Dark matter freeze-in produces large post-inflationary isocurvature

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In this Letter, we show that the nonthermal nature of dark matter freeze-in production leads to large, totally correlated dark matter-photon isocurvature perturbations, which are imprinted in anisotropies of the cosmic microwave background (CMB). Isocurvature is typically expected from inflationary physics, but the isocurvature from freeze-in arises post inflation. We compute the freeze-in of millicharged dark matter, generated from electron-positron annihilations in the early Universe. We find that current CMB observations from Planck exclude this scenario for dark matter masses between 1 MeV and 10 GeV at more than 2σ, whereas upcoming CMB experiments will have the sensitivity to reach at least the 4σ level. We anticipate any scenario in which dark matter is nonthermally produced to generically give rise to isocurvature. Our work opens a new avenue for exploring fundamental dark matter physics through its impact on cosmological observables.

Introduction. Cosmological and astrophysical observations provide incontrovertible evidence for the existence of dark matter (DM) [1–3]. A cold and collisionless DM component of the Universe describes the large-scale structure of the Universe remarkably well, as evidenced by the anisotropy of the cosmic microwave background (CMB) [3] and galaxy clustering [4, 5], for example.

One of the prominent theoretical descriptions for the nature of DM is the weakly-interacting massive particle (WIMP), for which DM has interactions with Standard Model particles beyond gravity. These interactions are weak enough for WIMPs to be considered collisionless for the purposes of structure formation and yet substantial enough for WIMPs to be in equilibrium with the thermal bath of Standard Model particles at very early times in cosmic history. As the Universe expands and cools, WIMPs eventually undergo thermal freeze-out, in which the comoving number density approaches a fixed value after chemical decoupling from the thermal bath.

A compelling alternative scenario for the cosmological production of DM is freeze-in [6]. For example, the annihilation of Standard Model particles can generate an abundance of feebly-interacting massive particles constituting DM, but the interaction strength is so weak that these particles never attain thermal equilibrium with the thermal bath [6]. A variety of well-motivated models can give rise to freeze-in dynamics in the early Universe and are considered compatible with the observed properties of DM [6–18].

Experimentally, freeze-in DM sets an important cosmological benchmark for direct detection experiments searching for evidence of DM interactions via electronic recoils [8, 19–31]. Typical WIMP-motivated nuclear-recoil experiments lose sensitivity for DM masses below O(GeV), while electronic-recoil experiments can currently probe DM masses down to O(MeV). For lower masses, thermal WIMPs generate too much relativistic energy density during big bang nucleosynthesis (BBN), altering predictions of the primordial element abundances [32–34]. Freeze-in provides a mechanism to produce such light DM candidates without running afoul of BBN constraints [35–38].

The nonthermal nature of freeze-in production has been previously studied in the context of their nonthermal phase space distribution function (PSD) and its impact on cosmological observables [15, 16, 39, 40]. These works considered the impact of out-of-equilibrium production on the background evolution of the Universe only.

In this Letter, we present the first analysis of how freeze-in impacts the initial conditions and evolution of DM perturbations. Under the standard cosmological framework, the cold DM component is assumed to be thermal, and thus its initial conditions are adiabatic (i.e., determined by the curvature perturbation). For nonthermal DM, there is no expectation that the initial conditions for DM must be adiabatic and thus DM may possess a non-adiabatic isocurvature component.

The generation of isocurvature is typically associated with exotic inflationary physics, such as multi-field inflation [41–44]. Alternatively, breaking the shift symmetry of axion-like particles before or during inflation can produce isocurvature if that symmetry is not subsequently restored [45–49]. The important difference for our work is that isocurvature from the epoch of inflation is primordial in nature, whereas the isocurvature from nonthermal DM is produced after inflation. Regardless of origin, isocurvature perturbations are constrained at the percent level by CMB anisotropy measurements from Planck [50–52].

For concreteness, we consider pure millicharged DM that freezes in via pair production from electron-positron annihilation in the early Universe. We compute the nonthermal PSD for DM and determine its impact on the initial conditions for DM perturbations. We find that the out-of-equilibrium process of freeze-in sources large post-inflationary DM-photon isocurvature perturbations that are totally correlated with the curvature perturbation by permitting non-trivial energy and momentum exchange...
between DM and the thermal bath. Since freeze-in DM never comes into thermal equilibrium with the rest of the Universe, the DM isocurvature perturbation persists and becomes imprinted on the CMB. We find that Planck 2018 CMB anisotropy measurements exclude this freeze-in scenario for DM masses in our region of interest, between 1 MeV and 10 GeV.

**Isocurvature.** Initial conditions for linear cosmological perturbations in Fourier space are defined during the radiation-dominated era, when all Fourier modes of interest for cosmological observables are on super-horizon scales: $x = k\tau \ll 1$, where $\tau$ is conformal time and $k$ is the mode wave number. Each mode evolves independently and can be decomposed into adiabatic and isocurvature modes [53], whose amplitudes are coupled [54].

A pure adiabatic mode (in either the synchronous or Newtonian gauge) describes a Universe in which there is a nonzero curvature perturbation and the cosmic medium temperature is inhomogeneous. It can be generated from single-field slow-roll inflation, which provides a simple description of the post-Planckian era consistent with observation [55]. After inflation, the Universe reheat into the radiation-dominated era in which the particle content of the Universe is presumably in thermal equilibrium. Traditional WIMPs would indeed be in equilibrium with the thermal bath at early times and evolve adiabatically; the freeze-out process suppresses isocurvature [56].

An isocurvature mode portrays a Universe with no curvature perturbation and a homogeneous temperature. Such a mode can be generated during inflation [41–47]; however, even large initial isocurvature perturbations are swiftly erased by a subsequent phase of common thermal equilibrium, which enforces adiabaticity between the different species that constitute the cosmic medium [56]. For freeze-in DM, which never attains thermal equilibrium with the thermal bath, the absence of significant isocurvature is not guaranteed, and a careful analysis is required to quantify the amount of post-inflationary isocurvature produced during freeze-in.

**Dark matter freeze-in.** As a benchmark model for freeze-in, we consider DM particles $\chi$ with millicharge $eQ_\chi$, where $e$ is the standard electromagnetic charge and $Q_\chi \ll 1$ [8, 57–62]. We assume the Universe reheat with a negligible amount of DM [6], and the DM abundance builds over time through the process $e^- e^+ \rightarrow \chi \bar{\chi}$, until it kinematically shuts off as the background temperature of the bath $T$ falls below the DM mass $m_\chi$. Omitting contributions from heavier fermions has negligible impact on our numerical calculation of isocurvature. We fix $Q_\chi \sim \mathcal{O}(10^{-11})$ so that frozen-in DM constitutes the total DM relic abundance $\Omega_\chi = 0.26$, inferred from Planck 2018 data [3]. For such a small value of $Q_\chi$, the DM production rate is lower than the rate of Hubble expansion.

We include millicharged DM with masses between 1 MeV and 10 GeV in our analysis, for which electron-positron annihilation is the dominant DM production channel. For $m_\chi \lesssim 1$ MeV, the effects of plasmon decay become important [15, 16]. For $m_\chi \gtrsim 10$ GeV, there are additional interaction channels that depend on the choice of UV completion of the millicharge model. Additionally, we note that the rate of the reverse annihilation process $\chi \bar{\chi} \rightarrow e^- e^+$ is negligible, because the DM number density is small during freeze-in compared to the number density of electrons.

The nonthermal nature of freeze-in DM is key to generating isocurvature, requiring us to calculate the DM PSD by solving the Boltzmann equations. The background PSD $\tilde{f}(\tau, p)$ is given by [63]

$$\frac{\partial \tilde{f}}{\partial \tau} - \mathcal{H} \frac{\partial \tilde{f}}{\partial \log p} = \frac{a}{E} \tilde{C},$$

(1)

where $E$ and $p$ are the physical energy and momentum of $\chi$, respectively; $a$ is the scale factor; $\mathcal{H}$ is the conformal Hubble expansion rate; and $\tilde{C}$ is the background collision term, given in Appendix A. We assume there is initially no DM after reheating such that $\tilde{f} = 0$ for all momenta $p$.

