Looking into the matter of light-quark hadrons

Abstract In tackling QCD, a constructive feedback between theory and extant and forthcoming experiments is necessary in order to place constraints on the infrared behaviour of QCD’s $\beta$-function, a key nonperturbative quantity in hadron physics. The Dyson-Schwinger equations provide a tool with which to work toward this goal. They connect confinement with dynamical chiral symmetry breaking, both with the observable properties of hadrons, and hence provide a means of elucidating the material content of real-world QCD. This contribution illustrates these points via comments on: in-hadron condensates; dressed-quark anomalous chromo- and electro-magnetic moments; the spectra of mesons and baryons, and the critical role played by hadron-hadron interactions in producing these spectra.

Keywords confinement · dynamical chiral symmetry breaking · Dyson-Schwinger equations · hadron spectrum · hadron elastic and transition form factors · in-hadron condensates

1 Introduction

Solving QCD presents a fundamental problem that is unique in the history of science. We have never before been confronted by a theory whose elementary excitations are not those degrees-of-freedom readily accessible via experiment; i.e., whose elementary excitations are confined. Moreover, QCD generates forces which are so strong that less-than 2% of a nucleon’s mass can be attributed to the so-called current-quark masses that appear in QCD’s Lagrangian; viz., forces capable of generating mass from nothing, a phenomenon known as dynamical chiral symmetry breaking (DCSB).

Neither confinement nor DCSB is apparent in QCD’s Lagrangian and yet they play the dominant role in determining the observable characteristics of real-world QCD. The physics of hadronic matter is ruled by emergent phenomena, such as these, which can only be elucidated and understood through the use of nonperturbative methods in quantum field theory. This is both the greatest novelty and the greatest challenge within the Standard Model. We must find essentially new ways and means to explain precisely via mathematics the observable content of QCD. Building a bridge between QCD and the observed properties of hadrons is one of the key problems for modern science.

The complex of Dyson-Schwinger equations (DSEs) is a puissant tool, which has been employed with success to study confinement and DCSB, and their impact on hadron observables. In this contribution, I will briefly describe selected recent highlights, some of which are covered more fully in recent reviews [1; 2; 3; 4].
Fig. 1 Left panel – An observable particle is associated with a pole at timelike-\(P^2\), which becomes a branch point if, e.g., the particle is dressed by photons. Middle panel – When the dressing interaction is confining, the real-axis mass-pole splits, moving into pairs of complex conjugate poles or branch points. No mass-shell can be associated with a particle whose propagator exhibits such singularity structure. Right panel – \(\Delta(k^2)\), the function that describes dressing of a Landau-gauge gluon propagator, plotted for three distinct cases. A bare gluon is described by \(\Delta(k^2) = 1/k^2\) (the dashed line), which is convex on \(k^2 \in (0, \infty)\). Such a propagator has a representation in terms of a non-negative spectral density. In some theories, interactions generate a mass in the transverse part of the gauge-boson propagator, so that \(\Delta(k^2) = 1/(k^2 + m_g^2)\), which can also be represented in terms of a non-negative spectral density. In QCD, however, self-interactions generate a momentum-dependent mass for the gluon, which is large at infrared momenta but vanishes in the ultraviolet [8]. This is illustrated by the curve labelled “IR-massive but UV-massless.” With the generation of a mass-function, \(\Delta(k^2)\) exhibits an inflexion point and hence cannot be expressed in terms of a non-negative spectral density.

2 Confinement

It is worth stating plainly that the potential between infinitely-heavy quarks measured in simulations of quenched lattice-QCD – the so-called static potential – is irrelevant to the question of confinement in the real world, in which light quarks are ubiquitous. In fact, it is a basic feature of QCD that light-particle creation and annihilation effects are essentially nonperturbative. It is therefore impossible in principle to compute a potential between two light quarks [5; 6]. Hence, in discussing this physics, linearly rising potentials, flux-tube models, etc., have no connection with nor justification via QCD.

On the other hand, confinement can be related to the analytic properties of QCD’s Schwinger functions; i.e., the dressed-propagators and -vertices. This perspective was laid out in Ref. [7]. Whilst there is a great deal of mathematical background to this observation, it is readily illustrated, Fig. 1. The simple pole of an observable particle produces a propagator that is a monotonically-decreasing convex function, whereas the evolution depicted in the middle-panel of Fig. 1 is manifest in the propagator as the appearance of an inflexion point at \(P^2 > 0\). To complete the illustration, consider \(\Delta(k^2)\), which is the single scalar function that describes the dressing of a Landau-gauge gluon propagator. Three possibilities are exposed in the right-panel of Fig. 1. The inflexion point possessed by \(M(p^2)\), visible in Fig. 2 entails, too, that the dressed-quark is confined.

