Extracting Dark Matter Properties Model–Independently from Direct Detection Experiments

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In this article I review model–independent procedures for extracting properties of Weakly Interacting Massive Particles (WIMPs) from direct Dark Matter detection experiments. Neither prior knowledge about the velocity distribution function of halo Dark Matter particles nor about their mass or cross sections on target nucleus is needed. The unique required information is measured recoil energies from experiments with different detector materials.

Keywords: Dark Matter; WIMP; direct detection.

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1. Introduction

Different astronomical observations and measurements indicate that more than 80% of all matter in our Universe are “dark” and this Dark Matter interacts at most very weakly with ordinary matter. Weakly Interacting Massive Particles (WIMPs) arising in several extensions of the Standard Model of particle physics with masses roughly between 10 GeV and a few TeV are one of the leading candidates for Dark Matter\textsuperscript{1,2,3,4}. Currently, the most promising method to detect different WIMP candidates is the direct detection of the recoil energy deposited by elastic scattering of ambient WIMPs off the target nuclei\textsuperscript{5,6}. The differential event rate for elastic WIMP–nucleus scattering is given by\textsuperscript{1}:

\[
\frac{dR}{dQ} = \left( \frac{\rho_0 \sigma_0}{2 m_t^2 m_r^2} \right) F^2(Q) f(v) \int_{v_{\text{min}}}^{v_{\text{max}}} \left[ \frac{f_1(v)}{v} \right] dv .
\]  

(1)

Here \( R \) is the direct detection event rate, i.e., the number of events per unit time and unit mass of detector material, \( Q \) is the energy deposited in the detector, \( \rho_0 \) is the
WIMP density near the Earth, $\sigma_0$ is the total cross section ignoring the form factor suppression, $F(Q)$ is the elastic nuclear form factor, $f_1(v)$ is the one–dimensional velocity distribution function of the WIMPs impinging on the detector, $v$ is the absolute value of the WIMP velocity in the laboratory frame. The reduced mass $m_{r,N}$ is defined by

$$m_{r,N} \equiv \frac{m_\chi m_N}{m_\chi + m_N},$$  \hspace{1cm} (2)$$

where $m_\chi$ is the WIMP mass and $m_N$ that of the target nucleus. Finally, $v_{\text{min}} = \alpha \sqrt{Q}$ is the minimal incoming velocity of incident WIMPs that can deposit the energy $Q$ in the detector with the transformation constant

$$\alpha \equiv \sqrt{\frac{m_N}{2m_{r,N}}},$$  \hspace{1cm} (3)$$

and $v_{\text{max}}$ is the maximal WIMP velocity in the Earth’s reference frame, which is related to the escape velocity from our Galaxy at the position of the Solar system, $v_{\text{esc}} \gtrsim 600 \text{ km/s}$.

The total WIMP–nucleus cross section $\sigma_0$ in Eq. (1) depends on the nature of WIMP couplings on nucleons. Through e.g., squark and Higgs exchanges with quarks, WIMPs could have a “scalar” interaction with nuclei. The total cross section for the spin–independent (SI) scalar interaction can be expressed as\(^1, 2\)

$$\sigma_{0}^{\text{SI}} = \left(\frac{4}{\pi}\right) m_{r,N}^2 \left[Z f_p + (A - Z) f_n\right]^2.$$  \hspace{1cm} (4)$$

Here $m_{r,N}$ is the reduced mass defined in Eq. (2), $Z$ is the atomic number of the target nucleus, i.e., the number of protons, $A$ is the atomic mass number, $A - Z$ is then the number of neutrons, $f_{(p,n)}$ are the effective scalar couplings of WIMPs on protons $p$ and on neutrons $n$, respectively. Here we have to sum over the couplings on each nucleon before squaring because the wavelength associated with the momentum transfer is comparable to or larger than the size of the nucleus, the so–called “coherence effect”.

