Evaluating Parameters for BS-Horizon Surface Generation Using Elevation Data

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Abstract: In order to create reliable DEM from limited altitude data, it is necessary to effectively use the curved surface estimation method. One of the methods available for estimating DEM using altitude data is BS-Horizon (Nonogaki et al., 2012) which is based on cubic B-spline and exterior penalty function. In this study, the parameter settings for the estimating DEM using the BS-Horizon surface interpolation method were investigated. The experiments were carried out using field topographic survey data as elevation constraint condition and also incorporating inequality constraints based contextual relationship denoted by topographic contours. The parameters investigated include numbers elevation points used for spline estimation, the degree of satisfaction of the data and the smoothness of curved surface. We examined the influence of each parameter setting in detail and evaluated appropriate parameter setting criteria for DEM generation.

Keywords: Surface generation, BS-Horizon, Bi-cubic spline interpolation, Equality-inequality constraints.

1. Introduction

Commonly used interpolation methods take into account only actual data points or contour lines which we refer to as equality constraints. Shiono et al. (2001) has developed an algorithm, namely Horizon2000, for determination of geological surfaces based on field survey observational including inequality (relationships such as “above” or “below”) and slope information. Noumi (2003) applied Horizon2000 method for generation of topographic surface using inter-contour height information expressed as relation of being higher or lower than the assigned contour values (inequality constraint). Horizon2000 approximates bi-linear functions within a grid cell and estimates the optimal surface using exterior penalty function method. In the exterior penalty function method, the constrained problem is transformed to unconstrained problem and optimal surface is determined by minimizing the error of this unconstrained problem by iterative calculation with a series of penalty parameter. The penalty approach does not exactly solve the constrained problem but it is reasonably close. An improved BS-Horizon method developed by Nonogaki et al. (2012) performs surface generation with equality-inequality constraints using bi-cubic spline interpolation and an exterior penalty function. Further, the integration of piece-wise bi-cubic spline algorithm in BS-Horizon enables surface generation for larger datasets and at finer spatial resolution. Effective use of equality-inequality constraints has also been demonstrated in estimation of geological surface. This method has also been suggested as being suited to estimate topographical surface from elevation data (Nonogaki et al., 2012). However, the effects of parameters settings on surface generation using BS-Horizon remains to be fully investigated. Tran et al. (2015; 2016) have conducted experiments on investigating the parameter values suitable for DEM generation using BS-Horizon. This study takes up a comprehensive investigation of parameter settings for BS-Horizon method and evaluating the effects on resultant topographic surface, with the aim to provide a better understanding of surface generation using equality-inequality point elevation data.

2. BS-Horizon Theory

BS-Horizon program (Nonogaki et al., 2012) was developed based on bi-cubic spline interpolation algorithm and exterior penalty function method. This program takes advantage of applying bi-cubic spline on smaller tiles (sub-domains) instead of the whole surface domain for better local approximation results. Method used in BS-Horizon program can
be briefly explained as below.

Suppose a situation that we determine a topographic surface \( z = f(x, y) \) in a Cartesian coordinate system. The elevation \( z_i \) at a point \((x_i, y_i)\) can be expressed by one of following three constraints:

\[
\begin{align*}
    f(x_i, y_i) - z_i &= 0 \quad (1) \\
    f(x_i, y_i) - z_i &< 0 \quad (2a) \\
    f(x_i, y_i) - z_i &> 0 \quad (2b)
\end{align*}
\]

Equality constraint (1) is used in cases when the surface passes through the known point elevation. Inequality constraint (2a) denotes that surface passes below and (2b) above a given elevation constraint. In order to distinguish three types of constraints, a parameter \( l \) is introduced with values as \( l = 0 \) for a constraint (1), \( l < 0 \) for (2a) and \( l > 0 \) for (2b).

Many feasible solutions that satisfy conditions (1), (2a) and (2b) may exist (Shiono et al., 2002). It is assumed that the topographic surface must be the smoothest one among the feasible solutions. Further, considering the surface determination as a constrained optimization problem, we need to find a surface \( f(x, y) \) that minimizes an objective function:

\[
J(f) = m \int_{\Omega} \left( \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \right)^2 \mathrm{d}x \mathrm{d}y / S + m \int_{\Omega} \left( \frac{\partial^2 f}{\partial x \partial y} + 2 \frac{\partial^2 f}{\partial y^2} \right)^2 \mathrm{d}x \mathrm{d}y \quad (3)
\]

In the equation (3), \( J(f) \) is the function for evaluating the smoothness of surface \( f(x, y) \). When \( J(f) \) is small, the surface smoothness is emphasized, and vice versa. \( \Omega \) denotes the rectangular domain of surface \( f(x, y) \) in the \( x \)-\( y \) plane and \( S \) represents the area of \( \Omega \). \( m, m_1, m_2 \) are parameters to control the balance between the first and the second terms. In order to solve the constrained optimization problem above, an augmented objective function is introduced as shown in equation (4)

\[
Q(f, \alpha) = J(f) + \alpha R(f) \quad (4)
\]

where \( Q(f) \) evaluates the smoothness of the surface. \( R(f) \) denotes an exterior penalty function for evaluating the goodness of fit. \( \alpha \) is a penalty parameter \((\alpha > 0)\) that controls the balance between \( J(f) \) and \( R(f) \). \( R(f) \) indicates the mean square of residuals for data which do not satisfy constraints. It is also considered as the degree of violations of constraints. \( R(f) \) is defined by equation (5):

\[
R(f) = \frac{1}{n} \sum_{i=1}^{N} E_i^2 \quad (5)
\]

where \( n \) is the number of elevation data which do not satisfy elevation constraints. \( N \) is the total number of data. \( n = N \) if only equality data are taken into account in interpolation. \( E_i \) is the residual between theoretical value and observed elevation at each observation point as below:

\[
E_i = f(x_i, y_i) - z_i \quad (6)
\]

In case that we use only equality constraints (1), the function \( f(x, y) \) minimizing the augmented objective function \( Q(f, \alpha) \) with a moderately large \( \alpha \) gives the optimal topographic surface. In case that we use all types of constraints (1) - (2b), we need to perform iterative calculations with increasing series of penalties \( \{\alpha_1, \alpha_2, \ldots, \alpha_NT\} \) to find the optimal solution (Shiono et al., 2001, 2002; Noumi, 2003; Masoud, 2003; Nonogaki et al., 2009, 2012). In BS-Horizon, increasing series of penalties are generated as follows:

\[
a_j = a_1 \times r^{j-1} \quad (i = 1, 2, \ldots, NT) \quad (7)
\]

where

\[
r = \left( \frac{\alpha_{NT}}{\alpha_1} \right)^{\frac{1}{NT-1}} \quad (8)
\]

\( NT \) is the number of iteration, \( \alpha_1 \) is an initial penalty \( \alpha_{min} \) and \( \alpha_{max} \) is the maximum value \( \alpha_{max} \). At the initial stage of the iteration, the value of \( J(f) \) is small and value of \( R(f) \) is large, because the surface \( f(x, y) \) does not satisfy the constraint conditions (Noumi, 2003). With the increase of \( \alpha \), the surfaces gradually tend to satisfy the constraint conditions. It is advisable to start with a small \( \alpha_{min} \) and increase \( \alpha \) slowly for stable convergence.

