Real-time forecasting of the COVID 19 using fuzzy grey Markov: a different approach in decision-making

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Abstract
The ongoing epidemic SARS-CoV-2 named Corona Virus Disease (COVID-19) is highly infectious and subsequently spread all over the world affecting millions of people. Humans have never seen such a deadly disease so far, and as there is no specific drug or vaccination, the mortality rate of the disease has been increasing exponentially. This current situation exacerbated people’s restlessness and fear. Because of this pandemic, the world is travelling on a different path. This world has recovered from many disasters, but this is entirely a different situation. Today’s world is struggling in many ways to get rid of this disease. On the other hand, the number of people recovering from this disease gives us comfort. Yet, we have to take urgent precautionary measures to control this disease in all possible ways. Therefore, forecasting is one of the ways to take the necessary precautionary measures. In this paper, using fuzzy–grey–Markov model, we predict the number of affected and recovered patient count, death count using real-time data in different approaches and compared with the real data. The study concludes with important recommendations for the Indian government to manage the COVID 19 critical situation in advance.

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1 Introduction

The origin of the novel coronavirus was first detected in a seafood market in Wuhan, the capital city of Hubei province, China in December 2019. The virus is named coronavirus because of its structure that takes the form of a crown with protrusions around it. Within a few months, this deadly virus spread rapidly from human to human worldwide. The World Health Organisation (WHO) declared it as a global pandemic on 11 March 2020. In India, this disease was first detected in Kerala on 30th January 2020 from a student who returned from Wuhan, and gradually, it spread to all the states in India. The government took several wartime measures to control this disease. As a first step, the government announced a 7-day complete lockdown. The lockdown included banning of public transport, closing schools and colleges and introducing online classes, closing all the government and private offices and factories, and asking officials to work from home; people were advised to come out from home only for essential needs between particular time intervals. Shops selling essential items were instructed to operate till noon, and all places of worship and tourist attractions were closed. All the places where the people could gather for entertainment were also closed. In the second step, the government has taken several health measures to protect every individual from this disease through social media. People were mandated to wear a face mask when going out for essential needs and instructed to observe social distance in all public places. People were instructed to clean their hands continuously with soap or sanitizer when they returned from the outdoors. Third, the government has taken several steps to cure COVID-19 patients: those who are having a cold, cough, fever, or breathing problem are advised to consult a doctor immediately and take the coronavirus test. To treat the COVID-19 patients, separate wards were opened in all the government and private hospitals. They were treated according to severity of the disease. If the severity was high, they were admitted to the hospital. Otherwise, they were instructed to self-isolate. The government also opened several isolation camps for COVID-19 patients. Finally, the government-appointed staff conducted flu screening daily in the cities. This lockdown extended till 31st May 2020 in India. On the other hand, to control the economic downturn, the government announced some relaxation in the lockdown from June 2020.

Several methods are available for analyzing uncertainty especially in disease management. Zadeh (1965) introduced fuzzy set as an extension of classical set (usually called crisp set) in 1965 for analysis of uncertainty. Fuzziness will arise in all the decision-making problems; especially in the medical field. Zadeh (1969) is the first researcher who recommended a fuzzy set for medical diagnosis. The nature of many diseases is subject to uncertainty, since two or more diseases have common symptoms. On the other hand, the pain suffered in any disease by a patient is usually expressed in fuzzy terms (linguistic expression), such as low, medium, high (too much), and very high (severe). Furthermore, in the initial stage of any medical diagnosing, we cannot get the crisp answer ‘yes’ or ‘no’. The physician wants many parameters such as patients’ history, symptoms, and laboratory reports to identify the disease. Since fuzzy set admits gradual membership and deals with uncertainty, it is very suitable to use in medical diagnosis. Innocent et al. (2005) applied various fuzzy methods, such as
fuzzy set aggregation, fuzzy clustering, and type-2 fuzzy sets for medical diagnosis. Ruben et al. (2016) used a fuzzy prototype for the treating of fuzzy disease. Using the concept of distance measures, Palash and Soumendra (2018) applied intuitionistic fuzzy sets for medical diagnosis. Arji et al. (2019) classified the fuzzy logic application in infectious disease. For prevention and identification of COVID-19, Nitesh and Sharma (2020) employed a fuzzy logic inference system. Moreover, to predict COVID-19 cases, Deepak et al. (2020) applied a fuzzy rule-based system, while Castillo and Melin (2020) used the combination of fractal dimension and fuzzy logic and Chowdhury et al. (2021) applied Long Short-Term Memory (LSTM) network and Adaptive Neuro-Fuzzy Inference System (ANFIS). On the other hand, Baz et al. (2020) performed fuzzy-AHP computational modelling for predicting the spread of COVID-19.

Grey models, especially the GM(1,1) forecasting model, have been applied in various fields including medical and promising results are available in the literature. Yang et al. (2018) applied the GM(1,1) model to forecast the incidence trend of typhoid and paratyphoid fever in Wuhan City, China. To predict the postprandial glucose in type-2 diabetes, Wang et al. (2016) used improved grey GM(1,1). Iqelan (2017) forecast and studied the performance of breast cancer referrals by applying the grey prediction model GM(1,1). For the COVID-19 patient count prediction, the grey Verhulst model was used by Zhao et al. (2020) and Sahin and Sahin (2020) applied the fractional nonlinear grey Bernoulli model. Zeng et al. (2020) investigated a novel grey prediction model GM(1,1) under incomplete information to predict natural gas consumption in China. Furthermore, Gao et al. (2020) forecasted the cumulative incidences of typhoid and paratyphoid fevers by employing predictive models such as grey model GM(1,1) and seasonal autoregressive integrated moving average (SARIMA) model. Nieszporska (2022) forecast the number of future patients of palliative care facilities in Poland by using the grey GM(1,1) model.

One of the superior forecasting models in probability theory is the Marko process; it is a stochastic process in which the future depends only on the current, independent of the past. Marfak et al. (2020) used Hidden Markov Model (HMM) for predicting COVID-19 spreading, and they predicted several affected, recovered, and death cases in the study. For COVID-19 mortality rate prediction in India, Dhamodharavadhani et al. (2020) applied a statistical neural network. To analyze COVID-19 transmission, Bherwani (2020) used the concept of Bayesian probabilistic modelling and the GIS-based Voronoi approach. To study COVID-19 outbreaks Overton et al. (2020) used statistical and mathematical modelling. Furthermore, Dattner and Huppert (2018) applied modern statistical tools for the inference and prediction of infectious diseases using mathematical methods. To forecast and control COVID-19 outbreaks, Sha et al. (2020) used a discrete stochastic model, namely, the SEIR model, and Roda et al. (2020) applied the SIR model. In another study, Malavika et al. (2020) applied the logistic growth curve model and SIR model for forecasting COVID-19 active cases, peak time, and to identify the impact of lockdown, they used the Time Interrupted Regression model. Zhou et al. (2021) established the grey–Markov forecasting model to predict the dust concentration in mines. Geng et al. (2015) used a fuzzy–grey–Markov model to forecast biofuel production.

