Electromagnetic Radiation and Motion of Dust Particle – A Simple Model

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Abstract. A simple model for motion of dust particle (meteoroid) under the action of (solar) electromagnetic radiation is presented. The particle of the form of plane mirror is taken into account and exact analytical results are presented. As for long-term orbital evolution, particle may spiral outwards the central body (Sun); initial conditions are important. As a consequence, motion of real dust particles may differ from that generally considered.

1. Introduction

The Poynting-Robertson effect (P-R effect) (Robertson 1937; Klačka 1992a – the most complete form of the P-R effect) is generally considered to be the real effect which causes inspiralling of interplanetary dust particles (IDPs), meteoroids, towards the Sun. (Other, more simple correct derivations may be found in Klačka’s papers: 1992b, 1993a, 1993b).

The most general case of the validity of the P-R effect requires that Eq. (120) (or, Eq. (122) for the moving particle) in (Klačka 1992a) holds. This may not be the case for the real nonspherical particle, as it was discussed in Klačka (1993c, 1993d) and applied in Klačka (1994a).

General equation of motion of IDP in terms of optical properties was presented by Klačka and Kocifaj (1994) – the paper does not present any quantitative calculation.

The aim of this paper is to make a detailed correct calculations for plane mirror particle as for its equation of motion and as for secular changes of orbital elements. It
is considered that particle rotates around an axis fixed in space. The advantage of this simple model is that it can be treated in an analytical way and it may show possible orbital evolution of real dust particles.

2. Equation of motion

Let us consider a particle of a plane form, both sides of which are covered with mirror. The area of each side is \( A \). When the particle’s position is characterized by radius vector \( r \) with respect to the source of electromagnetic radiation, it’s instantaneous velocity is \( v \) with respect to the source.

Density of the flux of the radiation energy of the source is \( S \) when measured in the rest frame of the source. If \( c \) is the speed of light (all occurs in vacuum) and \( \hat{S} \equiv r/|r| \), then the quantities measured in the rest frame of the particle (proper inertial frame of reference) are:

\[
\begin{align*}
\hat{S}'_i &= (1 + v \cdot \hat{S}/c) \hat{S} - v/c , \\
\hat{S}'_o &= \hat{S}'_i - 2 (\hat{n}' \cdot \hat{S}'_i) \hat{n}' , \\
S' &= (1 - 2 v \cdot \hat{S}/c) S ,
\end{align*}
\]

where unit vector \( \hat{S}'_o \) represents direction and orientation of the beam reflected from the plane mirror characterized by normal unit vector \( \hat{n}' \); all quantities are correct to the first order in \( v/c \).

If the plane mirror is turned at an angle \( \delta \) about an axis characterized by unit vector \( a \), the new normal unit vector \( \hat{n}'_n \) (coordinate system is fixed – it does not change) is

\[
\hat{n}'_n = \hat{n}' \cos \delta + a (a \cdot \hat{n}') (1 - \cos \delta) + (a \times \hat{n}') \sin \delta .
\]

We will consider that \( \hat{n}' = \pm \hat{x}' = \pm \hat{x} \) and the particle rotates around axis \( \hat{x}' \), for the sake of simplicity. The effective area for the incident radiation is

\[
A_{eff} = A |\cos \Theta| ,
\]

where \( \Theta \) is the angle between \( \hat{S} \) and \( \hat{x} \) \((r = x \hat{x} + y \hat{y} + z \hat{z})\). We can write, thus:

\[
\begin{align*}
\hat{n}' &= -\hat{x}' = -\hat{x} , \quad \Theta \in (-\pi/2, \pi/2) , \\
\hat{n}' &= +\hat{x}' = +\hat{x} , \quad \Theta \in (\pi/2, 3\pi/2) .
\end{align*}
\]

Equation of motion for the particle has the following form

\[
\begin{align*}
\frac{dp'}{dt} &= \frac{A_{eff} S'}{c} (\hat{S}' - \hat{S}'_o) , \\
\frac{dE'}{dt} &= 0 .
\end{align*}
\]
in the proper inertial frame of reference.

Using the previous relations (and Lorentz transformation to the first order in \(v/c\)), we finally obtain equation of motion in the reference frame of the source of electromagnetic radiation:

\[
\dot{v} = 2 \frac{A S}{m c} \left\{ \left(1 - \frac{v}{c} \cdot \hat{S} \right) \hat{S} \cdot \dot{x} - \frac{v}{c} \cdot \dot{x} \right\} \mid \cos \Theta \mid \dot{x},
\]

where \(m\) is mass of the particle and the dot denotes differentiation with respect to time.