From the perspective that confinement is related to the analytic properties of QCD’s Schwinger functions, the question of light-quark confinement may be translated into the challenge of charting the infrared behavior of QCD’s \(\beta\)-function. (The behavior of the \(\beta\)-function on the perturbative domain is well known.) This is a well-posed problem whose solution is a primary goal of hadron physics; e.g., Refs. [11; 12; 13]. It is the \(\beta\)-function that is responsible for the behavior evident in Figs. 1 and 2 and thereby the scale-dependence of the structure and interactions of dressed-gluons and -quarks. One of the more interesting of contemporary questions is whether it is possible to reconstruct the \(\beta\)-function, or at least constrain it tightly, given empirical information on the gluon and quark mass functions.

Experiment-theory feedback shows promise for providing the latter [14; 15; 16]. This is illustrated through Fig. 3 which depicts the running-gluon-mass, analogous to \(M(p)\) in Fig. 2, and the running-coupling determined by analysing a range of properties of light-quark ground-state, radially-excited and exotic scalar-, vector- and pseudoscalar-mesons in the rainbow-ladder truncation, which is leading order in a symmetry-preserving DSE truncation scheme [17]. Consonant with modern DSE- and lattice-QCD results [8], these functions derive from a gluon propagator that is a bounded, regular
3 Dynamical chiral symmetry breaking

Whilst the nature of confinement is still debated, Fig. 2 shows that DCSB is a fact. It is the most important mass generating mechanism for visible matter in the Universe, being responsible for roughly 98% of the proton’s mass \(20\). Indeed, the Higgs mechanism is (almost) irrelevant to light-quarks. In Fig. 2 one observes the current-quark of perturbative QCD evolving into a constituent-quark as its momentum becomes smaller. This behavior, and that illustrated in Figs. 1 has a marked influence, e.g., on the \(Q^2\)-dependence of hadron elastic and transition form factors, whose measurement is a large part of the programme planned at the upgraded JLab facility. Therefore, in combination with well-constrained QCD-based theory, such data can potentially be used to chart the evolution of the mass function on \(0.3 \lesssim p/\text{GeV} \lesssim 1.2\), which is a domain that bridges the gap between nonperturbative and perturbative QCD. This can assist in unfolding the relationship between confinement and DCSB.

The appearance of running masses for gluons and quarks is a quantum field theoretic effect, unrealisable in quantum mechanics. It entails, moreover, that: quarks are not Dirac particles; and the coupling between quarks and gluons involves structures that cannot be computed in perturbation theory. Recent progress with the two-body problem in quantum field theory \(27\) has enabled these facts to be established \(30\). One may now plausibly argue that theory is in a position to produce the first reliable symmetry-preserving, Poincaré-invariant prediction of the light-quark hadron spectrum \(28\).
4 Condensates are confined within hadrons

The connection between DCSB and the generation of hadron masses was first described in Ref. [51], wherein it was represented as a vacuum phenomenon. Two inequivalent classes of ground-state were identified in the mean-field treatment of a meson-nucleon field theory: symmetry preserving (Wigner phase); and symmetry breaking (Nambu phase). In the symmetry breaking class, each of an uncountable infinity of distinct configurations is related to every other by a chiral rotation. This is arguably the origin of the concept that strongly-interacting quantum field theories possess a nontrivial vacuum.

With the introduction of the parton model for the description of deep inelastic scattering (DIS), this notion was challenged via an argument [32] that DCSB can be realised as an intrinsic property of hadrons, instead of through a nontrivial vacuum exterior to the observable degrees of freedom. This perspective is tenable because the essential ingredient required for dynamical symmetry breaking in a composite system is the existence of a divergent number of constituents and DIS provided evidence for the existence within every hadron of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence within every hadron of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention. On the contrary, the introduction of QCD sum rules as a method to estimate the existence of a sea of low-momentum partons. This view has, however, received scant attention.
onto the origin in configuration space; that in Eq. (2) is its pseudo-scalar analogue; and quark condensate, meson’s mass. The initial step in extending the concept was a proof that the in-pseudoscalar-meson factor at zero momentum transfer, limiting cases were also established: 

\[ \kappa_Q \text{ at } \mathbf{q} \rightarrow \infty. \]

Panel Two – [\kappa_{qQ}]^{1/3} computed directly from Eqs. (3) and Eqs. (10), respectively. Panel Three – solid curve, [\tilde{f}_{P,Q}^2S_{P,Q}]^{1/3}; dashed curve, [\tilde{f}_{Q,Q}^2S_{Q,Q}]^{1/3}; and dash-dotted and dotted lines, respectively, the \( m_Q \rightarrow \infty \) values. Heavy-heavy systems. Panel Four – solid curve, [\tilde{f}_{Q,Q}^2S_{Q,Q}]^{1/3}; and dashed curve, [\tilde{f}_{Q,Q}^2S_{Q,Q}]^{1/3}. The dot-dashed and dotted curves show the \( m_Q = m_Q \rightarrow \infty \) limiting behaviours: \( f_{P,Q}^2S_{P,Q} \propto 1/m_Q; \) and the short horizontal dashed line indicates the value of \( 2kf_{Q,Q} \) at \( m_Q = 10 \text{GeV}. \) These four panels: illustrate that \( \chi := f^2S \) is a smoothly varying measure of DCSB for pseudoscalar-scalar- and vector-mesons, with values typical of those usually associated with this phenomenon; and confirm Eqs. (8). (NB. All results computed using the symmetry-preserving regularisation of a vector × vector contact-interaction described in Ref. 22, 23: \( m_w \) is fixed at 7 MeV; and the chiral-limit value of the in-pseudoscalar-meson condensate is \( \kappa^0 = (0.24 \text{GeV})^3. \)