Despite so many measures, the severity of the disease and the mortality rate could not be controlled. However, the number of people recovering from COVID-19 was increased day by day. It gave everyone a little comfort. According to the WHO, so far 1 million of people (during the study period) have lost their lives due to this pandemic and they warned that the death rate could rise to 2 million before a vaccine is found. We need to take urgent precautionary measures to deal with this situation. In a situation like this, infectious disease forecasting helps the public health department in many ways to decide measures in advance. Similarly, the public uses the infectious disease forecast to control the disease.
and take economic action experienced during the lockdown period. This issue inspired and motivated us to find a suitable approach to forecast the affected, recovered, and death count of the COVID-19 infectious disease. Therefore, to improve forecasting accuracy, this research adopted the fuzzy–grey–Markov model (FGMM) introduced by Geng et al. (2015), in which the fuzzy set is combined with grey–Markov. The advantage of the FGMM method is that the relative error is reduced when compared with existing forecasting methods. Thus, the prediction drawn from real-time data through this method is more accurate than using any other methods.

2 Related works and motivation

In some recent works on COVID-19 prediction, some mathematical perspectives of numerical and simulation study of COVID-19 have been seen in Boccaletti et al. (2020). Sun and Wang (2020) formulated a COVID-19 epidemic mathematical model by ordinary differential equation to mimic the number of patients in Heilongjiang Province in China. In Melin et al. (2020a), the authors summarized the self-organizing maps based on unsupervised neural networks and subsequently analyzed the spatial evolution of the COVID-19 pandemic around the world. Melin et al. (2020b) integrated the multiple ensemble neural network model with fuzzy response aggregation, and then reported the COVID-19 cases of Mexico. In Varela-Santos and Melin (2020), the authors reported supervised learning models to classify the COVID-19 patients based on medical images. Castillo and Melin (2021) outlined the hybrid intelligent approach based on fuzzy logic and fractal dimension to categorize the COVID-19 time-series data based on each country. Ding et al. (2021) predicted the transmission trajectory of the COVID-19 using the combined method of particle swarm optimization algorithm (PSO) and susceptible exposed infected recovered (SEIR). Ceylan (2021) proposed a hybrid model based on GM(1,1) and a PSO and utilized it to forecast the cumulative case number of COVID-19 in Germany, Turkey, and USA. Kumar and Susan (2021) considered fuzzy time-series and PSO to forecast the COVID-19 pandemic. Li et al. (2021) identified new infected COVID-19 patients in terms of small data and poor information using the improved GM(1,1) model. In all this previous related work, we can notice that to date, the FGMM has not been used for the forecasting of the COVID-19 pandemic.

Geng et al. (2015) indicated that their proposed fuzzy–grey–Markov model is relatively superior to the traditional prediction models. This hybrid model makes use of all the characteristics of the fuzzy, grey GM(1,1), and Markov model to enhance forecasting accuracy; however, the calculations are done in different ways. Using this greater forecasting potential model, we forecast in three angles via affected, recovered, and number of deaths from COVID-19, and the forecasting results are remarkable when compared with the actual Indian data. The forecasting values are validated graphically with the real data for COVID-19 with respect to India’s perspective. As a new approach for forecasting, the real data of each affected, recovered, and the death count data are divided into three parts, namely, left length, middle value, and right length. Taking logarithm for each of these values, the forecasting values are calculated for each case. We take the antilogarithm and the final forecasting value is obtained by combining all the left, middle, and right values using the centre of gravity method. As an application, this prediction in three angles will definitely help the Government to manage the COVID-19 critical situation in advance.

This paper is structured as follows: a review of the literature is given in Sect. 2 after the introduction. Section 3 is devoted to defining the methodology of this proposed model.
Forecasting in three angles of this COVID-19 disease is presented in Sect. 4 and Sect. 5 concludes the paper.

3 Models and methodology of the proposed work

In this section, some of the basic things needed for the forecasting have been described.

3.1 Grey GM(1,1) model

1. Consider a positive data collection that represent the affected, recovered, and death count of the COVID-19 disease.

\[ X^{(0)} = \left( x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n) \right), \quad n \geq 4. \]  

(1)

2. To avoid the vibration in the collected COVID-19 data, a new monotonically increasing data sequence \( X^{(1)} \) is formed from \( X^{(0)} \) called Accumulating Generation Operator (AGO). Then, Eq. (1) becomes

\[ X^{(1)} = \left( x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n) \right), \quad n \geq 4. \]  

(2)

where \( x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), \quad k = 1, 2, \ldots, n. \)  

(3)

3. The mean sequence is \( Z^{(1)} = \left( (z^{(1)}(1), z^{(1)}(2), \ldots, z^{(1)}(n)) \right) \) obtained from \( X^{(1)} \), where \( z^{(1)}(k) \) is the average of the adjacent values

\[ z^{(1)}(k) = \frac{x^{(1)}(k - 1) + x^{(1)}(k)}{2}, \quad k = 2, 3, \ldots, n. \]  

(4)

4. The grey GM(1,1) difference equation is

\[ x^{(0)}(k) + az^{(1)}(k) = b, \]  

(5)

where the parameters ‘a’ and ‘b’ are determined using least square method

5. The whitening differential equation of grey GM(1, 1) is

\[ \frac{dx^1(t)}{dt} + ax^1(t) = b, \]  

(6)

and the solution of Eq. (6) is the forecasting value of grey GM(1, 1)

\[ \hat{x}^{(1)}(k + 1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} + \frac{b}{a}, \quad k = 0, 1, 2, \ldots, n - 1. \]  

(7)

6. To convert the COVID-19 data into original form, we apply the Inverse Accumulating Generation Operator (IGAO) on Eq. (7)

\[ \hat{x}^{(0)}(k + 1) = \hat{x}^{(1)}(k + 1) - \hat{x}^{(1)}(k) \quad \text{where} \quad \hat{x}^{(0)}(1) = x^{(0)}(1) \]  

(8)

\[ \hat{x}^{(0)}(k + 1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} \left( 1 - e^{-ak} \right), \quad k = 0, 1, 2, \ldots, n. \]  

(9)
7. Error is calculated as the difference between the actual and the forecasting values of the COVID-19 data

\[ E(k) = x^{(0)}(k) - \hat{x}^{(0)}(k), \quad k = 1, 2, \ldots, n. \] (10)

8. The relative error for the forecasting value is defined as

\[ \Delta(k) = \frac{E(k)}{x^{(0)}} \times 100, \quad k = 1, 2, 3, \ldots, n. \] (11)

### 3.2 Fuzzy–grey–Markov model (FGMM)

To construct the FGMM, we follow the steps given below:

1. The relative error sequence \( \Delta(k) \) in grey GM(1, 1) is divided into number of classes and it is denoted by \( U \).

2. Allocate the states for the transition probability matrix using the relative error sequence belonging to the set of classes \( U \).

