Implications on $\eta$-$\eta'$-glueball mixing from $B_{d/s} \to J/\psi \eta^{(')}$ Decays

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Abstract

We point out that the recent Belle measurements of the $B_{d/s} \to J/\psi \eta^{(')}$ decays imply large pseudoscalar glueball contents in the $\eta^{(')}$ meson. These decays are studied in the perturbative QCD (PQCD) approach, considering the $\eta$-$\eta'$-$G$ mixing, where $G$ represents the pseudoscalar glueball. It is shown that the PQCD predictions for the $B_{d/s} \to J/\psi \eta^{(')}$ branching ratios agree well with the data for the mixing angle $\phi_G \approx 30^\circ$ between the flavor-singlet state and the pure pseudoscalar glueball. Extending the formalism to the $\eta$-$\eta'$-$\eta_c$ tetramixing, the abnormally large observed $B_d \to K \eta'$ branching ratios are also explained. The proposed mixing formalism is applicable to other heavy meson decays into $\eta^{(')}$ mesons, and could be tested by future LHCb and Super-B factory data.

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Recently, the Belle Collaboration reported their new measurements of the $B_d \to J/\psi \eta^{(l)}$ decays \cite{1}, and the first observations of the $B_s \to J/\psi \eta^{(l)}$ decays \cite{2} with the branching ratios

\[
\begin{align*}
\text{BR}(B_d \to J/\psi \eta)_{\text{Exp}} &= 12.3^{+1.9}_{-1.8} \times 10^{-6}, \\
\text{BR}(B_d \to J/\psi \eta')_{\text{Exp}} &< 7.4 \times 10^{-6}, \quad (90\% \text{ C.L.})
\end{align*}
\]

(1)

(2)

$$\text{BR}(B_s \to J/\psi \eta)_{\text{Exp}} = 5.10 \pm 0.50(\text{stat.}) \pm 0.25(\text{syst.}) \times 10^{-4},$$

(3)

$$\text{BR}(B_s \to J/\psi \eta')_{\text{Exp}} = 3.71 \pm 0.61(\text{stat.}) \pm 0.18(\text{syst.}) \times 10^{-4},$$

(4)

and the relation of the branching ratios \cite{2}

\[
R_{s}^{\text{Exp}} = \frac{\text{BR}(B_s \to J/\psi \eta')}{\text{BR}(B_s \to J/\psi \eta)} = 0.73 \pm 0.14(\text{stat.}) \pm 0.02(\text{syst.}).
\]

(5)

The updated result in Eq. (1), as stated in \cite{1}, is consistent with and supersedes the previous one, $\text{BR}(B_d \to J/\psi \eta) = 9.5^{+1.9}_{-1.9} \times 10^{-6}$, in 2007 \cite{3}. The accuracy of the above data is expected to be improved rapidly along with operation of LHCb experiments.

The $B_{d/s} \to J/\psi \eta^{(l)}$ decays were first investigated by Deandrea et al. \cite{4} under the naive factorization assumption without considering the $\eta-\eta'$ mixing. In Ref. \cite{5}, Skands analyzed the $B_{d/s} \to J/\psi \eta$ decays via SU(3) relations to the ”golden channel” $B_d \to J/\psi K^0$ including the $\eta-\eta'$ mixing. The predictions $\text{BR}(B_d \to J/\psi \eta) = 11.0 \times 10^{-6}$ and $\text{BR}(B_s \to J/\psi \eta) = 5.0 \times 10^{-4}$ (BR$(B_d \to J/\psi \eta') = 15.0 \times 10^{-6}$ and $\text{BR}(B_s \to J/\psi \eta') = 3.3 \times 10^{-4}$), corresponding to $\theta = -10^\circ$ ($\theta = -20^\circ$), were obtained \cite{5}, where $\theta$ is the angle between the flavor-singlet state $\eta_1$ and the flavor-octet state $\eta_8$. Note that the $B_{d/s} \to J/\psi \eta'$ branching ratios were not calculated in \cite{5}. If they were, the outcome of $\text{BR}(B_s \to J/\psi \eta')$ would be much larger than in Eq. (1). This must be the case in the conventional $\eta-\eta'$ mixing formalism as elaborated below.

