Unification Picture in Minimal Supersymmetric SU(5) Model with String Remnants

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Abstract

The significant heavy threshold effect is found in the supersymmetric SU(5) model with two adjoint scalars, one of which is interpreted as a massive string mode decoupled from the lower-energy particle spectra. This threshold related with the generic mass splitting of the basic adjoint moduli is shown to alter properly the running of gauge couplings, thus giving a natural solution to the string-scale grand unification as prescribed at low energies by LEP precision measurements and minimal particle content. The further symmetry condition of the (top-bottom) Yukawa and gauge coupling superunification at a string scale results in the perfectly working predictions for the top and bottom quark masses in the absence of any large supersymmetric threshold corrections.
Some euphoria caused by the first indication [1] of unification of gauge couplings extrapolated from their low-energy values under assumption of the simplest particle content corresponding to the Minimal Supersymmetric Standard Model (MSSM) seems to be over. In recent years new thorough tests of the Standard Model have been performed [2]. They show clearly that new states beyond those of MSSM should be included into play not to be in conflict with the present precision measurement data, if one contemplated the gauge coupling unification with not acceptably high SUSY soft-breaking scale ($M_{SUSY} \gg M_Z$) [3].

On the other hand, the unifying scale $M_U$ at which the couplings would meet lies one order of magnitude below than the typical string scale $M_{STR} \approx 5 \cdot 10^{17}$ GeV [4]. This mismatch, while often considered as a drawback of string phenomenology, may be interpreted as the most clear string motivation for a possible grand-unified symmetry beyond MSSM\footnote{The matching of those two scales might be qualified as a strong indication in favor of a pure string unification rather than somewhat superfluous string-scale Grand Unified Theory (GUT), unless there appears some more symmetry beyond the GUT at a string scale, e.g. gauge-Yukawa unification, and the like. We show below that it could be the case.}. In this scenario [4], the standard gauge couplings would still unify at $M_U$ to form the single coupling $g_U$, but then it would run up to the string scale $M_{STR}$ and unify with any other gauge ("hidden") and gravitational couplings.

Now, when pondering the possible string-inspired GUT candidates, the minimal supersymmetric $SU(5)$ model [5] should be considered at the first place, although it could be stringy realized at higher affine levels $k \geq 2$ only, or in the so-called "diagonal construction" within a $k$-fold product of the $SU(5)$'s at a level $k = 1$ [4]. The $SU(5)$ model with just the light MSSM particle spectra contributing to the evolution of the gauge couplings can initiate the additional logarithmic threshold corrections due to the superheavy states $\Sigma_8(5c,1)+\Sigma_3(3c,3p)$ and $H_c(3c,1)+H_c(\bar{3}c,1)$, which are left as the non-Goldstone remnants of the starting Higgs multiplets $SU(5)$ when $SU(5)$ breaks to $SU(3)_C \otimes SU(2)_W \otimes U(1)_Y$ and then to $SU(3)_C \otimes U(1)_{EM}$, respectively. At the same time the model has an additional severe restriction following from proton decay due to the color-triplet $H_c(\bar{H}_c)$ exchange [6]. While at the moment its mass $M_c$ can favorably be arranged inside of the narrow interval $2.0 \div 2.4 \cdot 10^{16}$ GeV (from the negative search for the proton decay and the detailed renormalization group (RG) analysis for the gauge couplings, respectively) [7], the model is apparently nearing to be ruled out [8] by a combination of the improved lower limits on the proton decay mode $p \rightarrow \bar{\nu}K^+$ from SuperKamiokande and on the superparticle masses at LEP2, expected in the short run.

In this Letter we argue that the aforementioned adjoint remnants $\Sigma_8$ and $\Sigma_3$, specifically their plausible generic mass splitting, can play an important part in superhigh-scale physics, thus giving new essential (yet missing) details to the unification picture in the minimal supersymmetric $SU(5)$ model to overcome the difficulties mentioned above.

