On the ambiguities in the tri-bimaximal mixing matrix and corresponding charged lepton corrections

Chandan Duarah\(^1\) and N Nimai Singh\(^2\)

\(^1\) Department of Physics, Dibrugarh University, Dibrugarh 786 004, India
\(^2\) Department of Physics, Gauhati University, Guwahati 781 014, India

E-mail: chandan.duarah@gmail.com

Received 2 June 2013
Accepted for publication 29 October 2013
Published 25 November 2013
Online at stacks.iop.org/PhysScr/88/065101

Abstract

Two negative signs naturally appear in the \(U_{\mu 1}\) and \(U_{\tau 2}\) elements of the tri-bimaximal (TBM) matrix for positive values of the mixing angles \(\theta_{12}\) and \(\theta_{23}\). Apart from this, in other TBM matrices negative signs are shifted to other elements in each case. They account for positive as well as negative values of \(\theta_{12}\) and \(\theta_{23}\). We discuss the sign ambiguity in the TBM matrix and find that the TBM matrices, in fact, can be divided into two groups under certain circumstances. Interestingly, this classification of the TBM matrices is accompanied by two different \(\mu - \tau\) symmetric mass matrices which can separately be related to the groups. To accommodate the non-zero value of \(\theta_{13}\) and deviate \(\theta_{23}\) toward the first octant, we then perturb the TBM mixing ansatz with the help of charged lepton correction. The diagonalizing matrices for the charged lepton mass matrices also possess sign ambiguity and respect the grouping of the TBM matrices. They are parameterized in terms of the Wolfenstein parameter \(\lambda\) and satisfy the unitarity condition up to the second order in \(\lambda\).

PACS number: 14.60.Pq

1. Introduction

Tri-bimaximal (TBM) mixing, also known as Harrison–Perkins–Scott mixing [1], is a specific lepton mixing ansatz which draws special interest in the search of the exact lepton mixing pattern. It respects \(\mu - \tau\) symmetry [2] and can also be realized from discrete symmetries such as \(A_4\), \(S_4\) [3–6]. These interesting facts add significant attention to the TBM mixing ansatz. Except the prediction \(\theta_{13} = 0\) on the reactor angle, the other two predictions on solar for angle \(\theta_{12}\) and atmospheric angle \(\theta_{23}\) of TBM mixing are attractively close to the existing global data. However, a small non-zero value of \(\theta_{13}\), confirmed by recent results from DAYA BAY [7], RENO [8] and DOUBLE CHOOZ [9] collaborations, indicates certain deviation of neutrino mixing from the exact TBM mixing ansatz. The global analysis of the 3\(\nu\) oscillation data [10] prefers the first octant for \(\theta_{23}\). A lot of works which discuss the deviations from TBM mixing can be found in the literature [11–13]. We address the issue of sign ambiguity in the TBM mixing matrix and suitable charged lepton correction to TBM mixing, which can accommodate non-zero \(\theta_{13}\) and \(\tan^2 \theta_{23} < 1\) as well.

In the TBM mixing ansatz, the neutrino mass eigenstate \(v_2\) is tri-maximally mixed between all of the three lepton flavors while the mass eigenstate \(v_3\) is bimaximally mixed between the \(v_\mu\) and \(v_\tau\) flavors. The consequences of this mixing ansatz are: \(\theta_{13} = 0\), \(\theta_{23} = \pm 45^\circ\) and \(\theta_{12} = \pm \sin^{-1}(\frac{1}{\sqrt{3}})\). When all of the elements of the mixing matrix are expressed in moduli squares, the TBM matrix has the form [1]

\[
|U_{\text{TBM}}|^2 = \begin{pmatrix}
\frac{2}{3} & \frac{1}{3} & 0 \\
\frac{1}{3} & \frac{1}{3} & \frac{1}{\sqrt{2}} \\
\frac{1}{3} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\]

(1)

