An upper limit to the electric dipole moment of the neutron from lattice QCD

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A linear increase with the volume of the topological susceptibility can signal spontaneous breaking of parity $P$ and time inversion $T$, due to a nonzero vacuum expectation value of the topological charge $Q$. Such a breaking would produce a nonzero electric dipole moment of the neutron, $d_n$. An upper limit to $d_n$ is derived from numerical simulations at increasing volumes.

1. INTRODUCTION

The QCD lagrangian

$$\mathcal{L}_{\text{QCD}} = \frac{1}{2} \text{Tr} \{ G_{\mu\nu} G^{\mu\nu} \} + \sum_f \overline{\psi}_f (i \mathbf{D} - m_f) \psi_f$$

is invariant under parity $P$ and time inversion transformations $T$. Adding to $\mathcal{L}_{\text{QCD}}$ a Chern–Simons term

$$\mathcal{L}_{\text{CS}} = \theta Q(x),$$

with $Q(x)$ the topological charge density

$$Q(x) = \frac{g^2}{64\pi^2} \varepsilon_{\mu\nu\rho\sigma} \text{Tr} \{ G_{\mu\nu} G_{\rho\sigma} \}$$

does not modify the equations of motion since $Q(x)$ is a divergence, $Q(x) = \partial_\mu K_\mu(x)$,

$$K_\mu = \frac{g^2}{16\pi^2} \varepsilon_{\mu\nu\rho\sigma} A_\nu^a \left( \partial_\rho A_\sigma^b - \frac{1}{3} g f^{abc} A_\rho^a A_\sigma^c \right).$$

The lagrangian $\mathcal{L}_{\text{CS}}$ has a dynamical effect, being sensitive to the boundary conditions, and its presence in $\mathcal{L} \equiv \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{CS}}$ results in an explicit breaking of $P$ and $T$. Indeed, since $Q \sim \vec{E} \cdot \vec{H}$ it is odd under both of them. This breaking can produce a nonzero value of $d_n$, the electric dipole moment of the neutron $[1]$, $d_n \approx 8 \cdot 10^{-16} \text{e}\cdot \text{cm}$. The experimental upper limit is $[2]$, $d_n|_{\text{exp}} < 6 \cdot 10^{-26} \text{e}\cdot \text{cm}$ which implies $\theta < \sim 10^{-10}$. This is a small number and its smallness is not protected by any known symmetry.

An alternative possibility for having $d_n \neq 0$ could be a spontaneous breaking of $P$ by a nonzero value $\langle 0|Q|0 \rangle$. This possibility however would be excluded by a theorem by Vafa and Witten $[3]$. Nevertheless this theorem is based on assumptions whose validity has been questioned. We have then addressed the issue of a spontaneous breaking of $P$ and $T$ by numerical simulations of QCD on the lattice $[4]$.

2. THE VAFA–WITTEN THEOREM

The statement is that in a theory of gauge fields and fermions like QCD which is Lorentz invariant and with vector interactions, the v.e.v. of any operator $O$ odd under parity is zero. The argument goes schematically as follows: let us consider the generalized partition function

$$Z[\theta] = \int D\varphi \; e^{-i(S(\varphi) + \theta \int O d^4x)},$$

*Presented the talk.
where $\varphi$ represents the generic field variables, and its continuation to the euclidean

$$Z_{\text{Euclid}}[\theta] \equiv e^{-V E(\theta)}, \quad (6)$$

where $E(\theta)$ is the free energy as a function of the $\theta$ parameter and $V$ is the spacetime volume.

After the analytic continuation the measure is positive since the action is real and the fermion determinant is positive. The operator $O$ instead acquires an extra factor $i$ since, to be odd under $P$ and $T$, it must have an odd number of invariant tensors $\epsilon_{\mu\nu\rho\sigma}$, at least if it depends only on the gauge fields like $Q_{\mu\nu\rho\sigma}$.

If $O$ is hermitean then the term $\exp \left(-i\theta \int O d^4 x\right)$ becomes a phase factor and since the rest of the integrand is positive, the inequality

$$e^{-VE(\theta)} \leq e^{-VE(0)} \quad (8)$$

derives. Consequently we have $E(\theta) \geq E(0)$, i.e. $\theta = 0$ is a minimum.

Since for small $\theta$, $E(\theta) \approx 1 - \theta \langle 0\vert Q\vert 0 \rangle$, it implies $\langle 0\vert Q\vert 0 \rangle = 0$.

There are a number of assumptions behind this result.

- Lorentz invariance, which is certainly true for QCD at zero temperature, is lost at finite temperature. Then, the breaking of parity, $\langle 0\vert Q\vert 0 \rangle \neq 0$ becomes plausible with possible observable consequences in heavy ion collisions.

- The argument can be nonvalid if $O$ is constructed not only by use of gauge fields but also with fermionic fields.

- Analyticity in $\theta$ of $Z[\theta]$ is assumed. This hypothesis has been debated in the literature and it is in fact the one which we have tried to test on the lattice for the topological charge operator.

Suppose indeed that $E(\theta)$ has a minimum at $\theta = 0$, but a discontinuous derivative (a cusp) at this point so that

$$\frac{dE(\theta)}{d\theta} \vert_{\theta=0^\pm} = \pm \alpha, \quad (9)$$

or

$$\langle 0\vert Q\vert 0 \rangle \vert_{\theta=0^\pm} = \pm V \alpha. \quad (10)$$

Then a spontaneous breaking of $P$ and $T$ would result.

