Quantization of the Superstring in Ramond-Ramond Backgrounds

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Talk presented at Strings ‘99 (Potsdam, Germany)

Abstract.
Sigma model actions are constructed for the Type II superstring compactified to four and six dimensional curved backgrounds which can contain non-vanishing Ramond-Ramond fields. These actions are N=2 worldsheet superconformally invariant and can be covariantly quantized preserving manifest spacetime supersymmetry. They are constructed using a hybrid version of superstring variables which combines features of the Ramond-Neveu-Schwarz and Green-Schwarz formalisms. For the $AdS_2 \times S^2$ and $AdS_3 \times S^3$ backgrounds, these actions differ from the classical Green-Schwarz actions by a crucial kinetic term for the fermions. Parts of this work have been done in collaborations with M. Bershadsky, T. Hauer, W. Siegel, C. Vafa, E. Witten, S. Zhukov and B. Zwiebach.
Quantization of the superstring in Ramond-Ramond backgrounds is currently of great interest because of the AdS-CFT conjectures. Although Ramond vertex operators are well understood using the results of Friedan, Martinec and Shenker \[1\] for the Ramond-Neveu-Schwarz (RNS) formalism, these vertex operators differ from Neveu-Schwarz vertex operators in breaking worldsheet supersymmetry and mixing matter and ghost variables. Without worldsheet supersymmetry as a guiding principle, it is difficult to guess the correct generalization of these vertex operators for constructing the superstring action in Ramond-Ramond backgrounds.

It has been proposed in \[2\] that spacetime supersymmetry may replace worldsheet supersymmetry as a guiding principle for constructing the action, e.g. in determining the ‘contact terms’ which are required to eliminate unphysical divergences \[3\]. However, spacetime supersymmetry is hard to verify in the RNS formalism since the worldsheet action is only expected to be spacetime-supersymmetric when the background fields are on-shell. This can be seen from the fact that spacetime supersymmetry transformations change the ‘picture’ of a vertex operator, but picture-changing can only be defined when the vertex operator is physical.

An alternative approach is to use the Green-Schwarz (GS) formalism for the superstring where spacetime supersymmetry is manifest. Although it is easy to construct a classical GS action for the superstring in a curved superspace background including Ramond-Ramond fields, the existence of fermionic second-class constraints makes it difficult to quantize this action unless the background admits a light-cone gauge choice. Furthermore, the lack of a Fradkin-Tseytlin term coupling the dilaton to worldsheet curvature makes it doubtful that the classical GS action is correct in arbitrary backgrounds.

The fermionic second-class constraints of the GS formalism are

\[
d_{\alpha} = p_{\alpha} - i\sigma_{\alpha\beta}^{m} \theta^{\alpha} \partial x_{m} + \ldots = 0
\]

where \((x_{m}, \theta^{\alpha})\) are the usual superspace variables, \(p_{\alpha}\) is the conjugate momentum for \(\theta^{\alpha}\), \(\sigma_{\alpha\beta}^{m}\) are the Pauli matrices, and \(\ldots\) refers to terms which are higher order in \(\theta^{\alpha}\). Siegel has suggested in \[4\] a modification of the GS formalism in which the above second class constraints are replaced by a suitable set of first-class constraints constructed from \(d_{\alpha}\). In this modified GS formalism, \(p_{\alpha}\) is treated as an independent worldsheet variable. Although he was unable to find an appropriate set of first-class constraints, he argued on general grounds that the massless open superstring vertex operator should have the form

\[
V = \int dz [\partial Y^{M} A_{M}(x, \theta) + d_{\alpha} W^{\alpha}(x, \theta)]
\]

where \(Y^{M} = (x^{m}, \theta^{\alpha})\), \(A_{M}\) are the superspace gauge fields, and \(W^{\alpha}\) are the superspace field-strengths.

