Constraining the strength of Dark Matter Interactions from Structure Formation

Céline Bœhm\textsuperscript{1}, Pierre Fayet\textsuperscript{2}, and Richard Schaeffer\textsuperscript{3} \textsuperscript{*}

\textsuperscript{1} Astrophysics Lab., Keble Road, Oxford, UK
\textsuperscript{2} Laboratoire de Physique Théorique de l’Ecole Normale Supérieure, UMR 8549 CNRS, 24 rue Lhomond, 75231 Paris Cedex 05, France
\textsuperscript{3} SPhT, CEA Saclay, 91191 Gif-sur-Yvette, France

\textsuperscript{*} Presented by Pierre Fayet

Abstract. We discuss the damping of primordial dark matter fluctuations, taking into account explicitly the interactions of dark matter – whatever their intensity – both with itself and with other particle species. Relying on a general classification of dark matter particle candidates, our analysis provides, from structure formation, a new set of constraints on the dark matter particle mass and interaction rates (in particular with photons and neutrinos).

This determines up to which cross sections the dark matter interactions may effectively be disregarded, and when they start playing an essential rôle, either through collisional damping or through an enhancement of the free-streaming scale. It leads us to extend the notions of Cold, Warm and Hot Dark Matter scenarios when dark matter interactions are no longer taken to be negligible. It also suggests the possibility of new scenarios of Collisional Warm Dark Matter, with moderate damping induced by dark matter interactions.

1 Introduction

We would like to approach here the question of the nature of the non-baryonic dark matter of the Universe – assuming it is made of particles – through the constraints which may be imposed, from structure formation, on the strength of dark matter interactions. Beyond the well-known list of dark matter candidates, we would like to know, in a more general way, which kinds of particles are admissible and which ones ought to be rejected, taking explicitly into account the effects of dark matter interactions not only with itself, but also with other particle species such as, most notably, photons and neutrinos. This presentation, which summarizes some of the main results obtained in Ref. \textsuperscript{[1]}, relies on a general and systematic discussion of dark matter properties in terms of a few specific parameters (typically its mass and interaction rates), independently of any specific underlying particle theory or model one may have in mind.
The interactions of dark matter particles were generally taken as negligible in the past, and therefore disregarded, when discussing structure formation. But such interactions should normally exist, and are even essential when estimating the relic density of dark matter particles having a sizeable mass, which should annihilate at a sufficient rate (otherwise their relic abundance would be too large). The importance of such annihilations is well-known, for example, in the familiar case of “Weakly-Interacting Massive Particles”, the most favored being the neutralino, whose stability results from the R-parity of supersymmetric theories. But even if such interactions are considered as “weak”, is it really legitimate to disregard them when discussing the theory of structure formation? The possible rôle of dark matter interactions is precisely what we would like to study, in a general way, irrespectively of the specific dark matter candidate one may have in mind.

2 Collisional damping and free-streaming scales

Indeed dark matter properties should be such that the primordial fluctuations corresponding to objects of masses larger than about $10^9 M_\odot$ (i.e. to sizes $\approx 100$ kiloparsecs) should not be erased. The damping of the primordial fluctuations may be due to the familiar free-streaming of dark matter particles; but also, prior to this free-streaming, to collisional damping effects resulting from dark matter interactions, when these are explicitly taken into account. Up to which point is it legitimate to consider dark matter particles as effectively free, and from which values of the cross-sections and interaction rates should dark matter interactions start playing a significant rôle?

2.1 Expressions of the damping scales

The free-streaming scale associated with dark matter particles is simply fixed by the distance – as evaluated today – travelled by a freely-propagating dark matter particle, i.e.:

$$l_{\text{free-streaming}} \approx \int_{t_{\text{dec}}}^{t_{\text{collapse}}} \frac{v_{\text{dm}}(t)}{a(t)} dt,$$

in which the integral runs from the time $t_{\text{dec}}$ at which dark matter particles start propagating freely, until the time of the gravitational collapse of the primordial fluctuations, $a(t)$ denoting the scale factor of the Universe at time $t$.

