A step size control scheme based on polynomial regression for continuation power flow calculation

Hua Sun¹, Quan Chen² and Xiaoming Dong²*

¹ Technician Department, Shandong Labor Vocational and Technology College, Jinan, 250022, China
² School of Electrical Engineering, Shandong University, Jinan, 250061, China
*Corresponding author’s e-mail: 1654116735@qq.com

Abstract. Continuation power flow is an effective approach to analyse static voltage stability by accurately calculating the maximum loading point. To improve the efficiency of continuation power flow model by decreasing its calculation burden, a new step size control scheme for predictor-corrector steps is proposed in this study. Firstly, the predictor tangent vector is obtained according to Jacobin matrix. Secondly, the optimal parameter with the largest sensitivity to load power change is identified by recognizing biggest component in tangent predictor vector. Thirdly, polynomial regression between the optimal parameter and the power increment parameter is analysed. Finally, the improved self-adaption step size control scheme is established in terms of the obtained polynomial regression model. Studies based 14-bus test system are used to demonstrate the efficacy of the proposed scheme. For this study, specific point-to-point maximum loading is calculated.

1. Introduction

In order to make the best use of power-transmission equipment, it is needed to estimate how much power can be transferred from a source to a load through the network, which can be called the maximum loading point [1]-[3]. Under heavy load, system state gets close to this kind of critical condition, which is defined as saddle-node bifurcation [4]-[7]. Particularly, with the increasement in both power generation and load demand, voltage magnitudes of the network buses generally decrease due to the lack of reactive power support. This happens because the series reactance of transmission line will consume large amount of reactive power when the transmission line carries heavy load power. Suffering from low voltage level and heavy reactive power demand, responsible network collapses [8]-[11] may accrue combining with a trail of cascading system emergencies, leading to large tracts of blackout. All the above analyses create the need to accelerate calculating the maximum loading point of power systems and estimating the stability of current system state by evaluating the “distance” between the present state to critical point.

Many methods are presented to evaluate this kind of distance in different point of views, including the power flow methods [12]. The methods in accordance with power flow aim at simulating the changes of system state variables from normal state to critical state by gradually increasing the system generation and power load. However, conventional power flow model faces convergence problems due to numerical ill-condition when the state gets close to the critical point. Therefore, continuation method is introduced to solve the power flow model to form the parameterized power balancing equations, which is defined as the continuation power flow model [13]-[16]. With the increasement in
the supplemented power increase parameter, the electric power transferred through network grows gradually approaching to the critical point. Its predictor-corrector steps can avoid the convergence problem by the parameter selection scheme and supplemented Jacobin matrix. Thus, continuation power flow is considered as an effective method to obtain the maximum loading point. However, because repeated computations to solve linear equations are needed in predictor-corrector steps, large computation burden limited the practical application of continuation power flow.

Step size control scheme [17]-[19] attracts more and more attentions for its efficiency to accelerate the calculation of continuation power flow by effectively decreasing the predictor-corrector steps. This study presents a new self-adopted step size control scheme based on polynomial regression.

The rest of the paper is organized as follows: The expression of power flow formulations is shown in Section 2. In Section 3, continuation power flow mode is expressed detailly. The improved step size control scheme is proposed in Section 4 and then in Section 5 case studies using the new scheme indicate its efficiency. The conclusions are drawn in section 6.

2. Power flow formulation expression

With all the parameters expressed in per-unit value, the conventional PF equations can be expressed as in (1), where \( P_i \) is the active power injection into bus \( i \); \( Q_i \) is the reactive power injection into bus \( i \); \( V_i \) is the voltage magnitude of bus \( i \); \( \theta_{ij} \) is the voltage phase angle between bus \( i \) and bus \( j \); \( G_{ii} \) is self-conductance of bus \( i \); \( B_{ii} \) is self-susceptance of bus \( i \); \( G_{ij} \) is mutual-conductance between bus \( i \) and \( j \); \( B_{ij} \) is mutual-susceptance between bus \( i \) and \( j \); \( n \) is the total number of network buses.

\[
\begin{align*}
\Delta P_i & = P_i - V_i^2 G_{ii} - \sum_{j=1}^{n} V_j V_j \left( G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij} \right) = 0 \\
\Delta Q_i & = Q_i + V_i^2 B_{ii} - \sum_{j=1}^{n} V_j V_j \left( G_{ij} \sin \theta_{ij} - B_{ij} \cos \theta_{ij} \right) = 0 \\
& \quad (i = 1, 2, \cdots, n - 1)
\end{align*}
\]

(1) can be abbreviated as follows:

\[
F(v, \theta) = 0
\]

(2)

with

\[
v = \left[ V_1 \quad V_2 \quad \cdots \quad V_{n-1} \right]^T
\]

(3)

\[
\theta = \left[ \theta_1 \quad \theta_2 \quad \cdots \quad \theta_{n-1} \right]^T
\]

