Strong decays $\Sigma^* \to \Sigma\pi, \Lambda\pi$ and related strong coupling constant

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Abstract

In this article, we calculate the strong coupling constant $g$ among the decuplet baryons, the octet baryons and the pseudoscalar mesons in the heavy baryon chiral perturbation theory with the light-cone QCD sum rules, and study the strong decays $\Sigma^* \to \Lambda\pi, \Sigma\pi$. The numerical value of the strong coupling constant $g$ is consistent with our previous calculation, the central values lead to small SU(3) breaking effects, less than 6%; and no definitive conclusion can be drawn due to the large uncertainties.

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1 Introduction

In the (heavy) baryon chiral perturbation theory, the resonant baryon states are usually assumed to be very heavy and integrated out, their effects are represented by a finite piece of counterterms. The mass difference between the decuplet baryons and the octet baryons is rather small, about 300 MeV, and the coupling constant among the decuplet baryons, the octet baryons and the pseudoscalar mesons is rather large [1]. For example, the $\Delta(1232)$ resonance dominates many nuclear phenomena at energies above the pion-production threshold. It is almost an ideal elastic $\pi N$ resonance, and decays into the nucleon and pion ($\Delta \to N\pi$) with the branching fraction about 99% [1]. It is useful to include the decuplet baryons as an explicit degree of freedom in the effective lagrangian. In the small scale expansion approach (which builds upon the heavy baryon chiral perturbation theory), the nucleon and $\Delta$ degrees of freedom are treated simultaneously [2].

The phenomenological chiral lagrangian can be written as

$$\mathcal{L} = -\mathcal{C} \left[ \bar{T}^\mu u_\mu B + \bar{B} u_\mu T^\mu \right],$$  \hspace{1cm} (1)
where

\[ u_\mu = \frac{i}{2} \{ \xi^i, \partial_\mu \xi \}, \]

\[ \xi = \exp\left( \frac{i\phi}{f_\pi} \right), \]

\[ \phi = \begin{pmatrix}
\frac{\pi^0 + \eta}{\sqrt{2}} & -\frac{\pi^+}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^+ \\
\pi^- & \frac{\pi^0 + \eta}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\
K^- & -\frac{2\sqrt{6} \eta}{\sqrt{2}} & -\frac{2\sqrt{6} \eta}{\sqrt{2}}
\end{pmatrix}, \]

\[ B = \begin{pmatrix}
\frac{\Sigma^0 + \Lambda}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\
\Sigma^- & \frac{\Sigma^0 + \Lambda}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\
\Xi^- & \frac{\Xi^0 + \Lambda}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & -\frac{2\sqrt{6} \Lambda}{\sqrt{2}}
\end{pmatrix}, \quad (2) \]

and

\[ T_{uuu} = \Delta^{++}, \quad T_{uud} = \frac{\Delta^+}{\sqrt{3}}, \quad T_{udd} = \frac{\Delta^0}{\sqrt{3}}, \quad T_{ddd} = \Delta^-, \quad T_{uss} = \frac{\Sigma^{*+}}{\sqrt{3}}, \]

\[ T_{uds} = \frac{\Sigma^{*0}}{\sqrt{6}}, \quad T_{dds} = \frac{\Sigma^{*-}}{\sqrt{3}}, \quad T_{uss} = \frac{\Xi^{*0}}{\sqrt{3}}, \quad T_{dss} = \frac{\Xi^{*-}}{\sqrt{3}}, \quad T_{sss} = \Omega^-, \quad (3) \]

the \( T_\mu \) are the Rarita-Schwinger fields of the decuplet baryons \( T_i \), the \( f_\pi \) is the decay constant of the \( \pi \). From the chiral lagrangian \( \mathcal{L} \), we can obtain the following relations,

\[ g_{\Delta^{++} p\pi^+} = \frac{C}{f_\pi}, \]

\[ g_{\Sigma^{*+} \Sigma^0 \pi^+} = -\frac{C}{\sqrt{6} f_\pi}, \]

\[ g_{\Sigma^{*+} \Lambda \pi^+} = -\frac{C}{\sqrt{2} f_\pi}, \quad (4) \]

the coupling constant \( C \) is a basic parameter, which can be fitted phenomenologically or calculated with some theoretical approaches, we introduce a parameter \( g \) with \( g = \frac{C}{f_\pi} \) to simplify the notation. We terminate the Taylor series \( \xi = \exp\left( \frac{i\phi}{f_\pi} \right) = 1 + \frac{i\phi}{f_\pi} + \frac{1}{2!}\left( \frac{i\phi}{f_\pi} \right)^2 + \cdots \) at leading order \( \mathcal{O}(\phi) \) and approximate \( u_\mu = \frac{i}{2} \{ \xi^i, \partial_\mu \xi \} \approx -\frac{\partial_\mu \phi}{f_\pi} \) to obtain the three relations in Eq.(4). The higher order terms of the Taylor series \( \xi = 1 + \frac{i\phi}{f_\pi} + \frac{1}{2!}\left( \frac{i\phi}{f_\pi} \right)^2 + \cdots \) have contributions to the strong coupling constants \( g_{\Delta^{++} p\pi^+}, \) \( g_{\Sigma^{*+} \Sigma^0 \pi^+} \) and \( g_{\Sigma^{*+} \Lambda \pi^+} \), their effects can be taken into account with the replacement \( \frac{C}{f_\pi} \rightarrow \frac{C}{f_\pi} \left( 1 + \frac{\alpha}{f_\pi} + \frac{\beta}{f_\pi} + \cdots \right) \), where the coefficients \( \alpha \) and \( \beta \) originate from the corresponding chiral loops. In this article, we take the leading order approximation.

Thereafter we will introduce the notations \( g_N, g_\Sigma \) and \( g_\Lambda \) to represent the strong coupling constant \( g \) from the \( g_{\Delta^{++} p\pi^+}, g_{\Sigma^{*+} \Sigma^0 \pi^+} \) and \( g_{\Sigma^{*+} \Lambda \pi^+} \) respectively, and study
the strong coupling constants $g_\Lambda$, $g_\Sigma$ and the $SU(3)$ breaking effects with the light-cone QCD sum rules.

From the Particle Data Group [1], we can see that the following strong decays are kinematically allowed,

$$\Delta \rightarrow p\pi,$$
$$\Sigma^* \rightarrow \Sigma\pi, \Lambda\pi,$$
$$\Xi^* \rightarrow \Xi\pi,$$(5)

the strong decays $\Sigma^* \rightarrow \Sigma\pi, \Lambda\pi$ are ideal channels to study the $SU(3)$ breaking effects as the constituent quark contents of the baryons $\Sigma^*$, $\Sigma$ and $\Lambda$ are $uds$ or $uus$.

In a previous work, we have calculated the strong coupling constant $g_N$ with the light-cone QCD sum rules, and studied the decay width $\Gamma_{\Delta \rightarrow p\pi}$ [3]. The strong coupling constants among the octet baryons, the vector and pseudoscalar mesons $g_{NNV}$ and $g_{NNP}$ have been calculated with the light-cone QCD sum rules [4, 5, 6, 7].

The light-cone QCD sum rules carry out the operator product expansion near the light-cone $x^2 \approx 0$ instead of the short distance $x^2 \approx 0$ while the nonperturbative hadronic matrix elements are parameterized by the light-cone distribution amplitudes instead of the vacuum condensates [8, 9, 10, 11, 12, 13]. The nonperturbative parameters in the light-cone distribution amplitudes are calculated with the conventional QCD sum rules and the values are universal [14, 15].

The article is arranged as: in Section 2, we derive the strong coupling constants $g_\Lambda$ and $g_\Sigma$ with the light-cone QCD sum rules; in Section 3, the numerical result and discussion; and Section 4 is reserved for conclusion.