Several parameters such as \( M, M_1, \alpha_{min}, \alpha_{max}, NT, m_1, m_2 \) need to be set for BS-Horizon. In summary, they can be explained as below:

- \( M \) and \( M_1 \): the numbers of tiles (sub-domains) that constitute the surface domain \( \Omega \) in which bi-cubic spline function is applied.
- \( \alpha_{min} \) and \( \alpha_{max} \): the range for penalty parameter \( \alpha \) in the exterior penalty function. \( \alpha \) controls the balance between smoothness of surface \( J(f) \) and goodness of fit to constraints \( R(f) \).
- \( NT \): number of iterations for a given \( \alpha \) range.
- \( m_1 \) and \( m_2 \): \( m_1 \) is parameter for minimizing slope of surface denoted by the first order partial derivative and \( m_2 \) is the one for minimizing curvature of surface denoted by the second order partial derivative of spline function as shown in equation (3).

3. Data Used

The elevation data covers lowland area of 16 square kilometers (4 by 4 km) obtained from the Department of Natural Resource and Environment, Danang City (Figure 1a). The test site is located in Danang City, Vietnam with the major 1A National Road across and Qua Giang and Cai rivers that flow through area (Figure 1b). The topography is relatively flat with elevation range from 0 to 7.2 m and 9,730 point elevations (Figure 1a) have been collected during field survey in 2009. The average distance between measured elevation points is around 40 m and the observational accuracy is ±10 cm.

Table 1 and Figure 2 describe the distribution density of
point elevation data for different tiles sizes ($M_x, M_y$). Data in Table 1 shows the frequency of point elevation in each tile for different cases of $M$ settings ($M_x = M_y = 50$ to $M_x = M_y = 400$). When $M = 50$, the data distribution varies from 0 to 12 points per tile. The most frequent concentration of data in case of $M = 50$ is 4 points per tile (Table 1). When $M$ size is increased, the distributed density becomes smaller. In cases of $M = 300$ and $M = 400$, elevation point data distribute only within 0, 1 or 2 points per tile (Figure 2). When $M$ increases the number of tile having no data also increases since the tile size is smaller (Table 1). From $M = 140$ and above, there are more than 50 percent of tiles having no data. The BS-Horizon surface generation in the tile having no data will take elevation information from surrounding tiles for estimation. It is observed that there is no observational point elevation along the rivers. The points adjacent to rivers also have low elevation. This condition easily causes the underestimated elevation in the interpolated surface which will be discussed in the following sections. In addition, the 1A National Road constructed in such lowland area with higher elevation than surrounding area and is one of the reasons of anomalous representation of interpolated surfaces that is also taken into account in further discussion.

![Figure 1. Location of study area including: (a) field survey point elevation, (b) Satellite RapidEye imagery in 2014.](image)

### Table 1. Distribution of field survey elevation points for cases of $M$

| $M_x (= M_y)$ | 50   | 70   | 90   | 100  | 120  | 140  | 150  | 200  | 250  | 300  | 400  |
|---------------|------|------|------|------|------|------|------|------|------|------|------|
| Tile size (m) | 80.00| 57.14| 44.44| 40.00| 33.33| 28.57| 26.67| 20.00| 16.00| 13.33| 10.00|
| Number of points |
| No data       | 0    | 116  | 545  | 1853 | 3010 | 6330 | 10880| 13525| 3012 | 52866| 80311|
| 1             | 177  | 1355 | 3644 | 4759 | 6611 | 7782 | 8260 | 9253 | 9539 | 9648 | 9696 |
| 2             | 342  | 1465 | 1902 | 1794 | 1279 | 869  | 676  | 229  | 94   | 41   | 17   |
| 3             | 481  | 939  | 550  | 375  | 161  | 67   | 38   | 5    | 1    |     |     |
| 4             | 486  | 415  | 131  | 54   | 17   | 1    | 1    |     |     |     |     |
| 5             | 380  | 135  | 14   | 7    | 2    | 1    |     |     |     |     |     |
| 6             | 261  | 36   | 5    | 1    |     |     |     |     |     |     |     |
| 7             | 134  | 4    | 1    |     |     |     |     |     |     |     |     |
| 8             | 63   | 5    |     |     |     |     |     |     |     |     |     |
| 9             | 35   | 1    |     |     |     |     |     |     |     |     |     |
| 10            | 18   |     |     |     |     |     |     |     |     |     |     |
| 11            | 5    |     |     |     |     |     |     |     |     |     |     |
| 12            | 2    |     |     |     |     |     |     |     |     |     |     |
| Total tiles   | 2500 | 4900 | 8100 | 10000| 14400| 19600| 22500| 40000| 62500| 90000| 160000|
using only point elevation, the estimated surface mostly passes through given points, but may have overshoots and undershoots since there is no elevation control in the areas where point data are not available (unconstrained). When inequality data is taken into account, the interpolated surface tends to be constrained between 0 m and 10 m elevation. Therefore, elevation range of the generated surface does not much exceed the elevation range of study area, as in case of using only point elevation (Figure 3).

Since the present study area is characterized by only one constraint from 0 m to 10 m, inequality data included X, Y coordinates, elevation Z and the relationship to the constrained elevation l was generated at desired spatial interval (l = +1 for elevation higher than 0 m, and l = −1 for elevation below 10 m). Next, the point elevation (l = 0 for equality point elevation data) was combined with inequality constraints in CSV format and used as input for BS-Horizon program.

Figure 3 is a depiction of equality and inequality constrained data used in this case study. Topographic map of Danang City (scale 1:25,000, 2010) covering the study area indicates elevation of this area ranges between 0 m and 10 m contours. Therefore, elevation from 0 m to 10 m was selected as inequality constraint for the study area. In Figure 3, point elevations which are referred as equality constraints are shown as circles and inequality elevation are shown as triangles. In case of

4. Evaluating Effect of Parameter Settings for BS-Horizon

In this study, the evaluation of parameters settings is conducted using 5 m resolution output DEM. Hengl (2006) has suggested appropriate resolution on interpolated surface based on the size of study area and number of point elevation. Based on the method proposed by Hengl (2006), it is evident that 5 m spatial resolution could be suitable for DEM generation using the present dataset.

