Probing extended Higgs sector through rare $b \to s \mu^+ \mu^-$ transitions

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We study the constraints on the contribution of new physics in the form of scalar/pseudoscalar operators to the average forward backward asymmetry $\langle A_{FB} \rangle$ of muons in $B \to K \mu^+ \mu^-$ and the longitudinal polarization asymmetry $A_{LP}$ of muons in $B_s \to \mu^+ \mu^-$. We find that the maximum possible value of $\langle A_{FB} \rangle$ allowed by the present upper bound on $B(B_s \to \mu^+ \mu^-)$ is about 1% at 95% C.L. and hence will be very difficult to measure. On the other hand, the present bound on $B(B_s \to \mu^+ \mu^-)$ fails to put any constraints on $A_{LP}$, which can be as high as 100% even if $B(B_s \to \mu^+ \mu^-)$ is close to its standard model prediction. The measurement of $A_{LP}$ will be a direct evidence for an extended Higgs sector, and combined with the branching ratio $B(B_s \to \mu^+ \mu^-)$ it can even separate the new physics scalar and pseudoscalar contributions.

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I. INTRODUCTION

The quark level flavor changing neutral interaction $b \to s\mu^+\mu^-$ is forbidden at the tree level in the standard model (SM) and can occur only at the one-loop level. Therefore it can serve as an important probe to test SM at loop level and also constrain many new physics models beyond the SM. This quark level interaction is responsible for the purely leptonic decay $B_s \to \mu^+\mu^-$ and also the semi-leptonic decays $B \to (K, K^*)\mu^+\mu^-$. The semi-leptonic decays have been observed by BaBar and Belle \cite{1, 2, 3} with the following branching ratios:

$$B(B \to K\mu^+\mu^-) = (5.7^{+2.2}_{-1.8}) \times 10^{-7},$$
$$B(B \to K^*\mu^+\mu^-) = (11.0^{+2.99}_{-2.69}) \times 10^{-7}. \quad (1)$$

These values are close to the SM predictions \cite{4, 5, 6}. However there is about 20% uncertainty in these predictions mainly due to the errors in the determination of the hadronic form factors and the CKM matrix element $|V_{ts}|$.

The decay $B_s \to \mu^+\mu^-$ is highly suppressed in SM. Its branching ratio is predicted to be $(3.35 \pm 0.32) \times 10^{-9}$ \cite{7, 8, 9}. This decay is yet to be observed experimentally. Recently the upper bound on its branching ratio has been improved to \cite{10}

$$B(B_s \to \mu^+\mu^-) < 5.8 \times 10^{-8} \quad (95\% \text{ C.L.}) , \quad (2)$$

which is still more than an order of magnitude above its SM prediction. $B_s \to \mu^+\mu^-$ will be one of the important rare $B$ decays to be studied at the upcoming Large Hadron Collider (LHC) and we expect that the sensitivity of the level of the SM prediction can be reached with $\sim 1 \text{ fb}^{-1}$ of data. \cite{11, 12}.

Many new physics models predict an order of magnitude enhancement or more in $B(B_s \to \mu^+\mu^-)$. These include theories with $Z'$ mediated vector bosons \cite{13}, as well as multi-Higgs doublet models that violate \cite{13} or obey \cite{14} natural flavor conservation. In \cite{15}, it was shown that the new physics mediated by vector bosons is highly constrained by the measured values of the branching ratio of $B \to (K, K^*)\mu^+\mu^-$. As a result, an order of magnitude enhancement in $B(B_s \to \mu^+\mu^-)$ from new physics vector or axial vector operators is ruled out. On the other hand, such an enhancement from the scalar/pseudoscalar new physics (SPNP) operators is still allowed,
since the most stringent bound on the SPNP operators comes from $B(B_s \to \mu^+ \mu^-)$ itself. In particular, multi-Higgs doublet models or supersymmetric (SUSY) models with large $\tan \beta$ can give rise to such an enhancement.

Apart from the branching ratios of the purely leptonic and semi-leptonic decays, there are other observables which are sensitive to the SPNP contribution to $b \to s$ transitions. These are forward-backward (FB) asymmetry $A_{FB}$ of muons in $B \to K \mu^+ \mu^-$ and longitudinal polarization (LP) asymmetry $A_{LP}$ of muons in $B_s \to \mu^+ \mu^-$. Both these are predicted to be zero in the SM. Therefore, any nonzero measurement of one of these asymmetries is a signal for new physics. In addition, these asymmetries are almost independent of form factors and CKM matrix element uncertainties, which makes them attractive candidates in searches for new physics. In this paper we investigate what constraints the recently improved upper bound on $B(B_s \to \mu^+ \mu^-)$ puts on the possible SPNP contribution to $A_{FB}$ and $A_{LP}$. Do SPNP operators enhance these observables to sufficiently large values to be measurable in future experiments?

The paper is organized as follows. In section II, we study the effect of possible SPNP contribution to $A_{FB}$. In section III, we calculate the possible $A_{LP}$ enhancement due to SPNP, and point out some interesting experimental possibilities. In section IV, we present our conclusions.

