Long-term Photometric Variability in Kepler Full-frame Images: Magnetic Cycles of Sun–like Stars

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Abstract

Photometry from the Kepler mission is optimized to detect small, short-duration signals like planet transits at the expense of long-term trends. This long-term variability can be recovered in photometry from the full-frame images (FFIs), a set of calibration data collected approximately monthly during the Kepler mission. Here we present f3, an open-source package to perform photometry on the Kepler FFIs in order to detect changes in the brightness of stars in the Kepler field of view over long time baselines. We apply this package to a sample of 4000 Sun–like stars with measured rotation periods. We find that $\approx 10\%$ of these targets have long-term variability in their observed flux. For the majority of targets, we find that the luminosity variations are either correlated or anticorrelated with the short-term variability due to starspots on the stellar surface. We find a transition between anticorrelated (starspot-dominated) variability and correlated (facula-dominated) variability between rotation periods of 15 and 25 days, suggesting the transition between the two modes is complete for stars at the age of the Sun. We also identify a sample of stars with apparently complete cycles, as well as a collection of short-period binaries with extreme photometric variation over the Kepler mission.

Key words: methods: data analysis – stars: activity – stars: solar-type – techniques: photometric

Supporting material: machine-readable table

1. Introduction

For more than two millennia, observations of the Sun have shown the presence of spots on its surface. Regular telescopic observations of these spots date back more than four centuries (Galilei et al. 1613). These spots have been known to vary on an 11 yr timescale (Schwabe 1844), with more than two dozen of these cycles now observed (Hathaway 2015 and references therein). This starspot cycle is likely produced by the solar dynamo (Charbonneau 2010). While the most obvious effect of the solar cycle is variation in the location and number of sunspots, it also correlates with the occurrence of coronal mass ejections (Gosling 1993) and a change in the total solar irradiance (Fröhlich & Lean 1998). Over the solar cycle, the Sun changes in brightness by approximately 0.1% in the optical, with times of increased starspot activity corresponding to an increase in luminosity due to the bright faculae that typically surround darker sunspots (Fröhlich & Lean 2004).

More recently, similar behavior has been seen on other Sun–like stars. Spectroscopic observations at Mt. Wilson showed that main-sequence stars from spectral types F5 to M2 have chromospheric variations in their atmospheres, providing evidence for analogs of the solar cycle (Wilson 1978; Baliunas et al. 1995). The level of chromospheric variation has been shown to correlate with stellar rotation (Noyes et al. 1984; Saar & Brandenburg 1999). Additional observations have shown that Sun–like stars also have photometric variations, with more rapidly rotating stars having larger variations in flux (Lockwood et al. 1997; Radick et al. 1998). Magnetic cycles have also been observed in time-series radial velocity (RV) planet searches, as magnetic activity affects the convective blueshift, leading to an apparent RV shift with the period of the magnetic cycle (Endl et al. 2016).

Observations of different stars at different ages have provided opportunities to detect the evolution of the stellar dynamo. Presumably older stars with slower rotation rates have longer magnetic cycles with lower amplitudes (Noyes et al. 1984; Baliunas et al. 1995). Observations of rotation periods and activity cycles have suggested that there are two branches of Sun–like stars: an “active” branch with longer magnetic cycles and an “inactive” branch with shorter cycles for a given rotation period (Böhm-Vitense 2007). Interestingly, some stars have shown behavior consistent with both branches. The Sun appears to fall between these two branches, both in the length of its magnetic cycle and in its observed chromospheric activity, suggesting that the solar dynamo may be in transition (Metcalfe et al. 2016). An additional transition has been proposed, with a transition from complex to smooth magnetic cycles suggested at an age between 2 and 3 Gyr (Oláh et al. 2016). This transition has also been suggested to lead to a change from starspot-dominated photometric variations to facula-dominated variations, although the details of any particular star depend on the star’s inclination and latitudinal distribution of spots (Shapiro et al. 2014).

Our understanding of the solar dynamo and its potential transition is limited by the small number of stars with detailed observations of their magnetic activity. Spectroscopic observations are expensive, and photometric variations at the sub-mmag level can be small and hard to detect from the ground for all but the brightest, nearest stars (Tregloan-Reed & Southworth 2013). Asteroseismic observations can be used to measure global changes in stellar oscillation modes due to changes in the overall stellar activity level, which have been used to detect stellar activity cycles with both CoRoT and Kepler (García et al. 2010; Salabert et al. 2016b). However, asteroseismic observations require high-precision photometry at a fast enough cadence to resolve the modes of pulsation. For
Sun–like stars, long-cadence observations are insufficient; short cadence is required but only present for a small fraction of these targets (Gilliland et al. 2010). Even over small spatial scales, absorption by the interstellar medium (ISM) can bias spectroscopic stellar activity observations: a star 100 pc away can have a bias in its \( R'_{\text{HK}} \) value of as much as 0.1 dex due to interstellar Ca II (Fossati et al. 2017). In total, there are only approximately 100 Sun–like stars with measured activity cycles (e.g., Hall et al. 2009).

Photometric tracers of magnetic activity can be obtained from space-based transit surveys like Kepler. Launched in 2009, Kepler targeted nearly 200,000 stars across 115 deg\(^2\) of the sky and was optimized to detect small changes in brightness due to the presence of planets transiting the disk of each star (Borucki et al. 2010). The primary data product, “long-cadence photometry,” includes observations of each star at approximately 30 minute intervals over a span of 4 yr (Jenkins et al. 2010). These data are ideal for detecting short-term changes in the brightnesses of these stars, such as starspots rotating into and out of view (Niesi et al. 2013; McQuillan et al. 2014), stellar flares (Hawley et al. 2014; Davenport 2016), and asteroseismic pulsations (Huber et al. 2013; Silva Aguirre et al. 2015). Mathur et al. (2014) observed changes in the starspot variability of F stars in Kepler data as a proxy for magnetic activity, an approach that Reinhold et al. (2017) extended across the main sequence. Changes in the frequency shift of asteroseismic modes have also provided an opportunity to probe magnetic activity (Salabert et al. 2016b). However, signals on longer timescales (~50 days or longer), such as brightness variations on the timescale of magnetic cycles, are both intentionally removed by the data-processing pipeline and overwhelmed by instrumental systematics due to the small aperture sizes relative to the size of the telescope point-spread function (PSF; Gilliland et al. 2011).