In case one can consider that dark matter particles, alone, form a one-component fluid (i.e. excluding particles of other species that might be in equilibrium with dark matter particles, influencing their damping properties), the collisional damping scale of dark matter fluctuations may be expressed as follows:

$$l_{\text{coll.-damping}}^2 \approx \int_{t_{\text{dec}}}^{t_{\text{collapse}}} \frac{v_{\text{dm}}^2(t)}{\Gamma_{\text{dm}}(t) a^2(t)} dt,$$

1 Unless of course these particles may be diluted by some appropriate inflation mechanism.
Here \( v_{\text{dm}}(t) \) denotes the average quadratic velocity (at time \( t \)) of a dark matter particle, and \( \Gamma_{\text{dm}}(t) \) the corresponding total interaction rate of this particle (the \( ... \) stand for additional factors which usually turn out to be of order unity, as well as an overall normalisation factor, which we disregard here for simplicity). The physical interpretation of this formula will be explained shortly.

However, in a number of situations it is essential to take into account that dark matter particles may remain in equilibrium with particles belonging to other species (generically denoted by an index \( i \), up to a decoupling time \( t_{\text{dec}}(\text{dm-i}) \). It is then possible to view the dark matter particles as forming, together with the particles of these other species, a single (multicomponent) fluid. It is the corresponding damping length of this composite fluid that will be relevant for us.

The collisional damping scale of the fluctuations may in fact be decomposed as a quadratic sum of different contributions, associated respectively with each of the species (including itself) which have been in equilibrium with dark matter before its decoupling. Indeed at any given time \( t \), the instantaneous contribution to the square of the damping length may be viewed as a sum of the corresponding contributions of the individual components of this fluid, with weight factors proportional to the energy densities \( \rho_i \) of particles of species \( i \). Integrating upon time up to the decoupling time of dark matter, we obtain the expression of the collisional damping scale \( l_{\text{c.d.}} \), which may be written in a slightly simplified way as follows (the \( ... \) standing for additional factors which are usually of order unity):

\[
\frac{l^2_{\text{coll.-damping}}}{\Gamma_i} \approx \int_{t_{\text{dec}}(\text{dark matter})}^{t_{\text{dec}}(\text{dark matter})} \sum_{\text{species } i = \{ \text{dark matter, other species} \}} \frac{\rho_i}{\rho} \frac{v_i^2(t) \, dt}{\Gamma_i(t) \, a^2(t)}, \tag{3}
\]

in which \( v_i(t) \) denotes the average quadratic velocity (at time \( t \)) of a particle of species \( i \), and \( \Gamma_i(t) \) the corresponding total interaction rate.

The physical meaning of this formula may be understood easily from the fact that for a particle of velocity \( v_i \) and total interaction rate \( \Gamma_i \) (i.e. average time between two successive collisions \( \tau_i = \Gamma_i^{-1} \)), the corresponding mean free path between two collisions – as it would be measured today – is \( v_i/(\Gamma_i \, a) \), the corresponding contribution to the mean squared distance travelled by the particle being \( v_i^2/(\Gamma_i^2 \, a^2) \). The average number of such collisions during a time interval \( \Delta t \) may be expressed as \( \Delta t/\tau_i \) i.e. \( \Delta t \, \Gamma_i \), resulting in a contribution \( v_i^2 \, \Delta t/(\Gamma_i \, a^2) \) to the mean squared distance travelled by the particle of species \( i \) considered. This contribution should be weighted by a factor proportional to the energy density \( \rho_i \) carried by the particles of species \( i \), divided by the total energy density of the fluid. One should then integrate upon time, up to the decoupling time of dark matter, which leads to equation (3).

The species with which dark matter may be in equilibrium include the dark matter itself. The corresponding contribution to the damping scale, called the
“self-damping scale”, is then given by the following expression

\[ l_{\text{self-damping}}^2 \approx \int_{t_{\text{dec.(dark matter)}}}^{t_{\text{dec.}}} \frac{\rho_{\text{dm}}}{\rho} \frac{\nu_{\text{dm}}^2(t)}{\Gamma_{\text{dm}}(t) a^2(t)} \, dt, \]  

in which \( \Gamma_{\text{dm}} \) denotes the total interaction rate of a dark matter particle (both with other ones and with other particle species). In a similar way, all other species (still denoted by the index \( i \)) lead collectively to the following contribution to the collisional damping scale, that we shall globally refer to as the “induced-damping scale”, given by:

\[ l_{\text{induced-damping}}^2 \approx \sum_{\text{other species \( i \)} \text{ coupled to dark matter}} \int_{t_{\text{dec.(dm-\( i \))}}}^{t_{\text{dec.}}} \frac{\rho_i}{\rho} \frac{\nu_i^2(t)}{\Gamma_i(t) a^2(t)} \, dt. \]  