(4)

\( z \) is the set combined with \( v \), \( \theta \):

\[
z = \left[ v^T \quad \theta^T \right]^T
\]

(5)

In order to solve the power balancing equations in (1), the Newton iteration formulations are established as shown in (6).

\[
\begin{align*}
F(z(k)) &= \frac{dF(z(k))}{dz(k)} \Delta z(k) = J z(k) \\
z(k + 1) &= z(k) + \Delta z(k)
\end{align*}
\]

(6)

where \( J \) represents the Jacobian matrix which can be expressed as (7).

\[
J = \begin{bmatrix}
\frac{\partial F}{\partial v} & \frac{\partial F}{\partial \theta}
\end{bmatrix}
\]

(7)
Following inequality can be used as the convergence standard of the Newton iteration process.

\[ \left\| \Delta z \right\| \leq \varepsilon \]  

where \( \varepsilon \) is a small positive number that given in advance.

3. Continuation Power Flow Model Expressions

In order to make a detailed expression for the relations of continuation power flow model, four departments are elaborated as below.

3.1. Parameterized power flow equation:
When parameter \( \lambda \) is introduced into power flow equations to represent the power increment, parameterized power flow equations in (9) is obtained.

\[
\begin{align*}
\Delta P_i(z) &= \Delta P_i(z) + \lambda K_{P_i} = 0 \\
\Delta Q_i(z) &= \Delta Q_i(z) + \lambda K_{Q_i} = 0
\end{align*}
\]

where \( K_{P_i} \) and \( K_{Q_i} \) represents the incremental rate of active and reactive power, which should be set in advance of continuation power flow calculation. (9) can be written as follows for abbreviation.

\[ \mathbf{R}(z, \lambda) = \mathbf{R}(x) = 0 \]

\[ \left( x = [z', \lambda] \right) \]

3.2. Predictor
Predictor process aims at getting an approximate solution (forecast point), which is the initial value of corrector iterative processes. The forecast point can be computed by (11) in predictor process.

\[ \mathbf{x}' = \mathbf{x} + \mu \mathbf{d} \mathbf{x} \]  

where \( \mathbf{x}' \) is forecast point of predictor process; \( \mathbf{x} \) is base point of predictor process; \( \mathbf{d} \mathbf{x} \) is tangent vector at base point; \( \mu \) is step size coefficient. \( \mathbf{d} \mathbf{x} \) in (11), can be obtained by solving the equation in (12).

\[
\mathbf{J}_m \mathbf{d} \mathbf{x} = \begin{bmatrix} \frac{\partial \mathbf{R}}{\partial \mathbf{x}} \\ \mathbf{e}_{(m)} \end{bmatrix} \begin{bmatrix} \mathbf{d} \mathbf{x} \end{bmatrix} = \begin{bmatrix} \mathbf{J} & \frac{\partial \mathbf{R}}{\partial \lambda} \end{bmatrix} \begin{bmatrix} \mathbf{d} \mathbf{x} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}
\]

with

\[ \mathbf{e}_{(m)} = \begin{bmatrix} 0 & \cdots & 0 & 1 & 0 & \cdots & 0 \end{bmatrix} \]

(13) represents the vector, in which the \( m \)th component is 1 while the others are zero. The parameter \( m \) can be calculated according to (14).

\[ m = \left\{ m \mid |d_{x_m}| = \max \{|dx_1|, |dx_2|, \ldots, |dx_{2n-1-1}|\} \right\} \]

3.3. Corrector:
In corrector process, forecast point is used as the initial value to solve (15).

\[ \mathbf{O}(x) = \begin{cases} \mathbf{R}(x) = 0 \\ x_m - x'_m = 0 \end{cases} \]

To solve (15), Newton iterative formats are established as below.
\[
\begin{align*}
O(x(k)) &= \frac{dO(x(k))}{dx(k)} \Delta x(k) = J_o \Delta x(k) \\
x(k + 1) &= x(k) + \Delta x(k)
\end{align*}
\] (16)

Following inequality can be used to judge the convergence of (16).
\[
\|\Delta x\|_o \leq \epsilon_o
\] (17)

where \(\epsilon_o\) is a small positive number that given in advance. While the Newton iteration of corrector is converged, the converged solution of the variables corresponding to the power flow state are obtained.

3.4. Step size control:
The coefficient \(\mu\) in (11) is set to control the predictor-corrector step size by determining the distance between current point and the forecast point. At the start of continuation power flow calculation, a large step size is needed to speed up the computation. When system is close to the maximum loading point the step size should decrease automatically. With this demand, this study presents the improved self-adaptation step size control scheme as shown in Section 4.