Long-term brightness variations can be recovered through the full-frame images (FFIs), a set of calibration data obtained approximately monthly during the Kepler mission. These observations are the only times the entire detector, encompassing all 4.5 million stars in the field of view, is downloaded. FFI data have been used to help confirm signals in long-cadence data and for visualization purposes (Lehmann et al. 2012; Jenkins et al. 2015; Gaidos et al. 2016). On their own, they contain the entire PSF for each star and a large number of reference stars nearby on the detector, enabling long-term brightness variations removed from the long-cadence data to be recovered (Montet & Simon 2016).

In this paper, we develop a method to recover long-term brightness variations for stars in the Kepler field from FFI data, building on previous work (Montet & Simon 2016). In Section 2, we explain in detail how to measure photometry from the FFIs using our publicly available code, then use it to explore long-term flux variations in a sample of more than 3000 Sun–like stars with measured rotation periods. In Section 3, we present the results of our search and verify that we are observing astrophysical variability from the target stars themselves. In Section 4, we discuss particularly interesting systems and present evidence for a shift in the relation between short- and long-term photometric variability at a rotation period of \( \approx 25 \) days, rather than at solar ages. In Section 5, we conclude with a discussion of the future prospects of extending our sample with data from other space missions.

2. Data Analysis

2.1. Sample of Sun–like Stars

In this work, our goal is to consider Sun–like stars. While spectroscopic surveys of the Kepler field are underway (De Cat et al. 2015; Guo et al. 2017), these surveys generally target the bright stars in the field. Photometric surveys are more complete (Brown et al. 2011; Huber et al. 2014) but provide larger uncertainties on the physical parameters of each individual star. As our sample is large and dominated by faint stars, we build a sample of stars using established photometric stellar parameters.

McQuillan et al. (2014) detected rotation periods in 34,000 main-sequence stars in the Kepler field and reported nondetections in nearly 100,000 additional stars. Since stellar rotation periods are correlated with the age of the star (Barnes 2007; Mamajek & Hillenbrand 2008; Angus et al. 2015), these stars provide us the opportunity to not only search for activity signatures but also to understand their evolution with stellar age.

We select all stars in the McQuillan et al. (2014) catalog with measured rotation periods, estimated \( T_{\text{rot}} \) within 150 K of the Sun, and log\((g) \geq 4.2. This particular catalog uses the stellar parameters of Brown et al. (2011). While more recent publications have updated stellar parameters (Huber et al. 2014), the Kepler Input Catalog (KIC) is more homogeneous. Pinsonneault et al. (2012) showed that this catalog systematically underestimates effective temperatures by approximately 200 K, which, combined with random uncertainties of approximately 100 K, suggests that our sample should largely be stars with spectral types F7 to G4.

The Bastien et al. (2016) catalog of stellar log\((g) \) inferred from the Kepler data is limited to bright Kepler stars with \( K_p < 13.5, \) while the majority of stars in our sample are fainter than that value, limiting our ability to intentionally exclude evolved stars. However, the requirement that each star have a measured rotation period should also ensure that the number of evolved stars with very slow rotation periods is minimal. The distribution of stellar effective temperatures and rotation periods is shown in Figure 1.

These cuts provide us with a total of 4876 stars. We additionally remove all stars that fall within 10 pixels of the detector edge, which would complicate aperture photometry calculations. We then select a similar number of stars with the same stellar parameter cuts but no detected rotation periods to use as a control sample. These stars have similar colors and brightnesses but are older, less active, and possibly evolved, so we should expect to see less variability. Differences between these two samples ensure that our observations are dominated by astrophysical information rather than underlying, unknown instrumental systematics. Once we have a list of KIC identifiers associated with the stars in the sample, we can perform photometry on each of them to search for long-term brightness variations.

2.2. Full-frame Images

2.2.1. Data Collection

The observations in this paper are taken from the 53 FFIs collected by Kepler during its primary mission. These data represent the only publicly available data of simultaneous
observations of the entire Kepler field of view. Eight FFIs, called the “golden FFIs,” were obtained over 34 hr during commissioning of the spacecraft before the start of the primary mission. Additional FFIs were obtained approximately once per month during the primary mission, immediately before the spacecraft turned toward Earth to transmit data. Two of these observations were missed because the spacecraft entered a safe-mode state before data downlink. Additionally, on two occasions, two FFIs were collected in succession. In one case, the telescope was mispointed by 15″, or approximately 4 pixels. We do not include this image in our analysis. On the second occasion, two acceptable images were obtained. All FFIs have integration times of 29.4 minutes, identical to that of a standard long-cadence frame during the Kepler and K2 missions. The observations are obtained and calibrated following the same procedure as that of long-cadence photometry but applied to every pixel rather than a small equal-sized region. We build a master postcard for each detector, they are shifted to ensure that we always consider an equal-sized region. We build a master postcard for each field by summing over all 52 usable FFIs, allowing for shifts of up to 1 pixel in either direction in each individual frame to account for apparent pointing variations induced by differential velocity aberration (Haas et al. 2010). With this image, we identify the 250 brightest sources on the postcard corresponding roughly to stars brighter than $K_p = 16.5$. This selection will include our target star, while we draw our photometric comparison stars from the collection of other nearby stars that we select, typically around 200 in number.

Once we have identified targets, the next step is to measure flux values for all targeted stars in each epoch. One possibility is to employ PSF modeling. Detailed PSF modeling is challenging with Kepler, as the underlying flat field is poorly understood. It has been shown that PSF modeling works for faint stars in crowded K2 fields but underperforms simple aperture photometry for bright ($K_p \lesssim 15.5$) isolated stars (Libralato et al. 2016). We also explore model PSFs as the sum of three Gaussian functions conditioned on all target stars in the postcard but find that the uncertainties in the model fit are larger than those we are able to achieve in simple aperture photometry. Therefore, we rely on aperture photometry in this

2.2.2. Data Reduction

In this work, we develop time-series photometry for each of the target stars as described in Section 2.1. The method used is broadly similar to that of Montet & Simon (2016) but is more completely automated and achieves a typical precision higher than that achieved by those authors by a factor of 2–3, depending on the target star and field. For all stars in our sample, we identify the row and column on the detector in which our target star appears using data from the Kepler Input Catalog, which contains position information for each star at arcsecond precision, more than sufficient for our purposes. We then select $300 \times 300$ pixel (20″) “postcard” regions around each star (Figure 2). In most cases, these regions are centered on the target star; when the target star falls near the edge of a detector, they are shifted to ensure that we always consider an equal-sized region. We build a master postcard for each field by summing over all 52 usable FFIs, allowing for shifts of up to 1 pixel in either direction in each individual frame to account for apparent pointing variations induced by differential velocity aberration (Haas et al. 2010). With this image, we identify the 250 brightest sources on the postcard corresponding roughly to stars brighter than $K_p = 16.5$. This selection will include our target star, while we draw our photometric comparison stars from the collection of other nearby stars that we select, typically around 200 in number.

