Search for $\eta$ and $\eta'$ → $\pi^+e^-\bar{\nu}_e$ + c.c. decays in $J/\psi \to \phi \eta$ and $\phi' \eta$.
Weak decays of quarkonium states such as $\eta$, $\eta'$, $\psi$, $\Upsilon$, etc., offer a window into what may lie beyond the standard model (SM)\cite{11,12}. The reason for the expected sensitivity is that the rates of the quarkonium weak decays are expected to be tiny in the framework of the SM\cite{13}. As originally pointed out by Singer\cite{14}, the weak decays $\eta \to \pi^+\pi^-$, $\eta' \to \pi^+\pi^-$ (where $\eta$ includes all $\eta$, $\eta'$, and charge conjugate states; $\mu$, and charge conjugate state implicitly included) are purely second class with a vector-type coupling in the SM (see Ref.\cite{15} for the definition of the second class current), and hence vanish in the limit of exact isospin symmetry. They occur in the SM in first order in the weak interaction, and hence vanish in the limit of exact isospin symmetry.

I. INTRODUCTION

Using a sample of 225.3 million $J/\psi$ events collected with the BESIII detector at the BEPCII $e^+e^-$ collider in 2009, searches for the decays of $\eta$ and $\eta' \to \pi^+\pi^-\nu\bar{\nu}$ and $\phi\phi'$ are performed. The $\phi$ signals, which are reconstructed in $K^+K^-$ final states, are used to tag $\eta$ and $\eta'$ semileptonic decays. No signals are observed for either $\eta$ or $\eta'$, and upper limits at the 90% confidence level are determined to be $7.3 \times 10^{-4}$ and $5.0 \times 10^{-4}$ for the ratios $\frac{B(\eta \to \pi^+\pi^-\nu\bar{\nu} + c.c.)}{B(\eta \to \pi^+\pi^- + c.c.)}$ and $\frac{B(\eta' \to \pi^+\pi^-\nu\bar{\nu} + c.c.)}{B(\eta' \to \pi^+\pi^- + c.c.)}$, respectively. These are the first upper limit values determined for $\eta$ and $\eta'$ semileptonic weak decays.

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for the branching fraction of $\eta \to \pi^+ l^- \bar{\nu}_l$ is predicted to be $2.6 \times 10^{-13}$. Therefore, any observation of $\eta \to \pi l \nu_l$ violating this bound would be a clear indication for new physics beyond the SM.

The decays $\eta \to \pi l \nu_l$ can be used to probe some types of possible new charged current interactions \cite{10, 11}. A rather old suggestion would be the introduction of a new second class vector current for the $\eta \to \pi$ transition \cite{12}. Scalar-type charged current four-fermion interactions can arise in gauge theories for example from the exchange of charged Higgs bosons in the two-Higgs-doublet model \cite{18, 19}. Also light leptoquarks \cite{20}, occurring naturally in grand unified theories and composite models, may enhance the $\eta \to \pi l \nu_l$ branching fraction considerably \cite{21}. For example, by considering scalar or vector type interaction, the branching fraction of $\eta \to \pi^+ l^- \bar{\nu}_l$ was estimated to be $10^{-8} - 10^{-9}$ \cite{22, 23}, which is a few order of magnitudes higher than that in the SM. Therefore, searches for the $\eta \to \pi^+ l^- \bar{\nu}_l$ and $\eta' \to \pi^+ l^- \bar{\nu}_l$ at the branching fractions level of $10^{-8} - 10^{-9}$ and below will provide information on the new physics beyond the SM. At present there is no experimental information on the decays $\eta \to \pi l \nu_l$. In this paper, we present measurements of branching fractions of $\eta$ and $\eta'$ into $\pi^+ e^- \bar{\nu}_e$ decays. This analysis is based on $(225.3 \pm 2.8) \times 10^6 J/\psi$ events \cite{24}, accumulated with the Beijing Spectrometer III (BESIII) detector \cite{25}, at the Beijing Electron Positron Collider II (BEPCII).