In addition, for the lightest supersymmetric neutralino, and for all WIMPs which interact primarily through Higgs exchange, the scalar couplings are approximately the same on protons and on neutrons: $f_n \simeq f_p$. Thus the “pointlike” cross section $\sigma_{0}^{\text{SI}}$ in Eq. (4) can be written as

$$\sigma_{0}^{\text{SI}} \simeq \left(\frac{4}{\pi}\right) m_{r,N}^2 A^2 |f_p|^2 = A^2 \left(\frac{m_{r,N}}{m_{r,p}}\right)^2 \sigma_{\chi p}^{\text{SI}},$$  \hspace{1cm} (5)$$

where $m_{r,p}$ is the reduced mass of the WIMP mass $m_\chi$ and the proton mass $m_p$, and

$$\sigma_{\chi p}^{\text{SI}} = \left(\frac{4}{\pi}\right) m_{r,p}^2 |f_p|^2$$  \hspace{1cm} (6)$$

is the SI WIMP–nucleon cross section. Here the tiny mass difference between a proton and a neutron has been neglected.
Table 1. List of the relevant spin values of the most used spin-sensitive nuclei. More details can be found in e.g., Refs. 1, 7, 8, 9.

| Isotope | Z  | J  |  \langle S_p \rangle | \langle S_n \rangle | \langle S_p \rangle / \langle S_n \rangle | \langle S_n \rangle / \langle S_p \rangle |
|---------|----|----|----------------------|------------------|--------------------|--------------------|
| ¹⁹F     | 9  | 1/2| 0.441                | -0.109           | 4.05               | -0.25              |
| ²³Na    | 11 | 3/2| 0.248                | 0.020            | -12.40             | 0.08               |
| ³⁵Cl    | 17 | 3/2| -0.059               | -0.011           | -5.36              | 0.19               |
| ³⁷Cl    | 17 | 3/2| -0.058               | 0.050            | 1.16               | -0.86              |
| ⁷³Ge    | 32 | 9/2| 0.030                | 0.378            | -0.08              | 12.6               |
| ¹²⁷I    | 53 | 5/2| 0.309                | 0.075            | -4.12              | 0.24               |
| ¹²⁷Xe   | 54 | 1/2| 0.028                | 0.359            | -0.08              | 12.8               |
| ¹³¹Xe   | 54 | 3/2| -0.009               | -0.227           | -0.04              | 25.2               |

On the other hand, through e.g., squark and Z boson exchanges with quarks, WIMPs could also couple to the spin of target nuclei, an “axial–vector” interaction. The spin-dependent (SD) WIMP–nucleus cross section can be expressed as:

$$\sigma^{SD}_0 = \left( \frac{32}{\pi} \right) G_F^2 m_{\chi,N}^2 \left( \frac{J + 1}{J} \right) \left[ \langle S_p \rangle a_p + \langle S_n \rangle a_n \right]^2.$$  \(7\)

Here $G_F$ is the Fermi constant, $J$ is the total spin of the target nucleus, $\langle S_{(p,n)} \rangle$ are the expectation values of the proton and neutron group spins, and $a_{(p,n)}$ are the effective SD WIMP couplings on protons and on neutrons. Some relevant spin values of the most used spin–sensitive nuclei are given in Table 1.

For the SD WIMP–nucleus interaction, it is usually assumed that only unpaired nucleons contribute significantly to the total cross section, as the spins of the nucleons in a nucleus are systematically anti-aligned. Under this “odd–group” assumption, the SD WIMP–nucleus cross section can be reduced to

$$\sigma^{SD}_0 = \left( \frac{32}{\pi} \right) G_F^2 m_{\chi,N}^2 \left( \frac{J + 1}{J} \right) \langle S_{(p,n)} \rangle^2 |a_{(p,n)}|^2.$$  \(8\)

And the SD WIMP cross section on protons or on neutrons can be given as

$$\sigma^{SD}_{\chi(p,n)} = \left( \frac{24}{\pi} \right) G_F^2 m_{\chi,(p,n)}^2 |a_{(p,n)}|^2.$$  \(9\)

Due to the coherence effect with the entire nucleus shown in Eq. (5), the cross section for scalar interaction scales approximately as the square of the atomic mass of the target nucleus. Hence, in most supersymmetric models, the SI cross section for nuclei with $A \gtrsim 30$ dominates over the SD one.

