Students’ Attitudes towards Discovery Learning / Constructivistic Approach using Computers as Cognitive Tools in Higher Mathematics Education

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Abstract—This paper analyses the discovery learning / constructivistic approach using cognitive tools in higher Mathematics education and focuses on electronic worksheets designed and implemented in Mathematica. The approach was applied at the University of Piraeus. Students from the Department of Statistics and Insurance Sciences participated in the research. The paper focuses on empirical research results on students’ attitudes towards the approach, concerning the cognitive tool used (Mathematica), the approach’s discovery learning and constructivistic characteristics and the development of higher order thinking skills supported by the approach. Regarding factors that could influence students’ attitudes, gender and experience in using computers are investigated. This paper uses quantitative methods in analyzing data collected via the use of a questionnaire and the research approaches used are the descriptive / investigative and the correlational approach.

Index Terms—Cognitive Tools; Discovery Learning; Constructivism; Higher Order Thinking Skills.

I. INTRODUCTION

The discovery learning / constructivistic approach using cognitive tools is theoretically based on Discovery learning, Constructivism, Social development theory, Andragogy and the theory of computers as cognitive tools. The theories of Discovery learning and Constructivism support learning environments in which students are engaged in a process in which they explore the natural or material world, they formulate and check conjectures and “discover” the various ideas and methods. The Social development theory points out the significance of the social interaction between students and between students and the teacher (social constructivism). Andragogy is a theory of adult learning that points out the differences in learning between adult learners and primary/secondary education students [1]. Instructional technology studies the utilization of computer based learning environments in teaching and learning; many researchers work on the conceptual foundation and others on the practical application of instructional technology in the classroom (on campus or virtually). The research on the field has taken two diverse directions, the one towards artificial intelligence in student modeling and the other towards constructivistic approaches promoting discovery learning and social interaction. Both these two directions support the use and utilization of computer based learning environments as “cognitive tools” [2]. Cognitive tools or mindtools are computer based tools that are used as “intellectual partners” of the students, as they are involved in discovery learning / constructivistic activities [3]. It is a practical and effective form of using computers since it does not require the development or use of purely educational software, but the selection and appropriate use of software already developed for educational or vocational purposes. Higher education, especially higher Mathematics and Science education, aims primarily in the development of higher order thinking skills. The problems and the problematic situations contemporary workers and employees have to deal with demand analysis, synthesis and evaluation of the problematic situations, also problem solving and decision making, since tasks demanding purely practical skills are now performed by automated procedures; students’ education should be pointed in that direction in order for them to be better prepared for their occupational demands [4]. In the literature, several instructional design interventions can be found that have been designed and implemented in order to engage learners in complicated cognitive activities, aiming in the development of higher order thinking skills [5].

This paper presents and analyses the discovery learning / constructivistic approach using cognitive tools in higher mathematics education and focuses on the design and implementation of electronic worksheets in Mathematica. The approach was applied at the University of Piraeus. The paper presents empirical research on students’ attitudes towards the approach, concerning the cognitive tool used (Mathematica), the discovery learning and constructivistic characteristics of the approach and the development of higher order thinking skills supported by the approach.

II. THEORETICAL BACKGROUND

A. Contemporary theories of learning

According to the theory of Discovery Learning, proposed by Bruner [6], [7], the teacher’s main role is to help and encourage his/her students to discover the various concepts and ideas and to develop an aspect of exploration and experimentation towards knowledge. According to the theory of Constructivism, proposed by Piaget, Von Glasersfeld and other contemporary theorists and researchers, students construct their knowledge actively, based on their past knowledge and experience [8]. According to Von Glasersfeld, knowledge is a process of
adaptation with the world of experiences and not the discovery of a pre-existent world, independent to the learner.

According to Vygotsky’s Social Development Theory, the learner initially learns, or is developed culturally, socially and then individually [9]. Humans use “tools” (such as speech, written speech, cognitive tools) in order to interact with others and develop higher order thinking skills by adopting the functions and the structure of the tools [10].

