Thermocapillary model of formation of surface nanostructure in metals at electron beam treatment

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Abstract. The paper presents the thermocapillary model for the formation of nanostructures in surface layers of materials. It is based on Navier-Stokes hydrodynamic equations, the thermal conductivity equation, the state equation and the boundary conditions. A search for the solutions in the form of a progressive wave has been carried out. The dispersion equation has been obtained and analyzed. The dependence of the instability increment on wavelength has been built. The values of the critical wavelength, at which thermocapillary instability for iron and titanium comes, have been obtained. Value comparison of the calculated wavelengths with cell sizes of crystallization has showed a satisfactory agreement.

1. Introduction

Earlier in the papers [1–3], it has been found out that the fatigue life of steels of different structural classes irradiated by low-energy high-current electron beams from 10 to 40 J/cm² increases in 2–5 times compared with the non-irradiated samples. One of the reasons for this increase, for example for pearlitic steel, is considered to be, firstly, the melting of the surface layer, leading to the formation of columnar structures. The transverse dimensions of the columns change insignificantly with the increase of the beam energy [3] ranging from 0.3 to 0.6 μm, and the longitudinal dimensions that characterize the thickness of the melted layer, undergo a significant increase. Secondly, there is a hardening of the surface layer due to the formation of a martensitic structure; thirdly, the formation in the crystallization cells of nanoscale martensite and, fourthly, the graphite precipitation at the cell borders.

The electron beam treatment of titanium alloys [4–6] also leads to the formation of a gradient nanostructures and strengthening of their surface layers. This leads to the increased wear resistance. Thermal effect modeling [6] of the electron beam on titanium, carburized by electric explosion has showed that carbon particles of nanometer range are dissolved in about 10 μs. This suggests that in the process of the combined treatment a graphitized carbon fiber should be used, in which the diameter of microfibrils is about of tens of nanometers.

The impact of the electron beams on the silumin has been studied in [7–9]. In [7] it has been found out that low-energy high-current electron beams contribute to the dissolution of large silicon particles in the layer with the thickness of from 40 to 55 μm with the formation of cellular-dendritic structure. Silumin hardening is due to the formation of substitution solid solution, as well as the dispersion of the structural components. In [8, 9] it has been shown that electron beams contribute to
the formation in the surface layers of a nanocrystalline structure, which also contributes to its hardening.

The formation models of gradient structures under the influence of concentrated energy fluxes have been considered in the works [10 – 15]. The main formation mechanism of nanoscale structures with this effect is the hydrodynamic instability of a shear flow of the molten layer relative to solid supports. In the papers [10, 11] it has been found out that the nanostructure formation in the surface layer of the material, when exposed to a heterogeneous plasma flows of electric explosion of conductors, is due to Kelvin-Helmholtz instability, which occurs due to the reversal of the plasma front and the emergence of the parallel flow field of plasma and molten metal. The dependence analysis of the instability increment on a wavelength [11] has showed that it has two maximums in nano- and microrange, and microwave mode is formed by the interaction of the layers of a perfect liquid with a surface tension at the border, but nanowave mode is formed due to viscosity. Another formation mechanism of periodic surface structures is that in the irradiated material by reason of the inhomogeneous heating along the depth thermocapillary and thermo-gravitational convections occur [12 – 15]. If the layer is thin enough, the thermocapillary convection mode is dominated; it is caused by the heterogeneity of the surface tension [12]. Flows of this type are characterized not only by the movement of a liquid along the free surface, but also the deformation of the surface [13]. In these works the approximation of incompressible liquid is not enough; that is why it is necessary to use a wide-range state level.

