Measuring the Surface Inhomogeneity of Metals on Accreting White Dwarfs

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Abstract. Due to the short settling times of metals in DA white dwarf atmospheres, any white dwarfs with photospheric metals must be actively accreting. It is therefore natural to expect that the metals may not be deposited uniformly on the surface of the star. We present calculations showing how the temperature variations associated with white dwarf pulsations lead to an observable diagnostic of the surface metal distribution, and we show what constraints current data sets are able to provide.

1. Astrophysical context

There are two main classes of white dwarf stars: those with hydrogen-rich atmospheres (spectral type DA) and those with helium-rich atmospheres (non-DA spectral types). The reason for this is that the high surface gravities of white dwarfs lead to efficient gravitational settling, with the lightest elements rising to the surface. In addition, some DA white dwarfs have spectra showing metal lines of elements such as Ca and Mg (Zuckerman et al. 2003), and these stars are referred to as DAZs; about 20\% of all DAs fall into this category. Recently, Dufour et al. (2007) have announced a new class of white dwarf with carbon-dominated atmospheres, the “hot DQ” stars, several examples of which have been found in the Sloan Digital Sky Survey (Liebert et al. 2003). The presence of metals in the DAZs is intriguing since the settling time scale for the metals may be many orders of magnitude shorter than the evolutionary age of these objects. Indeed, for DAZs with $T_{\text{eff}} \sim 12,000$ K, the settling time scale can be on the order of days or weeks, meaning that these objects are experiencing ongoing accretion (Koester & Wilken 2006). This ongoing accretion is consistent with the fact that nearly a dozen of these objects have detected dust disks (“debris disks,” see Tokunaga, Becklin & Zuckerman 1990; Becklin et al. 2005; Kilic et al. 2005; Reach et al. 2005; Kilic et al. 2006; von Hippel et al. 2007; Jura, Farhi & Zuckerman 2007a, Jura et al. 2007b; Farhi, Zuckerman & Becklin 2008) and these disks are assumed to be the sources of the metal lines seen in these white dwarf atmospheres. The best studied object of this DAZ class with an observed disk, G29-38, is also a multi-periodic variable white dwarf (DAV), pulsating in non-radial g-modes with periods of a few hundred to one thousand seconds.

The technique of asteroseismology uses the observed pulsation modes of a star to infer and constrain the interior structure of the star, thus obtaining information on the star’s structure as a function of radius (Bradley & Winget 1994; Kawaler & Bradley 1994; Metcalfe, Salaris &
In contrast to this, our approach in this paper is to use the different angular dependence of the pulsations to constrain the accretion process. Since the accretion is most likely occurring through a disk, it is natural to suppose that the metals may not be uniformly distributed across the star’s surface. We present calculations showing how we can place constraints on the non-uniformity of the accretion process.

2. Gravitational settling and horizontal diffusion

For all our calculations, we use the DAZ G29-38 as our template, since, given its brightness, it has the greatest potential for successful measurement of a surface inhomogeneity. In addition, we have archival data on this star appropriate to this application. In this section, we will therefore attempt to quantify the importance of gravitational settling and horizontal diffusion assuming a model with parameters similar to those of G29-38.

Bergeron et al. (2004) find \( T_{\text{eff}} = 11,820 \) K and \( \log g = 8.14 \) for this star, while Koester, Provenčal & Shipman (1997) find \( T_{\text{eff}} = 11,600 \) K and \( \log g = 8.05 \). Interpolating in the tables of Koester & Wilken for Ca yields a settling time of \( \sim 13 \) days for the first set of parameters\(^1\) and a settling time of \( \sim 23 \) days for the second. We note that while unseen helium in the atmosphere could lengthen these settling times considerably (e.g., García-Berro et al. 2007), its presence is inconsistent with the pulsation results for this star: such an amount would imply a much deeper surface convection zone, in conflict with that found by Montgomery (2005).

Since these stars should have surface convection zones, the dominant form of horizontal transport of Ca will be due to the turbulent viscosity. We can estimate the size of this diffusion coefficient as \( D \sim v_C l_h \), where \( v_C \) is a typical convective velocity and \( l_h \) is the assumed “mixing length” for convection. From our white dwarf evolution code (e.g., see Montgomery et al. 1999) we find for both sets of stellar parameters that \( D \approx 1.5 \cdot 10^{10} \) cm\(^2\)/sec.

