STUDENT LEARNING, CHILDHOOD & VOICES | RESEARCH ARTICLE

Relationship between mental computation and mathematical reasoning

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Abstract: Mental computation and mathematical reasoning are two intertwined top-level mental activities. In deciding which strategy to use when doing mental computing, mathematical reasoning is essential. From this reciprocal influence, the current study aims at examining the relationship between mental computation and mathematical reasoning. The study was carried out with 118 fifth-grade students (11–12-year-olds). As data collection tool, “mathematical reasoning test” and “mental computation test” were developed and used. In analyzing the data, Pearson's correlation coefficient ($r$) between participants’ scores of each test was computed. Some sample student responses to some questions in both tests were also presented directly. Evidence was found that there is a significant positive correlation between mental computation and mathematical reasoning. It is noteworthy that rather than exposing students to familiar classical problems, students need to be enabled to deal with exceptional/non-routine problems, and especially young children should be encouraged to do mental computing in order for developing both skills. On the other hand, students must be asked to write the strategies they use and on which grounds they preferred them while solving the problems.

Subjects: Assessment & Testing; By Subject; Early Years; Education Studies; Primary/Elementary Education

Keywords: mental computation; mathematical reasoning; mental computation-mathematical reasoning relation; fifth-grade students; problems requiring reasoning

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PUBLIC INTEREST STATEMENT

Mental computation and mathematical reasoning are two crucial skills that are essential for cultivating higher order thinking because they help individuals realize relationships among numbers while increasing their number sense. Mental computing may be needed not only at school but also in daily life since there is an evidence that daily mathematics in which four basic operations take part mainly is suggested to be more prone to human’s natural creativity and reasoning. The importance and need of mathematical reasoning on mental computation was referred before by many researches but there is no statistical evidence of relationship between these two skills in the researches. It is thought that present study will fill an important gap in the literature revealing the relationship between mental computation and mathematical reasoning of fifth-grade students as both statistical and qualitatively.
1. Introduction
Mental computation and mathematical reasoning are in the nature of mathematics (Ministry of National Education [MNE], 2013; National Council of Teachers of Mathematics [NCTM], 2000) which is called as Science of Numbers. Mental computation and mathematical reasoning skills begin in childhood years and continue throughout life. With the start of formal education, reasoning that starts by comparing assets at early ages develops and focuses on a specific field as age increases. This focus, in the field of mathematics, is known as mathematical reasoning. Mental computation which is a complex mental procedure involving a frontoparietal network of brain regions (Vansteensel et al., 2014) begins by calculating small numbers mentally, and, either by age or formal education, continues in different areas at a higher level. The purpose of mental computation is to make the calculation mentally without using tools such as paper, pencil or a calculator (Reys, 1984). Mathematical reasoning that is higher level of thinking aims to reach a reasonable result by considering all aspects of a problem or case (Erdem, 2011).

1.1. Mental computation
Mental computation is used for computing that is based on the anyone's understanding and knowledge of mathematical properties and relationships (McIntosh, Reys, & Reys, 1997) due to requiring him or her to calculate with their head rather than in their head (Linsen, Verschaffel, Reynvoet, & De Smedt, 2015; Sowder, 1992; Torbeyns & Verschaffel, 2013; Verschaffel, Greer, & De Corte, 2007). National curriculum (MNE, 2013), international (National Council of Teachers of Mathematics [NCTM], 1989, 2000) reforms which are related to mathematics education and many relevant researches (Almeida, Bruno, & Perdomo-Díaz, 2015; Blöte, Klein, & Beishuizen, 2000; Carvalho & da Ponte, 2013; Cobb & Merkel, 1989; Cooper, Heirdsfield, & Irons, 1996; Foxman & Beishuizen, 2002; Heirdsfield & Cooper, 2004a, 2004b; Hope & Sherrill, 1987; Ineson, 2007; Klein & Beishuizen, 1994; Linsen et al., 2015; Maclellan, 2001; Mardjetko & Macpherson, 2007; McIntosh et al., 1997; Reys, 1984, 1985; Reys, Reys, Nohda, & Emori, 1995; Sowder, 1992; Varol & Farran, 2007; Yang & Huang, 2014) emphasize the importance of mental computation. For example, Carvalho and da Ponte (2013) reported that systematic work with mental computation contributes to the development of students’ strategies, reasoning, critical skills and number and operations sense. Reys et al. (1995) pointed out that mental computation is regarded as both important and useful in everyday living as well as valuable in promoting and monitoring higher level mathematical thinking.

