The measurement research on government subsidy of urban rail transit PPP project under uncertainty of revenue

Sun Chunling¹,*, Ma Susut¹, and Xv Dieyuan¹

¹ School of management, Tianjin university of technology, Tianjin, 300384, China

* Corresponding author: hyscl@263.net

Abstract. In the process of promoting urban rail transit PPP projects, there are still obstacles in terms of laws, market mechanisms, risk allocation mechanisms and project revenue. This paper studies the impact of government subsidy on project revenue, and uses real options to build a government subsidy model of the rail transit PPP project. Meanwhile, the model is verified by an example. The results show that the two subsidy methods are more in line with the interests of both the government and the private sector. The aim is to provide a criterion and method for the government to determine the subsidy amount. Finally, under the framework of the subsidy combined pre-investment and operation period, the paper analyzes the nature of investment subsidy, operating period subsidy, passenger flow threshold and volatility, and expected return rate. The paper show some policy recommendations to provide a good policy environment for the development of rail transit.

1. Introduction

Urban rail transit, especially the subway, has a large investment and a long construction period. The price as the main income is also controlled by government because of its external effect greater than economic benefits. So it haves low profitability and then difficult to favored by private sector investment. On this occasion, the government adopts various subsidy methods to maintain the normal operation of enterprises. Subsidy mechanism is an important part of benefit distribution mechanism between government and private sector in PPP project cooperation, meanwhile is an improved interest return mechanism of the government[1]. It can avoid the failure of project cooperation due to excessive subsidy caused by the government lack the project construction funds. It is an important measure to attract private sector to invest PPP (short for “Public-Private Partnership”) projects.

Traditional discounted cash flow, NPV low and traditional real option method with relatively rigid and static features cannot effective to evaluate the economic value of the project. By contrast, the real option is an effect method because the uncertainty of PPP projects leads to the nature of options. Many scholars at home and abroad have done a lot of research on it. Li Yan[2] using binary tree and fuzzy theory derived the modified fuzzy real option pricing model by considering the potential strategic value of the project and took the subjective uncertainty factor of PPP project as the fuzzy number. Wibowo A[3] use the Monte Carlo method to price options by considering various requirements of investors. Liang Wei[4] combined the real option theory with the PPP project to deal with the difficulties caused by uncertainty in the negotiation, decision-making and execution of PPP projects. Liu Jicai[5] used the binary tree theory and through the practical case analysis, considered the traffic project demand (the uncertainty of the traffic flow), concluded the advantages of the real option theory over the cash flow discount method. Nicola[6] used the Monte Carlo simulation method constructed the profit decision-making model of BOT projects by considering the impact of uncertainty on the benefits.
Doanand[10] took the value of deferred investment options as the decision basis for the government to provide financial support for infrastructure projects by considering the high uncertainty and irreversibility of infrastructure projects.

In view of the future earning’s uncertainty and the positive externality characteristics of urban rail transit project, the research takes the PPP in urban rail transit project as the research object. Using delay options to quantitative analysis government subsidy and assess project value under uncertain earnings, and then getting a government subsidy model that determines government investment limit. The limits not only can private sectors gain profit but also guarantee smooth cooperation between government and private sector.

2. Analysis of government subsidy for urban rail transit PPP project
The starting point of China's current government subsidy mechanism design is to solve the loss status of urban rail transit operators and make up for operating losses. However, it ignores the proportion of public funds utilization and the effective provision of public services by operating companies, while also ensuring the quality of services. The subsidy model for China's urban rail transit needs a long-term, stable, continuous, and certain incentive effect, which can effectively improve the efficiency of purchasing public services. The existence of such a mechanism not only solves the long-term financial burden of the government, moreover, it can lay a good foundation for the development of operating enterprises and is an effective model for mutual benefit.

The investment of PPP projects is irreversible. Once private sector decides to invest in public projects, it cannot fully invest investment income, but private sector can choose to wait for the optimal timing of investment until the uncertainty in the project is weakened. The value brought by investment is the meaning of the delayed option in real options. It is also the analysis entry point of this study: government subsidy is used to weaken the uncertainty of the project so that private sector accepts the investment opportunity agreed by the government.

![Fig. 1. The impact of uncertainty on the value of investment.](image)

3. The government subsidy decision model
The PPP model is intended to form a long-term cooperation model that magnifies government funds and credits through the amplification of the company's capital structure and builds infrastructure together with social funds. As a company, debt financing is a channel to enhance the company's financial leverage, improve the company's asset structure, and expand the company's size and influence. However, the cost of low debt financing costs is higher risk, while government subsidy only obtains relatively stable and easy-to-use equity for the cost of additional debt, and increases the credibility and refinancing ability of the entire company. From this perspective, the role of the
government's financial subsidy under the PPP model has far exceeded the significance of the ordinary cash flow provided by the government guarantee under the traditional model. According to the order of government subsidy, government subsidy can be divided into pre-project investment subsidy and investment subsidy during project operation. The government subsidy method is divided into pre-project investment subsidy and investment subsidy during project operation.