Students who attend university courses have both the characteristics of adolescent students (studied by Pedagogy) and adult learners. According to Knowles’ Andragogy, adults are self-directed and want to participate in the programming and evaluation of their teaching. Teaching should take into consideration the different backgrounds of the learners. Adult learners’ motives to learn are basically internal and they learn better when the object of learning is directly related to their work or personal life [11].

B. Higher order thinking skills

The contemporary occupational necessities and demands in most fields include problem solving and decision making, so higher education students who are competent in tasks requiring higher order thinking skills (as analysis, synthesis or evaluation) are better prepared in order to use them in their workplace.

Social interaction is proposed by contemporary theories of learning as necessary for the construction of knowledge. Brierton, Wilson, Kistler, Flowers & Jones [4] investigated whether social constructivism promote students’ higher order thinking skills. They compared synchronous and asynchronous online discussion, with synchronous discussion proving to be more effective in promoting students’ higher order thinking skills.

The type of teaching intervention and the type of cognitive activities that can promote the development of students’ higher order thinking has been studied in a great extent in the literature. Lee & Choi [5] investigated learner factors that affect higher order thinking in learning environments using technology. The factor identified by the study as directly affecting higher order thinking was deep learning approaches; epistemological beliefs and attitudes towards the use of technology were identified as indirectly affecting higher order thinking.

Higher order thinking skills involve metacognition, reflective thinking, mindfulness, self regulation, critical thinking and a number of other thinking skills [12], [1].

Integrated Thinking is an interactive system that consists of four components [13], [3]:

- Content / Basic thinking: It represents skills required to learn information commonly accepted and to recall that information after we have learnt it (traditional learning).
- Critical thinking: It represents the dynamic reorganization of knowledge and includes three general skills: Evaluating, Analyzing and Connecting.
- Creative thinking: It presupposes the production of new knowledge and includes three skills: Synthesizing, Imagining and Elaborating on information.
- Complex thinking skills are in the center of the integrated thinking model, combining content thinking, critical thinking and creative thinking and includes three skills: Problem solving, Designing and Decision making.

C. Computers as cognitive tool

Cognitive tools are provided by the teaching–learning environment and are used by students in order to use and utilize symbols, interpretations and procedures to a problematic situation they are dealing with, which otherwise would be unavailable [2].

Jonassen [3] regards cognitive tools or mindtools as computer based learning environments that have been developed or adjusted, in order to function as “intellectual partners” of the students, in order to promote higher order thinking.

Cognitive tools have the following characteristics:

- They are generalizable computational tools that have simple, powerful formalism and are easily learnable.
- They support knowledge construction and transferable learning.
- They reorganize the way learners think and extend their thinking processes.
- They promote higher order thinking.
- They are not just accommodating tools, nor “fingertip” tools.

D. Discovery learning / constructivistic approach using electronic worksheets

The discovery learning / constructivistic approach usually includes the following steps [1]:

- Definition of a problem.
- Collection of data.
- Processing, organization and analysis of data.
- Formation and checking of conjectures.
- Formulation of conclusions.
- Discussion–Investigation of the results and conclusions.
- Reflection on the conclusion.

The approach can be effectively applied via the design and implementation of electronic worksheets in the environment of a cognitive tool which include [1]:

- Activities which follow the steps of the discovery learning / constructivistic approach, using actions that should be performed by the students and questions that should be answered by the students.
- Basic commands of the cognitive tool and the required programs for the subjects being studied.
- Suggestions about the subject being taught and the functions of the cognitive tool.
- Exercises that can be solved with the use and modification of the commands and programs that are already contained in the worksheet.

The lessons are advisable to take place in the computer laboratory, with the students working in groups of 2 or 3 students per computer. The students should cooperate with the members of their group, with members from other teams and with the teacher in dealing with the problematic situations they come across while working with the electronic worksheet. They should be given the opportunity
to set into discussion questions, conjectures and conclusions to the community of the class and to ask at any time the help of the teacher, regarding the understanding of the elements of the theory that are referred to at the lesson and the use, the syntax and the function of the software’s commands.

