Effects of fluid inertia and bearing flexibility on the performance of finite length journal bearing

Juliana Javorova¹ and Vassil Alexandrov²

¹Department of Applied Mechanics, University of Chemical Technology and Metallurgy, 8 Kliment Ohridski Boulevard, 1756 Sofia, Bulgaria
²Department of Mechanics, University of Transport, 158 Geo Milev Street, 1574 Sofia, Bulgaria

E-mail: july@uctm.edu

Abstract. The paper describes the theoretical study concerning the effect of lubricant inertia forces and deformability of the bearing elastic layer on the performance of a plane journal bearing. The problem is investigated for a Newtonian lubricant under isothermal and isoviscous conditions. The analysis considers the generalized Reynolds equation governing the flow of lubricant in the clearance space and the linear elasticity equation governing the displacement field in the bearing shell. An iterative numerical procedure with successive over relaxation is used to pressure distribution within the lubricated conjunction. Bearing performance characteristics have been presented for typically selected values of generalized Reynolds number Re* and elasticity parameters of the elastic liner. It has been observed that the combined effect of fluid inertia forces and bearing flexibility affects the performance characteristics of dynamically loaded journal bearing.

1. Introduction

It is well known that fluid inertia effect is of importance in modern bearing and seal design. This effect is most notable due to the prevailing trend towards light and compact turbomachinery operating at higher speeds, and the use of process liquids (low viscosity) and gases in fluid film bearings. Important examples include water and lubricant mixtures, liquid metals in the nuclear industry, and cryogenic fluids in space turbopumps.

The lubrication and performance characteristics of HD journal bearings are affected by the including of the lubricants inertia forces at the analysis. Some investigators have studied the performance characteristics of HD journal bearings taking into consideration the inertia effects. Among the few studies related to the fluid inertia effect must be mentioned the works of Reinhardt and Lund [1], Banerjee et al. [2], Chen and Chen [3], Kakoty and Majumdar [4, 5], Javorova et al. [6, 7], Bou-Said and Ehret [8], Prasad et al. [9], etc.

Along with that most of research on the design of HD journal bearings relies on the assumption that both the journal and bearing are rigid bodies. In some applications under high loading conditions of the bearings and/or when using bearings with layers on the contact surfaces, distortions of these contact surfaces are significant and cannot be ignored. There is increasing trend for using polymers and similar as bearings materials and layers in different applications, due to their good tribological properties [10].
The effect of elastic deformation of the bearing surface on the static and dynamic performance of HD journal bearings was studied by Fillon and co-authors [11, 12, 13], Osman [10], Elsharkawy [14], Javorova et al. [15], Ma [16], Attia et al. [17]. The major factors affecting the HD journal bearing are thermal and elastic deformation, which cause a reduction in pressure generated in the oil film. Their design requires careful attention because consideration has to be given to the steady load-carrying characteristics and also to the oil film dynamic performance. Furthermore, along last two decades many investigations are also focused on non-Newtonian, thermal, surface roughness effects, etc. as other important factors influenced on HD lubrication and performance characteristics of the bearings.

The present work is aimed at studying the effect of interaction of the lubricant inertia forces and elastic deformation of the bearing liner on the performance characteristics of HD journal bearings. The problem is investigated under isothermal and isoviscous conditions for a Newtonian lubricant. The bearing bush is covered with a thin resilient layer (figure 1), whose radial displacements are of the same order of magnitude as the film thickness. To take into account the inertia effects, the modified Reynolds equation for dynamic loading governing the film pressure is derived by averaged acceleration method using the Navier-Stokes equations. A finite difference scheme is implemented to solve the pressure distribution within the lubricated conjunction. The presented solution demonstrates the squeeze film effect, as is assumed vibration velocity \( \omega \) of the shaft centre in direction of the centres line.

2. Analysis

The coordinate system and the configuration of journal bearing are shown on a figure 1. It is assumed that the journal and bearing are circular and their surfaces are smooth, the load is applied in vertical direction, the groove is filled with a lubricant of a constant pressure, and the journal rotates with a constant angular velocity \( \omega \) about its axis. An elastic liner with elastic properties \( \mu \) and \( E \) is press-fitted in a rigid housing. The liner thickness \( d \) is assumed to be of the similar order of magnitude as the lubricant thickness. In the present analysis the following assumptions are also considered: the layers material is homogenous and isotropic, the variation of the pressure across the layer and the fluid film is negligible, and the thermal effects are not considered.

