Interacting viscous entropy-corrected holographic scalar field models of dark energy with time-varying $G$ in modified FRW cosmology

Farzin Adabi$^1$, Kayoomars Karami$^2$, Fereshte Felegary$^2$ and Zohre Azami$^2$

$^1$ Department of Physics, Sanandaj Branch, Islamic Azad University, Sanandaj, PO Box 618, Iran; $^2$ Department of Physics, University of Kurdistan, Pasdaran St., Sanandaj, PO Box 66177-15175, Iran; KKarami@uok.ac.ir

Received 2011 July 27; accepted 2011 September 11

Abstract We study the entropy-corrected version of the holographic dark energy (HDE) model in the framework of modified Friedmann-Robertson-Walker cosmology. We consider a non-flat universe filled with an interacting viscous entropy-corrected HDE (ECHDE) with dark matter. Also included in our model is the case of the variable gravitational constant $G$. We obtain the equation of state and the deceleration parameters of the interacting viscous ECHDE. Moreover, we reconstruct the potential and the dynamics of the quintessence, tachyon, K-essence and dilaton scalar field models according to the evolutionary behavior of the interacting viscous ECHDE model with time-varying $G$.

Key words: cosmology: dark energy — cosmological parameters

1 INTRODUCTION

The observed acceleration in the expansion rate of the universe (Riess et al. 1998; Perlmutter et al. 1999; Knop et al. 2003; de Bernardis et al. 2000) is usually attributed to the presence of an exotic kind of energy, called dark energy (DE). Although the nature and cosmological origin of DE are still enigmatic at present, a great variety of models have been proposed to describe DE (for a review, see Padmanabhan 2003; Peebles & Ratra 2003; Copeland et al. 2006; Li et al. 2011).

Recently a new DE candidate, namely holographic DE (HDE), based on the holographic principle, was proposed (’t Hooft 1993; Susskind 1995). Following Guberina et al. (2007), the HDE density can be derived from the entropy bound. In the thermodynamics of black holes (Bekenstein 1973, 1974, 1981, 1994; Hawking 1975, 1976), there is a maximum entropy in a box of size $L$, namely the Bekenstein-Hawking entropy bound $S_{BH} \sim M_P^2 L^2$, which scales as the area of the box $A \sim L^2$, rather than the volume $V \sim L^3$. Here $M_P$ is the reduced Planck mass $M_P^2 = 8\pi G$. In addition, for a macroscopic system in which self-gravitation effects can be disregarded, the Bekenstein entropy bound $S_B$ is given by the product of the energy $E \sim \rho_\Lambda L^3$ and the length scale (the IR cut-off) $L$ of the system. Here $\rho_\Lambda$ is the quantum zero point energy density caused by the UV cut-off $\Lambda$. Requiring $S_B \leq S_{BH}$, namely $EL \leq M_P^2 L^2$, one has $\rho_\Lambda \leq M_P^2 L^{-2}$. If the largest cut-off $L$ is taken for saturating this inequality, we get the HDE density as

$$\rho_\Lambda = 3c^2 M_P^2 L^{-2},$$

(1)
where $c$ is a numerical constant. Recent observational data, which have been used to constrain the HDE model, show that for the non-flat universe $c = 0.815^{+0.179}_{-0.139}$ (Li et al. 2009a), and for the flat case $c = 0.818^{+0.139}_{-0.097}$ (Li et al. 2009b). Li (2004) showed that the cosmic coincidence problem can be resolved by inflation in the HDE model, provided the minimal number of e-foldings exists. HDE models have been studied widely in the literature (Hsu 2004; Enqvist & Sloth 2004; Huang & Gong 2004; Huang & Li 2004; Gong 2004; Elizalde et al. 2005; Zhang & Wu 2005; Guberina et al. 2005; Shen et al. 2005; Wang et al. 2005; Beltrán Almeida & Pereira 2006; Guberina et al. 2006; Li et al. 2006; Zhang 2006; Zhang & Wu 2007; Xu 2009; Jamil et al. 2009a; Jamil & Farooq 2010a; Sheykhi 2009, 2010a,b; Karami et al. 2011). Indeed, the definition and derivation of HDE density depends on the entropy-area relationship

$$S_{BH} = \frac{A}{4G},$$

where $A \sim L^2$ is the area of the horizon. However, this definition can be modified from the inclusion of quantum effects, motivated from loop quantum gravity (LQG). These quantum corrections provided to the entropy-area relationship leads to the curvature correction in the Einstein-Hilbert action and vice versa (Banerjee & Modak 2009; Modak 2009; Banerjee et al. 2010). The corrected entropy takes the form (Banerjee & Majhi 2008a,b; Zhang 2008)

$$S_{BH} = \frac{A}{4G} + \tilde{\alpha} \ln \left( \frac{A}{4G} \right) + \tilde{\beta},$$

(2)

where $\tilde{\alpha}$ and $\tilde{\beta}$ are dimensionless constants of order unity. The exact values of these constants are not yet determined and are still an open issue in quantum gravity. These corrections arise in the black hole entropy in LQG due to thermal equilibrium fluctuations and quantum fluctuations (Rovelli 1996; Ashtekar et al. 1998; Meissner 2004; Medved & Vagenas 2004; Ghosh & Mitra 2005). Taking the corrected entropy-area relation (2) into account, and following the derivation of HDE (especially the one shown in Guberina et al. 2007), the HDE density will also be modified. On this basis, Wei (2009) proposed the energy density of the so-called “entropy-corrected HDE” (ECHDE) in the form

$$\rho_\Lambda = \frac{3c^2}{8\pi G L^2} + \frac{\alpha}{L^4} \ln \left( \frac{L^2}{8\pi G} \right) + \frac{\beta}{L^4},$$

(3)

where $\alpha$ and $\beta$ are dimensionless constants of order unity. In the special case $\alpha = \beta = 0$, the above equation yields the well-known HDE density (1). Since the last two terms in Equation (3) can be comparable to the first term only when $L$ is very small, the corrections make sense only at the early stage of the universe. When the universe becomes large, ECHDE reduces to the ordinary HDE model. The ECHDE models have aroused a lot of enthusiasm recently and are examined in detail by Khodam-Mohammadi & Malekjani (2011); Farooq et al. (2010); Jamil et al. (2010); Jamil & Farooq (2010b) and Karami et al. (2011).

