Research on machining nonorthogonal face gears by power skiving with tooth flank modification based on a six-axis machine tool

Han Zhengyang1 · Jiang Chuang2,3 · Deng Xiaozhong2,3

Received: 4 November 2021 / Accepted: 23 May 2022 / Published online: 16 June 2022 © The Author(s), under exclusive licence to Springer-Verlag London Ltd., part of Springer Nature 2022

Abstract
To solve the manufacturing difficulties of nonorthogonal face gears, an efficient gear machining method, referred to as power skiving, is proposed. The machining principle of power skiving and the relative position between the cutter tool and the workpiece were analyzed. Then, a mathematical model of machining nonorthogonal face gears by power skiving was established, and the tooth flank equation was obtained. The installation and movement mode of nonorthogonal face gears on a six-axis machine tool were analyzed, and the machining parameters were calculated precisely. A method of tooth flank modification by changing the machining parameters on the six-axis machine tool was presented. The meshing performance of the obtained nonorthogonal face gear was analyzed with an example. Finally, the processing test and the tooth flank measurement were carried out. The experimental results show that a nonorthogonal face gear can be machined and modified by power skiving on the proposed six-axis machine tool.

Keywords Nonorthogonal face gear · Power skiving · Tooth flank modification · Gear cutting

1 Introduction

Face gear drives are the precision mechanical transmission matching cylindrical gears to bevel gears. When the cone angle of the bevel gear is not 90°, it is called a nonorthogonal face gear and can be used in some nonorthogonal transmission mechanisms, such as helicopter tail transmission and ship propeller transmission. When the cone angle of the bevel gear is 90°, it is called an orthogonal face gear and its teeth are distributed in a plane. Because the pinion is a cylindrical gear, face gear drives inherit some advantages of cylindrical gear drives, such as insensitivity to installation error, and the axial position of the cylindrical gear has no influence on the contact performance. Face gear drives perform excellently in power splitting, large transmission ratios, low noise and high strength, so they can be used in helicopter main reduction boxes [1].

Face gear drives were first proposed by Buckingham [2] in 1949. In the 1990s, Professor Litvin et al. team [3–6] carried out a series of studies and discussions on the theoretical design and tooth surface modification of face gears and achieved considerable results. In addition, Wang et al. [7, 8] studied the honing and gridning method for face gears with tooth profile modification. Chu et al. [9] presented a novel method for grinding face gears along contact traces using a disk CBN wheel. Kawasaki et al. [10] clarified the effect of the helix angle on the composition of the surface contact lines and proposed a geometric design method that recognizes meshing singularity. At present, the design and calculation of face gears have a plentiful theoretical foundation. The machining technology of face gears mainly relies on the basic theory of generating surfaces by gear shaping. Machining methods include face gear hobbing [11], CONIFACE [12], gear milling at machining centers, gear shaping and powder metallurgy forming. However, these machining methods are mainly aimed at orthogonal face gears. Due to the particularity of the 90° cone angle of orthogonal face gears, it is easier to implement manufacture of the orthogonal face gear in
Cartesian coordinate system machine tools. For nonorthogonal face gears, the machine tool needs to be more flexible due to the diversity of its cone angle. Nonorthogonal face gears are machined mainly rely on the machining center. The tooth groove generated by the milling cutter at the machining center will be limited by the size of the milling cutter. Thus, it is difficult to manufacture a tooth root surface with a small transition arc.

Aiming at the difficulties of machining nonorthogonal face gears, this paper proposed machining nonorthogonal face gears by power skiving on the proposed six-axis machine tool. Power skiving, which was proposed by Pittler [13] as early as 1910, is a continuous efficient processing method for cylindrical gears. Power skiving requires an accurate rotating speed ratio between the cutter tool and the workpiece at high speed. However, due to the lack of proper processing equipment, this novel processing method was not applied in industrial manufacturing at that time. With the development of CNC machine tool technology and tool materials, power skiving is gradually favored by industrial development and has been studied by scholars around the world in recent years. For example, Guo et al. [14] conducted fundamental research on the cutting mechanism of cylindrical gear power skiving and proposed a method to design and calculate the skiving tool with the modification coefficient and tapered teeth for machining involute gears. Shih and Li [15] proposed a method of fitting B-spline curve to modify the generating gears which is conjugated with the work gear. A more accurate power skiving tool model compared with the traditional tool was obtained. This gear machining method is currently only applied to cylindrical gears, especially for internal gears. However, there is no relevant information about machining face gears by power skiving thus far.

In this paper, power skiving is used for machining nonorthogonal face gears. The processing motion conversion in the proposed six-axis machine tool is realized. An approach to realize tooth surface modification on the machine tool is also presented. An experiment is carried out finally. The calculation and experimental results show that power skiving is an effective machining method for nonorthogonal face gears. This processing method for machining nonorthogonal face gears can be realized on the six-axis machine tool proposed in this paper.

