Minimal Supergravity with $m_0^2 < 0$

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Abstract

We extend the parameter space of minimal supergravity to negative values of $m_0^2$, the universal scalar mass parameter defined at the grand unified scale. After evolving to the weak scale, all scalars can be non-tachyonic with masses consistent with collider constraints. This region of parameter space is typically considered excluded by searches for charged dark matter, since the lightest standard model superpartner is a charged slepton. However, if the gravitino is the lightest supersymmetric particle, the charged slepton decays, and this region is allowed. This region provides qualitatively new possibilities for minimal supergravity, including spectra with light sleptons and very heavy squarks, and models in which the lightest slepton is the selectron. We show that the $m_0^2 < 0$ region is consistent with low energy precision data and discuss its implications for particle colliders. These models may provide signals of supersymmetry in even the first year of operation at the Large Hadron Collider.

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I. INTRODUCTION

Supersymmetric models are theoretically motivated extensions of the standard model (SM) of particle physics that predict both direct and indirect signals in particle physics experiments. Most analyses of supersymmetric models assume the minimal supersymmetric standard model (MSSM), the supersymmetric extension of the SM that contains the minimal number of superpartners. Supersymmetry (SUSY) must also be broken. To make phenomenological analyses tractable, a moderately simple model for soft supersymmetry-breaking terms must be chosen.

By far the most studied supersymmetric model is minimal supergravity (mSUGRA) [1], which is specified by 6 parameters:

\[ m_0^2, M_{1/2}, A_0, \tan \beta, \text{sign}(\mu), \text{and } m_{3/2}. \]  

(1)

Here \( m_0^2 \) is the universal soft scalar mass squared, \( M_{1/2} \) is the universal soft gaugino mass, \( A_0 \) is the universal soft trilinear term, \( \tan \beta \) is the ratio of the vacuum expectation values of the up and down type Higgs bosons, \( \mu \) is the supersymmetric Higgs mass parameter, and \( m_{3/2} \) is the gravitino mass. The first three terms are defined at the grand unified theory (GUT) scale \( M_{\text{GUT}} \approx 2.4 \times 10^{16} \) GeV, where the gauge couplings unify. All superpartner masses and couplings are determined by these 6 parameters and renormalization group equations (RGEs). The lightest SM superpartner is typically either the lighter stau or the lightest neutralino.

Note that mSUGRA is typically thought to be determined by the first 5 parameters. When the gravitino is not the lightest supersymmetric particle (LSP), much of cosmology and all of particle phenomenology is insensitive to \( m_{3/2} \). However, if the gravitino is the LSP, both cosmology and particle phenomenology are sensitive to the gravitino mass, and \( m_{3/2} \) is an essential parameter of mSUGRA.

When \( R \)-parity is conserved, as we assume throughout this study, the LSP is stable. Commonly it is (implicitly) assumed that the gravitino is not the LSP. In this case, the region of parameter space in which the stau is the lightest SM superpartner is strongly disfavored, as it predicts an absolutely stable charged massive particle (CHAMP), which has not been found [2, 3]. Results of mSUGRA analyses are often displayed in the \( (m_0^2, M_{1/2}) \) plane. Null results from CHAMP searches then exclude from consideration a thin triangular wedge in this plane with small \( m_0^2 > 0 \) and, \textit{a fortiori}, the entire half plane with \( m_0^2 < 0 \).

As emphasized above, however, this line of reasoning relies on the assumption that the gravitino is not the LSP. There are no theoretical motivations for this assumption — the gravitino mass is a free parameter in mSUGRA. In this, as well as in other scenarios with high-scale supersymmetry breaking, it is naturally of the same order of magnitude as other superpartner masses, but it is not necessarily larger. In addition, recent work has established that there are also no phenomenological or cosmological reasons to exclude the gravitino LSP scenario [4–11]. In fact, high-scale supersymmetry breaking with a gravitino LSP has a number of novel implications and virtues. For example, if the gravitino is the LSP and the next-lightest supersymmetric particle (NLSP) is charged, the signal of SUSY at colliders will be metastable charged particles. Such particles have spectacular signatures [12–15]. They also make possible the investigation of gravitational interactions and the quantitative verification of supergravity in high energy physics experiments [16–20]. Cosmologically, gravitinos produced through decays naturally have the correct relic density to be superweakly-interacting massive particle (superWIMP) dark matter. For some parameters, gravitino dark matter produced in this way has features usually associated with warm
dark matter and may resolve controversial discrepancies in halo profiles and the formation of small scale structure [21–25]. Last, the late decays that produce gravitino dark matter also release electromagnetic and hadronic energy, with (possibly felicitous) implications for Big Bang nucleosynthesis [4, 6, 9, 11, 26–29] and the cosmic microwave background [4, 6, 30].

