COMPOSITE S-BRANE SOLUTIONS
ON PRODUCT OF RICCI-FLAT SPACES

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Abstract

A family of generalized $S$-brane solutions with orthogonal intersection rules and $n$ Ricci-flat factor spaces in the theory with several scalar fields and antisymmetric forms is considered. Two subclasses of solutions with power-law and exponential behaviour of scale factors are singled out. These subclasses contain sub-families of solutions with accelerated expansion of certain factor spaces. The solutions depend on charge densities of branes, their dimensions and intersections, dilatonic couplings and the number of dilatonic fields.
1 Introduction

The recent discovery of the cosmic acceleration [1, 2] was a starting point for a big number of publications on multidimensional cosmology giving some explanations of this phenomenon using certain multidimensional models [3], e.g. those of superstring or supergravity origin (see, for example [4] and references therein). These solutions deal with time-dependent scale factors of internal spaces (for reviews see [5, 6, 8, 9]) and contain as a special case the so-called S-brane solutions [10], i.e. space like analogues of $D$-branes [18], see for example [11, 12, 13, 14, 15, 16, 17] and references therein. For earlier S-brane solutions see also [19, 20, 21].

In our recent paper [4] we have obtained a family of cosmological solutions with $(n+1)$ Ricci-flat spaces in the theory with several scalar fields and multiple exponential potential when coupling vectors in exponents obey certain "orthogonality" relations. In [4] two subclasses of "inflationary-type" solutions with power-law and exponential behaviour of scale factors were found and solutions with accelerated expansion were singled out. In this paper we generalize "inflationary-type" solutions from [4] to $S$-brane configurations in models with antisymmetric forms and scalar fields. Two subclasses of these solutions with the power-law and exponential behaviour of scale factors in the synchronous time are singled out. These subclasses contain sub-families of solutions with accelerated expansion of certain factor spaces.

Here we deal with a model governed by the action

$$S_g = \int d^Dx \sqrt{|g|} \left\{ R[g] - h_{\alpha\beta} g^{MN} \partial_M \varphi^\alpha \partial_N \varphi^\beta - \sum_{a \in \Delta} \frac{\theta_a}{n_a} \exp[2\lambda_a(\varphi)](F^a)^2 \right\}$$

(1.1)

where $g = g_{MN}(x) dx^M \otimes dx^N$ is a metric, $\varphi = (\varphi^\alpha) \in \mathbb{R}^l$ is a vector of scalar fields, $(h_{\alpha\beta})$ is a constant symmetric non-degenerate $l \times l$ matrix ($l \in \mathbb{N}$), $\theta_a = \pm 1$, $F^a = dA^a = \frac{1}{n_a} F_{M_1...M_{n_a}}^a dz^{M_1} \wedge ... \wedge dz^{M_{n_a}}$ is a $n_a$-form ($n_a \geq 1$), $\lambda_a$ is a 1-form on $\mathbb{R}^l$; $\lambda_a(\varphi) = \lambda_{a\alpha} \varphi^\alpha$, $a \in \Delta$, $\alpha = 1, \ldots, l$. In (1.1) we denote $|g| = |\det(g_{MN})|$, $(F^a)^2_g = F_{M_1...M_{n_a}}^a F_{N_1...N_{n_a}}^a g_{M_1N_1} ... g_{M_{n_a}N_{n_a}}$, $a \in \Delta$. Here $\Delta$ is some finite set. For pseudo-Euclidean metric of signature $(-, +, \ldots, +)$ all $\theta_a = 1$.

