Applying Multi-qubit Correction to Frustrated Cluster Loops on an Adiabatic Quantum Computer

John E. Dorband
Department of Computer Science and Electrical Engineering
University of Maryland, Baltimore County
Maryland, USA
dorband@umbc.edu
February 18, 2019

Abstract
The class of problems represented by frustrated cluster loops, FCL, is a robust set of problems that spans a wide range of computational difficulty and that are easy to determine what their solutions are. Here, we use frustrated cluster loops to test the relative performance of the D-Wave without post-processing and the D-Wave with multi-qubit correction (MQC) post-processing. MQC post-processing has shown itself exceptionally beneficial in improving the performance of the D-Wave 2000Q when processing difficult FCL problems.

1 Introduction
The D-Wave\cite{4} is an adiabatic quantum computer\cite{2, 8} which supports the following objective function:

\[ F = \sum_i a_i q_i + \sum_i \sum_j b_{ij} q_i q_j \]  

where \( q_i \in \{-1, 1\} \) are the qubit values returned by the D-Wave, and \( a_i \in [-2, 2] \) and \( b_{ij} \in [-1, 1] \) are the coefficients given to the D-Wave associated with the qubits and the qubit couplers respectively.

An algorithm has been developed, multi-qubit correction, MQC, that reduces a set of D-Wave samples to a single sample that has an objective function value that is less-than or equal-to the objective function value of any sample in the D-Wave sample set. This algorithm presumes that the D-Wave samples contain groups of qubits that can be used to construct a more optimal solution to the objective function. This has borne out to be true, as has been presented in the paper \cite{1}.

The problem class designated as frustrated cluster loops have been used to show how well a quantum annealer’s performance compares to that of a simulated annealer running on a classical computer. Two such methods are described in \cite{3} and \cite{5}. Other similar problems have been constructed to demonstrate quantum speedup such as \cite{6} and \cite{7}, but will not be utilized here. These problems have been developed to show if and when a quantum annealer such as the D-Wave exceeds the performance of a classical simulated annealer.

The purpose here however is not to show how the D-Wave compares to classical computers, but to show how the D-wave performs with and without MQC post-processing. The two problem classes that will be used in the comparison will be designated as type 1 described in \cite{3} and type 2 described in \cite{5}. Granted this is not aimed at showing quantum supremacy, but is to show how
much faster the optimal result can be obtained on the D-Wave with MQC post-processing than with the D-Wave alone.

The advantage of using frustrated cluster loops (FCL) as a means of testing an adiabatic quantum computer is two fold, 1) the solution to an FCL problem is easily determined before it is run and 2) cases of FCL can be easily generated that have wide variations in their solution difficulty. The solution to an FCL problem (the global minimum of the objective function) is simply the sum of all the qubit and coupler coefficients. The difficulty of an FCL problem is measured by how many samples it takes for it to be solved. Figures 3 and 6 show plots of the $\log_2$ of how many samples it takes for the D-Wave to solve various cases of FCL problems. They range from 1 sample to $2^{13} = 8192$ samples. And in some cases the D-Wave never found the solution.

2 The Frustrated Cluster Loop Problem

A frustrated cluster loop (FCL) problem on the D-Wave consists of creating randomly generated loops over a group of qubits. A loop consists of a chain of qubits interleaved with couplers connected in a closed loop. These loops take random paths of random length around the group of qubits. The qubit coefficients are given a value of zero and the coupler coefficients are given a value of -1, except one random coupler which is given a coupler coefficient value of 1. A number of loops are generated for the group of qubits equal to the number of qubits in the group times a constant, $\alpha$. The qubit and coupler coefficients of the loops are summed together for each corresponding qubit and coupler in the group. This forms the problem to be optimized on the D-Wave. For the D-Wave, the groups of qubits are specified in terms of square regions of qubit cells. A region of size $L_c$ designates a qubit group of size $c$ by $c$ cells of 8 qubits each. Thus for a qubit group of size $L_4$, 128 qubits, and an $\alpha$ value of 0.1, 12 or 13 loops would be randomly generated for a single frustrated cluster loop problem.

The difference between a type 1 problem and a type 2 problem is how a minimum size problem is determined. A type 1 problem must contain at least 8 qubits, while for a type 2 problem a loop must contain qubits from more than one cell. This allows a type 2 problem to contain as few as 6 qubits.

An overlap factor, $R$, can also be specified. The overlap factor indicates the maximum number of loops that can include a specific coupler. If, as loops are being generated, a loop attempts to use a coupler that already is being used by $R$ loops, that loop must be discarded. $R$ is a metric of ruggedness. The larger $R$ is, the harder it is for hardware (D-Wave) to represent the problem coefficients accurately. The values of $R$ used here are 2, 3, and $\infty$ (unlimited).

