More on Ramond–Ramond, SYM, WZ couplings, and their corrections in IIA

Ehsan Hatefi\textsuperscript{1,2,3,a,b,c}

\textsuperscript{1} International Centre for Theoretical Physics, Strada Costiera 11, Trieste, Italy
\textsuperscript{2} Centre for Research in String Theory, School of Physics and Astronomy, Queen Mary University of London, Mile End Road, London E1 4NS, UK
\textsuperscript{3} National Institute for Theoretical Physics, School of Physics and Centre for Theoretical Physics, University of the Witwatersrand, Wits 2050, Johannesburg, South Africa

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Abstract We obtain the closed form of the correlation function of one current and four spin operators (with different chiralities) in type IIA superstring theory. The complete form of the S-matrix of one gauge, two fermions (with different chiralities) and one closed string Ramond–Ramond for all order $\alpha'$ type IIA has been explored. Moreover, we make use of a different gauge fixing to be able to find all the closed forms of $\langle V_C V_{\bar{A}} V_A V_{\bar{\psi}} \rangle$ correlators. An infinite number of $t$, $s$-channel fermion poles of this S-matrix in the field theory of IIA is generated. Unlike the closed form of the correlators of the same amplitude of IIB, for various $p$, $n$ cases in type IIA we do have different double poles in $t$, $(t+s+u)$ and in $s$, $(t+s+u)$ channels and we produced them. We also find new Wess–Zumino couplings of IIA with their third order $\alpha'$ corrections as well as a new form of higher derivative corrections (with different coefficient from its IIB one) to SYM couplings at third order of $\alpha'$. Using them, we are able to produce a $(t+s+u)$ channel scalar pole of the $(V_C V_A V_{\bar{\psi}} V_{\bar{\psi}})$ amplitude for the $p + 2 = n$ case. Finally we make some comments on $\alpha'$ corrections to IIA superstring theory.

1 Introduction

It is well known that the fundamental objects, the so called $D_p$-branes ($p$ is the spatial dimension of a brane) \cite{1–4} could play the most important role in both type IIA and IIB superstring theories as well as in AdS/CFT correspondence. Let us start by mentioning several crucial papers on superstring theories \cite{5–8} and refer to some pioneering papers that either pointed out the mathematical structures or dealt with various symmetries of the scattering amplitudes/gauge theories \cite{9–11}. To reveal the dynamics of a brane one needs to consider diverse transitions of open/closed strings, where several processes have been very well realized in \cite{12}.

For several reasons, we are not interested in applying dualities, however, it is always nice to have understanding of string dualities and clearly reveal most of the dual descriptions of IIA (IIB) theories that to our knowledge originated from \cite{13}.

In this paper we try to find the effective actions of type IIA for all BPS branes\textsuperscript{2} of two fermions (with different chiralities), one closed string Ramond–Ramond (RR), and a massless gauge field in the world volume space by making use of superstring scattering from different D-branes. One could view some part of those actions from the point of view of supergravity \cite{15}, for which the ADM formalism could be applied to IIB and the realization of both AdS and dS space brane worlds is achieved \cite{15}.

Concerning the complete form of the IIA S-matrix, we are able to first of all find the third order of $\alpha'$ higher derivative corrections to SYM couplings and secondly distinguish $\alpha'$ corrections from type IIB. Our computations show that the coefficients of the corrections should be different in type IIA, while in \cite{18} we have proven that even the general structure of IIA corrections of two fermions and two scalars is different from its IIB one.

In order to discuss the dynamical facet of various branes, we should focus on the proper effective actions. The bosonic effective actions in the presence of different configurations are addressed in Myers’ paper \cite{19} with some clarifications

\textsuperscript{a} e-mail: ehsanhatefi@gmail.com
\textsuperscript{b} e-mail: ehatefi@ictp.it
\textsuperscript{c} e-mail: ehsan.hatefi@cern.ch

\textsuperscript{1} As an instance of dual prescriptions, one might point out the mixed system of $D0/D4$ with some particular applications \cite{14}.
\textsuperscript{2} They accompany the RR charge as well.
\textsuperscript{3} In \cite{16,17} different boundary conditions are applied to actually observe the appearance of D-branes in flat de-compactification space.
concerning all order higher derivative corrections added in [20]. On the other hand, to our knowledge, the supersymmetrized actions of those bosonic actions have not yet been completely derived; however, let us address an original work in this topic [21]. For a single bosonic D_p-brane and for its supersymmetrized action it seems appropriate to refer to [22] and [23–27].

In order to avoid explaining all the details of the description of the world volume brane dynamics (Myers terms, the Chern–Simons, Wess–Zumino (WZ), and Born–Infeld actions) we just refer to the basic references [20,28–30]. Three standard approaches to the effective field theory of either the Myers terms or Taylor expansion/pull-back methods including the method for exploring all order \( \alpha' \) higher derivative corrections are explicitly explained in [31] (providing full details). Nevertheless, we argued in [20] that some other methods are needed to be able to construct new couplings of either BPS [20,32] or non-supersymmetric branes [33–37].

Some great results due to the efforts in [38–40] have already appeared. The first goal of this paper is just to employ the direct scattering amplitude process to see whether or not one could extract more information on the general structure of the effective actions in IIA; and the second goal is to describe in an efficient way the dynamical aspects of branes. We refer to [41–49] on the scattering of BPS branes or to provide more applications to the BPS/non-BPS branes [50–54]. Eventually having worked out new WZ and Myers terms [30,35–37,55–57], we can definitively get some of the corrections of string theory [58].

Given the fact that there is no derivation for the AdS/CFT correspondence, it is always good to have some theoretical methods at hand to get \( \alpha' \) higher derivative corrections of IIA superstring theory. Indeed there is a very direct correspondence both in AdS/CFT and in string theory between a closed and an open string so it should be an inevitable task to have embedded the corrections in a compact form. Thus if we deal with open–closed amplitudes of type II, we may hope to shed light on various features of corrections, dualities, and some other features. As a specific example, we have obtained some WZ couplings involving their \( \alpha' \) higher derivative corrections, and in particular it was shown how those corrections might be taken to interpret the \( N^3 \) entropy of M5 brane [14] so that it could be realized as a matter of dissolving branes with lower dimensions inside the higher dimensions. It also happens for the \( D(-1)/D3 \) system in which by applying those \( \alpha' \) corrections we could interpret the \( N^2 \) entropy relation. \( \alpha' \) corrections and new WZ couplings are also applied to flux vacua and M- or F-theory frameworks [59–62].

The organization of the paper is as follows.

We first apply conformal field theory techniques (CFT) to discover the compact/closed form of the correlation function of four spin operators with different chiralities and one current \( \langle V_{\psi} V_{\bar{\psi}} V_{\psi} V_{\bar{\psi}} \rangle \) in ten dimensions of space-time in type IIA superstring theory. The entire form of the S-matrix of one closed string RR, two fermion fields with different chiralities, and one gauge field of IIA with all order \( \alpha' \) closed form have also been explored. Let us stress the fact that due to several differences, we cannot obtain this S-matrix from its IIB one.

