Gabor is Enough: Interpretable Deep Denoising with a Gabor Synthesis Dictionary Prior

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Abstract—Image processing neural networks, natural and artificial, have a long history with orientation-selectivity, often described mathematically as Gabor filters. Gabor-like filters have been observed in the early layers of CNN classifiers and even throughout low-level image processing networks. In this work, we take this observation to the extreme and explicitly constrain the filters of a natural-image denoising CNN to be learned 2D real Gabor filters. Surprisingly, we find that the proposed network (GDLNet) can achieve near state-of-the-art denoising performance amongst popular fully convolutional neural networks, with only a fraction of the learned parameters. We further verify that this parameterization maintains the noise-level generalization (training vs. inference mismatch) characteristics of the base network, and investigate the contribution of individual Gabor filter parameters to the performance of the denoiser. We present positive findings for the interpretation of dictionary learning networks as performing accelerated sparse-coding via the importance of untied learned scale parameters between network layers. Our network’s success suggests that representations used by low-level image processing CNNs can be as simple and interpretable as Gabor filterbanks.

I. INTRODUCTION AND BACKGROUND

In recent years, deep neural network (DNN) and convolutional neural network (CNN) architectures have claimed state-of-the-art performance on low-level image processing tasks such as denoising [1]–[3]. However, most DNNs are constructed as black-box function approximators whose architectures are largely derived from trial and error [4], ignoring the theoretical signal processing backgrounds related to the task at hand. Such networks have been shown to suffer a catastrophic performance failure when a mismatch is present between the training and inference scenarios [5], [6]. More recently, there has been a growing desire for principled approach to DNN design that leads to better analysis and possible improvement of the network [6]–[12]. To this end, in [6] we introduced an architecture (CDLNet) derived from direct parameterization of an iterative algorithm solving the convolutional dictionary learning (CDL) problem. By relating the thresholds in CDLNet to the input image noise-level, this network showed near-perfect generalization at inference to noise-levels unseen during training. Notably, CDLNet achieves similar performance to other state-of-the-art fully convolutional neural networks (FCNNs) [1], [2] while sidestepping the use of feature-domain processing and limiting the representation to only analysis and synthesis convolutional dictionaries. Such directly-parameterized dictionary learning based networks [6], [8], [10], [13] challenge the notion of low-level image processing CNNs capturing complex signal representations in hidden feature domains, and suggest that much simpler mechanisms, such as subband representations, can account for their performance.

Many dictionary learning frameworks for low-level image processing tasks, especially denoising, learn “Gabor-like” dictionaries [6], [8], [14]. Neuroscientific investigations suggest that V1 simple cells in primary visual cortex behave like Gabor filters [15]–[17]. The early visual cortex responds maximally to lines or edges with a specific orientation which can be achieved with Gabor filter with a proper scale, a specific direction and a spatial frequency [18]. More recently, Gabor filters have attracted attention in CNN design, from preprocessing stage [19], to use of Gabor filters in early CNN layer [20], to combining Gabor and learned convolutional filters [21]. Fixed Gabor filters have also been shown to reduce the computational complexity of CNNs, however, with a significant performance reduction [22]. These works overall suggest that use of Gabor filters in CNN design leads to better performance, albeit with a more complex network, or reduced complexity with degraded performance.

The goal of this work is to quantitatively explore the Gabor-like behavior of learned filters in convolutional dictionary based neural networks and pose the question “can Gabor-like be replaced with Gabor?”. We tackle this question by replacing the filters of CDLNet [6] with parameterized 2D Gabor functions. In place of learned filters, this network learns sets of parameters that generate Gabor filters (Section II-B). Surprisingly, we find that this representation is sufficient for achieving near state-of-the-art performance in the natural-image denoising task (Section III-B) with respect to popular FCNNs such as DnCNN [1] and FFDNet [2]. We verify that the near-perfect denoising generalization of CDLNet to unseen noise-levels are retained with this parameterization (Section III-C). Finally, in [6] we demonstrated the importance of untying filter weights between network layers. With the proposed Gabor filterbank representation, we further explore untying of individual parameters of the Gabor function between layers and their contribution to denoising performance (Section III-E).