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work, leaving a detailed exploration of the possibility of PSF modeling for future efforts.

Aperture photometry requires accurate apertures that are large enough to capture the extent of the stellar PSF but small enough to only include the target star. To capture the Kepler PSF, the ideal aperture is asymmetric, especially near the edges of the detector (Bryson et al. 2010). Here we use the computer vision library mahotas to draw appropriate apertures around our targets and separate them from the background (Coelho 2013). We first choose all connected pixels with flux values larger than 1.5% of the brightest pixel’s flux, then we extend the aperture to include a border of 3 pixels outside this region. These selection criteria create a region of the detector large enough to encapsulate the entire PSF but small enough to isolate only the star in question. Any aperture where both of these criteria are true enables the light curve to be recovered. In cases where multiple comparison stars have apertures that collide with each other, we combine their apertures into one and treat them as a single target.

We visually inspect each aperture for our science targets to ensure they contain only one star, manually removing stars with nearby bright companions that encroach on the primary star’s PSF; due to the large PSFs on the detector, this removes approximately 10% of all possible stars from our analysis. Once our apertures are determined, we measure fluxes for all stars by summing over all pixels in each aperture.

For all stars, we notice a slow decrease in the observed flux. If this decrease were a function of color or magnitude, it would limit our ability to use any nearby star as a potential calibrating reference star. Fortunately, this does not appear to be the case: the magnitude of this decrease appears to be the same for all stars on the same detector (Figure 3). Therefore, we consider all nearby stars to share similar photometric systematics with our target star, making them potentially acceptable reference stars.

We invoke two methods to use the reference stars to measure the true long-term photometric behavior of each of our target stars. As a first pass, we simply eliminate all possible reference stars with excess variability (2.5σ from the mean) in more than 10% of frames. This should ensure that intrinsically variable stars and stars with poorly drawn apertures are removed from the sample of reference stars. For all images in a particular orientation, we then sum the flux of all reference stars and divide the observed flux from our target star by this value. We divide each observed flux value by the median flux value for the target star from all epochs in that orientation, effectively eliminating possible effects caused by variations in the flat field between detectors used at different orientations. This method is fast and enables us to quickly measure the brightness of each star to look for candidate active stars.

**Figure 2.** (Left) Typical postcard region of the detector from which reference stars are drawn. The target star, in this case KIC 8462852, is directly in the center of the image. (Right) Same postcard with apertures drawn over reference stars. The 250 brightest targets are considered: targets with apertures that touch the edge of the postcard are removed, and targets for which apertures overlap are combined and treated as one reference star.

**Figure 3.** Observed change in brightness in time for all stars in our sample as a function of $K_p$ magnitude and $g-r$ color, an instrumental effect observed in the FFI photometry and standard Kepler data. There is a slow decay that is instrumental and shared by all stars on the same part of the same detector. Red dashed lines represent a linear fit to the slope as a function of magnitude or color; we find that both are consistent with zero, suggesting that there is no instrumental systematic that specifically affects saturated stars or particularly red or blue wavelengths. We therefore use all nearby stars as potential reference stars, regardless of their stellar parameters.
For each of the candidate active stars, we then employ a probabilistic approach—similar in spirit to the “ubercalibration” method used in the Sloan Digital Sky Survey (SDSS) pipeline (Padmanabhan et al. 2008)—that enables us to measure the brightness of our target star and estimate an uncertainty on each observation. The full details of this method are given in the Appendix, but we summarize the important points here. We model the observed light curves of an ensemble of stars as noisy measurements of intrinsically variable time series observed with varying pixel responses and photometric zero points. We make this model tractable by assuming that nearby stars with similar properties sample a constant photometric zero point and that the pixel response for a particular source is constant in each season of observation. We fit for a distribution of zero points for each exposure and the pixel responses and amplitude of intrinsic variability for each target by maximizing the likelihood function defined by this model (Equation (7)). We use this maximum likelihood model to compute the detrended light curve and its uncertainties.

The end result is time-series photometry for all of our target stars conditioned on the observations of nearby reference stars on the detector, with underlying systematics between detectors accounted for. These light curves are considerably more sparsely sampled than Kepler’s long-cadence data but allow for long-term trends that are overwhelmed by instrumental systematics in the long-cadence data to be recovered. We make the code underlying the FFI photometry code publicly available for community use and additional development as the \textit{f3} (Full Frame Photometry) package. The uncertainties on each data point are heteroskedastic. We describe the method through which uncertainties are estimated in the Appendix. For each star, the photon noise observed in the primary Kepler mission is not the dominant source of uncertainty. Over short timescales, similar to a typical transit duration, this is true, but on longer timescales in the Kepler data changes in the detector properties, such as thermal variations and detector degradation, overwhelm the photon noise. This is evident from the long-term trends seen in standard Kepler data over a quarter. Indeed, inspecting our photometry over the “golden FFIs” collected across two days over which the pointing is stable shows lower variability than the point-to-point scatter across monthly observations, similar to what would be observed if, for example, one selected monthly points from a long-cadence light curve. Due to uncertainties in the flat-field and detector properties, combined with the sparse sampling of the FFIs, our photometric uncertainties on any individual point are considerably larger than those from relative detrended photometry from the Kepler mission.

For an upcoming mission like TESS (Ricker et al. 2014), with an FFI collected every 30 minutes, it will be possible to simultaneously model the detector properties and the photometry, similar to what is done in K2 fields (e.g., Luger et al. 2016), enabling a similar study to be accomplished but recovering a similar photometric precision to that achievable on the primary target stars.

2.2.3. Potential Systematics

While it does enable accurate measurements of long-term flux variations on target stars in the field of view, FFI photometry also comes with its own set of systematics. Many of these are familiar to users of Kepler and K2 data products, although they can manifest themselves in ways that may not be familiar. We show examples of each of these systematics in Figure 4.

The most common systematic is a sudden pixel sensitivity dropout (SPSD). In these events, a cosmic-ray hit causes a particular pixel to immediately decrease in sensitivity, leading to a decrease in the observed flux on that pixel without changing its neighbors (Smith et al. 2012). Pixels infected with SPSDs may recover after hours or days, while others remain degraded through the remainder of the mission.