II. THE BESIII EXPERIMENT AND MC SIMULATION

BEPCII/BESIII \cite{25} is a major upgrade of the BESIII experiment at the BEPC accelerator. The design peak luminosity of the double-ring $e^+e^-$ collider, BEPCII, is $10^{33} \text{cm}^{-2}\text{s}^{-1}$ at the center-of-mass energy of 3770 MeV. The BESIII detector has a geometrical acceptance of 93% of $4\pi$ and consists of four main components: (1) a small-celled, helium-based main drift chamber (MDC) with 43 layers, which provides measurements of ionization energy loss ($dE/dx$). The average single wire resolution is 135 \mu m, and the momentum resolution for charged particles with momenta of 1 GeV/c in a 1 T magnetic field is 0.5%; (2) an electromagnet calorimeter (EMC) made of 6240 CsI (TI) crystals arranged in a cylindrical shape (barrel) plus two end-caps. For 1.0 GeV photons, the energy resolution is 2.5% in the barrel and 5% in the end-caps, and the position resolution is 5 mm in the barrel and 9 mm in the end-caps; (3) a time-of-flight system (TOF) for particle identification (PID) composed of a barrel part made of two layers with 88 pieces of 5 cm thick and 2.4 m long plastic scintillators in each layer, and two end-caps with 96 fan-shaped, 5 cm thick, plastic scintillators in each end-cap. The time resolution is 80 ps in the barrel, and 110 ps in the end-caps, corresponding to a 2\sigma K/\pi separation for momenta up to about 1.0 GeV/c; (4) a muon chamber system made of 1000 m$^2$ of resistive-plate-chambers arranged in 9 layers in the barrel and 8 layers in the end-caps and incorporated in the return iron of the superconducting magnet. The position resolution is about 2 cm.

The optimization of the event selection and the estimation of physics backgrounds are performed using Monte Carlo (MC) simulated data samples. The GEANT4-based simulation software BOOST \cite{20} includes the geometric and material description of the BESIII detectors, the detector response and digitization models, as well as the track records of the detector running conditions and performance. The production of the $J/\psi$ resonance is simulated by the MC event generator KKMC \cite{27}; the known decay modes are generated by EVTGEN \cite{28} with branching ratios taken from the Particle Data Group (PDG) tables \cite{29} and determined by the Lundcharm model LUNDCHARM \cite{30} for the remaining unknown decays. The analysis is performed in the framework of the BESIII offline software system \cite{31} which takes care of the detector calibration, event reconstruction and data storage.

III. DATA ANALYSIS

A. Analyses for $\eta$ and $\eta' \to \pi^+ e^- \bar{\nu}_e$

In order to detect $\eta$ and $\eta' \to \pi^+ e^- \bar{\nu}_e$ decays, we use $J/\psi \to \phi \eta$ and $\phi \eta'$ decays. These two-body decays provide a very simple event topology, in which the $\phi$ signals can be reconstructed easily and cleanly decaying into $K^+K^-$. The reconstructed $\phi$ particles can be used to tag $\eta$ and $\eta'$ in order to allow a search for their semileptonic decays. In addition, the $\eta$ and $\eta'$ decays are easy to define in the lab system due to the strong boost of the $\phi$ from $J/\psi$ decay.

Charged tracks in the BESIII detector are reconstructed using track-induced signals in the MDC. We select tracks within $\pm10$ cm of the interaction point in the beam direction and within 1 cm in the plane perpendicular to the beam direction. The tracks must be within the MDC fiducial volume, $|\cos \theta| < 0.93$ ($\theta$ is the polar angle with respect to the $e^+$ beam direction). Candidate events require four charged tracks with net charge zero. The TOF and $dE/dx$ information are combined to form PID confidence levels for the $\pi$, $K$, and $e$ hypotheses; each track is assigned to the particle type that corresponds to the hypothesis with the highest confidence level. To suppress background from $J/\psi \to \phi \eta$ ($\eta'$), where $\eta$ ($\eta'$) decays into nonleptonic modes, the electron candidate is further identified with the ratio of deposited energy in the EMC to track momentum, $E/p$, which must be larger than 0.8. We further require that $E/p$ should be less than 0.8 for the pion candidate to suppress background from $J/\psi \to \phi \eta$ ($\eta \to \gamma \pi^+ e^-$) decay.

Showers identified as photon candidates must satisfy fiducial and shower-quality requirements. The minimum
energy is 25 MeV for EMC barrel showers ($|\cos \theta| < 0.8$) and 50 MeV for end-cap showers ($0.86 < |\cos \theta| < 0.92$). To eliminate showers produced by charged particles, a photon must be isolated from any charged track by more than 20° if not specified otherwise.

