are coupled and there appear an upper and a lower bound state with an internal gap of the order of \( 2\Delta \sqrt{1 - \tau} \), where \( \tau = 1/ \left[ 1 + (\epsilon_0^*/(2\Gamma^*))^2 \right] \) is the normal transmission at the Fermi energy. This is similar to the behavior of the AS’s in a point contact with finite reflection probability, except for the detachment from the continuous spectrum (see Fig. 1 b).

In order to analyze the voltage biased situation we shall assume that the description in terms of the renormalized parameters \( \epsilon_0^* \) and \( \Gamma^* \) is still valid in the voltage range \( (ev \ll \Delta) \) in which we are interested. Notice that in the Kondo regime this corresponds to \( ev \ll T_K \) and thus one can safely assume that the coherence associated with the Kondo effect will not be destroyed 12.

For calculating the current we use a combination of Keldysh and Nambu formalisms following Refs. 10. Explicit extensions of this approach to the case of interacting quantum dots are discussed in Refs. 8 13.

This procedure allows to evaluate the current in terms of Green functions as \( I(t,v) = \sum_n I_n(v) \exp[\phi_0(t)] \), where \( \phi(t) = 2evt/\hbar + \phi_0 \). In the present case we shall be interested in the dc component \( I_0(v) \).

We shall analyze first the electron-hole symmetric case \( (\epsilon_0^* = 0) \). The behavior of the dc current at low bias is shown in Fig. 2. When \( \Gamma^* \gg \Delta \) the IV characteristics of a ballistic quantum point contact is gradually recovered, exhibiting a divergent low bias conductance with a saturation of the current at \( v = 0 \) towards the value \( 4e\Delta/\hbar \) 8 10. As \( \Gamma^*/\Delta \) is reduced the low bias conductance is gradually suppressed. When \( \Gamma^* \sim \Delta \) oscillations with a period \( \Delta/n \), where \( n \) is an integer, starts to be observable in the IV characteristic. These oscillations arise from the opening of new Andreev channels. As discussed in Ref. 13, MAR processes of odd (even) order are enhanced
The probability for this transition is then given by

$$P(t) = \frac{1}{\hbar} \int_0^\infty d\omega \text{Im} \Sigma(\omega) \times \left| \int_0^t dt_1 e^{i \int_0^{t_1} dt_2 (\omega - v/2 - \omega_n(t_2))/\hbar} \right|^2.$$  (3)

Using the approximate form of the AS, $\omega_n \simeq \Delta \cos \phi/2$ (valid for large $\Gamma^*/\Delta$) and in the stationary limit ($t \to \infty$) one obtains the rate for this transition, $\gamma_T = \sum_{n \geq 0} \gamma_n$, with

$$\gamma_n = \frac{2\pi}{\hbar} J_n^2(\Delta/v) \text{Im} \Sigma(nv + v/2) \theta(nv + v/2 - \Delta),$$  (4)

where $J_n$ are the integer order Bessel functions and $\theta$ is the step function. In this decomposition the rates $\gamma_n$ correspond to processes in which $(2n+1)e$ are transferred, which allows to write the dc current as $I_0 = e \sum_n (2n + 1) \gamma_n$. In the limit $v \to 0$, $nv + v/2$ in Eq. (4) can be taken as a continuous variable $x$. From the asymptotic behavior of $J_n$ for large $n$ one can obtain a simplified expression for the current at low bias

$$I_0 = \frac{2e}{\Gamma^* \hbar} \int_\Delta^\infty dxx^2 \sqrt{x^2 - \Delta^2} e^{2\pi(i\hbar \alpha - \alpha)/v},$$  (5)

where $\cosh \alpha = x/\Delta$.

The predictions of Eq. (5) are compared with the full numerical results in Fig. 2. It should be stressed that the asymptotic expression is only valid in the voltage range $ev \leq (\Delta - \Delta)$. Within this range the current is suppressed according to Eq. (5), decreasing exponentially with the parameter $(\Delta - \Delta)/ev$. The agreement with the full numerical results support our interpretation in terms of the AS’s dynamics.

In the absence of electron-hole symmetry ($\epsilon_0^* \neq 0$) the dynamics of the AS’s become more complex due to interference effects. To establish a dc current now requires LZ transitions between the lower and the upper state in addition to the mechanism previously discussed. In contrast to the case of a quantum point contact (in which the AS’s merge with the continuous spectrum at $\phi = 2n\pi$), in the S-QD-S case there is a finite probability of preserving the system coherence after a complete cycle. As a consequence interference effects between subsequent LZ transitions may appear. The two main paths for this interference correspond to the evolution following the lower and upper AS, as schematically shown in Fig. 1 b. The phase difference between these two paths is thus simply given by $\int_\pi^{3\pi} d\phi \omega_n(\phi)/ev$. One would therefore expect oscillations of the form $\cos^2(2\Delta/ev)$ to be observable in the current in addition to the ones associated with the onset of new MAR processes.

FIG. 3: Evolution of the oscillations pattern in the IV characteristics as a function of $\epsilon_0^*$ for $\Gamma^*/\Delta = 5$ and $\epsilon_0^*/\Delta = 0, 0.5, 1.0, 1.5$ and 2 from top to bottom. Inset: $\Gamma^*/\Delta = 2$ and $\epsilon_0^*/\Delta = 0, 0.2, 0.4, 0.6$ and 0.8.

