Introduction. The introduction of safe high-speed railway traffic on the main roads of Ukraine in accordance with the «National Transport Strategy of Ukraine for the Period up to 2030» directly depends on the infrastructure and conditions for ensuring traffic speeds: the condition of the rolling stock, railway track and automation systems. The main executive element of the railway automation systems, providing the throughput, is the turnout switch. Since the mathematical modelling of the electric drive (ED) of the switch in railway transport has not received sufficient attention, there are practically no models of the electromechanical system (EMS) of the switch.

The goal of the work is the development of a mathematical and simulation model of a switch’s electric drive in the form of a two- and three-mass electromechanical system based on DC and AC motors as the main executive element of the railway automation system.

Research material. A serious problem in the study of switches’ electric drives by the method of mathematical modelling was that in the known models the load was not taken into account and they were considered as single-mass electromechanical systems. Switches SP-6, as well as modifications based on them, are the most common in the railway transport system of Ukraine. The main structural elements of the switch are shown in Fig. 1: 1 – the body; 2 – the electric motor; 3 – the reducer with frictional connection 4 built in the same block; 5 – the main shaft with gate gear; 6 – the autoswitch block; 7 – the gate with cam locking mechanism.

The design features of the SP-6 switch and the electromechanical system based on DC and AC motors as the main executive element of railway automation. The modelling is based on the parameters of the SP-6m switches as the main definition of the executive element of the Ukrainian domestic railways automation system. Methodology. The authors have presented the mechanical part of the railway switch as a three-mass construction scheme with serial connection of elastic masses. Special attention is paid to the character of the movement of the switch point tongues, when describing them in the form of two- and three-mass kinematic schemes, as well as to the mechanical state of the traction and connections as nodes, which are most sensitive to the negative effects of some modes of the railroad switch. Results. A simulation mathematical model of SP-6m switch for AC and DC motors, which allows to study the influence of various factors, has been developed and tested at the stand of the Automation Laboratory of the Ukrainian State University of Railway Transport, Kharkiv. The deviation in the basic parameters of the switch translation process is less than 5 %. Originality. For the first time a mathematical model of an electric drive of the railroad switch is developed, which takes into account elastic relations and load characteristics as an object of controlling the speed of movement of switch point tongues. Practical significance. We have created a mathematical model of the electric drive of the railroad switch which allows to examine the influence of various factors, including the influence of the environment on the operation of the switch. References 10, table 1, figures 10.

Key words: electric drive of the railroad switch; electromechanical switch system.
The indicated disadvantages can be conditionally divided into mechanical (friction clutch, technological clearance) and electrical (position sensor, uncontrolled switching). To study the processes occurring in the switch’s electric drive, it is proposed to consider it as a multi-mass electromechanical system divided into mechanical (friction clutch, technological clearance) and electrical (position sensor, uncontrolled switching). To study the processes occurring in the interconnected electromechanical system, it is advisable to consider a switch as a multi-mass electromechanical system that takes into account the masses of an electric motor, wits and elastic connections.

For a two-mass electromechanical system (Fig. 2,a), the first mass \( J_1 \) is the total moment of inertia of the motor, gearbox and gate. The second mass \( m_2 \) is the total mass of wits and inter-edge thrust. The working thrust from the gate to the first blade serves as an elastic connection \( c_{12} \). The transformation of the rotary motion of the motor shaft into the translational movement of the gate is carried out in the gearbox. The model takes into account the clearances in the kinematic transmission: the technological clearance \( \varphi_{23} \) and the clearances \( l_c \) in the cotter-pin connections of the thrust attachment with the gate and the blade. The presence of an interstellar thrust in a two-mass system means the existence of elastic connections between the first and second witses.

Therefore, further the authors consider the switch namely as a three-mass system (Fig. 2,b,c). The main reason for this approach is that the presence of elastic coupling elements has a serious impact on the dynamic processes in the mechanism [1-6], and the two-mass model is advisable to use for engineering calculations.

