Fostering Problem-based Learning Competence through Teaching the Generalization of Practical Problems on the Topic of Exponential and Logarithmic Functions

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Abstract Problem-based learning competence is one of the most important competencies of mathematical competence. Many researchers believe that this capacity is a factor that directly affects the development of students' competencies and qualities. Among the components of problem-based learning competencies, the generalization of the problem is one of the key factors. Especially generalization of problems about exponential functions and logarithmic functions is still an open issue, which is rarely interested in research. Our research is carried out to identify and analyze factors of problem-based learning competence through generalizing the actual problems on the topic of exponential and logarithmic functions in Vietnam. The questionnaire survey method with 47 high school teachers in Ho Chi Minh City, Vietnam, shows that 10.64% of teachers think that the teaching method of problem-based learning competence is of little importance; 25.53% think it is important and 63.83% think it is very important. We conducted experiments with 89 students and processed analytical data using SPSS software. From the results of qualitative and quantitative analysis, students showed interest and fascination with the generalized teaching method of the actual problem. The test results showed that the experimental class had a much higher score than the control class. The research also provides a table comparing the differences between problem-solving and discovery teaching with teaching to develop problem-based learning competence; the process of teaching generalizations as well as examples illustrating practical problems on the topic of exponents and logarithmic functions. Research results also show that students are excited, proactively discover and learn from real-life problems more than before and do more practical issues. The findings from the research have important implications in fostering problem-based learning competence through generalizing practical problems on the topic of exponents and logarithms for students.

Keywords Teaching Problem-based Learning Competence, Practical Math, Exponential Functions, Logarithmic Functions

1. Introduction

In the world, there are many pieces of research on competency as well as teaching towards capacity development, especially problem-solving capacity in mathematics and other subjects.

Deborah (2009) in the Doctoral Dissertation “A problem-posing intervention in the development of
Competency is a concept that many people are interested in researching. This is a concept with many different definitions, inconsistent with each other, but extremely important in life. Researchers have outlined the definition of competency from their perspectives, below are some definitions of competency from different perspectives of researchers:

Competency is the individual's ability to work whose performance parameters meet the requirements of the business, or organization. Because of this good response, the business or organization wants to hire him or her. It is a combination of available or learned abilities, skills as well as students' readiness to solve emerging problems and act responsibly, critically to come to solutions. Competence is the ability to effectively respond to
complex requirements in a particular context [9]. Competence is the ability to perform work based on mobilizing and using a combination of resources to effectively solve life situations, thereby reaping progress and success in life. [10]. Competency is the combination of a series of knowledge, abilities, skills, experiences, and behaviors that results in effective personal activities. Competency is measurable and can be developed through training. It can also be subdivided into smaller criteria [11].

In Vietnam, there are many authors with different points of view when studying the concept of competency, especially in the field of education. Competency is a characteristic of an individual demonstrating a degree of proficiency, i.e., being able to perform skillfully and with certainty, one or several types of activities [12]. Competency is a psychological and physiological quality that enables people to complete a certain type of activity with high quality [13]. Competency is a combination of psychological characteristics of a person (also known as a combination of psychological attributes of a personality). This feature combination operates for a certain purpose to produce the result of a certain activity [14]. Competency is a personal attribute that allows an individual to successfully perform certain activities and achieve the desired results under specific conditions [15]. Competency is a personal attribute formed and developed by existing qualities and the process of learning and training allows people to successfully perform a certain type of activity and achieve desired results under specific conditions [16]. Competence is the mobilization of knowledge, skills and other personal attributes such as interest, belief, will ... to perform a type of work in a certain context [17]. Competency is a personal attribute formed and developed by existing qualities and the process of learning and training allows people to mobilize a combination of knowledge, skills, and other personal attributes such as interest, belief, will, ... successfully perform a certain type of activity and achieve desired results under specific conditions [18].

From the above things, we recognize that the definitions of competence refer to the ability to successfully perform activities, the knowledge, skills and personal attributes. Some definitions do not refer to the certain context.

Based on the views of many authors given above, we conceive as follows: “Competence is the ability to successfully perform activities in a certain context thanks to the mobilization and synthesis of knowledge, skills, and other personal attributes such as excitement, belief, will ... Individual competence is assessed by the method and ability of that individual to deal with life problems.”

2.2. Mathematical Competence

Mathematical competence has been researched by scientists since the beginning of the twentieth century and one of the initiators of this concept is the French mathematician H. Poincaré. This mathematician believes that mathematical competency has a specificity (thinking and reasoning, posing and solving problem, etc) and that the most important component is mathematical intuition. According to psychologists, mathematical competencies have three forms: arithmetic competency, algebraic competency, and geometric competency. In the last century, A.E. Camenron, L.V. Commerel, H. Thomas, E.L. Thorndike, V. Haecker, and Th. Ziehen was interested in doing research and tried to come up with the composition of the mathematical competency. The most remarkable and still receiving much attention is the work of psychologist E.L. Thorndike. Besides, many psychologists and mathematicians published many studies on mathematical competencies. [19]

The concept of mathematical competency has been studied for a long time. However, the 2002 article on Mathematical Competencies and the Learning of Mathematics: The Danish KOM Project by Mogens Niss marked a strong development of research on this topic. Since that time, Mogens Niss has been considered by many people in the education field as an expert in mathematical competency. Also according to Mogens Niss, “Mathematical competence means the ability to understand, judge, do and use mathematics in many mathematical contexts and situations inside and outside mathematics”. A prerequisite for mathematical competency is a lot of practical knowledge and technical skills, which are necessary, but certainly not enough, just as vocabulary, spelling, and grammar are necessary but not sufficient prerequisites for literacy. He was also the first to elaborate on the eight elements of mathematical competency, namely, mathematical thinking, mathematical reasoning, problem discovery and problem-solving, mathematical communication, mathematical representation, modeling mathematically, using mathematical symbols and formulas, and using aids and tools (IT included). Mogens Niss said that eight types of mathematical competencies form two groups. The first group of competencies is to ask and answer mathematical questions including mathematical thinking, problem discovery and problem-solving, mathematical modeling (analysis and model building) as well as mathematical reasoning. The second group of mathematical competencies is the processing and management of languages and mathematical tools, including mathematical representation, using mathematical formulas and symbols, communicating mathematics, using tools and support facilities (including information technology) [20]. Mathematical competence is the ability to be ready to act to adapt to mathematical challenges in a defined context or situation [21].

It can be said that mathematical competency is not the ability to learn and master mathematical knowledge, but it
aims at students’ understanding, application, and development of skills, analytical, reasoning, and reasoning abilities, generalizing and discovering mathematical knowledge hidden within certain situations and events. Both competency and mathematical competency are concepts that are of special interest in the present period. Applying mathematics to practical situations is one of the key factors in capacity development-oriented teaching. Students will find that Mathematics is life-bound and truly beneficial.

