The mathematization process of students to understand the concept of vectors through learning realistic mathematics and ethnomathematics

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Abstract. The concept of vector was one of concepts that was difficult for students to understand. Realistic mathematics learning with ethnomathematics approach makes it easy for students to learn mathematics. The purpose of this study was to describe the mathematical process of students in understanding vector concepts through learning realistic mathematics and ethnomathematics. This research was the prototype stage of development research. The subjects of this study were 38 high school students in Bengkulu. Observation sheets, anecdotes and interview guides were the instruments of this research. Data was collected during the realistic mathematics learning process using the ethnomathematics approach. Data was analyzed qualitatively. The results of this study were the mathematical process of students in understanding the concept of vector by using selfie culture. Camera drones can stop in space. This was one of the multiplication properties of two perpendicular vectors. Multiplication of points was zero. The conclusion of this study was that the mathematical process of students using realistic selfie culture can achieve the multiplication of two vectors which form right angles.

1. Introduction
Based on a survey in math class, linear algebra was one of the mathematics learning materials at school. It was material that was often difficult for students to learn. One concept in linear algebra was vector. Also, the vector concept was one that was difficult for students to understand. According to Dorie [1] that teaching linear algebra was universally recognized as difficult. Students were overwhelmed by the many new definitions and lack of connection with prior knowledge. Teachers often feel frustrated when faced with the inability of their students to overcome ideas that they consider very simple. Usually, they burden the lack of practice in basic logic and establish theories or impossibilities for students to use geometric intuition.

Teaching linear algebra has always been a challenge for mathematics teachers. It was something that was very important for students to be introduced into complex and abstract mathematical systems. Also, learn concepts that can be successfully applied later in other mathematical topics [2]. Students have difficulty learning vector spaces. Like a collection of vectors and linear transformations obtained from a base vector image [3]. Therefore, it was important for teachers to better understand how students learn, and recognize and let the appropriate content, methods, and contexts differ in different environments [2]. In linear algebra learning, some teachers assign students with computer projects to do outside the classroom, and use computer demonstrations and class examples to enrich their learning
In mathematics learning, the need to acquire knowledge has become very clear. This was because it was relevant to everyday life and in various disciplines [5]. That was to give an idea that realistic learning was more appropriate to help students learn linear algebra. Because the learning environment was a context to make it easier for students to interact and develop concepts, then ethnomathematics was the right approach.

The term ethnomathematics was used to express the relationship between culture and mathematics. This term requires a dynamic interpretation because it describes concepts that themselves were not rigid or singular - that is, ethno and mathematics [6]. The ethnomathematics approach involves real world contexts and mathematics. It was a learning approach that was close to the thoughts and culture of students. Therefore, contextual learning media was applied in that approach.

Students were required to think creatively, invention or re-invention of mathematical concepts [7]. That was a manifestation of mathematics as a human activities [8]. Based on contextual problems with the ethnomathematics approach, students solve them through a horizontal mathematization process and vertical mathematization [9]. In realistic mathematics learning, learning mathematics means doing mathematics, which solves everyday life problems (contextual problems) was an important part. The other main principle was that students must be given the opportunity to rediscover mathematical concepts, and that the teaching-learning process becomes very interactive [10].

Treffers statement [9] was synthesized as follows. In the process of horizontal mathematization, students with their knowledge can organize and solve problems in real world situations. It was a process that moves from the real world to the world of symbols. The activity was carried out through social interaction between students. An example of horizontal mathematics was the identification, formulation and visualization of problems in different ways, transforming real-world problems into mathematical problems. Whereas in vertical mathematization was the process of reorganizing using mathematics itself or 'real world' was a source of mathematics and as a place to re-apply mathematical concepts. It was a process that moves from the world of symbols. Vertical mathematical examples were representations of relationships in formulas, smoothing and adjusting mathematical models, using different models, formulating mathematical models and generalizing. Therefore, realistic mathematics learning using the ethnomathematics approach makes it easy for students to learn mathematics.

The groups of students taught with realistic mathematical approaches and ethnomathematics had the ability to understand higher mathematical concepts than those taught by direct instruction [11]. If the two groups of students were given the ethnomathematics material, the students' mathematical communication skills were taught with a higher realistic learning approach used by traditional learning approaches [12]. Mathematical understanding of students taught using realistic mathematics learning was higher than those who taught using the conventional method. The mathematical understanding of students learned the ethnomathematics oriented materials was higher than those learned non-ethnomathematics oriented materials [13]. Through ethnomathematics learning, students can develop the ability to solve problems through self-reflection on planning, monitoring and evaluating the implementation of thought processes. Students can combine pieces of information about traditional parts of the house that have properties similar to geometric shapes [14]. Students who learn to use contextual learning media, produce patterns that can make it easier for students to construct conjectures and with vertical mathematical activities, students can achieve mathematical concepts and principles appropriately [15]. Abstract level student characters who teach the ethnomathematics approach were able to use all the statements given to solve the problem. He can explain the relationship of statements given with arguments in solving problems. The student was able to explain the usefulness of each statement used to solve the problem, as a result of a proven statement. He was able to explain statements that were arranged as a result of existing statements by using good arguments and drawing conclusions that have been made on paper and pencil [16]. These studies provide recommendations for applying mathematics learning with the ethnomathematics approach. It was an appropriate and suitable approach to mathematics learning for mathematics, of course it was also very appropriate for learning vector material. Thus, we were interested in describing the
mathematical process of students in understanding vector concepts through realistic mathematics learning and ethnomathematics.