Since the mass of neutrino is almost zero and it is invisible in the detectors, a one-constraint (1C) kinematic fit is performed to constrain the missing mass of the reconstructed tracks to be zero, and $\chi^2_{1C} < 200$ is required. The 1C fit improves the resolution of recoil mass of the $K^+K^-$ system by a factor of 2.5 for $\eta$ case or a factor of 1.6 for $\eta'$ case. After the 1C fit, the missing momentum $P_{\text{miss}} = |\vec{P}_{\text{miss}}|$ can be calculated; here, $\vec{P}_{\text{miss}} = -(\vec{P}_\phi + \vec{P}_{\pi^+} + \vec{P}_{\pi^-})$ in the rest frame of $J/\psi$, and we require that the missing momentum should be larger than 0.03 GeV/c to suppress backgrounds from final states with only four tracks, such as $J/\psi \rightarrow \phi \pi^+ \pi^- (\phi \rightarrow K^+ K^-)$. We count the number $N_{\text{shower}}$ of EMC showers that could originate from a $K_L$ or a photon, and require that $N_{\text{shower}}$ be zero in the region inside a cone of 0.3 (1.5) rad around the direction of the missing momentum for $J/\psi \rightarrow \phi \eta (\eta') \ (\eta (\eta') \rightarrow \pi^+ e^- \bar{\nu}_e)$. These requirements reject most $\eta$ and $\eta'$ decays into nonleptonic final states. They also eliminate most backgrounds from multi-body decays of $J/\psi \rightarrow \phi$ anything. The different requirements on the cone angle for the $\eta$ and $\eta'$ cases are made because of the following two reasons: firstly, in the $J/\psi \rightarrow \phi \eta (\eta')$ decays, the booster for $\eta$ is stronger than that for $\eta'$ in the central of mass energy of $J/\psi$, which leads to a larger open angle for the $\eta'$ decay products than that for the $\eta$ decay products in the detector. Secondly, the most dangerous backgrounds are from $\eta(\eta') \rightarrow \pi^+ \pi^- \gamma$ decay, in which one of the charged pions is mis-identified as an electron. Meanwhile, the decay rate for $\eta' \rightarrow \pi^+ e^- \bar{\nu}_e$ is more than 6 times larger than the rate for $\eta \rightarrow \pi^+ e^- \bar{\nu}_e$.[29]

Figures 1 (a) and (b) show the invariant mass distribution of $K^+K^-$ candidates, $m_{K^+K^-}$, after the above selections. Clear $\phi$ signals are seen. The invariant mass of $\pi^+ e^- \bar{\nu}_e$ can be obtained as

$$m_{\pi^+ e^- \bar{\nu}_e} = \sqrt{(E_{\pi^+} + E_{e^-} + E_{\nu})^2 - (\vec{P}_{\pi^+} + \vec{P}_{e^-} + \vec{P}_{\nu})^2},$$

where $E_{\phi} \equiv E_{\text{miss}} = |\vec{P}_{\text{miss}}|$ and $\vec{P}_{\phi} = \vec{P}_{\text{miss}}$. Figures 2 (a) and (b) show the $m_{\pi^+ e^- \bar{\nu}_e}$ distributions for events with $1.01 < m_{K^+K^-} < 1.03$ GeV/c$^2$ for the decays $J/\psi \rightarrow \phi \eta (\eta \rightarrow \pi^+ e^- \bar{\nu}_e)$ and $J/\psi \rightarrow \phi \eta' (\eta' \rightarrow \pi^+ e^- \bar{\nu}_e)$, respectively. No events are observed in the $\eta$ and $\eta'$ signal regions. The signal regions for $\eta$ and $\eta'$ are defined in the ranges $[0.51, 0.58]$ and $[0.92, 0.99]$ GeV/c$^2$, respectively, on the mass of $\pi^+ e^- \bar{\nu}_e$.

We use MC simulated events to determine selection efficiencies for the signal channels and study possible backgrounds. With phase space MC simulations, we obtain efficiencies of 17.9% and 17.4% for $\eta$ and $\eta'$ semileptonic decays, respectively. According to the study of the $J/\psi$ inclusive MC sample, more than 20 exclusive decay modes are identified as potential background modes, and are studied with full MC simulations in order to understand the backgrounds. The sources of backgrounds are divided into two classes. Class I: The background is from $J/\psi \rightarrow \phi \eta (\eta')$, $\phi \rightarrow K^+ K^-$ and $\eta (\eta')$ decays into other modes than the signal final states. We find that the expected number of background events from this class is $0.18 \pm 0.05 (0.58 \pm 0.09)$ in the signal region for $\eta (\eta')$. Class II: It is mainly from $J/\psi$ decays to the final states without $\eta$ or $\eta'$, such as $\phi \pi^+ \pi^-$, $\phi f_0(980)$ ($f_0(980) \rightarrow \pi^+ \pi^-)$, and $K^{*0} K^{*0}$ ($K^{*0} \rightarrow K^+ \pi^+\pi^-$). The expected number of background events from class II is $0.05 \pm 0.04 (0.45 \pm 0.13)$ in the signal region for the $\eta (\eta')$ case. The total number of background events is $0.23 \pm 0.06 (1.03 \pm 0.16)$ in the signal region for $\eta (\eta')$.

