Based-Ohmic-Reservoir non-Markovianity and Quantum Speed Limit Time

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The non-Markovianity and the quantum speed limit time (QSLT) are investigated in Jaynes-Cummings model coupling with the Ohmic reservoir at zero temperature when when $t = 0$ and $N = 1$. We obtain the non-Markovianity expressed by using the decoherence rate in the time-local master equation. We find that the non-Markovianity in the dynamics process can be witnessed by both of the negative decoherence rate and the positive derivative of the trace distance. The atom-cavity coupling is the main physical reasons of the transition from Markovian to non-Markovian dynamics and the quantum speed-up process of the atom, which the critical value of this sudden transition only depends on the Ohmicity parameter. The atom-cavity coupling and the appropriate reservoir parameters can effectively improve the non-Markovianity in the dynamics process and speed up the evolution of the atom. Moreover, the initial non-Markovian dynamics first turns into Markovian and then back to non-Markovian with increasing the atom-cavity coupling under certain condition. We also provide the physical interpretation.

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I. INTRODUCTION

As we known, the decoherence effect and the energy dissipation caused by coupling of system-environment will bring remarkable influences on the dynamical behaviour of the open system. The evolution process of the open system is Markovian if a quantum system is weakly coupled to a memoryless environment, while the evolution process is non-Markovian if a quantum system is strong coupled to a memory environment \cite{1,6}. The non-Markovian effect in the dynamics process can be described by the non-Markovianity. Many efforts have been made to define non-Markovianity, to measure it, and to take advantage of it \cite{7,14}. In recent years, the research on non-Markovianity in the dynamics process of the open system has attracted the attention of the community, both theoretically and experimentally \cite{15,18}.

On the other hand, quantum speed limit (QSL) has been considered a purely quantum phenomenon with no corresponding concept in classical mechanics, which sets a bound on the maximal evolution velocity that a quantum system needs to evolve between two distinguishable states. Driving a given initial state to a target state at the maximal evolution speed is one of fundamental and important tasks of quantum physics, thus QSL plays a significant role in the fields of quantum computation, quantum metrology, and so on \cite{10,23}. The minimal evolution time between two distinguishable states of a quantum system is defined as the quantum speed limit time(QSLT) \cite{23,28}. For closed systems, the QSLT is defined by $\tau = \max(\frac{\pi h}{2\Delta E}, \frac{\pi h}{2\Delta E})$, where $\Delta E$ in Mandelstam-Tamm(MT) bound \cite{29} and $E$ in Margolus-Levitin(ML) bound \cite{30} are the fluctuation and the mean value of the initial-state energy, respectively. For open systems, Deffner and Lutz obtained the unified bound of QSLT from the MT and ML types by using the Bures angle and showed that the non-Markovian effects could speed up the quantum evolution \cite{31}. In recent years, many efforts have been made in the study of QSLT of an open system \cite{32,38}.

In addition, much valuable effort has also been devoted to the relationships between the non-Markovianity and the QSL, such as quantum speedup in a memory environment \cite{39}, quantum speedup in open quantum systems \cite{40} and the relationship between the quantum speedup and the formation of a system-environment bound state \cite{11,12}. The authors in \cite{39} found that a classical field can effectively regulate the non-Markovianity and the QSLT of an open qubit. Namely, the strong coupling of qubit-environment and an external classical field can all realize the transformation from Markovian to non-Markovian dynamics and the speed-up evolution of the system. In this paper, we will investigate the non-Markovianity and the QSLT of the atom in Jaynes-Cummings model coupling with the Ohmic reservoir at zero temperature. The results show that the atom-cavity coupling and the appropriate reservoir parameters can improve the non-Markovianity in the dynamics process and accelerate the evolution of the atom.