The topological susceptibility

$$\chi = \int d^4 x \, \langle 0\vert Q(x)Q(0)\vert 0 \rangle = \frac{\langle 0\vert Q^2\vert 0 \rangle}{V} \quad (11)$$

would then contain the usual contribution coming from vacuum fluctuations (which in the present paper shall be called $\chi$), and another contribution derived from the nonzero vacuum expectation $\langle 0\vert Q\vert 0 \rangle$,

$$\chi = \alpha^2 V + \chi \quad (12)$$

and $\chi$ would be infrared divergent.

The electric dipole moment of the neutron $d_n$ can be estimated as in $\mathbb{I}$. It is proportional to the parameter $\alpha$ times a typical volume $m_n^{-4}$ times a factor $(m_x/m_n)^2$ which makes it vanish in the chiral limit,

$$d_n = \frac{\alpha}{m_n} \frac{m_x^2}{m_n^4}. \quad (13)$$

An extra factor $1/m_n$ is needed to assign the correct length units. From lattice we will get an upper limit for $\alpha$ and, from Eq. $\mathbb{I}$, an upper limit for $d_n$.

3. LATTICE ANALYSIS

Lattice is an UV regulator of the theory. The topological charge $Q$ can be determined either directly for the discretized version $Q_L$ of the charge operator (the so-called field-theoretical method) or from the counting of zero modes in the fermion fields which at the present state of the art is rather demanding in computer time, especially when one has to deal with large volumes as in our case.
In the quenched theory the general relations

\[ Q_L = ZQ, \quad \chi_L \equiv \frac{\langle Q^2_L \rangle}{V} = Z^2 a^4 \chi + M \quad (15) \]

hold. \( Z \) is a multiplicative constant \([13]\) that can be determined by the expression

\[ Z = \frac{1}{n} \langle Q_L \rangle_{Q=n} \quad (16) \]

where the v.e.v. is calculated in the topological charge sector \( Q = n \) where \( n \) is a nonzero integer. The procedure of selection of the set of configurations belonging to this sector is performed by cooling. On the other hand \( M \) is an additive renormalization due to singularities that arise after the limit \( x \to 0 \) in the integrand of Eq. (11). \( M \) is an UV effect that can be singled out by calculating \( \chi_L \) in the zero charge topological sector, \( M = \chi_L|_{Q=0} \) \([15]\).

The Ginsparg–Wilson formalism allows to preserve exact chiral symmetry in the theory. In this case \( Z = 1 \) and \( M = 0 \) \([16]\). In the field-theoretical method these renormalization constants must be calculated with the help of cooling–heating methods \([14,15]\).

We have adopted the field-theoretical method. The simulations have been done on volumes 16\(^4\), 32\(^4\) and 48\(^4\) at \( \beta = 6/g^2 \) bare \( = 6.0 \), i.e. at lattice spacing \( a \approx 0.1 \) Fermi. The statistics was \( 12 \cdot 10^4 \), \( 6 \cdot 10^4 \) and \( 5 \cdot 10^4 \) independent measurements for the three volumes respectively.

We have first determined the dependence of \( Z \) and \( M \) on the lattice size \( L = V^{1/4} \). In Figures 1 and 2 we show the corresponding results. Only the additive renormalization \( M \), which is an extensive quantity, displays a clear size dependence.

Then, knowing \( Z \), \( M \) and the lattice spacing \( a \) we have extracted \( \chi \) as a function of the volume \( V = L^4 \). The result is shown in Figure 3.

No linear dependence on \( L^4 \) is found within errors and this allows to put the upper bound (within 1–\( \sigma \))

\[ \alpha \leq \left( \frac{1}{4 \text{ Fermi}} \right)^4, \quad (17) \]

or, by use of Eq. (13),

\[ d_n \leq 3.5 \cdot 10^{-21} \text{ e} \cdot \text{cm}. \quad (18) \]

As a byproduct, we found that the physical topological susceptibility at \( a \approx 0.1 \) Fermi \( \bar{\chi} \) (or \( \chi \) if \( \alpha \) vanishes) is \((173.4(\pm 0.5)(\pm 1.2)(^{+1.1}_{-0.2}) \text{ MeV})^4\) where the errors are the statistical error from our data, the one derived from the value used for \( \Lambda_L \) and an estimate of the systematic error respectively. This result agrees with previous determinations \([17]\) and with the Witten–Veneziano formula \([18]\) needed to explain the large \( \eta' \) mass.

4. CONCLUDING REMARKS

Our upper limit on \( d_n \) is 5 orders of magnitude bigger than the experimental one. It can, however, be improved by increasing the lattice size and by decreasing the statistical errors.

We are repeating the computation at nonzero temperature to check the ideas of reference \([6]\).

The calculation was done on the APEmille facility of INFN in Pisa.

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Figure 2. $M$ versus $L$ for the lattice definition of the topological susceptibility $\chi_L$ adopted in the present paper at $\beta = 6.0$. The line is the result of the fit (shown in the legend) and the grey band is the 1–σ error.

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