Higgs ID at the LHC

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Abstract

We make a complete catalog of extended Higgs sectors involving SU(2)₇ doublets and singlets, subject to natural flavor conservation. In each case we present the couplings of a light neutral CP-even Higgs state h in terms of the model parameters, and identify which models are distinguishable in principle based on this information. We also give explicit expressions for the model parameters in terms of h couplings and exhibit the behaviors of the couplings in the limit where the deviations from the Standard Model Higgs couplings are small. Finally we discuss prospects for differentiation of extended Higgs models based on measurements at the LHC and ILC and identify the regions in which these experiments could detect deviations from the SM Higgs predictions.

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I. INTRODUCTION

The Standard Model (SM) of particle physics has provided a remarkably successful description of electroweak data up to present energies. While the minimal SM implementation of electroweak symmetry breaking (EWSB) relies on a single $SU(2)_L$ doublet Higgs field, the dynamics of the EWSB sector have not yet been directly probed and extensions of the SM allow for a wide variety of extended Higgs sectors consistent with all existing data. Indeed, models that address the hierarchy problem – the extreme instability of the SM Higgs mass parameter to radiative corrections – contain additional fundamental or composite scalar particles at or below the TeV scale.

If a Higgs-like state is discovered, the next priority will be to test the Higgs mechanism of mass generation by measuring its couplings to SM particles. Experimental sensitivity to the Higgs couplings comes from measurements of the Higgs production cross sections, decay branching fractions, and total width. Prospects for the extraction of Higgs couplings from experimental data have been studied for the CERN Large Hadron Collider (LHC) [8, 9, 10, 11], International Linear $e^+e^-$ Collider (ILC) (for a review and references see Ref. [12]), photon collider [13, 14], and muon collider [15]; for a recent review of Higgs boson production and decay at these machines see also Ref. [16]. These measurements can be used to test the consistency of the measured Higgs couplings with SM predictions, and, if a discrepancy is found, to constrain the possible nature of the extended model. Different models give rise to different patterns of Higgs coupling deviations; in other words, they occupy different “footprints” in the space of measurable Higgs couplings. By identifying the footprint of each different extended Higgs sector in the space of Higgs couplings, we can determine whether the models can be distinguished in principle and establish a framework to map back any observed pattern of Higgs coupling deviations onto the appropriate underlying model.

Our aim is to make a complete catalog of extended Higgs sectors based on the patterns of coupling shifts of a single neutral Higgs state $h$. In this paper we limit ourselves to models that contain only Higgs doublets and/or singlets. This allows us to avoid tree-level violation of custodial $SU(2)$ symmetry and the resulting stringent constraints from the $\rho$.

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1 Models with extra Higgs doublets and/or singlets can yield better agreement with electroweak precision measurements than the SM; cf. Refs [1, 2, 3, 4, 5, 6].
2 Determining the nature of a new observed spin-less state that is not a Higgs is also challenging.

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parameter that arise, e.g., in models containing Higgs triplets \cite{17}. We also limit our study to models that obey the Glashow-Weinberg-Paschos condition \cite{18, 19} for natural flavor conservation, which requires that all fermions of the same electric charge get their mass from exactly one Higgs doublet. This allows us to avoid tree-level flavor-changing neutral Higgs interactions and the resulting stringent constraints from low-energy flavor physics. We also neglect the possibility of CP violation \cite{20}, and assume that the state $h$ is pure CP-even. Of course, many viable extended Higgs models exist outside these categories; their inclusion into the framework of Higgs coupling footprints presented here would make an obvious future extension of this work.

Within the above constraints we enumerate the complete set of models that can arise and present formulas for the shifts in the couplings of $h$ to SM particles relative to their SM values. We consider only what can be learned from one Higgs state, $h$; of course, observation of additional Higgs states (CP-even, CP-odd, or charged) or other new particles will complement this information. We focus on the couplings that arise from dimension-four operators, in particular the couplings of $h$ to $W$ or $Z$ boson pairs and to fermion pairs. Note that the $hWW$ and $hZZ$ couplings are modified from their SM values by a common multiplicative factor in any model containing only Higgs doublets and singlets. Similarly, our assumption of natural flavor conservation implies that the $h\bar{u}_i u_i$ couplings for the three generations of up-type quarks are modified by a common multiplicative factor; the same holds for down-type quarks and for charged leptons. We do not give explicit results for the loop-induced Higgs couplings to gluon or photon pairs or $\gamma Z$ because new physics at higher scales can generate additional effective couplings to these final states comparable in strength to those induced by SM loops \cite{21}. These loop-induced couplings are nevertheless important because they provide experimental access to the relative signs of the dimension-four couplings. Similarly, higher scale physics can generate dimension-six $hWW$ and $hZZ$ operators \cite{23, 24, 25, 26, 27}; we do not include these effects in our analysis.

This paper is organized as follows. In the next section we briefly review the Higgs couplings in the SM and introduce the notation and general framework that will be used to describe the extended models. We then proceed to the discussion of the extended models in

\footnote{Indeed, efforts to distinguish new physics running in loops involved in production and decay of a lone Higgs state have been made for selected models \cite{22}.}
For each model we present the couplings of \( h \) in terms of its composition and the vacuum expectation values (vevs) of the doublets in the model; where possible we also invert these relations to find explicit expressions for the model parameters in terms of the Higgs couplings. We identify the coupling patterns that allow the different models to be distinguished and specify the sets of models that cannot be distinguished based on the couplings of only one state. We also give expansions for the couplings near the decoupling limit \[28, 29\] in which the deviations of the couplings from their SM values are small. In Sec. \[\text{VI}\] we discuss the implications radiative corrections would have on our results.

We finish in Sec. \[\text{VII}\] by comparing the predictions of the individual models to each other and to the expected experimental sensitivity of the LHC and ILC. We present a decision tree for identifying the underlying model based on the couplings of \( h \) and point out which models cannot be distinguished even if the couplings of \( h \) were known exactly. Overall we find 15 models (or sets of models) with extended Higgs sectors that are distinguishable in principle in at least part of their parameter spaces. We also discuss the prospects for model differentiation based on the expected accuracy of Higgs coupling measurements at the LHC and ILC. For representative models, we plot the regions in which \( h \) would appear SM-like given the expected experimental uncertainties at the LHC (Fig. 5) and ILC (Fig. 6). We also provide a summary table showing the decoupling behavior of the \( h \) partial widths. We end with a brief summary of our conclusions.

We find it most straightforward to organize the models on the basis of the structure of the Yukawa Lagrangian – in particular, the number of different Higgs doublet(s) involved in fermion mass generation. In Sec. \[\text{III}\] we consider models in which the fermion masses are generated by the vev of only one Higgs doublet. Models with this characteristic are:

- The SM, in which the Higgs sector consists of only one SU(2)\(_L\) doublet that gives masses to the \( W \) and \( Z \) bosons and all the quarks and charged leptons. This is the simplest realization of the Higgs mechanism.

- The SM extended with one or more singlet scalars \[5, 30, 31, 32, 33\]. This model yields an overall reduction in Higgs couplings due to doublet-singlet mixing and its phenomenology has been studied extensively. Models with similar light Higgs phenomenology include unparticle models in which Higgs-unparticle mixing can suppress couplings \[34\] and Randall-Sundrum models in which Higgs-radion mixing also leads
to reduced couplings [35, 36, 37].

- The Type-I two Higgs doublet model (2HDM), in which only one doublet couples to fermions, while both doublets are involved in the generation of the W and Z boson masses [38, 39]. The sharing of the vev between two doublets can have a dramatic effect on the coupling pattern of the Higgs boson.

- A Type-I 2HDM extended with one or more singlet scalars.

- A Type-I 2HDM extended with one or more additional doublets.

In Sec. IV we consider models in which the fermion masses are generated by the vevs of two different Higgs doublets. Models with this characteristic are:

- The Type-II 2HDM, in which one doublet generates the masses of the up-type quarks while a second doublet generates the masses of the down-type quarks and charged leptons [40, 41, 42, 43, 44, 45]. This fermion coupling structure appears at tree level in the minimal supersymmetric standard model (MSSM), contributing to the great popularity of the Type-II 2HDM. In the MSSM, radiative corrections involving supersymmetric particles can induce potentially significant couplings of the bottom quarks to the “wrong” Higgs doublet [46, 47]; while this feature formally puts the MSSM Higgs sector outside our requirement of natural flavor conservation, we nevertheless consider the features of this extension as well.

- A Type-II 2HDM extended with one or more singlet scalars [44, 48, 49, 50, 51, 52, 53, 54].

- A Type-II 2HDM extended with one or more additional doublets [56].

- The “flipped” and “lepton-specific” 2HDMs [62, 63, 64], in which the coupling assignments of the two Higgs doublets to up-type quarks, down-type quarks, and charged leptons are varied relative to the usual Type-II 2HDM. In the flipped 2HDM, one doublet generates the masses of the up-type quarks and charged leptons while the second doublet generates the masses of the down-type quarks. In the lepton-specific 2HDM,

\footnote{Singlet extensions of the Higgs sector are also popular in supersymmetric models [53, 55, 56, 57, 58, 59, 60, 61].}
one doublet generates the masses of both up-type and down-type quarks while the second doublet generates the masses of the leptons. We also consider extensions of these two models with additional doublets and/or singlet scalars.

In Sec. V we consider models in which the fermion masses are generated by the vevs of three different Higgs doublets. Models with this characteristic are:

- A “democratic” three Higgs doublet model (3HDM-D), in which one doublet generates the masses of up-type quarks, a second doublet generates the masses of down-type quarks, and a third doublet generates the masses of the charged leptons.

- The 3HDM-D extended with one or more singlet scalars.

- The 3HDM-D extended with one or more additional doublets.

The physical Higgs boson content of these models can be summarized as follows. Consider a model that contains \(N_d\) complex doublets and \(N_s\) singlets, \(N_c \leq N_s\) of which are complex. After removing the unphysical charged and neutral Goldstone bosons, this model contains \(N_d - 1\) charged Higgs bosons \(H_i^\pm\), \(N_d + N_s\) CP-even neutral Higgs bosons \(H_i^0\), and \(N_d + N_c - 1\) CP-odd neutral Higgs bosons \(A_i^0\). The state that we consider throughout is one of the CP-even neutral Higgs bosons, denoted \(h\). Our results can be applied to any of the CP-even neutral Higgs bosons \(H_i^0\). We make no assumptions about whether additional states can be discovered at the LHC.

II. HIGGS COUPLINGS IN THE STANDARD MODEL AND BEYOND

The Higgs doublet of the SM is given by

\[
\Phi = \begin{pmatrix}
\phi^+

(\phi^{0,r} + v_{SM} + i\phi^{0,i})/\sqrt{2}
\end{pmatrix},
\]

where the vev of the Higgs field is \(v_{SM} = 246 \text{ GeV}\). The couplings of the Higgs to SM fermions are given by the Yukawa Lagrangian,

\[
L_{Yuk} = -y_{e}\bar{e}_R\Phi^\dagger L_L - y_{d}\bar{d}_R\Phi^\dagger Q_L - y_{u}\bar{u}_R\Phi^\dagger Q_L + \text{h.c.},
\]
where $Q_L = (u_L, d_L)^T$, $L_L = (\nu_L, e_L)^T$, and $\bar{\Phi}$ is the conjugate Higgs multiplet,

$$\bar{\Phi} \equiv i\sigma^2 \Phi^* = \begin{pmatrix} (\phi^0_r + v_{SM} - i\phi^0_i)/\sqrt{2} \\ -\phi^- \end{pmatrix}. \quad (3)$$

These couplings generate the fermion masses, $m_f = y_f v_{SM}/\sqrt{2}$, and the Feynman rules for the couplings of the physical SM Higgs boson $h = \phi^0_r$ to the fermions, $-iy_f/\sqrt{2} = -im_f/v_{SM} \equiv -ig^f_{SM}$.

