Motion coordination for humanoid jumping using maximized joint power

Xuechao Chen¹, Wenxi Liao², Zhangguo Yu¹, Haoxiang Qi², Xinyang Jiang² and Qiang Huang¹

Abstract
Jumping capability of humanoid robots can be considered as one of the cruxes to improve the performance of future humanoid robot applications. This paper presents an optimization method on a three-linkage system to achieve a jumping behavior, which is followed by the clarification of the mathematical modeling and motor-joint model with practical factors considered. In consideration of the constraints of ZMP and the performance of the motor, the output power of the joint motors is maximized as much as possible to achieve a higher height. Finally, the optimization method is verified by the simulation and experiment. Different from other electric driven robots, which take the output power of the joint as the constraint, we maximize the output power of the joint to optimize the hopping performance of the robot. Realizing dynamic jumping of humanoid robots can also provide a solid foundation for further research on running, which can greatly enhance the environmental adaptability.

Keywords
Humanoid robots, vertical jump, motion control, optimization, electric actuator

Introduction
In the past few decades, the research of humanoid robots has made breakthroughs in stable walking and dexterous operation.¹–⁶ Lacking of diversified motion mode, humanoid robots are not qualified for operating in intricate occasion. For instance, without jumping ability, the robot will not able to pass through the discontinuous road. As for the aspect of the jumping motion, humanoid robots are still lagging behind, while jumping is one of the basic locomotion abilities of human beings. By means of jumping, human beings can avoid obstacles and adapt to discontinuous complex terrain promptly. Analogously, humanoid robots capable of jumping ability as human beings will greatly improve their adaptability to complex environments, thereby further promoting humanoid robots’ practical process. Hence, researchers have focused their interest intensely on multifarious jump in recent years.⁷–¹⁰

Many research works devoted to jumping robots have been reported. Some of them used hydraulic driving or pneumatic driving to enhance the actuating force of the robot. Raibert’s team from MIT studied hopping with hydraulically driven leg robots.¹¹ Based on the former, the Atlas robot developed by Boston Dynamics is capable of many motion tasks.¹²,¹³ Hosoda from Osaka University developed a lower limb jumping mechanism composed of pneumatic bionic muscles.¹⁴ Although

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hydraulic driving and pneumatic driving offer some advantages, there are still some issues to deal with. The hydraulic system is relatively complicated and heavy, which may cause hydraulic oil leakage accidents. Moreover, the sensitivity to oil temperature makes it a challenge to control stability. The drawbacks of the latter, pneumatic driving, are the low transmission efficiency and high noise.

Part of researches investigated the method by installing elastic energy storage elements to pre-store energy for jumping. Niiyama et al. added springs to the legs of the Mowgli robot as elastic links to achieve a jump of more than twice its own height in 2007. Curran and Orin and Ugurluet al. focused on the jumping mechanism with spring and other energy storage elements and its control strategy. These researches are to improve the jumping performance of robot through bionic mechanical design. Elastic element has merits in terms of energy storage and release, while it also brings certain limitations to other motion tasks.

Some research realized jumping by strengthening hardware. Okada and Takeda developed a design method of nonlinear profile of gear ratio to utilize a DC servo motor effectively for a jumping robot. The HRP3L-JSK humanoid robot developed by Tokyo University in Japan enhanced the joint driving ability of the motor to achieve a jump of more than twice its own height in 2007. Curran and Orin and Ugurlu et al. focused on the jumping mechanism with spring and other energy storage elements and its control strategy. These researches are to improve the jumping performance of robot through bionic mechanical design. Elastic element has merits in terms of energy storage and release, while it also brings certain limitations to other motion tasks.

Other research used the method of planning the trajectory for the humanoid robot to jump. Victor et al. presented a control method for planar vertical jump in 2005, which imposes the trajectory following for the distance between the foot and the total center of mass and maintains the CoM above the feet during the whole jump. Sakka and Yokoi dealt with the generation of motion pattern for humanoid robots’ vertical jump in 2006. The study concentrates on the landing phase of the jump, assuming the characteristics of the aerial phase are known. Based on previous work, they proposed a method for adapting human vertical jumping dynamics to humanoid robotic structure using precomputed reference trajectory. Taking the ground reaction force (GRF) as the model reference and bodies inertial forces as task constraints while optimizing energy to determine the humanoid robot posture and improve its jumping performance. Hong realized vertical jump on the Darwin-OP simulation model through CPG and evolutionary optimization. However, it is difficult to apply the method of small-scale robot directly to large-scale humanoid robot platform. Cho and Oh and Cho et al. from KAIST University designed PD position controller and attitude balance controller to realize the jumping of HUBO2 robot by planning the trajectory of the robot’s feet. In this way, HUBO2 can resist large disturbance in the jumping process, but the jumping height is not obvious. Most of the above methods only take the output power of the joint as the constraint, and we maximize the output power of the joint to optimize the jumping performance of the robot.