The paper is organized as following. In Section 2 we consider cosmological-type solutions with composite intersecting $S$-branes from [15, 9, 17] on product of Ricci-flat spaces obeying the "orthogonal" intersection rules. Section 3 is devoted to exceptional ("inflationary-type") $S$-brane solutions.
2 Cosmological-type solutions with composite intersecting \( p \)-branes

2.1 Solutions with \( n \) Ricci-flat spaces

Let us consider a family of solutions to field equations corresponding to the action (1.1) and depending upon one variable \( u \) (see also [7, 8]). These solutions are defined on the manifold

\[ M = (u_-, u_+) \times M_1 \times M_2 \times \ldots \times M_n, \tag{2.1} \]

where \((u_-, u_+), u \in \mathbb{R}\), and have the following form

\[ g = \left( \prod_{s \in S} [f_s(u)]^{2d(I_s)h_s/(D-2)} \right) \left\{ \exp(2c^0 u + 2\bar{c}^0) w du \otimes du + \right. \]

\[ \sum_{i=1}^n \left( \prod_{s \in S} [f_s(u)]^{-2h_s \delta_i I_s} \right) \exp(2c^i u + 2\bar{c}^i) g^i \right\}, \tag{2.2} \]

\[ \exp(\varphi^\alpha) = \left( \prod_{s \in S} f_s^{h_s \chi_s \lambda_\alpha_s} \right) \exp(c^\alpha u + \bar{c}^\alpha), \tag{2.3} \]

\[ F^a = \sum_{s \in S} \delta^a_{a_s} F^s, \tag{2.4} \]

\( \alpha = 1, \ldots, l; a \in \Delta \).

In (2.2) \( w = \pm 1 \), \( g^i = g_{m_i n_i}^i(y_i)dy^m_i \otimes dy^n_i \) is a Ricci-flat metric on \( M_i \), \( i = 1, \ldots, n \),

\[ \delta_{ij} = \sum_{I \in I} \delta_{ij} \tag{2.5} \]

is the indicator of \( i \) belonging to \( I \): \( \delta_{ii} = 1 \) for \( i \in I \) and \( \delta_{ij} = 0 \) otherwise.

The \( p \)-brane set \( S \) is by definition

\[ S = S_e \sqcup S_m, \quad S_v = \sqcup_{a \in \Delta} \{a\} \times \{v\} \times \Omega_{a,v}, \tag{2.6} \]

\( v = e, m \) and \( \Omega_{a,e}, \Omega_{a,m} \subset \Omega \), where \( \Omega = \Omega(n) \) is the set of all non-empty subsets of \( \{1, \ldots, n\} \). Here and in what follows \( \sqcup \) means the union of non-intersecting sets. Any \( p \)-brane index \( s \in S \) has the form

\[ s = (a_s, v_s, I_s), \tag{2.7} \]

where \( a_s \in \Delta \) is colour index, \( v_s = e, m \) is electro-magnetic index and the set \( I_s \in \Omega_{a_s, v_s} \) describes the location of \( p \)-brane worldvolume.
The sets $S_e$ and $S_m$ define electric and magnetic $p$-branes, correspondingly. In (2.3) \[ \chi_s = +1, -1 \] for $s \in S_e, S_m$, respectively. In (2.4) forms \[ F_s = Q_s f_s^{-2} du \wedge \tau(I_s), \] \[ s \in S_e, \] correspond to electric $p$-branes and forms \[ F_s = Q_s \tau(I_s), \] \[ s \in S_m, \] correspond to magnetic $p$-branes; $Q_s \neq 0$, $s \in S$. Here and in what follows \[ \bar{I} \equiv I_0 \setminus I, \quad I_0 = \{1, \ldots, n\}. \] All manifolds $M_i$ are assumed to be oriented and connected and the volume $d_i$-forms \[ \tau_i \equiv \sqrt{|g^i(y_i)|} \ dy_i^1 \wedge \ldots \wedge dy_i^{d_i}, \] and parameters \[ \varepsilon(i) \equiv \text{sign}(\det(g^i_{m,n})) = \pm 1 \] are well-defined for all $i = 1, \ldots, n$. Here $d_i = \dim M_i$, $i = 1, \ldots, n$; $D = 1 + \sum_{i=1}^{n} d_i$. For any set $I = \{i_1, \ldots, i_k\} \in \Omega$, $i_1 < \ldots < i_k$, we denote \[ \tau(I) \equiv \tau_{i_1} \wedge \ldots \wedge \tau_{i_k}, \] \[ d(I) \equiv \sum_{i \in I} d_i, \] \[ \varepsilon(I) \equiv \varepsilon(i_1) \ldots \varepsilon(i_k). \] The parameters $h_s$ appearing in the solution satisfy the relations \[ h_s = (B_{ss})^{-1}, \] where \[ B_{ss'} \equiv d(I_s \cap I_{s'}) + \frac{d(I_s)d(I_{s'})}{2 - D} + \chi_s \chi_{s'} \lambda_{\alpha a_s} \lambda_{\beta a_s'} h^\alpha \beta, \] \[ s, s' \in S, \] with $(h^{\alpha \beta}) = (h_{\alpha \beta})^{-1}$. Here we assume that \[ (i) \quad B_{ss} \neq 0, \] \[ (2.19) \]
for all \( s \in S \), and
\[
(ii) \quad B_{ss'} = 0, \tag{2.20}
\]
for \( s \neq s' \), i.e. canonical (orthogonal) intersection rules are satisfied.

