Statistical comparison of Schlumberger arrays using randomized complete block design

Oluseun Adetola SANUADE¹*, Joel Olayide AMOSUN², Kehinde David OYEYEMI³, Tokunbo Sanmi FAGBEMIGUN⁴, Jane Idowu FALOYO⁵

¹Geosciences Department, King Fahd University of Petroleum & Minerals, Dhahran, Saudi Arabia.
²,⁴Department of Geophysics, Federal University Oye-Ekiti, Ekiti State, Nigeria.
³Department of Industrial Physics, Covenant University, Ota, Ogun State, Nigeria.
⁵Department of Applied Geophysics, Federal University of Technology Akure, Ondo State, Nigeria.

*sheunsky@gmail.com

Abstract. In this study, statistical comparison of the resistivity data acquired using vertical electrical sounding (VES) technique with three arrays (conventional Schlumberger array, Hummel array and half-Hummel array) was performed. The objective is to assess the efficacy of the Hummel and half-Hummel arrays of VES as alternative arrays to the conventional Schlumberger array at areas with limited space for acquisition during groundwater exploration activities. A total of fifteen (15) VES data were acquired for the three arrays at five locations within Phase II of Federal University Oye-Ekiti, southwestern, Nigeria. The electrode spacing ($AB/2$) varies from 1 to 65 m during the acquisition. The raw VES data were subjected to statistical analysis to compare the three arrays. Randomized complete block design (RCBD) was used to establish if one array could be a perfect or near perfect substitution for other arrays. The responses obtained for all the arrays analysed are statistically the same at some locations which means measurements could either be taken using any of the three arrays. However, responses for the three arrays are different at a location which shows the three arrays are independent in this location. Pairwise comparison analysis at this location shows the difference in the three arrays. We therefore generated acceptance and rejection map of the study area which could be used as a guide prior to any geophysical data acquisition.

Keywords: RCBD, resistivity, array, Schlumberger.

1. Introduction

Geophysical techniques are useful in the estimating aquifer parameters and in general for groundwater exploration in many geological settings. The most frequent used geophysical techniques include magnetic, direct current (DC) resistivity surveys, electromagnetic and seismic refraction methods [1]. These geophysical techniques could provide reliable information that can be used effectively to identify and locate geological structures such as faults, joints, fractures, and weathered rock materials in the subsurface. However, DC electrical resistivity method is the most commonly used method in the basement complex because it can provide information such as the lithology, stratigraphic sequence and hydrogeological characteristics of the subsurface material [2-5]. Moreover, the frequently used electrode configuration of DC electrical resistivity survey for estimating aquifer parameters is the conventional Schlumberger array. This array involves symmetrical spread of electrodes on both sides of the array length [4-7; Fig. 1a]. However, in developed areas, this type of configuration could be difficult as a result of constraint to have electrode spread on both sides. This condition is also possible in areas where we have thick vegetation and also congested areas (Fig. 1b). This could lead to the incompleteness of field data which may result in ambiguities in the geophysical survey and as a result lead to wrong recommendations. This therefore led to the emergence of a modified Schlumberger
arrays which involve asymmetrical array of electrodes in VES [2-3; 8-9]. The modified arrays are called Hummel and half-Hummel arrays, which can be used to solve the problems highlighted above.

Fig. 1: Diagrammatic representation of electrode spread (a) symmetrical (conventional) arrangement (b) in congested area with limited access

To establish the relationship between the three Schlumberger arrays of data acquisition and to ascertain whether modified Schlumberger arrays can be used as alternative method to conventional Schlumberger array, a statistical approach called "Experimental Design" was employed. "Experimental design" is defined as the process that involves planning a study to meet specific objectives. The proper planning of experiment is essential to ensure that the right type of data and a sufficient sample size and power are provided to answer the research questions of interest as clearly and efficiently as possible [10]. Therefore this study investigates the statistical relationship between the conventional Schlumberger, Hummel and half-Hummel arrays of geophysical data acquisition using an experimental design statistical approach.

