Simulating QCD at finite density with static quarks

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We study lattice QCD in the limit that the quark mass and chemical potential are simultaneously made large, resulting in a controllable density of quarks which do not move; this is similar in spirit to the quenched approximation for zero density QCD. In this approximation we find that the deconfinement transition seen at zero density becomes a smooth crossover for any nonzero density at which we simulated, and that at low enough temperature chiral symmetry remains broken at all densities.

Lattice QCD with a nonzero density of quarks is a difficult problem since the fermion determinant becomes complex, thus we are led to consider approximations which hopefully capture some of the essential physics. Here we present a study of QCD at arbitrary quark density in an approximation where the dynamics of the quarks has been removed. This approach is analogous to the quenched approximation at zero density.

Our idea is to simultaneously take the limits of infinite quark mass and infinite chemical potential while the density of quarks remains fixed. This leaves us with quarks that can be present or absent at each lattice site, but which do not move in the spatial directions. The result is a much simpler fermion determinant such that gauge variables can be easily updated to equilibrium in the background of a prescribed density of quarks.

With a chemical potential included, the lattice Dirac operator using Kogut-Susskind quarks is

\[ M(x, y) = 2am_q \delta_{x,y} + \sum_{\nu=1,2,3} \left[ U_\nu(x) \eta_\nu(x) \delta_{x+\hat{\nu},y} - U_\nu^d(y) \eta_\nu(y) \delta_{x-\hat{\nu},y} \right] \]

\[ + \left[ e^{\mu a} U_1(x) \eta(x) \delta_{x+i,y} - e^{-\mu a} U_1^d(y) \eta(y) \delta_{x-i,y} \right] \]

Taking limits \( m \to \infty \) and \( \mu \to \infty \) simultaneously leaves \( 2ma \) along the diagonal, and the forward hopping terms, \( e^{\mu a} U_1 \). Each spatial point decouples from

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all others, and the fermion determinant is just a product of easily computed $SU(3)$ determinants:

$$
\det(M) = \prod_{x} e^{\lambda_{\mu} n_{\mu}} \det(P_{\vec{x}} + C1),
$$

$$
\det(P_{\vec{x}} + C) = C^3 + C^2 \text{Tr} P_{\vec{x}} + C \text{Tr} P_{\vec{x}}^* + 1. \quad (2)
$$

Here $P_{\vec{x}}$ is the Polyakov loop at spatial site $\vec{x}$, and $n_{\mu}$ is the number of time slices. The coefficient of the unit matrix, $C$, is $(2ma/e^\mu a)^{n_{\mu}}$, and is the fundamental parameter in our approximation, through which we fix the density. We are still left with a complex determinant, albeit a much simpler one, allowing us to generate high statistics. We estimate expectation values by taking the ratio

$$
\langle O \rangle = \frac{\langle O e^{i\theta} \rangle ||}{\langle e^{i\theta} \rangle ||} \quad (3)
$$

where $\theta$ is the phase of $\det(M)$ and $\langle \rangle ||$ indicates an expectation value in the ensemble weighted by the modulus of the determinant.

The physical quark density is obtained from eq. 2.

$$
\langle n \rangle = \frac{1}{a n_{\mu} V} \frac{\partial \ln(Z)}{\partial \mu} = \frac{1}{V} \left\langle \sum_{\vec{x}} \frac{C^2 \text{Tr} P_{\vec{x}} + 2C \text{Tr} P_{\vec{x}}^* + 3}{C^3 + C^2 \text{Tr} P_{\vec{x}} + C \text{Tr} P_{\vec{x}}^* + 1} \right\rangle \quad (4)
$$

where $V$ is the spatial volume. At $C = \infty$ the density is 0; at $C = 0$ the system is saturated with density 3 per site; $C = 1$ represents “half-filling” and the density is 3/2.

We have run simulations on $6^3 \times 2$, $8^3 \times 2$, $10^3 \times 2$, and $6^3 \times 4$ lattices as described in Ref. 2. The lattice spacing was set by measuring the rho mass. In figure 1 we summarize the behavior of the Polyakov loop magnitude ($|P|$) in the $T-$\rho plane, on $n_{t} = 2$ lattices, with a fitting function. At zero density, we see the strong first order transition at $T_c$. As the density increases, this transition becomes a smooth crossover for all nonzero values of the density at which we simulated (our smallest density just below the transition region was $\sim 0.02$ quarks/fm$^3$). We are currently investigating the way in which the first order transition disappears by examining very low densities near the transition. Since we see no systematic dependence of the crossover on the spatial size, except for the expected decrease of the Polyakov loop magnitude on cold lattices, we conclude that this rounding is not a finite size effect.

We use $\langle \bar{\psi} \psi \rangle$ evaluated for light quarks as an indicator of chiral symmetry breaking in the presence of a finite density of massive quarks. In a series of runs on $6^3 \times 4$ lattices, we began with $6/g^2 = 5.0$, which is a fairly cold lattice,
at a temperature less than half the zero density $T_c$. We find that $\langle \bar{\psi}\psi \rangle$ remains large at this (cold) value of $6/g^2$ even up to a density of 1.5 quarks/site (where $C = 1$). Maintaining this maximum density (1.5 quarks/site) and increasing the temperature, we find a crossover to restored chiral symmetry at $6/g^2 \approx 5.3$, a significantly lower temperature than the zero density $n_t = 4$ transition, which occurs at $6/g^2 \approx 5.7$.

Does this static approximation have anything to do with real QCD? Certainly the nature of the high temperature transition at zero density depends strongly on the presence of dynamical quarks. However, it is not a priori clear to us that a deconfinement transition or chiral symmetry restoration driven by high density should depend on the quarks moving, or whether the mere presence of the quarks would be enough. In particular, we had not expected to see the zero density first order transition disappear for very small quark densities, or the signal of chiral symmetry restoration to vanish. This suggests that we might want to re-examine the conventional wisdom that a high density of quarks causes a phase transition similar to that caused by high temperature.

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