The couplings of the Higgs to the $W$ and $Z$ bosons arise from the covariant derivative terms in the Lagrangian, $\mathcal{L} = |\mathcal{D}_\mu \Phi|^2$, where the covariant derivative is given by

$$\mathcal{D}_\mu = \partial_\mu - i \frac{g}{\sqrt{2}} \left( W^{+}_\mu T^{+} + W^{-}_\mu T^{-} \right) - i \sqrt{g^2 + g'^2} Z_\mu (T^3 - \sin^2 \theta_W Q) - ieA_\mu Q. \quad (4)$$

This term generates the $W$ and $Z$ boson masses,

$$m_W = \frac{g v_{SM}}{2}, \quad m_Z = \sqrt{g^2 + g'^2} v_{SM}, \quad (5)$$

and the Feynman rules for the couplings of $h$ to $W$ or $Z$ boson pairs, given by $ig^\nu_{SM} g_{\mu\nu}$, where

$$g^\nu_{SM} = \frac{g^2 v_{SM}}{2}, \quad g_{\mu\nu}^{SM} = \frac{(g^2 + g'^2) v_{SM}}{2}. \quad (6)$$

We now introduce our general framework for the extended models considered in this paper. In a model with multiple Higgs doublets, we can define the neutral, CP-even Higgs mass eigenstate under consideration as

$$h = \sum_i a_i \phi_i, \quad (7)$$

where $\phi_i \equiv \phi^0_r$ is the properly normalized real neutral component of doublet $\Phi_i$. The coefficients $a_i \equiv \langle h | \phi_i \rangle$ are constrained by the usual quantum mechanical requirement that $h$ be properly normalized:

$$\sum_i |a_i|^2 = 1. \quad (8)$$

In such a model, the vev that gives rise to the $W$ and $Z$ masses is in general shared among the doublets. We define the ratio of each doublet’s vev to $v_{SM}$ as

$$b_i \equiv \frac{v_i}{v_{SM}}, \quad \sum_i b_i^2 = 1, \quad (9)$$
where the second condition is required to obtain the correct $W$ and $Z$ masses. In the absence of CP violation, which we assume throughout, the quantities $b_i$ can all be chosen real and positive. Eq. 9 can also be thought of as a normalization requirement. In models containing only doublets, one can define a linear transformation to a “Higgs basis” in which only one doublet, $\Phi_v$, carries a nonzero vev. The ratios $b_i$ are then given by $b_i = \langle \phi_i | \phi_v \rangle$, such that $\phi_v = \sum_i b_i \phi_i$ and the condition $\sum_i b_i^2 = 1$ follows from unitarity.

In models containing both doublets and singlets, these relations are modified as follows. Because the Higgs state $h$ can contain a singlet admixture, the sums in Eqs. 7 and 8 must include the singlet states as well as the doublets:

$$h = \sum_{\text{doublets, singlets}} a_i \phi_i, \quad \sum_{\text{doublets, singlets}} |a_i|^2 = 1, \quad (10)$$

where now $\phi_i$ can also represent the real neutral component of a singlet. The $W$ and $Z$ masses, however, are generated only by the vevs of doublets, so that the sum in Eq. 9 is restricted to run over doublet vevs only:

$$\sum_{\text{doublets only}} b_i^2 = 1. \quad (11)$$

While singlet scalars can have vevs of their own, these singlet vevs play no role in our analysis and will be ignored.

In such an extended model, the couplings of the real neutral states $\phi_i$ to $W$ or $Z$ boson pairs arise from the covariant derivative terms for the doublets,

$$\mathcal{L} = \sum_{\text{doublets}} |\nabla_\mu \Phi_i|^2. \quad (12)$$

After the mixing in Eq. 10, the coupling of the physical state $h$ to $W$ or $Z$ boson pairs is controlled by the overlap of $h$ with the doublet $\phi_v$ that carries the vev in the Higgs basis,

$$g_V^h = g_V^{SM} \langle h | \phi_v \rangle, \quad (13)$$

where $V = W$ or $Z$. Inserting a complete set of states, we obtain,

$$g_V^h = g_V^{SM} \sum_i \langle h | \phi_i \rangle \langle \phi_i | \phi_v \rangle = g_V^{SM} \sum_{\text{doublets only}} a_i b_i, \quad (14)$$

where the restriction of the sum to run over only the doublets arises because $\phi_v$ cannot contain a singlet admixture. Note that $g_W^h/g_W^{SM} = g_Z^h/g_Z^{SM}$. 8
We note here that the familiar 2HDM sum rule \cite{69} for Higgs couplings to gauge bosons can be generalized to models containing arbitrary numbers of doublets and singlets. Summing over all CP-even neutral states and using Eq. \ref{eq:13} we have,

\[
\sum_{H^0_i}(g^H_i)^2 = (g^S_M)^2 \sum_{H^0_i}|\langle H^0_i | \phi_e \rangle|^2 = (g^S_M)^2,
\]

(15)

where the last equality is a consequence of completeness of the set of states $H^0_i$.

Under our assumption of natural flavor conservation \cite{18, 19}, the masses of each type of fermion are generated by only one doublet. For fermion species $f$, the Yukawa Lagrangian can be written in the general form

\[
\mathcal{L}_{Yuk} = -y_f \bar{f}_R \Phi_f^\dagger F_L + h.c.,
\]

(16)

where $F_L$ is the appropriate left-handed fermion doublet and $\Phi_f$ is the Higgs doublet that gives mass to fermion species $f$. If $f_R$ is an up-type quark then $\Phi_f^\dagger$ should be replaced by $\tilde{\Phi}_f^\dagger$ above. These couplings generate the fermion masses, $m_f = y_f v_f / \sqrt{2} = y_f b_f v_{SM} / \sqrt{2}$. Note that perturbativity of the Yukawa couplings, $y_f \lesssim \sqrt{4\pi}$, together with the known third-generation fermion masses, imposes lower bounds on $b_f$ in each fermion sector: $b_u \gtrsim 0.3$, $b_d \gtrsim 0.005$, and $b_\ell \gtrsim 0.003$. After the mixing in Eq. \ref{eq:10} the coupling of the physical state $h$ to an $f \bar{f}$ pair is given by

\[
g^h_f = \frac{y_f}{\sqrt{2}} \langle h | \phi_f \rangle = \frac{m_f}{b_f v_{SM}} a_f = g^S_M a_f b_f.
\]

(17)

A sum rule can also be constructed for the fermion couplings, as follows:

\[
\sum_{H^0_i}(g^H_i)^2 = \frac{y_f^2}{2} \sum_{H^0_i}|\langle H^0_i | \phi_f \rangle|^2 = \frac{m_f^2}{b_f^2 v_{SM}^2},
\]

(18)

where we have again used completeness of the set of states $H^0_i$. Note that, unlike for the gauge coupling sum rule in Eq. \ref{eq:15} the right-hand side is not known a priori and instead depends on $b_f$. We will show later that in some models, $b_f$ can be extracted from the couplings of a single state $h$.

For compactness we define barred couplings normalized to their corresponding SM values,

\[
\bar{g}_i \equiv \frac{g^h_i}{g^S_M}, \quad \bar{g} = \frac{\bar{g}_f}{b_f},
\]

(19)

so that $\bar{g} \rightarrow 1$ in the SM limit and

\[
\bar{g}_V = \sum_{\text{doublets}} a_i b_i, \quad \bar{g}_f = \frac{a_f}{b_f}.
\]

(20)
III. FERMION MASSES FROM ONE DOUBLET

In this section we consider extensions of the Standard Model in which the masses of all fermions arise from couplings to a single Higgs doublet. This class includes models containing additional doublets that do not couple to the fermions (so-called Type I models), as well as models containing one or more singlets.

A. Standard Model plus one or more singlets (SM+S)

The simplest way to extend the Standard Model Higgs sector is to add one real singlet scalar, $S$, which mixes with the usual SM Higgs boson to form two CP-even neutral Higgs mass eigenstates. The constraints of Eqs. (10, 11) become

$$a_f^2 + a_s^2 = 1, \quad b_f^2 = 1,$$

where the subscript $f$ refers to the Higgs doublet, which is responsible for all fermion masses, and $s$ refers to the singlet. As noted below Eq. (9) $b_f$ can be chosen real and positive, $b_f = 1$. Simultaneously, $a_f$ can be chosen real and positive through an appropriate rephasing of the mass eigenstate $h$; $a_s$ can then be chosen real and positive through a rephasing of the field $S$. In particular, we can write

$$a_f = \sqrt{1 - a_s^2} \equiv \sqrt{\xi}.$$

The couplings of $h$ to SM particles, normalized to their SM values as in Eq. (19), are then given by

$$\bar{g}_W = a_f b_f = \sqrt{\xi}, \quad \bar{g}_f = \frac{a_f}{b_f} = \sqrt{\xi}.$$ 

In particular, the couplings of $h$ to $W$ or $Z$ boson pairs and to fermion pairs are all scaled down by a common factor $\sqrt{\xi} \leq 1$ relative to their values in the SM. The production cross sections, partial decay widths, and total width of $h$ are then all suppressed by a factor of $\xi$ relative to those of the SM Higgs boson:

$$\frac{\Gamma_h}{\Gamma_{SM}} = \xi = 1 - a_s^2.$$ 

Note that a nonzero branching fraction of $h$ to invisible particles could mimic this effect by suppressing Higgs event rates in all visible channels by a common factor. This possibility can be tested through a dedicated search for $h \rightarrow \text{invisible}$ \cite{61,66,67,68}. 

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\[5\]
A measurement of any of these quantities allows a unique determination of the model parameters $a_f = \sqrt{\xi}$ and $a_s = \sqrt{1-\xi}$. Because all the Higgs partial widths scale the same way with $\xi$, the branching fractions of $h$ are the same as in the SM.

We also note that a decoupling limit can be defined in which $a_s \to 0$ and the couplings of $h$ approach those of the SM Higgs; defining a small decoupling parameter $\delta \equiv a_s$ we can write

$$\tilde{g}_f = \tilde{g}_W = \sqrt{1-\delta^2} \simeq 1 - \frac{1}{2}\delta^2, \quad \frac{\Gamma^h_i}{\Gamma^h_{SM}} = 1 - \delta^2. \quad (25)$$

These results can easily be extended to models containing two or more singlets by making the replacement

$$a^2_s \to \sum_{\text{singlets}} a^2_{s_i}. \quad (26)$$

The relations given above for $a_f$ continue to hold, and we conclude that models with more than one singlet cannot be distinguished from the one-singlet model on the basis of the $h$ couplings alone.

B. Type-I Two Higgs Doublet Model (2HDM-I)

The Type-I 2HDM [38, 39] has been extensively studied in the literature. This model contains two scalar SU(2)$_L$ doublets, which we denote by $\Phi_f$ and $\Phi_0$; $\Phi_f$ couples to fermions and $\Phi_0$ does not.

The constraints of Eqs. (10, 11) become

$$a^2_f + a^2_0 = 1, \quad b^2_f + b^2_0 = 1. \quad (27)$$

The couplings of $h$ to SM particles, normalized to their SM values as in Eq. (19) are then given by

$$\tilde{g}_W = a_f b_f + a_0 b_0, \quad \tilde{g}_f = \frac{a_f}{b_f}. \quad (28)$$

The vev ratios $b_f$ and $b_0$ can both be chosen real and positive. Simultaneously, $\tilde{g}_W$ can be chosen real and positive through an appropriate rephasing of the mass eigenstate $h$. There is no freedom left to choose $a_f$ positive, though; thus $\tilde{g}_f$ can have either sign.

Note in particular that while the couplings of $h$ to fermions of all three sectors are scaled by the same factor relative to the SM, $\tilde{g}_u = \tilde{g}_d = \tilde{g}_\ell$, the coupling of $h$ to $W$ or $Z$ boson pairs is scaled by a different factor unless $b_0 = 0$. This distinguishes the 2HDM-I from the
FIG. 1: Surface inhabited by the 2HDM-I in the plane of $\Gamma^h/\Gamma^{SM}_h$ versus $\Gamma^h/\Gamma^{SM}_W$. Here $\tan \beta \equiv v_f/v_0 = b_f/b_0$. Note the double covering of the plane for small values of $\Gamma^h/\Gamma^{SM}_f$ and $\Gamma^h/\Gamma^{SM}_W$.

SM plus a singlet discussed above. In the limit $b_0 \rightarrow 0$ we obtain $\bar{g}_W = \bar{g}_f = a_f$ and the couplings of $h$ in the 2HDM-I reduce to those in the SM+S model.

The constraint equations and coupling relations can be solved explicitly for the $b_i$ and $a_i$ factors in terms of the $h$ couplings:

$$b_f = \left[ \frac{1 - \bar{g}_W^2}{1 + \bar{g}_f^2 - 2\bar{g}_W \bar{g}_f} \right]^{1/2}, \quad b_0 = \sqrt{1 - b_f^2},$$

$$a_f = b_f \bar{g}_f, \quad a_0 = \frac{\bar{g}_W - b_f^2 \bar{g}_f}{\sqrt{1 - b_f^2}}. \quad (29)$$

Note that a full, unique solution is obtained if the relative signs of $\bar{g}_f$ and $\bar{g}_W$ are known. If the relative signs are not known, then there are two possible solutions, as illustrated in Fig. 1. Access to the relative signs of the couplings requires interfering the relevant amplitudes, e.g., by using $h \to \gamma\gamma$ or $h \to Z\gamma$.

