Lift estimation of Half-Rotating Wing in hovering flight

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Abstract. Half-Rotating Wing (HRW) is a new kind of flapping wing system with rotating flapping instead of oscillating flapping. Estimating approach of hovering lift which generated in hovering flight was important theoretical foundation to design aircraft using HRW. The working principle of HRW based on Half-Rotating Mechanism (HRM) was firstly introduced in this paper. Generating process of lift by HRW was also given. The calculating models of two lift mechanisms for HRW, including Lift of Flow Around Wing (LFAW) and Lift of Flow Dragging Wing (LFDW), were respectively established. The lift estimating model of HRW was further deduced, by which hovering lift for HRW with different angular velocity could be calculated. Case study using XFLOW software simulation indicates that the above estimating method was effective and feasible to predict roughly the hovering lift for a new HRW system.

1. Introduction
Flapping Wing Air Vehicle (FWAV) is widely applied in military and civil fields for its advantages. Some great results had been got in micro FWAV by lots of researchers in the world [1], [2]. However, the swing motion of flapping wing at high speed leads to large inertial force, by which the development of FWAV in large size had been further restricted. The movement forms of animal limbs are asymmetric swing in essence. Swing is a movement form in order to fit muscles for animal, while asymmetric motion is the basic condition to fly and walk. Based the above views, our work team had developed a rotating mechanism named Half-Rotating Mechanism (HRM) [3], which could not only generate asymmetrical motion but avoid swing. HRW flight mechanism was developed by improving HRM with lighter weight. It would make FWAV in large size possible for its simple structure, high efficiency and strong adaptability.

Lift prediction is a basic link to develop the bionic aircraft. Abundant research achievements [4-6] have been made on the lift of flapping flight. HRW aircraft as a new kind of flapping wing system has not yet researched about its lift. Therefore, the hovering lift of HRW aircraft was calculated in this paper based on LFAW and LFDW [7]. This research would provide primary quantitative analysis about the lift of HRW as well as guide the design and flight control for HRW aircraft.

2. Work principle of HRW and generating hovering lift by HRW

2.1. Motion and driving mechanism for HRW
The driving mechanism of HRW was a kind of simplified HRM which was suitable for flight. Figure 1 showed the driving mechanism which was consist of spindle, crank, blade of HRW, positioning slider and shaft with groove. The blade of HRW would turn half round anticlockwise when crank turned one
round anticlockwise. Herein, point $C$ was not only the center of shaft with groove but also the middle of blade of HRW. Point $H$ was the center of positioning slider, which was the fixed point in HRM. To avoid discontinuous movement caused by the interference when point $C$ moved to fixed point $H$, one edge of the crank was designed as shaft with groove. Furthermore, positioning slider was set at the location of point $H$ to make sure the blade of HRW have accurate moving rule. The combination of positioning slider and shaft with groove could realize simplified design of HRM. HRW could produce continuous and certain motion when motor drove the crank to rotate.

As shown in figure 2, HRW rotated counterclockwise. The blade of HRW was divided into two parts by fixed point $H$. The part of HRW on fixed point $H$ left moved downwards, which was equivalent to that wings of insect moved on downstroke. Thus this part could be called downstroke. Similarly, the other part of HRW on fixed point $H$ right could be called upstroke.

![Figure 1. HRW driving mechanism](image1)

![Figure 2. Motion interval division for HRW](image2)

### 2.2. The spanwise airflow around HRW

The effect of air on the blade was so complex during the whole motion of HRW that a simplified model that lift was generated by the relative motion between the air and wing was used. Generally, only chordwise airflow was around the flapping wing. Spanwise airflow herein arose from obvious telescopic movement of HRW along the spanwise direction, as shown in figure 3. The influence came from both chordwise and spanwise airflow on the lift estimation of the HRW should be considered. Typical flight states of aircraft included hovering flight and forward flight. The air relative to the HRW just produced spanwise airflow when the HRW aircraft was in the hovering flight. Not only the spanwise airflow but also chordwise airflow would be produced when HRW aircraft was in the forward flight. In this paper, the lift was studied only when HRW aircraft was in hovering flight, namely only spanwise airflow was around the wing.

