Rain Calms the Sea - The Impact of Entrained Air

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We propose a mechanism for the damping of short ocean gravity waves during rainstorms associated with the injection of air bubbles by rain drops. The mechanism is proposed as one of the possible explanations that ascribe to rain a calming effect on ocean surface waves. A model is developed that shows how wave attenuation increases with the presence of air bubbles in the upper reaches of the ocean. The model makes predictions of the effective wave dissipation coefficient, as a function of the volumetric ratio of air to water, as well as of the rain rate. The model predicts dissipation rates that are in line with experimental estimates of the effective wave damping rate.

I. INTRODUCTION

Mariners have known, for centuries, that “rain calms the sea.” The sea surface roughness can change quickly, once rain begins [1, 2]. Several mechanisms explaining the effect of rain on the short wind-waves have been proposed. In [3] the authors conjectured that a momentum exchange between the rain drops and the sea takes place upon impact. Rain induces a fluctuating free-surface pressure as well as changes in the boundary shear stresses, and these changes lead to an increase in wave damping affecting, primarily, the high frequency spectral composition of the sea surface. In high wind conditions the authors suggest that these changes in the free boundary condition could lead to damping as well as to an amplification of high frequency wave components, leading to smoother or rougher sea states, respectively. A model for impinging rain as a distributed pressure was proposed and investigated in [4]. Within the irrotational framework they use, the rain pressure distribution modifies the wave dispersion relation, affecting primarily the high frequency wave components. The authors also considered the effect of rain impact angle on the pressure distribution. They found that waves are only damped for vertically-falling rain, but can be amplified or damped otherwise.

Several laboratory experiments have been performed in which the wave amplitude was measured prior and after going through a zone of rain. Linear damping rates [5] were then estimated. In this framework, the crucial parameter to be estimated is \( \nu_e \), the dissipation coefficient in the wave dissipation term (of the form \( \exp[-2\nu_e k^2 t] \)), where \( t \) is time, and \( k \) the wavenumber of the wave. In [5] the dissipation rate was found to be \( 3 \times 10^{-5} \text{m}^2\text{s}^{-1} \), at rain rates of 140 mm h\(^{-1}\) (recalculated in [5]). The experiments in [5] yielded a damping rate due to rain of \( 8.5 \times 10^{-6} \text{m}^2\text{s}^{-1} \), approximately. In [6] experiments were performed using freshwater rain on waves produced in a synthetic salt water tank. It was observed that the rain drops cratered the free surface, created a stratified water column with sharp density gradients, and generated near-surface turbulence. The depth-integrated kinematic viscosity estimate, derived from these experiments, was about ten times larger than the value reported in [5]. However, as pointed out by the authors, there is a complex relation between the depth-integrated kinematic viscosity, based upon a kinetic energy dissipation rate, and wave damping that affects comparative interpretations of their results with the older experiments.

Here we propose an alternative, but by no means exclusive, mechanism for high frequency damping: the entrainment of air bubbles by the impact of rain drops. Air bubble creation due to impinging water drops and rain is well known (see [9] for a review. See also [10]). A (heavy) rain rate of the order of 100 mm h\(^{-1}\) produces a rainwater flux of \( 3 \times 10^{-6} \text{m}^3 \text{m}^{-2} \text{s}^{-1} \). Assuming 4 mm diameter rain drops (volume \( 3 \times 10^{-8} \text{m}^3 \)), this translates into 100 drops per \( 1 \times 10^{-3} \text{m}^2 \) \((i.e., 1 \text{ per } 10 \times 10 \text{ cm}^2 \text{ s}^{-1}) \). In [11] Woolf suggests that under these conditions air injected into the sea can be a few \( \text{cm}^3 \text{ m}^{-2} \text{ s}^{-1} \). In Section II we show how the entrainment of air leads to an increase in the effective viscosity of the upper ocean. We then use results from ocean acoustics (see [12], [13]) to compute the distribution of air bubbles for a given rain rate (Section III). How changes in the effective viscosity affect gravity waves is taken up in Section IV. Conclusions and a brief discussion appear in Section V.

II. THE DYNAMICS - DURING RAINFALL

To determine the impact of rain-induced air bubbles on wave attenuation, we perform homogenization on the
Navier-Stokes equations. We assume that air bubbles are distributed uniformly within the upper ocean. We seek to find the effective viscosity of the fluid averaged over a cell Ω, of size ℓ^4 (sub-wave scale), over which the distributions of the density and viscosity of the combined water and air bubble mixture are statistically stationary at wave scales.