In the calculation scheme with series connection of elastic masses (Fig. 2,b) of such a system, the first mass, as before, is made up of the moments of inertia of the motor shaft and the moment of inertia of the gearbox reduced to the motor shaft. The second and third masses are the first and second wits, respectively; the interstitial thrust acts as an additional elastic connection \( c_{23} \).

The kinematic diagram of the switch (Fig. 3) demonstrates that the working rod does not rest directly on the first wit, but is connected to it from the side of the second wit through the inter-turn rod. The authors consider it most expedient to consider the mechanical part of the switch as a three-mass design scheme with series connection of elastic masses, where: \( J_1 \) is the moment of inertia of the motor rotor; \( J_p \) is the moment of inertia of the gearbox, reduced to the motor shaft; \( J_{21} \) is the total moment of inertia of the first mass; \( M_{ds}, F_{dl} \) are the electromagnetic torque and thrust force of the motor, respectively; \( \varphi_{23}, l_c \) are the technological clearance and clearance in the tie rods, respectively; \( F_{12}, F_{23} \) are the elastic forces in the working and inter-shaft rods, respectively; \( F_{14}, F_{24} \) are the resistance forces of the first and second wits, respectively; \( c_{12}, c_{23} \) are the elasticity of the working and interstellar rods, respectively; \( \beta_{12}, \beta_{23} \) are the coefficients of internal viscous friction of the working and inter-shaft rods, respectively; \( m_2, m_3 \) are the second and third masses, respectively (Fig. 2,b). The representation of the switch as a four-mass system is impractical.

When switching (Fig. 4), the wits 2 and 7, the connecting (inter-wit) thrust 3, the working thrust 4, the control 5 and 6 rods of the switch head and fastening parts move. The force of switching is spent on overcoming: the resistance of the moved parts along the switching surfaces 1 and the resistance in the hinges.

The force required to overcome the friction of the moving parts of the switch is determined by the friction force that occurs during movement. Its value can be determined as follows. Divide the switch into \( n \) equal elements of length \( l \) each. Let \( q_1, q_2, \ldots q_n \) denote the weight of each element. The weight of the elements will be different both due to the change in the weight of the parts of the wits 2 and 7, and due to the presence or absence of connected rods, switch heads and other movable parts within the elements.
Fig. 4. General view of the mechanism for determining the effort of switching: 1 – switching surface; 2, 7 – wits; 3 – connecting thrust; 4 – working thrust; 5 – first control rod; 6 – the second control rod

Assuming that the wit lies not on separate pillows, but on a solid base and the coefficient of friction between the wit and the base is the same along the entire length of the wit and is equal to $\psi$, we find the elementary friction forces:

$$F_k = \psi \cdot q_k, \quad k = 1, 2, ..., K.$$  \hspace{1cm} (1)

Moments of elementary forces relative to the root of the wits:

$$M_k = \psi \cdot q_k \cdot l, \quad k = 1, 2, ..., K.$$  \hspace{1cm} (2)

Total moment of friction force:

$$M_T = \sum_{k=1}^{K} M_k = \frac{1}{2} \psi \cdot q \cdot l \cdot \sum_{k=1}^{K} q_k + 3q_2 + ... + (2k-1)q_k.$$  \hspace{1cm} (3)

Consequently, the force required to overcome the frictional forces of the moved parts, applied at a distance $L-a$ from the root (beginning) of the wit, is

$$F = \frac{M_T}{L-a} = \frac{1}{2} \psi \cdot q \cdot \frac{l}{L-a} \sum_{k=1}^{K} q_k + 3q_2 + ... + (2k-1)q_k.$$  \hspace{1cm} (4)

For approximate calculations of the force $F$ and assuming that $q_1 = q_2 = ... = q_k = q$, we obtain:

$$F = \frac{1}{2} \psi \cdot q \cdot \frac{l}{L-a} \sum_{k=1}^{K} 1 + 3 + ... + (2k-1) = \frac{1}{2} \psi \cdot q \cdot n^2 \cdot \frac{l}{L-a}.$$  \hspace{1cm} (5)

Since $L = l \cdot n$ and $Q = q \cdot n$, then

$$F = \frac{1}{2} \psi \cdot \frac{Q}{L-a}.$$  \hspace{1cm} (6)

where $L$ is the wit length, $Q$ is the total weight of the moving parts of the switch.

