Geometric Gait Design for a Starfish-Inspired Robot Using a Planar Discrete Elastic Rod Model

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A starfish-inspired robotic platform consisting of multiple soft fluidic bending actuator arms arranged with radial symmetry about a rigid hub is described. Intrinsic properties of the soft actuators are estimated via computer vision for varying input fluid pressures. The dynamic motion of individual arms and the full robot are modeled using the planar discrete elastic rod (PDER) theory. Locomotion gaits (periodic shape changes) that result in translation in the plane, separately considering fixed or rotating anchors at the end of each arm, are derived. Gait efficiency is defined as the displacement magnitude divided by a measure of the input control effort over each gait cycle, including a cost for anchor attachment. Through numerical computation, optimally efficient gaits are found and the desired motion with a pneumatic hardware prototype is demonstrated.

1. Introduction

The development of novel soft actuators that combine bending, twisting, and/or lengthening action presents new opportunities in robot design. The animal world provides many examples of organisms that achieve locomotion through periodic motion of soft appendages, and recent research has focused on developing robot designs that mimic the body plan and locomotory capabilities of marine invertebrates. Starfish-inspired robots have been created using shape-memory alloy (SMA) actuators, as well as designs driven by traditional motors with multiple rigid segments and with passive compliant arms. More recently, an octopus robot was developed using entirely soft components, powered by chemical reactions and controlled by microfluidic oscillator circuits.

Starfish, or sea stars, of class Asteroidea, along with the related brittle stars of class Ophiuroidea, serve as interesting model systems for control design due to the modularity inherent in their radial symmetry. In brittle stars, different arms take on a leading role as the direction of motion changes. A decentralized controller based on coupled oscillators was developed to achieve similar behavior in a brittle star-inspired robot. In our earlier work, we studied walking gait design for an idealized starfish robot with curvature-controlled circular arc arms. Here, we expand and improve upon the starfish robot gait design method using a discrete elastic rod (DER) dynamic model for each arm, based on existing fluidic bending actuators and demonstrate it experimentally.

Soft robot architectures with many underactuated degrees of freedom bring with them new challenges in mathematical modeling and control design. Methods from geometric control theory are a means to accommodate nonlinear locomotive systems with many degrees of freedom. A key concept in geometric control is the “local connection”, a mathematical operator that maps changes in the shape of a robot to its overall motion relative to the ground. In highly underactuated systems such as a flexible snake robot, it is possible to design efficient gaits by estimating the local connection experimentally.

This work addresses the problem of locomotion gait design for a radially symmetric starfish-inspired robot with soft fluidic bending actuators serving as its appendages. Legged locomotion is in general a “hybrid system” consisting of intervals of continuous-time dynamics alternating with discrete events when the ground contact conditions are updated. We use a model-based approach to study how controlled shape changes of the robot arms give rise to gross motion of the robot body under a set of holonomic constraints for ground contact. Each arm is given the capability of reversibly anchoring its distal end, or foot, to the ground surface, with either a fixed orientation or rotating connection. The mapping of shape change to body motion, i.e., the local connection, is optimized to yield a set of repeating continuous trajectories, which along with discrete updates to the ground constraints produces a candidate locomotion gait.

To model the dynamics of the individual soft actuator arms, which can be individually controlled via fluid pressure input to vary their intrinsic shape (curvature) and stiffness, we use a planar Discrete Elastic Rod (PDER) formulation. PDER is a spatial discretization of a continuous elastic rod model, such that the rod is represented by a series of point nodes connected by edges. The PDER approach has the advantage of being
computationally tractable and is amenable to actuator characterization via the vision-based tracking of points that are associated one to one with the nodes of the model. The dynamic forces acting on the nodes are derived via Euler–Lagrange methods from scalar elastic bending and stretching potential-energy functions. We experimentally characterize the material properties of an individual fluidic bending actuator and use the computed values in the complete model of a central rigid body coupled to five PDER actuators. PDER models have previously been used to model SMA actuators;[11] this work presents the first application of the PDER model to bellows-style fluidic bending actuators. The bellows-style or pneumatic network (pneu-net) actuators we use are based on the design from the study by Galloway et al.[12] The properties of bellows-style bending actuators have been studied via other models, such as elastica[13] and finite element models.[14] The PDER modeling approach presented here is general and may be applied to other soft robotic designs with different arm geometries or types of actuators.

Fluidic soft actuators such as the bellows-style actuators used in this work have desirable properties in terms of stiffness and range of motion, but they have the disadvantage of requiring bulky electrical components like pumps and valves to generate and distribute the desired fluid flow. For indoor and tabletop operation, such as the pneumatic system described in this article, a tethered design with onboard fluid control is convenient. However, there are examples of untethered pneumatic soft robots that are large enough to incorporate a rigid section to house electronics and pumps.[15,16] For hydraulically actuated soft robots, electric gear pumps[17] and peristaltic pumps[18] have been used. Alternatively, a chemical reaction may be used as a source of fluid pressure, allowing for the entire device to be made from soft materials as in the octobot.[9]

Although legged soft robots with open-loop control have achieved successful locomotion in a range of conditions, e.g., in untethered soft quadruped[19] and caterpillar[11] robots, the integration of feedback control via on-board shape sensors enables adaptive gaits for higher performance. This work describes a bench-top test bed featuring a tethered robot with pneumatic pressure inputs to experimentally demonstrate numerically optimized gait designs. Two forms of feedback control were implemented in the system: an inner-loop pressure controller regulates the timing of the pulse-width-modulated (PWM) valves in the pneumatic fluid control board to fill the individual arms with the desired pressure input and a vision-based tracking controller varies the input pressures until a desired robot motion is achieved, which is necessary due to unmodeled friction forces and variation in material parameters between multiple actuators. As a practical matter, we utilize vision-based tracking for shape sensing, though in ongoing and future works, we seek to enable shape-based feedback through onboard sensors, such as 3D-printable soft shape sensors,[19] optical shape sensors,[20] soft force sensors with localization based on magnetic microparticles,[21] touch sensing based on deformation of soft fluid channels,[22] or soft capacitive touch sensors.[23]

This article presents a novel starfish-inspired soft robot design and the synthesis, optimization, and demonstration of model-based gaits for legged locomotion. The contributions are as follows: 1) experimental characterization of the intrinsic shape and stiffness of a pneumatic bending actuator under varying internal air pressure; 2) application of a PDER model to describe the motion of a starfish-inspired robot with pneumatic bending actuator arms; 3) a mathematical description and numerical optimization of periodic walking gaits for the starfish robot through the use of fixed and rotating anchors at the end of the arms; and 4) experimental validation of the optimized gaits using closed-loop feedback control. Although we focus on a specific type of actuator and robot geometry, the process of gait design through actuator characterization and computation of the local connection based on the PDER mathematical model may be applied to any legged soft robot for which the ground contact dynamics are well defined.

