BFKL Pomeron at non-zero temperature
and integrability of the Reggeon dynamics in multi-colour QCD

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Abstract
We consider the QCD scattering amplitudes at high energies $\sqrt{s}$ and fixed momentum transfers $\sqrt{-t}$ in the leading logarithmic approximation at a non-zero temperature $T$ in the $t$-channel. It is shown that the BFKL Hamiltonian has the property of holomorphic separability. The Pomeron wave function for arbitrary $T$ is calculated using an integral of motion. In multi-colour QCD, the holomorphic Hamiltonian for $n$-reggeized gluons at temperature $T$ is shown to coincide with the local Hamiltonian of an integrable Heisenberg model and can be obtained from the $T = 0$ Hamiltonian by an unitary transformation. We discuss the wave functions and the spectrum of intercepts for the colourless reggeon states.

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In QCD the scattering amplitudes $A(s,t)$ in Regge kinematics for high energies $2E = \sqrt{s}$ and fixed momentum transfers $q = \sqrt{-t}$ are obtained in the leading logarithmic approximation $\alpha_s \ln s \sim 1$, $\alpha_s = \frac{g^2}{4\pi} \to 0$ ($g$ is the QCD coupling constant) by summing the largest contributions $\sim (\alpha_s \ln s)^n$ to all orders of perturbation theory within the approach of Balitsky, Fadin, Kuraev and Lipatov (BFKL) [1]. The BFKL Pomeron in the $t$-channel turns out to be a composite state of two reggeized gluons (it is valid also in the next-to-leading approximation [2]). Its wave function $\Psi(\bar{\rho}_1, \bar{\rho}_2)$ satisfies the Schrödinger equation in the two dimensional impact-parameter space $\bar{\rho}$

$$E \Psi(\bar{\rho}_1, \bar{\rho}_2) = H_{12} \Psi(\bar{\rho}_1, \bar{\rho}_2).$$

The intercept $\Delta$ of the Pomeron, related to the high energy asymptotics $\sigma_t \sim s^\Delta$ of the total cross-section, is proportional to the ground state energy $E$

$$\Delta = -\frac{\alpha_s N_c}{2\pi} E .$$

The kinetic part $H_{\text{kin}} = \ln |p_1|^2 + \ln |p_2|^2$ of $H_{12}$ is a sum of two gluon Regge trajectories and its potential part $H_{\text{pot}}$ is related by a similarity transformation to the two-dimensional Green function $\ln |\rho_{12}|^2$, where $\rho_{12} = \rho_1 - \rho_2$. [We introduced here the complex coordinates $\rho_r = x_r + iy_r$ and the corresponding momenta $p_r = i\partial_r$.]

The BFKL equation is used for the description of the deep-inelastic lepton-hadron scattering together with the DGLAP equation [3] (see for example [4]). It is invariant under the Möbius transformations [5]

$$\rho_r \to \frac{a \rho_r + b p_r}{c \rho_r + d p_r}$$

with arbitrary complex parameters $a, b, c, d$ and $H_{12}$ has the property of holomorphic separability (see [4] [6])

$$H_{12} = h_{12} + h_{12}^* , \quad h_{12} = \sum_{r=1}^2 \left[ \ln p_r + \frac{1}{p_r} \ln (\rho_{12}) p_r - \psi(1) \right] ,$$

where $\psi(z) = \Gamma'(z)/\Gamma(z)$.

The wave functions $\Psi$ belong to the principal series of unitary representations of the Möbius group with conformal weights $m = 1/2 + i\nu + n/2$, $\tilde{m} = 1/2 + i\nu - n/2$ expressed in terms of the anomalous dimension $\gamma = 1 + 2i\nu$ and the integer conformal spin $n$ for the local gauge-invariant operators [5]. The conformal weights are related to the eigenvalues $m(m-1)$, $\tilde{m}(\tilde{m}-1)$ of the Casimir operators $M^2$ and $M^{2*}$, where

$$M^2 = \left( \sum_{r=1}^2 M_{3}^{(r)} \right)^2 + \frac{1}{2} \left( \sum_{r=1}^2 M_+^{(r)} \sum_{s=1}^2 M_-^{(s)} + \sum_{r=1}^2 M_-^{(r)} \sum_{s=1}^2 M_+^{(s)} \right) = \rho_{12}^2 p_1 p_2 .$$

Here $\tilde{M}^{(r)}$ are the Möbius group generators

$$M_3^{(r)} = \rho_r \partial_r , \quad M_+^{(r)} = \partial_r , \quad M_-^{(r)} = -\rho_r^2 \partial_r$$

and $\partial_r = \partial/\partial \rho_r$. 

