Data-driven study of
the implications of anomalous magnetic moments
and lepton flavor violating processes of $e$, $\mu$ and $\tau$

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Abstract

We study anomalous magnetic moments and flavor violating processes of $e$, $\mu$ and $\tau$ leptons. We use a data driven approach to investigate the implications of the present data on the parameters of a class of models, which has spin-0 scalar and spin-1/2 fermion fields. We compare two different cases, which has or does not have a built-in cancelation mechanism. Our findings are as following. Chiral interactions are unable to generate large enough $\Delta a_e$ and $\Delta a_\mu$ to accommodate the experimental results. Although sizable $\Delta a_e$ and $\Delta a_\mu$ can be generated from non-chiral interactions, they are not contributed from the same source. Presently, the upper limit of $\mu \rightarrow e\gamma$ decay gives the most severe constraints on photonic penguin contributions in $\mu \rightarrow e$ transitions, but the situation may change in considering future experimental sensitivities. The $Z$-penguin diagrams can constrain chiral interaction better than photonic penguin diagrams in $\mu \rightarrow e$ transitions. In most of the parameter space, box contributions to $\mu \rightarrow 3e$ decay are subleading. The present bounds on $\Delta a_\tau$ and $d_\tau$ are unable to give useful constraints on parameters. In $\tau \rightarrow e (\mu)$ transitions, the present $\tau \rightarrow e\gamma (\mu\gamma)$ upper limit constrains the photonic penguin contribution better than the $\tau \rightarrow 3e (3\mu)$ upper limit, and $Z$-penguin amplitudes constrain chiral interaction better than photonic penguin amplitudes. Box contributions to $\tau \rightarrow 3e$ and $\tau \rightarrow 3\mu$ decays can sometime be comparable to $Z$-penguin contributions. The $\tau^- \rightarrow e^-\mu^+e^-$ and $\tau^- \rightarrow \mu^+e^-\mu^-$ rates are highly constrained by $\tau \rightarrow e\gamma$, $\mu \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$, $\mu \rightarrow e\gamma$ upper limits, respectively. We compare the current experimental upper limits, future sensitivities and bounds from consistency on various muon and tau LFV processes.
I. INTRODUCTION

The Large Hadron Collider completed run-2 in 2018 and is currently preparing for run-3. From the results of the searches, we see that New Physics (NP) signal is yet to be found (see, for example [1], for recent search results). It is therefore useful and timely to explore the high-precision frontier, where the NP at the scale beyond our reach may manifest in low energy processes via virtual effects. Indeed, there are some interesting experimental activities in the lepton sector in recent years.

The muon’s anomalous magnetic moment remains as a hint of contributions from NP since 2001 [2]. Presently the deviation of the experimental result $a_{\mu}^{\text{exp}}$ from the Standard Model (SM) expectation $a_{\mu}^{\text{SM}}$ is $3.7\sigma$ [3–5]:

$$\Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (27.06 \pm 7.26) \pm 10^{-10}. \quad (1)$$

For more details, see [6–8]. New experiments in Fermilab and J-PARC are on their way to improve the sensitivities [9].

In addition, in 2018, a measurements of the fine-structure constant $\alpha$ using the recoil frequency of cesium-133 atoms in a matter-wave interferometer, inferred a deviation on electron $g - 2$ from the SM prediction, [10]

$$\Delta a_{e} = a_{e}^{\text{exp}} - a_{e}^{\text{SM}} = (-0.88 \pm 0.36) \pm 10^{-12}. \quad (2)$$

In the tau sector, the experimental and the theoretical results of the anomalous magnetic moment are given by

$$-0.052 < a_{\tau}^{\text{exp}} < 0.013, \quad a_{\tau}^{\text{SM}} = (1.17721 \pm 0.00005) \times 10^{-3}, \quad (3)$$

respectively [4, 11]. The experimental sensitivity is roughly one order of magnitude from the SM prediction.

Furthermore, it is known that the SM contributions to lepton electric dipole moments are at four-loop level and, consequently, are highly suppressed. For example, the electron electric dipole moment was estimated to be $d_{e} \approx 8 \times 10^{-41} \text{e cm}$ [12]. The present experimental bounds on electric dipole moment of $e, \mu$ and $\tau$ are given by [13, 14]

$$|d_{e}| < 1.1 \times 10^{-29} \text{e cm}, \quad (4)$$

$$|d_{\mu}| < 1.9 \times 10^{-19} \text{e cm}, \quad (5)$$

and

$$|d_{\tau}| < 1.6 \times 10^{-18} \text{e cm}, \quad (6)$$

where the above limit on $d_{e}$ is used to constrain $d_{\tau}$ via $\Delta d_{e} = 6.9 \times 10^{-12}d_{\tau}$ [15].

It is known that SM prohibits charge lepton flavor violating (LFV) processes. Hence, they are excellent probes of NP. Indeed, they are under intensive searches. In 2016 the MEG collaboration reported the search result of $\mu \to e\gamma$ decay, [16]

$$\mathcal{B}(\mu^{+} \to e^{+}\gamma) \leq 4.2 \times 10^{-13}, \quad (7)$$
TABLE I: Present upper limits and future sensitivities of some muon and tau lepton flavor violating processes are listed [1, 16, 17, 21, 23].

| Process | Current Limit | Future Sensitivity |
|---------|---------------|--------------------|
| $B(\mu^+ \rightarrow e^+ \gamma)$ | $< 4.2 \times 10^{-13}$ | $6 \times 10^{-14}$ |
| $B(\mu^+ \rightarrow e^+ \gamma e^-)$ | $< 1.0 \times 10^{-12}$ | $10^{-16}$ |
| $B(\mu^- \rightarrow e^- \text{Ti})$ | $< 4.3 \times 10^{-12}$ | $10^{-17}$ |
| $B(\mu^- \rightarrow e^- \text{Au})$ | $< 7 \times 10^{-13}$ | $10^{-16}$ |
| $B(\mu^- \rightarrow e^- \text{Al})$ | $...$ | $10^{-17}$ |
| $B(\tau^- \rightarrow e^- \gamma)$ | $< 3.3 \times 10^{-8}$ | $3 \times 10^{-9}$ |
| $B(\tau^- \rightarrow \mu^- \gamma)$ | $< 4.4 \times 10^{-8}$ | $1 \times 10^{-9}$ |
| $B(\tau^- \rightarrow e^- \mu^+ e^-)$ | $< 2.7 \times 10^{-8}$ | $4.3 \times 10^{-10}$ |
| $B(\tau^- \rightarrow \mu^- \mu^+ \mu^-)$ | $< 1.7 \times 10^{-8}$ | $2.7 \times 10^{-10}$ |
| $B(\tau^- \rightarrow e^- \mu^+ e^-)$ | $< 1.5 \times 10^{-8}$ | $2.4 \times 10^{-10}$ |
| $B(\tau^- \rightarrow \mu^- \mu^+ \mu^-)$ | $< 2.1 \times 10^{-8}$ | $3.3 \times 10^{-10}$ |

and the upgrade is on the way to improve the sensitivity by roughly one order of magnitude [17]. Interestingly, $\mu \rightarrow e\gamma$ decay may be closely related to lepton anomalous magnetic moments and other LFV processes, such as $\mu^+ \rightarrow 3e$ decays and muon to electron conversions, $\mu^- N \rightarrow e^- N$ [18]. See [19] for a review on $(g - 2)_\mu$ and LFV processes. Note that LFV processes can sometime be related to cosmological effects, see for example [20].

Lepton flavor violating $\tau$ decays are also under intensive search. Current bounds on $\tau \rightarrow e\gamma, \mu\gamma, 3e, 3\mu, e\mu e, \mu\mu\mu$ decays was provided by $B$ factories. They are at the level of $10^{-8}$ and the sensitivities will be improved by two orders of magnitude in the updated $B$ factory [21, 22].

The current limits and future experimental sensitivities of various $l' \rightarrow l\gamma$, $l \rightarrow l'\mu\mu$ and $lN \rightarrow l'N$ processes are summarized in Table I.

Many popular NP scenarios or models are disfavored or even closed to being ruled out by data (see, for example, [11]). Given the present situation, it is worthy to use a data driven approach. It will be interesting to see where the present data lead us to. As a working assumption, we consider a general class of models that lepton anomalous magnetic moment and various lepton flavor violating processes, such as $\mu \rightarrow e\gamma, \mu \rightarrow 3e, \mu \rightarrow e$ conversions, $\tau \rightarrow e\gamma, \mu\gamma, 3e, 3\mu, e\mu e$ and $\mu\mu\mu$ decays are induced by loop diagrams via exchanging spin-0 and spin-1/2 particles in this work. Two cases are considered, which does not have any built-in cancellation mechanism or has some built-in mechanism, such as Glashow-Iliopoulos-Maiani or super-Glashow-Iliopoulos-Maiani mechanism. These two cases are complementary to each other and it will be interesting to compare them. This work is an updated and extended study of [24], where only $\mu$ decays were considered. Note that a similar setup, but in the quark sector, has been used in a study of the $b \rightarrow s\mu^+\mu^-$ decay [25].

We briefly give the framework in the next section. In Sec. III, numerical results will be presented, where data on $g - 2$, $d_t$ and upper limits of LFV rates will be used to constrain parameters and the correlations between different processes will be investigated. We give our conclusion in Sec. IV.
FIG. 1: Diagrams contributing to various processes. Penguin diagrams contributing to $\epsilon, \mu$ and $\tau, g-2, d_t, l' \rightarrow l\gamma, l' \rightarrow l\bar{l}l$ and $l'N \rightarrow lN$ processes are shown in Fig. 1 (a) and (b), while box diagrams contributing to the $\bar{l}' \rightarrow l\bar{l}l'$ process are shown in Fig. 1 (c) and (d). Note that we do not show diagrams involving self energy parts. Fig. 1 (d) is for the Majorana case.

Some formulas are collected in the Appendix.

II. FRAMEWORK

The generic interacting Lagrangian involving leptons ($l$), exotic spin-0 bosons ($\phi_i$) and spin-1/2 fermions ($\psi_n$) is given by

$$\mathcal{L}_{\text{int}} = \bar{\psi}_n (g_n^{il} P_L + g_{ni}^{il} P_R) l \phi_i^* + \bar{l} (g_l^{ni} P_R + g_{il}^{ni} P_L) \psi_n \phi_i,$$  

(8)

where indices, $l$, $i$ and $n$, are summed and these fields are in their mass bases. It can contribute to lepton $g-2, d_t$ and various LFV processes, such as $l' \rightarrow l\gamma, l' \rightarrow l\bar{l}l$ decays and $l'N \rightarrow lN$ transitions, via diagrams shown in Fig. 1. Some useful formulas can be found in ref. [24] and are collected in Appendix A.

As noted in the introduction, we consider two complementary cases. In case I there is no any built-in cancellation mechanism. A typical amplitude, $A$, may contain several sub-amplitudes, $A_j$, each comes from one of the loop diagrams (see Fig. 1) giving

$$A = \sum_{j=1}^{N} A_j.$$  

(9)
To constrain these sub-amplitudes from data, we will switch them on one at a time. Different sub-amplitudes are in principle independent from each other as there is no any built-in cancellation mechanism. However, in a realistic model calculation, it is likely to have several amplitudes to appear at the same time and interfere. Nevertheless, it is well known that interference effects can be important only when the amplitudes are of similar size. For amplitudes of different sizes, this analysis can constrain the most dominant amplitude. On the other hand, through investigating the sizes of different sub-amplitudes the analysis can also identify the region, where several sub-amplitudes are of similar sizes, and, hence, identify where interference can be potentially important.

The Wilson coefficients of a typical sub-amplitude can be obtained by using formulas in Appendix A, but with the following replacement,

$$g_{1M}^{ni} ightarrow g_{1M}.$$  \hspace{1cm} (10)

Terms contributing to various processes in case I are shown in Table II. Note that $\Delta T_{3\psi}$ is basically the difference of weak isospin quantum numbers of $\psi_R$ and $\psi_L$, while $\kappa_{R,L}$ are defined in Eq. (A12). Note that $\Delta T_{3\psi}$ is expected to be an order one quantity, while $\kappa_R$ is expected to be a small quantity. See Appendix A for more informations.

