Evolution of discharge and runner rotation speed along no-load curves of Francis turbines

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Abstract. In hydraulic turbines, no-load operation is among the most damaging conditions since unextracted swirl leads to the creation of highly energetic flow structures causing high pressure and strain fluctuations on the turbine components. To date, experimental and numerical studies typically focus on flow characteristics for a specific no-load condition of a specific turbine. However, for a given turbine, a unique no-load condition exists for every single guide vane opening, forming what is called a no-load curve. Few studies describe the evolution of engineering quantities such as discharge and speed along the no-load curve even if those quantities may highlight trends in no-load behavior that can be used to tailor numerical simulations according to specific flow conditions. This paper presents results from a project underway at Andritz Hydro Canada Inc., in collaboration with Université Laval, to analyze the extensive database of experimental no-load tests performed at model scale in order to identify the evolution of discharge and runner rotation speed following the guide vane opening.

1. Introduction
For hydraulic turbines, no-load (NL) is an operation where no net torque is extracted by the runner. In NL at a given head and cavitation level, the runner rotation speed varies according to the guide vane opening angle. These NL conditions form what is commonly called a NL curve or a runaway curve. Speed-no-load (SNL) corresponds to the guide vane opening yielding the synchronous speed. Results from the model Francis turbines investigated for this paper show that the SNL opening may be as low as 1.5°. NL conditions are an integral part of transients. They are the final state of start-up sequences [1–3], an important stage during load rejections [4] and the final condition of transient runaway events [5,6]. Full-gate opening NL conditions are particularly important as a turbine is free to reach its maximum rotation speed. This speed must stay lower than a limit value to ensure the safety of the generator and the shaft line. NL conditions are thus inevitable in a turbine life. They are also among the most damaging operating conditions for hydraulic turbines [7–10] since unextracted swirl leads to the formation of highly energetic flow structures. Although NL conditions are critical, to date, they are poorly understood. Experimental and numerical studies on NL conditions typically focus on flow characteristics for a specific NL operation of a specific turbine [11], often linked to a transient event [1,6]. Few studies describe the evolution of engineering quantities along the NL curve [12,13] even if those quantities may highlight trends in turbine NL behaviour that can be used, for example, to tailor numerical simulations.
Consequently, the main objective of this paper is to describe the evolution of the runner rotation speed and the discharge along a NL curve for Francis turbines with a wide range of specific speeds.

2. Methodology

2.1 Dimensionless parameters

The specific speed at the best efficiency point (BEP), \( n_{QE}^* \), is the main criterion used to classify the turbines in this study. It is defined according to the IEC 60193 standard for model tests of hydraulic turbines [14] as

\[
n_{QE}^* = \frac{nQ^{0.5}}{E^{0.75}}
\]

where \( n \) [rev/s] is the runner speed, \( Q \) [m³/s] is the discharge and \( E \) [J/kg] is the specific energy. The evolution of the speed and the discharge under a given specific energy is defined by the speed factor,

\[
n_{ED} = nD^2\sqrt{\frac{E}{\nu}}
\]

and the discharge factor,

\[
Q_{ED} = \frac{Q}{(D_2)^2\sqrt{E}}
\]

where \( D_2 \) is the reference diameter (the runner outlet diameter). The discharge coefficient, \( Q_{nD} \), is the ratio of \( Q_{ED} \) and \( n_{ED} \):

\[
Q_{nD} = \frac{Q_{ED}}{n_{ED}}
\]

\( Q_{nD} \) normalized to \( Q_{nD} \) at the BEP (\( Q_{nD}/Q_{nD}^* \)) is used to classify the flow topology at the runner outlet for normal operating conditions [15]. In hydraulic turbines, the Reynolds number is often defined with the runner rotation speed by

\[
Re = \frac{D_2^2\pi n}{\nu}
\]

where \( \nu \) [m²/s] is the kinematic viscosity. Without cavitation and at high Reynolds numbers, \( n_{ED} \) and \( Q_{ED} \) form a sufficient set of parameters to define operating points. In this paper, \( n_{ED} \), \( Q_{ED} \) and \( Q_{ad} \) at NL are identified with the index “NL” (\( n_{ED,NL} \), \( Q_{ED,NL} \), \( Q_{ad,NL} \)). The same dimensionless numbers at BEP are identified with the symbol “^” (\( n_{ED}^* \), \( Q_{ED}^* \), \( Q_{ad}^* \)). For the sake of comparison, dimensionless numbers at NL are normalized with the ones at BEP (\( n_{ED,NL}/n_{ED}^* \), \( Q_{ED,NL}/Q_{ED}^* \), \( Q_{ad,NL}/Q_{ad}^* \)).

