Behavior of strip footing rested on undrained clay using consistency limits-based constitutive law

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ABSTRACT

The behavior of undrained clay was extensively studied by many earlier researchers. A lot of constitutive models were developed to describe the behavior of undrained clay based on its mechanical properties. The aim of this research is to present an innovative constitutive model for undrained clay based on its consistency limits and water content. The main concept of this model is to estimate the mechanical properties of clay using earlier correlations with consistency limits, then implement the estimated mechanical properties in a hyperbolic model and calibrate the hyperbolic parameters to match the failure criteria of the undrained clay. To verify the validity of the developed constitutive model, it was applied on a standard problem which is a strip footing rested on undrained clay layer, the results confirmed the ability of the model to simulate the nonlinear behavior of undrained clay up to ultimate condition. The main advantage of this constitutive model is the ability to capture the reduction of mechanical properties of clay with the increase in its water content, which makes it ideal to study the impact of seepage on shallow foundation.

1. Introduction

During earthwork activities involving clay materials, the rate of loading and unloading determines their behavior after all (Bowles, 1986; Biswas et al., 2016). Undrained conditions always exist for clays due to their low hydraulic conductivity when subjected to fast static loads thereby sustaining stress changes (Sharma and Bora, 2003). It has been noted also that conventional model software has the default clay behavior for all clay types set on undrained condition (Wahls, 2019; Wang and Carter, 2003). This is because when clay under saturated conditions is loaded, pore water is not immediately let out and it remains undrained and this is when most of the failures occur in loaded clay materials (Bowles, 1997). The resistance to shear for clay under undrained conditions is estimated with effective friction angle tending to zero (φ’ = 0) (Wahls, 2019). Measuring shear strength in undrained clay materials is a complex exercise due to the gap between field and laboratory values. Loading conditions are complex in this sense due to the lag between isotropic loading and anisotropic loading conditions which are achieved in the laboratory and the field respectively (Wahls, 2019). Constitutive model operations are being executed to overcome these gaps to avoid unnecessary mathematical extrapolations and correlation difficulties in the application of constitutive models in undrained clay soils (Mayne, 1980). As credence is given field measurements and the anisotropic loading that exists under field conditions during the determination of undrained shear strength, the engineering geological history of the clay deposits are also important (Hanzawa and Kishida, 1982; Bell, 2000), because this gives an understanding of the characteristic stress history for efficient evaluation of the undrained shear. Hyodo et al. (1992) also showed that structural gap exists between isotropic loading achieved using the cyclic triaxial test and anisotropic loading conditions that exist in an in-situ testing condition. It further proved that undrained and partially drained clay materials react differently during loading and unloading due to the unique relationship that exists in the p’-q stress space (Hyodo et al., 1992). Also, Mayne (1980) emphasized in his work that critical-state pore pressure of fast-loaded clay materials is of importance in evaluating the undrained shear strength parameters of clay due to low dissipation rate as a result of low permeability. Also, Atterberg limits have been shown by Sharma and Bora (2003) to influence the undrained shear strength of clay materials. How this happens will be extensively discussed. Meanwhile, these soils under undrained conditions experience loading from various types of foundations common among which is the strip footing type (Ebid et al., 2021). Strip footings...
are area-based localized loading structures that subject soils within the soil-structure interaction envelope to static loading conditions (Bowles, 1997; Honne and Muguda, 2019; Craig, 2004; Das, 2010). This is why it has been a difficult exercise to model undrained clay soils subjected to loading from strip footings (Honne and Muguda, 2019). Strip footings under this case exhibit undrained bearing capacity for the yield locus of the failure envelopes under hydraulic influences or water carrying consistencies (Ouahab et al., 2020). The focus of this research work is to utilize constitutive law conditions to evaluate the behavior of a strip footing underlain by undrained clay using consistency limits (plastic and liquid limits and water content).

