Numerical Analysis of Rotor Dynamics of Dredge Pump Shafting

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Abstract. The operating condition of dredge pump is highly complicated, a technical issue is the shaft vibration problems, which has great effects on the operation safety and stability of pumps. This paper introduces a mathematical model of rotor dynamics for the dredge pump, which is used to numerically investigate critical speeds and mode shapes of the pump shaft under different bearing stiffness and damping. The effects of bearing stiffness and damping on shafting vibration characteristics are considered. The results show that the stiffness has great effect on 1st and 3rd order critical speed and the damping effect can be ignored. The stiffness has little effect on shafting axial and torsional natural frequency, but has great effect on lateral natural frequency. The analysis method of rotor dynamics dealt with in this paper can provide a foundation for the design of dredge pump shafting.

1. Introduction

Shaft system is very important for Rotating machinery and bears the load of the unit. Its operating conditions are pretty complex, and usually has vibration and noise problems[1]. Many people studied the vibration characteristics of the shaft system. Liao et al.[2] did dynamic characteristics research of a turbocharger rotor and the gyroscopic moment and stiffness of floating bearings were taken into account, the results shown the bearing stiffness can affect the critical speed but have little effect on the vibration mode.

Zhai et al.[3] did failure analysis of a diesel turbocharger rotor system by considering the effect of floating bearing stiffness coefficient and unbalanced mass, they found that the initial design of sensor will lead to the rubbing and failure of the rotor. Zhai[4] also analysed dynamic characteristics of a dredge pump shaft system and found that the influence of mud water should be counted, but the water sealing effect need not.

Fiori et al.[5] analyse the unstable motion of a rotor-bearing system and validate the nonlinear simulation of the fluid-induced instability, they found that the oil whirl phenomenon occurs when the journal bearings are at heavy-load condition.

In order to analyse the effect of bearing stiffness and damping on shafting dynamic characteristics, a
3D modal is established and the rotor dynamic research is conducted.

2. Model

![Dredge pump shafting modal](image)

**Figure 1.** Dredge pump shafting modal

A 3D model is established, as shown in Figure 1. The dredge pump shaft system is composed of gear, coupling, wheel, shaft, bearing and runner, details of the shaft system (such as holes in wheel, blades and narrow faces) are also considered, the rated rotational speed is 325 rpm. The material and parameters are shown in Table 1.

| Material   | Young's modulus (GPa) | Poisson ratio | Density (kg/m³) |
|------------|-----------------------|---------------|-----------------|
| Gear shaft | 40CrNiMoA             | 209           | 0.3             | 7830            |
| Coupling   | 40CrNiMoA             | 209           | 0.3             | 7830            |
| Wheel      | Q235-A                | 200           | 0.3             | 7850            |
| Shaft      | 40CrNiMoA             | 209           | 0.3             | 7830            |
| Runner     | High chromium cast iron | 200         | 0.3             | 7700            |

The 3D model is meshed and the finite element model is shown in Figure 2, the support is constrained by the bearing boundary condition, the stiffness is $150 \times 10^3$ N/mm and the damping are ignored. The support is the bushing of the shaft, in order to simply the calculation, the support is represented by bearing constraint.

![Finite element model](image)

**Figure 2.** Finite element model

3. Governing equations

Considering the gyroscopic effect, the rotor dynamics equation can be expressed as:

\[
[M]\ddot{u} + [B] + \Omega[G]\dot{u} + [K]u = 0
\]

(1)

Wherein $[M]$ is the mass matrix, $u$ is the displacement vector, $[B]$ is the damping matrix, $\Omega$ is the
rotational speed, \([G]\) is the gyroscopic matrix corresponding to the rotational speed \(\Omega\), \([K]\) is the generalized stiffness matrix.

Set:

\[
u = \phi e^{\lambda t}
\]

Wherein \(\phi\) is undetermined constant column vector, \(\lambda\) is complex frequency. Substituting (2) into (1), we have:

\[
\{\lambda^2[M] + \lambda [B] + \Omega[G] + [K][\nu]\} \phi = 0
\]

Then, the problem of natural vibration analysis of rotor system is transformed into the eigenvalue problem of the formula (3).

If the shaft system is undamped, the equation (1) can be rewritten as:

\[
[M]\{\dot{q}\} + \Omega[G]\{\dot{q}\} + [K]\{q\} = 0
\]

4. Results

The effects of bearing stiffness and damping on shafting critical speed are studied, and the modal analyses under different stiffness are conducted, the detailed calculation information is as follows.