2.2 Model measurements and data

The data used in this study were measured on two closed loop test stands (test stands #2 and #4) of Andritz Hydro Canada Inc. between 2010 and 2017 and cover seven Francis turbines with specific speed between 0.115 and 0.219. The data were measured at high Thoma numbers, \( \sigma \), with minimal or no cavitation. A list of Francis turbines studied in this paper is given in table 1.

| Name | \( n_{QE}^* \) | Test stand | Name | \( n_{QE}^* \) | Test stand |
|------|----------------|------------|------|----------------|------------|
| F7   | 0.115          | 4          | F32  | 0.219          | 4          |
| F8   | 0.136          | 4          | F36  | 0.140          | 4          |
| F9   | 0.117          | 4          | F38  | 0.179          | 2          |
| F15  | 0.204          | 2          |

All measurement procedures and equipment used on the test stands are calibrated in accordance with IEC 60193 standard [14]. During NL tests, the following parameters were recorded at a frequency of 1 Hz over at least 40 seconds for a constant guide vane opening:

- runner rotation speed;
- head;
- discharge using a venturi meter on test stand #4 and a magnetic flowmeter on test stand #2;
- water temperature (kept at 22°C ± 2°C by a cooling system).

The torque is measured by a digital torque meter composed of a sensor using strain gauges and a receiver-transmitter module mounted directly on the shaft. To perform NL tests, the runner rotation
speed is increased until the hydraulic torque is zero at a constant head value for a range of guide vane openings. The Reynolds number for all measurements presented in this study is above \(3.5\times10^6\), a value considered sufficient in the IEC 60193 standard to ensure that the results are independent of viscous effects for model acceptance tests.

Since the data at NL conditions were not intended for extensive analysis, limited information is available on measurement errors and uncertainties. Consequently, the uncertainty \(\varepsilon\) on \(n_{ED.NL}, Q_{ED.NL}\) and \(Q_{AD.NL}\) for both test stands was estimated by combining the worst possible calibration uncertainties with the worst repeatability errors encountered for each test stand. The uncertainties on \(n_{ED.NL}/n_{ED}^\wedge, Q_{ED.NL}/Q_{ED}^\wedge\) and \(Q_{AD.NL}/Q_{AD}^\wedge\) on test stand \#2 and test stand \#4 are presented in Table 2 where they are separated into \(Q_{AD}/Q_{AD}^\wedge\) ranges to consider the test stand responses to different flow conditions. Uncertainty on guide vane opening is evaluated to be less than 0.1° from experience.

**Table 2.** Total uncertainty on engineering quantities for Francis turbines on test stands \#2 and \#4 at NL conditions.

| Uncertainties \(\%\) | Test stand \#2 | Test stand \#4 |
|-----------------------|----------------|----------------|
| \(\varepsilon \; n_{ED.NL}/n_{ED}^\wedge\) | 0.84 | 0.44 | 0.48 | 0.48 |
| \(\varepsilon \; Q_{ED.NL}/Q_{ED}^\wedge\) | 2.86 | 1.48 | 2.16 | 1.25 |
| \(\varepsilon \; Q_{AD.NL}/Q_{AD}^\wedge\) | 2.98 | 1.54 | 2.21 | 1.34 |

3. Swirl during no-load operation

3.1 Conservation of angular momentum for no-load operation

There are some basic fluid mechanics laws and turbomachinery principles driving NL behavior. The Euler turbine equation states that the time averaged torque produced by the runner is equal to the change of angular momentum flux between the runner inlet (index 1) and the runner outlet (index 2):

\[
T_m = M_1 - M_2. \tag{6}
\]

In equation (6), \(M [\text{Nm}]\) is the angular momentum defined as follows:

\[
M_1 = \int_0^A (r \cdot C_u) \cdot \rho \cdot \vec{C} \cdot \vec{r} \cdot \vec{n} \cdot dA, \quad M_2 = \int_0^A (r \cdot C_u) \cdot \rho \cdot \vec{C} \cdot \vec{n} \cdot dA. \tag{7}
\]