is the meson’s Bethe-Salpeter amplitude, and \( S_{f_1,f_2} \) are the component dressed-quark propagators, with \( m_{f_1,f_2} \) the associated current-quark masses. The quantity in Eq. (11) is the pseudoscalar meson’s leptonic decay constant; i.e., the pseudovector projection of the meson’s Bethe-Salpeter wave-function onto the origin in configuration space; that in Eq. (12) is its pseudoscalar analogue; and \( m_{P,Q}^2 \) is the meson’s mass. The initial step in extending the concept was a proof that the in-pseudoscalar-meson quark condensate, \( \kappa^\zeta_{P_{1,2}} \), can rigorously be represented through the pseudoscalar-meson’s scalar form factor at zero momentum transfer, \( Q^2 = 0; \) viz., \( \langle m_{P,Q}^2 \rangle = m_{f_1}^2 + m_{f_2}^2 \)

\[
\begin{align*}
S_{P_{1,2}} := & -\langle P_{f_1,f_2} \rangle \frac{1}{2} (q_f^{-}, q_f^{+}) \langle P_{f_1,f_2} \rangle = \frac{\partial}{\partial m_{P_{1,2}}^2} m_{P_{1,2}}^2 = \kappa^\zeta_{P_{1,2}} f^S_{P_{1,2}} + m_{f_1}^2 m_{f_2}^2 \frac{\partial}{\partial m_{f_1,f_2}^2} \left[ \kappa^\zeta_{P_{1,2}} \right] (4)\end{align*}
\]

The second step employs a mass formula for scalar mesons, exact in QCD; viz. 39,

\[
\begin{align*}
f^S_{f_1,f_2} m^2_{P_{f_1,f_2}} &= -[m_{f_1} - m_{f_2}] \rho^\zeta_{S_{f_1,f_2}}, \\
f^S_{f_1,f_2} K_{\mu} &= Z_2 \text{trCD} \int_{k} A^{\rho} i\gamma_{\mu} S_{f_1}(k_{+}) T S_{f_1,f_2}(k; K) S_{f_2}(k_{-}) , \end{align*}
\]

which was used to prove that the in-scalar-meson quark condensate is, analogously and rigorously, connected with the scalar-meson’s scalar form factor at zero momentum transfer. Moreover the following limiting cases were also established:

\[
\begin{align*}
\tilde{f}_{P,S}^2 S_{P,S}^\zeta_{\text{chiral limit}} \kappa^0 &= -\langle qq \rangle^0 \text{ and } f^S_{P,S} S_{P,S}^\zeta_{\text{heavy quark(s)}} = 2\kappa^\zeta_{P,S}, (8)\end{align*}
\]
where \( \langle q\bar{q} \rangle^0 \) is precisely the quantity that is widely known as the \textit{vacuum quark condensate} through any of its definitions \([22]\).

With appeal to demonstrable results of heavy-quark symmetry in QCD, it was argued that the \( Q^2 = 0 \) value of vector- and pseudovector-meson scalar form factors also determine the in-hadron condensates in these cases, and that this expression for the concept of in-hadron quark condensates is readily extended to the case of baryons. Thus, through the \( Q^2 = 0 \) value of any hadron’s scalar form factor, one can extract the magnitude of a quark condensate in that hadron which is a reasonable and realistic measure of dynamical chiral symmetry breaking.

These ideas are illustrated in Fig. 4 using the symmetry-preserving regularisation of a vector \( \times \) vector contact interaction described in Refs. [26, 43]; the left two panels describe evolution from the chiral limit to heavy-light pseudoscalar and scalar systems; the third panel describes the same for heavy-light vector mesons and compares this with the kindred pseudoscalar’s behaviour; and the rightmost panel portrays evolution to heavy-heavy pseudoscalar- and vector-mesons. It is notable that the in-vector-meson condensate is \((1.13)^3\)-times greater than the in-pseudoscalar meson condensate in the neighbourhood of the chiral limit.

