From classical xenon fringes to hydrogen interferometry

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Keywords: Talbot–Lau interferometry, moiré deflectometer, hydrogen, xenon, classical–quantum, antiproton interferometer

Abstract

A setup with three equally spaced transmission gratings can be described as a moiré deflectometer in the classical regime and as a Talbot–Lau interferometer in the quantum regime. We successfully operate such a three-grating device with a de Broglie wavelength span of more than two orders of magnitude (20 fm to 2.2 pm), employing different particles such as xenon, krypton, argon, helium, and hydrogen. With that we directly observe the transition from the classical description with fast xenon atoms, to the characteristic quantum behaviour of the Talbot–Lau interferometer using hydrogen atoms with correspondingly long de Broglie wavelength. The systematic study of the interference pattern gives important insights in the feasibility of a proton or antiproton interferometer with this setup.

1. Introduction

The wave description of particles is a cornerstone of quantum physics and lies in the focus of many modern experiments. Since the seminal work of Davisson and Germer with electrons in 1927 [1] the concept of matterwave interference is confirmed for multiple kind of particles, such as positrons [2], neutrons [3], atoms [4] and complex molecules [5, 6].

A paradigmatic experimental setup for revealing matterwave interference is a setup with three equally spaced transmission gratings [4], which is also employed in this work (see insets in figure 1). For that, one can distinguish two regimes of operation. In the case of a well collimated beam i.e. transverse coherent source, the device is the implementation of a Mach–Zehnder type interferometer independent of the spacing between the gratings. For uncollimated beams i.e. spatially incoherent sources, also interferences can be observed, but the distances have to be multiples of the Talbot length

$$L_T = \frac{d^2}{\lambda}$$

with $$d$$ as the grating constant and $$\lambda$$ as the de Broglie wavelength. This operation is known as a three-grating Talbot–Lau interferometer [4, 6–8]. The operation can be understood as follows. The first grating establishes transverse coherence in the beam, the second lets the beam interfere and the third resolves the resulting nanoscopic fringe pattern. The same device can also be operated in the classical regime for small $$\lambda$$ and is known as moiré deflectometer shown for argon atoms [9] and antiprotons [10]: the particle trajectories of an uncollimated beam are restricted due to the first two gratings in such a way, that they form a periodic shadow structure in the plane of the third grating.

One can estimate the transition between those two regimes: the classical assumption of straight trajectories has to break down if the first-order diffraction

$$\sin(\theta_d) = \frac{\lambda}{d}$$

leads to a transverse coherence comparable to the grating constant on the following grating. Hence, a classical description is guaranteed under the condition of

$$L \sin(\theta_d) \approx L \theta_d \ll d \Rightarrow L \ll \frac{d^2}{\lambda} =: L_T.$$  (1)

The relevance of the Talbot length $$L_T$$ is illustrated in figure 1(b): behind a grating illuminated with a plane wave, self-images of the grating are observed at integer multiples of the Talbot length. It is important to note that in the plane of the third grating such self-images are indistinguishable from the pattern formed using classical trajectories if the grating distances match a multiple of the Talbot length [11]. A significant
Figure 1. Working principle of a three-grating moiré deflectometer and its quantum mechanical counterpart the Talbot–Lau interferometer: inset (a) principle of the classical moiré deflectometer: an uncollimated beam passing through two gratings forms a shadow image of the gratings in the plane of the third grating. Inset (b) near-field interference pattern with a plane wave illumination of the first grating. At integer multiples of the Talbot length \( L_T \) a grating’s self-image is observed. In both cases—classical and quantum—a nanoscopic pattern is formed in the plane of the third grating and resolved with a third grating. The full graph shows the visibility as a function of the de Broglie wavelength for a setup with a grating distance \( L = 14 \) cm and a grating pitch \( d = 257 \) nm, as realized in this work. The visibility peaks are at wavelengths corresponding to the Talbot rephasing. The maximal visibility is the same as expected in the classical limit \( \lambda \ll \lambda_{1T} \). The top graphs illustrate the progression from straight classical trajectories to the increasing influence of the quantum behaviour for longer wavelengths leading to so-called Talbot-carpets.

The difference between classical and quantum pattern is observed if the distance does not match multiples Talbot lengths, since there the periodic pattern disappears in the quantum regime, while persists in the classical limit.

