Application of grid-characteristic method to some seismic exploration problems in the Arctic

D I Petrov
1Moscow Institute of Physics and Technology
diapetr@gmail.com

Abstract. The aim of this work is simulation of wave propagation in the Arctic with the presence of different ice structures, using high-performance computing. In the given paper grid-characteristic method is applied, which provides correctly describing the contact and boundary conditions.

1. Introduction
The question of developing the Arctic in Russia is of current importance as there are eight hydrocarbon fields, and their supplies are estimated approximately 2,7 trillion m³. The considerable barrier on way of oil extraction in north seas is presence of different ice formations, particularly, ice ridges, icebergs. One of the main stages of planning geological survey works is mathematical modeling, that allows significantly bring down the cost of carrying out seismic exploration. In this work numerical experiments on solving problems of seismic exploration in the conditions of the Arctic shelf were carried out. and applied to calculate the fluxes in the finite volume discretization of the governing equations.

In this paper we present the results of a numerical experiment on the modeling of wave processes in water-soil systems by the grid-characteristic method. A form was obtained from clique from a reservoir with a hydrocarbon deposit at various initial conditions at significant depths. The experiment is performed for various configurations of geological sediments and two types initial perturbations. Also, wave processes in “ice-water-soil”, the soil contains a non-uniform oil inclusion.

2. Mathematical model of medium
A complete system of equations describing the state continuous linear-elastic medium and a complete system of equations describing sonic acoustic field are being solved. The components of the velocity \( \vec{v} \) and symmetric Cauchy stress tensor \( \sigma \) in a linearly elastic medium are described by the following system of equations [1]:

\[
\rho \frac{d \vec{v}}{dt} = -\nabla p \tag{1}
\]

\[
\frac{\partial \sigma}{\partial t} = \lambda (\nabla \cdot \vec{v}) I + \mu \left( \nabla \otimes \vec{v} + (\nabla \otimes \vec{v})^T \right) \tag{2}
\]

For numerical modeling of sea water and oil-containing inclusions, the ideal fluid approximation was used and the complete system of equations:
\[ \rho \frac{\partial \vec{v}}{\partial t} = -\nabla p \]
\[ \frac{\partial p}{\partial t} = -c^2 \rho (\nabla \cdot \vec{v}) \]  

(3)

where \( \lambda, \mu \) – Lamé parameters, which determine the properties of the elastic material, \( c \) denotes the speed of sound in an ideal fluid. The velocity of longitudinal waves in a linearly elastic medium can be found from

\[ c_p = \sqrt{\frac{\lambda + 2\mu}{\rho}} \]

and the velocity of transverse waves is calculated in accordance with

\[ c_s = \sqrt{\frac{\mu}{\rho}} \]

3. Numerical method

For the numerical solution of the given systems, a grid-characteristic method is used on curvilinear structural grids, which allows to build correct numerical algorithms for calculating boundary points and points lying on the interfaces of two media with different Lamé parameters and/or densities [2].

The system in the two-dimensional case can be represented in the form

\[ \frac{\partial q}{\partial t} + A_1 \frac{\partial q}{\partial x_1} + A_2 \frac{\partial q}{\partial x_2} + A_3 \frac{\partial q}{\partial x_3} = 0 \]

In the given expression, the vector \( q \) is understood to be the vector formed by of two velocity components and three components of a symmetric tensor of stresses

\[ q = \{v_1, v_2, \sigma_{11}, \sigma_{22}, \sigma_{12}\} \]

First, the spatial splitting method is applied coordinates, as a result of which we have two one-dimensional systems:

\[ \frac{\partial \hat{q}}{\partial t} = \Lambda_1 \frac{\partial \hat{q}}{\partial x_j} \]

Each of these systems is hyperbolic and has a complete set of eigenvectors with real eigenvalues, therefore each of the systems can be rewritten in the form

\[ \frac{\partial \hat{q}}{\partial t} = \Omega^{-1} \Lambda \Omega \frac{\partial \hat{q}}{\partial x_j} \]

where \( \Omega \) is a matrix composed of eigenvectors, \( \Lambda \) is a diagonal matrix, a matrix whose elements are eigenvalues (index \( j \) is omitted here and later). The general view of the matrix \( \Lambda \) will be

\[ \Lambda = \text{diag} \{c_p, -c_p, c_s, -c_s, 0\} \]
After changing the variables

\[ \vec{p} = \Omega \vec{q} \]

each of the systems decomposes on five independent scalar advection equations:

\[ \frac{\partial \vec{p}}{\partial t} + A \frac{\partial \vec{p}}{\partial x} = 0 \]

