Transient Quality Performance Evaluation of Multi-stage Flexible Manufacturing Systems

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Abstract. This paper aims at evaluation of the quality performance of multi-stage manufacturing system during transients. A Markovian method is constructed in order to evaluate transient propagation of quality at multi-stage manufacturing systems which have remote quality information feedback. With the derived Markovian model, analytical expressions of estimating quality metrics in transients are provided, such as settling time and dynamical quality. Case study at a factory is given to demonstrate the derived approach. The conclusions drawn illustrate the applicability of transient quality evaluation of multi-stage systems.

1. Introduction

1.1. Motivation

Steady state analysis of manufacturing systems has been numerously researched in the past few decades. It is effective in mass volume production since the transients can be ignored compared to the entire production. However, steady state analysis may become ineffective in flexible manufacturing systems, in which production is conducted with small volume and thus transients may become dominant in the manufacturing process. In flexible manufacturing systems multi-types of products are manufactured with batches on the same line. During the production, product quality is dominant by locating accuracy of flexible fixtures. Product changes may result in defect items caused by frequent fixture relocation errors. After product changeover, system quality suffer from transients due to condition such as the fixture relocating errors. Quality transients display the quality before arriving at steady state and are important. System quality mean is unstable during transients and different from steady state. However, compared to the huge results of steady state analysis, transient quality analysis of manufacturing systems maintains generally unexplored. Few analytical approaches regarding the relation between manufacturing systems and quality propagation during transients are researched in current works. This work aims at contributing to this part.

1.2. Literature Review

Numerous research has been put on flexible manufacturing systems which are widely applied in modern industry. It has been shown in recent research flexibility has effects on quality [1]. Among various modeling methods of manufacturing systems, the application of Markov models to manufacturing and service systems has been largely published recently (see [2]-[4]). However, these
results are largely focused with steady state of manufacturing systems and transient analysis is neglected. Recently an effort is paid to transient analysis in terms of throughput of manufacturing systems. The production throughput of lines with Bernoulli reliability machines in transients were studied using Markov model. Later on more reports are published in multi-stage Bernoulli systems, batch-based lines, assembly systems, closed production lines in [5]-[10]. Despite of these efforts, transient quality analysis of flexible systems is not fully understood. [11] deals with the special two-stage case of quality evaluation during transients. This work addresses the transient system quality in multi-stage systems.

2. Models and problem formulation

2.1. Descriptive Model

The assumptions of system quality transition in multi-stage manufacturing systems are as follows.

1. The manufacturing system has \( n \) stages with an inspection station at last stage.
2. Time slot equals to machine cycle time and only consider working period of system machines.
3. The product quality by stage \( M_i(i \geq 2) \) not only relies on states of \( M_i \) but coming part quality from upstream. There are both quality correction and quality degradation in the manufacturing system and quality may get better or worse after a certain stage.
4. Denote the stage of \( M_i(i = 1,2,...,n) \) which is in good state \( g_i \) or defective state \( d_i \) if producing a good or defective product.
5. Quality of coming parts in stage \( M_i(i \geq 2) \) at time \( t \) relies on upstream product quality state at time \( (t-1) \). The good state \( g_{i-1} \) or defective \( d_{i-1} \) of stage \( M_{i-1} \) means a good or defective coming part for \( M_i \) at time \( t \) respectively.
6. When \( M_i \) is in good state \( g_i \), the probability to transit to good state \( g_i \) is \( (1 - \alpha_i) \) and to defective \( d_i \) is \( \alpha_i \). If \( M_i \) is in defective state \( d_i \), the probability to transit to defective \( d_i \) is \( (1 - \beta_i) \) and to good state \( g_i \) is \( \beta_i \).

In terms of good coming part, if \( M_i(i \geq 2) \) is in good state \( g_i \), the probability to transit to good state \( g_i \) is \( (1 - \gamma_i) \) and to defective \( d_i \) is \( \gamma_i \). If \( M_i \) is in defective state \( d_i \), the probability to defective \( d_i \) is \( (1 - \mu_i) \) and probability to transit to good state \( g_i \) is \( \mu_i \).

In terms of defective coming part, if \( M_i(i \geq 2) \) is in good state \( g_i \), it has probability \( (1 - \eta_i) \) to transit to good state \( g_i \) and probability \( \eta_i \) to defective state \( d_i \). If \( M_i \) is in defective \( d_i \), it has probability \( \theta_i \) to transit to \( g_i \) and probability to \( d_i \) is \( (1 - \theta_i) \).

2.2. Mathematical Model

Under the previous assumptions, transient system quality are developed in two-stage condition and then expanded for multi-stage condition.

At certain time, the two-stage manufacturing system contains four quality states. (1) State \( g_1d_2 \), both \( M_1 \) and \( M_2 \) produce good products; (2) state \( g_1d_2 \), \( M_1 \) produces good product and \( M_2 \) is producing defective part; (3) state \( d_1g_2 \), \( M_1 \) produces defective product and \( M_2 \) is producing good; (4) state \( d_1d_2 \), both \( M_1 \) and \( M_2 \) produces defective products.