In this paper we consider the possibility of mSUGRA with a gravitino LSP and \( m_0^2 < 0 \). We define

\[
m_0 \equiv \text{sign}(m_0^2) \sqrt{|m_0^2|}.
\]

We show that the region with \( m_0 < 0 \) contains models consistent with all collider limits. We also consider precision measurements, analyzing the anomalous magnetic moment of the muon \( a_\mu \), \( B(b \to s\gamma) \), and \( B_s^0 \to \mu^+\mu^- \). We find that the current discrepancy in \( a_\mu \) between experiment and the SM prediction may be resolved for \( m_0 < 0 \) without disrupting the agreement for \( B(b \to s\gamma) \), and near future probes of \( B_s^0 \to \mu^+\mu^- \) will have significant reach in \( m_0 < 0 \) model space. Precision data do not currently favor one sign of \( m_0 \) over the other.

The simple modification of taking \( m_0 < 0 \) therefore “doubles” the viable mSUGRA parameter space and leads to qualitatively new possibilities. For example, in some regions of parameter space, the NLSP is not the stau, but the selectron. This overturns the common lore that Yukawa couplings in RGEs lower soft masses; when some scalars are tachyonic in part of the RG evolution, Yukawa terms may increase scalar masses. We also find that light charged sleptons can be produced for any value of \( M_1/2 \). This produces spectra where the charged sleptons have masses around 100 GeV, but all other superpartner masses are above 1 TeV, with squark and gluino masses around \( 3 - 4 \) TeV. Such spectra are not found for \( m_0 > 0 \) and have novel implications for the Large Hadron Collider (LHC) and International Linear Collider (ILC). Although all of these features may be found in general MSSM models, it is striking that we find them here in a framework with universal scalar and gaugino masses, motivated as these features are by simplicity, the SUSY flavor and CP problems, and gauge coupling unification.

Cosmologically, these models differ from conventional models with \( m_0 > 0 \) in several ways. As noted above, the \( m_0 < 0 \) models have superWIMP dark matter, as opposed to the conventional neutralino WIMP dark matter, with the implications mentioned above for Big Bang nucleosynthesis, the cosmic microwave background, and structure formation. In addition, there are possibly novel implications for vacuum stability [11] and gauge symmetry breaking at high temperatures. We defer discussion of cosmological issues [31], and focus here on implications for particle physics.

This study is organized as follows. In Sec. II we show that, even given \( m_0 < 0 \) at the GUT scale, all superpartner masses, when evolved to the weak scale, can have values consistent with current experimental bounds. We delineate the allowed regions and determine which regions of parameter space have stau and selectron NLSPs. The resulting superpartner masses in the \( m_0 < 0 \) region are discussed in Sec. III. Low-energy observables are analyzed in Sec. IV. In Sec. V, we show two representative superpartner spectra and briefly discuss the implications for the LHC and ILC. In Sec. VI, we conclude and indicate interesting avenues for further investigation.
FIG. 1: Regions of the \((m_0, M_{1/2})\) plane, extended to \(m_0 < 0\), for \(A_0 = 0, \mu > 0,\) and \(\tan \beta = 10\) (left) and \(\tan \beta = 60\) (right). The green (medium shaded) region is experimentally excluded, and the unshaded region is the conventional neutralino (N)LSP region. In the remaining regions, the gravitino is the LSP: in the yellow (light shaded) region, the stau is the NLSP, and in the thin magenta (dark shaded) region of the \(\tan \beta = 60\) plot, the selectron is the NLSP. The present Higgs mass bound \(m_h > 114.1\) GeV excludes regions below the solid contours. The symbols \(\oplus\) mark the location of benchmark Models A and B; their RG evolution is shown in Fig. 2.

II. REGIONS OF MSUGRA PARAMETER SPACE FOR \(m_0 < 0\)

In this section, we determine the allowed regions of mSUGRA parameter space with \(m_0 < 0\), and further classify the allowed parameter space according to what particles are the LSP and NLSP, since these play a large role in determining experimental signatures.