It is well known [4,9] that those states appear in many string models as continuous moduli which is why they can remain relatively light ($M_\Sigma \ll M_U$) and, as a result, push the unification scale $M_U$ up to $M_{STR}$ [7,9]. At the same time, while a prediction for $\alpha_s$ was found independent (at one-loop order) of $M_U$ [10] and $M_\Sigma$ [7,11], that is turned out to depend crucially on the mass-splitting between $\Sigma_8$ and $\Sigma_3$, as it is demonstrated clearly below. Whereas in the standard one-adjoint superpotential [5] such a splitting is generically absent at a GUT scale, with a new renormalizable superpotential proposed, which contains also the second adjoint $\Omega$ (interpreted as a massive string mode), the large generic mass-splitting appeared between $\Sigma_8$ and $\Sigma_3$ is found fairly ample for gauge coupling unification to be completely adapted with the present values of $\alpha_s(M_Z)$, weak mixing angle $\theta_W$ and top-quark pole mass ($M_T$) values [2]

\begin{equation}
\alpha_s(M_Z) = 0.119 \pm 0.004, \quad \sin^2\theta_W = 0.2313 \pm 0.0003, \quad m_t = 175.6 \pm 5.5, \quad (1)
\end{equation}

even though the SUSY soft-breaking scale $M_{SUSY}$ ranges closely to $M_Z$ and the color-triplet mass $M_c$ is taken at the GUT scale $M_U$. Both of cases of the low and high values of $\tan\beta$ (the ratio of the vacuum expectation values (VEVs) of the up-type and down-type Higgs doublets involved) therewith look in the model to be physically interesting.

We start recalling that, to one-loop order, gauge coupling unification is given by the three RG equations relating the values of the gauge couplings at the Z-peak $\alpha_i(M_Z)$ ($i = 1, 2, 3$), and the common gauge coupling $\alpha_U$ [1]:

\begin{equation}
\alpha^{-1}_U = \alpha^{-1}_i + \sum_p \frac{b^p}{2\pi} \ln \left( \frac{M_U}{M_p} \right) \quad (2)
\end{equation}

where $b^p_i$ are the three b-factors corresponding to the $SU(5)$ subgroups $U(1)_s$, $SU(2)_w$ and $SU(3)_c$, respectively, for the particle lebelled by $p$. The sum extends over all the contributing particles in the model, and $M_p$ is the mass threshold at which each decouples. All of the SM particles and also the second Higgs doublet of MSSM are already presented at the
starting scale $M_Z$. Next is assumed to be supersymmetry threshold associated with the decoupling of the supersymmetric particles at some single effective (lumped) scale $M_{\text{SUSY}}$ [3]; we propose thereafter the relatively low values of $M_{\text{SUSY}}, M_{\text{SUSY}} \sim M_Z$, to keep sparticle masses typically in a few hundred GeV region. The superheavy states, such as the adjoint fragments $\Sigma_3$ and $\Sigma_3$ at the masses $M_8$ and $M_3$, respectively, and the color-triplets $H_c$ and $H_c$ at a mass $M_c$ are also included in the evolution equations (2). As to the superheavy gauge bosons and their superpartners (X-states), they do not contribute to the Eqs.(2), for they are assumed to lie on the GUT scale $M_U (M_X = M_U)$, above which all particles fill complete $SU(5)$ multiplets.

Now, by taking the special combination of Eqs.(2) we are led to the simple relation between gauge couplings and logarithms of the neighboring threshold mass ratios

$$12\alpha_1^{-1} - 7\alpha_3^{-1} - 5\alpha_1^{-1} = \frac{3}{2\pi} (-2\ln \frac{M_X}{M_c} + \ln \frac{M_c}{M_3} - 7\ln \frac{M_3}{M_8} + \frac{19}{6} \ln \frac{M_{\text{SUSY}}}{M_Z}) \quad (3)$$