A Markov chain with above four states describe the two-stage system. At time \( t \) the Markov states of the system in matrix are

\[
Z_t = \begin{bmatrix} P(g_1g_2,t) & P(g_1d_2,t) & P(d_1g_2,t) & P(d_1d_2,t) \end{bmatrix}
\]

(1)

The system state transits between the four quality states according to the transition probability. Set the state transition probabilities together so as to build the transition matrix.

\[
B = \begin{bmatrix} (1-\alpha_1)(1-\gamma_2) & (1-\alpha_1)\mu_2 & \beta_1(1-\eta_2) & \beta_1\theta_2 \\ (1-\alpha_1)\gamma_2 & (1-\alpha_1)(1-\mu_2) & \beta_1\eta_2 & \beta_1(1-\theta_2) \\ \alpha_1(1-\gamma_2) & \alpha_1\mu_2 & (1-\beta_1)(1-\eta_2) & (1-\beta_1)\theta_2 \\ \alpha_1\gamma_2 & \alpha_1(1-\mu_2) & (1-\beta_1)(1-\eta_2) & (1-\beta_1)(1-\theta_2) \end{bmatrix}
\]

(2)

Define \( P(g_2,t) \) as the system probability to produce good products.
\[ P(g_2, t) = P(d_2, g_2, t) + P(g_2, g_2, t) \]  
\[ Z_z(t+1) = B_z Z_z(t) \]  
The evolution of \( P(d_2, t) \) and \( P(g_2, t) \) are calculated as

\[
y_z(t) = \begin{bmatrix} P(g_z, t) \\ P(d_z, t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} Z_z(t)
\]

3. Transient quality performance evaluation

3.1. Two-stage Case

For two-stage systems under definition of assumptions above, it is shown by the mathematical model that \( B_z \) is transition probability matrix of the Markovian chain. The matrix \( B_z \) is transformed to a diagonal matrix with a non-singular \( T \) matrix.

\[ TB_z T^{-1} = \text{diag} \left[ \lambda_2, \lambda_3, \lambda_4 \right] \]

Introduce the following substitution (6) and substitute equation (6) into equations (4)-(5),

\[
\tilde{Z}_z(t) = TZ_z(t)
\]

\[
\tilde{Z}_z(t+1) = \tilde{B}_z \tilde{Z}_z(t)
\]

\[
y_z(t) = \tilde{C} \tilde{Z}_z(t)
\]

The evolution of system states, \( P(g_2, t) \) can be denoted as

\[
P(g_2, t) = \tilde{P}(g_2) = \text{diag} \left[ \lambda_2', \lambda_3', \lambda_4' \right] \tilde{Z}_z(0)
\]

Equation (9) shows that Markov chain \( \tilde{Z}_2(t) \) arrives at the steady state denoted as exponential functions of \( \lambda_i \). Equation (10) shows that system quality \( P(g_2, t) \) is described by the exponential factors and eigenvalues \( \lambda_i \).

3.2. Multi-stage Case

The proposed method from section 3.1 can be expanded to multi-stage case. The iteration process is described by Figure 1. System quality of two-stage \( M_1 - M_2 \) is developed using the Markov method proposed from section 3.1. Then stage \( M_1 \) and \( M_2 \) are merged to a combined stage \( M'_2 \). And then system quality of fresh two-stage \( M'_2 - M_3 \) is developed, after which stage \( M'_2 \) and \( M_3 \) are merged to a merged stage \( M'_3 \). Conduct the iteration method and the first ever \((n-1)\) system stages are aggregated into a merged stage \( M'_{n-1} \). Finally we build the model of system quality in final two-stage \( M'_{n-1} - M_n \). The iteration method is as follows.

![Figure 1. Iteration method of multi-stage systems.](image1)

![Figure 2. A type of valve shell in case study.](image2)
First, any two-stage systems $M'_i - M_{i+1}$ has a total of six system transition parameters. These consist of parameters $\alpha_i(t), \beta_i(t)$ representing the characteristics of the new merged stage $M'_i$ and $\gamma_{i+1}, \eta_{i+1}, \mu_{i+1}, \theta_{i+1}$ representing the characteristics of the stage $M_{i+1}$. The parameters $\alpha_i(t), \beta_i(t)$ are calculated as follows.

$$\alpha_i(t) = \frac{P(g_{i+1}, g_{i+1}, t) + P(d_{i+1}, g_{i+1}, t) \eta_i}{P(g_{i+1}, g_{i+1}, t) + P(d_{i+1}, g_{i+1}, t)}$$

$$\beta_i(t) = \frac{P(g_{i+1}, d_{i+1}, t) \mu_i + P(d_{i+1}, d_{i+1}, t) \theta_i}{P(d_{i+1}, d_{i+1}, t) + P(g_{i+1}, d_{i+1}, t)}$$

