The research on bearing capacity of supports with annular section

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Abstract. Our research work on the physical and mechanical properties of centrifuged concrete, the strength and deformability of cylindrical and conical transmission towers made it possible to reveal some incorrect calculations according to the old and new standards. To identify discrepancies in the calculations for the supports with annular section according to the SNiP 2.03.01-84 and according to the SP 63.13330.2012, we carried out the numerical experiments on the example of the transmission tower supports. As the test sample, the cylindrical transmission tower according to the Certification System GOST 22687.2

Introduction
The reinforced concrete structures with annular section are widely used in road construction (non-pressure water pipes), energy construction (transmission towers, contact network supports), industrial construction (industrial buildings columns) etc.

The greatest contribution to the research of centrifuged concrete properties, as well as the strength and deformability of structures with annular section under bending and eccentric compression, was made by I.N. Akhverdov [1], A.P. Kudzis [2, 3], V.M. Batashev [5, 6], S.A. Dmitriev [6], T.M. Petsold [7], S.T. Androsov, T.F. Nagornaya [5].

The research works of these authors were the basis for the calculation of reinforced concrete structures with annular section according to the SNiP 2.03.01-84.

Experimental program and research results
In new standards of the SP 63.13330.2012, the methodology for calculating the elements with annular section has not undergone significant changes, however, in the basic equations for determining the relative height of the compressed zone $ξ_{cp}$ and the bearing capacity $M_{ult}$, the separate influence of prestressed reinforcement $A_{sp}$ and non-prestressed reinforcement $A_s$ is not taken into account (Figure 1)
**Figure 1.** Reinforcement scheme of an element with an annular section.

\[ r_1, r_2 - \text{outer and inner radius of the section; } \]
\[ r_{sp}, r_s - \text{radius of prestressed and non-prestressed reinforcement; } \]
\[ A_{sp}, A_s - \text{areas of prestressed and non-prestressed reinforcement, respectively.} \]

Our research work on the physical and mechanical properties of centrifuged concrete [8], the strength and deformability of cylindrical and conical transmission towers [9, 10] made it possible to reveal some incorrect calculations according to the old and new standards.

Let us examine the formulas for determining the relative height of the compressed zone of concrete \( \xi_{cir} \) of annular elements according to the SNiP 2.03.01-84 in the form (1) and according to the SP 63.13330.2012 in the form (2):

\[
\xi_{cir} = \frac{N + (\sigma_{sp} + \omega_{sp} r_{sp}) A_{sp} + \omega_s r_s A_s}{R_b A_b + (R_{scp} + \delta_{sp} \omega_{sp} r_{sp}) A_{sp} + (R_{sc} + \delta_s \omega_s r_s) A_s}
\] (1)

Where \( \delta_{sp(s)} = 1.5 + 6R_{sp(s)} 10^{-4} \)
\[
\omega_s = 1; \quad \omega_{sp} = 1.1 - \frac{\sigma_{sp}}{R_{sp}}
\]

\[
\xi_{cir} = \frac{N + R_s A_{s,tot}}{R_b A_b + (R_{sc} + 1.7 R_s) A_{s,tot}}
\] (2)

Where \( A_{s,tot} = A_{sp} + A_s \)

The comparison shows that in formula (1) the influence of prestressed reinforcement \( A_{sp} \) and non-prestressed reinforcement \( A_s \) is taken into account differentially with the corresponding influence coefficients \( \delta_{sp}, \omega_{sp} \), and in formula (2) there is no differentiated approach of taking into account the effect of prestressed and non-prestressed reinforcement.

