Experimental observation of Hardy-like quantum contextuality

Breno Marques, Johan Ahrens, Mohamed Nawareg, Adán Cabello & Mohamed Bourennane

1 Physics Department, Stockholm University, S-10691, Stockholm, Sweden.
2 Departamento de Física Aplicada II, Universidad de Sevilla, E-41012 Sevilla, Spain.

Contextuality is a fundamental concept for understanding quantum probabilities and the origin of the power of quantum systems for computation and information processing. A natural question is: Which is the simplest and cleanest version of quantum contextuality? It has been very recently found that there is a version that is analogous to “the simplest and cleanest” version of quantum non-locality, but only requires single three-level quantum systems (qutrits) rather than composite systems. Here we report the first experimental observation of this “Hardy-like” quantum contextuality. We measured the correlations between the outcomes of five combinations of two compatible measurements performed sequentially, in any possible order, on heralded photonic path-encoded qutrits. The experiment adopts a novel configuration which allows for performing two sequential measurements on the same photon with a high fidelity and allows for observing the independence of the correlations with respect to the order in which measurements are performed. The experimental results match the conditions needed to define a Hardy-like argument and the predictions of quantum theory. In addition, they violate the relevant non-contextuality inequality, even when the non-contextual bound is corrected to take into account experimental imperfections. The experimental observation of this form of quantum contextuality is of fundamental importance, since it connects the Kochen-Specker theorem with the simplest non-contextuality inequality. In addition, the method introduced here opens the door to applications requiring sequential quantum measurements on photonic systems.
1 Introduction

Quantum probabilities cannot be reproduced by a joint probability distribution over a single probability space. This implies that the outcomes of quantum measurements cannot be preassigned independently of the measurement context (i.e., the set of other compatible measurements that are simultaneously performed). Consequently, some predictions of quantum theory (QT) cannot be reproduced with non-contextual hidden variable (NCHV) theories. In this sense, QT is said to have contextuality.

This contextuality is the critical resource for universal quantum computation via ‘magic state’ distillation (the leading model for experimentally realizing fault-tolerant quantum computation) and for measurement-based quantum computations for calculating non-linear Boolean functions with a high probability. Contextuality is also the underlying property behind quantum non-locality and its applications for cryptography, reduction of classical communication complexity and randomness generation.

A natural question is: How can one explain the essence of quantum contextuality to the layman? Which is the simplest and cleanest explanation (in the sense of Ref. 4) of quantum contextuality? While Hardy’s is the “the simplest and cleanest” version of quantum non-locality, an equivalently simple and clean version of quantum contextuality has only been recently discovered. It can be formulated in terms of five imaginary boxes, numbered from 1 to 5, which can be either full or empty. Suppose that \( P_{|\psi\rangle}(0,1|i,j) \) denotes the joint probability that box \( i \) is empty and box \( j \) is full (and likewise for both boxes full and both empty) when the boxes are prepared in state \( |\psi\rangle \). If one assumes that the outcomes of opening the boxes are non-contextual, then the fact that

\[
\begin{align*}
P_{|\psi\rangle}(0,1|1,2) + P_{|\psi\rangle}(0,1|2,3) &= 1, \\
P_{|\psi\rangle}(0,1|3,4) + P_{|\psi\rangle}(0,1|4,5) &= 1,
\end{align*}
\]

would lead to the conclusion that

\[
P_{|\psi\rangle}(0,1|5,1) = 0.
\]

However, in QT, conditions (1a) and (1b) can occur while prediction (2) fails. This proves that some
predictions by NCHV theories are violated by QT. Instead of (2), QT predicts

$$P_{\psi}(0, 1|5, 1) = \frac{1}{9}. \quad (3)$$

Hereafter, we will refer to the non-zero probability in (3) as the “Hardy fraction”, because of the analogy with Hardy’s proof\(^5\).\(^6\)

As pointed out by Mermin, although Hardy-like proofs may “reign supreme in the gedanken realm”, they “provide a rather weak basis for a laboratory violation of the experimentally relevant inequality”\(^14\). This is so because any Hardy-like proof can be expressed as a violation of an “experimentally relevant” inequality: the Clauser-Horne-Shimony-Holt Bell inequality\(^15\), in case of Hardy’s proof of non-locality\(^14\), and the Klyachko-Can-Binicio˘ glu-Shumovsky (KCBS) non-contextuality inequality\(^7\) in case of the Hardy-like proof of contextuality\(^3\). The problem is that this violation is small compared to the one that can be achieved when the constraints characteristic of a Hardy-like proof are removed. Another difficulty is that a Hardy-like proof requires a very precise state preparation and very precise measurements. Consequently, an experimental test of a Hardy-like proof is more demanding than a standard test of non-locality or contextuality.

