Empirical Analysis of TOA and AOA in Hybrid Location Positioning Techniques

Fangliang Wang¹, a* and Xinhong Liu², b*
¹Shandong University, China.
²Shandong Jianzhu University, China
Email: wfl1057521777@icloud.com, wojiaoliukubi@gmail.com

Abstract. We explored the role of TOA and AOA in hybrid positioning algorithms in the LOS environment, as well as the effects of weight parameters. We built a model to simulate the experimental process. The simulation environment is an environment of a multi-base station single mobile terminal built on a cellular network. We observed the change of the positioning error by changing the weight of the base station TOA and AOA, so as to analyze the influence of the weight parameter on the accuracy. The final simulation results show that AOA has little effect on the positioning algorithm in LOS communication, and the positioning algorithm mainly depends on TOA. The results of others do not indicate the role of weight parameters in the positioning algorithm, and our research makes up for this vacancy.

1. Introduction
The popularity of navigation and positioning in mobile phones makes a rapid development of LBS(Location-Based Service)[1]. At the same time, the requirements for the accuracy and reliability of the pedestrian navigation and positioning technology are getting higher and higher. However, in a complex geographic environment, because of signal interference and occlusion, the positioning accuracy of Global Positioning System(GPS) is very poor or even impossible to locate[2]. However, GSM communication system[3] relies on stability and has developed into the most practical standard of communication.

The most important positioning technology is TOA[4] , AOA [5] [6] and hybrid TOA/AOA[7][8]. As traditional positioning methods, they have good performance in cellular network. However, if the ways between MS and BS are in Line-Of-Sight(LOS) environment, there is preeminent accuracy in TOA and AOA method, but in some situation, the accuracy is greatly reduced. Wang [9] enhanced TOA localization algorithm to reduce the error. Ding [10] introduced a cooperative localization algorithm that combines the TOA/AOA measurements in Line-of-Sight (LOS) and None-Line-Of-Sight(NLOS) cases, and simulation results prove that the proposed algorithm increased its accuracy in both conditions.

In the past, everyone thought AOA was a powerful complement to TOA. We have been discussing the best weighting method in the hybrid positioning scheme. The mainstream weighting method is 1/2 each[11]. To verify whether it is the most effective method, we explored the degree of action of TOA and AOA in the hybrid algorithm and the influence of the weight parameters on the accuracy in LOS condition. We take 0-20 of the weights of TOA and AOA, respectively. we explored the roles of TOA and AOA in the hybrid algorithm by studying the effects of weight parameters on the accuracy in LOS case. Therefore, only TOA should be used, eliminating people's long misconceptions about AOA.

The rest of this paper is organized as follows: Section 2 presents the experimental scheme, describing the models and principles. Afterwards, Section 3 demonstrates computer simulation results
using hybrid AOA/TOA method, then discusses the reason why the error cannot be reduced within AOA method. Finally Section 4 gives the conclusion.

2. Formatting the Title, Authors and Affiliations
Although there are various positioning systems and positioning methods, in essence, the location-related information parameters are collected by the positioning system, and then analyzed to obtain the estimated position of the target mobile station.

2.1. Time of Arrivals Technique
About the basic principles of TOA technology, the location of the mobile station (MS) is derived from the time it takes for the measurement signal to travel from multiple BS to the MS. The equation \( d = c \times t \) provides the distance between the MS and the BS.

In the TOA positioning method, by measuring the TOA value \( t_i \) at the signal base station \( BS_i \), it can be determined that the mobile station is in a circle centered on the base station BS and having a radius \( L_i \) \( (L_i = c \times t_i) \). Therefore, when the number of base stations participating in the same time is N, the possible position of the mobile station is at the intersection of the N circles constructed by the TOA value, as shown in the area \( A_{12}A_{13}A_{23} \) in the figure 1.

![Figure 1. Positional relationship between mobile station and base station.](image)

According to the TOA measured value \( t_i \), the following residual function can be obtained.

\[
 f_i(X) = L_i - \sqrt{(x - x_i)^2 + (y - y_i)^2}, \quad i = 1, 2, ..., N
\]  

(1)

where \((x, y)\) represents the coordinates of the MS and \((x_i, y_i)\) represents the coordinates of the \( BS_i \). Then, the following cost function can be derived:

\[
 F(x) = \sum_{i=1}^{N} r_i^2 f_i(X)^2, \quad i = 1, 2, ..., N
\]  

(2)

where \( r_i \) is the weight, indicating the reliability of the measured data.

