Extended Color Models with a Heavy Top Quark

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Abstract
We present a class of models in which the top quark, by mixing with new physics at a higher energy scale, is naturally heavier than the other standard model particles. We take this new physics to be extended color. Our models contain new particles with masses between 100 GeV and 1 TeV, some of which may be just within the reach of the next generation of experiments. In particular one of our models implies the existence of two right handed top quarks. These models demonstrate the existence of a standard model-like theory consistent with experiment, and leading to new physics below the TeV scale, in which the third generation is treated differently than the first two.

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1. Introduction

Upon examining the fundamental particle spectrum one is immediately impressed by how much heavier the top quark is than all the other quarks. In the standard model this is achieved in an unnatural way. One tunes most of the Yukawa couplings to be very small and allows one of them to be of order one. Here we propose a class of models in which the much higher mass of the top quark (with respect to the other fundamental fermions) is a result of the dynamics and symmetry breaking in the theory. The key feature of the models we propose is that while the top quark gets its mass from electroweak symmetry breaking, this quark also mixes with physics at a higher energy scale. We propose the physics of this higher energy scale to be an extended color group. In particular, we have in mind models like $SU(4)_c$ [1] and $SU(5)_c$ color [2]. The $SU(5)_c$ model was particularly successful in generating new phenomena at the multi-GeV energy scale while agreeing with experiment to the same extent as the standard model [3]-[8].

In the extended color model scenario, the constraint of reproducing the standard model at energies below 100 GeV led to $SU(5)_c$ as the only possibility [2],[9]. This constraint meant that none of the standard model particles received a contribution to their mass from the breaking of the extended color group. We are no longer restricted to the group $SU(5)_c$; however that is the group we will use to describe our scenario. In section 2 we present the basic ideas behind two different models that allow for a heavy top quark and discuss their common features. Sections 3 and 4 concentrate on the specific features of each of the two respective models. Finally in section 5 we present our criticisms and conclusions.

2. The models

We would like to reformulate the $SU(5)_c$ color model [2] in such a way that it leads to a heavy top quark and yet retains all its previous successes. Non-standard model alternatives to the color sector are especially important avenues to explore. For example, hadronic physics has nothing comparable to the stunning theoretical agreement with precision LEP measurements, whereas revising the leptonic sector would almost most certainly contradict LEP results (or force us to place the new physics at a much higher energy scale). Our models therefore always leave the leptonic sector unchanged.

One loophole which we exploit is that experiments have very little to say about the right handed top quark, $t_R$, since it is an $SU(2)_L$ singlet. It is actually $t_R$ which
we will couple to the extended color physics coming from the higher energy scale. This comes about because \( t_R \) is in a higher dimensional representation of \( SU(5)_c \), whereas the other quarks are in the fundamental. This is one of the major differences between these models and the original \( SU(5)_c \) of reference [2]. We present two possibilities for the \( SU(5)_c \) representation of \( t_R \), the 10 and the \( \overline{10} \).

Consider a theory with gauge group

\[
SU(5)_{\text{color}} \otimes SU(2)_L \otimes U(1)'_y.
\]  

(2.1)

As stated above the leptonic sector is unchanged, and under (2.1) they have the quantum numbers

\[
f_L \sim (1, 2, -1), \quad e_R \sim (1, 1, -2),
\]

(2.2)

while the first two quark generations have the form

\[
Q_L \sim (5, 2, y_Q), \quad u_R \sim (5, 1, y_Q + 1), \quad d_R \sim (5, 1, y_Q - 1),
\]

(2.3)

just as in the original \( SU(5)_c \) color model [2]. However the third quark generation, and in particular the \( SU(2)_L \) singlet top quark, have different quantum numbers. We will be studying two types of models. The “ten bar” model has \( t_R \) in the \( \overline{10} \) of \( SU(5)_c \),

\[
T_L \sim (5, 2, y_Q), \quad t_R \sim (\overline{10}, 1, y_{t_R}), \quad b_R \sim (5, 1, y_Q - 1),
\]

(2.4)

and the “ten” model has \( t_R \) in the 10 of \( SU(5)_c \),

\[
T_L \sim (5, 2, y_Q), \quad t_R \sim (10, 1, y_{t_R}), \quad b_R \sim (5, 1, y_Q - 1).
\]