Their work has certain significance for the research of modeling and control of jumping motion of humanoid robots, but the issue of motor output is not considered. In this paper, we will aim to maximize the output of the motor, while ensuring the stability of the take-off process to achieve effective jumping. For the process of jump, the critical part that needs to be controlled is the trajectory of the robots in the take-off phase and the initial velocity of the flight phase, while the latter is crucial to determine the maximal height that robot can reach. Certainly, the output power of the actuators is roughly proportional to the maximal height when the energy consumption rate is constant. In summary, a coordinated take-off is extremely vital for the whole jumping process.

In our previous work, we proposed a jumping planning method for robot with ideal motor model and did some simple simulation of vertical jump. Based on the previous work, this paper makes a more accurate model between the motors and the joints, rather than the ideal model previously assumed. In this paper, we simplify the BHR-6 humanoid model and obtain the simplified structure. On this basis, the kinematics and dynamics of the mathematical model and the joint model are illustrated. In terms of the aspect of planning trajectory, starting from the joint space, multiple joints are allowed to work together with the maximum torque allowed by the motor to achieve an effective vertical jump. Meanwhile, taking account of the constraint of ZMP point (zero moment point) on the soles of robot’s feet, the joint torque of robot is fine-tuned to ensure the dynamic stability of robot during take-off phase and achieve joint coordination. Experiment results are proposed to prove the effectiveness of the trajectory. From our perspective, this study is interesting for the modeling and trajectory planning of the humanoid robot vertical jump.

The remainder of this paper is organized as follows. In Section 2, we present reasonable assumptions and construct suitable simplified structure for the vertical jump of the humanoid, followed by the mathematical modeling and the joint model which considers the reducer efficiency, joint friction, and damping. Considering the constraints in hardware and physical, the trajectory planning method to realize vertical jump is clarified in
Section 3. We verify the validity of the proposed method in the experiment in Section 4. Finally, conclusions and perspectives are addressed in Section 5.

**Modeling**

In this section, we first make some assumptions for the vertical jump of humanoid robot and further simplify the BHR-6 robot’s structure for our analysis. (BHR-6 is the sixth generation of Beijing Institute of Technology Humanoid Robots.) Then, the mathematical model and the motor model in consideration of factors of reducer efficiency, joint friction, and damping are illustrated.

**Assumptions and Simplified structure**

For better modeling of the vertical jump of the humanoid robot, this paper puts forward several reasonable assumptions as follows:

1. (AS1) The whole vertical jump process is completed in one plane.
2. (AS2) The robot is completely symmetric with respect to the sagittal plane.
3. (AS3) In the take-off phase, there is no slipping between the robot’s feet and the ground. (This is satisfied when the ground friction constraint and ZMP constraint are satisfied, which will be discussed in detail later.)
4. (AS4) Each joint is individually driven by a motor.

According to AS1-2, the BHR-6 model can be simplified to the sagittal plane model. Thus, as shown in Figure 1, the simplified structure of an open-loop kinematic chain composed of four links and three joints are obtained. $Link_i$ ($i = 0, 1, 2, 3$) is the link of foot, shank, thigh, and the upper body of the humanoid robot, respectively. $d_i$ ($i = 0, 1, 2$) denotes the length of the foot, the horizontal distance from the ankle to the heel and the toe, respectively. $l_0$ is the distance from the ankle to the sole of the foot, and $m_0$ is the mass of the foot. $l_i$ ($i = 1, 2, 3$), $m_i$ ($i = 1, 2, 3$) represents the length and the mass of $Link_i$, respectively. The CoM of each link is located at its geometric center. As a sagittal model, $m_3$ is half the mass of the actual robot’s upper body.

**Mathematical modeling**

In the take-off phase of the vertical jump of the robot, since there is no relative movement between the foot and the ground, the simplified model of jump movement in Figure 1 can be further simplified to a three-linkage inverted pendulum fixed on the ground.