In equations (7), \(r [\text{m}]\) is the radius coordinate, \(C_u [\text{m/s}]\) the circumferential velocity component, \(\vec{C} [\text{m/s}]\) is the velocity vector, \(\vec{n} [-]\) is the unit vector normal to a section and \(A [\text{m}^2]\) the area of a section. The locations of the runner inlet and outlet, on a meridional contour of a Francis turbine, are shown in figure 1. As the time averaged value of \(T_m\) in equation (6) is zero at NL, the time averaged value of the angular momentum upstream and downstream of the runner are the same but non-zero, \(M_1 = M_2\). A dimensionless form of the angular momentum is used in different studies [16–20] to characterize the draft tube surge. The angular momentum parameter, called \(RC_{u11}\), is defined by

\[
(RC_{u11})_1 = D_2 M_1 / (\rho Q^2), \tag{8}
\]

where \(i\) is equal to 1 at the runner inlet and 2 at the runner outlet. \(RC_{u11}\) is an appropriate way of quantifying the swirl at NL because it is equal at the turbine runner inlet and outlet, \((RC_{u11})_1 = (RC_{u11})_2\). Considering \(RC_{u11}\) is approximately conserved between the guide vane outlet and the runner inlet, it is possible to evaluate it by using the time averaged velocity triangles for a flow with negligible viscous effects at the guide vane outlet to obtain:

\[
RC_{u11} = \frac{1}{\sum_{i=1}^n} D_2 B_0 \cdot \frac{1}{\tan(\alpha + 0)}. \tag{9}
\]

In this equation, \(B_0 [\text{m}]\) is the distributor height. The average flow angle on a plane at a short distance downstream of the guide vane trailing edge is \((\alpha + 0) [^\circ]\), as shown in figure 2. The component \(\alpha [^\circ]\) is the guide vane opening angle and the component \(0 [^\circ]\) represents the deviation between the flow angle and \(\alpha\). Cassidy [16] uses \(RC_{u11}\) in the form of equation (9) with a different referential for the flow angle to study the draft tube surging in models only composed of guide vanes and draft tubes. Equation (9) is
valid at the runner inlet for any turbine operation, but for NL, it also represents the swirl level at the runner outlet since \((R_{c_{11}})_1 = (R_{c_{11}})_2\).

**Figure 1.** Meridional contour of a Francis turbine showing the guide vane (GV) outlet, the runner inlet (index 1) and the runner outlet (index 2) positions for the control volume of the formulation of the angular momentum equation.

**Figure 2.** Velocity triangle at runner inlet. \(C\) [m/s] and \(W\) [m/s] are the absolute and relative flow velocities. \(U\) [m/s] is the runner circumferential velocity.

**Figure 3.** Theoretical value of \(R_{c_{11}}\) with \(\theta=0^\circ\) according to equation (9).

### 3.2 Consequences of a large swirl variation along a no-load curve

Different swirl levels along a NL curve lead to different flow topologies in a turbine. For example, in propeller and Kaplan turbines, for NL operation at small guide vane openings (smaller than the one at the BEP), Skotak [13], Půlpitel et al. [21] and Houde et al. [11] observed, experimentally or numerically, that the dominant source of pressure fluctuations on the runner blades are associated with columnar vortices extending from the vaneless space, upstream of the runner, to the draft tube conical diffuser. In contrast, for larger guide vane openings (larger than or equal to the one at the BEP), Skotak [13] and Fortin et al. [6] observed that the dominant structure at NL is a single corkscrew vortex attached to the runner hub extending deep into the draft tube.

Also, at NL, a backflow develops in the middle of the draft tube. A higher swirl level, or a lower guide vane opening, means a larger backflow in the draft tube. As the guide vane opening increases, the swirl level at the runner outlet decreases and the backflow diameter decreases along with it [13,22].

### 4. Results and analysis

#### 4.1 Evolution of discharge and speed along no-load curves

The runner speed and especially the discharge do not evolve in the same way for every Francis turbines. For most of the turbines, there is a mix of two different trends, which will be presented in this section.
Figure 4. $\frac{Q_{\text{ED,NL}}}{Q_{\text{ED}}}^\alpha$ and $\frac{n_{\text{ED,NL}}}{n_{\text{ED}}}^\alpha$ as a function of guide vane opening (uncertainties are in table 2).

