EFFECTIVE INTERACTIONS OF PLANAR FERMIONS
IN A STRONG MAGNETIC FIELD-THE EFFECT OF LANDAU LEVEL MIXING

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Abstract

We obtain expressions for the current operator in the lowest Landau level (L.L.L.) field theory, where higher Landau level mixing due to various external and interparticle interactions is systematically taken into account. We consider the current operators in the presence of electromagnetic interactions, both Coulomb and time-dependent, as well as local four-fermi interactions. The importance of Landau level mixing for long-range interactions is especially emphasized. We also calculate the edge-current for a finite sample.

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I. Introduction

The many-body problem of charged planar fermions is a fascinating problem in its own right. It is encountered, for example, in the quantum Hall phenomena [1] as well as in the theory of $c = 1$ strings, where the connection is not an obvious one [2]. A strong magnetic field creates a large gap between successive Landau levels of the single-particle spectrum. Specifically, the gap between the lowest Landau level (L.L.L.) and higher levels is of the order of the cyclotron frequency, which is high for a large magnetic field. This naturally indicates that a second quantized field theory of the lowest Landau level fermions is the most convenient tool for handling the problem in a strong magnetic field.

Since the seminal work of Girvin & Jach, [3], a lot of progress has been made in this direction [1],[5]. As the degenerate states of the L.L.L. do not form a complete set, the L.L.L. field operators do not satisfy the standard fermionic anticommutation relation. In fact, the field operators at equal time but at different spatial points are related. This feature makes the formulation of a second quantized theory a challenging task. Specifically, as has been demonstrated by Stone [1], the current operator for the L.L.L. fermions is not the operator one would expect in a more standard fermionic field theory. Recently, the appropriate current operators have been obtained, for the cases of an external local interaction and for a Coulomb interaction between the fermions [4],[5]. However, in these works, the effect of mixing with higher L.L. through the perturbing potentials has not been addressed. As indicated in [6], the effect of such mixing could be significant in some physical situations. In the sequel, we have constructed expressions for the current operator for the L.L.L. fermions in the presence of a variety of interactions, where Landau level mixing has been systematically accounted for. Our results are an explicit realization of the idea schematically discussed in [6].

The organization of this paper is as follows: in section II we set up the basic notation, in III we obtain an effective action for the L.L.L fermions and in the subsequent sections
we utilize this action to obtain the current operators in the presence of Coulomb (IV), electromagnetic (V) and local four-Fermi (VI) interaction. In VII we discuss the edge current in the case of a finite sample.

II. Notations and Conventions

The kinematics of electrons in the L.L.L. has already been discussed [3]. However, to make the discussion reasonably self-contained, we introduce our own notation [7] in the following.

The Lagrangian for planar electrons in a magnetic field normal to the plane is

\[ \int d\vec{x} \psi^\dagger(\vec{x},t) \left( i\partial_t - h_0 \right) \psi(\vec{x},t). \] (2.1)

The free single-particle Hamiltonian is given by

\[ h_0 = \frac{1}{2m}\left[ (\hat{\pi}_x)^2 + (\hat{\pi}_y)^2 \right], \] (2.2)

where \( \hat{\pi}_i \equiv \hat{p}_i - A^i(\hat{x},\hat{y}) \); \( \vec{A} \equiv \frac{B}{2}(y,-x) \) and the constant magnetic field is in the negative \( \hat{z} \) direction. We define

\[ \hat{\pi} \equiv \frac{1}{\sqrt{2B}}(\hat{\pi}_x - i\hat{\pi}_y) \] (2.3).

\( \hat{\pi}, \hat{\pi}^\dagger \) are dimensionless. Their commutator is

\[ [\hat{\pi}, \hat{\pi}^\dagger] = 1 \] (2.4)

In this notation,

\[ h_0 = \omega \hat{\pi}^\dagger \hat{\pi} \] (2.5),

where we have dropped the zero point fluctuation and \( \omega = \frac{B}{m} \), the cyclotron frequency.

