The Valence-Quark Distribution of the Kaon from Lattice QCD

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We present the first lattice-QCD calculation of the kaon valence-quark distribution functions using the large-momentum effective theory (LaMET) approach, a method that has been applied to a wide variety of isovector nucleon distributions and valence pion distributions. This is the first such lattice calculation with multiple pion masses with the lightest one around 220 MeV, 2 lattice spacings \(a = 0.06\) and \(0.12\) fm, \((M_s)_{\text{min}}L \approx 5.5\), and high statistics ranging from 11,600 to 61,312 measurements. We also find the valence-quark distribution of pion to be consistent with the FNAL E615 experimental results. Our ratio of the \(u\) quark PDF in the kaon to that in the pion agrees with the CERN NA3 experiment. We make predictions of the strange quark distribution of the kaon.

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I. INTRODUCTION

Light pseudoscalar mesons play a fundamental role in QCD since they are the Nambu-Goldstone bosons associated with dynamical chiral symmetry breaking (DCSB), and thereby provide a useful testing ground for our understanding of nonperturbative QCD. While studies of pion and kaon structures both reveal physics of DCSB, a comparison between them helps to reveal the relative impact of DCSB versus the explicit breaking of chiral symmetry by the quark masses.

The parton distribution functions (PDFs) are important quantities characterizing the structure of the pion and kaon. They can be measured by scattering a secondary pion (\(\pi\)) or kaon (\(K\)) beam over target nuclei (\(A\)), inducing the Drell-Yan process, \(\pi(K)A \rightarrow X\mu^+\mu^-\), where the muon (\(\mu\)) pair is detected but not the rest of the final state (\(X\)) [1,5]. With a combined analysis of \(\pi^+A\) and \(\pi^-A\) Drell-Yan on the same nuclear target, the valence and sea distributions can be separated [1], provided the nuclear PDF is known. Currently, the nuclear PDFs are approximated by a combination of proton and neutron PDFs; the remaining nuclear modifications must then be quantified by combined efforts from theory and experiment. The valence-quark PDF of the pion for momentum fraction \(x > 0.2\) has been determined reasonably well [1,2,5,7], subject to the systematic uncertainty in the PDF parametrization.

Combining \(K^-A\) and \(\pi^-A\) Drell-Yan data, the kaon valence PDF can be measured through the ratio [8]

\[
\frac{\bar{u}_K^V(x)/u_v^u(x)C(x)}{\bar{u}^\pi_\nu(x)/\bar{u}^\pi_\bar{u}(x)}
\]

where \(\bar{u}_K^V(x)\) denotes the valence anti-up distribution in the \(K^-\) (\(\pi^-\)). The function \(C(x)\) encodes the corrections needed due to the nuclear modification of the target PDFs, the omission of meson sea-quark distributions and the ignorance of the ratio \(s_K^V(x)/\bar{u}_K^V(x)\). In principle, the first two can be addressed by new experiments. For example, the valence and sea PDFs for the pion and kaon at \(x > 0.2\) can be separated in the \(\pi^\pm\) and \(K^\pm\) Drell-Yan experiments proposed by the COMPASS++/AMBER collaboration using the CERN M2 beamline [2]. Numerically, the biggest uncertainty in \(C(x)\) is due to ignorance of the \(s_K^V(x)/\bar{u}_K^V(x)\) ratio, and a reliable theoretical determination of this ratio, e.g. by lattice QCD, would greatly reduce the uncertainty in \(\bar{u}_K^V(x)/u_v^u(x)\).

To understand the importance of \(C(x)\), consider, for example, the PDFs at \(x = 0.9\); if one uses a conservative estimation \(1 < s_K^V(x)/\bar{u}_K^V(x) < 10\) then \(1 > C(x) > 0.85\), although \(C(x)\) is very close to unity at \(x = 0.2\) [3].