As shown in Figure 1, the origin of the world coordinate system $O - XY$ is established at the robot’s ankle joint, with the $Y$-axis vertically upward and the $X$-axis pointing to the front and remaining horizontal in the sagittal plane of the robot. According to the coordinate system $O - XY$, the generalized coordinate system $Q_i = (q_{1,i}; q_{2,i}; q_{3,s})$, as shown in Figure 1, is defined, and counterclockwise is positive. Defined by the coordinate system, $q_{2,i}$ is positive, $q_{1,i}$ and $q_{3,i}$ are negative, and the initial state of the robot is vertical standing state.

According to the Lagrange equation, the dynamic equation of the take-off phase is expressed as equation (1):

$$
D_s(Q_s)\dot{Q}_s + C_s(Q_s, \dot{Q}_s) \dot{Q}_s + G_s(Q_s) = B_su_s
$$

Where $D_s$ is the inertia matrix, $C_s$ is the Coriolis and centrifugal force term, and $G_s$ is the gravity term in the generalized coordinate system $Q_s$. $u_s = (\tau_{1,s}; \tau_{2,s}; \tau_{3,s})$ represents the torque of the ankle, knee, and hip, respectively. $B_s$ is the transformation matrix between the relative coordinate $Q_{rel,s} = (q_{1,s}; q_{2,s}; q_{3,s})$ and the generalized coordinate $Q_s$, as equation (2):

$$
B_s = \frac{\partial Q_{rel,s}}{\partial Q_s}
$$

Due to the factors of reducer efficiency, joint friction, and damping, the relationship between joint end torque and motor end torque is not simply multiplied by the deceleration ratio. After precise modeling of the motor model, the relationship between the motor torque and the joint torque is obtained as shown in equation (3).

$$
\begin{align*}
\tau_m &= K_i\dot{i}_c - J_i\dot{\theta}_m - b\dot{\theta}_m, \\
\tau_j &= N\tau_m - \tau_j, \\
\tau_f &= F_c\dot{i}_c + F_s\text{sgn}(\dot{\theta}_j),
\end{align*}
$$

Where $\tau_m$, $\tau_j$, $\tau_f$ represent the torques of the motor, joint, and reducer, respectively. $K_i$, $J_i$, and $b$ denote the torque constant, the moment of inertia, and damping coefficient of the motor, respectively. $F_c$ and $F_s$ are the friction coefficients of the reducer. $\theta_m$ and $\theta_j$ are the speeds of the motor and the joint, respectively. $i_c$ is the current and $N$ is the gear ratio.

To determine the time of the robot leaving the ground, it is necessary to calculate the acceleration of the CoM of the robot in the vertical direction. When it is equal to $-g$, there is no longer any interaction between the robot’s foot and the ground, that is, the robot enters the air.

From Figure 1, the position of the CoM of the robot in the take-off phase can be calculated as $P_{CoM,s} = (c_z,s; c_y,s)$. Furthermore, the acceleration of
the CoM is obtained by second derivative as equation (4):

\[ \ddot{P}_{\text{CoM},s} = \frac{d}{dt} \left( \frac{\partial P_{\text{com},s}}{\partial \dot{Q}_s} \right) \dot{Q}_s + \frac{\partial P_{\text{com},s}}{\partial Q_s} \ddot{Q}_s \]  

(4)

**Method**

In the take-off phase, the CoM of the humanoid robot is expected to be accelerated by high torque of the joints, and at the same time obtains a high initial speed for the flight phase.

In this section, based on the three-linkage inverted pendulum model of the take-off phase proposed in Section 2, the motor at the joints can do work sufficiently as far as possible to achieve stable and efficient vertical jump under the conditions of robot ZMP constraints and others.

**Constraints**

**Ground friction constraint.** We have presented the assumption that there is no slipping between the foot and the ground (AS3). The premise of this assumption is that the physical constraints of ground friction need to be met, that is, the force applied to the soles of the foot must meet the constraints of the ground friction cone. Supposing the friction coefficient of the ground is \( \mu \), the horizontal and vertical forces on the soles of the foot are \( F^T \) and \( F^N \) respectively, and the relations between them should be as equation (5):

\[ \frac{F^T}{F^N} \leq \mu; \quad (F^N > 0) \]  

(5)

**Motor constraint.** Like most humanoid robots, BHR-6 is equipped with the same Parker motor (K044100-6Y-4.98Ams) in the ankle, knee, and hip joint. In the take-off phase, with the increase of the joint speed, the torque provided by the motor to the joints is constrained by the motor properties, which means that the sustained torque and peak torque provided by the motor at a certain speed are limited. Since the robot takes off in a very short time, only \( 0.15 - 0.45s \), which is far less than the time allowed by the motor under the peak condition, the peak torque speed curve of the motor is adopted as the constraint curve of the motor properties of vertical jump movement. The motor properties curve is shown below.