First of all unlike the type IIB amplitude there is no \( u \)-channel scalar/gauge pole left over. Secondly, we will obtain various double poles in the \( (t, t + s + u) \) and in \( s, (t + s + u) \) channels of the closed form of \( \langle V_{\psi} V_{\bar{\psi}} V_{\psi} V_{\bar{\psi}} \rangle \) of IIA, but there is no double pole in IIB and the presence of these double poles cannot be confirmed by a duality transformation at all. We also generate all infinite massless fermion poles of \( t, s \)-channels of type IIA for diverse \( p, n \) (\( n \) is the rank of RR field strength) cases, and the extensions of several new Wess–Zumino couplings have been confirmed.

Most importantly their all order \( \alpha' \) higher derivative corrections (related to contact interactions out of fermion poles) without any ambiguity are found. Thus our computations are different from IIB; one could clearly confirm this stark conclusion by comparing both S-matrix computations of all two different theory. We also show that there are no corrections to two fermion–two gauge couplings. Having obtained the entire form of the S-matrix, we are able to discover several new Wess–Zumino couplings with their third order \( \alpha' \) correction and note that those couplings cannot be found by having the IIB form of the S-matrix. It is just the first \( (t + s + u) \)-channel scalar pole for the \( p + 2 = n \) case dictating us that the \( \alpha'^3 \) corrections of two fermions, one scalar, one gauge of IIA have to be accompanied with the same corrections but with different coefficients of their corrections in IIB. We take this fact as an advantage carrying out the direct CFT rather than applying duality transformation. More promising explanations are clarified in [20] and we have shown that one is not able to derive the entire result of \( \langle V_{\psi} V_{\bar{\psi}} V_{\psi} V_{\bar{\psi}} \rangle \) from \( \langle V_{\psi} V_{\bar{\psi}} V_{\psi} V_{\bar{\psi}} \rangle \) by using a T-duality transformation [30].

2 The complete form of \( CA\bar{\psi}\bar{\psi}S \)-matrix in IIA

To get the complete and closed form of the amplitude of one closed string RR, two fermions with different chiralities, and one gauge field in the world volume of BPS branes of IIA, one must use the direct CFT techniques. The reason for this stark conclusion will be clarified in the next sections. We try to address some of the higher point stable BPS functions [30,56,57,64–70] and also emphasize there being various non-BPS [31,34] computations. It is obvious that non-BPS
S-matrices cannot be derived by making use of any dualities (see [35]). The fermion vertex operators of type IIA are of the same as type IIB, however, their chirality in IIA must be changed; for completeness we just write them down:

\[
V_A^{(-2)}(y) = e^{-2\phi(y)} V_A^{(0)}(y),
\]

\[
V_A^{(0)}(x) = \xi_\alpha \left( \partial X^\alpha(x) + \alpha' i k \cdot \psi^\alpha(x) \right) e^{\alpha' i k \cdot X(x)},
\]

\[
V_A^{(-1/2)}(x) = u^\alpha \xi_\alpha e^{-\phi(x)/2} S_\beta(x) e^{\alpha' i q_k (x)},
\]

\[
V_A^{(-1/2)}(x) = u^\alpha e^{-\phi(x)/2} S_\beta(x) e^{\alpha' i q_k (x)}.
\]

\[
V_C^{(-1/2)}(z, \bar{z}) = (P_- H(n) M_p) \alpha\beta e^{-\phi(z)/2} S_{\bar{\alpha}}(z) e^{i \bar{\psi} \cdot X(z)}
\]

\[
\times e^{-\phi(\bar{z})/2} S_{\bar{\beta}}(\bar{z}) e^{i \bar{\psi} \cdot \bar{D} \cdot X(\bar{z})}.
\]

(1)

All on-shell conditions, the definitions of RR’s field strength and charge conjugation in type IIB and type IIA are given, respectively, in [18] and [32,71]. To work with holomorphic functions, we apply the doubling tricks to deal with the usual correlation functions for bosonic and fermionic world sheet fields. We have

\[
\langle X^\mu(z) X^\nu(w) \rangle = -\frac{\alpha'}{2} \eta^{\mu\nu} \log(z - w),
\]

\[
\langle \psi^\mu(z) \psi^\nu(w) \rangle = -\frac{\alpha'}{2} \eta^{\mu\nu} (z - w)^{-1},
\]

\[
\langle \phi(z) \phi(w) \rangle = -\log(z - w).
\]

(2)

To see more standard notations and further details we refer to [31,56].

Since the correlation function of two fermions and one gauge field in type IIA is exactly the same as type IIB, the S-matrices are also the same as they were computed in [72]. Hence, one could easily show that by taking into account these definitions, the \(V_A V_{\bar{\psi}} V_{\bar{\psi}}\) S-matrix could be generated in the same manner in the field theory of IIA as IIB; however, for mixed higher point functions the work gets complicated, as we comment on in a moment.

We put RR in the \((-1)\) picture \((V_R^{(-1/2)}(z, \bar{z}))\), fermions in \((V^{(-1/2)}(x) V^{(-1/2)}(x))\) (their standard pictures), and, since the ghost total charge must be \((-2)\), we then have to put the gauge field just in the zero picture \((V_A^{(0)}(x))\). We find the complete form of this S-matrix in IIA and we show that this S-matrix is different from IIB; thus all closed form of the corrections of type IIB cannot be used for IIA and vice versa.

For the first part of the S-matrix we need to work with the correlation function of four spin operators with different chiralities [73,74].6 Having said that, we find the first part of the mixed amplitude of IIA as follows:

\[
A_1 \sim \int dx_1 dx_2 dx_3 dx_4 dx_5 (P_- H(n) M_p) \phi^\beta
\]

\[
\times \xi_{\alpha} \bar{u}^\delta \left( x_{23} x_{24} x_{34} x_{35} x_{45} \right)^{-1/4} \left( \frac{x_{45} x_{23}}{x_{42} x_{43} x_{52} x_{53}} \right)^{1/4}
\]

\[
\left[ C_\alpha^\beta C_\beta^\delta \left( x_{43} x_{52} - 1 + \frac{1}{2} \left( y(\mu C)_{\alpha\beta} (y(\mu C)_{\delta\rho} \right) \right) \right] \bar{I}_1 \left( \lambda_1 \lambda_2 \lambda_3 \right).
\]

(3)

with

\[
I_1 = \left[ i k_2 \left( x_{42} + x_{52} \right) \right] \left( x_{43} + x_{53} \right) \left( x_{123} + x_{124} \right)
\]

\[
\times \left( x_{23} x_{24} x_{34} x_{35} \right)^2 \left( x_{23} x_{24} x_{34} x_{35} \right)^2 \left( x_{23} x_{24} x_{34} x_{35} \right)^2.
\]

(4)