In this paper, \( S \) represents the Cartesian coordinate system, \( M_{mn} \) represents the transformation matrix \((4 \times 4)\) from the \( S_n \) coordinate system to the \( S_m \) coordinate system (refer to Appendix). \( T_{mn} \) represents the transformation matrix \((3 \times 3)\) after removing the last row and the last column of \( M_{mn} \).

### 2 Machining scheme of nonorthogonal face gear

The relative cutting speed of power skiving is generated by the rotation of the cutter and the workpiece around their own axes, which are situated at a shaft angle. Taking the internal gears as an example, the shaft angle between the cutter axis and the internal gear axis is \( \Sigma_0 \) (see Fig. 1). \( a_1 \) is the axis of the internal gear, and \( a_c \) is the axis of the cutter. \( \omega_1 \) and \( \omega_c \) are the angular speeds of the internal gear and cutter, respectively. The power skiving cutter can be regarded as a specially treated helical gear. For example, the tooth crest can be designed as a cone to provide the relief angle of the cutter. The pitch circle of the cutter is tangent to the pitch cylinder of the internal gear at point \( O \). \( V_1 \) and \( V_c \) are the linear velocities generated by the internal gear and the cutter, respectively. The relative cutting speed \( V_{c1} \) provides the cutting motion for the processing. At the same time, the translational motion of the cutter along the direction of the axis of the internal gear provides feed motion. The following relation should be satisfied:

\[
z_c \omega_c = \omega_1 z_1 \quad (1)
\]

where \( z_c \) and \( z_1 \) are the number of teeth of the cutter and the internal gear, respectively.

On the above theoretical basis, we can further extend the processing of the face gear. Machining face gears by power skiving, as shown in Fig. 2, can be conceived. We assume that the tooth of the internal gear corresponds to the tooth of the face gear at point \( O \), and the pitch cone of the face gear will be tangent to the pitch circle cylinder.
of the internal gear. Thus, the position of the cutter and the face gear will be visualized. In Fig. 2, \(a_f\) is the axis of the face gear, and \(\omega_f\) is the angular speed of the face gear. At this point, the axis of the face gear will intersect the axis of the internal gear by a shaft angle \(\Sigma\). The feed motion is still provided by the translational motion of the cutter moving along the direction of the axis of the internal gear. The following relationships should be satisfied in the machining scheme:

\[
z_c \omega_c = \omega_1 z_1 = z_f \omega_f
\]

(2)

As seen in Eq. (2), the angular speed of the cutter and face gear only needs to meet the ratio of teeth. The internal gear, as a fictitious medium, no longer affects the material relationship between the cutter and face gear. In the machining process, only three movements are needed: rotation of the cutter, rotation of the workpiece and feed motion of the cutter along the axis of the fictitious internal gear.

3 Tooth surface mathematical model

3.1 Cutter mathematical model

A power skiving cutter can be regarded as a helical gear in which the end face provides the cutting edges, which are formed by the rake face and helical gear tooth surface (see Fig. 3a). The cutting edges of the cutter tool are distributed in the rake face, which is present as a circular conical surface (see Fig. 3b). The taper angle is defined as the complement angle of the front rake angle (expressed as \(\alpha_a\)). \(\beta_c\) is the helix angle of the cutter.

To simplify the model, the front rake angle is set to 0 in this paper. In this case, the two cutting edges are symmetrical to each other on the transverse plane of the cutter. As shown in Fig. 4, coordinate system \(c\) and coordinate system \(e\) are fixedly connected to the cutter. \(Z_c\) and \(Z_e\) are simultaneously coincident with the cutter axis. \(r_b\) is the radius of the basic circle of the cutter. \(r_a\) is the radius of the addendum circle. \(p_a\) is any point at the cutting edge. The coordinates of \(p_a\) can be represented in Eq. (3). in \(S_c\).

\[
\begin{bmatrix}
    r_c \cos \varphi \\
    r_c \sin \varphi \\
    z_c
\end{bmatrix} =
\begin{bmatrix}
    r_c (\cos \varphi + \varphi \sin \varphi) \cos \theta + K_c r_f (\sin \varphi - \varphi \cos \varphi) \sin \theta \\
    -K_c r_f (\cos \varphi + \varphi \sin \varphi) \sin \theta + K_c r_f (\sin \varphi - \varphi \cos \varphi) \cos \theta \\
    0
\end{bmatrix}
\begin{bmatrix}
    0 \\
    0 \\
    1
\end{bmatrix}
\]

(3)

where \(K_c = \pm 1\) represents the right cutting edge and left cutting edge, respectively. \(\varphi = \tan \alpha_p\) represents the profile parameter when the transverse pressure angle of the cutter is \(\alpha_p\).

\[
\theta = \pi/z_c/2 + \tan \alpha_i - \alpha_i
\]

(4)
where $\alpha_0$ is the transverse pressure angle at the pitch circle of the cutter.