II. PROBLEM STATEMENT AND PROPOSED METHOD

In this manuscript, we look at the denoising of natural images via convolutional neural networks (CNNs). We denote our observations contaminated with additive Gaussian white noise (AWGN), \( y = x + \nu \), where \( \nu \sim \mathcal{N}(0, \sigma I) \).
A. CDLNet Architecture

The Convolutional Dictionary Learning Network (CDLNet) architecture [6] is a denoising neural network derived from the Iterative Shrinkage Thresholding Algorithm (ISTA) [23]. Much like a traditional CNN, it employs layered convolutions and point-wise nonlinearities, as described below,

\[ \hat{x} = Dz^{(K)}, \quad z^{(0)} = 0, \quad k = 0, 1, \ldots, K - 1, \]

\[ z^{(k+1)} = \text{ST} \left( z^{(k)} - A^{(k)T} (B^{(k)} z^{(k)} - y), \tau^{(k)} \right) \]  

Here, \( A^{(k)T}, B^{(k)} \) refer to \( M \)-subband analysis and synthesis filterbank convolutions, respectively, with stride-\( s \), \( \text{ST}(x, \tau)[n] := \text{sign}(x[n]) \max(0, |x[n]| - \tau) \) is known as the element-wise soft-thresholding operator, and \( \tau^{(k)} \in \mathbb{R}^M \) are the networks learned non-negative thresholds. The final denoised image \( \hat{x} \) is given by convolution with the network’s synthesis filterbank (dictionary), \( D \).

CDLNet’s interpretation as an accelerated sparse-coding algorithm allows for consideration of a varying input noise-level via augmenting the thresholds of the network. In [6], it was shown that the following noise-adaptive thresholding,

\[ \tau^{(k)} = \tau_0^{(k)} + \tau_1^{(k)} \sigma, \quad \text{such that} \quad \tau_0^{(k)} \geq 0, \]

with \( \tau_0^{(k)}, \tau_1^{(k)} \in \mathbb{R}^M \) learned parameters, yields a near perfect generalization to input noise-levels outside the network’s training range at inference time. This behavior is in contrast to the catastrophic failures of black box DNNs observed outside their training ranges [5], [6].

B. Parameterized Gabor Filterbanks

The learned dictionary elements in [6] are “Gabor-like” filters. In this work, we consider the explicit parameterization of CDLNet’s filterbanks (both analysis and synthesis) as learnable 2D real Gabor filters,

\[ g(x; \phi) = \alpha e^{-\frac{\|x-x_0\|^2}{2\sigma^2}} \cos(\omega_0^T x + \psi), \]

with \( x \in \mathbb{R}^2 \) denoting spatial position, \( \phi = (\alpha, \sigma, \omega_0, \psi) \in \mathbb{R}^6 \). We refer to these as \( \alpha \in \mathbb{R} \) the scale, \( \sigma \in \mathbb{R}^2 \) the (root) precision, \( \omega_0 \in \mathbb{R}^2 \) the frequency, and \( \psi \in \mathbb{R} \) the phase parameters. These filters are orientation selective, as seen in their Fourier domain representation of conjugate symmetric Gaussians centered at frequency \( \pm \omega_0 \),

\[ F\{g(\cdot; \phi)\}(\omega) = \frac{\alpha}{2} \left( e^{j\psi} e^{-\frac{\|\omega - \omega_0\|^2}{2\sigma^2}} + e^{-j\psi} e^{-\frac{\|\omega + \omega_0\|^2}{2\sigma^2}} \right). \]

To further enable these filters to admit a sparse image representation, we consider parameterizing the filters of CDLNet as a mixture of Gabor (MoG) filters,

\[ d(x; \phi) = \sum_{i=1}^{S} g(x; \phi_i). \]  

A single MoG filter of order \( S > 0 \) is parameterized by \( \phi \in \mathbb{R}^{6S} \), and a MoG \( M \)-subband analysis/synthesis filterbank is parameterized by \( \Phi = [\Phi_A^{(k)} \cdots \Phi_B^{(k)}] \in \mathbb{R}^{6SM} \). This is in contrast to the unconstrained filters in CDLNet, parameterized in \( \mathbb{R}^{PM} \) for \( \sqrt{P} \times \sqrt{P} \) filters.