Anjorin and Olorunfemi [8] carried out an investigation on the comparative study of VES using conventional Schlumberger and half Schlumberger arrays in a typical Basement Complex area in Southwest Nigeria. The investigation was carried out to assess the usefulness of the half Schlumberger array as a substitute for the conventional Schlumberger array. Their comparison was done using crossplots of the interpreted geoelectrical parameters. Akintorinwa and Abiola [2] also compared conventional Schlumberger and modified-Schlumberger arrays VES interpretation results in part of Akure, Ondo State Nigeria. Their study used visual inspection of VES curves and geoelectric sections to compare the arrays. They also used crossplots where they determined the coefficient of correlation between the arrays. Oladunjoye and Jekayinfa [3] also study the comparison of the interpretation
results of VES data obtained with the conventional and modified Schlumberger arrays in order to assess the efficacy of the modified Schlumberger arrays as a substitute for the conventional Schlumberger arrays. Their study also employed visual inspection of VES curves and geoelectric sections and cross plots for the comparison of these arrays. Adetokunbo et al. [11] used randomized complete block design (RCBD) experiment to compare plus-minus and conventional reciprocal methods of seismic data processing using synthetic data. However, no study have used RCBD to compare arrays in electrical resistivity method, hence the uniqueness of this study.

2. Study area and its geology

The study area is located at the Phase II of the Federal University Oye-Ekiti, Ekiti State. The area lies within latitude 755250 to 755600 universal transverse Mercator (UTM) and longitude 860150 to 860450 UTM with the extent of about 2 km². The study area is accessible through network of roads and footpaths (Fig. 2).

Fig. 2: Location map of the study area.

The study area is underlain by the Basement Complex of Southwestern Nigeria. The basement rocks are concealed in places by a variably thick overburden. The major lithologic unit in the study area is migmatite gneiss (Fig. 3).
3. Methodology

3.1 Electrical Resistivity

Basically in DC resistivity survey, currents (I) are passed into the ground through a pair of current electrode and the resultant resistance (R) are obtained through pair of potential electrode. In VES vertical variations of resistivities in the ground are measured with depth with respect to a fixed point. To achieve this, inter-electrodes spacing are gradually increased at the center of the spread to investigate [13]. In conventional Schlumberger array, four electrodes (two current electrodes and two potential electrodes) system is used (Fig. 4). The four electrodes are usually arranged colinearly with different inter-electrode spacing. The potential electrodes (M and N) are partially fixed at the center of the spread while the current electrodes (A and B) are moved symmetrically about the center of the spread (Fig. 4). To obtain the apparent resistivity ($\rho_a$) of the subsurface, the measured R is multiplied by the geometric factor ($G_s$) as given by equation 1:

$$\rho_a = R G_s$$

where:

$$G_s = \pi \frac{a^2}{b}$$

Fig 3: Geological map of the area around Oye-Ekiti, Southwest Nigeria [12]
Fig. 4: Electrode configuration for conventional Schlumberger array
However, there is a fixed current electrode (A) placed perpendicular to the line of electrode spread at a
distance three times the current electrode spacing (3 × L) or equal to the current electrode spacing (L)
for the Hummel array and half-Hummel array, respectively (Fig. 5); that is, in modified Schlumberger
arrays (Hummel and half-Hummel), only B is moved while A is fixed.

Fig. 5: Electrode configuration for Hummel method (modified Schlumberger array).
The geometric factor of the modified Schlumberger arrays (GH) has been calculated to be twice of GS
[14].

\[ G_H = 2G_S \]

Therefore, apparent resistivity (ρa2) for modified Schlumberger arrays is given by equation 4:

\[ \rho_{a2} = \frac{R G_H}{4} \]

In this study, a total of fifteen (15) sounding data were collected at five locations using conventional
Schlumberger, Hummel and half-Hummel arrays within Oye-Ekiti metropolis, southwestern Nigeria to
compare the three different arrays. Data were collected using PASI Earth resistivity meter. The
current electrode (AB/2) spacing ranges from 1 to 65 m in all the three arrays

3.2 Statistical analysis
Statistical analysis was done to make direct comparison between the three arrays used to acquire data
for apparent resistivity. The objective is to investigate the effect of treatments (conventional
Schlumberger, Hummel and half Hummel) on apparent resistivity (response) so as to affirm if the type
of treatment has an influence on the response gathered. To achieve this, a fundamental design of
experiment called randomized complete block design (RCBD) was adopted. This approach is the
standard design for experiments where similar experimental units are grouped into blocks or
replicates. The data source (the three arrays) is known as the treatments while electrode spacing are the blocks.

3.2.1 Theory of RCBD

To compare 't' treatments and 'b' blocks in a given experiment shown in Table 1, there is one observation per treatment in each block, and the order of running the treatments within each block is randomly determined.