Further, define the guiding-centre coordinate operators as

\[ \hat{X} \equiv \hat{x} - \frac{1}{B} \hat{\pi}_y \; ; \; \hat{Y} \equiv \hat{y} + \frac{1}{B} \hat{\pi}_x \] (2.6).
We also define their holomorphic combinations
\[ \hat{a} \equiv \sqrt{\frac{B}{2}} (\hat{X} + i\hat{Y}) \quad \hat{a}^\dagger \equiv \sqrt{\frac{B}{2}} (\hat{X} - i\hat{Y}) \] (2.7)
for which the commutators are:
\[ [\hat{X}, \hat{Y}] = \frac{i}{B}, \quad [\hat{a}, \hat{a}^\dagger] = 1 \] (2.8).

Also,
\[ [\hat{a}, \pi] = [\hat{a}^\dagger, \pi] = [\hat{a}, \pi^\dagger] = [\hat{a}^\dagger, \pi^\dagger] = 0 \] (2.9).

Thus there are two pairs of independent canonical operators defined on this Hilbert space. Of them, \( \hat{a} \) and \( \hat{a}^\dagger \) commute with \( h_0 \) and thus characterize the degeneracy of the landau levels. Let us further define
\[ \hat{z} \equiv \sqrt{\frac{B}{2}} (\hat{x} + i\hat{y}) = \hat{a} - i\hat{\pi}^\dagger; \quad \hat{\bar{z}} = \hat{z}^\dagger; \quad [\hat{z}, \hat{\bar{z}}] = 0 \] (2.10)
We specify the coordinates of the Hilbert space by \( |\vec{x}\rangle \) or \( |n\rangle|\zeta\rangle \), where
\[ \hat{\pi}^\dagger \hat{\pi} |n\rangle = n|n\rangle \]
\[ \hat{a}|\zeta\rangle = \zeta|\zeta\rangle. \] (2.11)

Here \( |\zeta\rangle \equiv e^{\zeta \hat{a}^\dagger} |0\rangle \), \( \hat{a}|0\rangle = 0 \). These two are related by
\[ \langle \vec{x}|(|n\rangle|\zeta\rangle) = \sqrt{\frac{B}{2\pi}} \frac{i^n (z - \zeta)^n}{\sqrt{n!}} e^{-\frac{1}{2} |\vec{x}|^2 + \bar{\zeta} \zeta} \] (2.12),
where \( z \equiv \sqrt{\frac{B}{2}} (x + iy) \). From 2.12 it is clear that for \( n=0 \) the \( \zeta \) enables one to make the transition to the coordinate most easily.

We represent the second-quantized Schrödinger operator by \( |\psi\rangle \), where \( \hat{\psi}(\vec{x},t) \equiv \langle \vec{x},t|\psi\rangle \) is the corresponding field operator. We can write
\[ |\psi\rangle = \sum_n |\psi_n\rangle \] (2.13),
where $|\psi_n\rangle$ is the Schrödinger operator for the $n$th Landau level. It follows that $\hat{\pi}|\psi_0\rangle = 0$, which means that

$$\langle \vec{x}|\hat{\pi}|\psi_0\rangle = -(\partial_z + \frac{1}{2}\bar{z})\hat{\psi}_0(\vec{x}, t) = 0.$$  (2.14)

