Optical rotation of heavy hole spins by non-Abelian geometrical means

Hui Sun$^{1,2}$, Xun-Li Feng$^{1,2}$, Chunjie Wu$^{1,2}$, Jinming Liu$^4$, and C. H. Oh$^{1,2}$

$^1$ Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, Singapore 117543
$^2$ Department of Physics, National University of Singapore, 2 Science Drive 3, Singapore 117542
$^3$ State Key Laboratory of High Field Laser Physics, Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China
$^4$ Physics Department, East China Normal University, Shanghai, China

(Dated: September 1, 2009)

A non-Abelian geometric method is proposed for rotating of heavy hole spins in a singly positive charged quantum dot in Voigt geometry. The key ingredient is the delay-dependent non-Abelian geometric phase, which is produced by the nonadiabatic transition between the two degenerate dark states. We demonstrate, by controlling the pump, the Stokes and the driving fields, that the rotations about $y$- and $z$-axes with arbitrary angles can be realized with high fidelity. Fast initialization and heavy hole spin state readout are also possible.

PACS numbers: 78.67.Hc, 03.67.Lx, 03.65.Vf

I. INTRODUCTION

Electron spins in quantum dots (QDs) are promising candidate for implementations of qubits $^{1,2,3}$ because of their potential integration into microtechnology. The two spin states of electron can be mapped directly to the two operational states in quantum information processing (QIP). A key element for spin-based QIP is the coherent manipulation of the spin states $^4,5,6,7,8,9,10,11,12,13,14,15,16,17,18$. This QIP approach requires not only rotation of unknown spin states—the heart of spin-based QIP, but also the spin states initialized in a known state and readout of spins. There has been significant experimental progress in the demonstration of the key DiVincenzo requirements $^{19}$, for examples, efficient optical methods for initialization and readout of spins $^{20,21,22,23,24}$. Significant theoretical and experimental effort has been invested in optical manipulation of electron spin such as by using two Raman-detuned laser pulses $^{10}$. Abelian geometric phase induced by $2\pi$ pulses $^{12}$, resonant radio-frequency pulses $^{13}$, and so on. Using ultrafast optical pulses, Press et al. reported that they have controlled and observed the spin of a single electron in a semiconductor (over six Rabi oscillations between the two spin states) $^4$. Subsequently, the rotations of electron spins about arbitrary axes in a few picoseconds were also demonstrated in an ensemble of QDs $^{11}$.

In spin-based QIP, in addition to preparing the spin in a precisely defined state, this state should survive long enough to allow its manipulations. Therefore a long spin coherence time is necessary. Different from the conduction electron, a valence hole has an atomic $p$ orbital, which has negligible overlap with the nuclei. Consequently, the suppressed hyperfine interaction leads to a longer spin coherence time than that of electron. This may provide an attractive route to hole-spin-based applications free from the complications caused by the fluctuating nuclear spin system. In particular, Heiss et al. reported that the spin-relaxation times of holes are up to 270 microseconds in InGaAs QDs embedded in a GaAs diode structure $^{25}$. Besides long coherence times, an equally important requirement is the ability to manipulate spins coherently. More recently, the high-fidelity hole spin initialization by optical pumping $^{26}$, optical control and readout of hole spin $^{27}$ have been demonstrated experimentally. These works promote the spin of a hole in a semiconductor QD to be the best position to be a contender for the role of a solid-state qubit.

When a quantum system governed by a Hamiltonian with nondegenerate eigenstates undergoes some appropriate cyclic evolutions by adiabatically changing the controllable parameters, besides a dynamical phase, it may acquire a so-called geometric phase or Berry phase $^{28}$. Wilczek and Zee generalized the geometric phase to degenerate systems, i.e., non-Abelian geometric phase $^{29}$. The geometric phase differs from the dynamic phase in that the former depends only on the geometry of the path executed, being therefore insensitive to the local inaccuracies and fluctuations. They are thus expected to be particularly robust against noise $^{30,31,32,33,34,35}$.