2.3. Problem-based Learning Competence

Problem-based learning competence is one of the important core competencies needed by learners; It is also a concept with many different interpretations and representations. Problem-based learning competence is the ability of an individual to recognize, understand, and resolve problem situations without a clear resolution. There, the individual is willing to engage in similar situations to achieve his or her potential as an active and constructive citizen [22]. Problem-solving capacity is a flexible and organized combination of individual knowledge, skills, attitudes, emotions, ... to meet complex requirements in a defined context [23]. Mathematical problem-solving capacity is defined as the ability to solve real-world problems and transform problem-solving solutions through perception and technology, and the ability to solve problems using how to apply cognitive skills such as reasoning and logical thinking. Problem-solving is done in two phases, problem representation and problem-solving. [24]

In Vietnam, there are also many authors researching problem-based learning competence. Problem-solving capacity is the ability to effectively use an individual's cognitive processes, motivations, and emotions to solve problem situations where conventional solutions do not immediately resolve [25]. Problem-solving and creativity capacity is the individual's ability to effectively use cognitive processes, actions and attitudes, motives, and emotions to analyze, propose solutions, choose solutions, and perform solving learning and practical situations, problems in which no normal processes, procedures, or solutions are available; at the same time, to evaluate problem-solving solutions to adjust and apply flexibly in new circumstances and tasks. Problem-based learning competence demonstrates the individual's ability to think about problem situations, find and implement solutions to those problems when working in groups or alone [18].

From the above points of view, we agree with the point of view of the General Education Program in Vietnam (2018), it is possible to understand that problem-based learning competence is the ability of individuals to effectively use knowledge, skills, attitudes, ... to recognize, detect and resolve problem situations in a given context, in which there may or may not be a common process, procedure or solution.

2.4. Teaching According to the Orientation of Developing Problem-based Learning Competence

Teaching Mathematics contributes to forming and developing the quality and competence of students. Specifically, Mathematics contributes to the formation and development of specific and core competencies such as computing capacity, language capacity, communication and cooperation capacity, mathematical modeling capacity, problem-solving capacity, and creativity, ... Each capacity is a teaching method oriented to individual capacity development. And in this paper, we focus on teaching methods towards the development of problem-based learning competence.

Teaching methods towards the development of problem-based learning competence focus on developing the necessary competencies so that learners can apply knowledge to practical situations to prepare and develop the ability to detect and solve problems in life and work. Problem detection and solving teaching is a method of teaching by problem detection and problem-solving methods to find new knowledge. Here students can or may not do it while doing the problem detection and solving activity. That is, the students’ academic performance can be effective, or less effective, or not. Meanwhile, teaching that develops the ability of problem-based learning competence is teaching in which we make sure that students, after learning, will solve some similar problems effectively.

To clarify the above difference, we give our views and evaluation on teaching in the direction of developing problem-based learning competence; At the same time, we compare some of its basic characteristics with problem-solving and discovery teaching methods. We refer to the documents [26], [27], [28], [29] and give the following comparison table:
### Table 1. Comparison of problem detection and solving teaching methods with teaching towards developing problem-based learning competence

| Characteristics                  | Problem-solving and discovery teaching                                                                 | Teaching according to the orientation of developing the problem-based learning competence |
|----------------------------------|----------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| **Input conditions**             | Students acquire knowledge and develop the necessary skills in problem detection and solving.            | Students can detect and solve problems, know and perform the manipulations and skills of this competency. |
| **Output result**                | Forming new knowledge in the process of teaching problem detection and solving. Helping students master knowledge through problem-solving and discovery activities. | - Mastering knowledge through teaching activities oriented to develop problem-based learning competence. |
|                                  | - Helping students to acquire new knowledge through the results of the problem-solving process in an active and exploratory manner in the problem-solving process. | - Students know how to apply knowledge to real problems. |
|                                  | - Help students begin to get acquainted, develop skills such as problem detection and presentation, finding ways to solve problems, examining and evaluating results, and problem-solving methods. | - Learning results should be described in detail, observable, assessed, and able to show the progress of students. |
| **Educational goals**            | - Teachers base on the content of knowledge that needs to be conveyed to create problem situations and create excitement for students to actively and gradually discover new knowledge in the problem-solving process. | - To develop learners' problem-based learning competence to ensure the output quality of teaching. |
|                                  | - Content is specified in the program.                                                                   | - Focusing on the ability to apply knowledge and capacity in practical situations to prepare people for the capacity to solve situations and problems of life and work. |
| **Educational content**          | - Take knowledge standards as the basis for curriculum design and teaching content.                       | - Teachers select suitable content, associated with practical situations to achieve the specified outputs (developing problem-based learning competence). |
|                                  | - Teachers (or with students) create problem situations, organize and control students to discover, present problems, and solve problems so that students acquire knowledge, skills, and develop thinking development. | - The content is not specified in detail, only the main content is specified. |
|                                  | - Students are placed in a problem-provoking situation rather than passively informed knowledge in a form available. | - Taking capacity as a reference basis for organizing, designing programs, and learning content. |
|                                  | - Students do not fully actively mobilize their knowledge and ability to detect and solve problems.       | - Teachers are mainly the organizers, supporting students to autonomously and actively perform tasks of learning and acquiring knowledge; the learner is the center. |
| **Teaching methods**             | Mainly teaching in theory and depending on the independence of students, there are the following main forms of teaching: self-study learners, cooperative learners, asking and answering questions between teachers and students; teacher presentations on problem detection and resolution. | - Teachers pay close attention to each student to help them explore, discover, master, and apply knowledge to solve real-life problems and situations. |
|                                  | Creating problem-solving situations, directing students to discover problems, be self-aware, positive, and creative to solve problems; encouraging students to work together and interact with teachers to solve problems and build knowledge. | - Focusing on the development of communication skills, problem-solving ...; using a combination of active teaching perspectives, methods, and techniques; teaching methods of experiment and practice. |
| **Teaching mode**                | Students learn and perform under the guidance and suggestion of teachers and it depends on the student's independence and interest in learning. | Organizing diverse forms of learning; paying attention to social activities, extracurricular activities, scientific research, and creative experiences; promoting the application of information technology and communication in teaching and learning. |
| **Evaluating students' learning results** | - Evaluation criteria are based on learned knowledge that students have memorized and demonstrated.         | Evaluation criteria are based on the output capacity, the continual progress in the student's learning process, and focus on the applicability in practical situations. |
|                                  | - Evaluating the knowledge that students construct, comprehend, and apply new knowledge.                  | Facilitating positive interactions between students, between students and teachers; motivating and helping students to realize their problem-based learning competence through observation, communication, exploration, discovery, and creation. |
| **Teaching environment**         | Students can do learning activities on their own, group study, and practice in activities and through activities. | Teachers provide support only when needed. |
| **Advantages and disadvantages** | - Advantages: good effect in activating students' cognitive activities, increasing the interaction, dynamism, and creativity between teachers and students in teaching. ... | - Advantages: good quality management according to specified outputs, emphasizing students' problem-based learning competence. |
|                                  | - Disadvantages: time-consuming, requires the teacher to have professional qualifications, a lot of dedication and creativity, and must clearly define the conditions for effective problem-solving and discovery teaching. | - Disadvantages: it depends on the implementation process; if applying a bias to focus on competencies, not paying attention to teaching content, will lead to a lack of basic knowledge and affect the systematization of knowledge. |
2.5. The Purpose of Fostering Problem-based Learning Competence through Teaching the Generalization of Practical Problems with Exponential Functions and Logarithmic Functions

The purpose of fostering problem-based learning competence through teaching the topic of exponents and logarithms is to help students establish and formulate general formulas. Each general formula is a mathematical solution to problems of the same type (cell division; compound interest problem; continuous compound interest rate problem; capital contribution problem). Through forming the general formula, students will be trained and fostered in reasoning, logical thinking, creative thinking as well as problem-based learning competence. Finally, students know how to apply formulas for real problems to form their problem-solving skills.