Surface approximation for different parameter settings was carried out. Mx and My were examined from 50 to 400. α was selected to range from 1 to $10^2$ and $m_1$, $m_2$ range between 0 and 1. In order to evaluate the effect of parameter settings on the BS-Horizon surface generation, firstly, DEMs were generated using only equality constraints. Subsequently, inequality constraint has been integrated with equality constraint which are then used as input for the BS-Horizon program.

4.1. Equality and Inequality Constraints

Figure 3 is a depiction of equality and inequality constrained data used in this case study. Topographic map of Danang City (scale 1:25,000, 2010) covering the study area indicates elevation of this area ranges between 0 m and 10 m contours. Therefore, elevation from 0 m to 10 m was selected as inequality constraint for the study area. In Figure 3, point elevations which are referred as equality constraints are shown as circles and inequality elevation are shown as triangles. In case of

In case where only equality constraint is used, the interpolated surface is expected to satisfy the given elevation data. Therefore, iterative calculation is redundant when using only equality constraint as input. When inequality constraints are taken into account, the exterior penalty function is estimated by an iterative calculation using increasing sequence of penalty α {α1 = αm1, α2, …, αNT = αmNT}. The iterative calculation with a ratio r is introduced based on equation (7). Ratio should be small enough to observe the trend and at the same time not be too computationally intensive. This study uses a ratio as $r = 1.189$ for calculation. Applying this ratio and α range from 1 to $10^2$ into equation (8) the suitable iteration in this case study is $NT = 161$.

At the beginning of iterative calculation, the algorithm starts with a roughly approximated surface in which the smoothness is high (low J(f) values) and the goodness of fit is low (high R(f) values). This is because there are many control points that do not satisfy the constraint conditions and can be further used for better estimations (Masoud, 2003). After the first approximation, point data that satisfy the constraints are
excluded from the next approximation as these become equality height data, and the control points that do not satisfy the equality as well as inequality constraint conditions are extracted. These points are used for generation of new surface at the next $\alpha$ and calculate the goodness of fit $R(f)$ at that $\alpha$ value. When the calculation at each sequence of $\alpha$ is over, the height data that do not satisfy the constraint conditions are extracted for determination of $R(f)$ to evaluate the approximated surfaces. The calculation is repeated until the maximum number of iteration $NT$ is reached. Parameters involved in iterative calculation, which are $M$, $\alpha$, $m_1$, $m_2$ and different inequality constrained intervals are discussed in the following sections.

4.2. Effects of $M$ and $\alpha$ Setting

4.2.1. $M$ and $\alpha$ Setting in Case of Using Only Equality Constraints

Figure 4 shows the trend of $R(f)$, $J(f)$ and $Q(f, \alpha)$ in different cases of $M$ and $\alpha$ when using equality data. Table 2 is the calculated values of these functions and the statistical parameters for corresponding DEMs. This result is obtained at $m_1 = 0$ and $m_2 = 1$. It can be observed from Figure 4 that the starting points of $R(f)$, $J(f)$ and $Q(f, \alpha)$ in all cases of $M$ are almost same, meaning that at the initial $\alpha$ setting, $R(f)$, $J(f)$ and $Q(f, \alpha)$ values are always obtained similar (Table 2 at $\alpha = 1.0 \times 10^0$).

When $\alpha$ increases, $R(f)$ tends to decrease indicating that the degree of violation of constraints is reduced, or surface gradually fit to elevation constraints. $R(f)$ ranges from 0.494 at $\alpha = 1.0 \times 10^0$ to 0.3361 at $\alpha = 1.0 \times 10^8$ when $M = 50$ (Table 2). When $M = 100$, $R(f)$ is decreased from 0.4914 at $\alpha = 1.0 \times 10^0$ to 0.1044 at $\alpha = 1.0 \times 10^8$. The rate of decrease in $R(f)$ with increasing of $\alpha$ is different depending on $M$ settings. At $M = 50$, $R(f)$ does not change much, from 0.494 to 0.3361 (Table 2). $R(f)$ is reduced from 0.4914 to 0.1044 in case of $M = 100$ and minimized from 0.4908 to 0.0025 in case of $M = 400$. Comparing different cases of $M$ settings, $R(f)$ show similar values at $\alpha = 1.0 \times 10^0$, but significant reduces at larger $M$ settings.

The smoothness of surface is considered as the minimization of function $J(f)$. Table 2 and Figure 4 denote that $J(f)$ increases with the increase of $\alpha$. When $M = 50$ (Figure 4a), $J(f)$ increases from $5.58 \times 10^{-2}$ at $\alpha = 1.0 \times 10^0$ to $4.43 \times 10^3$ at $\alpha = 1.0 \times 10^8$. $J(f)$ steeply increases in case of $M = 100$ as shown in Figure 4b (from $5.69 \times 10^{-2}$ to $5.44 \times 10^4$). In cases of $M = 200$ and $M = 400$, $J(f)$ slowly increase from $5.71 \times 10^{-2}$ to $9.07 \times 10^1$ ($M = 200$) and from $5.71 \times 10^{-2}$ to $5.32 \times 10^1$ ($M = 400$).