Above we wrote the S-matrix in such a way that it clearly shows the property of SL(2,R) invariance of the amplitude. We make the change of variables \(x_{ij} = x_i - x_j, x_4 = z = x + iy, x_5 = \bar{z} = x - iy\), and we have

\[
s = -\frac{\alpha'}{2} (k_1 + k_2)^2, \quad t = -\frac{\alpha'}{2} (k_1 + k_2)^2,
\]

\[
u = -\frac{\alpha'}{2} (k_3 + k_2)^2.
\]

In order to explore the closed and complete form of the amplitude with their \(\alpha'\) corrections, one must take integrations on the location of RR closed string, and it means that we have to get rid of the position of open strings by having applied a very particular gauge fixing as \(x_1 = 0, x_2 = 1, x_3 = \infty\). If we do apply that gauge fixing, then we get the following:

\[
A_1^{C A^\alpha \bar{A}^\alpha} \sim (P_- H(n) M_p) \phi^\beta \xi_{\alpha} \bar{u} \eta_{\beta} \int \int d\bar{z} \bar{d}z \left( \frac{z - \bar{z}}{z - \bar{z}} \right)^{2 + 2s}
\]

\[
\times \left| 1 - z \right|^{2 + 2u - 1} \left( \frac{z - \bar{z}}{z - \bar{z}} \right)^{-2\left(t + s + u\right)} \left( \lambda_1 \lambda_2 \lambda_3 \right)
\]

\[
\times \left[ \frac{C_\alpha^\beta C_\beta^\delta}{1 - \bar{z}} - \frac{C_\alpha^\beta C_\beta^\delta}{1 - z} - \frac{1}{2} \left( \mu C \right)_{\alpha\beta} \left( \mu C \right)_{\delta\rho} \right] \left( z - \bar{z} \right)
\]

\[
\times \left( 2i k_2 \left( z + \bar{z} \right) \left( ik_2^2 + ik_4^2 \right) \right)
\]

\[
\times \left( x_{23} x_{24} x_{34} x_{35} \right)^2 \left( x_{23} x_{24} x_{34} x_{35} \right)^2 \left( x_{23} x_{24} x_{34} x_{35} \right)^2.
\]

We need to deal with several important results for the integrals on the upper half plane. For the fourth term above we need to employ the results for the integrals taken in [31] and for the other integrals in the above amplitude we use [75]. Thus one can write down the final result for the first part of the amplitude:

5 We have \(2\pi \alpha' T_P \text{Tr} (\psi \gamma^\alpha D_\alpha \psi)\). Let \(D^\alpha \psi = \partial^\alpha \psi - i [A^\alpha, \psi]\).

6 We have \(\left( S_0 (z) S_\beta (z) S^\beta (z) S^\delta (z) \right) = \left( \frac{x_{45} x_{23}}{x_{42} x_{43} x_{52} x_{53}} \right)^{1/4} \left[ \frac{C^\alpha C^\beta}{x_{43} x_{52}} + \frac{1}{2} \left( \eta(\mu C)_{\alpha\beta} (\eta(\mu C)_{\delta\rho} \right) \right] \).

\[A_1^{CA\bar{\psi}\psi} \sim (P_- \slashed{H}(\omega)M_p)^{a\bar{b}} \xi_{1a} \bar{u}^\gamma u^\delta \]
\[ \times \left\{ ik_2 \left[ sL_1(C_a \bar{C}_\gamma + C_\gamma \bar{C}_a) - 2sL_2(C_a \bar{C}_\beta - C_\beta \bar{C}_a) \right] - ik_2 \left[ tL_1(C_a \bar{C}_\gamma + C_\gamma \bar{C}_a) - 2tL_2(C_a \bar{C}_\beta - C_\beta \bar{C}_a) \right] \right\} \]
\[+ [ik_2 s - ik_2 t](\gamma^\mu C)_{a\bar{b}}(\bar{\gamma}_{\mu} C)_{\bar{\delta}\bar{\delta}}L_3 \right\} \text{Tr} (\lambda_1 \lambda_2 \lambda_3), \] with
\[L_1 = (2)^{-2(t+s+u)+1} \]
\[\times \frac{\Gamma(-u + \frac{3}{2}) \Gamma(-s) \Gamma(-t) \Gamma(-s - u + 1)}{\Gamma(-u - t + \frac{3}{2}) \Gamma(-s - u + 1)}, \]
\[L_2 = (2)^{-2(t+s+u)+1} \]
\[\times \frac{\Gamma(-u + 1) \Gamma(-s + \frac{1}{2}) \Gamma(-t + \frac{1}{2}) \Gamma(-t - s - u + 1)}{\Gamma(-u - t + \frac{3}{2}) \Gamma(-s - u + \frac{3}{2})}, \]
\[L_3 = (2)^{-2(t+s+u)-1} \]
\[\times \frac{\Gamma(-u + \frac{1}{2}) \Gamma(-s) \Gamma(-t) \Gamma(-s - u)}{\Gamma(-u - t + \frac{1}{2}) \Gamma(-s - u + 1)}, \]

Obviously, as we expected, the amplitude is antisymmetric with respect to interchanging the fermions. All the terms including \(L_2\) are related to contact interactions, on the other hand the terms carrying the coefficients of \(-sL_1\) \((-tL_1\) have infinite singularities in the \(r(s)\) channels.

In the closed form of the correlators of one RR, two fermions, and one gauge field of type IIB, we did not have any double pole, while here, as is clear from the closed form of the first part of the amplitude in type IIA, we do have various double poles. Basically \(-sL_3\) \((-tL_3\) showed us the fact that those parts of the S-matrix do include double poles in \(t\) and \((t + s + u)\) \(s\) and \((t + s + u)\) channels, respectively. Of course, if \(\mu\) gets a world volume (transverse) index then we will appropriately have a gauge for the \(n = p\) case (scalar for the \(n = p + 2\) case) poles, where the expansion is just the low energy expansion and all singularities/contact interactions would be generated by having sent all three Mandelstam variables to zero [30].

Now we derive the second part of the S-matrix in type IIA. In order to proceed one has to know the closed form of the correlation function of four spin operators (with different chiralities in ten dimensions) and one current. Indeed the current which consists of two fermion fields in one position comes from the second part of the vertex of the massless gauge field in zero picture.

