The turbulent destruction of clouds – I. A $k-\epsilon$ treatment of turbulence in 2D models of adiabatic shock–cloud interactions

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ABSTRACT
The interaction of a shock with a cloud has been extensively studied in the literature, where the effects of magnetic fields, radiative cooling and thermal conduction have been considered. In many cases, the formation of fully developed turbulence has been prevented by the artificial viscosity inherent in hydrodynamical simulations. This problem is particularly severe in some recent simulations designed to investigate the interaction of a flow with multiple clouds, where the resolution of individual clouds is necessarily poor. Furthermore, the shocked flow interacting with the cloud has been assumed to be completely uniform in all previous single-cloud studies. In reality, the flow behind the shock is also likely to be turbulent, with non-uniform density, pressure and velocity structure created as the shock sweeps over inhomogeneities upstream of the cloud (as seen in recent multiple cloud simulations). To address these twin issues we use a subgrid compressible $k-\epsilon$ turbulence model to estimate the properties of the turbulence generated in shock–cloud interactions and the resulting increase in the transport coefficients that the turbulence brings. A detailed comparison with the output from an inviscid hydrodynamical code puts these new results into context.

Despite the above concerns, we find that cloud destruction in inviscid and $k-\epsilon$ models occurs at roughly the same speed when the post-shock flow is smooth and when the density contrast between the cloud and intercloud medium, $\chi \lesssim 100$. However, there are increasing and significant differences as $\chi$ increases. The $k-\epsilon$ models also demonstrate better convergence in resolution tests than inviscid models, a feature which is particularly useful for multiple-cloud simulations.

Clouds which are over-run by a highly turbulent post-shock environment are destroyed significantly quicker as they are subject to strong ‘buffeting’ by the flow. The decreased lifetime and faster acceleration of the cloud material to the speed of the ambient flow leads to a reduction in the total amount of circulation (vorticity) generated in the interaction, so that the amount of vorticity may be self-limiting. Additional calculations with an inviscid code where the post-shock flow is given random, grid-scale, motions confirm the more rapid destruction of the cloud.

Our results clearly show that turbulence plays an important role in shock–cloud interactions, and that environmental turbulence adds a new dimension to the parameter space which has hitherto been studied.

Key words: hydrodynamics – shock waves – turbulence – ISM: clouds – ISM: kinematics and dynamics – supernova remnants.

1 INTRODUCTION
Circumstellar, interstellar and intergalactic environments are inhomogeneous, with clouds of various densities and temperatures embedded in a hotter, more tenuous, substrate. This substrate is often turbulent, in part due to the interaction of shocks, shells, winds and jets with these clouds. The nature of such interactions is interesting, because the evolution and morphology of large-scale flows can ultimately be regulated by objects of much smaller size.

In the interstellar medium (ISM), for instance, the interaction of supernova shock waves with interstellar clouds creates a
continuous interchange of mass and energy between various thermal phases (Cox & Smith 1974; McKee & Ostriker 1977). Several outcomes are possible. The shocks may destroy the clouds, and mix their material into their surroundings. Alternatively, the shocks may trigger the collapse of the clouds and the formation of new stars, thereby removing material (at least temporarily) from the ISM (Elmegreen & Lada 1977). The shocks themselves will slow and material behind them will cool to form thin dense shells. These shells may then fragment and form new clouds. Clouds which survive the passage of the shell subsequently find themselves either inside a hot, low-density bubble (if the bubble is energy conserving), or exposed to a fierce, high Mach number wind (if the shell is momentum driven). An important process is the generation of vorticity as the diffuse medium flows around the clouds. Understanding the interaction between shock/shells/winds/jets and interstellar clouds is therefore a key step in studies of the structure and evolution of the ISM (see the recent reviews by Elmegreen & Scalo 2004; Scalo & Elmegreen 2004).

A similar interaction occurs in massive, early-type, stellar systems where each star blows a powerful, clumpy, wind which collides with the other. The impact of the clumps causes the wind–wind collision region to become highly turbulent (Pittard 2007a), with implications for particle acceleration, the time-scales for equilibrium ionization and electron heating and the physical mixing of the winds.

In this work we consider the adiabatic interaction of a shock with a cloud. An extensive literature of analytical and numerical investigations of shock–cloud interactions now exists (see e.g. Nakamura et al. 2006, and references therein). The effects of ordered magnetic fields, radiative cooling and thermal conduction have all been considered, but not simultaneously until recently (Orlando et al. 2008). High-power laser experiments of shock–cloud interactions (e.g. Klein et al. 2003) have complemented this literature. While thermal conduction acts to prevent hydrodynamic instabilities, radiative cooling enhances them. The effect of ordered magnetic fields is more complicated: instabilities are prevented in cases where a magnetic field provides a high tension at the surface of the cloud, but the effects depend on the strength, orientation and scale of the magnetic field. Instabilities and vortical motions may be prevented in some directions, but not necessarily in others. Interestingly, in three-dimensional (3D) simulations a magnetic field may actually enhance the fragmentation of a cloud (Gregori et al. 1999; Shin, Stone & Snyder 2008), though the actual mixing of the cloud and ambient medium remains hindered. Even if the magnetic fields are not strong enough to directly affect the dynamics, they can significantly reduce the effects of thermal conduction in directions normal to the field lines, thus allowing instabilities to develop (Orlando et al. 2008).

Given the fact that in shock–cloud interactions the Reynolds number is typically high, and the above predisposition for instabilities to develop, the fluid velocity field around the cloud is expected to vary significantly and irregularly in both position and time. This ‘turbulence’ transports and mixes the cloud material much more effectively than a comparable laminar flow. Turbulence is also effective at ‘mixing’ the momentum of a fluid (i.e. accelerating material ripped off the cloud to the ambient flow speed), and at transferring heat. In turbulent flows of high Reynolds number there is a separation of scales – large-scale motions will be strongly influenced by the geometry of the cloud, and control the transport and mixing of cloud material, while the behaviour of small-scale motions is determined almost entirely by the rate at which they receive energy from the large scales, and by the viscosity. Hence these small-scale motions have a universal character, independent of the flow geometry.

While much insight has been gained from previous numerical investigations of shock–cloud interactions, the artificial viscosity inherent in all such simulations has the potential to prevent the formation of fully developed turbulence, to limit the turbulent mixing of cloud material into the surrounding medium and to hinder the destruction of the cloud. Furthermore, all previous simulations are highly idealized in the sense that the flow behind the shock is assumed to be perfectly smooth and uniform (i.e. laminar). In reality, random inhomogeneities in the ambient medium upstream of the cloud will deform the shock, and will cause velocity, density and pressure structures to develop in the post-shock flow. The post-shock flow will then also be ‘turbulent’, and it is expected that the destruction of the cloud will be more rapid in such conditions.

In this work we address whether fully developed turbulence has been prevented in all previous numerical works. For high Reynolds number flows, the only tractable method is a statistical approach i.e. to describe the turbulent flow, not in terms of a velocity field \( u(\mathbf{x}, t) \), but in terms of some statistics. A model based on such statistics can lead to a tractable set of equations, because statistical fields vary smoothly (if at all) in position and time. To do this we use a subgrid turbulent viscosity model, where an attempt is made to calculate the properties of the turbulence and the resulting increase in the transport coefficients. The most widely used is the so-called \( k-\epsilon \) model, where the properties of the turbulence are described by two variables, the turbulent energy per unit mass, \( k \), and the turbulent dissipation rate per unit mass, \( \epsilon \). The addition of viscous and diffusive terms in the fluid equations simulates the turbulent mixing of the cloud and intercloud medium due to shear instabilities. In this way the subgrid turbulence model emulates a high Reynolds number flow. Incorporating a \( k-\epsilon \) model into shock–cloud simulations should produce more realistic results than those from inviscid codes, where the viscosity is purely numerical and the size of shear instabilities is determined by the resolution of the numerical grid. A turbulent viscosity model differs from simply adding physical viscosity to the grid, because the turbulent viscosity is largest in shear layers and essentially vanishes in regions with little shear, whereas models with grid viscosity have the same viscosity everywhere.

The structure of this paper is as follows. The key physics of a shock–cloud interaction is reviewed in Section 2. Section 3 introduces the investigation and the numerical method used. The results are presented in Section 4, where the effects of turbulence on the cloud evolution are described. A discussion of the relevance of our results to shock–cloud and wind–cloud observations is given in Section 5. Section 6 summarizes the conclusions of this work, and possible future work is noted in Section 7.

2 THE INTERACTION OF A SHOCK WITH A CLOUD

The interaction of a shock with a cloud is a highly non-linear and complex phenomenon which can be vastly simplified if some assumptions are made. In this work we assume that the magnetic field is too weak to be dynamically important (though it must be strong enough to reduce the thermal conductivity and effective mean-free-path), and that the interaction is adiabatic. In the ISM, the typical magnetic field is \( \approx 5 \mu G \), and the magnetic pressure is typically a few times higher than the thermal pressure (e.g. Cox 2005). However, if the shock driven into the cloud is strong and adiabatic the post-shock thermal pressure becomes much higher than the post-shock...
magnetic pressure, justifying our assumption that the magnetic field is too weak to be dynamically important. We also assume that the magnetic field does not prevent the full mixing of initially disparate phases (i.e. that turbulence drives the reconnection needed to allow this mixing).

The effects of thermal conduction are also ignored in this work. The efficiency of thermal conduction in magnetized turbulent plasmas remains highly uncertain (see e.g. Pidard 2007b). X-ray observations of the hot intracluster medium (Ettori & Fabian 2000; Vikhlinin et al. 2001) and theoretical considerations (Narayan & Medvedev 2001; Asai, Fukuda & Matsumoto 2004; Chandran & Maron 2004) suggest that the coefficient of heat conduction is at least five times lower than the Spitzer value in the presence of tangled magnetic fields. Furthermore, if turbulent resistivity can reconnect field lines quickly enough, turbulent heat transport may be more efficient than thermal conduction (Cho et al. 2003; Chandran & Maron 2004; Lazarian 2006).

For the interaction to be adiabatic, radiative cooling must be unimportant. This is often the case for clouds in hot, low-density environments (e.g. in planetary nebulae, in bubbles blown around individual or groups of massive stars and in starburst and superwind environments). The behaviour of the cloud is more likely to be adiabatic if the cloud is small. Since the assumption of adiabaticity preserves the scale-free nature of the simulations, the interaction is then determined by whether there is a sufficient rate of collisions within the plasma to give it fluid properties, by the dominant mechanism for the damping of hydromagnetic waves, by the Reynolds number and by the way the turbulence is driven. Each of these issues is discussed below. In order that their importance can readily be determined, two different examples are considered. In the first scenario the cloud is ionized, and has a radius \( r_c = 1 \text{ pc} \), density \( n_c = 0.4 \text{ cm}^{-3} \) and temperature \( T_c = 8000 \text{ K} \). In the second scenario we consider a neutral cloud with \( r_c = 0.46 \text{ pc} \), \( n_c = 30 \text{ cm}^{-3} \) and \( T_c = 100 \text{ K} \). In both cases we imagine that the clouds are in approximate pressure equilibrium with a surrounding medium with \( n_s = 3 \times 10^{-3} \text{ cm}^{-3} \) and \( T_s = 10^8 \text{ K} \), and are struck by a high-speed Mach 10 shock with velocity \( v_b = 1.52 \times 10^8 \text{ cm s}^{-1} \). The shocked intercloud gas has a density \( \rho_s = 2.6 \times 10^{-26} \text{ g cm}^{-3} \), temperature \( T = 3.2 \times 10^7 \text{ K} \) and velocity \( \gamma \approx 0.75 v_b = 1.1 \times 10^8 \text{ cm s}^{-1} \). The parameters noted above are typical of small interstellar clouds (McKee & Ostriker 1977).

### 2.1 Collisional or collisionless?

The first consideration is whether the mean-free-path is short enough that a collisional treatment can be adopted (i.e. whether the plasma has fluid-like properties). If this is the case, complications due to collisionless wave–particle interactions and their associated effects such as Landau damping can be ignored (Parker 1979). The damping of hydromagnetic waves in a collisionless plasma is stronger than in a collisional plasma, since collisions serve to suppress the wave–particle interactions which damp the wave so strongly in the collisionless case.

The Coulomb collision cross-section for electron or ion scattering is \( \sigma \approx 10^{-12} T_{\text{eV}}^3 \text{ cm}^2 \), where \( T_{\text{eV}} \) is the temperature in electron volts. For neutral material the gas atomic cross-section is \( \sim 10^{-16} \text{ cm}^2 \).

Hence the mean-free-path within the ionized and neutral clouds is \( \lambda = 1/\sigma \approx 10^{22} \text{ cm} \) and \( 10^{14} \text{ cm} \), respectively. These values are much smaller than the cloud radii, so a fluid-like treatment is appropriate. On the other hand, the mean-free-path in the post-shock intercloud medium is \( \lambda \approx 6 \times 10^{20} \text{ cm} \), which is significantly larger than the radius of the clouds. However, even a small \( B \) field will significantly reduce the effective mean-free-path (the mean-free-path is then of order the gyroradius, which for \( B = 3 \mu \text{G} \) and \( T = 10^8 \text{ K} \) is \( r_g \approx 10^9 \text{ cm} \) for protons), so the shocked intercloud gas can also be considered to be collisional.

