QCD correction to single top quark production at the ILC

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Single top quark production at the ILC can be used to obtain a high precision measurements of the the $V_{tb}$ CKM matrix element as well as the effective $tbW$ coupling. We have calculated the QCD correction for the cross section in the context of an effective vector boson approximation. Our results show a $\sim 10\%$ increase due to the strong interaction.

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I. INTRODUCTION

The top quark stands out as the heaviest known elementary particle and its properties and interactions are one of the most important measurements for present and future high energy colliders\cite{1}. At the Tevatron and at the LHC the process of single Top quark production has been extensively studied\cite{2}.

The top quark is likely to provide us with the first clues of physics beyond the Standard Model\cite{3}. In fact, new physics effects are probably already manifest in the recent forward-backward asymmetry observed at the Tevatron\cite{4,5}.

The planned International Linear Collider (ILC) will collide electron and positron beams at an initial energy of 500 GeV and higher. It will provide a clean environment for the study of precision measurements.

The single top production processes at lepton and photon ($e^+e^-$, $e^-e^-$, $\gamma e$ and $\gamma\gamma$) colliders have been extensively studied at tree level in Ref.\cite{8}. The reaction $\gamma e^- \rightarrow \bar{t}b\nu_e$, is particularly suitable for precision studies, as it does not have the $t\bar{t}$ background. Compared to the ILC $e^+e^- \rightarrow \bar{t}b e^-\bar{\nu}_e$ process the $\gamma e^-$ reaction can yield a larger production rate and is
directly proportional to the $V_{tb}$ term. Further studies, have thus been done for this reaction. In particular, the QCD corrections have been studied in Ref. [9]. Their conclusion is that the QCD correction is not very large ($\sim 5\%$) so that this mode remains very well suited for a precise measurement of $V_{tb}$. The approach used by [9] is to use the effective vector boson approximation, also known as effective $W$-approximation [6] (EWA) and to compute the QCD loop corrections for the $W^{+}\gamma \rightarrow t\bar{b}$ fusion process. Then, the convolution with the $f_{W^{+}/e^{+}}(x)$ distribution function is applied to obtain the correction to the actual $e^{+}\gamma$ process. We would like to point out that the authors in Ref. [9] have made a very clear and thorough presentation of the calculation. In this work we use their analysis on the $W^{+}\gamma \rightarrow t\bar{b}$ process to estimate the QCD correction for the $e^{+}e^{-} \rightarrow t\bar{b}e^{-}\bar{\nu}_{e}$ process of the ILC. Here, in addition to the convolution with the $W^{+}$ boson we will use the effective photon (as well as the effective $Z$-boson) approximation to obtain the QCD correction. We will use the same input values for masses and coupling constants, except for the masses of top and bottom quarks we take $m_{t} = 173$ GeV and $m_{b} = 4.2$ GeV.

II. VECTOR BOSON CONTRIBUTIONS AT TREE LEVEL

At tree level there are 20 diagrams for the $e^{+}e^{-} \rightarrow t\bar{b}e^{-}\bar{\nu}_{e}$ process [8]. We can list them in three different types: (a) vector boson fusion, (b) vector boson exchange and (c) $e^{+}e^{-}$ annihilation (see FigureII). For the energy range we consider one of the diagrams actually corresponds to $t\bar{t}$ production, where one of the tops decays leptonically. In order to exclude $t\bar{t}$ production from the single top process we discard all events where the invariant mass of the decay products $(e^{-},\bar{\nu}_{e},\bar{b})$ falls inside an interval around the top mass $m_{t} - \Delta M \leq M_{e\nu\bar{b}} \leq m_{t} + \Delta M$. We take the value $\Delta M = 20$GeV as in Ref. [8].

