Buckling study of composite plates subjected to impact loading

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Abstract: Considering the stress wave effect, taking into account the axial inertia and the rotational inertia, the buckling control equation of the composite plate subjected to the impact load is obtained by the Hamilton principle. The equation is dimensionless and solved by the differential method, discuss the influence critical length, impact quality and lay-up angle other factors on the dynamic buckling of composite plates. It shows that the change of critical length, impact quality, and the lay-up angle, the modal number, half-wavelength and peak value of the buckling mode change, and the buckling high-order mode is changed. Excitation further illustrates that the stress wave effect and axial inertia have a significant effect on the dynamic buckling of composite plates.

1. Introduction

With the development of science and technology, composite materials have broad application prospects in many fields such as aviation, aerospace, mechanical engineering and nuclear engineering. As the most common basic structural components, plates are widely used in these fields, so buckling research of composite plates plays a significant role.

Cui[1] numerically investigates the dynamic buckling of thin imperfect rectangular plates subjected to intermediate-velocity impact loads. From numerical results obtained, a dynamic buckling and a dynamic yielding critical condition are defined, and the corresponding critical dynamic loads are estimated. Chen[2] numerically investigates the buckling loads and the absorbed energy of thin metallic plates stamped with V-grooves. By using the dynamic-explicit FEM Code, LS-DYNA together with the mass scaling technique and contact algorithm, the collapse processes of the plates subjected to various impact velocities were simulated. Stevens K A [3] obtained the postbuckling behavior of a flat, stiffened, carbon fiber composite compression panel has been studied, theoretically and experimentally. Kong C W [4] investigates the postbuckling behavior of graphite-epoxy laminated stiffened panels has been studied analytically and experimentally herein. Li Lekun[5] researched the stability of compression test and the finite element method for composite stiffened panel. Peng Ying[6] chose the spline function to describe the deflection. The governing equation of the dynamic buckling of imperfect rectangular plates are obtained by weighted residual method, the equations are solved by a fourth-order Runge-Kutta method, and the computational code is developed in FPRTRAN. The formed B-spline function is adaptable to any elastic restraints against rotation. The influences of initial geometric imperfection, load duration and elastic restraints are discussed. Peng Ying[7] studied...
Theoretically, the nonlinear dynamic response and dynamic buckling of stiffened plates under impact loading.

In conclusion, the study of static buckling has become more mature. In the existing research, the analytical expression of buckling cannot be obtained considering the axial inertia, and the numerical method of dynamic buckling does not consider the stress wave effect. In this paper, considering stress wave effect, axial inertia and rotational inertia, dynamic buckling control equations of composite plates subjected to impact loading is obtained by Hamilton principle. Control equations are dimensionless, and Matlab is used to calculate the calculations. Effects of different critical length, impact mass and lay-up angles on dynamic buckling are discussed.

2. The control equations of dynamic buckling
The composite plate is a simple at \( x=0, y=0 \), and \( y=b \), and it is solid at \( x=a \), is shown in Figure 1. Under the impact load, the stress wave propagation along the \( x \) direction, and internal forces in each section of plate are shown in Figure 2, where \( l_{ct} \) is the critical length, \( t \) is critical time, and \( c \) is stress wave’s velocity.

![Figure 1 Composite plate loading diagram.](image1)

![Figure 2 stress wave propagation diagram.](image2)

The propagation of stress waves in the plate can be expressed as:

\[
\sigma(x,t) = \begin{cases}
-\rho c v_0 e^{-\frac{\rho c t}{M + \rho c t}} & , \quad 0 \leq x \leq \frac{ct}{L} \\
0 & , \quad \frac{ct}{L} \leq x \leq L
\end{cases}
\]  (1)

Where \( c \) is the wave velocity, \( t \) is the stress wave propagation time, and \( x \) is the propagation distance of the stress wave propagation wave along the plate length.

