ON THE INFLUENCE OF ACOUSTIC WAVES ON COHERENT BREMSSTRAHLUNG IN CRYSTALS

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Abstract

We investigate the coherent bremsstrahlung by relativistic electrons in a single crystal excited by hypersonic vibrations. The formula for the corresponding differential cross-section is derived in the case of a sinusoidal wave. The conditions are specified under which the influence of the hypersonic is essential. The case is considered in detail when the electron enters into the crystal at small angles with respect to a crystallographic axis. It is shown that in dependence of the parameters, the presence of hypersonic waves can either enhance or reduce the bremsstrahlung cross-section.

Keywords: Interaction of particles with matter; coherent bremsstrahlung; physical effects of ultrasonics.

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1 Introduction

The processes converting the energy of relativistic electrons into flows of electromagnetic radiation with the help of single crystals are still of fundamental and practical interest in high-energy physics. In crystals the cross-sections of the high-energy electromagnetic processes can change essentially compared with the corresponding quantities for a single atom (see, for instance, Refs. [1, 2, 3, 4, 5, 6] and references therein). The momentum transfer between a highly relativistic interacting particle and the crystal can be small, especially along the direction of particle motion. When this longitudinal momentum transfer is small, the uncertainty principle dictates that the interaction is spread out over a distance, known as the formation length for radiation or, more generally, as the coherence length. If the formation length exceeds the interatomic spacing, the interference effects from all atoms within this length are important and they can essentially affect the

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corresponding cross-sections. From the point of view of controlling the parameters of
the high-energy electromagnetic processes in a medium it is of interest to investigate
the influence of external fields, such as acoustic waves, temperature gradient etc., on the
corresponding characteristics. The considerations of concrete processes, such as diffraction
radiation [7], transition radiation [8], parametric X-radiation [9], channeling radiation
[10], electron-positron pair creation by high-energy photons [11] have shown that the
external fields can essentially change the angular-frequency characteristics of the radiation
intensities.

The coherent bremsstrahlung of high-energy electrons moving in a crystal is one of the
most effective methods for producing of quasimonochromatic gamma-quanta and has been
intensively investigated either theoretically and experimentally over the last decade (see,
for instance, Refs. [1, 2, 3, 4, 5, 6]). Such radiation has a number of remarkable properties
and at present it has found many important applications. Among these is the generation
of intense positron beams. The basic source to creating positrons for high-energy electron-
positron colliders is the electron-positron pair creation by hard bremsstrahlung photons
produced when a powerful electron beam hits an amorphous target. One possible approach
to increase the positron production efficiency is to use a crystal target as a positron emitter
(see Refs. [12, 13] and references therein). When the crystal axis is aligned to the incident
beam direction, intense photons are emitted through the coherent bremsstrahlung process
and the channeling radiation process. These photons are then converted to electron-
positron pairs in the same crystal, or in the amorphous target behind the crystal.

The wide applications of the bremsstrahlung by relativistic particles motivate the im-
portance of investigations for various mechanisms of controlling the radiation parameters.
In the present paper we investigate the influence of a hypersonic wave on the coherent
bremsstrahlung by relativistic electrons in a crystal. We specify the conditions under
which the external deformation field changes noticeably the bremsstrahlung cross-section
compared to the case of an undeformed crystal and demonstrate the possibility for the
radiation yield enhancement. The plan of the paper is as follows. In Sec. 2 a formula
is derived for the coherent part of the bremsstrahlung by an electron in presence of the
sinusoidal deformation field generated by a hypersonic vibrations. The analysis of the
general formula and numerical results in the special cases when the electron enters into
the crystal at small angles with respect to crystallographic axes or planes are presented
in Sec. 3. The main results are summarized in Sec. 4.

