Hyperfine state entanglement of spinor BEC and scattering atom

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Abstract
Condensate of spin-1 atoms frozen in a unique spatial mode may possess large internal degrees of freedom. The scattering amplitudes of polarized cold atoms scattered by the condensate are obtained with the method of fractional parentage coefficients that treats the spin degrees of freedom rigorously. Channels with scattering cross sections enhanced by the square of the atom number of the condensate are found. Entanglement between the condensate and the propagating atom can be established by scattering. Entanglement entropy is analytically obtained for arbitrary initial states. Our results also give a hint for the establishment of quantum thermal ensembles in the hyperfine space of spin states.

Keywords: Bose–Einstein condensate, entanglement entropy, spin coupling

Quantum entanglement is the central issue in modern many-body physics. For instance, the entanglement between different parts of the system plays an important role in the thermalization of an isolated quantum system [1]. A new phase of matter such as the topological order may be distinguished by long-range entanglement [2]. On the other hand, generating and manipulating entanglement is the basis of the quantum computation and quantum information process [3, 4]. Remote entanglement between quantum objects is the essential ingredient for quantum networks [5]. In experiments, entanglement between an atom and a Bose–Einstein condensate (BEC) has been realized by intermediate coupling with photon [6].

Spinor Bose–Einstein condensates (SBEC) [7, 8] as giant quantum objects which possess large internal degrees of freedom while the spatial modes are frozen have attracted much interest for testing the fundamental concepts of quantum mechanics, for the implementation of qubit, and so on. Spin mixing experiments have shown that spin states of SBECs may remain coherent from microsecond to second, which is long enough for quantum mechanical manipulation. SBECs are such systems for which a feasible method for preparing the ground state may exist, but an effective method for the exact preparation and detecting of a hyperfine spin state is still lacking because the energy splitting of various hyperfine spin states is extremely small due to the tiny spin–spin interaction [9]. A possible solution would be to entangle the SBEC with a micro-system that is separated from the SBEC in distance, and then to select the wanted SBEC state by measuring the micro-system. In the state collapse of the measurement, the entropy of SBEC is transferred to the micro-system (or environment). Consequently, the SBEC goes to more specific states. The entangled state of a condensate which may consist of a million atoms and a single propagating atom is a perfect analog of the Schrödinger cat state. It would be interesting to implement the which-way experiment [10] and Wheeler’s delayed-choice experiment with SBEC-atom scattering [11, 12]. One simple way to entangle the BEC with a single atom is using the direct scattering process without any intermediate coupling. Though the entanglement production during the scattering of two particles has been studied [13–22], due to the large number of atoms and the internal degree of freedom, the precess of an SBEC scattered by an atom will give much richer physics.

In the experiment we proposed, a spin polarized flux of spin-1 atoms is scattered by an SBEC of the same species of...
atoms. The incident kinetic energy is assumed to be very tiny (e.g. the incident flux may come from another leaking SBEC) such that the spatial modes of the SBEC cannot be excited and the scattering is dominated by s-wave scattering. The present paper studies the entanglement generated by the scattering of SBEC and an atom of the same species. It is expected that the spin degrees of freedom of the condensate and the propagating atom will be entangled due to the spin-spin interaction. Spin conservation is also crucial for the entanglement. Therefore, one should go beyond the mean-field theory and deal with the spin degrees of freedom exactly. We calculate the scattering amplitudes for both elastic and inelastic channels in the single mode approximation (SMA) [23, 24] for the spatial degrees of freedom of the condensate but treating the spin states exactly via the method of fractional parentage coefficients. The entanglement entropy is obtained analytically. The following discussions will only consider the case that the incident atom has low kinetic energy which cannot excite the spatial mode of the condensate. Thus the condensate remaining in the spatial ground mode has only spin degrees of freedom.

Let \( \hat{c}_i^+ \) and \( \hat{c}_i \) respectively be the creation and annihilation operators of atom in the inelastic channels of \( \mu = 0 \) and 1 for the initial incident polarization \( \mu_i = -1 \). The solid (blue) and dashed (yellow) curves respectively are corresponding to \( \mu = 1 \) and \( \mu = 0 \), for the initial condensate spin \( s_i = N \). The short-dashed (green) and dash-dotted (red) curves respectively are \( B_{1i}^\pm \) and \( B_{2i}^\pm \) for \( s_i = \frac{N}{2} \). (b) The contour plot of the difference of branching ratios of two final condensate states, \( D_{0,k}^\pm - D_{0,k}^\mp \) (refer to equation (11) for the definition), for the initial incident polarization \( \mu^i = -1 \). The values labelling the contour lines are the differences. The initial condensate spin \( s_i \) runs from 0 to \( N \) along the horizontal line from the left point to the right edge of the triangle. The initial condensate magnetization \( m_i \) runs from \(-s_i \) to \( s_i \) along a vertical line from the lower edge to the higher edge of the triangle.