Figure 4 shows $\frac{Q_{\text{ED,NL}}}{Q_{\text{ED}}}^\alpha$ and $\frac{n_{\text{ED,NL}}}{n_{\text{ED}}}^\alpha$ along a NL curve for three Francis turbines of different specific speed, from $n_{\text{QE}}^\alpha=0.115$ to $n_{\text{QE}}^\alpha=0.204$. Considering the broad range of discharge and runner speed covered on a graph, the error bars are in a similar size than the markers on every curve. On figure 4, the discharge of each turbine evolves linearly with the guide vane opening with a correlation coefficient over 0.999 between the experimental data and the linear trendline. This linear relation between the discharge and the guide vane opening over the entire opening range is surprising considering that the flow topology and the dominant flow structures evolve along the NL curve (see section 3.2).

Not all turbines have a linear discharge evolution along a NL curve. In figure 5, the discharge evolution is only linear for the lower guide vane openings with correlation coefficients above 0.999. After a given opening, a transition appears. A grey zone on the graphs of figure 5 gives the approximate position of this transition region. After the transition, the discharge may be represented by either a linear equation for turbines F9 and F36 or by a second order polynomial equation for turbines F38 and F32 with regression coefficients above 0.999. The transition region exists for Francis turbines with low and
high specific speed (for example, F17 with $n_{QE} = 0.117$ and F32 with $n_{QE} = 0.219$), indicating that the effect is independent of specific speed.

![Graphs showing $Q_{ED.NL}/Q_{ED}$ and $n_{ED,NL}/n_{ED}$ as a function of the guide vane opening.](image)

**Figure 5.** $Q_{ED,NL}/Q_{ED}$ and $n_{ED,NL}/n_{ED}$ as a function of the guide vane opening (uncertainties are in table 2).
The graphs in figure 5 indicate that there seems to be a link between the evolution of the discharge and the runner rotation speed. A pronounced change of the runner speed evolution appears after the transition region previously identified on $Q_{\text{ED.NL}}/Q_{\text{ED}}^\wedge$ graphs. It is visible especially for turbines F9, F36 and F38, where the gradient of $n_{\text{ED.NL}}/n_{\text{ED}}^\wedge$ drastically drops after the transition guide vane openings. At this time, it is undetermined if the discharge choke and then causes a reduction of the runner rotation speed or if the saturation of the runner rotation speed causes a choking of the discharge. The existence of this tendency is interesting. For a given turbine, the particular geometrical features inducing this choking effect may potentially help to reduce the maximal runner rotation speed for the full-gate opening NL.

The discharge and the runner speed evolution along a NL curve have been compared for over 40 Francis turbines tested since 2007 at the laboratory of Andritz Hydro Canada Inc. In most of the cases, the discharge is not as linear as for turbines F7, F8 and F15. A transition zone appears as for turbines F9, F36, F38 and F32. However, the transition is less pronounced than for those turbines.

4.2 Link between runner speed and swirl

Figure 6 a) compares $R_{\text{Cu11}}$, evaluated from equation (9) with $\theta=0^\circ$, and the runner speed along the NL curve of turbine F38. Since the real value of $\theta$ is unknown, only the curve shapes are compared. A fast increase of the runner speed happens when the swirl level strongly drops at low guide vane openings. The swirl and the runner speed vary in opposition to one another. By comparing $R_{\text{Cu11}}$ and $1/(n_{\text{ED.NL}}/n_{\text{ED}}^\wedge)$, in figure 6 b) for F38, one can see that both curve shapes are highly similar. As shown in figure 7, other turbines show the same similarity between $R_{\text{Cu11}}$ and $1/(n_{\text{ED.NL}}/n_{\text{ED}}^\wedge)$. However, turbines with a linear evolution of the discharge with the guide vane opening (F7, F8 and F15) present slightly different evolutions between $R_{\text{Cu11}}$ and $1/(n_{\text{ED.NL}}/n_{\text{ED}}^\wedge)$.

![Figure 6. $R_{\text{Cu11}}$ as a function of the guide vane opening ($\theta=0^\circ$) compared to a) $n_{\text{ED.NL}}/n_{\text{ED}}^\wedge$ and b) $1/(n_{\text{ED.NL}}/n_{\text{ED}}^\wedge)$ for turbine F38.](image)

By definition, no-load conditions can be described by the following two statements:
1. The average positive and negative torque zones over the blades must be balanced to give a resulting torque equal to zero.
2. The swirl at the runner outlet, quantified by $R_{\text{Cu11}}$, must be equal to the one at the runner inlet.