Motivated by these work, we propose a method for manipulating arbitrary rotation of an unknown heavy hole (HH) spin state in a singly positive charged quantum dot. By applying an external magnetic field in Voigt geometry, a double tripod-shaped scheme can be configured. Most importantly, in contrast to the existing proposals based on electron spin, the HH spin rotations are realized in terms of the non-Abelian Berry phase, which is acquired by controlling the parameters along adiabatic loops, i.e., stimulated Raman adiabatic passage (STIRAP) $^{36}$ and fractional STIRAP $^{37}$. The STIRAP process can be used to transfer populations coherently between quantum states through “dark state” which efficiently suppress relaxation. The geometric phases accumulated during a STIRAP process were previously investigated for tripod systems $^{38}$ and double-A systems $^{39}$.
In this paper, after briefly reviewing non-Abelian geometric phase (Sec. III), we discuss the hole and electron energy levels of a singly positive charged QD in Voigt geometry and the selection rules in Sec. IIII and study the feasibility of initialization by optical pumping. In Sec. IV we show how to achieve a two-fold degenerate dark states, and how to implement the rotations about $y$- and $z$-axes by using the non-Abelian geometric phase produced by the nonadiabatically coupling between the two degenerate dark states. The fidelities of these rotations are also discussed in this section. The readout of spin state is discussed in Sec. V. In Sec. VI we end with some remarks.

II. NON-ABELIAN GEOMETRIC PHASE

Our propositions of rotating the HH spin about $y$- and $z$-axes are based on non-Abelian geometric phase [29], so we start by recalling the basic facts about non-Abelian geometric phase [30]. We consider an $n$-fold degenerate eigenspace of a Hamiltonian $H(\chi)$ ($\kappa = 1, 2, \ldots, N$) (i.e., the eigenspace information encoded) depending continuously on parameters $\chi$. Based on the time-dependent Schrödinger equation, we control the parameters along loops $O$ in an adiabatic fashion so that the initial preparation can evolve according to

$$\Psi(t) = U(O)\Psi(t = 0).$$

The transformation can be computed in terms of the Wilczek-Zee gauge connection [29]:

$$U(O)_{\lambda} = \mathcal{P} \exp \int_{O} \sum_{\kappa=1}^{N} A_{\kappa} d\chi_{\kappa}.$$  

$\mathcal{P}$ denotes the path-order operator. $A^{ab}_{\kappa}$ is called the gauge potential given by

$$A^{ab}_{\kappa} = \left< \psi^{a}(\chi) \left| \frac{\partial}{\partial \chi_{\kappa}} \right| \psi^{b}(\chi) \right>,$$  

with $\{\left< \psi^{a}(\chi) \right>_{a=1}^{n}$ being an orthonormal basis of the degenerate eigenspace. It is worth noting that the parameter-dependent Hamiltonian should evolve adiabatically so that the instantaneous state ($\Psi(t)$) does not overflow the state vector space spanned by $|\psi^{a}\rangle$. An intriguing feature of the gauge potential $A$ lies in that it depends only on the geometry of executed path in the space of degenerate states.

III. THE ENERGY LEVELS AND INITIALIZATION

In the scheme of rotating HH spin, the basic idea is to reconfigure a multi-level system with interacting Hamiltonian possessing two-fold degenerate dark states. By changing the Rabi frequencies in adiabatic fashion, we perform a loop in the parameter space. At both the beginning and the end of the cycle, we have only the HH spin states (up and down). But after a loop a non-Abelian geometric phase is accumulated at HH spin states. Based upon this property, arbitrary rotations of HH spin can be implemented.

