Low-energy limits on heavy Majorana neutrino masses from the neutrinoless double-beta decay and non-unitary neutrino mixing

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Abstract

In the type-I seesaw mechanism, both the light Majorana neutrinos ($\nu_1, \nu_2, \nu_3$) and the heavy Majorana neutrinos ($N_1, \cdots, N_n$) can mediate the neutrinoless double-beta ($0\nu\beta\beta$) decay. We point out that the contribution of $\nu_i$ to this $0\nu\beta\beta$ process is also dependent on the masses $M_k$ and the mixing parameters $R_{ek}$ of $N_k$ as a direct consequence of the exact seesaw relation, and the effective mass term of $\nu_i$ is in most cases dominant over that of $N_k$. We obtain a new bound $|\sum R_{ek}^2 M_k| < 0.23$ eV (or $< 0.85$ eV as a more conservative limit) at the $2\sigma$ level, which is much stronger than $|\sum R_{ek}^2 M_k^{-1}| < 5 \times 10^{-8}$ GeV$^{-1}$ used in some literature, from current experimental constraints on the $0\nu\beta\beta$ decay. Taking the minimal type-I seesaw scenario for example, we illustrate the possibility of determining or constraining two heavy Majorana neutrino masses by using more accurate low-energy data on lepton number violation and non-unitarity of neutrino mixing.

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I. INTRODUCTION

Almost undeniable evidence for finite neutrino masses and large neutrino mixing angles has recently been achieved from solar, atmospheric, reactor and accelerator neutrino oscillation experiments [1–4]. This exciting breakthrough opens a new window to physics beyond the standard model (SM), since the SM itself only contains three massless neutrinos (i.e., $\nu_e$, $\nu_\mu$ and $\nu_\tau$, corresponding to the mass eigenstates $\nu_1$, $\nu_2$ and $\nu_3$). The simplest way to generate non-zero but tiny neutrino masses $m_i$ for $\nu_i$ is to extend the SM by introducing at least two right-handed neutrinos and allowing lepton number violation. In this well-known (type-I) seesaw mechanism [5], the $SU(2)_L \times U(1)_Y$ gauge-invariant mass terms of charged leptons and neutrinos are given by

$$-\mathcal{L}_{\text{mass}} = \bar{l}_L Y_l H E_R + \bar{l}_L Y_\nu \tilde{H} N_R + \frac{1}{2} \sum_{k=1}^n \frac{R_{ek}^2}{M_k} N_R N_R + \text{h.c.},$$

where $\tilde{H} \equiv i\sigma_2 H^*$, $l_L$ denotes the left-handed lepton doublet, and $M_R$ is the mass matrix of right-handed neutrinos. After the spontaneous gauge symmetry breaking, we arrive at the charged-lepton mass matrix $M_l = Y_l v$ and the Dirac neutrino mass matrix $M_D = Y_\nu v$, where $v \approx 174$ GeV is the vacuum expectation value of the neutral component of the Higgs doublet $H$ and characterizes the Fermi scale of weak interactions. The mass scale of $M_R$ (or equivalently the seesaw scale $\Lambda_{SS}$) is crucial, because it is relevant to whether the seesaw mechanism itself is theoretically natural and experimentally testable. Although $\Lambda_{SS} \ll v$ is not impossible [6], it is in general expected that $\Lambda_{SS}$ should be much higher than the Fermi scale. In particular, the conventional seesaw mechanism works at a scale which is not far away from the scale of grand unified theories. Driven by the upcoming running of the Large hadron Collider (LHC), more and more attention has been paid to the TeV scale at which the unnatural gauge hierarchy problem of the SM may be solved or softened by new physics. If the TeV scale is really a fundamental scale, then we are reasonably motivated to speculate that possible new physics existing at this scale and responsible for the electroweak symmetry breaking might also be responsible for the origin of neutrino masses. In this sense, it is meaningful to investigate the TeV seesaw mechanism and balance its theoretical naturalness and experimental testability at the energy frontier set by the LHC [7].

A direct test of the type-I seesaw mechanism demands the experimental discovery of heavy Majorana neutrinos $N_k$ (for $k = 1, \cdots, n$) at the LHC, but two prerequisites must be satisfied: (a) their masses $M_k$ must be of $\mathcal{O}(1)$ TeV or smaller; and (b) their couplings to charged leptons $R_{\alpha k}$ (for $\alpha = e, \mu, \tau$ and $k = 1, \cdots, n$) must not be too small. The strongest bound on $M_k$ and $R_{ek}$ comes from the non-observation of the neutrinoless double-beta ($0\nu\beta\beta$) decay [8], as $N_k$ can mediate this lepton-number-violating process. Current experimental lower limit on the half-lifetime of the $0\nu\beta\beta$ decay is usually translated into

$$\left| \sum_{k=1}^n \frac{R_{ek}^2}{M_k} \right| < 5 \times 10^{-8} \text{ GeV}^{-1}$$

