Multi-periodic variations in the last 104 years light curve of the symbiotic star BF Cyg

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ABSTRACT
We analyze a light curve of the symbiotic star BF Cyg, covering 114 years of its photometric history. The star had a major outburst around the year 1894. Since then the mean optical brightness of the system is in steady decline, reaching only in the last few years its pre-outburst value. Superposed on this general decline are some 6 less intense outbursts of 1-2 magnitude and duration of 2000-5000 days. We find a cycle of 6376 days, or possibly twice this period, in the occurrence of these outbursts. We suggest that the origin of the system outbursts is in some magnetic cycle in the outer layers of the giant star of the system, akin to the less intense 8000 days magnetic cycle of our Sun. We further find, that in addition to its well known binary period of 757.3 days, BF Cyg possesses also another photometric period of 798.8 days. This could be the rotation period of the giant star of the system. If it is, the beat period of these two periodicities, 14580 days, is the rotation period of a tidal wave on the surface of the giant. A 4th period of 4436 days, the beat period of the 14580 and the 6376 cycles is possibly also present in the LC. We predict that BF Cyg will be at the peak of its next outburst around the month of May in the year 2007. The newly discovered 798.8 days period explains the disappearance of the orbital modulation at some epochs in the light curve. The 757.3 oscillations will be damped again around the year 2013.

Key words: stars:oscillations-binaries:symbiotic-stars:magnetic fields-stars:individual: BF Cyg.

1 INTRODUCTION
Symbiotic stars (SS) are a class of variable stars consisting of a cool giant, a hotter object, either a hot subdwarf or a compact object, and an emission nebula.

The optical variability of symbiotics may take different forms and time scales. One form is of cyclic variations, due to the varying aspects of the revolving binary system, with or without an apparent eclipse in the light curve (LC). Binary periods of SS are of the order of 1 to a few years. Another type of variability has an explosive character, in the form of a single outburst, as for symbiotic novae, or multiple events. The time scales of these variations are quite long: the decay times of the outburst of symbiotic novae range between a few months to more than a century. The cool giant in some symbiotic systems shows also intrinsic variability such as radial pulsations of Mira-type.

Variability of SS light on short time scale of minutes and hours has not been reported much in the literature. Recently, however, such variations on this time scale, reminiscing the flickering phenomenon in cataclysmic variables, have been discovered in symbiotics (Sokoloski et al. 2001).

We have analysed anew the historical light curve of BF Cyg. This analysis resulted in the discovery of new elements in the long-term light curve of the star which may give us new clues on the nature of this system.

In this paper we present this analysis of the long-term light curve of BF Cyg, covering 114 years of observations. The characteristics of the symbiotic system BF Cyg are described in Section 2, and the data sets used on our analysis are reviewed in Section 3. In Section 4 we describe the time series analysis and the periodicities detected. In Section 5 we discuss the physical interpretation of these periodicities.

2 BRIEF DESCRIPTION OF BF CYG
BF Cyg is a very bright object (V ≃ 9). This makes it a popular target for observations on a wide wavelength range.

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This is also probably the reason why the record of measurements of its magnitude goes back in history for over 115 years (see below). Similar to many SS stars, and in particular to the prototype Z And, the long-term light curve of BF Cyg shows two kinds of optical variability. One is a regular periodic modulation, with large changes in its amplitude. The other type of the long range variability of this star is of explosive character, taking the form of repeating outbursts (see Figure 1).

The cool component is a fairly normal M5 giant and the system is classified as an S-type symbiotic with near-IR colors consistent with those of normal cool giants (Kenyon & Fernandez-Castro, 1987; Munari et al. 1992). The IUE ultraviolet continuum suggests the presence of a hot subdwarf with a temperature $T > 60000K$ (González-Riestra, Cassatella & Fernandez-Castro 1990).

Photometric variability with an apparent modulation period of 754 days was first noted by Jacchia (1941). The system has been extensively studied in the optical and the ultraviolet wavelength ranges (Mikolajewska et al. 1989; Fernandez-Castro et al. 1990; González-Riestra et al. 1990; Skopal et al. 1997). Its optical and ultraviolet emission lines vary in phase or/and in anti phase with the photometric minima. The orbital nature of this variation is confirmed by infrared radial velocities data and the spectroscopic orbital period is 757.2, nearly identical to the photometric one (Fekel et al. 2001).