The outline of the paper is the following. In Section II, we introduce a physical model. In Section III, we introduce the non-Markovianity and the quantum speed limit time. Results and discussions are provided in Section IV. Finally, we give a brief summary in Section V.
II. PHYSICAL MODEL

We consider a dissipative Jaynes-Cummings model \cite{44, 45}, namely, an atom is in a leaky cavity that the leakage is usually modelled by coupling of the cavity mode to the bosonic modes of the reservoir. Supposing the atomic transition frequency is $\omega_0$ and the Pauli matrices of the atom are $\hat{\sigma}_z$ and $\hat{\sigma}_\pm$. $\hat{a}^\dagger (\hat{a})$ and $\hat{b}_k^\dagger (\hat{b}_k)$ express the creation (annihilation) operators of the cavity and the $k$-th mode of reservoir with the frequency $\omega_k$, respectively. $\Omega$ and $g_k$ are the coupling strength of the atom-cavity and the cavity-reservoir, respectively. The Hamiltonian of the total system is given by

$$\hat{H} = \hat{H}_{JC} + \hat{H}_{CR}, \quad (1)$$

where

$$\hat{H}_{JC} = \frac{1}{2}\omega_0 \hat{a}^\dagger \hat{a} + \omega_0 \hat{a}^\dagger \hat{a} + \Omega (\hat{a} \hat{\sigma}_+ + \hat{a}^\dagger \hat{\sigma}_-) \quad (2)$$

and

$$\hat{H}_{CR} = \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k + (\hat{a}^\dagger + \hat{a}) \sum_k g_k (\hat{b}_k^\dagger + \hat{b}_k). \quad (3)$$

In this work, let the reservoir be at zero temperature and the total system has only one initial excitation. Performing the Born-Markov and the rotating wave approximations, tracing out the freedom degrees of the reservoir in the interaction picture and then going back to the Schrödinger picture, one obtains the following master equations of a flat spectrum) by $\tau_B \approx \omega_c^{-1}$ and $\tau_R \approx \eta^{-1}$. $\omega_c < \omega_0$ implies that the spectrum of the reservoir does not completely overlap with the frequency of the cavity, that is, the reservoir is effectively adiabatic, so that the evolution behaviour of the system is essentially non-Markovian. While $\omega_c > \omega_0$ indicates the converse case, which the quantum information is quickly dissipated, the evolution behaviour of the system is Markovian. The smaller the value of $\eta$ is, the longer the reservoir correlation time is, and the more obvious the non-Markovian effect is \cite{32, 54}. Common values of $s$ are $\frac{3}{2}, 1$ and 3, inserting Eq. \ref{9} into Eq. \ref{5}, $\gamma_j(t)$ is written as

$$s = 1: \gamma_j(t) = -\frac{2\eta\omega_c}{\left(1 + \omega_c^2 t^2\right)^{\frac{s}{2}}} \sin(\omega_j t - \frac{\alpha_0}{2}), \quad (10)$$

$$s = 3: \gamma_j(t) = -\frac{2\eta\omega_c}{\left(1 + \omega_c^2 t^2\right)^{\frac{s}{2}}} \sin(\omega_j t - \frac{\alpha_0}{3}), \quad (12)$$

where $\alpha_0 = \arctan(\omega_c t)$. We consider the structured reservoir with an Ohmic spectral density

$$J(\omega) = \eta \omega^{s-1} e^{-\omega/\omega_c}, \quad (9)$$

and

$$\hat{\rho}(t) = \frac{1}{2} \sum_{j=1}^{2} e^{-i\omega_j t} e^{-\frac{i}{2}\beta_j}$$

$$\beta_j = \int_0^t \gamma_j(t') dt'.$$

$$\gamma_j(t) = 2Re\left[\int_0^t dt \int_{-\infty}^{+\infty} d\omega^{\prime} e^{i\omega^{\prime} \tau} J(\omega^{\prime})\right], \quad (5)$$

in which $J(\omega')$ is the spectral density of the reservoir.

We can acquire an analytical solution of the density operator $\hat{\rho}(t)$ in the dressed-state basis from Eq. \ref{4}, then the density matrix $\rho(t)$ of the atom in the standard basis $\{|\varphi_1, +\rangle, |\varphi_1, -\rangle, |\varphi_0\rangle\}$ is also obtained by means of the representation transformation and taking a partial trace over the freedom degrees of the cavity. Suppose the initial state is $\{\rho_{11}(0), \rho_{10}(0), \rho_{01}(0), \rho_{00}(0)\}$, the density matrix $\rho(t)$ \cite{48} of the atom at all time $t$ is expressed as

