Bubble fluctuations in $\Omega < 1$ inflation

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Abstract

In the context of the open inflationary universe, we calculate the amplitude of quantum fluctuations which deform the bubble shape. These give rise to scalar field fluctuations in the open Friedmann-Robertson-Walker universe which is contained inside the bubble. One can transform to a new gauge in which matter looks perfectly smooth, and then the perturbations behave as tensor modes (gravitational waves of very long wavelength). For $(1 - \Omega) \ll 1$, where $\Omega$ is the density parameter, the microwave temperature anisotropies produced by these modes are of order $\delta T/T \sim H(R_0\mu l)^{-1/2}(1 - \Omega)^{l/2}$. Here, $H$ is the expansion rate during inflation, $R_0$ is the intrinsic radius of the bubble at the time of nucleation, $\mu$ is the bubble wall tension and $l$ labels the different multipoles ($l > 1$). The gravitational backreaction of the bubble has been ignored. In this approximation, $G\mu R_0 \ll 1$, and the new effect can be much larger than the one due to ordinary gravitational waves generated during inflation (unless, of course, $\Omega$ gets too close to one, in which case the new effect disappears).

1 Introduction

The possibility of an open inflationary universe, in which the cosmological density parameter $\Omega$ is less than 1, has been intensively studied in recent years. According to this model, the universe is initially in a de Sitter phase, driven by the potential energy of a scalar field trapped in a false vacuum $\sigma_f$ (see Fig. 1). A bubble of the new vacuum $\sigma_t$ nucleates and its interior undergoes a second period of inflation. The homogeneity of our universe is then attributed to the O(3,1) symmetry of the bubble:
the interior of the light cone from the nucleation event is isometric to an open Friedman-Robertson-Walker (FRW) universe [8]. The second period of inflation has to be sufficiently long to generate the observed entropy, but if it is too long then $\Omega$ is driven exponentially close to 1. As a result, to obtain $\Omega < 1$ the parameters of the scalar field potential have to be fine tuned to some extent [1, 3]. Nevertheless, if observations ultimately determine that $\Omega$ is smaller than one, then open inflation may be regarded as a ‘natural’ scenario [2].

The cosmic microwave background anisotropies produced by a nearly massless scalar field in open inflation were analyzed in [3]. It was shown that a ‘super-curvature’ mode [4], which is not normalizable on the open FRW space-like sections, would give a significant contribution to the low multipoles if $\Omega < .1$ (see also [3, 4]). A more complete study of cosmological perturbations in open inflation requires the quantization of fields in the presence of a bubble. Quantum field theory in a bubble background was pioneered in [4] and further developed in [5, 6, 7]. As noted in [2], large contributions to microwave perturbations may result from the quantum fluctuations of the bubble wall itself [11]. The purpose of this paper is to calculate the amplitude of such fluctuations and their effect on the microwave background.

In Section 2 we briefly describe the bubble geometry. In Section 3 we calculate the amplitude of wall fluctuations for bubbles nucleated during inflation. This extends previous work for bubbles in flat space [11] (see also [7]). In Section 4 we show that the effect of wall perturbations can be described in terms of long wavelength tensor modes (analogous to gravitational waves), and we evaluate their impact on the microwave sky. Finally, in Section 5 we summarize our conclusions and compare them with recent related work [2, 12].

2 Bubble geometry

Before calculating the amplitude of bubble wall perturbations, it will be useful to summarize, in this section, some of the features of the spacetime containing the bubble. A conformal diagram is given in Fig. 2. The nucleation event is marked as N. The bubble wall is represented by the timelike hypersurface $w$ (solid line). In region I, which is the interior of the light-cone
from N, the line element is given by 

$$ds^2 = -dt^2 + a^2(t)d\Omega_{H^3},$$

(1)

where $d\Omega_{H^3}$ is the metric on the unit space-like hyperboloid

$$d\Omega_{H^3} = dr^2 + \sinh^2 r(d\theta^2 + \sin^2 \theta d\varphi^2).$$

(2)

Eq.(1) represents the geometry of an open FRW universe. This open universe inflates up to the time when the scalar field reaches the value $\sigma = \sigma_{rh}$ (see Fig. 1), and then reheats. After the usual radiation and matter dominated eras, it eventually becomes our observable universe. At all stages of expansion, the scale factor obeys the Friedman equation

$$(1 - \Omega) \dot{a}^2 = 1,$$

(3)

where $\Omega$ is the ratio of the matter density $\rho_m$ to the critical density $\rho_c = (3/8\pi G)(\dot{a}/a)^2$.