By taking the square root of each element of equation (1), the TBM matrix \(U_{\text{TBM}}\) can be obtained, where each element can assume either a positive or negative value. The choice of the sign is not unique; rather it arises from the particular model considered. A few familiar choices, available in the literature,
for $U_{\text{TB}}$ are
\begin{align}
\begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}
\end{pmatrix} ,
\end{align}
(2)
\begin{align}
\begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
-\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix} ,
\end{align}
(3)
\begin{align}
\begin{pmatrix}
\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix} ,
\end{align}
(4)
\begin{align}
\begin{pmatrix}
\sqrt{\frac{2}{3}} & -\frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix} ,
\end{align}
(5)
and
\begin{align}
\begin{pmatrix}
-\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\
\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}
\end{pmatrix} .
\end{align}
(6)
where the positions of negative signs are different in different matrices. Starting from the standard Particle Data Group (PDG) parameterization of the lepton mixing matrix, it can be shown that the above TBM matrices correspond to different choices of positive and negative values of the mixing angles $\theta_{12}$ and $\theta_{23}$. The sign ambiguity in the TBM matrix sometimes creates inconveniences in the phenomenological works [14], related to the parameterization of the neutrino mass matrices. Different choices of TBM matrices lead to different results. In order to avoid such inconveniences, we are motivated to place different TBM matrices in two groups viz. group-I and group-II. It is interesting to see that this classification is immediately followed by the identification of two different $\mu - \tau$ symmetric mass matrices which are separately associated with the groups. They differ from each other by a distinguishing character obeyed by the $m_{\text{eff}}$ and $m_{\tau\tau}$ elements of the mass matrix. Group I contains a single TBM matrix which accounts for positive values of both $\theta_{12}$ and $\theta_{23}$, while group-II contains the other TBM matrices which account for both the positive and the negative values of the mixing angles. With regard to the phenomenological works [14], this classification then directs us to relate the TBM matrix of group-I, say, only with the mass matrix that is associated with the same group. This omits misleading results in numerical analysis. The classification is also suitable in the discussion of charged lepton correction to TBM mixing. We find an appropriate form for the diagonalizing matrix of the charged lepton mass matrix which can generate a non-zero value of $\sin\theta_{13}$ and $\tan^{2}\theta_{23} < 1$. These charged lepton mass diagonalizing matrices also reflect sign ambiguity and two different diagonalizing matrices work separately for the two groups.

The paper is organized as follows: in section 2, we discuss the sign ambiguity and the classification of the TBM matrices. Section 3 presents the charged lepton correction to TBM mixing without CP effects. The discussion of charged lepton correction is reanalyzed in the presence of a CP violating phase in section 4. Finally, section 5 is devoted to the summary and discussion.

2. TBM mixing matrix and the sign ambiguity

In the standard PDG parameterization [15], the lepton mixing matrix, also known as Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix, is written as
\begin{equation}
U_{\text{PMNS}} =
\begin{pmatrix}
\cos\theta_{12} \cos\theta_{13} & \cos\theta_{12} \sin\theta_{13} e^{i\delta_{CP}} & s_{12} c_{13} e^{-i\delta_{CP}} \\
-s_{12} \cos\theta_{23} - c_{12} s_{23} s_{13} e^{i\delta_{CP}} & c_{12} \cos\theta_{23} - s_{12} s_{23} c_{13} e^{i\delta_{CP}} & -c_{12} s_{13} e^{-i\delta_{CP}} \\
-s_{23} s_{13} c_{12} e^{i\delta_{CP}} & -c_{23} s_{13} c_{12} e^{i\delta_{CP}} & c_{13}
\end{pmatrix}
P,
\end{equation}
(7)
where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ with $i, j = 1, 2, 3$, $\delta_{CP}$ is the Dirac CP phase and $P = \text{diag}(1, e^{i\alpha}, e^{i\beta})$ is the diagonal matrix which contains two Majorana CP phases $\alpha$ and $\beta$. In our discussion, we ignore the Majorana phases.

For TBM mixing $s_{13} = 0$ and under this condition equation (7) reduces to
\begin{equation}
U_{\text{PMNS}} =
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} c_{23} & c_{12} c_{23} & s_{23} \\
-s_{12} c_{23} & -c_{12} s_{23} & c_{23}
\end{pmatrix} .
\end{equation}
(8)
We would now like to classify different TBM matrices presented in equations (2)–(6). The TBM matrix in equation (2) is placed in group-I and we denote it as $U_{\text{TB}}^{I}$. This TBM matrix can be obtained from equation (8) for positive values of both $\theta_{12}$ and $\theta_{23}$. The remaining four TBM matrices in equations (3)–(6) are placed in group-II and we denote them as $U_{\text{TB}}^{IIa}$, $U_{\text{TB}}^{IIb}$, $U_{\text{TB}}^{IIc}$ and $U_{\text{TB}}^{IId}$, respectively. The TBM matrix $U_{\text{TB}}^{IIb}$ in equation (3) can be obtained from equation (8) for positive $\theta_{12}$ and negative $\theta_{23}$ and $U_{\text{TB}}^{IIa}$ in equation (5) can be obtained from equation (8) for negative values of both $\theta_{12}$ and $\theta_{23}$. The TBM matrix $U_{\text{TB}}^{IId}$ in equation (6) can be obtained from equation (8) under the transformations $\theta_{12} \rightarrow (\pi - \theta_{12})$ and $\theta_{23} \rightarrow (\pi - \theta_{23})$.