It is useful to make contact with the usual notation for the 2HDM-I [44]. We assume that the state $h$ under consideration is the lighter of the two neutral CP-even Higgs mass eigenstates,

$$h^0 = \sqrt{2} \left( \cos \alpha \text{ Re } \Phi_f^0 - \sin \alpha \text{ Re } \Phi_0^0 \right) = \cos \alpha \phi_f - \sin \alpha \phi_0, \quad (30)$$

with $-\pi/2 < \alpha < \pi/2$, so that $a_f = \cos \alpha$ and $a_0 = -\sin \alpha$. The ratio of the doublet vevs
can be defined according to
\[ \tan \beta \equiv \frac{v_f}{v_0} = \frac{b_f}{b_0} \]  
so that \( b_f = \sin \beta \) and \( b_0 = \cos \beta \). With these definitions, the couplings of \( h \) to SM particles become
\[ \bar{g}_W = \sin(\beta - \alpha), \quad \bar{g}_f = \frac{\cos \alpha}{\sin \beta} = \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha). \]  

We note that for \( \tan \beta < 1 \) the fermion couplings can be enhanced (\( \bar{g}_f > 1 \)). Perturbativity of the top quark Yukawa coupling requires \( \tan \beta \gtrsim 0.3 \); there is no upper bound on \( \tan \beta \). For fixed \( \tan \beta \) the maximum possible value of \( \frac{\Gamma_h}{\Gamma_f} \) is \( 1 + \cot^2 \beta \).

The decoupling limit of this model occurs when the mass eigenstate \( h \) coincides with the state \( \phi_v \), the vev-carrying doublet in the Higgs basis. In that case \( a_i \equiv \langle h | \phi_i \rangle = \langle \phi_v | \phi_i \rangle = b_i \), so that \( \bar{g}_f = a_f/b_f = 1 \) and \( \bar{g}_W = a_f b_f + a_0 b_0 = b_f^2 + b_0^2 = 1 \). Near the decoupling limit, we can parameterize the deviations of the couplings from their SM values in terms of a small parameter,
\[ \delta \equiv \cos(\beta - \alpha) = a_f b_0 - a_0 b_f. \]

We have, for the couplings of \( h \) to gauge bosons,
\[ \bar{g}_W = \sqrt{1 - \delta^2} \simeq 1 - \frac{1}{2} \delta^2, \quad \frac{\Gamma^h}{\Gamma^{SM}_W} = 1 - \delta^2. \]  
The couplings of \( h \) to fermions depend also on \( \tan \beta \):
\[ \bar{g}_f = \sqrt{1 - \delta^2} + \cot \beta \delta \simeq 1 + \cot \beta \delta - \frac{1}{2} \delta^2, \quad \frac{\Gamma^h}{\Gamma^{SM}_f} \simeq 1 + 2 \cot \beta \delta - \delta^2, \]  
where the terms of order \( \delta^2 \) must be kept if \( \cot \beta \) is very close to zero. Note that \( \delta \) can take either sign.

### C. 2HDM-I plus one or more singlets (2HDM-I+S)

We next consider the consequences of adding a real singlet scalar field, \( S \), to the 2HDM-I. The constraints of Eqs. (10) (11) become
\[ a_f^2 + a_0^2 + a_s^2 = 1, \quad b_f^2 + b_0^2 = 1, \]  
(36)
where \( a_s \equiv \langle h | S \rangle \) and the other \( a_i, b_i \) are defined as in the previous section. The couplings of \( h \) to SM particles, normalized to their SM values, are given as for the 2HDM-I by

\[
\bar{g}_W = a_f b_f + a_0 b_0, \quad \bar{g}_f = \frac{a_f}{b_f}. \tag{37}
\]

As before, \( b_f, b_0 \), and \( \bar{g}_W \) can be chosen real and positive, while \( \bar{g}_f \) can have either sign. The coefficient \( a_s \) can then be chosen real and positive by a rephasing of \( S \).

With five parameters and only four equations, the parameters of this model cannot be fully solved for in terms of the \( h \) couplings. In order to display the ambiguity we define

\[
\xi \equiv 1 - a_s^2 = a_f^2 + a_0^2, \tag{38}
\]

with \( 0 < \xi \leq 1 \) parameterizing the doublet content of \( h \). We then obtain,

\[
b_f = \left[ \frac{\xi - \bar{g}_W^2}{\xi + \bar{g}_f^2 - 2\bar{g}_W \bar{g}_f} \right]^{1/2}, \quad b_0 = \sqrt{1 - b_f^2},
\]

\[
a_f = b_f \bar{g}_f, \quad a_0 = \frac{\bar{g}_W - b_f^2 \bar{g}_f}{\sqrt{1 - b_f^2}}, \quad a_s = \sqrt{1 - \xi}, \tag{39}
\]

where \( \xi \) remains an undetermined parameter.

This model can be cast into the usual notation of the 2HDM-I as follows. We first parameterize the doublet-singlet mixing in terms of \( \xi \),

\[
h = \sqrt{\xi} h' + \sqrt{1 - \xi} S, \tag{40}
\]

where \( h' \) corresponds to our Higgs in the 2HDM-I in the limit of zero singlet admixture:

\[
h' = \cos \alpha \phi_f - \sin \alpha \phi_0. \tag{41}
\]

We then have \( a_f = \sqrt{\xi} \cos \alpha, \ a_0 = -\sqrt{\xi} \sin \alpha, \) and \( a_s = \sqrt{1 - \xi} \). The couplings are given by

\[
\bar{g}_W = \sqrt{\xi} \sin(\beta - \alpha), \quad \bar{g}_f = \sqrt{\xi} \frac{\cos \alpha}{\sin \beta}. \tag{42}
\]

In particular, the couplings of \( h \) to SM particles are all scaled down by a common factor \( \sqrt{\xi} \).

Regardless of whether the parameters of the model can be solved for uniquely in terms of the \( h \) couplings, this model would be distinguishable from the 2HDM-I if it occupied a different footprint in the space of observables; i.e., if one could obtain sets of couplings \((\bar{g}_W, \bar{g}_f)\) in this model that could not be obtained in the 2HDM-I. \textit{This is not the case.} Any
set of couplings \((\bar{g}_W, \bar{g}_f)\) that can be obtained in the 2HDM-I+S can also be obtained in the 2HDM-I, albeit from different underlying values of the parameters \(a_f, a_0, b_f,\) and \(b_0\). In particular, the models are identical when \(\sqrt{\xi} = 1\); away from this limit the ellipses in Fig. 1 are simply scaled down by \(\xi\) on both axes.

The presence of the singlet thus cannot be established through measurements of the couplings of \(h\) only. However, if the couplings of a second CP-even neutral Higgs state \((H)\) could be measured, nonzero singlet mixing would violate the usual 2HDM coupling sum rule \([69]\), \((g_W^h)^2 + (g_W^H)^2 = (g_W^{SM})^2\) (see Eq. 15). This violation would indicate the presence of a third CP-even state such that \(\sum_{i=1}^3 (g_W^{h_i})^2 = (g_W^{SM})^2\).

These results can easily be extended to models containing two or more singlets by making the replacement

\[
a_s^2 \rightarrow \sum_{\text{singlets}} a_s^2 = 1 - \xi. \tag{43}
\]

Again, such a model cannot be distinguished from the 2HDM-I on the basis of the \(h\) couplings alone.

We conclude that adding one or more singlets to the 2HDM-I results in a model that cannot be distinguished from the 2HDM-I on the basis of the \(h\) couplings alone.

**D. 2HDM-I plus additional doublet(s) (2HDM-I+D)**

Let us now consider the consequences of adding one or more additional Higgs doublets to the 2HDM-I. These additional doublets can carry vevs but, under our assumption of natural flavor conservation, they must not couple to fermions. We can denote the field content of the model as \(\Phi_f, \Phi_0i,\) with \(i = 1 \ldots n (n \geq 2)\) counting the doublets that do not couple to fermions.

We first define a linear combination \(\phi_0'\) of the neutral CP-even states \(\phi_0i\) such that

\[
h = a_f \phi_f + a_0' \phi_0', \quad a_f^2 + a_0'^2 = 1. \tag{44}
\]

The vev of \(\phi_0'\) is parameterized by \(b_0' \equiv \langle \phi_0'|\phi_v\rangle\), chosen to be positive; the phase of \(h\) is chosen to make \(\bar{g}_W = a_f b_f + a_0' b_0'\) positive. Eq. 11 is modified to read

\[
b_f^2 + b_0'^2 = \omega^2, \quad 0 < \omega \leq 1, \tag{45}
\]
where \( \omega < 1 \) indicates that a nonzero vev is carried by the linear combination(s) of \( \phi_0 \) orthogonal to \( h \). Again, this model has five parameters but only four constraint equations; the solution for the model parameters becomes

\[
b_f = \left[\frac{\omega^2 - \bar{g}_W^2}{1 + \omega^2 \bar{g}_f^2 - 2 \bar{g}_W \bar{g}_f}\right]^{1/2}, \quad b'_0 = \sqrt{\omega^2 - b_f^2},
\]

\[
a_f = b_f \bar{g}_f, \quad a'_0 = \frac{\bar{g}_W - b_f^2 \bar{g}_f}{\sqrt{\omega^2 - b_f^2}}, \quad (46)
\]

where \( \omega \) remains an undetermined parameter.

Translating into the usual 2HDM-I notation, we define \( \alpha \) as for the 2HDM-I and \( \tan \beta \) as

\[
\tan \beta \equiv \frac{b_f}{b'_0}, \quad (47)
\]

where now \( \sin \beta = b_f/\omega \) and \( \cos \beta = b'_0/\omega \). In this notation, the couplings of \( h \) become

\[
\bar{g}_W = \omega \sin(\beta - \alpha), \quad \bar{g}_f = \frac{1}{\omega} \frac{\cos \alpha}{\sin \beta}. \quad (48)
\]

The couplings of \( h \) to gauge bosons are scaled down by a factor \( \omega \leq 1 \) while the couplings of \( h \) to fermions are scaled up by a factor \( 1/\omega \geq 1 \). Again, this model occupies the same footprint in Higgs coupling space as the 2HDM-I (consider Fig. 1), and we conclude that adding one or more additional doublets to the 2HDM-I results in a model that cannot be distinguished from the 2HDM-I on the basis of the \( h \) couplings alone. This conclusion remains unchanged if singlets are added to the model as well.

IV. FERMION MASSES FROM TWO DOUBLETS

We now consider models in which the fermion masses arise from couplings to two different Higgs doublets. Imposing natural flavor conservation allows for three possible patterns of couplings of two Higgs doublets to the fermions [62, 63, 64]:

(i) the Type-II 2HDM, or Model II, in which one doublet generates the masses of the up-type quarks while the other generates the masses of the down-type quarks and charged leptons (this is the coupling structure present at tree level in the MSSM);

(ii) the “flipped” 2HDM, in which one doublet generates the masses of the up-type quarks and charged leptons while the other generates the masses of the down-type quarks; and
(iii) the “lepton-specific” 2HDM, in which one doublet generates the masses of up- and
down-type quarks while the other generates the masses of the charged leptons.

We consider here also extensions of these three models obtained by adding one or more
electroweak singlets or doublets that do not couple to fermions.

While the Higgs sector of the MSSM is a Type-II 2HDM at tree level, one-loop ra-
diative corrections involving supersymmetric particles can induce a significant coupling of
the bottom quark to the “wrong” Higgs doublet, encoded in an extra coupling parameter
$\Delta_b$ \cite{46,47,70,71}. This violates our assumption of natural flavor conservation; however,
for completeness, we consider the main features of this model here separately.

### A. Type-II Two Higgs Doublet Model (2HDM-II)

The Type-II 2HDM \cite{44,72,73} is perhaps the most widely studied extension of the SM
Higgs sector. The Higgs content and coupling structure are the same as in the MSSM at tree
level. This model contains two scalar SU(2)$_L$ doublets, which we denote by $\Phi_u$ and $\Phi_d$. $\Phi_u$
generates the masses of the up-type quarks while $\Phi_d$ generates the masses of the down-type
quarks and the charged leptons.

The constraints of Eqs. (10) and (11) become

\begin{equation}
a_u^2 + a_d^2 = 1, \quad b_u^2 + b_d^2 = 1. \tag{49}
\end{equation}

The normalized couplings of $h$ to SM particles are then given by

\begin{equation}
\bar{g}_W = a_u b_u + a_d b_d, \quad \bar{g}_u = \frac{a_u}{b_u}, \quad \bar{g}_d = \bar{g}_\ell = \frac{a_d}{b_d}, \tag{50}
\end{equation}

where $\bar{g}_u$, $\bar{g}_d$, and $\bar{g}_\ell$ denote the normalized couplings of $h$ to all three generations of up-type
quarks, down-type quarks, and charged leptons, respectively.

The vev ratios $b_u$ and $b_d$ can both be chosen real and positive. Simultaneously, we can
choose $\bar{g}_W$ to be real and positive through an appropriate rephasing of the mass eigenstate
$h$. There is no freedom left to choose the signs of the fermion couplings $\bar{g}_u$ and $\bar{g}_d$; depending
on the underlying values of the parameters they can take the signs $++$, $+-$, or $-+$; $\bar{g}_u$ and
$\bar{g}_d$ cannot both be negative.
The 2HDM-II has four parameters related by five constraints, resulting in a pattern relation among the three couplings of $h$: \[ P_{ud} \equiv \bar{g}_W (\bar{g}_u + \bar{g}_d) - \bar{g}_u \bar{g}_d = 1. \] (51)

This pattern relation provides a test of the 2HDM-II coupling structure: It defines a two-dimensional surface accessible by the model in the three-dimensional space of couplings $\bar{g}_W$, $\bar{g}_u$, and $\bar{g}_d$.

