Optimal Bidding of Price-Maker Retailers With Demand Price Quota Curves Under Price Uncertainty

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ABSTRACT This paper proposes an optimal bidding method of price-maker retailers in the electricity market with demand price quota curves (DPQC)–based probability distribution function (PDF) estimation of the market price. Different from traditional game-theory methods or agent-based methods, the proposed DPQC-based PDF estimation method unnecessity to have full knowledge of the strategies of each rival or the market operation. In detail, the DPQC method is applied to consider the impacts of the market clearing price from the price-maker retailers themselves, and the PDF model is utilized to consider the market price uncertainty. The technical key point of the proposed method is to amend the PDF along with the PQCs dynamically. Moreover, the DPQC-based PDF estimation with one-segment and multi-segment bidding rules are presented, respectively. The optimization model of the bidding problem is formulated then, and we use the genetic algorithm to solve it. The case study shows that the proposed method can help the price-maker retailers better to consider the price impacts from their bidding behaviors, and enable them to make a higher profit in the electricity market.

INDEX TERMS Demand price quota curve (DPQC), optimal bidding, price-maker, power market.

I. INTRODUCTION

This paper addresses the optimal bidding problem faced by a price-maker retailer in a pay-as-clear electric energy market. Bidding optimization is a traditional topic in the field of the electricity market. Generally, retailers purchase electricity from the wholesale market and sell it to the end-users. To gain higher operation profits, retailers need to optimize their participation strategy in the wholesale market. Over the past years, several studies have worked on developing optimal offering or bidding strategy of a generator or retailer in the competitive electricity markets. From the perspective that whether the influences of biddings/offers on clearing prices are considered, a generator or retailer can be defined as a price-taker or price-maker depending on its impact on the market clearing outcomes. A price-taker refers to a facility, which cannot change the market price by its bidding/offering actions. For these participants, the market clearing prices are often assumed known or forecasted [1], [2]. To consider the uncertainty of the market prices, the probability distribution and scenarios of the prices are modeled in [1] and [2], respectively. A price-maker refers to a facility, which is large enough that its action can alter market prices. Price-maker strategies are initially studied for generators, such as thermal generators [3]–[12], hydro generators [13], wind power [14], energy storage [15]–[17], and then applied to the demand side [18], [19].

Focused on optimal bidding or offering problems of the price-makers, we category the exiting research into four types: game theory-based, bi-level optimization, agent-based or simulation-based method, and price quota curve (PQC)-based method.

1) Game theory-based. Game theory is a classical method to find an equilibrium state of the bidding game. References [3] and [20] discuss the Nash equilibrium corresponding to the optimal bidding strategies achieved by the participants.
However, these methods are more suitable for analyzing strategic behavior rather than proposing a tool to develop bidding strategies [1]. References [4] and [5] present the conjectured supply function (CSF) method. In the CSF method, each market player maximizes its profit while taking account of the reactions of its competitors. The method can be viewed as a generalization of the Cournot models in that each generator is allowed to conjecture that rival firms will adjust their supplies in response to price changes.

(2) Bi-level optimization. Bi-level optimization method usually procedures to derive optimal bidding/offers relies on a bi-level programming model whose upper-level problem represents the profit maximization or cost minimization of the target price-maker while the lower-level one represents the market clearing process [10], [14], [17]. Concerning the solving algorithm, the bi-level optimization model is usually reduced to a mixed-integer linear programming problem using the duality theory and the Karush–Kuhn–Tucker optimality conditions, or through mathematical program equilibrium constraints (MPEC). MPEC approach often imposes a high computational burden due to its complicated mathematical formulation, which makes it challenging to solve for large-scale systems [22]. Focus on the monthly energy market in China, [12] presents a bi-level optimization model for developing optimal bidding strategies of power producer in the monthly sequential contract and balancing markets.

(3) Agent-based or learning-based simulation method. These methods simulate the market operation by several agents. For example, [19] presents a two-level game simulation for virtual power plant (VPP) to optimize its bidding in the day-ahead and real-time energy market. Reference [8] applies Q-learning method to simulate an oligopolistic electricity market and study the effects of bidding behaviors of the generators. Reference [9] proposes a CSF-based learning method for generator bidding.

(4) PQC-based method. Price quota curve (PQC), initially proposed by Sheblé in [21], is also known as Residual Demand Curves (RDC) [3]. PQC describes how the market-clearing price changes as the quota of the price maker changes. PQC is at the beginning used for bidding optimization of the generators [3], [6], [7], and now is also used for demand-side bidding [15]–[22]. To distinguish the PQCs for generators and demand-side retailers, they are named generation PQC (GPQC) and demand PQC (DPQC), respectively [16]. Reference [13] applied PQC method to a hydro-generator and a pumped storage plant. References [15] and [16] apply the GPQC and DPQC in the bidding optimization problem of the energy storage system. PQC-based method has a significantly simpler formulation and consequently, less computational burden compared to the MPEC approach [15]. Due to the inevitable error in the forecasts of the PQCs, the associated uncertainty needs to be incorporated in the scheduling problem to minimize the risk. Stochastic programming [17] or robust programming [22] has been applied to deal with the uncertainty issues in PQCs.

