Prolonged Decoupling

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ABSTRACT

We discuss decay of unstable particles and pair annihilation of stable heavy particles that occur in the cosmic medium, from the point of the fundamental microscopic theory. A fully quantum mechanical treatment shows that the effect of thermal environment on these processes cannot be described in terms of quantities on the mass shell alone, thus requiring an extension of the Boltzmann-like equation. The off-shell effect tends to prolong physical processes that take place subsequent to the decay.

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1. Introduction

Any important event in cosmology is a series of elementary processes that occur in the expanding universe. Some of these events can be understood in terms of a rather simple process or their combination. One should however keep in mind that the process does not occur in isolation, unlike a well-prepared experiment in laboratory. In the past the effect of environment on elementary processes has been dealt with in very simplified manners in practical computation in cosmology, and in many cases has been completely ignored.

We address this problem here and would like to fully incorporate the quantum mechanical principles. A subtle quantum mechanical effect might require a change of the cherished practice in this field, but we would like to think of the problem, going back to the basic point. It turns out that the familiar Boltzmann equation must be reexamined due to its neglect of the off-shell effect.[1]

Although no specific application is discussed in any detail here, it is presumably helpful to mention at the outset what I have in mind for applications. They are X-boson decay for baryogenesis, and WIMP or LSP annihilation for dark matter. In all these cases it is important that decay or annihilation products make up a part of the cosmic environment. Thus, when heavy particles are about to decay or decouple, inverse processes must be considered simultaneously. Interaction with thermal environment becomes the major concern for this problem.

A usual computational practice of the rate calculation for a particular process goes something like this; one first computes the elementary S-matrix element, $\langle f | S | i \rangle$, between the initial state $i$ and the final state $f$ on the mass shell, and then averages the probability in a thermal bath, $\langle (f | S | i) \rangle^2$. This rate is used in the Boltzmann equation in the expanding universe.

I can think of two basic problems in this approach; first, the states chosen $|i\rangle$ and $|f\rangle$ may not be necessarily on the mass shell, as will be explained more fully below, and secondly, even the on-shell matrix element

$$\langle f | T \exp \left( -i \int dt H_{\text{int}}(t) \right) | i \rangle$$

may contain propagation effects in thermal environment, different from those in vacuum.

I shall first repeat the argument already made on the first point.[2] For this it is
useful to look at the thermal Green function,

\[ G_{\text{th}}(\omega) = \Delta(\omega + i\epsilon(\omega)) - i\frac{\rho(|\omega|)}{e^{\beta|\omega|} - 1}, \]  

with \( T = 1/\beta \) the temperature and the spectral function is given by

\[ \rho(\omega) = \frac{2\Im\Pi(\omega)}{(\omega^2 - \omega_k^2 - \Re\Pi(\omega))^2 + (\Im\Pi(\omega))^2}, \]

with \( \omega_k = \sqrt{k^2 + M^2} \). Here \( \Delta \) is the Green function in vacuum. The proper self-energy \( \Pi(\omega) \) is in general a complicated object, and only in the weak coupling limit one may use the Breit-Wigner function for \( \rho(\omega) \), leading to the familiar form,

\[ \rho_{\text{BW}}(\omega) = \frac{4\omega_k \Gamma}{(\omega^2 - \omega_k^2)^2 + (2\omega_k \Gamma)^2} \rightarrow G_{\text{th}}(\omega) \approx \Delta - \frac{2\pi i \delta(\omega^2 - \omega_k^2)}{e^{\beta\omega_k} - 1}. \]

If one further assumes that the relevant \( \omega \) integral fully includes the peak region \( \approx \omega_k \) of width \( \Gamma \), then the Boltzmann equation follows.

