The Interaction Structure and Cosmological Relevance of Mass Scales in String Motivated Supersymmetric Theories

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Abstract

A string motivated, $N=1$ global supersymmetric, general Lagrangian is obtained that represents an inflaton interacting with the modes of a string. Focus is placed on the Lagrangian's relevance to warm inflation, which is a cosmology based on dissipative dynamics. Several interaction schemes are identified from the general Lagrangian that may have interesting consequences for dissipative dynamics. Distributed-mass-models, which have been shown to solve the horizon problem within a warm inflation regime, are identified in the general Lagrangian.

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I. INTRODUCTION

Warm inflation cosmology \[1,2\] has incited a detailed study of the nature of interactions in particle physics models at energies from the electroweak scale to the Planck scale. Warm inflation is comprised of non-isentropic expansion in the background cosmology \[3,4\] and thermal seeds of density perturbations \[5\]. Within the warm inflation regime, while the scale factor undergoes inflationary expansion, the radiation energy density is still substantial due to its constant production from conversion of vacuum energy. This expansion regime is intrinsically different from the supercooled inflation regime, since in the warm inflation case the inflationary period smoothly terminates into a subsequent radiation dominated regime, without a reheating period.

The simplest field theory description of the non-isentropic background cosmology is from Ginzburg-Landau overdamped relaxational kinetics \[4\]. The quasi-adiabatic regime of such kinetics implies it can be systematically computed in quantum field theory \[4,12\] and the overdamped limit has been explicitly demonstrated \[12\]. Such kinetics, in a cosmological warm inflation setting requires several additional and nontrivial consistency conditions between the macroscopic and microscopic dynamics. In a specific quantum field theory model which has been named the distributed-mass-model (DM-model) \[12,14\], it recently has been demonstrated \[13\] that all the consistency conditions can be satisfied with sufficient overdamping to solve the cosmological horizon problem. These models are atypical to particle physics due to a certain shifted interaction scheme which is detailed in Sect. III. Furthermore, the relevant DM models for the warm inflation results in \[12\] require a large number of fields, \[>10^4\].

The success of DM models to warm inflation cosmology immediately prompts questions about their fundamental origin and more generally exemplifies the importance of the specific interaction structure in the underlying particle physics model for the eventual realizability of a first principles warm inflation cosmology. In \[14,15\] it was mentioned that DM-models have heuristic similarity to string theory, but no explanation was offered. In general, for the energy scales of interest to cosmology, the obvious place to examine the variety of particle physics interaction schemes is effective supersymmetric theories derived from strings. This is a vast subject for which an ab initio approach at this moment would be impossible. As a first step, this paper attempts an interpretation of effective low energy SUSY theory motivated from string theory that is conducive to realizing warm inflation cosmology. The specific example of DM-models will be reviewed and we will demonstrate their origin through this interpretation. En route we will also note the variety of possible other particle physics models which may be relevant to warm inflation cosmology.

II. MOTIVATION FROM STRING THEORY

We will assume warm inflation takes place somewhere below the Planck scale \(M_{Pl} \sim 10^{19}\)GeV but above some GUT scale \(M_{GUT} \sim 10^{15}\)GeV. In string theory the region between these two scales is not devoid of states. Quite the contrary, the generic string scale is \(M_s = 5.3g_s \times 10^{17}\)GeV \[16\] so there can be \(O(10^2)\) mass levels between \(M_{GUT}\) and \(M_{Pl}\). Furthermore, as the mass level increases, the number of states per level grows rapidly \[17\].
The states at each mass level fall into representations of the symmetry of the theory, gauge, global and Lorentz \[18\]. As the temperature falls, these symmetries can change. Let us follow the gauge symmetry. The gauge group \( G \) may break to some smaller group \( G' \) as the temperature falls. Each G-irrep \( R \) at the N-th level of the theory will decompose into irreps that are a sum of \( G' \) irreps, \( \sum R'_i \), when the symmetry is broken. This creates a fine structure at each energy level. Similar arguments hold for global symmetry breaking and for reduction of the Lorentz symmetry via compactification.