![Figure 3. The spanwise airflow around HRW](image3)

### 2.3. How to generate hovering lift by HRW

The computation model of hovering lift by HRW was established. As shown in figure 4, $AB$ and $OC$ respectively represented the blade of HRW and crank, and the lengths of them were $2a$ and $R$. $C$ was the shaft with groove, $H$ denoted positioning slider. $P$ was instantaneous center of velocity at this time. The angular velocity of HRW was $\omega/2$ when the angular velocity of crank was $\omega$. $\phi$ represented the rotation angle of crank from original position where crank was in direction of horizontal-right. $\alpha$ denoted the angle of attack. It should be noted that all angles in counterclockwise direction were positive. The coordinate system which the origin was at point $C$ and the positive $X$ axis was along $AB$. 


was established. Then the range of downstroke and upstroke could be expressed by the bounds for \( x \) in \((-2 \sin \frac{\phi}{2}, a]\) and \([-a, -2 \sin \frac{\phi}{2}]\) respectively. The velocity of each point on the blade of HRW was so different that the relative velocity of airflow at corresponding points was different. For instance, the relative velocity of airflow at point \( D \) and velocity of point \( D \) were equal and opposite. Lift element was taken along the spanwise direction because the lift of each point on the blade of HRW was different, as shown in figure 5. Then angle of attack, LFAW and LFDW were calculated respectively. It should be noted that directions of lift generated by HRW in downstroke and upstroke were different.

\[
\alpha = \arcsin \left( \frac{x + 2R \sin \frac{\phi}{2}}{\sqrt{4R^2 + x^2 + 4Rx \sin \frac{\phi}{2}}} \right)
\]

Where \( R \) was the length of the crank, and \( \phi \) was the rotation angle of crank.

3. Lift estimation of HRW in hovering flight

3.1. Angle of attack for HRW flight

As the speed of each point on the wing was different when HRW rotated, its corresponding angle of attack was also different. The flow patterns of the airflow around the wing were different under the condition of small Angle of Attack (SAA) and Large Angle of Attack (LAA) when calculating the LFAW, and the lift estimation method was also different. Thus, the scope of angle of attack should be defined. The angle of attack of point \( C \) which was always \( \phi/2 \) could be regarded as the decision value. Considering the cyclicality of blade motion, the stroke of the crank was set to one round in this paper in order to make calculation and description easier. When \( \phi/2<\pi/9 \), namely \( \phi<2\pi/9 \), the lift of any point on the wing was calculated by the method of SAA. When \( \phi/2>\pi/9 \), namely \( \phi>2\pi/9 \), the lift of any point on the wing was calculated by the method of LAA. Therefore, \( \phi \) could be divided into four intervals that were \([0, \frac{2\pi}{9}), [\frac{2\pi}{9}, \pi), [\pi, \frac{16\pi}{9}), [\frac{16\pi}{9}, 2\pi] \). When \( \phi \in [0, \frac{2\pi}{9}) \) or \( \phi \in [\frac{16\pi}{9}, 2\pi] \), the wing’s angle of attack was SAA. When \( \phi \in [\frac{2\pi}{9}, \frac{16\pi}{9}) \), the wing’s angle of attack was LAA.

Wherever the blade of HRW was, the angle of attack for any point on the wing could be got:

\[
\alpha = \arcsin \left( \frac{x + 2R \sin \frac{\phi}{2}}{\sqrt{4R^2 + x^2 + 4Rx \sin \frac{\phi}{2}}} \right)
\]

3.2. LFAW estimation of HRW in hovering flight

3.2.1. LFAW estimation with SAA. Under the condition of SAA, air flowed around the wing could be regarded as air flowed around flat plate [8] then circulation around the wing which determined LFAW would be generated. Circulation at point \( C \) could be considered as the circulation of the whole wing, because the speed and circulation of each point on the wing was different.
The wing’s angle of attack was SAA when $\phi \in [0, \frac{2\pi}{9})$. According to the velocity direction of point $C$, it was known that clockwise circulation around the wing was as follows:

$$\Gamma = \pi v_c \cdot (2a) \sin \alpha_c$$

(2)

Where $2a$ was spanwise length of HRW, $v_c = \omega R$ was the velocity direction of point $C$, and $\alpha_c = \frac{\phi}{2}$ was the angle of attack of point $C$.

The blade was composed of infinite point vortexes according to the thin wing theory [8]. If each point vortex was the same, circulation was equivalent to uniform distribution. Linear density of circulation was $\frac{\Gamma}{2a}$, namely the circulation around spanwise length element of wing $dx$ was:

$$d\Gamma = \pi v_c \sin \alpha_c \, dx$$

(3)

Then the aerodynamic lift element was:

$$dL_s = \rho v d\Gamma = \pi \rho hv v_c \sin \alpha_c \, dx$$

(4)

Where $\rho$ was air density, $h$ was chord length of HRW, and $v$ was the relative velocity of airflow.