Over the last five years, a third approach has been developed which combines advantages of the RNS and GS approaches and has therefore been named the ‘hybrid’
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...approach. Using a field redefinition which preserves free-field OPE’s, one can map the worldsheet matter and ghost RNS variables into a set of superspace variables (including conjugate $p_\alpha$ variables) plus an internal sector. In fact, part of this map was already suggested in [1] where it was noticed that since the spacetime-supersymmetry generator in the $-\frac{1}{2}$ picture is given by $q_\alpha = \int dz e^{-\frac{i}{2}\phi} \Sigma_\alpha$, it is natural to define

$$\theta^\alpha = e^{\frac{i}{2}\phi} \Sigma^\alpha, \quad p_\alpha = e^{-\frac{i}{2}\phi} \Sigma_\alpha,$$

where $\phi$ comes from fermionizing the bosonic ghosts as $\beta = \partial \xi e^{-\phi}$, $\gamma = \eta e^\phi$ and $\Sigma^\alpha$ is the spin field of conformal weight $5/8$ constructed from the $\psi^m$ variables.

Unfortunately, these $\theta^\alpha$ variables are not all free fields if $\alpha = 1$ to $16$ as can be seen from the OPE

$$\theta^\alpha(y) \theta^\beta(z) \rightarrow (y - z)^{-1} \sigma^\alpha_m e^\phi \psi^m.$$

However, it is possible to choose a subset of these variables which are free fields. The subset that is most convenient depends on which subgroup of $SO(9,1)$ Lorentz invariance one wants to remain manifest. For example, using $U(5)$ notation, the subset $[\theta^{++++}, \theta^{−−++}]$ are free fields which leave manifest the subgroup $SO(3,1) \times U(3)$. This choice is most convenient for describing compactifications of the superstring to four dimensions on a Calabi-Yau three-fold [3] [4] [5] [6]. Another choice is the subset $[\theta^{++++}, \theta^{−−++}, \theta^{++−−}, \theta^{−−++}]$ which leaves manifest the subgroup $SO(5,1) \times U(2)$. This choice is most convenient for describing compactifications of the superstring to six dimensions on a Calabi-Yau two-fold [7] [10] [11]. A third choice is the subset $[\theta^{−−−−}, \theta^{−−−−}, \theta^{−−−−}, \theta^{−−−−}, \theta^{−−−−}]$ which leaves manifest a $U(5)$ subgroup of the Wick-rotated $SO(10)$ [12].

In section 2, the hybrid formalism for compactification to four dimensions is used to construct the superstring action in an $AdS_5 \times S^5$ background with Ramond-Ramond flux. In section 3, the hybrid formalism for compactification to six dimensions is used to construct the superstring action in an $AdS_5 \times S^3$ background with Ramond-Ramond flux. Work is in progress on using the $U(5)$ version of the hybrid formalism to construct the superstring action in an $AdS_5 \times S^5$ background with Ramond-Ramond flux.

### 2. Hybrid formalism for compactification to four dimensions

After defining $\theta^\alpha = [e^{\frac{i}{2}\phi} \Sigma^{++++}, e^{\frac{i}{2}\phi} \Sigma^{−−++}]$ and $p_\alpha = [e^{-\frac{i}{2}\phi} \Sigma^{−−−−}, e^{-\frac{i}{2}\phi} \Sigma^{−−−−}]$ to be fundamental variables for $\alpha = 1$ to $2$, one still has to define the remaining set of hybrid variables. It is convenient to define

$$\hat{\theta}^\alpha = [c \xi e^{-\frac{i}{2}\phi} \Sigma^{−−−−}, c \xi e^{-\frac{i}{2}\phi} \Sigma^{−−−−}],$$

$$\hat{p}_\alpha = [b \eta e^{\frac{i}{2}\phi} \Sigma^{++++}, b \eta e^{\frac{i}{2}\phi} \Sigma^{++++}]$$

to be the complex conjugates of $\theta^\alpha$ and $p_\alpha$. As discussed in [13], this definition of complex conjugation flips the picture so $\hat{q}^{\tilde{\alpha}}$ is naturally defined in the $+\frac{1}{2}$ picture as

$$\hat{q}^{\tilde{\alpha}} = \int dz [b \eta e^{\frac{i}{2}\phi} \Sigma^{\tilde{\alpha}} + e^{\frac{i}{2}\phi} \partial x^m \sigma^m \Sigma^\alpha].$$
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With this choice of picture for the four-dimensional supersymmetry generators, \( \{ q^\alpha, \hat{q}^{\dot{\alpha}} \} = \int dz \partial x^m \sigma^\alpha_m \) as desired for the supersymmetry algebra.