A standard characterization of a pattern is the visibility

\[
\nu = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}, \tag{2}
\]

with \( I_{\text{max}} \) and \( I_{\text{min}} \) the maximal and minimal intensity of the pattern. Figure 1 gives the visibility over a range of de Broglie wavelengths, accessible in our experiment, with the fixed grating distance of \( L = 14 \) cm, the grating period of \( d = 257 \) nm, and the open fraction of the grating (slit-width over period) \( \eta = 30\% \), as realized in this work. The visibility is calculated on the basis of the Wigner representation assuming ideal transmission gratings with grating function \( g(y) \in \{0, 1\} \) as discussed in [11]. Following the definition of the Talbot length (equation (1)) and the condition that \( L = n \cdot L_T \) \( (n \in \mathbb{N}) \), we have high visibility for the de Broglie wavelengths \( \lambda_{nT} = n \cdot 472 \) fm, as well as in the classical limit \( \lambda \ll 472 \) fm. For wavelengths not matching these conditions the visibility is drastically reduced. The top row of graphs in figure 1 shows the underlying Talbot carpets for the corresponding de Broglie wavelengths illustrating that the visibility peaks at positions where the slit pattern rephases.

2. Experimental results

Building on a robust electron cyclotron resonance (ECR) ion source [12] and a neutralization chamber [13] we realize different atomic beams such as xenon, krypton, argon, helium, and hydrogen with controlled energies as input for the three-grating device (see figure 2). The gratings have a periodicity of \( d = 257 \) nm, are separated by \( L = 14 \) cm and placed on a set of actuators to align them horizontally and rotationally. Single atoms are detected with a resolution of \( 10 \) \( \mu \)m using a microchannel plate (MCP), a phosphor screen, and a camera. In the case of an 11.6 keV Xe-beam and a 3.7 keV H-beam periodic patterns are observed...
Figure 2. Experimental setup. Based on the gas being fed into the electron cyclotron resonance (ECR) source, a beam with corresponding charged and neutral particles is produced. The ions are accelerated to the desired momentum and separated with a Wien filter and a pair of Helmholtz coils. A neutralization chamber flooded with nitrogen serves to neutralize the ion beam without significant energy loss. The following deflector plates separate the left over charged particles leading to a clean beam of neutral atoms. The experimental interferometer/moiré setup consists of three equally spaced ($L = 14\text{ cm}$) transmission gratings ($d = 257\text{ nm}$) which are placed on a set of actuators for alignment. The whole setup is placed inside a mu-metal to magnetically and electrically shield the device. The particles are detected with a microchannel plate, a phosphor screen, and a camera. A Faraday cup is used for beam diagnostics and a set of movable pinholes for the control of the beam divergence.

Figure 3. Classical versus quantum pattern. With the same three-grating setup a fringe pattern is observed with a xenon beam ($E = 11.6\text{ keV}$) and a hydrogen beam ($E = 3.7\text{ keV}$). Both patterns exhibit similar visibility ($\nu \approx 40\%$) but are due to different physical mechanisms. In the classical regime (xenon) it is a result of periodic shadowing while in the quantum regime (hydrogen) it is due to interference. The small structure in the left figure results from the channel size of the microchannel plate ($\approx 10\mu\text{m}$). The spatial change of the flux originates from the atomic beam inhomogeneity. The histograms on the right illustrate the fringe projection in the direction of the pattern.

(see figure 3) and correspond to the classical and quantum regimes respectively. The last grating is tilted with respect to the two others by $\approx 1\text{ mrad}$ to magnify the nanoscopic pattern using the moiré effect [14]. The histograms on the right of each graph reveal the spatially periodic pattern with a similar visibility $\nu \approx 40\%$.

From the mere observation of fringes it is not possible to distinguish between classical and quantum pattern. Hence, in order to reveal the difference between a classical shadow image and quantum interference fringes we use the dependence of the visibility on the energy of particles. The results are shown in figure 4. In the case of the xenon beam and the other heavy atoms such as krypton and argon, the visibility decreases monotonically with decreasing energy (velocity). For an ideal setup where only shadowing is present this is not expected, but in our setup forces within the slits of the grating lead to velocity dependence due to deviations from straight trajectories (more detailed discussion in section 3 interaction between particles and material gratings). The situation is very different for the helium and hydrogen beam. Here, we observe visibility maxima which are a clear indication of wave properties and hence the quantum regime. In the measurable range we observe one maximum for helium (390 eV), and four maxima in the case of hydrogen (230 eV, 390 eV, 950 eV, 2.8 keV).

