Absorption and phase retrieval in phase contrast imaging with non linear Tikhonov regularization

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Abstract. The X-ray phase contrast imaging technique relies on the measurement of the Fresnel diffraction intensity patterns associated to a phase shift induced by the object. The simultaneous recovery of the phase and of the absorption is an ill-posed non linear inverse problem. In this work, we investigate the resolution of this problem with non linear Tikhonov regularization and with the Iterative Gauss Newton method. The algorithm is evaluated using simulated noisy data.

1. Introduction

X-ray microtomography is used in many applications such as bone imaging and material science [1]. Yet, it is difficult to image soft tissues, especially within dense structures. For coherent X-rays, phase contrast can be achieved by letting the beam propagate in the free space after interaction with the object [2, 3] with a drastic increase in sensitivity. The Fresnel intensity maps recorded set the inverse problem to retrieve the phase shift induced by the object [1, 2, 3, 4, 5]. Several linear phase retrieval methods have already been investigated [1, 2, 3, 4, 5] and improved by non linear approaches recently [6]. Yet, all these approaches are based on a precise knowledge of the absorption. The simultaneous absorption and phase retrieval remains a challenge. This work investigates the resolution of the non linear ill-posed inverse problem associated to the simultaneous recovery of the absorption and of the phase. We successively present the direct problem of the image formation, the non-linear Tikhonov regularization of the inverse problem and results obtained from simulations on noisy data.

2. The direct problem of the image formation

Considering a thin object illuminated with coherent X-rays of wavelength $\lambda$, the interaction of X-rays with matter can be described by a transmittance function $T$ of the coordinates $X=(x,y)$ in a plane perpendicular to the propagation direction $z$ of the X-rays:

$$T(X) = \exp[-B(X) + i\varphi(X)] = a(X) \exp[i\varphi(X)]$$

where $B(X)$ is the absorption and $\varphi$ the phase induced by the object. The intensity detected at a distance $D$ is given by the squared modulus of the exit wave:

$$I_D(X) = |T(X) * P_D(X)|^2$$
where $*$ denotes the 2D convolution of the transmittance with the Fresnel propagator:

$$P_D(X) = \frac{1}{i\lambda D} \exp(i \frac{\pi}{\lambda D} |X|^2)$$  \hspace{1cm} (3)

The real and imaginary part of the refractive index can then be reconstructed by recording the intensity at different angles with tomographic reconstruction algorithms, when the phase shift $\varphi(X)$ and the absorption $B(X)$ are known. Most existing phase retrieval approaches are based on a linearization of the direct problem valid under some rather restrictive assumptions [1, 2, 3, 4, 5]. Recently, a non linear iterative approach based on the Fréchet derivative of the intensity has improved the phase retrieval accuracy [6]. Yet, in this approach, the absorption $B(X)$, obtained from the intensity measured for $D=0$, is assumed to be known. When imaging at high resolution, it is very difficult to obtain an absorption image without phase contrast sufficiently close to the detector. And this motivates the study of the joint absorption and phase retrieval algorithms, leading to improved solutions for the phase $\varphi(X)$ and the absorption $B(X)$.

3. The non linear inverse problem of the simultaneous absorption and phase recovery

In order to solve this non linear inverse problem, we have used a Tikhonov regularization method and the Iterative Gauss Newton method (IRGN).

3.1. Non linear Tikhonov regularization

In the following, we will consider that the phase and the absorption have a bounded support $\Omega$ and that the domain of the intensity operator $I_D(B, \varphi)$ belongs to the functional space $H \times H$ with $H = L^2(\Omega) \cap L^\infty(\Omega)$. The real intensity data will be denoted $I$ and the noisy intensity $I_\delta$. These intensities are assumed to be such that $\|I_\delta - I\|_{L^2(\Omega)} \leq \delta$. The aim of the non linear Tikhonov regularization is to minimize the functional $T_\alpha$ of the two components $u=(B, \varphi)$:

$$T_\alpha(u) = \frac{1}{2} \|I_D(u) - I_\delta\|_{L^2(\Omega)}^2 + \frac{\alpha}{2} \|u\|_{L^2(\Omega)}^2$$  \hspace{1cm} (4)

where $\alpha$ is a regularization parameter. A minimizer $u^*_\alpha$ always exists under the assumptions detailed in [7, 8]. If a minimum norm solution $u^*$ exists with $I_D(u^*)=I$, then, under these assumptions, for a sequence $(I_\delta)$, with $\|I_\delta - I\| \leq \delta$, an a priori regularization parameter choice $\alpha \sim \delta$, there exist constants $C$ and $E$ such that $\|u^* - u^*_\alpha\| \leq C\sqrt{\delta}$ and $\|I_D(u^*_\alpha) - I\| \leq E\delta$. It will be shown in a forthcoming paper that $I_D(B, \varphi)$ satisfies all former assumptions.