2.6. Scientific Basis of Fostering Problem-based Learning Competence through Teaching the Generalization of Practical Problems on the Topic of Exponential Functions and Logarithmic Functions

Since the Pythagorean era, the question of whether mathematics comes from reality has been raised. Up to now, over thousands of years of history, the above question has always been a topical question that is concerned by many people [30]. The most important task of teaching mathematics in high schools is to teach how to set up equations to solve quiz problems. The quiz problems including real problems [31]. First, teachers tend to think that knowledge of mathematics comes from real problems. Second, mathematics is a useful tool to help solve real problems effectively. Third, students can really have difficulties solving real-life problems in that mathematics is an abstract and coherent knowledge system, while real problems are often complex and trouble [32].

Thus, setting up several general formulas according to the teaching process-oriented to develop problem-based learning competence on the topic of exponential functions, logarithmic functions in particular and practical mathematics, in general, is always the topic of much attention and also a difficult topic in teaching. In our opinion, one of the main reasons students have difficulty solving real problems is that they do not have a common process, and knowledge of real math is diverse and rich.

2.7. Teaching Process of Generalizing Practical Problems on the Topic of Exponents and Logarithms

Through research on the topic of exponential functions and logarithmic functions, we offer a teaching process generalizing the real problem according to the development orientation of problem-based learning competence in figure 1.

![Figure 1. Real problem-solving process](image)

Here we would like to detail each step in the process above:

- **Step 1:** Convert from a real problem to a math problem
  + Find out the problem: reading the problem, summarizing the problem (determining the condition factor, information, conclusion requirements of the problem).
  + Select appropriate equations, exponential formulas, logarithmic functions to represent the conditions and facts of the problem.
  + Making appropriate mathematical models to solve real problems.

- **Step 2:** Solve math problems
  + Find plans and algorithms to solve the modeled problem.
  + Solve the modeled problem that was set up in step 1 by using mathematical tools, reasoning, and mathematical inference about exponential functions, logarithmic functions.
  + Convert math problems to actual conclusions. Make conclusions for the actual problem and adjust (if any).

- **Step 3:** Expand and exploit the actual problem
  Generalize, similarize, find the inverse problem for the original real problem, find other solutions, or you can apply this problem to solve other real problems, ...

2.8. An Illustrative Example of Problem-based Learning Competence through Teaching about Generalizing the Real Problem on the Topic of Exponents and Logarithmic Functions

Example 1. If we start with a unicellular yeast cell under favorable growth conditions, it will split into two identical "daughter cells" within one hour. Next, after an hour, each of these daughter cells will divide into two identical cells.

a). Set up the formula to represent the number of cells after \( n \) hours. Then find the number of identical daughter cells after 24 hours.
b). When will we have 1024 "daughter cells" identical to the original mother cell. \[33\]

We guide students to solve the above example according to the teaching process of generalizing the actual problem on the topic of exponents and logarithmic functions oriented to the development competence of problem-based learning in table 2 as follows:

| Teacher | Students |
|---------|----------|
| **Step 1: Convert from an actual problem to a math problem** | **Table 2. Teaching the generalization of the yeast cell problem** |
| What data does the topic give? What are the requirements and goals to look for? Are there any conditions? | **Teacher** |
| Read the problem and combine with figure 3.1, there will be the following summary: Initially \( t = 0 \) hour, there is 1 cell. After \( t = 1 \) hour, there are 2 cells. After \( t = 2 \) hours, there are 4 cells ... There are no additional conditions. And the topic requires finding: a) Number of daughter cells after \( n \) hours and after 24 hours. b) When will there be 1024 cells? |
| **Is there a general formula for this problem?** | We need to find a function or an expression that represents the growth of unicellular fungal cells. |
| **What can we do with the data from the topic? How do we come up with the required formula?** | We have the following argument: When \( t = 0 \) hour, there is \( 1 = 2^0 \) cell. After \( t = 1 \) hour, there are \( 2 = 2^1 \) cells. After \( t = 2 \) hours, there are \( 4 = 2^2 \) cells. After \( t = 3 \) hours, there are \( 8 = 2^3 \) cells. After \( t = 4 \) hours, there are \( 16 = 2^4 \) cells. After \( t = 5 \) hours, there are \( 32 = 2^5 \) cells... Find the function in \( t \) to represent the number of unicellular yeast cells after \( t \) hours. |
| **Please make your arguments and comment. Is there anything special to you about the number of cells? What is the relationship between time and the number of cells?** | The number of cells is calculated according to the power of the base 2. After the argument, we find that the relationship between the time and the number of cells is \( N(t) = 2^t \), this is also the general formula to find the number of cells after \( t \) hours. |

**Step 2: Solve the math problem**

| Is this problem strange or familiar? What kind of thing is it? Have you seen it? How do you use terms and data? Do you make full use of the given assumptions and conditions? What formula, theorem, or property can be used to solve that problem? | a) When \( t = n \) hours, replace \( t = n \) in the formula just found out, we will have \( N(n) = 2^n \). Then, \( t = 24 \), we have \( N(24) = 2^{24} = 16777216 \). b) When will we have 1024 daughter cells? This problem is new and there is no way or solution suitable for the problem. We can only solve it when we find the general formula for the number of cells after \( t \) hours is \( N(t) = 2^t \). Conversely, we can only find the time to reach \( x \) cells when we have the function \( t = \log_2 x \). After using all the facts for the problem and reasoning, we have: \( N(t) = 1024 = 2^1 \), find time \( t \), \( t = \log_2 1024 = 10 \). |

| What is the conclusion for the actual problem? Is there any adjustment? | a) After \( n \) hours, there will be \( N(n) = 2^n \) daughter cells. After 24 hours, we will have \( N(24) = 2^{24} = 16777216 \) daughter cells. b) After 10 hours, there will be 1024 daughter cells. |

**Step 3: Expand and exploit the actual problem**

| Compare the results of the above arguments with \( N(t) = t^2 \). Comment the comparison results when changing exponent and radix together. | Replace \( t = 6 \) into the formula \( N(t) = 2^t \), then \( N(6) = 2^6 = 64 \). Substitute \( t = 6 \) into the formula \( N(t) = t^2 \), then \( N(6) = 6^2 = 36 \). We have: \( N(t) = 2^t = 64 > N(t) = t^2 = 36 \). When the variable between the base and the exponent was changed, there was a significant change. |

**Note:** When a variable is exponential, even a small change in the variable can cause a significant change in the value of a function. | Listen attentively and absorb. |
Based on the problem just solved, we can give the following general problem:
If we start with a cell under favorable growth conditions, then in an hour it will divide into \( n \) "daughter cells". Next, after an hour, each of these daughter cells will divide into \( n \) "daughter cells".

