Nonlinear field-dependence of the Imaginary Squashing Mode of superfluid $^3$He at moderate magnetic fields

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Abstract. We present the results of transverse acoustics studies in superfluid $^3$He-B at fields up to 0.11 T. Using acoustic cavity interferometry, we observe the acoustic Faraday effect for a transverse sound wave propagating along the magnetic field, and we measure Faraday rotations of the polarization of the sound up to 2070°, significantly more extensive than has been previously seen. We use these results to extend previous calculations of the Landé $g$ factor of the imaginary squashing mode (ISQ), an order parameter collective mode with total angular momentum $J = 2$. We report nonlinear field splitting of the ISQ at fields as low as 0.05 T.

1. Transverse sound

Since its discovery in 1972, superfluid $^3$He has been widely studied using a variety of methods. A $p$-wave spin-triplet superfluid, $^3$He has many analogues to unconventional superconductors, and the underlying structure of its order parameter is of great interest. Among the tools that can best probe the order parameter is collisionless transverse sound, first predicted in $^3$He by Landau in 1956 [1] but not seen until further predictions by Moores and Sauls in 1993 [2]. The latter prediction showed that transverse sound couples to a collective mode of the order parameter, the $J = 2$ Imaginary Squashing Mode (ISQ), with angular momentum $J = 2$. This coupling, and the presence of transverse sound in superfluid $^3$He, was proven in 1999 by Lee et al. [3].

From that point on, transverse sound has been an excellent tool for probing the structure of the order parameter, particularly the collective mode spectrum focusing on the ISQ.

Due to its sensitive coupling, transverse sound is an excellent probe of any changes to the ISQ, such as those imposed by a magnetic field. As shown in Fig. 1a., the ISQ, with a zero-field energy of roughly $\sqrt{12/5}\Delta$, where $\Delta$ is the gap, splits into 5 Zeeman substates in a magnetic field, of which only two couple to transverse sound [2]. It was this splitting that proved the existence of transverse sound in [3], as right circularly-polarized (RCP) and left circularly-polarized (LCP) sound couple to opposite Zeeman states $m_J = \pm 1$, causing acoustic birefringence. When the field strength increases, the velocities of RCP and LCP split, resulting in a rotation of the sound polarization, a phenomenon known as the acoustic Faraday effect (AFE). As the transducers used to excite transverse sound into the superfluid have a single polarization axis, this rotation is seen as a sinusoidal envelope on the acoustic signal.
Figure 1. a. Schematic of collective mode energy spectrum, showing the $J = 2^-$ ISQ mode relative to the full gap, along with its field splitting. b. Energy spectrum for pressure sweep experiments, showing the sound frequency (red) crossing first the ISQ mode frequency (blue), then the pair-breaking frequency (green dashed); all frequencies are normalized to the zero-temperature gap.

With linear field effects taken into account, the coupling is described by the following dispersion relation, given by Sauls et al. [4]:

$$\frac{\omega^2}{q^2 v_F^2} = \Lambda_0 + \Lambda_2 - \frac{\omega^2 - \Omega_2^2}{q^2 v_F^2} - 2m_J g_{2-} \gamma_{\text{eff}} H \omega,$$

(1)

where $\omega$ is sound frequency, $q$ is the wavevector, $v_F$ is the Fermi velocity, and $\Omega_2 = a_2 - \Delta$, with $a_2 \simeq \sqrt{12/5}$. $\Lambda_0 = \frac{F_n^2}{15}(1-\lambda)(1 + \frac{F_s^2}{5})/(1 + \lambda F_s^2/5)$ is the quasiparticle restoring force, $\Lambda_2 = \frac{2F_n^2}{15} \lambda (1 + \lambda F_s^2)/(1 + \lambda F_s^2)$ is the superfluid coupling strength; the $F_n$ are Landau parameters, and $\lambda$ is the Tsuneto function. $\gamma_{\text{eff}}$ is the effective gyromagnetic ratio of $^3\text{He}$, $H$ is the external magnetic field, and $g_{2-}$ is the Landé $g$-factor. Nonlinear field effects on the ISQ are expected to come in as quadratic corrections to the denominator of Eq. 1, expressed as $-AH^2$; such effects have been seen at fields above 0.1 T using longitudinal sound [5], but without great precision, insufficient to quantify the onset of nonlinearity, or more than its rough magnitude. Subsequent transverse sound studies explored lower fields, below 0.04 T, and saw only linear effects [3, 6]. Our experiments probe the intermediate field region to explore the onset of quadratic behavior, and also to further explore $g_{2-}$.