3. Construct the transition probability matrix

\[ P(k) = \begin{bmatrix}
p_{11}^k & p_{12}^k & \ldots & p_{1n}^k \\
p_{21}^k & p_{22}^k & \ldots & p_{2n}^k \\
\vdots & \vdots & \ddots & \vdots \\
p_{m1}^k & p_{m2}^k & \ldots & p_{mn}^k
\end{bmatrix}. \] (12)

4. Using triangular method, the membership function for any relative error \( x \in U \) is defined by

\[ u_1(x) = \begin{cases} 
1 & \text{if } \mu_0 \leq x \leq \frac{\mu_0 + \mu_1}{2} \\
\frac{\mu_1 - \mu_2 - 2x}{\mu_2 - \mu_0} & \text{if } \frac{\mu_0 + \mu_1}{2} \leq x \leq \frac{\mu_1 + \mu_2}{2}
\end{cases} \] (13)

\[ u_i(x) = \begin{cases} 
\frac{2x - \mu_i - \mu_{i-1}}{\mu_{i+1} - \mu_{i-1}} & \text{if } \frac{\mu_{i-2} + \mu_{i-1}}{2} \leq x \leq \frac{\mu_{i-1} + \mu_i}{2} \\
\frac{\mu_{i+1} - \mu_i}{2} & \text{if } \frac{\mu_{i-1} + \mu_i}{2} \leq x \leq \frac{\mu_i + \mu_{i+1}}{2} \\
0 & \text{otherwise}
\end{cases} \] (14)

for \( i \neq 1, n \).

\[ u_n(x) = \begin{cases} 
\frac{2x - \mu_n - \mu_{n-2}}{\mu_n - \mu_{n-2}} & \text{if } \frac{\mu_{n-2} + \mu_{n-1}}{2} \leq x \leq \frac{\mu_{n-1} + \mu_n}{2} \\
\frac{\mu_{n-1} + \mu_n}{2} & \text{if } \frac{\mu_{n-1} + \mu_n}{2} \leq x \leq \mu_n \\
0 & \text{otherwise}
\end{cases} \] (15)

5. Define a fuzzy vector

\[ F(\Delta(k)) = \{ u_{A_1}(\Delta(k)), u_{A_2}(\Delta(k)), \ldots, u_{A_n}(\Delta(k)) \}, \] (16)

where \( u_{A_n}(\Delta(k)) \) is the membership function of \( \Delta(k) \) of the fuzzy set \( A_n \).

6. The fuzzified form of the COVID-19 forecasting error is defined as

\[ F(\Delta(k + 1)) = F(\Delta(k)) P(1) = \{ u_{A_1}(\Delta(k + 1)), u_{A_2}(\Delta(k + 1)), \ldots, u_{A_n}(\Delta(k + 1)) \}. \] (17)
7. The crisp value of the COVID-19 forecasting error is

\[ \Delta(k + 1) = \sum_{i=1}^{n} \frac{1}{2} u_{A_i} (\Delta_i (k + 1))(\Delta_{i-1} + \Delta_i), \]  

(18)

where \( \Delta_{i-1} \) and \( \Delta_i \) are the lower and upper bounds of the relative error in the class \( U \).

8. At the end, the FGMM forecasting values for the COVID-19 affected, recovered, and number of death cases is given by

\[ \hat{z}^{(0)}(k + 1) = \frac{\hat{x}^{(0)}(k + 1)}{1 - \Delta(k + 1)}. \]

(19)

Here, \( \hat{x}^{(0)}(k + 1) \) is the forecasting value of grey GM(1, 1).

### 3.3 Pseudo-code for the proposed methodology

The following steps involved to forecast separately the affected, recovered, and death count using FGMM for the COVID-19 disease:

**Step 1:** Collect the real-time affected, recovered and death cases of the COVID-19 disease for forecasting.

**Step 2:** Form the left length, right length by taking 5% of the each corresponding original data. Therefore, we have three values (namely left, middle, right) for each datum as well as for each case.

**Step 3:** Take the logarithm for the left length, middle value data and find the forecasting value using grey GM(1, 1). We will get the predicted value in terms of logarithm.

**Step 4:** Find the absolute relative error between the actual and forecasting logarithm values.

**Step 5:** Similarly using the logarithm of the left length and middle values, we follow the steps from 1 to 8 in Sect. 3.2 to find the forecasting values of FGMM.

**Step 6:** Take the antilogarithm to convert the data into original form.

**Step 7:** Since the left length is same as the right length, we have left length, middle and right length forecasting values in both grey GM(1, 1) and FGMM.

**Step 8:** Using the centre of gravity method, we convert all the left length, middle and right length forecasting values into a single forecasting value.

**Step 9:** To come to the conclusion, the mean absolute error is compared and we choose the model having less absolute relative error and the corresponding forecasting value is compared with the original value for model accuracy (Fig. 1).

### 4 Prediction of affected, recovered and death count of the COVID-19 disease based on the proposed methodology

We collected the 7-day real-time Indian data for this three-angle prediction. The period of collection is from August 9 to 15, 2020 (https://science.thewire.in/health/coronavirus-daily-updates-covid-19/), because this is a peak period of this COVID-19 disease. We separately forecast the number of affected, recovered and death count using the proposed approach. First, we concentrate on number of affected cases. The number of affected cases is given in Table 1.
Fig. 1 Pictorial representation of the proposed approach

Table 1 Actual affected cases of the COVID-19 disease

| August | Left   | Actual (middle) | Right  |
|--------|--------|-----------------|--------|
| 9      | 256,300| 269,789         | 283,278|
| 10     | 262,848| 276,682         | 290,516|
| 11     | 269,237| 283,407         | 297,577|
| 12     | 277,645| 292,258         | 306,871|
| 13     | 286,529| 301,609         | 316,689|
| 14     | 295,987| 311,565         | 327,143|
| 15     | 303,848| 319,840         | 335,832|
4.1 Forecasting of affected cases

We first forecast the affected cases for middle values using grey GM(1, 1). The number of affected people in the period of study is taken as $X_M^{(0)}$

$$X_M^{(0)} = [269789, 276682, 283407, 292258, 301609, 311565, 319840]. \quad (20)$$

Applying the AGO to the above $X^{(0)}$, we get

$$X^{(1)} = [269789, 546471, 829878, 1122136, 1423745, 1735310, 2055150]. \quad (21)$$

From Eq. (5), the grey difference equation is $x^{(0)}(k) + az^{(1)}(k) = b$; solving ‘$a$’ and ‘$b$’ using the method of least square, we get $a = -0.0024$ and $b = 5.4216$. From Eq. (9), the forecasting value of grey GM(1, 1) is given by

$$\hat{x}_M^{(0)}(k + 1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} \left( 1 - e^{-ak} \right), \quad k = 0, 1, 2, \ldots, n. \quad (22)$$

Using Eq. (22), the grey forecasting value for the affected case is given in Table 2.

Next, we forecast the affected cases using FGMM. For that, we divide the relative error sequence $\Delta(k)$ of grey GM(1, 1) into five states, as shown in Table 3.