### 2.2 Reynolds number, eddies and instabilities

The Reynolds number of flow past a cloud is \( \text{Re} = u r_c / \nu \), where \( u \) is the average flow velocity past the cloud, \( r_c \) is the radius of the cloud and \( \nu \) is the kinematic viscosity. For a fully ionized, non-magnetic gas of density \( \rho \) and temperature \( T \),

\[
v = 2.21 \times 10^{-15} \frac{T^{1/2} A^{1/2}}{Z^2 \rho \ln \Lambda} \text{ cm s}^{-1},
\]

where \( A \) and \( Z \) are the atomic weight and charge of the positive ions, and \( \ln \Lambda \) is the coulomb logarithm (Spitzer 1956). In a magnetized plasma, the kinematic viscosity is of the order of the mean-free-path (i.e. the particle gyroradius) times the typical thermal velocity.

The characteristic Reynolds number of the interaction is higher in the cloud material than the surrounding environment, since the cloud is considerably cooler. The shock driven into the cloud has a speed \( v = v_b / \sqrt{\chi} \), where \( \chi \) is the density contrast of the cloud with respect to its surroundings. For the ionized cloud considered in Section 2, the post-shock temperature is \( 2.6 \times 10^7 \text{ K} \) and \( \nu \approx 5 \times 10^{20} \text{ cm}^2 \text{s}^{-1} \). Hence \( \text{Re} \approx 7 \times 10^6 \). If the cloud is magnetized with a post-shock field \( B \sim 10 \mu \text{G} \), the kinematic viscosity for protons is \( \nu \sim 10^{15} \text{ cm}^2 \text{s}^{-1} \) and the Reynolds number is proportionally higher. In the neutral cloud the Reynolds number calculated from the kinematic molecular viscosity is similarly high. The Reynolds number of the flow around a cloud is also high. For a flow with \( B = 3 \mu \text{G} \) and \( T = 10^7 \text{ K} \), the kinematic viscosity is \( \nu \sim 10^{17} \text{ cm}^2 \text{s}^{-1} \). In all cases, the viscous stresses acting on the boundary layer which forms as the shock sweeps over the cloud are negligible, and a turbulent energy cascade ensues.

The smallest eddies have a length scale, \( l \), comparable in size to the cloud, while the smallest eddies where the turbulent energy is dissipated have a length scale \( \eta \sim \text{Re}^{-1/4} l \). Because of the non-linear term \( u \cdot \nabla u \) in the equation of motion, large eddies, created by instabilities in the mean flow, are themselves subject to inertial instabilities and rapidly ‘break-up’ or evolve into yet smaller vortices. Energy is transferred into vortices of about one-half their size in a time comparable to their ‘turnover time’ \( \tau = l/\nu \), where \( \nu \) is the characteristic velocity of eddies of size \( l \); Davidson 2004). Provided that energy is constantly injected at large scales, small-scale eddies are superimposed on larger eddies. The time-scale to set up a turbulent energy cascade is roughly twice the turnover time of the largest eddies (i.e. \( \tau = 2 l/\nu \)). Since \( l/\nu \sim r_c / \nu_c \), the set-up time is \( t \sim 2 r_c / \nu_c = t_{\text{sc}} \), where \( t_{\text{sc}} \) is the time-scale for the shock in the intercloud medium to sweep over the cloud. For dense clouds (\( \chi \gg 1 \)), this set-up time is much shorter than the survival time of the cloud, which is a few times the ‘cloud crushing’ time-scale,

\[
t_{\text{sc}} = \sqrt{1/2} r_c / v_b
\]

for the cloud to be crushed by the initial shock that is driven into it (Klein, McKee & Colella 1994).

Simultaneously with the top-down energy transfer from large to small eddies, Kelvin–Helmholtz (KH) and Rayleigh–Taylor (RT) instabilities inject energy from the bottom-up, since the smallest scale disturbances grow fastest. The KH and RT growth times are

\[
t_{\text{KH}} \sim \frac{t_{\text{sc}}}{k_z r_c}, \quad t_{\text{RT}} \sim \frac{t_{\text{sc}}}{(k_z r_c)^{1/2}},
\]
where $k_i$ is the wavenumber of the perturbation (Klein et al. 1994). The smallest scale of the instabilities is set by the scale at which the damping of hydromagnetic waves occurs, which was shown in Section 2.1 to be through particle collisions rather than wave–particle interactions.

For the unmagnetized ionized cloud the minimum scale due to viscous damping is $\eta_{\mathrm{vis}} \sim Re^{-3/4} r_c \sim 4 \times 10^{-5} r_c$. However, in ionized plasmas the thermal conductivity is even more effective at damping waves with a significant longitudinal component. The thermochromic conductivity $K = \kappa T/\rho$, where $U/T$ is the thermal energy per unit volume and $\kappa$ is the thermal conduction coefficient (Parker 1979). Here we find that $K \approx 40 \tau$, and thermal conduction prevents instabilities with a scale smaller than $\eta_{\mathrm{th}} \sim 6 \times 10^{-3} r_c$.

In magnetized ionized clouds the length-scale at which instabilities are damped is smaller if the magnetic field is sufficiently weak to be dynamically unimportant. On the other hand, if the magnetic field is strong enough to be dynamically important, magnetic tension increases the minimum length-scale at which instabilities occur.

A disturbance of wavelength $\lambda = 10^{-3} r_c$ grows in a time-scale of $t_{\mathrm{KH}} = 1.6 \times 10^{-4} t_c$ and $t_{\mathrm{RT}} = 1.3 \times 10^{-2} t_c$. With the ionized cloud parameters given above, $\chi = 133, t_c = 5.8 I_{-6}, and we obtain $t_{\mathrm{KH}} \approx 10^{-3} t_c$ and $t_{\mathrm{RT}} \approx 0.1 I_{-6}$. These time-scales are much faster than the one to set-up the turbulent energy cascade (which was shown above to be $\sim t_c$) and the (ensemble averaged) turbulent spectrum will differ from the classical Kolmogorov spectrum because it is driven by energy input at both large and small scales.

Photoionization at the surface of neutral clouds maintains most of the C in the form C$^+$ (e.g. Hartquist et al. 1998). An upper limit to the ratio of the ionized to neutral mass density, $\rho^+/\rho$, is therefore the fractional abundance by mass of carbon, which has a cosmic value of about $3 \times 10^{-3}$ (e.g. Dopita & Sutherland 2003). Kulsrud & Pearce (1969) note that for hydromagnetic waves with $\lambda < \lambda_1$, the charged particles move as if the neutrals were absent, while for $\lambda > \lambda_1$, the entire medium moves. In contrast, when $\lambda_1 < \lambda < \lambda_2$, the neutrals and ions move independently and friction is strong, and waves do not propagate. $\lambda_1$ corresponds to waves with angular frequencies comparable to the rate at which neutral transfer momentum to ions, while $\lambda_2$ is the corresponding rate for momentum transfer from ions to neutrals. Assuming that the magnetic field strength within our neutral cloud is $\sim 3 \mu$G, we find that $\lambda_1 \approx 4 \times 10^3$ cm and $\lambda_2 \approx 1.3 \times 10^8$ cm. In this case fully developed turbulence is prevented, since $\lambda_1/r_c > \eta_{\mathrm{vis}}$ when $Re \sim 10^6$, although instabilities continue to be driven from the bottom up. Fully developed turbulence may be obtained when the magnetic field is weaker, since $\lambda_1$ and $\lambda_2$ are both proportional to $B$, or when the cloud is larger.

In numerical simulations of shock–cloud interactions, the growth of KH and RT instabilities is closely related to the development of the slip surface around the cloud, and perturbations with wavelengths smaller than the thickness of the shear layer are stabilized (Nakamura et al. 2006). Thus, small-scale instabilities have been artificially suppressed in previous work on hydrodynamic shock–cloud interactions. Our use of a $k^{-\epsilon}$ turbulence model in this paper attempts to address this shortcoming.

### 2.3 The turbulent boundary layer

Hartquist & Dyson (1988) argued that the turbulent boundary layer which forms around a cloud has a thickness of the order of $r_c/\sqrt{Re}$, where $Re_c$ is an effective ‘turbulent’ Reynolds number arising from the fact that the turbulence itself gives rise to an effective viscosity. Since $Re_c \sim 10^3$ (Hartquist & Dyson 1988), the thickness of the turbulent boundary layer is a few per cent of the cloud radius.

More detailed calculations and laboratory experiments show that the opening angle of a turbulent mixing layer in a mildly supersonic flow (such as occurs behind a high Mach number shock) is of order $10^\circ$ (Cantó & Raga 1991) – we find good agreement with this (see Section 4.2). Convergence tests reveal that a minimum numerical resolution of about 120 cells per cloud radius is needed for convergence of various global quantities (see e.g. Nakamura et al. 2006, although the necessary resolution may to some extent also depend on the numerical scheme). This is consistent with simulations of this resolution and higher beginning to resolve the turbulent boundary layer.

### 2.4 Cloud destruction and mixing

The main stages in the destruction of a cloud by a shock and the subsequent mixing of its material into the surrounding flow are reviewed in Section 4.1. The most disruptive KH and RT instabilities are those with wavelengths of the order of the cloud radius. However, there are a number of ways in which the growth of KH and RT instabilities may be hindered or amplified. For instance, clouds with diffuse boundaries are less susceptible to KH instabilities and survive longer (Nakamura et al. 2006). Two-dimensional (2D) magnetohydrodynamic (MHD) simulations found that the growth of KH and RT instabilities is strongly inhibited when there is a dynamically important magnetic field, due to the field providing an additional tension at the interface between the cloud and the surrounding flow (Mac Low et al. 1994). Somewhat surprisingly, fully 3D MHD simulations of a wind–cloud interaction revealed that an ordered magnetic field can actually enhance hydrodynamic instabilities, as background field lines become trapped in deformations in the surface of the cloud (Gregori et al. 1999, though these were low-resolution calculations). Higher resolution shock–cloud simulations recently presented by Shin et al. (2008) show that irrespective of the field geometry, and the morphology of the cloud fragments which are produced in the interaction, the rate of mixing is reduced compared to the non-magnetic case. However, sufficiently weak magnetic fields have no dynamical influence, and only offer a potential reduction in the collision mean-free-path. Strong thermal conduction suppresses hydrodynamic instabilities (see Section 2.2 and also Marcolini et al. 2005; Orlando et al. 2005), but the degree of this effect is sensitive to the orientation of any magnetic field (Orlando et al. 2008).

KH and RT instabilities are always stronger when radiative cooling is important, and the cloud breaks up into numerous dense, cold fragments (Mellema, Kurk & Röttgering 2002; Fragile et al. 2004, 2005; Van Loo et al. 2007). In the corresponding simulations the fragments appear to survive for an appreciable time, but are poorly resolved, so the time-scales corresponding to their further evolution are somewhat uncertain. The way in which clouds are destroyed and mixed into their surroundings is sensitive also to the density contrast between the cloud and the surrounding medium, in that clouds with higher values of $\chi$ suffer direct stripping of material from their surfaces by hydrodynamic ablation (Klein et al. 1994, also compare Figs 4, 9 and 10 in this work).

Cloud destruction and the mixing of the cloud material into the surrounding flow are two distinct processes which were not always distinguished in previous work. In Nakamura et al. (2006), the cloud destruction time-scale is taken to be the time when the largest fragment drops below a certain fraction of the initial cloud mass, while the time-scale for the mixing of former cloud material into the surrounding medium is estimated by comparing the integrated mass above a particular threshold density with the initial cloud mass.
However, even weak magnetic fields can prevent the actual mixing of stripped material with the surrounding medium, and reconnection on small-scales is necessary if the plasmas are to mix fully. This may, of course, take some considerable time to achieve.

3 THE NUMERICAL SET-UP

3.1 The numerical scheme

The shock–cloud interaction is modelled by solving numerically the Euler equations of inviscid fluid flow, supplemented by a subgrid turbulent viscosity model as appropriate. When the subgrid model is included, the continuity, scalar, momentum, energy, turbulent energy and turbulent dissipation equations are, respectively,

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0, \]  
\[ \frac{\partial \rho u}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) - \nabla \cdot (\mu_T \nabla k) = 0, \]  
\[ \frac{\partial \mathbf{u}}{\partial t} + \nabla \cdot (\rho \mathbf{u} \mathbf{u}) + \nabla P - \nabla \cdot \tau = S_p, \]  
\[ \frac{\partial E}{\partial t} + \nabla \cdot [(E + P) \mathbf{u} - \mathbf{u} \cdot \tau] - \frac{\gamma}{\gamma - 1} \nabla \cdot (\mu_T \nabla T) = S_E, \]  
\[ \frac{\partial \rho k}{\partial t} + \nabla \cdot (\rho k \mathbf{u}) - \nabla \cdot (\mu_T \nabla k) = S_k, \]  
\[ \frac{\partial \rho \epsilon}{\partial t} + \nabla \cdot (\rho \epsilon \mathbf{u}) - \nabla \cdot (\mu_T \nabla \epsilon) = S_, \]  

Here \( \rho \) is the mass density, \( \mathbf{u} \) is the velocity, \( E \) is the total energy density,

\[ E = \frac{P}{\gamma - 1} + \frac{1}{2} \rho \mathbf{u}^2, \]  

\( P \) is the thermal pressure, \( k \) is the turbulent energy per unit mass, \( \epsilon \) is the turbulent dissipation rate per unit mass and the turbulent diffusion coefficients are

\[ \mu_T = \rho C_{\mu} \frac{k^2}{\epsilon}, \quad \mu_T = \frac{\mu_T}{\epsilon}, \]  

where \( C_{\mu} = 0.09, \) \( k \) is an advected scalar used to distinguish between cloud and ambient material.

