PARTON ENERGY LOSS IN COLLINEAR EXPANSION*

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We demonstrate that the $N=1$ rescattering contribution to the gluon radiation from a fast massless quark in $eA$ DIS vanishes in the collinear approximation. It is shown that the nonzero $N=1$ gluon spectrum obtained in the higher-twist approach by Guo, Wang and Zhang is a consequence of unjustified neglecting some important terms in the collinear expansion.

1. There are several approaches to the induced gluon emission from fast partons due to multiple scattering in cold nuclear matter and hot quark-gluon plasma. The most general approach to this phenomenon is the so-called light-cone path integral (LCPI) approach (for reviews, see [67]). This formalism reproduces the predictions of the BDMPS and GLV approaches in their applicability regions (at strong Landau-Pomeranchuk-Migdal suppression for massless partons, and thin plasmas, respectively). However, the relation between the LCPI approach and the higher-twist formalism by Guo, Wang and Zhang (GWZ) is not clear. The GWZ approach is based on the Feynman diagram formalism and collinear expansion. It includes only the $N=1$ rescattering and has originally been derived for the gluon emission from a fast quark produced in $eA$ DIS. The analyses neglect the quantum nonlocality in production of fast partons. In the nonlocal fast quark production and gluon emission have been treated on even footing. However, one can show that in the applicability region of the GWZ formalism the quantum nonlocality in the quark production is not important for gluon emission, and the gluon spectrum should coincide with the $N=1$ gluon spectrum in the LCPI approach. But this is not the case. The GWZ gluon spectrum predicted contains the logarithmically dependent nucleon gluon density, which is absent in the LCPI calculations.

We will demonstrate that the approximations used really lead to a disagreement with the LCPI approach. However, contrary to the results of the correct use of the collinear expansion gives a zero gluon spectrum. The nonzero spectrum obtained is a consequence of unjustified neglecting some important terms.

2. We consider the gluon emission from a fast quark produced in $eA$ DIS for the Bjorken variable $x_B$ and photon virtuality $Q$. The transverse momentum integrated distribution for the $gg$ final state can be described in terms of the semi-inclusive nuclear hadronic tensor $dW_\mu^\nu_A/dz$ (hereafter $z = \omega/E$, where $\omega$ is the gluon energy and $E$ is the struck quark energy). The spin effects in the rescatterings of fast partons can be neglected. This ensures that the spin structure of $dW_\mu^\nu_A/dz$ is the same as for the usual hadronic tensor $W_\mu^\nu_N$ in $eN$ DIS. It allows one to describe the

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gluon emission in terms of the scalar semi-inclusive quark distribution. Neglecting the EMC and shadowing effects it can be written as

\[
\frac{df_A}{dz} = \int d\mathbf{r} n_A(r) \frac{df_N(r)}{dz},
\]

(1)

where \( df_N(r)/dz \) is the in-medium semi-inclusive quark distribution for a nucleon located at \( r \), and \( n_A(r) \) is the nucleus number density.

In the LCPI approach\textsuperscript{[5]} the matrix element of the \( q \rightarrow gq' \) in-medium transition is written in terms of the wave functions of the initial quark and final quark and gluon in the nucleus color field (we omit the color factors and indices)

\[
\langle gq' | \hat{S} | q \rangle = ig \int dy \bar{\psi}_{q'}(y) \gamma^\mu A_\mu^*(y) \psi_q(y).
\]

(2)

Each quark wave function in (2) is written as

\[
\psi(y) = \exp(-ip^- y^-) \sqrt{2} \hat{u}_\lambda \phi(y^-, \vec{y}_T).
\]

The \( y^- \) evolution of the transverse wave functions can be described in terms of the Green’s function \( K \) for the Schrödinger equation (3). One can show that for the gauges with potential vanishing at large distances (say, covariant gauges, or Coulomb gauge) one can ignore the transverse potential \( \vec{A}_T \).