When math problems are solved using only pencil and paper without any reasoning, students, when faced with a problem in the same parallel, can solve the problem in a standard way (as they experienced before) without understanding the logic of the rule or the formulas. However, when the problems they face later cannot be solved with the problem-solving patterns existing in their minds, believing that they cannot solve the problem, they quit making the effort to solve the problem. Especially the calculations made by using only pencil and paper in the early childhood years can play a negative role in students’ understanding the numbers and developing a mental computation strategy (Clarke, Clarke, & Horne, 2006). For example, while conducting a mental calculating of 74–49, a calculation such as “in unit’s digits, we cannot subtract 4 from 9, I take a 10 from the neighbor digit and now it is 14, when we subtract 9 from 14, we get 5. Now, there are 6 ten in ten digits, 6–4 is 2 and the result is 25” is pencil and paper type of calculation interchanged into mental medium which is called mental image of pencil and paper algorithm (Heirdsfield & Cooper, 2004a; Varol & Farran, 2007) and this strategy is not in accordance with the core of mental calculation. Researchers consider mental image of pencil and paper algorithm to be an inefficient strategy (Carragher, Carragher, & Schliemann, 1987; Ginsburg, Posner, & Russell, 1981; Heirdsfield & Cooper, 2004a; Hope, 1985; Reys et al., 1995; Sowder, 1992). This strategy leads to inability to employ mathematical reasoning, which requires high-level thinking while doing mathematical calculations. Therefore, for the students to learn the nature and basics of mathematics in the early years of formal education process, it is necessary to make students face the necessity of mental computing and make sure that they can practice sufficiently.
### 1.2. Mathematical reasoning

Mathematical reasoning is defined as the process of reaching a decision by using critical, creative and logical thinking (Erdem & Gürbüz, 2015). In math, truths are arrived at through reasoning—not by experiment or observation (Umay & Kaf, 2005), and mathematical concepts and operations are associated via reasoning (Ball and Bass, 2003). The reasoning includes abilities like following and assessing chains of arguments, knowing what a proof is and how it differs from other kinds of reasoning, uncovering the basic ideas in a given line of argument, and devising formal and informal arguments (Niss, 2003). Thus, ones who can do mathematical reasoning about a case have enough knowledge in that case, thoroughly analyze a newly encountered knowledge and associate with previous knowledge, make reasonable estimates and assumptions, justify their thinkings, reach some conclusions and explain and defend them (Umay, 2003). People who reason and think analytically tend to note patterns, structure, or regularities in both real-world situations and symbolic objects; they ask if those patterns are accidental or if they occur for a reason; and they conjecture and prove (NCTM, 2000).

Lithner (2008) depicts that reasoning can be considered as thinking process, output of this process or both, and visualizes reasoning process in Figure 1: a vertex $v_n$ represents both a momentary state of knowledge and of the (sub)task. The reasoner makes a strategy choice among the edges leading from $v_n$. The strategy implementation is represented by a transition edge $e_{n,m}$. Here knowledge not already accessed in $v_n$ is recalled or constructed and added up to form the new knowledge state in $v_m$, where the task is partially resolved and therefore a new task state is formulated. A reason is the motivation supporting transitions between vertices (p. 257). To solve such a mathematical task or problem, it is necessary to visually process the given problem and to perform a number of calculation steps before coming to a decision and selecting the appropriate response (Vansteensel et al., 2014). Since the thinking strategy can vary in efficiency and elegance depending on the sophistication of the individual’s understanding (McIntosh et al., 1997), making right decision occurs through higher order thinking, namely person-specific mathematical reasoning. Indeed, mathematical reasoning is processed in solving non-routine, complex problems, that are not easily reach the conclusion, with the help of questions “why” and “how” by using critical, logical, and creative thinking styles.

In mathematics, reasoning can have many functions such as verification, explanation, systematization, discovery, communication, construction of theory, and exploration (Yackel & Hanna, 2003). So, reasoning skill is a good predictor of mathematics performance, as demonstrated by correlational evidence (Erdem, 2011; Primi, Ferraô, & Almeida, 2010; Singley & Bunge, 2014). For instance, Singley and Bunge (2014) reported that improvements in relational reasoning over childhood and adolescence support students’ ability to reason about increasingly complex mathematical relations and explained the relationship between relational reasoning and mathematical development in the following framework: Understanding fractions, for example, requires the representation of 1st-order relations between numerators and denominators. Comparing fractions requires evaluation of a 2nd-order relation. Pre-algebra tasks require representation of complex relations between known values, unknown values, and operations. Algebra problems include variables and complex systems of equations that must be solved in relation to each other (p. 34). This framework and model in Figure 2 shows that reasoning is a crucial ability that supports the development of mathematics proficiency.
1.3. Theoretical framework on relationship between mental computation and mathematical reasoning

Mental computation and mathematical reasoning are two crucial skills that are essential for cultivating higher order thinking because they help individuals realize relationships among numbers while increasing their number sense (Johnson & Partlo, 2014). Thinking strategies involve and overlap with other widely discussed topics, including mental computation, estimation, and number sense (McIntosh et al., 1997). Thus, mental computation training can be used as a vehicle for promoting thinking, conceptual understanding on numbers and operations, and developing number sense (McIntosh, 2005). For instance, in a study by Heirdsfield (2011), evidence was found that the teaching on mental computation strategies in early mathematics was not only helping children develop mental computation strategies, but also helping them develop higher order reasoning, critiquing, and making sense of numbers and operations.

