A Four-site Higgsless Model with Wavefunction Mixing

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Motivated by models of holographic technicolor, we discuss a four-site deconstructed Higgsless model with nontrivial wavefunction mixing. We compute the spectrum of the model, the electroweak gauge boson vertices, and, for brane-localized fermions, the electroweak parameters to $\mathcal{O}(M_W^2/M_p^2)$. We discuss the conditions under which $\alpha S$ vanishes (even for brane-localized fermions) and the (distinct but overlapping) conditions under which the phenomenologically interesting decay $a_1 \to W \gamma$ is non-zero and suppressed by only one power of $(M_W/M_p)$.

I. INTRODUCTION

Higgsless models of electroweak symmetry breaking may be viewed as “dual” to more conventional technicolor models and, as such, provide a basis for constructing low-energy effective theories to investigate the phenomenology of a strongly interacting symmetry breaking sector. One approach to constructing such an effective theory, the three-site model, includes only the lightest of the extra vector mesons typically present in such theories – the meson analogous to the $\rho$ in QCD. An alternative approach is given by “holographic technicolor”, which potentially provides a description of the first two extra vector mesons – including, in addition to the $\rho$, the analog of the $a_1$ meson in QCD.

In this note we consider a four-site “Higgsless” model illustrated, using “moose notation”, in fig. 1. We show how, once an $L_{10}$-like “wavefunction” mixing term for the two strongly-coupled $SU(2)$ groups in the center of the moose is included, we can reproduce the features of the holographic model – including the vanishing of the parameter $\alpha S$ for brane-localized fermions and the existence (whether or not $\alpha S = 0$) of the potentially interesting decay $a_1 \to W \gamma$.

II. THE MODEL

The Lagrangian for the model consists of several parts. First, the usual nonlinear sigma model link terms

$$\mathcal{L}_\pi = \frac{f_\pi^2}{4} \left[ \text{Tr} D^\mu \Sigma_1 D_\mu \Sigma_1^\dagger + \text{Tr} D^\mu \Sigma_3 D_\mu \Sigma_3^\dagger \right]$$

$$+ \frac{f_\pi^2}{4} \text{Tr} D^\mu \Sigma_2 D_\mu \Sigma_2^\dagger. \hspace{1cm} (1)$$

Next, the gauge-boson kinetic energies

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4} \left( \tilde{W}_{0 \mu}^2 + \tilde{W}_{1 \mu}^2 + \tilde{W}_{2 \mu}^2 + \tilde{W}_{3 \mu}^2 \right), \hspace{1cm} (2)$$

where we denote the weakly-coupled $SU(2) \times U(1)$ fields by $\tilde{W}_0$ and $\tilde{W}_3 \equiv B$ (by convention, $i = 3$ vanishes for the charged sector), and the strongly coupled $SU(2)$ fields by $\tilde{W}_{1,2}$. And finally, there is an $L_{10}$-like mixing between the middle two sites

$$\mathcal{L}_\varepsilon = -\frac{\varepsilon}{2} \text{Tr} \left[ \tilde{W}_{1 \mu \nu} \Sigma_2 \tilde{W}_{2 \mu \nu} \Sigma_1^\dagger \right], \hspace{1cm} (3)$$

where in this calculation we treat $\varepsilon$ as an $O(1)$ parameter. This model has a “parity” (more precisely, a $G$-parity) symmetry in the $g = g' = 0$ limit, under which $\tilde{W}_1^\mu \to \tilde{W}_2^\mu$, $\Sigma_1 \to \Sigma_3^\dagger$, and $\Sigma_2 \to \Sigma_1^\dagger$. In the limit $f_2 \to \infty$, this model reduces to the three-site model considered in [2].

In unitary gauge (with $\Sigma_1 = \Sigma_2 = \Sigma_3 \equiv I$), the $\mathcal{L}_\varepsilon$ term above corresponds to wavefunction-mixing of the fields $\tilde{W}_i$,

$$\mathcal{L} = -\frac{1}{4} \tilde{W}_{i \mu \nu} \tilde{Z}_{ij} \tilde{W}_{j \mu \nu} - \frac{1}{2} \tilde{W}_{i \mu} M_{ij}^2 \tilde{W}_{j \mu}^\dagger, \hspace{1cm} (4)$$

with

$$\tilde{Z} = \begin{pmatrix} 1 & \varepsilon & 0 \\ 1 & \varepsilon & 1 \\ 0 & 1 & 1 \end{pmatrix}. \hspace{1cm} (5)$$

To avoid ghosts, we require $\tilde{Z}$ to be positive-definite, and hence $|\varepsilon| < 1$. [For fixed values of $2/f_2^2 + 1/f_3^2$, see eqn. (4)].