The chart (1) covers only the interior of the light-cone from N. One can cover the outside by analytically continuing the coordinates $t$ and $r$ to the complex plane. By taking $t = i\tau$ and $r = \rho + i(\pi/2)$, where $\tau$ and $\rho$ are real, we have 

$$ds^2 = +d\tau^2 + R^2(\tau)d\Omega_{dS}.$$ 

(4)

Here $R(\tau) = -ia(i\tau)$, and

$$d\Omega_{dS} = -d\rho^2 + \cosh^2 \rho(d\theta^2 + \sin^2 \theta d\varphi^2)$$

(5)

is the metric of a (2+1) dimensional de Sitter space of unit ‘Hubble length’. Note that $\tau$ is now a space-like coordinate, playing the role of a proper radial distance from N. In this way, the spacetime outside the light-cone from N is foliated into (2+1)-dimensional de Sitter leaves with constant $\tau$. In each one of these leaves $\rho$ plays the role of time.

The scalar field $\sigma$ obeys the field equation

$$-\Box \sigma + \frac{dV(\sigma)}{d\sigma} = 0,$$

(6)

where $\Box$ is the covariant d’Alembertian and $V(\sigma)$ is a potential of the form depicted in Fig. 1. The bubble configuration is a solution of (6) of the form

$$\sigma = \sigma_0(\tau).$$
It has the shape of a kink that interpolates between the false vacuum $\sigma_f$ and the true one $\sigma_t$. The locus where the field is at the top of the barrier, $\sigma(\tau_w) = \sigma_m$, can be identified with the trajectory of the domain wall (solid time-like line in Fig. 2). There, we have actually drawn an eternal bubble. A real bubble would nucleate at a given moment of time, say $\rho = 0$, with intrinsic radius

$$R_0 = R(\tau_w). \tag{7}$$

The intrinsic radius would subsequently expand with $\rho$ as $R_0 \cosh \rho$. The scalar field in region I can be found by analytically continuing from $\tau$ back to $t$.

Both the bubble configuration and the corresponding spacetime metric enjoy an $O(3,1)$ symmetry inherited from the spherical symmetry of the instanton describing the tunneling. This is the group of isometries of the de Sitter leaves in region II and of the open hyperboloids in region I. For the scalar field, the symmetry simply means that $\sigma_0$ only depends on $\tau$ (or $t$).

### 3 Scalar field fluctuations

In this section we calculate the amplitude of small fluctuations for a bubble that nucleates during inflation. This extends previous results for bubbles in flat space. We shall work in the approximation in which the gravitational backreaction of the bubble can be ignored. In practice, this means that up to the time of reheating, the energy density of matter can be expressed as a large cosmological constant part $\Lambda/8\pi G$, plus a small part $\delta \rho_m$ which contains the barrier feature of the potential energy (see Fig. 1) plus the gradient and kinetic contributions of the scalar field. Then we take the limit

$$8\pi G \delta \rho_m \ll \Lambda. \tag{8}$$

The case when gravitational backreaction is included will be discussed elsewhere.

In our approximation, the geometry during inflation is that of de Sitter space. In region II the line element is given by

$$\begin{equation}
R(\tau) = H^{-1} \sin(H\tau), \quad (0 < \tau < \pi H^{-1}),
\end{equation} \tag{9}$$
where \( H = (\Lambda/3)^{1/2} \) is the de Sitter Hubble rate. Note that \( R \) vanishes both at \( \tau = 0 \) (the nucleation event N) and at \( \tau = \pi H^{-1} \) (the antipodal point A). To study the wall fluctuations, we expand the scalar field as

\[
\sigma(\tau, x^i) = \sigma_0(\tau) + \phi(\tau, x^i),
\]