We consider a singly positive charged GaAs/AlGaAs QD with growth direction $z$, which can be formed naturally by interface steps in narrow quantum well (QW). The electron and holes become localized into QD regions of the QW. In the absence of magnetic field, the lowest conduction-band (CB) level is two-fold degenerate with respect to the spin projection $\pm 1/2$. In the valence-band (VB), the hole has a total angular momentum of $3/2$, with the projection $m_{j} = \pm 1/2$ (“light hole” LH) doublet separated by more than 30-50 meV from the $m_{j} = \pm 3/2$ (HH) states due to confinement. It is very large compared to the bandwidth of the picosecond and femtosecond pulsed laser, so one should be able to separate the HH and LH excitations by picosecond pulsed fields in practical applications. The spin states of HH (spin up and down) trapped in the QD, which are denoted by $|0\rangle = |\uparrow\rangle = |\frac{3}{2}, \frac{1}{2}\rangle$ and $|1\rangle = |\downarrow\rangle = |\frac{3}{2}, -\frac{1}{2}\rangle$, are our qubit degrees of freedom. We will perform sequentially the optical initialization, rotations of this spin by non-Abelian geometrical means, and readout of a single hole spin. With $\sigma^{-}$ and $\sigma^{+}$ excitations, the only dipole allowed optical transitions from the valence HH states to the conduction electron states are $|\frac{3}{2}, \frac{1}{2}\rangle \rightarrow |\frac{3}{2}, \frac{3}{2}\rangle$ and $|\frac{3}{2}, -\frac{1}{2}\rangle \rightarrow |\frac{3}{2}, -\frac{3}{2}\rangle$, and the light hole states cannot be excited because of the frequency selection. The angular moment restriction inhibits optical coupling between the two HH spin states. Consequently, the structure can be

![Diagram of energy-level diagram](image-url)
FIG. 2: (Color online) The population of the states |0⟩ and |1⟩ as a function of time with σ− continuous illumination. Different initial population distributions are considered: (a) ρ₀₁₀(t = 0) = 0, ρ₁₁₀(t = 0) = 1.0; (b) ρ₀₁₀(t = 0) = ρ₁₁₀(t = 0) = 0.5. The values of parameters are explained in the text.

Conversely, the HH can also be prepared in spin down state |↓⟩ by only applying the pump field. To assess the efficiency of optical spin preparation, we have performed numerical simulations using the Liouville equation for the density matrix for the four-level system as shown in Fig. 2. We have assumed that four exciton recombining channels (from two CB electron states to two spin states) proceed incoherently, and they are equal to each other (1/2γ = 800 ps [26]). The magnetic field and the spin-flip rates for hole and electron are taken as $B_z = 55$ mT, $γ_{hh} = γ_{ee} = 1$ ms $^{-1}$, and the Rabi frequencies as 1.0γ. The HH spin state initialization with a fidelity close to 1 (~99.95%), we define the fidelity of the hole spin initialization as $(\rho_{00} - \rho_{11})/(\rho_{00} + \rho_{11})$ for σ+ polarization with $\rho_{00} (\rho_{11})$ being the population of state |0⟩ (|1⟩) is possible to be achieved within a few times the inverse of the exciton recombining rate (~1.6 ns) in the QD structure in Voigt geometry. The initialization of HH spin here is realized based on the fact that the magnetic field applied in Voigt geometry reconfigures the electron eigenstates, which is different from Refs. [17, 26], where the initializations are realized based on spin precession.

IV. ARBITRARY ROTATIONS BY NON-ABELIAN GEOMETRIC MEANS

It is well known that, with two noncommutable rotations about two axes, any rotation can be implemented as a composite rotation [22]. Here we design two noncommutable rotations about y- and z-axes and compose general rotations from them. In the following, based on nono-Aelian geometric phase, we first show how to rotate the HH spin about y-axis, and then explain how to control the rotation about z-axis with the relative phase between the Stokes and the driving fields. As a result, any rotations of the HH spin can be realized.

A. Rotation about y-axis

In order to rotate the HH spin about y-axis, we apply the pump field, the Stokes field and the driving field to excite the corresponding transitions (as shown in Fig. 1). The Hamiltonian in the interaction picture and in the rotating-wave approximation (RWA) is given by

$$H(t) = \hbar[(\Delta_\delta - \Delta_p)|1⟩⟨1| + (\Delta_\delta + \Delta_p)|a⟩⟨a| - \Delta_p|e_1⟩⟨e_1| - (\Delta_p + \Delta)|e_2⟩⟨e_2| - Ω_{p1}(t)|e_1⟩⟨e_1| - Ω_{e1}(t)|e_1⟩⟨e_1| - Ω_{e2}(t)|e_2⟩⟨e_2| - Ω_{p2}(t)|e_2⟩⟨e_2| + H.c.,$$

where the half Rabi frequency is defined as $\Omega_{jk}(t) = \langle e_k | \vec{μ} \cdot \vec{E}_j(t) | q \rangle / 2\hbar$ with $\vec{μ}$ being the dipole moment, $k = 1, 2$, $j = p(s,d)$ denoting the pump (the Stokes and the driving) field, and $q = 0, 1, a$. $\Delta_j = ω_j - (ω_{s1} - ω_{q})$ is the detuning and $\Delta = ω_{s1} - ω_{s2} = |g_{ee}| μ_B B_z / \hbar$ is the elec-
tron Zeeman splitting with $g_e^2$ and $\mu_B$ representing Landé factor of electron and Bohr magneton, respectively.

