COSMIC SHEAR FROM GALAXY SPINS

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ABSTRACT

We discuss the origin of galactic angular momentum and the statistics of the present-day spin distribution. It is expected that the galaxy spin axes are correlated with the intermediate principal axis of the gravitational shear tensor. This allows one to reconstruct the shear field and thereby the full gravitational potential from the observed galaxy spin fields. We use the direction of the angular momentum vector without any information of its magnitude, which requires a measurement of the position angle and inclination on the sky of each disk galaxy. We present the maximum likelihood shear inversion procedure, which involves a constrained linear minimization. The theory is tested against numerical simulations. We find the correlation strength of nonlinear structures with the initial shear field and show that accurate large-scale density reconstructions are possible at the expected noise level.

Subject headings: galaxies: statistics — large-scale structure of universe

1. INTRODUCTION

In the gravitational instability picture of structure formation, galaxy angular momentum arises from the tidal torquing during the early protogalactic stages (Hoyle 1949). Gravitational torquing results whenever the inertia tensor of an object is misaligned with the local gravitational shear tensor. One might thus expect that the observed galaxy spin field would contain some information about the gravitational shear field and possibly allow a statistical reconstruction thereof. Although the observational search has yet to detect any significant alignments of the galaxy spins so far (Han, Gould, & Sacket 1995), it has been shown that the sample size has to be increased in order to detect weak alignments (Cabanela & Dickey 1999, and references therein).

It was argued that angular momentum only arises at second order in perturbation theory (Peebles 1969). This picture involved a spherical protogalaxy, which at first order has no quadrupole moment and thus cannot be tidally torqued. Later it was realized that protogalaxies should be expected to have order unity variations from spherical symmetry and should be tidally torqued at first order (Doroshkevich 1970; White 1984). However, recent quantitative predictions of this picture (Catelan & Theuns 1996, hereafter CT) appear to be contradicted by simulations (Lemson & Kauffmann 1999), which leaves the field in a state of confusion.

The Eulerian angular momentum of a halo in a region $V_e$ relative to its center of mass in comoving coordinates, $L_{\text{E}}^\text{2nd} = \int_{V_e} \rho \mathbf{x} \times \mathbf{v} d^3\mathbf{x}$, can be written in terms of Lagrangian variables to second-order accuracy using the Zeldovich approximation:

$$L_{\text{E}}^\text{2nd} \propto -\int_{V_e} \tilde{\rho} q \times \nabla \phi d^3q,$$

where $V_e$ is a Lagrangian counterpart of $V_e$, and $\phi$ is the gravitational potential. Approximating $\mathbf{V}\phi$ in equation (1) by the first three terms of the Taylor series expansion about the center of mass, we obtain the first-order expression of the angular momentum (White 1984) in terms of the shear tensor $T = (T_{ij}) = (\delta_i \delta_j \phi)$ and the inertia tensor $I = (I_{ij}) = (\int \rho q_i q_j d^3q)$:

$$L_{\text{E}}^\text{1st} \propto \epsilon_{ijk} T_{ij} I_{kl}.$$

CT used equation (2) to calculate the linearly predicted angular momentum under the approximation that the principal axes of $I$ and $T$ are uncorrelated. They also discussed the small factor by which the neglect of the correlation between $I$ and $T$ overestimates the angular momentum in the context of the Gaussian-peak formalism. In § 2, we show by numerical simulations that this factor in fact is dominant.

For galaxies, the direction of angular momentum is relatively straightforward to observe, while the magnitude is very difficult to observe. We will thus concentrate on the statistics of the direction of the spin, dealing with unit spin vectors. We note that the spin in equation (2) does not depend on the trace of $I$ and $T$. So we can consider unit trace-free tensors $\hat{T}_{ij}$ ($\hat{T}_{ii} = 1$) and the one-point statistics of the spins. The most general quadratic relation between a unit spin vector and a unit traceless shear tensor is

$$\langle \vec{L} \vec{L} | T \rangle = Q_{ij} = \frac{1}{3} a \delta_{ij} - a \hat{T}_{ia} \hat{T}_{aj},$$

where $a \in [0, 1]$ is the correlation parameter, measuring how well aligned $I$ and $T$ are. If $I$ and $T$ are uncorrelated, $a = 1$. If they are perfectly correlated, $a = 0$. It is also a convenient parameterization of the nonlinear effects on the spin-shear correlation and must be measured in numerical simulations.

