RESEARCH ARTICLE

Generalized Kubelka’s theory for light transmission in multilayer materials and its application for UV light penetration in filtering facepiece respirators

Ramin Farnood1* | Housyn Mahmoud2 | John Gibson1© | Ted Mao1,3 | Conrad Odegaard4

1Department of Chemical Engineering and Applied Chemistry, University of Toronto, Toronto, Ontario, Canada
2ZAP Sterilization Solutions, Toronto, Ontario, Canada
3MW Technologies Inc, London, Ontario, Canada
4GAP EnviroMicrobial Services Ltd, London, Ontario, Canada

*Correspondence
Ramin Farnood, Department of Chemical Engineering and Applied Chemistry, University of Toronto, Toronto, Ontario, Canada.
Email: ramin.farnood@utoronto.ca

Abstract
The spread of SARS-CoV-2 has resulted in the shortage of filtering facepiece respirators (FFRs). As a result, the use of ultraviolet (UV) irradiation for disinfection and reuse of FFRs has been the topic of much investigation. In this article, a mathematical model is developed based on Kubelka’s theory to determine light transmission in multilayer materials, such as N95 masks. Using this model, the predicted UV transmittance and absorbance of a N95 mask layers were found to be in close agreement with the experimental values. In addition, when the mask was exposed to UV equally from both surfaces, the estimated minimum UV irradiance inside the N95 mask was 14.5% of the incident irradiance, suggesting a significant degree of light penetration. The proposed model provides a simple and practical methodology for the design and use of UV decontamination equipment for FFRs and other multilayer materials.

KEYWORDS
COVID-19, disinfection, N95 masks, ultraviolet light

INTRODUCTION

As of July 2022, there have been 550 million cases of COVID-19 and over 6.3 million deaths [1]. To help slow the spread of this global pandemic, filtering facepiece respirators (FFRs) have been recommended for use by the public and health care workers [2]. FFRs can intercept suspended droplets that contain the virus and can interrupt the airflow during a cough or sneeze, possibly reducing the transport distance [3]. A ubiquitous example of an FFR is the US National Institute for Occupational Health and Safety approved N95 mask.

However, there are number of challenges associated with these often single-use, disposable FFRs. Early in the pandemic, the sudden demand for these devices exceeded the supply [4]. In Tokyo, for example, 67% of registered clinics surveyed reported shortages of surgical masks in April 2020 [5]. In the United States, critical shortages of FFRs were reported in March 2020 [6]. Over time, FFR supply has increased, but it is likely that these increases are temporary. When the next pandemic
arrives, shortages are once again possible, as suppliers ramp up production. As such, it is important to research approaches that can effectively increase the supply of these masks in a crisis. One such strategy is the disinfection and reuse of these masks, which is the topic of this research.

A second challenge with single-use, disposal masks is their environmental impact. Discarded FFRs and other masks have been reported to play a key role in microplastics pollution [7], as a threat to the marine ecosystem [8], and are, for example, invading tourist beaches in Chile at a rate of three masks per day per kilometer of beachfront [9]. During the pandemic, the single-use mask was a trade-off many were willing to make to protect health care workers and public health. However, in the longer-term mask decontamination and reuse is a way to reduce the resources required to manufacture and distribute these masks, as well as some the negative consequences associated with their disposal.

It has been suggested the ideal FFR decontamination technology would preserve performance and fit, leave no residual toxicity, and be fast-acting, inexpensive, and readily available [10]. Decontamination with germicidal ultraviolet (UV) light was included in three technologies that approach these requirements, along with microwave-generated steam, and warm (−65°C) humid air [10]. This work showed a 4 to 6 log reduction in the H1N1 virus in droplets on FFR material after UV at an intensity of 1.6 to 2.0 mW/cm² for 15 minutes. In 2020 review of FFR decontamination, UV was included among three “promising” technologies [11]. Similarly, in a 2021 review of FFR decontamination focused on the COVID-19 pandemic, 60% (9 of 15) of the reported studies used UV light for decontamination [12]. In general terms, most hospitals have access to UV lamps and UV exposure of FFRs provides a simple and inexpensive FFR reuse solution.