This is the L.L.L. condition satisfied by the L.L.L. Schrödinger field

$$\hat{\psi}_0(\vec{x}, t) = \sqrt{\frac{B}{2\pi}}e^{-\frac{1}{2}|z|^2}\sum_{l=0}^{\infty} \frac{\bar{z}^l}{\sqrt{l!}}\hat{c}_l(t),$$  (2.15)

where $\{\hat{c}_l, \hat{c}_{l'}^\dagger\} = \delta_{l,l'}$. Let us further define

$$\hat{X}|x\rangle = x|x\rangle, \quad \hat{Y}|y\rangle = y|y\rangle,$$

where

$$\langle x|y\rangle = \sqrt{\frac{B}{2\pi}}e^{iBxy}$$  (2.16)

and

$$\langle x|\zeta\rangle = \left(\frac{B}{\pi}\right)^{\frac{1}{4}}e^{-\frac{1}{2}(Bx^2 + z^2) + \sqrt{2B}xz}\zeta,$$

$$\langle y|\zeta\rangle = \left(\frac{B}{\pi}\right)^{\frac{1}{4}}e^{-\frac{1}{2}(Bx^2 - z^2) - i\sqrt{2B}yz} - i\sqrt{2B}yz. \quad (2.17)$$

It is sometimes convenient to choose an angular momentum basis for the single-particle states, given by

$$\hat{a}^\dagger \hat{a}|l\rangle = l|l\rangle, \quad \langle l|l'\rangle = \delta_{l,l'}, \quad \langle l|\zeta\rangle = \frac{\zeta^l}{\sqrt{l!}}.$$  (2.18)

$\hat{c}_l$ in (2.15) above destroys a L.L.L. electron with angular momentum $l$.

Having established the notation, let us introduce an interaction $V(\vec{x}, t)$ into the single-particle problem. Thus,

$$h = h_0 + V.$$  (2.19)

Now, a general scalar potential $V$ can be written as

$$V(\vec{x}, t) = V(\hat{z}, \hat{z}^\dagger, t) = \sum_{m,n} \frac{1}{m!n!}(-i)^n(i)^m(\hat{\pi}^\dagger)^n(\hat{\pi})^m \partial_z^n \partial_{\bar{z}}^m V(z, \bar{z}, t)|\hat{\pi} = \hat{a}^\dagger, \hat{z} = \hat{a}^\dagger.$$  (2.20)
The ordering adopted for $\hat{\pi}, \hat{\pi}^\dagger$ automatically forces us to anti-normal order $\hat{a}, \hat{a}^\dagger$. This is indicated by $\hat{\pi}^\dagger$. We thus write
\[ V(\vec{x}, t) = \sum_{m,n=0}^{\infty} (\hat{\pi}^\dagger)^m (\pi)^n v_{m,n} \] (2.21)

Where,
\begin{align*}
v_{00} &= \frac{i}{\hbar} V(\hat{a}, \hat{a}^\dagger, t) \\
v_{10} &= -i \frac{i}{\hbar} \partial_z V = v_{01}^\dagger \\
v_{11} &= \frac{i}{\hbar} \partial_z \partial_{\bar{z}} V = v_{02}^\dagger \\
v_{20} &= \frac{(-i)^2}{2!} \frac{i}{\hbar} \partial_z^2 V = v_{02}^\dagger.
\end{align*}
(2.22)

III Effective Action for the L.L.L.

In the notation established above, the action is written as
\[ -S = \langle \psi | \hat{p}_t + \hat{h} | \psi \rangle. \] (3.1)

Here, $\langle t | \hat{p}_t \equiv -i \partial_t \langle t |$. More explicitly
\[ -S = \langle \psi_0 | \hat{p}_t + \hat{V} | \psi_0 \rangle + \sum_{n \neq 0} \left[ \langle \psi_0 | \hat{V} | \psi_n \rangle + \langle \psi_n | \hat{V} | \psi_0 \rangle \right] \]
\[ + \sum_{n,n' \neq 0} \langle \psi_n | \hat{p}_t + \hat{h} | \psi_{n'} \rangle, \] (3.2)
where we have used $\hat{h}_0 | \psi_0 \rangle = 0$. The L.L.L. truncation corresponds to keeping only the first term on the r.h.s. of (3.2). However, in order to incorporate the effect of Landau level mixing, we integrate the $n \neq 0$ modes out instead of simply dropping them from the problem. This leads to an effective action
\[ -S_{\text{eff}} = \langle \psi_0 | \hat{p}_t + \hat{V} | \psi_0 \rangle - \langle \psi_0 | \hat{V} \frac{1}{\hat{p}_t + \hat{h}_0 + \hat{V}''} \hat{V} | \psi_0 \rangle \] (3.3),
where $''$ indicates that $n = 0$ does not enter into the sum over intermediate states.
At this point, we choose to work with potentials that are slowly varying in space and
time. Further, the potentials are taken to be small in magnitude compared to the cylcotron
frequency.