3 THE LONG-TERM LIGHT CURVE OF BF CYG

We collected data from three large photometric measurement sets retrievable for this system, in order to reconstruct its historical light curve. A regular photometric monitoring of BF Cyg for the years 1890-1940 is available from the Harvard plates (Jacchia, 1941). While few plates were collected in the first ten years, the data are more frequent after the year 1900.

A second data set is composed of the photographic measurements of Skopal et al. (1995).

A third large data set is the visual magnitude estimates collected by the American Variable Stars Association (AAVSO). These are often daily estimates of the magnitude of the system. For homogeneity we averaged the AAVSO data over a time interval of 24 days, similar to the sampling interval of the Jacchia (1941) data set. The AAVSO at our hands is updated to Dec 2004.

In Fig. 1 we present the long-term light curve (LC) of BF Cyg from 1890 up to the present. In this figure, the AAVSO data have been scaled to the photographic scale, with a temperature $T > 6000K$ (González-Riestra, Cassatella & Fernandez-Castro 1990). The IUE ultraviolet continuum suggests the presence of a hot subdwarf with a temperature $T > 60000K$ (González-Riestra, Cassatella & Fernandez-Castro 1990).

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4 TIME SERIES ANALYSIS

The time series that we analyze in this work is the fading branch of the light curve of BF Cyg that follows the outburst of the star in 1894. This means that the few old measurements of the star magnitude prior to this event have been discarded. We have detrended the series from the long-term fading effect by subtracting from it a polynomial of 3d degree that was fitted to the data by the least squares method. The dots (photographic magnitudes) and crosses (scaled visual magnitudes) in Figure 2 are the detrended data points and they constitute the LC that we shall refer to in this work. As a first step we computed the power spectrum (PS) of

Table 1. The m$_{pg}$ or scaled m$_v$ data of Fig. 1

| Julian day | magnitude |
|-----------|-----------|
| 2411547.9 | 12.22     |
| 2411570.8 | 12.70     |
| 2411616.4 | 12.30     |
| 2411684.9 | 11.87     |

The system underwent a dramatic 3.5 magnitudes brightening event around the year 1894. This outburst is followed by slow fading of the star that continues until the very present time. This behavior is reminiscent of that of the small class of symbiotic novae, that have one single major outburst recorded in their historical light curve. Actually, there is hardly a typical light curve of symbiotic novae, and the fading from the outburst shows a large variety of behavior. It can be gradual, such as in RR Tel or AG Peg, or characterized by large light oscillations as in V1329 Cyg or by a deep minimum as in PU Vul (Vietti, 1993).

In BF Cyg, a few major events of sudden brightening of the system by 1-2 magnitudes are superposed on the decline from the major 1894 outburst. The duration of such an explosive event may last a few years, and may include episodes of short term flares of brightening by a few tenths of magnitude lasting a few days. In between outbursts, the system exhibits periods of relative quiescence that last for a few years. In the last eight years BF Cyg is in one of these quiescence states, while the system is now back to the brightness level measured by Jacchia (1941) on the few Harvard patrol plates obtained in the year 1890 before the large outburst.

A periodic oscillation of about 754 days was already recognized by Jacchia (1941), with amplitude $\sim 1$ magnitude. Throughout the years other values of the photometric orbital periodicity have been suggested by various investigators. For example P=757.3 days (Pucinskas, 1970), P=756.8 days (Mikolajewska et al. 1989), P=757.2 days (Fekel et al. 2001). As already noted by Jacchia (1941), however, there were epochs in the history of the star, during which the binary modulation was very small, sometime nearly disappearing entirely from the LC.
Multi-periodic variations of BF Cyg

Figure 1. A 114 years light curve of the symbiotic star BF Cyg, from the year 1890 up until Dec 2004. Dots refer to \( m_{\text{pg}} \) and crosses indicate visual data transformed to \( m_{\text{pg}} \). The solid line is a 3d degree polynomial, and a 9 term harmonic wave based on 3 independent periods, fitted to the data by least squares. See text for further explanations.

Figure 2. Dots (mpg magnitudes) and crosses (visual transformed to mpg) represent the detrended LC of BF Cyg that is analysed in this work. Solid line is a running means curve over 757 days of the data points.