The off-shell effect appears in two ways; first, in the effective cutoff of the \( \omega \) region, \( \omega > \omega_c \) (the threshold and taken to be \( < \omega_k \) ) for \( \omega \) integral. If the temperature is below the threshold, \( T < \omega_c \), the dominant region that contributes, \( \omega < T \), misses the peak region. Second, if the coupling is not very weak, one must use the correct form and not its Breit-Wigner approximation of the spectral function \( \rho(\omega) \). For instance, the behavior of \( \Im\Pi(\omega) \) near the threshold \( \omega_c \) may become important at low temperatures. A systematic treatment of the off-shell effect overcoming these points is called for.

Let us make clear the fundamental difference between the on-shell and the off-shell contributions. In elementary quantum mechanics the off-shellness appears in virtual states in perturbation formulas. It deals with the short-time behavior and thus represents any transition that does not conserve energy. If one considers a decay of unstable system of mass \( M \) and decay width \( \Gamma \), then the off-shell effect is essential at times of \( t \approx 1/M \) while the on-shell S-matrix element well describes the long-time behavior of \( t \geq 1/\Gamma \) (the lifetime). As is well known, the on-shell behavior is given by the S-matrix, while the off-shell effect must be discussed in terms of the more fundamental quantity, the Green function.
2. Model of unstable particle decay

We consider here the case in which one can clearly separate the small system in question and the cosmic environment. This means that one should be able to ignore mutual interaction among unstable particles and also to ignore the change of the environment due to the interaction with the unstable particle. Thus, all the nonlinear effect and the back reaction is ignored.

Within this linear approximation one can think of a harmonic environment of arbitrary spectral distribution as well as a bilinear interaction between the small system and the environment. We then take the following model Hamiltonian,

\[ H = H_c + H_b + H_{\text{int}} = \omega_k c^\dagger c + \int_{\omega_c}^{\infty} d\omega \omega b^\dagger(\omega)b(\omega) + \int_{\omega_c}^{\infty} d\omega \sqrt{\sigma(\omega)} \left( b^\dagger(\omega)c + c^\dagger b(\omega) \right). \] (4)

Here \( c^\dagger, c \) are creation and annihilation operators of the unstable particle of energy \( \omega_k = \sqrt{k^2 + M^2} \), and \( b^\dagger(\omega), b(\omega) \) are those of the environment state, usually taken to be two-body states of the decay product. The threshold \( \omega_c = \sqrt{k^2 + (m_1 + m_2)^2} \) and \( m_i \) is the masses of decay product. The instability condition is imposed; \( \omega_c < \omega_k \).

Although for definiteness we consider the unstable particle decay, one may choose with some generalization a pair of heavy stable particles for an initial state.

It should be reminded of that even in this linear model the irreversibility exists; it can occur because an unstable particle couples to an infinitely many degenerate environment states. A great virtue of this model is that it can be analytically solved \[3\] and in principle one has many explicit results that can test approximation schemes one always needs in practice. We shall discuss extension of this model later.

A way to make integrability explicit is to present the operator solution. One can find variables that diagonalize the Hamiltonian;

\[ B^\dagger(\omega) = b^\dagger(\omega) + F(\omega + i0^+) \left( -\sqrt{\sigma(\omega)} c^\dagger + \int_{\omega_c}^{\infty} d\omega' \frac{\sqrt{\sigma(\omega)\sigma(\omega')}}{\omega' - \omega - i0^+} b^\dagger(\omega') \right), \] (5)

where

\[ H = \int_{\omega_c}^{\infty} d\omega \omega B^\dagger(\omega)B(\omega). \]

The transformation from \( c,b(\omega) \) to \( B(\omega) \) (and their conjugates) is canonical. Even the operator inversion can be written down. The important function \( F(z) \) is analytic
except along the cut, \( z > \omega_c \), and explicitly
\[
F(z) = \frac{1}{-z + \omega_k - \int_{\omega_c}^{\infty} d\omega \frac{\sigma(\omega)}{\omega - z}}. \tag{6}
\]

We do not explain here the frequency and the wave function renormalization, for which we refer to our paper.\[^3\]