2.2. Angle of Arrivals Technique
In the AOA technique, the signal arrival angle of one or more BSs from the MS is measured by using an antenna array (or an adaptive antenna). The antenna array determines the incident angle of the signal according to the signal transmitted by the mobile station, and a radial connection, that is, the azimuth line of the mobile station, can be determined between the base station and the mobile station.
When using the AOA positioning method, usually only two sets of base stations are needed to determine the location of the mobile station.

According to figure 1, the following residual function can be determined from the AOA measurement at the primary base station.

\[
 f_i(X) = \alpha_m - \arctan\frac{y-y_i}{x-x_i}, i = 1,2, ..., N
\]  

(3)

where \((x, y)\) represents the coordinates of the MS and \((x_i, y_i)\) represents the coordinates of the \(BS_i\), \(\alpha_m\) is AOA measurement, \(\arctan\frac{y-y_i}{x-x_i}\) represents the actual value of AOA.

\[
 G(x) = \sum_{i=1}^{N} h_i^2 f_i(X)^2, i = 1,2, ..., N
\]  

(4)

where \(h_i\) is the weight, indicating the reliability of the measured data.

2.3. Hybrid TOA/AOA Location Positioning Technique

The hybrid positioning method is to mix and use the above methods to achieve the purpose of improving positioning accuracy and positioning efficiency. Because the hybrid positioning method absorbs the advantages of a single method, it is also a hot topic of current research, and it has a very bright prospect.

In the TOA positioning method, the solution to the positioning problem is often solved by nonlinear optimization. Not only is the solving process complicated, the calculation amount is large, and often a relatively accurate initial solution is needed to obtain better positioning accuracy. By using the TOA measurement equations of different base stations, the positioning problem can be converted into a linear optimization problem, so as to reduce the computational complexity.

Assuming that the coordinates of the \(BS_i\) are \((x_i, y_i)\), the measurement distance is \(L_i\), the coordinates of the \(BS_j\) are \((x_j, y_j)\), and the measurement distance is \(L_j\), the following equation can be obtained.

\[
 L_i^2 = (x - x_i)^2 + (y - y_i)^2
\]  

(5)

\[
 L_j^2 = (x - x_j)^2 + (y - y_j)^2
\]  

(6)

At the same time at the primary base station \(BS_1\), we can also determine the azimuth straight line based on the AOA measurements.

\[
 y - y_1 = (x - x_1) \tan \alpha_m
\]  

(7)

Subtracting equations (5) and (6) yields:

\[
 x(x_j - x_i) + y(y_j - y_i) = 0.5(L_i^2 - L_j^2 + K_j - K_i)
\]  

(8)

where \(K_i = x_i^2 + y_i^2\).
Therefore, the linear relationship between the base station and the mobile station can be determined, then \( N(N>3) \) base stations can obtain the \( C_N^3 \) positioning linear line, and at the same time, the mobile station is in the base station BS by the formula (7). It is on a straight line with an angle \( \alpha \), so a total of \( C_N^3 + N \) positioning straight lines can be obtained. The possible location of the mobile station is at the intersection of these lines. Due to the influence of NLOS error, the TOA measurement distance value is often larger than the actual distance, so the mobile station position must meet:

\[
L_i^2 \geq (x - x_i)^2 + (y - y_i)^2
\]

(9)

The solution to the problem can be transformed into the following matrix form:

\[
A = \begin{bmatrix}
(x_1 - x_2) & (y_1 - y_2) \\
\vdots & \vdots \\
(x_1 - x_n) & (y_1 - y_n) \\
\tan \alpha_m & -1
\end{bmatrix}
\]

\[
b = \begin{bmatrix}
(L_2^2 - L_1^2 + K_1 - K_2) \\
\vdots \\
(L_n^2 - L_1^2 + K_1 - K_n) \\
2(x_1 \tan \alpha_m - y_1)
\end{bmatrix}
\]

According to the least squares method, the coordinates of the mobile station can be determined as:

\[
x = (A^T A)^{-1} A^T b
\]

(10)

3. Results
As shown in the figure 3, in the cell, when positioning the mobile station, the TOA measurement method determines that the mobile station is in a circle with a radius of L centered on the base station by measuring the signal propagation distance L from the mobile station to the base station.
During the simulation, the cell radius $R$ is 1000m, and the coordinates of the base station are $(0,0)$, $(\sqrt{3}R,0)$, $(\sqrt{3}R/2,1.5R)$, $(-\sqrt{3}R,0)$, $(-\sqrt{3}R/2,1.5R)$, $(\sqrt{3}R/2,-1.5R)$.