We have the following argument:

When \( t = 0 \) cell division, there are \( n^0 \) cells.
After \( t = 1 \) cell division, there are \( n^1 \) cells.
After \( t = 2 \) cell division, there are \( n^2 \) cells.
After \( t = 3 \) cell division, there were \( n^3 \) cells.
After \( t = 4 \) cell division, there were \( n^4 \) cells.
After \( t = 5 \) cell division, there are \( n^5 \) cells ...

The number of cells is calculated according to the power of the base \( n \). Then the number of cells after \( t \) hours will be \( N(t) = n^t \) and to find the time to reach \( A \) cells, we use the function \( t = \log_n A \).

**Note:** Cell division time can be calculated by a number of times, phase, or time (days, hours, ...). The most common number of \( n \) cells is 2 and 4.

**Example 2.** *(Compound interest problem)* A person deposits VND 1 million into a bank at an annual interest rate of 7%. We know that if the money is not withdrawn from the bank, every year, the interest will be entered into the original capital (it is called compound interest). How much money does the person receive after \( n \) years \((n \in \mathbb{N})\), if there is no withdrawal within this period and the interest rate does not change? When \( n = 5 \) years, how much money will the person have? [34].

We guide students to solve the above example according to the teaching process of generalizing the actual problem on the topic of exponents and logarithmic functions oriented the development competence of problem-based learning in table 3 as follows:
### Table 3. Teaching the generalization of compound interest problem

| Teacher | Students |
|---------|----------|
| Step 1: Convert from an actual problem to a math problem | The amount of invested capital is VND1 million with an interest rate of 7% annually. Calculate the total amount after n years. Investing in the form of compound interest: interest will be added to the original capital and the interest itself will continue to generate interest. |
| What facts and requirements does the problem give us? Are there any conditions? | Because it is the general number n, it cannot be found immediately but requires finding the general formula. We need to find a function or some expression that represents the growth of unicellular fungal cells. |
| Is there a general formula for this problem? | Perform the accrual in this way: interest will be added to the original capital and the interest itself will continue to generate interest. |
| What can we do with the data and what we’ve just learned from the problem? | We have: \( P_0 = 1; r = 0.07 \). Arguments for the proceeds from each year: After the first year: Interest is \( T_1 = Pr = 1,07 = 0.07 \) (million dong). The amount earned (accumulated capital) is: \( P_1 = P_0 + T_1 = P_0 + Pr = P_0(1 + r) = 1,07 \) (million dong). After the second year: Interest is \( T_2 = Pr = 1,07,07 = 0,0749 \) (million dong). The amount received (accumulated capital) is: \( P_2 = P_1 + T_2 = P_1 + Pr \) \( = P_0(1 + r) \) \( = [P_0(1+r)](1+r) \) \( = P_0(1+r)^2 = (1,07)^2 \) \( \ldots \) Similarly, in general, after \( n \) years, the total value obtained is: \( P_n = P_0(1 + r)^n = (1,07)^n \) (million dong). |
| Make an argument and see if there is any special thing or relationship between the given facts? | After setting the above formula, we have: After \( n \) years, the total achieved value is: \( P_n = P_0(1 + r)^n = (1,07)^n \) (million dong). Replace \( n = 5 \) years, the total value obtained is: \( P_5 = P_0(1 + r)^5 = (1,07)^5 = 1,402551731 \) (million dong). |
| Step 2: Solve the math problem | After \( n \) years, the total value obtained is achieved: \( P_n = P_0(1 + r)^n = (1,07)^n \) (million dong). After \( n = 5 \) years, the total value achieved is \( 1,402551731 \) (million dong) or approximately \( 1402600 \) (million dong). |
| Is it possible to now calculate how much money that person has earned? Calculate the amount paid. | Step 3: Expand and exploit the actual problem |
| What is the conclusion for the actual problem? Is there any adjustment? | Call the initial capital \( P_0 \). The investor wants the interest rate \( r \) per period as compound interest. The investor invests money for \( n \) periods. We know that, at the end of each period, the investor withdraws interest and only leaves the capital, calculate \( P_n \) which is the total amount earned (including capital and interest) after \( n \) periods. The interest earned after \( n \) periods is \( P_n - P_0 \). Arguments for the proceeds each year: After the first year: The interest is \( T_1 = Pr \). The amount received (accumulated capital) is: \( P_1 = P_0 + T_1 = P_0 + P_0r = P_0(1 + r) \). After the second year: The interest is \( T_2 = Pr \). The amount earned (accumulated capital) is: \( P_2 = P_1 + T_2 = P_1 + Pr = P_1(1 + r) \) \( = [P_0(1+r)](1+r) = P_0(1+r)^2 \) \( \ldots \) Similarly, in general, after \( n \) years, the total value obtained is: \( P_n = P_0(1 + r)^n \) (compound interest formula). |
| Please state the problem and general formula for compound interest. |  |
Table 3. Continued

| For compound interest formula: | Listen and absorb. |
|-------------------------------|-------------------|
| + The time unit of each period can be a year, quarter, month, or day. | + Find time to get a certain amount. |
| + Interest is expressed as a decimal. | + Find interest. |
| Please state the inverse problem for the compound interest problem. | + Find the initial amount. |

Here is an example to help students learn how to set up an exponential formula to solve math problems. At the same time, it is also used by the Textbook of Analysis 12 in Vietnam to form the exponential definition as well as the compounding formula for students. During the solving process, the formulation of formulas is done by answering the teachers’ questions and requests. Through activities, students will gradually develop the ability to recognize, detect, and solve problems.

