Spectroscopy of doubly and triply-charmed baryons from lattice QCD

Padmanath M.

Department of Theoretical Physics,
Tata Institute of Fundamental Research,
Mumbai, India.

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In collaboration with R. G. Edwards, N. Mathur and M. Peardon.

Computations performed on computational facilities at DTP, TIFR, Mumbai, Jefferson Laboratory and TCHPC, Trinity College, Dublin.
$4 \ (u, d, s, c)$ degenerate flavors

We have one heavy and 2+1 light flavor quarks.
4 \((u, d, s, c)\) degenerate flavors

We have one heavy and 2+1 light flavor quarks.
Ensemble details

Calculations performed on lattices generated by **Hadron Spectrum Collaboration**.

- Dynamical configurations ($N_f = 2 + 1$).
- Anisotropic lattices with $\xi = a_s/a_t \sim 3.5$.
- Scale set via $m_\Omega : a_s = 0.12$ fm
- Lattice size : $16^3 \times 128$.
- Statistics : 96 cfgs and 4 time sources.
- **Clover** fermions : Non-perturbtive O(a) improvement.
- Spatial links are **stout smeared**.
- Quark fields are **distilled**.
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- Statistics : 96 cfgs and 4 time sources.
- **Clover** fermions : Non-perturbtive $O(a)$ improvement.
- Spatial links are **stout smeared**.
- Quark fields are **distilled**.
- **Caveat** : Pion mass $\sim 391$ MeV.
### Interpolating operators

$\Omega_{ccc}$

**Non-Rel: $SU(6) \otimes O(3)$**

| D  | J   | 1/2 | 3/2 | 5/2 | 7/2 |
|----|-----|-----|-----|-----|-----|
| 0  |     | 0   | 1   | 0   | 0   |
| 1  |     | 1   | 1   | 0   | 0   |
| $2_{hybrid}$ |     | 1   | 1   | 0   | 0   |
| 2  |     | 2   | 3   | 2   | 1   |

$\Omega_{cc}$ and $\Xi_{cc}$

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| 0  |     | 1   | 1   | 0   | 0   |
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| $2_{hybrid}$ |     | 3   | 3   | 1   | 0   |
| 2  |     | 6   | 8   | 5   | 2   |
**Interpolating operators**

\( \Omega_{ccc} \)

**Non-Rel:** \( SU(6) \otimes O(3) \)

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|----------------|-----|-----|-----|-----|
| 0              | 0   | 1   | 0   | 0   |
| 1              | 1   | 1   | 0   | 0   |
| \( 2_{hybrid} \) | 1   | 1   | 0   | 0   |
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**Whole operator set**

\( \Omega_{ccc} \) and \( \Xi_{cc} \)

**Non-Rel:** \( SU(6) \otimes O(3) \)

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|----------------|-----|-----|-----|-----|
| 0              | 1   | 1   | 0   | 0   |
| 1              | 3   | 3   | 1   | 0   |
| \( 2_{hybrid} \) | 3   | 3   | 1   | 0   |
| 2              | 6   | 8   | 5   | 2   |
Generalized eigenvalue problem

Using this large operator basis, with definite $J^P$ in the continuum limit, to build the correlation matrix

$$C_{ij}(t) = \langle 0|O_i(t)O_j^\dagger(0)|0\rangle = \sum_n \frac{Z_n^i Z_j^{n\dagger}}{2E_n} \exp^{-E_n t}$$

Solving the generalized eigenvalue problem for this correlation matrix

$$C_{ij}(t)\nu_j^{(n)}(t, t_0) = \lambda^{(n)}(t, t_0) C_{ij}(t_0)\nu_j^{(n)}(t, t_0)$$

- Principal correlators given by eigenvalues
  $$\lambda_n(t, t_0) \sim (1 - A_n) \exp^{-m_n(t-t_0)} + A_n \exp^{-m'_n(t-t_0)}$$
- Eigenvectors related to the overlap factors
  $$Z_i^{(n)} = \langle 0|O_i|n\rangle = \sqrt{2E_n} \exp^{E_n t_0/2} \nu_j^{(n)\dagger} C_{ji}(t_0)$$
Spin identification

Discretized space-time breaks rotational symmetry down to octahedral symmetry.

