An adiabatic transport of Bose–Einstein condensates in double-well traps

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Abstract
A complete adiabatic transport of Bose–Einstein condensate (BEC) in a double-well trap is investigated within the mean field approximation. The transfer is driven by the time-dependent (Gaussian) coupling between the wells and their relative detuning. The protocol successfully works in a wide range of both repulsive and attractive BEC interaction. The nonlinear effects caused by the interaction can be turned from detrimental into favourable for the transport. The results are compared with familiar Landau–Zener scenarios using the constant coupling. It is shown that the pulsed Gaussian coupling provides a new transport regime where coupling edges are decisive and convenient switch of the transport is possible.

1. Introduction
Nowadays the trapped Bose–Einstein condensate (BEC) is widely recognized as a source of new fascinating physics, see monographs [1, 2] and early [3–5] and last [6, 7] reviews. Between diverse directions of this field, a large attention is paid to dynamics of weakly bound condensates (or multi-component BEC) with emphasis to nonlinear effects caused by interaction between BEC atoms. The studies cover different aspects of tunnelling in a boson Josephson junction. Starting since early theoretical publications for two-well systems [8–10] and subsequent experiments [11], where main tunnelling regimes are explored, numerous new dynamical scenarios are proposed: nonlinear Landau–Zener tunnelling in double-well (two-level) [12–14] and triple-well [15] traps, Rosen–Zener tunnelling [16], asymmetric nonlinear tunnelling in optical lattices [6, 17–19], adiabatic transport in triple-well traps [14, 20–24], tunnelling driven by time-dependent modulation of the traps [25, 26], Rabi switch [27], population of topological states [28, 29], and many others. Both multilevel or multi-well systems are considered. In the first case, BEC is confined in a single-well trap and contains atoms in a few hyperfine levels, thus forming BEC components. The laser light can couple the components and initiate various regimes of the population transfer. In the second case, BEC atoms occupy the same hyperfine state but the trap is separated by laser-produced barriers into a series of weakly bound wells. The atoms can tunnel through the barriers and exhibit similar effects as in a multi-level system. In this case, BEC components are represented by populations of the wells.

The present work is devoted to complete irreversible transport of BEC in a two-well trap. We will consider the process when BEC, being initially in one of the wells, is then completely transferred in a controllable way to the target well and kept there. Such a transport can be realized in multi-well traps [11] and arrays of selectively addressable traps [30]. Being useful for a general manipulation of the condensate, the irreversible transport can also open intriguing prospects for producing new dynamical regimes [22], investigation of topological states [28, 29] and generation of various geometric phases [22, 31]. The latter is especially important since geometric phases are considered as promising information carriers in quantum computing [32–34].

In general, the BEC transport can be designed by various ways: Rabi switch [27], time-dependent potential modulation [25], Rosen–Zener (RZ) method [16] and adiabatic population...
transfer [14, 20–24]. Adiabatic methods seem to be especially effective for the transport because of their robustness to modest variations of the process parameters [35–37]. In particular, the stimulated rapid adiabatic passage (STIRAP) [38] is widely used for this aim [14, 20–24]. This method promises robust and complete population transfer of the ideal condensate but suffers from nonlinearity effects caused by interaction between BEC atoms. Then a natural question arises, is it possible to turn the nonlinearity from detrimental to favourable factor of adiabatic transport?

In this respect, the study of nonlinear Landau–Zener (LZ) tunnelling of BEC in the systems like two-band accelerated optical lattices give a useful and promising message [6, 18, 19]. It was shown that in such systems the tunnelling is asymmetric, i.e. it can be considerably enhanced or suppressed by the nonlinearity, depending on its sign. This means that the LZ scheme [39] can serve as a good basis for a nonlinear transport of BEC.