Several experiments have tested Hardy’s proof of non-locality\(^16\)\(^17\). The experiment to test Hardy-like contextuality faces an additional difficulty: It cannot be implemented by measuring the compatible observables on different subsystems of a composite system, as in Bell-inequality experiments, or different degrees of freedom of single systems\(^18\)\(^19\). It requires sequentially measuring compatible observables on the same system. Moreover, it requires measuring each observable using the same device in any context\(^20\) achieving that: (i) The probabilities in (1a) and (1b) sum up to 1 within the experimental error, (ii) the Hardy fraction is non-zero and in agreement with the quantum prediction in (3), and (iii) the probabilities violate the relevant non-contextuality inequality (the KCBS inequality).

2 Experimental setup

Our Hardy-like contextuality experiment adopted a novel method for performing two sequential measurements on the same photon. We encoded the outcome of the first (path) measurement in the polariza-
tion degree of freedom. This allowed us to experimentally reach the conditions for a Hardy-like proof of quantum contextuality.

Any experiment testing contextuality must satisfy two requirements: (I) Observables measured together must be compatible and (II) every observable must be measured using the same device in any context. Requirement (I) is sometimes difficult to fulfill and the implications of not having perfect compatibility have been extensively studied. Requirement (II) is neglected in some experiments.

In Bell-inequality experiments, no special action is needed to guarantee (I), since each observable is measured on a different subsystem. However, in experiments in which observables cannot be measured on different subsystems or different degrees of freedom of the same system, special care must be taken to enforce compatibility.

Recall that, by definition, in a general probabilistic theory, two observables $\mu_1$ and $\mu_2$ are compatible if there exists a third observable $\mu$ such that the outcome set of $\mu$ is the Cartesian product of the outcome sets of $\mu_1$ and $\mu_2$, and, for all states $\rho$, the outcome probability distributions for $\mu_1$ or $\mu_2$ are recovered as marginals of the outcome probability distribution of $\mu$.

In our experiment, we took advantage of the fact that, if $\mu_1$ and $\mu_2$ represent sharp quantum observables (i.e., quantum observables in the sense defined by von Neumann) on path degrees of freedom of a photon. In this case, there is an algorithm for constructing the optical device implementing any unitary transformation. If QT is correct, whenever two operators commute the corresponding devices implement compatible observables. We used this to construct devices for measuring compatible observables $\mu_1$ and $\mu_2$. Constructing a device for measuring $\mu$ can be done by noticing that it is equivalent to the device in which $\mu_1$ and $\mu_2$ are placed sequentially in any order (see Fig. 1). Compatibility can then be tested afterwards (thus removing the need of assuming that QT correctly represents compatible observables) by checking that the definition of compatibility is satisfied within the experimental error for a set of states.

Our approach consists of first constructing devices corresponding to sharp quantum observables, and then placing pairs of them which are compatible in both possible orders. This allows us to satisfy at the same time both requirements (I) and (II). Perfect compatibility is only limited by our ability to construct...
devices corresponding to the exact unitary transformations needed and by imperfections when combining
them. The aim of our setup is to implement a photonic version of a contextuality experiment in which
several sequential compatible measurements are performed on a quantum system as the one illustrated
in Fig. 1(a). In our experiment the physical systems are defined by single photons in a three-path setup.
The basis vectors $|0\rangle$, $|1\rangle$ and $|2\rangle$ correspond to finding the photon in path $a$, $b$ and $c$, respectively. To
perform two sequential measurements on the same photon we implement a novel scheme shown in Fig. 1(b) in which the outcome of the first measurement (that only addresses the spatial degrees of freedom)
is stored in the polarization of the photon before the second measurement (that also only addresses the
spatial degrees of freedom).

Each run of the experiment consists of preparing a single photon in a given state and measuring two
compatible observables, $i$ and $i + 1$ (or $i + 1$ and $i$), sequentially. Single photons are generated from a
heralded single photon source through a spontaneous parametric down-conversion (SPDC) process. The
twin photon is used as trigger. To exactly define the spatial and spectral properties of the photons, the
source is coupled into a single mode fiber (SMF) and passed through a narrowband interference filter (F).
Photons are initially horizontally polarized in order to allow us to encode the outcome of the first
measurement by simply rotating the polarization in one of the paths after the unitary transformation
 corresponding to the first observable. The initial state of the photon in the path degrees of freedom is

$$|\eta\rangle = \frac{1}{\sqrt{3}} (1, 1, 1)^T,$$

where $T$ means transposition. This state is prepared by combining two beam splitters (BSs), the first
with a reflectivity/transmitivity ratio of 33:66 and the second with a 50:50 ratio. The setup for the state
preparation is shown in Fig. 2.