### 3.2 Machining mathematical model

The tooth surface of the face gear is machined by the cutting edge of the power skiving cutter through the continuous cutting method. The coordinate data of the tooth surface can be obtained by a mathematical model. The coordinate data of the tooth surface are a necessary condition for measuring bevel gears. In this section, the mathematical model of a nonorthogonal face gear machined by power skiving is established. The tooth surface equation will support the subsequent measurement and evaluation tasks.

Figure 5 shows the coordinate system of machining a nonorthogonal face gear achieved by power skiving. The coordinate system $S_1$ is fixedly connected with the cutter. Coordinate system $S_2$, $S_3$, and $S_4$ are the auxiliary coordinate systems. $Z_1$ is parallel to the pitch cone generatrix of the workpiece, $Y_1$ is the angle between the cutter axis and the pitch cone generatrix of the workpiece. When the spiral angle of the face gear is 0, $Y_1$ will be equivalent to the helix angle of the cutter $\beta_c$. $X_1$ is parallel to $S_1$, $S_2$ overlaps $X_1$, $S_3$ is parallel to $S_4$, and $Y_3$ overlaps $Y_2$. $\Sigma$ is the shaft angle of the nonorthogonal face gear pair. Point $O$ is the intersection of the pitch circle of the cutter and $X_1$. Point $O_2\alpha$ is located in the pitch cone generatrix of the nonorthogonal face gear as well as in $X_1$. $d_s = O_2\alpha O_f$ denotes the spatial position of the cutter relative to the workpiece. The point $O_f$ is the projection of $O_2\alpha$ on $X_1$. The point $C$ is the projection of $O_c$ on $X_3$.

According to the theory of coordinate transformation, the equation of any point on the tooth surface of a nonorthogonal face gear in $S_f$ can be expressed as:

$$r_f(\varphi, \varphi_c, d_s) = M_f \cdot (\varphi_f, a_1, a_2, d_s, \Sigma, \gamma_c, \varphi_c) r_c(\varphi)$$

(5)

The transformation matrix is:

$$M_f(\varphi, a_1, a_2, \Sigma, \gamma_c, \varphi_c) = M_f(\Sigma)M_3(\alpha_1, a_2)M_2(\Sigma)M_1(\gamma_c)M_0(\varphi)$$

(6)

where

$$\begin{align*}
\alpha_1 &= \frac{d_s}{\Sigma} \sin \Sigma + (r_0 - \xi) \cos \Sigma \\
\alpha_2 &= \frac{d_s}{\Sigma} \cos \Sigma - (r_0 - \xi) \sin \Sigma \\
\varphi_f &= \frac{\varphi_c - \varphi_c(\Sigma)}{r_f} + \Delta d_s \sin \beta_f / (\pi m z_f)
\end{align*}$$

(7)

where $\xi$ indicates the excess in-feed (which will be used in the modification part below). $\Delta d_s$ is the variation of $d_s$. $r_0$ is the pitch radius of the cutter. The sign “±” indicates the hands of the nonorthogonal face gear.

The unit normal vector of the tooth surface is:

$$n_f(\varphi, \varphi_c, d_s) = \frac{dr_f}{d\varphi} \times \frac{dr_f}{d\varphi_c}$$

(8)

Equation (5) represents the tooth surface equation of the nonorthogonal face gear taking the vertex of the pitch cone as the origin of coordinates. This expression should meet the meshing equation as Eq. (9).

$$f_j(\varphi, d_s, \varphi_c) = \frac{dr_f}{d\varphi} \times \frac{dr_f}{d\varphi_c} \times \frac{dr_f}{d\varphi} = 0$$

(9)

The three-dimensional coordinates of any point on the tooth surface of the nonorthogonal face gear can be obtained by solving nonlinear equations when the axial and radial positions of the point are provided.

### 3.3 Disperse tooth flank

To research the gear flank more directly, the working flanks of the nonorthogonal face gear can be divided into discrete grid points. In Fig. 6, the working tooth flank is divided into $9 \times 5$ nodes on the rotating projection plane of the tooth surface. $b_p$, $b_r$, $b_o$, and $b_y$ represent the inward setovers of the four boundaries of the working tooth flank. This will be beneficial to the measurement of tooth flanks. The coordinates of grid points should meet the following formula:
Equations (10) and (9) can form a system of nonlinear equations containing three parameters. Exact solutions \((\varphi, \varphi_c, d_j)\) can be obtained by providing three initial values. The corresponding three-dimensional coordinates of any grid point on the tooth flank can be obtained by substituting the solutions into Eq. (5).

\[
\begin{align*}
    x_f(\varphi, d_i, \varphi_c) - X_f &= 0 \\
    \sqrt{y_f(\varphi, d_i, \varphi_c)^2 + z_f(\varphi, d_i, \varphi_c)^2} - R_f &= 0
\end{align*}
\]

Equations (10) and (9) can form a system of nonlinear equations containing three parameters. Exact solutions \((\varphi, \varphi_c, d_j)\) can be obtained by providing three initial values. The corresponding three-dimensional coordinates of any grid point on the tooth flank can be obtained by substituting the solutions into Eq. (5).