Reconstructing the HDE scalar field models of DE is one of the interesting issues that has been investigated in the literature (Zhang 2006, 2007, 2009; Zhang et al. 2007, 2008; Granda & Oliveros 2009; Karami & Fehri 2010b; Karami & Abdolmaleki 2010; Jamil & Saridakis 2010; Sheykhi & Jamil 2011; Wu et al. 2008). The HDE models originate from considerations of the features of the quantum theory of gravity. On the other hand, the scalar field models (such as quintessence, tachyon, K-essence and dilaton) are often regarded as an effective description of an underlying theory of DE (Wu et al. 2008). The scalar field models can mimic the cosmological constant at the present epoch and also alleviate fine-tuning and coincidence problems (Ali et al. 2009). It therefore becomes meaningful to reconstruct scalar field models from DE models that possess some of the significant features of LQG theory, such as ECHDE models.

Here our aim is to investigate the correspondence between the interacting viscous ECHDE and scalar field models of DE such as quintessence, tachyon, K-essence and dilaton scalar fields and obtain the evolutionary form of these fields with time-varying $G$. There are significant indications that by varying, $G$ can be a function of time or equivalently of the scale factor (degli’Innocenti
et al. 1996; Umezu et al. 2005; Nesseris & Perivolaropoulos 2006). In particular, observations of the Hulse-Taylor binary pulsar B1913+16 lead to the estimation $\dot{G}/G \sim 2 \pm 4 \times 10^{-12}\text{yr}^{-1}$ (Damour et al. 1988; Bisnovatyi-Kogan 2006), while helio-seismological data provide the bound $-1.6 \times 10^{-12}\text{yr}^{-1} < \dot{G}/G < 0$ (Guenther et al. 1998). Similarly, type Ia supernova observations give the best upper bound of the variation of $G$ as $-10^{-11}\text{yr}^{-1} \leq \dot{G}/G < 0$ (Gaztañaga et al. 2002), while asteroseismological data from the pulsating white dwarf star G117-B15A lead to $|\dot{G}/G| \leq 4.10 \times 10^{-11}\text{yr}^{-1}$ (Biesiada & Malec 2004).

This paper is organized as follows. In Section 2, we investigate the viscous ECHDE model with time-varying $G$ in the presence of interaction with DM and in the framework of modified Friedmann-Robertson-Walker (FRW) cosmology. In Section 3 we establish a correspondence between the interacting viscous ECHDE with time-varying $G$ and the quintessence, tachyon, K-essence and dilaton scalar field models of DE. Section 4 is devoted to our conclusions.

2 INTERACTING VISCOUS ECHDE WITH TIME-VARYING $G$ IN A MODIFIED FRW UNIVERSE

Within the framework of the FRW metric,

$$ds^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right),$$

for the non-flat FRW universe containing the ECHDE and dark matter (DM), the first modified Friedmann equation corresponding to the corrected entropy-area relation (2) is given by (Cai et al. 2008)

$$H^2 + \frac{k}{a^2} + \frac{\dot{\alpha}G}{2\pi} \left( H^2 + \frac{k}{a^2} \right)^2 \frac{8\pi G}{3} (\rho_m + \rho_\Lambda),$$

where $k = 0, 1, -1$ represent a flat, closed and open FRW universe, respectively. Also, $\rho_\Lambda$ and $\rho_m$ are the energy densities of the ECHDE and DM, respectively. Using the following definitions

$$\Omega_m = \frac{8\pi G \rho_m}{3H^2}, \quad \Omega_\Lambda = \frac{8\pi G \rho_\Lambda}{3H^2}, \quad \Omega_k = \frac{k}{a^2 H^2},$$

$$\Omega_\alpha = \frac{\dot{\alpha}GH^2}{2\pi} (1 + \Omega_k)^2,$$

one can rewrite the modified Friedmann Equation (5) as

$$1 + \Omega_k + \Omega_\alpha = \Omega_m + \Omega_\Lambda.$$

From the definition $\rho_\Lambda = 3H^2\Omega_\Lambda/(8\pi G)$ and using Equation (3), we get

$$\Omega_\Lambda = \frac{c^2}{L^2 H^2} \gamma_c,$$

where

$$\gamma_c = 1 + \frac{8\pi G}{3c^2 L^2} \left[ \alpha \ln \left( \frac{L^2}{8\pi G} \right) + \beta \right].$$

Note that to obtain an accelerating universe, following Huang & Li (2004) the IR cut-off $L$ for a non-flat universe should be defined as

$$L = a(t) \frac{\sin n \left( \sqrt{|k|}y \right)}{\sqrt{|k|}},$$
where

$$\frac{\sin n(\sqrt{|k|y})}{\sqrt{|k|}} = \begin{cases} 
\sin y, & k = 1, \\
y, & k = 0, \\
\sinh y, & k = -1, 
\end{cases} \quad (12)$$

and

$$y = \frac{R_h}{a(t)} = \int_t^\infty \frac{dt}{a(t)} = \int_0^r \frac{dr}{\sqrt{1 - kr^2}} = \begin{cases} 
\sin^{-1} r, & k = 1, \\
r, & k = 0, \\
\sinh^{-1} r, & k = -1. 
\end{cases} \quad (13)$$

Here $R_h$ is the radial size of the event horizon measured in the $r$ direction and $L$ is the radius of the event horizon measured on the sphere of the horizon (Huang & Li 2004). For a flat universe, $L = R_h$.