Table 1: Theory of RCBD

|       | \( \tau_1 \) | \( \tau_2 \) | \( \tau_3 \) | \( \ldots \) | \( \tau_t \) |
|-------|-------------|-------------|-------------|--------------|-------------|
| \( \text{B}_1 \) | \( y_{11} \) | \( y_{21} \) | \( y_{31} \) | \( \ldots \) | \( y_{11} \) |
| \( \text{B}_2 \) | \( y_{12} \) | \( y_{22} \) | \( y_{32} \) | \( \ldots \) | \( y_{12} \) |
| \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) | \( \ldots \) |
| \( \text{B}_b \) | \( y_{1b} \) | \( y_{2b} \) | \( y_{3b} \) | \( \ldots \) | \( y_{1b} \) |

The statistical model for RCBD is given in equation 5

\[
y_{ij} = \mu + \tau_i + B_j + \epsilon_{ij}
\]

where:

\( y_{ij} \) = response of treatment i in block j; \( \mu \) = overall mean; \( \tau_i \) = effect of the \( i^{th} \) treatment; \( B_j \) = effect of the \( j^{th} \) block; \( \epsilon_{ij} \) = identical, independently distributed random error term with mean of 0 and standard deviation of \( \sigma^2 \).

In an experiment involving RCBD, the null hypothesis (H\(_0\)) of no difference in treatment means (\( \mu_i \)) is tested using equation 6a. The alternate hypothesis is given by equation 6b:

\[
H_0: \mu_1 = \mu_2 = \ldots = \mu_m \quad \quad 6a
\]

\[
H_1: \text{At least one } \mu_i \neq \mu_j \quad \quad 6b
\]

For RCBD, the total sum of squares (SS\(_T\)) can be partitioned as given in equation 7 [10].

\[
SS_T = SS_{\text{treatments}} + SS_{\text{blocks}} + SS_E
\]

and \( \frac{SS_{\text{treatments}}}{\sigma^2} \), \( \frac{SS_{\text{blocks}}}{\sigma^2} \) and \( \frac{SS_E}{\sigma^2} \), are independently distributed chi-square random variables where SS\(_{\text{treatments}}\) is the sum of square of treatments, SS\(_{\text{blocks}}\) is the sum of square of blocks and SS\(_E\) is the sum of square of error.

If the sum of square is divided by its degree of freedom, we obtained the expected value of the mean squares (MS) as given in equations 8-10.

\[
E(\text{MS}_{\text{treatments}}) = \frac{\sigma^2 + \sum_{i=1}^m \tau_i^2}{m-1} \quad \quad 8
\]

\[
E(\text{MS}_{\text{blocks}}) = \frac{\sigma^2 + \sum_{j=1}^b B_j^2}{b-1} \quad \quad 9
\]

\[
E(\text{MS}_E) = \frac{\sigma^2}{\sigma^2} \quad \quad 10
\]

Therefore, to test the null hypothesis of no difference, the test statistic in equation 11 is used.

\[
F_0 = \frac{MS_T}{MS_E} \quad \quad 11
\]

This test statistic is distributed as \( F_{t-1, \frac{(t-1)(b-1)}{\sigma^2}} \). If \( F_0 > F_{\alpha, \frac{m-1}{(m-1)(b-1)}} \), we would reject the null hypothesis, otherwise we do not reject. Furthermore, p-value which is the smallest probability of accepting the null hypothesis can also be used. If the p-value is less than the level of significance (\( \alpha \)), we would reject the null hypothesis of no difference, otherwise we do not reject [10].
17 software was used to carry out the analysis of variance (ANOVA) models described above. However, prior to the analysis, the data were normalized by taking natural logarithm.