Figure 3. Power spectrum of the light curve of BF Cyg shown in Figure 2. Insert is the window function.

this LC (Scargle 1982). Figure 3 displays this PS in the frequency range corresponding to the period interval between 40000 and 100 days. The upper bound is the entire length of the LC. The lower bound is the period corresponding to one half of the Nyquist frequency of the series. The insert shows the window function created by the non uniform sampling of the LC by the available measurements. Two groups of high peaks are clearly identified in the PS. One is at the red end of the spectrum, around the frequency .00015 days\(^{-1}\), and the other around 0.0013 days\(^{-1}\). We shall discuss them in turns in the next two sections.

4.1 Periods between 1000 and 40000 days

Figure 4 is a blowup of the PS shown in Figure 3 in the period range between 1000 and 40000 days. We also considered a time series of 46 points obtained from the observed LC by binning it into 46 bins of 757 days width. A third LC that we also considered is the running mean shown in Figure 2 as a solid line. These two LCs have identical PS to that shown in Figure 4. Table 2 lists the frequencies, in days\(^{-1}\), of the highest peaks seen in Figure 4, as well as their corresponding periods, in days, and the corresponding peak power.

The highest peak (a2) in Figure 4, corresponding to the period of \( \approx 6400 \) days represents a periodicity in the outburst events of the star that is also apparent in Figure 1. This periodicity is well established at a high level of statistical confidence as demonstrated in Appendix A (available electronically).
The significance of the P5 periodicity will become apparent in Section 4.2. We show there that a similar relation 1/P2-1/P3=1/P5 exists between P5 and two other prominent periodicities in the LC of the star, P2 and P3. The peak marked b'/a4' is an alias of its two neighbours on both sides, due to the gap in the data, as discussed above.

Further evidence to the validity of our interpretation of the PS seen in Figure 3 is given in Appendix B (available electronically).

By least squares fitting, which will be described in the next section, we obtain the following best estimates of the values of the 3 periods P1, P4 and P5: P1=12750±400 days, P4=4436±40 days, P5=14580±400 days.

Figure 5 presents the detrended LC of BF Cyg (Figure 2), folded onto the period P1. The figure presents the cycle twice. The presence of the P1 peak in the PS is due to the apparent alternating height of the 6 recorded successive outbursts, as presented by the two unequal maxima in the P1 cycle.

The last recorded outburst reached its maximum light around JD 2448040. Based on our best fit values of the 3 periodicities P1, P2 and P3 (see Section 4.2), we predict that BF Cyg will reach the formal maximum point of its next outburst around JD 2454236. In practice it means that the system will be found at the height of an outburst in mid-year (~May) 2007.

### 4.2 Periods between 200 and 1000 days

Figure 6 is a blowup of the PS shown in Figure 3 in the period range 500 to 1000 days. Among the four peaks that dominate this figure, the one marked p2 corresponds to the known binary period of the system P2=757 days. The peak marked p3 corresponds to the period P3=798 days. The two unmarked neighbors of the p2 peak correspond to the periods 777 and 738 days. They are aliases of the p2 peak, created by the gap in the distribution of the observed points, discussed in Sections 3. Detailed explanation of this claim is given in appendix C (available electronically).

Best estimates for the value of the periods P2 and P3, as well as of P1, P4 and P5, were obtained in the following way. We present the observed LC by a 9 term Fourier series, consisting of the P1 period with its first 5 higher even harmonics and the P4 periodicity, along with the P3 and the P2 periods. This presentation has only 3 free parameters,
the values of P1, P2 and P3. The values of the other 6 periodicities are fixed by these 3. We find the value of these 3 parameters that give the 9 term series that fits best the observed LC in the least squares sense. The values of P1, P4 and P5 so obtained were given in Section 4.1. For P2 and P3 and their ephemeris we obtain:

\[
\begin{align*}
\text{Min}(P2) &= JD 2451443 \pm 20 + (757.3 \pm 0.9) \times E \\
\text{Min}(P3) &= JD 2451531 \pm 23 + (798.8 \pm 1.4) \times E
\end{align*}
\]

Here the JD numbers are times of minimum light and the error figures indicate half of a 95% confidence intervals around the corresponding values. They are computed by bootstrap (Efron and Tibshirani 1993) as detailed in Appendix D (available electronically).

Our P2 period is identical to the period already found by Pucinskas (1970), \( \text{Min}= JD 2415065 + 757.3 \times E \) and confirmed as the orbital one by Fekel et al. (2001).