For compactifications of the superstring which preserve four-dimensional supersymmetry, the six-dimensional compactification manifold can be represented by a \( c = 9 \) \( N=2 \) superconformal field theory whose \( N=2 \) generators will be denoted \([T_C, G^+_C, G^-_C, J_C]\). Up to some minor modifications which will be described below, the compactification manifold in the hybrid formalism is represented by the same \( c = 9 \) \( N=2 \) superconformal field theory. For the compactification-dependent worldsheet fields to have no singular OPE’s with \( \theta^\alpha \) and \( \hat{\theta}^{\dot{\alpha}} \), they need to be twisted by a factor \( e^{N\kappa} \) where \( \partial \kappa = \partial \phi + \eta \xi \) and \( N \) is the charge of the field with respect to \( J_C \). For example, for compactification on a six-torus, the worldsheet fields \( \Gamma_j = \psi^j + 3 + i \psi^{j+6} \) for \( j = 1 \) to 3 are redefined to \( \Gamma_j \rightarrow e^{\kappa} \Gamma_j \) which have no singular OPE’s with \( \theta^\alpha \) and \( \hat{\theta}^{\dot{\alpha}} \). One also needs to redefine the \( c = 9 \) \( N=2 \) generators to be \([T_C + 3/2(\partial \kappa)^2 - \partial \kappa J_C, e^{\kappa}G^+_C, e^{-\kappa}G^-_C, J_C + 3\partial \kappa]\) which still generates a \( c = 9 \) \( N=2 \) algebra.

There is one chiral boson \( \rho \) defined by \( \partial \rho = -3 \partial \phi + cb + 2 \xi \eta - J_C \)

which has no singular OPE’s with the other hybrid variables. This is the last free hybrid variable since the compactification-independent hybrid variables \([x^m, \theta^\alpha, \hat{\theta}^{\dot{\alpha}}, p_\alpha, \hat{p}^{\dot{\alpha}}, \rho]\) contain the same degrees of freedom as the compactification-independent RNS variables \([x^m, \psi^m, b, c, \xi, \eta, \phi]\) where \( m = 0 \) to 3. So any vertex operator constructed out of RNS variables can be rewritten in terms of hybrid variables and vice versa.

One now needs to define the physical state conditions in terms of the hybrid variables. In terms of the RNS variables, the physical state conditions for a vertex operator \( V \) are \( QV = q_{\text{ghost}} V = 0 \) where \( Q_{\text{BRST}} = \int dz j_{\text{BRST}} \) is the BRST charge and \( q_{\text{ghost}} = \int dz j_{\text{ghost}} \) is the ghost current. At least for Neveu-Schwarz integrated vertex operators in the zero picture, these conditions can be strengthened to require that \( V \) has no poles with \( j_{\text{BRST}} \), with \( j_{\text{ghost}} \), with the \( b \) ghost, and with the stress tensor \( T \). After adding a total derivative to \( j_{\text{BRST}} \) and defining \( j_{\text{ghost}} = bc + \xi \eta \) (which is equivalent to the standard definition \( j_{\text{ghost}} = bc + \partial \phi \) in the zero picture), one can show that \([T, j_{\text{BRST}}, b, j_{\text{ghost}}]\) form a twisted \( N=2 \) algebra. So the above definition of physical state conditions implies that \( V \) is a \( U(1) \)-neutral primary field with respect to these \( N=2 \) generators. Note that \( V \) has no poles with \( \eta \) since \( \{ b, e^{\int j_{\text{ghost}}} \} = \{ b, c\eta \} = \eta \). So it is independent of the \( \xi \) zero mode as desired for RNS physical vertex operators.

After performing a similarity transformation on the hybrid variables of the form \( Y \rightarrow e^R Y e^{-R} \) with \( R = -\int dz [\theta^\alpha \hat{\theta}^{\dot{\alpha}} \sigma^m_{\alpha \dot{\alpha}} \partial x_m + e^{-\rho} \theta^\alpha \hat{\theta}^{\dot{\alpha}} G^-_C] \), one finds that these twisted
N=2 generators are mapped to
\[ T = \frac{1}{2} \partial x^m \partial x_m + p_\alpha \partial \theta^\alpha + \hat{p}_\alpha \hat{\theta}^\alpha + \frac{1}{2} \partial \rho \partial \rho + \frac{1}{2} \partial^2 \rho + T_C, \]
\[ G^+ = j_{\text{BRST}} = d^\alpha d_\alpha e^\rho + G_C^+, \]
\[ G^- = b = d_\dot{\alpha} \dot{d}_\dot{\alpha} e^{-\rho} + G_C^-, \]
\[ J = j_{\text{ghost}} = \partial \rho + J_C, \]
where \( d_\alpha = p_\alpha + \frac{i}{2} \sigma_{\alpha\dot{\alpha}} \hat{\theta}^\alpha \partial x_m - \frac{1}{2} (\hat{\theta})^2 \partial \theta_\alpha + \frac{1}{4} \theta_\alpha \partial (\hat{\theta})^2 \) and \( \dot{d}_\dot{\alpha} = \hat{p}_\dot{\alpha} - \frac{i}{2} \sigma_{\dot{\alpha}\dot{\alpha}} \theta^\alpha \partial x_m - \frac{1}{2} (\theta)^2 \partial \hat{\theta}_{\dot{\alpha}} + \frac{1}{4} \hat{\theta}_{\dot{\alpha}} \partial (\theta)^2 \) are supersymmetric combinations of the four-dimensional superspace variables.

Note that the N=2 generators factorize into a set of four-dimensional N=2 generators and compactification-dependent N=2 generators. These N=2 generators provide first-class constraints for \( d_\alpha \) and \( \dot{d}_{\dot{\alpha}} \) that can replace the second-class constraints of the four-dimensional GS superstring.

For the open superstring, the massless compactification-independent states are described by the gluon and gluino of four-dimensional super-Yang-Mills. As shown in [5], the integrated vertex operator for these states is given by
\[ V = \int dz [\partial Y^M A_M + d_\alpha W^\alpha + \dot{d}_{\dot{\alpha}} \dot{W}^{\dot{\alpha}}] \]

exactly as proposed in [4] where \( Y^M = (x^m, \theta^\alpha, \hat{\theta}^\alpha) \), \( W^\alpha \) and \( \dot{W}^{\dot{\alpha}} \) are the chiral and anti-chiral superspace field-strengths, and \( A_M \) are the superspace gauge fields. One can check that in Wess-Zumino gauge, this vertex operator is mapped to the standard Neveu-Schwarz and Ramond vertex operators for the gluon and gluino where the Weyl gluino vertex operator is in the \(-\frac{1}{2}\) picture, the gluon vertex operator is in the 0 picture, and the anti-Weyl gluino vertex operator is in the \(+\frac{1}{2}\) picture.

Since the closed superstring vertex operator is the holomorphic square of the open superstring vertex operator, one obtains the following integrated vertex operator for the massless compactification-independent states of the Type II superstring [7]:
\[ S = \int d^2 \bar{z} [\partial Y^M \bar{\partial} Y^N (G_{MN} + B_{MN}) + d_a \bar{\partial} Y^N E_a^N + \partial Y^M \bar{d}_a E_a^\bar{a} + \bar{d}_\bar{a} \bar{d}_b F^{ab}] \]

where \( Y^M = (x^m, \theta^\alpha, \hat{\theta}^\alpha, \bar{\theta}^\dot{\alpha}, \bar{\hat{\theta}}^\dot{\alpha}) \), \( \bar{\theta}^\dot{\alpha} \) and \( \bar{\hat{\theta}}^\dot{\alpha} \) are the right-moving analogs of the left-moving \( \theta^\alpha \) and \( \hat{\theta}^\alpha \), \( a = (\alpha, \dot{\alpha}) \) and \( \bar{a} = (\bar{\alpha}, \bar{\dot{\alpha}}) \), and the lowest components of the superfields \( G_{mn}, B_{mn}, E_m^a, E_m^{\bar{a}}, F^{ab} \) are the graviton, the anti-symmetric tensor, the two gravitini, and the Ramond-Ramond bispinor field-strength.

