ON THERMAL COMPTONIZATION IN e\(\pm\) PAIR PLASMAS

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ABSTRACT

We study e\(\pm\) pair plasmas in pair equilibrium, which emit high energy radiation by thermal Comptonization of soft photons. We find that the maximum luminosity to size ratio of the source (i.e. the compactness) depends not only on the hot plasma temperature, but also on the spectral index of the resulting Comptonized spectrum. In the observationally interesting range, sources of same compactness can be hotter if their spectrum is steeper. Instruments observing in the 50–500 keV energy range, such as OSSE on board CGRO, and especially the future SAX satellite, can be more successful in detecting sources moderately steep, the flattest sources being characterized by an high energy cut–off at too low frequencies. For any given pair of values of spectral index and temperature, Comptonization theory alone fixes the ratio of the compactnesses in hard and soft photons. However, if the source is pair dominated, the absolute values of the two compactnesses are fixed. Therefore there is a a one–to–one correspondence between the physical parameters of the source (the compactnesses in soft and hard photons) and the observable quantities (spectral index and temperature). This correspondence can be extremely useful in interpreting the physical behaviour of the sources, especially during variations, and can help discriminating between different models for the high energy emission of compact sources.

Subject headings: radiation mechanisms: thermal – galaxies: Seyfert – gamma rays: theory – plasmas – X–rays: general

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1. INTRODUCTION

The recent observations of OSSE on the Compton Gamma Ray Observatory have shown a break or an exponential cut–off in the high energy X–ray spectrum of Seyfert galaxies (Maisack et al. 1993, Cameron et al. 1993, Madejski et al. in preparation).

These observations can be interpreted in the framework of thermal Comptonization models, even if a non–thermal origin of the high energy radiation cannot be ruled out (Zdziarski, 1994). Thermal models have been recently successful in explaining the X–ray background as the sum of the emission of Seyfert galaxies, even if it is not yet clear which type of Seyfert galaxies contribute the most (see e.g. Madau, Ghisellini & Fabian 1993; Zdziarski, Zycki & Krolik 1993).

The fact that there exists a relatively narrow range of temperatures of emitting plasmas in AGNs can be understood on the basis of the thermostat effect of electron–positron pairs, after the classic papers of Bisnovatyi-Kogan, Zel’dovich & Sunyaev (1971), Svensson (1982, 1983, 1984), Lightman (1982) and Zdziarski (1985).

The main result of these studies is that, for any given luminosity to size ratio of the source, there exists a maximum temperature at which the emitting plasma can be in steady state. If the temperature is greater than this maximum limit, photon–photon, photon–particle and particle–particle collisions produce pairs that cannot annihilate at the same rate. For the same reason, increasing the luminosity to size ratio (the compactness), the temperature is bound to decrease.

Furthermore there are indications that the high energy emission of AGNs in general, and Seyfert galaxies in particular, comes from the inner regions surrounding the black hole, and that the luminosity is greater than few per cent of the Eddington limit (Padovani 1989, Done & Fabian 1990). We therefore believe that AGNs are characterized by a relatively narrow range of compactnesses and, hence, of temperatures.

However, there can be differences among different AGNs, and, most notably, there are differences in the spectral indices of their X–ray emission. Although the 2–10 keV spectral indices $\alpha_x$ of Seyfert 1 galaxies and quasars have a small scatter around the mean value, $\alpha_x$ ranges from 0.4 to values steeper than 1 (Turner et al. 1990, Comastri et al. 1992).

In this paper we investigate the behaviour of sources in pair equilibrium characterized by different spectral indices by assuming thermal Comptonization of soft photons as the main radiation mechanism. In the framework of these models different spectral indices are due to different ratios $L_s/L_h$ between the luminosity in soft photons and the luminosity produced by the hot particles. We therefore investigate the equilibrium states of hot plasmas characterized by different compactnesses and different $L_s/L_h$ ratios.