SPSDs are visible in Kepler long-cadence light curves of many stars. Since larger apertures are required to ensure accurate FFI photometry, there are more possibilities for pixels to be affected by an SPSD than in typical Kepler data. SPSDs can be separated from true astrophysical variability in the FFIs because changes will manifest themselves in only one of the four telescope orientations, while the others will appear unchanged. Difficulties remain in separating astrophysical variability in the first or last quarter of observations from instrumental effects: this is especially true for the start of the
mission, as the “golden FFIs” are separated from the first FFI in the primary mission by 115 days.

The second most common systematic is apparent brightness variations caused by changes in the position of the star over the underlying unknown flat field. K2 observations have shown that there are significant interpixel and intrapixel variations in the flat field across the detector (Van Cleve et al. 2016). Some stars in the Kepler field of view can have proper motions as large as 0.1 pixels yr\(^{-1}\), and differential velocity aberrations can cause periodic motions at the pixel level. Flat-field variations can then manifest themselves as a long-term trend random, and the affect our results significantly. Variations to separate stars with long-term variability dominated by faculae or starspots, respectively (Radick et al. 1998).

As a proxy for starspot variability, we calculate the \( S_{ph} \) metric from the long-cadence data. The \( S_{ph} \) index measures a running standard deviation of points within five rotation periods of each cadence and has been shown to be a useful tracer of magnetic activity variations that requires only photometry (Mathur et al. 2014). We then compare this photometric activity index, \( S_{ph} \), to the observed bulk brightness variations to separate stars with long-term variability dominated by spots from those with long-term variability dominated by faculae.

### 3. Results

Of the total of 3845 stars targeted in this survey, we find that 463 of them have observed brightness variations at the 3σ level over the Kepler mission. Of course, this does not mean that the remainder are unvarying: their variability is either too small in amplitude or too low in frequency to be observed over the 4 yr baseline of Kepler’s FFIs. The stars with observed photometric variability are listed in Table 1, and a representative sample of observed variability is shown in Figure 5.

In almost all cases, we neglect stars with long-term trends in the photometry, only considering stars with nonlinear variability over the Kepler mission. This is due to the possibility that long-term trends could be affected by data artifacts and nonastrophysical events, as discussed in Section 4.4. We claim any other observed variability is intrinsic to the star and related to modulations in the overall stellar brightness induced by magnetic activity. In the following subsection, we attempt to rule out other explanations.

#### 3.1. Alternative Explanations

If the observed long-term variability were the result of an instrumental effect, then we should expect to see similar behavior on all stars with similar stellar properties. Fortunately, a control sample exists to test this idea. In addition to the stars with measured rotation periods observed by McQuillan et al. (2014), the authors of that paper also published a list of stars without measured rotation periods. We select a sample of 3000 stars that satisfy our same temperature and surface gravity cuts but have no measured rotation signal.

These stars, having similar magnitudes and colors as our target sample, should provide a reasonable control sample to compare against in a search for instrumental effects. We repeat our analysis on this control sample and search for photometric variability. We find that only 11 of the stars have observed variability, significantly lower than the rate observed in the target sample. Of these, closer inspection reveals that three stars’ (KIC 3383794, 4639329, and 4649300) light curves do indeed exhibit a rotation signature in some quarters, but the signal was not detected in enough segments of the data to be included as a bonafide detection by McQuillan et al. (2014). If we assume these rotation signals are real, then we have only
eight stars with FFI variability but no rotation signature, suggesting that the vast majority of our observed signals are not instrumental in nature.

Astrophysically, the control sample is not necessarily identical to the target sample, despite their similar inferred temperature and gravity. As they do not have observed rotation periods, they are likely on average to be less chromospherically active and older. A fraction of them may be nearly pole-on, while others may be slightly evolved relative to their counterparts with measured rotation, leading to their rotation signals being removed by the Kepler data-processing pipeline.

If the stars in the control sample are systematically more distant, then they might systematically have lower proper motions. Therefore, if the observed variability were caused by stars passing behind small-scale structure in the ISM, changing the extinction along our line of sight and modulating the brightness (Meyer & Blades 1996), then we might expect to not observe as much variability in the control sample. This scenario is not well represented by the data either. In this case, we would expect stars with higher proper motions to have higher rates of variability, which is not the case, as shown in Figure 6. Moreover, we would expect stars near the galactic plane, where extinction is higher, to preferentially display variability, while we detect signals across the Kepler field (Figure 7).

We also note that stars with observed variability are more likely to be detected in the GALEX survey of the Kepler field, which has a limiting NUV magnitude of 22.6 (Olmedo et al. 2015). Of the 463 target stars, 145 of them (31.3%) are detected in the UV. Of the other 3382 stars in our sample, 775 of them (22.9%) are detected in the UV, a 3.6:1 discrepancy. This is in line with expectations, as UV-bright stars are more likely to be chromospherically active (Findeisen et al. 2011).

These tests rule out certain alternate explanations for the observed long-term variability but do not prove that the brightness variations we are observing are magnetic in nature. The strongest evidence in favor of that explanation is through comparing the observed variability to other tracers of magnetic activity, such as starspots.

3.2. Photometry as Tracer of Magnetic Activity

Kepler long-cadence photometry can provide information on the evolution of starspots, a proxy of magnetic activity. Changes in the level of starspot variability have been shown to correlate with spectroscopic proxies of stellar activity (Mathur et al. 2014; Salabert et al. 2016a). Starspot variability has been shown to correlate with long-term brightness variations as detected through the $S_{ph}$ index; recently, C. Karoff et al. (submitted) showed that brightness variations observed through Kepler FFI photometry also correlate with spectroscopic and photometric proxies of magnetic activity for KIC 8006161. The same effect is seen for the Sun, where an increase in stellar chromospheric activity is visible through both an increase in the number of observed starspots and an overall increase in flux due to the presence of bright faculae on the surface of the star (e.g., Fröhlich & Lean 1998).

Here we do not have uniform spectroscopy for our target stars, but we do have uniform long-cadence photometry: as these stars are selected from their rotation properties, by definition they all are on silicon during the Kepler mission. We compare the FFI photometry to the $S_{ph}$ index for each of our stars with nonconstant flux over the mission. We find that 84% of stars exhibit behavior in their long-term brightnesses that corresponds to the variability observed in the $S_{ph}$ index. We label each star’s brightness fluctuations as either correlated or anticorrelated in time with the $S_{ph}$ index, meaning that the long-term variability is driven by bright faculae or dark sunspots, respectively. The majority (68%) are anticorrelated: increases in spot activity correspond to an overall decrease in brightness. This is the opposite of what is observed in the Sun (Noyes et al. 1984).