The momentum equation source term, \( S_p, \) is zero in Cartesians. In cylindrical symmetry it is

\[ S_p = \left[ \mu_T \left( \frac{2}{3r} \nabla \cdot \mathbf{u} - \frac{2u_r}{r^2} \right) + \frac{1}{r} \left( P + \frac{2}{3r} \rho k \right) \right] \]  

The \( k \) and \( \epsilon \) source terms are, respectively,

\[ S_k = \mu_T \frac{k}{\epsilon}, \]  
\[ S_\epsilon = \frac{\epsilon}{k} \left( C_1 \mu_T - C_2 \rho \epsilon \right), \]

where \( C_1 = 1.4 \) and \( C_2 = 1.94. \)

The turbulent production term

\[ P_t = \mu_T \left( \frac{\partial u_i}{\partial x_j} \frac{\partial u_j}{\partial x_i} + \frac{\partial u_j}{\partial x_i} \frac{\partial u_i}{\partial x_j} \right) - \frac{2}{3} \nabla \cdot \mathbf{u} (\rho k + \mu_T \nabla \cdot \mathbf{u}), \]  

where the summation convention is assumed. The terms involving \( \mu_T \) are due to the rate of working of the turbulent stresses, and the terms involving \( \nabla \cdot \mathbf{u} \) take into account volume changes on the turbulence. In cylindrical symmetry, the production term has to be complemented by an extra geometric term

\[ 2\mu_T \frac{u_i^2}{r^2}. \]  

The turbulent stress tensor, \( \tau, \) is

\[ \tau_{ij} = \mu_T \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \delta_{ij} (\mu_T \nabla \cdot \mathbf{u} + \rho k). \]  

All computations were performed in 2D cylindrical symmetry using an Eulerian adaptive mesh refinement (AMR) hydrodynamic code, with a linear Godunov solver and piecewise linear cell interpolation (see Falle 1991). Although it is of lower order than the piecewise parabolic method (PPM), it performs well in multidimensional problems, as it is only partially operator split, and it performs better than PPM for problems where there is rapid advection across the computational grid (Runacres & Owocki 2005).

The entire computational domain is covered by the two coarsest grids, \( G^0 \) and \( G^1. \) The solution at each position is calculated on all grids that exist there, and the difference between these solutions is used to control refinement [note that refinement criteria based on the local gradients of only selected variables (e.g. density) do not properly resolve turbulent flow – see Iapichino et al. 2008]. Finer grids are dynamically added where they are needed, and removed where they are not. Each refinement level increases the resolution in each of the spatial directions by a factor of 2, and the refinement is done on a cell-by-cell rather than on a patch basis. The time-step on grid \( G^0 \) is \( \Delta t_{0}/2n, \) where \( \Delta t_{0} \) is the time-step on \( G^0, \) in order to ensure Courant number matching at the boundaries between coarse and fine grids.

The \( k-\epsilon \) model is designed to model the mean flow in fully developed, high Reynolds number, turbulence. It has been calibrated by comparing the computed growth of shear layers with experiments of high Reynolds number flows (Dash & Wolf 1983). Although the flow in our problem is somewhat more complicated than in these experiments, it is described by exactly the same equations. Of course, any turbulence model can only be approximately correct, but it should give more reliable results than an inviscid calculation.

In the subgrid model, turbulent energy is generated by the action of the turbulent viscosity on the mean flow and is converted to heat at the dissipation rate, \( \epsilon. \) Since the turbulent energy resides mainly in large eddies, while the dissipation occurs in the small ones, one can think of \( k \) and \( \epsilon \) as describing the large-scale and small-scale turbulence, respectively. Since the aim of the subgrid model is to mimic a 3D turbulent flow, the effects of the turbulence (such as enhanced transport coefficients) are treated correctly, even though the grid is cylindrically symmetric. However, the turbulent motions that are resolved on the grid are actually vortex rings, and not eddies. Further details of the model implementation can be found in Falle (1994).

3.2 Initial and boundary conditions

We consider a Mach 10 shock interacting with a cloud with a density contrast \( \chi \) of either 10, 100 or 1000, and compute simulations with different spatial resolutions using either an inviscid code or one which includes a \( k-\epsilon \) turbulence model. The effect of different levels of turbulence in the post-shock gas is also explored. Table 1 summarizes the calculations performed. Most computations are for clouds with steep density profiles, but we have also examined the effect of a shallower density profile on the resulting evolution (see
Table 1. Summary of the shock–cloud simulations performed. The resolution is the number of cells per cloud radius on the finest grid. Models with ‘sh’ in their name were computed for clouds with a shallow density gradient. Model c1rth32 was computed using an inviscid code with grid-scale turbulence.

| Model     | $\chi$ | $p_1$ | Resolution | Turbulence |
|-----------|--------|-------|------------|------------|
| c1no      | $10^1$ | 10    | 32, 64, 128| No         |
| c1lo      | $10^1$ | 10    | 32, 64, 128| Low        |
| c1hi      | $10^1$ | 10    | 64, 128    | High       |
| c2no      | $10^2$ | 10    | 32, 64, 128| No         |
| c2lo      | $10^2$ | 10    | 32, 64, 128| Low        |
| c2hi      | $10^2$ | 10    | 32, 64, 128| Low        |
| c3no      | $10^3$ | 10    | 16, 32, 128| No         |
| c3lo      | $10^3$ | 10    | 16, 32, 128| Low        |
| c3hi      | $10^3$ | 10    | 32, 64, 128| High       |
| c2nosh    | $10^2$ | 1     | 64         | No         |
| c2losh    | $10^2$ | 1     | 64         | Low        |
| c2nish    | $10^2$ | 1     | 64         | High       |
| c3nosh    | $10^3$ | 1     | 64         | No         |
| c3losh    | $10^3$ | 1     | 64         | Low        |
| c3nishi   | $10^3$ | 1     | 64         | High       |
| c1rthb    | $10^1$ | 10    | 32         | Grid-scale |

Section 4.4). A calculation using an inviscid code with grid-scale post-shock turbulence is also presented (see Section 4.5).

The calculations are computed on an $r - z$ cylindrically symmetric grid, with a domain of $0 \leq r \leq 24$, $-94 \leq z \leq 6$ when $\chi = 10$, $0 \leq r \leq 24$, $-120 \leq z \leq 6$ when $\chi = 10^2$, and $0 \leq r \leq 48$, $-594 \leq z \leq 6$ when $\chi = 10^3$. This ensures that the cloud is well dispersed and mixed into the post-shock flow before the shock reaches the edge of the numerical grid. In this way the global quantities detailed in Section 3.4 are accurately computed. All calculations are for an ideal gas with $\gamma = 5/3$, and are scaled so that the fluid variables have values reasonably close to unity.

Several additional parameters must be specified when using a turbulence model. One of these is the maximum eddy size, which here is set equal to the cloud radius. Another choice concerns the initial level of turbulence in the gas, and two extreme cases are considered. In the first case (hereafter identified by ‘low $k-e$’, or ‘lo’ in the model name) the post-shock gas initially has an extremely low level of turbulence, with a ratio of turbulent energy density to thermal energy density $e_h/e_0 \sim 10^{-6}$. In the second case (identified by ‘high $k-e$’, or ‘hi’ in the model name) this ratio is 0.13 (values much higher than this cause the shock to accelerate too much and the interaction does not occur at the intended Mach number). High levels of post-shock turbulence may arise if the shock is propagating through an inhomogeneous medium (e.g. when there are density variations further upstream). As we shall see, this ratio is similar to the turbulent energy fraction attained by the cloud material after the shock encounter (see Section 4.6.7), so there is a degree of self-consistency in these models. In both cases, initially $e_h/e_0 = 0.04$ within the cloud (this is set low enough to not affect the dynamics – cf. the initial development of the interaction in models c3no128 and c3lo128), and the cloud and intercloud medium are set in pressure equilibrium.

3.3 Cloud density profile

Clouds in the ISM do not have infinitely sharp edges (see e.g. the discussion in Nakamura et al. 2006). The density profile adopted in this work is

$$\rho(r) = \rho_{amb} [\psi + (1 - \psi) \eta].$$

(18)

where

$$\eta = \frac{1}{2} \left(1 + \frac{\alpha - 1}{\alpha + 1}\right),$$

(19)

$\alpha = \exp[(\min [20.0, (r/r_c)^2 - 1])]$, and $r$ is the distance from the centre of the cloud of radius $r_c$, $\psi$ is adjusted to obtain a specific density contrast for the centre of the cloud with respect to the ambient medium, $\chi = \rho_{max}/\rho_{amb}$. The parameter $p_1$ controls the steepness of the profile at the edge of the cloud. Equation (18) tends to give a flatter density profile within the centre of the cloud, and a steeper profile as the cloud merges into the ambient medium, than profiles obtained using equation (1) in Nakamura et al. (2006). For $p_1 \gtrsim 5$, $\psi \approx \chi$. Clouds with reasonably sharp edges are obtained with $p_1 = 10$ (similar to those from equation 1 in Nakamura et al. 2006 with $n = 24$), while $p_1 = 1$ produces a much more extended cloud which is closer to the Nakamura et al. (2006) profile with $n = 2$ (see Fig. 1).

3.4 Global quantities

The cloud evolution is studied through various integrated quantities (see Klein et al. 1994; Nakamura et al. 2006). Averaged quantities $\langle f \rangle$ are constructed by

$$\langle f \rangle = \frac{1}{m_\beta} \int_{x \leq \beta} \kappa \rho f \, dV,$$

(20)

where the mass identified as being part of the cloud is

$$m_\beta = \int_{x \leq \beta} \kappa \rho \, dV.$$

(21)

$\kappa$ is an advected scalar, which has an initial value of $\rho/(\chi \rho_{amb})$ for cells within a distance of 2.25 $r_c$ from the centre of the cloud, and a value of zero at greater distances. Hence, $\kappa = 1$ in the centre of the cloud, and declines outwards. The above integrations are performed only over cells in which $\kappa$ is at least as great as the threshold value, $\beta$. Setting $\beta = 0.5$ probes only the densest parts of the cloud and its fragments, while setting $\beta = 2/\chi$ probes the whole cloud including its low-density envelope, and regions where only a small percentage of cloud material is mixed into the ambient medium.

Figure 1. Comparison of cloud density profiles obtained using equation (18) ($p_1 = 1$ and 10) and equation (1) in Nakamura et al. (2006) ($n = 24$) with $\chi = 10^2$. © 2009 The Authors. Journal compilation © 2009 RAS, MNRAS 394, 1351–1378.
To measure the shape of the cloud, effective radii normal to and along the axis of symmetry are defined, respectively, as
\[ a = \left( \frac{5}{2} \langle r^2 \rangle \right)^{1/2}, \quad c = \sqrt{5 \left( \langle z^2 \rangle - \langle z \rangle^2 \rangle \right)^{1/2}}. \] (22)

In inviscid calculations a measure of the turbulence of the cloud is obtained from the velocity dispersions in the radial and axial directions, defined, respectively, as
\[ \delta v_r = \langle r^2 \rangle^{1/2}, \quad \delta v_z = \left( \langle v_z^2 \rangle - \langle v_z \rangle^2 \right)^{1/2}. \] (23)

The mean density is defined as
\[ \langle \rho \rangle = \frac{m_\beta}{V_\beta}, \] (24)
where \( V_\beta \) is the volume of the region with \( \kappa \geq \beta \).

All quantities computed with \( \beta = 0.5 \) are identified with the subscript ‘core’ (e.g. \( a_{\text{core}} \)), while those computed with \( \beta = 2/\chi \) are given the subscript ‘cloud’ (e.g. \( a_{\text{cloud}} \)).

3.5 Time-scales

Several time-scales are obtained from the simulations. The characteristic radial expansion time-scale, \( t_{\text{exp}} \), is defined as the time at which the cloud radius normal to the axis of symmetry, \( a \), has increased to 90 per cent of its maximum value. The time for the cloud velocity relative to that of the post-shock ambient flow to decrease by a factor of 1/e is defined as the ‘drag time’, \( t_{\text{drag}} \). The ‘mixing time’, \( t_{\text{mix}} \), is defined as the time when the mass of the core of the cloud, \( m_{\text{core}} \), reaches half of its initial value. The zero-point of all the time measurements quoted in this work occurs when the intercloud shock is level with the centre of the cloud.

3.6 Convergence tests

It is important to demonstrate that the calculations are performed at spatial resolutions that are high enough to resolve key features of the interaction. Increasing the resolution in inviscid calculations leads to smaller scales of instabilities. Quantities which are sensitive to these small scales (such as the mixing rate between cloud and ambient gas) may not be converged, while quantities which are insensitive to gas motions at small scales (e.g. the shape of the cloud) are more likely to show convergence. Previous studies (e.g. Klein et al. 1994; Nakamura et al. 2006) have indicated that about 100 cells per cloud radius are needed for convergence of the simulations. Here we carry out a similar study for calculations which use a subgrid turbulence model.

Fig. 2 shows the evolution of the core mass and mean cloud velocity as a function of spatial resolution for both inviscid and \( k-\epsilon \) calculations. These parameters are a good test of the convergence, since both the rate of mixing and the momentum transfer between the cloud and the ambient flow are sensitive to small-scale instabilities. It is therefore not too surprising to see that these quantities are poorly converged in the low-resolution inviscid calculations (see also Shin et al. 2008). In contrast, the subgrid turbulence model leads to results which are much less dependent on the spatial resolution. This is also demonstrated in Fig. 3 which shows, for a number of parameters, the relative error defined as the fractional difference between the value measured at resolution \( N \) and the value at the finest resolution, \( f \):
\[ \Delta Q_N = \frac{|Q_N - Q_f|}{|Q_f|}. \] (25)

The convergence is much better for the \( k-\epsilon \) calculations (see Fig. 3), leading to much less variation with resolution as seen in Fig. 2. Both

**Figure 2.** Convergence tests for a shock–cloud interaction with \( \chi = 10^3 \) and \( p_1 = 10 \) using an inviscid (panels a and b) and \( k-\epsilon \) (panels c and d) code. The time evolution of the core mass (panels a and c) and the mean velocity of the cloud (panels b and d) are shown. Note the much tighter correlation of \( m_{\text{core}} \) and \((v_z, \text{cloud})\) from the \( k-\epsilon \) code.
inviscid and \( k-\varepsilon \) calculations demonstrate better convergence at \( \chi = 10^3 \) (not shown).