To calculate the \( N=1 \) rescattering contribution we need not use the path integral representation for the Green’s functions (which is used at the last stage of calculations\textsuperscript{[5]}). To obtain the \( N=1 \) contribution it is enough to expand \( K \) to the second order in the external potential. It allows one to describe the induced gluon emission in \( eA \) DIS in terms of the free Green’s functions \( K \) for fast partons and the gluon correlator \( \langle A^+(y_1) A^+(y_2) \rangle \) in the nucleus. Diagrammatically it is represented by a set of diagrams like shown in Fig. 1 in which the horizontal solid line corresponds to \( K (\rightarrow) \) and \( K^* (\leftarrow) \), the gluon line shows the gluon correlators, the vertical dashed line shows the transverse density matrices of the final quark and gluon at very large \( y^- \).

Figure 1.

The typical difference in the coordinate \( y^- \) for the upper and lower \( \gamma^*qq \) vertices (which gives the scale of the quantum nonlocality of the fast quark production) is given by the well known Ioffe length \( L_I = 1/m_{\pi x_B} \). For the nucleon quark distribution \( L_I \) is the dominating scale in the Collins-Soper formula\textsuperscript{[11]} \( f_N = \frac{1}{4\pi} \int dy^- e^{y^+ x_B} \frac{1}{y^-} \langle N| \bar{\psi}(y^-/2) \gamma^+ \psi(y^-/2)|N \rangle \). For the \( gq \) final
state the integration over the \( y^− \) coordinate of the \( \gamma^*qq \) vertex is affected by the integration over the positions of rescatterings and the \( q \to qq \) splitting. However, for moderate \( x_B \) when \( L_I \ll R_A \) one can neglect the effect of rescatterings on the integration over \( y^− \). For production of the final \( qq \) states with \( M_{qq}^2 \ll Q^2 \) the restriction on \( y^− \) from the splitting point can also be ignored. Indeed, the typical scale in integrating over the splitting points is given by the gluon formation length \( L_f \sim E/M_{qq}^2 \) which is much bigger than \( L_I \) at \( M_{qq}^2 \ll Q^2 \). This is valid for both the vacuum DGLAP and the induced gluon emission. Also, at \( L_I \ll L_f \) one can take for the lower limit of the integration over the splitting points for the upper and lower parts of the diagrams in Fig. 1 the position of the struck nucleon. Then the quark production and gluon emission become independent and the \( df_N(r)/dz \) can be approximated by the factorized form

\[
\frac{df_N(r)}{dz} \approx f_N \frac{dP(r)}{dz},
\]

where \( dP/dz \) is the induced gluon spectrum described by the right parts of the diagrams evaluated neglecting the quantum nonlocality of the fast quark production.

Due to confinement the typical separation of the arguments in the gluon correlators is of the order of the nucleon radius, \( R_N \). It allows one to replace the fast parton propagators between the gluon fields in the graphs like Fig. 1b by \( \delta \) functions in impact parameter space. This approximation is valid for parton energy \( \gg 1/R_N \). It follows from the Schrödinger diffusion relation for the parton transverse motion \( \rho^2 \sim L/E \). Also, the smallness of the fast parton diffusion radius at the longitudinal scale \( \sim R_N \) allows one to replace in other transverse Green’s functions the \( y^− \) coordinates by the mean values of the arguments of the vector potentials in the gluon correlators. This approximation corresponds to a picture with rescatterings of fast partons on zero thickness scattering centers (nucleons). The inequality \( \omega \gg 1/R_N \) for the emitted gluon is equivalent to \( R_N \ll L_f \). For this reason, in the picture of thin nucleons the contribution of the graphs like Fig. 1c,d with gluon correlators connecting the initial quark and final quark or gluon can be neglected since they are suppressed by the small factor \( R_N/L_f \). These approximations have been used in the original formulation of the LCPI approach \( \text{\cite{5}} \) (the BDMPS\( \text{\cite{11}} \) and GLV\( \text{\cite{22}} \) approaches use them as well). Note that (similarly to the case of the quark-gluon plasma\( \text{\cite{12}} \)) each gluon correlator appears only in the form of an integral over \( \Delta y^− = y_2^− - y_1^− \) and at \( y_2^+ = y_1^+ \). One can easily show that this ensures gauge invariance of the result (to leading order in \( \alpha_s \)).