**Example 3. (Continuous compound interest rate problem)** An amount of $5,000 is invested at an interest rate of 5% per annum. Form a formula for the sum of the money after t years and how long does it take for the amount to double if interest is exercised on continuous compounding? [35]

We guide students to solve the above example according to the teaching process of generalizing the actual problem on the topic of exponents and logarithmic functions oriented the development competence of problem-based learning in table 4 as follows:

| Table 4. Teaching the generalization of the continuous compounding problem |
| Teacher | Students |
|---------|---------|
| **Step 1: Convert from an actual problem to a math problem** | Available factors: \( P = 5000 \$; \ r = 0,05 \)
| Summarize the problem and represent it with mathematical notation. | Requirements: find a formula that calculates the sum of the money earned after t years and how long it will take for the amount to double.
| Is there a general formula for this problem? What is the form of continuous compounding? What is special about this form, different from the previous two forms of compounding? | Interest is calculated in the form of continuous compounding.
| What can we do with the data extracted from the topic? | The compound interest formula can be used as it relates to the compound interest form.
| Suppose that if you deposit an initial capital \( P \) at the annual interest rate \( r \%)\ in the form of compound interest, then after \( t \) years of deposit, what will be the return on both principal and interest? | Students think about the problem posed. Consider setting up a new recipe.
| Suppose that we divide each year into \( n \) periods to calculate interest and keep the annual interest rate \( r \)\%, then the interest per period will be \( \frac{r}{n} \) and what is the amount after \( t \) years? | Based on the factors given, students reason to find an exponent representing the money obtained in the form of continuous compounding.
| When increasing the number of periods \( n \) in a year, what will be the amount earned after \( t \) years? | In the form of compound interest after \( t \) years, the proceeds are: \( P_0 (1 + r)^t \).
| When increasing the number of periods \( n \) in a year, the amount earned after \( t \) years also increases. | The interest rate per period is \( \frac{r}{n} \) and the amount earned after \( t \) years (or after \( nt \) periods) is \( P_0 \left( 1 + \frac{r}{n} \right)^t \), in the form of compounding after \( nt \) periods.
| We see that the amount cannot grow indefinitely. The form of interest calculation when \( n \rightarrow +\infty \) is called continuous compounding. Thus, with the initial capital \( P \) with the annual interest rate \( r \) in the form of continuous compounding, it can be proved that after \( t \) years of deposit, the amount of both the principal and the interest will be: \( A(t) = Pe^{rt} \). | Listen and receive ideas.
| **Step 2: Solve the math problem** | With \( P = 5000 \$; \ r = 0,05 \), we have the formula to calculate the total amount after \( t \) years: \( A(t) = 5000e^{0,05t} \).|

We guide students to solve the above example according to the teaching process of generalizing the actual problem on the topic of exponents and logarithmic functions oriented the development competence of problem-based learning in table 4 as follows:

| Table 4. Teaching the generalization of the continuous compounding problem |
| Teacher | Students |
|---------|---------|
| **Step 1: Convert from an actual problem to a math problem** | Available factors: \( P = 5000 \$; \ r = 0,05 \)
| Summarize the problem and represent it with mathematical notation. | Requirements: find a formula that calculates the sum of the money earned after t years and how long it will take for the amount to double.
| Is there a general formula for this problem? What is the form of continuous compounding? What is special about this form, different from the previous two forms of compounding? | Interest is calculated in the form of continuous compounding.
| What can we do with the data extracted from the topic? | The compound interest formula can be used as it relates to the compound interest form.
| Suppose that if you deposit an initial capital \( P \) at the annual interest rate \( r \%)\ in the form of compound interest, then after \( t \) years of deposit, what will be the return on both principal and interest? | Students think about the problem posed. Consider setting up a new recipe.
| Suppose that we divide each year into \( n \) periods to calculate interest and keep the annual interest rate \( r \)\%, then the interest per period will be \( \frac{r}{n} \) and what is the amount after \( t \) years? | Based on the factors given, students reason to find an exponent representing the money obtained in the form of continuous compounding.
| When increasing the number of periods \( n \) in a year, what will be the amount earned after \( t \) years? | In the form of compound interest after \( t \) years, the proceeds are: \( P_0 (1 + r)^t \).
| When increasing the number of periods \( n \) in a year, the amount earned after \( t \) years also increases. | The interest rate per period is \( \frac{r}{n} \) and the amount earned after \( t \) years (or after \( nt \) periods) is \( P_0 \left( 1 + \frac{r}{n} \right)^t \), in the form of compounding after \( nt \) periods.
| We see that the amount cannot grow indefinitely. The form of interest calculation when \( n \rightarrow +\infty \) is called continuous compounding. Thus, with the initial capital \( P \) with the annual interest rate \( r \) in the form of continuous compounding, it can be proved that after \( t \) years of deposit, the amount of both the principal and the interest will be: \( A(t) = Pe^{rt} \). | Listen and receive ideas.
| **Step 2: Solve the math problem** | With \( P = 5000 \$; \ r = 0,05 \), we have the formula to calculate the total amount after \( t \) years: \( A(t) = 5000e^{0,05t} \).|
Based on the above constant compound interest formula, find time to double the amount? (During solving the problem, apply: \( \log_a b^x = x \log_a b \) and \( \log_a a = 1 \)).

After having the formula in the form of continuous compound interest \( A = Pe^r \). Apply the formula with \( P = 5000 \$ \); \( r = 0.05 \), we solve the exponential equation and find \( r \):

\[
10000 = 5000e^{0.05t} \iff e^{0.05t} = 2
\]

\[
\iff \ln e^{0.05t} = \ln 2
\]

\[
\iff 0.05t = \ln 2
\]

\[
\iff t = \frac{\ln 2}{0.05} \approx 13.86
\]

What is the conclusion for the actual problem? Is there any adjustment?

The formula to look for is \( A(t) = 5000e^{0.05t} \).

That amount doubled after 13.86 years, or roughly 13 years, 10 months, and 7 days.

Step 3: Expand and exploit the actual problem

General problem: With the initial capital \( P \) with the annual interest rate \( r \) in the form of continuous compounding, it can be proved that after \( t \) years of depositing, how much will be the return on both capital and interest be? (Provide the formula for continuous compound interest)

The proceeds of both capital and interest will be: \( A(t) = Pe^r \).

The formula \( A(t) = Pe^r \) is called the continuous compound interest formula.

Besides the requirement to find the sum of money and find the time, are there any other requirements for this problem?

This type of problem also requires finding the interest rate or finding the amount of capital.

Note:

+ Interest is expressed as a decimal.
+ It differs from the form of compounding that, in a year, there will be forms of compounding annual interest such as monthly, weekly, daily, or quarterly.

The above example can be used to form formula for the problem of continuous compound interest. Students and teachers work together to form a compound interest formula when \( n \to +\infty \). Students will discover connections between formulas, along with differences between mathematical types.

