Ground-state correlations and final state interactions in exclusive lepton scattering off few-nucleon systems

C. Ciofi degli Atti, L. P. Kaptari

Department of Physics, University of Perugia and INFN, Sezione di Perugia, via A. Pascoli, Perugia, I-06100, Italy

The two nucleon emission process off \(^3\text{He}\) induced by medium energy electrons has been theoretically analyzed using realistic three-nucleon wave functions and taking the final state interaction into account. Various kinematical conditions have been considered in order to clarify the question whether the effects of the final state interaction could be minimized by a proper choice of the kinematics.

1. Introduction: correlations and final state interactions in lepton scattering off nuclei

The investigation of Ground State Correlations (GSC) in nuclei, in particular those which originate from the most peculiar features of the Nucleon-Nucleon (NN) interaction, i.e. its strong short range repulsion and complex state dependence (spin, isospin, tensor, etc), is one of the most challenging aspects of experimental and theoretical nuclear physics and, more generally, of hadronic physics. The results of sophisticated many-body calculations in terms of realistic models of the NN interactions, show that the complex structure of the latter generates a rich correlation structure of the nuclear ground state wave function, but the investigation of such a structure is problematic due to the effects of the final state interaction (FSI); these, in fact, very often compete with the effects generated by GSC, so that the longstanding question \textit{Does FSI hinder the investigation of GSC?} has not yet been clearly answered. To-day the answer could probably be provided in a more reliable way, particularly in the case of few-body systems, for which accurate ground state wave functions are available and FSI effects can also be calculated in a satisfactory way (see e.g. [1], [2]). In this paper the process of two-nucleon emission off the three-nucleon systems, which is under intense experimental investigation (see e.g. [3], [4]) will be discussed, with the aim of providing another attempt at answering the above longstanding question. Realistic three-nucleon wave functions [2] corresponding to the AV18 interaction [5], will be used, and the effects from the FSI will be investigated at various levels of complexity.

\*Presented at the Third International Conference on Perspective in Hadronic Physics, Trieste, 7-11 May, 2001. To appear in Nuclear Physics.

†On leave from Bogoliubov Lab. Theor. Phys., JINR, Dubna, Russia
2. Two-nucleon emission off the three-nucleon systems

We consider the absorption of a virtual photon $\gamma^*$ by a nucleon bound in $^3He$ followed by two-nucleon emission, in particular the process $^3He(e, e'N_2N_3)N_1$, where $N_2$ and $N_3$ denote the two nucleons which are detected and $N_1$ the third one. In what follows $Q^2 = q^2 - \nu^2$ denotes the photon four-momentum transfer, $k_i$ the momenta of the bound nucleons before $\gamma^*$ absorption, and $p_i$ the momenta in the continuum final state.

Momentum and energy conservation require that

\[
\sum_{i=1}^{3} k_i = 0, \quad \sum_{i=1}^{3} p_i = q, \quad \nu + M_3 = \sum_{i=1}^{3} (M^2 + p_i^2)^{1/2}
\]

where $M$ and $M_3$ are the nucleon and the three-nucleon system masses, respectively.

In one-photon exchange approximation the cross section of the process reads as follows

\[
\frac{d^8\sigma}{d\nu d\Omega dP_1 dP_2 dP_3} = \sigma_{Mott} \cdot \sum_{j=1}^{6} v_j \cdot W_j \cdot \delta(q - \sum_{i=1}^{3} p_i) \delta(\nu + M_3 - \sum_{i=1}^{3} (M^2 + p_i^2)^{1/2}) \tag{2}
\]

where $v_j$ are well known kinematical factors, and $W_j$ the response functions, which have the following general form

\[
W_j \propto \left| \langle \Psi_j^-(p_1, p_2, p_3)\mid \hat{O}_j(q)\mid \Psi_i(k_1, k_2, k_3) \rangle \right|^2 \tag{3}
\]

In Eq. (3) $|\Psi_j^-(p_1, p_2, p_3)\rangle$ and $|\Psi_i(k_1, k_2, k_3)\rangle$ are the continuum and ground state wave functions of the three body system, respectively, and $\hat{O}_j(q)$ is a quantity depending on proper combinations of the components of the nucleon current operator $j^\mu$ (see e.g. [5]).

The various processes, in order of increasing complexity, which contribute to the reaction $^3He(e, e'N_2N_3)N_1$ are depicted in Fig. [1].

Let us introduce the relative, $t = \frac{p_2 - p_3}{2}$, and Center-of-Mass, $P = p_2 + p_3$, momenta of the detected pair, and the missing momentum $p_m = p_1 - q = -(p_2 + p_3)$. As already stated, we consider the process $^3He(e, e'p_1p_2)n$ ($^3He(e, e'p_1n)p_2$), in which $\gamma^*$ interacts with the neutron (proton) and the two protons (proton-neutron) correlated in the initial state are emitted and detected. Within the PWIA, i.e. when the final state rescattering between the two detected nucleons is taken into account (processes $a$ and $b$), but the interaction of the hit neutron(proton) with the emitted proton-proton (proton-neutron) pair is disregarded, the cross section (Eq. (2)) integrated over $P$ and the kinetic energy of $N_1$, has the following form (we take $q \parallel z$)

\[
\frac{d^8\sigma}{d\nu d\Omega d\Omega_{N_1} dt d\Omega_t} = \mathcal{K} \left( Q^2, \nu, p_m, t \right) \cdot \frac{1}{2} \sum_{M_3, \sigma, s_f, \mu_f} \left| \int \exp(i p_m \rho) \chi_{3M_3}^s (r) \Phi_{s_f}^{t(-)}(r) \Psi_{s_i}^s (r, \rho) \right|^2 \equiv \mathcal{K} \left( Q^2, \nu, p_m, t \right) \cdot M(p_m, t) \tag{4}
\]

where $p_m = -k_1$, $\mathcal{K}$ incorporates all kinematical factors, $\chi_{3M_3}^s \sigma$ represents the Pauli spinor for the hit particle, $\Phi_{s_f}^{t(-)}(r)$ is the two-nucleon wave function in the continuum, and $r$ and $\rho$ are usual Jacobi coordinates.
Figure 1. The various processes contributing to the reaction $^3\text{He}(e,e'N_2N_3)N_1$: (a) No FSI, (b) the NN rescattering, (c) the three-body rescattering. Note that in this paper, following Ref. [6], the sum of (a) and (b) is called The Plane Wave Impulse Approximation (PWIA), whereas in Ref. [1] PWIA is the same as our No FSI (process a)).

In the rest of the paper we will omit, for ease of presentation, all explicit summations over the quantum numbers and will denote the continuum two-nucleon wave function simply by $\Phi_{N_2N_3}$. The quantity $M(p_m,t)$ can then be cast in the following simple form

$$M(p_m,t) = M(k_1,t) = \left| \int \exp(i p_m \cdot \rho) I_{N_2N_3}^t(\rho) d^3\rho \right|^2$$  \hspace{1cm} (5) $$

where $I_{N_2N_3}^t(\rho)$ is the overlap integral between the three-nucleon ground state wave function and the two-nucleon continuum state, i.e.

$$I_{N_2N_3}^t(\rho) = \int \Phi_{N_2N_3}^{(-)}(r) \Psi_{3M_3}(r,\rho) d^3r \hspace{1cm} (6)$$

Within the No FSI approximation, i.e. when only process a) contributes to the reaction, one has $\Phi_{N_2N_3}^{(-)} \propto \exp(it \cdot r)$ and $\int_{N_2N_3}^t(\rho) = \int e^{itr} \Psi_{3M_3}(r,\rho) d^3r$, so that

$$M(k_1,t) = \left| \int \exp(i k_1 \cdot \rho) \exp(it r) \Psi_{3M_3}(r,\rho) d^3r d^3\rho \right|^2$$  \hspace{1cm} (7) $$

represents nothing but the square of the three-nucleon wave function in momentum space. When the PWIA is considered (process a) plus process b)), the direct correspondence between the three-nucleon wave function and the cross section is lost and the process is governed by the quantity $M(k_1,t)$ provided by Eq. (5). The integral of the latter over the direction of $t$, is related to the Spectral Function of nucleon $N_1$, namely

$$\frac{M|t|}{2} \int M(k_1,t) d\Omega_t = P_1(k_1,E^*)\hspace{1cm} (8)$$

\begin{align*}
Q^2 & \quad P_1=k_1+q \\
Q^2 & \quad P_2=k_2 \\
Q^2 & \quad P_3=k_3
\end{align*}
where $E^* = t^2/M$ is the "excitation energy" of the spectator pair $N_2 N_3$, which is related to the removal energy $E$ of nucleon $N_1$ by $E = E_3 + E^*$, where $E_3$ is the (positive) binding energy of the three nucleon system. If the Coulomb interaction is disregarded, the neutron Spectral Function in $^3\text{He}$ is the same as the proton Spectral Function in $^3\text{H}$. 