The observables $i$, with $i = 1, \ldots, 5$, needed for our experiment are those represented by the projec-
tors $|v_i⟩⟨v_i|$ on the following states:

\[
|v_1⟩ = \frac{1}{\sqrt{3}}(1, -1, 1)^T, \\
|v_2⟩ = \frac{1}{\sqrt{2}}(1, 1, 0)^T, \\
|v_3⟩ = (0, 0, 1)^T, \\
|v_4⟩ = (1, 0, 0)^T, \\
|v_5⟩ = \frac{1}{\sqrt{2}}(0, 1, 1)^T.
\]

(5a) (5b) (5c) (5d) (5e)

Their possible outcomes are 1 and 0. Each measurement $i$ consists of a unitary transformation $U_i$ to project the qutrit onto the two eigenspaces of $i$, followed by a recording of the outcome. In our case, the outcome is encoded in the polarization of the photon by adding a half wave plate (HWP) in one of the paths (see Fig. 2). Then, the inverse unitary transformation $U_i^\dagger$ is implemented in order to rotate back to the initial basis. Unitary transformations $U_i$ and $U_i^\dagger$, with $i = 1, \ldots, 5$, were implemented by mapping $|v_i⟩$, i.e., the quantum state corresponding to eigenvalue 1, to path $a$ and mapping the subspace corresponding to eigenvalue 0 to the remaining two paths $b$ and $c$. The devices implementing the unitary transformations needed for the five observables $i$ are shown in Fig. 3. Fig. 2 shows the complete experimental setup corresponding to the sequential measurement of observables 2 (in the first place) and 1.

State (4) and these measurements lead to the conditions needed for a Hardy-like proof of contextuality, namely:

\[
P_{[0]}(0, 1|1, 2) + P_{[1]}(0, 1|2, 3) = \frac{2}{3} + \frac{1}{3}, \\
P_{[0]}(0, 1|3, 4) + P_{[1]}(0, 1|4, 5) = \frac{1}{3} + \frac{2}{3}, \\
P_{[0]}(0, 1|5, 1) = \frac{1}{9}.
\]

(6a) (6b) (6c)

### 3 Experimental Hardy-like quantum contextuality

The experimental results are presented in Table 1. The errors come from Poissonian counting statistics and systematic errors. The main sources of systematic errors are the slight imperfections in the optical
interferometers due to nonperfect overlapping and intrinsic imperfections of the BSs and HWP s. As shown in Table [1], the results are in very good agreement with the predictions of QT for an ideal experiment and are essentially insensitive to the order in which the measurements are performed. This contrasts with previous photonic experiments where the independence of the order was not tested. 21, 22

Taking from Table [1] the experimental results needed for the Hardy-like proof for contextuality, we obtain

\[
P_{|\eta\rangle}(0, 1|1, 2) + P_{|\eta\rangle}(0, 1|2, 3) = 0.981 \pm 0.021, \quad (7a)
\]
\[
P_{|\eta\rangle}(0, 1|3, 4) + P_{|\eta\rangle}(0, 1|4, 5) = 0.987 \pm 0.012, \quad (7b)
\]
\[
P_{|\eta\rangle}(0, 1|5, 1) = 0.110 \pm 0.005. \quad (7c)
\]

These results were obtained by measuring \(i\) in the first place in half of the experimental runs and measuring \(i + 1\) in the first place in the other half. These results show a very good agreement with the predictions of QT for an ideal experiment and thus provide experimental evidence of the Hardy-like quantum contextuality as shown by the following facts: (i) \(P_{|\eta\rangle}(0, 1|1, 2) + P_{|\eta\rangle}(0, 1|2, 3)\) and \(P_{|\eta\rangle}(0, 1|3, 4) + P_{|\eta\rangle}(0, 1|4, 5)\) are 1 within the experimental error, as required for the Hardy-like proof, (ii) \(P_{|\eta\rangle}(0, 1|5, 1)\) is nonzero and in very good agreement with the value predicted by quantum theory given by (3), (iii) the joint probabilities are almost independent on the order in which the measurements were performed (see Table [1]), and (iv) the sum of the joint probabilities violates the KCBS inequality, namely,

\[
S = \sum_{i=1}^{5} P((0, 1|i, i + 1)^{\text{NCHV}}) \leq 2, \quad (8)
\]

where the sum is taken modulo 5 and “ \(\leq 2\)” indicates that 2 is the maximum value for NCHV theories. From the results in (7a)–(7c), we obtain

\[
S_{\text{exp}} = 2.078 \pm 0.038, \quad (9)
\]
in agreement with the quantum prediction for an ideal experiment.