2 Cross-section for the coherent bremsstrahlung

Let us consider the bremsstrahlung by a relativistic electron moving in a single crystal
excited by hypersonic vibrations. The corresponding cross-section can be presented in the
form (see, for example, [1, 3])

$$\sigma(q) \equiv \frac{d^4\sigma}{d\omega dq} = \left| \sum_n e^{iqr_n} \right|^2 \sigma_0(q),$$  \hspace{1cm} (1)

where $q = p_1 - p_2 - k$ is the momentum transferred to the crystal, $\sigma_0(q)$ is the cross-
section on an individual atom, $r_n$ are the positions of atoms in the crystal. Here and
below $p_1$ and $p_2$ are momenta of particle in the initial and final states, $\omega$ and $k$ are the
frequency and wave vector for the radiated photon (the system of units \( \hbar = c = 1 \) is used). The differential cross-section in a crystal, Eq. (1), differs from the cross-section on an isolated atom by the interference factor which is responsible for coherent effects arising due to periodical arrangement of the atoms in the crystal.

The positions of the atoms in a crystal can be presented as \( \mathbf{r}_n = \mathbf{r}_{n0} + \mathbf{u}_n \), where \( \mathbf{u}_n \) is the displacement of atoms with respect to the equilibrium positions \( \mathbf{r}_{n0} \) (by taking into account the crystal deformation due to the hypersonic wave) due to the thermal vibrations. After averaging on thermal fluctuations the cross-section takes the standard form

\[
\sigma(q) = \left\{ N_0 \left( 1 - e^{-q^2 \bar{u}_t^2} \right) + e^{-q^2 \bar{u}_t^2} \left| \sum_n e^{i \mathbf{q} \mathbf{r}_{n0}} \right|^2 \right\} \sigma_0(q), \tag{2}
\]

where \( \bar{u}_t^2 \) is the temperature dependent mean-squared amplitude of the thermal vibrations of atoms, \( N_0 \) is the number of atoms in the crystal, \( e^{-q^2 \bar{u}_t^2} \) is the Debye-Waller factor.

When external influences are present the positions of atoms can be written as

\[
\mathbf{r}_{n0} = \mathbf{r}_{ne} + \mathbf{u}_n, \tag{3}
\]

with \( \mathbf{r}_{ne} \) being the equilibrium positions of atoms in the situation without deformation, \( \mathbf{u}_n \) are the displacements of atoms caused by the acoustic wave. We will consider deformations with the sinusoidal structure

\[
\mathbf{u}_n = \mathbf{u}_0 \sin \left( \mathbf{k}_s \mathbf{r}_{ne} + \varphi_0 \right), \tag{4}
\]

where \( \mathbf{k}_s \) is the wave vector of the hypersonic wave. Here the dependence of \( \mathbf{u}_n \) on time (through the phase \( \varphi_0 \)) we can disregard, as for particle energies we are interested in, the characteristic time for the change of deformation field is much greater than the passage time of particles through the crystal. For the deformation field given by Eq. (4) the sum over the atoms in Eq. (2) can be transformed into the form

\[
\sum_n e^{i \mathbf{q} \mathbf{r}_{n0}} = \sum_{m = -\infty}^{+\infty} J_m(\mathbf{q}\mathbf{u}_0) e^{im\varphi_0} \sum_n e^{i \mathbf{q}_m \mathbf{r}_{ne}}, \quad \mathbf{q}_m = \mathbf{q} + m \mathbf{k}_s, \tag{5}
\]

where \( J_m(x) \) is the Bessel function. For a lattice with a complex cell the coordinates of the atoms can be written as \( \mathbf{r}_{ne} = \mathbf{R}_n + \rho_j \), with \( \mathbf{R}_n \) being the positions of the atoms for one of primitive lattices, and \( \rho_j \) are the equilibrium positions for other atoms inside \( n \)-th elementary cell with respect to \( \mathbf{R}_n \). Now the square of the modulus for the sum (5) can be presented as

\[
\left| \sum_n e^{i \mathbf{q} \mathbf{r}_{n0}} \right|^2 = \sum_{m, m' = -\infty}^{+\infty} J_m(\mathbf{q}\mathbf{u}_0) J_{m'}(\mathbf{q}\mathbf{u}_0) e^{i(m-m')\varphi_0} \sum_{n, n'} e^{i \mathbf{q}_m \mathbf{R}_n} e^{-i \mathbf{q}_{m'} \mathbf{R}_{n'}} S(\mathbf{q}_m) S^*({\mathbf{q}_{m'}}), \tag{6}
\]

