Research on Command Allocation Method for Flying Wing Aircraft

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Abstract. Flying wing has nice aerodynamic efficiency, good loading and stealth characteristics, it is an ideal aerodynamic configuration for reconnaissance, transportation and bombing type aircraft. However, flying wing aircraft suffers from poor stability and control characteristics, which makes it difficult to design and control. Due to elimination of horizontal and vertical tails, flying wing aircraft has poor longitudinal and directional dynamic characteristics. Unlike most of the conventional aircraft, flying wing aircraft usually have a large number of control effectors, the redundant effectors are designed and required to provide adequate lift, pitch and yawing force or moments. There fore, flying wing aircraft usually need to adopt reasonable control allocation algorithms to exploit the control capability of redundant effectors. Especially for large-aspect ratio flying wing with very high level of effector redundancy, the pursuit of performances, e.g., gust load alleviation control and maximum lift mode, makes control allocation extremely important and indispensable. This paper summarizes the typical characteristics of command allocation methods, and its applicability in different flight missions, providing a reference for the design of command allocation method for flying wing aircraft.

1. Introduction
When compared with conventional aircraft, the longitudinal and lateral static stability of flying wing aircraft is greatly degraded, the directional axis usually is weakly unstable, and the three-axis damping is significantly reduced. Hence the dynamics characteristics of flying wing aircraft is difficult to meet the flying quality requirements, and control augmentation system is necessity for flying wing. Since horizontal and vertical tails have been cancelled, flying wing aircraft usually adopts a combination of multiple sets of elevens and new type of drag rudder to meet the aircraft's requirements for control power.

The flying wing aircraft adopts redundant multi-control surface configuration to improve control power, but each control surface has multi-axis control capability and there is strong control coupling effect. In order to solve the problem of control surface redundancy and the axial function of the control surface is not clear, it is necessary to introduce a control allocation unit in the flight control system to map the control command required by the control law to the redundant control surfaces in real time, so that the aircraft can accurately complete the flight mission. Conventional aircraft usually manual decouples control surface function according to control characteristics of effectors, although this method can reduce the complexity of control allocation unit, which would limit the control surface's control effectiveness, especially for flying wing aircraft, and cannot achieve the maximum attainable moment set (AMS) of control surfaces. Therefore, it is necessary to design a reasonable control allocation unit for the allocation of control command in order to maximize the rudder surface control ability of the
aircraft.

This paper studies and summarizes common control allocation methods for flying wing aircraft, including generalized inverse, daisy chain, direct allocation, and non-linear optimization allocation, etc., which aims at providing a reference for the design of command allocation algorithms for flying wing aircraft.

2 Command allocation problem description

2.1 Flying wing command allocation

Conventional aircraft has three typical effectors, elevator, aileron, and rudder. They are used to generate three-axis control moments to control the flight attitude of the aircraft. Therefore, within the range that the three-axis moment can reach, the control deflections that the aircraft needed for maneuvering is uniquely determined, that is, the traditional control allocation problem can be expressed as follows:

\[
\begin{align*}
M &= qS \vec{e}(M_\alpha + M_\beta + M_\delta + M_\epsilon) \\
L &= qS \vec{b}(L_\alpha \beta + L_\beta \beta + L_\beta \alpha + L_\epsilon \epsilon) \\
N &= qS \vec{a}(N_\alpha \beta + N_\beta \beta + N_\beta \alpha + N_\epsilon \epsilon)
\end{align*}
\]

(1)

It can be seen that the desired moment \([L,M,N]\) corresponds to unique control vectors \([\delta_\alpha, \delta_\beta, \delta_\epsilon]\) and the three control effectors have a clear functional axis, and there is almost no control coupling in the vertical and horizontal axes.

Flying wing aircraft adopts multiple sets of new type of control surface configurations. In order to solve the problem that redundant control surfaces are more than the desired control axes and have no explicit axial control function, it is necessary to adopt command allocation unit to allocate the desired control moments required by the control law to relevant control surfaces optimally.

Due to the usage of innovative control effector of flying wing aircraft, especially the drag based control effector for the directional axis, such as the all-moving wing tip used on tailless fighter aircraft, as shown in Figure 1, it is found that for flying wing aircraft not only the longitudinal and lateral-directional axes under different flight conditions have strong coupling effects, but also the generated large amount of drag cannot be ignored. Therefore, drag, lift and side force should also be considered in command control allocation.