Taking the time derivative of Equation (11) and using Equation (9) yield

$$\dot{L} = \left(\frac{c^2 \gamma c}{\Omega_\Lambda}\right)^{1/2} - \cos n(\sqrt{|k|y}), \quad (14)$$

where

$$\cos n(\sqrt{|k|y}) = \begin{cases} 
\cos y, & k = 1, \\
1, & k = 0, \\
\cosh y, & k = -1. 
\end{cases} \quad (15)$$

Using Equations (6), (9), (11) and (12), one can rewrite Equation (15) as

$$\cos n(\sqrt{|k|y}) = 1 - \Omega_k \left(\frac{c^2 \gamma c}{\Omega_\Lambda}\right)^{1/2}. \quad (16)$$

Hence, Equation (14) yields

$$\dot{L} = \left(\frac{c^2 \gamma c}{\Omega_\Lambda}\right)^{1/2} \left[1 - \left(\frac{\Omega_\Lambda}{c^2 \gamma c} - \Omega_k\right)^{1/2}\right]. \quad (17)$$

Here, we would like to generalize our study to the case where the ECHDE model has viscosity properties. In an isotropic and homogeneous modified FRW universe, dissipative effects arise due to the presence of bulk viscosity in cosmic fluids. DE with bulk viscosity has a peculiar property which can cause accelerated expansion of phantom type in the late evolution of the universe (Brevik & Gorbunova 2005, 2008; Brevik et al. 2005, 2010). It can also alleviate several cosmological puzzles such as the age problem, coincidence problem and phantom crossing. The energy-momentum tensor of the viscous fluid is

$$T_{\mu\nu} = \rho_\Lambda u_\mu u_\nu + \tilde{p}_\Lambda (g_{\mu\nu} + u_\mu u_\nu), \quad (18)$$

where $u_\mu$ is the four-velocity vector, $g_{\mu\nu}$ is the background metric,

$$\tilde{p}_\Lambda = p_\Lambda - 3H\xi \quad (19)$$

is the effective pressure of DE and $\xi$ is the bulk viscosity coefficient (Zimdahl et al. 2001; Zimdahl & Pavón 2003; Chimento et al. 2003).

We further assume there is an interaction between viscous ECHDE and DM. The recent observational evidence provided by the galaxy cluster Abell A586 supports the interaction between DE and DM (Bertolami et al. 2007). In the presence of interaction, $\rho_\Lambda$ and $\rho_m$ are not separately conserved and the energy conservation equations for viscous ECHDE and DM are

$$\dot{\rho}_\Lambda + 3H\rho_\Lambda(1 + \omega_\Lambda) = 9H^2 \xi - Q, \quad (20)$$
\[ \dot{\rho}_m + 3H\rho_m = Q, \]  
(21)

where \( \omega_\Lambda = p_\Lambda/\rho_\Lambda \) is the equation of state (EoS) parameter of the interacting viscous ECHDE and \( Q \) stands for the interaction term. Following Pavón & Zimdahl (2005), we shall assume \( Q = 3b^2H(\rho_m + \rho_\Lambda) \) with the coupling constant \( b^2 \). This expression for the interaction term was first introduced in the study of the suitable coupling between a quintessence scalar field and a pressureless cold DM field (Zimdahl et al. 2001; Zimdahl & Pavón 2003; Chimento et al. 2003; Amendola 1999, 2000; Amendola & Tocchini-Valentini 2001; Amendola & Quercellini 2003). Using Equation (8), the interaction term \( Q \) can be rewritten as

\[ Q = 3b^2H\rho_\Lambda \left( \frac{1 + \Omega_k + \Omega_\Lambda}{\Omega_\Lambda} \right). \]  
(22)

Taking the time derivative of Equation (3) and using Equations (9), (10) and (17), one can obtain

\[ \dot{\rho}_\Lambda = -2H\rho_\Lambda \left[ 2Y + \frac{1}{\gamma_c} \left( \frac{\dot{G}}{2G} - Y \right) \left( 1 + \frac{8\pi G\alpha H^2\Omega_\Lambda}{3c^2\gamma_c} \right) \right], \]  
(23)

where

\[ 
Y = 1 - \left( \frac{\Omega_\Lambda}{c^2\gamma_c} \right)^{1/2} \cos n \left( \sqrt{|k|}y \right), \\
Y = 1 - \left( \frac{\Omega_\Lambda}{c^2\gamma_c} - \Omega_k \right)^{1/2},
\]  
(24)

and the prime denotes the derivative with respect to \( x = \ln a \).

Substituting Equations (22) and (23) in Equation (20), using Equation (9) and assuming \( \xi = \varepsilon H^{-1}\rho_\Lambda \) (Sheykhi & Setare 2010) where \( \varepsilon \) is a constant parameter, one can then obtain the EoS parameter of the interacting viscous ECHDE as

\[ \omega_\Lambda = -1 + 3\varepsilon + \frac{4}{3} Y + \frac{2}{3\gamma_c} \left( \frac{\dot{G}}{2G} - Y \right) \left( 1 + \frac{8\pi G\alpha H^2\Omega_\Lambda}{3c^2\gamma_c} \right) - b^2 \left( \frac{1 + \Omega_k + \Omega_\Lambda}{\Omega_\Lambda} \right). \]  
(25)

If we set \( \dot{\alpha} = 0 = \Omega_\Lambda \) and \( \varepsilon = \dot{G} = 0 \), then Equation (25) reduces to the EoS parameter of the interacting ECHDE in Einstein gravity (Karami et al. 2011)

\[ \omega_\Lambda = -1 - 2\frac{2Y}{3\gamma_c} \left( 1 - 2\gamma_c + \frac{8\pi G\alpha H^2\Omega_\Lambda}{3c^2\gamma_c} \right) - b^2 \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right). \]  
(26)