For \(m_0 < 0\), an immediate worry is that scalar masses will remain tachyonic even after RG evolution to the weak scale. As usual, gauge interactions raise the soft masses, and so the most problematic scalars are the right-handed sleptons, since these have only hypercharge interactions. The RGEs for conventional mSUGRA have been studied in great detail. A well-known approximate relation for the weak-scale right-handed slepton mass in terms of GUT-scale parameters is [37]

\[
m^2_{\tilde{e}_R} = m_0^2 + 0.15 M_{1/2}^2.
\]

(3)

This remains valid for \(m_0 < 0\). From this, we see that negative \(m_0^2\) can always be compensated by large \(M_{1/2}\) to make the right-handed sleptons, and with them the entire superpartner spectrum, non-tachyonic.

Allowed regions of the \((m_0, M_{1/2})\) plane are shown for two values of \(\tan \beta\) in Fig. 1. The SUSY spectra have been calculated with the software package ISAJET v7.71 [32], modified to accommodate \(m_0 < 0\). ISAJET includes 2-loop RGEs and 1-loop corrections to superpartner masses, and we choose a top quark mass of 175 GeV. The green (medium shaded) region is excluded. For \(m_0 > 0\) the boundary is determined by the LEP chargino mass limit \(m_{\tilde{\chi}^\pm} > 103.5\) GeV [33]. For \(m_0 < 0\), it is essentially determined by null searches for long-lived charged sleptons at LEP, leading to limits \(m_{\tilde{l}_R} > 99\) GeV [34]. For \(\tan \beta = 10\), the border for \(m_0 < 0\) follows to a reasonable approximation the tachyonic slepton line \(m_0 = -2.6 M_{1/2}\) one can derive from Eq. (3). For \(\tan \beta = 60\), the excluded region has a more complicated shape because large 1-loop corrections play an important role, as discussed below.
The allowed regions may be further divided according to what particles are the LSP and NLSP. The unshaded regions of Fig. 1 are the conventional regions in which the lightest SM superpartner is the lightest neutralino. It is either the LSP or, if the gravitino is the LSP, the NLSP.

In the rest of the allowed regions shown, the gravitino must be the LSP to avoid having charged dark matter, and the NLSP is a charged slepton. To determine which charged slepton is the NLSP, consider the RGEs for their soft mass parameters. At 1-loop, these are

$$\frac{dm^2_{\tilde{\tau}_L}}{dt} = \frac{2}{16\pi^2} \left[ -\frac{12}{5} g_1^2 M_1^2 \right]$$

$$\frac{dm^2_{\tilde{\tau}_R}}{dt} = \frac{2}{16\pi^2} \left[ -\frac{12}{5} g_1^2 M_1^2 + 2\lambda_\tau^2 \left( m_{\tilde{\tau}_L}^2 + m_{\tilde{\tau}_R}^2 + m_{H_d}^2 + A_{\tilde{\tau}}^2\right) \right]$$

where $t = \ln \left( Q^2/M_{GUT}^2 \right)$. As is well-known, when all mass parameters are non-tachyonic, Yukawa interactions lower soft masses, leading to the common lore that, given universal scalar boundary conditions, the lightest slepton is always the stau. In the present case, however, $m_0 < 0$, and so in evolving from the GUT scale, selectron masses initially rise slower than stau masses. Of course, for the spectrum to be viable, all physical scalar masses must eventually become non-tachyonic, and so will exert the conventional effect of Yukawa couplings as one approaches the weak scale. (The Higgs scalar mass parameters may remain negative.) The competition between the new and the conventional effects determines whether the NLSP is the selectron or the stau.

These effects are shown in Fig. 2 for the two benchmark models highlighted in Fig. 1. In the left panel, all scalar masses of Model A quickly become positive as they evolve from the GUT scale, and so the stau becomes the NLSP, as usual. In the right panel, however, the scalar masses of Model B are negative for much of the RG evolution, and $m_{H_d}^2$ becomes negative, leading to an inverted flavor spectrum with a selectron NLSP.