Mental computing may be needed not only at school but also in daily life since there is an evidence that daily mathematics in which four basic operations take part mainly is suggested to be more prone to human’s natural creativity and reasoning (Erdem, Gürbüz, & Duran, 2011). Tools such as calculator, paper and pencil may not be always available to make calculations in everyday life. In such cases, making predictions and using mental computing can help minimize the problems. In these cases, logical thinking is needed for the mental computing to be carried out correctly. This requires mathematical reasoning, which is defined as the process of reaching a rational conclusion by thinking all possible factors (Umay, 2003). For example, in the addition of 25 + 68 (also see Figure 3), student’s mental computing with the strategies is as: 20 + 60 = 80, 5 + 8 = 13, 80 + 13 = 93 (1010 strategy), and 20 + 60 = 80, 80 + 5 = 85, 85 + 8 = 93 (10s strategy), or 25 + 70 = 95 and 95–2 = 93 (N10C strategy). As stated in the literature (Blöte et al., 2000; Cooper et al., 1996; Heirdsfield & Cooper, 2004a, 2004b; Ineson, 2007; Lucangeli, Tressoldi, Bendotti, Bonanomi, & Siegel, 2003; Varol & Farran, 2007) such examples give a clue to the way that his or her mathematical reasoning has developed. In other words, student’s developing different and practical strategies and using them while carrying out mental computing suggest that this student’s mathematical reasoning is at a good level. If a student uses expressions such as “If … then …”, “because …” when developing strategies, it can be said that mathematical reasoning is at work. Mason (2001, p. 5) also remarked that reasoning ability required using the structure “If … then …”. The role of the mathematical reasoning on mental computation when a student is choosing a strategy is briefly explained in Figure 3.

Within that context, when students generate their own computation strategies, mental computation can be seen as higher order thinking. In such situations, rather than being taught standard algorithms that limit students’ thinking and reasoning (Yang, 2005; Yang & Huang, 2014), students are encouraged to generate strategies for computing based on their intuitive understanding of the numbers and action needed. For example, “to compute the cost of three erasers each priced 19 cents, a second grader might think, 20 and 20 and 20 is 60, but that’s 3 cents too much, so the cost is 57 cents” (Reys et al., 1995). Mental computation is framed as a higher order thinking process where the generation of a strategy is as important as the execution of the strategy (Sowder, 1992). Thus,
development of different strategies specifically in order for solving non-routine open-ended problems by students could be seen as an indication of good mathematical reasoning and high-level thinking.

Mental computing helps children understand numbers and mathematical concepts better and develop different strategies to solve mathematical problems (Cobb & Merkel, 1989; Klein & Beishuizen, 1994; Maclellan, 2001; Reys, 1984, 1985; Sowder, 1992; Varol & Farran, 2007) because the solution of a math problem requires not only some adjustment of the numbers but also a reflection on what adjustments are made before deciding on a reasonable estimate (McIntosh et al., 1997). Therefore, students with different mental computing strategies both understand basic math concepts better and make better mathematical judgments (Hope & Sherrill, 1987; Mardjetko & Macpherson, 2007). A similar effect reported by Ministry of National Education [MNE] (2013) is that students who acquired mathematical reasoning ability can make estimates about results of operations and measurements by using strategies “rounding”, “grouping proper numbers”, “first and last digits” or the ones they developed. When relationship between mental computation and mathematical reasoning is considered dually, a similar picture arises. Kasmer and Kim (2011) revealed an evidence for this, explaining that students in experimental group who have teaching program being carried out using prediction strategy found to be more successful in math test and reason well.

It is important to determine the relationship between mental computing and mathematical reasoning because of the following facts: (a) mental computing skill enables student to establish a connection between problem-solving and mathematical structure, concepts or subjects (Cobb &
Merkel, 1989; Hope & Sherrill, 1987; Klein & Beishuizen, 1994; Maclellan, 2001; Mardjetko & Macpherson, 2007; Reys, 1984, 1985; Sowder, 1992; Varol & Farran, 2007), (b) mathematical reasoning skill is used effectively in the problem-solving process (Briscoe & Stout, 2001; Erdem, 2011, 2015; Polya, 1997; Schoenfeld, 1985), (c) mental computation necessitates student’s mathematical reasoning, creativity, and problem-solving skills (Sexton, Gervasoni, & Brandenburg, 2009; Tsao, 2004; Yang & Huang, 2014). The importance and need of mathematical reasoning on mental computation was referred before by many researches but there is no statistical evidence of relationship between these two skills in the researches. It is thought that present study will fill an important gap in the literature revealing the relationship between mental computation and mathematical reasoning as both statistical and qualitatively. Mental computation training at early ages (6–11), that develops mathematical reasoning, will be emphasized through this relationship. From the literature-related aspects and reciprocal influence between both skills, the aim of this study is to determine the relationship between mental computation and mathematical reasoning.

2. Method

2.1. Research design
This study was carried out using the correlational model among relational survey models. In the model, relationships between the variables are searched for, and the levels of these relationships are determined (Fraenkel, Wallen, & Hyun, 2012).

2.2. Participants
The participants were 118 fifth-grade students (11–12-year-olds) studying at three primary schools that served low and middle socioeconomic areas in a city in Turkey. The schools were randomly selected by drawing their names from a box in which the names of all the primary schools in that city were written. These students were randomly selected from the schools and given code names such as “S1”, “S2”, “S3” ...

