Projection evolution and delayed-choice experiments

To cite this article: A Gód and K Stefaska 2008 J. Phys.: Conf. Ser. 104 012007

View the article online for updates and enhancements.
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Abstract. A theoretical description of Wheeler’s delayed–choice Gedankenexperiment is presented in the non–relativistic space–time in terms of quantum events. It turns out that the description, in which time is a dynamical variable, allows to explain the delayed–choice problem as a result of spread of quantum states in time.

1. Introduction

To explain the interference experiments, like Young’s double–slit experiment, with a single particle, one needs to assume that the particle has travelled through both paths (slits). In this case, the particle behaves like a wave rather than a particle. Several of the single–particle experiments confirmed the wave–particle duality of matter [1, 2, 3, 4, 5]. This striking feature is “dependent” on the detection method of the particle behind the slits. If one uses the screen as a detector which is unable to determine which slit the particle has passed through, one obtains the wave–like behaviour of particles, but, using the detector which can determine which slit the particle has passed through, one gets the particle picture.

An interesting question has been put forward by Wheeler [6, 7]. J. A. Wheeler proposed the “delayed-choice” Gedankenexperiment in which the choice of which property will be observed (wave or particle) is made after the particle has passed the slits. “Thus one decides the photon shall have come by one route or by both routes after it has already done its travel”[7].

Several delayed–choice experiments have been performed [8, 9, 10, 11]. The recent one [12] was done with the great precision. The authors claim that their experiment “demonstrates beyond any doubt the behavior of the photon in the interferometer depends on the choice of the observable which is measured, even when that choice is made at a position and a time such that it is separated from the entrance of the photon in the interferometer by a space–like interval”.

Theoretical explanations of this phenomenon are strongly dependent on the interpretations of quantum mechanics. Short reviews of some attempts can be found in [13, 14]. The analysis of these and other papers implies that there is no generally accepted explanation of the delayed–choice problem.

The purpose of this paper is to describe the delayed–choice problem in terms of the projection evolution approach [15]. Within this formalism, the delayed–choice phenomenon can be explained as a consequence of treating the time on the same footing as the other position observables. In this case, a quantum system knows about its potential history along the whole time line (the potential past, the potential present and the potential future) at the given step of the evolution. The spread of the wave function in time allows to explain the experimentally
observed phenomenon described by the Wheeler [7], because the quantum object is able to change its behavior within the appropriate time interval independently of our imagination about its motion based on classical mechanics. In this way, one can decide about wave or particle behaviour of photons, in the Wheeler’s type of experiments, even in the end of their travels.

In standard quantum mechanics, time is a parameter not a dynamical variable, thus not an observable. In this context, there are difficulties to consider events in quantum mechanics as phenomena in the space–time. For example, insertion of the second beam splitter into the interferometer at a given time is such an event. Another known problem related to events is the time of arrival of a particle to a given target. Also here, a good solution is to consider time as a dynamical variable. There are even more arguments that time should be treated in this way. For example, there are observations of interference phenomena in time (see [16]).

In the next section, a construction of quantum evolution which treats the time and spatial variables on the same footing is sketched. The idea of the so–called projection evolution was first described in the series of papers [17, 18, 15].

2. Projection evolution
In this section, we generalize the model of the projection evolution to make it more flexible. Our goal is to write practical rules for the projection evolution, not to construct the minimal mathematical system.

The projection evolution requires the inclusion of events in the evolution process, i.e., the time when an event happens should be an observable. It implies that the evolution should not be driven by physical time, but it should be based on ordering of related quantum events. For this purpose we assume, by definition, existence of a real, growing evolution parameter which enumerates subsequent “causally” related events. We will call the evolution parameter $\tau$.

The ordering of quantum events means here the order in which we have to include them in the evolution operator to reproduce experimental observations. It is a kind of quantum causality. Note that the causality has here a specific meaning and it is, in fact, an internal relation among events in the stream of real quantum events.

We would like to stress that the quantum evolution process should be understood not in terms of the time variable but as the subsequent, causally ordered (by the evolution parameter) “screen-shoots” of events in the space–time. In this context, the projection evolution can be applied to both non–relativistic and relativistic cases.