2. Reconstructing the one–dimensional velocity distribution function of halo WIMPs

As the first step of the development of these model–independent data analysis procedures, starting with a time–averaged recoil spectrum $dR/dQ$ and assuming that no

\(a\)However, more detailed nuclear spin structure calculations show that the even group of nucleons has sometimes also a non–negligible spin (see Table 1 and e.g., data given in Refs. 1, 7, 8, 9).
directional information exists, the normalized one–dimensional velocity distribution function of incident WIMPs, \( f_1(v) \), is solved from Eq. (1) directly as

\[
f_1(v) = N \left\{ -2Q \cdot \frac{d}{dQ} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] \right\}_{Q=v^2/\alpha^2},
\]

where the normalization constant \( N \) is given by

\[
N = \frac{2}{\alpha} \left\{ \int_{Q_{\min}}^{Q_{\max}} \frac{1}{\sqrt{Q}} \left[ \frac{1}{F^2(Q)} \left( \frac{dR}{dQ} \right) \right] dQ \right\}^{-1}.
\]

Note that the WIMP velocity distribution reconstructed by Eq. (10) is independent of the local WIMP density \( \rho_0 \) as well as of the WIMP–nucleus cross section \( \sigma_0 \).

However, in order to use the expressions (10) and (11) for reconstructing \( f_1(v) \), one needs a functional form for the recoil spectrum \( \frac{dR}{dQ} \). In practice this requires usually a fit to experimental data and data fitting will re–introduce some model dependence and make the error analysis more complicated. Hence, expressions that allow to reconstruct \( f_1(v) \) directly from experimental data (i.e., measured recoil energies) have been developed. Considering experimental data described by

\[
Q_n - \frac{b_n}{2} \leq Q_{n,i} \leq Q_n + \frac{b_n}{2}, \quad i = 1, 2, \ldots, N_n, \quad n = 1, 2, \ldots, B.
\]

Here the total energy range between \( Q_{\min} \) and \( Q_{\max} \) has been divided into \( B \) bins with central points \( Q_n \) and widths \( b_n \). In each bin, \( N_n \) events will be recorded. Since the recoil spectrum \( \frac{dR}{dQ} \) is expected to be approximately exponential, in order to approximate the spectrum in a rather wider range, the following exponential ansatz for the measured recoil spectrum (before normalized by the exposure \( E \)) in the \( n \)th \( Q \)–bin has been introduced:

\[
\left( \frac{dR}{dQ} \right)_{\text{expt},n} \equiv \left( \frac{dR}{dQ} \right)_{\text{expt},Q=Q_n} \equiv r_n e^{k_n(Q-Q_{s,n})},
\]

Here \( r_n = N_n/b_n \) is the standard estimator for \( \left( \frac{dR}{dQ} \right)_{\text{expt}} \) at \( Q = Q_n \), \( k_n \) is the logarithmic slope of the recoil spectrum in the \( n \)th \( Q \)–bin, which can be computed numerically from the average value of the measured recoil energies in this bin:

\[
\frac{Q_{n,i} - Q_n}{N_n} = \frac{1}{N_n} \sum_{i=1}^{N_n} \left( Q_{n,i} - Q_n \right) = \left( \frac{b_n}{2} \right) \coth \left( \frac{k_n b_n}{2} \right) - \frac{1}{k_n}.
\]

Then the shifted point \( Q_{s,n} \) in the ansatz (13), at which the leading systematic error due to the ansatz is minimal, can be estimated by

\[
Q_{s,n} = Q_n + \frac{1}{k_n} \ln \left[ \frac{\sinh(k_n b_n/2)}{k_n b_n/2} \right].
\]

Note that \( Q_{s,n} \) differs from the central point of the \( n \)th bin, \( Q_n \).