![Leading quark-level diagrams for the $B_{d/s} \to J/\psi \eta^{(l)}$ decays.](image)

\[\text{FIG. 1: (Color online) Leading quark-level diagrams for the } B_{d/s} \to J/\psi \eta^{(l)} \text{ decays.}\]

The $\eta^{(l)}$ mesons are produced via their nonstrange components in the $B_d$ meson decays at leading level, while they are produced via their strange components in the $B_s$ meson decays, as easily seen in Fig. 1. Under the conventional mixing, the physical states $\eta$ and $\eta'$ are related to the quark-flavor states $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$ through a mixing matrix with an angle $\phi$ \cite{6},

\[
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix}
= \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix}
\begin{pmatrix}
\eta_q \\
\eta_s
\end{pmatrix}.
\]

(6)
Various experimental and theoretical constraints have yielded $\phi < 45^\circ$ (for a recent detailed discussion, see Ref. [7]), for which one observes [8]

$$R_d \equiv \frac{\text{BR}(B_d \to J/\psi \eta')}{\text{BR}(B_d \to J/\psi \eta)} \approx \tan^2 \phi < 1 ,$$

$$R_s \equiv \frac{\text{BR}(B_s \to J/\psi \eta')}{\text{BR}(B_s \to J/\psi \eta)} \approx \cot^2 \phi > 1 .$$

It is obvious that only the first relation meets the data, and the second one does not as confronted with Eq. (5). As speculated in [2], the small measured ratio $R_{s \text{Exp}}$ indicates additional flavor-singlet components in the $\eta'$ meson other than the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ pairs, or violation of the $\eta$-$\eta'$ mixing scheme.

Inspired by the Belle measurements, we analyze the $B_{d/s} \to J/\psi \eta(0)$ decays in the more complete $\eta$-$\eta'$-$G$ mixing formalism, $G$ denoting a pseudoscalar glueball. Adopt the construction in [9],

$$
\begin{pmatrix}
|\eta\rangle \\
|\eta'\rangle \\
|G\rangle
\end{pmatrix} = U_3(\theta)U_1(\phi_G) \begin{pmatrix}
|\eta_s\rangle \\
|\eta_i\rangle \\
|g\rangle
\end{pmatrix},
$$

where the matrices

$$U_1(\phi_G) = \begin{pmatrix}
1 & 0 & 0 \\
0 & \cos \phi_G & \sin \phi_G \\
0 & -\sin \phi_G & \cos \phi_G
\end{pmatrix}, \quad U_3(\theta) = \begin{pmatrix}
\cos \theta & -\sin \theta & 0 \\
\sin \theta & \cos \theta & 0 \\
0 & 0 & 1
\end{pmatrix},$$

represent rotations around the axis along the $\eta_s$ meson and the unmixed glueball $g$, respectively. Equation (9) is based on the assumption that $\eta_s$ does not mix with the glueball, under which two mixing angles $\theta$ and $\phi_G$ are sufficient.

The octet and singlet states are related to the flavor states through

$$
\begin{pmatrix}
|\eta_s\rangle \\
|\eta_i\rangle \\
|g\rangle
\end{pmatrix} = U_3(\theta_i) \begin{pmatrix}
|\eta_q\rangle \\
|\eta_s\rangle \\
|g\rangle
\end{pmatrix},
$$

where $\theta_i$ is the ideal mixing angle with $\cos \theta_i = \sqrt{1/3}$, i.e., $\theta_i = 54.7^\circ$. The flavor states are then transformed into the physical states via the matrix

$$U(\phi, \phi_G) = U_3(\theta)U_1(\phi_G)U_3(\theta_i) ,$$

$$= \begin{pmatrix}
\cos \phi + \sin \theta \sin \theta_i \Delta_G & -\sin \phi + \sin \theta \cos \theta_i \Delta_G & -\sin \theta \sin \phi_G \\
\sin \phi - \cos \theta \sin \theta_i \Delta_G & \cos \phi - \cos \theta \cos \theta_i \Delta_G & \cos \theta \sin \phi_G \\
-\sin \theta_i \sin \phi_G & -\cos \theta_i \sin \phi_G & \cos \phi_G
\end{pmatrix},$$

with the angle $\phi = \theta + \theta_i$ and the abbreviation $\Delta_G \equiv 1 - \cos \phi_G$. $U$ has been written in the form, that approaches the mixing matrix in Eq. (6) in the $\phi_G \to 0$ limit. We mention that a different mixing matrix has been constructed by diagonalizing a mass matrix for pseudoscalar bound states, which was derived from an effective QCD Lagrangian [10].