The present paper is devoted to the realization of this idea for an adiabatic transport of interacting BEC between two potential wells. In addition to the familiar LZ case with a constant coupling $\Omega$ between the wells [39], we will inspect the nonlinear transport with time-dependent coupling $\Omega(t)$ of a Gaussian form. Actually this is a generalization of both LZ (constant coupling and linear bias of diabatic energies) [39] and RZ (time-dependent coupling and constant diabatic energies) [40] methods. Such generalization is partly motivated by Stark chirped rapid adiabatic passage (SCRAP) method [41]. Being based on the LZ protocol with a pulsed laser coupling, SCRAP turned out to be quite effective in atomic physics. Note also that a constant coupling is somewhat artificial and its time-dependent version is more realistic.

As shown below, the nonlinear transfers with constant and time-dependent couplings have much in common (e.g. asymmetric impact of nonlinearity, determined by the interaction sign) and both of them can result in an effective adiabatic transport at strong nonlinearity. However, the time-dependent (pulsed) protocol is more flexible and has interesting peculiarities. For example, we get a new transport regime where the coupling edges play a decisive role. Besides, the pulsed coupling allows us to switch on/off the transport in much wider interval of the detuning rate $\alpha$ and control the switch value $\alpha_s$ through the coupling width $\Gamma$.

In the present paper, these peculiarities are studied by using the stationary spectra, in particular, the loops arising there due to the nonlinearity. Although the transport problem is not stationary and we actually deal with time-dependent Gross–Pitaevskii equation [42], such analysis is known to be very instructive [12, 17, 18]. It gives a clear intuitive treatment of quite complicated processes and allows for getting simple analytical estimations.

The paper is outlined as follows. In section 2, the mean-field formalism is outlined. In section 3, we present the transport protocol. In section 4, the results of the calculations are discussed. The summary is done in section 5.

2. Model

2.1. General formalism

The calculations have been performed in the mean-field approximation by using the nonlinear Schrödinger, or Gross–Pitaevskii equation [42]

$$i\hbar \dot{\Psi}(\vec{r}, t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + V_{ext}(\vec{r}, t) + g_0|\Psi(\vec{r}, t)|^2 \right] \Psi(\vec{r}, t),$$

where the dot means the time derivative, $\Psi(\vec{r}, t)$ is the classical order parameter of the system, $V_{ext}(\vec{r}, t)$ is the external trap potential involving both (generally time-dependent) confinement and coupling, $g_0 = 4\pi a/m$ is the parameter of interaction between BEC atoms, $a$ is the scattering length and $m$ is the atomic mass.

The order parameter of BEC in a double-well trap can be expanded as [9]

$$\Psi(\vec{r}, t) = \sqrt{N}(\psi_1(t)\Phi_1(\vec{r}) + \psi_2(t)\Phi_2(\vec{r})),$$

where $\Phi_k(\vec{r})$ is the static ground-state solution of (1) for the isolated kth well ($k = 1, 2$) and

$$\psi_k(t) = \sqrt{N_k(t)} e^{i\phi_k(t)}$$

is the corresponding amplitude expressed via the relative population $N_k(t)$ and phase $\phi_k(t)$. The total number of atoms $N$ is fixed

$$\int d\vec{r} |\Psi(\vec{r}, t)|^2/N = N_1(t) + N_2(t) = 1. \quad (4)$$

Substituting (2) into (1) and integrating out the spatial distributions $\Phi_k(\vec{r})$ we get [9]

$$i\dot{\psi}_1 = [E_1(t) + UN|\psi_1|^2]\psi_1 - \Omega(t)\psi_2, \quad (5)$$

$$i\dot{\psi}_2 = [E_2(t) + UN|\psi_2|^2]\psi_2 - \Omega(t)\psi_1, \quad (6)$$

where

$$\Omega(t) = -\frac{1}{\hbar} \int d\vec{r} \left[ \frac{\hbar^2}{2m} \nabla\Phi_1^* \cdot \nabla\Phi_2 + \Phi_1^* V_{ext}(t) \Phi_2 + \Phi_1^* \Phi_2 V_{ext}(t) \Phi_1^* \Phi_2 \right]$$