(10)

where \( x^i \) are the coordinates on the 2+1-dimensional de Sitter leaves \( \mathbb{S} \). The small perturbation \( \phi \) is promoted to a quantum operator \( \hat{\phi} \), which is then expanded into a sum over modes times the usual creation and annihilation operators as \( \hat{\phi} = \sum \phi_{klm} a_{klm} + h.c. \) As noted in [6], the spacelike surface \( \rho = 0 \), connecting the nucleation event N with its antipodal A, is a good Cauchy surface for the entire spacetime (see Fig. 2). Therefore we shall normalize our modes on that hypersurface. Some of them, the so-called super-curvature modes [3, 4], may not be normalizable on the open hyperboloids of the Friedman Robertson Walker chart \( \mathbb{H} \), but this is just because the hyperboloids are not good Cauchy surfaces. As we shall see, the perturbations of the bubble wall are super-curvature.

The equation of motion for small perturbations is

\[
[-\Box + m^2(\sigma_0)]\phi_{klm} = 0,
\]

(11)

where \( m^2(\sigma_0) = d^2V/d\sigma^2|_{\sigma=\sigma_0(\tau)} \). With the ansatz

\[
\phi_{klm} = R^{-1}(\tau) F_k(\tau) Y_{klm}(x^i),
\]

(12)

this separates into

\[
[-(3)\Box + k^2]Y_{klm}(x^i),
\]

(13)

and

\[
-\frac{d^2 F_k}{d\eta^2} + R^2[m^2(\sigma_0) - 2H^2]F_k = (k^2 - 1)F_k.
\]

(14)

Here \( (3)\Box \) stands for the covariant d’Alembertian on the 2+1 dimensional de Sitter leaves \( \mathbb{S} \), \( k^2 \) is a separation constant and the conformal ‘radial’ coordinate \( \eta \) is defined through the relation \( R(\tau)d\eta \equiv d\tau \),

\[
cosh \eta \equiv \frac{1}{\sin(H\tau)} = \frac{1}{HR(\tau)}.
\]

(15)

Equations (13) and (14) have a familiar interpretation [10, 11]. The first one tells us that \( Y_k \) behave as scalar fields of mass \( k^2 \) living in a 2+1-dimensional unit de Sitter space. The masses \( k^2 \) are determined as the
eigenvalues of equation (14), which is simply a one dimensional Schrodinger equation with effective potential

\[ U_{\text{eff}} = R^2[m^2(\sigma_0) - 2H^2]. \tag{16} \]

Note also that the modes \( \phi_{klm} \) must obey the Klein-Gordon normalization condition

\[ -i \int \phi_{klm} \bar{\phi}_{k'l'm'} d\Sigma^\mu = \delta_{kk'}\delta_{ll'}\delta_{mm'}, \tag{17} \]

where \( \Sigma \) is the hypersurface \( \rho = 0 \). If we choose the \( Y_{klm} \) to be Klein-Gordon normalized on the 2 + 1 dimensional de-Sitter leaves, then (17) reduces to

\[ \int_{-\infty}^{+\infty} F_k F_{k'} d\eta = \delta_{kk'}, \tag{18} \]

which is the usual normalization condition for eigenfunctions of the Schrodinger equation.

The effective potential (16) is schematically represented in Fig. 3. The height of \( U_{\text{eff}} \) at \( \eta = 0 \) (which corresponds to \( R(\tau) = H^{-1} \)) is basically given by \( (m_f^2 H^2 - 2) \), where \( m_f \) is the scalar field mass in the false vacuum. The narrow well on the left corresponds to the location of the bubble wall, where \( m^2(\sigma) \) is negative and large in absolute value. The equation of motion (6) for \( \sigma_0 \) written in terms of the conformal coordinate \( \eta \) is

\[ \sigma_0'' + \frac{2R'}{R} \sigma_0' - \frac{dV(\sigma_0)}{d\sigma} = 0, \]

where primes denote derivatives with respect to \( \eta \). Taking one more derivative with respect to \( \eta \) it is straightforward to show that

\[ F_{-3} \equiv N\sigma_0'(\eta) \tag{19} \]