2.3. Data collection
The “Mathematical Reasoning Test (MRT: Appendix 1)” and the “Mental Computation Test (MCT: Appendix 2)”, which were developed using the literature, have been used as data collection tools. In the MCT, which consists of 16 questions, there are four questions per each basic arithmetic operation (Addition, subtraction, multiplication, and division). In the MRT, which consists of 14 questions, there are two-tier questions that consist of a multiple-choice part and an open-ended response. Two mathematics teachers and two mathematics educators confirmed the validity of the instruments. Because the numbers in questions in the MCT are a maximum of three-digit numbers, they can be said to be appropriate for students at this level due to the curriculum (MNE, 2013). The pilot test was performed with 38 fifth-grade students who did not participate in the actual study. The pilot study revealed that questions in both tests were understandable and clear for 5th graders. However, having figured out that the 30-s period which was given to the students to examine the question and the 2-min period which was given for mental computing are too much during the MCT pilot project, the question review time was reduced to 10 s, and mental computing time to 60 s. In addition, some of the questions in the test were re-arranged. For example, because it was observed that the students had difficulty in mental computing of multiplying three-digit numbers, these questions were excluded. As it was observed that the 40-min period of the MRT given to students, is too long in the pilot phase, it was reduced to 30 min. Also, the Cronbach’s alpha reliability coefficient of the MRT was found to be .86, and that of the MCT was found to be .82.

2.4. Procedure
First some explanations were made to the students regarding the purpose of study, and then brief information was given about the contents of the MRT and the MCT. Students were asked to remove the label on each question in the MCT (Figure 4(a)) during a certain period of time (10 s) and then,
after placing the label back on the question, they were expected to sit back and calculate the result mentally (Figure 4(d)). After the allocated period of time (60 s), the students were told to write down the results they found mentally and the strategies they used in reaching this conclusion on a paper and sit back again (Figure 4(b)). They were also told that they would not be able to change their written response. In order to prevent students from changing the answers they had written, all students were given the same type of pen (non-volatile). Because the application process of the MCT had been recorded, the students who did not observe the application directive of the test were identified and excluded from the evaluation process. While the class was being arranged for the MCT, the students were observed to be eager and excited. During the application of this test, the students were observed to answer the questions in the test as if they were participating in a game. In fact, the students were observed to be staring at each other suggesting, “What are we doing?”, “What is this?”, “How’s it going?”, “Fun!”, etc. with their body language. As for the MRT, it was given to the same students in the classical written exam format the next day for 30 min. All the students were encouraged to answer all questions in each test. In addition, students were seated so as not to be able to see what other students wrote (Figure 4(c)).

2.5. Data analysis
In analyzing the data, students’ answers were classified based on the levels on Table 1. Since four external mathematics educators who had experience in analyzing qualitative data initially categorized the data separately, they discussed the consistency of the categorization. There was high
Table 1. The rubric developed and used for MCT and MRT and students’ sample responses