This naturally leads to the expansion

\[
\left( \frac{1}{\hat{p}_t} + \hat{h}_0 + \hat{V} \right) ^{-1} = \frac{1}{\hat{h}_0} - \frac{1}{\hat{h}_0} (\hat{p}_t + \hat{V}) \frac{1}{\hat{h}_0} + \ldots .
\]  

In what follows we drop the subscript 0 and the karets over the operators as they should
be clear from the context. We also drop the “” and the absence of \( n = 0 \) is to be tacitly
understood. Thus,

\[
-S_{\text{eff}} = \langle \psi | (1 + V \frac{1}{\hbar_0^2} V) p_t | \psi \rangle + H^{(0)}_{\text{eff}} + H^{(1)}_{\text{eff}} + H^{(2)}_{\text{eff}} + \ldots .
\]

Here,

\[
H^{(0)}_{\text{eff}} = \langle \psi | V | \psi \rangle
\]

\[
H^{(1)}_{\text{eff}} = -\langle \psi | V \frac{1}{\hbar_0} V | \psi \rangle
\]

\[
H^{(2)}_{\text{eff}} = -i \langle \psi | V \frac{1}{\hbar_0^2} \partial_t V | \psi \rangle
\]

\[
H^{(3)}_{\text{eff}} = \langle \psi | V \frac{1}{\hbar_0} V \frac{1}{\hbar_0} V | \psi \rangle.
\]

At this point we see that if we keep the magnetic field fixed in value and work with
a large cyclotron frequency, we have to let \( m \to 0 \). Then \( m/\sqrt{B} \) is a natural expansion
parameter. We obtain \( S_{\text{eff}} \) to \( O(m/\sqrt{B}) \), where \( H^{(0)}_{\text{eff}} \), which corresponds to a L.L.L.
truncation is of \( O(1) \). One observes that to order \( O(m/\sqrt{B}) \) the symplectic structure of
the effective action retains its original form and the fermion field need not be renormalized
to regain its original symplectic structure.

Thus, the effective action to \( O(m/\sqrt{B}) \) is

\[
S_{\text{eff}} = -\langle \psi | p_t | \psi \rangle - H^{(0)}_{\text{eff}} - H^{(1)}_{\text{eff}}
\]  

(3.8).
Explicitly,

\[ S_{\text{eff}} = - \int dt \int d\vec{x} \psi(\vec{x}, t) \partial_t \psi(\vec{x}, t) - \int dt \int d^2 z \rho(z, \bar{z}, t) [V(z, \bar{z}, t) \]

\[ - \frac{1}{\omega} \sum_{n=1}^{\infty} \frac{1}{n!} \sum_{m=0}^{\infty} \frac{(-1)^m}{m!} (\partial_z^{n+m} V)(\partial_{\bar{z}}^{n+m} V). \]  

(3.9)

Thus \( H_{\text{eff}} \) can be cast in the form

\[ H_{\text{eff}} \equiv \int d\vec{x} \rho(\vec{x}, t) U(\vec{x}, t) \]  

(3.10)

where

\[ U(\vec{x}, t) = V(z, \bar{z}, t) - \frac{1}{\omega} \sum_{n=1}^{\infty} \sum_{m=0}^{\infty} \frac{(-1)^m}{m! n!} (\partial_z^{n+m} V)(\partial_{\bar{z}}^{n+m} V) \]  

(3.11).

