Behaviour of spin-$\frac{1}{2}$ particle around a charged black hole

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Dirac equation is separable in curved space-time and its solution was found for both spherically and axially symmetric geometry. But most of the works were done without considering the charge of the black hole. Here we consider the spherically symmetric charged black hole background namely Reissner-Nordström black hole. Due to presence of the charge of black-hole charge-charge interaction will be important for the cases of incoming charged particle (e.g. electron, proton etc.). Therefore both gravitational and electromagnetic gauge fields should be introduced. Naturally behaviour of the particle will be changed from that in Schwarzschild geometry. We compare both the solutions. In the case of Reissner-Nördstrom black hole there is a possibility of super-radiance unlike Schwarzschild case. We also check this branch of the solution.

04.20.-q, 04.70.-s, 04.70.Dy, 95.30.Sf

I. INTRODUCTION

Chandrasekhar separated Dirac equation in Kerr geometry into radial and angular parts [1] in 1976. His separation method can be extended to Schwarzschild geometry and corresponding separated equations can be found. But he did not consider the charge of the black hole. If we consider the black hole as a charged one then electromagnetic interaction is important for incoming particle with charge. To study the behaviour of spin-$\frac{1}{2}$ particle, Dirac wave is treated as a perturbation in the space-time which is asymptotically flat [1]. Far away from the black hole its influence on particle is not significant. As it comes closer, feels the curvature of the space-time and corresponding behaviours start to change with respect to that of flat space. Their behaviour around black-hole without charge have been studied in the past by several authors [1-6]. In this paper, we will introduce charge in the black hole. Here, we study a simpler problem to have a feeling about the solution when the black hole is non-rotating but charged. Here we have to solve Dirac equation in electromagnetic field around a Reissner-Nordström black-hole. Thus we will study the particle in crossed electromagnetic and gravitational field. It is very clear that the potential felt by the incoming Dirac wave will be different from that for Schwarzschild black hole [5]. For the incoming uncharged particle like neutron, electromagnetic field does not play any part and the Dirac equation will be reduced to same as Schwarzschild case except the re-definition of horizon. For charged incoming particle like electron, proton etc. electromagnetic gauge field should be introduced. One also can study the neutrino wave whose behaviour is known for Kerr geometry [7]. In the next Section, we present the basic Dirac equations and separate them in this crossed field. In §3, we study the
II. DIRAC EQUATION AND IT’S SEPARATION

By introducing electromagnetic interaction and gravitational effect the covariant derivatives take the form as

\[ D_\mu = \partial_\mu + iq_1 A_\mu + q_2 \Gamma_\mu. \]  

The derivative of spinor \( P^A \) can be written as

\[ D_\mu P^A = \partial_\mu P^A + iq_1 A_\mu P^A + q_2 \Gamma^A_{\mu \nu} P^\nu, \]  

where, \( q_1 \) and \( q_2 \) are coupling constants. \( q_1 \) is the charge of the incoming particle (say \( q_1 = q \)) and \( q_2 \) is chosen throughout 1. \( A_\mu \) and \( \Gamma^A_{\mu \nu} \) are electromagnetic and gravitational gauge (spin coefficients) fields respectively. Thus, following [7] the Dirac equation in Newman-Penrose formalism can be written as

\[ \sigma^\mu_{AB'} D_\mu P^A + i\mu_p \bar{Q}^\nu \epsilon_{C'B'} = 0, \]  

\[ \sigma^\mu_{AB'} D_\mu Q^A + i\mu_p \bar{P}^\nu \epsilon_{C'B'} = 0, \]  

where, for any vector \( X_i \), according to the spinor formalism [7] \( \sigma^i_{AB'} X_i = X_{AB'}; A, B = 0, 1 \). Here, we introduce a null tetrad \((\vec{l}, \vec{n}, \vec{m}, \vec{\bar{m}})\) to satisfy orthogonality relations, \( \vec{l}.\vec{n} = 1, \vec{m}.\vec{\bar{m}} = -1 \) and \( \vec{l}.\vec{m} = \vec{n}.\vec{\bar{m}} = \vec{l}.\vec{\bar{m}} = \vec{n}.\vec{m} = 0 \) following Newman & Penrose [8]. \( 2\pi \mu_p \) is the mass of the Dirac particle. In terms of this new basis in Newman-Penrose formalism Pauli matrices can be written as

\[ \sigma^\mu_{AB'} = \frac{1}{\sqrt{2}} \begin{pmatrix} l^\mu & m^\mu \\ \bar{m}^\mu & n^\mu \end{pmatrix}. \]  