With this estimation of \(C(x)\), \(\bar{u}_K^V(x) < \bar{u}_\pi^\pi(x)\) is observed for \(x > 0.7\). The smallness of \(\bar{u}\) in \(K^-\) relative to \(\pi^-\) at large \(x\) can be understood in the limit where the strange-quark mass is very heavy (\(m_s \rightarrow \infty\)). In this heavy-quark limit, no strange sea quarks are produced, and all the momentum of \(K^-\) must be carried by the valence strange quark, so \(\bar{u}\) is highly skewed towards small \(x\). At physical \(m_s\), the effects of the strange sea quarks are not negligible. Although one can still think about a dressed quark as an effective valence quark, it is not clear whether this notion of effective valence quark is a good approximation to the valence quark in full QCD. Therefore, whether \(\bar{u}_K^V(x) < \bar{u}_\pi^\pi(x)\) at large \(x\) for physical quark masses is an interesting and nontrivial theoretical problem that we would like to address using lattice QCD.

Another experiment that could measure the pion and
kaon PDFs is tagged deep inelastic scattering (TDIS), such as $p + p \rightarrow e^+ (n \text{ or } Y) X$. By tagging a neutron ($n$) or hyperon ($Y$) with specific kinematics in the final state of an electron ($e$) scattering on a proton ($p$), one can select events of the Sullivan process, where an electron scatters off an intermediate $t$-channel pion or kaon. Unlike the Drell-Yan experiment, whose theoretical error can be systematically studied by QCD factorization, the Sullivan-process approach relies on the validity of the meson-cloud model. It is unclear how to quantify the theoretical uncertainty systematically in this approach. Although the meson-cloud model was made systematic using chiral perturbation theory for inclusive processes such as DIS, it is not clear whether it can be applied to TDIS processes as well. Approved experiments at JLab aim to determine $\bar{n}/n(01 \rightarrow K^0)$. The RI/MOM renormalization scale is set to $\mu = 1$ with $\bar{\alpha}_S(a_0)$.

## II. KAON PDF FROM LAMET LATTICE CALCULATION

To see how the quark PDF in the kaon can be obtained within LaMET, we begin with the following operator definition:

$$q^K(x) = \frac{d\lambda}{4\pi} e^{-ix_{\lambda n} \cdot P_h} \langle \bar{\psi}_q(\lambda n) | W(\lambda n, 0) | \psi_q(0) \rangle K(P),$$

where $|K(P)\rangle$ denotes a kaon state with momentum $P^\mu = (P_0, 0, 0, P_z)$, $\bar{\psi}_q, \psi_q$ are the quark fields of flavor $q$, $n^\mu$ is a unit direction vector and

$$W(\zeta n, \eta n) = \exp \left( ig \int_0^\zeta d\rho n \cdot A(\rho n) \right).$$

is the gauge link inserted to ensure gauge invariance. For later convenience, we have used a subscript $\parallel$ on $h$ to denote the Dirac structure sandwiched between the quark fields. If we choose lightlike $n = \eta = (1, 0, 0, -1)/\sqrt{2}$, then Eq. 1 defines the usual quark PDF with $x$ denoting the fraction of kaon momentum carried by the quark. The support of $x$ is $[-1, 1]$ with the negative $x$ part corresponding to the antiquark distribution: $\bar{q}^K(x) = -q^K(-x)$ for $x > 0$. One can define the valence-quark distribution for the positive range as $q^K(x) = q^K(x) - \bar{q}^K(x)$, which satisfies $\int_0^1 dx q^K(x) = 1$ for a quark of the appropriate flavor.

On the other hand, if we choose spacelike $n = \tilde{n} = (0, 0, 0, -1)$, then Eq. 1 becomes a Euclidean correlator known as quasi-PDF, which can be calculated in lattice QCD. The idea behind LaMET is that for a given momentum $P_z \gg A_{QCD}$, the quasi-PDF has the same infrared physics as the PDF, so the two quantities can be connected via a factorization formula. Such a factorization can be done with either bare or renormalized correlators. We will follow the latter, since it facilitates the conversion from lattice results to results in the continuum.

On the lattice, we first calculate the quasi-PDF matrix element in coordinate space, and then renormalize it nonperturbatively in a conventional scheme such as the regularization-independent momentum-subtraction (RI/MOM) scheme. To avoid potential mixing with scalar operators, we replace the Dirac structure $\bar{\psi}$ with $\bar{\psi}_{\tilde{n}}$, where $\hat{n}_i = (1, 0, 0, 0)$. The RI/MOM renormalization...
factors $Z$ can then be determined by demanding that it cancels all the loop contributions for the matrix element in an off-shell external quark state at a specific momentum $[30, 36]:$

$$h_{\alpha, R}(\lambda \tilde{n}) = Z^{-1}(\mu R, 1/a, \mu_R)h_{\alpha, R}(\lambda \tilde{n}),$$