2. Planar Discrete Elastic Rod Model for a Starfish Robot

Consider a planar starfish robot consisting of a rigid central hub with \( n_s \) flexible arms attached to the hub with radial symmetry. We model each flexible arm as a PDER,[11] based on Bergou et al.’s Discrete Elastic Rod (DER) theory,[24,25] which models a flexible rod as a series of nodes with conservative spring-like forces arising from potential energy functions for elastic stretching and bending.

The center hub is modeled as a rigid body in the shape of a regular polygon with \( n_s \) edges, centered at \( r_i = (x_i, y_i)^T \in \mathbb{R}^2 \), with orientation \( \theta_i \in \mathbb{S} \) in an inertial frame. Let \( \beta = 2\pi/n_s \) represent the angular offset between neighboring arms at the hub. Each of the \( n_s \) arms is modeled as a PDER consisting of \( n + 1 \) nodes labeled from 0 to \( n_s \). Arms are labeled from 1 to \( n_s \) in a counterclockwise order around the hub. Let \( r_{ij} \in \mathbb{R}^2 \) be the position of the \( j \)th node of the \( i \)th arm in the plane. Let \( e_{ij} = r_{ij+1} - r_{ij} \) represent an edge vector between two adjacent nodes. Define edge length \( l_{ij} \) and edge orientation \( \theta_{ij} \) such that \( e_{ij} = l_{ij} (\cos \theta_{ij}, \sin \theta_{ij})^T \). The turning angle at a node \( \phi_{ij} \) is the difference in orientation of neighboring edges, such that \( \phi_{ij} = \theta_{ij} - \theta_{i-1,j} \), in the range \((-\pi, \pi)\). The (unitless) discrete curvature at a node is defined as \( \kappa_{ij} = 2 \tan(\phi_{ij}/2) \).

The first two nodes of each arm are rigidly attached to the center hub, allowing for transfer of forces and torques from the arms to the hub and vice versa. Parameters \( x_0 \), \( y_0 \), and \( l_0 \) determine the locations of the first two nodes of each arm relative to the hub, with radial symmetry, i.e.,

\[
\begin{align*}
\mathbf{r}_{0,i} &= r_i + R(\theta_i + i\beta) \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} \\
\mathbf{r}_{1,i} &= r_i + R(\theta_i + i\beta) \begin{pmatrix} x_0 + l_0 \\ y_0 \end{pmatrix}
\end{align*}
\]

where \( R(\theta) = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \) is a 2D rotation matrix.

We define an arm-centric coordinate frame fixed to the hub for each arm, such that node 0 lies at the origin, and node 1 lies on the positive \( x \)-axis. Let \( a_{ij} \in \mathbb{R}^2 \) be the location of the \( j \)th node of arm \( i \) in the arm-centric frame for arm \( i \), with the following mapping from one frame to another.
\[ r_{ij} = r_i + R(\theta_i + i\varphi)(a_{ij} + (x_0, y_0)) \]  
(2)

Let \( \alpha_{ij} \) be the orientation of edge \( j \) in the arm-centric frame, with
\[ \alpha_{ij} = \theta_{ij} - \theta_x - i\varphi, \quad j = 1, \ldots, n \]  
(3)

Let \( a_{ij} \in \mathbb{R}^{2(n+1)} \) be the (column) vector formed by stacking each \( a_{ij} \) for \( j = 0, \ldots, n \).

Consider two types of ground attachment modes for the feet (distal ends) of the arms: rotating anchors and fixed anchors. During the course of locomotion, each foot may be attached or detached from the ground at will, e.g., via retractable magnets. For rotating anchors, the last node of an anchored arm is fixed in position in the inertial frame, with associated forces of constraint. For fixed-anchors, the last two nodes of an anchored arm are fixed in position and the forces at the two nodes act together to create torque at the second-to-last node.

### 2.1. Kinematics of a Free Arm

Here we describe the reference (undeformed) configuration of a rod with no external forces acting on it. The rest configuration of a rod with a free end is a function of the intrinsic lengths \( l_{ij}^* \) of each edge and the intrinsic curvature \( \kappa_{ij} \) at each internal node, \( j = 1, \ldots, n-1 \). These two quantities may vary in time as a function of a scalar input variable such as the fluid pressure \( p \), in a bellows actuator.

In the arm-centric frame, the first node \( a_{i0} \) is fixed at the origin and the first edge lies along the positive x-axis, so that \( \alpha_{i0} = 0 \). For each internal node, the intrinsic turning angle is \( \Theta_{ij} = 2 \tan^{-1}(k_{ij}/2) \). The edge orientations for the free arm at rest are
\[ \pi_{ij} = \pi_{ij-1} + \Phi_{ij} = \sum_{k=1}^{j} \Phi_{ik} \]  
(4)

The edge vectors and node positions are
\[ a_{ij} = \sum_{k=0}^{j-1} l_{ik} \left( \cos \pi_{ik} \right) \left( \sin \pi_{ik} \right), \quad j = 1, \ldots, n \]  
(5)

We refer to \( \pi_{i}(p) \) as the “natural shape” of actuator \( i \) for input pressure \( p \). In the special case that each edge has identical length \( l_i^* \), and all nodes have identical curvature \( \kappa_{i0}^* \), with turning angle \( \Theta_{i0} = 2 \tan^{-1}(k_{i0}/2) \), the nodes lie on a circle of radius \( \frac{l_i^*}{2} \csc(\Theta_{i0}/2) \) with
\[ a_{i} = \frac{l_i^*}{2} \left( \cot(\Theta_{i0}/2) \right) \left( 1 - \csc(\Theta_{i0}/2) \right) \left( \sin(\Theta_{i0}/2) \right) \left( \sin(\Theta_{i0}/2) \right) \]  
(6)

In that case, as the number of nodes in the arm increases, the arm shape converges toward a smooth circular arc. An idealized starfish robot with arms modeled as continuous circular arcs was studied previously.[16]

### 2.2. Dynamics of Planar Discrete Elastic Rods

The dynamics of the starfish robot are derived from conservative elastic forces and nonconservative damping forces acting on the individual nodes in the arm rods. In effect, we have a system of one rigid body and a number of point-mass particles coupled by linear and torsional springs.

The total elastic potential \( E_{\text{elastic}} = E_{\text{bend}} + E_{\text{stretch}} \) in a single PDER is the sum of stored elastic energy due to bending and axial stretching, with
\[ E_{\text{bend}} = \frac{1}{2} \sum_{i=1}^{n-1} \frac{EI}{C_0} (k_i - k_1)^2 \]  
(7)

and
\[ E_{\text{stretch}} = \frac{1}{2} \sum_{i=0}^{n-1} \frac{EA}{C_0} \left( \frac{l_i}{l_i^*} - 1 \right)^2 l_i \]  
(8)

where the geometric properties, \( k_i \), and \( l_i^* \) and material stiffnesses, \( EI \) and \( EA \), may each be a function of the input pressure.

