COUPLING COEFFICIENTS OF DIFFERENT CYLINDRICAL DIELECTRIC RESONATORS IN THE OPEN SPACE

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Introduction

Cylindrical DRs are applying today in various microwave devices [1 - 6]. For calculation and optimization of such devices is more convenient to use electrodynamic modeling with sufficient accuracy [7, 8]. Calculation of the systems, containing multiple DRs in the open space, is based on computation of mutual coupling coefficients. The coupling coefficients of the Cylindrical DRs did not studied in full measure even in case of the lowest modes. The goal of the present work is the calculation and analysis of the Cylindrical DR coupling coefficients with main modes, located in the open space, in the general case of its arbitrary spatial orientation as well as for cases of various shapes and materials.

Coupling coefficient calculating

Allocation microresonators side by side with each other leads to the coupling oscillations appearance. The fields and frequencies of the DR oscillations are defined by values of the mutual coupling coefficients. In the common case, the coupling coefficient can be determined as a surface integral [9]:

$$\kappa_{sn} = \frac{i}{2\omega_0 w_n (1 + \delta_{sn})} \oint_{s_n} \left\{ \left[ \bar{\varepsilon}_s, \bar{h}_s^+ \right] + \left[ \bar{\varepsilon}_n^+, \bar{h}_s^- \right] \right\} \hat{n} ds,$$

expressed via the eigenmode field (\(\bar{\varepsilon}_s, \bar{h}_s^-\)) of one (s-th) DR on the surface of another (n-th) DR. Here \(s, n = 1, 2\); and \(\hat{n}\) - is the normal to the surface \(s_n\) of n-th DR, \(\omega_0\) - is the resonance frequency of the resonators; \(w_n\) - is the energy, stored in the dielectric.

http://radap.kpi.ua/radiotechnique/article/view/1223
Fig. 1. A - sketch of two different Cylindrical DRs in the AA position in Open space. The real part of the coupling coefficients $\kappa_{12}$ (solid lines); $\kappa_{21}$ (dotted lines) (b, d - f) of two Cylindrical DRs with $H_{10}^1$ modes; ($m = 0$): $\varepsilon_{r_1} = 36$; $\varepsilon_{r_2} = 81$; $\Delta_1 = L_1 / 2r_1 = 0.4$; $\Delta_2 = L_2 / 2r_2 = 0.8$. Imaginary part of the coupling coefficients versus coordinates of the DR centers (c, g - i). B, c, f, i: $k_0 \Delta x = k_0 \Delta y = 0$; d, g: $k_0 \Delta y = 0$; $k_0 \Delta z = l, k_0 (r_1 + r_2)$; e, h: $k_0 \Delta x = 0$; $k_0 \Delta z = l, k_0 (r_1 + r_2)$.

As follow from (1), in the general case of unequal DRs: $\kappa_{11} \neq \kappa_{22}$; $\kappa_{12} \neq \kappa_{21}$. A direct calculation of the integral (1) in most cases is not possible, so we will use the well-known expressions for the mutual coupling coefficients found for the DR in a rectangular waveguide [10]. Then the same coupling coefficients for the open space can be simply obtained using the integral transformation of the known analytical expression based on assumption that the transmission line metal walls
have been "removed" to the infinity.

Using necessary expressions, for example, 5.2 of the [10], as well as integrals [9, 11], after simplifications obtain:

For the AA position (see fig. 1, a) in the case of two different Cylindrical DRs, with mode $H_{1,0,1}^+$, the mutual coupling coefficients can be obtained in the form:

in the area: $\Delta z \geq r_1 + r_2$:

\[
\kappa_{1,2} = \kappa_0 \cdot \int_0^\infty \varphi_1(\sqrt{1 - \xi^2}) \cdot \varphi_2(\sqrt{1 - \xi^2})^* H_0^{(2)}(k_0\Delta \rho) \sqrt{1 - \xi^2} \cos(k_0 \Delta x \xi) d \xi; \quad (\Delta \rho = \sqrt{\Delta y^2 + \Delta z^2})
\]

