Angle Error-tracking Iterative Learning Control for Pneumatic Artificial Muscle Systems

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ABSTRACT In this paper, a novel angle error-track adaptive iterative learning control scheme is proposed to solve the angle tracking problem for a pneumatic artificial muscle-actuated mechanism with nonzero initial errors. Lyapunov synthesis is used to design the adaptive learning controller and analyze the stability of closed-loop PAM system. Firstly, the system modeling for the PAM-actuated mechanism is introduced as a preparation of controller design. Then, the reference error trajectory is constructed to deal with the initial position problem of iterative learning control. The parametric uncertainties in the controlled system are estimated by using difference learning method. Robust control strategy is used to deal with nonparametric uncertainties and disturbances. By making the system error follow the desired error trajectory over the whole time interval as the iteration number increases, we derive the accurate tracking from the system state to the reference trajectory during the predetermined part time interval. Simulation results show the effectiveness of the propose angle error-tracking adaptive ILC scheme.

INDEX TERMS Pneumatic artificial muscles; adaptive iterative learning control; error-tracking strategy; Lyapunov approach

I. INTRODUCTION

Pneumatic artificial muscle (PAM) is a novel nonlinear biomimetic actuator, whose structure and functionality differs significantly from traditional actuators, such as motors, hydraulic actuators and gas cylinders. PAM is usually made of an elastic bladder which is enclosed in a double helically braided sleeve. Both bladder and sleeve are held airtight at their ends by using mechanical fixtures. If a PAM is injected with compressed air, the "muscle" can shorten and generate an axial contractile force; through reducing the internal pressure, the "muscle" may stretch and return to its original length [1], [2]. In this way, PAM systems can act roughly like human muscles by performing contractile or extensional motions actuated by pressurized air. An example of the PAM prototype is shown in Fig. 1.

Due to the advantages of rapid response, cheapness and high power-weight ratio, PAM systems have recently attracted great attentions and have been widely applied in various situations, such as healthcare, entertainment robotics, etc. In PAM systems, there exist some complicated inherent
characteristics caused by pressed air, such as high nonlinearities, hysteresis, time-varying characteristics, which brings much difficulty to the system modeling and control system design [3].

In the past 20 years, great progresses have been made in the field of controller design for nonlinear uncertain systems [4]-[7], which strongly promote the investigation and development of PAM control techniques. In [8], Andrikopoulos et al. proposed a nonlinear PID-based antagonistic scheme to solve the positioning problems for PAM systems. In [9], Lilly et al. tackled the tracking problem for PAM systems by using sliding mode control strategy. In [10], Cao et al. presented a neural network based nonlinear model predictive control algorithm to achieve tracking control of a PAM-driven lower limb exoskeleton for passive gait training. In [11], Zhang et al. put forward an active model-based control scheme to compensate for the uncertainties in the dynamics model of PAM systems. In [12], Cai et al. designed an adaptive backstepping controller for PAM systems. In [13], Xie et al. investigated the fuzzy control algorithm for the PAM-driven rehabilitation robot. In [14], Zhao et al. applied the active disturbance rejection control technique to improve the control precise and response rapidity for the PAM-actuated mechanism. So far, many existing PAM control schemes belong to model-based control algorithms. Due to the existing of inherent hysteresis, high nonlinearities and creep characteristics, it is hard to get the accurate model via theoretic analysis and system identification. For the control task of PAM systems, the control schemes which largely depend on system models can not perform better in accuracy and applicability. It is of great significance to propose advanced control schemes with less dependence on the system model.