The constraint equations and coupling relations can be solved explicitly for the $b_i$ factors in terms of the $h$ couplings:

\[
\begin{align*}
  b_u &= \left( \frac{\bar{g}_W - \bar{g}_d}{\bar{g}_u - \bar{g}_d} \right)^{1/2} = \left[ \frac{1 - \bar{g}_d^2}{\bar{g}_u^2 - \bar{g}_d^2} \right]^{1/2} \\
  b_d &= \left( \frac{\bar{g}_W - \bar{g}_u}{\bar{g}_d - \bar{g}_u} \right)^{1/2} = \left[ \frac{1 - \bar{g}_u^2}{\bar{g}_d^2 - \bar{g}_u^2} \right]^{1/2}
\end{align*}
\] (52)

where in the second relation the dependence on $\bar{g}_W$ has been removed using the pattern relation. We also obtain the $a_i$ factors,

\[ a_u = b_u \bar{g}_u, \quad a_d = b_d \bar{g}_d. \] (53)

If the relative signs of $\bar{g}_u$, $\bar{g}_d$, and $\bar{g}_W$ are known, then the solution for the model parameters is unique. Note also that, unlike in the 2HDM-I, a unique solution for $b_u$ and $b_d$ can be obtained even if the signs of the couplings are not known, by using the second set of equalities in Eq. (52). In the absence of information on the signs of the couplings, the magnitudes of $a_u$ and $a_d$ can also be determined uniquely but their relative signs cannot.

Let us now make contact with the usual notation for the 2HDM-II. We assume that the state $h$ under consideration is the lighter of the two neutral CP-even Higgs mass eigenstates,

\[ h^0 = \sqrt{2} \left( \cos \alpha \text{ Re } \Phi_u^0 - \sin \alpha \text{ Re } \Phi_d^0 \right), \] (54)

so that $a_u = \cos \alpha$ and $a_d = -\sin \alpha$. The ratio of the doublet vevs can be defined according to $\tan \beta \equiv v_u/v_d = b_u/b_d$, so that $b_u = \sin \beta$ and $b_d = \cos \beta$.\(^7\) With these definitions, the

---

\(^6\) One can define an additional pattern relation involving $\bar{g}_W$, $\bar{g}_u$ and $\bar{g}_t$ according to $P_{ut} \equiv \bar{g}_W (\bar{g}_u + \bar{g}_t) - \bar{g}_u \bar{g}_t = 1 = P_{ud}$.

\(^7\) The constraint $\bar{g}_W \geq 0$ corresponds to $\beta - \pi \leq \alpha \leq \beta$. Perturbativity of the top quark Yukawa coupling requires $\tan \beta \gtrsim 0.3$; perturbativity of the bottom quark Yukawa coupling requires $\tan \beta \lesssim 200$.\(^18\)
The decoupling limit of this model occurs when the mass eigenstate $h$ coincides with the state $\phi_v$, the vev-carrying doublet in the Higgs basis. Near the decoupling limit we parameterize the deviations of the couplings from their SM values in terms of a small parameter

$$\delta \equiv \cos(\beta - \alpha) = a_u b_d - a_d b_u.$$
FIG. 3: Surface inhabited by the 2HDM-II in the plane of $\Gamma_u^h/\Gamma_u^{SM}$ versus $\Gamma_d^h/\Gamma_d^{SM}$.

The couplings and corresponding partial widths, normalized to their SM values, become

$$
\bar{g}_W = \sqrt{1 - \delta^2} \simeq 1 - \frac{1}{2} \delta^2, \\
\bar{g}_u = \sqrt{1 - \delta^2 + \cot \beta \delta} \simeq 1 + \cot \beta \delta, \\
\bar{g}_d = \bar{g}_t = \sqrt{1 - \delta^2 - \tan \beta \delta} \simeq 1 - \tan \beta \delta, \\
\frac{\Gamma_W^h}{\Gamma_W^{SM}} \simeq 1 - \delta^2, \\
\frac{\Gamma_u^h}{\Gamma_u^{SM}} \simeq 1 + 2 \cot \beta \delta, \\
\frac{\Gamma_d^h}{\Gamma_d^{SM}} \simeq 1 - 2 \tan \beta \delta. \quad (58)
$$

Note that $\delta$ can take either sign.

B. 2HDM-II plus one or more singlets (2HDM-II+S)

We now consider the consequences of adding a real singlet scalar field, $S$, to the 2HDM-II. The constraints of Eqs. [10] [11] become

$$
a_u^2 + a_d^2 + a_s^2 = 1, \\
b_u^2 + b_d^2 = 1, \quad (59)
$$

where $a_s \equiv \langle h|S \rangle$ and the other $a_i$, $b_i$ are defined as in Sec. [IV A] The couplings of $h$ to SM particles, normalized to their SM values, are given in terms of $a_{u,d}$ and $b_{u,d}$ as for the 2HDM-II by Eq. [50]. As before, $b_u$, $b_d$, and $\bar{g}_W$ can be chosen real and positive. The coefficient $a_s$ can be chosen real and positive by a rephasing of $S$.

Because of the presence of the additional parameter $a_s$, the pattern relation of the 2HDM-
II no longer holds. Instead we obtain

\[ P_{ud} \equiv \bar{g}_W (\bar{g}_u + \bar{g}_d) - \bar{g}_u \bar{g}_d = \xi \leq 1, \quad (60) \]

where \( \xi \equiv 1 - a_s^2 = a_u^2 + a_d^2 \) parameterizes the doublet content of \( h \). In particular, \( \xi \) can be determined by applying the pattern relation to measurements of the couplings \( \bar{g}_W, \bar{g}_u \) and \( \bar{g}_d \). The solutions for the rest of the model parameters then become

\[
\begin{align*}
  b_u &= \left( \frac{\bar{g}_W - \bar{g}_d}{\bar{g}_u - \bar{g}_d} \right)^{1/2} = \left( \frac{\xi - \bar{g}_d^2}{\bar{g}_u^2 - \bar{g}_d^2} \right)^{1/2}, \\
  b_d &= \left( \frac{\bar{g}_W - \bar{g}_u}{\bar{g}_d - \bar{g}_u} \right)^{1/2} = \left( \frac{\xi - \bar{g}_u^2}{\bar{g}_d^2 - \bar{g}_u^2} \right)^{1/2}, \\
  a_u &= b_u \bar{g}_u, \quad a_d = b_d \bar{g}_d, \quad a_s = \sqrt{1 - \xi}, \quad (61)
\end{align*}
\]

where in the expressions for \( b_u, b_d \) we have shown how the dependence on \( \bar{g}_W \) can be traded for dependence on \( \xi \) using Eq. (60).

Clearly, if the relative signs of \( \bar{g}_W, \bar{g}_u \) and \( \bar{g}_d \) are known, then this model can be distinguished from the 2HDM-II using the pattern relation (\( P_{ud} \) is equal to one in the 2HDM-II and less than one in the 2HDM-II+S) and the solution for the model parameters is unique. If, however, the relative signs of \( \bar{g}_W, \bar{g}_u \) and \( \bar{g}_d \) are not known, then \( \xi \) cannot be obtained uniquely and there will be discrete ambiguities in the solutions for all the parameters. In this situation the pattern relation can still be used to test for the presence of the singlet in the model; if no combination of signs of the Higgs couplings gives \( P_{ud} = 1 \), then the model cannot be the minimal 2HDM-II.

This model can be cast into the usual notation for the 2HDM-II as follows. We first parameterize the doublet-singlet mixing in terms of \( \xi \),

\[ h = \sqrt{\xi} h' + \sqrt{1 - \xi} S, \quad (62) \]

where \( h' \) corresponds to the Higgs state considered in the 2HDM-II in the limit of zero singlet admixture:

\[ h' = \cos \alpha \phi_u - \sin \alpha \phi_d. \quad (63) \]

We then have \( a_u = \sqrt{\xi} \cos \alpha \) and \( a_d = -\sqrt{\xi} \sin \alpha \). The couplings are given by

\[
\begin{align*}
  \bar{g}_W &= \sqrt{\xi} \sin(\beta - \alpha), \quad \bar{g}_u = \sqrt{\xi} \frac{\cos \alpha}{\sin \beta}, \quad \bar{g}_d = \bar{g}_d = -\sqrt{\xi} \frac{\sin \alpha}{\cos \beta}. \quad (64)
\end{align*}
\]
In particular, the couplings of $h$ to SM particles are all scaled down by a common factor $\sqrt{\xi} \leq 1$. This means that the 2HDM-II+S lives on a volume in the three-dimensional parameter space of $\bar{g}_W$, $\bar{g}_u$, and $\bar{g}_d$, consisting of the surface inhabited by the 2HDM-II (corresponding to $\xi = 1$) together with all lines that connect points on that surface to the origin (corresponding to $0 \leq \xi < 1$). Clearly, the 2HDM-II+S occupies a different footprint in coupling space than the 2HDM-II, and it can thus be distinguished from the 2HDM-II. This is different from the case of the 2HDM-I+S; the reason is that the Type-II fermion coupling structure yields a third observable coupling related nontrivially to the other two.

The decoupling limit comprises $\delta \equiv \cos(\beta - \alpha) \rightarrow 0$ and $\epsilon \equiv \sqrt{1 - \xi} \rightarrow 0$. (Note that $\delta$ can have either sign while $\epsilon$ is chosen positive.) The couplings and corresponding partial widths, normalized to their SM values, become

$$\bar{g}_W = \sqrt{1 - \delta^2} \sqrt{1 - \epsilon^2} \simeq 1 - \frac{1}{2} \delta^2 - \frac{1}{2} \epsilon^2,$$

$$\Gamma_h^W/\Gamma_{SM}^W \simeq 1 - \delta^2 - \epsilon^2 \quad (65)$$

$$\bar{g}_u = \left[\sqrt{1 - \delta^2} + \cot \beta \delta\right] \sqrt{1 - \epsilon^2} \simeq 1 + \cot \beta \delta - \frac{1}{2} \epsilon^2,$$

$$\Gamma_h^u/\Gamma_{SM}^u \simeq 1 + 2 \cot \beta \delta - \epsilon^2,$$

$$\bar{g}_d = \bar{g}_l = \left[\sqrt{1 - \delta^2} - \tan \beta \delta\right] \sqrt{1 - \epsilon^2} \simeq 1 - \tan \beta \delta - \frac{1}{2} \epsilon^2,$$

$$\Gamma_h^d/\Gamma_{SM}^d \simeq 1 - 2 \tan \beta \delta - \epsilon^2.$$

Note that in the limit $\delta \rightarrow 0$ with $\epsilon$ finite, the deviations of the $h$ couplings from their SM values become identical to those in the SM+S.

These results can easily be extended to models containing two or more singlets by making the replacement

$$a_s^2 \rightarrow \sum_{\text{singlets}} a_s^2 = 1 - \xi. \quad (66)$$

Such a model cannot be distinguished from the 2HDM-II+S (with only one singlet) on the basis of the $h$ couplings alone.

**C. 2HDM-II plus additional doublet(s) (2HDM-II+D)**

We now consider the consequences of adding an additional Higgs doublet $\Phi_0$ to the 2HDM-II. The additional doublet can carry a vev, but under our assumption of natural flavor conservation it must not couple to fermions. The constraint equations become,

$$a_u^2 + a_d^2 + a_0^2 = 1, \quad b_u^2 + b_d^2 + b_0^2 = 1, \quad (67)$$
where \( a_0 \equiv \langle h | \phi_0 \rangle \) and \( b_0 \equiv v_0/v_{SM} \). The normalized couplings of \( h \) to SM particles are given by

\[
\begin{align*}
\bar{g}_W &= a_u b_u + a_d b_d + a_0 b_0, \\
\bar{g}_u &= a_u b_u, \\
\bar{g}_d &= \bar{g}_\ell = a_d b_d.
\end{align*}
\] (68)

All three \( b_i \) parameters can be chosen real and positive; \( \bar{g}_W \) can also be chosen positive through an appropriate rephasing of \( h \). Any combination of signs is then possible for \( \bar{g}_u \) and \( \bar{g}_d \); in particular, both can be negative (for \( a_0 b_0 > |a_u b_u + a_d b_d| \)) in contrast to the 2HDM-II or 2HDM-II+S.

Because of the presence of the two additional parameters \( a_0 \) and \( b_0 \), the model is underconstrained and the parameters \( a_i, b_i \) cannot be extracted in terms of the \( h \) couplings. However, the model *is* distinguishable from the 2HDM-II because the pattern relation of Eq. (51) no longer holds. In some parts of parameter space, this model can also be distinguished from the 2HDM-II+S.