![All moving wing tip](image)

**Figure 1.** Multi-axis coupling effect of a full-motion wingtip

The control surface aerodynamic nonlinear characteristics of the flying wing aircraft are very strong, which leads to the traditional simple linear allocation method no longer applicable [1]. Besides, the drag based control effector usually adopts a single-sided large deflection strategy, such as the all-moving wing tip deflection range is [0, 60] degree, the split drag rudder is even in the range [0, 90] degree, and so the dynamics of the actuator cannot be ignored, including the frequency and rate of deflection. Here only the speed limit \(\dot{\delta}\) of the actuator is considered, namely,

\[
\rho_{\text{min}} < \dot{\delta} < \rho_{\text{max}} \quad (i = 1 \ldots n)
\]

(2)

Where \(\rho_{\text{min}}\) and \(\rho_{\text{max}}\) are the minimum and maximum deflection rate of each control surface. Therefore, the command allocation problem for tailless aircraft is described as follows,
\[ g(u(t)) = M_A(t) \]
\[ \text{s.t. } \delta_i \leq \delta_{i \text{min}} \leq u \leq \delta_{i \text{max}} \]  

(3)

Where \( g(.) \) represents the nonlinear function of mapping control surface deflection \( u(t) \) into control moments, \( M_A(t) \) are the desired three-axis control moments, \( \delta_{i \text{min}} \) and \( \delta_{i \text{max}} \) are the lower and upper position limits for the \( i_{th} \) control surface. For a multi-control surface flying wing aircraft usually has \( n \) control surfaces. And \( u \) is,

\[ u = [\delta_1, \delta_2, \ldots, \delta_n](n > 3) \]  

(4)

The control surface position and rate limit can be approximated as follows, the upper and lower limits of the position within a single sampling period,

\[
\begin{align*}
\delta_{\text{max}}^{i} & = \max (\delta_{\text{max}}^{i}, \delta_{\text{max}}^{i}(t-T)+T \delta_{\text{max}}^{i}) \\
\delta_{\text{min}}^{i} & = \min (\delta_{\text{min}}^{i}, \delta_{\text{min}}^{i}(t-T)-T \delta_{\text{min}}^{i})
\end{align*}
\]  

(5)

2.2 Attainable moment set

Under different control effectiveness matrix and control effector constraints, the maximum attainable moment set \([M] \) are quite different. Large AMS means that the aircraft has good controllability and may achieve maneuverability.

In the three dimensional vector space \( \mathbb{R}^3 \) composed of aircraft three-axis moment coefficients, the AMS of conventional aircraft is a space hexahedron, which corresponds to the three control surfaces elevator, aileron and rudder, respectively. But the AMS of flying wing aircraft usually is space polyhedron (greater than six), and it is a spatial convex body if nonlinear aerodynamic effects are considered. As shown in Figure 2, the AMS of flying wing aircraft has strong influence on aircraft total drag.

![Figure 2](image_url)

(a) A large aspect ratio flying wing (b) Boeing747 aircraft

\( H = 20000 \text{m}, \text{Ma} = 0.6 \) cruise \( H = 6000 \text{m}, \text{Ma} = 0.65 \) cruise

(c) A small aspect ratio flying wing (d) F-4 aircraft

\( H = 4572 \text{m}, \text{Ma} = 0.6 \) cruise \( H = 10668 \text{m}, \text{Ma} = 0.6 \) cruise

**Figure 2.** Attainable moment set of several aircraft [2]
2.3 Solution situation
Control allocation problem can be treated as find solution to nonlinear equations \( g(u) = M_d \) with constraints \( \underline{u} \leq u \leq \overline{u} \). Because the number of control surfaces is more than the expected control variables, the nonlinear equation of the command allocation problem is underdetermined. As shown in Figure 3, there are two cases for the solution of this problem.

![Diagram](image)

**Figure 3.** Solution to the instruction allocation problem

When the expected moment exceeds the AMS range, in order to accurately control the difference between the magnitude and direction of the realized moment and the expected moment, the expected moment must first be converted into attainable moment according to performance index, and then convert it into optimal exact solution.

3. Control allocation method

3.1 Generalized inverse
The generalized inverse method is the most classic and simple control allocation method, which is only suitable for linear control allocation problem. By solving the pseudo inverse of the control effectiveness matrix, we can obtain the minimum norm solution \( u \) of the control vectors, i.e.

\[
\mathbf{u} = \mathbf{B}^T \left( \mathbf{B} \mathbf{B}^T \right)^{-1} \cdot \mathbf{M}_d \Rightarrow \min \| \mathbf{u} \|^2
\]  

(6)

Where \( M_d \) is the desired moment, \( B \) is the control effectiveness matrix.