Velocity and position are denoted by \( \mathbf{u} = (u, v, w) \), \( \mathbf{r} = (x, y, z) \), respectively. The free surface is denoted by \( \eta \) and \( z = 0 \) corresponds to the quiescent sea level. Gravity \( g \) points upwards, along increasing \( z \). Velocity is scaled by \( u_0 \), the typical wave phase speed, space by \( \ell \), time by \( u_0/\ell \), density by \( \rho_w \), the density of water, pressure by \( \rho_w u_0^2 \). We then define a Reynolds number \( \alpha = u_0\ell/\nu \), where \( \nu \) is the viscosity of water. Also define a Froude number \( 1/\sqrt{\gamma} = u_0/\sqrt{g\ell} \), and a scale parameter \( \epsilon \ll 1 \) that represents the ratio (of spatial scales) of the microscale to the large scale. The scaling leads to

\[
\alpha \mathbf{u}_\epsilon + \mathbf{a} \mathbf{u} \cdot \nabla \mathbf{u} = -\alpha \gamma \nabla \Pi + \nabla \cdot [D(\mathbf{r})\nabla \mathbf{u}] \quad (1)
\]

\[
\nabla \cdot \mathbf{u} = 0. \quad (2)
\]

Let \( \mathbf{R} = (X, Y, Z) \) be the large-scale space variable, such that \( \nabla = \nabla + \epsilon \nabla \mathbf{R} \), and assume slow time \( \frac{\partial}{\partial T} \approx \delta \frac{\partial}{\partial t} \). Also, assume that \( \Pi(r, R, T) = p - z + \epsilon^2 p_1 + \epsilon^4 p_2 + ... \), \( \mathbf{u} = (u_0 + \epsilon u_1 + \epsilon^2 u_2 + ...) \) and \( \eta = (\eta_0 + \epsilon \eta_1 + \epsilon^2 \eta_2 + ...) \). The orders are \( \mathcal{O}(\epsilon^2 \), \( \delta = \mathcal{O}(\epsilon) \), and \( \gamma = \mathcal{O}(\epsilon^{-1}) \).

In the absence of rain, the dimensional \( D(\mathbf{r}) \) is equal to \( \nu_w \), the kinematic viscosity of water and the dissipative term in the momentum balance is equal to \( \nabla \cdot \nu_w \nabla \mathbf{u} \). However, when rain is present and intense enough, the dissipative term becomes equal to \( \nabla \cdot K \nabla \mathbf{u} \), where \( K \) is an effective viscosity. This change in the momentum balance, it is proposed, comes about by the injection of gas bubbles by rain in the near-surface ocean. Specifically, the gas voids in the ocean are created by the impact of the rain drops on the ocean surface. If the rain is intense enough, the voids can cause a significant change in viscous forces, within the momentum balance. Under the assumption that the subsurface air bubble distribution is spatially homogeneous we can then propose a periodic arrangement of sub-wave cells.

Collecting by orders in \( \epsilon \), the momentum equation is,

- \( \mathcal{O}(\epsilon) \):
  \[
  \nabla \cdot [D(\mathbf{r})\nabla \mathbf{u}] = 0.
  \]

Upon integrating and using periodicity it is clear that \( \mathbf{u}_0 = \mathbf{u}_0(\mathbf{R}, T) \).

- \( \mathcal{O}(\epsilon^2) \):
  \[
  \nabla \cdot [D(\mathbf{r})\nabla \mathbf{u}] + \nabla \cdot [D(\mathbf{r})\nabla \mathbf{R} \mathbf{u}_0] + \mathbf{R} \cdot [D(\mathbf{r})\nabla \mathbf{u}_0] = 0. \quad (3)
  \]

Integrating twice over the cell \( \Omega \), using the divergence theorem, and using periodicity we obtain

\[
0 = -\Omega \nabla \mathbf{R} \mathbf{u}_0 + \int_\Omega \frac{d\mathbf{r}}{D(\mathbf{r})} C_1(\mathbf{R}). \quad (4)
\]

Hence, the tensor

\[
C_1(\mathbf{R}) = \left[ \frac{1}{\Omega} \int_\Omega \frac{d\mathbf{r}}{D(\mathbf{r})} \right]^{-1} \nabla \mathbf{R} \mathbf{u}_0.
\]