is the coupling between BEC fractions,

$$E_k(t) = \frac{1}{\hbar} \int d\vec{r} \left[ \frac{\hbar^2}{2m} |\nabla\Phi_k|^2 + \Phi_k^* V_{ext}(t) \Phi_k \right]$$

is the depth of the kth well, and

$$U_k = \frac{g_0}{\hbar} \int d\vec{r} |\Phi_k|^4$$

labels the interaction between BEC atoms in the same well. Here we assume that $U_1 = U_2 = U$. The values $\Omega(t)$, $E_k(t)$ and $U$ have the dimension of frequency.

In the present study, we use a Gaussian coupling with an amplitude $K$

$$\Omega(t) = K \tilde{\Omega}(t), \quad \tilde{\Omega}(t) = \exp \left\{ -\frac{1}{2\Gamma^2}(t - \bar{t})^2 \right\},$$

where $\bar{t}$ and $\Gamma$ are centroid and width parameters. Then scaling (5) and (6) by $1/(2K)$ gives

$$i\dot{\psi}_1 = [\bar{E}_1(t) + \Lambda|\psi_1|^2]\psi_1 - \frac{1}{2}\tilde{\Omega}(t)\psi_2, \quad (10)$$

$$i\dot{\psi}_2 = [\bar{E}_2(t) + \Lambda|\psi_2|^2]\psi_2 - \frac{1}{2}\tilde{\Omega}(t)\psi_1, \quad (11)$$
\[ i \psi_2 = [\tilde{E}_2(t) + \Lambda |\psi_2|^2] \psi_2 - \frac{1}{2} \Omega(t) \psi_1, \]  

where \[ \tilde{E}_k(t) = \frac{E_k(t)}{2K}, \quad \Lambda = \frac{UN}{2K} \]  

and the scaled time, \( 2Kt \rightarrow t \), is dimensionless. In (11) and (12), the key parameter \( \Lambda \) is responsible for the interplay between the coupling and the interaction.

Substituting (3) into (11) and (12) leads to the system of equations for the populations \( N_k(t) \) and phases \( \phi_k(t) \):

\[ \dot{N}_k = -\tilde{\Omega}(t) \sqrt{N_j N_k} \sin(\phi_j - \phi_k), \]  

\[ \dot{\phi}_k = -[\tilde{E}_k(t) + \Lambda N_k] + \frac{1}{2} \tilde{\Omega}(t) \sqrt{\frac{N_j}{N_k}} \cos(\phi_j - \phi_k) \]  

with \( j \neq k \). Considering \( N_1 \) and \( -\phi_k \) as conjugate variables and using the linear canonical transformation [22]

\[ z = N_1 - N_2, \quad Z = N_1 + N_2 = 1, \]  

\[ \theta = \frac{1}{2}(\phi_1 - \phi_2), \quad \Theta = -\frac{1}{2}(\phi_1 + \phi_2), \]  

one may extract the integral of motion \( Z \) and corresponding total phase \( \Theta \) from equations (14) and (15) and thus reduce the problem to a couple of equations for new unknowns, population imbalance \( z \) and phase difference \( \theta \). These equations read

\[ \dot{z} = -\tilde{\Omega}(t) \sqrt{1 - z^2} \sin 2\theta \]  

\[ \dot{\theta} = \frac{1}{2} \left[ \Delta(t) + \Delta z + \tilde{\Omega}(t) \frac{z}{\sqrt{1 - z^2}} \cos 2\theta \right] \]  

where

\[ \Delta(t) = E_1(t) - E_2(t) = \alpha t \]  

is the detuning (difference in well depths). In accordance with LZ practice we assume for \( \Delta(t) \) a linear time dependence with the rate \( \alpha \).

