The Model of Visual Thinking of Prospective Math Teacher in Understanding the Formal Definition of Convergent Sequences Based on Gender Differences

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Abstract. The model of visual thinking is interesting to learn because the concepts are organized based on the meaning of forming mental imagery in the mind. Thinking by using mental imagery (visual information) is called visual thinking. This paper discusses the model of visual thinking of prospective mathematics teachers in understanding formal definition of convergent sequences based on gender differences as a result of descriptive-explorative research with a qualitative approach. The visual thinking activities of male and female students are found around three aspects: bringing up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. To bring up mental imagery, the mind summons mental imagery or makes mental imagery. To determine mental imagery, the mind collects mental imagery and selects mental imagery. To improve mental imagery, the mind justifies mental imagery or completes mental imagery. To utilize mental imagery, the mind uses mental imagery or represents mental imagery. Processing mental imagery has done according to the needs and purpose of the activity in understanding the formal definition of the convergent sequence.

1. Introduction

Students of mathematics education courses are candidates for mathematics teachers. The characteristics of students entering the program with a high readiness to learn, bringing a lot of experience to the class, less flexible in interacting, want to be treated by maturity and have initiative and independence [1]. The six behaviors of students learning are self-determining the direction and purpose of education, having a set of life experiences, goal-oriented, and relevance, tending to be practical and requiring rewards in learning [2].

One of the materials that must be understood by the students of prospective mathematics teacher is the course of Real Analysis. One of the concepts in the real analysis course is the concept of real numbers sequence. The sequence is the function of the set of natural numbers to a set of real numbers. Each expert can use different notations and symbols in defining. Some notations are \( \{a_n\}_{n=1}^{\infty} \) [3], \( (x_n)_{n=1}^{\infty} \) [4], \( \{a_n\} \) [5], \( \{a_n\} \) [6]. In this paper, the sequence of real numbers is denoted by \( \{a_n\}_{n=1}^{\infty} \).

One of the real numbers of sequence material that often makes it difficult for students is a convergent sequence concept. Some experts have defined the concept of convergent sequence. If the sequence of real numbers \( \{a_n\}_{n=1}^{\infty} \) has the limit \( L \), we say that \( \{a_n\}_{n=1}^{\infty} \) is convergent to \( L \) [3]. Let \( X = \{a_n\}_{n=1}^{\infty} \) be a sequence of real numbers. A real number \( a \) is said to be a limit of \( X \) if for each neighborhood \( V \) of \( a \) there is a natural number \( K(V) \) such that for all \( n \geq K(V) \), the terms \( a_n \) belong to \( V \) [4]. A sequence \( \{a_n\}_{n=1}^{\infty} \) is said to converge to \( a \in R \) if given \( \varepsilon > 0 \), there exist a positive integer \( m \)
such that \( n \geq m \) implies \(|a_n - a| < \varepsilon\) [5]. The sequence \( \{a_n\}_{n \geq 1} \) is converged to \( L \) and written \( \lim_{n \to \infty} a_n = L \), if for each positive number \( \varepsilon \) there is a corresponding positive number \( N \). Such that \( n \geq N \Rightarrow |a_n - L| < \varepsilon \). In this paper, the sequence of real numbers \( \{a_n\}_{n \geq 1} \) is said to be convergent sequence if and only if for every \( \varepsilon > 0 \) there is \( a \in R \) and \( n_0(\varepsilon) \in \mathbb{N} \) so for \( n \geq n_0(\varepsilon) \) apply \(|a_n - a| < \varepsilon\).

To understand definition means to understand the concepts underlying the definitions. Understanding the formal definition of a convergent sequence can be a problem or difficulty for some individuals so it is necessary to look for an immediate solution. One of the goals of mathematics lessons is to understand the concept of mathematics on the content standards behind the concepts [7]. Prolonged difficulty brings a negative impact on brain function. Therefore, it needs creative effort to understand the definition immediately.

One of the creative endeavors to understand is by visualization. To visualize the formal definition of convergent lines, it is necessary to represent the convergent lines and attributes that exist in the formal definition of convergent lines. Visualization requires visual thinking. However, students were found to still lacking in mental imagery [8]. Research results show that vocational secondary students were lacking in visual thinking [9]. Besides, learning is still too focused on formalities.

Visual thinking is information processing in the mind by using mental imagery. Mental imagery is modalities or mental representation (a form of information in the memory) that can be presented in the form of images/graphs. The internal representation of visual codes is mental imagery. Mental imagery is the mental image [10]. Mental imagery is image information or visual knowledge or visual imaging or images or visual imagery [11]. Mental imagery can be called as a picture for perception [12]. Some writers or other researchers call mental imagery as mental images, mental images, shadows or images of the mind, visualization, imagination, imagination, imagery, or visual imagery.