Clearly, the relative errors do not lie in the fuzzy state 1 and 5, so we consider only three states 2, 3, and 4. We construct the membership function for the fuzzy states using Eqs. (13),

**Table 2** Forecasting affected cases using grey GM(1, 1)

| Days | $X_M^{(0)}$ | $\log(X_M^{(0)})$ | $\hat{x}_M^{(0)}$ | $\Delta(k)$ | State |
|------|-------------|-------------------|-------------------|-------------|-------|
| 1    | 269,789     | 5.4310            | 5.4346            | 0           | 4     |
| 2    | 276,682     | 5.4419            | 5.4477            | -0.1050     | 4     |
| 3    | 283,407     | 5.4524            | 5.4607            | -0.1518     | 2     |
| 4    | 292,258     | 5.4657            | 5.4739            | -0.1486     | 3     |
| 5    | 301,609     | 5.4794            | 5.487             | -0.1377     | 3     |
| 6    | 311,565     | 5.4935            | 5.5002            | -0.1209     | 4     |
| 7    | 319,840     | 5.5049            | 5.5134            | -0.1536     | 2     |
| Mean absolute relative error (MAPE) | | | | 0.1362 |

**Table 3** State description for FGMM

| State | Affected count | Interval   |
|-------|----------------|------------|
| 1     | Very less      | $(-\infty, -0.16)$ |
| 2     | Less           | $(-0.16, -0.15)$ |
| 3     | Medium         | $(-0.15 - 0.13)$ |
| 4     | High           | $(-0.13, 0)$ |
| 5     | Very high      | $(0, \infty)$ |
(14) and (15)

\[ u_2(x) = \begin{cases} 
1 & -0.16 \leq x \leq -0.155 \\
\frac{-0.0028-2x}{0.0003} & -0.155 \leq x \leq -0.14 \\
0 & \text{otherwise}
\end{cases} \]  
(23)

\[ u_3(x) = \begin{cases} 
\frac{2x+0.0031}{0.0003} & -0.155 \leq x \leq -0.14 \\
\frac{-0.0013-2x}{0.0015} & -0.14 \leq x \leq -0.065 \\
0 & \text{otherwise}
\end{cases} \]  
(24)

\[ u_4(x) = \begin{cases} 
\frac{2x+0.0028}{0.0015} & -0.14 \leq x \leq -0.065 \\
1 & -0.065 \leq x \leq 0 \\
0 & \text{otherwise}
\end{cases} \]  
(25)

The fuzzy states for the day-1 is calculated as follows: the relative error for the day-1 from GM\((1, 1)\) is 0. For this relative error, the membership value \(u_2(x) = 0, u_3(x) = 0\) and \(u_4(x) = 1\). Therefore, the fuzzy state for the day-1 is \((0, 0, 1)\). Similarly, we calculate the fuzzy states for the remaining days. The fuzzy states for the days 1–7 are given in Table 4.

For the GM\((1, 1)\) relative error sequence, we allocate the states using the state description in Table 3 and it is given in Table 4 for the days 1–7. For instance, the transition from state 2 to state 3 is one, state 2 to state 4 is zero and the transition from state 4 to state 4 is one. Similarly, preceding, we get the transition for all the states. The state transition between the states is given in Table 5.

### Table 4 Fuzzy states for the days 1–7

| Day | Relative error | State | Fuzzy state (state 2, state 3, state 4) |
|-----|----------------|-------|---------------------------------------|
| 1   | 0              | 4     | \((0, 0, 1)\)                          |
| 2   | −0.1050        | 4     | \((0, 0.533, 0.466)\)                  |
| 3   | −0.1518        | 2     | \((1, 0, 0)\)                         |
| 4   | −0.1486        | 3     | \((0.5733, 0.4266, 0)\)               |
| 5   | −0.1377        | 3     | \((0, 0.9693, 0.0306)\)               |
| 6   | −0.1209        | 4     | \((0, 0.7453, 0.2546)\)               |
| 7   | −0.1536        | 2     | \((0.9066, 0.0933, 0)\)               |

### Table 5 State transition between the three states

| States | Total |
|--------|-------|
|        | 2     | 3     | 4     |     |
| States |       |       |       |
| 2      | 0     | 1     | 0     | 1    |
| 3      | 0     | 1     | 1     | 2    |
| 4      | 1     | 0     | 1     | 2    |
Using Eq. (12), the one-step transition probability matrix is

\[
P^{(1)} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \end{pmatrix},
\]

(26)

Now, we forecast the number of affected cases for the day-2 using FGMM. The fuzzy vector for the day-1 is \((0, 0, 1)\). The vector of membership degree for the day-2 is

\[
(0, 0, 1) \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0.5 & 0.5 \\ 0.5 & 0 & 0.5 \end{pmatrix} = (0.5, 0, 0.5).
\]

(27)

The relative error for the day-2 is

\[
\Delta(2) = \frac{1}{2} (0.5) (-0.16 - 0.15) + \frac{1}{2} (0) (-0.15 - 0.13) + \frac{1}{2} (0.5) (-0.13 + 0) = -0.11.
\]

(28)

The forecasting value for the affected case in day-2 is given by

\[
\hat{z}_M^{(0)}(k + 1) = \frac{\hat{x}_M^{(0)}(k + 1)}{1 - \Delta(k + 1)} = \frac{5.4477}{1.0011} = 5.4417.
\]

(29)

Similarly, the forecasting value of the affected patient count with the corresponding relative error for the days 1–7 is given in Table 6.

Since the MAPE value (0.1145) (Table 6) of FGMM is less compared to MAPE value 0.1362 (Table 2) of grey GM(1, 1), we forecast the affected cases for the next 8 days and the forecasting values are given in Table 7.

Next, following the same procedure, we forecast the affected cases using the left length value of the real-time affected data. We calculate the left length value for the day-1

\[
X_M^{(0)} - (0.05 \times X_M^{(0)}) = 269789 - (0.05 \times 269789) = 256300
\]

(30)

Now the left length value for the day-1 = 269789 – 256300 = 13489.

(31)

| Day | \(X_M^{(0)}\) | \(\log(X_M^{(0)})\) | \(\hat{z}_M^{(0)}\) | \(\Delta(k)\) (FGMM) | Antilog \(\hat{z}_M^{(0)}\) |
|-----|-------------|----------------|-----------------|-----------------|-----------------|
| 1   | 269,789     | 5.4310         | 5.4346          | 0               | 272,019         |
| 2   | 276,682     | 5.4419         | 5.4417          | -0.1100         | 276,889         |
| 3   | 283,407     | 5.4524         | 5.4549          | -0.1059         | 284,031         |
| 4   | 292,258     | 5.4657         | 5.4662          | -0.1400         | 292,807         |
| 5   | 301,609     | 5.4794         | 5.4802          | -0.1240         | 301,774         |
| 6   | 311,565     | 5.4935         | 5.4946          | -0.1027         | 311,004         |
| 7   | 319,840     | 5.5049         | 5.5077          | -0.1044         | 320,345         |

Mean absolute relative error (MAPE) \(0.1145\)
Similarly, we calculate the left length for the remaining days. Now, the $X^{(0)}$ for the left length is

$$X_L^{(0)} = \{13489, 13834, 14170, 14613, 15080, 15578, 15992\}. \quad (32)$$

Solving ‘$a$’ and ‘$b$’ for the grey difference equation using least square method, we get $a = -0.0031$ and $b = 4.1206$. The forecasting value of grey GM(1,1) is

$$\hat{x}_L^{(0)}(k + 1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} \left( 1 - e^{-ak} \right), \quad k = 0, 1, 2, \ldots, n.$$

$$= 4.1334e^{0.0031k}, \quad k = 0, 1, 2, \ldots, n. \quad (33)$$

Similarly, proceeding as middle value, we get the grey GM(1,1) and FGMM forecasting values for the left value of the affected cases. The forecasting values are given in Table 8.