![Figure 1](image_url)

**Figure 1.** (a) The spin-flipped branching ratios \( B_{1i}^\pm \) among the inelastic channels of \( \mu = 0 \) and 1 for the initial incident polarization \( \mu_i = -1 \). The solid (blue) and dashed (yellow) curves respectively are corresponding to \( \mu = 1 \) and \( \mu = 0 \), for the initial condensate spin \( s_i = N \). The short-dashed (green) and dash-dotted (red) curves respectively are \( B_{1i}^\pm \) and \( B_{2i}^\pm \) for \( s_i = \frac{N}{2} \). (b) The contour plot of the difference of branching ratios of two final condensate states, \( D_{0,k}^\pm - D_{0,k}^\mp \) (refer to equation (11) for the definition), for the initial incident polarization \( \mu^i = -1 \). The values labelling the contour lines are the differences. The initial condensate spin \( s_i \) runs from 0 to \( N \) along the horizontal line from the left point to the right edge of the triangle. The initial condensate magnetization \( m_i \) runs from \(-s_i \) to \( s_i \) along a vertical line from the lower edge to the higher edge of the triangle.

step 2, and the vertical one is \( m \), running from \(-s \) to \( s \). In low-energy scattering, only the s-wave component of the propagating mode has an important contribution. The atom propagating in the s-wave is specified by the wave number \( k \). Therefore, the bases of the hyperfine space of the composite system are \( \{ |k| w \} \), where \( |w| = |\mu; s, m \rangle \) with \( \mu \) the spin component of the propagating atom. It has energy \( \varepsilon_m + E_s \) with \( E_s \) the kinetic energy of the propagating mode and \( E_s \) the energy of the condensate, which only depends on the total spin of the condensate.

Denote the creation and annihilation operators of atom in the s-wave state with wave number \( k \) and the spin component \( \mu \) as \( \hat{b}_{k\mu}^+ \) and \( \hat{b}_{k\mu} \) respectively. Applying the effective Hamiltonian for the s-wave scattering [26, 27], the interaction responsible to the SBEC-atom scattering reads

\[
H_{sc} = 2 \sum_{F=0,2} g_F \int d\mathbf{k}_1 \hat{c}_{i1}^+ \hat{b}_{k_1\mu_1}^+ \hat{b}_{k_1\mu_1} \hat{b}_{k_2\mu_2} \hat{c}_{i2} (\hat{p}^e)_{F,\mu_1\mu_2 \rightarrow \mu_2 \mu} \]

(1)

where \( \hat{p}^e \) is the projection operator onto a two-body state with the total spin \( F \), defined as \( \hat{p}^e_{F,\mu_1\mu_2 \rightarrow \mu_2 \mu} = \sum_{\alpha_1} \langle F,\mu_1\mu_2 |C_{1\alpha_1,1\mu_2}^F |C_{1\alpha_1,1\mu} \rangle \) with \( C_{1\alpha_1,1\mu}^F \) the Clebsch–Gordan coefficients (CGs). The channel \( F = 1 \) is absent due to the permutation symmetry of bosons. The couplings \( g_F = \frac{4\varepsilon_m\varepsilon_m}{k_h} \) with \( k_h \) the scattering lengths [8]. The overall factor 2 is due to the scattering of identical particles.

Consider a polarized initial state \( |i\rangle = |k_i\rangle |w_i\rangle \) with \( |w_i\rangle = |\mu; s_i, m_i\rangle \) the spin state. Since the incident atom has spin one particle, the scattered final spin of the condensate can be \( s_f = s_i + 2\sigma \) for \( \sigma = 0, \pm 1 \). Denote the final state as \( |f\rangle = |k_f\rangle |w_f\rangle \) with \( \sigma \) labels the spin of the condensate and \( \mu \) the spin polarization of the outgoing atom. There are nine possible scattering channels, corresponding to \( |w_f\rangle = |\mu; s_f, m_f\rangle \) for \( \sigma = 0, \pm 1 \) and \( m = m_i + \mu \) due to the conservation of the spin component. The outgoing s-wave of the \( \sigma \)-channel has the wave number \( k_s \), which is determined by the energy conservation \( \varepsilon_m + E_s = \varepsilon_m + E_s \). The channel \( s_0 = s_i \) and \( \mu = \mu_i \) is the elastic scattering channel. The eight other channels are inelastic channels.