where \( S(\mathbf{q}) = \sum_j e^{i \mathbf{q} \rho_j} \) is the structure factor of an elementary cell. For thick crystals the sum over cells can be presented as a sum over the reciprocal lattice:

\[
\sum_n e^{i \mathbf{q}_m \mathbf{R}_n} = \frac{(2\pi)^3}{\Delta} \sum_\mathbf{g} \delta(\mathbf{q}_m - \mathbf{g}), \tag{7}
\]
where $\Delta$ is the unit cell volume, and $g$ is the reciprocal lattice vector. By taking into account the $\delta$-function, the quantity $q'_{m'}$ can be written as $q'_{m'} = g + (m' - m)k_s$ and, hence, we receive

$$\sum_{n'} e^{-i q'_{m'} R_{n'}} = \sum_{n'} e^{-i (m' - m)k_s R_{n'}} = \frac{(2\pi)^3}{\Delta} \sum_g \delta((m' - m)k_s - g). \quad (8)$$

As in the case of the electron-positron pair creation by high-energy photons [11], it can be seen that in the sum over $m$ the main contribution comes from the terms for which $mk_s u_0 \lesssim gu_0$, or equivalently $m \lesssim \lambda_s/a$, where $\lambda_s = 2\pi/k_s$ is the wavelength of the external excitation, and $a$ is the lattice constant. Further, under the condition $u_0/\lambda_s \ll 1$ the contribution of the terms with $m \neq m'$ in the sum (6) is small compared to the diagonal terms (see analogous discussion in Ref. [11]). In the case $m = m'$ the sum in the left hand side of (8) is equal to the number of cells, $N$, in a crystal and the square of the modulus for the sum on the left of Eq. (6) can be written as

$$\left| \sum_n e^{i q r_{n0}} \right|^2 = N \frac{(2\pi)^3}{\Delta} \sum_{m=-\infty}^{+\infty} J_m^2 (gu_0) |S(q_m)|^2 \sum_g \delta(q_m - g). \quad (9)$$

Note that in this case we have no dependence on the phase $\varphi_0$.

In formula (2) the first two terms in figure braces do not depend on the direction of the vector $q$ and correspond to the contribution of incoherent effects. The third summand depends on the orientation of crystal axes with respect to the vector $q$ and determines the contribution of coherent effects. The corresponding part of the cross-section is known as an interference term. By taking into account the formula for $\sigma_0(q)$ (see, e.g., [1, 3]), in the region $q \ll m_e$ for the values of the momentum transfer this term can be written as

$$\sigma_c = \frac{e^2}{8\pi^3 E_1^2} \frac{|q|^2}{q_{||}} |u_q|^2 \left( 1 + \frac{\omega \delta}{m_e^2} - \frac{2\delta}{q_{||}} + \frac{2\delta^2}{q_{||}^2} \right) e^{-\frac{q_{||}^2}{l_e^2}} \left| \sum_n e^{i q r_{n0}} \right|^2, \quad (10)$$

where $E_1$ is the energy of the initial electron, $q_{||}$ and $q_{\perp}$ are the parallel and perpendicular components of the vector $q$ with respect to the direction of the initial electron momentum $p_1$, $u_q$ is the Fourier-transform of the atomic potential, $\delta = 1/l_e$ is the minimum longitudinal momentum transfer, and $l_e = 2E_1E_2/(\omega m_e^2)$ is the formation length for the bremsstrahlung, with $E_2$ being the energy of the final electron. Usually one writes the quantity $u_q$ in the form $4\pi Ze^2 [1 - F(q)]/q^2$, where $Z$ is the number of electrons in an atom, and $F(q)$ is the atomic form-factor. For the exponential screening of the atomic potential one has $u_q = 4\pi Ze^2/(q^2 + R^2)$, with $R$ being the screening radius of the atom.

