Electric dipole moments as signals of supersymmetric unification

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Abstract

If supersymmetric unification is true, we show how the combined effort of several experiments under way to try to measure an electric dipole moment of the electron or of the neutron has a significant chance not only of producing a positive signal but also of providing crucial information to understand the physical origin of the signal itself.

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As recently pointed out [1, 2], the electric dipole moments (EDMs) of the electron, $d_e$, and of the neutron, $d_N$, represent a very significant signature for supersymmetric unification. In a typical supersymmetric GUT with supersymmetry breaking transmitted by supergravity couplings, the heaviness of the top quark induces a splitting between the sfermion masses of the third generation with respect to the masses of the first two generations [3, 4]. Such splitting, together with the CKM-like mixing angles and phases appearing in the gaugino-matter interactions, manifests itself, through one loop radiative corrections, in electron and neutron EDMs which are at the level of the current limits for large CP-violating phases and for sparticle masses visible at LHC [1, 2]. This observation justifies the believe that the electron and neutron EDMs can be considered among the few characteristic signatures of supersymmetric unification and should therefore be vigorously searched for. This view is strengthened by the fact that the discovery of the EDMs, if indeed originated by the unified interactions, must be accompanied by the observation of processes with violation of lepton flavour, such as $\mu \rightarrow e\gamma$ or $\mu \rightarrow e$ conversion in atoms, with typical rates again “around the corner” [3, 4].

 Needless to say, however, as always in the case of radiative corrections effects, the discovery of an EDM would not allow an immediate identification of its physical origin. In general one would have to discriminate between sources of EDMs inside or beyond the Standard Model (SM) or even, within a definite extension of the SM, between alternative mechanisms that can produce an EDM.

This last case is of relevance to the theories of interest to this paper. In a generic supersymmetric extension of the SM with minimal particle content one can identify four different sources of CP violation and, eventually, of EDMs:

i. the CKM phase in the charged current interactions;

ii. the strong $\theta_{\text{QCD}}$-angle;

iii. the phases appearing in the soft terms of the supersymmetry breaking Lagrangian (“complex soft terms” case);

iv. other CKM-like phases entering the fermion-sfermion-gaugino (higgsino) interactions (“unified theory” case).

The first two sources are in common with the SM; the third one might be present in any softly broken supersymmetric Lagrangian; the last one is present, at a significant level, in unified theories like SO(10). It is therefore at least the last case that one wants to discriminate against the others. We intend to show under which circumstances this might be possible: special attention must be payed to compare the results expected in cases iii. versus iv.
On the experimental side, the search for EDMs is being pursued by working on different systems: the paramagnetic atoms with open shells of unpaired electron spins, the diamagnetic atoms with closed shells of paired electron spins, and the neutron. As it will be immediately clear, this diversity of experimental searches is essential.

From a microscopic point of view, or more precisely in terms of the physics at the Fermi scale, all the aforementioned sources of CP violation can affect the experiments on the EDMs in a significant way only through the electron EDM, $d_e$, the EDM of the up quark, $d_u$ and of the down quark, $d_d$, generically denoted by $d_q$, the chromoelectric dipole moments of the same quarks, $d_{qQCD}$, and the $G_{µν}G^{µν}$ operator in the QCD Lagrangian. Other contributions from the three-gluon operator or from four-fermion interactions do not play any significant role in the present discussion. It is on the other hand well known that the three kinds of experiments considered are affected in a different way by the electron EDM and by the quark dipole moments, electric or chromoelectric.

Taking three among the most significant cases, one per category, one has for the respective EDMs

\begin{align}
\text{paramagnetic:} & \quad d_{Tl} = -600d_e + O(10^{-4})d_q + O(10^{-3})d_{qQCD} + O(10^{-3})(\theta/10^{-9})d_{1995}^{Tl} \quad (1a) \\
\text{diamagnetic:} & \quad d_{Xe} = 10^{-3}d_e + O(10^{-4})d_q + O(10^{-3})d_{qQCD} + O(10^{-1})(\theta/10^{-9})d_{1995}^{Xe} \quad (1b) \\
\text{neutron:} & \quad d_N = 1.6(d_e - \frac{1}{2}d_u) + O(10^{-1})d_{qQCD} + O(10^{-1})(\theta/10^{-9})d_{1995}^{N} \quad (1c)
\end{align}

where we have expressed the contribution from strong CP-violation involving the $\theta_{QCD}$ parameter in terms of the current upper bounds respectively.

