Microscopic mean field approximation and beyond with the Gogny force

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Abstract

We review the main results of several works using the finite range Gogny interaction within mean field–based approaches. Starting from static mean field description, a GCM-like method including rotational degrees of freedom, namely the five-dimension collective Hamiltonian, is applied. The theoretical results are used to interpret the shell evolution along the $^N_1 = 16$ isotonic chain. The quasiparticle random-phase-approximation formalism is introduced and used to simultaneously describe high- and low-energy spectroscopy as well as collective and individual excitations. After a discussion on the role of the intrinsic deformation in giant resonances, the appearance of low-energy dipole resonances in light nuclei is analysed. Finally, a comparison of the low-energy spectroscopy obtained with these two extensions of static mean field is performed for $^N_2$ states of $^N_1 = 16$ isotopes.

Keywords: collective levels, giant resonances, transition probabilities, excitation spectra, Hartree–Fock–Bogoliubov, random-phase approximation, configuration mixing

(Some figures may appear in colour only in the online journal)

1. Introduction

One challenge in theoretical nuclear physics is the development of a single approach to describe the excited states of all nuclear systems with the same accuracy. Even if ground state properties are theoretically obtained via Hartree–Fock–Bogoliubov (HFB) static mean field calculations, most of these properties are deduced experimentally from decays of excited states; hence a proper theoretical treatment of nuclear excitations is needed. Extending the description of the nuclear states beyond the HFB formalism improves the nuclear structure models and allows us to describe the spectroscopy, i.e., energies and wave functions of the excited states. Two methods able to describe nuclear spectroscopy and some related studies using the Gogny D1S effective force [1] are presented. Then results on the low-lying $^2_2$ states obtained with both models are compared to each other.

2. Mean field–based approaches

The extension beyond the static mean field can be done in various ways, depending on studied phenomena, leading to different assumptions and approximations. Let us first introduce a method using shape-constrained potential energy surfaces to describe low-energy quadrupole spectroscopy: the five-dimension collective Hamiltonian (5DCH) [2, 3]. The 5DCH can be understood as a mixture of triaxial constrained HFB solutions for different shapes, which describes rotational and vibrational degrees of freedom on the same footing. This approach is well suited to the spectroscopy of transitional nuclei (mixing between rotor and vibrator) and to the shape coexistence description. Systematics of 5DCH predictions for D1S Gogny force can be found in [4, 5]. Nevertheless, such theoretical framework does not provide information on high-energy spectra or non collective motion.
As an alternative to the 5DCH, models based on the random-phase-approximation (RPA) [6] are well suited in rigid nuclei as they describe on the same footing both individual and collective excited states. This formalism has been found to be successful in predicting low-lying multipole vibrations as well as giant resonances [7]. Pairing correlations have been included within the quasiparticle RPA (QRPA) formalism to describe excited states of open-shell nuclei [8] and deformed (Q)RPA models have been developed [9–11]. In order to avoid inconsistencies [12], many calculations are now performed using the same effective nucleon-nucleon interaction for the mean field ground state and the (Q)RPA excited states [10, 12–14]. The only parameters of these fully self-consistent HF(B) + (Q)RPA approaches are those of the effective interaction. In order to describe giant resonances along a consistent line, it became essential to microscopically take into account the intrinsic deformation of the nuclei. Thus assuming axial symmetry, QRPA calculations could be performed on top of deformed HFB solutions. In an axially symmetric-deformed nuclear system, the response function of given total angular momentum and parity contains different \( K^* = 0^+, \pm 1^+, \ldots \pm J^+ \) components, where \( K \) is the projection of \( J \) onto the symmetry axis. In spherical nuclei, all these components are degenerated in energy, then the response functions associated to any multipolarity can be obtained from \( K^* = 0^+ \) results only. In QRPA calculations [10], phonons are coherent states of two-quasiparticle excitations. Quasiparticle states which are solutions of HFB calculations are expanded on a finite harmonic oscillator (HO) basis. Consequently the positive energy continuum is discretized. The number of major shells involved in the HO basis depends on the number of nucleons of studied nuclei: from 9 major shells for light nuclei, Mg and Si isotopes for example, up to 13 major shells for actinides. A reasonable choice of the size of the HO basis guarantees the feasibility of complete QRPA calculation without any cut in excitation energy. In the \( K^* = 0^+ \) calculations we have 5002, 26894 and 52432 two-quasiparticle configurations for 9, 13, and 15 major shells, respectively. Recently, axially symmetric-deformed-QRPA calculations have been performed to describe multipolar responses up to the octupole in a deformed and heavy nucleus, namely \(^{238}\text{U}\) [15]. In order to treat such a heavy deformed system, a new implementation of the numerical code, based on parallel computation, was needed. Thus a master-slave algorithm adapted to QRPA matrices has been developed, thus around 80 years of computation time has been dispatched to 256 or 512 processes in two- or five-hour runs. The present scalable scheme has been checked and used up to 4096 processes. Since, the axially symmetric-deformed HFB+QRPA method has been applied to nuclei for which photo-absorption data exist using the D1M Gogny force [16].

