Correlation Structure and Fat Tails in Finance: a New Mechanism.

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Fat tails in financial time series and increase of stocks cross-correlations in high volatility periods are puzzling facts that ask for new paradigms. Both points are of key importance in fundamental research as well as in Risk Management (where extreme losses play a key role).

In this paper we present a new model for an ensemble of stocks that aims to encompass in a unitary picture both these features. Equities are modelled as quasi random walk variables, where the non-Brownian components of stocks movements are led by the market trend, according to typical trader strategies.

Our model suggests that collective effects may play a very important role in the characterization of some significantly statistical properties of financial time series.

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I. INTRODUCTION

One of the most important goals in the application of Physics to Finance and in Risk Management, is to model accurately financial time series. Indeed physicists have become more involved in the study of financial systems and financial time series, among all complex systems, due to the huge amount of available data in electronic format, a situation that makes easier to check paradigms and theories.

On the other hand, practitioners working in finance need to understand financial time series in order to: (i) assess correctly empirical distributions when large events occur, (ii) make a more rational price evaluation of derivative assets in financial market with an emphasis on asset returns. On the other hand, from a theoretical point of view, in recent years an increasing attention has been devoted to the study of ensembles of stocks, we under-estimate the probability of simultaneous losses and consequently underestimate the possible portfolio claims. On the other hand, from a theoretical point of view, in recent years an increasing attention has been devoted to the study of ensembles of stocks traded in the same financial market, with an emphasis on correlation structure characterization.

Starting from this context, we introduce a new model.
for a set of stocks that aims to encompass in a unitary picture some of the most important features of financial time series, that is: time evolution of volatility, fat tails and correlation structure. For this model we show that a super-diffusive process characterizes the time scaling of volatility for intermediate time intervals (with a critical exponent $3/4$) while a standard diffusive regime takes place for long times (par. [III B]). In paragraph [III C] we discuss how fat tails can emerge in the system. In paragraph [III D] a Monte Carlo simulation approach is used to investigate the correlation structure of the model. Finally Section [V] is devoted to discuss briefly some conclusions.

II. THE MODEL

As discussed in the Introduction, one of the most interesting characteristics of financial markets is the increase of cross-correlations when large market events occur. Empirical evidence shows that for large market returns almost 90% of the equities have the same sign as that of the market [21]. In other words during market crisis or running days most of stocks follow the general trend. This suggests that the probability for an individual stock to have a positive (negative) return can be affected by a market increase (decrease). Basing on this consideration we introduce a one step model for an ensemble of $N$ equities, $\{S_i\}_{i=1}^N$, where each stock movement, $\delta S_i = \pm s$, follows a quasi random walk, with a hopping probability, $P_S$, depending on the previous market return. Precisely:

$$\mathcal{P}_{\delta S_i} (M^{(t-\Delta t)}) = \frac{1}{2} + \frac{1}{2} \frac{\delta S_i}{s} g(M^{(t-\Delta t)}) , \quad (1)$$

where:

$$M^{(t-\Delta t)} = \frac{1}{N} \sum_{i=1}^{N} \frac{\delta S_i^{(t-\Delta t)}}{s} , \quad \Delta t$$

is the normalized market return, $\Delta t$ is the time required to complete one step and $g$ is a function such that $| g(M) | \leq 1$. (To simplify the formulas in the following we have chosen $s = 1$ and $\Delta t = 1$.)

The factor $1/2$, in equation (1), represents the usual random walk contribution (i.e. a 50% chance to perform a positive or a negative step). The function $g$ incorporates the typical trader strategy: when market blows up some traders (the bulls) assume that the market will continue to rise (this behaviour is modelled by imposing the following condition on $g$: $\lim_{M \to 0} \frac{dg}{dM} = 1$). On the opposite hand other agents will consider this positive trend a good opportunity to realise a profit by selling their positions (thus we require that $| g(M) | < | M |$ for $M \neq 0$).

Finally, a rising market reflects into an upward general tendency for the stocks belonging to that market (as in crash periods most of equities drop dramatically as the market does); hence $g$ must have the same sign of $M$.

Since for large $N$ the normalized market return, $M$, is expected to be small compare to 1, it turns out that:

(i) the function $g$, satisfying all the above constraints, can be well approximated as:

$$g(M) = M - \gamma M^3 \quad 0 < \gamma \leq 1 ; \quad (3)$$

(ii) $g$, in eq. (1), is indeed a correction respect to the random walk contribution.

