Spin-isospin resonances: A self-consistent covariant description

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For the first time a fully self-consistent charge-exchange relativistic RPA based on the relativistic Hartree-Fock (RHF) approach is established. The self-consistency is verified by the so-called isobaric analog state (IAS) check. The excitation properties and the non-energy weighted sum rules of two important charge-exchange excitation modes, the Gamow-Teller resonance (GTR) and the spin-dipole resonance (SDR), are well reproduced in the doubly magic nuclei 48Ca, 90Zr and 208Pb without readjustment of the particle-hole residual interaction. The dominant contribution of the exchange diagrams is demonstrated.

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At present, spin-isospin resonances become one of the central topics in nuclear physics and astrophysics. Basically, a systematic pattern of the energy and collectivity of these resonances could provide direct information on the spin and isospin properties of the in-medium nuclear interaction, and the equation of state of asymmetric nuclear matter. Furthermore, a basic and critical quantity in nuclear structure, neutron skin thickness, can be determined indirectly by the sum rule of spin-dipole resonances (SDR) 4, 5 or the excitation energy spacing between isobaric analog states (IAS) and Gamow-Teller resonances (GTR) 6. More generally, spin-isospin resonances allow one to attack other kinds of problems outside the realm of nuclear structure, like the description of neutron star and supernova evolutions, the importance of full self-consistency was stressed 9, and the existence of exotic odd-odd nuclei 10 and the efficiency of a solar neutrino detector 11.

It was realized long ago that the Random Phase Approximation (RPA) is an appropriate microscopic approach for charge-exchange giant resonances 8, 9. The importance of full self-consistency was stressed 9, and Skyrme-RPA calculations of charge-exchange modes exist for about 30 years 10. Recently, a fully self-consistent charge-exchange Skyrme-RPA model has been developed 11. Self-consistency is an extremely important requirement for the analysis of long isotopic chains extending towards the drip lines. On the relativistic side, so far the charge-exchange (Q)RPA model based on the relativistic mean field (RMF) theory has been developed 3, 4, 6, 12, 13, 14.

However, the self-consistency of the RMF+RPA is not completely fulfilled for the following reasons. First, the isovector pion plays an important role in the relativistic description of spin-isospin resonances. Because of the parity conservation this degree of freedom is absent in the ground-state description under the Hartree approximation. Therefore, the pion is out of control in this best-fitting effective field theory. Second, to cancel the contact interaction coming from the pseudovector pion-nucleon coupling, a zero-range counter-term is needed with the strength $g' = 1/3$ exactly 13. However, in order to reproduce the excitation energies of the GTR, $g'$ must be treated as an adjustable parameter in RMF+RPA model with the value $g' \approx 0.6 12, 14$.

In this Letter, for the first time a fully self-consistent charge-exchange relativistic RPA based on the relativistic Hartree-Fock (RHF) approach 13, 16. The two major advantages of this RHF+RPA approach are that the pion is included in both the ground-state description and the particle-hole (p-h) residual interaction, and that the zero-range pionic counter-term with $g' = 1/3$ is maintained self-consistently. Without any adjusted p-h residual interaction or re-fitting process, we expect the present RHF+RPA approach to be reliable and to have predictive power.

For a self-consistent calculation, the RPA p-h residual interaction must be derived from the same Lagrangian as the nucleus-nucleon interactions read

\begin{align}
V_\sigma(1, 2) &= -[g_\sigma\gamma_0][g_\sigma\gamma_0]2D_\sigma(1, 2), \\
V_\omega(1, 2) &= [g_\omega\gamma_\mu\gamma^\mu][g_\omega\gamma_\mu\gamma^\mu]2D_\omega(1, 2), \\
V_\rho(1, 2) &= [g_\rho\gamma_\mu\gamma^\mu\gamma^\nu][g_\rho\gamma_\mu\gamma^\mu\gamma^\nu]2D_\rho(1, 2), \\
V_\pi(1, 2) &= -[f_\pi]m_\pi\vec{\tau}\gamma_0\gamma_5\vec{\gamma}\partial_0[1 + [f_\pi]m_\pi\vec{\tau}\gamma_0\gamma_5\gamma^\nu\partial_0]2D_\pi(1, 2),
\end{align}

where $D_i(1, 2)$ denotes the finite-range Yukawa type propagator

\begin{equation}
D_i(1, 2) = \frac{1}{4\pi} \frac{e^{-m_i|\mathbf{r}_1 - \mathbf{r}_2|}}{|\mathbf{r}_1 - \mathbf{r}_2|}. \quad (2)
\end{equation}

As discussed before, a zero-range pionic counter-term with $g' = 1/3$ must be included,

\begin{equation}
V_\pi^\delta(1, 2) = g'[f_\pi]m_\pi\vec{\tau}\gamma_0\gamma_5\gamma^\nu[1 + [f_\pi]m_\pi\vec{\tau}\gamma_0\gamma_5\gamma^\nu]2\delta(\mathbf{r}_1 - \mathbf{r}_2). \quad (3)
\end{equation}
In the present RHF+RPA framework, both direct and exchange terms must be taken into account and therefore the isoscalar mesons also play their role in spin-isospin resonances via the exchange terms. This is another distinct difference from the RMF+RPA model.