As stated above, compatibility was enforced by choosing \(i\) or \(i + 1\) to be devices represented in QT by commuting operators. In addition, we checked compatibility in two ways. First, we checked that the
probabilities do not depend on the order in which measurements were performed. Second, we counted the detections in the detector corresponding to \((1, 1|i, i + 1)\). These detections would never occur in an ideal situation, since the eigenstates of \(i\) and \(i + 1\) with eigenvalue 1 are orthogonal. Our experiment was very close to this ideal situation, since these events only occurred with probabilities in the range 0.002±0.001–0.006±0.001 for 9 out of the 10 configurations. For the most complex configuration (the one shown in Fig. 2) these events occurred with probability 0.021±0.001.

The non-contextual upper bound of the KCBS inequality, namely, 2 is derived under the assumption that the two observables measured sequentially are perfectly compatible. However, experimental imperfections make this assumption only approximately satisfied. To show the significance of the experimental results in this case, we followed the approach used in previous experimental violations of non-contextuality inequalities with sequential measurements \(^{22}\) (there are other approaches to extract conclusions about contextuality from experimental data \(^{23,24}\)). It consists of assuming that the non-contextual upper bound of the inequality is valid for some fraction \((1−\epsilon)\) of the experimental runs, but must be corrected (increased) assuming the most adversarial scenario for the other fraction \(\epsilon\). As in previous experiments \(^{22}\), here \(\epsilon\) is defined as the average of \(P(1, 1|i, i + 1)\) for the 10 experimental configurations tested. In an ideal experiment \(\epsilon\) would be zero. Our assumption is that the non-contextual upper bound of the KCBS inequality (namely 2) is valid only for a fraction \(1−\epsilon\) of the runs, while for the remaining fraction \(\epsilon\) we assume the worst-case scenario in which the algebraic bound of the KCBS inequality (namely 5) is reached. In our experiment, \(\epsilon = 0.0062\). Therefore, the non-contextual upper bound of the KCBS inequality shifts from 2 to \(2(1−\epsilon)+5\epsilon = 2.019\). Nevertheless, the experimental value in (9) still violates this bound.

4 Conclusions

Using a novel method for performing two sequential measurements on the same photon, we have reported the first experimental observation of Hardy-like quantum contextuality on qutrits, which is the conceptually simplest version of quantum contextuality.

We have sharply observed, for the first time in sequential measurements on photons, that correlations
between the outcomes of observables represented by commuting operators are independent of the order in which the sequential measurements are performed, as predicted by QT. Moreover, we have shown that the experimental results violate the relevant NC inequality even when experimental imperfections are taken into account.

The experimental observation of this Hardy-like quantum contextuality is of fundamental importance, since it connects the simplest NC inequality violated by qutrits\textsuperscript{7} with the Kochen-Specker theorem\textsuperscript{2}, thus providing the missing link between these two fundamental results\textsuperscript{3}. At the same time, the method introduced here for sequential measurements on photonic systems opens the door to applications for quantum information processing requiring sequential quantum measurements on physical systems that can be also used for transmission of quantum information between spatially separated locations.

References

1. Bell, J. S. On the problem of hidden variables in quantum mechanics. Rev. Mod. Phys. 38, 447–452 (1966).
2. Kochen, S. & Specker, E. P. The problem of hidden variables in quantum mechanics. J. Math. Mech. 17, 59–87 (1967).
3. Cabello, A., Badziąg, P., Terra Cunha, M. & Bourennane, M. Simple Hardy-like proof of quantum contextuality. Phys. Rev. Lett. 111, 180404 (2013).
4. Mermin, N. D., The best version of Bell’s theorem. Ann. N. Y. Acad. Sci. 755, 616–623 (1995).
5. Hardy, L., Quantum mechanics, local realistic theories, and Lorentz-invariant realistic theories. Phys. Rev. Lett. 68, 2981–2984 (1992).
6. Hardy, L., Nonlocality for two particles without inequalities for almost all entangled states. Phys. Rev. Lett. 71, 1665–1668 (1993).
7. Klyachko, A. A., Can, M. A., Binicioğlu, S. & Shumovsky, A. S. Simple test for hidden variables in spin-1 systems. Phys. Rev. Lett. 101, 020403 (2008).
8. Howard, M., Wallman, J.J., Veitch, V. & Emerson, J. Contextuality supplies the ‘magic’ for quantum computation. *Nature* **510**, 351–355 (2014).