(3)

with

$$Z(\mu R, 1/a, \mu_R) = \frac{\text{Tr}[\mathcal{P} \sum_s \langle p, s | \psi_f(\lambda \tilde{n}) \bar{\psi}_f(\lambda \tilde{n}) | 0 \rangle | p, s \rangle]}{\text{Tr}[\mathcal{P} \sum_s \langle p, s | \psi_f(\lambda \tilde{n}) \bar{\psi}_f(\lambda \tilde{n}) | 0 \rangle | p, s \rangle_{\text{tree}}]} e^{\frac{2}{\mu_R^2} \lambda^2},$$

(4)

After renormalization and taking the continuum limit, $h_{\alpha, R}(\lambda \tilde{n})$ can be converted to the lightcone PDFs via the factorization

$$q^K(x, \tilde{n}, \tilde{\mu}) = \int \frac{dy}{y} C(\frac{x}{y}, \frac{\tilde{n}}{y}, \frac{\tilde{\mu}}{y \mu}, \frac{P_z}{p_z}) q^K(y, n_+, \mu) + O\left(\frac{m_q^2}{P_z^2}, \frac{A_{1 \rightarrow CD}}{x^2 P_z^2}\right),$$

(5)

where $C$ is a perturbative matching kernel converting the RI-MOM renormalized quasi-distribution to the one in MS scheme used in our previous works $[32, 34, 35, 38].$ In this work we use clover valence fermions with $N_f = 2 + 1 + 1$ (degenerate up/down, strange and charm) flavors of highly improved staggered dynamical quarks (HISQ) $[54]$ in the sea, on ensembles generated by MILC Collaboration $[55].$ We use one step of hypercubic (HYP) smearing on the gauge links $[57]$ to suppress discretization effects, and the fermion-action parameters are tuned to recover the lowest pion mass of the staggered quarks in the sea. Details can be found in Refs. $[53, 61].$ We note that no exceptional configurations have been found among all the ensembles we use in this work $[53, 58, 61].$ The multigrid algorithm $[62, 63]$ in the Chroma software package $[64]$ is used to speed up the clover fermion inversion of the quark propagators. We use Gaussian momentum smearing $[65]$ for both the light- and strange-quark fields $\psi(x) + \alpha \sum_j U_j(x)e^{ik_\mu \cdot \bar{x}} \psi(x + \bar{e}_j),$ where $k$ is the input momentum-smearing parameter. $U_j(x)$ are the gauge links in the $j$ direction, and $\alpha$ is a tunable parameter as in traditional Gaussian smearing. Table $[6]$ summarizes the momenta, source-sink separations, and statistics used in this work.

On the lattice, we calculate both two-point and three-point correlators. We test on the meson unpolarized quasi-PDF measurements on the lattice:

$$C_{2pt}(t_{\text{sep}}, \vec{P}) = \langle 0 | \int d^3 \vec{y} e^{i\vec{P} \cdot \vec{y}} \mathcal{M}_{ps}(\vec{y}, t_{\text{sep}}) M_{ps}(0, 0) | 0 \rangle, $$

(6)

$$C_{3pt}(z, t, t_{\text{sep}}, \vec{P}) = \langle 0 | \int d^3 \vec{y} e^{i\vec{P} \cdot \vec{y}} \mathcal{M}_{ps}(\vec{y}, t_{\text{sep}}) \times \bar{q}(z, t) \Gamma \prod_{z=0}^{z-1} U_z(x, t) q(0, t) M_{ps}(0, 0) | 0 \rangle,$$

(7)

where $C_{3pt}$ is the three-point correlator with $q = \{l, s\}$ quarks, $C_{2pt}$ is the two-point correlator, $M_{ps} = \bar{q}_i \gamma_5 q_2$ is the pseudoscalar meson operator with $q_1, q_2$ being either the light- or strange-quark operator, $z$ is the length of the Wilson line, $U_{\mu}(x, t)$ is a lattice gauge link. For the unpolarized meson PDFs, we choose Dirac spinor matrices $\Gamma = \gamma_i$ here as suggested in Refs. $[66, 67]$ to avoid mixing with the scalar matrix element. $t$ and $t_{\text{sep}}$ are the operator-insertion time and source-sink separation. We choose the meson boost momentum $\vec{P}$ to lie along the $z$ direction and denote its magnitude $P_z.$ All the source locations are randomly selected for each configuration; we shift to $t = 0$ for convenience before the measurements are averaged.