The effective-W approximation relies on the fact that the vector fusion diagrams become dominant when heavy particles are produced at very high energy collisions [6]. In general, 3 conditions should be met for the EWA to work well: (1) The mass of the vector boson ($M_{W}$ or $M_{Z}$) should be much smaller than its energy, and this can be met if we require $M_{V} \ll \sqrt{s}/2$, (2) for $q\bar{q}$ production $m_{q} \gg M_{V}$, this is true for the top quark but not for the bottom quark, and (3) One polarization mode should be dominant so that interference effects can be neglected. Fortunately, in our case the mode $W\gamma \rightarrow t\bar{b}$ dominates for longitudinal $W$, and the modes with the $Z$ boson $WZ \rightarrow t\bar{b}$ give even lower contributions.
As expected, this method works very well for $t \bar{t}$ production at high $\sqrt{s}$ and to a lesser degree for single top, which in our case can be seen as $t \bar{b}$ production. In Ref. [9] the QCD correction to the process $e^+\gamma \rightarrow t \bar{b}\nu_e$ was calculated by doing first the QCD correction to the $W^+\gamma$ fusion into $t \bar{b}$ and then by taking the convolution with an effective $W^+$ coming from the initial positron (see Figure 2). We follow the same approach by doing the one loop QCD correction to $W^+\gamma \rightarrow t \bar{b}$ as well as $W^+Z \rightarrow t \bar{b}$ and then convolute with the effective distribution functions for $W^+$, $\gamma$ and $Z$:

$$\sigma(e^+e^- \rightarrow t \bar{b}\nu_e e^-) = \sum_{W_L,W_T} \int_{x_{W}^{min}}^{1} dx_W \frac{f_{W^+e^+}(x_W)}{x_W} \int_{0}^{1} dx_\gamma f_{\gamma/e^-}(x_\gamma) \sigma(W^+\gamma \rightarrow t \bar{b})(\hat{s})$$

$$+ \sum_{W_L,T,Z_L,T} \int_{x_{W}^{min}}^{1} dx_W \frac{f_{W^+e^+}(x_W)}{x_W} \int_{x_{Z}^{min}}^{1} dx_Z f_{Z/e^-}(x_Z) \sigma(W^+Z \rightarrow t \bar{b})(\hat{s})$$

Where, $x_{W}^{min} = 2M_V/\sqrt{s}$, $\hat{s} = x_W x_\gamma s$ and the structure functions can be found in [6]. The tree level cross section for the single top production at the ILC is shown in Fig. 3. The exact Born level calculation for the $e^+e^- \rightarrow t \bar{b}\nu_e e^-$ process is obtained with Calchep [10] and is shown by the solid line. We can see that the prediction of the EWA (dot-dashed curve) is in very good agreement with the exact result for center of mass energies above 1.5 TeV. However, for the energy range of the ILC the EWA values can be significantly lower. In particular, for $\sqrt{s} = 1000\text{GeV}$ there is a 15% difference and for $\sqrt{s} = 500\text{GeV}$ the EWA result be about one half of the exact value.

There is one aspect of the calculation that is worth mentioning. Because of the kinematics of the $W^+Z \rightarrow t \bar{b}$ process, we run into a divergent behavior as we integrate over the Mandelstam variable $t$ (or the polar angle of the outgoing quark). At a certain value of $t$ the massive $Z$ boson can actually decay into $b\bar{b}$ and this makes the bottom quark propagator to hit a pole at this value. We were able to avoid this singularity by setting $k_Z^2 = 0$ instead of $M_Z$. This is completely justifiable in the context of the EWA. Let’s understand more the importance of the assumption $M_V \ll \sqrt{s}/2$. In the complete process (like $e^+e^- \rightarrow W^{++}Z^* \rightarrow t \bar{b}\nu_e$) the virtual $Z$ gets a space-like momenta $k_Z^2 \leq 0$ and is always far from the on-shell condition. In fact, the EWA works better when the initial state vector boson momentum square is set equal to zero: $k_Z^2 = 0$, $k_W^2 = 0$ (see Ref. [7] for a detailed discussion). Nevertheless, when dealing with a process like $t \bar{t}$ production one may set $k_Z^2 = M_Z^2$ as this introduces only a small error of order $m_{Z}/\sqrt{s}$. It is customary to set the external massive $W^+$ and $Z$ on-shell
for convenience. However, for the single top process the fact that $Z$ is heavy enough to decay into $b\bar{b}$ is prompting us to implement the $k^2_Z = 0$ condition in order to avoid the divergent behavior. Notice that a similar situation does not apply to the $W^+$ boson as it cannot decay into $t\bar{b}$. Therefore in our study we choose to keep the on-shell condition $k^2_W = M^2_W$ for the initial state $W^+$ but impose $k^2_Z = 0$ for the $Z$ boson. For the case of the $W^+$ we have checked that indeed by setting $k^2_W = 0$ we don’t find a significant change in the result.

Below, we will describe the QCD corrections to the $W^+\gamma$ and $W^+Z$ processes, including the Dipole substraction method of infrared divergencies. We have followed closely the analysis done for the $W^+\gamma$ mode done by Kuhn et.al. in Ref. [9].

III. QCD CORRECTION TO THE $W^+\gamma(Z) \rightarrow t\bar{b}$ PROCESS.

The QCD loop correction to the $W^+\gamma(Z) \rightarrow t\bar{b}$ process is given by 9 Feynman diagrams (see Fig.2 of [9]). The renormalization procedure involves only the quark’s wave function and mass parameter. Specific formulas can be found in [9]. Concerning the renormalization scale dependence we have also set $\alpha_s$ at the scale $\mu = \sqrt{s}$ for our numerical calculation.
FIG. 3: The contributions from $W^+\gamma$ and $W^+Z$ fusion to the $e^+e^- \rightarrow t\bar{t}e^-\bar{\nu}_e$ process. The solid line shows the exact calculation.

(it becomes $\sqrt{s}$ under the convolution). The extraction of IR singularities is done with the substraction method of the dipole formalism \[11\]. This method consists of adding and substracting a so-called dipole term:

$$
\sigma^{NLO}(W^+\gamma \rightarrow t\bar{t}) = \int_{tbg} [(d\sigma^R)_{\epsilon=0} - (d\sigma^B \otimes dV_{dipole})_{\epsilon=0}] + \int_{tb} [d\sigma^V + d\sigma^B \otimes I]_{\epsilon=0} \quad (2)
$$

Where $d\sigma^R$ comes from the real emission $W^+\gamma(Z) \rightarrow t\bar{t}g$ process and $d\sigma^B \otimes dV_{dipole}$ is the substracting dipole term that matches point-wise the singularities associated to the soft and/or collinear gluon. Both terms are calculated in $d = 4$ dimensions. In the second integral the same dipole term has been partially integrated in the gluon phase space and then added to the virtual correction $d\sigma^V$. This sum is performed in $d = 4 - 2\epsilon$ dimensions (consistent with the dimensional regularization).

The general formula for the dipole term is found in Eq. (5.16) of \[11\]. The specific expression in our case is:

$$
d\sigma^B \otimes dV_{dipole} = \frac{\langle V_{gt,b} \rangle}{2k_g \cdot k_t} |\mathcal{M}_0(\vec{k}_{gt}, \vec{k}_b)|^2 + \{t \leftrightarrow b\}, \quad (3)
$$

where

$$\langle V_{gt,b} \rangle = 8\pi\alpha_s C_F \left\{ \frac{2}{1 - \tilde{z}_t(1 - y_{gt,b})} - \frac{\tilde{v}_{gt,b}}{v_{gt,b}} \left[ 1 + \tilde{z}_t + \frac{m_t^2}{k_g \cdot k_t} \right] \right\},$$

$$
\tilde{z}_t = \frac{k_t \cdot k_b}{(k_t + k_g) \cdot k_b}, \quad y_{gt,b} = \frac{2k_g \cdot k_t}{xs_{tb}}, \quad \tilde{v}_{gt,b} = \frac{\lambda_{tb}}{x_{tb}},
$$

$$
v_{gt,b} = \sqrt{(1 + a_{gt,b})^2 - a_{gt,b}^2 z_b}, \quad a_{gt,b} = \frac{2z_b}{x_{tb}(1 - y_{gt,b})},
$$

$$
\tilde{k}_b = \frac{x_b}{2} P + \frac{\lambda_{tb}}{\lambda_{gt}} (k_b - \frac{P \cdot k_b}{\sqrt{s}} P), \quad \tilde{k}_{gt} = P - \tilde{k}_b, \quad P = k_W + k_\gamma,
$$
and \(M_0(k_{gt}, \bar{k}_b)\) is the Born level \(W^+ \gamma \to t\bar{b}\) amplitude with one modification: the final state momenta \(k_t\) and \(k_b\) have been replaced by \(\bar{k}_{gt}\) and \(\bar{k}_b\) respectively.