(2.5)

Just as in [2] we break \( SU(5)_c \) with the Higgs

\[
\chi \sim (10, 1, 2y_Q).
\]

(2.6)

However we use a colored Higgs, \( H \), to break the electroweak symmetry, and give masses to the \( W \) and \( Z \) bosons and the top quark. Its quantum numbers are

\[
H \sim (\overline{10}, 2, y_Q - y_{t_R}),
\]

(2.7)

for \( t_R \sim \overline{10} \) and

\[
H \sim (\overline{3}, 2, y_Q - y_{t_R}),
\]

(2.8)

for \( t_R \sim 10 \). The Yukawa Lagrangian is

\[
L_{\text{Yuk}} = \lambda \overline{T}_L H t_R + \lambda \overline{T}_L \chi (T_L)^c + \lambda_1 \overline{Q}_L \chi (Q_L)^c + \lambda_2 \overline{d}_R \chi (d_R)^c + \text{h.c}.
\]

(2.9)
Consider the $SU(5)$ tensor products
\begin{align}
\bar{5} \times \overline{10} &= 10 + 40 \\
\bar{5} \times 10 &= 5 + 45,
\end{align}
and the following $SU(5)_c$ branching rules under $SU(2)' \times SU(3)_c \times \tilde{U}(1)$.
\begin{align}
10 &= (1, 1)(6) + (1, \overline{3})(-4) + (2, 3)(1) \\
40 &= (2, 1)(9) + (1, \overline{3})(-4) + (2, 3)(1) + (3, \overline{3})(-4) + \cdots \\
5 &= (2, 1)(3) + (1, 3)(-2) \\
45 &= (2, 1)(3) + (1, 3)(-2) + (3, 3)(-2) + (1, \overline{3})(8) + \cdots.
\end{align}

In order for the Higgs $H$ to give a mass to the top, the $\tilde{U}(1)$ charges must satisfy the constraint $\tilde{y}_H - \tilde{y}_{T_L} + \tilde{y}_{t_R} = 0$. Eqs. (2.9), (2.10), (2.11) show that we have no choice but to pick $H$ in the $\bar{10}$ for $t_R$ in the $\bar{10}$. If we take $t_R$ in the 10, we can pick $H$ to be either in the $\bar{5}$ or $\overline{10}$ representation; however we will only consider $H$ in the $\bar{5}$.

The symmetry breaking pattern for the ten bar model is:
\begin{align}
SU(5)_c &\otimes SU(2)_L \otimes U(1)'_y \\
\downarrow \langle \chi \rangle &\sim w \\
SU(2)' \otimes SU(3)_c &\otimes SU(2)_L \otimes U(1)_y \\
\downarrow \langle H \rangle &\sim u
\end{align}

That for the ten model is:
\begin{align}
SU(5)_c &\otimes SU(2)_L \otimes U(1)'_y \\
\downarrow \langle \chi \rangle &\sim w \\
SU(2)' \otimes SU(3)_c &\otimes SU(2)_L \otimes U(1)_y \\
\downarrow \langle H \rangle &\sim u
\end{align}

The important point is that unlike the standard model, electroweak symmetry breaking takes place with a Higgs that carries color. Within the context of this model the problem of fermion masses becomes the question of why the other five quarks and leptons have small nonzero masses (in relation to the electroweak symmetry breaking scale). We do not seek a fundamental solution to this problem in
this paper. Instead we will add the usual standard model Higgs $\phi$ and imagine that its VEV is fifty times smaller than the VEV of $H$. This offers a partial explanation of light fermion masses, and maybe renormalization explains why $\langle H \rangle$ is much greater than $\langle \phi \rangle$ since the parameters for $H$ run faster. We write the standard model Yukawa couplings to generate these masses,

$$L_0 = \lambda_3 f_L \phi e_R + \lambda_4 Q_L \phi d_R + \lambda_5 Q_L \phi^c u_R + \lambda' T_L \phi b_R + \text{h.c.} \quad (2.14)$$

At this point we mention that all the hypercharge assignments have been normalized in such a way that the standard model Higgs hypercharge is one, i.e. $\phi \sim (1, 2, 1)$ under eq.(2.1).