In order to verify the model self-consistency, we perform a so-called IAS check according to the following property about IAS degeneracy: it is expected that the IAS would be degenerate with its isobaric multiplet partners if the nuclear Hamiltonian commutes with the isospin lowering $T_-$ and raising $T_+$ operators, which is true when the Coulomb field is switched off. While this degeneracy is broken by the mean field approximation, it can be restored by the self-consistent RPA calculation\(^\text{[10]}\). As an example we calculate the IAS in $^{208}\text{Pb}$ with the parametrization PKO1\(^\text{[16]}\) and we find the unperturbed excitations between $-10.46$ MeV and $-8.96$ MeV when the Coulomb interaction is put to zero, thus showing the isospin symmetry breaking. Then, the RHF+RPA calculation leads to $E_{\text{IAS}} = 4$ keV within the single-particle energy truncation $[-M, M+80\text{ MeV}]$. This restoration of the IAS degeneracy indicates that the present approach is fully self-consistent. Furthermore, it should be emphasized that the pion also plays its role in this restoration process. Therefore, the coefficient $g'$ is not a free parameter. Changing the value of $g'$ would destroy the symmetry restoration process. For example, $g' = 0$ leads to $E_{\text{IAS}} = -801$ keV.

Taking the doubly magic nuclei $^{48}\text{Ca}$, $^{90}\text{Zr}$ and $^{208}\text{Pb}$ as test cases, and using the Gamow-Teller (GT) operator $F_\pm^{\text{GT}} = \sum_i \sigma(i) r_\pm(i)$, the excitation energies and strengths calculated with the fully self-consistent RHF+RPA approach using the parametrizations PKO1, PKO2, PKO3\(^\text{[17]}\) are summarized in Table II. These three parametrizations correspond to different sets of coupling strengths and meson masses in Eqs. (1)-(3). In PKO2 the pion is not included (i.e. $f_\pi = 0$), whereas PKO1 and PKO3 have different constraints on the density dependence of $f_\pi$. A good agreement with empirical energies is obtained without any re-adjusted parameter. All calculated strengths correspond to the main peak and contain 60-70% of the Ikeda sum rule.

We can understand the different physical mechanisms between the present RHF+RPA and other RMF+RPA approaches by the following analysis. On the one hand, it has been shown that the $\pi NN$ interaction and its zero-range counter-term ($g' \approx 0.6$) are the dominant ingredients in $p$-$h$ residual interaction for the GT mode in the case of RMF+RPA\(^\text{[12]}\). On the other hand, in the present RHF+RPA calculations, three parametrizations PKO1, PKO2 and PKO3 lead to similar results for the GTR excitation energies. It should be emphasized that the pion is not included in PKO2. This hints to the fact that the pion interaction is not the only dominant ingredient for the GT excitations in this framework. The GT strength distribution in $^{208}\text{Pb}$ with PKO1 is shown in Fig. 1. It is compared with the calculation in which the pion is excluded ($f_\pi = 0$) in the $p$-$h$ channel, the calculation including only $\sigma + \omega$-$p$-$h$ residual interactions, and the unperturbed (HF) case. One can conclude that the isoscalar $\sigma$- and $\omega$-mesons play an essential role via the exchange terms, whereas the pion just stands on a marginal position in determining the GTR strength distribution.

The relativistic RPA is equivalent to the time-dependent RMF in the small amplitude limit if the $p$-$h$ configuration space includes not only the pairs formed from the occupied and unoccupied Fermi states but also the pairs formed from the Dirac states and occupied Fermi states\(^\text{[18]}\). Based on this idea, a relativistic reduction mechanism of the Gamow-Teller strength due to the effects of the Dirac sea states was pointed out\(^\text{[19]}\). This kind of reduction mechanism appears in both nuclear matter\(^\text{[12]}\) and finite nuclei\(^\text{[13]}\). In Table II we show the contributions to the Ikeda sum rule values coming from the Fermi ($S^F$) and Dirac ($S^D$) sectors of both $T_\pm$ channels. It is explicitly shown that the Ikeda sum rule

$$S^-_{\text{GT}} - S^+_{\text{GT}} = 3(N - Z)$$

\hspace{1cm} (4)

can be 100% exhausted only when the effects of the Dirac sea are included. The reduction factors, $1 - (S^F - S^D)/(S^-_{\text{GT}} - S^+_{\text{GT}})$, of $^{48}\text{Ca}$, $^{90}\text{Zr}$, $^{208}\text{Pb}$ by the present self-consistent approach indicate to which extent the nucleon degrees of freedom play a role. It has been proposed that the neutron skin thickness could be extracted via the spin-dipole sum rule\(^\text{[1]}\).