Figs 2 and 3 support claims from previous studies that of order 100 cells per cloud radius are needed for convergence. However, the neglect of the effect of turbulent eddies on the mean flow means that it is not clear that inviscid simulations, particularly at high values of \( \chi \), are actually converging to a physically realistic solution.

4 RESULTS

4.1 Stages

The main stages of the interaction of a shock of velocity \( v_b \) with a uniform cloud of density contrast \( \chi \), in the adiabatic, unmagnetized, non-conducting case are (1) an ‘initial transient stage’, where the incident shock propagates into the cloud with velocity \( v_b = v_b/\chi^{1/2} \), and a bow shock or bow wave propagates upstream into the ambient medium; (2) the ‘compression stage’, where the cloud is compressed mainly in the \( z \)-direction by the transmitted shock and by a shock driven into the back of the cloud; (3) the ‘expansion stage’, where the cloud is destroyed and its material mixed into the surrounding flow. In other situations, for example when there is efficient cooling, the evolution can be significantly different (see Section 2.4). In all work to date the cloud is destroyed by the shock. The addition of gravitational forces is likely to be needed if the cloud is to survive.

4.2 Cloud morphology and turbulence

In Fig. 4, snapshots of the density distribution at different times are shown for an inviscid calculation with 128 grid cells per cloud radius (model c3no128). The evolution of the cloud broadly follows the stages outlined above. The first two stages last until \( t \approx t_{cc} \) (i.e. the top five panels in Fig. 4). The expansion of the cloud in stage 3 is supersonic with respect to the sound speed within the cloud, and a low-density interior surrounded by a higher density shell forms (see the snapshot at \( t = 1.83 t_{cc} \)). The high-density shell then collapses in on itself (see the snapshot at \( t = 2.31 t_{cc} \)). Material is continuously ablated off the surface of the cloud by the fast-flowing surroundings, and a turbulent wake with prominent RT and KH instabilities forms. Fig. 7 shows that at later times the mass loss from the cloud resembles a single tail-like structure (this is in contrast to models with lower values of \( \chi \) – see Section 4.3).

Fig. 5 shows that the initial interaction of the shock with the cloud in the low \( k-\varepsilon \) case (model c3lo128) is similar to the inviscid case, since the initial post-shock turbulence is low and \( k \) and \( \varepsilon \) are small. However, the viscosity introduced by the subgrid turbulence model prevents the subsequent development of the resolution-dependent RT and KH instabilities seen in Fig. 4. Instead, the loss of material from the cloud occurs much more smoothly. This is exactly as expected given that the purpose of the \( k-\varepsilon \) model is to approximate the time-averaged flow.

Simulations with a high initial level of post-shock turbulence are different again, as seen in Fig. 6 where the results of model c3hi128 are displayed. The transport/diffusion coefficients are considerably higher in this simulation, and this leads to a faster rate of ablation from the cloud, and ultimately its more rapid destruction. The high
The turbulent destruction of clouds

Figure 4. Snapshots of the density distribution from an inviscid calculation of a Mach 10 adiabatic shock hitting a cloud with a density contrast of $10^3$ with respect to the ambient medium and a density profile with $p_1 = 10$ (model c3no128). The resolution is 128 cells per cloud radius. The evolution proceeds left to right and top to bottom with $t = 0.0, 0.1, 0.3, 0.49, 0.87, 1.35, 1.83, 2.31, 2.79$ and $3.75 \ t_{cc}$.

level of upstream turbulence also smooths/broadens the bow shock ahead of the cloud and the tailshock which forms downstream, so that their time-averaged positions are represented.

Fig. 7 compares the morphology of the clouds in these three simulations at $t = 5.66 \ t_{cc}$. The global features of models c3no128 and c3lo128 are reasonably comparable at this time, but the cloud in model c3hi128 is clearly at a more advanced stage of destruction (cf. Table 2 and Fig. 17). Interestingly, material stripped from the cloud lies off-axis in model c3hi128. This develops from the off-axis density peak seen at $t = 1.83$ and $3.75 \ t_{cc}$ shown in Fig. 6, though the initial disturbance occurs at even earlier times. Clearly the turbulence in this model affects the properties of the shocks driven into the cloud and the slip surface that forms around it, with small differences at early stages being amplified during the subsequent evolution.

The development of the subgrid turbulence in model c3lo128 is shown in the three left-most panels of Fig. 8, where the turbulent energy per unit mass, $k$, is displayed. $k$ is created in regions of
Figure 5. As Fig. 4 but for a $k-\epsilon$ calculation with low initial post-shock turbulence (model c3lo128). The times of the snapshots are $t = 0.1, 0.49, 0.87, 1.83$ and $3.75\,t_{cc}$.

Figure 6. As Fig. 4 but for a $k-\epsilon$ calculation with high initial post-shock turbulence (model c3hi128). The times of the snapshots are $t = 0.1, 0.49, 0.87, 1.83$ and $3.75\,t_{cc}$.

high shear, particularly in a thin turbulent boundary layer along the slip surface around the cloud (see the left-most panel of Fig. 8). The turbulent intensity quickly saturates at a level that is almost independent of its initial value.

The $\nabla \cdot \mathbf{u}$ terms in the production term for $k$ (see Section 3.1) means that $k$ is also generated behind shocks, as can be seen in four specific regions in the snapshot at $t = 0.1\,t_{cc}$: behind the incident shock sweeping through the ambient medium; behind the reflected shock formed as the incident shock converges on the axis behind the cloud; behind the bow shock formed upstream of the cloud and behind the slow shock driven into the cloud.

The reflected shock on the axis behind the cloud becomes increasingly oblique as the point of convergence moves away from the rear of the cloud, and interacts with the incident shock to create
a double Mach reflected shock that propagates along the axis (Klein et al. 1994). A powerful supersonic vortex ring forms just behind the Mach reflected shock, in which a region of high turbulence is generated (see the second from left-hand panel of Fig. 8). While the turbulence generated behind shocks decays very rapidly, the turbulence associated with the vortex ring is much more persistent, as is the turbulence generated at the slip surface around the cloud.

Fig. 8 shows that at later times the turbulence generated at the slip surface proceeds to develop into a highly turbulent wake with a radius comparable to the initial cloud radius. The set-up time for the wake is $\sim t_{cc}$, and the subgrid turbulent energy of the cloud material grows and then dissipates as the cloud is mixed into its surroundings (see Figs 17g and 22). Note that the core of the cloud has very little turbulence associated with it.

The finite time-scale for the development of significant turbulence means that simulations with the $k-\epsilon$ subgrid turbulence model with a low initial level of post-shock turbulence produce similar morphologies to those obtained from inviscid calculations at early times ($t \lesssim 0.5 t_{cc}$). However, the increase in the transport coefficients in regions of high turbulence leads to increasing divergence from inviscid calculations at later times, and ultimately to a faster destruction of the cloud.

In contrast, a high level of environmental turbulence immediately affects the evolution of the cloud, since the transport coefficients around the cloud are also high. The two right-most panels of Fig. 8 show the highly turbulent post-shock flow engulfing the cloud in model c3hi128. Both the limb of the cloud and the bow shock upstream of the cloud become broader and less distinct as the high level of turbulence leads to strong diffusion across these boundaries (see Fig. 6). Another major difference compared to model c3lo128 is that the turbulence downstream of the cloud at $t = 0.87 t_{cc}$ is roughly as strong as that in the post-shock flow. Hence the turbulent wake which is seen so clearly in the centre panel of Fig. 8 is indistinguishable from the surroundings in the right-hand panel of Fig. 8. Note also that the value of $k$ created downstream of the incident shock (see the second from right-hand panel of Fig. 8) is much smaller than the initial post-shock value. This reflects the
Table 2. Dependence of the global cloud and core properties on the level of turbulence and the density contrast of the cloud (see Table 1). In each case the cloud was hit by a Mach 10 shock. The time-dependent quantities are evaluated at $t = t_{\text{mix}}$ rather than $t = t_m$ (cf. Nakamura et al. 2006), since $a_{\text{cloud}}$ continues to rise in some simulations. Values in parentheses are obtained from integrations over the 'core' mass rather than the 'cloud' mass. Model names containing 'sh' were computed using a shallow density profile ($p_1 = 1$). Model c1rt32 was computed using an inviscid code and grid-scale post-shock turbulence.

| Model   | $t_{\text{drag}}/t_{\text{cc}}$ | $t_{\text{mix}}/t_{\text{cc}}$ | $t_{\text{in}}/t_{\text{cc}}$ | $a/r_c$ | $c/r_c$ | $c/a$ | $(\rho)_{\text{max}}/\rho_{\text{max}}$ | $(v_z)/v_b$ |
|---------|---------------------------------|---------------------------------|--------------------------------|--------|--------|------|---------------------------------|----------------|
| c1no128 | 1.03 (1.10)                     | (6.82)                          | 3.82                           | 1.74 (1.72) | 2.82 (2.92) | 1.62 (1.70) | 0.65 (0.87) | 0.634 (0.685) |
| c1hi128 | 1.04 (1.11)                     | (5.96)                          | 3.57                           | 1.66 (1.74) | 2.20 (2.40) | 1.33 (1.38) | 0.68 (1.03) | 0.579 (0.629) |
| c2no128 | 0.72 (0.86)                     | (4.37)                          | 3.52                           | 1.88 (1.90) | 1.36 (1.30) | 0.72 (0.68) | 0.56 (0.66) | 0.621 (0.651) |
| c2lo128 | 3.06 (3.10)                     | (5.05)                          | 4.53                           | 4.36 (4.21) | 5.07 (3.58) | 1.16 (0.85) | 0.073 (0.130) | 0.635 (0.606) |
| c2hi128 | 2.47 (2.65)                     | (5.73)                          | –                              | 4.05 (4.27) | 6.24 (1.66) | 1.54 (0.39) | 0.058 (0.078) | 0.606 (0.590) |
| c3no128 | 6.58 (9.48)                     | (8.59)                          | 10.23                          | 2.60 (1.22) | 75.9 (12.6) | 29.2 (10.3) | 0.0081 (0.0401) | 0.394 (0.189) |
| c3lo128 | 6.84 (7.15)                     | (7.83)                          | –                              | 4.01 (2.82) | 49.1 (6.88) | 12.2 (2.44) | 0.0078 (0.0248) | 0.430 (0.271) |
| c3hi128 | 4.58 (4.93)                     | (5.95)                          | 7.25                           | 6.84 (7.79) | 38.2 (5.57) | 5.58 (0.71) | 0.0064 (0.0122) | 0.584 (0.578) |
| c2no64  | 4.05 (5.51)                     | (7.92)                          | 8.60                           | 3.09 (2.75) | 11.7 (2.33) | 3.80 (0.85) | 0.104 (0.392) | 0.455 (0.402) |
| c2lo64  | 4.00 (4.33)                     | (7.37)                          | 9.50                           | 2.89 (2.21) | 10.8 (2.02) | 3.74 (0.91) | 0.103 (0.446) | 0.441 (0.377) |
| c2hi64  | 3.08 (3.85)                     | (4.87)                          | –                              | 2.49 (1.46) | 6.50 (0.98) | 2.61 (0.67) | 0.114 (0.476) | 0.379 (0.313) |
| c3no64  | 6.72 (9.84)                     | (7.86)                          | –                              | 5.16 (2.42) | 48.6 (8.64) | 9.41 (3.57) | 0.0124 (0.145) | 0.370 (0.216) |
| c3lo64  | 7.56 (14.44)                    | (8.99)                          | 5.16                           | 3.89 (1.05) | 64.2 (6.12) | 16.5 (5.86) | 0.0121 (0.085) | 0.302 (0.150) |
| c3hi64  | 6.55 (8.36)                     | (4.95)                          | 11.82                          | 2.63 (2.13) | 22.8 (2.94) | 8.67 (1.38) | 0.0241 (0.144) | 0.204 (0.155) |
| c1rt32  | 0.66 (0.72)                     | (5.84)                          | –                              | 2.21 (2.58) | 1.29 (0.90) | 0.58 (0.35) | 0.61 (0.80) | 0.614 (0.666) |

Figure 8. Snapshots of the turbulent energy per unit mass, $k$, from the $k$–$\epsilon$ calculations with $\chi = 10^3$ and $p_1 = 10$. The three left-most panels show the evolution with low initial post-shock turbulence (model c3lo128) proceeding left to right with $t = 0.1, 0.3$ and $0.87 t_{\text{cc}}$. The two right-most panels show the evolution with high initial post-shock turbulence (model c3hi128) proceeding left to right with $t = 0.1$ and $0.87 t_{\text{cc}}$. The white regions in the middle panel are artefacts of the plotting routine.

The fact that such high levels of post-shock turbulence are not naturally generated by a shock sweeping through a perfectly homogeneous medium.

The two right-most panels in Fig. 13 show the turbulent energy per unit mass, $k$, on a linear scale at $t = 1.81 t_{\text{cc}}$ for low $k$–$\epsilon$ calculations with $\chi = 10^3$ and different density profiles (models c3lo128 and c3losh64). This highlights the fact that the strongest turbulence is generated where the shear velocity is high, that the central region of the wake immediately behind the cloud has a somewhat lower level of turbulence, and that further downstream the turbulence has penetrated throughout the wake. The opening angle of the turbulent layer in model c3lo128 is estimated as $\approx 13^\circ$, which is in good agreement with experimental results (see Cantó & Raga 1991, and references therein), where an opening angle of $\approx 11^\circ$ is obtained for a Mach 1.3 flow past a stationary medium (the gas behind a Mach 10 shock has a Mach number of 1.31). Future work will examine whether this agreement with experiment persists as the Mach number is varied.