Considering the authority of different control axes, a weighted generalized inverse method is proposed [3]. As is shown in equation (7), this method can prevent the control surface with high control efficiency from being saturated in advance.

\[
u = N(BN)^T \left( BN(BN)^T \right)^{-1} \Rightarrow \min N^{-1} \| \mathbf{u} \|^2
\]  

(7)

This method has been developed for a very long time since it is simple and easy to implement. However, this method cannot reach all the values in AMS. More importantly, if using this method the deflection of the control surface and the rate constraint may cause great errors and pose uncertain risks.
to the control system.

In order to reduce the above-mentioned risk, a redistributed pseudo-inverse algorithm is proposed, that is, re-allocating the control surface that is not saturated in previous allocation and redistribute command to it until it is fully saturated [4]. But this improved method based on generalized inverse still cannot reach all the values in AMS.

3.2 Daisy chain

In daisy chain method [5], before the allocation routine is started, the control surfaces are divided into several groups based on their functions, such as the primary control surface and the secondary control surface combination (or the conventional control surface and the non-conventional control surface). Then the control allocation routine will first use the primary control surfaces, when primary control surfaces are saturated, the secondary control surfaces will be used. This method can maximize the use of conventional control surfaces and avoid using non-conventional control surfaces.

\[
M = [g_1(u_1) \ g_2(u_2)] [u_1] \rightarrow M_d \tag{8}
\]

Where \( B_1 \) and \( u_1 \) are the control effectiveness matrix and control vector of the primary control surfaces, \( B_2 \) and \( u_2 \) are the control effectiveness matrix and control vector of the secondary control surfaces.

The daisy chain allocation algorithm is to find the pseudo inverse of each group of the control matrices in turn, as described below,

\[
\begin{align*}
M_i &= B_i u_i \\
u_i &= B_i^T [B_i B_i^T]^T (M_d - \sum_{j=1}^{i-1} M_j)
\end{align*}
\tag{9}
\]

Different groups of control surfaces can also use different algorithms to allocate control command. This allocation method is suitable for aircraft that performs simple tasks during level flight states.

3.3 Direct allocation

Direct allocation method is first proposed by Durham [6], it is a geometric space method based on the maximum usable AMS. This method searches for the boundary element of AMS to find the intersection point of the desired moment, that is,

\[
\dot{M} = M_d \ I |M_d| \\ M = \rho \dot{M} \tag{10}
\]

In case \( \rho \geq 1 \), the expected moment is reachable, otherwise, the expected moment is unreachable, reducing the moment \( M \) proportionally to intersect AMS boundary, because we assume the aircraft dynamics as linear system, the amount of control deflection is reduced in proportion, which ensures that the realized moment is in the same direction as the desired moment.

Because the direct allocation method can reach all the values in AMS and has clear physical meaning, there are many control allocation algorithms are developed based on this method. Like for example, Durham has proposed a face search algorithm[7], edge search algorithm [8] and adjoint surface search algorithm[9]. Besides, direct allocation method can also be realized by linear programming method[10,11]. Currently direct allocation methods are limited to linear control allocation problems.

3.4 Optimization based (nonlinear programming) method

Due to the emergence of advanced aerodynamic configuration and innovative control effectors, the problem of non-linear control allocation is becoming increasingly prominent. At the same time, in order to effectively solve the control allocation problem when the expected moment exceeds the AMS range, control allocation method based on mathematical programming have been rapidly developed. Optimization based allocation method can be simply described as,
\[ \min f(u) \]  
S.t \[ g(u) = \frac{M_j}{\bar{u}_{\text{min}}} \leq u \leq \bar{u}_{\text{max}} \]  

Where \( f(u) \) is optimization goals, \( g(u) \) is the control effectiveness function of control surfaces, \([\bar{u}_{\text{min}}, \bar{u}_{\text{max}}]\) is the position limit of control surfaces.

Assume equation (13) is in the feasible region of optimization problem, if the feasible solution does not exist, that is, when the expected moment is outside the AMS, then the equation constraint will be relaxed and the unreachable moment need to be constrained in the objective function.