4 Tooth flank modification

4.1 Tooth Flank topology

Modifying the tooth flank improves the meshing performance of gear drives. Bevel gear drives are usually designed as point contacts to avoid tooth edge contact. If the tooth flank of the face gear obtained by the generating cylindrical gear, which has the same number of teeth as the pinion, engages with the matching pinion, conjugate line contact will occur. This will make the face gear pair extremely sensitive to the assembly error and easily cause edge contact. For the tooth flank of the face gear generated by the gear shaping method, the point contact is usually obtained by the generating gear with more than 1 ~ 3 teeth than the matching pinion. The meshing performance of this kind of tooth flank is less affected by the installation position.

To describe the tooth flank structure of the face gear by power skiving, we can take the tooth flank by shaping (TFS) as the referenced surface to analyze the topological structure of the tooth flank by power skiving (TFP). Since the mathematical model and derivation process of TFS have been studied by many scholars [6], this paper did not repeat the method of obtaining the gear flank.

We selected the data in Table 1 as an example for the topological analysis of the tooth flank. The tooth number of the generating cylindrical gear of TFS is equal to the tooth number of the cutter. Figure 7 shows the topology deviation between TFP without modification and TFS.

From topological deviation, we find that the tooth profile deviation of TFP compared with TFS is mainly reflected in the change in the pressure angle. In the inner part of the tooth, the pressure angle is larger than that of TFS. In the outer part of the tooth, the pressure angle is smaller than that of TFS. This pressure angle change is approximately linear from the inner end to the outer end according to the variation in the lead deviation at the tip and the root of the tooth deviation. This phenomenon is mainly caused by the tilted installation of the cutter and the workpiece. If the helical angle of the cutter is 0, it means that TFP will turn into TFS. However, this type of tilted installation with a nonzero helical angle cutter is unavoidable in power skiving. The distorted distribution of the topology deviation of TFP refers to TFS easily causing edge contact and heavy noise. Thus,
it is significant to find a modification method to make TFP approximate to TFS.

4.2 profile modification

It can be seen from the above section that the topological deviation of the tooth flank is the pressure angle deviation. In addition, the deviation of the two flanks is symmetric, so we can correct the pressure angle of the two flanks simultaneously by the angle between the cutter axis and the pitch cone generatrix of the workpiece. Profile correction can be realized by turning the cutter at an angle to the X-tw-axis (in Fig. 5). At this time, the angle between the cutter axis and the pitch cone generatrix of the workpiece can be represented as:

$$\gamma_c = \beta_c + d_\gamma$$  \hspace{1cm} (11)

Figure 8b shows the projections of the cutting edges in the tooth length direction. $E_0$ is the uncorrected position of the cutting edge. $E_a$ and $E_b$ are the corrected positions of the cutting edges. When $d_\gamma > 0$, the projected pressure angle of the cutter profile will decrease; otherwise, it increases. However, the sign of the projected pressure angle change does not necessarily guarantee a consistent increment of the pressure angle of the face gear obtained, but a change in the gear tooth pressure angle is inevitable. In view of the difference in pressure angle deviation between the inner and outer ends of the gear tooth, $\gamma_c$ should constantly change at different spatial positions of cutter $d_\gamma$, $d_\gamma$ can be expressed as $d_\gamma(\gamma_c)$.

To avoid interference, the cutter needs to be designed with a relief angle $\gamma_b$ which should meet $\gamma_b > |d_\gamma|_{\max}$.

Functional expression of $d_\gamma(\gamma_c)$ requires further study. First, the influence of $d_\gamma$ on the pressure angle should be confirmed. Figure 9 shows the topological variation of the tooth flank at $d_\gamma = d_\gamma = 0.5^\circ$ with respect to TFP without any modification. When the angle $\gamma_c$ increases, the pressure angle of the face gear tooth also increases. Nonetheless, the magnitude of influence at the outer end and inner end is different, and its change trend is approximately linear. Therefore, we can fit the profile change of a certain position into a linear sensitivity function.

The sensitive factor of the profile deviation of the gear tooth can be defined as:

$$\sigma_{Ki} = \frac{e_{Ki(\delta_i)} - e_{Ki(\delta_i)}}{d_{\gamma_0}}$$  \hspace{1cm} (12)

where $K = L$ or $R$ represents the left flank or the right flank, respectively. In fact, Fig. 9 shows $\sigma_{Ki} = \sigma_{Li} = \sigma_i$, $i = 1 - 9$ maps 9 different positions of the cutter relative to the workpiece along tooth length. Thus, the variate $d_\gamma$ can be represented as $d_\gamma(K_i)$.