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4.3 Dependence on cloud density contrast

A range of density contrasts between the cloud and the ambient medium is expected. For instance, $\chi \sim 10^2$ for cold atomic clouds embedded in the warm neutral or photoionized medium where $T \sim 10^4$ K, or for warm clouds embedded in the coronal gas where $T \sim 10^6$ K. For molecular clouds embedded in warm gas, $\chi \sim 10^3$, while cold atomic clouds embedded in coronal gas have $\chi \sim 10^5$.

Fig. 9 shows the destruction of a cloud with a density contrast $\chi = 10$, while Fig. 10 shows the corresponding case for a cloud with $\chi = 10^2$, both computed with an inviscid code. The colour scaling in both figures is identical to that in previous figures for easier comparison. The time-scale for the cloud material to mix into the ambient flow scales roughly with the cloud crushing time-scale, $t_{cc}$, in agreement with previous works (Klein et al. 1994; Nakamura et al. 2006). However, the normalized growth time-scale for RT and KH instabilities decreases with increasing $\chi$, as is apparent from a comparison of Figs 4, 9 and 10. The normalized drag time, $t_{drag}/t_{cc}$, increases with $\chi$, as does the axial stretching of the cloud, $c/t_0$ (see Table 2). Figs 15–17 also reveal that the ratio of the velocity dispersion in the axial to normal directions, $\delta v_z/\delta v_r$, increases with $\chi$. These results are all in agreement with earlier works.

The effect of a highly turbulent post-shock flow is greatest at high $\chi$ (e.g. compare the values of the mean cloud and core velocities in Table 2 for the ‘high $k$–$\epsilon$’ models against the ‘low $k$–$\epsilon$’ and inviscid models as a function of $\chi$). This is because clouds with a high density contrast survive for a considerable time after the initial passage of the shock, and thus are subject to considerable ‘buffeting’ by the highly turbulent post-shock environment, whereas at lower values of $\chi$, the cloud is destroyed relatively quickly after the initial passage of the shock. This is also manifest in the increasing disparity with $\chi$ in the evolution of various global quantities from the ‘hi’ models on the one hand, and the ‘no’ and ‘lo’ models on the other hand, as shown in Figs 15–17.

4.4 Dependence on cloud profile

In Fig. 11, snapshots of the density distribution at different times are shown for an inviscid calculation of a Mach 10 shock hitting a cloud with a shallow density gradient (model c3nosh64; see Table 1). While the resolution is such that $r_c$ is equal to the width of 64 cells on the finest grid, the cloud in fact extends to $r \gtrsim 2 r_c$, so the effective resolution is similar to the previous models (see Fig. 1).

The interaction of a shock with a smooth cloud was previously studied by Nakamura et al. (2006) for the case where $\chi = 10$. In this case the cloud offered little impediment to the oncoming shock, with the result that the transmitted shock and the intercloud shock had similar mean speeds. As a result, the intercloud shock did not converge on the $z$-axis behind the cloud, and the shock compression from the downstream side was weak, leading to a slow lateral expansion of the cloud. In contrast, the cloud is a much more robust obstacle when $\chi = 10^3$, and we find that it maintains many aspects of the evolution seen in sharper-edged clouds (cf. Figs 4 and 11). Fig. 12 compares the density structure at $t = 2.77 t_{cc}$ for inviscid and low $k$–$\epsilon$ calculations (models c3nosh64 and c3losh64). At later times the material stripped from the cloud resembles a single tail-like structure, as was also the case for a cloud with sharper edges.

Nevertheless, the shallower density gradient does lead to a milder interaction. This is also manifest as a slower growth of turbulence around the cloud (cf. Figs 8 and 13 for $k$–$\epsilon$ models with low initial turbulence), with the turbulent wake not completely forming until $t \approx 2 t_{cc}$. An exact comparison of the respective time-scales is complicated by the fact that the cloud with the shallow density gradient is also larger and more massive, and so the true value of $t_{cc}$ will be...
Figure 10. Same as Fig. 9 but for a density contrast of $\chi = 10^2$ (model c2no128). The evolution proceeds left to right with $t = 0.0, 0.46, 0.91, 1.82$ and $3.65 \, t_{cc}$.

Figure 11. Snapshots of the density distribution from an inviscid calculation of a Mach 10 adiabatic shock hitting a cloud with a density contrast of $10^3$ with respect to the ambient medium and with a shallow density gradient, $p_1 = 1$ (model c3nosh64). The formal resolution is 64 cells per cloud radius, but due to the shallow profile the effective resolution is roughly 128 cells per cloud radius. The evolution proceeds left to right with $t = 0.08, 0.27, 0.46, 0.85$ and $1.81 \, t_{cc}$. Note the different spatial scale of the plots compared to Fig. 4.

different between the models. In fact, we find that the forward and rear shocks driven into the cloud converge at $t \approx 0.8 \, t_{cc}$ in model c3no128, and at $t \approx 1.0 \, t_{cc}$ in model c3nosh64 (we continue to calculate $t_{cc}$ using equation 2 with $\chi = 10^3$ and $r_c = 1$, despite this equation being applicable only to clouds with sharp edges). Since these times are not too discrepant, it becomes clear that the growth of turbulence around the smoother cloud is indeed slower, in agreement with the statement by Nakamura et al. (2006) that it takes more time to form the slip surface around the cloud. The maximum turbulent energy per unit mass (i.e. $k_{\text{max}}$) is also higher in model c3o128.
4.5 A non-uniform post-shock flow

To obtain further confirmation of the previous results, we have simulated the destruction of a cloud hit by a shock with non-uniform post-shock flow. To generate the necessary random motions of the post-shock flow, perturbations in the density and velocity are mapped on to the flow and allowed to evolve. These initial perturbations produce normalized standard deviations of 0.18, 0.26 and 0.13 in the density, pressure and velocity of the post-shock flow. The perturbations subsequently decay as the shock sweeps up the smooth intercloud ambient medium and because of the viscosity inherent in any numerical code, and the standard deviations of the fluctuations noted above decrease. We use a non-AMR set-up (i.e. a fine, rather than a coarse, $G^0$ grid), since the perturbations downstream of the shock would cause a large amount of grid refinement, thus erasing the benefits that are usually possible with AMR. These two issues (decay of the post-shock turbulence, and the inability to effectively use AMR) further highlight the benefits of using a subgrid turbulence model.

In Fig. 14 we show the initial interaction of a shock with grid-scale post-shock turbulence and a cloud with a density contrast, $\chi = 10$. The bow shock around the cloud is much less distinct, and the symmetry of the interaction present in the previous figures is broken. The shock shows small-scale curvature along its surface and is marginally faster than its counterpart with smooth post-shock flow (the effective Mach number of the shock is about 5 per cent higher, and reflects the extra ‘turbulent’ energy mapped into the initial post-shock flow). However, the most important difference is that the highly turbulent environment destroys the cloud much more rapidly than when the post-shock flow is smooth (cf. Figs 9 and 14, and also examine Fig. 15), confirming the results obtained with the $k-\epsilon$ subgrid turbulence model.
4.6 Cloud statistics

Figs 15–17 show the evolution of various global quantities of the cloud as a function of numerical resolution for the inviscid and $k-\epsilon$ models with $\chi = 10, 10^2$ and $10^3$. In addition, the results from a simulation with $\chi = 10^3$ and a shallow density profile are shown in Fig. 18. Various numerical quantities from these simulations are noted in Table 2. As already shown, the solutions that the $k-\epsilon$ simulations attain depend on the initial level of turbulence in the post-shock flow which overruns the cloud. For models with low initial turbulence, the evolution of the cloud is similar to that attained from the inviscid code, particularly at lower values of $\chi$. However, it is clear from the plots in Figs 15–18 that when the environment surrounding the cloud is turbulent itself, the mixing of cloud material into the surrounding flow proceeds at a much faster pace (as shown, for instance, by the more rapid growth in the cloud size (see panels a and c) and reduction in the core mass (see panel g) due to the enhanced transport and diffusion coefficients. It is also clear that the cloud with the shallow density gradient is typically less susceptible to high levels of environmental turbulence than clouds with sharper boundaries.

A detailed discussion of the statistics is presented in the following subsections. The results obtained from models with $\chi = 10^2$ are discussed initially in each subsection, and then a comparison is made to results obtained with lower and higher density contrasts and with a shallower density profile. Results from the inviscid model with grid-scale post-shock turbulence (model c1rt32) are also discussed where appropriate.

4.6.1 Cloud shape

Figs 16(a)–(f) show the time evolution of the rms cloud and core radii, $a$ and $c$, and their ratios, for simulations with $\chi = 10^2$. The transverse dimensions, $a_{\text{cloud}}$ and $a_{\text{core}}$, decrease during the shock compression stage, and then increase during the expansion stage. $a_{\text{cloud}}$ reaches a maximum of $\approx 4 \, a_c$ (at $t \approx 6 \, t_{cc}$) in all three models, and stays at roughly this level until at least $t = 12 \, t_{cc}$. Although $a_{\text{core}}$ reaches a similar maximum, its value subsequently drops precipitously once the mass within the core becomes a small fraction ($\lesssim 15$ per cent) of its initial value.

After the initial compression stage, which lasts until $t \approx t_{cc}$, the cloud becomes increasingly elongated in the direction of the propagation of the shock. The values of $c_{\text{cloud}}$ and $c_{\text{cloud}}/a_{\text{cloud}}$ are still growing at $t = 12 \, t_{cc}$, when the simulations were stopped. At this point the cloud material is dispersed over a distance of $\sim 15 \, a_c$ in the axial direction in all three simulations. However, Fig. 10 shows that even at earlier times the material stripped from the cloud is highly fragmented and does not resemble a single tail-like structure (in contrast, the stripped material better resembles a tail when $\chi = 10^3$ – see Fig. 7).

The axial radius of the core, $c_{\text{core}}$, displays the same initial behaviour as $c_{\text{cloud}}$, though its rate of expansion is less rapid. Thereafter $c_{\text{core}}$ behaves in a similar way to $a_{\text{core}}$: it reaches a maximum in all three simulations by $t \lesssim 8 \, t_{cc}$, and then declines as material which was formerly within the core is mixed into the surrounding flow to the extent that it can no longer be identified as ‘core’ material. The values of $a_{\text{core}}$ and $c_{\text{core}}$ can show abrupt changes as the cloud fragments, and are generally (though not always) smaller than the corresponding values of $a_{\text{cloud}}$ and $c_{\text{cloud}}$.

The core remains compressed in the axial direction (i.e. $c_{\text{core}}/a_{\text{core}} < 1$) in simulations c2no128 and c2lo128 until $t \approx 6 \, t_{cc}$, after which $c_{\text{core}}/a_{\text{core}}$ rapidly increases to values much larger than unity. In contrast, in model c2hi128, $c_{\text{core}}/a_{\text{core}} < 1$ at all times, and declines to a minimum value of 0.32. The core is always less elongated than the shocked cloud (i.e. has a smaller value of $c/a$) in the c2hi128 simulation, but this is true only for times...
prior to $\approx 6 t_{cc}$ and $\approx 8 t_{cc}$ in the c2lo128 and c2no128 simulations, respectively.

Clouds with $\chi = 10$ do not expand as much in the lateral direction, with the values of $a_{cloud}$ and $a_{core}$ remaining below $2 r_{cc}$ as the cloud is destroyed (Figs 15a and b). In contrast, $a_{cloud}$ continues to increase in the latter stages of the interaction in model c1rt32, due to the strong diffusion in this simulation. The enhanced diffusion from the model with grid-scale turbulence probably reflects the larger eddy sizes in this simulation, and reveals that the process of cloud destruction is sensitive to the properties of any turbulence in the surrounding medium.

$c_{cloud}$ increases to a value of 10 in model c1no128 at the time that the simulation is terminated ($t = 28 t_{cc}$), while $c_{cloud}/a_{cloud}$ asymptotes at $\approx 5.5$ at $t = 25 t_{cc}$. In model c1lo128, $c_{cloud}/a_{cloud}$ follows fairly closely the behaviour of model c1no128 until $t = 17 t_{cc}$, after which there is a dramatic plunge as mixing of the material reduces $\chi$ below the threshold for material to be identified as ‘cloud material’. In model c1hi128, $c_{cloud}/a_{cloud}$ reaches a much lower maximum of $1.5$ at $t = 12 t_{cc}$.

Clouds with $\chi = 10^3$ show different behaviour again (Figs 17a–e). In models c3no128 and c3lo128, $a_{cloud}$ and $a_{core}$ are comparable to the initial cloud radius until a considerable time into the evolution.

Figure 15. Time evolution of the cloud shape, mass, volume, mean density, mean axial velocity, velocity dispersion and circulation for inviscid and $k$–$\epsilon$ models with a density contrast $\chi = 10$, a steep density gradient ($\rho_1 = 10$) and 128 cells per cloud radius.
(t \lesssim 6 t_{cc}), a_{\text{cloud}} eventually reaches values of \approx 4 and 6 r_c in models c3lo128 and c3hi128, respectively. In contrast, the core and the more extended cloud grow much more rapidly with time in model c3hi128, due to the enhanced transport coefficients in this case, and the lateral extent of the cloud eventually exceeds 10 times the initial cloud radius. However, the faster dispersal of the ablated material into the cloud’s surroundings does not last, and eventually the growth rate of \rho_{\text{cloud}} and \rho_{\text{core}} slows below that from models c3no128 and c3lo128 as efficient mixing with the surrounding flow prevents the most dispersed material from being identified as material from the original cloud. For this reason, the value of \chi_{\text{cloud}} eventually falls below the values obtained in the inviscid and ‘low \kappa–\epsilon’ models for each of the cloud density contrasts considered (this change occurs at 4.5 \lesssim t/t_{cc} \lesssim 7). This process of mixing also limits the maximum value of \chi_{\text{core}} obtained in the simulations.