When the pump, the Stokes, and the driving fields are tuned to match the conditions $\Delta_p = \Delta_s = \Delta_d \neq -\Delta/2$ (three fields are tuned to three-photon resonance, but they are not at the middle point of the two electron Zeeman splitting levels) and $\Omega_{11}(t)/\Omega_{12}(t) = C$ (for simplicity, we choose $C = 1$ and denote $\Omega_{11}(t) = \Omega_{12}(t) = \Omega_j(t)$), one can easily find from the interaction Hamiltonian \ref{interaction} that the interacting system has two degenerate dark states

\begin{align}
|D_1⟩ &= \cos \theta(t)|1⟩ - \sin \theta(t)|a⟩, \\
|D_2⟩ &= \cos \varphi(t)|0⟩ - \sin \varphi(t) \sin \theta(t)|1⟩ - \sin \varphi(t) \cos \theta(t)|a⟩,
\end{align}

where the mixing angle $\theta(t)$ and the additional mixing angle $\varphi(t)$ are defined as

$$
\tan \theta(t) = \frac{\Omega_a(t)}{\Omega_d(t)}, \quad \tan \varphi(t) = \frac{\Omega_p(t)}{\sqrt{\Omega_d^2(t) + \Omega_s^2(t)}}.
$$

It is well known that the exciton recombination occurs only if there is electron excited to the CB states $|e_{1,2}⟩$. The two-fold degenerate dark states $|D_{1,2}⟩$, which are known as trapped states, receive no contributions from the CB electron states [see Eqs. \ref{lens} and \ref{sun}]. Hence the rotation of HH spin about $y$-axis is robust against the exciton recombination process, and thus leading to high fidelity operations. It is also worth noting that, in the absence of the three fields, i.e., all the parameters (actually the angles $\theta(t)$ and $\varphi(t)$) are fixed to zero, the previous eigenstates coincide with the two spin states $|D_1(0)⟩ = |↑⟩$ and $|D_2(0)⟩ = |↓⟩$. When the three fields are applied adiabatically and hence the angles $\theta(t)$ and $\varphi(t)$ change adiabatically, the non-Abelian geometric connection components can be calculated, according to Eq. \ref{ping}, and we have

$$
A = A_0 d\theta = -i \sin \varphi(t) \sigma_y d\theta,
$$

with $\sigma_y$ being the $y$-component Pauli matrix. The related unitary operation is

$$
U(\mathcal{O}) = \exp \left(-i \sin \varphi(t) \int_{\mathcal{O}} \sin \varphi(t) d\theta \right) = R_y(\beta),
$$

where the rotating angle $\beta$ is given by

$$
\beta = \int_{\mathcal{O}} \sin \varphi(t) d\theta.
$$

Similar degenerate dark states and non-Abelian geometric connection have been realized for ion trap \ref{ion} and atoms \ref{atom}. However, in the QD structure under consideration, the non-Abelian geometric connection $A$ is based on HH spin states.

For a quantitative analysis of the rotating angle $\beta$ about $y$-axis, we assume that the pump, the Stokes and the driving fields have Gaussian shapes as $\Omega_d(t) = \Omega_d^0 \exp[-(t + \tau_0)^2/\tau^2]$, $\Omega_p(t) = \Omega_p^0 \exp(-t^2/\tau^2)$, and $\Omega_s(t) = \Omega_s^0 \exp[-(t - \tau_0)^2/\tau^2]$ with $\tau$ and $\tau_0$ being, respectively, the pulse widths and the delay. Figure \ref{three} shows that the evolution of the rotating angle $\beta$ about $y$-axis in unit of $\pi$ as a function of delay $\tau_0$. In the interaction, the time-dependent Hamiltonian should be performed sufficiently slowly. According to the condition for adiabatic passage \ref{passage}, we take $\Omega_p^0 \tau = \Omega_s^0 \tau = \Omega_d^0 \tau = 50 \gg 1$, which ensures no transition between the dark and bright states (not given in this paper). Figure \ref{nine} shows clearly that the rotating angle $\beta$ is delay-dependent. It goes up successively with increasing value of the delay, and then reaches its maximum value, $\pi/2$, when the delay $\tau_0$ is large.