Some research was undertaken by focusing on visual thinking within learning process. Thus, some thoughts regarding this topic are emerging. One such influence is the clarity and stability of their beliefs about the role of visualization [13]. In 2003, the research has examined the increase in limit learning outcomes with visualization [14]. However, the research in 2006 shows that students are still struggling to use visualizations to improve perception [15]. Mental imagery only benefits the learning process’ [16]. In 2016, I have researched the visual profile of student thinking with visual [17] and auditory [18] learning style in solving the problem. The stages of students in processing mental shading have not been researching.

In general learning in schools such as SD / MI (Elementary School), SMP / MTS (Junior High School), SMA / SMK (Senior/Vocational High School), even to college. The teachers/educators distinguish students until college students based on gender differences. Gender refers to the concept of male or female based on socio-cultural and psychological dimensions [19]. Gender is distinguished by sex involving the biological dimensions of women or men. Therefore, the university knows for male (vigorouls) college students and female college students. Regarding visual thinking scheme, the understanding of the formal definition of convergent sequences based on interesting gender differences are discussed to provide a picture of organizing the concepts of male and female prospective mathematics teachers. By this knowledge, lecturers can give them necessary explanations in understanding the formal definition of convergent sequences.

2. Research methods

The model of visual thinking of prospective math teachers in understanding the formal definition of convergent sequences were presented in complete form with words or sentences according to empirical data. Empirical evidence obtained is natural as they are and focus more on the process than on results. Therefore, a descriptive-explorative research was used with the qualitative approach in this present study.

To obtain the data, a male and a female college students of mathematics education study program of a university in Madiun city, East Java, were selected as the participants of this research. Then, the criteria for selecting the participants are based on some points: 1) having taken the course of real analysis courses, 2) have a GPA above 2.75, 3) The grade/score of the course of calculus value and introduction
to basic mathematics minimum B, 4) communicative, honest, and willing to be the participants of research.

The primary instrument in this study is the researchers themselves. Researchers are required: 1) have responsive, adaptive, and holistic nature; 2) aware of the context; and 3) able to immediately process, clarify, summarize, explore, and understand the answers. The research supporting instrument is a task-based interview guide. The task is given by using a duty sheet that contains the formal definition of convergent rows and has validated the expert.

Data were naturally collected by task-based interviews. The task given to students is explaining how they understand the formal definition of the sequence of real numbers converges. The formal definition of the sequence of real numbers converges as follows.

"The sequence of real numbers \( \{a_n\}_{n \geq 1} \) is said to be convergent sequence if and only if for every \( \varepsilon > 0 \) there is \( a \in \mathbb{R} \) and \( n_0(\varepsilon) \in \mathbb{N} \) so for \( n \geq n_0(\varepsilon) \) apply \( |a_n - a| < \varepsilon \)

Interviews are conducted in-depth with the semi-structured format. This format is chosen to know the honesty and openness of the participant in conveying information. The visualization of sequence \( \{-\frac{1}{n}\}_{n \geq 1} \) is

![Figure 1. The understanding of the convergent sequence concept of \( \{-\frac{1}{n}\}_{n \geq 1} \)](image_url)

Participants being interviewed are given the freedom to follow their inclination of mind including in determining the direction of the topic of speech to form the focus of the conversation. All activities are recorded with handy cams for the preparation of transcripts.

The result of the interview on the participant is the date of the interview result. The data of the interview result are then triangulated and validated to obtain valid and accurate data. The validity of the data in this study is emphasized more on internal validity test or credibility test derived from extending observation, increasing persistence, triangulation, negative case analysis, and member check. If the data is not valid yet, it is necessary to re-interview, and the existing data are triangulated and validated again.

The steps of data analysis include reduction, categorization, synthesis, and conclusion. Data reduction activity is the activity of identifying the "unit" that is the smallest part found in the data and has meaning when it associated with the focus of the research problem. After it obtained, the next step is to create a "coding" that gives the code on each unit to remain traceable source data. Categorization is the activity of sorting each unit into parts that have similarities. Each classification named after a "label." Synthesis is an exercise of looking for links between one category with another category. The association of one category to another category is given the name/label again. The conclusion was made by formulating a statement that provides the visual thinking scheme of mathematics teacher candidates in understanding the formal definition of the gender-convergent sequence.

3. **Research results and discussions**

The results of the data analysis show that male and female students understand the formal definition of a convergent sequence by recognizing, imagining, showing (showing the definition picture, showing the attribute definition), and concluding. The male and female participant representation of the formal definition of a convergent sequence is as follows.
To recognize, the male participant summarizes the definitions and describes what can describe. To imagine, the male participant makes the general formula, create a help table, and make a general overview. To show a definition picture, the male participant creates a special sequence formula, auxiliary tables, and plotting the members of the row. To show the definition attribute, the male subject specifies $a$ value, determines the value of $\varepsilon$, determines the value of $n_0(\varepsilon)$, draws the $n_0(\varepsilon)$, and draws the line $\varepsilon$. To conclude, the male participant matches and concludes.