For the AB position (fig. 2, a) in the area: $\Delta z \geq r_1 + r_2$:

\[
\kappa_{1,2} = \kappa_0 \cdot \int_0^\infty \int_0^\infty e^{-iyk_0\Delta z} \varphi_1(\sqrt{\eta^2 + \gamma^2}) \cdot \varphi_2(\sqrt{\xi^2 + \gamma^2})^* \sin(k_0 \Delta x \xi) \sin(k_0 \Delta y \eta) \cdot \xi \cdot d \xi d \eta;
\]

For the AC position (see fig. 3, a) in the area: $\Delta z \geq r_1 + L_2 / 2$:

\[
\kappa_{1,2} = \kappa_0 \cdot \int_0^\infty \int_0^\infty e^{-iyk_0\Delta z} \varphi_1(\sqrt{\eta^2 + \gamma^2}) \cdot \varphi_2(\sqrt{\xi^2 + \eta^2})^* \sin(k_0 \Delta x \xi) \cos(k_0 \Delta y \eta) \cdot \xi \cdot d \xi d \eta;
\]

\[
(\xi^2 + \eta^2 + \gamma^2 = 1);
\]

For the CC position (see fig. 4, a) in the area: $\Delta z \geq L_1 / 2 + L_2 / 2$:

\[
\kappa_{1,2} = \kappa_0 \cdot \int_0^\infty \frac{e^{-iyk_0\Delta z}}{\gamma} \varphi_1(\kappa) \cdot \varphi_2(\kappa)^* J_0(k_0\Delta \rho \kappa) \kappa d \kappa,
\]

\[
(\kappa^2 + \gamma^2 = 1; \Delta \rho = \sqrt{\Delta x^2 + \Delta y^2});
\]

where; $\Delta x = x_1 - x_2$; $\Delta y = y_1 - y_2$; $\Delta z = |z_1 - z_2|$; $(x_1, y_1, z_1)$ – are the rectangular coordinates of the DR' centers;

\[
\kappa_0 = \frac{8i}{v_2 r_2 \beta_1} \frac{\beta_2}{\beta_2} \frac{\beta_2}{\beta_2} \frac{(e_{1z} - 1)(e_{2z} - 1)}{e_{2r}}
\]

\[
v_2 = [J_1(p_{2z}) - J_0(p_{2z})J_2(p_{2z})](2p_{2z} + \sin 2p_{2z}).
\]
Fig. 2. AB position of two Cylindrical DRs in Open space. Mutual coupling coefficients $k_{12}$ (solid lines); $k_{21}$ (dotted lines) (b - e) as a function coordinates of the DR centers with $\varepsilon_{1r} = 36; \varepsilon_{2r} = 81; \Delta_1 = L_1 / 2r_1 = 0.4; \Delta_2 = L_2 / 2r_2 = 0.8$. B, d: $k_0\Delta y = 2$; c, e: $k_0\Delta x = k_0\Delta y = 2$.

Here

$$
\varphi_s(\eta) = \frac{[\frac{p_{s\perp}}{q_{s\perp}} J_0(p_{s\perp})J_1(q_{s\perp}) - \eta J_1(p_{s\perp})J_0(q_{s\perp})]}{[(\frac{p_{s\perp}}{q_{s\perp}})^2 - \eta^2]}
$$

and $p_{s\perp} = \beta_x r_s; p_{sz} = \beta_z L_s / 2; q_{s\perp} = k_0 r_s; q_{sz} = k_0 L_s / 2; \beta_x, \beta_z$ - are the wave numbers of the eigenoscillation field of the $s$ - th Cylindrical DR [10]; $r_s$ - is the radius, $L_s$ - is the height of the $s$ - th DR ($s = 1, 2$); $H_n^{(2)}(\beta_0\rho)$ are the Hankel functions of the second kind; $J_n(x)$ - are the Bessel functions of the first kind of the $n$ - th order; $k_1 = \sqrt{\varepsilon_{1r}} k_0; k_0 = \omega_0 / c; \omega_0$ - is the circular resonance frequency; $c$ -
is the light velocity; $\varepsilon_{sr}$ - is the relative dielectric permittivity of the $s$-th DR material.