The perturbed Boltzmann equation for the first-order correction $\delta f$ to the background PSD is [53, 63, 64]

$$\frac{\partial \delta f}{\partial \tau} + \frac{i p k}{E} (\hat{k} \cdot \hat{n}) \delta f + \frac{a f}{\partial \log p} \left[ \frac{\partial \eta}{\partial \tau} - \left( \frac{k \cdot n}{2} \right) \left( \frac{\partial h}{\partial \tau} + 6 \frac{\partial \eta}{\partial \tau} \right) \right] = \frac{a}{E} \delta \tilde{C},$$

(2)

where $\eta$ and $h$ are the metric perturbations in synchronous gauge; $k = k/k$ and $\hat{n} = p/p$ are the directions of the mode wave number and momentum, respectively; and $\delta \tilde{C}$ is the perturbed collision term, given in Appendix A. We perform a Legendre expansion for $\delta \tilde{f}$ and $\delta \tilde{C}$ and solve the perturbed Boltzmann equations for the lowest few multipole moments $\ell$; we neglect higher moments, since they are suppressed by $k\tau$.

In the top panel of Fig. 1, we show the nonthermal background PSDs for different DM masses. In the bottom panel, we show the first two moments of the perturbed PSD for the adiabatic and isocurvature modes. The background PSDs and the moments of the perturbed PSDs are narrower and shifted to lower momenta, compared to the Fermi-Dirac distribution of a species in thermal equilibrium. Our calculations assume the only relevant collision term is that from annihilation. We expect the elastic scattering process $e^- e^+ \rightarrow e^- e^+$, which is proportional to the elastic cross section

$$\bar{\sigma}_e = \frac{e^4 Q_\chi^2}{\pi m_e^2 \alpha^4} \left( \frac{m_e m_\chi}{m_e + m_\chi} \right)^2,$$

(3)

where $m_e$ is the electron mass and $\alpha = e^2/(4\pi)$, during and after freeze-in to have negligible impact on our results (see Appendix B for details).

Our work is the first to perform the calculation of the perturbed PSD for freeze-in DM, from the onset of nonthermal production until the comoving PSD becomes fixed, as shown in Fig. 1. With the background
and perturbed PSDs, we are able to compute the initial conditions for and evolution of macroscopic DM fluid properties (e.g., density and pressure perturbations, velocity divergence, anisotropic stress, etc.) from first principles, without imposing any assumptions about the fluid [63, 65, 66]. Although the DM fluid exhibits nonstandard properties during the formation stage, we find numerically that DM post freeze-in is indistinguishable from standard cold DM with primordial isocurvature [53, 63, 64].

**Isocurvature from freeze-in.** We follow the formalism developed by Bardeen and Kodama-Sasaki [54, 67, 68] to solve for the evolution of the curvature perturbation $\mathcal{R}$ and isocurvature perturbation $S_{\chi, r}$ between DM and radiation. These perturbations control the amplitudes of the adiabatic and isocurvature modes, respectively. At early times, the Universe contains effectively two fluids: DM and radiation, consisting of tightly-coupled baryons, photons, and neutrinos. We treat radiation as a perfect, relativistic fluid with no entropy or anisotropic stress perturbations. Since freeze-in results in negligible energy loss from the radiation bath, we ignore any backreaction due to DM formation.

During the period of DM formation, the equations governing the evolution of the curvature and isocurvature perturbations at leading order in $x$ are [54, 67, 68]

$$\frac{d^2 \Delta}{dx^2} - \frac{2}{x^2} \Delta \simeq 0, \quad \frac{d^2 S_{\chi, r}}{dx^2} + \frac{2}{x} \frac{dS_{\chi, r}}{dx} + \frac{S_{\chi, r}}{3} \simeq \frac{3 dE_{\chi, r}}{x} - \frac{F_{\chi, r}}{x},$$

where $\Delta$ is the gauge-invariant total density perturbation, and $E_{\chi, r}$ and $F_{\chi, r}$ parameterize the energy and momentum exchange, respectively, [54, 68] between DM and radiation at the perturbative level. The solution to Eq. (4a) scales as $\Delta \propto x^2 \mathcal{R}$ and matches the solution obtained for a Universe with radiation and standard cold DM; thus, our assumption that freeze-in leaves the evolution for radiation unaltered is consistent with our numerical results. The energy- and momentum-exchange terms in Eq. (4b), which source the isocurvature perturbation, scale as $E_{\chi, r} \propto x^3 \mathcal{R}$ and $F_{\chi, r} \propto x \mathcal{R}$ at leading order in $x$ for freeze-in DM. We find that $E_{\chi, r}$ is the dominant source term for generating isocurvature.