in some literature [9]. In obtaining Eq. (2), one has ignored the contribution of three light Majorana neutrinos $\nu_i$ (for $i = 1, 2, 3$) to the $0\nu\beta\beta$ decay.
The first purpose of this paper is to point out that the constraint in Eq. (2) is not always useful for the type-I seesaw mechanism either at a superhigh-energy scale or at the electroweak or TeV scale. The reason is simply that the contribution of $\nu_i$ to the $0\nu\beta\beta$ decay is in most cases dominant over the contribution of $N_k$ to the same process, leading to a much stronger bound on $M_k$ and $R_{ek}$ through the exact seesaw relation:

$$\left| \sum_{k=1}^n R_{ek}^2 M_k \right| < 0.23 \text{ eV} \quad \text{(or < 0.85 eV)}$$

at the $2\sigma$ level, which is equivalent to $\langle m \rangle_{ee} < 0.23 \text{ eV}$ (or < 0.85 eV as a more conservative bound) [10,11] for the effective mass of the $0\nu\beta\beta$ decay mediated by $\nu_i$. The second purpose of this paper is to look at whether the future measurements of lepton number violation and non-unitarity of neutrino flavor mixing are possible to shed light on $M_k$. Taking the minimal type-I seesaw scenario [12] for example, we shall illustrate the possibility of determining or constraining two heavy Majorana neutrino masses by using more accurate low-energy data on the $0\nu\beta\beta$ decay and non-unitary neutrino mixing and CP violation.

II. STRONGER BOUND ON THE $0\nu\beta\beta$ DECAY

After the spontaneous gauge symmetry breaking (i.e., $SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$), the mass terms in Eq. (1) turn out to be

$$-\mathcal{L}_{\text{mass}} = E_L M_L E_R + \frac{1}{2} (\nu_L N_R) \begin{pmatrix} 0 & M_D & M_R \\ M_D^T & 0 & \nu_L \\ M_R^T & \nu_R & 0 \end{pmatrix} + h.c.$$  

where $E$ and $\nu_L$ stand respectively for the column vectors of $(e, \mu, \tau)$ and $(\nu_e, \nu_\mu, \nu_\tau)_L$. Without loss of generality, one may take $M_l = \text{Diag}\{m_e, m_\mu, m_\tau\}$. The overall $(3+n) \times (3+n)$ neutrino mass matrix in Eq. (4) can be diagonalized by a unitary transformation; i.e.,

$$\begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} = \begin{pmatrix} V & 0 \\ S & U \end{pmatrix} \begin{pmatrix} \tilde{M}_L & 0 \\ 0 & \tilde{M}_N \end{pmatrix} \begin{pmatrix} V & 0 \\ S & U \end{pmatrix}^T,$$

where $\tilde{M}_L = \text{Diag}\{m_1, m_2, m_3\}$ and $\tilde{M}_N = \text{Diag}\{M_1, \cdots, M_n\}$. After this diagonalization, the flavor states of three light neutrinos ($\nu_e, \nu_\mu, \nu_\tau$) can be expressed in terms of the $(3+n)$ mass states of light and heavy neutrinos ($\nu_1, \nu_2, \nu_3$ and $N_1, \cdots, N_n$), and thus the standard charged-current interactions between $\nu_\alpha$ and $\alpha$ (for $\alpha = e, \mu, \tau$) can be written as

$$-\mathcal{L}_{cc} = \frac{g}{\sqrt{2}} (e\mu\tau)_L \gamma^\mu \left[ V \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}_L + R \begin{pmatrix} N_1 \\ \vdots \\ N_n \end{pmatrix}_L \right] W^-_\mu + h.c.$$  

in the basis of mass states. So $V$ is just the neutrino mixing matrix responsible for neutrino oscillations, while $R$ describes the strength of charged-current interactions between $(e, \mu, \tau)$ and $(N_1, \cdots, N_n)$. $V$ and $R$ are correlated with each other through $VV^\dagger + RR^\dagger = \mathbf{1}$. Hence $V$ itself is not exactly unitary in the type-I seesaw mechanism and its deviation from unitarity is simply characterized by non-vanishing $R$ [13].
Note that $V$ and $R$ are also correlated with each other through the exact seesaw relation

$$V \tilde{M}_\nu V^T + R \tilde{M}_N R^T = 0,$$

which can directly be derived from Eq. (5). Taking the $(ee)$-elements for both terms on the left-hand side of Eq. (7), we immediately arrive at

$$\left( V \tilde{M}_\nu V^T \right)_{ee} = \sum_{i=1}^3 V_{ei}^2 m_i = - \left( R \tilde{M}_N R^T \right)_{ee} = - \sum_{k=1}^n R_{ek}^2 M_k.$$

This simple but interesting result implies that the effective mass of three light Majorana neutrinos in the $0\nu\beta\beta$ decay is directly associated with the masses, mixing angles and CP-violating phases of heavy Majorana neutrinos in the type-I seesaw mechanism:

$$\langle m \rangle_{ee} \equiv \left| \sum_{i=1}^3 V_{ei}^2 m_i \right| = \left| \sum_{k=1}^n R_{ek}^2 M_k \right|.$$