The eigenfunctions of $H_{12}$ can be considered as the three-point functions of a two-dimensional conformal field theory and have the property of holomorphic factorization [5],

$$f_{m,\tilde{m}}(\tilde{p}^1_1, \tilde{p}^2_2; \tilde{p}^0_0) = \langle 0 | \varphi(\tilde{p}^1_1) \varphi(\tilde{p}^2_2) O_{m,\tilde{m}}(\tilde{p}^0_0) | 0 \rangle = \left( \frac{\rho_{12}}{\rho_{10} \rho_{20}} \right)^m \left( \frac{\rho_{12}^*}{\rho_{10}^* \rho_{20}^*} \right)^{\tilde{m}}. \quad (3)$$

One can calculate the energy putting this Ansatz in the BFKL equation[1]

$$E_{m,\tilde{m}} = \varepsilon_m + \varepsilon_{\tilde{m}} , \quad \varepsilon_m = \psi(m) + \psi(1-m) - 2\psi(1) . \quad (4)$$

The minimum of $E_{m,\tilde{m}}$ is obtained at $m = \tilde{m} = 1/2$ leading to a large intercept $\Delta = 4 \frac{g_2}{\pi} N_c \ln 2$ of the BFKL Pomeron. In the next-to-leading approximation the intercept is comparatively small ($\Delta \sim 0.2$ for the QCD case) [7].

2. On the other hand, now a significant interest is devoted to the quark-gluon plasma (QGP) generation in heavy nucleus collisions (see for example [9]). Current theoretical understanding suggests that the QGP thermalizes via parton-parton scattering. The QGP is understood to cool down by hydrodynamic expansion till the temperature reaches the hadronization scale $\sim 160\text{MeV}$. One interesting phenomena is the suppression of the $\psi$-meson production in the heavy nucleus collisions due to the disappearance of the confining potential between $q$ and $\bar{q}$ at high temperature [9]. A similar effect should exist for glueballs constructed from gluons. Because the Pomeron is considered as a composite state of reggeized gluons, the influence of the temperature on its properties is of great interest. In this paper we construct the BFKL equation at temperature $T$ in the center-mass system of the $t$-channel (where $\sqrt{t} = 2\epsilon$) and investigate the integrability properties of the BFKL dynamics in a thermostat for composite states of $n$ reggeized gluons in multi-colour QCD.

Let us consider the Regge kinematics in which the total particle energy $\sqrt{s}$ is asymptotically large in comparison with the temperature $T$. In this case one can neglect the temperature effects in the propagators of the initial and intermediate particles in the direct channels $s$ and $u$. But the momentum transfer $|q|$ is considered to be of the order of $T$ (note, that $q_\mu$ is the vector orthogonal to the initial momenta $q_\mu \approx q^\perp_\mu$). As it is well known [8], the particle wave functions $\psi(x^\mu)$ at temperature $T$ are periodic in the euclidean time $\tau = it$ with period $1/T$.

We introduce the temperature $T$ in the center of mass frame of the $t$-channel. Thus, the euclidean energies of the intermediate gluons in the $t$-channel become quantized as

$$k^{(t)}_4 = 2\pi l T .$$

In the $s$-channel the invariant $t$ is negative and therefore the analytically continued 4-momenta of the $t$-channel particles can be considered as euclidean vectors. It means, that at temperature $T$, the wave functions for virtual gluons are periodic functions of the holomorphic impact-parameter $\rho = x + iy$ with imaginary period $\frac{1}{T}$. Also, the canonically conjugated momenta $p$ have their imaginary part quantized,

$$\rho \to \rho + \frac{i}{T}, \quad p = \text{Re}p + \pi il T . \quad (5)$$

with integer $l$ (note that $p = (p_1 + ip_2)/2$).