The coefficient taking into account the influence of friction forces in the joints is usually taken equal to 1.1 [7]. We determine the calculated effort of switching with a pivot or inlay-onlay mount according to the formula:

$$F = 1.1 \cdot \frac{1}{2} \psi \cdot \frac{Q}{L-a} = 0.55 \cdot \psi \cdot \frac{Q}{L-a}.$$  \hspace{1cm} (7)

From (7) it follows that the switching force depends on the weight of the moving parts of the switch, the type of rails, the place of fixing the switching rods and the coefficient of friction of the wits on the pillows. All specified quantities, except for the last, are determinable.

The value of the coefficient of friction can range from 0.05 to 0.3 and depends on the condition of the switch, the quality of lubrication of the switch pads, the cleanliness of the treatment of their surfaces, as well as the soles of the wit and other factors.

Maintaining the clearances in the kinematic line (Fig. 5) of the switching within the limits set by the instructions plays one of the key roles in ensuring the safety of the switching of wit.

Fig. 5. Clearances in the kinematic line of the switch: 1, 2, 3 – at the points of attachment of the rods, 4 – technological clearance

Such a distance for the safe movement of the wheels of the rolling stock should be no more than 4 mm [8, 9], otherwise emergency situations may arise, such as «cutting the switch», which lead to the derailment of the rolling stock and the destruction of both the roadbed and the accompanying infrastructure. Due to the impact during the development of a technological clearance, works on friction and unregulated switching speed – the linkages of the rods gradually break and the clearances increase. Studies carried out on the railways of Latvia [8] have shown that the rate of increase in the size of the clearance is on average 0.05 mm per month (Table 1).

| Connection place                        | Maximum increase in backlash (mm) | Average increase in backlash (mm) |
|----------------------------------------|-----------------------------------|----------------------------------|
| Connecting inter-wits thrust – shackle | 0.07                              | 0.04                             |
| Connecting inter-wits thrust – working thrust | 0.06                             | 0.04                             |
| Working thrust – gate of the electric drive | 0.09                             | 0.05                             |
The design feature of SP-6m switches is the presence of a 46° technological clearance between the third and fourth stages of the gearbox (Fig. 4, 5), which is necessary to facilitate the acceleration of the motor and its stock of some kinetic energy in order to break the wits at the beginning of the switching process [7]. The simulation of the mechanical part of the electric drive is carried out taking into account the real values of the clearances and the presence of the force of internal viscous friction.

The total elastic-viscous force is determined as:

\[ F_{12} = \begin{cases} F_{tr} + F_{12}, & \text{if } \text{sign} \Delta l \neq \text{sign} \Delta v \text{ and } |F_{tr}| > |F_{12}|; \\ 0, & \text{if } \text{sign} \Delta l = \text{sign} \Delta v \text{ and } |F_{tr}| \leq |F_{12}|, \end{cases} \]

where \( F_{12} = c_{12}F_{1}(l) \) is the elastic force; \( F_{tr} = \beta_{12} \Delta v F_{2}(l) \) is the viscous friction force; \( F_{1}(l) \) and \( F_{2}(l) \) are the nonlinear functions:

\[ F_{1}(l) = \begin{cases} 0 & \text{at } |\Delta l| \leq \frac{\Delta l_1}{2}; \\ |\Delta l| - \frac{\Delta l_1}{2} & \text{at } |\Delta l| > \frac{\Delta l_1}{2}; \\ |\Delta l| + \frac{\Delta l_1}{2} & \text{at } |\Delta l| < -\frac{\Delta l_1}{2}, \end{cases} \]

\[ F_{2}(l) = \begin{cases} 0 & \text{at } |\Delta l| \leq \frac{\Delta l_2}{2}; \\ 1 & \text{at } |\Delta l| > \frac{\Delta l_2}{2}. \end{cases} \]

In the three-mass model (Fig. 2,b), the elastic-viscous force \( F_{23} \) is determined in a similar way.

