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An IT2FS-PT³ based emergency response plan evaluation with MULTIMOORA method in group decision making

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A B S T R A C T

The eruption of COVID-19 at the beginning of 2020 has sounded the alarm, making experts pay more attention to public health emergency events. A suitable emergency response plan plays a vital role in handling emergency events. Therefore, this paper focuses on the evaluation of emergency response plans among a set of group in the comprehensive prospect, and an emergency decision making method integrated with the interval type-2 fuzzy information based on the third generation prospect theory (PT³) and the extended MULTIMOORA method is proposed. Individuals express their preferences using some given linguistic terms set. Furthermore, considering the conflicts may occur in the group, a convergent iterative algorithm is designed for group consensus reaching. Then, the stochastic multi-criteria acceptability analysis (SMAA) method and the Borda Count (BC) method are generated to combine the results instead of the dominance theory in MULTIMOORA system. Finally, based on the background of the COVID-19 pandemic from Wuhan, a case study about the selection of emergency response plan and the corresponding sensitivity and comparative analysis are exhibited to explain the effectiveness of the proposed method.

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1. Introduction

Emergency events refer to any situation arising from sudden and unforeseen catastrophe that may cause casualties, economic losses, environmental damage, and serious social harm [1]. In China, kinds of emergency events have had severely negative impact on people's life and social development, especially for the public health emergency events, e.g. the outbreak of SARS in 2003, the Wenchuan Earthquake in Sichuan, China on May 12, 2008. Recently, the new corona virus (COVID-19) in the end of 2019 caused huge loss of lives and properties. More seriously, the confirmed and suspected cases of COVID-19 have been increasing without ending not only in Wuhan, Hubei Province, but also China and the world. As early as January 30, 2020, the World Health Organization (WHO) announced COVID-19 as the eruption a public health emergency of international concern. In the meanwhile, China declared a travel quarantine of Wuhan and other 16 cities, encompassing a population of 45 million [2]. By October 18, 2021, a total of 4,512 fatalities and 68,303 laboratory-confirmed cases had been reported in Hubei Province. With the continuous exponential increase of the number of pandemic cases, the Chinese government has taken powerful preventive measures followed by the highest level of class A infectious diseases based on the Law of the People's Republic of China on Prevention and Treatment of Infectious Diseases (LPTID) [3]. China has imposed strict restrictions on public activities containing a large number of people, thus reducing the possibility of virus transmission, which exists a negative impact on economy development and caused the unemployment on a certain scale.

Therefore, researchers proposed the corresponding emergency response plans to minimize destructive consequences for people's healthy life and the normal development of society. Apparently, decision makers (DMs) from emergency department are supposed to select the suitable and desirable emergency response plan when tackling with these devastating hazards. Generally speaking, the assessments of emergency response plans are complicated, owing to the uncertain and partial information of the catastrophic scenarios and the multi-perspectives of DMs. Hence, many researchers have focused on this topic and made great contributions [1,4–9]. For instance, Hämäläinen et al. [1] used a multi-attribute utility theory analysis in a simulated nuclear emergency, Levy and Taji [4] discussed the hazard planning and emergency management in the group analytic network process (GANP). Ferreira et al. [5] presented a new urban fire emergency plan assessment method using the integrated geographical information system (GIS) tool. Shamim et al. [6] integrated the
Delphi technique with a mathematical model to qualify the performance of emergency planning response (EPR) in a real case of a major accident in the process industry. Liu et al. [9] dealt with the decision making in emergency response plans about the simulation of H1N1 infectious diseases based on Fault Tree Analysis. These studies mentioned above provided various decision making methods for the selection of the optimal emergency response plan. Whereas, in the existing literatures about emergency response plans evaluation, the characteristics of the DMs are seldomly considered. It is worth noting that the decision making process in emergency events tend to be vague and uncertain due to the inadequate information. A substantial body of psychological studies on human behaviors show that DMs incline to express the reference dependence, preference reversal and other typical characteristics in the circumstance of fuzzy and uncertain events [10–13]. Hence, the psychological behaviors of DMS should be taken into consideration.

Since, Kahneman and Tversky [10] introduced the alternative model, named prospect theory (PT) to replace the expected utility theory considering human behaviors, the behavioral decision making theories showed rapid development, such as the regret theory [14]. Afterwards, they extended the original PT called cumulative prospect theory (CPT) with using the cumulative decision weights [12], inspired by the rank and sign dependent utility(RSDU) by Luce and Fishburn [15]. Then, Schmidt et al. [13] founded that both PT and CPT have a common limitation: the reference points in the prospects are supposed to be certainties, which cannot be applied to solve this type of situations: DMs purchased lotteries and had the chance to sell or exchange them. Hence, they expanded these theories and proposed the third generation prospect theory (PT3), which retained the power of previous version of PT and increased new proposals: the value-maximal buying prices (WTP) and minimal selling prices (WTA). It is easy to see that PT3 has the wider range to solve various decision making problems in risk and uncertain environment [16–18]. For instance, Wang et al. [17] constructed the three-way decision model based on PT3 and Z-numbers to solve the task assessment in human–machine collaboration. Feng et al. [18] applied PT3 to illustrate the reduced demand of U.S. corn and soybean producers. Thus, the idea to incorporate PT3 into the assessments of emergency response plans deserves more attention. Simultaneously, one should consider the features of public health emergency events affected by time series. The consequences caused by COVID-19 are changed over time. Wuhan switched from a high-risk area in March into a low-risk area in June. Hence, the time factor is supposed to be considered for the formation and updating of dynamic reference points. We introduce the prediction method of reference points in the time line referred in [19], which proposed a parsimonious formula to predict reference points.

Furthermore, in a real emergency scenario, DMs often face the incomplete and fuzzy information, which means DMs prefer to make judgments on linguistic terms than single numbers in the complex situations. Then type-2 fuzzy sets (T2FSS) is regarded as the ideal tool to qualify terms, which offers capabilities to handle higher level uncertain problems. In light of the computational complexity mathematical calculation, researchers intend to choose interval type-2 fuzzy sets (IT2FSS), known as the special type of T2FSS [20–26], which are characterized by the membership values of numerical intervals, the benefits of IT2FSS can be summarized as: IT2FSSs are the extension of type-1 fuzzy sets (T1FSSs), they can handle higher degrees of uncertainty and ambiguity when the preference information expressed linguistically. Moreover, IT2FSSs are relatively simpler among the higher order fuzzy sets [22]. As a result, in this proposed model, the linguistic terms set in the form of IT2FSSs is served as the evaluation systems for DMs. Meanwhile, the evaluation of emergency response plans by a set of DMs can be viewed as multiple criteria group decision making (MCGDM) process. Some integrated hazard assessments using GANP [4], fuzzy analytic hierarchy process (FAHP) [27], multiple multi-objective optimization by ratio analysis (MULTIMOORA) [28] and other multi-criteria decision making (MCDM) methods see in [27,29]. Among these approaches, MULTIMOORA, initially introduced by Brauers and Zavadska [30], is an effective decision making method for the benefits of computational time, the simplicity and stability for mathematical calculations [31]. There are wide applications of MULTIMOORA method, for instance, the green supply chain management [32], healthcare management [33] and other field referred in [34]. Since [30] first proposed the Multi-Objective Optimization by Ratio Analysis (MOORA) method, which contained two major parts: Ratio System (RS) and Reference Point Theory (RPT). Afterwards, the Full Multiplicative Form (FMF) method is considered, then the MULTIMOORA (MOORA plus FMF) method is constructed in [35], which derives the three subordinate rankings. At present, there are abundant ranking aggregation techniques to fuse these results [24,32,36], the dominance theory is the classical integration tool in the initial MULTIMOORA method [35]. Furthermore, other aggregation tools are proposed to take place the theory which emerge the better robustness. For instance, Celik et al. [24] applied the dominance directed graph, rank position method and the Borda Count (BC) method [37] to integrate the three results. Besides using the improved BC method as the fusion tool, Mi et al. [32] applied the stochastic multi-criteria acceptability analysis (SMAA) to increase the stochastic uncertain factors for the input of MULTIMOORA. The more fusion tools on MULTIMOORA method refer to Ref. [36]. Inspired by these above studies, we choose the improved BC method to aggregate the three subordinate methods, instead of adding the rankings of these three methods directly, we consider the integration of the utility values, which are based on the cardinal numbers and further conform the Arrow’s opinion [38], that is, a cardinal utility implies an ordinal preference but not vice versa. Besides, considering the advantages of SMAA in dealing with uncertain problems, we introduce the combination of SMAA and MULTIMOORA by disturbing the weights of criteria. SMAA is an efficient method to deal with decision problems where little or no weight information is available, which is suitable to assist DMs for tackling the corresponding criteria weights of emergency response plans. The detailed content on SMAA refers to Refs. [39–41]. Hence, in this study, owing to the randomness and contingency of emergency events, we integrate SMAA method with MULTIMOORA method to form the extended MULTIMOORA method to deal with the issue of information uncertainty in the assessment of emergency response plans. Furthermore, in the evaluation process of emergency response plans, all DMs should reach consensus to avoid conflicts and obtain higher-quality decision result with timely feedback [4,42]. Generally, there are two steps to reach group consensus: (i) aggregate individual decision information into a group decision result, and (ii) verify whether the result have reach consensus, if not, use the relative algorithm to modify the group decision result. Moreover, different distance functions are usually applied to reflect consensus measures in [43,44]. In this paper, a standard Euclidean-based distance measurement is proposed to calculate the degree of group consensus. Simultaneously, the distance threshold is given as referred to Refs. [43,45] and the corresponding convergent iterative algorithm is put forward to modify the group decision result. Hence, on the basis of considering group consensus, we combine the IT2FSS, PT3, and the extended MULTIMOORA method to construct the evaluation system of emergency response plans in the group set. Then a novel
IT2FS-PT\(^3\) with the extended MULTIMOORA method in a group emergency response plan evaluation is proposed.