- Continuum spin operators subduced to lattice irreps.

- $G_1$, $H$ and $G_2$: $O_h$ irreps representing half spin.

| $\Lambda$ | $d_\Lambda$ | $J$          |
|----------|-------------|--------------|
| $G_1$    | 2           | 1/2, 7/2, 9/2, ... |
| $H$      | 4           | 3/2, 5/2, 7/2, ... |
| $G_2$    | 2           | 5/2, 7/2, 9/2, ... |

- Subduced operators carry a memory of the continuum spin $J$.

- An operator of spin $J$ overlaps mainly with states of spin $J$. Overlap factors to identify spin of states.
ccc correlation matrix plot \((H^g; \text{at } t=5)\): \(C_{ij} / \sqrt{C_{ii}C_{jj}}\)

\[
\begin{align*}
nr - nh &= \text{non} - \text{relativistic} \& \text{non} - \text{hybrid} \\
nr - h &= \text{non} - \text{relativistic} \& \text{hybrid} \\
r - nh &= \text{relativistic} \& \text{non} - \text{hybrid} \\
r - h &= \text{relativistic} \& \text{hybrid}
\end{align*}
\]
Spin identification using overlap factors: \((ccc, G_1^g)\)

| Operators | States |
|-----------|--------|
| \(nr - nh\) | 0, 1, 2, 3, 4, 8, 10, 11 |
| \(nr - h\) | 0, 1, 2, 3, 4, 8, 10, 11 |
| \(r - nh\) | 0, 1, 2, 3, 4, 8, 10, 11 |
| \(r - h\) | 0, 1, 2, 3, 4, 8, 10, 11 |

\(nr - nh\) = non-relativistic & non-hybrid

\(nr - h\) = non-relativistic & hybrid

\(r - nh\) = relativistic & non-hybrid

\(r - h\) = relativistic & hybrid
Spin identification across multiple irreps: $7/2^+$

$7/2^+$

$\begin{array}{c}
G_1^g: a_t E = 0.950(3) \\
H^g: a_t E = 0.943(10) \\
G_2^g: a_t E = 0.949(3)
\end{array}$
$\Omega_{ccc}$ spectrum

arXiv:1307.7022 [hep-lat]
## Interpolating operators

### $\Omega_{\text{ccc}}$

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|---|----|-----|-----|-----|-----|
| 0 |    | 0   | 1   | 0   | 0   |
| 1 |    | 1   | 1   | 0   | 0   |
| $2_{\text{hybrid}}$ | | 1   | 1   | 0   | 0   |
| 2 |    | 2   | 3   | 2   | 1   |

### $\Omega_{\text{cc}}$ and $\Xi_{\text{cc}}$

**Non-Rel:** $SU(6) \otimes O(3)$

| D | J  | 1/2 | 3/2 | 5/2 | 7/2 |
|---|----|-----|-----|-----|-----|
| 0 |    | 1   | 1   | 0   | 0   |
| 1 |    | 3   | 3   | 1   | 0   |
| $2_{\text{hybrid}}$ | | 3   | 3   | 1   | 0   |
| 2 |    | 6   | 8   | 5   | 2   |
$\Omega_{ccc \ (3/2^+)}$ ground state: discretization errors

This work ILGTI-2013 PACS-CS-2012

$0.06 \ 0.09 \ 0.12 \ 0.15 \ 0.18 \ 0.21$

$\Omega_{ccc,g}$

$- 3/2 m_{J/ψ}$ (GeV)

$c_{SW}=1.35$

$c_{SW}=2.0$

$a=0.0582 \text{fm}$

$a=0.0888 \text{fm}$

$a=0.09 \text{fm}$

Excited charm baryon spectroscopy from lattice QCD

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Excited charm baryon spectroscopy from lattice QCD

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Ω_{cc} spectrum
Excited charm baryon spectroscopy from lattice QCD