Also, in the absence of correction terms (\( \alpha = \beta = 0 \)), from Equation (10) we have \( \gamma_c = 1 \) and Equation (26) recovers the EoS parameter of interacting HDE in Einstein gravity (Wang et al. 2005)

\[ \omega_\Lambda = -1 - \frac{1}{3} - \frac{2}{3c^2\gamma_c} \Omega_\Lambda^{1/2} \cos n \left( \sqrt{|k|}y \right) - b^2 \left( \frac{1 + \Omega_k}{\Omega_\Lambda} \right). \]  
(27)

Note that as we have already mentioned, at the very early stage when the universe undergoes an inflationary phase, the correction terms in the ECHDE density (3) become important. After the end of the inflationary phase, the universe subsequently enters into the radiation and then matter-dominated eras. In these two epochs, since the universe is much larger, the entropy-corrected terms to ECHDE, namely the last two terms in Equation (3), can safely be ignored. During the early inflation era when the correction terms make sense in the ECHDE density (3), the Hubble parameter \( H \) is constant and \( a = a_0 e^{Ht} \). Hence the Hubble horizon \( H^{-1} \) and the future event horizon \( R_h = a \int_t^\infty \frac{dt}{a^2} \) will coincide, i.e. \( R_h = H^{-1} = \text{const} \). On the other hand, since an early inflation era leads to a flat universe, we have \( L = R_h = H^{-1} = \text{const} \). Also from Equations (9) and (16) we have \( \frac{\Omega_\Lambda}{c^2\gamma_c} = 1 \).
and \( \cos n \left( \sqrt{k} |y| \right) = 1 \), hence Equation (24) gives \( Y = 0 \). Therefore, during the early inflation era, Equation (25) reduces to

\[
\omega_\Lambda = -1 + 3\varepsilon + \frac{c^2}{3\Omega_\Lambda} \frac{\dot{G}}{G} \left( 1 + \frac{8\pi G \alpha H^2}{3c^2} \right) - \frac{b^2}{\Omega_\Lambda} \left( 1 + \tilde{\alpha} GH^2 \right),
\]

(28)

where

\[
\Omega_\Lambda = c^2 \gamma_c = c^2 + \frac{8\pi G \alpha H^2}{3} \left[ \alpha \ln \left( \frac{1}{8\pi GH^2} \right) + \beta \right].
\]

(29)

Taking the time derivative of Equation (9) and using \( \Omega_\Lambda = H \Omega'_\Lambda \), one can get the equation of motion for \( \Omega_\Lambda \) as

\[
\Omega'_\Lambda = -2\Omega_\Lambda \left( \frac{\dot{H}}{H^2} + \frac{\dot{L}}{LH} - \frac{\gamma_c}{2H\gamma_c} \right),
\]

(30)

where

\[
\frac{\dot{H}}{H^2} = \left( \frac{1 + \Omega_k}{1 + \Omega_k + 2\Omega_\Lambda} \right) \left( \frac{3}{2} \left( b^2 - 1 \right) + \frac{\dot{G}}{2G} (1 + \Omega_k) \right) + \left[ \frac{3}{2} b^2 - 2Y - \frac{1}{\gamma_c} \left( \frac{\dot{G}}{2G} - Y \right) \left( 1 + \frac{8\pi G \alpha H^2 \Omega_\Lambda}{3c^4 \gamma_c} \right) \right] \Omega_\Lambda \}
\]

and

\[
-\frac{\gamma_c}{2H\gamma_c} = \frac{1}{\gamma_c} \left( \frac{\dot{G}}{2G} - Y \right) \left( 1 - \gamma_c + \frac{8\pi G \alpha H^2 \Omega_\Lambda}{3c^4 \gamma_c} \right).
\]

(31)

(32)

Using Equations (9), (17), (31) and (32), one can rewrite (30) as

\[
\frac{\Omega'_\Lambda}{\Omega_\Lambda} = -2 \left( \frac{1 + \Omega_k}{1 + \Omega_k + 2\Omega_\Lambda} \right) \left( \frac{3}{2} \left( b^2 - 1 \right) + \frac{\dot{G}}{2G} (1 + \Omega_k) \right) + \left[ 2Y + \frac{1}{\gamma_c} \left( \frac{\dot{G}}{2G} - Y \right) \left( 1 + \frac{8\pi G \alpha H^2 \Omega_\Lambda}{3c^4 \gamma_c} \right) \Omega_\Lambda \right] \right)
\]

\[
+ \frac{\dot{G}}{G} - 2\Omega_k.
\]

(33)

For completeness, we give the deceleration parameter

\[
q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2}.
\]

(34)

When the deceleration parameter is combined with the Hubble parameter and the dimensionless density parameters, it forms a set of useful parameters for the description of the astrophysical observations. Substituting Equation (31) in (34) gives

\[
q = \left( \frac{1 + \Omega_k}{1 + \Omega_k + 2\Omega_\Lambda} \right) \left( \frac{3}{2} \Omega_m - \frac{3}{2} b^2 (1 + \Omega_k + \Omega_\Lambda) - \frac{\dot{G}}{2G} (1 + \Omega_k) \right)
\]

\[
+ \left[ 2Y + \frac{1}{\gamma_c} \left( \frac{\dot{G}}{2G} - Y \right) \left( 1 + \frac{8\pi G \alpha H^2 \Omega_\Lambda}{3c^4 \gamma_c} \right) \Omega_\Lambda \right]
\]

\[
- (1 + \Omega_k).
\]

(35)