On the other hand, iterative learning control (ILC) is an effective control strategies for repeated tracking control or periodic disturbance rejection problems [16]-[23]. For ILC systems, good control performance may be achieved even where it is difficult to carry out system modeling. In the earlier stage of ILC research, ILC techniques are mainly developed on the basis of contraction mapping method (contraction-mapping ILC), which leads to the in-depth study and application of D-type ILC [24], [25], P-type ILC [26]-[30], PD-type ILC [31], [32] and PID-type ILC [33]. Later on, with the continuous development of ILC technique [34], [35], some other ILC design methods have been widely investigated. Among them, the ILC method based on Lyapunov-like functional, the so-called adaptive ILC, is one of the most interesting and important developments [36], [37]. In contraction-mapping ILC, the control input is directly updated by using the information of error and input in the previous iteration. For example [38], a P-type learning law is usually expressed as

\[ u_{i+1}(t) = u_i(t) + \Gamma e_i, e_i = r(t) - y_i(t), \]

where \( i \) is the iteration index, \( u(t) \) is the control input, \( r(t) \) is the reference trajectory, \( y(t) \) is the output, and \( \Gamma \) is a proper constant learning gain matrix. In adaptive ILC, the control parameters are tuned between successive iterations by using the system error in the previous iteration. The typical expression of control input in adaptive ILC can be seen in equation (26)-(28). Through the continuous efforts over the last three decades, many significant ILC theoretical results have been obtained, which promotes the practical applications of ILC technique [39]-[44]. For those PAM-actuated mechanisms performing repeated tasks in rehabilitation, assembly lines and other situations, ILC is a proper technique to implement high-precision control. In traditional ILC algorithms, the initial value of system error is assumed to be zero at each iteration. If this assumption can not be met, a slight nonzero initial error may lead to the divergence of tracking error, which is called the initial position problem in ILC area. Due to the limitation of physical resetting, the assumption of zero initial error cannot be met in real occasions. Hence, for applying ILC technique into industrial processes, researchers have to investigate how to remove the assumption of zero initial error in iterative learning controller design. This problem is called the initial position problem of ILC. At present, there mainly exist four solutions to initial position problem of ILC: time-varying boundary layer [45], [46], error-tracking strategy [47], initial rectifying action [48], alignment condition [49]. In [50], Yang et al. designed a neural network-based error-track iterative learning controller to solve trajectory tracking problem for tank gun control systems. In [51], Yan et al. proposed an error-tracking iterative learning control scheme to tackle the position tracking problem for robot manipulators with random initial errors and iteration-varying reference trajectories. Up to now, the literature results on adaptive iterative learning control of PAM systems are few. How to develop an error-tracking adaptive ILC scheme for PAM systems with nonzero initial error is still unclear.

Motivated by the above discussion, this work focuses on the adaptive ILC algorithm design for PAM systems with nonzero initial errors. The main results and contributions are given as follows.

1) Error-tracking adaptive ILC is proposed to solve the angle tracking for PAM systems, which can guarantee the performance and overcome the initial position problem of ILC.

2) A novel construction method of desired error trajectory is presented for the implementing of error-tracking adaptive ILC. Signal replacement mechanism is applied to handle complicated uncertainties in the PAM system.

3) With the proposed ILC, all the signals of the closed-loop PAM system are proved to be bounded, the closed-loop system error during the preset operation time interval converge to a tunable residual set as the iteration number increases.

The paper is organized as follows. The system model and problem formulation is introduced in Section II. The detailed procedure of controller design is addressed in Section III. The convergence analysis of closed-loop PAM systems is given in Section IV. In Section V, the simulation results are illustrated to verify the effectiveness of the proposed control scheme. Finally, Section VI concludes this work.
II. PROBLEM FORMULATION