LFAW elements on downstroke and upstroke were both:

$$dF_{LAS} = dL_s \sin \left(\frac{\phi}{2} + \alpha\right)$$

(5)

LFAW element was integrated along the span of the wing to get the relationship between LFAW of whole wing and rotation angle $\phi$:

$$F_{LAS} = \int_{-a}^{a} dF_{LAS} = 2\pi a \rho h \omega \, R^2 \sin \frac{\phi}{2} \sin \phi$$

(6)

Where $\omega$ was the angular velocity of crank.

Lift estimation method was the same as above when $\phi \in [\frac{16\pi}{9}, 2\pi]$. The relationship between LFAW of whole wing and rotation angle $\phi$ was got:

$$F_{LAS} = \int_{-a}^{a} dF_{LAS} = -2\pi a \rho h \omega \, R^2 \sin \frac{\phi}{2} \sin \phi$$

(7)

3.2.2. LFAW estimation with LAA. Under the condition of LAA, the airflow was detached on the back of the wing. Thus, circulation around the wing could not be generated. The approximate calculation formulas of aerodynamic force originated from flowing around a flat at LAA [9] were used to estimate LFAW and LFDW with LAA. The pressure drag coefficient $C_D$ measured by experiment was 4.6 in this paper.

The wing’s angle of attack was LAA when $\phi \in [\frac{2\pi}{9}, \pi)$. The estimation of aerodynamic lift element was in this form:

$$dL_s = \frac{1}{4} C_D \rho h v^3 \sin 2\alpha \, dx$$

(8)

Where $C_D$ was pressure drag coefficient.

LFAW element on downstroke was:
LFAW element on upstroke was:

$$dF_{LAL_2} = -dL_s \sin \left( \frac{\phi}{2} + \alpha \right)$$  \hspace{1cm} (10)

The wing’s angle of attack was LAA when $\phi \in \left[ \pi, \frac{16\pi}{9} \right)$. The estimation of aerodynamic lift element was in this form:

$$dL_s = \frac{1}{4} C_v \rho h v^2 \sin 2(\pi - \alpha)dx = -\frac{1}{4} C_v \rho h v^2 \sin 2\alpha dx$$  \hspace{1cm} (11)

LFAW element on downstroke was:

$$dF_{LAL_1} = -dL_s \sin \left( \frac{\phi}{2} + \alpha \right)$$  \hspace{1cm} (12)

LFAW element on upstroke was:

$$dF_{LAL_2} = -dL_s \sin \left( \frac{\phi}{2} + \alpha \right)$$  \hspace{1cm} (13)

LFAW element was integrated along the span of the wing to get LFAW of whole wing:

$$F_{LAL} = \int_{-a}^{a} dF_{LAL_1} + \int_{-a}^{a} dF_{LAL_2}$$

$$= \frac{1}{4} C_v \rho h v^2 R \int_{-2R \sin \frac{\phi}{2}}^{2R \sin \frac{\phi}{2}} \left[ x^2 \cos^2 \frac{\phi}{2} + 3Rx \sin \phi \cos \frac{\phi}{2} + 2R^2 \sin^2 \phi \right]^{1/2} dx$$

$$= \frac{1}{4} C_v \rho h v^2 R \int_{-2R \sin \frac{\phi}{2}}^{2R \sin \frac{\phi}{2}} \left[ 4R^2 + x^2 + 4Rx \sin \frac{\phi}{2} \right]^{1/2} dx$$ \hspace{1cm} (14)

3.2.3. LFDW estimation of HRW in hovering flight. The lift was mainly derived from LFAW produced by circulation and LFDW could be ignored when the wing’s angle of attack was SAA. However, LFDW made a significant contribution to lift in the range of LAA. The estimation of aerodynamic drag element was in this form:

$$dD = \frac{1}{2} C_v \rho h v^2 \sin^2 \alpha dx$$  \hspace{1cm} (15)

LFDW elements on downstroke and upstroke were both:

$$dF_{Lb} = -dD \cos \left( \frac{\phi}{2} + \alpha \right)$$  \hspace{1cm} (16)

LFDW element was integrated along the span of the wing to get LFDW of whole wing:
\[ F_{lw} = \int_{a}^{\infty} dF_{lw} = \frac{1}{8} C_d \rho \omega^2 \int_{a}^{\infty} \left( x + 2R \sin \frac{\phi}{2} \right)^2 \cdot \frac{2R \cos \phi - x \sin \frac{\phi}{2}}{4R^2 + x^2 + 4Rx \sin \frac{\phi}{2}} \, dx \]  

(17)

4. Simulation verification

4.1. Case study about lift calculation

To verify the results, the lift formulas obtained above were used to estimate the lift of HRW prototype which had been developed before. The dimensions of HRW were \( R = 0.06 \text{ m} \), \( a = 0.141 \text{ m} \), \( h = 0.2 \text{ m} \), and the air density \( \rho = 1.225 \text{ kg/m}^3 \). The lift values of HRW at different positions within one round were calculated when \( \omega \) was 6\( \pi \) rad/s, 10\( \pi \) rad/s, 20\( \pi \) rad/s, 30\( \pi \) rad/s respectively.