The moduli functions read
\[
f_s(u) = R_s \sinh(\sqrt{C_s}(u - u_s)), \quad C_s > 0, \quad h_s \varepsilon_s < 0; \tag{2.21}
\]
\[
R_s \sin(\sqrt{|C_s|}(u - u_s)), \quad C_s < 0, \quad h_s \varepsilon_s < 0; \tag{2.22}
\]
\[
R_s \cosh(\sqrt{C_s}(u - u_s)), \quad C_s > 0, \quad h_s \varepsilon_s > 0; \tag{2.23}
\]
\[
|Q^s||h_s|^{-1/2}(u - u_s), \quad C_s = 0, \quad h_s \varepsilon_s < 0, \tag{2.24}
\]
where \( R_s = |Q_s||h_sC_s|^{-1/2}, \) \( C_s, \) \( u_s \) are constants, \( s \in S \).

Here
\[
\varepsilon_s = (\varepsilon[g])^{(1-\chi_s)/2}\varepsilon(I_s)\theta_{a_s}, \tag{2.25}
\]
\( s \in S, \) \( \varepsilon[g] \equiv \text{sign}(|\det(g_{MN})|) \). More explicitly (2.25) reads: \( \varepsilon_s = \varepsilon(I_s)\theta_{a_s} \) for \( v_s = e \) and \( \varepsilon_s = -\varepsilon[g]\varepsilon(I_s)\theta_{a_s} \) for \( v_s = m \).

Vectors \( c = (c^A) = (c^i, c^\alpha) \) and \( \bar{c} = (\bar{c}^A) \) obey the following constraints
\[
\sum_{i \in I_s} d_i c^i - \chi_s \lambda_{a_s \alpha} c^\alpha = 0, \quad \sum_{i \in I_s} d_i \bar{c}^i - \chi_s \lambda_{a_s \alpha} \bar{c}^\alpha = 0, \quad s \in S, \tag{2.26}
\]
\[
c^0 = \sum_{j=1}^n d_j c^j, \quad \bar{c}^0 = \sum_{j=1}^n d_j \bar{c}^j, \tag{2.27}
\]
\[
\sum_{s \in S} C_s h_s + h_{a\beta} c^\alpha c^\beta + \sum_{i=1}^n d_i(c^i)^2 - \left( \sum_{i=1}^n d_i c^i \right)^2 = 0. \tag{2.28}
\]

Here we identify notations for \( g^i \) and \( \hat{g}^i \), where \( \hat{g}^i = p^*_s g^i \) is the pullback of the metric \( g^i \) to the manifold \( M \) by the canonical projection: \( p_i : M \rightarrow M_i, \) \( i = 1, \ldots, n \). An analogous agreement will be also kept for volume forms etc.

Due to (2.9) and (2.10), the dimension of \( p \)-brane worldvolume \( d(I_s) \) is defined by
\[
d(I_s) = n_{a_s} - 1, \quad d(I_s) = D - n_{a_s} - 1, \tag{2.29}
\]
for \( s \in S_e, S_m \), respectively. For a \( p \)-brane we have \( p = p_s = d(I_s) - 1 \).

