Challenge of lepton pair production in peripheral collisions of relativistic ions

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Abstract

The new approach to the lepton pair production in the Coulomb field of two highly relativistic nuclei was developed. Solving the operator equation for lepton scattering in arbitrary Coulomb field, we obtain the amplitudes for lepton scattering in the Coulomb potential in terms of light cone variables. Using the Watson expansion for the amplitude of lepton scattering on two centers we propose prescription which allows one to construct the amplitude for lepton pair production in the Coulomb field of two highly relativistic ions. We show that for the certain sums of finite terms of the Watson series numerous cancellations lead to infrared stability of amplitude.

1 Introduction

For many years it has been known that the yield of lepton pairs in the collision of two charged relativistic particles grows with energy as the third power of logarithm [1, 2]. The present interest in the process of lepton pair production off the Coulomb fields of two highly relativistic ions with charge numbers \( Z_1 \) and \( Z_2 \)

\[
Z_1 + Z_2 \rightarrow e^+e^- + Z_1 + Z_2,
\]

is aroused mainly by operation of heavy ion colliders as RHIC (Lorentz factor \( \gamma = \frac{E}{M} = 100 \)) and LHC (\( \gamma = 3000 \)). At such energies the cross section

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of process (1) becomes huge (tens kilobarns at RHIC, hundreds kilobarns at LHC energies) so that its precise knowledge becomes a pressing

In the last years a lot of work [4] - [14] has been done on the problem of Coulomb corrections (CC) in process (1) (multiple photon exchanges between produced lepton pair and the Coulomb fields of colliding ions). For the collisions of heavy ions the relevant parameter \( Z\alpha \) (\( \alpha = \frac{e^2}{4\pi} \) is the fine structure constant) is not small (for instance, for lead \( Z\alpha \approx 0.6 \)), thus the contribution of CC can be essential.

Due to Lorentz contraction the Coulomb fields of colliding ultrarelativistic ions can be considered as very thin discs. This gives hope that at relativistic energies one would obtain the relative simple expression for the amplitude of process (1) with regard Coulomb corrections.

The authors of [5] solved the Dirac equation for the electron propagating in the Coulomb field of two highly relativistic nuclei and using obtained the amplitude of the process (1). The result was striking: allowance for the Coulomb correction leads to multiplication of the Born amplitude by phase factor, i.e. CC have not any impact on the cross section.

This result was criticized in the works [8, 9, 10]. In fact, it is well known [15] that the contribution of Coulomb corrections to the process of lepton pair photoproduction off the Coulomb field is given by a series in fine structure constant and there are no reasons for disappearance of CC in ion collisions.

The next step was made in the number of works [11, 12, 13, 14], where the multiphoton exchanges between the produced pair and the ions Coulomb fields were calculated in perturbation theory. Using the Sudakov technique authors computed Feynman diagrams relevant to the first terms of the perturbative expansion (up to the fifth order in \( \alpha \)) and tried to conjecture the general structure of higher order terms [14]. Their main results can be formulated as follows:

1) Taking in account the Coulomb corrections does not reduce to the simple phase factor, as is argued in [5, 7].

2) Any term in the perturbation expansion of the amplitude \( M_m^n \sim (Z_1\alpha)^m(Z_2\alpha)^n \) is infrared stable, i.e. does not depend on infrared regularization parameter (screening radius) \( \lambda \).

The last property was proved in [11, 14] only for the first terms of the perturbative expansion \( m, n \leq 2 \). On the other hand generalization to higher order done in [14] leads to terms depending on the regularization parameter \( \lambda \), thus the further work in this direction is necessary.

In the present work we propose a new approach to this issue. We construct the amplitude of process (1) in the form of the Watson series, thus expressing it through the operators relevant to the lepton scattering on separate centres. We solve the corresponding equations in the limit of infinite energies and
obtain the amplitudes for lepton scattering off any external field. The problem of the regularization is investigated. Considering the specific sets of the Watson expansion, we show that the relevant amplitudes are infrared stable. The prescription is proposed which allows one to calculate the cross section of process (1) with high precision.