It has been verified that the contribution from the gluonic distribution amplitudes in the $\eta^{(i)}$ meson is negligible for $B$ meson transition form factors [11]. Hence, the $\eta$ and $\eta'$
mesons are still produced via the nonstrange (strange) component in the $B_d$ ($B_s$) meson decays under the $\eta$-$\eta'$-$G$ mixing. Equation (12) then simply modifies Eqs. (7) and (8) into

$$R_d^{\text{Th}} \approx \left( \frac{\sin \phi - \cos \theta \sin \theta \Delta_G}{\cos \phi + \sin \theta \sin \theta \Delta_G} \right)^2,$$

$$R_s^{\text{Th}} \approx \left( \frac{\cos \phi - \cos \theta \cos \theta \Delta_G}{-\sin \phi + \sin \theta \cos \theta \Delta_G} \right)^2,$$

respectively. Since the angle $\theta$ is negative, both the numerator and the denominator in Eq. (13) decrease, and the ratio $R_d^{\text{Th}}$ could remain smaller than unity. On the other hand, the numerator in Eq. (14) decreases, while the magnitude of denominator increases. Therefore, the ratio $R_s^{\text{Th}}$ might drop from above unity to below unity for a sufficiently large angle $\phi_G$.

We compute the $B_{d/s} \rightarrow J/\psi\eta^{(s)}$ decays explicitly in the $\eta$-$\eta'$-$G$ mixing formalism, employing the perturbative QCD (PQCD) approach [12] at next-to-leading order (NLO) of the strong coupling constant. Replacing the kinematic variables and distribution amplitudes of the $K^0$ meson by those of the $\eta(s)$ meson, we derive the $B_d \rightarrow J/\psi\eta^{(s)}$ decay amplitudes from the $B_d \rightarrow J/\psi K$ ones [13]. Further substituting $B_s$ for $B_d$, we arrive at the $B_s \rightarrow J/\psi\eta^{(s)}$ decay amplitudes. Throughout this work it is assumed that there are no final state interactions and no isospin violation. The input parameters such as the QCD scale (in units of GeV), masses (GeV), decay constants (GeV), and $B$ meson lifetime (ps) are set to [12, 14]

$$\Lambda_{\overline{MS}}^{(f=4)} = 0.287, \quad m_W = 80.41, \quad m_{B_d} = 5.28, \quad m_{B_s} = 5.37;$$

$$m_{J/\psi} = 3.097, \quad m_b = 4.8, \quad m_c = 1.50, \quad f_{J/\psi} = 0.405;$$

$$f_{B_d} = 0.21, \quad f_{B_s} = 0.23, \quad \tau_{B_d} = 1.53, \quad \tau_{B_s} = 1.47;$$

$$f_q = f_\pi, \quad f_s = 1.3 f_\pi, \quad \phi = 43.7^\circ, \quad \phi_G = 33^\circ;$$

$$m_0^{\eta} = 1.50, \quad m_0^{\eta'} = 2.00, \quad f_\pi = 0.13. \quad (15)$$

Following the observation in [15], we have chosen a value for $m_0^{\eta}$ slightly higher than the chiral scale for the pion, $m_0^{\eta} \approx 1.3 \sim 1.4$ GeV. The best fit on the mixing angle $\theta$ from experiment data converged in the range of $[-11^\circ, -17^\circ]$ [16]. Our choice $\phi = 43.7^\circ$ in Eq. (15) is equivalent to $\theta = \phi - \theta_1 = -11^\circ$, close to $\theta = -10^\circ$ in [5], which corresponds to a sizable gluonic admixture. It is also consistent with $\phi = (44^{+6}_{-2})^\circ$ determined from the $B_d \rightarrow J/\psi\eta/\eta^0$ data [17]. The angle $\phi_G$, as summarized in [9], varies in a wide range $10^5 \lesssim \phi_G \lesssim 30^\circ$. We have taken the central value of $\phi_G = 33^\circ \pm 13^\circ$ recently extracted in [17, 18]. For the Cabibbo-Kobayashi-Maskawa matrix elements, we employ the Wolfenstein parametrization with the updated parameters $A = 0.832, \lambda = 0.2246, \bar{\rho} = 0.130, \text{and} \bar{\eta} = 0.350$ [14].

