An Improved Population Migration Algorithm Introducing the Local Search Mechanism of the Leap-Frog Algorithm and Crossover Operator

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Abstract
The population migration algorithm (PMA) is a simulation of a population of the intelligent algorithm. Given the prematurity and low precision of PMA, this paper introduces a local search mechanism of the leap-frog algorithm and crossover operator to improve the PMA search speed and global convergence properties. The typical test function verifies the improved algorithm through its performance. Compared with the improved population migration and other intelligent algorithms, the result shows that the convergence rate of the improved PMA is very high and its convergence is proved.

Introduction
The population migration algorithm (PMA) was proposed by Zhou et al in 2003 [1–2]. The PMA is a simulated population migration theory global optimization algorithm. The PMA is also a simulation mechanism that involves population along with economic center transfer and population pressure diffusion in the field. In other words, people relocate to a preferential region that has a high level of economic development and employment opportunities. When the preferential region has relative overpopulation, the population pressure constantly increases. When population pressure exceeds a certain limit, people will move to a more suitable preferential region. Population pressure makes the PMA engage in a better regional search. To a certain extent, population proliferation can prevent the PMA from falling into a local optimal solution. The whole algorithm shows alternate features that localized search and scatter search have in the search process.

In 2004, Xu gave an improved PMA (IPMA), which can be summarized into three forms: population flow, population migration, and population proliferation [3]. Population flow states that the population residing in the region is spontaneous and the overall flow is uncertain. Population migration is based on the mechanism across a wide range of the selection movement, in which people flow to the rich place. When the population pressure is very high, population proliferation is a selective movement from a preferential area to a no preferential area. It also reflects the people’s pioneering spirit.

Recent, various kinds of intelligent algorithms have been improved by many researchers. Wang et al improved the PMA with the steepest descent operator and the Gauss mutation [4]. Ouyang et al introduced the simplex in the PMA to improve the search performance of the algorithm [5]. Guo et al increased performance of the population migration by improving the migrations’ efficiency [6]. Karaboga improved the bee swarm [7]. Li et al introduced extremal optimization to improve the performance of the shuffled frog leaping algorithm (SFLA) [8]. However, the results of all these studies show low accuracy or fall into the local optimal solution. To further improve the local search ability of the PMA and speed up convergence, we introduce a local search mechanism of frog leaping and crossover. This paper is improved and uses a number of classic functions for the simulation to show that the algorithm is effective and feasible.

Population Migration Algorithm
Consider the following unconstrained single-objective optimization problem:

\[ \min f(X) \quad (1) \]
\[ s.t \; X \in S \]

In which \( f : S \rightarrow \mathbb{R}^1 \) is a real valued mapping, \( X \in \mathbb{R}^n \), \( S = \prod_{i=1}^{n} [a_i, b_i] \) is a search space, \( a_i < b_i \). We assume that problem (1) has a constant solution, namely, global optimal solution exists and the set \( H \) of the global optimal solution is not empty. Seeking the most value problem can be transformed into the form of problem (1). In the algorithm, \( X \) is an individual and stands for the population and various kinds of information. \( X_i = (X_{i1}, X_{i2}, \ldots, X_{in}) \) is the \( ith \) individual in the solution space, \( X^i \in \mathbb{R}^n; X_{ij} \) is the \( ith \) individual of the \( jth \) weight; \( \delta^i \in \mathbb{R}^n; \delta^i_j \) is the \( jth \) weight of the \( ith \) region’s radius \( \delta^i \), in which, \( \delta^i_j > 0, i = 1, 2, \ldots, N; j = 1, 2, \ldots, m; \) and \( N \)
holds true, and then we stop. Continue the local search in the groups until the termination rule is satisfied. All groups have completed the depth of local search, all the frogs of the group instead of the original worst frog. The updated process is:

\[ Y_i^j = a_i X_i^j + (1 - a_i) X_i^{j-1}, \quad j = 1, 2, \ldots, n. \]

Methods

In the PMA, the population flow includes extensive information exchange. In practice, population information exchange decides the direction of the migration under certain conditions, so we should give full attention to population flow. When under pressure, population migration is the random population flow in the range of the population pressure, which does not have a significant role in further improving the algorithm.