The method for obtaining this correlation function has been completely explained in [32] and it has been mentioned how to build various combinations of the gamma matrices where we have also used Appendices A.1, A.3, B.3 of [74]. Basically one has to first take the OPE of the current with one spin operator (the result is just a spin operator) and then replace it inside the main correlation function (four spin operators) and carry out the same approaches for the other cases. Eventually we re-compose different combinations of various symmetric and antisymmetric gamma matrices. Given the comprehensive explanations in [18,71], one can find the entire and closed form of the correlation function of four spin operators with different chiralities with one current in ten dimensions of space-time in type IIA as follows:

\[\langle \psi^a \psi^d (z_1) S^\gamma (z_2) S^\delta (z_3) S_a (z_4) S_b (z_5) \rangle = \]
\[\frac{(x_{45} x_{23})^{-\frac{3}{4}}}{(x_{14x_15} x_{12x_13})^{-1/4}} \times \left[ -\frac{1}{2} (\gamma^a \bar{\gamma}^d C^\mu_a C^\mu_b \frac{x_{13x_15} x_{45x_23}}{x_{53}} \right. \]
\[+ \frac{1}{2} (\bar{\gamma}^a C^\delta_{a\bar{b}} C^\delta_{b\bar{a}})^{\bar{a}} x_{14x_15} x_{45x_23} x_{34} \]
\[\left. - \frac{1}{2} (\bar{\gamma}^d C^\mu_{d\bar{a}} C^\mu_{a\bar{d}})^{\bar{d}} x_{14x_15} x_{34} x_{23} x_{45} \right]. \]

Hence, one can write down the second part of the S-matrix:

\[A_2^{CA\bar{\psi}\psi} \sim \int dx_1 dx_2 dx_3 dx_4 dx_5 \left( P_- \slashed{H}(\omega)M_p \right)^{a\bar{b}} \xi_{1a} \bar{u}^\gamma u^\delta \times \left( 2ik_{1b} \bar{u}^\gamma u^\delta \right) \langle \psi^a \psi^d (x_1) S^\gamma (x_2) S_a (x_4) S_b (x_5) \rangle \right\} \text{Tr} (\lambda_1 \lambda_2 \lambda_3), \] with

\[I = |x_{12}|^{\alpha_2 k_1} |x_{13}|^{\alpha_2 k_2} |x_{14x_15}|^{-\frac{3}{2}} k_{1p} \times |x_{23}|^{\alpha_2 k_3} |x_{24x_25}|^{-\frac{3}{2}} k_{2p} \times |x_{34x_35}|^{-\frac{3}{2}} k_{3p} |x_{45}|^{\frac{1}{2}} k_{Dp}. \]

Now by replacing the closed form of the \(\langle \psi^b \psi^d (x_1) : S^\gamma (x_2) : S^\gamma (x_3) : S_a (x_4) \rangle \) correlator, gauge fixing it as we did for the first part of the amplitude, and taking the integrations on the position of the RR, we get the final result of the amplitude in type IIA:

\[A_2^{CA\bar{\psi}\psi} \sim (P_- \slashed{H}(\omega)M_p)^{a\bar{b}} \xi_{1a} (2ik_{1b}) \bar{u}^\gamma u^\delta \times \left( A_{21} + A_{22} + A_{23} + A_{24} + A_{25} \right) \text{Tr} (\lambda_1 \lambda_2 \lambda_3), \]
such that
\[
\mathcal{A}_{21} \sim \left\{ -\frac{1}{2} (y^b \tilde{y}^a)_{Aa} \tilde{C}_a \left[ L_5 + \frac{s}{2} L_6 \right] + \frac{1}{2} (y^b \tilde{y}^a)_{Aa} \tilde{C}_a \left[ L_5 - \frac{s}{2} L_6 \right] \right\} \frac{1}{- (s - u + \frac{1}{2})} \\
\mathcal{A}_{22} \sim \left\{ \frac{1}{2} (y^b \tilde{y}^a)_{Aa} \tilde{C}_a \left[ - L_5 + \frac{t}{2} L_6 \right] - \frac{1}{2} (y^b \tilde{y}^a)_{Aa} \tilde{C}_a \left[ - L_5 - \frac{t}{2} L_6 \right] \right\} \frac{1}{- (t - u + \frac{1}{2})} \\
\mathcal{A}_{23} \sim - \frac{1}{2} (y^b \tilde{y}^a)_{Aa} \tilde{C}_a \left[ s L_3 + \frac{1}{2} L_4 \right] \\
\mathcal{A}_{24} \sim L_4 \left[ - \frac{1}{4} (y^b \tilde{y}^a)_{Aa} \tilde{C}_a \left[ (y^a \tilde{y}_a)_{Aa} \tilde{C}_a \right] \tilde{y}^a \right] \\
\mathcal{A}_{25} \sim - (t - s) L_3 \left[ \frac{1}{4} (\tilde{y}^b \tilde{y}^a \tilde{y}_a) \tilde{C}_a \right] (\gamma^a \tilde{C}_a)_{Aa} \right\}
\] (9)

with the following definitions:

\[
L_4 = (2)^{-2(t + s + u) \pi} \frac{\Gamma(-u) \Gamma(-s + \frac{1}{2}) \Gamma(-t + \frac{1}{2}) \Gamma(-t - s + u + \frac{1}{2})}{\Gamma(-u - t + \frac{1}{2}) \Gamma(-t - s + 1) \Gamma(-s - u + \frac{1}{2})} \tag{10}
\]

such that \(L_5 = -u L_4\) and it does not involve any singularity and also \(L_6 = 4(t - s - u) L_3\).

From the above relations we understand that \(L_4\) has infinite singularities in the \(u\)-channel and \(-s L_6\) \((-t L_6)\) has infinite \(t(s)\)-channel fermion poles and finally \(-s L_3 (t L_3)\) has double poles in the \(t,(t + s + u)\) and \(s,(t + s + u)\) channels, respectively.

Therefore unlike the IIB amplitude, here we do have various double poles in different channels, which cannot be explained by any duality transformations. Notice that if we apply momentum conservation, then we obtain

\[
t + s + u = -p^a p_a;
\]

thus this clearly shows that the expansion is a low energy expansion and all \(s,t,u\) and \(p^a p_a\) should be sent to zero, in order to be able to find all \(\alpha'\) corrections.

More importantly we obtained the complete form of the S-matrix in type IIA to be able to look for all the corrections to super Yang–Mills couplings and in particular remove the ambiguities related to the corrections by fixing the coefficients of new couplings. Once more from the above we can see that the second part of the amplitude is also antisymmetric with respect to fermions.

Let us first deal with singularities and then get the desired \(\alpha'\) corrections of SYM and WZ couplings of type IIA superstring theory.

3 Infinite singularities for \(\langle V_C V_A V_p \tilde{V}_p \rangle\) of IIA

In [32] for the closed form of \(\langle V_C V_A V_p \tilde{V}_p \rangle\) in type IIB we have shown that there are infinite \(u\)-channel singularities for both massless gauge field and massless scalar field and in particular for the same S-matrix with different chiralities, we have seen that there are only the infinite massless scalar fields of IIA. On the other hand, after having done further computations and having provided physical kinematic reasons we confirmed that \(\langle V_C V_A V_p \tilde{V}_p \rangle\) of type IIB suggested to us that it just involves infinite massless scalar poles. Now for the same S-matrix (of course with different chiralities) of IIA, here we have used various identities and observed that neither there are gauge nor scalar poles. Therefore this clearly shows that the sum of the second term of \(A_{23}\), the fourth term of \(A_{23}\), and all the terms appearing in \(A_{24}\) will be canceled. Let us just provide the physical reasons in favor of showing that there are no \(u\)-channel gauge/scalar poles left over.