In Fig. 2 the nucleon Spectral Function calculated with and without the NN rescattering is shown. It can be seen that there is a region where the FSI (NN rescattering) does not play any role. This is the so called two-nucleon correlation region, where the relation $E^* = t^2/M \approx k_1^2/4M$ holds (see e.g. Ref. [8]). The existence of such a region is a general feature of any Spectral Function, independently of the two-nucleon interaction and of the method to generate the wave function. This is illustrated in Fig. 3 where the Spectral Function obtained with the variational wave function of Ref. [2], corresponding to the AV18 interaction, is shown for several values of $k \equiv k_1$.

From the figures we have exhibited one expects that if the kinematics is properly chosen...
(i.e. \( E^* = t^2/M \simeq k_1^2/4M \)) the NN rescattering can be strongly reduced; on the contrary, if it is chosen improperly (in particular corresponding to an initial state characterized by \( k_1 \simeq 0 \)), the NN rescattering fully distorts the No FSI predictions. This is demonstrated in Fig. 4 where Eq. (5) is shown and compared with the No FSI approximation (Eq. (7)). In this calculation, we have fixed the two-nucleon relative energy \( E^* = t^2/M = 50 MeV \) and have plotted, for a given values of \( k_1 \), the dependence of \( M(k_1, t) \) upon the angle \( \theta_t \) between \( t \) and \( q \), the latter being chosen along \( k_1 \). The four values of \( k_1 \) correspond to three relevant regions of the Spectral Function, viz : 1. \( E^* > k_1^2/4M \) (\( k_1 = 0.5 fm^{-1} \) and \( 1 fm^{-1} \)); 2. \( E^* \simeq k_1^2/4M \) (\( k_1 = 2.2 fm^{-1} \)), the correlation region; 3. \( E^* < k_1^2/4M \) (\( k_1 = 3 fm^{-1} \)).

It can be seen that in the first region the two nucleon rescattering is very large (cf. Figs. 4 and 5), whereas in the two other regions, it is very small.

We have also investigated the effect of NN rescattering on a particular kinematics, namely that which corresponds to the initial state in which \( N_2 \) and \( N_3 \) are correlated with momenta \( k_2 = -k_3 \) and \( k_1 = 0 \), so that, after \( \gamma^* \) absorption, \( N_1 \) is emitted with momentum \( q \), and \( N_2 \) and \( N_3 \) are emitted back-to-back with momenta \( p_2 = -p_3 \) (\( p_m = 0 \)).

The results are presented in Fig. 4 where it can be seen that, as expected, the effect
Figure 4. The quantity $M(k_1, t)$ calculated at fixed value of $E^* = 50$ MeV, versus the angle $\theta_t$ between the relative momentum of the emitted nucleons $t$ and the momentum transfer $q$. The full line includes the two nucleon rescattering and the dashed line represents the No FSI result, i.e. the three-body wave function in momentum space. The three values of $k_1$ which have been chosen, correspond to four different regions of the Spectral Function (see text). Three-nucleon wave function from [2]; AV18 interaction [9].

The NN rescattering is large. We have repeated this calculation in the correlated region and found, obviously, that the rescattering, in this case, has negligible effects [11].

We have eventually considered the three-body rescattering, e.g. process c) of Fig. 1, by treating the rescattering of $N_1$ with the interacting pair $N_2N_3$ within an extended Glauber-type approach [10]. The details of the calculation will be presented elsewhere [11].

In Figs. 5 and 6 we show our preliminary results for the quantity (dot-dashed line)

$$M^D(p_m, t) = \left| \int \Phi^{P_m}_{N_1N_2N_3}(r, \rho) I^{t}_{N_2N_3}(\rho) d^3rd^3\rho \right|^2$$ (9)

which is the generalization of Eq. (5) to take into account, via the quantity $\Phi^{P_m}_{N_1N_2N_3}(r, \rho)$, the rescattering of $N_1$ with the interacting pair $N_1N_2$. In the Figure, $M^D(p_m, t)$ is plotted vs the missing momentum ($p_m \neq k_1$) for a fixed value of $t$; in the same Figure we also show the results corresponding to the case when only the NN rescattering is active (full line) and to the case when all FSI's are switched off (dashed line). In this calculation we have considered high values of $|q|$, such that the asymptotic values of those quantities
Figure 5. The quantity $M^D(p_m, t)$ (Eq. (9), dot-dashed line) calculated with $p_2 = -p_3$ and $p_m = 0$, which for the processes a) and b) corresponds to the kinematics where, in the initial state, nucleons $N_2$ and $N_3$ were correlated with momenta $k_2 = -k_3$ and $k_1 = 0$. The dashed line corresponds to the No FSI case (Eq. (4)), whereas the full line includes the two nucleon rescattering (Eq. (5)). Three-nucleon wave function from [2]; AV18 interaction [9].

