Comments on “Plasma oscillations and nonextensive statistics”

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The paper, authored by J. A. S. Lima et al, was published in Phys. Rev. E in 2000 has discussed the dispersion relation and Landau damping of Langmuir wave in the context of the nonextensive statistics proposed by Tsallis. It has been cited by many authors because the dispersion relation in Tsallis formalism present a good fit to the experimental data when \( q < 1 \), while the classical result based on Maxwellian distribution only provides a crude description. However, the results obtained in this paper are problematic. In this comments on the paper we shall derive the correct analytic formulas both for the dispersion relation and Landau damping in Tsallis formalism. We hope that this comments will be useful in providing the correct results.

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I. INTRODUCTION

Over the last few years, it has been proven that systems which present long-range interactions, long-time memory, fractality of the corresponding space-time, or intrinsic inhomogeneity are intractable within the conventional Boltzmann-Gibbs statistics [1, 2]. So there has been an increasing focus on the new statistical approach, i.e. nonextensive statistical mechanics (NSM), in recent years. For \( q \neq 1 \), it gives power-law distribution and only when the parameter \( q \rightarrow 1 \) Maxwellian distribution is recovered [3]. NSM has been successfully applied to stellar polytropes [4], two dimensional Euler and drift turbulence in a pure electron plasma column [5], as well as to the peculiar velocity function of galaxies clusters [6]. In particular, Liu et al. [7] showed a reasonable indication for the non-Maxwellian velocity distribution from plasma experiments.

Dispersion relations are fundamental and important for studying the wave in the plasma. According to the dispersion relations, we can study the problem of instability, propagation, refraction and absorption of the plasma wave. Recently, there has been a great deal of interest in studying the dispersion property in plasmas in the context of the nonextensive statistics. The paper authored by J. A. S. Lima et al [8], has a considerable hold over the field of plasma physics, have studied the dispersion relation of Langmuir wave based on \( q \)-distribution, the results show that nonextensive formalism presents a good fit to the experimental data, while the standard Maxwellian distribution only provides a crude description. However, the results obtained in this paper are problematic. The reason is that the author investigates the propagation of electrostatic waves by using the one-dimensional equilibrium distribution. But the equilibrium distribution in the dielectric function should be marginal distribution (quasi one-dimensional distribution), which is different with one-dimensional distribution in the context of nonextensive statistics unlike the classical Boltzmann-Gibbs statistics. In this comments on the paper we will derive the correct analytic formulas both for the dispersion relation and Landau damping in detail with Tsallis formalism. It is our hope that the discussion here will be useful in the field of plasma physics.

The paper is organized as follows. In Section II we briefly introduce the nonextensive distribution function. The generalized dispersion relation and Landau damping for Langmuir wave are obtained in Section III. Finally, the summary is given in Section IV.

II. NONEXTENSIVE DISTRIBUTION FUNCTION

First let us recall some basic facts about Tsallis statistics. In Tsallis statistics, the entropy has the form [3] of

\[
S_q = k_B \frac{1 - \sum_i p_i^q}{q - 1},
\]

where \( k_B \) is the Boltzmann constant, \( q \) is a parameter quantifying the degree of nonextensivity, \( p_i \) is the probability of the \( i \)-th microstate. The B-G entropy is recovered in the limit \( q \rightarrow 1 \). The basic property of Tsallis entropy is the nonadditivity or nonextensivity for \( q \neq 1 \). For example, for two systems A and B, the rule of composition [3] reads

\[
S_q (A + B) = S_q(A) + S_q(B) + (1 - q)S_q(A)S_q(B).
\]

In the nonextensive description, the three-dimensional equilibrium distribution function can be written as [9]
Eq. (3) reduces to the Boltzmann distribution function\[ q < q_{\text{max}} \]

for particles and particle number density. As one may check, the Boltzmann constant, temperature of particles, mass of particles, and momentum of particles, thermal speed, according to the normalizing condition

\[
\int f_q(p) \frac{1}{(2\pi)^3} dp = n_0, \]

the normalization constant reads

\[
A_q = \frac{L_q}{(m v_T)^3 n_0},\]

in which

\[
L_q = \frac{\Gamma \left( \frac{1}{1-q} \right)}{\Gamma \left( \frac{1}{1-q} - \frac{3}{2} \right) \Gamma \left( \frac{1}{q-1} + \frac{3}{2} \right)}, \quad 0 < q < 1
\]