The CP-averaged branching ratios for the $B_{d/s} \rightarrow J/\psi\eta^{(s)}$ decays in the standard model then read as,

$$\text{BR}(B_d \rightarrow J/\psi\eta) = 11.2^{+2.8}_{-2.1}(\omega_{B_d})^{+1.9}_{-1.3}(a_2)^{+1.5}_{-1.4}(f_{J/\psi}) \times 10^{-6},$$

$$\text{BR}(B_s \rightarrow J/\psi\eta') = 6.5^{+1.2}_{-1.0}(\omega_{B_s})^{+1.1}_{-0.9}(a_2)^{+0.9}_{-0.8}(f_{J/\psi}) \times 10^{-6},$$

$$\text{BR}(B_d \rightarrow J/\psi\eta') = 5.14^{+1.45}_{-1.10}(\omega_{B_s})^{+1.0}(a_2)^{+0.77}_{-0.72}(f_{J/\psi}) \times 10^{-4},$$

$$\text{BR}(B_s \rightarrow J/\psi\eta') = 3.68^{+1.04}_{-0.78}(\omega_{B_s})^{+0.78}_{-0.55}(a_2)^{+0.51}_{-0.46}(f_{J/\psi}) \times 10^{-4},$$

4
with the ratios $R_{d}^{\mathrm{Th}} \approx 0.58$ and $R_{s}^{\mathrm{Th}} \approx 0.72$. We have kept the $\eta'$ meson mass in the phase-space factor for the $B_{d/s} \to J/\psi \eta'$ decay rates, and neglected masses of other light mesons. The theoretical uncertainties arise from the variation of the shape parameter $\omega_{B} = 0.40 \pm 0.04$ ($\omega_{B_{s}} = 0.50 \pm 0.05$) GeV for the $B_{d}$ ($B_{s}$) meson wave function $[19, 20]$, of the $J/\psi$ meson decay constant $f_{J/\psi} = 0.405 \pm 0.014$ GeV $[21]$, and of the Gegenbauer coefficient $a_{2} = 0.44 \pm 0.22$ for the leading-twist $\eta_{\eta(s)}$ distribution amplitude. Obviously, the theoretical branching ratios are in good agreement with the existing data and upper bound after considering the $\eta$-$\eta'$-$G$ mixing.

A remark is in order. The $B_{s} \to J/\psi\eta^{(0)}$ decays have been proposed $[22]$ to explore the $\eta$-$\eta'$-$G$ mixing with KLOE’s parametrization for the mixing matrix $[23]$, namely, assuming the absence of the glueball content in the $\eta$ meson. It was observed that the angles $\phi \approx 40^\circ$ and $\phi_{G} \approx 20^\circ$ determined by KLOE lead to $R_{s} \approx 1$, consistent with the previous Belle data $[24]$. Hence, the updated data $R_{s} < 1$ indeed imply larger glueball components in the $\eta^{(0)}$ meson as explained before. In principle, $R_{s} < 1$ can be achieved in both KLOE and our parametrizations by tuning the angles. Our parametrization is close to that in $[17, 18]$ with a nonvanishing glueball component in the $\eta$ meson, so we chose the larger $\phi_{G} = 33^\circ$ extracted in $[17, 18]$, and obtained $R_{s} < 1$ naturally. Taking $\phi_{G} = 22^\circ$ $[23]$, the central values of the branching ratios become

$$BR(B_{d} \to J/\psi\eta) = 11.7 \times 10^{-6}, \quad BR(B_{d} \to J/\psi\eta') = 8.2 \times 10^{-6},$$
$$BR(B_{s} \to J/\psi\eta) = 5.00 \times 10^{-4}, \quad BR(B_{s} \to J/\psi\eta') = 4.28 \times 10^{-4},$$

and the consistency with the data deteriorates. Compared to Eqs. $[17]$ and $[19]$, we find that $BR(B_{d/s} \to J/\psi\eta)$ are less sensitive to $\phi_{G}$ than $BR(B_{d/s} \to J/\psi\eta')$, a result attributed to the smaller glueball component in the $\eta$ meson. Another remark is that as far as the $B_{d} \to J/\psi\eta^{(0)}$ decays are concerned, their branching ratios can be accommodated in the conventional $\eta$-$\eta'$ mixing by tuning the angle $\phi$. Fitting Eq. $[17]$ to our predictions in Eqs. $[16]$ and $[17]$ yields $\phi \approx 37.3^\circ$, close to $\phi = 39.3^\circ \pm 1.0^\circ$ in $[6]$.