$$\rho(t) = \begin{pmatrix} |p(t)|^2 \rho_{11}(0) & p(t)\rho_{10}(0) \\ p(t)^*\rho_{01}(0) & 1 - |p(t)|^2 \rho_{11}(0) \end{pmatrix}, \quad (6)$$

where the probability amplitude $p(t)$ can be given by

$$p(t) = \frac{1}{2} \sum_{j=1}^{2} e^{-i\omega_j t} e^{-\frac{i}{2}\beta_j}$$

$$\beta_j = \int_0^t \gamma_j(t') dt'.$$

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where the probability amplitude $p(t)$ can be given by

$$p(t) = \frac{1}{2} \sum_{j=1}^{2} e^{-i\omega_j t} e^{-\frac{i}{2}\beta_j}$$

$$\beta_j = \int_0^t \gamma_j(t') dt'.$$
We cannot get the analytical expressions for $\beta_j$ from Eq. (8) and Eqs. (10)-(12), but we can calculate mathematically $\beta_j$ for the sub-Ohmic, Ohmic and super-Ohmic spectra, respectively.

In view of Eq. (6), we can also write a time-local master equation [55] for the density operator $\rho(t)$ as

$$\frac{d}{dt}\rho(t) = \mathcal{L}\rho(t)$$

$$= -\frac{i}{\hbar}S(t)[\hat{\sigma}_-, \rho(t)] + \Gamma(t)\{\hat{\sigma}_-\rho(t)\hat{\sigma}_+ - \frac{1}{2}\rho(t)\hat{\sigma}_+\hat{\sigma}_-\},$$

where the Lamb frequency shift $S(t)$ and the decoherence rate $\Gamma(t)$ can be respectively expressed as

$$S(t) = -2\Im\left[\frac{\hat{\beta}(t)}{\rho(t)}\right],$$

and

$$\Gamma(t) = -2\Re\left[\frac{\hat{\beta}(t)}{\rho(t)}\right],$$

$S(t)$ describes the contribution from the unitary part of the evolution under dynamical decoherence. $\Gamma(t)$ characterizes the dissipation and the feedback of the information of the system. $\Gamma(t) > 0$ indicates that quantum information flows from the system to its environment, i.e., Markovian process. $\Gamma(t) < 0$ expresses that quantum information flows back from its environment to the system, i.e., non-Markovian process.

### III. NON-MARKOVIANITY AND QUANTUM SPEED LIMIT TIME

#### A. Non-Markovianity

In the dynamics process of an open system, the non-Markovianity can describe the total backflow of information to the system from its environment. Among the different measurement of the non-Markovianity, the method based on the time rate of change of the trace distance is more commonly used at present. The method based on the time rate of change of the trace distance can be expressed as

$$\mathcal{D}(\rho(t)) = \frac{d}{dt}\rho(t)$$

$$\sigma(t, \rho_1(t)) = \frac{d}{dt}\mathcal{D}(\rho(t)) = \frac{d}{dt}\mathcal{D}(\rho(t), \rho_2(t)), \quad \sigma(t, \rho_1(t)) \leq 0$$

that is, there is non-Markovian in the dynamical process if $\Gamma(t) < 0$ because $|p(t)|$ is non-negative. Eq. (19) shows that the non-Markovianity can also be described by the decoherence rate in the time-local master equation. In the dissipative JC model, the quantum information will be exchanged between the reservoir with the cavity and between the cavity with the atom. Because we only care about the dynamics of the atom, both the cavity and its outside reservoir are regarded as the atomic environment. From Eqs. (7) and (19), we can see that the non-Markovianity $\mathcal{N}$ is determined by all environment parameters (including the atom-cavity coupling $\Omega$, the cavity-reservoir coupling $\eta$, the cut-off frequency $\omega_c$ and the value of $s$). The non-Markovianity $\mathcal{N}$ is larger, the information from the environment feeding back to the atom is more.