In order to directly compare our experimental observations with the theoretical predictions (figure 1) we plot in figure 5 the visibility as a function of the inferred de Broglie wavelength. Considering the Talbot–Lau interferometer in our experiment, fringes with high contrast are expected to appear whenever
the de Broglie wavelength is a multiple of 472 fm, i.e. $\lambda_{IT} = n \cdot 472$ fm. We use a Gaussian fit to extract the wavelength of maximum visibility and find:

Hydrogen:

$$\lambda_{IT} = 474$ fm $\pm 2$ fm $= (1.004 \pm 0.004) \cdot 472$ fm
$$
$$\lambda_{IT} = 951$ fm $\pm 2$ fm $= (2.015 \pm 0.004) \cdot 472$ fm
$$
$$\lambda_{IT} = 1444$ fm $\pm 10$ fm $= (3.059 \pm 0.021) \cdot 472$ fm
$$
$$\lambda_{IT} = 1875$ fm $\pm 5$ fm $= (3.972 \pm 0.011) \cdot 472$ fm
$$

Helium: $\lambda_{IT} = 470$ fm $\pm 3$ fm $= (0.996 \pm 0.006) \cdot 472$ fm.

The peak values obtained by fitting the data agree with the theoretical prediction within 2%, thus supporting the claim that the detected contrast maxima can be ascribed to the first up to the forth Talbot length for hydrogen, and to the first Talbot length for helium.

### 3. Interaction between particles and material gratings

The measured visibility reveals a striking and noteworthy difference in comparison to the calculated dependence in the ideal transmission grating limit. First, heavy particles with wavelengths below 100 fm (classical) show a stronger dependence on the de Broglie wavelength in visibility as theoretically predicted [see figure 6(a), black line], second, the visibility maxima for the Talbot condition (quantum regime) are
Figure 5. The quantum structure is revealed plotting the visibility as a function of the de Broglie wavelength. With that the peaked visibility structures for different species such as helium and hydrogen fall on top of each other expected from the underlying quantum mechanical interference. The visibility dependence in the classical regime does not collapse on one curve since this behaviour depends on specific interaction of the particles within the massive nanometric gratings. Histograms on top show the raw data for the indicated positions.

Figure 6. Influence of charges within in the material transmission gratings. (a) Charges lead to a strong deflection within a slit of the gratings. With this additional force the dependence on the de Broglie wavelength can be explained. A good agreement with the data is obtained if the mean charges $Q$ are assumed to be Ar: $Q = 170e$, Kr: $Q = 190e$, Xe: $Q = 100e$. The shaded areas correspond to $Q \pm 20e$. (b) In the quantum regime the visibility peak at the first and second Talbot order gets narrower with increasing intra-grating interactions. The best agreement with the simulation is found for a charge of $Q = 700e$. The shaded area corresponds to $[300e, 1400e]$.

narrower than expected [see figure 6 (b), black line]. This can be understood as a result of particle grating interaction. Due to the finite thickness $l$ of the material gratings a significant deviation from straight paths is expected influencing the performance of the three grating setup.

Assuming the grating walls to be parallel and straight, the interaction is only significant within the slits, and captured via a potential $V(y, z)$ as well as the eikonal approximation [11], the modified grating function is given by

$$
\tilde{g}(y) = g(y) \exp \left( -i \frac{\hbar}{lv_z} \int dz \, V(y, z) \right) = g(y) \exp \left( -i l \frac{\hbar}{v_z} V(y) \right).
$$

(3)

For simplicity, in the last step we assumed that the interaction potential is independent of the $z$-position inside the grating (i.e. $V(y, z) \equiv V(y)$) and that the longitudinal velocity $v_z$ is much greater than the transverse, and thus $v_z \approx v$.

As a model for the strength of the intra-grating interactions we propose an interaction potential due to implanted charges $Q$ on the inner wall of the grating bars, which induce a dipole moment in the atoms.
resulting in a potential of
\[
V(y) = -\frac{\alpha Q^2}{16\pi^2 \varepsilon_0} \left( \frac{1}{(y + s/2)^2} - \frac{1}{(y - s/2)^2} \right)^2 \quad \text{for} \quad y \in \left( -\frac{s}{2}, \frac{s}{2} \right),
\]
with \( s \) the slit width of the gratings and \( \alpha \) the static polarisibility of the corresponding atom (\( \alpha_{\text{Kx}} = 452 \times 10^{-42} \text{cm}^2\text{V}^{-1} \), \( \alpha_{\text{Kr}} = 277 \times 10^{-42} \text{cm}^2\text{V}^{-1} \), \( \alpha_{\text{Ar}} = 184 \times 10^{-42} \text{cm}^2\text{V}^{-1} \), \( \alpha_{\text{He}} = 23 \times 10^{-42} \text{cm}^2\text{V}^{-1} \), \( \alpha_{\text{H}} = 75 \times 10^{-42} \text{cm}^2\text{V}^{-1} \) [15]).