The parameter $\tau$ is an ordering parameter only, not an additional dimension of the space–time. The domain for the parameter $\tau$ does not even need to have a metric or more reach structure. However, for practical reason, one can always treat it as a subset of the real numbers.

For further purpose, we need to introduce a subset of quantum operations, which we will call the projection evolution mappings. The formalism of quantum operations was invented around 1983 by K. Krauss [19], who relied on the earlier mathematical works of M. D. Choi [20]. It has the advantage that it expresses operations which transform density operators (quantum states) into density operators.

The projection evolution mapping at the evolution step $\tau$ is defined as the family of quantum operations $F(\tau; \nu, \rho)$, labelled by the appropriate quantum numbers $\nu$:

$$F(\tau; \nu, \rho) = \sum_k E(\tau; \nu, k)\rho E(\tau; \nu, k)^\dagger,$$

where for the fixed $\nu$ the Krauss–type operators $E(\tau; \nu, k)$ (in most practical cases they are projection or unitary operators) are labelled by some “intrinsic” quantum numbers $k$. The operators $E(\tau; \nu, k)$ we call the projection evolution operators.
The mappings have to preserve the “probability”:
\[
\sum_\nu \text{Tr}[\mathcal{F}(\tau; \nu, \rho)] = 1, \quad \text{for all quantum states } \rho.
\] (2)

In the following, we assume that the set of quantum numbers \( \nu \) labels possible states of the system uniquely.

Now, let us assume that the evolution steps can be enumerate by a set of integers. There exists a mechanism (chooser) which chooses randomly the next state of the quantum system. A new state is chosen randomly from the following states:
\[
\rho(\tau_n; \nu_n) = \frac{\mathcal{F}(\tau_n; \nu_n, \rho(\tau_{n-1}; \nu_{n-1}))}{\text{Tr}(\mathcal{F}(\tau_n; \nu_n, \rho(\tau_{n-1}; \nu_{n-1})))}
\] (3)

where \( \nu_n \) runs over all sets of quantum numbers allowed by the projection evolution mapping.

The probability distribution for the chooser depends on the previous state of the quantum system and it is defined by the following relation:
\[
\text{Prob}(\tau_n; \nu_{n-1}, \nu_n) = \text{Tr}(\mathcal{F}(\tau_n; \nu_n, \rho(\tau_{n-1}; \nu_{n-1}))).
\] (4)

It is, in principle, the conditional probability of choosing the next state from a set of alternatives offered by the evolution operators available at the evolution parameter \( \tau_n \). It can be interpreted in this way, because the eq.(4) describes the probability of choosing the next state, under the condition that the system at the previous evolution parameter was in the state \( \rho(\tau_{n-1}; \nu_{n-1}) \).

In this way, one can construct possible scenarios of the evolution and their probabilities.

It is important to notice that this method allows to treat space and time coordinates on the same footing. All calculations have to be done in the space of states having space–time interpretation. The time is no longer a parameter of the system; it is a dynamical variable which allows for construction of an appropriate observable, i.e., the time operator.

3. Delayed–choice problem

The description of Young’s experiment for a single photon is crucial for the development of quantum mechanics. This “simple” idea can also be used for realization of Wheeler’s delayed–choice experiment. The simplest experimental setup consists of an interferometer having two 50/50 beam splitters (see Fig. 1).

A single particle enters the first channel of the first beam splitter BS1. In the absence of the second beam splitter BS2, the particle can either stay in the first channel or it moves to the second channel. Both processes can happen with equal probabilities. Installing the beam splitter BS2, the information about choice of the channel by the particle is lost and one can observe the “interference fringes” in the final detectors, either D1 (first channel) or D2 (second channel).

The decision about inserting the second beam splitter BS2 or not can be made before or after the particle has passed BS1. The main point of the Wheeler’s Gedankenexperiment is how the result of the experiment depends on that instant of time.

To describe this situation, we have to define the state space for the interferometer.

Let us introduce two spaces \( \mathcal{K}^{(0)} \) and \( \mathcal{K}^{(1)} \). The Hilbert space \( \mathcal{K}^{(0)} \) is spanned by a vacuum state and \( \mathcal{K}^{(1)} \) denotes a single particle state space. We assume that the single particle state vectors \( |\psi\rangle \in \mathcal{K}^{(1)} \) admit the space–time representation \( \psi(x) = \psi(t, \vec{x}) = \langle t, \vec{x}|\psi\rangle \).