2. SIMULATIONS

We ran the $N$-body simulations using the PM (particle-mesh) code described by Klypin & Holtzman (1997) with 128$^3$ particles on a 256$^3$ mesh in a periodic box of size $L_n = 80 h^{-1}$ Mpc for the cold dark matter model ($\Omega_0 = 1$, $\sigma_8 = 0.55$, and $h = 0.5$). We identified halos in the final output of $N$-body runs (the total number of halos $N_h$ is 2975) by the standard friends-of-friends algorithm with a linking length of 0.2. The angular momentum of each halo was measured from the positions and
velocities of the component particles in the center-of-mass frame. We also calculated the initial shear tensor by taking a second derivative of the gravitational potential at the Lagrangian center of mass of each halo.

Through the simulations, we first tested the validity of the linear perturbation theory by calculating the average correlations among the first- and second-order Lagrangian and the Eulerian angular momentum obtained from simulations. We have found that \( \langle \mathbf{L}_{\text{lin}} \cdot \mathbf{L}_{\text{lin}} \rangle = 0.55 \), \( \langle \mathbf{L}_{\text{lin}} \cdot \mathbf{L}_{\text{eul}} \rangle = 0.51 \), and \( \langle \mathbf{L}_{\text{eul}} \cdot \mathbf{L}_{\text{eul}} \rangle = 0.62 \). It shows that nonlinear effects only add a factor of 2 scatter to the linearly predicted spin-shear correlation (Sugerman, Summers, & Kamionkowski 2000).

Rotating the frame into the shear-principal axes, we find the optimal estimation formula for the parameter \( a \) from equation (3):

\[
a = 2 - 6 \sum_{i=1}^{3} \lambda_i^2 / | \lambda_i |^3, \tag{4}
\]

where \( \lambda_i \) are the three eigenvalues of the trace-free unit shear tensor, satisfying \( \sum \lambda_i = 1 \) and \( \sum \lambda_i = 0 \). Using equation (4) along with the initial shear tensors and the angular momentum of final dark halos measured from the simulations, we calculated the average value of \( a \) to be 0.237 ± 0.023, showing 10\( \sigma \) deviation [\( a = 8/(5N) \)] from the value of 0. The inferred value of \( a \) results in a significant correlation of the shear vector, but poor for predicting its magnitude (Lemson & Kauffmann 1999) due to the strong correlation between the shear and inertia tensors. We have shown by simulations that the shear and inertia tensors are misaligned with each other to a detectable degree, generating net tidal torques even at first order, although they are quite strongly correlated (it was found by the simulations that \( \langle \mathbf{L}_{\text{lin}} \rangle = -0.85 \), \( \langle \mathbf{L}_{\text{eul}} \rangle = 0.11 \), and \( \langle \mathbf{L}_{\text{lin}} \rangle = 0.74 \) for the trace-free unit inertia tensor \( \mathbf{I} \) in the shear principal axes frame). Thus we conclude that the CT approximations are useful for determining the direction of the spin vector, but poor for predicting its magnitude (Lemson & Kauffmann 1999) due to the strong correlation between \( \mathbf{I} \) and \( \mathbf{T} \). The strong correlation can be understood physically if one considers \( \mathbf{I} \) to be the collection of particles that have shell crossed, in which case it is identical to \( \mathbf{T} \).