| Levels                        | Explanation                                                                 | Assessment criteria (1st–2nd phase)                                                                 | Score | Sample responses                                                                 |
|-------------------------------|-----------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------|-------|------------------------------------------------------------------------------------|
| Correct justification        | Answers that encompass all aspects of the valid justification               | Correct answer—correct justification                                                              | 5     | Q2: \(48 + 57 = 105\)<br>\(40 + 50 = 90\)<br>\(8 + 7 = 15\)<br>\(90 + 15 = 105\)<br>Q1: Choice “d”, because there are nine numbers between the first page and ninth page; six numbers (15 - 10 + 1) nine numbers between the tenth page and fifteenth page; and twelve numbers (6 * 2 = 12) because here are two numbers in each number. In total, 9 + 21 = 30 numbers were used | MCT   | MRT    |
|                               |                                                                             | Q11: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q10: Choice “c” because the length of a side of the garden is between 6 and 7 m. Like that: \(\sqrt{36(6^2)} < \sqrt{39} < \sqrt{49(7^2)}\) |
| Partially correct justification| Answers that do not encompass all aspects of the valid justification        | Correct answer—partially correct justification                                                      | 3     | Q6: \(75 - 49 = 26\)<br>\(70 - 50 = 20\)<br>\(5 - 1 = 4\)<br>\(20 + 4 = 24\)<br>Q1: Choice “d”, because there are nine numbers in the first nine pages and six numbers in the next pages. Thus, in total, 9 + 6 = 15 numbers are used |
|                               |                                                                             | Q5: \(53 - 28 = 25\)<br>\(50 - 20 = 30\)<br>\(6 - 3 = 5\)                                       | 4     | Q6: Choice “c”, because chickens have two feet and sheep have four feet |
| Wron justi cation             | Answers that contain incorrect knowledge                                    | Correct answer—wrong justification                                                                | 1     | Q13: Choice “c”, because the closest number to 39 is 6 |
|                               |                                                                             | Q14: \(78/3 = 26\)<br>\(3 \times 7 = 21\)<br>\(8 - 3 = 5\)<br>\(21 + 5 = 26\)                   | 0     | Q14: Choice “c”, here (“?”), multiplication of its previous two numbers goes, \(2 \times 3 = 6\) |
| No justification             | Correct, incorrect or blank answers with no justifications written          | Correct answer—no justification                                                                  | 1     | Q4: Choice “a”, because, 1/4 is 3 times of 1/12 |
|                               |                                                                             | Q12: \(58 \times 64 = 3,600\)<br>\(60 \times 60 = 3,600\)<br>\(3 \times 7 = 21\)<br>\(8 - 3 = 5\)<br>\(21 + 5 = 26\)          | 0     | Q4: Choice “a”, because, 1/4 is 3 times of 1/12 |
|                               |                                                                             | Q16: \(285/5 = 59\)<br>\(300/5 = 60\)<br>\(15/5 = 3\)<br>\(60 - 3 = 57\)                          | 4     | Q2: Choice “a”, dolphin is 3 m under the water level. After jumping 3 m, it will be at the level of water and after jumping 8 m in total, it will be 5 m on the water level |
|                               |                                                                             | Q7: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
|                               |                                                                             | Q8: \(75 - 49 = 26\)<br>\(70 - 50 = 20\)<br>\(5 - 1 = 4\)<br>\(20 + 4 = 24\)                     | 3     | Q1: Choice “d”, because there are nine numbers between the first page and ninth page; six numbers (15 - 10 + 1) nine numbers between the tenth page and fifteenth page; and twelve numbers (6 * 2 = 12) because here are two numbers in each number. In total, 9 + 21 = 30 numbers were used |
|                               |                                                                             | Q9: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
|                               |                                                                             | Q10: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
|                               |                                                                             | Q11: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
|                               |                                                                             | Q12: \(58 \times 64 = 3,600\)<br>\(60 \times 60 = 3,600\)<br>\(3 \times 7 = 21\)<br>\(8 - 3 = 5\)<br>\(21 + 5 = 26\)          | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
|                               |                                                                             | Q13: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
|                               |                                                                             | Q14: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
|                               |                                                                             | Q15: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
|                               |                                                                             | Q16: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
|                               |                                                                             | Q17: \(52 \times 69 = 3,588\)<br>\(70 \times 50 = 3,500\)<br>\(2 \times 70 = 140\)<br>\(140-50=90\)<br>\(90 - 2 = 88\)<br>\(3,500 + 88 = 3,588\) | 4     | Q4: Choice “d”, because (3/4)/ (1/12) = (3/4) * (12) = 9 |
agreement, approximately 85–90%, in most of the categorization. The assessment test consisted of two phases (1st phase, multiple choice, and 2nd phase, open-ended), and, therefore, the assessment criteria also consisted of two phases.

Student responses were analyzed using statistical package for the social sciences (SPSS). The correlation between the total scores of the students taken from the both tests was computed using Pearson correlation coefficient (r). To explain this relationship in detail, some of the students’ answers to some of the questions in each test were discussed in the study. For this reason, the self-adhesive labels on the test papers were removed and the students’ answers were discussed in more detail, and a detailed assessment was made. In scoring questions on the tests, the grading scale which was developed from the studies of Erdem (2011), Gürbüz (2010), and Gürbüz and Birgin (2012) was used (see Table 1). Reys and his colleagues (1995) who are experts in analysis of mental computation used a similar classification: “Each student response was coded either correct, correct with signs of written computation, incorrect, incorrect with signs of written computation, or no response”. Sample answers, which were given on Table 1, are the responses of the students who used strategy in the MCT and wrote justifications in the MRT.

3. Findings

After analysis, it was found out that there is a relationship between mental computation and mathematical reasoning of the students involved in the study as shown in Table 2.

When Table 2 is examined, it can be seen that there is a significant relationship between the students’ mental computation and mathematical reasoning (r = .654, p = .000). In literature, only a correlation of .65 or higher will allow individual predictions that are reasonably accurate for most purposes (Fraenkel et al., 2012). Hence, a student with a high level of mental computation can be said to have a high level of mathematical reasoning. In addition, when some students’ answers to questions in both tests are examined in detail and correlated, it could be seen that there is a highly positive relationship between mental computation and mathematical reasoning. Below are some of the students’ answers to some of the questions in each test; the students’ answers in each test are interpreted in relation to each other.

In Figure 5(a), when the student’s answer to Q1 in the MRT is analyzed, it can be said that answer 15 was given as a result of the student’s poor judgment, as the student thought that each page is represented by a number. However, when the student’s response was analyzed, it is visible that two digits are used on each page from page 10 on. However, the student gave the wrong answer because she was unable to reason her answer. Although this student was so close to the right answer, it can be said that she could not write the justification because she was unable to analyze the answer she had written. The average score that S17 got out of MRT is calculated to be 2.33.

