Nonlinear stochastic drillstrings vibrations: parametric study for stick-slip suppression

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Abstract. Unstable vibrations caused by the transition between static and kinetic frictions produces stick- slip motions which is one of the primary causes of poor drilling efficiency and mechanism failure in the drilling process. Coupled lateral and torsional drill string dynamics are explored numerically and experimentally within this article. A reduced order model able to capture the non- linear responses, observed during dry-friction whirling, is presented along with scaled experimental apparatus tests. To reduce stick-slip oscillations, the control strategies are mostly focused on the angular velocity of the upper part of the drill string, the torque on bit and the torque of friction. These parameters are investigated, and the simulation results are seen to be in good agreement with experimental observations.

1. Introduction

Rotary drilling systems employs rotational drill bit, which is one of the most important component impacting, directly, on the rock surface. Drill bit is driven from the surface by a slender structure of pipes, called the drill string, which transports the energy from the motor to the drill bit [1-3]. Structurally, the drill-string components are housed inside a drill pipe and includes the drill string, drill collar, and drill bit. Based on the functions of the drill collar and bit, the combination of these two parts is referred to as the Bottom-Hole-Assembly (BHA).

Most structural failures occurring on the BHA are due to bending and torsion motions experienced by the drill string. Downhole/ surface monitoring have indicated that drill string exhibits severe stick-slip vibrations causing violent torsional motion and whirling leading to lateral vibrations with large amplitudes [3-7].

Stick-slip phenomenon has been subjected to many prior efforts, including theoretical analysis and field measurement. Several models have been investigated to explain this type of dysfunction, such as the model of mass-spring on a moving belt [8] and the torsional pendulum model [9,10].

Drilling wells production have opened new challenges for experts to investigate the parameters and phenomenon enhancing drilling vibration and affecting drilling costs. Before the 1960’s, research efforts
were focused on the drillstring’s material strength components, the challenges have since changed to emphasize on its dynamic behavior [11]. As a result, various control strategies appeared to compensate the drill string stick-slip phenomenon, such as manipulating the top rotary system in soft manner, and thus, helping the absorption of torsional harmful waves at the surface. This solution has been used in different drilling wells and it has significantly reduced the bit stick-slip oscillations [12]. In order to understand the reasons of this type of vibrations, laboratory drilling experiments have been carried out [13] and theoretical models have been discussed. Kyllingstad and Halsey [14] used single Degree of Freedom 1DOF pendulum model in which the drillstring is assumed to be suspended at a rotary table. Constant coulomb friction torque is applied to the drill bit to analyze its stick-slip motion. Classic Coulomb torque model has provided a good estimation of torsional vibrations in many cases: Jansen and van den Steen [15] employed this model and divided the torque-on-bit (TOB) into static and sliding. Stick slip vibrations were observed for different conditions. Leine et al. used two-degree-of-freedom model to study stick-slip and whirl vibrations of the drill string. The BHA was modeled as a rigid disk at the end of a mass less flexible drill pipe. 

Based on drilling parameters supervision, many control methodologies have appeared to compensate drill string non-linear complex vibration. A workshop sponsored by the International Association of Drilling Contractors (IADC) regarding stick-slip mitigation was held in 2010 to help the petroleum industry.

In this work, stick-slip control and suppression is focused on. The organization of the article is as follow: in sec 2, a reduced order dry friction model of the drilling system using closed loop control system is proposed, numerical studies conducted with this model are presented. Simulation results are compared for different input parameters and presented in Sec 3. In Sec 4, qualitative comparison of torsional behavior between the experimental observations and numerical results is carried out, then, concluding remarks are collected in Sec.5

2. Dynamic model development

The drill string consists of the BHA and drill pipes screwed end to end to each other to form a long rod. As shown in Figure 1 (a), the BHA englobe the bit which is the cutting tool, the stabilizers, which prevent the drill string from unbalancing, and a connected pipe sections known as drill collars. The length of the BHA is almost constant, however, the length of drill pipes depends on the borehole depth. The drillstring is persistently driven by a rotating table from which the energy is transferred to the drill bit.

