Finite Temperature Fractional Quantum Hall Effect

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Abstract

We investigate fractional quantum Hall effect at finite temperature using a fermion Chern-Simons field theoretical approach. In the absence of impurity scattering, the essential aspects of fractional quantum Hall effect, such as the quantization of Hall conductance, as well as quasi-particle charge and statistics are unrenormalized by thermal fluctuations. On the other hand, we find that the low energy excitation spectrum at finite $T$ may undergo some qualitative changes as temperature raises. Possible experimental consequences are discussed.

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Introduction – The novel phenomenon of fractional quantum Hall effect (FQHE) may be understood theoretically as a manifestation of certain two dimensional (2D) highly correlated fermion states [1,2]. Most of the theoretical efforts thus far have been given to the study of zero temperature properties, where the ground state is known to be incompressible, separated from higher energy states by a gap $\Delta$ produced by strong electron-electron interaction. On the other hand, experimental measurements are done, of course, at finite temperature. But from the above mentioned property of the ground state, which has been established firmly through more than a decade of extensive study, it is reasonable to expect that at finite temperature the effect of thermal fluctuations would not be important, as long as $kT \leq \Delta$.

However, for a quantitative comparison between theory and experiment, there are several issues that need to be addressed concerning the finite (low) temperature properties of the FQHE: the first one is the fundamental question about the accuracy of the FQHE at non-zero $T$. How do thermal fluctuations affect the quantization of the Hall conductance? Another important question is the $T$ dependence of the quasi-particle gap which can be explicitly computed at $T = 0$ in Laughlin’s theory [1,2] for the ‘fundamental states’ and in Jain’s composite fermion theory [3] for general filling fractions, and can be implicitly measured experimentally through the $T$ dependence of the longitudinal resistivity [2,4]. In this paper we study FQHE at finite temperature. In particular, we investigate the $T$ dependence of low energy collective excitations. At zero $T$, the existence of a special roton-like excitation in the Laughlin states was first demonstrated by Girvin et al through the Feynman-Bijl approach [5], and later by Zhang et al using a Chern-Simons Landau Ginzburg theory [6]. For general filling fraction $\nu$, this problem was investigated within the composite fermion picture aided with the method of ‘Chern-Simons transformation’ [7]-[10]. Our approach is a fermion Chern-Simons field theory formalism, which has been used previously by Lopez and Fradkin [7] to study FQHE at zero $T$, and by Halperin, Lee and Read in their study of the $\nu = 1/2$ state [8]. Presented in an equivalent quantum many-body language, Simon and co-workers [9,10] have studied this Chern-Simons–composite fermion approach in great detail. An important point addressed in Ref. [8]-[10] is the problem of mass renormalization.
of composite fermions. Without treating it properly, this fermion Chern-Simons theory approach will set the system at a spurious energy scale $\hbar\bar{\omega}_c$ (see below), which is on the order of (although less than) the bare cyclotron energy, while the true physical energy scale of the problem is given by electron-electron interactions. However, the question which concerns us here is the temperature dependence of the collective excitations rather than evaluation of the precise value of the energy gap. Thus while the issue of mass renormalization is important and its phenomenological Fermi-liquid treatment (discussed in Refs. [8]-[10]) may be straightforwardly generalized to the finite $T$ case, it will not be consider here. This issue, along some details of the present work, will be discussed somewhere else [11].

**Approach** – In the spirit of composite fermion approach, we describe 2D electrons of band mass $m_b$ in a magnetic field at temperature $kT = 1/\beta$ by a coherent functional integral with action (in the unit $\hbar = 1$):

$$S = \int_0^\beta d\tau \int d^2 r \left\{ \psi^\dagger (\partial_\tau - ia_0 - \mu)\psi + \frac{1}{2m_b} \left| (-i\vec{\nabla} + \frac{e}{c}\vec{A} - \vec{a})\psi \right|^2 + \frac{i}{4\pi\phi} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda \right\}$$

$$+ \frac{1}{2} \int_0^\beta d\tau \int d^2 r \int d^2 r' \psi^\dagger (\vec{r} - \vec{r}') \psi (\vec{r}') \nu (\vec{r}) \psi^\dagger \psi (\vec{r}) ,$$