In order to illustrate these features, we cast the model into the usual notation for the 2HDM-II. We first parameterize the mixing of the third doublet in terms of an angle \( \theta \),

\[
h = \cos \theta h' + \sin \theta \phi_0,
\] (69)

where \( h' \equiv \cos \alpha \phi_u - \sin \alpha \phi_d \) corresponds to the Higgs in the 2HDM-II in the limit of zero mixing with the extra doublet. We then have \( a_u = \cos \theta \cos \alpha \), \( a_d = -\cos \theta \sin \alpha \), and \( a_0 = \sin \theta \). We also define \( \tan \beta \equiv v_u/v_d = b_u/b_d \) and \( \cos \Omega \equiv \sqrt{b_u^2 + b_d^2} \), \( \sin \Omega = b_0 \), where the angle \( 0 \leq \Omega < \pi/2 \) parameterizes the amount of vev carried by \( \Phi_0 \). The couplings of \( h \) are then given by

\[
\begin{align*}
\bar{g}_W &= \cos \Omega \cos \theta \sin(\beta - \alpha) + \sin \Omega \sin \theta, \\
\bar{g}_u &= \frac{\cos \theta \cos \alpha}{\cos \Omega \sin \beta}, \\
\bar{g}_d &= \bar{g}_\ell = \frac{-\cos \theta \sin \alpha}{\cos \Omega \cos \beta}.
\end{align*}
\] (70)

We note the features of two limiting cases:

(i) When \( b_0 = 0 \) (i.e., \( \cos \Omega = 1 \)), the \( h \) couplings reduce to those of the 2HDM-II+S, with \( \sqrt{\xi} \) replaced by \( \cos \theta \). This happens because in this limit, \( \phi_0 \) does not couple to fermion pairs or gauge boson pairs and the physics is simply that of the 2HDM-II with mixing of a “sterile” state into \( h \). The pattern relation in this special case becomes

\[
P_{ud} \equiv \bar{g}_W (\bar{g}_u + \bar{g}_d) - \bar{g}_u \bar{g}_d = \cos^2 \theta \leq 1.
\] (71)

(Note that the 2HDM-II result \( P_{ud} = 1 \) is recovered in the limit \( \cos^2 \theta \to 1 \).)
(ii) When $a_0 = 0$ (i.e., $\cos \theta = 1$, so $h = h'$) there is no mixing of the new doublet into $h$, but the vev of $\phi_0$ is nonzero so that the total vev carried by the two doublets that couple to fermions is reduced. The fermion Yukawa couplings must thus be enhanced in order to yield the required fermion masses, while the coupling of $h$ to $W$ or $Z$ pairs is suppressed. In this case the couplings of $h$ become

$$
\bar{g}_W = \cos \Omega \sin (\beta - \alpha), \quad \bar{g}_u = \frac{1}{\cos \Omega} \frac{\cos \alpha}{\sin \beta}, \quad \bar{g}_d = -\frac{1}{\cos \Omega} \frac{\sin \alpha}{\cos \beta}, \quad (72)
$$

and the pattern relation in this special case becomes

$$
P_{ud} \equiv \bar{g}_W (\bar{g}_u + \bar{g}_d) - \bar{g}_u \bar{g}_d = 1 + \tan^2 \Omega \frac{\sin \alpha \cos \alpha}{\sin \beta \cos \beta}, \quad (73)
$$

In particular, $P_{ud} > 1$ whenever $\sin \alpha \cos \alpha > 0$, i.e., whenever $\bar{g}_d$ and $\bar{g}_u$ have opposite signs. Furthermore, when $\sin \alpha \cos \alpha < 0$ (i.e., when $\bar{g}_d$ and $\bar{g}_u$ have the same sign), small values of $\cos \Omega$, $\sin \beta$, and/or $\cos \beta$ can yield $P_{ud} < 0$. Either of these situations allows the 2HDM-II+D to be distinguished from both the 2HDM-II and the 2HDM-II+S. (Note that the 2HDM-II result $P_{ud} = 1$ is recovered in the limit $\cos \Omega \to 1$.)

In the general case of both $\cos \theta < 1$ (nonzero mixing of $\phi_0$ into $h$) and $\cos \Omega < 1$ (nonzero vev of $\phi_0$), the footprint of the 2HDM-II+D covers a three-dimensional volume in the space of couplings ($\bar{g}_W, \bar{g}_u, \bar{g}_d$). Part of this volume overlies the footprint of the 2HDM-II+S (when mixing dominates, or when $\bar{g}_u$ and $\bar{g}_d$ have the same sign), but part is unique to the 2HDM-II+D (when vev sharing dominates, or when $\bar{g}_u$ and $\bar{g}_d$ are both negative as discussed before). Thus the model is distinguishable in general from the 2HDM-II, and is distinguishable from the 2HDM-II+S in some regions of parameter space.

We now describe the approach to decoupling in this model. The decoupling limit corresponds to $\langle h|\phi_v \rangle \to 1$. Deviations from this limit can be parameterized by writing

$$
h = c_\parallel \phi_v + c_\perp \phi_\perp, \quad \langle \phi_\perp |\phi_v \rangle = 0, \quad (74)
$$

where $c_\parallel^2 + c_\perp^2 = 1$ and $\phi_v$ is given in our parameterization by

$$
\phi_v = \cos \Omega (\sin \beta \phi_u + \cos \beta \phi_d) + \sin \Omega \phi_0. \quad (75)
$$

The component of $h$ orthogonal to $\phi_v$ can be constructed as follows. We first define two states orthogonal to $\phi_v$ and to each other:

$$
\phi_{\perp 1} = \cos \beta \phi_u - \sin \beta \phi_d, \quad (76)
$$
which lies in the $\phi_u - \phi_d$ plane, and

$$\phi_{\perp 2} = - \sin \Omega (\sin \beta \phi_u + \cos \beta \phi_d) + \cos \Omega \phi_0. \quad (77)$$

Then $\phi_{\perp}$ can be parameterized in terms of a new mixing angle $\gamma$,

$$\phi_{\perp} = \sin \gamma \phi_{\perp 1} + \cos \gamma \phi_{\perp 2}$$

$$= (\sin \gamma \cos \beta - \cos \gamma \sin \Omega \sin \beta) \phi_u + (- \sin \gamma \sin \beta - \cos \gamma \sin \Omega \cos \beta) \phi_d$$

$$+ (\cos \gamma \cos \Omega) \phi_0. \quad (78)$$

Defining the decoupling parameter $\delta \equiv c_{\perp}$, we obtain the couplings of $h$:

$$\tilde{g}_W = \langle h | \phi_v \rangle = \sqrt{1 - \delta^2}$$

$$\tilde{g}_u = \frac{\langle h | \phi_u \rangle}{b_u} = \sqrt{1 - \delta^2} + \delta \left[ \sin \gamma \frac{\cot \beta}{\cos \Omega} - \cos \gamma \tan \Omega \right]$$

$$\tilde{g}_d = \tilde{g}_\ell = \frac{\langle h | \phi_d \rangle}{b_d} = \sqrt{1 - \delta^2} + \delta \left[ - \sin \gamma \frac{\tan \beta}{\cos \Omega} - \cos \gamma \tan \Omega \right]. \quad (79)$$

Letting $\delta$ take either sign, we can fix $0 \leq \gamma < \pi$. Note that for $\sin \gamma = \cos \Omega = 1$, these formulas reduce to those for the 2HDM-II given in Eq. (58). The decoupling limit corresponds to $\delta \to 0$.

This analysis can be extended to the 2HDM-II plus two or more doublets in a straightforward way. We denote the doublets that do not couple to fermions as $\Phi_{0,i}$, with $i = 1 \ldots n$ ($n \geq 2$). As in Sec. III D, we first define a linear combination $\phi'_0$ of the neutral CP-even states $\phi_{0,i}$ such that

$$h = a_u \phi_u + a_d \phi_d + a'_0 \phi'_0, \quad a_u^2 + a_d^2 + a'_0^2 = 1. \quad (80)$$

The vev of $\phi'_0$ is parameterized by $b'_0 \equiv \langle \phi'_0 | \phi_v \rangle$, and Eq. (11) becomes $b_u^2 + b_d^2 + b'_0^2 \leq 1$, where the inequality accounts for the vev carried by the linear combination(s) of $\phi_{0,i}$ orthogonal to $h$. While the underlying physics of this model differs from that of the 2HDM-II plus one extra doublet, the footprint of the model in coupling space is the same. This can be seen straightforwardly by noting that the pattern relation $P_{ud}$ can take any value in the 2HDM-II+D, leaving no room for a larger footprint when additional extra doublets are added. Thus, while it is possible to tell from the couplings of $h$ alone that (at least) one additional doublet has been added to the 2HDM-II, it is not possible to tell how many.

The addition of singlet(s) to the 2HDM-II+D can be parameterized in a similar way. We first note that as far as the couplings of $h$ are concerned, adding an additional Higgs doublet
with zero vev is indistinguishable from adding a singlet. We again obtain Eq. 80 in which \( \phi_0' \) now denotes the appropriate linear combination of \( \phi_0 \) and the singlets. The constraint equation for the doublet vevs remains as given in Eq. 67. We see that the model occupies the same footprint in \( h \) coupling space as the 2HDM-II+D and thus it is not possible on the basis of \( h \) couplings alone to tell whether the 2HDM-II+D also contains additional singlets.

## D. Flipped 2HDM, lepton-specific 2HDM, and their extensions

The flipped and lepton-specific two Higgs doublet models comprise the two possible alternate assignments of fermion couplings of the two-doublet models considered here. These models were introduced in Refs. [62, 63, 64]. Some early studies of their phenomenology have been made in Refs. [76, 77]. Much can be extrapolated in a straightforward way from existing results for the usual 2HDM-I and 2HDM-II.

In the flipped 2HDM, one doublet \( \Phi_u \) generates the masses of the up-type quarks and the charged leptons while the other doublet \( \Phi_d \) generates the masses of the down-type quarks. The constraint equations remain identical to those of the 2HDM-II as given in Eq. 49 while the normalized couplings of \( h \) to SM particles are given by

\[
\bar{g}_W = a_u b_u + a_d b_d, \quad \bar{g}_u = \frac{a_u}{b_u}, \quad \bar{g}_d = \frac{a_d}{b_d}. \tag{81}
\]

The distinguishing feature of this model is the behavior of \( \bar{g}_\ell \). The quark coupling results carry over unchanged from the 2HDM-II model and its extensions by extra doublets and/or singlets.

In the lepton-specific 2HDM\(^8\), one doublet \( \Phi_q \) generates the masses of all flavors of quarks while the other doublet \( \Phi_\ell \) generates the masses of the charged leptons. The constraint equations become

\[
a_q^2 + a_\ell^2 = 1, \quad b_q^2 + b_\ell^2 = 1. \tag{82}
\]

The normalized couplings of \( h \) to SM particles are given by

\[
\bar{g}_W = a_q b_q + a_\ell b_\ell, \quad \bar{g}_u = \bar{g}_d = \frac{a_q}{b_q}, \quad \bar{g}_\ell = \frac{a_\ell}{b_\ell}. \tag{83}
\]

\(^8\) LHC phenomenology for \( h \) in the lepton-specific 2HDM was also discussed in Ref. 78. Ref. 79 also makes use of this fermion coupling structure.
Note that in the quark sector, this model is identical to the 2HDM-I. Its distinguishing feature, however, is again the behavior of $\bar{g}_q$; the pattern relation and all other results for the 2HDM-II and its extensions by extra doublets and/or singlets carry over to this model with the replacements

$$\bar{g}_u \rightarrow \bar{g}_q, \quad \bar{g}_d \rightarrow \bar{g}_t.$$  \hfill (84)

E. MSSM (2HDM-II with $\Delta_b$)

At tree level, the Higgs sector of the MSSM is a Type-II 2HDM. The natural flavor conservation structure of the Yukawa couplings is enforced by the analyticity of the superpotential. Beyond tree level, however, radiative corrections involving loops of supersymmetric particles can induce additional couplings of right-handed fermions to the “wrong” Higgs doublet \[46, 47, 70, 71\]. Thus, beyond tree level the Higgs sector of the MSSM is technically a Type III 2HDM\(^9\). This violation of natural flavor conservation is a consequence of supersymmetry breaking; because of this, the loop-induced wrong-Higgs couplings do not decouple as all SUSY mass parameters are simultaneously taken large \[82, 83\].

The most important loop-induced wrong Higgs couplings of this type arise in the bottom-quark sector from loops involving bottom squarks and gluinos (involving the large QCD gauge coupling) and from loops involving top squarks and charginos (involving the large top Yukawa coupling). In particular, the effect of the wrong-Higgs coupling on $\bar{g}_b$ is enhanced by $\tan \beta$, meaning that even though it is a one-loop effect, it can be important at large $\tan \beta$.

The radiatively-corrected couplings can be parameterized by an effective Lagrangian \[84\],

$$- \mathcal{L}_{\text{eff}} = \epsilon_{ij} h_b \bar{b}_R H_u^i Q_L^j + \Delta h_b \bar{b}_R Q_L^{k_s} H_u^{k_s} + \text{h.c.,}$$  \hfill (85)

with $H_u$ and $H_d$ defined in the usual way for the MSSM with opposite hypercharges. Note that we absorb into $h_b$ any SUSY radiative corrections to the “right-Higgs” coupling.