### 2.2 Constraints on the damping scales

For a dark matter candidate to be acceptable it is necessary to require that the primordial fluctuations in the dark matter density (believed to be at the origin of the formation of the structures) have not been erased as an effect of collisional damping and free-streaming. We shall then obtain constraints on the possible dark matter candidates which may be allowed, by demanding that both the collisional damping and the free-streaming scales be smaller than a scale \( l_{\text{struct.}} \), defined as the scale associated with the smallest objects known to be of primordial origin. I.e.:

\[ \begin{align*}
   l_{\text{coll.-damping}} &\leq l_{\text{struct.}}, \\
   l_{\text{free-streaming}} &\leq l_{\text{struct.}}.
\end{align*} \]  

Here we shall consider that this scale \( l_{\text{struct.}} \) is of the order of 100 kiloparsecs, corresponding to masses of the order of \( 10^9 \) solar masses, e.g. the mass of a small galaxy. Indeed one can evaluate the collapsing mass associated with a scale which was, at collapse time, 100 kpc \( a_c \) to be

\[ \simeq \frac{4\pi}{3} (100 \text{ kpc} a_c)^3 \rho_c \Omega_m/a_c^3 \simeq 1.16 \, \Omega_m h^2 \, 10^9 M_\odot, \]  

(i.e. about \( 0.5 \Omega_m 10^9 M_\odot \) for \( h_0 \approx 0.65 \)), \( \rho_c \) being the critical density of the Universe. Of course if objects of smaller masses were found to be of primordial origin, the constraints discussed here, coming from the inequality (8), could become significantly more stringent.

In practice we shall consider that the constraints (8) may be expressed as

\[ \begin{align*}
   l_{\text{coll.-damping}} &\leq 100 \text{ kpc}, \quad \text{i.e.} \quad l_{\text{self-damping}} &\leq 100 \text{ kpc}, \\
   l_{\text{free-streaming}} &\leq 100 \text{ kpc}.
\end{align*} \]  

We shall therefore evaluate and discuss the different damping scales, in the various possible conceivable situations. For this purpose we shall be led to introduce of general classification of dark matter candidates, according to their mass and interaction rates, as discussed later in section 3.
2.3 A first comparison of self-damping and free-streaming scales

It is already instructive, before presenting such a classification and making more precise statements, to compare the self-damping and free-streaming scales. The free-streaming scale \( l_{\text{free-streaming}} \approx \int_{t_{\text{dec}}}^{t_{\text{collapse}}} \frac{v_{\text{dm}}(t)}{a(t)} dt \), obtained by integrating \( v_{\text{dm}}/a \) between the decoupling time of dark matter and the time of the gravitational collapse, may be roughly estimated \(^2\) as

\[
l_{\text{free-streaming}} \approx \pi \text{ Max.} \left. \frac{v_{\text{dm}}}{a} \right|_{t_{\text{dec}}(\text{dm})}. \tag{9}\]

The accumulated collisional damping scale, on the other hand, turns out, in the relevant cases, to be dominated by the contribution of the late epochs, for which the time \( t \) gets close to the decoupling time of dark matter. The interaction rate of a dark matter particle \( \Gamma_{\text{dm}} \) then gets close to the expansion rate of the Universe \( H = \dot{a}/a \), taken at the decoupling time of dark matter. The self-damping scale given by eq. (4) then verifies, approximately,

\[
l_{\text{self-damping}} \approx \pi \sqrt{\frac{\rho_{\text{dm}}}{\rho}} \text{ Max.} \left. \frac{v_{\text{dm}}}{a} \right|_{t_{\text{dec}}(\text{dm})} \lesssim \pi \left. \frac{v_{\text{dm}}}{a} \right|_{t_{\text{dec}}(\text{dm})}. \tag{10}\]

It is, at most, of the same order as the free-streaming scale (9) (provided of course dark matter particles may actually be in the free-streaming regime after their decoupling, i.e. provided they actually decouple before the gravitational collapse). As a result one generally gets the following inequality between the self-damping and free-streaming scales:

\[
l_{\text{self-damping}} \lesssim l_{\text{free-streaming}}, \tag{11}\]

the two scales being in fact of the same order in a number of situations, that we shall discuss later in section \(^3\).