The main results of this work are: a relation between the spectral index $\alpha_x$ and the temperature $T$ of plasmas in pair equilibrium, and a one–to–one correspondence between $\alpha$, $T$ and the compactnesses in hard and soft photons.

These results can have important consequences:

i) for predicting a correlation between spectral index and cut–off energy in pair dominated AGNs, in the sense already observed for NGC 4151 (Jourdain et al. 1992
and Maisack et al. 1993, Zdziarski, Lightman & Maciolek-Niedźwiecki, 1993) and IC 4329A (Madejski et al. 1994);

ii) for predicting the high energy emission, in the OSSE range, of sources with different spectral indices in the 2–10 keV band;

iii) for interpreting the behaviour of the overall spectrum of individual sources during spectral and flux variations.

2. THERMAL COMPTONIZATION AND PAIR PRODUCTION

Thermal Comptonization has been extensively studied in the past (see e.g. Sunyaev & Titarchuk 1980, Pozdnyakov, Sobol’ & Sunyaev 1983).

We assume that soft photons are homogeneously distributed throughout a sphere of radius $R$, with a diluted black body spectral distribution of dimensionless temperature $\Theta_{BB} \equiv kT_{BB}/(m_e c^2)$. The sphere is homogeneously filled with hot plasma, with dimensionless temperature $\Theta$, producing, via Comptonization, a luminosity $L_h$ corresponding to a compactness $\ell_h \equiv L_h \sigma_T/(Rmc^3)$ where $\sigma_T$ is the Thomson cross section. Analogously, $\ell_s$ is the compactness of the soft photon source.

The scattering optical depth, $\tau_T$, and the temperature, $\Theta$, of the hot plasma uniquely determine the Comptonized photon energy distribution, characterized by a power law and a cut–off at $h\nu/(m_e c^2) \sim \Theta$. Therefore we can treat the temperature $\Theta$ and the spectral index $\alpha$ (instead of $\Theta$ and $\tau_T$) as the two variables.

Although sufficient to completely determine the shape of the Comptonized photons, $\alpha$ and $\Theta$ do not yield its normalization, but only the amplification factor, i.e. the ratio $\ell_h/\ell_s$. Symbolically:

$$(\alpha, \Theta) \rightarrow \ell_h/\ell_s$$  \hspace{1cm} (1)

Note that the reverse is not true, since the same amplification can be achieved with a range of values of $\alpha-\Theta$.

In order to determine the absolute value of $\ell_h$ and $\ell_s$ one has to consider the effects of $e^\pm$ pair production.

The main result of hot plasma studies is that the compactness of a source in equilibrium at a given temperature cannot reach arbitrary large values. In fact, as long as the compactness is very small, an increase in the heating rate corresponds to an increased mean energy per particle, and therefore to an increased temperature. But when the temperature starts to be relativistic, pair production is important, and the number of particles in the thermal bath increases. Since the available energy is now shared among more particles, the temperature in this regime decreases as the heating rate (the compactness) increases. The equilibrium and steady state corresponds to pair balance: pairs are destroyed at the same rate at which they are created. For any given temperature, there is a maximum compactness allowing pair equilibrium, and for any given compactness, there is a maximum allowed temperature. The precise value of the function $\ell_{h,max}(\Theta)$ depends on the detail of the emission mechanism (bremsstrahlung,
Comptonization, cyclosynchrotron, and so on). Hereafter we use the symbol $\ell_h$ to indicate the maximum allowed value $\ell_h$.

If the main radiation mechanism is thermal Comptonization, any pair of values of $\alpha$ and $\Theta$ determines the maximum allowed compactness $\ell_h$. Symbolically we have:

$$(\alpha, \Theta) \iff (\ell_h, \ell_s)$$ (2)

Note that now there is a one-to-one relation between the physical parameters of the source ($\ell_h$ and $\ell_s$) and the observable quantities ($\alpha$ and $\Theta$).

