One-way surface states due to nonreciprocal light-line crossing

Fereshteh Abbasi, Arthur R Davoyan and Nader Engheta
Department of Electrical and Systems Engineering, University of Pennsylvania, Pennsylvania 19104, Philadelphia
E-mail: engheta@ee.upenn.edu

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Abstract
We revisit the problem of the surface plasmon polariton (SPP) propagation at a metal–magneto-optical dielectric interface and predict a conceptually different regime of one-way surface wave propagation. We show that in the presence of magnetization, the nonreciprocal crossing of the light-line by dispersion curve of SPPs is possible. This leads to the formation of leaky-wave surface states in one of the propagation directions, whereas in other direction SPPs are confined to the interface. We show that such a regime of surface plasmon propagation is fundamentally different from one-way regimes of surface waves predicted earlier.

1. Introduction
Rapid progress in the fields of plasmonics and metamaterials has paved the way to create desired exotic electromagnetic properties in engineered structures. Such a possibility, enables a variety of prospective applications in different frequency regimes. For example, one can mention focusing and guiding of light beyond the diffraction limit [1, 2], ultra-compact plasmonic nanoantennas [3] and optical circuitry and signal handling similar to that in electronics [4]. In particular, several electromagnetic analogues of key electronic components have been demonstrated recently, including interconnects [5], splitters [6], filters [7], as well as optical counterparts of inductors and capacitors (the so-called optical metatronic lumped elements [4, 8]). Several phenomena were experimentally shown in microwave and THz regimes as well, including cloaking [9, 10], perfect absorbers [11], and THz devices [12]. Nevertheless, the success of all the aforementioned concepts crucially depends on: (1) the ability to tune dynamically the electromagnetic networks and circuits, and (2) the signal isolation analogous to the functionality of the diode in electronics. Several approaches have been proposed recently for passive and active control over the propagation of electromagnetic waves at the nanoscale, including pump-probe techniques [13], nonlinear self-tuning [14], spatio-temporal waveguide modulation [15], as well as electric and magnetic field control over interaction of fields and matter [16, 17]. In particular, magneto-optical (MO) material response has been exploited recently for tuning the light propagation in plasmonic and metamaterial structures [18–23]. Furthermore, magneto-active materials may break the time reversal symmetry of Maxwell’s equations, which may also imply the nonreciprocal wave propagation, i.e. break of invariance with respect to the direction of the wave propagation [24]. The latter effect is widely employed in ferromagnetic microwave systems, most well-known examples being isolators, circulators, and in general, nonreciprocal transmission lines [25–29]. More recently, this concept was extended to optical isolation in magneto-photonic structures [30–36]. In waveguiding systems with a reduced dimensionality, e.g. metal-dielectric interfaces and surfaces of magneto-photonic crystals, one-way regimes of surface wave propagation may be achieved [30, 33, 37], i.e. the surface wave propagation is possible only in one direction and not in the opposite. The key ingredient for such a one-way propagation regime is the split of wave ‘existence’ regions, which is related to the break of the symmetry in the wave dispersion in forward and backward directions of propagation. Typically such a split occurs near a geometry-related resonance (e.g. near the waveguiding cut-off frequencies, frequencies of surface Fano resonances, etc). For example, in a single-interface plasmonic metal-dielectric waveguide, different regimes of surface plasmon polariton (SPP) propagation have been predicted close to the epsilon-near-zero resonances [37]. Also, in a similar geometry, one-way regimes of SPP propagation occur due to a direction-
dependent split of the SPP resonance frequency in the presence of magnetization (this regime is shown in figure 1(b)) [30]. The frequency bandwidth of the one-way regime is proportional to the strength of MO activity, which is typically weak, hence significantly limiting the frequency range where such a one-way regime of surface wave propagation is observed. Furthermore, occurring in the proximity of the resonance, one-way wave propagation is highly sensitive to the material losses. As we show in this paper, the magnetically induced symmetry breaking in the wave dispersion is pronounced not only near the resonances, but also can be employed for a nonresonant one-way wave guiding. We propose a concept for a different regime of one-way SPP propagation at the interface of a MO dielectric and a metal. We theoretically demonstrate that the symmetry breaking between surface waves propagating in forward and backward directions is crucial even at frequencies much below the resonance of SPP and explore a nonresonant nonreciprocal light-line crossing and one-way SPP propagation.