FIG. 1: The “moose” diagram for the $SU(2)^3 \times U(1)$ model considered in this note. The solid circles represent $SU(2)$ groups; the dashed circle, a $U(1)$ group; the “links”, $SU(2) \times SU(2)/SU(2)$ non-linear sigma models. In order to be phenomenologically realistic, we work in the limit $g, g' \ll g$; in this limit the model also has an approximate parity symmetry. We consider brane-localized fermions, which couple only the the $SU(2) \times U(1)$ at the ends of the moose, and add an $L_{10}$-like “wavefunction mixing” term to mix the two strongly-coupled $SU(2)$ groups in the middle two sites.
III. MASSES AND MIXING ANGLES

The eigenstates corresponding to the quadratic part of Lagrangian in eqn. (1) satisfy the generalized eigenvalue equation

\[ M^2 \vec{v}_n = m^2_n \vec{Z} \vec{v}_n , \]

(6)

where \( \vec{v}_n \) is a vector in site-space with components \( v^i_n \). The superscript \( i \) labels the sites, running from 0 to 2 for charged-bosons \( (n = W^\pm, \rho^\pm, \alpha_1^\pm) \), and 0 to 3 for neutral ones \( (n = Z^0, \rho^0, \alpha_1^0, \gamma) \). If we choose eigenvectors normalized by \( \vec{v}^\dagger_n \vec{Z} \vec{v}_m = \delta_{nm} \), the gauge-eigenstate \( (W^\nu_n) \) and mass-eigenstate \( (W^\nu_{nm}) \) fields are related by

\[ W^\nu_n = \sum_n v^i_n W^\nu_{nm} . \]

(7)

A. The \( g = g' = 0 \) Limit

Consider first the \( g = g' = 0 \) limit, in which we can determine the leading contributions to the heavy gauge-boson masses. Due to the parity symmetry in this limit, we expect the eigenvectors to be proportional to \( \vec{W}^\nu_1 \pm \vec{W}^\nu_2 \). Applying the normalization condition \( \vec{v}^\dagger_n \vec{Z} \vec{v}_m = \delta_{nm} \), we find a parity-even eigenvector (the “\( \rho \)”)

\[ \vec{\rho}^\nu = \frac{1}{\sqrt{2(1 + \varepsilon)}} \left( \vec{W}^\nu_1 + \vec{W}^\nu_2 \right) , \]

(8)

with mass

\[ m^2_\rho = \frac{\tilde{g}^2}{4} \frac{f_1^2}{1 + \varepsilon} , \]

(9)

and a parity-odd eigenvector (the “\( \alpha_1 \)”)

\[ \vec{\alpha}_1^\nu = \frac{1}{\sqrt{2(1 - \varepsilon)}} \left( \vec{W}^\nu_1 - \vec{W}^\nu_2 \right) , \]

(10)

with mass

\[ m^2_{\alpha_1} = \frac{\tilde{g}^2}{4} \frac{f_1^2 + 2f_2^2}{1 - \varepsilon} . \]

(11)

We note that the \( \rho \) and \( \alpha_1 \) are degenerate for

\[ \varepsilon = - \frac{f_2^2}{f_1^2 + f_2^2} , \]

(12)
a value satisfying the constraint \( |\varepsilon| < 1 \). As \( \varepsilon \) becomes more negative, the \( \alpha_1 \) becomes lighter than the \( \rho \).

B. The Photon

Examining the eigenvalue eqn. (6) we see that the wavefunction factor \( \vec{Z} \) affects the normalization of a massless eigenvector, but not the orientation. We see, therefore, that the photon must be of the form

\[ A_\mu = \frac{e}{g} W^3_{0\mu} + \frac{e}{g} W^3_{1\mu} + \frac{e}{g} W^3_{2\mu} + \frac{e}{g'} B_\mu , \]

(13)

or

\[ (v_\gamma)^T = \left( \frac{e}{g} , \frac{e}{g} , \frac{e}{g} , \frac{e}{g'} \right) . \]

(14)

The electric charge \( e \) is, then, determined from the normalization condition to be

\[ \frac{1}{g^2} = \frac{1}{g^2} + \frac{1}{g'^2} + 2(1 + \varepsilon) . \]

(15)

Examining the photon-couplings, we see that the unbroken gauge-generator has the expected form \( Q = T^3 + T_1^1 + T_2^3 + Y \).