In order to give both components of the top the correct electric charge, we must have $y_{t_R} = 2 - 2y_Q$ for $t_R$ in the $\overline{10}$ and $y_{t_R} = (3 - y_Q)/2$ for $t_R$ in the $10$. The electromagnetic charge operator is

$$Q_{em} = I_3 + \frac{1}{2}(Y' + (1 - 3y_Q)T) \quad (2.15)$$

One can show that $Y = Y' + (1 - 3y_Q)T$ gives the familiar values for the hypercharges of the color triplet quarks. $T$ is an $SU(5)_c$ generator in $\tilde{U}(1)$ with the following representation when acting on the five colors of ordinary plus exotic quarks

$$T = \text{Diag} \left( \frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -\frac{1}{2}, -\frac{1}{2} \right) \quad (2.16)$$

Further characteristics of the $\overline{10}$ and the $10$ model differ substantially and we will discuss each one separately in the following two sections.

3. The $\overline{10}$ model

In this section we study in more detail the $t_R \sim (\overline{10}, 1, 2 - 2y_Q)$ model. When the $SU(2)' \times SU(3)_c$ singlet piece of $\chi$ and $H$ gets a VEV we are left with $SU(2)' \times SU(3)_c \times U(1)_{em}$ as the unbroken gauge group. $SU(2)'$ is the same exotic force sector discussed in [2] and in much more detail in [8]. The $SU(3)_c$ singlet quarks that come from the $5$ of $SU(5)_c$ have charges $(5y_Q - 1 \pm 2)/4$. These quarks are confined by the $SU(2)'$ force and will lead to charge $0, \pm 1, 10y_Q \pm \frac{1}{2}, 10y_Q - \frac{3}{2}$ exotic mesons.

The field $t_R$ in the $\overline{10}$ will contain a charge $2/3$ color triplet which is just the usual right handed top quark. The exotics that come from the $t_R$ have charges $(3 - 5y_Q)/2$ for the singlets and $(13 - 15y_Q)/12$ for the $SU(2)'$ doublet color antitriplets, which will be bound into a charge-$0$ meson.
The $[SU(2)_L]^2 U(1)'$ anomaly equation implies $y_Q = 1/5$. Thus all the exotic mesons will have integer charge. However since $t_R$ is in the $\mathbf{10}$ of $SU(5)_c$, cancellation of the color anomaly is no longer automatic. This forces us to add exotics. Recall that the anomaly of the $5$ and $10$ of $SU(5)$ are equal. Our philosophy is to associate the higher dimensional representations of $SU(5)_c$ with the higher energy physics. Thus we cancel the anomaly by adding two electroweak singlets in the $10$. We also need to give large mass to the exotic components of $t_R$ and the other exotic fields. One possible exotic particle content is

\[ a_R \sim (10, 1, y_a) \quad c_R \sim (10, 1, y_c) \]

\[ E_R \sim (24, 1, -2) \quad P_R \sim (75, 1, -y_a + 2/5) \quad Q_R \sim (75, 1, -y_c + 2/5), \]

with the following Yukawa terms

\[ \lambda_t \chi E_R^c t_R + \lambda_a \chi^c P_R^c a_R + \lambda_c \chi Q_R^c c_R + \]

\[ \lambda_e \rho E_R^c E_R + \lambda_p \rho_p P_R^c P_R + \lambda_q \rho Q_R^c Q_R + \text{h.c.}. \]

(3.2)

It is straightforward to check that $a_R, c_R$ and the exotic components of $t_R$ get masses of order the $SU(5)_c$ scale without affecting the mass of the ordinary color triplet quarks. The $\rho$ Higgses are $SU(5)_c, SU(2)_L$ singlets whose $U(1)'$ charges are chosen so as to allow the Yukawa coupling above. We imagine these $\rho$’s getting a VEV at the same time as $H$ thus giving $E_R, P_R, Q_R$ masses of order the top quark mass.

We still have to cancel the $[SU(5)_c]^2 U(1)'$ and $[U(1)']^3$ anomalies. This will determine the hypercharges $y_a$ and $y_c$. The equations that correspond to cancelling the above two anomalies are

\[ -\frac{118}{5} + 47(y_a + y_c) = 0 \]

\[ \frac{3752}{25} + 36(y_a + y_c) - 90(y_a^2 + y_c^2) + 65(y_a^3 + y_c^3) = 0. \]

(3.3)

These equations can be combined to give a quadratic equation whose solutions are irrational and are given approximately by $y_a = 1.64$ and $y_c = -1.14$.