2. Analysis

Figure 1 (a) shows the geometry of our problem. In particular, we consider wave propagation along the interface of a metal and a magnetized MO dielectric. We assume that the direction of magnetization is in the $-z$ direction and is perpendicular to the wave propagation direction, i.e., Voigt configuration. In this case we can write the relative permittivity of the MO dielectric in a tensorial form [38]:

$$\varepsilon = \begin{pmatrix} \varepsilon_{MO} & i \delta & 0 \\ -i \delta & \varepsilon_{MO} & 0 \\ 0 & 0 & \varepsilon_{\perp} \end{pmatrix}$$

(1)

where $\delta$ is the off-diagonal term of the relative permittivity tensor (in further analysis without loss of generality we assume $\delta > 0$), responsible for the magnetically induced coupling of $x$ and $y$ components of the electric field. In such a geometry, transverse magnetic and transverse electric waves can be studied independently [24]. However, only TM surface waves can propagate along the metal-dielectric interface, hence in our further analysis we only consider TM surface waves (we note that by duality of Maxwell equations a similar analysis is applicable for surface magnons at the interface of a ferromagnetic medium [39]). Representing fields as

$$H = H_{MO} \exp (+i k_0 \beta x) \exp (-k_0 \alpha_{MO} y) \hat{z} \text{ in MO dielectric and}$$

$$H = H_{met} \exp (+i k_0 \beta x) \exp (+k_0 \alpha_{met} y) \hat{z} \text{ in metal (} k_0 \text{ being the free space wavenumber), we write the}$$
surface wave dispersion equation [37]:

$$\frac{\epsilon_{\text{met}}}{\delta} = \frac{-\epsilon_{\text{eff}}}{\epsilon_{\text{MO}} \beta + \alpha_{\text{MO}}}$$

(2)

where $\epsilon_{\text{met}}$ is the relative permittivity of the metal and $\epsilon_{\text{eff}} \equiv \frac{\epsilon_{\text{MO}}^2 - \delta^2}{\epsilon_{\text{MO}}}$, $\beta$ is the normalized propagation constant along the interface and $\alpha_{\text{MO}} = \sqrt{\beta^2 - \epsilon_{\text{eff}}}$ and $\alpha_{\text{met}} = \sqrt{\beta^2 - \epsilon_{\text{met}}}$ are the transverse wavenumbers (normalized to free space wavenumber) in MO medium and metal, respectively. In equation (2) all quantities are relative and dimensionless. Generally speaking, all the relative permittivities in this dispersion equation are frequency-dependent. However, in order to get a deeper insight into the possible regimes of nonreciprocal wave propagation, we begin with a phenomenological approach, neglecting the frequency dependence of material parameters and assuming the material to be lossless. Later in this manuscript, we bring into consideration the frequency dispersion of the materials as well as material losses. In the absence of MO activity, i.e. when $\delta = 0$, equation (2) simplifies to the well-known dispersion equation of SPP at the interface between metal and dielectric [40]. In this case the SPP resonance at $\beta \to \pm \infty$ is defined by a simple relation between the metal and dielectric permittivities: $\epsilon_{\text{met}} = -\epsilon_{\text{MO}}$. When magnetized (i.e. $\delta > 0$), the symmetry between the wave propagation in forward and backward directions is broken, leading to the direction-dependent split of the SPP resonance condition: $\epsilon_{\text{met}} = -\epsilon_{\text{MO}} - \delta$ for backward ($\beta \to -\infty$), and $\epsilon_{\text{met}} = -\epsilon_{\text{MO}} + \delta$ for forward directions of propagation ($\beta \to +\infty$). Consequently, in the range of parameters $-\epsilon_{\text{MO}} - \delta < \epsilon_{\text{met}} < -\epsilon_{\text{MO}} + \delta$, SPP propagates in forward direction only, and its propagation in backward direction is prohibited [30]. Figure 1(b) shows the dispersion curve of SPP in such a geometry for $\epsilon_{\text{MO}} = 1.18$, $\delta = 0.3$, with the variation of the metal permittivity. Clearly for values of metal permittivity bound between $-\epsilon_{\text{MO}} - \delta = -1.48$ and $-\epsilon_{\text{MO}} + \delta = -0.88$, i.e. the shaded region, one-way-regime of SPP propagation is observed.