In the \( m_Q \to \infty \) limit, this ratio rises to \((1.43)^3\), an outcome which finds explanation in the results: \( m_\text{v}_{\bar{q}q} \approx m_\text{P}_{\bar{q}q} \) but \( f_\text{v}_{\bar{q}q}^2 \approx f_\text{P}_{\bar{q}q}^2 / m_Q \), \( f_\text{v}_{\bar{q}q}^2 \approx f_\text{P}_{\bar{q}q}^2 / m_Q \), with \( f_\text{v} \neq f_\text{P} \). In QCD, on the contrary \([11]\): \( f_\text{v} = f_\text{P} \), and hence the solid and dashed curves in this panel would merge. In fact, based on the information in Refs. [25, 40] one estimates \( \kappa_B = (0.60 \pm 0.10 \text{ GeV})^3 \) and \( \kappa_B^\text{max} = (0.59 \pm 0.05 \text{ GeV})^3 \), values which may be compared with \( \kappa_B^\text{max} = (0.27 \text{ GeV})^3 \). In the case of heavy-heavy mesons, the vector \( \times \) vector contact-interaction produces \( m_\text{v}_{\bar{q}q}^4 \text{V}_{\bar{q}q}^2 = m_\text{QQ} \kappa_{\bar{q}q} = \text{constant} \), \( f_\text{v}_{\bar{q}q}^2 \text{F}_{\bar{q}q}^2 = m_\text{QQ} \kappa_{\bar{q}q} = \text{constant} \), where “constant” is independent of \( m_\text{QQ} \). In QCD, on the contrary, the result is \( m_\text{V,P}_{\bar{q}q} f_\text{P}_{\bar{q}q}^2 = m_\text{QQ} \kappa_{\bar{q}q} = \text{constant} \times m_\text{QQ}^4 \), where “constant” is independent of \( m_\text{QQ} \), and hence, again, one would compute curves that merge.

### 5 Dressed-quark anomalous chromo- and electro-magnetic moments

The appearance and behaviour of \( M(p) \) in Fig. 2 are essentially quantum field theoretic effects, unrealisable in quantum mechanics. The running mass connects the infrared and ultraviolet regimes of the theory, and establishes that the constituent-quark and current-quark masses are simply two connected points on a single curve separated by a large momentum interval. QCD’s dressed-quark behaves as a constituent-quark, a current-quark, or something in between, depending on the momentum of the probe which explores the bound-state containing the dressed-quark. These remarks should make clear that QCD’s dressed-quarks are not simply Dirac particles.

This is emphasised by a consideration of their anomalous magnetic moments. In QCD the analogue of Schwinger’s one-loop calculation can be performed to find an anomalous \textit{chromo}-magnetic moment for the quark. There are two diagrams in this case: one similar in form to that in QED; and another owing to the gluon self-interaction. One reads from Ref. [13] that the perturbative result vanishes in the chiral limit. This is consonant with helicity conservation in perturbative massless-QCD, which entails owing to the gluon self-interaction. One reads from Ref. [47] that the perturbative result vanishes in the chiral limit, this ratio rises to \((1.43)^3\), an outcome which finds explanation in the results: \( \langle q\bar{q} \rangle^0 \approx m_\text{P}_{\bar{q}q} \) but \( f_\text{v}_{\bar{q}q}^2 \approx f_\text{P}_{\bar{q}q}^2 / m_Q \), with \( f_\text{v} \neq f_\text{P} \). In QCD, on the contrary \([44]\): \( f_\text{v} = f_\text{P} \), and hence the solid and dashed curves in this panel would merge. In fact, based on the information in Refs. [25, 40] one estimates \( \kappa_B = (0.60 \pm 0.10 \text{ GeV})^3 \) and \( \kappa_B^\text{max} = (0.59 \pm 0.05 \text{ GeV})^3 \), values which may be compared with \( \kappa_B^\text{max} = (0.27 \text{ GeV})^3 \). In the case of heavy-heavy mesons, the vector \( \times \) vector contact-interaction produces \( m_\text{v}_{\bar{q}q}^4 \text{V}_{\bar{q}q}^2 = m_\text{QQ} \kappa_{\bar{q}q} = \text{constant} \), \( m_\text{v}_{\bar{q}q}^2 \text{F}_{\bar{q}q}^2 = m_\text{QQ} \kappa_{\bar{q}q} = \text{constant} \), where “constant” is independent of \( m_\text{QQ} \). In QCD, on the contrary, the result is \( m_\text{V,P}_{\bar{q}q} f_\text{P}_{\bar{q}q}^2 = m_\text{QQ} \kappa_{\bar{q}q} = \text{constant} \times m_\text{QQ}^4 \), where “constant” is independent of \( m_\text{QQ} \), and hence, again, one would compute curves that merge.