To get the superstring action in a curved background, one simply interprets the superfields \([G_{MN}, B_{MN}, E_m^a, E_m^{\bar{a}}, F^{ab}]\) appearing in \( S \) as background superfields. This action is manifestly super-reparameterization invariant and is expected to be invariant under N=2 worldsheet superconformal transformations when the background fields satisfy the appropriate equations of motion. Note that the first term in \( S \) is the usual GS action in a curved background, but the other terms in \( S \) proportional to \( d_a \) and \( \bar{d}_{\bar{a}} \) are required for quantization since they provide kinetic terms for the fermions.
Of course, the complete action also contains a contribution from the compactification-dependent fields but, at string tree-level, one can consistently choose the four-dimensional background superfields to be compactification-independent. So the compactification-dependent contribution can be chosen to be the same as in the flat four-dimensional case. One also needs an action for the chiral boson $\rho$, but this term similarly decouples from the four-dimensional background. Finally, one needs to add a Fradkin-Tseytlin term to the action to couple the dilaton to worldsheet curvature. As discussed in [7], this term can be constructed using chiral and twisted-chiral spacetime and worldsheet superfields. Although it has not yet been checked for the Type II superstring that $N=2$ worldsheet superconformal invariance at one-loop sigma model implies the expected superspace equations of motion for the background superfields, this has been checked for an analogous sigma model action for the heterotic superstring [14].

To obtain the action for the superstring in an $AdS_2 \times S^2$ background with constant Ramond-Ramond flux $F^{ab} = \delta^{ab}$, one simply plugs the appropriate values for the background fields into the action. Because $d_a$ and $\bar{d}_a$ are auxiliary fields in the presence of constant Ramond-Ramond flux, they can be integrated out to produce the action

$$S_{AdS} = \int d^2z [\eta_{cd} J^c_z J^d_z + \frac{1}{2} \delta_{ab} (J^a_z J^b_{\bar{z}} - J^a_{\bar{z}} J^b_z) + \delta_{ab} (J^a_z J^b_{z} + J^a_{\bar{z}} J^b_{\bar{z}})]$$

where $(J^a, J^\bar{a})$ and $J^c$ are the eight fermionic currents and four bosonic currents $(g^{-1}dg)^A$ constructed from a coset supergroup $g$ taking values in $PSU(1,1|2)/U(1) \times U(1)$ [8]. The first line of this action comes from the $\partial Y^M \partial Y^N (G_{MN} + B_{MN})$ term and is therefore identical to the GS action on $AdS_2 \times S^2$ [13] [14]. However, the second line of this action is crucial for quantization and is absent from the GS action. Using standard techniques, it was confirmed to one-loop sigma model in [8] that the above action is conformally invariant as expected.

3. Hybrid formalism for compactification to six dimensions

For compactifications to six dimensions, it is convenient to choose

$$\theta^a = [e^{\frac{1}{2} \phi} \Sigma^{+++++}, e^{\frac{1}{2} \phi} \Sigma^{++++-}, e^{\frac{1}{2} \phi} \Sigma^{++-++}, e^{\frac{1}{2} \phi} \Sigma^{++-+-}],$$

$$p_\alpha = [e^{-\frac{1}{2} \phi} \Sigma^{--------}, e^{-\frac{1}{2} \phi} \Sigma^{+++---}, e^{-\frac{1}{2} \phi} \Sigma^{++-+-}, e^{-\frac{1}{2} \phi} \Sigma^{++---}]$$