Note that various other factors exist, which affect the pattern visibility, for instance grating misalignments or independent vibrations of gratings [16, 17]. However, they merely reduce the visibility by a constant factor for our wavelength range. Hence for the analysis of the overall dependence of the visibility we rescale its absolute values.

Figure 6(a) shows the data for de Broglie wavelengths below 100 fm (classical regime) compared with the best fitting values for implanted charges \( Q \). It can be clearly seen that the data points drop down at much smaller value for \( \lambda \), as expected for the undisturbed case, i.e. \( Q = 0 \) (black line) where the quantum interference (dephasing) leads to the visibility reduction. The best agreement of the experimental data with the simulation is obtained for the depicted values of \( Q \) in the graph. While for argon a charge of 170 times the elementary charge \( e \) is found to reproduce the experimental observation, the result for krypton is \( Q = 190e \) and for xenon \( Q = 100e \). The shaded areas indicate the variability of the signal for charges \( Q \pm 20e \). Also in the quantum regime, i.e. \( \lambda \gg 472 \text{ fm} \) the influence of implanted charges is very prominent, specifically the width of the visibility peak at the first and second Talbot orders. As can be seen in figure 6(b), the simulation agrees with \( Q = 700e \) (the shading indicates implanted charges ranging from \( Q = 300e \) up to \( Q = 1400e \)). A similar result is obtained for the helium data shown in figure 5 \( (Q = 740e) \). The difference in estimated charges \( Q \) (H, He, Ar, Kr, and Xe) might be a consequence of the corresponding histories of the measurements (alignment, exposure of charged ion beams, etc; during the data taking over three weeks). Nevertheless it shows the robustness of the interference phenomenon at exactly the Talbot length, as the intra-grating interaction does not destroy the rephasing.

4. Outlook: proton- and antiproton-interferometry

This study has been initiated within the AEgIS-Collaboration [10, 18] where one of the goals is the implementation of an antimatter interferometer. Thus, building on our results on Talbot–Lau interferometer for hydrogen atoms, we discuss in the following the interferometry with the same setup for protons and antiprotons.

The main difference, the charges, gives rise to a number of experimental issues due to the sensitivity to electric and magnetic fields. While external magnetic and electric fields can be sufficiently shielded the fields, inside the material gratings and hence intra-grating interactions due to previously mentioned implanted charges may be the critical point of this setup. Using the Coulomb potential
\[
V(y) = -\frac{Qq}{4\pi\varepsilon_0 \left( y + \frac{s}{2} \right)} \left( y - \frac{s}{2} \right) \quad \text{for} \quad y \in \left( -\frac{s}{2}, \frac{s}{2} \right),
\]
instead of the charge–dipole interaction in equation (3) and the estimation of the implanted charges in the previous section, we simulate the visibility for the present Talbot–Lau interferometer when utilized with a proton or antiproton beam. Figure 7 shows the calculated visibility dependence on wavelengths for different implanted charges \( Q \) compared to the charge-free case. As it can be recognized the peak around the first Talbot order gets significantly narrower with the increasing strength of the intra-grating interactions. For instance, an implanted charge of \( Q = 100e \) requires to match the wavelength for the first Talbot order \( \lambda_{1T} = 471.78 \text{ fm} \) with a precision better than \( \Delta \lambda = 0.25 \text{ fm} \). This implies the control of the kinetic energy with a precision better than \( \Delta E = 4 \text{ eV} \) for the mean kinetic energy of 3652 eV. Nevertheless an energy spread of the beam of 1% still allows to observe an interference pattern but with a reduced visibility of 20%. For a precisely given de Broglie wavelength this means for the described setup \( (L = 14 \text{ cm}, d = 257 \text{ nm}) \) an absolute alignment of the grating distance better than \( \Delta L = 74 \mu \text{m} \) and the grating pitch known to a level of \( \Delta d = 68 \text{ pm} \).