According to the motor manual, the function relationship between the peak torque of the motor and the speed can be established, and the value of the peak torque \( \tau^\text{max}_m \) that the motor can output at different motor speeds \( \theta_m \) can be calculated, as shown in equation (6), see Figure 2 below

\[ \tau^\text{max}_m = f(\theta_m) \]  

(6)

Neglecting the factors of reducer efficiency, joint friction, and damping, if the reducer with a deceleration ratio of \( N \) is configured between the motor and the joint, then the rotational speed \( \theta_j \) of the joint and the
rotational speed $\theta_m$ at the motor have a relationship as shown in equation (7).

$$\theta_m = \theta_j N$$

(7)

According to equation (3), the torque $\tau_j$ of the joint and the torque $\tau_m$ at the motor have a relationship as shown in equation (8).

$$\tau_j = \tau_m N - \tau_f = g(\theta_j N)$$

(8)

**ZMP constraint.** In order to ensure that the soles of the robot remain relatively fixed with the ground during the take-off process, the ZMP range of the robot should be limited to prevent the soles from overturning. The position of ZMP $P_{ZMP} = (p_x, p_y)$ of the robot can be calculated from the position of the CoM $P_{CoM} = (c_x, c_y)$ and the equation of the velocity and acceleration (equation (4)) of the CoM, as equation (9).\[29\]

$$P_{ZMP} = \left(\begin{array}{c} p_x \\ p_y \end{array}\right) = \left(\begin{array}{c} c_x - \frac{(c_x - p_x)c_x}{c_y + g} \\ 0 \end{array}\right)$$

(9)

In the high-speed motion mode, the ZMP point of the robot may be close to the edge of the sole of the foot, which may cause the robot to be prone to toppling. For the sake of the dynamic stability of the robot in the take-off phase, the position of ZMP of the humanoid robot needs to be constrained. Figure 3 is the top view of the robot, point $O$ is the projection of the ankle joint, and $O - XZ$ is the reference coordinate system of the fixed base. In the figure, the red shaded area is the ZMP restricted area, that is, the ZMP point needs to be within the shaded range, which can be expressed as equation (10). Once the ZMP point is out of range, an adjustment amount $\Delta p_x$ is applied to it so that it can return to the ZMP constraint zone again.

$$- d_1 + \Delta p_x \leq p_x \leq d_2 - \Delta p_x; \Delta p_x > 0$$

(10)

Where $\Delta p_x$ is the stability margin constant of ZMP.

**Trajectory planning strategy**

As mentioned in Section 2, in take-off phase, the CoM of the robot accelerates rapidly until the acceleration equals $g$ in the vertical direction, which means the robot enters flight phase. On account of the fixed third-order inverted pendulum model, the robot can’t leave the ground. When the robot has a tendency to leave the ground, it will be pulled downward by the ground. Therefore, the jump determination condition of the robot is equation (11).

$$\ddot{c}_y, s \leq -g$$

(11)

From the velocity $\dot{c}_y, \text{takeoff}$ of the vertical direction at the instant of the robot leaving the ground, the maximal height reached can be calculated as shown in equation (12).

$$h = \frac{(\dot{c}_y, \text{takeoff})^2}{2g}$$

(12)

The position of ZMP can be expressed as a function of joint angular acceleration $\dot{q}$. Thus, the total differential of ZMP can be calculated to get the following relationship between the variation of ZMP $\delta p_x$ and the adjustment quantity of joint angular acceleration $\delta q$:

$$\delta p_x = \sum_{i=1}^{3} \left(\frac{\partial p_x}{\partial q_i}\right) \delta q_i$$

(13)

When the current ZMP exceeds the constraint, it can be changed by joint angular acceleration $\dot{q}$, as shown in Figure 3. According to equations (10) and (13), the
adjustment quantity $\delta\dot{q}$ should satisfy the following equation.