In this paper, we try to combine the idea of DPQC with the probability distribution function (PDF) of the market price estimation. Such a combination can reflect the price impacts from the bidding behavior of the retailer itself and reflect the forecast uncertainty of the market clearing price. The advantages of the proposed method can be summarized as followings:

1. Unnecessity to have full knowledge of the strategies of each rival and the operation parameters of the market clearing process. Game theory method, bi-level optimization, and agent-based simulation methods usually necessitate that each strategic market player has full knowledge of the strategies of each rival and the operating parameters of the market clearing process. This unrealistic assumption, along with the modeling and computational complexities, renders such approaches less applicable for conducting practical use of the bidders. On the contrary, the proposed DPQC-based method only assumes an overall DPQC estimation of the market.

2. Incorporate the forecast uncertainty of the market price. The traditional DPQC-based method is based on a strong assumption that the overall DPQC estimation derived from the market clearing price is precise without any uncertainty. However, prediction uncertainty is inevitable. To overcome the drawbacks, the proposed method combines DPQC with the PDF to incorporate the forecast uncertainty of the market clearing price.

To sum up, we consider the price impacts from the price-maker retailer by DPQC method and incorporate the forecast uncertainty of the market clearing price by the PDF model. The main contributions of this paper are as follows.

1. We propose a DPQC-based PDF method to reflect the price impacts from the bidding behavior of the price-maker retailers. In the existing literature, both probability distribution based price-taker bidding methods and PQC-based price-maker bidding methods have been separately studied, but the combined approach of the above two methods is seldom seen. The technical key point in the combination is to amend the PDF along with the PQCs dynamically.

2. We build up an optimal bidding model for retailers in China’s monthly energy market. Both the one-segment and multi-segment bidding rules are considered for the pay-as-clear market, and the solving algorithm for the proposed model is presented.

The remaining of this paper is organized as follows: Section II presents the DPQC-based PDF estimation of the market price. This section introduces China’s monthly energy market at first, then gives the DPQC model, and proposes the one-segment and multi-segment DPQC-based PDF model, respectively. Section III presents the optimal bidding model and the solving algorithm. In Section IV, we carry out a case study to verify the effectiveness of the proposed method, and section V presents the conclusions.
II. DPQC-BASED DISTRIBUTION ESTIMATION OF THE MARKET PRICE

A. INTRODUCTION OF CHINA’S MONTHLY ENERGY MARKET

China has witnessed a profound revolution in the electricity market, and the yearly and monthly markets have been implemented in recent four years. Since the new round of power industry reform in 2015, the trading electricity volume in China has constantly been increasing [12]. In the first half of 2019, the total traded electrical energy reached over 884.7 TWh, experiencing an annual growth of 40% [23]. With the development of the monthly energy market, the numbers of retailers are quickly increasing at the same time. Given this fact, we focus on retailers in China’s monthly energy market in this paper. Note that the method in this paper is widely applicable in different kinds of pay-as-clear markets, which might be not only limited to the given case. At present, electricity transaction in China is mainly conducted by long-term (yearly) and medium-term (monthly) market. The yearly market is operated based on bilateral negotiations and the monthly market is operated based on pay-as-clear mechanism. Besides, some regions, such as Guangdong and Zhejiang provinces, have spot electricity markets (day-ahead and real-time) in trial operation. FIGURE 1 presents the structure of the monthly energy market in China.

As retailers with large capacity in electricity markets can influence clearing prices by strategic bidding, a large-scaled retailer is often assumed to be a price maker in the markets.

For the monthly market, the month-by-month operation provides plenty of available information. Such information allows small market players (price-takers) to forecast next-month market-clearing prices and also enables price-makers to predict their corresponding price quota curves. For a retailer, the transactions with the end-users are usually contracted based on a bilateral negotiation or standard form contract, and it needs to minimize the purchasing cost in the wholesale market.

As to the bidding rules, they are different in different regions or provinces. One of the differences is the number of bidding segments. With stepwise bidding, retailers can divide their required demand quantities into several segments and bid these segments with monotonic decreasing prices.

However, the numbers of bidding segments are different in different regions or provinces. For example, Jiangsu province only allows 1-segment biddings, Guangdong province allows 3-segment and Zhejiang province allows 6-segment biddings. FIGURE 2 shows the illustrative figure of the 1-segment and 3-segment bidding curves, respectively. In FIGURE 2 (b), $q_{b,n}$ is the incremental bidding quantity of the No. n segment. With different allowed bidding segment numbers, different bidding strategies may be adopted by a generation company [24]. As to the retailer, the answers are probably the same. Regarding that the bidding strategies are highly dependent on the segment bidding rules, the models with 1-segment and multi-segment are separately modeled in section II.C and II.D, respectively.

B. DPQC MODEL

For a given bidding period, the quota of a price maker is the amount of power it contributes to serving the demand in that period. If the price maker exercises its market power by retaining production (of a generator) or retaining demand (of a retailer), the market clearing price increases or decrease, respectively. The curve that expresses how the market-clearing price (for the whole market) changes as the quota of the price maker changes is called the residual demand curve or price quota curve [21].