The constitutive equation for a composite plate is:

\[
\begin{bmatrix}
N_x \\
N_y \\
N_{xy} \\
M_x \\
M_y \\
M_{xy}
\end{bmatrix} = 
\begin{bmatrix}
A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\
A_{12} & A_{22} & A_{26} & B_{21} & B_{22} & B_{26} \\
A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\
B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\
B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\
B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy} \\
k_x \\
k_y \\
k_{xy}
\end{bmatrix}
\]  (2)

Where \( A_{ij} \) stand for tensile stiffness, \( B_{ij} \) denotes Coupling stiffness, \( D_{ij} \) is Bending stiffness, they all defined by the following formulas\(^8\).
\[(A_y, B_y, D_y) = \sum_{m=1}^{n} Q_{y(m)} \left( h_m - h_{m-1}, \frac{h_m^2 - h_{m-1}^2}{2}, \frac{h_m^3 - h_{m-1}^3}{3}\right) \]  

(3)

For isotropic symmetric plates, special orthotropic symmetric plates, and regular symmetric orthogonally laid plates, here are \(A_{16}=A_{26}=D_{16}=D_{26}=B_y=0^\circ\). Considering the moment of inertia and taking into account the axial inertia, the dynamic buckling control equations of composite plates are as follows:

\[\rho_0 h u_t = A_{11} u_{xx} + A_{66} u_{yy} + \frac{1}{4} \rho_0 h^2 w_{txx} \]

(4)

\[D_{11} w_{txxx} + (2D_{12} + 4D_{66}) w_{txyy} + D_{22} w_{tyyy} - A_{11} (u_{xx} \cdot w_x + w_{xx} \cdot u_x) \]

\[+ \rho_0 h w_t - \frac{1}{4} \rho_0 h^2 u_{txx} - \frac{\rho_0 h^3}{12} (w_{xxx} + w_{yyyy}) = 0 \]

(5)

3. Boundary conditions and dimensionless forms of control equations

Take the following dimensionless quantities:

\[\bar{w} = \frac{w}{l_{cr}}, \bar{u} = \frac{u}{l_{cr}}, \bar{x} = \frac{x}{l_{cr}}, \bar{y} = \frac{y}{l_{cr}}, D_{11} = l_{cr}^2 A_{11}, c = \sqrt{\frac{A_{11}}{\rho h l_{cr}}}, t_{cr} = ct_{cr} \]

(6)

\[\bar{t} = \frac{t}{l_{cr}}, \bar{A}_2 = \alpha, A_{11}, A_{66} = \alpha_2 A_{11}, A_{22} = \alpha_3 A_{11}, h = \alpha_4 \]

(7)

The dimensionless form of the control equations are calculated by bringing formula (6), (7) into formula (4), (5):

\[\bar{u}_t = \bar{u}_{xx} + \alpha_2 \bar{u}_{yy} + \frac{1}{4} \alpha_4 w_{txx} \]

(8)

\[-\bar{w}_{txxx} + (2\alpha_1 + 4\alpha_2) \bar{w}_{txyy} + \alpha_3 \bar{w}_{tyyy} - \left(\bar{u}_{xx} \cdot \bar{w}_x + \bar{w}_{xx} \cdot \bar{u}_x \right) \]

\[+ \bar{w}_t - \frac{1}{4} \alpha_4 \bar{u}_{xx} - \frac{1}{12} (\bar{w}_{xxx} + \bar{w}_{yyyy}) = 0 \]

(9)

The dynamic buckling control equations of composite plates satisfy the following initial conditions and boundary conditions:

\[
\begin{aligned}
\bar{u}(0, y) &= \bar{u}(l_{cr}, y) = 0 \\
\bar{u}''(0, y) &= \bar{u}''(l_{cr}, y) = 0 \\
\bar{u}(x, 0) &= \bar{u}(x, b) = 0 \\
\bar{u}''(x, 0) &= \bar{u}''(x, b) = 0
\end{aligned}
\]

(10)

\[
\begin{aligned}
\bar{w}(0, y) &= \bar{w}(l_{cr}, y) = 0 \\
\bar{w}''(0, y) &= \bar{w}''(l_{cr}, y) = 0 \\
\bar{w}(x, 0) &= \bar{w}(x, b) = 0 \\
\bar{w}''(x, 0) &= \bar{w}''(x, b) = 0
\end{aligned}
\]

(11)

The axial initial displacement is calculated as formula (12).

\[\bar{u}_0 = -\frac{MV_0}{\rho c l_{cr}^3} e^{-\frac{\rho c l_{cr}^3 t}{M}} + \frac{MV_0}{\rho c l_{cr}^3} \]

(12)

Cite w-direction initial displacement\(^ Cynthia\) as formula (13).

\[\bar{w}_0 = c_0 \left\{ \sin(n \pi \bar{x}) - \frac{n}{n+2} \sin((n+2) \pi \bar{x}) \right\} \sin\left(\frac{\pi y_{cr}}{b}\right) \]

(13)

Where \(M\) is impact mass, \(v_0\) denotes impact velocity, \(A\) stand for cross-sectional area, \(\rho\) is material.
density, $c$ is a parameter, and $n$ is transverse mode number, $m$ is circumferential mode number.

