Radiative contribution to the effective potential in a composite Higgs model

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1. The Promised Land: SU(5)/SO(5) sigma model via SU(4) gauge theory (Ferretti 1404.7137)

2. Latticeland: SU(4) gauge theory, $N_f = 2$ sextets $\rightarrow$ global SU(4) $\rightarrow$ SO(4)

3. First ingredient of the Higgs potential via $\Pi_{LR}(q^2)$
   — much in common with $\pi^\pm - \pi^0$ EM mass splitting in QCD (JLQCD 2008)
COMPOSITE HIGGS – BASICS

The aim: a light Higgs boson, protected *naturally* from high-energy scales

- **Hypercolor**: new strong sector with scale \( f \gg v \)

- Hypercolor theory has spontaneous symmetry breaking: Goldstone bosons will include the Higgs multiplet so \( m_h = 0 \) and in fact \( V(h) = 0 \)

- Couple to gauge bosons/fermions of SM, generate

\[
V_{\text{eff}}(h) = (\alpha - 4\beta)(h/f)^2 + O(h^4)
\]

  - gauge bosons: \( \alpha = (3g^2 + g'^2)C_{LR} \) \( > 0 \) (Witten 1983) today’s talk
  - top quark: \( \beta = -(y_t^2/2)C_{\text{top}} \) probably \( > 0 \) [S.E.P.]

- If \( 4\beta > \alpha \) then we have EWSB (and maybe \( v = \sqrt{2}\langle h\rangle \ll f \))
THE MODEL

\textbf{(Ferretti 2014)}

Wanted: $\text{SU}(5)/\text{SO}(5)$ sigma model for Goldstone bosons $\Rightarrow \text{SO}(5) \supset [\text{SU}(2)_L \times \text{SU}(2)_R]_{EW}$

$\rightarrow$ demands \textbf{real} representation of hypercolor for fermions

$\rightarrow$ restricts color group severely if asymptotically free

Solution: $\text{SU}(4)$ gauge theory with \textbf{sextet} fermions $\rightarrow$ \textbf{a real} representation

With $N_f$ Dirac flavors we would have $\text{SU}(2N_f) \rightarrow \text{SO}(2N_f)$;

$\quad$ but with 5 \textbf{Majorana} fermions we can have $\text{SU}(5) \rightarrow \text{SO}(5)$ as desired.

THE LATTICE MODEL (for today)

\[ N_f = 2 \] Dirac fermions: $\text{SU}(4) \rightarrow \text{SO}(4)$

\textbf{(BONUS: add} $\square$ for partially composite top quark --- \textit{see next talk by W. Jay})
CALCULATING THE HIGGS POTENTIAL — gauge contribution

\[ C_{LR} = \int_0^\infty dq^2 q^2 \Pi_{LR}(q^2) \]

where

\[ (q^2 \delta_{\mu\nu} - q_\mu q_\nu) \Pi_{LR}(q^2) = -\int d^4x e^{iqx} \langle J^L_\mu(x) J^R_\nu(0) \rangle \]

Analytical guess — saturate with lowest resonances ("Minimal Hadron Approx"): \[
\Pi_{LR}(q^2) \approx \frac{f^2}{q^2} - \frac{f^2_\rho}{q^2 + m^2_\rho} + \frac{f^2_{a_1}}{q^2 + m^2_{a_1}}
\]

LATTICE CALCULATION of \( C_{LR} \)

- Dynamical fermions: Wilson–clover fermions, nHYP smearing with suppression of dislocations
- Currents calculated from propagators of overlap fermion kernel with valence mass \( m_\nu \neq 0 \)
- Take chiral limit \( m_\nu \to 0 \) — currents satisfy exact chiral Ward identities, give \( C_{LR} \).
LATTICE CALCULATION

- $\Pi_{LR}$ is the transverse part of

\[
\frac{1}{2} \delta_{ab} \Pi_{\mu\nu}^{\text{lat}}(q) = - \sum_x e^{iqx} \langle J^L_{\mu a}(x) J^R_{\nu b}(0) \rangle
\]

- Direct summation:

\[
C_{LR}(m_v) = \frac{16\pi^2}{V} \sum_{q\mu} \Pi_{LR}(q_{\mu})
\]

while modeling pole at $q = 0$ via

\[
\Pi_{LR}(q_{\mu}) \simeq p + \frac{f^2}{q^2}
\]

- The constant we want is $\lim_{m_v \to 0} C_{LR}(m_v)$.

- Discrepancy between the two ensembles . . .
LATTICE CALCULATION

- A hint: Integrating the *Minimal Hadron Approx* (in the chiral limit*) gives

\[ C_{LR} \approx f_\pi^2 \frac{m_{a_1}^2 m_\rho^2}{m_{a_1}^2 - m_\rho^2} \log \left( \frac{m_{a_1}^2}{m_\rho^2} \right), \]

i.e., \( C_{LR} \propto f_\pi^2 \).

* via the Weinberg sum rules

(two ensembles — two *sea* actions):
LATTICE CALCULATION

- A hint: Integrating the *Minimal Hadron Approx* (in the chiral limit*) gives
  
  \[ C_{LR} \approx f_\pi^2 \frac{m_{a_1}^2 m_\rho^2}{m_{a_1}^2 - m_\rho^2} \log \left( \frac{m_{a_1}^2}{m_\rho^2} \right), \]

  i.e., \( C_{LR} \propto f_\pi^2 \).

- so look at the ratio \( C_{LR}/f_\pi^2 \)

*(two ensembles — two sea actions):*

\[ C_{LR}/f_\pi^2 \]

\[ m_\nu \]

---

* via the Weinberg sum rules
ALTERNATIVE PATH to $C_{LR}$ — Fit to the MINIMAL HADRON formula, integrate $\int dq^2 q^2 \Pi_{LR}$

$$\Pi_{LR}(q^2) \approx \frac{f^2_\pi}{q^2} - \frac{f^2_\rho}{q^2 + m^2_\rho} + \frac{f^2_{a_1}}{q^2 + m^2_{a_1}} \quad (5 \text{ free parameters})$$

Ray along one axis:

Diagonal ray:
Result of $\int dq^2 q^2 \Pi_{LR}$:

- Consistent with direct summation in the chiral limit (hatched points).
- Individual fits: Excellent $\chi^2$ with $MHA$ for $m_v$ not too large.
- $f_\pi$ consistent with spectroscopic value used above. $\rho$, $a_1$ not so much.
- We have studied systematics of the fit/integration procedures.
- Consistent results for $C_{LR}/f_\pi^2$ between ensembles, but no continuum limit yet.
SUPPLEMENTAL
CALCULATING THE HIGGS POTENTIAL — gauge contribution

Gauge tadpole on Higgs propagator

becomes in the gauge theory

where “+” is a vertex to the GB. (Hyperglue not shown.)

In the chiral Lagrangian for the GB’s, all \( n \)-GB amplitudes are related — including the zero-GB amplitude!

SO

\[
C_{LR} = \int_0^\infty dq^2 q^2 \Pi_{LR}(q^2)
\]

where

\[
(q^2 \delta_{\mu\nu} - q_\mu q_\nu) \Pi_{LR}(q^2) = - \int d^4 x e^{i q x} \langle J_\mu^L(x) J_\nu^R(0) \rangle
\]