For convenience, let us now represent these choices of TBM matrices as different sign conventions where, for example, we obtain the convention
\begin{equation}
\begin{pmatrix}
+ & + & 0 \\
- & + & + \\
+ & - & +
\end{pmatrix} ,
\end{equation}
for the TBM matrix in equation (2). We would like to use a bold face notation $U_{\text{TB}}^{I}$ in correspondence to $U_{\text{TB}}^{II}$, for this sign convention. Similarly we obtain sign conventions $U_{\text{TB}}^{IIa}$, $U_{\text{TB}}^{IIb}$, $U_{\text{TB}}^{IIc}$ and $U_{\text{TB}}^{IId}$ for the TBM matrices in equations (3)–(6), respectively.

The underlying motivation for this classification of the TBM matrices is basically extracted from phenomenological works [14], based on parameterization of $\mu - \tau$ symmetric mass matrices. The general $\mu - \tau$ symmetric mass matrix is
which leads to maximal atmospheric mixing and zero reactor angle, leaving behind the solar angle arbitrary. On choosing the diagonalizing matrix

$$U = \begin{pmatrix}
c_{12} & s_{12} & 0 \\
-\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
-\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix},$$

for the mass matrix in equation (9), we obtain the important relation

$$\tan 2\theta_{12} = \frac{2\sqrt{2}B}{A - C - D},$$

which allows us to fix the value of the solar angle at any desired value by the choice of the elements of the mass matrix. This relation has significant implications in the works done in [14]. Here, we have followed the diagonalization relation $m_{\mu\tau}^{\text{Max}} = U m_{\mu\tau}^{\text{II}} U$ and for the diagonal matrix in equation (10) we follow the sign convention $U_{\text{aTB}}^{\mu\tau}$. The inconvenience due to sign ambiguity in the TBM matrix is that if we choose the sign convention $U_{\text{TB}}^{\mu\tau}$, for example, for the diagonalizing matrix $U$ to diagonalize the mass matrix in equation (9), we obtain the undesired relation for $\tan 2\theta_{12}$ instead of equation (11).

We note that $U_{\text{TB}}^{\mu\tau}$ in group I predicts that both the mixing angles $\theta_{12}$ and $\theta_{23}$ are positive while the TBM matrices in group II predict that either one of the mixing angles is negative or both are negative. The $\mu - \tau$ symmetric mass matrix which is consistent with the positive mixing angles is given by [2]

$$m_{\mu\tau}^{\text{I}} = \begin{pmatrix}
A & -B & B \\
-B & C & D \\
B & D & C
\end{pmatrix}.$$ 

This mass matrix is different from $m_{\mu\tau}^{\text{II}}$ by the distinguishing character $m_{ee} = -m_{\mu\tau}$. We therefore associate this mass matrix with the TBM matrix of group I. Then the diagonalizing matrix in equation (10), along with sign convention $U_{\text{TB}}^{\mu\tau}$, leads to the desired expression

$$\tan 2\theta_{12} = \frac{2\sqrt{2}B}{A - C + D}.$$ 

The mass matrix $m_{\mu\tau}^{\text{II}}$ does not guarantee positive mixing angles and it works for the TBM matrices of group II. We obtain the same expression for $\tan 2\theta_{12}$, given in equation (11), when we follow any sign convention $U_{\text{TB}}^{\mu\tau}$ ($i = a, b, c, d$) for the diagonalizing matrix $U$ in equation (10).

3. Charged lepton correction to the TBM matrix

Charged lepton corrections [16–19] to neutrino mixing may be defined through the relation

$$U_{\text{PMNS}} = U_{\text{IL}} U_{\nu},$$

where $U_{\text{PMNS}}$ is the lepton mixing matrix, $U_{\text{IL}}$ and $U_{\nu}$ are the diagonalizing matrices for the charged lepton and left-handed Majorana neutrino mass matrices, respectively. They are defined through the relations: $m_1 = U_{\text{IL}} m_1 \text{diag} V_{1R}$ and $m_\nu = U_{\nu} m_\nu \text{diag} V_{1R}$, where $m_2 \text{diag} = \text{Diag}(m_\nu, m_{\mu}, m_{\tau})$ and $m_\nu \text{diag} = \text{Diag}(m_1, m_2, m_3)$. In the basis where the charged lepton mass matrix $m_1$ is diagonal, $U_{\text{PMNS}} = U_{\nu}, U_{\text{IL}}$ being the identity matrix. The effects of the charged lepton correction in this basis can be absorbed in the left-handed Majorana mass matrix as $m_{\nu} = U_{\nu} m_1 U_{\nu}^\dagger$.