#### B. Quantum speed limit time

The bound of the minimal evolution time from an initial state $\rho(0)$ to a final state $\rho(\tau)$ is defined as the quantum speed limit time (QSLT) of a system, where $\tau$ is an actual evolution time. If the initial state is $\rho(0) = |\psi_0\rangle\langle\psi_0|$ and its target state $\rho(\tau)$ satisfies the master equation $\rho(t) = \mathcal{L}\rho(t)$, the positive generator of the dynamical semigroup, the QSLT can be expressed as

$$\mathcal{QSL} = \sin^2\beta[\rho(0), \rho(\tau)]/\mathcal{L}$$

according to the unified lower bound derived by Deffner and Lutz, where $\beta[\rho(0), \rho(\tau)] = \arccos\sqrt{|\psi_0\rangle\langle\psi_0|B}$ indicates the Bures angle between $\rho(0)$ and $\rho(\tau)$, and $

$$\mathcal{L}_{\infty} = \tau^{-1}\int_0^\tau \mathcal{L}||B||dt$$

equal to the largest eigenvalue of $\sqrt{B^T B}$ [31]. When
\[ \rho(0) = |e\rangle\langle e|, \]
we can obtain the QSLT from Eq. (6) as
\[ \frac{\tau_{\text{QSL}}}{\tau} = \frac{1 - |p(t)|^2}{\int_0^\tau \partial_t |p(t)|^2 dt}, \quad (20) \]

For the dynamics process from \( \rho(0) \) to \( \rho(\tau) \), the non-Markovianity is also written as
\[ \mathcal{N} = \frac{1}{2} \int_0^\tau \left| \partial_t |p(t)|^2 \right| dt + |p(\tau)|^2 - 1, \quad (21) \]

From Eqs. (20)-(21), the relationship between the QSLT and the non-Markovianity can be obtained as
\[ \frac{\tau_{\text{QSL}}}{\tau} = \frac{1 - |p(\tau)|^2}{1 - |p(\tau)|^2 + 2\mathcal{N}}, \quad (22) \]

Eq. (22) shows that the QSLT is equal to the actual evolution time when \( \mathcal{N} = 0 \), but the QSLT is smaller than the actual evolution time when \( \mathcal{N} > 0 \). That is, the non-Markovianity in the dynamics process can lead to the faster quantum evolution and the smaller QSLT.

**IV. RESULTS AND DISCUSSIONS**

In this section, we will analyse the relations between the trace distance with its derivative, between the non-Markovianity with the decoherence rate and the derivative of the trace distance, as well as between the non-Markovianity with the quantum speed limit time. We will also study the influence of the atom-cavity coupling and the reservoir parameters on the non-Markovianity and the quantum speed limit time.

In Fig.1, we draw the curve of the trace distance and its derivative, the decoherence rate and the non-Markovianity when \( s = 1 \) (Ohmic spectrum), \( \Omega = 3\omega_0 \) and \( \frac{\omega}{\omega_0} = 2 \). We find that, the trace distance degenerates to zero from 1,0, and the derivative of the trace distance becomes negative and the decoherence rate simultaneously increases from zero. Then the trace distance again increases from zero, and the derivative of the trace distance becomes positive and the decoherence rate suddenly becomes negative at the same time.

In addition, we can see that the non-Markovianity is equal to zero when the decoherence rate is positive (i.e. the derivative of the trace distance is negative), in which the quantum information flows from the system to its environment due to the dissipation of environment. The non-Markovianity is larger than zero when the decoherence rate is negative (i.e. the derivative of the trace distance is positive), where the quantum information flows back from its environment to the system because of the memory and feedback of environment. Therefore, once \( \mathcal{D}(t) \) increases, the positive value of \( \sigma(t) \) and the negative value of the decoherence rate will appear at the same time, the non-Markovianity in the dynamics process can be witnessed.

Fig.2(a) shows the dynamical properties of the derivative of the trace distance under different atom-cavity coupling \( \Omega \). \( \sigma(t) \) changes from zero to negative when \( \Omega > \omega_0 \), thus the shaded area with positive \( \sigma(t) \) is missing which means \( \mathcal{D}(t) \) is nonincreasing during the whole evolution, shown as the green dotted line in Fig.2(a). \( \sigma(t) \) changes from zero to negative and then again to zero when \( \Omega = 1.55\omega_0 \), as shown as the brown dashed line in Fig.2(a). The non-Markovianity \( \mathcal{N} \) is plotted in Fig.2(b) versus \( \Omega/\omega_0 \). For the region with \( \Omega < 1.55\omega_0 \), \( \mathcal{N} \) is always zero, which means the derivative \( \sigma(t) \) can never give a positive value, an example with \( \Omega = \omega_0 \) is the green dot (\( \mathcal{N} = 0 \)) in Fig.2(b) corresponding to the shaded area with positive \( \sigma(t) \) of the green line in Fig.2(a).