In\( \text{\cite{3H}} \) the gluon emission in eA DIS is described by the diagrams like shown in Fig. 2.

![Figure 2](image)

The lower soft part is expressed in terms of the matrix element \( \langle A|\bar{\psi}(0)A^+(y_1)A^+(y_2)\psi(y_3)|A \rangle \), and the upper hard parts are calculated perturbatively. Due to conservation of the large \( p^− \) momenta of fast partons in the Feynman propagators only the Fourier components with \( p^− > 0 \) are important. It means that the Feynman propagators are effectively reduced to the retarded (in \( y^− \) coordinate) ones. One can show that the Feynman diagram treatment of\( \text{\cite{3H}} \) is equivalent
to that in terms of the transverse Green’s functions. Indeed, using the representation

\[ K(\vec{y}_{T,2}, y_2^- | \vec{y}_{T,1}, y_1^-) = i \int \frac{dp^+ d\vec{p}_T}{(2\pi)^3} \exp \left[ -ip^+ (y_2^- - y_1^-) + i\vec{p}_T (\vec{y}_{T,2} - \vec{y}_{T,1}) \right] \]  

one can write the retarded quark propagator as

\[
G_r(y_2 - y_1) = \frac{1}{4\pi} \int_0^\infty \frac{dp^-}{p} e^{-ip^-(y_2^+ - y_1^+)} \left[ \sum_\lambda \tilde{u}_\lambda \bar{u}_\lambda K(\vec{y}_{T,2}, y_2^- | \vec{y}_{T,1}, y_1^-) + i\gamma^- \delta(y_2^+ - y_1^+) \delta(\vec{y}_{T,2} - \vec{y}_{T,1}) \right]. \tag{6}
\]

Here \( \tilde{u}_\lambda \) and \( \bar{u}_\lambda \) act on the variables with indices 2 and 1, respectively. The last term in (6) is the so-called contact term. It does not propagate in \( y^- \) and can be omitted in calculating the nuclear final-state interaction effects for fast partons. Using (6) and a similar representation for the gluon propagator the hard parts in the higher-twist method can be represented in terms of the transverse Green’s functions automatically includes all the processes in the GWZ approach (hard-soft, double-hard, and interferences in the terminology of \( 3\)).

3. The calculation of the diagrams like shown in Fig. 1 is simplified by noting that the free transverse Green’s function can be written as

\[ K(\vec{y}_{T,2}, y_2^- | \vec{y}_{T,1}, y_1^-) = \theta(y_2^- - y_1^-) \sum_{\nu_T} \phi_{\nu_T}(\vec{y}_{T,2}, y_2^-) \phi^\nu_{\nu_T}(\vec{y}_{T,1}, y_1^-). \tag{7} \]

where \( \phi_{\nu_T}(\vec{y}_{T}, y^-) \) is the plane wave solution to the Schrödinger equation for \( A^\mu = 0 \) with the transverse momentum \( \vec{p}_T \). It allows one to represent the upper and lower parts of the diagrams shown in Fig. 1 in the form \( \int dy^- d\vec{y}_T \phi^\nu_{\nu_T}(\vec{y}_T, y^-) \phi^\nu_{\nu_T}(\vec{y}_T, y^-) \phi_{\nu_T}(\vec{y}_T, y^-) \) where the outgoing and incoming wave functions have the form of the plane waves with sharp changes of the transverse momenta at the points of interactions with the external gluon fields. This method has previously been used in 13 for investigation of the role of the finite kinematical boundaries. All the hard parts evaluated with the help of the plane waves agree with that obtained in Refs. 34.