To date, for NL, a complete characterization of the load repartition over the blades depending on the different flow topologies along NL curves for Francis turbines is missing in the literature. However, some studies give indications on how the different flow topologies at NL may affect the load repartition over the blades. According to Skotak [13] and Bettocchi et al. [22], the backflow size decreases along a NL curve, from low to large guide vane openings. At SNL, the backflow is known to enter the runner of Francis turbines [1,4,23–25]. The backflow affects the load distribution over the blade. Melot et al. [26] show the distribution of the blade loading with pressure curves at specific blade spans for a Francis turbine operated at SNL. The backflow generates a negative torque zone near the blade trailing edge,
towards the hub. For a guide vane opening near the one at the BEP, Hosseinimanesh [25] also identifies a negative torque zone near the blade trailing edge engendered by the backflow entering the runner of a Francis turbine. For a full-gate opening NL condition, Fortin et al. [27] show that the backflow in a Francis turbine decreases in size and does not necessarily enter the runner. Consequently, the backflow cannot produce a negative torque near the blade trailing edges for some full-gate opening NL conditions. The distribution of the positive and negative torque zones over the blades is different at full-gate opening NL than at SNL conditions.

| Condition | \( F_7 \) \( n_{Q_e} = 0.115 \) | \( F_9 \) \( n_{Q_e} = 0.117 \) | \( F_8 \) \( n_{Q_e} = 0.136 \) | \( F_36 \) \( n_{Q_e} = 0.140 \) | \( F_{15} \) \( n_{Q_e} = 0.204 \) | \( F_{32} \) \( n_{Q_e} = 0.219 \) |
|-----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( R_Cu_{11} \) | ![Graph](image1) | ![Graph](image2) | ![Graph](image3) | ![Graph](image4) | ![Graph](image5) | ![Graph](image6) |
| \( 1/(n_{ED,NL}/n_{ED}^*) \) | ![Graph](image7) | ![Graph](image8) | ![Graph](image9) | ![Graph](image10) | ![Graph](image11) | ![Graph](image12) |

**Figure 7:** \( R_Cu_{11} \) and \( 1/(n_{ED,NL}/n_{ED}^*) \) as a function of the guide vane opening.

According to statement 1 above, the averaged distribution of the positive torque and the negative torque over the blades must be balanced to result in a zero net motor torque. The NL runner speed for a given guide vane opening under a given head is reached when the equilibrium point is reached. Thus, the backflow size and the different flow instabilities evolving around the backflow affect the distribution of the blade loading along the NL curve. Understanding the dynamics between the swirl level, the flow
instabilities and the blade loading is the key to better understanding the runner speed evolution along a NL curve.

4.3 Impact of the dependency between the NL speed and the swirl on experimental and numerical studies at NL

The variation of the runner rotation speed related to the guide vane opening is larger for small openings, where the swirl level also changes rapidly (figure 6). Thus, a small change in the guide vane opening induces a larger variation of the runner speed around SNL than at full-gate opening NL condition. This entails that precision in the guide vane opening angle and measurement on a test stand is more critical near SNL than at full-gate opening NL. The same observation can be made for numerical simulations. A strong correlation on the guide vane opening and geometry between the physical and the numerical models must be achieved to have relevant numerical predictions at SNL.

5. Conclusion

This paper shows the evolution of the discharge and the runner rotation speed along the NL curve for seven Francis turbines covering a large range of different specific speed, from $n_{Qe}^a=0.115$ to $n_{Qe}^a=0.219$. For those turbines, the discharge evolves linearly along the NL curve either over the entire guide vane opening range or up to a transition zone. After the transition region, the discharge changes behavior but can still vary linearly with a different slope.

The NL runner speed increases inversely to the swirl with the guide vane opening. At low guide vane openings, when the swirl drops fast as the guide vanes open, the variation in the NL speed is the highest. The size of the backflow, also dependent on the swirl level, is suspected of playing a role in this link between the swirl and the NL runner speed. Numerical simulation of the flow for conditions along a NL curve could help in understanding how the backflow affects the blade loading and, consequently, the NL speed.

Thus, for low guide vane openings, such as for SNL, the runner speed is known to be highly sensitive to the guide vane opening. Consequently, for Francis turbines, the prediction of the SNL conditions on model turbines requires precise geometries and precise measurements of the guide vanes angles. This observation also translates to numerical simulations.

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