To emphasize the reasons for the novel combination of IT2FS, PT\(^3\) and MULTIMOORA method, it can be illustrated as follows: considering that it is difficult to acquire enough essential information in real emergency events and the evolution of diseases is hard to estimate. Thus, it is unreasonable for DMs to assign the accurate numerical assessments to the emergency response plans. Hence, we apply the linguistic terms to assist DMs to make evaluations with IT2FSs served as the quantitative tool. In the meanwhile when dealing with catastrophes, PT\(^3\) has a good performance in dealing with the subjectivity of decision making process. Furthermore, the dynamic reference points suit well to the timeliness of emergency events. In the end, it is vital to choose an effective MCDM method and the reasonable establishment of the relative criteria weight due to the huge pessimistic impact brought by emergency events, the MULTIMOORA method is more robust and objective from integrating three utility functions than other MCDM methods, such as AHP in [46], TOPSIS in [46], VIKOR in [47], etc. In addition, SMAA method can be used with uncertain other MCDM methods, such as AHP in [46], TOPSIS in [46], VIKOR.

Based on the above discussion, the major contributions of the proposed model can be summarized as follows.

- We take into consideration the personality characteristics of DMs, then introduce the PT\(^3\) combined the extended MULTIMOORA method to make assessments to the emergency response plans. In detail, we associate the time line with the setting of dynamic reference points, which is in line with time factor of emergency events and the SMAA method is applied to randomize the criteria weight to increase the stability and robustness of the final ranking results.
- We apply the linguistic evaluation matrix to make DMs more flexible in expressing their preferences, and for the qualification of the IT2FSs-based linguistic terms in PT\(^3\), there are six possible cases to construct the corresponding prospect matrix. Meanwhile, we design a standard Euclidean-based distance formula to measure the level of agreement between the individuals and the group, the relative convergent consensus iterative algorithm is given as well.
- We present an emergency response plan assessments case based on COVID-19 erupted in Wuhan, China. The corresponding risk states can be referred from Wuhan Municipal Health Commission (wjw.wuhan.gov.cn), which further illustrates the effectiveness and practicality of this method.

The remainder of this paper is arranged as follows. Section 2 briefly exhibits the knowledge about IT2FSs, PT\(^3\) and the extended MULTIMOORA method. Section 3 describes the main model of this paper and the solution procedures of the optimal emergency response plan selection. Section 4 gives a case study on COVID-19 infectious diseases to demonstrate the feasibility of this proposed model. In the meanwhile, sensitivity analysis and comparative analysis are taken into consideration as well. Section 5 presents a further discussion. Finally, Section 6 summarizes conclusions, limitations, and future studies.

2. Preliminaries

In this section, the basic concepts about IT2FSs, PT\(^3\) and the extended MULTIMOORA method are briefly discussed.

2.1. Interval type-2 fuzzy sets

IT2FSs, served as a kind of special T2FSs have a wider application for its computational simplicity [48]. The important associated concepts are given as below.

![Fig. 1. An example of the IT2FS MF in a 3-D plane.](image)

**Definition 1 ([49]).** Let \(\tilde{\mu}_A\) be a general T2FS, characterized by type-2 membership function (MF) \(\tilde{\mu}_A(x)\) in the universe of discourse \(X\) it can be expressed as follows.

\[
\tilde{\mu}_A = \int_{x \in X} \int_{u \in J_x} \mu_A(x, u)/(x, u)
\]

where \(J_x\) is the primary MF, satisfied \(0 \leq J_x \leq 1\), \(\mu_A(x, u) = 1\) stands for the secondary MF with \(0 \leq \mu_A(x, u) \leq 1\) and \(\int\int\cdot\cdot\cdot\) represents the union contained all admissible \(x\) and \(u\).

**Definition 2 ([49]).** If all \(\bar{\mu}_A(x, u) = 1\) then, an IT2FS can be expressed as follows.

\[
\tilde{\mu}_A = \int_{x \in X} \int_{u \in J_x} 1/(x, u) \quad J_x \in [0, 1]
\]

where Fig. 1 displays an example of IT2FS MF in a 3-D plane, which the construction of IT2FS is intuitively presented, and the lower MF (LMF) and upper MF (UMF) of the IT2FS are type-1 MF, respectively.

**Definition 3 ([50]).** Let \(c_l\) and \(c_r\) be the left and right end-points of the centroid of an IT2FS satisfying the following equations.

\[
\sum_{i=1}^{N} x_i \theta_i = \min \quad \forall (x_i) \in [\mu_1(\bar{\mu}_2(\tilde{\mu}_A(x)))\]

\[
\sum_{i=1}^{N} \theta_i = \max \quad \forall (x_i) \in [\mu_1(\bar{\mu}_2(\tilde{\mu}_A(x)))]
\]

where \(x \in X\), \(\theta(x) = (i = 1, \ldots, N)\) is the value of MF \(\tilde{\mu}_A\) IT2FS, \(\bar{\mu}_2(\tilde{\mu}_A(x))\) is the value of LMF of the IT2FS and \(\mu_1(\bar{\mu}_2(\tilde{\mu}_A(x)))\) denotes the UMF, which can be seen in Fig. 1. We can find the optimal values of \(c_l\) and \(c_r\) by KM algorithm, for the detail procedures about KM algorithm see in [51].

**Remark 1.** In order to better understand the operation mechanism of IT2FSs, there exhibits a numerical example for illustration with its MFS in Fig. 1: The IT2FSs \(\tilde{\mu}_A = (\bar{\mu}_A^l, \bar{\mu}_A^r)\) are \(((\bar{a}_1^l, \bar{a}_2^l, \bar{a}_3^l, H(\bar{A}^l), (\bar{a}_1^u, \bar{a}_2^u, \bar{a}_3^u, \bar{a}_4^u, H(\bar{A}^u)))))\), where \(\bar{A}^l\) and \(\bar{A}^u\) are the TIFSSs, and \(\bar{a}_i^l, \bar{a}_i^u, d_i^l, d_i^u, \bar{a}_i^l, \bar{a}_i^u\) are the reference points of \(\bar{A}\) on \(x\) axis, and \(H(\bar{A}^l), H(\bar{A}^u)\) \((0, 1)\) represent the membership values in the LMF \(\bar{\mu}_2(\tilde{\mu}_A(x))\) and UMF \(\bar{\mu}_1(\tilde{\mu}_A(x))\) respectively. In Fig. 1, \(a_1^u = 1, a_1^l = 2, a_2^l = 3, a_2^u = 5, a_3^u = 6, a_4^u = 7, H(\bar{A}^l) = 0.4\) and \(H(\bar{A}^u) = 0.8\).
2.2. Third generation prospect theory

There are some prerequisites about PT$^3$ as given follows.

Definition 4 ([13]). Let $S = \{s_i | i = 1, 2, \ldots, n\}$ be a finite state space, containing the states $Pl = \{\pi_i \geq 0, \sum_{i=1}^{n} \pi_i = 1\} = 1, 2, \ldots, n\}$ be the objective probability set associated with $S$, $X = \{x_i | i = 1, 2, \ldots, n\}$ is the result of state under probability $Pl, F$ be the set of all acts, and acts $f, h \in F$ are the functions from $S$ to $X$, satisfied $f(s_i) \in X, h(s_i) \in X$, where $h$ is the reference act of $f$, the value function used in the PT$^3$ can be expressed in the following form.

$$v(f, h) = \begin{cases} \rho(f(s_i) - h(s_i))^a & f \geq h \\ \rho(-\sigma(h(s_i) - f(s_i))^a) & f < h \end{cases}$$

where the parameters $\alpha$ and $\sigma$ are strictly positive, and $\alpha$ associated with the curvature of the value functions of gain and losses, $\sigma$ is the risk-aversion parameter, which controls the DMs' attitudes to gain and loss.

Definition 5 ([19]). Let $N = \{1, \ldots, i, \ldots, n\}$ be the time series set, $Y = \{y_1, \ldots, y_{i-1}, y_i\}$ be the outcome set in the time line, and $\pi_{n, i}$ be the relative weight function of outcome $y_i$. The reference point $r_{n+1}$ in period $n + 1$ can be calculated in the following form.

$$r_{n+1} = \rho + \sum_{i=1}^{n} \pi_{n,i} y_i$$

where $\rho$ is a built-in parameter, the estimate value of $\rho$ in [19] is 5.2. And $\sum_{i=1}^{n} \pi_{n,i}$ represents the weight information of $i$-th time, which can be expressed as a weighting function $w(\frac{1}{n})$ and Eq. (6) can be in the following form as well.

$$r_{n+1} = \rho + \sum_{i=1}^{n} \left[w\left(\frac{i}{n}\right) - w\left(\frac{i-1}{n}\right)\right] y_i$$

where the weight function $w(\cdot)$ can be any continuous and increasing function with $w(0) = 0$ and $w(1) = 1$. Through the experiments in [19], the specific form of $w(\cdot)$ is represented in Eq. (8).

$$w(x) = e^{-((1-\gamma)x)^\xi}$$

where the value of $\gamma$ is 0.2 and 0.26, the value of $\xi$ is 0.9, 1.7 and 2.1, from Fig. 2, it can be intuitively see that the shape of $w(x)$ is reverse S-shaped, with steep sides and quite flat in the middle and the parameter $\gamma$ determines the curvature of $w(x)$, while the parameter $\xi$ controls the elevation of $w(x)$.