(1)

where we attached even flux quanta per fermion through the well-known ‘Chern-Simons term’ with statistical gauge field $a_\mu$. In the above expression, $\tilde{\phi} = 2p$, and $p$ is an integer. $\vec{A}$ is the vector potential for the magnetic field and the chemical potential $\mu$ fixes the Landau level (LL) filling fraction at $\nu$. $\nu (\vec{r})$ is a two-body interaction potential. For Coulomb interaction, $\nu (\vec{r}) = e^2/\epsilon r$, where $\epsilon$ is the dielectric constant. A similar Euclidean action has been studied in Ref. [13] in the context of anyon superconductivity.

While this action is equivalent to the usual one without the statistical gauge field, it provides us a convenient starting point for approximations: consider the homogeneous liquid saddle point solution for the Chern-Simons field $a_\mu = \vec{a}_\mu$, such that

$$|\vec{\nabla} \times \vec{a}| = 2\pi \tilde{\phi} \hat{\rho} , \quad \vec{a}_0 = 0 ,$$

(2)

where $\hat{\rho}$ is the average particle density. In this mean field theory, an electron feels an effective magnetic field
\[ B_{eff} \equiv |\mathbf{\nabla} \times \mathbf{A}_{eff}| = |\mathbf{\nabla} \times (\mathbf{A} - \frac{e}{c} \mathbf{a})| \quad . \tag{3} \]

For the filling fraction \( \nu \) such that \( 1/(1/\nu - 2p) = n \), an integer, the mean-field theory possesses a ground state of \( n \) filled LL, which is thus incompressible and stable against weak perturbations. The filling fraction \( \nu \) which satisfies this condition is precisely the value where FQHE arises. This correspondence between the fractional and the integer QHE is precisely the basic idea of Jain’s composite fermion theory \([3]\) which underlines the present approach. Fluctuation corrections may be systematically build around this (stable) saddle point solution \([1]\).

At finite \( T \), we shall also take such a homogeneous saddle point solution as our starting place and consider only fluctuation corrections on the one-loop level. The implicit assumption is that at thermal equilibrium the composite fermion picture remains a good description, hence effects of the *thermal disintegration* of composite fermions, i.e., the unbinding of particles and (even number of) vortices are not important for the temperature range under consideration. Experimentally it has been found \([12]\) that the \( \nu = 1/2 \) anomaly persists to some temperature range in which the FQHE at other filling fractions are smeared out by thermal fluctuations, which suggests strongly \([12]\) that composite fermions \([8]\) are rather robust against thermal fluctuations, and the disintegration temperature \( T_d \) of composite fermions lies well above the (zero \( T \) ) energy gap of the incompressible FQH states.

With Eqns.\((2)\) and \((3)\), the mean field action is given by

\[ S_0 = \int_0^\beta \! d\tau \int d^2r \left\{ \psi^\dagger (\partial_\tau - \mu) \psi + \frac{1}{2m_b} |(-i\mathbf{\nabla} + \frac{e}{c} \mathbf{A}_{eff})\psi|^2 \right\} \quad . \tag{4} \]

This action describes a system of non-interacting fermions with magnetic field \( B_{eff} \), in which the energy spectrum is given by the effective LL \( \epsilon_l = \tilde{\omega}_c (l + \frac{1}{2}) \), where \( \tilde{\omega}_c = eB_{eff}/m_b c \), is the effective cyclotron frequency. The chemical potential \( \mu \) is determined by the condition