II. FORWARD-BACKWARD ASYMMETRY IN $B \to K \mu^+ \mu^-$

There are numerous studies in literature of the FB asymmetry of leptons in the SM and its possible extensions. In the SM, the FB asymmetry of muons in $B \to K \mu^+ \mu^-$ vanishes (or to be more precise, is negligibly small) because the hadronic current for $B \to K$ transition does not have any axial vector contribution. However this asymmetry can be nonzero in multi-Higgs doublet models and supersymmetric models with large $\tan \beta$, due to the contributions from Higgs bosons. Therefore FB asymmetry in $B \to K \mu^+ \mu^-$ is expected to serve as an important probe to test the existence and importance of an extended Higgs sector. Any nonzero measurement of this asymmetry will be a clear signal of new physics.
The average (or integrated) FB asymmetry of muons in $B \rightarrow K\mu^+\mu^-$, which is denoted by $\langle A_{FB} \rangle$, has been measured by BaBar \cite{2} and Belle \cite{23, 26} to be

$$\langle A_{FB} \rangle = (0.15^{+0.21}_{-0.23} \pm 0.08) \quad \text{(BaBar)},$$

$$\langle A_{FB} \rangle = (0.10 \pm 0.14 \pm 0.01) \quad \text{(Belle)}.$$  

These measurements are consistent with zero. But on the other hand, they can be as high as $\sim 40\%$ within $2\sigma$ error bars.

### A. Calculation of $A_{FB}$

We consider new physics in the form of scalar/pseudoscalar operators. The effective Lagrangian for the quark level transition $b \rightarrow s\mu^+\mu^-$ can be written as

$$L(b \rightarrow s\mu^+\mu^-) = L_{SM} + L_{SP},$$

where

$$L_{SM} = \frac{\alpha G_F}{\sqrt{2\pi}} V_{tb} V_{ts}^* \left\{ C_9 (\bar{s}\gamma_\mu P_L b) \bar{\mu} \gamma_\mu \mu + C_{10} (\bar{s}\gamma_\mu P_L b) \bar{\mu} \gamma_\mu \gamma_5 \mu \\
- 2 \frac{C_7^{\text{eff}}}{q^2} m_b (\bar{s}i\sigma_{\mu\nu} q^\nu P_R b) \bar{\mu} \gamma_\mu \mu \right\},$$

$$L_{SP} = \frac{\alpha G_F}{\sqrt{2\pi}} V_{tb} V_{ts}^* \left\{ R_S (\bar{s} P_R b) \bar{\mu} \mu + R_P (\bar{s} P_R b) \bar{\mu} \gamma_5 \mu \right\}.$$  

Here $P_{L,R} = (1 \mp \gamma_5)/2$ and $q_\mu$ is the sum of 4-momenta of $\mu^+$ and $\mu^-$. $R_S$ and $R_P$ are the new physics scalar and pseudoscalar couplings respectively. In our analysis we assume that there are no additional CP phases apart from the single CKM phase. Under this assumption, $R_S$ and $R_P$ are real. Within SM, the Wilson coefficients in eq. (6) have the following values:

$$C_7^{\text{eff}} = -0.310, \quad C_9^{\text{eff}} = +4.138 + Y(q^2), \quad C_{10} = -4.221,$$

where the function $Y(q^2)$ is given in \cite{27, 28}.

The normalized FB asymmetry is defined as

$$A_{FB}(z) = \frac{\int_0^1 d\cos\theta \frac{d\Gamma}{d\cos\theta} - \int_{-1}^0 d\cos\theta \frac{d\Gamma}{d\cos\theta}}{\int_0^1 d\cos\theta \frac{d\Gamma}{d\cos\theta} + \int_{-1}^0 d\cos\theta \frac{d\Gamma}{d\cos\theta}}.$$
In order to calculate the FB asymmetry, we first need to calculate the differential decay width. The decay amplitude for $B(p) \to K(p')\mu^+(p_+)\mu^-(p_-)$ is given by

$$M(B \to K\mu^+\mu^-) = \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^*$$

$$\times \left[ \langle K(p') | \bar{s} \gamma_\mu b | B(p) \rangle \left\{ C^{-\text{eff}}_9 \bar{u}(p_+)^\gamma_\mu v(p_-) + C_{10} \bar{u}(p_+)^\gamma_\mu \gamma_5 v(p_-) \right\} - 2 \frac{C_{17}^{\text{eff}}}{q^2} m_b \langle K(p') | \bar{s} i \sigma_\mu q^\nu b | B(p) \rangle \bar{u}(p_+)^\gamma_\mu v(p_-) + \langle K(p') | \bar{s} b | B(p) \rangle \left\{ R_S \bar{u}(p_+)^\gamma_\mu v(p_-) + R_P \bar{u}(p_+)^\gamma_5 v(p_-) \right\} \right],$$

(10)

where $q_\mu = (p - p')_\mu = (p_+ + p_-)_\mu$. The relevant matrix elements are

$$\langle K(p') | \bar{s} \gamma_\mu b | B(p) \rangle = (2p - q)_\mu f_+(z) + \left( \frac{1 - k^2}{z} \right) q_\mu [f_0(z) - f_+(z)],$$

(11)

$$\langle K(p') | \bar{s} i \sigma_\mu q^\nu b | B(p) \rangle = - \left[ (2p - q)_\mu q^2 - (m_B^2 - m_K^2) q_\mu \right] \frac{f_T(z)}{m_B + m_K},$$

(12)

$$\langle K(p') | \bar{s} b | B(p) \rangle = \frac{m_B (1 - k^2)}{m_b} f_0(z).$$

(13)

Here, $k \equiv m_K/m_B$, $z \equiv q^2/m_B^2$ and $m_b \equiv m_b/m_B$. In this paper, we approximate $m_b$ by 1.