It is interesting to examine whether $D, D_{s}$ decays into $\eta^{(0)}$ mesons, such as $D, D_{s} \to \eta^{(0)}\ell^{+}\nu$, reveal the similar implication on the mixing mechanism $[7, 25, 26]$. The ratios of the branching ratios of these semileptonic decays have been expressed as

$$R_{d}' \equiv \frac{BR(D^{+} \to \eta'\ell^{+}\nu)}{BR(D^{+} \to \eta\ell^{+}\nu)} = \tilde{R}_{D}\tan^{2}\phi,$$
$$R_{s}' \equiv \frac{BR(D_{s} \to \eta'\ell^{+}\nu)}{BR(D_{s} \to \eta\ell^{+}\nu)} = R_{D}\cot^{2}\phi,$$

where the factors $\tilde{R}_{D} \approx R_{D}$ collect the information on the $D_{d/s} \to \eta_{b/s}$ transition form factors and the corresponding phase space. The estimate of $R_{D}$ depends on how to model the $q^{2}$ dependence of the form factor, $q^{2}$ being the lepton-pair invariant mass squared, and suffers theoretical uncertainty. Taking $R_{D} \approx 0.28$ $[27]$, it was found that the measured values $R_{d}^{\exp} \approx 0.19 \pm 0.05 < R_{D} [28]$ and $R_{s}^{\exp} = 0.37 \pm 0.10 > R_{D} [14]$ exhibit a pattern in agreement with the conventional $\eta$-$\eta'$ mixing. However, we stress that the above observation is not in conflict with the $\eta$-$\eta'$-$G$ mixing formalism advocated in this work, viewing the potential uncertainties in the estimate of $R_{D}$ and in the assumption $\tilde{R}_{D} \approx R_{D}$. In addition, it is not sure that the contributions from the $D, D_{s}$ transitions to pseudoscalar glueballs, which were referred as the weak annihilation process in $[7]$, are
negligible as in the \(B_{d/s}\) meson decays. The inclusion of these channels would modify the analysis of the \(D, D_s \rightarrow \eta(\ell^+\nu)\) decays.

At last, we have confirmed that the larger angle \(\phi_G = 33^\circ\) generates the pseudoscalar glueball mass \(m_G \approx 1.49\) GeV, following the \(\eta-\eta'-G\) mixing formalism in [9]. Namely, the postulation [9] that the \(\eta(1405)\) meson is a leading candidate for the pseudoscalar glueball is not altered. A larger \(\phi_G\) also makes an impact on the \(B \rightarrow K\eta(\ell^+\nu)\) branching ratios in the \(\eta-\eta'-G-\eta_c\) tetramixing formalism [15]. Substituting \(\phi_G = 33^\circ\) into the formula in Sec. III D of [15], \(BR(B^0 \rightarrow K^0\eta') = (6.61 \pm 3.1) \times 10^{-6}\) and \(BR(B^0 \rightarrow K^0\eta) = (1.12^{+0.30}_{-0.28}) \times 10^{-6}\) can also be understood.

In summary, most of studies in the literature on \(B_{d/s}\) decays into \(\eta(\ell^+)\) mesons were performed under the conventional \(\eta-\eta'\) mixing. The recent \(B_{d/s} \rightarrow J/\psi\eta(\ell^+)\) data provided a strong implication on the sizable pseudoscalar glueball contents in the \(\eta(\ell^+)\) meson, which motivated our investigation in the \(\eta-\eta'-G\) mixing formalism. We have verified this implication by computing explicitly the \(B_{d/s} \rightarrow J/\psi\eta(\ell^+)\) branching ratios in the NLO PQCD approach: the outcomes from a large angle \(\phi_G \approx 30^\circ\) were found to be well consistent with the current measurements and upper bounds. The abnormally large observed \(B \rightarrow K\eta'\) branching ratios were also accommodated in the \(\eta-\eta'-G-\eta_c\) tetramixing formalism with the same \(\phi_G\). Our work suggests that complete understanding of dynamics in \(\eta(\ell^+)-involved\) processes demands the \(\eta-\eta'-G\) mixing scheme. The resultant predictions for other \(B_{d/s} \rightarrow \eta(\ell^+)\) decays could be tested by future data of LHCb and/or Super-\(B\) factories.

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