In this paper, the angle tracking problem of a PAM-actuated mechanism is studied [52], which is mainly comprised of an industrial control computer, an air compressor, two proportional valves, two PAM actuators and an angle sensor. The control structure of the mechanism is shown in Fig. 2. The charging or discharging of two PAM actuators are controlled by opening and closing of two pressure proportional valves according to the instructions of computer, respectively. The charging or discharging of two PAM actuators are determined by

\begin{align*}
P(t) &= P_0 + \Delta P = \alpha_0(u_{p0} + \alpha_u u), \\
P(t) &= P_0 - \Delta P = \alpha_0(u_{p0} - \alpha_u u),
\end{align*}

where \(P_0\) is the initial internal pressure of PAM actuators, \(P(t)\) is the internal pressure of two PAM actuators, and \(\alpha_0\) is the proportional coefficient of the control voltage and output pressure. The pulling forces may be derived as follows:

\begin{align*}
F_1(t) &= F_1(t)(\lambda_1 \varepsilon_1^2(t) + \lambda_2 \varepsilon_1^2(t) + \lambda_3) + \lambda_4, \\
F_2(t) &= F_2(t)(\lambda_1 \varepsilon_2^2(t) + \lambda_2 \varepsilon_2^2(t) + \lambda_3) + \lambda_4,
\end{align*}

where \(F_1\) and \(F_2\) represent two pulling forces of PAM actuators, \(\varepsilon_1(t)\) and \(\varepsilon_2(t)\) denote two shrinkage rates of the PAM actuators, and \(\lambda_1, \lambda_2, \lambda_3\) and \(\lambda_4\) are four parameters of PAM model. \(\varepsilon_1(t)\) and \(\varepsilon_2(t)\) may be calculated according to the following equations:

\begin{align*}
\varepsilon_1(t) &= \varepsilon_0 + r \varepsilon_0^{-1} \theta(t), \\
\varepsilon_2(t) &= \varepsilon_0 - r \varepsilon_0^{-1} \theta(t).
\end{align*}

where \(\theta(t)\) represents the deflection angle of the mechanism, \(\varepsilon_0\) and \(l_0\) are the initial shrinking angle and initial length of PAM actuators, respectively.

The driving moment of the PAM-actuated mechanism is determined by

\begin{equation}
T_M(t) = J\dot{\theta}(t) + b_v \dot{\theta}(t) = F_1(t) d_1 - F_2(t) d_2 + d_v(t),
\end{equation}

where \(J\) and \(b_v\) are the moment of inertia and damping coefficient, respectively, and \(d_v(t)\) denotes the external disturbances and unmodeled dynamics. In the PAM-actuated mechanism, \(d_1 = d_2 = r\), where \(r\) is the radius of pulley. Combing (3)-(5) with (6) leads to

\begin{align*}
T_M(t) &= \alpha_0 u_{p0} r(4 \lambda_1 \varepsilon_0 r l_0^{-1} + 2 \lambda_2 r l_0^{-1}) \theta(t) \\
&+ \alpha_0 \alpha_u r(2 \lambda_1 (\varepsilon_0 + 2 \lambda_1 (r \theta(t) l_0^{-1})^2 + 2 \lambda_2 \varepsilon_0 + 2 \lambda_3) u(t) + d_v(t).
\end{align*}

It follows from (6) and (7) that

\begin{equation}
\dot{\theta}(t) = -\frac{b_v}{J} \dot{\theta}(t) + 2 \frac{\alpha_0 u_{p0} r^2 (2 \lambda_1 \varepsilon_0 + \lambda_2) l_0^{-1}}{J} \theta(t) \\
+ 2 \frac{\alpha_0 \alpha_u r(\lambda_1 \varepsilon_0^2 + \lambda_2 \varepsilon_0 + \lambda_3)}{J} u(t) + d_v(t),
\end{equation}

where \(d_v = J^{-1} d_v(t)\). By letting \(x_1(t) = \theta(t)\) and \(x_2(t) = \dot{\theta}(t)\), we get the simplified system model at the \(k\)th iteration as

\begin{align*}
\dot{x}_{1,k}(t) &= x_{2,k}(t), \\
\dot{x}_{2,k}(t) &= \xi_1 x_{1,k}(t) + \xi_2 x_{2,k}(t) + g u_k(t) + d_v(t),
\end{align*}

where \(k\) is the iteration index, \(\xi_1 = \frac{2 \alpha_0 u_{p0} r^2 (2 \lambda_1 \varepsilon_0 + \lambda_2) l_0^{-1}}{J}\), \(\xi_2 = -\frac{b_v}{J}\) and \(g = \frac{2 \alpha_0 \alpha_u r(\lambda_1 \varepsilon_0^2 + \lambda_2 \varepsilon_0 + \lambda_3)}{J}\).