Following the scattering amplitude theorem, the exact scattering amplitude reads [28]

\[
f_{\sigma,\mu}(\mathbf{r}) = f^\mu - \frac{M}{2\pi\hbar^2} \langle f^-|H_{sc}|f^+\rangle / f^\mu
\]

(2)

where \( f^\mu \) is the scattering amplitude of the optical trapping potential \( V(r) \) that is assumed to be spin-independent. Since we are only interested in spin-flip processes, the scattering due to \( V(r) \) is irrelevant and \( f^\mu \) can be skipped. In (2), \( |f^\mu\rangle \) is the outgoing state of the Hamiltonian which contains \( V(r) \) but without \( H_{sc} \), while \( |f^-\rangle \) is the exact incoming state with \( |f\rangle \) as the initial state. In the distorted wave Born approximation [28], \( (f^-) \) and \( |f^\mu\rangle \) are replaced by the approximative ones, \( \langle f^-| \) and \( |f^\mu\rangle \), respectively. The transition matrix is given by

\[
(f^-)|H_{sc}|f^+\rangle \approx \langle w_f|s_i \rangle \langle k_f|H_{sc}|k_i \rangle \langle w_i|f^\mu\rangle
\]

(3)

The annihilation operator \( c_{i1} \) should contract with each creation operator \( c^+ \) in the factor \( \langle (c^+)^\mu \rangle_{s,m} \), which results \( N \)
identical terms. Thus, one only needs to contract \( \hat{C}_N \) with a specified \( c^+ \) on its r.h.s., the latter should be extracted from the normalized operator \([c^+]_N \) _m\_m. This can be done with the method of fractional parentage coefficients, which enables to rewrite the normalized operator as

\[
[(c^+)_N]_m = \frac{a^{(N)}_i}{\sqrt{N}} \sum_{\mu} C_{x+1,m-\mu,\mu}(c^+)_{N-1}v_{x+1,m-\mu,\mu}^+ \\
+ \frac{b^{(N)}_i}{\sqrt{N}} \sum_{\mu} C_{x-1,m-\mu,\mu}(c^+)_{N-1}v_{x-1,m-\mu,\mu}^+ \\
\tag{4}
\]

where the factors \( a^{(N)}_i \) and \( b^{(N)}_i \) are the fractional parentage coefficients (FPCs) known as \([29, 30]\). They are nonzero only if \( N - S \) is even and the sum of squares of them is one when \( N - S \) is even.

The calculation of contractions involving \( c^+_i \) is similar. All contractions contribute a factor \( N^2 \) to the transition matrix. Both the initial and final states of the condensate have an extra normalization factor \( \frac{1}{\sqrt{N}} \) as seen in \((4)\). Therefore the transition matrix is enhanced by an overall factor \( N \) due to the coherent scattering of \( N \) atoms. For large \( N \) the condensate scattering will be dominant and the scattering due to the trapping potential is relatively unimportant and will be neglected.

With the transition matrix calculated with the FPCs, we obtain the scattering amplitudes \( f_{\sigma,\mu}^i \) for the channels \( |w_{\sigma,\mu}\rangle \) as

\[
f_{0,\mu}^i = -4N \sum_{\mu_i,v_i,F} a_F v_{\mu_i,v_i,F} \delta_{m-\mu,m_i-v_i} \cdot [(c^+)_{N} v_{x+1,m-\mu,\mu}]^2 C_{x+1,m-\mu,\mu}^+(c^+)_{N-1}v_{x+1,m-\mu,\mu}^+ \\
+ (b_{N}^{(N)})^2 C_{x-1,m-\mu,\mu}^+(c^+)_{N-1}v_{x-1,m-\mu,\mu}^+ \tag{7}
\]

\[
f_{\pm,\mu}^i = -4N \sum_{\mu_i,v_i,F} a_F v_{\mu_i,v_i,F} \delta_{m-\mu,m_i-v_i} \cdot (c^+)_{N} v_{x+1,m-\mu,\mu}^2 C_{x+1,m-\mu,\mu}^+(c^+)_{N-1}v_{x+1,m-\mu,\mu}^+ \\
+ (b_{N}^{(N)})^2 C_{x-1,m-\mu,\mu}^+(c^+)_{N-1}v_{x-1,m-\mu,\mu}^+ \tag{8}
\]