\[ n = \sum_l f(\epsilon_l - \mu) \quad , \quad f(\epsilon_l - \mu) = \frac{1}{e^{\beta(\epsilon_l - \mu)} + 1} \quad . \tag{5} \]
To study the Gaussian fluctuations in the Chern-Simons gauge field, we adopt the approach of \([8,9]\) by taking a transverse gauge such that \(\bar{a}_T \parallel \hat{y}, \bar{A}_T \parallel \hat{y}\) and choose \(\vec{q} \parallel \hat{x}\). In this case, \(j_x(q, \omega)\) is simply given by \(\frac{i\omega}{q}\rho(q, \omega)\). Shifting \(a_\mu\) by \(\bar{a}_\mu\), i.e., \(a_\mu \to \bar{a}_\mu + a_\mu\), we can express the kernel of the Gaussian action for the statistical field \(a_\mu\) in terms of a \(2 \times 2\) matrix. At finite frequency, it is given by

\[
D^{-1} = \frac{1}{2\pi\phi} \begin{pmatrix}
\tilde{n}q^2/\tilde{\omega}_c\Sigma_0 & iq(1 - \tilde{n}\Sigma_1) \\
-iq(1 - \tilde{n}\Sigma_1) & \tilde{\omega}_c(\frac{k_B}{2}\bar{l}_0 q + \tilde{n}(\Sigma_2 + 1))
\end{pmatrix},
\]

with \(\tilde{n} = 2pn\) and

\[
\Sigma_j = \frac{e^{-x}}{n} \sum_{m<l} \frac{l-m}{(l-m)^2} \frac{m!}{l!} x^{l-m-1} \{f(\epsilon_m - \mu) - f(\epsilon_l - \mu)\} \left[\frac{L^{l-m}(x)}{(l-m+x)^{l-m}} + 2x \frac{dL^{l-m}(x)}{dx}\right]^j.
\]

In the above equation, \(i\omega_n\) is the Matsubara frequency, \(L^l_m\) is the Laguerre polynomial, \(x = (\bar{l}_0 q)^2/2\), and \(\bar{l}_0 = \sqrt{c/eB_{eff}}\) is the effective magnetic length. \(\mu_s\) is the ratio of \(\bar{l}_0\) to the Bohr radius \(a_0 = \epsilon/mq^2\). To obtain electromagnetic response, we consider a fluctuation in \(A_\mu\) such that \(A_\mu \to A_\mu + \delta A_\mu\). Integrating out the statistical field \(a_\mu\), we arrive the effective action

\[
\tilde{S}_{\text{Gaussian}}(\delta A_\mu) = \frac{1}{2} \sum_{q,i\omega_n} \left( i\delta A_0(-q,-i\omega_n) \frac{\epsilon}{c} \delta A(-q,-i\omega_n) \right) K(q,i\omega_n) \begin{pmatrix}
\delta A_0(q,i\omega_n) \\
\frac{\tilde{\Sigma}_0}{\tilde{\omega}_c}
\end{pmatrix},
\]

with the kernel \((i\omega_n \neq 0)\)

\[
K(q,i\omega_n) = \frac{n}{2\pi |\det|} \begin{pmatrix}
\tilde{n}q^2/\tilde{\omega}_c\Sigma_0 & -iq\Sigma_s \\
-iq\Sigma_s & \tilde{\omega}_c\Sigma_r
\end{pmatrix}.
\]

In the above equation, \(\Sigma_s = \Sigma_1(1 - \tilde{n}\Sigma_1) + \tilde{n}\Sigma_0(\Sigma_2 + 1), \Sigma_r = 1 + \Sigma_2 + n\mu_s \bar{l}_0 q(\Sigma_1^2 - \Sigma_0(\Sigma_2 + 1))\) and \(det = (1 - \tilde{n}\Sigma_1)^2 - \Sigma_0(n\mu_s \bar{l}_0 q + (\tilde{n})^2(\Sigma_2 + 1))\). The electromagnetic response is obtained by the usual procedure of substituting the Matsubara frequency \(i\omega_n\) by \(\omega - i\eta\). The above
results differ from the zero $T$ calculations \cite{7,9} by the appearance of the fermion distribution $f(\epsilon_l - \mu)$, hence the necessity of summing up the additional LLs in the expression for $\Sigma_j$.