Following Zdziarski (1985, hereafter Z85) we calculated $\ell_h(\Theta)$ for different values of $\alpha$, limiting ourselves to temperatures $\Theta < 10$ ($kT < 5$ MeV), where particle–particle and particle–photon pair production processes are less important than photon–photon interactions, and can therefore be neglected. The formulae used are described in Z85; the Comptonized spectrum is described as the sum of a power law spectrum with an exponential cut–off at $\Theta$ and a Wien spectrum at $\Theta$: $\ell_h = \ell_{pl} + \ell_W$. The formulae of Z85 well describe the Comptonized spectrum at high energies, but give a poor representation of the spectrum at low energies. To derive $\ell_h$ for $\alpha > 1$ and $\ell_s$ (to be derived by photon number conservation), a more detailed description of the Comptonized spectrum at low energies is required.

We therefore computed the thermally Comptonized spectra in spherical geometry by means of the full relativistic kernel (Jones 1968, Coppi & Blandford 1990). This gives the exact value $\ell_h/\ell_s$ for any $(\tau_T, \Theta)$. We then computed a lower integration limit, $x_1$, for equation (1) in Z85 so that the integrated photon spectrum and luminosity coincide with the exact values. We checked that, with the given $\tau_T$ and $\Theta$, the spectral indices used following the prescription of Z85 were correct. We used $kT_{BB} = 10$ eV and, neglected, for simplicity, pair escape. Note that in the absence of pair escape there is an absolute maximum in the allowed temperature ($\Theta_{max} = 24$) derived considering particle–particle interactions.

Our calculations are presented in Fig. 1. As can be seen the curves $\ell_h(\Theta)$ have a sort of pivot at $\Theta \sim 1$, being flatter for flatter spectral indices.

To qualitatively understand this behaviour, let us consider first a fixed value of $\Theta$ in the region corresponding to $\ell_h > 1$. As $\Theta < 1$, the only photons effective in pair production are in the Wien tail of the spectrum (see e.g. fig. 4a of Z85).

Furthermore, if $\tau_T > 1$, simple radiative transfer assures that the compactness of the Wien component is fixed (it depends only on $\Theta$), independently of $\tau_T$. Increasing the total compactness then means to increase the compactness of the power law component. But the two compactnesses are related by the value of $\tau_T$: their ratio $\ell_{pl}/\ell_W$ decreases as $\tau_T$ increases (more photons are driven to the Wien peak). Only a lower $\tau_T$ (i.e. steeper $\alpha$) therefore allows a greater compactness of the power law component.

Consider now the region corresponding to $\ell_h < 1$. In this case both the total luminosity and the pair production process are determined by $\ell_{pl}$, $\ell_W$ playing no role. For fixed $\Theta$, $\ell_h$ can increase if:

i) the new equilibrium state yields a greater $\tau_T$ and therefore a flatter spectral index. In this case the luminosity is increased at high energies increasing the pair production rate, which in turn yields the required increased $\tau_T$. 4
ii) the new equilibrium state yields a much lower value of $\tau_T$ and a much steeper spectrum. In this case the luminosity is increased at low energies and decreased at high energies, the pair production rate is decreased, and the corresponding $\tau_T$ decreases.

The first solution gives $\alpha < 1$ while the second one yields $\alpha > 1$. Although both these solutions are consistent with pair balance and Comptonization, the latter one exists only in a range of $\Theta$. This range becomes larger for small values of the minimum energy of the Comptonized spectrum $x_1$ (where most of the luminosity is for $\alpha > 1$). For the adopted $\Theta_{BB}$, the two solutions exist only in the small range $1 < \Theta < 2$, as shown in Fig. 1.

Fig. 1 shows that in the interesting (from the observational point of view) parameter range $0.1 < \Theta < 1$ pair dominated sources with same compactness should show a clear correlation between the spectral index and the cut–off energy: the flatter the spectral index, the lower the temperature.

Note that, as shown in Fig. 1, the maximum possible compactnesses for $kT \sim 50$ keV are extremely large, but these corresponds to completely pair dominated sources: if the hot emitting plasma has also a ‘normal’, electron–proton component, the compactness can be smaller (or the temperature can be smaller than shown in Fig. 1, for the same compactness).