A simple way to understand the PQC is that to treat all the other participants as a whole, then there is an initial cleared point by demand curve and supply curve (which does not include the biddings/offers of the target price-maker participant). For a generator, when the cleared quantity of the price-maker increases gradually, the supply curve shifts right, and a set of new cleared points to come out. The more the large participant offers, the lower the cleared price is. The construction method of the GPQCs can be found in [15], and DPQCs can be obtained in a similar way. The general method requires the database of supply curves, market clearing price and demand curves, which can be estimated by the participators from the historical data or experience.

Therefore, the generation PQC (GPQC) is a stepwise decreasing curve that shows the market price as a function of the total generation of the unit. In contrast, for a retailer, the demand PQC (DPQC) is a stepwise increasing curve that shows the market price as a function of the total demand of the demand buyer. The illustration figures of GPQC and DPQC are shown in FIGURE 3. In this paper, we assume the PQCs of
the target retailer is known. The detailed construction method of the PQCs can be seen in [25].

The PQCs are usually in discretized form for the convenience of the formulation. Assuming the discretized PQCs has \( K \) blocks. For each bidding quantity, there is a corresponding clearing price. In some paper, the bidding quantity and price can be modeled as a linear formulation by introducing binary variables, as shown by (1)-(4) [15]. In this paper, we model it as (5).

\[
\lambda_c(Q_b) = \sum_{k=1}^{K} u_k \cdot \lambda_{ck} \quad (1)
\]

\[
Q_b = \sum_{k=1}^{K} u_k \cdot q_k \quad (2)
\]

\[
0 \leq q_k < u_k \cdot q_k^{\text{max}} \quad (3)
\]

\[
\sum_{k=1}^{K} u_k = 1, \quad u_k \in \{0, 1\} \quad (4)
\]

where \( \lambda_c \) is the clearing price, \( Q_b \) is the bidding quantity, \( u_k \) is a binary variable indicating whether the bidder will choose block \( k \), \( q_k^{\text{max}} \) is the maximal quantity of block \( k \), \( \lambda_{ck} \) is the clearing price of the block \( k \) of the price quota curve.

\[
\lambda_c(Q_b) = \begin{cases} 
\lambda_{c1} & \text{if } 0 < Q_b < q_1 \\
\lambda_{c2} & \text{if } q_1 < Q_b < q_2 \\
\vdots & \vdots \\
\lambda_{cb} & \text{if } q_{K-1} < Q_b < q_K \\
\lambda_{cK} & \text{if } q_K < Q_b < q_K 
\end{cases} \quad (5)
\]

**C. ESTIMATING DPQC-BASED PDF WITH ONE-SEGMENT BIDDING RULE**

We define \( \lambda_c \) and \( \hat{\lambda}_c \) to be the energy prices for the cases without and with the price-maker retailer’s bidding. Without the bidding behavior of the price-maker retailer, the energy price is represented by \( \lambda_c \). When the price-maker retailer bids into the market, the energy price is represented by \( \hat{\lambda}_c \).

Here, we assume that the \( \lambda_c \) satisfies the normal probability, as shown by (6). Note that the proposed method is also appropriate to other kinds of distributions because we finally use the discretized PDF as an input in the optimal bidding model.

\[
f_p(\lambda_c; \mu_c, \sigma_c) = \frac{1}{\sqrt{2\pi}\sigma_c} \exp \left( -\frac{(x - \mu_c)^2}{2\sigma_c^2} \right) \quad (6)
\]

where \( \lambda_c \) is the clearing energy price, \( \mu_c \) and \( \sigma_c \) are the expected value and the standard deviation of the distribution, respectively.

For a given amount of the bidding quantity, the DPQC-based PDF in one-segment bidding can be estimated by the following three steps, as shown in FIGURE 4.

Step 1: Move. Assuming that the PDF of the energy price \( f_p(\lambda_c) \) is the blue curve in FIGURE 4 (a). If the price-maker retailer succeeds in bidding, the market clearing price could be pulled up to a certain degree. If we don’t consider the uncertainty of the market clearing price, such an increase in the price can be represented by the black arrow in the DPQC subfigure for a given amount of the bidding quantity. When considering the uncertainty of the price, we can infer that the PDF will generally move right at a certain distance, as shown.
by the green PDF curve. In other sentences, the green PDF curve can be obtained by a simple movement of the blue curve if neglecting the change in the shape of the PDF. Moreover, the degree of the movement can be approximately treated as the value in the step increase of the DPQC for a given amount of the bidding quantity.