Korres, Psycharis and Makri-Botsari [14] studied the issue of software-realized scaffolding in Mathematics and Science Higher Education, using the methodology of the computational experiment via electronic worksheets in Mathematica. The approach was found to have a positive impact on students’ scores. Also the students were very positive regarding Mathematica and the computational experiment. Kyriazis, Psycharis and Korres [15] studied the computational experiment via electronic worksheets in Mathematica and presented applications for selected domains of Physics. Also they studied the change in students’ beliefs and scores as a result of the application of the approach.

III. METHODS

The study involved the application of discovery learning / constructivist approach using electronic worksheets to the undergraduate course Calculus II, at the University of Piraeus, at the Department of Statistics and Insurance Sciences. The cognitive subjects selected are partial derivatives and multiple integrals, since they are basic subjects in the curriculum of Mathematics’ and Sciences’ Departments, Economic Studies’ Departments and Polytechnic Departments. Also they are basic tools and have many useful applications in domains directly or indirectly related to them.

The paper studies two research questions:

1. What are the students’ attitudes towards the cognitive tool used (Mathematica), the characteristics of the approach relatively to the principles of discovery learning and constructivism and the development of higher order thinking skills supported by the approach?
2. Do gender and experience in using computers influence students’ attitudes?

As a sample we used the students of the course Calculus II that had attended the application of the approach in the computer laboratories and they had filled a questionnaire of attitudes towards the approach.

We used a 7-grade scale of evaluation (Likert scale) to almost all the questions of the questionnaire, in order to have the most accurate information possible concerning students’ attitudes and the least influence possible in students’ answers.

The research approaches used are the descriptive / investigative and the correlational approach [16]. Regarding factors that could influence students’ attitudes, gender and experience in using computers were investigated. We used non parametric criteria in combination with tables of mean ranks for the variables to which differences occurred. In particular we used the Mann–Whitney test at the grouping according to gender and the Kruskal–Wallis test at the grouping according to experience in the use of computers.

IV. RESULTS

A. General characteristics of the students that participated

The students that participated in the study were 54. Regarding gender, 19 (35.2 %) were males and 35 (64.8 %) were females.

Regarding the students’ experience in using computers, 50.0 % used computers for more than 3 years and 30.8 % from 1 to 3 years. Regarding the students’ interest in using computers, 88.4 % of the students showed positive attitudes (5 and above).

B. Students attitudes towards the cognitive tool used (Mathematica)

Students expressed positive attitudes (5 and above) towards almost all questions regarding the cognitive tool used (Mathematica), except the question whether the students can use the software independently of the presence of the teacher, in which answers are almost equally distributed, slightly towards negative attitudes (55.6 %) (Table I).

| TABLE I: STUDENTS’ ATTITUDES TOWARDS COGNITIVE TOOL USED (MATHEMATICA) |
|--------------------------|-----------------|-----------------|
| Positive attitudes | Neutral attitudes | Negative attitudes |
| 1. Is the software easy for someone to operate? | 72.3 % | 20.4 % | 7.4 % |
| 2. Is the software easy for someone to learn? | 78 % | 17.1 % | 4.9 % |
| 3. Does the software have simple formalism? | 63.4 % | 34.1 % | 2.4 % |
| 4. Does the software have dynamic formalism? | 85.4 % | 14.6 % | 0 % |
| 5. Is the software generalizable? | 63 % | 22.2 % | 14.8 % |
| 6. How would you characterize the clarifications and explanations of the software regarding errors in data entry? | 64.9 % | 18.5 % | 16.7 % |
| 7. How would you characterize the Help Browser of the software? | 64.8 % | 22.2 % | 13 % |
| 8. Can the learners use the software independently of the presence of the teacher? | 44.4 % | – | 55.6 % |

C. Students attitudes towards the characteristics of the approach relatively to the principles of discovery learning / constructivism

Students expressed positive attitudes (5 and above) towards all questions regarding the characteristics of the teaching approach relatively to the principles of discovery learning / constructivism (Table II).
Students: Males had a lower mean rank (21.92). Experience had the second higher mean rank (24.20), and those with lower experience (Less than 1 year: 12.44, 1 to 3 years: 19.87, More than 3 years: 13.41) had a lower mean rank (20.82) than females (29.77).