3. Electromagnetic radiation and gravitation

Taking into account gravitational acceleration of the central body of mass \(M\), we can write the complete equation of motion in the form

\[
\dot{v} = -\frac{\mu}{r^3} r + 2 \beta \frac{\mu}{r^2} \left\{ \left(1 - \frac{v}{c} \cdot \hat{S} \right) \hat{S} \cdot \dot{x} - \frac{v}{c} \cdot \dot{x} \right\} \mid \cos \Theta \mid \dot{x},
\]

\[
\beta \frac{\mu}{r^2} = \frac{AS}{mc},
\]

where \(\mu = G M\) and \(G\) is gravitational constant.

4. Secular changes of orbital elements

Let the initial position and velocity vectors lie in the \(xy\)-plane. Eq. (7) yields that the orbital plane conserves. Secular changes of other orbital elements can be also calculated in an analytical way. The results are summarized in Eqs. (8) – (10) for semimajor axis \(a\), eccentricity \(e\) and longitude of pericenter \(\omega\) (the quantity \(p\) is defined \(p = a \left(1 - e^2\right)\)).

\[
< \frac{d a}{dt} > = \frac{4}{\pi} \frac{\beta \mu}{c} \frac{2}{3} + \left[ \frac{6 \cos(2 \omega)}{5} - 1 \right] \frac{e^2}{a \left(1 - e^2\right)^{3/2}}
\]

\[
< \frac{d e}{dt} > = \frac{2}{\pi} \frac{\beta \sqrt{\mu}}{a^{3/2}} \left\{ \frac{4}{15} e \left(1 - e^2\right) \sin(2 \omega) + 
\right.
\]

\[
\left. + (1 - e^2) \sum_{n=2}^{\infty} e^{2n-1} \int_{-\pi/2}^{\pi/2} \cos^n \Theta \sin \Theta \cos^{2n}(\Theta - \omega) \ d \Theta + 
\right.
\]

\[
+ \left( \frac{\sqrt{\mu}}{\pi} \frac{p}{c} \right) \left[ \frac{2}{5} - \frac{22}{15} \sin^2 \omega + \frac{4}{15} \cos(2 \omega) + 
\right.
\]

\[
+ 2 \left(1 - e^2\right) \int_{-\pi/2}^{\pi/2} \cos^n \Theta \sin \Theta \cos(\Theta - \omega) \sin(\Theta - \omega) \ d \Theta \right\}
\]

\[
< \frac{d \omega}{dt} > = -\frac{2}{\pi} \frac{\beta \sqrt{\mu}}{a^{3/2} e} \left\{ \frac{2}{3} \cos \omega + 
\right.
\]

\[
\left. \frac{4}{5} \cos \omega \right\}
\]
\[
+ \sin (2 \omega) \sum_{n=0}^{\infty} (e \sin \omega)^{2n} \left[ \frac{1}{2 n + 1} + \frac{\cos (2 \omega)}{2 n + 3} \right] + \\
+ \cos (2 \omega) \sum_{n=0}^{\infty} \frac{e^{2n}}{2 n + 3} \int_{-\pi/2 - \omega}^{\pi/2 - \omega} \cos^{2n+1} x \, dx - \\
- \frac{\sqrt{\mu / p}}{c} \, e \left[ \frac{3}{5} \sin (2 \omega) + 4 \sin^3 \omega \cos (2 \omega) \sum_{n=0}^{\infty} \frac{(e \sin \omega)^{2n}}{2 n + 3} \right] \right\} 
\]

Eq. (10) shows that advance of pericenter (perihelion, in the case of Solar System) exists. If the relativistic terms proportional to \( \sqrt{\mu / p} / c \) would not exist, then \( < d\omega / dt > \) would be zero for \( \cos \omega = 0 \), i.e. for \( \omega = \pm \pi / 2 \). The relativistic terms yield that \( < d\omega / dt > \) < 0 for \( \omega = +\pi / 2 \) and \( < d\omega / dt > \) > 0 for \( \omega = -\pi / 2 \).

The stable situation occurs at \( \omega = -\pi / 2 + \varepsilon \), where \( \varepsilon \) is small positive number (depends on the value of eccentricity). Eq. (9) yields that \( < de / dt > \) < 0. Eq. (8) yields that \( < da / dt > \) < 0 for \( e > \sqrt{10/33} \) (approximately) and \( < da / dt > \) > 0 for \( e < \sqrt{10/33} \) (approximately). Thus, if the initial eccentricity is greater than \( e > \sqrt{10/33} \) (approximately), the particle spiral towards the central body, its eccentricity still decreases. When \( e < \sqrt{10/33} \) (approximately) occurs, the particle spiral outwards the central body.

6. Conclusion

We have shown, in an analytical way, that special type of particle exhibits orbital evolution which is not consistent with that for the Poynting-Robertson effect (moreover, the behaviour depends on initial conditions). Therefore, the P-R effect cannot be simply applied to the study of motion of real particles. (Numerical calculations for real particle are in preparation.)

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