3.3. A Transition in the Stellar Dynamo

While the majority of stars have brightness variations anticorrelated with their starspots, this is not true at all rotation periods. This is the case because the majority of stars in our sample have rotation periods considerably shorter than the Sun’s. Figure 8 shows this effect. At rotation periods between...
10 and 15 days, 11% of stars show detectability over the 4 yr of 
Kepler observations; spot-dominated, anticorrelated variability 
is an order of magnitude more common here than facula-
dominated, correlated variability. However, at a rotation period 
of 15 days, the occurrence of spot-dominated variability 
sharply decreases and facula-dominated variability rises, 
consistent with a change in the driver of the long-term 
brightness variations from spots to faculae at a rotation period 
of approximately 24 days. We do not detect any stars with spot-
dominated variability with rotation periods beyond 26 days, 
while detectable facula-dominated variability continues to be 
increasingly common at rotation periods slower than that of the 
Sun. The distribution of spot-dominated stars peaks at a 
rotation period of 13 days; it has a mean of 12.8 days and 
standard deviation of 5.6 days. The distribution of facula-
dominated stars is visually distinct: it has a mode of 30 days, a 
mean of 27.5 days, and a standard deviation of 9.4 days.

We can convert observed stellar parameters into Rossby 
numbers \( R_0 \) following the prescription of Noyes et al. (1984), 
who related a star’s \( B - V \) color to its convective turnover 

\[ \text{Figure 5.} \text{ Representative sample of 12 stars displaying variability over the Kepler mission. Different stars exhibit different amplitudes of variability, with different timescales and different structure from that of the observed variability. The levels of variability shown here are not observed in our control sample of stars with no observed rotation period, eliminating instrumental false positives.} \]

\[ \text{Figure 6.} \text{ Proper motions of stars (top) displaying variable behavior in the FFIs and (bottom) not displaying variable behavior in the FFIs, for all stars with listed proper motions in the Kepler Input Catalog. The two distributions are consistent with each other, suggesting that we are not seeing extinction from small-scale structure in the ISM on preferentially high proper motion stars.} \]
time. We use the $B - V$ colors of Everett et al. (2012), who performed a $UBV$ survey of the $Kepler$ field largely complete to $V \approx 19$. The typical uncertainty in each bandpass is 0.02 mag, leading to an uncertainty of 0.03 mag in the color and therefore \~2 days in the convective turnover time. We see the same result in this space: stars with lower values of $R_0$ are considerably more likely to have spot-dominated variability, while no stars with $R_0 > 1.7$ are spot-dominated. For reference, the Sun has $R_0 = 2.05$ based on its color and rotation period. The relation between spots and faculae as the primary cause of observed long-term photometric variability is more strongly observed in Figure 8 as a function of stellar rotation period than Rossby number. This effect may be due to noise in the calculation of the Rossby number. The distribution of $B - V$ colors for our sample has a standard deviation of 0.05 mag, comparable in magnitude to the typical uncertainty of 0.03 mag on the $B - V$ color for any individual star. Additionally, a 0.03 mag uncertainty on $B - V$ leads to a 50% uncertainty on the convective turnover time, and thus the Rossby number. At the limit of our photometric precision, given that our uncertainties on $B - V$ are compatible with the spread of $B - V$ colors in our sample, the Rossby number is effectively a noisy estimator of stellar rotation period, meaning that rotation period is a more suitable parameter to look for fundamental changes in the stellar activity of this particular narrowly defined sample.

Rotation period correlates with stellar age (Barnes 2007). Mamajek & Hillenbrand (2008) showed that for main-sequence stars, the two are related such that

$$P = A^* \times c(B - V - c)^b,$$

where $B - V$ is the color of the star in the Johnson–Cousins filters, $A$ is the star’s age in Myr, $P$ is the rotation period of the star in days, and $a$, $b$, and $c$ are constants such that $n = 0.566 \pm 0.008$, $a = 0.407 \pm 0.021$, $b = 0.325 \pm 0.024$, and $c = 0.495 \pm 0.010$. For the Sun, $B - V = 0.653 \pm 0.003$ (Ramírez et al. 2012). The stars in our sample have $B - V$ values approximately normally distributed with a mean of 0.64 and standard deviation of 0.04. In this case, Equation (1) reduces to

$$P = (0.229 \pm 0.021) \times A^{0.566 \pm 0.008}.$$

Therefore, assuming that the gyrochronological relations are accurate across the entire span of observed rotation periods, the peak occurrence of detectable spot-dominated variability occurs at an age of approximately 1200 Myr and begins to fall off significantly at an age of 1600 Myr. Stars with an age of 3500 Myr are equally likely to exhibit long-term behavior dominated by either spots or facula, and we do not detect any stars with spot-dominated variability older than 4200 Myr.

4. Discussion

4.1. Transition or Two Populations?

Understanding this putative transition means understanding the stars in our sample. One help would be additional observations: it is possible that 13% of stars with a rotation period of 13 days have observable spot-dominated variability, or it is possible that all such stars have a signal, but it is only detectable over a 4 yr time baseline when viewed from a random angle 13% of the time. Continued photometric observations at this cadence and precision would be helpful and will be obtained as described in Section 5.2.

Upcoming parallax observations from $Gaia$ will also be important for characterizing these systems. The photometric-based effective temperature estimates for our stars have uncertainties similar in magnitude to the range of temperatures chosen, suggesting that our Sun–like stars span the F7 to G4 spectral classes. This cut will affect different rotation periods differently. Although there is not a large spectral range in our sample, there is a significant difference between the typical rotation period of an average main-sequence F star and G star (Nielsen et al. 2013). Therefore, when dividing our initial sample by rotation period, the group of stars with faster rotation periods is likely slightly biased in favor of F stars and vice versa. The sample of stars with rotation periods of \~10 days includes a higher fraction of F stars, which slow down more slowly and have a shorter main-sequence lifetime; the sample at a rotation period of \~30 days will have a larger fraction of mid-G stars. As stated in the previous section, we do not detect any stars with spot-dominated variability older than 4200 Myr; this sample by definition cannot contain stars with main-sequence lifetimes shorter than 4200 Myr. A more detailed characterization of individual systems and the evolution of the stellar dynamo of G2 stars in particular will require improved stellar parameters, which will become more attainable with measured parallaxes to these stars.

In this work, we take the gyrochronological relations at face value in our analysis that the transition between spot-dominated and facula-dominated variability occurs at an age younger than the Sun. Comparisons to solar twins at similar ages to the Sun show the Sun’s rotation period to be typical (dos Santos et al. 2016). Recently, van Saders et al. (2016) showed that older stars can have significantly weakened magnetic braking compared to younger stars, leading to a discrepancy between rotation periods and age estimates, especially for stars more evolved than the Sun.