For these simple exploratory calculations we assume azimuthal symmetry for both the accretion and the surface metal distribution, i.e., \( Z = Z(\theta, t) \) and \( S = S(\theta, t) \), where \( Z(\theta, t) \) is the metal abundance, \( S(\theta, t) \) is the source function of metals accreting onto the white dwarf, \( \theta \) is the co-latitude of a point on the star’s surface, and \( t \) is time.

Since convection will uniformly mix material vertically, we can treat the convective region as a single zone and write an equation for the time evolution of \( Z \) as a function of \( \theta \) and \( t \):

\[
\frac{\partial Z(\theta, t)}{\partial t} = -\gamma Z(\theta, t) + D \nabla^2_h Z(\theta, t) + S(\theta, t),
\]

where \( \gamma \equiv 1/(\text{settling time}) \) is the settling rate, \( \nabla^2_h \) is the horizontal part of the Laplacian operator, and the other variables are as defined above. An estimate of the relative importance of sinking to spreading is \( \eta \equiv \gamma R_\star^2/D \), where \( R_\star \) is the radius of the white dwarf: \( \eta \approx 50 \) using the Bergeron et al. (2004) values whereas \( \eta \approx 30 \) for those of Koester et al. (1997). When \( \eta \gg 1 \) the metals will sink before they have a chance to diffuse horizontally, while for \( \eta \ll 1 \) the metals will have a chance to mix thoroughly horizontally before sinking, producing a nearly uniform surface distribution.

In Figure 1 we show the metal distributions which arise from solutions of equation 1. The dashed curve is the equilibrium distribution which results from constant accretion centered at the poles and the solid curve is that which results from constant accretion centered on the equator.

3. The diagnostic

The flux variations observed in pulsating white dwarfs are due almost entirely to temperature changes on the surface of the stars (Robinson, Kepler & Nather 1982). These same temperature

\(^1\) Due to a mis-reading of Table 2 in Koester & Wilken (2006), von Hippel & Thompson (2007) calculated a settling time of \( \sim 7 \) days.
changes will also affect the equivalent widths (EWs) of any spectral lines, and in particular the EWs of metal lines. These metals may not be uniformly distributed across the star’s surface and since the temperature variations are also non-uniform, we hope to be able to constrain the surface metal distribution.

The relevant diagnostic we have developed, denoted by $R$, is the ratio of the fractional EW amplitude to the fractional flux amplitude:

$$R \equiv \frac{\delta\langle EW \rangle}{\delta F_X/F_X} = \frac{A_{EW}}{A_{Flux}},$$

where $A_{EW}$ is the fractional amplitude of EW variations, and $A_{Flux}$ is the amplitude of the fractional flux variations observed in the given passband $X$. In Figure 2 we show separately the amplitudes of the photometric (top panels) and equivalent width (lower panels) variations as a function of the inclination angle.

To compute $R$, we assume a particular surface temperature perturbation of the form $\delta T/T \propto Y_{\ell m}(\theta, \phi)$. We then use model atmospheres to turn this into EW and flux variations on the surface of the star, taking into account the fact that the EW of the Ca lines will be directly proportional to the local abundance of Ca (e.g., the curves in Figure 1). For the passband $X$ we assume a wavelength response appropriate to the Argos CCD with a BG40 filter on the 2.1m telescope at McDonald Observatory (Nather & Mukadam 2004); the wavelength range is taken to be 3000 Å to 7000 Å, with a peak response at 5400 Å. Finally, we integrate the result across the visible surface of the star. Limb darkening is automatically taken into account by this procedure.

We note for a uniform distribution of metals that $R = 2.71$ for $\ell = 1$ and $R = 2.59$ for $\ell = 2$. Thus, even by measuring $R$ for a single mode it may be possible to tell whether metals are uniformly distributed on the star’s surface. To obtain further constraints, we need $R$ determinations from other modes and/or additional information such as the inclination angle of the star.
Figure 2. Left panels: The flux amplitude (top panel) and the EW amplitude (lower panel) as a function of $\theta_i$ for the polar distribution of metals shown in Figure 1. Right panels: the same as the left panels but for the case of the equatorial distribution of metals in Figure 1.