Definition 6 ([13]). Let $m^+$ be the number of the states of weak gains, $N$ be the total number of states and $m^- = N - m^+$ be the number of the states of strict loss. The decision weight function $w(s_i, f, h)$ assigned to state $s_i$ when $f$ is evaluated from $h$ in PT$^3$ can be represented as follows.

$$w(s_i, f, h) = \begin{cases} w^+(\pi_i) & i = N \\ w^+\left(\sum_{j=1}^{i-1} \pi_j\right) - w^+\left(\sum_{j=i}^{N} \pi_j\right) & 1 \leq i < N \\ w^-\left(\sum_{j=1}^{i-1} \pi_j\right) - w^-\left(\sum_{j=1}^{N} \pi_j\right) & 1 \leq i \leq m^- \\ w^-(\pi_i) & i = 1 \end{cases}$$

where $w^+(\cdot)$ is the probability weight function for the gain domains and $w^-(\cdot)$ is the probability weight function for loss domains, according to the Ref. [11], the form of $w^+(\cdot)$ and $w^-(\cdot)$ are denoted as follows.

$$w^+(\pi_i) = \left(\frac{\pi^\delta}{\pi^\delta + (1-\pi)^\delta}\right)^{1/\delta} \tag{10}$$

$$w^-(\pi_i) = \left(\frac{\pi^\delta}{\pi^\delta + (1-\pi)^\delta}\right)^{1/\delta} \tag{11}$$

where $\delta$ and $\varepsilon$ are model parameters controlling the shape of the weighting function.

Remark 2. For any $f, h$ pair, there is a weak gain in a state $s_i$ if $f(s_i) \geq h(s_i)$, and a strict loss if $f(s_i) < h(s_i)$.

Definition 7 ([13]). Let $M$ be the set of weak gains, $G$ denotes the strict loss. The prospect value of PT$^3$ can be expressed as follows.

$$V(f, h) = \sum_{i \in M} v(f, h) \times w(s_i, f, h) - \sum_{i \in G} v(f, h) \times w(s_i, f, h) \tag{12}$$

Thus, the function $V(f, h)$ can be applied to construct the prospect value matrix of each DM.

2.3. The extended MULTIMOORA method

In this subsection, the major part in MULTIMOORA method is presented. Furthermore, the SMAA method [39] and BC method [37,52] are applied to replace the dominance theory to determine the final ranking results, where the BC method is named and proposed by the French mathematician and physicist Jean-Charles de Borda [53], which is inspired the voting paradox first introduced by Condorcet [54]. BC method can be regarded as the generalization of the majority-voting rule. To a certain degree, it can be defined as a mapping from individuals ranking results to the integrated ranking result to the most relevant decision.

(1) **Ratio System**

There are the decision matrix $X = (x_{ij})_{m \times n}$ for alternative set $A = \{a_1, a_2, \ldots, a_m\}$ and the criteria set $c = \{c_1, c_2, \ldots, c_n\}$ with its corresponding weight set $w_c = \{w_{c_1}, w_{c_2}, \ldots, w_{c_n}\}$ and $\sum_{j=1}^{n} w_{c_j} = 1$, where $x_{ij}$ is the evaluation value of alternative $a_i$ on the criteria $c_j$. Then
standardization $X$ is completed in the form [55].

$$X^*_j = \frac{X_j}{\sum_{j=1}^{m} X^*_j} \quad (13)$$

After that,

$$y^*_j = \sum_{j=1}^{r} w_c x^*_j - \sum_{j=g+1}^{n} w_c x^*_j \quad (14)$$

where $g$ and $n - g$ respectively denote the numbers of benefit criteria and cost criteria, the parameter $y^*_j$ represents the normalized evaluation of alternative $a_i$ related to all objectives. The optimal alternative $a_{RS}^*$ under RS can be acquired as

$$a_{RS}^* = \{a_i | \max_j y^*_j \} \quad (15)$$

(2) **Reference Point Theory**

The first step is to find the reference point for each alternative using the standardized data obtained by Eq. (13), then the reference point $r_j$ can be defined as follows.

$$r_j = \begin{cases} \max_j x^*_j & j \leq g \\ \min_j x^*_j & j > g \end{cases} \quad (16)$$

Based on this, a Tchebycheff Min–Max metric to calculate the deviation between the assessment value of each alternative and the reference point in Eq. (17) as below.

$$d_i = \{\max_j w_c | r_j - x^*_j | \} \quad (17)$$

where $i \in [1, n]$ and the optimal alternative in reference point theory can be obtained as

$$a_{RP}^* = \{a_i | \min_j d_i \} \quad (18)$$

(3) **Full Multiplicative Form Method**

The utility value of the alternative can be written as follows.

$$U_i = \prod_{j=1}^{g} x^*_j w_c \quad \prod_{j=g+1}^{m} (x^*_j)^{-w_c} \quad (19)$$

where $\prod_{j=1}^{g} x^*_j$ stands for the product of evaluation value of all benefit criteria and $\prod_{j=g+1}^{m} x^*_j$ is the product of the evaluation values of all cost criteria. After that, the optimal alternative under the MFM method can be expressed as $a_{MFM} = \{a_i | \max_i U_i \}$.

(4) **The BC method**

After the establishment of the ranking results of the three subordinate methods, it is need to aggregate these results to obtain the final rankings, all of these methods in MULTIMOORA are non-correlated objectives [56]. The original dominance theory determines the overall rankings by integrating the subordinate rankings, when the number of alternatives is in a small scale, the original dominant theory can quickly obtain the overall rankings. However, in the large amount of alternatives, its operation efficiency will reduce. Furthermore, it fails to consider the utility value of each alternative in the three methods. Based on these analysis, BC method as a substitute for aggregation of the results. The vector normalization method in Eq. (13) is used to normalize the three sub utility values, which proved to be the suitable choice for normalization [55]. The aggregation process can be seen in Eq. (20)

$$b_{ci} = n(a_{RS}) - n(a_{RP}) + n(a_{MFM}) \quad (20)$$

where $n(\cdot)$ denotes the normalized utility values, $a_{RS}$, $a_{RP}$ and $a_{MFM}$ are the values of alternative $x_i$ under the three utility functions. The larger value of the $b_{ci}$ indicates that the corresponding alternative has the better performance.

(5) **SMAA method**

There are two feasible space of $W$ and $X$, where $W$ is the space of criteria weights, while $X_{m \times n}$ presents the evaluation matrix of alternatives under a set of criteria. Firstly the ranking of alternative $x_i$ can be obtained in Eq. (21).

$$rank(x_i, \psi, w) = 1 + \sum_{k=1}^{m} \rho(u(\psi(x_i), w) > u(\psi(x_k), w)) \quad (21)$$

where $\psi$ represents the stochastic value of alternative on a criterion and $w$ denotes a random giving criterion weight value, satisfying the uniform distribution in [0, 1] and $\sum w_j = 1$. $u(\cdot)$ is the related utility function of alternative, then $u(\psi(x), w) = \sum_{j=1}^{n} w_j \psi_j$. $\rho(\cdot)$ is a binary function, if $u(\psi(x_1), w) > u(\psi(x_2), w)$, then $\rho(\cdot) = 1$, else $\rho(\cdot) = 0$. Then $W^*_i(\psi) = \{w | w \in W \land rank(x_i, \psi, w) = r \}$ is the set of rank weights making alternative $x_i$ in the rank $r$.

There are three measurements of SMAA to evaluate the final rankings, the rank acceptability indexes, the central weight vectors and the confidence factors. Let $b^*_i$ be the rank acceptability index of alternative $x_i$ being on $r - th$ position. Where the expression of $b^*_i$ will be defined as follows.

$$b^*_i = \int_{\psi \in X} f_\psi(\psi) \int_{w \in W_i(\psi)} f_w(w \, dw \, d\psi) \quad (22)$$

where $f_\psi$ and $f_w$ are the probability density functions of $\psi$ and $w$.

Let $w_{central}$ be the central weight vector of alternative $x_i$ being on the first rank. The value of $w_{central}$ is displayed as.

$$w_{central} = \frac{1}{b^*_i} \int_{\psi \in X} f_\psi(\psi) \int_{w \in W_i(\psi)} f_w(w \, dw \, d\psi) \quad (23)$$

where the rank acceptability index $b^*_i$ indicates $x_i$ is on the first rank, and $W_i(\psi) = \{w | w \in W \land rank(x_i, \psi, w) = 1 \}$. The confidence factor $p^i_{central}$ stands for the possibility of alternative $x_i$ in the first position with determined central weight vector, which is shown as follows.

$$p^i_{central} = \int_{\psi \in X \land rank(x_i, \psi, w) = 1} f_\psi(\psi) \, d\psi \quad (24)$$

The SMAA-MULTIMOORA method is based on Monte Carlo simulation to calculate the results of the measurements, the detail procedures are illustrated in Algorithm 1.

3. **Solution procedures for assessment of emergency response plan**

In this section, we develop a novel emergency response plans evaluation method based on GDM. Firstly, the IT2FS-PT method is represented, then we integrate the extended MULTIMOORA method, and the group consensus is considered as well by designing a convergent iterative algorithm to gain the consentaneous group decision result. Finally, the solution process of emergency response plan selection is illustrated. In the evaluation process depicted in Fig. 3, the linguistic terms are quantified as the centroid intervals of IT2FSs through KM algorithm, then interval-based decision matrix of each DM is constructed. In the PT framework, we input the concept of time series to predict the reference points in different periods for the setting of dynamic reference points, which is line with the timeliness of emergency
response plans. Simultaneously, there are six possible cases of the relationship between interval-based evaluation information and the related reference point, shown in Table 1. Therefore, we construct the decision matrix in the level of reaching group consensus, and the final ranking results can be determined by the extended MULTIMOORA method.

In this study, we assume that all the DMs make their judgments on the aspect of emergency response plans evaluations by using the same linguistic terms set.

### 3.1. Description of IT2FS-PT³ based emergency response plan evaluation

We can see that the assessments for emergency response plans can be regarded as a MCGDM problem in the uncertain and fuzzy environment. First, the relative parameters are expressed as follows.

