Chiral Dynamics with Quark Degrees of Freedom

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Abstract. Possibility to detect DCC fluctuations is discussed. It is shown that interactions with quark background and dissipative effects due to interactions in the chiral field may result in damping of fluctuations. Since the magnitude of fluctuations depends strongly on the initial state and speed of chiral phase transition accurate evaluation of all modifying processes is required to predict observability of DCCs.

The possibility of producing quark-gluon plasma in relativistic heavy ion collisions is exciting especially from the point of view of observing the chiral and deconfinement phase transitions. Rajagopal and Wilczek have suggested that if chiral restoration is of second order, nonequilibrium dynamics can generate transient domains in which macroscopic pion fields develop. As the chiral field relaxes to the true vacuum in such a domain, it may lead to coherent emission of pions \cite{Rajagopal}. This kind of phenomenon is called a disoriented chiral condensate (DCC). Bjorken and others pointed out that DCC can lead to fluctuations in the charged and neutral pion spectra \cite{Bjorken}. If we can observe this phenomenon we can say the quark-gluon plasma has been produced, and we see the signs of its hadronization.

The possibility to resolve fluctuations of neutral and charged pions critically depends on the size and energy content of these domains. If the domains are roughly pion sized, the effect of DCC is too small to be resolved in experiments. If domains are large enough, the effects of DCC can have measurable consequences. All these signals: the isospin fluctuations, the enhancement of the pion spectrum at low $p_T$, and the suppression of HBT correlations are characteristics of any large coherent source \cite{Most}.

Most dynamical studies \cite{Most, Molnar, Csernai, Feng, Molnar, Csernai} of DCC have been carried out within...
the framework of the linear sigma model. Using this model, one can study the possibility of DCC formation and the domain size of DCC.

The Lagrangian density of the linear sigma model can be written as:

$$L = L_q + L_{SM} = L_q + \frac{1}{2} \left( \partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \partial^\mu \vec{\pi} \right) - U(\sigma, \vec{\pi}),$$

(1)

where

$$U(\sigma, \vec{\pi}) = \frac{\lambda}{4} \left( \sigma^2 + \vec{\pi}^2 - v^2 \right)^2 - H \sigma + \left( m^2_{\pi} f^2_{\pi} - \frac{m^2_{\sigma}}{4\lambda} \right),$$

(2)

is the so-called Mexican Hat potential, and $L_q$ is the contribution of quarks,

$$L_q = \bar{q} \left[ i\gamma_\mu \partial^\mu - g(\sigma + i\gamma_5 \vec{\tau} \vec{\pi}) \right] q.$$  

(3)

Here $q$ stands for the light quark fields, $(u, d)$, while $\sigma$ and $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ are the scalar meson and vector meson fields which together form a 4-component chiral field $\Phi = (\sigma, \vec{\pi})$. The last constant term is included to secure that the minimum of the potential is $U_{\text{min}} = 0$. In the linear sigma model, the fields $\sigma$ and $\vec{\pi}$ are treated independently without any constraint, and coupled to the fermion field as in the above equation. If we impose the constraint $\sigma^2 + \vec{\pi}^2 = f^2_{\pi}$, we will have the nonlinear sigma model. Without the term $H \sigma$ this Lagrangian is invariant with respect to the $SU_L(2) \otimes SU_R(2)$ chiral transformations.

The parameters in this Lagrangian are chosen in such a way [12] that in normal vacuum at $T = 0$ chiral symmetry is spontaneously broken and expectation values of the meson fields are

$$\langle \sigma \rangle = f_\pi, \quad \langle \vec{\pi} \rangle = 0,$$

(4)

where $f_\pi = 93$ MeV is the pion decay constant. In the vacuum with broken chiral symmetry pions represent soft “azimuthal” excitations of the chiral field. To have the correct pion mass in vacuum, $m_\pi = 138$ MeV, one should take

$$v^2 = f^2_{\pi} - \frac{m^2_{\pi}}{\lambda}, \quad H = f_\pi m^2_{\pi}.$$  

(5)

The parameter $\lambda$ is related to the sigma mass, $m^2_{\sigma} = 2\lambda f^2_{\pi} + m^2_{\pi}$, which can be chosen to be about 0.6 GeV (then $\lambda \approx 20$). Sigmas represent stiff, “radial” excitations of the chiral field. The remaining coupling constant $g$ can be fixed by the requirement that the effective quark mass, $(m^2_q = g(\sigma^2 + \vec{\pi}^2))$, in broken vacuum, $m_q = g\langle \sigma \rangle = gf_\pi$, coincides with the constituent quark mass in hadrons, about 1/3 of the nucleon mass $m = m_N/3$. This gives $g \approx (m_N/3)/f_\pi \approx 3.3$.

