Distance between a Vertical Solid Wall and a Falling Insulating Sphere in a Conductive Liquid under the Imposition of a Horizontal Static Magnetic Field

Kazuhiko IWAI,¹ Keisuke KUMAZAWA² and Ippei FURUHASHI³

¹) Eco-Topia Institute, Nagoya University, Furo-cho, Chikusa-ku, Nagoya, 464-8603 Japan. E-mail: d42859a@cc.nagoya-u.ac.jp
²) Formerly Graduate Student of Nagoya University. Now at Panasonic Corporation.
³) Formerly Graduate Student of Nagoya University.

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For the clarification of falling behavior of an electrically insulating spherical second phase in the vicinity of a wall under the imposition of a horizontal static magnetic field, distance between the wall and a falling Polytetrafluoroethylene spherical ball in a saturated sodium chloride aqueous solution was observed under the imposition of the horizontal magnetic field where magnetic field intensity and its direction, and electrical conductivity of the wall were chosen as experimental parameters. The obtained distances were statistically analyzed. As the results, the distance when the horizontal magnetic field direction was parallel to the wall, and the distance when the wall was electrically insulating, were the same with the distance when the magnetic field was not imposed. On the other hand, the distance was smaller than the distance when the magnetic field direction was perpendicular to the electrically conductive wall.

KEY WORDS: magnetic field; second phase; electromagnetic force; solid wall; electrical conductivity.

1. Introduction

An electrically insulating second phase in molten steel such as a gas bubble and an inclusion is usually separated by their density difference in a continuous casting process. Separation of a small second phase is difficult because the rising (falling) velocity is inversely proportional to the square of the second phase radius. This induces its trap into products. Thus, motion of the molten steel is controlled by an electromagnetic brake in an industrial process. For precise control of the molten steel flow or for decrease of defects, clarification of second phase behavior under the imposition of a horizontal static magnetic field is required.

A water model experiment is a traditional experimental method for elucidation of the second phase behavior. However, an electromagnetic force can not be excited in this experimental system because water is essentially an insulating material. On the other hand, C. Zhang, S. Eckert et al.¹,² observed argon gas behavior in a molten alloy with low melting point under the imposition of a magnetic field in which the electromagnetic force was induced. They measured velocity of gas bubbles using ultrasonic wave. Furthermore, they investigated difference of the alloy velocity distributions in a mold with and without the magnetic field imposition using a simulator of a continuous casting machine.³ Furuhashi et al.⁴,⁵ constructed a model experimental system for observation of the second phase in which a strong magnetic field produced by a super-conducting magnet was imposed on a transparent conductive liquid, and they observed an argon gas behavior. Numerical simulation is another tool to clarify the second phase behavior. Thus, bubble behaviors in a conductive liquid under the imposition of the magnetic field have been investigated.⁶,⁷

The second phase behavior in the vicinity of an interface between a liquid and a solid is also important to reduce trap of the second phase. Thus, a lot of investigations have been done.⁸–¹¹ Focused parameters in these investigations were temperature distribution, concentration distribution, curvature, gradient of interfacial tension and so on. However, a magnetic field has not been taken up as a parameter. Under the imposition of the magnetic field, the second phase behavior in the vicinity of the solid-liquid interface in the continuous casting process of the steel must be affected by the difference of the electrical conductivity between the solid and the liquid, and the direction of the magnetic field and so on, because they affect electromagnetic force induced by the motion of the second phase.

In this study, an electrically insulating second phase falling behavior in an electrically conductive liquid in the vicinity of a solid wall has been investigated to clarify the effect of electrical conductivity of the solid wall and direction of the magnetic field.