After all selection criteria are applied, no event survives in the $\eta$ and $\eta'$ signal regions. The signal components and the expected background shapes are projected and compared to data for both $\eta$ and $\eta'$ cases, as shown in Figs. 2 (a) and (b). We set an upper limit at the 90% confidence level (C.L.) to be $N_{UL} = 2.36 (N_{UL}^{\eta} = 1.59)$ for $\eta(\eta')$, using the POL++ program [32] with the Feldman-Cousins frequentist approach [33]. The information used to obtain the upper limit includes the number of observed events in the signal region, and the expected number of background events and their uncertainty.
B. Analyses for $\eta (\eta') \rightarrow \pi^+\pi^-\pi^0 (\eta)$

The $\eta \rightarrow \pi^+\pi^-\pi^0$ and $\eta' \rightarrow \pi^+\pi^-\eta$ decays are also studied in $J/\psi \rightarrow \phi \eta$ and $\phi \eta'$ processes, in order to obtain the ratio of $B(\eta (\eta') \rightarrow \pi^+\pi^-\eta)$ to $B(\eta \rightarrow \pi^+\pi^-\pi^0)$. The advantage of measuring the ratios of semileptonic weak decays over hadronic decays is that the uncertainties due to the total number of $J/\psi$ events, tracking efficiency, PID for kaon and one pion, the number of the charged tracks, and residual noise in the EMC cancel.

The selection criteria for the charged tracks are the same as those for the $J/\psi \rightarrow \phi \eta (\eta')$, $\eta (\eta') \rightarrow \pi^+\pi^-\nu_\ell$ decays except for the electron identification requirement. The candidate events are required to have two charged kaons and two charged pions with opposite charge. In addition, two photon candidates are required to be isolated from all charged tracks by more than 10$^\circ$ which is different from the selection criteria for the $J/\psi \rightarrow \phi \eta (\eta')$, $\eta (\eta') \rightarrow \pi^+\pi^-\nu_\ell$ decays in order to improve the efficiency of $\pi^0$ reconstruction. A four-constraint (4C) energy-momentum conservation kinematic fit is performed to the $J/\psi \rightarrow K^+K^-\pi^+\pi^-\gamma\gamma$ hypothesis, and only events with $\chi^2_{4C}$ < 200 are accepted. For events with more than two photon candidates, the combination with the minimum $\chi^2_{4C}$ is selected. After the 4C fits, the $\pi^0$ and $\eta$ signal windows on the $\gamma\gamma$ invariant mass distributions are defined in the ranges 0.115 < $m_{\gamma\gamma}$ < 0.150 GeV/c$^2$ and 0.518 < $m_{\gamma\gamma}$ < 0.578 GeV/c$^2$, respectively.

The numbers of $J/\psi \rightarrow \phi \eta (\eta')$, $\eta (\eta') \rightarrow \pi^+\pi^-\pi^0 (\eta)$ events are obtained from an unbinned extended maximum likelihood (ML) fit to the $K^+K^-$ versus $\pi^+\pi^-\pi^0 (\eta)$ invariant mass distributions. The projection of the fit on the $m_{KK}$ ($m_{\pi^+\pi^-\pi^0}$ and $m_{\pi^+\pi^-\eta}$) axis is shown in Figs. 3 (a) and 4 (a). The PDFs for the $\eta$ and $\eta'$ cases, respectively. In the ML fits, we require that 0.99 GeV/c$^2$ < $m_{KK}$ < 1.09 GeV/c$^2$ and 0.50 GeV/c$^2$ < $m_{\pi^+\pi^-\pi^0}$ < 0.60 GeV/c$^2$ (0.87 GeV/c$^2$ < $m_{\pi^+\pi^-\eta}$ < 1.07 GeV/c$^2$) for the $\eta (\eta')$ case. The signal shape for $\phi$ is modeled with a relativistic Breit-Wigner (RBW) function convoluted with a Gaussian function that represents the detector resolution. In the fit, the width of $\phi$ is fixed at the PDG value, and its central mass value is floated, and the width of the Gaussian is free: the signal shape for $\eta (\eta')$ is described by a Crystal Ball (CB) function, and its parameters are floated. The backgrounds are divided into three categories: non-$\phi\eta (\eta')$-peaking background (i.e., $J/\psi \rightarrow \pi^+\pi^-\pi^0 K^+K^-$); non-$\phi$-peaking background [i.e., $J/\psi \rightarrow K^+K^-\eta (\eta')$]; and non-$\eta (\eta')$-peaking background (i.e., $J/\psi \rightarrow \phi\eta (\eta')\pi^0$). The probability density functions (PDF) for non-$\phi$-peaking background in the $m_{KK}$ distribution is parameterized by