$$
-d_1 + \Delta p_x - p_x \leq \sum_{i=1}^{3} \left( \frac{\partial p_x}{\partial q_i} \delta \dot{q}_i \right) \leq d_2 - \Delta p_x - p_x
$$

(14)

For greater energy output from each joint of the robot in the take-off phase, maximal torque will be provided to each joint as much as possible under every constraint. The relationship between the variation of joint torque $\delta\tau$ and the adjustment of joint angular acceleration $\delta\dot{q}$ is shown in equation (15).

$$
\delta\tau = D_i(q)\delta\dot{q}
$$

(15)

Due to the motor constraint, the variation of joint torque $\delta\tau$ should be a reduction contrary to the peak torque provided, which can be expressed as equation (16).

$$
|\delta\tau + \tau| \leq |\tau|
$$

(16)

Where $\tau$ is the maximal allowable joint torque of the motor calculated from equation (8).

Since $\delta\tau$ is the reduction of each joint torque, $|\delta\tau|\dot{q}$ is the reduction of each joint power. According to the total power reduction of three joints, the following cost function is constructed. The smaller the value of equation (17) is, the more energy the robot consumes in vertical jump, and the higher the maximal height it can reach.

$$
J(\delta\dot{q}) = |\delta\tau|\dot{q}
$$

(17)

As shown in the control flow chart in Figure 4, the optimal adjustment of the angular acceleration of joints will be calculated to minimize the cost function (17) and satisfy the ZMP constraint equation (14) and motor constraint equation (15) at the same time.

### Experiments and results

With the three constrains described in the previous section, the trajectory of the take-off phase is obtained using maximized joint power. To verify the effectiveness of the vertical jump trajectory of humanoid robot proposed in this paper, verification needs to be carried out through the simulation and experiment. Due to the dozens of DOF (Degree of Freedoms) and multiple sensors of the humanoid robot, which are expensive to manufacture and difficult to maintain, in this section, we firstly use a combination of MATLAB and V-rep dynamics simulation software for simulation. Then, we do confirmatory experiment on a simple three-link platform.

In the simulation, when the motor model is assumed as ideal, the BHR-6 model can jump on a vertical barrier of 0.6 m. In consideration of reducer efficiency, joint friction and damping factors, the precision motor-joint model is adopted according to equation (3). As a result, the robot can jump on a 0.2 m high platform. After that, a simple jump experimental platform is designed and made for preliminary experiment with a control system based on the EtherCAT bus. Then, in the experiment, the effectiveness of the motion coordination strategy proposed is verified.

### Simulation

In this section, MATLAB and V-rep are adopted to establish the dynamic simulation environment. The controller is written in MATLAB in terms of the control flow chart in Figure 4. Considering the simplified
model of jump movement model in Figure 1, the BHR-6 model is built in V-rep. The dynamic parameters of simplified BHR-6 model are set as shown in Table 1. In addition, the foot parameters of BHR-6 model are shown in Table 2. Every period, MATLAB Controller sends a set of joint control parameters to V-rep for real-time simulation, and gets the joint state parameters returned.

Table 1. The dynamic parameters of simplified BHR-6 model.

| Link   | Length (m) | CoM (m) | Mass (kg) | Inertia (kgm²) |
|--------|------------|---------|-----------|----------------|
| Link₁  | 0.33       | 0.148   | 5.32      | 0.05547        |
| Link₂  | 0.33       | 0.157   | 5.48      | 0.05216        |
| Link₃  | 0.73       | 0.363   | 14.84     | 0.68425        |

Table 2. The foot parameters of BHR-6 model.

| Foot Length | d₀ (m) | Distance from ankle to heel d₁ (m) | Distance from ankle to toe d₂ (m) |
|-------------|--------|-----------------------------------|----------------------------------|
| 0.30        | 0.10   | 0.20                              |

Figure 5. Simulation of BHR-6 model: (a) jumping onto a 0.6 m step and (b) jumping onto a 0.2 m step.

The BHR-6 model is transformed from a standing upright position to a squat initial take-off posture through the PVT interpolation. According to the control flow chart that we mentioned before, the motion trajectory of the model of every period can be obtained.

As shown in Figure 5, ignoring the factors of reducer efficiency, joint friction, and damping, the robot can
jump onto a 0.6m platform. When precise joint model is adopted, the height is reduced to 0.2m.