2. Experiment

2.1. Experimental Method

Experimental setup is shown in Fig. 1. Two vessels were prepared. The first one was an acrylic vessel with 54 mm inner width and 64 mm inner depth. The second vessel was the same dimension with the first one while one side wall
of 54 mm width was made of brass with 2 mm thickness while the other parts were made of acryl. The electrical conductivity of the brass is $1.6 \times 10^7$ S/m while the acryl is an electrically insulating material. A pipe of 180 mm length with an inner square cross section of 4 mm*4 mm was set at the top of the side wall of 54 mm width. One of its side faces was the side wall of the vessel made of acryl or brass and the other three side faces were made of acryl as shown in Fig. 1. A saturated sodium chloride aqueous solution whose electrical conductivity is 22 S/m was filled in the vessel. Its free surface was 165 mm below from the top of the vessel. Thus, the lower 15 mm part from the bottom of the pipe was submerged in the solution. Because there were many holes in the side walls of the pipe, the solution can easily flow across the side walls of the pipe. We dropped a Polytetrafluoroethylene spherical ball with 3.17 mm diameter into the vessel through the top of the pipe with or without the imposition of a horizontal magnetic field. Intensity of the magnetic field was 7.5T or 0T while its direction was parallel (B//) or perpendicular (B┴) to the side wall made of acryl or brass. The falling motion of the Polytetrafluoroethylene spherical ball was recorded using a video camera through the front wall of 64 mm inner depth.

The distance between the side wall and the ball was between 0 mm and 0.83 mm at the pipe bottom. This gradually increased with the drop of the ball under the all experimental conditions. The horizontal position, x from the side wall to the ball at the horizontal level of 30 mm below from the pipe bottom was measured using the recorded data. Reynolds number in this experiment was roughly a few hundreds, thus the behavior of the falling ball was in transition regime from the hydrodynamic viewpoint.

2.2. Experimental Results

Distributions of the measured horizontal position of the ball from the wall are shown as histograms in Fig. 2 with the dropping numbers of the Polytetrafluoroethylene spherical ball in the case that the magnetic field was parallel to the 54 mm width side wall, while those when the magnetic field was perpendicular to the side wall are shown in Fig. 3. The minimum position was 1 mm while the maximum position was 7 mm. The former was observed only when the 7.5T magnetic field was imposed perpendicular to the brass side wall. On the other hand, the maximum value of 7 mm was obtained except the condition that the 7.5T magnetic field was imposed perpendicular to the side wall. Since the horizontal position at the bottom of the pipe was less than 0.83 mm, some force acted on the Polytetrafluoroethylene spherical ball in the direction to increase the distance between the wall and the ball, though we do not know what force acted on the ball. These data were statistically analyzed using analysis of variance.

3. Analysis of Variance

3.1. Homogeneity of Variance

Analysis of variance¹ is a general method to judge whether differences among more than three average values are significant or not. Both of random sampling and normal distribution of population must be satisfied in this experiment. Thus, we firstly examined whether homogeneity of
variance was valid or not using Bartlett’s test.\(^{14}\) The result is shown in Table 1. As the conclusion, homogeneity of variance was valid at the significant level of \(\alpha=0.05\) in the both cases of the parallel and perpendicular imposition of the magnetic field to the side wall, because the calculated \(\chi^2\) square statistics were smaller than the \(\chi^2_{0.05} = 7.81\). Therefore, the data obtained in this experiment were satisfied precondition for the analysis of variance.

3.2. Analysis of Variance

Analytical method of variance depends on whether sample numbers in each group are equivalent or not, and also depends on whether correspondence of data in each group exists or not. The method we adopted was that sample numbers in each group were not equivalent and correspondence of data did not exist, in which we considered two factors of the magnetic field imposition and of the wall material.

First, null hypothesis that averages of the all groups were equivalent was assumed. Then \(F\) statistic was calculated. And the critical \(F\) statistics \(F_{0.05}\) at the significant level of \(\alpha=0.05\) and \(F_{0.001}\) at the significant level of \(\alpha=0.001\) were obtained from a \(F\)-distribution table. The critical \(F\) statistic, \(F_{0.05}\) is 3.91 and \(F_{0.001}\) is 11.3. Then these were used to judge whether the null hypothesis was rejected or not. The results are shown in Table 2. The “****” in this Table means that the averages in each group are not equivalent at the significant level of \(\alpha=0.001\).