The total expression for the bremsstrahlung cross-section can be presented in the form

$$d\sigma = N_0 (d\sigma_n + d\sigma_c), \quad (11)$$

where $d\sigma_n$ and $d\sigma_c$ are the cross-sections for the non-coherent and coherent bremsstrahlung in a crystal per single atom. Using formulae (9) and (10) and integrating over $q$, the cross-section for the coherent part can be presented as

$$\frac{d\sigma_c}{d\omega} = \frac{e^2 N}{N_0 E_1^2 \Delta} \sum_{m,g} \frac{g_{m||}^2}{|g_{m\perp}|^2} \left[ 1 + \frac{\omega^2}{2E_1E_2} - 2 \frac{\delta}{g_{m||}} \left( 1 - \frac{\delta}{g_{m||}} \right) \right] |u_{g_m}|^2 \times \quad (12)$$

$$\times J_m^2 (g_m u_0) |S(g)|^2 e^{-g_{m\perp}^2/\omega}, \quad g_m = g - mk_s,$$
where the summation goes under the constraint
\[ g_{m\parallel} \geq \delta. \] (13)

For the simplest crystal with one atom in the elementary cell one has \( N = N_0 \) and \( S(g) = 1 \). Formula (12) differs from the corresponding formula for the bremsstrahlung in an undeformed crystal (corresponding to the summand with \( m = 0 \) and \( u_0 = 0 \), see, for instance, [1, 3]) by replacement \( g \rightarrow g_m \), and additional summation over \( m \) with weights \( J^2_m(g_m u_0) \). This corresponds to the presence of an additional one dimensional superlattice with period \( \lambda_s \) and the reciprocal lattice vector \( m k_s \), \( m = 0, \pm 1, \pm 2, \ldots \). Note that by taking into account the \( \delta \)-function in Eq. (9), the momentum conservation law can be written down as
\[ p_1 = p_2 + k + g - m k_s, \] (14)
where \( -m k_s \) stands for the momentum transfer to the external field.

3 Discussion of the general formula and numerical results

First of all let us specify the conditions under which the influence of the external excitation on the bremsstrahlung cross-section is noticeable. In formula (12) the main contribution comes from the terms with \( g_{m\parallel} \sim \delta \). It follows from here that the external excitation with a wave vector \( k_s \) will influence on the process of the bremsstrahlung if \( m k_s \parallel \gg \delta \). As a consequence of the well-known properties of the Bessel function, in the sum over \( m \) the main contribution is due to the summands with \( m \lesssim g_m u_0 \sim g u_0 \sim \frac{2\pi u_0}{a} \). (15)

From these relations it follows that it is necessary to take into account the influence of external fields on the bremsstrahlung if
\[ \frac{u_0}{\lambda_s} \gtrsim \frac{a}{(2\pi)^2 l_c} = \frac{am_e m_e \omega}{8\pi^2 E_1 E_2}. \] (16)

It should be noted that at high energies \( a/l_c \ll 1 \) and condition (16) does not contradict to the condition \( u_0/\lambda_s \ll 1 \).

