VIM Time-domain Simulation on a Semi-submersible Floater Using
Wake Oscillator Model

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Summary

This paper presents a time-domain simulation method using the wake oscillator model for prediction of Vortex-Induced Motion (VIM) of a four columns semi-submersible. VIM effect should be assessed in an appropriate manner since the VIM causes fatigue damage of the floating structure’s mooring lines. There is no way, however, to assess VIM phenomenon on semi-submersible type floating structures in time-domain. Then this paper represents the method to simulate the VIM motion to be able to use in the time-domain simulation of a moored floater in ocean engineering topics. This method for a four columns floater is proposed by modifying the VIM simulation method on a cylindrical column type floater. Some empirical parameters in the method are obtained from the systematic model tests used many type semi-submersible floaters. This simulation method is easy to use and to understand since the method is based on a rational physical modeling. The validity of this method is shown comparing the simulated results of in-line and transverse VIM amplitudes with experimental ones.

1. Introduction

Semi-submersible type offshore floating structures, i.e. semi-submersibles, are often used for drilling and production supports in the development of oil and gas fields all over the world. In Japan, the semi-submersibles are also expected to be used in the coastal area and at sea off Japan for promoting resource exploitation and development, and for renewable energy facilities, e.g. as a floating body for a wind power generation plant etc. Considering the operation of the semi-submersibles for a long-term, VIM evaluation is the necessary requirement for the safety assessment of mooring lines.

In the ISO and API standards1,2), the VIM assessment of the floaters is treated as an important element for the safety assessment of mooring lines. VIM investigations of individual semi-submersibles have been usually conducted in the experimental tanks3~6). Although VIM simulations of a semi-sub have been tried using the potential theory and the CFD calculations7~9) in some cases, it seems to be difficult to calculate in a short-term under reliable conviction at the initial design stage of it. Then a reasonable and simple VIM simulation method of semi-submersibles, which becomes effective in the design sea condition, is needed for the safety assessment of semi-sub operation and mooring lines.

To answer this request, the author and the others had conducted the VIM model tests on the various types of semi-sub models with four circular form columns with/without lower hulls to understand basic VIM specification of semi-submersibles10). As lower hull volume had influence on the VIM amplitude largely, the estimation equations on maximum in-line and transverse VIM amplitudes to be able to obtain the VIM amplitude in the directed current velocity deterministically were proposed using the column ratio parameter of the semi-sub.

Using the specification of the model test results of VIM phenomenon on the semi-submersible, the time-domain simulation method with wake oscillator model of a semi-sub is proposed in this paper. The method is based on the one cylindrical floater, VIM simulation model proposed by the author11), and VIM amplification qualities of the semi-submersibles obtained from model tests are treated in the simulation method. This method is effective to do the time-domain simulation of a semi-sub floater with a mooring system in wind, waves and current. That is to say, the method is able to contribute the strength assessment of mooring lines in the heavy design weather conditions and the fatigue assessment of them from the safety aspect of semi-submersibles’ long-term operation.

At first in this paper, the VIM time-domain simulation method of a semi-sub using the wake oscillator model is introduced. Here, the semi-sub simulation targets four columns type of a floater with/without lower hulls since the semi-sub is major in the worldwide for offshore engineering12,13). Some empirical parameters in the motion equations of the simulation are obtained from the systematic model tests used eight semi-submersibles. In the final stage, comparing the simulated results of VIM amplitudes with experimental ones, the validities of this method are shown clearly.

2. VIM time-domain simulation method on a semi-sub using wake oscillator model

2.1 Basic concept

A four columns semi-sub was selected as a subject from the recent semi-sub rig built trend12,13). The semi-sub equips round form columns as safe side consideration for the reason that VIM seemed to be caused easily by unstable vortex shedding at each column.

On the other hand, it is well known that the maximum VIM amplitude level seems to be about representative length3~6,10,12,13), here it is the diameter of each column, not four times column diameter. In this paper, the VIM amplitude is treated as averaged semi-sub motion against one column diameter to get a simple solution. At this time, the motion equations of a semi-sub are

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the 1/2 non-dimensional average length of wake oscillator $\Gamma^*$, that is 1.10, are used in the equations. These parameters, $f^*$ and $\Gamma^*$, may be changed by the effect of the column-column and column-lower hull interactions. As the previous paper by the author\textsuperscript{(11)}, however, showed that the constant $f^*$ and $\Gamma^*$ values were effective for some types of floaters and current velocity range, these parameters were used as same and fixed values continuously.