Fig. 3 shows the numerical results for the low bias...
dc current in a non-symmetric situation. As can be observed, when increasing \( q \) there is an overall suppression of the current together with the onset of an oscillatory behavior of increasing amplitude. These curves correspond to a large \( \Gamma^\ast / \Delta \) value in order to disentangle these oscillations from the MAR induced ones. The overall suppression is governed by the LZ probability \( p \). The superimposed oscillations exhibit the expected behavior associated with the interference between AS’s due to multiple LZ transitions. A simple argument suggests that the amplitude of these oscillations should scale as \( p(1-p) \) and therefore be maximum for \( p = 1/2 \) (i.e. \( e \nu / \Delta \approx \pi \tau / \ln 2 \)), as confirmed by the numerical results. When \( \Gamma^\ast / \Delta \) is reduced one can observe a superposition of the oscillations arising from the two mechanisms discussed above. The inset of Fig. 3 illustrates the transition from one type of oscillations to the other in a reduced voltage range.

The results presented above indicate that the IV curves of a S-QD-S in the Kondo regime are extremely non-linear in the low bias limit and thus the system cannot be characterized just by its zero-bias conductance. In an actual experiment the observability of the predicted behavior will be limited by several possible factors like the multilevel structure of the QD, environmental effects and inelastic relaxation processes which could be of importance at finite temperature. In the present calculations we have assumed sufficiently small temperatures so that the inelastic relaxation is negligible within the voltage range considered. In order to attempt a comparison with the results of Ref. [13] one should take into account that the actual voltage resolution is limited to some minimum value \( \delta v \) in such a way that the measured zero-bias conductance is \( G_S \approx I_0(\delta v) / \delta v \). In these experiments \( \delta v \) can be estimated to be around 2.5\( \mu \)V, which corresponds to \( e \delta v / \Delta \approx 0.025 \). The zero-bias conductance defined by this finite voltage resolution is plotted in Fig. 4 as a function of \( T_K / \Delta \). When \( T_K / \Delta \gg 1 \) the conductance \( G_S \) tends to saturate to a value which can be several times larger than the normal conductance. The actual value depends on \( \delta v \) and is very sensitive to slight deviations from perfect transmission (deviations as small as 2% from \( \tau = 1 \) produce variations by a factor \( \sim 10 \) in \( G_S / G_N \)). On the other hand, the conductance exhibits an exponential suppression for \( T_K / \Delta \leq 1 \). All these predictions are in qualitative agreement with the results presented in Fig. 4 of Ref. [3].

In conclusion we have shown that the transport properties of a S-QD-S in the Kondo regime can be understood from the dynamics of the Andreev states under an applied bias voltage. The quasiparticle current at low bias is controlled by inelastic transitions between the AS’s and the extended states in the continuous spectrum. On the other hand, we predict novel interference effects due to multiple Landau-Zener transitions between Andreev states which could be worth to explore in future experiments.

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[1] D.L. Cox and M.B. Maple, Phys. Today. 48, 32 (1995).
[2] D. Goldhaber-Gordon et al., Nature 391, 156 (1998); S. M. Cronenwett et al., Science 281, 540 (1998); J. Nygärd et al., Nature 408, 342 (2000).
[3] M.R. Buitelaar et al., Phys. Rev. Lett. 89, 256801 (2002).
[4] L.I. Glazman and K.A. Matveev, JETP Lett. 49, 659 (1989); B. I. Spivak and S.A. Kivelson, Phys. Rev. B 43, 3740 (1991); C.W.J. Beenakker and H. van Houten, cond-mat/0111505; A.V. Rozhkov and D.P. Arovas, Phys. Rev. Lett. 82, 2788 (1999); A.A. Clerk and V. Ambegaokar, Phys. Rev. B 61, 9109 (2000).
[5] E. Vecino et al., Phys. Rev. B 68, 035105 (2003).
[6] See monograph issue on “Mesoscopic Superconductivity” in Superlattices Microstruct. 25, No. 5/6 (1999).
[7] Y. Avishai et al., Phys. Rev. B 63, 134515 (2001); Phys. Rev. B 67, 041301 (2003).
[8] C.W.J. Beenakker, Phys. Rev. Lett. 67, 3836 (1991).
[9] D. Averin and A. Bardas, Phys. Rev. Lett. 75, 1831 (1995).
[10] J.C. Cuevas et al., Phys. Rev. B 54, 7366 (1996); A. Martín-Rodero et al., Superlattices Microstruct. 25, 927 (1999).
[11] An equivalent description in terms of renormalized parameters is obtained within the slave-boson mean field approach. See for instance last reference in [7].
[12] A. Kaminski et al. Phys. Rev. B 62, 8154 (2000); A. Rosch et al. Phys. Rev. Lett. 87, 156802 (2001).
[13] A.L. Yeyati et al., Phys. Rev. B 55, R6137 (1997); G. Johansson et al., Physica C 293, 77 (1997).
[14] C. Schönenberger and M.R. Buitelaar, private communication.