Now, substituting the ansatz (13) into Eq. (10) and then letting \( Q = Q_{s,n} \), we can obtain that

\[
f_{1,\text{rec}} \left( v_{s,n} = \alpha \sqrt{Q_{s,n}} \right) = N \left[ \frac{2Q_{s,n} r_n}{F^2(Q_{s,n})} \right] \left[ \frac{d}{dQ} \ln F^2(Q) \right]_{Q=Q_{s,n}} - k_n.
\]
Here the normalization constant $N$ given in Eq. (11) can be estimated directly from the data:

$$N = \frac{2}{\alpha} \left[ \sum a \frac{1}{\sqrt{Q_a F^2(Q_a)}} \right]^{-1},$$

where the sum runs over all events in the sample.

3. Determining the WIMP mass and the SI WIMP–nucleon coupling

By using expressions (10) and (11) for reconstructing the WIMP velocity distribution function, not only the overall normalization constant $N$ given in Eq. (11), but also the shape of the velocity distribution, through the transformation $Q = v^2/\alpha^2$ in Eq. (10), depends on the WIMP mass $m_\chi$ involved in the coefficient $\alpha$. It is thus crucial to develop a method for determining the WIMP mass model–independently.

From Eq. (10) and using the exponential ansatz in Eq. (13), the moments of the normalized one–dimensional WIMP velocity distribution function can be estimated by

$$\langle v^n \rangle = \int_{v(Q_{\text{min}})}^{v(Q_{\text{max}})} v^n f_1(v) \, dv$$

$$= \alpha^n \left[ \frac{2Q_{\text{min}}^{(n+1)/2} r(Q_{\text{min}})/F^2(Q_{\text{min}})}{2Q_{\text{min}}^{1/2} F^2(Q_{\text{min}})} + I_n(Q_{\text{min}}, Q_{\text{max}}) \right].$$

Here $v(Q) = \alpha \sqrt{Q}$, $Q_{(\text{min}, \text{max})}$ are the experimental minimal and maximal cut–off energies,

$$r(Q_{\text{min}}) \equiv \left( \frac{dR}{dQ} \right)_{\text{expt}, Q=Q_{\text{min}}} = r_1 e^{k_1(Q_{\text{min}}-Q_{\text{X,1}})}$$

is an estimated value of the measured recoil spectrum $(dR/dQ)_{\text{expt}}$ (before the normalization by the exposure $\mathcal{E}$) at $Q = Q_{\text{min}}$, and $I_n(Q_{\text{min}}, Q_{\text{max}})$ can be estimated through the sum:

$$I_n(Q_{\text{min}}, Q_{\text{max}}) = \sum a \frac{Q_a^{(n-1)/2}}{F^2(Q_a)},$$

where the sum runs again over all events in the data set. Note that by using Eq. (18) $\langle v^n \rangle$ can be determined independently of the local WIMP density $\rho_0$, of the WIMP–nucleus cross section $\sigma_0$, as well as of the velocity distribution function of incident WIMPs, $f_1(v)$.

By requiring that the values of a given moment of $f_1(v)$ estimated by Eq. (18) from two experiments with different target nuclei, $X$ and $Y$, agree, $m_\chi$, appearing in the prefactor $\alpha^n$ on the right–hand side of Eq. (18) can be solved as:

$$m_\chi |_{\langle v^n \rangle} = \sqrt{\frac{m_X m_Y - m_X (R_{n,X}/R_{n,Y})}{R_{n,X}/R_{n,Y} - \sqrt{m_X/m_Y}}},$$

where the sum runs over all events in the sample.
where

\[
R_{n,X} \equiv \left[ \frac{2Q_{\text{min},X}^{(n+1)/2} r_X(Q_{\text{min},X})/F_X^2(Q_{\text{min},X}) + (n + 1)I_{n,X}}{2Q_{\text{min},X}^{1/2} r_X(Q_{\text{min},X})/F_X^2(Q_{\text{min},X}) + I_{0,X}} \right]^{1/n},
\]