Using equation (2), (3a), (4) and choosing \( B = 0 \) and subsequently \( B = 1 \) we get

\[ l^\mu(\partial_\mu + iqA_\mu)P^0 + \bar{m}^\mu(\partial_\mu + iqA_\mu)P^1 + (\Gamma_{1000'} - \Gamma_{0010'})P^0 + (\Gamma_{1100'} - \Gamma_{0110'})P^1 - i\mu_p \bar{Q}^{1'} = 0, \]  

\[ m^\mu(\partial_\mu + iqA_\mu)P^0 + n^\mu(\partial_\mu + iqA_\mu)P^1 + (\Gamma_{1001'} - \Gamma_{0011'})P^0 + (\Gamma_{1101'} - \Gamma_{0111'})P^1 + i\mu_p \bar{Q}^{0'} = 0, \]  

Next by taking complex conjugation of equation (3b), writing various spin coefficients by their named symbol [7] and choosing

\[ P^0 = F_1, P^1 = F_2, \bar{Q}^{1'} = G_1, \bar{Q}^{0'} = -G_2 \]

we get

\[ l^\mu(\partial_\mu + iqA_\mu)F_1 + \bar{m}^\mu(\partial_\mu + iqA_\mu)F_2 + (\epsilon - \rho)F_1 + (\pi - \alpha)F_2 = i\mu_p G_1, \]  

\[ m^\mu(\partial_\mu + iqA_\mu)F_1 + n^\mu(\partial_\mu + iqA_\mu)F_2 + (\mu - \gamma)F_2 + (\beta - \tau)F_1 = i\mu_p G_2, \]  

\[ l^\mu(\partial_\mu + iqA_\mu)G_2 - m^\mu(\partial_\mu + iqA_\mu)G_1 + (\epsilon^* - \rho^*)G_2 - (\pi^* - \alpha^*)G_1 = i\mu_p F_2, \]  

behaviour of the potential and possibilities of super-radiance. In §4, we present a complete solution. Finally, in §5, we draw our conclusions.
\[ n^\mu (\partial_\mu + i q A_\mu) G_1 - \bar{m}^\mu (\partial_\mu + i q A_\mu) G_2 + (\mu^* - \gamma^*) G_1 - (\beta^* - \tau^*) G_2 = i \mu_p F_1, \quad (6d) \]

These are the Dirac equations in Newman-Penrose formalism in curved space-time with the presence of electromagnetic interaction.

Now we write the basis vectors of null tetrad in terms of elements of the Reissner-Nordström geometry \([7,9]\) as,

\[ l^\mu = \frac{1}{\Delta}(r^2, \Delta, 0, 0), \quad (7a) \]

\[ n^\mu = \frac{1}{2r^2}(r^2, -\Delta, 0, 0), \quad (7b) \]

\[ m^\mu = \frac{1}{r\sqrt{2}}(0, 0, 1, icosec\theta), \quad (7c) \]

\[ \bar{m}^\mu = \frac{1}{r\sqrt{2}}(0, 0, 1, -icosec\theta), \quad (7d) \]

where, \( \Delta = r^2 - 2Mr + Q^2 \) and \( G = \hbar = c = 1 \) are chosen. Here \( M \) is mass of the black hole, \( Q \) is charge of the black hole, \( G \) is gravitational constant, \( h \) is Plank’s constant, \( c \) is speed of light.

We consider the spin-\( \frac{1}{2} \) wave function as the form of \( e^{i(\sigma t + m\phi)} f(r, \theta) \) where, \( \sigma \) is the frequency of the incoming wave and \( m \) is the azimuthal quantum number. The temporal and azimuthal dependencies are chosen same but radial and polar dependencies are chosen different for different spinors. Thus we write,

\[ f_1 = e^{i(\sigma t + m\phi)} r F_1, \quad f_2 = e^{i(\sigma t + m\phi)} F_2, \quad g_1 = e^{i(\sigma t + m\phi)} G_1, \quad g_2 = e^{i(\sigma t + m\phi)} r G_2 \quad (8) \]

Now we strictly consider the static field so the magnetic potentials are chosen zero, i.e., \( A^\mu = (A^t, 0, 0, 0) \). \( A^t \) is nothing but corresponding scaler potential of the field as (in this spherically symmetric space-time)

\[ A^t = \frac{qQ^*}{r - r_+}, \quad (9) \]

where, \( r_+ = \) location of the horizon \( = M + \sqrt{M^2 - Q^2} \).