9. Raussendorf, R. Contextuality in measurement-based quantum computation. *Phys. Rev. A* **88**, 022322 (2013).

10. Bell, J. S. On the Einstein Podolsky Rosen paradox. *Physics* **1**, 195–200 (1964).

11. Ekert, A. K. Quantum cryptography based on Bell’s theorem. *Phys. Rev. Lett.* **67**, 661–663 (1991).

12. Brukner, Č., Žukowski, M., Pan, J.-W. & Zeilinger, A. Bell’s inequalities and quantum communication complexity. *Phys. Rev. Lett.* **92**, 127901 (2004).

13. Pironio, S. *et al.* Random numbers certified by Bell’s theorem. *Nature* **464**, 1021–1024 (2010).

14. Mermin, N. D. What’s wrong with this temptation? *Phys. Today* **47**(6), 9–11 (1994).

15. Clauser, J. F., Horne, M. A., Shimony, A. & Holt, R. A. Proposed experiment to test local hidden-variable theories. *Phys. Rev. Lett.* **23**, 880–884 (1969).

16. Torgerson, J. R., Branning, D., Monken, C. H. & Mandel, L. Experimental demonstration of the violation of local realism without Bell inequalities. *Phys. Lett. A* **204**, 323–328 (1995).

17. Boschi, D., Branca, S., De Martini, F. & Hardy, L. Ladder proof of nonlocality without inequalities: Theoretical and experimental results. *Phys. Rev. Lett.* **79**, 2755–2758 (1997).

18. Michler, M., Weinfurter, H. & Žukowski, M. Experiments towards falsification of noncontextual hidden variable theories. *Phys. Rev. Lett.* **84**, 5457–5461 (2000).

19. Hasegawa, Y. *et al.* Violation of a Bell-like inequality in single-neutron interferometry. *Nature* **425**, 45–48 (2003).

20. Amselem, E. *et al.* Comment on “State-independent experimental test of quantum contextuality in an indivisible system”. *Phys. Rev. Lett.* **110**, 078901 (2013).
21. Amselem, E., Rädmark, M., Bourennane, M. & Cabello, A. State-independent quantum contextuality with single photons. *Phys. Rev. Lett.* **103**, 160405 (2009).

22. D’Ambrosio, V. *et al.* Experimental implementation of Kochen-Specker set of quantum tests. *Phys. Rev. X* **3**, 011012 (2013).

23. Gühne, O. *et al.* Compatibility and noncontextuality for sequential measurements. *Phys. Rev. A* **81**, 022121 (2010).

24. Winter, A. What does an experimental test of quantum contextuality prove or disprove? *arXiv:1408.0945*

25. Reck, M., Zeilinger, A., Bernstein, H. J. & Bertani, P. Experimental realization of any discrete unitary operator. *Phys. Rev. Lett.* **73**, 58–61 (1994).
Acknowledgements: We thank K. Blanchfield and P. Mataloni for discussions. This work was supported by the Swedish Research Council, Ideas Plus (Polish Ministry of Science and Higher Education Grant No. IdP2011 000361), CAPES (Brazil), Project No. FIS2011-29400 (MINECO, Spain) with FEDER funds and the FQXi large grant project “The Nature of Information in Sequential Quantum Measurements”. A. C. thanks NORDITA and M. B. for their hospitality at Stockholm.

Author contributions: A.C. and M.B. designed the experiment and supervised the project. B.M. carried out the experiment and analyzed the data. J.A. and M.N. contributed to the experiment in its earlier stages. All authors discussed the results and agreed on the conclusions. A.C., B.M. and M.B. wrote the manuscript.