In the second case (case II), there is a built-in cancellation mechanism. Now some sub-amplitudes in Eq. (9) are related intimately. They need to be grouped together to allow the cancellation mechanism to take place, and the resulting grouped amplitude should be viewed as a new sub-amplitude. To constrain these new sub-amplitudes from data, we will turn on one at a time. To be specific, we consider the following replacement,

$$g_{1M}^{ni} ightarrow g_{1M}^i = g_{1M} \Gamma_{M}^{ii}.$$  \hspace{1cm} (11)

where $g_{1M}$ is real (as the phase is absorbed into $\Gamma_M$) and we have $M = L, R$. These $\Gamma$ satisfy the following relations:

$$\Gamma_{M}^{ii} m_{1T}^2 \Gamma_{N}^{ii'} = (m_{\phi}^2)_{MN}, \quad \Gamma_{M}^{ii} \Gamma_{N}^{ii'} = \delta_{ii'} \delta_{MN},$$  \hspace{1cm} (12)
TABLE III: Terms contributing to various processes in case II.

| Processes | γ-penguin | γ-penguin | Z-penguin | Box |
|-----------|-----------|-----------|-----------|-----|
| $\Delta \rho_{\ell}$ | $Q_{\phi,\psi}|g_{\ell R(L)}|^2$ | $Q_{\phi,\psi} \Re(g_{\ell R(L)}\delta_{RL}^{\ell L})$ | | |
| $d_{i}$ | | $Q_{\phi,\psi} \Im(g_{\ell R(L)}\delta_{RL}^{\ell L})$ | | |
| $\mu^{+} \to e^{+} \gamma$ | $Q_{\phi,\psi} g_{\mu M} g_{\mathrm{M}} \delta_{MM}^{\mu}$ | $Q_{\phi,\psi} g_{\mu R(L)} g_{M L(R)} \delta_{RL}^{\mu}$ | $g_{\mu M} g_{\mathrm{M}} g_{\mu N} g_{\mathrm{N}} \delta_{MM}^{\mu}$ | |
| $\mu^{+} \to e^{-} e^{-}$ | $Q_{\phi,\psi} g_{\mu M} g_{\mathrm{M}} \delta_{MM}^{\mu}$ | $Q_{\phi,\psi} g_{\mu R(L)} g_{M L(R)} \delta_{RL}^{\mu}$ | $g_{\mu M} g_{\mathrm{M}} g_{\mu N} g_{\mathrm{N}} \delta_{MM}^{\mu}$ | |
| $\mu^{-} N \to e^{-} N$ | $Q_{\phi,\psi} g_{\mu M} g_{\mathrm{M}} \delta_{MM}^{\mu}$ | $Q_{\phi,\psi} g_{\mu R(L)} g_{M L(R)} \delta_{RL}^{\mu}$ | $g_{\mu M} g_{\mathrm{M}} g_{\mu N} g_{\mathrm{N}} \delta_{MM}^{\mu}$ | |
| $\tau^{-} \to e^{-} e^{-}$ | $Q_{\phi,\psi} g_{\tau M} g_{\mathrm{M}} \delta_{MM}^{\tau}$ | $Q_{\phi,\psi} g_{\tau R(L)} g_{M L(R)} \delta_{RL}^{\tau}$ | $g_{\tau M} g_{\mathrm{M}} g_{\mu N} g_{\mathrm{N}} \delta_{MM}^{\tau}$ | |
| $\tau^{-} \to \mu^{-}$ | $Q_{\phi,\psi} g_{\tau M} g_{\mathrm{M}} \delta_{MM}^{\tau}$ | $Q_{\phi,\psi} g_{\tau R(L)} g_{M L(R)} \delta_{RL}^{\tau}$ | $g_{\tau M} g_{\mathrm{M}} g_{\mu N} g_{\mathrm{N}} \delta_{MM}^{\tau}$ | |
| $\tau^{-} \to \mu^{-} \mu^{-}$ | $Q_{\phi,\psi} g_{\tau M} g_{\mathrm{M}} \delta_{MM}^{\tau}$ | $Q_{\phi,\psi} g_{\tau R(L)} g_{M L(R)} \delta_{RL}^{\tau}$ | $g_{\tau M} g_{\mathrm{M}} g_{\mu N} g_{\mathrm{N}} \delta_{MM}^{\tau}$ | |
| $\tau^{-} \to e^{-} \mu^{+}$ | | | $g_{\tau M} g_{\mathrm{M}} g_{\mu N} g_{\mathrm{N}} \delta_{MM}^{\tau}$ | |

where the $\delta$s are Kronecker deltas. Typical terms in a Wilson coefficient given in Appendix A should now be replaced accordingly:

$$
\sum_{i} g_{\ell M}^{i} f(m_{\phi}, m_{\phi}) g_{\ell N} \rightarrow m_{\phi}^{2} \frac{\partial}{\partial m_{\phi}} f(m_{\phi}^{2}, m_{\phi}^{2}) g_{\mu M} g_{\mu N} \delta_{M N}^{\ell L}, \quad (13)
$$

where $m_{\phi}^{2}$ is the average of the mass squared of $\phi_{i}$ and $\delta_{M N}^{\ell L}$ is the mixing angle defined in the usual way (do not confuse it with the Kronecker delta): [20]

$$
\delta_{M N}^{\ell L} = \frac{1}{m_{\phi}^{2}} (m_{\phi}^{2} - m_{\phi}^{2}) \frac{\Gamma_{M N}^{\ell L}}{m_{\phi}^{2}} = \frac{(m_{\phi}^{2})_{M N}^{\ell L}}{m_{\phi}^{2}}. \quad (14)
$$

Terms contributing to various processes in case II are shown in Table III.

III. RESULTS

In this section we present the numerical results for cases I and II. Experimental inputs are from refs. [4] [16] [17] [21] [23] and are shown in Table I. Further inputs not listed in the table are from ref. [4].

A. Case I

In Table IV we present the constraints on parameters in case I using $x = m_{\phi}/m_{\phi} = 1$ and $m_{\phi} = 500$ GeV. Results for other $m_{\phi}$ can be obtained by scaling the results with a factor for $Q_{\phi,\psi} g_{\ell}^{i} g_{R L}$ and $(m_{\phi}/500)$ for other quantities. Results in [...] are obtained by using the future experimental sensitivities. Both results for the cases of Dirac and Majorana fermion are given, where results in {...} are for the Majorana case. Note that some of the results are unphysical. For example, the values of $Q_{\phi,\psi}|g_{\ell R}|^{2}$ and $Q_{\phi,\psi}|g_{R L}|^{2}$ required to produce large enough $\Delta a_{\ell}$ and
TABLE IV: Constraints on parameters in case I using \( x = m_\phi/m_\psi = 1 \) and \( m_\psi = 500 \) GeV from various processes are shown. Results are applicable with \( L \) and \( R \) interchanged. Results for other \( m_\psi \) can be obtained by scaling with a \( (m_\psi/500 \text{GeV})^2 \) or \( m_\phi/500 \text{GeV} \) factor, where the latter is for \( Q_\phi \psi g_R^* \psi g_R \). Results in [...] are obtained by using the future experimental sensitivities, results in {...} are for the Majorana case.

| Processes | \( \Delta a_e \) | \( \Delta a_\mu \) | \( \Delta a_\tau \) | \( d_e, d_\mu, d_\tau \) |
|----------|----------------|----------------|----------------|----------------|
| \( Q_\psi |g_R|^2 \) | \( -1597 \pm 653 \) | \( -1597 \pm 653 \) | \( -4.1 \pm 1.6 \) \( \times 10^{-4} \) | \( 2.0 \pm 0.8 \) \( \times 10^{-4} \) |
| \( Q_\psi |g_R|^2 \) | \( 115 \pm 31 \) | \( -115 \pm 31 \) | \( 6.1 \pm 1.6 \) \( \times 10^{-3} \) | \( -3.0 \pm 0.8 \) \( \times 10^{-3} \) |
| \( Q_\psi |g_R|^2 \) | \( -7 \sim 2 \times 10^6 \) | \( -2 \sim 7 \times 10^6 \) | \( -7 \sim 2 \times 10^3 \) | \( -8 \sim 3 \times 10^3 \) |
| \( Q_\psi |g_R|^2 \) | \( 2.6 \times 10^{-10} \) | \( 1.3 \times 10^{-10} \) | \( 4.6 \) (38.3) | \( 2.3 \) (19.1) |
| \( \phi \) | \( 0.002 \) \( [0.0008] \) | \( 0.002 \) \( [0.0008] \) | \( 11 \) \( [4] \) \( \times 10^{-8} \) | \( 6 \) \( [2] \) \( \times 10^{-8} \) |
| \( \phi \) | \( 0.046 \) \( [0.0005] \) | \( 0.030 \) \( [0.0003] \) | \( 224 \) \( [2] \) \( \times 10^{-8} \) | \( 112 \) \( [1] \) \( \times 10^{-8} \) |
| \( \phi \) | \( 0.020 \) \( [0.0002] \) | \( 0.016 \) \( [0.0002] \) | \( 236 \) \( [3] \) \( \times 10^{-8} \) | \( 118 \) \( [1] \) \( \times 10^{-8} \) |
| \( \phi \) | \( 0.051 \) \( [0.0008] \) | \( 0.046 \) \( [0.0007] \) | \( 569 \) \( [9.6] \) \( \times 10^{-8} \) | \( 284 \) \( [0.4] \) \( \times 10^{-8} \) |
| \( \phi \) | \( 0.00010 \) | \( 0.00009 \) | \( 1.1 \) \( \times 10^{-8} \) | \( 0.5 \) \( \times 10^{-8} \) |
| \( \phi \) | \( 393 \) \( [4] \) \( \times 10^{-6} \) | \( 115 \) \( [1] \) \( \times 10^{-6} \) | \( 0.01 \) \( \{\} \) \( [1] \) \( \times 10^{-4} \) \( \{\} \) \( 7 \) \( [7] \) \( \times 10^{-3} \) \( [7] \) \( [7] \) \( \times 10^{-5} \) |
| \( \phi \) | \( 492 \) \( [6] \) \( \times 10^{-7} \) | \( 145 \) \( [2] \) \( \times 10^{-7} \) | \( 0.01 \) \( \{\} \) \( [1] \) \( \times 10^{-4} \) \( \{\} \) \( 7 \) \( [7] \) \( \times 10^{-3} \) \( [7] \) \( [7] \) \( \times 10^{-5} \) |
| \( \phi \) | \( 1718 \) \( [3] \) \( \times 10^{-7} \) | \( 5049 \) \( [8] \) \( \times 10^{-8} \) | \( 0.01 \) \( \{\} \) \( [1] \) \( \times 10^{-4} \) \( \{\} \) \( 7 \) \( [7] \) \( \times 10^{-3} \) \( [7] \) \( [7] \) \( \times 10^{-5} \) |
| \( \phi \) | \( 4 \) \( \{\} \) \( [1] \) \( \times 10^{-7} \) | \( 1 \) \( \{\} \) \( [1] \) \( \times 10^{-7} \) | \( 0.01 \) \( \{\} \) \( [1] \) \( \times 10^{-4} \) \( \{\} \) \( 7 \) \( [7] \) \( \times 10^{-3} \) \( [7] \) \( [7] \) \( \times 10^{-5} \) |
| \( \phi \) | \( 1.4 \) \( [0.4] \) | \( 1.4 \) \( [0.4] \) | \( 13 \) \( [4] \) \( \times 10^{-4} \) | \( 6 \) \( [2] \) \( \times 10^{-4} \) |
| \( \phi \) | \( 13.2 \) \([1.7] \) | \( 10.0 \) \([1.3] \) | \( 11 \) \( [1] \) \( \times 10^{-3} \) | \( 56 \) \( [7] \) \( \times 10^{-4} \) |
| \( \phi \) | \( 0.15 \) \( [0.02] \) | \( 0.05 \) \( [0.006] \) | \( 4.3 \) \( [0.5] \) \( \{\} \) \( 2.9 \) \( [2.9] \) \( [0.4] \) \( [0.4] \) |
| \( \phi \) | \( 1.7 \) \( [0.3] \) | \( 1.7 \) \( [0.3] \) | \( 15 \) \( [2] \) \( \times 10^{-4} \) | \( 7 \) \( [1] \) \( \times 10^{-4} \) |
| \( \phi \) | \( 30.7 \) \( [3.9] \) | \( 12.5 \) \( [1.6] \) | \( 21 \) \( [3] \) \( \times 10^{-3} \) | \( 11 \) \( [1] \) \( \times 10^{-3} \) |
| \( \phi \) | \( 0.14 \) \( [0.02] \) | \( 0.04 \) \( [0.005] \) | \( 3.8 \) \( [0.5] \) \( \{\} \) \( 2.5 \) \( [2.5] \) \( [0.3] \) \( [0.3] \) |
| \( \phi \) | \( 3.2 \) \( \{\} \) \( [0.4] \) \( \{\} \) | \( 2.3 \) \( [2.3] \) \( [0.3] \) \( [0.3] \) | \( 6.4 \) \( [6.4] \) \( [0.8] \) \( [0.8] \) |
| \( \phi \) | \( 3.4 \) \( \{\} \) \( [0.4] \) \( \{\} \) | \( 2.4 \) \( [2.4] \) \( [0.3] \) \( [0.3] \) | \( 6.8 \) \( [6.8] \) \( [0.9] \) \( [0.9] \) |
\[\Delta a_e\]

\[
\Delta a_e = Q_\phi|g_\tau|^2 - 811 \pm 332 \\
1059 \pm 433 (-2.3 \pm 1.0) \times 10^{-4} \\
(1.5 \pm 0.7) \times 10^{-4}
\]

\[
\Delta a_\mu = Q_\psi|\mu_\tau|^2 - 58 \pm 16 \\
-76 \pm 20 (3.5 \pm 0.9) \times 10^{-3} \\
(-2.3 \pm 0.6) \times 10^{-3}
\]

\[
\Delta a_\tau = Q_\psi|g_\tau|^2 - (4 \sim 1) \times 10^6 \\
(1 \sim 5) \times 10^6 (-4 \sim 1) \times 10^3 \\
(-0.7 \sim 3) \times 10^3
\]