**Example 4. (Capital contribution problem)** A person deposits the same amount in the bank every month as VND3,000,000 (at the beginning of each term), a 1-month term with an interest rate of 0.67% per month. After \( t \) years, how much capital and interest will that person receive? [36]

We guide students to solve the above example according to the teaching process of generalizing the actual problem on the topic of exponents and logarithmic functions oriented the development competence of problem-based learning in table 5 as follows:

| Teacher | Students |
|---------|----------|
| Summarize the facts given. | The capital contribution amount is \( a = 3 \) million VND. The monthly interest rate is \( r = 0.0067 \). What is the amount of capital and interest after \( n \) months? That is, look for \( P_n \). |
| Is there a general formula for this problem? | We do not have a formula for the capital contribution problem yet, so let’s set up a formula for this type of math problem. |
| What can we do with the data and what we just extracted from the problem? So at the end of the first month, how much money does the investor get? | At the end of the first month, the investor will have an amount: \( P_1 = a + ar = a(1+r) = 3(1+0.0067) = 3,0201 \). |
| At the beginning and end of the second month, how much are the investor’s earnings? | At the beginning of the second month, the amount is: \( P_1 + a = a(1+r)+a = a(1+(1+r)) = 6,0201 \). |
| | At the end of the second month, the amount is: \( P_2 = P_1 + Pr = a + a(1+r) + [a + a(1+r)]r = a[1+r + (1+r)] = 6,0604 \). |

| Table 4. Continued |
|-------------------|-------------------|
| Based on the above constant compound interest formula, find time to double the amount? | After having the formula in the form of continuous compound interest \( A = Pe^r \). Apply the formula with \( P = 5000 \$ \); \( r = 0.05 \), we solve the exponential equation and find \( r \):
| \[
10000 = 5000e^{0.05t} \iff e^{0.05t} = 2
\]
| \[
\iff \ln e^{0.05t} = \ln 2
\]
| \[
\iff 0.05t = \ln 2
\]
| \[
\iff t = \frac{\ln 2}{0.05} \approx 13.86
\] |
| What is the conclusion for the actual problem? Is there any adjustment? | The formula to look for is \( A(t) = 5000e^{0.05t} \). That amount doubled after 13.86 years, or roughly 13 years, 10 months, and 7 days. |
| Step 3: Expand and exploit the actual problem | The proceeds of both capital and interest will be: \( A(t) = Pe^r \). The formula \( A(t) = Pe^r \) is called the continuous compound interest formula. |
| General problem: With the initial capital \( P \) with the annual interest rate \( r \) in the form of continuous compounding, it can be proved that after \( t \) years of depositing, how much will be the return on both capital and interest be? (Provide the formula for continuous compound interest) | Besides the requirement to find the sum of money and find the time, are there any other requirements for this problem? This type of problem also requires finding the interest rate or finding the amount of capital. |
| Note: | List and absorb |
| + Interest is expressed as a decimal. | |
| + It differs from the form of compounding that, in a year, there will be forms of compounding annual interest such as monthly, weekly, daily, or quarterly. | |
Continue to execute the above argument for \( n \) months. Find out relationships between the given facts.

At the beginning of the third month, Mr. X has the amount of:

\[
P_2 + a = a\left[(1 + r) + (1 + r)r^2\right] + a \\
= a\left[1 + (1 + r) + (1 + r)^2\right].
\]

At the end of the third month, Mr. X has the amount:

\[
P_3 = P_2 + Pr_3 \\
= a\left[1 + (1 + r) + (1 + r)^2\right] + a\left[1 + (1 + r) + (1 + r)^2\right]r \\
= a\left[(1 + r)^3 + (1 + r)^2 + (1 + r)\right].
\]

... At the end of the \( n \)th month, Mr. X has the amount:

\[
P_n = a\left[(1 + r)^n + (1 + r)^{n-1} + \ldots + (1 + r)^2 + (1 + r)\right].
\]

Can you reduce the formula \( P_n \)? Do you notice anything special about the expression in square brackets?

The terms in square brackets are the sequence of the exponentiation with the first term \( u_e = 1 + r \) and the ratio \( q = 1 + r \). The expression in square brackets is the sum of the first \( n \) terms of the exponential, so we have:

\[
S_n = a \frac{q^n - 1}{q - 1} = \frac{(1 + r)^n - 1}{r}.
\]

So, we can reduce it as follows:

\[
P_n = a\left[(1 + r)^n + (1 + r)^{n-1} + \ldots + (1 + r)^2 + (1 + r)\right] \\
\Leftrightarrow P_n = a\left[(1 + r)^n - 1\right] \cdot \frac{1}{r}.
\]

Step 2: Solve the math problem

Using all the given assumptions and conditions, work on solving the problem. We use the argument with the unit of time which is months, so what if we are asked to find the sum after \( t \) years?

After \( t \) years, ie \( n = 12t \) months, replace \( n = 12t \) and other data into the new formula, we have:

\[
P_{12t} = 3(1 + 0.0067)\left(1 + 0.0067\right)^{12t} - 1 \\
= \frac{30201\left(1 + 0.0067\right)^{12t} - 1}{67} \\
= \frac{450.76\left(1 + 0.0067\right)^{12t} - 1}{67}.
\]

What is the conclusion for the actual problem? Is there any adjustment?

After \( t \) years, that person receives the principal and interest:

\[
P_{12t} \approx \frac{450.76\left(1 + 0.0067\right)^{12t} - 1}{67} \text{ million dong}.
\]

Step 3: Expand and exploit the actual problem

Give a general description of the installment problem and its formula.

A person deposits the same amount in the bank monthly as \( a \) dong (at the beginning of each term), for a 1-month term with an interest rate of \( r\% \) per month. What is the amount of capital and interest after \( n \) months?

With the same argument as in step 1 and generalizing the formula found, we have: After \( n \) months, he receives the capital and interest:

\[
P_n = a\left[(1 + r)^n - 1\right] \cdot \frac{1}{r}.
\]

In addition to the requirement to find the amount after a certain time, what are the other requirements?

+ Find time to contribute capital.
+ Find interest.
+ Find the amount of recurring installment.

The formula for the monthly capital contribution problem is established through the argument about monthly accrual: contributing capital at the beginning of the month, making interest at the end of the month, arguing continuously until finding the final formula in the transition step from an actual problem to a math problem. Through it, students are trained in skills such as: using mathematical language, methods of inductive mathematics, logical reasoning, problem discovery (discovering the relationship between numbers in the amount expression at the end of the month), relating to old knowledge.

Thus, through this example, we see that setting up the general formula for the actual problem in step 1 is converting the actual problem into the math problem. Specifically, students need to identify given factors, exploit data, perform math, and find out the special connections and relationships between them to create a general formula for the problem. Next, students apply the formula just established to solve the problem. Finally, the formula will be repeated by the teacher in step 3, and students will be working with new problems. Through these steps, in addition to formulating formulas, practicing math solving skills, students are also trained in problem-based learning competence, logical reasoning,
2.9. Methodology

2.9.1. Purpose, Requirements, and Duties of the Pedagogical Experiment

The purpose of our pedagogical experiment is to test the feasibility and effectiveness of fostering problem-based learning competence through generalizing the actual problem on the topic of exponential and logarithmic functions. The pedagogical experiment must ensure objectivity and science and it must be suitable for students and actual teaching situation and conditions. Teachers must compile lesson plans and conduct the proposed pedagogical experiment, know how to collect, process, analyze, and evaluate experimental results to verify the feasibility and effectiveness of the proposed pedagogical experiment.

