A Computational Fluid Dynamic Study on Efficiency of a Wavy Microchannel/Heat Sink Containing Various Nanoparticles

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Abstract: In this paper, a common and widely used micro-heat sink (H/S) was redesigned and simulated using computational fluid dynamics methods. This H/S has a large number of microchannels in which the walls are wavy (wavy microchannel heat sink: WMCHS). To improve cooling, two (Al2O3 and CuO) water-based nanofluids (NFs) were used as cooling fluids, and their performance was compared. For this purpose, studies were carried out at three Reynolds numbers (Re) of 500, 1000, and 1500 when the volume percent (ϕ) of the nanoparticles (NPs) was increased to 2%. The mixture two-phase (T-P) model was utilized to simulate the NFs. Results showed that using the designed WMCHS compared to the common H/S reduces the average and maximum temperatures (T-Max) up to 2°C. Moreover, using the Al2O3 NF is more suitable in terms of WMCHS temperature uniformity as well as its thermal resistance compared to the CuO NF. Increasing the ϕ is desirable in terms of temperature, but it enhances the pumping power (PP). Besides, the Figure of Merit (FOM) was investigated, and it was found that the value is greater at a higher volume percentage.

Keywords: wavy microchannel heat sink; two-phase model; nanofluid; numerical study

1. Introduction

Nowadays, many available devices are intelligent and contain a variety of electronic components. One of the substantial obstacles in these electronic devices is concerned with the overheating issue during operation. These devices get hot by consuming electricity and performing their functions at the same time. This situation results in the consumption of electricity. In these components, overheating causes damage to the devices or burns these components. Therefore, it is essential to find a way to cool these devices. Using H/S is one of the best solutions for cooling different electronic components [1,2]. Heat sinks are established in a wide range of shapes and applications. Many of these heat sinks are cooled by airflow, and they have various applications [3,4]. However, in some devices with very high heating, the airflow is not sufficient for the cooling process. Water and fluids are utilized for the cooling process. Therefore, researchers have used different fluids in their research according to the conditions of their problem. [5–7]. According to the results of various research, the application of nanofluid (NF) can improve the thermal efficiency in heat exchangers [8–11]. Nanotechnology has been considered by researchers for many years [12–16]. The thermal conductivity in NF is higher than in fluids such as...
Several researchers have employed NFs in their investigations [21–24]. Heat sinks are not an exception, and NFs are commonly used in heat sinks [25–29]. Bahraie and Heshmatian [30] employed commercial software to numerically simulate the flow of a specific NF in a H/S. They changed the percentage of nanoparticles (NPs) and the inlet flow rate. The results revealed that growing the Reynolds number (Re) and volume percentage of the NPs decreases the maximum H/S temperature. By increasing the one percent by volume of water-based Na, the maximum temperature (T-Max) has dropped by 2.21 °C.

Ambreen and Kim [31] developed a numerical solution, and they investigated the thermal efficiency of a rectangular micro-H/S using titanium oxide NFs in water. In this study, several pin fins with different shapes were placed on the H/S. They found that circular pin fins, especially in higher Re numbers, could cool the H/S more than other shapes of pin fins. Additionally, an increase in the Re leads to the enhancement of the heat transfer from the H/S to the fluid. Alfaryjat et al. [32] employed a numerical method to investigate a micro-H/S containing various microchannels. They utilized different types of NPs in their study, including SiO₂, CuO, and Al₂O₃. In this study, water fluid was used as the base fluid. They evaluated the effects of these three types of NFs on a micro-H/S. This procedure was performed by changing the Re in the slow flow range. These three types of NPs had different diameters. The results demonstrated that the temperature gradient formed in the H/S is affected by this diameter. Wu et al. [33] conducted a numerical study on a rectangular H/S considering many microchannels. The thermal efficiency of the H/S was analyzed by flowing various NFs in the H/S. Similar to many other heat sinks mentioned above, constant heat flux was applied from the floor. The analysis results demonstrated that the use of a hybrid NF of copper and alumina in water dramatically dropped the temperature of the H/S in comparison with applying other NFs. The results of studies on the channels and pipes revealed that the use of wavy walls instead of a flat wall improves heat transfer [34–38]. In one of these articles, Vanaki et al. [38] studied the impact of wavy walls on a macro-sized channel. They utilized silicon oxide NPs in the study. They found that an intensification in the NF flow rate leads to enhancement of the heat transfer. However, this process simultaneously intensifies the pressure drop, which is an undesirable phenomenon.