In Figure 5(b), when the student’s answer to Q11 in the MCT is analyzed, we can see that the student used a correct strategy when doing mental computing. However, she could not reach the correct answer as due to lack of judgment and faulty mental calculation. What is necessary to do at the

| Table 2. Correlation between mental computation and mathematical reasoning |
|-----------------------------------------------|
| Mental computation | Pearson correlation | .654(**) | Mathematical reasoning |
| Sig. (2-tailed) | .000 | 1 |
| N | 118 | 118 |
| Mathematical reasoning | Pearson correlation | .654(**) | 1 |
| Sig. (2-tailed) | .000 | 1 |
| N | 118 | 118 |

**Correlation is significant at the 0.01 level (2-tailed).
very basics in mental calculation is to finalize the process by dividing it into the simplest sub-proces-
ses. When the strategy the student used is analyzed, it can be seen that she guessed the answer
to be around 3,500 by performing a simpler rounding operation, but she could not do the operations,
which require further mental computing and mathematical reasoning. The average score that S17
got from MCT is calculated to be 2.59.

In Figure 6(a), when the student’s response to Q2 in MRT is analyzed, it can be said that he made
an accurate mathematical reasoning. This student not only tried to give a results-oriented idea so as
to be the most straightforward but he also detailed the strategy. This student even supported his
strategy with visuals drawn. The average score that S115 got from MRT is calculated to be 3.88.

In Figure 6(b), it is seen that the student reached the immediate correct answer by using the right
strategy and the right reasoning during the mental computation when the student’s answer to the
question in the MCT is analyzed. The student detailed the strategy he used for the question in the
MRT as well. For example, he added the numbers after rounding the number 57–50 and 48–40
(40 + 50 = 90) using “1,010” strategy. Besides that, he added the numbers 7 and 8 among them-
selves (8 + 7 = 15). The average score that S115 got from MCT is calculated to be 3.75.

In Figure 7(a), when the student’s response to Q14 in the MRT is analyzed, it can be said that he
made an accurate mathematical reasoning. However, this student made a short justification in or-
der for the solution to be the most direct. For example, this student wrote a correct but too short
justification for the answer, “It went by adding the next number in line.” However, the pattern of this
problem started only after the first two elements. Yet, the student did not go into such a detail in his
justification. Moreover, the student did not detail his pattern like 1 + 1 = 2; 1 + 2 = 3; 2 + 3 = 5; 3 +? = 8.
The average score that S88 got out of MRT is calculated to be 4.52.

In Figure 7(b), when the same student’s answer to Q15 in the MCT test is analyzed, it is seen that
he reached the immediate correct answer by using the right strategy and the right reasoning when
doing mental computing. As in the MRT, it can be noted that the student did make an effort to reach his conclusion hastily. It can be said that he used the strategy such “N10” strategy because he rounded 112 to 100. But he did not detail his strategy in the following: “112 = 100 + 12; 100/4 = 25; 12/4 = 3 and 25 + 3 = 28”. The average score that S88 got from MCT is calculated to be 4.69.

In summary, when students’ performances in the MRT and the MCT are analyzed either by the software program or individually by hand, it was clearly seen that there is a linear relationship between mental computation and mathematical reasoning.

4. Discussion and conclusions
The present study examines the relationship between mental computation and mathematical reasoning. As a result of the analysis, it has been concluded that there is a significant positive relationship between mental computation and mathematical reasoning ($r = .654$, $p = .000$). Correlation values at and above .65 level in educational research shows that it represents the relationship correctly (Fraenkel et al., 2012). Hence, a student with a high level of mental computation can be said to have a high level of mathematical reasoning. On analyzing the strategies several students used to answer questions in MRT and MCT collectively (in the study, the students 17, 88 and 115 were mentioned for their answers used as examples), this result has been confirmed. Answers given to some questions by some students showing different level performance in both tests were discussed in association with literature in order to help readers get more detailed information on this result.

For example, a student answered the Q1 in MRT as “1–2–3–4–5–6–7–8–9–10–11–12–13–14–15” by writing the number of pages on the book and checked 15 items. The situation that necessitates student’s reasoning in this question is to notice that different number of digits is used on pages with one-digit and two-digit numbers. It could not be said that there is no reasoning in students’ answer. However, failure to think that two digits are used in every page starting from page 10 lead us to think that the student has not displayed expected reasoning. Also, as can be understood from such an answer, it could be stated that the students could not provide reasons for what they wrote. Students’ answers at this level could be seen in previous research by Erdem (2011, 2015), Erdem and Gürbüz (2015).

When the same student’s answer to Q11 in the MCT is analyzed, we can see that the student used a correct strategy when doing mental computing. However, she could not reach the correct answer as a result of lacking in her judgment and faulty mental calculation. She guessed the answer to be around 3,500 by performing a simpler rounding operation, but she could not do the operations, which require further mental computing and mathematical reasoning. This result is supported by various studies (Blöte et al., 2000; Chusnul, 2014; Cooper et al., 1996; Foxman & Beishuizen, 2002; Heirdsfield, 2000, 2004; Heirdsfield & Cooper, 2004a, 2004b; Ineson, 2007; Linsen et al., 2015; Lucangeli et al., 2003; Reys, 1984, 1985; Reys et al., 1995; Varol & Farran, 2007; Yang & Huang, 2014) which mention that when students cannot find any answer, they tend to round it to a close number.
On the other hand, while the average score the student got from MRT is 2.33, her average score is calculated to be 2.59 in MCT. Answers given in both questions and average scores on both tests suggest that student’s performances in both tests are close.