Using the above matrix elements, the double differential decay width can be calculated as

$$\frac{d^2\Gamma}{dz d\cos\theta} = \frac{G_F^2 \alpha^2}{2^9 \pi^5} |V_{tb} V_{ts}^*|^2 m_b^5 \phi^{1/2}(1, k^2, z) \beta_\mu$$

$$\times \left[ \left( |A|^2 \beta_\mu^2 + |B|^2 \right) z + \frac{1}{4} \phi(1, k^2, z) \left( |C|^2 + |D|^2 \right) (1 - \beta_\mu^2 \cos^2\theta) \right.$$

$$+ 2 \tilde{m}_\mu (1 - k^2 + z) \text{Re}(BC^*) + 4 \tilde{m}_\mu^2 |C|^2$$

$$+ 2 \tilde{m}_\mu \phi^{1/2}(1, k^2, z) \beta_\mu \text{Re}(AD^*) \cos\theta \left. \right],$$

(14)
where
\[
A \equiv \frac{1}{2}(1 - k^2)f_0(z)R_S ,
\]
\[
B \equiv -\hat{m}_\mu C_{10}\left\{f_+(z) - \frac{1 - k^2}{z}(f_0(z) - f_+(z))\right\} + \frac{1}{2}(1 - k^2)f_0(z)R_P ,
\]
\[
C \equiv C_{10}f_+(z) ,
\]
\[
D \equiv C_{\text{eff}}^e f_+(z) + 2C_{\text{eff}}^T \frac{f_T(z)}{1 + k} ,
\]
\[
\phi(1, k^2, z) \equiv 1 + k^4 + z^2 - 2(k^2 + k^2 z + z) ,
\]
\[
\beta_\mu \equiv (1 - \frac{4\hat{m}_\mu^2}{z}) .
\]
(15)

Also, \(\hat{m}_\mu = m_\mu/m_B\) and \(\theta\) is the angle between the momenta of \(K\) meson and \(\mu^-\) in the dilepton centre of mass frame. The kinematical variables are bounded as
\[
-1 \leq \cos \theta \leq 1 ,
\]
\[
4\hat{m}_\mu^2 \leq z \leq (1 - k)^2 .
\]

The form factors \(f_{+,0,T}\) can be calculated in the light cone QCD approach. Their \(q^2\) dependence is given by [18]
\[
f(z) = f(0) \exp(c_1 z + c_2 z^2 + c_3 z^3) ,
\]
(16)
where the parameters \(f(0), c_1, c_2\) and \(c_3\) for each form factor are given in Table I.

|       | \(f(0)\)    | \(c_1\)  | \(c_2\)  | \(c_3\)  |
|-------|-------------|----------|----------|----------|
| \(f_+\) | 0.319\(^{+0.052}_{-0.041}\) | 1.465    | 0.372    | 0.782    |
| \(f_0\)  | 0.319\(^{+0.052}_{-0.041}\) | 0.633    | -0.095   | 0.591    |
| \(f_T\)  | 0.355\(^{+0.016}_{-0.055}\) | 1.478    | 0.373    | 0.700    |

TABLE I: Form factors for the \(B \rightarrow K\) transition [18].

The calculation of FB asymmetry gives
\[
A_{FB}(z) = \frac{2\Gamma_0 \hat{m}_\mu a_1(z) \phi(1, k^2, z) \beta_\mu^2 R_S}{d\Gamma/dz} ,
\]
(17)
where
\[
\Gamma_0 = \frac{G_F^2 \alpha^2}{2\pi^5} |V_{tb}V_{ts}^*|^2 m_B^5 ,
\]
(18)
\[ a_1(z) = \frac{1}{2}(1 - k^2)C_9 f_0(z) f_+(z) + (1 - k)C_7 f_0(z) f_T(z), \quad (19) \]

\[
\frac{1}{\Gamma_0} \frac{d\Gamma}{dz} = \frac{1}{2}(1 - k^2) \beta_\mu \phi^\dagger z \ f_0^2(z) \ (R_P^2 + \beta_\mu^2 R_S^2) + 2(1 - k^2) \tilde{m}_\mu \ C_{10} f_0(z) \beta_\mu \phi^\dagger(z) (1 - k^2 + z) R_P \\
- 2(1 - k^2) \tilde{m}_\mu \ C_{10} \beta_\mu z \phi^\dagger f_0(z) \left\{ f_+(z) - \frac{1 - k^2}{z} (f_0(z) - f_+(z)) \right\} R_P \\
+ 2\tilde{m}_\mu^2 C_{10}^2 \beta_\mu \phi^\dagger(z) \left\{ f_+(z) - \frac{1 - k^2}{z} (f_0(z) - f_+(z)) \right\}^2 \\
+ 8\tilde{m}_\mu^2 C_{10}^2 \beta_\mu \phi^\dagger(z) f_+(z) \\
+ \frac{1}{3}(1 + \frac{2\tilde{m}_\mu^2}{z}) \beta_\mu \phi^\dagger(z) \times \\
\left\{ (C_{10}^2 + C_9^{eff}) f_0^2(z) + \frac{4C_7^{eff}}{(1 + k)^2} f_T^2(z) + \frac{4C_9^{eff} C_T^{eff}}{(1 + k) f_+(z) f_T(z)} \right\} \\
- 4\tilde{m}_\mu^2 C_{10}^2 f_+(z) \beta_\mu (1 - k^2 + z) \phi^\dagger(z) \times \\
\left\{ f_+(z) - \frac{1 - k^2}{z} (f_0(z) - f_+(z)) \right\}. \quad (20) \]