For our case $U_{\nu}$ is to be given by $U_{\text{TB}}$. We then propose a possible form for the charged lepton mass diagonalizing matrix $U_{\text{IL}},$ parameterized in terms of Wolfenstein parameter $\lambda$ [20], which can generate non zero $\theta_{13}$ as well as $\tan^2 \theta_{23} < 1$. In our analysis, we are preferring the first octant for $\theta_{23}$, motivated by the global analysis data [10]. The diagonalizing matrix is given by

$$U_{\text{IL}}^{\mu\tau} = \begin{pmatrix}
1 - \frac{\lambda^2}{8} & \frac{1}{2} & \frac{\lambda}{2} & \frac{\lambda^2}{8} \\
-\frac{\lambda}{2} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} & \frac{\lambda}{8} \\
\frac{\lambda}{2} & \frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} \\
\frac{\lambda^2}{8} & \frac{\lambda}{8} & -\frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8}
\end{pmatrix},$$

which works for the TBM matrix of group-I. The diagonalizing matrix that works for TBM matrices of group-II is given by

$$U_{\text{IL}}^{\mu\tau} = \begin{pmatrix}
1 - \frac{\lambda^2}{8} & \frac{1}{2} & \frac{\lambda}{2} & \frac{\lambda^2}{8} \\
-\frac{\lambda}{2} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} & \frac{\lambda}{8} \\
\frac{\lambda}{2} & \frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} \\
\frac{\lambda^2}{8} & \frac{\lambda}{8} & -\frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8}
\end{pmatrix}.$$ 

The diagonalizing matrices in equations (15) and (16) satisfy the unitarity condition up to the second order in $\lambda$. Their structures can be derived from the diagonalizing matrix considered in [19], which is

$$U_{\text{IL}}^\dagger = \tilde{R}_{23} \tilde{U}_{\text{IL}}.$$ 

Here

$$\tilde{R}_{23} = \begin{pmatrix}
1 & 0 & 0 \\
0 & \tilde{c}_{23} & \tilde{s}_{23} \\
0 & -\tilde{s}_{23} & \tilde{c}_{23}
\end{pmatrix}$$

and

$$\tilde{U}_{\text{IL}} = \begin{pmatrix}
\frac{\lambda}{2} & -\frac{\lambda^2}{8} & -\frac{\lambda^2}{8} \\
\frac{1}{2} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} \\
\frac{\lambda}{2} & -\frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8}
\end{pmatrix}.$$ 

Equations (17)–(19) then give

$$U_{\text{IL}} = \begin{pmatrix}
1 - \frac{\lambda^2}{8} & \frac{1}{2} & \frac{\lambda}{2} & \frac{\lambda^2}{8} \\
-\frac{\lambda}{2} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} & \frac{\lambda}{8} \\
\frac{\lambda}{2} & \frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8} & -\frac{\lambda^2}{8} \\
\frac{\lambda^2}{8} & \frac{\lambda}{8} & -\frac{\lambda^2}{8} & 1 - \frac{\lambda^2}{8}
\end{pmatrix}.$$
Under the approximations $\delta_{23} \approx \lambda^2$ and $\tilde{c}_{23} \approx 1$, equation (20) leads to

$$U_{\text{IL}} \approx \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \frac{\lambda}{2} & -\frac{\sqrt{2}}{2} \\ (1 - \lambda^2) \frac{\lambda}{2} & \frac{\lambda}{2} + (1 - \frac{\lambda^2}{8}) & -\frac{\lambda}{2} \\ (1 + \lambda^2) \frac{\lambda}{2} & -\frac{\lambda}{2} + \lambda^2 (1 - \frac{\lambda^2}{8}) & -\frac{\lambda}{2} + (1 - \frac{\lambda^2}{8}) \end{pmatrix}$$

or

$$U_{\text{IL}}^\dagger = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \frac{\lambda}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\lambda}{2} & 1 - \frac{\lambda^2}{2} - \frac{\lambda}{2} \lambda^2 & \frac{\lambda}{2} \\ -\frac{\lambda}{2} & \frac{\lambda}{2} \lambda^2 & 1 - \frac{\lambda^2}{8} \end{pmatrix} + O(\lambda^{n+2}).$$

The structure of the matrix on the right-hand side of equation (22) is what was considered for $U_{\text{IL}}^\dagger$ or $U_{\text{IL}}^{\dagger\dagger}$.