which can be viewed as the basis for giving the qualitative constraints to the $\alpha_s(M_Z)$ depending on the present (very precise) measurement of $\sin^2 \theta_W$ (4) and superheavy mass splitting, when one goes beyond the MSSM limit ($M_X = M_c = M_3 = M_8$). One can see from Eq.(3) that $\alpha_s$ increases with $\frac{M_3}{M_8}$ and decreases with $\frac{M_3}{M_8}$, and, especially, with $\frac{M_3}{M_8}$ (the largest coefficient before logarithm). On the other hand, with an "alive" color triplet $H_c (M_c < M_X)$ one can raise the GUT scale $M_X$ by lowering the masses of the $\Sigma$ remnants (say, $M_3$ with $M_8$ then calculated) without affecting $\alpha_s$, as theory predicts (to one-loop order) the entire mass combination $M_3^2/M_3$ in addition to $M_c$ and common gauge coupling $\alpha_U$ (that can be more clearly viewed from the other running coupling combinations $5\alpha_i^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1}$ and $3\alpha_2^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1}$ extracted from Eqs.(2) respectively [7,10,11]). When the color triplet is taken at a GUT scale ($M_c = M_X$), both of masses $M_X$ and $M_3$ and $\alpha_U$ are then predicted. And finally, when all thresholds are taken to be degenerate ($M_c = M_3 = M_X$), besides the $M_X$ and $\alpha_U$, the strong coupling $\alpha_s(M_Z)$ depending on the generic mass ratio $M_3/M_8$ can also be predicted – somewhat naive, while truly predictive, ansatz of grand unification leading in the standard case ($M_3/M_8 = 1$ at a GUT scale $M_X$) [5] to the unacceptably high values of $\alpha_s(M_Z)$ for the

\footnote{As its best, the $\alpha_U$ could be predicted from the string coupling $\alpha_{\text{STR}}$, if the latter was somehow fixed at the $M_{\text{STR}}$ and then taken to run down to the $M_X$.}

physically most interesting SUSY soft-breaking scale area [3].

However, a somewhat different (more string-motivated) version of the minimal supersymmetric $SU(5)$ model, we are coming to, with a generically large mass splitting between $\Sigma_3$ and $\Sigma_8$ suggests an alternative unification picture. This follows essentially from a general renormalizable two-adjoint superpotential of $\Sigma$ and $\Omega$ satisfying also the reflection symmetry ($\Sigma \to -\Sigma, \Omega \to \Omega$)

$$W = \frac{1}{2} m\Sigma^2 + \frac{1}{2} M_p\Omega^2 + \frac{1}{2} h\Sigma^2\Omega + \frac{1}{3} \Omega^3 + W' \quad (4)$$

where the second adjoint $\Omega$ can be considered as a state originated from the massive string mode with the (conventionally reduced) Planck mass $M_P = (8\pi G_N)^{-1/2} \approx 2.4 \cdot 10^{18}$ GeV, while the basic adjoint $\Sigma$ is left (relatively) light when going from the string scale to lower energies, $m << M_P$. The superpotential includes also the ordinary Higgs-doublet containing fundamental chiral supermultiplets $H$ and $\bar{H}$ presented in $W'$, which will be discussed just below. It is particularly remarkable that there are no any generically massless non-trivial string modes, apart from those corresponding to the standard supersymmetric $SU(5)$ GUT [5].

One can find now from the vanishing $F$-terms of the adjoints $\Sigma$ and $\Omega$ that their basic supersymmetric vacuum configurations, besides the trivial (symmetry-unbroken) case and the case when the superheavy adjoint $\Omega$ alone develops the VEV, include also the desired case when both of them develop the parallel VEVs which break $SU(5)$ down to $SU(3) \otimes SU(2) \otimes U(1)$

$$\Sigma = \text{diag}(1, 1, 1, -3/2, -3/2)\sigma, \quad \sigma = \sqrt{\frac{8mM_P}{h}} \left(1 - \frac{m}{M_P h} \right)^{1/2} \approx \sqrt{\frac{8mM_P}{h}}, \quad (5)$$

$$\Omega = \text{diag}(1, 1, 1, -3/2, -3/2)\omega, \quad \omega = \frac{2m}{h} \quad (6)$$

with the hierarchically large VEV ratio $r = (2M_P/m)^{1/2}$ inverted to their masses. After symmetry breaking the non-Goldstone remnants of $\Sigma$ ($\Sigma_3$ and $\Sigma_8$) survive, while being a little mixed ($\sim r^{-1}$) with $\Omega_3$ and

\footnote{This is assumed to be the gauge type discrete symmetry $Z_2$ inherited from superstrings [4] and stable under the gravitational corrections.}