There are also works devoted to the modeling of thermocapillary liquid flows on the surface of the material [16 – 19]. The paper [16] gives the numerical investigation of the flow in a double-layer medium with the neglecting of gravity force influence. It has been established that the introduction of viscous encapsulant leads to the intensity reduction of thermocapillary flow in an encapsulated layer. Interface deformation is small if it passes between the liquid and solid. The increase of viscous encapsulant leads to the formation of pressure gradient in this layer and interface deformation, which differs qualitatively from the observed liquid on a free surface at the lack of this layer. The structure of the encapsulated layer and the interface deformation depends on the viscosity and the thickness. In [17] the influence of the moving local heat source on the flow structure in a thin liquid layer on a horizontal padding has been theoretically analyzed. A two-dimensional problem in the approximation of the layer has been considered. For the case of the small value of Reynold’s number in an attendant coordinate system a steady-state equation has been received; it describes the deformation of a liquid film. The received equation is applicable, particular, for the conditions of the horizontal padding. In the paper [18] the mathematical model of crystallization process has been offered and substantiated; it reflects experimentally established properties of structure formation in the instability zone. Its essence is reduced to the design of a mathematical object including mathematical Kahn-Hillard models [20], heat-and-mass transfer as Stephen task generalization [21]. The results of numerical experiments have shown that in a isotropic surface tension a banded heterogeneity is formed. It on the strength of plane front instability and the development of the recrystallization processes undergoes the deformation. The similar situation is observed in the eutectic crystallization process, when one of the phases disintegrates on the separate fine cells [22]. Further development of the banded structure is the result of the origin of surface waves, which are similar to Marangoni surface. The works [19, 23, 24] are devoted to the research of the long wave instability in the task on thermocapillary convection in a horizontal layer with a free deforming border and a solid bottom. In these works the dispersed equations have been received, from which the equations to define critical values of the parameters for all three main types of stability loss have been obtained. With the fixed values of frequency and amplitude of the oscillations neutral curves of monotonic and oscillatory instability in the form of the dependence of Marangoni number on wave number have been built. The areas of parametric resonances, corresponding to synchronous and subharmonic modes have been defined. A frequency rate, at which a yield to the high-frequency asymptotics occurs, has been found. It has been shown that
the longitudinal vibrations have no effect on convective instability and the transverse ones lead to stabilization of the interface.

Thus, it should be concluded that the analysis of the dispersion relations of thermocapillary waves in the nanoscale range has not been practically conducted. Therefore, the aim of this work is to obtain and analyze the dispersion relations in the nanoscale range of wavelengths.

2. Another section of your paper

To solve the given problem we will consider a viscous heat-conducting incompressible fluid, which has the layer thickness \( h \) on the free surface \( z = \eta(x,y,t) \), and absorbs heat. At the values of thermal flow \( \sim 10^5 \text{ W/cm}^2 \), used in the experiments [1 – 9], the approximation of an incompressible fluid should be considered justified [25]. After the electron beam impact in the liquid layer the temperature profile \( T = T_0(z) \) is set. This temperature is called undisturbed. If the wave vector of the perturbation is directed in the plane \( XOY \), then they depend on coordinates \( x, y \) and the time according to the law \( \exp(i(\omega t - k (mx + ly))) \). We shall only consider the short-wave perturbations. The behavior of such waves is determined by the temperature variations in the surface layers, where \( |z| \leq \kappa^{-1} \), \( \kappa^{-1} \)– the depth of wave attenuations. In this case, the gradient of the unperturbed temperature along the \( z \)-axis will be considered constant.

Write the linearized Navier-Stokes equations:

\[
\frac{\partial u}{\partial t} = -\frac{\partial p}{\rho \partial x} + v \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right), \quad \frac{\partial v}{\partial t} = -\frac{\partial p}{\rho \partial y} + v \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right),
\]

\[
\frac{\partial w}{\partial t} = -\frac{\partial p}{\rho \partial z} + v \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right), \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{1} \]

\[
\frac{\partial T}{\partial t} + \gamma w \frac{dT_0}{dz} = \chi \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right).
\]

Boundary conditions on the melt surface \( z = 0 \) have the form of:

\[
-p + 2\nu \frac{\partial w}{\partial y} = \sigma \left( \frac{\partial^2 \eta}{\partial x^2} + \frac{\partial^2 \eta}{\partial y^2} \right), \quad \rho \nu \left( \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \right) = \frac{\partial \sigma}{\partial x}, \quad \rho \nu \left( \frac{\partial \sigma}{\partial z} + \frac{\partial w}{\partial y} \right) = \frac{\partial \sigma}{\partial y}, \tag{2} \]

\[
\frac{\partial T}{\partial z} = 0, \quad \frac{\partial \eta}{\partial t} = \gamma w,
\]

Adherence conditions and the absence of temperature perturbations on the boundary «melt – solid» \( z = -h \) will have the following form:

\[
u = v = w = 0, \quad T = 0. \tag{3} \]