The physical bottom quark mass is given by

$$m_b = \frac{h_b v_d}{\sqrt{2}} + \frac{\Delta h_b v_u}{\sqrt{2}} = \frac{h_b v_{SM} \cos \beta}{\sqrt{2}} \left(1 + \frac{\Delta h_b \tan \beta}{h_b}\right) = \frac{h_b v_{SM} \cos \beta}{\sqrt{2}} (1 + \Delta_b).$$  \hfill (86)

\(^9\) The phenomenology of the general Type III 2HDM has been reviewed in Ref. \[80\]. In this model the basis chosen for the two Higgs doublets is somewhat arbitrary; basis-independent methods have been developed in Refs. \[65, 81\].
(Note the factor of $\tan \beta$ that is absorbed into the definition of $\Delta_b$.) Similarly, the $h^0 b \bar{b}$ coupling becomes

$$g_b^h = -\sin \alpha \frac{h_b}{\sqrt{2}} + \cos \alpha \frac{\Delta h_b}{\sqrt{2}}, \quad (87)$$

where the mixing angle $\alpha$ is defined as in Eq. 54 for the 2HDM-II. This coupling can be written in terms of $m_b$ and $\Delta_b$ by noting that

$$h_b \sqrt{2} = m_b v_{SM} \cos \beta 1 + \Delta_b,$$

$$\Delta h_b \sqrt{2} = m_b v_{SM} \sin \beta 1 + \Delta_b. \quad (88)$$

Inserting these relations into $g_b^h$ and normalizing by the SM coupling we obtain

$$\bar{g}_b = \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha) \frac{1 - \cot^2 \beta \Delta_b}{1 + \Delta_b}. \quad (89)$$

The $\Delta_b$ corrections are typically the only large SUSY radiative corrections to the Higgs Yukawa couplings – in particular, the analogous corrections to the Higgs couplings to top quarks are not $\tan \beta$ enhanced, and those to the Higgs couplings to tau leptons involve only the small electroweak gauge couplings. Thus the SUSY corrections to these couplings can generally be neglected, and the usual 2HDM-II relations are recovered:

$$\bar{g}_W = \sin(\beta - \alpha),$$

$$\bar{g}_u = \frac{\cos \alpha}{\sin \beta} = \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha),$$

$$\bar{g}_t = -\frac{\sin \alpha}{\cos \beta} = \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha). \quad (90)$$

The model parameters can be obtained as in the 2HDM-II by using the couplings that are unaffected by $\Delta_b$:

$$\tan \beta = \left[ \frac{\bar{g}_u^2 - 1}{1 - \bar{g}_t^2} \right]^{1/2}, \quad \cos \alpha = \bar{g}_u \left[ \frac{1 - \bar{g}_t^2}{\bar{g}_u^2 - \bar{g}_t^2} \right]^{1/2}, \quad \sin \alpha = \bar{g}_t \left[ \frac{1 - \bar{g}_u^2}{\bar{g}_t^2 - \bar{g}_u^2} \right]^{1/2}. \quad (91)$$

The value of $\Delta_b$ can also be extracted from the $h$ couplings using

$$\Delta_b = \frac{\bar{g}_b - \bar{g}_t}{\bar{g}_u - \bar{g}_b}. \quad (92)$$

We first note that the pattern relation of the 2HDM-II among the couplings $\bar{g}_W$, $\bar{g}_u$ and $\bar{g}_t$ survives:

$$P_{u \ell} \equiv \bar{g}_W (\bar{g}_u + \bar{g}_t) - \bar{g}_u \bar{g}_t = 1. \quad (93)$$

This allows the MSSM Higgs sector to be distinguished from the more general three-Higgs-doublet models discussed in the next section and allows one to test for the presence of additional singlets or doublets that mix with $h$ or carry nonzero vevs.

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However, the $\Delta_b$ correction to the Higgs coupling to bottom quarks leads to $\bar{g}_d \neq \bar{g}_\ell$, such that the pattern relation among the $W$, $u$ and $d$ couplings is violated:

$$P_{ud} \equiv \bar{g}_W (\bar{g}_u + \bar{g}_d) - \bar{g}_u \bar{g}_d = 1 - \cos^2(\beta - \alpha) \frac{\Delta_b (1 + \cot^2 \beta)}{1 + \Delta_b}.$$  \hspace{1cm} (94)

Depending on the sign of $\Delta_b$, the right-hand side can be greater or less than one.

In the decoupling limit the deviations of the $h$ couplings from their SM values can be parameterized in terms of $\delta \equiv \cos(\beta - \alpha)$. We have,

$$\bar{g}_W = \sqrt{1 - \delta^2} \simeq 1 - \frac{1}{2} \delta^2,$$

$$\bar{g}_u = \sqrt{1 - \delta^2} + \cot \beta \delta \simeq 1 + \cot \beta \delta,$$

$$\bar{g}_b = \sqrt{1 - \delta^2} - \tan \beta' \delta \simeq 1 - \tan \beta' \delta,$$

$$\bar{g}_\ell = \sqrt{1 - \delta^2} - \tan \beta \delta \simeq 1 - \tan \beta \delta.$$

$$\frac{\Gamma_h}{\Gamma_{SM}} = 1 - \delta^2,$$

$$\frac{\Gamma_{\bar{g}_u}}{\Gamma_{SM}} \simeq 1 + 2 \cot \beta \delta,$$

$$\frac{\Gamma_{\bar{g}_b}}{\Gamma_{SM}} \simeq 1 - 2 \tan \beta' \delta,$$

$$\frac{\Gamma_{\bar{g}_\ell}}{\Gamma_{SM}} \simeq 1 - 2 \tan \beta \delta.$$ \hspace{1cm} (95)

Note that $\delta$ can take either sign. These expressions are identical to those given in Eq. 58 for the 2HDM-II except that in the bottom quark couplings we have replaced $\tan \beta$ with $\tan \beta' \equiv \tan \beta \frac{1 - \cot^2 \beta \Delta_b}{1 + \Delta_b}$. \hspace{1cm} (96)

V. FERMION MASSES FROM THREE DOUBLETS

Finally we consider a model in which the fermion masses arise “democratically” from couplings to three different Higgs doublets – i.e., models in which the masses of the up-type quarks, down-type quarks, and charged leptons are generated by couplings to three different Higgs doublets $\Phi_u$, $\Phi_d$, and $\Phi_\ell$, respectively. Such a model was considered in Ref. [64]. We also consider extensions of this model obtained by adding one or more singlets or doublets that do not couple to fermions.\(^{10}\)

A similar Higgs-fermion coupling structure has recently been proposed in the “Private Higgs” model \cite{85, 86}, which introduces one Higgs doublet for each of the six flavors of quarks in order to address the hierarchy of quark masses. Here, however, we limit the

\(^{10}\)Models with three or more doublets introduce the possibility of new CP-violating parameters in the $n \times n$, $n \geq 3$ mixing matrices of the charged scalars \cite{64}. Again, we neglect the possibility of CP-violating effects in this work.
discussion to models in which the masses of fermions of a given electric charge are generated
by their couplings to only one Higgs doublet; in particular, we do not allow the Higgs
coupling structure to differ by fermion generation. We also make no assumptions about the
structure of the Higgs potential. The discussion here can be extended to models in which
the three generations are treated differently but care must be taken to avoid Higgs-mediated
flavor-changing neutral currents.

A. Democratic Three Higgs Doublet Model (3HDM-D)

In this model the up-type quarks, down-type quarks, and charged leptons each get their
mass from a different Higgs doublet, denoted $\Phi_u$, $\Phi_d$, and $\Phi_\ell$, respectively. The constraints
of Eqs. (10, 11) become

\[ a_u^2 + a_d^2 + a_\ell^2 = 1, \quad b_u^2 + b_d^2 + b_\ell^2 = 1. \] (97)

The normalized couplings of $h$ to SM particles are then given by

\[ \bar{g}_W = a_u b_u + a_d b_d + a_\ell b_\ell, \quad \bar{g}_u = \frac{a_u}{b_u}, \quad \bar{g}_d = \frac{a_d}{b_d}, \quad \bar{g}_\ell = \frac{a_\ell}{b_\ell}. \] (98)

The vev ratios $b_u$, $b_d$, and $b_\ell$ can all be chosen real and positive. Simultaneously, we can
choose $\bar{g}_W$ positive through an appropriate rephasing of the mass eigenstate $h$. There is no
freedom left to choose the signs of the fermion couplings $\bar{g}_u$, $\bar{g}_d$, or $\bar{g}_\ell$; depending on the
underlying values of the parameters they can take any combination of signs so long as at
least one of them is positive.

The constraint equations and coupling relations can be solved explicitly for the $b_i$ factors
in terms of the $h$ couplings:

\[ b_u = \left[ \frac{1 - \bar{g}_W (\bar{g}_d + \bar{g}_\ell) + \bar{g}_d \bar{g}_\ell}{(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)} \right]^{1/2}, \]
\[ b_d = \left[ \frac{1 - \bar{g}_W (\bar{g}_u + \bar{g}_\ell) + \bar{g}_u \bar{g}_\ell}{(\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)} \right]^{1/2}, \]
\[ b_\ell = \left[ \frac{1 - \bar{g}_W (\bar{g}_u + \bar{g}_d) + \bar{g}_u \bar{g}_d}{(\bar{g}_\ell - \bar{g}_u)(\bar{g}_\ell - \bar{g}_d)} \right]^{1/2}. \] (99)

We also obtain solutions for the $a_i$ factors,

\[ a_u = b_u \bar{g}_u, \quad a_d = b_d \bar{g}_d, \quad a_\ell = b_\ell \bar{g}_\ell. \] (100)
As in the other solvable models, if the relative signs of \( \bar{g}_u, \bar{g}_d, \bar{g}_\ell \) and \( \bar{g}_W \) are known, then the solution for the model parameters is unique. However, if the relative signs are not known, discrete ambiguities arise in the solutions for the \( b_i \) and \( a_i \) factors.

The key feature that distinguishes the democratic 3HDM from the previous models considered is that \( \bar{g}_u \neq \bar{g}_d \neq \bar{g}_\ell \). While this is also true in the MSSM with \( \Delta_b \) corrections, the MSSM couplings satisfy the pattern relation \( P_{u\ell} = 1 \) involving the couplings \( \bar{g}_W, \bar{g}_u, \) and \( \bar{g}_\ell \) (Eq. 93); this relation does not hold in the 3HDM-D.

The features of this model can be clarified by examining its parallels with the 2HDM-II+D. In particular, the behavior of \( \bar{g}_W, \bar{g}_u \) and \( \bar{g}_d \) is identical to that in the 2HDM-II+D, while now \( \bar{g}_\ell \) behaves differently with the model parameters and serves to distinguish the current model. As in the 2HDM-II+D, we parameterize the mixing according to \( h = \cos \theta h' + \sin \theta \phi_\ell \), with \( h' \equiv \cos \alpha \phi_u - \sin \alpha \phi_d \), yielding \( a_u = \cos \theta \cos \alpha, a_d = -\cos \theta \sin \alpha, \) and \( a_\ell = \sin \theta \). We also define \( \tan \beta \equiv v_u/v_d = b_u/b_d \) and \( \cos \Omega \equiv \sqrt{b_u^2 + b_d^2} \), \( \sin \Omega = b_\ell \). The couplings of \( h \) are then given by

\[
\bar{g}_W = \cos \Omega \cos \theta \sin(\beta - \alpha) + \sin \Omega \sin \theta, \\
\bar{g}_u = \frac{\cos \Omega \cos \alpha}{\cos \Omega \sin \beta}, \quad \bar{g}_d = -\frac{\cos \Omega \sin \alpha}{\cos \Omega \cos \beta}, \quad \bar{g}_\ell = \frac{\sin \theta}{\sin \Omega}.
\]

The decoupling relations can be parameterized exactly as in the 2HDM-II+D (Eq. 79) except for \( \bar{g}_\ell \), which is given by

\[
\bar{g}_\ell = \frac{\langle h | \phi_\ell \rangle}{b_\ell} = \sqrt{1 - \delta^2} + \delta [\cos \gamma \cot \Omega],
\]

where \( \phi_\ell \) is the third doublet and \( \gamma \) is defined as in Eq. 78 with \( \phi_0 \to \phi_\ell \).