and

\[
L_q = \frac{3q - 1}{2} \left( \frac{1}{q-1} \right)^{-3/2} \frac{\Gamma \left( \frac{1}{q-1} + \frac{3}{2} \right)}{\Gamma \left( \frac{1}{q-1} \right)}, \quad q \geq 1
\]

where \( p, v_T = \sqrt{k_B T/m}, k_B, T, m, \) and \( n_0 \) denotes, respectively, the momentum of particles, thermal speed, Boltzmann constant, temperature of particles, mass of particles and particle number density. As one may check, for \( q < 1/3 \), the q-distribution is unnormalizable. For \( 1/3 < q \leq 1 \), the momentum of the particles can take any value. For \( q \geq 1 \), the distribution function Eq. (3) exhibits a cutoff on the maximum value allowed for the momentum of the particles, which is given by

\[
p_{\text{max}} = \sqrt{2/(q-1)m v_T},\]

We see that in the limit \( q \to 1 \), \( p_{\text{max}} \) goes to infinity and Eq. (3) reduces to the Boltzmann distribution function

\[
f_{q=1}(p) = \frac{(2\pi)^3}{(m v_T)^3 n_0} \exp(-\frac{p^2}{2m v_T^2}).\]

In order to define the temperature of the system which is described by the nonextensive distribution, we will calculate the average kinetic energy below. For \( 1/3 < q \leq 1 \),

\[
\langle E_q \rangle = \frac{\langle p^2 \rangle}{2m} = \frac{\int p^2 f_q(p) \frac{1}{(2\pi)^3} dp}{\int f_q(p) \frac{1}{(2\pi)^3} dp} = \frac{L_q}{(2\pi)^3 (m v_T)^3 n_0},
\]

where Eq. (10) has been calculated using the integral formula [10, p325], that is,

\[
\int_0^\infty \frac{p^2}{2m}[1 - (q-1) \frac{p^2}{2mv_T^2}]^{-\frac{1}{q-1}} dp = \frac{L_q}{\sqrt{2\pi} (m v_T)^3 n_0} \int_0^\infty p^4[1 + \frac{1-q}{2m v_T^2}]^{-\frac{1}{q-1}} dp
\]

\[
= \frac{2}{5} \frac{3}{2} n_0 m v_T^2 = \frac{2}{5} \frac{3}{2} n_0 k_B T. \]

For \( q \geq 1 \),

\[
\langle E_q \rangle = \frac{\langle p^2 \rangle}{2m} = \frac{\int p^2 f_q(p) \frac{1}{(2\pi)^3} dp}{\int f_q(p) \frac{1}{(2\pi)^3} dp} = \frac{L_q}{(2\pi)^3 (m v_T)^3 n_0},
\]

where Eq. (12) has been calculated using the transformation \( t = \frac{q-1}{2} p^2/2m v_T^2 \) and integral formula [10, p324], that is,

\[
\int_0^\infty \frac{t^2}{2m}[1 - (q-1) \frac{t^2}{2mv_T^2}]^{-\frac{1}{q-1}} dt = \frac{L_q}{\sqrt{2\pi} (m v_T)^3 n_0} \int_0^{\infty} t^3/2 \langle 1 - t \rangle^{-\frac{1}{q-1}} dt
\]

\[
= \frac{2}{5} \frac{3}{2} n_0 m v_T^2 = \frac{2}{5} \frac{3}{2} n_0 k_B T. \]
\[
\int_0^1 x^{\nu-1} (1 - x^\lambda)^{\nu-1} \, dx = \frac{1}{\lambda} B \left( \frac{\mu}{\lambda}, \nu \right)
\]  
(14)

with \( Re\mu > 0, Re\nu > 0, \lambda > 0 \). So the average kinetic energy can be expressed as

\[
\langle E_q \rangle = \left\langle \frac{p^2}{2m} \right\rangle = \frac{2}{5q - 3} \frac{3}{2} n_0 k_B T = \frac{3}{2} n_0 k_B T_q,
\]
(15)

where \( T_q = 2T/(5q - 3) \) is the physical temperature of the nonextensive system. We see that in the limit \( q \to 1 \), \( T_q = T \) and the average kinetic energy reduces to \( \langle E_{q=1} \rangle = 3n_0 k_B T/2 \), which is the standard result in B-G statistics.