According to the review of previous articles, the temperature of electronic components such as CPU increases as a result of their operation, which damages them. Hence, due to their micro and millimeter dimensions, cooling them is very complicated. Thus, in this paper, the geometry of a MHS was investigated by changing its microchannel walls. Two widely used NFs of Al₂O₃ and CuO/waters were utilized as the operating fluid. The effect of using these two NFs was examined. The two-phase (T-P) method was employed to better simulate the NFs; the effect of NFs diameter was also studied. The output and loss exergies, as well as the second law efficiency, are the other issues studied. In the end, the thermal efficiency of the MHS was investigated by increasing the volume percent (ϕ) of each NP up to 2% in the Re range of 500 to 1500. The innovation of the present article is the use of wavy channel walls in heat sink using alumina/water and copper oxide/water nanofluids.

2. Problem Definition

As displayed in Figure 1, the micro-H/S has a circular output and input placed on its top. The material of this H/S is aluminum, and it includes the principal part and upper part. The principal part of the H/S consists of 20 microchannels. The walls of these microchannels are wavy. The H/S has heights of 0.24 and 0.36 mm without and with considering the door, respectively. The inlet and outlet of the fluid have the same diameter of 0.19 mm. In the H/S, two different states are considered: (I) flowing copper/water oxide NF and (II) flowing water/alumina. The range of volume percentage of NPs was from 0 to 2%. These values were evaluated by the Re of 500, 1000, and 1500. A region under the H/S receives a constant flux from the operation of a Core i7 CPU.
the door, respectively. The inlet and outlet of the fluid have the same diameter of 0.19 mm. In the H/S, two different states are considered: (I) flowing copper/water oxide NF and (II) flowing water/alumina. The range of volume percentage of NPs was from 0 to 2%. These values were evaluated by the Re of 500, 1000, and 1500. A region under the H/S receives a constant flux from the operation of a Core i7 CPU.

Figure 1. The model and boundary conditions applying to the geometry of the problem.

3. Governing Equations

3.1. T-P Mixture Equations

The equations inside the WMCHS are in the form of a T-P mixture model, which are presented here. These equations have been written for the permanent flow of a Newtonian and incompressible fluid. The equations of conservation of mass, momentum, and energy in the T-P mixture model are solved for the mixture as the same as the volume percent equation for the second phase [39].

\[ \nabla \cdot (\rho_{mi} \vec{v}_{mi}) = 0 \]  

\[ \nabla \cdot (\rho_{mi} \vec{v}_{mi} \nabla \vec{v}_{mi}) = -\nabla P_{mi} + \nabla \cdot \left( \mu_{mi} \nabla \vec{v}_{mi} + \sum_{k=1}^{n} \phi_k \rho_k \vec{v}_{dr,k} \vec{v}_{dr,k} \right) \]  

\[ \nabla \cdot \left( \sum_{k=1}^{n} \phi_k \rho_k c_p \vec{v}_{k} T \right) = \nabla \cdot (k_{mi} \nabla T) \]  

where \( \vec{v}, T \) and \( P \) referred to velocity, temperature, and pressure, \( \rho \) and \( k \) are density and thermal conductivity, and \( \mu \) and \( c_p \) are viscosity and heat capacity. The subscript \( mi \) indicates mixture. The amount of the average mixture velocity, density, viscosity, and thermal conductivity can be obtained by the following equations [40]:

\[ \vec{v}_{mi} = \frac{\sum_{k=1}^{n} \phi_k \rho_k \vec{v}_{k}}{\rho_{mi}} \]  