The implementation of the mathematical model was made in the environment for creating engineering applications Simulink of the MATLAB code. The model was created block by block, in full accordance with the calculation schemes (Fig. 2): motor block, reducer block, block of the first wit, block of the second wit. Clearances – technological one and two clearances in the joints reduced to the fastening of the first and second rail are marked with separate blocks.

The numerical values of the moments of inertia of the gear stages are calculated based on their weight and dimensions. For modelling, mechanical parameters of the working and connecting thrusts are also required – rigidities and viscous friction coefficients \( c_{12}, \beta_{12}, c_{23}, \beta_{23} \), respectively. The thrust is a metal rod, the deformation of which occurs along the longitudinal axis:

\[ c_{12} = c_{23} = E \left( \frac{d}{2} \right)^2 \frac{l}{E}, \]

where \( E \) is the modulus of elasticity for steel; \( d \) is the thrust diameter; \( l \) is the thrust length.

The rigidity of the working thrust in the model is taken to be equal to the rigidity of the direct connecting thrust due to the fact that these thrusts have the same length and diameter. On the basis of the research results [10], the value of the internal viscous friction coefficient is taken within the limits \( 1 \cdot 10^{3} \ldots 1 \cdot 10^{4} \).

At present, schemes with both a DC motor (DCM) and an induction motor (IM) are widely used. Due to the possibility of powering the frequency converter (FC) both with DC directly from an autonomous voltage inverter and with AC of a rectifier, the system under development can be integrated into a switch control circuit based on both DCM, for example, sequential excitation motors of the types MSP-0,1, MSP-0,15 and MSP-0,25, which are widely used in Ukraine, and also IM.

Figure 6 shows a block diagram of a three-mass model of a switch with various configurations of a drive motor – DCM and IM.

The equations that describe the electromagnetic and electromechanical processes in the DCM of sequential excitation have the form:

\[ u = (L_a + L_v) \frac{di}{dt} + (r_a + r_v)i + e_a; \]

\[ J \frac{d\omega}{dt} = M_d - M_s; \]

\[ e_a = k \cdot \omega \cdot \Phi_v; \]

\[ M_d = k \cdot i \cdot \Phi_v, \]

where \( k = \frac{pN}{2\pi} \) is the design factor.
The model presented in this paper demonstrates ways of replacing the electric drive of a switch from a DCM to an IM with frequency control. As a solution, we propose a mathematical model of a switch based on a structural diagram (Fig. 6), where instead of a DCM/IM block, a FC-IM block is used. An induction motor is a nonlinear multidimensional object with a rather complex structure, therefore, the mathematical description and modelling of an electric drive with an IM differs from the given system of equations (12) for DCM. The equivalent circuit of the IM in the electric drive is shown in Fig. 7.

![Fig. 7. Phase equivalent circuit of an induction motor](image)

In the mathematical description of the IM, the adopted coordinate system, in which the mathematical description of the object is performed, and the configuration of the model, which depends on the content of the input and output signals of the model, the structure of the control system, and on the characteristics of the motor power source, are of great importance.

The basis for modelling of the configuration of an electric drive in this study was a coordinate system oriented along the rotor flux linkage. When carrying out the simulation, the following assumptions were made: the stator and rotor of a three-phase induction motor have harmonic signals in multiphase coordinates to a air gap is sinusoidally distributed, the axes of the stator and rotor windings do not coincide, forming an arbitrary angle. To move from a mathematical description of harmonic signals in multiphase coordinates to a mathematical description in orthogonal two-phase coordinates, the concept of a generalized vector is used:

\[
\alpha = e^{j2\pi/3}, \quad \beta = e^{j4\pi/3} \]

where \( \alpha \) and \( \beta \) are the vectors taking into account the spatial displacement of the windings;

\[
i_A = I_m \cos \alpha, \quad i_B = I_m \cos(\alpha - \frac{2\pi}{3}), \quad i_C = I_m \cos(\alpha + \frac{2\pi}{3})
\]

is the three-phase symmetrical system of stator currents.