Although many authors have demonstrated that UV works for FFR decontamination, there have been studies examining the fundamental science and optics needed for the effective implementation of this technology. Abdalrhman et al [13] presented a Monte Carlo simulation of light transport in several FRR types and suggested that UV viral decontamination may be improved by exposing FFRs to an isotropic UV irradiation. Similarly, Lilge et al [14] examined the numerical simulation of light propagation through several FFRs and found that the minimum Fluence rate in their study was in the nW/cm² range. Others used a combination of theoretical and experimental approach to estimate the minimum dose received by the FFR to be 1 J/cm² [15].

Here, we present an analytical model to study the interaction of UV light with FFRs by extending the well-known Kubelka’s theory that describes light penetration in absorbing media [16]. The generalized form of Kubelka’s theory developed in this study allows us to estimate UV light transmission in multilayer materials such as FFRs. This analytical model was then combined with a simple experimental approach to determine the UV irradiance profile within the layers of an N95 mask based on measurements of UV transmittance and reflectance of individual mask layers.

1.1 | Experimental

The facepiece filtering respirator used in this study was a SPERIAN ONE-Fit W1400 N95 mask. A 6 × 6 cm sample was cut out of the mask for analysis. There were five layers of filter materials in the sample, labeled sequentially from L1 to L5, with L1 representing the outside layer and L5 representing the inside, face-touching layer.

A custom-made collimated beam device (GAP Environmental Services Ltd.) equipped with three low-pressure germicidal UV lamps (Philips TUV30W) was used for irradiation. This device generated a 10-cm diameter collimated beam of UV light. Irradiance measurements were conducted using an International Light IL 1700 Research Radiometer (International Light Technologies) equipped with a SED240 vacuum photodiode detector (185-320 nm with 240 nm peak response) fitted with a 25.15 mm internal hemispherical solid quartz diffuser (model W) with cosine response from 200 to 2100 nm and a 25 mm neutral density filter.

For UV transmission measurements (Figure 1A), the detector was located immediately below the sample and at the center of the collimated beam. The incident radiation (0.124 mW/cm²) was determined without a sample in place.
For the reflection measurements, detector was positioned 5.3 cm above the sample and 7.5 cm away from the center line of the collimated light beam while sample was placed on a black backing (Figure 1B). All samples were irradiated both facing up or down and the average values are reported.

From Figure 1B, given the finite field of view of the detector in this study, only a fraction of the reflected UV can be captured during the measurements. To correct for this, assuming that sample is an ideally diffuse (Lambertian) reflector, and by applying cosine-to-the-fourth approximation, the irradiance measured by the detector for layer $i$, that is, $E_{d,i}$, is estimated from [16]:

$$E_{d,i} = \frac{2\pi r_s^2 \cos^4 \theta}{r_s^2 + r_d^2 + s_{sd}^2 + \left[(r_s^2 + r_d^2 + s_{sd}^2)^2 - 4r_s^2r_d^2\right]^{1/2}} E_{R,i}$$  \hspace{1cm} (1)

Here, $E_{R,i}$ is the reflected irradiance, $r_s$ is the radius of the illuminated area, $r_d$ is the radius of the detector, $s_{sd}$ is the vertical distance between detector and the sample, and $\theta$ is the offset angle defined as:

$$\theta = \tan^{-1}\left(\frac{b}{s_{sd}}\right)$$  \hspace{1cm} (2)

where $b$ is the off-axis distance between the detector and the center line of collimated beam. For the experimental set up used in this study, $r_s = 5.0$ cm, $r_d = 1.25$ cm, $s_{sd} = 5.3$ cm, and $b = 7.5$ cm. Hence, the corrected reflected irradiance could be calculated from:

$$E_{R,i} = 6.2 E_{d,i}$$  \hspace{1cm} (3)

For transmitted irradiance, given that samples were resting on the detector, the measured irradiance values were used without any correction.