There is, however, one possible exception to this statement (11): if dark matter were sufficiently strongly interacting so as to remain coupled until the time of the gravitational collapse, there would be no free-streaming at all of dark matter particles! In this rather extreme case, for which dark matter would remain collisional until (and after) collapse, the collisional damping scale is the only one to be considered.

With the exception of this particular situation, the inequality (11) would seem, naïvely, to indicate that no new constraint is to be expected from the consideration of the collisional scale associated with self-damping effects, bounded from above by the free-streaming scale, so that these effects of the interactions could seem irrelevant.

This too naïve conclusion, however, should be corrected, because the evaluation of the free-streaming scale is itself modified, in the presence of interactions. As a result even by considering free-streaming effects only, one may get, in the
case of dark matter particles which decouple after becoming non-relativistic (cf. regions II and III to be discussed later), stronger constraints on the dark matter particle mass by considering its interactions. And this, despite the fact that dark matter particles enter later in the free-streaming regime!

Furthermore, returning to self-damping effects it would be incorrect to disregard them as irrelevant, even if strictly-speaking they do not bring in any additional constraint as compared to free-streaming. Because collisional damping effects are at work before free-streaming, the smaller scale fluctuations are in any case erased by collisional damping, up to a scale \( l_{\text{coll.-damping}} \) (with in a number of cases \( l_{\text{self-damping}} \) as large as \( l_{\text{free-streaming}} \), before free-streaming could have a chance to do it.

## 3 Classification of dark matter particle candidates

To go further and specify the explicit expressions of the various damping lengths and subsequent constraints of the characteristics of dark matter particles, we need to know 1) whether dark matter particles are still relativistic or not at the time of their decoupling; and 2) whether their non relativistic transition and their decoupling transition occur while the Universe is still radiation-dominated (with \( a(t) \approx \sqrt{t/t_{\text{rad}}} \)), or already matter-dominated (with \( a(t) \approx (t/t_{\text{mat}})^{2/3} \)).

This leads us to introduce a general classification of the dark matter particle candidates, according to these criteria – i.e. essentially on their mass \( m_{\text{dm}} \) and the strength of their interactions, which involves their interaction rate \( \Gamma_{\text{dm}} \) evaluated at their decoupling time. We therefore consider the following three characteristic times:

\[
\begin{align*}
  t_{\text{dec}}, & \quad \text{the decoupling time of dark matter,} \\
  t_{\text{nr}}, & \quad \text{at which dark matter particles become non-relativistic,} \\
  t_{\text{eq}}, & \quad \text{the “standard” time of equality between matter and radiation,}
\end{align*}
\]

the corresponding scale factors being denoted by \( a_{\text{dec}}, a_{\text{nr}} \) and \( a_{\text{eq}} \), respectively. \( a_{\text{dec}} \) and \( a_{\text{nr}} \) are both unknown and depend mostly on the strength of the interactions of dark matter particles and on their mass, respectively. \( a_{\text{eq}} \) is in fact defined as a fixed reference time, given as in standard cosmology by \( a_{\text{eq}} = \rho_{\gamma+\nu}(T_{0})/\rho_{\text{matter}}(T_{0}) \), and roughly equal to \( 10^{-4} \Omega_{\text{matter}}^{-1} \Omega_{\text{H}} \) (for a value of the Hubble expansion parameter \( H_{0} \approx 65 \text{ km/s/Mpc} \)).

From the ordering of these three characteristic times or, equivalently, of the corresponding scale factors, we can define six general categories of particles, labelled from I to VI. They may be represented graphically in a plane in which the horizontal axis corresponds to decreasing values of \( t_{\text{nr}} \) \( \square \), i.e. increasing values of the dark matter particle mass \( m_{\text{dm}} \); and the vertical axis to increasing

\[2 \quad \text{On the horizontal axis, we represent the time } t_{\text{nr}} \text{ of the non-relativistic transition of dark matter by plotting a quantity proportional to } 1/a_{\text{nr}} \text{. } t_{\text{nr}} \text{ is defined as the mo-} \]
values of the decoupling time \( t_{\text{dec}} \). It can be considered, very roughly, as an “interaction strength versus mass” representation plane: cf. Figure 1 of Ref. [1].