When another student’s answer to Q2 in the MRT is analyzed, it can be said that he made an accurate mathematical reasoning. This student not only tried to give a results-oriented idea so as to be the most straightforward but he also detailed the strategy. The student gave an explanation such as “normally, there is 3 meters to sea level. When the dolphin jumps 8 meters, it will reach sea level after 3 meters and later it will reach \(8 - 3 = 5\) meters above sea level”. This student even supported his strategy with visuals drawn (See Figure 6(a)). This student reached the immediate correct answer by using the right strategy and the right reasoning during the mental computation when the student’s answer to the Q2 in the MCT is analyzed. The student detailed the strategy he used for the question in the MRT as well. The student made necessary explanation and the answer for the question verbally in a correct way: “Like in the previous question, we get 90 after 40 + 50. Then, we calculate \(8 + 7 = 15\) and finally we add both these numbers \(90 + 15 = 205\)”. When student’s answer is expressed more definitely, he added the numbers after rounding the number 57-50 and 48-40 (\(40 + 50 = 90\)) using “1,010” strategy. Besides, he added the numbers 7 and 8 among themselves (\(8 + 7 = 15\)). While the average score this student got from MRT is 3.88, his average score is calculated to be 3.75 in MCT. When the student’s answers and average scores on both tests are analyzed, it can be said that there is a linear relationship between mathematical reasoning and mental computing.

Finally, a high performing student has displayed correct mathematical reasoning in Q14 in MRT, but the student made a correct but too short justification for the answer, “It went by adding the next number in line.” However, the pattern of this problem started only after the first two elements. Yet, the student did not go into such a detail in his justification. Moreover, the student did not detail his pattern like \(1 + 1 = 2; 1 + 2 = 3; 2 + 3 = 5; 3 + ? = 8\). Student’s failure to provide a detailed explanation in this question despite his short explanation for the correct answer is not an indication that his reasoning is not good. Lack of students’ explanation for what they think could be related to language development. This conclusion supports research that emphasize language development (Erdem & Gürbüz, 2015; Ford & Kuhs, 1991; Gibbs & Orton, 1994; Gürbüz, 2010; Gürbüz & Birgin, 2012; Kazimo, 2006; Tatsis, Kafoussi, & Skoumpourdi, 2008). This result in the current study could be attributed to that the participant students in this research study at schools located in a socioeconomically disadvantageous region in Turkey. Education level of students’ parents and their contacts, economic conditions and other factors could influence language development and the level one can express his or her feelings and thoughts. For example, the study by Baya’a (1990) revealed that students of the high Socio-Economic Status (SES) demonstrated a significantly higher level of mathematics achievement than students of the low SES. When the same student’s answer to Q15 in the MCT is analyzed, it is seen that he reached the immediate correct answer by using the right strategy and the right reasoning when doing mental computing. As in the MRT, it can be noted that the student did make an effort to reach his conclusion hurriedly. It can be said that he used the strategy such “N10” strategy because he rounded 112–100.

There are many studies (Blöte et al., 2000; Chusnul, 2014; Cooper et al., 1996; Foxman & Beishuizen, 2002; Heirdsfield, 2000; 2004; Heirdsfield & Cooper, 2004a, 2004b; Ineson, 2007; Linsen et al., 2015; Lucangeli et al., 2003; Reys, 1984, 1985; Reys et al., 1995; Varol & Forran, 2007; Yang & Huang, 2014) focusing on students who give correct answers by making mental calculations through using this strategy. 588 did not detail his strategy in the following example: “\(112 = 100 + 12; 100:4 = 25; 12:4 = 3\) and \(25 + 3 = 28\)”. From the student’s explanation: “I took 112 as 100. I added what I got after I subtracted 100 from 112 and divided the result to 4”, it could be deduced that he found the correct answer by using this strategy. The student has verbally explained his thoughts in this question without using mathematical language. While the average score this student got from MRT is 4.52, his average score is calculated to be 4.69 in MCT. Answers given to both questions and average scores on both tests show that this student has displayed a good performance that is close to one another in both tests.
Another issue that needs to be discussed in this research is students who use mental image of pencil and paper algorithm. The reason why this issue is emphasized is that there are quite a number of students (N = 24, about % 20) who used this algorithm. As a result of careful analysis of students’ answers, it could be noted that mental calculation of students who use mental image of pencil and paper algorithm is not so efficient. Many studies also consider mental image of pencil and paper algorithm to be an inefficient strategy (Carraher et al., 1987; Ginsburg et al., 1981; Heirdsfield & Cooper, 2004a; Hope, 1985; Reys et al., 1995; Sowder, 1992). The reason behind this is that mental calculation necessitates carrying out operations mentally by using different strategies instead of carrying out operations by copying paper medium into mind. In other words, the logic of mental computation is based on calculating short cut by developing different strategies rather than using the mental image of pencil and paper algorithm. This finding supports research that emphasizes mental computation requires children to calculate with their head rather than in their head (Linsen et al., 2015; Sowder, 1992; Torbeyns & Verschaffel, 2013; Verschaffel et al., 2007).