Then the relations $U_{\text{PMNS}}^1 (U_{\text{TB}}^\dagger)^i (U_{\text{TB}}^\dagger)^j (U_{\text{PMNS}}^j) = (U_{\text{PMNS}}^j) (U_{\text{TB}}^\dagger)^i (U_{\text{TB}}^\dagger)^j (U_{\text{PMNS}}^i)$ lead to the PMNS matrices

$$U_{\text{PMNS}}^1 = \begin{pmatrix} \sqrt{\frac{3}{5}} (1 - \frac{\lambda^2}{4}) & \frac{\lambda}{\sqrt{15}} (1 - \frac{\lambda^2}{4}) & -\frac{\lambda}{\sqrt{2}} \\ -\frac{\lambda}{\sqrt{6}} (1 - \lambda + \frac{\lambda^2}{2}) & \frac{\lambda}{\sqrt{15}} (1 + \frac{\lambda^2}{2} + \lambda^2) & \frac{\lambda}{\sqrt{2}} (1 - \frac{\lambda^2}{4}) \\ \frac{1}{\sqrt{6}} (1 + \lambda - \lambda^2) & -\frac{1}{\sqrt{3}} (1 - \frac{\lambda^2}{2} - \lambda^2) & \frac{1}{\sqrt{2}} (1 + \frac{\lambda^2}{4}) \end{pmatrix},$$

$$U_{\text{PMNS}}^{\dagger \dagger} = \begin{pmatrix} \sqrt{\frac{3}{5}} (1 - \frac{\lambda^2}{4}) & \frac{\lambda}{\sqrt{15}} (1 - \frac{\lambda^2}{4}) & -\frac{\lambda}{\sqrt{2}} \\ -\frac{\lambda}{\sqrt{6}} (1 - \lambda + \frac{\lambda^2}{2}) & \frac{\lambda}{\sqrt{15}} (1 + \frac{\lambda^2}{2} + \lambda^2) & \frac{\lambda}{\sqrt{2}} (1 - \frac{\lambda^2}{4}) \\ \frac{1}{\sqrt{6}} (1 + \lambda - \lambda^2) & -\frac{1}{\sqrt{3}} (1 - \frac{\lambda^2}{2} - \lambda^2) & \frac{1}{\sqrt{2}} (1 + \frac{\lambda^2}{4}) \end{pmatrix},$$

$$U_{\text{PMNS}}^{\dagger} = \begin{pmatrix} \sqrt{\frac{3}{5}} (1 - \frac{\lambda^2}{4}) & \frac{\lambda}{\sqrt{15}} (1 - \frac{\lambda^2}{4}) & -\frac{\lambda}{\sqrt{2}} \\ -\frac{\lambda}{\sqrt{6}} (1 - \lambda + \frac{\lambda^2}{2}) & \frac{\lambda}{\sqrt{15}} (1 + \frac{\lambda^2}{2} + \lambda^2) & \frac{\lambda}{\sqrt{2}} (1 - \frac{\lambda^2}{4}) \\ \frac{1}{\sqrt{6}} (1 + \lambda - \lambda^2) & -\frac{1}{\sqrt{3}} (1 - \frac{\lambda^2}{2} - \lambda^2) & \frac{1}{\sqrt{2}} (1 + \frac{\lambda^2}{4}) \end{pmatrix},$$

and

$$U_{\text{PMNS}}^{\dagger \dagger} = \begin{pmatrix} \sqrt{\frac{3}{5}} (1 - \frac{\lambda^2}{4}) & \frac{\lambda}{\sqrt{15}} (1 - \frac{\lambda^2}{4}) & -\frac{\lambda}{\sqrt{2}} \\ -\frac{\lambda}{\sqrt{6}} (1 - \lambda + \frac{\lambda^2}{2}) & \frac{\lambda}{\sqrt{15}} (1 + \frac{\lambda^2}{2} + \lambda^2) & \frac{\lambda}{\sqrt{2}} (1 - \frac{\lambda^2}{4}) \\ \frac{1}{\sqrt{6}} (1 + \lambda - \lambda^2) & -\frac{1}{\sqrt{3}} (1 - \frac{\lambda^2}{2} - \lambda^2) & \frac{1}{\sqrt{2}} (1 + \frac{\lambda^2}{4}) \end{pmatrix}.\tag{27}$$

Table 1. Best fit, $1\sigma$ and $3\sigma$ ranges of the parameters for the normal hierarchy (NH) obtained from the global analysis by Forero et al [10].

| $\sigma$ range | $1\sigma$ range | $3\sigma$ range |
|---------------|----------------|----------------|
| $\tan^2 \theta_{23}$ | 0.42 - 0.50 | 0.37 - 0.587 |
| $\tan^2 \theta_{13}$ | 0.74 - 0.85 | 0.56 - 2.125 |
| $\sin^2 \theta_{13}$ | 0.024 - 0.0275 | 0.017 - 0.033 |

respectively. All of these matrices predict

$$\sin^2 \theta_{13} = \left| \frac{\lambda}{\sqrt{2}} \right|^2,$$

$$\tan^2 \theta_{23} = 0.5,$$

$$\tan^2 \theta_{13} = \left| \frac{1 - \frac{\lambda^2}{2}}{1 + \frac{\lambda^2}{2}} \right|^2.$$

For $\lambda = 0.225$ we obtain $\sin^2 \theta_{13} \approx 0.025$ and $\tan^2 \theta_{23} \approx 0.81$. These predictions on $\sin^2 \theta_{13}$ and $\tan^2 \theta_{23}$ are consistent with the $1\sigma$ range of global data (table 1).