Let

\[
K = \left[ \frac{1}{\Omega} \int_\Omega \frac{d\mathbf{r}}{D(\mathbf{r})} \right]^{-1}. \quad (5)
\]

Returning to (3),

\[
\nabla \mathbf{u}_1 = -\nabla \mathbf{R} \mathbf{u}_0 + \frac{K}{D(\mathbf{r})} \nabla \mathbf{R} \mathbf{u}_0, \quad (6)
\]

thus,

\[
\nabla \mathbf{R} \cdot (D \nabla \mathbf{u}_1) = -\nabla \mathbf{R} \cdot (D \nabla \mathbf{R} \mathbf{u}_0) + \nabla \mathbf{R} \cdot (K \nabla \mathbf{R} \mathbf{u}_0). \quad (7)
\]

- \( \mathcal{O}(\epsilon^3) \): the momentum balance reads

\[
\frac{\partial \mathbf{u}_0}{\partial T} + \mathbf{u}_0 \cdot \nabla \mathbf{u}_0 + \mathbf{u}_0 \cdot \nabla \mathbf{u}_1 + \nabla \mathbf{R} p_0 =
\]

\[
+ \nabla \cdot [D(\mathbf{r})\nabla \mathbf{R} \mathbf{u}_1] + \nabla \mathbf{R} \cdot [D(\mathbf{r})\nabla \mathbf{R} \mathbf{u}_0] + \nabla \cdot [D(\mathbf{r})\nabla \mathbf{u}_2] + \mathbf{R} \cdot [D(\mathbf{r})\nabla \mathbf{u}_1]. \quad (8)
\]

Using (7) in the last term in (8) and averaging all quantities over \( \Omega \), leads to

\[
\frac{\partial \mathbf{u}_0}{\partial T} + \mathbf{u}_0 \cdot \nabla \mathbf{R} \mathbf{u}_0 + \nabla \mathbf{R} p_0 = \nabla \mathbf{R} \cdot [K \nabla \mathbf{R} \mathbf{u}_0]. \quad (9)
\]

The homogenized incompressibility condition is

\[
\nabla \mathbf{R} \cdot \mathbf{u}_0 = 0. \quad (10)
\]

The value of \( K \) is equal to \( \nu_w \), prior to a rainstorm, and it becomes the enhanced value

\[
K = \frac{\nu_w}{1 - \Theta (1 - \frac{\nu_w}{\nu_w})}, \quad (11)
\]

computed via (5), where \( \nu_w \) is the kinematic viscosity of water and \( \nu_w \), its counterpart for air. The volumetric ratio of air to water is \( \Theta \). Air is about 100 times less viscous, hence, the viscosity of water with air bubbles produces a larger effective viscosity \( K \), than the viscosity of water. (If the ocean flow is turbulent, the ocean viscosity would be better described by the eddy viscosity, which can be substantially larger than the molecular viscosity of water). Figure 11 describes how the effective dissipation coefficient \( K \) changes, as a function of the volumetric ratio \( \Theta \).

III. AIR ENTRAINMENT DUE TO RAIN

III.1. Rain Distribution

The density of rain drops is assumed to follow the Marshall-Palmer distribution \([14]\). The density \( N_r \) of
rain drops of radius $r$ [mm] per unit volume [number m$^{-3}$ mm$^{-1}$] is

$$N_r(r,R) = N_0 \exp(-\Lambda(R)r),$$

(12)

where $N_0 = 8 \times 10^3$ m$^{-3}$ mm$^{-1}$, $\Lambda(R) := 8.2R^{-0.21}$ [mm$^{-1}$]. where $R$ [mm h$^{-1}$] is the rain rate. Using $N_r$ we can then define the drop rate density (DRD), which describes the rate of falling rain drops of radius $r$ per surface area [number m$^{-2}$ s$^{-1}$ mm$^{-1}$]. Namely,

$$\text{DRD}(r, R) = w_a(r)N_r(r, R),$$

(13)

where $w_a$ is the terminal velocity of drops in the air, approximated as

$$w_a \sim 4.6\sqrt{2r}.$$  

(14)

III.2. Bubble Production

Rain drops generate subsurface air bubbles in the neighborhood of the sea surface (see [13] for a review). Small rain drops produce small air bubbles with a very narrow distribution of radii. Intermediate rain drops, with a radius between 0.55 - 1.1 mm, were shown not to produce air bubbles. [13]. For these the impinging rain drops do not have the kinetic energy necessary to produce the requisite conical crater and jetting of the sea surface that engulfs air. Large rain drops, with radii larger than 1.1 mm, create a crater and a canopy on the sea surface, which by collapsing produces a downward liquid jet at the bottom of the crater, followed by the generation of an air bubble. At high rain rates the larger rain drops are responsible for the bulk of the gas injection, creating bubbles with a varied distribution of radii.

