Research on Dynamic Calibration Technology of Shock Accelerometer Based on Model Method

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Abstract. High-g shock accelerometers usually use model-based dynamic calibration to obtain their dynamic characteristics. Starting from the dynamic calibration technology chain, a different model linearization algorithm from ISO16063-43 is used for accelerometer model establishment and parameter identification, and the Monte Carlo method is used to calculate the uncertainty of model parameters. In order to verify the proposed method, simulated signals were constructed under different shock excitation amplitudes, and the parameter identification and uncertainty evaluation were done on the simulated signals to verify the proposed algorithm. At the same time, the algorithm is tested by using the actual shock accelerometer calibration signal. The results show that the algorithm has achieved good results without increasing the amount of calculation, and has practical significance for the dynamic calibration of accelerometers and other sensors in practical engineering applications.

Keywords: model-based dynamic calibration; Monte Carlo; parameter identification

1. Introduction
The core function of the sensor is measurement. If the dynamic characteristics of the sensor are not good, it will not be able to quickly and accurately reflect the changes being measured. In order to make the measurement results of the sensor more reliable, the sensor characteristics must be accurately obtained. The sensor characteristics include the static and dynamic characteristics. For sensors with harsh application environments and high reliability requirements such as high-g shock accelerometer, it is more important to obtain dynamic characteristics than static characteristics. To obtain the static or dynamic characteristics of the sensor, it must be calibrated[1].

The current shock accelerometer calibration methods are primary calibration and comparison calibration, no matter which method is adopted, the ultimate goal is to obtain the dynamic characteristics of the sensor. The dynamic calibration of shock accelerometer mainly uses peak sensitivity to describe the dynamic characteristics of the sensor. Sensitivity reflects the linear relationship between the electrical output of the sensor and the actual measured value. However, the concept of sensitivity is meaningful only when it is discussed within the working range of the sensor, and it is usually hoped that the electrical output of the sensor meets a linear relationship with the actual measurement signal. In the actual using, the sensor only behaves as approximately linear in a part of the interval, and the overall performance is non-linear. Therefore, the best way to describe the dynamic characteristics of the sensor is to obtain its model.
According to the model, the dynamic performance of the sensor can be correctly evaluated, and its performance can be estimated directly according to the measured curve of the dynamic calibration of the sensor. It will inevitably be interfered by various noises and may get incorrect results[2]. Through the fitting of the model, the errors can be partly eliminated. Commonly used models include differential equations, ARMA models, frequency transfer functions, etc. ISO16063-43 describes a model-based parameter identification method for the calibration of vibration and shock transducers[3]. It solves the application of shock accelerometer in broadband measurement, such as shock or other highly dynamic transient motion measurement.

A more important goal of dynamic calibration is to evaluate the uncertainty of dynamic calibration results. Since the traditional accelerometer calibration results give a single variable of peak sensitivity, the uncertainty evaluation is from the GUM file[4], through the analysis of the source of error, each uncertainty component is given. But for dynamic calibration, the various characteristics of the shock accelerometer show dependence on time or frequency. GUM's supplementary files S1 and S2 provide a Monte Carlo uncertainty evaluation method, which can effectively supplement the above problems[5,6]. ISO16063-43 also provides an evaluation method for the uncertainty of the model parameters.

Many researches have been carried out on the model analysis technology of dynamic calibration. Link.A proposed the use of a spring mass damping model to establish a shock accelerometer model, and use frequency domain deconvolution method to obtain the transfer function and the model parameters[7], related methods are partly applied in ISO 16063-43. Eichstadt introduced the latest results of the EMPIR 14SIP08, and constructed an open source software package - PyDynamic for dynamic measurement analysis, which makes it easy to apply its functions for dynamic measurement analysis[8]. Hu used a narrow pulse shock excitation device to fully excite the high-frequency part of the accelerometer, and used a recurrence equation as the accelerometer parameter model. At the same time, a compensation filter was established[9]. Wang added a nonlinear term to the second-order model of the shock accelerometer. The parameters are solved directly from the transfer function itself [10].

Based on the work of the aforementioned, this article focuses on the research on the model calibration technology of shock accelerometer, referring to the overall implementation framework of ISO16063-43, and realizes model construction and optimization through the improved model linearization technology, and uses the weighted least square method to identify the parameters of the accelerometer transfer function. The transfer process of uncertainty in the dynamic calibration chain is constructed, and the Monte Carlo method is used to calculate the uncertainty. Finally, the validity of the algorithm is verified by the analysis results of the constructed simulation signal and the measured signal.