**III. THE GENERALIZED DISPERSION RELATION AND LANDAU DAMPING**

For the longitudinal wave propagating in an unmagnetized, collisionless, isotropic plasma, the longitudinal dielectric function of electron can be written as [11]

\[
\varepsilon^l_k = 1 + \frac{4\pi e^2}{k^2} \int \frac{1}{\omega - k \cdot \mathbf{v} + i \delta} \left( \mathbf{k} \cdot \frac{\partial f_q (p)}{\partial p} \right) \frac{dp}{(2\pi)^3},
\]
(16)

we consider the direction of wave vector \( \mathbf{k} \) to be along x-axis, Eq. (16) becomes

\[
\varepsilon^l_k = 1 + \frac{4\pi e^2}{k^2} \int \frac{dp_x}{2\pi} \frac{k_x}{\omega - kv_x} \frac{\partial f_q (p_x)}{\partial p_x} \frac{dp_x}{2\pi},
\]
(17)

where \( e \) is the electron charge, \( i \delta \) comes from Landau rules(\( \delta \to 0^+ \)) [12]. Note that \( f_q (p_x) \) is the marginal distribution in the nonextensive framework, which is given by

\[
f_q (p_x) = \int f_q (p) \frac{dp_y dp_z}{(2\pi)^2}.
\]
(18)

Next we will derive the expression of the marginal distribution. Substituting Eq. (18) into Eq. (15), for \( 3/5 < q \leq 1 \), we obtain

\[
f_q (p_x) = \frac{4L_q}{(mv_T)^3} \frac{n_0}{\sqrt{2\pi}}
\]

Then the integral in Eq. (19) over \( p_y \) is

\[
\int_0^\infty \left[ 1 - (q - 1) \frac{p_y^2 + p_y^2 + p_z^2}{2m^2 v_T^2} \right]^{1/\nu} \, dp_y
\]

\[
= \int_0^\infty \left\{ \frac{2m^2 v_T^2}{1 - (q - 1) (p_x^2 + p_y^2)} \right\}^{1/\nu - 1/\nu} \, dp_y
\]

\[
= \left\{ \frac{2m^2 v_T^2}{1 - (q - 1) (p_x^2 + p_y^2)} \right\}^{1/\nu - 1/\nu} \, dp_y
\]

\[
= \frac{\sqrt{\pi} \Gamma \left( \frac{1}{\nu - 1/\nu} \right) \Gamma \left( \frac{1}{\nu - 1/\nu} \right) \pi^{1/2}}{2 \Gamma \left( \frac{1}{\nu - 1/\nu} \right)} \left( 2m^2 v_T^2 \right)^{1/\nu - 1/\nu}.
\]
(20)

where Eq. (20) has been calculated using the integral formula [11], and substituting Eq. (20) into Eq. (19), according to the same method we can calculate the integral over \( p_z \). Finally Eq. (19) becomes

\[
f_q (p_x) = \frac{L_q}{q \sqrt{2\pi n_0}} \left[ 1 - (q - 1) \frac{p_x^2 + p_y^2 + p_z^2}{2m^2 v_T^2} \right]^{1/\nu - 1/\nu}.
\]
(21)

For \( q \geq 1 \), Substituting Eq. (18) into Eq. (15), we obtain

\[
f_q (p_x) = \frac{4L_q}{(mv_T)^3} \frac{n_0}{\sqrt{2\pi}}
\]

\[
\int_0^{p_{x,\text{max}}} dp_x \int_0^{p_{y,\text{max}}} \left[ 1 - (q - 1) \frac{p_x^2 + p_y^2 + p_z^2}{2m^2 v_T^2} \right]^{1/\nu - 1/\nu} \, dp_y,
\]
(22)

then the integral in Eq. (22) over \( p_y \) becomes

\[
\int_0^{p_{y,\text{max}}} \left[ 1 - (q - 1) \frac{p_x^2 + p_y^2 + p_z^2}{2m^2 v_T^2} \right]^{1/\nu - 1/\nu} \, dp_y
\]