2.1. Modified Reynolds equation

For an incompressible viscose lubricant flow under assumption that a fluid film is thin compared with the journal radius and because of which neglecting the film’s curvature, the field equations (1) governing the motion of the lubricant given in Cartesian coordinates can be reduced to the following form

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}
\]  

(1a)
\begin{equation}
0 = \frac{\partial p}{\partial y}
\end{equation}
(1b)

\begin{equation}
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \frac{\partial^2 w}{\partial y^2}
\end{equation}
(1c)

\begin{equation}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\end{equation}
(1d)

The boundary conditions at the bearing surface are:
\begin{equation}
u(x, 0, z, t) = u(x, 0, z, t) = 0
\end{equation}
(2)

The boundary conditions at the journal-fluid film interface are
\begin{equation}
u(x, h, z, t) = v_n = \omega r \frac{\partial h}{\partial x} + \dot{\varepsilon} \cos \theta + e \gamma \sin \theta
\end{equation}
\begin{equation}
w(x, h, z, t) = w_n = 0
\end{equation}
(3)

Integrating equations (1a) and (1c) by applying the above related boundary conditions, the velocity components can be derived as follows:
\begin{equation}
u(x, y, z) = \frac{1}{2} F_z (y - h) y + \frac{u_n}{h} y \quad \nu(x, y, z) = \frac{v_n}{h} y \quad w(x, y, z) = \frac{1}{2} F_0 (y - h) y
\end{equation}
(4)

Here
\begin{equation}F_z (x, z, t) = \frac{1}{\eta} \frac{\partial p}{\partial x} + \frac{1}{\nu} a_{x, av} \quad F_0 (x, z, t) = \frac{1}{\eta} \frac{\partial p}{\partial z} + \frac{1}{\nu} a_{z, av}
\end{equation}
(5)

\begin{math}a_{x, av} = \frac{1}{h} \int \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} \right) dy \quad a_{z, av} = \frac{1}{h} \int \left( \frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} \right) dy\end{math}
(6)

Integrating the continuity equation (1d) with respect to \(y\) using boundary conditions (2) and (3), a modified Reynolds equation can be derived in a form [7, 18]:
\begin{equation}
\frac{\partial}{\partial x} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{\eta} \frac{\partial p}{\partial z} \right) = 6 \omega r \frac{\partial h}{\partial x} + 12 \frac{\partial h}{\partial t} - \frac{1}{\nu} \left[ \frac{\partial (h^3 a_{x, av})}{\partial x} + \frac{\partial (h^3 a_{z, av})}{\partial z} \right]
\end{equation}
(7)

At substitution of the velocity components (4) in the averaged accelerations expresses (6) and introduction of the non-dimensional variables, the dimensionless form of (7) can be obtained. From it, after serious transformations and by neglecting the higher-order terms, the modified Reynolds equation takes the final dimensionless form: / Detailed derivation of this equation is presented in [6]./
where \( H = h / c \); \( \Pi = p.(c / r)^2 / 6 \omega \); \( \theta = x / r \); \( z_i = z / (L / 2) \); \( \alpha = 2r / L \); \( \beta = c / r \); \( \varepsilon = e / c \); \( \tau = t(\omega / 2) \); \( \Re^* = \rho \omega / \nu \); \( \Re^* = \beta \Re^* \); \( \varepsilon_i = \varepsilon / \omega \); \( \varepsilon_i = \varepsilon_i / \omega \); \( \bar{u}_h = \bar{u}_h + \varepsilon_i \sin \theta - \varepsilon_i \cos \theta \); \( \bar{v}_h = (\partial H / \partial \theta) + \varepsilon_i \cos \theta + \varepsilon_i \sin \theta \); \( \dot{H} = \partial H / \partial \tau \). All of the terms of above equation (8), in which the generalized Reynolds number \( \Re^* \) participates as a coefficient, take into account the contribution of lubricants inertia forces to the hydrodynamic pressure gradient.

