QUANTUM VERSION OF GAUGE INVARIANCE AND NUCLEON INTERNAL STRUCTURE

Fan Wang and Wei-Min Sun
Department of Physics, Nanjing University, Nanjing, 210093, China

Xiao-Fu Lü
Department of Physics, Sichuan University, Chengdu, 610064, China

The conflict between canonical commutation relation and gauge invariance, which both the momentum and angular momentum of quark and gluon should satisfy, is clarified. The quantum version of gauge invariance is studied. The gauge independence of the matrix elements of quark momentum and angular momentum operators between physical states are proved. We suggest to use the canonical quark momentum and angular momentum distributions to describe the nucleon internal structure in order to establish an internal consistent description of hadron spectroscopy and hadron structure. The same problem for the atomic spectroscopy and structure is discussed.

I. CONFLICT BETWEEN CANONICAL COMMUTATION RELATION AND GAUGE INVARIANCE

The nucleon (atom) is a QCD (QED) gauge field system. The momentum and angular momentum of the nucleon (atom) is the sum of contributions from quark (electron) and gluon (photon) respectively:

\[ \vec{P} = \int d^3x \psi^\dagger \vec{\nabla} \psi + \int d^3x E_i \vec{\nabla} A_i. \]  

\[ \vec{J} = \frac{1}{2} \int d^3x \psi^\dagger \vec{\Sigma} \psi + \int d^3x \psi^\dagger \vec{r} \times \vec{\nabla} \psi + \int d^3x \vec{E} \times \vec{A} + \int d^3x E_i \vec{r} \times \vec{\nabla} A_i \]  

In the above equations, \( \psi \) is the quark (electron) field, \( \vec{E} \) is the color electric (ordinary electric) fields, \( \vec{A} \) is the vector potential. In QCD case a summation over color indices is understood. The good side of the above decomposition is that each term in Eq.(1,2) satisfies the canonical commutation relation of the momentum and angular momentum operator, so they are quark and gluon momentum, quark spin and quark orbital angular momentum, gluon spin and gluon orbital angular momentum, respectively. However they are not gauge invariant individually except the quark (electron) spin term.

Alternatively one can derive a gauge invariant decomposition,

\[ \vec{P} = \int d^3x (\psi^\dagger \vec{D} \psi + \vec{E} \times \vec{B}), \]  

\[ \vec{J} = \int d^3x (\frac{1}{2} \psi^\dagger \vec{\Sigma} \psi + \psi^\dagger \vec{r} \times \vec{D} \psi + \vec{r} \times (\vec{E} \times \vec{B})). \]

The good side of this decomposition is that each term is gauge invariant. However they do not satisfy the canonical commutation relation individually except the quark (electron) spin term.\[ ^1 \]

In classical gauge field theory, only gauge invariant quantities are physically meaningful. In the study of nucleon internal parton momentum and angular momentum structure, also only the gauge invariant operators related to quark and gluon momenta, spin and orbital angular momenta are appreciated\[ ^2 \]. In hadron spectroscopy, partial wave analysis and multi-pole radiation are widely used where the gauge non-invariant canonical momentum and orbital angular momentum must be used accordingly.

Canonical momentum and orbital angular momentum have been used in describing atomic structure for almost a century already. Are the atomic electron momentum and orbital angular momentum not measurable ones? Can these operators be used to describe the nucleon internal structure? In this report we show that the canonical quark (electron) momentum and orbital angular momentum have gauge independent matrix elements between physical states and so is observable, which should be used to establish an internal consistent description of hadron spectroscopy and hadron internal structure.
II. QUANTUM VERSION OF GAUGE INVARIANCE

F. Strocchi and A.S. Wightman studied the quantum version of gauge invariance. A gauge (or a quantization scheme) in a quantum gauge field theory is specified by:

(a) field operators: \( A_\mu \), the gauge potential; \( j_\mu \), the gauge interaction current; \( \psi \), the fermion field and other fields of the gauge in a Hilbert space \( H \);

(b) a representation \( U \) of the Poincaré group in \( H \);

(c) a sesquilinear form (Gupta scalar product) \( \langle \Phi, \Psi \rangle \) on \( H \) with respect to which \( U \) is unitary, \( \Phi \) and \( \Psi \) are vectors in \( H \);

(d) a distinguished subspace \( H' \subset H \) such that

(i) The restriction of the sesquilinear form to \( H' \) is bounded and nonnegative

\[ \langle \Psi, \Psi \rangle \geq 0 \quad \text{for} \quad \Psi \in H'. \]

(ii) The analogue of the Maxwell equation holds in the sense that

\[ \langle \Phi, (\partial_\mu F^{\mu\nu} - j^\nu) \Psi \rangle = 0 \quad (5) \]

for all \( \Phi, \Psi \in H' \).

(iii) \( H' \) has a subspace \( H'' \) consisting of vectors \( \Phi \) in \( H' \) of zero length \( \langle \Phi, \Phi \rangle = 0 \). The physical Hilbert space is \( H_{\text{phys}} = H'/H'' \).

There exists a unique vector \( \Psi_0 \), called the vacuum, which is invariant under the translation subgroup of the Poincaré group. The vector \( \Psi_0 \) lies in \( H' \).