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
Processes & \multicolumn{4}{c|}{constraints} \\
\hline
& \multicolumn{2}{c|}{\text{TABLE V}: Same as Table IV, but with } x = m_\phi/m_\psi = 0.5. \\
\hline
& constraints & \multicolumn{3}{c|}{constraints} \\
\hline
\Delta a_e & $Q_\phi|g_\tau|^2$ & $Q_\psi|g_\mu|^2$ & $Q_\phi \text{ Re}(g_{\tau R}^* g_{\mu L})$ & $Q_\psi \text{ Re}(g_{\tau R}^* g_{\mu L})$ \\
& $-811 \pm 332$ & $1059 \pm 433$ & $(-2.3 \pm 1.0) \times 10^{-4}$ & $(1.5 \pm 0.7) \times 10^{-4}$ \\
\hline
\Delta a_\mu & $Q_\phi|\mu_\tau|^2$ & $Q_\psi|g_\mu|^2$ & $Q_\phi \text{ Re}(g_{\tau R}^* g_{\mu L})$ & $Q_\psi \text{ Re}(g_{\tau R}^* g_{\mu L})$ \\
& $58 \pm 16$ & $-76 \pm 20$ & $(3.5 \pm 0.9) \times 10^{-3}$ & $(-2.3 \pm 0.6) \times 10^{-3}$ \\
\hline
\Delta a_\tau & $Q_\psi|g_\tau|^2$ & $Q_\psi|g_\mu|^2$ & $Q_\phi \text{ Re}(g_{\tau R}^* g_{\mu L})$ & $Q_\psi \text{ Re}(g_{\tau R}^* g_{\mu L})$ \\
& $(-4 \sim 1) \times 10^6$ & $(-1 \sim 5) \times 10^6$ & $(-4 \sim 1) \times 10^3$ & $(-0.7 \sim 3) \times 10^3$ \\
\hline
\hline
$d_\psi, d_\mu, d_\tau$ & $|Q_\phi \text{ Im}(g_{\mu R}^* g_{\psi L})|$ & $|Q_\phi \text{ Im}(g_{\mu R}^* g_{\psi L})|$ & $|Q_\phi \text{ Im}(g_{\mu R}^* g_{\psi L})|$ & $|Q_\phi \text{ Im}(g_{\mu R}^* g_{\psi L})|$ \\
& $1.5 \times 10^{-10}$ & $1.0 \times 10^{-10}$ & $2.6 (22.0)$ & $1.8 (14.9)$ \\
\hline
$\mu^+ \rightarrow e^+\gamma$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ \\
& $0.001 [0.0004]$ & $0.001 [0.0005]$ & $7 \times 10^{-8}$ & $4 \times 10^{-8}$ \\
\hline
$\mu^+ \rightarrow e^+e^-$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $129 [1] \times 10^{-8}$ & $87 [0.9] \times 10^{-8}$ \\
& $0.024 [0.0002]$ & $0.021 [0.0002]$ & $136 [2] \times 10^{-8}$ & $92 [1] \times 10^{-8}$ \\
\hline
$\mu^- \rightarrow e^-\mu$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $327 [0.5] \times 10^{-8}$ & $222 [0.3] \times 10^{-8}$ \\
& $0.008 [0.0001]$ & $0.013 [0.0002]$ & $649 [1] \times 10^{-7}$ & $[4.2 \times 10^{-9}]$ \\
\hline
$\mu^- \rightarrow e^-\tau$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $3 \times 10^{-7}$ & $[1 \times 10^{-7}]$ \\
& $0.022 [0.0003]$ & $0.038 [0.0006]$ & $3 \times 10^{-7}$ & $[1 \times 10^{-7}]$ \\
\hline
$\mu^- \rightarrow e^-\mu^+$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $|Q_\phi g_{\mu R} g_{\psi L}|$ & $3 \times 10^{-7}$ & $[1 \times 10^{-7}]$ \\
& $0.011 [0.001]$ & $0.06 [0.007]$ & $2.5 \times 10^{-4}$ & $1.1 \times 10^{-4}$ \\
\hline
$\tau^- \rightarrow e^-\gamma$ & $|Q_\phi g_{\tau R} g_{\phi L}|$ & $|Q_\phi g_{\tau R} g_{\phi L}|$ & $|Q_\phi g_{\tau R} g_{\phi L}|$ & $|Q_\phi g_{\tau R} g_{\phi L}|$ \\
& $0.7 [0.2]$ & $1.0 [0.3]$ & $7 \times 10^{-4}$ & $5 \times 10^{-4}$ \\
\hline
$\tau^- \rightarrow e^-e^-$ & $|Q_\phi g_{\tau R} g_{\phi L}|$ & $|Q_\phi g_{\tau R} g_{\phi L}|$ & $65 [8] \times 10^{-4}$ & $44 [6] \times 10^{-4}$ \\
& $6.8 [0.9]$ & $6.9 [0.9]$ & $65 [8] \times 10^{-4}$ & $44 [6] \times 10^{-4}$ \\
\hline
$\tau^- \rightarrow e^-\mu^-$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ \\
& $16.7 [2.1]$ & $8.9 [1.1]$ & $12 \times 10^{-3}$ & $8 \times 10^{-3}$ \\
\hline
$\tau^- \rightarrow \mu^-\mu^+$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ \\
& $0.09 [0.01]$ & $0.05 [0.006]$ & $2.2 \times 10^{-4}$ & $1.0 \times 10^{-4}$ \\
\hline
$\tau^- \rightarrow e^-\mu^+$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ \\
& $1.8 \times 2.0 [0.1]$ & $0.9 \times 0.9 [0.1]$ & $1.9 \times 1.9 [0.1]$ & $1.9 \times 1.9 [0.1]$ \\
\hline
$\tau^- \rightarrow e^-\mu^+$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ & $|Q_\phi g_{\tau R} g_{\mu L}|$ \\
& $2.0 \times 2.0 [0.1]$ & $1.0 \times 1.0 [0.1]$ & $2.1 \times 2.1 [0.1]$ & $2.1 \times 2.1 [0.1]$ \\
\hline
\end{tabular}
\end{table}

\(\Delta a_\mu\) as required by data are much larger than 4\(\pi\). Perturbative calculation breaks down and the results are untrustworthy, hence, unphysical. We state these naïve results simply to indicate that
TABLE VI: Same as Table IV but with $x \equiv m_\phi/m_\psi = 2$.

| Processes | $Q_{\phi|g_{\ell R}}|^2$ | $Q_{\phi|g_{\ell R}}|\Delta a_e^{\psi}$ | $Q_{\phi|g_{\ell R}}|\Delta a_\mu^{\psi}$ | $Q_{\phi|g_{\ell R}}|\Delta a_\tau^{\psi}$ |
|-----------|-------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| $\Delta a_e$ | $-4234 \pm 1732$ | $3247 \mp 1328$ | $-9.4 \pm 3.8 \times 10^{-4}$ | $(3.2 \pm 1.3) \times 10^{-4}$ |
| $\Delta a_\mu$ | $305 \pm 82$ | $-234 \mp 63$ | $(14.0 \pm 3.7) \times 10^{-3}$ | $(-4.8 \pm 1.3) \times 10^{-3}$ |
| $\Delta a_\tau$ | $(-20 \sim 5) \times 10^{6}$ | $(-4 \sim 16) \times 10^{6}$ | $(-16 \sim 4) \times 10^{3}$ | $(-1 \sim 5) \times 10^{3}$ |

| $d_e, d_\mu, d_\tau$ | $Q_{\phi|g_{\ell R}}|g_{\ell R}^{\psi}$ | $Q_{\phi|g_{\ell R}}|g_{\ell R}^{\psi}|\Delta T_{\psi}$ | $Q_{\phi|g_{\ell R}}|g_{\ell R}^{\psi}|\Delta T_{\psi}$ | $Q_{\phi|g_{\ell R}}|g_{\ell R}^{\psi}|\Delta T_{\psi}$ |
|------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| $\mu^+ \to e^+\gamma$ | $0.060 [0.002]$ | $0.004 [0.002]$ | $26 [10] \times 10^{-8}$ | $9 [3] \times 10^{-8}$ |
| $\mu^+ \to e^+e^-$ | $0.120 [0.001]$ | $0.056 [0.0006]$ | $516 [5] \times 10^{-8}$ | $177 [2] \times 10^{-8}$ |
| $\mu^- \to e^-\gamma$ | $0.059 [0.0007]$ | $0.024 [0.0003]$ | $542 [6] \times 10^{-8}$ | $187 [2] \times 10^{-8}$ |
| $\mu^- \to e^-e^-$ | $0.151 [0.0002]$ | $0.069 [0.0001]$ | $1309 [2] \times 10^{-8}$ | $450 [7] \times 10^{-8}$ |
| $\mu^- \to e^-\tau$ | $0.0003$ | $0.0001$ | $2.5 \times 10^{-8}$ | $0.9 \times 10^{-8}$ |

contributions from $Q_{\phi,\psi|g_{\ell R}}|^2$ and $Q_{\phi,\psi|g_{\mu R}}|^2$ cannot generate the desired results on $\Delta a_e$ and $\Delta a_\mu$. Results for $x = 0.5$ and 2 are given in Tables IV and V respectively.

In Fig. 2(a) and (b), we show the allowed parameter space for $Q_{\phi,\psi|g_{\ell L}}|^2$, $Q_{\phi,\psi|\text{Re}(g_{\ell R}^* g_{\ell L})}$.
FIG. 2: We show in (a) and (b), allowed parameter space for $\pm Q_{\psi,\phi}|g_{e R}|^2$, $\pm Q_{\psi,\phi}\text{Re}(g_{e R}^\ast g_{e L})$ and $|Q_{\psi,\phi}\text{Im}(g_{e R}^\ast g_{e L})|$ constrained by $\Delta a_e$ and $d_e$, in (c) and (d), allowed parameter space for $\pm Q_{\psi,\phi}|g_{\mu L(R)}|^2$, $\pm Q_{\psi,\phi}\text{Re}(g_{\mu R}^\ast g_{\mu L})$ and $|Q_{\psi,\phi}\text{Im}(g_{\mu R}^\ast g_{\mu L})|$ constrained by $\Delta a_\mu$ and $d_\mu$, in (e) allowed parameter space for $|Q_{\psi,\phi}|\text{Re}(g_{\mu R}^\ast g_{\mu L})\text{Re}(g_{\mu L(R)}^\ast)|^{1/2}$ to produce $\Delta a_e$ and $\Delta a_\mu$. These results are given for $m_\psi = 500$ GeV. For other $m_\psi$, plots in (a) and (c) scale with $(500\text{ GeV}/m_\psi)^2$, while plots in (b), (d) and (e) scale with $500\text{ GeV}/m_\psi$.

and $|Q_{\psi,\phi}\text{Im}(g_{e R}^\ast g_{e L})|$ constrained by $\Delta a_e$ and $d_e$. In Fig. 2(c) and (d), allowed parameter space for $\pm Q_{\psi,\phi}|g_{\mu L(R)}|^2$, $\pm Q_{\psi,\phi}\text{Re}(g_{\mu R}^\ast g_{\mu L})$ and $|Q_{\psi,\phi}\text{Im}(g_{\mu R}^\ast g_{\mu L})|$ constrained by $\Delta a_\mu$ and $d_\mu$ are shown.
FIG. 3: Parameter space excluded or projected by various experimental bounds or expected sensitivities on $\mu \to e$ LFV processes from photonic penguin, $Z$-penguin and box contributions.
FIG. 4: Same as Fig. 3 but for $\tau \rightarrow e$ transition.

In Fig. 3 (e) the allowed parameter space for $|Q_{\phi, \psi} g_{R(L)}^* g_{eL(R)}|$ constrained by $\mu \rightarrow e\gamma$ and the
FIG. 5: Same as Fig. 3, but for $\tau \rightarrow \mu$ transition.

parameter space on $|Q_{\phi,\psi}| |\text{Re}(g_{eR}g_{eL})| |\text{Re}(g_{\mu R}g_{\mu L})|^{1/2}$ to produce $\Delta a_e$ and $\Delta a_\mu$ are presented. These
results are given for \( m_\psi = 500 \text{ GeV} \). For other \( m_\psi \), scale plots in (a) and (c) with \((500 \text{ GeV}/m_\psi)^2\), and scale plots in (b), (d) and (e) with \(500 \text{ GeV}/m_\psi\).

In Figs. 3, 4, 5 the parameter space excluded or projected by various bounds or expected sensitivities on \( \tau^{-} \rightarrow e^{-} \mu^{+} e^{-}, \mu^{-} e^{+} \mu^{-} \) processes from box contributions. In Fig. 6 the parameter space excluded or projected by using various bounds or projected sensitivities on \( \tau^{-} \rightarrow e^{-} \mu^{+} e^{-}, \mu^{-} e^{+} \mu^{-} \) processes through contributions from box diagrams are shown.
From these results we can extract several messages. First we note that chiral interactions \((g_L \times g_R = 0)\) are unable to generate large enough \(\Delta a_e\) and \(\Delta a_\mu\) to accommodate the experimental results, Eqs. (1) and (2). From Tables IV, V, VI, Fig. 2(a) and (c), we see that \(Q_{\phi,\psi}|g_{\mu R(L)}|^2\) and \(Q_{\phi,\psi}|g_{\mu R(L)}|^2\) need to be unreasonably large to produce the experimental value of \(\Delta a_e\) and \(\Delta a_\mu\). This implies the incapability of chiral interactions to generate large enough \(\Delta a_e\) and \(\Delta a_\mu\) to accommodate the experimental results.

Although non-chiral interactions are capable to generate \(\Delta a_e\) and \(\Delta a_\mu\) successfully accommodating the experimental results, they are not contributed from the same source. From Tables IV, V and VI we see that, for \(x = 0.5, 1\) and 2, \(Q_{\phi,\psi}\text{Re}(g^*_{e R}g_{e L})\) and \(Q_{\phi,\psi}\text{Re}(g^*_{\mu R}g_{\mu L})\) of orders \(10^{-4}\) and \(10^{-3}\), respectively, are able to produce the experimental values of \(\Delta a_e\) and \(\Delta a_\mu\). However, the contributions cannot come from the same source, i.e. from diagrams involving the same set of \(\phi\) and \(\psi\). The reasons are as follows. If \(\Delta a_e\) and \(\Delta a_\mu\) are generated from the same set of \(\phi\) and \(\psi\), the very same set of \(\psi\) and \(\phi\) will also generate \(\mu \to e\gamma\) decay with rate exceeding the experimental bound. Indeed, from Fig. 2(e) we see that the \(\mu \to e\gamma\) data constrains \(|Q_{\phi,\psi}g^*_{\mu R|g_{e R}|g_{e L}}|\) to be less than \(10^{-7}\) to \(10^{-6}\), but experimental data on \(\Delta a_e\) and \(\Delta a_\mu\) require \(|Q_{\phi,\psi}||\text{Re}(g^*_{e R}g_{e L})\text{Re}(g^*_{\mu R}g_{\mu L})|^{1/2}\) to be of the order of \(10^{-3}\) to \(10^{-1}\), which is larger than the constrain from \(\mu \to e\gamma\) by more than 4 orders of magnitude. Hence, the contributions to \(\Delta a_e\) and \(\Delta a_\mu\) do not come from the same source. Our finding agrees with ref. [27], where a common explanation of \(\Delta a_e\) and \(\Delta a_\mu\) was investigated.

Presently, the upper limit in \(\mu \to e\gamma\) decay gives the most severe constraints on photonic penguin contributions in \(\mu \to e\) transitions, but the situation may change when we include future experimental sensitivities in the analysis. From Tables IV, V, VI and Fig. 3(a) to (d), we see that the present \(\mu \to e\gamma\) bound constrains the \(|Q_{\phi,\psi}g^*_{\mu R}g_{e R}|\) and \(|Q_{\phi,\psi}g^*_{\mu R}g_{e L}|\) much better than the present \(\mu \to 3e\) and \(\mu N \to eN\) upper limits. In fact, the bounds obtained from \(\mu \to e\gamma\) decay are better than those from other processes by at least one order of magnitude. The situation is altered when considering future experimental searches. From the tables and the figures, we see that, on the contrary, in near future experiments the \(\mu \to 3e\) and \(\mu N \to eN\) processes may be able to probe the photonic penguin contributions from \(|Q_{\phi,\psi}g^*_{\mu R}g_{e R}|\) and \(|Q_{\phi,\psi}g^*_{\mu R}g_{e L}|\) better than the future experiment search on \(\mu \to e\gamma\) decay.