## 2 Strong coupling constants $g_\Lambda$ and $g_\Sigma$ with light-cone QCD sum rules

In the following, we write down the two-point correlation functions $\Pi^{\Lambda/\Sigma}_{\mu}(p, q)$,

$$\Pi^{\Lambda/\Sigma}_{\mu}(p, q) = i \int d^4x e^{-i q \cdot x} \langle 0| \{ J_{\Lambda/\Sigma}(0)%J_\mu(x) \} |\pi(p)\rangle,$$ (6)

$$J_\Lambda(x) = \sqrt{\frac{2}{3}} \epsilon_{abc} \left[ u_a^T(x) C \gamma_\mu s_b(x) \gamma_5 s_c(x) - d_a^T(x) C \gamma_\mu s_b(x) \gamma_5 u_c(x) \right],$$

$$J_\Sigma(x) = \sqrt{2} \epsilon_{abc} \left[ u_a^T(x) C \gamma_\mu s_b(x) \gamma_5 \gamma^\mu u_c(x) + d_a^T(x) C \gamma_\mu s_b(x) \gamma_5 \gamma^\mu s_c(x) \right],$$

$$J_\mu(x) = \epsilon_{abc} \sqrt{3} \left[ 2u_a^T(x) C \gamma_\mu s_b(x) u_c(x) + u_a^T(x) C \gamma_\mu u_b(x) s_c(x) \right],$$ (7)

where the baryon currents $J_\Sigma(x)$, $J_\Lambda(x)$ and $J_\mu(x)$ interpolate the octet baryons $\Sigma$, $\Lambda$ and the decuplet baryon $\Sigma^*$, respectively [16, 17, 18, 19], the external state $\pi$ has the four momentum $p_\mu$ with $p^2 = m_\pi^2$. The correlation functions $\Pi^{\Lambda/\Sigma}_{\mu}(p, q)$ (sometime
we will smear the indexes \( \Lambda \) and \( \Sigma \) for simplicity) can be decomposed as

\[
\Pi_\mu(p, q) = \Pi\sigma_{\alpha\beta}p^\alpha q^\beta p_\mu + \Pi A_1 p_\mu + \Pi A_2 q_\mu + \Pi B_1 q_\mu + \Pi B_2 \bar{q}p_\mu + \Pi B_3 \bar{q}p_\mu + \Pi B_4 \sigma_{\alpha\beta}p^\alpha q^\beta q_\mu + \Pi C_1 \gamma_\mu + \Pi C_2 \bar{q}p_\mu + \Pi C_3 \bar{q}p_\mu + \Pi C_4 \epsilon_{\mu\nu\alpha\beta}p^\nu q^\alpha q^\beta
\]

due to the Lorentz invariance, where the \( \Pi \) and \( \Pi \) are Lorentz invariant functions of \( p \) and \( q \). In this article, we choose the tensor structure \( \sigma_{\alpha\beta}p^\alpha q^\beta q_\mu \) for analysis.

Basing on the quark-hadron duality \([14, 15]\), we can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators \( J_{\Lambda/\Sigma}(x) \) and \( J_\mu(x) \) into the correlation functions \( \Pi_\mu(p, q) \) to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the baryons \( \Lambda/\Sigma \) and \( \Sigma^* \), we get the following results,

\[
\Pi^{\Lambda}_\mu(p, q) = \frac{\langle 0| J_\Lambda(0)|\Lambda(q + p)\rangle\langle \Lambda(q + p)|\Sigma^*(q)\pi(p)\rangle\langle \Sigma^*(q)|\bar{J}_\mu(0)|0 \rangle}{\{M^2_{\Lambda} - (q + p)^2\} \{M^2_{\Sigma^*} - q^2\}} + \ldots
\]

\[
\Pi^{\Sigma}_\mu(p, q) = \frac{\langle 0| J_\Sigma(0)|\Sigma(q + p)\rangle\langle \Sigma(q + p)|\Sigma^*(q)\pi(p)\rangle\langle \Sigma^*(q)|\bar{J}_\mu(0)|0 \rangle}{\{M^2_{\Sigma} - (q + p)^2\} \{M^2_{\Sigma^*} - q^2\}} + \ldots
\]

where the following definitions have been used,

\[
\langle 0| J_{\Lambda/\Sigma}(0)|\Lambda/\Sigma(p)\rangle = \lambda_{\Lambda/\Sigma} U(p, s),
\]

\[
\langle 0| J_\mu(0)|\Sigma^*(p)\rangle = \lambda_{\Sigma^*} U_\mu(p, s),
\]

\[
\sum_s U(p, s)\bar{U}(p, s) = \not{p} + M_{\Lambda/\Sigma},
\]

\[
\sum_s U_\mu(p, s)\bar{U}_\nu(p, s) = -\langle \not{p} + M_{\Sigma^*} \rangle \left\{ g_{\mu\nu} - \frac{\gamma_\mu\gamma_\nu}{3} - \frac{2p_\mu p_\nu}{3M^2_{\Sigma^*}} + \frac{p_\mu\gamma_\nu - p_\nu\gamma_\mu}{3M^2_{\Sigma^*}} \right\},
\]

\[
\langle \Lambda/\Sigma(q')|\Sigma^*(q)\pi(p)\rangle = ig_{\Sigma^*\Lambda/\Sigma\pi}\bar{U}(q', s')U_\mu(q, s)p^\mu.
\]

The current \( J_\mu(x) \) couples not only to the isospin \( I = \frac{3}{2} \) and spin-parity \( J^P = \frac{3}{2}^+ \) states, but also to the isospin \( I = \frac{3}{2} \) and spin-parity \( J^P = \frac{1}{2}^- \) states. For a generic \( \frac{1}{2}^- \) resonance \( \Sigma^* \) \([20]\),

\[
\langle 0| J_\mu(0)|\Sigma^*(p)\rangle = \lambda^*_\mu(\gamma_\mu - 4\frac{p_\mu}{M^*_\mu})U^*(p, s),
\]

where \( \lambda^* \) is the pole residue and \( M^*_\mu \) is the mass. The spinor \( U^*(p, s) \) satisfies the usual Dirac equation \( (\not{p} - M^*_\mu)U^*(p, s) = 0 \). If we take the phenomenological lagrangian,

\[
\mathcal{L}(x) = g_{\Sigma^*/\Lambda/\Sigma\pi} \left\{ \bar{\Sigma^*}(x)\Lambda/\Sigma(x)\pi(x) + \bar{\Lambda}/\Sigma(x)\Sigma^*(x)x(x) \right\},
\]

(13)
which corresponds to \( \langle \Lambda / \Sigma (q') | \tilde{\Sigma}^\ast(q) \pi (p) \rangle = g \tilde{\Sigma}^\ast \Lambda / \Sigma \pi \overline{U}(q', s') U^\ast(q, s) \), the contributions from the \( \frac{1}{2}^- \) states can be written as

\[
\Pi_{\mu/\Sigma}^\Lambda (p, q) = \frac{g \tilde{\Sigma}^\ast \Lambda / \Sigma \pi \lambda^\ast \{ M^2_{\Lambda / \Sigma} - (q + p)^2 \} \{ M^2_{\pi} - q^2 \} \{ (q' + M_{\Lambda / \Sigma})(q' + M_{\pi})(\gamma_\mu - 4 q_\mu / M_{\pi}) \}}{M^2_{\Lambda / \Sigma} - (q + p)^2 \{ M^2_{\pi} - q^2 \}} + \cdots
\]

where the \( \Pi_i \) are Lorentz invariant functions of \( p \) and \( q \). If we choose the tensor structure \( \sigma_{\alpha\beta} p_\alpha q_\beta p_\mu \), the \( \tilde{\Sigma}^\ast \) has no contaminations.