So using equations (7), (8) and (9) and writing various spin coefficients in terms of the Reissner-Nordström metric elements (actually in terms of basis vectors) \([7]\) equation (6)’s reduce to

\[ D_0 f_1 + 2^{-\frac{1}{2}} L_\frac{1}{2} f_2 = i \mu_p r g_1 \quad (10a) \]

\[ \Delta D_\frac{1}{2} f_2 - 2^{-\frac{1}{2}} L_\frac{1}{2} f_1 = -2i \mu_p r g_2 \quad (10b) \]

\[ D_0 g_2 - 2^{-\frac{1}{2}} L_\frac{1}{2} g_1 = i \mu_p r f_2 \quad (10c) \]

\[ \Delta D_\frac{1}{2} g_1 + 2^{-\frac{1}{2}} L_\frac{1}{2} g_2 = -2i \mu_p r f_1 \quad (10d) \]

where,

\[ D_n = \frac{d}{dr} + \frac{ir^2 \sigma}{\Delta} + \frac{iqQ^* r^2}{\Delta(r - r_+)} + 2n \frac{r - M}{\Delta}, \quad D_\frac{1}{2} = \frac{d}{dr} - \frac{ir^2 \sigma}{\Delta} - \frac{iqQ^* r^2}{\Delta(r - r_+)} + 2n \frac{r - M}{\Delta}, \quad (11) \]
\[ \mathcal{L}_n = \frac{d}{d\theta} + Q + n\cot\theta, \quad \mathcal{L}^\dagger_n = \frac{d}{d\theta} - Q + n\cot\theta, \quad Q = m\csc\theta. \]  

Now considering \( f_1(r, \theta) = R_{-\frac{1}{2}}(r)S_{-\frac{1}{2}}(\theta) \), \( f_2(r, \theta) = R_{\frac{1}{2}}(r)S_{\frac{1}{2}}(\theta) \), \( g_1(r, \theta) = R_{\frac{1}{2}}(r)S_{-\frac{1}{2}}(\theta) \), \( g_2(r, \theta) = R_{-\frac{1}{2}}(r)S_{\frac{1}{2}}(\theta) \)

and following Chandrasekhar [7] we can separate the Dirac equation into radial and angular parts as

\[ \Delta^{1/2} \mathcal{D}_0 R_{-1/2} = (\lambda + im_pr)\Delta^{1/2} R_{1/2}, \]  
\[ \Delta^{1/2} \mathcal{D}_0^\dagger \Delta^{1/2} R_{1/2} = (\lambda - im_pr)R_{-1/2}, \]

\[ \mathcal{L}_{1/2} S_{1/2} = -\lambda S_{-1/2}, \]  
\[ \mathcal{L}^\dagger_{1/2} S_{-1/2} = \lambda S_{1/2}. \]

Here, \( m_p \) is the normalised rest mass of the incoming particle and \( \lambda \) is the separation constant.

### III. NATURE OF THE POTENTIAL IN DECOUPLED SYSTEM

The equations (14a-b) are same as the angular equation in Schwarzschild geometry whose solution is given in [2,10,11] as

\[ \lambda^2 = \left( l + \frac{1}{2} \right)^2, \quad R_{\pm \frac{1}{2}} = \text{standard spherical harmonics} = \pm \frac{1}{2} Y^l_m(\theta). \]  

It is clear that the separation constant depends on orbital angular momentum quantum number \( l \).