$$B(m_{KK}) = (m_{KK} - 2m_K)^a \cdot e^{-bm_{KK} - cm_{KK}^2},$$

(1)

where $a$, $b$, and $c$ are free parameters, and $m_K$ is the nominal mass value of the charged kaon from the PDG. The shape for the non-$\eta (\eta')$-peaking background in the $m_{\pi^+\pi^-\pi^0 (\eta)}$ distribution is modeled by a first-order Chebychev polynomial function $B(m_{\pi^+\pi^-\pi^0 (\eta)})$. All parameters related to the background shape are floated in the fit to data. Totally, 14 parameters including signal and background yields are floated in the fit. The PDFs for signal and backgrounds are combined in the likelihood function $L$, defined as a function of the free parameters $N^{\eta}$, $N^{\eta\phi}_{bkg}$, $N^{\eta\non\phi}_{bkg}$, and $N^{\eta\non\eta}_{bkg}$.

$$L = \frac{e^{-(N^0 + N^{\eta\phi}_{bkg} + N^{\eta\non\phi}_{bkg} + N^{\eta\non\eta}_{bkg})}}{N!} \times \prod_{i=1}^{N} \left[ N^0 RBW(m_{KK}) \times CB(m_{\pi^+\pi^-\pi^0}) + N^{\eta\phi}_{bkg} B(m_{KK}) \times CB(m_{\pi^+\pi^-\pi^0}) + N^{\eta\non\phi}_{bkg} B(m_{KK}) \times CB(m_{\pi^+\pi^-\pi^0}) + N^{\eta\non\eta}_{bkg} RBW(m_{KK}) \times CB(m_{\pi^+\pi^-\pi^0}) \right],$$

(2)

where $N^0$ is the number of $J/\psi \rightarrow \phi\eta$, $\phi \rightarrow K^+K^-$, $\eta \rightarrow \pi^+\pi^-\pi^0$ events, and $N^{\eta\phi}_{bkg}$, $N^{\eta\non\phi}_{bkg}$, and $N^{\eta\non\eta}_{bkg}$ are the numbers of the corresponding three kinds of backgrounds. The fixed parameter $N$ is the total number of selected events in the fit region, and $m_{KK}$ ($m_{\pi^+\pi^-\pi^0}$) is the value of $m_{KK}$ ($m_{\pi^+\pi^-\pi^0}$) for the $i$th event. We use the product of the PDFs, since we have verified that $m_{KK}$ and $m_{\pi^+\pi^-\pi^0}$ are uncorrelated for each component. The negative log-likelihood ($-\ln L$) is then minimized with respect to the extracted yields. The resulting fitted signal and background yields are summarized in Table I. We also obtain the results for the $\eta'$ case by replacing $\eta$ and $\pi^0$ with $\eta'$ and $\eta$ in Eq. (2). The fitted results for $\eta (\eta') \rightarrow \pi^+\pi^-\pi^0 (\eta)$ are shown in Fig. 3 (b) and 4 (b). The detection efficiencies are determined with MC simulations to be 20.37% and 20.89% for $\eta$ and $\eta'$, respectively.

IV. SYSTEMATIC UNCERTAINTIES

Contributions to the systematic error on the ratios are summarized in Table I. The uncertainty, due to the requirement of no neutral showers in the EMC inside a cone spanning 0.3 (1.5) rad around the direction of the missing momentum for $\eta (\eta')$ decay is obtained using the control sample of decays $J/\psi \rightarrow \phi\eta'$, $\phi \rightarrow K^+K^-$, $\eta' \rightarrow \gamma\pi^+\pi^-$. We calculated the missing momentum of the $K^+K^-\pi^+\pi^-$ system, and define the same cones around the direction of the missing momentum as in the $\eta$
We also used the control sample of $J/\psi \rightarrow \phi \eta'$, $\phi \rightarrow K^+K^-$, $\eta' \rightarrow \pi^+\pi^-\eta$ to obtain the uncertainty due to the requirement on the missing momentum $P_{\text{miss}} > 0.03$ GeV/$c$ for both $\eta$ and $\eta'$ cases. Thus we calculated the missing momentum of the $K^+K^-\pi^+\pi^-$ system. The ratios of events with the requirement on the missing momentum $P_{\text{miss}} > 0.03$ GeV/$c$ to events without the requirement are obtained for both data and MC simulation. The difference 2.5% is considered as a systematic error for both $\eta$ and $\eta'$ cases.

The phase space MC is used to generate $\eta$ ($\eta'$) semileptonic analysis. The ratios of events with the requirement on the number of extra photons to events without the requirement are obtained for both data and MC simulation. The difference 0.1% (1.1%) is considered as a systematic error for the $\eta$ ($\eta'$) case.