In the following, we briefly outline the operator product expansion for the correlation functions \( \Pi_\mu(p, q) \) in perturbative QCD theory. The calculations are performed at the large space-like momentum regions \( (q + p)^2 \ll 0 \) and \( q^2 \ll 0 \), which correspond to the small light-cone distance \( x^2 \approx 0 \) required by the validity of the operator product expansion approach. We write down the "full" propagator of a massive light quark in the presence of the quark and gluon condensates firstly [8, 15],

\[
\langle 0 | T[q_a(x) q_b(0)] | 0 \rangle = \frac{i \delta_{ab} x^\mu}{2 \pi^2 x^4} - \frac{\delta_{ab} m_q}{4 \pi^2 x^2} \frac{1}{12} \langle \bar{q} q \rangle + \frac{i \delta_{ab}}{48} m_q \langle \bar{q} q \rangle \frac{1}{192} \langle \bar{q} g_\sigma \gamma_\mu G q \rangle + \frac{i \delta_{ab} x^2}{1152} m_q \langle \bar{q} g_\sigma G q \rangle \frac{x^\mu}{x^2} - \frac{i}{16 \pi^2 x^2} \int_0^1 dv \left[ (1 - v) g_\sigma G_{\mu\nu}(vx) \right] x^\mu x^\nu + v g_\sigma G_{\mu\nu}(vx) x^\mu x^\nu \frac{x^\nu}{x^2} \right] + \cdots,
\]

then contract the quark fields in the correlation functions \( \Pi_\mu(p, q) \) with the Wick
theorem, and obtain the following results,

\[
\Pi^\mu_\mu(p, q) = \frac{2\sqrt{2}}{3} i\varepsilon_{abc}c_{\alpha'\nu'} \int d^4xe^{-iq\cdot x}
\left\{ Tr \left[ \gamma_\mu C S_{\nu'}(0) U_{\alpha'\nu'}(0) \right] \gamma_5 \gamma_\alpha (0) d_c(0) \bar{u}_c(x) |\pi(p)\rangle \right.
\]

\[
- Tr \left[ \gamma_\mu C S_{\nu'}(0) U_{\alpha'\nu'}(0) \right] \gamma_5 \gamma_\alpha (0) |\pi(p)\rangle \gamma_\mu U_{\nu'\nu}(0) (0) \bar{u}_c(x) |\pi(p)\rangle
\]

\[
- \gamma_5 \gamma_\alpha (0) d_c(0) \bar{u}_c(x) \gamma_\mu U_{\nu'\nu}(0) (0) |\pi(p)\rangle \gamma_\mu C S_{\nu'}(0) (0) \bar{u}_c(x) |\pi(p)\rangle
\]

\[
+ \gamma_5 \gamma_\alpha (0) d_c(0) \bar{u}_c(x) \gamma_\mu C S_{\nu'}(0) (0) \bar{u}_c(x) |\pi(p)\rangle \gamma_\mu U_{\nu'\nu}(0) (0) |\pi(p)\rangle
\]

\[
+ \gamma_5 \gamma_\alpha (0) d_c(0) \bar{u}_c(x) \gamma_\mu U_{\nu'\nu}(0) (0) |\pi(p)\rangle C_{\alpha'\nu'}(0) S_{\nu'}(0) |\pi(p)\rangle \right\}, \quad (16)
\]

\[
\Pi^\mu_\nu(p, q) = \frac{2\sqrt{2}}{\sqrt{3}} i\varepsilon_{abc}c_{\alpha'\nu'} \int d^4xe^{-iq\cdot x}
\left\{ Tr \left[ \gamma_\mu C S_{\nu'}(0) U_{\alpha'\nu'}(0) \right] \gamma_5 \gamma_\alpha (0) d_c(0) \bar{u}_c(x) |\pi(p)\rangle \right.
\]

\[
+ Tr \left[ \gamma_\mu C S_{\nu'}(0) U_{\alpha'\nu'}(0) \right] \gamma_5 \gamma_\alpha (0) |\pi(p)\rangle \gamma_\mu U_{\nu'\nu}(0) (0) \bar{u}_c(x) |\pi(p)\rangle
\]

\[
- \gamma_5 \gamma_\alpha (0) d_c(0) \bar{u}_c(x) \gamma_\mu U_{\nu'\nu}(0) (0) |\pi(p)\rangle \gamma_\mu C S_{\nu'}(0) (0) \bar{u}_c(x) |\pi(p)\rangle
\]

\[
- \gamma_5 \gamma_\alpha (0) d_c(0) \bar{u}_c(x) \gamma_\mu C S_{\nu'}(0) (0) \bar{u}_c(x) |\pi(p)\rangle \gamma_\mu U_{\nu'\nu}(0) (0) |\pi(p)\rangle
\]

\[
- \gamma_5 \gamma_\alpha (0) d_c(0) \bar{u}_c(x) \gamma_\mu U_{\nu'\nu}(0) (0) |\pi(p)\rangle C_{\alpha'\nu'}(0) S_{\nu'}(0) |\pi(p)\rangle \right\}. \quad (17)
\]

Perform the following Fierz re-ordering to extract the contributions from the two-particle and three-particle \(\pi\)-meson light-cone distribution amplitudes respectively,

\[
q_\alpha^a(0) q_\beta^b(x) = -\frac{1}{12} \delta_{ab} \delta_{\alpha\beta} \bar{q}(x) q(0) - \frac{1}{12} \delta_{ab} (\gamma^\mu)_{\alpha\beta} \bar{q}(x) \gamma_\mu q(0)
\]

\[
- \frac{1}{24} \delta_{ab} (\sigma^{\mu\nu})_{\alpha\beta} \bar{q}(x) \sigma_{\mu\nu} q(0) + \frac{1}{12} \delta_{ab} (\gamma^\mu \gamma_5)_{\alpha\beta} \bar{q}(x) \gamma_\mu \gamma_5 q(0)
\]

\[
+ \frac{1}{12} \delta_{ab} (i\gamma_5)_{\alpha\beta} \bar{q}(x) i\gamma_5 q(0) \quad (18)
\]

\[
q_\alpha^a(0) q_\beta^b(x) G_{\lambda^r}(v x) = -\frac{1}{4} \delta_{ab} \bar{q}(x) G_{\lambda^r}(v x) q(0) - \frac{1}{4} (\gamma^\mu)_{ab} \bar{q}(x) \gamma_\mu G_{\lambda^r}(v x) q(0)
\]

\[
- \frac{1}{8} (\sigma^{\mu\nu})_{ab} \bar{q}(x) \sigma_{\mu\nu} G_{\lambda^r}(v x) q(0) + \frac{1}{4} (\gamma^\mu \gamma_5)_{ab} \bar{q}(x) \gamma_\mu \gamma_5 G_{\lambda^r}(v x) q(0)
\]

\[
+ \frac{1}{4} (i\gamma_5)_{ab} \bar{q}(x) i\gamma_5 G_{\lambda^r}(v x) q(0) \quad (19)
\]

and substitute the hadronic matrix elements (such as the \(\langle 0|\bar{u}(x) \gamma_\mu \gamma_5 d(0)|\pi(p)\rangle\), \(\langle 0|\bar{u}(x) g_\sigma g_\gamma 5 G_{ab}(v x) d(0)|\pi(p)\rangle\), \(\langle 0|\bar{u}(x) \sigma_{\mu\nu} \gamma_5 d(0)|\pi(p)\rangle\), etc.) with the correspond-
ing π-meson light-cone distribution amplitudes, finally we obtain the spectral densities at the coordinate space. Once the spectral densities in the coordinate space are obtained, we can translate them into the momentum space with the \( D = 4 + 2\epsilon \) dimensional Fourier transform,