Optimization based control allocation method can also achieve multi-target, variable-target control allocation, such as minimal drag during cruise[12], maximum lift during take-off and landing phase or the smallest radar reflection cross-sectional area during combat phase[13]. Nelson et al also proposes to maximize the priority of the pitching moment under any maneuver[14] to solve the problem of the expected moment beyond the maximum AMS under optimization based control allocation, which has gain positive supports from pilots. But at present, the classic quadratic programming method can only solve the case where the control effector model is linear.

3.5 Dynamic allocation method

In an actual control system, control allocation does not statically allocate the desired moment at current time, but needs to allocate the expected moment command that changes over a period of time required by the control law. Härkegård et al. considered this issue as dynamic control allocation [15], this method has been initially applied on a tailless aircraft [16] and developed the generalized method for dynamic control allocation,

\[ \min J = \|W_1(u(t) - u_\text{eq}(t))\|^2 + \|W_2(u(t) - u(t - T))\|^2 \]  

Where \( W_1 \) and \( W_2 \) is the weighting factor, and \( W = (W_1^2 + W_2^2)^{1/2} \), then

\[ u(t) = E u_\text{eq}(t) + F u(t - T) + GM_j(t) \]  

Where \( E, F \) and \( G \) satisfy

\[
\begin{align*}
E &= (I - GB)W^2W_1^2 \\
F &= (I - GB)W^2W_2^2 \\
G &= W^{-1}(BW^{-1})
\end{align*}
\]  

Dynamic allocation considers the relationship between control allocation command of current time and the allocation result of previous time step \( u(t) = f(M_j(t), u(t - T)) \) to prevent the abrupt jump of control surfaces. It can be seen that when \( W_2 = 0 \), dynamic control allocation is the same as generalized inverse method.

Considering the dynamic characteristics of multiple control surfaces as second-order system, only damping ratio \( \xi_j \) and frequency \( \omega_j \) are modeled, then we will have simplified control surface model,

\[ \delta_j = \frac{\omega_j^2}{s^2 + 2\xi_j \omega_j s + \omega_j^2} \delta_j,_{\text{des}} \]  

Since the dynamic characteristics of each control surface may be different, Venkataraman et al. adopt a Fourier transform to the optimization target (moment error, etc.) to consider the constraints and optimization problem in frequency domain [17]. The optimization result uses the low-frequency control effectors to complete the low-frequency moment command and improve the tracking accuracy of the commanded moment.
| Allocation method | Integration | Control model | Implementing moment sets | Allocation target | Consider dynamic allocation | When expected moment ≠ AMS | Algorithm | Applicable flight phase |
|-------------------|-------------|---------------|--------------------------|-------------------|-----------------------------|-----------------------------|-----------|------------------------|
| Generalized inverse method | √ | √ | | | √ | Uncertainty risk | Simple (weighted) pseudo-inverse method, redistribution pseudo-inverse method | Difficult to apply directly |
| Daisy chain | | √ | √ | | | | Depends on different algorithms | Multi-level generalized inverse cruise |
| Direct allocation | √ | √ | | √ | | Holding torque direction | Area search, edge search, linear programming, etc. | Maneuver |
| Optimization based allocation | √ | √ | √ | √ | √ | | Achieving optimization goals | Linear programming, quadratic programming, penalty function method, etc. | Field / Cruise / Maneuver |

It can be seen from Table 1 that in different flight phase and tasks of the aircraft, it is not possible to rely on a single command allocation method, and the actual aircraft command allocation has become an optimization problem. Therefore, the command allocation scheme for flying wing aircraft should consider the different flight phases, from the perspective of optimization, it can be converted to achieve different expected performance indicators under different flight phases.

4. Conclusion
This paper studies and summarizes common command control allocation algorithms, including generalized inverse allocation, daisy chain allocation, direct allocation, and optimization based allocation for flying wing aircraft. The generalized inverse method is simple to implement, but the saturation of the control surface may bring uncertainty risks to control system. Daisy chain allocation can be treated as an improved method of the generalized inverse method, which can deal with control surface saturation, but cannot reasonably allocate command based on the dynamic characteristics of the control surface. The direct allocation method is based on the geometric space method, which is relatively simple to implement, but can only be applied to linear control effectiveness model. The optimization based allocation method is a more ideal method for the command allocation of flying wing aircraft in the future. It can achieve the command allocation that meet specific optimization goals according to different mission requirements, but the nonlinear optimization algorithm is currently difficult to solve in real time. Hence, according to the control effector characteristics of flying wing aircraft, we should select reasonable command allocation algorithms to maximize the control power of redundant effectors for flying wing aircraft.

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