Let us consider the case of the simplest crystal with the orthogonal lattice and one atom in the elementary cell assuming that the electron enters into the crystal at small angle \( \theta \) with respect to the crystallographic axis \( z \). The corresponding reciprocal lattice vector components are \( g_i = 2\pi n_i/a_i \), \( n_i = 0, \pm 1, \pm 2, \ldots \), where \( a_i \), \( i = 1, 2, 3 \) are the lattice constants in the corresponding directions. For the longitudinal component we can write
\[ g_m \parallel = g_{mz} \cos \theta + (g_{my} \cos \alpha + g_{mx} \sin \alpha) \sin \theta, \] (17)
where \( \alpha \) is the angle between the projection of the vector \( p_1 \) on the plane \((x, y)\) and axis \( y \). For small angles \( \theta \) the main contribution into the cross-section comes from the summands with \( g_z = 0 \) and we receive
\[ \frac{d\sigma_c}{d\omega} \approx \frac{e^2}{E_1^2 \Delta} \sum_{m,g_x,g_y} g^2_{m\parallel} \left[ \frac{\omega^2}{2E_1 E_2} + 1 - \frac{2\delta}{g_{m\parallel}} \left( 1 - \frac{\delta}{g_{m\parallel}} \right) \right] |u_{g,m\parallel}|^2 J^2_m(g_m, u_0), \] (18)
where \( g_{\perp}^2 = g_x^2 + g_y^2 \), and the summation goes over the region \( g_{m\parallel} \geq \delta \) with
\[
g_{m\parallel} \approx -mk_z + (g_y \cos \alpha + g_x \sin \alpha) \theta. \tag{19}
\]
Note that in the argument of the Bessel function \( g_{m\parallel}u_0 \approx g_{\perp}u_0 \). It follows from here that if the displacements of the atoms in the acoustic wave are parallel to the axis \( z \) then the main contribution into the cross-section is due to the summation with \( m = 0 \) and the influence of the acoustic wave is small. The most promising case is the transversal acoustic wave propagating along the \( z \)-direction. If the electron moves far from the crystallographic plane (the angles \( \alpha \) and \( \pi/2 - \alpha \) are not small) the expression under the sum is a smooth function on \( g_x \) and \( g_y \), and the summation over these variables can be replaced by integration:
\[
\sum_{g_x, g_y} \rightarrow [a_1/a_2/(2\pi)^2] \int dg_x dg_y, \quad \text{and one receives}
\]
\[
\frac{d\sigma_c}{d\omega} \approx \frac{e^2}{4\pi^2 E_1^2 a_1} \sum_m \int dg_x dg_y \frac{g_{\perp}^2}{g_{m\parallel}^2} \left[ \frac{\omega^2}{2E_1E_2} + 1 - 2\frac{\delta}{g_{m\parallel}} \left( 1 - \frac{\delta}{g_{m\parallel}} \right) \right] |u_{g_m}|^2 J_m^2(g_mu_0).
\tag{20}
\]

Figure 1: Coherent bremsstrahlung cross-section, \( (m^2\omega/Z^2e^6)\frac{d\sigma_c}{d\omega} \), as a function of \( \omega/E_1 \) for \( 2\pi u_0/a_2 = 0 \) (dashed curve), 2.2 (full curve), \( \theta = 1 \) mrad (left panel) and \( 2\pi u_0/a_2 = 0 \) (dashed curve), 3 (full curve), \( \theta = 0.5 \) mrad (right panel). The values for the other parameters are as follows: \( a_2/\lambda_s = 5 \cdot 10^{-4}, a_2/2\pi R = 1, m_e a_2/(2\lambda_s E_1) = 0.001. \)

We now assume that the electron enters into the crystal at small angle \( \theta \) with respect to the crystallographic axis \( z \) and near the crystallographic plane \( (y, z) \) (\( \alpha \) is small). In this case with an increase of \( \delta \) some sets of terms in the sum will fall out. This can essentially change the cross-section. Two cases have to be distinguished. Under the condition \( \delta \sim 2\pi\theta/a_2 \), in Eq. (18) for the longitudinal component one has
\[
g_{m\parallel} \approx -mk_z + \theta g_y \geq \delta. \tag{21}
\]
This relation does not depend on the component \( g_x \) and the summation over this component can be replaced by integration \( \sum_{g_x} \rightarrow (a_1/2\pi) \int dg_x. \) When \( u_0 \parallel a_1 \), for exponential
screening the corresponding integral is expressed in terms of the hypergeometric functions. Here we will consider in detail the simpler case $u_0 \parallel a_2$ when the variable $g_x$ does not enter in the argument of the Bessel function. For the exponential screening after the elementary integration over $g_x$ we obtain

$$\frac{d\sigma}{d\omega} \approx \frac{4\pi^2Z^2e^6}{E_1^2a_2a_3} \sum_{m,g_y} \frac{2g_y^2 + R^{-2}}{g_m^2(g_y^2 + R^{-2})^{3/2}} \left[ \frac{\omega^2}{2E_1E_2} + 1 - 2\frac{\delta}{g_m}(1 - \frac{\delta}{g_m}) \right] J_m^2(g_yu_0),$$