The \(Z\)-penguin diagrams can constrain chiral interaction better than photonic penguin diagrams in \(\mu \to e\) transitions. From Tables IV, V, VI, Fig. 3(a), (b), (e) and (f) we see that the bounds on \(|g^*_{\mu R}g_{e R}\Delta T_{3\psi}|\) and \(|g^*_{\mu R}g_{e R}\Delta T_{3\psi}\) from \(Z\)-penguin contributions are more severe (by two orders of magnitude) than bounds on \(|Q_{\phi,\psi}g^*_{\mu R}g_{e R}|\) from photonic penguin contributions. In addition, from Fig. 3(e) and (f) we see that \(\mu N \to eN\) transitions give better constraints on \(|g^*_{\mu R}g_{e R}\Delta T_{3\psi}|\) and \(|g^*_{\mu R}g_{e R}\Delta T_{3\psi}|\) than the \(\mu \to 3e\) decay.

In case I, either in the Dirac or Majorana case, box contributions to \(\mu \to 3e\) decay are sub-leading. Furthermore, there are cancelation in box contributions in the Majorana fermionic case making the contributions even smaller. Fig. 3(g) and (h) show the bounds on \(|g^*_{\mu R}g_{e R}\Delta T_{3\psi}|\) and \(|g^*_{\mu R}g_{e R}\Delta T_{3\psi}|\) obtained by considering box contributions to \(\mu \to 3e\) decay. Note that the constraint on \(|g^*_{\mu R}g_{e R}\Delta T_{3\psi}|\) obtained from \(\mu Au \to eAu\) upper limit and perturbativity is much severe than the \(|g^*_{\mu R}g_{e R}|\) bound by one to two orders of magnitude, while \(|Q_{\phi,\psi}g^*_{\mu R}g_{e R}|\) obtained from \(\mu \to e\gamma\), \(\Delta a_e\) and \(d_e\) experimental results is much severe than the \(|g^*_{\mu R}g_{e R}|\) bound by more than 8 orders of magnitude. One can also use the values in Tables IV, V, VI to
obtain similar findings. These results imply that box contributions to $\mu$ in case I on various muon and tau LFV processes. Experimental bounds are from \[4, 16, 21–23\].

TABLE VII: Current experimental upper limits, future sensitivities and bounds from consistency in case I on various muon and tau LFV processes. Experimental bounds are from \[4, 16, 21–23\].

| bound | current limit (future sensitivity) | consistency bounds | remarks |
|-------|-----------------------------------|--------------------|---------|
| $B(\mu^+ \rightarrow e^+\gamma)$ | $\leq 4.2 \times 10^{-13}$ (6 $\times$ $10^{-14}$) | $\leq 4.2 \times 10^{-13}$ | input |
| $B(\mu^+ \rightarrow e^+\mu^-\mu^+)$ | $< 1.0 \times 10^{-12}$ (10 $\times$ $10^{-16}$) | $< 1.3 \times 10^{-14}$ | from $\mu \rightarrow e\gamma$ bound |
| $B(\mu^-\Ti \rightarrow e^-\Ti)$ | $\leq 4.3 \times 10^{-12}$ (10 $\times$ $10^{-17}$) | $< 1.6 \times 10^{-14}$ | from $\mu\Au \rightarrow e\Au$ bound |
| $B(\mu^-\Au \rightarrow e^-\Au)$ | $\leq 7.0 \times 10^{-13}$ (10 $\times$ $10^{-16}$) | $< 1.1 \times 10^{-13}$ | from $\mu \rightarrow e\gamma$ bound |
| $B(\mu^-\Al \rightarrow e^-\Al)$ | $\cdots$ (10 $\times$ $10^{-17}$) | $< 7.0 \times 10^{-13}$ | input |
| $B(\tau^- \rightarrow e^-\gamma)$ | $< 3.3 \times 10^{-8}$ (3 $\times$ $10^{-9}$) | $< 3.3 \times 10^{-8}$ | input |
| $B(\tau^- \rightarrow e^-\mu^-\mu^+)$ | $\leq 2.7 \times 10^{-8}$ (4.3 $\times$ $10^{-10}$) | $< 1.2 \times 10^{-9}$ | from $\tau \rightarrow e\gamma$ bound |
| $B(\tau^- \rightarrow \mu^-\gamma)$ | $< 4.4 \times 10^{-8}$ (1 $\times$ $10^{-9}$) | $< 4.4 \times 10^{-8}$ | input |
| $B(\tau^- \rightarrow \mu^-\mu^-\mu^+)$ | $\leq 2.1 \times 10^{-8}$ (3.3 $\times$ $10^{-10}$) | $< 1.2 \times 10^{-9}$ | from $\tau \rightarrow \mu\gamma$ bound |
| $B(\tau^- \rightarrow e^-\mu^-\mu^+)$ | $\leq 1.7 \times 10^{-8}$ (2.7 $\times$ $10^{-10}$) | $\lesssim 7 \times 10^{-11}$ | from $\tau \rightarrow e\gamma$, $\mu \rightarrow e\gamma$ bounds |
| $B(\tau^- \rightarrow e^-\mu^-\mu^-)$ | $\leq 1.5 \times 10^{-8}$ (2.4 $\times$ $10^{-10}$) | $\lesssim 7 \times 10^{-11}$ | from $\tau \rightarrow e\gamma$, $\mu \rightarrow e\gamma$ bounds |

From Tables IV, V and VI, we see that the present bounds on $\Delta a_\tau$ cannot constrain $Q_{\phi,\psi}|g_{\tau R(L)}|^2$ and $Q_{\phi,\psi}\Re(g_{\tau R(L)}^*g_{\tau L})$ well. Even the bound on $d_\tau$ cannot give good constraints on $Q_{\phi,\psi}\Im(g_{\tau R(L)}^*g_{\tau L})$. There is still a long way to go.

In $\tau \rightarrow e$ ($\mu$) transitions, the $\tau \rightarrow e\gamma$ ($\mu\gamma$) upper limit constrains photonic penguin contributions better than the $\tau \rightarrow 3e$ ($3\mu$) upper limit, and Z-penguin constrains chiral interaction better than photonic penguin. From Tables IV, V, VI, Fig. 4(a) to (d) and Fig. 5(a) to (d), we see that bounds on $|Q_{\phi,\psi}g_{\tau R}^*g_{\tau L}|$ and $|Q_{\phi,\psi}g_{\tau R}^*g_{\tau L}|$ are constrained by the $\tau \rightarrow e\gamma$ ($\mu\gamma$) data more severely than by the $\tau \rightarrow 3e$ ($3\mu$) upper limit. Note that the bounds of these parameters using the proposed sensitivities on $\tau \rightarrow 3e$ and $\tau \rightarrow 3\mu$ decays by Belle II are superseded by the bounds using the present limits of $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ decays. From Tables IV, V, VI, Fig. 4(e), (f) and Fig. 5(e), (f), we see that bounds on $g_{\tau R}^*g_{\tau L}^*|e\gamma|$ and $g_{\tau R}^*g_{\tau L}^*|e\gamma|$ from Z-penguin contributions are more severe (by one order of magnitude) than those on $|Q_{\phi,\psi}g_{\tau R}^*g_{\tau L}|$ from photonic penguin contributions. Hence, Z-penguin constrains chiral interaction better than photonic penguin.

Box contributions to $\tau \rightarrow 3e$ and $\tau \rightarrow 3\mu$ decays can sometime be comparable to Z-penguin contributions. In Fig. 4(g), (h) and Fig. 5(g), (h) we show the bounds on $|g_{\tau R}^*g_{\tau L}^*|_{\Delta T_{2\tau}}$ obtained by considering box contributions to $\tau \rightarrow 3e$ ($3\mu$) decay. Note that the constraint on $g_{\tau R}^*g_{\tau L}^*|e\gamma|$ obtained from Z-penguin contributions to $\tau \rightarrow 3e$ ($3\mu$) decay and perturbativity is much severe than the $|g_{\tau R}^*g_{\tau L}^*|_{\Delta T_{2\tau}}$ bound from box contributions for $x \gtrsim 0.4$, but it is the other way around for $x \lesssim 0.4$. The bound on $|Q_{\phi,\psi}g_{\tau R}^*g_{\tau L}|_{\Delta T_{2\tau}}$ obtained using $\tau \rightarrow e\gamma$ ($\mu\gamma$), $\Delta a_\tau$ and $d_\tau$ experimental results
is much severe than the $|g^{*}_{\tau R}g_{e(\mu)L}g^{*}_{e(\mu)L}|$ bound from box contributions by five to seven (one to three) orders of magnitude. One can also obtain similar results using the values in Tables [IV][V][VI]. These findings imply that box contributions to $\tau \to 3e$ (3$\mu$) can sometime be comparable to $Z$-penguin contributions.

The $\tau^{-} \to e^{-}\mu^{-}e^{-}$ rate is highly constrained by $\tau \to e\gamma$ and $\mu \to e\gamma$ upper limits. From Fig. 6 (a), (c), (e) and Tables [IV][V][VI] we see that the bounds on $|g^{*}_{\tau R}g_{eR}g^{*}_{\mu R}g_{eR}|$, $|g^{*}_{\tau R}g_{eL}g^{*}_{\mu R}g_{eL}|$ and $|g^{*}_{\tau R}g_{eR}g^{*}_{\mu L}g_{eL}|$, obtained from the upper limit of the $\tau^{-} \to e^{-}\mu^{-}e^{-}$ rate, are larger than the bounds on $|Q_{\phi,\psi}g^{*}_{\tau R}g_{eR}|$, $|Q_{\phi,\psi}g^{*}_{\tau R}g_{eL}|$, $|Q_{\phi,\psi}g^{*}_{\tau R}g_{eL}|$, $|Q_{\phi,\psi}g^{*}_{\tau R}g_{eR}|$ and $|Q_{\phi,\psi}g^{*}_{\tau R}g_{eL}|$, obtained from the upper limits of $\tau \to e\gamma$ and $\mu \to e\gamma$ rates, by several orders of magnitude. Note that the $\tau^{-} \to e^{-}\mu^{-}e^{-}$ rate is constrained to be smaller than the proposed sensitivity. Hence, the $\tau^{-} \to e^{-}\mu^{-}e^{-}$ rate is highly constrained by the present $\tau \to e\gamma$ and $\mu \to e\gamma$ upper limits.

The $\tau^{-} \to \mu^{-}e^{+}\mu^{-}$ rate is also highly constrained by $\tau \to \mu\gamma$ and $\mu \to e\gamma$ upper limits. From Fig. 6 (b), (d), (f) and Tables [IV][V][VI], we see that the bounds on $|g^{*}_{\tau R}g_{\mu R}g^{*}_{eR}g_{\mu R}|$, $|g^{*}_{\tau R}g_{\mu L}g^{*}_{eR}g_{\mu L}|$ and $|g^{*}_{\tau R}g_{\mu R}g^{*}_{eL}g_{\mu L}|$, obtained from the upper limit of the $\tau^{-} \to \mu^{-}e^{+}\mu^{-}$ rate, are larger than the bounds on $|Q_{\phi,\psi}g^{*}_{\tau R}g_{\mu R}|$, $|Q_{\phi,\psi}g^{*}_{\tau R}g_{\mu L}|$, $|Q_{\phi,\psi}g^{*}_{\tau R}g_{\mu L}|$ and $|Q_{\phi,\psi}g^{*}_{\tau R}g_{\mu R}|$, obtained from the upper limits of $\tau \to e\gamma$ and $\mu \to e\gamma$ rates, by several orders of magnitude. Hence, the $\tau^{-} \to \mu^{-}e^{+}\mu^{-}$ rate is highly constrained by $\tau \to \mu\gamma$ and $\mu \to e\gamma$ upper limits. In fact, the $\tau^{-} \to \mu^{-}e^{+}\mu^{-}$ rate is constrained to be smaller than the proposed sensitivity.

In Table [VII], we compare the current experimental upper limits, future sensitivities and bounds from consistency for case I on various muon and tau LFV processes. We see that the present $\mu \to e\gamma$ upper limit requires the bounds on $\mu \to 3e$, $\mu Ti \to e Ti$ and $\mu Au \to e Au$ be lower by two orders of magnitude, more than one order of magnitude and almost one order of magnitude, respectively, from their present upper limits, and the $\mu Al \to e Al$ rate is predicted to be smaller than $6 \times 10^{-14}$. These bounds can be further pushed downward by one order of magnitude if we still cannot observed $\mu \to e\gamma$ decay in MEG II. It is interesting that the future sensitivities of $\mu \to 3e$ and $\mu N \to e N$ are much lower than the above limits based on consistency, giving them good opportunity to explore these LFV processes. We find that the situation is similar but the bounds are slightly relaxed when the $\mu Au \to e Au$ upper limit instead of the present $\mu \to e\gamma$ upper limit is used as an input. Similarly, using the present $\tau \to e\gamma$ ($\mu\gamma$) upper limit as input, the $\tau \to 3e$ (3$\mu$) bound is smaller than its present upper limit by one order of magnitude. Finally, the $\tau^{-} \to \mu^{-}e^{+}\mu^{-}$ and $\tau^{-} \to e^{-}\mu^{+}e^{-}$ bounds are lower than their present upper limits by two orders of magnitude as required from the present $\tau \to \mu\gamma$, $e\gamma$ and $\mu \to e\gamma$ upper limits. These limits are lower than the proposed future sensitivities.

B. Case II

We now turn to the second case, where we have a built-in cancellation mechanism.