It is obvious that the rate of increase in $J(f)$ by the increase of

![Figure 4. Calculations of $R(f)$, $J(f)$ and the resulting $Q(f, \alpha)$ for cases of $M$ and $\alpha$ using equality constrained data](image-url)
| $M_x (= M_y)$ | $\alpha$  | $R(f)$  | $J(f)$  | $Q = J + \alpha^*R$ | Minimum  | Maximum  | Mean    | STD     | Variance |
|-------------|------------|---------|---------|----------------------|-----------|-----------|---------|---------|----------|
|             | 1.00×10^0 | 0.4940  | 5.58×10^{-2} | 5.49×10^{-1} | 0.1652 | 5.8912 | 2.3617 | 1.0219 | 1.0444 |
|             | 1.00×10^1 | 0.4010  | 3.45×10^{-1} | 4.35×10^{0}  | 0.1152 | 6.6204 | 2.3549 | 1.0871 | 1.1817 |
|             | 1.00×10^2 | 0.3527  | 1.48×10^{0}  | 3.67×10^{1}  | 0.5530 | 6.8349 | 2.3438 | 1.1354 | 1.2892 |
|             | 1.00×10^4 | 0.3378  | 9.70×10^{-3} | 3.39×10^{3}  | 11.6713 | 10.8697 | 2.3141 | 1.3246 | 1.7547 |
|             | 1.00×10^6 | 0.3655  | 1.22×10^{-2} | 3.37×10^{5}  | 66.4742 | 144.3825 | 2.2852 | 3.3532 | 11.2439 |
|             | 1.00×10^8 | 0.3614  | 4.43×10^{-5} | 3.35×10^{7}  | 27.73150 | 724.8530 | 2.0416 | 25.0283 | 626.4170 |
|             | 1.00×10^0 | 0.4914  | 5.69×10^{-2} | 5.48×10^{1}  | 0.1658 | 5.8918 | 2.3614 | 1.0221 | 1.0446 |
|             | 1.00×10^1 | 0.3836  | 4.01×10^{-1} | 4.24×10^{0}  | 0.1088 | 6.6389 | 2.3542 | 1.0876 | 1.1829 |
|             | 1.00×10^2 | 0.2787  | 2.97×10^{0}  | 3.08×10^{1}  | -0.4214 | 7.0120 | 2.3435 | 1.1390 | 1.2973 |
|             | 1.00×10^4 | 0.1499  | 6.33×10^{-1} | 1.56×10^{3}  | -3.6240 | 9.3363 | 2.3240 | 1.2746 | 1.6247 |
|             | 1.00×10^6 | 0.1169  | 1.33×10^{1}  | 1.18×10^{3}  | -48.1260 | 424.3933 | 2.2227 | 2.9601 | 8.7623 |
|             | 1.00×10^8 | 0.1044  | 5.44×10^{-4} | 1.05×10^{7}  | -3564.4590 | 473.9600 | 2.3254 | 19.1727 | 367.5920 |
| 200         | 1.00×10^0 | 0.4909  | 5.71×10^{-2} | 5.48×10^{1}  | 0.1660 | 5.8906 | 2.3614 | 1.0221 | 1.0446 |
|             | 1.00×10^1 | 0.3802  | 4.12×10^{-1} | 4.21×10^{0}  | 0.1089 | 6.6318 | 2.3541 | 1.0875 | 1.1827 |
|             | 1.00×10^2 | 0.2572  | 3.39×10^{0}  | 2.91×10^{1}  | -0.3604 | 6.9755 | 2.3436 | 1.1380 | 1.2950 |
|             | 1.00×10^4 | 0.0292  | 5.38×10^{-1} | 3.46×10^{2}  | -2.4126 | 7.9255 | 2.3323 | 1.1987 | 1.4369 |
|             | 1.00×10^6 | 0.0031  | 7.59×10^{-1} | 3.18×10^{3}  | -4.1085 | 7.9832 | 2.3314 | 1.2061 | 1.4547 |
|             | 1.00×10^8 | 0.0025  | 9.07×10^{-1} | 2.50×10^{5}  | -7.9285 | 10.7990 | 2.3287 | 1.2091 | 1.4620 |
| 400         | 1.00×10^0 | 0.4908  | 5.71×10^{-2} | 5.48×10^{1}  | 0.1659 | 5.8904 | 2.3614 | 1.0220 | 1.0446 |
|             | 1.00×10^1 | 0.3793  | 4.15×10^{-1} | 4.21×10^{0}  | 0.1089 | 6.6318 | 2.3541 | 1.0875 | 1.1826 |
|             | 1.00×10^2 | 0.2522  | 3.48×10^{0}  | 2.87×10^{1}  | -0.3465 | 6.9626 | 2.3437 | 1.1375 | 1.2940 |
|             | 1.00×10^4 | 0.0194  | 4.47×10^{-1} | 2.38×10^{2}  | -1.1456 | 7.8430 | 2.3340 | 1.1906 | 1.4175 |
|             | 1.00×10^6 | 0.0026  | 5.30×10^{-1} | 2.65×10^{3}  | -1.6891 | 7.8966 | 2.3336 | 1.1936 | 1.4247 |
|             | 1.00×10^8 | 0.0025  | 5.32×10^{-1} | 2.50×10^{5}  | -1.6966 | 7.6967 | 2.3317 | 1.1919 | 1.4208 |
Figure 5. DEMs generated from equality constrained data using different $M$ and $\alpha$ settings (Continued).

$M = 400$

$M = 200$

$M = 100$

$M = 50$

$\alpha_{01} = 0$

$\alpha_{01} = 0$

$\alpha_{01} = 0$
Figure 5. DEMs generated from equality constrained data using different $M$ and $\alpha$ settings.
The undershoot $\alpha$ values are small (around $10^{-3}$), $R(f)$ is gradually reduced by $\alpha = 50$ to $\alpha = 200$ and $R(f)$ tends to be minimized. This zone is usually in the middle $\alpha$ range. The elevation ranges are over or under estimation anomalies in the generated surface.

In all cases of $M$, DEMs generated at $\alpha = 1.0 \times 10^{0}$ are the smoothest surfaces since $J(f)$ values are minimized. With the increase in $\alpha$, surface smoothness is gradually lost since $J(f)$ is increased. In cases of $M$ values from $M = 50$ to $M = 100$, although $R(f)$ is gradually reduced by $\alpha$, $J(f)$ values are significantly increased. Therefore, the interpolated surfaces tend to be more undulated with a large number of undershoot and overshoot elevations presented on DEMs, especially for larger $\alpha$ settings (Figure 5 and Table 2). The minimum and maximum elevations of DEMs at upper $\alpha$ range, in case of $M = 50$ and $M = 100$, also exceed the elevation range of study area (Table 2). The elevation ranges are from −773.71 m to 924.85 m when $M = 50$ and $\alpha = 1.0 \times 10^{3}$, and in between −564.46 m to 473.96 m when $M = 100$ and $\alpha = 1.0 \times 10^{4}$. The elevation ranges are continuously extended with more overshoot and undershoot when $\alpha$ increases to $1.0 \times 10^{12}$. These elevations largely exceed the elevation range of study area. In case of $M = 200$ and $M = 400$, surfaces are not so much undulated (Figure 5) since $J(f)$ values slowly increase and $R(f)$ values become stable. Elevation ranges do not differ significantly from elevation range of input data, as in case of $M = 50$ and $M = 100$. The undershoot and overshoot elevations are still observed on the generated surface in case of $M = 200$ and $M = 400$, however the number constraint violations are less than in previous cases. In the following section, we discuss about incorporating inequality data and discuss the results.