The local search mechanism of frog-leaping algorithm and the crossover operator are introduced into the PMA in the process of population flow, which can well improve the original search results. IMPA organically made the frog-leaping combine with the crossover operator in the PMA. The process of the algorithm is as follows:

**Initialization**

Input the population size \( N \), the initial radius \( \delta(0) \), the vigilance parameter of population pressure \( \pi(0) \), the number of the population flow \( l \), the number of local search \( L_s \), and the maximum number of iterations \( T \). Randomly generate \( N \) individuals in the search space \( S, X^1, X^2, \ldots, X^N \). The \( i \)th regional center \( center^i = X^i \) determines the upper and lower \( center^i \pm \delta^i \) bounds of the \( i \)th region, in which \( \delta^i > 0 \), \( i = 1, 2, \ldots, N \). The extraction method \( \delta^i \) makes \( \delta^i \) equal, so we cancel the superscript of \( \delta^i \) in the following steps. We obtain an initial search space \( \Omega(0) = \bigcup_{i=1}^{N} B(X^i(0), 0, \delta(0)) \), which stands for the initial residential area. The set \( B(X, \delta) \) is taken as the sphere, in which \( X \) is the center and \( r \) is the radius. The following variables appearing that way are the same. \( f(X^0) = \max_{1 \leq i \leq N} f(X^i) \) and \( X_{best}(0) = X^0 \). We set iteration counter \( t = 0 \).

**Evolutionary**

1. Preparation: \( \eta(t) = \delta(t) \)
2. Population flow: In every \( B(X^i(t), \eta(t)) \), we randomly generate \( l_i \) individuals, so we can obtain the group \( Y(t) \) in \( \Omega(t) \), which has \( N \) individuals.
3. Population migration
4. Next introduce the local search mechanism of frog-leaping algorithm and crossover operator. At this time, the role of population has been transformed to the frog. The search area is seen as the frog’s wetland. The formed \( N \) local areas are seen as the \( N \) groups. The individual of every local area is seen as the frog.

(a) We choose \( N \) best frogs in \( Y(t) \), which form the intermediate frog group \( Y_{best}(t) = \{ Y_1^{(t)}, Y_2^{(t)}, \ldots, Y_N^{(t)} \} \).
(b) Forming \( N \) regions: \( \{ B(Y_1^{(t)}(t), \eta(t)), i = 1, 2, \ldots, N \} \).
(c) In every group, we produce a frog body flow. In other words, \( l_i \) individuals are randomly generated in \( B(Y_{best}^{(t)}(t), \eta(t)) \).
l and the adaptive value of \( Y_{best}^{(t)} \) are proportional.
(d) We set the group counter \( iN = 1 \), which is compared with the total of groups \( N \); the local evolution counter \( im = 1 \), which is compared with the number of local search \( L_s \).

Crossover Operators

By two leaner combinations, we can obtain two new individuals. Arithmetic crossover operation is often used to express individuals by floating individual coding. Hypothesis: The two parents are \( X_a = (X_{a,1}, X_{a,2}, \ldots, X_{a,n}) \) and \( X_b = (X_{b,1}, X_{b,2}, \ldots, X_{b,n}) \). Then, \( n \) random numbers are randomly generated, in which \( a_{ij} \in [0, 1], \ i = 1, 2, \ldots, n \). After hybrid implementation, two new progenies are generated:

\[ Y_a = (Y_{a,1}, Y_{a,2}, \ldots, Y_{a,n}), \quad Y_b = (Y_{b,1}, Y_{b,2}, \ldots, Y_{b,n}) \]
An Improved Population Migration Algorithm

Termination of Inspection

If the new generation of population \(X_{t+1}\) contains the eligible solution, we stop and output the best solution in \(X_{t+1}\). Otherwise, we narrow \(\delta(t)\) and \(\varepsilon(t)\) properly and set \(t \leftarrow t + 1\). If the maximum numbers of iterations for the algorithm are not met, we turn to the second step.

Convergence Analysis

This paper uses the axiom model to analyze and prove the convergence of IMPA.

Definition 1 Optimization variables \(X\) have limited-length characters and have the form \(A = a_1a_2...a_n = X = (x_1, x_2, ..., x_n)\). \(n\) is called the code string length of \(X\), \(A\) is called the code of \(X\), and \(x_i\) is called the decode of the character \(A\). \(a_i\) is considered a genetic gene. All possible values of \(a_i\) are called allelic gene. \(A\) is a chromosome made up of \(n\) genes [11].