In Fig. 1, the $\tilde{\tau}_1$ is the NLSP in the yellow (light shaded) region, and $\tilde{e}_R$ is the NLSP in the magenta (dark shaded) region. The current experimental limits force sleptons to be not just non-tachyonic, but significantly so, and so the viable selectron NLSP region is reduced to a thin sliver near the upper, left-hand corner in the $\tan \beta = 60$ plot. Its exact location is therefore rather sensitive to small corrections and depends on the implementation of RG evolution and loop-level corrections to the superpartner mass spectrum. The mere possibility that universal slepton masses can lead to sleptrons lighter than staus, however, is a robust physics effect; it is novel and never realized in conventional mSUGRA. The selectron NLSP region may be much larger in even slightly more general models. For example, motivated by SO(10) unification, one may consider models in which the matter scalar masses are determined by the parameter $m^2_0$, but the Higgs scalar masses are unified at a different GUT-scale parameter $m^2_H$. In these non-universal Higgs mass models [35], by choosing $m^2_H < m^2_0 < 0$, the stau mass will receive large positive contributions from $m_{H_d}^2$ that are absent for the selectron, as can be seen in Eq. (5), and the selectron NLSP region will be much larger. Last, we note that in the mSUGRA models considered here, the possibility of a non-tachyonic sneutrino (N)LSP also exists at low $M_{1/2}$, but this lies entirely in the excluded region.

As noted above, the excluded region can have a rather complicated shape. For $\tan \beta = 60$, for example, as can be seen in the right-hand panel of Fig. 1, the excluded region has an interesting shape. This results from the remarkable fact that the light charged slepton masses do not increase monotonically as one increases $M_{1/2}$ for constant $m_0$. In fact, this is
FIG. 2: The RG evolution of soft scalar masses in Model A, the stau NLSP point with \( m_0 = -40 \) GeV, \( M_{1/2} = 300 \) GeV, \( \tan \beta = 10 \) (left) and in Model B, the selectron NLSP point with \( m_0 = -700 \) GeV, \( M_{1/2} = 1900 \) GeV, \( \tan \beta = 60 \) (right). In both cases, \( A_0 = 0 \) and \( \mu > 0 \).

FIG. 3: Regions of the \( (m_0, M_{1/2}) \) plane, extended to \( m_0 < 0 \), for \( A_0 = 0, \mu > 0, \) and \( \tan \beta = 60 \) as in the right-hand panel of Fig. 1, but with 1-loop corrections to sparticle masses neglected. This behavior results from 1-loop corrections present in the slepton mass matrix

\[
\begin{pmatrix}
M^2_{LL} + \delta M^2_{LL} & M^2_{LR} + \delta M^2_{LR} \\
M^2_{RL} + \delta M^2_{RL} & M^2_{RR} + \delta M^2_{RR}
\end{pmatrix},
\]

where \( M^2 \) are tree-level contributions and \( \delta M^2 \) are 1-loop corrections. If 1-loop corrections are neglected, the slepton mass monotonically increases for increasing \( m_0 \) and fixed \( M_{1/2} \) (or vice versa), as illustrated in Fig. 3. However, for large \( \tan \beta \) the loop corrections have the proper sign and magnitude to lower the mass eigenvalues of the stau below the experimental bounds in part of the parameter space. Of course, the 1-loop corrections are physical, and we include them in all plots and results below.

To conclude this section, the impression that neutralinos are the lightest SM superpartners in most of mSUGRA parameter space is artificial: it follows only if one requires \( m_0 > 0 \). Allowing \( m_0 < 0 \) extends the viable region of mSUGRA parameter space significantly and shows that staus are the lightest SM superpartners in much of mSUGRA parameter space.
FIG. 4: Squark masses for mSUGRA extended to $m_0 < 0$ for $A_0 = 0$, $\mu > 0$, and $\tan \beta = 10$ (left) and $\tan \beta = 60$ (right). The up-type squarks (top) are $\tilde{u}_L$ (solid black), $\tilde{u}_R$ (dotted black), $\tilde{t}_1$ (dot dashed black), and $\tilde{t}_2$ (dashed black). Similarly the down-type squarks (bottom) are $\tilde{d}_L$ (solid black), $\tilde{d}_R$ (dotted black), $\tilde{b}_1$ (dot dashed black), and $\tilde{b}_2$ (dashed black). The gluino mass (cyan, solid light) is presented on all 4 plots. The contours are for masses 1 TeV, 2 TeV, and 3 TeV from bottom to top.

In addition, allowing $m_0 < 0$ leads to other new phenomena, such as the possibility that the selectron is the lightest SM superpartner. In the next section, we explore the implications of $m_0 < 0$ for the sparticle spectrum more fully.

