A new strategy for probing the Majorana neutrino CP violating phases and masses

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We propose a new strategy for detecting the CP-violating phases and the effective mass of muon Majorana neutrinos by measuring observables associated with neutrino-antineutrino oscillations in \( \pi^\pm \) decays. Within the generic framework of quantum field theory, we compute the non-factorizable probability for producing a pair of same-charged muons in \( \pi^\pm \) decays as a distinctive signature of \( \nu_\mu - \bar{\nu}_\mu \) oscillations. We show that an intense neutrino beam through a long baseline experiment is favored for probing the Majorana phases. Using the neutrino-antineutrino oscillation probability reported by MINOS collaboration, a new stringent bound on the effective muon-neutrino mass is derived.

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I. INTRODUCTION.

Detecting CP violating phases in the lepton sector is one of the most challenging problems in the study of neutrino mixing. In the basis of diagonal charged lepton mass matrix, the neutrino mass matrix is parameterized by Dirac phase, three angles: solar angle \( \theta_{12} \), atmospheric angle \( \theta_{23} \), and Chooz angle \( \theta_{13} \).

To measure the effective electron-neutrino mass in the neutrinoless double beta decay \((0\nu\beta\beta)\) experiments, we can only restrict the two Majorana CP violating phases present in the PMNS mixing matrix. The effective electron-neutrino mass \( \langle m_{ee}\rangle \) is given by

\[
|\langle m_{ee}\rangle| = \left| \sum_i U_{e1}^2 m_{\nu_i} \right|
= \left| m_{\nu_1} U_{e1}^2 + m_{\nu_2} U_{e2}^2 + m_{\nu_3} U_{e3}^2 \right|.
\]

This effective mass parameter depends on the angles \( \theta_{12} \) and \( \theta_{13} \), the neutrino masses \( m_{\nu_i} \), Dirac CP phase, and Majorana phases \( \alpha_i \). There are several studies on using the results of \((0\nu\beta\beta)\) together with the new data from terrestrial and astrophysical observation to restrict the Majorana neutrino CP violating phases.\([1,2,3]\). However, this analysis is model dependent and quite sensitive to the ansatz of the neutrino mass spectrum: quasi-degenerate, normal or inverted hierarchies. In this respect, it is not possible to measure the Majorana neutrino CP phases from \((0\nu\beta\beta)\) experiment. This may be expected since in the \((0\nu\beta\beta)\) one measures the lifetime of the decay of two neutrons in a nucleus into two protons and two electrons, which is a CP conserving quantity.

On the other hand, direct bounds on other effective neutrino mass parameters \( \langle m_{\mu\mu}\rangle \equiv \sum_i U_{\mu i}^2 m_{\nu_i} \) from present experimental data are very poor. Currently, the strongest bound for the muon-neutrino case from the \( K^+ \rightarrow \pi^+ \mu^+\mu^+ \) branching fraction \([4]\) is only \( \langle m_{\mu\mu}\rangle \leq 0.04 \text{ TeV} \) \([5]\), which leads to a negligible constraint on the neutrino masses and CP violating phases. Therefore, it is commonly believed that direct bounds from other \( \Delta L = 2 \) decays are only of academic interest and can not fix the neutrino mixing parameters \([6]\). Some attempts to detect CP violation based on the difference between oscillation probabilities of neutrinos and antineutrinos can be found in Ref.\([7]\).

Here we propose a mechanism, based on neutrino-antineutrino oscillation \([8,9,10]\), which would allow to derive a strong bound on the effective mass of the muon-neutrino. In addition, it provides a method for detecting the Majorana neutrino CP violating phases through measuring the CP asymmetry of the \( \pi^\pm \) decay where neutrino-antineutrino oscillation take place. Using the bound on the neutrino-antineutrino oscillation probability reported by the MINOS Collaboration \([11]\), we derive a bound on \( \langle m_{\mu\mu}\rangle \) which improves existing bounds by several orders of magnitude.