One-dimensional advection equations are solved by the method of characteristics or by ordinary finite-difference schemes. After all the components have been transferred, the solution may be restored:

\[ \vec{q} = \Omega^{-1} \vec{p} \]

The solution of system in the two-dimensional case is carried out similarly.

The program implements the application of TVD-difference schemes of the 2nd order of accuracy, 15 different limiters, the superbee limiter and grid-characteristic schemes of 2-4 order of accuracy [3].

4. Boundary conditions

Let in a part of the integration region \( a \) a system of equations (1), (2), and in the part \( b \) of the integration region \( b \) the system of equations (3) are being solved. Consider the contact condition between them. System (1), (2) has three outgoing characteristics in the three-dimensional case, and system (3) in the three-dimensional case has one outgoing characteristic. Thus, in order to find all four outgoing characters, it is necessary to fulfill the following contact conditions:

\[ p^{(b,n+1)} = - (\sigma^{(a,n+1)} \cdot \vec{p}) \cdot \vec{p} \]  \hspace{1cm} (4)

\[ \sigma^{(a,n+1)} \cdot \vec{p} - (\sigma^{(a,n+1)} \cdot \vec{p}) \cdot \vec{p} = 0 \]  \hspace{1cm} (5)

\[ \nu^{(a,n+1)} \cdot \vec{p} = \nu^{(b,n+1)} \cdot \vec{p} \]  \hspace{1cm} (6)

The condition (4) is the equality of the normal component surface density of forces on the part of a solid body to the pressure in ideal fluid, condition (5) ensures that the tangential component of the surface density of forces on the side of the body, and expression (6) gives the equality of the normal components velocities in an ideal fluid and a solid. In (4) - (6) for \( p \) is noted the outer normal to the solid body, which is the inner normal to the liquid. The velocity vector \( \nu \) such that the conditions (4)-(6) are fulfilled. Further, for a rigid body, the boundary condition is used with a given velocity, and for a liquid, a boundary condition with given by the normal component of velocity.

5. Result of the numerical simulation

We consider a region for integration 1200 m wide and 600 m depth, representing multilayered geological medium under conditions of the Arctic shelf. The medium consists of ice, sea water, rock and carbohydrate reservoir. The density of water was assumed equal to 1000 kg/m³, the speed of sound is 1500 m/s. The ground has a density of 2500 kg/m³, the longitudinal propagation velocity of the disturbance is 4500 m/s, the transverse velocity is 2250 m/s. The following parameters of the hydrocarbon layer were used: density 2000 kg/m³, longitudinal speed the propagation of sound is 4000 m/s, the transverse speed - 1250 m/s. The source of the initial Rieker wavelet with dominant frequency equals 40 Hz is placed in the center of the ice field.
The following grid parameters are taken: spatial step is 0.2 m, time step is $3 \times 10^{-5}$ s. 15 000 iterations were calculated. Absorbing conditions are set at the sides and at the bottom of the region. Free boundary condition is set on the top side of the region.

**Figure 1.** A model of geological medium. From the top to bottom: air, ice, water, rock, reservoir, rock.

**Figure 2.** A model of geological medium. From the top to bottom: air, ice, water, rock. No layer of carbohydrates.

**Figure 3.** The wave field in the medium after initial perturbation. A response of reservoir is being observed.

**Figure 4.** The wave field in the medium after initial perturbation.

**Figure 5.** Synthetic seismogram corresponding the first definition.

**Figure 6.** Synthetic seismogram corresponding the second definition.
6. Conclusion
In this paper, we considered the solution of elastic-acoustic problems of shelf seismic exploration using grid-characteristic method. A boundary condition was developed between a part of the integration in which a system of equations is solved that describes the state of an infinitesimal volume of a continuous linearly elastic and the part of the region of integration in which the equations describing the acoustic field.

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