Let us assume possible improvements in the sensitivities of the various experiments by one or two orders of magnitude at most. This excludes the detection of an EDM generated by the CKM phase. On the other hand, in view of eq.s (1a), (1b) and (1c) and of the range of values taken by $d_e$, $d_q$, $d_{qQCD}$ in the theories under consideration,

i. the measurement of $d_{Tl}$, or of the EDM for other paramagnetic systems, can be viewed as a search for an electron EDM;

ii. the measurement of $d_N$ might reveal a quark EDM or a strong CP-violation effect;

iii. the EDM of a diamagnetic system, like the Xe atom, might be influenced by all the three sources of CP-violation, from i. to iv., listed before.

In view of these considerations, we focus on the correlations between $d_e$, which can be considered as a direct observable, and $d_N$.

3

Since the contribution to $d_N$ from strong CP-violation, for given $\theta_{QCD}$, is known, within a decent approximation, we concentrate on $d_e$ and $d_N$ as arising from CP-violating phases in the soft supersymmetry breaking terms (source iii. of section) or from CKM-like phases in loops of sfermions and gauginos-higgsinos (source iv. of section). More precisely, in connection with case iii., we consider the MSSM with complex soft terms and universal initial conditions at the GUT scale. In this case, it is well known that only two phases have a physical meaning. We choose them to be the phase of the universal $A$-term and of the $\mu$-parameter

\begin{equation}
A = |A|e^{i\beta A}, \quad \mu = |\mu|e^{-i\beta \mu}, \quad \text{with} \quad B\mu = |B\mu|.
\end{equation}

As a prototype example of case iv., we consider the “minimal” SO(10) theory with no other phases than in the Yukawa couplings and with universal initial conditions on the soft terms at the Planck scale.

Both the electron and the quark EDMs, as the chromoelectric dipole moments, are produced by one loop vertex diagrams with sfermions and gaugino-higgsinos as internal lines. In turn, the calculation of such diagrams involves the knowledge of the full Lagrangian at the Fermi scale, which is the relevant scale. How the MSSM parameters are renormalized from their initial conditions at $M_G$ is too well known to be recalled here. In the precise case of “minimal” SO(10) with a large top Yukawa coupling, the rescaling to low energy of the various parameters is done in ref.
In terms of these parameters, the various EDMs are readily computed by means of the following formulæ.

In the “soft complex terms” case one has

\[
d_c = + \frac{e \alpha_e m_e}{4 \pi \cos^2 \theta_W} m_e \text{Im}(A_a^* + \mu \cot \beta) \sum_{n=1}^{4} \frac{H_n b_n}{M_N} (H_n b_n + H_n \tilde{W}_e \cot \theta_W) G_2(\tilde{e}_L, \tilde{e}_R) + \\
+ \frac{e \alpha_e m_e}{4 \pi \cos^2 \theta_W \cos \beta \sin \theta_W} \sum_{n=1}^{4} \frac{H_n b_n}{M_Z M_N} \left[ H_n b_n g_2(m^2_{\tilde{b}_n}) - \frac{1}{2} (H_n b_n + H_n \tilde{W}_e \cot \theta_W) g_2(m^2_{\tilde{b}_n}) \right] + \\
+ \frac{e \alpha_e m_e}{4 \pi \sin^2 \theta_W \cos \beta \cot \theta_W} \sum_{i=1}^{2} \frac{H_i^* b_i b_n}{M_Z M_{\tilde{b}_n}} h_2(m^2_{\tilde{b}_n})
\]

\[
d_u = - \frac{16 e \alpha_3 m_u}{9 4\pi M_3} m_u \text{Im}(A_u^* + \mu \cot \beta) G_2(\tilde{u}_L, \tilde{u}_R, 3)
\]