The production of air bubbles by rain can be measured by acoustic means [13]. Oguz and Prosperetti [12] classify air bubble production by falling rain drops in two regimes. Small rain drops, of radii between 0.41 and 0.55 mm, create type I air bubbles. These have a radius of 0.22 mm and are created at the bottom of a conical crater created on the ocean surface by the impinging rain drop.

These air bubbles have a narrow acoustic spectrum with a spectral peak at 14 Hz. The acoustic spectrum for these type I air bubbles was found to be insensitive to the rain rate. (Medwin et al. [14] observed that when rain hits the ocean surface at an angle, owing to strong winds or very steep wave conditions, the acoustic spectrum peak shifts downward and there is a broadening of the spectrum at higher frequencies). The type I air bubbles do not contribute significantly to total submerged gas volume and their contribution will be ignored, in what follows.

Rain drops with a radius greater than 1.1 mm produce type II air resonant bubbles of varying radius. The relationship between the rain drop radius $r$ [mm] and the peak acoustic emission frequency $f_0$ due to trapped air bubbles was empirically determined [15] as

$$f_0 = \frac{160}{8r^3} + 0.6.$$  

(15)

The approximate relation between the near surface air bubble radius $a$ [mm] and the peak acoustic emission is [17]:

$$f_0 \sim 3.25 \times 10^3 / a.$$  

(16)

The relation between the entrained air bubble radius $a(r)$ and the incident rain drop radius $r$, for large rain drops, thus reads

$$a(r) = 3.25 \times 10^3 \left(\frac{160}{8r^3} + 0.6\right)^{-1}, \text{ for } r > 1.1.$$  

(17)

Small rain drops ($r < 0.55$ mm ) almost always produce type I air bubbles. However, this is not the case for the larger rain drops. The distribution (%) of the number of air bubbles produced, as a function of the rain drop radius $r$, was found by Medwin et al., [15], and is depicted in Figure 2.

III.3. Bubble Distribution in the Surface Boundary Layer

We denote with $P_r(r)$ the probability of production of an air bubble by a rain drop of radius $r$ (see Figure 2).
We also define the rate of air bubbles produced by rain drops of radius \( r \) [number m\(^{-2}\) s\(^{-1}\) mm\(^{-1}\)] as

\[
Q_a(r) = (\text{DRD} \times P_r)(r).
\] (18)

The distribution of bubbles in the water is a balance between (i) the production of bubbles by rain \((Q_a)\), (ii) a sink term that captures the loss of air bubbles that burst at the surface, (iii) vertical advection, (iv) attenuation of bubbles by diffusion. See [18] and references therein.

It is assumed that the injection of bubbles occurs at a fixed depth, near the sea surface, that the bubble distribution is vertically homogeneous (which cancels advection), and that diffusion takes place on longer times than the characteristic time of bursting of a bubble (the typical time of diffusion is 100 times longer than the upward motion). Armed with these assumptions the following balance equation

\[
Q_a = w_w N(r, R)
\] (19)
is obtained, where

\[
w_w(a) \sim \frac{2}{9} \frac{g \rho_w (a \times 10^{-3})^2}{\mu_w}
\] (20)
is the terminal velocity of a bubble in water of dynamic viscosity \( \mu_w \) \([\text{m}^2 \text{s}^{-1}]\), and \( N \) is the density of bubbles produced by drops of radius \( r \), per unit volume [number m\(^{-3}\) m\(^{-1}\)]. Non-stationary conditions descriptions are also possible, by including the mixing due to wave turbulence (see [20, 21]).

The desired volumetric ratio of air to water is obtained as an integration of the density of bubbles, times their volume (assuming a spherical bubble), over the distribution of rain drops, \( \text{viz.} \)

\[
\Theta(R) = \int_{1,1}^{2,3} \frac{4\pi}{3} [a(r) \times 10^{-3}]^3 \frac{\text{DRD} \times P_r)(r)}{w_w[a(r)]} dr,
\] (21)

where the limits of integration encompass the volumetric contributions of type II bubbles.