The reason why cellular networks are widely used is the result of a mathematical conclusion that the planes are covered by a circle of the same radius, when the center of the circle is at the center of each regular hexagon of the regular hexagonal grid, that is, when the center of the circle is in the square of the regular triangle. The number of circles used is the least. In communications, it is often reasonable to use circular to express practice requirements, so a positive triangle mesh or a simple hexagonal mesh is the best choice for saving equipment construction costs.

The mobile station is located at any position within the range of 1/6 cell where the three base stations intersect, including the primary base station. The algorithm is simulated when the number of positioning base stations is 3, 5, and 7. In order to determine the effects of TOA and AOA in different environments, different TOA and AOA measurement errors were set up for independent experiments, each time independently testing 1000 times. During simulation, the AOA measurement error obeys a mean of 0, and the standard deviation is Gaussian distribution of 2, 3, 5, and 7. The TOA measurement error obeys a mean of 0, and the standard deviation is a Gaussian distribution of 5, 20, 50, and 100 m. There are 48 simulation environments in total.

We are based on the cost function of the TOA and AOA hybrid algorithm as follows:

$$F(x,y) = \sum_{i=1}^{N} \alpha \left( L_i - \sqrt{(x-x_i)^2 + (y-y_i)^2} \right) + \sum_{i=1}^{N} \beta \left( \theta_i - \arctan \frac{y-y_i}{x-x_i} \right), \quad i = 1, 2, ..., N$$  \hspace{1cm} (11)

$\alpha$ and $\beta$ represent the weights of TOA and AOA, respectively, and we take ten values for interval 1 from 0 to 20. In this way, we have a total of 400 combinations of values to perform simulation experiments in different simulation environments. Measurement error is expressed by root mean square error(RMSE). It is the square root of the ratio of the square of the deviation between the observed value and the true value and the number of observations.
As shown in the figure 4, the experimental results for three kinds of base stations in the case where the TOA error is 100 meters and the AOA error is 2 degrees. It can be seen that in the experimental environment with different number of base stations, the respective RMSE values are stable at 123, 81, 69 respectively. The number of base stations has a significant impact on the experimental accuracy, and the RMSE of the seven base stations is the smallest and the accuracy is the highest. However, changing the weight of TOA and AOA has little effect on the overall experimental results.

We separately analysed the experimental results of seven base stations with a TOA error of 100 meters and an AOA error of 2 degrees. Figure 5 shows the trend of experimental error RMSE with AOA weight.

It can be seen from the trend of figure 5 that although the fluctuation of RMSE is not large as a whole, the AOA weight is large, the experimental RMSE is large, and the result is not accurate.
Similarly, in the case of three base stations and five base stations, different TOA and AOA simulation experiments showed similar results. The greater the proportion of AOA, the higher the RMSE. Therefore, it can be said that AOA has a negative effect on the accuracy of base station positioning.

Overall, in the hybrid algorithm, the role of AOA has no significant effect on TOA, the error of TOA will have a greater impact on the accuracy of positioning, while the role of AOA is more stable and will not cause too much. Great fluctuations, but the effect is not obvious.

4. Conclusion
We explored the extent of TOA and AOA in hybrid algorithms in the LOS communication environment, and the study of the influence of weight parameters on accuracy. Simulation results show that AOA has little effect on TOA in LOS communication, and the localization algorithm mainly relies on TOA.

Our shortcoming is that we have not detected our method in NLOS communication. In reality, most environments are NLOS. In this case, the role of AOA cannot be ignored, and LOS and NLOS are two completely different environments. We did not perform simulation experiments on NLOS, so we cannot determine the respective roles of TOA and AOA in this environment.

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