In Table [VIII], we show the constraints on parameters in case II using $x = m_{\phi}/m_{\psi} = 1$ and $m_{\psi} = 500$ GeV. Constraints for other $m_{\psi}$ can be obtained by scaling the results in the table by a $(m_{\psi}/500$ GeV)$^2$ or a $(m_{\psi}/500$ GeV)$^{-1}$ factor, where the latter is for $Q_{\phi,\psi}g^{*}_{(\phi,\psi)R}g_{(\phi,\psi)L}$. Results in [... ] are obtained by using the projected sensitivities for future experiments. For box contributions both results of
TABLE VIII: Same as Table IV \((x = 1)\), but for case II.

| Processes | constraints | constraints | constraints | constraints |
|-----------|-------------|-------------|-------------|-------------|
| \(\Delta a_e\) | \(Q_\phi|g_{eR}|^2\) | \(Q_\psi|g_{eR}|^2\) | \(Q_\psi \text{Re}(g_{eR}^* g_{eL} \delta_{RL}^{e\mu})\) | \(Q_\psi \text{Re}(g_{eR}^* g_{eL} \delta_{RL}^{e\mu})\) |
|           | \(-1597 \pm 653\) | \(1597 \pm 653\) | \((8 \pm 3) \times 10^{-4}\) | \((-8 \pm 3) \times 10^{-4}\) |
| \(\Delta a_\mu\) | \(Q_\phi|g_{\mu R}|^2\) | \(Q_\psi|g_{\mu R}|^2\) | \(Q_\psi \text{Re}(g_{\mu R}^* g_{\mu L} \delta_{RL}^{\mu\mu})\) | \(Q_\psi \text{Re}(g_{\mu R}^* g_{\mu L} \delta_{RL}^{\mu\mu})\) |
|           | \(115 \pm 31\) | \(-115 \pm 31\) | \((-12 \pm 3) \times 10^{-3}\) | \((-12 \pm 3) \times 10^{-3}\) |
| \(\Delta a_\tau\) | \(Q_\phi|g_{\tau R}|^2\) | \(Q_\psi|g_{\tau R}|^2\) | \(Q_\psi \text{Re}(g_{\tau R}^* g_{\tau L} \delta_{RL}^{\tau\tau})\) | \(Q_\psi \text{Re}(g_{\tau R}^* g_{\tau L} \delta_{RL}^{\tau\tau})\) |
|           | \((-7 \sim 2) \times 10^6\) | \((-3 \sim 13) \times 10^3\) | \((-13 \sim 3) \times 10^3\) | |
| \(d_e, d_\mu, d_\tau\) | \(|Q_\phi \text{Im}(g_{eR}^* g_{eL} \delta_{RL}^{e\mu})|\) & \(5.3 \times 10^{-10}\) & \(5.3 \times 10^{-10}\) & \(9.1 (76.5)\) & \(9.1 (76.5)\) |
| \(\mu^+ \rightarrow e^+\gamma\) | \(|Q_\phi g_{\mu R} g_{R^* R} \delta_{RR}^{\mu\mu}\) | \(|Q_\phi g_{\mu R} g_{R^* R} \delta_{RL}^{\mu\mu}\) | \(|Q_\phi g_{\mu R} g_{R^* R} \delta_{RL}^{\mu\mu}\) | \(|Q_\phi g_{\mu R} g_{R^* R} \delta_{RL}^{\mu\mu}\) |
| \(\mu^+ \rightarrow e^- e^-\) | \(0.04 [0.0014]\) & \(0.005 [0.0020]\) & \(23 [9] \times 10^{-8}\) & \(23 [9] \times 10^{-8}\) |
| \(\mu^- \rightarrow e^- e^-\) | \(0.077 [0.0008]\) & \(0.085 [0.0008]\) & \(448 [4] \times 10^{-8}\) & \(448 [4] \times 10^{-8}\) |
| \(\mu^- Au \rightarrow e^- Au\) | \(0.028 [0.0003]\) & \(0.074 [0.0009]\) & \(471 [6] \times 10^{-8}\) & \(471 [6] \times 10^{-8}\) |
| \(\mu^- Ti \rightarrow e^- Ti\) | \(0.072 [0.0001]\) & \(0.219 [0.0003]\) & \(1137 [2] \times 10^{-8}\) & \(1137 [2] \times 10^{-8}\) |
| \(\mu^- Al \rightarrow e^- Al\) | \([0.0001]\) & \([0.0004]\) & \([2 \times 10^{-8}]\) & \([2 \times 10^{-8}]\) |
| \(\mu^+ \rightarrow e^- e^-\) | \(|g_{eR}^* g_{R^* R} T_{3 \phi} \delta_{RR}^{\mu\mu}\) | \(|g_{eR}^* g_{R^* R} \delta_{RR}^{\mu\mu}\) | \(|g_{eR}^* g_{R^* R} \delta_{RR}^{\mu\mu}\) | \(|g_{eR}^* g_{R^* R} \delta_{RR}^{\mu\mu}\) |
| \(\mu^- \rightarrow e^- e^-\) | \(118 [1] \times 10^{-5}\) & \(0.04 [0.04]\) & \(4 [4] \times 10^{-4}\) & \(3 [6] \times 10^{-4}\) |
| \(\mu^- Au \rightarrow e^- Au\) | \(148 [2] \times 10^{-6}\) | \(148 [2] \times 10^{-6}\) | \(148 [2] \times 10^{-6}\) | \(148 [2] \times 10^{-6}\) |
| \(\mu^- Ti \rightarrow e^- Ti\) | \(5155 [8] \times 10^{-7}\) | \(5155 [8] \times 10^{-7}\) | \(5155 [8] \times 10^{-7}\) | \(5155 [8] \times 10^{-7}\) |
| \(\mu^- Al \rightarrow e^- Al\) | \([1 \times 10^{-6}]\) | \([1 \times 10^{-6}]\) | \([1 \times 10^{-6}]\) | \([1 \times 10^{-6}]\) |
| \(\tau^- \rightarrow e^-\gamma\) | \(|Q_\phi g_{R^* R} g_{eR} \delta_{RL}^{e\mu}\) | \(|Q_\psi g_{R^* R} g_{eR} \delta_{RL}^{e\mu}\) | \(|Q_\psi g_{R^* R} g_{eR} \delta_{RL}^{e\mu}\) | \(|Q_\psi g_{R^* R} g_{eR} \delta_{RL}^{e\mu}\) |
| \(\tau^- \rightarrow e^- e^-\) | \(2.4 [0.7]\) & \(3.6 [1.1]\) & \(26 [8] \times 10^{-4}\) & \(26 [8] \times 10^{-4}\) |
| \(\tau^- \rightarrow e^- e^-\) | \(22.2 [2.8]\) & \(27.3 [3.5]\) & \(22 [3] \times 10^{-3}\) & \(22 [3] \times 10^{-3}\) |
| \(\tau^- \rightarrow e^- e^-\) | \(0.46 [0.06]\) & \(17.2 [17.2]\) & \(22 [2.2]\) & \(22 [2.2]\) |
| \(\tau^- \rightarrow e^- e^-\) | \(12.2 [24.3]\) & \(12.2 [24.3]\) & \(12.2 [24.3]\) & \(12.2 [24.3]\) |
| \(\tau^- \rightarrow \mu^-\gamma\) | \(|Q_\phi g_{R^* R} g_{eR} \delta_{RL}^{e\mu}\) | \(|Q_\psi g_{R^* R} g_{eR} \delta_{RL}^{e\mu}\) | \(|Q_\psi g_{R^* R} g_{eR} \delta_{RL}^{e\mu}\) | \(|Q_\psi g_{R^* R} g_{eR} \delta_{RL}^{e\mu}\) |
| \(\tau^- \rightarrow \mu^-\mu^+\) | \(2.8 [0.4]\) & \(4.2 [0.6]\) & \(30 [4] \times 10^{-4}\) & \(30 [4] \times 10^{-4}\) |
| \(\tau^- \rightarrow \mu^-\mu^+\) | \(19.5 [2.4]\) & \(24.1 [3.0]\) & \(20 [2] \times 10^{-3}\) & \(20 [2] \times 10^{-3}\) |
| \(\tau^- \rightarrow \mu^-\mu^+\) | \(0.41 [0.05]\) & \(15.2 [15.2]\) & \(10.7 [21.4]\) & \(10.7 [21.4]\) |
| \(\tau^- \rightarrow \mu^-\mu^+\) | \(32.0 [16.0]\) & \(15.1 [22.7]\) & \(21.4 [24.1]\) & \(21.4 [24.1]\) |
| \(\tau^- \rightarrow \mu^-\mu^+\) | \(34.1 [17.1]\) & \(16.1 [24.1]\) & \(22.7 [22.7]\) & \(22.7 [22.7]\) |

Dirac and Majorana fermion are given, where results in \(\{\}\) are for the Majorana case. Results for \(x = 0.5\) and 2 are given in Tables IX and X respectively.
TABLE IX: Same as Table VIII but with $x \equiv m_\phi/m_\psi = 0.5$.

| Processes | constraints | constraints | constraints | constraints |
|-----------|-------------|-------------|-------------|-------------|
| $\Delta a_e$ | $Q_\psi |g_eR|^2$ | $Q_\psi |g_eR|^2$ | $Q_\psi \text{Re}(g_{\psi R}^* g_{eL} \delta_{eL}^{\mu \mu})$ | $Q_\psi \text{Re}(g_{\psi R}^* g_{eL} \delta_{eL}^{\mu \mu})$ |
|           | $-812 \pm 332$ | $1059 \mp 433$ | $(8 \mp 3) \times 10^{-4}$ | $(-13 \pm 6) \times 10^{-4}$ |
| $\Delta a_\mu$ | $Q_\psi |g_\mu R|^2$ | $Q_\psi |g_\mu R|^2$ | $Q_\psi \text{Re}(g_{\psi R}^* g_{\mu L} \delta_{\mu \mu}^{\mu \mu})$ | $Q_\psi \text{Re}(g_{\psi R}^* g_{\mu L} \delta_{\mu \mu}^{\mu \mu})$ |
|           | $58 \pm 16$ | $-76 \pm 20$ | $(-1.2 \mp 0.3) \times 10^{-2}$ | $(-20 \pm 5) \times 10^{-3}$ |
| $\Delta a_\tau$ | $Q_\psi |g_\tau R|^2$ | $Q_\psi |g_\tau R|^2$ | $Q_\psi \text{Re}(g_{\psi R}^* g_{\tau L} \delta_{\tau \tau}^{\mu \mu})$ | $Q_\psi \text{Re}(g_{\psi R}^* g_{\tau L} \delta_{\tau \tau}^{\mu \mu})$ |
|           | $(-4 \sim 1) \times 10^6$ | $(-1 \sim 5) \times 10^6$ | $(-3 \sim 13) \times 10^3$ | $(-2 \sim 5) \times 10^3$ |

| $d_e$, $d_\mu$, $d_\tau$ | $|Q_\psi \text{Im}(g_{\psi R}^* g_{eL} \delta_{eL}^{\mu \mu})|$ | $|Q_\psi \text{Im}(g_{\psi R}^* g_{eL} \delta_{eL}^{\mu \mu})|$ | $|Q_\psi \text{Im}(g_{\psi R}^* g_{eL} \delta_{eL}^{\mu \mu})|$ | $|Q_\psi \text{Im}(g_{\psi R}^* g_{eL} \delta_{eL}^{\mu \mu})|$ |
| $\mu^+ \rightarrow e^+ \gamma$ | $|Q_\psi g^*_{\mu R} g^*_{eL} T_{\psi \phi} \delta_{eL}^{\mu \mu} |$ | $|Q_\psi g^*_{\mu R} g^*_{eL} T_{\psi \phi} \delta_{eL}^{\mu \mu} |$ | $|Q_\psi g^*_{\mu R} g^*_{eL} T_{\psi \phi} \delta_{eL}^{\mu \mu} |$ | $|Q_\psi g^*_{\mu R} g^*_{eL} T_{\psi \phi} \delta_{eL}^{\mu \mu} |$ |
| $\mu^+ \rightarrow e^- e^+$ | $142 \{1 \times 10^{-5}$ | $0.04 \{0.01\}$ | $22 \{8\} \times 10^{-8}$ | $4 \{1\} \times 10^{-7}$ |
| $\mu^- \rightarrow \mu^- \mu^+$ | $178 \{2\} \times 10^{-6}$ | $0.015 \{0.0002\}$ | $449 \{5\} \times 10^{-8}$ | $777 \{9\} \times 10^{-8}$ |
| $\mu^- \rightarrow \tau^+ \tau^-$ | $6226 \{9\} \times 10^{-7}$ | $0.040 \{0.0006\}$ | $1084 \{2\} \times 10^{-8}$ | $1875 \{3\} \times 10^{-8}$ |
| $\mu^- \rightarrow \mu^- \mu^+$ | $1 \{6\} \times 10^{-6}$ | $0.00001$ | $2 \{0\} \times 10^{-8}$ | $4 \{0\} \times 10^{-8}$ |

| $\tau^- \rightarrow \tau^- \gamma$ | $|Q_\psi g^*_{\tau R} g^*_{eL} \delta_{eL}^{\mu \mu} |$ | $|Q_\psi g^*_{\tau R} g^*_{eL} \delta_{eL}^{\mu \mu} |$ | $|Q_\psi g^*_{\tau R} g^*_{eL} \delta_{eL}^{\mu \mu} |$ | $|Q_\psi g^*_{\tau R} g^*_{eL} \delta_{eL}^{\mu \mu} |$ |
| $\tau^- \rightarrow \mu^- \gamma$ | $1.9 \{0.6\}$ | $4.7 \{1.4\}$ | $24 \{7\} \times 10^{-4}$ | $42 \{13\} \times 10^{-4}$ |
| $\tau^- \rightarrow \mu^- \mu^+$ | $18.0 \{2.3\}$ | $36.8 \{4.6\}$ | $21 \{3\} \times 10^{-3}$ | $37 \{5\} \times 10^{-3}$ |

| $\tau^- \rightarrow \mu^- \mu^+$ | $5.06 \{0.07\}$ | $16.4 \{4.5\} \{2.1\} \{0.6\}$ | $5.0 \{6.4\} \{0.6\} \{0.8\}$ | $5.0 \{6.4\} \{0.6\} \{0.8\}$ |

| $\tau^- \rightarrow \mu^- \mu^+$ | $2.2 \{0.3\}$ | $5.4 \{0.8\}$ | $28 \{4\} \times 10^{-4}$ | $49 \{7\} \times 10^{-4}$ |
| $\tau^- \rightarrow \mu^- \mu^+$ | $15.8 \{2.0\}$ | $32.4 \{4.1\}$ | $18 \{2\} \times 10^{-3}$ | $33 \{4\} \times 10^{-3}$ |

| $\tau^- \rightarrow \mu^- \mu^+$ | $0.49 \{0.06\}$ | $14.4 \{4.0\} \{1.8\} \{0.5\}$ | $4.4 \{5.6\} \{0.6\} \{0.7\}$ | $4.4 \{5.6\} \{0.6\} \{0.7\}$ |

| $\tau^- \rightarrow \mu^- \mu^+$ | $41.9 \{6.1\} \{6.3\} \{0.8\}$ | $7.5 \{8.6\} \{1.0\} \{1.1\}$ | $10.7 \{10.7\} \{1.3\} \{1.3\}$ | $59.3 \{29.6\} \{7.5\} \{3.7\}$ |

| $\tau^- \rightarrow \mu^- \mu^+$ | $44.6 \{6.5\} \{5.6\} \{0.8\}$ | $8.0 \{9.2\} \{1.0\} \{1.2\}$ | $11.3 \{11.3\} \{1.4\} \{1.4\}$ | $63.1 \{31.5\} \{7.9\} \{4.0\}$ |

In Fig. 1 the allowed parameter space for (a) $\pm Q_\phi \psi \text{Re}(g_{\psi R}^* g_{eL} \delta_{eL}^{\mu \mu})$ and (b) $\mp Q_\phi \psi \text{Re}(g_{\psi R}^* g_{\mu L} \delta_{\mu \mu}^{\mu \mu})$ constrained by $\Delta a_\mu$ and $d_e$, respectively, and (b) $\mp Q_\phi \psi \text{Re}(g_{\psi R}^* g_{\mu L} \delta_{\mu \mu}^{\mu \mu})$ and (b) $\mp Q_\phi \psi \text{Re}(g_{\psi R}^* g_{\mu L} \delta_{\mu \mu}^{\mu \mu})$.  