In the next section, the single aspect of string theory we need is that there are many mass levels. To our knowledge, the effective theory in the next section is the first to make use of the higher levels of strings for phenomenological purposes. As such, we will focus only on the basic point and leave detailed model building for future work.

Between gauge symmetry breaking, SUSY breaking and compactification, there are many fields that could be considered as potential inflatons. Several of these possibilities have been recently reviewed (see [19]). A few fields of particular interest are singlet scalar fields involved in the Fayet-Iliopoulos SUSY breaking mechanism, singlets involved in F-term spontaneous SUSY breaking via the O’Raifeartaigh mechanism, gauge singlet superfields in flat direction of the superpotential, possible singlet winding modes associated with Wilson loop gauge symmetry breaking in the string theory, etc., not to mention the universal favorite, the dilaton. Rather than make a choice, here is will be assumed that we have started with a fundamental string theory, and we then study only the effective SUSY gauge theory below the compactification scale, but above \( M_{GUT} \), where an inflaton has already emerged at a high field value along with a tower of states in representations of the gauge group, all weakly coupled to the inflaton. There can be many fields that do not mix with the inflaton. As these are of no interest for the following discussion, terms involving them will not be displayed. As the inflaton descends through the tower of states causing dissipation, it engenders the warm inflation scenario.

Before writing down a specific SUSY model with the desired warm inflation properties, two remarks about supersymmetry are in order (For a complete review of supersymmetry, please see [20, 21]). First, as long as global supersymmetry is a good symmetry, F-terms are not renormalized (meaning that once we choose the parameters in the superpotential, they stay fixed until SUSY is broken). Because of this nonrenormalization theorem [22], the fixing of F-term parameters is natural in the technical sense, and for that reason has been invoked to alleviate difficulties like the hierarchy problem. (D-terms in general are renormalized and can therefore run with energy scale, but this is not always true [23].) Amongst other things, the nonrunning of the F-terms allows one to fix flat directions in the superpotential. The second comment about SUSY is the SUSY Hamiltonian, \( H \), can be written as the sum of the squares of the supercharges \( Q_\alpha \). SUSY is unbroken if the vacuum \( \Omega \) is invariant under SUSY transformations, \( Q_\alpha |\Omega\rangle = 0 \). In the case that SUSY is unbroken, it therefore implies the vacuum energy is zero, \( \langle H \rangle_\Omega \equiv \langle \Omega |H|\Omega \rangle = 0 \). As such, a necessary and sufficient condition for SUSY to be spontaneously broken is a nonvanishing vacuum energy \( \langle H \rangle_\Omega > 0 \) [24].

Since we plan to deal only with the effective global SUSY theory, we will not need to make a specific choice of where the inflaton lives within the overlying string theory but as we have noted, many possibilities for its origin exist. Even after SUSY breaking by F-terms, mass sum rules [23] exist and constrain the running of masses. This provides useful knowledge as the gauge group \( G \) breaks and the states rearrange into representations of \( G' \). Although in
general the sum rules are altered by D-term SUSY breaking, under certain circumstances anomaly cancellation requires their preservation.

III. EFFECTIVE LAGRANGIAN

Let us consider the general form of an effective N=1 global SUSY theory representing an inflaton interacting with the modes of a string. Such a theory consists of a single chiral superfield \( \Phi \) which represents the inflaton and a set of chiral superfields \( X_i, i = 1, \ldots, N_M \) representing the string modes. All the superfields have their antichiral superfields \( \bar{\Phi}, \{ \bar{X}_i \} \) appearing in kinetic and Hermetian conjugate (h.c.) terms. In the chiral representation the expansion of the superfields in terms of the Grassmann variable \( \theta \) is

\[
\Phi = \phi + \theta \psi + \theta^2 F \quad \text{and} \quad X_i = \chi_i + \theta \psi_i + \theta^2 F_i, \quad i = 1, \ldots, N_M.
\]