The equations (13a-b) are in coupled form. Following Chandrasekhar’s [7] and Mukhopadhyay & Chakraborti’s [5] approach we can decouple it as

\[ \left( \frac{d^2}{dr^2} + \sigma^2 \right) Z_\pm = V_\pm Z_\pm, \]  

where,

\[ \hat{r}_* = r_* + \frac{1}{2\sigma} \tan^{-1} \frac{m_pr}{\lambda} + \frac{qQ_*}{\sigma} \left[ \log(r - r_-) + \left\{ \frac{2r_+}{r_+ - r_-} - \frac{r_+^2}{(r_+ - r_-)^2} \right\} \log \left( \frac{r - r_+}{r - r_-} \right) - \frac{r_+^2}{(r - r_-)(r - r_+)} \right], \]  

\[ r_* = r - 3M + \frac{r_+^2}{r_+ - r_-} \log(r - r_+) - \frac{r_-^2}{r_+ - r_-} \log(r - r_-), \]  

\[ r_\pm = M \pm \sqrt{M^2 - Q_*^2}, \quad Z_\pm = \Delta^{1/2} R_{1/2} e^{i\Theta/2} \pm R_{-1/2} e^{-i\Theta/2}, \quad \Theta = m_pr. \]

In the extreme case when \( M = Q^* \), expression for \( \hat{r}_* \) and \( r_* \) are given as,

\[ \hat{r}_* = r_* + \frac{1}{2\sigma} \tan^{-1} \frac{m_pr}{\lambda} + \frac{qQ_*}{\sigma} \left[ \log(r - M) - \frac{2M^2}{(r - M)^2} - \frac{2M}{(r - M)} \right], \]  
\[ r_* = r - M + 2M \log(r - M) - \frac{M^2}{(r - M)}. \]
Here, $\hat{r}_*$ is varying from $-\infty$ to $+\infty$ (cartesian coordinate). If we compare equation (16) with one dimensional Schrödinger equation in cartesian coordinate system the energy $E$ of the incoming particle can be written as $E \propto \sigma^2$ and the potential ($V_\pm$) felt by the particle is given as

$$V_\pm = \frac{\Delta (\lambda^2 + m_p^2 r^2)}{r^2(\lambda^2 + m_p^2 r^2) \left(1 + \frac{Q_\pm q}{(r-r_+)^2} \right) + \frac{\Delta \lambda m_p}{2 \sigma} + \frac{\Delta \lambda m_p}{2 \sigma}} \pm \frac{\Delta (\lambda^2 + m_p^2 r^2)}{r^2(\lambda^2 + m_p^2 r^2) \left(1 + \frac{Q_\pm q}{(r-r_+)^2} \right) + \frac{\Delta \lambda m_p}{2 \sigma} + \frac{\Delta \lambda m_p}{2 \sigma}}$$

$$\{ r^2(\lambda^2 + m_p^2 r^2) \left(1 + \frac{Q_\pm q}{(r-r_+)^2} \right) + \frac{\Delta \lambda m_p}{2 \sigma}, \} (\lambda^2 + m_p^2 r^2)^{1/2} \frac{(r-M)(\lambda^2 + m_p^2 r^2) + 3\Delta m_p^2}{2} - \frac{\Delta^{1/2}(\lambda^2 + m_p^2 r^2)^{3/2}}{2}$$

$$\{ 2r(\lambda^2 + m_p^2 r^2) \left(1 + \frac{Q_\pm q}{(r-r_+)^2} \right) + 2r^3 m_p^2 \left(1 + \frac{Q_\pm q}{(r-r_+)^2} \right) - 2(\lambda^2 + m_p^2 r^2) \frac{Q_\pm q}{(r-r_+)^2} + \frac{(r-M)\lambda m_p}{\sigma} \}. (20)$$

From the expression of $V_\pm$ it is very clear that potential strictly depends on charge of the particle as well as of black hole. More precisely it depends on Coulomb interaction between charge of black hole and incoming particle. When charge of the black hole or particle or both are chosen zero the potential reduces to same as that in Schwarzschild geometry [5]. When factor $\frac{Q_\pm q}{\sigma}$ is positive potential varies smoothly. When $\frac{Q_\pm q}{\sigma}$ becomes negative $V_\pm$ diverges at a certain location $r = \alpha$. For the second case factor $\left(1 + \frac{Q_\pm q}{(r-r_+)^2} \right)$ vanishes at $r = r_+ - \frac{Q_\pm q}{\sigma} > r_+$ and then becomes negative. At $r = \alpha > r_+$ denominator of $V_\pm$ vanishes. For all other cases $\alpha < r_+$ always, so there is no scope to diverge the potential. Thus for the positive energy solution when the electro-magnetic scalar potential in the field is of attractive nature corresponding potential diverges again for negative energy solution potential diverges for repulsive electro-magnetic scalar potential. For the integral spin particle, it is found that when potential diverges energy extraction is possible i.e., super-radiation is occurred in the space-time [7]. On the other hand for the case of spin-half particle in Kerr geometry although at a certain parameter region potential diverges but super-radiation does not exist [7]. In the case of spherically symmetric Schwarzschild geometry potential does not diverge at all and no scope of super-radiation [5]. Here it is interesting to note that although our space-time is spherically symmetric but due to presence of electromagnetic interaction term, the region exist which is expected to be super-radiant.