Step 2: Cut and combine. In step 1, the green PDF curve is obtained based on an assumption – the price-maker retailer has succeeded in bidding; otherwise, the price-maker retailer will not impact the market price and the PDF of the market clearing price will keep the same. Thus, when constructing the DPQC-based PDF, two situations need to be considered at the same time: succeed or fail in bidding. For a retailer, if its bidding price is higher than or equal to the clearing price, i.e. \( \lambda_b \geq \lambda_c \), it will succeed in bidding and need to move PDF, as shown by the part I of the shadow area in FIGURE 4(b). The second situation is the bidding price is lower than the clearing price, i.e. \( \lambda_b < \lambda_c \), it will fail in bidding, and the need to keep the original PDF, as shown by the part II of the shadow area. Finally, part I and part II together formed the preliminary estimated DPQC-based PDF.

Step 3: Standardize. After step 2, the PDF is reshaped and the integration is probably not equal to 1. To make it a complete PDF, the curve got in step 2 needs to be standardized, as shown in (7) and illustrated by FIGURE 4(c).

\[
f_{\hat{\lambda}_c}(\lambda_c) = \begin{cases} 
\frac{1}{\sqrt{2\pi} \sigma_c} \exp \left( -\frac{(x - \mu_{c,0})^2}{2\sigma_c^2} \right) / P_{sum} & \text{if } \hat{\lambda}_c < \lambda_b \\
\frac{1}{\sqrt{2\pi} \sigma_c} \exp \left( -\frac{(x - \mu_{c,0})^2}{2\sigma_c^2} \right) / P_{sum} & \text{if } \hat{\lambda}_c \geq \lambda_b
\end{cases}
\]  

(7)

where \( \mu_{c,0} \) and \( \mu_{c,1} \) are the expected value of the PDF without and with considering DPQC impacts, respectively.

\[
P_{sum} = \int_{-\infty}^{\lambda_b} f_{p}(\hat{\lambda}_c, \mu_c, \sigma_c) \, d\hat{\lambda}_c,1 \\
+ \int_{\lambda_b}^{\infty} f_{p}(\hat{\lambda}_c, \mu_c, \sigma_c) \, d\hat{\lambda}_c,0 
\]  

(8)

D. ESTIMATING DPQC-BASED PDF WITH MULTI-SEGMENT BIDDING RULE

Similar to the estimating method of the DPQC-based PDF with one-segment bidding rule, the DPQC-based PDF with multi-segment bidding rule can be estimated in the same way. The key difference is that the bidding results in multi-segment bidding have more situations compared to one-segment bidding. Assuming the number of the bidding segment is \( N \), the bidding results can, therefore, be divided into \( N + 1 \) different situations. The first situation is that the bidding prices of all segments are lower than the clearing price and none of the biddings succeeds. The second situation is that the first one bidding segment succeeds while the others fail. The \( n^{th} \) situation is that the first \( n-1 \) bidding segments succeed while the others fail. In different situations, the quantities of successful biddings are different, which leads to different DPQC-based PDF curves, as shown in FIGURE 5 (a). Similar to the 3 steps “Move, Cut and combine, Standardize” in the estimating method of the DPQC-based PDF with one-segment bidding rule, the DPQC-based PDF with multi-segment bidding rule can be estimated as the same way. Assume the \( N + 1 \) bidding prices satisfy that \( \lambda_{b, n+1} \leq \lambda_{b,n}, \text{i.e., } \lambda_{b, 3} \leq \lambda_{b, 2} \leq \lambda_{b, 1} \) when \( N = 3 \). The reshaped DPQC-based PDF after step 2 is illustrated in FIGURE 5 (b). Note that the results after step 3 are not illustrated in this figure. (9) and (10) gives the PDF calculation formula when \( N = 3 \).

\[
f_{p}(\hat{\lambda}_c) = \begin{cases} 
\frac{1}{\sqrt{2\pi} \sigma_c} \exp \left( -\frac{(x - \mu_{c,3})^2}{2\sigma_c^2} \right) / P_{sum} & \text{if } \hat{\lambda}_c \leq \lambda_{b,3} \\
\frac{1}{\sqrt{2\pi} \sigma_c} \exp \left( -\frac{(x - \mu_{c,2})^2}{2\sigma_c^2} \right) / P_{sum} & \text{if } \lambda_{b,3} \leq \hat{\lambda}_c \leq \lambda_{b,2} \\
\frac{1}{\sqrt{2\pi} \sigma_c} \exp \left( -\frac{(x - \mu_{c,1})^2}{2\sigma_c^2} \right) / P_{sum} & \text{if } \lambda_{b,2} \leq \hat{\lambda}_c \leq \lambda_{b,1} \\
\frac{1}{\sqrt{2\pi} \sigma_c} \exp \left( -\frac{(x - \mu_{c,0})^2}{2\sigma_c^2} \right) / P_{sum} & \text{if } \hat{\lambda}_c \geq \lambda_{b,1}
\end{cases}
\]  

(9)

\[
P_{sum} = P_3 + P_2 + P_1 + P_0 \\
= \int_{-\infty}^{\lambda_{b,3}} f_{p}(\hat{\lambda}_c, 3; \mu_c, 3, \sigma_c) \, d\hat{\lambda}_c,3 \\
+ \int_{\lambda_{b,3}}^{\lambda_{b,2}} f_{p}(\hat{\lambda}_c, 2; \mu_c, 2, \sigma_c) \, d\hat{\lambda}_c,2 \\
+ \int_{\lambda_{b,2}}^{\lambda_{b,1}} f_{p}(\hat{\lambda}_c, 1; \mu_c, 1, \sigma_c) \, d\hat{\lambda}_c,1 \\
+ \int_{\lambda_{b,1}}^{\infty} f_{p}(\hat{\lambda}_c, 0; \mu_c, 0, \sigma_c) \, d\hat{\lambda}_c,0 
\]  

(10)

where \( \mu_{c,0} \) is the expected value of the PDF without considering DPQC impacts, and \( \mu_{c,n} \) is the expected value of the PDF of No. \( n \) segment with DPQC impacts.