We also use the control sample of $J/\psi \rightarrow \phi \eta'$, $\phi \rightarrow K^+K^-$, $\eta' \rightarrow \gamma\pi^+\pi^-$ to obtain the uncertainty due to the requirement on the missing momentum $P_{\text{miss}} > 0.03$ GeV/$c$ for both $\eta$ and $\eta'$ cases. Thus we calculated the missing momentum of the $K^+K^-\pi^+\pi^-$ system. The ratios of events with the requirement on the missing momentum $P_{\text{miss}} > 0.03$ GeV/$c$ to events without the requirement are obtained for both data and MC simulation. The difference 2.5% is considered as a systematic error for both $\eta$ and $\eta'$ cases.

The phase space MC is used to generate $\eta$ ($\eta'$) semileptonic analysis. The ratios of events with the requirement on the number of extra photons to events without the requirement are obtained for both data and MC simulation. The difference 0.1% (1.1%) is considered as a systematic error for the $\eta$ ($\eta'$) case.

Since the uncertainties on the PID of the electron and one of the pions do not cancel in the ratio, the efficiencies for pion and electron PID are obtained with the control samples of $J/\psi \rightarrow \pi^+\pi^-\eta$ and radiative Bhabha scattering $e^+e^- \rightarrow e^+e^-$ (including $J/\psi \rightarrow \gamma e^+e^-$), respectively. Samples with backgrounds less than 1.0% are obtained [37]. The differences between data and MC for the efficiencies of pion and electron PID are about 1.0% and 1.2%, respectively, which are taken as systematic errors. Using the same control samples,

(\eta') semileptonic analysis. The ratios of events with the requirement on the number of extra photons to events without the requirement are obtained for both data and MC simulation. The difference 0.1% (1.1%) is considered as a systematic error for the $\eta$ ($\eta'$) case.

FIG. 4: The (a) $m_{KK}$ and (b) $m_{\pi^+\pi^-\eta}$ distributions with fit results superimposed for $J/\psi \rightarrow \phi \eta'$, $\phi \rightarrow K^+K^-$, $\eta' \rightarrow \pi^+\pi^-\eta$. Points with error bars are data. The (black) solid curves show the results of the total fits, and the (black) short-dashed curves are for signals. The (blue) dotted-dash curve shows non-$\eta$-peaking backgrounds, the (red) dashed curve shows the non-\phi-peaking background, and the (green) dotted-short-dash curve shows non-\phi\eta'-peaking backgrounds.
we estimate the uncertainty due to the requirement of $E/p$ for the electron selection to be 3.5% (3.4%) for the $\eta (\eta')$ case, and the uncertainty due to the requirement of $E/p$ for pion selection in the $\eta$ semileptonic decay is estimated to be 0.8%. The systematic uncertainty due to the requirements of $\phi$ and $\eta (\eta')$ mass windows are estimated to be 1.4% and 0.04% (0.2%) by using the control sample of $J/\psi \rightarrow \phi \eta (\eta')$, $\eta (\eta') \rightarrow \pi^+ \pi^- \pi^0 (\eta)$.

The uncertainty in the background determination of the numbers of observed events for $J/\psi \rightarrow \phi \eta [\eta \rightarrow \pi^+ \pi^- \pi^0 (\pi^0 \rightarrow \gamma \gamma)]$ and $J/\psi \rightarrow \phi \eta' [\eta' \rightarrow \pi^+ \pi^- \eta (\eta \rightarrow \gamma \gamma)]$ decays are estimated on the basis of earlier published results. The photon detection efficiency and its uncertainty are studied by three different methods in Ref. [37]. The systematic error of photon detection is estimated to be 1.0% per photon. In the fit to the $\phi$ mass distribution, the mass resolution is fixed to the MC simulation; the level of possible discrepancy is determined with a smearing Gaussian, for which a non-zero $\sigma$ would represent a MC-data difference in the mass resolution. The uncertainty associated with the difference determined in this way is 0.03% (0.06%) for the $\eta (\eta')$ case. The systematic uncertainty due to the choice of parameterization for the shape of the non-$\phi \eta (\eta')$-peaking background is estimated by varying the order of the polynomial in the fit; we find the relative changes on the $\eta (\eta')$ signal yield of 1.3% (0.8%), which is taken as the uncertainty due to the background shapes. The systematic errors from $\pi^0 (\eta)$ reconstruction from $\gamma \gamma$ decays is determined to be 1.0% per $\pi^0 (\eta)$ by using a high purity control sample of $J/\psi \rightarrow \pi^0 p\bar{p}$ ($J/\psi \rightarrow \eta p\bar{p}$) decay [38]. The branching fractions for the $\pi^0$ and $\eta \rightarrow \gamma \gamma$ decays are taken from the PDG [29]. The uncertainties on the branching fractions are taken as a systematic uncertainty in our measurements. The total systematic error $\sigma_{\eta}^{sys}$ ($\sigma_{\eta'}^{sys}$) on the ratio is 5.6% (7.4%) for $\eta (\eta')$, as summarized in Table II.