(22)

where the summation goes under the condition (21). Note that in this case the cross-section does not depend on the lattice constant $a_1$. In Fig. 1 we have plotted the coherent part of the bremsstrahlung cross-section evaluated by formula (22) as a function on $\omega/E_1$ for $a_2 = a_3$, $a_2/\lambda_s = 5 \cdot 10^{-4}$, $a_2/2\pi R = 1$, $m_ea_2/(2\lambda_cE_1) = 0.001$, where $\lambda_c$ is the Compton wavelength of an electron. For the lattice constant $a_2 \approx 1.89 \cdot 10^{-8}$ cm the value of the last combination corresponds to the electron energy $E_1 = 20$ GeV. On the left panel of Fig. 1 the graphics are plotted for $\theta = 1$ mrad, $2\pi u_0/a_2 = 0$ (dashed curve), 2.2 (full curve) and for the right panel we have chosen $\theta = 0.5$ mrad, $2\pi u_0/a_2 = 0$ (dashed curve), 3 (full curve). The parameters are taken in such a way to illustrate the fact that the hypersonic wave can either enhance or reduce the cross-section. In Fig. 2 we give the dependence of cross-section (22) on the amplitude of the hypersound for $\omega/E_1 = 0.325$, $\theta = 1$ mrad (full curve) and $\omega/E_1 = 0.25$, $\theta = 0.2$ mrad (dashed curve).

![Figure 2: Bremsstrahlung cross-section, $(m_e^2\omega/Z^2e^5)d\sigma_c/d\omega$, evaluated by formula (22), as a function of $2\pi u_0/a_2$ for $\omega/E_1 = 0.325$, $\theta = 1$ mrad (full curve) and $\omega/E_1 = 0.25$, $\theta = 0.2$ mrad (dashed curve). The values for the other parameters are the same as in Fig. 1.](image)

Now we will assume that $\delta \sim 2\pi \theta \alpha/a_1$. The main contribution into the sum in Eq. (18) is due to the terms with $g_y = 0$ and we have two summations: over $m$ and $n_1$, $g_x = 2\pi n_1/a_1$. For the corresponding cross-section one receives:

$$\frac{d\sigma}{d\omega} \approx \frac{e^2}{E_1^2\Delta} \sum_{m,n_1} \frac{g_x^2}{g_m^2} \left[ \frac{\omega^2}{2E_1E_2} + 1 - 2\frac{\delta}{g_m}(1 - \frac{\delta}{g_m}) \right] |u_{gm}|^2 J_m^2(g_xu_{0x}),$$

(23)
where
\[ g_m \parallel \approx -mk_z + g_x \psi, \quad \psi = \alpha \theta, \] (24)
and summation goes over the values \( m \) and \( n_1 \) satisfying the condition
\[ |n_1 \psi - ma_1/\lambda_s| \geq \frac{m^2 a_1}{4\pi E_1 E_2}. \] (25)
In this case the most favorable conditions to have an influence on the bremsstrahlung cross-section due to the hypersonic vibrations are \( \mathbf{u}_0 \parallel \mathbf{a}_1 \) (to have large values for \( m \)) and \( \mathbf{k}_s \parallel \mathbf{a}_3 \) (to have large values for \( mk_z \)).