Figure 6. The quantity $M^D(p_m, t)$ (Eq. (9), dot-dashed line) calculated at fixed value of the relative momentum $|t| = 240\text{ MeV}/c$, versus the missing momentum $p_m$. The full line includes the two nucleon rescattering (Eq. (5)), whereas the dashed line represents the No FSI result (Eq. (3)). Three-nucleon wave function from [2]; AV18 interaction [9].

which enter the calculation (e.g. the total NN cross section, the ratio of the imaginary to real parts of the forward scattering amplitude, etc.) have been adopted. A more correct calculation, along the line of Ref. [10], will be presented elsewhere [11]. It can be seen in Fig. 2 that in the two-nucleon correlation region and at high values of the missing momentum, the full FSI merely reduces to a change of the amplitude, without appreciably distorting the missing momentum distributions calculated without any type of FSI. Such a result appears to be a very promising one in the investigation of the correlated part of the three-body wave function.

3. Summary

We have investigated the effects of the Final State Interaction in the process of two-nucleon emission off $^3\text{He}$ induced by medium energy electrons. To this end, we have used
realistic three-nucleon wave functions, which, being the exact solution of the Schroedinger equation, incorporate all types of correlations, in particular the short-range and tensor ones generated by modern NN potentials. Using these wave functions, we have investigated the process $^3\text{He}(e,e'pp)n$ ($^3\text{He}(e,e'npp)p$) which can be generated by two main mechanisms: i) the absorption of $\gamma^*$ by a non correlated nucleon, followed by the emission of two "high" momenta nucleons, namely the ones which were correlated in the initial state, and ii) the absorption of $\gamma^*$ by a nucleon of a correlated pair, followed by the emission of a "high" momentum nucleon, which was the second correlated nucleon in the initial state, and a "low" momentum nucleon, corresponding to the spectator one in the initial state. A very specific kinematics corresponding to case i), with the active nucleon at rest in the initial state leaving the nucleus with momentum $p_1=q$, followed by the back-to-back emission of the two correlated nucleons with momenta $p_2=-p_3$, was also analyzed. We have taken into account the final state interaction both between the two detected nucleons, as well as between these and the active nucleon which absorbed $\gamma^*$. We have investigated the above process in different kinematical regions governed by various types of correlations. The so called two-nucleon correlation region, leading to process i), turns out to be of particular interest, for, in such a region, Final State Interaction effects can be minimized

REFERENCES

1. J. Golak, H. Kamada, H. Witala, W. Gloeckle and S. Ishikawa Phys. Rev. C63 (1995) 1638.
2. A. Kievsky, S. Rosati and M. Viviani, Phys. Rev. Lett. 82 (1999) 3759 and private communication.
3. D. L. Groep et al, Phys. Rev. C63 (2000) 014005.
4. Tjlab Experiment E89-027
5. S. Boffi, C. Giusti and F.D. Pacati, Phys. Rep. 226 (1993) 1.
6. C. Ciofi degli Atti, E. Pace and G. Salme in Lecture Notes in Physics 86 (1978) 315; Phys. Rev. 21 (1980) 805.
7. R.V. Reid Jr., Ann. Phys. 50 (1968) 411;
8. C. Ciofi degli Atti, S. Simula, L. L. Frankfurt and M. I. Strikman, Phys. Rev. C44 (1991) R7. 
C. Ciofi degli Atti and S. Simula, Phys. Rev. C53 (1996) 1689.
9. R. B. Wiringa, R. A. Smith and T. L. Ainsworth, Phys. Rev. C29 (1984) 1207.
10. C. Ciofi degli Atti, L. Kaptari and D. Treleani, Phys. Rev. C63 (2001) 044601.
11. C. Ciofi degli Atti, L. P. Kaptari and H. Morita, to be published