The cross sections are proportional to the absolute square of the scattering amplitudes. The branch ratios are given by

\[
A_{\sigma,\mu}^i = \frac{(f_{\sigma,\mu}^i)^2}{\sum_{\sigma',\mu'}(f_{\sigma',\mu'}^i)^2}. \tag{9}
\]

The large \( N \) behaviors of the amplitudes can be obtained from the CGs and FPCs that have explicit expressions. Firstly, the amplitudes of elastic scatterings are always proportional to \( N \) as a consequence of coherent scattering. When \( \frac{N^2}{\sqrt{N}} \ll 1 \), the values of CGs and FPs are of order one, therefore the inelastic amplitudes are also proportional to \( N \). From \((5)\) it can be seen that \( a^{(N)}_i \sim \frac{1}{\sqrt{N}} \) when \( N - s_i \sim O(1) \), thus the associate terms can be neglected. If both \( s_i \) and \( |m| \) are of order \( N \), all inelastic amplitudes could be neglected as their scattering branch ratios with respect to the elastic amplitudes are vanishing as \( \frac{1}{\sqrt{N}} \) or even faster. When \( s_i \) is of order \( N \) but \( |m| \) of order one, the inelastic amplitudes of the \( \sigma = 0 \) channel is also proportional to \( N \). Four example channels with initial states located on three corners of the hyperfine space are given in the following (only channels with amplitudes proportional to \( N \) are retained and the difference between \( a^0_i \) and \( a^0\) has been neglected).

(i) The initial states with \( s_i = m_i = N \) and \( \mu_i = 0 \), -1 have only elastic scattering amplitude proportional to \( N \). Amplitudes of all inelastic channels are at most proportional to \( \sqrt{N} \) therefore have negligible contribution. This initial state may be easier to be realized for ferromagnetic SBEC.

(ii) For \( s_i = N \) and \( m_i = \mu_i = 0 \), there are three possible final spin states: \(|0; N, 0\rangle \) and \(|\pm 1; N, \mp 1\rangle \). The first one is from the elastic scattering, having normalized amplitude \( \frac{1}{\sqrt{10}} \). Each of the other two has \( \frac{1}{2\sqrt{5}} \). It is argued in the text following equation \((12)\) that the ferromagnetic SBEC will tend to this state in steady scattering of zero-polarized beam.

(iii) For \( s_i = m_i = 0 \) and \( \mu_i = -1 \), the elastic scattering final spin state is \(|2; 0, 0\rangle \), having normalized amplitude \( \frac{2\sqrt{2}}{3} \), while the inelastic scattering final spin states are \( |0; 2, 0\rangle \) and \(|\pm 1; 2, \mp 1\rangle \) with normalized amplitudes \( \frac{1}{3\sqrt{5}}, \frac{1}{\sqrt{10}} \) respectively. This and the following case would be easier to be implemented with antiferromagnetic SBEC.

(iv) For \( s_i = m_i = 0 \) and \( \mu_i = +1 \), the elastic scattering final spin state is \(|-1; 0, 0\rangle \), having normalized amplitude \( \frac{2\sqrt{2}}{3} \); while the inelastic scattering final spin states are \( |-1; 2, 0\rangle \), \(|0; 2, -1\rangle \) and \(|1; 2, -2\rangle \), having normalized amplitudes \( \frac{1}{3\sqrt{10}}, \frac{1}{\sqrt{30}}, \frac{1}{\sqrt{15}} \) respectively.

The scattering is dominated by the elastic channel that has over 85% branching ratio for any initial state. Since the elastic scattering does not change the state of the condensate, it is less interesting. To exclude the elastic scattering, one can use a spin filter, such as Stern–Gerlach apparatus, that separates the outgoing beam into three ones according to their spin polarization and absorbs the beam of the same spin polarization as the incident flux. The left two beams will all come from inelastic scattering. Coherent superpositions of the left two spin-flipped beams can be recovered by another Stern–Gerlach apparatus that combines two beams and erases the information of paths which any beam had taken. The effect of the spin filter is just decreasing the brightness of the incident flux. We can subtract the elastic scattering wave from the entire outgoing wave and concentrate on the spin-flipped scattering. On that account, we introduce a spin-flipped branching ratio (SFBR) as the cross section ratio of a spin-flipped channel over all of those.