From eq. (17), it is clear that \( A_{FB} \) is proportional to \( \tilde{m}_\mu \approx 0.02 \), and to the scalar new physics coupling \( R_S \). In the minimal supersymmetric standard model (MSSM) and two Higgs doublet models, \( R_S \) itself is proportional to \( \tilde{m}_\mu \) and \( \tan^2 \beta \). Hence a large FB asymmetry is possible only for exceptionally large values of \( \tan \beta \).

The average FB asymmetry is obtained by integrating the numerator and denominator of eq. (17) separately over dilepton invariant mass, which leads to

\[
\langle A_{FB} \rangle = \frac{2\Gamma_0 \tilde{m}_\mu \beta_\mu^2 R_S \int dz \ a_1(z) \phi(1, k^2, z)}{\Gamma(B \rightarrow K^{\mu^+ \mu^-})} = \frac{2\tau_B \Gamma_0 \tilde{m}_\mu \beta_\mu^2 R_S \int dz \ a_1(z) \phi(1, k^2, z)}{B(B \rightarrow K^{\mu^+ \mu^-})}. \quad (21) \]

where \( B(B \rightarrow K^{\mu^+ \mu^-}) \) is the total branching ratio of \( B \rightarrow K^{\mu^+ \mu^-} \). The numerator in eq. (21) can be calculated to be

\[
2\tau_B \Gamma_0 \tilde{m}_\mu \beta_\mu^2 R_S \int dz \ a_1(z) \phi(1, k^2, z) = (5.25 \times 10^{-9})(1 \pm 0.20) R_S, \quad (22) \]

whereas the total branching ratio, including the contribution of SPNP operators, is given by [20]

\[
B(B \rightarrow K^{\mu^+ \mu^-}) = [5.25 + 0.18(R_S^2 + R_P^2) - 0.13 R_P] (1 \pm 0.20) \times 10^{-7}. \quad (23) \]

In the SM calculation of \( B(B \rightarrow K^{\mu^+ \mu^-}) \), two vector form factors, \( f_0 \) and \( f_+ \), as well as the tensor form factor \( f_T \) appear. The SPNP contribution, on the other hand,
\[ G_F = 1.166 \times 10^{-5} \text{ GeV}^{-2} \]
\[ \alpha = 7.297 \times 10^{-3} \]
\[ \tau_{B_s} = (1.437^{+0.031}_{-0.030}) \times 10^{-12} \text{s} \]
\[ \tau_{B_d} = 1.53 \times 10^{-12} \text{s} \]
\[ m_\mu = 0.105 \text{ GeV} \]
\[ m_K = 0.497 \text{ GeV} \]
\[ m_{B_s} = 5.366 \text{ GeV} \]
\[ m_B = 5.279 \text{ GeV} \]
\[ V_{tb} = 1.0 \]
\[ V_{ts} = (40.6 \pm 2.7) \times 10^{-3} \]
\[ f_{B_s} = (0.259 \pm 0.027) \text{ GeV} \]

TABLE II: Numerical inputs used in our analysis. Unless explicitly specified, they are taken from the Review of Particle Physics [30].

is only through \( f_0 \). We have made the assumption that the fractional uncertainties in all the form factors are the same. The \(|V_{ts}|\) dependence in the numerator and denominator of eq. (21) cancels completely, whereas the errors due to the form factors uncertainties cancel partially. We conservatively take the net error in \( \langle A_{FB} \rangle \) to be 30%, leading to

\[
\langle A_{FB} \rangle = \frac{5.25 \times 10^{-9} R_S}{[5.25 + 0.18(R_S^2 + R_P^2) - 0.13R_P] \times 10^{-7}} (1 \pm 0.3) . \tag{24}
\]

**B. Constraints on \( \langle A_{FB} \rangle \) from \( B(B_s \rightarrow \mu^+ \mu^-) \)**

We now want to see what constraints the present upper bound on \( B(B_s \rightarrow \mu^+ \mu^-) \) puts on the maximum possible value of \( \langle A_{FB} \rangle \). The present experimental upper limit on \( B(B_s \rightarrow \mu^+ \mu^-) \) is an order of magnitude larger than the SM prediction. In such a situation, the SM amplitude for this decay will be much smaller than the new physics amplitude and hence can be neglected in determining the constraints on new physics couplings, \( R_S \) and \( R_P \). In other words, we will assume that SPNP operators saturate the present upper limit. Therefore we need to consider only the contribution of \( L_{SP} \) to the decay rate of \( B_s \rightarrow \mu^+ \mu^- \).
The decay amplitude for $B_s \to \mu^+ \mu^-$ is given by

$$M(B_s \to \mu^+ \mu^-) = \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \langle 0 | \bar{s}\gamma_5 b | B_s \rangle \left[ R_S \bar{u}(p_\mu) v(p_\mu) + R_P \bar{u}(p_\mu) \gamma_5 v(p_\mu) \right].$$  