The MoG representation allows for a wider class of filters to be learned with the same computational complexity in the forward pass of the network, while maintaining the interpretability of the Gabor parameterization over unconstrained filters. In the digital realization of these Gabor filters, we limit their spatial extent to \( \sqrt{P} \times \sqrt{P} \) and evaluate (3) at lattice points. Interestingly, the spatial extent of these filters has no effect on their parameter count. We refer to the CDLNet architecture with parameterized mixture of Gabor filters as the Gabor Dictionary Learning Network (CDLNet). Its parameters are given by \( \Theta = \{ \Phi_A^{(k)} \cdots \Phi_B^{(k)} \}_{k=0}^{K-1} \), where \( \Phi_A^{(k)}, \Phi_B^{(k)} \) refer to the Gabor parameters of the analysis, synthesis filterbanks of the \( k \)-th layer, and similarly \( \Phi_D \) refers to the Gabor parameters of the synthesis dictionary \( D \) (see (1)). Figure 1 shows an overview of the CDLNet architecture with MoG filterbanks.

III. EXPERIMENTAL RESULTS

A. Training and Inference Setup

We follow the training and inference setup of CDLNet [6]. All models are trained on the BSD432 dataset [24] under supervised learning with the mean-squared-error loss,

\[ \minimize_{\Phi_A^{(k)}, \Phi_B^{(k)}, \Phi_D} \sum_{(x_i, y_i) \in D} \| x_i - Dz^{(K)}(y_i) \|_2^2, \]

where \( (x_i, y_i) \in D \) represent ground-truth and observed images of the dataset. Data-augmentation, as well as additive noise, is applied online. Noise-level \( \sigma \in \sigma^{\text{train}} \) is sampled uniformly for each mini-batch sample. Noise-level adaptive models are given the ground truth noise-level at both training and inference, though common noise-level estimators [25], [26] may be applied instead at inference (see [6] for details).
C. Blind Denoising and Generalization

Architectures: Suffixes (-S, -B) are used to denote models trained on a single noise-level (-S) or over a noise-level range without adaptive thresholds (-B). Unless otherwise specified, table I shows the hyperparameters of our proposed GDLNet. Code available at [27].

Initialization: As in [6], all filterbanks are initialized with the same parameters and subsequently normalized by their spectral norm, \( L = \|D^T D\|_2 \). For the Gabor filters, this normalization takes place via the scale parameter, \( \alpha \leftarrow \alpha / \sqrt{L} \).

Gabor Filters: The Gabor filter parameterization requires that we recompute the filters from their parameters after each gradient update of the optimizer. This offers a minor computational overhead during training, however the filter weights may be computed and stored for faster inference once training is complete.

B. Single Noise-level Denoising

Single noise-level grayscale denoising results, as compared to state-of-the-art popular FCNNs, are shown in Table II. The non-learned BM3D method [28] is given as baseline. GDLNet-S (MoG 1) performs competitively in the low parameter count regime, outperforming CSCNet [8] and small-CDLNet-S [6]. The use of higher order MoG filters in GDLNet-S (MoG 3) allows it to perform on par with DnCNN [1] and FFDNet [2] while using less than half the learned parameters.

| Model                  | Params | Noise-level (σ) |
|------------------------|--------|-----------------|
| BM3D [28]              |        | 30.07 28.57 25.62 |
| CSCNet [8]†            | 6k     | 31.57 29.11 26.24 |
| small-CDLNet-S [6]     | 6k     | 31.60 29.11 26.19 |
| GDLNet-S (MoG 1)       | 6k     | 31.59 29.13 26.21 |
| FFDNet [2]             | 48k    | 31.63 29.19 26.29 |
| DnCNN [1]              | 55k    | 31.72 29.22 26.23 |
| CDLNet-S [6]           | 50k    | 31.74 29.26 26.35 |
| GDLNet-S (MoG 3)       | 188k   | 31.68 29.22 26.30 |

MoG order and filter-size hyperparameters used in Table II are verified in Tables III and IV. An important boost in performance is given by MoG orders \( \geq 1 \), though diminishing returns are observed above order 3. Minimal performance gains are observed as the filter size increases beyond \( 11 \times 11 \).