It is worth noting that the probability of a process associated to neutrino oscillation is usually assumed to be factorized into three independent parts: the production process, the oscillation probability and the detection cross section. In Ref.\([12]\), this approximation was avoided and a generic framework based on quantum field theory was proposed to get a simple expression for the CP asymmetry. Here, we adopt the S-matrix amplitude
method described in \[12\].

**II. NEUTRINO-ANTINEUTRINO OSCILLATION**

Let us start by considering a positive charged pion which decays into a virtual neutrino at the space-time location \((x, t)\) together with a positive charged muon. After propagating, the neutrino can be converted into an antineutrino which produces a positive charged muon at the point \((x', t')\) when it interacts with a target, as shown in Fig. 1. For definiteness, we illustrate this process with the production of the neutrino in \(\pi^+\) decay and its later detection via its weak interaction with a target nucleon \(N\)

\[
\pi^+(p_1) \rightarrow \mu^+(p_2) + \nu_\mu(p) \\
\rightarrow \pi(p) + N(p_N) \rightarrow N'(p_{N'}) + \mu^+(p_1)
\]

where the superscript \(s(d)\) refers to the virtual neutron (antineutrino) at the source (detection) vertex. This \(\Delta L = 2\) process is a clear signal for neutrino-antineutrino oscillations of the muon type and its amplitude should be proportional to neutrino Majorana masses.

If one ignores other flavors, the time evolution of the \(\nu_\mu - \overline{\nu}_\mu\) system would be analogous to that of the \(K^0 - \overline{K}^0\) or \(B^0 - \overline{B}^0\) systems. Instead, we prefer to use the formalism developed in Ref. \[12\], where the whole reaction includes the production and detection processes of neutrinos. The decay amplitude becomes (for simplicity we assume that leptonic flavor is conserved at the production and detection vertices):

\[
T_{\nu_\mu - \overline{\nu}_\mu}(t) = (2\pi)^4 \delta^4(p_\mu - p_N + p_{N'} + p_2 - p_1) \\
\times (G_F V_{ud})^2 (J_{N'N})_\mu f_\pi \\
\times \sum_{i} \overline{u}_\mu(p_1) \gamma^\mu (1 + \gamma_5) \, \phi_1 v(p_2) \\
\times U_{\mu i} U_{\nu_i}(m_{\nu_i}) \frac{e^{-iE_{\nu_i}t}}{2E_{\nu_i}},
\]

where the relation \(\nu_k = \sum U_{k\alpha} \nu_\alpha\) between flavor \(k\) and mass \(\alpha\) neutrino eigenstates has been used, \(f_\pi = 130.4\) MeV is the \(\pi^+\) decay constant, and \(J_{N'N}\) parametrizes the interaction with the nucleon. Note that, contrary to the case of neutrino oscillations \[12\], only the neutrino mass term survives in this case. For simplicity, one assumes that

\[
(J_{N'N})_\mu = \overline{\nu}_{N'}(p_{N'}) \gamma_\mu (g_\nu + g_A \gamma_5) u_N(p_N)
\]

where we use \(g_\nu = g_\nu(q^2 = 0) = 1\) and \(g_A = g_A(q^2 = 0) \approx -1.27\) \[13\]. If we neglect terms of \(O(m_\mu / m_{N,N'})\), one obtains

\[
|T_{\nu_\mu - \overline{\nu}_\mu}(t)|^2 = \left( \frac{2\pi}{\sqrt{2}} \right)^4 \delta^4(p_\mu - p_N + p_{N'} + p_2 - p_1) (G_F V_{ud})^4 \\
\times \left| \sum_{i} \overline{u}_\mu(p_1) \gamma^\mu (1 + \gamma_5) \, \phi_1 v(p_2) \\
\times U_{\mu i} U_{\nu_i}(m_{\nu_i}) \frac{e^{-iE_{\nu_i}t}}{2E_{\nu_i}} \right|^2.
\]

where \(E_{\nu_i}\) and \(\nu_i\) are, respectively, the initial (final) muon and the pion energies and \(\Delta E_{\nu_i} = E_{\nu_i} - E_{\nu_i}\). The functions \(F(g_\nu)\) and \(G(g_A)\) are given by:

\[
F(g_\nu) = \frac{g_\nu^2 + 1}{g_A (g_A - 1)}, \quad G(g_A) = \frac{g_A^2 + 1}{g_A - 1}.
\]