\[
B_{\mu}^i = \frac{\sum_{\sigma} (f_{\sigma,\mu}^i)^2}{\sum_{\sigma,\mu'} (f_{\sigma,\mu'}^i)^2}. \tag{10}
\]
for $\mu = \mu_i$. This quantity is measurable in principle. We further introduce a branching ratio for different final spin of the condensate among spin-flipped channels,

$$D^i_{\mu} = \frac{\sum_{\mu_1 = \mu} |f_{\mu_1}^i|^2}{\sum_{\mu_1, \mu_2 = -1, 0, 1} |f_{\mu_1, \mu_2}^i|^2}. \quad (11)$$

When the elastic scattering background is subtracted, the scattering amplitudes would depend on $d\sigma_i$ significantly. In the following we will consider the condensate of $^{87}$Rb that has $a_0 = 101.8$ and $a_2 = 100.4$ (a.u.)

First, consider the incident flux with $\mu_i = -1$. The SFBRs $B_{\mu}^i$ of $\mu = 0$, 1 versus $\frac{N}{m}$ for $s_i = \frac{N}{2}$ and $N$ are given in figure 1(a). It is clear that the final condensate magnetization $m \leq m_i$ because the conservation of spin component associate with $\mu_i = 1$. For $s_i = N$, the solid and dashed curves of figure 1(a), $B_{\mu}^i$ of $\mu = 0$ ($\mu = 1$) is a monotonic decreasing (increasing) function of $m_i$. In the large $N$ limit and neglecting the difference between $a_0$ and $a_2$, one has

$$B_{\mu}^i = \frac{1}{2} + \frac{(2 - \mu)(m_i \mu - \frac{m_i}{N} - 1)}{2(3 - \frac{m_i}{N})} \quad (12)$$

for $s_i = N$ and $\mu = 1, 0$. For general $s_i$, e.g. $s_i = \frac{N}{2}$ shown as the short-dashed and dash-dotted curves in figure 1(a), the SFBRs have complicated behavior. The SFBRs of two channels have reflecting symmetry with respect to the value $\frac{N}{2}$ since they are normalized. To compare the transition probabilities for spin increasing and decreasing, the differences of the $\sigma = \pm 1$ channels, $D^{+1}_{\mu} - D^{-1}_{\mu}$, are plotted in figure 1(b). The triangle is the hyperfine space of the condensate where the left apex has $s = m = 0$ and the vertical line on the right has $s = N$ and $m$ running from $-N$ to $N$ from the lower end to the upper end. The condensate spin tends to increase in the positive regime and tends to decrease in the negative regime. Combining the information of figures 1(a) and (b), one can have the following picture of state evolution of condensate under the steady scattering of $\mu_i = -1$ incident flux: the states in the upper-right regime flow to the left and approach the horizontal axis of $m = 0$; after crossing the axis to the regime of negative $m$ the states will flow to the lower-right apex and finally the condensate maximizes its magnetization to $-N$. If the incident flux has $\mu_i = 1$ the condensate will reversely go to the state of $s = m = N$.

Second, consider the scattering of the zero-polarized incident flux with $\mu_i = 0$. The SFBRs, $B_{\mu}^i$, of $\mu = \pm 1$, are given in figure 2(a) for $s_i = \frac{N}{2}$ and $N$. For $s_i = N$ and neglecting the difference between $a_0$ and $a_2$ the SFBRs simply have expressions in the large $N$ limit,

$$B_{\mu}^i = \frac{1}{2} + \frac{m_i \mu}{1 + \left(\frac{N}{m_i}\right)^2}. \quad (13)$$

Figure 2(a) shows that $B_{\mu}^i$ of the $\mu = 1$ is larger (smaller) than that of the $\mu = -1$ for $m_i > 0$ ($m_i < 0$). Therefore the magnetization of the condensate tends to vanish in the scattering of the zero-polarized incident flux. Figure 2(b) shows $D^i_{\mu}$ for the initial magnetization $m_i = 0$ and $\frac{N}{2}$. The curves of $\sigma = \pm 1$ channels have observable difference only in the small regime near to $s_i = 0$. It is consistent with the prejudice of hard-to-distinguish states with tiny energy difference. In steady scattering, the states will approach the $m = 0$ axis from both sides in the hyperfine space. The condensate tends to increase its total spin in the large spin regime but very slowly.