The matrix elements for the meson PDF are then extracted using multiple source-sink separations $t_{\text{sep}},$ removing excited-state contamination by performing "two-simRR" fits $[61]:$

$$C_{2pt}^{3pt}(P_z, t, t_{\text{sep}}) = |A_0|^2 |O_T(0)| e^{-E_0 t_{\text{sep}}} + A_1 A_1^* \langle |O_T(0)| e^{-E_1 (t_{\text{sep}} - t)} e^{-E_0 t} + A_0 A_1^* \langle |O_T(0)| e^{-E_0 (t_{\text{sep}} - t)} e^{-E_1 t} + |A_1|^2 \langle |O_T(0)| e^{-E_1 t_{\text{sep}}} + \ldots $$

(8)

at each meson boost momentum. The $E_0$ ($E_1$) and $A_0$ ($A_1$) are the ground- (excited-) state nucleon energy and overlap factors, extracted from the two-point correlators by fitting them to the form

$$C_{2pt}^{3pt}(P_z, t_{\text{sep}}) = |A_0|^2 e^{-E_0 t_{\text{sep}}} + |A_1|^2 e^{-E_1 t_{\text{sep}}} + \ldots $$

(9)

A few selected fits to the three-point ratio

$$R_{\psi}(t_{\text{sep}}, t) = \frac{C_{3pt}(t_{\text{sep}}, t)}{C_{2pt}(t_{\text{sep}})},$$

(10)

are plotted from a subset of data on all three ensembles with $P_z = 5 \times 2\pi/L$ from the a12m220L ensemble in Fig. [7] using different $t_{\text{sep}},$ with source-separ separations from 0.72 fm to 1.08 fm. The upper-left plot shows "two-simRR" fits, while the rest of the plots show fits without the (1/|O_T(0)|) term; the extracted ground-state matrix elements are consistent between these two analysis methods. We also examine the fitted ground-state matrix elements from a two-state fit to each $t_{\text{sep}}$ in the upper-right and bottom plots. The extracted ground-state matrix elements are also consistent among different $t_{\text{sep}},$ and agree with the simultaneous fits using all $t_{\text{sep}}.$ The signal-to-noise ratio deteriorates significantly as $t_{\text{sep}}$ is increased, even though we have increased the number of measurements at larger...
source-sink separations. One can clearly see that the simultaneous fits well describe data from all $t_{\text{sep}}$, and the errors in the final extracted ground-state matrix-element extraction are not over-constrained by the smallest $t_{\text{sep}}$ data. For the remainder of the paper, we only use the “two-sim” fits to obtain ground-state matrix elements for further processing. An example result from one of the ensembles is shown in Fig. 2. We find that the extractions of the ground-state matrix elements are stable across different fit-range choices among two-point and three-point correlators.

### III. RESULTS AND DISCUSSION

In this work, we choose $\mu^R = 2.4$ GeV and $p_z^R = 0$ to compute the RI/MOM renormalization factors defined in Eq. 4. With $p_z^R = 0$, the renormalization factors are real. They are calculated from all three ensembles and are shown in Fig. 3. The renormalization factors are counterterms to cancel the UV divergence of the bare matrix elements; hence, they are sensitive to the UV cutoff or lattice spacing but not sensitive to the pion mass.

We summarize the renormalized kaon valence-quark matrix elements in Fig. 4. Only the real parts of the matrix elements are shown, because $q^K(|x|) = q^K(|x|) - q^K(-|x|)$, and by Eq. 4 only the real parts of the matrix elements contribute to the valence distribution. The matrix elements shown here are normalized by the corresponding mean value of the calculated vector charge so that one can easily compare the $zP_z$ dependence. The pion-mass dependence of the matrix elements is small for the two $a \approx 0.12$ fm ensembles with 220 and 310 MeV pion mass points. The lattice-spacing dependence between $a \approx 0.06$ and 0.12 fm is benign in most regions of $zP_z$, but the trend starts to diverge between these two lattice spacing results for $zP_z > 5$. They remain within two standard deviations. For the rest of this work, we will neglect the lattice-spacing dependence.