4. The 10 model

We now consider $t_R$ in the $10$ of $SU(5)_c$. One key difference between the ten and ten bar model is that $H$ also breaks the $SU(2)'$ subgroup of $SU(5)_c$. Thus the unbroken gauge group is the same as in the standard model. Since the coupling in $SU(2)'$ is given by the strong coupling constant, the three heavy gauge bosons
from that sector will be slightly heavier than the Z. Also these $SU(2)'$ gauge bosons will not couple to ordinary matter at tree level so that their production in hadron colliders will be suppressed.

This model has two right handed top quarks with charge 2/3 that couple to the broken $SU(2)'$ generators. The singlet and anti-triplet exotics of $t_R$ have charges $(5y_Q + 1)/4$ and $(13 - 15y_Q)/12$, respectively. As in the ten bar model, the $[SU(2)_L]^2U(1)'$ anomaly equation implies $y_Q = 1/5$. While exotics are no longer needed to cancel the color anomaly, we need them to cancel the $U(1)'$ anomalies and to give the exotic components of $t_R$ a mass. Thus we add the following particle content

$$E_R \sim (75, 1, -1) \quad P_R \sim (24, 1, y_P) \quad Q_R \sim (75, 1, y_Q)$$  \hspace{1cm} (4.1)

with the following Yukawa terms

$$\lambda t X E_R^c t_R + \lambda e \rho e E_R^c E_R + \lambda f \rho P_P P_R + \lambda q \rho Q_Q Q_R + \text{h.c.}.$$  \hspace{1cm} (4.2)

The $\rho$ Higgses behave in the same way as they did in the ten bar model.

The values for $y_P$ and $y_Q$ are determined from anomaly cancellation, which can be summarized in the following equations:

$$\begin{align*}
47 - 10y_P - 50y_Q &= 0 \\
\frac{1067}{25} - 24y_P^3 - 75y_Q^3 &= 0.
\end{align*}$$  \hspace{1cm} (4.3)

This has one real irrational solution given approximately by $y_P = -1.67$ and $y_Q = 1.27$.

5. Conclusions

We have presented two models which lead to a heavy top quark within the framework of extended color models [2].

In our approach, the first two generations are exact copies of each other. However the third-generation right-handed top quark is in a nonfundamental representation of the extended color group. It is this property which we use to characterize the physics at the higher energy scale.

Both models lead to new particles at the color symmetry breaking scale. Even without a detailed calculation, one expects that this scale will satisfy the same experimental bounds of the original $SU(5)_c$ model [2]. These bounds can be as low as 300 GeV [3]–[7]. Certain theoretical prejudices may put it at the 1-10 TeV
scale [8], yet the exotic $SU(2)'$ force sector has been sufficiently changed in our “ten” model, that the constraints from [8] do not apply. In any case we find it more interesting to consider the possibility of a multi-GeV color breaking scale, which would then naturally lead to our exotics having masses below a TeV. The ten model in particular will have a top-like right handed quark with the same electric charge, which gets its mass by coupling to the exotic sector only.

However, independent of how high one chooses the color symmetry breaking scale, both models predict that the $E_R, P_R, Q_R$ exotics will have masses of order the top quark mass. There are obvious ways around this prediction, such as choosing a different mass generating mechanism than the one suggested here, or considering a different exotic particle content.

Another feature of our models is the irrational electric charges of some of the exotic particles. This was forced upon us by anomaly cancellation, and by the desire to keep the number of new fields to a minimum. Irrational charges certainly run against current folklore, though it leads to no contradiction with accelerator experiments. We do not know if irrational charges are a generic feature of such models or if perhaps some cleverer model builder will be able to construct an exotic sector with rational charges only.

Unfortunately our model offers no insights into the origin of mass for the lighter generations, nor does it explain why higher dimensional representations should involve higher energy physics. Instead we present this model as an existence proof for standard model-like theories in which the third generation is treated differently than the first two. The models we presented are consistent with experiment and lead to new physics at the 100 GeV to 1 TeV scale. We hope these models can serve as a possible direction of study into the origin of fermion masses.

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