The essential reason why this model is integrable is saturation of unitarity by the elastic one. Analogy to the scattering problem is useful and is indeed made very evident by considering the energy eigenstate,
\[
|\omega\rangle_S = B^\dagger(\omega)|0\rangle,
\]
where \(|0\rangle\) is the perturbative (and also the true) vacuum with \( c|0\rangle = 0 \). Clearly, this state \(|\omega\rangle_S\) is a linear superposition of \( c^\dagger|0\rangle \) and \( b^\dagger(\omega)|0\rangle \). When \( \omega_k > \omega_c \), the original one-particle state \( c^\dagger|0\rangle \) becomes a resonance, or the unstable particle, which is \(|\omega\rangle_S\). The overlap probability
\[
|\langle 0|c|\omega\rangle_S|^2 \equiv H(\omega)
\]
is very fundamental. It is related to the discontinuity along the cut,
\[
F(\omega + i0^+) - F(\omega - i0^+) = 2\pi i \sigma(\omega)|F(\omega + i0^+)|^2 = 2\pi i H(\omega). \tag{7}
\]

In terms of the spectral function
\[
H(\omega) = \frac{\sigma(\omega)}{(\omega - \omega_k + \Re\Pi(\omega))^2 + (\pi \sigma(\omega))^2}, \tag{8}
\]
where
\[
\Pi(z) = \int_{\omega_c}^{\infty} d\omega \frac{\sigma(\omega)}{\omega - z}.
\]
From the discontinuity structure it follows that a simple pole exists in the second Riemann sheet extended via this discontinuity formula; the original state pole on the real axis at \( \omega = \omega_k \) moves into the second sheet and acquires an imaginary part, \( \approx \Gamma = 2\pi \sigma(\epsilon_r) \) where \( \epsilon_r \) is the real part of the pole (in the weak coupling limit, \( \epsilon_r \approx \omega_k \) after renormalization). This pole describes the exponential decay law; the non-decay amplitude \( \approx e^{-i\epsilon_r t - \Gamma t/2} \).

It can also be shown \[^3\] that this quantity \( H(\omega) \) is directly related to the Lehmann spectral function \( \rho_L(\omega) \) by
\[
H(\omega) = \frac{(\omega + M)^2}{2M} \rho_L(\omega).
\]
The decay law of the unstable particle state $|1\rangle = c^\dagger |0\rangle$ in vacuum is precisely described as follows. One notes that the non-decay amplitude,

$$\langle 1|e^{-iHt}|1\rangle = \langle 0|c e^{-iHt} c^\dagger |0\rangle,$$

is calculable if one finds the Heisenberg evolution, $c(t) = e^{iHt} c e^{-iHt}$. Since the operator relation is known, this is given by

$$c(t) = -\int_{\omega_c}^\infty d\omega \sqrt{\sigma(\omega)} F(\omega + i0^+) e^{-i\omega t} B(\omega).$$

(9)

It is useful to rewrite the Heisenberg operator in terms of the original variables;

$$c^\dagger(t) = g(t) c^\dagger + i \int_{\omega_c}^\infty d\omega \sqrt{\sigma(\omega)} h(\omega, t) e^{i\omega t} b^\dagger(\omega).$$

(10)

The basic function here is $g(t)$;

$$g(t) = \int_{\omega_c}^\infty d\omega H(\omega)e^{i\omega t} = \frac{1}{2\pi i} \int_{C_0+C_1} dz F(z) e^{izt},$$

(11)

$$h(\omega, t) = i F^*(\omega + i0^+) - ik(\omega, t), \quad k(\omega, t) = i e^{-i\omega t} g(t).$$

(12)

The boundary condition for $k(\omega, t)$ is $k(\omega, t) \to 0$ as $t \to \infty$, hence

$$k(\omega, t) = \frac{1}{2\pi i} \int_{C_0+C_1} dz \frac{e^{i(z-\omega)t}}{z-\omega} F(z).$$

(13)