The joints angle curve of the simulation during the take-off phase of the jump is shown in Figure 6, and the velocity change of the CoM is shown in Figure 7. Among them, “ideal” refers to the ideal motor-joint model; “precise” refers to the precise motor-joint model. In both figures, the curves overlap in the first 3.4s because the same PVT interpolation method is used to transform the robot from a standing position to a take-off preparation posture. This is a period of preparation, while the time between 3.4s and 4s is the take-off phase. During the period of latter, the robot joints start to accelerate to push up the CoM. While the CoM velocity reaches the maximum, the angular velocity of the joints is also the largest. At this time, the robot has taken off, and the joint will gradually decelerate to zero to facilitate the next adjustment.

It can be inferred from Figure 7 that when the ideal motor model is adopted, the robot’s velocity at the take-off time is 3.193m/s; when the precise motor model is adopted, the take-off velocity is decreased to 1.859m/s. According to equation (12), the maximum height that the robot can reach in two cases is 0.52m and 0.176m, respectively. The data of latter will be further validated in the experiment. In the simulation, we adjust the hip of the robot after taking off and retract its legs to jump up the steps.

In the simulation, joint velocity of the take-off phase increases monotonously, and the curve is shown in Figure 8. At the same time, the curve of joint torque versus the joint velocity in this phase is shown in Figure 9, which is directly connected to the motor characteristics. Along the direction of speed change, the area enclosed by the torque curve and the horizontal coordinate axis of the joint velocity is the output power by the motor. It can be clearly seen that the surrounding area of the curve corresponding to the ideal motor model is larger than that of the precise motor model, which means the output power of the ideal motor model during the take-off phase is obviously greater than that of the precise motor model. This is mainly attributed to the different actual loss of the precise motor model.

In this process, the output power of the precise motor-joint model is smaller than ideal model, which is confirmed by the Figure 9. Owing to this, the robot takes a longer time to take off, which is confirmed by the Figure 6. On the other hand, the maximum vertical velocity of the CoM is also smaller, which is confirmed by the Figure 7. For the same reason, the joint angle of precise model changes less than the ideal model, which is confirmed by the Figure 8.
Experiment

The experiment of the vertical jump is carried out on a simple jump experimental platform, which was built in terms of BHR-6 simplified structure. The mechanical structure drawing of it is shown in Figure 10. Parker motors K044100-6Y with reduction ratio of 100 is installed in each joint. To enhance the structural strength of the robot, structural reinforcement parts are added between the left and right leg plates. Meanwhile, the arc design on the leg board improves the motion range of joints. The parameter of the simple humanoid robot jump experimental platform is shown in Table 3.

The control system needs to coordinate multiple motors to work together. At the same time, in order to ensure that the trajectory executed by the joint meets the desired dynamic characteristics, the control period needs to be as short as possible. Therefore, the EtherCAT bus network is adopted on the simple experiment platform for jumping motion, and the Platinum Maestro multi-axis controller is used as the master station to convert the control instructions into joint control instructions and send them to each Gold Twitter drive. Adopting distributed control system, the driver itself controls the position loop, speed loop, and current loop, and returns the sensor information requested by the multi-axis controller for further transmission to the PC for motion control and planning. In the end, the system achieved a control period of 1 ms with a period error < 1μs, which met the real-time requirements of the experimental robot for motion control.

Based on the simple jump experiment platform of humanoid robot, the take-off trajectory of the robot is optimized in light of control flow chart in Figure 4.
Figure 11 shows the whole process of the robot jumping. In the experiment, the robot first moves to the initial state of the take-off phase by PVT interpolation. Figure 11 (1) shows the initial state. Then the robot follows the planned joint trajectory to achieve the vertical jump. In Figure 11 (4), the robot is about to leave the ground. After that, the robot enters to the air. From Figure 11(6), it can be seen the robot eventually reach the height of about 0.16 m (in the experiment, the projection point of the ankle on the sole of the foot is used as the measurement point of the height of the robot).

| Table 4. The experiment results of vertical jump. |
|-----------------------------------------------|
| Lift time (s) | Theoretical Height (m) | Actual Height (m) |
|---------------|------------------------|------------------|
| 0.246         | 0.1760                 | 0.16             |

The whole process takes 0.246 s. Figure 12 shows the changes of joint angles during the whole process. The “desired” curve is the planned trajectory, while the “actual” is obtained from real experiments. The two curves are roughly consistent. It can be inferred from Figures 11 and 12 that the initial hip angle of the robot is relatively large, so the hip can do work sufficiently to help the robot off the ground. The experimental results are shown in Table 4. In the process of vertical jump, the error between the actual and expected angles of the three joints is small, so the height is only 0.016 m lower than the simulation value, which shows the effectiveness of the planned trajectory of humanoid robot proposed in this paper.

