Strength function under the absorbing boundary condition

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Abstract. The strength function of the linear response by the external field is calculated in the formalism of the absorbing boundary condition (ABC). The dipole excitation of a schematic two-body system is treated in the present study. The extended completeness relation, which is assumed on the analogy of the formulation in the complex scaling method (CSM), is applied to the calculation of the strength function. The calculation of the strength function is successful in the present formalism and hence, the extended completeness relation seems to work well in the ABC formalism. The contributions from the resonance and the non-resonant continuum is also analyzed according to the decomposition of the energy levels in the extended completeness relation.

1. Introduction

In unstable nuclei, one of interesting subjects is the dynamics of continuum states above the particle decay threshold. Neutron-drip line nuclei are weak binding systems and hence, they are easily excited to unbound continuum states. Therefore, the breakup reaction is an important tool to investigate the dynamical property of neutron-excess systems.

Theoretically, the so-called strength function is important to handle the breakup of weak binding systems. The complex scaling method (CSM) is one of powerful methods to calculate the strength function [1] because, in CSM, the scattering problem of the unbound states can be transformed into the bound-state-like problem. In CSM, the so-called extended completeness relation is basic and important principle in the calculation of the strength function. In fact, we can see the successful application of CSM in the three body problem [2].

The method of the absorbing boundary condition (ABC) is another powerful tool to treat the resonant and continuum states [4, 5, 6, 7]. In the present study, we assume the extended completeness relation on the wave functions under the absorbing boundary and demonstrate that the ABC method can describe the strength function in a similar manner to CSM.

2. Framework

In the present calculation, we consider a schematic two-body problem, which interacts by a simple Gaussian potential. The detailed settings of the Hamiltonian $H$ is same as those shown in Ref. [1]. First, we calculate the energy eigenvalues and their wave functions by applying the basis expansion method. The shifted Gaussian is used as the basis function [3]. The distance parameter $S$ in the shifted Gaussian is set in the range from $S_{\text{min}} = 0.1$ fm to $S_{\text{max}} = 32.1$ fm.
with a constant mesh of $\Delta S = 0.4$ fm. In the diagonalization of the Hamiltonian, we introduce the absorbing potential $-i\eta W$ with the strength $\eta$. Here the functional form of the absorbing potential is set to be the shifted polynomial in the radial coordinate $r$, $W(r) = \theta(r - r_a)(r - r_a)^\beta$, with $r_a = 7$ fm and $\beta = 4$ [4].

Secondly, we calculate the strength function for the two body breakup (or capture) by the dipole field, according to the following prescription. The strength function $S_\lambda(E)$ is generally defined by the imaginary part of the response function, $\mathcal{R}_\lambda(E)$, which are given by

$$S_\lambda(E) = \frac{-1}{\pi} \Im [\mathcal{R}_\lambda(E)] \quad (1)$$

$$\mathcal{R}_\lambda(E) = \int \int d r d r' \psi_i^*(r) \hat{O}_\lambda \mathcal{G}(E, r, r') \hat{O}_\lambda \psi_i(r') \quad . \quad (2)$$

Here $\psi_i$ and $\hat{O}_\lambda$ are an initial state and the external field with a multipole $\lambda$. $\mathcal{G}(E, r, r')$ denotes the coordinate representation of the Green function, which is given by

$$\mathcal{G}(E, r, r') = \left( r \left| \frac{1}{E - H} \right| r' \right) \quad . \quad (3)$$

In the calculation employing the ABC method, the absorbing potential, $-i\eta W$ is added to the total Hamiltonian $H$ in the Green function, Eq. (3) and hence, $H \rightarrow H^n = H - i\eta W$.

We assume the extended completeness relation for the energy levels calculated by diagonalizing the Hamiltonian with an absorber, $H - i\eta W$.

$$\mathcal{R}_\lambda(E) = \sum_{B}^{n_B} |\psi_B\rangle <\hat{\psi}_B|R|^2 + \sum_{R}^{n_R(\eta)} |\psi^\eta_R\rangle <\hat{\psi}_R|R|^2 + \sum_{C}^{n_C(\eta)} |\psi^\eta_C\rangle <\hat{\psi}_C|^2 \quad . \quad (4)$$

Here $\hat{\psi}_i$ shows the wave function calculated under the absorbing boundary with the strength $\eta$. The subscript of $i$ denotes a kind of energy levels: $i = B$ (bound states), $R$ (resonant states), $C$ (continuum states). In Eq. (4), tilde in $\hat{\psi}_i$ means that the complex conjugate is not performed.

By inserting the extended completeness relation in Eq. (4) into the response function in Eq. (2), the strength function can be easily calculated from the eigenstates with the absorbing boundary. This formulation is similar technique proposed in CSM [1].