### 3. SHEAR RECONSTRUCTION

As in the CT prescription, the probability distribution of \( P(\mathbf{L} \mid \mathbf{T}) \) can be described as Gaussian, and its directional part \( P(\mathbf{L} \mid \mathbf{T}) \) is calculated by

\[
P(\mathbf{L} \mid \mathbf{T}) = \int P(\mathbf{L} \mid \mathbf{T}) P(\mathbf{T}) \, d\mathbf{T} = \frac{1}{4\pi} \mathbf{Q}^{1/2} \mathbf{L}^{-3/2} \mathbf{Q}^{1/2}, \tag{5}
\]

where \( \mathbf{Q} \) is given from equation (3). According to Bayes’ theorem, \( P(\mathbf{T} \mid \mathbf{L}) = P(\mathbf{L} \mid \mathbf{T}) P(\mathbf{T}) / P(\mathbf{L}) \). Here, \( P(\mathbf{T}) = P(\mathbf{T}(x)) \), \( \mathbf{T}(x_1), \ldots \) is a joint random processes linking different points with each other. In the standard picture of galaxy formation from random Gaussian fields, \( P(\mathbf{T}) \) is given as \( P(\mathbf{T}) = N \exp(-\mathbf{T}^\mathbf{C}^{-1}\mathbf{T}/2) \), where \( \mathbf{C} = \langle \mathbf{C}_{ij} \rangle = \langle \mathbf{T}(x) \mathbf{T}(x + \mathbf{r}) \rangle \) is the two-point covariance matrix of shear tensors:

\[
C_{ijk}(\mathbf{r}) = (\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \left( \frac{J_3}{6} - \frac{J_3^2}{10} \right) + (\delta_{il} \delta_{jk} + \delta_{ik} \delta_{jl} + \delta_{ij} \delta_{lk}) \left( \frac{J_3}{2} - \frac{7J_3^2}{2} \right) + (\delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} + \delta_{il} \delta_{jk}) \left( \frac{J_3}{2} - \frac{5J_3^2}{2} \right). \tag{6}
\]

Here \( \mathbf{r} = \mathbf{r}/r \), \( J_n = \int_0^\infty \xi(r) r^{-n-1} dr \), and \( \xi(r) = \langle \delta(x) \delta(x + \mathbf{r}) \rangle \) is the density correlation function.

We would like to compute the posterior expectation value, for example \( \langle \mathbf{T} \mid \mathbf{L} \rangle \). By symmetry, both the expectation value and maximum likelihood of occur at \( \mathbf{T} = 0 \), which is not the solution we are looking for. We must thus consider the constrained expectation, with the constraint that \( \int \mathbf{T}^i \mathbf{T}^j \, d^3x = 1 \). In the limit that \( P(\mathbf{T} \mid \mathbf{L}) \) is Gaussian, this is given by the maximum eigenvector of the posterior correlation function \( \xi_{ij}(x, x') = \langle \mathbf{L}(x) \mathbf{L}(x') \rangle / \mathbf{L} \). The solution satisfies

\[
\int \xi_{ij}(x, x') \mathbf{T}_j(x') \, d^3x' = \Delta \mathbf{T}_{ij}(x), \tag{7}
\]

where \( \Delta \) is the largest eigenvalue for which equation (7) holds. In the asymptotic case that \( a \ll 1 \) as in our simulations results, we have a simple expression for the \( \mathbf{L} \) dependent part of the posterior correlation function in terms of the traceless shear, \( \hat{T}_j = T_j - \delta_{ij}T_i/3 \):

\[
\hat{\xi}_{ij}(x, x') = - \hat{C}_{ij}(x - x') \hat{C}_{iab}(x - x') \hat{L}_a(x) \hat{L}_b(x') \, d^3x'. \tag{8}
\]

where \( \hat{C} \) is now the trace-free two-point covariance matrix of the shear tensors, related to \( C_{ij} \) by \( C_{ij} = C_{ijkl} - \delta_{ik}C_{jll} - \delta_{jl}C_{iik} + \delta_{ij}C_{ll} \). Substituting equation (3) into equation (8) explicitly satisfies a modified version of equation (7). It is readily described in Fourier space:

\[
\int \hat{\xi}_{ij}(k, k') \hat{P}(k) \hat{P}(k') \hat{T}_j(k') \, dk' = \Delta \hat{T}_i(k). \tag{9}
\]

Equation (9) differs from equation (7) by the appropriate Wiener filter due to small \( a \), since we have assumed no noise.