Figs 18(a)–(e) show the evolution of the cloud shape for a much shallower initial density profile of the cloud but with the same \chi = 10^3 as for Fig. 17. The behaviour of the cloud shape is

Figure 16. As Fig. 15 but for \chi = 10^2.
The turbulent destruction of clouds

Figure 17. As Fig. 15 but for $\chi = 10^3$.

generally fairly similar to the models of the sharper-edged cloud (see Fig. 17).

Figs 15–18 and Table 2 show that the values of $c_{\text{cloud}}/\alpha_{\text{cloud}}$ and $c_{\text{core}}/\alpha_{\text{core}}$ generally increase with $\chi$ and decrease with increasing levels of turbulence in the model. Since clouds with a high-density contrast survive a long time, $c_{\text{cloud}}/\alpha_{\text{cloud}}$ can exceed 20 when $\chi = 10^3$.

Figs 15(a), 16(a) and 17(a) also show the predicted lateral expansion of the cloud determined from equation (6.1) of Klein et al. (1994). This equation is only appropriate for $t \lesssim t_m$ (i.e. until the lateral expansion of the cloud reaches a maximum). A mismatch at early times is also expected, since this expression does not properly account for the initial compression stage. With these caveats in mind, the predicted rate of expansion appears to be slightly too fast compared to the model results with $\chi = 10$ and $10^3$, but slower than is obtained from the models with $\chi = 10^2$. The most striking observation is that the lateral expansion of the cloud in model c3h128 is very much faster than the theoretical prediction over the period...
4 \lesssim t/t_{cc} \lesssim 8$, which reveals that the highly turbulent surroundings diffuse the cloud material at a speed faster than the shocked cloud’s internal sound speed.

### 4.6.2 Cloud mass

Panel (g) in Figs 15–18 shows the time evolution of the core mass, which reaches half its initial value at $t = t_{\text{mix}}$. The $k$-$\epsilon$ models with low initial turbulence show similar evolution to the inviscid models. In contrast, the models with high initial turbulence show a much faster decline in the core mass. We again note that the results from the subgrid turbulence models are much less sensitive to spatial resolution than those from inviscid models (Fig. 2). This is also revealed in the mixing time, $t_{\text{mix}}$, which, for instance, is 8.19, 8.50 and 7.93 $t_{cc}$ for models c3lo32, c3lo64 and c3lo128, but is 6.81, 11.47 and 8.69 $t_{cc}$ for models c3no32, c3no64 and c3no128 (i.e. the latter three models show much greater spread in $t_{\text{mix}}$).

Fig. 19 shows how the cloud material mixes with the surrounding gas in models c3no128, c3lo128 and c3hi128. The histograms indicate the fraction of cloud mass over a range of density bins of
width 0.1 $\rho_{\text{max}}$. For $p_1 = 10$, about 80 per cent of the cloud mass has a density $\approx 0.1 \rho_{\text{max}}$ at the start of each simulation. At $t = 0.38 \, t_{\odot}$, the shock driven into the cloud has increased the density of the gas it has encountered by a factor of $\sim 4$, while a substantial part of the cloud remains unaffected and at its initial density. Eventually the transmitted shock sweeps through the entire cloud, and densities more than 10 times higher than the initial core density occur as the transmitted shock interacts with the shock driven into the back of the cloud. At $t \approx t_{\odot}$, the mass distribution function is approximately flat over a wide range in density. As the shocked cloud expands the density decreases. The turbulent mixing of cloud material into the surrounding flow causes the density of the cloud material to eventually approach the shocked intercloud density ($\rho \approx 4 \times 10^{-3} \rho_{\text{max}}$). This evolution proceeds slightly faster in model c3hi128, as previously noted.

The double-peaked structure of the mass spectrum at $t = 1.83 \, t_{\odot}$ in models c3no128 and c3lo128 is consistent with previous findings (Nakamura et al. 2006), with the higher density peak at $\rho \approx 2.5 \rho_{\text{max}}$ representing the main core, and the lower density peak at $\rho \approx 0.7 \rho_{\text{max}}$ representing material stripped from it and mixed with surrounding material. While there is little difference between the evolution of the mass spectrum in models c3no128 and c3lo128, in contrast there is no sign of a double-peaked mass spectrum in model c3hi128. This is further evidence that the destruction and mixing of interstellar clouds depends on the level of turbulence present.

The rate of mass loss from the core in models c2no128, c2lo128 and c2hi128 can be compared to the analytical formula for hydrodynamic ablation given by Hartquist et al. (1986). To make such a comparison our simulations must be appropriately scaled. We therefore assume that the cloud is ionized, has a radius $r_c = 2 \, r_{\odot}$, a core density $\rho_c = 4 \times 10^{-25}$ g cm$^{-3}$ and a temperature of 8000 K, and is in pressure equilibrium with surroundings of density $4 \times 10^{-22}$ g cm$^{-3}$ and temperature $8 \times 10^4$ K. Both the cloud and its surroundings are assumed to have solar abundances. The Mach 10 shock then travels at a speed of $1360 \, \text{km s}^{-1}$ through the ambient medium, heating the medium to $T = 2.6 \times 10^7$ K and giving it a velocity and Mach number of $\approx 1000 \, \text{km s}^{-1}$ and 1.3, respectively. The rate of mass loss from a cloud of mass $M_0$ through hydrodynamic ablation is $M_{\text{ab}} = M_0 / t \approx F \rho v_{\text{exp}}$, where $t$ is a characteristic destruction/mixing time-scale, $\rho_0$ is the characteristic density of ablated material within a mixing region of size $F$ around the cloud and $v_{\text{exp}}$ is the expansion speed of material off the surface of the cloud. Momentum conservation requires that $\rho v_{\text{exp}} \approx \rho_0 v$ where $\rho$ and $v$ are the density and velocity of the surrounding flow. If the surrounding flow is supersonic, $v_{\text{exp}} \approx c_v$, since material cannot leave the cloud much faster than the sound speed of the cloud, $c_v$. Hence $M_{\text{ab}} \approx (M_0 c_v) / (\rho_0 v)^{1/3}$. With the above parameters, $M_{\text{ab}} \approx 1.2 \times 10^{-6} \, M_\odot \, \text{yr}^{-1}$, and the cloud survives for approximately $1.6 \times 10^5$ yr. In comparison the cloud crushing time-scale, $t_{\text{cc}} = 1.4 \times 10^4$ yr, so that the cloud survives for about 10 cloud crushing time-scales before being destroyed.

Fig. 20(a) compares $M_{\text{ab}}$ with the numerically determined mass-loss rates, where it is apparent that there is good basic agreement between the analytical and numerical mass-loss rates. Not surprisingly, the mass-loss rates from the numerical models are time dependent: they decrease as the cloud is compressed by the shocks driven into it, and then increase significantly as the cloud re-expands. The higher mass-loss rates compared to $M_{\text{ab}}$ during the latter period reflect the increased surface area of the cloud which is not taken into account in the analytical theory. The mass-loss rate in model c2hi128 is higher than in models c2no128 and c2lo128 for the first $t \approx 55 \, 000$ yr of the interaction – this is due to the increased transport coefficients which cause the stripping of material at a faster rate. During this time, the mass-loss rate from model c2hi128 is also remarkably constant. The maximum mass-loss rate in all three models is $\approx 6 \times 10^{-6} \, M_\odot \, \text{yr}^{-1}$, or five times the rate predicted from the formula in Hartquist et al. (1986). In model c2hi128 this occurs at the end of the cloud’s life. In contrast, the peak rate of mass-loss occurs at about 70 000 yr ($t \approx 5 \, t_{\odot}$), in models c2no128 and c2lo128, which is similar to the time at which half of the core mass has been mixed (see Table 2). In models c2lo128 and (especially) c2no128, the cloud enjoys a more sedate ending of its life, with the mass-loss rate declining as the core mass decreases. The time at which half of
the core is mixed (\(t_{\text{mix}}\)) and the destruction time of the core (when \(m_{\text{core}} = 0\)) are more widely separated in these simulations.

Fig. 20(b) shows an identical analysis for a cloud with a smoother density profile (models c2nosh64, c2losh64 and c2hish64). The mass of the cloud is 2.9 times greater, and the predicted mass-loss rate is twice as high. The cloud is therefore expected to survive for 40 per cent longer (i.e. 230 000 yr). The initial rate of mass loss from the simulations is approximately three times as high as the predicted value, which reflects the ease at which the tenuous outer layers of the cloud are removed. The variations in the rate of mass loss also appear to be reduced, again consistent with a milder interaction. Otherwise, similar behaviour to the previous case of a sharp-edged cloud is seen, though the cloud survives appreciably longer than predicted in models c2nosh64 and c2losh64.

To conclude this section, we note that the cloud in model c2hi128 is destroyed in about 65 per cent of the predicted time, while in models c2no128 and c2lo128 the destruction time is in good agreement with the analytical prediction. Clouds with a smoother density profile survive for about 30 per cent longer than the predicted time, though if they are overrun by a highly turbulent environment they may survive only 60 per cent of the predicted time. A similar analysis for the simulations with \(\chi = 10^3\) reveals that in model c3hi128 the cloud is destroyed more than twice as fast as predicted by the analytical theory appropriate for a smooth post-shock flow. In model c3lo128 it is destroyed in about 75 per cent of the predicted time, while in model c3no128 the destruction time is in good agreement with the analytical prediction. The Mach number dependence of the mass-loss rate is examined in a subsequent paper (Pittard, in preparation).

4.6.3 Cloud volume and mean density

In each simulation the volume of the cloud core increases after the initial compression stage to a maximum value, then decreases as the core material is gradually ablated and mixed into the surrounding flow. The time of this reversal is typically just prior to \(t_{\text{mix}}\). The strong turbulence present in models c2hi128 and c3hi128 vigorously rips material off the surface of the cloud and advects it downstream, so that the \(k = 0.5\) isosurface is greatly extended. This leads to a rapid increase in the volume of the ‘core’, \(v_{\text{core}}\), and a commensurate decrease in the core density, \(\rho_{\text{core}}\) (see Figs 16h and j and 17h and j). In contrast, the destruction of the cloud when \(\chi = 10\) is so rapid that there is not enough time for the highly turbulent surroundings to produce much of an effect (see Figs 15h and j), whereas a smooth density profile tends to delay the time at which the effects from highly turbulent surroundings become obvious (see Figs 18h and j).

4.6.4 Cloud velocity

The acceleration of the cloud occurs in two stages. The cloud is first accelerated to a velocity \(v_{\text{c}}\) by the shock driven into it. The flow of shocked intercloud gas then further accelerates the cloud until they have the same velocity (i.e. 0.725 \(v_{\text{c}}\) when \(M = 10\)). Since \(v_{\text{c}}/v_{\text{b}} = 1/\chi^{1/2}\), the second stage dominates when \(\chi \gtrsim 7.5\). If the cross-section of the cloud were to remain constant, the characteristic drag time for sharp-edged clouds would be \(t_{\text{drag,0}} \sim \chi^{1/2} t_{\text{cc}}\) (Klein et al. 1994). However, since the cloud expands laterally, the actual drag time is considerably shorter (Klein et al. 1994, see also Table 2).

Fig. 16(k) shows that the acceleration of the cloud is virtually identical in models c2no128 and c2lo128 for \(t \lesssim 4 t_{\text{cc}}\). This is also the case for the core acceleration (Fig. 16l). In contrast, the stronger turbulent mixing in model c2hi128 increases the momentum coupling of the surface of the cloud with the ambient flow so that the cloud accelerates more quickly to the post-shock intercloud speed. This effect is even more pronounced in model c3hi128 (see Fig. 17k). The acceleration of clouds with smooth density profiles is more gentle (cf. Figs 17k and 18k), as expected. In all cases the cloud accelerates faster than the core (i.e. \(\langle v_{z,\text{cloud}} \rangle > \langle v_{z,\text{core}} \rangle\)), as expected.

Klein et al. (1994) and Nakamura et al. (2006) presented a semi-analytic theory for the acceleration of sharp-edged and smooth-edged clouds, respectively. The formulation from Klein et al. (1994) converted to the rest frame of the initial cloud is shown in Figs 16(k) and 17(k), where the value of \(t_{\text{cc}}\) obtained from models c2no128 and c3no128 is used. In each case, the results obtained with three different values for the drag coefficient are shown: \(C_D = 0.2, 0.5 \text{ and } 1.0\). Klein et al. (1994) and Nakamura et al. (2006) adopt \(C_D = 1.0\), but it is not clear that this is the best choice. For solid spheres, experiments have determined that \(C_D = 1.0\) occurs when \(Re \approx 100\) (Landau & Lifshitz 1959), which is a far lower Reynolds number than occurs in the astrophysical settings that we are interested in (see Section 2.2). However, interstellar clouds are compressible and are ablated in shock–cloud...
interactions, so one would not necessarily expect the drag coefficient to be similar to the solid sphere case. Nevertheless, clouds with a high-density contrast should, at least initially, behave somewhat like solid spheres. If this is the case, more appropriate values for \( C_D \) would be \( \approx 0.5 \) when \( 2 \times 10^4 < Re < 2 \times 10^7 \), and \( \approx 0.2 \) for \( Re > 2 \times 10^9 \).