**B. 3HDM-D plus one or more singlets (3HDM-D+S)**

We now consider the consequences of adding a real singlet scalar field, \( S \), to the 3HDM-D. The constraints of Eqs. [10] [11] become

\[
a_u^2 + a_d^2 + a_\ell^2 + a_s^2 = 1, \quad b_u^2 + b_d^2 + b_\ell^2 = 1.
\]

The formulae for the normalized couplings of \( h \) to SM particles are identical to those of the 3HDM-D given in Eq. 98 The vev ratios \( b_u, b_d \) and \( b_\ell \) and the coupling \( \bar{g}_W \) can all be chosen real and positive; \( a_s \) can then be chosen real and positive by an appropriate rephasing of \( S \).
Because of the presence of the additional parameter $a_s$, this model is distinguishable from the 3HDM-D in part of its parameter space, as we now show. First we define the following three combinations of $h$ couplings,

$$
X_u = \frac{1 - \bar{g}_w(\bar{g}_d + \bar{g}_\ell) + \bar{g}_d\bar{g}_\ell}{(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)} = b_u^2 + \frac{a_s^2}{(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)},
$$

$$
X_d = \frac{1 - \bar{g}_w(\bar{g}_u + \bar{g}_\ell) + \bar{g}_u\bar{g}_\ell}{(\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)} = b_d^2 + \frac{a_s^2}{(\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)},
$$

$$
X_\ell = \frac{1 - \bar{g}_w(\bar{g}_u + \bar{g}_d) + \bar{g}_u\bar{g}_d}{(\bar{g}_\ell - \bar{g}_u)(\bar{g}_\ell - \bar{g}_d)} = b_\ell^2 + \frac{a_s^2}{(\bar{g}_\ell - \bar{g}_u)(\bar{g}_\ell - \bar{g}_d)}. \tag{104}
$$

Here $X_u + X_d + X_\ell = 1$ by construction. Note that if these formulae were applied to the 3HDM-D, they would yield $b_u^2$, $b_d^2$ and $b_\ell^2$, respectively (cf. Eq. 99; in the current model this is recovered in the limit $a_s \to 0$). In particular, the values of all three $X_i$ would necessarily lie between zero and one. However, in part of the parameter space of the 3HDM-D+S, one of the $X_i$ can be negative. In this part of the parameter space, if one were to (incorrectly) assume the 3HDM-D and attempt to solve for the $b_i$, Eq. 99 would fail to yield a solution. Thus we see that the footprint of the 3HDM-D+S in the space of $h$ couplings is larger than that of the 3HDM-D, and therefore the model with an additional singlet can be distinguished from the 3HDM-D in part of its parameter space.

A negative value for one of the $X_i$ can occur because exactly one of the three denominators in Eq. 104 is negative. This allows us to obtain a lower limit on $a_s^2$ when one of the $X_i$ is negative. We first define

$$
Y = \begin{cases} 
(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)X_u & \text{if } X_u < 0, \\
(\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)X_d & \text{if } X_d < 0, \\
(\bar{g}_\ell - \bar{g}_u)(\bar{g}_\ell - \bar{g}_d)X_\ell & \text{if } X_\ell < 0. 
\end{cases} \tag{105}
$$

Note $0 < Y < 1$ by construction for parameter points where $Y$ is defined. These expressions are entirely determined in terms of the $h$ couplings. The lower limit on $a_s^2$ is then given by $a_s^2 \geq Y$.

For completeness, we give here the relations for the parameters $b_i$ and $a_i$ in terms of the $h$ couplings and $\xi \equiv 1 - a_s^2$:

$$
b_u = \left[ \frac{\xi - \bar{g}_w(\bar{g}_d + \bar{g}_\ell) + \bar{g}_d\bar{g}_\ell}{(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)} \right]^{1/2},
$$

$$
b_d = \left[ \frac{\xi - \bar{g}_w(\bar{g}_u + \bar{g}_\ell) + \bar{g}_u\bar{g}_\ell}{(\bar{g}_d - \bar{g}_u)(\bar{g}_d - \bar{g}_\ell)} \right]^{1/2},
$$

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\[
\begin{align*}
\ell = & \left[ \xi - \bar{g}_W (\bar{g}_u + \bar{g}_d) + \bar{g}_u \bar{g}_d \right]^{1/2}, \\
a_u = & \, b_u \bar{g}_u, \quad a_d = b_d \bar{g}_d, \quad a_\ell = b_\ell \bar{g}_\ell. \quad (106)
\end{align*}
\]

Because \( a_s \) cannot be uniquely determined in this model, the model is underconstrained and the parameters \( b_i \) and \( a_i \) cannot be uniquely extracted.

These results can easily be extended to models containing two or more singlets by making the replacement
\[
a_s^2 \rightarrow \sum_{\text{singlets}} a_{s_s}^2 = 1 - \xi. \quad (107)
\]

We see that it is not possible to tell whether only one singlet or more than one singlet has been added to the 3HDM-D on the basis of \( h \) couplings alone.

**C. 3HDM-D plus additional doublet(s)**

We finally consider the consequences of adding an additional Higgs doublet \( \Phi_0 \) to the 3HDM-D. The additional doublet carries a vev but does not couple to fermions. The constraint equations become,
\[
a_u^2 + a_d^2 + a_\ell^2 + a_0^2 = 1, \quad b_u^2 + b_d^2 + b_\ell^2 + b_0^2 = 1. \quad (108)
\]

The normalized couplings of \( h \) to SM particles are given by
\[
\bar{g}_W = a_u b_u + a_d b_d + a_\ell b_\ell + a_0 b_0, \quad \bar{g}_u = \frac{a_u}{b_u}, \quad \bar{g}_d = \frac{a_d}{b_d}, \quad \bar{g}_\ell = \frac{a_\ell}{b_\ell}. \quad (109)
\]

All four \( b_i \) parameters and \( \bar{g}_W \) can be chosen real and positive, while now \( \bar{g}_u \), \( \bar{g}_d \), and \( \bar{g}_\ell \) can have any combination of signs; in particular, all three of these couplings can be negative if \( a_0 b_0 \) is big enough to keep \( \bar{g}_W \) positive.

Like the 3HDM-D+S, this model is distinguishable from the 3HDM-D in part of its parameter space; the parameters and couplings of the current model reduce to the form of the 3HDM-D+S in the limit \( b_0 \rightarrow 0 \). However, the footprint of the current model in \( h \) coupling space is larger than that of the 3HDM-D+S, so that in part of the parameter space the presence of the extra doublet can be detected, as we now show.

We again define \( X_u \), \( X_d \) and \( X_\ell \) in terms of the \( h \) couplings as in Eq. [104]. In terms of the underlying model parameters, these can be expressed as
\[
X_u = b_u^2 + \frac{a_u^2 + b_0^2 \bar{g}_d \bar{g}_\ell - a_0 b_0 (\bar{g}_d + \bar{g}_\ell)}{(\bar{g}_u - \bar{g}_d)(\bar{g}_u - \bar{g}_\ell)},
\]
Again, $X_u + X_d + X_\ell = 1$ by construction. In the limit $b_0 \to 0$, these expressions reduce to those for the 3HDM-D+S given in Eq. (104); in that limit the numerator of the second term is just $a_0^2$, which must lie between zero and one. When $b_0 \neq 0$, however, the numerator of the second term can be less than zero or greater than one.

In the part of parameter space with one negative $X_i$ we again construct the quantity $Y$ as given in Eq. (105). In the 3HDM-D+S, $Y$ provided a lower bound for $a_0^2$; in particular $0 \leq Y \leq 1$ always. In the current model, however, one can also obtain $Y < 0$ (when $X_i$ is negative due to the numerator of the second term being negative) or $Y > 1$ (when $X_i$ is negative due to the denominator of the second term being negative and the numerator of the second term is sufficiently greater than one). Neither of these possibilities can occur in the 3HDM-D+S and they therefore allow the current model to be distinguished in part of its parameter space.

The analysis can easily be extended to the 3HDM-D plus two or more doublets. We have seen in the case of the 2HDM-I plus an additional doublet (Sec. III D) and the 2HDM-II plus additional doublets (Sec. IV C) that, once the model already contains one doublet that does not couple to fermions, adding additional doublets that do not couple to fermions does not change the model footprint in $h$ coupling space. The same is true for the 3HDM-D plus additional doublets. Adding one additional doublet changes the model footprint as we have seen. Adding a second additional doublet, however, does not further change the model footprint; thus it is not possible to tell how many additional doublets have been added to the 3HDM-D based only on the $h$ couplings.

The addition of singlet(s) to the 3HDM-D plus a doublet can be dealt with in a similar way. As far as the couplings of $h$ are concerned, adding a singlet is indistinguishable from adding an additional Higgs doublet with zero vev. We see then that it is not possible to tell whether singlets have been added to the 3HDM-D plus a doublet based only on the $h$ couplings.
VI. RADIATIVE CORRECTIONS

In order to translate between the tree-level Lagrangian parameters $\bar{g}_W$, $\bar{g}_u$, $\bar{g}_d$, and $\bar{g}_\ell$ and experimentally observable $h$ production cross sections and decay partial widths, radiative corrections must be included. This program has been carried out in great detail for the SM Higgs as well as for the MSSM. For more general multi-Higgs-doublet models, however, detailed results are lacking; such corrections would be needed for a translation between observables and the underlying Lagrangian parameters at the few-percent level. We can however make the following general observations.

QCD corrections are universal and can be taken over from the SM, assuming that no new strongly-interacting particles contribute. In the MSSM, for example, squarks and gluinos yield large flavor-specific radiative corrections; integrating out these contributions into an effective Lagrangian yields the $\Delta_b$ formalism but leads to a violation of the underlying natural flavor conservation of the MSSM Higgs sector.

Electroweak radiative corrections are not universal and in principle must be computed for each model. These depend on the model content – both the extended Higgs sector and any additional new physics that may be present. Some parts of these corrections can be simply absorbed into our parameterization; for example, the largest electroweak corrections to the MSSM Higgs sector from top quark and top squark loops can be absorbed into an effective Higgs sector mixing angle $\alpha$. However, vertex corrections remain an issue for precision parameter extraction.

Experimental determination of the relative signs of $h$ couplings is also potentially problematic. These signs are accessible only through the interference of competing amplitudes in loops, and thus nonstandard sign combinations can be masked or faked by additional new contributions to loop-induced couplings.

Throughout we choose the phase of $h$ such that $\bar{g}_W$ is positive. The sign of $\bar{g}_u$ is then accessible through the $h\gamma\gamma$ coupling: in the SM, the $W$ loop dominates while the top quark loop interferes destructively, reducing the $h \rightarrow \gamma\gamma$ partial width by $\sim 30\%$. The relative sign of $\bar{g}_d$ and $\bar{g}_u$ is in principle accessible from the $ggh$ coupling: again in the SM the top quark loop dominates, while top-bottom interference is about a $10\%$ effect for moderate Higgs masses; however, the QCD scale uncertainty is still of this order. The sign of $\bar{g}_\ell$ will be even more difficult, since its contribution to $h\gamma\gamma$ is extremely small. The loop-induced $h\gamma Z$
coupling would provide additional information, but its experimental detection does not seem feasible at the moment. The best strategy may be to examine all possible sign combinations for $h$ couplings and enumerate their implications for the underlying model parameters and the size of the possible new physics contributions to loop-induced $h$ couplings.

VII. DISCUSSION AND CONCLUSIONS

Our ultimate aim in this work is to provide a framework for distinguishing among competing models for the Higgs sector. To this end we have studied the patterns of tree-level couplings of a single CP-even state $h$ in all models that can be constructed out of SU(2)$_L$ doublets and/or singlets, subject to the requirement of natural flavor conservation. Distinguishing one model from another relies not only on the underlying theoretical distinctions between the values taken by the $h$ couplings, but also on the experimental and theoretical precision with which those couplings can be measured. Here we collect our theoretical results, then turn to the question of model discrimination based on experimental data.

Our theoretical results can be conveniently summarized in the form of a decision tree, as follows. We assume a deviation from the SM; $n \geq 1$ counts additional singlets (S) and $m \geq 1$ counts additional doublets (D) that do not couple to fermions. We denote the pattern relation involving fermion couplings $\bar{g}_i$ and $\bar{g}_j$ as $P_{ij} \equiv \bar{g}_W (\bar{g}_i + \bar{g}_j) - \bar{g}_i \bar{g}_j$. The $X_i$ factors are defined in Eq. 104 and $Y$ is defined in Eq. 105.

(i) $\bar{g}_u = \bar{g}_d = \bar{g}_\ell$ (Type-I–like)
   
   (a) $\bar{g}_W = \bar{g}_f$: SM+$nS$; 2HDM-I when $\langle \Phi_0 \rangle = 0$.
   (b) $\bar{g}_W \neq \bar{g}_f$: 2HDM-I; 2HDM-I+$nS$; 2HDM-I+$mD$; 2HDM-I+$nS+mD$.

(ii) $\bar{g}_d = \bar{g}_\ell \neq \bar{g}_u$ (Type-II–like)
   
   (a) $P_{ud} = P_{u\ell} = 1$: 2HDM-II.
   (b) $0 \leq P_{ud} = P_{u\ell} \leq 1$: 2HDM-II+$nS$; 2HDM-II+$mD$; 2HDM-II+$nS+mD$.
   (c) $P_{ud} = P_{u\ell} > 1$ or $P_{ud} = P_{u\ell} < 0$: 2HDM-II+$mD$; 2HDM-II+$mD+nS$.