If we set \( \tilde{\alpha} = 0 = \Omega_\gamma, \beta = \dot{G} = 0 \) and \( \alpha = \beta = 0 \) then from Equation (10) we have \( \gamma_c = 1 \) and Equation (35) reduces to the deceleration parameter of interacting HDE in Einstein gravity (Wang et al. 2006)

\[
q = \frac{\Omega_\Lambda}{2} + \frac{1}{2} \frac{1}{1 + 3b^2} (1 + \Omega_k) - \frac{\Omega_\Lambda^{3/2}}{c} \cos n \left( \sqrt{k} |y| \right).
\]

(36)
3 CORRESPONDENCE BETWEEN THE INTERACTING VISCOUS ECHDE AND THE SCALAR FIELD MODELS OF DE

Here, we suggest a correspondence between the interacting viscous ECHDE model and the quintessence, tachyon, K-essence and dilaton scalar field models in the context of modified FRW cosmology. To establish this correspondence, we compare the ECHDE density (3) with the corresponding scalar field model density and equate the equations of state for these models with the EoS parameter given by Equation (25).

3.1 The Interacting Viscous ECHDE Quintessence Model

The quintessence scalar field model of DE was proposed to justify the late-time acceleration of the universe. For the quintessence scalar field $\phi$, the energy density and pressure are given by (Copeland et al. 2006)

$$\rho_Q = \frac{1}{2}\dot{\phi}^2 + V(\phi), \quad (37)$$

$$p_Q = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad (38)$$

and the EoS parameter is obtained as

$$\omega_Q = \frac{p_Q}{\rho_Q} = \frac{\dot{\phi}^2 - 2V(\phi)}{\dot{\phi}^2 + 2V(\phi)}. \quad (39)$$

Equating Equation (39) with the EoS parameter (25), $\omega_Q = \omega_\Lambda$, and also equating Equation (37) with (3), $\rho_Q = \rho_\Lambda$, we have

$$\dot{\phi}^2 = (1 + \omega_\Lambda)\rho_\Lambda, \quad (40)$$

$$V(\phi) = \frac{1}{2}(1 - \omega_\Lambda)\rho_\Lambda. \quad (41)$$

Substituting Equations (3) and (25) into Equations (40) and (41), the kinetic energy term and the potential energy of the quintessence scalar field are obtained as follows

$$\dot{\phi}^2 = \frac{3H^2\Omega_\Lambda}{8\pi G}\left[3\varepsilon + \frac{4}{3}Y + \frac{2}{3\gamma_c}\left(\frac{\dot{G}}{2G} - Y\right)\left(1 + \frac{8\pi G\alpha H^2\Omega_\Lambda}{3c^4\gamma_c}\right)\right.
- b^2\left(1 + \frac{\Omega_k + \Omega_\Lambda}{\Omega_\Lambda}\right)], \quad (42)$$

$$V(\phi) = \frac{3H^2\Omega_\Lambda}{16\pi G}\left[2 - 3\varepsilon - \frac{4}{3}Y - \frac{2}{3\gamma_c}\left(\frac{\dot{G}}{2G} - Y\right)\left(1 + \frac{8\pi G\alpha H^2\Omega_\Lambda}{3c^4\gamma_c}\right)\right.
+ b^2\left(1 + \frac{\Omega_k + \Omega_\Lambda}{\Omega_\Lambda}\right)]. \quad (43)$$

Integrating Equation (42) with respect to the scale factor $a$ yields the evolutionary form of the quintessence scalar field as

$$\phi(a) - \phi(a_0) = \int_{a_0}^{a} \frac{da}{a} \left[\frac{3\Omega_\Lambda}{8\pi G}\left[3\varepsilon + \frac{4}{3}Y + \frac{2}{3\gamma_c}\left(\frac{\dot{G}}{2G} - Y\right)\left(1 + \frac{8\pi G\alpha H^2\Omega_\Lambda}{3c^4\gamma_c}\right)\right.
- b^2\left(1 + \frac{\Omega_k + \Omega_\Lambda}{\Omega_\Lambda}\right)\right]^{1/2}, \quad (44)$$

where $a_0$ is the scale factor at the present time.
### 3.2 The Interacting Viscous ECHDE Tachyon Model

It has been suggested that the tachyon scalar field model, in a class of string theories, can act as a source of DE depending on the form of the tachyon potential. A rolling tachyon has an interesting EoS whose parameter smoothly interpolates between $-1$ and $0$ (Gibbons 2002). This has led to a flurry of attempts to construct viable cosmological models using the tachyon as a suitable candidate for the inflation at high energy. For the effective Lagrangian density of the tachyon field as (Sen 1999, 2002a,b; Bergshoeff et al. 2000; Padmanabhan 2002; Padmanabhan & Choudhury 2002)

$$\mathcal{L} = -V(\phi)\sqrt{1 + \partial_{\mu}\phi\partial^{\mu}\phi},$$  

(45)

the energy density and pressure are given by

$$\rho_T = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}},$$  

(46)

$$p_T = -V(\phi)\sqrt{1 - \dot{\phi}^2},$$  

(47)

where $V(\phi)$ is the tachyon potential. The EoS parameter of the tachyon scalar field is obtained as in

$$\omega_T = \frac{p_T}{\rho_T} = \dot{\phi}^2 - 1.$$  

(48)

If we establish the correspondence between the interacting viscous ECHDE and tachyon DE, then equating Equation (48) with (25), $\omega_T = \omega_\Lambda$, and also equating Equation (46) with (3), $\rho_T = \rho_\Lambda$, we obtain

$$\dot{\phi}^2 = 3\varepsilon + \frac{4}{3}Y + \frac{2}{3\gamma_c}\left(\frac{\dot{G}}{2G} - Y\right)\left(1 + \frac{8\pi G\alpha H^2 \Omega_\Lambda}{3c^4\gamma_c}\right) - b^2\left(1 + \Omega_k + \Omega_\alpha\Omega_\Lambda\right)^{-1/2},$$  

