Behaviour of interacting Ricci dark energy in \textit{logarithmic} $f(T)$ gravity

Rahul Ghosh$^1$ and Surajit Chattopadhyay$^2$

$^1$Department of Mathematics, Bhairab Ganguly College, Kolkata-700 056, India.

$^2$Pailan College of Management and Technology, Bengal Pailan Park, Kolkata-700 104, India.

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In the present work we have considered a modified gravity dubbed as “logarithmic $f(T)$ gravity” [1] and investigated the behavior of Ricci dark energy interacting with pressureless dark matter. We have chosen the interaction term in the form $Q \propto H \delta \rho_m$ and investigated the behavior of the Hubble parameter $H$ as a function of the redshift $z$. For this reconstructed $H$ we have investigated the behavior of the density of the Ricci dark energy \( \rho_{RDE} \) and density contribution due to torsion \( \rho_T \). All of the said cosmological parameters are seen to have increasing behavior from higher to lower redshifts for all values of \( c^2 \) pertaining to the Ricci dark energy. Subsequently, we observed the equation of state parameter \( w_{RDE} \) in this situation. The equation of state parameter is found to behave like phantom for all choices of \( c^2 \) in the Ricci dark energy.

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I. INTRODUCTION

Accelerated expansion of the universe is well-established by the works of [2, 3]. The “dark energy” (DE), characterized by negative pressure, is responsible for this cosmic acceleration [4–7]. Importance of modified gravity for late acceleration of the universe has been reviewed by [8, 9]. Various modified gravity theories have been proposed so far. These include, $f(R)$ [10, 11], $f(T)$ [12–15], $f(G)$ [16, 17], Horava-Lifshitz [18, 19] and Gauss-Bonnet [20, 21] theories. One of the newest extended theories of gravity is the so-called $f(T)$ gravity, which is a theory formulated in a spacetime possessing absolute parallelism [13]. Some fundamental aspects of $f(T)$ theories have been studied in the works of [22] and [23]. In this theory of modified gravity, the teleparallel Lagrangian density described by the torsion scalar $T$ has been promoted to a function of $T$, i.e.,
$f(T)$, in order to account for the late time cosmic acceleration \([24, 25]\).

Models of DE include quintessence \([26]\), phantom \([28]\), quintom \([27]\), Chaplygin gas \([29]\), tachyon \([30]\), hessence \([31]\) etc. All DE models can be classified by the behaviors of equations of state as following \([27]\):

(i) Cosmological constant: its EoS is exactly equal to \(-1\), that is \(w_{DE} = -1\);

(ii) Quintessence: its EoS remains above the cosmological constant boundary, that is \(w_{DE} \geq -1\);

(iii) Phantom: its EoS lies below the cosmological constant boundary, that is \(w_{DE} \leq -1\) and

(iv) Quintom: its EoS is able to evolve across the cosmological constant boundary. Inspired by the holographic principle \([32, 33]\), Gao et al.\([34]\] took the Ricci scalar as the IR cut-off and named it the Ricci dark energy (RDE), in which they take the Ricci scalar \(R\) as the IR cutoff. With proper choice of parameters the equation of state crosses \(-1\), so it is a ‘quintom’ \([35]\). The Ricci scalar of FRW universe is given by \(R = -6\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)\), where \(H\) is the Hubble parameter, \(a\) is the scale factor and \(k\) is the curvature. The energy density of RDE is given by \(\rho_{RDE} = 3c^2\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)\).

In flat FRW universe, \(k = 0\) and hence \(\rho_{RDE} = 3c^2\left(\dot{H} + 2H^2\right)\).

Interacting DE models have gained immense interest in recent times. Works in this direction include \([36–41]\). In a recent work by Jamil et al.\([40]\] examined the interacting dark energy model in \(f(T)\) cosmology assuming dark energy as a perfect fluid and choosing a specific cosmologically viable form \(f(T) = \beta\sqrt{T}\). Interacting RDE was considered in \([36]\), where the observational constraints on interacting RDE were investigated. In another recent work, Pasqua et al.\([42]\] reconstructed the potential and the dynamics of the tachyon, K-essence, dilaton and quintessence scalar field models according to the evolutionary behavior of the interacting entropy-corrected holographic RDE model. In the present work, we have considered an interacting RDE in the “logarithmic \(f(T)\) gravity” proposed by \([1]\). In the said form of \(f(T)\) gravity, the form of \(f(T)\) is proposed as \(f(T) = \beta T_0 \left(\frac{qT_0}{T}\right)^{-1/2} \ln \left(\frac{qT_0}{T}\right)\), where \(\beta = \frac{1-\Omega_0^m}{2q^{1/2}}\).

