Neutrino mass and muon $g-2$ explanation in $U(1)_{L_\mu-L_\tau}$ extension of left-right theories

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Abstract

We consider a gauged $U(1)_{L_\mu-L_\tau}$ extension of the left-right symmetric theory in order to simultaneously explain neutrino mass, mixing and the muon anomalous magnetic moment. We get sizeable contribution to muon $(g-2)$ anomaly ($\Delta a_\mu$) from the interaction of the new light gauge boson $Z_{\mu\tau}$ of the $U(1)_{L_\mu-L_\tau}$ symmetry with muons. The other positive contributions to $\Delta a_\mu$ come from the interactions of singly charged gauge bosons $W_L$, $W_R$ with heavy neutral fermions and that of neutral CP-even scalars with muons. The interaction of $W_L$ with heavy neutrino is facilitated by inverse seesaw mechanism which allows large light-heavy neutrino mixing and explains neutrino mass in our model. The results show that the model gives a small but non-negligible contribution to $\Delta a_\mu$ thereby narrowing down the deviation in theoretical prediction and experimental result of muon $(g-2)$ anomaly. We have briefly presented a comparative study for symmetric and asymmetric left-right symmetric model in context of various contribution to $\Delta a_\mu$. We also discuss how the generation of neutrino mass is affected when left-right symmetry breaks down to Standard Model symmetry via various choices of scalars.

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I. INTRODUCTION

While most of the theoretical predictions by Standard Model (SM) have been experimentally found to be correct to a very high precision, there lies a wide gap between SM’s prediction of muon anomalous magnetic moment, \( a_\mu = \frac{g_\mu - 2}{2} \) and its measurement. The SM prediction can be summed up as \( a^{\text{SM}}_\mu = (11659183.0 \pm 4.8) \times 10^{-10} \) \(^1\) \(^2\) whereas, the value obtained by Brookhaven National Laboratory (BNL) is \( a^{\text{exp}}_\mu = (11659209.1 \pm 6.3) \times 10^{-10} \) \(^3\) \(^4\) with \( \Delta a_\mu = (26.1 \pm 7.9) \times 10^{-10} \) \(^4\). While a 3.3\( \sigma \) deviation is achieved by BNL yet \(^3\), a nearly 5\( \sigma \) deviation is expected in the near future by Fermilab E989 \(^5\) and of similar precision by J-PARC \(^6\). In principle the \( a_\mu \) predicted by SM is a sum of contributions coming from QED, electroweak and hadronic sectors;

\[
a^{\text{SM}}_\mu = a^{\text{QED}}_\mu + a^{\text{electroweak}}_\mu + a^{\text{hadronic}}_\mu
\]

Among these three contributions, the theoretical uncertainty is believed to be coming from the hadronic loop contributions \(^7\) \(^9\) since the other two contributions have been verified with a high precision \(^10\) \(^11\). A proposed experiment, namely MUonE \(^12\) aspires to reduce this theoretical uncertainty by determining the hadronic vacuum polarization more precisely. All these recent developments in the experimental muon sector surely ignites theoretical research that aim at eliminating or narrowing down this wide gap in the prediction and measurement. Therefore recently many new physics scenarios have been explored in this context, for an incomplete list of which one may refer \(^13\) \(^27\).

Many of these new physics scenarios focus on \( U(1)_{L^\mu - L^\tau} \) symmetry to address the anomaly because of the phenomenology associated with its gauge boson \( Z_{\mu\tau} \). The total lepton number, \( L \), is a sum of individual lepton numbers \( L_e, L_\mu, L_\tau \) and one can always choose the difference between any two individual lepton numbers like \( L_e - L_\mu, L_\mu - L_\tau, L_e - L_\tau \) and gauge it to obtain an anomaly free theory. However, the gauged \( U(1)_{L^\mu - L^\tau} \) symmetry is the most chosen one due to the fact that the associated \( Z_{\mu\tau} \) gauge boson is not constrained by lepton and hadron colliders since it doesn’t couple to electrons and quarks. Moreover, as per the constraints given by neutrino-trident experiments \(^28\) a low mass of \( \mathcal{O}(100 \text{ MeV}) \) can be allowed for this new gauge boson \( Z_{\mu\tau} \) for a coupling as low as \( g_{\mu\tau} \leq 10^{-3} \).

The \( U(1)_{L^\mu - L^\tau} \) extension of SM has been extensively studied for explaining several issues like muon \( (g - 2) \) anomaly \(^29\), dark matter \(^30\), orbital energy loss of a neutron star \(^31\). Several other works have explained how the associated \( Z_{\mu\tau} \) gauge boson can ameliorate the tension in the late time and early time determination of Hubble constant \(^32\) and the unexpected dip in the energy spectrum of high energy cosmic neutrinos reported by the IceCube Collaboration \(^33\). Ref \(^29\) also
explains how this gauge boson can possibly mediate interactions between dark matter particles and muons inside a neutron star. The possible detection of this light $Z_{\mu\tau}$ boson has been discussed in Ref. [34, 35]. However the $U(1)_{L_\mu - L_\tau}$ extension of SM can not accommodate neutrino mass until and unless one adds a right-handed neutrino to the model. Such an attempt has been made in ref. [30, 36], where the authors explain neutrino mass by adding three right-handed neutrinos to the SM.

With the motivation of explaining neutrino mass, mixing and muon $(g-2)$ anomaly in a single framework we reach for the left-right symmetric model (LRSM) which naturally hosts a right handed neutrino and we augment it with the $U(1)_{L_\mu - L_\tau}$ symmetry. In manifest LRSM neutrino mass can be explained by canonical seesaw mechanism, but it can’t be verified by collider experiments since a very high right-handed scale ($10^{14}$ GeV) is associated with the mechanism. Thus in general extra particles are added to LRSM in order to generate neutrino mass by various low-scale seesaw mechanisms like linear seesaw, inverse seesaw, double seesaw etc. In particular, we take interest in inverse seesaw in our extended LRSM to explain neutrino mass which also allows large light-heavy neutrino mixing and thus leads to sizeable contributions to the muon anomalous magnetic moment via purely left-handed gauge boson mediation with heavy neutrino exchange. Apart from the usual fermions and scalars present in a manifest LRSM, the model contains three sets of extra sterile fermions and two extra scalars while the extra sterile fermions helping in creating the plot for inverse seesaw, the extra scalars help in breaking the $U(1)_{L_\mu - L_\tau}$ symmetry and also in implementing the inverse seesaw in the model. The $Z_{\mu\tau}$ boson originated from the breaking of $U(1)_{L_\mu - L_\tau}$ symmetry helps in ameliorating $\Delta a_\mu$ when gets mass around 150 MeV. Moreover our predictions on the mass of $Z_{\mu\tau}$ and its coupling $g_{\mu\tau}$ lies well below the constraint given by ref. [59]. We also discuss various symmetry breaking chains from LRSM to SM with different choices of scalars to see how it affects the generation of neutrino mass.

The rest of the paper is organised as follows. In Sec II we present the particle content of the extended LRSM and discuss the symmetry breaking of $U(1)_{L_\mu - L_\tau}$ symmetry and left-right symmetry down to low energy theory. We also discuss two different scenarios of neutrino mass generation with the help of doublet scalars in II A and triplet scalars in II B. In Sec III we discuss the generation of neutrino mass and mixing via extended inverse seesaw mechanism. In Sec IV we analytically study the new contributions to $\Delta a_\mu$ arising from different vector bosons and scalars present in the model. In Sec V we estimate the contributions numerically and present the results. This section also contains several plots of $\Delta a_\mu$ vs mass of mediators to check the sensitivity of the theoretical result to experimental bounds. In Sec VI we summarize and conclude the work.
II. THE MODEL

The model is an extension of manifest left-right theory with additional $U(1)$ gauge symmetry where the difference between muon and tau lepton numbers is gauged. Within manifest LRSM which is based on the gauge group $SU(2)_{L} \times SU(2)_{R} \times U(1)_{B-L} \times SU(3)_{C}$ and consists of usual quarks ($q_{L,R}$), leptons ($\ell_{L,R}$), Higgs bidoublet $\Phi$ and triplets $\Delta_{L,R}$ the light neutrino masses can be generated by type-I+II seesaw mechanism [46, 47, 50, 51, 54, 60, 61]. But these seesaw variants are not experiment friendly because of the very heavy right-handed scale associated with it. Therefore many other variants of LRSM have been explored in literature[56, 62–67] where spontaneous symmetry breaking is implemented with scalar bidoublet having $B - L = 0$ and Higgs doublets having $B - L = 1$ which leads to neutrino mass being generated by either simple Dirac mass terms or low scale seesaw mechanisms like inverse seesaw, linear seesaw etc.

| Fields | $SU(2)_{L}$ | $SU(2)_{R}$ | $B - L$ | $SU(3)_{C}$ |
|--------|-------------|-------------|--------|-------------|
| Fermions | $q_{L}$ | 2 | 1 | 1/3 | 3 |
| | $q_{R}$ | 1 | 2 | 1/3 | 3 |
| | $\ell_{L}$ | 2 | 1 | -1 | 1 |
| | $\ell_{R}$ | 1 | 2 | -1 | 1 |
| Scalars | $\Phi$ | 2 | 2 | 0 | 1 |
| | $\Delta_{L}$ | 3 | 1 | 2 | 1 |
| | $\Delta_{R}$ | 1 | 3 | 2 | 1 |

TABLE I: Particle content of the manifest left-right symmetric theories.

In our model, neutrino mass is explained via low scale inverse seesaw mechanism. We take interest in inverse seesaw mechanism since it allows large light-heavy neutrino mixing and this mixing facilitates the interaction of singly charged vector boson with heavy neutrinos which contributes positively to $\Delta a_{\mu}$. The model has three sets of extra sterile fermions and extra scalars apart from the usual particle content. While the extra sterile fermions are needed to make the theory anomaly free, the extra scalars help in breaking the extra $U(1)_{L_{\mu}-L_{\tau}}$ symmetry and also help in achieving inverse seesaw. The particle content of the model is given in Table II. The model is governed by the gauge group,

$$G_{LR}^{\mu\tau} \equiv SU(2)_{L} \times SU(2)_{R} \times U(1)_{B-L} \times SU(3)_{C} \times U(1)_{L_{\mu}-L_{\tau}}$$ (2)
At first, the spontaneous symmetry breaking (SSB) of $G_{\mu \tau}^{LR}$ down to left-right theory $G_{LR}$ is achieved by assigning a non-zero VEV to a scalar $\chi$ which is singlet under left-right symmetry but charged under $U(1)_{L\mu - L\tau}$, with non-zero value of $L_\mu - L_\tau$ but singlet under usual left-right gauge symmetry. Further, the SSB of LRSM to SM can happen in the following three ways:

- with Higgs doublets $H_L \oplus H_R$,
- with scalar triplets $\Delta_L \oplus \Delta_R$
- with the combination of doublets and triplets $H_L \oplus H_R$ and $\Delta_L \oplus \Delta_R$.

Now, as usual the SSB of SM to low energy theory occurs when the scalar bidoublet $\Phi$ takes non-zero vev and that generates masses for charged leptons and quarks. Before we move on to the working of inverse seesaw mechanism in the considered model, let’s have a clear picture of how the generation of neutrino mass is affected within various symmetry breaking of LRSM-SM chains.

| Fields | $SU(2)_L$ | $SU(2)_R$ | $U(1)_{B-L}$ | $SU(3)_C$ | $U(1)_{L\mu - L\tau}$ |
|--------|-----------|-----------|-------------|-----------|---------------------|
| Fermions | \(\ell_{e_L}\) | 2 | 1 | -1 | 1 | 0 |
|         | \(\ell_{\mu_L}\) | 2 | 1 | -1 | 1 | 1 |
|         | \(\ell_{\tau_L}\) | 2 | 1 | -1 | 1 | -1 |
|         | \(\ell_{e_R}\) | 1 | 2 | -1 | 1 | 0 |
|         | \(\ell_{\mu_R}\) | 1 | 2 | -1 | 1 | 1 |
|         | \(\ell_{\tau_R}\) | 1 | 2 | -1 | 1 | -1 |
| Scalars | $\Phi$ | 2 | 2 | 0 | 1 | 0 |
|         | $H_L$ | 2 | 1 | 1 | 1 | 0 |
|         | $H_R$ | 1 | 2 | 1 | 1 | 0 |
|         | $\chi$ | 1 | 1 | 0 | 1 | 1 or 2 |

TABLE II: Particle content of left-right theories extended with $U(1)_{L\mu - L\tau}$ gauge symmetry where fermion sector is limited to leptons and scalar sector contains the bidoublet $\Phi$, doublets $H_{L,R}$ and a singlet $\chi$.

A. Neutrino Masses with LRSM-SM symmetry breaking via $H_R, H_L$

In this case, $H_R$ breaks the LR symmetry while $H_L$ is required for left-right invariance. The other two symmetry breaking steps are done with $\chi$ and $\Phi$ as mentioned earlier. The leptons and
scalars are displayed in Table I. Even though this framework holds a minimal (in terms of $SU(2)$ representation) scalar spectrum it cannot provide Majorana mass for light neutrinos and thus forbids any signature of lepton number violation. The scalar bidoublet $\Phi$ gives masses to charged fermions. The allowed Yukawa interactions for leptons are given by,

$$
-\mathcal{L}_{Yuk} \supset \ell_e L \left[ Y_{\ell} \Phi + \tilde{Y}_{\ell} \tilde{\Phi} \right] \ell_e R + \ell_{\mu} L \left[ Y_{\ell} \Phi + \tilde{Y}_{\ell} \tilde{\Phi} \right] \ell_{\mu} R + \ell_{\tau} L \left[ Y_{\ell} \Phi + \tilde{Y}_{\ell} \tilde{\Phi} \right] \ell_{\tau} R + \text{h.c.}
$$

The charged fermion as well as light neutrino mass matrices are found to be diagonal in structure due to presence of $U(1)_{L_{\mu} - L_{\tau}}$ gauge symmetry. The non-zero masses for light neutrinos (which are Dirac fermions) can be explained by adjusting Yukawa couplings through the non-zero VEVs of scalar bidoublet. From the Yukawa interactions given in Eq. (3); with $Y_{\ell} \ll \tilde{Y}_{\ell}, v_2 \ll v_1$, the masses for charged leptons and the light neutrinos can be expressed as,

$$
M_{\ell} \simeq \tilde{Y}_{\ell} v_1^2, \quad M_{\nu}^D \simeq v_1 \left( Y_{\ell} + M_{\ell} v_2 v_1^2 \right).
$$

B. Neutrino Masses with LRSM-SM symmetry breaking via $\Delta_R, \Delta_L$

In Table II if we replace the doublets $H_L, H_R$ by triplets $\Delta_L$ and $\Delta_R$ then the model offers a better possibility from phenomenology point of view since in this case Majorana masses can be generated for light and heavy neutrinos. If the symmetry breaking occurs at few TeV scale, these Majorana neutrinos can mediate neutrinoless double beta decay process whose observation would confirm total lepton number violation in nature. Lepton number violation can also be probed via smoking-gun same-sign dilepton signatures at collider experiments. The interaction terms involving scalar triplets and leptons in the left-right theories with extra $U(1)$ symmetry are given by

$$
-\mathcal{L}_{Yuk} \supset \ell_e L \left[ Y_{\ell} \Phi + \tilde{Y}_{\ell} \tilde{\Phi} \right] \ell_e R + \ell_{\mu} L \left[ Y_{\ell} \Phi + \tilde{Y}_{\ell} \tilde{\Phi} \right] \ell_{\mu} R + \ell_{\tau} L \left[ Y_{\ell} \Phi + \tilde{Y}_{\ell} \tilde{\Phi} \right] \ell_{\tau} R + \text{h.c.}
$$

$$
+ \left[ f_{ee} (\ell_e \ell_e)^c \ell_e \ell_e + f_{\mu\tau} (\ell_{\mu} \ell_{\tau})^c \ell_{\tau} \ell_{\tau} + f_{\tau\mu} (\ell_{\tau} \ell_{\mu})^c \ell_{\mu} \ell_{\mu} \right] \Delta_L
$$

$$
+ \left[ f_{ee} (\ell_e \ell_e)^c \ell_e \ell_e + f_{\mu\tau} (\ell_{\mu} \ell_{\tau})^c \ell_{\tau} \ell_{\tau} + f_{\tau\mu} (\ell_{\tau} \ell_{\mu})^c \ell_{\mu} \ell_{\mu} \right] \Delta_R + \text{h.c.}
$$

Using Eq. (5), the structure of the masses for neutral leptons in the basis $(\nu_L, N^c_R)$ can be written as,

$$
\mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix},
$$

where, $M_D$ represents Dirac neutrino mass matrix, $M_L(M_R)$ denotes Majorana mass matrix arising from the non-zero VEV of LH (RH) scalar triplet. The masses for $M_D, M_L$ and $M_R$ can be written
explicitly as follows (considering \( f_{\mu\tau} = f_{\tau\mu} \) and \( f_{\mu\mu} = f_{\tau\tau} \)),

\[
M_D = \begin{pmatrix}
Y_{11}v_2 + \tilde{Y}_{11}v_1 & 0 & 0 \\
0 & Y_{22}v_2 + \tilde{Y}_{22}v_1 & 0 \\
0 & 0 & Y_{33}v_2 + \tilde{Y}_{33}v_1
\end{pmatrix} = \begin{pmatrix}
a & 0 & 0 \\
0 & b & 0 \\
0 & 0 & c
\end{pmatrix},
\]

\[
M_{L,R} = \begin{pmatrix}
f_{ee} & 0 & 0 \\
0 & f_{\mu\tau} & v_{L,R}/\sqrt{2} \\
0 & f_{\mu\tau} & v_{L,R}/\sqrt{2}
\end{pmatrix}
\]

(7)

Now using seesaw approximation \( M_R \gg M_D \) and \( M_L \to 0 \), the light neutrino mass can be generated via type-I seesaw formula as shown below.

\[
m^I_\nu = -M_D M_R^{-1} M_D^T
\]

\[
= \begin{pmatrix}
a & 0 & 0 \\
0 & b & 0 \\
0 & 0 & c
\end{pmatrix} \cdot \begin{pmatrix}
f_{ee}v_R/\sqrt{2} & 0 & 0 \\
0 & 0 & f_{\mu\tau}v_R/\sqrt{2} \\
0 & f_{\mu\tau}v_R/\sqrt{2} & 0
\end{pmatrix}^{-1} \cdot \begin{pmatrix}
a & 0 & 0 \\
0 & b & 0 \\
0 & 0 & c
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\sqrt{2a^2f_{ee}v_R} & 0 & 0 \\
0 & 0 & \sqrt{2bcf_{\mu\tau}v_R} \\
0 & 0 & \sqrt{2bcf_{\mu\tau}v_R}
\end{pmatrix}
\]

(8)

From this light neutrino mass matrix \( m^I_\nu \), the corresponding mass eigenvalues for light neutrino mass eigenstates can be obtained which are, \( \{-\sqrt{2bcf_{\mu\tau}v_R}, \sqrt{2bcf_{\mu\tau}v_R}, \sqrt{2a^2f_{ee}v_R}\} \). However, two mass eigenstates with eigenvalues \( \sqrt{2bcf_{\mu\tau}v_R} \) (ignoring the negative sign) are degenerate here, which implies either the solar neutrino mass (\( \Delta m^2_{\text{sol}} \)) or the atmospheric neutrino mass (\( \Delta m^2_{\text{atm}} \)) vanishes. This contradicts the neutrino oscillation data.

This degeneracy can be wiped out by introducing another pair of triplet scalars \( \Delta'_L \oplus \Delta'_R \) with \( L_{\mu-\tau} = 2 \). Now we can write additional Yukawa terms allowed by the \( U(1)_{L_{\mu-\tau}} \) symmetry as,

\[
-\mathcal{L}_{\text{ yuk}}^\text{new} \supset f_{\mu\mu}(\ell_{LR}\Delta'_{LR}\ell_{LR} + \ell_{\tau R}\Delta'_{\tau R}\ell_{\tau R}) + R \leftrightarrow L
\]

(9)

With these new permissible terms in the Yukawa sector, we can write the corresponding \( M'_{L,R} \) matrix as,

\[
M'_{L,R} = \begin{pmatrix}
f_{ee}v_{L,R}/\sqrt{2} & 0 & 0 \\
0 & f_{\mu\mu}v_{L,R}/\sqrt{2} & f_{\mu\tau}v_{L,R}/\sqrt{2} \\
0 & f_{\mu\tau}v_{L,R}/\sqrt{2} & f_{\mu\mu}v_{L,R}/\sqrt{2}
\end{pmatrix}
\]

(10)
where $v'_{L,R} = \langle \Delta_{L,R}' \rangle$. Now using the seesaw approximation $M'_{R} \gg M_D$ and $M'_{L} \to 0$, the light neutrino mass matrix can be expressed via type-I seesaw formula as,

$$ m_{\nu}^{T} = -M_D M_{R}^{-1} M_{D}^{T} $$

$$ = \begin{pmatrix}
\sqrt{2} a^2 f_{\nu e} v_R & 0 & 0 \\
0 & \sqrt{2} b^2 f_{\mu e} v'_{R} & \sqrt{2} b e f_{\mu e} v_R \\
0 & \sqrt{2} b e f_{\mu e} v_R & -f_{\mu e} v_R + f_{\mu e} v_R' \\
\end{pmatrix} \tag{11} $$

Here the mass eigenvalues which are non-degenerate are, $\{\sqrt{2} a^2 f_{\nu e} v_R, m^I_{\nu A} \pm m^I_{\nu B}\}$ with

$$ m^I_{\nu A} = -b^2 f_{\mu e} v'_{R} - c^2 f_{\mu e} v''_{R} $$

$$ m^I_{\nu B} = \sqrt{2}(b^2 f_{\mu e} v''_{R} - f_{\mu e} v_B') $$

Though the introduction of two extra scalar triplets $\Delta_{L}' \oplus \Delta_{R}'$ saves us from apparent inconsistency in the explanation of current-day neutrino oscillation data, the particle content of the model becomes crowded and it no more remains minimal.

### III. LRSM INVERSE SEESAW (LISS) FOR NEUTRINO MASSES

We have already discussed in previous section that the sub-eV scale neutrino masses can be generated either by canonical see-saw mechanism which requires a very high ($> 10^{14}$ GeV) seesaw scale and therefore cannot be verified by colliders or with very much suppressed value of Dirac neutrino Yukawa coupling. As an alternative, we explain one of the low scale seesaw mechanism, i.e, LRSM inverse seesaw (LISS) in our model where the left-right symmetry breaking occurs at few TeV. This symmetry breaking generates TeV scale masses for $W_R, Z_R$ gauge bosons which fall within the LHC range and the inverse seesaw mechanism provides large light-heavy neutrino mixing.

As a result the mixing of sub-TeV scale heavy neutrinos with sub-eV scale light neutrinos offers,

- sizeable contribution to muon $g-2$ anomaly arising form purely left-handed currents with the exchange of sub-TeV masses for sterile neutrinos in LISS scheme,
- dominant contribution to lepton flavour violating (LFV) decays, non-unitarity effects in leptonic sector,
- interesting collider signatures verifiable at LHC.

For implementing LISS, we consider an extra sterile neutrino $S_L$ per generation along with the usual leptons, scalars (bidoublet $\Phi$, doublets $H_{L,R}$ and $\chi$) presented in Table II. The relevant Yukawa
interaction Lagrangian for LISS invariant under $U(1)_{L_{\mu}-L_{\tau}}$ symmetry is given as sum of different components,

$$-\mathcal{L}_{\text{LISS}} = \mathcal{L}_{\nu N_R} + \mathcal{L}_{N_R S_L} + \mathcal{L}_{S_L S_L},$$

where, the individual components are given as follows:

**Generic Dirac Neutrino Mass Matrix $\mathcal{L}_{\nu N_R}$:**

The usual Dirac Yukawa interaction Lagrangian that allows Dirac mass terms for charged leptons and neutrinos) consistent with the $U(1)_{L_{\mu}-L_{\tau}}$ gauge symmetry is given by,

$$\mathcal{L}_{\nu N_R} = \ell \bar{L} (Y \Phi + \tilde{Y} \tilde{\Phi}) \ell_R$$

$$= \bar{\ell}_L [M_i^L]_{\ell \tau} \ell_{\tau R} + \bar{\ell}_L [M_i^\nu]_{\ell \tau} \ell_{\tau R} + \bar{\ell}_L [M_i^D]_{\ell \tau} \ell_{\tau R}$$

(13)

where, $M_i = M_L$, $M_D^\nu \equiv M_D$ are the corresponding Dirac mass matrices for charged leptons and neutrinos respectively. The imposition of extra $U(1)_{L_{\mu}-L_{\tau}}$ symmetry to the left-right theories results in diagonal Dirac mass matrices for charged leptons and neutrinos as,

$$M_L = \begin{pmatrix} Y_{11} v_1 + \tilde{Y}_{11} v_2 & 0 & 0 \\ 0 & Y_{22} v_1 + \tilde{Y}_{22} v_2 & 0 \\ 0 & 0 & Y_{33} v_1 + \tilde{Y}_{33} v_2 \end{pmatrix}$$

$$M_D = \begin{pmatrix} Y_{11} v_2 + \tilde{Y}_{11} v_1 & 0 & 0 \\ 0 & Y_{22} v_2 + \tilde{Y}_{22} v_1 & 0 \\ 0 & 0 & Y_{33} v_2 + \tilde{Y}_{33} v_1 \end{pmatrix} = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

(14)

**Dirac Mass term between $N_R$ and $S_L$, $\mathcal{L}_{N_R S_L}$:**

The corresponding Yukawa term gives rise to the mixing matrix $M$ between $N_R$ and $S_L$ as,

$$\mathcal{L}_{N_R S_L} = Y_{RS} \bar{\ell}_R S_L = Y_{RS} \langle \tilde{H}_R \rangle \left[ \bar{\ell}_{\tau R} S_{\tau L} + \bar{\ell}_{\mu R} S_{\mu L} + \bar{\ell}_{\ell R} S_{\ell L} \right]$$

(15)

The corresponding mixing matrix is also found to be diagonal as,

$$M = \begin{pmatrix} M_{11} & 0 & 0 \\ 0 & M_{22} & 0 \\ 0 & 0 & M_{33} \end{pmatrix}$$

whose diagonal entries are proportional to $\langle \tilde{H}_R \rangle = v_R$.

**Bare Majorana Mass term for $S_L$, $\mathcal{L}_{S_L S_L}$:**

Now, we focus on the generation of bare Majorana mass term for sterile neutrinos and the $U(1)_{L_{\mu}-L_{\tau}}$
gauge group allows the terms involving extra sterile neutrinos as,

\[
\mathcal{L}_{S_L S_L} = \mu S_L^T S_L \\
= \left[ \mu_{ee} S_L^T S_{eL} + \mu_{\mu\tau} S_L^T S_{\mu L}, \mu_{\mu\tau} S_L^T S_{\tau L} \right]
\]

So the bare Majorana mass matrix structure for extra sterile neutrinos can be expressed as,

\[
\mu = \begin{pmatrix}
\mu_{ee} & 0 & 0 \\
0 & 0 & \mu_{\mu\tau} \\
0 & \mu_{\mu\tau} & 0
\end{pmatrix}
\]

(17)

Thus, the complete \(9 \times 9\) neutral fermion mass matrix in the basis of \((\nu_L, N_R, S_L)\) is read as,

\[
\mathcal{M} = \begin{pmatrix}
0 & M_D & 0 \\
M_D^T & 0 & M_T \\
0 & M & \mu
\end{pmatrix}
\]

(18)

Using eq. 18 with mass hierarchy \(M > M_D >> \mu\), we can write the expression for Majorana mass \((m_\nu)\) for light neutrinos and pseudo-Dirac mass term \((m_H)\) for heavy neutrinos in LISS as,

\[
m_\nu = \left( \frac{M_D}{M} \right) \mu \left( \frac{M_D}{M} \right)^T
\]

(19)

\[
m_H = - (\pm M - \mu/2)
\]

(20)

Even with large \(M(\sim \text{TeV scale})\), we can have sizeable light-heavy neutrino mixing \((M_D/M)\) which can give rise to large lepton-flavor violating (LFV) decay channels as \(\mu \rightarrow e\gamma, \tau \rightarrow \mu\). Now from eq. 19 in this LRSM inverse seesaw approximation, we can express the light neutrino mass matrix as,

\[
m_\nu^{\text{LISS}} = \begin{pmatrix}
\frac{\sigma_{\mu ee}}{M_{11}} & 0 & 0 \\
0 & 0 & \frac{\sigma_{\mu\mu\tau}}{M_{22} M_{33}} \\
0 & \frac{\sigma_{\mu\mu\tau}}{M_{22} M_{33}} & 0
\end{pmatrix}
\]

(21)

which delivers light neutrino mass eigenstates with degenerate eigenvalues \(\{\frac{\sigma_{\mu ee}}{M_{11}}, \frac{\sigma_{\mu\mu\tau}}{M_{22} M_{33}}, \frac{\sigma_{\mu\mu\tau}}{M_{22} M_{33}}\}\) similar to the previous situation [II.B]. Since the mass matrices \(M_D\) and \(M\) are diagonal in structure, the non-degenerate light neutrino masses consistent with observed values \(\Delta m_{\text{sol}}^2\) and \(\Delta m_{\text{atm}}^2\) can be achieved with the possible modification in \(\mu\) matrix. The modification in the matrix structure of \(\mu\) matrix can be implemented with the inclusion of extra terms in the \(\mu\) matrix which may be either of off-diagonal or diagonal in nature. Therefore, the extra singlet scalar \(\chi\) with non-zero \(U(1)_{L_\mu-L_\tau}\) charge which was originally introduced for spontaneous symmetry breaking of \(U(1)_{L_\mu-L_\tau}\) symmetry
can remove this degeneracy without affecting the usual left-right symmetry. We call this scenario as ‘Extended LRSM with Inverse Seesaw (ELISS)’. The introduction of $\chi$ allows additional Yukawa-like terms in the Lagrangian and now the total Lagrangian for ELISS scenario becomes,

$$\mathcal{L}_{\text{ELISS}} = \mathcal{L}_{\text{LISS}} + \mathcal{L}_\chi$$

where $\mathcal{L}_\chi$ is the correction terms to the LISS lagrangian due to the introduction of new scalar $\chi$.

**$\mathcal{L}_\chi$ responsible for off-diagonal correction to $\mu$ matrix:**

Considering the extra scalar $\chi$ with $U(1)_{L_\mu-L_\tau}$ charge 1, the modified Lagrangian with Yukawa-like terms can be written as,

$$\mathcal{L}_\chi \supset \mu_{e\mu} S^T_{e\mu} S_{\mu L} \chi^* + \mu_{e\mu} S^T_{e\mu} S_{\tau L} \chi + \mu_{e\mu} S^T_{e\mu} S_{e L} \chi^* + \mu_{e\mu} S^T_{e\mu} S_{e L}$$

which modifies the structure of the light neutrino mass matrix now looking like,

$$m_{\nu}^{\text{ELISS}} = \begin{pmatrix}
\frac{a^2_{\mu\tau}}{M_{11}^2} & \frac{a_{\mu\tau}}{M_{11} M_{22}} & \frac{a_{\mu\mu}}{M_{11} M_{33}} \\
\frac{a_{\mu\tau}}{M_{11} M_{22}} & \frac{b_{\mu\tau}}{M_{22} M_{33}} & 0 \\
\frac{a_{\mu\mu}}{M_{11} M_{33}} & \frac{b_{\mu\tau}}{M_{22} M_{33}} & 0
\end{pmatrix}$$

Now, if we consider $M_D$ and $M$ as constant identity mass matrices i.e., $M_D = a I_{3\times3}$ and $M = M_{11} I_{3\times3}$, then $m_{\nu}^{\text{ELISS}} \sim \mu$. Since light neutrino mass matrix can be diagonalised by $U_{\text{PMNS}}$ matrix \[1\],

$$|U_{\text{PMNS}}| \approx \begin{pmatrix}
0.814 & 0.554 & 0.147 \\
0.329 & 0.572 & 0.717 \\
0.432 & 0.555 & 0.742
\end{pmatrix}$$

we can diagonalise $\mu$ by $U_{\text{PMNS}}$ and rewrite the mass matrix as,

$$m_{\nu}^{\text{ELISS}} = \begin{pmatrix}
\frac{a^2_{\mu\tau}}{M_{11}^2} & \frac{a^2_{\mu\tau}}{M_{11}^2} & \frac{a^2_{\mu\mu}}{M_{11}^2} \\
\frac{a^2_{\mu\tau}}{M_{11}^2} & 0 & \frac{a^2_{\mu\tau}}{M_{11}^2} \\
\frac{a^2_{\mu\mu}}{M_{11}^2} & \frac{a^2_{\mu\tau}}{M_{11}^2} & 0
\end{pmatrix}$$

whose corresponding eigenvalues are $\{-a^2_{\mu\tau}, m_{\nu}^{\text{ELISSa}} \pm m_{\nu}^{\text{ELISSb}}\}$ with

$$m_{\nu}^{\text{ELISSa}} = \frac{a^2}{2M_{11}} (\mu_{ee} + \mu_{\mu\tau}),$$

$$m_{\nu}^{\text{ELISSb}} = \frac{a^2}{2M_{11}} \sqrt{\mu_{ee}^2 + 8\mu_{e\mu}^2 - 2\mu_{ee}\mu_{\mu\tau} + \mu_{\mu\tau}^2}$$

**$\mathcal{L}_\chi$ responsible for diagonal correction to $\mu$ matrix:**

Similarly, if we consider $\chi$ with $U(1)_{L_\mu-L_\tau}$ charge 2, then the Lagrangian can be written as,

$$\mathcal{L}_\chi \supset \mu_{\mu\mu} \left[ S^T_{\mu L} S_{\mu L} \psi^* + S^T_{\tau L} S_{\tau L} \psi \right]$$

(27)
We consider same couplings for both the families ($S_\mu$ and $S_\tau$) for simplicity. Now the modified light neutrino mass matrix in this framework can be expressed as,

$$m^{\text{ELISS}}_\nu = \begin{pmatrix}
\frac{a^2_{\mu\mu}}{M_{11}^2} & 0 & 0 \\
0 & \frac{b^2_{\mu\mu}}{M_{22}^2} & \frac{bc_{\mu\tau}}{M_{22}M_{33}} \\
0 & \frac{bc_{\mu\tau}}{M_{22}M_{33}} & \frac{c^2_{\mu\mu}}{M_{33}^2}
\end{pmatrix}$$  \hspace{1cm} (28)

with mass eigenvalues \(\left\{\frac{a^2_{\mu\mu}}{M_{11}^2}, m^{\text{ELISS}}_\nu^a \pm m^{\text{ELISS}}_\nu^b\right\}\) where,

$$m^{\text{ELISS}}_\nu^a = \frac{c^2 M_{22}^2 \mu_{\mu\mu}^2 + b^2 M_{33}^2 \mu_{\mu\mu}}{2 M_{22}^2 M_{33}^2},$$

$$m^{\text{ELISS}}_\nu^b = \sqrt{c^4 M_{22}^4 \mu_{\mu\mu}^4 - 2b^2 c^2 M_{22}^2 M_{33}^2 \mu_{\mu\mu}^2 + b^4 M_{33}^4 \mu_{\mu\mu}^2 + 4b^2 c^2 M_{22}^2 M_{33}^2 \mu_{\mu\tau}^2}.$$

We found that, both the cases i.e with the diagonal as well as off-diagonal corrections to $\mu$-matrix successfully explain current-day neutrino oscillation data by generating non-degenerate light neutrino masses.

**IV. PREDICTION ON MUON ($g-2$) ANOMALY**

For a comprehensive review on new physics scenarios explaining muon ($g-2$) anomaly one may refer [13, 14, 68]. Most of these works predict that new light gauge bosons and light neutral scalars are good candidates for addressing the anomaly since they contribute positively to $\Delta a_{\mu}$. In our model, new contributions to muon ($g-2$) anomaly arise from the interactions of;

- singly charged gauge bosons with heavy neutral fermions,
- neutral vector boson with singly charged fermions,
- singly charged scalars with neutral fermion,
- neutral scalars with muons,
- extra light new gauge boson $Z_{\mu\tau}$ with muons.

In the following we study analytically all these new physics contributions to $\Delta a_{\mu}$ and numerically estimate the individual contributions in the next section. Notably, for the calculation of $\Delta a_{\mu}$ we neglect the flavor mixing as they give negligible correction to the anomaly [68]. Another important point to recall here is that inverse seesaw mechanism which explains neutrino mass in this model also allows large light-heavy neutrino mixing due to which the contribution coming from the charged gauge boson interaction with heavy neutral fermion becomes sizeable.
A. Gauge boson contribution

Before moving on to the Feynman diagrams mediated by gauge bosons, we write the basic charge current (CC) interaction Lagrangian for leptons within left-right theories.

$$\mathcal{L}_{cc}^\ell = \sum_{\alpha = e, \mu, \tau} \left[ \frac{g_L}{\sqrt{2}} L_{\alpha L} \gamma_\beta \ell_{\alpha L} W_L^\beta + \frac{g_R}{\sqrt{2}} L_{\alpha R} \gamma_\beta \ell_{\alpha R} W_R^\beta \right] + \text{h.c.} \quad (29)$$

For Inverse Seesaw (ISS) mechanism\,[69, 70], the flavour eigenstates $\nu_L$ and $N_R$ can be expressed in terms of admixture of mass eigenstates ($\nu_i$, $\xi_j$) as follows,

$$\nu_{\mu L} = V_{\mu i}^{\nu\nu} \nu_i + V_{\mu j}^{\nu\xi} \xi_j \quad (30)$$

$$N_{\mu R} = V_{\mu i}^{N\nu} \nu_i + V_{\mu j}^{N\xi} \xi_j \quad (31)$$

where $i = 1, 2, 3$ goes over physical states for light neutrinos and $j = 1, 2, ..., 6$ runs over heavy states forming three pairs of pseudo-Dirac neutrinos. Using eq. (30) in the charge current interaction lagrangian given in eq. (29), we present the vector and axial vector couplings ($g_v$ and $g_a$) in Table III.

| Interaction Vertex | $g_v$ | $g_a$ | Interaction Vertex | $g_v$ | $g_a$ |
|-------------------|-------|-------|-------------------|-------|-------|
| $\overline{\nu}_1 \mu W_L^{+}$ | $\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $-\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $\overline{\nu}_1 \mu W_R^{+}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ |
| $\overline{\nu}_2 \mu W_L^{+}$ | $\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $-\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $\overline{\nu}_2 \mu W_R^{+}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ |
| $\overline{\nu}_3 \mu W_L^{+}$ | $\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $-\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $\overline{\nu}_3 \mu W_R^{+}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ |
| $\overline{\nu}_4 \nu W_L^{+}$ | $\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $-\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $\overline{\nu}_4 \nu W_R^{+}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ |
| $\overline{\nu}_5 \nu W_L^{+}$ | $\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $-\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $\overline{\nu}_5 \nu W_R^{+}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ |
| $\overline{\nu}_6 \nu W_L^{+}$ | $\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $-\frac{g_L}{\sqrt{2}} V_{\mu i}^{\nu\nu}$ | $\overline{\nu}_6 \nu W_R^{+}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ | $\frac{g_R}{\sqrt{2}} V_{\mu i}^{N\nu}$ |

**TABLE III:** Relevant vector and axial vector couplings for muon with $W_L$, $W_R$ gauge bosons and physical neutral fermion states within the inverse seesaw (ISS) scenario.

In inverse seesaw scheme, the light neutrinos are Majorana in nature while heavy neutrinos are pseudo-Dirac. Alternatively, in extended inverse seesaw scenario (EISS)\,[69, 70] both light neutrino
νL as well as heavy neutrinos S_L, N_R are purely Majorana in nature. Thus, the flavour eigenstates ν_L and N_R can be expressed in terms of admixture of mass eigenstates (ν_i, S_i, N_i) in the following way,

\[ \nu_{\mu L} = V^{\nu\nu}_{\mu i} \nu_i + V^{\nu S}_{\mu i} S_i + V^{\nu N}_{\mu i} N_i \]  \hspace{1cm} (32)

\[ N_{\mu R} = V^{N\nu}_{\mu i} \nu_i + V^{NS}_{\mu i} S_i + V^{NN}_{\mu i} N_i \]  \hspace{1cm} (33)

where i = 1, 2, 3 goes over physical states. For EISS we present the vector and axial vector couplings in Table IV.

| Interaction Vertex | \( g_{\nu 2} \) | \( g_{\mu 2} \) | Interaction Vertex | \( g_{\nu 1} \) | \( g_{\mu 1} \) |
|---------------------|----------------|----------------|---------------------|----------------|----------------|
| \( \nu_1 \mu W^+_L \) | \( \frac{g_L}{2\sqrt{2}} V^{\nu\nu}_{\mu 1} \) | - \( \frac{g_L}{2\sqrt{2}} V^{\nu S}_{\mu 1} \) | \( \nu_1 \mu W^+_R \) | \( \frac{g_R}{2\sqrt{2}} V^{N\nu}_{\mu 1} \) | \( \frac{g_R}{2\sqrt{2}} V^{N S}_{\mu 1} \) |
| \( \nu_2 \mu W^+_L \) | \( \frac{g_L}{2\sqrt{2}} V^{\nu\nu}_{\mu 2} \) | - \( \frac{g_L}{2\sqrt{2}} V^{\nu S}_{\mu 2} \) | \( \nu_2 \mu W^+_R \) | \( \frac{g_R}{2\sqrt{2}} V^{N\nu}_{\mu 2} \) | \( \frac{g_R}{2\sqrt{2}} V^{N S}_{\mu 2} \) |
| \( \nu_3 \mu W^+_L \) | \( \frac{g_L}{2\sqrt{2}} V^{\nu\nu}_{\mu 3} \) | - \( \frac{g_L}{2\sqrt{2}} V^{\nu S}_{\mu 3} \) | \( \nu_3 \mu W^+_R \) | \( \frac{g_R}{2\sqrt{2}} V^{N\nu}_{\mu 3} \) | \( \frac{g_R}{2\sqrt{2}} V^{N S}_{\mu 3} \) |
| \( S_1 \mu W^+_L \) | \( \frac{g_L}{2\sqrt{2}} V^{\nu S}_{\mu 1} \) | - \( \frac{g_L}{2\sqrt{2}} V^{\nu S}_{\mu 1} \) | \( S_1 \mu W^+_R \) | \( \frac{g_R}{2\sqrt{2}} V^{N\nu}_{\mu 1} \) | \( \frac{g_R}{2\sqrt{2}} V^{N S}_{\mu 1} \) |
| \( S_2 \mu W^+_L \) | \( \frac{g_L}{2\sqrt{2}} V^{\nu S}_{\mu 2} \) | - \( \frac{g_L}{2\sqrt{2}} V^{\nu S}_{\mu 2} \) | \( S_2 \mu W^+_R \) | \( \frac{g_R}{2\sqrt{2}} V^{N\nu}_{\mu 2} \) | \( \frac{g_R}{2\sqrt{2}} V^{N S}_{\mu 2} \) |
| \( S_3 \mu W^+_L \) | \( \frac{g_L}{2\sqrt{2}} V^{\nu S}_{\mu 3} \) | - \( \frac{g_L}{2\sqrt{2}} V^{\nu S}_{\mu 3} \) | \( S_3 \mu W^+_R \) | \( \frac{g_R}{2\sqrt{2}} V^{N\nu}_{\mu 3} \) | \( \frac{g_R}{2\sqrt{2}} V^{N S}_{\mu 3} \) |
| \( \overline{N}_1 \mu W^+_L \) | \( \frac{g_L}{2\sqrt{2}} V^{\nu N}_{\mu 1} \) | - \( \frac{g_L}{2\sqrt{2}} V^{\nu N}_{\mu 1} \) | \( \overline{N}_1 \mu W^+_R \) | \( \frac{g_R}{2\sqrt{2}} V^{N N}_{\mu 1} \) | \( \frac{g_R}{2\sqrt{2}} V^{N N}_{\mu 1} \) |
| \( \overline{N}_2 \mu W^+_L \) | \( \frac{g_L}{2\sqrt{2}} V^{\nu N}_{\mu 2} \) | - \( \frac{g_L}{2\sqrt{2}} V^{\nu N}_{\mu 2} \) | \( \overline{N}_2 \mu W^+_R \) | \( \frac{g_R}{2\sqrt{2}} V^{N N}_{\mu 2} \) | \( \frac{g_R}{2\sqrt{2}} V^{N N}_{\mu 2} \) |
| \( \overline{N}_3 \mu W^+_L \) | \( \frac{g_L}{2\sqrt{2}} V^{\nu N}_{\mu 3} \) | - \( \frac{g_L}{2\sqrt{2}} V^{\nu N}_{\mu 3} \) | \( \overline{N}_3 \mu W^+_R \) | \( \frac{g_R}{2\sqrt{2}} V^{N N}_{\mu 3} \) | \( \frac{g_R}{2\sqrt{2}} V^{N N}_{\mu 3} \) |

TABLE IV: Relevant vector and axial vector couplings for muon with \( W_L, W_R \) gauge bosons and physical neutral fermion states within the extended inverse seesaw (EISS) scenario.

The diagrams in Fig. 1 are mediated by singly charged right-handed and left-handed gauge bosons \( W_R, W_L \) interacting with muons. Here \( \xi \) represents the heavy neutrino states in mass basis for inverse seesaw case. The contribution arising from singly charged vector boson bosons are discussed in [71][74].

Fig.1(a): Due to \( W_R \) mediated contribution \( \Delta a_\mu(\xi, W_R) \).

For calculating its contribution, we start by sorting out the relevant interaction terms for this diagram.

\[ \mathcal{L}_{\text{int}} = g_{\nu 1} W^+_\mu \bar{\nu}_\mu \gamma^\mu \mu + g_{\mu 1} W^+_\mu \bar{\nu}_\mu \gamma^\mu \gamma^5 \mu + h.c. \]  \hspace{1cm} (34)
FIG. 1: Feynman diagrams for the interaction of singly charged vector bosons: in left-panel due to the mediation of singly charged right-handed gauge boson $W_R$ with heavy neutrinos and in right-panel due to the mediation of singly charged left-handed gauge boson $W_L$ with exchange of heavy neutrinos. The $W_L$ mediated diagram with exchange of heavy neutrinos gives sizeable contribution in ISS scheme.

The contribution arising from this diagram to the anomalous magnetic moment can be determined by the following expression.

$$\Delta a_\mu(\xi, W_R) \simeq \frac{1}{8\pi^2} \frac{m_\mu^2}{m_{W_R}^2} \int_0^1 dx \frac{g_{\nu_1}^2 P_{\nu_1}(x) + g_{a1}^2 P_{a1}(x)}{e^2 \lambda^2 (1 - x) (1 - \epsilon^2 x) + x}$$

(35)

where, $m_\mu$ is the mass of muon, $m_{W_R}$ is the mass of right-handed charged gauge boson $W_R$, $\epsilon \equiv (m_\nu/ m_\mu)$, $\lambda \equiv (m_\mu/ m_{W_R})$, and

$$P_{\nu_1}(x) = 2x^2(1 + x - 2\epsilon) - \lambda^2(1 - \epsilon)^2 x(1 - x)(x + \epsilon)$$

$$P_{a1}(x) = 2x^2(1 + x + 2\epsilon) - \lambda^2(1 + \epsilon)^2 x(1 - x)(x - \epsilon)$$

After simplifying the integrations the expression can be rewritten as,

$$\Delta a_\mu(\xi, W_R) \simeq \frac{1}{4\pi^2} \frac{m_\mu^2}{m_{W_R}^2} \left[ |g_{\nu_1}|^2 \left( \frac{5}{6} - \epsilon \right) + |g_{a1}|^2 \left( \frac{5}{6} + \epsilon \right) \right]; \quad \text{with} \quad m_{W_R} \gg m_\mu. \quad (36)$$

Here we have, $|g_{\nu_1}| = |g_{a1}| = \frac{g_R}{2\sqrt{2}}$ (as given in Table III with $\mathcal{O}(1)$ neutrino mixing) and with these values we can rewrite Eq.36 as,

$$\Delta a_\mu(W_R) \simeq 2.3 \times 10^{-11} \left( \frac{g_R}{g_L} \right)^2 \left( \frac{1 \text{ TeV}}{m_{W_R}} \right)^2 \sum_{i=1,\ldots,6} |V_{\mu i}^{N \xi}|^2$$

(37)

Fig.1(b): Due to $W_L$ and $\xi$ mediation with large light-heavy neutrino mixing $\Delta a_\mu(\xi, W_L)$

Similarly, for $W_L$ interacting with heavy neutrino the contribution to muon anomalous magnetic moment can be expressed as,

$$\Delta a_\mu(\xi, W_L) \simeq \frac{1}{4\pi^2} \frac{m_\mu^2}{m_{W_L}^2} \left[ |g_{\nu_2}|^2 \left( \frac{5}{6} - \epsilon \right) + |g_{a2}|^2 \left( \frac{5}{6} + \epsilon \right) \right]$$

(38)
with $m_W \gg m_\mu$ and $\epsilon \equiv \left( \frac{m_\nu}{m_\mu} \right)$. The couplings for this interaction are given in Table III.

Since the ISS scenario allows large mixing between light and heavy neutrinos, moving from flavor to mass basis we can see that for $O(0.01)$ light-heavy neutrino mixing, heavy neutrinos with mass $\sim$ few GeV play a significant role in context of muon $g - 2$ anomaly by interacting with $W_L$. Also, in the next section we will see that this gives positive contribution to $\Delta a_\mu$.

**Fig. 2: Due to extra neutral gauge boson $Z_R$ mediation $\Delta a_\mu(Z_R)$**

The new contribution for muon anomalous $g - 2$ arising from exchange of right-handed neutral gauge boson $Z_R$ neutral, as shown in Fig. 2, is derived from the neutral current interaction as,

$$\frac{g_L}{\sqrt{1 - \delta \tan^2 \theta_W}} \bar{\mu} \gamma_\beta (g_v - g_a \gamma^5) \mu Z_R^\beta$$

with the couplings

$$g_v = \frac{1}{4} [3 \delta \tan^2 \theta_W - 1]$$
$$g_a = \frac{1}{4} [1 - \delta \tan^2 \theta_W]$$

where $\delta = \frac{g_L^2}{g_R^2}$ and $\theta_W$ is the Weinberg angle.

The Lagrangian for the charged fermions which interact with the SM leptons via a neutral vector boson ($Z_R$) can be written as

$$\mathcal{L}_{\text{int}} = g_v \bar{Z}_R \gamma^\mu \mu + g_a \bar{Z}_R \gamma^\mu \gamma^5 \mu + h.c.$$
Using Eq. 40 the contribution arising from $Z_R$ to the muon anomalous magnetic moment can be expressed as,

$$\Delta a_\mu(Z_R) \simeq \frac{1}{8\pi^2} \frac{m_\mu^2}{m_{Z_R}^2} \int_0^1 dx \frac{g_{\nu 3}^2 P_{\nu 3}(x) + g_{a 3}^2 P_{a 3}(x)}{(1-x)(1 - \lambda^2 x) + \lambda^2 x}$$

with $\lambda \equiv \left( \frac{m_\mu}{m_{Z_R}} \right)$, and

$$P_{\nu 3}(x) = 2x^2(1-x)$$
$$P_{a 3}(x) = 2x(1-x)(x-4) - 4\lambda^2 x^3$$

By simplifying the integrations the contribution is found to be,

$$\Delta a_\mu(Z_R) \simeq -\frac{1}{4\pi^2} \frac{m_\mu^2}{m_{Z_R}^2} \left[ \left( -\frac{1}{3} \right) |g_{\nu 3}^\mu|^2 + \left( \frac{5}{3} \right) |g_{a 3}^\mu|^2 \right]; \quad \text{with} \quad m_{Z_R} \gg m_\mu.$$ 

where the couplings $g_{\nu 3}, g_{a 3}$ are same as $g_{\nu}, g_{a}$ respectively as in Eq. 39 and depending on the values of these vector and axial couplings the contribution can be either positive or negative.

**B. Scalar sector contribution**

The Yukawa Lagrangian involving scalars can be written as,

$$\mathcal{L}_{\text{yuk}} = \bar{\ell}_L (Y_{22} \Phi + \tilde{Y}_{22} \tilde{\Phi}) \ell_R + \bar{\ell}_R (Y_{22} \Phi^* + \tilde{Y}_{22} \tilde{\Phi}^*) \ell_L$$

where the scalar bidoublet $\Phi$ contains two charged scalars $h_3^-, h_4^-$, two neutral CP-even scalars $h_1^0, h_2^0$ and two neutral CP-odd scalars $\phi_1^0, \phi_2^0$.

$$\Phi = \begin{pmatrix} v_1 + h_1^0 + i\phi_1^0 & h_3^+ \\ h_4^- & v_2 + h_2^0 + i\phi_2^0 \end{pmatrix}$$

and

$$\tilde{\Phi} = \sigma^2 \Phi^* \sigma^2 = \begin{pmatrix} v_2 + h_2^0 - i\phi_2^0 & -h_4^+ \\ -h_3^- & v_1 + h_1^0 - i\phi_1^0 \end{pmatrix}$$

The Feynman diagrams of these scalars interacting with muons are shown in Figures 3, 4, 5 respectively. We later find out in Sec-V that among these only the neutral CP-even scalars $h_1^0, h_2^0$ contribute positively to $\Delta a_\mu$. Now by considering only muon family with

$$\ell_L = \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \quad \ell_R = \begin{pmatrix} N_{\mu R} \\ \mu_R \end{pmatrix},$$
the expanded Yukawa Lagrangian can be written as,
\[
\mathcal{L}_{\text{yuk}} = \left[ \overline{\nu} \left( Y_{22} (v_1 + h_1^0 + i \phi_1^0) + \tilde{Y}_{22} (v_2 + h_2^0 - i \phi_2^0) \right) N_{\mu} + \overline{\nu} \left( Y_{22} h_4^+ - \tilde{Y}_{22} h_3^- \right) \right] \frac{(1 + \gamma_5)}{2} 
+ [\overline{\nu} \left( Y_{22} h_4^- - \tilde{Y}_{22} h_3^+ \right) N_{\mu} + \overline{\nu} \left( Y_{22} (v_2 + h_2^0 + i \phi_2^0) + \tilde{Y}_{22} (v_1 + h_1^0 - i \phi_1^0) \right) \mu] \frac{(1 + \gamma_5)}{2} 
+ [\overline{\nu} \left( Y_{22} h_3^+ - \tilde{Y}_{22} h_4^- \right) \nu_{\mu} + \overline{\nu} \left( Y_{22} (v_2 + h_2^0 - i \phi_2^0) + \tilde{Y}_{22} (v_1 + h_1^0 + i \phi_1^0) \right) \mu] \frac{(1 - \gamma_5)}{2} 
\]

(44)

The relevant terms in the Yukawa Lagrangian for the Feynman diagrams given in Fig.3 are as follows,
\[
\mathcal{L}_{\text{yuk}} (h_3^+, h_4^+) = \overline{\nu} \left[ Y_{22} h_3^+ - \tilde{Y}_{22} h_4^+ \right] \mu \frac{(1 + \gamma_5)}{2} + \overline{\nu} \left[ Y_{22} h_4^+ - \tilde{Y}_{22} h_3^+ \right] \nu_{\mu} \frac{(1 - \gamma_5)}{2} 
\]

(45)

The same equation can be written in mass basis using [30] as,
\[
\mathcal{L}_{\text{yuk}}^{\text{mass}} (h_3^+, h_4^+) = [V_{\mu_1}^{\nu\nu} \nu_1 + V_{\mu_2}^{\nu\nu} \nu_2 + V_{\mu_3}^{\nu\nu} \nu_3 + V_{\mu_1}^{\nu S_1} S_1 + V_{\mu_2}^{\nu S_2} S_2 + V_{\mu_3}^{\nu S_3} S_3 + V_{\mu_1}^{\nu N_1} N_1 
+ V_{\mu_2}^{\nu N_2} N_2 + V_{\mu_3}^{\nu N_3} N_3] \left[ Y_{22} h_3^+ - \tilde{Y}_{22} h_4^+ \right] \mu \frac{(1 + \gamma_5)}{2} + \overline{\nu} \left[ Y_{22} h_4^+ - \tilde{Y}_{22} h_3^+ \right] 
\]

\[
[V_{\mu_1}^{\nu \nu} \nu_1 + V_{\mu_2}^{\nu \nu} \nu_2 + V_{\mu_3}^{\nu \nu} \nu_3 + V_{\mu_1}^{\nu S} S_1 + V_{\mu_2}^{\nu S} S_2 + V_{\mu_3}^{\nu S} S_3 + V_{\mu_1}^{\nu N} N_1 + V_{\mu_2}^{\nu N} N_2 
+ V_{\mu_3}^{\nu N} N_3] \frac{(1 - \gamma_5)}{2} 
\]

(46)

The diagrams in Fig.3 represent the interactions mediated by singly charged scalars $h_3^-$ and $h_4^+$.

**FIG. 3:** Feynman diagrams for the interaction of singly charged scalars $h_3^-$, $h_4^+$ with muons contributing to the muon anomalous $g - 2$.

**Fig.3(a):** The relevant interaction terms involving singly charged scalar with scalar coupling ($g_{s1}$) and pseudo-scalar coupling ($g_{p1}$) are given by,
\[
\mathcal{L}_{\text{int}} = g_{s1} h_3^- \overline{\nu} \mu + g_{p1} h_3^- \overline{\nu} \gamma^5 \mu + h.c. 
\]

(47)
In general, the contribution of a singly charged scalar to the muon anomaly can be expressed as,

$$\Delta a_\mu(h^+) \simeq \frac{1}{8\pi^2} \frac{m_\mu^2}{m_{h^+}^2} \int_0^1 dx \frac{g_{s1}^2 P_{s1}(x) + g_{p1}^2 P_{p1}(x)}{\epsilon^2 \lambda^2 (1 - x)(1 - \epsilon^{-2}x) + x}$$  \tag{48}$$

with $\epsilon \equiv \left( \frac{m_\nu}{m_\mu} \right)$, $\lambda \equiv \left( \frac{m_\mu}{m_{h^+}} \right)$ and

$$P_{s1}(x) = -x(1-x)(x + \epsilon)$$

$$P_{p1}(x) = -x(1-x)(x - \epsilon)$$

So, in this case the extra contribution is found to be,

$$\Delta a_\mu(h^+) \simeq -\frac{1}{4\pi^2} \frac{m_\mu^2}{m_{h^+}^2} \left[ |g_{s1}^\mu|^2 \left( \frac{1}{12} + \frac{\epsilon}{4} \right) + |g_{p1}^\mu|^2 \left( \frac{1}{12} - \frac{\epsilon}{4} \right) \right] \quad \text{with} \quad m_{h^+} \gg m_\mu, m_\nu. \tag{49}$$

Fig.3(b): Similarly the interaction terms involving $h^+_4$ with scalar coupling ($g_{s2}$) and pseudo-scalar coupling ($g_{p2}$) are,

$$\mathcal{L}_{int} = g_{s2}h^+_4 \overline{\nu_\mu} \mu + g_{p2}h^+_4 \overline{\nu_\mu} \gamma^5 \mu + h.c. \tag{50}$$

The expression for the contribution arising from this scalar to the muon anomaly can be written as,

$$\Delta a_\mu(h^+_4) \simeq -\frac{1}{4\pi^2} \frac{m_\mu^2}{m_{h^+_4}^2} \left[ |g_{s2}^\mu|^2 \left( \frac{1}{12} + \frac{\epsilon}{4} \right) + |g_{p2}^\mu|^2 \left( \frac{1}{12} - \frac{\epsilon}{4} \right) \right] \tag{51}$$

with $m_{h^+_4} \gg m_\mu, m_\nu$, $\epsilon \equiv \left( \frac{m_\nu}{m_\mu} \right)$, $\lambda \equiv \left( \frac{m_\mu}{m_{h^+_4}} \right)$.

The couplings for the above two cases can be found from Eq.46 and are given in Table \text{V}. The diagrams in Fig.4 are mediated by neutral scalars $h^+_1$ and $h^+_2$.

![Fig. 4: Feynman diagrams for the interaction of neutral CP-even scalars $h^+_1, h^+_2$ with muons.](image-url)

**Fig.4(a):** In general if extra electrically neutral scalar fields are present in a model, they induce a shift in the leptonic magnetic moments via the following interactions:

$$\mathcal{L}_{int} = g_{s3}h^0_1 \overline{\nu_\mu} \mu + ig_{p3}h^0_1 \overline{\nu_\mu} \gamma^5 \mu$$  \tag{52}$$
TABLE V: Relevant couplings associated with the Feynman diagrams involving $h_3^+, h_4^-$ given in Fig. 3

From Eq.(52) one can see that scalar and pseudo-scalar couplings shift $(g - 2)_\mu$ by

$$\Delta a_\mu(h_1^0) \approx \frac{1}{4\pi^2 m_{h_1^0}^2} \int_0^1 dx \frac{g_{s3}^2 P_{s3}(x) + g_{p3}^2 P_{p3}(x)}{(1-x)(1-\lambda^2 x) + \lambda^2 x} \quad (53)$$

with $\lambda \equiv \left( \frac{m_0}{m_{h_1^0}} \right)$ and $P_{s3}(x) = x^2(2-x)$, $P_{p3}(x) = -x^3$.

So, from here we have the extra contribution to the anomalous magnetic moment as,

$$\Delta a_\mu(h_1^0) \approx \frac{1}{4\pi^2 m_{h_1^0}^2} \left[ |g_{s3}|^2 \left( -\frac{7}{12} - \log \lambda \right) + |g_{p3}|^2 \left( \frac{11}{12} + \log \lambda \right) \right] ; \quad \text{with} \quad m_{h_1^0} \gg m_\mu. \quad (54)$$

The result in Eq.(54) is for general neutral scalars with scalar and pseudo-scalar couplings in the regime $m_{\text{Neutral Scalar}} \gg m_\mu$. The contribution coming from pure scalar can be derived from Eq.(54) by setting the pseudo-scalar coupling ($g_p$) to zero and that from pseudo-scalar by setting the scalar coupling ($g_s$) to zero. By comparing with Eq.(44) we have the couplings $g_{s3} = \bar{Y}_{22}$, $g_{p3} = 0$.

**Fig.4(b):** For this diagram the interaction Lagrangian can be written as

$$\mathcal{L}_{\text{int}} = g_{s4} h_2^0 \bar{\mu}_\mu + ig_{p4} h_2^0 \bar{\nu}_\mu \gamma^5 \mu \quad (55)$$

Similar to the previous case its contribution to the anomalous magnetic moment can be written as,

$$\Delta a_\mu(h_2^0) \approx \frac{1}{4\pi^2 m_{h_2^0}^2} \left[ |g_{s4}|^2 \left( -\frac{7}{12} - \log \lambda \right) + |g_{p4}|^2 \left( \frac{11}{12} + \log \lambda \right) \right] \quad (56)$$
with \( m_{\phi_2} \gg m_\mu, \lambda \equiv \left( \frac{m_\mu}{m_{\phi_2}} \right) \). From comparison with Eq. 44 the couplings are \( g_{s4} = Y_{22}, \ g_{p4} = 0 \).

**Fig. 5(a):** In this case the interaction Lagrangian is given by,

\[
L_{\text{int}} = g_{s5} \phi_1^0 \mu \mu + i g_{p5} \phi_1^0 \mu \gamma^{5} \mu
\]  
(57)

As in the case 4(a), here we will have the extra contribution to the anomalous magnetic moment as,

\[
\Delta a_\mu(\phi_1^0) \simeq \frac{1}{4\pi^2} \frac{m_\mu^2}{m_{\phi_1}^2} \left[ |g_{s5}^\mu|^2 \left( -\frac{7}{12} - \log \lambda \right) + |g_{p5}^\mu|^2 \left( \frac{11}{12} + \log \lambda \right) \right]
\]  
(58)

with \( m_{\phi_1} \gg m_\mu, \lambda \equiv \left( \frac{m_\mu}{m_{\phi_1}} \right) \). The couplings here are \( g_{s5} = 0, \ g_{p5} = -\tilde{Y}_{22} \).

**Fig. 5(b):** For this interaction the Lagrangian can be written as,

\[
L_{\text{int}} = g_{s6} \phi_2^0 \mu \mu + i g_{p6} \phi_2^0 \mu \gamma^{5} \mu
\]  
(59)

and its contribution to \( \Delta a_\mu \) is,

\[
\Delta a_\mu(\phi_2^0) \simeq \frac{1}{4\pi^2} \frac{m_\mu^2}{m_{\phi_2}^2} \left[ |g_{s6}^\mu|^2 \left( -\frac{7}{12} - \log \lambda \right) + |g_{p6}^\mu|^2 \left( \frac{11}{12} + \log \lambda \right) \right]
\]  
(60)

with \( m_{\phi_2} \gg m_\mu, \lambda \equiv \left( \frac{m_\mu}{m_{\phi_2}} \right) \). The couplings for this case are \( g_{s6} = 0, \ g_{p6} = Y_{22} \).

**Fig. 6:** Due to extra gauge boson \( Z_{\mu\tau} \) mediation \( \Delta a_\mu(Z_{\mu\tau}) \)

This diagram comes from the interaction of the new gauge boson \( Z_{\mu\tau} \) associated with \( U(1)_{L_\mu - L_\tau} \) symmetry with muons. We have the terms in the Lagrangian

\[
\sum_{\alpha = e, \mu, \tau} \left[ \bar{\ell}_\alpha L \gamma^\mu D_\mu \ell_\alpha L + \bar{\ell}_\alpha R \gamma^\mu D_\mu \ell_\alpha R \right]
\]  
(61)
FIG. 6: Feynman diagram for the interaction of new light gauge boson $Z_{\mu\tau}$ with muons.

with covariant derivative $D_\beta = \partial_\beta + ig_{\mu\tau}qZ_{\beta}^{\mu\tau}$, where $g_{\mu\tau}$ is the gauge coupling of $U(1)_{L_\mu - L_\tau}$ symmetry and $q$ is the corresponding $L_\mu - L_\tau$ charge ($q_{\mu,\nu_\mu} = 1$, $q_{\nu,\nu_\tau} = -1$). By expanding this term explicitly for $\mu$-family we will get $g_{\mu\tau}\bar{\nu}\gamma^\beta\mu Z_{\beta}^{\mu\tau}$ and this term contributes to muon (g-2) anomaly.
So, the interaction Lagrangian can be written as,

$$L_{\text{int}} = g_{\mu\tau}Z_{\mu\tau}^{\mu\gamma^\mu\mu}$$  \hspace{1cm} (62)

Defining the parameter $\lambda \equiv \left(\frac{m_\mu}{m_{Z_{\mu\tau}}}\right)$, its contribution to the anomaly can be written as,

$$\Delta a_\mu(Z_{\mu\tau}) \simeq \frac{g_{\mu\tau}^2 m_\mu^2}{8\pi^2 m_{Z_{\mu\tau}}^2} \int_0^1 dx \frac{2x^2(1-x)}{(1-x)(1-\lambda^2 x) + \lambda^2 x} \hspace{1cm} (63)$$

After simplifying the integrations its contribution can be written as,

$$\Delta a_\mu(Z_{\mu\tau}) = \frac{g_{\mu\tau}^2 m_\mu^2}{12\pi^2 m_{Z_{\mu\tau}}^2}; \text{ with } \lambda \equiv \left(\frac{m_\mu}{m_{Z_{\mu\tau}}}\right) \hspace{1cm} (64)$$

V. NUMERICAL ESTIMATION OF MUON ANOMALY

Using the analytical expressions for different Feynman diagrams given in Sec IV, we estimate the individual contributions of different particles to $\Delta a_\mu$ in this section. For this we have taken the values of the model parameters as follows,

$$m_Z \simeq 91 \text{ GeV}, \quad m_\mu \simeq 105 \text{ MeV}, \quad m_{W_L} \simeq 80 \text{ GeV}, \quad m_{W_R} \simeq 4 \text{ TeV} \hspace{1cm} [75],$$

$$m_{Z_R} \simeq 6 \text{ TeV}, \quad m_{h_1} \simeq 5 \text{ TeV} = m_{h_2}, \quad m_{h_3} \simeq 5 \text{ TeV} = m_{h_4} = m_{\phi_1} = m_{\phi_2},$$

$$m_{Z_{\mu\tau}} \simeq \mathcal{O}(100) \text{ MeV}, \quad g_{\mu\tau} \simeq \mathcal{O}(10^{-4})$$
With these values of model parameters we present the individual contributions of different diagrams in Table VI and VII.

| Particles | Contribution to anomaly |
|-----------|-------------------------|
| $\Delta a_{\mu}(W_L)$ | $2.18 \times 10^{-12}$ |
| $\Delta a_{\mu}(h_3^\pm)$ | $-0.282 \times 10^{-12}$ |
| $\Delta a_{\mu}(h_1^\pm)$ | $-0.282 \times 10^{-12}$ |
| $\Delta a_{\mu}(h_1^0)$ | $29.357 \times 10^{-12}$ |
| $\Delta a_{\mu}(h_2^0)$ | $29.357 \times 10^{-12}$ |
| $\Delta a_{\mu}(\phi_1^0)$ | $-26.962 \times 10^{-12}$ |
| $\Delta a_{\mu}(\phi_2^0)$ | $-26.962 \times 10^{-12}$ |
| $\Delta a_{\mu}(Z_{\mu\tau})$ | $14.8 \times 10^{-12}$ |

TABLE VI: Estimated values of the individual contributions coming from different vector bosons and scalars except $W_R$ and $Z_R$ in extended LRSM. Apart from the usual scalar mediated contributions in left-right theories and the extra gauge boson $Z_{\mu\tau}$ mediated contribution, the purely left handed current mediated via $W_L$ with the exchange of heavy neutrinos within low scale (inverse) seesaw mechanism can give substantial large contribution to muon anomaly.

It may also be noted here that, usually in a left-right symmetric theory the left-handed and right-handed gauge coupling are equal, i.e. $g_L = g_R$. But, if it so happens that the Parity symmetry breaks earlier than that of the $SU(2)_R$ symmetry then the left-handed and right-handed gauge couplings become unequal, i.e. $g_L \neq g_R$. Such a model is called asymmetric LRSM, which was first proposed in [76] and more about this can be found in [77–82].

Since we have seen earlier that the couplings of some of our Feynman diagrams are dependent on $g_L$ and $g_R$ here we have considered two different cases for calculating the contributions.

Case I : $g_L = g_R = 0.632$
Case II : $g_L = 0.632, \ g_R = 0.39$

The tables show that the contributions coming from the interactions mediated by singly charged vector boson $W_L, W_R$, neutral scalars $h_1^0, h_2^0$ and the new gauge boson $Z_{\mu\tau}$ are positive while all other
\[
\begin{array}{|c|c|c|}
\hline
\text{Particles} & g_L = g_R \text{ (Case I)} & g_L \neq g_R \text{ (Case II)} \\
\hline
\Delta a_\mu(W_R) & 1.45 \times 10^{-12} & 0.55 \times 10^{-12} \\
\Delta a_\mu(Z_R) & -0.397 \times 10^{-12} & 0.184 \times 10^{-12} \\
\hline
\end{array}
\]

TABLE VII: Estimated values of the individual contributions coming from \(W_R\) and \(Z_R\) in extended LRSM for the cases \(g_L = g_R\) and \(g_L \neq g_R\).

Contributions are negative. After summing up all the contributions, we get

\[
\Delta a_\mu^{\text{ELRSM}} = 0.2226 \times 10^{-10} \quad \text{for} \quad g_L = g_R \\
\Delta a_\mu^{\text{ELRSM}} = 0.2194 \times 10^{-10} \quad \text{for} \quad g_L \neq g_R
\]

Even though the difference in results for the two cases are very little it comes from the interaction of neutral vector boson \(Z_R\) with muons which contributes negatively in the case \(g_L = g_R\) and positively in the case \(g_L \neq g_R\) and also from \(W_R\) which varies for both cases.

We know that the deviation of the experimental value for SM is \(14\),

\[
\Delta a_\mu^{\text{SM}} = 26.1 \times 10^{-10}
\]

By comparing this value with the result obtained in our model we get,

\[
\Delta a_\mu = \Delta a_\mu^{\text{SM}} - \Delta a_\mu^{\text{ELRSM}} = 25.8774 \times 10^{-10} \quad \text{for} \quad g_L = g_R \\
\Delta a_\mu = \Delta a_\mu^{\text{SM}} - \Delta a_\mu^{\text{ELRSM}} = 25.8806 \times 10^{-10} \quad \text{for} \quad g_L \neq g_R
\]

This means the considered model can ameliorate the muon \((g-2)\) anomaly by 0.8% for both the cases \(g_L = g_R\) and \(g_L \neq g_R\) as compared to SM prediction. In other words we can say the model gives a small but non-negligible extra contribution to \(\Delta a_\mu\) and thus slightly narrows down the deviation in theoretical prediction and the experimental result of muon anomalous magnetic moment.

Based on the discussion given in section IV, we present below the contributions coming from different particle sectors to muon \((g-2)\) anomaly in plots. For the standard results in the graphs the solid green line represents the projected bound on \(\Delta a_\mu\) and the dotted green line represents the current bound on \(\Delta a_\mu\). The red solid and red dotted lines represent the current and projected 1\(\sigma\) bound for \(\Delta a_\mu\). The values of these standard results [68] are given below.

\[
\Delta a_\mu^{\text{Current Bound}} = (295 \pm 81) \times 10^{-11} \\
\Delta a_\mu^{\text{Projected Bound}} = (295 \pm 34) \times 10^{-11}
\]
\[ \Delta a_\mu(1\sigma \text{ Current Bound}) = 81 \times 10^{-11} \]
\[ \Delta a_\mu(1\sigma \text{ Projected Bound}) = 34 \times 10^{-11} \]

FIG. 7: Plot showing the contribution of charged vector boson \( W_R \) to \( \Delta a_\mu \) for the case \( g_L = g_R \). The blue line represents the contribution of \( W_R \) when purely axial vector-like coupling is considered and magenta line represents the contribution when both vector-like and axial vector-like couplings are considered non-zero. It shows with purely axial vector-like coupling \( W_R \) with mass 2 TeV can address the anomaly whereas the case with combination of both couplings doesn’t satisfy the bound on \( W_R \) mass.

The contributions arising from charged gauge boson \( W_R \) for the cases (i) \( g_L = g_R \), (ii) \( g_L \neq g_R \) are presented in figures 7 and 8 respectively.

(i) For the case \( g_L = g_R \), if we consider purely axial-vector like coupling i.e. \( |g_v| = 0 \) and \( |g_a| = 0.22 \) then the gauge boson \( W_R \) with mass around 2 TeV can address the anomaly. This is represented by the blue solid line in Fig.7. But if we consider non-zero values for both couplings; \( |g_v| = 0.22 \) and \( |g_a| = 0.22 \), then the mass of \( W_R \) lies around 200 GeV (magenta line) which does not satisfy the allowed mass range.

(ii) Similarly for the case \( g_L \neq g_R \), when purely axial-vector like coupling is considered i.e. \( |g_v| = 0 \) and \( |g_a| = 0.14 \) then \( W_R \) with mass around 1 TeV can explain the anomaly and the same is represented by blue line in Fig.8. But for \( |g_v| = 0.14 \) and \( |g_a| = 0.14 \) the mass of \( W_R \) lies around 100 GeV (magenta line) which is not allowed.

Moreover from the experimental side, where \( W_R \) interacts only with right handed neutrinos, i.e for \( g_a = -g_v \) the LEP bound on \( \frac{g_v}{m_{W_R}} \) reads as \( \frac{g_v}{m_{W_R}} < 4.8 \times 10^{-3} \text{ GeV}^{-1} \) [83]. In our case \( \frac{g_v}{m_{W_R}} \sim 5.1 \times 10^{-5} \text{ GeV}^{-1} \) which clearly saturates the bound.

Figures 9 and 10 show the contributions coming from the right-handed neutral vector boson \( Z_R \) for the cases \( g_L = g_R \) and \( g_L \neq g_R \) respectively.
FIG. 8: Plot showing the contribution of charged vector boson $W_R$ to $\Delta a_\mu$ for the case $g_L \neq g_R$. The blue line represents the contribution of $W_R$ when purely axial vector-like coupling is considered and it is sensitive to both current and projected bound on $\Delta a_\mu$ when $W_R$ mass lies around 1 TeV. The magenta line represents the contribution when both vector-like and axial vector-like couplings are considered non-zero which is not sensitive to the current and projected bound on $\Delta a_\mu$ and it doesn’t even satisfy the bound on $W_R$ mass.

(i) For the case ($g_L = g_R$), only the purely vector-like contribution (black line in Fig.9) is the viable one to satisfy the $1\sigma$ bound on $\Delta a_\mu$. The other two contributions i.e. purely axial-vector like (blue line) and combination of both (magenta line) are negative and thus we have plotted the absolute values of these contributions. However plot shows that even for the purely vector-like contribution (black line) to satisfy the $1\sigma$ bound on $\Delta a_\mu$, the mass of $Z_R$ lies around 70 GeV which is not allowed.

(ii) For the case ($g_L \neq g_R$), all the choices on couplings i.e. purely vector-like (black line in Fig.10), purely axial-vector-like (blue line) as well as combination of both (magenta line) contribute positively to $\Delta a_\mu$ but they fail to saturate the present bound on $Z_R$ mass. Agreeably ref. [83] argues that a 95% C.L upper bound from LEP measurements applies for $g_v = g_a$ and $m_{Z_R} > \sqrt{s}$ that puts $\frac{g_v}{m_{Z_R}} < 2.2 \times 10^{-4}$ GeV$^{-1}$ and thus discards the idea of a single $Z_R$ boson explaining the anomaly. Some more bounds are given in ref. [83, 84].

Figure 11 shows the contributions coming from the charged scalars $h_3^+, h_4^+$ for three different choices of the couplings; $|g_s| = 0.2868$ and $|g_p| = 0$ (purely scalar), $|g_s| = 0$ and $|g_p| = 0.2868$ (purely pseudo-scalar) and $|g_s| = 0.2868$ and $|g_p| = 0.2868$ (combination of both). We have already discussed in Sec V that $h_3^+, h_4^+$ contribute negatively to $\Delta a_\mu$, and thus we have plotted the absolute value of these contributions in Log-Log plot. Here the black and blue lines representing purely scalar and
FIG. 9: Plot showing the contribution of neutral vector boson $Z_R$ to $\Delta a_\mu$ for the case $g_L = g_R$. The black line represents the contribution of $Z_R$ when purely vector-like coupling is considered and the magenta line represents the contribution when both vector-like and axial vector-like couplings are considered non-zero. The contribution with purely axial vector-like coupling is negative and doesn’t show in the plot. Even though the contribution from $Z_R$ is positive with purely vector-like coupling, all the cases fail to satisfy the current bound on $Z_R$ mass.

purely pseudoscalar couplings respectively have merged together since the contributions coincide. The plot shows that the masses of the charged scalars lie around $\mathcal{O}(50)$ GeV which satisfies the collider bounds on masses given in ref. [84]. Also from the results it can be concluded that singly charged scalars are not good candidates for explaining muon $(g-2)$ anomaly since they give negative or suppressed contribution.

Figure 12 shows the contributions coming from all the neutral scalars present in our model, namely $h_1^0, h_2^0, \phi_1^0, \phi_2^0$. While the y-axis bears the numerical values of the contributions to $\Delta a_\mu$, the x-axis bears the mass of the scalars. We have mentioned earlier that the contribution to muon anomaly coming from either pure scalar or pseudo-scalar or both can be easily derived from their couplings. In this case we can see that the neutral CP-even scalars $h_1^0$ and $h_2^0$ with mass around 1 TeV can explain the anomaly if we consider pure scalar couplings for them (represented by black line). The result obtained from this plot also matches with our numerical estimation where we have used 1 TeV mass with purely scalar coupling, $g_s = 0.8$ for these scalars. If we consider purely pseudo-scalar coupling; i.e. $g_s = 0$ and $g_p = 0.8$ then the contribution becomes negative. However if we take non-zero values for both the scalar and pseudo-scalar couplings, i.e. $g_s = 0.8$ and $g_p = 0.8$ (represented by the magenta line), then a neutral scalar with 150 GeV mass can address the anomaly since it is sensitive to all the current and projected bounds on $\Delta a_\mu$. 

27
FIG. 10: Plot showing the contribution of neutral vector boson $Z_R$ to $\Delta a_\mu$ for the case $g_L \neq g_R$. The black line, blue line and magenta line represent the contributions of $Z_R$ when purely vector-like, purely axial vector-like and the combination of both couplings are considered non-zero respectively. Even though all contributions are positive and sensitive to the current and projected bounds on $\Delta a_\mu$, none of the cases satisfy the bound on $Z_R$ mass.

FIG. 11: Plot showing the contributions of singly charged scalars to $\Delta a_\mu$ for three different choices of couplings; purely scalar, purely pseudo-scalar and combination of both. All the contributions are negative and thus are plotted in log-log plot. The contributions coming from purely scalar and purely pseudo-scalar couplings are super-imposed and represented by the blue line. The magenta line represents the contribution from the combination of both couplings.

It is to be noted that neutral scalars are constrained by LEP searches for four-lepton contact interactions which requires $\frac{g_s}{M_\phi} < 2.5 \times 10^{-4}$ GeV$^{-1}$ for $M_\phi > \sqrt{s}$. For our case $\frac{g_s}{M_\phi} = 1.6 \times 10^{-4}$ GeV$^{-1}$ which clearly satisfies the LEP search bound.

Figure 13 shows the contribution coming from the new neutral vector boson $Z_{\mu\tau}$ in our model.
FIG. 12: Plot showing the contributions of CP-even and CP-odd neutral scalars to $\Delta a_\mu$ under three assumptions about the couplings; purely scalar or purely pseudo-scalar or the combination of both. The black line represents purely scalar contribution coming from $h_1^0, h_2^0$, while the magenta line represents the combination of both couplings. For purely pseudo-scalar coupling the contribution becomes negative and thus it couldn’t be plotted.

FIG. 13: Plot showing the contribution coming from new light gauge boson $Z_{\mu\tau}$ vs mass of $Z_{\mu\tau}$. The blue line shows the contribution of $Z_{\mu\tau}$ with coupling $g_{\mu\tau} = 8 \times 10^{-4}$ can address the anomaly with $Z_{\mu\tau}$ mass lying around 150 MeV.

The plot shows that for coupling $g_{\mu\tau} = 8 \times 10^{-4}$, the neutral vector boson $Z_{\mu\tau}$ having mass nearly 150 MeV can address the anomaly (magenta line). Although $\mathcal{O}(100$ MeV) mass for $Z_{\mu\tau}$ is allowed, its coupling strength ($g_{\mu\tau}$) is strongly constrained to be less than $\simeq 10^{-3}$ from the measurement of neutrino trident cross section by experiments like CHARM-II [85] and CCFR [86] which is satisfied in our case.

In general, the individual contribution to muon anomaly arising due to mediation of various gauge
bosons and scalars is related to its mass by the relation,
\[ \Delta a_\mu \propto \frac{1}{m^2_{\text{mediator}}} \]  

Thus, these contributions to $\Delta a_\mu$ becomes negligible if we consider heavy mass for the mediators (scalars and gauge bosons except $W_L$ and $Z_{\mu\tau}$). We have mentioned earlier that the inverse seesaw mechanism allows large light-heavy neutrino mixing and thus facilitates the interaction of $W_L$ with heavy neutrino. In that case, the only significant contribution to muon anomaly (satisfying current day experimental as well as $1\sigma$ bound) comes from the $W_L$ mediation in the model if we consider large light-heavy neutrino mixing. In the figure [14] we have plotted the contribution coming from $W_L$ to $\Delta a_\mu$ vs $V^{\nu\xi}$. Here, we vary this mixing from $10^{-2}$ to 1. The magenta line represents the
dependence of $\Delta a_\mu$ on light-heavy mixing and we have found that $V^{\nu\xi}$ should be of $O(0.3 - 1)$ in order to saturate current and projected bound on $\Delta a_\mu$.

In figure 15 we vary the contribution of $W_L$ with the mass of heavy neutrino. For plotting the dependence of $\Delta a_\mu(W_L)$ on mass of heavy neutrino, we have considered four different values of mixing i.e. 0.01, 0.1, 0.3 and 1 represented by blue, magenta, cyan and black line respectively. However for the numerical estimation of $\Delta a_\mu(W_L)$ we have considered this mixing to be $O(0.01)$ in order to have compatible value for theoretical estimation of muon anomaly.

VI. CONCLUSION

We have studied for the first time the $U(1)_{L_\mu - L_\tau}$ extension of left-right symmetric model which explains non-zero neutrino mass, mixing and muon anomalous magnetic moment simultaneously. Neutrino mass is generated in the model through inverse seesaw mechanism that allows large light-heavy neutrino mixing and thereby contributes positively to the muon anomaly. We have also discussed how the choice of scalars in various LRSM-SM symmetry breaking chains affect the generation of neutrino mass. We have calculated the individual contributions to muon anomaly arising from various vector bosons and scalars interacting with muons in the model. While the largest positive contribution arises from singly charged right-handed gauge boson $W_R$, other major contributions come from neutral scalars $h_1^0, h_2^0$ and the new gauge boson $Z_{\mu\tau}$. The mass of $Z_{\mu\tau}$ is taken 150 MeV, while other scalars and vector bosons are assigned few TeV mass. With this the total contribution to $\Delta a_\mu$ in the model becomes $0.22 \times 10^{-10}$ for both the cases $g_L = g_R$ and $g_L \neq g_R$ which means in our model the anomaly is ameliorated by 0.8% in both the cases as compared to SM. We have also plotted the individual contributions to $\Delta a_\mu$ coming from charged scalars, neutral scalars and vector bosons. We have shown that when a heavy mass scale is considered for various scalars and vector bosons except $W_L$ and $Z_{\mu\tau}$, their contribution to muon anomaly becomes negligible. However when the light-heavy neutrino mixing is considered to be $O(0.3 - 1)$ then the individual contribution from $W_L$ mediation can saturate the present and projected bounds on muon anomaly.

VII. ACKNOWLEDGEMENT

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