Here \( u, v, w \) are the components of the perturbed flow velocity along the surface (along the axes \( x \) and \( y \)) and perpendicular to it; \( \rho, \nu, \chi, \sigma(T) \) – density, kinematic viscosity, thermal diffusivity and coefficient of surface tension of the melt; \( p \) – pressure in the fluid layer; \( T \) – temperature perturbation \( T_0(z) \). The value perturbations, which are included in (1) and (2) have the following form:

\[
u(x,y,z,t) = U(z) \exp(\alpha t - i(mx + ly)), \quad v(x,y,z,t) = V(z) \exp(\alpha t - i(mx + ly)), \quad w(x,y,z,t) = W(z) \exp(\alpha t - i(mx + ly)), \tag{4} \]

\[
u(x,y,z,t) = T(z) \exp(\alpha t - i(mx + ly)), \quad \eta(x,y,t) = \eta_0 \exp(\alpha t - i(mx + ly)) \]
where \( \omega \) – perturbation frequency, \( k = (m, l) \) – wave vector, \( i \) – imaginary unit. Substitution (4) into (1) – (3) subject to \( \begin{bmatrix} \frac{\partial \sigma}{\partial x} & \frac{\partial \sigma}{\partial y} & \frac{\partial \sigma}{\partial z} \\ \frac{\partial T}{\partial x} & \frac{\partial T}{\partial y} & \frac{\partial T}{\partial z} \end{bmatrix} \) allows obtaining the system of the ordinary differential equations:

\[
P(z) = i \frac{\rho v}{k^2} (W^\ast(z) - k_1^2 W'(z))
\]

\[
W^\ast(z) - k_1^2 W'(z) - \frac{1}{\rho v} P'(z) = 0
\]

\[
T^\ast(z) - k_2^2 T(z) - G_0 \frac{W(z)}{\chi} = 0
\]

where \( k_1^2 = \frac{\omega}{v} + k^2, \quad k_2^2 = \frac{\omega}{\chi} + k^2, \quad G_0 = \frac{dT_0}{dz} \). The equation system (5) is reduced to one equation, containing \( W(z) \)

\[
W^{IV}(z) - (k^2 - k_1^2) W''(z) + k^2 k_1^2 W(z) = 0
\]

We use (2) – (4) to get boundary conditions

\[
- P(0) + 2 \rho v W'(0) = -\eta_0 k^2 \sigma_0
\]

\[
\rho v (U'(0) - i m W(0)) = -i m \sigma_T \Sigma_0
\]

\[
\rho v (V'(0) - i l W(0)) = -i l \sigma_T \Sigma_0
\]

\[
T'(0) = 0, \quad W(0) = \eta_0 \omega, \quad W'(-h) = W(-h) = T(-h) = 0
\]

where \( \sigma_T = \frac{d\sigma}{dT} = const, \quad \Sigma_0 = G_0 \eta_0 + T(0) \). After the transformation (7) will have the form of:

\[
W^\ast(0) - (k_1^2 - 2k^2) W'(0) - k^3 \frac{\omega_\varepsilon^2}{\omega \omega_{\eta}} W(0) / \omega \omega_{\eta} = 0
\]

\[
W^\ast(0) - k^2 W(0) - k^2 \frac{\omega_\eta^2}{\omega \omega_{\eta}} (W(0) + (\omega / G_0)T(0)) / \omega \omega_{\eta} = 0
\]

\[
T'(0) = 0, \quad W'(-h) = W(-h) = T(-h) = 0
\]

where \( \omega_\varepsilon^2 = k^3 \sigma_0 / \rho, \quad \omega_\eta^2 = k^2 \sigma_T G_0 / \rho, \quad \omega = \nu k^2 \).

3. Results and discussion

The solution of (6) which satisfies the conditions at the boundary of «melt – solid», will be written in the form:

\[
W(z) = W_1(z) + W_2(z).
\]

where

\[
W_1(z) = A_1 k_1 sh (k (z + h)) + A_2 ch (k (z + h)),
\]

\[
W_2(z) = -A_1 k sh (k_1 (z + h)) - A_2 ch (k_1 (z + h)).
\]