\footnote{One could integrate out $\Omega$ and consider the high-order effective adjoint superpotential of $\Sigma$ only, however it would not look more instructive for the analysis presented.}
\( \Omega_s \), respectively. Remarkably, to the obviously good approximation \( \left( \frac{\lambda}{h^2} \gg \frac{h^2}{M_8^2} \right) \) for any reasonable values of the couplings \( h \) and \( \lambda \), already used in Eq. (3), the (physical) mass ratio of the \( \Sigma_3 \) and \( \Sigma_8 \) is definitely fixed (at a GUT scale) as

\[
M_3 = 10 m, \quad M_8 = \frac{5}{2} m, \quad \frac{M_3}{M_8} = 4 \quad (7)
\]

in contrast to \( M_3/M_8 = 1 \) in the standard one-adjoint superpotential [5]. Another distinctive feature of the superpotential considered (which can easily be viewed from Eqs. (3) and (4)) is the quite moderate values of the adjoint coupling \( h \) even in the case of the pushed string-scale unification: \( h \sim 0.1 \) instead of somewhat fine-tuned adjoint coupling value \( \sim 10^{-5} \) in the ordinary one-adjoint case [7,11].

Let us turn now to the electroweak symmetry breaking in the model induced by the fundamental Higgs supermultiplets \( H \) and \( \bar{H} \) in \( W' \) Eq. (4). It is apparent that they must necessarily interact with the basic adjoint \( \Sigma \), since another (superheavy) adjoint \( \Omega \) develop too small VEV (3) to give a reasonable order of mass to the color triplets \( H_c \) and \( \bar{H}_c \) after a fine-tuning (to make the accompanied doublets light) occurs. That is the \( HH \) pair, along with \( \Sigma \), has to change sign (\( HH \to -\bar{H}H \)) under reflection symmetry\(^3\), owing to which their mass term can not be included in the superpotential. Instead, the singlet superfield \( S \) ("the 25th component" of the \( \Sigma \) with \( S \to -S \) defined) should be introduced to make a fine-tuning required. This part \( W' \) of the reflection-invariant superpotential Eq. (4) has a general form

\[
W' = \bar{H}(\lambda_1 \Sigma + \lambda_2 S)H + \frac{1}{2} m' S^2 + \lambda' SS\Omega \quad (8)
\]

As it can routinely be established from a total superpotential \( W \) (4), there always appears possibility in a new supersymmetric vacuum configuration (\( H = \bar{H} = 0 \))

\[
S = -\frac{\lambda'}{m'} Tr(\Sigma \Omega) \simeq -15q \sigma \quad (q \equiv \frac{\lambda'}{h} \frac{m'}{m}) \quad (9)
\]

(see \( \sigma \) in Eq. (3)) to pick a right order of the coupling-and-mass ratio \( q \) so that, from the one hand, not to disturb noticeably the adjoint vacuum solutions (4) and masses \( M_3 \) and \( M_8 \) (5) and, from the other hand, to attain (after a fine-tuning \( 10q = \lambda_1/\lambda_2 \)) the desired order of the color-triplet mass \( M_c \) in the vicinity of the unification scale \( M_X \), \( M_c = \frac{2}{3} g \alpha M_X \) \( (M_X = \frac{1}{2} g \alpha \sigma, \ g \alpha \) is the unified coupling constant).

So, with the observations made we are ready now to carry out the standard two-loop analysis (with conversion from \( MS \) scheme to \( DR \) one included) [1,12] for gauge (\( \alpha_1, \alpha_2, \alpha_3 \)) and Yukawa (\( \alpha_4, \alpha_5, \alpha_6 \)) in a self-evident notation for top- and bottom-quarks and tau-lepton) coupling evolution depending on, apart from the single-scale (\( M_{\text{SUSY}} \)) supersymmetric threshold corrections mentioned above, the heavy \( \Sigma \) threshold only. This varies, in turn, from the GUT scale \( M_X \) (\( M_3 = M_X, M_8 = \frac{1}{2} M_X \)) down to some intermediate value \( O(10^{14}) \) GeV pushing thereafter the \( M_X \) up to the string scale \( M_{\text{STR}} \). The mass splitting between weak triplet \( \Sigma_3 \) and color octet \( \Sigma_8 \) noticeably decreases in itself, while \( M_3 \) and \( M_8 \) run from \( M_X \) down to the lower energies, as it results from their own two-loop RG evolution, which is also included in the analysis. On the other hand, the color triplets \( H_c(\bar{H}_c) \) are always taken at \( M_X \), for the strings seem to say nothing why any states, other than the adjoint moduli \( \Sigma_3 \) and \( \Sigma_8 \), could left relatively light.