C. Blind Denoising and Generalization

We consider the blind denoising and generalization scenarios, and compare GDLNet models of different MoG order equipped with adaptive thresholds. In Figures 2a and 2b, we show the performance of the models trained on the noise range \( \sigma^{train} = [1, 20] \) and \( [20, 30] \), respectively, and tested on different noise-levels \( \sigma^{test} \in [5, 50] \) for grayscale images.

The blind denoising version of DnCNN [1], DnCNN-B, and FFDNet trained on \( \sigma^{train} = [01, 20] \) and \( [20, 30] \), are as reported in [6]. The single points on these plots show the performance of GDLNet-S (MoG 3) model with single noise-level training (i.e. \( \sigma^{train} = \sigma^{test} \)). As shown in Figures 2a and 2b, all networks perform closely over the training noise-range. Conversely, we observe the catastrophic failure of models without adaptation when generalizing outside the training noise-level. GDLNet nearly matches the performance of the models trained for a specific noise-level (GDLNet-S) across the test range. We observe a slight improved performance for GDLNet with higher order. Note that the CDLNet model architecture disentangles the noise generalization from the dictionary structure. Consequently, GDLNets of different order show similar generalization capability when equipped with adaptive thresholds. Visual comparisons in Figures 2c and 2d show superior performance of GDLNet compared to other models.

D. Comparison of Learned Dictionaries

Figure 3 shows the final synthesis dictionaries of CDLNet, GDLNet, and GDLNet’s initial dictionary. The use of parameterized Gabor filters adds to the interpretability of GDLNet as we can understand the subband representations simply as (a sum of) orientation features. GDLNet has cleaner looking filters and does not exhibit phase-shifted copies of filters, as in seen CDLNet Fig. 3a. This is because each filter in GDLNet is centered, by construction. As shown in Fig. 3a, dictionary in GDLNet MoG order 3 exhibits complex structures that are not simply a single orientation, showing that the additional flexibility of the MoG representation is
contributing to the performance gain. The random initialization of Gabor parameters \( \phi \) yields a good starting point for the dictionary (Fig. 3d) and the network is free to learn different orientations and precisions as desired (Fig. 3c).

E. Ablation Study of Gabor Filterbanks

The success of CDLNet, in contrast to CSCNet [8] and others [9], [31], is largely attributed to untying of the weights between layers of the network. However, CDLNet and other directly parameterized unrolled networks [7]–[9], [31] are often interpreted as performing an accelerated sparse coding, either via a learned preconditioning and/or learned step-sizes of the original algorithm [32], [33]. Untying the weights between layers makes this interpretation more difficult as the network is able to vary arbitrarily from layer-to-layer.

GDLNet allows us to examine the relationship of weight tying more closely by only coupling certain parameters of the Gabor filters. Figure 4 shows the performance of GDLNet as we progressively untie parameters between layers. While untying the phase parameter \( \psi \) shows increased performance compared to baseline, very significant increase is seen by simply untying the scale parameter \( \alpha \). To the benefit of the sparse-coding interpretation of the network, the scale parameter \( \alpha \) corresponds most closely to having learned step-sizes within the original algorithm. The minimal performance improvement observed when phase and scale are untied further verifies the sparse-coding interpretation of the network. In contrast, untying the phase and frequency alone does not yield as great of an improvement. This can be justified by considering that phase and frequency parameters may have a considerable effect on the overall magnitude of the filter and thus can indirectly effect the implicit learned step-sizes.

IV. CONCLUSION

In this work, we replaced the filters of an unrolled dictionary learning network (CDLNet) with parameterized 2D real Gabor functions. By using a mixture of Gabor filters, we achieved near state-of-the-art denoising performance at a fraction of parameter count and retained the network’s generalization behavior in unseen noise-levels. Progressively untying the Gabor filter parameters allowed us to understand the contribution of each component to denoising performance and strengthened the sparse-coding interpretation of our network. In future, we aim to further investigate the use of Gabor representation in imaging inverse problems. The simplicity of this representation is suited for a better understanding of the network and may also be exploited for novel capabilities.
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