The non-decay amplitude is given by

$$\langle 1|e^{-iHt}|1\rangle = g^*(t) = \int_{\omega_c}^\infty d\omega \sigma(\omega) |F(\omega + i0^+)|^2 e^{-i\omega t}.$$  

(14)

The second form of the $\omega$ integral for $g(t)$ along $C_0 + C_1$ makes separation of the pole term explicit; the contour $C_0$ encircles the pole (conjugate to the one already discussed, due to $e^{izt}$ factor instead of $e^{-izt}$) in the second sheet, while $C_1$ consists of two lines parallel to the imaginary axis, one in the first and the other in the second sheet. It is clear that the $C_1$ integral gives the non-exponential decay law which is important both at early and late times of the decay. For instance, at very late times the important part of $\omega$ integral comes from the threshold region, and if one parametrizes this region using the rate $\Gamma$,

$$\sigma(\omega) \approx \frac{\Gamma}{2\pi} \left( \frac{\omega - \omega_c}{Q} \right)^\alpha,$$

(15)

the non-decay amplitude is given by

$$\langle 1|e^{-iHt}|1\rangle \approx -i \frac{\Gamma}{2\pi Q} \frac{\Gamma(\alpha + 1)}{(Qt)^{\alpha+1}} e^{-i\omega_{c,t} - i\pi\alpha/2},$$

(16)
with $Q = \omega_k - \omega_c$. The decay at late times thus follows the power law;

$$\approx \left( \frac{\Gamma}{2\pi Q} \right)^2 (Qt)^{-2(\alpha+1)}.$$ 

3. Decay in cosmic medium

Amusingly, this model and the basic technique outlined is taken over to derive the decay law in thermal medium.[1], [3] In the thermal medium one is interested in time evolution of the occupation number, $f(t)$, to be given by the thermal average over a given mixed state,

$$f(t) \equiv \langle c^\dagger(t)c(t) \rangle_i = \text{tr} \left( c^\dagger(t)c(t) \rho_i \right),$$

where $\rho_i$ is the density matrix of the mixed state.

We take for $\rho_i$ the thermal density matrix,

$$\rho_i = e^{-\beta(H_b+H_c)}/(\text{tr} e^{-\beta(H_b+H_c)}),$$

where $H_b$ and $H_c$ are the environment and the subsystem parts of the Hamiltonian. It is perhaps necessary to explain why we take for the initial mixed state this one and not the one like $e^{-\beta H}$ using the total Hamiltonian $H$. The cosmic environment we consider is not a stationary state; it adiabatically changes with the cosmic expansion. With the momentum redshift a time lag occurs during the decay between the unstable particle and its lighter decay products. Light decay products have their own stronger interaction not written in the Hamiltonian above, and they tend to keep in thermal equilibrium with the well-known temperature variation, $T(t)a(t) = \text{constant}$, with $a(t)$ the cosmic scale factor. On the other hand, the decay and its inverse process cannot keep pace with the cosmic expansion. We model this circumstance by postulating that the environment by itself is always in thermal equilibrium, while the unstable particle starts to lose its thermal correlation with the environment. This is the reason we take the initial density matrix uncorrelated between the unstable and the decay product particles, although at much higher temperatures they may be in thermal contact. Thus, we take for the initial state average

$$\langle c^\dagger c \rangle = f(0), \quad \langle c^\dagger b(\omega) \rangle_i = 0, \quad \langle b^\dagger(\omega)b(\omega') \rangle_i = \delta(\omega - \omega') \frac{1}{e^{\beta\omega} - 1}. \quad (18)$$
For the moment we ignore the effect of the cosmic expansion. Time evolution of the occupation number \( f(t) \) can be derived using the explicit form of \( c^\dagger(t) \), eq.(10):

\[
f(t) = |g(t)|^2 f(0) + \int_{\omega_c}^{\infty} d\omega \frac{\sigma(\omega)|h(\omega,t)|^2}{e^{\beta\omega} - 1}.
\]