**Restrictions on \( p \)-brane configurations.** The solutions presented above are valid if two restrictions on the sets of composite \( p \)-branes are
satisfied \[7\]. These restrictions guarantee the block-diagonal form of the energy-momentum tensor and the existence of the sigma-model representation (without additional constraints) \[22\].

The first restriction reads
\[(R1) \quad d(I \cap J) \leq d(I) - 2, \quad (2.30)\]
for any \(I, J \in \Omega_{a,v}, \ a \in \triangle, \ v = e, m \) (here \(d(I) = d(J)\)).

The second restriction is following one
\[(R2) \quad d(I \cap J) \neq 0, \quad (2.31)\]
for \(I \in \Omega_{a,e} \) and \(J \in \Omega_{a,m}, \ a \in \triangle\).

### 2.2 Minisuperspace-covariant notations

Here we consider the minisuperspace covariant relations from \[5, 22\] for the sake of completeness. Let
\[(\bar{G}_{AB}) = \left( \begin{array}{cc} G_{ij} & 0 \\ 0 & h_{\alpha\beta} \end{array} \right), \quad (\bar{G}^{AB}) = \left( \begin{array}{cc} G^{ij} & 0 \\ 0 & h^{\alpha\beta} \end{array} \right) \quad (2.32)\]
be, correspondingly, a (truncated) target space metric and inverse to it, where (see \[23\])
\[G_{ij} = d_i \delta_{ij} - d_i d_j, \quad G^{ij} = \delta^{ij} d_i + \frac{1}{2 - D}, \quad (2.33)\]
and
\[U^s a^A = \sum_{i \in I_s} d_i c^i - \chi_s \lambda_{a,\alpha} c^\alpha, \quad (U^s_A) = (d_i \delta_{I_s}, -\chi_s \lambda_{a,\alpha}), \quad (2.34)\]
are co-vectors, \(s = (a, v_s, I_s) \in S\) and \((c^A) = (c^i, c^\alpha)\).

The scalar product from \[22\] reads
\[(U, U') = \bar{G}^{AB} U_A U_B', \quad (2.35)\]
for \(U = (U_A), U' = (U_A') \in \mathbb{R}^N, \ N = n + l\).

The scalar products for vectors \(U^s\) were calculated in \[22\]
\[(U^s, U^{s'}) = B_{ss'}, \quad (2.36)\]
where \( s = (a_s, v_s, I_s) \), \( s' = (a_{s'}, v_{s'}, I_{s'}) \) belong to \( S \) and \( B_{ss'} \) are defined in (2.18). Due to relations (2.20) \( U^s \)-vectors are orthogonal, i.e.

\[
(U^s, U^{s'}) = 0,
\]

for \( s \neq s' \).

The linear and quadratic constraints from (2.26) and (2.28), respectively, read in minisuperspace covariant form as follows:

\[
U^s_A c^A = 0, \quad U^s_A c^B = 0,
\]

(2.38)

\( s \in S \), and

\[
\sum_{s \in S} C_s h_s + \bar{G}_{AB} c^A c^B = 0.
\]

(2.39)

## 3 Special solutions

Now we consider a special case of classical solutions from the previous section when \( C_s = u_s = c^i = c^\alpha = 0 \) and

\[
B_{ss} \varepsilon_s < 0,
\]

(3.1)

\( s \in S \).

We get two families of solutions written in synchronous-type variable with:

A) power-law dependence of scale factors for \( B \neq -1 \),

B) exponential dependence of scale factors for \( B = -1 \),

where

\[
B = \sum_{s \in S} h_s \frac{d(I_s)}{D - 2}.
\]

(3.2)

Remind that \( h_s = (B_{ss})^{-1} \).