On the other hand in the limit $\epsilon_{\text{met}} \to -\infty$, equation (2) reduces to:

$$-\beta \frac{\delta}{\epsilon_{\text{MO}}} = \sqrt{\beta^2 - \epsilon_{\text{eff}}}$$

(3)

clearly underlining the nonreciprocal nature of the surface waves propagating in the presence of magnetization in this limit. In particular, the solutions of the dispersion equation (3) are possible only when $\beta \delta < 0$, implying that in this limit, for $\delta > 0$, only surface waves propagating in backward direction exist. Hence, the dispersion curve for waves propagating in forward direction (i.e. with $\beta > 0$) crosses the light-line of the MO medium, $\beta = \sqrt{\epsilon_{\text{eff}}} \equiv k_{\text{MO}}$, at some value of metal permittivity. For the specific parameters used to plot the dispersion curve in figure 1(b), this crossing happens at $\epsilon_{\text{met}} \approx -10$. Physically, the crossing of the MO light-line corresponds to the energy leaking into the ambient MO dielectric medium, i.e. the surface waves are not bound to the interface anymore. Therefore for any $\epsilon_{\text{met}} < -10$ bound surface waves propagate only in backward direction, and surface waves in forward direction become leaky.

From equation (2) the condition for the light-line crossing can be found as:

$$\sqrt{\frac{\epsilon_{\text{eff}} - \epsilon_{\text{met}}}{\epsilon_{\text{met}}^{-1}}} = -\frac{\delta}{\epsilon_{\text{MO}}}$$

(4)

3. Results and discussion

To analyze the relation in equation (4), in figure 2(a) we plot the variation of $\epsilon_{\text{met}}$ as function of $\epsilon_{\text{MO}}$ for different values of MO activity $\delta$. For optical materials we normally have $|\delta| \ll \epsilon_{\text{MO}}$, as a result the corresponding condition of light-line crossing holds only for very large negative values of metal permittivity, i.e. $|\epsilon_{\text{met}}| \gg \epsilon_{\text{MO}}$, which are observed at terahertz and microwave frequency ranges. With increase of the ratio $|\delta| / \epsilon_{\text{MO}}$ and consequently stronger symmetry breaking, the required value of metal permittivity, i.e. $|\epsilon_{\text{met}}|$, decreases. Note that, in principle, such high values of MO activity may be observed near the cyclotron resonances of biased plasmas (i.e. semiconductors or metals) [41], or may be achieved with proper engineering of metamaterials [37].

We proceed further by accounting for the frequency dependence of the relative permittivities of the materials. In this case, the diagonal and off-diagonal elements of permittivity tensor of a MO material are written in the frame of Drude–Lorentz model [42]:

$$\epsilon_{\text{MO}} = \epsilon_{\text{b}} \left[ 1 - \frac{\omega^2 \beta_1 (\omega + i \gamma)}{\omega (\omega + i \gamma)^2 - \omega^2} \right],$$