(19)

Its time derivative can be written in the following form,

\[
\frac{df}{dt} = -\Gamma(t) |g(t)|^2 f(0) + \int_{\omega_c}^{\infty} d\omega \sigma(\omega) \frac{2\Re(h^*e^{-i\omega t})}{e^{\beta\omega} - 1},
\]

(20)

where the time dependent decay rate is defined by

\[
\Gamma(t) = -2\Re\frac{\dot{g}}{g} = -\frac{d}{dt} \ln |g|^2.
\]

(21)

The rate \( \Gamma(t) \) is time independent during the pole dominant epoch, but at late times it decreases as \( \Gamma(t) \to \frac{2(\alpha+1)}{t} \), using the spectral function having the threshold behavior [15]. An equivalent form for the time evolution equation is obtained by eliminating the initial value \( f(0) \) dependence;

\[
\frac{df}{dt} + \Gamma(t)f = \int_{\omega_c}^{\infty} d\omega \sigma(\omega) \frac{2\Re(h^*e^{-i\omega t}) + \Gamma(t)|h|^2}{e^{\beta\omega} - 1}.
\]

(22)

This form is more convenient to application to cosmology.

The first form of time evolution equation (20) has a clear interpretation if one retains only the on-shell contribution; by using the pole dominance for \( g(t) = e^{i\omega_k t - \Gamma t/2} \), the equation is reduced to

\[
\frac{df}{dt} = -\Gamma e^{-\Gamma t} \left( f(0) - \frac{1}{e^{\beta\omega_k} - 1} \right).
\]

(23)

There are oscillatory terms such as \( e^{-\Gamma t/2} \times (\cos \text{ or } \sin(\omega - \omega_k)t) \), which averages out in the \( \omega \) integral, leading to this result. Thus, if the initial occupation \( f(0) \) is chosen to be that of the thermal one, then there is no change, while its deviation from the equilibrium value is relaxed back by the rate \( \Gamma \).

The second form of evolution equation (22) suggests that irrespective of the pole dominance the occupation number has an asymptote;

\[
f_\infty = \int_{\omega_c}^{\infty} d\omega \frac{H(\omega)}{e^{\beta\omega} - 1} \approx \int_{\omega_c}^{\infty} d\omega \frac{\sigma(\omega)}{(\omega - \omega_k)^2 + (\pi\sigma(\omega))^2} \frac{1}{e^{\beta\omega} - 1},
\]

(24)

since both of \( g(t) \) and \( k(t) \) in \( h(\omega,t) \) asymptotically vanish and

\[
h(\omega,t) \to iF^*(\omega + i0^+).
\]
This asymptotic form reveals the essence of our result; only when the full Breit-Wigner region for $H(\omega)$ contributes, the on-shell result gives a good description for the asymptotic occupation number. Otherwise, $f_\infty \neq \frac{1}{e^{\beta \omega_k} - 1}$ (on-shell result).

The particle number density is calculated by summing the occupation number over independent momenta;

$$n(t) = \frac{1}{2\pi^2} \int_0^\infty dk \, k^2 \, f(t,k).$$

Let us work out its asymptotic expression $n_\infty$, using $f_\infty$ for $f(t,k)$. When the pole term dominates, this gives a well-known result; for the non-relativistic region,

$$n_\infty \approx \left(\frac{\alpha + 2}{\alpha + 1}\right)^{\frac{3}{2}} e^{-\Gamma M/T}.$$

On the other hand, at low temperatures the threshold region dominates in the $\omega$ integral, and one has a power of temperature;

$$n_\infty \approx A(\alpha) \frac{\Gamma}{M} \right(\frac{T}{M}\left)^{\alpha + 1} \frac{1}{T^3}, \quad A(\alpha) = \frac{\zeta(\alpha + 4)\Gamma(\alpha + 4)\Gamma(\frac{\alpha}{2} + 1)}{16\pi^2 \sqrt{\pi} \Gamma(\frac{\alpha}{2} + \frac{3}{2})}.$$

Thus, the low temperature behavior is much enhanced compared to that given by the on-shell contribution alone.