The first three regions (I to III) in this plane correspond to dark matter particles which get non-relativistic before the standard time of matter/radiation equality (which is in these cases the actual equality time), i.e. for which

\[
a_{\text{nr}} < a_{\text{eq}} \simeq 10^{-4} \Omega_{\text{matter}}^{-1} .
\]  

(13)

This corresponds, typically, to dark matter particle candidates heavier than a few eV’s. Their classification is given in Table 1, which may be read from bottom to top, according to increasing interaction strength (and therefore later decoupling) of dark matter particles. We also mention for completeness the three other situations corresponding to what we call regions IV, V and VI, for which \( a_{\text{eq}} < a_{\text{nr}} \).

This concerns rather unconventional situations, usually excluded, of very light dark matter particles of mass \(< \sim \) a few eV’s, that would get non-relativistic only after the “standard” matter-radiation equality time, and lead in general to excessive damping.

4 Comparison of the self-damping and free-streaming scales

We now concentrate on regions I, II and III – in the order of increasing dark matter interaction strength.

Within region III one may even consider, at least theoretically, the possibility that dark matter particles might be so strongly interacting that they would remain coupled until (and after) the time of the gravitational collapse (for a scale factor \( a \) typically \( \sim 1/10 \)). We shall refer to this specific subdomain of the plane for which the dark matter temperature drops down to \( T_{\text{dm}}(t_{\text{nr}}) = m_{\text{dm}}/3 \).

The dark matter and photon temperatures may be parametrized as \( T_{\text{dm}}(t) = T_0/\kappa_{\text{dm}}(t) a(t) \) and \( T(t) = T_0/\kappa(t) a(t) \), respectively (where \( \kappa_{\text{dm}} \) and \( \kappa \) essentially constant by intervals, account for the effects of the effective numbers of interacting degrees of freedom, and \( T_0 \simeq 2.73 \) K). We ultimately plot, on this horizontal axis, the product \( m_{\text{dm}} \kappa_{\text{dm}} = 3 T_0 a_{\text{nr}} \simeq 7 \) eV \( 10^{-4}/a_{\text{nr}} \). The boundary between regions IV-VI and I-II-III corresponds to \( a_{\text{nr}} = a_{\text{eq}} \simeq 10^{-4} \Omega_{\text{matter}}^{-1} \) and therefore \( m_{\text{dm}} \kappa_{\text{dm}} \simeq 7 \) eV \( \Omega_{\text{matter}}^{-1} \), so that regions I-II-III normally concern particles heavier than a few eV’s.

The total interaction rate of a dark matter particle \( \Gamma_{\text{dm}} \), evaluated at the dark matter decoupling time \( t_{\text{dec}} \), is equal to the corresponding value of the Hubble parameter \( H = \dot{a}/a \). The more strongly a dark matter particle is coupled, the later it decouples, and the smallest is its total interaction rate at decoupling \( \Gamma_{\text{dm}}(t_{\text{dec}}) \equiv H(t_{\text{dec}}) \). It is natural to normalize the total interaction rate \( \Gamma_{\text{dm}} \) relatively to the inverse of the volume of a comoving cell, i.e. relatively to \( 1/a^3 \). Increasing values of \( \Gamma_{\text{dm}} a^3 \) (evaluated at \( t_{\text{dec}} \)) now correspond to more strongly coupled particles, which decouple later. More specifically, we plot on the vertical axis \( \Gamma_{\text{dm}} a^3 \), evaluated at the decoupling time of dark matter (or collapse time if dark matter particles were still coupled at that epoch).
Table 1. Classification of Dark Matter candidates according to the strength of their interactions (for Dark Matter particles heavier than about a few eV's).

| Region        | Condition                | Dark Matter Status                                                                 |
|---------------|--------------------------|------------------------------------------------------------------------------------|
| III           | $a_{nr} < a_{eq} < a_{dec}$ | Dark Matter particles get non-relativistic before equality, but decouple after equality |
| II            | $a_{nr} < a_{dec} < a_{eq}$ | Dark Matter particles first get non-relativistic, then decouple, before equality (e.g. LSP neutralino, ...) |
| I             | $a_{dec} < a_{nr} < a_{eq}$ | Dark Matter particles first decouple, then get non-relativistic, before equality (e.g. gravitino of $\sim$ keV, or massive neutrinos slightly heavier than a few eV's) |

“Very strongly interacting” dark matter particles

“No interactions”, i.e. quasi-free dark matter particles

The parameter space as region III'. In this case no free-streaming at all of dark matter particles is to be considered, the only possible damping of primordial dark matter in this extreme case being collisional damping.