Figure 1 shows behaviour of potential $V_\pm$ for different values black hole charges, where $\sigma = 0.8$, $m_p = 0.8$, $l = \frac{1}{2}$, $q = 1$ are chosen; $\alpha < r_+$. When $Q_* = 0$ (solid curve), potential reduces to same as Schwarzschild case shown in Fig. 2 by Mukhopadhyay & Chakraborti [5]. It is also seen that with the increment of charge of the black hole, barrier height decreases. Increment of black hole charge indicates the increment of electro-magnetic coupling and corresponding repulsive scalar potential opposes the attractive gravitational field. So net effect decreases. Figure 2 shows the change of potential barrier for different values of particle charge, where $\sigma = 0.8$, $m_p = 0.8$, $l = \frac{1}{2}$, $Q_* = 0.6$ are chosen; $\alpha < r_+$. Solid curve indicates the potential felt by the neutron like particle.

Now come to the cases when $\frac{Q_\pm q}{\sigma}$ is negative. For these cases $\hat{r}_* - r$ relation attends multivalues. For both $r \to \infty$ and $r \to r_+$, $\hat{r}_* \to \infty$. As explained above net potential barrier diverges at a certain location in this parameter region. From equation (20) it is very clear that near $r = \alpha$, potential varies as $\frac{1}{(r-\alpha)^2}$. So it has two branches, one repulsive and another attractive on each side of the singular point. As a result super-radiation is absent for the case of Reissner-Nordström geometry as in other cases [5,7]. We can choose any combination of $Q_*$, $q$ and $\sigma$ in such a way that $\frac{Q_\pm q}{\sigma}$ is negative.

In Fig. 3 we show how nature of the potential ($V_\pm$) changes with rest mass of the incoming particle where, $\sigma = 0.8$, $Q_* = 0.5$, $l = \frac{1}{2}$, $q = 1$ are chosen. Solid curve shows nature for neutrino wave. It is very clear from the figure that with the increase of rest mass of the incoming particle gravitational interaction increases and corresponding potential barrier attains high value.
IV. THE COMPLETE SOLUTION

Now we will find spatially complete solution. As we mentioned earlier that solution of the angular part is known which is same as Schwarzschild case [5,10,11]. For radial solution we need to solve decoupled radial equation. The solution of equation (16) for potential $V_+$ and $V_-$, using Instantaneous WKB Approximation (in short IWKB) method [5,6] can be written as

$$Z_+ = \sqrt{T_+[k_+(\hat{r}_*)]exp(iu_+)} + \sqrt{R_+[k_+(\hat{r}_*)]exp(-iu_+)},$$

(21a)

$$Z_- = \sqrt{T_-[k_-(\hat{r}_*)]exp(iu_-)} + \sqrt{R_-[k_-(\hat{r}_*)]exp(-iu_-)},$$

(21b)

where,

$$k_\pm(\hat{r}_*) = \sqrt{\sigma^2 - V_\pm},$$

(22)

$$u_\pm(\hat{r}_*) = \int k_\pm(\hat{r}_*)d\hat{r}_* + \text{constant},$$

(23)

with

$$T_+(r) + R_+(r) = 1, \quad T_-(r) + R_-(r) = 1 \quad \text{instantaneously.}$$

(24)

Here, $k$ is the wavenumber of the incoming wave and $u$ is the Eiconal, $T_\pm$ and $R_\pm$ are instantaneous transmission and reflection coefficients [5] respectively. Using this method at each location, instantaneously, WKB method is applied. This solution is valid when $\frac{1}{k} \frac{dk}{d\hat{r}_*} \ll k$, otherwise different method [5] should be used.