III. SUSY MASS SPECTRA FOR $m_0 < 0$

The squark and gluino masses are presented in the $(m_0, M_{1/2})$ plane in Fig. 4. In the allowed region with $m_0 < 0$, they are relatively insensitive to $m_0$, since they are dominated by the RG contributions of the gaugino masses. The left-handed down and up squarks are approximately degenerate, as are the right-handed up and down squarks. The gluino is always heavier than all squarks in the $m_0 < 0$ allowed region.

Slepton masses are shown in Fig. 5. In contrast to squark masses, slepton masses are extremely sensitive to $m_0$ in the allowed $m_0 < 0$ region. Contours of constant slepton mass switch from concave down for $m_0 > 0$ to concave up for $m_0 < 0$. As a result, light sleptons near their experimental limit may be found for any value of $M_{1/2}$. This is true even though
FIG. 5: Slepton masses for mSUGRA extended to $m_0 < 0$ for $A_0 = 0$, $\mu > 0$, and $\tan \beta = 10$ (left) and $\tan \beta = 60$ (right). The contours are for $\tilde{\tau}_1$ (dashed), $\tilde{e}_L$ (solid) and $\tilde{e}_R$ (dotted). The contours are for masses 200 GeV, 500 GeV, and 1 TeV from bottom left to top right, with the exception that in the right panel, because the slepton masses drop and then rise again for fixed $M_{1/2}$ and increasing $m_0$, there are two sets of 200 GeV contours for both $\tilde{e}_R$ and $\tilde{\tau}_1$. For both of these particles, the leftmost 200 GeV contour is barely visible in the selectron NLSP region. Throughout the parameter space, the sneutrinos and $\tilde{\tau}_2$ are almost degenerate with the $\tilde{e}_L$.

FIG. 6: Neutralino and chargino masses for mSUGRA extended to $m_0 < 0$ for $A_0 = 0$, $\mu > 0$, and $\tan \beta = 10$ (left) and $\tan \beta = 60$ (right). The lightest neutralino $\tilde{\chi}^0_1$ (dashed) has values from bottom to top of 200 GeV, 500 GeV, and 800 GeV. The lightest chargino $\tilde{\chi}^+_1$ (dotted) has values from bottom to top of 500 GeV, 1 TeV, and 1.5 TeV, while $\mu$ (solid) has values 500 GeV, 1 TeV, and 2 TeV from bottom to top.

the masses of all other superpartners becomes large for large $M_{1/2}$, as may be seen in Model B, the selectron NLSP model shown in the right panel of Fig. 2.

In Fig. 6, we present contours for neutralino and chargino masses and for the Higgsino mass parameter $\mu$. In the $m_0 < 0$ region, $|\mu|$ is always much larger than the electroweak gaugino masses $M_1$ and $M_2$. As a result, the lighter chargino and lighter two neutralinos are nearly pure gauginos, with $\tilde{\chi}^0_1 \approx \tilde{B}$, $\tilde{\chi}^0_2 \approx \tilde{W}^0$, $\tilde{\chi}^\pm_1 \approx \tilde{W}^\pm$, and $m_{\tilde{\chi}^0_1} \approx M_1$ and $m_{\tilde{\chi}^0_2} \approx m_{\tilde{\chi}^\pm_1} \approx M_2 \approx 2M_1$.

Last, Higgs boson masses are given in Fig. 7. The masses of the Higgs bosons, $h^0$, $A^0$, $H^0$, $H^0$, $H^0$, $H^0$, and $H^0$, are
FIG. 7: Higgs boson masses for mSUGRA extended to $m_0 < 0$ for $A_0 = 0$, $\mu > 0$, and $\tan \beta = 10$ (left) and $\tan \beta = 60$ (right). The SM-like Higgs boson $h^0$ mass contours (solid) have values 114 GeV, 118 GeV, and 122 GeV from bottom to top. The $A^0$ mass contours (dashed) have values 500 GeV, 1 TeV, and 1.5 TeV from bottom to top. The heavy CP-even and charged Higgs scalars are approximately degenerate with the $A^0$. and $H^\pm$ are all insensitive to $m_0$ for $m_0 < 0$. The predicted value of $m_{h^0}$ is above 114 GeV throughout the allowed $m_0 < 0$ region, and so consistent with current bounds. It increases with increasing $M_{1/2}$, rising to approximately 122 GeV for $M_{1/2} = 1$ TeV. $A^0$, $H^0$, and $H^\pm$ are all approximately degenerate for $m_0 < 0$.