is a solution of (14) with eigenvalue \( k^2 = -3 \). This is analogous to what happens for bubbles in flat space [11]. Note that \( \sigma_0' = R\dot{\sigma}_0 \), where a dot indicates derivative with respect to the ‘radial’ variable \( \tau \). But in order for the instanton to be smooth, we must have \( \dot{\sigma}_0 \to 0 \) both at the nucleation event \( (\eta \to -\infty) \) and at the antipodal point \( (\eta \to +\infty) \) [8]. From (13), \( R(\eta) \) also vanishes exponentially at \( \eta \to \pm\infty \). Therefore it is clear that the mode (13) is normalizable and that its eigenvalue \( k^2 = -3 \) belongs to the spectrum. In addition, since \( \sigma_0 \) is a monotonous function interpolating
between true and false vacuum, the mode (19) has no nodes and is the eigenstate of lowest eigenvalue. Although all higher modes will contribute to density perturbations and microwave temperature distortions [6], for the remainder of this paper we shall focus on the lowest mode. This mode has a clear geometrical interpretation as deformations of the bubble shape, which is the effect we are concentrating on. To linear order we can write the perturbed field as

\[ \sigma_0(\tau) + \phi_{-3}(\tau, x^i) \approx \sigma_0(\tau + NY_{-3lm}(x^i)). \]

Therefore, like in the case of flat space [11], the perturbations associated with \( k^2 = -3 \) correspond to deformations that shift the position of the bubble wall in a \( x^i \) dependent way, without altering the ‘radial’ profile function \( \sigma_0(\tau) \).

The normalization constant in (19) will eventually determine the magnitude of the effect. In order to calculate it we use (18), in the form

\[ N^2 \int R(\tau) \dot{\sigma}_0^2 d\tau = 1. \]  

(20)

The integral can be numerically evaluated for any particular type of bubble, but its meaning is best illustrated in the thin wall case. In this case \( \dot{\sigma}_0 \) is very small except in a small region of size comparable to the width of the narrow well in Fig. 3, which is centered around the location of the wall, at \( \tau = \tau_w \). The factor \( R(\tau_w) \) can be pulled out of the integral and what remains is simply the wall tension \( \mu \). Therefore

\[ N^2 = \frac{1}{R_0\mu}, \]

Here, as in (14), \( R_0 = R(\tau_w) \) is the radius of the bubble at the moment of nucleation. In the general case, the denominator in the right hand side is just shorthand for the integral in (21).

In the thin wall case, the explicit expression for the radius of the bubble at nucleation is given by (see e.g. [15]),

\[ R_0 = \frac{3\mu}{(9\mu^2H^2 + \epsilon^2)^{1/2}}, \]  

(21)

where \( \epsilon \) is the jump in energy density between the true and the false vacuum. As mentioned before, we have neglected the bubble’s gravitational backreaction. For the approximation to be valid, we need on one hand that \( G\epsilon << \Lambda \)
[see (8)]. On the other hand we are neglecting the gravity of the wall. As is well known, the gravitational field of a domain wall is characterized by a Rindler-type horizon distance 

\[ l_w \equiv \frac{1}{8\pi G \mu} \]  

(22)

We need this distance to be much larger than the radius of the bubble at nucleation

\[ G \mu R_0 \ll 1. \]  

(23)

So far we have found the modes describing quantum fluctuations outside the light cone from N

\[ \phi_{-3lm} = \frac{\dot{\sigma}_0(\tau)}{(R_0\mu)^{1/2}} Y_{-3lm}(x^i). \]  

(24)

The modes \( Y_{-3lm} \) are those of a scalar field of tachyonic \( (mass)^2 = k^2 = -3 \) living in a (2+1) dimensional de Sitter space (5). If we want to preserve the \( O(3,1) \) symmetry of the bubble solution then we need to choose the Bunch-Davies vacuum in the lower dimensional time-like \( \tau = const \) sections. The corresponding normalized modes are (11)

\[ Y_{-3lm} = -\left( \frac{\pi \Gamma(l - 1)}{4\Gamma(l + 3)} \right)^{1/2} \frac{1}{\cosh \rho} R_t^2(\tanh \rho) Y_{lm}(\theta, \varphi), \]  

(25)

where \( Y_{lm} \) are the spherical harmonics and \( R_t^\lambda(x) \equiv P_\lambda^\lambda(x) - (2i/\pi)Q_\lambda^\lambda(x) \). Here \( P \) and \( Q \) are the Legendre functions on the cut \(-1 < x < 1\).