This logarithmic \(f(T)\) gravity model is basically constructed based on phenomenological approach. Therefore, the motivation to examine this model is that if we consider the interaction between the Ricci dark energy and dark matter in this model, we can obtain some desirable cosmological consequence.
II. AN OVERVIEW OF $f(T)$ GRAVITY

In the framework of $f(T)$ theory, the action of modified TG is given by

$$I = \frac{1}{16\pi G} \int d^4x \sqrt{-g} [f(T) + L_m]$$

where, $L_m$ is the Lagrangian density of the matter inside the universe. We consider a flat Friedmann-Robertson-Walker (FRW) universe filled with the pressureless matter. Choosing $(8\pi G = 1)$ the modified Friedman equations in the framework of $f(T)$ gravity are given by

$$H^2 = \frac{1}{3}(\rho + \rho_T)$$

$$2\dot{H} + 3H^2 = -(p + p_T)$$

where,

$$\rho_T = \frac{1}{2}(2Tf_T - f - T)$$

$$p_T = -\frac{1}{2}\left[-8\dot{H}f_T + (2T - 4\dot{H})f_T - f + 4\dot{H} - T\right]$$

Where

$$T = -6\left(H^2\right)$$

As we are considering interaction between pressureless dark matter and RDE, we shall have $\rho = \rho_m + \rho_{RDE}$ and $p = p_{RDE}$ in the equations (2) and (3). In the logarithmic $f(T)$ gravity

$$f(T) = \beta T_0 \left(\frac{qT_0}{T}\right)^{-1/2} \ln \left(\frac{qT_0}{T}\right)$$

where, $\beta = \frac{1-\Omega_m(0)}{2q^{1/2}}$ and $q$ is a positive constant. In [1], Bamba et al. have shown that for the said form of $f(T)$ gravity, the EoS parameter stays above $-1$, when plotted against redshift $z$. In the present work, we shall investigate the behavior of the EoS parameter when interacting RDE is considered in the said form of modified gravity. In the next section, we shall briefly describe the mathematical background of the RDE.

III. INTERACTING RDE

The metric of a spatially flat homogeneous and isotropic universe in FRW model is given by

$$ds^2 = dt^2 - a^2(t) \left[dr^2 + r^2(d\theta^2 + sin^2 \theta d\phi^2)\right]$$
where \(a(t)\) is the scale factor. The conservation equation is given by

\[
\dot{\rho} + 3H(\rho + p) = 0
\]  

(9)

As we are considering interaction between RDE and dark matter, the conservation equation will take the following form

\[
\dot{\rho}_{\text{total}} + 3H(\rho_{\text{total}} + p_{\text{total}}) = 0
\]

(10)

where, \(\rho_{\text{total}} = \rho_{\text{RDE}} + \rho_{\text{m}}\) and \(p_{\text{total}} = p_{\text{RDE}}\) (as we are considering pressureless dark matter, \(p_{\text{m}} = 0\)). It is already stated that

\[
\rho_{\text{RDE}} = 3c^2\left(\dot{H} + 2H^2 + \frac{k}{a^2}\right)
\]

(11)

As in the case of interaction the components do not satisfy the conservation equation separately, we need to reconstruct the conservation equation by introducing an interaction term \(Q\). Considering the interaction term \(Q\) as \(Q = 3H\delta\rho_{\text{m}}\) \([43, 44]\), where \(\delta\) is the interaction parameter, the conservation equation takes the form

\[
\dot{\rho}_{\text{RDE}} + 3H(\rho_{\text{RDE}} + p_{\text{RDE}}) = 3H\delta\rho_{\text{m}}
\]

(12)

and

\[
\dot{\rho}_{\text{m}} + 3H\rho_{\text{m}} = -3H\delta\rho_{\text{m}}
\]

(13)

In the subsequent section, we shall discuss the behaviors of the various cosmological parameters when the interacting RDE is considered in the logarithmic \(f(T)\) gravity.