III. OPTIMAL BIDDING MODEL

A. MATHEMATIC FORMULATION

This model is formulated based on the DPQC-based PDF estimation of the market clearing price described in Section II.B and II.C. The objective of a retailer is to maximize its expected value of profit for selling energy, as shown by (11). The reason to maximize the expected value is that the retailer’s revenue is probabilistic when the market clearing price is probabilistic. Note that the DPQC-based PDF estimation depends on the decision variable – the bidding quantity.

\[
\text{max } EP = \int_{-\infty}^{\infty} PROFIT \cdot f (PROFIT) \cdot d (PROFIT) 
\]  

(11)

where \( EP \) is the expected value of profit, \( PROFIT \) is the profit for selling energy.
The PROFIT is supposed to be calculated by subtracting the purchasing costs in the wholesale market from the expected revenues with the end-users, as shown by (12).

\[
\text{PROFIT} = \sum_{j=1}^{J} RVE_j - PC
\]  

(12)

where \( RVE_j \) is the revenue from the end-user \( j \), and \( PC \) is the purchasing costs in the wholesale market.

Note that although the quantity and the price of the transaction energy with the end-users are often previously agreed in the bilateral contract, the realized transaction quantity and price could be different depending on the bidding results of the retailer. The retailer and the end-users usually make a particular clause in the contract in case of the bidding failures in the wholesale market. Such failures could be failing to bid the whole quantity with the one-segment bidding rule or just failing to bid a few segments with the multi-segment bidding rule. When the retailer fails to buy a certain part of the energy sold, they can turn to other markets for the remaining part, e.g., day-ahead or real-time energy markets if there are. An alternative way is to curtail some load based on the prearranged clauses in the contract, which is similar to demand response. Moreover, they could probably get some compensation depending on the detailed clauses in the contract. For example, they may get a total fixed amount of compensation or a discount coupon for future transactions. For the retailers, they can take advantage of such kind of a clause to make strategic bidding instead of bidding the whole quantity in the wholesale energy market. In this paper, we assume that such compensation is at a fixed price per MWh for simplification. Such an assumption will not impact the validity of the proposed method.

With one-segment bidding rule, the PROFIT is modeled as (13). With one-segment bidding rule, the calculation of PROFIT can be divided into two different situations. The first situation is that the bidding price is lower than the clearing price, in which the retailer fails to buy any electricity in the monthly energy market and has to undertake the loss in compensation to the end-users. The second situation is that the bidding price is higher or equal to the clearing price, in which the retail succeeds to buy all the electricity in the monthly energy market, and the profit can be calculated as the formula (13).

\[
\text{PROFIT} = \begin{cases} 
-Q_s \lambda_{cps} & \text{if } \lambda_b < \lambda_c \\
\sum_{j=1}^{J} Q_{s,j} \cdot \lambda_{s,j} - Q_c \hat{\lambda}_c & \text{if } \lambda_b \geq \lambda_c 
\end{cases}
\]  

(13)

where \( \lambda_b \) is the bidding price of the retailer, \( \hat{\lambda}_c \) is the market clearing price, \( \lambda_{s,j} \) is the selling price to the end-user \( j \), \( \lambda_{cps} \) is the compensation price for the retailer if a certain portion of the energy is not served. \( Q_c \) is the clearing quantity of the energy bidding, \( Q_{s,j} \) is the selling quantity of the energy with the end-user \( j \).

Noted that the sum of the selling quantities with all the end-users equals the total clearing quantity of the energy bidding, thus the (13) can be simplified as (14)-(15).

\[
\text{PROFIT} = \begin{cases} 
-Q_s \lambda_{cps} & \text{if } \lambda_b < \lambda_c \\
Q_b \cdot (\lambda_s - \hat{\lambda}_c) & \text{if } \lambda_b \geq \lambda_c 
\end{cases}
\]  

(14)

\[
\lambda_s = \frac{1}{Q_c} \sum_{j=1}^{J} Q_{s,j} \cdot \lambda_{s,j}
\]  

(15)

where \( \lambda_s \) is the weighted average price of the selling price.

Then, the probability distribution of PROFIT, i.e., \( f(\text{PROFIT}) \), can be obtained by a linear transformation of the probability distribution of \( \hat{\lambda}_c \).