In order to assess the effect of these fluctuations in our Universe today, we must first analytically continue to the interior of the light-cone,

\[ \phi_{-3lm} = \frac{\dot{\sigma}(t)}{(R_0\mu)^{1/2}} Y_{-3lm}(r, \theta, \varphi) \]  

(26)

where now the analytically continued harmonics can be cast, after some algebra (13), into the form

\[ Y_{-3lm} = \left( \frac{\Gamma(3 + l)\Gamma(l - 1)}{2} \right)^{1/2} \frac{P_{3/2}^{-l-1/2}(\cosh r)}{\sqrt{\sinh r}} Y_{lm}(\theta, \varphi). \]  

(27)
Here we have used Eqns. 8.738.2 and 8.732.5 of Ref. [13]. The Legendre functions can be given in terms of elementary functions. For \( l = 0, 1 \) and \( 2 \) they are

\[
P_{3/2}^{-1/2} = (2\pi \sinh r)^{-1/2} \sinh(2r),
\]

\[
P_{3/2}^{-3/2} = \frac{1}{\Gamma(5/2)} \left( \frac{\sinh r}{2} \right)^{3/2},
\]

\[
P_{3/2}^{-5/2} = \left( \frac{2}{\pi} \right)^{1/2} \frac{1}{8(\sinh r)^{5/2}} \left[ \frac{1}{12} \sinh(4r) - \frac{2}{3} \sinh(2r) + r \right],
\]

and for higher \( l \) they can be obtained through the well known recurrence relations.

Several comments should be made. First of all, for \( l = 0 \) and \( l = 1 \) the normalization factor in (27) diverges. This is not a problem, since these modes do not contribute to observables. They simply correspond to space-time translations of the nucleation event [11]. Second, the modes are real on the open chart, and hence their Klein-Gordon norm vanishes there. This is not a problem either, because the open hyperboloids are not good Cauchy surfaces [6]. Finally, the analytic continuation of equation (13) with \( k^2 = -3 \) tells us that

\[
\Delta Y_{-3lm} = +3Y_{-3lm},
\]

and so the eigenvalue of the Laplacian has the ‘wrong’ sign. Not surprisingly, the modes diverge exponentially for large \( r \). As we shall see, this divergence does not appear in the physical effect.

### 4 From scalar to tensor modes

The wall fluctuations induce scalar field fluctuations of the form (26) in the open FRW universe. These will locally advance or retard by an amount

\[
\delta t = \frac{\phi_{-3lm}}{\dot{\sigma}} = (R_0 \mu)^{-1/2} Y_{-3lm}(r, \theta, \varphi)
\]

the time at which the universe reheats. Deformations of the reheating surface generically induce density fluctuations and perturbations in the microwave background. It turns out that the particular modes (23) do not cause density perturbations [12], but, as we shall see, they do affect the microwave background just like gravitational waves do.
A nice framework to study deformations of the constant scalar field surfaces is that of Ref. [14]. One defines a ‘fluid’ velocity
\[ u_\mu \equiv \frac{\sigma_\mu}{(-\sigma_\mu \sigma_\mu)^{1/2}} \] (30)
orthogonal to the constant field surfaces, and projects its covariant derivative \( u_{\mu;\nu} \) onto these surfaces:
\[ u_{\mu;\nu} \equiv (\delta_\mu^\rho + u_\mu u^\rho)u_{\rho;\nu}. \]
One can separate \( u_{\mu;\nu} \) into a symmetric and an antisymmetric part. The antisymmetric part is called vorticity and it can be shown that it vanishes for a four vector of the form (30). As a consequence, the projected covariant derivative \( u_{\mu;\nu} \) coincides with the intrinsic covariant derivative on the surfaces \( \sigma = \text{const} \) [14]. The symmetric tensor \( K_{\mu\nu} = u_{\mu;\nu} \) is also known as the extrinsic curvature. Its trace \( K_\mu^\mu \equiv \Theta \) is the expansion, and in the unperturbed FRW \( \Theta = 3(\dot{a}/a) \). A straightforward calculation shows that under perturbations of the form (26) the expansion does not change [11].
The traceless part of \( K_{\mu\nu} \) is the shear tensor \( \sigma_{\mu\nu} = K_{\mu\nu} - (\Theta/3)(g_{\mu\nu} + u_\mu u_\nu) \). To linear order in perturbations and in the coordinate system (1), we have
\[ u_\mu = (-1, Y_i), \] (31)
where \( Y \) stands for \( Y_{klm}(x^i) \), with \( x^i = (r, \theta, \phi) \). Then \( \sigma_{00} = \sigma_{0i} = 0 \) and
\[ \sigma_{ij} = Y_{ij} - Y \gamma_{ij}. \] (32)
Here \( \gamma_{ij} \) is the metric on the unit spacelike hyerboloid (2). In addition to being traceless, the shear tensor for \( k^2 = -3 \) is transverse \( \sigma_{ijj} = 0 \), just like a tensor mode [3].