(22)

and \(R_{n,Y}\) can be defined analogously\(^b\). Here \(n \neq 0\), \(m_{(X,Y)}\) and \(F_{(X,Y)}(Q)\) are the masses and the form factors of the nucleus \(X\) and \(Y\), respectively, and \(r_{(X,Y)}(Q_{\text{min}},(X,Y))\) refer to the counting rates for detectors \(X\) and \(Y\) at the respective lowest recoil energies included in the analysis. Note that the general expression (21) can be used either for spin–independent or for spin–dependent scattering, one only needs to choose different form factors under different assumptions.

On the other hand, by using the theoretical prediction that the SI WIMP–nucleus cross section dominates, and the fact that the integral over the one–dimensional WIMP velocity distribution on the right–hand side of Eq. (1) is the minus–first moment of this distribution, which can be estimated by Eq. (18) with \(n = -1\), one can easily find that\(^{12}\)

\[
\rho_0|f_1|^2 = \frac{\pi}{4\sqrt{2}} \left( \frac{1}{\mathcal{E} A^2/\mathcal{M}^2} \right) \left[ 2Q_{\text{min}}^{1/2} r(Q_{\text{min}})/F^2(Q_{\text{min}}) + I_0 \right] (m_X + m_N).
\]

(23)

Note that the exposure of the experiment, \(\mathcal{E}\), appears in the denominator. Since the unknown factor \(\rho_0|f_1|^2\) on the left–hand side above is identical for different targets, it leads to a second expression for determining \(m_\chi\):\(^{12}\)

\[
m_\chi \bigg|_{\sigma} = \frac{(m_X/m_Y)^{5/2} m_Y - m_X (R_{\sigma,X}/R_{\sigma,Y})}{R_{\sigma,X}/R_{\sigma,Y} - (m_X/m_Y)^{5/2}}.
\]

(24)

Here \(m_{(X,Y)} \propto A_{(X,Y)}\) has been assumed and

\[
R_{\sigma,X} \equiv \frac{1}{\mathcal{E}_X} \left[ 2Q_{\text{min},X}^{1/2} r_X(Q_{\text{min},X})/F_X^2(Q_{\text{min},X}) + I_{0,X} \right].
\]

(25)

Remind that the basic requirement of the expressions for determining \(m_\chi\) given in Eqs. (21) and (24) is that, from two experiments with different target nuclei, the values of a given moment of the WIMP velocity distribution estimated by Eq. (18) should agree. This means that the upper cuts on \(f_1(v)\) in two data sets should be (approximately) equal\(^c\). Since \(v_{\text{cut}} = \alpha \sqrt{Q_{\text{max}}}\), it requires that\(^{12}\)

\[
Q_{\text{max},Y} = \left( \frac{\alpha_X}{\alpha_Y} \right)^2 Q_{\text{max},X}.
\]

(26)

Note that \(\alpha\) defined in Eq. (3) is a function of the true WIMP mass. Thus this relation for matching optimal cut–off energies can be used only if \(m_\chi\) is already known. One possibility to overcome this problem is to fix the cut–off energy of the

\(^b\)Hereafter, without special remark all notations defined for the target \(X\) can be defined analogously for the target \(Y\) and eventually for the target \(Z\).

\(^c\)Here the threshold energies of two experiments have been assumed to be negligibly small.
experiment with the heavier target, determine the WIMP mass by either Eq. (21) or Eq. (24), and then estimate the cut–off energy for the lighter nucleus by Eq. (26) algorithmically.

Furthermore, by combining two or three data sets with different target nuclei and making an assumption for the local WIMP density $\rho_0$, we can use Eq. (23) to estimate the squared SI WIMP coupling on protons (nucleons), $|f_p|^2$. It is important to note that $|f_p|^2$ and $m_\chi$ can be estimated separately and from experimental data directly with neither prior knowledge about each other nor about the WIMP velocity distribution.