For the region with \( \Omega > 1.55\omega_0 \), there always exists a positive value of \( \mathcal{N} \), the red dot (\( \mathcal{N} = 0.948 \)) in Fig.2(b) expresses an example of \( \Omega = 3\omega_0 \) which corresponds to the shaded area with the red line in Fig.2(a). The critical point with \( \Omega = 1.55\omega_0 \) shows the situation of the transition from Markovian to non-Markovian, which the brown dot (\( \mathcal{N} = 0 \)) in Fig.2(b) corresponds to the shaded area with positive \( \sigma(t) \) of the brown line in Fig.2(a). Namely, the atom-cavity coupling is the main physical reasons of the transition from Markovian to non-Markovian dynamics and enhancing the non-Markovianity in the dynamics process.

In Fig.3, we exhibit the curves of the non-Markovianity
and the QSLT as functions of the coupling strength $\Omega$ when $s = 1$ and $\frac{\omega_c}{\omega_0} = 2$ for different coupling constant $\eta$, respectively. Fig.3(a) shows that $N$ is always zero when $\Omega < \Omega_c$ and $N$ will increase with $\Omega$ enlarging when $\Omega > \Omega_c$. Namely, there is a critical value $\Omega_c$ that $N$ steeply increases from zero and the critical value is same for different coupling constant $\eta$. However, the increasing rate of $N$ depends on the value of $\eta$, i.e., the smaller the coupling $\eta$, the stronger the non-Markovianity. The dependence of QSLT on the coupling $\Omega$ and $\eta$ is dotted in Fig.3(b), we find that $\tau_{QSLT}$ is always equal $\tau$ when $\Omega < 1.55\omega_0$ and $\tau_{QSLT}$ will decrease with $\Omega$ enlarging when $\Omega > 1.55\omega_0$. Namely, there is a critical value $\Omega_c$ of a sudden transition from no speed-up to speed-up and the critical value is same for different coupling constant $\eta$. But the decreasing rate of $\tau_{QSLT}$ depends on the value of $\eta$, i.e., the smaller coupling $\eta$ is corresponding to the more obvious speedup process. This shows that, in addition to the atom-cavity coupling $\Omega$, the cavity-reservoir coupling $\eta$ can also regulate the non-Markovianity in the dynamics process and the speedup evolution process of the atom.

In Fig.4, we describe the dependence relation of the non-Markovianity and the QSLT on the coupling $\Omega$ and the cut-off frequency $\omega_c$ when $s = 1$ and $\eta = 0.9$. Fig.4(a) gives the non-Markovianity as a function of the coupling $\Omega$ for different values of $\omega_c$. If $\frac{\omega_c}{\omega_0} = 2$, $N$ is always zero when $\Omega < \Omega_c$, and $N$ will increase with $\Omega$ enlarging when $\Omega > \Omega_c$. It should be noted that, if $\frac{\omega_c}{\omega_0} = 1$ or $\frac{\omega_c}{\omega_0} = 0.5$, the non-Markovian dynamics occurring for $\Omega = 0.1\omega_0$ turns into Markovian and then back to non-Markovian by increasing $\Omega$, which such a behaviour has been also observed in different structured systems [57, 58]. But the critical value $\Omega_c$ is same for different values of $\omega_c$.

Besides, we also find that, the smaller the value of $\frac{\omega_c}{\omega_0}$, the bigger the initial value of $N$, and the bigger the value of $N$ in areas with $\Omega > \Omega_c$. Fig.4(b) shows the QSLT as a function of the coupling $\Omega$ for different values of $\omega_c$. When $\frac{\omega_c}{\omega_0} = 2$, $\tau_{QSLT}$ is always equal $\tau$ when $\Omega < \Omega_c$, and $\tau_{QSLT}$ will decrease with $\Omega$ enlarging when $\Omega > \Omega_c$. In particularly, when $\frac{\omega_c}{\omega_0} = 1$ or $\frac{\omega_c}{\omega_0} = 0.5$, $\tau_{QSLT}$ will increase from a certain value to one and then again quickly decrease from one with $\Omega$ enlarging and there is a same critical value $\Omega_c$ for different values of $\omega_c$. In addition, we can see that, the smaller the value of $\frac{\omega_c}{\omega_0}$, the smaller the initial value of $\tau_{QSLT}$, and the smaller the value of $\tau_{QSLT}$ in areas with $\Omega > \Omega_c$. Therefore, not only the atom-cavity coupling $\Omega$ but also cut-off frequency $\omega_c$ can enhance the non-Markovianity in the dynamics process and speed up the evolution of the atom.