E. Students attitudes and gender

Groups of males – females, according to the Mann–Whitney test, can be considered to be subsets of populations with the same distribution functions regarding their attitudes to most questions concerning the cognitive tool used, the discovery learning / constructivistic characteristics of the approach and the development of higher order thinking skills supported by the approach (Table IV).

| Differences occurred to the questions (Table IV): |
|-------------------------------------------------|
| • The software has simple formalism: Males had a higher mean rank (26.29) than females (17.25). |
| • Conversation between students: Males had a lower mean rank (21.21) than females (30.91). |
| • Designing actions and assessing their results: Males had a lower mean rank (20.82) than females (29.77). |

F. Students attitudes and experience in the use of computers

Groups of students according to their experience in the use of computers, according to the Kruskal–Wallis test, can be considered to be subsets of populations with the same distribution functions regarding their attitudes to most questions concerning the cognitive tool used, the discovery learning / constructivistic characteristics of the approach and the development of higher order thinking skills supported by the approach (Table IV).

| Differences occurred to the questions (Table IV): |
|-------------------------------------------------|
| • The software has simple formalism: Students with greater experience in computers had higher mean ranks than those with lower experience (Less than 1 year: 12.44, 1 to 3 years: 19.87, More than 3 years: 24.67). |
| • Conversation between students: Students with 1 – 3 years experience in computers had the higher mean rank (35.38), students with less than 1 year experience had the second higher mean rank (24.20), while students with more than 3 years experience had the lower mean rank (21.92). |

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**TABLE II: Students’ attitudes towards the approach’s characteristics relatively to the principles of discovery learning / constructivism**

| Question                                                                 | Positive attitudes | Neutral attitudes | Negative attitudes |
|--------------------------------------------------------------------------|--------------------|-------------------|-------------------|
| 1. Did the approach provoke your interest for the lesson?                | 96.3 %             | 0 %               | 3.7 %             |
| 2. Did the use of the software provoke your interest for Calculus as a subject? | 90.7 %             | 3.7 %             | 5.6 %             |
| 3. Did the approach permit you to participate actively to the lesson?    | 94.4 %             | 5.6 %             | 0 %               |
| 4. Did the approach permit you to self-act?                              | 79.6 %             | 13 %              | 7.4 %             |
| 5. Did the approach permit you to experiment with the concepts?          | 75.9 %             | 20.4 %            | 3.7 %             |
| 6. Did the approach provide opportunities for the formulation and checking of conjectures? | 87 %               | 13 %              | 0 %               |
| 7. Did the approach permit you to reflect on the concepts and the activities you got involved with? | 85.2 %             | 14.8 %            | 0 %               |
| 8. Did the approach permit you to create personal representations of the concepts taught? | 74.1 %             | 25.9 %            | 0 %               |
| 9. Did the approach give the opportunity for conversation between teacher–students? | 100 %              | 0 %               | 0 %               |
| 10. Did the approach give the opportunity for conversation between students? | 92.6 %             | 7.4 %             | 0 %               |

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**TABLE III: Students’ attitudes towards the development of higher order thinking skills supported by the approach**

| Question                                                                 | Positive attitudes | Neutral attitudes | Negative attitudes |
|--------------------------------------------------------------------------|--------------------|-------------------|-------------------|
| 1. Did the approach permit you to evaluate information and ideas?        | 71.1 %             | 21.2 %            | 7.7 %             |
| 2. Did the approach permit you to analyze information or ideas?          | 79.6 %             | 16.7 %            | 3.7 %             |
| 3. Did the approach permit you to combine information or ideas?          | 96.3 %             | 3.7 %             | 0 %               |
| 4. Did the approach permit you to elaborate on information or ideas?     | 87 %               | 5.6 %             | 7.4 %             |
| 5. Did the approach permit you to synthesize ideas?                      | 83.3 %             | 13 %              | 3.7 %             |
| 6. Did the approach permit you to imagine ideas?                         | 90.7 %             | 5.6 %             | 3.6 %             |
| 7. Did the approach permit you to deal with problem solving?             | 88.8 %             | 5.6 %             | 3.8 %             |
| 8. Did the approach permit you to design actions                         | 88.5 %             | 7.7 %             | 3.8 %             |