\[
d_d = + \frac{8 e \alpha_3 m_d}{9 4\pi M_3} m_d \text{Im}(A_d^* + \mu \tan \beta) G_2(\tilde{d}_L, \tilde{d}_R, 3)
\]

\[
d_u^{QCD} = + \frac{3 g_{3} \alpha_3}{2 4\pi M_3} m_u \text{Im}(A_u^* + \mu \cot \beta) [H_2(\tilde{u}_L, \tilde{u}_R, 3) + \frac{2}{9} G_2(\tilde{u}_L, \tilde{u}_R, 3)]
\]

\[
d_d^{QCD} = + \frac{3 g_{3} \alpha_3}{2 4\pi M_3} m_d \text{Im}(A_d^* + \mu \tan \beta) [H_2(\tilde{d}_L, \tilde{d}_R, 3) + \frac{2}{9} G_2(\tilde{d}_L, \tilde{d}_R, 3)]
\]

where $H$, $H^+$, $H^-$ are the mass-eigenstate interaction-eigenstate rotation matrices for the neutral $N_n$, the positively charged and the negatively charged $\chi_i$ gauginos-higgsinos respectively, and

\[
g_2(r) = \frac{1}{2(2r - 1)^2} \left[ r^2 - 1 - 2r \ln r \right], \quad h_2(r) = \frac{1}{2} - g_2(\frac{1}{r}) \]

\[
G_2(a, b, n) = \frac{g_2(m^2_{\tilde{a}_n}/M^2_{\tilde{a}_n}) - g_2(m^2_{\tilde{b}_n}/M^2_{\tilde{b}_n})}{m^2_{\tilde{a}_n} - m^2_{\tilde{b}_n}}
\]

\[
H_2(a, b, n) = \frac{h_2(m^2_{\tilde{a}_n}/M^2_{\tilde{a}_n}) - h_2(m^2_{\tilde{b}_n}/M^2_{\tilde{b}_n})}{m^2_{\tilde{a}_n} - m^2_{\tilde{b}_n}}
\]
In the “unified theory” case the electric dipoles have already been computed in [2, 4], to which we refer.

The relationship between these contributions is the following. In the “complex soft terms” case all the contributions to \(d_e\) not proportional to the combination \((A_e^1 + \mu \tan \beta)\) originate from the phase in the gaugino-higgsino mass matrices, through the \(\mu\)-term. These terms, which actually dominate the overall \(d_e\), are of course not present in the “unified theory” case. In such case, since all effects arise from flavour mixings in the gaugino-matter interactions, they are subject to a potential GIM-like cancellation. Finally, at least in the “minimal” SO(10) case considered here, there is no contribution to the up-quark DMs, either electric or chromoelectric, because the mass matrices for the \(Q = 2/3\) quarks and squarks may be simultaneously diagonalized without introducing any relative rotation.

We can now make a numerical computation, in the space of the parameters, of \(d_e\) and \(d_N\) in the two cases. As we have said, most important is the correlation between \(d_e\) and \(d_N\). For this reason, the results are presented as a scatter plot in the plane \((d_e, d_N)\) in fig.s 1. As mentioned, the “complex soft terms” case depends on two phases \(\phi_A\) and \(\phi_B\), whereas the “unified theory” case considered here depends on one combination of phases only, \(\phi\), which includes the standard CKM phase entering the normal charged-current weak-interaction vertex. In fig. 1a we have taken a uniform random distribution of \(\phi_A\), \(\phi_B\) and \(\phi\) between 0 and \(2\pi\). In fig. 1b the CP violating phases are uniformly distributed in logarithmic scale, with the two soft-term phases, \(\phi_A\) and \(\phi_B\), kept within the same order of magnitude. The other parameters are made to vary in such a way that

\[
45 \text{ GeV} < m_{\tilde{e}_R} < 500 \text{ GeV}, \quad -3 < \frac{A_e}{m_{\tilde{e}_R}} < 3, \quad 1.5 < \tan \beta < 5, \quad 0 < M_2 < m_{\tilde{e}_R}.
\]

This easily covers the range of values for the relevant sparticle masses explorable at LHC by direct pair production. Both squarks and gluinos go above 1 TeV. We have checked that the results do not change in any significant way if the sampling of the parameters is uniformly distributed at the level of the initial conditions at the large scale or at the level of the “low energy” parameters.