\[
6\sqrt{2}\Pi_\Lambda = \frac{f_\pi}{\pi^2} \int_0^1 du u \phi_\pi(u) \frac{\Gamma(\epsilon)}{(-Q^2)\epsilon} + \frac{3f_\pi m_\pi^2}{4\pi^2} \int_0^1 du u A(u) \frac{\Gamma(1)}{(-Q^2)^1} \\
-4m_s\langle \bar{s}s \rangle f_\pi \int_0^1 du u \phi_\pi(u) \frac{\Gamma(2)}{(-Q^2)^2} \\
+ \frac{2m_s\langle g_s\sigma G \bar{s}s \rangle f_\pi}{3} \int_0^1 du u \phi_\pi(u) \frac{\Gamma(3)}{(-Q^2)^3} \\
-8[\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]f_\pi m_\pi^2 \int_0^1 du u \phi_\pi(u) \frac{\Gamma(2)}{(-Q^2)^2} \\
+ \frac{2[\langle \bar{q}g_s\sigma G q \rangle - \langle \bar{s}g_s\sigma G s \rangle]f_\pi m_\pi^2}{9[m_u + m_d]} \int_0^1 du u \phi_\pi(u) \frac{\Gamma(3)}{(-Q^2)^3} \\
- \frac{m_s f_\pi m_\pi^2}{3\pi^2[m_u + m_d]} \int_0^1 du u \phi_\pi(u) \frac{\Gamma(1)}{(-Q^2)^1} \\
- \frac{2f_\pi}{3} \left( \frac{\alpha_s GG}{\pi} \right) \int_0^1 du u \phi_\pi(u) \frac{\Gamma(2)}{(-Q^2)^2} \\
+ \frac{f_\pi [4\langle \bar{q}q \rangle + \langle \bar{s}s \rangle]}{12} \int_0^1 du \int_0^1 dv \int_0^1 \beta \int_0^1 \alpha \int_0^{1-\alpha_g} d\alpha \int_0^{1-\alpha_u} d\alpha \phi_{3\pi} \left( \alpha_u, \alpha_g, 1 - \alpha_u - \alpha_g \right) \\
\Gamma(2) \left|_{u=\alpha_u+v\alpha_g} \phi_{3\pi} \left( \alpha_u, \alpha_g, 1 - \alpha_u - \alpha_g \right) \right. \\
\left. \right|_{u=\alpha_u+v\alpha_g} \\
-[13(1-3v)\nu + 12(1-2v)\nu A + 12(1-v)\nu A] \left( \alpha_u, \alpha_g, 1 - \alpha_u - \alpha_g \right) \\
+ \frac{6f_\pi m_\pi^2}{\pi^2} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_u \int_0^{1-\alpha_u} d\alpha \frac{\Gamma(1)}{(-Q^2)^1} \left|_{u=\alpha_u+v\alpha_g} \right. \\
\left. \right|_{u=\alpha_u+v\alpha_g} \\
-[\nu + \nu + (1-2v)(\nu A + \nu A)] \left( \alpha, \alpha_g, 1 - \alpha - \alpha_g \right) \\
- \frac{6f_\pi m_\pi^2}{\pi^2} \int_0^1 dv (1-v) \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\beta \int_0^{1-\alpha_u} d\alpha \frac{\Gamma(1)}{(-Q^2)^1} \left|_{u=1-(1-v)\alpha_g} \right. \\
\left. \right|_{u=1-(1-v)\alpha_g} \\
- \frac{6f_\pi m_\pi^2}{\pi^2} \int_0^1 dv (1-v) \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\beta \int_0^{1-\alpha_u} d\alpha \frac{\Gamma(1)}{(-Q^2)^1} \left|_{u=1-(1-v)\alpha_g} \right. \\
\left. \right|_{u=1-(1-v)\alpha_g} \\
\left[ \nu + \nu + (1-2v)(\nu A + \nu A) \right] \left( \alpha, \beta, 1 - \alpha - \beta \right), \quad (20)
\]
\[ 2\sqrt{6}\Pi_{\Sigma} = -\frac{f_{\pi}}{3\pi^2} \int_0^1 duu \phi_{\pi}(u) \frac{\Gamma(\epsilon)}{(-Q^2)^\epsilon} + \frac{f_{\pi} m_{\pi}^2}{4\pi^2} \int_0^1 duu A(u) \frac{\Gamma(1)}{(-Q^2)^1} \]
\[ -\frac{4m_s \langle ss \rangle f_{\pi}}{3} \int_0^1 duu \phi_{\pi}(u) \frac{\Gamma(2)}{(-Q^2)^2} \]
\[ + \frac{2m_s \langle g_s \sigma G \bar{s}s \rangle f_{\pi}}{9} \int_0^1 duu \phi_{\pi}(u) \frac{\Gamma(3)}{(-Q^2)^3} \]
\[ + \frac{8[\langle \bar{q}q \rangle - \langle \bar{s}s \rangle]}{9[m_u + m_d]} \int_0^1 duu \phi_{\sigma}(u) \frac{\Gamma(2)}{(-Q^2)^2} \]
\[ - \frac{2[\langle q \bar{g}_s \sigma G q \rangle - \langle \bar{q}g_s \sigma G s \rangle]}{9[m_u + m_d]} f_{\pi} m_{\pi}^2 \int_0^1 duu \phi_{\pi}(u) \frac{\Gamma(3)}{(-Q^2)^3} \]
\[ + \frac{m_s f_{\pi} m_{\pi}^2}{3\pi^2[m_u + m_d]} \int_0^1 duu \phi_{\pi}(u) \frac{\Gamma(1)}{(-Q^2)^1} \]
\[ - \frac{2f_{\pi}}{9} \frac{\langle \alpha_s G G \rangle}{\pi} \int_0^1 duu \phi_{\pi}(u) \frac{\Gamma(2)}{(-Q^2)^2} \]
\[ + \frac{f_{\pi} [8\langle \bar{q}q \rangle + 11\langle \bar{s}s \rangle]}{12} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_u \Gamma(2) \left|_{u = \alpha_u + \alpha_g} \phi_{3\pi}(\alpha_u, \alpha_g, 1 - \alpha_u - \alpha_g) \right| \]
\[ + \frac{f_{\pi} m_{\pi}^2}{4\pi^2} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_u \int_0^{\alpha_u} d\alpha \Gamma(1) \left|_{u = \alpha_u + \alpha_g} \right| \]
\[ [5(1 - 3v) V_{\perp} + 4(1 - 2v) A_{\parallel} + 4(1 - v) A_{\perp}] \right|_{(\alpha_u, \alpha_g, 1 - \alpha_u - \alpha_g)} \]
\[ + \frac{2f_{\pi} m_{\pi}^2}{\pi^2} \int_0^1 dv \int_0^1 d\alpha_g \int_0^{1-\alpha_g} d\alpha_u \int_0^{\alpha_u} d\alpha \Gamma(1) \left|_{u = \alpha_u + \alpha_g} \right| \]
\[ [V_{\parallel} + V_{\perp} + (1 - 2v)(A_{\parallel} + A_{\perp})] \right|_{(\alpha, \alpha_g, 1 - \alpha - \alpha_g)} \]
\[ - \frac{2f_{\pi} m_{\pi}^2}{\pi^2} \int_0^1 dv(1 - v) \int_0^1 d\alpha_g \int_0^{\alpha_g} d\beta \int_0^{\beta} d\alpha \Gamma(1) \left|_{u = 1 - (1 - \alpha)\alpha_g} \right| \]
\[ [V_{\parallel} + V_{\perp} + (1 - 2v)(A_{\parallel} + A_{\perp})] \right|_{(\alpha, \beta, 1 - \alpha - \beta)} , \quad (21) \]

where \( Q_\mu = q_\mu + up_\mu \) and \( Q^2 = (1 - u)q^2 + u(p + q)^2 - u(1 - u)m_{\pi}^2 \). The \( \epsilon \) is a small positive quantity, after taking the double Borel transform, we can take the limit \( \epsilon \to 0 \).