Then the boundary value problem for the temperature equation has the form:

\[
T^\ast(z) - k_2^2 T(z) - G_0 \frac{W(z)}{\chi} = 0, \quad T'(0) = 0, \quad T(-h) = 0.
\]
Its solution is

\[ T(z) = \frac{G_0}{\chi} \left( \frac{W_1(z)}{k^2 - k_1^2} + \frac{W_2(z)}{k_1^2 - k_2^2} - S_1 \frac{\text{sh}(k_2(z + h))}{\text{ch}(k_2h)} - S_2 \frac{\text{ch}(k_2z)}{\text{ch}(k_2h)} \right), \quad (12) \]

where

\[ S_1 = \frac{1}{k_2} \left( \frac{W_1'(0)}{k^2 - k_2^2} + \frac{W_2'(0)}{k_1^2 - k_2^2} \right), \quad S_2 = W_1(-h) \left( \frac{1}{k^2 - k_2^2} - \frac{1}{k_1^2 - k_2^2} \right). \]

Subject to the relations of

\[ k^2 - k_2^2 = -\frac{\omega_\chi}{\chi}, \quad k_2^2 - k_1^2 = \frac{\omega(1 - \varepsilon)}{\chi}, \quad \varepsilon = \frac{\nu}{k_2} - \text{Prandtl number}. \]

Convert \( S_1 \) and \( S_2 \) to the form

\[ S_1 = \frac{\chi}{\omega k_2} \left( -W_1'(0) + \frac{\varepsilon}{(1 - \varepsilon)} W_2'(0) \right), \quad S_2 = -\frac{\chi}{\omega(1 - \varepsilon)} A_2. \quad (13) \]

Then (11) will have the form:

\[ T(z) = \frac{G_0}{\omega} \left( -W_1(z) + \frac{\varepsilon}{(1 - \varepsilon)} W_2(z) + \left( W_1'(0) - \frac{\varepsilon}{(1 - \varepsilon)} W_2'(0) \right) \frac{\text{sh}(k_2(z + h))}{k_2 \text{ch}(k_2h)} - \frac{\text{ch}(k_2z)}{(1 - \varepsilon)\text{ch}(k_2h)} A_2 \right) \quad (14) \]

at \( z=0 \)

\[ T(0) = \frac{G_0}{\omega} \left( -W_1(0) + \frac{\varepsilon}{(1 - \varepsilon)} W_2(0) + \left( W_1'(0) - \frac{\varepsilon}{(1 - \varepsilon)} W_2'(0) \right) \frac{\text{th}(k_2h)}{k_2} - \frac{A_2}{(1 - \varepsilon)\text{ch}(k_2h)} \right) \quad (15) \]

Subject to (14) convert the system (8) to the form

\[ \omega_\chi \left( k_1^2 + k_1^2 \right) W_1'(0) + 2k^2 W_2'(0) + k^3 \omega_\chi W_1(0) + W_2(0) = 0 \]

\[ 2k^2 \omega_\chi W_1(0) + \left( \omega_\chi \left( k_1^2 + k_1^2 \right) - \omega_\chi^2 k_1^2 / (1 - \varepsilon) \right) W_2(0) + \frac{\omega_\chi^2 k_2^2 A_2}{(1 - \varepsilon)\text{ch}(k_2h)} - \]

\[ - \omega_\chi^2 k_2^2 \left( W_1'(0) - \frac{\varepsilon}{(1 - \varepsilon)} W_2'(0) \right) \frac{\text{th}(k_2h)}{k_2} = 0 \quad (16) \]

\[ 2k^2 \omega_\chi W_1(0) + \left( \omega_\chi \left( k_1^2 + k_1^2 \right) - \omega_\chi^2 k_1^2 / (1 - \varepsilon) \right) W_2(0) + \frac{\omega_\chi^2 k_2^2 A_2}{(1 - \varepsilon)\text{ch}(k_2h)} - \]

\[ - \omega_\chi^2 k_2^2 \left( W_1'(0) - \frac{\varepsilon}{(1 - \varepsilon)} W_2'(0) \right) \frac{\text{th}(k_2h)}{k_2} = 0 \]