This immediately suggests going to a new coordinate system which straightens out the constant scalar field surfaces, while still remaining in a synchronous gauge
\[ t' = t + \delta t, \quad x'^i = x^i - \gamma_{ij} \frac{\dot{a}}{a} \delta t_{|j}. \]
Here \( \delta t \) is given by (29). In this new gauge \( \sigma = \sigma(t') \) is constant on \( t' = \text{const} \) surfaces, but the metric reads
\[ ds^2 = -dt'^2 + (\gamma_{ij} + h_{ij})dx^idx^j. \]
Here
\[ h_{ij} = -2E\sigma_{ij}, \] (33)
and, during inflation
\[ E = \frac{\dot{a}}{a}(R_0\mu)^{-1/2}. \] (34)
Once in the new transverse and traceless gauge, the matter distribution looks perfectly smooth. It is now legitimate to evolve \( h_{ij} \) with the usual equation for tensor perturbations through the entire cosmic evolution
\[ \ddot{h}_{ij} + 3\frac{\dot{a}}{a}\dot{h}_{ij} - \frac{1}{a^2} (\Delta h_{ij} + 2h_{ij}) = 0. \] (35)
Note that although \( \Delta Y = +3Y \), the corresponding tensor mode, derived from (33) and (32) satisfies
\[ \Delta h_{ij} = -3h_{ij}, \]
with the ‘correct’ sign for the Laplacian eigenvalue. For \( l = 0 \) and \( l = 1 \), it can be readily checked that \( \sigma_{ij} = 0 \), and so the tensor mode only exists for \( l > 1 \), as expected of gravitational waves.

Introducing (33) in (35) we can calculate the evolution of the amplitude \( E \) throughout the different stages of expansion. In terms of conformal time \((d\eta = a^{-1}dt)\), we have
\[ E'' + 2\frac{a'}{a}E + E = 0. \]
During inflation \( E \) is given by (34) and it tends to a constant,
\[ E = \frac{H}{\sqrt{R_0\mu}}, \]
where \( H = (\Lambda/3)^{1/2} \). In the radiation era \( \Omega = 1 \) to very high accuracy, and it can be checked that as a result \( E \) stays constant.

During the matter era \( a \sim t^{2/3} \sim \eta^2 \), so
\[ E'' + \frac{4}{\eta}E' + E = 0. \]
This can be solved in terms of Bessel functions. For small \( \eta \)
\[ E = \frac{H}{\sqrt{R_0\mu}}(1 - \frac{\eta^2}{8} + \ldots). \]

11
From (3) we have \( \eta = 2(1 - \Omega)^{1/2} \), and in what follows we shall concentrate in the case \( (1 - \Omega) \ll 1 \). (The general case can be treated numerically along the lines of Refs. [4, 3]).

The amplitude of microwave fluctuations due to \( h_{ij} \) is given by the well-known Sachs-Wolfe formula

\[
\frac{\delta T}{T} = \frac{1}{2} \int_0^{r_{ls}} \frac{d h_{rr}}{d \eta} d r,
\]

where \( r_{ls} \approx 2(1 - \Omega)^{1/2} \) is the comoving distance to the surface of last scattering and \( h'_{rr} \) is evaluated at \( \eta = r_{ls} - r \). Since \( r_{ls} \) is assumed to be small, we can use the asymptotic form of the Legendre functions in (27) to obtain

\[
Y_{-3lm} \approx \Gamma_l 2^{-(l+1)} r^l Y_{lm},
\]

where \( \Gamma_l = \frac{\Gamma(3 + l) \Gamma(l - 1)}{\Gamma(l + 3/2)} \). Using this form in (32) and (33) one immediately obtains

\[
\frac{\delta T}{T} \approx \frac{H}{\sqrt{R_0 \mu}} \frac{\Gamma_l}{8} (1 - \Omega)^{l/2}, \tag{36}
\]

which for low spatial curvature is dominated by the quadrupole \( l = 2 \).