C. The \( W \)-boson

Next, we consider a perturbative evaluation of the electroweak boson eigenvectors and eigenvalues, computed in powers of \( x = g/g' \). We start with the \( W \)-boson; the charged-boson mass matrix is given by

\[ M^2_W = \frac{\tilde{g}^2}{4} \frac{f_1^2}{2(f_1^2 + 2f_2^2)^2} \begin{pmatrix} x^2f_1^2 & -xf_1^2 & 0 & 0 \\ -xf_1^2 & f_1^2 + f_2^2 & -f_2^2 & -f_2^2 \\ 0 & -f_2^2 & f_1^2 + f_2^2 & -x \tan \theta f_1^2 \\ 0 & 0 & -x \tan \theta f_1^2 & x^2 \tan^2 \theta f_1^2 \end{pmatrix} . \]

(16)

To \( \mathcal{O}(x^2) \) we find

\[ v^0_W = \left[ 1 - f_1^4 + 2(1 + \varepsilon)f_1^2f_2^2 + 2(1 + \varepsilon)f_2^4 \right] x^2 , \]

\[ v^1_W = x \frac{f_1^2 + f_2^2}{f_1^2 + 2f_2^2} W_1 , \]

\[ v^2_W = x \frac{f_2^2}{f_1^2 + 2f_2^2} W_2 , \]

where we have computed, but do not display, the corrections of \( \mathcal{O}(x^3) \) to the last two components. For the corresponding eigenvalue we find

\[ m^2_W = \frac{\tilde{g}^2}{4} \frac{f_1^2f_2^2}{f_1^2 + 2f_2^2} \left[ 1 - f_1^4 + 2(1 + \varepsilon)f_1^2f_2^2 + 2(1 + \varepsilon)f_2^4 \right] x^2 . \]

(18)

D. The \( Z \)-boson

The neutral gauge-boson mass matrix is

\[ M^2_Z = \begin{pmatrix} x^2f_1^2 & -xf_1^2 & 0 & 0 \\ -xf_1^2 & f_1^2 + f_2^2 & -f_2^2 & 0 \\ 0 & -f_2^2 & f_1^2 + f_2^2 & -x \tan \theta f_1^2 \\ 0 & 0 & -x \tan \theta f_1^2 & x^2 \tan^2 \theta f_1^2 \end{pmatrix} . \]

(19)
where we have defined the angle $\theta$ by $g'/g \equiv \tan \theta$. Note that $\theta$ is the leading order weak mixing angle; we will later define a weak mixing angle $\theta_Z$ that is better suited to comparison with experiment. We have computed the $Z$-boson eigenvector to $O(x^3)$ – as the result is complicated, and the algebra unilluminating, we do not reproduce it here. For the $Z$-boson mass, we find

$$m^2_Z = \frac{g^2}{4 \cos^2 \theta} \left[ f_1^2 f_2^2 \left(1 - \frac{(3 - \varepsilon) f_1^4 + 4(1 + \varepsilon)(f_1^2 f_2^2 + f_2^4) + (1 + \varepsilon)(f_1^2 + 2 f_2^2)^2 \cos 4\theta x^2 \sec^2 \theta}{4(f_1^2 + 2 f_2^2)^2} \right) \right].$$ (20)

### IV. THE ELECTROWEAK PARAMETERS

From eqn. (7), we can compute the couplings of the mass-eigenstate $W$-boson couplings $g^f_W = g_0 v_W e^{\theta^i} B_i$, and the $Z$-boson couplings $g^f_Z = g v^2_0 I_3 + g' v^2_0 Y = g I_3 (v^2_0 - \tan \theta v^2_Z) + g' v^2_Z Q$. (22)

We may then compute the on-shell precision electroweak parameters at tree-level to $O(x^3)$, using the definitions and procedures outlined in [10][11]. The values of electric charge, eqn. (15), and $m^2_Z$, eqn. (20), are given above, and we find the Fermi constant

$$\sqrt{2} G_F = \frac{2}{v^2} = \frac{2}{f_1^2} + \frac{1}{f_2^2},$$ (23)

where $v \approx 246$ GeV.

The only non-zero precision electroweak parameter parameter is $\alpha S$, which we tract

$$\alpha S = \frac{\varepsilon f_1^4 + 4(1 + \varepsilon)(f_1^2 f_2^2 + f_2^4) + (1 + \varepsilon)(f_1^2 + 2 f_2^2)^2 \cos 4\theta x^2 \sec^2 \theta}{(f_1^2 + 2 f_2^2)^2},$$ (24)

As expected, we can choose $\varepsilon$ so that $\alpha S$ vanishes for any given value of $f_1/f_2$

$$\varepsilon \rightarrow -\frac{2(f_1^2 + f_2^2 f_2^2 + 2 f_2^2)}{f_1^2 + 2 f_2^2 f_2^2 + 2 f_2^2},$$ (25)

while satisfying $|\varepsilon| < 1$.

Note, however, that the value of the low-energy parameter $|\varepsilon|$ that makes $\alpha S$ vanish is of order one, larger than would be expected by naive dimensional analysis. This result is consistent with investigations of continuous 5d effective theories [14][15], and with investigations of plausible conformal technicolor “high-energy completions” of this model using Bethe-Salpeter methods [16][17], both of which suggest that $\alpha S > 0$ and that it may not be possible to achieve very small values of $\alpha S$.