Some considerations can be made on the model defined above. First of all we observe that the model (with the choice (3)) contains a symmetry between upward and downward movements. Actually, as pointed out in the literature (see for instance [21]), the increase of correlations for large market events is thought to be asymmetric, with a stronger interlink dependence between equities during crash periods. Indeed, it is very simple to incorporate this feature in our model by considering two different values of $\gamma$: a $\gamma^+$ for upward moves and a lower value, $\gamma^-$, for downward moves. However in order to simplify the analysis we maintain $\gamma^+ = \gamma^-$. Secondly, from a heuristic point of view the model proposed has a strong analogy with a critical Ising model in infinite dimensions, it is therefore not surprising that critical (i.e. power law) behaviours can emerge. Finally, it is clear that the model we have introduced is quite schematic: we treat all the equities on the same foot and apart from correlations induced by the market growth (see par. [III D]), no explicit correlations are considered. However we can expect that a more complete model would maintain the basic features we are going to discuss.

III. RESULTS AND DISCUSSION

In this section we present, relatively to our model, some results regarding fat tails and correlation structures. We resort to analytical calculations (paragraphs [III A] and [III B]) as well as Monte Carlo simulations (par. [III C]).

A. Probability distribution of market returns

The model discussed in the previous section, can be regarded as a system of $N$ random walkers with a hopping probability depending on the previous mean hopping. If we indicate with $P(j, t, M)$ the probability to find a generic walker at site $j$, after a time $t$, when the mean hopping at time $t-1$ is $M$, then the time evolution master equation for $P$ is given by:

$$P(j, t + 1, \tilde{M}) = \sum_{M = -1}^{1} G(M, \tilde{M}) \cdot$$
where:

\[ G(M, \tilde{M}) = \frac{N!}{(N : \frac{1}{2} \tilde{M})! (N : \frac{1}{2} M)!} \cdot [P_{\delta S = s}(M)]^{N \frac{1}{2} \tilde{M}} \cdot [P_{\delta S = -s}(\tilde{M})]^{N \frac{1}{2} M} \]  

(5)

and \( P(j, t, M) \) obeying to the initial condition:

\[ P(j, t = 0, M) = \delta_{j,0} \cdot \delta_{M,0} \]

Interestingly, \( G(M, \tilde{M}) \) can be regarded as a sort of transition matrix from a state characterized by a market return \( M \) to a state \( \tilde{M} \).

The probability distribution of market returns over a time horizon of one step is given by \( P_0(t + 1, M) = \sum_j P(j, t, M) \). From eq. (4), it follows that:

\[ P_0(t + 1, \tilde{M}) = \sum_{M = -1}^{1} G(M, \tilde{M}) P_0(t, M) . \]  

(6)

After a transient period, the solution of eq. (3) converges to a stationary state. In fig. 1, the solution obtained by solving numerically the eq. (3) is presented (solid line)

FIG. 1. The distribution of market returns over a time horizon \( \Delta t \) (i.e. one step) for \( N = 200 \) and \( \gamma = 0.2 \). Solid line: exact result; dotted line: analytical expression (3); points: Monte Carlo simulation.

An analytical solution of eq. (3) can be worked out, in the limit of large \( N \), by approximating the operator \( \tilde{T}(f(M)) \triangleq \sum_M G(M, \tilde{M}) f(M) \) through differential operators. Indeed by using the Stirling formula and considering the Taylor expansion of \( f(M) \) at the second order in \( \delta M = M - \tilde{M} \), it is possible to show that:

\[
\sum_{M = -1}^{1} G(M, \tilde{M}) f(M) \approx f(\tilde{M}) + \frac{\partial}{\partial \tilde{M}} \left[ \gamma \tilde{M}^2 f(\tilde{M}) \right] + \frac{1}{2N} \frac{\partial^2}{\partial \tilde{M}^2} \left[ \left( 1 - \tilde{M}^2 \right) f(\tilde{M}) \right],
\]  

(7)

where the above approximation holds in the limit of large \( N \) and under the following restrictions on the generic function \( f \):

\[
\gamma M^2 \ll 1 \quad \forall M : f(M) \text{ is significantly } \neq 0,
\]  

(8a)

\[
\frac{1}{f(M)} \frac{d^2 f(M)}{dM^2} \ll N.
\]  

(8b)

Setting \( f = P_0 \) and substituting eq. (3) in eq. (3), we obtain a second order differential equation for \( P_0 \); its analytical solution is given by:

\[
P_0(M) = C \left( 1 - M^2 \right)^{\gamma N - 1} e^{\gamma N M^2} \approx C e^{-\frac{N}{M^4 + M^2}},
\]  

(9)

where \( C \) denotes a constant such that \( \sum_M P_0(M) = 1 \).