It should be noted that the tensor mode \( h_{ij} \) is pure gauge during inflation (as it should since we are neglecting the bubble’s backreaction.) However, the overall configuration taking the scalar field into account is not. The \( \sigma = \text{const.} \) surfaces have non-vanishing shear. As we evolve \( h_{ij} \) past the reheating surface, it gradually ceases to be pure gauge, as can be checked by expressing the evolved mode in the longitudinal gauge [17]. Also, it can be checked that although the scalar modes \( Y_{-3lm} \) diverge exponentially at large distances, the corresponding tensor components \( h_{rr} \) entering the Sachs-Wolfe formula do not.

5 Conclusions

We have calculated the amplitude of small fluctuations in the shape of bubbles nucleated during inflation. In the case of thin bubbles, the result takes the simple form (26). Since the de Sitter modes \( Y_{-3lm} \) grow like the intrinsic radius of the bubble \( r = R_0 \cosh \rho \) (\( \rho R_0 \) is the proper time coordinate on the
bubble wall), the relative amplitude of proper local radial wall displacements is given by

$$\frac{\delta r}{r} \sim (R_0^3 \mu)^{-1/2},$$

where $R_0$ is the radius of the bubble at the time of nucleation, given by (7), and $\mu$ is the wall tension. This coincides by order of magnitude with the estimate of Ref. [2] in the case when the size of the bubbles at the time of nucleation is much smaller than the de Sitter horizon $H^{-1}$.

The propagation of wall perturbations to the interior of the light cone gives rise to shear deformations of the reheating surface, and consequently, of the surface of last scattering. The simplest way to study the cosmological evolution of such perturbations is to use a synchronous coordinate system in which matter looks perfectly smooth. Somewhat surprisingly, the metric fluctuations in this gauge are transverse and traceless, just like usual gravitational waves. This transmutation of scalar into tensor-like modes is only possible because of the peculiar (supercurvature) eigenvalue of the Laplacian for the scalar modes corresponding to wall fluctuations, $k^2 = -3$ [6].

For $(\Omega - 1) \ll 1$, the anisotropies of the microwave sky produced by these waves are given in (36). The dominant effect is in the quadrupole, with

$$\frac{\delta T}{T} \sim \frac{H}{\sqrt{R_0 \mu}} (1 - \Omega).$$

Within the limits of validity of our approximation [see (23)], and unless $\Omega$ is too close to 1, this can be much larger than the distortion produced by usual gravitational waves produced during inflation, which is of order $G^{1/2} H$. The case with strongly gravitating domain walls may yield a different result [12], and is currently under investigation.

Finally, we would like to compare our results with those of Ref. [12]. There it is claimed that the $k^2 = -3$ modes produce no distortions of the microwave background. Here we have calculated the amplitude of such perturbations and their nonvanishing effect. In Ref. [12] the amplitude of a $k = 0$ homogeneous fluctuation is estimated by considering the small change $\Delta S$ in the instanton action when the radius of the bubble is changed by an amount $\delta a$. This is used to estimate the ‘typical deviation’ $\delta a$ as the one that corresponds to $\Delta S \sim 1$. However, the physical meaning of that prescription is unclear, because the radius of the bubble cannot have a homogeneous fluctuation. In flat space, this would violate energy conservation, and a similar
argument can be applied in curved space. The only homogeneous radial fluctuations allowed in the thin wall limit are time translations of the nucleation point.

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References

[1] M. Bucher, A. Goldhaber and N. Turok, Nucl. Phys. B, Proc. Suppl. 43, 173 (1995); M. Bucher and N. Turok, Phys. Rev. D52, 5538 (1995).