As to the Yukawa coupling evolution, we consider the two possible cases of low and large values of \( tan\beta \) leading to the proper bottom-tau Yukawa unification [5,13] with their mass ratio \( R = m_t/m_\tau \) within the experimental region \( R_{\text{exp}}(M_Z) = 1.60 \pm 0.25 \) [2,13] required. The first case corresponds to the large enough value of \( \alpha_t \) at a unification scale \( M_X \), \( \alpha_t > 0.1 \) (while \( \alpha_6(M_X) \) is significantly smaller), evolving rapidly towards its infrared fixed point. From the observable values of the \( t \) quark mass \( m_t \) the proper values of \( tan\beta \) are then predicted. The second case, while generally admitted over the whole area for the starting values for \( \alpha_t \) and \( \alpha_6 \) at \( M_X \), is favorably advanced to the physically well-motivated (with or without the underlying SO(10) gauge symmetry of no concern) top-bottom unification case [13] with the relatively low, though still providing the fixed-point solution, \( \alpha_t \) and \( \alpha_6 \) values at \( M_X \), \( \alpha_t(M_X) = \alpha_6(M_X) = 0.02 \pm 0.1 \), required. Here, not only \( tan\beta \), but also \( m_t \) could distinctively be predicted (from \( m_b \) and \( m_\tau \), if there were a more detailed information about superparticle mass spectrum. Generally the large supersymmetric loop contributions to the bottom mass are expected which make uncertain the top mass prediction as well [13] unless the SUSY parameter sector is arranged in such a way (low values of \( \mu \) and \( m_{1/2} \) and large values of \( m_0 \)) to make the above radiative

\(^3\)Those lead to the (relatively) light gluinos and higgsinos and heavy squarks and sleptons which could be good for the SUSY-inspired proton decay [6-8] and radiative bottom decay \( (b \to s\gamma) \) [13], the both enormously enhanced in the large \( tan\beta \) case. On the other hand, such a non-uniform superspectrum contradicts, in principle, neither the low SUSY soft-breaking effective scale \( M_{\text{SUSY}} \) [3] considered here, nor the radiative (though being technically unnatural) electroweak symmetry breaking [13]. While other, less tight, superspectra are also
corrections to be negligible [13].

Our results, as appeared after numerical integration of all the RG equations listed above, are largely summarized in Figs. 1a and 1b. One can see from them that the $\alpha_s(M_Z)$ values predicted (with a percent accuracy due to the precise value of $\sin^2\theta_W$ [1] used and Yukawa couplings appropriately fixed at $M_X$) are in a good agreement with the World average value [1] for the unification mass $M_X$ ranging closely to $M_{\text{STR}}$.

The values of $\alpha_s(M_Z)$ on the very left of Fig.1a correspond to the case when $M_3 = M_X$ (thresholds are degenerate). This value, $\alpha_s(M_Z) = 0.116 \pm 0.001$ (for the $\alpha_s(M_X) = 0.3$ taken), can be considered as a naive threshold-neglecting prediction of the present model in the contrast to the analogous value $\alpha_s(M_Z) = 0.125 \pm 0.001$ in the standard $SU(5)$ under the same conditions.

In Fig.1b the predicted values of the top quark pole mass are also exposed. It is readily seen that the experimentally favorable intervals for $m_t$ and $\alpha_s$ (1) correspond in much to the same area of $M_X$ in the vicinity of $M_{\text{STR}}$. Interestingly, the unification mass region allowed is automatically turned out to largely be safe for proton decay through the Higgs color-triplet exchange [6-8].