As the vast majority of stars in our sample have ages younger than the Sun, and the transition of interest occurs at
These results suggest that the long-term photometric variability is driven by starspots for younger stars and faculae for older stars and that the transition appears to occur at a similar age for most Sun-dominated variability increases in time, with the two equal at a rotation period of 24 days. Beyond 26 days, we do not detect any stars with spot-dominated variability. Kepler long-cadence data. Of all Sun-dominated signals. The occurrence drops off due to the limited time baseline and photometric precision from asteroseismic ages for stars in our sample with observed subject to the resolution of this discrepancy. Where possible, the previous subsection should be treated with some caution. Certainly possible that some of our stars are affected. Therefore, not significantly affected by this discrepancy, although it is certainly possible that some of our stars are affected. Therefore, the age estimates derived through gyrochronology in this and the previous subsection should be treated with some caution subject to the resolution of this discrepancy. Where possible, asteroseismic ages for stars in our sample with observed variability would be useful. As the Kepler telescope is no longer able to point at the Kepler field, and these measurements require short-cadence data, additional data is likely needed for the vast majority of our sample in order for such an analysis to come to fruition. In the meantime, spectroscopic activity indicators would be useful to probe the putative transition and compare to the spectroscopic transition considered by previous studies, such as that of Metcalfe et al. (2016).

4.2. Comparison with Previous Work

Metcalf et al. (2016) suggested that spectroscopic diagnostics indicate that the solar dynamo may be in transition due to the loss of the large-scale stellar magnetic field. Although we find a possible transition at a younger age, around 3 Gyr, these two results are not necessarily inconsistent. The transition described in that paper, which matches the theoretical work of van Saders et al. (2016), would suggest that weakened magnetic braking in older stars would lead to a pileup of rotation periods as a function of spectral type. If the collection of spot-dominated stars we observe with rotation periods of ≈15 days were mostly late F stars at a variety of ages, and the facula-dominated stars with rotation periods of ≈25 days were mostly G stars, then this would support these predictions. As before, improved stellar properties and ages would be useful to better understand the relation between the potential transition discussed here and the spectroscopic transition discussed in that paper.

Oláh et al. (2016) found that there is a transition between smooth and complex magnetic cycles at an age of 2–3 Gyr. This is in line with our analysis, as we find a rapid decrease in the fraction of stars with spot-dominated variability during this age range, which corresponds to rotation periods of 17–22 days. It is plausible that the more rapidly rotating, spot-dominated stars contain short-period and long-period cycles (e.g., Brandenburg et al. 2017), leading to a more complex appearance in ground-based data. Additional long-term observations of these stars, leading to a detection of longer-timescale magnetic cycles in these same stars, would support this claim. Saar & Brandenburg (1999) and Böhm-Vitense (2007) divided stars with observed cycles into “active” and “inactive” branches. Some of these stars have cycles that would be observable over the relatively short time baseline of Kepler. These shorter-cycle periods are limited by observational biases from the ground and are not well sampled. Data from Kepler provide an opportunity to identify systems to compare to the ground-based detections of magnetic cycles along both of these branches.

4.3. Particularly Interesting Systems

4.3.1. Possibly Complete Magnetic Cycles

A 4 yr time baseline does not preclude the detection of magnetic cycles. Previous studies have shown the existence of magnetic cycles lasting 2–3 yr on stars (e.g., Saar & Brandenburg 1999). We identify by eye 28 systems that appear to exhibit at least one cycle over the baseline of Kepler. Brandenburg et al. (2017) showed that stars with longer cycles can have additional, shorter cycles with periods of 1–3 yr. Therefore, it is not implausible that some of our stars may have considerably longer cycles in addition to the ones observed in Kepler FFI photometry that will reveal themselves through continued observations.

The observed cycles and rotation periods for these stars are plotted in Figure 9, along with similar stars in the literature. We see that many of these stars follow the “active” and “inactive” branches of Böhm-Vitense (2007). We estimate the scatter by calculating the rms distance from each of the 38 points to the closer of the two lines representing the active and inactive branches in this figure and compare it against the rms distance from 38 randomly drawn points in the same plane. We draw rotation periods following the distribution of rotation periods in
our sample and draw activity cycle periods uniformly over the range shown in Figure 9. The distribution of randomly drawn points is significantly further away from the two branches than the observed potential magnetic cycles. In 500,000 simulated data sets, only 24 of them (0.0048%) have an rms scatter smaller than the real data set. If we only consider the 28 points in our sample in this figure, ignoring the 10 points from previous analyses, then 260 of 500,000 simulated data sets have rms scatter smaller than the real data set. Both of these are significant at the $3\sigma$ level. The difference between these two subsamples is due to the increased variance in the random distribution when selecting a smaller number of test samples: the difference in scatter between our own targets and the two branches compared to the stars from previous studies is not significant.

Additional photometry of these stars to confirm these cycles, better measure their periods, and search for longer cycles that are also possibly present can provide constraints for future models of stellar dynamos. In Table 2, we list those systems observed to have apparently complete magnetic cycles, with an estimate of the observed cycle period. Two of these stars, each with very short periods and likely binary systems, have their cycle observed and period inferred from $S_{ph}$ variability.

### 4.3.2. Short-period Binaries

Several of the stars observed with the most extreme long-term brightness variations have, on inspection of their long-cadence light curves, been revealed as short-period eclipsing binaries. It has long been considered that contact binaries could have extreme magnetic activity that leads to an evolution of their brightness (Applegate 1992). Such brightness variations have been detected, as has indirect evidence for short-period magnetic cycles on contact binaries (Ibanoğlu et al. 2001; Borkovits et al. 2005).

Recently, Marsh et al. (2017) analyzed nearly 10,000 contact binaries observed in the Catalina Real-Time Transient Survey (Djorgovski et al. 2011), finding that 20% of them undergo a linear change in brightness during the survey.

In this work, we largely ignore systems with long-term trends observed in the Kepler FFI photometry. While we can eliminate most false positives for other stars with variability, meaning the results in Table 1 should be dominated by stars with magnetic activity variations, some false positives remain for systems with long-term trends. One false positive that may be common is faint stars in the field of view. The typical Kepler PSF has a radius of 6″ (Bryson et al. 2010); the Kepler Input Catalog only includes targets down to 21st magnitude and is incomplete at the faint end (Brown et al. 2011).