After DM freezes in and becomes nonrelativistic, we have

$$\frac{d^2 \Delta}{dx^2} - \frac{2}{x^2} \Delta \simeq 0, \quad \frac{d^2 S_{\chi, r}}{dx^2} + \frac{1}{x} \frac{dS_{\chi, r}}{dx} \simeq 0.$$

The evolution of $\Delta$ is unaffected, but the DM isocurvature perturbation becomes approximately constant, fixed to the value it reaches at the end of freeze-in.

In practice, we fully solve the set of coupled differential equations for $\Delta$ and $S_{\chi, r}$ [68]. We find $S_{\chi, r} \propto \mathcal{R}$, indicating that the DM isocurvature perturbation is totally correlated with the curvature perturbation. Furthermore, the ratio $S_{\chi, r}/\mathcal{R}$ is approximately scale-independent; hence, $S_{\chi, r}$ inherits its statistical properties from $\mathcal{R}$, and their power spectra differ only by a rescaling of the amplitude. Therefore, we do not expect any non-Gaussianity.

We show $S_{\chi, r}/\mathcal{R}$ as a function of DM mass in Fig. 2. The top horizontal axis shows the thermal bath temperature $T_\theta$ at which DM freeze-in ends. The mass dependence of $S_{\chi, r}/\mathcal{R}$ is due to changes in the effective number of relativistic degrees of freedom $g_*$ in the thermal bath, which alters the evolution of $T_\theta$ in a manner consistent with entropy conservation [69].

**CMB constraints.** Having established that millicharged DM produces the same mode evolution as cold DM with inflationary DM isocurvature, we can use existing bounds on isocurvature to constrain millicharged DM freeze-in. We show the 95% confidence level (CL) upper bound of $S_{\chi, r}/\mathcal{R} < 0.0308$ for totally correlated DM.
isocurvature from Planck 2018 data [52] in Fig. 2. In the mass range we consider, freeze-in DM produces an isocurvature perturbation that exceeds this bound and is thus excluded. The temperature and polarization fluctuations in isocurvature modes are suppressed approximately by a factor of $\ell^{-2}$ with respect to the adiabatic modes at small scales [70, 71]; thus, the constraining power of CMB anisotropies is dominated by low multipoles $\ell \lesssim 1000$. Planck is cosmic variance limited in this range of multipoles for temperature, but not polarization, anisotropies.

The upcoming CMB experiments, Simons Observatory [74] (SO) and CMB-S4 [75], will be nearly cosmic variance limited in both temperature and polarization at low multipoles and will thus have more constraining power than Planck. We perform a Fisher forecast for the expected sensitivity of SO and CMB-S4 to detect totally correlated isocurvature perturbations. Using temperature and polarization (temperature, polarization, and lensing), the expected error is $\sigma_{S/R} = 6.2 \times 10^{-3}$ ($\sigma_{S/R} = 6.0 \times 10^{-3}$) and $\sigma_{S/R} = 5.9 \times 10^{-3}$ ($\sigma_{S/R} = 5.5 \times 10^{-3}$) for SO and CMB-S4, respectively.

In Fig. 3, we compare our constraints on the millicharged DM-electron cross section from isocurvature produced during freeze-in to existing constraints. As evident from Fig. 2, Planck rules out the scenario in which freeze-in millicharged DM produces the total DM relic abundance for the range of masses shown. Therefore, we cover the freeze-in line with the excluded orange region in Fig. 3 for the mass range of our analysis. Cross sections just above (below) the freeze-in line correspond to millicharges $Q$ that overproduce (underproduce) the observed DM relic abundance. For the case of under-production, we can consider freeze-in DM constituting a fraction of the total DM abundance, with a standard cold DM component making up the rest.