3.1. Presupposition conditions of model
The research adopt continuous time model with irreversible investment (means D-P model) to solve the optimal stop problem.

(1) The investment of private sector in the construction period and operation period of urban rail transit projects is partial or completely irreversible investment, and at the initial stage of project construction.

(2) Regard the unimplemented state of the rail transit project as the initial state. For the initial state, the following state must be the optimal strategy. Urban rail transit project passenger traffic growth is affected by mode of residents travel, public transport service level and stability factors of economic development. Urban rail transit project passenger flow Q obeys geometric Brownian motion (GBM)

\[
\frac{dQ}{Q} = (u-\delta)dt + \sigma dz.
\]

In this formula, \( u, \alpha \) and \( \delta \) are constants; \( u \) represent the expected rate of return; \( \alpha \) represent expected growth rate of passenger traffic; \( \delta \) represent operating project load rate. \( \delta \) repesents the degree of fluctuation in passenger flow. \( dz \) is the increment of the Wiener process, obeying a normal distribution. \( \delta = u-\alpha \). \( E(dz) = 0 \), \( E(dz)^2 = dt \).

(3) The parameter definition of the model
\( u \) represents the expected return rate of private sector (\( u > \alpha \)), indicating that the expected return rate of private sector is greater than the growth rate of passenger flow, \( \rho \) represents the risk-free rate of return, \( \alpha \) represents the expected growth rate of passenger flow, \( \sigma \) represents the degree of fluctuation in passenger flow \( T \) indicates the operating period of private sector projects, \( Re \) is the commercial income of private sector management projects, \( R \) is the rent of private sector using public resources, \( P \) is the project charging price, \( Qt \) is the passenger flow in the \( t \)-year, \( c \) is the operating cost, \( I \) is the project investment quota.

Note: Private sector is government partner. If Set up project company, it will be called project company, if not, it will be an enterprise that actually fulfills the responsibility of the “project company”.

Under the above assumptions, the project value function \( F(V) \) of the private sector investment urban rail transit project can be expressed as:

\[
F(V) = \int_0^T e^{-\rho s} e^{-u(s)}(Re + PQ_s - R - c)ds - I \tag{2.1}
\]

According to D-P option pricing model, the private sector option value \( f(Q) \) is expressed as:

\[
\frac{1}{2} \sigma^2 Q^2 \frac{\partial^2 f(Q)}{\partial Q^2} + (u-\delta) \frac{\partial f(Q)}{\partial Q} - uf(Q) = 0 \tag{2.2}
\]

The general solution of the equation is also the value matching condition is

\[
f(Q) = A_1Q^\delta + A_2Q^\beta \tag{2.3}
\]

3.2. Combined subsidy decision model for government subsidy in the previous investment and operation period
This section analyzes the state of operation of the project in which the government simultaneously compensates private sector in the pre-project and operation periods. The public facilities and sites used
in the project belong to the government (similar to the nature of the land, the collective concept), so the private sector needs to pay rent to the government when exercising the project management rights. The government is responsible for the purpose of encouraging private sector. The actual passenger flow situation has a flexible rent preferential policy for the project company. At this time, the project value function \( F(V) \) of the private sector investment urban rail transit project can be expressed as:

\[
F(Q) = E \left[ \int_0^T e^{\alpha s} e^{\beta s} (R + PQ + PQ - R - c) ds - I_1 \right] \quad (3.1)
\]

\( I_1 \) in formula (3.1) represents the amount of private sector investment.

Moreover, when private sector invests at \( t = T_\pi \), \( T_\pi = \inf(t / Q \geq Q_\pi) \), the project value function \( F(Q) \) is expressed as:

\[
F(Q) = \frac{(P + P_i)Q_\pi + R e - R - c}{\delta} (1 - e^{-\beta_1 T}) - I_1 \quad (3.2)
\]

According to the initial conditions of the project, value matching and smooth pasting condition are expressed as:

\[
\begin{cases}
F(0) = 0 \\
A_1(Q_\pi) = \frac{(P + P_i)Q_\pi + R e - R - c}{\delta} (1 - e^{-\beta_1 T}) - I_1 \\
\beta_1 A_1(Q_\pi)^{\beta_1 - 1} = 1
\end{cases} \quad (3.3)
\]

The purpose of project subsidy is to reduce the demand threshold value \( Q_\pi \) of the investment demand, \( Q_\pi \leq Q \). The evaluation is based on the predicted value \( Q \).

\[
Q_\pi = \frac{\delta \beta_1 I_1}{(P + P_i)(\beta_1 - 1)(1 - e^{-\beta_1 T})} + \frac{R + e - R e}{(P + P_i)} \quad (3.4)
\]

The difference between government subsidy value and the project value before government subsidy is \( V_i(Q) - V(Q) \). The benefit that the enterprise can obtain from government subsidy is \( I_1 + P(Q - V_i(Q) + V(Q)) \).