to be fundamental variables where $\alpha = 1$ to 4. For compactifications of the superstring which preserve six-dimensional supersymmetry, the four-dimensional compactification manifold can be represented by a $c = 6$ $N=2$ superconformal field theory whose $N=2$ generators will be denoted $[T_C, G_C^+, G_C^-, J_C]$. As before, for the compactification-dependent worldsheet fields to have no singular OPE’s with $\theta^a$ and $\bar{\theta}^\alpha$, they need to be
twisted by a factor $e^{N\kappa}$ where $\partial \kappa = \partial \phi + \eta \xi$ and $N$ is the charge of the field with respect to $J_C$. Furthermore, the $c = 6$ N=2 generators need to be redefined as

$$[T_C, G^+_C, G^-_C, J_C] \rightarrow [T_C + \partial^2 \kappa - \partial \kappa J_C, e^\kappa G^+_C, e^{-\kappa} G^-_C, J_C + 2\partial \kappa]$$

which still generates a $c = 6$ N=2 algebra.

There are two chiral bosons, $\rho$ and $\sigma$, defined by

$$\partial \rho = -2\partial \phi - \xi \eta - J_C, \quad \partial \sigma = ibc$$

which have no singular OPE’s with the other fields. Since $[x^m, \theta^\alpha, p_\alpha, \rho, \sigma]$ contain the same degrees of freedom as the compactification-independent RNS variables $[x^m, \psi^m, b, c, \beta, \gamma]$ where $m = 0$ to 5, any vertex operator constructed out of RNS variables can be rewritten in terms of hybrid variables and vice versa. After performing a similarity transformation on the hybrid variables, the twisted N=2 generators described in the previous section are mapped to

$$T = \frac{1}{2} \partial x^m \partial x_m + p_\alpha \partial \theta^\alpha + \frac{1}{2} \partial \rho \partial \rho + \frac{1}{2} \partial \sigma \partial \sigma + \frac{3}{2} \partial^2 (\rho + i\sigma) + T_C,$$

$$G^+ = j_{BRST} = -e^{-2\rho-i\sigma}(p)^4 + \frac{i}{2}e^{-\rho}p_\alpha p_\beta \partial x^{\alpha\beta}$$

$$+ e^{i\sigma}(\frac{1}{2} \partial x^m \partial x_m + p_\alpha \partial \theta^\alpha + \frac{1}{2} \partial (\rho + i\sigma) \partial (\rho + i\sigma) - \frac{1}{2} \partial^2 (\rho + i\sigma)) + G^+_C,$$

$$G^- = b = e^{-i\sigma} + G^-_C,$$

$$J = j_{ghost} = \partial (\rho + i\sigma) + J_C,$$

where $(p)^4 = \frac{1}{24} e^{\alpha\beta\gamma\delta} p_\alpha p_\beta p_\gamma p_\delta$, $x^m$ has been written in bispinor notation as $x^{\alpha\beta} = (\sigma_m)^{\alpha\beta} x^m$, and $(\sigma_m)^{\alpha\beta}$ are the six-dimensional Pauli matrices satisfying

$$(\sigma_m)^{\alpha\beta}(\sigma_n)^{\beta\gamma} + (\sigma_n)^{\alpha\beta}(\sigma_m)^{\beta\gamma} = 2\eta_{mn}\delta^\alpha_\gamma$$

with $(\sigma_m)^{\alpha\beta}$ defined as $(\sigma_m)^{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\gamma\delta} (\sigma_m)^{\gamma\delta}$.

Although physical vertex operators can be defined as U(1)-neutral primary fields with respect to the above N=2 generators, they would not be manifestly spacetime-supersymmetric since they depend on only half of the usual eight $\theta^{\alpha j}$ variables of six-dimensional superspace where $j = \pm$. If one calls $\theta^{\alpha-} = \theta^\alpha$ where $\theta^\alpha$ is defined above, the complex conjugates $\theta^{\alpha+}$ are missing. To make all six-dimensional supersymmetries manifest, one therefore needs to add $\theta^{\alpha+}$ to the hybrid variables, as well as their conjugate momentum $p_{\alpha+}$. Since these variables are not related by a field redefinition to RNS variables, one needs to simultaneously introduce new fermionic gauge invariances which allow $\theta^{\alpha+}$ and $p_{\alpha+}$ to be gauged away \[11\]. These new variables will be defined to have no singular OPE’s with the other fields and to satisfy $\theta^{\alpha+}(y)p_{\beta+}(z) \rightarrow (y - z)^{-1}\delta^\alpha_\beta$.