**IV Current Operators and Landau Level Mixing**

In this section we utilize \( S_{\text{eff}} \) to derive expressions for the current operators beyond leading order. To this end we use the equation of continuity in conjunction with the Heisenberg equation of motion.

\[ \partial_t \rho(\vec{x}, t) = -i[\rho(\vec{x}, t), H_{\text{eff}}] \]

\[ = -i \int d\vec{x}' \{ \rho(\vec{x}, t), \rho(\vec{x}', t) \} U(\vec{x}', t) \]

\[ = -i \sum_{n=1}^{\infty} \frac{1}{n!} \{ \partial_z^n \rho(\vec{x}, t) \partial_{\bar{z}}^n U(\vec{x}, t) \} - \partial_{\bar{z}}^n \{ \rho(\vec{x}, t) \partial_z^n U(\vec{x}, t) \} \]  

(4.2)

where we have used

\[ [\rho(\vec{x}, t), \rho(\vec{x}', t)] = \sum_{n=1}^{\infty} \frac{1}{n!} \{ \partial_z^n \rho(\vec{x}, t) \partial_{\bar{z}}^n \delta(\vec{x} - \vec{x}') \} - \partial_{\bar{z}}^n \{ \rho(\vec{x}, t) \partial_z^n \delta(\vec{x} - \vec{x}') \} \]  

(4.1)

which has been derived in [2],[5]. Using

\[ \partial_t \rho(\vec{x}, t) = -\partial_z J(\vec{x}, t) - \partial_{\bar{z}} \bar{J}(\vec{x}, t) \]  

(4.3)
and combining (4.1) and (4.2), we get
\[
J(\vec{x}, t) = i \sum_{p=0}^{\infty} \sum_{n=1}^{\infty} \sum_{q=1}^{\infty} \frac{(-1)^p}{n!q!p!} \partial_z^{n-1} \rho(\vec{x}, t) \partial_z^n \{V(\vec{x}, t) - \frac{1}{\omega} (\partial_z^{q+p} V)(\partial_z^{q+p} V)\} \]
(4.3),
to \(O(m/\sqrt{B})\). The first term, obtained solely from \(H_{\text{eff}}^{(0)}\), does not contain any information about Landau level mixing, has been obtained earlier [5]. It is the second term which in principle contains the effect of all higher Landau levels, that is interesting for the study of mixing. For simplicity, we assume the potential to be slowly varying in space-time and drop terms containing more than two derivatives.

Thus, to leading order,
\[
J(\vec{x}, t) \approx i \rho(\vec{x}, t) \partial_z [V(\vec{x}, t) - \frac{1}{\omega} (\partial_z V)(\partial_z V)]
\]
(4.4).

From these we easily obtain
\[
J^i(\vec{x}, t) = \frac{1}{B} \rho(\vec{x}, t) \epsilon^{0ij} \partial_j [V - \frac{m}{2B^2} (\nabla V)^2]
\]
(4.5).

V. Coulomb Interaction between the Electrons

Let us consider a potential of the form
\[
V(\vec{x}, t) = V_0(\vec{x}, t) + a_0(\vec{x}, t)
\]
(5.1).

Here, \(a_0\) is a potential leading to an electric field. We imagine that there is no perturbative magnetic fields acting on the electrons. Thus we set \(a_i\) to zero. The potential \(a_0\) has an associated Maxwell term given by \(\frac{1}{2} f_{i0} f^{i0}\). At this point one should bear in mind that even though the problem is planar, the electrons move in a two-dimensional subspace of a three-dimensional space. Consequently, the Coulomb potential between charges is of a \(\frac{1}{r}\) form rather than a log \(r\) form. To implement this analytically, we extend the problem to three spatial dimensions. Namely,
\[
\rho(x, y, t) \rightarrow \rho(x, y, z, t) \equiv \rho(x, y, t) \delta(z)
\]
\[
a_0(x, y, t) \rightarrow a_0(x, y, z, t) \equiv a_0(x, y, t) \delta(z).
\]
(5.2)
The equation of motion for $a_0$ as obtained from the Hamiltonian, including the Maxwell term is given by

$$\nabla^2 a_0 = -\rho - \frac{m}{B^2} [\rho \nabla^2 a_0 + \nabla \rho \cdot \nabla a_0] \quad (5.3)$$