(25)

On substituting

$$\langle 0 | \bar{s}\gamma_5 b | B_s \rangle = -i \frac{f_{B_s} m_{B_s}^2}{m_b + m_s},$$  

(26)

we get

$$M(B_s \to \mu^+ \mu^-) = -i \frac{\alpha G_F}{2\sqrt{2}\pi} V_{tb} V_{ts}^* \frac{f_{B_s} m_{B_s}^2}{m_b + m_s} \left[ R_S \bar{u}(p_\mu) v(p_\mu) + R_P \bar{u}(p_\mu) \gamma_5 v(p_\mu) \right],$$  

(27)

where $m_b$ and $m_s$ are the masses of bottom and strange quark, respectively. The calculation of the branching ratio $B(B_s \to \mu^+ \mu^-)$ gives

$$B(B_s \to \mu^+ \mu^-) = \frac{G_F^2 \alpha^2 m_{B_s}^3 \tau_{B_s}}{64\pi^3} |V_{tb} V_{ts}^*|^2 f_{B_s}^2 (R_S^2 + R_P^2).$$  

(28)

Here we have neglected terms of order $m_s/m_b$ and approximated $m_{B_s}/m_b$ by 1. Taking $f_{B_s} = (0.259 \pm 0.027)$ GeV, we get

$$B(B_s \to \mu^+ \mu^-) = (1.43 \pm 0.30) \times 10^{-7} (R_S^2 + R_P^2).$$  

(29)

Equating the expression in eq. (29) to the present 95% C.L. upper limit in eq. (2), we get the inequality

$$(R_S^2 + R_P^2) \leq 0.70,$$  

(30)

where we have taken the $2\sigma$ lower bound for the coefficient in eq. (29). Thus, the allowed region in the $R_S-R_P$ parameter space is the interior of a circle of radius $\approx 0.84$ centered at the origin.

In [31], it was shown that the SPNP operators cannot lower $B(B \to K\mu^+\mu^-)$ below its SM prediction. Therefore from eq. (24), the maximum value of $\langle A_{FB} \rangle$ with the current upper bound on $B(B \to K\mu^+\mu^-)$ is 1.34% at $2\sigma$. If $B(B_s \to \mu^+ \mu^-)$ is bounded to $10^{-8}$, the $2\sigma$ maximum value of $\langle A_{FB} \rangle$ will be 0.56%.

A naive estimation suggests that the measurement of an asymmetry $\langle A_{FB} \rangle$ of a decay with the branching ratio $B$ at $n\sigma$ C.L. with only statistical errors require

$$N \sim \left( \frac{n}{B \langle A_{FB} \rangle} \right)^2$$  

(31)
number of events. For $B \to K \mu^+ \mu^-$, if $\langle A_{FB} \rangle$ is 1% at 2$\sigma$ C.L., then the required number of events will be as high as $10^{18}$! Therefore it is very difficult to observe such a low value of FB asymmetry in experiments. Hence FB asymmetry of muons in $B \to K \mu^+ \mu^-$ will play no role in testing SPNP.

III. LONGITUDINAL POLARIZATION ASYMMETRY IN $B_s \to \mu^+ \mu^-$

The longitudinal polarization asymmetry of muons in $B_s \to \mu^+ \mu^-$ is a clean observable that depends only on SPNP operators. It vanishes in the SM, whereas its value is nonzero if and only if the new physics contribution is in the form of scalar operator. Therefore any nonzero measurement of this observable $A_{LP}$ will confirm the existence of an extended Higgs sector. The observable $A_{LP}$ was introduced in ref. [17], though the corresponding analysis in the context of $K_L \to \mu^+ \mu^-$ had been carried out earlier [32, 33, 34, 35]. In this section, we will determine the allowed values of $A_{LP}$ consistent with the present upper bound on $B(B_s \to \mu^+ \mu^-)$, and explore the correlation between these two quantities.

The most general model independent form of the effective Lagrangian for the quark level transition $b \to s \mu^+ \mu^-$ that contributes to the decay $B_s \to \mu^+ \mu^-$ has the form [36, 37]

$$L = \frac{G_F \alpha}{2\sqrt{2} \pi} (V_{ts}^* V_{tb}) \left\{ R_A (\bar{s} \gamma_\mu \gamma_5 b)(\bar{\mu} \gamma^\mu \gamma_5 \mu) \\
+ R_S (\bar{s} \gamma_5 b)(\bar{\mu} \mu) + R_P (\bar{s} \gamma_5 b)(\bar{\mu} \gamma_5 \mu) \right\} , \quad (32)$$

where $R_P$, $R_S$ and $R_A$ are the strengths of the scalar, pseudoscalar and axial vector operators respectively. Note that the effective Lagrangian in eq. (32) is essentially the same as the effective Lagrangian given in eq. (5). Here we have dropped $C_7$ and $C_9$ terms which do not contribute to $B_s \to \mu^+ \mu^-$. In addition, the $R_A$ in eq. (32) is the sum of SM and new physics contributions.