The light-cone distribution amplitudes \( \phi_{\pi}(u) \), \( \phi_{\sigma}(u) \), \( A(u) \), \( \phi_{3\pi}(\alpha_i) \), \( A_{\parallel}(\alpha_i) \), \( A_{\perp}(\alpha_i) \), \( V_{\perp}(\alpha_i) \) and \( V_{\parallel}(\alpha_i) \) of the \( \pi \) meson are presented in the appendix [21][22][23][24], the nonperturbative parameters in the light-cone distribution amplitudes are scale dependent, in this article, the energy scale is taken to be \( \mu = 1 \text{ GeV} \).

Taking double Borel transform with respect to the variables \( Q_1^2 = -q^2 \) and \( Q_2^2 = -(p + q)^2 \) respectively (i.e. \( \frac{\Gamma[\pi]}{\mu[1-(1-u)Q_1^2+(1-u)Q_2^2]} = \frac{M^{2(2-n)}}{M_1^2 M_2^2} e^{-\frac{(1-u)m_{\pi}^2}{M^2}} \delta(u - u_0) \)), then subtract the contributions from the high
resonances and continuum states by introducing the threshold parameter $s_0$ (i.e. $M^{2n} \rightarrow \frac{1}{F_{0\pi}} \int_{0}^{s_0} ds s^{n-1} e^{-\frac{s}{\Lambda^2}}$), finally we obtain two sum rules for the strong coupling constants $g_\Lambda$ and $g_\Sigma$ respectively,

$$g_\Lambda = \frac{1}{\lambda_\Lambda \lambda_\Sigma} \exp \left\{ \frac{M^2}{M_0^2} + \frac{M_2^2}{M_0^2} - \frac{u_0 (1 - u_0) m_s^2}{M^2} \right\} \left\{ \frac{u_0}{2\pi^2} M^4 E_1(x) f_\pi \phi_\pi(u_0) ight.$$ 

$$- \frac{3u_0}{8\pi^2} M^2 E_0(x) f_\pi m_s^2 A(u_0) + \frac{u_0}{3} \frac{\alpha_s G G}{\pi} f_\pi \phi_\pi(u_0)$$

$$+ 2u_0 m_s \langle \bar{s}s \rangle f_\pi \phi_\pi(u_0) - \frac{u_0 m_s \langle \bar{s}g_s G g_s \rangle f_\pi \phi_\pi(u_0)}{9\left[m_u + m_d\right]}$$

$$+ \frac{4u_0 \langle \bar{q}q \rangle - \langle \bar{s}s \rangle}{24} f_\pi m_s^2 \phi_\sigma(u_0) + \frac{u_0 m_s M^2 E_0(x) f_\pi m_s^2 \phi_\sigma(u_0)}{6\pi^2 \left[m_u + m_d\right]}$$

$$- \frac{u_0}{9M^2 \left[m_u + m_d\right]} \left[ 12(1 - \frac{u_0 - \alpha_u}{\alpha_g}) A_\| + 12(1 - \frac{u_0 - \alpha_u}{\alpha_g}) A_\perp ight]$$

$$- 13(1 - 3 \frac{u_0 - \alpha_u}{\alpha_g}) V_\perp \right] (\alpha_u, \alpha_g, 1 - \alpha_u - \alpha_g)$$

$$- \frac{3}{\pi^2} M^2 E_0(x) f_\pi m_s^2 \left[ \int_{0}^{1-u_0} d\alpha_g \int_{u_0}^{u_0} d\alpha_u \int_{0}^{\alpha_u} d\alpha \right]$$

$$+ \left[ \int_{1-u_0}^{1} d\alpha_g \int_{0}^{1-\alpha_g} d\alpha_u \int_{0}^{\alpha_u} d\alpha \right] \frac{1}{\alpha_g}$$

$$\left[ V_\| + V_\perp + (1 - 2 \frac{u_0 - \alpha_u}{\alpha_g}) (A_\| + A_\perp) \right] (\alpha, \alpha_g, 1 - \alpha - \alpha_g)$$

$$+ \frac{3}{\pi^2} M^2 E_0(x) f_\pi m_s^2 \left(1 - u_0\right) \left[ \int_{1-u_0}^{1} d\alpha_g \frac{1}{\alpha_g^2} \int_{0}^{\alpha_g} d\beta \int_{0}^{1-\beta} d\alpha \right]$$

$$\left[ V_\| + V_\perp - (1 - 2 \frac{1 - u_0}{\alpha_g}) (A_\| + A_\perp) \right] (\alpha, \beta, 1 - \alpha - \beta) \right\} , \quad (22)$$
The input parameters are taken as $m_u = m_d = (0.0056 \pm 0.0016)$ GeV, $f_\pi = 0.130$ GeV, $m_\pi = 0.138$ GeV, $\lambda_3 = 0.0$ (which appears in the coefficient of the

$$
\begin{align*}
g_\Sigma &= \frac{1}{\lambda_3 \lambda_\Sigma} \exp \left\{ \frac{M^2_u}{M^2_1} + \frac{M^2_d}{M^2_2} - \frac{u_0(1 - u_0)m^2_\pi}{M^2} \right\} \left\{ \frac{u_0}{2\pi^2} M^4 E_1(x) f_\pi \phi_\pi(u_0) - \frac{3u_0}{8\pi^2} M^2 E_0(x) f_\pi m^2_\pi A(u_0) + \frac{u_0}{3} \langle \frac{\alpha_s G G}{\pi} \rangle f_\pi \phi_\pi(u_0) \right. \\
&\quad \left. + 2u_0 m_s \langle \bar{s}s \rangle f_\pi \phi_\pi(u_0) \right\} \\
&\quad - \frac{4u_0 \langle \bar{q}q \rangle + \langle \bar{s}s \rangle}{3 [m_u + m_d]} f_\pi m^2_\pi \phi_\pi(u_0) \\
&\quad + \frac{u_0}{3 [m_u + m_d]} \left( \langle \bar{q}g_s \sigma G g \rangle - \langle \bar{s}g_s \sigma G s \rangle \right) f_\pi m^2_\pi \phi_\pi(u_0) \\
&\quad - \frac{3u_0}{8\pi^2} M^2 E_0(x) f_\pi m^2_\pi \int^{u_0}_0 d\alpha_u \int^{1 - u_0}_{u_0 - \alpha_u} d\alpha_g \frac{u_0 - \alpha_u}{\alpha_g^2} \phi_3\pi(\alpha_u, \alpha_g, 1 - \alpha_u - \alpha_g) \\
&\quad - \frac{8\langle \bar{q}q \rangle + 11\langle \bar{s}s \rangle}{8} f_\pi \int^{u_0}_0 d\alpha_u \int^{1 - u_0}_{u_0 - \alpha_u} d\alpha_g \frac{\alpha_g}{\alpha_g^2} \phi_3\pi(\alpha_u, \alpha_g, 1 - \alpha_u - \alpha_g) \\
&\quad - \frac{3u_0}{8\pi^2} M^2 E_0(x) f_\pi m^2_\pi \int^{u_0}_0 d\alpha_u \int^{1 - u_0}_{u_0 - \alpha_u} d\alpha_g \frac{1}{\alpha_g} \\
&\quad \left[ 4(1 - 2\frac{u_0 - \alpha_u}{\alpha_g}) A_{||} + 4(1 - \frac{u_0 - \alpha_u}{\alpha_g}) A_{\perp} + 5(1 - 3\frac{u_0 - \alpha_u}{\alpha_g}) V_{||} \right] (\alpha_u, \alpha_g, 1 - \alpha_u - \alpha_g) \\
&\quad + \frac{3}{\pi^2} M^2 E_0(x) f_\pi m^2_\pi \left[ \int^{1 - u_0}_0 d\alpha_g \int^{u_0}_{u_0 - \alpha_g} d\alpha_u \int^{u_0}_{u_0 - \alpha_g} d\alpha \right] \\
&\quad + \int^{1 - u_0}_1 d\alpha_g \int^{1 - \alpha_g}_{u_0 - \alpha_g} d\alpha_u \int^{\alpha_u}_{0} d\alpha \left[ \frac{1}{\alpha_g} \right] \\
&\quad \left[ V_{||} + V_{\perp} + (1 - 2\frac{u_0 - \alpha_u}{\alpha_g})(A_{||} + A_{\perp}) \right] (\alpha, \alpha_g, 1 - \alpha - \alpha_g) \\
&\quad + \frac{3}{\pi^2} M^2 E_0(x) f_\pi m^2_\pi (1 - u_0) \int^{1 - u_0}_0 d\alpha_g \frac{1}{\alpha_g^2} \int^{\alpha_g}_{0} d\beta \int^{1 - \beta}_{0} d\alpha \left[ V_{||} + V_{\perp} - (1 - 2\frac{1 - u_0}{\alpha_g})(A_{||} + A_{\perp}) \right] (\alpha, \beta, 1 - \alpha - \beta) \right\} , \tag{23}
\end{align*}
\]