### 3.1 Power-law solutions

Let us consider the solution corresponding to the case \( B \neq -1 \). The solution reads

\[
g = w d\tau \otimes d\tau + \sum_{i=1}^n A_i \tau^{2\nu_i} \hat{g}^i,
\]

(3.3)

\[
\varphi^\alpha = \frac{1}{B + 1} \sum_{s \in S} \chi_s h_s \lambda^\alpha_a \ln \tau + \varphi_0^\alpha,
\]

(3.4)
where $\tau > 0$, 

$$\nu_i = -\frac{1}{B+1} \sum_{s \in S} h_s \left( \delta_{iI_s} - \frac{d(I_s)}{D-2} \right),$$  

(3.5) 

$i = 1, \ldots, n$ and 

$$|h_s| \left( \prod_{i \in I_s} A_i^{d_i} \right) \exp(2\chi_s \lambda_{a_s,\alpha} \varphi^a_0) = Q_s^2 |B + 1|^2,$$  

(3.6) 

$s \in S$; and $A_i > 0$ are arbitrary constants. 

The elementary forms read 

$$F^s = \frac{|h_s| A^{1/2}}{Q_s(B + 1)|B + 1|} \tau^{-(B+2)/(B+1)} d\tau \wedge \tau(I_s),$$  

(3.7) 

$s \in S_e$, (for electric case) and forms 

$$F^s = Q_s \tau(I_s),$$  

(3.8) 

$s \in S_m$, (for magnetic case). Here and in what follows $w = \pm 1$, $Q_s \neq 0$, $s \in S$, and $A = \prod_{i=1}^n A_i^{d_i}$. 

We see that these solutions depend on charged densities of branes, their dimensions and intersections, dilatonic couplings and the number of dilatonic fields. 

In the special case of electric $S$-branes of maximal dimension $d(I_s) = D - 1$ the metric and scalar fields are coinciding (up to notations) with the solutions obtained in [4] when signature restrictions (3.1) are obeyed. Since solutions from [4] contain a subfamily of solutions with accelerated expansion of factor spaces, we are led to non-empty set of solutions with ”acceleration” in the model under consideration [24]. 

### 3.2 Solutions with exponential scale factors

Here we consider the solution corresponding to the case $B = -1$. The solution reads 

$$g = w d\tau \otimes d\tau + \sum_{i=1}^n A_i \exp(2M \mu_i \tau) g^i,$$  

(3.9) 

$$\varphi^a = -M \tau \sum_{s \in S} h_s \chi_s \lambda_{a_s}^a + \varphi^a_0,$$  

(3.10)
where
\[ Q_s^2 \exp(-2\chi_s \lambda_a \alpha_s \phi^a_0) = |h_s| M^2 \prod_{i \in I_s} A_i^d_s, \quad (3.11) \]

\[ s \in S, \]

\[ \mu_i = \sum_{s \in S} h_s \left( \delta_i I_s - \frac{d(I_s)}{D-2} \right), \quad (3.12) \]

\( M \) is parameter and \( A_i > 0 \) are arbitrary constants, \( i = 1, \ldots, n \).

The elementary forms read
\[ \mathcal{F}_s = \frac{|h_s|^2 A_s^{1/2}}{Q_s} M^2 e^{M \tau} d\tau \wedge \tau(I_s), \quad (3.13) \]

for \( s \in S_e \), and \( \mathcal{F}_s = Q_s \tau(\bar{I}_s) \) for \( s \in S_m \).

In the cosmological case \( w = -1 \) we get an accelerated expansion of factor space \( M_i \) if and only if \( \mu_i M > 0 \) [24]. We see again that these solutions depend on charge densities of branes, their dimensions and intersections, dilatonic couplings and the number of dilatonic fields.

4  Conclusions

In this paper we considered generalized S-brane solutions with orthogonal intersection rules and \( n \) Ricci-flat factor spaces in the theory with several scalar fields and antisymmetric forms. We singled out subclasses of solutions with power-law and exponential behaviour of scale factors depending in general on charge densities of branes, their dimensions and intersections, dilatonic couplings and the number of dilatonic fields. These subclasses contain sub-families of solutions with accelerated expansion of certain factor spaces [24], e.g. those considered in our earlier paper [4] (with signature restrictions imposed).

We note that in our approach the intersection rules for composite S-branes have a minisuperspace covariant form, i.e. they are formulated in terms of scalar products of brane \( U \)-vectors and generally (see [15]) are classified by Cartan matrices of (semi-simple) Lie algebras. The intersection rules considered in this paper correspond to the Lie algebra \( A_1 + \ldots + A_1 \).

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