The comparison of the self-damping and free-streaming scales (cf. eqs. (9-11) and section 5) is given in Table 2. In regions II and III (excluding III'), for which dark matter particles decouple only after becoming non-relativistic (but before the gravitational collapse), the self-damping and free-streaming scales may be of the same order. In this case

\[
\text{collisional damping } \quad \text{(not free-streaming)}
\]

is actually at the origin of the damping of most of the primordial fluctuations!

Collisional damping then appears as an efficient mechanism at work to erase the primordial fluctuations, even in situations where these fluctuations were previously believed to be erased by free-streaming effects!
Table 2. Comparison of self-damping and free-streaming lengths.

| Region            | Description                                      | Equation                                      |
|-------------------|--------------------------------------------------|-----------------------------------------------|
| Region III'       | Collisional damping only, no free-streaming !    |                                               |
| Region III (excluding III') |                      | $l_{\text{self-damping}} \approx l_{\text{free-streaming}} \approx \pi \frac{v_{\text{dm}} t}{a} \left| t_{\text{dec}} \right| \approx \pi \frac{v_{\text{dm}} t}{a} \left| t_{\text{eq}} \right|$ |
| Region II         |                      | $l_{\text{self-damping}} \lesssim l_{\text{free-streaming}} \approx \pi \frac{v_{\text{dm}} t}{a} \left| t_{\text{dec}} \right|$ |
| Region I          |                      | $l_{\text{self-damping}} < l_{\text{free-streaming}} \approx \pi \frac{c t}{a} \left| t_{\text{nr}} \right|$ (in connection with the early decoupling of dark matter, before it gets non-relativistic) |

5 The enhancement of the free-streaming scale, as a result of the interactions

Furthermore, the explicit expression of the free-streaming scale (1) is itself modified, when the effects of dark matter interactions are taken into account. Indeed the velocity of a non-relativistic dark matter particle of mass $m_{\text{dm}}$ decreases more slowly with time if it is interacting ($v_{\text{dm}} \propto \sqrt{T_{\text{dm}}(t)} \propto 1/\sqrt{a(t)}$) than if it propagates freely ($v_{\text{dm}} \propto 1/a(t)$).

This leads to an enhancement of the free-streaming scale, for particles in regions II and III (as compared to region I). Indeed from the time $t_{\text{dec}}$ such dark matter particles start propagating freely (after getting non-relativistic at $t_{\text{nr}}$), they actually do so with a higher velocity than if they were propagating freely since the time $t_{\text{nr}}$ of their non-relativistic transition. The corresponding enhancement factor of their velocity at and after the decoupling time is roughly $\sqrt{a_{\text{dec}}/a_{\text{nr}}}$.

As a result the free-streaming scale of a dark matter particle of a given mass $m_{\text{dm}}$ turns out to be larger in regions II or III (excluding III') than in region I, despite the fact that in these regions the particle remains longer in the collisional regime!

In region I the integral (1) (with $v_{\text{dm}}(t) \approx c a_{\text{nr}}/a(t)$ once the particle gets non-relativistic) yields a free-streaming scale roughly of the order of $\pi c t_{\text{nr}}/a_{\text{nr}} \approx \pi c t_{\text{nr}} a_{\text{nr}}$. It increases with $t_{\text{nr}}$ roughly as $\sqrt{a_{\text{nr}}}$, i.e. as $a_{\text{nr}}$, or $1/m_{\text{dm}}$.

In region II, the free-streaming scale is roughly of the order of $\pi v_{\text{dm}} t/a$ evaluated at $t_{\text{dec}}$, with $v_{\text{dm}}(t_{\text{dec}}) \approx c \sqrt{a_{\text{nr}}/a_{\text{dec}}}$. It is therefore the order of $\pi c t_{\text{rad}} \sqrt{a_{\text{nr}} a_{\text{dec}}}$ (which may also be expressed as $\pi c (t_{\text{nr}} t_{\text{dec}}/a_{\text{nr}} a_{\text{dec}})^{1/2}$),
i.e. larger than in I, precisely by an enhancement factor $\approx \sqrt{a_{\text{dec}}/a_{\text{nr}}}$. The free-streaming length increases with $t_{\text{dec}}$ roughly as $(t_{\text{dec}})^{1/4}$, or $\sqrt{a_{\text{dec}}}$.