where

$$
E_n(x) = 1 - (1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}) e^{-x},
$$

$$
x = \frac{s_0}{M^2}.
$$

3 Numerical result and discussion

The input parameters are taken as $m_u = m_d = (0.0056 \pm 0.0016)$ GeV, $f_\pi = 0.130$ GeV, $m_\pi = 0.138$ GeV, $\lambda_3 = 0.0$ (which appears in the coefficient of the
three-particle light-cone distribution amplitude $\phi_{3\pi}(\alpha_i)$, one can consult Ref. [24] for the definition), $f_{3\pi} = (0.45 \pm 0.15) \times 10^{-2}$ GeV$^2$, $\omega_3 = -1.5 \pm 0.7$, $\omega_4 = 0.2 \pm 0.1$, $a_2 = 0.25 \pm 0.15$, $a_1 = 0$, $\eta_4 = 10.0 \pm 3.0$ [21][22][23][24], $\langle \bar{q}q \rangle = -(0.24 \pm 0.01)$ GeV$^3$, $\langle \bar{ss} \rangle = (0.8 \pm 0.2) \langle \bar{q}q \rangle$, $\langle \bar{q}g_s \sigma Gq \rangle = m_0^2 \langle \bar{q}q \rangle$, $\langle \bar{s}g_s \sigma Gs \rangle = m_0^2 \langle \bar{ss} \rangle$, $m_0^2 = (0.8 \pm 0.2)$ GeV$^2$, $\langle \alpha_G^G \rangle = (0.33$ GeV$)^4$ [14][15], $M_{\Sigma_s} = 1.3828$ GeV, $M_{\Sigma_u} = 1.1926$ GeV, $M_A = 1.1157$ GeV [1], $\lambda_\lambda = (2.7 \pm 0.2) \times 10^{-2}$ GeV$^3$, $\lambda_{\Sigma} = (2.8 \pm 0.2) \times 10^{-2}$ GeV$^3$ and $\lambda_{\Sigma^*} = (3.7 \pm 0.2) \times 10^{-2}$ GeV$^3$ [16][17][18][19].

In this article, we neglect the perturbative $O(\alpha_s)$ corrections to the strong coupling constants $g_{\Sigma^*/\Sigma \pi}$, and take the values of the pole residues $\lambda_{\Sigma}$, $\lambda_{\lambda}$ and $\lambda_{\Sigma^*}$ without perturbative $O(\alpha_s)$ corrections for consistency.

The threshold parameter $s_0$ is chosen to be $s_0 = (3.6 \pm 0.1)$ GeV$^2$ to avoid possible contamination from the contributions of the high resonance states, and it is large enough to take into account the contribution of the decuplet baryon $\Sigma^*$. Although the $P_{13}$ state $\Sigma(1840)$ and the $P_{11}$ states $\Sigma(1770)$, $\Sigma(1880)$ are below the threshold, they have no contaminations due to the mismatch of the isospin and spin [1].

The Borel parameters are chosen as $\frac{M_{\Sigma^*}^2}{M_{\Sigma^*}^2} = \frac{M_{\lambda/\Sigma}^2}{M_{\Sigma^*}^2}$ and $M^2 = \frac{M_{\Sigma^*}^2 M_{\lambda/\Sigma}^2}{M_{\Sigma^*}^2 + M_{\lambda/\Sigma}^2} = (2.2 - 3.2)$ GeV$^2$, in those regions, the value of the strong coupling constants $g_{\lambda}$ and $g_{\Sigma}$ are rather stable with variation of the Borel parameter $M^2$.

The theoretical values of the $a_2$ vary in a large range ($a_2 = 0.10 \sim 0.40$) at the energy scale $\mu = 1$ GeV [24], we can take smaller uncertainty, say 30% (i.e. $a_2 = 0.25 \pm 0.08$), which is the typical uncertainty in the QCD sum rules. The value obtained by Ball, Braun and Lenz with the QCD sum rules is $a_2 = 0.28 \pm 0.08$ [24], which has the typical uncertainty. In this article, we present the results with two sets of parameters, the parameters characterized by $a_2 = 0.25 \pm 0.08$ and $a_2 = 0.28 \pm 0.08$ are denoted as PI and PII respectively, because other parameters have the same values.

In calculation, we observe the main uncertainties come from the two parameters $a_2$ and $\eta_4$, the uncertainty originates from the parameter $\omega_4$ is also considerable, which are shown in Figs.1-3.

The dominant contributions come from the two-particle light-cone distribution amplitudes $\phi_\pi(u)$ and $A(u)$; the contributions from the terms involving the three-particle (quark-antiquark-gluon) light-cone distribution amplitudes are of minor importance, about 7% and 12% of the contribution from the term $\frac{m_0^2}{2\pi^2} M^4 E_1(x) f_{\pi} \phi_{\pi}(u_0)$ for the $g_{\lambda}$ and $g_{\Sigma}$ respectively.

The shapes of the light-cone distribution amplitudes $\phi_{\pi}(u)$ and $A(u)$ have significant impacts on the values of the $g_{\lambda}$ and $g_{\Sigma}$, because only the values of the special point $u = u_0$ are involved. This case is in contrast to the light-cone QCD sum rules for the hadronic form-factors, where the momentum fraction $u$ is integrated out, dependence on the shapes is mild. For example, the $\phi_{\pi}(u)$ has been analyzed with the light-cone QCD sum rules and (non-local condensates) QCD sum rules confronting with the high precision CLEO data on the $\gamma \gamma^* \rightarrow \pi^o$ transition form-factor [25][26][27][28][29][30][31][32], where the $\phi_{\pi}(u)$ is expanded in terms of the
Figure 1: The strong coupling constants $g_\Lambda$ and $g_\Sigma$ with variation of the Borel parameter $M^2$ and the coefficient $a_2$.

Figure 2: The strong coupling constants $g_\Lambda$ and $g_\Sigma$ with variation of the Borel parameter $M^2$ and the nonperturbative parameter $\eta_4$.

Gegenbauer polynomials $C_n^{3/2}(2u - 1)$, truncations at the order $n = 2$ or $n = 4$ both lead to satisfactory results.