2.9.2. Content of pedagogical experiment

The experimental contents are the following specific contents:
- Carrying out teaching in the experimental class according to the lesson "Practice exercises, chapter II: Exponentiation functions, exponential functions and Logarithmic functions" of Advanced 12th-grade Calculus of Vietnam according to the teaching process of generalizing practical problems on the subject of exponential functions, logarithmic functions.
- Experimental lessons are exercises with the content of Chapter II "Exponentiation functions, exponential functions, and logarithmic functions".
- After the experiment, conduct the test and assessment of the experimental class and the control class in the form of an essay; analyze the evaluation results of the experimental class and the control class to check the feasibility and effectiveness of the experiment.

2.9.3. Experimental Objects

We consulted 47 high school teachers in Ho Chi Minh City, Vietnam in a questionnaire survey: “In your opinion, is it important to develop problem-based learning competence on the topic of exponents and logarithms for students?”. We obtained the results in table 6.

The data table shows that the majority of teachers believe that teaching to develop problem-based learning competence on the topic of exponential and logarithms for students is very important (63.83%) or important (25.53%). This result shows that teachers appreciate the importance of developing problem-based learning competence on the topic of exponential and logarithms for students in teaching.

Next, we conduct experiments at Nguyen An Ninh High School (District 10, Ho Chi Minh City, Vietnam) according to the advanced study program in Vietnam for the 2020-2021 school year.
- Experimental class 12A9 includes 45 students. Math teacher: Nguyen Thi Ly.
- Control class 12A13 includes 44 students. Math teacher: Nguyen Thi Ly.

We designed lesson plans and implemented teaching content "Practice exercises in chapter II" for both classes. For the experimental class, the teacher will teach according to lesson plans on practical problems oriented to foster problem-based learning competence. For the control class, the teacher will teach according to the normal program distribution. Practical problems must ensure the following:
- Define clearly key knowledge, skills to be achieved.
- Appropriate in terms of time and general knowledge of students.
- Activities in the teaching process must help students foster their problem-based learning competence through practical problems.

2.9.4. Process of Pedagogical Experiment

- Investigate and evaluate the learning situation of students in experimental and control classes.
- Prepare documents and compile experimental lesson plans according to the teaching process that generalizes the actual problem on the topic of exponential and logarithmic functions.
- Teachers give lessons according to the compiled lesson plan.
- After the teaching is completed, the teacher gives the experimental class and the control class a 45-minute test.
- Collect, analyze, and evaluate test results of experimental and control classes.

### Table 6

|                      | Not important | Less important | Important | Very important |
|----------------------|---------------|----------------|-----------|---------------|
| Number of people     | 0             | 5              | 12        | 30            |
| %                    | 0%            | 10.64%         | 25.53%    | 63.83%        |

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2.9.5. Methods of Experimental Evaluation

In the thesis, we use the following experimental evaluation methods:

- **Classroom observations**: to receive students' feedback on lessons about excitement, positive attitude, cognitive level, and applicability.

- **Interview**: We use the method of interviewing by talking with students to clarify information about the level of interest that is difficult to determine through observation. We interviewed teachers to get the teacher's assessment and comments on students' interest and perception in the experiment.

- **Essay test**: aims to assess students' ability to acquire knowledge through the lessons. Test the individual knowledge of the experimental class and the control class through the self-essay test after the experiment. Test content is based on the lesson plan's goals and we pay special attention to exercises to evaluate the effectiveness of students' use of problem-based learning competence. Scores of the tests are scored on a 10-point scale.

- **Mathematical statistics method**: After marking students' tests (rounded to one decimal), the test score data will be collected and processed using SPSS Statistics software. In which, we calculate the characteristic parameters of the statistics based on the following formulas:

\[
\bar{X} = \frac{1}{n} \sum_{i=1}^{k} n_i X_i
\]

\[
S^2 = \frac{1}{n-1} \sum_{i=1}^{k} n_i (X_i - \bar{X})^2
\]

\[
S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{k} n_i (X_i - \bar{X})^2}
\]

In which, \(\bar{X}\) is the average score; \(X_i\) is the score achieved; \(n_i\) is the number of papers (the number of students) achieving a corresponding \(X_i\) score in each test; \(k\) is the number of different groups of points; \(n\) is the sample size (total number of students tested).

Standard deviation indicates the dispersion of the score data around the mean. The lower the \(S\)-index, the less dispersion around the mean and the higher the concentration around the mean.

+ Test hypotheses to compare two mean values of two independent samples by SPSS Statistical software.

2.9.6. Analyze Before Conducting the Experiment

We surveyed the math performance of two classes through their homeroom teacher and class subject teacher and we obtained the most recent test scores for both classes for the 2020-2021 school year. (See Table 7).

And we have figure 3 comparing the results of the most recent tests of students before conducting a pedagogical experiment between experimental class and control class based on Table 7 as follows:

![Figure 3](image.png)

From Figure 3, we see that the height difference of the columns is not much, showing that the score frequencies of the two classes are nearly the same. The distribution of scores for the two classes is nearly equal.

| Scores | Total |
|--------|-------|
| 5.0    | 2     |
| 5.5    | 2     |
| 6.0    | 0     |
| 6.5    | 0     |
| 7.0    | 3     |
| 7.5    | 23    |
| 8.0    | 11    |
| 8.5    | 9     |
| 9.0    | 3     |
| 9.5    | 2     |
| Total  | 44    |
Besides, from the data in Table 7, we obtain the typical parameters of the statistics as in Table 8.

**Table 8.** Table of typical parameters of Mathematics test scores for experimental and control classes (pre-experimental) in SPSS

| Class        | Mean | Variance | Standard Deviation |
|--------------|------|----------|--------------------|
| Experimental | 7.27 | .94      | .97                |
| Control      | 7.53 | 1.10     | 1.05               |

From the results for Table 8 test scores, we get:
- The average grade of the two classes is: $\bar{X}_{TN} = 7.27$; $\bar{X}_{DC} = 7.53$.
- Variance: $S_{TN}^2 = 0.94$; $S_{DC}^2 = 1.10$
- Comment: the difference between the mean and variance of the two classes is not large. We can say that the scores of the two classes are nearly equal.

To accurately assess the equivalent difference between the average scores of the two classes that we experimented with, we test the hypothesis $H_0$: “The average score of the latest Math test of the experimental class and the control class was equivalent”, with a significant level $\alpha = 0.05$ by T-test table (table 9).

From Table 9, we have the following results:
- According to the Levene test, we have a value of Sig = 0.002, so the variance of the two classes is equivalent, using the results of the Independent-samples T-test corresponding to the case where the variance of two classes is equal.
- According to the Independent-samples T-test, we have Sig = 0.968, so we accept the hypothesis $H_0$, the average score of the closest Math test of the experimental class and the control class are equivalent.