![Figure 1. (a) Schematic of Typical Drill String (b) Mechanical model of a drill string](image)

In this section, a discrete system is developed to model the qualitative dynamics causing stick slip motions. Same assumptions are those used in previous studies [8] are used. A representative mechanical model which can capture axial and torsional model in presented in Figure 1(b). The drilling system is
assumed to be a torsional pendulum system driven by a top motor described by a lumped parameter model with two-degree-of-freedom with an equivalent torsional stiffness $K$ and an equivalent rotational mass moment of inertia $I_b$. The interaction Bit-rock is modeled by a dry friction model. The presented model is similar to the ones given in the literature with the difference that, here, the TOB and the torque of drill string are considered, and their effects is analyzed. The equation of motion of the two-degree-of-freedom model of drill string can be written as:

$$J_d \ddot{\varphi}_d + k(\varphi_d - \varphi_b) + c(\varphi_d - \varphi_b) = T_M(t) - T_d(\varphi_d)$$ (1)

$$J_b \ddot{\varphi}_b + k(\varphi_d - \varphi_b) - c(\varphi_d - \varphi_b) = -T_b(\varphi_b)$$ (2)

Such as $\varphi_d$ and $\varphi_b$ are, respectively, the angular displacement of the rotary top driving system and the angular displacement of the BHA. $J_d$ is the drive moment inertia and $J_b$ is the inertia of the drill string associated with the bottom hole assembly described by:

$$J_b = \rho \frac{\pi}{96}[L_p(D^4_p - d^4_p) + L_c(D^4_c - d^4_c)]$$ (3)

The driving motor torque is presented by $T_M$, the dry friction torque by $T_b$ and the viscous damping torque by $T_d$, where:

$$T_b(\varphi_b) = c_b \dot{\varphi}_b + T_{fb} \text{sgn} (\dot{\varphi}_b)$$ (4)

$$T_d(\varphi_d) = c_d \dot{\varphi}_d + T_{cd} \text{sgn} (\dot{\varphi}_d)$$ (5)

The damping viscous coefficient associated with top driving system are given by the coefficients $c_d$ and $c_b$, $T_{cd}$ is the coulomb friction torque and $T_{fb}$ is the friction torque which is the variation of the stribeck torque with the static dry friction described as:

$$T_{fb}(X) = \begin{cases} T_e(X) & \text{if } \dot{\varphi}_b = 0 \text{ and } |T_e| \leq T_{sf} \\ T_{sf} \text{sgn} (T_e(\gamma)) & \text{if } \dot{\varphi}_b = 0 \text{ and } |T_e| > T_{sf} \\ T_{sf} \frac{\mu_b(\varphi_b)}{\mu_{sf}} & \text{if } \dot{\varphi}_b \neq 0 \end{cases}$$ (6)

Such as $X$ is the system state vector, $T_e$ is the external torque facing the static friction torque $T_{sf}$ to move the bit:

$$T_e(X) = c(\varphi_d - \varphi_b) + k(\varphi_d - \varphi_b) - c_b \dot{\varphi}_b$$ (7)

To describe the zero-speed friction, an exponential decaying law at the sliding phase is expressed by the dry friction coefficient:

$$\mu_b(\varphi_b) = [\mu_{cf} + (\mu_{sf} - \mu_{cf})e^{-\gamma_b|\dot{\varphi}_b|}] \text{sgn} (\varphi_b)$$ (8)

Where, $\gamma_b$ defines the velocity decrease rate ($0 < \gamma_b < 1$) and $\mu_{cf}$ and $\mu_{sf}$ are the coulomb and static friction coefficients.
\[ T_{sf} = r_b W_{ob} \mu_{sf} \quad T_{cf} = T_{sf} \frac{\mu_{cf}}{\mu_{sf}} \]  

(9)

where \( r_b \) is the bit radius and \( W_{ob} \) is the Work On Bit (WOB). \( \dot{\theta}_s \) is the stribeck speed and \( \dot{\phi}_b \in [-\dot{\phi}_s, \dot{\phi}_s] \).