**FQHE at finite temperature** – To answer the fundamental question concerning the accuracy of FQHE at finite temperature, one has to consider dissipative processes such as phonon or impurity scattering. In the absence of them one should expect the same transport properties at finite temperature as those at $T = 0$. In our calculation, this may be seen directly from the zero $q$ response obtained from

$$
\Sigma_j(q = 0, \omega) = \frac{1}{n} \sum_{m=0}^{\infty} \frac{m + 1}{(\omega_c)^2 - 1} \{f(\epsilon_m - \mu) - f(\epsilon_{m+1} - \mu)\} = \frac{1}{(\omega_c)^2 - 1},
$$

where in the last equality we have used Eqn.(5). The zero frequency response needs to be calculated separately. Straightforward calculation shows that at finite $T$ the compressibility $\kappa$ acquires an exponential correction \cite{11}. Now we consider the charge and statistics of the quasi-particle (which is defined by excitations from the thermal equilibrium state at a given $T$ \cite{14}) at finite temperature. Following Laughlin’s gauge argument \cite{2}, which can be readily generalized to the finite $T$ case if one assumes states in each (pseudo) LL are uniformly occupied with probability $f(\epsilon_l - \mu)$, it is reasonable to expect the same fractionalization of quasi-particle charge at finite temperature. Indeed, if one lets $\delta A$ be a thread of unit flux passing through the origin, one finds that total charge thus induced is $\nu e$, independent of $T$. For $\nu = 1/(2p + 1)$, we equate this change to the charge of a quasi-particle, which is in agreement with Laughlin’s gedanken experiment \cite{1,2} at zero $T$. In the general case, the charge of a quasi-particle may be obtained by examining the the gauge invariant (finite temperature) one particle Green function. Following an argument similar to that of Fradkin \cite{15}, one obtains $eB_{\text{eff}} = e^* B$, which gives the quasi-particle charge $e^* = e/(1 + 2pn)$. This is the same as the zero temperature result \cite{3,15}. Statistics of quasi-particles may be obtained from gradient expansion of $\Pi^0$ \cite{15}. In the small $q$ limit, the effective Chern-Simons coupling is given by $1/\tilde{\phi}_{\text{eff}} = 1/\tilde{\phi} + n$. Counting the original statistical phase of bare fermion, we find that the statistical phase (relative to the boson) of quasi-particles is $\pi(1 - 2p/(1 + 2pn))$. All these results stated here, which involve only the physics at large length scale, are $T$
independent and are given by those obtained at $T = 0$. This is a consequence of the fact that at the long wavelength limit, temperature appears in our formalism only through the sum $\sum_l f(\epsilon_l - \mu)$, which is constant due to (5). Also as a result of this, the asymptotic behavior of quasi-particle amplitude distribution remains the same as that at zero $T$, which can be readily demonstrated by computing responses of the system to point-like external charge and external flux thread [7,14,17]. This result is significant, since the statistics of quasi-particles described by the effective Chern-Simons term obtained above corresponds to the phase gain from adiabatic interchange of two particles separated infinitely apart. Changes in the quasi-particle amplitude profile at finite $T$ will then lead to the corrections to the statistics of quasi-particles [16] due to their overlap at finite distance.

Collective Excitations – Collective modes are obtained from the poles in $K(q, \omega)$ which describes the electromagnetic response of the system at a given $T$. At $q = 0$, the cyclotron mode $\omega_c = \frac{\mu}{e} \tilde{\omega}_c$ saturates the $f$-sum rule, in accordance with the Kohn’s theorem [18] which can be readily generalized to the finite $T$ case. As pointed out in the introduction, the approach adopted here sets a spurious energy scale $\hbar \tilde{\omega}_c$ in the problem. Furthermore, once the saddle point solution Eqn.(2) is taken, the interaction $v(\vec{r})$ plays only a nominal role in the perturbation expansion used here. Since FQHE occurs as a result of strong electron-electron correlations, this artifact of our approach seems quite disturbing. One can use the following rationale to justify the fermion Chern-Simons field theory approach [3]: although one needs interaction to create a composite fermion (i.e., the bound state of a bare fermion and $2p$ vortices), hence FQHE, once it is formed the residual interaction among the composite fermions becomes unimportant. Since the presence of $v(\vec{r})$ does not change the qualitative physics and is deceiving about the role of interaction in this formalism, we shall set $\mu_s$ to zero hereafter.