As is obvious from \(A_{23}\), the trace is just non-zero for the \(p = n\) case and Ramond–Ramond (\(C_{p-1}\)) is allowed, or \(C_{p-1} \wedge F\) makes sense for this part of the S-matrix. On the other hand, in order to have \(u\)-channel gauge poles in the field theory of IIA, we need to have Ramond–Ramond (\(C_{p-3}\)) and employ \(C_{p-3} \wedge F \wedge F\). Thus obviously there are no \(u\)-channel gauge poles. Hence the following rule:

\[
\mathcal{A} = V_a(C_{p-3}, A_1, A) G_{a b} (A) V_{b} (A, \tilde{\Psi}_1, \Psi_2), \tag{11}
\]

and in particular the following corrections:

\[
i \frac{\lambda^2 \mu_p}{(p - 2)!} \int q^{p+1} \alpha \sum_{n=-1}^{\infty} b_n (\alpha')^{n+1} \text{Tr} \left( C_{(p-3)} \wedge D^0 \cdots D^{p-3} F \wedge D_{a_0} \cdots D_{a_p} F \right) \tag{12}
\]

should not be taken into account for mixed RR, fermion/gauge of type IIA. Let us further discuss not having a \(u\)-channel scalar pole either. From \(A_{23}\), the trace is just non-zero for the \(p = n\) case and Ramond–Ramond (\(C_{p-1}\)) is allowed and, in particular, to have \(u\)-channel scalar poles in the field theory of IIA, we need to have Ramond–Ramond (\(C_{p-1}\)) and consider \(\partial_1 C_{p-1} \wedge F \phi^7\), while this is satisfactory, we should note that to have scalar poles in the closed form of the

\[7\] To work with the field theory of a RR and scalar fields, the Wess–Zumino (WZ) terms [19] or pull-back or Taylor expansion [31] are entirely explained.
amplitude, we need to have the kinematic factor of $\bar{u} \gamma^i u$, which does not appear in our $S$-matrix. Thus unlike the closed form of $\langle V_c V_4 V_\Psi V_\phi \rangle$ of IIB here we do not have any scalar $u$-channel poles. The above arguments hold even for $A_{24}$, therefore we realize there are no gauge/scalar poles for the closed form of $\langle V_c V_4 V_\Psi V_\phi \rangle$ of IIA.

Hence the following rule:

$$A = V_\alpha (C_{p-1}, A_1, \phi) G_{a_\beta}(\phi) V_\beta (\phi, \bar{\Psi}_1, \Psi_2);$$

(13)

more significantly the following corrections of type IIB cannot hold for type IIA:

$$i \frac{\lambda^2 \mu_p}{p!} \int \frac{d^{p+1}\sigma}{V_{\left(2i k_1 b\right)}\bar{u} \gamma^i u}$$

$$\times \left\{ \partial_i C_{(p-1)} \wedge D^{a_0} \ldots D^{a_p} F D_{a_0} \ldots D_{a_p} \phi \right\},$$

(14)

Let us now emphasize the infinite fermion poles.

Having settled on the fact that the total super ghost charge must be satisfied, we reveal that one can write down all fermion $t$-channel poles as follows:

$$A \sim \left\{ \frac{-s L_6}{(4s - u + 2)} \right\}$$

$$\times \left\{ \left(\gamma^b \gamma^a C \right)_{a_\beta} \gamma^b \gamma^a C_{a_\beta} + \left(\gamma^b \gamma^a C \right)_{a_\beta} \gamma^b \gamma^a C_{a_\beta} \right\},$$

where all $s$-channel fermion poles should be read off as

$$A \sim \left\{ \frac{t L_6}{(4t - u + 2)} \right\}$$

$$\times \left\{ \left(\gamma^b \gamma^a C \right)_{a_\beta} \gamma^b \gamma^a C_{a_\beta} + \left(\gamma^b \gamma^a C \right)_{a_\beta} \gamma^b \gamma^a C_{a_\beta} \right\}.$$
Concerning (20), one finds the entire and all order $\alpha'$ corrections of $V_\alpha(C_{p-1}, \Psi_3, \bar{\Psi})$ as follows:

$$V_\alpha(C_{p-1}, \Psi_3, \bar{\Psi}) = \frac{(2i \pi \alpha')\mu_p H_{a_0\cdots a_{p-2}}^c \sqrt{t} \gamma^c (\epsilon)^{a_0\cdots a_{p-2}}}{(p)!} \times \left( \frac{\lambda^\alpha_{\mu_\nu}}{\lambda^\alpha_{\mu_\nu}} \right)^{1/4} \sum_{n=1}^{\infty} \frac{\beta_n}{n!} \left( \alpha' (k_3 k_2 + k_1 k_3) \right)^{n+1} \cdot \tag{21}$$

If we replace (21) and (17) in (16) and use fermion’s equations of motion, most notably we apply momentum conservation just in the world volume direction $(k_1 + k_2 + k_3 + p)^a = 0$ (in the transverse direction it is arbitrary) then we are precisely able to re-construct all orders in the $\alpha'$ t(s)-channel fermion poles. Hence, we were able to clarify the fact that for the RR $p-1$ form field one has proposed the same $\alpha'$ corrections to two fermion fields of IIA.

We just highlight the point that these corrections are the key ingredient tools for all higher point functions; therefore, by applying them, we could either overcome the full world-sheet integrals of the higher point functions or find all their singularities and therefore resolve the singularities of the string theory.

Let us end this section by producing all order $\alpha'$ higher derivative corrections of a RR $p-1$ form field, two fermions of type IIA, and a gauge field that would correspond to contact interactions extracted out of expansions where we just write down the final result. We have

$$\sum_{p,n,m=0}^{\infty} e_{p,n,m}(\alpha')^{2m+n-2} \left( \frac{a'}{2} \right)^p \frac{(2\pi \alpha')^2 \mu_p}{p!} \int d^{p+1} \sigma \epsilon^{a_0\cdots a_{p-2}} \text{Tr}$$

$$\times \left( C_{a_0\cdots a_{p-2}}(D^a D_b)^p D^{a_1} \cdots D^{a_6} (D_{a_1} \cdots D_{a_6} A_{i_1} D_{a_{k_1}} \cdots D_{a_{k_2}} \Psi) \times \gamma^c D_{a_1} \cdots D_{a_6} (D^{a_1} \cdots D^{a_6} A_{i_1} D_{a_{k_1}} \cdots D_{a_{k_2}} \Psi) \right)$$

the connection parts could be confirmed if we would have the result of higher point functions.

Indeed these contact interactions are produced by comparison with string theory S-matrix of type IIA.

3.1 New Wess–Zumino couplings of IIA and their $\alpha'^3$ corrections

The aim of this section is to actually overcome double poles, new Wess–Zumino couplings of IIA and their $\alpha'^3$ corrections. Unlike the closed form of the correlation functions for various $p, n$ cases of one RR, two fermions, and a gauge field of type IIB, here in IIA we discover some double poles and we are able to advance our knowledge about the closed form of SYM/WZ couplings. On the other hand we are going to emphasize, going into details, that the following fact holds.