In general, $d_\gamma$ corresponding to two flanks on the same node are not the same. To balance the modification of the two flanks, the variable $d_\gamma$ can be averaged:

$$d_{\gamma_i} = (d_{\gamma Ri} + d_{\gamma Li}) / 2$$  \hspace{1cm} (13)

The corresponding adjustment angle at each position of the cutter is:

$$d_\gamma = \frac{e_{RL(\delta_i)} - e_{RL(\delta_i)}}{2\sigma_{RL}} + \frac{e_{LR(\delta_i)} - e_{LR(\delta_i)}}{2\sigma_{LR}}$$  \hspace{1cm} (14)

where $e_{RL}$ represents the profile error in Column $i$.

The linear sensitivity function can be represented as:

$$d_\gamma(\gamma_c) = A(d_\gamma - m_\gamma / \sin \Sigma) + B$$  \hspace{1cm} (15)

where $A$ and $B$ are the coefficients of the linear sensitivity function. They can be obtained by the least square method with arrays $d_{\gamma_i}$ and $d_{\gamma_i}$. The additional item $-m_\gamma / \sin \Sigma$ obtains more accurate fitting coefficients.

Substituting Eqs. (15) and (11) into Eq. (5) can we obtain new tooth flank. Figure 10a shows the topological deviation after fitting the linear sensitivity function by one calculation. Compared with Fig. 7, the distorted distribution of profile deviation has decreased obviously.
The pressure angle of the gear tooth of each column is slightly smaller than that of TFP. However, the deviation of the modified TFP is still up to 7 μm. To solve this problem, we can take the deviations in Fig. 10a as the profile error $e_{JK}$ and substitute it into Eq. (14) once again. The new deviation of TFP will be obtained, as shown in Fig. 10b. The deviation is reduced to -2.9 μm. In addition, the deviations at the inner end and outer end are all negative, which is conducive to contact transmission.

A tooth contact analysis (TCA) was conducted for the second modified TFP (as shown in Fig. 11). The transmission errors are -2.5 μrad and -4.42 μrad in the right flank and left flank, respectively. In fact, the transmission error of TFS is 0. The nonzero transmission error is caused by the deviation of the modified TFP. The contact pattern obtained is too long and easily causes uneven stress distribution. Therefore, lead crown relief also needs to be considered.

### 4.3 Lead crown relief

Lead crown relief can effectively reduce the sensitivity of the contact area to gear pair installation. The length of the contact area is usually controlled in this way. For gear shaping, more cutter teeth can be effectively modified for lead crown relief. To modify face gear flanks more flexibly, alterable in-feed along the tooth depth can be conducted.

This will lead to thinning down the tooth thickness in the deeper in-feed position. In Fig. 5, $O_cO$ will be greater than $O_cO_a$ at some selective positions. As shown in Fig. 12, the modification curve will increase the curvature of the tooth surface of the face gear along the tooth length direction. Because the in-feed along the tooth depth can be set by the CNC system precisely, the changes in curvature along the tooth length direction can be more flexibly controlled.

In Fig. 12, $\alpha_r$ is the top relief angle of the cutter. $\rho_0$ indicates the position of the cutter when $\xi=0$. $\rho_\xi$ indicates the position of the cutter at an additional in-feed of $\xi$. $B$ is the tooth width. The modification curve can be designed as a variety of two-dimensional curves, such as parabolas, ellipses, arcs or other complex curves. Various modification methods can be obtained by providing the corresponding coefficients of these curves. In this paper, a parabola was chosen as an example. The change of additional in-feed $\xi$ can be expressed as:

$$\xi(d_\circ) = a_{prb}(\rho_\xi - \rho_0)^2 = a_{prb}\left(\frac{d_\circ}{2 \sin \Sigma} - d_\circ - \rho_0\right)^2$$  \(16\)

where $a_{prb}$ is the coefficient of the parabola. Its value will directly affect the extent of modification.
The mathematical model of the modified tooth flank can be obtained by substituting Eq. (16) into Eq. (5).

The cutter moves along the modified curve when the cutter is moving from the outer end to the inner end of the workpiece. This means that the tooth root curve of the non-orthogonal face gear is also bent accordingly. Since the end face of the cutter is perpendicular to the pitch cone generatrix of the nonorthogonal face gear, the tooth root curve will be parallel to the modified curve. It can be seen from Fig. 6 that when the cutter reaches the inner end of the workpiece, the condition “α_r > α_c” should be guaranteed to avoid interference. The upper limit of the parabolic coefficient can be expressed as:

$$a_{prb} < \frac{\tan \alpha_r}{2(B - \rho_0)}$$  \hspace{1cm} (17)

By substituting Eq. (17) into Eq. (16) and taking $\rho_\xi = B$, the maximum modification can be obtained:

$$\xi_{max} = \frac{(B - \rho_0) \tan \alpha_r}{2}$$  \hspace{1cm} (18)