The internal forces acting on each node are equal to the partial derivative of the corresponding potential energy with respect to the node position. We refer the reader to the study by Goldberg et al.[11] for a full derivation of the planar DER equations of motion. For nodes within a rod, i.e., nodes that are not constrained to move with a rigid body or constrained by ground anchoring, the equations of motion are
\[ m_{i,j} \ddot{x}_{ij} = -\frac{\partial E_{\text{elastic}}}{\partial x_{ij}} - d \dot{x}_{ij}, \quad m_{i,j} \ddot{y}_{ij} = -\frac{\partial E_{\text{elastic}}}{\partial y_{ij}} - d \dot{y}_{ij} \]  
(9)

where \( d \) is the viscous damping coefficient.

For the rigid body hub, the equations of motion for translation and rotation are
\[ m_x \ddot{x} = -\sum_{i=1}^{n_s} \left( \frac{\partial E_{\text{elastic}}}{\partial x_{i0}} + \frac{\partial E_{\text{elastic}}}{\partial x_{i1}} \right), \]  
\[ m_y \ddot{y} = -\sum_{i=1}^{n_s} \left( \frac{\partial E_{\text{elastic}}}{\partial y_{i0}} + \frac{\partial E_{\text{elastic}}}{\partial y_{i1}} \right) \]  
(10)

\[ I_x \ddot{\theta} = -\sum_{i=1}^{n_s} \left( (x_{i0} - x_{i1}) \frac{\partial E_{\text{elastic}}}{\partial y_{i0}} - (y_{i0} - y_{i1}) \frac{\partial E_{\text{elastic}}}{\partial x_{i0}} \right) - \sum_{i=1}^{n_s} \left( (x_{i1} - x_{i0}) \frac{\partial E_{\text{elastic}}}{\partial y_{i1}} - (y_{i1} - y_{i0}) \frac{\partial E_{\text{elastic}}}{\partial x_{i1}} \right) \]

such that the motion of the hub is governed by the computed elastic forces acting on the PDER nodes that are fixed to the hub.

For simulations of the PDER starfish model, we use an implicit Euler integration scheme with fixed step size \( h \), as in the study by Goldberg et al.[11] i.e.
\[ \dot{q}(t_{k+1}) = q(t_{k+1}) - q(t_k), \]
\[ M(q(t_{k+1}) - q(t_k)) = \dot{h}F(t_{k+1}, q(t_{k+1}), \dot{q}(t_{k+1})) \]  
(11)

where the configuration vector \( q \) is a concatenation of the coordinates of the nodes of \( n_s \) arms along with \( x_r, y_r, \) and \( \theta_r, \) and \( M \) is the appropriate diagonal mass matrix.
3. Experimental Characterization of a Soft Bending Actuator

We created an experimental test-bed for planar gait design in the form of a five-arm radially symmetric starfish robot. Pressure is supplied to each arm individually from a pneumatic control board via PWM valve timing. Each arm is a unidirectional bellows-style actuator, also known as a pneu-net actuator, cast in Smoothsil silicone rubber.

To determine the necessary parameters for the PDER model of the bellows actuator, we performed two sets of experiments with a range of pressure inputs: free-end experiments and pinned-end experiments. In both setups, custom MATLAB computer-vision code tracks the positions of marker dots painted along the neutral axis of the actuator to determine the edge lengths and curvatures at each marker as a function of the input pressure. The marker dots are positioned such that each dot is associated with a node in the PDER mathematical model. In the free-end experiment, the measured curvatures and lengths are equal to the intrinsic curvatures and lengths at each node and pressure. In the pinned-end experiment, we fix the location of the last node in the y-direction while allowing it to rotate and slide in the x-direction. A load cell is used to determine the force $F_i$ acting at the last node, which allows us to estimate the bending stiffness by comparing the curvatures of the deformed arm to the intrinsic curvatures of the free arm.

3.1. Experimental Methods

Ten trials each of the free- and pinned-end configurations were performed with actuators cast in Smoothsil 950 and Smoothsil 960 platinum-cure silicone rubbers from Smooth-On, Inc. The pressure is controlled through pulse-width modulation (PWM), where the duty cycle represents the fraction of each open-close cycle that the valve is open. In the open-valve position, high pressure air from the pump flows into the actuator and, in the closed-valve position, air in the actuator is vented to ambient pressure. In each trial, valve duty cycles were swept from zero to a maximum value in 5% increments with valve switching frequency of 40 Hz. Each duty cycle value was held constant for 5 s before a snapshot was taken with the overhead camera. For the Smoothsil 950 actuator, the maximum duty cycle used was 60% and, for the Smoothsil 960 actuator, the maximum duty cycle was 75%. At duty cycles higher than those used, the tip of the actuator came into contact with the base, which contradicts the zero tip-force assumption necessary to measure the intrinsic curvatures of the free arm.

3.2. Experimental Results

In experiments with a single actuator connected to the PWM fluid valve and pump system, the mapping from duty cycle to average measured fluid pressure was monotonic and highly repeatable between trials, with little variation between the two material types and test types.

3.2.1. Intrinsic Curvature

For the free-end trials, we find an approximately linear relationship between input pressure and curvature at each node, as shown in Figure 1C,D. Due to the way the nodes were painted by hand, some display a nonzero curvature at zero pressure. Thus, for our linear fits of intrinsic curvature versus pressure, we allow for a nonzero constant coefficient. To estimate the curvature at the first node, an additional virtual node is added such that its position is 1 cm to the left of the first node in the initial zero-input-pressure snapshot. The virtual node is associated with node 0 of the PDER model of the arm for use in simulations and, similarly, the painted marker points are associated with nodes 1 through 11. In previous studies of pneu-net style bellows actuators, the curvature was found to diminish at nodes toward either end of the rod[13] but this effect was not significant for the actuator design considered here.

3.2.2. Intrinsic Length and Stretching Stiffness

We find that the intrinsic length of the actuator grows as pressure increases in the free-end trials. As the change in length is quite small, we estimate only the total length change and assume that the axial strain is evenly distributed along the length of the actuator. We perform a linear fit to determine the axial-strain coefficient $\varepsilon$, such that the length of a segment of the actuator with resting length $l_0$ varies with pressure as

$$l(p) = l_0(1 + \varepsilon p)$$

For Smoothsil 950, $\varepsilon = 9.83 \times 10^{-4}$ kPa$^{-1}$ from a linear fit with coefficient of determination 0.957, and Smoothsil 960 has $\varepsilon = 8.12 \times 10^{-4}$ kPa$^{-1}$ from a linear fit with coefficient of determination 0.984.