From the damping spring model, the drill string torque and the TOB are obtained

\[ T_{DS} = K \int (\dot{\phi}_d - \dot{\phi}_b) \, dt + c \, \dot{\phi}_b \]  

(10)

\[ T_{OB} = T_{DS} - \mu_b (\dot{\phi}_b) \]  

(11)

3. Parametric studies and results

To understand the phenomenon of stick-slip, also known as micro-stalling, a close loop system has been developed. Interactive graphical editor blocks of MATLAB/ SIMULINK have been used to the numerical simulation of the proposed model given by equation (1) – (11).

| Parameter | Value | Units |
|-----------|-------|-------|
| \( D_r \) | 0.15  | m     |
| \( D_s \) | 0.18  | m     |
| \( L \)   | \( 2 \times 10^3 \) m |
| \( J_d \) | 500   | Kg m² |
| \( J_b \) | 300   | Kg m² |
| \( K \)   | \( 500 \times 10^3 \) Nm⁻¹ |
| \( c \)   | 0.1   | N.ms.rad⁻¹ |
| \( \mu_{sf} \) | 80    | -     |
| \( \mu_{cf} \) | 50    | -     |
| \( \Omega \) | 60-200 | RPM  |
| \( G \)   | \( 25 \times 10^9 \) Pa |
| \( E \)   | \( 70 \times 10^9 \) Pa |

Using linear stability analysis of the two-degree-of-freedom model, varied stability charts are obtained. Different torque profiles showing the stick slip vibration are presented below:

![Figure 2](image)

(a) (b)

Figure 2. Torque variation during stick-slip at \( \Omega = 100 \) RPM (a) Top drive motor torque (black line) and TOB (red line) (b) Toque of friction (blue line) and drill string torque (brown line).

Figure 2 shows the variation of the different system torques for stick-slip vibration. For the maximum of driving torque, an increased demand from the bit to achieve penetration, that cannot be met by the drilling
motor, causing a complete standstill during stick-slip, then, the bit alternately rotates faster than the driving speed, and the torque on bit increase rapidly which can damage the drill string.

During stick slip vibration, the dry friction curve, presented in Figure 2(b) shows a stribeck effect resulting on an instable steady rotation and uneven loading across the bit rock interface creating an increased friction forces.

Figure 3 illustrate phase plan simulations of the drill string stick-slip vibrations. The horizontal axis presents the twisting force of the drill string and the vertical axis presents the angular velocity of the bit. Initially, the bit is sticking, and the friction torque balances the drilling torque [1]. When the friction torque exceeds the maximum static friction, the bit starts sliding [2-3]. The friction torque decreases, and the bit speed reaches its maximum [4]. The behavior of the drill string dynamics under different driving conditions have been studied. The torsional drill string model discussed in the previous part has been used to investigate the influencing parameters on stick slip behavior. A range of selected parameters have been selected for different simulation runs.

3.1 The effect of driving speed on stick slip
Low and high motor/upper-hole driving speed values has been selected. The simulation shows the same trend for the different speeds, however, more the driving speed is high, smoother the drill string torque is. The stick-slip vibration are highly observed for low velocities. For low RPM’s, the bit motion become stationary for certain period (long sticking period). For high RPM’s the stick-slip vibration are transformed to regular torsional oscillations, the bit speed decreases without stoppage.

3.2 The Effect of stiffness on stick slip
The effect of torsional stiffness of drill string on bit-rock interaction was rarely investigated in the literature.

Figure 4. Effect of Torsional drill string stiffness on stick slip vibration Ω= 100 RPM (a) K=500Nm⁻¹ (b) K=10⁹ Nm⁻¹
As shown in Figure 4, the stiffness effect was tested for three different values of K using the base case parameters. The drill string torque and the TOB increase with high stiffness and stick-slip vibration are transferred to torsional rolling with more stability regions. From the simulation results it can be also noted that for high stiffness, the amplitude (peak to peak) of the drill string torque decreases which can reduce the severity of sticking.

4. Experimental Design and Arrangement

Laboratory scaled drilling apparatus has been constructed to simulate the working conditions experienced by drill string. The BHA was modeled by a rigid disk/rotor, the borehole by a rigid cylinder/stator and the drill string by a long aluminum shaft. The upper end of the shaft was driven by a servo motor with controlled rotation speed.