When we take the mean-field approximation, ignore all loop contributions and consider $\sigma$ and $\vec{\pi}$ as classical fields, the equation of motion of chiral field can be written as:

$$\partial_\mu \partial^\mu \sigma(x) + \lambda \left[ \sigma^2(x) + \vec{\pi}^2(x) - v^2 \right] \sigma(x) - H = -g \rho_\sigma(x),$$

$$\partial_\mu \partial^\mu \vec{\pi}(x) + \lambda \left[ \sigma^2(x) + \vec{\pi}^2(x) - v^2 \right] \vec{\pi}(x) = -g \rho_{\vec{\pi}}(x).$$

(6)
Here $\rho_S = \langle \bar{q}q \rangle$ and $\rho_P = i \langle \bar{q}\gamma_5 \tau q \rangle$ are scalar and pseudoscalar quark densities, which should be determined self-consistently from the motion of $q$ and $\bar{q}$ in background meson fields. The scalar and pseudoscalar densities can be represented as

$$\rho_S(x) = g a(x) \sigma(x), \quad \rho_P(x) = g a(x) \vec{\pi}(x),$$

where $a(x)$ is expressed in terms of the momentum distribution of quarks and antiquarks $f(x, p)$,

$$a(x) = \nu_q \int \frac{d^4p}{(2\pi \hbar)^4} 2\delta(p^\mu p_\mu - m^2(x)) f(x, p)$$

$$\rightarrow \frac{\nu_q}{(2\pi \hbar)^3} \int \frac{d^3p}{E(x, p)} [n_q(x, p) + n_{\bar{q}}(x, p)].$$

Here $\nu_q$ is the degeneracy factor of quarks. Let us assume that the quark distribution is an ideal Fermi distribution at the freeze-out time $\tau_0$ ($\tau_0 \sim 7 \text{fm}/c$) and the subsequent distribution for time $\tau > \tau_0$ is given by the solution of the Vlasov equation \[9\]. The integral of this source term $a(x)$ in the equation of motion is evaluated for the $\mu = 0, m_q = 0$ case analytically earlier in ref.\[9\]. For $m_q \neq 0$,

$$a(x) = 2\nu_q \frac{1}{(2\pi)^2} \int_0^\infty \frac{q}{e^{\beta \sqrt{q^2 + \alpha^2 m^2}} + 1} \arcsin \sqrt{1 - \frac{\alpha^2}{1 + \frac{\alpha^2 m^2}{q^2}}} dq,$$

where $\alpha = \tau_0/\tau$, $m = m(\tau)$ and $m_0 = m(\tau_0)$ are the quark masses at $\tau$ and $\tau_0$. This integral can be numerically evaluated by using nine point Laguerre integral formula. This way we can follow the solution of the equation of motion numerically from arbitrary initial condition at $\tau_0$ during the whole symmetry breaking process. In ref. \[9\] only an estimate was given for the initial growth rate of the instability.

Considering an $N$-dimensional uniform scaling expansion, ($N = 0, 1, 2, 3$) the differential operator, $\partial_\mu \partial^\mu$, in terms of the variables, proper time, $\tau$, and approximate $N$-dimensional rapidity, $\eta$, takes the form

$$\left[ \partial_\tau^2 + \frac{N}{\tau} \partial_\tau - \frac{1}{\tau^2} \partial_\eta^2 \right],$$

where the term, $-N\tau^{-1} \partial_\tau$, describes the damping caused by the collective expansion of the system. A similar damping can occur due to interactions, particularly due to thermal interactions if our system is interacting with heat-bath. To describe such damping we might add an additional damping term which is present even in the absence of collective expansion ($N = 0$): $-\gamma \partial_\tau$.

Assuming further that our initial condition satisfies the scaling symmetry, i.e., it depends only on $\tau$ but not on $\eta$, such an $\eta$ independence is conserved by the equations of motion. Thus for the studies we describe in the following we drop the
rapidly dependent terms in the equations of motion. The arising equations in the 
\((\sigma, \vec{\pi})\) coordinates are

\[
\begin{align*}
\partial^2_\tau \sigma &= \left( -\frac{N}{\tau} \partial_\tau - \gamma_\sigma \partial_\tau - \lambda (\sigma^2 + \vec{\pi}^2) + \lambda v^2 \right) \sigma + H - g \rho S, \\
\partial^2_\tau \vec{\pi} &= \left( -\frac{N}{\tau} \partial_\tau - \gamma_\pi \partial_\tau - \lambda (\sigma^2 + \vec{\pi}^2) + \lambda v^2 \right) \vec{\pi} - g \rho P.
\end{align*}
\] (10)

In the above equation, \(\gamma_\pi\) and \(\gamma_\sigma\) are damping parameters of \(\pi\) and \(\sigma\) fields.