In fact, we find that there is poor agreement between the theoretical acceleration and the results from model c2no128 for values of \( C_D \lesssim 1 \) (Fig. 16k). The closest match is obtained with \( C_D \approx 1.0 \), though the theoretically estimated acceleration is too high for \( t \lesssim 4 t_{cc} \), and too low at later times. This discrepancy is the opposite of that noted in Klein et al. (1994). The poor match is at least in part due to the poor agreement between the predicted and model evolution of \( a_{\text{cloud}} \) (Fig. 16a). Since \( C_D \approx 0.2–0.5 \) for high Reynolds number flow over a solid sphere, we conclude that clouds with \( \chi = 10^3 \) are a poor approximation to a hard sphere.

In contrast, Fig. 17(k) shows that the theoretical acceleration of the cloud agrees much better with the numerical results from models c3no128 and c3lo128 where \( \chi = 10^5 \). The best agreement is obtained with \( C_D \approx 0.5 \), which is consistent with the assumption of fully developed turbulence in the \( k-e \) turbulence model. We therefore conclude that clouds with \( \chi = 10^3 \) are a better approximation to a hard sphere.

4.6.5 Cloud velocity dispersion

The interaction of shocks with clouds produces substantial vorticity and velocity dispersion, which may be a key mechanism for generating turbulent motions in the ISM (e.g. Kornreich & Scalo 2000; Mac Low & Klessen 2004). Although the subgrid turbulence model deals with turbulent motions and mixing on scales smaller than the cell size of the numerical grid, larger scale turbulent motions can be directly measured from the velocity dispersions in the axial and radial directions, \( \delta v_z \) and \( \delta v_r \), respectively. The time evolution of these quantities is shown in panels (m) and (n) of Figs 15–18.

Our results for model c2no128 can be compared to figs 9 and 10 in Klein et al. (1994). The level of qualitative and quantitative agreement is basically good. In particular, we confirm that the radial velocity dispersion is generally somewhat less than the axial velocity dispersion. However, we find that the maximum values of \( \delta v_z \) and \( \delta v_r \) occur somewhat later in our simulations. Comparing models c2no128, c2lo128 and c2hi128, we see that \( \delta v_z \) is more sensitive to a higher initial level of turbulence in the environment around the cloud than \( \delta v_r \).

A comparison with the \( \chi = 10 \) and \( 10^3 \) simulations reveals that \( \delta v_z / \delta v_r \) increases with \( \chi \), as is also apparent in the work of Klein et al. (1994). \( \delta v_r \) peaks at about the same value in c2hi128 and c3hi128, indicating that the random instability-induced grid-scale motions which develop are limited by the high viscosity introduced by the subgrid turbulence model in these simulations. \( \delta v_z \) peaks at higher values in models c3no128 and c3lo128, compared to models c2no128 and c2lo128, due to greater growth of RT and KH instabilities resulting from the longer drag and mixing time in the former models. The velocity dispersion appears to be slightly reduced for clouds with smooth boundaries, with some sign that the time evolution of \( \delta v_z \) attains a broader maximum. In all simulations, \( \delta v_z \) peaks slightly prior to the time of the maximum radial extent of the cloud, \( t_{\text{max}} \).

Fig. 21 shows the mass distribution function in velocity space within the entire cloud integrated along the \( z \)-axis for models with \( \chi = 10^3 \). The histograms indicate the mass contained within a corresponding velocity bin with a width of \( 0.01 u_{\text{esc}} \), where \( u_{\text{esc}} \) is the post-shock ambient velocity. The shock initially driven into the cloud has a speed \( v_b \sqrt{T} = 0.042 u_{\text{esc}} \). By \( t = 3.74 t_{cc} \), the majority of the mass within the cloud in models c3no128 and c3lo128 has been further accelerated by additional shocks and momentum transfer from the surrounding flow to roughly twice this speed. In contrast, the material stripped off the surface of the cloud moves at speeds up to \( u_{\text{esc}} \), with a small fraction exceeding \( u_{\text{esc}} \) as a result of turbulent motions. Further stripping and momentum transfer results in a gradual acceleration of the majority of the cloud material to speeds near \( u_{\text{esc}} \) as time progresses. Not unexpectedly, the acceleration of cloud material in model c3lo128 is more rapid than in model c3no128, and even more so in model c3hi128, where even the slowest moving cloud material has a speed in excess of \( 0.6 u_{\text{esc}} \) at \( t \approx 12 t_{cc} \).

4.6.6 Cloud vorticity

A key feature of the interaction of a shock with a cloud is the development of powerful vortex rings. Vorticity \( \omega = \nabla \times \mathbf{u} \) can be produced (or destroyed) when pressure and density gradients are not aligned (i.e. by a curl in the acceleration), and by viscosity. The corresponding circulation, \( \Gamma = \int \omega \cdot d\mathbf{A} \). Klein et al. (1994) showed that the vorticity production can be classified into four components. Vorticity at the interface between the cloud and the surrounding flow is produced by the initial passage of the shock (\( \Gamma_{\text{shock}} \)) and the subsequent post-shock flow (\( \Gamma_{\text{post}} \)). The third component is connected with the triple points associated with the Mach-reflected shocks behind the cloud (\( \Gamma_{\text{ring}} \)), while the fourth component is the vorticity produced in the cloud (\( \Gamma_{\text{ics}} \)) and is smaller than the other components by a factor \( \sim \chi^{-1/2} \). Nakamura et al. (2006) showed that the vorticity produced by a shock overrunning a cloud with smooth boundaries is qualitatively similar to that produced from clouds with sharp boundaries. Klein et al. (1994) constructed analytical expressions for three of the above components which we reproduce below:

\[
\Gamma_{\text{shock}} \approx -\frac{9}{4} (1 - \chi^{-1/2}) v_b r_c, \tag{26}
\]

\[
\Gamma_{\text{post}} \approx \frac{9}{64} \left( \frac{\chi^{1/2} t_{\text{drag}}}{t_{cc}} \right) v_b r_c, \tag{27}
\]

\[
\Gamma_{\text{ring}} = \frac{3}{4} v_b r_c. \tag{28}
\]

\( \Gamma_{\text{post}} \) is the dominant contribution to the circulation when \( \chi \) is large. Klein et al. (1994) did not attempt to model \( \Gamma_{\text{cloud}} \), since this component is small. For clouds with smooth boundaries, \( \Gamma_{\text{post}} \) must be further multiplied by \( (1 - \chi^{-1/2})^2 \) (Nakamura et al. 2006).

The time evolution of the circulation from our numerical models is shown in panel (o) of Figs 15–18. Table 3 lists values estimated for \( \Gamma_{\text{post}} \) and the total circulation, \( \Gamma_{\text{ics}} = \Gamma_{\text{shock}} + \Gamma_{\text{post}} + \Gamma_{\text{ring}} \). In each case we use the values for \( t_{\text{drag}} \) in Table 2. Obviously, the circulation computed using the cloud and core drag times is similar when \( t_{\text{drag}} \) for the cloud and the core are also similar (see e.g. models of sharp-edged clouds with \( \chi = 10 \) and 100). However, when \( t_{\text{drag}} \) for the cloud and the core differ (as occurs for models with a shallow cloud density profile), we find that the numerically determined circulation is in much better agreement with theoretical estimates if the drag time for the core is used. Figs 15–18 indeed show that the mean velocity of the cloud and core as a function of time become increasingly disparate with increasing \( \chi \). Clearly
Figure 21. Mass distribution as a function of \( v_z \) for models c3no128 (panels a–c), c3lo128 (panels d–f) and c3hi128 (panels g–i) at \( t = 3.74, 7.58 \) and 11.9 \( t_{cc} \). The histograms denote the mass contained within a velocity bin of width 0.01 \( u_{cc} \) for a line of sight parallel to the \( z \)-axis. The integrated mass is the mass of the cloud, \( m_c \).

Table 3. Theoretical estimates for the total circulation and the component produced by the post-shock flow. In each case the calculated value uses the drag time for the cloud (core), as given in Table 2. \( -\Gamma_{\text{shock}} = 1.54, 2.03 \) and 2.18 when \( \chi = 10, 10^2 \) and \( 10^3 \), respectively, while the value of \( \Gamma_{\text{ring}} \) is always 0.75.

| Model    | \( -\Gamma_{\text{post}} \) | \( -\Gamma_{\text{tot}} \) |
|----------|-----------------------------|-----------------------------|
| c1no128  | 0.45 (0.49)                 | 1.24 (1.28)                 |
| c1lo128  | 0.47 (0.49)                 | 1.26 (1.28)                 |
| c1hi128  | 0.32 (0.38)                 | 1.11 (1.17)                 |
| c2no128  | 4.30 (4.35)                 | 5.58 (5.63)                 |
| c2lo128  | 4.33 (4.35)                 | 5.61 (5.63)                 |
| c2hi128  | 3.47 (3.73)                 | 4.75 (5.01)                 |
| c3no128  | 29.2 (42.1)                 | 30.6 (43.5)                 |
| c3hi128  | 30.4 (31.8)                 | 31.8 (33.2)                 |
| c3no128  | 20.4 (21.8)                 | 21.8 (23.2)                 |
| c2no128  | 4.61 (6.28)                 | 5.89 (7.56)                 |
| c2lo128  | 4.56 (4.93)                 | 5.84 (6.21)                 |
| c2hi128  | 3.51 (4.39)                 | 3.08 (3.85)                 |
| c3no128  | 28.0 (41.0)                 | 29.4 (42.4)                 |
| c3hi128  | 31.5 (60.2)                 | 32.9 (61.6)                 |
| c3hi128  | 27.3 (34.9)                 | 28.7 (36.3)                 |

it is the motion of the core with respect to the post-shock ambient medium which dominates the generation of circulation – after all, this is where the highest velocity shear occurs.

Table 3 reveals that the model results and the theoretical estimates are generally in good agreement. The contribution of the three main components is best seen in Fig. 15(o). The initial rise to maximum is caused by the initial passage of the shock (\( \Gamma_{\text{shock}} \)), while the subsequent drop is caused by the formation of the supersonic vortex ring behind the cloud, \( \Gamma_{\text{ring}} \). The increase in circulation after this minimum (until \( t \approx 2 t_{cc} \)) reveals the vorticity production due to the post-shock flow over the cloud.

The total circulation in the models is often in excellent agreement with the predictions (for instance, the peak circulation in models c3lo128, c3hi128, c3losh64 and c3lohsh64 are 33.8, 23.3, 65.2 and 40.0, while the theoretical predictions are 33.2, 23.2, 61.6 and 36.3, respectively). In some models (e.g. c2no128, c2lo128, c3no128 and c3nosh64) the circulation is somewhat higher than predicted.

Spurious vorticity can be generated at the boundary of grids of different refinement levels (Plewa & Müller 2001), but this does not seem to be the case here (since, for instance, one would expect a larger discrepancy for model c3lo128 than for model c2lo128).

The total circulation in models with \( \chi = 10^3 \) tends to reach a lower maximum when there is high post-shock turbulence, due to the more rapid destruction of the cloud in such circumstances. Models where the cloud has a shallow density profile generate more circulation, principally because of the larger cloud mass and the longer drag times in these models. The decay in \( \Gamma_{\text{tot}} \) at later times in some models may reflect the inherent viscosity diffusing vorticity as turbulence is dissipated into heat.

4.6.7 Energy fractions

As the cloud material accelerates it gains kinetic energy, while its thermal energy increases due to shock heating, adiabatic
The turbulent destruction of clouds

5 DISCUSSION

The interaction of shocks, winds and jets with clouds, and the collapse and/or destruction of the cloud, is of fundamental importance to studies on star formation, the ISM, feedback and galaxy evolution and the evolution of diffuse astrophysical sources, such as planetary nebulae, wind-blown-bubbles, supernova remnants (SNRs), H II regions, galactic winds and the intracluster medium.

This work has examined the interaction of a shock with a cloud. However, clouds with a high-density contrast easily survive the initial passage of the shock, and then find themselves immersed in a subsonic or mildly supersonic post-shock flow which to all intents and purposes resembles a wind of the same Mach number. Hence the simulations presented in this work are also relevant to scenarios where clouds are ablated by a wind. In a future paper we will examine in detail the differences between shock–cloud and wind–cloud interactions.

In the following subsections we discuss shock–cloud interactions in SNRs, and the tails formed behind clouds in various types of wind–cloud interactions. Jet–cloud interactions, though not discussed in this work, can be seen in Herbig–Haro objects and active compression and heat transport from turbulent mixing with the hotter surrounding flow. There is also likely to be some numerical transport of heat. The turbulent energy increases as shear motions generate a turbulent cascade. Eventually, the cloud material should acquire the same kinetic and thermal energy density as the ambient medium, and the turbulent energy should dissipate as heat. For a strong adiabatic shock, the ratio of kinetic to thermal energy in the post-shock flow, $E_k/E_{th} = \gamma M_p^2/3$, where $M_p$ is the post-shock Mach number. In our simulations $M_p = 1.32$, so at late times we expect that $E_k/E_{th} \approx 1$.

Fig. 22 shows the time evolution of these energies and the fraction of the total energy in small-scale (subgrid) turbulent motions for simulations with $\chi = 10, 10^2$ and $10^3$. In models c3no128 and c3lo128, the thermal energy of the cloud generally exceeds its kinetic energy at any particular instant in time, while in model c3hi128 the kinetic energy exceeds the thermal energy during the period $\sim t/t_{cc} \lesssim 11$. In model c3hish64, the kinetic energy always exceeds the thermal energy. It is clear that the cloud material has still not fully mixed into the surrounding flow at the end of most of the simulations, since the thermal energy often significantly exceeds the kinetic energy at this point (obvious exceptions are models c2hi128, c3hi128 and c3hish64).