4.2.2. $M$ and $\alpha$ Settings for Equality-inequality Constraints

Figure 6 shows the graph representing values of $J(f)$ and $R(f)$ for different $M$ settings. In cases of $M$ values from 50 to 200 (Figure 6a, 6b, 6c), there are three different zones of $J(f)$, $R(f)$ and $Q(f, \alpha)$ for $\alpha$ values ranging from $1.0 \times 10^{0}$ to $1.0 \times 10^{12}$. Zone A shows the steady increase of $J(f)$, $Q(f, \alpha)$ and steady decrease of $R(f)$ which corresponds to the initial $\alpha$ values. In zone B, $J(f)$ and $Q(f, \alpha)$ continuously increase and $R(f)$ tends to be minimized. This zone is usually in the middle $\alpha$ range. In zone C, all of the functional values show spikes especially $R(f)$ which reflects the degree of violation of constraints becomes maximum.

In cases of $M$ settings from $M = 250$ to $M = 400$, there are no $C$ zone observed (Figure 6d). $J(f)$ and $R(f)$ become constant in the upper range of $\alpha$, and $Q(f, \alpha)$ still continuously increases. The degree of constraint violation $R(f)$ is also minimized with value around 0.0025 in zone B (Figure 6). $R(f)$ is obtained as minimum value and the B zone does not change from $M = 300$. Figure 7 and Table 3 show DEMs generated from equality-inequality constrained data using different $M$ and $\alpha$ settings. At initial $\alpha$ settings ($\alpha = 1.0 \times 10^{0}$ and $\alpha = 1.19 \times 10^{5}$), $J(f)$, $Q(f, \alpha)$ and $R(f)$ in all cases of $M$ obtain similar values and statistical parameters for generated surfaces are almost similar. DEM surfaces at $\alpha = 1.0 \times 10^{0}$ are almost flat surface with elevation range from 0 m to 0.1 m. In the second iteration settings ($\alpha = 1.19 \times 10^{5}$) the generated surfaces gradually express the topographical characteristics such as higher or lower elevation, and the contours slightly become undulated (Figure 7). Surfaces in case of $\alpha = 1.19 \times 10^{0}$ are the smoothest ones as shown in Figure 7. In case of $\alpha = 1.06 \times 10^{5}$ the generated surfaces are still smooth since $J(f)$ values are small (around $4.6 \times 10^{-2}$, Table 3). DEMs in case of $\alpha = 1.0 \times 10^{3}$ and $\alpha = 1.12 \times 10^{5}$ are more clearly representing topographic features, for example the roads and rivers. The generated contours show more details compared to previous cases (Figure 7). The increase of $\alpha$ leads to the reduction of $R(f)$ and increase in $J(f)$ values, $Q(f, \alpha)$ is also increases exponentially (Table 3). Surface smoothness is lost since $J(f)$ increases with increase in $\alpha$ (Figure 7). In cases of $M = 50$ to $M = 200$, $R(f)$ values become spikes for higher $\alpha$ values (Figure 6, from $\alpha = 1.0 \times 10^{8}$ to $\alpha = 1.0 \times 10^{12}$) and the generated surfaces depict anomalies with number of undershoot and overshoot elevation as shown in Figure 7 and Table 3. DEM generated at $M = 200$ and $\alpha = 1.19 \times 10^{5}$.
has the elevation range from −504.51 m to 539.33 m which are also much exceeding the elevation range of the study area. In case of \( M = 250 \) and above, surfaces generated at \( \alpha \) from \( \alpha = 1.0 \times 10^8 \) to \( \alpha = 1.0 \times 10^{12} \) are almost similar since \( R(f) \) and \( J(f) \) become constant (Figure 6d). \( R(f) \) is minimized to value around 0.0025 and does not change after \( \alpha = 1.0 \times 10^8 \) as shown in Table 3 and Figure 6d when \( M = 400 \).

### 4.2.3. Evaluation of \( M \) and \( \alpha \) Settings

In the present study, DEMs are generated using 9,730 point elevations with the observational accuracy at ±10 cm. If the generated surface has the Root Mean Square Error (RMSE = \( \sqrt{R(f)} \)) of 0.1 m or the Mean of Square Error \( R(f) = (0.1)^2 = 0.01 \) m$^2$, it can satisfy the given elevation constraints. However, in the exterior penalty function, the minimization of \( R(f) \) results in the maximization of \( J(f) \) and therefore the smoothness of surface can be sacrificed when \( R(f) \) is 0.01 m. In order to balance the degree of constraint violation and the smoothness of surface, a RMSE = 0.5 or \( R(f) = (0.5)^2 = 0.25 \) has been proposed for evaluating the results of BS-Horizon surface generation. The surfaces having \( R(f) \leq 0.25 \) are considered satisfying elevation constraint and being appropriately smooth. Figure 8 shows the surfaces which have \( R(f) \leq 0.25 \) in both cases using equality and equality-inequality constraints, and Table 4 is the statistical results of the corresponding surfaces. In case of \( M = 50 \), there is no surface satisfying the constraints since \( R(f) \) is always more than 0.3 (Table 2 and Table 3). The degree of constraint violation \( R(f) \) is not much reduced, hence \( M = 50 \) is not considered for BS-Horizon DEM generation in this case study. When using \( M = 100 \) (Table 4), DEM generated from equality data obtains \( R(f) = 0.2485 \) at \( \alpha = 2.0 \times 10^7 \), and the surface from equality-inequality data has \( R(f) = 0.2482 \) at \( \alpha = 2.66 \times 10^4 \). \( J(f) \) values in this case are around 5.24 as shown in Table 4. However, \( R(f) \) and \( J(f) \) are even improved at larger \( M \) settings. In case of \( M = 200 \), \( R(f) \) is obtained as \( R(f) = 0.2461 \) at \( \alpha = 1.20 \times 10^2 \) (Equality input) and \( R(f) = 0.2463 \) at \( \alpha = 1.58 \times 10^4 \) (Equality-inequality input) (Table 4). In this case, \( J(f) \) values are around 4.00 which is slightly smaller compared to the case \( M = 100 \). When \( M = 400 \), \( R(f) = 0.2461 \) at \( \alpha = 1.10 \times 10^7 \) (Equality input) and \( R(f) = 0.2407 \) at \( \alpha = 1.58 \times 10^4 \) (Equality-inequality input). DEMs in this case have \( J(f) \) around 3.80 to 4.09 which are not much different from the case \( M = 200 \). Comparing surfaces having similar values of \( R(f) \leq 0.25 \) (Table 4), it is observed that DEMs at \( M = 100 \) are generated at larger \( \alpha \) setting (\( \alpha = 2.00 \times 10^7 \) and \( \alpha = 2.66 \times 10^4 \)), therefore \( J(f) \) and \( Q(f, \alpha) \) values are obtained as larger than in the cases of \( M = 200 \) and \( M = 400 \). The elevation ranges in case \( M = 100 \) show several negative undershoot both in the case of...
### Table 3. DEMs from equality-inequality data for different $M$ and $\alpha$