In an analogous way in region III, the free-streaming scale is still roughly $\approx \pi v_{\text{dm}} t/a$ evaluated at $t_{\text{dec}}$, as in region II. But we may now evaluate this quantity, just as well, at the earlier time $t_{\text{eq}}$. It may then be expressed as $\approx \pi c t_{\text{rad}} \sqrt{a_{\text{nr}} a_{\text{eq}}}$ (also $\approx \pi c (t_{\text{eq}} v_{\text{nr}} a_{\text{eq}})^{1/2}$), and is larger than in region I by a factor $\approx \sqrt{a_{\text{eq}}/a_{\text{nr}}}$ (the enhancement factor of the velocity evaluated at $t_{\text{eq}}$, which behaves roughly like $(t_{\text{eq}}/t_{\text{nr}})^{1/4}$). This enhancement factor may be significantly larger than unity, especially in the case of heavy particles, which get non-relativistic very early. Altogether:

In regions II and III (excl. III'):

the free-streaming length is actually enhanced as an effect of the interactions!

(15)

As a result (and despite the fact that the constraints from self-damping may be at most as strong as the ones obtained from free-streaming only, cf. eqs. (9-11) and Table 2), taking explicitly into account the effects of interactions finally turns out to be essential, in both regions II and III, to derive the constraints on the characteristics of dark matter.

6 Redefining the Cold, Warm and Hot Dark Matter scenarios, in the presence of interactions

In region I, for which dark matter particles are so weakly interacting that they decouple before getting non-relativistic, the discussion is simple, and the result finally does not depend on the effects of the interactions. Since $l_{\text{self-damping}} < l_{\text{free-streaming}}$, the self-damping constraint may be ignored in view of the free-streaming one. Furthermore the expression of the free-streaming length itself is not significantly affected by the effects of the interactions and the exact decoupling time of dark matter (which occurs before dark matter particles get non-relativistic). The free-streaming scale of a particle in region I finally reads

$$l_{\text{free-streaming}} \approx \frac{\pi c t_{\text{rad}}}{a_{\text{nr}}} \approx \frac{\pi c t_{\text{rad}}}{a_{\text{nr}}} \approx \frac{\pi c t_{\text{rad}}}{3 T_{\text{o}}/m_{\text{dm}} \kappa_{\text{dm}}}.$$  

(16)

It behaves roughly as the inverse of the mass of the dark matter particle. To get a first idea on the value of this scale we may use $c t_{\text{rad}} \approx c t_{\text{o}}/\sqrt{a_{\text{eq}}} \approx 10^{28} \text{ m} \approx 3 \times 10^{8} \text{ kpc}$. With $3 T_{\text{o}} \approx 7 \times 10^{-4} \text{ eV}$ we then get $l_{\text{f.s.}} \approx 100 \text{ kpc}$ $(10 \text{ keV}/m_{\text{dm}} \kappa_{\text{dm}})$ as a rough order of magnitude estimate.

Particles heavier than about $\sim 10 \text{ keV}$, approximately, would then lead to acceptable values of the free-streaming scale less than about 100 kpc. In the lower part of the allowed mass range, this corresponds to “warm dark matter” scenarios, as when interactions were disregarded; and to “cold dark matter”, in the case of heavier particles.

We recover at this stage, in the lower part of region I, the usual notions of Cold, Warm and Hot dark matter, as they may be classified according to the dark
matter particle mass or to the corresponding value of the free-streaming scale \(a\) or \(v\). “Hot dark matter” (made of, e.g., light massive neutrinos, ...) is excluded due to excessively large free-streaming scales, “warm dark matter” corresponding to free-streaming scales nearly as large as the allowed value taken of about \(\sim 100\) kpc, and “cold dark matter” to significantly smaller values of this scale.

We are now able to extend these notions by taking into account, systematically, the effects of dark matter interactions, which may both 1) enhance the value of the free-streaming scale and 2) provide physically-interesting collisional damping effects, either from self-damping or from induced-damping. The boundary between region I and region II, which obeys the equation \(a_{\text{dec}} = a_{\text{nr}}\), indicates the level at which dark matter interactions start modifying significantly the damping of the primordial dark matter fluctuations. A dark matter particle with a mass of \(\sim 1\) MeV for example (assuming appropriate dilution or annihilations to have occurred, for a suitable relic abundance), which would normally have been considered as leading to relatively small damping compared to the allowed scale of 100 kpc, may actually become a warm dark matter candidate, as a result of the enhancement of the free-streaming scale in region II, by a factor \(\sqrt{a_{\text{dec}}/a_{\text{nr}}}\).