To estimate the stretching stiffness, we measure elongation from a 1 kg weight attached at the pin in a vertical configuration. Stretching due to the weight of the actuator itself was negligible. For the Smoothsil 950 actuator, the rest length from the first painted marker to the last (at the pin) was 101 mm. Under a tension force of 9.81 N (1 kg under gravity), the stretched length
was measured to be 110 mm. The stretching stiffness was estimated as

\[ EA = F_l/(l_{\text{stretched}} - l_0) \approx 110 \text{ N} \]  \hspace{1cm} (13)

Similarly for the Smoothsil 960 actuator, we measure resting and stretched lengths to be 100 and 104 mm, respectively, to give an estimated stretching stiffness of 245 N. As we are only able to measure the value at zero pressure, we assume stretching stiffness \( EA \) remains constant regardless of pressure in the simulations.

### 3.2.3. Bending Stiffness

For the pinned-end trials, we use the fact that the measured force at the last node is only in the normal (positive y-axis) direction to compute the externally applied moment at each node along the length as

\[ M_{i,\text{ext}} = F_y(x_{n+1} - x_i) \]  \hspace{1cm} (14)

In PDER theory, the internal moment at a node is proportional to the variation in curvature, with the bending stiffness \( EI/\ell \) as the proportionality constant, i.e.\(^{[27]}\)

\[ M_{i,\text{int}} = EI(\kappa_i - \bar{\kappa}_i) \]  \hspace{1cm} (15)

When the rod is in the steady state, the internal and external moments must balance at each node.

For our analysis, assume that the bending stiffness of the actuator is equal at each node (invariant with position) for each input fluid pressure. The curvature difference at each node is found by subtracting the discrete curvature measured in the pinned-end trial from the linear model curvature for that fluid pressure based on the free-end data. For a given node and pressure, we estimate the bending stiffness by rearranging (15), setting internal and external moments equal

\[ \hat{E}I = \frac{M_{i,\text{ext}}(p)}{\kappa_i - \bar{\kappa}_i} \]  \hspace{1cm} (16)

These estimates are combined between all nodes and all trials to fit a linear model of bending stiffness as a function of input pressure.

We exclude data with an average pressure of less than 10 kPa, as at these low pressures, the tip force and curvature difference are quite small and noisy. Any points with a negative value for
calculated stiffness are obviously spurious and are excluded. The last two nodes (at the tip end) displayed some anomalous stiffness values, which may have been caused by the slightly different rod geometries at the tip, combined with the small moment arm yielding lower applied moments. We thus exclude the measurements from nodes 9 and 10.

For Smoothsil 950, the linear best fit for bending stiffness is \(E I(p) = (2.48 \times 10^{-3} \text{N-m}^2 \text{kPa}^{-1})p + 5.88 \times 10^{-4} \text{N-m}^2\). For Smoothsil 960, the linear best fit for bending stiffness is \(E I(p) = (3.10 \times 10^{-3} \text{N-m}^2 \text{kPa}^{-1})p + 7.99 \times 10^{-4} \text{N-m}^2\). As expected, the bending stiffness is found to be higher in Smoothsil 960, which has a higher Shore hardness rating.

In the pinned-end trials, the measured normal tip force appears to follow a quadratic trend for both material types tested, which is expected as the intrinsic curvature and bending stiffness both increase roughly linearly with input fluid pressure. Figure 1B shows tip force \((F, \text{ in N})\) versus average pressure \((P, \text{ in kPa})\) with quadratic fits overlaid, for \(F(P) = c_1 P^2 + c_2 P + c_3\). For the Smoothsil 950 arm, the least-squares quadratic fit is \(c_1 = 1.49 \times 10^{-4}, c_2 = 2.73 \times 10^{-3}\), and \(c_3 = -0.0265\) with coefficient of determination 0.9955. For the Smoothsil 960 arm, the fit is \(c_1 = 8.76 \times 10^{-5}, c_2 = 0.005\), and \(c_3 = -0.0435\) with coefficient of determination 0.9989.

4. Gait Design for Starfish Robot with Fixed Anchors

This section defines a framework to parameterize gaits for a robot with feet that can be anchored to the ground with a fixed position and fixed orientation, which is achieved by constraining the position of the final two nodes of the arm. Suppose that only one anchor is active at any given time, with instantaneous switches between anchored arms at discrete times. In the case of a fixed attachment, the motion of the center of the robot is completely constrained by the motion of the anchored arm. The curvatures of the free unanchored arms do not contribute at all to the motion and may be ignored until such time that a new arm becomes anchored.

The typical pneu-net actuator bends only in one direction, with increased pressure, bringing about an increase in curvature. The two modes of pneu-net action are the power stroke (pressure increase) and the recovery stroke (pressure decrease). Change in pressure not only changes the intrinsic curvature (shape of the unloaded arm), but also the bending stiffness (or flexural rigidity, \(EI\)). The consequence of this property is that, when friction is present, a depressurized arm can only apply a small amount of force due to decreased stiffness. Thus, we focus on the design of gaits that utilize a power stroke to achieve motion of the center hub of the robot, though the mathematical gait descriptions presented later are sufficiently general to apply to bidirectional actuators as well.

A mathematical description of a starfish robotic gait is as follows. Gait \(G = \{S_m\}, m = 1, \ldots, M\) is defined as a set of one or more steps, \(S_m = \{p_m^{-}, p_m^{+}, \Delta m\}\), with each step \(S_m\) defined by three parameters: the beginning and ending pressures for the active (anchored) arm, \(p_m^{-}, p_m^{+} \in \mathbb{R}\), and the index offset \(\Delta m\) that specifies the next active arm. A single gait cycle is a trajectory comprising steps \(S_1\) through \(S_M\) in succession.

Consider a gait with a single step \(G = \{(a, b, c)\}\) and suppose, at the start of the step, arm \(i\) activates with initial pressure \(p_i = a\), starting in its natural shape \(a_i = \bar{a}_i(p_i)\). During the step, the pressure in arm \(i\) increases to \(p_j = b\), which causes the arm to bend to the natural shape associated with the new pressure, causing the center of the robot to move relative to the anchored foot. Once arm \(i\) reaches the natural shape for its desired ending pressure, arm \(j = (i + c) \mod M\) brings its own pressure to \(p_j = a\) and becomes active once the arm settles to its natural shape, anchoring its foot and restarting the process for the next step. To increase speed, the next arm may begin moving toward its desired pressure \(p^+\) before the active arm reaches \(p^+\), but this does not affect the displacement for the step.