Although the general structure at third order of $\alpha'$ for the corrections to two on-shell fermions and one off-shell scalar and one on-shell gauge field (SYM) of IIA is the same, however, the coefficient of those corrections are completely different from type IIA, which means that one has to first derive the S-matrix of type IIA and then by comparison those elements on the field theory side, the coefficient of the field theory couplings should have been fixed without any ambiguity. We explain this further.

In order to get the above conclusion, let us first write down the S-matrix of type IIA. Hence, we need to consider the first and the third term of $A_{23}$ and in particular consider the world volume component of $\lambda = a$ in both terms of $A_{25}$; finally we need to add them up and extract the related traces, such that after having considered all the simplifications, one could explore the S-matrix as below:

$$A = \frac{\alpha^{3} i k_{1 b} \pi^{-1/2} \mu_p \epsilon^{a_0 \cdots a_{p-1} b} H_{a_0 \cdots a_{p-1} \xi a} \bar{u} \gamma^{c}}{p!} \times (\gamma^a) \bar{\psi} \psi^{b} (t) L_3 \text{Tr} (\gamma_1 \gamma_2 \gamma_3 \lambda_3) \cdot \tag{22}$$

Note that we have the same S-matrix for $-s L_3$, but the terms inside that S-matrix can be precisely generated by inter-changing the fermions. Most importantly, since the amplitude is antisymmetric under the inter-changing of the fermions, we conclude that this part has to vanish for IIB with the same chirality, which is consistent with our expectation of IIA.

The expansion of $t_3 L_3$ has been given before, but for completeness we present it here:

$$t_3 L_3 = \frac{\sqrt{\pi}}{2} \left( \frac{-1}{s(t + s + u)} + \frac{4 \ln(2)}{s} + \left( \frac{\pi^2}{6} - 8 \ln(2)^2 \right) \left( \frac{s + t + u}{s} \right) + \cdots \right). \tag{23}$$

As we can see from the expansion we have a double pole in the $s, (s+t+u)$ – channels (there is no double pole in type IIB). If we make use of type IIA effective field theory and RR’s kinematic relations, then we can discover the following in IIA:

$$A = V_\alpha(C_{p-1}, A) G_{ab}(A) V_\beta(A, \psi_1, \psi_2) G(\psi_2) \times V(\bar{\psi}_2, \psi_1, A^{(1)}) \tag{24}$$

where the vertex of one RR and an Abelian gauge field should have been read from the Chern–Simons term in which it has been fixed and there is no correction to that coupling so that $V_\alpha(C_{p-1}, A) = (2\pi \alpha')\mu_p \rho_1^{p+1} \epsilon^{a_0 \cdots a_{p}} \epsilon_{a_0 \cdots a_{p}}$; meanwhile the gauge propagator must be derived from the kinetic term of gauge field which has no correction either. We have

$$\int d^{p+1} \sigma \frac{1}{p!} \text{Tr} \left( C_{p-1} \wedge F \right). \tag{25}$$
\[ C^{ab}_{\alpha\beta}(A) = \frac{-i\delta^{ab}}{T_p(2\pi\alpha')^2(t + s + u)}. \]  \hspace{1cm} (26)

Now one may argue that the vertex of an on-shell/off-shell fermion and one off-shell gauge field has no contribution to this S-matrix. We need to extract the covariant derivative of fermion fields, however, notice that these fermions originated from two different D-branes, so the following vertex makes sense in the presence of different overlapping branes (note that similar couplings have been discovered for non-supersymmetric cases in brane–antibrane systems):

\[ V_b(A, \bar{\psi}_1, \psi_2) = T_p(2\pi\alpha')\bar{u}^\gamma \gamma_{b\gamma}. \]  \hspace{1cm} (27)

It is also discussed that the kinetic term of fermion has no correction. Concerning momentum conservation, one explores the fermion propagator as well as \( V(\bar{\psi}_2, \psi_1, A^{(1)}) \). We have

\[ G(\psi) = \frac{-iy^a(k_1 + k_3)}{T_p(2\pi\alpha')s} \]
\[ V(\bar{\psi}_2, \psi_1, A^{(1)}) = T_p(2\pi\alpha')u^\delta \bar{\gamma}_{\delta} \xi_{1c}. \]  \hspace{1cm} (28)

Now if we substitute (26), (27), (28), and the above Chern–Simons term into the above rule (24), we obtain

\[ A = \frac{4ik_{1b}\mu_p}{p!(s)(t + s + u)} e^{\alpha_{0a}a_{p+1}^{-1}} H_{a_{0a}a_{p+1}^{-1}} \bar{\xi}_{1a} u^\gamma \]
\[ \times (\gamma^a)_{\bar{\gamma}} u^\delta \bar{\gamma} \delta \left( \lambda_1 \lambda_2 \lambda_3 \right). \]  \hspace{1cm} (29)

Hence we have generated the first double pole of the type IIA, where fermion field equations of motion have been applied to the above field theory side as well.

The natural question one may ask is how to deal with the second term of the expansion, which is a simple s-channel pole. Replacing it in the S-matrix one might explore the second term of the S-matrix as

\[ A = \frac{ln(2)\alpha'_2ik_{1b}\mu_p}{2sp!} e^{\alpha_{0a}a_{p+1}^{-1}} H_{a_{0a}a_{p+1}^{-1}} \bar{\xi}_{1a} u^\gamma (\gamma^a)_{\bar{\gamma}} u^\delta \bar{\gamma} \delta \left( \lambda_1 \lambda_2 \lambda_3 \right). \]  \hspace{1cm} (30)

Let us begin with generating new WZ couplings as follows. In order to actually produce the above S-matrix, one has to take into account the following rule:

\[ A = V(C_{p-1}, \bar{\Psi}_1, \Psi_2)G(\Psi_2) V(\bar{\Psi}_2, \Psi_1, A^{1}) \]  \hspace{1cm} (31)

with a new WZ coupling:

\[ \frac{(2\pi\alpha')\mu_p}{p!} \beta_1^2 \left( C_{a_{0a}a_{p+1}} \bar{\Psi}_1 \gamma^b D_b \Psi_2 \right) (a_{0a}a_{p+1})^b, \]  \hspace{1cm} (32)

whereas the fermion propagator and the structure of \( V(\bar{\Psi}_2, \Psi_1, A^{1}) \) are kept fixed.\(^{12}\) Therefore, if we extract the needed vertex from (32) as

\[ \frac{(2\pi\alpha')\mu_p}{p!} \beta_1^2 H_{a_{0a}a_{p+1}} \bar{u}^\gamma (\gamma^b)_{\bar{\gamma}} (a_{0a}a_{p+1})^b \]  \hspace{1cm} (33)

with \( \beta_1 = (2\ln2/(\pi\alpha'))^{1/2} \) and replace the given vertices appearing in (31), we will precisely obtain (30) and this means that not only can we confirm the new form of the WZ coupling but also we fix its normalization constant. One might wonder about the third simple pole, because it is again a simple s-channel pole and we have already produced it. However, the answer is hidden in the higher derivative correction of the new WZ coupling that we just discovered right now.