### 3. Results for light nuclei

With the development of experimental techniques and the emergence of large facilities, the production of more and more exotic elements and the study of their spectroscopy helped to highlight the disappearance of some magic numbers and the emergence of new ones. It is still relevant to reproduce such singular numbers and to understand their evolution as functions of isospin difference. Three criteria for shell closure occurrence have been defined for the analysis of the vanishing of magic numbers \( N = 20 \) and \( N = 28 \) [17] and used in the analysis of the \( N = 16 \) ph gap evolution [18]; for a given shape (i) the nuclear energy is minimum, (ii) there is a significant gap above the Fermi level, and (iii) pairing correlations vanish. The potential energy surfaces have a minimum for spherical shape, for all \( N = 16 \) studied nuclei. For stable nuclei, the \( N = 16 \) spherical gap between particle and hole states (ph gap) is not large enough to ensure magic shell closure, but there are ph gaps for prolate shapes as illustrated in [18]. For exotic isotones, the spherical gap becomes efficient, and neutron pairing correlation vanishes. Such mean field results suggest a new magic number. However, such static description does not provide evidence of shell closure, and low energy spectroscopy needs to be predicted. Thus, configuration mixing with 5DCH formalism is applied for even–even \( N = 16 \) isotones, except for the doubly magic \( ^{24}\text{O} \). For this rigid nucleus, RPA formalism has been used to complete predictions along the isotonic chain. Results can be found in table 1. For all the \( N = 16 \) isotones under study, the agreement between theory and experimental data is satisfactory. For the heaviest isotones, from \( Z = 14 \) to \( Z = 18 \), excitation energies and transition probabilities are constant. In opposite, from \(^{26}\text{Mg} \) to \(^{24}\text{O} \), the excitation energy of the first \( 2^+ \) state increases when the related transition probability decreases. This evolution of the first excited state as a function of the proton number characterizes the widening of the spherical ph gap. Thus, the present study predicts that \( N = 16 \) is a spherical shell closure for exotic nuclei in agreement with the excitation energy of the first \( 2^+ \) state of \(^{24}\text{O} \) [20].

The first application of the axially symmetric-deformed QRPA approach with the Gogny force [10] was devoted to the analysis of the impact of deformation on giant resonances in Mg and Si isotopes. Light nuclei have been chosen because there are many different intrinsic deformations along isotopic chains: when only two neutrons or two protons are added or

| \( ^A\text{X} \) | \( E(2^+) \) (MeV) | \( B(E2) e^2\text{fm}^4 \) |
|---|---|---|
| \(^{34}\text{Si} \) | 1.98 | 2.09 | 306 | 240(40) |
| \(^{36}\text{Ar} \) | 1.78 | 2.23 | 254 | 300(13) |
| \(^{28}\text{Si} \) | 2.11 | 2.23 | 220 | 215(10) |
| \(^{24}\text{Mg} \) | 1.50 | 1.47 | 202 | 350(5) |
| \(^{30}\text{Ne} \) | 2.19 | 2.02 | 86 | 228(41) |
| \(^{32}\text{O} \) | 3.81 | 4.65 (±0.14) | 15 | — |