\[ \rho_{mi} = \sum_{k=1}^{n} \phi_k \rho_k \]
\[ \mu_{mi} = -0.4892 + \frac{26.9036}{\phi} + 0.6837 \phi + \frac{24.1141}{\phi^2} - 0.1785 \phi^2 + 0.1818 \phi^3 + 27.015 \phi^4 + 0.0132 \phi^3 - 2940.1775 \phi^4 (Al_2O_3) \]

\[ \mu_{mi} = -0.4262 + \frac{8.4312}{\phi} + 0.898 \phi + \frac{524.7147}{\phi^2} - 0.2217 \phi^2 - 4.7329 \phi^3 + 70.3105 \phi^4 + 0.0176 \phi^3 - 5559.4641 \phi^4 (CuO) \]

\[ \frac{k_m}{T} = 1.0 + 1.0112 \phi + 2.4375 \phi \left( \phi \frac{\phi}{\phi} \right) - 0.0248 \phi \left( \phi \frac{\phi}{\phi} \right) (Al_2O_3, CuO) \]

The equation of volume percentage is formulated as follows:

\[ \nabla \cdot \left( \phi \rho_p \nu_{mi} \right) = - \nabla \cdot \left( \phi \rho_p \nu_{dr,p} \right) \]  

(7)

Additionally, drift viscosity is represented for NP as follows (it is for the \( k \)-th phase):

\[ \nu_{dr,k} = \nu_{pf} - \sum_{i=1}^{n} \frac{\phi_i \rho_i \nu f_k}{\rho_{mi}} \]  

(8)

The sliding velocity is defined as the second phase velocity that is dependent on the first phase.

\[ \nu_{pf} = \nu_f - \nu_f \]  

(9)

\[ \nu_{pf} = \frac{\rho_p \nu_f^2 (\rho_p - \rho_{mi})}{18 \mu_{f,drag} \rho_p} \]  

(10)

\[ f_{drag} = \begin{cases} 1 + 0.15 Ra^{0.687}, & Ra_{f} \leq 1000 \\ 0.0183 Ra^{0.687}, & Ra_{f} > 1000 \end{cases} \]  

(11)

In the above equations, the \( p \) and \( f \) indices are related to the NP and base fluid. Additionally, the gravitational acceleration is defined as follows:

\[ a = g - \left( \overrightarrow{\nu}_{mi} \nabla \right) \overrightarrow{\nu}_{mi} \]  

(12)

The value of \( g \) is not considered if the gravitational acceleration is ignored.

The standard \( k \)-\( \varepsilon \) turbulence model is used to simulate turbulent flow. This model is suitable for turbulent flow with low velocity. In the following, the governing equations of the standard \( k \)-\( \varepsilon \) turbulence model are presented:

\[ \frac{\partial (u \rho k)}{\partial x} = \frac{\partial}{\partial x} \left[ (\mu + \mu_t) \frac{\partial k}{\partial x} \right] + \mu_t \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) \frac{\partial v}{\partial x} - \rho \varepsilon \]  

(13)

\[ \frac{\partial (u \rho \varepsilon)}{\partial x} = \frac{\partial}{\partial x} \left[ (\mu + \mu_t) \frac{\partial \varepsilon}{\partial x} \right] + C_1 \frac{\varepsilon}{k_f} \mu_t \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right) \frac{\partial v}{\partial x} - \rho C_2 \varepsilon \frac{\varepsilon^2}{k_f} \]  

(14)

where \( \mu_t \) is the dynamic viscosity for turbulent flow (kg/m-s), \( k \) and \( \varepsilon \) turbulence kinetic energy and turbulent dissipation rate, respectively. \( C_1 \) and \( C_2 \) were constant values that are equal to 1.44 and 1.92, respectively. \( \sigma_t \) and \( \sigma_k \) are the turbulent flow Prandtl number and are 1.3 and 1.0, respectively.