For the correct mathematical reasoning in mathematical education to develop, students must be enabled to deal with exceptional/non-routine problems, and young children, especially, must be encouraged to do mental computing. The students must be asked to write the strategies they use and on which grounds they preferred them while solving the problems. In a similar study with less students, the relationship between mathematical reasoning and mental computing can be explored qualitatively with a small number of students. Moreover, the reasons why students who use mental image of pencil and paper algorithm choose to use this algorithm could be analyzed in detail through interviews.

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### Appendix 1. Mathematical reasoning test (MRT).

| Q1 | Q2 | Q3 | Q4 | Q5 |
|----|----|----|----|----|
| **Pages of a 15-page journal are to be numbered starting from 1. How many numerals will have been used after this process is finished? Explain** | **A dolphin has jumped 8 meters out of a 3-meter deep pool. How many meters has the dolphin gone from the water level? Explain** | **There are 5 schools with 8 classes in each school. Each class has 30 students. If we equally divide these students into 16 schools in another city, how many students will be in each school? Explain** | **The shape above was created by bringing 3 1/4 strips in fraction team together. How many 1/12 strips do you think can create the size of that shape? Explain** | **Yılmaz family is expecting a baby. Their first 5 children are boys. Which one the following is correct for the sixth child? Why?** |
| (a) 15 (b) 17 (c) 19 (d) 21 | (a) 5 m (b) 11 m (c) 17 m (d) 24 m | (a) 60 (b) 70 (c) 75 (d) 80 | (a) 3 (b) 6 (c) 9 (d) 12 | (a) The sixth will probably be a girl (b) The sixth will probably be a boy (c) The possibility of the sixth child to be girl or boy is equal |

| Q6 | Q7 | Q8 | Q9 | Q10 |
|----|----|----|----|----|
| **There are 19 sheep and 33 chickens in a farm. Based on this, what is the total number of feet of all the sheep and chickens in this farm? Explain** | **The area filled with water on earth is more than the area that is filled with land. From time to time, some meteors fall to any place on earth. Do you think the possibility of these meteors falling on land is higher than falling on water? Why?** | **Young people generally drive fast. While the old drive slowly, they have a attention deficit problem. It is observed that out of 35 car accidents that happened in the past one month in Adıyaman, the drivers of 25 accidents were young. Do you think the driver in a possible 36th accident will be young or old? Why?** | **1/6 of eggs in a basket are broken. How many eggs will be left after 2/5 of the remaining eggs are sold? Explain** | **How many meters is the length of one of the sides of a square-shaped garden if the area of the garden is 39 m? Explain** |
| (a) 140 (b) 141 (c) 142 (d) 143 | (a) The possibility of falling on land is higher (b) The possibility of falling on water is higher (c) The possibility of falling on land and water is equal | (a) Driver is probably young (b) Driver is probably old (c) The possibility of the driver to be old and young is equal | (a) 10 (b) 15 (c) 20 (d) 25 | (a) Between 4 and 5 m (b) Between 5 and 6 m (c) Between 6 and 7 m |

| Q11 | Q12 |
|----|----|
| **In questions Q11–Q12, mark the choice with the shape you think will come after the given pattern of shapes. Explain** | **Specify the pattern between the numbers given in questions Q13 and Q14 and mark the choice you think can replace "?" Explain** |
| ![Shapes](image1.png) | ![Shapes](image2.png) |

| Q13 | Q14 |
|----|----|
| **Specify the pattern between the numbers given in questions Q13 and Q14 and mark the choice you think can replace "?" Explain** | ![Pattern](image3.png) |
| 1/16, 1/8, 1/4, ?, 1, 2 | 1, 1, 2, 3, ?, 8 |
| (a) 1/5 (b) 1/2 (c) 1/3 (d) 1/6 | (a) 4 (b) 5 (c) 6 (d) 7 |
Appendix 2. Mental Computation Test (MCT)

| Q1 | Q2 | Q3 | Q4 |
|----|----|----|----|
| 59 | + | 86 | |
| 48 | + | 57 | |
| 658 | + | 142 | |
| 25 | + | 679 | |

| Q5 | Q6 | Q7 | Q8 |
|----|----|----|----|
| 53 | - | 82 | |
| 75 | - | 49 | |
| 528 | - | 230 | |
| 65 | - | 82 | |

| Q9 | Q10 | Q11 | Q12 |
|----|-----|-----|-----|
| 32 | x | 9 | |
| 46 | x | 7 | |
| 52 | x | 69 | |
| 58 | x | 64 | |

| Q13 | Q14 | Q15 | Q16 |
|-----|-----|-----|-----|
| 52 | : | 4 | |
| 78 | : | 3 | |
| 112 | : | 4 | |
| 285 | : | 5 | |

**Note:** Compute mentally without any calculator, pen/pencil and write down your strategy in bubbles under each operation.