Evaluate, coming into (15) values \( A_2, W_2(0), W_2'(0) \) through \( W_1(0), W_1'(0) \), as a result one gets

\[ 2k^2 \omega_\chi W_1(0) + \left( \omega_\chi \left( k_1^2 + k_1^2 \right) - \omega_\chi^2 k_1^2 / (1 - \varepsilon) \right) \left( b_{11} W_1(0) + b_{12} W_1'(0) \right) + \]

\[ + \frac{\omega_\chi^2 k_2^2 \text{ch}(kh)}{(1 - \varepsilon)\text{ch}(k_2h)} \left( W_1(0) - W_1'(0) \text{th}(kh) / k \right) - \]

\[ - \omega_\chi^2 k_2^2 \left( W_1'(0) - \frac{\varepsilon}{(1 - \varepsilon)} \left( b_{21} W_1(0) + b_{22} W_1'(0) \right) \right) \frac{\text{th}(k_2h)}{k_2} = 0, \]
\[
\begin{align*}
\left( \omega \nu \left( 2k^2 + (k^2 + k_1^2)b_{11} \right) + \frac{\omega_0^2}{(1 - \varepsilon)} \left( \frac{\cosh (kh)}{b_{11} + \frac{b_{21} \varepsilon \tanh (kzh)}{k_2} - b_{11}} \right) \right) W_1(0) + \\
\left( \omega \nu \left( k^2 + k_1^2 \right)b_{12} \right) - \frac{\omega_0^2}{(1 - \varepsilon)} \left( \frac{\sinh (kh)}{kzh} + \left( (1 - \varepsilon) - \varepsilon b_{22} \right) \frac{\tanh (kzh)}{k_2} \right) W_1'(0) = 0
\end{align*}
\]  
\tag{17}

where

\[
\begin{align*}
b_{11} &= (k \sinh (kh)) \sinh (kh) - k \cosh (kh) \cosh (kh)/k_1,  \\
b_{12} &= (k \cosh (kh)) \sinh (kh) - k \sinh (kh) \cosh (kh)/(kk_1),  \\
b_{21} &= (k \cosh (kh)) \sinh (kh) - k \sinh (kh) \cosh (kh),  \\
b_{22} &= (k \sinh (kh)) \sinh (kh) - k \cosh (kh) \cosh (kh)/k.
\end{align*}
\tag{18}

Substitution (18) into (17) leads to the system

\[
A_{11} W_1(0) + A_{12} W_1'(0) = 0; \quad (A_{21} + B_1) W_1(0) + (A_{22} + B_1) W_1'(0) = 0.
\tag{19}
\]

Where

\[
\begin{align*}
A_{11} &= 2k^2 \omega \nu \left( 2k^2 + k_1^2 \right) + k^3 \omega_0^2 \left( 1 + b_{11} \right), \\
A_{12} &= \omega \nu \left( 2k^2 + k_1^2 \right), \\
A_{21} &= \omega \nu \left( 2k^2 + k_1^2 \right), \\
A_{22} &= \omega \nu \left( 2k^2 + k_1^2 \right)b_{12}, \\
B_1 &= \frac{\omega_0^2}{(1 - \varepsilon)} \left( \frac{\cosh (kh)}{kzh} + \frac{b_{21} \varepsilon \tanh (kzh)}{k_2} - b_{11} \right),  \\
B_2 &= \frac{\omega_0^2}{(1 - \varepsilon)} \left( \frac{kzh + \frac{b_{21} \varepsilon \tanh (kzh)}{k_2} + \left( (1 - \varepsilon) - \varepsilon b_{22} \right) \frac{\tanh (kzh)}{k_2}}{kzh} \right).
\end{align*}
\tag{20}
\]

Equating the determinant of the system (17) to zero we get the dispersion equation for the finite layer

\[
\Delta = \Delta_1 + \Delta_2, \quad \Delta_1 = A_{11}A_{22} - A_{12}A_{21}, \quad \Delta_2 = A_{11}B_2 - A_{12}B_1.
\tag{21}
\]

Consider the case of short waves \( k h \gg 1 \), then the approximation of a semi-infinite layer is fair. As a result we will get the dispersion equation for such layer:

\[
\begin{align*}
\omega^2 \left( \frac{\delta (\omega_0^2 + \delta k^2v^2) - 4k_1 k^2v^2 + \omega_0^2}{(1 + \delta(1 - k_1/k_2)) (\omega_0^2 + 2k^2v^2 + \omega_0^2) - k / k_2 (2k_1 k^2v^2 + \omega_0^2)} \right) &= 0.
\end{align*}
\tag{22}
\]