The $\omega_h$ was decided from the trend of the model test data like as follows:

$$
\omega_h = \frac{2\pi S}{D} \sqrt{V} \quad \text{in} \ V_e < V_{\text{cin}},
$$

$$
\omega_h = \frac{2\pi S}{D} \left[0.2(V - V_{\text{cin}}) + V_{\text{cin}}\right] \quad \text{in} \ V_e \geq V_{\text{cin}},
$$

$$
V_e = \frac{1}{T_N} D, \\
V_{\text{cin}} = D V_{\text{cin}} / T_N, \quad V_{\text{cin}} = 9.0, \\
S_v = 1 / V_{\text{cin}},
$$

(3)

where the $V_e$ is reduced velocity, the $V$ is current velocity and the $S_v$ is the Strouhal number on a stationary cylinder relating to the reduced velocity in the vortex shedding lock-in situation, $V_{\text{cin}}$, which is $V_{\text{cin}} = 9.0$.

2. 2. 2 Motion equations of a semi-sub

The motion equations of a semi-sub are represented using VIM external force $F$ with subscriptions of each direction as follows:

$$
(M + M_c)\ddot{x} + C_{c_{x}} \dot{x} + K_c x = F_{c_{x}},
$$

$$
(M + M_c)\ddot{y} + C_{c_{y}} \dot{y} + K_c y = F_{c_{y}},
$$

(4)

where the $M_c$, $M_e$ are mass and added mass of an object floaters, the $C_c$, namely $C_{c_{x}}$, $C_{c_{y}}$, are damping coefficient, that is decided from free surge and sway tests etc. for instance, and the $K_c$ is restoring coefficient of mooring stiffness.

The external force $F$ consists of the subscript characters caused by lift $L$ and drag $D$, that is:

$$
F_x = F_{Lx} + F_{Dx}, \\
F_y = F_{Ly} + F_{Dy}.
$$

(5)

Considering $x$, $y$-directions separately, the x-directional induced drag $F_{Lx}$, caused by lift relating to the wake, is also assumed to be proportional to $(\alpha - \theta)$, where the $\theta$ is the relative angle for current velocity.

$$
F_{Lx} = -\frac{1}{2\pi} \rho D^2 (V + \dot{x})^2 \left(f^* (\alpha - \theta)\right)^2.
$$

(6)

Here, the $\rho$ means the water density. The $F_{Lx}$ is included the effect of four times column number and half volume of a mirror image on wing lifting theory.

The x-directional drag $F_{Dx}$ acting on a floater is represented as follows:

$$
F_{Dx} = -\frac{1}{2} \rho A_{WF} C_{Dx} (V + \dot{x})^2.
$$

(7)

The $A_{WF}$ is frontal projected area in the water and the $C_{Dx}$ is
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drag coefficient for in-line direction.

The y-directional lift \( F_{ly} \), based on the wake is also assumed to be proportional to \( (\alpha - \theta) \), then,

\[
F_{ly} = -2\rho Dc_{Dy} (V + \dot{x})^2 \cdot f \cdot (\alpha - \theta) .
\]

Here, the \( d_c \) means draught of the column. This \( F_{ly} \) is also considered for four times column effect.

The drag \( F_{Dy} \) in y-direction in the VIM is represented as follows:

\[
F_{Dy} = -\frac{1}{\rho} A_{eff} C_{Dy} \left( V + \dot{x} \right)^2 \sqrt{V + \dot{x}^2} + \frac{1}{2} \frac{\alpha}{\rho} \left( \alpha + 2n^2 \dot{y} \right).
\]

(9)

where the \( A_{eff} \) is lateral projected area in the water, and the \( C_{Dy} \) is drag coefficient for transverse direction, but it is difficult to decide the \( C_{Dy} \) in the VIM oscillation. Then in this paper, the \( C_{Dy} \) is set as the constant representative value, 1.0.