The maximum amount of isocurvature allowed by data scales inversely with the freeze-in abundance. Thus, for a given $m_\chi$ in Fig. 2, we can determine the lowest abundance of freeze-in DM that would still be ruled out by Planck and extend the orange exclusion region in Fig. 3 appropriately. This extension corresponds to millicharged DM making up a maximum fraction of all of DM between 3% and 65% for the millicharged DM mass range shown in Fig. 2. A similar procedure is used to produce the red region in Fig. 3 for the sensitivity range of SO and CMB-S4, corresponding to a maximum fraction between 1% and 25%.

**Conclusions.** In this Letter, we calculate the post-inflationary DM isocurvature produced during the freeze-in of millicharged DM. Our analysis focuses on DM masses between 1 MeV and 10 GeV, the range in which electron-positron annihilation dominates the production of DM. We find that DM formation generates a large amount of isocurvature, which becomes fixed on superhorizon scales. The isocurvature produced during freeze-in is fully correlated with the curvature perturbation. The scale dependence of the isocurvature power spectrum mimics that of the curvature power spectrum, and thus the isocurvature field inherits the statistical prop-
eerties of the curvature field. Given the magnitude of the isocurvature-to-curvature ratio, Planck rules out millicharged DM as the main component of the total DM for most of the mass range we explored. Future CMB experiments can further tighten constraints on the fraction of millicharged DM to the 10% level.

We emphasize that the biased energy exchange from the thermal bath to the DM fluid is responsible for sourcing large isocurvature during freeze-in DM production. Such a biased exchange is a feature of all DM freeze-in models, independent of production channel (e.g., decay, heavy mediators), which suggests that they are subject to constraints that were never considered before. The same argument applies to other models with feebly interacting particles (e.g., Dodelson-Widrow or resonance production of sterile neutrinos), regardless of whether DM is created relativistically or not.

Furthermore, we do not expect DM self-thermalization (e.g., via coupling to a dark radiation bath) during or after its production to have sizeable impact on the amplitude of the generated isocurvature. Even changes to the DM PSD (e.g., from thermalization with the Standard Model) post production could have minimal impact on isocurvature. We will study these interesting possibilities in future work.

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Appendix A: Collision terms

In this section, we provide the expressions for the background and perturbed collision terms appearing in the Boltzmann equations for the dark matter (DM) fluid. We work in a regime in which the relevant process contributing to the collision terms is from electron-positron annihilation into DM and anti-DM: $e^- e^+ \rightarrow \chi \bar{\chi}$. We denote the electron mass and DM mass as $m_e$ and $m_\chi$, respectively. DM has millicharge $eQ_\chi$, where $e$ is the standard electromagnetic charge. The collision terms are written in terms of the physical energy $E$ and momentum $p$ of $\chi$.

The background collision term in Eq. (1) is

$$\tilde{C}[p] = \frac{Q_\chi^2 e^4 T}{24 \pi^3 p} \int ds e^{-\frac{s^2 E}{2m_\chi}} \sinh \left( \frac{p \sqrt{(s - 4m_\chi^2)}}{2m_\chi T} \right) \times \sqrt{1 - \frac{4m_\chi^2}{s}} \left( 1 + \frac{2m_e^2}{s} \right) \left( 1 + \frac{2m_\chi^2}{s} \right), (A1)$$

where $T$ is the background temperature of the thermal bath, and the integral over $s$ is restricted to $s > \max\{4m_e^2, 2m_\chi(E + m_\chi)\}$.