The scalar product in \( \mathcal{K}^{(1)} \) is defined as
\[
\langle \psi'|\psi\rangle = \int_{\mathbb{R}^4} d^4x \psi^*(x) \psi(x).
\] (5)
Figure 1. Interferometer and the schema of Wheeler’s delayed-choice experiment. A single particle enters the first port of the first beam splitter (BS1). In the absence of the second beam splitter (BS2), the detector D=(D1,D2) is able to “check” the path along which the particle travels. After insertion of the second beam splitter, the “Welcher-Weg” information is irreversibly lost, and one can observe interference phenomena in the detector D, which can be interpreted as the particle has travelled both paths. According to Wheeler, one can decide “whether to put in the second beam splitter or take it out at very last minute. Thus one decides whether the photon shall have come by one route or by both routes after it has already done its travel”. Note the integration over time which is not present in the standard approach.

Because the beam splitter (also the interferometer) has two input and two output ports, there are two possibilities for input and output states: the particle is in the first port and the second one is empty and vice versa (see §5.1.3 [23]). These two alternatives will be called channels. They can be described by tensor product of both spaces $\mathcal{K}^{(n)} \otimes \mathcal{K}^{(1-n)}$, where $n = 0, 1$. Since the particle number is preserved, the spaces $\mathcal{K}^{(0)} \otimes \mathcal{K}^{(0)}$ and $\mathcal{K}^{(1)} \otimes \mathcal{K}^{(1)}$ cannot occur.

Because both channels are orthogonal, the state space of the system is:

$$\mathcal{K} = (\mathcal{K}^{(1)} \otimes \mathcal{K}^{(0)}) \oplus (\mathcal{K}^{(0)} \otimes \mathcal{K}^{(1)}),$$

(6)

To distinguish the channels in which a particle can move, one needs to introduce the channel number $\gamma = 1, 2$. An arbitrary vector belonging to the state space $\mathcal{K}$ can be thus written as:

$$|\Psi\rangle = \sum_{\gamma=1,2} \sum_{\nu} a_{\gamma \nu} |\gamma, \nu\rangle$$

(7)

where $|\gamma = 1, \mu\rangle \equiv |\mu\rangle \otimes |0\rangle$ and $|\gamma = 2, \mu\rangle \equiv |0\rangle \otimes |\mu\rangle$ and the vectors $|\mu\rangle$ form an arbitrary orthonormal basis in $\mathcal{K}^{(1)}$.

For simplifications of formulae, let us assume that the source Z occupies the same region of space as the beam splitter BS1, and the detector D=(D1,D2) occupies the same region of space as the beam splitter BS2.

To describe the particle moving in the setup shown on Fig. 1, one needs to construct the evolution operators for its elements.

Let us assume that the source Z produces particles in the states

$$|Z; \gamma_z = 1, \nu\rangle = |\phi^{(Z)}_\nu \rangle \otimes |0\rangle$$

$$|Z; \gamma_z = 2, \nu\rangle = |0\rangle \otimes |\phi^{(Z)}_\nu \rangle,$$

(8)
where $\gamma_z$ labels the entrance channel of the particle. The first step of the evolution, $\tau = \tau_0$, describes a particle created in one of the possible states (8):

$$\mathcal{E}(Z; \gamma_z, \nu) = |Z; \gamma_z, \nu\rangle\langle Z; \gamma_z, \nu|.$$  \hspace{1cm} (9)

For boson like particles, beam splitters are traditionally described by making use of the channel mixing matrix $S$ [21, 22, 23]:

$$S = e^{i\alpha_0} \begin{bmatrix} S_{11} = \cos \theta e^{i\alpha_\rho} & S_{12} = \sin \theta e^{i\alpha_\rho} \\ S_{21} = -\sin \theta e^{-i\alpha_\rho} & S_{22} = \cos \theta e^{-i\alpha_\rho} \end{bmatrix}.$$  \hspace{1cm} (10)

For 50/50 beam splitters, the angle $\theta = \pi/4$. The phases $\alpha_\rho$ and $\alpha_\rho$ describe possible phase shifts. The common phase, without loss of generality, can be omitted, i.e., $\alpha_\rho = 0$.