The diagonalizing matrices in equations (15) and (16) also possess sign ambiguity and their identification for the two groups is analogous to the case of the $\mu - \tau$ symmetric mass matrices. We want to emphasize that if we follow the relation $U_{\text{PMNS}}^1 = (U_{\text{IM}}^1)^i (U_{\text{TB}})_{ii} (U_{\text{PMNS}}^j)$ instead of $U_{\text{PMNS}}^1 = (U_{\text{PMNS}}^j) (U_{\text{TB}})^i (U_{\text{TB}})^j (U_{\text{PMNS}}^i)$ say, it alters all of the predictions presented in equations (28)–(30).

It is important to note here that the PMNS matrix in any of the equations (23)–(27) when compared with the TBM-Cabibbo mixing matrix ($U_{\text{BC}}$) proposed by King [21], we find that it can predict $\tan^2 \theta_{23} < 1$ along with no zero $\theta_{13}$ while $U_{\text{BC}}$ predicts non zero $\theta_{13}$ keeping the solar and atmospheric angles fixed at TBM values.

4. CP violation

The PMNS mixing matrices in equations (23)–(27) conserve CP symmetry. In this section we would like to analyze the effects of a CP violating phase $\delta$ on the predictions of the PMNS matrix after charged lepton correction. To introduce the phase $\delta$ in $U_{\text{PMNS}}^1$ we follow the TBM-Cabibbo mixing matrix $U_{\text{BC}}$ [21], given by

$$U_{\text{BC}} = \begin{pmatrix} \sqrt{\frac{3}{5}} (1 - \frac{\lambda^2}{4}) & \frac{1}{\sqrt{15}} (1 - \frac{\lambda^2}{4}) & -\frac{\lambda}{\sqrt{2}} e^{-i\delta} \\ -\frac{\lambda}{\sqrt{6}} (1 - \lambda + \frac{\lambda^2}{2}) & \frac{1}{\sqrt{15}} (1 + \frac{\lambda^2}{2} + \lambda^2) & \frac{\lambda}{\sqrt{2}} e^{i\delta} \\ \frac{1}{\sqrt{6}} (1 + \lambda - \lambda^2) & -\frac{1}{\sqrt{3}} (1 - \frac{\lambda^2}{2} - \lambda^2) & \frac{1}{\sqrt{2}} (1 + \frac{\lambda^2}{4}) \end{pmatrix}.\tag{26}$$

If we ignore the phase $\delta$ in equation (31) we obtain $U_{\text{BC}} = U_{\text{IL}}^{\dagger} U_{\text{IL}}$, where $U_{\text{IL}}$ is defined in equation (19). Then, from equation (17) we find that the expression $U_{\text{PMNS}} = U_{\text{IL}}^{\dagger} U_{\text{IL}}$ is equivalent to $U_{\text{PMNS}} = R_{23}^{-1} U_{\text{BC}}$, where $R_{23}$ is given by equation (18). To incorporate the phase $\delta$ in the PMNS matrix we therefore employ the relation $U_{\text{PMNS}} = R_{23}^{-1} U_{\text{BC}}$ such that $U_{\text{BC}}$ is now given by equation (31) and the approximations $\delta_{23} \approx \lambda^2$ and $\tilde{c}_{23} \approx 1$ should be considered in addition.
\[ U_{\text{TBC}} \text{ in equation (31) follows the sign convention } U_{\text{TB}}. \text{ We thus obtain} \]
\[ U_{\text{PMNS}}^{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}}(1 - \frac{\lambda^2}{4}) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda^2}{4}) & -\frac{\lambda}{\sqrt{6}} e^{-\text{i}\delta} \\ -\frac{1}{\sqrt{6}}(1 - \lambda e^{\text{i}\delta} + \lambda^2) & \frac{1}{\sqrt{3}}(1 + \frac{3}{2} e^{\text{i}\delta} + \lambda^2) & \frac{1}{\sqrt{2}}(1 - \frac{\lambda}{2} \lambda^2) \\ \frac{1}{\sqrt{6}}(1 + \lambda e^{\text{i}\delta} - \lambda^2) & -\frac{1}{\sqrt{3}}(1 - \frac{\lambda}{2} e^{\text{i}\delta} - \lambda^2) & \frac{1}{\sqrt{2}}(1 + \frac{\lambda}{2} \lambda^2) \end{pmatrix}. \]