In SM, the scalar and pseudoscalar couplings $R_{SM}^S$ and $R_{SM}^P$ receive contributions from the penguin diagrams with physical and unphysical neutral scalar exchange and are highly suppressed:

$$R_{SM}^S = R_{SM}^P \propto \frac{(m_\mu m_b)}{m_W^2} \sim 10^{-5} . \quad (33)$$
Also, $R_{A}^{\text{SM}} = Y(x)/\sin^2\theta_W$, where $Y(x)$ is the Inami-Lim function \[38\]

$$Y(x) = \frac{x}{8} \left[ \frac{x-8}{x-1} + \frac{3x}{(x-1)^2} \ln x \right], \quad (34)$$

with $x = (m_t/M_W)^2$. Thus, $R_{A}^{\text{SM}} \simeq 4.3$.

The calculation of the branching ratio gives [17, 36]

$$B(B_s \to \mu^+ \mu^-) = a_s \left[ 2m_\mu R_A - \frac{m_{B_s}^2}{m_b + m_s} R_P \right]^2 + \left( 1 - \frac{4m_\mu^2}{m_{B_s}^2} \right) \left| \frac{m_{B_s}^2}{m_b + m_s} R_S \right|^2, \quad (35)$$

where

$$a_s \equiv \frac{G_F^2 \alpha^2}{64\pi^2} |V_{ts}^* V_{tb}|^2 \tau_{B_s} f_{B_s}^2 m_{B_s} \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}}. \quad (36)$$

Here $\tau_{B_s}$ is the lifetime of $B_s$. Eq. (35) represents the most general expression for the branching ratio of $B_s \to \mu^+ \mu^-$. We now derive an expression for the lepton polarization. In the rest frame of $\mu^+$, we can define only one direction $\hat{\mathbf{p}}_-$, the three momentum of $\mu^-$. The unit longitudinal polarization 4-vectors along that direction are

$$\hat{s}_{\mu}^{\pm} = (0, \hat{\mathbf{e}}_L^{\pm}) = \left(0, \pm \frac{\hat{\mathbf{p}}_-}{|\hat{\mathbf{p}}_-|}\right). \quad (37)$$

Transformation of unit vectors from the rest frame of $\mu^+$ to the center of mass frame of leptons (which is also the rest frame of $B_s$ meson) can be accomplished by the Lorentz boost. After the boost, we get

$$\hat{s}_{\mu}^{\pm} = \left(\frac{\hat{\mathbf{p}}_-}{m_\mu}, \pm \frac{E_\mu \hat{\mathbf{p}}_-}{m_\mu |\hat{\mathbf{p}}_-|}\right), \quad (38)$$

where $E_\mu$ is the muon energy.

The longitudinal polarization asymmetry of muons in $B_s \to \mu^+ \mu^-$ is defined as

$$A_{LP}^{\pm} = \frac{\Gamma(\hat{\mathbf{e}}_L^{\pm}) - \Gamma(-\hat{\mathbf{e}}_L^{\pm})}{\Gamma(\hat{\mathbf{e}}_L^{\pm}) + \Gamma(-\hat{\mathbf{e}}_L^{\pm})}. \quad (39)$$

Thus we get [17]

$$A_{LP} = 2 \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \left[ \frac{m_{B_s}^2}{m_b + m_s} R_S \left( 2m_\mu R_A - \frac{m_{B_s}^2}{m_b + m_s} R_P \right) \right] \left| 2m_\mu R_A - \frac{m_{B_s}^2}{m_b + m_s} R_P \right|^2 + \left( 1 - \frac{4m_\mu^2}{m_{B_s}^2} \right) \left| \frac{m_{B_s}^2}{m_b + m_s} R_S \right|^2, \quad (40)$$
with $A_{LP}^+ = A_{LP}^- \equiv A_{LP}$. It is clear from eq. (10) that $A_{LP}$ can be nonzero if and only if $R_S \neq 0$, i.e. for $A_{LP}$ to be nonzero, we must have contribution from SPNP operators. Within the SM, $R_S \simeq 0$ and hence $A_{LP} \simeq 0$.

Using eq. (35), we can eliminate $R_A$ and $R_P$ from eq. (40) in favour of the physical observables $B(B_s \to \mu^+ \mu^-)$ and $a_s$. We get [17]

$$A_{LP} = \pm \frac{2a_s}{B(B_s \to \mu^+ \mu^-)} \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2}} \times \frac{m_{B_s}^2 R_S}{m_b + m_s} \sqrt{\frac{B(B_s \to \mu^+ \mu^-)}{a_s}} - \left(1 - \frac{4m_\mu^2}{m_{B_s}^2} \right) \left| \frac{m_{B_s}^2 R_S}{m_b + m_s} \right|^2 .$$  \hspace{1cm} (41)

Eq. (41) represents a general relation between the longitudinal polarization asymmetry $A_{LP}$ and the branching ratio of $B_s \to \mu^+ \mu^-$. 