A description of Wheeler’s experiment requires the beam splitters being placed somewhere in the space–time. Let us denote the space–time region where the beam splitter is inserted by $B$.

We need an operator $\hat{S}(B)$ acting in the state space $K$ which act as the mixing matrix in the region $B$ and as the unit operator $I$ outside of $B$. For this purpose we have to define the operator describing beam splitter by its action on the basis (7) of the state space $K$:

$$\hat{S}(B)|\gamma = 1, \mu\rangle = (\mathbb{1} + (S_{11} - \mathbb{1})\chi_B(\hat{x}^{(1)})|\gamma = 1, \mu\rangle + S_{12}\chi_B(\hat{x}^{(2)})|\gamma = 2, \mu\rangle$$

$$\hat{S}(B)|\gamma = 2, \mu\rangle = S_{21}\chi_B(\hat{x}^{(1)})|\gamma = 1, \mu\rangle + (\mathbb{1} + (S_{22} - \mathbb{1})\chi_B(\hat{x}^{(2)})|\gamma = 2, \mu\rangle.$$  \hspace{1cm} (11)

The operator characteristc function of the subset $B$ is defined by the relation: $\chi_B(\hat{x}^{(\gamma)})f(\hat{x}^{(\gamma)}) = f(\hat{x}^{(\gamma)})$ for $\hat{x}^{(\gamma)} \in B$ and 0 otherwise, for an arbitrary function $f(\hat{x}^{(\gamma)})$. The operators $\hat{x}^{(\gamma)} = (\hat{x}^{(\gamma)}, \hat{y}^{(\gamma)}, \hat{z}^{(\gamma)})$ denote here the position operators in the channel $\gamma = 1, 2$ [15].

The particle produced by the source $Z$ passes 3 regions of the interferometer: the first beam splitter $B_1$, the region of free flight between $B_1$ and $B_2$ and the second beam splitter $B_2$. The free flight region will be described by the unitary operator $\hat{U}_F$ (see below).

Combining these operators, we get the following projection evolution operator $\mathcal{E}(MZ)$ for the interferometer ($\tau = \tau_1$):

$$\mathcal{E}(MZ) = \hat{S}(B_2)\hat{U}_F\hat{S}(B_1),$$  \hspace{1cm} (12)

where $B_k$ denotes the space–time region occupied by the $k$-th beam splitter.

The last step of the evolution is the detection of particles ($\tau = \tau_2$). Let us assume that the evolution operator responsible for detection of a particle in the detector $D$ can be expressed as a set of projection operators projecting onto the space–time regions $D_\mu$:

$$\mathcal{E}(D; \gamma_D, \mu) = \int_{D_\mu} d^4x |\gamma_D, x\rangle\langle \gamma_D, x|,$$  \hspace{1cm} (13)

where $|\gamma_D, x\rangle$ are generalized eigenstates of the position operator $\hat{x}^{(\gamma_D)} = (\hat{\rho}^{(\gamma_D)}, \hat{\gamma}^{(\gamma_D)}, \hat{\rho}^{(\gamma_D)}, \hat{z}^{(\gamma_D)})$ in the channel $\gamma_D = 1, 2$.

Now we are ready to describe the evolution of a particle from the source to the detector: for $\tau = \tau_0$, the evolution is described by (9); for $\tau = \tau_1$, the evolution operator is given by (12); and finally for $\tau = \tau_2$, we have (13).

Let us now assume that the source $Z$ places a particle in the first channel of BS1 only, i.e., $\gamma_z = 1$.