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Of course, it does, and it is now known that the effect is modulated by the strong momentum dependence of the dressed-quark mass-function \([31]\). In fact, dressed-quarks possess a large, dynamically-generated anomalous chromomagnetic moment, which produces an equally large anomalous electromagnetic moment that has a material impact on nucleon magnetic form factors \([15]\) and also very likely on nucleon transition form factors. Furthermore, given the magnitude of the muon “\( g_\mu - 2 \) anomaly” and its assumed importance as an harbinger of physics beyond the Standard Model \([49]\), it might also be worthwhile to make a quantitative estimate of the contribution to \( g_\mu - 2 \) from the quark’s DCSB-induced anomalous moments following, e.g., the computational pattern indicated in Ref. [50].
6 $a_1$-$\rho$ mass splitting

Following Ref.\ [27] it is now possible to construct a symmetry-preserving kernel for the Bethe-Salpeter equation given any form for the dressed-quark-gluon vertex, $\Gamma_\mu$. Owing to the importance of symmetries in forming the spectrum of a quantum field theory, this is a pivotal advance. One may now use all available, reliable information to construct the best possible vertex Ansatz. The preceding section illustrated that this enables one to incorporate crucial nonperturbative effects, which any finite sum of contributions is incapable of capturing, and thereby prove that DCSB generates material, momentum-dependent anomalous chromo- and electro-magnetic moments for dressed light-quarks.

Reference [28] computes ground-state masses using the method detailed in Ref.\ [51], which ensures one need only solve the gap- and Bethe-Salpeter-equations at spacelike momenta. This simplifies the numerical problem and is illustrated for the $\rho$- and $a_1$-channels in Fig.\ 5. More important in this figure, however, are the predicted masses for the mesons. The $\rho$- and $a_1$-mesons have been known members of the spectrum for more than thirty years and are typically judged to be parity-partners; i.e., they would be degenerate if chiral symmetry were manifest in QCD. Plainly, they are not, being split by more than 400 MeV (i.e., $> m_\rho/2$). It is suspected that this large splitting owes to DCSB. Hitherto, however, no symmetry-preserving treatment of bound-states could explain the splitting. This is illustrated by the curves labelled “RL,” which show that whilst a good estimate of $m_\rho$ is readily obtained at leading-order in the systematic DSE truncation scheme of Ref.\ [18], the axial-vector masses are much underestimated. The defect persists at next-to-leading-order\ [53; 54].

The analysis in Ref.\ [28] points to a remedy for this longstanding failure. Using the Poincaré-covariant, symmetry preserving formulation of the meson bound-state problem enabled by Ref.\ [27], with nonperturbative kernels for the gap- and Bethe-Salpeter-equations, which incorporate effects of DCSB that are impossible to capture in any step-by-step procedure for improving upon the rainbow-ladder truncation, it provides realistic estimates of axial-vector meson masses. In obtaining these results, Ref.\ [28] showed that the vertex Ansatz used most widely in studies of DCSB, $\Gamma_\mu^{BC}$\ [52], is inadequate as a tool in hadron physics. Used alone, it increases both $m_\rho$ and $m_{a_1}$ but yields $m_{a_1} - m_\rho = 0.21$ GeV, qualitatively unchanged from the rainbow-ladder result (see Fig.\ 5). A good description of axial-vector mesons is only achieved by including interactions derived from the dressed-quark anomalous chromomagnetic moment\ [30]. This emphasises again that the dressed-quark-gluon and -quark-photon vertices involve structures that cannot be computed in perturbation theory.

7 Unification of meson and baryon properties

Owing to the importance of DCSB, full capitalisation on the results of forthcoming experimental programmes is only possible if the properties of meson and baryon ground- and excited-states can be correlated within a single, symmetry-preserving framework, where symmetry-preserving means that all relevant Ward-Takahashi identities are satisfied. Constituent-quark-like models, which cannot incorporate the momentum-dependent dressed-quark mass-function, fail this test. That is not to say, however, such models are worthless: they continue to be valuable because there is nothing that is yet providing a bigger picture. Nevertheless, such models have no connection with quantum field theory.
Fig. 6 Comparison between DSE-computed hadron masses (filled circles) and: bare baryon masses from Ref. [58] (filled diamonds) and Ref. [59] (filled triangles); and experiment [46], filled-squares. For the coupled-channels models a symbol at the lower extremity indicates that no associated state is found in the analysis, whilst a symbol at the upper extremity indicates that the analysis reports a dynamically-generated resonance with no corresponding bare-baryon state. In connection with Ω-baryons the open-circles represent a shift downward in the computed results by 100 MeV. This is an estimate of the effect produced by pseudoscalar-meson loop corrections in Δ-like systems at a s-quark current-mass.

and therefore not with QCD; and they are not “symmetry-preserving” and hence cannot veraciously connect meson and baryon properties.

An alternative is being pursued within quantum field theory via the Faddeev equation. This analogue of the Bethe-Salpeter equation sums all possible interactions that can occur between three dressed-quarks. A simplified equation [52] is founded on the observation that an interaction which describes color-singlet mesons also generates nonpointlike quark-quark (diquark) correlations in the color-antitriplet channel [52]. The dominant correlations for ground state octet and decuplet baryons are scalar (0+ ) and axial-vector (1+ ) diquarks because, e.g., the associated mass-scales are smaller than the baryons’ masses and their parity matches that of these baryons. (Recent studies have confirmed the fidelity of the nonpointlike diquark approximation within the nucleon three-body problem; e.g. Ref. [57].) On the other hand, pseudoscalar (0−) and vector (1−) diquarks dominate in the parity-partners of those ground states [42]. This approach treats mesons and baryons on the same footing and, in particular, enables the impact of DCSB to be expressed in the prediction of baryon properties.