Substituting the values of instantaneous currents into equation (13), we find a mathematical description of the spatial vector of the stator current:

\[
i_S = \frac{2}{3} I_m (\cos \alpha + \frac{2\pi}{3} \cdot \cos(\alpha - \frac{2\pi}{3}) + \frac{2\pi}{3} \cdot \cos(\alpha + \frac{2\pi}{3})) = I_m e^{j2\alpha/3}.
\]

Thus, for a coordinate system rotating at an arbitrary speed, the following system of equations can be written:

\[
\begin{align*}
\frac{d\Psi}{dt} &= \frac{d\Psi}{dt} + j \omega \Psi; \\
\frac{d\Psi}{dt} &= \frac{d\Psi}{dt} + j (\omega_0 - \omega) \Psi; \\
\Psi &= L_S i_S + L_{sl} i_B; \\
\Psi &= L_m i_S + L_{rm} i_B; \\
M &= \frac{3}{2} p k_R \cdot Mod(\Psi \times i_S); \\
J \frac{d\omega_m}{dt} &= M - M_n,
\end{align*}
\]

where \( \omega = p \omega_n; \) \( p \) is the number of pole pairs in the machine; \( J \) is the moment of inertia of the rotor.

For a squirrel-cage motor, the vectors \( \Psi, \Psi \) differ from each other due to the presence of scattering of the stator and rotor windings. In this regard, the following is true for them:

\[
\Psi_S = (L_m + L_{sl}) \cdot i_S + L_{sl} i_B;
\]

\[
\Psi_R = (L_m + L_{rm}) \cdot i_B + L_{rm} i_S;
\]

where \( L_m \) is the main leakage inductance; \( L_{sl}, L_{rm} \) are the leakage inductance of windings.

Here, for the inductances of the stator \( L_S \) and rotor windings \( L_R \) we have:

\[
L_S = (L_m + L_{sl});
\]

\[
L_R = (L_m + L_{rm}).
\]

When constructing real systems of an AC electric drive, coordinate converters are almost always included in the control system, which are necessary to recalculate real three-phase variables into two-phase model variables and vice versa. For the coordinate system \( \alpha, \beta \) the direct transformation is as follows:

\[
i_A = i_\alpha, i_B = -\frac{1}{2} i_\alpha + \frac{\sqrt{3}}{2} i_\beta, i_C = -\frac{1}{2} i_\alpha - \frac{\sqrt{3}}{2} i_\beta,
\]

and the opposite one:

\[
i_\alpha = i_A, \quad i_\beta = i_B - i_C \cdot \frac{1}{\sqrt{3}}.
\]

For modelling, the parameters of the SP-6m switch as the main modification of the executive element of the automation system for the domestic railways of Ukraine were taken as the initial data.

For further work with the simulation model, an assessment of its adequacy to the real object was carried out. For this, at the laboratory of the Ukrainian State University of Railway Transport, a Siemens FC was connected to the SP-6 drive and started with a rated load up to a rated speed according to a given tachogram. The results of the experiment on the stand (a) and on the model (b) are shown in Fig. 8. Analysis of the graphs allows to conclude that the operation of the model and the real object is identical, since the discrepancy in the coordinates of the current and speed is less than 5%.

The obtained results of modelling of the operation of an electric switching drive with an AC motor are shown in Fig. 9.
The occurrence of oscillations in the kinematic line of the switch is confirmed by studies of the state of the switching surfaces, where wave-like traces are visible, which are formed as a result of elastic oscillations of the wits in the process of their switching (Fig. 10).

The characteristic mats present on both pillows indicate the emerging elastic vibrations in both thrusts and lead to the vibrations of both wits, which confirms the need to consider the electric drive of the switch as a three-mass EMS. The depth and width of undulating formations, according to the observations of the service personnel, depends on the presence and magnitude of «backlashes» in the kinematic line of the switch.