Second, put transition probabilities in matrix form

$$B_{i+1}(t) = \begin{bmatrix} (1-\alpha_i(t))(1-\gamma_{i+1}) & (1-\alpha_i(t))(1-\eta_{i+1}) & \beta_i(t)(1-\gamma_{i+1}) & \beta_i(t)(1-\eta_{i+1}) \\ (1-\alpha_i(t))(1-\gamma_{i+1}) & (1-\alpha_i(t))(1-\eta_{i+1}) & \beta_i(t)(1-\gamma_{i+1}) & \beta_i(t)(1-\eta_{i+1}) \\ \alpha_i(t)(1-\gamma_{i+1}) & \alpha_i(t)(1-\gamma_{i+1}) & (1-\beta_i(t))(1-\gamma_{i+1}) & (1-\beta_i(t))(1-\eta_{i+1}) \\ \alpha_i(t)(1-\gamma_{i+1}) & \alpha_i(t)(1-\gamma_{i+1}) & (1-\beta_i(t))(1-\gamma_{i+1}) & (1-\beta_i(t))(1-\eta_{i+1}) \end{bmatrix}$$

The matrix of system state probabilities at certain time is described as follows,

$$Z_{i+1}(t) = \begin{bmatrix} P(g_{i+1}, g_{i+1}, t) & P(g_{i+1}, d_{i+1}, t) & P(d_{i+1}, g_{i+1}, t) & P(d_{i+1}, d_{i+1}, t) \end{bmatrix}$$

The evolution of $Z_{i+1}(t)$ can be described by the linear equation,

$$Z_{i+1}(t+1) = B_{i+1}(t)Z_{i+1}(t)$$

$$P(g_{i+1}, g_{i+1}, t) + P(d_{i+1}, g_{i+1}, t) + P(g_{i+1}, d_{i+1}, t) + P(d_{i+1}, d_{i+1}, t) = 1$$

In multi-stage systems, final system quality equals to that $M_{i+1}$ produces a good quality part. Thus we define $P(g_{i+1}, t)$ as system probability to produce good quality.

$$P(g_{i+1}, t) = P(d_{i+1}, g_{i+1}, t) + P(g_{i+1}, d_{i+1}, t)$$

### 3.3. Settling time

Settling time denotes system lasting time of transients for quality performance evaluation in transients. The settling time is defined when quality performance arrives at and keeps in ±3% of steady state metric. Calculate the settling time as below.

$$t_s = \frac{P(g_{i+1}, t) - P(g_{i+1}, t)}{P(g_{i+1}, t)_S} \leq 3\%$$

### 4. Case study

Here demonstrates a case study in manufacturing system for valve shell to verify the effectiveness of the proposed model. The valve shell is shown in Figure 2. The production process has five stages, i.e., OP10 to OP50 in Figure 3. These five stages are correlated. For example, the flatness variation in OP10 may put effects on clamping precision of downstream OP30. The hole in the upstream OP10 can be degraded or corrected in downstream OP50. Quality propagation of products is explored in the five-stage system.

*Figure 3. Valve shell manufacturing system.*
After statistical analysis of processing data, transiting data of this five-stage system are given by quality failure and quality repair probabilities. The transition probability parameters are calculated $\alpha_1 = 0.05$, $\beta_2 = 0.9$, $\gamma_1 = [0.05, 0.1, 0.05, 0.05]$, $\mu_i = [0.8, 0.8, 0.9, 0.9]$, $\eta_i = [0.5, 0.5, 0.4, 0.5]$, $\theta_i = [0.4, 0.3, 0.2, 0.4]$.

Using transient analysis of quality in this paper, we can calculate the transient quality performance, settling time and quality of steady state in this multi-stage system. The quality of steady state of each two-stage merged system are 91.36%, 84.85%, 88.25%, 89.07%, respectively. Settling time lasts 8 slots. And during transients the evolution of system quality $P(g_i, t)$ of each two-stage merged system are plotted in Figure 4. The results are consistent to real data at the plant floor and prove the effectiveness of the constructed model.

The evolution of system states of the final two-stage merged system $M'_4 - M_5$ is illustrated in Figure 5 (a). The dynamics of transition probability parameters corresponding to the merged stage $M'_4$ are shown in Figure 5 (b). It indicates that transition probabilities of merged stage also converge as time evolves. In the future analysis of monotonicity and sensitivity will be carried out and investigate how system parameters changes will pay influence on quality performance in multi-stage systems during transients, which will provide the guidance for quality improvement.

5. Conclusion
This paper aims at evaluation of the quality performance of multi-stage manufacturing system during transients. A Markovian method is constructed in order to evaluate transient propagation of quality at multi-stage manufacturing systems which have remote quality information feedback. With the derived Markovian model, analytical expressions of estimating quality metrics in transients are provided, such as the settling time and dynamical quality. Case study at a factory is given to demonstrate the derived approach. The conclusions drawn illustrate the applicability of transient quality evaluation of multi-
stage systems. Future works can be pointed to continuous improvement through analysis of structural properties, and extension of the approaches to modeling and evaluation of multi-types of products during transients.

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