We now explore the correlation between $A_{LP}$ and $B(B_s \to \mu^+ \mu^-)$. It is quite obvious that when $B(B_s \to \mu^+ \mu^-) \gtrsim 10^{-8}$, we can neglect the SM contribution in obtaining the bounds on $R_S$ and $R_P$. However if $B(B_s \to \mu^+ \mu^-)$ is of the order of the SM prediction, then we will have to take into account the SM contribution as well. Therefore it is reasonable to consider both the cases separately.
We first consider the constraints on $A_{LP}$ coming from the present upper bound on $B(B_s \to \mu^+\mu^-)$. Fig. 1 shows the plot between $A_{LP}$ and $R_S$ for three different values of $B(B_s \to \mu^+\mu^-) \gtrsim 10^{-8}$. Fig. 2 is a plot between $|A_{LP}|$ and $B(B_s \to \mu^+\mu^-)$ for various allowed values of $R_S$. The bands in Figs. 1 and 2 are mainly due to the uncertainties in CKM matrix element $|V_{ts}|$ and decay constant $f_{B_s}$.

We see from Fig. 1 that the maximum possible value of $A_{LP}$ consistent with the present upper bound on $B(B_s \to \mu^+\mu^-)$ is 100%, i.e. the present upper bound of $B(B_s \to \mu^+\mu^-)$ does not put any constraint on $A_{LP}$. Indeed, $B(B_s \to \mu^+\mu^-)$ will be unable to put any constraint on $A_{LP}$ even if it is as low as $10^{-8}$.

Thus we see that the recently improved upper bound on the branching ratio of $B_s \to \mu^+\mu^-$, which provides the most stringent bound on SPNP couplings, fails to put any bound on $A_{LP}$. Therefore $A_{LP}$ is more sensitive to SPNP operators as compared to $B(B_s \to \mu^+\mu^-)$. Any nonzero measurement of $A_{LP}$ will be evidence for an extended Higgs sector.
We would like to emphasize another important point: The measurement of $B(B_s \rightarrow \mu^+\mu^-)$ will only give the allowed range for the values of the SPNP couplings $R_S$ and $R_P$. However the simultaneous determination of $B(B_s \rightarrow \mu^+\mu^-)$ and $A_{LP}$ will allow the determination of new physics scalar coupling $R_S$ (see Fig. 2) and this in turn will enable us to determine the new physics pseudoscalar coupling $R_P$.

B. $B(B_s \rightarrow \mu^+\mu^-) \lesssim 10^{-8}$

LHC is expected to reach the SM sensitivity in $B_s \rightarrow \mu^+\mu^-$. In fact, it may even go $5\sigma$ below the SM prediction [11]. Therefore it is worth considering the case when $B(B_s \rightarrow \mu^+\mu^-)$ is of the order of the SM prediction. In this section we study the correlation between $A_{LP}$ and $B(B_s \rightarrow \mu^+\mu^-)$ under the assumption that $B(B_s \rightarrow \mu^+\mu^-)$ is close to its SM prediction.

Taking $R_A = R_A^{SM}$, eq. (35) gives

$$B(B_s \rightarrow \mu^+\mu^-) = a_s \left[28.8(R_S^2 + R_P^2) - 9.7R_P + 0.81\right],$$

which leads to

$$R_S^2 + (R_P - 0.165)^2 = \frac{0.035 B(B_s \rightarrow \mu^+\mu^-)}{a_s}.\quad (43)$$

This corresponds to a circle in $R_S - R_P$ plane with centre at $(R_S = 0, R_P = 0.165)$ and radius given by $r = \sqrt{0.035 B(B_s \rightarrow \mu^+\mu^-)/a_s}$.

Fig. 3 shows the plot between $A_{LP}$ and $R_S$ for three different values of $B(B_s \rightarrow \mu^+\mu^-) \lesssim 10^{-8}$. Fig. 4 is a plot between $|A_{LP}|$ and $B(B_s \rightarrow \mu^+\mu^-)$ for various allowed values of $R_S$. It is obvious from fig. 3 that $A_{LP}$ can be 100% even if $B(B_s \rightarrow \mu^+\mu^-)$ is close to its SM prediction.

We now consider three exciting experimental possibilities, all of which can be accounted for with SPNP.

1. $B(B_s \rightarrow \mu^+\mu^-)$ is consistent with SM but $A_{LP} \neq 0$

It is possible to have a non-zero value of $A_{LP}$ even if $B(B_s \rightarrow \mu^+\mu^-)$ is equal to its SM prediction. We can re-write eq. (35) in the following form:

$$B(B_s \rightarrow \mu^+\mu^-) = a_s [(b_{SM} - b_P)^2 + b_S^2],$$

(44)
FIG. 3: $A_{LP}$ vs $R_s$ plot for $B(B_s \rightarrow \mu^+\mu^-) = (5.5, 3.5, 1.5) \times 10^{-9}$

FIG. 4: Plot between $|A_{LP}|$ and $B(B_s \rightarrow \mu^+\mu^-)$ for different $R_S$ values, when $B(B_s \rightarrow \mu^+\mu^-) \lessapprox 10^{-8}$. The vertical shaded band corresponds to $1\sigma$ theoretical prediction within the SM. The region $B(B_s \rightarrow \mu^+\mu^-) \gtrapprox 5 \times 10^{-9}$ is not ruled out; here we just concentrate on the region near the SM prediction.
where
\[ b_{SM} = 2m_\mu R_{SM}^A, \quad b_P = \frac{m_{B_s}^2 R_P}{m_b + m_s}, \quad b_S = \sqrt{1 - \frac{4m_\mu^2}{m_{B_s}^2 m_b + m_s} R_S} . \] (45)