As a result, the motion equations caused by VIM on a semi-sub are represented as:

\[
\ddot{X} + \left[ \eta \left( \frac{g + n V}{\pi} \right) A_{eff} C_{Dy} \frac{f^2 \alpha}{D_d} \right] \dot{X} = -n^2 \frac{V}{\pi} A_{eff} C_{Dy} \frac{f^2 \alpha}{D_d} \left( \alpha + 2n^2 \dot{y} \right).
\]

(10)

\[
\ddot{Y} + \left[ \frac{2\eta}{\pi} \left( \frac{n V}{\pi} + \frac{2n X}{\pi} \right) A_{eff} C_{Dy} \frac{f^2 \alpha}{D_d} \right] \dot{Y} = -4 \frac{fn vy^2}{\pi} \left( \frac{v}{\pi} + 2 \dot{X} \right).
\]

(11)

where the \( X \) is non-dimensional in-line position \((x / D)\) of the column as same as the \( Y \), and the \( \eta \) \((= C_{Dx}(2d_c(M + M_c)))\) is non-dimensional damping coefficient of a floater, the \( n \) \((= \rho Dc_{Dy} \alpha / (2(M + M_c)))\) is mass ratio, the \( \Lambda \) \((= \pi d_c / D)\) is constant parameter that means the aspect ratio of a column. Here, the damping coefficients \( C_{Dy} \) were set as the same value in the \( x \), \( y \)-directions since the measurement of the \( C_{Dx} \) had not been conducted in the model test and the two ‘P’ models shown as later only comes under the influence.

Moreover, this study made clear that the effects on the interaction of \( X \), \( Y \) terms and high order term of the wake oscillator angle, square term \( \alpha^2 \), are small for contribution against VIM amplitudes (See Appendix). Then omitting the interaction of \( X \), \( Y \) terms and \( \alpha^2 \) term of the wake oscillator angle, the motion equations are rewritten as follows:

\[
\ddot{X} + \left[ \eta + C_{Dx} \frac{A_{eff} C_{Dy} n V}{\pi} \right] \dot{X} = -C_{Dx} \frac{A_{eff} n V^2}{\pi^2} \dot{X} ,
\]

(12)

\[
\ddot{Y} + \left[ 2\eta + \frac{f^2 \alpha}{D_d} \right] \dot{Y} + Y = -4 \frac{fn vy^2}{\pi} \left( \frac{v}{\pi} + 2 \dot{X} \right).
\]

(13)

2.3 Parameter settings based on the model test of semi-subs

In the previous study\(^{(39)}\), it was clear that the semi-sub VIM amplitudes are affected by the column volume ratios in the whole displacement. The summary of the model test to be conducted to obtain the relationship between the VIM amplitudes and the column volume ratio is presented in this section.

2.3.1 Sample models

Semi-sub model test on VIM investigation was conducted in our research institute, NMRI. Plural semi-sub models were prepared for the model test.

Table 1 shows the model specifications, and Fig. 2 shows the appearances of the models. Each column has 0.2m diameter \( D \). The columns were connected by oval sectioned thin braces to avoid making drag and flow turbulence. Two types of column intervals were set using different length braces. Short brace semi-subs were named as ‘C05’ series, which had 0.5m column interval, and relatively long brace ones were ‘C08’ series, which had 0.8m one. Moreover, two types of lower hulls were prepared in the tests, that one was a parallel lower hull, added the character ‘P’ in the model name, and the other was a square shaped lower hull ‘S’.

![Fig. 2 Model forms on the C05 (left) and C08 (right) series (The plans represent S14 conditions).](image)
Each lower hull can be changed its thickness, 0.07m and 0.14m. The column ratio \( R_{CLM} \) in the table, that has an important role to assess the VIM characteristics, means the ratio that total columns displacement \( V_C \) divided by the whole displacement \( V \). The \( T_N \) is the natural period of mooring condition without current. As a basic characteristic of the models, damping ratios of each model against critical damping are also shown in the table. Here, the damping ratio \( \gamma \) was fitted to the function \( e^{\frac{t}{2}} \) using duration time \( t \). The test set-up of model mooring condition in the tank is shown in Fig. 3. It is expected to refer to the Reference 10 for the details about the information of the semi-sub model test.