In a similar manner, we obtain the PMNS matrices for group II as
\[ U_{\text{PMNS}}^{\text{TBM}} = \begin{pmatrix} \sqrt{\frac{2}{3}}(1 - \frac{\lambda^2}{4}) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda^2}{4}) & \frac{\lambda}{\sqrt{6}} e^{-\text{i}\delta} \\ -\frac{1}{\sqrt{6}}(1 - \lambda e^{\text{i}\delta} + \lambda^2) & \frac{1}{\sqrt{3}}(1 + \frac{3}{2} e^{\text{i}\delta} + \lambda^2) & -\frac{1}{\sqrt{2}}(1 - \frac{\lambda}{2} \lambda^2) \\ -\frac{1}{\sqrt{6}}(1 + \lambda e^{\text{i}\delta} - \lambda^2) & \frac{1}{\sqrt{3}}(1 - \frac{\lambda}{2} e^{\text{i}\delta} - \lambda^2) & \frac{1}{\sqrt{2}}(1 + \frac{\lambda}{2} \lambda^2) \end{pmatrix}. \]

All of the PMNS matrices in equations (32)–(36) yield the same predictions of the mixing angles as given in equations (28)–(30). Furthermore, they all lead to a similar expression for the rephasing invariant quantity, defined as \( J_{\text{CP}} = \text{Im} \{ U_{e2} U_{\mu 3} U_{e3}^* U_{\mu 2}^* \} \), which is
\[ |J_{\text{CP}}| = \frac{1}{6} \lambda (1 + \lambda^2) \left( 1 - \frac{\lambda^2}{4} \right) \left( 1 - \frac{5}{4} \lambda^2 \right) \sin \delta. \]

For maximal CP violation we obtain \( |J_{\text{CP}}| \approx 0.0364. \)

5. Summary and discussion

We discuss the sign ambiguities in the TBM mixing matrix which arise due to different choices of positive and negative values of the mixing angles \( \theta_{12} \) and \( \theta_{23} \). Such sign ambiguities sometimes create inconveniences in the phenomenological works and the numerical analysis. To avoid the inconveniences we find it useful to divide different TBM matrices into two groups. Group-I contains a single TBM matrix which accounts for the positive values of both of the mixing angles. The other TBM matrices are placed in group-II. Few of them account for positive as well as negative values of \( \theta_{12} \) and \( \theta_{23} \). Some others are found to obey certain quadrant transformations. This grouping of the TBM matrices is followed by two \( \mu - \tau \) symmetric mass matrices, separately associated with the groups. They differ by the fact that for the mass matrix associated with group-I we have \( m_{\text{eq}} = -m_{\tau} \) while for the other, associated with group-II, we have \( m_{\text{eq}} = m_{\tau} \). The classification is also useful in the discussion of charged lepton correction to TBM mixing. We find a possible form of the charged lepton mass diagonalizing matrix \( U_{\text{CL}} \) which can generate non zero \( \theta_{13} \) and \( \tan^2 \theta_{23} < 1 \) consistent with the latest global analysis data. We can identify two diagonalizing matrices, which also reflect the sign ambiguities, for the two groups of TBM matrices such that they separately work to obtain the desired results. The discussion of the sign ambiguities and related classifications may help the authors in systematic phenomenological analysis. This work points out that it is useful to conduct phenomenological studies related to the TBM mixing ansatz under two groups where the TBM matrix which predicts the positive mixing angles can be isolated from the other TBM matrices.

References

[1] Harrison P F, Perkins D H and Scott W G 2002 Phys. Lett. B 530 167
Harrison P F and Scott W G 2002 Phys. Lett. B 535 163
[2] Lam C S 2001 Phys. Lett. B 507 214
Harrison P F and Scott W G 2002 Phys. Lett. B 547 219
Lam C S 2005 Phys. Rev. D 71 093001
[3] Ma E and Rajasekaran G 2001 Phys. Rev. D 64 113012
Babu K S, Ma E and Valle J W F 2003 Phys. Lett. B 552 207
[4] Low C I and Volkas R R 2003 Phys. Rev. D 68 033007
[5] Ma E 2004 Phys. Rev. D 70 031901
Ma E 2004 arXiv:hep-ph/0409075
Altarelli G and Feruglio F 2005 Nucl. Phys. B 720 64
Ma E 2006 Phys. Rev. D 73 057304
Feruglio F, Hagedorn C, Lin Y and Merlo L 2007 Nucl. Phys. B 775 120
Altarelli G and Feruglio F 2010 Rev. Mod. Phys. 82 2701
[6] An F P et al (DAYA-BAY Collaboration) 2012 Phys. Rev. Lett. 108 171803
[7] Ahn J K et al (RENO Collaboration) 2012 Phys. Rev. Lett. 108 191802
[8] Abe Y et al (DOUBLE-CHOOZ Collaboration) 2012 Phys. Rev. Lett. 108 131801
[9] Forero D V, Tortola M and Valle J W F 2012 Phys. Rev. D 86 073012
[10] Flambaum V, Tortola M and Valle J W F 2012 Phys. Rev. D 87 073012
[11] Plentinger F and Rodejohann W 2005 arXiv:hep-ph/0507143v3
Chen A H, Fritzsch H, Luo S and Xing Z Z 2007 Phys. Rev. D 76 073009
Pakvasa S, Rodejohann W and Weiler T J 2008 Phys. Rev. Lett. 100 111801
Boudjema S and King S F 2009 Phys. Rev. D 79 033001
King S F 2009 Phys. Lett. B 675 347
[12] Haba N, Watanabe A and Yoshioka K 2006 Phys. Rev. Lett. 97 041601
Goswami S, Petcov S T, Ray S and Rodejohann W 2009 Phys. Rev. D 80 053015