The influences of $\Omega$ and $s$ on the non-Markovianity and the QSLT are shown in Fig.5. when $\eta = 0.9$ and $\frac{\omega_c}{\omega_0} = 2$. From Fig.5(a), we know that, for the Ohmic spectrum ($s = 1$) and the super-Ohmic spectrum ($s = 2$), $N$ is always zero when $\Omega < \Omega_c$ and $N$ will increase with $\Omega$ enlarging when $\Omega > \Omega_c$, and their critical values are different. However, for the sub-Ohmic spectrum ($s = \frac{3}{2}$), the non-Markovian dynamics occurring for $\Omega = 0.1\omega_0$ also

FIG. 2: (Color online) The dependence of the non-Markovianity $N$ on the derivative $\sigma(t)$ of the trace distance and the coupling strength $\Omega$ when $s = 1$ (Ohmic spectrum) and $\frac{\omega_c}{\omega_0} = 2$. (a) The curves of the derivative $\sigma$ for different $\Omega$ values: the green dotted line is an example with $\Omega = \omega_0$, the brown dashed line indicates an example with $\Omega = 1.55\omega_0$ and the red solid line represents $\Omega = 3\omega_0$ where the positive region of $\sigma(t)$ is shaded. (b) Non-Markovianity as a function of the coupling strength $\Omega$, in which the green dot is corresponding to the green line in (a), the brown dot corresponds to the brown line in (a) which is the transition point from Markovian to non-Markovian dynamics, and the red dot corresponds to the red line in (a). The other parameters are $\eta = 0.1, \omega_0 = 1$ and $\omega_c = 2$.

FIG. 3: (Color online) Non-Markovianity and QSLT as a function of the coupling strength $\Omega$ when $s = 1$ and $\frac{\omega_c}{\omega_0} = 2$ for different coupling constant $\eta$, respectively. $\eta = 0.1$, red dotted line; $\eta = 0.5$, blue dashed line; $\eta = 0.9$, green solid line. (a) Non-Markovianity as a function of $\Omega$; (b) QSLT as a function of $\Omega$. The other parameters are $\omega_0 = 1$ and $\omega_c = 2$. 
turns into Markovian and then back to non-Markovian by increasing $\Omega$, and the critical value under the sub-Ohmic spectrum is obvious less than that under the Ohmic spectrum. From Fig.5(b), we discover that, for the Ohmic spectrum ($s = 1$) and the super-Ohmic spectrum ($s = 3$), $\tau_{QSLT}$ is always zero when $\Omega < \Omega_c$ and $\tau_{QSLT}$ will decrease with $\Omega$ enlarging when $\Omega > \Omega_c$, and their critical values are different. However, for the sub-Ohmic spectrum ($s = \frac{1}{2}$), $\tau_{QSLT}$ will increase from 0.3 to one and then again quickly decrease from one with $\Omega$ enlarging, and the critical value under the sub-Ohmic spectrum is obvious less than that under the Ohmic spectrum. Namely, the atom-cavity coupling $\Omega$ and the Ohmicity parameter $s$ can effectively control the non-Markovianity in the dynamics process and speed up the evolution of the atom.