![Table I](image1)

### Table I

| Quantity | Value       |
|----------|-------------|
| $N^0(\eta')$ | $3850 \pm 73$ 1623 ± 44 |
| $N_{bkg}^{non-\phi} (N_{bkg}^{non-\phi'})$ | $24 \pm 8$ 49 ± 10 |
| $N_{bkg}^{non-\phi} (N_{bkg}^{non-\phi'})$ | $367 \pm 43$ 22 ± 17 |
| $N_{bkg}^{non-\phi} (N_{bkg}^{non-\phi'})$ | $88 \pm 14$ 61 ± 12 |
| $\epsilon^{\eta'}$ | 20.37\% 20.89\% |

### Table II

| Sources | Sys. error(%) | $\eta$ | $\eta'$ |
|---------|--------------|-------|-------|
| Requirement on $N_{shower}$ | 0.1 | 1.1 |
| Requirement on $P_{miss}$ | 2.5 | 2.5 |
| Signal model | 1.0 | 5.0 |
| Electron PID | 1.2 | 1.2 |
| Requirement on $E/p$ for $e$ | 3.5 | 3.4 |
| Requirement on $E/p$ for $\pi$ | 0.8 | 0.2 |
| $\phi$ mass window | 1.4 | 1.4 |
| $\eta (\eta')$ mass window | 0.0 | 0.5 |
| Photon efficiency | 2.0 | 2.0 |
| $\pi$ PID | 1.0 | 1.0 |
| Signal shapes for $\eta(\eta') \rightarrow \pi^+ \pi^- \pi^0 (\eta)$ | 0.0 | 0.1 |
| Background shape for $\eta(\eta') \rightarrow \pi^+ \pi^- \pi^0 (\eta)$ | 1.3 | 0.8 |
| $\pi^0 (\eta)$ reconstruction | 1.0 | 1.0 |
| Cited $B(\pi^0 (\eta) \rightarrow \gamma \gamma)$ | 0.0 | 0.5 |
| Total | 5.6 | 7.4 |

### V. RESULTS

The upper limit on the ratio of branching fractions of the semileptonic decay $B(\eta \rightarrow \pi^+ e^- \bar{\nu}_e + c.c.)$ over the hadronic decay $B(\eta \rightarrow \pi^+ \pi^- \pi^0)$ is calculated with

$$\frac{B(\eta \rightarrow \pi^+ e^- \bar{\nu}_e + c.c.)}{B(\eta \rightarrow \pi^+ \pi^- \pi^0)} < \frac{N^0_{UL}/\epsilon_{\eta}^{SL}}{N^0/\epsilon_{\eta} (1 - \sigma_{\eta})},$$

where $N^0_{UL}$ is the 90% upper limit of the observed events for $J/\psi \rightarrow \phi \eta, \phi \rightarrow K^+ K^- \eta, \eta \rightarrow \pi^+ e^- \nu_e$ decay, $\epsilon_{\eta}^{SL}$ is the MC determined efficiency for the signal channel, $N^0$ is the number of events for the $J/\psi \rightarrow \phi \eta, \phi \rightarrow K^+ K^- \eta, \eta \rightarrow \pi^+ \pi^- \pi^0 (\pi^0 \rightarrow \gamma \gamma)$ decay, $\epsilon_{\eta}$ is the MC determined efficiency for the decay mode, and $\sigma_{\eta} = \sqrt{(\sigma_{\eta}^{sys})^2 + (\sigma_{\eta}^{stat})^2} = 5.9\%$, where $\sigma_{\eta}^{sys}$ and $\sigma_{\eta}^{stat}$ are the total relative systematic error for the $\eta$ case from Table II and the relative statistical error of $N^0$, respectively. For $\eta'$, $\sigma_{\eta'} = \sqrt{(\sigma_{\eta'}^{sys})^2 + (\sigma_{\eta'}^{stat})^2} = 7.9\%$.