One can easily check that Eq. \(1\) is not factorizable into (production)×(propagation)×(detection) subprocesses due to the terms proportional to \(p_\mu \cdot p_2 = E_2 E_\nu - |p_1|^2 |p_2|^2 \cos \alpha\), where \(\alpha\) is the angle between the directions of \(\mu^+\) particles. This is an important difference with respect to the case of neutrino-antineutrino (\(\Delta L = 0\)) oscillations where it was shown in Ref. \[12\] that the S-matrix formalism reproduces the hypothesis of factorization of the probabilities.

After integration over kinematical variables, it is possible to write the rate of the complete process as

\[
\Gamma_{\nu_\mu - \overline{\nu}_\mu} = \left| \sum_i U_{\mu i}^2 m_{\nu_i} \frac{e^{-iE_{\nu_i}t}}{2E_{\nu_i}} \right|^2 \times F(M, \phi),
\]

where \(F(M, \phi)\) denotes the kinematical function

\[
F(M, \phi) = \frac{\pi}{2E_p} (G_F V_{ud})^4 \left| \int \frac{d^3p_2}{2E_2} \frac{d^3p_1}{2E_1} \delta(E_2 - E_p) f_\nu(E_p + E_N - E_{N'} - E_1 - E_2) \right|^2.
\]

The functions \(I_a\) for \(a = 1, \ldots, 4\) can be obtained from the following integral:

\[
I_a = \int \frac{d^3p_2}{2E_2} \frac{d^3p_1}{2E_1} \delta(E_2 - E_p) f_\nu(E_p + E_N - E_{N'} - E_1 - E_2),
\]
with \( f_1 = 1, \ f_2 = E_2, \ f_3 = E_1, \) and \( f_4 = (p_1 \cdot p_2) \) and
\[
f_5 = \frac{(p_1 \cdot p_2)}{E_2 - E_1}.
\]

There are two interesting limits to this process. At very short times, which means that the detection is very close to the production vertex (short-baseline neutrino experiment), one has, assuming that the \( E_\nu \simeq E_\nu, \) that
\[
\Gamma_{\nu_\mu - \bar{\nu}_\mu} \simeq \left| \frac{\langle m_{\mu\mu} \rangle^2}{E_\nu} \right| \times F(M, \phi) \tag{8}
\]
where \( \langle m_{\mu\mu} \rangle \) is the effective Majorana mass for the muon neutrino. In the long time limit which corresponds to a long-baseline neutrino experiment, the oscillation terms cannot be neglected and this process depends on a new combination of phases, mixing angles and masses which could give us complementary information on the neutrinoless double beta decays or on any process that depends exclusively on the effective Majorana mass of the neutrinos. Using this expression for the rate it is possible to get the CP asymmetry which will depend explicitly on Majorana phases.