Table 1 illustrates the properties of pure fluid and alumina and copper NPs. In this case, \( d \) is the diameter of the NPs (its unit is nm).

| Properties          | H_2O     | Al_2O_3 | CuO  |
|---------------------|----------|---------|------|
| \( C_p \) (J/kg-K)  | 4179     | 765     | 540  |
| \( k \) (W/m-K)     | 0.613    | 40      | 18   |
| \( \rho \) (kg/m^3) | 997.1    | 3970    | 6500 |
| \( \mu \) (kg/m-s)  | 0.001    | -       | -    |
| \( d_f \) (nm)      | -        | 13      | 100  |
3.2. Boundary Condition

The existing boundary conditions for the H/S are displayed in Figure 1. The inlet and outlet of the fluid have the same diameter of 0.19 mm. In this situation, the fluid enters the H/S with a constant velocity at the temperature of 300\(^{\circ}\) Kelvin, and then it exits from the H/S at the atmospheric pressure. A constant heat flux of 88,000 W/m\(^2\) is applied to the aluminum H/S in the surface area of 14.71 cm\(^2\) (Figure 1). These thermal boundary conditions operate as the cooling system for an Intel Core i7 CPU and its similar components. It is assumed that all the outer walls of the H/S are insulated, except the region subjected to the constant heat flux.

4. Numerical Process, Mesh Independency and Validation

After designing the geometry and applying mesh, the control volume and SIMPLE algorithm solved the velocity-pressure coupling. After activating the T-P mixture model, the first-order upwind scheme was employed to solve the momentum and energy equations. The convergence criterion was \(10^{-7}\) for the equations. To use the models for thermal conductivity and viscosity, an in-house UDF code has been added to the software to calculate the values of thermal conductivity and viscosity correlations more accurately.

By examining various grids on the geometry, the grid with 1,220,000 elements was chosen as the grid. Figure 2 shows the comparisons between different factors (e.g., average temperature and T-Max of H/S) for selecting mesh according to the number of elements considering two Re numbers and 2% percent by volume of NPs. According to this figure, the mentioned mesh was chosen for this geometry.

![Figure 2. Grid independency study comparing the output temperature of the WMCHS based on the number of grids.](image)

To validate and investigate the accuracy of the performed numerical work results, a comparison between the current research work and a number of similar articles was carried out. For this goal, the findings were compared to a T-P numerical method by Moraveji and Ardehali [39] and an experimental work by and Ho and Chen [43]. As displayed in Figure 3, the difference between the findings of the T-P method used in this work and ref [43] is small.
5. Thermal Analysis

In the following, the parameters, which are utilized to scrutinize the thermal efficiency of the WMCHS, are introduced. One of the most important of these parameters is the heat transfer coefficient expressed as follows [30]:

\[
h = \frac{q''}{T_{\text{Mid}} - T_m}
\]  

(15)

where \( T_m \) is \( \frac{T_{\text{in}} + T_{\text{out}}}{2} \) (\( T_{\text{in}} \): input temperature of the fluid, \( T_{\text{out}} \): output temperature of the fluid). \( T_{\text{Mid}} \) is the average temperature of the part of the WMCHS constant flux and \( q'' \) is the heat flux applying on the WMCHS, which is equal to 88,000 W/m².

The pumping power (PP) indicating the energy required to move the fluid in the WMCHS is introduced in the below relations.

\[
PP = \dot{Q} \Delta P
\]

(16)

where \( \dot{Q} \) denotes the volume flow rate of the fluid and \( \Delta P \) is the pressure difference at the input and output of the WMCHS. The other parameter investigated is the Figure of Merit (FOM). This relation is applied when the NF is used instead of a normal fluid; also, it is presented as follows.

\[
\text{FOM} = \frac{h_{\text{nf}}}{h_f} \frac{\Delta P_{\text{nf}}}{\Delta P_f}
\]

(17)

The other two important parameters which are useful to investigate the thermal efficiency of the WMCHS are the thermal resistance value and its temperature uniformity. The first parameter is an indication of thermal resistance of WMCHS against heat transfer, and the second one is the temperature distribution of the WMCHS. The relations concerning the aforementioned parameters are as follows, respectively. In fact, according to several investigations, the lower the value of these two parameters, the lower the thermal resistance of the WMCHS as well as the higher the temperature uniformity.

\[
R = \frac{T_{\text{Mid}} - T_{\text{in}}}{q''}
\]

(18)

\[
\theta = \frac{T_{\text{Max}} - T_{\text{min}}}{q''}
\]

(19)

In the above relations, the subscripts Max and Min indicate the T-Max and minimum temperature on the surface of the WMCHS constant flux.
The Re, which is written as follows, is utilized to show the fluid velocity.

\[ \text{Re} = \frac{\rho v l}{\mu_l} \]  

(20)

6. Results and Discussion

In the simulation process, the NFs water/Al\(_2\)O\(_3\) and water/CuO oxide were compared to the pure water at the Res of 500, 1000, and 1500. Furthermore, the \(\varphi\) of NPs was increased to 2\% and the results were expressed as follows.