The fraction of the cloud energy in small-scale subgrid turbulent motions increases with $\chi$ for the ‘low–$k$–$\epsilon$’ models, with maxima of 0.0062, 0.017 and 0.029 for models c1lo128, c2lo128 and c3lo128, respectively. When the environment is highly turbulent, the turbulent energy fraction is 0.079, 0.107 and 0.157 for models c1hi128, c2hi128 and c3hi128, respectively. The latter value is similar to the fractional energy in turbulence in the post-shock flow prior to its impact with the cloud, and demonstrates how high levels of upstream turbulence may be maintained through sequential shock–cloud interactions. Clouds with smoother density profiles produce a slightly smaller peak fractional energy in turbulent motions (0.020 and 0.140 for models c3losh64 and c3hish64, respectively).
galactic nuclei (AGNs). In some sources, clouds are more commonly referred to as bullets, clumps, globules or knots.

5.1 SNR–cloud interactions

The study of shock–cloud interactions has typically focused on SNRs. In most cases the shocks driven into the clouds are radiative, so such interactions are not directly comparable to the adiabatic simulations presented in this paper. Nevertheless, in recent years there has been much progress in unravelling the nature of these interactions and it is worth discussing some of their interesting features.

An interesting observation is that some clouds show clear signs of a bow shock (for instance, the Cygnus Loop’s south-east cloud, Graham et al. 1995, and also clouds on the western limb, Levenson, Graham & Walters 2002), while others do not (e.g. the A cloud in the Cygnus Loop, Danforth, Blair & Raymond 2001, the south-western cloud of the Cygnus Loop, Patnaude et al. 2002, and Vela FilD, Miceli et al. 2006). Possible explanations for the lack of a bow shock are the earliness of the interaction (Klein et al. 1994), a smooth cloud density profile (Nakamura et al. 2006), or a high ellipticity of the cloud (Miceli et al. 2006). Thermal conduction can also reduce the visibility of bow shocks (Orlando et al. 2005; though Marcolini et al. 2005 show that in the case of a wind–cloud interaction it can enhance its visibility), while cosmic rays can smooth shocks out (e.g. Wagner et al. 2006). In addition to these mechanisms, we note that a highly turbulent post-shock flow may also hinder the formation of a clearly defined bow shock (see e.g. Fig. 14). Variations in pre-shock density due to interstellar turbulence have been estimated to be ~20 per cent (Raymond et al. 2007). For SNRs, post-shock turbulence could be generated by pre-shock interstellar turbulence, or, at least in young SNRs like SN 1006, by clumps of ejecta and/or long RT fingers affecting the contact discontinuity and blast shock. Particle acceleration also produces turbulent motions (see e.g. Jones & Ellison 1991, and references therein). Finally, we note that although thermal conduction has been invoked as necessary to explain the hot corona surrounding the Vela FilD cloud (Miceli et al. 2006), it is possible that the turbulent transport of heat could instead account for this.

It is also interesting to note that all of the numerical simulations of shock–cloud interactions have assumed that the interaction takes place in the so-called ‘small-cloud’ limit, where the size of the cloud is sufficiently small that the properties of the post-shock gas do not change significantly in the time that it takes for the cloud to be crushed or destroyed. This is unlikely to be the case for denser clouds. In addition, the interaction of a superbubble with a cloud is better described as a shell–cloud interaction, and will be the subject of a forthcoming work.

5.2 Wind–cloud interactions: tails

Material stripped off clouds by the passage of a shock or fast wind is frequently manifest as a long tail behind the cloud (see e.g. Fig. 7). Although this work does not specifically focus on tails, the tails themselves are interesting as their properties may allow a diagnosis of the flow past the cloud (e.g. Dyson, Hartquist & Biro 1993; Dyson et al. 2006). Examples of resolved tails include those in NGC 7293 (the Helix nebula; O’Dell, Henney & Ferland 2005; Hora et al. 2006; Matsuura et al. 2007), the complex substructure of knots and wakes seen in the Orion Molecular Cloud OMC1 (e.g. Allen & Burton 1993; Schultz et al. 1999; Tedds, Brand & Burton 1999; Kaifu et al. 2000; Lee & Burton 2000) and the radial protrusions from the bipolar nebula surrounding the supermassive star η Carinae (Weis, Duschl & Chu 1999; Currie et al. 2000; Redman, Meaburn & Holloway 2002). Comet-like tails are also seen extending from the Galactic Centre source IRS 7, a red supergiant (Serabyn, Lacy & Achtermann 1991; Yusef-Zadeh & Morris 1991), and from behind Mira, an asymptotic giant branch star (Martin et al. 2007).

Tail-like structures are also seen in galactic winds (Cecil et al. 2001; Ohyama et al. 2002), and resemble similar features seen in simulations (e.g. Strickland & Stevens 2000; Cooper et al. 2008). Observations and simulations of galactic winds suggest that the clumps at the heads of the filaments are material which is ripped out of the galactic disc as the galactic wind develops. Beautiful filamentary structures are also seen in some galaxy clusters, of which the most prominent example is in the Perseus cluster (Conselice, Gallagher & Wyse 2001). Numerical simulations indicate that a high-velocity wind is necessary for these to form, otherwise material stripped from the cloud sinks under gravity almost as quickly as the cloud itself (Pope et al. 2008). Tails in the intracluster medium may help to regulate the heating of the central regions of galaxy clusters by dissipating some of the energy injected by the central AGN, and therefore better couple the AGN to the intracluster medium.

The Reynolds number is likely to be high enough in all of the above cases that the tail is fully turbulent. Indeed, the emission in Mira’s tail can be explained if molecular hydrogen is excited by the turbulent mixing of cool molecular gas and shock-heated gas (Martin et al. 2007).

6 SUMMARY AND CONCLUSIONS

This is the first in a series of papers investigating the turbulent destruction of clouds. Here we have investigated the destruction of a cloud by an adiabatic shock using a hydrodynamical code which incorporates a subgrid $k$–$\epsilon$ turbulence model, in which an attempt is made to calculate the properties of the turbulence and the resulting increase in the transport coefficients. The results are compared against those from a hydrodynamical code which solves only the inviscid Euler equations of fluid motion. The motivation for this study is that fully developed turbulence is prevented by the artificial viscosity in all numerical codes, though it is expected in astrophysical environments where the Reynolds number of the shock–cloud interaction is $\gtrsim 10^5$. The effect of a highly turbulent environment sweeping over the cloud is also investigated – all other works studying the interaction of a shock with a single cloud have assumed that the post-shock environment is completely smooth, though this is clearly not the case when there are upstream inhomogeneities. Our main results are summarized below.

(i) The evolution and destruction of clouds with inviscid and $k$–$\epsilon$ models occurs at roughly the same speed when the post-shock flow is smooth. This is because the turbulence takes some time to be generated and for its effects to be subsequently felt. However, it is clear that there are increasing differences between such models as the density contrast, $\chi$, between the cloud and intercloud medium increases. We also show that the $k$–$\epsilon$ model results are far less dependent on the spatial resolution than calculations with an inviscid code. This behaviour is attractive given the recent interest in simulations with multiple clouds.

(ii) Turbulence is mainly generated at the slip surface around the cloud, though a small amount is also generated behind shocks. The set-up time to form a turbulent wake behind the cloud is $\approx t_{\text{cc}}$. There is slower growth of turbulence around clouds with a smooth density profile. The opening angle of the turbulent mixing layer...
is consistent with experimental results. The fraction of energy in small-scale subgrid turbulent motions initially increases with time as turbulence continues to be generated by the velocity shear of flow past the cloud, but then decays as turbulent energy subsequently dissipates as heat. The peak value of the turbulent energy fraction increases with $\chi$ due to the longer drag time of denser clouds and the higher velocity shear at the cloud surface, and decreases as the boundary of the cloud becomes smoother. The interaction of shocks with smooth clouds is generally milder, as previously reported.

(iii) Clouds which are subject to a highly turbulent post-shock environment are destroyed significantly quicker than those within a smooth flow, due to the enhanced transport and diffusion coefficients. This effect increases with $\chi$, since high-density clouds are subject to a longer period of ‘buffeting’. Clouds with small density contrasts (e.g. $\chi = 10$) are destroyed so quickly after the initial shock passage that they experience very little ‘buffeting’. Strong environmental turbulence increases the momentum coupling between the cloud and its surroundings, which increases the acceleration of the cloud. The degree by which the destruction of the cloud speeds up should depend on the strength of the turbulence imposed on the post-shock flow, but here our intention is simply to demonstrate that it does, in fact, occur more rapidly.

(iv) The rate of mass loss from a cloud overrun by a Mach 10 shock due to hydrodynamic ablation is found to be broadly consistent with theoretical expectations, but shows large variations over the destruction period: the peak mass-loss rate is about five times higher than the time-independent value from a simplified theory. Clouds in a highly turbulent environment can be destroyed approximately twice as fast. However, significant differences between theoretical and numerically determined mass-loss rates exist at higher Mach numbers (Pittard, in preparation).

(v) Material stripped off the cloud is fragmented and irregular in structure when $\chi \lesssim 10^2$. In contrast, the mass loss from models with $\chi = 10^3$ better resembles a single tail-like feature. The length-to-width ratio of the tail increases with $\chi$ (denser clouds live longer, allowing material to be more dispersed in the axial direction), but decreases with the level of environmental turbulence.

(vi) The vorticity generated in the interaction is broadly similar in models with differing levels of post-shock turbulence when $\chi \gtrsim 10^2$. However, models with high environmental turbulence generate substantially less total circulation when $\chi = 10^3$ compared to models with a smooth post-shock environment, due to the more rapid destruction of the cloud. This may self-limit the total circulation that can be generated in such interactions.

(vii) Confirmation of the general speeding up of cloud destruction in a highly turbulent post-shock environment is attained by additional calculations where strong grid-scale motions and inhomogeneities are imposed in the post-shock flow of an inviscid calculation. Differences in the resulting evolution compared to those from the subgrid turbulence models with high post-shock turbulence reveal that the interaction is sensitive to the details of the turbulence, such as the maximum eddy size. The degree by which the destruction speeds up should again depend on the strength and properties of the turbulence imposed on the post-shock flow. Future simulations with a non-smooth intercloud medium would also be of interest.

7 FUTURE WORK

Irrespective of the shortcomings of the $k$–$\varepsilon$ model (see e.g. Davidson 2004), it is clear that in shock–cloud interactions, turbulence (both pre-existing and newly generated) plays an important role, and adds a new dimension to the parameter space that has hitherto been studied. An obvious extension of this work is to three dimensions: previous work has shown that many features seen in 2D simulations are unstable in three dimensions (Stone & Norman 1992). Future work will also examine the Mach number dependence of shock–cloud interactions and the interaction of winds and dense shells with clouds.

Synthetic signatures of the interaction should also be compared against observations. A comparison of the synthetic emission from models against X-ray observations has, for instance, already been carried out by Marcolini et al. (2005) and Miceli et al. (2006). Other observational signatures also need to be examined in greater detail: Westmoquette et al. (2007a, b) have recently concluded that the broad emission wings to Hα line profiles which are observed throughout the central regions of starburst galaxies arise in a turbulent boundary layer at the interface between hot gas flowing past cold gas stripped from clouds (Melnick, Tenorio-Tagle & Terlevich 1999), but the range in velocities may instead reflect the acceleration of material along the tail, rather than the turbulent motions within the mixing layer.

In many environments flows interact not with a single cloud, but with many clouds. Hydrodynamic simulations of the complex interactions between multiple clouds and a tenuous flow have been studied by Jun, Jones & Norman (1996), Poludnenko, Frank & Blackman (2002), Steffen & López (2004), Melioli, de Gouveia Dal Pino & Raga (2005), Pittard et al. (2005), Tenorio-Tagle et al. (2006), Sutherland & Bicknell (2007), Cooper et al. (2008) and Yirak et al. (2008). Most of these works focused on the changes to the global flow, but the interaction between two or more long-lived clouds in close proximity has been examined in detail by Pittard et al. (2005). In all situations, mass injection into a flow due to the destruction of clouds enhances the thermal pressure of the flow at the expense of the flow’s ram pressure. If the flow is slow and pressurized enough, it may induce the gravitational collapse of clouds and trigger new star formation. This process could be a central mechanism for feedback in the ISM (e.g. in starburst regions), and a self-consistent hydrodynamical model of it is a long-term goal.

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REFERENCES

Allen D. A., Burton M. G., 1993, Nat, 363, 54
Asai N., Fukuda N., Matsumoto R., 2004, ApJ, 606, L105
Cantó J., Raga A. C., 1991, ApJ, 372, 646
Cecil G., Bland-Hawthorn J., Veilleux S., Filippenko A. V., 2001, ApJ, 555, 338
Chandran B. D. G., Maron J. L., 2004, ApJ, 602, 170
Cho J., Lazarian A., Henneb K., Knapen B., Kassinos S., Moin P., 2003, ApJ, 589, L77
Consello C. J., Gallagher J. S., Wyse R. F. G., 2001, ApJ, 122, 2281
Cooper J. L., Bicknell G. V., Sutherland R. S., Bland-Hawthorn J., 2008, ApJ, 674, 157
Cox D. P., 2005, ARA&A, 43, 337
Cox D. P., Smith B. W., 1974, ApJ, 189, L105
Currie D. G. et al., 2000, ESO Messenger, 100, 12
Danforth C. W., Blair W. P., Raymond J. C., 2001, AJ, 122, 938
Dash S. M., Wolf D. E., 1983, AIAA Paper 83-0704