Similarly, let us compare the formulas for determining the bearing capacity according to the old standards [11] in the form of (3) and to the new standards [12] in the form of (4):

\[
M_{ult} \leq \left( R_b A_b r_m + R_{scp} A_{sp} r_{sp} + R_{sc} A_s r_s \right) \frac{\sin(\pi \xi_{cir})}{\pi} + \left( R_{sp} A_{sp} \varphi_{sp} z_{sp} + R_s A_s \varphi_s z_s \right)
\] (3)

Where \( \varphi_{sp(s)} = \omega_{sp(s)} (1 - \delta_{sp(s)} \xi_{cir}); \)
\[
z_{sp} = (0.2 + 1.3 \xi_{cir}) r_{sp(s)}, \text{ but } 0.15 \leq \xi_{cir} \leq 0.6
\]

In the particular case, when the prestressed and non-prestressed reinforcement are located with the same radius (Figure 1) \( r_s = r_{sp} \), the lever arm for the prestressed and non-prestressed reinforcement will be constant \( z_s = z_{sp} \). However, the parameters \( \varphi_{sp} \) and \( \varphi_s \) will differ significantly.

\[
M_{ult} \leq \left( R_b A_b r_m + R_{sc} A_{s,tot} r_s \right) \frac{\sin(\pi \xi_{cir})}{\pi} + R_s A_{s,tot} (1 - 1.7 \xi_{cir}) z_s
\] (4)
Where $z_s = (0.2 + 1.3 \xi_{cir}) r_s$.

The comparison shows that the first terms of equation (3) and (4) are identical with the equality $r_s = r_{sp}$, however, the relative height of the compressed zone $\xi_{cir}$ will differ. The second term of equations (3) and (4), which reflects the effect of reinforcement on the bearing capacity of the section, varies significantly.

It should be noted that according to formulas (2) and (4), it is not possible to analyze the influence of the relation between the prestressed reinforcement $A_{sp}$ and non-prestressed reinforcement $A_s$ explicitly, as well as the effect of the prestressed reinforcement level $\sigma_{sp}$.

To identify discrepancies in the calculations for the supports with annular section according to the SNiP 2.03.01-84 [11] and according to the SP 63.13330.2012 [12], we carried out the numerical experiments on the example of the transmission tower supports. As the test sample, the cylindrical transmission tower according to the Certification System GOST 22687.2, the parameters of which are given in Table 1, was adopted.

**Table 1. Parameters of the investigated transmission tower**

| Type of transmission tower according to GOST | Diameter of support, mm | Wall thickness, mm | Class and amount of reinforcement | Level of reinforcement prestress |
|---------------------------------------------|-------------------------|-------------------|----------------------------------|----------------------------------|
| Cylindrical support 20.2-1.0                | 800, 640                | 80                | Prestressed A-IV (A600) Non-prestressed A-IV (A600) | 0.8R_{sp,n} |

The amount of reinforcement corresponds to the most prestressed section of the support in a restrained condition.

The bending stiffness of support $D$ and the critical force $N_{cr}$ were determined, respectively, according to the formulas (5) and (6):

$$D = \frac{0.15 E_b l_b}{\varphi_l(0.3 + \delta_e)} + 0.7E_d l_d \tag{5}$$

$$N_{cr} = \frac{\pi^2 D}{T_0} \tag{6}$$

Where $\varphi_l = 1$ with short-term load;

$\delta_e = \frac{\varepsilon_0}{H}$ relative eccentricity of external force

$N$, a H- outer diameter of the support.

In a numerical experiment, the value of longitudinal force $N$ in formulas (1) and (2) varied in the range (0.1-1.0) $N_{cr}$, and the value $\delta_e =$0.15-0.75.

This allowed us to analyze in a numerical experiment the change in bending stiffness $D$ for the support under study, critical strength $N_{cr}$, load bearing capacity $M_{sh}$, in a real area of possible loads, all other things being equal (section geometry, reinforcement percent, concrete classes and reinforcement).

Table 2 shows the calculation results of $\xi_{cir}$ according to the SNiP 2.03.01-84 [11], and Table 3 shows the calculation results of $\xi_{cir}$ according to the SP 63.13330 / 2012 [12], depending on the $N/N_{cr}$ and $\delta_e$ ratios. Figure 2 shows the graphs of the variation of $\xi_{cir}$ at $\delta_e = 0.15; 0.45; 0.75$ depending on the ratio $N/N_{cr}$.