The following notations are used in the paper: \( e, m \) are the electron charge and mass; \( u(p'), v(p) \) are electron and positron spinors; \( \gamma_\mu \) are Dirac matrices and \( \gamma_\pm = \gamma_0 \pm \gamma_z \). We use the light cone definition of four momenta and coordinates \( k_\pm = k_0 \pm k_z, x_\pm = x_0 \pm x_z \). Throughout the paper the transverse components of momenta and coordinates are defined as two dimensional vectors. For instance, \( \vec{b}_j \) are the impact parameters of ions whereas \( \vec{x}_i, \vec{k}_i \) are transverse coordinates and momenta of leptons. The index \( j=1,2 \) is reserved for quantities attached to relevant ions \( Z_1, Z_2 \).

## 2 Watson expansion

The amplitude of lepton pair production in an arbitrary external electromagnetic field created by two potentials \( A_\mu = A^{(1)}_\mu + A^{(2)}_\mu \) is given by

\[
M = \bar{u}(p')T(p', -p)v(p)
\]

The operator \( T(p, p') \) can be cast in the form of an infinite Watson series [16], which allows one to express the amplitude of particle scattering on two centers through the scattering amplitudes of each of them \( T_j(p, p') \); \( j = 1, 2 \). In short notation the Watson series for scattering on two centers reads

\[
T = T_1 + T_2 - T_1 \otimes G \otimes T_2 - T_2 \otimes G \otimes T_1 \\
+ T_1 \otimes G \otimes T_2 \otimes G \otimes T_1 + T_2 \otimes G \otimes T_1 \otimes G \otimes T_2 + \ldots
\]

\( T_j \) are the separate amplitudes of lepton scattering in the Coulomb field of ions \( Z_1 \) and \( Z_2 \) and \( G(k) \) is the lepton casual Green function. In Fig.1 we depicted the possible exchanges in lepton pair production in accordance with various terms of the Watson expansion. The thick lines attached to ions \( Z_1, Z_2 \) represent the full set of photon exchanges between the lepton (electron or positron) and the ion.

The amplitudes \( T_j(p, p') \) satisfy the well-known (see e.g. [17]) operator equation

\[
T_j = V_j - V_j \otimes G \otimes T_j \\
V_j(p, p') = e\gamma_\mu A^{(j)}_\mu(p - p')
\]

\(^1\)After substituting (3) in (2) the first two terms in (3) are identically zero, therefore later on we begin numbering from the third term in (3).
Equation (4) for single amplitude can be solved in the case of ultrarelativistic energies. At such energies due to Lorentz contraction the Coulomb field of nucleus looks like very thin disc for which the Coulomb potential in moving system takes a simple form. In this case the solution of equation (4) is found and can be presented in the following form:

\[ T_1(p, p') = \frac{(2\pi)^2}{2} \delta(p_+ - p_+')(\theta p_+ f^+_1(\vec{p} + \vec{p}') - \theta(-p_+) f^+_1(\vec{p} - \vec{p}')) \gamma^+ \]

\[ T_2(p, p') = \frac{(2\pi)^2}{2} \delta(p_- - p_-')(\theta p_- f^+_2(\vec{p} - \vec{p}') - \theta(-p_-) f^+_2(\vec{p} - \vec{p}')) \gamma_- \]

\[ f_j^\pm(\vec{q}) = \frac{i}{2\pi} \int d^2x e^{i\vec{q}\vec{x}} [1 - S_j^\pm(\vec{x}, \vec{b}_j)]; S_j^\pm(\vec{x}, \vec{b}_j) = \exp(\pm i\chi_j(\vec{x}, \vec{b}_j)) \]

The Coulomb potentials of ions \( \Phi_j \) and the eikonal-type amplitudes for electron and positron scattering depend on the impact parameters of colliding ions \( \vec{b}_1, \vec{b}_2 \). On the other hand, as a result of translation invariance, the square of amplitude (2) depend only on the difference \( \vec{b} = \vec{b}_1 - \vec{b}_2 \).

We adopt such normalization of the amplitude that the cross section of the process (1) is

\[ d\sigma = \frac{1}{(2\pi)^{10}} \int |M(b, p, p')|^2 d^2b d^3p d^3p' \]

Substituting the expressions (5) for \( T_j \) in the Watson expansion (3), one can calculated the cross section of process (1). Every consequent term in the Watson series begins with higher order in the parameter \( Z\alpha \), thus one can calculate the cross section with a desirable precision. This is true for the screened Coulomb potential, for instance in the case of interaction of relativistic atoms. But heavy ion colliders deal with ions whose Coulomb fields are unscreened and for which the problem of amplitude regularization demands special consideration.