| $M$ (= $M_r$) | $\alpha$ | $\xi_f$ | $\varphi_f$ | $J(\xi_f)$ | $Q(\varphi_f)$ | Minimum | Maximum | Mean | STD | Variance |
|---------------|----------|---------|-------------|-------------|---------------|---------|---------|------|------|---------|
| 1             | 1.00×10^0 | 2.6200  | 2.62×10^0  | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 |
| 2             | 2.00×10^0 | 2.6200  | 2.62×10^0  | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 |
| 5              | 1.00×10^0 | 2.6200  | 2.62×10^0  | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 |
| 10             | 2.00×10^0 | 2.6200  | 2.62×10^0  | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 |
| 20             | 1.00×10^0 | 2.6200  | 2.62×10^0  | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 |
| 50             | 2.00×10^0 | 2.6200  | 2.62×10^0  | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 |
| 100            | 1.00×10^0 | 2.6200  | 2.62×10^0  | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 |
| 200            | 2.00×10^0 | 2.6200  | 2.62×10^0  | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 |
| 400            | 1.00×10^0 | 2.6200  | 2.62×10^0  | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 | 0.0017 0.0190 | 0.0360 0.0208 | 2.6200  | 2.62×10^0 |

Evaluating Parameters for BS-Horizon Surface Generation Using Elevation Data
Figure 7. DEMs generated from equality-inequality constrained data using different $M$ and $\alpha$ settings (Continued)
Figure 7. DEMs generated from equality-inequality constrained data using different $M$ and $\alpha$ settings.
using equality and equality-inequality constraints. Therefore, 
$M = 200$ and $M = 400$ can be considered as more appropriate parameters since they can lead to satisfying the elevation constraints and can minimize the objective functions $J(f)$ and $Q(f, \alpha)$. In these cases ($M = 200$ and $M = 400$) the generated DEM surfaces have very similar values in both $R(f)$, $J(f)$ and $Q(f, \alpha)$, and the statistics of generated surfaces are not much different (Table 4). However $M = 200$ setting consumes less processing time than the case of $M = 400$. Therefore, DEMs in case of $M = 200$ that having $R(f) \leq 0.25$ ($\alpha = 1.20 \times 10^2$ in case of equality data and $\alpha = 1.58 \times 10^4$ in case of equality-inequality data) can be considered as satisfying the elevation constraints for the present dataset.

4.3. Effect of Inequality Constrained Interval

In this section, we examine the effect of intervals of inequality constraint on the surface generation. Firstly, inequality data was created at 10 m, 5 m, 2.5 m and 1 m intervals that were then combined with equality points to generate input data. Subsequently, DEMs were generated at spatial resolution as the inequality data intervals. These DEMs were then re-interpolated to 5 m resolution using the estimated elevation data obtained from BS-Horizon program. A FORTRAN interpolation program named APP4OPT developed by Nonogaki et al. (2009) was used for this processing. APP4OPT allows interpolation of surface obtained in BS-Horizon method to different resolutions by specifying output region and grid size. Table 5 shows results of the experiment for creating 5 m DEMs using input data of different intervals and statistics of each DEM. In this case, $M_x = M_y = 200$, $m_1 = 0$, $m_2 = 1$ and $\alpha$ are various based on the condition that $R(f) \leq 0.25$.

Statistical results from Table 5 indicate that there are no significant differences between 5 m DEMs generated from different inequality constraint intervals. All of these DEMs have $R(f) \leq 0.25$, $J(f)$ ranges from 3.91 to 4.25, and other statistical parameters such as minimum, maximum and mean are closely similar (Table 5). $Q(f, \alpha)$ values are different since $\alpha$ settings change with the change in constraint intervals. Considering distribution of field data and statistical results of DEMs generated from different inequality constraints, 5 m resolution is confirmed to be suitable for BS-Horizon DEM generation with the present dataset.

![Figure 8. Surfaces generated from equality constraints (up) and equality-inequality constraints (down) with $R(f) \leq 0.25$](image)
4.4. Evaluating Effect of $m_1$ and $m_2$ Settings

In equation (3), $m_1$ and $m_2$ are weighting coefficients to control the balance between the first and the second order partial derivative terms in $J(f)$ which represent the slope and curvature of surface respectively. $m_1$ and $m_2$ were set at different values starting from $m_1 = 0.0$, $m_2 = 1.0$ to $m_1 = 0.9$, $m_2 = 0.0$ with increment step of 0.1. These $m_1$, $m_2$ settings were applied for different cases of $M$ and $\alpha$ to evaluate behavior of interpolated surfaces.

Figure 9 are illustrations of DEMs derived from equality constraints in case of $m_1 = 0.0$, $m_2 = 0.5$, $m_1 = 0.9$ and $m_2 = 1.0$ and Table 6 are statistical parameters of these DEMs. In Figure 9a and Table 6a, DEMs are generated at $M = 200$ and $\alpha = 1.20 \times 10^2$. Figure 9b and Table 6b show the surfaces having $R(f) \leq 0.25$ in different $m_1$, $m_2$ settings. It can be seen that the generated surfaces gradually change from $m_1 = 0.0$ to $m_1 = 0.9$, but significantly different in cases of $m_1 = 1.0$. Table 6a indicates that from $m_1 = 0.0$ to $m_1 = 0.9$, $R(f)$ reduces from $R(f) = 0.2461$ to $R(f) = 0.1086$. $J(f)$ also reduces from $J(f) = 4.00$ to $J(f) = 2.42$ and the statistical parameters show the gradual extension of elevation ranges to minimum and maximum. In case of $m_1 = 1.0$, $m_2 = 0.0$, $R(f)$ is drastically minimized to value of $R(f) = 0.0067$, $J(f)$ is also drastically reduced to $J(f) = 0.207$ (Table 6a) and the surface represents many spikes as shown by densely spaced contours in some places (Figure 9a). This is because there is no weight for the second term of $J(f)$ in case of $m_1 = 1.0$, $m_2 = 0.0$. The surfaces generated at $m_1 = 0.0$ to $m_1 = 0.5$ are almost similar (Figure 9a). Surface in case of $m_1 = 0.9$ is propagated by several flat topographic parts which shown as rounded and widen contours where point elevations are located (Figure 9a). At the transitions or boundaries between different topographic areas, the surface becomes steep with dense contours covering (Figure 9a). In general, the increase of $m_1$ setting tends to flatten surfaces and produce steep surfaces in the transition zones.