Building on lessons from meson studies [4], a unified spectrum of u, d-quark hadrons was obtained using a symmetry-preserving regularization of a vector x vector contact interaction [41]. That study simultaneously correlates the masses of meson and baryon ground- and excited-states within a single framework. In comparison with relevant quantities, the computation produces rms =13%, where rms is the root-mean-square-relative-error/degree-of freedom. As evident in Fig. 6 the prediction uniformly overestimates the PDG values of meson and baryon masses [40]. Given that the employed truncation deliberately omitted meson-cloud effects in the Faddeev kernel, this is a good outcome, since inclusion of such contributions acts to reduce the computed masses.

Following this line of reasoning, a striking result is agreement between the DSE-computed baryon masses [43] and the bare masses employed in modern coupled-channels models of pion-nucleon reactions [58, 59], see Fig. 6 and also Ref. [60]. The Roper resonance is very interesting. The DSE study [43] produces a radial excitation of the nucleon at 1.82±0.07 GeV. This state is predominantly a radial excitation of the quark-diquark system, with both the scalar- and axial-vector diquark correlations in their ground state. Its predicted mass lies precisely at the value determined in the analysis of Ref. [58]. This is significant because for almost 50 years the “Roper resonance” has defied understanding. Discovered in 1963, it appears to be an exact copy of the proton except that its mass is 50% greater. The mass was the problem: hitherto it could not be explained by any symmetry-preserving QCD-based tool. That has now changed. Combined, see Fig. 7, Refs. [43, 58] demonstrate that the Roper resonance is indeed the proton’s first radial excitation, and that its mass is far lighter than normal for such an
excitation because the Roper obscures its dressed-quark-core with a dense cloud of pions and other mesons. Such feedback between QCD-based theory and reaction models is critical now and for the foreseeable future, especially since analyses of CLAS data on nucleon-resonance electrocouplings suggest strongly that this structure is typical; i.e., most low-lying $N^*$-states can best be understood as an internal quark-core dressed additionally by a meson cloud \[16\]. Additional analysis \[60\] suggests a fascinating new feature of the Roper. The nucleon ground state is dominated by the scalar diquark, with a significantly smaller but nevertheless important axial-vector diquark component. This feature persists in solutions obtained with more sophisticated Faddeev equation kernels (see, e.g., Table 2 in Ref. \[61\]). From the perspective of the nucleon’s parity partner and its radial excitation, the scalar diquark component of the ground-state nucleon actually appears to be unnaturally large. One can nevertheless understand the structure of the nucleon. As with so much else, the composition of the nucleon is intimately connected with DCSB. In a two-color version of QCD, the scalar diquark is a Goldstone mode, just like the pion. (This is a long-known result of Pauli-Gürsey symmetry.) A “memory” of this persists in the three-color theory and is evident in many ways. Amongst them, through a large value of the canonically normalized Bethe-Salpeter amplitude and hence a strong quark+quark−diquark coupling within the nucleon. (A qualitatively identical effect explains the large value of the $\pi N$ coupling constant.) There is no such enhancement mechanism associated with the axial-vector diquark. Therefore the scalar diquark dominates the nucleon. The effect on the Roper is striking, with orthogonality of the ground- and excited-states forcing the Roper to be constituted almost entirely from the axial-vector diquark. One may reasonably expect this to have a material impact on the momentum-dependence of the nucleon-to-Roper transition form factor.

8 Epilogue

Dynamical chiral symmetry breaking (DCSB) is a fact in QCD. It is manifest in dressed-propagators and vertices, and, amongst all other things, it is responsible for: the transformation of the light-current-quarks in QCD’s Lagrangian into heavy constituent-like quarks, in terms of which order was first brought to the hadron spectrum; the unnaturally small values of the masses of light-quark pseudoscalar mesons and the $\eta^-$ splitting \[62\]; the unnaturally strong coupling of pseudoscalar mesons to light-quarks − $g_{\pi qq} \approx 4.3$; and the unnaturally strong coupling of pseudoscalar mesons to the lightest baryons − $g_{\pi NN} \approx 12.8 \approx 3g_{\pi qq}$.