Then the minimum investment amount of the project \( I_1 \) is

\[
I_1 = \frac{(P + P_i)Q_\pi + R e - R - c}{\delta \beta_1} (\beta_1 - 1)(1 - e^{-\beta_1 T}) \quad (3.5)
\]

There is a minimum amount of investment subsidy for the private sector given by the government in the early stage of the project \( I_2 \).

\[
I_2 = I_1 - I = I - \frac{(P + P_i)Q_\pi + R e - R - c}{\delta \beta_1} (\beta_1 - 1)(1 - e^{-\beta_1 T}) \quad (3.6)
\]

In formula (3.7), \( I_2 \) represents the amount of investment the government has when the demand forecast is \( Q \).

The relationship between the government investment subsidy for private sector \( I_2 \) and the input subsidy \( P_1 \) in the early stage of the project is

\[
I_2 = I_1 - \frac{(P + P_i)Q_\pi + R e - R - c}{\delta \beta_1} (\beta_1 - 1)(1 - e^{-\beta_1 T}) \quad (3.7)
\]

\[
or \quad P_1 = \frac{\delta \beta_1 (1 - I)}{Q(\beta_1 - 1)(1 - e^{-\beta_1 T})} + \frac{R e - R e}{Q} \quad (3.8)
\]

3.3. Presupposition conditions of model

This article takes Beijing Metro Line 4 as an example. The project is divided into two parts. The first part (main facilities such as stations, tunnels, and other mechanical and electrical equipment such as
elevators, water supply and drainage, etc.) is funded by the government (about 70% of project investment). The second part (vehicles and communication, signal and other mechanical and electrical equipment) is funded by the private sector using the BOT model (about 30% of project investment).

3.3.1. Project instance data
The results of the data required for the study are as follows:

\[ I = 153 \text{ million yuan}, \quad I_1 = 46 \text{ million yuan}, \quad I_2 = 107 \text{ million yuan}, \quad \sigma = 0.02, \quad \alpha = 4\%, \quad Q = 4.1 \text{ million people year}, \quad P = 2.29 \text{ yuan per person}, \quad c = 6 \text{ million yuan}, \quad R_e = 1 \text{ million yuan}, \quad \text{Rent: } R = 0.425 \text{ million yuan}, \quad u = 0.0723, \quad \delta = u - \alpha = 0.0323, \quad \beta = \frac{1}{2} - \frac{u - \delta}{\sigma^2} + \sqrt{\left(\frac{u - \delta}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2u}{\sigma^2}} \approx 1.2, \quad T = 30 \text{ year.}

3.3.2. Data result analysis.
The government compensates private sector in the pre-project and operation period. The relationship between the government's investment subsidy for private sector \( I_2 \) and the input subsidy \( P_1 \) in the early stage of the project is:

\[
P_1 = \frac{\delta \beta \left(1 - I_1\right)}{Q \left(\beta - 1\right) \left(1 - e^{-\sigma T}\right)} + \frac{R + c - R_e}{Q} - P \approx 0.49 \text{ yuan/person}
\]

In 2013, the operating cost of Beijing Subway was 8.56 yuan per person, while the per capita operating income was 1.87 yuan per person. The difference between operating cost and operating income was financially compensated. This is equivalent to a citizen taking a kilometer subway. The financial department will have to pay 0.446 yuan per person, which is close to the actual situation.

It can be seen that the government combination subsidy method can better reflect the economic benefits of the project's overall value. In the case of insufficient government financial resources, it is conducive to the successful negotiation between the government and private sector projects.

4. Discussion and enlightenment on model solutions
There are some discussion and enlightenment of the nature of the solution of government combination subsidy decision model.

(1) Volatility is positively correlated with investment critical passenger flow and government investment subsidy, indicating that the greater the uncertainty of the project, the higher the threshold for private sector investment projects, so the government departments should bear the necessary uncertainty.

(2) The rate of return is positively correlated with investment critical passenger flow and government investment subsidy. The higher the expected return rate of private sector investment, the higher the "threshold" of private sector investment projects, so the government should provide financial and administrative support.

(3) The government's input subsidy has a negative correlation with government investment subsidy, indicating that the more government investment subsidy in the early stage of the project, the less subsidy input is invested in the project operation period. Therefore, the government departments should maintain a fair distribution mechanism, firmly implement the spirit of the contract, effectively supervise and supervise the operation of the project, effectively supervise and supervise the operation of the project, ensure the continuous and stable operation of the project facilities, and ensure that the public interest is not infringed.

5. Conclusion
Government subsidy directly determines the success or failure of urban rail transit PPP project. A reasonable government subsidy decision is crucial to the successful operation of urban rail transit PPP project. By contrast, the development of real options provides a theoretical tool to measure the uncertain value of a project, which can better solve the problem of uncertainty and flexibility in an investment project. This research builds a government subsidy decision model of urban rail transit PPP
project, based on the real option theory and benefit flow of project. The model is used to solve
government subsidy question of transportation PPP project under uncertain income. The best
combination scheme of the government subsidy is obtained by analyzing pre-project investment and
operation period investment subsidy. And then, the paper analyzed the relationship between customer
flow, volatility, expected return rate, pre-project investment subsidy and project operation period
investment subsidy under fixed operating period and operating price. At least, the paper shared some
policy suggestions on creating a fair and transparent policy environment for the development of PPP
model.

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