Again, the MAPE value (0.1225) (Table 8) of FGMM is less compared to MAPE value 0.1573, (Table 8) of grey GM(1,1). Therefore, we forecast using FGMM for the next 8 days. The forecasting values are given in Table 9.

Finally, we cannot separately find out the forecasting values for the right length of the affected cases, because the left length is equal to the right length. Here, the lower value is

### Table 7 Forecasting affected cases for the middle value using FGMM

| Day | $X_M^{(0)}$ | Forecasting value |
|-----|-------------|-------------------|
| 8   | 331,146     | 340,722           |
| 9   | 342,473     | 352,752           |
| 10  | 358,692     | 363,635           |
| 11  | 362,824     | 372,041           |
| 12  | 383,218     | 382,789           |
| 13  | 391,967     | 391,478           |
| 14  | 396,453     | 395,432           |
| 15  | 415,678     | 414,452           |

### Table 8 Grey GM(1, 1) and FGMM forecasting values

| Day | $X_L^{(0)}$ | log($X_L^{(0)}$) | $\hat{x}_L^{(0)}$ | $\Delta(k)$(GM (1, 1)) | $\hat{z}_L^{(0)}$ | $\Delta(k)$ (FGMM) | Antilog $\hat{z}_L^{(0)}$ |
|-----|-------------|------------------|-------------------|------------------------|-------------------|-------------------|-----------------------|
| 1   | 13,489      | 4.1299           | 4.1334            | 0                      | 4.1334            | 0                 | 13,596                |
| 2   | 13,834      | 4.1409           | 4.1462            | -0.1268                | 4.1404            | -0.1394           | 13,815                |
| 3   | 14,170      | 4.1513           | 4.1591            | -0.1859                | 4.1551            | -0.0960           | 14,289                |
| 4   | 14,613      | 4.1647           | 4.172             | -0.1744                | 4.1651            | -0.1650           | 14,625                |
| 5   | 15,080      | 4.1784           | 4.185             | -0.1576                | 4.1783            | -0.1604           | 15,083                |
| 6   | 15,578      | 4.1925           | 4.198             | -0.1307                | 4.1946            | -0.0818           | 15,652                |
| 7   | 15,992      | 4.2039           | 4.211             | -0.1688                | 4.2071            | -0.0928           | 16,101                |

Mean absolute relative error (MAPE) 0.1573

0.1225
Table 9 FGMM forecasting for the left value of the affected cases

| Day | $X_L^{(0)}$ | Forecasting value |
|-----|------------|-------------------|
| 8   | 16,557     | 16,904            |
| 9   | 17,124     | 17,759            |
| 10  | 17,955     | 18,433            |
| 11  | 18,436     | 18,329            |
| 12  | 18,576     | 18,465            |
| 13  | 19,435     | 19,234            |
| 14  | 20,078     | 19,987            |
| 15  | 21,367     | 21,009            |

Table 10 Centre-of-gravity values for the affected cases using FGMM

| Day | Lower  | Middle | Upper  | COG    | Actual |
|-----|--------|--------|--------|--------|--------|
| 1   | 258,423| 272,019| 285,615| 272,019| 269,789|
| 2   | 263,073| 276,889| 290,704| 276,889| 276,682|
| 3   | 269,742| 284,031| 298,320| 284,031| 283,407|
| 4   | 278,182| 292,807| 307,432| 292,807| 292,258|
| 5   | 286,690| 301,774| 316,857| 301,774| 301,609|
| 6   | 295,352| 311,004| 326,657| 311,004| 311,565|
| 7   | 304,244| 320,345| 336,445| 320,345| 319,840|
| 8   | 323,819| 340,722| 354,540| 340,722| 331,146|
| 9   | 334,993| 352,752| 370,511| 352,752| 342,473|
| 10  | 345,202| 363,635| 382,068| 363,635| 358,692|
| 11  | 357,834| 374,578| 386,712| 373,041| 362,824|
| 12  | 375,468| 384,589| 395,634| 385,230| 383,218|
| 13  | 382,361| 394,154| 399,876| 392,130| 391,967|
| 14  | 386,734| 399,675| 406,734| 397,714| 396,453|
| 15  | 408,921| 416,750| 423,678| 416,450| 415,678|

the difference between the FGMM forecasting values of the middle and the left length and similarly for upper. Using the method of Centre of Gravity (COG), we combine all the lower, middle and upper value of the affected cases, and compare with the real-time Indian affected data, and the comparison is given in Table 10 (Fig. 2).

4.2 Prediction of recovered count

Using the proposed approach, we can predict the number of recovered patient count. The number of recovered cases is given in Table 11.

We first forecast the recovered cases for middle values using grey GM(1, 1). The number of affected people in the period of study is taken as $X_M^{(0)}$ (Table 12)
Fig. 2 Comparison of actual affected patient count with FGMM forecasting values using COG

Table 11 Actual recovered cases of the COVID-19 disease

| Days | August | Left    | Actual (middle) | Right    |
|------|--------|---------|-----------------|----------|
| 9    | 452,559| 476,378 | 500,197         |          |
| 10   | 470,740| 495,516 | 520,292         |          |
| 11   | 489,617| 515,386 | 541,155         |          |
| 12   | 507,890| 534,621 | 561,352         |          |
| 13   | 525,797| 553,471 | 581,145         |          |
| 14   | 542,887| 571,460 | 600,033         |          |
| 15   | 562,430| 592,032 | 621,634         |          |

Table 12 Forecasting recovered cases using grey GM(1, 1)

| Days | $X_M^{(0)}$ | log($X_M^{(0)}$) | $\Delta M^{(0)}$ | $\Delta(k)$ | State |
|------|-------------|------------------|------------------|--------------|-------|
| 1    | 476,378     | 5.6779           | 5.6892           | 0            | 4     |
| 2    | 495,516     | 5.6950           | 5.7046           | −0.1675      | 2     |
| 3    | 515,386     | 5.7121           | 5.72             | −0.1377      | 3     |
| 4    | 534,621     | 5.7280           | 5.7354           | −0.1283      | 3     |
| 5    | 553,471     | 5.7430           | 5.751            | −0.1376      | 3     |
| 6    | 571,460     | 5.7569           | 5.7665           | −0.1652      | 2     |
| 7    | 592,032     | 5.7723           | 5.7821           | −0.1689      | 2     |

Mean absolute relative error (MAPE) 0.1508
The grey difference equation is $x^{(0)}(k) + az^{(1)}(k) = b$; solving for ‘$a$’ and ‘$b$’ using the method of least square, we get $a = -0.0027$ and $b = 5.6739$. The forecasting value of grey GM(1, 1) is given by

$$\hat{x}^{(0)}_M(k + 1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} \left( 1 - e^{-ak} \right), \quad k = 0, 1, 2, \ldots, n. = 5.6899e^{0.0027k}. \quad (35)$$

Next, we forecast the recovered cases using FGMM. For that, we divide the relative error sequence $\Delta(k)$ of grey GM(1, 1) into five states, as shown in Table 13.