Let \(x_{1,d}(t)\) denotes the reference angle \(\theta_d\). Define \(x_{2,d}(t) = \dot{x}_{1,d}(t)\) and \(x_d = [x_{1,d}(t), x_{2,d}(t)]^T\). The control task of this work is to let \(x_{1,k}(t)\) track \(x_{1,d}(t)\) as the iteration number increases. Note that zero initial error is not required in this work, i.e. \(x_{1,0}(0) \neq x_d(0)\).

III. CONTROL SYSTEM DESIGN

Define \(e_k = [e_{1,k}(t), e_{2,k}(t)]^T = x_k(t) - x_d(t)\). Due to \(e_k(0) = 0\) is not guaranteed, it is of need to overcome the nonzero initial error during ILC system design. As a solution, we want to drive \(e_k(t)\) to follow the reference error trajectory \(e_k^e(t) = [e_{1,k}^e(t), e_{2,k}^e(t)]^T\) over \([0, T]\), whose detailed construction of \(e_k^e(t)\) will be introduced in the next subsection.

A. CONSTRUCTION OF DESIRED ERROR TRAJECTORY

The reference error trajectory \(e_k^e(t)\) is constructed as follows. While for \(t < t \leq T\),

\begin{equation}
e_k^e(t) = 0;
\end{equation}
while $0 \leq t < t_\delta$,\newpage
\begin{align}
\dot{e}_{1,k}(t) &= e_{1,k}(0) + (e_{2,k}(0))h_1(t), \\
\dot{e}_{2,k}(t) &= e_{2,k}(0)h_1(t) + e_{2,k}(0)h_2(t),
\end{align}
\tag{11}
\begin{align}
\dot{r}_{r,k}(t) &= e_{1,k}(0)h_1(t) + e_{2,k}(0)h_2(t),
\end{align}
\tag{12}
where $t_\delta$ is a time point between 0 and $T$,
\begin{align}
h_1(t) &= \frac{1}{2} \cos\left(\left(\cos(\frac{\pi t}{2}) + 1\right)\pi\right) + \frac{1}{2}, \\
h_2(t) &= \frac{1}{2} \cos\left(\left(\cos(\frac{\pi t}{2}) + 1\right)\pi\right) + \frac{1}{2}.
\end{align}
\tag{13}
\tag{14}

From the above constructions, we can see that $e_{r,k}(0) = e_{k}(0)$ and $e_{r,k}(t)$ is continuously differentiable for $0 < t < T$. If $e_k(t)$ can follow $e_{r,k}(t)$ over $[0, T]$, then the precise tracking from $x_k(t)$ to $x_{d}(t)$ may be achieved during $[t_\delta, T]$. In the next step, we will design an iterative learning controller to realize this objective. For the sake of brevity, in this paper, arguments are sometimes omitted when no confusion is likely to arise.

### B. Controller Design

From (9), we have
\begin{align}
\begin{cases}
\dot{z}_{1,k} = e_{1,k}, \\
\dot{z}_{2,k} = \xi_{1}x_{1,k} + \xi_{2}x_{2,k} + g u_k + d_{\omega,k} - \ddot{x}_d, 
\end{cases}
\tag{15}
\end{align}

By defining $z_{1,k} = e_{1,k} - e_{r,k}$ and $z_{2,k} = e_{2,k} - e_{2,k}$, from (9), we obtain
\begin{align}
\dot{z}_{1,k} = z_{2,k}, \\
\dot{z}_{2,k} = \xi_{1}x_{1,k} + \xi_{2}x_{2,k} + g u_k + d_{\omega,k} - \ddot{x}_d + \dot{e}_{r,k}.
\end{align}
\tag{16}