Next, we perform a chiral extrapolation to obtain the

| Ensemble ID | $\alpha$ (fm) | $N_f \times N_t$ | $M_{\text{val}}^\text{RI} \text{ (MeV)}$ | $M_{\text{val}}^\text{RI} \text{ (MeV)}$ | $M_{\text{val}}^\text{RI} \text{ L}$ | $t_{\text{sep}}/a$ | $P_z$ | $N_{\text{cfg}}$ | $N_{\text{meas}}$ |
|-------------|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|-------|----------------|-----------------|
| a12m310     | 0.12          | $24^3 \times 64$ | 310             | 683             | 4.55            | $\{6,7,8,9\}$  | $\{3\}$ | 958           | (22922, 45984, 45984, 61312) |
| a12m220L    | 0.12          | $40^3 \times 64$ | 217             | 687             | 5.5             | $\{6,7,8,9\}$  | $\{4,5\}$ | 840           | (13440, 26800, 26800, 53760) |
| a06m310     | 0.06          | $48^3 \times 96$ | 319             | 690             | 4.52            | $\{12,14,16,18\}$ | $\{3\}$ | 725           | (11600, 23200, 23200, 46400) |
FIG. 2: Comparison of the fitted kaon ground-state matrix elements as functions of Wilson-line length \( z \) (in lattice units) from the a12m220L ensemble with \( P_z \approx 5 \times 27 \) and pion mass of 220 MeV using “two-sim” fits and varying the fit range of the two-point (\( t_{\text{skip}}^{\text{pt}} \)) corresponding to fit range of \( \{t_{\text{min}}^{\text{pt}}, N_f/4\} \) and three-point correlators (\( t_{\text{skip}}^{\text{pt}} \)) corresponding to fit range of \( \{t_{\text{skip}}^{\text{pt}}, t_{\text{skip}} - t_{\text{skip}}^{\text{pt}}\} \).

FIG. 3: The inverse renormalization constant from all three ensembles as functions of Wilson-line displacement \( z \) with RI/MOM renormalization scales \( \mu_R = 2.4 \) GeV and \( p_R^2 = 0 \). The dependence of the renormalization constants on pion mass is negligible, but the dependence on lattice spacing is significant.

FIG. 4: The renormalized matrix elements of the light (top) and strange (bottom) valence-quark contributions to the kaon PDFs as functions of dimensionless \( z P_z \). The renormalization scales are fixed at \( \mu_R = 2.4 \) GeV and \( p_R^2 = 0 \). We observe no significant pion-mass nor lattice-spacing dependence in most regions of the calculated \( z P_z \).

renormalized matrix elements at physical pion mass. We use a simple ansatz to combine our data from 220, 310 and 690 MeV: \( h_i^L(P_z, z, M_x) = c_{0,i} + c_1,i M_x^2 \) with \( i = K, \pi \). The extrapolation plots can be found in Fig. 3 where the results from individual pion masses are shown as lines, while the extrapolated results at physical pion mass are shown as pink bands. Overall, the extrapolated matrix elements are consistent with the 310- and 220-MeV results, but can be significantly different from the 690-MeV ones.

With the matrix elements at physical pion mass, we obtain the pion and kaon PDFs through Eq. 5 as proposed for the pion valence PDF in Ref. 68. We take the commonly used analytical form

\[
    f_{m,n}(x) = \frac{x^m(1-x)^n}{B(m+1, n+1)},
\]

where \( B(m+1, n+1) = \int_0^1 dx x^m(1-x)^n \) is the beta function, which normalizes the distribution such that the area under the curve is unity. The RI/MOM renormalized quasi-PDF can be matched to the \( \overline{\text{MS}} \) renormalized PDF using the matching kernel defined in Ref. 38 with the meson-mass correction from Ref. 25.