Here \( \phi = (\phi_1 + i\phi_2)/\sqrt{2} \) and \( \chi_i = (\chi_1 + i\chi_2)/\sqrt{2} \) are complex scalar fields, \( \psi \) and \( \{ \psi_i \} \) are Weyl spinors, and \( F \) and \( \{ F_i \} \) are auxiliary fields. By definition, the inflaton \( \Phi \) characterizes the state of the vacuum energy through a nonzero amplitude in the bosonic sector \( \langle \phi \rangle \equiv \varphi_c \neq 0 \). Warm inflation cosmology specifically recognizes that the interaction of the inflaton with the other fields can lead to nontrivial dissipative effects that both convert vacuum energy into radiation energy and slow the motion of \( \varphi_c \).

The general Lagrangian for this system of fields is composed of a kinetic and interacting part

\[
L = L_K + L_I.
\]

The kinetic energy portion of the Lagrangian is

\[
L_K = \int d^4x d^2\theta d^2\bar{\theta} \left[ \Phi \bar{\Phi} + \sum_{i=1}^{N_M} X_i \bar{X}_i \right],
\]

where the fields \( F, \{ F_i \} \) do not appear, thus in fact are auxiliary fields. The interaction Lagrangian is

\[
L_I = \int d^4x d^2\theta W(\Phi, \{ X_i \}) + \text{h.c.} + \text{interaction D − terms}.
\]

Considering only F-terms, the general superpotential for this system is

\[
W(\Phi, \{ X_i \}) = 4m\Phi^2 + \lambda\Phi^3 + \sum_{i=1}^{N_M} \left[ 4\mu_i X_i^2 + f_i X_i^3 + \lambda_i'\Phi^2 X_i + \lambda_i''\Phi X_i^2 \right].
\]

If \( \{ X_i \} \) represent the modes of a string, then the mass parameters \( \{ \mu_i \} \) will range over all energy levels of the string from the string scale \( M_S \) to the Planck scale \( M_{Pl} \) and beyond. The energy levels can represent both the coarse splittings between mass levels of the symmetry unbroken theory and the fine structure splitting after some symmetry breaking. Here we will not commit to a specific model. The auxiliary fields \( F, \{ F_i \} \) can be eliminated in Eqs. (3), (3) and (4) through the “field equations” \( \partial W/\partial F = \partial W/\partial F_i = 0 \), with the resulting Lagrangian only in terms of the physical fields \( \phi, \psi \), and \( \{ \chi_i, \psi_i \} \).
A. Distributed Mass Model

The general Lagrangian involves several interesting interaction schemes between the inflaton and the mode-fields. As an example, the DM-models will be obtained from the general Lagrangian Eqs. (1) – (4). First, let us take a moment to review the DM-model; these are models where the fields \( \{\chi_i, \psi_i\} \) that couple to the inflaton \( \phi \), have masses that are dependent on the inflaton’s amplitude \( \varphi_c \) and for any given \( \varphi_c \), there is a wide range of mass scales for the system of fields \( \{\chi_i, \psi_i\} \). Such models are obtained by the shifted couplings \( g^2(\phi - M_i)^2 \chi_i^2, g(\phi - M_i)\bar{\psi}_i\psi_i \) (for Weyl fermions mass terms are of the form \( g(\phi - M_i)\bar{\psi}_i\psi_i \) and \( g(\phi - M_i)\bar{\psi}_i\psi_i \) for bosons \( \chi_i \) and fermions \( \psi_i \) respectively, with \( \{M_i\} \) ranging over mass scales that can be both coarsely and finely split. The important reference energy scale during warm inflation is the temperature \( T \) of the universe. When the mass of a specific \( \chi_i(\psi_i) \) field \( m_{\chi_i(\psi_i)} \approx g|\varphi_c - M_i| \sim T \), that field is thermally excited and can contribute to the dissipative dynamics of the inflaton, but outside this region it cannot [12,13]. By having many mass scales \( \{M_i\} \), the \( \varphi_c \)-amplitude region for active dissipative dynamics is increased. This directly translates to an increased duration of warm inflation, thus increasing the total e-foldings.