2. Background

Prior to the 1940s, constitutive relations have been proposed for undrained clay soils using soil physic-mechanical relationships as well as coupled hydro-mechanical conditions (Sharma and Bora, 2003). Different values of undrained shear strength at the liquid limit (LL) of clay were proposed depending on the type of experimental requirement followed to arrive at it (Vardanega and Haigh, 2014). While Casagrande suggested an average \((Cu)\) of 2.65 kN/m\(^2\) considering the spread of the values depending of the apparatus of determination of LL, other authorities on the subject had proposed a range between 0.7 kN/m\(^2\) and 2.8 kN/m\(^2\) and suggested the use of the Casagrande apparatus for determining LL (Sharma and Bora, 2003). It was remarked that the failure of undrained clay is self-weight-induced and the use of in-situ experimental setups to validate constitutive relations becomes difficult due to the impact of the weight of cone on the failure of clay (Spagnoli and Feinendegen, 2017). Note, mechanical properties \((E_s, \nu)\) of clay have been used in the previous studies as well as hydro-mechanical properties (LL, PL, and WC). In the present work, Boussinesq’s theoretical formula about stress distribution was adopted due to the isotropic and semi-infinite conditions of the undrained clay. While Sharma and Bora (2003) presented an experimental correlation between consistency limits and shear strength (LL, PL, WC and Cu), the proposed new constitutive relation is based on consistency limits by estimating Cu from LL, PL, and WC using Sharma and Bora’s equation (Sharma and Bora, 2003), estimate \(E_s\) and \(\nu\) using regression equations for typical values in handbooks, and using \(E_s\) and \(\nu\) in the new constitutive relation presented to calculate the vertical stress using Boussinesq’s formula. The advantage of the new constitutive relation is its simplicity because it doesn’t require mechanical properties but just the consistency limits and it correlates the clay strength with its water content (WC), which is an innovative feature. Also, the geotechnical parameters can justify the gap in multi-correlation accuracy from consistency limits to mechanical properties and the proposed constitutive model is valid for strip footing (Kayabali et al., 2015).

3. Methodology

The proposed approach depends on the following constitutive conditions:

1. The strip footing is flexible and rested on a semi-infinite and isotropic medium (Boussinesq’s theoretical condition for stress distribution). Accordingly, 2D plain strain analysis is used, the thickness of the
The considered soil mass is large enough (about four times the footing width) to include the deformations of the whole effective depth.

2- As any numerical approach, the soil mass is meshed into small elements. Simple orthogonal mesh is considered in this approach. Mesh size was selected smaller than half the width of the footing to maintain acceptable accuracy, as shown in Figure 1.

3- Because the soil mass is considered semi-infinite and isotropic, Boussinesq’s formulas showed in Figure (2) were used to calculate the vertical stress ($\sigma_z$), and because the vertical stresses is calculated using Boussinesq’s formula, there is no need for any fixation around the considered zone.

4- The ultimate shear strength of the soil (undrained cohesion strength of clay) ($C_u$) in (kN/m²) is calculated from the consistency limits and actual water content using Binu Sharma and Padma K. Bora formula as shown in Eq. (1) (Sharma and Bora, 2003);

$$\log(C_u) = \frac{2 \log(L/L_c)}{\log(L/L_D)} + 0.77$$

Figure 4. Sample for Boussinesq’s vertical stress distribution ($\sigma_z$).

Figure 5. Sample for calculated vertical strain ($\varepsilon_z$).
Both elastic modules ($E_s$) and Poisson’s ratio ($\nu$) of soil are estimated using Eqs. (2) and (3). These two equations are regression for the typical values given in Bowles (1986, 1997);

$$E_s (t / m^3) = Cu (2.7 Cu + 80) \tag{2}$$

$$\nu = \sqrt{1/(20 Cu)} > 0.5 \tag{3}$$

The vertical strain ($\varepsilon_z$) of each element was calculated assuming a hyperbolic behavior with vertical stress ($\sigma_z$) as shown in Figure (3). Accordingly, the vertical strain ($\varepsilon_z$) could be calculated as per Eq. (4)

$$\varepsilon_z = \frac{\sigma_z a}{(1-b \sigma_z)} \tag{4}$$

At failure, ($\sigma_z = \sigma_{ult}$) and ($\varepsilon_z = \varepsilon_{ult}$), accordingly (b) value could be calculated from Eq. (5) considering ($\varepsilon_{ult} = \nu$) and ($\sigma_{ult} = 5Cu$) as follows:

$$b = \frac{1}{5Cu} - \frac{1}{Es \cdot \nu} \tag{5}$$

Finally, the proposed constitutive relation is illustrated in Eq. (6), where ($Cu$), ($Es$) & ($\nu$) are calculated from Eqs. (1), (2), and (3) respectively, and ($\sigma_z$) is calculated using Boussinesq’s formula

$$\varepsilon_z = \frac{\sigma_z}{Es \left(1 - \frac{\sigma_z}{Es \cdot \nu}ight)} \tag{6}$$

The deformation of each element was calculated by multiplying the vertical strain by the vertical dimension of the element.