The optimal bidding model is then formulated as (16). The constraints represent that the bidding price has to be not less than the floor price and not larger than the cap price set by the market.

\[
\max \text{ } EP = \int_{-\infty}^{\infty} \text{PROFIT} \cdot f(\text{PROFIT}) \cdot d(\text{PROFIT})
\]

s.t. \( (7) \), \( \lambda_{b,\text{floor}} \leq \lambda_b \leq \lambda_{b,\text{cap}} \)  

(16)

where \( \lambda_{b,\text{floor}} \) and \( \lambda_{b,\text{cap}} \) are the price floor and price cap of the allowable bidding price, which are often set in the market bidding rules.

With multi-segment bidding rule, the PROFIT is modeled as (17), shown at the bottom of the next page. Assume that the number of the bidding segment is \( N \). The calculation of PROFIT can be divided into \( N + 1 \) different situations. The first situation is that the bidding prices of all segments are lower than the clearing price, in which the
In each time of the optimization, the problem will be simplified. To fix $Q_b$, we select the genetic algorithm (GA) to solve this problem.

GA is initially proposed by John Holland in 1960, and the theory is based on the concept of Darwin’s theory of evolution. Afterward, David E. Goldberg extended GA in 1989 [26]. GA is a metaheuristic inspired by the process of natural selection that belongs to the larger class of evolutionary algorithms (EA). Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover, and selection. The flow chart of solving the proposed model is given in FIGURE 6.

IV. CASE STUDIES
This section mainly provides a comparison in terms of profit and bidding strategy obtained by a retailer under non-PQC-based PDF method and the PQC-base PDF method.
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TABLE 1. Case settings.

| Case                | Non DPQC-based | DPQC-based |
|---------------------|----------------|------------|
| 1-segment bidding   | Case 1         | Case 2     |
| 3-segment bidding   | Case 3         | Case 4     |

TABLE 2. Parameters of the simulation.

| Symbol/Unit | RMB/kWh | Symbol/Unit | MWh |
|-------------|---------|-------------|-----|
| $\lambda_{bf}$ | 0       | $g_{b,n}$    | 0   |
| $\lambda_{cu}$ | 0.5     | $g_{c,n}$    | $10^4$ |
| $\lambda_{s}$   | 0.385   | $Q_s$        | $10^4$ |
| $\lambda_{cp}$  | 0       |             |     |
| $\lambda_c$      | N(0.350, 0.034$^2$) | | |

TABLE 3. Parameters of GA.

| Parameter                  | Value  |
|----------------------------|--------|
| Number of populations      | 50     |
| Number of Generations      | 500    |
| Length of the gene (bit)   | 10     |
| Generation gap             | 0.9    |
| Crossover percentage       | 0.7    |
| Mutation percentage        | 0.009  |

FIGURE 7. DPQC parameters.

The former one represents the price-taker retailers and the latter one represents the price-maker retailers. Moreover, some interesting results are discussed.

A. CASE AND DATA SETTINGS

We design four basic cases based on two factors, i.e., the number of bidding segment and whether it is DPQC-based PDF method. The details of the four cases are presented in TABLE 1. Case 1 and Case 3 are with Non-DPQC-based PDF method, and Case 2 and Case 4 are with DPQC-based PDF method. Data for the considered retailer as well as market data are given in TABLE 2, and the parameters of the GA are given in TABLE 3. We assume that the DPQC curve can be estimated, and it is assumed as a six-step curve shown in FIGURE 7.

B. BASIC RESULTS

TABLE 4 presents the results of the four cases. The dash lines “-” indicate the bidding results of the corresponding segments are not unique. From the results, we can see that optimal biddings in the Non-DPQC-based cases (case 1 and case 3) are to bid all the needed quantity ($10 \times 10^4$ MWh in total), and the market clearing prices are 0.345 yuan/kWh. While the optimal biddings in the DPQC-based cases (case 2 and case 4) are to bid fewer ($9 \times 10^4$ MWh in total), and the market clearing prices are 0.340 yuan/kWh. It can be seen that the optimal bidding strategy of the price-maker is to bid less quantity to pull down the market clearing prices ($\hat{\lambda}_c$ in PQC). In case 3 and case 4, they can decrease their bidding quantities by $1 \times 10^4$ MWh and increase the clearing price by 0.01 yuan/kWh, and increase the total expected profit by about 22% finally. In conclusion, the price-maker retailer can better take advantage of its market power with DPQC-based PDF method and make a higher profit compared to the one with non-DPQC-based PDF method.

Furthermore, it can be found some patterns of the optimal bidding strategies from the optimized results. For price-takers, the optimal bidding strategies with both 1-segment and 3-segment bidding rules are to bid at their cost price, i.e., the selling price, and the optimal bidding quantities are the total quantity they need. For price-makers, the bidding price of the first segment tends to be close to the cost price, while the bidding prices of the second and third segments are lower. By such a strategy, they can make a basic-level profit from the first-segment bidding, and make a high-level profit from the second and third-segment biddings.