Indeed the above rule, \( A = V(C_{p-1}, \bar{\Psi}_1, \Psi_2)G(\Psi_2) V(\bar{\Psi}_2, \Psi_1, A^{1}) \), with the same fermion propagator, and likewise \( V(\bar{\Psi}_2, \Psi_1, A^{1}) \), should be applied for the third term of the expansion, while the vertex \( V(C_{p-1}, \bar{\Psi}_1, \Psi_2) \) gets modified. Let us propose the following higher derivative correction to a \((p - 1)\)-RR form field and one off-shell/on-shell fermion field:

\[ \left( \frac{\pi^2}{6} - 8ln2^2 \right) \left( \frac{i\alpha'_2\mu_p}{p!} \right) \]
\[ \times \left( C_{a_{0a}a_{p+1}} D^a D_b (\bar{\Psi}_1 \gamma^b D_b \Psi_2) \right) (a_{0a}a_{p+1})^2, \]  \hspace{1cm} (34)

and take momentum conservation for all external states in the world volume direction \( s + t + u = -p^a p_a \) so that

\[ V(C_{p-1}, \bar{\Psi}_1, \Psi_2) = a^2 \mu_p \left( \frac{\pi^2}{6} - 8ln2^2 \right) \]
\[ \times \frac{1}{p!} H_{a_{0a}a_{p+1}} \bar{u}^\gamma (\gamma^b)_{\bar{\gamma}} (t + s + u) (a_{0a}a_{p+1})^b. \]  \hspace{1cm} (35)

Having replaced (35) and the same vertex/propagator inside (31), we get

\[ A = \left( \frac{\pi^2}{6} - 8ln2^2 \right) \left( t + s + u \right) \xi_{1a} i k_{1b} \mu_p \]
\[ \times H_{a_{0a}a_{p+1}} \bar{u}^\gamma (\gamma^b)_{\bar{\gamma}} (a_{0a}a_{p+1})^b \]  \hspace{1cm} (36)

we have also used the fermion field \( k_{3a} \gamma^a u = 0 \), thus the third pole is also precisely produced. In order to show that the coefficients of the higher derivative corrections to SYM couplings of two fermion fields, one off-shell scalar, and one on-shell gauge of type IIA are different from the type IIB ones, we just consider the last two terms of the first part of the amplitude:

\[ A \sim (P_\beta H_{(n)} M_p) a_{0a} \xi_{1a} \bar{u}^\gamma (i k_{3a} - i k_{3b} t) \]
\[ \times (\gamma^b)_{a0} \bar{\gamma} \gamma^a L_3 \bar{\gamma}, \]  \hspace{1cm} (37)

\(^{12}\) We should drop the connection part inside the above WZ coupling (32).
If we consider the sum of the fourth terms of the expansions of $s_{L_3}$ and $-tL_3$ in the above S-matrix,\footnote{We have $s_{L_3} = \sqrt{\pi} \frac{\pi^2 \mu_p}{2} (\frac{-1}{(t+u+s)} + \frac{4 \ln(2)}{t} + (\frac{\pi^2}{6} - 8 \ln(2))^2) + \frac{\pi^2}{3} \frac{s}{(t+u+s)} + \ldots$ and $-tL_3 = \sqrt{\pi} \frac{\pi^2 \mu_p}{2} (\frac{1}{s(t+u+s)} - \frac{4 \ln(2)}{s} + (\frac{\pi^2}{6} - 8 \ln(2))^2) + \frac{\pi^2}{3} \frac{s}{(t+u+s)} + \ldots$.} we derive the trace for the transverse component of $(\mu = i)$ and for the $p + 2 = n$ case with a proper normalization constant, we derive the S-matrix as follows:

$$A = \frac{-i\alpha^3 \pi^2 \mu_p (\pi/2)(\pi/2)}{12(t + s + u)(p + 1)!} \left( x - sk_2 \xi_1 + tk_3 \xi_1 \right) \left( \lambda_1 \lambda_2 \lambda_3 \right).$$

(40)

This S-matrix with three momenta (without counting RR’s momentum) has to be produced by the following rule of the effective field theory:

$$A = V_i^a(C_{p+1}, \phi) G_{ij}^a(\phi) V_j^b(\phi, \bar{\psi}, \psi, A_1).$$

(41)

For the RR’s kinetic term one has proposed the constraint that just a massless scalar can be propagated to generate the $(t + s + u)$-pole. Hence one might wonder whether or not the couplings that we found for two fermions and one scalar/gauge of type IIB\footnote{We have $C^{nm} = \pi^2 \alpha \mu_p + \frac{3}{T_p} \left( \psi f_i D_{ab} \Psi D^a \phi_i + \bar{\psi} f_i D_{ab} \Psi D^a \phi_i \right) + h.c. \right) + \frac{3}{T_p} \left( \psi f_i D_{ab} \Psi D^a \phi_i + \bar{\psi} f_i D_{ab} \Psi D^a \phi_i \right) \right)$.} even at third order in $\alpha'$, $n = m = 0$ could be applied to IIA.

Let us first consider the third order corrections of two on-shell fermions, one off-shell scalar and one on-shell gauge of type IIB. We have

$$T_p(2\pi \alpha')^3 \left[ \left( \psi \gamma^i D^a \Psi D^a \phi_i + \bar{\psi} f_i D_{ab} \Psi D^a \phi_i \right) \right].$$

(42)

The scalar propagator should be derived from its kinetic term and the vertex of one RR $(p + 1)$-form field and an off-shell scalar is given in [32] but for completeness we mention both once more. We have

$$G_{ij}^{\alpha}(\phi) = \frac{-i\alpha^3 \pi^2 \mu_p (\pi/2)(\pi/2)}{T_p(2\pi \alpha')^3 k^2} \left( x - sk_2 \xi_1 + tk_3 \xi_1 \right) \left( \lambda_1 \lambda_2 \lambda_3 \right).$$

(43)

Taking (42) and considering the orderings of $\text{Tr} \left( \lambda_2 \lambda_3 \lambda_3 \lambda_\beta \right)$ and $\text{Tr} \left( \lambda_2 \lambda_3 \lambda_1 \lambda_\beta \right)$ for the first and second couplings of (42), respectively, we get

$$V_\beta^i(C_{p+1}, \phi) = \frac{-i\alpha^3 \pi^2 \mu_p (\pi/2)(\pi/2)}{2(t + s + u)(p + 1)!} \left( x - sk_2 \xi_1 + tk_3 \xi_1 \right) \left( \lambda_1 \lambda_2 \lambda_3 \right).$$

(44)

where all the connection parts or commutator terms should have been ignored in the those corrections and momentum conservation has to be used.