Thus we have shown that the rotation of hole spin about $y$-axis can be implemented by using the non-Abelian geometric phase. The rotation is determined only by the global property and does not depend upon the details of the evolution path in the parameters space.

### B. Rotation about $z$-axis

As suggested in Ref. \ref{nonabelian}, by setting $\Omega_p(t) = 0$ and changing adiabatically $\Omega_s(t)$ and $\Omega_d(t)$, the rotation about $z$-axis can be achieved by making use of the Abelian geometric phase in our QD system. However, by involving the non-Abelian geometric phase, here we suggest another method for rotating the hole spin about $z$-axis. An important feature of this rotation is that the rotating angle about $z$-axis is not geometric phase-dependent, it is controlled by the relative phase between the Stokes and the driving fields. To do so, we set $\Omega_p(t) = 0$ so that the spin down state $|0⟩$ is decoupled, and assume the phase of the driving field is zero, the relative phase $\phi$ is therefore the phase of the Stokes field. The time-dependent Hamiltonian $H(t)$ in the interaction picture and RWA takes the form

$$
H(t) = \hbar (|\Delta_d - \Delta_s⟩⟨a| - |\Delta_d⟩⟨e_1|e_1⟩ - (\Delta_d + \Delta)⟨e_2|e_2⟩ - \Omega_d(t)⟨e_1|a⟩ + ⟨e_2|a⟩) - \Omega_s(t) \exp(-i\phi)(⟨e_1|1⟩ + ⟨e_2|1⟩) + \text{H.c.},
$$

where $\phi$ is the phase of the driving field.

![FIG. 3: (Color online) The rotating angle $\beta$ about $y$-axis in units of $\pi$ as a function of the delay $\tau_0$. The values of parameters are explained in the text.](image)
where $\phi$ is the relative phase between the Stokes and the drive fields. In the derivation of Hamiltonian (11), the condition $\Omega_{11}/\Omega_{22} = C = 1$ is applied. When the Stokes and the driving fields are controlled to satisfy two photon resonance, only one dark state ($|D_1(t)\rangle$) exists. The hole spin cannot be rotated by non-Abelian geometrical means. Fortunately, however, when the two fields are tuned to the middle point of the two electron Zeeman splitting levels, there is another dark state $|D_2(t)\rangle$. The two-fold degenerate dark states are

$$|D_1(t)\rangle = \cos\theta(t)e^{i\phi}|1\rangle - \sin\theta(t)|a\rangle, \quad (12)$$

$$|D_2(t)\rangle = \frac{1}{\sqrt{2}}[\cos\varphi(t)(|e_1\rangle - |e_2\rangle) + \sin\varphi(t)\cos\theta(t)|a\rangle + \sin\varphi(t)\sin\theta(t)e^{i\phi}|1\rangle], \quad (13)$$

where the mixing angle $\theta(t)$ and the additional mixing angle $\varphi(t)$ related to the electron Zeeman energy splitting $\Delta$ are respectively defined as

$$\tan\theta(t) = \frac{\Omega_s(t)}{\Omega_d(t)}, \quad \tan\varphi(t) = \frac{\Delta/2}{\sqrt{2(\Omega_s^2(t) + \Omega_d^2(t))}}. \quad (14)$$

A similar form of double degenerate dark states in double-$\Lambda$ atomic scheme has been obtained [39]. However, in our QD structure the degenerate HH spin-based dark states stem from electron Zeeman splitting.