4. Determining ratios between different WIMP–nucleon cross sections

4.1. Determining the ratio between two SD WIMP couplings

Assuming that the SD WIMP–nucleus interaction dominates and substituting the expression (7) for $\sigma_0^{SD}$ into Eq. (1) for two target nuclei $X$ and $Y$, the ratio between two SD WIMP–nucleon couplings can be solved analytically as

$$\frac{a_n}{a_p}^{SD}_{\pm,n} = \frac{\langle S_p \rangle_X \pm \langle S_p \rangle_Y (R_{J,n,X}/R_{J,n,Y})}{\langle S_n \rangle_X \pm \langle S_n \rangle_Y (R_{J,n,X}/R_{J,n,Y})},$$

for $n \neq 0$. Here I have defined

$$R_{J,n,X} \equiv \left( \frac{J_X}{J_X + 1} \right) \frac{R_{\sigma,X}}{R_{n,X}} \right)^{1/2},$$

with $R_{n,X}$ and $R_{\sigma,X}$ defined in Eqs. (22) and (25).

Note that, firstly, the expression (27) for $a_n/a_p$ is independent of the WIMP mass $m_\chi$ and the ratio can thus be determined from experimental data directly without knowing the WIMP mass. Secondly, because the couplings in Eq. (7) are squared, we have two solutions for $a_n/a_p$ here; if exact “theory” values for $R_{J,n,(X,Y)}$ are taken, these solutions coincide for

$$\frac{a_n}{a_p}^{SD}_{+,n} = \frac{a_n}{a_p}^{SD}_{-,n} = \begin{cases} \frac{-\langle S_p \rangle_X}{\langle S_n \rangle_X}, & \text{for } R_{J,n,X} = 0, \\ \frac{-\langle S_p \rangle_Y}{\langle S_n \rangle_Y}, & \text{for } R_{J,n,Y} = 0, \end{cases}$$

which depend only on the properties of target nuclei (see Table 1). Moreover, it can be found from Eq. (27) that one of these two solutions has a pole at the middle of two coincident values, which depends simply on the signs of $\langle S_n \rangle_X$ and $\langle S_n \rangle_Y$: since $R_{J,n,X}$ and $R_{J,n,Y}$ are always positive, if both of $\langle S_n \rangle_X$ and $\langle S_n \rangle_Y$ are positive or negative, the “$-$” solution $(a_n/a_p)^{SD}_{-,n}$ will diverge and the “$+$” solution $(a_n/a_p)^{SD}_{+,n}$ will be the “inner” solution; in contrast, if the signs of $\langle S_n \rangle_X$ and $\langle S_n \rangle_Y$ are opposite, the “$-$” solution $(a_n/a_p)^{SD}_{-,n}$ will be the “inner” solution.
4.2. Determining the ratio between two WIMP–proton cross sections

Considering a general combination of both the SI and SD cross sections given in Eqs. (5) and (7), we can find that\(^{16,18}\)

\[
\frac{\sigma_{SD}^{SI}}{\sigma_{SI}^{SD}} = \left(\frac{32}{\pi}\right) C_p^2 m_{r,p}^2 \left(\frac{J + 1}{J}\right) \left[\frac{\langle S_p \rangle + \langle S_n \rangle (a_n/a_p)}{A}\right]^2 \frac{a_p^2}{\sigma_{SI}^{SD}} = C_p \left(\frac{\sigma_{SD}^{SI}}{\sigma_{SI}^{XP}}\right), \tag{30}
\]

where \(\sigma_{SI}^{SD}\) given in Eq. (9) has been used and

\[
C_p \equiv \frac{4}{3} \left(\frac{J + 1}{J}\right) \left[\frac{\langle S_p \rangle + \langle S_n \rangle (a_n/a_p)}{A}\right]^2. \tag{31}
\]

