Effects of magnetic field applied on leads

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1. Introduction

During the past decade, there has been considerable interest in quantum transport phenomena in devices where the spin degree of freedom of the electron plays a crucial role e.g. the spin-transistor. Of particular interest are mesoscopic systems where the conductance is sensitive not only to the sample’s specific configuration but also to the spin population in the leads attached to the sample.

One of the special characteristics of mesoscopic systems is the fluctuating nature of transport coefficients such as the conductance. Such fluctuations do not depend on the details of the sample and the strength of disorder but are usually determined by the symmetry and the dimensionality of the system. The standard classification is detailed in Table I.

Table I. Universality class and corresponding symmetry of the system and the properties of scattering matrix.

| Universality class | Symmetry | Properties of S matrix |
|--------------------|----------|------------------------|
| Orthogonal         | yes      | yes, unitary symmetric |
| Symplectic         | yes      | no, unitary self-dual  |
| Unitary            | no       | irrelevant, unitary    |

The two relevant symmetries are time reversal symmetry (TRS) and spin rotation symmetry (SRS). TRS is broken by an applied magnetic field or by magnetic scattering due to magnetic impurities or magnetic domain walls. If TRS is broken, systems are classified as unitary, regardless of whether or not SRS is broken. The spin-orbit interaction breaks SRS but preserves TRS, and in this case systems are classified as symplectic.

The relation between the Hamiltonian of the sample and its corresponding universality class is well understood. However, the properties of mesoscopic systems that are measured in transport experiments depend not only on the sample but also on the leads through which currents flow in and out of the sample and through which voltages are measured.

In this contribution we address the question of whether or not the universality class can change as a result of an asymmetric spin population in the leads i.e. when the number of up and down spin channels in the leads becomes asymmetric as a result of Zeeman splitting. We show that even if the sample is unchanged, the universality class can change under certain conditions.

2. Model

We consider a two dimensional (2D) sample connected to two electron reservoirs by ideal leads. The 2D system is constructed in the xy-plane with the x direction taken as the direction of current flow. The sample region contains both a random potential and a spin-orbit interaction described by the Ando Hamiltonian

\[ \mathcal{H} = \sum_{i,\sigma} W_i c_i^\dagger \sigma c_i^\sigma - \sum_{\langle i,j \rangle, \sigma, \sigma'} V_{i,\sigma,j,\sigma'} c_i^\dagger \sigma c_j^\sigma' \]  

(1)

where

\[ V_{i,\sigma,j,\sigma'} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \]

(2)

and

\[ V_{i,\sigma,j,\sigma'} = \begin{pmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{pmatrix} \]

(3)

Here \( c_i^\dagger (c_i^\sigma) \) denotes the creation (annihilation) operator of an electron on site \( i \) with spin \( \sigma \). \( W_i \) denotes the random potential on the site \( i \), distributed uniformly on \([-W/2, W/2]\). The hopping is restricted to nearest neighbours. The unit vector of the \( x(y) \)-direction is denoted by \( \hat{x}(\hat{y}) \). The parameter \( \theta \) denotes the strength of the spin-orbit interaction. The strength of the hopping is taken to be the energy unit.

We impose the fixed boundary condition in the transverse direction \( y \). The transverse energy of the channel \((i, \sigma)\) in the lead \( \varepsilon_i^\sigma \) is given by

\[ \varepsilon_i^\sigma = -2 \cos \left( \frac{i \pi}{n+1} \right) - Z \sigma \quad (i = 1, 2, \ldots, n) \]  

(4)

where \( n \) denotes the number of sites in \( y \)-direction, and \( Z \) denotes the strength of Zeeman splitting and \( \sigma(= \pm 1) \) is the spin index. The channel \((i, \sigma)\) is a propagating mode if \( |E_F - \varepsilon_i^\sigma| < 2.0 \), \( E_F \) being the Fermi energy. For example, when we set \( E_F = -1.1 \) and \( Z = 1.0 \), the number of up (down) spin channels becomes 27 (15) for the sample of width 30 sites.

In the following simulation, we set to be \( W = 2.0 \), \( \theta = 0, \pi/4 \) and \( E_F = -1.1 \). The system size is set to be \( 30 \times 30 \) in units of the lattice spacing.

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3. Results

We extend the transfer matrix method\textsuperscript{1} to the case where there is a spin-orbit interaction in the sample and the population of up and down spins in the leads is asymmetric. We calculate the transmission matrix $t$ in the two terminal configuration. The population of up and down spins in one lead is always set to be symmetric ($Z = 0$), and that in the other lead is varied ($Z = 0, 1, 2$).

To detect any change of universality class, we investigate the spacing distribution $P(s)$ where $s$ is the interval between neighboring transmission eigenvalue $\tau$ (the eigenvalues of $tt^\dagger$). An ensemble of about $10^6$ samples is simulated for the estimation of the level statistics.

Figure 1 shows the spacing distribution $P(s)$ of the transmission eigenvalue $\tau$ for the sample with a spin-orbit interaction. In the absence of Zeeman splitting in the leads (o) $P(s)$ is close to the Wigner surmise for the Gaussian symplectic ensemble (GSE). On the other hand, with Zeeman splitting in the leads (•) $P(s)$ is close to the surmise for the Gaussian unitary ensemble (GUE). The reason for this crossover is that the asymmetry of the population of up and down spins in the lead destroys the self-dual property of the scattering matrix.

Figure 2 shows the spacing distribution of the transmission eigenvalue $\tau$ for the sample with $\theta = 0$ i.e. no spin-orbit interaction. In contrast to Fig. 1 the level statistics do not depend on the states of the leads and are well fitted by the formula for the Gaussian orthogonal ensemble (GOE). We summarize these results in Table II.

In summary, we have shown that the level statistics of the transmission coefficient is sensitive to the asymmetry of the spin population in the lead. We can thus control the conductance fluctuations without changing the sample parameters. For example, the variance of conductance is suppressed by 1/2 when we introduce the asymmetry of the spin population in a lead. (Note that the GUE in the presence of the asymmetry is spin non-degenerate, and the variance of conductance is 1/4 the spin degenerate value.) This behavior is essentially different from that of the asymmetry of the number of channels between the leads where the universality class is not changed.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
SO in a sample & Zeeman in a lead & Universality class \\
\hline
yes & no & Symplectic \\
yes & yes & Unitary \\
no & no & Orthogonal \\
no & yes & Orthogonal \\
\hline
\end{tabular}
\caption{Effects of Zeeman splitting in a lead}
\end{table}

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