In adiabatic limit (the time derivative of the mixing angles $\theta(t)$ and $\varphi(t)$ are small compared with the splitting of eigenvalues, given by $2\sqrt{2(\Omega_s^2(t) + \Omega_d^2(t)) + (\Delta/2)^2}$), only the transitions between the degenerate dark states $|D_1(t)\rangle$ and $|D_2(t)\rangle$ should be taken into account, and the nonadiabatic coupling of states $|D_1(t)\rangle$ and $|D_2(t)\rangle$ to other states (the expressions have not given in this paper) can be safely ignored [38]. $\langle D_2|D_1\rangle = -\sin\varphi(t)\theta(t)$ also exhibits that a nonadiabatic transition between the two-fold degenerate dark states may occur. Although the dark state $|D_2(t)\rangle$ receives contribution from the CB electron states [see Eq. (13)], we note that the electron magnetic-dependent Zeeman energy splitting is independent of time. At the beginning and end of the adiabatic process, we have $\Delta^2 \gg \Omega_s^2(t) + \Omega_d^2(t)$, which therefore leads to $|\varphi| = \pi/2$. The CB electron states have no influence on the final dark state $|D_2(\infty)\rangle$. In other words, there is no electron in the CB states when the interaction is finished.

Next, we will show how to translate the phase $\phi$ into the HH spin state $|\uparrow\rangle$, thus leading to the rotation about $z$-axis with angle $\phi$. Recalling the fact that, in the subspace, HH is prepared with spin up, namely $|D_1(0)\rangle = |1\rangle$. The Stokes and the driving fields are applied in the counterintuitive order, while they terminate with a constant ratio of their amplitude so that the phase $\phi$ can be introduced into the HH spin state. This extension of STIRAP is called fractional STIRAP, which has been suggested to create the coherent atomic superpositions in a robust way [37]. As shown in Fig. 4(a), the driving field consists of two parts—one with the same time dependence as the Stokes field and the other coming earlier, for example

$$\Omega_s(t) = \Omega_s^0\exp(-t^2/\tau^2), \quad (15)$$

$$\Omega_d(t) = \Omega_d^0\{\exp[-(t + \tau_0)^2/\tau^2] + \exp(-t^2/\tau^2)\}. \quad (16)$$

Here $\Omega_s^0$ and $\Omega_d^0$ are amplitudes of the Stokes and the driving fields, $\tau$ and $\tau_0$ are, respectively, pulse widths and delay between the two parts of driving field. During the interaction, the mixing angle $\theta(t)$ varies from 0 to $\pi/4$. According to the theory of non-Abelian geometric phase, after the interaction, we have [38, 39]

$$|\Psi(\infty)\rangle = \frac{1}{\sqrt{2}}[e^{i\phi}(\sin\gamma_f + \cos\gamma_f)|1\rangle + (\sin\gamma_f - \cos\gamma_f)|a\rangle], \quad (17)$$

where $\gamma_f$ is given by

$$\gamma_f = \int_0^{\Delta/2} \frac{\Delta/2}{\Omega_s^2(t) + \Omega_d^2(t)} \frac{\Omega_s(t)d\Omega_p(t) - \Omega_p(t)d\Omega_s(t)}{\sqrt{2(\Omega_s^2(t) + \Omega_d^2(t)) + (\Delta/2)^2}}. \quad (18)$$

Equation (17) exhibits that the relative phase $\phi$ translates to the HH spin state $|\uparrow\rangle$ via fractional STIRAP. If $\gamma_f$ can be accumulated to $\pi/4$ by controlling the Stokes and drive fields, after the cycle evolution, the HH spin will return to the spin up state with the phase $\phi$. As a result, the rotation of HH spin about $z$-axis is realized, and the varying rotating angle can be obtained by changing the relative phase $\phi$, namely the phase of the Stokes field.

It should be noted that $\gamma_f$ is gauge invariant, it depends upon the delay $\tau_0$ [38, 39]. In Fig. 4(b), we plot the
The evolution of $\gamma_f$ in units of $\pi$ as a function of $\tau_0$. We take the Rabi frequency amplitudes as: $\Omega_R^z = \Omega_d^z = 0.5$ ps$^{-1}$, the pulse width parameter $\tau = 100$ ps. The magnetic field is taken as $B_x = 55$ mT (we take the in-plane $g$ factor of electron as $g^e_z = -0.21$ [40], then the electron Zeeman splitting $\Delta \approx 1$ GHz). With these parameters, the adiabatic condition is hold and the governing Hamiltonian evolves adiabatically. As shown in Fig. 4(b), $\gamma_f$ increases from 0 by degrees with the growing of the delay, and reaches its maximum value, $\pi/4$, when the delay $\tau_0$ is large. Thus, the relative phase $\phi$ can be used to control the HH spin rotation about z-axis.