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TABLE X: Same as Table VIII but with $x \equiv m_\phi/m_\psi = 2$.

| Processes       | $|Q_{\phi}^\star g_{\tau R}^\star g_{\tau R}^\star \delta^{\text{ee}}_{\tau R}|$ | $|Q_{\phi}^\star g_{\tau R}^\star g_{\tau R}^\star \delta^{\text{ee}}_{\tau L}|$ | $|Q_{\phi}^\star g_{\tau R}^\star g_{\tau R}^\star \delta^{\text{ee}}_{\tau R}|$ | $|Q_{\phi}^\star g_{\tau R}^\star g_{\tau R}^\star \delta^{\text{ee}}_{\tau L}|$ |
|-----------------|--------------------------|--------------------------|--------------------------|--------------------------|
| $\Delta a_e$    | $Q_{\phi}^\star g_{\mu R}^\star g_{\mu R}^\star \delta^{\mu e}_{\mu R}$ | $Q_{\phi}^\star g_{\mu R}^\star g_{\mu R}^\star \delta^{\mu e}_{\mu R}$ | $Q_{\phi}^\star g_{\mu R}^\star g_{\mu R}^\star \delta^{\mu e}_{\mu R}$ | $Q_{\phi}^\star g_{\mu R}^\star g_{\mu R}^\star \delta^{\mu e}_{\mu R}$ |
| $\Delta a_\mu$  | $305 \pm 82$            | $-234 \pm 63$           | $(13 \pm 6) \times 10^{-4}$ | $(12 \pm 3) \times 10^{-3}$ |
| $\Delta a_\tau$ | $(21 \sim 5) \times 10^6$ | $(-4 \sim 16) \times 10^6$ | $(-6 \sim 23) \times 10^{-3}$ | $(-13 \sim 3) \times 10^3$ |
| $d_e$, $d_\mu$, $d_\tau$ | $7.8 \times 10^{-10}$ | $5.0 \times 10^{-10}$ | $15.0 (126.2)$ | $8.7 (73.0)$ |
| $\mu^+ \rightarrow e^+ \gamma$ | $0.07 \{0.0027\}$ | $0.07 \{0.0027\}$ | $38 \{14\} \times 10^{-8}$ | $22 \{8\} \times 10^{-8}$ |
| $\mu^+ \rightarrow e^- e^-$       | $0.152 \{0.0015\}$ | $0.103 \{0.0010\}$ | $739 \{7\} \times 10^{-8}$ | $427 \{4\} \times 10^{-8}$ |
| $\mu^- \rightarrow e^- e^-$       | $0.069 \{0.0008\}$ | $0.064 \{0.0008\}$ | $777 \{9\} \times 10^{-8}$ | $449 \{5\} \times 10^{-8}$ |
| $\mu^- \rightarrow e^- e^-$       | $0.177 \{0.0003\}$ | $0.183 \{0.0003\}$ | $1875 \{3\} \times 10^{-8}$ | $1084 \{2\} \times 10^{-8}$ |

constrained by $\Delta a_\mu$ and $d_\mu$, respectively, are shown. These constrains are obtained using $m_\psi = 500$ GeV. For other $m_\psi$, apply a $(500\text{GeV})/m_\psi$ factor to the plots.
FIG. 7: Allowed parameter space for (a) $\pm Q_{\phi,\psi}\Re(g_{eRg_{eL}}^{ee}\delta_{eL}^{ee})$ and $Q_{\phi,\psi}\Im(g_{eRg_{eL}}^{ee}\delta_{eL}^{ee})$ constrained by $\Delta a_\mu$ and $d_e$, respectively, and (b) $\mp Q_{\phi,\psi}\Re(g_{\mu Rg_{\mu L}}^{\mu\mu}\delta_{RL}^{\mu\mu})$ and $Q_{\phi,\psi}\Im(g_{\mu Rg_{\mu L}}^{\mu\mu}\delta_{RL}^{\mu\mu})$ constrained by $\Delta a_\mu$ and $d_\mu$, respectively. These constraints are obtained using $m_\psi = 500$ GeV, for other $m_\phi$, apply $(100\text{GeV})/m_\psi$ to the plots.

In Figs. 8 and 9 we show the parameter space constrained by using various experimental bounds or expected sensitivities on $\mu \to e$, $\tau \to e$ and $\tau \to \mu$ lepton flavor violating processes. Contributions from photonic penguin, Z-penguin and box diagrams are considered. In Fig. 10 the parameter space constrained by using various bounds or expected experimental sensitivities on $\tau^- \to e^-\mu^+e^-\mu^-$ processes through contributions from box contributions are shown.

There are several messages we can extracted from these results. First we note that, comparing to case I, the built-in cancelation has more prominent effects in penguin amplitudes than in box amplitudes. Furthermore, the cancelation affects small-$x (x = m_\phi/m_\psi)$ region more effectively. We can see this clearly in the above figures by noting that the curves corresponding to penguin contributions bend upward in the small-$x$ region, hence, relaxing the constraints.

Similar to case I, we note that chiral interactions ($g_L \times g_R = 0$) are unable to generate large enough contributions to $\Delta a_e$ and $\Delta a_\mu$ to accommodate the experimental results, Eqs. (1) and (2). This can be seen in Tables IV, IX and X where $Q_{\phi,\psi}|g_{e,R(L)}|^2$ and $Q_{\phi,\psi}|g_{\mu,R(L)}|^2$ need to be unreasonably and unacceptably large to produce the experimental values of $\Delta a_e$ and $\Delta a_\mu$.

Again similar to case I, we find that although non-chiral interactions are capable to generate $\Delta a_e$ and $\Delta a_\mu$ successfully accommodating the experimental results, they are contributed from different sources. From Tables VIII, IX and X we see that $Q_{\phi,\psi}\Re(g_{eRg_{eL}}^{ee}\delta_{eL}^{ee})$ and $Q_{\phi,\psi}\Re(g_{\mu Rg_{\mu L}}^{\mu\mu}\delta_{RL}^{\mu\mu})$ of orders $10^{-3}$ and $10^{-2}$ or larger, are able to produce the experimental values of $\Delta a_e$ and $\Delta a_\mu$. As $\Delta a_e$ is generated from $Q_{\phi,\psi}\Re(g_{eRg_{eL}}^{ee}\delta_{eL}^{ee})$, while $\Delta a_\mu$ is generated from $Q_{\phi,\psi}\Re(g_{\mu Rg_{\mu L}}^{\mu\mu}\delta_{RL}^{\mu\mu})$, the contributions are not from the same source (meaning the same $\psi$ and $\phi$). We also note that these values are larger than $Q_{\phi,\psi}\Re(g_{eRg_{eL}}^{ee})$ and $Q_{\phi,\psi}\Re(g_{\mu Rg_{\mu L}}^{\mu\mu})$ in case I by roughly one order of magnitude. This is reasonable as we have cancellation in this case. Furthermore, comparing Figs. 2(b), (d) and Fig. 7(a) and (b), we can clearly see the relaxation in the small-$x$ region.

The upper limit in $\mu \to e\gamma$ decay gives the most severe constraints on photonic penguin contributions in $\mu \to e$ transitions, but the constraints on parameters are relaxed, especially in the small-$x$ region, comparing to case I. From Tables VIII, IX and X and Fig. 8(a) to (d), we see that the
FIG. 8: Same as Fig. 3 but for case II.

bounds on $|Q_{\psi,\psi} g_{\mu R}^* g_{e R} \delta_{\mu e}^{\mu e}|$ and $|Q_{\phi,\psi} g_{\mu R}^* g_{e L} \delta_{\mu e}^{\mu e}|$ are severely constrained by the $\mu \to e\gamma$ upper
FIG. 9: Same as Fig. 4 but for case II.

limit. Indeed, the $\mu \to e\gamma$ bound is more severe than the $\mu \to 3e$ and $\mu N \to eN$ bounds. The
situation is altered when considering future experimental searches. From the tables and the figures,
we see that, on the contrary, the \( \mu \to 3e \) and \( \mu N \to eN \) processes can probe the photonic penguin contributions from \( |Q_{\phi,\psi}g_{\mu R}g_{eR}\delta_{RR}^{\mu e}| \) and \( |Q_{\phi,\psi}g_{\mu R}g_{eL}\delta_{RL}^{\mu e}| \) better than the \( \mu \to e\gamma \) decay in near future experiments.

Similar to case I, the \( Z \)-penguin diagrams can constrain chiral interaction better than photonic penguin diagrams in \( \mu \to e \) transitions. From Tables VII, IX, X, Fig. 8(a), (b), and (e) we see that the bounds on \( |g_{\mu R}g_{eR}\Delta T_{3\phi}\delta_{RR}^{\mu e}| \) from \( Z \)-penguin contributions are more severe (by one to two orders of magnitude) than the bounds on \( |Q_{\phi,\psi}g_{\mu R}g_{eR}\delta_{RR}^{\mu e}| \) from photonic penguin contributions. In addition, from Fig. 8(e) we see that the upper limits of \( \mu N \to eN \) transitions give better bounds

FIG. 11: Same as Fig. 6 but for case II.
on $|g_{\mu R}^*g_{e R}\Delta T_{3\psi}\delta_{RR}^e|$ than the $\mu \rightarrow 3e$ bound.

For $x$ larger than 0.2, box contributions to $\mu \rightarrow 3e$ decay are subleading comparing to $Z$ penguin contributions, but the former can be important for $x \lesssim 0.2$. In Fig. 8(f) and (g) we show the bounds on $|g_{\mu R}^*g_{e R}\Delta T_{3\psi}\delta_{RR}^e g_{e R}^e R|$ and $|g_{\mu R}^*g_{e R}\Delta T_{3\psi}\delta_{RR}^e g_{e L} g_{e L}|$ obtained by considering box contributions to $\mu \rightarrow 3e$ decay. Note that the constraint on $|g_{\mu R}^*g_{e R}\Delta T_{3\psi}\delta_{RR}^e g_{e R}^e R||g_{e R(L)}^e g_{e R(L)}|$ obtained from $\mu Au \rightarrow e Au$ upper limit and perturbativity is much severe than the $|g_{\mu R}^*\Delta T_{3\psi}\delta_{RR}^e g_{e R}^e R||g_{e R(L)}^e g_{e R(L)}|$ bound. However for $x$ smaller than 0.2, box contributions can be important. This is different from case I, as penguin contributions have larger cancellation in the small-$x$ region in the present case and, as a result, box contributions become relatively important in this region.

From Tables VII, IX and X, we see that similar to case I the present bound on $\Delta a_\tau$ cannot constrain $Q_{\phi,\psi}|g_{e R(L)}^e|^2$ and $Q_{\phi,\psi}\text{Re}(g_{e R(L)}^e)^2$ well. Even the bound on $d_\tau$ cannot give good constraints on $Q_{\phi,\psi}\text{Im}(g_{e R(L)}^e)^2$.

In $\tau \rightarrow e (\mu)$ transitions, the $\tau \rightarrow e\gamma (\mu\gamma)$ upper limit constrains photonic penguin contributions better than the $\tau \rightarrow 3e (3\mu)$ upper limit, and the $Z$-penguin constrains chiral interaction better than the photonic penguin. From Tables VII, IX, X Fig. 9(a) to (d) and Fig. 10(a) to (d), we see that bounds on $|Q_{\phi,\psi}\Delta T_{3\psi}\delta_{RR}^e g_{e R}^e R|^2$ and $|Q_{\phi,\psi}\Delta T_{3\psi}\delta_{RL}^e g_{e R(L)}^e R|^2$ are constrained by the $\tau \rightarrow e\gamma (\mu\gamma)$ data more severely than by the $\tau \rightarrow 3e (3\mu)$ upper limit. Note that even the bounds using the proposed sensitivities on $\tau \rightarrow 3e$ and $\tau \rightarrow 3\mu$ decays in Belle II are superseded by the bounds using the present limits of $\tau \rightarrow e\gamma$ and $\tau \rightarrow \mu\gamma$ decays in most of the parameter space. From Tables VII, IX, X Fig. 9(e) and Fig. 10(e), we see that bounds on $|Q_{\phi,\psi}\Delta T_{3\psi\tau}\delta_{RR}^e (\mu\gamma)|$ from photonic penguin contributions are more severe (by one order of magnitude) than those on $|Q_{\phi,\psi}\Delta T_{3\psi\tau}\delta_{RR}^e (\mu\gamma)|$ from photonic penguin contributions. Hence, $Z$-penguin constrains chiral interaction better than photonic penguin. These features are similar to case I, but comparing Fig. 4, 5, 9 and 10 we can clearly see that the bounds are significant relaxed in the small-$x$ region in the present case.