Finally, the total settlement at any element was calculated by integrating the vertical deformation of all elements below that element down to the bottom boundary.

### 4. Results interpretation

The developed constitutive relation was used to calculate the settlement of three strip footings with widths of 3.0, 4.0 and 5.0 m rested on different clays.

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**Figure 6.** Sample for settlement below the footing.

**Figure 7.** Load-settlement curves for different footing widths rested on the same clay.

**Figure 8.** Load-settlement curves for the same footing width rested on different clays.
undrained clay layer with LL = 75% and PL = 25%. Three values for water content (WC) were used (WC = 25%, 30% and 35%), which are corresponding to undrained cohesion of 17.0, 7.9 and 4.2 t/m² respectively. The results are presented in Figures 4, 5, 6, 7, 8, 9, 10, and 11 as follows:

- Figures 4, 5, and 6 illustrate the outputs of sample case of 3.0 m width strip footing rested on clay with water content of 35% (Cu = 4.2 t/m²) and subjected to vertical stress of 12.4 t/m² (3 Cu). Figure 4 shows the Boussinesq’s vertical stress distribution (σz), while Figure 5 shows the calculated vertical strain for each element (εz), finally, Figure 6 shows the settlement below the footing.

- Figure 7 presents the load-settlement curve for the three footings on the same clay (WC = 35%). The results showed that all load-settlement curves followed the hyperbolic pattern and the ultimate bearing capacity of undrained clay is independent from the footing width which agreed with previous studies where (σzult = Cu. Nc).

- Figure 8 presents the load-settlement curve for the same footing with width of 5.0 m rested on the three considered clays (WC = 25, 30 & 35%). The results showed the impact of water content (WC) on both stiffness and strength of clay. It could be noted that for the same load the settlement increases with increasing the (WC). Also, it could be noted that the ultimate bearing capacity of undrained clay equals 5.2 Cu, which agreed with previous studies where Nc is varied between 5.0 and 5.7

- Figure 9 presents the average settlement of the three footings on the same clay (WC = 35%) at different vertical stresses (2Cu, 4Cu & 5Cu). The results showed that the average settlement increases linearly with increasing the footing width because the effective depth depends on the footing width (varied between twice the footing width for square and circular footings to four times the footing depth for strip footing) (Ahmed et al., 2020). Also, the rate of increasing settlement with increasing the footing width depends on the vertical stress value which reflects the hyperbolic behavior of the clay where the stiffness reduces with increasing the applied stress and almost vanished (flow condition) when the applied stress equals the ultimate strength of clay.

- Figure 10 presents the settlement distribution below sample footing with 5.0 m width rested on clay layer with water content of 35% and subjected to at different vertical stresses (2Cu, 4Cu & 5Cu). The results showed that the difference between the settlements at the center and the edge of the footing increases with increasing the vertical stress value which also agreed with previous studies.

- Finally, Figure 11 summarized the load-settlement curves for all the nine cases in the parametric study. The effect of (WC) on both stiffness and strength of undrained clay is very clear in terms of initial slope of the load-settlement curve and the maximum bearing pressure respectively. Also, the figure indicated that the footing width has no effect on the maximum bearing pressure, but it is directly affecting the settlement value.

5. Conclusions

The results of this study could be concluded in the following points:

- The behavior of strip footing rested on undrained clay using the proposed constitutive relation matched the previous studies as follows:
  - The ultimate bearing stress is independent from footing width
  - The ultimate bearing stress is about 5.2 times the undrained cohesion of the clay
  - The average settlement increased linearly with footing width
  - The difference between settlements at the center and the edge of the footing increases with vertical stresses

- Based on previous points, the proposed constitutive relation can successfully simulate the behavior of strip footing rested on undrained clay

- The proposed constitutive relation is much simpler than earlier ones because it depends on consistency limits and water content not the mechanical properties.

- A coupled model of the proposed constitutive relation with seepage constitutive relation can be able to simulate the settlement changes below the footings due to seepage.

- Further studies may consider the influence of multiple adjacent strip footings on the global behavior a structure rested on undrained clay.
Declarations

Author contribution statement

Ahmed M. EBID, Kennedy C. ONYELOWE: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

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