Interestingly, we observe that the above solution satisfies, for \( N \rightarrow \infty \), both conditions (8). As a definitive check we compare, in figure 1, the probability distribution of market returns obtained with different methodologies, that is: (i) by solving numerically the exact equation (3); (ii) by using the analytical expression (3) and (iii) by resorting to a Monte Carlo simulation. All the three curves are quite close (the analytical and the exact solutions are almost indistinguishable).

It is interesting to compare the analytical solution (3) with the corresponding expression for a set of \( N \) independent equities. In such a case the market return is the sum of \( N \) uncorrelated variables assuming the values \( \pm s \). By applying the central limit theorem, the probability distribution of market variations turns out to be the usual Gaussian function:

\[
P_0^{\text{uncorr}}(M) = \frac{1}{\sqrt{2\pi N}} e^{-\frac{M^2 N}{2}}.
\]  

(10)

As one can realise immediately, the market volatility, \( \sigma_M \), computed from the above probability distribution, scales with \( N \) as \( N^{-\frac{1}{2}} \), in contrast with our model where \( \sigma_M \sim N^{-\frac{1}{4}} \). This means that the inter-dependence among equities has the effect to maintain the market volatility still very large even for large value of \( N \). As a consequence the extreme market price movements in our model happen far more often than would be expected by chance. This characteristic is one of the statistical properties of real financial indexes pointed out in literature [21].

B. Power law in time scaling of volatility

The volatility of a single stock over a horizon \( t \), \( \sigma_S(t) \), (which, in our model, is indeed the same for all equities) is given by:

\[
\sigma_S^2(t) = \sum_j \sum_M f^2(j, t, M).
\]

By using the eq. (3), it turns out that \( \sigma_S(t) \) satisfies the following finite-difference equation:
\[ \sigma_S^2(t + 1) - \sigma_S^2(t) = 1 + 2 \cdot \sum_{M=-1}^{1} g(M) \cdot P_1(t, M), \]

(11a)

where \( P_1(t, M) \overset{def}{=} \sum_j j P(j, t, M) \) must obey to:

\[ P_1(t + 1, M) = \sum_{M=-1}^{1} G(M, \tilde{M}) P_1(t, M) + \tilde{M} \tilde{P}_0(\tilde{M}), \]

(11b)

with the initial condition \( P_1(t = 0, M) = 0 \).

Note that if we substitute to the matrix \( G(M, \tilde{M}) \) its approximated expression (7) and represent the finite difference of market trend on stock returns. If this last term has the first contribution represents the usual diffusive process while the second term is originated by the influence of market trend on stock returns. If this last term has a power law time dependence \( t^\delta \) (\( \delta > 0 \)) then the stock volatility is characterized by a super-diffusive process.

In order to investigate the time scaling behaviour of equations (11a), we set \( P_1(t, M) = h(t, M) \cdot \tilde{P}_0(M) \). Substituting (8) into (11), the equation for \( h \) reads:

\[ \frac{\partial h}{\partial t} = -\gamma M^3 \frac{\partial h}{\partial M} + \frac{1}{2N} (1 - M^2) \frac{\partial^2 h}{\partial M^2} + M. \]

(12)

Interestingly, we note that the analysis of the finite-difference version of eq. (12) shows that \( P_1 \) can be always expressed as a finite sum of the form \( P_1(t, M) = \sum_{k=1}^{i} a_{k,t} M^{2k-1} \tilde{P}_0(M) \). Now it is simple to prove that \( M^{\delta} \tilde{P}_0(M) \) fulfil the conditions (8) only for \( \sqrt{n/\gamma} \ll 1 \) and therefore we can expect that \( P_1 \) satisfies the relations (8) only in the range of times \( t \) such that \( \sqrt{T/N} \ll 1 \). Hence the Partial Differential Equation (PDE) (12) can be regarded as a good approximation for the evolution of \( h \) only for \( t \ll N/\gamma \). As a result the time evolution of \( \sigma_S(t) \) is characterized by three regimes:

(i) \( \sqrt{\frac{N}{\gamma}} \ll t \ll \frac{N}{\gamma} \), \( h(t, M) \) satisfies the PDE (12). The long time scaling behaviour of \( h \) is investigated in Appendix A.