IV. PRECISION EXPERIMENTAL CONSTRAINTS

We now consider constraints on these models from precision experimental data. We focus on three processes that are well-known to have significant sensitivity to supersymmetric contributions: the anomalous magnetic moment of the muon $a_\mu$ and the rare decays $b \to s\gamma$ and $B^0_s \to \mu^+\mu^-$. These contributions have been calculated using the software package micrOMEGAs, v1.3.6 [38].

A. Anomalous Magnetic Moment of the Muon

Determining the SM value of the anomalous magnetic moment of the muon is not straightforward because of hadronic contributions to higher order loop processes. These hadronic loop contributions are usually estimated from measurements of $e^+e^- \to$ hadrons or $\tau \to$ hadrons. The resulting SM predictions for $a_\mu$ are $a_\mu = 116592018 (63) \times 10^{-11}$ if the $\tau$ data are used, and $a_\mu = 116591835 (69) \times 10^{-11}$ if the $e^+e^-$ data are used [39, 40]. Given theoretical assumptions required to use the $\tau$ data, the $e^+e^-$ value is generally judged to be more reliable [41]. These should be compared to the measured value $a_\mu = 116592080 (60) \times 10^{-11}$ from the Muon $(g - 2)$ Collaboration [42, 43]. Taking the SM value for $a_\mu$ using the $e^+e^-$ data, there is a discrepancy between theory and experiment of $\delta a_\mu = 245 \times 10^{-11}$, a deviation of approximately $3\sigma$.

The SUSY contribution to $a_\mu$ in mSUGRA with $m_0 < 0$ is shown in Fig. 8. A deviation consistent with the discrepancy between experiment and the $e^+e^-$ SM prediction may be
FIG. 8: The SUSY contribution to $a_\mu$ for mSUGRA extended to $m_0 < 0$ for $A_0 = 0$, $\mu > 0$, and $\tan \beta = 10$ (left) and $\tan \beta = 60$ (right). From bottom to top the contour values are 300, 250, 200, 150, and 100 in units of $10^{-11}$.

obtained for $\mu > 0$ and light neutralinos and sleptons or light charginos and sneutrinos. As noted in Sec. III, sleptons are light along the entire $m_0 < 0$ experimentally excluded border, but the gauginos increase in mass as $M_{1/2}$ increases. A large SUSY contribution to $a_\mu$ is therefore found only for relatively small $M_{1/2}$. The 3$\sigma$ deviation mentioned above may be explained, for example, for $\tan \beta = 10$ and $M_{1/2} \sim 300$ GeV.

B. $B(b \rightarrow s\gamma)$

The flavor changing neutral current transition $b \rightarrow s\gamma$ has a branching fraction measured to be

$$B(b \rightarrow s\gamma) = \begin{cases} 3.21 \pm 0.43 \pm 0.27^{+0.18}_{-0.10} \times 10^{-4} \text{ (CLEO) [44]} \\ 3.88 \pm 0.36 \pm 0.37^{+0.43}_{-0.23} \times 10^{-4} \text{ (BABAR) [45]} \\ 3.55 \pm 0.32 \pm 0.30^{+0.11}_{-0.07} \times 10^{-4} \text{ (BELLE) [46]} \end{cases} \ .$$

The SM prediction is $3.79^{+0.36}_{-0.53} \times 10^{-4}$ [47]. The $m_0 < 0$ mSUGRA predictions for $B(b \rightarrow s\gamma)$ are shown in Fig. 9. The experimental and SM theory values agree within errors, and so $b \rightarrow s\gamma$ may, in principle, eliminate models with light squarks and gauginos. As can be seen in Fig. 9, however, supersymmetric effects in the plotted $m_0 < 0$ region are never large enough to create a discrepancy between these mSUGRA models and experiment.