2. Dynamic calibration of shock accelerometer

2.1. Model-based dynamic calibration

Figure 1 shows the process of dynamic calibration of shock accelerometer implemented by ISO16063-43. The process of dynamic calibration includes: the collection of standard acceleration signal and accelerometer output signal, DFT of the standard acceleration signal and sensor output signal, frequency domain deconvolution and parameter identification. The uncertainty is transferred from the standard acceleration signal and the sensor output signal to the model parameters. The algorithm framework is adapted to the primary method, the comparison method or other shock excitation devices[11].
Figure 1. Model-based dynamic calibration of shock accelerometer

It can be seen from the figure that after the dynamic calibration, the model parameters and their uncertainty are used as the dynamic calibration results. The results can be used to construct a deconvolution filter or other means for dynamic compensation, in addition to the uncertainty transmission of the whole process, a complete dynamic measurement chain is realized.

2.1.1. Shock accelerometer model. The shock accelerometer is usually equivalent to a linear time-invariant dynamic system, and its structure is shown in Figure 2.

Figure 2. Single-degree-of-freedom spring mass damping system

Suppose the displacement of the mass $m$ relative to the shell is $x_0(t)$, and the displacement of the sensor base relative to the ground is $x_1(t)$. According to Newton's second law, the sensor motion equation can be obtained:

$$c\ddot{x}_0(t) + kx_0(t) = m(\dot{x}_1(t) - \dot{x}_0(t))$$  \hspace{1cm} (1)

The differential equation is:

$$\ddot{x}(t) + 2\delta\omega_0\dot{x}(t) + \omega_0^2x(t) = \rho a(t)$$  \hspace{1cm} (2)

Where $\delta = c/2\sqrt{k/m}$, $\omega_0 = \sqrt{k/m}$, $\rho$ represent the physical model parameters of damping coefficient, resonance frequency, and conversion factor, respectively.

Do Laplace transformation on the differential equation (2) to get the transfer function:

$$G(\omega) = \frac{\rho}{\omega_0^2 - \omega^2 + 2j\delta\omega\omega_0}$$  \hspace{1cm} (3)

Where $\rho$ can also be expressed as $S_0\omega_0^2$.

In the actual measurement process, the transfer function can be obtained by deconvolution in the frequency domain:

$$G(\omega) = \frac{Y(\omega)}{X(\omega)}$$  \hspace{1cm} (4)

$$Y(\omega) = \text{DFT}\{y(n)\}$$  \hspace{1cm} (5)

$$X(\omega) = \text{DFT}\{x(n)\}$$  \hspace{1cm} (6)

2.1.2. Model linearization and parameter identification. The real and imaginary parts of the transfer function of the second-order model of the shock accelerometer is:

$$G(\omega) = \frac{\rho}{\omega_0^2 - \omega^2 + 2j\delta\omega\omega_0} = A + jB$$  \hspace{1cm} (7)
Where \( A = (a_1, a_2, \cdots, a_N) \) is the real part of the transfer function, \( B = (b_1, b_2, \cdots, b_N) \) is the imaginary part of the transfer function.

Model parameter identification is equivalent to finding the optimal solution of the transfer function under a certain cost function. Usually, the solution of the optimization uses gradient to calculate. In order to avoid nonlinear solutions in the identification process, the model is linearized as:

\[
\rho = (a_i\omega_0^2 - a_1\omega_1^2 - 2b_i\delta\omega_1\omega_0) + j(b_i\omega_0^2 - b_1\omega_1^2 + 2a_i\delta\omega_1\omega_0)
\]

(8)

So:

\[
S_0 = (a_i - a_i\omega_1^2 \frac{1}{\omega_0^2} - b_i\omega_1\frac{2\delta}{\omega_0}) + j(b_i - b_i\omega_1^2 \frac{1}{\omega_0^2} + a_i\omega_1\frac{2\delta}{\omega_0})
\]

(9)

In the parameter model constructed by the spring mass damping system, \( S_0 \) is equivalent to the amplification factor, which is a real number, so:

\[
s_0 = a_i - a_i\omega_1^2 \frac{1}{\omega_0^2} - b_i\omega_1\frac{2\delta}{\omega_0}
\]

(10)

\[
0 = b_i - b_i\omega_1^2 \frac{1}{\omega_0^2} + a_i\omega_1\frac{2\delta}{\omega_0}
\]

(11)

Simplified:

\[
a_i = u_1 + a_i\omega_1^2 u_2 + b_i\omega_1 u_3
\]

(12)

\[
b_i = b_i\omega_1^2 u_2 - a_i\omega_1 u_3
\]

(13)

Where

\[
[u_1, u_2, u_3] = \left[ S_0, \frac{1}{\omega_0^2}, \frac{2\delta}{\omega_0} \right]
\]

(14)

