Towards Initial Mass Functions for Asteroids and Kuiper Belt Objects

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Introduction: The initial creation of primitive bodies (asteroids, KBOs, etc) from freely-floating nebula particles remains problematic. Traditional growth-by-sticking models encounter a formidable "meter-size barrier" in turbulent nebulae, but nonturbulent nebulae form large asteroids too quickly to explain long spreads in formation times, or the dearth of melted asteroids [1]. Other puzzles (and clues) include (a) a wide range (at least a half Myr) in formation ages of individual chondrules lying in the same chondrite [2, and references therein], and (b) an apparent mass peak in the pre-depletion, pre-erosion mass distribution of asteroids [3-6]. We recently advanced a scenario [7] to help resolve some of these puzzles, which also naturally explains the peculiar and characteristic chondrule-dominated fabric of most primitive meteorites [8]. In this scenario, dense clumps of size-sorted particles, formed in turbulence, can under certain conditions shrink inexorably on 100-1000 orbit timescales and form sandpile planetesimals. These conditions impose a threshold size for primary bodies which is intriguingly close to the proposed dominance of the primordial asteroid belt by 100km diameter objects [6]. Primitive bodies in the outer solar system pose a similar set of puzzles [4] - KBOs show a diversity in volatile compositions and melting state which might be explained by a range in formation time [9] and also seem to show a preferred modal size in the 100km diameter range [10,11]. Here and in [4] we further explore the primary accretion "Initial Mass Function" (IMF) in both the inner and outer solar system.

Cascade model and thresholds: The outcome of turbulent concentration [8] can be captured statistically in a cascade model [7,12] which predicts probability distribution functions \(P(\Phi, S)\) for dense particle clumps. Here \(\Phi\) is the ratio of particle mass density \(\rho_p\) to gas mass density \(\rho_g\), and \(S\) is the local entrophy \(\omega^2\), with \(\omega\) a local vorticity or eddy frequency. \(P(\Phi, S)\) is a function of level \(N\) in the turbulent cascade which transfers energy from large to small scales; each level characterizes a nebula lengthscale \(l = 2^{-N/3} L\) where \(L\) is the largest eddy scale: \(L = H \alpha^{1/2}\) with \(H\) the gas vertical scale height and \(\alpha\) the nebula turbulent viscosity parameter [4,7]. Figure 1a shows cascade model predictions of \(P(\Phi, S)\) (contours), for two different levels \(N\) (or lengthscales \(l\)). Clumps can be stabilized by sufficiently large \(\Phi\) against disruption while they slowly contract into sandpile planetesimals. Thresholds are associated with self-gravity overcoming (a) local coriolis force (\(\Phi > \Phi_1\)), and (b) ram pressure disruption by the headwind \(\beta V_K\) from the nebula gas, which orbits a small amount \(\beta \ll 1\) slower than Keplerian velocity \(V_K\):

![Figure 1: Top (a): contours of occurrence probability, from our cascade model, for two different levels (different values of \(l\)) are shown in red and blue. At each level, there is a different peak value of \(P^*(\Phi, S)\) and a corresponding value of enclosed mass \(M = \pi \Phi \rho_g l^3/6\). Also, straight lines are the different thresholds for survival at the same two levels. Bottom (b): Initial Mass Functions (IMFs) determined using the approach of [4] - basically plotting \(P^*\) vs. the diameter of a solid body of mass \(M\). See http://spacescience.arc.nasa.gov/users/cuzzi/IMF.ppt for a better demonstration.](image-url)
(Φ > Φ_2), while (c) the local enstrophy can be no smaller than some global value S_{min} [4]. The thresholds Φ_1(S), Φ_2, and S_{min}, like the PDFs \( P(\Phi, S) \), vary with cascade level or nebula lengthscale (figure 1a), and depend on nebula density, turbulent intensity, and distance from the sun [4]. Figure 1a shows that each level or lengthscale \( l \) has a (different) maximum \( P(\Phi, S) = P^* \) in the region exceeding the thresholds Φ_1(S), Φ_2, and S_{min}; this value \( P^* \) is plotted in figure 1b against the diameter of an object having the mass of the associated clump \( \pi \Phi \rho_{\text{d}} l^3/6 \). The well-defined modal peak contains most of the planetesimal mass (in figure 1b, around 40km diameter) for a given set of nebula properties, but objects with other masses are being formed at the same time, in lower abundances. This approach is clearly very simplified, and only a first attempt at quantifying this scenario of primary accretion.

**Model results:** Figures 2a and 2b show IMF’s for a variety of nebula parameter choices. There is generally a well-defined modal planetesimal diameter, which ranges between tens and hundreds of km at both 2.5 and 30 AU depending on parameters. These sizes lie between different estimates of pre-runaway, pre-erodional primary body diameters by others [5,6]. A key assumption here is that the formation timescale of a planetesimal is the sedimentation or shrinkage time (100-1000 orbits); other timescale assumptions can be adopted [13] which greatly increase the mass production rates (lower the “goal lines”). This and other aspects of the model are worthy subjects for future study. All of our arguably “successful” models at both 2.5 and 30AU, including those shown, require a significant enhancement in the abundance of nebula solids relative to hydrogen, by a factor of 10 or more. We suggest that a combination of radial and vertical global-scale decoupling of solids from gas is capable of providing the needed solid enhancement [see 4].

References: [1] Cuzzi JN and Weidenschilling SJ (2006) in Meteorites and the Early Solar System II, Univ. Arizona Press. [2] Kurahashi et al G.C.A. 72, 3865-3882; [3] Bottke et al 2005, Icarus 175, 111; [4] Cuzzi, JN, RC Hogan, and WF Bottke 2010 Icarus, submitted [5] Morbidelli et al Icarus in press; [6] Weidenschilling 2009 LPSC [7] Cuzzi JN et al 2008 ApJ 687, 1432; [8] Cuzzi JN et al 2001 ApJ 546, 496; [9] McKinnon et al (2008) in “The Solar System Beyond Neptune”, Univ. of Az. Press, 213-241 [10] Bernstein GM et al 2004 AJ 128, 1364; [11] Kenyon S and Bromley B 2004 AJ 128, 1916 [12] Hogan RC and JN Cuzzi 2007, Phys Rev E 75, 056305 [13] Chambers J, submitted. This work was supported by NASA’s Planetary Geology and Geophysics program.

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**Figure 2:** Top (a): representative IMFs at 2.5AU, assuming a global solids enhancement near the midplane by 10x over cosmic abundance and different combinations of \( \alpha, \rho_{\text{d}}, \) and \( \beta \). The short vertical dashes at the top/right of each plot indicate “goal” values of \( P^* \) which, under debatable assumptions, lead to creation of a plausible amount of mass in planetesimals [4]. The proximity of the IMF peaks to their “goals” indicates how close the parameter set comes to producing \( 2M_{\oplus} \) between 2-4AU in 2Myr; blue “case a” comes the closest. Bottom (b): IMFs at 30 AU, assuming a nebula radial surface density profile \( \sigma(r) = \sigma_o(r/r_o)^{-p} \) and a solids enhancement of 10x. Here most of the cases fall a little short of the “goal” of creating \( 50M_{\oplus} \) between 40-60AU in 2Myr [4]. With a factor of only three, additional solids enhancement most cases robustly achieve their goals. A significantly lower value of \( \beta \) than commonly assumed \((\sim 10^{-3}) \) seems indicated; several, but not all, combinations of parameters are consistent with observed nebula as accretion rates for typical solar-mass protoplanetary nebulae.