**Conclusions.** For the first time, a mathematical and simulation model of an electric switching drive has been developed in the form of a two- and three-mass electromechanical system based on DC and AC motors as the main executive element of the automation system of Ukrainian railways. The initial data of the most common turnout switch SP-6m were taken as the object of modelling. The necessity of taking into account, when modelling an electromechanical system of a switch, the size of the clearances and the dynamics of their change in order to take into account the processes taking place in the mechanical subsystem, is shown. The spread of the deviation values of the main coordinates of the electric switching drive during transients, obtained by the method of simulation and experimentally, does not exceed 5 %.

**REFERENCES**

1. Kuznetsov B.I., Nikitina T.B., Kolomiets V.V., Bovdui I.V. Improving of electromechanical servo systems accuracy. *Electrical engineering & electromechanics*, 2018, no. 6, pp. 33-37. doi: 10.20998/2074-272X.2018.6.04.

2. Buriakovskiy S., Smirnov V., Asmolova L., Obruch I., Rafalskiy O., Maslii A. Analysis of optimization criteria for the process of switch displacement in a DC railroad turnout. *Eastern-European Journal of Enterprise Technologies*, 2019, vol. 6, no. 2 (102), pp. 58-69. doi: 10.15587/1729-4061.2019.187580.
3. Sokol Y.I., Buryakovskiy S.G., Masliy Ar.S. Energy-efficient electric drive of multifunctional turnout. Problemy Kolejnictwa, 2014, no. 165, pp. 99-107.

4. Lagos R.F., San Emeterio A., Vinolas J., Alonso A., Aizpun M. The influence of track elasticity when travelling on a railway turnout. Proceedings of the Second International Conference on Railway Technology: Research, Development and Maintenance, 2014, p. 11. doi: 10.4203/ecp.104.208.

5. Papacef K. Turnout of new construction. International Railway Journal, 2000, no. 3, pp. 30-32.

6. Krishnan R. Switched reluctance motor drives. Modeling, simulation, analysis, design and applications. CRC Press, 2001. 432 p. doi: 10.1201/9781420041644.

7. Resnikov J.M. Strelochnye elektroprivody elektricheskoj i gorochnoj centralizacii [Switch electric drives of electric and mining centralization]. Moscow. Transport Publ., 1975. – p.152.

8. Latvian Railways Newsletter. Available at: http://ita.times.lv/Automatika/Otzimi1.html (assessed 19 February 2020).

9. Gluzberg B.E. Klassifikacija defektov i povrezhdenij elementov stroelchnykh perevodov [Classification of defects and damage to the elements of the switchgears]. Moscow. Transport Publ., 1996. 87 p. (Rus).

10. Akimov L.V., Kolotilo V.I., Markov V.S. Dinamika dvumassovykh sistem s netradicionnymi reguljatorami skorosti i nabljudateliami sostojanija [Dynamics of two-mass systems with non-traditional speed controllers and state observers]. Kharkiv, KhSPU Publ., 2000. 93 p. (Rus).

Received 22.05.2020

S.G. Buriakovski, Doctor of Technical Science, Professor,
A.S. Masli, Candidate of Technical Science, Associate Professor,
O.V. Pasko, Candidate of Technical Science, Associate Professor,
V.V. Smirnov, Candidate of Technical Science,
1 Scientific-&-Research Planning-&-Design Institute «Molniya», National Technical University «Kharkiv Polytechnic Institute», 2, Kyrychova Str., Kharkiv, 61002, Ukraine, e-mail: sergbyr@i.ua
2 Ukrainian State University of Railway Transport, 7, Feierbakh Square, Kharkiv, 61050, Ukraine, e-mail: a.masliy@ukr.net, zamdek@kart.edu.ua
3 Company «ELAKS», 5a, build. 12, Ac. Proskura Str., Kharkiv, 61085, Ukraine, e-mail: v.smirnov@elaks.kharkov.ua

How to cite this article:
Buriakovski S.G., Maslii A.S., Pasko O.V., Smirnov V.V. Mathematical modelling of transients in the electric drive of the switch – the main executive element of railway automation. Electrical engineering & electromechanics, 2020, no. 4, pp. 17-23. doi: 10.20998/2074-272X.2020.4.03.