Note that both light Majorana neutrinos $\nu_i$ and heavy Majorana neutrinos $N_k$ can mediate the $0\nu\beta\beta$ decay, as shown in Fig. 1. When the contribution of $N_k$ is least suppressed [14], the overall decay width of the $0\nu\beta\beta$ process in the type-I seesaw scenario can approximately be expressed as

$$\Gamma_{0\nu\beta\beta} \propto \left| \sum_{i=1}^3 V_{ei}^2 m_i - \sum_{k=1}^n \frac{R_{ek}^2 M_k^2}{M_A^2} \mathcal{F}(A, M_k) \right|^2 \left| \sum_{k=1}^n R_{ek}^2 M_k \left[ 1 + \frac{M_k^2}{M_A^2} \mathcal{F}(A, M_k) \right] \right|,$$

where $A$ is the atomic number, $\mathcal{F}(A, M_k) \simeq 0.1$ depending mildly on the decaying nucleus, and $M_A \simeq 900$ MeV [14]. Given $M_k \gtrsim 10^2$ GeV, the second term in the square brackets of Eq. (10) turns out to be $\lesssim 8.1 \times 10^{-6}$. Hence this term is negligible in most cases, unless the contribution of $\nu_i$ is vanishing or vanishingly small due to a contrived cancellation among three different $V_{ei}^2 m_i$ terms (or equivalently, among $n$ different $R_{ek}^2 M_k$ terms), which is in principle not impossible. Let us consider two limits in which the contributions of light and heavy Majorana neutrinos to $\Gamma_{0\nu\beta\beta}$ are decoupled.

- In the limit of $\sum_{k=1}^n R_{ek}^2 M_k^{-1} \mathcal{F}(A, M_k) \to 0$, which is almost a realistic case, Eq. (10) is directly simplified to

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1Current calculations of the nuclear matrix elements of the $0\nu\beta\beta$ decay are quite uncertain due to our poor knowledge of the nucleon wave functions [8]. For point-like nucleons, a Yukawa-type potential has been used to estimate the nuclear matrix elements [14]. The relevant result remains true even if the finite size of the nucleons is taken into account [14]. We thank the referee for calling our attention to these points.
\[ \left( \Gamma_{0\nu\beta\beta} \right)^{1/2} \propto \langle m \rangle_{ee} = \left| \sum_{k=1}^{n} R_{ek} M_k \right| . \]  

(11)

Current experimental data on the 0\(\nu\beta\beta\) decay yield an upper bound on this effective mass term, \(\langle m \rangle_{ee} < 0.23 \text{ eV}\) at the 2\(\sigma\) level [11], which has extensively been used to constrain the masses, flavor mixing angles and Majorana CP-violating phases of three light neutrinos in the unitary limit of \(V\). However, it should be kept in mind that this upper bound corresponds to some “favorable” values of the relevant nuclear matrix elements [8]. If their “unfavorable” values are used, one may also arrive at \(\langle m \rangle_{ee} < 0.85 \text{ eV}\) at the 2\(\sigma\) level [11].

- In the limit of \(\sum_{k=1}^{n} R_{ek} M_k \rightarrow 0\), which is rather contrived, Eq. (10) can be simplified to

\[ \left( \Gamma_{0\nu\beta\beta} \right)^{1/2} \propto \langle m \rangle'_{ee} \equiv M_A^{2} \left| \sum_{k=1}^{n} \frac{R_{ek}^2}{M_k} \mathcal{F}(A, M_k) \right| \]  

(12)

as a rough approximation. Imposing the bound \(\langle m \rangle'_{ee} < 0.23 \text{ eV}\) and inputting \(M_A \simeq 900 \text{ MeV}\) and \(\mathcal{F}(A, M_k) \simeq 0.1\) [14], for example, we obtain

\[ \left| \sum_{k=1}^{n} \frac{R_{ek}^2}{M_k} \right| < 2.8 \times 10^{-9} \text{ GeV}^{-1}, \]  

(13)

which is a bit stronger than the upper bound shown in Eq. (2). If the more conservative bound \(\langle m \rangle'_{ee} < 0.85 \text{ eV}\) is taken, one will arrive at \(\sum_{k=1}^{n} R_{ek}^2 M_k^{-1} < 1.0 \times 10^{-8} \text{ GeV}^{-1}\), much closer to the result given in Eq. (2). Such rough bounds have been used by a number of authors in their preliminary studies of possible collider signatures of heavy Majorana neutrinos [9].

Note again that we have ignored the mild dependence of \(\mathcal{F}(A, M_k)\) on the decaying nuclei in the above discussions. Otherwise, different 0\(\nu\beta\beta\) decays should be separately analyzed.