If we would replace (44) and (43) in (41) we would obtain the following field theory amplitude of type IIA:

$$A = \frac{-i\alpha^3 \pi^2 \mu_p (\pi/2)(\pi/2)}{T_p(2\pi \alpha')^3 k^2} \left[ \left( \psi \gamma^i D^a \Psi D^a \phi_i + \bar{\psi} f_i D_{ab} \Psi D^a \phi_i \right) \right].$$

(46)

The scalar propagator and the vertex of one RR $(p + 1)$-form field and an off-shell scalar remain invariant; now taking into account (46) and considering the orderings of $\text{Tr} \left( \lambda_2 \lambda_3 \lambda_\beta \right)$ and $\text{Tr} \left( \lambda_2 \lambda_3 \lambda_\beta \lambda_\alpha \right)$ for the first and second couplings of (46) accordingly, we obtain

$$V_\beta^i(\phi, \bar{\psi}, \psi, A_1) = \frac{-i\alpha^3 \pi^2 \mu_p (\pi/2)(\pi/2)}{T_p(2\pi \alpha')^3 k^2} \left[ \left( \psi \gamma^i D^a \Psi D^a \phi_i + \bar{\psi} f_i D_{ab} \Psi D^a \phi_i \right) \right].$$

(47)

where all the connection parts or commutator terms are removed in the corrections and momentum conservation was used.

If we would replace (47) and (43) in (41) we would find the following field theory amplitude of type IIA:

$$A = \frac{-i\alpha^3 \pi^2 \mu_p (\pi/2)(\pi/2)}{12(t + s + u)(p + 1)!} \left( x - sk_2 \xi_1 + tk_3 \xi_1 \right) \left( \lambda_1 \lambda_2 \lambda_3 \right).$$

(48)
which is precisely the string theory S-matrix of (40), and this dictates that we have correctly derived the third corrections of SYM to type IIA. It is shown that all order $\alpha'$ corrections of type IIB for fermionic amplitudes and bosonic amplitudes could be derived by applying a universal prescription to their leading order couplings [57] (see [30,56]), however, it is not obvious whether that kind of rule holds for type IIA.

Let us end this section by discussing some other couplings that one could expect to see in type IIA. Namely for the $p = n$ case, one might hope to obtain the other SYM couplings of two on-shell fermions and one off-shell/an on-shell gauge, then fix their coefficients in both type IIA and IIB. First of all one has to write down all possible couplings with three momenta as follows:

$$\bar{\Psi}^a D_a \Psi F_{bc} F^{bc} \quad \bar{\Psi}^a D_a \Psi F_{ac} F^{bc} \quad \bar{\Psi}^a D_b \Psi F_{bc} F^{ac}. \quad (49)$$

If we take two different orderings of $\text{Tr} (\lambda_2 \lambda_3 \lambda_1 \lambda_\beta)$ and $\text{Tr} (\lambda_2 \lambda_3 \lambda_\beta \lambda_1)$ with Abelian gauge field $\lambda_\beta$, then we conclude that the contribution of the second and third term to the effective field theory is the same so we just need to keep track of one of them. On the other hand the first coupling produces the following terms:

$$v^\beta_\nu (\bar{\Psi}_2, \Psi_3, A_1, A) = \bar{u} \gamma^\alpha u(-ik_{3a}) \times \left[-(t+s)\xi^b + 2k^b_{\xi}(-k_2, \xi_1 - k_3, \xi_1)\right] \text{Tr} (\lambda_1 \lambda_2 \lambda_3 \lambda_\beta), \quad (50)$$

where, having taken the on-shell condition for the fermion field, we realize that the first coupling of (49) does not play any role in the effective field theory of this S-matrix. What about the second coupling of (49)? If we consider that coupling, apply momentum conservation, take all possible orderings, and re-consider the on-shell condition for fermions, we obtain the following terms:

$$v^\alpha_\nu (\bar{\Psi}_2, \Psi_3, A_1, A) = (i)\bar{u} \gamma^\alpha u \left[k_{1a}\left(\frac{t}{2} \xi_{1c}(k_2, \xi_1 + k_3, \xi_1)k_{3c} - \frac{t}{2} \xi_{1c}k_{1c}\right) - \left(\frac{t}{2} + \frac{s}{2}\right) \xi_{1a}k_{3c} \right. \left.+ k_{1a}\left(-\frac{s}{2} \xi_{1c} - k_3, \xi_1 k_{1c}\right) + \frac{s}{2} k_{2, \xi_1} - \frac{t}{2} k_{3, \xi_1}\right] \times \text{Tr} (\lambda_1 \lambda_2 \lambda_3 \lambda_\beta), \quad (51)$$

for which these terms do not appear in the S-matrix elements of one RR, two fermion fields (with different chiralities), and one gauge field of IIA. Therefore we can definitely conclude that there exists even no first order of $\alpha'$ correction to those SYM couplings of type IIA.

### 4 Concluding remarks

In this paper, first we obtained the compact/closed form of the correlation function of four spin operators with different chiralities and one current $\langle Vc V^\beta_\nu V^\alpha_\nu V_A \rangle$ in ten dimensions of space-time in type IIA superstring theory. Conformal field theory techniques have been used to explore the entire form of the S-matrix of one closed string RR, two fermion fields with different chiralities, one gauge field of IIA. Concerning the complete form of the IIA S-matrix, we are able to first of all find the third order of $\alpha'$ higher derivative corrections to SYM couplings and secondly distinguish $\alpha'$ corrections from type IIB. Our computations show that the coefficients of two fermions (with different chiralities), one scalar, and one gauge of IIA corrections should be completely different from type IIB, while in [18] we have proven not only the coefficients of two fermions and two scalars, but also the general structure of those corrections of IIA is entirely different from the IIB corrections at all order of $\alpha'$.

The other result that we obtained is that, although in the S-matrix we have the $u$-channel pole, if we use various identities those scalar/gauge singularities will disappear; so, unlike the closed form of $\langle Vc V^\beta_\nu V^\alpha_\nu V_A \rangle$ of type IIB, we have no $u$-channel singularity in IIA. Note also that unlike the computations of IIB for the same S-matrix, in IIA we do have several double poles in the $t, (t + s + u)$ and in the $s, (t + s + u)$ channels where the presence of these double poles cannot be confirmed by a duality transformation at all. We have produced all infinite massless fermion poles of $t, s$-channels of type IIA for diverse $p, n$ cases. Most importantly, their all order $\alpha'$ higher derivative corrections and their extensions have been discovered and constructed without any ambiguities.

Having found the entire form of the S-matrix, we are able to discover several new Wess–Zumino couplings with their third order $\alpha'$ corrections. Note that those new WZ couplings cannot be found by having the S-matrix of IIB. It was just the first $(t + s + u)$-channel scalar pole that dictated that the $\alpha'^3$ corrections of IIA have to be accompanied with the same corrections but with different coefficients of IIB. We take this fact as a benefit of having done direct conformal field theory techniques rather than applying duality transformation. By comparison of the field theory amplitude with string theory elements we also explored the idea that there is not even a single $\alpha'$ correction to gauges/two fermions of type IIA.

Finally, we used CFT for propagators and, as is well known, the RR has two sectors and it is not obvious how to get involved its second sector $\alpha_n$ but one might apply the ideas appearing in [76] for further developments so that clearly the analytic continuation must be made. In the other words, background fields should be seen as functions of SYM. We could observe background fields as some composite states of the...
open strings where a certain definition of a Taylor expanded function is necessarily made [19].

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