By combining the above rotations about $y$- and $z$-axis, any rotation can be implemented. For example, rotation about $x$-axis can be realized by $R_x(\phi) = R_{y}^{\dagger}(\pi/2)R_z(\phi)R_y(\pi/2)$. Our rotation procedure is sensitive to the non-Abelian geometric phase and the relative phase between the Stokes and the driving fields. Moreover, the non-Abelian geometric phase is independent of pulse areas and dependent on the ratios $\tau_0/\tau$, $\Omega_R^y/\Delta$, and $\Omega_d^z/\Delta$. Thus it is robust against the fluctuation of the pulse shapes, pulse areas and noise.

### C. Fidelity

Based on the non-Abelian geometric phase, arbitrary rotations of HH spin are possible. But how about the degree to which our approximate description matches the actual behavior of the system? The fidelity is a measure of how accurate the target gate is implemented and it is defined as $F(U) = \langle \Psi | U^{\dagger} U_{\text{target}} | \Psi \rangle^2$, where $U_{\text{target}}$ is the target operation, $U$ is the actual operation, and the average is taken over all input spin states $|\Psi\rangle$. By numerical simulation of the density matrix equations, the fidelity can be calculated. Typical rotation fidelities, listed in Table I, are on the order of $99.9\%$. The rotations of HH spin are realized with high fidelity.

### V. READOUT

The accurate measurement of the spin state of each qubit is essential in a quantum computation scheme. In our scheme, the $\sigma^+$-polarized Stokes (the $\sigma^-$-polarized pump) field can only excite the HH state with spin down (up). The measurement of the HH spin states therefore can be achieved by applying a $\sigma^+$ or $\sigma^-$ polarized continuous laser field. When the $\sigma^+$-polarized field is applied, for example, if the spin is rotated to $|\downarrow\rangle$, the QD will emit a single photon from the $|\psi_{1,2} \rightarrow |\downarrow\rangle$ transitions, which can be detected using a single-photon counter [4].

### VI. CONCLUSIONS

To summarize, we consider a singly positive charged quantum dot, and demonstrate sequentially the initialization, the optical rotations of HH spin with non-Abelian geometrical means, and readout of a single hole spin. Together with an magnetic field applied in Voigt geometry, the quantum dot system can be reconfigured as a double tripod scheme. When the pump, the Stokes and the driving fields are tuned to satisfy certain conditions, the QD system has two-fold degenerate dark states. Based on the non-Abelian geometric phase produced by the nonadiabatic coupling between the two dark states, not only can the HH spin be rotated about $y$-axis with stimulated Raman adiabatic passage, but also the relative phase between the Stokes and the driving fields can be translated into the hole spin state with fractional stimulated Raman adiabatic passages, leading to the implementation of rotation about z-axis. Therefore the key step of optical arbitrary rotations of HH spin with high fidelity for QIP can be implemented by non-Alelian geometrical means. It is in principle useful for spin-based quantum information processing.

### Acknowledgments

This work is supported by the National Research Foundation and Ministry of Education, Singapore under academic research grant No. WBS: R-710-000-008-271. One of the authors (H. S.) would also like to acknowledge the support of the National Basic Research Program of China (973 Program) under Grant No. 2006CB921104, the author (S. Q. G.) acknowledges funding from the National Natural Science Foundation of China through Grant No. 10874194 and the author (J. M. L.) acknowledges funding from the National Natural Science Foundation of China under Grant No. 60708003.

### Table I: Fidelity of selected rotations of HH spin.

| $R_{x}(\phi)$ | $\tau_0/\tau$ | Fidelity |
|---------------|---------------|----------|
| $R_{y}(\pi/2)$ | 1.5           | 99.96\%  |
| $R_{z}(\pi/2)$ | 6.5           | 99.99\%  |
| $R_{z}(\pi/2)$ |               | 99.94\%  |