1.2 | Extending Kubelka’s theory of light transmission to multilayer materials

Following Kubelka [16], light penetrating in a two layer flat material is partly absorbed and partly transmitted by
Photographs of individual layers of N95 mask used in this study, with L1 being the outer most layer, facing away from the face, and L5 being the inner most layer, touching the face.

**Table 1** Transmitted and reflected irradiance for mask layers and N95 mask

| Layer            | Transmitted (mW/cm²) | Reflected (mW/cm²) |
|------------------|----------------------|--------------------|
| L1               | 5.7E-02              | 2.0E-02            |
| L2               | 1.0E-02              | 1.3E-02            |
| L3               | 4.7E-02              | 4.3E-02            |
| L4               | 4.0E-02              | 4.7E-02            |
| L5               | 7.5E-02              | 1.6E-02            |
| Mask (L1-L5)     | 4.3E-04              | 2.0E-02            |

*Note: Incident radiation: 0.124 mW/cm²*

Each layer (Figure 2A). The overall reflectance, $R_{1,2}$, and transmittance, $T_{1,2}$, of such a material is:

$$R_{1,2} = R_1 + \frac{T_1^2 R_2}{1 - R_1 R_2}$$  \hspace{1cm} (4)

$$T_{1,2} = \frac{T_1 T_2}{1 - R_1 R_2}$$  \hspace{1cm} (5)

where $R_i$ and $T_i$ are reflectance and transmittance of the $i$th layer, and $R_{1,2}$ and $T_{1,2}$ are the reflectance and transmittance of the two-layer material.

Below, we extend Kubelka’s theory to composite materials with an arbitrary number of layers. Consider a three-layer material as in Figure 2B. We replace layers 1 and 2 with a single hypothetical layer with reflectance and transmittance of $R_{1,2}$ and $T_{1,2}$, as provided by Equations (4) and (5). In this way, the three-layer material can be treated as a pseudo bilayer material, such that:

$$R_{1,3} = R_{1,2} + \frac{T_{1,2}^2 R_3}{1 - R_{1,2} R_3}$$  \hspace{1cm} (6)

where $R_{1,3}$ is the reflectance of the three-layer structure, and similarly:

$$T_{1,3} = \frac{T_{1,2} T_3}{1 - R_{1,2} R_3}$$  \hspace{1cm} (7)

where $T_{1,3}$ denotes the transmittance of the three-layer structure.

Continuing the above procedure, the following recursive relationships for reflectance and transmittance of composite flat structures containing $n > 2$ layers is obtained:

$$R_{1,n} = R_{1,(n-1)} + \frac{T_{1,(n-1)}^2 R_n}{1 - R_{1,(n-1)} R_n}$$  \hspace{1cm} (8)

$$T_{1,n} = \frac{T_{1,(n-1)} T_n}{1 - R_{1,(n-1)} R_n}$$  \hspace{1cm} (9)

Here, $R_{1,n}$ and $T_{1,n}$ are reflectance and transmittance of an $n$-layer material irradiated from layer 1.

Now, we determine irradiance at the interface of layers $m$ and $m + 1$ in an $n$-layer structure, herein referred to as the interlayer irradiance. For this, let us partition the material into two sections (Figure 2C). The top section containing layers 1 to $m$ and the bottom section containing layers $m + 1$ to $n$. The reflectance and transmittance of the top section (ie, layers 1 to $m$) are $R_{1,m}$, $T_{1,m}$, and for the bottom section (ie, layers $m + 1$ to $n$) are $R_{m+1,n}$ and $T_{m+1,n}$, respectively. Referring to Figure 2C, at this interface, light bounces between layers $m$ and $m + 1$ infinite number of times, reducing in intensity after each reflection. Hence, it can be shown that the irradiance at the interface of these two layers, $E_{m,m+1}$, is given by:

$$E_{m,m+1} = T_{1,m} + T_{1,m} R_{m+1,n} + T_{1,m} R_{m+1,n} R_{1,m} + T_{1,m} R_{m+1,n} R_{1,m}^2 + ...$$  \hspace{1cm} (10)

where $E_o$ is the irradiance impinging on layer 1. Hence, the interlayer irradiance between layers $m$ and $m + 1$ in a $n$-layer material can be determined from:
TABLE 2  Observed UV reflectance, transmittance, and absorbance of each of the five layers of the N95 mask

| Layer | Transmittance ($T_i$) | Reflectance ($R_i$) | Absorbance ($A_i$) |
|-------|-----------------------|---------------------|--------------------|
| L1    | 46%                   | 16%                 | 38%                |
| L2    | 8%                    | 11%                 | 81%                |
| L3    | 38%                   | 35%                 | 27%                |
| L4    | 32%                   | 38%                 | 30%                |
| L5    | 60%                   | 13%                 | 27%                |
| Mask (L1-L5) | 0.3%       | 16%                 | 84%                |

\[ T_i = \frac{E_{T,i}}{E_0} \]

\[ R_i = \frac{E_{R,i}}{E_0} \]

where, $E_o$ was measured to be 0.124 mW/cm² (see Control in Table 1). Results summarized in Table 2 show that L2 had the largest absorption (81%) among all layers of this N95 mask. It is worth noting that this layer constituted 40% of the N95 mask by weight.

2.1  |  Model validation

The $T$ and $R$ for individual layers were measured and using Equations (6) and (7) the $T$ and $R$ for different two- and three-layered materials was predicted. This was compared to the measured $T$ and $R$ for these multiple-layer stacks in Figure 4 below. Overall, there is very good agreement between measured and predicted values with an $R^2$ value of .97.

2.2  |  Case studies

Using the generalized Kubelka’s theory [16] described earlier, here we examine the effect of layered structure of mask on UV propagation for the following cases.

2.2.1  |  Case 1: Mask is irradiated from outside layer (L1)

Here, we stack individual layers of the mask one at a time creating increasingly thicker structures (ie, L1, L1 + L2, L1 + L2 + L3, L1 + L2 + L3 + L4, and finally L1 + L2 + L3 + L4 + L5). By increasing the number of layers, $n$, the predicted UV transmittance of the resulting material, $T_{1,n}$, decreased from 16.4% for $n = 1$ (only L1) to 18.7% for $n = 5$ (all five layers stacked) while UV absorbance, $A_{1,n}$, increased from 37.8% to 81.0% (Table 3). These results also confirm that L2 contributed the most to the absorbance of UV in this sample.
2.2.2 | Case 2: Mask is irradiated from inside layer (L5)

Here, once again we stack the individual layers, however, in the reverse order, namely L5, L5 + L4, L5 + L4 + L3, L5 + L4 + L3 + L2, and finally L5 + L4 + L3 + L2 + L1. Although the predicted trends in this case were like those of Case 1, transmittance, reflectance, and absorbance values were not the same. Moreover, compared to Case 1, the calculated UV reflectance increased to 28.9% while transmittance and absorbance decreased to 0.28% and 70.8%, respectively. This phenomenon is due to the anisotropy in the layered structure of this mask.

2.2.3 | Case 3: Mask is irradiated from both sides

We can now estimate the interlayer irradiance, if the mask were irradiated identically from both sides.