Here we have taken \( R_A = R_{SM}^A \), i.e. we have considered new physics only through the SPNP operators. Now if \( B(B_s \to \mu^+\mu^-) \) is equal to its SM prediction, then
\[ a_s[(b_{SM} - b_P)^2 + b_S^2] = a_s b_{SM}^2 , \] (46)

which leads to
\[ (b_P - b_{SM})^2 + b_S^2 = b_{SM}^2 , \] (47)
or
\[ R_S^2 + \left[ R_P - \frac{(m_b + m_s)b_{SM}}{m_{B_s}^2} \right]^2 = \left( \frac{m_b + m_s}{m_{B_s}^2} b_{SM} \right)^2 . \] (48)

Eq. (48) represents a circle in \( R_S - R_P \) plane with center at \((0, (m_b + m_s)b_{SM}/m_{B_s}^2)\).

The interference between the SPNP and SM operators can decrease the branching ratio \( B(B_s \to \mu^+\mu^-) \) far below its SM prediction. In fact it can be seen from eq. (35), \( B(B_s \to \mu^+\mu^-) \) can even vanish, provided the following conditions are satisfied simultaneously:
\[ R_S = 0, \quad R_P = \frac{2m_\mu m_b}{m_{B_s}^2} R_A = 0.04 R_A . \] (49)

From Fig. 4 it can be seen that for low \( R_S \) values, it is indeed possible to suppress \( B(B_s \to \mu^+\mu^-) \) much below its SM value.

2. **LHCb fails to find \( B_s \to \mu^+\mu^- \)**

If LHCb fails to find \( B_s \to \mu^+\mu^- \) or puts an upper bound on its branching ratio which is smaller than \( 2 \times 10^{-9} \) (5\( \sigma \) below SM prediction), this scenario can still be accommodated within the SPNP.

The interference between the SPNP and SM operators can decrease the branching ratio \( B(B_s \to \mu^+\mu^-) \) far below its SM prediction. In fact it can be seen from eq. (35), \( B(B_s \to \mu^+\mu^-) \) can even vanish, provided the following conditions are satisfied simultaneously:
\[ R_S = 0, \quad R_P = \frac{2m_\mu m_b}{m_{B_s}^2} R_A = 0.04 R_A . \] (49)
3. Both $B(B_s \to \mu^+ \mu^-)$ and $A_{LP}$ are consistent with the SM

The lepton polarization asymmetry is a result of the interference of the scalar term with pseudoscalar / axial vector, as can be seen from eq. (10). Therefore it vanishes when either $b_S$ or $(b_P - b_{SM})$, defined in eq. (15), vanishes. Thus there exists the interesting possibility of nontrivial SPNP even when both $B(B_s \to \mu^+ \mu^-)$ and $A_{LP}$ are consistent with the SM. This occurs when

$$b_S = 0, \quad b_P = 2b_{SM}, \quad (50)$$

or

$$b_S = b_{SM}, \quad b_P = b_{SM}, \quad (51)$$

as can be confirmed from eq. (17). Therefore, the absence of SPNP is not guaranteed simply by the consistency of these observables with the SM; more channels need to be examined to rule out this possibility completely.

IV. CONCLUSIONS

An order of magnitude enhancement in $B(B_s \to \mu^+ \mu^-)$ is possible only due to SPNP operators. Apart from $B(B_s \to \mu^+ \mu^-)$, observables such as FB asymmetry of muons in $B \to K\mu^+ \mu^-$ and LP asymmetry of muons in $B_s \to \mu^+ \mu^-$ are also sensitive to SPNP operators. In this paper we consider the constraints on possible SPNP contribution to these observables coming from the present upper bound on $B(B_s \to \mu^+ \mu^-)$.

We find that $B(B_s \to \mu^+ \mu^-)$ puts very stringent constraint on SPNP contribution to $\langle A_{FB} \rangle$ and restricts its value to be less than $\sim 1\%$. Such a small FB asymmetry is almost impossible to be measured in experiments. In the literature, $\langle A_{FB} \rangle$ of muons in $B \to K\mu^+ \mu^-$ has been considered a promising measurement for probing SPNP operators. Our results show that the present upper bound on $B(B_s \to \mu^+ \mu^-)$ makes searching for SPNP through $\langle A_{FB} \rangle$ a futile exercise.

On the other hand, the present upper bound on $B(B_s \to \mu^+ \mu^-)$ does not put any constraint on $A_{LP}$. Indeed, $A_{LP}$ can be 100% even if $B(B_s \to \mu^+ \mu^-)$ is close to its SM prediction. $A_{LP}$ is sensitive only to SPNP operators and hence its nonzero value will give direct evidence for a non-standard Higgs sector.
A simultaneous determination of $B(B_s \to \mu^+\mu^-)$ and $A_{LP}$ will enable us to separate the new physics scalar and pseudoscalar contributions. Therefore it is worth considering this observable in experiments to probe the extended Higgs sector.

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