Since we have discarded the constant diagonal component that does not depend on \( \hat{L}_i \), equation (8) is not positive definite, and in fact has zero trace. We can nevertheless use a power iteration to quickly obtain the eigenvector corresponding to the largest eigenvalue. One starts with an initial guess \( \hat{T}_j(x) \), and defines iterates \( \hat{T}_j^{n+1} = \hat{C}_{ij} \hat{T}_i^{n} \) and \( \hat{T}_j^{n+1} = \hat{T}_j^{n+1} / \langle \hat{T}_j^{n+1} \rangle^{1/2} / \langle \hat{C} \rangle^{1/2} \). This effectively eliminates the negative eigenvectors from the iteration and converges to the correct solution. After \( n \) iterations, the fractional error is proportional to \( (\Delta_2/\Delta_1)^n \), where \( \Delta_2 \) and \( \Delta_1 \) are the largest and second largest eigenvalues, respectively. In our experience, 50 iterations converge the result to within 1% of the exact solution. In practice, one can expect the correlation to drop off rapidly
at large separations, requiring only an evaluation of a sparse matrix-vector multiplication.

The final step is the reconstruction of the density given the traceless shears. It is convenient to consider the full shear as an orthonormal reparametrization in terms of trace and trace-free components into a vector

\[ T_i = \{ \delta \sqrt{3}, v_2, v_3, \sqrt{2} T_{12}, \sqrt{2} T_{23}, \sqrt{2} T_{31} \} , \]

where

\[ \delta = T_{11} + T_{22} + T_{33} , \]

\[ v_2 = \frac{(-3 - \sqrt{3}) T_{11} + 2 \sqrt{3} T_{22} + (3 - \sqrt{3}) T_{33}}{6} , \]

and

\[ v_3 = \frac{(3 - \sqrt{3}) T_{11} + 2 \sqrt{3} T_{22} + (-3 - \sqrt{3}) T_{33}}{6} . \]

The two-point correlation function (eq. [6]) is just linearly transformed into these new variables, which we will denote \( \langle T_i(x) T_j(x + r) \rangle = C_{ij}(r) \), where the indices \( i, j \) run from 1 to 6. The inverse correlation will be denoted \( D = (C^{-1}) \). From \( D \) we extract a scalar correlation \( D_{11}(r) \) and a five-component vector correlation \( E_i = \{ D_{12}, D_{13}, D_{14}, D_{15}, D_{16} \} \). Similarly, we will use a Greek index \( T_i \) to denote the five-component vector of \( T_j \), where \( i > 1 \). We then obtain the expression for the density \( \langle \delta T_i^* \rangle = D_{ij} E_j \). If the galaxies are uniformly spaced on a lattice, one could also use a fast Fourier transform to rapidly perform the shear reconstruction iterations and the projection of traceless shear to density.

To test the reconstruction procedure, we have made a large stochastic spin field on a 128^3 lattice and applied the reconstruction procedure with a Wiener filtering scale of \( k = 32 \) for the case of a power-law spectrum, \( P(k) = k^{1.5} \). Figure 1 plots the correlation coefficient \( \rho = \langle \hat{\delta}(k) \hat{\delta}(-k) \rangle / \langle P(k) \rangle^{1/2} \) between the reconstructed density field \( \delta \), and the original density field \( \delta \). We see that for \( a = 1 \), corresponding to the case of uncorrelated shear and inertia tensor, the reconstruction is very accurate on large scale (small \( k \)) and noisy at small scale (large \( k \)) as expected. Even for the realistic case of \( a \approx 0.2 \) given the nonlinear effects, Figure 1 shows that a reconstruction is still quite possible but with less accuracy. Note that the reconstructed signal automatically becomes Wiener filtered on large scale.

4. OBSERVATIONAL EFFECTS

Let us now mention several complications that might arise with real data and speculations on how they might be addressed.