**IV. DISCUSSION**

Solving the conservation equation (15) we get under interaction

\[
\rho_{\text{m}} = \rho_{m0}a^{-3(1+\delta)}
\]

(14)

Using equations (6) and (7) we get \(\rho_T\) from equation (4) and use it in (2) along with (14) and get the following differential equation

\[
(15 + 6c^2)H^2 + \left(\frac{6\beta H_0}{\sqrt{q}}\right)H + 3c^2\dot{H} + \rho_{m0}a^{-3(1+\delta)} = 0
\]

(15)
FIG. 1: This figure plots the Hubble parameter $H$ for interacting RDE under logarithmic $f(T)$ gravity. We have taken $\Omega_m^0 = 0.26$ and $\delta = 0.05$.

From equation (15) we get the new $H$ under the said interaction considered in logarithmic $f(T)$ gravity and plot against redshift $z = a^{-1} - 1$ in figure 1 for $c^2 < 0.5$, $= 0.5$ and $> 0.5$. The three cases are indicated by solid, dashed and dotted lines respectively in the figure. This is followed in the subsequent figures also. Following [1], we take $\Omega_m^0 = 0.26$ while plotting $H$. As we approach from higher to lower redshifts, we observe that $H(z)$ is exhibiting an increasing pattern. The rate of increase is sharper in the case of $c^2 < 0.5$ than the other two cases. To view the behavior of the logarithmic $f(T)$ we have plotted it against $z$ in figure 2. From figure 2 we observe that $f(T)$ is

FIG. 2: This figure plots the $f(T)$ against redshift $z$. We have taken $\Omega_m^0 = 0.26$ and $\delta = 0.05$. 
FIG. 3: This figure plots the $\rho_{RDE}$ against redshift $z$. We have taken $\Omega_m^0 = 0.26$ and $\delta = 0.05$.

FIG. 4: This figure plots the density contribution due to torsion i.e. $\rho_T$ against redshift $z$. We have taken $\Omega_m^0 = 0.26$ and $\delta = 0.05$.

increasing with evolution of the universe.

In figure 3, we have plotted the density $\rho_{RDE}$ under the interaction in the proposed model of $f(T)$ gravity. We observe that for all of the choices of $c^2$, the density of RDE is increasing with the evolution of the universe. This is consistent with the fact that the universe has evolved from matter dominated to DE dominated phase. In figure 4, we plot the density contribution due to torsion and we observe that like $\rho_{RDE}$, the $\rho_T$ is increasing as the universe evolves from higher to lower redshifts. In figures 5 – 7 we have plotted the EoS parameter for $c^2 < 0.5$, $c^2 = 0.5$ and $c^2 > 0.5$. 
In all of the figures we observe \( w_{RDE} < -1 \) that indicates phantom-like behavior [48].

V. CONCLUDING REMARKS

In the present work we have considered a modified gravity dubbed as “logarithmic \( f(T) \) gravity” and investigated the behavior of Ricci dark energy interacting with pressureless dark matter. We have chosen the interaction term in the form \( Q \propto H \delta \rho_m \) and investigated the behavior of the Hubble parameter \( H \) as a function of the redshift \( z \). For this reconstructed \( H \) we have investigated the behavior of the density of the Ricci dark energy \( \rho_{RDE} \) and density contribution due to torsion \( \rho_T \). All of the said cosmological parameters are seen to have increasing behavior from higher to lower redshifts for all values of \( c^2 \) pertaining to the Ricci dark energy. Subsequently, we observed the equation of state parameter \( w_{RDE} \) in this situation. In [1], the motivation behind this study, the logarithmic \( f(T) \) gravity was found not to cross the phantom-divide. In the present paper we considered interacting Ricci dark energy in the said form of gravity and here also we found that the phantom-divide can not be realized. Rather, the equation of state parameter
\(w_{RDE}\) is found to stay below \(-1\) that indicates phantom-like behavior. In [49] it was reported that the Ricci dark energy behaves like phantom in Einstein gravity for \(c^2 < 0.5\). However, in the present work, where we consider interacting Ricci dark energy in a modified gravity in the form of logarithmic \(f(T)\) gravity, the equation of state parameter behaves like phantom for all choices of \(c^2\).

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