For the EDMs characteristic of the supersymmetric unified theory, the Yukawa coupling of the top quark at the unification scale, \(\lambda_{tG}\), plays a crucial role [4]. In fig.s 1, \(\lambda_{tG}\) is taken to vary between 0.5 and 1.4. From extrapolation of the top Yukawa coupling in the “low energy” range, we know that \(\lambda_{tG}\) should be bigger than \(0.5 \pm 0.6\) and that its preferred value from bottom-tau unification is above unity. For values greater than one, \(\lambda_{tG}\) rapidly reaches an infrared fixed point value in its behavior from \(M_{Pl}\) to \(M_G\): 1.36 is such a value for an SO(10) \(\beta\)-function coefficient of \(-3\) [4].
Two facts are apparent from figs. 1. If the various phases are not constrained to be small, the EDMs generated by the complex soft terms are generally not consistent with the present bounds, unlike the case for the effect in the unified theories. Most important for the purpose of this paper is the rather significant correlation that exists between $d_e$ and $d_N$ when they are generated by the phases in supersymmetry breaking terms: the ratio of $d_N$ over $d_e$ is most often between 2 and 10 and is generally significantly bigger than in the unified theory case irrespective of the values of the CP violating phases (see the distributions of $d_N/d_e|_{\text{GUT}}$ and of $d_N/d_e|_{\text{soft terms}}$ in fig. 2). The ratio between the present upper bounds on $d_N$ and $d_e$ is $d_N^{1995}/d_e^{1995} \approx 20$.

The qualitative reasons for the different behavior of the ratio $d_N/d_e$ in the two cases are the following.

The lower correlation between $d_N$ and $d_e$ in the “grand unified” case, apparent from figs. 1, is an effect of the top Yukawa coupling, which affects the low energy parameters related to the third generation in an important way, and amplifies their dependence on the initial conditions. One rests here on the assumption of universality for the sfermion masses at the large scale, that will have to be eventually checked by direct mass measurements.

The relative value of $d_N/d_e|_{\text{SO(10)}}$ versus $d_N/d_e|_{\text{soft terms}}$ has a definite pattern. The EDMs generated by the complex soft terms are proportional to the light fermion masses, unlike the case for the unified theory, so that

$$d_N/d_e|_{\text{SO(10)}} \approx d_N/d_e|_{\text{soft terms}} \approx \frac{m_d}{m_e} \frac{m_b}{m_\tau} \approx 5 \pm 2.$$  

Furthermore, in the unified theory case, the GIM-like cancellation is more effective in suppressing the quark DMs, relative to the electron EDM, because of a gluino focussing effect: the strong radiative corrections to the squark masses, proportional to the gluino mass, are family independent and, as such, counteract the splitting effect induced by the top Yukawa coupling. This is apparent in fig. 3 where we show a scatter plot of the ratio $d_N/d_e$ versus the ratio $M_2/m_{\tilde{e}_R}$ between the SU(2)$_L$ gaugino mass term $M_2$ and the right-handed selectron mass, both computed at the Fermi scale. The gluino mass is $M_3 = 3.6M_2$. In this plot all the points correspond to values of both $d_e$ and $d_N$ not excluded by the present bounds. Fig. 3 shows that $d_N/d_e|_{\text{SO(10)}}$ may in fact overlap or even exceed $d_N/d_e|_{\text{soft terms}}$ with a significant probability only for relatively light gluinos. This could of course also be another handle to try to distinguish the physical origin of the effects that might be observed.

Finally, as far as the strong CP-violation source is concerned, it is clear from eq. (1), (2) that it could only show up in a $d_N$ signal but not in $d_e$.

To conclude, we think to have shown that the combined efforts of several experiments under way to try
to measure an EDM have a significant chance not only of producing a positive signal but also of providing crucial information to understand the physical origin of the signal itself. This supports our view that the EDM experiments are among the few crucial experiments, that can be conceived at all, able to provide evidence for supersymmetric unification.

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