We now derive the displacement of the group variables representing the translation and rotation of the center hub for a single step \(S_m\) as a function of the step parameters. Let the initial state at time \(t^−\) be described by \((r_i, \theta_i^-)\), with shape \(a_i(t^-) = \bar{a}_i(p_i^-)\) in the active arm, and similarly for the final state at time \(t^+\), \((r_i, \theta_i^+)\), with active arm shape \(a_i(t^+) = \bar{a}_i(p_i^+)\). We solve for the displacements \(\Delta \theta_m = \theta_i^+ - \theta_i^-\) and \(\Delta r_m = r_i^+ - r_i^-\) via Equation (2) and (3). Note that the locations of the final two nodes \(r_{i,n-1}\) and \(r_{i,n}\) are constant during a step for a fixed-foot anchored arm, which implies that the orientation of the final edge \(\theta_{i,n}\) is also constant. We have

\[
\begin{align*}
\theta_{i,n}(t^-) &= \theta_{i,n}(t^+), \\
\theta_i^- + \beta + \bar{a}_i(p^-) &= \theta_i^+ + \beta + \bar{a}_i(p^+), \\
\Delta \theta_m &= \theta_i^+ - \theta_i^- = \bar{a}_i(p^-) - \bar{a}_i(p^+) \\
\end{align*}
\]

Therefore

\[
\begin{align*}
r_{i,n}(t^-) &= r_{i,n}(t^+), \\
r_i^- + R(\theta_i^- + i\beta)\left(\bar{a}_i(p^-) + \left(\frac{x_0}{y_0}\right)\right) &= r_i^+ + R(\theta_i^+ + i\beta)\left(\bar{a}_i(p^+) + \left(\frac{x_0}{y_0}\right)\right) \\
\end{align*}
\]

Through rearranging and substitution, we find

\[
\begin{align*}
\Delta r_m^m &= r_i^+ - r_i^- \\
&= R(\theta_{i,n})\left[R(-\bar{a}_i(p^-))\left(\bar{a}_i(p^-) + \left(\frac{x_0}{y_0}\right)\right) - R(-\bar{a}_i(p^+))\left(\bar{a}_i(p^+) + \left(\frac{x_0}{y_0}\right)\right)\right] \\
\end{align*}
\]

Note that the direction of \(\Delta r_m\) depends on the orientation \(\theta_{i,n}\) of the anchored edge.

At the end of step \(S_m\), arm \(j = (i + \Delta m) \mod M\) activates the anchor at its foot. The difference in orientations for successive foot anchors determines whether a given gait follows a straight line or a curved trajectory over multiple gait cycles. Let \(\Delta \theta_j^m\) be the difference in foot edge orientations from arm \(i\) to arm \(j\) at the end of step \(S_m\), i.e.
\[ \Delta \theta_{\text{pp}} = \theta_{\text{pp}} - \theta_{\text{in}} = \left( \theta_{x}(t) + i \beta + \alpha_{m}(p_{m}^{(m+1)}) \right) - \left( \theta_{x}(t) + i \beta + \alpha_{m}(p_{m}^{(m-1)}) \right) \]

\[ = \beta \Delta_{m} + \alpha_{m}(p_{m}^{(m+1)}) - \alpha_{m}(p_{m}^{(m-1)}) \]  

For locomotion over long distances, it is necessary for repeated cycles of a walking gait to proceed in the same direction. We now describe conditions on the gait parameters that guarantee straight-line motion for fixed-anchor gaits.

**Theorem 1.** For a starfish robot with identical arms all having natural shape defined by \( \alpha(p) \), if the steps in a gait satisfy the condition

\[ \cos \left( \sum_{m=1}^{M} \beta \Delta_{m} + \alpha_{m}(p_{m}^{(m-1)}) - \alpha_{m}(p_{m}^{(m+1)}) \right) = 1 \]  

then the net displacement in position for each gait cycle will be parallel over repeated gait cycles.

**Proof:** We show that condition (21) implies that the orientation of the anchored edge at the start of each cycle is the same, which further implies that the displacement vectors for subsequent cycles are parallel. Let \( j(m) \) be the active arm for step \( m \) in the gait, such that \( j(m+1) = (j(m) + \Delta_{m}) \mod n \), with \( j(1) = 0 \). The difference in stance-foot orientation from one gait cycle to the next, denoted \( \Delta \theta_{\text{pp}} \), is the sum of differences for each step in a gait cycle

\[ \Delta \theta_{\text{pp}} = \sum_{m=1}^{M} \Delta \theta_{m} = \sum_{m=1}^{M} \beta \Delta_{m} + \alpha_{m}(p_{m}^{(m+1)}) - \alpha_{m}(p_{m}^{(m-1)}) \]  

Since the steps in the next gait cycle are the same as the steps in this cycle, \( p_{m}^{(m+1)} = p_{m+1}^{(m-1)} \). In addition, as we sum over all steps, we can change the index in a term without changing the value of the sum, thus

\[ \Delta \theta_{\text{pp}} = \sum_{m=1}^{M} \beta \Delta_{m} + \alpha_{m}(p_{m}^{(m-1)}) - \alpha_{m}(p_{m}^{(m+1)}) \]  

Condition (21) constrains the net orientation change of the stance foot over the course of a gait cycle to be zero. Having the same stance-foot orientation at the start of successive gait cycles implies that the net motion is purely translational (up to a renaming of the identical arms).

### 4.1. Performance Metrics for Gait Design

By choosing a suitable performance metric, we can solve for the optimal value of the gait parameters for gaits of a particular form, given the geometry and material parameters of the system. For our analysis, consider the following two objectives: maximum displacement per gait cycle and maximum efficiency, where efficiency is defined as the displacement magnitude divided by the measure of the input control effort over each gait cycle.

There are several possible metrics by which to measure the input effort. The appropriate effort metric for a given application depends on the particular power source, such as the types of pumps and valve systems used in the robot. For instance, in a hydraulic system with an electric peristaltic pump, the change in volume may be more relevant than the pressure difference. In the pneumatic system used for our hardware experiments, the pump is always on, and pressure is regulated through PWM valves, so the power input does not depend strongly on the choice of gait.

For a more general measure of energy input, we define a metric based on the change in elastic energy of the PDER as the intrinsic parameters of length and curvature are changed. We consider the energy cost of a step to be the stored elastic energy of the system evaluated at the initial shape and final pressure, which represents the amount of elastic potential that would be added if the input pressure changed from its initial to final value for a step instantaneously, i.e.,

\[ E_{\text{step}}(p_{1}^{1}, p_{1}^{2}) = \frac{1}{2} \sum_{p=1}^{2} \left( \frac{E_{I}(p_{1}^{1})}{l(p_{1}^{1})} \right) \left( \alpha_{m}(p_{1}^{1}) - \alpha_{m}(p_{1}^{2}) \right)^{2} \]

\[ + \frac{1}{2} \sum_{p=0}^{2} \left( \frac{E_{A}(l(p_{1}^{1}) - 1) l(p_{1}^{1})}{l(p_{1}^{1})} \right)^{2} \]

Efficiency of a gait is defined as the total displacement distance divided by the total energy input over a gait cycle.