C. $B(B^0_s \rightarrow \mu^+\mu^-)$

The branching fraction for $B_s^0$ decaying to two leptons is an important measurement for constraining supersymmetric models with large $\tan \beta$ [48–52]. The decay is enhanced by $(\tan \beta)^6$ for large $\tan \beta$. The current experimental bound is $B(B_s^0 \rightarrow \mu^+\mu^-) < 1.5 \times 10^{-7}$ from CDF II, based on 364 $\text{pb}^{-1}$ of data [53], while the SM prediction is $3.42(54) \times 10^{-9}$ [54]. For small $\tan \beta$, mSUGRA with $m_0 < 0$, along with other SUSY models, predicts deviations far below current experimental bounds. These deviations will not be probed until the LHC. However, for large $\tan \beta$, observable deviations are predicted even in Tevatron data. As shown in Fig. 10, the current Tevatron data do not exclude additional parameter space.
FIG. 9: $B(b \rightarrow s\gamma)$ for mSUGRA extended to $m_0 < 0$ for $A_0 = 0$, $\mu > 0$, and $\tan \beta = 10$ (left) and $\tan \beta = 60$ (right). The values from bottom to top are 3.10, 3.25, 3.40, and 3.55 in units of $10^{-4}$. In the $\tan \beta = 10$ panel, $B(b \rightarrow s\gamma)$ does not exceed $3.59 \times 10^{-4}$.

FIG. 10: $B(B^0_s \rightarrow \mu^+\mu^-)$ for mSUGRA extended to $m_0 < 0$ for $A_0 = 0$, $\mu > 0$, and $\tan \beta = 60$. The lower contour is CDF II’s experimental upper bound $1.5 \times 10^{-7}$, and the upper contour is for $B(B^0_s \rightarrow \mu^+\mu^-) = 1.0 \times 10^{-8}$.

Nevertheless, future improvements to sensitivities of $\sim 10^{-8}$ will probe the $m_0 < 0$ region all the way up to $M_{1/2} \sim 1.8$ TeV for $\tan \beta = 60$, and will also be sensitive to models with more moderate values of $\tan \beta$.

V. COLLIDER SIGNALS

As is well-known, the LHC will provide an extremely powerful probe of weak-scale supersymmetry in the next few years. Here we discuss the implications of $m_0 < 0$ mSUGRA for the LHC and the proposed ILC.

The prototypical signature of ($R$-parity conserving) supersymmetry at hadron colliders is missing transverse energy. In the case of mSUGRA with $m_0 < 0$, however, all SUSY events result in the production of two metastable charged sleptons. These have lifetimes of seconds to months, and so pass through collider detectors without decaying. These models
FIG. 11: Superpartner spectra for Model A, the stau NLSP point \(m_0 = -40\) GeV, \(M_{1/2} = 300\) GeV, \(\tan \beta = 10\) (left) and for Model B, the selectron NLSP point \(m_0 = -700\) GeV, \(M_{1/2} = 1900\) GeV, \(\tan \beta = 60\) (right). In both cases, \(A_0 = 0\) and \(\mu > 0\). These models correspond to the parameter points highlighted with \(\oplus\) symbols in Fig. 1; the RGEs of their scalars are displayed in Fig. 2, and their weak-scale parameters are given in Table I.

| ISAJET Specification | Parameter | Model A | Model B |
|----------------------|-----------|---------|---------|
| MSSMA                | \(m_{\tilde{g}}\) | 720.70 395.95 | 3964.95 1994.33 |
| MSSMA                | \(m_A\) | 436.09 10.00 | 1274.81 60.00 |
| MSSMB                | \(m_{\tilde{q}}\) \(m_{\tilde{d}}\) \(m_{\tilde{u}}\) | 652.21 601.12 603.80 | 3352.61 3178.77 3198.46 |
| MSSMB                | \(m_{\tilde{l}}\) \(m_{\tilde{e}}\) | 199.04 103.81 | 1027.84 67.57 |
| MSSMC                | \(m_{\tilde{q}}\) \(m_{\tilde{b}}\) \(m_{\tilde{t}}\) | 578.76 598.21 502.38 | 2987.96 2860.45 2727.32 |
| MSSMC                | \(A_t\) \(A_b\) \(A\) | 198.49 101.56 | 1040.40 255.73 |
| MSSMD                | \(m_{\tilde{q}}\) \(m_{\tilde{s}}\) \(m_{\tilde{c}}\) | Same as MSSMB (default) | Same as MSSMB (default) |
| MSSME                | \(M_1\) \(M_2\) | 120.17 231.47 | 831.43 1527.66 |
| MSSME                | \(\delta_{\mu}\) | 296 \times 10^{-11} | 78.1 \times 10^{-11} |
| MSSME                | \(B(b \to s\gamma)\) | 3.49 \times 10^{-4} | 3.57 \times 10^{-4} |
| MSSME                | \(B(B_d^0 \to \mu^+\mu^-)\) | 3.21 \times 10^{-9} | 8.84 \times 10^{-9} |

TABLE I: Mass parameters in GeV and the predicted values for precision observables for benchmark Models A and B. The masses in category MSSMA are physical masses; all other masses listed are soft SUSY-breaking parameters specified at the electroweak scale.