To recognize, the female participant encapsulates, determines the keywords, and describes the keywords. To imagine, the female participant remembers about convergent and makes the coordinate axis. To show a definition picture, the female participant specifies a row instance, enrolls a member of a row, and plots a member of the row. To show the definition attribute, the female participant defines the value of $\varepsilon$, determines the value of $a$, describes the value of $\varepsilon$ and $a$, draws the line of $a \pm \varepsilon$, determines the value of $n_0(\varepsilon)$, and draws the line of $n_0(\varepsilon)$. To conclude, the female participant makes the first conclusion, makes a second conclusion, and makes the third conclusion.

Some male and female activities in understanding the formal definition of the convergent sequence are identified involving the processing of mental imagery for visual thinking. Some male student activities that identified involving the processing of mental imagery are drawing a general picture, creating a unique series formula, plotting, assigning $a$ value, drawing a line $n_0(\varepsilon)$, and drawing a line $\varepsilon$. Some female student activities that identified involving the processing of mental imagery are remembering about convergent, drawing coordinate axis, determining convergence sequence sample formulas, plotting members of the sequence, determining value $\varepsilon$, determining the value of $a$, drawing the line of $a \pm \varepsilon$, determining the value of $n_0(\varepsilon)$, and drawing the line of $n_0(\varepsilon)$.

To drawing a general picture, the male participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to draw a general picture are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. To bring up mental imagery, the male participant remembers the mental imagery of convergent lines and making mental imagery of other converging functions that are still in the form of a conjunctive graph. To processing mental imagery, the male participant collect all mental imagery obtained and justifies it so that it is a plot of points. After that chooses or focuses on mental imagery, and then the male participant utilizes or represents it as a general description.

To create a particular sequence of formulas, the male participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to create a particular sequence are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. To bring up mental imagery, the male participant summoning the mental imagery of the general picture and creating another mental imagery. To processing mental imagery, the male participant collect and complete the mental imagery by adding a sequence of rows. Mental imagery by focusing on one mental imagery, then using or using information on selected mental imagery to determine the formula of the unique sequence.

The male participant activity to plot is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to plot are bring up mental imagery,
processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. To bring up mental imagery, the male participant pays attention or summoning mental imagery of the coordinate axis and making mental imagery plot sequences, and then utilizing or representing mental imagery in the form of a plot of lines.

To determine the value of $a$, the male participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to determine the value of $a$ are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. To bring up mental imagery, the male participant observes or summoning the real mental imagery of the series. Then by creating mental imagery of the plot of the sequence, processing mental imagery collects mental imagery of the emerging plots. Finally, the participant utilizes mental imagery information to determine the value of $a$.

To draw the line of $n_0(\varepsilon)$, the male subject activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to draw the line of $n_0(\varepsilon)$ are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. The sequence of activities or visual thinking activities to draw the line of $n_0(\varepsilon)$ is: to bring up mental imagery by paying attention to or summoning mental imagery of the existing image/graph and making mental imagery of line $n_0(\varepsilon)$, based on visual input $n_0(\varepsilon)$ then processing mental imagery by collecting mental imagery of the $n_0(\varepsilon)$, line appearing and selecting one of the mental imagery, and utilizing or representing the mental representation in the form of a line image $n_0(\varepsilon)$.

To draw the line $\varepsilon$, the male participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to draw the line $\varepsilon$ are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. The sequences of activities or visual thinking activities to draw the line $\varepsilon$ are: to generate mental imagery by watching or calling mental imagery of the existing image/graph and making mental imagery of another line of $\varepsilon$. Then processing or determining mental imagery by choosing mental imagery of the line of $\varepsilon$, and then use or represent the selected mental imagery to be expressed in the form of a line drawing $\varepsilon$.

To remember the convergent term, the female participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to remember the convergent term are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. The sequence of activities or visual thinking activities to remember the convergent term is: to bring up mental imagery by remembering or summoning mental imagery of convergent lines and creating mental imagery of other convergent lines. Then cultivate the mental imagery by collecting mental imagery that appears and focuses on attention or chooses mental imagery of certain convergent lines. Furthermore, utilizing or using information on mental imagery so that it can give notes to the summary that the convergent is centered. Then processing or determining mental imagery in the form of a line drawing $\varepsilon$.

To draw the coordinate axis, the female participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to draw the coordinate axis are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. A series of activities or visual thinking activities to draw the coordinate axis are: to generate mental imagery by remembering or calling for the mental representation of convergent functions, and then utilizing or representing the axis of mental imagery coordinates.