(iii) $\bar{g}_u = \bar{g}_\ell \neq \bar{g}_d$ (flipped 2HDM–like)
   
   (a) $P_{ud} = P_{\ell d} = 1$: flipped 2HDM.
(b) \( 0 \leq P_{ud} = P_{\ell d} \leq 1 \): flipped 2HDM+nS; flipped 2HDM+mD; flipped 2HDM+nS+mD.

(c) \( P_{ud} = P_{\ell d} > 1 \) or \( P_{ud} = P_{\ell d} < 0 \): flipped 2HDM+mD; flipped 2HDM+mD+nS.

(iv) \( \bar{g}_u = \bar{g}_d \neq \bar{g}_\ell \) (lepton-specific 2HDM–like)

(a) \( P_{u\ell} = P_{d\ell} = 1 \): lepton-specific 2HDM.

(b) \( 0 \leq P_{u\ell} = P_{d\ell} \leq 1 \): lepton-specific 2HDM+nS; lepton-specific 2HDM+mD; lepton-specific 2HDM+nS+mD.

(c) \( P_{u\ell} = P_{d\ell} > 1 \) or \( P_{u\ell} = P_{d\ell} < 0 \): lepton-specific 2HDM+mD; lepton-specific 2HDM+mD+nS.

(v) \( \bar{g}_u \neq \bar{g}_d \neq \bar{g}_\ell \)

(a) \( P_{u\ell} = 1 \): MSSM with \( \Delta_b \).

(b) \( P_{u\ell} \neq 1 \)

i. \( 0 \leq X_i \leq 1 \): 3HDM-D; 3HDM-D+nS; 3HDM-D+mD; 3HDM-D+nS+mD.

ii. One of \( X_i < 0 \) and \( 0 \leq Y \leq 1 \): 3HDM-D+nS; 3HDM-D+mD; 3HDM-D+nS+mD.

iii. One of \( X_i < 0 \) and \( Y < 0 \) or \( Y > 1 \): 3HDM-D+mD; 3HDM-D+mD+nS.

In particular, we count 15 models (or sets of models) that are distinguishable in principle based on the couplings of \( h \). Explicit formulae for \( h \) partial widths (equivalently couplings squared) in the decoupling limit, i.e., for small deviations from the SM predictions, are collected in Table I.

We now give a first comparison of our theoretical results to anticipated LHC and ILC measurements of Higgs couplings-squared. In Fig. 4 we illustrate the behavior of the partial widths (equivalently couplings squared) of \( h \) to \( W \) or \( Z \) boson pairs, down-type quarks, up-type quarks, and charged leptons, normalized to their SM values, as a function of the decoupling parameter \( \delta \). Because we consider the full range \(-1 < \delta < 1\), we use the exact formulae from the text rather than the decoupling limit approximations of Table I. We show results for the SM plus a singlet (Eq. 25), the Type-I 2HDM (Eqs. 34 and 35), the Type-II 2HDM (Eq. 58), the flipped and lepton-specific 2HDMs, and the democratic 3HDM. In all
TABLE I: Behavior of the Higgs partial widths (equivalently couplings squared) near the decoupling limit, $|\delta| \ll 1$. For the 2HDM-II+S we also require $\epsilon^2 \ll 1$. The other parameters are defined as $t_\beta \equiv \tan \beta = v_f/v_0$ in the 2HDM-I, $v_u/v_d$ in the 2HDM-II, flipped 2HDM, and 3HDM-D, and $v_q/v_\ell$ in the lepton-specific 2HDM. For the MSSM we define $t'_\beta \equiv \tan \beta' \equiv v_u(1 - \cot^2 \beta \Delta_b)/v_d(1 + \Delta_b)$.

For the 2HDM-II+D and 3HDM-D we also define $c_\Omega \equiv \cos \Omega = \sqrt{v_u^2 + v_d^2}/v_{SM}$ and $\gamma$ is the remaining mixing angle that parameterizes the state $h$.

For models except the SM+S we set $\tan \beta = 5$; for the democratic 3HDM we also set $\sin \Omega = 0.2$ (corresponding to $v_\ell = 50$ GeV) and $\cos \gamma = 0.5$.

On the right-hand side of each plot in Fig. 4 we also show the expected 1$\sigma$ measurement uncertainties of the squared Higgs couplings at the LHC from Refs. [11, 87] (summarized in Table II), assuming SM coupling strengths and a Higgs mass of 120 GeV. The coupling fit of Refs. [11, 87] was based on anticipated Higgs production and decay rate measurements from the LHC using 300 fb$^{-1}$ of integrated luminosity times two detectors. Vector boson fusion channels (which have only been studied for 30 fb$^{-1}$ to date) are scaled to 100 fb$^{-1}$ to account for potential degradation at high luminosity running. The fit assumes SM rates in all channels and allows for an unobserved component of the Higgs total width as well as nonstandard contributions to the $ggh$ and $h\gamma\gamma$ vertices. It further assumes $g_W^2 = g_Z^2 \leq 1.05$, which is valid for models containing only doublets and/or singlets. Theoretical uncertainties

| Model                  | $\Gamma^h_W/\Gamma^SM_W$ | $\Gamma^h_d/\Gamma^SM_d$ | $\Gamma^h_u/\Gamma^SM_u$ | $\Gamma^h_\ell/\Gamma^SM_\ell$ |
|------------------------|---------------------------|---------------------------|---------------------------|---------------------------------|
| SM                     | 1                         | 1                         | 1                         | 1                               |
| SM+S                   | $1 - \delta^2$            | $1 - \delta^2$            | $1 - \delta^2$            | $1 - \delta^2$                  |
| 2HDM-I                 | $1 - \delta^2$            | $1 + 2\delta/t_\beta$    | $1 + 2\delta/t_\beta$    | $1 + 2\delta/t_\beta$           |
| 2HDM-II                | $1 - \delta^2$            | $1 - 2t_\beta\delta$     | $1 + 2\delta/t_\beta$    | $1 - 2t_\beta\delta$            |
| 2HDM-II+S              | $1 - \delta^2 - \epsilon^2$ | $1 - 2t_\beta\delta - \epsilon^2$ | $1 + 2\delta/t_\beta - \epsilon^2$ | $1 - 2t_\beta\delta - \epsilon^2$ |
| 2HDM-II+D              | $1 - \delta^2$            | $1 - 2\delta(s_\gamma t_\beta/c_\Omega + c_\gamma t_\Omega)$ | $1 + 2\delta(s_\gamma/c_\Omega t_\beta - c_\gamma t_\Omega)$ | $1 - 2\delta/s_\gamma/c_\Omega t_\beta - c_\gamma t_\Omega$ |
| Flipped 2HDM           | $1 - \delta^2$            | $1 - 2t_\beta\delta$     | $1 + 2\delta/t_\beta$    | $1 + 2\delta/t_\beta$           |
| Lepton-specific 2HDM   | $1 - \delta^2$            | $1 + 2\delta/t_\beta$    | $1 + 2\delta/t_\beta$    | $1 - 2t_\beta\delta$            |
| MSSM                   | $1 - \delta^2$            | $1 - 2t'_\beta\delta$    | $1 + 2\delta/t_\beta$    | $1 - 2t_\beta\delta$            |
| 3HDM-D                 | $1 - \delta^2$            | $1 - 2\delta(s_\gamma t_\beta/c_\Omega + c_\gamma t_\Omega)$ | $1 + 2\delta(s_\gamma/c_\Omega t_\beta - c_\gamma t_\Omega)$ | $1 + 2\delta c_\gamma/t_\Omega$ |

models except the SM+S we set $\tan \beta = 5$; for the democratic 3HDM we also set $\sin \Omega = 0.2$ (corresponding to $v_\ell = 50$ GeV) and $\cos \gamma = 0.5$. On the right-hand side of each plot in Fig. 4 we also show the expected 1$\sigma$ measurement uncertainties of the squared Higgs couplings at the LHC from Refs. [11, 87] (summarized in Table II), assuming SM coupling strengths and a Higgs mass of 120 GeV. The coupling fit of Refs. [11, 87] was based on anticipated Higgs production and decay rate measurements from the LHC using 300 fb$^{-1}$ of integrated luminosity times two detectors. Vector boson fusion channels (which have only been studied for 30 fb$^{-1}$ to date) are scaled to 100 fb$^{-1}$ to account for potential degradation at high luminosity running. The fit assumes SM rates in all channels and allows for an unobserved component of the Higgs total width as well as nonstandard contributions to the $ggh$ and $h\gamma\gamma$ vertices. It further assumes $g_W^2 = g_Z^2 \leq 1.05$, which is valid for models containing only doublets and/or singlets. Theoretical uncertainties
We set $\tan \beta = 5$ for all models except the SM+S; for the 3HDM-D we also set $\sin \Omega = 0.2$ and $\cos \gamma = 0.5$. At the right of each panel we show the expected $1\sigma$ LHC measurement uncertainties for the Higgs couplings-squared to $WW$ (blue), $bb$ (red), $tt$ (green), and $\tau \tau$ (orange) for $m_h = 120$ GeV and SM event rates, taken from Table II. Note the log scale on the $y$ axis.
TABLE II: Expected uncertainties on Higgs coupling-squared measurements at the LHC and ILC, assuming $m_h = 120$ GeV and SM rates for all processes involved. See text for details.

|       | $g_{W}^2$ | $g_{b}^2$ | $g_{t}^2$ | $g_{\tau}^2$ |
|-------|---------|---------|---------|---------|
| LHC [11] | 22%     | 43%     | 32%     | 27%     |
| ILC [16]  | 2.4%    | 4.4%    | 6.0%    | 6.6%    |

on Higgs production rates due to QCD scale uncertainty were also included.

The results of Refs. [11, 87] will likely change when updated experimental and theoretical results are included. Updates of all experimental channels are now available in the CMS Physics TDR [88] and the ATLAS Computing System Commissioning (CSC) Notes [89]. In particular, the critical $tth, h \rightarrow b\bar{b}$ channel is dead; more work is needed on the experimental side to evaluate the potential of newly-proposed $bb$ final state channels like $Wh, h \rightarrow b\bar{b}$ [90]. Progress has also been made on the higher-order corrections to the $gg \rightarrow h$ cross section [91].

Nevertheless, we sketch the current situation as follows. We scan over model parameters and compute a $\chi^2$ relative to the SM prediction according to

$$\chi^2 = \sum_{i=W,b,t,\tau} \frac{(\Gamma_i - \Gamma_{SM}^i)^2}{[\delta \Gamma_{SM}^i]^2},$$

using the LHC uncertainties in the partial widths from Refs. [11, 87] as summarized in Table II. We make no attempt here to account for the correlations in the extracted couplings. One-, two- and three-sigma contours are shown in Fig. 5 for the LHC. For comparison, we show the corresponding ILC expectations in Fig. 6.

The SM plus a singlet contains only one additional parameter that universally shifts the partial widths to all SM decay modes, while the 2HDM models listed contain $\delta$ and $\tan \beta$ as free parameters that describe the Higgs coupling. Since the 3HDM-D model has four free parameters, $\delta$, $\tan \beta$, $\Omega$, and $\gamma$, we marginalize over $\Omega$ and $\gamma$ by evaluating a Markov Chain Monte Carlo (MCMC) [92, 93, 94] following the procedure of Ref. [94].

In summary, we have provided a first roadmap to determine the underlying model of electroweak symmetry breaking under the assumption that only Higgs doublets and/or singlets participate and the Glashow-Weinberg-Paschos condition for natural flavor conservation holds. Our approach is based on the couplings of a single identified CP-even Higgs
FIG. 5: Regions of parameter space with combined $h$ couplings within one, two and three $\sigma$ corresponding to the inner, middle and outer contours, respectively, of the SM limit for various models, based on the expected LHC sensitivities given in Table II. Values of $\chi^2$ are calculated according to Eq. 111. The 3HDM-D model contains four free parameters, $\delta$, $\tan \beta$, $\Omega$, and $\gamma$; we marginalize over $\Omega$ and $\gamma$ by evaluating a Markov Chain Monte Carlo (MCMC). The minor wiggles in the shapes of the contours is due to the numerical precision of the MCMC.
FIG. 6: Same as Fig. 5 but for the ILC, using the precisions on couplings squared given in Table II.

state without regard to other Higgs particles that may appear in the spectrum. We restrict our considerations to tree-level decays of the Higgs boson to avoid complications from new physics that may appear in loop-mediated decays. We described 15 classes of models and
compared their predictions for the shifts in the Higgs couplings relative to the SM. In each case, we presented formulae for the couplings of a single CP-even state $h$ to $W$ or $Z$ boson pairs, up-type quarks, down-type quarks, and charged leptons as a function of the model parameters at tree level. Where possible, we also inverted those relations to provide explicit formulae for the model parameters in terms of the $h$ couplings. We summarized our results in a decision tree that can be used to differentiate among the models. While extraction of the couplings of $h$ with sufficient precision at the LHC will be challenging, our results provide a starting point for a more detailed study of model discrimination based on future experimental results.

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