### 3 The challenge of infrared stability

The Coulomb phases (6) in the case of the screened Coulomb potential reads

\[ \chi_j(b) = e \int \Phi_j(\sqrt{b^2 + z^2})dz = 2Z_j\alpha K_0(b\lambda_j) \to -2Z_j\alpha(ln(b\lambda_j) + C) \]

Here \( \lambda_j \) are the screening radii, which can differ for different ions. Substituting expressions (5,6) in the Watson expansion (3) leads to the products of
$S^\pm$-matrix elements some of which do not depend on the screening parameter, for instance

$$S_j^+(\vec{x})S_j^-(\vec{x}') = \exp\left(2iZ_j\alpha \ln\frac{|\vec{x}' - \vec{b}_j|}{|\vec{x} - \vec{b}_j|}\right) \quad (9)$$

Nevertheless the majority of these products are oscillating functions of $\lambda_j$. On the other hand, our experience from photoproduction \[15\] and perturbation theory \[12, 14\] tell us that the amplitude of the process under consideration must be infrared stable, so all oscillating products have to be cancelled in the full amplitude.

To follow these cancellations, consider firstly the case where one of the ions, for instance $Z_2$, is light so that one can expand the amplitude in the parameter $Z_2\alpha$. In the general case Watson series (3) is infinite and there are no reasons to truncate it. However it is automatically cut off if one considers the finite number of exchanged photons attached to one of the nuclei (with any number of exchanges with another nuclei).

Denoting the transverse momenta of leptons in intermediate states by $\vec{k}_i$ and the transverse momenta of exchanged photons by $\vec{q}_i$ (see Fig.1) we introduce the following notations:

$$\Omega_j(\vec{q}, \vec{q}') = \frac{1}{(2\pi)^2} \int d^2x d^2x' \exp (i\vec{q}\vec{x} + i\vec{q}'\vec{x}') \left(1 - S_j^+(\vec{x})S_j^-(\vec{x}')\right)$$

$$f_j^+ = -\sum_{n=1}^{\infty} \frac{(-i)^{n+1}}{n!} f_j^{(n)}; \quad f_j^- = \sum_{n=1}^{\infty} \frac{1}{2\pi} \int d^2x e^{i\vec{q}\vec{x}} \chi^{(n)}(\vec{b}_2 - \vec{x}) \quad (10)$$

Note that expressions (11) are nothing but expansion of amplitudes from (6).

To obtain the sum of all terms from Watson expansion (3) relevant to the first order exchange in $Z_2\alpha$ and any exchanges with $Z_1$, it’s enough to calculate the terms which are linear in $T_2$. These terms correspond to the first three diagrams of Fig.1, with only replacement of the thick line attached to the ion $Z_2$ by a single photon exchange.

Using the above expressions after a lengthy but well known algebra, we get:

$$M^{(1)} = \sum_{n=1}^{\infty} M_n^{(1)} = \frac{i}{8\pi} \int \frac{\bar{u}(p')\gamma_+\nu_1\gamma_-\nu_2\gamma_+v(p)}{\mu_1 p_+ + \mu_2 p'_+} \chi^{(1)}(q_2)\Omega_1(q_1, q_3)d^2k_1 d^2k_2$$

$$\nu_i = m - \vec{k}_i\gamma; \quad \mu_i = m^2 + \vec{k}_i^2 \quad (12)$$

This expression does not depend on the regularization parameter $\lambda_1$ as a result of nontrivial cancellations among the different terms of the Watson
As in the previous case this expression does not depend on the regularization parameter \( \lambda_2 \). To investigate the problem of regularization in the general case, we consider the set of terms from the Watson series corresponding to two photons attached to the ion \( Z_2 \) (any number of exchanged photons with \( Z_1 \)). This contribution is provided by the first four diagrams of Fig.1, with obvious replacement of a set of photon exchanges attached to ion \( Z_2 \) by one and two photon exchanges. The result of our calculations reads:

\[
M^{(2)} = \sum_{n=1}^{\infty} M_{n}^{(2)} = -\frac{i}{(4\pi)^2} \int \frac{\bar{u}(p')\gamma_+ \nu_1 \gamma_+ \nu_2 \gamma_+ \nu_3 v(p)}{\mu_1 p_+ + \mu_2 p'_+} f_2^{(2)}(q_2) \Omega_1(q_1, q_3) \ln \left( \frac{\mu_1 p_+}{\mu_2 p'_+} \right) d^2k_1 d^2k_2 \\
- \frac{i}{(4\pi)^3} \int \frac{\bar{u}(p')\gamma_+ \nu_1 \gamma_+ \nu_2 \gamma_+ \nu_3 v(p)}{\mu_1 \mu_3 + \mu_2 p'_+ p_-} f_2^{(1)}(q_2) f_2^{(1)}(q_4) \Omega_1(q_1, q_3) \\
\times \left[ \ln \left( \frac{\mu_1 \mu_3}{\mu_2 p'_+ p_-} \right) + i\pi \right] \frac{d^2k_1 d^2k_2 d^2k_3}{\mu_1 \mu_3 + \mu_2 p'_+ p_-} \\
- \frac{i}{(4\pi)^3} \int \frac{\bar{u}(p')\gamma_+ \nu_1 \gamma_+ \nu_2 \gamma_+ \nu_3 v(p)}{\mu_1 \mu_3 + \mu_2 p'_+ p_-} f_2^{(1)}(q_1) f_2^{(1)}(q_3) \Omega_1(q_2, q_4) \left[ \ln \left( \frac{\mu_1 \mu_3}{\mu_2 p'_+ p_-} \right) + i\pi \right] d^2k_1 d^2k_2 d^2k_3. \quad (13)
\]

As in the previous case this expression does not depend on the regularization parameter \( \lambda_2 \). We do not cite here the next sets of Watson terms corresponding to three and four photons attached to the ion \( Z_2 \) in view of their inconvenience, but we verified that they also do not depend on the regularization parameter \( \lambda_2 \).

To investigate the problem of regularization in the general case, we consider the first six terms of Watson expansion (3) (diagrams a-f in Fig.1). It can be shown that this set consists of the infrared stable term \( M_s \) and the term \( M_u \) depending on \( \lambda_j \), i.e.

\[
\sum_{n,m=1}^{\infty} M_n^m = M^s + M^u \quad (14)
\]

We calculated the infrared stable part \( M_s \) with the following result:

\[
M^s = \frac{i}{(4\pi)^2} \int d^2k_1 d^2k_2 d^2k_3 \bar{u}(p') [\gamma_+ \nu_1 \gamma_+ \nu_2 \gamma_+ \nu_3 \gamma_- \Omega_1(q_1, q_3) \Omega_2(q_2, q_4)] \left[ \ln \left( \frac{\mu_1 \mu_3}{\mu_2 p'_+ p_-} \right) + i\pi \right] \frac{d^2k_1 d^2k_2 d^2k_3}{\mu_1 \mu_3 + \mu_2 p'_+ p_-} \\
+ \gamma_- \nu_1 \gamma_+ \nu_2 \gamma_+ \nu_3 \gamma_+ \Omega_2(q_1, q_3) \Omega_1(q_2, q_4) \left[ \ln \left( \frac{\mu_1 \mu_3}{\mu_2 p'_+ p_-} \right) + i\pi \right] v(p) \quad (15)
\]

As to the unstable part \( M_u \) it turns out to be of the order \( (Z_1 Z_2 \alpha^2)^3 \), i.e. a higher degree in fine structure constant than (15) and has to be exactly cancelled when one considers the next terms of the Watson series.
4 Conclusions

The problem of taking account of the Coulomb corrections in the process of lepton pair production in the Coulomb field of two highly relativistic nuclei turns out to be much more complicated than seemed decade ago. The hope that due to Lorentz contraction of the Coulomb potential at ultrarelativistic energies one can obtain a simple expression for the amplitude was not come true. The investigation in perturbation theory has shown that the terms of higher order in fine structure constant become more and more complex and bulky and one needs some general algorithm to construct them. The receipt proposed in leads to the infrared unstable result and thus cannot be considered as a final one.

We developed a new approach to this problem using the Watson series and solving the well-known operator equations for amplitudes relevant to lepton scattering in the Coulomb field of relativistic nuclei and proposed the prescription allowing one to construct the amplitude of process (1) to any order in . We show that in the case of a finite number of photon exchanges between the lepton pair and one of the ions the amplitude does not depend on the regularization parameter relevant to this ion.

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Figure 1: The diagrams relevant to first six terms of Watson series