2. Method
We conducted a large research through the Master Mathematics Education project at the FKIP-Universitas Bengkulu in Indonesia. That was a development research. This research was the prototype stage of development research. The subjects of this study were 38 high school students in Bengkulu. Subjects were selected through the intact group technique. That was one class that we call the ethnomathematics class. In the class, realistic mathematics learning was applied with the ethnomathematics approach. During and after learning with this approach, we conducted in-depth interviews with selected students. The instruments of this study were observation sheets, anecdotes and interview guides. These were tools for collecting data through observation during learning, filling in anecdotes about student activities, and in-depth interviews with these students. To ensure data accuracy, all these activities were recorded using an audio-visual device. That was a data collection technique during the realistic mathematics learning process with the ethnomathematics approach. Data was analyzed qualitatively (data reduction, data display and conclusion drawing / verification)

3. Results and Discussion
The research process was carried out in Bengkulu High School. The selection made through the intact group produced a class which was taught using a realistic mathematical approach with ethnomathematics. It was learning mathematics by implementing a prototype learning device in the development process. We describe data specifically for vector concept learning.

To implement this, we use contextual problems with ethnomathematics in the form of a drone camera. It was a culture for high school students. In a high school in the district of Kaur, there were learning tools (one of which was a drone camera) which has become a habit for students. That indeed has become a culture, when drones become ubiquitous, cultural differences affect human-drone interactions. That was illustrating how users will interact intuitively with drones [17]. Notice Figure 1, was a picture of the excitement of students flying a camera drone.

![Figure 1](http://atikahblogsman10pentagonkaur.blogspot.com/2017/02/)

The culture of using a drone camera can not only be shown in Figure 1, but students in the school also have the ability to make and improve it. It was an activity that they have done as a habit. They were used to it in their daily lives. It was supported by a paper that civil drones were very popular. They were getting cheaper, smaller and stronger. Because of their mobility and mediality, they were deeply entangled in spatial and power relations. The broad idea such as the Drone Culture considers
the quality of making a drone (civil) era and related to popular imagination about innovation, risk, and supervision [18]. That was an activity as can be seen in Figure 2.

Figure 2. The ability of Pentagon high school students to assemble a camera drone
(Source: https://regional.kompas.com/read/2016/04/07/07000041/)

At the school, students and teachers were very happy to fly a drone camera on their school yard. This shows that the drone camera was indeed their habit. Academic atmosphere was going very well. They learn not in the classroom, they do it outside the classroom. According to one article (by Stahl), the dramatic growth of the drone war in the last decade meant the arrival of a new type of image of war in civilian life: a view through a drone camera. That was a common thing, drones were not only weapons, but also media that appear to represent conflict. Drone cameras can choose images, which can be interpreted, framed and used in public and popular culture [19]. That was a realistic mathematical learning approach with ethnomathematics in the form of a drone camera (see Figure 3, and Figure 4).

Figure 3. Learning outside the classroom using a drone camera
(Source: http://smanpentagon.blogspot.com/2015/01/)
The description above shows that high school students in Bengkulu have made the drone camera something ordinary. That was a culture. Therefore, we use it as a media for contextual learning with the ethnomathematics approach. We developed this learning tool to teach vector material. We applied the learning prototype to one of the high schools in the Kota Bengkulu.

Our students invite out of class to fly a camera drone. They observed and filled out the student activity sheets that we had compiled. Students were very enthusiastic in carrying out the learning process. Based on observations and assessments, the results of data analysis on learning effectiveness were obtained. According to Slavin the effectiveness of learning can be measured by four indicators, namely the quality of learning, the suitability of the level of learning, incentives (how much effort the teacher motivates students) and the time needed to complete learning activities [20]. The data shows the level of effectiveness of mathematics learning through realistic mathematical approaches and ethnomathematics in the form of a drone camera as follows.

![Figure 4](http://smanpentagon.blogspot.com/2015/01/)

**Figure 4.** Students learn outside the classroom using a drone camera

(Source: [http://smanpentagon.blogspot.com/2015/01/](http://smanpentagon.blogspot.com/2015/01/))

![Figure 5](image)

**Figure 5.** Level of effectiveness of learning
Based on Figure 5, all indicators of learning effectiveness for vector material by applying realistic mathematical approaches and ethnomathematics of camera drones, show a very high level. The learning quality reaches 86.50%, conformity of learning levels was 84.70%, incentives were 85.80%, and the time used effectively was 97.50%.