We construct the membership function for the fuzzy states using Eqs. (13), (14) and (15). The fuzzy states for the days 1–7 are given in Table 14 and the state transition between the states is given in Table 15.

**Table 13** State description for FGMM

| State | Affected count | Interval     |
|-------|----------------|--------------|
| 1     | Very less      | $(-\infty, -0.17)$ |
| 2     | Less           | $(-0.17, -0.15)$ |
| 3     | Medium         | $(-0.15 - 0.13)$ |
| 4     | High           | $(-0.13, 0)$ |
| 5     | Very high      | $(0, \infty)$ |

$X^{(0)}_M = \{476378, 495516, 515386, 534621, 553471, 571460, 592032\}$. \quad (34)

**Table 14** Fuzzy states for the days 1–7

| Day | Relative error | State | Fuzzy state (state 2, state 3, state 4) |
|-----|----------------|-------|---------------------------------------|
| 1   | 0              | 4     | (0, 0, 1)                              |
| 2   | -0.1675        | 2     | (1, 0, 0)                              |
| 3   | -0.1377        | 3     | (0, 0.9693, 0.0306)                    |
| 4   | -0.1283        | 3     | (0, 0.844, 0.156)                      |
| 5   | -0.1376        | 3     | (0, 0.968, 0.032)                      |
| 6   | -0.1652        | 2     | (1, 0, 0)                              |
| 7   | -0.1689        | 2     | (1, 0, 0)                              |

**Table 15** State transition between the three states

| States | 2 | 3 | 4 | Total |
|--------|---|---|---|-------|
| States | 2 | 1 | 1 | 0     | 2     |
|        | 3 | 1 | 0 | 1     | 2     |
|        | 4 | 1 | 1 | 0     | 2     |
\(u_2(x) = \begin{cases} 
1 & -0.17 \leq x \leq -0.16 \\
\frac{-0.0028-2x}{0.0004} & -0.16 \leq x \leq -0.14 \\
0 & \text{otherwise} 
\end{cases} \) \hspace{1cm} (36)

\(u_3(x) = \begin{cases} 
\frac{2x+0.0032}{0.0004} & -0.16 \leq x \leq -0.14 \\
\frac{-0.0015-2x}{0.0015} & -0.14 \leq x \leq -0.065 \\
0 & \text{otherwise} 
\end{cases} \) \hspace{1cm} (37)

\(u_4(x) = \begin{cases} 
1 & -0.14 \leq x \leq -0.065 \\
\frac{2x+0.0028}{0.0015} & -0.065 \leq x \leq 0 \\
0 & \text{otherwise} 
\end{cases} \) \hspace{1cm} (38)

Using Eq. (12), the one-step transition probability matrix is

\[ P^{(1)} = \begin{pmatrix} 
0.5 & 0.5 & 0 \\
0.5 & 0 & 0.5 \\
0.5 & 0.5 & 0 
\end{pmatrix} \] \hspace{1cm} (39)

Now, we forecast the number of recovered cases for the day-2 using FGMM. The fuzzy vector for the day-1 is

\((0, 0, 1)\) \hspace{1cm} (40)

The relative error for the day-2 is

\[
\Delta(2) = \frac{1}{2}(0.5)(-0.17 - 0.15) + \frac{1}{2}(0.5)(-0.15 - 0.13) + \frac{1}{2}(0)(-0.13 + 0).
\]

\[\Delta(2) = -0.0015 \] \hspace{1cm} (41)

The forecasting value for the recovered case in day-2 is given by

\[
\hat{x}^{(0)}_M (k + 1) = \frac{\hat{x}^{(0)}_M (k + 1)}{1 - \Delta(k + 1)} = \frac{50467}{1.0015} = 50467. \] \hspace{1cm} (42)

Similarly, the forecasting value of the revered patient count with the corresponding relative error for the days 1–7 is given in Table 16.

Since the MAPE value (0.1401) (Table 16) of FGMM is less compared to MAPE value 0.1508 (Table 12) of grey GM(1, 1), we forecast the recovered cases for the next 3 days and the forecasting values are given in Table 17.

We forecast the recovered cases using the left length value of the real-time recovered data. Now, the \(X^{(0)}_L\) for the left length is

\[X^{(0)}_L = \{23819, 24776, 25769, 26731, 27674, 28573, 29602\}. \] \hspace{1cm} (43)

Solving \(a\) and \(b\) for the grey difference equation using least square method, we get \(a = -0.0035\) and \(b = 4.3727\). The forecasting value of grey GM(1, 1) is

\[
\hat{x}^{(0)}_L (k + 1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} \left( 1 - e^{-ak} \right), \hspace{0.5cm} k = 0, 1, 2, \ldots, n.
\]
Table 16 FGMM forecasting value for the recovered cases

| Day | $X^{(0)}_M$ | $\log(X^{(0)}_M)$ | $z^{(0)}_M$ | $\Delta(k)$ (FGMM) | Antilog $\zeta^{(0)}_M$ |
|-----|-------------|-------------------|-------------|---------------------|---------------------|
| 1   | 476,378     | 5.6779            | 5.6892      | 0                   | 488,877             |
| 2   | 495,516     | 5.6950            | 5.6956      | -0.1575             | 496,169             |
| 3   | 515,386     | 5.7121            | 5.7110      | -0.1575             | 514,050             |
| 4   | 534,621     | 5.7280            | 5.7284      | -0.1211             | 535,132             |
| 5   | 553,471     | 5.7430            | 5.7437      | -0.1258             | 554,334             |
| 6   | 571,460     | 5.7569            | 5.7595      | -0.1212             | 574,804             |
| 7   | 592,032     | 5.7723            | 5.7730      | -0.1575             | 592,936             |

Mean absolute relative error (MAPE) = 0.1401

Table 17 Forecasting recovered cases for the middle value using FGMM

| Day | $X^{(0)}_M$ | Forecasting value |
|-----|-------------|-------------------|
| 8   | 612,815     | 622,537           |
| 9   | 635,757     | 640,128           |
| 10  | 653,751     | 663,270           |
| 11  | 667,324     | 674,311           |
| 12  | 692,135     | 695,189           |
| 13  | 704,523     | 716,637           |
| 14  | 735,443     | 736,771           |
| 15  | 754,311     | 755,644           |

$7 = 4.38801e^{0.0035k}$, $k = 0, 1, 2, \ldots, n.$ (44)

Similarly, proceeding with the middle value, we get the grey GM(1, 1) and FGMM forecasting values for the left value of the recovered cases. The forecasting values are given in Table 18.

Here also, the MAPE value(0.1796)(Table 18) of FGMM is less than for MAPE value 0.1924 (Table 18) of grey GM(1, 1). Therefore, we forecast using FGMM for the next 8 days. The forecasting values are given in Table 19.

Similarly as in affected cases, we combine all the predicted values and the comparison of COG with actual recovered cases, as shown in Table 20 (Fig. 3).

4.3 Forecasting of death count

Finally, we predict the death count using the proposed models. The actual death count in the study period is given in Table 21.