(49)

$$V(\phi) = \frac{3H^2\Omega_\Lambda}{8\pi G}\left[1 - 3\varepsilon - \frac{4}{3}Y - \frac{2}{3\gamma_c}\left(\frac{\dot{G}}{2G} - Y\right)\left(1 + \frac{8\pi G\alpha H^2 \Omega_\Lambda}{3c^4\gamma_c}\right)\right.\left.\frac{1 + \Omega_k + \Omega_\alpha\Omega_\Lambda}{\Omega_\Lambda}\right]^{1/2}.$$  

(50)

From Equation (49), the evolutionary form of the tachyon scalar field is obtained as

$$\phi(a) - \phi(a_0) = \int_{a_0}^{a} \frac{da}{H_a}\left[3\varepsilon + \frac{4}{3}Y + \frac{2}{3\gamma_c}\left(\frac{\dot{G}}{2G} - Y\right)\left(1 + \frac{8\pi G\alpha H^2 \Omega_\Lambda}{3c^4\gamma_c}\right)\right.\left.\frac{1 + \Omega_k + \Omega_\alpha\Omega_\Lambda}{\Omega_\Lambda}\right]^{1/2}.$$  

(51)

### 3.3 The Interacting Viscous ECHDE K-essence Model

The scalar field model known as K-essence is also used to explain the observed late-time acceleration of the universe. It is well known that K-essence scenarios have attractor-like dynamics, and therefore avoid the fine-tuning of the initial conditions for the scalar field. K-essence is characterized by a scalar field with a non-canonical kinetic energy. The scalar field action of the K-essence is a function of $\phi$ and $\chi = \dot{\phi}^2/2$ as in (Garriga & Mukhanov 1999; Chiba et al. 2000; Armendariz-Picon et al. 1999, 2000, 2001)

$$S = \int d^4x\sqrt{-g}p(\phi, \chi).$$  

(52)
where 
\[ p(\phi, \chi) = f(\phi)(-\chi + \chi^2), \] 
(53)
is the pressure density of the K-essence field. Also the energy density of the field \( \phi \) is given by 
\[ \rho(\phi, \chi) = f(\phi)(-\chi + 3\chi^2). \] 
(54)
The EoS parameter of the K-essence scalar field is obtained as 
\[ \omega_K = \frac{p(\phi, \chi)}{\rho(\phi, \chi)} = \frac{\chi - 1}{3\chi - 1}. \] 
(55)
Equating Equation (55) with the EoS parameter (25), \( \omega_K = \omega_\Lambda \), yields the solution for \( \chi \) as 
\[ \chi = \frac{2 - 3e - \frac{4}{3}Y - \frac{2}{\sqrt{\gamma}} \left( \frac{G}{2\alpha'} - Y \right) \left( 1 + \frac{8\pi G a H^2 \Omega_\Lambda}{3c^2 \gamma_e} \right) + b^2 \left( \frac{1 + \Omega_\Lambda + \Omega_\Lambda}{\Omega_\Lambda} \right)}{4 - 9e - 4Y - \frac{2}{\sqrt{\gamma}} \left( \frac{G}{2\alpha'} - Y \right) \left( 1 + \frac{8\pi G a H^2 \Omega_\Lambda}{3c^2 \gamma_e} \right) + 3b^2 \left( \frac{1 + \Omega_\Lambda + \Omega_\Lambda}{\Omega_\Lambda} \right)}. \] 
(56)
Using \( \dot{\phi}^2 = 2\chi \) and Equation (56), one can get the evolutionary form of the K-essence scalar field as 
\[ \phi(a) - \phi(a_0) = \int_{a_0}^{a} da \frac{1}{H a} \times \left[ 4 - 6e - \frac{4}{3}Y - \frac{4}{\sqrt{\gamma}} \left( \frac{G}{2\alpha'} - Y \right) \left( 1 + \frac{8\pi G a H^2 \Omega_\Lambda}{3c^2 \gamma_e} \right) + 2b^2 \left( \frac{1 + \Omega_\Lambda + \Omega_\Lambda}{\Omega_\Lambda} \right) \right]^{1/2}. \] 
(57)

3.4 The Interacting Viscous ECHDE Dilaton Model

The dilaton scalar field model is also an interesting attempt to explain the origin of DE using string theory. This model appears from a four-dimensional effective low-energy string action and includes higher-order kinetic corrections to the tree-level action in low energy effective string theory. For the dilaton DE model, the pressure and energy densities are given by (Gasperini et al. 2002; Arkani-Hamed et al. 2004; Elizalde et al. 2008)