2.2. Oil-film thickness

The approach used in the present study aims to superimpose the deformation of the layer on the bearing bush (the other components of the bearing and the journal will be treated as rigid), caused by hydrodynamic pressure generated onto the oil film thickness. The gap thickness is then modified to account for the estimated elastic deformation as follows:

\[
h = c + e \cos \theta + \delta
\]

The last term of this equation takes into account the influence of the elastic layer deformation. In current paper the liner’s surface points radial displacements are determined according to plain strain hypothesis (column or Winkler model) [13, 14, etc.]

\[
\delta = \frac{(1 + \mu)(1 - 2\mu)}{(1 - \mu)} \frac{d}{E} p
\]

Here must be remarked that the column or the Winkler model is applicable when lining is thin compared to the dimensions of the bearing [12, 13, etc.]. The applicability of this formula for mechanical deformations is also verified by comparison with the results obtained using a full deformation model.

2.3. Bearing characteristics

Once the film pressure distribution is known, the hydrodynamic film forces acting on the system can be determined by integrating the film pressure over the journal surface. Expressing in terms of dimensionless quantities give

\[
\bar{W}_1 = -\int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \Pi \cos \theta d\theta dz_i \quad \bar{W}_2 = \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} \Pi \sin \theta d\theta dz_i
\]

where \( \bar{W}_1 \) and \( \bar{W}_2 \) are the components along and perpendicular to the line of centers, respectively.

Then the resultant dimensionless load carrying capacity \( \bar{W} \) can be calculated by

\[
\bar{W} = \sqrt{\bar{W}_1^2 + \bar{W}_2^2} = \frac{\beta^2}{6\eta\omega L} W
\]

The Sommerfeld number \( S \) is defined as:

\[
S = \frac{1}{6\eta\omega L} W
\]
Integrating the shear stress $\tau_{xy}$ around the journal surface, the dimensionless friction force acting on the journal can be derived, subsequently the friction factor can be calculated and written as

$$F = \int \int_{-1}^{1} \frac{\partial \bar{u}}{\partial y} \, d\theta dz \quad C_f = \frac{F}{W}$$  \hspace{1cm} (14)$$

3. Simulations procedure and results

3.1. General remarks

The solution of the problem for simultaneous influence of lubricants inertia forces and bearing’s layer elastic deformations on the performance characteristics of finite journal bearings is done numerically by original program system. The system consists of two basic sections. First of them is concerned with the EHD part of the problem (elastic deformations of bearing liner) and second to the performance characteristics calculation. The pressure distribution, the film shape within the lubricant film region, and elastic deformations distribution are the outputs of the numerical solution, which is done by using a program code FULLINER. Then with program module PERFORMANCES the performance parameters such as the load carrying capacity, Sommerfeld number, attitude angle, side leakage flow and friction factor can be calculated. This program possesses convenient interface, as the output results are arranged in a suitable structure to integration in other kind of studies (for example, stability tests).

3.2. EHD solution – Algorithm procedure

Considering the Reynolds boundary conditions, it is clear that the hydrodynamic pressure is symmetrical in axial direction about the middle plane of the bearing. In the circumferential direction it is assumed that the positive pressure terminates where the pressure gradient angle is zero, as it represents the Reynolds boundary conditions for film pressure.

An iterative numerical procedure by applying the finite difference method is used for solving the problem. The successive over-relaxation is adopted in order to improve the convergence rate [4, 5, etc.]. Negative pressure values are immediately put to zero to be satisfying the Reynolds boundary conditions. The film shape is estimated as being due to the eccentricity of the journal with respect to the bearing, in addition to the elastic deformation of the liner on the bearing bush as result of the pressure generated in the oil film. The following steps are performed [10, 7, etc.]: a) The value of the oil film thickness all over the bearing is estimated as it is assumed an absence of deformations on the bearing layer; b) For the estimated film thickness, equation (8) can be solved numerically to obtain the generated pressure in oil film; c) For the generated pressure, the elastic deformation can be calculated in a direction normal to the bearing surfaces using the equation (10); d) The film shape, equation (9), is then modified to account for the estimated elastic deformation; e) Steps (b) to (d) should be repeated until convergence of solution is obtained.

The film domain is divided by the grid spacing. Different mesh sizes have been tried and a mesh with 72 intervals in the circumferential direction and 20 intervals across the bearing width is used. This size gives a rapid rate of convergence and agreeable computer working time. It was observed that increasing of the number of grid points almost does not affect the results. The convergence criterion adopted for pressure is \(\left|1 - \sum \Pi_{new}/\sum \Pi_{old}\right| \leq 1.10^{-7}\).