$$S^-_{\text{SD}} - S^+_{\text{SD}} = \frac{9}{4\pi} \langle N \langle r^2 \rangle_n - Z \langle r^2 \rangle_p \rangle,$$

\hspace{1cm} (5)

with the SD operator $F_{\pm}^{\text{SD}} = \sum_i [r_i \sigma Y_1(i) \otimes \sigma(i)]_{j = 0,1,2 \tau_{\pm}}(i)$. While experimental results in

\hspace{1cm} FIG. 1: (color online) Strength distribution of GTR in $^{208}\text{Pb}$ calculated by RHF+RPA with PKO1 (solid line). The unperturbed (HF) strength (dotted line), the calculation with only $\sigma + \omega$-$p$-$h$ residual interaction (dashed line), and the calculation excluding pion ($f_\pi = 0$) in the $p$-$h$ residual interaction (dash-dotted line) are also shown. A Lorentzian smearing parameter $\Gamma = 1$ MeV is used.
The dash-dotted, dotted, dashed lines show the 0-, 1-, 2- contributions respectively, while the solid line shows their sum. The experimental data are shown as filled symbols [2]. A Lorentzian smearing parameter $\Gamma = 2$ MeV is used.

The reduction factor, $1 - (S_-^F - S_+^F)/(S_- - S_+)$, is given in the last column.

Since the neutron skin thickness of $^{208}$Pb is important in many aspects of nuclear physics and astrophysics, it is worthwhile to investigate the SDR properties of $^{208}$Pb. From Fig. 3 and the second part of Table III, the present approach predicts a dominant resonance structure at $E = 20 \sim 30$ MeV in the $T_-$ channel and a small bump at $E = 5 \sim 10$ MeV in the $T_+$ channel. Furthermore, a reduction factor of about 5.5% is predicted by the three different parametrizations. In conclusion, for the first time, a fully self-consistent charge-exchange relativistic RPA model based on the RHF approach is established. The IAS degeneracy broken by the RHF approximation can be accurately restored in the present self-consistent RPA calculations.
TABLE III: SD sum rule values and neutron skin thickness of $^{90}$Zr, $^{208}$Pb in RHF+RPA approach. Neutron and proton rms radii and corresponding data from SD experiment are given for comparison. $S(r_n, r_p)$ stands for the RHS of Eq. 1.

|         | $r_n$ (fm) | $r_p$ (fm) | $\delta_{np}$ (fm) | $S(r_n, r_p)$ (fm$^2$) | $S^p - S^p_0$ (fm$^2$) | $S_0 - S_0^p$ (fm$^2$) | reduction |
|---------|------------|------------|---------------------|------------------------|------------------------|------------------------|-----------|
| $^{90}$Zr | PKO1 4.289 4.188 0.082 153.5 | 143.8 | 153.6 | 6.4% |
|         | PKO2 4.264 4.184 0.080 149.6 | 139.7 | 149.4 | 6.5% |
|         | PKO3 4.271 4.192 0.079 149.8 | 140.3 | 149.9 | 6.5% |
| $^{208}$Pb | PKO1 5.691 5.457 0.234 1174 | 1111 | 1174 | 5.4% |
|         | PKO2 5.655 5.461 0.194 1134 | 1071 | 1135 | 5.6% |
|         | PKO3 5.658 5.456 0.202 1141 | 1077 | 1141 | 5.6% |

FIG. 3: (color online) Same as Fig. 2 but for the nucleus $^{208}$Pb.

Compared with RMF+RPA, the isoscalar mesons ($\sigma$, $\omega$) are found to play an essential role in spin-isospin resonances via the exchange terms. The GTR excitation energies and their strengths can be reproduced in the present self-consistent RPA calculation while maintaining $g' = 1/3$ in the contact counter-term. The SDR strength distributions up to the giant resonance region in both channels are well reproduced in $^{90}$Zr. Furthermore, a spin-dipole sum rule reduction mechanism due to the effects of the Dirac sea is found. The SD reduction factor for $^{90}$Zr is around 6.5%. Finally, the dominant structures of the SD strength distribution in $^{208}$Pb are shown and a strength reduction of about 5.5% is obtained.

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