Following the above arguments, realization of a proton interferometer with the current setup would need a control of the beam energy with precision of 1 eV. In principle this is feasible with the current setup using a double multipole Wien filter [19] for necessary energy selection. Thus, for antiproton interferometry the main constraint is the production of an antiproton beam with a tunable energy with the necessary precision.
Figure 7. The effect of a charge $Q$ within the massive transmission gratings for proton interference. The force inside the slits due to the Coulomb interaction exceeds the induced dipole interaction by more than six orders of magnitude. Nevertheless, the visibility maxima at the Talbot rephasing still survives. But it results in a very narrow peak around the wavelength corresponding to the first Talbot length. For an implanted charge of $Q = 100e$ the Talbot order has to be matched sufficiently better as $\Delta \lambda = 0.25 \text{ fm}$, which corresponds to the beam’s energy precision of $\Delta E = 4 \text{ eV}$ at $E = 3652 \text{ eV}$.

5. Conclusion

The characteristic behaviour of a Talbot–Lau interferometer for different de Broglie wavelengths over more than two orders of magnitude has been demonstrated, directly revealing the connection between classical shadowing and quantum interference. We observe up to the fourth order of the Talbot-effect and the transition to the classical regime for $\lambda \ll \frac{d}{2L}$. The measurements in the classical as well as in the quantum regime are in good agreement with the assumption of intra-grating interaction due to implanted charges inside the material gratings. The presence of such charges and their estimated value set constraints on the feasibility of showing proton/antiproton interference with the present setup. We conclude that proton and antiproton interferometry is possible even with implanted charges, but with corresponding small energy spread of the impinging beam.

Acknowledgments

This publication is based on the PhD thesis by S R M [20], which includes a more detailed discussion of the underlying theory and of the experimental setup. The authors express their grateful thanks to Markus Arndt providing the nanometric gratings. For the help coating the gratings with AuPd and taking SEM images we thank the research group of Rasmus Schröder, especially Anne Kast and Lisa Veith. Special thanks go to Andrea Demetrio and Pierre Lansonneur for the efforts in the early stage of the experiment. Furthermore the authors are very thankful for the discussions within the AEgIS Collaboration. This work was supported by the Deutsche Forschungsgemeinschaft (Grant No. OB164/10-1).

References

[1] Davisson C and Germer L H 1927 Phys. Rev. 30 705
[2] Sala S, Ariga A, Ereditato A, Ferragut R, Giammarchi M, Leone M, Pistillo C and Scampoli P 2019 Sci. Adv. 5 eaav7610
[3] Rauch H, Treimer W and Bonse U 1974 Phys. Lett. A 47 369
[4] Cronin A D, Schmiedmayer J and Pritchard D E 2009 Rev. Mod. Phys. 81 1051
[5] Arndt M, Nairz O, Vos-Andreae J, Keller C, Van der Zouw G and Zeilinger A 1999 Nature 401 680–2
[6] Fein Y Y, Geyer P, Zwick P, Kiałka F, Pedalino S, Mayor M, Gerlich S and Arndt M 2019 Nat. Phys. 15 1242–5
[7] Clauser J F and Li S 1994 Phys. Rev. A 49 R2213–6
[8] Gromniger G, Barwick B and Batelaan H 2006 New J. Phys. 8 224
[9] Oberthaler M K, Bernet S, Rasel E M, Schmiedmayer J and Zeilinger A 1996 Phys. Rev. A 75 3165–76
[10] Aghion S et al 2014 Nat. Commun. 5 4538
[11] Nimmrichter S and Horßberger K 2008 Phys. Rev. A 78 023612
[12] Sortais P, Lamy T, Méard J, Angot J, Latrasse L and Thuillier T 2010 Rev. Sci. Instrum. 81 02B314
[13] Pauly H 2000 Atom, Molecule, and Cluster Beams II (Berlin: Springer)
[14] Oster G, Wasserman M and Zwerling C 1964 J. Opt. Soc. Am. 54 169–75
[15] Hoinkes H 1980 Rev. Mod. Phys. 52 933–70
[16] Stibor A, Hornberger K, Hackermüller L, Zeilinger A and Arndt M 2005 Laser Phys. 15 10
[17] Demetrio A, Müller S R, Lansonneur P and Oberthaler M K 2017 Phys. Rev. A 96 063604
[18] Bräunig P H M 2014 Atom optical tools for antimatter experiments PhD Thesis Heidelberg University https://archiv.ub.uni-heidelberg.de/volltextserver/18044/
[19] Martínez G and Tsuno K 2004 Ultramicroscopy 100 105–14
[20] Müller S R 2020 From classical xenon fringes to hydrogen interferometry PhD Thesis Heidelberg University https://archiv.ub.uni-heidelberg.de/volltextserver/27912/