The DM-models are contained in the class of Lagrangians from Eqs. (1) – (4) in which \( \chi_i = 0, i = 1, \ldots, N_M \). For this case writing \( L = \int d^4x \mathcal{L} \), we obtain

\[
\mathcal{L} = \mathcal{L}_\phi + \sum_{i=1}^{M} \mathcal{L}_{X_i-\text{kinetic}} + \sum_{i=1}^{M} \mathcal{L}_{X_i-\text{mass}} + \mathcal{L}_I
\]

(5)

where

\[
\mathcal{L}_\phi = \partial_\alpha \phi^\dagger \partial^\alpha \phi - 4m^2 \phi^\dagger \phi
\]

\[
- \frac{3m\lambda}{2} (\phi^\dagger \phi^2 + \phi^\dagger 2\phi) - \frac{9}{16} \lambda^2 (\phi^\dagger \phi)^2
\]

\[
+ \frac{i}{4} \psi \overset{\leftrightarrow}{\partial} \bar{\psi} + \frac{m}{2} (\psi \bar{\psi} + \bar{\psi} \psi) + \frac{3}{8} \lambda (\phi \psi \psi + \phi^\dagger \bar{\psi} \bar{\psi}),
\]

(6)

\[
\mathcal{L}_{X_i-\text{kinetic}} = \partial_\alpha \chi_i^\dagger \partial^\alpha \chi_i + \frac{i}{4} \psi_i \overset{\leftrightarrow}{\partial} \bar{\psi}_i,
\]

(7)

\[
\mathcal{L}_{X_i-\text{mass}} = -(2\mu_i + \frac{\lambda_i''}{2} \phi^\dagger) (2\mu_i + \frac{\lambda_i''}{2} \phi) \chi_i^\dagger \chi_i
\]

\[
- (\frac{\lambda_i''}{2} m \phi + \frac{3\lambda_i''}{16} \phi^\dagger 2\phi) \chi_i^2 (\phi^\dagger 2\phi)^2
\]

\[
+ [\frac{\mu_i}{2} + \frac{\lambda_i''}{8} \phi] \psi_i \psi_i + [\frac{\mu_i}{2} + \frac{\lambda_i''}{8} \phi] \bar{\psi}_i \bar{\psi}_i,
\]

(8)

and

\[
\mathcal{L}_I = \sum_{i=1}^{N_M} \frac{\lambda_i''}{4} (\psi_i \chi_i + \bar{\psi}_i \chi_i^\dagger) + \frac{3f_i}{16} (\chi_i \psi_i + \chi_i^\dagger \bar{\psi}_i)
\]

(8)
\[ -\frac{3}{2}\mu_if_i(\chi_i^\dagger\chi_i + \chi_i^\dagger\chi_i^2) - \frac{9}{16}f_i^2\chi_i^\dagger\chi_i^2 - \frac{3f_i\lambda''_{i}}{16}(\phi^\dagger\chi_i^\dagger\chi_i^2 + \phi\chi_i\chi_i^\dagger^2) \]
\[ - \frac{1}{16}\sum_{ii'}^{NM}\lambda''_{i}\chi_i^\dagger\chi_i^2. \]

\( \mathcal{L}_{\chi_i-\text{mass}} \) contains all the terms that contribute to the mass of the \( \chi_i, \psi_i \) fields for arbitrary inflaton amplitude \( \varphi_c \). The DM-model is realized for the case \( \mu_i = gM_i/2 \) and \( \lambda''_{i} = -2g \).

For this case the masses of the \( \chi_i, \psi_i \) fields are respectively

\[ m_{\chi_i}^2 = g^2(\varphi_c - M_i)^2 - 2gm\varphi_c - \frac{3g\lambda}{4}\varphi_c^2 \]  

and

\[ m_{\psi_i}^2 = g^2(\varphi_c - M_i)^2. \]