From the above results, we can confirm that the Mathematics skills of the two classes are similar.

**2.9.7. Analyze Experimental Results**

We conducted experimental teaching while following up, observing the progress, and interviewing to give some comments as follows:
- Before experimenting, students in both classes have not had much access to the practical problem on the topic of exponents and logarithms, so even though they are interested in it, students are still afraid and have not solved the actual problem yet.
- During the experiment, we found:
  - Experimental class atmosphere is positive; students are interested in practical problems so they voluntarily participate in activities; high teaching efficiency.
  - Compared with the control class, students in the experimental class operate and learn by more activities, acquire knowledge more effectively.
- After experimenting, students also have some changes as follows:
  - Students in the experimental class have a better grasp of the contents of practical problems than the control class, from which they can solve problems quickly and begin to be more confident with their abilities; they actively learn from real-life problems more than before.

Through the above comments and assessments, we believe that the proposed experiment is initially effective in teaching. However, to ensure objectivity about the effectiveness of the experiment, we conduct quantitative analysis and evaluation by Mathematical statistics through SPSS software.

**Table 9.** Average T-test checklist of Mathematics test scores of experimental and control classes (pre-experimental) in SPSS

|                  | Levene's Test for Equality of Variances | t-test for Equality of Means |
|------------------|----------------------------------------|-----------------------------|
|                  | F          | Sig. | T   | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference |
|                  |            |      |     |    |               |                |                   | Lower                  | Upper                  |
| Scores           |            |      |     |    |               |                |                   | 0.002                  | 0.968                  |
| Equal variances  | 0.002      | 0.968| -1.251 | 87 | 0.214 | -0.26742 | 0.21383 | -0.69244 | 0.15759 |
| assumed          |            |      |     |    |               |                |                   |                        |                        |
| Equal variances  |            |      |     |    |               |                |                   | -1.250 | 86.126 | 0.215 | -0.26742 | 0.21402 | -0.69288 | 0.15803 |
| not assumed      |            |      |     |    |               |                |                   |                        |                        |
Fostering Problem-based Learning Competence through Teaching the Generalization of Practical Problems on the Topic of Exponential and Logarithmic Functions

Table 10. Table of the frequency distribution of test scores for experimental and control classes (post-experimental) in SPSS

| Class       | 5.5 | 6.0 | 6.5 | 7.0 | 7.5 | 8.0 | 8.5 | 9.0 | 9.5 | 10.0 | Total |
|-------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|--------|
| Experimental| 0   | 3   | 1   | 7   | 6   | 10  | 6   | 5   | 5   | 2   | 45     |
| Control     | 4   | 6   | 5   | 5   | 3   | 8   | 2   | 6   | 4   | 1   | 44     |
| Total       | 4   | 9   | 6   | 12  | 9   | 18  | 8   | 11  | 9   | 3   | 89     |

To test the feasibility and evaluate the effectiveness of the experiment, we gave students in the experimental class and the control class a 45-minute test (essay exam). Quantitative analysis is based on the test results. First of all, we create table 10.

From the data in Table 10, we have figure 4 comparing the scores of the two classes:

Figure 4. Bar chart comparing test scores of experimental and control classes (after the experiment) in SPSS

From Figure 4, we can see that there are differences in the height of the grade columns and the distribution of scores of the two classes. The scores of the experimental classes range from 6 to 10 points and most are 7-9.5 points. The distribution of scores of the control class is from 5.5 to 10 points and a large number is from 5.5 to 8 points. Besides, we also obtain the typical parameters of the statistics as table 11:

Table 11. Table of characteristic parameters of the statistics on the test scores of experimental and control classes (post-experimental) in SPSS

| class     | Mean | Variance | Standard Deviation |
|-----------|------|----------|--------------------|
| Experimental | 8.04 | 1.10     | 1.05               |
| Control   | 7.52 | 1.76     | 1.32               |

Reading the above test result data, we have:

The average grade of the two classes is: \( \bar{X}_{EN} = 8.04 \); \( \bar{X}_{DC} = 7.52 \).

Variance: \( S_{EN}^2 = 1.10; S_{DC}^2 = 1.76 \).

Comment: the mean and variance of the two classes have a big difference. The GPA of the experimental class is higher; The standard deviation and variance are lower, so the concentration around the mean is higher than that of the control. We can say that the experimental class's scores are higher than those of the control class.

To accurately assess the difference (or the high, low) between the average points of the two classes, we test the average between the two classes, with a significance level of \( \alpha = 0.05 \) through Table 12 with the following two assumptions:

Hypothesis \( H_0 \): "The average score of the experimental class and the control class are similar".

\( H_1 \): "The average score of the experimental class is higher than that of the control class".

Table 12. Average T-test checklist of test scores of experimental and control classes (post-experimental) in SPSS

| Independent Samples Test | Levene's Test for Equality of Variances | t-test for Equality of Means |
|--------------------------|----------------------------------------|------------------------------|
|                          | F          | Sig. | t          | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | 95% Confidence Interval of the Difference |
| Score                    |            |      |            |    |                |                |                        |                                    |
| Equal variances assumed  | 5.060      | .027 | 2.062      | 87 | .042            | .52172          | .25300                 | .01886 to 1.02458            |
| Equal variances not assumed | 2.057 | .043 | 81.824     |    | .52172          | .25300          | .01709                 | 1.02634                      |
From the result sheet, we have:

- According to the Levene test, we have the value Sig = 0.05; so the variance of the two classes is not equal, using the results of the Independent-samples T-test corresponding to the case where the variance of the two samples is not equal.
- According to the Independent-samples T-test, we have Sig = 5, so we reject hypothesis H0, accept hypothesis H1. So the average score of the experimental class is 5% higher than that of the control class.

Thus, by the method of testing between classes with equivalent learning, the results show that the experimental class, after being taught according to the teaching process of generalizing the actual problem on the topic of the exponential and logarithmic function, the test and the GPA is higher than those of the control class. It can be seen that the experiment applied to the experimental class is completely feasible and effective in teaching.

3. Conclusions

From the results in the process of conducting pedagogical experiments, it shows that the implementation of fostering problem-based learning competence through teaching the generalization of practical problems on the topic of the exponential function and logarithmic function is feasible and effective. We answered four questions in introduction section. First is to compare problem detection and solving teaching methods with teaching towards developing problem-based learning competence. Second is to give a process of teaching generalizing practical problems on the topic of exponents and logarithms. Third is to give some illustrative examples of problem-based learning competence through teaching about generalizing the real problem on the topic of exponents and logarithmic functions. Fourth is to prove the effectiveness of teaching about generalizing the real problem on the topic of exponents and logarithmic functions. Attention is paid to pedagogical activities and especially the pedagogical experimental organization process is focused. The experimental results are processed using SPSS software. However, the paper has not mentioned problem-based learning competence in the direction of applying information technology in teaching practical problems on the topic of exponent and logarithmic functions. We look forward to your interest and discovery in this research direction.

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