### 3. Results

In Fig. 1, the strength functions calculated from the extended completeness relation in Eq. (4) are shown by the solid curves. The strength of the absorber is set to be $\eta = 10^{-5}$. To check the ABC calculation, we also calculate the strength function from the scattering wave function, $\psi_E^{(+)}$, at a scattering energy $E$. The explicit form of the strength function with the scattering wave function of $\psi_E^{(+)}$ becomes

$$S^{(+)}_\lambda(E) = |<\psi_E^{(+)}|\hat{O}_\lambda|\psi_i>|^2 \quad . \quad (5)$$

Here the scattering wave function $\psi_E^{(+)}$ is solved under the normal scattering boundary condition, the incoming wave plus outgoing wave with the amplitude of the scattering matrix. The results of Eq. (5) are shown by the dotted curves. In the scattering calculation, we have applied the Fox-Goodwin method with a mesh of $\delta = 0.01$ fm and the matching distance of $r_m = 12$ fm.

Because the normalization condition in the ABC calculation is different from the scattering calculation, the peak magnitude of the latter calculation is re-normalized to that of the former calculation. In both panels, we can confirm the strength functions calculated by ABC completely reproduce the results of the scattering solution. Thus, the assumption of extended completeness relation to the ABC method is considered to nicely work in the pragmatic calculation of the strength function. This result is consistent to the previous study of CSM [1].
According to Eq. (4), the strength function can be decomposed into three parts, the bound ($\psi_B^\eta$), resonance ($\psi_R^\eta$) and continuum ($\psi_C^\eta$) states. In the present ABC calculation, two resonances are separated from the non-resonant continuum in the complex energy plane. The energy and width of these two resonances are the same as those calculated by using CSM [1].

The individual contributions from the resonances and non-resonances are shown in Fig. 2. In this figure, the contributions from the first resonance (dashed curve) and the second resonance (dotted curve) are dominant, but the continuum states (dotted-dashed curve) give considerable contribution in the total strength function (solid curve), especially at the higher energy region of $E \geq 2$ MeV.

It should be noticed that the decomposed strengths are not necessarily positive values although the total strength must be defined by the positive value because it is an observable quantity. In the low energy region of $E \leq 1$ MeV, the positive contribution from the second resonance (dotted curve) cancels out the negative contribution from the first resonance (dashed curve). As a result of this cancellation, the total strength becomes almost zero value in the lower energy region. Moreover, the contribution from the continuum states are negative in a whole energy region. Thus, the continuum contributions considerably reduce the resonant contribution at the higher energy region.

These features of the cancellation are consistent to the previous study by using CSM [1]. In the CSM calculation, the two resonances are separated from the continuum line when the rotation angle passes $\theta = 10^\circ$. At this rotation angle, the total strength function is decomposed into the contributions from two resonances and the continuum states. The individual contributions in ABC, presented in Fig. 2, are almost same as those obtained in the CSM calculation [1].

In the ABC calculation, the strength of the absorber, $\eta$, is an external parameter in calculating the resonant and continuum states. The imaginary part of the resonant states is almost invariant with respect to the variation of $\eta$ if once the resonant levels are separated from the continuum. However, the maximum value of the imaginary part of the continuum states depends on $\eta$ and hence, they are controlled by this parameter [4, 5, 6].

The sensitivity of the strength function to $\eta$ is investigated in Fig. 3. In this figure, the various strength functions obtained by changing $\eta$ in the absorbing potential. The solid curve...
shows the strength function obtained from the ABC method, while the dashed curve shows the strength function calculated from the scattering wave function in the final state, according to Eq. (5).

In the individual panels of Fig. 3, \( \eta \) denotes the strength of the absorber used in the ABC calculation. Many spikes appear in the case of the smaller strength, \( \eta = 10^{-8} \) (top panel), but the appearance of the spikes is suppressed more and more as \( \eta \) becomes stronger: \( \eta = 10^{-7} \) (second panel from the top) and \( \eta = 10^{-6} \) (third panel). Finally, the smooth strength function is obtained at \( \eta = 10^{-5} \) (bottom panel). The smoothed strength function reproduces the strength function calculated from the scattering wave function in Eq. (5).

We have checked the \( \eta \) dependence of the smoothed strength function. In the range of \( \eta = 4.0 \times 10^{-6} \) to \( \eta = 4.0 \times 10^{-4} \), the distribution of the strength function keeps its original form shown in the bottom panel of Fig. 3. Thus, the strength function is insensitive and stable to the variation of \( \eta \) over about two order of the magnitudes after the spikes are completely smeared out. If \( \eta \) becomes further stronger, the distribution of the ABC strength function begins to deviate from the result of the scattering calculation. This is because the reflection effect from the absorber becomes prominent in the case that the strength of the absorber is much stronger than the optimal value.

4. Summary

In the present study, we have assumed the extended completeness relation, proposed in the study of CSM, to the energy eigenstates under the absorbing boundary condition. The schematic two-body problem has been handled, and the extended completeness relation is applied to the calculation of the strength function. We have found that the pragmatic calculation of the strength function is successful, and the obtained results are completely consistent to the previous study of CSM [1].

In the present report, we have shown the decomposition of the strength function into the resonances and the non-resonant continuum according to the extended completeness relation. In the ABC method, however, the non-resonant states are classified into three type of continuum [4, 5]. The further analysis on the continuum states is very interesting because the feature of the “triple-continuum states” in ABC is in marked contrast to the “single continuum state” in CSM. The analysis of the continuum state is now under progress.

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