Below Eq. (10), we have pointed out that the contribution of three light Majorana neutrinos \(\nu_i\) to \(\Gamma_{0\nu\beta\beta}\) is dominant in most cases. This observation is especially true for the conventional type-I seesaw mechanism with superhigh \(M_k\) (e.g., \(\max(M_k) \sim 10^{15} \text{ GeV}\)) and extremely small \(R_{\alpha k}\) (e.g., \(|R_{\alpha k}| \sim 10^{-13}\) [15]). When the seesaw mechanism is realized at the electroweak or TeV scale to generate experimentally accessible signatures of heavy Majorana neutrinos \(N_k\) at the LHC, however, one usually has to require \(\max(M_k) \lesssim \mathcal{O}(1) \text{ TeV}\) and \(|R_{\alpha k}| \gtrsim 10^{-3}\) up to \(\mathcal{O}(0.1)\) [7], which imply a terrible cancellation in every term of Eq. (8) so as to give rise to tiny masses of \(\nu_i\). Because such a terrible cancellation in \(\langle m \rangle_{ee}\) does not necessarily mean the same cancellation in \(\langle m \rangle'_{ee}\), it is possible to get \(\langle m \rangle'_{ee} \gg \langle m \rangle_{ee}\) as a special case, as already discussed in Eqs. (12) and (13). But the situation might become quite subtle if the masses of heavy Majorana neutrinos are degenerate [16]. Since the function \(\mathcal{F}(A, M_k)\) depends both on the atomic number \(A\) and the heavy Majorana neutrino masses \(M_k\), \(\langle m \rangle'_{ee}\) might be exceedingly small for one decaying nucleus in the \(\langle m \rangle_{ee} \rightarrow 0\) limit but
not for another in the same limit. A careful analysis of the relative magnitudes of \( \langle m \rangle_{ee} \) and \( \langle m \rangle'_{ee} \) for different 0\( \nu \beta \beta \) decays is nevertheless beyond the scope of the present paper and will be done elsewhere.

For \( n = 3 \), \( R \) can be parametrized in terms of nine rotation angles \( \theta_{ij} \) and nine phase angles \( \delta_{ij} \) (for \( i = 1, 2, 3 \) and \( j = 4, 5, 6 \)) [13]. In this representation,

\[
\langle m \rangle_{ee} = \left| M_{1}^{2} c_{14}^{2} c_{16}^{2} e^{-2i\delta_{14}} + M_{2} s_{15}^{2} c_{16}^{2} e^{-2i\delta_{15}} + M_{3} s_{16}^{2} e^{-2i\delta_{16}} \right| \\
\approx \left| M_{1}^{2} + M_{2} s_{15}^{2} e^{2i(\delta_{14}-\delta_{15})} + M_{3} s_{16}^{2} e^{2i(\delta_{14}-\delta_{16})} \right| ,
\]

(14)

where \( s_{ij} \equiv \sin \theta_{ij} \) and \( c_{ij} \equiv \cos \theta_{ij} \). The approximation made in Eq. (14) is very reasonable because \( |RR| \) is at most of \( \mathcal{O}(10^{-2}) \) [17] and thus all the mixing angles of \( R \) must be very small (at most at the \( \mathcal{O}(0.1) \) level). Given \( \langle m \rangle_{ee} \lesssim 1 \) eV and \( M_{1} \approx M_{2} \approx M_{3} \approx v \) or \( \mathcal{O}(1) \) TeV, for instance, the constraint in Eq. (14) implies that two phase differences \( \delta_{14}-\delta_{15} \) and \( \delta_{14}-\delta_{16} \) should be very close to \( \pm \pi/2 \) in order to assure significant cancellations among three terms. Fixing \( M_{i} \sim 10^{2} \) GeV (for \( i = 1, 2, 3 \)) as an example, we find that the level of fine-tuning is at least of \( \mathcal{O}(10^{-9}) \) with \( s_{12}^{2} \sim 10^{-2} \) or \( \mathcal{O}(10^{-7}) \) with \( s_{12}^{2} \sim 10^{-4} \) (for \( j = 4, 5, 6 \)). Such unnatural cancellations seem to be unavoidable in the type-I seesaw models at the electroweak or TeV scale, unless the relevant mixing angles are extremely small. Current experimental data can only provide us with a rough bound \( s_{14}^{2} + s_{15}^{2} + s_{16}^{2} \lesssim 1.1 \times 10^{-2} \) [17], unfortunately. In a specific type-I seesaw model with the fine-tuning conditions \( (M_{D})_{3i} \propto (M_{D})_{2i} \propto (M_{D})_{1i} \) (for \( i = 1, 2, 3 \)) and \( (M_{D})_{2i}^{2}/M_{1} + (M_{D})_{12}/M_{2} + (M_{D})_{13}/M_{3} = 0 \) [18], one may obtain \( M_{\nu} = -M_{D} M_{R} M_{R}^{\dagger} = 0 \), which in turn leads to \( \langle m \rangle_{ee} = 0 \). Then non-zero but tiny \( M_{\nu} \) and \( \langle m \rangle_{ee} \) can be achieved by introducing a small perturbation to the texture of \( M_{D} \). If the magnitudes of \( M_{k} \) are too big or those of \( R_{\alpha k} \) are too tiny, of course, there will be no hope to produce and detect heavy Majorana neutrinos and test the seesaw mechanism at the LHC [19].