FIGURE 8, FIGURE 9, and FIGURE 10 present the bidding curves during the iteration process of optimization with the four cases. It can be seen that the results in
one-segment bidding rule (case 1 and case 2) converge within 50 iteration times, and the results are unique. As to case 3 and case 4, the objective function, i.e., the expected profit, converges within 200 times. However, the price bidding curves and quantity bidding curves are not entirely converged, indicating that the optimized results are not unique, which are consistent with the results in TABLE 4. In detail, in case 3, the quantities of each segment are not converged. In case 4, the first segment is nearly converged to $3.42 \times 10^4$ MWh, and the summary of the second and third segments is nearly converged to $5.58 \times 10^4$ MWh. When focusing on the bidding prices, all three segments in case 3 converge to 0.385 yuan/kWh. In case 4, the first segment is nearly converged to 0.387 yuan/kWh, and the second and third segments converge to 0.364 yuan/kWh. The non-uniqueness of the bidding quantities is because the bidding prices of the corresponding segments are the same. In such a situation, the increase or decrease of bidding quantities will not affect the expected profit on condition that their summary stays the same. Therefore, the optimized results of some segments are not unique.

### TABLE 5. Sensitivity analysis of the price distribution (case 2).

| $\lambda_c$ (yuan/kWh) | $Q_c$ (10^4 MWh) | $\lambda_s$ (yuan/kWh) | $EP$ (10^6 yuan) | $\hat{\lambda}$ in PQC (yuan/kWh) |
|-------------------------|-----------------|--------------------------|-----------------|----------------------|
| (0.350, 0.01)           | 9.0             | 0.333                    | 4.47            | 0.340                |
| (0.350, 0.02)           | 9.0             | 0.354                    | 4.46            | 0.340                |
| (0.350, 0.03)           | 9.0             | 0.361                    | 4.57            | 0.340                |
| (0.350, 0.03^4)         | 9.0             | 0.365                    | 4.64            | 0.340                |
| (0.350, 0.04)           | 9.8             | 0.365                    | 4.76            | 0.340                |
| (0.350, 0.05)           | 9.8             | 0.370                    | 5.01            | 0.348                |

### TABLE 6. Sensitivity analysis of the price distribution (case 4).

| $\lambda_c$ (yuan/kWh) | $Q_c$ (10^4 MWh) | $\lambda_s$ (yuan/kWh) | $EP$ (10^6 yuan) | $\hat{\lambda}$ in PQC (yuan/kWh) |
|-------------------------|-----------------|--------------------------|-----------------|----------------------|
| (0.350, 0.01^4)        | 3.42            | -                        | -               | -                    |
| (0.350, 0.02^4)        | 3.42            | -                        | -               | -                    |
| (0.350, 0.03^4)        | 3.60            | -                        | -               | -                    |
| (0.350, 0.03^4)        | 3.42            | -                        | -               | -                    |
| (0.350, 0.04^4)        | 3.45            | -                        | -               | -                    |
| (0.350, 0.05^4)        | 3.58            | -                        | -               | -                    |

### C. SENSITIVITY ANALYSIS

The sensitivity analysis of the price distribution, selling price, and compensation price are provided in this section. The results of different PQCs are compared as well. The bold data is the case in the base case.

1. **Sensitivity analysis of the distribution of the original market clearing price ($\lambda_c$).** TABLE 5 and TABLE 6 present the bidding results of different standard deviations of $\lambda_c$ with one-segment and 3-segment bidding rules, respectively. A higher standard deviation indicates a higher uncertainty of the market clearing price. The results show that with both 1-segment and 3-segment bidding rules, the optimal bidding strategy of a price-maker is to increase the total bidding quantity to hedge the price uncertainty, and the advantage of the market power becomes less obvious. Meanwhile, the optimal bidding strategy with one-segment bidding rule is to increase the bidding price. With 3-segment bidding rule, the optimal bidding strategy is to decrease the bidding price of the first segment a little bit while increasing the bidding prices of the second and third segments. In other words, the results show that the gaps of the bidding prices between different segments become smaller with higher price uncertainty.

2. **Sensitivity analysis of the selling price ($\lambda_s$).**

   TABLE 7 and TABLE 8 present the bidding results of selling prices with one-segment and 3-segment bidding rules, respectively. Higher selling prices indicate less unit cost of the retailer. Therefore, the retailer can increase the bidding price to increase the probability of successful bidding and increase the expected profit at the same time. As to the bidding quantity, the results show that the bidding quantities are not sensitive to the selling price.