We note also that the result is consistent with the expectation of [18][19], since the value of $\varepsilon$ required for $\alpha S$ to vanish results in axial-vector mesons which are lighter than the vector mesons.²

### V. TRIPLE BOSON VERTICES

#### A. Electroweak Vertices

Consider the electroweak vertices $\gamma WW$ and $ZWW$. To leading order, in the absence of CP-violation, the triple gauge boson vertices may be written

$$\mathcal{L}_{TGV} = -i e^{c_Z} s_Z \left[ (1 + \Delta \kappa_Z) W^+ \gamma W^- s_{Z}^\dagger (W^+ W^-) \right] - i e^{c_Z} s_Z \left[ (1 + \Delta \theta^Z) (W^+ \gamma W^- - W^- \gamma W^+) \right] - i (W^+ \gamma W^- - W^- \gamma W^+) A_\nu,$$ (26)

where the two-index tensors denote the Lorentz field-strength tensor of the corresponding field. In the standard model, $\Delta \kappa_Z = \Delta \kappa = \Delta \theta^Z = 0$. Note that the expressions for $\kappa_Z$ and $g^Z_1$ involve $c_Z \equiv \cos \theta_Z$ and $s_Z \equiv \sin \theta_Z$, as defined by

$$c^2_Z = \frac{e^2}{4 \sqrt{2} G_F M_Z^2},$$ (27)

rather than the leading order mixing angle $\theta$.

Let us begin with the coupling of the photon of the form $(W^+ \gamma W^- - W^- \gamma W^+) A_\nu$. In terms of the wavefunctions $v_\gamma, v_\nu$, this coupling is proportional to

$$g_\gamma = \sum_{i,j} g_i v^i_\gamma v^j_W Z_{ij} v^j_W.$$ (28)

From eqn. (14), we have $g_i v^i_\gamma = e$ and therefore, by applying the normalization condition $v^T_W Z v_W = 1$, we

² An alternative approach, Degenerate BESS [20][21], produces degenerate vector and axial mesons and $\alpha S = 0$ using a different theory without unitarity delay [10] – see “case I” described in [22].
obtain $g_v \equiv e$ independent of any choice of the four-site parameters — as required by gauge-invariance and consistent with the form of eqn. (26).

Next, we evaluate $\Delta \kappa_\gamma$, with

$$e [1 + \Delta \kappa_\gamma] = \sum_{i,j} g_i (v_i^x)^2 \tilde{Z}_{ij} v_j^x = e \sum_{i,j} \frac{g_i}{g_j} (v_i^x)^2 \tilde{Z}_{ij} ,$$

for which we calculate

$$\Delta \kappa_\gamma = \frac{\varepsilon f_1^4}{(f_1^2 + 2f_2^2)^2} x^2 = \frac{\varepsilon v_4^4}{f_2^2} x^2 .$$

(29)

Note that this vanishes in the absence of wavefunction mixing ($v \rightarrow 0$), and also in the “three-site” limit ($v/f_2 \rightarrow 0$), as consistent with [6].

Similarly we may compute $\Delta \kappa_{\gamma Z}$ and $\Delta \kappa_{\mu \nu}$, and find

$$\Delta g_1^2 = \Delta \kappa_{\gamma Z} + \frac{\varepsilon f_1^4}{(f_1^2 + 2f_2^2)^2} x^2,$$

$$= - \frac{(\varepsilon s_2^2 f_1^4 + (1 + \varepsilon) f_2^4 + 4 f_2^4) x^2}{(f_1^2 + 2f_2^2)^2 \cos(2\theta_Z)} ,$$

(31)

where the difference between $\theta$ and $\theta_Z$ is irrelevant to this order. Note that $\Delta g_1^2 - \Delta \kappa_{\gamma Z}$ vanishes when $\varepsilon \rightarrow 0$, and also, as expected [6], in the “three-site” limit $f_2 \rightarrow \infty$.

VI. SUMMARY

We have introduced a deconstructed Higgsless model with four sites and non-trivial wavefunction mixing, and have shown that it exhibits key features of holographic technicolor [3, 7]. The electroweak parameter $\alpha S$ vanishes for a value of the wavefunction mixing at which the $a_1$ is lighter than the $\rho$ — even if all fermions are brane-localized. Furthermore, the model includes the decay $a_1 \rightarrow W \gamma$, suppressed by only one power of $(M_W/M_\rho)$, in contrast with an $(M_W/M_\rho)^3$ suppression of the decay $\rho \rightarrow W \gamma$. These decays are of potential phenomenological interest at LHC.

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