(5)
where $\omega_{p1}$ and $\omega_{g1} = \frac{eB}{m}$ are plasma frequency and cyclotron frequency, respectively, $\epsilon_b$ is the background permittivity of the MO medium, $\gamma$ is the damping factor. Metal permittivity, in turn, is modeled as
\[
\varepsilon_{\text{met}} = 1 - \frac{\omega_{p2}^2\omega_g}{\omega^2 + i\gamma^2},
\]
where $\omega_{p2}$ is the plasma frequency of metal and $\gamma_2$ is the damping factor (note that such a Drude model gives a very good description at lower frequency range). The relative permittivities of these materials versus frequency are plotted in Figure 2(b). To be more specific, as an example, we consider here a low-loss limit and take the following material parameters:

- for InAs, $\varepsilon_{\text{met}} = 0.46$ and $\gamma_1 = 0.001$ (comparable relative values of plasma and gyrotropic frequencies may be found in semiconductors, e.g. in InAs for which $\omega_{p1}$ is 2π 1.8 THz yielding $\omega_{g1} = 0.46 \omega_{p1} = 2\pi 0.82$ THz. Note that the value of $\omega_{g1}$ depends on the doping and applied magnetic bias. $\gamma_1$ for InAs is in the order of $2\pi 0.75$ THz which we neglect at this point, but later take into consideration [42]). For the metal permittivity we assume $\varepsilon_{\text{met}} = 1000 \omega_{p2}$ and $\gamma_2 = 0.001 \omega_{p2}$ (comparable values can be seen in multiple noble metals, for example in silver for which $\omega_{p2} = 2\pi 2.18$ THz and $\gamma_2 = 2\pi 4.35$ THz [43]). It should be noted that if a biasing magnetic field is applied to create the MO activity (as opposed to using material with intrinsic MO activity), the magnetization may extend to the metallic region as well. However, since the crossing of light-line and the proposed one-way regime occur at highly negative values of $\varepsilon_{\text{met}}$, the penetration of the surface waves into the metallic medium is extremely small. As a result, the effect of magnetization of metal on the propagation of surface waves, is extremely small and can be neglected. Solving equation (4) with the parameters mentioned, we find that at a critical frequency $\omega_c \approx 2.01$, the condition for light-line crossing is obtained; below this frequency only surface waves in backward direction are bound to the surface, whereas waves in forward direction become leaky.

![Figure 2](image-url) (a) Variation of required $\varepsilon_{\text{met}}$ in terms of $\varepsilon_{\text{MO}}$ (for different $|\delta|$) in order to have light-line crossing. (b) Plot of frequency dependence of the relative permittivities of metal and MO medium. The dashed line denotes the parameter values at which the light-line crossing occurs. The values of $\delta$ are multiplied by 10 in order to be seen more clearly.

We plot the dispersion characteristics of the surface wave at around the critical frequency $\omega_c$ in figures 3(a) and (b). Figure 3(a) shows three different, but hardly distinguishable dispersion curves, corresponding to three different cases. First in absence of MO activity, i.e. $\omega_{g1} = 0$, and next, in presence of MO activity, i.e. when $\omega_{g1} = 0.46 \omega_{p1}$, which leads to two different (but very close) curves for SPP in forward and backward directions. In order to highlight the difference between the three dispersion curves that we mentioned, and to reveal the dynamics close to the point of crossing, in the inset of figure 3(a) we plot the three dispersion curves, normalized to $k_{\text{MO}}$, the light-line in MO material (which by itself is a function of frequency as a result of material dispersion). The dashed curve shows the normalized SPP dispersion in absence of MO activity. Once MO activity is introduced, i.e. when $\omega_{g1} = 0.46 \omega_{p1}$, the symmetry breaking in the dispersion of SPP propagating in forward and backward directions occurs. Note that all the three curves are normalized to $k_{\text{MO}}$ with the MO activity present. Such symmetry breaking increases as the frequency decreases. The curve corresponding to backward direction (the red curve) is always greater than unity implying the existence of bound surface waves travelling in backward direction at every frequency in the plotted range. In contrast, for the waves travelling in the forward direction (the green curve), the normalized dispersion curve below $\omega_c$ crosses 1, corresponding to the leakage of...
the forward surfaces wave into the MO medium. To gain a deeper understanding in this nonreciprocal transformation, we examine the confinement of surface waves propagating in forward and backward directions near the interface, i.e. we study the dispersion of the transverse wavenumber, $\alpha_{MO}$ (see figure 3(b)). The split in the localization with MO activity is clearly observed. With the frequency decrease the split grows and below $\omega_k$ a bifurcation in the localization of forward and backward surface waves occurs. Below $\omega_k$ the real part of $\alpha_{MO}$ for surface waves in forward direction becomes almost infinitesimally small, implying that there is no confinement to the surface, which is consistent with the dynamics shown in figure 3(a) and our phenomenological predictions.