Figure 9b and Table 6b are the comparisons of DEMs generated in different $m_1$, $m_2$ under the condition of $R(f) \leq 0.25$. In this case, $M = 200$ and $\alpha$ values are selected so that $R(f)$ values are similar to each other. It is observed that $J(f)$ and $Q(f, \alpha)$ gradually decrease with the increase of $m_1$ from 0.0 to 0.9 and the minimum elevation of DEMs become closer to elevation 0 m. (Table 6b). There is almost no difference between generated surfaces (Figure 9b). The surface generated at $m_1 = 1.0$, $m_2 = 0.0$ is significantly different to other surfaces as shown by differences in $J(f)$, $Q(f, \alpha)$ and elevation range (Table 6b). The contours in this DEM are also different from contours in other surfaces (Figure 9b).

We considered DEM generated at $m_1 = 0.5$, $m_2 = 0.5$ (the middle values between $m_1 = 0.0$ and $m_1 = 1.0$) as the appropriate surface for BS-Horizon DEM generation, since the values of $R(f)$, $J(f)$ and $Q(f, \alpha)$ are reduced and surface representation is reasonably expressed.

Effect of $m_1$ and $m_2$ above has been considered in case of $M = 200$ from $\alpha = 0.65 \times 10^3$ to $\alpha = 1.20 \times 10^3$. Larger $\alpha$
settings (from $\alpha = 1.0 \times 10^6$ and above) in cases of $M$ values from 50 to 100 lead to many overshoot or undershoot in the DEM surfaces as discussed in previous section. Therefore, effect of $m_1$ and $m_2$ parameters is not considered in these cases. When $M = 150$ and above, the topographic change from $m_1 = 0.0$ to $m_1 = 1.0$ are obvious if $\alpha$ ranging from $1.0 \times 10^0$ to $1.0 \times 10^6$. In the upper range of $\alpha$ ($\alpha = 1.0 \times 10^6$ to $\alpha = 1.0 \times 10^2$) surfaces are not much difference from $m_1 = 0.0$ to $m_1 = 0.9$ since the RMS error ($\sqrt{\mathcal{R}(f)}$) are similar and the surfaces fit to most of constraint elevations. It is noted that DEM in case of $m_1 = 1.0$ always generates the roughest surface.

Effect of parameters $m_1$ and $m_2$ on interpolated surface using equality-inequality constraints is similar as in case of using equality data. Surfaces show smaller $R(f)$, $J(f)$ and $Q(f, \alpha)$, and become flat when $m_1$ increases and $m_2$ decreases. In case of small $\alpha$ ($\alpha = 1.0 \times 10^6$ to $\alpha = 1.0 \times 10^0$) the change in surfaces representations based on different $m_1$, $m_2$ can be observed clearly. At larger $\alpha$ setting, topographic surfaces show almost no change with variation of $m_1$, $m_2$ since $R(f)$ is minimized and does not change for different cases. The problems also arise in

![Image](image-url)

Figure 9(a). DEM generated from equality constraints for $m_1$ and $m_2$ settings (Parameter $M = 200$, $\alpha = 1.2 \times 10^2$)

Table 6(a). Statistics of 5 m DEMs for different $m_1$ and $m_2$ settings

| $m_1$ | $m_2$ | $R(f)$ | $J(f)$ | $Q(f, \alpha)$ | Minimum | Maximum | Mean | STD | Variance |
|-------|-------|--------|--------|----------------|---------|---------|------|------|----------|
| 0.0   | 1.0   | 0.2461 | 4.0066 | $3.35 \times 10^1$ | -0.4417 | 7.0235  | 2.3428 | 1.1415 | 1.3031   |
| 0.1   | 0.9   | 0.2397 | 3.9679 | $3.27 \times 10^1$ | -0.4835 | 7.0506  | 2.3425 | 1.1437 | 1.3080   |
| 0.2   | 0.8   | 0.2325 | 3.9122 | $3.18 \times 10^1$ | -0.5284 | 7.0796  | 2.3420 | 1.1458 | 1.3129   |
| 0.3   | 0.7   | 0.2243 | 3.8636 | $3.08 \times 10^1$ | -0.5766 | 7.1108  | 2.3416 | 1.1482 | 1.3184   |
| 0.4   | 0.6   | 0.2147 | 3.7905 | $2.95 \times 10^1$ | -0.6279 | 7.1443  | 2.3410 | 1.1509 | 1.3246   |
| 0.5   | 0.5   | 0.2033 | 3.6942 | $2.81 \times 10^1$ | -0.6821 | 7.1805  | 2.3404 | 1.1538 | 1.3313   |
| 0.6   | 0.4   | 0.1894 | 3.5614 | $2.63 \times 10^1$ | -0.7373 | 7.2199  | 2.3397 | 1.1577 | 1.3403   |
| 0.7   | 0.3   | 0.1715 | 3.3654 | $2.39 \times 10^1$ | -0.7884 | 7.2629  | 2.3389 | 1.1622 | 1.3506   |
| 0.8   | 0.2   | 0.1471 | 3.0459 | $2.07 \times 10^1$ | -0.8188 | 7.3094  | 2.3378 | 1.1677 | 1.3637   |
| 0.9   | 0.1   | 0.1086 | 2.4216 | $1.55 \times 10^1$ | -0.7570 | 7.3577  | 2.3367 | 1.1747 | 1.3799   |
| 1.0   | 0.0   | 0.0067 | 0.2070 | $1.01 \times 10^1$ | -0.9906 | 7.3070  | 2.3406 | 1.1385 | 1.2963   |
5. Discussion

5.1. Comparing BS-Horizon DEM Generation

Considering surface behaviors using only equality and combining equality-inequality data, it can be observed in DEM derived from equality data that elevation constraints are satisfied immediately and RMS errors at initial $\alpha$ setting are small ($R(f)$ around 0.49 at $\alpha = 1.0 \times 10^0$). The elevation range of DEM generated at $\alpha = 1.0 \times 10^0$ is also similar to the elevation range for the study area (Table 2). In case of DEM derived from equality-inequality constraints, surfaces generated at initial $\alpha$ values usually do not satisfy constraints. Table 3 shows $R(f)$ around 2.62 and $J(f)$ around $2.7 \times 10^{-5}$ at $\alpha = 1.0 \times 10^0$ in all cases of $M$ settings, elevation ranges between $-0.0017$ m to 0.119 m which is mostly flat surfaces. With increase in $\alpha$, the estimated surface gradually satisfies constraint conditions.
Although DEMs generated from only equality data tend to satisfy most of given elevation constraints and the RMS errors are small at the first $\alpha$ setting, the surfaces still show strong oscillations since the number of elevation constraints are limited. In the area where point elevations are not available, the estimated results can significantly exceed the elevation range of study area. As shown in Table 2, the minimum and maximum elevation of DEM from equality data are $-7.9285$ m and $10.799$ m when $M = 200$ and $\alpha = 1.0 \times 10^5$. In case of DEM derived from equality-inequality data, the inequality constraints have controlled the estimated elevations and therefore the variations in elevation are not as much as in case of using only equality constraints. The elevation range of DEM generated from equality-inequality constraints is from $-0.0202$ m to $7.983$ m when $M = 200$ and $\alpha = 1.0 \times 10^5$ (Table 3).