Let
\begin{align}
s_{z,k} = c z_{1,k} + z_{2,k}
\tag{17}
\end{align}

with $c > 0$. Then, combining (16) with (17) yields
\begin{align}
\dot{s}_{z,k} = c z_{2,k} + \xi_{1}x_{1,k} + \xi_{2}x_{2,k} + g u_k + d_{\omega,k} - \ddot{x}_d - \dot{e}_{r,k}.
\end{align}
\tag{18}

Define a candidate Lyapunov function at the $k$th iteration as
\begin{align}
V_k = \frac{1}{2g} s_{z,k}.
\end{align}
\tag{19}

Taking the time derivative of $V_k$ leads to
\begin{align}
\dot{V}_k = s_{z,k} \left[ \frac{1}{g} (c z_{2,k} - \ddot{x}_d + \dot{e}_{r,k}) + \xi_{1}x_{1,k} + \xi_{2}x_{2,k} + g u_k + d_{\omega,k} - \ddot{x}_d + \dot{e}_{r,k} \right].
\end{align}
\tag{20}

Without loss of generality, we assume
\begin{align}
\frac{1}{g} d_{\omega,k} = d_{g1}(x_k) + d_{g2}(x_k, t),
\end{align}
\tag{21}
in which $d_{g1}(x_k)$ is Lipschitz continuous, i.e.,
\begin{align}
|d_{g1}(x_k) - d_{g1}(x_d)| \leq l_g \|x_k - x_d\|
\tag{22}
\end{align}

with $l_g$ an unknown positive constant, and $d_{g2}(x_k, t)$ represents the sum of random perturbations, whose upper bound is $d_{gm}(t)$. According to (20) and (21), we have
\begin{align}
\dot{V}_k = s_{z,k} \left[ \frac{1}{g} (c z_{2,k} - \ddot{x}_d - \dot{e}_{r,k}) + \xi_{1}x_{1,k} + \xi_{2}x_{2,k} + g u_k + d_{g2}(x_k, t) \right] + s_{z,k} d_{g1}(x_k).
\end{align}
\tag{23}

It follows from (22) that
\begin{align}
s_{z,k} d_{g1}(x_k) \leq s_{z,k} d_{g1}(x_d) + l_g |s_{z,k}||e_k|.
\end{align}
\tag{24}

By using the above inequality, the iteration-dependent function $d_{g1}(x_k)$ is replaced by $d_{g1}(x_k)$ in a certain way, $d_{g1}(x_k)$ is iteration-independent, so it can be estimated by using difference learning approach. According to (24), handling $s_{z,k} d_{g1}(x_k)$ with Combining (23) with (24) yields
\begin{align}
\dot{V}_k \leq s_{z,k} \left[ \frac{1}{g} (c z_{2,k} - \ddot{x}_d - \dot{e}_{r,k}) + \xi_{1}x_{1,k} + \xi_{2}x_{2,k} + g u_k + d_{g1}(x_d) \right] + l_g |s_{z,k}||e_k| + |s_{z,k}|d_{gm}.
\end{align}
\tag{25}