3.3. Results and discussions

The present analysis showed that the effect of lubricants inertia forces can be presented by generalized Reynolds number $Re^*$, whereas the effect of deformability of the bearing’s layer - by parameters
\( \mu \) and \( E \). Then, considering the mathematical model, the governing parameters are eccentricity ratio \( \varepsilon \), diameter to length ratio \( \alpha \), \( \text{Re}^{*} \) and elastic layer parameters \( \mu, E \). The results are obtained for \( \alpha \) equal to 0.5; 1.0; 2.0; \( \varepsilon \) was varied from 0.1 to 0.9. To establish the validity of the solution algorithm and the computer code employed in the current study, comparison between the results of the present numerical solution and the available theoretical results from previous studies were conducted. For the purpose of the comparison the generalized Reynolds number is set to 0; 0.56; 1.4 at different elasticity parameters: \( E = 2 \times 10^{11} \) [Pa], \( \mu = 0.25 \) (rigid case); \( E = 7.33 \times 10^{7} \) [Pa], \( \mu = 0.4 \) (soft case).

The effect of fluid inertia and surfaces deformability on the pressure profile is presented on figures 2 and 3. Some examples are plotted: considering fluid inertia (\( \text{Re}^{*}=1.4 \)) and deformability (soft case), as well as neglecting acceleration terms (\( \text{Re}^{*}=0 \)) and deformability (rigid case). It has been observed that the maximum pressure values increase with the values of the generalized Reynolds

**Figure 2.** Distribution of pressure \( \Pi \) with/without inertia terms.

**Figure 3.** Time variation of \( \Pi \) with radial velocity.

**Figure 4.** Friction factor versus eccentricity ratio.

**Figure 5.** Variation of \( S \) with \( \text{Re}^{*} \).
number but reduce at larger deformability values. When the generalized Reynolds number approaches zero, the pressure distribution represents the pure EHD lubrication case (elastic deformations of bearing liner) and if deformations are ignored the lubrication is hydrodynamically.

The influence of inertia terms is shown on a figure 4, where the friction factor is displayed as a function of eccentricity ratio $\varepsilon$ for different values of Re* (soft case). As seen in this figure, fluid inertia yields reduced values of $C_f$, such the effect of inertia terms is significantly apparent especially at low eccentricities. The dependence of Sommerfeld number on the generalized Reynolds number for rigid and soft cases is given on a figure 5. It is found that the value of load-carrying capacity coefficient decreases with reducing of Re* and the effect is more pronounced for the so called “rigid case”.

A comparison between the variation of dimensionless load capacity with eccentricity ratio $\varepsilon$ at $\alpha = 1.0$, for solutions with consideration of fluid inertia and without inertia terms respectively is presented in table 1.

| Eccentricity ratio $\varepsilon$ | Dimensionless load-carrying capacity $\bar{W}$ |
|---------------------------------|------------------------------------------|
|                                 | Re* = 0 | Re* = 0.56 | Re* = 1.4 |
| 0.1                            | 0.2980  | 0.3232     | 0.3448    |
| 0.2                            | 0.3965  | 0.4268     | 0.4528    |
| 0.3                            | 0.5427  | 0.5804     | 0.6128    |
| 0.4                            | 0.7703  | 0.8191     | 0.8610    |
| 0.5                            | 1.1498  | 1.2154     | 1.2718    |
| 0.6                            | 1.8467  | 1.9390     | 2.0183    |
| 0.7                            | 3.3267  | 3.4656     | 3.5850    |
| 0.8                            | 7.3517  | 7.5846     | 7.7850    |
| 0.9                            | 16.2333 | 16.7441    | 17.1833   |

It is found a good agreement with results, reported in [4, 3] although some theoretical treatments are different. Values which are calculated here are slightly lower, particularly at smaller eccentricities. However, the obtained results for the so called “inertia solution” (Re* = 0.56; 1.4) seem to be higher than these for “inertia-free solution” for the whole range of eccentricity ratio. Such a trend has been indicated in the literature by various investigators [3, 4, 18, etc].

4. Conclusions
The research numerically analyzes the performance of HD journal bearings in consideration of lubricant inertia forces and deformability of the elastic bearing layer. In order to render an account the inertia effect the modified Reynolds equation is solved simultaneously with elasticity equation, using FDM. On the base of HD pressure and film thickness values the load carrying capacity, Sommerfeld number and friction coefficient are calculated.