\[
= \int_0^{p_{y,\text{max}}} \left[ \frac{2m^2 v_T^2}{1 - (q - 1) (p_x^2 + p_y^2)} \right]^{1/\nu - 1/\nu} \, dp_y
\]

\[
= \frac{\sqrt{\pi} \Gamma \left( \frac{1}{\nu - 1/\nu} \right) \Gamma \left( \frac{1}{\nu - 1/\nu} \right) \pi^{1/2}}{2 \Gamma \left( \frac{1}{\nu - 1/\nu} \right)} \left( 2m^2 v_T^2 \right)^{1/\nu - 1/\nu}.
\]
\[
\int_0^{p_y \text{ max}} \left( 1 - \frac{(q - 1) \left( p_z^2 + p_y^2 \right)}{2m^2v_T^2 - (q - 1) \left( p_z^2 + p_y^2 \right)} \right)^{\frac{1}{q-1} + \frac{1}{2}} \, dp_y
\]

\[
= \left( \frac{2m^2v_T^2 - (q - 1) \left( p_z^2 + p_y^2 \right)}{2m^2v_T^2 - (q - 1) \left( p_z^2 + p_y^2 \right)} \right)^{\frac{1}{q-1} + \frac{1}{2}}.
\]

\[
\int_0^1 t^{-\frac{1}{q}} (1 - t)^{\frac{1}{q-1} + \frac{1}{2}} \, dt
\]

\[
= \frac{\sqrt{\pi}}{2} \frac{\Gamma \left( \frac{1}{q-1} \right)}{(q - 1)^{\frac{1}{2}} \Gamma \left( \frac{1}{q-1} + \frac{3}{2} \right)}.
\]

\[
\left( \frac{2m^2v_T^2 - (q - 1) \left( p_z^2 + p_y^2 \right)}{2m^2v_T^2} \right)^{\frac{1}{q-1} + \frac{1}{2}}.
\]

where Eq. (23) has been calculated using the transformation \( t = (q - 1) p_y^2 / (2m^2v_T^2 - (q - 1) \left( p_z^2 + p_y^2 \right)) \) and integral formula (14), then substituting Eq. (23) into Eq. (22), according to the same method we can calculate the integral over \( p_z \). Finally Eq. (22) becomes

\[
f_q(p_x) = \frac{L_q}{q} \sqrt{\frac{2\pi n_0}{mv_T}} [1 - (q - 1) \frac{p_x^2}{2m^2v_T^2}]^{\frac{1}{q-1} + \frac{1}{2}}.
\]

(24)

Obviously, the marginal distribution Eqs. (21) and (24) are different with one-dimensional distribution in the context of nonextensive statistics [9]

\[
f_q(p_x) = B_q \frac{\sqrt{2\pi n_0}}{mv_T} [1 - (q - 1) \frac{p_x^2}{2m^2v_T^2}]^{\frac{1}{q-1} + \frac{1}{2}}
\]

in which

\[
B_q = \frac{\Gamma \left( \frac{1}{q-1} \right)}{\left( \frac{1}{q-1} \right)^{\frac{1}{2}} \Gamma \left( \frac{1}{q-1} - \frac{1}{2} \right)}, \quad -1 < q \leq 1
\]

(26)

and

\[
B_q = \frac{1 + q \left( \frac{1}{q-1} \right)^{-\frac{1}{2}} \Gamma \left( \frac{1}{q-1} + \frac{3}{2} \right)}{\Gamma \left( \frac{1}{q-1} \right)}, \quad q \geq 1
\]

(27)

unlike the classical Boltzmann-Gibbs statistics. It is the reason why the results obtained by J. A. S. Lima et al, are problematic.

Substituting the marginal distribution Eqs. (21) and (24) into the dielectric function Eq. (17), we obtain

\[
\varepsilon'_k = 1 + \frac{\omega_{pc}^2}{k^2v_T^2} \left[ \frac{3q - 1}{2} - Z_q(x) \right],
\]

(28)

where \( \omega_{pc} = \sqrt{4\pi n_0 e^2/m} \) is the plasma frequency, \( x \) is the dimensionless parameter, namely, \( x = \omega / \sqrt{\pi} kv_T \). \( Z_q(x) \) is the generalized plasma dispersion function in the context of Tsallis statistics,

\[
Z_q(x) = L_q \frac{x}{\sqrt{\pi}} \int_0^1 \left[ 1 - (q - 1) \xi^2 \right]^{\frac{1}{q-1} + \frac{1}{2}} \, d\xi
\]