The analysis of these data shows that with an increase of $\delta_e$, the values of $\xi_{cir}$ decrease, but at the same time, the obtained values approximate to each other as a result of calculation according to the SNiP and the SP. For example, when calculating by the SNiP [11] with the ratio $N/N_{cr}=0.5$, the value of $\xi_{cir}$ changes from 0.4260 ($\delta_e = 0.15$) to 0.3879 ($\delta_e = 1$). And when calculating by the SP [12] with...
the same parameters, the value of $\xi_{cir}$ changes from 0.3631 ($\delta_c = 0.15$) to 0.3282 ($\delta_c = 1$). This pattern is clearly visible in the graphs of Figure 2.

It should be noted that the value of $\xi_{cir}$ in the SNIp [11] exceeds their values in the SP [12], while the graphs are almost parallel and their convergence with the increase of $\delta_c$ takes place.

**Table 2.** Change of $\xi_{cir}$ depending on the N/N_cr and $\delta_c$ ratios (SNIp)

| $\delta_c$ | 0.1  | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  | 1   |
|------------|------|------|------|------|------|------|------|------|------|-----|
| 0.15       | 0.48  | 0.37  | 0.39  | 0.40  | 0.42  | 0.42  | 0.44  | 0.46  | 0.48  | 0.51 |
| 0.3        | 0.35  | 0.36  | 0.38  | 0.39  | 0.41  | 0.42  | 0.46  | 0.47  | 0.46  | 0.46 |
| 0.45       | 0.35  | 0.36  | 0.37  | 0.38  | 0.40  | 0.41  | 0.45  | 0.45  | 0.45  | 0.46 |
| 0.6        | 0.35  | 0.35  | 0.37  | 0.38  | 0.39  | 0.40  | 0.42  | 0.42  | 0.43  | 0.43 |
| 0.75       | 0.35  | 0.35  | 0.37  | 0.38  | 0.39  | 0.40  | 0.41  | 0.41  | 0.42  | 0.42 |
| 0.9        | 0.35  | 0.35  | 0.37  | 0.38  | 0.39  | 0.40  | 0.41  | 0.41  | 0.41  | 0.42 |
| 1          | 0.35  | 0.35  | 0.37  | 0.38  | 0.39  | 0.40  | 0.41  | 0.41  | 0.42  | 0.42 |

**Table 3.** Change of $\xi_{cir}$ depending on the N/N_cr and $\delta_c$ ratios (SP)

| $\delta_c$ | 0.1  | 0.2  | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  | 0.9  | 1   |
|------------|------|------|------|------|------|------|------|------|------|-----|
| 0.15       | 0.29  | 0.31  | 0.33  | 0.34  | 0.36  | 0.37  | 0.39  | 0.41  | 0.42  | 0.44 |
| 0.3        | 0.29  | 0.30  | 0.32  | 0.33  | 0.35  | 0.36  | 0.38  | 0.39  | 0.40  | 0.41 |
| 0.45       | 0.29  | 0.30  | 0.32  | 0.33  | 0.35  | 0.36  | 0.38  | 0.39  | 0.40  | 0.41 |
| 0.6        | 0.29  | 0.30  | 0.32  | 0.33  | 0.35  | 0.36  | 0.38  | 0.39  | 0.40  | 0.41 |
| 0.75       | 0.29  | 0.30  | 0.32  | 0.33  | 0.35  | 0.36  | 0.38  | 0.39  | 0.40  | 0.41 |
| 0.9        | 0.29  | 0.30  | 0.32  | 0.33  | 0.35  | 0.36  | 0.38  | 0.39  | 0.40  | 0.41 |
| 1          | 0.29  | 0.30  | 0.32  | 0.33  | 0.35  | 0.36  | 0.38  | 0.39  | 0.40  | 0.41 |

Table 4 shows the calculation results of the bearing capacity of $M_{ult}$ supports according to the SNIp 2.03.01-84 [11], and Table 5 shows the calculation results of $M_{ult}$ according to the SP 63.13330 / 2012 [12], depending on the N/N_cr and $\delta_c$ ratios. Figure 3 shows the graphs of the variation of $M_{ult}$ at $\delta_c = 0.15; 0.45: 0.75$ depending on the N/N_cr ratio.