We compute the perturbed collision term due to small temperature fluctuations in the cosmic medium. The phase space distribution function (PSD) $f_e$ of electrons/positrons is perturbed with respect to the standard Fermi-Dirac distribution and it reads as

$$f_e(T) = \left( e^{E/T} + 1 \right)^{-1} \sim \left( e^{E/T} + 1 \right)^{-1} + \frac{E}{T^2 (e^{E/T} + 1)^2} \delta T, (A2)$$

where $\delta T$ is the temperature fluctuation in Fourier space. The perturbations in the PSD induce a perturbed collision term which (in Fourier space) is given by

$$\delta \tilde{C} = \frac{e^4 Q_\chi^2 \delta T}{6(2\pi)^3 p} \int ds \sqrt{1 - \frac{4m_\chi^2}{s}} \left( 1 + \frac{2m_e^2}{s} \right) \left( 1 + \frac{2m_\chi^2}{s} \right) \times \left[ \left( 1 + \frac{E_{\min}}{T} \right) e^{-E_{\min}/T} - \left( 1 + \frac{E_{\max}}{T} \right) e^{-E_{\max}/T} \right], (A3)$$

where

$$E_{\min} = \frac{s}{2m_\chi^2} \left[ E - p \sqrt{1 - \frac{4m_e^2}{s}} \right],$$
$$E_{\max} = \frac{s}{2m_\chi^2} \left[ E + p \sqrt{1 - \frac{4m_\chi^2}{s}} \right],$$

and $s > \max\{4m_e^2, 2m_\chi(E + m_\chi)\}$.

Appendix B: Elastic scattering

In this section, we justify why neglecting elastic scattering processes $e^- \chi \rightarrow e^- \chi$ when solving for the PSD is a good approximation. During freeze-in the collision term $\tilde{C}_{\text{el}}$ due to elastic scattering scales with the annihilation collision term in Eq. (A1) as

$$\tilde{C}_{\text{el}} \lesssim \tilde{C} \left| \frac{M_t^2}{M_s^2} \right| f_\chi f_e. \quad (B1)$$
The scattering amplitude for the $t$-channel process
\[ |M_t|^2 = \frac{e^4 Q^2}{2t^2} \left[ s^2 + u^2 - 4(m_e^2 + m_\chi^2)(s + u) + 6(m_e^2 + m_\chi^2)^2 \right], \] (B2)
is enhanced for small $t$ compared to the scattering amplitude of the $s$-channel annihilation process
\[ |M_s|^2 = \frac{e^4 Q^2}{2s^2} \left[ t^2 + u^2 + 4s(m_e^2 + m_\chi^2) - 2(m_e^2 + m_\chi^2)^2 \right]. \] (B3)

However, the $t$-channel scattering enhancement does not beat the suppression in relative abundance encoded in the ratio of the PSDs $f_\chi/f_e$. The suppression is weakest towards the end of freeze-in when $s \sim 4m_\chi^2$, where
\[ f_\chi/f_e \sim z_{eq}/z_{fi} \sim 10^{-6} \left( \frac{\text{MeV}}{m_\chi} \right). \] (B4)

At the end of freeze-in we approximate the ratio of scattering amplitudes as
\[ \frac{|M_t|^2}{|M_s|^2} = \frac{s^2}{t^2} \leq \frac{s^2}{m_D^2} \approx 10^4, \] (B5)
where $m_D^2 = e^2 T^2 / 3$ is the Debye plasma mass that accounts for screening of the electromagnetic field in a charged, relativistic plasma [76]. Plugging Eq. (B5) and Eq. (B4) in Eq. (B1), we find
\[ \bar{C}_{\text{el}} / \bar{C} \lesssim 0.01 \left( \frac{\text{MeV}}{m_\chi} \right) \left( \frac{s^2/m_D^4}{10^4} \right), \] (B6)
which shows that the evolution of the PSD described by the differential Eq. (1) is dominated by the collision term in Eq. (A1).

After freeze-in, we expect the rate of momentum transfer between DM and the baryon fluid to be sufficiently small, such that the PSD remains unaltered. Based on CMB studies of millicharged DM scattering [73, 77, 78], for $m_\chi \gtrsim 1$ MeV, the momentum-transfer rate coefficient between DM and electrons/protons is $\lesssim \mathcal{O}(0.01)$ of the Hubble expansion rate pre recombination for $Q_\chi \gtrsim \mathcal{O}(10^{-9})$. Since we are interested in millicharges $Q_\chi$ at least an order of magnitude smaller, we expect elastic scattering to have a negligible impact on the DM velocity distribution.