(ii) \( t \) comparable to \( \frac{N}{\gamma} \). The PDE (12) fails in describing the time evolution of \( h \). The numerical solution of equation (11b) shows that \( P_1(t, M) \) converges towards a stationary state (see fig. 2). As a consequence \( \sigma_S(t) \sim t^{\frac{1}{2}} \) and therefore the super-diffusive scaling behaviour breaks down;

(iii) an intermediate regime between the two, where volatility scales faster than \( t^{\frac{1}{2}} \) but without a sharp power law behaviour.

The behaviour reported above for the time evolution of volatility is indeed typical of financial time series (see [1], par. 7.1).

C. Fat tails

Using the above results we can have some insights on the presence of fat tails in the probability distribution of stock price returns. Indeed, we can expect that for \( N \) sufficiently large, most of the time the market growth, \( M \), is relatively small (from eq. (9)) \( \lim_{N \to \infty} P_0(M) = 0 \) and therefore the majority of stocks follow a geometric Brownian motion \( (P_{GS}(M) \approx 1/2) \). On the other hand during extreme market movements (crashes or running days), the walkers have a higher probability, respect to a simple random walk, to move far from the origin (extreme events). Therefore is reasonable to imagine that the central part of the stock price variations distribution is approximately near to a standard Gaussian distribution while for the tails a fatter non-Gaussian shape is expected. Indeed the results of subsection [11b] show that the stock volatility grows faster than the Brownian term \( \sqrt{t} \) (at least until \( t \) is of order \( N/\gamma \)). This is coherent with
the hypothesis of fat tails in the distribution of stock returns, which indeed provide an extra-contribution to the volatility growth. Since the time evolution of volatility reaches a standard diffusive behaviour for $t$ comparable to $N/\gamma$ (as indicated by the numerical simulation reported in fig. 2) we can suppose that also fat tails would disappear on long time scales, reducing the process to a pure Gaussian stochastic random walk. This is in accordance with the empirical evidence observed in financial time series, where a crossover between a leptokurtic distribution for short times and a Gaussian behaviour for long time intervals takes place.

D. Cross-correlations during turmoil

As we have seen in the introduction, ensembles of stocks are characterized by an increase of correlations during critical periods. This behaviour is qualitatively well reproduced within our model. In figures 3 and 4 some Monte Carlo results are given.

FIG. 3. Conditional probability for a stock to have the same sign of the market return for different values of the time horizon: $t = 10; 100$ (curve A and B). In the figure is also reported the probability distribution corresponding to a set of independent stocks over a time horizon $t = 100$ (curve C). The Monte Carlo results are obtained considering 200 samples, $N = 200$ and $\gamma = 0.2$. On the x-axis the market returns (over a time $t$) are measured in unit of market standard deviation (over a horizon $t$).

Specifically, in fig. 3, we present the conditional probability for stocks to have the same sign as that of the market. The results show a strong sign dependence as a function of the market return. Interestingly, that dependence increases further considering larger time horizons (compare curve A and B in fig. 3). On the other hand, for a set of uncorrelated stocks, the sign conditional probability is substantially weaker (see fig. 3) and remains stable increasing time.

The above results are consistent with the empirical obser-

vations reported by Bouchaud et al [21]. They consider a set of 450 U.S. equities, obtaining the same shape of fig. 3, with a 90% of the stocks have the same sign as that of the market return for very large market movements. Figure 4 clearly shows that correlations between stocks are enforced during high-volatility periods. Indeed in a one step model for an ensemble of $N$ stocks, it is always true that the mean stocks cross-correlation, $\rho$, is related to market variance, $\sigma^2_M = \langle M^2 \rangle - \langle M \rangle^2$ (over a horizon of one step), through the relation: $\rho \approx \sigma^2_M - \frac{1}{N}$. However in our model the spreading of market volatility is enormously accentuated respect to the uncorrelated case (see fig. 4 for a comparison). As a consequence, in our model, we have effectively periods characterized by high volatility and strong cross-correlations alternated to normal market activities.

IV. SUMMARY AND CONCLUSIONS

In this paper we have proposed a new model that aims to capture some of most puzzling aspects of statistical properties of financial time series. Specifically, we have focalsied our attention on two open problems: (i) leptokurtic behaviours in the probability distribution of stock returns and (ii) inter-dependence between market volatility and correlation structure when an ensemble of equities is considered. Both aspects cannot be taken into account by the standard approach commonly used in finance to describe correlated risk factors (that is the covariate Gaussian model).