The analysis of these data shows that, with an increase of $\delta_c$, the values of $M$ decrease, but at the same time, the obtained values approximate to each other as a result of calculation according to the SNIp and the SP. For example, when calculating by the SNIp [11] with the ratio N/N_cr = 0.5, the value of the moment changes from 1288.8 kN*m ($\delta_c=0.15$) to 1271.9 kN*m ($\delta_c=1$). And when calculating by the SP [12] with the same parameters, the value of the moment changes from 1330.29 kN*m ($\delta=0.15$) to 1297.86 kN*m ($\delta=1$). This pattern is clearly visible in the graphs of Figure 3.
Figure 2. Change of $\xi_{cir}$ depending on the $N/N_{cr}$ and $\delta_e$ ratios

**Table 4.** Change of $M$ depending on the $N/N_{cr}$ and $\delta_e$ ratios according to the SNiP

| $\delta_e$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|
| $1$        |     |     |     |     |     |     |     |     |     |   |
| $2$        |     |     |     |     |     |     |     |     |     |   |
| $3$        |     |     |     |     |     |     |     |     |     |   |
| $4$        |     |     |     |     |     |     |     |     |     |   |
| $5$        |     |     |     |     |     |     |     |     |     |   |
| $6$        |     |     |     |     |     |     |     |     |     |   |
| $7$        |     |     |     |     |     |     |     |     |     |   |
| $8$        |     |     |     |     |     |     |     |     |     |   |
| $9$        |     |     |     |     |     |     |     |     |     |   |
| $10$       |     |     |     |     |     |     |     |     |     |   |
| $11$       |     |     |     |     |     |     |     |     |     |   |

**Table 5.** Change of $M$ depending on the $N/N_{cr}$ and $\delta_e$ ratios according to the SP

| $\delta_e$ | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 | 1 |
|------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|---|
| $1$        |     |     |     |     |     |     |     |     |     |   |
| $2$        |     |     |     |     |     |     |     |     |     |   |
| $3$        |     |     |     |     |     |     |     |     |     |   |
| $4$        |     |     |     |     |     |     |     |     |     |   |
| $5$        |     |     |     |     |     |     |     |     |     |   |
| $6$        |     |     |     |     |     |     |     |     |     |   |
| $7$        |     |     |     |     |     |     |     |     |     |   |
| $8$        |     |     |     |     |     |     |     |     |     |   |
| $9$        |     |     |     |     |     |     |     |     |     |   |
| $10$       |     |     |     |     |     |     |     |     |     |   |
| $11$       |     |     |     |     |     |     |     |     |     |   |

5
0.6 1251.6 1267.2 1281.5 1294.7 1306.6 1317.3 1326.7 1334.7 1341.5 1347.0
0.75 1250.5 1265.1 1278.7 1291.2 1302.6 1313.0 1322.3 1330.4 1337.4 1343.3
0.9 1249.7 1263.5 1276.5 1288.5 1299.5 1309.6 1318.8 1326.9 1334.0 1340.1
1.0 1249.2 1262.9 1275.3 1287.0 1297.9 1307.8 1316.8 1324.9 1332.1 1338.3

Figure 3. Change of M depending on the N/N_cr and δ_e ratios

It should be noted that the bearing capacity of M_{ult} according to the SNiP [11] is constantly increasing with the increase of N/N_cr ratio, and according to the SP [12], a decrease of M_{ult} is observed at certain values of N/N_cr depending on the relative eccentricity of δ_e. So, for example, at δ_e = 0.15, M_{ult} according to the SP [12] begins to decrease at N/N_cr ≥0.6, and at δ_e = 0.45, the decrease begins at N/N_cr ≥0.85 (See graphs in Figure 3).

The reason for this incorrectness in the calculations of the bearing capacity of supports with annular section should be explained by the fact that the new standards [12] do not have a differentiated approach to taking into account the effects of prestressed reinforcement A_{sp} and non-prestressed reinforcement A_s when determining the relative value of the compressed zone ξ_{cir} by the formula (2) and M_{ult} by the formula (four). As it follows from the graphs of changes in the bearing capacity of the supports (Figure 4), the convergence in the calculations noticeably improves with the increase of δ_e.

This improvement in convergence is also associated with the change in stiffness of the support D according to the formula (5) and the critical force N_{crc} according to the formula (6).