Herein we have illustrated the dramatic impact that DCSB has upon observables in hadron physics. A “smoking gun” for DCSB is the behaviour of the dressed-quark mass function. The momentum dependence manifest in Fig. 2 is an essentially quantum field theoretical effect. Exposing and elucidating its consequences therefore requires a nonperturbative and symmetry-preserving approach, where the latter means preserving Poincaré covariance, chiral and electromagnetic current-conservation, etc. The Dyson-Schwinger equations (DSEs) provide such a framework. Experimental and theoretical studies are underway that will identify observable signals of $M(p^2)$ and thereby confirm and explain the mechanism responsible for the vast bulk of visible mass in the Universe.
There are many reasons why this is an exciting time in hadron physics. We have focused on one. Namely, through the DSEs, we are positioned to unify phenomena as apparently diverse as: the hadron spectrum; hadron elastic and transition form factors, from small- to large-$Q^2$; and parton distribution functions. The key is an understanding of both the fundamental origin of nuclear mass and the far-reaching consequences of the mechanism responsible; namely, DCSB. These things might lead us to an explanation of confinement, the phenomenon that makes nonperturbative QCD the most interesting piece of the Standard Model.

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References
1. Roberts, C. D., Bhagwat, M. S., Höll, A. Wright, S. V.: Aspects of hadron physics. Eur. Phys. J. (Special Topics) 140, 53–116 (2007)
2. Roberts, C. D.: Hadron Properties and Dyson-Schwinger Equations. Prog. Part. Nucl. Phys. 61, 50–65 (2008)
3. Holt, R. J., Roberts, C. D.: Nucleon and pion distribution functions in the valence region. Rev. Mod. Phys. 82, 2991–3044 (2010)
4. Chang, L., Roberts, C. D., Tandy, P. C.: Selected highlights from the study of mesons. arXiv:1107.4003 [nucl-th]
5. Bali, G. S. et al.: Observation of string breaking in QCD. Phys. Rev. D 71, 114513/1-26 (2005)
6. Chang, L. et al.: Exploring the light-quark interaction. Chin. Phys. C 33, 1189–1196 (2009)
7. Roberts, C. D., Williams, A. G., Krein, G.: On The Implications Of Confinement. Int. J. Mod. Phys. A 7, 5607–5624 (1992)
8. Boucaud, Ph. et al.: The Infrared Behaviour of the Pure Yang-Mills Green Functions. arXiv:1109.1936 [hep-ph]
9. Bhagwat, M. S., Tandy, P. C.: Analysis of full-QCD and quenched-QCD lattice propagators. AIP Conf. Proc. 842, 225–226 (2006)
10. Bowman, P. O. et al.: Unquenched quark propagator in Landau gauge. Phys. Rev. D 71, 054507/1-7 (2005)
11. Qin, S. x., Chang, L., Liu, Y. x., Roberts, C. D., Wilson, D. J.: Interaction model for the gap equation. arXiv:1108.0603 [nucl-th]
12. Brodsky, S. J., de Teramond, G. F., Deur, A.: Nonperturbative QCD coupling and its β-function from light-front holography. Phys. Rev. D 81, 096010/1-13 (2010)
13. Aguilar, A. C., Binosi, D., Papavassiliou, J.: QCD effective charges from lattice data, JHEP 1007, 002/1-24 (2010)
14. Roberts, C. D.: Opportunities and Challenges for Theory in the N* program. arXiv:1108.1030 [nucl-th]
15. Gothe, R. W.: Experimental Challenges of the N* Program. arXiv:1108.4703 [nucl-ex]
16. Azaïr, I. G., Burkert, V. D., Mokeev, V. I.: Nucleon resonance electrocouplings from the CLAS meson electroproduction data. arXiv:1108.1125 [nucl-ex]
17. Qin, S. x., Chang, L., Liu, Y. x., Roberts, C. D., Wilson, D. J.: Commentary on rainbow-ladder truncation for excited states and exotics. arXiv:1109.3438 [nucl-th]
18. Bender, A., Roberts, C. D., von Smialkiewicz, L.: Goldstone theorem and quark confinement beyond rainbow ladder approximation. Phys. Lett. B 380, 7–12 (1996)
19. Bowman, P. O. et al.: Unquenched gluon propagator in Landau gauge. Phys. Rev. D 70, 045009/1-4 (2004)
20. Aguilar, A. C., Binosi, D., Papavassiliou, J., Rodriguez-Quintero, J.: Non-perturbative comparison of QCD effective charges. Phys. Rev. D80, 085018/1-18 (2009)
21. Skullerud, J. I., et al.: Nonperturbative structure of the quark gluon vertex. JHEP 0304, 047/0-15 (2003)