[13] Barry J and Rodejohann W 2010 Phys. Rev. D 81 093002
He X G and Zee A 2011 Phys. Rev. D 84 053004

[14] Singh N N, Rajkhowa M and Borah A 2007 Pramana J. Phys. 69 533
Francis N K and Singh N N 2012 Nucl. Phys. B 863 19
Singh N N, Rajkhowa M and Borah A 2007 J. Phys. G: Nucl. Part. Phys. 34 345

[15] Nakamura K et al (Particle Data Group) 2010 J. Phys. G: Nucl. Part. Phys. 37 075021

[16] Xing Z Z 2002 Phys. Lett. B 533 85

[17] Frampton P H, Petcov S T and Rodejohann W 2004 Nucl. Phys. B 687 31
Mohapatra R N and Rodejohann W 2005 arXiv:hep-ph/0507312v2
Antusch S and King S F 2005 Phys. Lett. B 631 42
Hochmuth K A, Petcov S T and Rodejohann W 2007 Phys. Lett. B 654 177

[18] Altarelli G, Feruglio F and Merlo L 2009 J. High Energy Phys. JHEP08(2009)020

[19] Toorop R A, Bazzocchi F and Merlo L 2010 J. High Energy Phys. JHEP08(2010)001
Morisi S, Patel K M and Peinado E 2011 Phys. Rev. D 84 053002
Marzocca D, Petcov S T, Romanino A and Spinrath M 2011 J. High Energy Phys. JHEP11(2011)009
Altarelli G, Feruglio F, Merlo L and Stamou E 2012 J. High Energy Phys. JHEP08(2012)021
Bazzocchi F and Merlo L 2012 arXiv:1205.5135v1 [hep-ph]
Dorame L, Morisi S, Peinado E and Valle J W F 2012 arXiv:1203.0155v1 [hep-ph]
Antusch S and Maurer V 2011 Phys. Rev. D 84 117301
Antusch S, Gross C, Maurer V and Sluka C 2012 arXiv:1205.1051v2 [hep-ph]
Acosta J A, Aranda A, Buen-Abad M A and Rojas A D 2012 arXiv:1207.6093v1 [hep-ph]
Varzielas I M 2012 J. High Energy Phys. JHEP01(2012)097
Cooper I K, King S F and Luhn C 2012 J. High Energy Phys. JHEP06(2012)130
Varzielas I M and Ross G G 2012 arXiv:1203.6636v3 [hep-ph]

[20] Altarelli G, Feruglio F and Merlo L 2009 J. High Energy Phys. JHEP08(2009)020

[21] Wolfenstein L 1983 Phys. Rev. Lett. 51 1945

Toorop R A, Bazzocchi F and Merlo L 2010 J. High Energy Phys. JHEP08(2010)001
Morisi S, Patel K M and Peinado E 2011 Phys. Rev. D 84 053002
Marzocca D, Petcov S T, Romanino A and Spinrath M 2011 J. High Energy Phys. JHEP11(2011)009
Altarelli G, Feruglio F, Merlo L and Stamou E 2012 J. High Energy Phys. JHEP08(2012)021
Bazzocchi F and Merlo L 2012 arXiv:1205.5135v1 [hep-ph]
Dorame L, Morisi S, Peinado E and Valle J W F 2012 arXiv:1203.0155v1 [hep-ph]
Antusch S and Maurer V 2011 Phys. Rev. D 84 117301
Antusch S, Gross C, Maurer V and Sluka C 2012 arXiv:1205.1051v2 [hep-ph]
Acosta J A, Aranda A, Buen-Abad M A and Rojas A D 2012 arXiv:1207.6093v1 [hep-ph]
Varzielas I M 2012 J. High Energy Phys. JHEP01(2012)097
Cooper I K, King S F and Luhn C 2012 J. High Energy Phys. JHEP06(2012)130
Varzielas I M and Ross G G 2012 arXiv:1203.6636v3 [hep-ph]