Remarkably enough, the presently testable (SUSY threshold neglecting) top-bottom unification is turned out to work well in the model, thus giving the good prediction of top-quark mass. Furthermore, the low starting values of $\alpha_t$ and $\alpha_b$ at $M_X$, as well as the closeness of the unification mass $M_X$ to the string scale, allow one to make a next step towards the most symmetrical case which can be realized in the present string-motivated $SU(5)$ - Yukawa and gauge coupling superunification at a string scale:

$$\alpha_t(M_X) = \alpha_b(M_X) = \alpha_f(M_X) = \alpha_U ,$$

$$M_X = M_{\text{STR}}$$

(10)

This conjecture certainly concerns the third-family Yukawa couplings solely, since those ones can naturally arise from the basic string-inspired interactions, whereas masses and mixing of the other families seem to be caused by some more complex and model-dependent dynamics showing itself at lower energies. Due to a crucial reduction of a number of the fundamental parameters the gauge-Yukawa coupling superunification leads immediately to a series of the very distinctive predictions (of the $\alpha_s$ in general, while masses in absence of any large supersymmetric threshold corrections mentioned):

$$\alpha_s(M_Z) = 0.120 \pm 0.001 , \quad m_t = 181 \pm 1 , \quad m_b = 180 \pm 0.01 , \quad \tan \beta = 52 \pm 0.2 (11)$$

in a surprising agreement with experiment [2]. In Fig.2 the superunification of gauge and Yukawa couplings is demonstrated.

The two concluding remarks concern the further salient features of the superpotential $W$ proposed.

The first one is that a generic adjoint mass ratio $\langle \rangle$, while underlying the self-consistent minimal supersymmetric $SU(5)$ model presented, remains in a general $SU(N)$ theory broken to $SU(5)$ by the set of the $N-5$ additional fundamental supermultiplets $\phi^{(k)}$ and $\bar{\phi}^{(k)}$ (k=1, ..., N-5), which interact with the adjoint $\Sigma$ via the general invariant couplings of type $\delta \Sigma \phi$ (other terms are forbidden by the above reflection symmetry$^3$ $\delta \Sigma \rightarrow -\delta \Sigma$, $\Sigma \rightarrow -\Sigma$) included in the superpotential $W$ [1]. This is to say that one can equally well start from the $SU(N)$ GUT and drive at the same unification picture.

The second one refers to the possible corrections to the superpotential $W$ arising from the high-dimension operators [14] induced at the Planck scale. Fortunately, due to the same basic reflection symmetry$^3$ of the model, such operators, if appeared for scalars $\Sigma$ and $S$ developing the principal VEVs, should have dimension six and higher, $\delta L = \frac{c}{M_p^4} \text{Tr}(GG\Sigma^2) + ...$ ($G$ is the gauge field-strength matrix of the $SU(5)$, $c = O(1)$), whose influence on the present model predictions seems to be negligible in contrast to the standard $SU(5)$ where they can largely be smeared out [14].

Those and the other (yet applied above) attractive features of the superpotential $W$ seem to open the way to the natural string-scale grand unification, as prescribed at low energies by the the gauge coupling precision measurement and the minimal particle content.

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Figure Captions

**Fig.1** The predictions in the present model (the solid lines) and in the standard supersymmetric $SU(5)$ model (the dotted lines) of $\alpha_s(M_Z)$ as a function of the grand unification scale $M_X$ for the two cases of small $\tan\beta$ values with top-Yukawa coupling $\alpha_t(M_X) = 0.3$ taken (a) and large $\tan\beta$ values corresponding to top-bottom unification under $\alpha_t(M_X) = \alpha_b(M_X) = 0.05$ (b). In the latter case, the predicted top-quark pole mass values are also exposed (in the same way) for both of models. The unification mass $M_X$ varies from the MSSM unification point ($M_U^0 = 10^{16.3} GeV$) to the string scale ($M_U = M_{STR} = 10^{17.8} GeV$ for the Kac-Moody level $k = 2$; properly, $M_{\Sigma_3} = 4M_{\Sigma_8} \approx 10^{14} GeV$ is taken at a string scale), while the color-triplet mass is assumed to be at unification scale in all cases ($M_c = M_X$). The all-shaded areas on the left of the figures (a) and (b) are generally disallowed by the present bound [2] on nucleon stability.

**Fig.2** The superunification of gauge ($\alpha_1$, $\alpha_2$, $\alpha_3$) and Yukawa (top, bottom, tau) couplings at the string scale (the solid and dotted lines, respectively).
Fig. 1    Chkareuli
Fig. 2    Chkareuli