According to the dimensions of HRW prototype, the model was established in fluid simulation software XFLOW and the related parameters were set. Similarly, numerical calculation for the lift of HRW at four different angular velocities was also carried out.

4.2. Comparison between theoretical results and simulation results

Lift values from theoretical calculation and numerical calculation were processed by Matlab. When \( \omega \) was 6\( \pi \) rad/s, 10\( \pi \) rad/s, 20\( \pi \) rad/s and 30\( \pi \) rad/s, the comparison results were shown respectively in figure 6.

| \( \omega = 6\pi \text{ rad/s} \) | \( \omega = 10\pi \text{ rad/s} \) | \( \omega = 20\pi \text{ rad/s} \) | \( \omega = 30\pi \text{ rad/s} \) |
|---|---|---|---|
| \( \phi \) (rad) | Lift (N) | theoretical result | simulation result | \( \phi \) (rad) | Lift (N) | theoretical result | simulation result | \( \phi \) (rad) | Lift (N) | theoretical result | simulation result | \( \phi \) (rad) | Lift (N) | theoretical result | simulation result |
| 0 | 0 | | | 0 | 0 | | | 0 | 0 | | | 0 | 0 | | |
| \( \pi \) | 1.5 | theoretical result | simulation result | \( \pi \) | 1.5 | theoretical result | simulation result | \( \pi \) | 1.5 | theoretical result | simulation result | \( \pi \) | 1.5 | theoretical result | simulation result |
| \( 2\pi \) | 3 | theoretical result | simulation result | \( 2\pi \) | 3 | theoretical result | simulation result | \( 2\pi \) | 3 | theoretical result | simulation result | \( 2\pi \) | 3 | theoretical result | simulation result |
| \( 3\pi \) | 4.5 | theoretical result | simulation result | \( 3\pi \) | 4.5 | theoretical result | simulation result | \( 3\pi \) | 4.5 | theoretical result | simulation result | \( 3\pi \) | 4.5 | theoretical result | simulation result |
| \( 4\pi \) | 6 | theoretical result | simulation result | \( 4\pi \) | 6 | theoretical result | simulation result | \( 4\pi \) | 6 | theoretical result | simulation result | \( 4\pi \) | 6 | theoretical result | simulation result |
| \( 5\pi \) | 7.5 | theoretical result | simulation result | \( 5\pi \) | 7.5 | theoretical result | simulation result | \( 5\pi \) | 7.5 | theoretical result | simulation result | \( 5\pi \) | 7.5 | theoretical result | simulation result |
| \( 6\pi \) | 9 | theoretical result | simulation result | \( 6\pi \) | 9 | theoretical result | simulation result | \( 6\pi \) | 9 | theoretical result | simulation result | \( 6\pi \) | 9 | theoretical result | simulation result |

**Figure 6.** Comparison between theoretical results and simulation results at different angular velocities

It could be seen from figure 6 that:

The changed trends of theoretical lift and simulation lift were consistent and corresponding lift values were also close, which could verify the validity of the theoretical calculation method about the lift of HRW in this paper.

The reasons for the difference between the two kinds of lift values:

1) The measuring accuracy of the pressure drag coefficient \( C_d \) in experiment.

2) Due to the complexity of airflow in HRW flight, the treatments of airflow were so different in theoretical calculation and numerical calculation that the calculation results might be different.
3) Flexible wing was used to measure the pressure drag coefficient $C_D$ in experiment, while the rigid wing was used to calculate lift in simulation. The lift calculated in simulation was smaller than the lift obtained from theoretical calculation, for which the pressure drag coefficient of flexible wing was bigger than that of rigid wing.

5. Conclusions
1) According to motion characteristics of HRW driving mechanism, the blade of HRW was divided into downstroke and upstroke. It was illustrated that spanwise airflow was the main source of hovering lift. The theoretical estimation formulas of LFAW and LFDW were deduced respectively after establishing the lift estimating model for spanwise airflow, which were verified by XFLOW. The research in this paper provided the theory support for lift prediction and parameter design of HRW.

2) The velocity of each point along HRM spanwise direction was different. Hence, LFAW and LFDW were obtained after analyzing and integrating lift elements. The lift on downstroke and upstroke needed to be analyzed respectively because lift directions of them were different.

3) The lift of HRW was proportional to the square of angular velocity for the crank, and changed with the rotational angle of crank. The lift value could reach the maximum when the blade of HRW flapped down horizontally.

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