The three-dimensional spin axis of a disk galaxy can in principle be determined (Han et al. 1995), but it requires knowledge of the rotation curve as well as morphological information such as extinction maps or the direction of spiral arms. A much easier observational task is to measure only the position angle \( \alpha \) on the sky and the projected axis ratio \( r \). Assuming a flat disk geometry for the light and the z coordinate along the radial direction, this corresponds to observation of \( r = |L| \) and \( \tan(\alpha) = L_z / L_r \). We immediately note two discrete degeneracies: \( L \rightarrow -L \), and \( \{ L_r, L_z \} \rightarrow \{-L_r, L_z \} \), giving four possible spin orientations consistent with the observables. Since we only care about the spatial orientation of the spin axis, but not its sign, only a twofold degeneracy exists, which we call \( \hat{L}_r, \hat{L}_z \). We can readily take that into account. Noting that \( P(\hat{L}_r \cdot \hat{L}_z) = P(\hat{L}_r | \hat{T}) + P(\hat{L}_z | \hat{T}) - P(\hat{L}_r \cdot \hat{L}_z | \hat{T}) \), the last term being zero, we simply replace all occurrences of \( \hat{L}_r, \hat{L}_z \) in equation (8) with \( \hat{L}_r, \hat{L}_z \).

The reconstruction itself is quite noisy for an individual galaxy, so we will treat as noise the displacement between the observable galaxy positions in Eulerian redshift space and their Lagrangian positions used in our analysis. If galaxies are randomly displaced from their origin by \( \sigma_g \), we simply convolve the two-point density correlation function \( \xi(r) \) with a Gaussian of variance \( \sigma_g^2 \) and appropriately update the shear correlation matrix. We furthermore note that the use of redshift for distance introduces an anisotropy in the smoothing Gaussian, which can also be readily taken into account. This procedure could in principle be inverted to measure the nonlinear length scale by measuring \( \sigma_g \) if one knows the intrinsic power spectrum. One could vary \( \sigma_g \) until one maximized the eigenvalue \( \Delta \) in equation (7).

The shear reconstruction correlator (eq. [8]) (L7) is only accurately defined at each galaxy position, so the integral should be replaced by a sum over galaxies. The eigenvector iteration scheme works the same way with the discretized matrix. The density is recovered from the trace-free shear in the same fashion as the continuum limit, where again we only reconstruct the density at the galaxy positions. The density at any other point is reconstructed in a similar fashion using the posterior expectation value given the Gaussian random field.

The observed spin vectors are of the luminous materials. They can be observed out to large radii, tens of kiloparsecs, through radio emission of the gas. While some warping of the disks is seen, the direction of the spin vector generally changes only very modestly as one moves to larger radii. This suggests...
that the luminous angular momentum is well correlated with the angular momentum of the whole halo. The standard galaxy biasing is not expected to affect the reconstruction, since we have not used the galaxy densities but only the spin directions. Merging of galaxies is also taken into account by this reconstruction procedure, since the resulting spin vectors are expected to align with the constituent orbital angular momentum vectors. The latter is again predicted by the same shear formula. The merger effects are included in the $N$-body simulations presented above.

5. CONCLUSIONS

We have presented a direct inversion procedure to reconstruct the gravitational shear and density fields from the observable unit galaxy spin field. The procedure is algebraically unique, dealing only with linear algebra, and is computationally tractable when done iteratively. We have shown that the angular momentum–shear correlation signal in simulations is strong enough to allow useful reconstructions. Direct $N$-body simulations suggest that nonlinear collapse effects only reduce the linearly predicted spin-shear correlation by a factor of 2. The deprojection degeneracy can be incorporated in a straightforward fashion. Large galaxy surveys with a million galaxy redshifts are coming on-line soon. This procedure may allow a density reconstruction that is completely independent of any other method that has thus far been proposed. We have demonstrated its effectiveness in simulated spin catalogs. Due to the nature of this Letter, only the key results essential to the reconstruction algorithm have been presented. Detailed intermediate steps and justifications will be shown elsewhere (J. Lee & U. Pen 2000, in preparation).

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