Conclusion

Humanoid robot has important scientific research significance and practical value. However, the weak adaptability to complex environment restricts the practical process of humanoid robot. Currently, most rigid humanoid robots driven by electricity are difficult to achieve natural and effective jump, which makes them cannot adapt to complex environments such as ditches and platforms. Therefore, this paper presents a trajectory planning approach to make humanoid robot perform vertical jump.

Firstly, a dynamic model of third-order inverted pendulum is established based on the BHR-6 model. Considering factors of reducer efficiency, joint friction and damping, a precise joint model is built. Then, with three constraints for the jump motion put forwarded, the cost function for maximizing the power output of joints is developed. Finally, our planned trajectory is validated on simulation and experiment. From the results that the robot takes 0.246 s to reach a height of 0.16 m in the experiment, we can infer that it is beneficial to consider the joint model for precise motion results in dynamic jumping.

The contributions of this paper are summarized as follows:

1. Considering factors of reducer efficiency, joint friction and damping, a precise joint model for the humanoid jumping is developed.
2. Combined with physical constraints, motor constraints, and ZMP constraints, a planning
strategy to maximize the joint power during the jump is proposed.

(3) A simple experimental platform for humanoid robot jumping is designed and made, in which the method proposed in this paper is verified.

In the future, we will consider of the whole-body dynamics to implement the algorithm for bipedal robot and validate the theory on BHR-6 robot. In addition, we will also try to develop the algorithm of continuous jumping motion, which would be preliminary research for the development of robot’s running.

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References
1. Hirose M and Ogawa K. Honda humanoid robots development. Philos Trans A Math Phys Eng Sci 2007; 365(1850): 11–19.
2. Huang Q, Yokoi K and Kajita S. Planning walking patterns for a biped robot. IEEE T Robot Autom 2001; 17(3): 280–289.
3. Hirai K, Hirose M, Haikawa Y, et al. The development of Honda humanoid robot. In: IEEE international conference on robotics and automation, Leuven, Belgium, 20–20 May 1998, pp.1321–1326. IEEE.
4. Kajita S, Kanehiro F, Kaneko K, et al. Biped walking pattern generation by using preview control of zero-moment point. In: IEEE international conference on robotics and automation, Taipei, Taiwan, 14–19 September 2003, pp.1620–1626. IEEE.
5. Huang Q, Kajita S, Koyachi N, et al. A high stability, smooth walking pattern for a biped robot. In: IEEE international conference on robotics and automation, Detroit, MI, USA, 10–15 May 1999. IEEE.
6. Da X, Harib O and Hartley R. From 2D design of underactuated bipedal gaits to 3D implementation: walking with speed tracking. IEEE Access 2016; 4: 3469–3478.
7. Hong Y and Lee B. Evolutionary optimization for optimal hopping of humanoid robots. IEEE Trans Ind Electron 2017; 64(2): 1279–1283.
8. Nunez V, Drakunov S and Nadjar-Gauthier N. Control strategy for planar vertical jump. In: IEEE international conference on Advanced Robotics, Seattle, WA, USA, 18–20 July 2005, pp.849–855. IEEE.
9. Raibert M, Blankespoor K and Nelson G. Bigdog, the rough-terrain quadruped robots. IFAC 2018; 41(2): 10822–10825.
10. Kajita S, Nagasaki T and Kaneko K. A hop towards running humanoid biped. In: IEEE international conference on robotics and automation, New Orleans, LA, USA, 26 April–1 May 2004, pp.629–635. IEEE.
11. Raibert M. Legged robots that balance. Cambridge: MIT press, 1986.
12. Griffin LD. The atlas structure of images. IEEE Trans Pattern Anal Mach Intell 2017; 41(1): 234–245.
13. Atmeh GM, Ranatunga I, Popa DO, et al. Implementation of an adaptive, model free, learning controller on the Atlas robot. In: 2014 American control conference, Portland, OR, USA, 4–6 June 2014, pp.2887–2892. IEEE.
14. Hosoda K, Takuma T and Nakamoto A. Biped robot design powered by antagonistic pneumatic actuators for multimodal locomotion. Robot Auton Syst 2008; 56(1): 46–53.
15. Kaneko K, Kamimura N and Sakaguchi T. Humanoid robot HRP-5P: an electrically actuated humanoid robot with high power and wide range joints. Robot Auton Lett 2019; 4(2): 1431–1438.
16. Niiyama R, Nagakubo A and Kuniyoshi Y. Mowgli: a bipedal jumping and landing robot with an artificial musculoskeletal system. In: IEEE international conference on robotics and automation, Rome, Italy, 10–14 April 2007, pp.2546–2551. IEEE.
17. Curran S and Orin D. Evolution of a jump in an articulated leg with series-elastic actuation. In: IEEE international conference on robotics and automation, Pasadena, CA, USA, 19–23 May 2008, pp.352–358. IEEE.
18. Ugurlu B, Saglia J and Tsegarakis N. Hopping at the resonance frequency: a trajectory generation technique for bipedal robots with elastic joints. In: IEEE international conference on robotics and automation, Saint Paul, MN, USA, 14–18 May 2012, pp.1436–1443. IEEE.
19. Okada M and Takeda Y. Optimal design of nonlinear profile of gear ratio using noncircular gear for jumping robot. In: IEEE international conference on robotics and automation, Saint Paul, MN, USA, 14–18 May 2012, pp.1958–1963. IEEE.
20. Kojima K, Kojo Y, Ishikawa T, et al. Drive-train design in JAXON3-P and realization of jump motions: impact mitigation and force control performance for dynamic motions. In: International conference on intelligent robots and systems (IROS), Las Vegas, NV, USA, 24 October–24 January 2021, pp.3747–3753. IEEE.
21. Victor N, Drakunov S and Nadja-Gauthier N. Control strategy for planar vertical jump. In: ICAR conference on advanced robotics, Seattle, WA, USA, 2005, pp.849–855. IEEE.
22. Sakka S and Yokoi K. Motion pattern for the landing phase of a vertical jump for humanoid robots. In: IEEE international conference on intelligent robots and systems, Beijing, 2006, pp.5477–5483.
23. Sakka S and Yokoi K. Humanoid vertical jumping based on force feedback and inertial forces optimization. In: IEEE
Appendix