Therefore provide a conventional setting for what might otherwise be considered to be rather exotic signals, such as highly ionizing tracks and time-of-flight signatures [12–15]. Even a few events will provide unmistakable signals.

As examples, let us consider the benchmark models indicated in Fig. 1. The RGEs for scalars in these models were shown in Fig. 2. In Fig. 11, we display the full superpartner spectrum for each of these models, and in Table I we list all mass parameters, which define these models at the electroweak scale.

In Model A, the stau NLSP model of the right panel in Fig. 11, all superpartners have
masses under 1 TeV. This model is an excellent benchmark model. It explains the 3σ deviation in $\alpha_\mu$, and preserves the agreement between the SM and experimental values of $B(b \to s\gamma)$. In addition, electroweak symmetry is broken radiatively and naturally, with $\mu \sim 400$ GeV.

This stau NLSP model has a total SUSY cross section at the LHC of $\sigma_{\text{LHC}}(14 \text{ TeV}) = 13.6 \text{ pb}$, as determined by ISAJET [32]. Even with an integrated luminosity of 100 pb$^{-1}$, this implies over 1000 SUSY events, each with two metastable sleptons. In many of these, the sleptons will be slow enough to be seen as highly-ionizing tracks, providing a spectacular signal of new physics in even the first year of LHC operation. With more luminosity, large numbers of sleptons may be collected and their decays studied, making possible a variety of measurements with implications for cosmology, astrophysics and supergravity [4, 6, 9, 11, 16–28, 30].

Model B, the selectron NLSP model indicated in Fig. 1, is a complementary benchmark model. As seen in Fig. 11, the model has squarks and gluinos around $3-4$ TeV, neutralinos, sneutrinos, non-SM type Higgs bosons, and left-handed sleptons around $1-2$ TeV, a 210 GeV stau, 160 GeV selectron and smuon, and finally a 124 GeV Higgs boson. The squarks and gluinos are too heavy to be produced with large cross section at the LHC. The biggest sources of SUSY particles at the LHC are direct Drell-Yan production of NLSP pairs, leading to 2 metastable charged sleptons, and Drell-Yan production of the heavier sleptons, leading to even more unusual events with 2 taus, 2 muons/electrons, and 2 metastable charged sleptons. The total SUSY production cross section for this model is $\sigma_{\text{LHC}}(14 \text{ TeV}) = 41 \text{ fb}$. This signal will be challenging to find in the first year of LHC running, but at the target luminosity of 100 fb$^{-1}$/yr, the LHC will produce 4100 SUSY events per year.

At the ILC, the total SUSY production cross sections are $\sigma_{\text{ILC}}(500 \text{ GeV}) = 1.35 \text{ pb}$ for the stau NLSP Model A and $\sigma_{\text{ILC}}(500 \text{ GeV}) = 137 \text{ fb}$ for the selectron NLSP Model B. Although squarks and gluinos are out of reach, the ILC will produce large numbers of sleptons. All the usual advantages of the ILC will allow detailed studies of the SUSY parameter space. In addition, the ability to produce sleptons at low velocities implies that they may be more easily trapped and studied than at the LHC.

VI. CONCLUSIONS

In this study, we have extended the well-known framework of mSUGRA to $m_0 < 0$. To our knowledge, this part of parameter space has not been considered previously, perhaps because it contains a charged slepton as the lightest SM superpartner. If the gravitino is the LSP, however, cosmological difficulties with CHAMPs are avoided, and this extended parameter space is allowed. We have noted that it is consistent with all limits from direct searches and constraints from low-energy precision measurements. In addition, we find models with qualitatively novel mSUGRA superpartner spectra, which predict spectacular signals with metastable charged sleptons at future colliders.

Some of the cosmology of these models will be considered in a future work. It would be interesting to extend this work to more general models. For example, as argued above, we expect that the selectron may emerge as the NLSP generically in models with non-unified Higgs masses, and there may well be other interesting phenomena. It would also be worthwhile to determine the reach of the LHC for various luminosities in the $m_0 < 0$ parameter space; given how spectacular the signal of metastable charged particles will be, these models provide a welcome example in which supersymmetry may be discovered and
studied in even the first year of LHC running.

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