When the magnetic field was parallel to the side wall, main effect of the magnetic field imposition, main effect of the wall material and interaction of the magnetic field imposition and the wall material were not significant. On the other hand, all of these effects were significant when the magnetic field was perpendicular to the side wall at the significant level of \(\alpha=0.001\).

Simple main effect test was done to examine whether significant difference existed or not among average values in each group when the magnetic field was perpendicular to the wall. The results are shown in Table 3. The effect of the magnetic field imposition was not significant when the wall material was acryl, while it was significant at the significant level of \(\alpha=0.001\) when the wall material was brass. And the effect of the wall material was not significant when the magnetic field was not imposed, while it was significant at the significant level of \(\alpha=0.001\) when the 7.5T magnetic field was imposed. Therefore, we can conclude that the horizontal position of the Polytetrafluoroethylene spherical ball from the side wall decreased in the case that the 7.5T magnetic field was imposed perpendicular to the brass wall in comparison with other experimental conditions, even though the change of the distance between the ball and the side wall from 0 mm to 0.83 mm at the pipe bottom was included in this analysis. Thus, some force might act on the ball in the direction to decrease the distance between the wall and the ball under this condition.

4. Discussions

The horizontal position of the Polytetrafluoroethylene spherical ball was small only when the magnetic field was imposed perpendicular to the brass side wall. This implies that electromagnetic force affected the motion of the Polytetrafluoroethylene spherical ball only in this condition. Its magnitude and direction must change during the dropping. However, its impulse essentially suppressed the increase of the distance between the ball and the wall. Thus, we discuss the effect of setting direction of a wall and its conductivity on eddy current path around a sphere because it induces the electromagnetic force.

Velocity potential, \(\Phi\) for a system in which a sphere of radius \(R\) falls with velocity \(U\) in an infinitely extended fluid is given as\(^{15}\)

\[
\Phi = -\frac{1}{2}(U \cdot r)\frac{R^3}{|r|} \quad \text{(1)}
\]

where \(r\) is a positional vector.

The stream line is schematically shown in Fig. 4. The fluid motion is upflow around the side of the sphere while it is downflow around the top and the bottom of the sphere. From this result and the eddy current distribution numerically calculated by Takatani,\(^{5}\) eddy current distribution around an insulating sphere falling in an electrically conductive liquid with a horizontal static magnetic field is esti-

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**Table 1.** \(\chi^2\) square statistic obtained by Bartlett’s test.

| Direction of magnetic field | B/side wall | B \(\perp\) side wall |
|----------------------------|-------------|----------------------|
| \(\chi^2\)                | 3.3         | 6.5                  |

**Table 2.** Calculated \(F\) statistics.

| Factor                          | B/side wall | B \(\perp\) side wall |
|--------------------------------|-------------|----------------------|
| Main effect of magnetic field   | 0.086       | 39.0****             |
| Main effect of wall material    | 0.263       | 14.6****             |
| Interaction of magnetic field and wall material | 0.473 | 13.4**** |

****\(\alpha=0.001\)

**Table 3.** Simple main effect test.

| Level                | Factor       | Horizontal position |
|----------------------|--------------|---------------------|
| Wall material: acryl | Magnetic field | 3.35                |
| Wall material: brass | Magnetic field | 49.1****           |
| Magnetic field: 0T   | Wall material | 0.012               |
| Magnetic field: 7.5T |              | 27.9****            |

****\(\alpha=0.001\)
mated as shown in Fig. 5. The eddy current flows from around the position A in Fig. 5 toward around the position B horizontally, and returns to around the position A through around the position C or around the bottom of the sphere. Because direction of the electromagnetic force caused by the interaction between the imposed magnetic field and the eddy current is opposite to the direction of the fluid motion around the positions B and C, the falling velocity of the sphere might be suppressed under the imposition of the horizontal magnetic field.