3. ‘MAPPING’

Thermal Comptonization together with pair plasma theory gives a one–to–one relation between $\alpha–\Theta$ and $\ell_s–\ell_h$. This means that we can ‘map’ the plane $\alpha–\Theta$ into the plane $\ell_s–\ell_h$ (or equivalently $\ell_h/\ell_s–\ell_h$), and viceversa.

This ‘mapping’ links physical parameters, such as the soft and the hard compactness, which completely characterize the source, with observable quantities, such as the spectral index and cut–off energy.

Fig. 2 shows the results. In the ‘starvness–compactness’ plane (i.e $\ell_h/\ell_s–\ell_h$) we have mapped the ‘temperature–spectral index’ plane, plotting curves for constant $\Theta$ and for constant $\alpha$. We have restricted our analysis to the range which is observationally interesting (i.e. $0.5 < \alpha < 1.5$ and $0.1 < \Theta < 1$ corresponding to $51 < kT < 511$ keV).

It can be seen that $\ell_h/\ell_s$ is not exactly constant for a given spectral index, but it has some weak dependence also on the temperature. Moving along a curve with constant $\Theta$ in the direction of steeper $\alpha$ we have that $\ell_h/\ell_s$ decreases (as expected), while $\ell_h$ increases (as explained in the previous section, and also shown in Fig. 1).

In this plane it is very easy to see what the equilibria states are when the source varies. For instance, suppose that a source initially in point labelled $A$ in Fig. 2 increases its power ($\ell_h$) by a factor 3. The compactness in soft photons, $\ell_s$, can either remain constant or vary together with $\ell_h$. The latter case may indicate that there is some feedback between the hot plasma producing the hard luminosity and the relatively cold plasma producing the seed photons, as in a cold disk illuminated by a hot corona.

The final states in the two cases are labelled $B$ and $C$ in Fig. 2. As can be seen,
point $B$ corresponds to a temperature lower by a factor 1.5 and slightly steeper spectral index $\alpha$, while point $C$ corresponds to an even lower temperature (factor 2) and a flatter $\alpha$. All the possible intermediate cases $5 < \ell_h/\ell_s < 15$ are between points $B$ and $C$.

Another possibility is that the soft compactness varies even without any change in $\ell_h$. This is possible only if reprocessing (as a source of soft photons) is not important. Point labelled $D$ corresponds to the final state of the source initially in $A$, after an increase by a factor 3 of $\ell_s$. The resulting equilibrium temperature is larger by a factor 1.2 and the spectrum has steepened.

In conclusion, knowing how the spectral index and the temperature change, one can know if $\ell_s$ is bound to follow the variation of $\ell_h$. This is extremely important, as we can test models in which an important role is played by reprocessing of the high energy radiation by cold matter located near the hot gas.

4. DISCUSSION

One of the main results of pair plasma studies is that the maximum compactness allowed by pair equilibrium decreases as the temperature increases. This however refers to sources with fixed spectral index, which roughly corresponds to fixed $\ell_h/\ell_s$.

We have shown that, from thermal Comptonization and pair plasma theories, one can derive a one-to-one correspondence between the observable quantities $\alpha$ and $kT$ and the physical parameters $\ell_h$ and $\ell_s$. The knowledge of any two of these 4 quantities univocally determines the other two.

The spectral behaviour of a varying X-ray source depends on whether the ratio $\ell_h/\ell_s$ or $\ell_s$ remains constant when $\ell_h$ varies. If $\ell_h/\ell_s$ is constant, the spectral index slightly steepens for increasing hard compactness and the temperature decreases. If $\ell_s$ remains constant, the spectral index flattens for increasing $\ell_h$ and the temperature decreases by a larger amount.