Figure 13 shows the topological variation and TCA of the tooth flanks after lead crown relief modification. $\rho_0$ is 0.5, and $a_{prb}$ has two chosen values to make a contrast. The reference tooth flank is TFS. The deviations of the working tooth flank at the outer side and the inner side increase significantly with increasing $a_{prb}$. For example, when $a_{prb} = 0.0001$, the tip error of the outer end is -28.1 μm in the left flank, but when $a_{prb} = 0.00025$, the error of the same position will be -53.4 μm. This will shorten the length of the tooth surface contact area. From the TCA result, we find that increasing the coefficient $a_{prb}$ has little effect on transmission errors and the contact trace. The transmission errors on the right flank and the left flank remain -2.8 μrad and -4.4 μrad, respectively. The length of the contact area changes significantly, and a large numerical value of $a_{prb}$ results in a short contact area.

5 Machining approach

5.1 Six-axis machine tool

From the analysis of Sect. 2, the machining of nonorthogonal face gears by power skiving requires three basic motions, that is, the rotation of the cutter, the rotation of the workpiece and the feed motion of the cutter relative to the workpiece. However, in view of the diversity of the shaft angle of nonorthogonal face gears, the structure of the machine tool with variable
axis angles in Fig. 14 can be a suitable choice. The machine tool consists of three linear motions (X, Y, Z) and three rotary motions (A, B, C). In the process of machining, the three linear motions can achieve feed motion and in-feed adjustment along any direction of three-dimensional space through interpolation. The two rotary motions (A, B) can meet the required position and speed of the cutter and workpiece at any time. Axis C is used to adjust the angle between Axis A and Axis B. The power skiving cutter is installed on Axis A, and the nonorthogonal face gear is installed on Axis B. The rotational speed or angular coordinate at any time of the two can be controlled by the CNC system.

5.2 Machine tool movement conversion

The three motions should be strictly controlled during the process. For the rotation movement of the cutter and the workpiece, rotary geometric axes A and B can meet this condition. However, the feed movement of the cutter along the pitch cone generatrix of the workpiece needs to be converted to the linear geometric axis of the six-axis machine tool.

The positions of the workpiece and cutter on the six-axis machine tool are shown in Fig. 15. Coordinate system $S_{m1}$ is parallel to $S_1$ and its origin overlaps the origin of the machine tool coordinate system $S_{m}$. The unit vector of the axis of the workpiece can be expressed in $S_1$ as:

$$e_{fm}^i(d_i) = T_{m1} T_{12} T_{23} e_{4}^i = \begin{bmatrix} \sin \Sigma & \sin \chi \sin \gamma - \cos \chi \cos \Sigma \\ \sin \gamma \cos \Sigma & \cos \chi \cos \Sigma \\ \cos \gamma \cos \Sigma & 0 \end{bmatrix}$$

where the superscript “$fi$” in $e_{fi}^i$ 1 represents the direction from $O_i$ to $O_f$. The subscript “1” represents that it is expressed in coordinate system $S_1$. The same below.

The unit vector of the workpiece axis can be expressed in the machine tool coordinate system as:

$$e_{fm}(d_f) = T_{m1} T_{12} T_{23} T_{34} e_{4}^i = \begin{bmatrix} \sin \chi \sin \gamma \cos \Sigma + \sin \chi \sin \gamma - \cos \chi \cos \Sigma \\ \sin \chi \sin \gamma \cos \Sigma - \sin \chi \sin \gamma - \cos \chi \cos \Sigma \\ \cos \chi \cos \Sigma \end{bmatrix}$$

where

$$e_{4}^i = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Similarly, the unit vectors of other corresponding directions can be expressed as:

$$e_{ma}^i(d_m) = T_{m1} T_{12} T_{23} T_{34} e_{4}^i = \begin{bmatrix} \sin \chi \sin \gamma \sin \Sigma - \cos \chi \cos \Sigma \\ \sin \chi \sin \gamma \sin \Sigma + \cos \chi \sin \gamma \sin \Sigma \\ \cos \chi \sin \Sigma \end{bmatrix}$$

$$e_{ma} = T_{m1} T_{12} e_{2}^a = \begin{bmatrix} -\cos \chi \\ \sin \chi \\ 0 \end{bmatrix}$$

$$e_{ma}^i(d_m) = T_{m1} T_{12} T_{23} T_{34} e_{4}^i = \begin{bmatrix} -\sin \chi \sin \gamma \cos \Sigma - \cos \chi \sin \gamma \cos \Sigma \\ -\sin \chi \sin \gamma \cos \Sigma - \cos \chi \sin \gamma \cos \Sigma \\ -\cos \gamma \cos \Sigma \end{bmatrix}$$

where $e_{af} m$ is the unit vector of the feed motion direction of the cutter, $e_{ia} m$ is the auxiliary unit vector, and $e_{ac} m$ is the unit vector of the in-feed motion direction.