3.2. The Iterated Gauss-Newton method

We have used the well-known Iterated Gauss-Newton method to minimize the regularization functional given in Eq.4. [9, 10, 11, 12]. Denoting $I_D(u_k)'$ the Fréchet derivative of the intensity at the current point, the discrepancy term in Eq.4 is replaced by its approximation at the current point $u_k$, $T_\alpha(u) = \frac{1}{2} \|I_D(u_k) - I_\delta + I_D(u_k)'(u - u_k)\|_{L^2(\Omega)}^2$ providing a strongly convex quadratic functional. The algorithm has been generalized by the introduction of a line search procedure with a variable step size $\tau_k$ [9, 10, 11, 12]. The update formula becomes:

$$u_{k+1} = u_k - \tau_k (I_D(u_k))' (I_D(u_k) - y) + \alpha (u_k - u_*)$$  \hspace{1cm} (5)

where $F'(u_k)$ is the adjoint of the Fréchet derivative. The step length parameter $\tau_k$ is chosen in order to minimize the Tikhonov’s functional along the descent direction with a backtracking strategy [9, 10, 11, 12]. The descent direction $d_k = (I_D(u_k))'(I_D(u_k) - y) + \alpha (u_k - u_*)$ is obtained with the linear system $A_k d_k = -g_k$ from the symmetric positive definite
The Fréchet derivative of the operator $I(B, \varphi)$ is defined by the relation $I'(B_k, \varphi_k)(\varepsilon_1, \varepsilon_2) = I(B_k + \varepsilon_1, \varphi_k + \varepsilon_2) - I(B_k, \varphi_k) + O(||\varepsilon_1, \varepsilon_2||^2)$

$$I'(B_k, \varphi_k)(\varepsilon_1, \varepsilon_2) = 2\text{Re} \{ [(-\varepsilon_1 - i\varepsilon_2)(\exp(-B_k - i\varphi_k)) * P_D] \{ \exp(-B_k + i\varphi_k) \} \}$$

(6)

The adjoint $I'(B_k, \varphi_k)^*: H \to H \times H$ is then defined by $I'(B_k, \varphi_k)^*(u) = 2\text{Re} \{ v_1, v_2 \}$ with:

$$v_1 = \{ [-u(\exp(-B_k)(\exp(-i\varphi_k) * P_D)] \exp(-B_k) \exp(i\varphi_k) \}$$

(7)

$$v_2 = \{ [u(\exp(-B_k)(\exp(-i\varphi_k) * P_D)] \exp(-B_k) \exp(i\varphi_k) \}$$

(8)

4. Results

4.1. Simulation of the diffracted intensity

Following [3], one phantom was defined with an absorption and a phase map displayed in Figures 1(a) and 1(b). This phantom consists of the projections of two spheres, each of homogeneous composition, the real and complex parts of the refractive index being proportional, with a different proportionality constant in each sphere. The absorption and phase values were discretized on a regular grid of size $N = 75 \times 75$ with a pixel size of 15 $\mu$m. Propagation in free space was simulated using Eq.2 for $D=1.4$ m and $\lambda = 0.5166\text{Å}$. A Gaussian white noise with peak-to-peak signal-to-noise ratio (PPSNR) of 24 dB was added to the image. Our phase and absorption retrieval algorithms are not globally convergent algorithms. An approximate absorption map was first chosen, with increased absorption values and larger spheres. Then, the phase obtained with the linear Mixed approach presented in [3] has been used as the starting point of our simulations. The initial absorption and phase root mean square error are around 45%. This a priori guess of the solution ensures the convergence of the algorithms.

4.2. Results

Figure 2(a) and 2(b) display the normalized mean square error for the phase and the absorption as a function of the number of iterations, $\|\varphi^+ - \varphi\|_{L^2(\Omega)}/\|\varphi^+\|_{L^2(\Omega)}$ and $\|B^+ - B\|_{L^2(\Omega)}/\|B^+\|_{L^2(\Omega)}$. The final phase and absorption error maps are displayed in Figures 1(c) and 1(d) respectively. Some errors are still present and a local minimum is obtained but the errors on the phase and absorption have been significantly reduced with this algorithm. We have no proof that the source condition is satisfied for the absorption and phase map considered here. Yet, our results demonstrate the efficiency of the proposed algorithm.

5. Conclusion

In this work, a non-linear iterative approach for the simultaneous phase and absorption retrieval has been proposed. This problem is ill-posed and thus Tikhonov regularization and the iterative Gauss Newton method are included to obtain stable solutions. The method was tested with simulated data in the presence of noise. Significant decrease of the phase and absorption root-mean-square errors are obtained. This method will be tested on experimental data and can be applied in various fields, such as biomedical science and material science.
Figure 1. (a) Original absorption map, (b) Original phase map, (c) Absorption error map and (d) Phase error map at convergence.

Figure 2. (a) Evolution of the phase RMSE with the iterations and (b) Evolution of the absorption RMSE with the iterations.

References

[1] Cloetens P, Barrett R, Baruchel J, Guigay J P, Schlenker M J. Phys. D 1996 13 133
[2] Paganin D M 2006 Coherent X ray optics (Oxford University Press, New York)
[3] Langer M, Cloetens P, Guigay J P, Peyrin F 2008 Med. Phys 35 4556
[4] Gureyev T E, Nugent K A J. Opt. Soc. Am. A 1996 A 13 1670
[5] Wilkins S W , Gureyev T E, Gao D, Fogany A, Stevenson A W Phys. Rev. Lett 1996 77 2961
[6] Davidou B, Sixou B, Langer M and Peyrin F 2011 Opt. Express 19 22809
[7] Dicken V 1999 Inverse Problems 15 931
[8] Engl H, Kunish K, Neubauer A 1989 Inverse Problems 5 523
[9] Smirnoval A, Renaut A , Khan T 2007 Inverse Problems 23 1547
[10] Bakushinsky A B 1992 Comput. Math. Math. Phys 32 1353
[11] Blaschke B, Neubauer A, Scherzer O 1999 IMA, Journal of Numerical Analysis 17 1997
[12] Bakushinsky A B, Smirnova A 2005 Numer. Funct. Anal. Optim 26 35