Our fit using Eq. 11 and Eq. 5 is good with \( \chi^2/\text{dof} = 0.27 \). The leading moments from the pion distribution are \( \langle x \rangle_v = 0.289(23), \langle x^2 \rangle_v = 0.144(18), \langle x^3 \rangle_v = 0.088(15) \), which are consistent with the traditional moment approach done by ETMC using \( N_f = 2 + 1 + 1 \) twisted-mass fermions with pion masses in the range of 230 to 500 MeV, renormalized at 2 GeV; see Table V in Ref. 69 with \( \langle x \rangle \) ranging 0.23–0.29 and \( \langle x^2 \rangle \) ranging 0.11–0.18. Figure 6 shows our final results for the pion valence distribution at physical pion mass \( (u^c_v) \) multiplied by Bjorken-\( x \) as a function of \( x \). We evolve our results to a scale of 27 GeV\(^2\) using the NNLO DGLAP equations from the Higher-Order Perturbative Parton Evolution Toolkit (HOPPET) 70 to compare with other results. When comparing with the results from experimental data, our result is consistent with the original
analysis of the FNAL-E615 experiment data [5] with our result approaching large-$x$ with $\sim (1-x)^{1.01}$, whereas there is tension with the $x > 0.6$ distribution from the re-analysis of the FNAL-E615 experiment data using next-to-leading-logarithmic threshold resummation effects in the calculation of the Drell-Yan cross section [7] (labeled as “ASV’10”), which agree better with the distribution from Dyson-Schwinger equations [71], both prefer $\sim (1-x)^2$ as $x \to 1$. An independent lattice study of the pion valence-quark distribution [72], also extrapolated to physical pion mass, using the “lattice cross sections” (LCSs) [73], reported similar results to ours. Our lowest 3 moments at the scale of 27 GeV$^2$ are 0.23(18), 0.102(13), 0.057(10), which are consistent with the moments (0.23, 0.094, 0.048) from chiral constituent quark model [73].

Figure 6 shows the ratios of the light-quark distribution in the kaon to the one in the pion ($u^K+/u^+\pi$). When comparing our result with the original experimental determination of the valence quark distribution via the Drell-Yan process by NA3 Collaboration [1] in 1982, we found good agreement between our results and the data. Our result approaches 0.4 as $x \to 1$ and agrees nicely with other analyses, such as constituent quark model [71], the Dyson-Schwinger equations (DSE) approach (“DSE’11”) [75], and basis light-front quantization with color-singlet Nambu–Jona-Lasinio interactions (“BLFQ-NJL’19”) [76]. Our lowest 3 moments for $u^K+$ are 0.193(8), 0.080(7), 0.042(6), respectively, which are close to the QCD model estimates of 0.23, 0.091, 0.045 from chiral constituent quark model [73] and 0.28, 0.11, 0.048 from Dyson-Schwinger equations [71]. Our prediction for $xu^K_+$ is also shown in Fig. 6 with the lowest 3 moments of $s^K_+$ being 0.267(8), 0.123(7), 0.070(6), respectively; the moment results are within the ranges of the QCD model estimates from chiral constituent quark model [73] (0.24, 0.096, 0.049) and Dyson-Schwinger equations [71] (0.36, 0.17, 0.092).
FIG. 7: (Top) The ratio of the light-quark valence distribution of kaon to that of pion from this work (labeled “MSULat’20”), along with the results of the CERN NA3 experiment [1], the next-leading-order of Gluck-Reya-Stratmann model [74] (“GRS’97”), the DSE approach (“DSE’11”) [75], and one obtained from basis light-front quantization with NJL interactions (“BLFQ-NJL’19”) [76]. (Bottom) Our result for $x_s K_v(x)$ (labeled “MSULat’20”) as function of $x$, along with that from chiral constituent quark model [77] (“XCQ’17”), the one obtained from basis light-front quantization with NJL interactions [77] (“BLFQ-NJL’19”), and one obtained from using rainbow-ladder truncation of QCD Dyson-Schwinger equations [75] (“DSE’18”).

IV. CONCLUSION

In this work, we presented the first direct lattice-QCD calculation of the $x$ dependence of the kaon parton distribution functions using two lattice spacings, multiple pion masses ($M_{\pi, \text{min}} = 217$ MeV) and $M_\pi L \in \{4.5, 5.5\}$ with high statistics, $N_{\text{meas}} \in \{11, 61\}$ thousands and $N_{\text{cfg}} \in \{725, 958\}$. Our valence-quark pion distribution is in good agreement with the one obtained by JLab/W&M group using LSC methods and extrapolated to the physical pion mass. The ratios of the light-quark valence distribution in the kaon to the one in pion, $u^K_K / u_p$, were found to be consistent with the original CERN NA3 experiments. We made predictions for the strange-quark valence distribution of the kaon, $s^K_K(x)$, determining that it is close to the DSE result [78].

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