Equations of motion (18) and (19) are invariant under transformations

\[ \Lambda \rightarrow -\Lambda, \quad \alpha \rightarrow -\alpha, \quad \theta \rightarrow -\theta + \frac{\pi}{2} \]  

or

\[ \alpha \rightarrow -\alpha, \quad z \rightarrow -z, \quad \theta \rightarrow -\theta. \]  

The former relates the transport protocols for repulsive and attractive BEC in one direction, while the latter connects the transport in opposite directions (with corresponding interchange of the initial conditions).

Note that equations (18) and (19) allow a classical analogy with \( z \) and \( \theta \) treated as conjugate variables. It is easy to verify that these equations can be recast into the canonical form

\[ \dot{z} = -\frac{\partial H_{cl}}{\partial \theta}, \quad \dot{\theta} = \frac{\partial H_{cl}}{\partial z} \]  

with the classical Hamiltonian

\[ H_{cl} = \frac{1}{2} \left[ \Delta(t)z + \frac{\Lambda}{2} z^2 - \tilde{\Omega}(t) \sqrt{1 - z^2} \cos 2\theta \right] \]  

and the chemical potential [43]

\[ \mu = H_{cl} + V_{int} = \frac{1}{2} \left[ \Delta(t)z + \Lambda z^2 - \tilde{\Omega}(t) \sqrt{1 - z^2} \cos 2\theta \right] \]  

where \( V_{int} = \Delta z^2/4 \).

It is easy to see that, up to notation, equations (18), (19) and (24) coincide with those in [9]. However, unlike the previous studies of the oscillating BEC fluxes in traps with constant parameters [9, 27], we will deal with irreversible BEC transport by monitoring time-dependent parameters \( \dot{E}_k(t) \) and \( \dot{\Omega}(t) \).

2.2. Stationary states

Since we are interested in the adiabatic (very slow) transport, the analysis of stationary states can be useful. These states are defined by the condition

\[ \dot{z} = \dot{\theta} = 0. \]  

Despite the fact that our Hamiltonian and thus the variables \( z \) and \( \theta \) actually depend on time, this dependence is assumed to be slow enough to make (26) relevant.

Under condition (26) equations (18) and (19) give

\[ \theta = \frac{\pi}{2} n, \]  

\[ \Delta(t) + z \left( \Lambda \pm \sqrt{\frac{\tilde{\Omega}(t)}{\sqrt{1 - z^2}}} \right) = 0, \]  

where \( n \) is an integer number. Equation (28) has ‘+’ or ‘−’ for even and odd \( n \), respectively. The analysis of (28) finds two real roots for \( |\Lambda| < \bar{\Omega} \) and four real roots for \( |\Lambda| > \bar{\Omega} \), i.e. at high nonlinearity.

The substitution of the stationary solutions of (27) and (28) into (25) gives the chemical potentials \( \mu_{-}(t) \) and \( \mu_{+}(t) \), representing the eigenvalues of the stationary states, i.e. (3) casts into \( \psi_1(t) = \sqrt{N_1(t)} e^{-i\mu t} \). In the four-root case, the repulsive interaction (\( \Lambda > 0 \)) leads to one solution for \( \mu_{+}(t) \) and three solutions for \( \mu_{-}(t) \), and vice versa for attractive interaction (\( \Lambda < 0 \)).

As shown below, three roots form a nonlinear loop in the stationary spectra. For \( |\Lambda| > \bar{\Omega} \), the time length of the loop reads [12]

\[ t_c = \frac{(|\Lambda|^{3/2} - (\bar{\Omega}(t_c))^{3/2})^{3/2}}{\alpha}, \]  

where \( \bar{\Omega}(t_c) \rightarrow 1 \) for the constant coupling. It is seen that the length increases with the magnitude of nonlinearity \( \Lambda \) and decreases with detuning rate \( \alpha \). The nonlinear structures (single loops, double loops, butterfly, . . . ) can appear in the stationary spectra for smaller nonlinearity \( |\Lambda| < \bar{\Omega} \) as well, see discussion in [44].