\[
a_{CP} = \frac{\Gamma_{\nu_\mu - \bar{\nu}_\mu} - \Gamma_{\nu_\mu - \nu_\mu}}{\Gamma_{\nu_\mu - \nu_\mu + \Gamma_{\nu_\mu - \nu_\mu}}} = \frac{\sum_{i>j} \text{Im} \left( U_{\mu i}^* U_{\mu j} U_{\mu j}^* U_{\mu i} \right) m_{\nu_\mu} m_{\nu_\mu} \sin \gamma}{\sum_{i>j} \text{Re} \left( U_{\mu i}^* U_{\mu j} U_{\mu j}^* U_{\mu i} \right) m_{\nu_\mu} m_{\nu_\mu} \cos \gamma} \tag{10}
\]
where \( \gamma = \frac{\Delta m^2_{23}\ell}{2E_\nu (\text{GeV})}. \) Here \( \Delta m^2_{23} \) is the difference in the squares of second and third eigenstate neutrino masses, \( \Delta m^2_{23} = (2.43 \pm 0.13) \times 10^{-3} \text{ eV}^2, \) and \( L \) is the distance between production and detection vertices. Finally, \( E_\nu \) is the energy of the neutrino beam. It is worth mentioning that the time evolution amplitude for the CP-conjugate process corresponds to the observation of \( \mu^- \) at the source and at the detector. Therefore, the associated nucleon weak vertex is given by \( (J_{N'} \gamma_\mu) = \overline{\tau}_N (p_N) \gamma_\mu (g_V + g_A \gamma_5) u_N \gamma_\mu (p_N'). \) Estimating the CP asymmetry in Eq. (10), we have assumed that \( J_{N'/N} \simeq J_{N'/N} \).

In the limit of \( \theta_{13} = 0, \) the Majorana phases \( \alpha_{12} \) are the only sources of CP violation and hence \( \text{Im} (U_{\mu i}^* U_{\mu j} U_{\mu j}^* U_{\mu i}) \propto \sin (\alpha_i - \alpha_j). \) For \( i = 2 \) and \( j = 3 \) one finds
\[
a_{CP} \simeq \frac{1}{2} |2(\alpha_2 - \alpha_3)| \sin \gamma. \tag{11}
\]

Thus, in the case of long-baseline neutrino experiment like MINOS where the distance \( L \) is given by \( L = 735 \text{ km} \) and the energy \( E_\nu \) is typically around \( 2 - 3 \text{ GeV} \) \cite{14,15}, one finds that \( \sin \gamma \sim O(1). \) Thus, measuring CP asymmetry will be unavoidable indication for large CP violating Majorana phases.

### III. APPLICATION TO MINOS RESULTS ON NEUTRINO-ANTINEUTRINO OSCILLATIONS

The last two decades have witnessed several experiments that investigate the neutrino-antineutrino transitions. It started in 1982 when the BEBC bubble chamber in the CERN SPS neutrino beam set a limit on \( \nu_\mu \rightarrow \bar{\nu}_e \) and \( \nu_e \rightarrow \bar{\nu}_e \) through the search for \( \bar{\nu}_e \) appearance. Recently, MINOS \cite{13} has measured the spectrum of \( \nu_\mu \) events which are missing after travelling 735 km. It is these missing events which are the potential source of \( \bar{\nu}_e \) appearance. In their preliminary analysis, they were able to put a limit on the fraction of muon neutrinos transition to muon anti-neutrinos \cite{11}:
\[
P(\nu_\mu - \bar{\nu}_\mu) < 0.026 \text{ (90\% c.l.)}. \tag{12}
\]

Assuming CPT, this limit can be written as
\[
\frac{\Gamma_{\nu_\mu - \bar{\nu}_\mu}}{\Gamma_{\nu_\mu - \nu_\mu}} < 0.026. \tag{13}
\]

Using our expression for \( \Gamma_{\nu_\mu - \bar{\nu}_\mu}, \) and the corresponding rate for neutrino oscillations \cite{12}, one gets
\[
\left| \sum_i U_{\mu i}^2 m_{\nu_i} e^{iE_{\nu_i}t} \right|^2 \lesssim 0.001. \tag{14}
\]

In the limit of ultrarelativistic neutrinos, \( E_\nu \simeq E_\nu (1 + m_{\nu_i}^2 /2E_\nu), \) and keeping the leading terms in the \( m_{\nu_i} / E_\nu \) terms, we get
\[
\left| \sum_i U_{\mu i}^2 m_{\nu_i} e^{i\ell^2 m_{\nu_i}^2 /2E_\nu} \right|^2 \lesssim 0.001 \times E_\nu^2. \tag{15}
\]