4. Results and Discussion

4.1 Geophysical Analysis

The geophysical data obtained in the study area using the three arrays is given in Table 2. The table shows resistivity data acquired using conventional Schlumberger (M1), Hummel (M2) and half-Hummel (M3) arrays against the electrode spacing.

| Location 1 | Location 2 | Location 3 | Location 4 | Location 5 |
|------------|------------|------------|------------|------------|
| Resistivity | Resistivity | Resistivity | Resistivity | Resistivity |
| Electrode spacing | M1 | M2 | M3 | Electrode spacing | M1 | M2 | M3 | Electrode spacing | M1 | M2 | M3 | Electrode spacing | M1 | M2 | M3 | Electrode spacing | M1 | M2 | M3 |
| 1 | 106 | 80 | 84 | 1 | 110 | 74 | 78 | 1 | 96 | 77 | 31 | 1 | 153 | 87 | 83 | 1 | 39 | 42 | 40 |
| 2 | 96 | 104 | 112 | 2 | 111 | 79 | 79 | 2 | 65 | 33 | 26 | 2 | 204 | 81 | 88 | 2 | 57 | 36 | 26 |
| 3 | 128 | 119 | 116 | 3 | 122 | 68 | 70 | 3 | 79 | 37 | 59 | 3 | 182 | 59 | 68 | 3 | 76 | 40 | 20 |
| 4 | 138 | 126 | 116 | 4 | 151 | 101 | 101 | 4 | 68 | 55 | 60 | 4 | 533 | 106 | 136 | 4 | 65 | 40 | 70 |
| 6 | 150 | 147 | 119 | 6 | 184 | 158 | 141 | 6 | 187 | 113 | 130 | 6 | 214 | 158 | 124 | 6 | 124 | 124 | 102 |
| 8 | 141 | 151 | 141 | 8 | 141 | 151 | 141 | 8 | 180 | 211 | 231 | 8 | 106 | 80 | 131 | 8 | 106 | 80 | 131 |
| 12 | 260 | 294 | 294 | 12 | 305 | 424 | 496 | 12 | 260 | 339 | 339 | 12 | 294 | 294 | 339 | 12 | 294 | 294 | 339 |
| 15 | 159 | 248 | 248 | 15 | 265 | 318 | 282 | 15 | 424 | 354 | 424 | 15 | 159 | 248 | 248 | 15 | 424 | 354 | 401 |
| 25 | 295 | 442 | 344 | 25 | 368 | 491 | 491 | 25 | 540 | 246 | 295 | 25 | 295 | 442 | 344 | 25 | 540 | 737 | 982 |
| 32 | 845 | 483 | 322 | 32 | 483 | 805 | 805 | 32 | 845 | 965 | 1126 | 32 | 322 | 644 | 885 | 32 | 322 | 644 | 885 |
| 40 | 603 | 754 | 855 | 40 | 603 | 754 | 855 | 40 | 603 | 704 | 754 | 40 | 603 | 704 | 754 | 40 | 603 | 704 | 754 |
| 65 | 1327 | 930 | 1062 | 65 | 1527 | 2655 | 2655 | 65 | 1327 | 930 | 1062 | 65 | 598 | 1859 | 1859 | 65 | 730 | 797 | 1062 |

Table 2: Resistivity data at location 1
4.2 Statistical Analysis

From Figs. 6 - 10, the normal probability plot in the form of Anderson Darling test shows that the response variable for the parameters considered are normal. The histograms equally portray normality. The plot for the residual versus fit confirmed the homogeneity of the variance as no scattered plot was observed. The plots for the residual versus order show the independent relationship that exist between points as no particular trend was developed.

Fig. 6: Residual plots for resistivity at location 1
Fig. 7: Residual plots for resistivity at location 2

Fig. 8: Residual plots for resistivity at location 3

Fig. 9: Residual plots for resistivity at location 4
The results of the two way analysis of variance (ANOVA) are shown in Tables 3-7. From the tables, the treatments (i.e. arrays) have p-values of 0.234, 0.981, 0.010, 0.976 and 0.687 respectively for locations 1, 2, 3, 4, and 5. It was noted that the p-value of locations 1, 2, 4 and 5 are greater than the 0.05 level of significance used for the analysis. This therefore, suggests that the null hypothesis of no difference should be accepted and thus at locations 1, 2, 4, and 5, Hummel and half-Hummel arrays can be used as substitutes for the conventional Schlumberger array as the same statistical responses were obtained.