In order to analyse the bearing stiffness effect and compare the shafting sensitivity to stiffness of three bearings, one bearing’s stiffness is set to vary from 50000 N/mm to 300000 N/mm while other bearings’ stiffness keep constant (150000 N/mm), the bearing1 stiffness effect cases are shown in Table 2, and the bearings’ location are shown in Figure 2.

| Case   | Stiffness       | support1 | bearing1 | bearing2 | bearing3 | support2 |
|--------|----------------|----------|----------|----------|----------|----------|
| Case1  | K (N/mm, \times 10^3) | 150      | 50       | 150      | 150      | 150      |
| Case2  | K (N/mm, \times 10^3) | 150      | 100      | 150      | 150      | 150      |
| Case3  | K (N/mm, \times 10^3) | 150      | 150      | 150      | 150      | 150      |
| Case4  | K (N/mm, \times 10^3) | 150      | 200      | 150      | 150      | 150      |
| Case5  | K (N/mm, \times 10^3) | 150      | 250      | 150      | 150      | 150      |
| Case6  | K (N/mm, \times 10^3) | 150      | 300      | 150      | 150      | 150      |

The damping has few effects on shafting modal frequency and its effect on critical speed are analysed. One bearing’s damping is set to vary from 2 N/(mm/s) to 12 N/(mm/s) while other bearings damping at 0, the bearing1 damping effect cases are shown in Table 3.

| Case   | Damping       | support1 | bearing1 | bearing2 | bearing3 | support2 |
|--------|---------------|----------|----------|----------|----------|----------|
| Case1  | B (N/(mm/s))  | 6        | 2        | 6        | 6        | 6        |
| Case2  | B (N/(mm/s))  | 6        | 4        | 6        | 6        | 6        |
| Case3  | B (N/(mm/s))  | 6        | 6        | 6        | 6        | 6        |
| Case4  | B (N/(mm/s))  | 6        | 8        | 6        | 6        | 6        |
| Case5  | B (N/(mm/s))  | 6        | 10       | 6        | 6        | 6        |
| Case6  | B (N/(mm/s))  | 6        | 12       | 6        | 6        | 6        |

4.1. Modal analysis

In order to analyse model shape of shafting, modal analysis is conducted when rotational speed is 0. The bearing damping is not taken into account, only the stiffness effect is considered. The variation of the 1st – 3rd order lateral and axial natural frequency of bearing1, bearing2 and bearing3 with stiffness are shown in Figure 3 (a) and Figure 3 (b). However, due to the 3rd torsional natural frequency is far
more than the second one, it is not considered in the calculation. The variation of the 1<sup>st</sup> and 2<sup>nd</sup> torsional natural frequency with stiffness is shown in Figure 3 (c).

The stiffness has little effect on shafting axial and torsional natural frequency, but has great effect on lateral natural frequency. The variation of the 1<sup>st</sup> order natural frequency with stiffness is more than the 2<sup>nd</sup> and 3<sup>rd</sup> natural frequency, and bearing3’s stiffness has the greatest effect.

![Figure 3](image)

**Figure 3.** Bearing stiffness effect on natural frequency

When the bearing1’s stiffness is $50 \times 10^3$ N/mm and other bearings’ stiffness are $150 \times 10^3$ N/mm, the modal shapes are shown in Figure 4 ~ Figure 6.

The 1<sup>st</sup> ~ 3<sup>rd</sup> order lateral vibration modes are shown in Figure 4, the 1<sup>st</sup> order lateral vibration mode is the inclination of the shafting and the vibration of the gear shaft, the 2<sup>nd</sup> order lateral vibration mode is mainly the vibration of the runner and the 3<sup>rd</sup> order lateral vibration mode is mainly the vibration of the gear shaft.
The 1st ~ 3rd order axial vibration modes are shown in Figure 5, the 1st order axial vibration mode is the axial endplay of the shafting, especially the runner. The 2nd and the 3rd order axial vibration mode are mainly the vibration of the runner.

The 1st and 2nd order torsional vibration modes are shown in Figure 6, the 1st order torsional vibration mode is mainly the torsional deformation of the coupling and the 2nd order torsional vibration mode is mainly the torsional deformation of the wheel.
4.2. Critical speed analysis

4.2.1. Stiffness effect

The critical speed increases with the increasing of bearing stiffness, the variation of the first three critical speeds with bearing stiffness of bearing 1, bearing 2 and bearing 3 are shown in Figure 7.

It can be seen that the bearing stiffness has great influence on 1$^{st}$ and 3$^{rd}$ order critical speed, but bearing 2’s stiffness has little effect on shafting 1$^{st}$ order critical speed and the 2$^{nd}$ order critical speed has little variation with the increasing of the stiffness. The bearing 3’s stiffness has greatest effect on shafting 3$^{rd}$ order critical speed.

![Figure 7. Bearing stiffness effect on critical speed](image)

4.2.2. Damping effect

The bearing damping have little effect on the critical speed, as shown in Figure 8. It can be seen that with the damping increasing, the natural frequency almost keeps constant, i.e., the damping effect can be ignored when it is far smaller than stiffness (the comparation is meaningful when the units of the stiffness and damping are N/mm and N/(mm/s) respectively).
5. Conclusions

The effect of the bearing stiffness and damping on critical speed are studied and the modal analysis under different stiffness is conducted, the following conclusions are obtained.

1) The shafting critical speed increases with the increasing of the stiffness, but the 2nd order critical speed has little variation. The bearing3’s stiffness has greatest effect on the 3rd critical speed. The damping effect can be ignored because of the little variation of the critical speed.

2) The shafting axial and torsional natural frequency have little variation with the increasing of the bearing stiffness, but the lateral natural frequency has great variation.

3) The critical speed is far more than the rated rotational speed (5.42 Hz), i.e., the shafting works in a sufficient safe speed range, but the bearing stiffness should be in a safe area to ensure the 1st critical speed not be less than the rated speed.

References

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