Current distribution around the sphere might change if the sphere falls down in the vicinity of a wall as shown in Fig. 6. When the wall is set parallel to the magnetic field, the current path does not change since the current hardly flows in the vicinity of the wall. The current path when the insulating wall is set perpendicular to the magnetic field is essentially the same path with that when the wall does not exist since the current can not flow in the wall. Therefore, behavior of the sphere is not affected by setting a wall in these cases. On the other hand, when the conductive wall is set perpendicular to the magnetic field, the current path changes because some of the current returns through the wall. Therefore, behavior of the sphere might be affected by setting a conductive wall perpendicular to a magnetic field. The consideration mentioned here agrees with the experimental results.

We propose a simple two dimensional model in which an insulating second phase falls in the vicinity of an electrically conductive wall under the magnetic field imposition perpendicular to the wall. Its physical model and equivalent electrical circuit are shown in Fig. 7. Electrically conductive fluid between the wall and the second phase flows outward from the page. Thus, the electromotive force, $V_1$ between the wall and the second phase induced by the interaction between the fluid motion and the magnetic field causes current flow from the position $P$ to the position $Q$. Some of the current flowing from the positions $P$ to $Q$ returns through the path I while the other goes back through the path II. The current flowing in the path I corresponds to the current flowing in the vicinity of the top or the bottom of the second phase. That in the path II corresponds to the current flowing in the conductive wall. $V_0$ indicates the electromotive force of the path I, while $R_1$, $R_0$, and $R_2$ indicate the resistances between the positions $P$ and $Q$ and those of the path I and the path II, respectively. The currents $I_0$ and $I_2$ flowing in the paths I and II were obtained using Kirchhoff’s law.

$$I_0 = \frac{V_0}{R_0} \cdot \frac{1}{\frac{R_1}{q_1} + \frac{R_2}{q_2} + q_1 q_2} \quad \text{(2)}$$

$$I_2 = \frac{V_0}{R_0} \cdot \frac{1 - \frac{R_1}{q_1} - \frac{R_2}{q_2} + q_1 q_2}{\frac{R_1}{q_1} + \frac{R_2}{q_2} + q_1 q_2} \quad \text{(3)}$$

where $q_1$ and $q_2$ are non-dimensional resistance, and $\omega$ is non-dimensional electromotive force, respectively, and these are defined as follows.

$$q_1 = \frac{R_1}{R_0} \quad \text{(4)}$$

$$q_2 = \frac{R_2}{R_0} \quad \text{(5)}$$

The electromotive forces $V_1$ and $V_0$ must be positive under the consideration of the velocity distribution mentioned above. That is, the non-dimensional electromotive force

$$\omega = \frac{V_1}{V_0} = \frac{V_0}{V_0 + V_1} \quad \text{(6)}$$
force \( \omega \) ranges from zero to unity. Thus, the current \( I_0 \) is always positive.

The current \( I_2 \) is expressed as a function of non-dimensional resistance \( q_2 \) as shown in Fig. 8. The current \( I_2 \) is zero if Eq. (7) is satisfied.

\[
\omega (1+q_1) = 1 \tag{7}
\]

When the left hand side of Eq. (7) is smaller than unity, the current \( I_2 \) is positive. On the other hand, the current \( I_2 \) is negative when it is larger than unity. In both the cases, the magnitude of the current \( I_2 \) decreases with increase of the non-dimensional resistance \( q_2 \).

The distribution of the current \((I_0+I_2)\) flowing in the gap between the wall and the second phase is shown in Fig. 9. The current is always positive. However, it monotonically decreases with increase of the non-dimensional resistance \( q_2 \) when the left hand side of Eq. (7) is smaller than unity, while it monotonically increases when it is larger than unity.