The strong coupling constant $g$ can serve as an excellent subject for determining the shapes of the light-cone distributions amplitudes $\phi_\pi(u)$ and $A(u)$, perturbative $O(\alpha_s)$ corrections should be taken into account before confronting with the experimental data. To my knowledge, only the leading order contributions to the strong coupling constants $g_{NNV}$ and $g_{NNP}$ have been calculated with the light-cone QCD sum rules [4, 5, 6, 7], where the $N$, $V$ and $P$ denote the octet baryons, the vector mesons and the pseudoscalar mesons, respectively.

Taking into account all the uncertainties, finally we obtain the numerical results.
Figure 3: The strong coupling constants $g_\Lambda$ and $g_\Sigma$ with variation of the Borel parameter $M^2$ and the nonperturbative parameter $\omega_4$.

for the strong coupling constants $g_\Lambda$ and $g_\Sigma$, which are shown in Figs.4-5,

$$g_\Lambda = (13.6 \pm 4.6) \text{ GeV}^{-1},$$
$$g_\Sigma = (12.9 \pm 4.2) \text{ GeV}^{-1},$$
$$g_N = (13.5 \pm 5.4) \text{ GeV}^{-1},$$

(24)

and

$$g_\Lambda = (12.6 \pm 4.7) \text{ GeV}^{-1},$$
$$g_\Sigma = (11.8 \pm 4.1) \text{ GeV}^{-1},$$
$$g_N = (12.5 \pm 5.5) \text{ GeV}^{-1},$$

(25)

for the parameters PI and P II respectively, here we also present the value of the strong coupling constant $g_{\Delta p\pi}$ with the light-cone QCD sum rules [3]. We calculate uncertainties $\delta$ with the formula $\delta = \sqrt{\sum_i \left(\frac{\partial f}{\partial x_i}\right)^2 (x_i - \bar{x}_i)^2}$, where the $f$ denote the strong coupling constants $g_\Lambda$, $g_\Sigma$ and $g_N$, the $x_i$ denote the input parameters $m_u$, $m_d$, $a_2$, $f_{3\pi}$, · · · . The average values are

$$g = 13.3 \pm 4.7 \text{ GeV}^{-1},$$
$$C = 1.7 \pm 0.6,$$

(26)

and

$$g = 12.3 \pm 4.7 \text{ GeV}^{-1},$$
$$C = 1.6 \pm 0.6,$$

(27)

for the parameters PI and P II respectively.
Figure 4: The strong coupling constants $g_\Lambda$ and $g_\Sigma$ with variation of the Borel parameter $M^2$ for the parameters PI. The uncertainties $\delta$ are calculated with the formula $\delta = \sqrt{\sum_i \left( \frac{\partial f}{\partial x_i} \right)^2 (x_i - \bar{x}_i)^2}$, where the $f$ denote the strong coupling constants $g_\Lambda$ and $g_\Sigma$, the $x_i$ denote the input parameters $m_u$, $a_2$, $f_{3\pi}$, \ldots.

The uncertainties are rather large, larger than 30%, and no definitive conclusion can be drawn for the $SU(3)$ breaking effects. If we take the central values as the input parameters, the $SU(3)$ breaking effects are rather small, less than 6%. The uncertainties may result in larger $SU(3)$ breaking effects, furthermore, we have neglected the perturbative $O(\alpha_s)$ corrections, which may also contribute to the $SU(3)$ breaking effects.

The strong coupling constants $g^*_{\Lambda/\Sigma \pi}$ have the following relation with the decay widths $\Gamma_{\Sigma^\ast \rightarrow \Lambda/\Sigma \pi}$,

$$\Gamma_{\Sigma^\ast \rightarrow \Lambda/\Sigma \pi} = \frac{g^2_{\Sigma^\ast \Lambda/\Sigma \pi} p_{cm}}{32\pi M^2_{\Sigma^\ast}} \sum_{ss'} |U(p', s)p_{\pi}(p'', s')|^2,$$

$$p_{cm} = \frac{\sqrt{[M^2_{\Sigma^\ast} - (M_{\Lambda/\Sigma} + m_{\pi})^2][M^2_{\Sigma^\ast} - (M_{\Lambda/\Sigma} - m_{\pi})^2]}}{2M^2_{\Sigma^\ast}}. \quad (28)$$

If we take the experimental data as the input parameters, $\Gamma_{\Sigma^\ast \rightarrow \Sigma \pi} = 4.19$ MeV, $\Gamma_{\Sigma^\ast \rightarrow \Lambda \pi} = 31.15$ MeV and $\Gamma_{\Delta \rightarrow \pi \pi} = 118.0$ MeV \cite{1}, we can obtain the values $g_\Sigma \approx 17.4$ GeV$^{-1}$, $g_\Lambda \approx 12.8$ GeV$^{-1}$ and $g_N \approx 15.6$ GeV$^{-1}$. The average value is about $g \approx 15.3$ GeV$^{-1}$, and the $SU(3)$ breaking effects are about $(12\% - 18\%)$. The values $\Gamma_{\Sigma^\ast \rightarrow \Sigma \pi} = 4.19$ MeV and $\Gamma_{\Sigma^\ast \rightarrow \Lambda \pi} = 31.15$ MeV are estimated (not fitted or averaged) by the Particle Data Group \cite{1}; more accurate data may result in smaller $SU(3)$ breaking effects.

In the region $M^2 = (2.2 - 3.2)$ GeV$^2$, $\frac{\alpha_s(M^2)}{\pi} \sim 0.10 - 0.12$ \cite{33}. If the radiative $O(\alpha_s)$ corrections to the leading perturbative terms are companied with large numerical factors, just like in the case of the QCD sum rules for the mass of the proton.
Figure 5: The strong coupling constants $g_\Lambda$ and $g_\Sigma$ with variation of the Borel parameter $M^2$ for the parameters P II. The uncertainties $\delta$ are calculated with the formula

$$\delta = \sqrt{\sum_i \left( \frac{\partial f}{\partial x_i} \right)^2 \left( x_i - \bar{x}_i \right)^2}$$

where the $f$ denote the strong coupling constants $g_\Lambda$ and $g_\Sigma$, the $x_i$ denote the input parameters $m_u, a_2, f_3, \cdots$.

$34$, $1 + \left( \frac{53}{12} + \gamma_E \right) \frac{\alpha_s(M)}{\pi} \sim 1 + (0.53 - 0.62)$, the contributions of the order $O(\alpha_s)$ are large. Furthermore, the pole residues $\lambda_\Lambda, \lambda_\Sigma$ and $\lambda_\Sigma^*$ also receive contributions from the perturbative $O(\alpha_s)$ corrections, if they are taken into account properly, we can improve the value of the strong coupling constant $g$.