To determine the formulas of convergent sequence example, the female participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to determine the formulas of convergent sequence examples are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. A sequence of activities or visual thinking activities to determine the formulas of convergent sequence examples are: bring up mental imagery by watching or calling mental imagery of the coordinate axis and creating mental imagery of the graph and its functions. Then cultivate mental imagery by collecting the emerging mental imagery and selecting the mental imagery corresponding to
the shadow of the convergent sequence. Furthermore, utilize or use the information on mental imagery to determine the convergence sequence instance formula.

To plot members of the series, the female participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to plot members of the sequence are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. A sequence of activities or visual thinking activities to plot members of the series are: to generate mental imagery by paying attention to or summoning mental imagery of the coordinate axis and making mental imagery plot sequences, then utilizing or representing mental imagery as a sequence of lines.

To determine the value of $\varepsilon$, the female participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to determine the value of $\varepsilon$ are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. The sequence of activities or visual thinking activities to determine the value of $\varepsilon$ is: to bring up mental imagery by paying attention to or summoning mental imagery of existing images/graphs and making mental imagery of the line $\varepsilon$. Then, participant collect mental imagery of the line $\varepsilon$, choose mental imagery of the line $\varepsilon$, utilize or use mental imagery of the line $\varepsilon$ to determine the value of $\varepsilon$.

To determine the value of $a$, the female participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to determine the value of $a$ are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. The sequence of activities or visual thinking activities to determine the value of $a$: to generate mental imagery by noticing or summoning existing mental imagery of the sequence of lines and making mental imagery of the continuation of the sequence of lines. Then, participant cultivates or collect mental imagery plot sequences that appear, and utilize or use the information on mental imagery to determine the value of $a$.

To illustrate the line $a \pm \varepsilon$, The female participant activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to illustrate the line $a \pm \varepsilon$ are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. A series of activities or visual thinking activities to illustrate the line $a \pm \varepsilon$, namely: generating mental imagery with summoning mental imagery while paying attention to images/graphs that have no line $a \pm \varepsilon$. Then, the participant creates mental imagery when it describes the line $a \pm \varepsilon$, utilizing or representing mental imagery when describing the lines $a + \varepsilon$ and $a - \varepsilon$.

To determine the value of $n_0(\varepsilon)$, the female subjek activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to determine the value of $n_0(\varepsilon)$ are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. The sequence of activities or visual thinking activities to determine the value of $n_0(\varepsilon)$ is: to generate mental imagery by paying attention to or summoning mental imagery of existing images/graphs and making mental imagery of the line $n_0(\varepsilon)$. Cultivate mental imagery by collecting mental imagery that appears and choosing certain mental imagery. Then use the information on the mental imagery to determine the value of $n_0(\varepsilon)$.

To illustrate the line of $n_0(\varepsilon)$, the female subjek activity is identified involving the processing of mental imagery. Based on data of deep interviews, visual thinking activities to illustrate the line $n_0(\varepsilon)$ are bring up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. To bring up mental imagery, the female subjek pay attention to or summoning mental imagery of existing images/graphs and making mental imagery of the line $n_0(\varepsilon)$. Then use or represent mental imagery of line $n_0(\varepsilon)$.

Based on the analysis of the purpose and usefulness of mental imagery processing, it can be concluded that the visual thinking activities of male and female students are brought up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. To bring up mental imagery, the mind summons mental imagery or makes mental imagery. To determine mental imagery, the mind collects mental imagery and selects mental imagery.
To improve mental imagery, the mind justifies mental imagery or completes mental imagery. To utilize mental imagery, the mind uses mental imagery or represents mental imagery. Processing mental imagery has done according to the needs and purpose of the activity in understanding the formal definition of the convergent sequence. It is in line with the theory of information processing is directed or controlled by a particular purpose [20].

This stage of visual thinking is important because it can assist students in the construction of students in giving meaning to the mathematical concepts of the formal level [21]. Mental imagery is helpful to students in building concepts [22]. Thinking using mental imagery (visual thinking) is very useful in supporting intuition when learning mathematics [23]. Students who are rich with mental shading will be greatly assisted in cognitive development, representing, and proofing [24]. The use of mental imagery (visual thinking) is very important for students during the transition to the mathematical study from concrete to abstract (formal proof) [25].

4. Conclusion
To conclude, visual thinking activities of male and female students are brought up mental imagery, processing mental imagery (determining mental imagery, improving mental imagery), and utilizing mental imagery. To bring up mental imagery, the mind summons mental imagery or makes mental imagery. To determine mental imagery, the mind collects mental imagery and selects mental imagery. To improve mental imagery, the mind justifies mental imagery or completes mental imagery. To utilize mental imagery, the mind uses mental imagery or represents mental imagery. Processing mental imagery has done according to the needs and purpose of the activity in understanding the formal definition of convergent sequence.

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