**III. THE MINIMAL SEESEAW SCENARIO**

The exact seesaw relation in Eq. (7) allows us to determine \( M_{k} \) in terms of \( m_{i} \) and the mixing parameters of \( V \) and \( R \). To illustrate this point, let us focus on the minimal type-I seesaw scenario which contains only two heavy Majorana neutrinos [12]. In this simpler case, it is easy to obtain two real and linear equations of \( M_{1} \) and \( M_{2} \) from Eq. (8) \(^2\):

\[
\sum_{k=1}^{2} \text{Re} R_{ek}^{2} M_{k} = -\sum_{i=1}^{3} \text{Re} V_{ei}^{2} m_{i} , \\
\sum_{k=1}^{2} \text{Im} R_{ek}^{2} M_{k} = -\sum_{i=1}^{3} \text{Im} V_{ei}^{2} m_{i} .
\]

(15)

\(^2\)Similar equations of \( M_{1} \) and \( M_{2} \) can also be obtained from the exact seesaw relation in Eq. (7). A detailed analytical calculation and numerical analysis of the dependence and consequences of such equations will be presented elsewhere [20].
Note that either \( m_1 = 0 \) or \( m_3 = 0 \) must hold in the minimal type-I seesaw model [12], and thus the non-vanishing neutrino masses can be determined from current experimental data on two independent neutrino mass-squared differences \( \Delta m^2_{21} \equiv m_2^2 - m_1^2 \) and \( \Delta m^2_{32} \equiv m_3^2 - m_2^2 \) corresponding to solar and atmospheric neutrino oscillations. After a simple calculation, we arrive at

\[
M_1 = \frac{\text{Re} R^2_{e1} \sum_{i=1}^{3} \text{Im} V_{ei}^2 m_i - \text{Im} R^2_{e1} \sum_{i=1}^{3} \text{Re} V_{ei}^2 m_i}{\text{Re} R_{e1} \text{Im} R_{e2}^2 - \text{Im} R_{e1} \text{Re} R_{e2}^2},
\]

\[
M_2 = -\frac{\text{Re} R^2_{e1} \sum_{i=1}^{3} \text{Im} V_{ei}^2 m_i - \text{Im} R^2_{e1} \sum_{i=1}^{3} \text{Re} V_{ei}^2 m_i}{\text{Re} R_{e1} \text{Im} R_{e2}^2 - \text{Im} R_{e1} \text{Re} R_{e2}^2}.
\]

(16)

Using the exact and convenient parametrization of \( V \) and \( R \) advocated in Ref. [13], we have

\[
V_{e1} = c_{12} c_{13} c_{14} c_{15}, \quad V_{e2} = s_{12} c_{13} c_{14} c_{15} e^{-i\delta_{12}}, \quad V_{e3} = s_{13} c_{14} c_{15} e^{-i\delta_{13}},
\]

\[
R_{e1} = s_{14} c_{15} e^{-i\delta_{14}}, \quad R_{e2} = s_{15} e^{-i\delta_{15}}.
\]

(17)

for the minimal type-I seesaw scenario, where \( c_{1i} \equiv \cos \theta_{1i} \) and \( s_{1i} \equiv \sin \theta_{1i} \) (for \( i = 2, \ldots, 5 \)). There are at least two phenomenological merits of this parametrization for our present discussions: (1) it can automatically reproduce the standard (unitary) parametrization of the light neutrino mixing matrix [21] when the non-unitary mixing angles of \( R \) are switched off; and (2) it can lead to a very simple result of \( \langle m \rangle_{ee} \), which is equal to the standard (unitary) expression of \( \langle m \rangle_{ee} \) multiplied by a factor \( r_{14}^2 c_{15}^2 \) [13]. Substituting Eq. (17) into Eq. (16), we obtain the explicit results of \( M_1 \) and \( M_2 \) for two different patterns of the light neutrino mass spectrum.

A. Normal hierarchy \((m_1 = 0)\)

In this case, it is straightforward to obtain \( m_2 = \sqrt{\Delta m^2_{21}} \approx 8.8 \times 10^{-3} \text{ eV} \) and \( m_3 = \sqrt{\Delta m^2_{21} + |\Delta m^2_{32}|} \approx 5.0 \times 10^{-2} \text{ eV} \), where we have typically input \( \Delta m^2_{21} = 7.7 \times 10^{-5} \text{ eV}^2 \) and \( |\Delta m^2_{32}| = 2.4 \times 10^{-3} \text{ eV}^2 \) [11]. The expressions of \( M_1 \), \( M_2 \) and \( \langle m \rangle_{ee} \) are given by

\[
M_1 = -\frac{m_2 s_{12}^2 c_{13}^2 \sin (\phi_2 + \phi) + m_3 s_{13}^2 \sin (\phi_2 - \phi)}{\sin (\phi_2 - \phi_1)} \cdot c_{14}^2, \quad M_2 = +\frac{m_2 s_{12}^2 c_{13}^2 \sin (\phi_1 + \phi) + m_3 s_{13}^2 \sin (\phi_1 - \phi)}{\sin (\phi_2 - \phi_1)} \cdot c_{14}^2 c_{15}^2, \quad (18)
\]

together with

\[
\langle m \rangle_{ee} = c_{14}^2 c_{15}^2 \sqrt{m_2^2 s_{12}^4 c_{13}^4 + m_3^2 s_{13}^4 + 2m_2 m_3 s_{12}^2 c_{13}^2 s_{13}^2 \cos 2\phi},
\]