The death count in the middle value is taken as $X^{(0)}_M$

$$X^{(0)}_M = \{21129, 21604, 22123, 22674, 23174, 23727, 24309\}.$$ (45)
Table 18 Grey GM(1, 1) and FGMM forecasting values

| Day | $X^{(0)}$   | log ($X^{(0)}$) | $\hat{x}^{(0)}_{L}$ | $\Delta(k)$ (GM(1,1)) | $\hat{z}^{(0)}_{L}$ | $\Delta(k)$ (FGMM) | Antilog $\hat{z}^{(0)}_{L}$ |
|-----|-------------|-----------------|----------------------|------------------------|----------------------|------------------------|-----------------------------|
| 1   | 23,819      | 4.3769          | 4.388                | 0                      | 4.388                | 0                      | 24,434                      |
| 2   | 24,776      | 4.3940          | 4.4033               | -0.2110                | 4.3934               | -0.225                 | 24,741                      |
| 3   | 25,769      | 4.4111          | 4.4188               | -0.1745                | 4.4101               | -0.1671                | 25,710                      |
| 4   | 26,731      | 4.4270          | 4.4343               | -0.1645                | 4.4258               | -0.1599                | 26,662                      |
| 5   | 27,674      | 4.4420          | 4.4498               | -0.1741                | 4.4412               | -0.1634                | 27,619                      |
| 6   | 28,573      | 4.4559          | 4.4654               | -0.2119                | 4.4569               | -0.19                  | 28,637                      |
| 7   | 29,602      | 4.4713          | 4.4811               | -0.2188                | 4.4725               | -0.1721                | 29,667                      |

Mean absolute relative error (MAPE) 0.1924 0.1796

Table 19 FGMM forecasting for the left value of the recovered cases

| Day | $X^{(0)}_{L}$ | Forecasting value |
|-----|---------------|--------------------|
| 8   | 30,641        | 30,019             |
| 9   | 31,788        | 31,140             |
| 10  | 32,688        | 32,016             |
| 11  | 33,674        | 33,546             |
| 12  | 34,234        | 33,987             |
| 13  | 36,547        | 35,498             |
| 14  | 35,349        | 34,987             |
| 15  | 35,987        | 35,645             |

Solving for ‘$a$’ and ‘$b$’ for the grey difference equation is $x^{(0)}(k) + az^{(1)}(k) = b$; using the method of least square, we get $a = -0.0023$ and $b = 4.3189$. The forecasting value of grey GM(1, 1) is given by

$$\hat{x}^{(0)}_{M}(k + 1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} \left( 1 - e^{-ak} \right), \quad k = 0, 1, 2, \ldots, n. = 4.3288e^{0.0023k},$$

and forecasting values are given in Table 22. Since the MAPE value, 0.0790 (Table 22) of FGMM is less compared to MAPE value 0.0829 (Table 22) of grey GM(1, 1).

Using FGMM, we forecast the death count for the next 8 days and the forecasting values are given in Table 23.

Next, using left length, we forecast the death count from COVID-19 for this period (Table 24)

$$X^{(0)}_{L} = \{1056, 1080, 1106, 1134, 1159, 1186, 1215\}$$
Table 20 Centre-of-gravity values for the recovered case using FGMM

| Day | Lower        | Middle       | Upper        | COG        | Actual     |
|-----|--------------|--------------|--------------|------------|------------|
| 1   | 464,443      | 488,877      | 513,312      | 488,877    | 476,378    |
| 2   | 471,428      | 496,169      | 520,910      | 496,169    | 495,516    |
| 3   | 488,340      | 514,050      | 539,760      | 514,050    | 515,386    |
| 4   | 508,470      | 535,132      | 561,794      | 535,132    | 534,621    |
| 5   | 526,715      | 554,334      | 581,953      | 554,334    | 553,471    |
| 6   | 546,166      | 574,804      | 603,441      | 574,804    | 571,460    |
| 7   | 563,268      | 592,936      | 622,603      | 592,936    | 592,032    |
| 8   | 592,518      | 622,537      | 652,556      | 622,537    | 612,815    |
| 9   | 608,989      | 640,128      | 671,269      | 640,129    | 635,757    |
| 10  | 631,255      | 663,270      | 695,287      | 663,271    | 653,751    |
| 11  | 654,512      | 674,789      | 693,633      | 674,311    | 667,324    |
| 12  | 675,623      | 694,267      | 715,678      | 695,189    | 692,135    |
| 13  | 695,467      | 719,867      | 734,578      | 716,637    | 704,523    |
| 14  | 715,689      | 737,845      | 756,778      | 736,771    | 735,443    |
| 15  | 735,612      | 758,965      | 772,356      | 755,644    | 754,311    |

Fig. 3 Comparison of actual recovered count with FGMM forecasting values using COG
Table 21: Actual death count of the COVID-19 disease

| August | Left  | Actual (middle) | Right |
|--------|-------|-----------------|-------|
| 9      | 20,073| 21,129          | 22,185|
| 10     | 20,524| 21,604          | 22,684|
| 11     | 21,017| 22,123          | 23,229|
| 12     | 21,540| 22,674          | 23,808|
| 13     | 22,015| 23,174          | 24,333|
| 14     | 22,541| 23,727          | 24,913|
| 15     | 23,094| 24,309          | 25,524|

Table 22: Forecasting death count using grey GM(1, 1) and FGMM

| Days | \(X_M^{(0)}\) | \(\hat{x}_M^{(0)}\) | \(\Delta(k)\) (GM (1, 1)) | \(\hat{z}_M^{(0)}\) | \(\Delta(k)\) (FGMM) | Antilog \(\hat{z}_M^{(0)}\) |
|------|---------------|-----------------|----------------------------|-----------------|----------------|----------------|----------------|
| 1    | 21,129        | 4.3288          | 0                         | 4.3288          | 0              | 21,131         |
| 2    | 21,604        | 4.3387          | -0.0961                   | 4.3347          | -0.09          | 21,617         |
| 3    | 22,123        | 4.3487          | -0.0887                   | 4.3452          | -0.0788        | 22,145         |
| 4    | 22,674        | 4.3587          | -0.0728                   | 4.3553          | -0.0759        | 22,667         |
| 5    | 23,174        | 4.3688          | -0.0870                   | 4.3654          | -0.0773        | 23,196         |
| 6    | 23,727        | 4.3788          | -0.0813                   | 4.3754          | -0.0760        | 23,740         |
| 7    | 24,309        | 4.3889          | -0.0714                   | 4.3855          | -0.0760        | 24,298         |
| MAPE | 0.0829        |                 |                           |                 |                | 0.0790         |

Table 23: Forecasting death count for the middle value using FGMM

| Day | \(X_M^{(0)}\) | Forecasting value |
|-----|---------------|-------------------|
| 8   | 24,915        | 25,424            |
| 9   | 25,602        | 26,024            |
| 10  | 26,273        | 26,640            |
| 11  | 27,389        | 27,595            |
| 12  | 25,112        | 25,389            |
| 13  | 26,437        | 27,557            |
| 14  | 27,321        | 28,477            |
| 15  | 27,431        | 27,661            |