2. Methods

Eq. 1 depends implicitly on temperature and pressure, and, with the explicit dependencies of $\Delta$ and $\Omega_2$, changing either $T$ or $P$ changes both the dispersion and the energy spectrum. Our experiments use a transverse transducer set at a single frequency, and sweeping $P$ sweeps that frequency through the energy spectrum, as shown in Fig. 1b.; the frequency starts out below the ISQ frequency, where transverse sound does not propagate, and through a depressurization it moves into the region between the ISQ and $2\Delta$, before crossing the pair-breaking threshold. All our experiments were performed between 650 – 900 $\mu$K, sweeping the pressure from $\sim 6 - 3$ bar.

Due to the changes in the dispersion from changes in pressure, the transverse sound velocity is also changing during a sweep, and thus, in a magnetic field, the rotation angle from the AFE.
This angle, $\theta$, can be measured, as mentioned above, from the amplitude of the acoustic signal, and can also be simply related to the wavevectors of RCP and LCP sound by $\theta = 2D\delta q$. $2D$ is the distance traveled by the sound, and $\delta q = |q_+ - q_-|/2$, where $q_{\pm}$ is found by solving Eq. 1 for $q$ setting $m_J = \pm 1$. As all the other dispersion components are known, this allows the relation of data to $g_{2-}$.

3. Results

Analysis of our data quickly showed that Eq. 1 was insufficient to describe the field dependence of the ISQ. As non-linear effects are expected to appear in sufficiently high fields, and as our experiments reached fields of 0.11 T, the logical conclusion was to include non-linear effects in the dispersion. This was done by introducing a quadratic term into the denominator of Eq. 1, turning the ISQ frequency into

$$\Omega_{2-}^2(H) = \Omega_{2-}^2(0) + 2m_Jg_{2-}\gamma_{\text{eff}}H\omega + A_{2-}H^2,$$

where $A_{2-}$ contains all the quadratic character of the mode splitting. The results are displayed in Fig. 2, alongside re-analyzed data from [6] (blue squares), plotted against $(\omega^2 - \Omega_{2-}^2)/\omega^2$, the natural scaling from Eq. 1. While there is clearly some discrepancy between the previous and current results, the overall trend is the same, $g_{2-}$ decreases linearly away from the mode. The discrepancy is possibly due to error in the magnetic field magnitude in the data of [6], and will be more fully explored in future publications. These values for $A_{2-}$ correspond to nonlinear effects becoming significant around 0.05 T, and provide fits to the data shown in Fig. 3.

Of particular interest is the general trend of $g_{2-}$, decreasing with increasing energy, or, equivalently, with decreasing pressure. Sauls & Serene predicted [7] that $g_{2-}$ should increase with increasing temperature or decreasing pressure, which is the opposite behavior from what is seen in Fig. 2. This is likely due to the fact that the calculations in [7] were done assuming the sound frequency to be equal to the mode frequency, and so effects caused by performing experiments away from the mode frequency, such as the divergence of the Tsuneto function, change the overall expected behavior of $g_{2-}$. Work is underway to verify this conjecture, but the current data can be extrapolated to the mode frequency to conform to the previous theory. When done, this yields a value of $x_3^{-1}$, the parameter quantifying the effect of $l = 3$ interactions on the order parameter, of $-0.85$, corresponding to a slightly attractive interaction.

![Figure 2](image-url)  
**Figure 2.** Results for $g_{2-}$ and $A_{2-}$, with red circles and green triangles calculated using Eq. 2 to correct Eq. 1, and blue squares re-analyzed from [6]. $2\Delta$ lies at $(\omega^2 - \Omega_{2-}^2)/\omega^2 \simeq 0.39$. 


4. Conclusion
We have shown results for magnetic fields ranging from 0.02 – 0.11 T, yielding values of $g_2$- and $A_2$- that approach 0.63 and 0, respectively, at the ISQ. The overall trend of $g_2$, while not that expected from theoretical calculations from [7], can be explained as stemming from the difference between the sound and mode frequencies. It is important to note that $A_2$ has not yet been identified within a theoretical framework such as that suggested by Schopohl et al., giving the ISQ frequency as $\Omega_2 (H) = \Omega_2 (0) + 2 \alpha mJH + \beta m^2J^2H^2 - \Gamma H^2$ [8]. Experiments on Zeeman substates $m \neq \pm 1$ must be done to differentiate between parameters such as $\beta$ and $\Gamma$.

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Figure 3. Experimental values of $\theta$ with fits from the $g_2$- and $A_2$- values shown above, for $H = 0.02 - 0.11$ T. Open circles are data points, and lines are experimental fits.