- **D** = {d₁, d₂, . . . , d₄}: the set of DMs from hospitals, public health departments and other related sectors, the corresponding weight of experts λ₄ satisfies \( \sum_{i=1}^{4} \lambda_i = 1 \).
- **A** = {a₁, a₂, . . . , a₇}: the set of l emergency response plans (alternatives) needs to find the optimal one in an emergency event.
- **C** = {c₁, c₂, . . . , c₆}: the set of J criteria is served as evaluation indexes of emergency events, which can be divided in 3 types: the beneficial criteria, the cost criteria and the neutral criteria and \( w_j \) is the weight of criterion \( c_j \), satisfying \( w_j \in [0, 1] \) and \( \sum_{j=1}^{6} w_j = 1 \).
- **L** = \{1, 2, . . . , 6\}: the set of linguistic terms, these linguistic terms are expressed in the form of IT2FSs in this proposed model.
- **S** = \{s₁, s₂, . . . , sₙ\}: the set of n states of a public health emergency event, where \( s_n \) is the n-th state. Generally, the classification of \( S \) can be obtained by the previous similar emergency events.
- **P** = \{p₁, p₂, . . . , p₆\}: the probability set for the state \( S \), where \( p_n \in [0, 1] \) and \( \sum_{n=1}^{6} p_n = 1 \), \( p_n \) means the probability of state \( s_n \) occurring in the future.
- **X** = \((x_{ij})_{s}^{J} \): the decision making matrix from a DM, where \( x_{ij} \) denotes the value of alternative \( a_i \) under the criteria \( c_j \).
- **O** = \{0, 1, 2, . . . , 6o\}: the time line set for an emergency event, where \( \text{length}(O) = W \), \( \text{length}(o₁) = \cdot \cdot \cdot = \text{length}(o₆) \) and \( o₆ \) stands for the \( w \)-th time period.
- **H** = \{H¹, H², . . . , H₆\}: the uncertain reference point vector set, where \( H^w(s_n) \) means the reference point vector in the state \( s_n \) at period \( o_w \) and can be expressed as \( H^w(s_n) = (h^w(s_{n1}), h^w(s_{n2}), \ldots , h^w(s_{n6})) \).
- **V** = \{V₁(1), V₂(1), . . . , V₆(1)\}: the illustration of value matrix set from DMs, where \( V_{ij}(w) \) is the prospect value matrix from DM \( d_i \) and \( v_{ij}(w) \) presents the prospect value of alternative \( a_i \) under the criteria \( c_j \) of DM \( d_i \).
Remark 3. In the process of a real-world emergency response plans evaluation, due to the complexity and inadequate information of the emergency event, the selection of evaluation indexes is supposed to be summarized from all relative aspects. These indexes are abstract and cannot be explained by numeric entries [28,42]. Consequently, the expressions of linguistic terms are more reasonable for DM making assessments. When for the setting of parameters $\alpha$, $\sigma$, $\varepsilon$ and $\delta$ of PT3 mentioned in Section 2.2, we refer the values of these parameters in Ref. [11], the median exponent of value function $\alpha = 0.88$, $\sigma = 2.25$ and the median values of $\varepsilon = 0.61$, $\delta = 0.69$.

In this paper, we develop a novel IT2FS-PT3 with the extended MULTIMOORA method to evaluate and select the optimal emergency response plan. Firstly, the process of the IT2FS-PT3 method in an emergency event can be described in Fig. 4, which shows in detail the construction of the prospect decision matrix of each DM in different states, then forms the consensus reached prospect decision matrix.

In the process of group consensus reaching, the group prospect value matrix can be obtained by aggregating the single DM’s prospect value matrix through the additive weighted aggregation (AWA) operator, which can be expressed as follows.

$$V = \sum_{t=1}^{T} \lambda_t V(t)$$  \hspace{1cm} \text{(25)}

where $\lambda_t$ denotes the weight of $t$-th DM and $V(t)$ is the prospect value of $t$-th DM. To measure the level of similarity of prospect value matrix between the individuals and the group, the distance function is expressed as.

$$d \left( V(g), V \right) = \frac{1}{mn} \left( \sum_{i=1}^{m} \sum_{j=1}^{n} v_{ij}(t) - v_{ij} \right)^{1/2}$$  \hspace{1cm} \text{(26)}

where $d \left( V(g), V \right)$ stands the similarity degree of $V(g)$ and $V$. It is worth noting that $d \left( V(g), V \right)$ satisfies the attributes of general distance function as: $0 \leq d \left( V(g), V \right) \leq 1$, $d \left( V(t), V \right) = d \left( V, V(t) \right)$ and $d \left( V, V \right) = 0$.

Definition 8.

Let $\eta$ be the threshold of acceptable consensus level, which can be determined by DMs in advance.

$$d \left( V(g), V \right) \leq \eta$$  \hspace{1cm} \text{(27)}

If $\forall t \in \{1, \ldots, T\}$, Eq. (27) is satisfied, which means that the group has reached consensus.

Definition 9. If $\exists t \in \{1, \ldots, T\}$ such that $d \left( V(t), V \right) > \eta$, then matrices $V(t)$ and $V$ are of unacceptable similarity. The Eq. (28) is given to reconstruct these matrices in a convergent iterative form.

$$v_{ij}^{(t+1)} = \begin{cases} \mu v_{ij}^{(t)} + (1 - \mu) v_{ij}^{(l)} & \text{if } d \left( V(t), V \right) > \eta \\ v_{ij}^{(l)} & \text{otherwise} \end{cases}$$  \hspace{1cm} \text{(28)}

where $\mu$ is a constant satisfied $0 < \mu < 1$ and $l$ be the $l$-th of iterative time.

Theorem 1. Under the above hypotheses, there is $d \left( V(t), V^{l+1} \right) \leq d \left( V(t), V \right)$.

Proof. if $d \left( V(t), V \right) > \eta$, then

$$d \left( V(t), V^{l+1} \right) = \frac{1}{mn} \left( \sum_{i=1}^{m} \sum_{j=1}^{n} \left( v_{ij}^{(t+l+1)} - v_{ij}^{(l+1)} \right)^{1/2} \right)^{1/2}$$

$$= \frac{1}{mn} \left( \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \left( \lambda_t v_{ij}^{(t+l+1)} - \lambda_t v_{ij}^{(l+1)} \right)^{1/2} \right) \right)$$

$$= \frac{1}{mn} \left( \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \lambda_t v_{ij}^{(t+l+1)} - \lambda_t v_{ij}^{(l+1)} \right) \right)^{1/2}$$

From Eq. (28), we have $v_{ij}^{(t+l+1)} - v_{ij}^{(l+1)} = \mu v_{ij}^{(l)} + (1 - \mu) v_{ij}^{(l)}$. Then:

$$d \left( V(t), V^{l+1} \right) = \frac{1}{mn} \left( \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \mu v_{ij}^{(l)} - \lambda_t v_{ij}^{(l+1)} \right)^{1/2} \right)^{1/2}$$

$$= \frac{\mu}{mn} \left( \sum_{i=1}^{m} \sum_{j=1}^{n} \left( \mu v_{ij}^{(l)} - \lambda_t v_{ij}^{(l+1)} \right) \right)^{1/2}$$

which completes the proof of Theorem 1.
3.2. The IT2FS-PT³ integrated with the extended MULTIMOORA method

For this section, we focus on the main model proposed in this paper, in what follows, the specific procedures are given below.

(1) Data Processing

In the evaluation system, there are three types of criteria: the benefit criteria, the neural criteria and the cost criteria. For these types of criteria, we assign different parameter values in the subsequent value functions. Then, the initial linguistic evaluation matrix given by each DM is transformed into IT2FSs-based decision matrix according to the given UMF and LMF of each linguistic term. Through the KM algorithm mentioned in Section 2, we can calculate the centroids of the IT2FSs. Supposed that, the values of linguistic terms follow uniform distribution in the centroid intervals of the IT2FSs, then the expression of linguistic terms is.

\[ l_m = \left[ c^m_l, c^m_u, g(x) \right] \]

where \( l_m \) is the \( m \)-th linguistic term, for instance, the linguistic term "Low" can be expressed as \([c^l_{low}, c^u_{low}, g(x)]\). \( g(x) \) is the corresponding probability density function expressed as follows.

\[ g(x) = \begin{cases} \frac{1}{m^u - m^l} & c^m_l \leq x \leq c^m_u \\ 0 & \text{otherwise} \end{cases} \]

where \( x \) is an arbitrary value satisfied \( x \in [c^m_l, c^m_u] \).

(2) Determination of the value function from DMs

According to the psychological characteristics of each DM, we give the corresponding value functions and the determination of the parameters is based on the research in Ref. [11]. The value functions of the benefit criteria and neutral criteria are expressed as follows.

\[ v\left( \Delta x_{ji}(t) \right) = \begin{cases} (-\Delta x_{ji}(t))^a x_{ji}(t) \geq h(s_n)_j & \text{Gain} \\ -\sigma (-\Delta x_{ji}(t))^a x_{ji}(t) < h(s_n)_j & \text{Loss} \end{cases} \]

and the value function of the cost criteria can be expressed as.

\[ v\left( \Delta x_{ji}(t) \right) = \begin{cases} (\Delta x_{ji}(t))^a x_{ji}(t) \leq h(s_n)_j & \text{Gain} \\ -\sigma (\Delta x_{ji}(t))^a x_{ji}(t) > h(s_n)_j & \text{Loss} \end{cases} \]

where \( \Delta x_{ji}(t) = h(s_n)_j - x_{ji}(t) \) and \( x_{ji}(t) \) denotes \( t \)-th DM’s assessment result of the \( i \)-th emergency response plan under the \( j \)-th criterion in the state \( s_n \), \( h(s_n)_j \) is the reference value of the \( j \)-th criterion in the state \( s_n \). And for the benefit and cost criteria, the loss aversion parameter \( \sigma \) is 2.55, \( \alpha \) is 0.88, while for the neural criteria, \( \alpha = 1 \) and \( \alpha = 0.88 \). Simultaneously, considering the features of public health emergency events affected by time series, Wuhan switched from a high risk area in March into a low risk area in April. Hence, the time factor is supposed to taken into consideration for the formation and updating of dynamic reference point. We introduce the prediction method of reference point formation in the time line referred in [19] as follows.

\[ h(s_n)_j = \rho + \sum_{w=1}^{W} \left[ f\left( \frac{w}{W} \right) - f\left( \frac{w - 1}{W} \right) \right] \cdot y_w \]

where \( f(\cdot) \) is the continuously increasing function of the variable in the interval [0,1], satisfying \( f(0) = 0 \) and \( f(1) = 1 \). The form of \( f(\cdot) \) is shown in Eq. (8). And \( y_w \) is the mean vector of DMs for all criteria in the \( w \)-th time period, which can be written as.

\[ y_w = \text{avg} \left( \sum_{i} \lambda_i x_{ji}(t) \right) \]

where \( \text{avg}(\cdot) \) is applied to calculate the average value of the column in the matrix calculations. Therefore, considering that \( x_{ji}(t) \) and \( h(s_n)_j \) are interval numbers, there are six possible situations for \( \Delta x \), as shown in the Table 1, and the procedures of these cases are illustrated in Appendix A.