### 2. 3. 2 Model test results and decided parameters

The maximum values of the transverse VIM ratio of the C05 series, divided by the model column diameter, \( A_p / D \), are shown in Fig. 4 as one example. Here, the ‘maximum’ plotted data mean the maximum values in peak VIM amplitudes measured in one-time history test data, where peak VIM amplitudes were counted about 20~30 times averagely, that is about 200 s duration time in one model test.

From Fig. 4, it became clear that lower hull volume in the displacement of semi-subs had an important role in the development of VIM amplitude in the current. Using the column volume ratio in the whole displacement, \( R_{CLM} \), maximum amplitude ratios of the transverse VIM in \( V_{cr} \leq 16 \) are summarized as shown in Fig. 5.

![Fig. 3 Mooring condition of the model in Ocean engineering tank in NMRI (upper) and side view of mooring condition of the model (lower).](image)

### Table 2 Damping coefficients of the sample semi-sub models obtained from the model tests.

| Model   | a   | b   |
|---------|-----|-----|
| C05d15  | 0.779 | -0.671 |
| C05d1SP07 | 0.886 | 0.0 |
| C05d1S07 | 0.991 | 0.0 |
| C05d1S14 | 0.908 | 0.0 |
| C08d15  | 1.294 | -0.139 |
| C08d1SP07 | 0.569 | -0.537 |
| C08d1SP07 | 0.547 | -0.573 |
| C08d1S14 | 0.693 | -0.377 |

The transverse VIMs have a linear relationship with the \( R_{CLM} \) generally. In the same way, VIM amplitude is closely connected with the lift force parameter \( C_{L0} \). The \( C_{L0} \) is, therefore, newly set as the following form using the experimental parameter.

\[
C_{L0} = 0.045 + 0.030 R_{CLM}
\]

where the \( V_{CLM} \) is total column displacement, and the \( V_{ALL} \) is the whole one of a semi-sub. The \( C_{L0} \) is used in Eq. (1), and has an influence on wake angle. As a result, VIM amplitude is affected from the interaction effect of columns and lower hulls through the wake angle. This \( C_{L0} \) is simple formulation using the \( R_{CLM} \). It is originally assumed that the tendencies of the \( C_{L0} \) are different by lower hull shapes, however, the influence of the shapes did not
The coefficients, $a$ and $b$, are shown in Table 2.

As it was previously mentioned, strictly speaking, some treatments lack and are not enough in the simulation, which is the interaction effect of the wake for columns interval with free surface disturbance, column and lower hull form specifications, draft etc. For example, the model wake of fore side columns shown in Section 2.2.1 contacts with the aft contacts. From the results of Fig. 5, however, it seems that the VIM amplitudes have fortunately dull trend except for the effect of the $R_{c00}$ that has an important role for the VIM amplitude rather than the column interval effect. Some problems in the formulations are placed as future works recognizing this simulation method represented in this paper is useful for the typical semi-sub forms in the present situation.

3. **Confirmation of the accuracy of the simulation method**

At first, the time history of calculated results of the in-line, transverse displacements and the wake oscillator angle $\alpha$ compared with the model test ones on the C05d15 in $V_s=9.1$ are shown in Fig. 6. The simulation, starting from no current and no VIM, was conducted by the 4th order Runge-Kutta method with 0.05 s time step, 1500 s in model scale, which was able to get the steady floater motion. The $\alpha = 0.001$ rad in $t=0$ s as the initial disturbance was set in all calculation. As a careful point, indicated initial time, 0 s, in the horizontal axis is a reference point, not the start time for measurement of the model test and simulation.

On the time history of in-line motion in the model test, the amplitude greatly varies to the difficulty level to find the maximum and minimum values of the amplitude as shown in the figure unlike it of the calculated result in no current disturbance. In the model test, the current disturbance, that is about 20% of the mean current velocity, seems to affect the floater’s motion. Free surface disturbance effect caused from the Reynolds number effect. This effect is averagely included in the drag inclination term in Eq. (15), but it is shown that to catch condition that changes moment by moment is difficult.