where \( \delta = \frac{\varepsilon}{1 - \varepsilon} \). It can be reduced to an algebraic equation of the 16-th degree. This equation depends on three dimensionless parameters: the Prandtl number and two squares of frequency relations \( a^2, b^2 \) \(( a = \omega_0 / \omega_\nu, b = \omega_T / \omega_\nu \)). For numerical calculations we consider that \( k_0 = 10^7 m^{-1}, h = 10^{-5} m, G_0 = (T_b - T_m)/h ) \approx 10^8 K/m, T_b, T_m \) – the melting and boiling temperatures. Using these values, one will calculate the key parameters for iron and titanium by the formulas

\[
a_0^2 = \frac{\sigma_0}{v^2 \rho k_0}, \quad b_0^2 = \frac{\sigma_T G_0}{v^2 k_0^2}.
\tag{23}
\]
These calculations are shown in the table, from which it is visible what can be used as a first approximation $\varepsilon \approx 0$ (the Prandtl number is equal to zero). Then the algebraic equation splits into two equations

\[
(z^3 + z^2 + 3z - 1)(z^2 - 1)^2(z - 1) - b^2(z^2 - 1)(z + 1)^2 + a^2(z^2 - 1)^2 - 2a^2b^2 = 0
\]

(24)

\[
(z^3 + z^2 + 3z - 1)(z^2 - 1) - b^2(z - 1) + a^2(z + 1) = 0
\]

Here it is denoted $z = \frac{k_1}{k}, a^2 = \frac{k_0^2}{k}, b^2 = \frac{k_0^2}{k^2}$.

We select only such roots, in which the real part is greater than zero (Re(z)>0). On the detected values of the roots of $z=x+iy$ we find the expression for the complex frequency

\[
\omega = \omega_v(z^2 - 1) = \omega_v(x^2 - y^2 - 1) + 2i\omega_vxy
\]

(25)

Decrement is calculated by the formular

\[
\alpha = \text{Re}(\omega_v(z^2 - 1)) = \omega_v(x^2 - y^2 - 1)
\]

(26)

The mentioned procedure allows determining the dependence of the decrement on the wave number $k$. Figures 1 a and b show the dependences $\alpha(k/k_0)$ for iron and titanium.

### Table 1. Physical parameters

| Parameter                                      | Iron     | Titanium |
|------------------------------------------------|----------|----------|
| Density, $\rho$, $10^3$ kg/m$^3$               | 7.87-7.02| 4.5-4.1  |
| Surface tension coefficient, $\sigma$, N/m    | 1.872    | 1.650    |
| $d\sigma/dT$, $10^{-3}$N/K·m                   | -0.49    | -0.26    |
| Coefficient of viscosity $\nu$, $10^{-7}$ m$^2$/s at 100-2000ºC | 4.0      | 13       |
| Coefficient of thermal conductivity $\chi$, $10^{-7}$ m$^2$/s | 150      | 40       |
| Melting temperature, ºC                        | 1536     | 1682     |
| Boiling temperature, ºC                        | 2862     | 3260     |
| Prandtl number $Pr = \nu/\chi$                 | 0.02     | 0.3      |
| $\omega_v = \nu k_0^2$, $\mu$s$^{-1}$          | 40       | 130      |
| Nondimensional capillary number, $a_0$         | 12.2     | 4.7      |
| Nondimensional thermodcapillary number, $b_0$  | 55.3     | 12.4     |
Figure 1. Dependencies of instability decrement on wave number for iron (a) and titanium (b).

From the analysis of the represented Fig. 1. it can be seen that the critical value of the nondimensional wave number for iron is equal to \( k = 5 \). This corresponds to the critical wavelength \( \lambda = 125 \text{ nm} \). As mentioned in the Introduction, the experimental values of cell sizes of crystallization are approximately of 0.3 \( \mu \text{m} \) [3], which is in three times more than the calculated wavelength. This discrepancy can be explained by the fact that the experiment has been carried out with steel containing 0.76 \% of carbon. Such small additives have an effect on the magnitude of the surface tension of liquid iron [26].

For titanium (Figure 1 b) the critical wavelength is 419 nm; that does not contradict the experimental values [4–6].

4. Conclusions
Thermocapillary model of the impact of low-energy high-current electron beams on metals has been developed. The dispersion equation, which is given by the algebraic equation of the 16-th degree, and in the approximation of short waves splits into two equations, has been obtained. For iron and titanium the critical wavelength, at which thermocapillary instability comes, has been defined.

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