Definition 2 (Individual Space) \(A\) is an allelic gene. \(n\) is given the code string length. The set \(H_1 = \{a_1a_2...a_n | a_i \in \Gamma, i = 1, 2, ..., n\}\) is called individual, in which \(\forall Y\) is a natural number and \(H_N = H_1 \times H_1 \times ... \times H_1\) is the Norder of population space [11].

Definition 3 (Satisfaction Set) If \(\forall X \in B, Y \in B\), then \(f(X) > f(Y)\). \(B, X\) is called the satisfaction set. The number of individuals in the set \(B, X\) is used for \([B(X)\]. \(X \in H^N\) is any individual in the population and \(X_k\) is the best individual in the set \(X\). The satisfaction set induced by \(X_k\) called satisfaction set of the population \(X\), which is marked by \(B_k(X) = \{X \in H_1, f(X) \geq f(X)\}\). All of the satisfaction sets’ intersections are formed with the global optimal solution \(P^*\) [11].

Definition 4 (Selection Operator) The selection operator \(S\) is a random mapping, \(S: H^N \rightarrow H^N\). It meets the following two conditions. [1] For \(\forall X \in H^N\), we can obtain \([SX], i = 1, 2, ..., N\) in which \([SX]\) stands for the random individual in the \(S\); [2] Every population \(X\) that meets \(B(X) < \text{Nhas} P\{[B(SX)] + \{N - M\} > B(X)\} > 0\). The first condition states that selected individuals should be in the selected population. The second condition states that the selection operator may increase the number of optimal individuals in the population, in addition to variations of the individuals’ numbers between selected population and original population [11].

Definition 5 (Reproduction Operator) Reproduction operator \(R\) is a random mapping \(R: H^N \rightarrow H^N\). If any population \(X \in H^N\), which meets \(P\{R(\tilde{X}) \cap B_k(\tilde{X}) \neq \emptyset\} > 0\), the new population \(R(\tilde{X})\) which \(X\) becomes under the action of the operator \(R\), may contain more satisfied individuals than the original population [11].

Definition 6 (Mutation Operator) Mutation operator \(M\) is a random mapping \(M: H^N \rightarrow H^N\). \(X \in H^N\) meets \(P\{M(M_\circ X) \cap B^* \neq \emptyset\} > 0\). Mutation operator is the reproduction operator that has a much better nature. If the population contains satisfied individuals, then after mutation, the population should contain satisfied individuals. If the population does not have satisfied individuals, then after mutation, the population may contain satisfied individuals [11].

In improved population migration, extended operator \(L_0: H^N_1 \rightarrow H^N_1\) meets the following expressions:

\[
L_0(X) = \left(\begin{array}{c}
X^{(1)}_1 \cdots X^{(1)}_N,
\cdots,
X^{(N)}_1 \cdots X^{(N)}_N
\end{array}\right)
\]

and

\[
H^N_1 = H_1 \times H_1 \times \cdots \times H_1
\]

Therefore, we can use the operator \(M, L_0: H^N_1 \rightarrow H^N_1\) to stand for the process of population flow. The selection operator \(S\) stands...
for the process of step (a) in population migration, where we chose \( N_i \), the best individuals from population \( Y\). The reproduction operator \( R_i \) stands for the progress from step (b) to the process of final output \( \mathcal{X}_{\text{end}} \) in step (b). The reproduction operator \( R\) stands for the process from population proliferation to finally generate \( X(t+1) \). Therefore, the IPMA can be expressed as:

\[
X(t+1) = R \ast X(t) \ast L_0(X(t)), t=0,1,2,\ldots \tag{8}
\]

\[Y(t) = M \ast L(X(t)), S = \left\{ S_1 \right\}, R = \left\{ R_1 \right\}, L_0 = R \ast R_1, t=1,2,\ldots \tag{9}\]

After both sides have the same function of operator \( M_i \ast L_0 \) for (8), we can find that \( Y(t+1) = R \ast S(Y(t)) \). The general simulated evolution algorithm can be composed of a series of selection operators \( \left\{ S_i \right\} \) and reproduction operators \( \left\{ R_i \right\} \) to be represented abstractly as follows: \( \mathcal{X}(t+1) = R_i \ast S_i(X(i)) \).

**Lemma 1** In IPMA, \( S \) is a selection operator and \( R \) is a reproduction operator. The proof can be obtained from reference [11].