For the measured data, each sampling frequency point has a set of equations (12) and (13), so the \( n \) sets of equations are expressed in matrix form:

\[
\begin{bmatrix}
1 & \text{Re}(G) & \text{Im}(G)
0 & \text{Im}(G) & \text{Re}(G)
\end{bmatrix}
\begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix} =
\begin{bmatrix}
\text{Re}(G) \\
\text{Im}(G)
\end{bmatrix}
\]

(15)

Using weighted least squares for parameter identification, the cost function in the identification process is defined as:

\[
\chi^2 = \sum_n \frac{(\text{Re}(G) - D\hat{u})^2}{u^2[\text{Re}(G)]} + \frac{(\text{Im}(G) - D\hat{u})^2}{u^2[\text{Im}(G)]}
\]

(16)

Where

\[
D = \begin{bmatrix}
1 & \text{Re}(G) & \text{Im}(G)
0 & \text{Im}(G) & -\text{Re}(G)
\end{bmatrix}
\]

\[
u = \begin{bmatrix}
u_1 \\
u_2 \\
u_3
\end{bmatrix}
\]

Let the partial derivative of the cost function with respect to the parameters to be identified in the model be zero, so that the loss function is minimized:

\[
\frac{\partial\chi^2}{\partial u} = 2D^T \text{Re}(G - D\hat{u}) + 2D^T \text{Im}(G - D\hat{u}) = 0
\]

(17)

The estimated parameter value is:

\[
\hat{u} = \text{argmin} \chi^2 = (D^T U_G^{-1} D)^{-1} D^T U_G^{-1} G
\]

(18)

2.2. Transmission of Uncertainty in Dynamic Calibration

From the entire dynamic calibration chain shown in Figure 1, the uncertainty evaluation includes two aspects: the uncertainty of the dynamic calibration signal source and the uncertainty transmission during the dynamic calibration process.

The uncertainty of the dynamic calibration signal source is usually determined by the measurement device and the charge amplifier acquisition device used. The uncertainty transmission in the dynamic...
 calibration process includes three parts: the uncertainty transmission in the DFT process, the uncertainty transmission in the deconvolution process, and the uncertainty transmission in the model parameter identification.

Since there are definite mathematical expressions in the DFT process and the deconvolution process, the uncertainty transfer in this process is calculated by calculating its sensitivity matrix (Jacobian matrix), and the specific formula is derived in reference 8.

Similarly, the parameter identification process can also be calculated according to the GUM linear transfer according to ISO16063-43. The uncertainty of the model parameters after identification is:

\[ V_u = (D^T U_u^{-1} D)^{-1} \]  

Due to the advantages of the Monte Carlo algorithm itself, compared with the GUM method, the Monte Carlo method is more suitable for evaluating the uncertainty in the process of model parameter identification. The algorithm of the evaluation process is shown in Table 1:

Table 1. Monte Carlo uncertainty evaluation in parameter identification

| Algorithm: Monte Carlo measurement uncertainty evaluation |
|----------------------------------------------------------|
| Initialization: Set Monte Carlo sampling number K, given parameter identification model \( \tilde{u} = f(G) = (D^T U_u^{-1} D)^{-1} D^T U_u^{-1} G \), and its covariance matrix is \( \Sigma \). For \( k = 1 \) to \( K \) do: |
| According to the transfer function \( G \) and its covariance matrix, generate a set of random variables conforming to the multivariate normal distribution: \( \varepsilon(G_j), j = 1, \ldots, N \); |
| Calculate the multivariate normal distribution: \( \tilde{G}^k = G^k + G^k \cdot \varepsilon(G^k) \); |
| For each group \( G_j \), calculate the identified parameters: \( \tilde{u}_i = f(\tilde{G}^k) \); |
| Get K group identification results: \( \tilde{U} = (\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_K) \); |
| The calculated mean \( \tilde{U} \) is used as an estimate of \( u: \tilde{u} = \frac{1}{K} \sum \tilde{u}_i \); |
| Calculated the uncertainty of \( \tilde{u}: V_u = \text{sqrt} \left( \text{diag} \left( \text{COV} \left( \tilde{U} \right) \right) \right) \) |

3. Experiment and analysis

3.1. Simulation experiment

In order to verify the proposed parameter identification and uncertainty evaluation method, this paper conducts a simulation experiment to verify the algorithm. The sensor parameters and their uncertainty are selected as:

\[ S_0 = 0.2254, u_{S_0} = 0.001; \]
\[ \delta = 0.021, u_\delta = 0.0002; \]
\[ f_0 = 42800, u_{f_0} = 320. \]

Since the resonance frequency of the verified sensor is 42.8kHz, the pulse width of the impulse excitation signal is selected 10\(^3\)s, and 5 different impulse excitation amplitudes are selected: 1000g, 3000g, 4000g, 7000g and 9000g. Add random noise with a mean value of 0 and a relative standard deviation of 0.01 to the excitation signal. According to the selected sensor model parameters, an IIR digital filter is designed to replace the actual sensor model. The simulated excitation signal and sensor response signal are shown in the figures 3 and figures 4 below.
3.2. Simulation experiment results
According to the shock accelerometer excitation signal and accelerometer response signal generated by the simulation, the model linearization and the parameter identification method given above is used to identify the sensor model parameters and evaluate the uncertainty.