In estimating thermal damping of small amplitude fluctuations we may take advantage of the estimated viscosity coefficient of the quark-gluon plasma [10] which was used earlier in estimating thermal nucleation rates [11]. These yield a damping of about

\[
\gamma \approx 0.2 - 10 \frac{c}{\text{fm}} \quad \Gamma = 50 - 2000 \text{ MeV}
\] (11)
for \(T \approx 100\) MeV.

| \(m_\sigma\) (MeV) | \(a\) | \(b\) |
|------------------|------|------|
| 400              | 0.0307 | 3.706 |
| 600              | 0.0526 | 3.446 |
| 800              | 0.0700 | 3.547 |

Table 1. Fit parameters \(a\) and \(b\) to estimate the pion-width as given in eq. (12), for three sigma masses including contributions of one- and two-loop self-energy diagrams, based on ref. [12].

In finite temperature field theory we can estimate the damping of a field using the imaginary-time propagators. The damping or width is characterized by the imaginary part of the self-energy. These hadronic thermal widths should be interpreted as the damping coefficients of wave packets propagating through a dispersive medium.

In the linear \(\sigma\)-model one-vertex one-loop graphs do not contribute an imaginary part to the self energy [13] thus the first contributions are given by two-vertex one-loop diagrams [14]. The damping of the \(\pi\)-field was evaluated in [12] by including a two-vertex one-loop self-energy graph with a dressed sigma-meson propagator, plus two two-loop graphs and two two-loop four-vertex graph contributions to the pion self-energy.

The resulting pion width can be approximated with

\[
\Gamma_\pi = \left( aT \right)^b
\] (12)

where \(\Gamma_\pi\) and \(T\) are measured in [MeV] and the parameters \(a\) and \(b\) are given in Table 1.
For $T = 100$ MeV and $m_\sigma = 600$ MeV, this yields $\Gamma_\pi \approx 300$ MeV, which is comparable with the value above based on analogies of damping of waves in a viscous fluid.

At the last moment before quark freeze-out we have a thermal distribution both for the quarks and the background fields. This moment was estimated in ref. [9] to be around $\tau_0 = 7$ fm and $T_0 \approx 130$ MeV. As an illustration we used the same initial condition as ref. [8] in our studies presented here.

\[\begin{align*}
\sigma(\tau_0) &= 0, & \vec{\pi}(\tau_0) &= 0, \\
\dot{\sigma}(\tau_0) &= 1\text{MeV/fm}, & \pi_1(\tau_0) &= 5\text{MeV/fm}.
\end{align*}\] (13)

**Fig. 1.** Proper time evolution of $\sigma$ and $\pi_1$ fields for $H = 0$, without quark-source terms and without damping.

Note that this initial condition is purely radial excitation of $\Phi$ from the origin and thus it leads to correlated $\sigma$ and $\vec{\pi}$ fluctuations.

**Fig. 2.** Proper time evolution of $\sigma$ and $\pi_1$ fields for $H \neq 0$, without quark-source terms and without damping.

For the above initial condition, we calculate the proper time evolution of sigma and pion fields. The features of different scenarios are shown in Fig. 1, Fig. 2, Fig. 3, and Fig. 4.
Fig. 3. Proper time evolution of $\sigma$ and $\pi_1$ fields for $H \neq 0$, with quark-source terms but without damping.

Fig. 4. Proper time evolution of $\sigma$ and $\pi_1$ fields for $H \neq 0$, with quark-source terms and with damping, $\gamma_\sigma = 5.0c/fm$, $\gamma_\pi = 0.3c/fm$.

In Fig. 1, we solve the equation of motion without symmetry breaking term, $H$, and do not consider the effect of quark source terms and damping. In this case, we exclude the background field. The sigma and pion fields do not couple to the quark field and they have complete symmetry. The oscillating frequencies of sigma and pion fields are exactly the same. They oscillate around different values due to different initial conditions and the motion is radial in the $(\sigma, \pi_1)$ plane.

In Fig. 2, we include the symmetry breaking term, $H$. The dynamics is quite different compared with Fig. 1. In Fig. 2 the sigma field oscillates nonlinearly around $f_\pi$, and the pion field oscillates around zero, their frequencies are also different. The sigma field takes about 2 fm proper time to grow from zero to reach the true vacuum expectation value $f_\pi$. On the other hand the pion field eventually tends to zero when the proper time gets large enough.

Compared to Fig. 2, we include the quark source terms in Fig. 3. The evolution of sigma field does not change too much and the oscillating amplitude of pion field is smaller due to the source term.
If $H \neq 0$ the transition is gradual, but fast. The source term leads to a modification of the dynamics. Compared to the earlier dynamical studies with a quenched initial condition, however, the difference is not large and we get almost the same rapid rollover transition.

In Fig. 4 we include the thermal damping term. The sigma field takes a longer time to grow from zero to the true vacuum, then quickly tends to $f_\pi$ almost without oscillation. The fluctuation of the pion field is damped out quickly. It takes about 20 fm proper time for the pion field to get to zero. For the observability it is important that the fluctuations are rapidly damped out.

If we add the thermal damping term in the equation of motion, we can see that the decrease of the amplitude of field oscillation is very fast. It is a question if this later one is a realistic approach, because for small systems both damping and fluctuations should be considered simultaneously.

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Notes

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Dedicated to Prof. Károly Nagy on his 70th birthday

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