Table 3: ANOVA for Transformed resistivity at location 1

| Source            | DF | Adj SS | Adj MS | F-Value | P-Value |
|-------------------|----|--------|--------|---------|---------|
| Electrode spacing | 11 | 12.1577| 1.10525| 53.21   | 0.000   |
| Arrays            | 2  | 0.0645 | 0.03224| 1.55    | 0.981   |
| Error             | 22 | 0.4570 | 0.02077|         |         |
| Total             | 35 | 12.6792|        |         |         |

Table 4: ANOVA for Transformed resistivity at location 2

| Source            | DF | Adj SS | Adj MS | F-Value | P-Value |
|-------------------|----|--------|--------|---------|---------|
| Electrode spacing | 11 | 33.7559| 3.06872| 59.82   | 0.000   |
| Arrays            | 2  | 0.0020 | 0.00101| 0.02    | 0.981   |
| Error             | 22 | 1.1286 | 0.05130|         |         |
| Total             | 35 | 34.8865|        |         |         |
Table 5: ANOVA for Transformed resistivity at location 3

| Source               | DF | Adj SS   | Adj MS   | F-Value | P-Value |
|----------------------|----|----------|----------|---------|---------|
| Electrode spacing    | 11 | 41.1663  | 3.74239  | 48.40   | 0.000   |
| Arrays               | 2  | 0.8899   | 0.44495  | 5.75    | 0.010   |
| Error                | 22 | 1.7011   | 0.07732  |         |         |
| Total                | 35 | 43.7573  |          |         |         |

Table 6: ANOVA for Transformed resistivity at location 4

| Source               | DF | Adj SS   | Adj MS   | F-Value | P-Value |
|----------------------|----|----------|----------|---------|---------|
| Electrode spacing    | 11 | 22.7705  | 2.07004  | 7.00    | 0.000   |
| Arrays               | 2  | 0.0141   | 0.00705  | 0.02    | 0.976   |
| Error                | 22 | 6.5044   | 0.29565  |         |         |
| Total                | 35 | 29.2890  |          |         |         |

Table 7: ANOVA for Transformed resistivity at location 5

| Source               | DF | Adj SS   | Adj MS   | F-Value | P-Value |
|----------------------|----|----------|----------|---------|---------|
| Electrode spacing    | 11 | 52.3178  | 4.75617  | 43.08   | 0.000   |
| Arrays               | 2  | 0.0843   | 0.04216  | 0.38    | 0.687   |
| Error                | 22 | 2.4287   | 0.11039  |         |         |
| Total                | 35 | 54.8308  |          |         |         |

However, at location 3, the p-value is less than the 0.05 level of significance, so the null hypothesis of no difference should be rejected. Therefore, at location 3, Hummel and half-Hummel cannot be used as a substitute for conventional Schlumberger array. To solve this problem so as to confirm which arrays are different from each other at this location, we performed pairwise comparison using Tukey method. The result of the pairwise comparison is shown in Table 8.

Table 8: Tukey Pairwise Comparisons for location 3

| Difference of Arrays | Difference of Means | SE of Difference | Simultaneous 95% CI | T-Value | Adjusted P-Value |
|----------------------|---------------------|------------------|---------------------|---------|------------------|
| M2 – M1              | -0.320              | 0.114            | (-0.605, -0.035)    | -2.82   | 0.026            |
| M3 – M1              | -0.346              | 0.114            | (-0.631, -0.061)    | -3.04   | 0.016            |
| M3 – M2              | -0.026              | 0.114            | (-0.311, -0.259)    | -0.23   | 0.972            |

Individual confidence level = 98.01%

The table shows that conventional Schlumberger array (M1) differ significantly from Hummel (M2) and half-Hummel (M3) at location 3 although both M2 and M3 shows no significant difference between them.
Therefore, acceptance (AR) and rejection (RR) map of the study area was generated as shown in Fig. 11. The figure indicates the locations of the acceptance and rejection locations in the study area. This map could be used as a guide before any data acquisition.

Fig. 11: Acceptance and rejection map of the study area.

5. Conclusions

Electrical resistivity survey at five (5) locations have been carried out at Phase II of the Federal University Oye-Ekiti, Nigeria using the conventional Schlumberger, Hummel, and half-Hummel arrays at each location of investigation. Statistical comparison was done on all the three arrays using a statistical approach called randomized complete block design (RCBD) in order to establish if the measurements taken using Hummel or half-Hummel array could be a perfect or near perfect substitution for the measurements taken using conventional Schlumberger array and vice-versa. The responses obtained for the three arrays at locations 1, 2, 4 and 5 are statistically the same; however, at locations 3 the three arrays differ significantly. The result of pairwise comparison shows that conventional Schlumberger array differ significantly from Hummel and half-Hummel at location 3 although both Hummel and half-Hummel shows no significant difference between them. Acceptance and rejection map of the study area was thus generated which could be used as a guide prior to any data acquisition.
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