Based on (25), the ILC law and learning laws are given as follows:
\begin{align}
u_k = -\gamma_1 s_{t,k} - \varpi_k \varphi_k - \chi_k \psi_k,
\end{align}
\tag{26}
\begin{align}
\varpi_k &= \text{sat}_{\varpi,k}(\varpi_{k-1}) + \gamma_2 s_{\varpi,k}, \\
\chi_k &= \text{sat}_{\chi,k}(\chi_{k-1}) + \gamma_3 s_{\chi,k},
\end{align}
\tag{27}
\tag{28}
where $\gamma_1 > 0$, $\gamma_2 > 0$, $\gamma_3 > 0$, $\varphi_k = [c z_{2,k} - \ddot{x}_d - \dot{e}_{r,k}, x_{1,k}, x_{2,k}, 1]^T$, $\psi_k = [\|e_k\|, \text{sat}_{-1,1}(\frac{e_k}{\delta}), \text{sat}_{-1,1}(\frac{\ddot{e}_k}{\delta})]^T$, $\phi$ is a small positive number, and sat(_) is defined as follows: for a scalar $\hat{a}$,
\begin{align}
\text{sat}_{\hat{a}}(\hat{a}) = \begin{cases}
\hat{a} & \hat{a} > \hat{a} \\
\hat{a} & \hat{a} \leq \hat{a} \\
\hat{a} & \hat{a} < \hat{a}
\end{cases}
\end{align}
for a vector $\hat{a} = [\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_m] \in \mathbb{R}^m$, $\text{sat}_{\hat{a}}(\hat{a}) = [\text{sat}_{\hat{a}}(\hat{a}_1), \text{sat}_{\hat{a}}(\hat{a}_2), \ldots, \text{sat}_{\hat{a}}(\hat{a}_m)]^T$.

### IV. Convergence Analysis

**Theorem 1:** Consider the PAM system (1) satisfying the assumption (21). The proposed adaptive error-tracking iterative learning controller (26)-(28) guarantees the tracking performance and system stability as follows:

(1) $|s_{z,k}(t)| \leq \phi$ holds for $t \in [0, T]$ as the iteration number $k$ increases, which means
\begin{align}
|e_{1,k}(t)| \leq \frac{\phi}{c}, \quad t \in [t_\delta, T].
\end{align}
\tag{29}

(2) All system variables are ensured to be bounded in the closed loop PAM system.

Proof: While $|s_{z,k}| > \phi$, it follows from (25) that
\begin{align}
\dot{V}_k \leq s_{z,k} \varpi_k \varphi_k + \chi_k \psi_k + u_k,
\end{align}
\tag{30}
where $\varpi = [\frac{1}{g} \xi_1, \frac{1}{g} \xi_2, d_{g1}(x_d)]^T$, $\chi = [l_g, d_{gm}]^T$. Due to $V_k(0) = 0$, from (30), we can see
\begin{align}
\dot{V}_k \leq -\gamma_1 \int_{t_\delta}^{t} s_{z,k} \varpi_k \varphi_k + \chi_k \psi_k + u_k dt
\end{align}
\tag{31}
holds while \( |s_{x,k}| > \phi \). Define a Lyapunov functional at the kth iteration as

\[
L_k = V_k + \frac{1}{2\gamma_2} \int_0^t \dot{\omega}_k^T \dot{\omega}_k \, d\tau + \frac{1}{2\gamma_3} \int_0^t \dot{\chi}_k^T \dot{\chi}_k \, d\tau, \tag{32}
\]

where \( \dot{\omega}_k = \dot{\omega} - \omega_k \) and \( \dot{\chi}_k = \dot{\chi} - \chi_k \). From (31) and (32), we have

\[
L_k - L_{k-1} = V_k - V_{k-1} + \frac{1}{2\gamma_2} \int_0^t (\dot{\omega}_k^T \dot{\omega}_k - \dot{\omega}_{k-1}^T \dot{\omega}_{k-1}) \, d\tau \\
+ \frac{1}{2\gamma_3} \int_0^t (\dot{\chi}_k^T \dot{\chi}_k - \dot{\chi}_{k-1}^T \dot{\chi}_{k-1}) \, d\tau \\
\leq -\gamma_1 \int_0^t s_{z,k}^2 \, d\tau + \int_0^t s_{z,k} (\dot{\omega}_k \varphi_k + \dot{\chi}_k \psi_k) \, d\tau \\
+ \frac{1}{2\gamma_2} \int_0^t (\dot{\omega}_k^T \dot{\omega}_k - \dot{\omega}_{k-1}^T \dot{\omega}_{k-1}) \, d\tau \\
+ \frac{1}{2\gamma_3} \int_0^t (\dot{\chi}_k^T \dot{\chi}_k - \dot{\chi}_{k-1}^T \dot{\chi}_{k-1}) \, d\tau - V_{k-1}
\]

(33)

for \( k > 0 \).