The following subsections present fixed-anchor gaits optimized for displacement and efficiency in two categories: single-step rolling gaits and two-step alternating-arm walking gaits.

### 4.2. Single-Step Rolling Gaits with Fixed Anchors

For a robot with \( n \) arms, there is a family of single-step gaits \( G_{\text{rol}} = \{ (p_{1}^{1} - p_{1}^{2}, \Delta_{1}) \} \) that satisfy the parallel-motion constraint (21) for each value of \( \Delta_{1} \in \{ 1, \ldots, n - 1 \} \). For a chosen value of \( \Delta_{1} \), the difference in foot edge orientation for the active arm is constrained to be \( \beta \Delta_{1} \) for clockwise rotation, with the opposite sign for counterclockwise rotation. The family of gaits is characterized by a single parameter \( p_{1}^{1} - p_{1}^{2} \), with \( p_{1}^{1} \) constrained by \( \alpha_{m}(p_{1}^{1}) - \alpha_{m}(p_{1}^{2}) = \beta \Delta_{1} \), assuming that the edge orientations of the natural shape vary monotonically with input pressure.

Figure 2A–C shows the displacement, effort, and efficiency, respectively, for two families of single-step rolling gaits with varying initial pressure \( p_{1}^{1} \). We compute efficiency based on the input-energy metric defined in (24). The \( \Delta_{1} = 1 \) family has the lowest displacement, but highest efficiency overall, and the \( p_{1}^{1} = 0 \) gait performs the best in each of the gait families. If we impose a constant energy cost on the attachment of an anchor point, then the efficiency calculation adds a new term to its denominator. Figure 2D shows how the efficiency of the various gait types changes as the anchor cost increases. For low anchor costs, \( \Delta_{1} = 1 \) is best, but as the anchor cost increases, \( \Delta_{1} = 2 \) is best. In the limit of a very high anchor cost, the gait with a maximum displacement per step is the most efficient. Simulation snapshots of the two gait types over several steps are shown in the inset of Figure 2D.

### 4.3. Two-Step Walking Gaits with Fixed Anchors

Gaits with two steps are defined by six parameters, \( G = \{ (p_{1}^{m}, p_{1}^{m+1}, \Delta_{1}), (p_{2}^{m}, p_{2}^{m+1}, \Delta_{2}) \} \). Two gait with the same steps in a different order are equivalent. A symmetric walking
gait is a two-step gait with alternating arms, such that \( \Delta b = C_0 \Delta a \). If we want to constrain each step of the two-step gait to use a power stroke (increasing pressure), then, for the unidirectional pneu-net actuators we have considered so far, it is necessary for the two alternating arms to be mounted with opposite orientations, so that \( \Delta a = -\Delta b \).

We find through numerical optimization that for symmetric walking gaits satisfying the parallel motion constraint (21), the highest per-cycle displacement and highest locomotion efficiency is achieved by gaits of the form \( G = \{(p^+, p^-, \Delta), (p^-, p^+, -\Delta)\} \).

Figure 3 shows how the center-hub displacement (A), change in hub orientation (B), and efficiency (C) of optimal gaits vary as the anchor attachment cost changes. As the anchor attachment cost approaches zero, the efficiency of optimal walking gaits approaches infinity with infinitesimally small steps. As the anchor attachment cost increases, the maximum-efficiency gaits

![Image](https://example.com/image1.png)

**Figure 2.** Plots of A) center hub displacement magnitude, B) energy input, and C) efficiency per step for rolling gaits with fixed-foot anchors as a function of starting pressure \( p_0 \) for the five-arm starfish model. For either value of \( \Delta_1 \), a starting pressure of \( p_0 = 0 \) yields the lowest displacement and highest efficiency. D) Efficiency for fixed-foot rolling gaits with varying anchor-attachment cost, all with the maximum displacement gait (starting pressure \( p_0 = 0 \)). The crossover point from the \( \Delta_1 = 1 \) gait to the \( \Delta_1 = 2 \) gait is shown as a dashed line at \( E_{\text{anchor}} \approx 0.083 \). Inset: Illustration of the maximum efficiency rolling gaits for five-arm starfish, for \( \Delta_1 = 1 \) in blue and \( \Delta_1 = 2 \) in red with anchored nodes shown as a black dots.

![Image](https://example.com/image2.png)

**Figure 3.** Optimal fixed-foot walking gaits for different pairs of arms offset at an angle \( \Delta = \Delta_1 \). Plots on the left show gait parameters for maximum-efficiency gaits as a function of anchor attachment cost. A) displacement magnitude; B) change in body orientation in a single step; C) efficiency of the gait. As the anchor cost approaches zero, efficiency of walking gaits approaches infinity with infinitesimally short steps. For any given cost, the \( \Delta = 3 \) optimized gait has the highest displacement and efficiency. D) Snapshots of maximum-displacement walking gaits for \( \Delta = 1, 2, 3, 4 \). Black lines show the trajectory of the hub center point during the steps and unused arms are hidden for visual clarity. All plots here use the same color scheme.
approach the maximum-displacement gaits. Simulation snapshots for gaits optimized for maximum displacement per step are shown in Figure 3D for \( \Delta = \{1, 2, 3, 4\} \). At any anchor cost, the \( \Delta = 3 \) optimal walking gait has the highest displacement and efficiency. Comparing the fixed-foot rolling gaits to the \( \Delta = 3 \) fixed-foot walking gaits, we find that the maximum efficiencies of the two are very close for a range of foot anchor costs.

5. Gait Design for Starfish Robot with Rotating Anchors

This section describes gaits for a robot with rotating-foot anchors, such that only the end node is fixed in position in the inertial frame, rather than the final two nodes, as in the fixed anchor case. With rotating anchors, two arms must have their feet anchored at any given time to control the motion of the system. Otherwise, with only one anchor point, the whole robot may rotate freely about the single anchor. To achieve the desired motion of the hub, the pressure change in the two arms must be coordinated.

The gait design for rotating feet is equivalent to the design for fixed feet, except that now instead of considering motion relative to an inertial frame fixed at a single foot we use a stance frame \( B_{ab} \) defined by the position of the two anchored feet \( r_{a,n} \) and \( r_{b,n} \). Let the origin of \( B_{ab} \) be located at the midpoint between the feet, \( O_{ab} = (r_{a,n} + r_{b,n})/2 \), oriented such that its \( x \)-axis is in the direction of the baseline vector from foot \( a \) to foot \( b \). For gait design, consider the translation and rotation of the center hub in this frame.