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### Table 2

| KIC ID     | Semiamplitude (%) | Period (yr) |
|------------|-------------------|-------------|
| 2694810    | 0.6               | 2           |
| 3236788    | 2.0               | 3.5         |
| 3743810    | 0.3               | 3           |
| 4555566    | 1.2               | 2           |
| 4726114    | 0.3               | 4           |
| 5352687    | 0.9               | 4           |
| 5450764    | 0.7               | 4           |
| 6038355    | 0.8               | 2.5         |
| 6263983    | 1.0               | 3           |
| 6708110    | 1.2               | 3           |
| 7272437    | 0.1               | 3           |
| 7432092    | 0.3               | 4           |
| 7433192    | 0.2               | 3           |
| 7678238    | 0.6               | 2.5         |
| 8041424    | 1.8               | 1.8         |
| 8043142    | 0.5               | 4           |
| 8345997    | 0.4               | 3           |
| 8750594    | 0.2               | 2.5         |
| 8804069    | 0.9               | 3           |
| 9306271*   | ...               | 1.6         |
| 10087863   | 0.6               | 3           |
| 10122937   | 0.9               | 3.5         |
| 10921242*  | ...               | 3           |
| 11014223   | 1.5               | 2           |
| 11034343   | 0.6               | 2           |
| 11415049   | 0.3               | 2           |
| 11873617   | 0.7               | 2.5         |
| 12417799   | 1.2               | 4.5         |

Note:

* Cycle observed and period inferred from $S_{ph}$ variability.
Photometry with FFI data could be used for detailed modeling of the evolution of these stars. Uncertainties correspond only to the photometric terms in this work, displaying long-term variability that may be evidence for extreme magnetic cycles in these stars. Uncertainties correspond only to the photometric variations from stellar contamination to dilute features in time. KIC 3853405 is an example of a system that is very short-period binaries observed in this work, displaying long-term variability that may be evidence for extreme magnetic cycles in these stars. Uncertainties correspond only to the photometric uncertainties on the individual observations, not to intrinsic stellar variability on the host star itself. For example, KIC 7691547 displays 4% variability peak to peak during each 8 hr orbit, which can be seen as excess scatter in the FFI data but is not accounted for in the uncertainties. Combining long-cadence photometry with FFI data could be used for detailed modeling of the evolution of these systems.

![Figure 10](image_url)  
**Figure 10.** $\beta$ time-series photometry from five short-period binaries observed in this work, displaying long-term variability that may be evidence for extreme magnetic cycles in these stars. Uncertainties correspond only to the photometric uncertainties on the individual observations, not to intrinsic stellar variability on the host star itself. For example, KIC 7691547 displays 4% variability peak to peak during each 8 hr orbit, which can be seen as excess scatter in the FFI data but is not accounted for in the uncertainties. Combining long-cadence photometry with FFI data could be used for detailed modeling of the evolution of these systems.

A faint M dwarf with a high proper motion could then be found to be expanding its PSF into the aperture for any target, leading to a long-term trend in the data. With the current data, we are unable to test this. Detailed PSF modeling of each star may be able to improve this in the future: if certain stars have changing levels of contamination, then their PSFs should be slightly varying relative to other nearby stars.

In some cases, long-term trends can also be caused by varying levels of extinction. This may be especially true for contact binaries or stars near the end of their life cycles, where shells of ejected material are expanding and decreasing in opacity in time. KIC 3853405 is an example of a system that displays this behavior. This contact binary increases in brightness by more than 5% over the Kepler mission (Figure 10). While described in the previous subsection as potential magnetic variability, the variation could also be explained by a change in the opacity of circumstellar dust ejected in a recent outburst or by an actual increase in luminosity as the two stellar cores prepare to merge (Tylenda et al. 2011; Molnar et al. 2017). Detailed modeling of both the long-cadence light curves and FFI photometry combined with additional follow-up observations could be used to better understand this system but is beyond the scope of this paper.

In Table 1 and throughout this work, we reject nearly all systems with long-term trends, as they cannot be uniquely shown to be related to changes in the magnetic activity of the stars. The only exceptions are stars in which the $S_{ph}$ index is also changing in a linear way throughout the Kepler mission. As the apertures used in long-cadence photometry are substantially smaller than ours, we do not necessarily expect FFI brightness variations from stellar contamination to dilute the long-cadence light curves in the same way, so we argue that these changes are likely to be astrophysical. We specifically note these systems in Table 1 and also note that their inclusion or exclusion does not significantly affect the results of our analysis.

5. Conclusions

5.1. FFI as a Probe of Long-term Astrophysical Variability

In this paper, we have presented the method behind the $f_3$ package to produce very long-cadence light curves from Kepler FFI data. These light curves provide photometry at a monthly cadence at 52 epochs across the Kepler mission and generally avoid the removal of astrophysical variations on month-long or slower timescales inherent in the Kepler long-cadence data.

Accurate relative photometry on long timescales is important, as it can be used to probe the long-term behavior of the brightness of astrophysical objects typically removed in transit surveys like Kepler. Here we develop light curves for a sample of Sun–like stars with observed rotation periods to probe changes in stellar activity in time, finding that we do observe evidence for magnetic activity in a substantial fraction of Sun–like stars, especially those that are young and rapidly rotating. We find that there appears to be a transition in the stellar dynamo at a stellar rotation period of 15–25 days, corresponding to a gyrochronological age of 1.5–3.5 Gyr; however, this may reflect a change in stellar parameters, with younger, hotter stars in our sample more likely to be rapidly rotating than older, cooler stars. Stars with rotation periods faster than 15 days have photometric variations that are typically dominated by starspots, while more slowly rotating stars have variations dominated by faculae. Additionally, we find that the more rapidly rotating stars are typically significantly more active than the slowly rotating stars, in line with previous work. We also identify 28 stars with apparently complete short-period magnetic cycles, finding that they are consistent with the “active” and “inactive” branches of stars of Saar & Brandenburg (1999).

Although it is our application in this work, this method is not exclusive to observing magnetic cycles. In principle, photometry from $\beta$ could be used to probe intrinsic brightness variations from variable stars, variable extinction through the ISM, or the evolution of binary stars or evolved stars that affects their observed brightness. To explore this data set to the fullest extent possible, we make our code publicly available for community use and development.

5.2. Future Prospects

Future missions will enable similar measurements on even larger samples of stars. TESS (Ricker et al. 2014) will obtain an
FFI every 30 minutes, enabling aperture photometry with a large number of reference stars at a high cadence. However, each field will only be observed for 1 month at a time, and the continuous viewing zone will only span 1 yr of observations: longer time baselines will only be possible with an extended mission. Stars in the continuous viewing zone will fall on 13 different pixels for 1 month each during the mission, so long-term changes like those observed in this field will require detailed knowledge of the instrument’s flat field. We encourage the TESS team to make every effort to understand the flat field both before launch and during the mission.