We have numerically evaluated the pair creation cross-section by making use of formula (23) for various values of parameters \( \psi, u_0, \lambda_s \) and the energy of the electron. As in the previous case, the corresponding results show that, in dependence of these parameters, the external excitation can either enhance or reduce the cross-section. As an illustration in Fig. 3 we have depicted the quantity \((m_e^2 \omega/Z^2 e^6)d\sigma_c/d\omega\) as a function of \( \omega/E_1 \) in the case of cubic lattice \((a_1 = a_2 = a_3)\) and exponential screening of the atomic potential for \( u_0 = 0 \) (dashed curve), \( 2\pi u_0/a_1 = 2 \) (full curve) and \( \psi = 0.00045 \) (left panel, both angles \( \alpha \) and \( \theta \) are measured in radians). For the right panel \( \psi = 0.00035, u_0 = 0 \) (dashed curve), \( 2\pi u_0/a_1 = 2 \) (full curve). The values for the other parameters are the same as in Fig. 1. In Fig. 4 we have presented the cross-section evaluated by Eq. (23) as a function of \( 2\pi u_0/a_1 \) for the photon energy corresponding to \( \omega/E_1 = 0.1 \) and for \( \psi = 0.00062 \) (full curve), \( \psi = 0.00017 \) (dashed curve). The values for the other parameters are the same as in Fig. 1.

![Figure 3: Coherent bremsstrahlung cross-section, \((m_e^2 \omega/Z^2 e^6)d\sigma_c/d\omega\), evaluated by formula (23), as a function of \( \omega/E_1 \) for \( 2\pi u_0/a_1 = 0 \) (dashed curve), 2 (full curve), \( \psi = 0.00045 \) (left panel) and \( 2\pi u_0/a_1 = 0 \) (dashed curve), 2 (full curve), \( \psi = 0.00035 \) (right panel). The values for the other parameters are the same as in Fig. 1.](image)

4 Conclusion

As a possible mechanism to control the spectral-angular characteristics of the bremsstrahlung by relativistic electrons in a crystal we have investigated the influence of the hypersonic
Figure 4: Bremsstrahlung cross-section, \((m_e^2\omega/Z^2e^6)d\sigma_c/d\omega\), evaluated by formula (23), as a function of \(2\pi u_0/a_1\) for \(\omega/E_1 = 0.1\), \(\psi = 0.00062\) (full curve) and \(\omega/E_1 = 0.1\), \(\psi = 0.00017\) (dashed curve). The values for the other parameters are the same as in Fig. 1.

vibrations on the corresponding cross-section. If the displacements of the atoms in the crystal under the influence of hypersound have the form (4), the coherent part of the cross-section per single atom, averaged on thermal fluctuations, is given by formula (12). To compared with the cross-section in an undeformed crystal this formula contains an additional summation over the reciprocal lattice vector \(m\mathbf{k}_s\) of the one dimensional superlattice induced by the hypersonic wave. The contribution for a given \(m\) is weighted by factor \(J^2_m(g_m, u_0)\), where the vector \(g_m\) is defined as in Eq. (12). We have argued that the influence of the hypersound on the cross-section can be remarkable under the condition (16). It should be emphasized that for \(u_0 \gtrsim a\) this condition is less restrictive than the naively expected one \(l_c \gtrsim \lambda_s\). In Sec. 3 we have considered in detail the most interesting case when the electron enters into the crystal at small angle with respect to a crystallographic axis (axis \(z\) in our consideration). The main contribution into the coherent part of the cross-section comes from the crystallographic planes, parallel to the chosen axis. The behaviour of this cross-section as a function on the photon energy essentially depends on the angle between the projection of the electron momentum on the plane \((x, y)\) and a crystallographic plane. If the electron moves far from crystallographic planes, the summation over the perpendicular components of the reciprocal lattice vector can be replaced by integration and the coherent part of the cross-section is given by formula (20). When the electron enters into the crystal near a crystallographic plane, two cases have to be distinguished. For the first one \(\theta \sim a_2/2\pi l_c\) and the summation over \(g_x\) can be replaced by integration. The corresponding integral is easily evaluated in the case when the displacements of atoms in the hypersonic wave are parallel to the incidence plane, and the corresponding cross-section takes the form (22). The numerical results for this case are presented in Figs. 1 and 2. In the second case \(\psi = a\theta \sim a_1/2\pi l_c\), and the main contribution into the cross-section comes from the crystallographic planes parallel to the incidence plane. The corresponding formula has the form (23). The results of the numerical evaluations for this case are depicted in Figs. 3 and 4. They show that in dependence
of the values for the parameters the presence of the hypersonic wave can either enhance or reduce the cross-section.

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