Recall that a fixed anchor allows for a single degree of freedom of the hub motion, whereas two rotating anchors permit two degrees of freedom in the hub motion as the two pressures in the anchored arms are varied. In contrast with the fixed-foot case, the configuration of the anchored arm is given by the natural shape; in the rotating-foot case, we must make use of the full PDER model to calculate the deformed shape of the anchored arms due to reaction forces at the anchors. For our analysis, assume quasistatic motion, such that as the pressures change, the state of the robot is in equilibrium at all times with all forces balanced. The mapping of the rate of change of input pressures to the rate of change of the hub pose (group variables \( x_c, y_c, \theta_c \)) is the local connection \( G \). \( G \) is a nonlinear function of the full configuration of the robot, including all node positions in each PDER arm, i.e.

\[
\begin{pmatrix}
\dot{x}_c \\
\dot{y}_c \\
\dot{\theta}_c
\end{pmatrix} = G(x_c, y_c, \theta_c, r_a, r_b, p_a, p_b) \begin{pmatrix}
p_a \\
p_b
\end{pmatrix}
\]  

(25)

Once we know the local connection, we can predict the configuration of the system at a future time \( t_{1} \) given the configuration at time \( t_{0} \) and (differentiable) input trajectories \( p_a(t), p_b(t) \) for \( t_0 \leq t \leq t_1 \):

\[
\begin{pmatrix}
x_c(t_1) \\
y_c(t_1) \\
\theta_c(t_1)
\end{pmatrix} = \begin{pmatrix}
x_c(t_0) \\
y_c(t_0) \\
\theta_c(t_0)
\end{pmatrix} + \int_{t_0}^{t_1} G(x_c, y_c, \theta_c, r_a, r_b, p_a, p_b) \begin{pmatrix}
p_a \\
p_b
\end{pmatrix} dt
\]

(26)

Note that the trajectories of the hub are highly path dependent, as multiple equilibrium configurations may exist with the same foot distance and arm pressures.

Through numerical PDER simulations, we estimate the value of the local connection for a given foot distance \( d_{foot} \) through the following procedure. Start at a reference configuration where both arms are in their natural shape (zero forces at the anchors). Next, linearly increase or decrease one of the input pressures up to a difference of a constant \( \Delta p \) and run the simulation until the starfish settles into an equilibrium state. By repeating this process in each direction (increasing and decreasing \( p_a \) and \( p_b \) in increments of \( \Delta p \)), build up the space of possible equilibrium configurations along with the structure of how the robot moves through that space via the local connection. Figure 4A shows the

![Figure 4](https://www.advintellsyst.com)

**Figure 4.** A) Range of motion of the center hub with two rotating anchors (58) at foot distance \( d_{foot} = 8.35 \) cm. Red lines (resp. blue) represent paths traced out by varying the pressure in arm \( b \) (resp. \( a \)) while keeping the pressure in arm \( a \) (resp. \( b \)) constant. The thick black line represents a trajectory for a rotating-rolling gait. B) Snapshots of the state of the starfish robot at the start and end of a single step of the rotating-rolling gait.
possible range of motion based on the computed local connection for the five-arm starfish robot with two rotating anchors on neighboring arms for $d_{\text{feet}} = 8.35$ cm.

The gait description for rotating anchors requires six parameters for each foot: $S_m = (p_m^+, p_m^-, p_m^+, p_m^-, \alpha \Delta_m, \Delta_m)$, where $p_m^+, p_m^-$, and $\alpha p_m^+$ refer to the starting and ending pressures of the two active arms, $i_m(m)$ and $i_m(m)$, respectively, $\alpha \Delta_m$ is the index offset between the stance arms, such that $i_m(m) = (i_m(m) + \alpha \Delta_m) \mod n$, and $\Delta_m$ is the index offset for the next step, such that $i_m(m + 1) = (i_m(m) + \Delta_m) \mod n$.

### 5.1. Single-Step Rolling Gaits with Rotating Anchors

As an analog to the single-step rolling gait for fixed-foot anchors, we describe single-step gaits where the anchored foot distance and orientation of successive stance frames are the same. In the simplest case of $\alpha \Delta = \Delta = 1$, arm $b$ of one step remains anchored and becomes arm $a$ of the next step. To construct such a gait, we search the space of configurations that allow the next arm to anchor its foot at the desired location to achieve straight-line motion. In other words, if the anchors of arm $a$ and $b$ are located at points $(0, 0)^T$ and $(0, d_{\text{feet}})^T$, respectively, we seek configurations where for some pressure the foot of the next arm is located at $(0, 2d_{\text{feet}})^T$. Through numerical optimization, we find that for a given foot distance, there exists a one-parameter family of single-step rolling gaits. For the geometry of the starfish robot, the maximum foot distance for which a gait exists without the arms contacting each other is $d_{\text{feet}} = 8.35$ cm. In Figure 4A, the black line on the range of motion plot represents the trajectory of the center hub over a single step of the rotating-anchor rolling gait with minimum anchor force for the maximum displacement. Figure 4B shows the configuration of the starfish robot at the start and end of the step.

### 5.2. Snap-Through Buckling Gaits with Rotating Anchors

When two arms with rotating anchors are mounted on the robot with opposite orientations, such that arm $a$ has a negative intrinsic curvature and arm $b$ has positive intrinsic curvature, we observe that the robot system may have multiple stable configurations corresponding to one set of input pressures. If the pressure in the arms passes a threshold, the robot displays a snap-through buckling behavior, taking it from one branch of equilibrium configuration to another in rapid motion. By constraining the pressures in the two anchored arms to be equal, the motion of the hub is constrained to the perpendicular bisector of the line from one foot anchor to the other, with no change in hub orientation. Figure 5A shows how the vertical position of the hub varies with arm pressure for a range of foot distances, and Figure 5B shows snapshots of the starfish configuration before and after snap-through buckling for a particular foot distance of $d_{\text{feet}} = 19.0$ cm. Snap-through buckling and bistability can be observed in simulations with foot distance $d_{\text{feet}} \leq 23.3$ cm.

The snap-through buckling effect can be used in a symmetric lunging-type locomotion gait. The two arms are anchored starting at a low pressure $p^-$ (determining $d_{\text{feet}}$), then the pressure in both arms is increased past the buckling threshold $p_{\text{buck}}$. At this point, two other arms anchor, the buckling arms detach, and the sequence is repeated when the buckling arms reattach with pressure $p^+$. We find that the maximum buckling displacement of 12.2 cm occurs for $p^+ = 0$, which corresponds to $d_{\text{feet}} = 16.1$ cm, with $p_{\text{buck}} = 76$ kPa. As the foot distance increases, the buckling displacement and threshold pressure both decrease, until the bifurcation point, where the upper and lower branches come together to form a single branch and buckling no longer occurs. Unlike the other gaits described so far, the buckling gait features motion on a fast timescale, increasing speed of locomotion.