[2] A. Linde and A. Mezhlumian, Phys. Rev. D52, 6789 (1995).

[3] K. Yamamoto, M. Sasaki and T. Tanaka, ‘Large angle CMB anisotropies in an open universe ...’, Preprint, KUNS-1309 (1995); K. Yamamoto and E. Bunn, ‘Observational tests of one bubble open inflationary cosmological models’, Preprint, KUNS-1357 (1995).

[4] D. Lyth and A. Wosckiyna, ‘Large scale perturbations in the open universe’, Phys. Rev. D (in press); J. Garcia-Bellido, A. Liddle, D. Lyth and D. Wands, ‘The open universe Grishchuk-Zeldovich effect’, SUSSEX-AST95/8-1 (1995).

[5] T. Tanaka and M. Sasaki, Phys. Rev. D50, 6444 (1994).

[6] M. Sasaki, T. Tanaka, K. Yamamoto and J. Yokoyama, Phys. Lett. B317, 510 (1993); M. Sasaki, T. Tanaka and K. Yamamoto, Phys. Rev. D51, 2979 (1995).

[7] T. Hamazaki, M. Sasaki, T. Tanaka and K. Yamamoto, ‘Self excitation of the tunneling scalar field in false vacuum decay’ Preprint KUNS 1340 (1995).
[8] S. Coleman and F. de Luccia, Phys. Rev. D21, 3305 (1980). A. Guth and E. Weinberg, Nucl. Phys. B212, 321 (1983).

[9] B. Ratra and P. Peebles, Phys. Rev. D52, 1837 (1995).

[10] T. Vachaspati and A. Vilenkin, Phys. Rev. D43, 3846 (1991); V. Rubakov, Nucl. Phys. B245, 481 (1984).

[11] J. Garriga and A. Vilenkin, Phys. Rev. D44, 1007 (1991); Phys. Rev. D45, 3469 (1992).

[12] J. Garcia-Bellido, ‘Metric perturbations from quantum tunneling in open inflation’, SUSSEX-AST 95/10-1.

[13] I.S. Gradshtein and I.M. Ryzhik, Tables of integrals, series and products (Academic, New York, 1980).

[14] M. Bruni, P. Dunsby and G.F.R. Ellis, Class. Quant. Grav. 9, 921 (1992).

[15] J. Garriga, Phys. Rev. D49, 6327 (1994).

[16] A. Vilenkin, Phys. Lett. 133B, 117 (1983); J. Ipser and P. Sikivie, Phys. Rev. D30, 712 (1984).

[17] V. Mukhanov, R. Brandenberger and H. Feldman, Phys. Rep. 215, 203 (1992).

[18] B. Allen, Phys. Rev. D51, 5491 (1995).

**Figure captions**

- Fig. 1 A scalar field potential $V(\sigma)$ which leads to open inflation. The universe is initially in the false vacuum phase $\sigma_f$ when a bubble of $\sigma_t$ nucleates. Then the scalar field slowly rolls down the hill until it reaches $\sigma_{rh}$, at which point the universe reheats. We separate the potential into a large part $\Lambda/8\pi G$ and a small part with the barrier feature, since we want to neglect the self gravity of the bubble. The top of the barrier is denoted by $\sigma_m$. 
Fig. 2 A conformal diagram of spacetime in the presence of a bubble. The nucleation event is marked as N. Region I corresponds to an open FRW which inflates and eventually becomes our observable universe. The trajectory of the domain wall is marked as $w$. It lies in Region II, which is covered by the chart $[4]$. The spacelike hypersurface $\rho = 0$, connecting N with the antipodal point A, is a good cauchy surface for the entire spacetime. Clearly, no such surfaces exist in Region I. Region III, the interior of the light-cone from A, is uninteresting for our purposes.

Fig. 3 The effective potential $U_{\text{eff}}$ as a function of conformal radius $\eta$. The narrow well at $\eta_w$ corresponds to the location of the bubble wall, where the effective mass $m^2(\sigma)$ is negative.
\[ V(\sigma) \]

\[ \Lambda/8\pi G \]

\[ \sigma_f \quad \sigma_m \quad \sigma_t \quad \sigma_r \]

\[ \eta \quad \eta_w \]

Fig. 1

Fig. 2

Fig. 3