The probability (4) of finding the particle in the detector channel $\gamma_D$ at the space–time region $D_\mu$ can be written as:

$$\text{Prob}(\gamma_z, \nu; \gamma_D, \mu) = \text{Tr} \left( \mathcal{E}(D; \gamma_D, \mu)\mathcal{E}(MZ)\mathcal{E}(Z; \gamma_z, \nu)\rho_0\mathcal{E}(Z; \gamma_z, \nu)\mathcal{E}(MZ)\mathcal{E}(D; \gamma_D, \mu)\right)$$  \hspace{1cm} (14)
Using $\gamma_z = 1$, the probability (14) can be rewritten as:

$$
\text{Prob} \{ \gamma_z = 1, \nu, \gamma_D, \mu \} = \langle Z; \gamma_z = 1, \nu | \rho_0 | Z; \gamma_z = 1, \nu \rangle \int_{D,\nu} d^4x \left| \int_{\mathbb{R}^4} d^4\vec{x}' G_0(t, \vec{x}; t', \vec{x}') \right|^2
$$

\[
\begin{align}
&\left\{ \left[ 1 + (S_{11} - 1) \chi B_2(x) \right] \left[ 1 + (S_{11} - 1) \chi B_1(x') \right] \\
&+ S_{12} S_{21} \chi B_2(x) \chi B_1(x') \right\} \langle \gamma = 1, t, x' | Z; \gamma_z = 1, \nu \rangle^2, & \text{for } \gamma_D = 1,
\end{align}
\]

(15)

The matrix elements $\langle \gamma'', x'' | \hat{U}_F | \gamma', x' \rangle = \delta_{\gamma, \gamma'} G_0(t'', x''; t', \vec{x})$ can be interpreted as transition amplitudes between the space-time points $\vec{x} \rightarrow x''$.

To show the result of our considerations in more illustrative way, let us assume the Schrödinger–type approximation for the amplitude $G_0(t, \vec{x}; t', \vec{x})$.

For this purpose, we use the nearly usual form of the unitary evolution operator $\hat{U}_F$ as a coupling between space and time coordinates

$$
\langle \gamma'', x'' | \hat{U}_F | \gamma', x' \rangle = \delta_{\gamma, \gamma'} G_0(t'', x''; t', \vec{x}) = \delta_{\gamma, \gamma'} \delta(t' - t') G_c(t'', x''; t', \vec{x}),
$$

(16)

where $\hat{U}_F$ denotes the time operator in the $\gamma$th channel [15], the Hamiltonian $\hat{H}_0$ is the usual, non–relativistic Hamiltonian for free particles acting in the appropriate channel, and the function $G_c(t'', x''; t', \vec{x})$ is the standard free particle non–relativistic propagator:

$$
G_c(t'', x''; t', \vec{x}) = \left[ \frac{m}{2 \pi i \hbar (t'' - t')} \right]^{3/2} \exp \left[ \frac{i m (x'' - \vec{x})^2}{2 \hbar (t'' - t')} \right].
$$

(17)

The coupling (16) between space and time is local in time and modifies the $\vec{x}$–dependence by shifting it to the actual dynamical time according to the traditional Schrödinger’s picture.

To simplify the calculation presenting the delayed–choice idea, without loss of generality, we are going to work in a two dimensional space–time $\mathbb{R}^2$ rather than in $\mathbb{R}^4$. Additionally the first beam splitter is assumed to be fixed in the space time i.e., $B_1 = \mathbb{R} \times [b_1, b_1]$, and as it was mentioned earlier, it occupies the same space $[b_1, b_1]$ as the source of the particles.

The second beam splitter is inserted at the time $t = t_b$, and it occupies the same space $[b_2, b_2]$ as the detector D.

The time axis is split into small intervals $\mathbb{R} = \bigcup_{\mu = -\infty}^{\infty} [\mu \delta_T, (\mu + 1) \delta_T]; \mu$ runs over integers. In the following, we will often label the time by the index $\mu$ which describes the interval in which particle can be detected.

In order to avoid not fully overlapping regions during integrations, one can assume the time of insertion $t_b = n \delta_T$, where $n$ is an integer. The same assumption is valid for other time parameters.

The region of final integration is given by $D_{\mu} = [\mu \delta_T, (\mu + 1) \delta_T] \times [b_2, b_2]$.

The intervals $[\mu \delta_T, (\mu + 1) \delta_T]$ determine the time resolution of the detector. A particle can be localized in time within these time intervals only. In addition we assume that the source produces particles described by the states spatially localized in the interval $[b_1, b_1]$.