According to the obtained results, the conclusions can be described as follows. The fluid inertia effect is not so much significant as a whole, but the maximum values of the HD pressure increase. As the value of the generalized Reynolds number Re* approaches zero, the problem reduces to the classical EHD lubrication case. Concerning to the Sommerfeld number, a tendency to increasing is evident, but friction factor is reduced under “inertia solutions”. By the other hand the pressure and load carrying capacity coefficient increase upon decreasing of deformability of the bearing surfaces. The lubricants inertia forces and elastic deformations of the bearing surfaces must be taken into consideration at the solution of similar kind of problems, because this change of HD pressure values affects the bearing performance characteristics and respectively the stability of rotor-bearing system.
Acknowledgments
The authors would like to thank for the support provided by Research and Development Sectors at University of Chemical Technology and Metallurgy - Sofia and University of Transport - Sofia.

References
[1] Reinhardt E and Lund J 1975 The influence of fluid inertia on the dynamic properties of journal bearings J. Lubr. Technol. 97 159-167
[2] Benerjee M, Shandil R, Katjal S, Dube G, Pal T and Benerjee K 1986 A nonlinear theory of hydrodynamic lubrication J. Math. Analysis Appl. 117 48-56
[3] Chen C H and Chen C K 1989 The influence of fluid inertia on the operating characteristics of finite journal bearings Wear 131 229-240
[4] Kakoty S K and Majumdar B C 1997 The influence of fluid inertia on the steady-state characteristics and stability of journal bearings Proc. 9th Nat. Conf. on Machines and Mechanism, India B15-B26
[5] Kakoty S K and Majumdar B C 2000 Effect of fluid inertia on the dynamic coefficients and stability of journal bearings Proc. Inst. Mech. Engrs. 214-J 229-242
[6] Javorova J G 2005 EHD lubrication of journal bearings with consideration of fluid inertia Proc. 10th Congress on Th. and Appl. Mechanics Bulgaria 1 51-57
[7] Javorova J G, Alexandrov V A, Stanulov K G and Tzvetkov T 2005 Journal bearings stability with consideration of fluid inertia Proc. World Tribology Congress III Washington D.C.
[8] Bou-Said B and Ehret P 1994 Inertia and shear-thinning effects on bearing behavior with impulsive loads ASME J. of Trib. 116 535-540
[9] Prasad E S, Nagaraju T and Sagar J P 2012 Thermohydrodynamic performance of a journal bearing with 3D-surface roughness and fluid inertia effects Int. J. of Appl. Research in Mech. Eng. (IJARME) 2 1 18-24
[10] Osman T A 2004 Effect of lubricant non-Newtonian behaviour and elastic deformation on the dynamic performance of finite journal plastic bearings Tribol. Lett. 17 1 31-40
[11] Monmousseau P and Fillon M 2000 Transient thermoelastohydrodynamic analysis for safe operating conditions of tilting-pad journal bearing during start-up Tribol. Intern. 33 3 225-231
[12] Glavatskih S and Fillon M 2006 TEHD analysis of thrust bearings with PTFE-faced pads ASME J. of Tribol. 128 49-58
[13] Kuznetsov E, Glavatskih S and Fillon M 2011 THD analysis of compliant journal bearings considering liner deformation Tribol. Intern. 44 1629-41
[14] Elsharkawy A 2005 Effects of lubricant additives on the performance of hydrodynamically lubricated journal bearings Tribol.Lett. 18 1 63-73
[15] Javorova J G, Alexandrov V A and Stanulov K G 2009 Static and dynamic performance of EHD journal bearings in turbulent flow Proc. Int. Conf. “Power Transmissions ’09” Greece 347-350
[16] Ma Y 2008 Performance of dynamically loaded journal bearings lubricated with couple stress fluids considering the elasticity of the liner J. Zhejiang Univ. Sci A 9 7 916-921
[17] Attia H, Bouaziz S, Maatbar M, Fakhfakh T and Haddar M 2010 Hydrodynamic and elasto-hydrodynamic studies of a cylindrical journal bearing J. of Hydrodynamics 22 2 155-163
[18] Pincus O and Sternlicht L 1961 Theory of Hydrodynamic Lubrication (New York: Mc Grow-Hill)