3. Transport protocol

In the standard LZ protocol without nonlinearity [39], the final probabilities of two competing processes, Landau–Zener tunnelling between adiabatic levels (\( P_{LZ} \)) and adiabatic following (\( P \)), read

\[ P_{LZ} = \exp \left( -\frac{\pi \bar{\Omega}^2}{2\alpha} \right), \quad P = 1 - P_{LZ}. \]
Both processes are controlled by a constant coupling $\Omega$ and detuning rate $\alpha$. A complete adiabatic transfer $P = 1$ takes place in the adiabatic limit $\alpha \rightarrow 0$. As shown in [12, 17, 18], the nonlinearity caused by the interaction between BEC atoms essentially modifies the transfer (30). Namely, the diabatic LZ tunnelling becomes possible even at $\alpha \rightarrow 0$. Moreover, the impact of nonlinearity is asymmetric, i.e. it favours or suppresses the LZ tunnelling and inversely affects the adiabatic following, depending on the interaction sign [18]. This suggests a principle possibility to use the LZ scheme for the complete adiabatic transport in the nonlinear case.

In the present study, this point is scrutinized for a constant and time-dependent Gaussian coupling. The relevant protocols are illustrated in figure 1 where the detuning (20) is modelled by

$$E_1(t) = \frac{1}{2}\alpha t, \quad E_2(t) = -\frac{1}{2}\alpha t$$

(31)

with $\alpha > 0$ and $-t_s < t < t_s$. We assume that at early time $t \leq -t_s$, the diabatic energies are tuned from the symmetry values $E_1 = E_2 = 0$ to the values $E_i(-t_s)$ and $E_i(t_s)$, then exhibit the linear evolution (31) at $-t_s < t < t_s$ and finally come back to the zero symmetry values. In the experimental setup such manipulations can be produced by varying the well depths. In what follows, we will present the results only for $-t_s < t < t_s$.

The conventional LZ scheme is illustrated in figure 1(a). Despite its artificial properties (constant coupling and infinite diabatic energies at $t \rightarrow \pm \infty$), it is known to give quite realistic transition probabilities (30). The reason is that the LZ transfer actually occurs only during a finite time of close approaching the diabatic energies near $t \approx 0$, when $\Delta(t) < \Omega$. The lateral regions of early and late times, where the coupling impact is negligible, are irrelevant. Hence the LZ artefacts are not essential.

However, if anyway LZ works only for a finite time, then it is natural to use a time-dependent coupling of a certain duration. In this connection, we propose an effective and flexible transport protocol where a time-dependent coupling $\Omega(t)$ of a Gaussian form (10) is used, see figure 1(b). Then, in addition to the usual LZ control parameters, detuning rate $\alpha$ and coupling amplitude $K$, we get a new one, the Gaussian width $\Gamma$. As discussed below, this protocol allows a rapid and complete switch of the adiabatic BEC transport in a wider range of $\alpha$ and provides a new transport regime where not the centre but edges of the coupling $\Omega(t)$ are important. This makes the new protocol more effective and flexible. Hereafter it is referred to as a Gaussian Landau–Zener (GLZ) protocol. Obviously, the GLZ is also a generalization of the Rosen–Zener scheme [40] which exploits a time-dependent coupling but constant diabatic energies.

In the present calculations, we use the Gaussian coupling (10) with the centroid at $\bar{t} = 0$ and width parameter $\Gamma = 5$, apart from figure 7 where various $\Gamma$ are applied. The Gaussian full width at half maximum is $\text{FWHM} = 2.355 \Gamma = 11.78$. The monitoring of the coupling $\Omega(t)$ which is actually the penetrability of the barrier between the wells, can be produced in experiment by the variation of the spacing between the wells. For a time-dependent coupling, the adiabatic following takes place under the condition [35]:

$$\left(\frac{1}{2}\Omega(t)\Delta(t) - \dot{\Omega}(t)\Delta(t)\right) \ll |\Omega(t) + \Delta^2(t)|^{1/2}$$

(32)

which requires slow rates and large values of the coupling and/or detuning.