The results of observations and assessments of Figure 5, show that there were 86.50% of the information presented so students can easily learn it. It shows the completeness of student learning about vector concepts. The learning rate conformity was 84.70%. It was stated that the teacher has ensured the level of readiness of students in receiving material about vectors. There was 85.80% was the value of incentives. That was how much effort the teacher motivates students to complete or do tasks and study vector material. Finally, there were 97.50% of time students use to carry out tasks in studying vectors. All of them use the ethnomathematics approach in a realistic mathematical learning framework. That was the time needed to complete the learning activities. That was to show a very high level of learning effectiveness. These results support prior research such as Widada et al. States that after participating in ethnomathematics learning, trans students build thematic relationships between actions, processes, objects, and other schemes. It took the previous scheme. This scheme can be used to solve mathematical problems and related characters. This scheme was used to classify objects selectively [20]. By utilizing contextual learning media (such as drones), there were 78% of students able to arrange definitions correctly, and the level of mastery learning in mathematics was 86.5%, even found 14% of students were able to increase as far as three levels. Students who can reach the highest level were able to store their new knowledge (construction of mathematical concepts and principles) in a mature scheme [15].

During and after mathematics learning realistic with the ethnomathematics drone camera approach, we conducted interviews with several students selected purposively. We can present interview data as follows: (Q: interviewer, Bg: student).

**Footage 1:**

**Q:** What do you think about today’s learning process?

Bg.01: ... yes ... I am very happy ... this was interesting learning and I feel constantly challenged to explore all the meanings of this activity ...

**Q:** ... Ok ... can you explain some of your learning results today?

Bg.02: ... I fly a drone ... and the drone can stop in space ...

**Q:** What do you think about that?

Bg.03: ... it’s interesting ... why can it stop and not fall ...

**Q:** ... yeah ... ok ...

Bg.04: ... that means there was a magnitude that pulls up and the amount holding back to keep moving forward ...

**Q:** ... hmmmm ... ok ... how do you mean?

Bg.05: yes ... that quantity has an upward direction ... that's the first ... and the other one was a quantity that has a forward direction ... it's a vector ... let's say a was an upward vector and b vector was forward ...

**Q:** ... then ...

Bg.06: ... because the drone stops which means it doesn’t move up and doesn't move forward ... it means zero ... I can say that a.b = 0 ...

**Q:** Ok ...

Bg.07: ... it turns out that the angle between vector a and vector b was right ... I bias the drone image (see Figure 6 and Figure 7).
Figure 6. Drone 1 stops and $a \cdot b = 0$

Figure 7. Drone 2 stops, also $a \cdot b = 0$

Footage 1 above gives the meaning that the mathematical process of students starts from pleasant activities. It was flying a drone on the school yard. They play drones to take pictures through the attached camera. This was when the drone stops in space. Students then think and discuss to answer "why can the drones stop in the air and not fall?" The results of the interview with Bg produce a complete picture that students have good cognitive processing abilities. Bg states that drones do not fall because there were two magnitudes that have perpendicular directions. The magnitude produces that the multiplication of the two vectors was the multiplication of points and the result was zero. That was the reason why the drone stopped. Bg produces a mature statement in the form of the following formula.

$$\cos \alpha = \frac{a \cdot b}{|a||b|} \quad (\alpha = 90 \Rightarrow a \cdot b = 0).$$

It was a mathematical process of students in understanding the concept of vectors by using selfie culture using camera drones. Camera drones can stop in space. This was one of the multiplication properties of two perpendicular vectors. Multiplication of points was zero. Other research shows that the results of the application of contextual learning media, then there were more than 82% of students able to reach the concepts and principles correctly [15]. Herawaty, et.al. recommend that teachers and researchers of mathematics education continue to implement mathematical learning approaches that were oriented towards ethnomathematics [11]. According to him learning with the ethnomathematics approach can improve mathematical abilities. Thus, the mathematical process of students using
realistic selfie culture can achieve the multiplication of two vectors which form right angles. Realistic learning with ethnomathematics approach can improve the effectiveness of mathematics learning, especially about vector concepts.

4. Conclusion
The conclusion of this study was that the implementation of realistic mathematics learning with the ethnomathematics approach increases the effectiveness of learning. It was using the drone camera in vector concept learning. Camera drones can stop in space. This was the multiplication properties of two perpendicular vectors. Multiplication of points was zero. Students were able to reach a mature scheme that multiplies two vectors as zero, if the two verbs were perpendicular to each other. It was a very good achievement of vector concepts and principles. We were recommend to implementing the ethnomathematics learning during mathematics learning at school.

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