**4.2.1 Further discussion of periods P2 and P3**

In order to better analyze the photometric variability of the star in the frequency range of the binary cycle 1/P2, we removed from the observed LC all the variations on time scale shorter than the P2 period. This was done by computing the running mean curve of the observed LC, averaging all points within a window of width 757 days. This curve is shown as a solid line in Figure 2. The dots in Figure 7 are the residuals of the observed LC after removing the running mean curve from it. The solid line in Figure 7a represents a 2 term harmonic series of the P2 and P3 periods.

In order to enable detailed examination, Figure 7b and 7c present, respectively, zooms on the first and on the second subsections of the data set of Figure 7a. Figure 7d is a blowup of the last 5000 days of the light curve. The second, thin solid curve is the best fitted harmonic wave to the data, with Fekel’s et al. (2001) binary period derived from radial velocity measurements.

Figure 7 demonstrates that the P2 orbital modulation, as well as the P3 oscillations, are present in the LC at quiescence states, as well as during the outburst events. Except for the modulation due to the beat phenomenon, the structure and amplitude are independent of the luminosity of the system. In particular, the apparent binary variation has the same amplitude at outburst maximum as during quiescence. It has also at the present epoch the same average magnitude as it had nearly a century ago, when the star was 3 magnitudes brighter.

The oscillations of the system with the P3 periodicity, simultaneously with P2, explain in particular the epoch in the history of the star, around JD 2427400, at which the 757 days oscillations have all but disappeared from the light curve (see Figure 7b). This phenomenon was already noted by Jacchia (1941) more than 60 years ago. A similar episode of a nearly disappearance of the orbit modulation is apparent again around JD 2442000. At this time the system was at the middle of one of its outburst events (Figure 2) and the intense activity of the star is masking the phenomenon to some extent, but it is still noticeable in Figure 7c. These two epochs of near disappearance of the 757 days variability are nodes of the beat between the P2 and P3 oscillations. The beat period is P5=14577, which is equal to the cycle of the beat between the 6376 and the 4436 periods discussed in Section 4.1.

Based on our ephemeris of the two neighboring periods P2 and P3 we predict that the 757 variability of the system will be damped again around the year 2013.

The presence of the second period P3 in the LC in addition to P2 also provides an alternative explanation to the apparent shift in some of the minima of the binary cycle. Jacchia (1941) already noted that an O-C diagram indicates an apparent change in position of minima, which he thought to be correlated with the brightness of the star. Skopal (1998) reports on a similar effect in more recent parts of the LC. Skopal et al. (1997) report also on the presence of a possible secondary minimum in some of the 757 days cycles of the star.

Figure 7 a,b and c show that most of these variations in the profile structure of the \( \sim 757 \) days oscillations find a natural explanation in the ever changing phase difference between the P2 and P3 frequencies. The figures show that not only the two phases of damped oscillations are well explained by the contribution of the P3 periodicity. At epochs of quiescence of the star, when the \( \sim 757 \) days oscillations are not perturbed by an outburst activity, e.g. at JD 2415000 < t < JD 2418000 or JD 2448000 < t <JD 2453000, the 2 periods LC fits the detailed structure of the observed oscillations quite well. It traces faithfully the minima of the oscillations and it also gives an apparent second minimum to some of the cycles. The interference of the two periods may also explain the rather different values that investigators of this star have derived from photometric time series at different epochs in the history of the star (Jacchia 1941; Skopal 1998).

In particular we point out, that due to this interference, the minimum brightness times of individual cycles of the combined periodicity do not fall necessarily at the minimum times of the binary period. Therefore, the times of least brightness do not overlap exactly the times of inferior conjunction of the giant star. One example is near the conjunction JD 2451395.2, determined by Fekel et al. (2001) from radial velocity curves. From our synthetic LC we find that minimum light of the system was reached only 48 days later, on JD 2451443. From the presently available observed data, it is difficult to determine the time of minimum light with the accuracy required to distinguish between these 2 dates.

Figure 7d shows the last 5000 days of the observed LC, along with our best fit 2 periods curve. The second, thin solid line is the best fit to the data of a harmonic wave with
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Figure 7. Dots in all frames are the residuals of the observed light curve after removing the running mean curve obtained with a 757 days wide window, shown as solid line in Figure 2. (a) Solid line is the best fitted 3 terms series with the periods P2, P3 and P2/2. (b) Zoom on the first section of the plot in frame a. (c) Zoom on the second section of plot a. (d) Blowup of the last 5000 days of the light curve. The second thinner solid line is the best fitted harmonic wave with the Fekel’s (2001) period of 757.2 days derived from radial velocity data.