It is convenient to rescale the transverse coordinates and corresponding momenta as follows

$$\rho \to \frac{1}{2\pi T} \rho , \quad p \to 2\pi T p .$$
In these dimensionless variables one obtains

$$0 < \text{Im} \rho < 2\pi, \quad \text{Im} p = \frac{l}{2}.$$ 

The calculation of the Regge trajectory $1 + \omega(t)$ of the gluon at temperature $T$ in the $t$-channel, in one-loop approximation reduces to the integration over the real part $k_1$ of the transverse momentum $\vec{k}_\perp$ of the virtual gluon and to the summation over its imaginary part $k_2 = l$. In such a way we obtain the following result for the trajectory having the separability property [cf. [6]],

$$\omega(-\vec{q}^2) = -\frac{g^2}{8\pi^2} N_c \Omega(-\vec{q}^2), \quad \Omega(-\vec{q}^2) = \Omega(q) + \Omega(q^*).$$

Here,

$$\Omega(q) = \frac{\pi T}{\lambda} + \frac{1}{2} \left[ \psi(1 + iq) + \psi(1 - iq) - 2\psi(1) \right],$$

where we regularized the infrared divergence for the zero mode $l = 0$ introducing a mass $\lambda$ for the gluon (see [1]).

A similar divergence appears in the Fourier transformation $G(\vec{\rho}_{12})$ of the effective gluon propagator $(\vec{k}_\perp^2 + \lambda^2)^{-1}$ contained in the product of the effective vertices $q_1 k^{-1} q_2^*$ for the production of a gluon with momentum $k_\mu$ (cf. [4])

$$G(\vec{\rho}_{12}) = -\frac{\pi T}{\lambda} + \ln \left( 2 \sinh \frac{\rho_{12}}{2} \right) + \ln \left( 2 \sinh \frac{\rho_{12}^*}{2} \right).$$

Therefore, the divergence at $\lambda \to 0$ cancels in the sum of kinetic and potential contributions to the BFKL equation and the Hamiltonian $H_{12}$ for the Pomeron in a thermostat has the property of holomorphic separability with the holomorphic Hamiltonian given below [cf. eq.(2)]

$$h_{12} = \sum_{r=1}^{2} \left[ \frac{1}{2} \psi(1 + ip_r) + \frac{1}{2} \psi(1 - ip_r) + \frac{1}{p_r} \ln \left( 2 \sinh \frac{\rho_{12}}{2} \right) p_r - \psi(1) \right]. \quad (6)$$

3. The Hamiltonian (6) is a periodic function of $\rho_{12}$. Therefore its eigenfunctions are quasi-periodic functions of this variable. The behaviour of $h_{12}$ for small $\rho_{12}$ corresponds to the low temperature regime. Hence, the eigenfunctions of $h_{12}$ behave for $\rho_{12} \to 0$ as the holomorphic part of the zero-temperature wave functions eq.(3),

$$\Psi_m(\rho_{12}) \overset{\rho_{12} \to 0}{\Rightarrow} \rho_{12}^m \quad (7)$$

Notice that $\Psi_{1-m}(\rho_{12})$ is an eigenfunction too.

We find the small-$T$ expansion of $h_{12}$ near its singularities $\rho_{12} = 2\pi il$. For example, for small $\rho_{12}$ and large $p_1, p_2$ we have

$$h_{12} = h^0_{12} + \sum_{k=1}^{\infty} \frac{B_{2k}}{2k} \sum_{r=1,2} \left[ \frac{(-1)^{k+1}}{p_r^{2k}} + \frac{1}{p_r (2k)!} p_r \right],$$

where $h^0_{12}$ is the holomorphic BFKL Hamiltonian at a zero temperature [given by eq.(2)] and the $B_{2k}$ are the Bernoulli numbers. This representation for $h_{12}$ permits us to find the
small-$T$ expansion of its eigenfunctions. For example, at a vanishing momentum transfer $Q = p_1 + p_2$ we obtain for the eigenfunction $\Psi_m(\rho_{12})$,

$$\Psi_m(\rho_{12}) = \rho_{12}^\frac{m}{24} \left[ 1 - \frac{1}{24} \frac{m(m-1)}{2m+1} \rho_{12}^2 + \frac{1}{5760} \frac{m(m-1)(5m^2 + 7m + 6)}{(2m+1)(2m+3)} \rho_{12}^4 + \mathcal{O}(\rho_{12}^6) \right].$$

These eigenfunctions are parametrized by the conformal weight $m$.