Table 6 shows the calculation results of the stiffness (D) of the support according to the SP 63.13330 / 2012 [12] depending on the eccentricity (δ_e). Figure 4 shows the graphs of the change in D concrete, reinforcement and the total stiffness of the support for various values of δ_e from 0.15 to 1.5.
Table 6. Change of the support stiffness depending on the eccentricity

| $\delta_e$ | $D_{\text{concrete}} \times 10^{-7}$ N*mm$^2$ | $D \times 10^{-7}$ N*mm$^2$ |
|------------|-------------------------------------------|--------------------------|
| 0.15       | 1525.29                                   | 2332.06                  |
| 0.30       | 1143.97                                   | 1950.74                  |
| 0.45       | 915.17                                    | 1721.95                  |
| 0.60       | 762.64                                    | 1569.42                  |
| 0.75       | 653.70                                    | 1460.47                  |
| 0.90       | 571.98                                    | 1378.76                  |
| 1.00       | 527.98                                    | 1334.76                  |
| 1.05       | 508.43                                    | 1315.20                  |
| 1.20       | 457.59                                    | 1264.36                  |
| 1.35       | 415.99                                    | 1222.76                  |
| 1.50       | 381.32                                    | 1188.10                  |

The analysis of these data shows that with an increase of $\delta_e$, the values of $D_{\text{concrete}}$ decrease, while $D_{\text{reinforcement}}$ remains constant and equal to 806.77 N * mm$^2$. Thus, with an increase in eccentricity, the stiffness of the support decreases, which is clearly shown in Figure 4.

![Figure 4. Change in stiffness according to $\delta_e$.](image)

Considering the formulas (5) and (6) and the graphs of changes in the stiffness of the supports (Figure 4), it is not difficult to verify that the graphs of the change in the critical force $N_{\text{crc}}$ of the function $\delta_e$ will be similar to the graphs $D = f(\delta_e)$.

For this reason, the convergence of $\xi_{\text{circ}}$ according to the formulas (1) and (2) improves (See Figure 2), and the convergence of $M_{\text{ult}}$ according to the formulas (5) and (6) (See Figure 2) improves with the increase of $\delta_e$ and $N/N_{\text{crc}}$.

Summary

1. The relative height value of the supports compressed zone with annular section $\xi_{\text{circ}}$ determined according to the Construction Standards and Regulations [11], significantly exceeds its value according to the Code Specification [12].
2. The quantity $\xi_{cr}$ decreases with the increase in the relative eccentricity of the longitudinal force $\delta_e$ while with the increase in $\delta_e$ and $N/N_{cr}$ the graphs $\xi_{cr} = f(\delta_e)$ approximate to each other (Fig. 2).

3. The bearing capacity of supports with annular section $M_{ult}$ according to the Construction Standards and Regulations [11] is constantly rising with the increase of $N/N_{cr}$, but according to the Code Specification [12] a decrease of $M_{ult}$ is observed at certain values $N/N_{cr}$ (Fig. 3). Moreover, the convergence of the graphs $M_{ult} = f(N/N_{cr})$ according to different standards improves with the increase of $\delta_e$.

4. The incorrectness in the calculations of bearing capacity of supports with annular section according to the standards [11] and [12] revealed in the numerical experiment is due to the fact that the new standards [12] do not have a differentiated effect on prestressed reinforcement $A_{pre}$ and non-prestressed reinforcement $A_e$ when determining $\xi_{cr}$ by the formula (2), as well as $M_{ult}$ by the formula (4).

5. The stiffness of supports with annular section $D$ according to the formula (5) significantly decreases with the increase in the eccentricity of the longitudinal force $\delta_e$ (Fig. 4). This change in stiffness proportionally affects the critical force $N_{cr}$ according to the formula (6). An increase in the convergence of $\xi_{cr}$ and $M_{ult}$ according to the standards [11] and [12] with an increase in $\delta_e$ and $N_{cr}$ (Fig. 2, Fig. 3) is associated with the nature of the change in the stiffness of the supports $D$ and $N_{cr}$.

6. The new standards [12] for calculating supports with annular section contain higher reliability coefficients in comparison with the standards [11]. However, when making scientific experiments and studies with elements of the circular section, standards should be preferred over experimental results with theoretical ones [11].

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