Figure 1 shows that in the linear case both LZ and GLZ protocols give very similar results. The only tiny difference is that GLZ adiabatic energies $\mu_{\pm}$, calculated with (25), less deviate from $\bar{E}_{1,2}$ at early and late times. In both protocols, the transfer is determined by the region where $|\mu_{-} - \mu_{+}| \approx \Omega(t = 0) = 1$. As shown below, much more serious difference between LZ and GLZ appears in the nonlinear case.

4. Results and discussion

The main results of our calculations are presented in figures 2–8.
In figure 2, the final LZ and GLZ populations of the second well, \( P = N_2(t = +\infty) \), are shown as a function of \( \alpha \) for different values of the nonlinear parameter \( \Lambda \). In all the cases, the BEC is initially in the first well, i.e. \( N_1(t = -\infty) = 1, N_2(t = -\infty) = 0 \). At first glance, the LZ and GLZ transfers look quite similar. Without the interaction (\( \Lambda = 0 \)) the processes are identical. For the repulsive interaction \( \Lambda = 4 \) and \( 20 \), both protocols produce a complete transfer within a wide \( \alpha \)-plateau. The stronger the interaction, the wider the plateau. At sufficiently high \( \alpha \), the process becomes too rapid and adiabatic transfer naturally fails, leading to decreasing \( P \). In this region, behaviour of \( P \) depends on \( \Lambda \) but is the same for LZ and GLZ.

It is remarkable that, unlike the STIRAP case [22], the repulsive interaction (\( \Lambda > 0 \)) does not spoil but even favours the transport by forming the \( \alpha \)-plateau. The robustness and completeness of the transport is illustrated in figures 3(a) and (b) for the particular case of \( \Lambda = 4 \) and \( \alpha = 3 \). Instead, following figures 2(g)–(j) and figures 3(c) and (d), the attractive interaction (\( \Lambda < 0 \)) damages the adiabatic transport and the stronger the interaction, the less the final population \( P \). So, in accordance with [16, 18], we obtain the asymmetric interaction effect when the transfer crucially depends on the interaction sign.

The origin of this effect is explained in figure 4 in terms of GLZ stationary eigenvalues \( \mu_\pm \) for the repulsive and attractive BEC. In both cases BEC is initially placed at the lower level, i.e. \( N_1(t = -\infty) = 1 \) and \( N_2(t = -\infty) = 0 \). The nonlinearity causes the loops in \( \mu_+ (\Lambda = -4) \) and \( \mu_- (\Lambda = 4) \) levels. For \( \Lambda = -4 \), the loop takes place at the BEC transfer path and so a part of the condensate arrives to the loop terminal point. There is no way further and so BEC is enforced to tunnel to upper or lower levels as indicated by arrows in plot (a). This results in leaking the population to the upper level \( \mu_- \) and hence the depletion of the adiabatic transfer. Instead, for \( \Lambda = 4 \), the transfer path does not meet this obstacle and so is robust.

The previous analysis mainly concerned the common features of LZ and GLZ protocols. Let us now consider the difference between these two cases, which takes place at small rates \( \alpha \). As seen from figure 2, for both protocols the nonlinearity results in a window \( 0 \leq \alpha < \alpha_0 \) where the adiabatic transfer completely vanishes. However, the GLZ window is much wider than the LZ one, which is a consequence of a finite duration of the Gaussian coupling.