| TABLE 3 | Calculated UV transmittance and reflectance for a multilayer material using the extended Kubelka model |
|-----------------|-----------------|-----------------|-----------------|
| **Case 1: Irradiated L1-L5** | **Structure** | **Reflectance, $\bar{R}_{1,n}$** | **Transmittance, $\bar{T}_{1,n}$** | **Absorbance, $\bar{A}_{1,n}$** |
| n | L1 | 16.4% | 45.8% | 37.8% |
| 2 | L1 + L2 | 18.6% | 3.8% | 77.6% |
| 3 | L1 + L2 + L3 | 18.7% | 1.5% | 79.8% |
| 4 | L1 + L2 + L3 + L4 | 18.7% | 0.53% | 80.8% |
| 5 | L1 + L2 + L3 + L4 + L5 | 18.7% | 0.33% | 81.0% |
| **Case 2: Irradiated L5-L1** | **Structure** | **Reflectance, $\bar{R}_{5,n}$** | **Transmittance, $\bar{T}_{5,n}$** | **Absorbance, $\bar{A}_{5,n}$** |
| n | L5 | 12.8% | 60.4% | 26.8% |
| 4 | L4 + L5 | 27.3% | 20.5% | 52.2% |
| 3 | L3 + L4 + L5 | 28.9% | 7.4% | 63.7% |
| 2 | L2 + L3 + L4 + L5 | 28.9% | 0.60% | 70.4% |
| 1 | L1 + L2 + L3 + L4 + L5 | 28.9% | 0.28% | 70.8% |

*Predicted from Equations (8) and (9).

**FIGURE 5** Normalized interlayer UV irradiance ($E_{m,m+1}$) within the N95 mask predicted using Equation (11). Case 1: mask was irradiated from outside, Case 2: mask was irradiated from inside, and Case 3: mask was irradiated identically from both sides.
Knowing the interlayer irradiance for outside illumination, \([E_{m,m+1}]_{\text{Case } 1}\), and that for inside illumination, \([E_{m,m+1}]_{\text{Case } 2}\), the total interlayer irradiance in this case can be obtained from:

\[
[E_{m,m+1}]_{\text{Case } 3} = [E_{m,m+1}]_{\text{Case } 1} + [E_{m,m+1}]_{\text{Case } 2}
\]  

Figure 5 shows the normalized interlayer irradiance, that is, \(E_{m,m+1}/E_o\), for the above three cases. When irradiating from one side only, the predicted value for interlayer irradiance decreased exponentially with distance from the irradiated surface reached to about 0.3% of the incident light intensity as light exited the mask material. However, in Case 3, the decrease in interlayer UV irradiance was compensated by the irradiation from both sides, such that the graph exhibits a minimum normalized irradiance (14.5%) between layers 2 and 3, that is, \(E_{2,3}\).

In practice, one can only measure the UV irradiance impinging on the surface of the mask. Therefore, to ensure the proper disinfection of this N95 mask, the irradiation time should be adjusted such that the delivered dose at the interface between layers two and three reaches the desired value. In this case, if the minimum dose required to achieve disinfection is denoted by \(D_{min}\), the minimum irradiance time should be set at:

\[
t_{\text{min}} = 6.7 \frac{D_{\text{min}}}{E_o}
\]

## 3 | SUMMARY AND CONCLUSIONS

These findings illustrate the utility of the generalized Kubelka’s theory developed in this work to predict the UV irradiance inside a multilayer material, such as an N95 mask, from the measurement of the transmittance and reflectance of individual layers. Using the proposed methodology, this study offers a simple and practical spreadsheet-based tool (Supporting Information) for the effective design and operation of UV equipment for decontamination of FFRs and other multilayer materials.

## NOMENCLATURE

\[A_i\] absorbance of \(i\)th layer  
\[E_o\] irradiance impinging on the surface of the mask  
\([E_{m,m+1}]\) interlayer irradiance at the interface of layers \(m\) and \(m+1\)  
\([R_i]\) reflectance of the \(i\)th layer  
\([R_{1,2}]\) combined reflectance of layers 1 and 2  
\([R_{1,n}]\) reflectance of an \(n\)-layer material  
\([T_i]\) transmittance of the \(i\)th layer  
\([T_{1,2}]\) combined transmittance of layers 1 and 2  
\([T_{1,n}]\) transmittance of an \(n\)-layer material

## ACKNOWLEDGMENTS

Authors would like to acknowledge support from Ontario Centers for Innovation (OCI) VIP program and the Michael and Theresa Wu COVID-19 Research Fund for financial support.

## DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available on request from the corresponding author.

## ORCID

Ramin Farnood [https://orcid.org/0000-0003-0915-8130](https://orcid.org/0000-0003-0915-8130)  
John Gibson [https://orcid.org/0000-0002-2680-0036](https://orcid.org/0000-0002-2680-0036)

## REFERENCES

[1] World Health Organization, COVID-19 Weekly Epidemiological Update. Report No.: 76, WHO, Geneva, Switzerland 2022.
[2] World Health Organization, Advice on the use of masks in the community, during home care and in healthcare settings in the context of the novel coronavirus (COVID-19) outbreak [Internet], WHO, Geneva, Switzerland 2020.
[3] Y. Bu, R. Ooka, H. Kikumoto, W. Oh, Sustain. Cities Soc. 2021, 73, 103106.
[4] Centers for Disease Control and Prevention, Healthcare Workers [Internet], https://www.cdc.gov/coronavirus/2019-ncov/hcp/ppe-strategy/decontamination-reuse-respirators.html (accessed: Feb 2022).
[5] M. Inaba, H. Naito, T. Sakata, A. Nakao, Acute Med. Surg. 2020, 7, e527.
[6] M. L. Ranney, V. Griffeth, A. K. Jha, N. Engl. J. Med. 2020, 382, e41.
[7] O. O. Fadare, E. D. Okollo, Sci. Total Environ. 2020, 737, 140279.
[8] S. Dharmaraj, V. Ashokkumar, S. Hariharan, A. Manibharathi, P. L. Show, C. T. Chong, C. Ngamcharussrivichai, Chemosphere 2021, 272, 129601.
[9] M. Thiel, D. de Veer, N. L. Espinoza-Fuenzalida, C. Espinoza, C. Gallardo, I. A. Hinojosa, T. Kiessling, J. Rojas, A. Sanchez, F. Sotomayor, N. Vasquez, R. Villablancia, Sci. Total Environ. 2021, 786, 147486.
[10] B. K. Heimbuch, W. H. Wallace, K. Kinney, A. E. Lumley, C.-Y. Wu, M.-H. Woo, J. D. Wander, Am. J. Infect. Control 2011, 39, e1.
[11] A. Polkinghorne, J. Braney, J. Hosp. Infect. 2020, 105, 663.
[12] K. Seresirikachorn, V. Phoophiboon, T. Chobarporn, K. Tiankanon, S. Aeumjaturapat, S. Chusakul, K. Sridvongs, Infect. Control Hosp. Epidemiol. 2021, 42, 25.
[13] A. S. Abdalrhman, C. Wang, A. Manalac, M. Weersink, A.-A. Yassine, V. Betz, B. Barbeau, L. Lilge, R. Hofmann, J. Biophotonics 2021, 14, e202100135.
[14] L. Lilge, A. Manalac, M. Weersink, F. Schwiegelshohn, T. Young-Schultz, A. S. Abdalrhman, C. Wang, A. Ngan, F. X. Gu, V. Betz, R. Hofmann, J. Biophotonics 2020, 13, e202000232.
[15] I. Kohli, A. B. Lyons, B. Golding, S. Narla, A. E. Torres, A. Parks-Miller, D. Ozog, H. W. Lim, I. H. Hamzavi, Photochem. Photobiol. 2020, 96, 1083.
[16] P. Kubelka, J. Opt. Soc. Am. 1954, 44, 330.

**SUPPORTING INFORMATION**
Additional supporting information can be found online in the Supporting Information section at the end of this article.

**How to cite this article:** R. Farnood, H. Mahmoud, J. Gibson, T. Mao, C. Odegaard, J. Biophotonics 2022, 15(10), e202200068. [https://doi.org/10.1002/jbio.202200068](https://doi.org/10.1002/jbio.202200068)