Fig. 1 shows the evolution of the lowest few branches of the collective modes in the $\nu = 1/3$ case as $T$ is raised from zero. At finite $T$, each branch of the zero temperature modes splits into two, where the upper one of the two retains all the weight at small $q$ value. This splitting is due to the finite probability of occupying higher (pseudo) LLs at non-zero
Inspecting the lowest mode, we see that the energy of the roton minimum $\omega_{min}$ is rather insensitive to $T$, while its position $q_{min}$ has stronger temperature dependence. This situation is depicted in Fig. 2. In general, increasing $T$ causes red shift of $q_{min}$. While its precise physical picture is unclear at the present time, this red shift may be understood roughly as a result of weakening of the roton-roton binding energy at small $q$ due to thermal fluctuations. A recent optical experiment measured directly the long wavelength ($q = 0$) collective modes \[19\]. The present calculation suggests that this experiment has actually not detected the lowest branch of the collective mode, which only exists at finite $T$ and vanishes at small $q$. On the other hand, this lowest mode can in principle be measured through the recently suggested experiment using evanescent field Raman scattering \[20\]. Since the spectrum at $q \to \infty$ limit is independent of $T$, our work also provides a justification for the fitting of $\sigma_{yy}$ data with a constant activation energy $\Delta$ \[2,4\].

For FQH states other than the ‘fundamental’ one, the zero $T$ calculation shows that there are more than one roton minimum, and the number of minima corresponds to the number of filled LLs in the composite fermion picture \[9\]. In Fig. 3 we show the lowest branch of the collective mode in the $\nu = 3/7$ state. At zero $T$, there are three roton minima. As $T$ is raised, the weakest one, located at a large wavevector ($\approx 4.75\tilde{l}_0q$), is smeared out first by thermal fluctuations. The second one disappears subsequently at higher $T$, leaving only one roton minimum at sufficiently high temperature. This is a general feature for all the $\nu$ values we have examined. While the position (and the number) of roton minima has not yet been subjected to direct experimental measurement, it does have physical implications according to one of the proposed scenario of FQH state–Wigner crystal transition \[2,5\]. In this picture, such a transition is directly related to the softening of the roton minimum and the position of the (softened) roton minimum corresponds to the reciprocal wavevector of the Wigner crystal near the transition. Such a scenario is not supported by the present calculation since the lattice constant of Wigner crystal should be determined by the electron density alone, and hence should be independent of $T$.

**Summary** – In the absence of impurity scattering, essential aspects of FQHE, such as
the quantization of Hall conductance as well as quasi-particle charge and statistics are unaffected by thermal fluctuations. Within the fermion Chern-Simons field theory approach, we show that the magnitude of energy gap in the collective excitation spectrum is weakly $T$ dependent, while the number and position of roton minima are quite sensitive to $T$; The lowest branch of the $T = 0$ collective modes splits into two when temperature is raised, where the lower one vanishes at long wavelength limit. Our results call for more extensive investigations on the effect of thermal fluctuations to the FQHE.

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FIGURES

Figure 1. Lowest few collective modes at \( \nu = 1/3 \). The width of the curve is proportional to \( q^2 \) times the weight of the pole in \( K_{00} \). As \( T \) is raised from zero, each mode splits to two.

Figure 2. Energy and position of the roton minimum for the \( \nu = 1/3 \) state as function of \( T \). Initially, \( q_{\text{min}} \) is roughly unchanged as \( T \) is increased from zero, then decreases faster as \( T \) is further increased. \( \omega_{\text{min}} \) on the other hand is quite insensitive to \( T \) for the whole range of temperature considered.

Figure 3. Evolution of the lowest collective mode in the case of \( \nu = 3/7 \). As \( T \) is increased from zero, this lowest mode also splits into two. The number of roton minima decreases as \( T \) is increased so that for sufficiently large \( T \) only one minimum remains.