This point is scrutinized in figure 5 where behaviour of nonlinear loops in different transport regimes is demonstrated. In the first regime (plots (a) and (b)), the loops are absent because of a small nonlinearity \( \Lambda < 1 \). The LZ and GLZ stationary spectra and transport features look similar. In particular, in both cases the adiabatic transport is possible only in a narrow interval of small rates \( \alpha \). In the second regime with a larger nonlinearity \( \Lambda > \Omega \) (plots (c)–(f)), the stationary spectra gain the loops but their length is still less than the Gaussian width, i.e. \( 2t_c \leq \text{FWHM} \) or \( t_c < \Gamma \). Then the LZ and GLZ spectra remain to be similar and both protocols give a

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\[ \text{Small nonlinearity} \quad |\Lambda| < 1 \quad \text{can also result in various nonlinear structures (loops, double-loops, etc); see discussion in [44].} \]
robust adiabatic transport \( P \approx 1 \) within common \( \alpha \) intervals inside the plateau in figure 2. Note that according to (29), the loop length \( 2t_c \) rises with \( \Lambda \) and decreases with \( \alpha \) as illustrated in figure 5. In other words, the loop behaviour is determined not only by the nonlinearity but the detuning rate as well. The latter becomes decisive in the third regime that determines the \( P \approx 0 \) windows at low rates \( \alpha \) in figure 2. In this regime (plots (g)–(j)), the loop becomes comparable or longer than the coupling width, i.e. \( t_c \gg \Gamma \). Then the edges of the Gaussian coupling come to play and a new regime, which is absent in LZ, arises. At the edges we have \( \Omega(t) \ll 1 \). So, in accordance to (29), the GLZ loop is extended and becomes longer than the

![Figure 5](image_url)

**Figure 5.** LZ (left) and GLZ (right) adiabatic energies \( \mu_\pm \) (solid curves) at different nonlinearity \( \Lambda \) and detuning rate \( \alpha \). The diabatic energies (dash lines) are given for the convenience of comparison. In plot (c), the loop length limits \( (-t_c, t_c) \) are marked.

![Figure 6](image_url)

**Figure 6.** GLZ evolution of adiabatic energies \( \mu_\pm \) (left) and population \( N_2 \) (right) for the detuning rates \( \alpha = 0.328 \) and 0.348. In all the panels \( \Lambda = 4 \) is used. The vertical arrows indicate the minimal energy gap between \( \mu_- \) (upper curve) and loop edge of \( \mu_+ \) (lower curve).

The previous analysis suggests that at \( t_c \geq \Gamma \) the success of adiabatic following is determined by the minimal gap between \( \mu_+ \) and \( \mu_- \). Figure 6 illustrates this point for two small detuning rates which hinder and favour the transfer by giving \( N_2 \sim 0.02 \) and \( \sim 1 \), respectively. This example shows that the Gaussian coupling provides a complete transfer switch by a tiny tuning of \( \alpha \) and this switch is determined by the lateral gaps between \( \mu_+ \) and \( \mu_- \). This property can be used for effective coherent control of BEC transport.

The Gaussian coupling delivers a new control parameter \( \Gamma \). Dependence of the transport on \( \Gamma \) is illustrated in figure 7. It is seen that increasing \( \Gamma \) makes the Gaussian coupling closer to the LZ case and leads to the downshift of the left plateau edge. Instead, decreasing \( \Gamma \) upshifts the plateau edge. Hence by changing \( \Gamma \) one can shift the switch value \( \alpha_s \) which, following a simple estimation, reads

\[
\alpha_s \simeq \frac{\Lambda}{2\Gamma}. \tag{33}
\]

The figure shows that already at \( \Gamma < 3 \) the coupling becomes too short to support a slow adiabatic evolution and transport tends to vanish. Note that the value of \( \Gamma \) influences only the left side of the plateau and does not affect its right side.