Moving further we study the excitation and propagation of the surface waves numerically with the help of the finite-element method implemented in COMSOL Multiphysics (TM) package. We place a two-dimensional (2D) small electric line dipole source, with its dipole moment oriented in $y$-direction, in close proximity of the metal-MO interface, ensuring the evanescent coupling and efficient excitation of the surface waves. Figure 4(a) shows the 2D magnetic field profile, i.e. $H_y$, of the excited electromagnetic field for three different cases of magnetization, i.e. different values of $\alpha_{MO}$, in the low loss limit at a constant frequency. As expected, the surface wave excitation and propagation is symmetric in the absence of magnetization (when $\alpha_{MO} = 0$). With the increase of the cyclotron frequency $\omega_c$ to 0.1 $\omega_p$ as shown in figure 4(a), the symmetry breaking in localization of forward and backward waves increases. In the last panel in figure 4(a), when the value of magnetization reaches the condition of the light-line crossing, bound surface waves are excited only in backward direction of propagation. In the forward direction of the propagation we observe only emission and the 'leakage' of the radiation into the bulk of the MO medium. Finally, we investigate the influence of losses on the wave propagation and its excitation. Figure 4(b) shows the wave excitation with the material parameters, similar to those in figure 4(a), but for larger damping frequency in MO material, i.e. $\gamma = 0.1 \omega_p$ (which is comparable with realistic semiconductors, e.g. InAs [42]). One-way surface wave excitation and propagation in backward direction is still preserved; however, the amplitude of the wave steeply decays with propagation, as expected. In figure 4(c) we plot the decay rate of SPP in backward direction with respect to the collision frequency $\gamma$ in the MO medium for several different values of the cyclotron frequency. We observe that with the increase of $\omega_c$, the propagation distance slightly decreases. This is due to the increase of localization of waves in backward direction near the interface for stronger values of MO activity. (We note that due to extremely small penetration of the light wave in the MO medium, the losses in the metal hardly have any effect on surface wave propagation.) The influence of losses can be mitigated with the use of a subwavelength MO coating, instead of a semi-infinite medium. In this case a strong symmetry breaking is also possible.

Finally we note that in our analysis we have neglected the carrier concentration variation due to the Schottky barrier. It is anticipated that a barrier width of about a few hundred nanometers would not affect the predicted effects. On the other hand, a possible way to avoid this is to have a very thin dielectric layer separating the metal and the semiconductor. If the thickness of this layer is much smaller than the wavelength at which the system
operates (for example a few hundred nanometers in THz frequencies), the effect of the dielectric layer on the surface waves is negligible and at the same time it prevents the formation of a Schottky barrier.

4. Conclusion

In conclusion, we have explored a conceptually different regime of one-way surface plasmon propagation at the interface of an MO material and a metal. We have theoretically demonstrated that due to the nonreciprocal light-line crossing, the bound surface wave propagation is possible only in one direction, whereas in the other propagation direction the waves become leaky. We also have studied the influence of loss on such symmetry breaking and demonstrated that the predicted one-way propagation is feasible even in the presence of relatively high losses. We believe that our idea may be of use in a variety of potential applications ranging from electromagnetic signal isolation, to novel type of nonreciprocal leaky wave antennas.

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