4. Improved Step size Control Scheme
At the very beginning, \(\mu\) could be initialized with a small enough positive number corresponding to the step serial number \(s=0\). While 0\(<s<3\), the iteration loops of corrector calculation are used to increase and decrease \(\mu\) according to the relations given in (18).
\[
\begin{align*}
\mu(s) &= \mu(s-1) \times 2 \quad (k<3) \\
\mu(s) &= \mu(s-1) / 2 \quad (k \geq 3)
\end{align*}
\] (18)

where \(k\) represents the loops of Newton iterations in corrector. When \(s\geq3\), last step and the step before last step are selected to form relation (19).
\[
\begin{bmatrix}
1 & x_m(s) & x_m^2(s) \\
1 & x_m(s-1) & x_m^2(s-1) \\
1 & x_m(s-2) & x_m^2(s-2)
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
a_2
\end{bmatrix}
= \begin{bmatrix}
\lambda(s) \\
\lambda(s-1) \\
\lambda(s-2)
\end{bmatrix}
\] (19)

Then the regression coefficients for the quadratic polynomial expressed by (20) can be obtained according to relation (19).
\[
\lambda = a_0 + a_1 x_m + a_2 x_m^2
\] (20)

As a result, the predicted maximum value of power increment parameter \(\lambda\) can be calculated according to the relation in (21).
\[
\lambda_{pre} = \frac{4a_0a_2 - a_1^2}{4a_2}
\] (21)

Then since \(s\geq3\), step size coefficient \(\mu\) in (11) could be calculated according to the relation given in (22) for improvement.
\[
\mu = \frac{\lambda_{pre} - \lambda}{d\lambda}
\] (22)

The flow diagram for the entire improved continuation power flow is given in figure 1. First of all, the power flow data such as transmission line parameters and transformer parameters should be input. Then variables for power flow calculation are initialized. Secondly, the Newton iterations for power flow calculation is activated. The power flow solution is obtained after the iteration is identified to be
converged according to the judgement standard. Thirdly, the process goes into the predictor-corrector steps which include the improved step size control scheme. The variables for continuation power flow calculation are initialized in accordance with the solution of power flow calculation. The step size control scheme is deployed between the processes of predictor and corrector. Finally, that $\lambda<0$ indicates that the saddle node bifurcation or the maximum loading point is reached.

5. Case study

In this section, the proposed step size control scheme is applied to the case study based on IEEE 14-Bus test system as shown in figure 2. The test system consists of 5 generation buses and 11 load points, interconnected by 17 lines and transformers. As shown in figure 2, the power transfer limit of interest is point-to-point, between the generator bus 1 and the load at bus 10. Thus, $K_P$ for bus 3 is set to 1 while $K_P$ for bus 10 is set to -1. Then the continuation power flow calculation is proceeded by repeating the predictor-corrector step. Figure 3 shows the changes of voltage magnitude at bus-10 and $\mu$ with the changes of $\lambda$. At the beginning, the calculation goes with large step size, while at the end the step size could be decreased automatically. As a result, the large step size accelerates computation speed, and the small step size guarantees the accuracy of critical point result. Furthermore, the detailed results of step size calculation are elaborated in Table 1.
Figure 2. IEEE 14-Bus Test System

Figure 3. Calculation results of improved continuation power flow method

Table 1. Detailed results for step size control scheme

| s  | $\lambda$ | $\lambda_{pec}$ | $a_0$   | $a_1$   | $a_2$   |
|----|-----------|-----------------|---------|---------|---------|
| 3  | 0.028     | 0.565           | -224.020| 448.539 | -223.955|
| 4  | 0.565     | 1.106           | -112.575| 214.394 | -100.970|
| 5  | 1.106     | 1.511           | -73.776 | 134.016 | -59.350 |
| 6  | 1.511     | 1.920           | -44.776 | 75.902  | -30.246 |
| 7  | 1.920     | 2.213           | -29.775 | 47.018  | -16.348 |
| 8  | 2.213     | 2.391           | -21.482 | 31.826  | -9.396  |
| 9  | 2.391     | 2.483           | -16.356 | 23.007  | -5.607  |
| 10 | 2.469     | 2.518           | -13.425 | 18.287  | -3.710  |
| 11 | 2.509     | 2.530           | -11.685 | 15.643  | -2.706  |
| 12 | 2.526     | 2.533           | -10.665 | 14.161  | -2.168  |
| 13 | 2.531     | 2.534           | -10.030 | 13.272  | -1.856  |
| 14 | 2.533     | 2.534           | -9.633  | 12.728  | -1.670  |
| 15 | 2.534     | 2.534           | -9.416  | 12.436  | -1.572  |
| 16 | 2.534     | 2.534           | -9.297  | 12.277  | -1.519  |
6. Conclusions
In this paper, a new step size control scheme for continuation power flow is presented. It captures the nonlinear relationship between system state variables and power increment parameter by polynomial regression. Then, using the obtained polynomial formulation, the self-adaptation step size control scheme is established. Case studies based on IEEE 14 bus test system demonstrate the efficiency of proposed scheme in accelerating the continuation power flow calculation as well as ensuring the accuracy of critical point results.

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