In the following, the physical interpretation of the results above is given. Because the cavity coupling with the reservoir can be regarded as the environment of the atom, the energy and information can flow back from the environment to the atom through regulating the coupling strength $\Omega$. The larger the atom-cavity coupling $\Omega$, the more information the cavity flows back to the atom. Thus, the non-Markovianity will increase and the QSLT will decrease with $\Omega$ enlarging when $\Omega$ is bigger than the critical value. On the other hand, the influence of the cavity on the atom is obviously greater than that of the reservoir on the atom, so the critical value of sudden transition is mainly determined by $\Omega$. From Eq. (9), we know that a smaller value of $\eta$ corresponds to a longer correlation time of the reservoir thus the non-Markovianity $\mathcal{N}$ is bigger and the QSLT is smaller. Moreover, the smaller value of $\frac{\omega_t}{\omega_0}$ corresponds to the less overlap of the spectrum of the reservoir with the frequency of the cavity, that is, the reservoir is more effectively adiabatic and the non-Markovian effect is more obvious and the evolution of the atom is quicker. The smaller the Ohmicity parameter $s$ is, the smaller the peak and the width of the Ohmic spectral density are, the more obvious the non-Markovian effect is. So the smaller value of $s$ will lead to the larger non-Markovianity and the smaller QSLT. Besides, Eq. (22) shows that the information flows irreversibly from the atom to the environment so that the atom evolves at the actual speed and the QSLT is equal to the actual evolution time when $\mathcal{N} = 0$. The information flows back from the environment to the atom thus the atom evolution is accelerated and the QSLT is smaller than the actual evolution time when $\mathcal{N} > 0$.

V. CONCLUSION

In this work, we investigated the non-Markovianity and the QSLT of the atom in Jaynes-Cummings model coupling with the Ohmic reservoir at zero temperature when the total excitation number $N = 1$. We obtain the non-Markovianity expressed by using the decoherence rate in the time-local master equation (see Eq. (19)), which the non-Markovianity can be explained reasonably by the decoherence rate, namely, the dynamical process is non-Markovian if the decoherence rate is negative. We studied in detail the influence of the atom-cavity coupling and the reservoir parameters on

\[\eta \approx \frac{\omega_t}{\omega_0} \quad \text{and} \quad s = \frac{1}{2} \quad \text{to} \quad s = 3, \quad \omega_t = \frac{1}{2} \omega_0 \quad \text{and} \quad \omega_t = \frac{1}{2} \omega_0 \quad \text{and} \quad \omega_c = \frac{1}{2} \omega_0, \quad \text{respectively.} \]
the the non-Markovianity and the QSLT. The results show that, the negative decoherence rate and the positive derivative of the trace distance can witness the non-Markovianity in the dynamics process, and the atom-cavity coupling is the main physical reasons of the transition from Markovian to non-Markovian dynamics and the quantum speed-up process of the atom, which the critical value of this sudden transition only depends on the Ohmicity parameter. The appropriate reservoir parameters, such as the cavity-reservoir coupling $\eta$, the cut-off frequency $\omega$, and the Ohmicity parameter $s$, can improve the non-Markovianity in the dynamics process and speed up the evolution of the atom. On the other hand, we also found that the non-Markovian dynamics occurring for $\Omega = 0.1\omega_0$ turns into Markovian and then back to non-Markovian by increasing $\Omega$ when $\frac{\omega}{\omega_0} = 1$, $\frac{s}{\omega_0} = 0.5$ and $s = \frac{1}{2}$ (the sub-Ohmic spectrum). These results will provide interesting perspectives for future applications of open quantum systems in quantum physics [59][62].