Table 1. Theoretical (HFB+5DCH with D1S Gogny force) and experimental values of \( B(E2) \) and excitation energy for \( 2^+ \) states. Data are taken from [19, 20].
suppressed, the shape of the ground state becomes either prolate or oblate. Crossing the $N = 16$ new shell closure, magnesium and silicon isotopic chains contain spherical nuclei ($^{28}\text{Mg}$ and $^{30}\text{Si}$), which are taken as reference. As expected, the dipole response of deformed nuclei is found to be split into two components along two directions: the symmetry axis and the transverse one. These two components have been found to be separated according to their $K$ values. A hierarchy between $K$ components is obtained, depending on the sign of the quadrupole moment, which also remains for other multipolarities. For deformed nuclei, the monopole response is also split thanks to the $K = 0$ quadrupole-monopole coupling. To illustrate these features, predicted electric transition probabilities $B(E\lambda)$ as functions of QRPA energies for monopole ($\lambda = 0$) and quadrupole ($\lambda = 2$) modes are given in figure 1 for prolate oblate and spherical Mg isotopes. For $^{24}\text{Mg}$ and $^{26}\text{Si}$ stable nuclei, experimental monopole and quadrupole data are available on [21]. The predicted averaged ISGMR energy values are in agreement with measurements, since for $^{24}\text{Mg}$ $E_{\text{QRPA}} = 21.06$ MeV and $E_{\text{Exp}} = 21.0 \pm 0.6$ MeV, and for the $^{26}\text{Si}$ $E_{\text{QRPA}} = 21.76$ MeV and $E_{\text{Exp}} = 21.25 \pm 0.38$ MeV. However, the QRPA averaged values for quadrupole resonances are larger than the experimental ones. One obtains for the $^{24}\text{Mg}$ nucleus $E_{\text{QRPA}} = 20.54$ MeV and $E_{\text{Exp}} = 16.9 \pm 0.6$ MeV, and for the $^{28}\text{Si}$ nucleus $E_{\text{QRPA}} = 20.41$ MeV and $E_{\text{Exp}} = 18.54 \pm 0.25$ MeV. This overestimation was already found in the study of giant resonances for doubly magic nuclei [13]. The present approach being free from adjustable parameters, the comparison between experimental data and QRPA predictions is satisfactory and shows that the Gogny interaction qualitatively reproduces resonances without major adjustment of its parametrization. In the same region of masses, the occurrence of low-energy dipole states in exotic nuclei has been considered. Starting from the ‘pygmy’ state observed in $^{26}\text{Ne}$ [22], a systematic study has been done for all Ne isotopes and $N = 16$ isotones. The results are presented in details in [23] where the microscopical structure of these low-lying resonances, as well as the behaviour of proton and neutron transition densities are analysed. Deduced motion does not coincide with the usual isoscalar representation of the Pygmy dipole resonance obtained in many theoretical calculations but for heavier nuclei [24]. From comparison between particle-hole and QRPA transition probabilities [23] it has been concluded that to generate the low-lying resonance, many transitions contribute in addition to the dominant one. This small but finite collective behaviour characterizes all the $N = 16$ isotones analysed.

4. 5DCH versus QRPA for spherical nuclei

In order to compare the two aforementioned approaches, we recall here their own features. On the one hand, 5DCH
formalism provides good predictions for low-energy quadrupole spectra for most even-even nuclei, including shape coexistence, or shape isomers [4, 5, 25]. While the low-energy spectroscopy of soft and transitional nuclei is well reproduced, the 5DCH fails to reproduce spherical and rigid nuclear spectra, which is dramatic for double-closed shell energy collective states, like giant resonances. However, this formalism does not describe rotational spectra, and before experimental excitation energies illustrates the role of rotational degrees of freedom. These results show that both approaches are able to provide structure informations in harmonic nuclei with quite the same accuracy. Nevertheless, the 5DCH approach is not able to describe nuclei which are too rigid against deformation.

5. Summary and perspectives

The 5DCH and the QRPA approach, well suited to axially symmetric-deformed nuclei, have been applied to light exotic nuclei in the vicinity of $N = 16$ shell closure. Results on monopole, dipole, and quadrupole modes are in good agreement with experimental data. For spherical nuclei, the QRPA formalism completes the 5DCH predictions, including the same effective interaction.

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