The relative statistical error of $N^0 (N^0')$ is 1.9% (2.7%). We also obtain the upper limit on the ratio of $B(\eta' \rightarrow \pi^+ e^- \bar{\nu}_e + c.c.)$ to $B(\eta' \rightarrow \pi^+ \pi^- \eta)$ by replacing $\eta$ and $B(\pi^0 \rightarrow \gamma \gamma)$ with $\eta'$ and $B(\eta \rightarrow \gamma \gamma)$, respectively, in Eq. (3). Since only the statistical error is considered when we obtain the 90% upper limit of the number of events, to be conservative, $N^0_{UL}$ and $N^0_{UL}$ are shifted up by one sigma of the additional uncertainties ($\sigma_{\eta}$ or $\sigma_{\eta'}$).

Using the numbers in Table III the upper limits on the ratios $\frac{B(\eta \rightarrow \pi^+ e^- \bar{\nu}_e + c.c.)}{B(\eta \rightarrow \pi^+ \pi^- \pi^0)}$ and $\frac{B(\eta' \rightarrow \pi^+ e^- \bar{\nu}_e + c.c.)}{B(\eta' \rightarrow \pi^+ \pi^- \eta)}$ are obtained at the 90% C.L. of $7.3 \times 10^{-4}$ and $5.0 \times 10^{-4}$, respectively.
TABLE III: The numbers used in the calculations of the ratios in Eq. (3), where \(N_{UL}^{\eta} (N_{UL}^{\eta'})\) is the upper limit of the signal events at the 90% C.L., \(\epsilon_{UL}^{\eta} (\epsilon_{UL}^{\eta'})\) is the selection efficiency, \(N_{UL}^{\eta} (N_{UL}^{\eta'})\) is the number of the events of \(J/\psi \to \phi \eta (\eta')\), \(\phi \to K^+K^-\), \(\eta \to \pi^+\pi^-\pi^0\) and \(\pi^0 \to \gamma \gamma (\eta' \to \pi^+\pi^-\eta\) and \(\eta \to \gamma \gamma\), \(\epsilon_{UL}^{\eta} (\epsilon_{UL}^{\eta'})\) is its selection efficiency, \(\sigma_{\eta}^{\text{stat}} (\sigma_{\eta'}^{\text{stat}})\) is the relative statistical error of \(N_{UL}^{\eta} (N_{UL}^{\eta'})\), and \(\sigma_{\eta} (\sigma_{\eta'})\) is the total relative error.

| Quantity | \(\eta\) Value | \(\eta'\) |
|----------|----------------|----------|
| \(N_{UL}^{\eta} (N_{UL}^{\eta'})\) | 2.36 | 1.59 |
| \(\epsilon_{UL}^{\eta} (\epsilon_{UL}^{\eta'})\) | 17.9% | 17.4% |
| \(N_{UL}^{\eta} (N_{UL}^{\eta'})\) | 3850 ± 73 | 1623 ± 44 |
| \(\epsilon_{UL}^{\eta} (\epsilon_{UL}^{\eta'})\) | 20.37% | 20.89% |
| \(\sigma_{\eta}^{\text{stat}} (\sigma_{\eta'}^{\text{stat}})\) | 1.9% | 2.7% |
| \(\sigma_{\eta} (\sigma_{\eta'})\) | 5.9% | 7.9% |

VI. SUMMARY

A search for the semileptonic weak decay modes \(\eta (\eta') \to \pi^+\pi^-\bar{\nu}_e\) has been performed for the first time in the process of \(J/\psi \to \phi \eta (\eta')\) using the \((225.3 \pm 2.8) \times 10^6\) \(J/\psi\) events measured at BESIII. We find no signal yields for the semileptonic weak decays of \(\eta\) and \(\eta'\). The upper limits at the 90% C.L. are \(7.3 \times 10^{-4}\) and \(5.0 \times 10^{-4}\) for the ratios of semileptonic over hadronic decay modes \(\frac{B(\eta \to \pi^+\pi^-\bar{\nu}_e + c.c.)}{B(\eta \to \pi^+\pi^-\eta)}\) and \(\frac{B(\eta' \to \pi^+\pi^-\bar{\nu}_e + c.c.)}{B(\eta' \to \pi^+\pi^-\eta)}\), respectively. The advantage of measuring the ratios instead of the branching fractions of the semileptonic weak decays is that many uncertainties cancel. Using the hadronic branching fraction values of \(\eta \to \pi^+\pi^-\pi^0\) and \(\eta' \to \pi^+\pi^-\eta\) as listed by PDG, we determine the semileptonic decay rates to be \(B(\eta \to \pi^+\pi^-\bar{\nu}_e + c.c.) < 1.7 \times 10^{-4}\) and \(B(\eta' \to \pi^+\pi^-\bar{\nu}_e + c.c.) < 2.2 \times 10^{-4}\) at the 90% C.L..

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