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**Interpolating operators**

$$\Omega_{ccc}$$

**Non-Rel:** $SU(6) \otimes O(3)$

| D  | J   | 1/2 | 3/2 | 5/2 | 7/2 |
|----|-----|-----|-----|-----|-----|
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| $2_{hybrid}$ | | 1   | 1   | 0   | 0   |
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**$\Omega_{cc}$ and $\Xi_{cc}$**

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| 0  |     | 1   | 1   | 0   | 0   |
| 1  |     | 3   | 3   | 1   | 0   |
| $2_{hybrid}$ | | 3   | 3   | 1   | 0   |
| 2  |     | 6   | 8   | 5   | 2   |
cc(q) ground states

\[ \Omega_{ccs}(1/2^+) \]
\[ \Omega_{ccs}(3/2^+) \]

\[ \Xi_{ccu}(1/2^+) \]
\[ \Xi_{ccu}(3/2^+) \]

- → This work
- □ → Other lattice results
- □ → EFT and model calculations
- ★ → Experiment

Excited charm baryon spectroscopy from lattice QCD

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Spin-Orbit interactions inversly proportional to $m_q^2$.
Vanishes in the heavy quark limit.
Degeneracy lifts : a measure of heavyness of the quark mass.

Binding energy quark mass dependence.
Mass of a hadron with n heavy quarks: $M_{H_{nq}} = nm_Q + A + B/m_Q + O(1/m_Q^2)$.
Energy splittings : $a + b/m_Q + O(1/m_Q^2)$.
Fits with heavy quark inspired functional forms.

From energy splittings $(\Xi_{cc}^* - D_c, \Omega_{cc}^* - D_s$ and $\Omega_{ccc} - \eta_c)$ and $(\Xi_{cc}^* - D_c^*, \Omega_{cc}^* - D_s^*$ and $\Omega_{ccc} - J/\psi)$, we extrapolate to bottom mass and get $B_c^* - B_c = 80 \pm 8$ MeV and $\Omega_{ccb}^* = 8050 \pm 10$ MeV.
Spin-Orbit splittings in $\Omega$ like baryons

$|E(1/2^+) - E(3/2^+)|$
$|E(5/2^+) - E(3/2^+)|$
$|E(7/2^+) - E(3/2^+)|$
$|E(5/2^+) - E(3/2^-)|$

$\Delta m \text{ (GeV)}$

$H_{g,1,S} \otimes D^{[2]}_{L=2,S}$
$G_{1g,1,M} \otimes D^{[2]}_{L=2,M}$
$G_{1g,1,M} \otimes D^{[1]}_{L=1,M}$

$u$ and $s \rightarrow$ Edwards, et. al., Phys. Rev. D 87, 054506 (2013)
$b \rightarrow$ S. Meinel, Phys. Rev. D 85, 114510 (2012)
Quark mass dependence of Ω like baryons

$\Delta m$ (MeV) vs $m_{ps}^2$ (GeV$^2$)

1/2$^+$

3/2$^+$

5/2$^+$

7/2$^+$

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Quark mass dependence

$\Delta m$ (MeV) vs. $m_{ps}^2$ (GeV$^2$)

$1/2^-$

$3/2^-$

$5/2^-$

$7/2^-$

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Summary and conclusions

- Non-perturbative calculation for excited state spectroscopy of $\Omega_{ccc}$, $\Omega_{cc}$ and $\Xi_{cc}$.

- Non-relativistic spectrum pattern observed up to the second energy band.

- Identification of the spin and spatial structure of the states using the overlap factors.

- SO splittings: The degeneracy more or less satisfied for $m_c$.

- Energy splittings: Heavy quark inspired form gives good fit with $m_b$, $m_c$ as well as $m_s$. For some, the fits even pass through $m_l$ also.

- Extrapolations to bottom sector: $B_c^* - B_c = 80 \pm 8$ MeV and $\Omega_{ccb}^* = 8050 \pm 10$ MeV.

- No multi-hadron operators being used: Further works required to see their effects.

- Singly charm baryons under investigation.