The velocity field for the Al\(_2\)O\(_3\) and CuO water-based NFs at the Res of 500 and 1500 on the middle plane of the WMCHS is presented in Figure 4. As can be seen, the maximum velocity in the WMCHS increased. This maximum velocity occurs at the input and output parts of the fluid, where the crossing section of the fluid is smaller than the other parts. The fluid’s velocity becomes lower when it passes through the input region and reaches the wider WMCHS area. Moreover, the fluid flows faster in the middle microchannels compared to the side ones. In fact, in comparison with the side microchannels, more fluid passes through the middle microchannels of the WMCHS.

![Figure 4](image_url). The velocity field for the Al\(_2\)O\(_3\) and CuO water-based NFs at the Res of 500 (a–c) and 1500 (d–f) on the middle plane of the WMCHS.

Velocity vectors at the fluid inlet and between the microchannels are shown in Figure 5. It can be seen that there is a high velocity change at the inlet due to the sudden enhancement in the flow rate. Initially, at the inlet, the fluid velocity is high due to the narrow passage of the fluid, leading to long vectors. With the change of flow direction in the heat sink and the enhancement of the flow area, the size of the vectors becomes smaller, indicating a decrease in velocity. The fluid motion between the microchannels and its collision with the walls show that the fluid has more collision with smooth walls in the corrugated walls.
Figure 5. Velocity vectors at the fluid inlet and between the microchannels.

Figure 6 shows the temperature contour at the middle plane (upper) and the part underneath the WMCHS (bottom) for the pure water and the water/Al$_2$O$_3$ and water/CuO.
NFs at the Re numbers of 500 and 1500. As observed, augmenting the Re decreases the T-Max and overall temperature of the WMCHS. In addition, increasing the velocity of the fluid leads to improving heat transfer and better cooling the WMCHS, which are observable in all the fluids. Compared to pure water, using NFs better improves the cooling process of the WMCHS. It should also be mentioned that between two NFs used in the current research, the water/Al₂O₃ NF has better performance in the cooling process. Actually, the presence of the NPs in the fluid flow leads to heightening its thermal conduction resulting in improvement of heat transfer, and finally, better cooling in the WMCHS. WMCHS has a lower temperature in its middle part where the fluid passes faster, while it has a higher temperature in its side parts where the fluid passes more slowly.

Figure 6. Cont.
Figure 6. Three-dimensional temperature contour at the middle plane (upper) and the part underneath the WMCHS (bottom) for Al₂O₃ and CuO water-based NFs at Re = 500 (a–c) and 1500 (d–f).

Figure 7 indicates the thermal resistance of the WMCHS for water/Al₂O₃ and water/CuO NFs at three different Res of 500, 1000, and 1500. As can be seen, the addition of both NPs decreases the value of the WMCHS thermal resistance, which is desirable. Compared to the water/CuO Na, using the Al₂O₃ NPs in the base fluid creates lower thermal resistance in the WMCHS. Increasing the Re also decreases the thermal resistance of the WMCHS. The average temperature decrease in the WMCHS leads to a decline in its thermal resistance by enhancing the Re and adding NPs.

![Figure 7. WMCHS thermal resistance for Water/Al₂O₃ and Water/CuO NFs at three different Res of 500, 1000, and 1500.](image)

Figure 8 shows the temperature uniformity of the WMCHS for two different NFs at three Res. The addition of the NPs to the base fluid leads to improving the temperature uniformity as well as closing the T-Max and minimum temperature of the WMCHS to each other. Moreover, according to the graph illustrated in Figure 8, the water/Al₂O₃ NF has better operation than the water/CuO one. Therefore, it creates better uniformity. The lower value of θ means closing the T-Max and minimum temperature of the WMCHS to each other, which is desirable in the WMCHS design. Increasing the Re decreases the θ values, which is the reason for the severe decrease in the T-Max.
density of the nanofluid, which can lead to an enhancement in the system pressure drop.