\[ p_D = -\chi + \epsilon' e^{\lambda' \phi} \chi^2, \] 
(58)
\[ \rho_D = -\chi + 3\epsilon' e^{\lambda' \phi} \chi^2, \] 
(59)
where \( \epsilon' \) and \( \lambda' \) are positive constants and \( \chi = \dot{\phi}^2/2 \). The EoS parameter of the dilaton scalar field is obtained as
\[ \omega_D = \frac{p_D}{\rho_D} = -1 + \epsilon' e^{\lambda' \phi} \chi. \] 
(60)
Equating Equation (60) with the EoS parameter (25), \( \omega_D = \omega_\Lambda \), we find the following solution
\[ \epsilon' e^{\lambda' \phi} = \frac{2 - 3e - \frac{4}{3}Y - \frac{2}{\sqrt{\gamma}} \left( \frac{G}{2\alpha'} - Y \right) \left( 1 + \frac{8\pi G a H^2 \Omega_\Lambda}{3c^2 \gamma_e} \right) + b^2 \left( \frac{1 + \Omega_\Lambda + \Omega_\Lambda}{\Omega_\Lambda} \right)}{4 - 9e - 4Y - \frac{2}{\sqrt{\gamma}} \left( \frac{G}{2\alpha'} - Y \right) \left( 1 + \frac{8\pi G a H^2 \Omega_\Lambda}{3c^2 \gamma_e} \right) + 3b^2 \left( \frac{1 + \Omega_\Lambda + \Omega_\Lambda}{\Omega_\Lambda} \right)}, \] 
(61)
then using \( \dot{\phi}^2 = 2\chi \), we obtain
\[ e^{\gamma' \phi} = \frac{1}{\sqrt{e'}} \left[ 4 - 6e - \frac{4}{3}Y - \frac{4}{\sqrt{\gamma}} \left( \frac{G}{2\alpha'} - Y \right) \left( 1 + \frac{8\pi G a H^2 \Omega_\Lambda}{3c^2 \gamma_e} \right) + 2b^2 \left( \frac{1 + \Omega_\Lambda + \Omega_\Lambda}{\Omega_\Lambda} \right) \right]^{1/2}. \] 
(62)
Integrating with respect to $a$, we get
\[ e^{\frac{\lambda \phi(a)}{2}} = e^{\frac{\lambda \phi(a_0)}{2}} + \frac{\lambda}{2\sqrt{c'}} \int_{a_0}^{a} \frac{da}{Ha} \times \left[ \frac{4 - 6\varepsilon - 8Y - \frac{4G}{\sigma a} - Y}{4 - 9\varepsilon - 4Y - \frac{2G}{\sigma a} - Y} \left( 1 + \frac{8\pi G a H^2 \Omega_\Lambda}{3c^2} \right) + 2\beta^2 \left( \frac{1+\Omega_m+\Omega_\Lambda}{\Omega_\Lambda} \right) \right]^{1/2}. \] (63)

Finally, the evolutionary form of the dilaton scalar field is obtained as
\[ \phi(a) = \frac{2}{\lambda} \ln \left\{ e^{\frac{\lambda \phi(a_0)}{2}} + \frac{\lambda}{2\sqrt{c'}} \int_{a_0}^{a} \frac{da}{Ha} \times \left[ \frac{4 - 6\varepsilon - 8Y - \frac{4G}{\sigma a} - Y}{4 - 9\varepsilon - 4Y - \frac{2G}{\sigma a} - Y} \left( 1 + \frac{8\pi G a H^2 \Omega_\Lambda}{3c^2} \right) + 2\beta^2 \left( \frac{1+\Omega_m+\Omega_\Lambda}{\Omega_\Lambda} \right) \right]^{1/2} \right\}. \] (64)

4 CONCLUSIONS

Among various candidates to explain cosmic accelerated expansion, only the HDE model is based on the entropy-area relation. It should be noted that the entropy-area relation depends on the gravity theory. When applying the curvature corrections to the gravity theory, this yields quantum corrections to the entropy-area relation. The ECHDE density is obtained by adding the correction terms to the HDE density, which is motivated from the LQG.

Here, we investigated the entropy-corrected version of HDE in the framework of modified FRW cosmology. We considered a spatially non-flat universe filled with the interacting viscous ECHDE with DM. Then, we extended our study to the case where the gravitational constant $G$ varies with time. A time-varying $G$ has some theoretical advantages, such as alleviating the DM problem (Goldman et al. 1992), the cosmic coincidence problem (Jamil et al. 2009b,c) and the discrepancies in the Hubble parameter value (Bertolami et al. 1993). We derived an exact differential equation that determines the evolution of the ECHDE density parameter. Furthermore, we obtained the EoS and deceleration parameters of the interacting viscous ECHDE model.

We also established a correspondence between the interacting viscous ECHDE model and the quintessence, tachyon, K-essence and dilaton scalar field models of DE in the modified FRW scenario including time-varying $G$. We adopted the viewpoint that these scalar field models of DE are effective theories of an underlying theory of DE. Thus, we should be capable of using these scalar field models to mimic the evolving behavior of the ECHDE and reconstructing the scalar field models according to the evolutionary behavior of the ECHDE. Finally, we reconstructed the potentials and the dynamics of these scalar field models, which describe quintessence, tachyon, K-essence and dilaton cosmology.

We hope that future high-precision astronomical observations like the type Ia supernovae surveys, the shift parameter of the cosmic microwave background given by the Wilkinson Microwave Anisotropy Probe observations, and the baryon acoustic oscillation measurement from the Sloan Digital Sky Survey may be capable of determining the fine properties of the interacting viscous ECHDE model in modified FRW cosmology and consequently reveal some significant features of the underlying theory of DE.

Acknowledgements This work has been financially supported by the Department of Physics, Sanandaj Branch, Islamic Azad University, Sanandaj, Iran.
References