DEMs generated from only equality data usually show significant undershooting. These undershoots mainly locate on rivers where there are no field data or at the transition between two different land-use mainly at the boundaries between paddy field and rivers and around roads or other man-made structures (Figure 5). These areas usually have significant differences in elevation between the adjacent areas. The integration of inequality data into BS-Horizon enhances the number of elevation constraints, and hence the undershooting elevation is reduced in the approximation results. However, DEMs generated from equality-inequality constraints still retain places with undershoots, especially when $\alpha$ settings are increased. It is recognized that spline algorithms have problem in representing discrete transitions and often 'undershoot' at the edges of floodplains or other breaks in slopes (Hengl and Reuter, 2008).

5.2. Selection of Parameters for BS-Horizon DEM Generation

Experiments with different parameter settings result in different representations of the generated surface. In general, BS-Horizon program generates surfaces which tend to satisfy the elevation constraints. Equality-inequality data provide better input elevation data for DEM generation as discussed in previous section. Considering the criterion that surface having $R(f) \leq 0.25$ appropriately satisfies the elevation constraints in this case study, $M = 50$ cannot be used since there is no DEM satisfy the given criterion. $M = 100$ setting can lead to generation a surface with $R(f) \leq 0.25$. However better $J(f)$ and other statistical parameters can be obtained in the larger $M$ settings. Surfaces generated in cases of $M = 200$ and $M = 400$ are very similar in all statistical parameters. Therefore $M = 200$ is considered appropriate for BS-Horizon DEM generation in this case study considering the achievement of $R(f) \leq 0.25$ and the acceptable processing time.

On $\alpha$ parameter which controls the balance between the smoothness of surface $J(f)$ and the goodness of fit $R(f)$, it has been observed that DEMs generated at $\alpha = 1.0 \times 10^5$ are the smoothest surfaces since $J(f)$ values are smallest. With increase in $\alpha$, although $R(f)$ decreases, $J(f)$ and $\alpha$ increase exponentially and hence the values of augmented objective function $Q(f, \alpha)$ also increase exponentially. Increase in $\alpha$ results in loss of surface smoothness and appearance of artifacts in the generated surfaces. Comparing surfaces generated at different $\alpha$ in Figure 5, Figure 7 and Figure 8, it can be suggested that for the present dataset, DEM at $\alpha = 1.20 \times 10^7$ and $M = 200$ is the appropriate surface when we use only equality constraints and the corresponding parameters for DEM generated from equality-inequality constraints are $\alpha = 1.58 \times 10^7$ and $M = 200$.

Parameters $m_1$ and $m_2$ also should be appropriately selected. All of $R(f)$, $J(f)$ and $Q(f, \alpha)$ are reduced when $m_1$ increases and $m_2$ decreases. However, surfaces tend to become flat in areas where point elevation available and steep in the transition between different topographic areas. Considering the effect of minimizing the degree of constraint violations, the maximization of surface smoothness and the representation of reasonable topographical characteristics, we have selected $m_1 = 0.5$ and $m_2 = 0.5$ as parameter settings for surface generation.

The reported results represent a comprehensive attempt to evaluate effects of various parameters in the bi-cubic spline algorithm implemented in BS-Horizon program. The BS-Horizon program provides the distinct advantage of combining equality and inequality constraints and affords better topographic surface representation. The parameters selected for BS-Horizon DEM generation in this case study are equality-inequality constrained input data at 5 m resolution, $M_s = M_e = 200$, $\alpha = 1.58 \times 10^7$, $m_1 = 0.5$ and $m_2 = 0.5$. The parameters in BS-Horizon may be different for other datasets depending on the satisfaction of elevation criteria and topographic characteristics of study area. It is therefore suggested that suitable parameters need to be carefully examined, considering the characteristics of particular study area and applications of the DEM. In order to use the BS-Horizon program effectively, the parameter $M$ should be investigated based on the assumed criteria at first. Subsequently, $\alpha$ need to be selected considering the need for the balance between $J(f)$ and $R(f)$. Finally, $m_1$ and $m_2$ need to be appropriately set for better representation of topographic surface.

6. Conclusion

This study presents a new procedure for DEM generation for lowland area in Danang City, Vietnam. The present experiment uses field survey point elevation data and bi-cubic
spline algorithm implemented in the BS-Horizon program which offers advantage of defining equality-inequality constraints to interpolate topographic surface. The generated 5 m resolution DEM provides reliable data for use in further applications such as flood inundation mapping. Evaluation of parameter settings for BS-Horizon topographic surface generation has provided a better understanding of effects of parameters such as $M$, $\alpha$, $m_1$ and $m_2$ that are involved in the generation. The results of these experiments are very important in effectively using available data for generating topographic surface considering local conditions.

As a future work, there is a need to further investigate the effects of parameter settings in areas of moderate and high relief. This would provide fuller understanding on appropriate parameter settings considering local topography and would help developing ways and means for adaptive parameter settings based on local topography. The results of the present paper will also be applied in generating high-resolution DEM for evaluation flood prone areas in larger tracts of the current area of our research in Central Vietnam.

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要旨

標高データを用いた BS-Horizon地形面推定のためのパラメータの検討

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限られた標高データから信頼できる DEM 作成するためには、曲面推定法を有効に利用する必要がある。標高データを用いて DEM を推定する方法の 1 つとして、3 次 B- スプラインと外点ペナルティ関数に基づく BS-Horizon（Nonogaki et al., 2012）がある。本研究では、BS-Horizon を用いて実際の標高データから DEM を推定する際の各種パラメータの効果を検討した。野外測量で観測した標高データのみを等式標高制約条件とした場合と、それに面の上下関係を示す不等式制約条件を加えた場合の 2 通りを実施した。パラメータとしては、スプラインの格子数 $M_x$, $M_y$、データの充足度 $\alpha$, および曲面の滑らかさ $m_1$, $m_2$ について検討した。各パラメータ設定の影響を詳細に検証し、デジタル標高モデル (DEM) 作成における適切なパラメータ設定基準を考察した。

キーワード：曲面推定、BS-Horizon、3 次 B- スプライン補間、等式・不等式制約条件