By the property \((a - b)^2 - (a - \tilde{b})^2 \leq (a - b)^2 - (a - \text{sat}_2(\tilde{b}))^2\), it follows from (27) that

\[
\frac{1}{2\gamma_2} (\dot{\omega}_k \varphi_k - \dot{\omega}_{k-1} \varphi_{k-1}) + s_{z,k} \dot{\omega}_k \varphi_k \\
\leq \frac{1}{2\gamma_2} (\dot{\omega}_k - \dot{\omega}_{k-1} + \text{sat}_\omega(\dot{\omega}_k) - \text{sat}_\omega(\dot{\omega}_{k-1}))^T \left( \text{sat}_\omega(\dot{\omega}_k) - \dot{\omega}_k \right) \\
+ s_{z,k} \dot{\omega}_k \varphi_k
\]

\[
\leq \frac{1}{\gamma_2} (\dot{\omega} - \dot{\omega}_k)^T (\text{sat}_\omega(\dot{\omega}_k) - \dot{\omega}_k) + \gamma_2 s_{z,k} \varphi_k
\]

(34)

Similarly, it follows from (28) that

\[
\frac{1}{2\gamma_3} (\dot{\chi}_k \psi_k - \dot{\chi}_{k-1} \psi_{k-1}) + s_{z,k} \dot{\chi}_k \psi_k \\
\leq \frac{1}{2\gamma_3} (\dot{\chi}_k - \dot{\chi}_{k-1} + \text{sat}_\chi(\dot{\chi}_k) - \text{sat}_\chi(\dot{\chi}_{k-1}))^T \left( \text{sat}_\chi(\dot{\chi}_k) - \dot{\chi}_k \right) \\
+ s_{z,k} \dot{\chi}_k \psi_k
\]

\[
\leq \frac{1}{\gamma_3} (\dot{\chi} - \dot{\chi}_k)^T (\text{sat}_\chi(\dot{\chi}_k) - \dot{\chi}_k) + \gamma_3 s_{z,k} \psi_k
\]

\( = 0 \).

(35)

Substituting (34) and (35) into (33), we have

\[
L_k - L_{k-1} \leq -\gamma_1 \int_0^t s_{z,k}^2 \, d\tau - V_{k-1}\]

which means

\[
\begin{align*}
L_k & \leq L_{k-1} - V_{k-1}, \\
L_{k-1} & \leq L_{k-2} - V_{k-2}, \\
& \vdots \\
L_1 & \leq L_0 - V_0.
\end{align*}
\]

Adding the two sides of above k inequations, respectively, we can assert

\[
L_k \leq L_0 - \sum_{j=0}^{k-1} V_j = L_0 - \frac{1}{2\gamma_2} \sum_{j=0}^{k-1} s_{z,j}^2
\]

(38)

According to the continuity of Lyapunov functional, we can see that \( L_0(t) \) is bounded for \( t \in [0, T] \). Note that \( |s_{z,k}| > \phi \) is the precondition of (31) and (38). Assume that \( |s_{z,k}| > \phi \) can hold as the iteration number increases. Then \( L_k < 0 \) will happen since \( L_0(t) \) is bounded and positive, which is in contradiction with the non-negativity of Lyapunov functional. Therefore,

\[
|s_{z,k}| \leq \phi
\]

(39)

will happen after several iteration numbers, which leads to

\[
|e_{1,k}(t)| \leq \left( 2e \right)^\phi \frac{\phi}{c}, t \in [t_\delta, T], i = 0, 1,
\]

(40)

as the iteration number increases [53]. According to the property of \( z_k \), from (40), we can obtain that

\[
|e_{1,k}(t)| \leq \phi \frac{\phi}{c}, t \in [t_\delta, T].
\]

(41)

Hence, higher control performance may be obtained by choosing proper constants \( \phi \) and \( c \).