Fig. 3. Position AC of the Cylindrical DRs in Open space. Mutual coupling coefficients $\kappa_{12}$ (solid lines); $\kappa_{21}$ (dotted lines) (b - e) as a function coordinates of the DR centers with $\varepsilon_{r1} = 36; \varepsilon_{r2} = 81; \Delta_1 = L_1 / 2r_1 = 0.4; \Delta_2 = L_2 / 2r_2 = 0.8$. B, d: $k_0\Delta y = 0$; $k_0\Delta z = 1,3k_0(r_1 + L_2 / 2)$; c, e: $k_0\Delta x = 2; k_0\Delta y = 0$.

The integral convergence provides by choice of the radical signs for $\xi > 1$: $\sqrt{1 - \xi^2} = -i\sqrt{\xi^2 - 1}$ in the (2.4), as well as for $\kappa > 1$: $\sqrt{1 - \kappa^2} = -i\sqrt{\kappa^2 - 1}$ in the (5).

Note also that the function $\varphi_s(\eta)$ has no singularities in the region $0 \leq \eta < \infty$. It makes a major contribution to the integrals (2-6) only for small values $\eta$. Expand the function $f_s(\eta) = \varphi_s(\eta) / \eta$ in a series of $\eta$ at 1 and taking into account that $f_s(1) = \varphi_s(1)$ obtain:

$$\varphi_s(\eta) = \varphi_s(1) \cdot \eta + \frac{d}{d\eta}\left[\frac{\varphi_s(\eta)}{\eta}\right]_{\eta=1} \eta(\eta - 1) + ...$$

Next, using the approximation:

$$\varphi_s(\eta) \approx \varphi_s(1) \cdot \eta.$$ 

we obtain
Fig. 4. CC position of two Cylindrical DRs in Open space. Mutual coupling coefficients (b - g) as a function coordinates of the DR centers with \( \varepsilon_{1r} = 36; \varepsilon_{2r} = 81; \Delta_1 = L_1 / 2r_1 = 0.4; \Delta_2 = L_2 / 2r_2 = 0.8 \). B, c, e, g: \( k_0 \Delta x = k_0 \Delta y = 0 \); d, f: \( k_0 \Delta z = 1.7k_0(L_1+L_2) / 2 \).

For the AA position:

\[
\kappa_{1,2} \approx \kappa_0 \cdot \varphi_1(1)\varphi_2(1) \left( \frac{\Delta \rho}{\Delta r} \right)^2 h_0^{(2)}(k_0 \Delta r) - \left[ 1 - 3 \left( \frac{\Delta x}{\Delta r} \right)^2 \right] h_1^{(2)}(k_0 \Delta r) \left( \Delta \rho = \sqrt{\Delta y^2 + \Delta z^2} \right) ; \tag{9}
\]

For the AB position:

\[
\kappa_{1,2} \approx -\kappa_0 \cdot \varphi_1(1)\varphi_2(1) \cdot \frac{\Delta x \Delta y}{\Delta r^2} \cdot h_2^{(2)}(k_0 \Delta r) ; \tag{10}
\]

For the AC position:
For the CC position:

\[ \kappa_{1,2} = \kappa_0 \cdot \varphi_1^{(1)}(l) \varphi_2^{(1)}(l)^* \frac{\Delta x \Delta z}{\Delta r^2} h_2^{(2)}(k_0 \Delta r) ; \]  

(11)

In the (9 - 12) \( \Delta r = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2} ; \) \( h_n^{(2)}(z) \) - are the spherical Hankel functions of the second kind [12]. Here we have also used the generalized Sommerfeld's integral [11].

Found relations allow us to calculate the coupling coefficients of different Cylindrical DRs in the open space. It is interesting that at the same time restrictions on the range DR coordinates (2 - 5) are removed. In the particular case of identical DRs the relations (2, 5) coincide with [13].

**Coupling coefficient analysis**

Fig. 1, 4, b - c shows the dependence of the coupling coefficients of the DR center coordinates, calculated according to the formulas (9, 12) (solid curves) as well as the numerical formulas (2, 5) (dashed curves). As can be seen from these curves, the use of approximation (8) gives a very good accuracy.