Solving ‘\(a\)’ and ‘\(b\)’ for the grey difference equation using least square method, we get \(a = -0.0033\) and \(b = 3.0186\). The forecasting value of grey GM(1, 1) is

\[
\hat{x}_L^{(0)}(k + 1) = \left[ x^{(0)}(1) - \frac{b}{a} \right] e^{-ak} (1 - e^{-ak}), \quad k = 0, 1, 2, \ldots, n.
\]

\[
= 3.0285 e^{0.0033k}, \quad k = 0, 1, 2, \ldots, n.
\] (48)

The forecasting death count using FGMM for the days 8–10 is given in Table 25.
Table 24 Grey GM(1, 1) and FGMM forecasting death count for the left length

| Day | $X_L^{(0)}$ | log ($X_L^{(0)}$) | $\hat{X}_L^{(0)}$ | $\Delta(k)$ (GM (1, 1)) | $\hat{z}_L^{(0)}$ | $\Delta(k)$ (FGMM) | Antilog $\hat{z}_L^{(0)}$ |
|-----|-------------|-------------------|---------------------|--------------------------|------------------|---------------------|--------------------------|
| 1   | 1056        | 3.0238            | 3.0285              | 0                        | 3.0285           | 0                   | 1068                     |
| 2   | 1080        | 3.0335            | 3.0385              | −0.1646                  | 3.0336           | −0.16               | 1081                     |
| 3   | 1106        | 3.0438            | 3.0485              | −0.1539                  | 3.0438           | −0.1518             | 1106                     |
| 4   | 1134        | 3.0544            | 3.0586              | −0.1342                  | 3.0540           | −0.1282             | 1133                     |
| 5   | 1159        | 3.0639            | 3.0687              | −0.1543                  | 3.0643           | −0.1415             | 1160                     |
| 6   | 1186        | 3.0742            | 3.0788              | −0.1492                  | 3.0742           | −0.1464             | 1186                     |
| 7   | 1215        | 3.0847            | 3.089               | −0.1381                  | 3.0845           | −0.1254             | 1215                     |

Mean absolute relative error (MAPE) 0.1491 0.1422

Table 25 FGMM forecasting for the left value of the death cases

| Day | $X_L^{(0)}$ | Forecasting value |
|-----|-------------|-------------------|
| 8   | 23,669      | 24,157            |
| 9   | 24,322      | 24,724            |
| 10  | 24,959      | 25,314            |
| 11  | 25,475      | 25,312            |
| 12  | 25,943      | 25,567            |
| 13  | 26,123      | 26,008            |
| 14  | 27,324      | 27,400            |
| 15  | 28,765      | 28,546            |

Now, we combine all the predicted values using COG. Comparison of COG with actual death count is given in Table 26 (Fig. 4).

The relative error comparison of grey GM(1, 1) with FGMM (0.1112 < 0.1233) for Middle and (0.1481<0.1662) for left and the analytical comparison between the models is given in Tables 27 and 28 and depicted in Figs. 5 and 6.

5 Conclusion

In this paper, a new method imposed on FGMM to forecast the affected, recovered and the death count of the infectious disease COVID-19. As a new innovation and also to improve the forecasting accuracy, all the actual data are divided into three categories, namely left, middle and right. In this proposed method, all the FGMM forecasting values coincide with the real-time Indian COVID-19 data and it is evident clearly from the graphs. In this situation, as all the nations are now facing the second wave of the COVID-19 pandemic, this study is very useful to all the Governments, as well as for the public to take the precautionary measures from all the angles. The scope of this work can be extended to neutrosophic grey–Markov model for forecasting the infectious disease COVID-19 as well as traffic volume problems in future.
### Table 26 Centre-of-gravity values for the death count using FGMM

| Day | Lower   | Middle  | Upper   | COG     | Actual  |
|-----|---------|---------|---------|---------|---------|
| 1   | 20,063  | 21,131  | 22,199  | 21,131  | 21,129  |
| 2   | 20,537  | 21,617  | 22,698  | 21,617  | 21,604  |
| 3   | 21,039  | 22,145  | 23,251  | 22,145  | 22,123  |
| 4   | 21,534  | 22,667  | 23,799  | 22,667  | 22,674  |
| 5   | 22,037  | 23,196  | 24,356  | 23,196  | 23,174  |
| 6   | 22,533  | 23,740  | 24,926  | 23,740  | 23,727  |
| 7   | 23,083  | 24,298  | 25,513  | 24,298  | 24,309  |
| 8   | 24,157  | 25,424  | 26,691  | 25,424  | 24,915  |
| 9   | 24,724  | 26,024  | 27,324  | 26,024  | 25,602  |
| 10  | 25,314  | 26,640  | 27,966  | 26,640  | 26,273  |
| 11  | 26,568  | 27,540  | 27,595  | 27,595  | 27,389  |
| 12  | 24,189  | 25,543  | 26,435  | 25,389  | 25,112  |
| 13  | 26,568  | 27,782  | 28,321  | 27,557  | 26,437  |
| 14  | 27,327  | 28,452  | 28,477  | 28,477  | 27,321  |
| 15  | 26,549  | 27,679  | 28,756  | 27,661  | 27,431  |

**Fig. 4** Comparison of forecasting death count using FGMM with the actual values
Table 27 Comparison of relative error between grey GM(1,1) and FGMM

| Relative error $\Delta(k)$ | Grey GM(1,1) | FGMM |
|--------------------------|-------------|------|
|                          | Middle      | Left | Middle | Left  |
| Affected                 | 0.1362      | 0.1573 | 0.1145 | 0.1225 |
| Recovered                | 0.1508      | 0.1924 | 0.1401 | 0.1796 |
| Death                    | 0.0829      | 0.1491 | 0.0790 | 0.1422 |
| Average of $\Delta(k)$   | 0.1233      | 0.1662 | **0.1112** | **0.1481** |

Table 28 Analytical comparison of grey GM(1,1) with FGMM

| Grey GM(1,1) | FGMM |
|--------------|------|
| The predicted latest data are seen in the increasing smooth curve. $\hat{x}^{(0)}(k+1) = \left[x^{(0)}(1) - \frac{b}{a}\right]e^{-ak}(1 - e^{-a})$ gives the forecasting value | It is the combination of the fuzzy, grey–Markov and GM(1,1) includes the fuzzy elements to avoid ambiguity and uncertainty in the data. Triangular method is used to define the membership functions. The following formula $\hat{y}^{(0)}(k+1) = \frac{\hat{x}^{(0)}(k+1)}{1+\delta(k+1)}$ gives the predicted value |
| Relative error is calculated from the variations between the original and the forecasting values | The forecasting error is calculated using $\delta(k+1) = \sum_{i=1}^{n} \frac{1}{2} A_i (\delta(k+1)) (\delta_{i-1} + \delta_i)$ |
| It is inappropriate for the data classifications that are naturally vibrant. Hence, it needs to be improved to address real-time sequences more efficiently though the forecasting accuracy is lower | Membership value is considered to forecast the COVID-19 cases and also the relative error is comparatively less than grey GM (1,1) |

![Fig. 5 Comparison of middle value relative errors of grey GM(1,1) with FGMM](image-url)
Fig. 6 Comparison of left value relative errors of grey GM(1, 1) with FGMM

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Declarations

Conflict of interest The authors declare no conflict of interest.

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