In this work, error-tracking strategy is adopted for dealing with the initial position problem in the iterative learning controller design for the PAM system. Partial saturation learning method is applied to estimate unknown time-invariant constants and unknown time-varying parameters.

V. NUMERICAL SIMULATION

To illustrate the effectiveness of the proposed ILC algorithm, we make simulation experiment on the system (9) where \( d_{\omega,k} = 0.3 \sin(0.5t) + 0.2 \sin(x_{2,k}) \), and the model parameters are listed in TABLE 1. The control objective is to make \( \dot{x}_k \) track \( \dot{x}_{d}(t) = [t^2 + 0.2t + 1.2 \cos(\frac{2\pi}{T}t) + 0.4, 2t + 0.2 - 1.8 \pi \sin(\frac{2\pi}{T}t)]^T \). The initial state at the kth iteration is \( x_k(0) = [4 + 0.1 \text{rand}(1), -2 + 0.05 \text{rand}(2)]^T \), where \( \text{rand}(1) \) and \( \text{rand}(2) \) represent two random numbers between 0 and 1. It is easy to check that \( x_k(0) \) is not equal to \( x_{d}(0) \).

TABLE 1: Parameters of the PAM system model

| Parameters | \( \alpha_0 = 0.2 \) | \( \varepsilon_0 = 0.5 \) | \( \lambda_1 = 5 \) | \( \lambda_2 = 3.5 \) |
|------------|-----------------|----------------|--------------|--------------|
| \( \lambda_3 = 2.5 \) | \( \alpha_u = 1.5 \) | \( r = 4mm \) | \( l_0 = 20mm \) |
| \( u_{\phi} = 2.5V \) | \( b_0 = 3 \) | \( J = 10kg \cdot cm^2 \) |

The desired error trajectory \( e_d(t) \) is constructed according to (10)-(14) with \( t_s = 0.8, T = 4 \). The control law (26) and learning laws (27)-(28) are used for the simulation, with control parameters and gains chosen as \( \gamma_1 = 10, \gamma_2 = 2, \gamma_3 = \ldots \)
0.05, \( \overline{\omega} = -40, \bar{\overline{\omega}} = 40, \bar{\chi} = 0, \bar{\chi} = 10, c = 2, \phi = 0.001 \). The simulation results are given in Figs. 3-10. The angle position tracking result and the angle velocity tracking result at the 15th iteration, are shown in Figs. 3-4, from which, we can see that angle position signal \( x_1 \) and angle velocity signal \( x_2 \) can accurately track the reference angle position trajectory and the reference angle velocity trajectory for \( t \in [t_\delta, T] \), respectively. The profiles of angle position tracking error and desired angle position error trajectory at the 15th iteration are shown in Fig. 5, with the difference between the angle position tracking error and the desired angle position error shown in Fig. 7. The profiles of angle velocity tracking error and desired angle velocity error trajectory are shown in Fig. 6, with the difference between the angle velocity tracking error and the desired angle velocity error trajectory shown in Fig. 8. The profile of control input at the 15th iteration is shown in Fig. 9. The convergence history of \( s_{z,k} \) is shown in Fig. 10, where \( J_k \triangleq \max_{t \in [0,T]} |s_{z,k}(t)| \). The above simulation results show the effectiveness of our proposed angle error-tracking ILC algorithm.
scheme is effective to solve the tracking problem for the considered PAM system, which also offers a reference to the iterative learning controller design for some other systems, such as robot manipulators, inverted pendulum systems and linear motor systems.

VI. CONCLUSION
This work investigates the angle tracking problem for PAM systems under nonzero initial errors. An adaptive error-tracking iterative learning controller is designed by using Lyapunov synthesis. The detailed procedure of controller design and convergence analysis are presented, and a novel construction of desired error trajectory is given to deal with the initial position problem of ILC. Numerical simulation further demonstrates the effectiveness of theoretical results.

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