Then the expression (1) for the differential event rate should be modified to

\[
\left(\frac{dR}{dQ}\right)_{\text{expt}, \, Q=Q_{\text{min}}} = \mathcal{E} A^2 \left(\frac{\rho_0 \sigma_{SI}^{SI}}{2 m_{X,Y}^2 c_p^2}\right) \left[F_{SI}^2(Q_{\text{min}}) + \left(\frac{\sigma_{SD}^{SI}}{\sigma_{SI}^{XP}}\right) C_p F_{SD}^2(Q_{\text{min}})\right] - C_p X m_{r,p}^2 \int_{v(Q_{\text{max}})}^{v(Q_{\text{min}})} \left[\frac{f_1(v)}{v}\right] dv, \tag{32}
\]

where I have used Eq. (5) again. Now by combining two targets \(X\) and \(Y\) and assuming that the integral over the WIMP velocity distribution function in Eq. (32) estimated by Eq. (18) for each target with suitable experimental maximal and minimal cut-off energies should be (approximately) equal, the ratio of the SD WIMP–proton cross section to the SI one can be solved analytically as\(^{16,17,18}\)

\[
\frac{\sigma_{SD}^{SI}}{\sigma_{SI}^{XP}} = \frac{F_{SI,Y}^2(Q_{\text{min},Y})(R_{m,X}/R_{m,Y}) - F_{SI,X}^2(Q_{\text{min},X})}{C_p X F_{SD,Y}^2(Q_{\text{min},Y})(R_{m,X}/R_{m,Y})}, \tag{33}
\]

where I have assumed \(m_{(X,Y)} \propto A_{(X,Y)}\) and defined

\[
R_{m,X} \equiv \frac{r_X(Q_{\text{min},X})}{\mathcal{E} X m_{X}^2}. \tag{34}
\]

Similarly, the ratio of the SD WIMP–neutron cross section to the SI one can be given analogously\(^4\):

\[
\frac{\sigma_{SD}^{SI}}{\sigma_{SI}^{XP}} = \frac{F_{SI,Y}^2(Q_{\text{min},Y})(R_{m,X}/R_{m,Y}) - F_{SI,X}^2(Q_{\text{min},X})}{C_n X F_{SD,Y}^2(Q_{\text{min},Y})(R_{m,X}/R_{m,Y})}, \tag{35}
\]

with the definition

\[
C_n \equiv \frac{4}{3} \left(\frac{J + 1}{J}\right) \left[\frac{\langle S_p \rangle (a_p/a_n) + \langle S_n \rangle}{A}\right]^2. \tag{36}
\]

Note here that one can use expressions (33) and (35) \textit{without} a prior knowledge of the WIMP mass \(m_X\). Moreover, \(\sigma_{SD}^{SI}/\sigma_{SI}^{XP}\) are functions of only \(R_{m,(X,Y)}\), or, equivalently, the counting rate at the experimental minimal cut-off energies,

\(^4\)Here I assumed that \(\sigma_{SI}^{SI} \approx \sigma_{SI}^{XP}\).
Eqs. (37), (38a), and (38b) are functions of only energy ranges.

On the other hand, for the general combination of the SI and SD WIMP–nucleon cross sections, by introducing a third nucleus with only the SI sensitivity: \( \langle S_p \rangle_Z = \langle S_n \rangle_Z = 0 \), i.e., \( C_{p,Z} = 0 \). The \( a_n/a_p \) ratio can in fact be solved analytically as\(^{16,17,18} \):

\[
\frac{(a_n/a_p)^{\text{SI+SD}}}{J_s} = \frac{(- (c_{p,X} s_{n/p,X} - c_{p,Y} s_{n/p,Y}) \pm \sqrt{c_{p,X} s_{n/p,X} - c_{p,Y} s_{n/p,Y}})}{c_{p,X} s_{n/p,X}^2 - c_{p,Y} s_{n/p,Y}^2}. \tag{37}
\]