(3) Calculation for the decision weight function

The decision weight function of the value of emergency response plan in the criterion expressed as below.

\[ w\left( s_n, x_{ji}(t) \right), h_j(s_n) \]

which \( w^+(p_n) \) and \( w^-(p_n) \) in Ref. [11] can be expressed as follows.

\[ w^+(p_n) = \frac{p_n^\epsilon}{(p_n^\epsilon + (1 - p_n)^\epsilon)^1/\epsilon} \]

and

\[ w^-(p_n) = \frac{p_n^\delta}{(p_n^\delta + (1 - p_n)^\delta)^1/\delta} \]

where the value of \( \epsilon \) is 0.61 and \( \delta \) is 0.69, which are the same settings in Ref. [11].

(4) Construction of the prospect value matrix of each DM

Through the calculation of Eq. (38), we can obtain the prospect value matrix of each DM for the emergency response plans in different criteria.

\[ V_{ij} = \sum_{i \in M} v\left( x_{ji}(t) \right) \times w^+(p_n) - \sum_{i \in G} v\left( x_{ji}(t) \right) \times w^-(p_n) \]

where \( V_{ij} \) is the prospect value matrix from DM \( d_i \).

(5) The group prospect matrix in an admissible consensus level

After attaining the prospect matrices of individual DMs, AWA operators is used to formalize the group prospect matrix as exhibited in Eq. (39).

\[ V^* = \sum_{i=1}^{T} \lambda_i V_{ij} \]

where \( V^* \) is the group prospect matrix, if the obtained group prospect matrix does not satisfy the acceptable consensus condition given in Eq. (27), then Eq. (28) is applied to recalculate the group prospect matrix until satisfying the group consensus. Thereafter, Eqs. (40) and (41) are applied to standardize \( V^* \).

\[ v_{ij}^* = \frac{v_{ij}^* - \min_j v_{ij}^*}{\max_j v_{ij}^* - \min_j v_{ij}^*} \]

and

\[ v_{ij}^* = \frac{\max_j v_{ij}^* - v_{ij}^*}{\max_j v_{ij}^* - \min_j v_{ij}^*} \]
The COVID-19 as a potential deadly coronavirus has caused a level of global illness unseen in numbers and rapidity since it occurred in late 2019. Through the report of National Health Commission of People’s Republic of China, there are three potential states of risk levels $S = \{s_1, s_2, s_3\}$ in COVID-19, i.e. $s_1$: high risk area, $s_2$: medium risk area and $s_3$: low risk area. The specific explanations of these status is shown in Table 2. Simultaneously, we collect the statistics of cumulative confirmed cases from Wuhan Municipal Health Commission website (wjw.wuhan.gov.cn) on March 5th, March 24th, April 7th and April 28th, which the corresponding data is shown in Appendix A. Fig. 5 presents the distribution of Wuhan epidemic risk level map at four time points, which can be vividly seen that the change of risk level is affected by time factors, and Fig. 5 also indicates the rationality of setting three states in the evaluation of an emergency event.

Meanwhile, it can be known that Wuhan successfully converted from full high risk areas to full low risk areas in nearly about two months owing to the effective prevention and controllability of the government, we set the time set $O = \{o_1, o_2, o_3\}$, the interval between them is set to 14 days. In the time point $o_1$ the corresponding states is $s_1$, in the time point $o_2$ is connected with the state $s_2$ and the state in the time point $o_3$ is $s_3$. Accordingly, the corresponding probability of the above states are designed as follows: $p_1 = 0.2$, $p_2 = 0.3$ and $p_3 = 0.5$. Suppose that there are 3 representative DMs $D = \{d_1, d_2, d_3\}$ from hospitals, disease control and prevention centers and other clinical institutions their corresponding weights are set as $\lambda_1 = 0.4$, $\lambda_2 = 0.3$, $\lambda_3 = 0.3$, 5 emergency response plans $A = \{a_1, a_2, a_3, a_4, a_5\}$ for DMs to make evaluations. Considering the complexity of emergency response plans evaluation, we select 7 criteria $C = \{c_1, c_2, c_3, c_4, c_5, c_6, c_7\}$ in the report of National Health Commission of People’s Republic of China, displayed in Table 3. And $c_1, c_2, c_3$ and $c_4$ are the cost criteria, $c_5$ and $c_7$ are the benefit criteria, $c_6$ is the neutral criterion. In general, the value of the consensus threshold level $\eta$ is 0.05, and the parameter coefficient $\mu$ during the iteration process is set as 0.5.

4.2. Procedures of this proposed model

Some required conditions and specific solution steps are displayed in this section. Table 4 exhibits the linguistic terms set and...
Step 1. Linguistic evaluation information process

According to Table 4, KM algorithm is used to calculate the centroid intervals of each linguistic term, follows as: VUI: [0.0963,0.1037], Ul: [0.2926,0.3074], SI: [0.4926,0.5074], I: [0.6926,0.7074] and VI: [0.8963,0.9037]. Therefore, the linguistic evaluation information of DMs can be transformed into the interval-based decision matrices.

Step 2. Calculation of the prospect values

Firstly, in the state of each criterion vector is the reference point vector \( h(s_1)^{a1} \). According to Eq. (34), \( h(s_1)^{a1} \) is equal to \( y_1 \), then following by the Eq. (33), \( h(s_1)^{a2} \) can be obtained as \( h(s_1)^{a2} = \rho + f (1) y_1 \), and \( h(s_1)^{a3} \) expressed as \( h(s_1)^{a3} = \rho + f (2) y_1 + f (1) - f (2) \), where \( y_1 \) and \( y_2 \) are the mean vectors of DMs for criteria in the \( a1 \) time and \( a2 \) time respectively. The value of \( y \) and \( x \) are 0.26 and 1.7. The estimation of \( \rho \) value in the research [19] is 5.2, the magnitude of variables in [19] is 10^5, while in this proposed study the magnitude of variables is between -1 and 0, then the value of \( \rho \) is set as 0.0052. Thus the reference vector \( h(s_1)^{a1}, h(s_2)^{a2} \) and \( h(s_3)^{a3} \) can be denoted in the following form.

\[
\begin{align*}
\begin{bmatrix}
0.61 & 0.63 & 0.49 & 0.50 & 0.42 & 0.44 & 0.71 & 0.72 & 0.41 & 0.42 & 0.51 & 0.52 & 0.69 & 0.70 \\
0.62 & 0.63 & 0.49 & 0.51 & 0.43 & 0.44 & 0.71 & 0.72 & 0.41 & 0.42 & 0.51 & 0.53 & 0.69 & 0.70 \\
0.59 & 0.60 & 0.52 & 0.53 & 0.44 & 0.45 & 0.69 & 0.70 & 0.42 & 0.43 & 0.52 & 0.53 & 0.67 & 0.69
\end{bmatrix}
\end{align*}
\]

Then, according to the six possible situations shown in Table 1, the corresponding \( 4x \) value obtained by comparing the reference points with the DMs' evaluation values. Thus, the gain or loss situation of each DM can be obtained. Table 6 exhibits the DM's evaluation values in state \( s_1 \), the situations in state \( s_2 \) and state \( s_3 \) are represented in Appendix A. For better comprehension, here is an example: in state \( s_1 \), the linguistic evaluation term given by DM: for emergency response plan \( a_1 \) under the benefit criterion \( c_4 \) is VI: [0.8963,0.9037], and \( h(s_1)^{a_1} (4) = [0.7056, 0.7184] \), which in accordance with Case 6, the assessed value is acquired as: 0.7184 - 0.5(0.8963 + 0.9037) = -0.1816. Secondly, calculate the prospect value and relative probability weight of each emergency response plan under the different criteria in Eqs. (31), (32) and (35). Hereafter, based on Eq. (38) the prospect value matrices of \( d_1, d_2 \) and \( d_3 \) can be attained as follows.

\[
\begin{align*}
V_{(1)} &= \begin{bmatrix} 0.0486 & -0.1531 & 0.0090 & 0.1745 & 0.0382 & 0.0239 & -0.2137 \\
-0.0179 & -0.2989 & -0.0254 & -0.1459 & -0.0195 & 0.0673 & -0.1326 \\
-0.1091 & 0.0154 & 0.0440 & 0.0422 & 0.0200 & -0.1083 & 0.0844 \\
0.0404 & -0.1531 & -0.1665 & 0.1745 & -0.1265 & -0.0073 & 0.0083 \\
-0.1266 & 0.0425 & -0.0046 & -0.2021 & -0.2450 & -0.0159 & 0.0767 \\
\end{bmatrix} \\
V_{(2)} &= \begin{bmatrix} 0.0232 & -0.0241 & -0.0140 & 0.0474 & -0.0351 & -0.0073 & -0.0426 \\
-0.0366 & -0.2393 & -0.2346 & -0.0683 & 0.0387 & -0.1083 & 0.0160 \\
-0.0242 & 0.0373 & -0.0110 & 0.0474 & 0.0067 & 0.0388 & -0.0948 \\
0.0130 & -0.0130 & 0.0236 & 0.0267 & 0.0167 & 0.0374 & 0.0239 \\
-0.1125 & 0.6409 & -0.0389 & 0.0328 & 0.0164 & 0.0142 & -0.0824 \\
\end{bmatrix} \\
\end{align*}
\]
Table 6
The value of evaluation from DMs in the state of s1.