In the discussion above, the successful left-to-right transport of repulsive BEC (\( \Lambda > 0 \)) was considered. This protocol is depicted in figure 8(a). However, by using the symmetry (21), the similar transport can be produced for the attractive condensate (\( \Lambda < 0 \)) if we change the sign of the detuning rate \( \alpha \) (figure 8(b)). Further, following the symmetry (22), the right-to-left transport is also possible for the attractive
adiabatic population transfer schemes are indeed very robust protocols, the transport demonstrates high effectiveness and perfectly adiabatic. However, being driven by adiabatic Pitaevskii equation (1) within the two-mode approximation an additional analysis. In particular, we used the Gross–Pitaevskii equation. This work presents transport protocols should be checked by quantum particles, the quantum fluctuations are important. Thus, the these points is possible. However, at small number of N atoms while the latter is generally valid for small condensates [45]. Following our estimations, the compromise between these points is possible. However, at small number of particles, the quantum fluctuations are important. Thus, the present transport protocols should be checked by quantum calculations beyond the Gross–Pitaevskii equation. This work is in progress.

5. Conclusions

We propose a simple and effective adiabatic population transfer protocol for the complete and irreversible transport of Bose–Einstein condensate (BEC) in a double-well trap by using a pulsed coupling of a Gaussian form. Being mainly based on the familiar Landau–Zener (LZ) scheme with a constant coupling and having much in common with that scheme, our protocol is, nevertheless, more realistic and flexible. The protocol delivers the additional control parameter, the coupling width Γ', which allows us to switch adiabatic transport at the detuning rate α beyond the LZ adiabatic limit α → 0, and, what is important, provides a new transport regime when not the centre but edges of the coupling are decisive. As a result, an effective control of the BEC transport, e.g. its rapid complete switch, becomes possible. In spite of asymmetric impact of nonlinearity, the protocol is quite universal and can be applied to both direct and inverse transports of BEC with both repulsive and attractive interaction. In other words, the protocol can always be cast to enforce the nonlinearity not to hamper but favour the adiabatic transfer. The stronger the nonlinearity, the wider the range of the detuning rate α where the complete transfer is possible. The protocol is actually a generalization of both Landau–Zener and Rosen–Zener schemes.

Our findings can open new opportunities in BEC dynamics like excitation of topological states [28, 29], exploration of diverse geometric phases generated in BEC transport [22], etc. The latter is especially interesting in relation to perspectives of using geometric phases in quantum computing [32–34].

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Figure 8. Transport schemes for repulsive (\(\Lambda > 0\), black dot) and attractive (\(\Lambda < 0\), grey dot) BEC in directions indicated by arrows. Note relation of the sign of \(\alpha\) with initial relative positions of the well depths.

(at \(\alpha > 0\)) and repulsive (at \(\alpha < 0\)) BEC, as is shown in figures 8(c) and (d). So our protocol is quite universal. It can also be used for other kinds of multicomponent BEC, e.g. for two-component BEC in a single-well trap.

As was mentioned in the previous sections, the GLZ is also a generalization of the Rosen–Zener method [40] where the coupling is time dependent but diabatic energies are constant. However, unlike the GLZ, the nonlinear RZ protocol totally blocks the adiabatic transfer [16].

Note that a possible (though weak) diabatic leaking \(P_{LZ}\), appearance of the loops in nonlinear stationary spectra, and finite rates \(\alpha\) signify that the actual transport is not perfectly adiabatic. However, being driven by adiabatic protocols, the transport demonstrates high effectiveness and completeness in a wide range of nonlinearity. This proves that adiabatic population transfer schemes are indeed very robust and promising.

Nevertheless, some approximations used in our study need an additional analysis. In particular, we used the Gross–Pitaevskii equation (1) within the two-mode approximation (2). These two points of the calculation scheme are somewhat contradictory, since the former assumes a large number of atoms \(N\) while the latter is generally valid for small condensates [45]. Following our estimations, the compromise between these points is possible. However, at small number of particles, the quantum fluctuations are important. Thus, the present transport protocols should be checked by quantum calculations beyond the Gross–Pitaevskii equation. This work is in progress.
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