Fekel’s period of 757.2 days. Figure 7d seems to indicate that the data support the notion that the later of the two dates, resulting from our 2 periods presentation of the LC is closer to the true minimum of the system.

We may summarize this section with the claim that all the gross features of the 1894-2004 LC of BF Cyg, following its outburst of 1894, are well accounted for by a slow continuous decay, and 3 distinct periods P1=12750 days (+5 even harmonics), P2=757.3 days and P3=798.8 days. A P4=14580 days period is also of significance although it is not presented explicitly in the LC. It is the beat of the P2 and P3 periods and it is manifested in the LC through the appearance of the P5=4436 d periodicity. The solid line in Figure 1 is a plot of the 9 term series of the 4 periods P1, P2, P3 and P4, with their corresponding harmonics, superposed on a 3d degree polynomial representing the slow decay from the 1894 outburst.

5 DISCUSSION

5.1 The outbursts cycle

The 6 recorded outbursts of BF Cyg in the last 104 years occurred with a constant time interval of ~6376 days between them. Periodic or quasi-periodic variations in the light of stars, with periods of thousands of days and amplitudes of 2 or 3 magnitudes are known to exist in the class of semi-regular giants (Mattei et al. 1988; Kiss et al. 1999).

It seems, however, that the 6376 days periodic variability of BF Cyg is of a different nature. The structure of the LC shows the characteristics of outbursts, rather than of pulsations and so it is indeed interpreted by most researchers in the field (e.g. González-Riestra et al. 1990; Skopal et al. 1997).

In an attempt to understand the origin of the outbursts phenomenon and the nature of the clock that regulates their appearances, the clock of the solar cycle, with its similar time constant of ~8000 days, comes to mind. We hypothesize that the origin of the cyclic outbursts of BF Cyg lies in some magnetic activity, driven by a magnetic dynamo process in the outer layers of the cool giant component of this stellar system. It is known that the 11/22 year solar cycle modulates the mass flux of the solar wind, among other measured parameters of the Sun. The basic process originates in an interaction between differential rotation and convection motions (Babcock 1961; Ulrich & Boyden 2005).

Solar-like cycle in Asymptotic Giant Branch stars has been proposed by Soker (2000) as an explanation to the morphology of a few planetary nebulae. Soker (2002) has also proposed that giant stars that are members of symbiotic systems may harbor a magnetic dynamo process.

The giant in BF Cyg may indeed possess one important ingredient of such a process, namely, a relatively fast rotation. It is classified as a M5 star (Kenyon & Fernández-Castro 1987; Mürset & Schmid 1999). No data are available for the rotation velocity of such late type stars in the De Medeiros et al. (2000) tables, but from the general trend of the velocity distribution of cool stars, very small mean velocity (2-3 km/sec) is expected. Fekel et al (2001), on the other hand, derive a projected rotational velocity of the giant in BF Cyg of ~4.5 kms⁻¹. This makes the giant of BF Cyg a fast rotator relative to field giants of similar spectral type. We note also that differential rotation, another important ingredient of the magnetic dynamo mechanism,
has been recently measured in some active K-type giants (Weber, Strassmeier & Wasuettl 2003).

In the magnetic dynamo scenario, the repeating intensifications in the optical luminosity of the system that take the form of the outbursts, are due to periodic enhancement of the stellar wind from the cool giant of the system, regulated by this dynamo process. This results in an enhancement of the mass accretion rate onto the compact star of the system. The intense optical luminosity originates in the vicinity of the hot component, probably in a bloated gaseous shell around the WD star (Munari 1989).

We proposed in the past a similar solar-like cycle as an explanation for the ~ 8400 days (~23 years) cycle that we discovered in the LC of another symbiotic star - Z And (Formiggini & Leibowitz 1994). A comparison between BF Cyg and Z And reveals similarities between these two systems and in their behavior both in quiescence and at outbursts. Their cool component is a M5 star (Mürset & Schmid 1999) and the orbital period is almost the same: 758.8 days for Z And (Formiggini & Leibowitz 1994) and 757.3 for BF Cyg (section 4.2). Both systems belong to the classical symbiotics family, and during outbursts their hot component seems to maintain a constant bolometric luminosity while expanding in radius (Mikolajewska & Kenyon 1992).