The effect of the cosmic expansion is readily incorporated for the evolution of the number density. The easiest way is to write it for the dimensionless yield $Y \equiv \frac{n}{T^3}$, using the dimensionless time variable, $\tau \equiv \Gamma t$;

$$\frac{dY}{d\tau} = - \int_0^\infty dk \frac{k^2}{2\pi T^3} \gamma(t,k) \left( f_k(t) - \int_k^\infty d\omega \frac{\sigma(\omega,k)}{(\omega - \omega_k)^2 + (\Gamma/2)^2} \right),$$

$$Y = \frac{1}{T^3} \int_0^\infty dk \frac{k^2}{2\pi^3} f_k(t),$$

$$\gamma(t,k) = -2 \Re \frac{d}{dt} \ln(g_0(t,k) + g_1(t,k)) = -\left(\frac{d}{dt} \ln|g_0(t,k) + g_1(t,k)|^2\right).$$

The momentum dependent spectral is $\sigma(\omega, \vec{k}) = \frac{M}{\omega_k} \sigma(\sqrt{\omega^2 - \vec{k}^2})$, where $\sigma(\omega)$ is the one in the rest frame. The usual time-temperature relation $T \propto 1/\sqrt{t}$ should be used in this equation.

The asymptotic solution for this equation is readily found, ignoring the time dilatation effect ($M/\omega_k \to 1$);

$$Y \to \frac{(\alpha + 2)\zeta(\alpha + 4)\Gamma(\alpha + 3)\Gamma(\frac{\alpha}{2} + 1)}{8\pi^2 \sqrt{\pi} \Gamma(\frac{\alpha}{2} + \frac{5}{2})} \frac{\Gamma}{M} \left(\frac{T}{M}\right)^{\alpha + 1}. \quad (30)$$
The important point is that it has a power behavior of the cosmic temperature, as is guessed from \( n_\infty \) of eq. (26). The power \( \alpha + 1 \) is related to the threshold behavior of the spectral function \( \sigma(\omega) \).

I refer to our recent paper [3] on the detailed time evolution, but roughly \( Y \) follows initially the thermal behavior, \( (\frac{M}{2\pi T})^{3/2} e^{-M/T} \), and after a short transient epoch it approaches the asymptotic form above. The transition epoch is characterized by two temperature scales, \( T_* \) and \( T_{\text{eq}} \) given by

\[
\frac{T_*}{M} \approx \sqrt{\frac{\eta}{(\alpha + 4) \ln \frac{M}{T}}}, \quad \eta = \sqrt{\frac{45}{16\pi^3 N}} \frac{m_{\text{pl}} \Gamma}{M^2},
\]

\[
\frac{T_{\text{eq}}}{M} \approx \left( \ln \frac{M}{\Gamma} \right)^{-1},
\]

where \( N \) is the number of massless particle species contributing to the energy density. The quantity \( \eta \) is the decay rate relative the Hubble rate calculated at \( T = M \). Usually, \( T_* > T_{\text{eq}} \). The higher temperature \( T_* \) is given by the time when the exponential pole contribution is changed to the off-shell power contribution, while \( T_{\text{eq}} \) is the one when the two temperature dependent terms in the stationary number density \( n_\infty \) compete equally;

\[
(\frac{M}{2\pi T_{\text{eq}}})^{3/2} e^{-M/T_{\text{eq}}} \approx \frac{\Gamma}{M} (\frac{T_{\text{eq}}}{M})^{\alpha+1}.
\]

Importance of the off-shell effect is determined by the remnant density at \( T = T_{\text{eq}}; \) \( (n/T^3)_{T=T_{\text{eq}}} \). Besides a weak logarithmic dependence, this quantity has the factor, \( \Gamma/M \). The effect is thus more pronounced for a larger rate \( \Gamma \), or a stronger coupling.