On the other hand, from the above expansion it is possible to verify, that the holomorphic Hamiltonian has the non-trivial integral of motion:

$$A = 4 \sinh^2 \frac{\rho_{12}}{2} p_1 p_2 \quad [A, h_{12}] = 0. \quad (8)$$

Therefore, instead of solving the Schrödinger equation we can search for the eigenfunctions of the operator $A$. For non-zero $Q$ one can write the holomorphic wave function as a product of a plane wave depending on $R = (\rho_1 + \rho_2)/2$ times a solution of the following equation for the relative motion of two gluons

$$\frac{Q^2}{4} + \frac{\partial^2}{\partial \rho^2} \Psi(\rho, Q) = \frac{m(m-1)}{4 \sinh^2 \frac{\rho}{2}} \Psi(\rho, Q), \quad \rho = \rho_{12}, \quad t = -4 |Q|^2.$$  

The two independent solutions of the above differential equation can be expressed in terms of hypergeometric functions

$$\Psi_1^{(m)}(\rho, Q) = e^{iQ\rho} (e^\rho - 1)^m F(iQ + m, m; 2m; 1 - e^\rho), \quad \Psi_2^{(m)}(\rho, Q) \equiv \Psi_1^{(1-m)}(\rho, Q). \quad (9)$$

For $\rho \to 0$ eq.(7) holds and the singularities of $\Psi^{(r)}(\rho, Q)$ at $1 - e^\rho = 1$ and $1 - e^\rho = \infty$ correspond to the points $\rho = -\infty$ and $\rho = \infty$, respectively.

The analytic continuation of $\Psi^{(r)}$ along the imaginary axes from $\rho = 0$ to $\rho = 2\pi i$ is equivalent to the continuation of these eigenfunctions in a circle passed in a clock-wise direction around the singularity at $\rho = -\infty$. The monodromy matrix expressing the analytically continued solutions in terms of the initial ones can be easily calculated.

The Pomeron wave functions can be written as a bilinear combination of holomorphic and anti-holomorphic eigenfunctions $\Psi^{(r)}(\rho, Q)$ and $\Psi^{(r)}(\rho^*, Q^*)$. The property of single-valuedness in the cylinder topology corresponding to the periodicity on the boundaries of the strip $0 < \text{Im} \rho_{12} < 2\pi$ is easily imposed to such Pomeron wave function using the monodromy matrix for $\Psi^{(r)}(\rho, Q)$. The resulting wave function can be written as

$$\Psi^{(m, \bar{m})}(\tilde{\rho}, \bar{Q}) = \chi_1^{(m)}(\rho, Q) \chi_1^{(\bar{m})}(\rho^*, Q^*) - (-1)^N \chi_2^{(m)}(\rho, Q) \chi_2^{(\bar{m})}(\rho^*, Q^*), \quad (10)$$

where,

$$\chi_1^{(m)}(\rho, Q) = 2^{1-2m} \frac{\Gamma(m+iQ)}{\Gamma(m+1)} \Psi_1^{(m)}(\rho, Q), \quad \chi_2^{(m)}(\rho, Q) = \chi_1^{(1-m)}(\rho, Q)$$

and $N = 2 \text{Im} Q$ is an integer.

4. The Pomeron wave function can be constructed directly in coordinate space. For this purpose we use the conformal transformation

$$\rho_r = \ln \rho'_r, \quad (11)$$
and the integral of motion eq.(8) becomes
\[ A = - (\rho_{12}')^2 \frac{\partial}{\partial \rho_1'} \frac{\partial}{\partial \rho_2'} . \]
Thus, \( A \) coincides in the variables \( \rho' \) with the Casimir operator of the conformal group whose eigenfunctions are well known (see [5]). Thus, the Pomeron wave function at non-zero temperature having the property of single-valuedness and periodicity takes the form
\[ \Psi^{(m,\bar{m})}(\bar{\rho}, \rho_1') = \left( \frac{\sinh \frac{\rho_{12}}{2}}{2 \sinh \frac{\rho_{10}}{2} \sinh \frac{\rho_{20}}{2}} \right)^m \left( \frac{\sinh \frac{\rho_{12}}{2}}{2 \sinh \frac{\rho_{10}}{2} \sinh \frac{\rho_{20}}{2}} \right)^{\bar{m}} . \] (12)
The orthogonality and completeness relations for these functions can be easily obtained from the analogous results for \( T = 0 \) (see [5]) using the above conformal transformation. These wave functions are proportional to the Fourier transformation of the wave functions \( \Psi^{m,\bar{m}}(\bar{\rho}, \bar{Q}) \).