Observationally, there are indications that in some sources, during large variations of the X-ray flux, the X-ray spectral index remains approximately constant (e.g. Nandra et al. 1991), or slightly steepens (e.g. Yaqoob & Warwick 1991). This indicates that $\ell_h/\ell_s$ is nearly constant during variations, strongly favoring models in which the soft component is dominated by the reprocessed flux (see e.g. Haardt & Maraschi 1991). This in turn would imply the presence of cold matter [cloudlets (Celotti, Fabian & Rees 1992, Sviron & Tsuruta 1993) or a cold accretion disk (Lightman & White 1988, George & Fabian 1991, Matt, Perola & Piro, 1991)] very close to the illuminating X-ray source. The above conclusion does not depend strictly on the importance of pairs, which instead can fix the temperature and the temperature change during variations of the source.

Observations of the cut-off energy of the X-ray spectrum is of crucial importance to establish if a source is pair dominated, and directly yield an upper limit on the compactness as illustrated in Fig. 1. In addition, if variations of the the flux corresponds to variations of the high energy cut-off as illustrated in Fig. 2, we can derive the compactness of the source, not only an upper limit, and therefore the size of the emitting
region. Hence it is very important to coordinate observations in the 2–10 keV band (which should yield the spectral index) and in the 50–500 keV band (which should yield the value of the cut–off energy). A good opportunity to pursue this program is presently offered by ASCA and OSSE missions, and in future, by the SAX satellite.

If a class of sources, such as Seyfert galaxies, are pair dominated with similar compactnesses, there should be a correlation between spectral index and temperature. In the range of compactness between 10 and 100, steeper spectra should correspond to larger temperatures. For $\ell_h = 100$, the maximum temperature is 70, 200 and 400 keV for spectral indices $\alpha = 0.5, 1$ and 1.5, respectively.

At present, only two Seyfert galaxies have their spectral index and cut–off energy reliably measured, i.e. NGC 4151 and IC 4329A, and it is encouraging that the differences in their $\alpha_x$ and $kT$ are in the sense discussed in this paper. NGC 4151 has $\alpha = 0.5$ and $kT \sim 50$ keV, while IC 4329A has $\alpha \sim 1$ and $kT > 150$ keV.

The correlation discussed in this paper should be taken into account when interpreting the X–ray background as due to Seyfert galaxies (and quasars) of different spectral indices, since for each $\alpha$ a different maximum temperature is indicated.

Finally, it is worth to point out that if a source has a flat $\alpha_x$ in the 2–10 keV band, it may have a cut–off at relatively small energies, and therefore it may be invisible in the 100–200 keV band (i.e. the OSSE band). On the other hand, very steep sources ($\alpha_x > 1$) have a very small flux in the OSSE band, and are therefore difficult to detect even if the cut–off energy is large. As a consequence, the probability to detect sources at 100 keV should peak for sources moderately steep ($\alpha_x \sim 1$) in the 2–10 keV band.

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FIGURE CAPTIONS

Fig. 1: For any given temperature $\Theta \equiv kT/(m_e c^2)$ there exists a maximum possible compactness $\ell_h$ of a plasma in pair equilibrium. The exact shape of the function $\ell_h(\Theta)$ depends on the spectral index resulting from the Comptonization process. Flatter spectral indices correspond to flatter $\ell_h(\Theta)$. Note that the curves (the labels indicate the value of the spectral index) cross for $\ell_h \sim 1$. For $\ell_h > 1$, sources with the same compactness have larger temperatures if their spectrum is steeper, and viceversa.

Fig. 2: In the plane $\ell_h/\ell_s$ vs $\ell_h$ (the starvness vs hard compactness plane) we have drawn the curves for constant $\Theta$ (solid lines) and for constant $\alpha$ (dashed lines). The temperature increases from right to left, $\alpha$ increases (the spectrum steepens) from top to bottom, as labelled. As illustration, consider a source, initially in point $A$, which increases $\ell_h$ by a factor 3. Its final equilibrium state will be between points $B$ and $C$, if $\ell_s$ remains constant or increases by the same amount of $\ell_h$. If, instead, $\ell_s$ increases by a factor 3 without variations in $\ell_h$, the final equilibrium state will correspond to the point labelled $D$. 