Here I have defined

\[
c_{p,X} = \frac{4}{3} \left( \frac{J_X + 1}{J_X} \right) \left[ \frac{\langle S_p \rangle_X}{A_X} \right]^2 \times \left[ F_{\text{SI},X}^2(Q_{\text{min},Z}) \frac{R_{m,Y}^2}{R_{m,Z}} - F_{\text{SI},Y}^2(Q_{\text{min},Y}) \right] F_{\text{SD},X}^2(Q_{\text{min},X}); \tag{38a}
\]

\[
c_{p,Y} = \frac{4}{3} \left( \frac{J_Y + 1}{J_Y} \right) \left[ \frac{\langle S_p \rangle_Y}{A_Y} \right]^2 \times \left[ F_{\text{SI},Y}^2(Q_{\text{min},Z}) \frac{R_{m,X}^2}{R_{m,Z}} - F_{\text{SI},X}^2(Q_{\text{min},X}) \right] F_{\text{SD},Y}^2(Q_{\text{min},Y}); \tag{38b}
\]

and \( s_{n/p,X} = \langle S_n \rangle_X / \langle S_p \rangle_X \). Note that, firstly, \((a_n/a_p)^{\text{SI+SD}}\) and \( c_{p,(X,Y)} \) given in Eqs. (37), (38a), and (38b) are functions of only \( r_{(X,Y,Z)}(Q_{\text{min},(X,Y,Z)}) \), which can be estimated with events in the lowest available energy ranges. Secondly, while the decision of the inner solution of \((a_n/a_p)^{\Sigma,n}\) depends on the signs of \( \langle S_n \rangle_X \) and \( \langle S_n \rangle_Y \), the decision with \((a_n/a_p)^{\text{SI+SD}}\) depends not only on the signs of \( s_{n/p,X} = \langle S_n \rangle_X / \langle S_p \rangle_X \) and \( s_{n/p,Y} = \langle S_n \rangle_Y / \langle S_p \rangle_Y \), but also on the order of the two targets.

Moreover, since in the expression (33) for the ratio of two WIMP–proton cross sections there are four sources contributing statistical uncertainties, i.e., \( C_{p,(X,Y)} \) and \( R_{m,(X,Y)} \), in order to reduce the statistical error, one can choose at first a nucleus with only the SI sensitivity as the second target: \( \langle S_p \rangle_Y = \langle S_n \rangle_Y = 0 \), i.e., \( C_{p,Y} = 0 \). Then the expression in Eq. (33) can be reduced to\(^{18} \):

\[
\frac{\sigma_{\text{SI}}^{\text{SD}}}{\sigma_{\text{SI}}^{\text{PD}}} = \frac{F_{\text{SI},Y}^2(Q_{\text{min},Y})(R_{m,X} / R_{m,Y}) - F_{\text{SI},X}^2(Q_{\text{min},X})}{C_{p,X} F_{\text{SD},X}^2(Q_{\text{min},X})}. \tag{39}
\]

Secondly, one chooses a nucleus with (much) larger proton (or neutron) group spin as the first target: \( \langle S_p \rangle_X \gg \langle S_n \rangle_X \simeq 0 \). Now \( C_{p,X} \) given in Eq. (31) becomes (almost) independent of \( a_n/a_p \):

\[
C_{p,X} \simeq \frac{4}{3} \left( \frac{J_X + 1}{J_X} \right) \left[ \frac{\langle S_p \rangle_X}{A_X} \right]^2. \tag{40}
\]
5. Summary and conclusions

In this article I reviewed the data analysis procedures for extracting properties of WIMP–like Dark Matter particles from direct detection experiments. These methods are model–independent in the sense that neither prior knowledge about the velocity distribution function of halo Dark Matter nor their mass and cross sections on target nucleus is needed. The unique required information is measured recoil energies from experiments with different target materials.

Once two or more experiments observe a few tens recoil events (in each experiment), one could in principle already estimate the mass and the SI coupling on nucleons as well as ratios between different cross sections of Dark Matter particles. All this information (combined eventually results from collider and/or indirect detection experiments) could then allow us to distinguish different candidates for (WIMP–like) Dark Matter particles proposed in different theoretical models and to extend our understanding on particle physics.

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