Moreover, the pair BFKL Hamiltonian \( h_{12} \) can be expressed in terms of the BFKL Hamiltonian at zero temperature in the new variables
\[ h_{12} = \ln(p_1' p_2') + \frac{1}{p_1'} \log(\rho_{12}') p_1' + \frac{1}{p_2'} \log(\rho_{12}') p_2' - 2\psi(1), \] (13)
where \( p_1' = i \frac{\partial}{\partial \rho_1'} \). In the course of the derivation the following operator identity (see [4])
\[ \frac{1}{2} \left[ \psi \left( 1 + z \frac{\partial}{\partial z} \right) + \psi \left( -z \frac{\partial}{\partial z} \right) \right] = \ln z + \ln \frac{\partial}{\partial z} \]
was used to transform the kinetic part as well as properties of the \( \psi \)-function.

In summary, the exponential mapping eq.(11) which in dimensional variables takes the form
\[ \rho' = \frac{1}{2\pi T} e^{2\pi T \rho} , \]
maps the reggeon dynamics from zero temperature to temperature \( T \). This mapping explicitly exhibits a periodicity \( \rho \to \rho + \frac{1}{T} \) for a thermal state. It must be noticed that such class of mappings are known to describe thermal situations for quantum fields in accelerated frames and in black hole backgrounds[23].

5. As it is well known [10], the BFKL equation at \( T = 0 \) can be generalized to composite states of \( n \) reggeized gluons. In the multi-colour limit \( N_c \to \infty \) the BKP equations are significantly simplified thanks to their conformal invariance [5], holomorphic separability [6] and integrals of motion [11]. The generating function for the holomorphic integrals of motion coincides with the transfer matrix for an integrable lattice spin model [12] [13]. The transfer matrix is the trace of the monodromy matrix
\[ t(u) = L_1(u) L_2(u) \ldots L_n(u) , \]
satisfying the Yang-Baxter equations [13]. The integrability of the \( n \)-reggeon dynamics in multi-colour QCD is valid also at non-zero temperature \( T \), where, according to the above arguments we should take the \( L \)-operator in the form
\[ L_k = \begin{pmatrix} u + p_k & e^{-p_k} p_k \\ -e^{p_k} p_k & u - p_k \end{pmatrix} . \]
In particular, the holomorphic Hamiltonian is the local Hamiltonian of the integrable Heisenberg model with the spins being unitarily transformed generators of the Möbius group (cf. [14][15])

\[ M_k = \partial_k , \quad M_+ = e^{-\rho_k} \partial_k , \quad M_- = -e^{\rho_k} \partial_k . \]

Because the Hamiltonian at non-zero temperature can be obtained by an unitary transformation from the zero temperature Hamiltonian, the spectrum of the intercepts for multi-gluon states is the same as for zero temperature [16]-[20] and the wave functions of the composite states can be calculated by the substitution \( \rho_k \to e\rho_k \).

Furthermore, the non-linear Balitsky-Kovchegov equation [21] can be generalized to the case of non-zero temperature as follows,

\[
\frac{\partial N_{\vec{\rho}_1,\vec{\rho}_2}}{\partial Y} = \bar{\alpha}_s \int \frac{d^2\rho_0}{2\pi} \frac{\left| \sinh \frac{\rho_{12}}{2} \right|^2}{4 \left| \sinh \frac{\rho_{10}}{2} \right| \left| \sinh \frac{\rho_{20}}{2} \right|^2} \left( N_{\vec{\rho}_1,\vec{\rho}_0} + N_{\vec{\rho}_2,\vec{\rho}_0} - N_{\vec{\rho}_1,\vec{\rho}_2} - N_{\vec{\rho}_1,\vec{\rho}_0} N_{\vec{\rho}_2,\vec{\rho}_0} \right) , \tag{14}
\]

where \( N_{\vec{\rho}_1,\vec{\rho}_2} \) is the amplitude of finding a dipole with the impact parameters \( \vec{\rho}_1 \) and \( \vec{\rho}_2 \) in a hadron and the integration over \( \rho_0 \) is performed over the strip \( 0 < \text{Im} \rho_0 < 2\pi \). Note, however, that in this equation one takes into account only fan diagrams for the Pomeron interactions among all possible diagrams for reggeized gluons appearing in the high energy effective action [22].

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