It is easy to verify that the coupling coefficients found (9 - 12) are proportional to the respective magnetic field components of the first resonator in the axis of symmetry direction of the second resonator.

Given this observation, we can assume that, in general, mutual coupling coefficient of two different cylindrical DR with the mode \( H_{1,0,1}^{+} \) will be represented as:

\[ \kappa_{1,2} = \kappa_0 \cdot \varphi_1^{(1)}(l) \varphi_2^{(1)}(l)^* \cdot (\vec{h}_1(\Delta r, \theta_1, \phi_1), \vec{n}_2) \]  

(13)

where \( \vec{h}_1(\Delta r, \theta_1, \phi_1) \) - is the magnetic field of the first DR in the center of the second one; \( \vec{n}_2 = \vec{n}_2(\theta_2, \phi_2) \) is the unit vector directed from the DR' center along the axis of second DR (fig. 5, a).

The relation (13) is exactly the same (9-12) in the case of AA; AB; AC and CC DR position. Fig. 5, b - e shows mutual coupling coefficients as a function of arbitrary relative DR orientation.
Fig. 5. General position of the Cylindrical DRs in the Open space. Mutual coupling coefficients (b - e) as a function of the relative DR orientation: \( k_1 \Delta r = 3 \); \( b, c: \phi_1 = \phi_2 = 0 \); curve 1: \( \theta_1 = \pi / 2 \); curve 2: \( \theta_1 = \pi / 4 \); curve 3: \( \theta_1 = \pi / 8 \); \( d, e: \phi_1 = 0 \); \( \theta_2 = \pi / 4 \); curve 1: \( \theta_1 = \pi / 2 \); curve 2: \( \theta_1 = \pi / 4 \); curve 3: \( \theta_1 = \pi / 8 \).

Conclusions

Analytical relationships for mutual coupling coefficients for the \( H_{1,0,1}^+ \) modes of different Cylindrical DR in the Open space has been obtained and investigated. It stated, that mutual coupling coefficients are determined by the dependencies on the DR magnetic field and relative orientation of the DR axes.

The resulting ratio can be used for calculations of the DR natural oscillations, as well as the scattering parameters of the various element gratings in the communication devices with dielectric resonators.

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Трубін О. О. Коефіцієнти зв'язку різних циліндричних діелектричних резонаторів у відкритому просторі. Приведені результати розрахунків коефіцієнтів взаємного зв'язку циліндричних діелектричних резонаторів різних відносних розмірів з магнітними типами коливань. Знайдено загальну формулу для коефіцієнтів взаємного зв'язку для довільної орієнтації циліндричних ДР у відкритому просторі. Розглянуті основні закономірності зміни зв'язку при варіації відносного положення резонаторів.

**Ключові слова:** циліндричні діелектричні резонатори різних розмірів; коефіцієнт зв'язку; магнітні типи коливань.

Трубін А. А. Коэффициенты связи различных цилиндрических диэлектрических резонаторов в открытом пространстве. Приведены результаты расчетов коэффициентов взаимной связи цилиндрических диэлектрических резонаторов различных относительных размеров с магнитными типами колебаний. Получена общая формула для коэффициентов взаимной связи для произвольной ориентации цилиндрических ДР в открытом пространстве. Рассмотрены основные закономерности изменения связи при вариации относительного положения резонаторов.

**Ключевые слова:** цилиндрические диэлектрические резонаторы разных размеров, коэффициенты связи, магнитные типы колебаний

Trubin A. A. *Coupling coefficients of different cylindrical dielectric resonators in the open space*. The calculation results of the mutual coupling coefficients of different relative sizes cylindrical dielectric resonators with magnetic modes are presented. A general formula for the mutual coupling coefficients for an arbitrary orientation of the cylindrical DRs in the open space are obtained. The basic patterns of coupling coefficient changing with the variation of the relative position of the resonators are examined.

**Keywords:** different cylindrical dielectric resonator, coupling coefficient, magnetic mode.