3. **Sensitivity analysis of the compensation price ($\lambda_{csp}$).**

   TABLE 9 and TABLE 10 present the bidding results of compensation prices with one-segment and 3-segment bidding rules, respectively. A higher compensation price indicates...
TABLE 7. Sensitivity analysis of the selling price (case 2).

| $\lambda_5$ (yuan/kWh) | $Q_y (10^4$ MWh) | $\lambda_6$ (yuan/kWh) | $EP (10^6$ yuan) | $\hat{\lambda}$ in PQC (yuan/kWh) |
|------------------------|------------------|------------------------|------------------|--------------------------|
| 0.320                  | 9.0              | 0.321                  | 0.66             | 0.340                    |
| 0.340                  | 9.0              | 0.338                  | 1.53             | 0.340                    |
| 0.360                  | 9.0              | 0.352                  | 2.77             | 0.340                    |
| 0.380                  | 9.0              | 0.361                  | 4.25             | 0.340                    |
| 0.385                  | 9.0              | 0.365                  | 4.64             | 0.340                    |
| 0.4                    | 9.8              | 0.369                  | 5.92             | 0.348                    |

TABLE 8. Sensitivity analysis of the selling price (case 4).

| $\lambda_5$ (yuan/kWh) | $Q_y (10^4$ MWh) | $\lambda_6$ (yuan/kWh) | $EP (10^6$ yuan) | $\hat{\lambda}$ in PQC (yuan/kWh) |
|------------------------|------------------|------------------------|------------------|--------------------------|
| 0.320                  | 2.90 - - - 9.0   | 0.322                  | 0.322            | 0.321                    |
| 0.340                  | 3.20 - - - 9.0   | 0.339                  | 0.339            | 0.337                    |
| 0.360                  | 3.50 - - - 9.0   | 0.359                  | 0.359            | 0.349                    |
| 0.380                  | 3.80 - - - 9.0   | 0.380                  | 0.380            | 0.360                    |
| 0.385                  | 3.40 - - - 9.0   | 0.387                  | 0.387            | 0.364                    |
| 0.4                    | 3.40 - - - 10.0  | 0.392                  | 0.392            | 0.365                    |

TABLE 9. Sensitivity analysis of the compensation price (case 2).

| $\lambda_{cpr}$ (yuan/kWh) | $Q_y (10^4$ MWh) | $\lambda_6$ (yuan/kWh) | $EP (10^6$ yuan) | $\hat{\lambda}$ in PQC (yuan/kWh) |
|-----------------------------|------------------|------------------------|------------------|--------------------------|
| 0                           | 9.0              | 0.365                  | 4.64             | 0.340                    |
| 0.01                        | 9.8              | 0.367                  | 4.48             | 0.348                    |
| 0.02                        | 9.8              | 0.370                  | 4.36             | 0.348                    |
| 0.03                        | 10.0             | 0.372                  | 4.29             | 0.350                    |
| 0.04                        | 10.0             | 0.374                  | 4.21             | 0.350                    |
| 0.05                        | 10.0             | 0.377                  | 4.14             | 0.350                    |

TABLE 10. Sensitivity analysis of the compensation price (case 4).

| $\lambda_{cpr}$ (yuan/kWh) | $Q_y (10^4$ MWh) | $\lambda_6$ (yuan/kWh) | $EP (10^6$ yuan) | $\hat{\lambda}$ in PQC (yuan/kWh) |
|-----------------------------|------------------|------------------------|------------------|--------------------------|
| 0                           | 3.40 - - - 9.0   | 0.387                  | 0.364            | 0.364                    |
| 0.01                        | 3.60 - - - 9.8   | 0.399                  | 0.365            | 0.365                    |
| 0.02                        | 3.59 - - - 10.0  | 0.403                  | 0.363            | 0.363                    |
| 0.03                        | 3.58 - - - 10.0  | 0.411                  | 0.368            | 0.368                    |
| 0.04                        | 3.60 - - - 10.0  | 0.423                  | 0.374            | 0.374                    |
| 0.05                        | 3.60 - - - 10.0  | 0.477                  | 0.373            | 0.373                    |

FIGURE 11. DPQC parameters for sensitivity analysis.

D. COMPUTATION EFFICIENCY

The experiments are performed on a PC with Intel(R) Core(TM) i5-7400 3.00 GHz and 8 GB of memory. The algorithms are implemented in MATLAB. The optimality gap is set as 1e-6. The average solution time for each iteration is 0.0085s for case 1, 2 and 3, and 0.0141s for case 4.

V. CONCLUSIONS

This paper proposes an optimal bidding method of price-maker retailers in the electricity market with DPQC-based PDF estimation of the market price. The main conclusions are:

1. We propose a DPQC-based PDF method to reflect the price impacts from the bidding behavior of the price-maker retailers and consider the price uncertainty at the same time. Moreover, we build up an optimal bidding model for retailers in China’s monthly energy market. Both the one-segment and multi-segment bidding rules are considered for the pay-as-clear market.

2. The case study shows that the proposed method can help the price-maker retailers better to consider the price impacts from their bidding behaviors. In detail, the optimal bidding strategy of the price-maker is to bid appropriate few quantities to pull down the market clearing prices. Meanwhile, the optimal bidding price is to bid close to the cost price for the first segment and to bid lower prices for the second and third segments with 3-segment bidding rule. By such a strategy, they can make a basic-level profit from the first-segment bidding, and make a high-level profit from the second and third-segment biddings.

3. In multi-segment bidding, it is common to see the non-uniqueness of the bidding results when the bidding prices of some segments are the same.

The proposed method assumes that the DPQCs can be estimated and the uncertainty and the market clearing price satisfies the normal probability. To estimate the DPQCs and the consider a realized probability distribution will be our future work.
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