4 Conclusion

In this article, we calculate the strong coupling constant $g$ among the decuplet baryons, the octet baryons and the pseudoscalar mesons in the heavy baryon chiral perturbation theory with the light-cone QCD sum rules, and study the strong decays $\Sigma^* \to \Lambda \pi, \Sigma \pi$. The numerical value of the strong coupling constant $g$ is consistent with our previous calculation, the central values lead to small $SU(3)$ breaking effects, less than 6%; and no definitive conclusion can be drawn due to the large uncertainties. The perturbative $O(\alpha_s)$ corrections may improve the results further.
Appendix

The light-cone distribution amplitudes of the $\pi$ meson are defined by \[21, 22, 23, 24\]

$$
\langle 0 | \bar{u}(x) \gamma_\mu \gamma_5 d(0) | \pi(p) \rangle = i f_\pi p_\mu \int_0^1 du e^{-iu p \cdot x} \left\{ \phi_\pi(u) + \frac{m_\pi^2 x^2}{16} A(u) \right\} 
$$

$$
+ \frac{i}{2} f_\pi m_\pi^2 \frac{x_\mu}{p \cdot x} \int_0^1 du e^{-iu p \cdot x} B(u) 
$$

$$
\langle 0 | \bar{u}(x) i \gamma_5 d(0) | \pi(p) \rangle = \frac{f_\pi m_\pi^2}{m_u + m_d} \int_0^1 du e^{-iu p \cdot x} \phi_p(u) 
$$

$$
\langle 0 | \bar{u}(x) \sigma_{\mu \nu} \gamma_5 | \pi(p) \rangle = i (x_\mu p_\nu - x_\nu p_\mu) \frac{f_\pi m_\pi^2}{6(m_u + m_d)} \int_0^1 du e^{-iu p \cdot x} \phi_p(u) 
$$

$$
\langle 0 | \bar{u}(x) \gamma_5 g_s G_{\alpha \beta} (v x) d(0) | \pi(p) \rangle = f_3 \pi \{ (p_\mu p_\alpha g_{\nu \beta} - p_\nu p_\alpha g_{\mu \beta}) - (p_\mu p_\beta g_{\nu \alpha} - p_\nu p_\beta g_{\mu \alpha}) \} \int D\alpha_i \phi_{3\pi}(\alpha_i) e^{-ip \cdot x(a_u + v a_g)} 
$$

$$
\langle 0 | \bar{u}(x) \gamma_5 g_s \tilde{G}_{\alpha \beta} (v x) d(0) | \pi(p) \rangle = f_\pi m_\pi^2 p_\mu \frac{p_\alpha x_\beta - p_\beta x_\alpha}{p \cdot x} \int D\alpha_i A_{\parallel}(\alpha_i) e^{-ip \cdot x(a_u + v a_g)} 
$$

$$
+ f_\pi m_\pi^2 (p_\beta g_{\alpha \mu} - p_\alpha g_{\beta \mu}) \int D\alpha_i A_{\parallel}(\alpha_i) e^{-ip \cdot x(a_u + v a_g)} 
$$

$$
\langle 0 | \bar{u}(x) \gamma_5 g_s \tilde{G}_{\alpha \beta} (v x) d(0) | \pi(p) \rangle = f_\pi m_\pi^2 p_\mu \frac{p_\alpha x_\beta - p_\beta x_\alpha}{p \cdot x} \int D\alpha_i V_{\parallel}(\alpha_i) e^{-ip \cdot x(a_u + v a_g)} 
$$

$$
+ f_\pi m_\pi^2 (p_\beta g_{\alpha \mu} - p_\alpha g_{\beta \mu}) \int D\alpha_i V_{\parallel}(\alpha_i) e^{-ip \cdot x(a_u + v a_g)} 
$$

$$
(29)
$$

where $g^{\mu \nu} = g_{\mu \nu} - \frac{p_\mu p_\nu + p_\nu p_\mu}{p \cdot x}$, $G_{\mu \nu} = \frac{1}{2} \epsilon_{\mu \nu \alpha \beta} G^{\alpha \beta}$ and $D\alpha_i = d\alpha_d d\alpha_d d\alpha_d d\alpha_d (1 - \alpha_u - \alpha_d - \alpha_d - \alpha_d)$. The light-cone distribution amplitudes of the $\pi$ meson are parameterized as \[21\]
\[
\begin{align*}
\phi_\pi(u) &= 6u(1-u) \left\{ 1 + a_1 C_1^3(\xi) + a_2 C_2^3(\xi) \right\}, \\
\phi_p(u) &= 1 + \left\{ 30 \eta_3 - \frac{5}{2} \rho^2 \right\} C_2^3(\xi) \\
&\quad + \left\{ -3 \eta_3 \omega_3 - \frac{27}{20} \rho^2 - \frac{81}{10} \rho^2 a_2 \right\} C_4^3(\xi), \\
\phi_\sigma(u) &= 6u(1-u) \left\{ 1 + \left[ 5 \eta_3 - \frac{1}{2} \eta_3 \omega_3 - \frac{7}{20} \rho^2 - \frac{3}{5} \rho^2 a_2 \right] C_2^3(\xi) \right\}, \\
\phi_3 \pi(\alpha_i) &= 360 \alpha_u \alpha_d a^2_g \left\{ 1 + \lambda_3 (\alpha_u - \alpha_d) + \omega_3 \frac{1}{2} (7 \alpha_g - 3) \right\}, \\
V_\parallel(\alpha_i) &= 120 \alpha_u \alpha_d a_g \left( v_{00} + v_{10} (3 \alpha_g - 1) \right), \\
A_\parallel(\alpha_i) &= 120 \alpha_u \alpha_d a_g a_{10} (\alpha_d - \alpha_u), \\
V_\perp(\alpha_i) &= -30 a_g \left\{ h_{00} (1 - \alpha_g) + h_{01} \left[ \alpha_g (1 - \alpha_g) - 6 \alpha_u \alpha_d \right] \\
&\quad + h_{10} \left[ \alpha_g (1 - \alpha_g) - \frac{3}{2} (\alpha_u^2 + \alpha_d^2) \right] \right\}, \\
A_\perp(\alpha_i) &= 30 a_g^2 (\alpha_u - \alpha_d) \left\{ h_{00} + h_{01} \alpha_g + \frac{1}{2} h_{10} (5 \alpha_g - 3) \right\}, \\
A(u) &= 6u(1-u) \left\{ \frac{16}{15} + \frac{24}{35} a_2 + 20 \eta_3 + \frac{20}{9} \eta_4 \\
&\quad + \left\{ -\frac{1}{15} + \frac{1}{16} - \frac{7}{27} \eta_3 \omega_3 - \frac{10}{27} \eta_4 \right\} C_2^3(\xi) \\
&\quad + \left\{ -\frac{11}{210} a_2 - \frac{4}{135} \eta_3 \omega_3 \right\} C_4^3(\xi) \right\} + \left\{ -\frac{18}{5} a_2 + 21 \eta_4 \omega_4 \right\} \\
&\quad \left\{ 2u^3 (10 - 15u + 6u^2) \log u + 2\bar{u}^3 (10 - 15\bar{u} + 6\bar{u}^2) \log \bar{u} \\
&\quad + u\bar{u} (2 + 13u\bar{u}) \right\}, \\
g(u) &= 1 + g_2 C_2^3(\xi) + g_4 C_4^3(\xi), \\
B(u) &= g(u) - \phi_\pi(u),
\end{align*}
\]
where

\[
\begin{align*}
    h_{00} &= v_{00} = -\frac{\eta_4}{3}, \\
    a_{10} &= \frac{21}{8} \eta_4 \omega_4 - \frac{9}{20} a_2, \\
    v_{10} &= \frac{21}{8} \eta_4 \omega_4, \\
    h_{01} &= \frac{7}{4} \eta_4 \omega_4 - \frac{3}{20} a_2, \\
    h_{10} &= \frac{7}{2} \eta_4 \omega_4 + \frac{3}{20} a_2, \\
    g_2 &= 1 + \frac{18}{7} a_2 + 60 \eta_3 + \frac{20}{3} \eta_4, \\
    g_4 &= -\frac{9}{28} a_2 - 6 \eta_3 \omega_3, \\
\end{align*}
\]

(31)

\[\xi = 2u - 1, \text{ and } C_2^1(\xi), C_4^3(\xi), C_1^3(\xi), C_2^3(\xi) \text{ are Gegenbauer polynomials, } \eta_3 = \frac{f_{3u}}{f_\pi} \frac{m_u + m_d}{m_\pi} \text{ and } \rho^2 = \left(\frac{(m_u + m_d)}{m_\pi}\right)^2 21, 22, 23, 24.\]

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