4. Some application and extention

Physical processes that occur after this transient time are prolonged, compared to the instantaneous decay in the exponential law. One of the important effects of this sort is baryogenesis at the GUT epoch. In the scenario of a heavy \( X \) boson decay the out-of-equilibrium condition was considered to impose a rather severe constraint on particle physics and cosmology [4] This constraint arises due to that the abundance of \( X \) and \( \bar{X} \) bosons should not be too much suppressed when the asymmetry generation starts. The on-shell kinematics requires that the temperature \( T_d \) at the Hubble \( H = \Gamma \) must not be too much larger than the mass of \( X \) boson;
thus $T_d < m_X$. Otherwise, the inverse decay can take place frequently, to suppress the number density of $X$ boson. This leads to the lower bound of the $X$ boson mass, $m_X > O[1] \times \frac{\alpha_X m_{pl}}{\sqrt{N}} \approx 10^{16}$ GeV.

This constraint is however based on the on-shell kinematics; thermal quarks and leptons are energetically difficult to produce heavy $X$ bosons when the cosmic temperature $T < m_X$. But, as we discussed here, heavy $X$ bosons that were once abundant do not disappear immediately due to the off-shell effect. Thus, one can anticipate that the above lower bound of the $X$ mass is considerably relaxed.\footnote{For details of this problem, I refer to our forthcoming paper.\cite{5}}

As an extension of the model so far discussed, it is instructive to treat the case of pair annihilation model. The model can describe two particle annihilation into lighter particles,

$$c_1 c_2 \rightarrow b(\omega),$$

again $b(\omega)|0\rangle$ taken to be two-body states of light particles. Its model Hamiltonian is

$$H = \omega_1 c_1^\dagger c_1 + \omega_2 c_2^\dagger c_2 + \int_{\omega_c}^\infty d\omega \omega b^\dagger(\omega)b(\omega) + \int_{\omega_c}^\infty d\omega \sqrt{\sigma(\omega)} \left( b^\dagger(\omega)c_1 c_2 + (\text{h.c.}) \right).$$

This time the model does not allow exact solution, but one can use the method of Feynman and Vernon \footnote{\cite{6}} to integrate the effect of environment variables. We leave details of this technical part to our future paper, but let me discuss how one may derive a Boltzmann-like equation. The key for this is to neglect a long-time correlation and to take the $t \rightarrow \infty$ limit in order to erase the off-shell effect. After cutting off the long-time correlation, we find that

$$\frac{d}{dt}\langle c_i^\dagger(t) c_i(t) \rangle = \int_{\omega_c}^\infty d\omega \sigma(\omega) \int_0^t ds \left[ 2 \cos(\omega - \omega_1 - \omega_2)s \left( -f_1 f_2 (f_{th} + 1) + (f_1 + 1)(f_2 + 1)f_{th} \right) \right],$$

where $f_{th}(\omega) = 1/(e^{\beta\omega} - 1)$ and $f_i(\omega_i)$ are occupation numbers of $i$. One may then use

$$\lim_{t \rightarrow \infty} \int_0^t ds \left[ 2 \cos(\omega - \omega_1 - \omega_2)s = 2\pi \delta(\omega - \omega_1 - \omega_2), \right]$$

to get

$$\frac{d}{dt}\langle c_i^\dagger(t) c_i(t) \rangle = \Gamma \left( -f_1(\omega_1) f_2(\omega_2)(f_{th} + 1) + (f_1(\omega_1) + 1)(f_2(\omega_2) + 1)f_{th} \right),$$

(37)
with \( f_{\text{th}} = f_{\text{th}}(\omega = \omega_1 + \omega_2) \). This is the Boltzmann equation with the rate averaged in thermal medium. Our formalism can thus describe the kinetic equation for the pair annihilation in thermal medium, once the thermal distribution of two-body states \( f_{\text{th}}(\omega) \) is given.

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