(19)

where \( \phi \equiv \delta_{13} - \delta_{12}, \phi_1 \equiv 2\delta_{14} - (\delta_{12} + \delta_{13}) \) and \( \phi_2 \equiv 2\delta_{15} - (\delta_{12} + \delta_{13}) \). Note that \( M_1 > 0 \) and \( M_2 > 0 \) require that three CP-violating phases \( \phi, \phi_1 \) and \( \phi_2 \) should not all be vanishing; instead, they must take proper and nontrivial values.
The above results can be simplified in the limit of $s_{13} \to 0$, which actually has no conflict with current experimental data:

$$M_1 = -\frac{\langle m \rangle_{ee}}{s_{14}^2 c_{15}^2} \frac{\sin (\phi_2 + \phi)}{\sin (\phi_2 - \phi_1)}$$

$$M_2 = +\frac{\langle m \rangle_{ee}}{s_{15}^2} \frac{\sin (\phi_1 + \phi)}{\sin (\phi_2 - \phi_1)},$$

\quad (20)

and $\langle m \rangle_{ee} = s_{12}^2 c_{14} c_{15}^2 m_2$. We have the following observations: (a) the magnitude of $\langle m \rangle_{ee}$ is of $\mathcal{O}(10^{-3})$ eV [22], and thus it is experimentally inaccessible in the near future; (b) the signs of $\sin(\phi_2 + \phi)$ and $\sin(\phi_2 - \phi_1)$ must be different, while the signs of $\sin(\phi_1 + \phi)$ and $\sin(\phi_2 - \phi_1)$ must be the same; and (c) it will be possible, at least in principle, to obtain the lower bounds on $M_1$ and $M_2$ if some constraints on the non-unitary mixing angles and CP-violating phases become available at low energies. To achieve $M_1 \sim M_2 \gtrsim 10^2$ GeV, for example, Eq. (20) implies that $s_{14}^2 c_{15}^2 \sin(\phi_2 - \phi_1) \sim s_{15}^2 \sin(\phi_2 - \phi_1) \lesssim 10^{-14}$ must hold. Current experimental bounds can only provide us with $s_{14}^2 + s_{15}^2 \approx 1.1 \times 10^{-2}$ [17]. Hence there is a long way to go before we can constrain $s_{14}, s_{15}$ and even the relevant CP-violating phases to a much better degree of accuracy.

\textbf{B. Inverted hierarchy ($m_3 = 0$)}

In this case, it is easy to obtain $m_1 = \sqrt{|\Delta m_{32}^2| - \Delta m_{21}^2} \approx 4.8 \times 10^{-2}$ eV and $m_2 = \sqrt{|\Delta m_{32}^2|} \approx 4.9 \times 10^{-2}$ eV, where $\Delta m_{21}^2 = 7.7 \times 10^{-5}$ eV$^2$ and $|\Delta m_{32}^2| = 2.4 \times 10^{-3}$ eV$^2$ [11] have typically been input. After a straightforward calculation, the explicit expressions of $M_1$, $M_2$ and $\langle m \rangle_{ee}$ are given as follows:

$$M_1 = -m_1 c_{12}^2 \sin (\phi_2' - \phi') + m_2 s_{12}^2 \sin (\phi_2' + \phi') \frac{c_{13}^2}{s_{14}^2},$$

$$M_2 = +m_1 c_{12}^2 \sin (\phi_1' - \phi') + m_2 s_{12}^2 \sin (\phi_1' + \phi') \frac{c_{13}^2}{s_{14}^2},$$

\quad (21)

and

$$\langle m \rangle_{ee} = c_{13}^2 s_{14}^2 s_{15}^2 \sqrt{m_1^2 c_{12}^4 + m_2^2 s_{12}^4 + 2m_1 m_2 c_{12}^2 s_{12}^2 \cos 2\phi'},$$

\quad (22)

where $\phi' \equiv -\delta_{12}, \phi_1' \equiv 2\delta_{14} - \delta_{12}$ and $\phi_2' \equiv 2\delta_{15} - \delta_{12}$. The requirement of $M_1 > 0$ and $M_2 > 0$ implies that three CP-violating phases $\phi', \phi_1'$ and $\phi_2'$ cannot all be vanishing; instead, they must take proper and nontrivial values.