22. Bhagwat, M. S., Tandy, P. C.: Quark-gluon vertex model and lattice-QCD data. Phys. Rev. D 70, 094039/1-6 (2004)
23. Roberts, C. D., McKellar, B. H. J.: Critical coupling for dynamical chiral symmetry breaking. Phys. Rev. D 41, 672–678 (1990)
24. Bloch, J. C. R.: Multiplicative renormalizability and quark propagator. Phys. Rev. D 66, 034032/1-15 (2002)
25. Bashir, A., Pennington, M. R.: Gauge independent chiral symmetry breaking in quenched QED. Phys. Rev. D 50, 7679–7689 (1994)
26. Roberts, H. L. L., et al.: π- and ρ-mesons, and their diquark partners, from a contact interaction. Phys. Rev. C 83, 065206/1-12 (2011)
27. Chang, L., Roberts, C. D.: Sketching the Bethe-Salpeter kernel. Phys. Rev. Lett. 103, 081601/1-4 (2009)
28. Chang, L., Roberts, C. D.: Tracing masses of ground-state light-quark mesons. arXiv:1104.4821 [nucl-th].
29. Flambaum, V. V. et al.: σ-terms of light-quark hadrons. Few Body Syst. 38, 31–51 (2006)
30. Chang, L., Liu, Y. x., Roberts, C. D.: Dressed-quark anomalous magnetic moments. Phys. Rev. Lett. 106, 072001/1-4 (2011)
31. Nambu, Y., Jona-Lasinio, G.: Dynamical Model of Elementary Particles Based on an Analogy with Superconductivity. 1. Phys. Rev. 122, 345-358 (1961)
32. Casher, A., Susskind, S.: Chiral magnetism (or magnetohadrochironic). Phys. Rev. D 9, 436–460 (1974).
33. Turner, M. S.: Dark energy and the new cosmology. [astro-ph/0108103]
34. Brodsky, S. J., Shrock, R.: Condensates in quantum chromodynamics and the cosmological constant. Proc. Nat. Acad. Sci. 108, 45–50 (2011)
35. Burkardt, M.: Dynamical vertex mass generation and chiral symmetry breaking on the light-front. Phys. Rev. D 58, 096015/1-11 (1998).
36. Brodsky, S. J., Shrock, R.: Maximum wavelength of confined quarks and gluons and properties of quantum chromodynamics. Phys. Lett. B 666, 95–99 (2008)
37. Turner, M. S.: Dark energy and the new cosmology. astro-ph/0108103.
38. Brodsky, S. J., Shrock, R.: Condensates in quantum chromodynamics and the cosmological constant. Proc. Nat. Acad. Sci. 108, 45–50 (2011)
39. Chang, L., Roberts, C. D., Tandy, P. C.: Expanding the concept of in-hadron condensates. arXiv:1109.2903 [nucl-th]
40. Maris, P., Roberts, C. D., Tandy, P. C.: T(r)opical Dyson-Schwinger Equations. AIP Conf. Proc. 1354, 110–117 (2011)
41. Jegerlehner, F., Nyffeler, A.: The Muon $g - 2$. Phys. Rept. 477, 1–110 (2009)
42. Gyulassy, T., Fischer, C. S., Williams, R.: Hadronic light-by-light scattering in the muon $g - 2$: a Dyson-Schwinger equation approach. Phys. Rev. D 83, 094006/1-14 (2011)
43. Bhagwat, M. S., et al.: Schwinger functions and light-quark bound states. Few Body Syst. 40, 209–235 (2007)
44. Ball, J. S., Chiu, T. W.: Analytic Properties Of The Vertex Function In Gauge Theories. 1. Phys. Rev. D 22, 2542–2549 (1980)
45. Watson, P., Cassing, W., Tandy, P. C.: Bethe-Salpeter meson masses beyond ladder approximation. Few Body Syst. 35, 129–153 (2004)
46. Fischer, C. S., Williams, R.: Probing the gluon self-interaction in light mesons. Phys. Rev. Lett. 103, 122001/1-4 (2009)
47. Cahill, R. T., Roberts, C. D., Praschikfa, J.: Baryon structure and QCD. Austral. J. Phys. 42, 129–145 (1989)
48. Cahill, R. T., Roberts, C. D., Praschikfa, J.: Calculation of diquark masses in QCD. Phys. Rev. D 36, 2804-2812 (1987).
49. Eichmann, G.: Nucleon electromagnetic form factors from the covariant Faddeev equation. Phys. Rev. D 84, 014014/1-23 (2011)
50. Suzuki, N. et al., Disentangling the dynamical origin of $P_{11}$ nucleon resonances. Phys. Rev. Lett. 104, 042302/1-4 (2010)
51. Gasparian, A. M., Haidenbauer, J., Hanhart, C., Speth, J.: Pion nucleon scattering in a meson exchange model. Phys. Rev. C 68 045207/1-16 (2003)
52. Roberts, C. D., Cloët, I. C., Chang, L., Roberts, H. L. L.: Dressed-quarks and the Roper resonance. arXiv:1108.1327 [nucl-th]
53. Cloët, I. C. et al., Survey of nucleon electromagnetic form factors. Few Body Syst. 46, 1–36 (2009)
54. Bhagwat, M. S. et al., Flavour symmetry breaking and meson masses. Phys. Rev. C 76, 045203/1-10 (2007)
55. Nguyen, T., Bashir, A., Roberts, C. D., Tandy, P. C.: Pion and kaon valence-quark parton distribution functions. Phys. Rev. C 83, 062201(R)/1-5 (2011)

[hep-th]