The sum of the complete set of the diagrams contributing to the \( N = 1 \) spectrum can be written as

\[ \frac{dP(r)}{dz} = \int_{r_3}^{\infty} dz n_A(\vec{r}_T, r_3 + \xi) \frac{d\sigma(z, \xi)}{dz}. \tag{8} \]

Here \( d\sigma(z, \xi)/dz \) is the cross section of gluon emission from the fast quark produced at distance \( \xi \) from the scattering nucleon. At \( z \ll 1 \) (we consider the soft gluon emission just to simplify the formulas) for massless partons it reads

\[ \frac{d\sigma(z, \xi)}{dz} = 2\alpha_s^2(1 + (1 - z)^2) \int d\vec{p}_T \frac{dk_T}{k_T^2} x dG(k_T^2, x) H(\vec{k}_T, z, \xi), \tag{9} \]

\[
H(\vec{p}_T, \vec{k}_T, z, \xi) = \left[ \frac{1}{\vec{p}_T^2} - \frac{(\vec{\rho}_T - \vec{k}_T)\vec{p}_T}{\vec{p}_T^2(\vec{\rho}_T - \vec{k}_T)^2} \right] \left[ 1 - \cos \left( \frac{i\vec{\rho}_T^2 \xi}{2Ez(1 - z)} \right) \right]. \tag{10} \]

Here the limit \( x \to 0 \) is implicit, \( dG(k_T^2, x)/dk_T \) is the unintegrated nucleon gluon density in the Collins-Soper form 11, which at \( x \ll 1 \) can also be written as

\[ \frac{dG(k_T^2, x)}{dk_T} = \frac{N_c^2 - 1}{x32\pi^4\alpha_s C_F} \int d\rho \exp (-i\vec{k}_T \vec{\rho}) \nabla^2 \sigma(\rho), \tag{11} \]
where $\sigma(\rho)$ is the well known dipole cross section.

The collinear expansion corresponds to replacement of the hard part $H$ by its second order expansion in $k_T$ (we suppress all the arguments except for $k_T$ for clarity)

\[
H(k_T) \approx H(k_T = 0) + \frac{\partial H}{\partial k_T^a} \bigg|_{k_T = 0} k_T^a + \frac{\partial^2 H}{\partial k_T^a \partial k_T^b} \bigg|_{k_T = 0} \frac{k_T^a k_T^b}{2}.
\]

(12)

Only the second order term in (12) is important, which gives to a logarithmic accuracy $d\sigma(z, \xi)/dz \propto \int d\tilde{p}_T xG(p_T^2, x)\nabla^2_{k_T} H(\tilde{p}_T, \tilde{k}_T, z, \xi) |_{k_T = 0}$. But from (10) one can easily obtain $\nabla^2_{k_T} H |_{k_T = 0} = 0$

It is also seen from averaging of the hard part over the azimuthal angle of $k_T$ which gives $\langle H(\tilde{p}_T, \tilde{k}_T, z, \xi) \rangle \propto \theta(k_T - p_T)$. Thus, contrary to the expected dominance of the region $k_T \lesssim p_T$ only the region $k_T > p_T$ contributes to the gluon emission, and formal use of the collinear expansion gives completely wrong result with zero gluon spectrum.

The zero $N = 1$ gluon spectrum in the collinear approximation agrees with prediction of the harmonic oscillator approximation in the BDMPS and LCPI approaches. The oscillator approximation in corresponds to the quadratic parametrization of the dipole cross section $\sigma(\rho) = C \rho^2$. This parametrization is equivalent to the approximation of the vector potential by the linear expansion $A^+(y^-, \tilde{y}^+, \tilde{p}) \approx A^+(y^-, \tilde{y}^+) + \rho \nabla_{y^+} A^+(y^-, \tilde{y}^+)$ which can be traced to the collinear expansion in momentum space. The first term in the expansion in the relative momentum of the spectrum in the oscillator approximation in the BDMPS and LCPI approaches corresponds to $N = 2$, and the term with $N = 1$ rescattering is absent. In terms of the representation absence of the $N = 1$ contribution in the oscillator approximation is a consequence of the fact that in this case $d\sigma/dk_T \propto \delta(k_T)$ (as one sees from (11)).

