Research on Static Rolling Angular Stiffness of Linear Motion Ball Guide

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Abstract: The purpose of this study is to investigate the static rolling angular stiffness characteristics of the linear motion ball guide (LMBG). Firstly, a deformation compatible equation of the LMBG under rolling moment is derived by deformation compatible hypothesis, and then the model of static stiffness for the LMBG is developed using the Hertz contact theory, the deformation compatible equation and force equilibrium equation. Finally, the static rolling stiffness of the LMBG was analyzed systematically via a case study, and the simulated results were verified by the experimental data.

1. Introduction
The linear motion ball guideway (LMBG) has been widely applied in mechanical engineering. The contact stiffness of the rolling ball guideway has a significant impact on the performance of machinery. By far, two methods have been proposed to obtain the contact stiffness of the ball guideway.

1) Experimental testing: Some scholars developed the experiential setups to test the linear stiffness and the angular stiffness of the guideway by applying external forces on the ball guideway and measuring the displacements due to those forces [1-4].

2) Theoretical modelling: The contact stiffness of the linear ball guideway has been calculated using Hertz contact theory [5-8] or FEM methods [9-10]. However, the existing models mainly focus on vertical and horizontal stiffness of the LMBG. Models of angular stiffness for the LMBG have been still hardly available by now.

Comparing to the parameters identification, the theoretical modelling for contact stiffness is preferred, this can be attributed to the fact that theoretical model is convenient for the parameter design and sensitivity analysis, which are needed for the analysis of machines. On the contrary, the parameters identification method is time-consuming and costly.

This paper proposes the modeling method of the static rolling angular stiffness of the LMBG. Firstly, the physical equation is present using the Hertz contact theory. Secondly, the LMBG deformation compatible equation is derived. Thirdly, the force equilibrium equation of guideway is built. Finally, the simulated results are compared with the experimental ones via a case study to verify the proposed model.

2. Theoretical model of Rolling Angular Stiffness $k_{\theta x}$

2.1 Definition of stiffness parameter
Figure 1 shows a schematic view of the ball guideway. When the ball guideway is subjected to external
loads \([F_y, F_z, M_x, M_y, M_z]\), the displacements of carriage relative to rail are \([\delta_y, \delta_z, \theta_x, \theta_y, \theta_z]\). The stiffness of the ball guideway includes 2 line stiffness and 3 angular stiffness, which are defined as:

\[
\begin{align*}
  k_y &= \frac{dF_y}{d\delta_y} \\
  k_z &= \frac{dF_z}{d\delta_z} \\
  k_{\theta_x} &= \frac{dM_x}{d\theta_x} \\
  k_{\theta_y} &= \frac{dM_y}{d\theta_y} \\
  k_{\theta_z} &= \frac{dM_z}{d\theta_z}
\end{align*}
\]

![Carriage and Rail](image)

Fig.1 LMBG subjected to external loads

2.2 Stiffness modeling of LMBG

To simplify the calculation of LMBG stiffness, the following assumptions are made when modeling:

1) The contact bodies are isotropic and all the contact deformations occur in the elastic range.
2) The contact area dimensions are small compared with the radii of curvature of the bodies under load.
3) The effect of surface roughness can be neglected.
4) The friction of the ball guideway can be neglected.

2.2.1 Deformation compatible equation. Fig. 2 shows a cross-section of the LMBG. Point O is the origin of overall coordinate system for the LMBG. The center of the raceway is \(O\), and the raceway groove curvature centers of the carriage and rail are \(O_c\) and \(O_r\). The suffix \(i\) refers to the raceway groove number \(i\). The length and width of rectangle \(O_1O_2O_3O_4\) are \(2L_1\) and \(2L_2\), respectively. The reference diameter (diameter with zero-oversize) of the ball is \(d_0\). The oversize of balls is \(d_0\), the distances between \(O_c\) and \(O_r\) are \(A_0\) (without external load) and \(A_i\) (with external load). The conformity factor of the raceway is

\[
r_c = r_r = f \cdot d_0
\]
The following relationship can be obtained from literature\[5\]:

$$A_0 = m_0 + \delta_0 \quad (2)$$

where $\delta_0$ is the oversize of balls, $m_0$ is the reference distance between the raceway groove curvature centers. They are defined as

$$\delta_0 = d_b - d_0 \quad (3)$$

$$m_0 = r_c + r_r - d_b \quad (4)$$

The rolling moment $M_x$ will produce an angular displacement $\theta$. Fig. 3 shows the deformation schematic of the LMBG.

$$L = \sqrt{[L_1 - (f - 0.5)d_b \cos \beta_0]^2 + [L_2 - (f - 0.5)d_b \sin \beta_0]^2} \quad (5)$$
\[ \varphi = \arctan \left( \frac{L_1 - (f - 0.5)d_s \cos \beta_0}{L_2 - (f - 0.5)d_s \sin \beta_0} \right) \] (6)

\[ A_i = [(A_i \cos \beta_{i0} \pm L_i \theta_i \cos \varphi)^2 + (A_i \sin \beta_{i0} \mp L_i \theta_i \sin \varphi)^2]^{1/2} \] (7)

The positive sign of the former sign is for \( i=1,3 \) and the negative sign is for \( i=2,4 \). And the latter is conversely. The contact angle \( \beta_i \) is expressed as

\[ \cos \beta_i = \frac{A_i \cos \beta_{i0} \pm L_i \theta_i \cos \varphi}{A_i} \] (8)

The positive sign of the radical sign is for \( i=1,3 \) and the negative sign is for \( i=2,4 \).