![Figure 5. 3D Design of the experimental apparatus](image)

4.1 Numerical/experimental comparative study

The intent of this part is to present the experimental observations and measurement to make model predictions for qualitative comparison with the numerical findings. Experimental data of the torsional strain during dry friction whirl for different drive speed has been recorded assuming that only the first torsional mode of vibration is excited, so that the strain measurement is directly proportional to the torsional displacement. For low driving speed, torsional vibrations are irregular with high fluctuations leading to sticking phases. However, when the table speed increase, these vibrations are converted to regular torsional oscillations over time.

![Figure 6. Experimentally observed torsional strain at Ω= 60 RPM (red line) and Ω= 100 RPM (black line)](image)

In fact, an increase in RPM leads to torsional strain reduction and so reduced torsional displacement/vibration. These results compare well with numerical simulation (sec. 3.1).
5. Concluding Remarks
In this paper, the authors have presented a dynamic two-degree-of-freedom model for torsional drill string focusing on bit-rock stick-slip interactions. Simulations were carried based on nonlinear differential equations considering nonlinear dry friction forces. Resulting periodic self-excited stick-slip vibration was observed in the bit, drill string and top torques. The influence of torsional stiffness and upper driving speed was studied. In order to explain the observed phenomena, experimental tests was carried out and qualitative comparison with the model predictions was done.

References
[1] Teale, R., 1965, The concept of specific energy in rock drilling, Int. J. Rock Mech. Min. Sci & Geom., 2, 57–73.
[2] Batako, A. D., Babitsky, V.I. and Halliwell, N.A., 2003, A self-excited system for percussive-rotary drilling, J. Sound Vib. 259, 97–118.
[3] Bailey, J.J. and Finnie, I., 1960, An analytical study of drillstring vibration. Trans. J. Eng. Ind., 82, 122-28.
[4] Denny, M., 2004, Stick-slip motion: an important example of self-excited oscillation, Eur. J. Phys. 25 311-22.
[5] Berloiz, A., Der Hagopian D., Dufour, J. R. and Draoui, E., 1996, Dynamic behavior of a drill-string: experimental investigation of lateral instabilities, J. Vib. Acoust. acoustics 118, 292–98.
[6] Macdonald, K.A. and Bjune, J. V., 2007, Failure analysis of drill strings, Engineering Failure Analysis 14, 1641–1666.
[7] Navarro-Lopez, E. and Cortes, D., 2007, Sliding-mode of a multi-DOF oil well drillstring with stick-slip oscillations. Proceedings of American Control Conference. New York City, USA, 3837-42.
[8] Liao, C.-M., Balachandran, B. and Karkoub, M., 2007, Drill String Dynamics: Reduced-Order Models., J. Vib. Acoust.2009-10339-349.
[9] Mihajlovic, N., Van de Wouw, N., Rosielle, P.C., and Nijmeijer, H., 2007, Interaction Between Torsional and Lateral Vibrations in Flexible Rotor Systems with Discontinuous Friction, Nonlinear Dyn., 50, 679-699.
[10] Leine, R. I, Nijmeijer, H., 2004, Dynamics and Bifurcations of Non-Smooth Mechanical Systems, Lecture Notes in Applied and Computational Mechanics, 18, Springer-Verlag, Berlin.
[11] Whittington, T. De Weegh, A. O. Penetration rates optimized by Aligning Torque and Rotary speed, J. Oil& Gas, 97, 55-57.
[12] Saukoine, P. Kmorthma, O., and white, D.B., 1992, A Field Method for Controlling Drillstring Torsional Vibrations, IADCLSPE Drilling Conference, New Orleans, Louisiana,43-45.
[13] Westermann, H., Gorelik, I. Rudat, J., Moritz, C. Neubaur, M., Wallaschrek, J., and Hon, O. 2010, A new test rig of experimental studies of drillstring vibrations, SPE Drilling & Completion, 30,119-128.
[14] Kyllingstad Edwards S., Lees A., and Friswell M.,1999, The influence of torsion on rotor/stator contact in rotating machinery. J Sound Vib.,767–78.
[15] Jansen, J.D., Van den Steen, L., 1995, Active Damping of Self-Excited Torsional Vibrations in Oil Well Drillstrings, J. Sound. Vib., 179, 647–668.

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