Ali, A., Sami, M., & Sen, A. A. 2009, Phys. Rev. D, 79, 123501
Amendola, L. 1999, Phys. Rev. D, 60, 043501
Amendola, L. 2000, Phys. Rev. D, 62, 043511
Amendola, L., & Quercellini, C. 2003, Phys. Rev. D, 68, 023514
Amendola, L., & Tocchini-Valentini, D. 2001, Phys. Rev. D, 64, 043509
Arkani-Hamed, N., Creminelli, P., Mukohyama, S., & Zaldarriaga, M. 2004, Journal of Cosmology and Astroparticle Physics, 04, 001
Armendariz-Picon, C., Damour, T., & Mukhanov, V. 1999, Physics Letters B, 458, 209
Armendariz-Picon, C., Mukhanov, V., & Steinhardt, P. J. 2000, Physical Review Letters, 85, 4438
Armendariz-Picon, C., Mukhanov, V., & Steinhardt, P. J. 2001, Phys. Rev. D, 63, 103510
Ashker, A., Baer, J., Corrigan, A., & Kraus, K. 1998, Physical Review Letters, 80, 904
Banerjee, R., Gangopadhyay, S., & Modak, S. K. 2010, Physics Letters B, 686, 181
Banerjee, R., & Majhi, B. R. 2008a, Physics Letters B, 662, 62
Banerjee, R., & Majhi, B. R. 2008b, Journal of High Energy Physics, 6, 95
Banerjee, R., & Modak, S. K. 2009, Journal of High Energy Physics, 5, 63
Bekenstein, J. D. 1973, Phys. Rev. D, 7, 2333
Bekenstein, J. D. 1974, Phys. Rev. D, 9, 3292
Bekenstein, J. D. 1981, Phys. Rev. D, 23, 287
Bekenstein, J. D. 1994, Phys. Rev. D, 49, 1912
Beltran Almeida, J. P., & Pereira, J. G. 2006, Physics Letters B, 636, 75
Bergshoeff, E. A., de Roo, M., de Wit, T. C., Eyras, E., & Panda, S. 2000, Journal of High Energy Physics, 5, 9
Bertolami, O., Gil Pedro, F., & Le Delliou, M. 2007, Physics Letters B, 654, 165
Bertolami, O., Mourão, J. M., & Pérez-Mercader, J. 1993, Physics Letters B, 311, 27
Biesiada, M., & Malec, B. 2004, MNRAS, 350, 644
Bisnovatyi-Kogan, G. S. 2006, International Journal of Modern Physics D, 15, 1047
Brevik, I., & Gorbunova, O. 2005, General Relativity and Gravitation, 37, 2039
Brevik, I., & Gorbunova, O. 2008, European Physical Journal C, 56, 425
Brevik, I., Gorbunova, O., & Sáez-Gómez, D. 2010, General Relativity and Gravitation, 42, 1513
Brevik, I., Gorbunova, O., & Shaido, Y. A. 2005, International Journal of Modern Physics D, 14, 1899
Cai, R.-G., Cao, L.-M., & Hu, Y.-P. 2008, Journal of High Energy Physics, 8, 90
Chiba, T., Okabe, T., & Yamaguchi, M. 2000, Phys. Rev. D, 62, 023511
Chimento, L. P., Jakubi, A. S., Pavón, D., & Zimdahl, W. 2003, Phys. Rev. D, 67, 083513
Copeland, E. J., Sami, M., & Tsujikawa, S. 2006, International Journal of Modern Physics D, 15, 1753
Damour, T., Gibbons, G. W., & Taylor, J. H. 1988, Physical Review Letters, 61, 1151
de Bernardis, P., Ade, P. A. R., Bock, J. J., et al. 2000, Nature, 404, 955
degl’Innocenti, S., Fiorentini, G., Raffelt, G. G., Ricci, B., & Weiss, A. 1996, A&A, 312, 345
Elizalde, E., Jhiang, S., Nojiri, S., et al. 2008, European Physical Journal C, 53, 447
Elizalde, E., Nojiri, S., Odintsov, S. D., & Wang, P. 2005, Phys. Rev. D, 71, 103504
Enqvist, K., & Sloth, M. S. 2004, Physical Review Letters, 93, 221302
Farooq, M. U., Jamil, M., & Rashid, M. A. 2010, International Journal of Theoretical Physics, 49, 2278
Garriga, J., & Mukhanov, V. F. 1999, Physics Letters B, 458, 219
Gasperini, M., Piazza, F., & Veneziano, G. 2002, Phys. Rev. D, 65, 023508
Gaztañaga, E., García-Berro, E., Isern, J., Bravo, E., & Domínguez, I. 2002, Phys. Rev. D, 65, 023506
Ghosh, A., & Mitra, P. 2005, Phys. Rev. D, 71, 027502
Gibbons, G. W. 2002, Physics Letters B, 537, 1
Goldman, T., Pérez-Mercader, J., Cooper, F., & Nieto, M. M. 1992, Physics Letters B, 281, 219
Gong, Y. 2004, Phys. Rev. D, 70, 064029
Sheykhi, A. 2009, Physics Letters B, 681, 205
Sheykhi, A. 2010a, Physics Letters B, 682, 329
Sheykhi, A. 2010b, Classical and Quantum Gravity, 27, 025007
Sheykhi, A., & Jamil, M. 2011, Physics Letters B, 694, 284
Sheykhi, A., & Setare, M. R. 2010, International Journal of Theoretical Physics, 49, 2777
Susskind, L. 1995, Journal of Mathematical Physics, 36, 6377
’t Hooft, G. 1993, arXiv:gr-qc/9310026
Umezu, K.-I., Ichiki, K., & Yahiro, M. 2005, Phys. Rev. D, 72, 044010
Wang, B., Abdalla, E., & Su, R.-K. 2005, Physics Letters B, 611, 21
Wang, B., Lin, C.-Y., & Abdalla, E. 2006, Physics Letters B, 637, 357
Wei, H. 2009, Communications in Theoretical Physics, 52, 743
Wu, J.-F., Ma, D.-Z., & Ling, Y. 2008, Physics Letters B, 663, 152
Xu, L. 2009, Journal of Cosmology and Astroparticle Physics, 09, 016
Zhang, J. 2008, Physics Letters B, 668, 353
Zhang, J., Zhang, X., & Liu, H. 2007, Physics Letters B, 651, 84
Zhang, J., Zhang, X., & Liu, H. 2008, European Physical Journal C, 54, 303
Zhang, X. 2006, Phys. Rev. D, 74, 103505
Zhang, X. 2007, Physics Letters B, 648, 1
Zhang, X. 2009, Phys. Rev. D, 79, 103509
Zhang, X., & Wu, F.-Q. 2005, Phys. Rev. D, 72, 043524
Zhang, X., & Wu, F.-Q. 2007, Phys. Rev. D, 76, 023502
Zimdahl, W., & Pavón, D. 2003, General Relativity and Gravitation, 35, 413
Zimdahl, W., Pavón, D., & Chimento, L. P. 2001, Physics Letters B, 521, 133