**IV. THE EFFECT OF RAIN ON GRAVITY WAVES**

The derivation of infinitesimal gravity waves dynamics, starting from Navier Stokes, appears in \([9]\), Article 349, hence we will be brief. We revert back to dimensional quantities in what follows. The solution \( u_s \) satisfies the linear version of \([9]\) and \([10]\) and the two stress conditions at the surface. Namely,

\[
K[\partial_X v_0 + \partial_Y u_0] = 0, \quad \text{at} \quad z = 0,
\] (22)

\[
-\partial_Y \phi + \left( g - \frac{\tau}{\rho_w} \partial_X \phi \right) + 2K \partial_Y v_0 = 0, \quad \text{at} \quad z = 0,
\] (23)

\[
\phi = Ae^{kZ+i(kX-\sigma T)} \exp[-2Kk^2T],
\]

\[
\psi = \frac{2Kk^2A}{\sigma} e^{kZ+i(kX-\sigma T)} \exp[-2Kk^2T],
\]

\[
\eta = \frac{kA}{\sigma} e^{i(kX-\sigma T)} \exp[-2Kk^2T],
\] (24)

where \( \phi \) is the velocity potential, \( \tau \) is the surface tension at the atmosphere/water interface and \( \rho_w \) is the density of water. The solution of the system, assuming a vanishing velocity at depth, is

\[
\phi = Ae^{kZ+i(kX-\sigma T)} \exp[-2Kk^2T],
\]

\[
\psi = \frac{2Kk^2A}{\sigma} e^{kZ+i(kX-\sigma T)} \exp[-2Kk^2T],
\]

\[
\eta = \frac{kA}{\sigma} e^{i(kX-\sigma T)} \exp[-2Kk^2T],
\] (24)

where \( \psi \) is the streamfunction and \( \sigma \) the angular freency. The solution \([21]\) represents infinitesimal progressive waves traveling in the \( X \) direction. The dispersion relation for the waves is

\[
\sigma^2 = gk + \tau' k^3,
\]

where \( \tau' \) is the effective surface tension. The effective wave dissipation \( 2K \) appears in the exponential factor \( \exp[-2Kk^2T] \). When the rainstorm is sufficiently intense, the wave dissipation \( 2K \) will increase from \( 2\nu_w \) to a higher value, depending on how much air is injected by the impinging rain drops (see Figure \([1]\)). The dependence of the effective wave dissipation on the volumetric ratio and on the rain rate, via \([11]\), is depicted in Figure \([3]\). (The rain rate range in \([3]\) corresponds to a volumetric range 0-0.4). As can be surmised from \([21]\), increases in the wave dissipation \( 2K \) leads to greater wave attenuation. The attenuation is most prominent in the high frequency components of the wave spectrum due to the dispersive nature of the waves.

**V. DISCUSSION AND CONCLUSIONS**

The mechanism whereby rough seas may be made smoother when exposed to rainfall is not fully understood. Several conjectures have been proposed. These range from distributed pressure fluxes to changes in the shear in the neighborhood of the sea surface, induced
micro-turbulence and fluctuations in buoyancy forces. Here we add another possible mechanism, namely, the injection of air bubbles by rain which then increases gravity wave damping. The model for how this occurs relies on homogenization theory, and on well established theory and observations of the process whereby rain drops inject air into the ocean. The model describes the connection between rain rates, air bubble distribution and wave damping rates. Our analysis does not take into account changes in the structure of the turbulent boundary layer by the presence of the air bubbles. Our model could be extended to incorporate vertical structure under commonly-used adiabatic assumptions for boundary layers. Future improvements on the model would account for the effect of wind, rain impact direction, in addition to bubble distribution heterogeneity.

Experiments using very high rain rates suggest that the effective dissipation can increase 3-10 times when rain is present, as compared to without rain. Our model exhibits increases of about half of that. A distinguishing characteristic of the proposed mechanism is that it can only dampen high frequency waves. Alternative mechanisms that connect rain to fluctuations in the shear and pressure gradients, however, can dampen as well as increase the presence of high frequency ocean waves under some situations. The possibility that several agents are responsible for the phenomenon of calming seas by rain cannot be discounted without further analysis and experiments.

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