### 6. Experimental Starfish Gait Demonstrations

This section describes experimental demonstrations of gait designs on a five-arm tethered pneumatic starfish robot. Identical bellows actuator arms are attached to a 3D-printed rigid hub with dimensions of $x_0 = 3$ cm, $b_0 = 1$ cm, and $y_0 = 0.5$ cm.
as defined in Equation (1). The hub has a roller bearing mounted below its center to reduce friction during motion across a smooth table top. We implement pressure and feedback control via pressure sensors mounted on the fluid control board and arm-tip position feedback based on visual tracking from an overhead camera. For fixed-foot anchors, we demonstrate a single-step rolling gait and, for rotating-foot anchors, we implement both rolling and snap-through buckling-type gaits.

6.1. Rolling Gait with Fixed Anchors

We implement the single-step rolling gait with \( \Delta_1 = 1, p^{1^-} = 0 \) in our five-arm starfish robot through the use of real-time vision-based feedback on the foot positions. To achieve the effect of a fixed anchor in the hardware prototype, we place the end of one of the arms in a stationary clamp to serve as the anchored foot.

A proportional controller varies the pressure input to the anchored arm until the foot of the next arm to be anchored has reached its desired position. The closed-loop feedback control is necessary due to stick-slip friction of the starfish on the table, which requires a pressure higher than the expected open-loop value to overcome the static friction and initiate motion. The pressure in the newly anchored arm starts at \( p^{1^-} = 0 \) at the end of the previous step. Once the previous anchored foot is detached, the closed-loop pressure control in the anchored arm is activated to start the new step. A feedforward term of the expected open-loop pressure sets the anchored arm pressure to \( p^{1^-} \) at the initial time.

Our control law runs in discrete time at 1 Hz, with time step limited by how long it takes to automatically track the foot position via the color thresholding computer vision code. The input pressure is updated after each measurement according to

\[
p(t_{m+1}) = p(t_m) + k_p(y_{foot} - y_{des})
\]

\[
p(t_1) = p^{1^-}
\]

(27)
for \( y_{foot} \) measured in image pixels. This control leads to a gait where the foot anchors are along the line \( y = y_{des} \) in the camera image.

**Figure 6A** shows snapshots at the start and end of two steps of the fixed-rolling gait. In Figure 6B, the \( y \) position of the next foot to be anchored and the input pressure of the anchored arm are plotted versus time, showing that the foot position converges to the desired value.

### 6.2. Rolling Gait with Rotating Anchors

To demonstrate the gait design for rotating anchors, we ran an experiment to recreate one step of the highest-displacement rolling gait where the arms do not interfere with each other. The description of the single step of the gait is \( a p^{1-} = 76 \text{ kPa}, a p^{1-} = 10 \text{ kPa}, b p^{1-} = 26.5 \text{ kPa}, \) and \( b p^{1+} = 76 \text{ kPa}, \) with \( \Delta a_1 = \Delta b_1 = 1. \)

When using the pneumatic pressure control system with a single arm, there is a one-to-one mapping between valve pulse-width and pressure output. However, with multiple arms operating at once there is a nonlinear coupling between the pressures. To overcome this challenge, we implemented closed-loop pressure control on the Arduino microcontroller attached to the valves. The pressure control consists of an independent PD controller running on each output channel. Proportional and derivative gains were tuned so that the pressure settles within two seconds with minimal overshoot to avoid the risk of puncturing the actuators with too high of a pressure input.

For the experiment, the pressure in each arm is incremented from its starting to ending value over four equal time segments of 8 s each, with a photo taken at the end of each segment. **Figure 7** shows the tracked positions of the arms and center hub at the end of each segment (A), along with time series of the pressure in each arm (B). Although the end of the free arm does not reach the exact desired location by the end, motion qualitatively matches the expected configuration from the Figure 4D inset. The discrepancy can be attributed to friction in the system, as well as variations in the properties of the three arms due to hand-made manufacturing.

### 6.3. Snap-Through Buckling Gait with Rotating Anchors

We demonstrate the snap-through buckling phenomenon for two arms with rotating-foot anchors at a distance \( d_{foot} \approx 19.4 \text{ cm}. \) The desired pressure input is maintained the same in both arms and increases from zero to 60 kPa in equal steps over roughly 3 s intervals. Measurements of pressure and lateral foot force are taken throughout. **Figure 8** shows the results of the experiment, showing tracked arm positions after each interval with images of the arms just before and after buckling (A) and pressure and force measurements over time (B). Buckling occurs when the average pressure in the arms approaches 60 kPa, which is close to the expected value from simulations of 62 kPa. At buckling, pressure remains steady, whereas foot force undergoes a sharp downward jump, as the robot transitions from a state of high-to-low stored elastic energy. The fast buckling motion appears to overcome the friction in the system, suggesting that gaits that feature this fast time-scale motion may be useful for real-world locomotion tasks in high-friction environments.

### 7. Conclusion

This article describes the motion characteristics and physical properties of a pneumatic soft bending actuator. A dynamic DER model simulates the motion of a starfish-inspired robot with bending actuator arms. Numerical optimization identifies periodic walking gaits that maximize displacement and efficiency for both fixed- and rotating-foot anchors. Finally, these gaits are demonstrated on a hardware test bed with feedback control.

The rod model and gait design used here are not limited to a specific actuator and robot geometry. Any soft actuator that operates by changing its intrinsic curvature and/or length may
be modeled with this approach, such as fluidic fiber-reinforced actuators\cite{12} or actuators based on phase changes in SMAs\cite{11,29}.

Ongoing and future works consider frictional contact with the ground rather than relying on idealized anchors and applies the DER modeling approach to three dimensions to model out-of-plane bending. In addition, we are working on applying the planar DER model to an underwater system with hydrodynamic forces, such as a eel-like swimming robot.

Acknowledgements

The authors thank Michael Bell, Kaitlyn Becker, and Robert Wood at Harvard University, Oliver O’Reilly at the University of California, Berkeley, and Carmel Majidi at Carnegie Mellon University for helpful discussions. This work was funded in part by ONR grant no. N000141712063.

Conflict of Interest

The authors declare no conflict of interest.

Keywords

feedback controls, fluidic soft actuators, legged locomotions, motion plannings, soft robotics

Received: December 30, 2019
Revised: February 9, 2020
Published online: March 10, 2020

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