Here we have set $V_0$ to zero for simplicity. This yields, to $O(1)$,

$$a_0(\vec{x}, t) = \frac{1}{4\pi} \int d\vec{x}' \frac{\rho(\vec{x}', t)}{|\vec{x} - \vec{x}'|} \quad (5.4).$$

Putting this back into (5.3), we obtain, to $O(m/\sqrt{B})$,

$$\nabla^2 a_0 = -\rho_{\text{eff}} \quad (5.5),$$

where

$$\rho_{\text{eff}} \equiv \rho - \frac{m}{4\pi B^2} [\rho^2 - \vec{\nabla} \rho \cdot \int d\vec{x}' \nabla \frac{1}{|\vec{x} - \vec{x}'|} \rho(\vec{x}')] \quad (5.6).$$

This can be readily solved to obtain

$$a_0(\vec{x}, t) = \frac{1}{4\pi} \int d\vec{x}' \frac{1}{|\vec{x} - \vec{x}'|} \rho_{\text{eff}}(\vec{x}', t) \quad (5.7).$$

The terms in $\rho_{\text{eff}}$ beyond the first show the effects of Landau level mixing.

Putting $a_0$ from (5.7) into (4.5), one obtains $J^i$ in the presence of mixing.

$$J^i(\vec{x}, t) = \frac{1}{4\pi B} \rho(\vec{x}, t) e^{\delta_{ij}} \partial_j \left[ \int d\vec{x}' \frac{1}{|\vec{x} - \vec{x}'|} \rho(\vec{x}', t) - \frac{m}{B^2} \{ \rho^2(\vec{x}', t) \right. \left. - \vec{\nabla}' \rho(\vec{x}', t) \int d\vec{y}' \frac{1}{|\vec{x}' - \vec{y}'|} \rho(\vec{y}', t) + \frac{1}{4\pi} \left( \int d\vec{x}' \nabla \frac{1}{|\vec{x} - \vec{x}'|} \rho(\vec{x}', t))^2 \right\} \right]. \quad (5.8)$$

The first term agrees with that obtained in [4], as it should. The terms beyond that are the corrections to $O(m/\sqrt{B})$, as mentioned in [5].

**VI. Minimal Coupling to Electromagnetism**

The effective action for planar electrons in a strong magnetic field coupled minimally to perturbative slowly-varying electromagnetic fields has already been given in [6],[7]. Here,
we discuss the structure of the current operators arising from this effective action as a natural extension to what has been discussed in the earlier sections. From [6], we know that the effective action is given by

$$S_{\text{eff}} = \int dt \int d\vec{x} \left[ \overline{\psi}(\vec{x}, t)i\partial_t \psi(\vec{x}, t) - \rho(\vec{x}, t) \left\{ a_0(\vec{x}, t) + \frac{1}{B} \epsilon_{0ij}(a^i \partial^j a^0 - \frac{1}{2} a^i \partial^0 a^j) \right\} \right]$$  \hspace{1cm} (6.1)

If we also consider the Maxwell term governing $a^\mu$, we can readily obtain the Maxwell equations for the potentials. Here, we choose to work in the $\nabla \vec{A} = 0$ gauge, where, $A \equiv \frac{1}{\sqrt{2B}}(a^1 + ia^2)$ may be written as

$$A = -i \partial_z \phi$$
$$\vec{A} = i \partial_z \phi.$$  \hspace{1cm} (6.2)

The Maxwell equations are,

$$\left( \partial_t^2 - \nabla^2 + \frac{i}{2B} \partial_t \rho \right) \partial_z \phi = -i \partial_z \partial_t a_0$$ \hspace{1cm} (6.3)

and

$$\nabla^2 a_0 = -\rho + \frac{1}{B}(\nabla \cdot \rho \nabla \phi + \rho \nabla^2 \phi)$$ \hspace{1cm} (6.4).