Fig. 15 Position of cutter and workpiece
Limited by the structure of the machine tool, the axis of the workpiece should only be parallel to the plane \(X_mZ_m\). This means that the \(Y\) component of the unit vector of the workpiece axis \(e_f m\) must be 0. To meet the placement conditions of the workpiece on the six-axis machine tool, the auxiliary angle \(\chi\) can be obtained as:

\[
\chi(d_z) = \arctan\left(\frac{\sin \gamma_c}{\tan \Sigma}\right) \quad (25)
\]

The angle between the workpiece axis and the cutter axis, that is, the root angle of the machine tool, can be expressed as:

\[
\delta_m(d_z) = \arccos(e_f^1 \cdot e_c^1) = \arccos(\cos \gamma_c \cos \Sigma) \quad (26)
\]

where

\[
e_c^1 = [0 \ 0 \ 1]^T \quad (27)
\]

According to the principle of vector superposition, the coordinates of the cutter center \(O_c\) in the machine tool coordinate system can be expressed as:

\[
r_m^c(d_z) = (L_fi - L_{mf})e_f^m + L_{mc}e_m^m + L_{ac}e_ac^m \quad (28)
\]

where \(L_{mf}\) depends on the fixture length and machine tool constant.

\[
\begin{align*}
L_{mc}(d_z) &= r_0 - \xi(d_z) \\
L_{fi}(d_z) &= d_z \cos \Sigma \\
L_{ac}(d_z) &= d_z \sin \Sigma
\end{align*} \quad (29)
\]

It can be seen from Eq. (26) that the motorial direction involved in the processing of the nonorthogonal face gear is only related to the shaft angle, which is between the axis of the cutter and the fictitious internal gear, and the pitch cone angle of the workpiece. When the pitch cone angle of the workpiece is zero, the nonorthogonal face gear becomes a cylindrical gear, and the root angle of the machine tool is the shaft angle. At this time, the processing of the nonorthogonal face gear has become the processing of the cylindrical gear. When the pitch cone angle of the workpiece is 90°, meaning the workpiece is orthogonal face gear, the root angle of the machine tool will be 90°. The above formulas explain the smooth transition from an orthogonal face gear to a cylindrical gear.

The movements of each motion axis of the machine tool are shown in Fig. 16.

### 6 Processing test

This section conducts the processing test of a nonorthogonal face gear by power skiving and carries out the flank measurement. The machining equipment selected is the self-developed machine tool YK2260MC, which adopts the structure shown in Fig. 14. The range of C-axis motion is from 0 to 90°. The workpiece axis and cutter tool axis can be accurately linked by an electronic gearbox (EGB). The material of the power skiving cutter is high-speed steel, and the rake angle is 0. The shape of the cutting edge is standard involute. The measuring equipment is GELEASON 650GMS, which can detect the flank error of

![Fig. 17 Machining experiment](image1)

![Fig. 18 Tooth surface measurement](image2)
the corresponding point through the given position vector and unit normal vector. The spatial feed position in the process of power skiving can be calculated by Eq. (28). Figure 17 shows the machining process of the nonorthogonal face gear. The feed rate is 0.08 mm for each turn of the workpiece. Multiple strokes are used to avoid tool wear. The machining takes approximately 32 min. Figure 18 shows the tooth surface measurement. The measurement results are shown as in Fig. 19.

In Fig. 19a, the maximum error of the tooth flank processed by power skiving is 24.4 μm compared with TFS, which is in the right tooth flank. The left tooth flank error is 14.5 μm. Usually, these errors appear due to vibration of the machine tool, an installation error of the cutter and workpiece, manufacturing errors of the cutter tool or other problems. The tooth surface finish will also be affected by these negative factors. However, the flank errors are within reasonable limits. Figure 19b shows the topological error of the modified tooth flank with lead crown relief. The negative errors that mainly appear at the outer side and inner side are distributed as approximately arc-shaped along the tooth length direction. This is consistent with the calculation results in Fig. 13b. The experiment verifies the feasibility of machining nonorthogonal face gears by power skiving, and the modification method proposed in this paper can be realized by the six-axis machine tool with the structure involved in this paper.