Figure 3. The diagnostic $R$ as a function of inclination angle $\theta_i$. The left panel shows the results for the polar distribution of metals shown in Figure 1 and the right panel shows the results for the equatorial distribution. The different curves are labeled by the $|\ell m\rangle$ values of the relevant pulsation modes.

In Figure 3 we show the $R$ diagnostic as a function of inclination angle $\theta_i$. The different curves are labeled according to their $\ell$ and $m$ values as $|\ell m\rangle$. The left-hand plot is for the polar distribution of metals given in Figure 1 and the right-hand plot is for the equatorial
Figure 4. Values of $R$ for modes in the DAV G29-38 as obtained from 4 hours of Keck data in 1999. The values for the first 6 highest amplitude modes are shown, with the period in seconds indicated above each point. Only the first two modes have a signal-to-noise ratio large enough to be useful.

distribution. The thin horizontal boxed region shows the value of $R$ expected if the metal distribution is uniform; we see that a non-uniform distribution is very unlikely to produce a value of $R$ in this range.

4. Comparison with observations
In 1996 Clemens & van Kerkwijk obtained over 4 hours of time-resolved spectroscopy of the DAV G29-38 (Clemens, van Kerkwijk & Wu 2000; van Kerkwijk, Clemens & Wu 2000; Clemens et al. 1999). While not originally intended for this purpose, we can use this as an example data set for the technique proposed in the previous section. We take the amplitudes for the EW variations from the analysis of von Hippel & Thompson (2007) and and we use the amplitudes of the broadband (5200–5500 Å) flux variations as determined by van Kerkwijk et al. (2000). We calculate $R = A_{EW}/A_{Flux}$ for each of the modes, taking into account the errors on all quantities.

We show the results of this procedure in Figure 4. Of the 6 modes we identified, only the first two have $R$ values with small enough error bars to provide any constraint on the Ca distribution. The 614 s mode provides the most convincing evidence for a non-uniform Ca distribution, since its $R$ value is a full $3\sigma$ above the range produced by a uniform distribution. The 818 s mode may also provide some evidence, although it is only about $1.3\sigma$ above the value expected from a uniform distribution.

Even though this wavelength range is much narrower than that used in the previous section for Figure 3, the central wavelengths of the two passbands are nearly the same and the results obtained are virtually indistinguishable from one another.
Comparison of the observed $R$ value for the 614 s mode in G29-38 (shaded region) to the value expected for a polar, an equatorial, and a uniform distribution of Ca, as a function of inclination angle $\theta_i$. The data are only consistent with the equatorial case.

With only one statistically significant value of $R$, we cannot hope to infer anything further about the Ca distribution. Fortunately, we do have additional information. Clemens et al. (2000) used time-resolved spectroscopy to determine that this mode has $\ell = 1$. In addition, Montgomery (2005), by modeling the nonlinear pulse shape of this mode, was able to further specify that it is an $\ell = 1, m = 1$ mode. With just this one additional constraint, we are somewhat surprisingly able to constrain our models of the Ca distribution. Figure 5 shows the $R$ values derived using an equatorial, a polar, and a uniform distribution, for a variety of inclination angles. The 614 s measurement is inconsistent with a polar distribution of metals, and is most consistent with an equatorial distribution.

5. Conclusions
We have shown how the temperature variations due to stellar pulsation can be used to constrain the metal distribution on the surface of a white dwarf, and we have shown that data currently in hand for the star G29-38 suggests that the metal distribution, and therefore the accretion, may be equatorial. If further studies support an equatorial distribution for Ca in G29-38, this argues against magnetic accretion onto spots near the poles. Non-magnetic equatorial accretion would further imply that the inner edge of the disk is much thinner than the white dwarf’s radius. A physically thin disk is consistent with observations to date (e.g., Jura et al. 2007a; von Hippel et al. 2007). Thus, the pulsations allow us to probe the physics of accretion and gravitational settling in these systems.
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