One difference between the 2 systems is that in addition to its sequence of 6 "small" outbursts, BF Cyg underwent the dramatic 1894 event of large outburst, quite distinct from the 6 that followed. No such event has been recorded in the history of Z And. In view of the 1894 event, BF Cyg should perhaps be classified as symbiotic nova, as already suggested by Skopal et al. (1997). That event seems to be of the scale and nature that are different from those of the small outbursts. The abrupt increase by 4 magnitudes in its luminosity, and the slow decay lasting over 100 years afterwards, are probably a signature of a thermonuclear runaway process, similar to the events that characterize systems such as AG Peg, V1016 Cyg as symbiotic novae (Mikolajewska & Kenyon 1992).

5.2 The brightness oscillations

Near the P2=757.3 days binary period of BF Cyg we discovered the period P3=798.8 days. This could be the periodicity of oscillations of the M giant star of this system. However, we consider this interpretation unlikely for the following reason. The giant of BF Cyg is not a Mira type star. It does not show modulations in the near infrared as for Mira. In the (J-H) vs. (H-K) color diagram (Whitelock 1994) it does not lie in the region occupied by Miras.

We suggest that the P3 periodicity is the rotation period of the giant of the system. The rotation of the giant component of SS’s was already discussed in the literature (e.g. Munari 1988). In general, the giant stars in symbiotic systems are assumed to rotate synchronously with their binary revolution. This assumption is based on theoretical considerations (Zahn 1977), taking into account the synchronization time scale expected from the values of the radii, masses and binary separations that are typical of symbiotics on one hand, and the estimated ages of these binary systems on the other. The period P3 is close enough to P2 and therefore would not be an unusual exception to this commonly believed rule.

Modulation of the star light at the rotation period of the giant is indeed expected in models explaining the binary photometric variations on the basis of the reflection effect (Kenyon 1986, Formiggini & Leibowitz 1990). If the giant star is not strictly isotropic in its photometric characteristics, e.g. if there are dark spots on its photosphere or if there are areas of different reflectivity on its surface, rotation of this star will modulate the luminosity of the system in the rotation frequency.

Modulations at the rotation frequency are expected also in models whereby the photometric binary variations are due to variation in the optical depth towards the main light source of the system that are coupled to the orbital revolution. In such models the WD and its close vicinity are seen by the observer through different regions of the stellar wind of the giant star (Gonzáles-Riestra et al. 1990). The stellar wind of the giant is not necessarily isotropic with respect to the giant center. The non isotropic structure of the wind may also be coupled to the giant rotation through the effect of the stellar magnetic field. In such a case, the opacity toward the main light source of the system will vary at the giant rotation frequency, in addition to its variation at the binary frequency.

The P5=14580 days periodicity is the beat period of the binary orbital period P2 and the P3 period. If the latter is indeed the giant rotation period, P5 is the period of the tidal wave that propagates in the outer layers of the giant, in the coordinate system of the rotating star. We suggested above that a magnetic dynamo process may be the driving mechanism responsible for the outbursts cycle. It is not unreasonable to speculate that a tidal wave in differentially rotating convecting layers may also modulate the magnetic field of the star at the tidal wave frequency, in addition to the modulation by the main cycle of the dynamo. The apparent P4 periodicity, if found to be real, may be the beat period of these two magnetic cycles in the outer layers of the giant.

Finally we note that as mentioned in Section 4.2.1 the structure and the amplitude of the P2 and P3 oscillations are independent of the average luminosity of the system. In particular it appears that the amplitudes of these modulations at maximum light of outbursts are not significantly different from those during quiescence states of the system.

This fact is consistent with the reflection interpretation of the binary modulation (Kenyon 1986, Formiggini & Leibowitz 1990). To first approximation the reflection and the reprocessing of the hot component’s radiation in the giant atmospheric layers, give rise to optical luminosity that is a given fraction of the hot component luminosity. An increase in the later will result in the same relative increase in the luminosity of the heated hemisphere of the giant, hence the independence of the amplitude of the P2 variations, expressed in magnitude units, on the luminosity of the system.