|   | c1   | c2   | c3   | c4   | c5   | c6   | c7   |
|---|------|------|------|------|------|------|------|
| DM1|      |      |      |      |      |      |      |
| a1 | 0.1136 | -0.1971 | 0.1249 | -0.1816 | 0.105 | -0.3771 | 0.1859 |
| a2 | -0.0736 | -1.4131 | -0.2609 | 0.0057 | 0.105 | 0.0091 | -0.2019 |
| a3 | 0.1136 | -0.1971 | -0.0609 | -0.1816 | -0.081 | -0.1771 | -0.2019 |
| a4 | -0.2736 | 0.3891 | -0.0609 | 0.2056 | -0.281 | 0.2091 | -1.2291 |
| DM2|      |      |      |      |      |      |      |
| a1 | 0.1136 | 0.1891 | 0.1249 | 0.0057 | 0.105 | 0.0091 | -0.2019 |
| a2 | -0.2736 | -1.4131 | -0.2609 | -0.1816 | 0.305 | 0.2091 | 0.1859 |
| a3 | 0.1136 | -0.1971 | -0.0609 | -0.1816 | 0.105 | 0.0091 | -0.2019 |
| a4 | -0.0736 | -5.4275 | -0.2609 | 0.2056 | -0.081 | 0.0091 | -0.2019 |
| DM3|      |      |      |      |      |      |      |
| a1 | 0.1136 | -1.4131 | 0.1249 | 0.0057 | 0.305 | 0.0091 | 0.3859 |
| a2 | -0.0736 | 0.1891 | -0.0609 | -0.1816 | 0.105 | 0.0091 | -5.1581 |
| a3 | 0.5136 | -0.3971 | -0.0609 | 0.0057 | -0.281 | 0.0091 | 0.3859 |
| a4 | -0.0736 | -1.4131 | 0.1249 | 0.0057 | -0.081 | 0.4091 | -0.2019 |

Then through the standardization of Eqs. (40) and (41), the modified prospect matrices of d1, d2 and d3 can be obtained.

\[ V_{(3)} = \begin{bmatrix}
0.0545 & -0.1743 & 0.0103 & 0.4413 & 0.0389 & 0.0388 & -0.0351 \\
-0.0016 & 0.0154 & -0.0695 & 0.0408 & 0.0387 & 0.0256 & -0.0037 \\
-0.1266 & -0.2598 & 0.0418 & -0.1276 & 0.0374 & 0.0680 & \\
0.0683 & -0.2131 & -0.2319 & -0.0796 & -0.1355 & -0.0176 & -0.3817 \\
-0.1828 & -0.2061 & 0.0231 & 0.5083 & -0.1317 & -0.0771 & 0.0680
\end{bmatrix} \]

Finally, the group prospect matrix is calculated by Eq. (38) given as below.

\[ V = \begin{bmatrix}
0.0165 & 0.4928 & 0.1448 & 0.9684 & 0.3000 & 0.8071 & 0.3598 \\
0.3674 & 0.5224 & 0.5539 & 0.1392 & 0.0819 & 0.6659 & 0.6340 \\
0.6975 & 0.3358 & 0.0403 & 0.5595 & 0.2375 & 0.5964 & 0.7000 \\
0.0415 & 0.5302 & 0.7000 & 0.6691 & 0.6224 & 0.6811 & 0.5979 \\
1.0000 & 0.5414 & 0.1856 & 0.5622 & 0.7843 & 0.4603 & 0.7202
\end{bmatrix} \]

Step 3. Consensus reaching process

For this part, we measure the distances between the individual matrices and the group matrix by Eq. (26), the results are as follows: \( d(V_{(1)}^{mod},V) = 0.0436, d(V_{(2)}^{mod},V) = 0.0620 \) and \( d(V_{(3)}^{mod},V) = 0.0529, \) which can be seen that \( d(V_{(2)}^{mod},V) \) and \( d(V_{(3)},V) \) exceed the threshold \( \eta. \) Hence, Eq. (28) is used to modify \( V_{(2)}^{mod}, V_{(3)}^{mod} \) and \( V, \) the modified results are given below.

\[ V' = \begin{bmatrix}
0.0132 & 0.5236 & 0.1492 & 0.9477 & 0.2400 & 0.7963 & 0.2878 \\
0.3698 & 0.6179 & 0.5091 & 0.1412 & 0.1063 & 0.7232 & 0.5616 \\
0.7379 & 0.2866 & 0.3222 & 0.3773 & 0.2156 & 0.4771 & 0.7600 \\
0.0426 & 0.5536 & 0.7600 & 0.7353 & 0.6142 & 0.6599 & 0.6273 \\
1.0000 & 0.4331 & 0.1947 & 0.4497 & 0.8275 & 0.4735 & 0.7710
\end{bmatrix} \]

Again, we recalculate the level of similarity of the individual matrix and the group matrix, \( d(V_{(1)}',V) = 0.0349, d(V_{(2)},V) = 0.0370 \) and \( d(V_{(3)'},V) = 0.0308, \) all of them satisfying the condition shown in Eq. (27). Therefore, \( V' \) is the consensus group prospect matrix.

Step 4. SMAA-MULTICOMA method

Let the consensus group prospect matrix be used in the extended MULTICOMA method, calculate the rank acceptability index in Eq. (22) and the central weight vector in Eq. (23) of each emergency response plan respectively.

Eventually, Fig. 6 exhibits the distribution of the rank acceptability indexes, it can be obtained the final ranking results of
Table 7: Emergency response plans’ rank acceptability indexes.

| 1st | 2nd  | 3rd  | 4th  | 5th  |
|-----|------|------|------|------|
| $a_1$ | 0.0070 | 0.0830 | 0.3300 | 0.2720 | 0.3080 |
| $a_2$ | 0.0060 | 0.1610 | 0.2260 | 0.1680 | 0.4390 |
| $a_3$ | 0.0000 | 0.1730 | 0.1830 | 0.3910 | 0.2530 |
| $a_4$ | 0.5540 | 0.2330 | 0.0750 | 0.1380 | 0.0000 |
| $a_5$ | 0.4330 | 0.3500 | 0.1860 | 0.0310 | 0.0000 |

Table 8: Emergency response plans’ central weight vectors.

| $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | $c_6$ |
|-------|-------|-------|-------|-------|-------|
| $a_1$ | 0.0160 | 0.0880 | 0.0410 | 0.5170 | 0.0230 | 0.2880 | 0.0270 |
| $a_2$ | 0.1130 | 0.0860 | 0.1020 | 0.0420 | 0.0430 | 0.5210 | 0.0920 |
| $a_3$ | NE    | NE    | NE    | NE    | NE    | NE    | NE    |
| $a_4$ | 0.0640 | 0.1320 | 0.1780 | 0.1760 | 0.1330 | 0.1710 | 0.1450 |
| $a_5$ | 0.1930 | 0.1250 | 0.0980 | 0.1030 | 0.1810 | 0.1340 | 0.1660 |

1 NE denotes “Not Exist”, that is, $a_1$ has got no possibility to rank the first place.

4.3. Sensitivity analysis

In this part, we explore the influences of related parameters $\gamma$ and $\xi$ on the reference point vector and the final ranking results. From the aforementioned in Section 2, parameters $\gamma$ and $\xi$ in Eq. (8) determine the curvature and the elevation of $f(x)$ respectively, or in another perspective, $\gamma$ and $\xi$ control the importance level of the current evaluation matrices of DMs in this period.

It can be intuitively that the change of the parameter $\gamma$ and $\xi$ merely have direct impacts on the reference point $h(s_j)^{o_3}$. Table 9 exhibits the values of $h(s_j)^{o_3}$ and the ranking sequence of emergency response plans in different values of $\gamma$ and $\xi$. And Fig. 7 displays the distribution of the rank acceptability indexes of emergency plans with different $\gamma$ and $\xi$.

In the following discussion, we elaborate on the changes of $h(s_j)^{o_3}$ and the final ranking results from Table 9 separately, which are described as.

- For the values of $h(s_j)^{o_3}$
  When the value of $\gamma$ is fixed, through increasing the value of $\xi$, the $h(s_j)^{o_3}$ displays a distinct upward trend. Take an example, when $\gamma = 0.2$, $\xi = 0.9$ the value of $h(s_j)^{o_3}(1)$ is $0.5675, 0.5799$, while $\gamma = 0.2$, $\xi = 2.1$ $h(s_j)^{o_3}(1)$ is $0.5803, 0.6029$; meanwhile, if the value of $\xi$ is constant, the value of $h(s_j)^{o_3}(1)$ has been slightly increased, and there is basically no change in the length of the intervals.

- For the final results
  It can be seen from Table 9 that parameters $\gamma$ and $\xi$ caused an unobviously change in the ranking of emergency response plans with keep the optimal alternative being $a_4$ and the second optimal alternative being $a_5$. This indicates the parameters presents less sensitive to the final ranking results in the proposed study.
The linguistic evaluation information of DMs in the state of $s_2$. 

| c₁   | c₂   | c₃   | c₄   | c₅   | c₆   | c₇   |
|------|------|------|------|------|------|------|
| d₁   | UI   | SI   | VUI  | I   | UI   | I   | SI   |
| d₂   | SI   | I    | SI   | I   | SI   | I   | VI   |
| d₃   | UI   | SI   | UI   | SI   | VUI  | VI  | VI   |
| d₄   | VUI  | SI   | I    | I   | VI   | SI  | I    |
| d₅   | VI   | UI   | SI   | I   | I    | SI  | I    |
| DM₁  |      |      |      |      |      |      |      |
| d₁   | SI   | I    | SI   | I   | VI   | UI  | SI   |
| d₂   | I    | VI   | VI   | I   | UI   | VUI | I    |
| d₃   | UI   | SI   | SI   | I   | SI   | I   | I    |
| d₄   | I    | SI   | SI   | I   | UI   | SI  | VI   |
| d₅   | VI   | I    | UI   | I   | UI   | I   | SI   |
| DM₂  |      |      |      |      |      |      |      |
| d₁   | UI   | SI   | UI   | SI   | VUI  | I   | VI   |
| d₂   | I    | SI   | I    | SI   | VUI  | SI  | SI   |
| d₃   | VI   | I    | VUI  | SI   | SI   | SI  | SI   |
| d₄   | UI   | I    | I    | I    | I    | I   | VUI  |
| d₅   | VI   | UI   | SI   | VI   | SI   | UI  | I    |
| DM₃  |      |      |      |      |      |      |      |

4.4. Comparative analysis

In what follows, we conduct the comparative analysis with the existed MCDM methods [42,57] in emergency situation to verify the feasibility and comprehensiveness of this study. The corresponding calculations and analysis are all based on the same scenario mentioned above.