The reduced density of matrix of the condensate after scattering is given by tracing off the outgoing states,

$$\rho_i = \sum_{\sigma, \mu} A_{\sigma \mu}^i |s_\sigma, m \rangle \langle s_\sigma, m| \quad (14)$$

where $A_{\sigma \mu}^i$ is the branching ratio of $(\sigma, \mu)$ channel. If the elastic scattering channel is excluded, $A_{\sigma \mu}^i$ should be the SFBR. Correspondingly, the entanglement entropy of the system is given by

$$S_i = -\sum_{\sigma, \mu} A_{\sigma \mu}^i \log(A_{\sigma \mu}^i). \quad (15)$$

The final state is an entangled state if $S_i = 0$ otherwise not entangled. The final state of the case (i) discussed previously has $S_i = 0$ therefore it is not an entangled state. The case (ii) has entanglement entropy 0.3944 when all channels, including the elastic channel, are considered. If the elastic channel is excluded, the entanglement entropy of the case (ii) becomes ln(2). Excluding the corresponding elastic channels, the case (iii) and (iv) have entanglement entropies 1.0889 and 0.8979 respectively. The dependence of the entanglement entropy on the initial states, with the elastic component subtracted, is presented in figure 3: for $\mu_i = 0$ (a) and for $\mu_i = -1$ (b). The case of $\mu_i = 1$ can be obtained as a reflection of figure 3(b) with respect to the horizontal axis.

Generally $S_i$ is nonzero, implying that the outgoing atom and the condensate is entangled. A deterministic measurement of the polarization of the outgoing beam will cause the state of
the condensate collapse. Whenever such an outgoing state is measured or operated quantum-mechanically, the condensate will collapse to a specific state instantly. This suggests a method for hyperfine state preparation. On the other hand, the condensate will be described by the non-pure state with a probability for each spin state given by (14) if the information of outgoing atoms is completely lost.

Let us discuss the conditions for experimental implementation. Consider a spherical harmonic optical potential trap with the vacuum energy level above the trap bottom by $W$. The harmonic potential is characterized by frequency $\omega$ and the lowest energy mode has energy $\frac{1}{2} \hbar \omega$ (negleting the spin coupling energy). Practically $W \gg \hbar \omega$. The initial total energy of the condensate and the incident atom is $\epsilon_k + W + \delta \hbar \omega$. Due to the weak coupling and dilute gas, only two-body scattering is significant and should be considered. Thus the final state could only contain (1) no propagating atom, (2) two propagating atoms, and (3) one propagating atom. Firstly, the condensate has little probability to absorb the incident atom due to the momentum conservation. Secondly, if the final state contains two propagating atoms, say with kinetic energies $\epsilon_k$ and $\epsilon_k$, respectively, the final energy will be $\epsilon_k + \epsilon_k + 2W + \frac{3N - 1}{2} \hbar \omega$. The energy conservation gives $\epsilon_k = \epsilon_k + \epsilon_k + W - \frac{3}{2} \hbar \omega$. Hence the channel of two final propagating atoms is impossible if $\epsilon_k < W - \frac{3}{2} \hbar \omega$, which is a rather loose condition. The only crucial condition for our model that has only one outgoing atom is that the spatial mode of the condensate should remain in the ground band during scattering. It is $\epsilon_k < \hbar \omega$ since the minimum energy to excite the ground spacial mode is about $\hbar \omega$. For a typical trap of $\omega = 100$ Hz, the incident atom beam should have an equivalent temperature smaller than $nK$. It seems feasible to obtain such an incident beam by atom evaporation from another condensate of such temperature.

In summary, we have obtained the scattering amplitudes of condensate-atom scattering, dealing the spin degrees of freedom strictly by the method of fractional parentage coefficients. The elastic scattering channel as well as some inelastic scattering channels are enhanced by the square of the atom number of the condensate, thus are much easier to be observed. The scattering branching ratios imply that the zero-polarized incident flux drives the condensate to the disorder state, while the nonzero-polarized incident flux maximizes the magnetization. In steady scattering, the flow of states in the hyperfine space of the condensate is outlined. The flow only weakly depends on the interaction, but is dominated by the spin symmetry. It provides a method to create reversed occupation states in the hyperfine space. For some special cases, such as cases (ii) to (iv), the quantum collapse in measuring the outgoing atom can select a hyperfine state of the condensate. The analytical expression for the entanglement entropy as a function of the initial states is obtained. For generic initial states, the entanglement entropy is generally nonzero, implying that the scattered atom and the condensate is entangled. The experiment would be implemented at temperatures of sub-nano Kelvin.

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