The fermionic gauge invariances will be generated by the first-class constraints

$$D_\alpha = d_{\alpha+} - e^{\rho-i\sigma} d_{\alpha-} = 0$$
where $d_{a-} = p_a$ and $d_{a+} = p_{a+} - i\theta^\beta - \partial x_{a\beta} + \frac{1}{8} \epsilon_{\alpha\beta\gamma\delta} \theta^\gamma - \partial \theta^\delta$. Since $\{D_\alpha, \theta^{\beta+}\} = \delta^\beta_\alpha$, this gauge invariance can be used to fix $\theta^{a+} = 0$ and the constraint then fixes $p_{a+} = e^{-\rho-i\sigma} p_{a-} + i\theta^\beta - \partial x_{a\beta}$. Furthermore, the N=2 constraints can be modified to commute with the $D_\alpha$ constraints by defining

$$T = \frac{1}{2} \partial x^m \partial x_m + p_{\alpha j} \partial \theta^{\alpha j} + \partial \theta^{\alpha+} D_\alpha + \frac{1}{2} \partial \rho \partial \rho + \frac{1}{2} \partial \sigma \partial \sigma + \frac{3}{2} \partial^2 (\rho + \sigma) + T_C,$$

$$G^+ = -\frac{1}{24} \epsilon^{\alpha\beta\gamma\delta} D_\alpha (D_\beta (D_\gamma (D_\delta (e^{2\rho+3i\sigma})))) + G_+^C,$$

$$G^- = e^{-i\sigma} + G_-^C,$$

$$J = \partial (\rho + i\sigma) + J_C,$$

where $D_\alpha (Y)$ denotes the contour integral of $D_\alpha$ around $Y$. One can check that these N=2 constraints agree with the ones defined earlier in the gauge $\theta^{a+} = 0$. These N=2 generators, together with the $D_\alpha$ constraints, provide first-class constraints for $d_{\alpha j}$ that can replace the second-class constraints of the six-dimensional GS superstring.

For the open superstring, the massless compactification-independent states are described by the gluon and gluino of six-dimensional super-Yang-Mills. As shown in [1], the integrated vertex operator for these states is given by

$$V = \int dz [\partial Y^M A_M + d_{\alpha j} W^{\alpha j}]$$

exactly as predicted by Siegel where $Y^M = (x^m, \theta^{\alpha j})$, $W^{\alpha j}$ is the superspace field-strength, and $A_M$ are the superspace gauge fields. In Wess-Zumino gauge when $\theta^{a+} = 0$, this vertex operator is mapped to the standard Neveu-Schwarz and Ramond vertex operators for the gluon and gluino where the gluino vertex operator with polarization $W^{a-}$ is in the $-\frac{1}{2}$ picture, the gluon vertex operator is in the $0$ picture, and the gluino vertex operator with polarization $W^{a+}$ is in the $+\frac{1}{2}$ picture [11].

Since the closed superstring vertex operator is the holomorphic square of the open superstring vertex operator, one obtains the following integrated vertex operator for the massless compactification-independent states of the Type II superstring:

$$S = \int d^2 z [\partial Y^M \partial Y^N (G_{MN} + B_{MN}) + d_{\alpha j} \bar{\partial} Y^N E_{\alpha j}^N + \partial Y^M d_{\alpha j} E_{\alpha j}^M + d_{\alpha j} d_{\beta k} F^{\alpha j \beta k}]$$

where $Y^M = (x^m, \theta^{\alpha j}, \bar{\theta}^{\alpha j})$, $\bar{\theta}^{\alpha j}$ is the right-moving analog of the left-moving $\theta^{\alpha j}$, and the lowest components of the superfields $G_{mn}, B_{mn}, E_{mn}^{\alpha j}, F_{mn}^{\alpha j}, F^{\alpha j \beta k}$ are the graviton, the anti-symmetric tensor, the two gravitini, and the Ramond-Ramond bispinor field strengths.