One can see that $M_1$, $M_2$ and $\langle m \rangle_{ee}$ are independent of the small mixing angle $\theta_{13}$. Given $\theta_{12} \approx 34^\circ$ [11], for example, the maximal value of $\langle m \rangle_{ee}$ in Eq. (22) is achievable at $\phi' \approx 0$; i.e., $\langle m \rangle_{ee}$ can maximally amount to $(m_1 c_{12}^4 + m_2 s_{12}^4) \approx 4.9 \times 10^{-2}$ eV [22], which is experimentally accessible in the near future. Again, we may optimistically argue that it is in principle possible to obtain the lower bounds on $M_1$ and $M_2$ if some constraints on the non-unitary mixing angles and CP-violating phases become available at low energies.
It is worth remarking that the non-unitarity of $V$ is signified by non-vanishing $R$, whose mixing angles and CP-violating phases are quite possible to have some nontrivial observable effects. For example, an appreciable CP-violating asymmetry up to the percent level is expected to show up between $\nu_\mu \rightarrow \nu_\tau$ and $\overline{\nu}_\mu \rightarrow \overline{\nu}_\tau$ oscillations in some medium- or long-baseline experiments at a neutrino factory [23], just as a consequence of non-vanishing $R$. A neutrino telescope could also be a useful tool to probe the non-unitary effect in ultrahigh-energy cosmic neutrino oscillations [24].

IV. SUMMARY

We have carefully examined the contributions of both light Majorana neutrinos $\nu_i$ with masses $m_i$ (for $i = 1, 2, 3$) and heavy Majorana neutrinos $N_k$ with masses $M_k$ (for $k = 1, \cdots, n$) to the $0\nu\beta\beta$ decay in the type-I seesaw mechanism, in which the light neutrino mixing matrix $V$ is non-unitary due to the non-vanishing coupling matrix $R$ between $N_k$ and charged leptons. The exact seesaw relation allows us to establish a straightforward relationship between $(m_i, V_{ai})$ and $(M_k, R_{ak})$. We have pointed out that the constraint $|\sum R_{ek}^2 M_k^{-1}| < 5 \times 10^{-8}$ GeV$^{-1}$ used in some literature is in most cases too loose for a type-I seesaw mechanism either at a superhigh-energy scale or at the electroweak or TeV scale, because the contribution of $\nu_i$ to the $0\nu\beta\beta$ decay is in most cases dominant over the contribution of $N_k$ to the same process. Such an observation leads us to a much stronger bound on $M_k$ and $R_{ek}$; i.e., $|\sum R_{ek}^2 M_k| < 0.23$ eV (or < 0.85 eV) at the $2\sigma$ level, extracted from the present experimental upper bound on the $0\nu\beta\beta$ decay.

We have also looked at whether the future measurements of lepton number violation and non-unitarity of neutrino flavor mixing at low energies are possible to shed light on the masses of heavy Majorana neutrinos. Taking the minimal type-I seesaw scenario for example, we have illustrated the possibility of determining or constraining two heavy Majorana neutrino masses by using more accurate low-energy data on the $0\nu\beta\beta$ decay and non-unitary neutrino mixing and CP violation. Such an analysis can simply be extended to the more general cases of the type-I seesaw mechanism with three or more heavy Majorana neutrinos.

As stressed in Ref. [13], testing the unitarity of the light neutrino mixing matrix $V$ in neutrino oscillations and searching for the signatures of heavy Majorana neutrinos $N_k$ at TeV-scale colliders can be complementary to each other, both qualitatively and quantitatively, in order to deeply understand the intrinsic properties of Majorana particles. We optimistically expect that some experimental breakthrough in this aspect will pave the way towards the true theory of neutrino mass generation and flavor mixing.