(29)

where \( \xi = v_x / \sqrt{\pi} v_T \), in the limit \( q \to 1 \), it is reduced to the standard form in B-G statistics [11]

\[
Z_{q=1}(x) = \frac{x}{\sqrt{\pi}} \int_0^1 \left[ 1 - (q - 1) \xi^2 \right]^{\frac{1}{q-1} + \frac{1}{2}} \, d\xi.
\]

(30)

Using the Plemelj formula [11]

\[
\frac{1}{z \pm i0} = \phi \frac{1}{z} \mp i\pi \delta(z),
\]

(31)

where \( \phi \) denotes the principal value, then the generalized plasma dispersion function Eq. (29) can be written as

\[
Z_q(x) = L_q \frac{x}{\sqrt{\pi}} \int_0^1 \left[ 1 - (q - 1) \xi^2 \right]^{\frac{1}{q-1} + \frac{1}{2}} \, d\xi
\]

\[
i L_q \sqrt{\pi} x \left[ 1 - (q - 1) \xi^2 \right]^{\frac{1}{q-1} + \frac{1}{2}},
\]

(32)

when \( \omega \gg kv_T \), namely \( x \gg 1 \), the real part of Eq. (32) becomes

\[
L_q \frac{x}{\sqrt{\pi}} \phi \int_0^1 \left[ 1 - (q - 1) \xi^2 \right]^{\frac{1}{q-1} + \frac{1}{2}} \, d\xi
\]

\[
= \frac{L_q}{\sqrt{\pi}} \phi \int_0^1 \left[ 1 - (q - 1) \xi^2 \right]^{\frac{1}{q-1} + \frac{1}{2}} \left( 1 + \frac{\xi}{x^2} + \frac{\xi^2}{x^4} + \cdots \right) \, d\xi,
\]

(33)

thus Eq. (32) can be expressed as

\[
Z_q(x) \approx \frac{3q - 1}{2} + \frac{1}{2x^2} + \frac{2}{5q - 3} \frac{3}{4x^4} -
\]

\[
i L_q \sqrt{\pi} x \left[ 1 - (q - 1) \xi^2 \right]^{\frac{1}{q-1} + \frac{1}{2}}.
\]

(34)
When $\omega \ll kv_T$, namely $x \ll 1$, introducing the transformation $\xi = \eta + x$, then the real part of Eq. (32) can be written as

$$L_q \frac{x}{\sqrt{\pi}} \varphi \int \frac{1}{x - \xi} \left[ 1 - (q - 1) \xi^2 \right] \frac{\varphi}{\eta} \, d\xi,$$

$$= L_q \frac{x}{\sqrt{\pi}} \varphi \int \left[ 1 - (q - 1) (\eta^2 + 2\eta x + x^2) \right] \frac{\varphi}{\eta} \, d\eta \approx -L_q \frac{x}{\sqrt{\pi}} \varphi \int \left[ 1 - (q - 1) \eta^2 \right] \frac{\varphi}{\eta} = 0,$$  

(35)

then Eq. (32) can be expressed as

$$Z_q (x) \approx -iL_q \sqrt{\pi} x \left[ 1 - (q - 1) x^2 \right] \frac{\varphi}{\eta},$$  

(36)

which can be used in investigating the low-frequency wave, such as the ion acoustic waves. It should be noted that the process is not pinpoint in Eq. (35), the real part should be a very small quantity, which may be obtained by numerical method. However, the small quantity can be neglect when Eq. (32) Substituted into the dielectric function Eq. (28).