To get the superstring action in a curved background, one again interprets the superfields $[G_{MN}, B_{MN}, E_{\alpha j}^M, E_{\alpha j}^M, F^{\alpha j \beta k}]$ appearing in $S$ as background superfields. This action is manifestly super-reparameterization invariant and is expected to be N=2 worldsheet superconformally invariant when the background fields satisfy the appropriate equations of motion. The first term in $S$ is the usual GS action in
a curved background, but the other terms in $S$ proportional to $d_\alpha$ and $\bar{d}_{\bar{\alpha}}$ are required for quantization. The complete action also contains a contribution from the compactification-dependent fields, from the chiral bosons, and from a Fradkin-Tseytlin term constructed in a manner similar to that of the four-dimensional action [11].

To obtain the action for the superstring in an $AdS_3 \times S^3$ background with constant Ramond-Ramond flux $F^{\alpha j \bar{k}} = \epsilon^{j k} \delta^{\alpha \beta}$, there are two approaches. The first approach [11] resembles the four-dimensional case where one simply plugs the appropriate values for the background fields into the action. After integrating out $d_\alpha$ and $\bar{d}_{\bar{\alpha}}$, one obtains the action

$$S_{AdS} = \int d^2 z [\eta_{cd} J^c_z J^d_{\bar{z}} + \frac{1}{2} \epsilon_{jk} \delta_{\alpha j \bar{\alpha} k} (J^{\alpha j} J^{\bar{k}} - J^{\bar{\alpha} j} J^{\bar{k}})$$

$$+ \epsilon_{jk} \delta_{\alpha j \bar{\alpha} k} (J^{\alpha j} J^{\bar{k}} + J^{\bar{\alpha} j} J^{\bar{k}})]$$

where $(J^{\alpha j}, J^{\bar{k}})$ and $J^c$ are the sixteen fermionic currents and six bosonic currents $(g^{-1} dg)^A$ constructed from a coset supergroup $g$ taking values in $PSU(1,1|2) \times PSU(2|2)/SU(2) \times SU(2)$. The first line of this action comes from the $\partial Y^M \overline{\partial Y}^N (G_{MN} + B_{MN})$ term and is therefore identical to the GS action on $AdS_3 \times S^3$ [13] [17]. However, the second line of this action is crucial for quantization and is absent from the GS action. Using standard techniques, it was confirmed to one-loop sigma model in [8] that the above action is conformally invariant as expected.

A second approach for constructing the action on $AdS_3 \times S^3$ is to first use the $D_\alpha$ and $\bar{D}_{\bar{\alpha}}$ constraints to solve for $d^{\alpha+}$ and $\bar{d}^{\bar{\alpha}+}$ and to gauge-fix half of the fermionic parameters of the coset supergroup. After integrating out the remaining $d^{\alpha-}$ and $\bar{d}^{\bar{\alpha}-}$ variables, one obtains the action [10]

$$S_{AdS} = \int d^2 z [\eta_{cd} J^c_z J^d_{\bar{z}} - (1 - e^{-\rho - i\sigma + \bar{\rho} + i\bar{\sigma}})^{-1} (J^{\alpha -}_z + e^{-\rho - i\sigma} J^{\alpha +}_z) (J^{\alpha +}_{\bar{z}} + e^{\bar{\rho} + i\bar{\sigma}} J^{\alpha -}_{\bar{z}})]$$

where $(J^{\alpha -}, J^{\alpha +})$ and $J^c$ are the eight fermionic and six bosonic left-invariant currents $(g^{-1} dg)^A$ constructed from a supergroup $g$ taking values in $PSU(2|2)$. It was proven to all orders in sigma model loops that this action is conformally invariant [10].

Acknowledgments

I would like to thank my collaborators for their contributions, the organizers of Strings ‘99 for an enjoyable conference, and CNPq grant 300256/94-9 for partial financial support.

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