The higher density of copper oxide than alumina nanoparticles, the amount of velocity
heat sink, which ultimately leads to an increase in the pressure drop. Additionally, due to
the addition of nanoparticles enhances the amount of pressure drop in the heat sink. An
increment in the Reynolds number means an increment in the velocity gradient in the
heat sink, which ultimately leads to an increase in the pressure drop. Besides, increasing
the NPs dimensions requires a higher PP than the smaller NPs. It is then observed that the
highest required PP is related to the 2 vol% water/CuO NF at Re = 1500.

Figure 9 illustrates the PP values required for moving the fluid having different Re
numbers for the water fluid and water/Al₂O₃ and water/CuO NFs at two various volume
percentages. The PP greatly enhances by increasing the Re owing to the simultaneous
increase in the flow rate and pressure drop in the WMCHS. Moreover, the addition of the
NPs increases the PP due to heightening the pressure drop. As can be seen, there is a drastic
pressure drop at the higher Res by adding the NPs to the base fluid. On the other hand,
adding the NPs and increasing them heightens the pressure drop. Besides, increasing the
NPs dimensions requires a higher PP than the smaller NPs. It is then observed that the
highest required PP is related to the 2 vol% water/CuO NF at Re = 1500.

Table 2 shows the pressure drop at different Reynolds numbers for two different
nanoparticles and different volume percentages of nanofluids. It can be seen that the
addition of nanoparticles enhances the amount of pressure drop in the heat sink. An
increment in the Reynolds number also increases the amount of pressure drop in the
heat sink. An enhancement in the Reynolds number means an increment in the velocity
gradient in the heat sink, which ultimately leads to an increase in the pressure drop.
Additionally, due to the higher density of copper oxide than alumina nanoparticles, the
amount of velocity difference or lift velocity created for nanoparticles in the two-phase
mixture method is different. Copper oxide nanoparticles have a higher density, and as
a result increase the density of the nanofluid, which can lead to an enhancement in the
system pressure drop.
Table 2. Pressure drop in heat sink at different Reynolds numbers for two different nanoparticles.

| ϕ   | Re = 500 | Re = 1000 | Re = 1500 |
|-----|----------|-----------|-----------|
| 0%  | 74.78    | 231.2     | 466       |
| 1%  | 90.12    | 360.4     | 721.47    |
| 2%  | 110.25   | 389.31    | 835.52    |

For 1% and 2 vol% Al_2O_3 and CuO NFs at three various Res of 500, 1000, and 1500. As can be generally seen using smaller dimensions, NPs having a higher thermal conduction coefficient creates a higher FOM value than the other NPs. This is as a result of greater enhancement of the heat transfer coefficient using this NP and its lower pressure drop, which leads to improving the FOM of the Al_2O_3 NP compared to the CuO one. Hence, the maximum FOM is related to the Al_2O_3 NF at 2 vol%. Adding more NPs improves the FOM, which shows using higher volume percentages of NPs is more desirable.
1. Using both the investigated NFs instead of the pure water decreases the thermal resistance of the WMCHS and creates more temperature uniformity in it. Moreover, using the Al₂O₃ NP is more suitable in the cooling operation of the H/S;

2. The addition of the NPs, especially Al₂O₃, increases the heat transfer coefficient. Furthermore, the convection heat transfer coefficient increase is more noticeable at the higher Res;

3. The addition of the higher diameter NPs, especially CuO, leads to increasing the PP required for moving the fluid. Compared to other NPs, using these NPs has also increased the power consumption cost significantly;

4. Increasing the ϕ increases the Figure of Merit value so that its maximum value occurs at a 2 vol% Al₂O₃ NF;

5. Increasing the Re decreases the thermal resistance of the WMCHS, increases the PP required for the heat transfer coefficient and improves the temperature uniformity in the WMCHS.

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