4. The vanishing $N = 1$ spectrum is in a clear contradiction with the nonzero result of. This discrepancy is strange enough since Eqs. (6) - (10) are completely equivalent to the formulation of in the approximation of thin nucleons, and the effects beyond this approximation cannot be evaluated in the formalism. This puzzle has a simple solution. In the nonsecond derivative of the hard part comes from the graph shown in Fig. 2b (at $z < 1$). The authors use for the integration variable in the hard part of this graph the transverse momentum of the final gluon, $\tilde{l}_T$. The $\tilde{l}_T$-integrated hard part obtained in (Eq. 15 of) reads (up to an unimportant factor)

\[
H(k_T) \propto \int \frac{d\tilde{l}_T}{(\tilde{l}_T - \tilde{k}_T)^2} R(y^-, y_1^-, y_2^-, \tilde{l}_T, \tilde{k}_T),
\]

(13)

where

\[
R(y^-, y_1^-, y_2^-, \tilde{l}_T, \tilde{k}_T) = \frac{1}{2} \exp \left[ \frac{i y^-(\tilde{l}_T - \tilde{k}_T)^2 - (1 - z)(y_1^- - y_2^-)(\tilde{k}_T^2 - 2\tilde{l}_T \tilde{k}_T)}{2q^- z(1 - z)} \right] \times \left[ 1 - \exp \left( \frac{i(y_1^- - y^-)(\tilde{l}_T - \tilde{k}_T)^2}{2q^- z(1 - z)} \right) \right] \times \left[ 1 - \exp \left( \frac{i y_2^- (\tilde{l}_T - \tilde{k}_T)^2}{2q^- z(1 - z)} \right) \right]
\]

(14)

is an analog of the last factor in the square brackets in for $y^- \neq 0$, $y_1^- \neq y_2^-$ (or channel gluons, our $z$ equals $1 - z$ in). In calculating $\nabla^2_{k_T} H(\tilde{k}_T)$ the authors differentiate only the factor $1/(\tilde{l}_T - \tilde{k}_T)^2$. However, the omitted terms from the factor $R$ are important. After the $\tilde{l}_T$ integration they almost completely cancel the contribution from the $1/(\tilde{l}_T - \tilde{k}_T)^2$ term. Indeed, after putting $y_1^- = y_2^-$ and changing the variable $\tilde{l}_T \to (\tilde{l}_T + \tilde{k}_T)$ the right-hand part of does not depend on $\tilde{k}_T$ at all. We emphasize that even without the change of the variable leads to $\nabla^2_{k_T} H |_{\tilde{k}_T = 0} = 0$ if one performs differentiating correctly which is not done in. The difference between $y_1^-$ and $y_2^-$
gives some nonzero contribution to the $\nabla_{k_T}^2 H|_{k_T=0}$ suppressed by the small factor $\sim (R_N/L_f)^2$. Such contributions may be viewed as zero since they are beyond predictive accuracy of the approximations used in [3,4].

5. In summary, we have demonstrated that the collinear expansion fails in the case of gluon emission from a fast massless quark produced in $eA$ DIS. In this approximation the $N = 1$ rescattering contribution to the gluon spectrum vanishes. The nonzero gluon spectrum obtained in [3,4] is a consequence of unjustified neglecting some important terms in the collinear expansion. The established facts demonstrate that the GWZ approach [3,4] is wrong. Its predictions for $eA$ DIS and jet quenching in $AA$ collisions do not make sense.

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* Keeping the correction from the nonzero $y^-$ in (14) also does not make sense under the approximations used in [3,4].