Notations

- $\delta \dot{q}$: adjustment quantity of joint angular acceleration
- $\delta \tau$: adjustment quantity of joint torque
- $\Delta p_x$: stability margin constant of ZMP
- $\delta p_x$: variation of ZMP
- $\mu$: friction coefficient
- $\tau_f$: torque of reducer
- $\tau_j$: torque of joint
- $\tau_m$: torque of motor
- $\theta_i$: speed of joint
- $\theta_m$: speed of motor
- $A S$: assumptions
- $b$: damping coefficient of motor
- $B_i$: transformation matrix between relative coordinate and generalized coordinate
- $B H R$: Beijing Institute of Technology Humanoid Robot
- $C_s$: Coriolis and centrifugal force term
- $\text{CoM}$: center of Mass
- $D_x$: length of foot, horizontal distance from ankle to heel, and horizontal distance from ankle to toe
- $d_i(i = 0, 1, 2)$: Inertia matrix
- $D$: degree of Freedom
- $DOF$: ground support force
- $F^N$: ground friction
- $F^T$: friction coefficients of the reducer
- $G_x$: gravity term
- $GRF$: ground reaction force
- $i_c$: current
- $J_t$: moment of inertia of motor
- $K_t$: torque constant
- $l_i(i = 1, 2, 3)$: length of $\text{Link}_i$
- $m_i(i = 0, 1, 2, 3)$: mass of $\text{Link}_i$
- $N$: gear ratio
- $O - XY$: coordinate system
- $P_{\text{CoM}} = \{x_c, y_c\}$: position of CoM
- $P_{\text{ZMP}} = \{x_p, y_p\}$: position of ZMP
- $P_{\text{ZMP}}$: position-velocity-time interpolation
- $Q_{s} = \{q_1, q_2, q_3\}$: vector of generalized coordinate
- $Q_{rel} = \{q_1, q_2, q_3, \theta_1, \theta_2, \theta_3\}$: vector of relative coordinate
- $u_r = \{\tau_1, \tau_2, \tau_3\}$: torque of ankle, knee, and hip
- $ZMP$: zero moment point