The frequency range is selected from 0 to 80kHz, and the algorithm proposed in this paper and ISO16063-43 are used. The amplitude-frequency results for one of the signal are shown in Figures 5 and 6. The identification parameters and uncertainty are as follows as shown in Table 2, the identification results under different excitation amplitudes in the Table are the average of 20 simulation results.
Table 2. Simulation experiment results

| Excitation | Item          | Set value     | ISO          |
|------------|---------------|---------------|--------------|
| /          | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.001         | 0.001        |
| 1000g      | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.00098       | 0.00099      |
|            | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.00002       | 0.00002      |
| 3000g      | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.00101       | 0.000199     |
|            | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.00099       | 0.001999     |
| 4000g      | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.00099       | 0.00198      |
|            | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.001         | 0.00198      |
| 7000g      | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.00099       | 0.0002       |
|            | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.0009        | 0.0002       |
| 9000g      | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.001         | 0.001998     |
|            | parameter     | 0.225         | 0.225        |
|            | uncertainty   | 0.00098       | 0.00202      |

Figure 6. ISO16063-43 identification results
It can be seen from Table 2 that in the case of different shock excitation amplitudes, the calculation results given in this paper and ISO16063-43 are almost equivalent to the set values, which verifies the effectiveness of the algorithm. For the specific parameter analysis results of $S_0$ and $\delta$, the identification results and uncertainty of the two methods are basically consistent with the set values; in the analysis results of $f_0$, the uncertainty obtained by ISO16063-43 is slightly larger than the results of this article, mainly because ISO16063-43 adopts the z-domain transfer function, and its identification accuracy is related to the sampling frequency. Because uses the Monte Carlo algorithm for the uncertainty evaluation, the value obtained under 95% confidence probability.

3.3. shock accelerometer calibration experiment

In order to fully verify the algorithm in this paper, the shock accelerometer calibration experiment is carried out on the device of Changcheng Institute of Metrology and Measurement. The experiment adopts the comparison method of shock accelerometer calibration. The standard sensor is Endevco2270, the calibrated sensor is BK4384, and the impact amplitude is 1000g.

The uncertainty of the standard acceleration signal obeys a normal distribution with relative standard deviation of 0.05, and the uncertainty of the sensor output signal obeys a normal distribution with relative standard deviation of 0.07. Before data processing, it is necessary to first perform alignment and noise reduction processing, and then model the sensor and calculate the uncertainty of related model parameters according to the aforementioned algorithm. Using the weighted least square method based on uncertainty to identify the parameter of the model, the identification results are shown in Table 3. The amplitude-frequency response of the sensor and its uncertainty envelope are shown in Figure 7.

| Table 3. Identification parameter results and uncertainty |
|----------------------------------------------------------|
| $S_0$ | $\delta$ | $f_0$ (Hz) |
|-------|---------|------------|
| Model parameter | 0.845 | 0.01454 | 41632.837 |
| Uncertainty | 5.834e-05 | 4.191e-05 | 299.231 |

Figure7. The amplitude-frequency response of the sensor and its uncertainty envelope

According to the application manual of the BK4384 sensor, the installed resonant frequency is 42kHz. Considering that the input signal uncertainty distribution has not been fully described and the actual installation situation, the identification result is basically reliable.

4. Conclusion and outlook
Since in the actual measurement process, the sensor output signal and excitation signal are extremely susceptible to various environmental factors, the sensor need the dynamic calibration. In this paper, a second-order spring mass damping system is used as the sensor model, the numerator and denominator are linearized, the least square method is used to identify the parameters of the linearized model, and the Monte Carlo method is used to evaluate the uncertainty. Finally, the simulation signal and the measured sensor signal are used to verify the effectiveness of the algorithm.

According to the range of the resonant frequency of the calibrated sensor, the selection of the standard sensor in the comparison method has received certain restrictions. The resonant frequency range of the standard sensor should be greater than the resonant frequency of the calibrated sensor. Sometimes this condition is difficult to achieve. Based on the above problems, the next step will continue to carry out research on model calibration technology based on laser interferometry for standard signal acquisition conditions, compare the advantages and differences between the two methods, and further improve the research on model-based dynamic calibration technology.

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