We can solve these iteratively in $m/\sqrt{B}$. To O(1),

$$a_0 \approx \frac{1}{4\pi} \int d\vec{y} \frac{1}{|\vec{x} - \vec{y}|} \rho(\vec{y}, t)$$  \hspace{1cm} (6.5).

Further,

$$\left( \partial_t^2 - \nabla^2 \right) \phi = -\frac{1}{4\pi} \int d\vec{y} \frac{1}{|\vec{x} - \vec{y}|} \partial_t \rho(\vec{y}, t)$$ \hspace{1cm} (6.6).

We can solve (6.6) in momentum space. We get,

$$a^1(\omega, \vec{k}) = \frac{i \omega k^2}{2|k|(|\omega^2 + k^2|)} \rho(\omega, \vec{k})$$
$$a^2(\omega, \vec{k}) = -\frac{-i \omega k^1}{2|k|(|\omega^2 + k^2|)} \rho(\omega, \vec{k})$$ \hspace{1cm} (6.7).
Putting (6.7) back in (6.4), we obtain
\[ a_0(\omega, \vec{k}) = \frac{1}{2|\vec{k}|} \left[ \rho(\omega, \vec{k}) + \frac{1}{2B} \int \frac{d\zeta d\vec{p}}{(2\pi)^{3/2}} \frac{\zeta \vec{k} \delta \vec{p} \rho(\omega - \zeta, \vec{k} - \vec{p}, t) \rho(\zeta, \vec{p}, t)}{\vec{p}^2 + (\zeta^2 + \vec{p}^2)} \right] \] (6.8).

The first term is the Coulomb term and the subsequent terms are corrections due to Landau level mixing. Now the appropriate current operator as obtained from (6.1) is
\[ J = i \rho \partial_z \left[ a_0 + \frac{1}{B} \epsilon_{0ij} \{ a^i \partial^j a^0 - \frac{1}{2} a^i \partial_0 a^j \} \right] \] (6.9).

From this it is quite clear that we need \( a_0 \) beyond the leading order only in the first term in (6.9), and \( a^i \) is never required beyond what is given in (6.7). Thus using (6.7), (6.8) and (6.9), we obtain the current operator in this case beyond the L.L.L. approximation.

**VII. Including a local four-fermi term**

A local four-fermi term is the ultralocal limit of a density-density type interaction.
\[ H_{\text{int}} = \frac{g}{2} \int d\vec{x} \left[ (\psi(\vec{x})^\dagger \psi(\vec{x}))^2 \right] \] (7.1).

In the previous two sections, we have utilised the Maxwell equations to obtain corrections to a density-density type of interaction even though the fermionic part of the theory was bilinear in the fermionic fields. In this case, we may use the extremely well-known auxilliary field method to render the theory bilinear in the fermionic field. Using an auxilliary field \( \sigma(\vec{x}, t) \), we rewrite the above as
\[ H_{\text{int}} \to \int d\vec{x} \left[ (\psi(\vec{x})^\dagger \psi(\vec{x})) \sigma(\vec{x}) - \frac{1}{2g} \sigma^2(\vec{x}) \right] \] (7.2).

Thus as far as the fermionic integration is concerned, we are shifting \( V \) in (3.9) to \( V + \sigma \). For simplicity, we set \( V \to 0 \) in the effective action. The Euler-Lagrange equation for \( \sigma \) may be solved iteratively as in the previous cases. To leading order, \( \sigma(\vec{x}, t) = g \rho(\vec{x}, t) \), as anticipated. When this is re-inserted in the equation of motion for \( \sigma \), we obtain
\[ \sigma = g \left[ \rho + \frac{g}{\omega} \{ \partial_z (\rho \partial_z \rho) + \partial_z (\rho \partial_z \rho) \} \right] \] (7.3).
This in turn, inserted into (4.5), yields the correction to the current operator in the presence of the four-fermi term due to mixing with higher Landau levels.