To illustrate the usefulness of this relation, let us assume the case of two flavor neutrinos. In this case, one finds
\[
0.001 \times E_\nu^2 \gtrsim |\langle m_{\mu\mu} \rangle|^2 - 4 \text{ Re} \left( U_{\mu 2}^2 U_{\mu 3}^2 \right) m_{\nu_2} m_{\nu_3} \sin^2 \frac{\gamma}{2} - 2 \text{ Im} \left( U_{\mu 2}^2 U_{\mu 3}^2 \right) m_{\nu_2} m_{\nu_3} \sin \gamma. \tag{16}
\]

From this equation, it is possible to get a bound on the effective muon-neutrino Majorana mass, only depending on the values of the Majorana phases as the oscillation terms cannot be neglected. If \( \pi - \gamma /2 < 2(\alpha_2 - \alpha_3) \leq 2\pi - \gamma /2 \) and using \( E_\nu \approx 2 \text{ GeV}, \) one gets the following conservative bound on
\[
|\langle m_{\mu\mu} \rangle| \lesssim 64 \text{ MeV.}
\]

If \( -\gamma/2 \leq 2\alpha_2 - \alpha_3 < 0, \) the conservative bound on |
\(|\langle m_{\mu\mu} \rangle| \) is given by
\[
|\langle m_{\mu\mu} \rangle| \lesssim 109 \text{ MeV.}
\]

If \( 0 \leq 2\alpha_2 - \alpha_3 \leq \pi - \gamma /2, \) it is not possible to get a conservative bound on \(|\langle m_{\mu\mu} \rangle| \) but the equation (16) could be used to bound Majorana parameters (masses and phases) which appear in \(|\langle m_{\mu\mu} \rangle| \).

So, within the hypothesis done on Majorana phases, the limits obtained improve by various order of magnitude the actual limit on \(|\langle m_{\mu\mu} \rangle| \) coming from direct search.
in $K^+ \rightarrow \pi^+\mu^+\mu^+$ decay. Also, one could expect improvement in experimental analysis in the next years and we could expect that with a specially designed neutrino experiment to measure these kind of processes, it should be possible to improve these limits by a few orders of magnitude in future experiments.

In MINOS and in general in all long-baseline neutrino experiments, the oscillation terms are not negligible. So, it means that such analysis could not only give us information on the neutrino effective Majorana mass but it could be used to determine parameters of the mixing matrices and bound on the absolute value of Majorana masses. Also, if in a long-baseline neutrino experiment, neutrino detectors are located at different distances from the source, it should be possible to get enough constraints on the mixing parameters and Majorana masses to fix them.

### IV. CONCLUSIONS

If Majorana neutrinos do exist, $|\Delta L| = 2$ processes like neutrino-antineutrino oscillations can occur. The production of leptons with same charges at the production and detection vertices of neutrinos will be a clear manifestation of these processes. In this paper we have used the S-matrix formalism of quantum field theory to describe these oscillations in the case of muon neutrinos produced in $\pi^+ \rightarrow \mu^+\mu^+$ decays which convert into muon antineutrinos that are detected via inverse beta decay on nucleons.

One interesting result is that the time evolution probability of the whole process is not factorizable into production, oscillation and detection probabilities, as is the case in neutrino oscillations \[12\]. We find that, for very short times of propagation of neutrinos, the observation of $\mu^+\mu^+$ events would lead to a direct bound on the effective mass of muon Majorana neutrinos. In the case of long-baseline neutrino experiments, the CP rate asymmetry for production of $\mu^+\mu^+/\mu^-\mu^-$ events would lead to direct bounds on the difference of CP-violating Majorana phases. Finally, using the current bound on muon neutrino-antineutrino oscillations reported by the MINOS Collaboration we are able to set the bound $\langle m_{\mu\mu} \rangle < \sim 64$ MeV, which is several orders of magnitude below current bounds reported in the literature.

As a consequence of these results, neutrino experiments aiming to measure neutrino-antineutrino oscillations with different short- and long-baseline setups can be useful to get direct and complementary constraints on the masses and phases of Majorana neutrinos.

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