This independence is also consistent with the modulating optical depth interpretation. A given optical depth of an absorbing layer reduces the flux of the emerging radiation by a given percentage of the flux of the radiation impinging on it, i.e. it has the same amplitude in magnitude units.
6 SUMMARY

We identify 3 independent periodicities in the LC of the last 104 years of the symbiotic star BF Cyg. One is a 6376 days (or twice this value) periodicity in the occurrence of outbursts on the decaying branch of the luminosity of the system, following its major outburst in 1894. The second is the well known binary period of the system of 757.3 days. The third one is 798.8 days, which we suggest is the rotation period of the giant star of this symbiotic system. A 4th period which is a beat of the 6370 days cycle with the beat period of the giant star of this symbiotic system. A 4th period which is a beat of the 6370 days cycle may also be present in the light curve.

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APPENDIX A: ESTIMATION OF CONFIDENCE LEVEL FOR P~6400 DAYS

We applied the bootstrap statistical test (Efron & Tibshirani, 1993 and see the Appendix D) on the 46 points LC mentioned in Section 4.1, which averages all variations on time scale shorter than 757 days. We find that even in that LC of merely 46 data points, the probability to obtain at random a distribution with a PS that has a peak as high as the one seen in Figure 4 is less than 1/300. The periodicity of ~6400 days in the occurrence of outbursts of BF Cyg is therefore well established at a high level of statistical confidence.

APPENDIX B: EVIDENCE FOR ALIASES AROUND THE FREQUENCY .00015 DAYS⁻¹

We also create an artificial LC by superposing the P1 period with its 5 higher even harmonics, and the P4 period on random noise with a standard deviation that is equal to that of the real data. We sample this LC at the times of the real data. The PS of this time series is very similar to that of the real data, including the high alias peaks a2' and b'/a4' seen in Figure 4.

APPENDIX C: EVIDENCE FOR ALIASES AROUND THE FREQUENCY .00013 DAYS⁻¹

We create an artificial LC consisting of a 799 days and 757 days harmonic waves superposed on white noise, and by sampling it at the times of the real LC. The PS of this time series is similar to that of the real data. In particular, it contains the two prominent unmarked peaks as in the real data, in addition to the high peaks corresponding to the two planted periods p2 and p3.

We also computed separately the PS of each of the two subgroups of data points that constitute the LC seen in Figure 2. The first group, LC1, consists of all points at times smaller than JD 2429986. The second group, LC2, consists of all points with t>JD 2434486. Each of these 2 LCs covers less than half the duration of the entire LC and has about half the number of data points of the complete LC. The uncertainty in the frequency of the highest peaks in the PS of LC1 and LC2 is therefore larger than in the whole LC. The two highest peaks in the PS of LC1 correspond to the periods 761 and 813 days, which within uncertainties of less than 2 sigma, as determined by the bootstrap method (see Appendix D), are consistent with the P2 and P3 periods. Likewise, the highest peaks in the PS of LC2 correspond to the periods 791 and 754 days, which are also consistent with the P2 and P3 periodicities.

The presence of the two periodicities in each half of the data set is evidence to the reality of these two periods in the LC of the star. Furthermore, the prominent unmarked peaks in the PS of the full LC seen in Figure 6 are absent from the PS of either of the two halves of the data set. This demonstrates that the unmarked high peaks are indeed aliases of the p2 and p3 frequencies, created by the gap in the data.

APPENDIX D: ESTIMATION OF THE UNCERTAINTY IN THE FREQUENCIES OF PEAKS IN A PS

The estimate of the uncertainty in the values of the components of the vector $F$, consisting of frequencies of oscillations identified in an observed light curve $(t, y)$ is based on the statistical "bootstrap" technique (Efron & Tibshirani 1993). Let $W_F(t)$ be a harmonic series of the components of the frequency vector $F$. We find by least squares the linear coefficients with which $W_F$ is best fitted to the observed $y$ values. For each point $(t_i, y_i)$ we obtain the residual $d_i = y_i - W_F(t_i)$. We now create a pseudo-observed LC, by adding to each synthetic value $W_F(t_i)$ an element $d_i$ that is randomly selected from the population $d$ of the residuals. By least square fitting we find the vector of frequencies $F'$ for which the series $W'_{F'}(t)$ fits best to the observed $y$ values. After repeating this process $N$ times, we have for each element $F_k$ of the frequency vector $F$, a sample of $N$ values of $F'_k$. In the histogram of all $F'_k$ values we find the narrowest interval that contains the fraction $q$ of all $N$ $F'_k$ values. We consider this as the full uncertainty interval $\Delta F_k$ of statistical significance $q$ around the frequency $F_k$. The corresponding interval for the period is $\Delta P_k = P_k \times \Delta F_k$.

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