Additional information: Reprints and permissions information is available online at [http://npg.nature.com/](http://npg.nature.com/) reprints and permissions. Correspondence and requests for materials should be addressed to M.B.
Figure captions

Figure 1. Sequential measurements on the same photon. (a) The system is prepared in the initial state $|\eta\rangle$ and then submitted to two measurements, one after the other. The measurement of observable $i$ is implemented by means of a unitary operation $U_i$ that maps the eigenstate of $i$ with eigenvalue 1 into the desired path, followed by an operation that stores the outcome in a detector, followed by the inverse unitary operation $U_i^\dagger$ that rotates to the basis used in the state preparation. The measurement of observable $j$ is implemented similarly. (b) A photonic implementation of the scheme in (a). The initial state and the measurements refer to the photon’s path degrees of freedom. The outcome of observable $i$ is encoded in the photon’s polarization. Then, the unitary operation $U_j$ corresponding to observable $j$ is applied. Then, the outcome of observable $i$ is decoded using polarizing beam splitters (PBSs). The outcomes of observables $i$ and $j$ are given by the detector that clicks (since each detector corresponds to a combination of outcomes). There is no need to implement $U_j^\dagger$, since no further measurements will be performed on the photon.

Figure 2. Experimental setup for measuring sequentially observables 2 (in the first place) and 1, given by (5), on the state (4). It starts with the preparation of state (4). Then, the paths corresponding to $|0\rangle$ and $|1\rangle$ are injected into a Sagnac interferometer to control the phase difference. The phase shifter consists of two quarter wave plates and one HWP in between them. The result of the unitary operation $U_2$ is given by the Sagnac outputs (which have the right phase because of the phase shifter) together with the third path. Then, the output of observable 2 is encoded by rotating the polarization in one of the paths with the help of a HWP at $45^\circ$. Then, $U_2^\dagger$ is applied to go back to the basis used in the state preparation and the unitary operation $U_1$ is implemented in a similar way. The outcome of observable 2 is decoded using PBSs and the single photon detector that clicks (there is one Silicon avalanche photodiode at the end of each of the six possible paths) gives the outcomes of 2 and 1: Detector D1 clicks when event $(1, 1|2, 1)$ happens, D2 clicks for $(0, 1|2, 1)$, D3 or D6 click for $(1, 0|2, 1)$, and D4 or D5 click for $(0, 0|2, 1)$. All coincidence counts between the trigger detector and the measurement detectors are registered using an eight-channel coincidence logic with a time window of 1.7 ns. The number of detected photons was
approximately $2 \times 10^3$ per second and the total time used for each experimental configuration was 10 s.

**Figure 3.** Experimental setups for the unitary operations $U_i$ (in red) and $U_i^\dagger$ (in blue) for $i = 1, 2, 3$. These setups consist of combinations of BSs with 33:66 and 50:50 splitting ratios. The setups for $U_4$ and $U_4^\dagger$ are the same as those for $U_3$ and $U_3^\dagger$, respectively, but with the following relabeling of paths: $|0\rangle_{U_3} \rightarrow |1\rangle_{U_4}$, $|1\rangle_{U_3} \rightarrow |2\rangle_{U_4}$ and $|2\rangle_{U_3} \rightarrow |0\rangle_{U_4}$. The setups for $U_5$ and $U_5^\dagger$ are the same as those for $U_2$ and $U_2^\dagger$, respectively, but with the following relabeling of paths: $|0\rangle_{U_2} \rightarrow |1\rangle_{U_5}$, $|1\rangle_{U_2} \rightarrow |2\rangle_{U_5}$ and $|2\rangle_{U_2} \rightarrow |0\rangle_{U_5}$. 
Table 1: **Experimental results for the probabilities involved in the Hardy-like proof of contextuality.** The second and third column shows the probabilities when the measurements are performed in direct and reverse order, respectively. The fourth column shows the prediction of QT for an ideal experiment.

| $P_{\eta}(0,1|i,i+1)$ | $P_{\eta}(0,1|i+1,i)$ | Ideal |
|------------------------|------------------------|-------|
| $P_{\eta}(0,1|1,2)$    | 0.635 ± 0.020          | 0.661 ± 0.011 | 0.6667 |
| $P_{\eta}(0,1|2,3)$    | 0.332 ± 0.008          | 0.331 ± 0.005 | 0.3333 |
| $P_{\eta}(0,1|3,4)$    | 0.330 ± 0.004          | 0.339 ± 0.003 | 0.3333 |
| $P_{\eta}(0,1|4,5)$    | 0.650 ± 0.008          | 0.656 ± 0.011 | 0.6667 |
| $P_{\eta}(0,1|5,1)$    | 0.111 ± 0.003          | 0.109 ± 0.004 | 0.1111 |

Figure 1:
Figure 2:
Figure 3: