Hydraulic resistance due to unsteady flow in river channels: numerical simulation results

Tatyana Dyakonova¹, Anna Klikunova¹ and Alexander Khoperskov¹
¹Volgograd State University, Volgograd, 400062, Russia
E-mail: dyakonova@volsu.ru

Abstract. The hydraulic resistance to flow in a river-type channel is due to the action of a large number of factors. We created numerical models of unsteady water dynamics in an inhomogeneous channel by varying its geometric and hydraulic characteristics. We focused on the study of flows at small values of the Manning roughness coefficient, when the water movement is unsteady. The transitional regime from a pulsating flow to a laminar flow is characterized by a complex system of large vortices covering the entire channel. The nonstationarity factor enhances the hydrological resistance to flow, and thus provides an additional contribution to the effective Manning coefficient.

1. Introduction
The shallow water model is based on two-dimensional hydrodynamic equations for vertically averaged flow parameters and is actively used to solve a wide range of problems, in particular, to describe the dynamics of surface waters in river beds [1, 2, 3]. Numerical modeling of realistic river flows requires consideration of hydraulic resistance, depending on a large number of physical factors, many of which require a phenomenological approach due to hydrostatic approximation in the vertical direction. These factors also include the interaction of the liquid flow with the underlying surface, internal friction and turbulence, dispersion and the actions of the air wind. Direct consideration of the viscous stress tensor significantly complicates the numerical model while maintaining uncertainty in the choice of the turbulent viscosity model [1, 4].

Nonstationary flow can be an important factor in changing the morphology of the river bottom and the overall structure of the riverbed (meandering, sandbank, branching, islands) [5]. Turbulent mixing is crucial for correctly modeling sediment transport, flood events, organic transfer in ecosystems [4, 6, 7, 8].

River flow turbulence is a fundamentally three-dimensional motion, consisting of a complex hierarchical system of vortices. However, 3D modeling of extended river systems on inhomogeneous topography is a practically unsolvable problem, therefore 2D shallow water models are a standard tool for applied research [2, 9, 10].

The transition from 3D-models to 2D-models of shallow water based on vertically averaged equations impoverishes the physics of the flow and does not allow studying small-scale structures with size $\ell \ll H$ ($H$ is water depth). Physical factors on small scales in the shallow water model are taken into account using subgrid methods and/or phenomenological parameters. A typical example is the widely used effective Manning roughness coefficient $n_M$ with which researchers try to describe a variety of phenomena such as flow interaction with the bottom, sediment...
transport, transient flow, turbulence, meandering, medium-scale non-uniformity of the bottom, etc. [11]. In this paper, we examined the spatial and temporal structure of flows in the channel, depending on the roughness factor \( n_M \), canal bed slope \( I \), discharge \( Q \), the characteristic width of the channel \( L \), etc.

2. Models and Results
2.1. Numerical model of shallow water
We use the standard Saint-Venant equations with friction in the Chezy approximation

\[
\frac{\partial H}{\partial t} + \nabla (H \vec{u}) = q, \quad (1)
\]

\[
\frac{\partial H \vec{u}}{\partial t} + \nabla (H \vec{u} \otimes \vec{u}) = -gH \nabla (b + H) - \frac{1}{2} H \Lambda |\vec{u}| \cdot \vec{u}, \quad (2)
\]

where \( H(x, y, t) \) is the depth, \( \vec{u}(x, y, t) = \{u_x, u_y\} \) is the 2D velocity vector, \( \vec{\nabla} = \{\partial/\partial x, \partial/\partial y\} \) is the operator nabla, \( b(x, y) \) is the function of river bed, \( q(x, y) \) is the water source density, \( Q = \int q \, dS \) is the discharge (the water volume rate), \( g \) is the gravitational acceleration, \( \Lambda \) characterizes the hydraulic resistance and it is equal to \( \Lambda = 2g n_M^2 / H^{1/3} \) in the model Chezy, \( n_M \) is the Manning coefficient (roughness coefficient). The dimension of \( n_M \) is defined as \([n_M] = \text{sec} \cdot \text{m}^{-1/3}\), but by tradition we will not write the dimension of \( n_M \) in the text below.

The parameter \( n_M \) is a multifactor effective characteristic that allows you to simulate a large number of different physical phenomena [12, 13]. We focus on the effects of unsteady flow on the hydrological resistance and we can write

\[ n_M = n_M^{(0)} + n_M^{(t)}, \quad (3) \]

where \( n_M^{(0)} \) is the roughness coefficient of the channel bottom, which is explicitly specified in the equation (2), the term \( n_M^{(t)} \) is the equivalent contribution of flow nonstationarity in hydraulic resistance. We consider different flow regimes in the channel, identifying the conditions for the transition between the laminar flow and the quasistationary flow depending on the roughness coefficient \( n_M^{(0)} \).

Our numerical scheme Combined Smooth Particle Hydrodynamics - Total Variation Diminishing (CSPH-TVD) for integrating equations (1) - (2) and parallel implementation features for GPUs are described in detail in works [9, 14, 15]. The CSPH - TVD numerical algorithm provides a set of good properties, such as the conservativeness, the well-balance, the end-to-end modeling of the moving interfaces between wet and dry bottom, the second order of accuracy.

The derivative \( \frac{\partial b}{\partial x} = I = \text{const} \) sets the canal bed slope \( (I) \), which determines the horizontal force acting on the water. Figure 1 demonstrates the geometry of the problem and the general flow structure for \( n_M^{(0)} = 0.005 \). The channel has a characteristic V-shaped profile \( b(y) \) with a half-width \( L \) (See Fig. 1). The figure shows only part of the channel for modeling, since its total length is 40 km. A spatially distributed source of water with discharge \( Q \) is located on the left boundary of the computational domain in the channel model. We set the free boundary for water outflow on the right border \( (x = 40 \text{ km}) \) and the details of this procedure are described in [16]. The channel is filled with water and we get a stationary or quasistationary flow depending on the numerical values of the free parameters.

The velocity vector is constant \( \vec{u} = u_x \cdot \vec{e}_x \) in the case of laminar flow, therefore the balance of forces is determined primarily by the right side of the equation (2), for which we have simple
Figure 1. Flow structure for the case $n^{(0)}_M = 0.005$, $I = 0.00007$, $Q = 1500\text{ m}\cdot\text{sec}^{-1}$, $L = 250\text{ m}$: (a) — the distribution of free water surface $\xi(x, y) = b + H$ (the bottom cross-section shown in the upper left insert). (b) — the velocity field for the channel fragment (See frame in figure a), (c) — the vorticity distribution $\Omega(x, y)$ ($\Omega \cdot \vec{e}_z = \text{rot(} \vec{u} \text{)}$).

expression after averaging along the channel cross section:

$$\langle u \rangle = \frac{\langle H^{2/3} \rangle}{n^{(0)}_M} \sqrt{\langle \frac{\partial (b + H)}{\partial x} \rangle},$$

(4)

where $\langle \ldots \rangle$ is averaging along $y$-coordinate. The formula (4) for $\partial H/\partial x = 0$ is called the Chezy formula and is used to estimate the parameter $n_M$ from the experimentally measured velocity $\langle u \rangle$ [17].
Figure 2. Typical time dependences of the depth $h(t)$ (a) and the velocity modulus $|\vec{u}| = \sqrt{u_x^2 + u_y^2}$ (b) at the observation points $(x_{p1}, y_{p1}) = (15000 \, \text{m}, 0 \, \text{m})$ (red lines), $(x_{p2}, y_{p2}) = (15000 \, \text{m}, 100 \, \text{m})$ (grey lines) (see figure 1). The inserts at the top show time dependencies in the small observation interval and indicate characteristic oscillation times.

Figure 3. Unsteady flow in the model with $n_{M}^{(0)} = 0.001$, $I = 0.00007$, $Q = 1500 \, \text{m} \cdot \text{sec}^{-1}$, $L = 250 \, \text{m}$. a) The velocity field and the time dependences of the depth $H(t)$ and the velocity modulus $V(t)$ at a fixed point on the fairway. b) The global structure of the flow channel.

2.2. Numerical simulation results

Let us discuss the structure of flows in the channels when varying the free parameters in fairly wide limits: $n_{M}^{(0)} = 0 \div 0.04$, $I = (1 \div 10) \cdot 10^{-5}$, $Q = (500 \div 4000) \cdot 10^{3} \, \text{m} \cdot \text{sec}^{-1}$, $L = 80 \div 300 \, \text{m}$. These parameters correspond to typical large plain rivers such as the Rhine, Don, Danube, Dnieper, Vistula, etc.

Channel flow is unstable at ultralow values of the Manning coefficient $n_{M}^{(0)} \lesssim 0.002$ and as a result a pulsating jet is formed in which the velocity has a random component $V(t) = |\vec{u}| = V_0 + \delta V(t)$ at each point of the channel (Figure 3).
Figure 4. Laminar flow in model with $n_M^{(0)} = 0.02$, $I = 0.00007$, $Q = 1500 \text{m-sec}^{-1}$, $L = 250 \text{m}$ with $u_y = 0$.

Figure 5. Dependencies of the dimensionless parameter $n_M/n_M^{(0)}$ on $n_M^{(0)}$ with fixed values of $I$, $Q$, $L$.

For large values of $n_M^{(0)} \gtrsim 0.02$, the water motion in the channel is stationary with straight lines of current flow $\vec{u} = u_x \cdot \vec{e}_x$ ($u_y = 0$) and any disturbances are dissipated efficiently (Figure 4). The flow velocities in numerical simulations agrees well with the formula (4).

In the intermediate range $0.002 \lesssim n_M^{(0)} \lesssim 0.02$ we have a quasi-periodic mode due to the sequence of vortices, which are located in pairs along the fairway (fig. 3). A characteristic feature of such a flow is a quasi-periodic system of vortices to the right and left of the fairway (See the velocity field in the figure 1). The vortex structure is quasi-stationary with small random pulsations of velocity and depth. Vortices move downstream with a characteristic stream velocity. Non-stationarity of flow is a factor that increases hydraulic resistance, for which we estimated the corresponding Manning coefficient $n_M^{(t)}$ (Figure 5).
3. Discussion and conclusions
The structure of the flow in the channel essentially depends on the effective Manning coefficient $n_M$, which is widely used in the shallow water model for describing flow resistance due to various physical factors. We used the results of numerical simulations of shallow water in the channel to determine the critical values of the roughness coefficient $n_M^{(0)}$, which share three characteristic types of motion:

(i) Flows with a large coefficient $n_M^{(0)} \geq 0.02$ are laminar and the constant velocity is directed exactly along the channel.

(ii) Flows with a very small parameter $n_M^{(0)} \leq 0.002$ are non-stationary, in which there are random velocity pulsations. This feature of the flow provides additional hydraulic resistance to flow, which is approximately doubled.

(iii) Flows with intermediate values $n_M^{(0)} \approx 0.002 \div 0.02$ are characterized by an almost regular vortex structure, when we have two vortices across the flow. Such a system of vortices is quasi-stationary and also provides additional hydraulic resistance to flow.

Acknowledgments
This work was supported by the Ministry of Science and Higher Education of the Russian Federation (government task 2019-0730).

References
[1] Wu W, Wang P, Chiba N 2004 Archives of Hydro-Engineering and Environmental Mechanics 51(2) 183–200
[2] Agafonnikova E O, Klikunova A Yu, Khoperskov A V 2017 Bulletin of the South Ural State University, Series: Mathematical Modelling, Programming and Computer Software 10(3) 148–55
[3] Bulatov O V and Elizarova T G 2016 Computational Mathematics and Mathematical Physics 56(4) 661–79
[4] Gharehbaghi A, Kaya B and Saadatnejadgharahassanlou H 2017 Arabian Journal for Science and Engineering 42(3) 999–1011
[5] van Dijk W M, Hiatt M R, van der Werf J J and Kleinhans M G 2019 Journal of Geophysical Research: Earth Surface 124 195–215
[6] Gushchin V A, Sukhinov A I, Chistyakov A E, Nikitina A V and Semenyakina A A 2018 Computational Mathematics and Mathematical Physics 58(8) 1316–33
[7] Chanson H and Trevelyan M 2010 Atmospheric turbulence, meteorological modeling and aerodynamics ed P R Lang and F S Lombardo (New York: Nova Science Publishers) pp 167–204
[8] Alekseyuk A I and Belikov V V 2017 Computational Mathematics and Mathematical Physics 57(2) 318–39
[9] Khrapov S, Pisarev A, Kobelev I, Zhumaliev A, Agafonnikova E, Losev A and Khoperskov A 2013 Advances in Mechanical Engineering 2013 787016
[10] Garcia-Navarro P, Murillo J, Fernandez-Pato J, Echeverribar I and Morales-Hernandez M 2019 Environmental Fluid Mechanics 1–18
[11] Dyakonova T and Khoperskov A 2018 Journal of Physics: Conference Series 1128 01204
[12] De Doncker L, Troch P, Verhoeven R, Bal K, Meire P and Quintelier J 2009 Environmental fluid mechanics 9 549–67
[13] Hadiani M, Asl S J, Banafsheh M R and Dinpajouh Y 2013 World Applied Sciences Journal 22 307–12
[14] Dyakonova T, Khoperskov A and Khrapov S 2016 Communications in Computer and Information Science 687 132–45
[15] Khoperskov A and Khrapov S 2018 Numerical Simulations in Engineering and Science ed S Rao (London: InTechOpen) pp 237–54
[16] D’yakonova T A, Khrapov S S and Khoperskov A V 2016 Vestnik Udmurtskogo Universiteta: Matematika, Mekhanika, Komp’yuternye Nauki 26(3) 401–17
[17] Xia J, Lin B, Falconer R A and Wang Y 2012 Proceedings of the Institution of Civil Engineers 165(7) 377–91