Some special specifics of dimensioning of a hydraulic cylinder as an executive device of an electrohydraulic actuator system

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Abstract The hydraulic cylinder is dimensioned based on a static or dynamic criterion, assuming the desired operating pressure. The paper further analyzes the dimensioning of cylinders according to different load categories. The sizing of the hydraulic cylinder affects the choice of control algorithm parameters but also the stability of the system. How the cylinder dimension and the gain of the distributor affect the choice of the integral constant in the control algorithm in the case when we have the PI control law is especially considered. Then we have by definition an unstable system $1 / s^2$ to which we provide stability through feedback.

Keywords: • hydraulic cylinder • hydraulic cylinder sizing • cylinder load modelling • integral member • structural load •


1 Introduction

The calculation and modelling of electrohydraulic actuation systems have been well-known engineering design techniques for many years. Here, in a special way, the calculation and modelling of the hydraulic cylinder as the executive body of the electrohydraulic actuation system and the influence of its basic dimensions on the dynamics of the actuation system are considered separately. It is clear that the hydraulic cylinder itself is a simple motor construction with linear motion and is therefore similarly treated when it comes to calculation and modelling. The matter becomes much more complicated when the cylinder is viewed as a loaded member of the actuation system, which is the only one that is important for design in practice. The cylinder stroke and the corresponding installation measure are not subject to calculation. The calculation of the cylinder basic dimensions is based on the requirements for providing the required force. It is a misinterpretation that the cylinder is dimensioned according to rod speed, because that is always the case with an unloaded cylinder. The cylinder is defined according to the force, where the influence of the speed or the desired flow can be observed within a higher or lower working pressure within the defined nominal pressure.

In this paper, the influence of the diameter of the piston and connecting rod, as the basic parameters of the cylinder on the responsible for obtaining the given dynamic characteristics is considered. The paper does not consider the structural elements of the cylinder structure, such as connecting rod buckling, stresses in the cylinder shell, bearing capacity of threaded joints and the like. The structure of the actuation system is observed by position or by force.

2 Basic elements for the initial calculation

The hydraulic cylinder is characterized by an integral nature, which means that a nominal operating point cannot be defined in an open control circuit. This means that actuation systems with a hydraulic cylinder cannot be modelled by deviations, but only in total coordinates. The cylinder is at the same time a type of oil spring, so that its own frequency depends on the position of the piston within the working stroke. In general calculations, the inertial load of the cylinder is reduced to the end of the connecting rod (reduced mass) and the external force. The first assumption that is adopted in this case is that the natural frequency of
the mechanical parts of the system, the connection of the rod of the cylinder to
the load is large, that is, the high stiffness of the mechanical part in relation to
the cylinder is assumed. Based on this, we can accept that the hydraulic natural
frequency is sufficient for all calculations, i.e. that we do not have a structural
load. By structural load we assume an inertial load of low stiffness, so that its
natural frequencies can be close to the hydraulic natural frequency, which must
be taken into account when calculating and modeling the electrohydraulic
actuation system. An example of a structural load for an electrohydraulic
actuation system is in flight control - the construction of control surfaces which,
due to the requirement for the mass to be as small as possible, have a relatively
low stiffness. Here, within the basic calculation, we present the initial criteria for
defining the dimensions of a hydraulic cylinder:

- Criterion (1) based on reduced force at the end of the connecting rod

\[ \frac{p_s \times \pi (D^2 - d^2)}{4} = F \Rightarrow (D^2 - d^2) = \frac{4F}{p_s \times \pi} = \frac{3D^2}{4} = \frac{4F}{p_s \times \pi} \Rightarrow \]

\[ D = \sqrt{\frac{16F}{3(p_s \times \pi)}} \]  

(1)

- Criterion (2) on the basis of inertial mass for dynamic requirements with
  a bandwidth of more than 4 Hz, a cylinder with a double rod [1].

\[ A = \frac{(f \times 4)^2 \times \pi^2 \times H \times m}{g \times \beta} \]  

(2)

- Criterion (3) based on the known natural frequency of the cylinder with
  inertial load, selection of an adequate proportional directional control
  valve or verification of the natural frequency of the actuation system
  based on a simple mechanical model mass-spring, Figure 1.

\[ \omega_h = \sqrt{\frac{4\beta_e A_0^2}{M_t V_t}} \]  

(3)

In (3) we have the influence of the dimensions of the cylinder on the natural
frequency through the work surface and over the stroke (working volume)
The hydraulic natural frequency, which depends on the dimensions of the hydraulic cylinder, can also be used for the initial selection of the basic control algorithm in the synthesis of the control system when comparing natural frequency of cylinder with inertial load to the natural frequency of the proportional directional control valve. Based on Table 1, it is possible to determine the initial approach when choosing the initial control algorithm. Of particular interest is case 1, where we have the proportional directional control valve which is "faster" than the cylinder and this leads to the requirement for operation in the state space, because the proportional directional control valve performs a certain destabilization of the actuation system, so stabilization is required and consequently additional feedbacks (load pressure, velocity or by the phase state variables).

Table 1: Initial approach when choosing the initial control algorithm

| Case | Ratio of cylinder natural frequency and directional control valve natural frequency | Recommended control algorithms | Remark |
|------|----------------------------------------------------------------------------------|-------------------------------|--------|
| Case 1 | $\frac{\omega_c}{\omega_R} \approx 0.3$ | State variables | Additional feedbacks, pressure and flow (velocity) |
| Case 2 | $\frac{\omega_c}{\omega_R} \approx 1$ | Proportional control | |
| Case 3 | $\frac{\omega_c}{\omega_R} \approx 3$ | Proportional-Differential control | |
3 Standard models of electrohydraulic actuator system

Figure 3 shows a simple structural approach to modelling where we do not have a direct influence of cylinder dimensions, which leads us directly into the state space, which arises from the mechanical analogy that a cylinder with inertial load is represented by a spring-mass model. This approach to modelling, Fig.3, allows us to easily arrive at a force-driven actuation system. This model took into consideration the main nonlinearity of the mathematical type that defines the flow characteristic, the square root type, while other primarily static nonlinearities were not considered.