Box contributions to $\tau \rightarrow 3e$ and $\tau \rightarrow 3\mu$ decays can sometime be comparable to $Z$-penguin contributions. We show in Fig. 9(g), (h) and Fig. 10(g), (h) the bounds on $|g_{\mu R}^*g_{e R}^e R\Delta T_{3\psi}\delta_{RR}^e g_{e R}^e R|^2$ and $|g_{\mu R}^*g_{e R}^e R\Delta T_{3\psi}\delta_{RL}^e g_{e R(L)}^e R|^2$ obtained by considering box contributions to $\tau \rightarrow 3e (3\mu)$ decay. Note that the constraint on $|g_{\mu R}^*g_{e R}^e R\Delta T_{3\psi}\delta_{RR}^e g_{e R(L)}^e R|$ obtained from $Z$-penguin contributions to $\tau \rightarrow 3e (3\mu)$ decay and perturbativity is much severe than the $|g_{\mu R}^*g_{e R}^e R\Delta T_{3\psi}\delta_{RR}^e g_{e R(L)}^e R|$ bound for $x \gtrsim 0.6$, but it is the other way around for $x \lesssim 0.6$. One can also obtain these results using the values in Tables VII, IX, X. These results imply that box contributions to $\tau \rightarrow 3e, 3\mu$ can sometime be comparable to $Z$-penguin contributions. This is similar to case I, but in different region of $x$.

The $\tau^- \rightarrow e^-\mu^+e^-$ rate is constrained by $\tau \rightarrow e\gamma$ and $\mu \rightarrow e\gamma$ upper limits. The bounds on $|g_{\mu R}^*g_{e R}^e R\Delta T_{3\psi}\delta_{RR}^e g_{e R}^e R|^2$, $|g_{\mu R}^*g_{e R}^e R\Delta T_{3\psi}\delta_{RL}^e g_{e R(L)}^e R|^2$, and $|g_{\mu R}^*g_{e R}^e R\Delta T_{3\psi}\delta_{RR}^e g_{e L} g_{e L}|$ obtained from constraining box contributions using the upper limit of the $\tau^- \rightarrow e^-\mu^+e^-$ rate are shown in Fig. II (a), (c), (e) and Tables VII, IX, X. They are larger than the bounds on $|Q_{\phi,\psi}\Delta T_{3\psi}\delta_{RR}^e (\mu\gamma)|$, $|Q_{\phi,\psi}\Delta T_{3\psi}\delta_{RL}^e (\mu\gamma)|$, $|Q_{\phi,\psi}\Delta T_{3\psi}\delta_{RR}^e (\mu\gamma)|$ and $|Q_{\phi,\psi}\Delta T_{3\psi}\delta_{RL}^e (\mu\gamma)|$ obtained by using the upper limits of $\tau \rightarrow e\gamma$ and $\mu \rightarrow e\gamma$ rates. Note that for $x \gtrsim 0.2$ even the proposed sensitivity on $\tau^- \rightarrow e^-\mu^+e^-$ rate is constrained. Hence, the $\tau^- \rightarrow e^-\mu^+e^-$ rate is constrained by the present $\tau \rightarrow e\gamma$ and $\mu \rightarrow e\gamma$ upper limits. This is
we see that the order of magnitude from their present upper limits, while the $\mu$ is highly constrained. This is similar to case I, but the constraints obtained using $\tau$ upper limit as input, the $\tau$ and $\tau$ and $\tau$ and $\tau$ upper limit are relatively relaxed. Similarly the $\tau \to e^-e^+\mu^-$ rate is constrained by $\tau \to \mu\gamma$ and $\mu \to e\gamma$ upper limits. From Fig. (b), (d), (f) and Tables VII, VIII, IX, X, we see that the bounds on $|g^*_{\tau R}g_{\mu R}^\delta_{RR}g^*_{\epsilon R}\epsilon_{RL}|$ and $|g^*_{\tau R}g_{\mu R}^\delta_{RR}g^*_{\epsilon R}\epsilon_{RL}|$ obtained from the upper limit of the $\tau \to e^-e^+\mu^-$ rate are larger than the bounds on $|Q_{\phi,\psi}g^*_{\tau R}g_{\mu R}^\delta_{RR}||Q_{\phi,\psi}g^*_{\epsilon R}\epsilon_{RL}|$ and $|Q_{\phi,\psi}g^*_{\tau R}g_{\mu R}^\delta_{RR}||Q_{\phi,\psi}g^*_{\epsilon R}\epsilon_{RL}|$ obtained from the upper limits of $\tau \to \mu\gamma$ and $\mu \to e\gamma$ rates. Hence, the $\tau \to e^-e^+\mu^-$ rate is constrained by $\tau \to \mu\gamma$ and $\mu \to e\gamma$ upper limits. Note that for $x \gtrsim 0.2$ even the proposed sensitivity on $\tau \to e^-e^+\mu^-$ rate is highly constrained. This is similar to case I, but the constraints obtained using $\tau \to \mu\gamma$ and $\mu \to e\gamma$ upper limits are relatively relaxed.

In Table XI we compare the current upper limits, future sensitivities and bounds from consistency for case II on various muon and tau LFV processes. We see that the present $\mu \to e\gamma$ upper limit requires the bounds on $\mu \to 3e$ and $\mu Ti \to e Ti$ be lower by more than one order of magnitude from their present upper limits, while the $\mu Au \to e Au$ bound is close to its present limit and the $\mu Al \to e Al$ rate is predicted to be smaller than $3 \times 10^{-13}$. Comparing to case I we see that the $\mu \to 3e$, $\mu Au \to e Au$ and $\mu Al \to e Al$ bounds are relaxed, while the $\mu Ti \to e Ti$ bound is tighten. We find that the situation is similar when the present $\mu Au \to e Au$ upper limit instead of the present $\mu \to e\gamma$ upper limit is used as an input. Using the present $\tau \to e\gamma (\mu\gamma)$ upper limit as input, the $\tau \to 3e (3\mu)$ bound is smaller than its present upper limit by one order of magnitude. These bounds are relaxed compared to those in case I. Finally, the $\tau \to e^-e^+\mu^-$ and $\tau \to e^-e^+\mu^-$ bounds are similar to their present upper limits when the present $\tau \to e\gamma, e\gamma$ and $\mu \to e\gamma$ upper limits are used. These limits are significant relaxed compared to those in case I.

| TABLE XI: Same as Table [VII] but for case II. |
|-----------------------------------------------|
| current limit (future sensitivity) consistency bounds | remarks |
| $B(\mu^+ \to e^+\gamma)$ | $< 4.2 \times 10^{-13}$ ($6 \times 10^{-14}$) | $< 4.2 \times 10^{-13}$ from $\mu \to e\gamma$ bound input |
| $B(\mu^+ \to e^+e^-e^-)$ | $< 1.0 \times 10^{-12}$ ($10^{-16}$) | $< 2.2 \times 10^{-14}$ from $\mu \to e\gamma$ bound input |
| $B(\mu^- Ti \to e^- Ti)$ | $< 4.3 \times 10^{-12}$ ($10^{-17}$) | $< 5.2 \times 10^{-14}$ from $\mu \to e\gamma$ bound input |
| $B(\mu^- Au \to e^- Au)$ | $< 7.0 \times 10^{-13}$ ($10^{-16}$) | $< 6.2 \times 10^{-13}$ from $\mu \to e\gamma$ bound input |
| $B(\mu^- Al \to e^- Al)$ | $\cdots$ ($10^{-17}$) | $< 3.2 \times 10^{-13}$ from $\mu \to e\gamma$ bound input |
| $B(\tau^- \to e^-\gamma)$ | $< 3.3 \times 10^{-8}$ ($3 \times 10^{-9}$) | $< 3.3 \times 10^{-8}$ input |
| $B(\tau^- \to e^-e^+e^-)$ | $< 2.7 \times 10^{-8}$ ($4.3 \times 10^{-10}$) | $< 1.9 \times 10^{-9}$ from $\tau \to e\gamma$ bound input |
| $B(\tau^- \to \mu^-\gamma)$ | $< 4.4 \times 10^{-8}$ ($1 \times 10^{-9}$) | $< 4.4 \times 10^{-8}$ input |
| $B(\tau^- \to \mu^-\mu^+\mu^-)$ | $< 2.1 \times 10^{-8}$ ($3.3 \times 10^{-10}$) | $< 2.5 \times 10^{-9}$ from $\tau \to \mu\gamma$ bound input |
| $B(\tau^- \to \mu^-e^+\mu^-)$ | $< 1.7 \times 10^{-8}$ ($2.7 \times 10^{-10}$) | $< 1.3 \times 10^{-8}$ from $\tau \to \mu\gamma, \mu \to e\gamma$ bounds |
| $B(\tau^- \to e^-\mu^+e^-)$ | $< 1.5 \times 10^{-8}$ ($2.4 \times 10^{-10}$) | $< 1 \times 10^{-8}$ from $\tau \to e\gamma, \mu \to e\gamma$ bounds |

27
IV. CONCLUSION

We study anomalous magnetic moments and lepton flavor violating processes of e, \( \mu \) and \( \tau \) leptons in this work. We use a data driven approach to investigate the implications of the present data on the parameters of a class of models, which has spin-0 scalar and spin-1/2 fermion fields and can contribute to \( \Delta a_l \) and LFV processes. We compare two different cases, case I and case II, which does not have or has a built-in cancelation mechanism, respectively. Our findings are as following.

- Parameters are constrained using the present data of \( \Delta a_l \), \( d_l \) and lepton flavor violating processes of e, \( \mu \) and \( \tau \) leptons.
- The built-in cancelation has more prominent effects in penguin amplitudes than in box amplitudes. Furthermore, the cancelation affects amplitudes in small-x \((x \equiv m_\phi/m_\psi)\) region more effectively.
- Chiral interactions are unable to generate large enough \( \Delta a_e \) and \( \Delta a_\mu \) to accommodate the experimental results.
- Although \( \Delta a_e \) and \( \Delta a_\mu \) can be successfully generated to accommodate the experimental results by using non-chiral interactions, they are not contributed from the same source.
- Presently, the upper limit in \( \mu \to e\gamma \) decay gives the most severe constraints on photonic penguin contributions in \( \mu \to e \) transitions, but the situation may change in considering future experimental sensitivities. In fact, the future \( \mu \to 3e \) and \( \mu N \to eN \) experiments may probe the photonic penguin contributions better than the future \( \mu \to e\gamma \) experiment.
- The Z-penguin diagrams can constrain chiral interaction better than photonic penguin diagrams in \( \mu \to e \) transitions. In addition, \( \mu N \to eN \) transitions constrain Z-penguin contributions better \( \mu \to 3e \) decay.
- In case I, either in the Dirac or Majorana case, box contributions to \( \mu \to 3e \) decay are subleading. Furthermore, there are cancelation in box contributions in the Majorana fermionic case making the contributions even smaller. In case II, we find that for \( x \gtrsim 0.2 \), box contributions to \( \mu \to 3e \) decay are subleading comparing to Z penguin contributions, but they can be important for \( x \lesssim 0.2 \).
- The present bounds on \( \Delta a_\tau \) and \( d_\tau \) are unable to give useful constraints on parameters.
- In \( \tau \to e \) (\( \mu \)) transitions, the \( \tau \to e\gamma \) (\( \mu\gamma \)) upper limit constrains photonic penguin contributions better than the \( \tau \to 3e \) (\( 3\mu \)) upper limit, and Z-penguin constrains chiral interaction better than photonic penguin. Note that even the bounds using the proposed sensitivities on \( \tau \to 3e \) and \( \tau \to 3\mu \) decays by Belle II are superseded by the bounds using the present limits of \( \tau \to e\gamma \) and \( \tau \to \mu\gamma \) decays for most of the parameter space. Bounds are significant relaxed in small-x region in case II.
• Box contributions to $\tau \rightarrow 3e$ and $\tau \rightarrow 3\mu$ decays can sometime be comparable to $Z$-penguin contributions.

• The $\tau^- \rightarrow e^- \mu^+ e^-$ rate is highly constrained by $\tau \rightarrow e\gamma$ and $\mu \rightarrow e\gamma$ upper limits. Note that in case I even the proposed sensitivity on $\tau^- \rightarrow e^- \mu^- e^-$ rate is highly constrained, but in case II, for $x \lesssim 0.2$ the constraints are relaxed.

• The $\tau^- \rightarrow \mu^- e^+ \mu^-$ rate is also highly constrained by $\tau \rightarrow \mu\gamma$ and $\mu \rightarrow e\gamma$ upper limits. Note that in case I even the proposed sensitivity on $\tau^- \rightarrow \mu^- e^- \mu^-$ rate is highly constrained, but in case II, for $x \lesssim 0.2$ the constraints are relaxed.

• We compare the current experimental upper limits, future sensitivities and bounds from consistency on various muon and tau LFV processes:
   (a) In case I, the present $\mu \rightarrow e\gamma$ upper limit requires the bounds on $\mu \rightarrow 3e$, $\mu \text{Ti} \rightarrow e\text{Ti}$ and $\mu \text{Au} \rightarrow e\text{Au}$ be lower by two orders of magnitude, more than one order of magnitude and almost one order of magnitude, respectively, from their present upper limits, and the $\mu \text{Al} \rightarrow e\text{Al}$ rate is predicted to be smaller than $6\times10^{-14}$. In case II, the $\mu \rightarrow 3e$, $\mu \text{Au} \rightarrow e\text{Au}$ and $\mu \text{Al} \rightarrow e\text{Al}$ bounds are relaxed, while the $\mu \text{Ti} \rightarrow e\text{Ti}$ bound is tighten.
   (b) We find that the situation is similar but the bounds are slightly relaxed when the $\mu \text{Au} \rightarrow e\text{Au}$ upper limit instead of the present $\mu \rightarrow e\gamma$ upper limit is used as an input.
   (c) Using the present $\tau \rightarrow e\gamma$ ($\mu\gamma$) upper limit as input, the $\tau \rightarrow 3e$ ($3\mu$) bound is smaller than its present upper limit by one order of magnitude.
   (d) In case I, the $\tau^- \rightarrow \mu^- e^+ \mu^-$ and $\tau^- \rightarrow e^- \mu^+ e^-$ bounds are lower than their present upper limits by two orders of magnitude as required from the present $\tau \rightarrow \mu\gamma$, $e\gamma$ and $\mu \rightarrow e\gamma$ upper limits. These limits are lower than the proposed future sensitivities. In case II, the $\tau^- \rightarrow \mu^- e^+ \mu^-$ and $\tau^- \rightarrow e^- \mu^+ e^-$ bounds are similar to their present upper limits when the present $\tau \rightarrow \mu\gamma$, $e\gamma$ and $\mu \rightarrow e\gamma$ upper limits are used. These limits are significant relaxed compared to those in case I.