Substituting Eq. (33) into the dielectric function Eq. (28), according to the longitudinal dispersion relation $\text{Re} \varepsilon'_k = 0$, thus the generalized dispersion relation of Langmuir wave is obtained,

$$\omega^2 = \omega_{pe}^2 + \frac{2}{5q-3} 3k^2 v_T^2,$$

$$= \omega_{pe}^2 + 3k^2 v_T^2,$$  

(37)

where $v_{Tq} = \sqrt{k_B T_q/m}$ is the physical thermal speed, $T_q$ is the physical temperature defined in Section 2. As expected, in the limit $q \rightarrow 1$, Eq. (37) reduces to

$$\omega^2 = \omega_{pe}^2 + 3k^2 v_T^2,$$  

(38)

being the standard result in B-G statistics [11]. Thus the dispersion relation of Langmuir wave obtained by J. A. S. Lima et al [8]

$$\omega^2 = \omega_{pe}^2 + \frac{2}{5q-1} 3k^2 v_T^2,$$

is problematic. According to Fig.1 in their paper, we can see that the dispersion relation for Tsallis formalism presents a good fit to the experimental data when $0.7 < q < 0.85$, obviously, it should be $0.82 < q < 0.91$ based on the correct result.

Next we will derive the expression of Landau damping. The Landau damping rate can be written as [11]

$$\gamma_k^l = -\frac{\text{Im} \varepsilon_k'}{\text{Re} \varepsilon_k'} \left| \frac{\partial \omega}{\partial \omega} \text{Re} \varepsilon_k' \right|_{\omega = \omega^l},$$  

(39)

according to Eqs. (37), (28) and (33), we have that $\omega \approx \omega_{pe}$, $\text{Re} \varepsilon_k' = 0 - \omega_{pe}^2/\omega^2$, 

$$\left( \partial/\partial \omega \right) \text{Re} \varepsilon_k' = 2/\omega_{pe}, \text{Im} \varepsilon_k' = L_q \sqrt{\pi}/2 (\omega_{pe}^2) / (k v_T)^3 \cdot [1 - (q - 1) \omega^2/(2k^2 v_T^2)]^{1/(q-1)}$$

combined with Eq. (39), we obtain the generalized Landau damping as

$$\gamma_k^l = -L_q \sqrt{\pi} \omega_{pe} \left( \frac{k_d}{k} \right)^3,$$

$$\left[ 1 - (q - 1) \left( \frac{k_d^2}{2k^2} + \frac{3}{5q-3} \right) \right] \frac{\varphi}{\eta},$$  

(40)

where $k_d = \omega_{pe}/v_T$ is the electronic Debye wave number. In the limit $q \rightarrow 1$, Eq. (40) reduces to

$$\gamma_k^l = -\sqrt{\pi} \omega_{pe} \left( \frac{k_d}{k} \right)^3 \exp \left( -\frac{k_d^2}{2k^2} - \frac{3}{2} \right),$$  

(41)

which is the classical Landau expression for the damping decrement in the framework of B-G statistics [11].

IV. SUMMARY

In this comments, we have discussed the dispersion property and Landau damping of Langmuir wave in an unmagnetized, collisionless, isotropic plasma with the nonextensive distribution in Tsallis statistics. The correct generalized dispersion relation and Landau damping are obtained. In the limiting case ($q \rightarrow 1$) the classical results based on the B-G statistics are recovered. It is our hope that the discussion here will serve as a useful introduction to the field of plasma physics.

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[1] C. Tsallis, in *New Trends in Magnetism, Magnetic Materials and Their Applications*, edited by J. L. Moran-Lopez and J. M. Sanchez (Plenum, New York, 1994).
[2] C. Tsallis, Chaos, Solitons Fractals 6, 539 (1995).
[3] C. Tsallis, J. Stat. Phys. 52, 479 (1988).
[4] A. Plastino and A. R. Plastino, Phys. Lett. A 177, 177 (1993).
[5] B. M. Boghosian, Phys. Rev. E 53, 4754 (1996).
[6] A. Lavagno, G. Kaniadakis, M. Rego-Monteiro, P. Quarati and C. Tsallis, Astrophys. Lett. 35, 449 (1998).
[7] J. M. Liu, J. S. De Groop, J. P. Matte, T. W. Johnston and R. P. Drake, Phys. Rev. Lett. 72, 2717 (1994).
[8] J. A. S. Lima, R. Silva Jr. and Janilo Santos, Phys. Rev. E 61, 3260 (2000).
[9] R. Silva Jr., A. R. Plastino, and J. A. S. Lima, Phys. Lett. A 249, 401 (1993).
[10] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series, and Products*, edited by A. Jeffrey and D. Zwillinger (Elsevier, New York, 2007).
[11] X. Q. Li, *Collapsing Dynamics of Plasmons* (Chinese Science and Technology Press, Beijing, 2004).
[12] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981).