3 Testing MQC With Frustrated Cluster Loops

The frustrated cluster loop problem is used here to compare the performance of the D-Wave to the D-Wave with MQC post-processing. MQC takes a group of D-Wave samples and reduces them to a single sample as described in [1]. Since each sample takes a fixed time, about $20\mu s$, the number of samples needed to solve a problem will be used in place of a metric of time.

For the D-Wave without MQC post-processing, a solution is determined to be found within a number of samples, if one of those samples is a solution. For the D-Wave with MQC post-processing, a solution is determined to be found within a number of samples, if those samples have been reduced to a solution.
3.1 Initial Test

The purpose of this initial test is to show the diversity in the complexity and difficulty of the frustrated cluster loop class of problems. The difficulty of a case of this problem class is demonstrated by how quickly the D-Wave solves it or whether it solves it at all. Figure 1d is an example of a case that the D-Wave solves easily, while figure 1a is an example of a hard case that the D-Wave did not solve within 81920 samples. The initial test was performed for the following problem generation parameters, $R = \infty$, $c = 16$, and for $\alpha$ between 0.1 to 0.5 in increments of 0.1. 50 cases were randomly generated, 10 cases for each of the 5 values of $\alpha$. 81920 samples were requested from a 2048 qubit D-Wave 2000Q for each case.

Figures 1 and 2 show arbitrary examples of FCL problems that demonstrate the diversity of relative behavior of the D-Wave with MQC post-processing and without MQC post-processing. All the cases in 1 where created with a value of $\alpha = 0.1$. In each of these case a probability of 1.0 indicates that for every group of size $2^n$ samples, a sample was attained with an objective function value equal to the global minimum. A probability of 0.5 indicates that only half of the groups of size $2^n$ samples attained the global minimum. In figure 1a the D-Wave attained the global minimum consistently for sample groups larger than $2^{11} = 2048$ samples while the D-Wave with MQC post-processing attained the global minimum consistently for sample groups of size $2^7 = 128$ or larger. Therefore MQC post-processing increased the speed by a factor of 16. In figure 1d out of 81920 samples the D-Wave never found the global minimum, while MQC consistently found the global minimum with 1024 samples. Figure 1 shows a range of difficulty of cases, from 1a which is moderately hard to 1d which was very hard. Figure 2 shows cases with different values of $\alpha$ where 2c and 2d are very easy and gain very little benefit from the use of MQC and 2a and 2b which are much harder to the extent that MQC finds the global minimum for 2a only 90% of the time with a group size of 8192 samples and the D-Wave alone only 40% of the time.
3.2 Problem Class Survey

In section 3.1 for each case, 81920 samples were obtained from the D-Wave for analysis. This section surveys a much larger parameter space so only 8192 samples were obtained for each case. The survey is performed over various ranges of the parameters $R$, $L_c$, and $\alpha$. The values of $R$ are 2, 3, and unlimited ($\infty$), the values of $c$ for $L_c$ are 2, 3, 4, ..., 16, and the values of $\alpha$ range from 0.05 to 0.5 in increments of 0.05. There were 100 cases generated for each $R$, $L_c$, and $\alpha$. For each case, 8192 samples were requested from a 2048 qubit D-Wave 2000Q.

For each case the 8192 samples were divided into groups, 8192 groups of size 1, 4096 groups of size 2, 2048 groups of size 4, and so forth up to 2 groups of size 4096 and 1 group of size 8192, in powers of two. Each group of D-Wave samples is considered to have succeeded if at least one sample in the group has an objective function value that is equal to the global minimum. Each group of D-Wave samples, post-processed with MQC, is considered to have succeeded if its resultant sample has an objective function value that is equal to the global minimum.

Figure 3 (D-Wave alone) and figure 4 (D-Wave w/ MQC) present the results for type 1 FCL problems as a family of curves where each line represents a different size partial D-Wave from a 2x2 cell partition to the full 2048 qubit, 16x16 cell D-Wave. When the problem is easier, such as a 2x2 cell partition or when $\alpha$ is between 0.3 and 0.5, MQC post-processing gives very little improvement over the D-Wave results alone. However when the problems get harder, when using the entire D-Wave and $\alpha$ is around 0.1, MQC post-processing gives as much as a 10 to 20 times improvement over the D-Wave alone. Figure 5 was added to show a clearer distinction between the behavior of the D-Wave alone and the D-Wave with MQC post-processing on type 1 FCL on a full 2048 qubit D-Wave. Note that there is no difference in the behavior for a 32 ($c = 2$) qubit partial D-Wave.
Figure 5: The number of samples from the D-Wave needed to solve a type 1 frustrated cluster loop problem. Comparing only D-Wave sizes of $c = 2$ ($L_c = 32$) and $c = 16$ ($L_c = 2048$).

Figure 6: The number of samples from the D-Wave needed to solve a type 2 frustrated cluster loop problem.