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**TABLE IV: Results of Mann–Whitney and Kruskal–Wallis test between groups of students according to gender and experience in the use of computers respectively**

| Gender (Mann–Whitney) | Experience in using computers (Kruskal–Wallis) |
|-----------------------|-----------------------------------------------|
| Easy to operate       | H = 0.309, p-value = 0.857                    |
| Operation independently of the teacher | H = 1.229, p-value = 0.541                    |
| Easy to learn         | H = 2.536, p-value = 0.281                    |
| Simple formalism      | H = 6.931, p-value = 0.031                    |
| Dynamic formalism     | H = 2.038, p-value = 0.361                    |
| Generalizable         | H = 2.444, p-value = 0.295                    |
| Interest for the lesson | H = 0.604, p-value = 0.739                    |

**TABLE IV:**

| Difference | Value |
|------------|-------|
| U = 268   | 0.229 |
| U = 263.5 | 0.147 |
| U = 170   | 0.342 |
| U = 114   | 0.013 |
| U = 166   | 0.289 |
| U = 271   | 0.242 |
| U = 247   | 0.088 |

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V. CONCLUSIONS

This paper analyzed the discovery learning / constructivistic approach using computers as cognitive tools, focusing on the design and implementation of electronic worksheets. Moreover, the paper presents research results from the application of the approach at the University of Piraeus, at the Department of Statistics and Insurance Sciences, focusing on students’ attitudes towards the approach.

Students expressed positive attitudes towards almost all questions regarding the cognitive tool used (Mathematica), except the question whether the students can use the software independently of the presence of the teacher, in which answers are almost equally distributed, slightly towards negative attitudes. Students expressed positive attitudes towards all questions regarding the characteristics of the teaching approach relative to the principles of discovery learning / constructivism. Also students expressed positive attitudes towards all questions regarding the development of higher order thinking skills supported by the approach.

Regarding factors that could influence students’ attitudes, similarities and differences regarding gender and experience in using computers were investigated. Regarding gender, males and females had similar attitudes to most questions. Differences occurred only to the questions whether the software has simple formalism (males had a more positive attitude than females), conversation between students (females had a more positive attitude than males) and designing actions and assessing their results (females had a more positive attitude than males). Regarding experience in using computers, groups with less than 1 year, 1 – 3 years and more than 3 years’ experience had similar attitudes to most questions. Differences occurred only to the questions whether the software has simple formalism (greater experience in computers corresponded to a more positive attitude) and conversation between students (students with 1 – 3 years’ experience showed the most positive attitude, students with less than 1-year experience showed the second most positive attitude).

The teaching approach’s innovation is the design and implementation of electronic worksheets in the environment of a cognitive tool, in order to be used in the discovery learning / constructivistic approach. These electronic worksheets include activities, basic commands and the required programs of the cognitive tool, suggestions about the subject being taught and the functions of the tool and exercises that can be solved with the use and modification of the commands and programs already contained in the worksheet. The research approach aims in evaluating the characteristics of the cognitive tool used (Mathematica), the discovery learning and constructivistic characteristics of the approach and whether higher order thinking skills are developed by the students via the evaluation of the students’ attitudes towards the approach. The author’s future directions include the combination of qualitative and quantitative research methods in evaluating whether students develop higher order thinking skills as they are engaged in the discovery learning / constructivistic approach using electronic worksheets and which thinking skills are developed and data analysis combining multivariate statistics’ methods.

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