The assumptions listed above allow simplification of the expression (15) for the probability of the detection of the particle in the channel $\gamma_D$ at the time $\mu$, i.e., the time belonging to the
Similarly, we cannot detect the particle before time $t$ only within this time interval. The detection probability for times $t \in (0, T)$ is generally not equal to 1, because the time is a dynamical variable.

It is important to notice that the sum of detection probabilities in both channels at the given time $\mu$ is not equal to 1, because the time $\tau$ is a dynamical variable.

Now we are in a position to analyze the delayed-choice experiment using our device. According to our assumptions, the particle can be localized at the interval $[0, T]$ if and only if the second beam splitter exists. In this case, both characteristic functions are equal to 0, and the particle can be detected in both channels with equal probability.

We can distinguish three cases:

1. If the time of detection is $(\mu + 1)\delta T \leq t_b$, the particle is detected before insertion of the second beam splitter. In this case, both characteristic functions are equal to 0, and the particle can be detected in both channels with equal probability.
2. If the time of detection is $(\mu + 1)\delta T > t_b$ and $(\mu + 1)\delta T < T$, the particle is detected only in the second channel. This behavior is independent of the actual value of $t_b$ as it is observed in delayed-choice experiments.
The first two cases have nothing in common with the “delayed–choice” and describe natural behavior of the particle.

The third case shows, however, that the model of quantum mechanics with dynamical time describes the delayed–choice as the phenomenon arising due to spread of the particle states over the time domain. The actual action, e.g., insertion of the second beam splitter, has influence also on the potential past of the particle and changes it. In this way, the particle behaves like a wave or a solid particle independently of the time of insertion of the second beam splitter.

Another interesting feature is that the equation (20) allows to calculate the probabilities $\text{Prob}(\gamma_z, \nu, \gamma_D, \mu)$ of the detection of particles at a given time $\mu$, i.e., in the time interval $[\mu \delta_T, (\mu + 1) \delta_T]$ as it is shown on Fig. 2.

Figure 2. The probabilities of particle detection in channels 1 and 2 as functions of time $\mu$. The atomic like units are used: $\hbar = 1$, the electron mass $m = m_e = 1$, the unit length $a_0 = 1$ cm and the unit time $a_t = \frac{a_0^2 m_e}{\hbar} = 0.864$ s. In this unit $b_1 = 0$, $b_1' = 1$, $b_2 = 801$, $b_2' = 802$, $T = 1000$, $\delta_T = 2$, $t_b = 225$.

From Fig. 2 one clearly sees that there exist minima in which the detection probability drops close to zero. There are also instants of time $\mu$ when the detection of the particle is the most probable. It is interesting to analyze dependence of the most probable detection time $\mu_P$ as a function of the distance $L$ between the source and the detector. It is plotted on Fig. 3.

Figure 3. The dependence of the most probable detection time $\mu_P$ as a function of distance $L$ between the source and the detector. $L$ and $\mu_P$ in the same units as in the Fig. 2.

The result shows, as expected, that the particle is moving with constant velocity. Despite the fact that the time is an observable, we get classical relation $L = v_0 (\mu \delta_T) = v_0 t_b$. 
4. Conclusions
The projection evolution approach allows the calculation of states and transition probabilities among them. It is possible to obtain not only the standard but also the temporal relations unavailable in the standard formalism. The most important difference is the introduction of the dynamical time into the formalism.

In case of the delayed-choice problem the latter feature is crucial. The spread of quantum states in time leads to non–local behaviour in time. Changes made at any time in the temporal part of the quantum state lead to changes in history of the physical system.

In our case, after production in the source, the particle travels through the interferometer varying within the space–time (the second beam splitter is inserted at time $t_b$). The third step of the evolution is detection of the particle at a given time interval labelled by $\mu$. It is a kind of stochastic process. The detection can happen at any time with a given probability. It is now obvious that if the third step of the evolution – localization of the particle in the time $\mu$ – happens before insertion of the second beam splitter, we have no influence of the beam splitter on the particle. In the opposite case, however, one can expect the interference effects. The result is a natural choice made randomly by the system, but traditional, macroscopic causality is always satisfied.

The “delayed–choice problem” is a natural phenomenon caused by spread of states in time.

Acknowledgment
The one of the authors (AG) would like to thank to Jim Buck for careful reading of the manuscript.

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