The other variables are defined as in [9]: \(\mu_q = m_q/\sqrt{s}, \ z_q = \mu_q^2, \ x_t = 1 + z_t - z_b, \ x_b = 1 + z_b - z_t, \ x_{tb} = 1 - z_t - z_b, \ \lambda_{tb} = \lambda(1, z_t, z_b), \ \lambda_{gt} = \lambda(1, (k_g + k_t)^2/s, z_b),\) and \(\lambda(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2xy - 2xz - 2yz}.

For the real emission correction we have prepared a Fortran program that integrates the cross section for the \(W^+ \gamma \to t\bar{b}g\) process along with dipole substraction. As it turns out, the substraction term defined by the dipole formalism in the first integral of Eq. (2) is actually a very good approximation to the real emission cross section in an important part of the \(t\bar{b}g\) phase space, so that the numerical results we obtained were very small: about two orders of magnitude below the values obtained for the virtual correction.

The expression for the dipole term in the virtual correction is:

\[
\frac{d\sigma^B}{\sigma} \otimes \mathbf{I} = |M_d(W^+ \gamma \to t\bar{b})|^2 \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1 - \epsilon)} \left( \frac{4\pi \mu^2}{s} \right) ^\epsilon (I_{gt,b} + I_{gh,t}),
\]

where \(M_d(W^+ \gamma \to t\bar{b})\) is the Born level amplitude in \(d = 4 - 2\epsilon\) dimensions (the flux term of the \(t\bar{b}\) phase space is understood). The dipole function is given by \(I_{gt,b} = C_{F} [2I^{eik} + I^{coll}].\) (also \(I_{gh,t} = I_{gt,b} \{ t \leftrightarrow b \}\), where \(I^{eik}\) and \(I^{coll}\) are given by Eqs. (5.34) and (5.35) in [11]:

\[
I^{eik} = \frac{x_{tb}}{\lambda_{tb}} \left\{ \frac{\ln \rho}{2\epsilon} + \frac{\pi^2}{6} - \ln \rho \ln [1 - (\mu_t + \mu_b)^2] - \frac{1}{2} \ln^2 \rho_t - \frac{1}{2} \ln^2 \rho_b + 2Li_2(-\rho) - 2Li_2(1 - \rho) - \frac{1}{2} Li_2 (1 - \rho_t^2) - \frac{1}{2} Li_2 (1 - \rho_b^2) \right\}
\]

\[
I^{coll} = \frac{1}{\epsilon} + 3 + \ln \mu_t + \ln (1 - \mu_t) - 2 \ln [(1 - \mu_b)^2 - z_t] - \frac{\mu_b}{1 - \mu_b}
\]

\[
- \frac{2}{x_{tb}} \left[ \mu_b(1 - 2\mu_b) + z_t \ln \frac{\mu_t}{1 - \mu_b} \right]
\]

where \(\rho^2 = (x_{tb} - \lambda_{tb})/(x_{tb} + \lambda_{tb}), \ \rho_t = (x_{tb} - \lambda_{tb} + 2z_t)/(x_{tb} + \lambda_{tb} + 2z_t),\) and \(\rho_b = \rho_t \{ t \leftrightarrow b \}\). These formulas also appear in [9], except that in their Eq. (4.14) \(I^{coll}\) the constant term should not be 5 but 3.

Concerning the calculation of \(d\sigma^V\), the details can be found in Ref. [9]. We actually worked out this same computation before doing the case for the \(Z\) boson. As expected from the results shown in Fig. 3 the contribution from the \(W^+Z\) fusion is much smaller than the one from \(W^+\gamma\). In fact, we only considered the correction for the polarizations \(W^+\) longitudinal and \(Z\) transversal as the other possibilities are negligible.
Our results are shown in Fig. 4. The QCD correction for the single top production in the $e^+e^-$ collision process is of order 10% of the Born level cross section. It will be interesting to compare this result based on the effective W-approximation with a future more robust calculation based on the complete $e^+e^- \rightarrow t\bar{b}e^-\bar{\nu}_e$ process.

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