Figure 7: The number of samples from the D-Wave needed for MQC to solve a type 2 frustrated cluster loop problem.

Figure 8: The number of samples from the D-Wave needed to solve a type 2 frustrated cluster loop problem. Comparing only D-Wave sizes of $c = 2$ ($L_c = 32$) and $c = 16$ ($L_c = 2048$).
In all such cases that were run, MQC post-processing found the answer.

Figure 9: The number of type 1 cases out of 100 cases that the D-Wave was unable to solve within 8192 samples. It appears to be worse for type 2 problems than for type 1 problems. That is, if the problems are easy, MQC shows very little improvement but if the problem is hard MQC may find the answer 10 to 20 times faster. Figure 8 was added for the same reason Figure 6 was, to show more clearly the differences in the behavior of a full 2048 qubit D-Wave on type 2 FCL problems.

In some cases, the problems are so hard that the D-Wave is unable to find the answer within 8192 samples. Figure 9 (type 1) and figure 11 (type 2) show the number of cases out of 100 that D-Wave does not solve within 8192 samples. It appears to be worse for type 2 problems than for type 1 problems. This only occurred for D-Wave partitions sizes corresponding to c = 10 or greater. In all such cases that were run, MQC post-processing found the answer.

Figures 10 (type 1) and 12 (type 2) show the number of cases needed on average for MQC to solve a type 1 FCL that the D-Wave was unable to solve.

Figure 11: The number of type 2 cases out of 100 cases that the D-Wave was unable to solve within 8192 samples.

\[ R = 2 \]

\[ R = 3 \]

\[ R = \infty \]
post-processing to find a solution that the D-Wave alone did not solve with 8192 samples. Note that, in all the cases that the D-Wave was unable to solve in 8192 samples, the D-Wave with MQC post-processing was able to solve, and in most case in less than 1000 samples.

4 Conclusion

The FCL problem class has been used to test the relative performance of the D-Wave with and without MQC post-processing. FCL problems were generated with a wide variety of difficulty, from problems that the D-Wave always solved to problems that it never solved. MQC post-processing has shown exceptional performance over the D-Wave without post-processing. MQC post-processing provided little benefit if the problem was easy, as one would expect. However, MQC post-processing has demonstrated a 10 to 20 times improvement in execution speed for problems of moderate difficulty (problems that the D-Wave can solve) and has solved problems the D-Wave was unable to solve alone even though 81920 samples were requested. These unsolved problems were often solved by MQC post-processing with less than D-Wave 1000 samples. The D-Wave with MQC post-processing was able to solve all cases of FCL, that were attempted.

Acknowledgment

The author would like to thank Michael Little and Marjorie Cole of the NASA Advanced Information Systems Technology Office for their continued support for this research effort under grant NNH16ZDA001N-AIST16-0091 and to the NASA Ames Research Center for providing access to the D-Wave quantum annealing computer. In addition, the author thanks the NSF funded Center for Hybrid Multicore Productivity Research, D-Wave Systems for their support and access to their computational resources, and for quantum computing system resources supported by the Quantum Computing Institute at Oak Ridge National Laboratory.

References

[1] J. E. Dorband. Improving the Accuracy of an Adiabatic Quantum Computer. ArXiv e-prints, May 2017.

[2] E. Farhi, J. Goldstone, S. Gutmann, and M. Sipser. Quantum Computation by Adiabatic Evolution. eprint arXiv:quant-ph/0001106, January 2000.
[3] I. Hen, J. Job, T. Albash, T. F. Rønnow, M. Troyer, and D. A. Lidar. Probing for quantum speedup in spin-glass problems with planted solutions. 92(4):042325, October 2015. doi: 10.1103/PhysRevA.92.042325.

[4] D-Wave Systems Inc. The D-Wave 2X Quantum Computer: Technology Overview. http://www.dwavesys.com/resources/publications, 2013. [Online; accessed 18-May-2016].

[5] A. D. King, T. Lanting, and R. Harris. Performance of a quantum annealer on range-limited constraint satisfaction problems. ArXiv e-prints, February 2015.

[6] J. King, S. Yarkoni, J. Raymond, I. Ozfidan, A. D. King, M. M. Nevisi, J. P. Hilton, and C. C. McGeoch. Quantum Annealing amid Local Ruggedness and Global Frustration. ArXiv e-prints, January 2017.

[7] S. Mandrà and H. G. Katzgraber. A deceptive step towards quantum speedup detection. Quantum Science and Technology, 3(4):04LT01, October 2018. doi: 10.1088/2058-9565/aac8b2.

[8] Giuseppe Santoro and Erio Tosatti. Optimization using quantum mechanics: quantum annealing through adiabatic evolution. Journal of Physics A: Mathematical and Theoretical, 41(20):209801, 2008. URL http://stacks.iop.org/1751-8121/41/i=20/a=209801