This is a generalization of the Gupta-Bleuler quantization scheme of QED. A generalized gauge transformation is an ordered pair consisting of two gauges

\[ < A_1, H_1, \cdot, \cdot >_1, \cdot, H'_1 > \]

and

\[ < A_2, H_2, \cdot, \cdot >_2, \cdot, H'_2 > \]

together with a bijection \( g \) of \((H)_{1\text{phys}}\) onto \((H)_{2\text{phys}}\)

\[ [\Psi_2] = g[\Psi_1], [\Phi_2] = g[\Phi_1] \]

\[ [\Psi_{20}] = g[\Psi_{10}] \]

Note that in the quantum version there is no need of the form of classical gauge invariance, such as \( F_1^{\mu\nu} = F_2^{\mu\nu} \). Only under some special gauge transformation, one has such a gauge invariant form. Instead the gauge invariance of an operator is classified into four categories, i.e., gauge independence, weak gauge invariance, gauge invariance, and strict gauge invariance.

An operator \( O \), mapping \( H \) into \( H \), is called gauge independent if

\[ \langle \Phi, O\Psi \rangle = \langle \Phi + \chi_1, O(\Psi + \chi_2) \rangle \quad (6) \]

for all \( \Phi, \Psi \in H' \) and all \( \chi_1, \chi_2 \in H'' \). In other words, the matrix elements \( \langle \Phi, O\Psi \rangle \) for \( \Phi, \Psi \in H' \) depend only on the equivalence classes \( \Phi, \Psi \in H_{\text{phys}} \). Such a gauge independent operator is an observable. The stronger restricted operators, weakly gauge invariant, gauge invariant and strictly gauge invariant ones are all gauge independent ones, obviously they are observable. The classically gauge invariant ones belong to the strictly gauge invariant category. Quantum gauge field theory includes more observable operators than the classical one.

III. QUARK (ELECTRON) MOMENTUM AND ORBITAL ANGULAR MOMENTUM ARE OBSERVABLE

To prove the quark (electron) momentum and orbital angular momentum are observable, one has to prove they are gauge independent, i.e., one has to prove

\[ \langle (\Phi + \chi_1), \vec{P}_q(\Psi + \chi_2) \rangle = \langle \Phi, \vec{P}_q\Psi \rangle, \]

\[ \langle (\Phi + \chi_1), \vec{J}_q(\Psi + \chi_2) \rangle = \langle \Phi, \vec{J}_q\Psi \rangle, \quad (7) \]

or equivalently to prove

\[ \langle \chi_1, \vec{P}_q\chi_2 \rangle = 0, \quad \langle \Psi, \vec{P}_q\Psi \rangle = 0, \quad \langle \chi_1, \vec{P}_q\Psi \rangle = 0, \quad (8) \]
and the same for $J_q$, where $Ψ \in H$ and $χ_1, χ_2 \in H'$. All states $χ$ of $H'$ can be expressed as $\sum_n a_n (\partial µ A^µ)^n |Ψ_{phys}\rangle$, so one has to consider

$$\text{out}\langle Φ| \int \! d^3 y \psi^\dagger(y) \frac{\vec{∇}}{\vec{l}} \psi(y) \partial µ A^µ(x) |Ψ_{in}\rangle,$$

which in the interaction representation can be written as

$$\langle Φ| T( \int \! d^3 y \psi^\dagger(y) \frac{\vec{∇}}{\vec{l}} \psi(y) \partial µ A^µ(x) S) |Ψ\rangle,$$

where $T$ is time-ordering operator, $S$ is the scattering operator. Expanding the scattering operator as usual, one has

$$S = \sum_n \frac{(-i)^n}{n!} \int dx_1 \cdots dx_n T \mathcal{H}_I(x_1) \cdots \mathcal{H}_I(x_n),$$

where $\mathcal{H}_I(x) = -j_µ A^µ(x)$. The physical states contain only transverse gluons (photons), so the operator $\partial µ A^µ(x)$ must be contracted with one $A^ν(y)$ in the interaction term $\mathcal{H}_I(x)$. This will give rise to a term

$$\partial^µ A^µ(x) A^ν(y) = \partial^µ \{ g^{µν} D(x - y) \} = \partial^ν D(x - y) = -\partial^ν D(x - y),$$

here the symbol $\overbrace{A^µ(x) A^ν(y) = g^{µν} D(x - y)}$ means contraction. Then using integration by parts one can move the differential operator $\partial_ν$ to act on $j_ν$ in the interaction term $\mathcal{H}_I(y)$ and use current conservation $\partial_ν j^ν = 0$ to prove Eq.(10) = 0. The other terms of Eq.(8) can be proved to be zero in the same way. Thus one has proved the quark (electron) momentum and orbital angular momentum operators are gauge independent and so are observable.

IV. CONCLUSION

The conflict between canonical commutation relation and gauge invariance of the quark (electron) momentum and angular momentum operators in a nucleon (atom) can be remedied by using gauge non-invariant canonical quark (electron) momentum and orbital angular momentum operators because they are gauge independent ones. This problem has been discussed by us since 1998[4]. Here we verified the results there by an alternative argument.

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