In the steady situation of the transverse amplitude in the model test, although the transverse amplitude also fluctuates to some extent, the calculated result amplitudes generally coincides with the model test ones of the maximum level. The profile in the wake angle is asymmetry slightly. This seems to be caused by the relative velocity of the transverse motion between the floater and wake.

Figs. 7 and 8 show the calculated results of maximum amplitude ratios of the in-line, transverse VIMs of the C05 and C08 series models compared with the experimental ones. The test
The time-domain VIM simulation method on a wide variety form of four columns semi-subs in the current has been proposed on the basis of the single cylindrical floater’s wake oscillator model. This method is the first trial to easily evaluate in-line and transverse VIMs on semi-subs. The lift force coefficient with the column ratio parameter, used in the simulation and has an important role, is presented originally on the basis of the model test results. This method is remarkable one that can calculate VIM motion from the outline external form of a semi-sub since there was not a method to evaluate VIM of the semi-subs so far.

As a result, calculated results of the in-line and transverse VIM amplitudes have generally shown good agreement for the model test results conducted in the experimental basin in the range of about \( V_r \leq 14 \). It is demonstrated that this simulation method is an effective technique for the VIM simulation of semi-subs, and the method can be applied to the time-domain analysis of a semi-sub in the fluctuating current and quasi-static one recommended in the ISO 19901-7 etc. in the design stage of the semi-subs.

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Appendix

Contribution of the X, Y interaction terms and high order terms of wake oscillator angle \( \alpha \)

Effects on the interaction of \( X \), \( Y \) terms and high order terms of wake oscillator angle \( \alpha \) were investigated in this section. The original motion equations of a floater are represented as follows:

\[
\begin{align*}
\ddot{X} &+ 2\eta + \frac{n\nu}{\pi^2} \left[ \frac{A_{FF}}{D_d} C_{Dn} + \frac{f^2 \alpha}{\Lambda} \left( \alpha + \frac{2\pi^2}{\nu} \right) \right] \dot{X} + X \\
&= -\frac{n\nu^2}{\pi^2} \left[ \frac{A_{FF}}{D_d} C_{Dn} + \frac{f^2 \alpha}{\Lambda} \left( \alpha + \frac{2\pi^2}{\nu} \right) \right], \\
\ddot{Y} &+ 2\eta + \frac{n\nu}{\pi^2} \left[ \frac{A_{WF}}{D_d} C_{Dn} + \frac{f^2 \alpha}{\Lambda} \left( \alpha + \frac{2\pi^2}{\nu} \right) \right] \dot{Y} + Y \\
&= -\frac{4fn\nu^2}{\pi^2} \left( \frac{\nu}{\pi} + 2X \right). \\
\end{align*}
\]

(10)

(11)

Omitting the interaction of \( \dot{X}, \dot{Y} \) terms and \( \alpha^2 \) term of wake oscillator angle, the motion equations are rewritten as follows:

\[
\begin{align*}
\ddot{X} &+ \frac{A_{FF}}{D_d} C_{Dn} + \frac{f^2 \alpha}{\Lambda} \left( \alpha + \frac{2\pi^2}{\nu} \right) \dot{X} + X \\
&= -\frac{A_{FF}}{D_d} C_{Dn} + \frac{f^2 \alpha}{\Lambda} \left( \alpha + \frac{2\pi^2}{\nu} \right), \\
\ddot{Y} &+ \frac{A_{WF}}{D_d} C_{Dn} + \frac{f^2 \alpha}{\Lambda} \left( \alpha + \frac{2\pi^2}{\nu} \right) \dot{Y} + Y \\
&= -\frac{4fn\nu^2}{\pi^2} \left( \frac{\nu}{\pi} + 2X \right). \\
\end{align*}
\]

(12)

(13)

VIM amplitudes of in-line and transverse were shown in Fig. A1 respectively comparing with the effect of the terms of the interaction of \( X, Y \) and wake oscillator angle \( \alpha^2 \). The results of ‘w/o inter.’ mean the calculated ones from Eqs. 10, 11, and those of ‘w/ inter.’ were from Eqs. 12, 13. It can be said that the differences of the results between the two set equations are very small. Then in this paper the motion equations, 12 and 13, omitting the interaction terms of the \( X, Y \) and high order wake oscillator angles, were used as simple representations.