VIII. Edge Current in a Quantum Hall Droplet

Given the expression for the current operator obtained earlier, one could directly address the issue of the edge-current in a finite quantum Hall droplet. The idea of using a confining potential to create a finite droplet has already been elaborated upon in [6]. Let us consider the potential to be of the form $V = V_0 + U$, where $V_0 = Ex$. Further, imagine momentarily that $U$ has been set to zero. In that case, the effective Hamiltonian to leading order is given by

$$H^{(0)}_{\text{eff}} = E \int dX \psi^\dagger(X) X \psi(X)$$  \hspace{1cm} (8.1)

Here, we have used the basis where $\hat{X}$ is diagonal. Further,

$$\{\psi(X), \psi^\dagger(X')\} = \delta(X - X')$$  \hspace{1cm} (8.2)

We define the droplet to be such that all the single-particle states up to the zero-energy state are filled. Thus, $X = 0$ is the Fermi surface. The corresponding many-body ground state is

$$|G\rangle \equiv \prod_{X \leq 0} \psi^\dagger(X) |0\rangle$$  \hspace{1cm} (8.3)

As discussed in [6], the density operator, normal-ordered with respect to this new ground state is given by

$$\rho(z, \bar{z}, t) = \int_{-\infty}^{\infty} dX \int_{-\infty}^{\infty} dX' [\psi^\dagger(X, t) \psi(X', t) : + \Theta(-X) \delta(X - X')] \langle X|z\rangle \langle \bar{z}|X'\rangle e^{-|z|^2}$$  \hspace{1cm} (8.4)

The first term gives the contribution of the interior. The second term is the edge contribution. From (4.5) and (8.4), we see that

$$J_{\text{edge}}^y(x, y, t) \simeq -\frac{E}{B} : \rho(x, y, t) :$$  \hspace{1cm} (8.5)
where : $\rho$ : denotes the second term in (8.4). From (2.17) it is clear that : $\rho$ : is peaked sharply at $x = 0$. Thus,

$$J_\text{edge}^y(y,t) \simeq -\frac{E}{B} : \psi(y,t)\psi(y,t) :$$  \hspace{1cm} (8.6)

to leading order, where

$$\{\psi(y), \psi(y')\} = \delta(y - y')$$  \hspace{1cm} (8.7).

This is just a chiral current moving with a velocity $v = \frac{E}{B}$ in agreement with [7], [8].

### IX. Conclusion

In this paper, we have studied how the interactions of fermions in the L.L.L. get modified due to mixing with higher Landau levels. Specifically, we have concentrated on the current operator and have deduced its form for a variety of interactions, including the long-range Coulomb interaction, electromagnetic interactions and contact interaction between the fermions. We have given a method of systematically including corrections arising from Landau level mixing. For electromagnetic interactions, these corrections introduce $1/r^2$, $1/r^3$ and higher modifications to the fundamental $1/r$ interaction and this is reflected in the current and density profiles of fermions, e.g. in the quantum Hall effect [6]. It is interesting to note that Landau level mixing introduces three-body and higher interactions between the fermions when the interactions are at most two-body in the absence of mixing. In conclusion we should also mention that the expression for the L.L.L. current has enabled us to extract the form of the edge-current operator in the case of a Hall sample of finite geometry.

### Acknowledgements

R.R. wishes to express his sincere appreciation of the discussions he had on this subject with V.P. Nair. He also thanks B. Sakita for introducing him to the exciting topic of L.L.L. fermions. G.G. and R.R. thank the National Science Foundation for partial support while this work was in progress.
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