Figure 3: General structural model of EHAS [2].
From Figure 3, we see that we have three integrators in the system, and for that reason that when we use the model from the input-output space for control synthesis, then we choose matching with the so-called "poor integrator", Figure 2.

The model in Figure 4 shows us the approach in which we have an inertial load without additional external force, which gives us the ability to clearly see the value of constant along the integral member, which depends on the working surface of the cylinder and the flow gain. Basically, when we define the PI control algorithm, we have two integrators in the system, the system is unstable in the open circuit, and we need to design a stable actuation system in the closed circuit. Here, the value of integral constant of the integrator in the cylinder is important, because it determines the order of value of the integral constant in the PI algorithm, i.e. it should be approximately ten times smaller. We do not have this possibility if we define an external constant or variable force in the actuation system in addition to the inertial load, Figure 5. Then we cannot estimate the order of value of the integral gain in the PI control algorithm at the beginning.

\[
\begin{align*}
T_c &= \frac{A_p}{K_{QX}} \\
T_n &= \sqrt{\frac{m}{c_1}} \\
K_1 &= \frac{K_{Qp} \times c}{K_{QX} \times A_p}, c = 2A_p^2 \frac{E_c}{V_0}
\end{align*}
\]
The models in Figure 3 and Figure 4 refer to throttling control in a hydraulic actuation system. Structurally, the diagram is similar to the variant with volumetric control. Also, feedback gain depends on cylinder working area.

![Figure 5: Structure of a mathematical model with external force and inertial load][4]

### 4 Basic linear model of hydraulic cylinder

A linear model with a load in the form of a transfer function is a third-order transfer function with an integral term. Depending on the degree of damping, we will have a pair of real poles or a pair of conjugate complex poles. We also show one phenomenon, and that is that if we have internal springs for centering in the hydraulic chambers, there is no longer an integral member.

\[
W(s) = \frac{K}{s(s^2 + 2\xi \omega_s + 1)}, \quad 0 < \xi < 1
\]  
(5)

\[
W(s) = \frac{K}{s(s-a_1)(s-a_2)}, \quad \xi > 1
\]  
(6)

\[
W(s) = \frac{K}{(T_3s^3 + T_2s^2 + T_1s + 1)}, \quad K = \frac{A_p}{c \times r}
\]  
(7)

According to equation (7) the model is with the positional load and the spring in the cylinder. Symbol \(r\) is the proportionality factor for the flow and \(c\) is the general stiffness.
5 Structural load cylinder model

The structural load of the cylinder means that we have close natural frequencies of the load with the hydraulic natural frequency, for instance of a gate, Figure 6. This means that we have to observe the load through natural frequencies. The model with structural load is given in Figure 7. We see that the structural load increases the order of the model by at least 4. The shape of the model allows us to show the influence of structural load on the connecting rod position or in another form to observe direct displacement of the structure. In addition to the aerodynamic control surfaces in aviation, the structural load for hydraulic cylinders is also represented by the gates within the hydromechanical equipment used on dams, canals and navigation locks in hydropower. With a structural load of this type, we have a significantly higher impact of friction, as we also have sliding friction, sealing rubber and rolling friction, wheels for lateral guide of the gate, where friction can be up to 30 % or more in intensity compared to inertial load, see Figure 6. This is not the case with aerodynamic surfaces where the friction component at the hinge moment is significantly smaller. In this case, we have a variable positional load (dependence of the attack point of the aerodynamic force on the angle of attack), while the gate has a variable force from the water.

**Figure 6: Slide gate model with loads acting on the rod of hydraulic cylinder [5].**
The question is how to analyse and model the structural load. One possibility is to use pressure feedback that will prevent the appearance of resonant frequencies when moving the gate, either lifting or lowering. Due to the rationality of making mechanical constructions of gates, they do not have sufficient rigidity, and in the control of their synchronous movement, ever greater dynamic requirements, such as speed of lifting and lowering or the smallest possible synchronous movement error, are constantly required.

![Block diagram of an electrohydraulic actuation system with two masses](image)

Figure 7: Block diagram of an electrohydraulic actuation system with two masses [6].

Figure 7 shows that the working surface of the hydraulic cylinder appears twice in the block diagram and the working volume of the chambers of the hydraulic cylinder appears once. The structural load model in Figure 7 allows us to easily define the transfer function of the structural load in a linear mathematical model if the previous structural load is calculated by the finite element method (FEM) from which we can directly obtain the frequency of the first harmonic and several higher as well as attenuation degrees. Then we have a complete linear model of high enough order that it can be used for dynamic analysis of the actuation system through which we can check the influence of the dimensions of the hydraulic cylinder on the dynamics of the movement of the gate. Figure 7 shows a dynamic load model with two masses. One mass is the mass of the gate and the other mass is the mass of the moving parts of the cylinder (connecting rod). Figure 7 shows a model of a single cylinder because the gate is symmetrical so that one half of the structure can be observed. The movement of the gate without the dynamics of synchronous movement is observed here.
6 Conclusion

All steps in the analysis and synthesis of the actuation system are visibly shown, where the dimensions of the hydraulic cylinder have an influence on the dynamic properties of the electrohydraulic actuation system. In this way, the designer of the actuation system, knowing in advance the required dynamic properties of the electrohydraulic actuation system, can consider any change in the dimensions of the cylinder comprehensively within the electrohydraulic system he is designing.

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