The nonorthogonal face gear processed by power skiving is not the same as TFS, but there are some similarities. Under special circumstances, we can assume that the shaft angle, in this paper referring to the helix angle of the power skiving cutter, is 0. The relative motion of the cutter and the workpiece can be equivalent to the gear shaping process. Thus, the tooth surface of the cutter, called the generating cylindrical gear, and the tooth surface of the nonorthogonal face gear are completely conjugated. However, at the same time, it will also lead to the failure of producing the relative cutting speed at the tangent point of the cutter and the workpiece. The cutting ability of the cutter will lose. The difference is that in this paper, after sacrificing the condition of complete conjugation with a cylindrical gear, the tooth surface of the nonorthogonal face gear is enveloped by the cutting edge of the cutter with angle $\gamma_c$. This kind of tooth flank with additional transmission error can be modified to be TFS accurately with the proposed modification method. The variation in the transmission error of the modified TFP is also insensitive to lead crown relief.
7 Conclusions

In this paper, a method for machining nonorthogonal face gears by power skiving is proposed. After calculation analysis and experimental verification, the following conclusions were obtained:

1. The spatial position relationship between the power skiving cutter and the nonorthogonal face gear was analyzed. The motions of the cutter and workpiece have been equivalently converted to a six-axis machine tool.
2. A mathematical model of machining nonorthogonal face gears by power skiving has been established. The tooth surface data of the nonorthogonal face gear were obtained.
3. Based on the structure of a six-axis machine tool, a modification method that modifies the tooth flanks by power skiving to the tooth flanks by gear shaping has been proposed to optimize the tooth flank structure. A nonorthogonal face gear flank with a small transmission error was obtained.
4. The feasibility of machining nonorthogonal face gears by power skiving and the effectiveness of flank modification have been indicated by a processing test.

Appendix

\[
M_{1c} = \begin{bmatrix}
\cos \varphi_c & \sin \varphi_c & 0 & 0 \\
-\sin \varphi_c & \cos \varphi_c & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
M_{21} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \beta_c - \sin \beta_c & 0 & 0 \\
0 & \sin \beta_c & \cos \beta_c & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

\[
M_{32} = \begin{bmatrix}
\sin \Sigma & 0 & \cos \Sigma & 0 \\
0 & 1 & 0 & 0 \\
-\cos \Sigma & 0 & \sin \Sigma & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

\[
M_{43} = \begin{bmatrix}
1 & 0 & a_2 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 1
\end{bmatrix}
\]

\[
M_{4f} = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \varphi_f & \sin \varphi_f & 0 \\
0 & -\sin \varphi_f & \cos \varphi_f & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

Authors' contributions Not applicable.

Funding This research is supported by the National Natural Science Foundation of China (Grant No. 51975185 and No. 52005157) and China Postdoctoral Science Foundation (No. 2021M690051).

Availability of data and material Not applicable.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication Not applicable.

Conflicts of interest/competing interests Not applicable.

References

1. Heath G, Slaughter S, Morris M, Fetty J, Fisher D (2009) Face gear development under the rotorcraft drive system for the 21st century program. Ann Forum Proc AHS Int 2:1011–1030
2. Buckingham E (1949) Analytical mechanics of gears. The McGraw-Hill Book
3. Litvin F, Zhang Y, Wang J (1992) Design and geometry of face-gear drives. J Mech Des Trans ASME 114(4):642–647
4. Litvin F, Wang J, Bosssler R, Chen Y, Heath G, Lewicki D (1992) Application of face-gear drives in helicopter transmissions. J Mech Des Trans ASME 116(3):672–676
5. Litvin F, Egelja A, Tan J, Heath G (1998) Computerized design, generation and simulation of meshing of orthogonal offset face-gear drive with a spur involute pinion with localized bearing contact. Mech Mach Theory 33(1–2):87–102
6. Litvin F, Fuentes A (2004) Gear geometry and applied theory. Cambridge University Press
7. Wang Y, Hou L, Lan Z, Zhang G (2016) Precision grinding technology for complex surface of aero face-gear. Int J Adv Manuf Technol 86(5–8):1263–1272
8. Wang Y, Lan Z, Hou L, Chu X, Yin Y (2017) An efficient honing method for face gear with tooth profile modification. Int J Adv Manuf Technol 90(1–4):1155–1163
9. Chu X, Wang Y, Du S, Huang Y, Su G (2020) An efficient generation grinding method for spur face gear along contact trace using disk CBN wheel. Int J Adv Manuf Technol 110(5–6):1179–1187
10. Kawasaki K, Tsuji I, Gunbara H (2018) Geometric design of a face gear drive with a helical pinion. J Mech Sci Technol 32(4):1653–1659
11. Kin'ichi S, Masafumi S, Md. Rezaur R (1978) Development of a new type cutter “Revolving Type Pinion Cutter” for the hobbing of the face gear. Bull JSME 21(155):899–906
12. Hermann J (2011) CONIFACE face gear cutting and grinding. Am Gear Manuf Assoc Fall Tech Meet 1–14
13. Pittler WV (1910) Verfahren zum Schneiden von Zahnradern mittels eines zahnradartigen, an den Stirnflächen der Zähne mit Schneidkanten versehenen Schneidwerkzeugs, Deutsche Patentschrift Nr. 243514
14. Guo E, Hong R, Huang X, Fang C (2014) Research on the design of skiving tool for machining involute gears. J Mech Sci Technol 28(12):5107–5115
15. Shih Y, Li Y (2018) A novel method for producing a conical skiving tool with error-free flank faces for internal gear manufacture. J Mech Des 140(4):043302

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