At \( \varphi_c = 0 \), the masses of the \( \chi_i, \psi_i \) pair are equal in Eqs. (10) and (11) as required by supersymmetry. On the other hand, a nonzero inflaton field amplitude, \( \varphi_c \neq 0 \), implies a breaking of supersymmetry, which in turn permits the differences in the \( \chi_i \) and \( \psi_i \) masses in Eqs. (10) and (11) respectively. In this case, \( \varphi_c \) is out of equilibrium, thus SUSY breaking here is not due to one of the standard equilibrium processes, for example O’Raifeartaigh \([26]\). However for this case, when the field amplitude comes into equilibrium, \( \varphi_c = 0 \), SUSY is restored. These breaking terms could be potentially dangerous to the bosonic sector of the mode fields, if their size is much larger than the temperature scale \( T \). In such a case, these bosonic mode fields would be too massive to contribute to the dissipative dynamics of \( \varphi_c \). However, this problem will not arise for the wide set of cases obtained in \([15]\), in which the parameter regime required \( m = m_T \sim \sqrt{\lambda T} \) and \( \lambda\varphi_c^2 = m_T^2 < T^2 \). For these values of the parameters, from inspection of Eq. (10), it follows that the soft breaking terms are smaller than \( T^2 \).

Based on the analysis in \([15]\), the amplitude of the inflaton \( \varphi_c \) and mass parameters \{\( \mu_i \)\} are expected to be generically much larger than \( T \). In this case, terms in the interaction Lagrangian Eq. (9) involving \( \varphi_c \) and \{\( \mu_i \)\} could be very large and lead to interesting dissipative effects. This deserves further investigation, but it will not be done here. It is worth adding that the interaction terms in Eq. (9) involving \( \varphi_c \) and \( \mu_i \) couple through the parameter \( f_i \). Thus the effect of these terms can be controlled and if desired they can be made negligible by choosing \( f_i \) sufficiently small.

**IV. CONCLUSION**

In the previous section we obtained the DM-models from the general SUSY Lagrangian Eqs. (1) – (4). Warm inflation cosmology based on DM-models recently has been shown to solve the horizon problem \([17]\). Thus the results in the previous section highlight a specific class of potential SUSY warm inflation models.

The general SUSY Lagrangian Eqs. (1) – (4) contains other useful features for warm inflation cosmology. For example, the inflaton potential in Eq. (6) has cubic \( \phi^3 \) terms.
which can produce plateau regions that have significant vacuum energy and reasonably flat potential surfaces. The dual effect of a flat potential and strong dissipation increases the duration of warm inflation, which is being driven by the vacuum energy that the plateau maintains. Another interesting feature of the SUSY Lagrangian Eqs. (1) – (4) is for $\lambda_i' \neq 0$ there are $\phi^3 \chi_i$ couplings in which the bosonic mode fields $\chi_i$ couple linearly to the inflaton. Such terms could be treated nonperturbatively in deriving the effective $\varphi_c$-equation of motion using methods similar to those for solving Caldeira-Leggett models [27,2]. An exact nonperturbative derivation of the effective $\varphi_c$-equation of motion would not be possible, since there still would be additional terms in the Lagrangian that would require a perturbative treatment. Nevertheless, such interactions provide a novel prospect for dissipative dynamics.

Finally in regards the general SUSY Lagrangian Eqs. (1) – (4), D-terms generically are products of chiral and antichiral fields, such as the kinetic terms eq. (2) and interaction terms from a gauge symmetry. Interaction D-terms are not required to obtain the DM-model mass scale structure. However, where present, they modify the Higgs potential, adding robustness to the model but at the same time constraining its parameters. For dissipative dynamics, the enhanced interactions amongst the mode fields due to interaction D-terms will increase the mode field decay widths, thus thermalization rates [7,9,10,12].

In summary, we have argued that DM-models arise when we consider effective $N=1$ SUSY Lagrangians obtained from a fundamental string. For observationally interesting warm inflation solutions in [12], the relevant DM models require a large number of fields, $> 10^4$. The tower of states between the string scale and Planck scale are made to order for such a realization of warm inflation. This is virtually independent of the field that behaves as the inflaton. All we require is the existence of such a state. A more complete analysis would derive these results from string theory, but this is currently out of reach and beyond the modest objectives of this work as well.

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