We have introduced a new model that encompasses in a coherent picture both points (i) and (ii). The mechanism we have proposed is based on the interplay between equities movements and market trend.

The results obtained suggest two considerations: (i) fat tails and correlation structure of extreme returns could
be indeed strictly connected; (ii) an ensemble of stocks can be regarded as a “critical system”, where the collective inter-dependences among equities play a key role.

Our analysis may be of interest, from a theoretical point of view, in clarifying statistical properties of ensembles of stocks and, from an applied perspective, in improving portfolio market risk estimations.

APPENDIX A:

This short appendix is devoted to determine the long time scaling behaviour of the solution of PDE (12), satisfying the initial condition: \( h(M, t = 0) = 0 \).

For \( 1/N = 0 \) a close solution of equation (12) can be worked out:

\[
h^{(0)}(M, t) = \sqrt{\frac{2}{\gamma}} \left| \frac{M}{M} \right| \left( \sqrt{t + \frac{1}{2\gamma M^2}} - \sqrt{\frac{1}{2\gamma M^2}} \right).
\]

(A1)

On the other hand, although for finite \( N \) an exact solution cannot be found, it is still possible to investigate the behaviour of \( h \) in the long time regime. This is done by considering a perturbative solution of equation (12) respect to parameter \( \frac{1}{\gamma} \): \( h(M, t) = \sum_j (\frac{1}{\gamma})^j h^{(j)}(M, t) \).

It is a matter of algebra to show that each function \( h^{(j)} \) contains only terms of the form \( [t + 1/(2\gamma M^2)]^{\frac{j}{2} - k} \), \( 0 \leq k \leq 2j \); this suggests to group together equal terms and look for a solution of the form:

\[
h(M, t) = \frac{M}{|M|} \sum_{k=0}^{\infty} f_k(M) \cdot \left[ \left( t + \frac{1}{2\gamma M^2} \right)^{\frac{j}{2} - k} - \left( \frac{1}{2\gamma M^2} \right)^{\frac{j}{2} - k} \right].
\]

(A2)

It turns out that each function \( f_k \), which indeed depends on \( N \), satisfies the recursive equation:

\[
-2\gamma N \frac{M^3}{1-M^2} \frac{df_k}{dM} + \frac{d^2f_k}{dM^2} + \left( \frac{3}{2} - k \right) \left[ \frac{3}{\gamma M^4} f_{k-1} - \frac{2}{\gamma M^3} \frac{df_{k-1}}{dM} \right] + \left( \frac{3}{2} - k \right) \left( \frac{5}{2} - k \right) \frac{1}{\gamma M^6} f_{k-2} = 0,
\]

(A3)

supplemented with the global condition:

\[
\sum_{k=0}^{\infty} (2\gamma)^{k+\frac{1}{2}} \left( \frac{1}{2} - k \right) M^{2k} f_k(M) = 1.
\]

(A4)

It is possible to demonstrate the following statements about \( \{f_k\} \):

(i) \( f_0 \) must be a constant;

(ii) for any given \( M \) and \( k \geq 1 \): \( \lim_{N \to \infty} f_k(M) = 0 \);

(iii) for any given \( M \): \( \lim_{N \to \infty} f_{2k+1}(M)/f_{2k}(M) = 0 \);

(iv) each \( f_k \) is a regular function apart from a divergence in \( M = 0 \): \( f_k \sim M^{-2k} - k \geq 1 \). (Hence for \( M \to 0 \) all the terms in eq. (A2) behave like \( 1/M \))

Basing on the above properties, it is straightforward to argue that the long time scaling behaviour of the solution of PDE (12) is given by the dominant term \( f_0 \) (i.e. the term corresponding to \( N = \infty \) apart from a multiplicative coefficient), even for small values of \( M \). In the language of Renormalization Group (RG), we say that the term in eq. (12) proportional to \( 1/M \) is irrelevant. (An overview of the application of RG ideas to PDEs is given in [20].)

To summarise, for large times compare to \( 1/\gamma M^2 \), the solution of PDE (12) behaves like \( h^{(0)}(M, t) \). Hence for \( t \gg 1/\gamma M^2 \), we have that \( h(M, t) \sim t^2 \) and a power law behaviour emerges in the model.

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