Parallel components of the currents \((I_0+I_2)\), \( I_2 \) and \( I_0 \) induce attractive or repulsive force among them. This force is inversely proportional to the square of the distance. The distance between the second phase and the wall was less than 0.83 mm at the bottom of the pipe which corresponds to the distance between the currents \((I_0+I_2)\) and \( I_2 \), while the second phase radius was 1.585 mm which corresponds to the distance between the currents \((I_0+I_2)\) and \( I_0 \) in this experiment. Thus, we consider only the interaction between the currents \((I_0+I_2)\) and \( I_2 \). Their product is proportional to the force acting between the currents \((I_0+I_2)\) and \( I_2 \). It is expressed as Eq. (10), and its values when the non-dimensional resistance \( q_2 \) approaches zero and that when \( q_2 \) approaches infinity are expressed as Eqs. (11) and (12).

\[
I_2(I_0+I_2) = \frac{V_0^2}{R_0^2 q_1^2} \frac{(q_1 \omega - 1)(\omega - q_2 - 1)}{\omega^2 (q_1 + q_2 + q_1 q_2)} \tag{10}
\]

\[
I_2(I_0+I_2) \bigg|_{q_2=0} = \frac{V_0^2}{R_0^2 q_1^2} \frac{(\omega - 1)}{\omega^2} \tag{11}
\]

\[
\lim_{q_2 \to +\infty} I_2(I_0+I_2) = 0 \tag{12}
\]

The product is shown in Fig. 10 as a function of the non-dimensional resistance \( q_2 \). The \( I_2(I_0+I_2) - q_2 \) curve depends on the non-dimensional electromotive force \( \omega \) and the non-dimensional resistance \( q_1 \). The force is always repulsive when the left hand side of Eq. (7) is smaller than unity. On the other hand, the force is always attractive when the left hand side of Eq. (7) is larger than unity. The latter condition is satisfied if the electromotive force between the second phase and the wall is small. That is, slow velocity between the second phase and the wall is suitable for excite the attractive force. This attractive force region is divided into two regions.

When Eq. (13) is satisfied, the attractive force monotonically decreases with increase of the non-dimensional resistance \( q_2 \), while the attractive force has a extremal value when Eq. (14) is valid.

\[
\frac{1}{1+q_1} < \omega < \frac{1}{2} \left(1 + \frac{1}{1+q_1}\right) \tag{13}
\]

\[
\frac{1}{1+q_1} < \omega < \frac{1}{2} \left(1 + \frac{1}{1+q_1}\right) \tag{14}
\]
\[ \frac{1}{2} \left( 1 + \frac{1}{1 + q_1} \right) < \omega < 1 \] ................. (14)

The attractive or repulsive force approaches zero if resistance of the wall drastically increases in all cases. In the latter two cases, the attractive force acts between the wall and the fluid in the gap between the wall and the second phase. Thus, the second phase is also attracted toward the wall. The experimental condition where the magnetic field was perpendicular to the conductive wall might be satisfied this condition.

5. Conclusions

For the clarification of falling behavior of an insulating spherical second phase in the vicinity of a wall under the imposition of a horizontal magnetic field, distance between the wall and a Polytetrafluoroethylene sphere falling in a saturated sodium chloride aqueous solution was observed under the imposition of the horizontal magnetic field where imposition of a magnetic field, setting direction of a wall and its electrical conductivity were chosen as experimental parameters.

The obtained results are as follows.

(1) The distance under the imposition of the horizontal magnetic field parallel to the wall was the same with the distance when the horizontal magnetic field was not imposed. And it was independent of the wall conductivity.

(2) The distance under the imposition of the horizontal magnetic field perpendicular to the insulating wall was the same with the distance when the horizontal magnetic field was not imposed.

(3) The distance under the imposition of the horizontal magnetic field perpendicular to the conductive wall was smaller than the distance when the horizontal magnetic field was not imposed.

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