In Fig. 4, the comparison of instantaneous reflection and transmission coefficients in between Schwarzschild and Reissner-Nordström geometry are shown. The parameters chosen are given in Figure Caption. With the decrease of barrier height the transmission coefficient increases as well as reflection coefficient decreases. As it is seen that by introduction of the electromagnetic coupling, potential barrier heights reduce so corresponding transmission probability increase with respect to that of Schwarzschild case (the behaviour for Schwarzschild case is graphically shown in [5]) for a particular set of parameter. So the presence of the charge of black hole decreases the curved nature of space-time.

Now recombining $Z_+$ and $Z_-$ one easily can find out original radial Dirac wave functions $R_\pm^{\frac{1}{2}}$ and $R_\mp^{\frac{1}{2}}$ [5]. Finally we will have complete solution as $J(r, \theta) = R_\pm^{\frac{1}{2}}(r)S_\pm^{\frac{1}{2}}(\theta)$.

V. CONCLUSIONS

In this paper, we have studied analytically the scattering of spin-half particles off Reissner-Nordström black hole. Our main motivation is to show analytically how the spin-half particles behave in the presence of electromagnetic interaction in curved space-time. We introduced the gravitational and electromagnetic gauge fields. Since no such kind of study had been carried out previously we started from scratch. Firstly, we wrote corresponding dynamical equation of spin-half particle namely Dirac equation in combined gravitational and electromagnetic background. Due to curvature of the space-time gravitational gauge field (here, spin coefficients for Reissner-Nordström geometry) was introduced. The electromagnetic interaction comes into the game because of charge of the black hole. Here, we have considered steady-state problem and corresponding components of electromagnetic vector potential to zero. We then
separated the equation into radial and angular parts. It is seen that in case of spherically symmetric space-time, presence of charge of the gravitating object does not affect the behaviour of the incoming particles in polar direction. Only the radial part of the equation is influenced. We then decoupled the radial Dirac equation. Now the potential is dependent on charge-charge coupling in the space-time. If the charge of the black hole reduces to zero, the potential reduces to that of Schwarzschild case. With the presence of repulsive (or attractive) charge-charge interaction for positive (or negative) energy solution magnitude of curvature effect reduces. This is because of opposing nature of two simultaneous interactions.

There is one interesting sector of the solution (which sector was absent in uncharged spherically symmetric space-time). If the charge-charge interaction is of attractive nature for positive energy solution (or repulsive for negative energy solution) then potential at a certain location (\(r = \alpha\)) diverges. But because of \(\frac{1}{(r-\alpha)^3}\) variance of potential super-radiation is absent.

Here we study the behaviour of potential by varying charge of the black hole, charge of the incoming particle, rest mass of the incoming particle. We also study the space-dependent reflection and transmission coefficients and show graphically for one set of physical parameter. It is seen that as potential barrier height decreases corresponding transmission probability increases. We solve the radial Dirac equation by IWKB method.

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FIGURE CAPTIONS

Fig. 1: Behaviour of potential for different values of the black hole charge. Fixed parameters are, \(\sigma = 0.8, m_p = 0.8, l = \frac{1}{2}\) and \(q = 1\). From upper to lower curves the charge \(Q^*\) of the black holes are chosen as 0, 0.2, 0.4, 0.6, 0.8, 0.998.

Fig. 2: Behaviour of potential for different values of the incoming particle charge. Fixed parameters are, \(\sigma = 0.8, m_p = 0.8, l = \frac{1}{2}\) and \(Q^* = 0.6\). From upper to lower curves the charge \(q\) of the particles are chosen as 0, 0.2, 0.4, 0.6, 0.8, 1.
Fig. 3: Behaviour of potential for different values of the rest mass of incoming particle. Fixed parameters are, $\sigma = 0.8$, $Q_\ast = 0.5$, $l = \frac{1}{2}$ and $q = 1$. From upper to lower curves the mass $m_p$ of the particles are chosen as 0.4, 0.3, 0.2, 0.1, 0.

Fig. 4: Instantaneous reflection (R) and transmission (T) coefficients for Reissner-Nordström (solid curves) and Schwarzschild (dotted curves) black holes. Physical parameters are chosen as $\sigma = 0.8, m_p = 0.8, l = \frac{1}{2}, q = 1$. For Reissner-Nordström case $Q_\ast = 0.5$. 