Firstly, Wang et al. [42] considered and emphasized the psychological behaviors of DMs in risk and uncertainty environment for solving group emergency decision making problem. Therefore, they applied PT into decision making process, simultaneously, the judgments provided by DMs were expressed in the interval-based linguistic terms. Then Wang et al. [57] discussed the MULTIMOORA method under IT2FS fuzzy environment. They used the IT2FSs-based linguistic terms to deal with the uncertainty and fuzzy evaluations which is the same as the proposed model, after that they calculated the ranking results by MULTIMOORA method. Table 10 represents the ranking results in different methods, it can be clearly seen that the results obtained by this proposed model are closely to the method in [42], while are different from those obtained by method in [57].

The above two studies have both considered the issue of setting criteria weights and proposed the distance based methods for determining the criteria weights. And for the GDM scenario, both of them merely used AWA operators to aggregate the individual’s assessments, and the group consensus reaching process is not reflected. Compared with these methods, the features of the proposed model can be summarized in the following aspects:

- Compared with Wang et al.’s [42] method, this study introduces PT² and develops the dynamic reference points associated with time series, which is more flexible than PT and has wider application in dealing with emergency events. Meanwhile, this proposed model selects the IT2FSs-based linguistic terms set which is more reasonable for solving emergency response plans evaluation problems than interval-based linguistic terms.
- Compared to Wang et al.’s [57] research, this paper considers the bounded rationality of DMs, and combines PT² and the SMAA-MULTIMOORA method to calculate the results in a multi perspective. The introduction of SMAA method allows the stochastic input data of MULTIMOORA framework, which can increase the consistency of the final ranking results.
- In addition, in GDM process, this paper considers the group consensus and designs a consensus iterative algorithm to promote group consensus reaching. Moreover, we use Monte Carlo simulation to produce the criteria’s weights randomly, thereafter, the three measurements in SMAA method are applied to testify the ranking results. It increases the robustness of results compared with the establishment of the criteria weights in the previous two studies [42,57].

Therefore, through the comparative analysis, the propose method can be applied in the process of emergency response plan evaluation in a more comprehensive perspective.
Based on the above discussions, through the comparative analysis, it can reflect the reliability of the method from the side. Based on the above discussions, the major novelty and advantages of the proposed model can be highlighted as follows.

- The proposed model uses the IT2FSs as the quantitative tool for linguistic terms for DMs making evaluations. Meanwhile, the six possible cases of dealing with the centroid intervals of IT2FSs are designed, which can make the final qualified evaluation values are closer to the real DMs' assessments.
- PT³ is applied to construct the decision making matrix of DMs, considering the bounded rationality in handling with emergency events of DMs. Furthermore, the setting of dynamic reference points corresponds to the timeliness of the development of emergency events as well.
- In the GDM situation, the distance formula in Eq. (26) is designed to examine the group consensus, and we put forward the related iterative algorithm to help consensus reaching, which is not considered in [42,57].
- The extended MULTIMOORA method is presented. Specifically, the BC method is applied to fuse the utility values of the three sub functions in MULTIMOORA framework, then the Monte Carlo simulation is constructed to randomize the criteria weights, and the corresponding indexes in the SMAA method are supposed to verify the final rankings, which increase robustness of results and make the model more suitable to deal with real cases.

5. Further discussion

In this proposed model, we have proposed an integrated MCGDM method, which takes the IT2FSs, PT³ and MULTIMOORA method into consideration. The major parts of this research are as follows: firstly, individual DMs make judgments on the emergency response plans in PT³, then through the consensus iterative algorithm, obtain the group evaluation information. Thereafter, with the application of the extended MULTIMOORA method to calculate the final ranking results.

Generally, in the assessments of emergency response plans, linguistic terms are usually applied to express the preferences of DMs. It is difficult to obtain sufficient information in emergency events, so we use IT2FSs to quantify the expressions of terms and reduce the loss in the data processing. We consider the characteristics of DMs and introduced the PT³. In the extended MULTIMOORA framework, the BC method is arranged to aggregate the utility values of the three subordinate methods instead of the original dominance theory, and we apply the SMAA method to randomize the criteria weight to increase the robustness of the final results.

After that, as shown in Table 9, by modifying the relevant parameters for sensitivity analysis, it can be seen that the changes of the parameters have a little effects on the final evaluation results, which illustrates the robustness of this model. Furthermore, through the comparative analysis, it can reflect the reliability of the method from the side. Based on the above discussions, the major novelty and advantages of the proposed model can be highlighted as follows.

| Table 12 |
The value of evaluation from DMs in the state of s₂.
| c₁ | c₂ | c₃ | c₄ | c₅ | c₆ | c₇ |
|-----|----|----|----|----|----|----|
| a₁ | 0.3188 | 0.0001 | 0.3301 | 0.0108 | 0.1102 | −0.1719 | 0.1911 |
| DM₁ | a₂ | 0.1188 | −0.1919 | −0.0557 | 0.0108 | −0.0758 | −0.3719 | −0.0006 |
| a₃ | 0.3188 | 0.0001 | 0.1301 | 0.4108 | 0.1102 | 0.1413 | −0.1967 |
| a₄ | 0.5188 | 0.0001 | −0.2557 | 0.0108 | −0.4758 | 0.0143 | −0.0006 |
| a₅ | −0.2684 | 0.1943 | 0.1301 | 0.2108 | −0.2758 | 0.0143 | −0.0006 |

| Table 13 |
The linguistic evaluation information of DMs in the state of s₁.
| c₁ | c₂ | c₃ | c₄ | c₅ | c₆ | c₇ |
|-----|----|----|----|----|----|----|
| a₁ | VUI | VI | SI | I | VUI | SI | UI |
| DM₁ | a₂ | I | I | UI | UI | SI | VI | UI |
| a₃ | VI | SI | UI | I | UI | VUI | VI |
| a₄ | SI | VI | I | I | SI | UI | I |
| a₅ | I | VUI | SI | UI | VI | SI | VI |

| a₁ | UI | I | UI | VI | SI | UI | I |
| DM₂ | a₂ | SI | VI | I | SI | VUI | VUI | I |
| a₃ | I | VUI | SI | VI | VUI | VI | UI |
| a₄ | SI | SI | UI | VI | VUI | I | I |
| a₅ | I | I | SI | UI | SI | SI | |

| a₁ | UI | SI | SI | I | UI | VI | I |
| DM₃ | a₂ | SI | SI | I | VI | VUI | I | I |
| a₃ | I | VUI | SI | UI | I | VI |
| a₄ | VUI | VI | VI | SI | I | UI | VUI |
| a₅ | VI | I | UI | I | UI | VI | |

6. Conclusions, limitations, and future studies

In the recent years, owing to the harmfulness caused by emergency events, researches on emergency response plans evaluation has attracted many scholars. The related studies can be seen in Refs. [58–61]. In this paper, we have developed an emergency response plans assessment method in a comprehensive way, owing to the complexity of emergency events and the bounded rationality of DMs, this proposed model combines PT³ and the extended MULTIMOORA method with considering the group consensus reaching, the whole process is described as follows: DMs use the given linguistic terms set to make judgments on the emergency response plans, then based on centroids of IT2FSs, each term can be qualified into interval numbers, formalize the interval-based evaluation matrices. When comparing with the related reference point vectors, six possible cases are presented to measure DMs' expectation of gains or losses of alternatives. It is worth noting that we provide a formula for the formation of dynamic reference point vectors over time. Thereafter, we calculate the prospect matrix of each DM. Afterwards, through the consensus conditions, we substitute the agreeable group prospect decision matrix into the extended MULTIMOORA method, through 1000 times of Monte Carlo simulation, the related indexes in SMAA are calculated to determine the final ranking results of these emergency response plans. Finally, the emergency response plans evaluation case of COVID-19 happened in Wuhan, China further...
Table 14
The value of evaluation from DMs in the state of s1.

| DM     | c1   | c2   | c3   | c4   | c5   | c6   | c7   |
|--------|------|------|------|------|------|------|------|
| DM1    |      |      |      |      |      |      |      |
| a1     | 0.4858 | −0.3702 | −0.0476 | −1.3037 | 0.3153 | 0.0194 | 0.3744 |
| a2     | −0.1017 | −0.1702 | 0.1385 | 0.3908 | −0.071 | −0.3671 | 0.3744 |
| a3     | −0.3017 | 0.0157 | 0.1385 | −1.3037 | 0.1153 | 0.4194 | −0.2131 |
| a4     | 0.0858 | −0.3702 | −0.2476 | −1.3037 | −0.071 | 0.2194 | −0.0131 |
| a5     | −0.1017 | 0.4157 | −0.0476 | 0.3908 | −0.471 | 0.0194 | −0.2131 |
| DM2    |      |      |      |      |      |      |      |
| a1     | 0.2858 | −0.1702 | 0.1385 | −0.1962 | −0.071 | 0.2194 | −0.0131 |
| a2     | 0.0858 | −0.3702 | −0.2476 | 0.1908 | 0.3153 | 0.4194 | −0.1131 |
| a3     | −0.1017 | 0.4157 | −0.0476 | −0.1962 | 0.3153 | −0.3671 | 0.3744 |
| a4     | 0.0858 | 0.0157 | 0.1385 | −0.1962 | 0.3153 | −0.1671 | −0.2131 |
| a5     | −0.1017 | −0.1702 | 0.0476 | −1.3037 | 0.1153 | 0.0194 | 0.1744 |
| DM3    |      |      |      |      |      |      |      |
| a1     | 0.2858 | 0.0157 | −0.0476 | −12.168 | 0.1153 | −0.3671 | −0.1131 |
| a2     | 0.0858 | 0.0157 | −0.2476 | −0.1962 | 0.3153 | −0.1671 | 0.3744 |
| a3     | −0.1017 | −0.1702 | 0.3385 | 0.1908 | 0.1153 | 0.1671 | −0.2131 |
| a4     | 0.4858 | −0.3702 | −0.4476 | 0.1908 | −0.271 | 0.2194 | 0.5744 |
| a5     | −0.3017 | −0.1702 | 0.1385 | −12.168 | −0.271 | 0.2194 | −0.2131 |

Appendix A

To better explain the six cases in Table 1, and make judgment for the losses or gains under different types of criteria, the main process is given as follows.