The total Hertzian contact deformation \( \delta_{ij} \) of the \( j \)th ball in the \( i \)th raceway groove can be expressed as

\[ \delta_{ij} = A_i - m_0 \] (9)

2.2.2 Physical equation. The physical equation of conventional LMBG is as following:

\[ \alpha_n P_i^{2/3} = \delta_{ij} \] (10)

\( \alpha_n \) is Hertzian deformation factor. It is expressed as

\[ \alpha_n = \frac{2K(e)}{\pi} \left[ \frac{(1-e^2)\pi}{2E(e)} \right]^{1/3} \left( \frac{3}{2} \sum \rho \right)^{2/3} \] (11)

The detailed derivation process and the meaning of each physical parameter are shown in literature[5].

2.2.3 Rolling moment equilibrium equation. The rolling moment equilibrium equation of the carriage is as follows:

\[ M_i + \sum_{j=1}^{x} P_i L_i \sin \beta_i - \sum_{j=1}^{x} P_i L_i \cos \beta_i - \sum_{j=1}^{x} P_i L_i \sin \beta_2 + \sum_{j=1}^{x} P_i L_i \cos \beta_2 \]

\[ - \sum_{j=1}^{x} P_i L_i \sin \beta_3 + \sum_{j=1}^{x} P_i L_i \cos \beta_3 + \sum_{j=1}^{x} P_i L_i \sin \beta_4 - \sum_{j=1}^{x} P_i L_i \cos \beta_4 = 0 \] (12)

Due to the symmetry of the structure and the anti-symmetry of the load, we can get

\[ P_1 = P_3, P_2 = P_4 \]

\[ \beta_1 = \beta_3, \beta_2 = \beta_4 \] (13)

Eq. (14) can be simplified as

\[ M_i + 2z P_4 L_i \sin \beta_1 - 2z P_4 L_i \cos \beta_1 + 2z P_4 L_i \cos \beta_4 - 2z P_4 L_i \sin \beta_4 = 0 \] (14)

The expression of the rolling angular stiffness of the LMBG is as following:

\[ k_{\theta_s} = \frac{dM}{d\theta_s} = \frac{d}{d\theta_s} (-2z P_4 L_i \sin \beta_1 + 2z P_4 L_i \cos \beta_1 - 2z P_4 L_i \cos \beta_4 + 2z P_4 L_i \sin \beta_4) \] (15)

The unknown parameters \( P_1, P_4, \beta_1, \beta_4 \) and \( \theta_s \) can be determined by simultaneously solving Eqs. (7)-(10) and (14) with the Newton iteration method.

3. Result and Discussion

In this case, the linear motion ball guide (product model: HGH45CA) is selected as the object, its static rolling stiffness is calculated to verify the proposed model. The simulated results are compared with experimental results by Heng in Ref. [2]. The main geometric and physical parameters of the LMBG are listed in Table 1.
Table 1 Parameters of linear motion ball guide (HGH45CA)

| Item                           | Value  | Item                           | Value  |
|--------------------------------|--------|--------------------------------|--------|
| Ball diameter, $d_b$ (mm)      | 4.7625 | Geometric dimension, $L_1$ (mm)| 28     |
| Conformity factor, $f$         | 0.52   | Geometric dimension, $L_2$ (mm)| 7.6    |
| Number of balls in a raceway, $z$ | 13     | Young’s modulus, $E$ (Gpa)    | 207.9  |
| Preload, $\delta_0$ (μm)      | 2      | Poisson’s ratio, $\nu$        | 0.27   |

Fig. 5 shows the angular deformation of guideway with the moment $M_x$. It can be seen that the angular deformation $\theta$ will increase significantly with the moment $M_x$. The angular deformation increases from $0.857 \times 10^{-5}$ rad to $8.70 \times 10^{-5}$ rad as the moment increases from 1.5Nm to 15Nm. As it is also shown, the theoretic results between loads and angular deformations agree with the experimental results. Moreover, the theoretic values are closer to the experimental data when the moments are smaller, it is attributed to the fact that a slight shift will occur for the center O of the carriage with the increasing of the moment.

4. Conclusion

This paper studies the modeling method of static rolling stiffness of LMBG. By a case study, the static rolling stiffness of the LMBG is analyzed systematically, and the theoretical values are verified by the experiment. Based on the discussion, the following conclusions can be drawn:

(1) The deformation compatible equation of LMBG is derived by the deformation compatible hypothesis, and then the static rolling stiffness of guideway can be modeled using the Hertz contact theory, the deformation compatible equation and force equilibrium equation of ball guideway.

(2) This study has proposed a modelling method for the stiffness of LMBG. With the modelling method, it becomes convenient to perform a forward design for a machine with the LMBG.

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