4. Difference format for control equations
Bring central differential and backward differential formats into control equations, the difference format of the control equations are obtained as follows.

$$
\overline{u}_{i,j,t+1} = \left( \overline{u}_{xx} + \alpha_2 \overline{u}_{yy} \right) \cdot (t)^2 + \overline{u}_{i,j,t-1} - 2 \overline{u}_{i,j,t}
$$

(14)

$$
\overline{w}_{i,j,t+1} = \left( \overline{w}_{xxxx} + \left( 2\alpha_1 + 4\alpha_2 \right) \overline{w}_{xxyy} + \alpha_2 \overline{w}_{yyyy} - \overline{u}_{xx} \cdot \overline{w}_x - \overline{w}_{xx} \cdot \overline{u}_x + \frac{1}{4} \overline{w}_{txx} \right)
$$

(15)

Where $i, j$ correspond to $x$ and $y$ spatial points of the plate, $t$ indicating time points. $\overline{x}$, $\overline{y}$ is a dimensionless space displacement step and $\overline{t}$ stand for a dimensionless time step.

5. Numerical Simulation
In this paper, dynamic buckling of carbon/epoxy composite plates is discussed and material parameters are as shown in Table 1 [8]. The influence of different critical length, impact mass and lay-up angle on dynamic buckling of composite plates is discussed by programming formula (14), (15) with Matlab. The impact mass is $M$, the critical speed is $v_0$, take displacement step $\Delta x=0.088$, $\Delta y=0.044$, and time step $\Delta t=0.025$. Then, obtained result is converted to the actual displacement by $w=\overline{w} \times l_{cr}$.

| Parameter | Value |
|-----------|-------|
| $E_1/(Gpa)$ | 139 |
| $E_2=E_3/(Gpa)$ | 9.4 |
| $\mu_{12}=\mu_{13}$ | 0.3095 |
| $\mu_{23}$ | 0.33 |
| $G_{12}=G_{13}/(Gpa)$ | 4.5 |
| $G_{23}/(Gpa)$ | 2.98 |
| $\rho/(kg/m^3)$ | 1583 |

Table 1 Composite material parameters.

Figure 3 expresses that dynamic buckling modes diagram of different critical length when impact mass and lay-up angle are constant.
Figure.3 Buckling waveforms of composite plates at different critical lengths. Figure.3 shows that with stress wave propagation, buckling is constantly expanding and growing. The number of buckling modes increases, higher-order modes are excited, and peak values increase, while the first half of the buckling wavelength decreases gradually. It also indicates that stress wave effects have a significant impact on dynamic buckling of the plates.

Figure.4 expresses the dynamic buckling mode diagram of different impact mass when critical length and lay-up angle are constant.

Figure.4 Buckling waveform of composite sheet at different impact mass. Figure.4 shows that the impact mass increases, the buckling continues to grow, the number of buckling modes increases, the higher-order mode of the buckling is excited, and the first half-wavelength of the buckling decreases gradually, and the corresponding peak values increase. It can be seen from equation (12) that the change in impact quality causes a change in axial inertia, which affects the dynamic buckling of the composite plates.

Figure.5 remarks that the buckling mode of different lay-up angles when critical length and impact mass are constant.
6. Conclusions

Based on theoretical research and numerical calculations, the paper draws the following conclusions:

1. Considering stress wave effect, axial inertia and rotational inertia are calculated, and dynamic buckling control equation of cylindrical shell is obtained by Hamilton principle based on Donnell thin shell theory. The control equations are dimensionless which solved by central difference and backward difference.

2. The Matlab is used to program the differential equations of the governing equations, and the effects of impact velocity, critical length, impact quality, and layup angle on the buckling mode peaks are discussed. The results show that with the increase of impact velocity, critical length, impact quality and layup angle, the peak value of buckling mode diagram increases, and the higher-order mode of buckling is excited. The stress wave and axial inertia have significant effects on dynamic buckling of composite plates.

Acknowledgments

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