Another possibility to detect magnetic cycles on a large number of stars is provided by the Gaia mission (Gaia Collaboration et al. 2016). While its photometric performance is often overlooked relative to its astrometric potential, Gaia will return ≈1 mmag photometry for all stars with $G < 14$ and ≈2 mmag photometry for all stars with $G < 16$ (Jordi et al. 2010). With the typical star observed 70 times over 10 yr, Gaia will provide FFI-like photometry at FFI-like cadence for millions of stars for a decade. Gaia long-term photometry combined with rotation periods from simultaneous observations with K2, TESS, and possibly PLATO (Rauer et al. 2014) will enable detailed photometric studies of magnetic cycles for millions of stars across the galaxy, allowing us to understand the interplay between magnetic activity and photometric modulations across the H-R diagram.

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Software: numpy (Van Der Walt et al. 2011), matplotlib (Hunter 2007), Theano (Theano Development Team 2016), mahotas (Coelho 2013), K2fov (Mullally et al. 2016), f3.

Facility: Kepler.

Appendix

Probabilistic Calibration Model

We model the relative flux $f_{n,t}$ of star $n$ at time $t$ as

\[ \hat{f}_{n,t} = r_{n,m(t)} z_t, \]

where the hat indicates that this is the predicted model value of $f_{n,t}$. $r_{n,m(t)}$ is the total response of the pixels in the aperture for the star at the season $m(t)$ of exposure $t$, and $z_t$ is the time-variable mean zero point of exposure $t$. Since we use aperture photometry to measure the flux with different apertures on different detectors in each season, the relative photometric calibration between seasons is not known a priori. We capture this effect by fitting for the parameters $r_{n,s(t)}$, one for each target in each season. Then, for this model, we assume that the mean zero point for a set of nearby targets varies systematically, and we capture this variability by fitting for a parameter $z_t$ that is shared by all targets at a single exposure $t$. Unlike the Kepler long- and short-cadence light curves, the dominant source of systematic variability on the timescales relevant for the FFIs is not pointing variation. Instead, it is longer-timescale trends—like temperature variations and detector degradation—that affect targets similarly. To complete our probabilistic model, we must also specify a noise model for each measurement. As usual, there is a contribution from the intrinsic measurement uncertainty $\sigma$, but that is not the dominant noise source. We must also take the intrinsic variability of the star and any variance introduced by misspecification of the calibration model into account. To capture these effects, we model the variance for each observation as

\[ \sigma_{n,t}^2 = V_n^2 f_{n,t}^2 + S_r^2 r_{n,s(t)}^2 + \sigma^2, \]

where $V_n$ is the amplitude of the intrinsic variability of star $n$, $S_r$ is the scatter in the zero point of the exposure $t$, and $\sigma$ is the photometric noise level. In this model, we simultaneously fit for the parameters $V_n$, $S_r$, and $\sigma$ along with the calibration parameters. This model can be interpreted as a quantification of the intuition that variable stars should contribute less weight to the calibration and that some epochs will be intrinsically noisier. To break the degeneracy between the pixel responses and the zero points, we regularize the fit by selecting Gaussian priors with unit mean and variance $10^{-4}$ for both $r_{n,m(t)}$ and $z_t$.

The model described in the previous paragraph assumes that the measurements in the light curve are independent. Since the observations are not uniformly distributed in time, this can lead to biased estimates of the pixel responses for targets with long-term variability—exactly the case that we are interested in. To mitigate this issue, we model the true flux of the target star using a Gaussian process with a Matérn-3/2 kernel. This implies a log-likelihood function for the target star of

\[ \mathcal{L}_0(\theta) = -\frac{1}{2} (f_0 - \hat{f}_0)^T K^{-1} (f_0 - \hat{f}_0) - \frac{1}{2} \log \det K - \frac{T}{2} \log 2 \pi, \]

where we have labeled the target star with $n = 0$ and the elements of the covariance matrix $K$ are given by

\[ K_{ij} = \delta_{ij}(\sigma^2 + S_r^2 r_{0,s(t)}^2) + \alpha \left( 1 + \sqrt{3} \frac{|t_i - t_j|}{\tau} \right) \left( \exp \left( -\sqrt{3} \frac{|t_i - t_j|}{\tau} \right) - 1 \right), \]

where $\delta_{ij}$ is the Kronecker delta, $T$ is the number of cadences in the light curve, and $\alpha$ and $\tau$ are parameters of the model.
Applying this model to a target star (labeled \( n = 0 \)) and set of \( N \) reference stars observed at \( T \) cadences, we obtain the log-likelihood function

\[
\mathcal{L}(\theta) = \mathcal{L}_0(\theta) - \frac{1}{2} \sum_{n=1}^{N} \sum_{t=1}^{T} \left( \frac{(f_{n,t} - \hat{f}_{n,t})^2}{\sigma_{n,t}^2} + \log(2 \pi \sigma_{n,t}^2) \right) - \frac{1}{2} \sum_{t=1}^{T} \left( \frac{\varepsilon_t - 1}{\sigma_t^2} \right)^2 - \frac{1}{2} \sum_{n=0}^{N} \sum_{m=1}^{4} \frac{(r_{n,m} - 1)^2}{10^{-4}},
\]

(7)

where \( \theta \) refers to the set of the following parameters:

1. \( 4(N + 1) \) parameters \( r_{n,m}(t) \),
2. \( T \) parameters \( \varepsilon_t \),
3. \( N \) parameters \( V_n \),
4. \( T \) parameters \( S_t \),
5. a single noise parameter \( \sigma \), and
6. the two hyperparameters of the Gaussian process, \( \alpha \) and \( \tau \).

To fit for the maximum likelihood parameters \( \theta^* \), we build this model using \textit{Theano} (Theano Development Team 2016) and maximize Equation (7) using 2000 iterations of the \textit{Adam} algorithm (Kingma & Ba 2014).

Conditioned on these maximum likelihood parameters, the detrended flux for star \( n \) at time \( t \) is given by

\[
\hat{f}_{n,t} = f_{n,t} \ast \hat{r}_{n,s(t)} \ast \hat{S}_{n,t} \ast \hat{r}_{n,m(t)}\ast \hat{\sigma}^2
\]

with predictive variance

\[
\hat{S}^* \hat{r}_{n,s(t)} \ast \hat{S}_{n,t} \ast \hat{r}_{n,m(t)}\ast \hat{\sigma}^2.
\]

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