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Appendix A: Formulas for various processes

Formulas in this Appendix are taken from ref. [24] and are updated. In the weak bases of $\psi_{Lp}$, $\psi_{Rp}$, $\phi_{La}$ and $\phi_{Ra}$, the interacting Lagrangian is given by

$$\mathcal{L}_{\text{int}} = (g_{1L}^{Lp} \bar{\psi}_{Lp}^L \phi_{La}^* + g_{1R}^{Lp} \bar{\psi}_{Lp}^R \phi_{Ra}^*) + h.c., \quad (A1)$$
where $\phi_{L(R)}$ are scalar fields coupling to $l_{L(R)}$ and $p, a$ indicate weak quantum numbers. Fields in the weak bases can be transformed into those in the mass bases,

$$\phi_i = U_{L}^{L} \phi_{La} + U_{R}^{R} \phi_{Ra}, \quad \psi_{nL(R)} = V_{np}^{L(R)} \psi_{L(R)p},$$

(A2)

with the help of mixing matrices, $U$ and $V$. It is useful to define

$$g_{ni}^{L(R)} \equiv g_{np}^{L(R)} U_{ia}^{L(R)} U_{ia}^{L(R)}$$

(A3)

and, consequently, the interacting Lagrangian can be expressed as in Eq. (8).

The effective Lagrangian for various precesses is given by

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\nu l} + \mathcal{L}_{l' l'q} + \mathcal{L}_{l'qq}$$

(A4)

with $l^{(\nu)} = e, \mu, \tau$ denoting leptons and $q$ denoting quarks. For $l' \neq l$, we have

$$\mathcal{L}_{\nu l} = \bar{\nu}_L \sigma_{\mu\nu} l_R F^{\mu\nu} A_{L'R} + \bar{l}_R \sigma_{\mu\nu} l_L F^{\mu\nu} A_{RL} + \text{h.c.},$$

(A5)

and

$$A_{LR} = A_{R'L}, \quad A_{RL} = A_{L'R}.$$  

(A6)

while for $l' = l$, the additional hermitian conjugated terms in Eq. (A5) are not required. These $A$s are from the so-called dipole photonic penguin. The relevant effective Lagrangians responsible for $\bar{l} \to \bar{l}' l'$ decays and $l' \to l$ conversion processes are given by [18]

$$\mathcal{L}_{l' l'q} = \sum_q \left[ g_{l l q} \bar{\nu}_L \gamma_{\mu} l_R + g_{R l q} \gamma_{\mu} l_L \bar{l}_R \right] h.c.,$$

(A7)

where

$$g_{MNOP} \equiv e^2 Q_i g_{M' M}^{\gamma} \delta_{MN} \delta_{OP} \delta_{ll'} + g_{M' M}^{Z} g_{O Q}^{Z} \delta_{MN} \delta_{OP} \delta_{ll'} + g_{M' M}^{B},$$

$$g_{M' M}^{Z}(q) = e^2 Q_i g_{M' M}^{\gamma} + \frac{1}{2} g_{M' M}^{Z} (g_{l l}^{Z} + g_{l l}^{Z}),$$

$$g_{X}^{Z} \equiv \frac{e}{\sin \theta_W \cos \theta_W} (T_3 - \sin^2 \theta_W Q) X,$$

(A9)

with $M, N, O, P = L, R$, $g_{M' M}^{\gamma}$ from the non-dipole photonic penguin, $g_{M' M}^{Z}$ from the Z-penguin, $g_{M' M}^{B}$ from the box diagrams and $X = l_L, l_R, q_L, q_R$ and so on.

Using Eq. (8), the Wilson coefficients for $\mathcal{L}_{\nu l}$ in Eq. (A5) can be calculated to be [21]

$$A_{M' N}^{\gamma} = \frac{e}{32 \pi^2} \left[ (m_{l} g_{M' M}^{n s} n_{l} + m_{\psi} g_{M' M}^{n s} n_{\psi}) Q_{\phi_{i}} F_{1}(m_{\phi_{i}}^{2}, m_{\phi_{i}}^{2}) - Q_{\psi} F_{1}(m_{\phi_{i}}^{2}, m_{\phi_{i}}^{2}) + m_{\psi} g_{M' M}^{n s} n_{\psi} Q_{\phi_{i}} F_{3}(m_{\phi_{i}}^{2}, m_{\phi_{i}}^{2}) - Q_{\psi} F_{2}(m_{\phi_{i}}^{2}, m_{\phi_{i}}^{2}) \right],$$

(A10)

for $M$ different from $N$, and $F_i$ are loop functions with the explicit forms to be given below. The Wilson coefficients for $\mathcal{L}_{l' l'q}$ and $\mathcal{L}_{l'qq}$ in Eq. (A9) are given by

$$g_{l' l' q}^{\gamma} = \frac{1}{16 \pi^2} \left[ g_{l' l' q}^{n s} n_{l} Q_{\phi_{i}} G_{2}(m_{\phi_{i}}^{2}, m_{\phi_{i}}^{2}) + Q_{\phi_{i}} G_{1}(m_{\phi_{i}}^{2}, m_{\phi_{i}}^{2}) \right].$$

(A10)
TABLE XII: The overlap integrate parameters and total capture rates $\omega_{\text{capt}}$ taken from [28, 29] are collected.

|        | $D(m_\mu^{5/2})$ | $V^p(m_\mu^{5/2})$ | $V^n(m_\mu^{5/2})$ | $\omega_{\text{capt}}(10^6 s^{-1})$ |
|--------|------------------|------------------|------------------|-------------------------------|
| $^{77}_{13}$Al | 0.0362           | 0.0161           | 0.0173           | 0.7054                        |
| $^{48}_{22}$Ti  | 0.0864           | 0.0396           | 0.0468           | 2.59                          |
| $^{197}_{79}$Au | 0.189            | 0.0974           | 0.146            | 13.07                         |
| $^{205}_{81}$Tl | 0.161            | 0.0834           | 0.128            | 13.90                         |

\[
g_{RR}^Z = +m_{\psi_n}(m_{\nu_R}g_{\nu_L}^{n\nu_L}g_{R}) + m_{\nu_R}g_{\nu_L}^{ni\nu_L}\{Q_{\psi_n}G_3(m_{\nu_n}, m_{\nu_n}^2) + Q_{\psi_l}G_3(m_{\psi_n}, m_{\psi_n}^2)\}
\]

\[
g_{RR}^B = \frac{1}{16\pi^2m_{\bar{Z}}^2 \sin 2\theta_W^Z} \left\{\frac{3}{2}g_{\nu_R}^{ni\nu_R}g_{\nu_R}^{ni\nu_R}F_{Z}(m_{\nu_n}, m_{\nu_n}^2, m_{\psi_n}^2, m_{\psi_n}^2)\right\}
\]

\[
g_{RR}^B = 1\left[\frac{1}{16\pi^2} \left\{\frac{1}{4}G(m_{\nu_n}, m_{\nu_n}^2, m_{\psi_n}^2, m_{\psi_n}^2)\right\} + \frac{1}{16\pi^2} \right\}
\]

\[
\kappa_{L(R)}ijmn \equiv \sin 2\theta_W^Z (g_{\psi_R(L)}^Z \delta_{ij} \delta_{mn} - g_{\psi_R(L)}^Z \delta_{ij} \delta_{mn}^Z) / 2e
\]

\[
\Delta T_{3\psi mn} = V_{mp}^R T_{3\psi R} + V_{mp}^L T_{3\psi L} \equiv \Delta T_{3\psi mn} = -\Delta T_{3\psi mn}^L
\]

$\eta = 1(0)$ for Majorana (Dirac) fermionic $\psi$ and the loop functions $F(Z)$ and $G_{(L,Z)}$ will be given shortly. Other $g$ can be obtained by exchanging $R$ and $L$. Note that $\Delta T_{3\psi}$ is basically the difference of weak isospin quantum numbers of $\psi_R$ and $\psi_L$ and in the case of no mixing, $\kappa_{L,R}$ are vanishing. Therefore, we expect $\Delta T_{3\psi}$ to be an order one quantity, while $\kappa$ to be a much smaller quantity. Note that in case II the leading order contributions to the Z penguin amplitudes are at the level of $\delta_{LR} \delta_{RL}$, which is beyond the accuracy of the this analysis and their contributions are, hence, neglected.

The above loop functions are defined as [24]

\[
F_1(a,b) = \frac{1}{12(a-b)^3} \left(2a^3 + 3a^2b - 6ab^2 + b^3 + 6a^2b\ln b^a\right),
\]

\[
F_2(a,b) = \frac{1}{2(a-b)^3} \left(-3a^2 + 4ab - b^2 - 2a^2\ln b^a\right),
\]

\[
F_3(a,b) = \frac{1}{2(a-b)^3} \left(a^2 - b^2 + 2ab\ln b^a\right),
\]
\[
\begin{align*}
G_1(a,b) &= \frac{1}{36(a-b)^2} \left( -(a-b)(11a^2 - 7ab + 2b^2) - 6a^3 \ln \frac{b}{a} \right), \\
G_2(a,b) &= \frac{1}{36(a-b)^2} \left( -(a-b)(16a^2 - 29ab + 7b^2) - 6a^2(2a-3b) \ln \frac{b}{a} \right), \\
G_3(a,b) &= \frac{1}{36(a-b)^3} \left( -(a-b)(17a^2 + 8ab - b^2) - 6a^2(a+3b) \ln \frac{b}{a} \right), \\
F_Z(a_1,a_2,b,b,c) &= \frac{a_1(2\sqrt{a_1a_2} - a_1)}{2(a_1 - a_2)(a_1 - b)} \ln \frac{a_1}{c} + \frac{a_2(2\sqrt{a_1a_2} - a_2)}{2(a_1 - a_2)(a_2 - b)} \ln \frac{a_2}{c} - \frac{b(2\sqrt{a_1a_2} - b)}{2(a_1 - b)(a_2 - b)} \ln \frac{b}{c}, \\
F_Z(a,a,b_1,b_2,c) &= -\frac{3}{4} + \frac{a^2}{2(a-b_1)(a-b_2)} \ln \frac{a}{c} - \frac{b_1^2}{2(a-b_1)(b_1-b_2)} \ln \frac{b_1}{c} + \frac{b^2}{2(a-b_2)(b_1-b_2)} \ln \frac{b}{c}, \\
G_Z(a_1,a_2,b) &= \frac{a_1\sqrt{a_1a_2}}{(a_1-a_2)(a_1-b)} \ln \frac{a_1}{b} - \frac{a_2\sqrt{a_1a_2}}{(a_1-a_2)(a_2-b)} \ln \frac{a_2}{b}, \\
F(a,b,c,d) &= \frac{b\sqrt{ab}}{(a-b)(b-c)(b-d)} \ln \frac{b}{a} - \frac{c\sqrt{ab}}{(a-c)(b-c)(c-d)} \ln \frac{c}{a} + \frac{d\sqrt{ab}}{(a-d)(b-d)(c-d)} \ln \frac{d}{a}, \\
G(a,b,c,d) &= -\frac{b^2}{(a-b)(b-c)(b-d)} \ln \frac{b}{a} + \frac{c^2}{(a-c)(b-c)(c-d)} \ln \frac{c}{a} - \frac{d^2}{(a-d)(b-d)(c-d)} \ln \frac{d}{a},
\end{align*}
\]

(A13)

We do not need the generic expression of \(F_Z(a_1,a_2,b_1,b_2,c)\), since only \(a_1 = a_2 = a\) and/or \(b_1 = b_2 = b\) are used in this work.

Comparing the generic expressions in Eq. [A5] to the following effective Lagrangians,

\[
\mathcal{L}_{a-2} = -\frac{eQ}{4m_t} \Delta a_i \bar{\sigma}_{\mu}\nu F^{\mu\nu}, \quad \mathcal{L}_{EDM} = -\frac{i}{2} d_i \bar{\sigma}_{\mu\nu \gamma} \gamma_5 F^{\mu\nu},
\]

(A14)

the \(\Delta a_i\) and \(d_i\) can be readily obtained as

\[
\Delta a_i = -\frac{4m_t}{eQ_i} \text{Re}(A_{RL}), \quad d_i = 2 \text{Im}(A_{RL}).
\]

(A15)

The \(l' \to \bar{l}\gamma\) decay rate is related to the above \(A_{M',N}\),

\[
\Gamma(l' \to \bar{l}\gamma) = \frac{(m_{l'}^2 - m_t^2)^3}{4\pi m_{l'}^3} \left( |A_{L'}|^2 + |A_{R'}|^2 \right),
\]

(A16)

the \(l' \to l''l\) decay rate is governed by the following formula, [18]

\[
\Gamma(l' \to l''l) = \frac{m_{l'}^5}{3(8\pi)^3} \left[ \frac{g_{RRLL}}{8} + 2|g_{RRRR}|^2 + |g_{RLLL}|^2 + 32 \delta_{ll'} \left| \frac{eA_{RL}}{m_{l'}} \right|^2 \log \left( \frac{m_{l'}^2}{m_t^2} - \frac{11}{4} \right) \right] + 16 \delta_{ll'} \text{Re} \left( \frac{eA_{RL}g_{RLLL}}{m_{l'}} \right) + 8 \delta_{ll'} \text{Re} \left( \frac{eA_{RL}g_{RLLL}}{m_{l'}} \right) + L \leftrightarrow R,
\]

(A17)

while the \(l' \to l\) conversion rate ratio is given by

\[
\mathcal{B}_{l'N \to lN} = \frac{\omega_{\text{conv}}}{\omega_{\text{capt}}},
\]

(A18)
with

\[ \omega_{\text{conv}} = \left| \frac{A_{R}^{*}D}{2m_{\nu}} + 2[2g_{LV}^*(u) + g_{LV}^*(d)]V^{(p)} + 2[g_{LV}^*(u) + 2g_{LV}^*(d)]V^{(n)} \right|^2 + L \leftrightarrow R, \quad (A19) \]

and the numerical values of \( D, V \) and \( \omega_{\text{capt}} \) are taken from [28, 29] and are collected in Table XII for completeness.

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