For case 1: \( x'_{ij} < h^0_i < h^0_j \), the relationship can be visually seen from Fig. 8.

It can be seen that \( h_l = [h^0_j, h^0_i] \) completely exceeds \( x_{ij} = [x^c_{ij}, x^v_{ij}] \), which indicates \( x_j < h_l, \Delta x_{ij} > 0 \), for the cost criteria there are weak gains to DMs, while for the benefit criteria there are strict losses to DMs. Let \( x_{ij} \) be a random variable in interval \([x^c_{ij}, x^v_{ij}]\) satisfied a uniform distribution, the form of probability density function \( f(x'_{ij}) \) is referred in Eq. (30). Thereafter, \( \Delta x_{ij} \) can be obtained as.

\[
\Delta x_{ij} = h^0_j - x'_{ij} = h^0_j - x'_{ij} + \int_{x'_{ij}}^{x^v_{ij}} (x'_{ij} - x_{ij}) f(x_{ij}) \, d(x_{ij})
\]

\[
= h^0_j - x'_{ij} + \int_{x'_{ij}}^{x^v_{ij}} \left( (x'_{ij} - x_{ij}) \cdot \frac{1}{x^v_{ij} - x^c_{ij}} \right) \, d(x_{ij})
\]

\[
= h^0_j - x'_{ij} + \frac{1}{x^v_{ij} - x^c_{ij}} \int_{x'_{ij}}^{x^v_{ij}} (x^v_{ij} - x_{ij}) \, d(x_{ij})
\]

\[
= h^0_j - x'_{ij} + \frac{1}{2} (x_{ij} - x'_{ij})^2
\]

\[
= h^0_j - h^0_j - \frac{1}{2} (x'_{ij} - x_{ij})^2
\]

For case 2: \( x^v_{ij} < h^0_j < h^0_i < h^0_j \), which is portrayed Fig. 9. In this situation, there are overlapping parts in intervals \( x_{ij} \) and \( h_l \), the value of the overlapping part is equal to 0, which means there is neither loss nor gain for DMs, \( \Delta x > 0 \), the same conclusion as in case 1. And \( \Delta x \) in case 2 presented as.

\[
\Delta x_{ij} = h^0_j - x'_{ij} = \int_{x^c_{ij}}^{h^0_j} (h^0_j - x'_{ij}) f(x_{ij}) \, d(x_{ij})
\]

\[
= \int_{x'_{ij}}^{x^v_{ij}} (h^0_j - x'_{ij}) \, d(x_{ij})
\]

\[
= \int_{x'_{ij}}^{x^v_{ij}} \left( h^0_j x_{ij} - \frac{1}{2} (x_{ij})^2 \right) \, d(x_{ij})
\]

\[
= \frac{1}{2} (x^v_{ij} - x'_{ij})^2 - \frac{1}{2} (x^v_{ij} - x'_{ij})^2.
\]
For case 3: $h_j^r < x_j^b < x_j^r < h_j^f$, see in Fig. 10. Interval $h_j$ contains interval $x_j$, it can be expressed as $[x_j^b, x_j^r] \subseteq [h_j^r, h_j^f]$, in this case, $\Delta x_j = 0$ which means no gains or losses for DMs.

For case 4: $h_j^r < x_j^b < h_j^f < x_j^r$, presented in Fig. 11. Compare with case 4 there is an overlapping part as well, the difference is, $\Delta x < 0$, which denotes that DMs feel weak gains under the benefit criteria evaluation, strict losses under the cost criteria evaluation. The main construction of $\Delta x$ is shown as follows.

$$\Delta x_j = h_j^r - x_j^r = \int_{h_j^r}^{x_j^r} (h_j^r - x_j^r) f(x_j^r) \, d(x_j^r)$$

$$= \frac{1}{x_j^r - x_j^b} \int_{h_j^r}^{x_j^r} (h_j^r - x_j^r) \, d(x_j^r)$$

$$= \frac{1}{x_j^r - x_j^b} \left( h_j^r x_j^r - \frac{1}{2} (x_j^r)^2 \right) \bigg|_{h_j^r}^{x_j^r}$$

$$= -\frac{(h_j^r - x_j^r)^2}{x_j^r - x_j^r}$$

For case 5: $x_j^b < h_j^r < h_j^f < x_j^r$, see in Fig. 12. This time, interval $x_j$ contains interval $h_j$, let $x_j^b, x_j^r$ be uniformly distributed values on the interval $x_j$ which satisfy $x_j^b \in [x_j^b, h_j^r]$ and $x_j^r \in [h_j^f, x_j^r]$. Divide $\Delta x_j$ into two parts $\Delta x_j^b$ and $\Delta x_j^r$. $\Delta x_j = \Delta x_j^b + \Delta x_j^r$ the solving steps are as follows.

$$\Delta x_j^b = \int_{x_j^b}^{h_j^r} (h_j^r - x_j^r) f(x_j^r) \, d(x_j^r)$$

$$= \frac{1}{x_j^r - x_j^b} \int_{x_j^b}^{h_j^r} (h_j^r - x_j^r) \, d(x_j^r)$$

$$= \frac{1}{x_j^r - x_j^b} \left( h_j^r x_j^r - \frac{1}{2} (x_j^r)^2 \right) \bigg|_{x_j^b}^{h_j^r}$$

$$= \frac{1}{x_j^r - x_j^b} \left( h_j^r x_j^r - \frac{1}{2} (x_j^r)^2 \right) - \frac{1}{x_j^r - x_j^b} \left( h_j^r x_j^r - \frac{1}{2} (x_j^r)^2 \right)$$

$$= \frac{(h_j^r - x_j^r)^2}{x_j^r - x_j^b}$$

For case 6: $h_j^r < h_j^f < x_j^b < x_j^r$, see in Fig. 13. It can be seen that $x_j = [x_j^b, x_j^r]$ completely exceeds $h_j = [h_j^r, h_j^f]$, which indicates $h_j < x_j$, $\Delta x_j < 0$, the situation of case 6 is the opposite of case 1. $\Delta x_j$ is presented as (see Tables 11–18).

$$\Delta x_j = h_j^r - x_j^r = h_j^f - x_j^f + \int_{x_j^f}^{x_j^r} (x_j^r - x_j^f) f(x_j^r) \, d(x_j^r)$$

$$= h_j^f - x_j^f + \frac{1}{x_j^r - x_j^f} \int_{x_j^f}^{x_j^r} (x_j^r - x_j^f) \, d(x_j^r)$$

$$= h_j^f - x_j^f + \frac{1}{x_j^r - x_j^f} \left( x_j^r x_j^r - \frac{1}{2} (x_j^r)^2 \right) \bigg|_{x_j^f}^{x_j^r}$$

Table 15

| Distribution of COVID-19 states in Wuhan on March 5th. |
|-----------------------------------------------------|
| High risk area | Medium risk area | Low risk area |
|---------------|------------------|--------------|
| Qingsheshan   | 0                | 0            |
| Hanhan        |                  |              |
| Jiangan       |                  |              |
| Hanyang       |                  |              |
| Qiaokou       |                  |              |
| Jianghan      |                  |              |
| Wuchang       |                  |              |
| Hongshan      |                  |              |
| Xinzhou       |                  |              |
| Huangpi       |                  |              |
| Jiangxia      |                  |              |
| Caidian       |                  |              |
| Dongxihu      |                  |              |

Fig. 12. The situations of interval.

Fig. 13. The situations of interval.
Table 16  Distribution of COVID-19 states in Wuhan on March 24th.

| High risk area | Medium risk area | Low risk area |
|----------------|-----------------|--------------|
| 0              | Qingshan        | Xinhou       |
|                | Hannan          | Huangpi      |
|                | Jiangan         | Jiangxia     |
|                | Hanyang         | Caidian      |
|                | Qiaokou         | Dongxihu     |
|                | Jiangan         | Wuchang      |
|                | Hongshan        |              |

Table 17  Distribution of COVID-19 states in Wuhan on April 7th.

| High risk area | Medium risk area | Low risk area |
|----------------|-----------------|--------------|
| 0              | Qiaokou         | Qingshan     |
|                | Hannan          | Hanyang      |
|                | Jiangan         | Jiangnan     |
|                | Wuchang         | Huangpi      |
|                | Hongshan        | Xinzhou      |
|                |                 | Huangpi      |
|                |                 | Jiangxia     |
|                |                 | Caidian      |
|                |                 | Dongxihu     |

Table 18  Distribution of COVID-19 states in Wuhan on April 28th.

| High risk area | Medium risk area | Low risk area |
|----------------|-----------------|--------------|
| 0              | 0 Qingshan      | Hannan       |
|                |                 | Jiangan      |
|                |                 | Hanyang      |
|                |                 | Qiaokou      |
|                |                 | Jiangnan     |
|                |                 | Wuchang      |
|                |                 | Hongshan     |
|                |                 | Xinzhou      |
|                |                 | Huangpi      |
|                |                 | Jiangxia     |
|                |                 | Caidian      |
|                |                 | Dongxihu     |

\[
h^*_j = h^* - \frac{1}{2} x^*_j (x^*_j - x^*_j)^2 = h^*_j - \frac{1}{2} x^*_j (x^*_j + x^*_j)
\]

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