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REFERENCES

[1] SNO Collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89, 011301 (2002).
[2] For a review, see: C.K. Jung et al., Ann. Rev. Nucl. Part. Sci. 51, 451 (2001).
[3] KamLAND Collaboration, K. Eguchi et al., Phys. Rev. Lett. 90, 021802 (2003).
[4] K2K Collaboration, M.H. Ahn et al., Phys. Rev. Lett. 90, 041801 (2003).
[5] P. Minkowski, Phys. Lett. B 67, 421 (1977); T. Yanagida, in Proceedings of the Workshop on Unified Theory and the Baryon Number of the Universe, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979), p. 315; S.L. Glashow, in Quarks and Leptons, edited by M. Lévy et al. (Plenum, New York, 1980), p. 707; R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).
[6] See, e.g., A. de Gouvêa, Phys. Rev. D 72, 033005 (2005); arXiv:0706.1732; A. de Gouvêa, J. Jenkins, and N. Vasudevan, Phys. Rev. D 75, 013003 (2007).
[7] For a brief review with many recent references, see: Z.Z. Xing, Int. J. Mod. Phys. A 23, 4255 (2008); Plenary talk given at ICHEP2008, August 2008, Philadelphia, USA; arXiv:0905.3903; Z.Z. Xing and S. Zhou, arXiv:0906.1757.
[8] For the latest review with extensive references, see: F.T. Avignone III, S.R. Elliot, and J. Engel, Rev. Mod. Phys. 80, 481 (2008).
[9] For two latest comprehensive works with extensive references, see: F. del Aguila and J.A. Aguilar-Saavedra, Nucl. Phys. B 813, 22 (2009); A. Atre, T. Han, S. Pascoli, and B. Zhang, JHEP 0905, 030 (2009).
[10] CUORICINO Collaboration, C. Arnaboldi et al., Phys. Rev. C 78, 035502 (2008).
[11] G.L. Fogli et al., Phys. Rev. D 78, 033010 (2008).
[12] P.H. Frampton, S.L. Glashow, and T. Yanagida, Phys. Lett. B 548, 119 (2002). For a review with extensive references, see: W.L. Guo, Z.Z. Xing, and S. Zhou, Int. J. Mod. Phys. E 16, 1 (2007).
[13] Z.Z. Xing, Phys. Lett. B 660, 515 (2008).
[14] W.C. Haxton and J. Stephenson, Prog. Part. Nucl. Phys. 12, 409 (1984).
[15] For some recent reviews with extensive references, see: H. Fritzsch and Z.Z. Xing, Prog. Part. Nucl. Phys. 45, 1 (2000); Altarelli and F. Feruglio, New J. Phys. 6, 106 (2004); Z.Z. Xing, Int. J. Mod. Phys. A 19, 1 (2004); S.F. King, Rept. Prog. Phys. 67, 107 (2004); R.N. Mohapatra and A.Yu. Smirnov, Ann. Rev. Nucl. Part. Sci. 56, 569 (2006); A. Strumia and F. Vissani, hep-ph/0606054.
[16] G. Bélanger, F. Boudjema, D. London, and H. Nadeau, Phys. Rev. D 53, 6292 (1996).
[17] S. Antusch, C. Biggio, E. Fernandez-Martinez, M.B. Gavela, and J. López-Pavón, JHEP 0610, 084 (2006); A. Abada, C. Biggio, F. Bonnet, M.B. Gavela, and T. Hambye, JHEP 0712, 061 (2007).
[18] See, e.g., J. Bernabeu et al., Phys. Lett. B 187, 303 (1987); W. Buchmuller and D. Wyler, Phys. Lett. B 249, 458 (1990); W. Buchmuller and C. Greub, Nucl. Phys. B 363, 345 (1991); A. Pilaftsis, Z. Phys. C 55, 275 (1992); A. Datta and A. Pilaftsis, Phys. Lett. B 278, 162 (1992); G. Ingelman and J. Rathsman, Z. Phys. C 60, 243 (1993); C. A. Heusch and P. Minkowski, Nucl. Phys. B 416, 3 (1994); D. Tommasini, G. Barenboim, J. Bernabeu, and C. Jarlskog, Nucl. Phys. B 444, 451 (1995); J. Gluza,
Acta Phys. Polon. B 33, 1735 (2002); J. Kersten and A. Yu. Smirnov, Phys. Rev. D 76, 073005 (2007); W. Chao, S. Luo, Z.Z. Xing, and S. Zhou, Phys. Rev. D 77, 016001 (2008).

[19] Z.Z. Xing, arXiv:0905.3903; and references therein.

[20] X.G. Wu and Z.Z. Xing, work in progress.

[21] Particle Data Group, C. Amsler et al., Phys. Lett. B 667, 1 (2008).

[22] S.M. Bilenky et al., Phys. Lett. B 465, 193 (1999); Z.Z. Xing, Phys. Rev. D 65, 077302 (2002); Phys. Rev. D 68, 053002 (2003); Phys. Lett. B 618, 141 (2005); S. Pascoli, S.T. Petcov, and W. Rodejohann, Phys. Lett. B 558, 141 (2003).

[23] E. Fernandez-Martinez, M.B. Gavela, J. López-Pavón, and O. Yasuda, Phys. Lett. B 649, 427 (2007); Z.Z. Xing, Phys. Lett. B 660, 515 (2008); S. Luo, Phys. Rev. D 78, 016006 (2008); S. Goswami and T. Ota, Phys. Rev. D 78, 033012 (2008); G. Altarelli and D. Meloni, Nucl. Phys. B 809, 158 (2009); Z.Z. Xing, arXiv:0901.0209; M. Malinsky, T. Ohlsson, and H. Zhang, arXiv:0903.1961; S. Antusch, M. Blennow, E. Fernandez-Martinez, and J. López-Pavón, arxiv:0903.3986; W. Rodejohann, arXiv:0903.4590; M. Malinsky, T. Ohlsson, Z.Z. Xing, and H. Zhang, arXiv:0905.2889.

[24] Z.Z. Xing and S. Zhou, Phys. Lett. B 666, 166 (2008); Z.Z. Xing, Nucl. Instrum. Meth. A 602, 58 (2009).
FIG. 1. A schematic Feynman diagram for the lepton-number-violating $0
\nu\beta\beta$ decay, in which “×” stands for either light Majorana neutrinos $\nu_i$ (for $i = 1, 2, 3$) or heavy Majorana neutrinos $N_k$ (for $k = 1, \cdots, n$) in the type-I seesaw mechanism.