Acknowledgments

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[33] delCampo, A., Egusquiza, I.L., Plenio, M.B., Huelga, S.F.: Quantum speed limits in open system dynamics. Phys. Rev. Lett. 110, 050403 (2013)
[34] Wu, S.X., Yu, C.S.: Quantum speed limit for a mixed initial state. Phys. Rev. A 98, 042132 (2018)
[35] Zhang, Y.J., Han, W., Xia, Y.J., Cao, J.P., Fan, H.: Quantum speed limit for arbitrary initial states. Sci. Rep. 4, 4890 (2014)
[36] S.-X. Wu, Y. Zhang, C.-S. Yu and H.-S. Song, J. Phys. A 48, 045301(2015).
[37] Liu, J., Segal, D., Hanna, G. Hybrid quantum-classical simulation of quantum speed limits in open quantum systems. J Phys A: Math. Theor. 52, 215301 (2019)
[38] Cianciaruso, M., Maniscalco, S., Adesso, G. Role of non-Markovianity and backflow of information in the speed of quantum evolution. Phys. Rev. A 96, 012105 (2017)
[39] Z.-Y. Xu, S. Luo, W. L. Yang, C. Liu, and S. Zhu, Quantum speedup in a memory environment, Phys. Rev. A 89, 012307 (2014).
[40] Liu, H.B., Yang, W.L., An, J.H., Xu, Z.Y.: Mechanism for quantum speedup in open quantum systems. Phys. Rev. A 93, 020105(R) (2016).
[41] Wang, J., Wu, Y.N. Xie, Z.Y. Role of flow of information in the speed of quantum evolution. Sci. Reps. 8, 16870 (2018)
[42] Ahansaz, B., Ektesabi, A. Quantum speedup, non-Markovianity and formation of bound state. Sci. Reps. 9, 14946 (2019)
[43] Zhang, Y.J., Han, W., Xia, Y.J., Cao, J.P., Fan, H.: Classical-driving-assisted quantum speed-up. Phys. Rev. A 91, 032112 (2015).
[44] Jaynes, E.T., Cummings, F.W.: Comparison of quantum and semiclassical radiation theories with application to the beam maser. Proc. IEEE 51, 89 (1963).
[45] Shore, B.W., Knight, P.L.: The Jaynes-Cummings model. J. Mod. Opt. 40, 1195 (1993).
[46] Scala, M., Militello, B., Messina, A., Pillo, J., Maniscalco, S.: Microscopic derivation of the Jaynes-Cummings model with cavity losses. Phys. Rev. A 75 013811 (2007).
[47] Zou, H.M., Fang, M.F.: Analytical solution and entanglement swapping of a double Jaynes-Cummings model in non-Markovian environments. Quantum Inf Process, 14, 2673-2686 (2015).
[48] Hong-Mei Zou, Jianhe Yang, Danping Lin, Mao-Fa Fang, Quantum speed-up process of atom in dissipative cavity. arXiv:1911.04839 (2019).
[49] Ming-Liang Hu, Heng Fan, Quantum-memory-assisted entropic uncertainty principle, teleportation, and entanglement witness in structured reservoirs. Phys. Rev. A 86, 032338 (2012).
[50] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M. P. A. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. 59, 1 (1987).
[51] Claudia Benedetti, Fahimeh Salari Sehdaran, Mohammad H. Zandi, and Matteo G. A. Paris, Quantum probes for the cutoff frequency of Ohmic environments. Phys. Rev. A 97 , 012126 (2018).
[52] Hong-Mei Zou, Mao-Fa Fang, You-Neng Guo and Bai-Yuan Yang, Quantum discord of the two-atom system in non-Markovian environments. Phys. Scr. 90, 035104 (2015).
[53] Eckel J, Reina J H and Thorwart M, Coherent control of an effective two-level system in a non-Markovian biomolecular environment. New J. Phys. 11, 085001 (2009).
[54] Cui W, Xi Z R and Pan Y, Non-Markovian entanglement dynamics between two coupled qubits in the same environment. J. Phys. A: Math. Theor. 42, 155303 (2009).
[55] Breuer, H.P., Petruccione, F.: The theory of open quantum systems. Oxford University Press, Oxford (2002).
[56] T. Baumgratz, M. Cramer, and M. B. Plenio, ?Quantifying coherence?, Phys. Rev. Lett. 113, 140401 (2014).
[57] Z.-X. Man, Y.-J. Xia, R. Lo Franco, Harnessing non-Markovian quantum memory by environmental coupling. Phys. Rev. A 92, 012315 (2015).
[58] Z.-X. Man, Y.-J. Xia, R. Lo Franco, Cavity-based architecture to preserve quantum coherence and entanglement. Sci. Reps. 5, 13843 (2015).
[59] Varcoe, B.T.H., Brattke, S., Weidinger, M., Walther, H.: Preparing pure photon number states of the radiation field. Nature (London) 403, 743 (2000).
[60] Jonathan, D., Plenio, M.B.: Light-shift-induced quantum gates for ions in thermal motion. Phys. Rev. Lett. 87, 127901 (2001).
[61] You, J.Q., Nori, F.: Atomic physics and quantum optics using superconducting circuits. Nature (London) 474, 589(2011).
[62] Prawer, S., Greentree, A.D.: Diamond for quantum computing. Science 320, 1601 (2008).