In region III the enhancement factor of the free-streaming scale, \(\sqrt{a_{\text{eq}}/a_{\text{nr}}}\), may be as large as \(\sim 10^3\) in the case of a \(\sim 10\) MeV particle, which may then acquire a just-acceptable damping length of \(\sim 100\) kpc (as compared to \(\sim 100\) pc for a particle of the same mass, in region I, according to equation (16)).

Beyond that, we can propose a further redefinition of the notions of cold, warm and hot dark matter by considering not only the free-streaming scale alone (even modified by the effects of the interactions), but the whole damping scale obtained by combining the collisional (self-damping plus induced damping) and free-streaming scales. In this way we take into account the various possible dark matter interactions, both with itself and with other particle species such as photons and neutrinos. CDM will then correspond to values of the damping scales significantly smaller than \(l_{\text{struct}}\), and WDM to values nearly as large as the allowed value.

Before turning to induced-damping effects, for which the damping of dark matter fluctuations is due to the transport of energy and momentum by other particle species, we refer to Fig. 1 of Ref. [1]. In this plane dark matter interaction rate \(I_{\text{dm}} a^4\) at decoupling versus dark matter particle mass \(m_{\text{dm}}\) (times \(\kappa_{\text{dm}}\)), we represent the regions excluded by both self-damping and free-streaming

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\(^4\) One should also not forget the familiar relic density constraints. This normally excludes regions of the parameter space corresponding to heavy particles too weakly coupled to annihilate sufficiently, so that their resulting relic abundance would be too large (unless they have been appropriately diluted by some prior inflation phase). This is particularly important in a large part of region I, corresponding to heavy dark matter particles that would decouple before getting non-relativistic — in contrast with the standard case of heavy neutralinos for example, which normally decouple after getting non-relativistic and belong to region II.
effects. These excluded regions may be viewed as corresponding to extensions of “hot dark matter” scenarios to the new cases for which collisional damping as well as interaction-enhanced free-streaming effects are taken into account. The regions relatively close to the excluded ones, for which the damping scale is not much lower than the scale \( l_{\text{struct}} \) of the smallest primordial objects, may be considered as corresponding to extensions of “warm dark matter” scenarios.

7 Induced-damping by photons or neutrinos

One should also consider the additional damping effects induced by other particle species to which dark matter may be coupled, such as the photons and the neutrinos. The magnitude of these effects is given by the induced-damping scale associated with a species \( i \), as expressed by eq. (5), the integral over time running until the decoupling time of dark matter from this species \( i \).

Photons and neutrinos may be particularly relevant, not only because they are relativistic \((v_i = c)\), but also because they could interact long enough with dark matter, if their interaction cross-sections with dark matter particles turned out to be sufficiently large. The corresponding induced-damping scales, given by

\[
    l_{\text{photon-induced damp.}} \approx \int_{t_{\text{dec}}}^{t_{\text{dec}}+t_{\gamma}} \frac{\rho_\gamma}{\rho} \frac{c^2 \, dt}{I_\gamma(t) \, a^2(t)},
\]

for photons, and similarly for neutrinos, should both be smaller than the length \( l_{\text{struct}} \), here taken to be \( \sim 100 \text{ kpc} \). In particular, the constraint \( l_{\text{photon-induced damp.}} \lesssim 100 \text{ kpc} \) leads us to upper limits on the interaction rates of dark matter particles with photons at the corresponding decoupling time, and therefore to upper limits on the dark-matter/photon interaction cross-sections. (The more dark matter particles interact with those of a given species, the longer the latter can influence their properties, and the larger is the resulting contribution to the induced-damping length of the primordial dark matter fluctuations.) The moment at which the photons cease to communicate their damping to the dark matter fluid should occur before the recombination epoch in order to avoid prohibitive damping effects. The damping induced by neutrinos is normally expected to be very small, excepted in special situations for which dark matter would decouple from neutrinos at a temperature \( T < 1 \text{ MeV} \). In that case “freely-propagating neutrinos” could influence the properties of dark matter and induce a non-negligible “collisional damping” of its primordial fluctuations. We refer the reader to Refs. [1,3] for a more detailed discussion of these photon or neutrino-induced damping effects.

References

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