**Theorem 1** The IPMA is a simulated evolutionary algorithm. In the IPMA, the characteristic number of the preferred selection operator, the selection pressure of the operator \( S_i \), the selection intensity \( \alpha \), and the IPMA just takes the preferred selection operator. According to reference [3], the selection pressure and the selection intensity are defined to calculate the characteristic number of the operator \( S \). The proof can be obtained from reference [11].

**Lemma 2** In IPMA, the characteristic number of the reproduction operator, the operator \( R : H^N \rightarrow H^{N + N} \), then the absorption and scattering rate of the reproduction meet the following estimates: 
\[
AE \geq Nl \beta_i (1-p(C_{t+1}))^{N_1} [\alpha(C(z)\beta_{t+1})^{N_1}]^N = O(1/t) \text{, where we know the divergence of the harmonic series, so } \sum_{t=1}^{\infty} A_{R(t)} = \infty. \text{ According to lemma 3: } S^R_k = 0.
\]

According to lemma 2: \( \alpha_{(0)} = 1 \), so \( \lim_{t \rightarrow \infty} (1 - S^R_k \alpha_{(0)}) / A_{R(t)} = 0 \). The three conditions of lemma 3 are met. The IPMA probably weakly approaches the global optimal solution.

**Results and Discussion**

**Simulation Examples and Experimental Parameter Setting**

To verify the performance of the IPMA, this paper performs an experiment. The IPMA is compared with the basic population algorithm and other improved algorithms. In the simulation experiment, we choose 10 classic test functions to test the algorithm. According to the performance of the functions, they are divided into a single minimum (single) and a number of local minima (multimodal) into two categories. The following lists the 10 functions’ definition, variable range, and global optimal values of the theory and optimal solution of the theory. The function \( f_1 \) is a unimodal function used to examine the accuracy of the algorithm. The function \( f_2 \) which contains infinite local optimal solution and has infinite local maximum near suboptimal solution, is a strong shock multimodal function. General optimization algorithms easily fall into local optimal solution. Thus, the ability of the algorithm is used to avoid falling into a local optimization algorithm. The function \( f_3 \) is a unimodal function. It has about 10*D (D is dimension) local optimal solution. The function \( f_6 \) is a deep and local minimum multimodal function that has many local optimal solutions. The functions \( f_7 \) and \( f_8 \) are complex nonlinear used to test global search performance of algorithms. The function \( f_9 \) is a complex and classical function used to test the optimization efficiency of algorithms. In a certain number of iterations, this paper estimates the performance of IPPM by the test function of the best value, the worst value, average value, variance, average running time, and convergence rate.

1. Sphere Model function: \( \min f_1 = \sum x^2_i, |x| \leq 100 \). The optimal solution is \( (X_1, X_2, \ldots, X_n) = (0,0,\ldots,0) \) and the smallest optimal value is 0, when \( n = 30 \).

| Function | \( N \) | \( \delta \) | \( l \) | \( L_0 \) | \( J \) | \( \alpha \) | \( K \) |
|----------|------|-----|-----|-----|-----|-----|-----|
| 1        | 3    | 10  | 5   | 0.1 | 1E-6| 30  |
| 2        | 10   | 10  | 5   | 0.1 | 1E-7| 30  |
| 3        | 2    | 10  | 5   | 0.1 | 1E-7| 30  |
| 4        | 0.5  | 10  | 5   | 0.1 | 1E-9| 30  |
| 5        | 1.6  | 10  | 5   | 0.15| 1E-8| 30  |
| 6        | 0.512| 10  | 5   | 0.1 | 1E-9| 30  |
| 7        | 0.206| 10  | 5   | 0.1 | 1E-9| 30  |
| 8        | 0.15 | 10  | 5   | 0.15| 1E-9| 30  |
| 9        | 5    | 10  | 5   | 0.1 | 1E-10| 30 |
| 10       | 0.2  | 10  | 5   | 0.01| 1E-9| 30  |
| 11       | 6    | 10  | 5   | 0.1 | 1E-9| 30  |
| 12       | 10   | 50  | 5   | 0.1 | 1E-6| 30  |

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Table 2. Comparison performance of the IPMA in the ten functions.

| Function | Algorithm | Best value | Worst value | Average value | Variance | Average running time | Convergence rate |
|----------|-----------|------------|-------------|--------------|----------|----------------------|------------------|
| 1        | [12]      | 1.52E−02   | 5.47E−02    | 2            | 2        | 26.8                 | −                |
|          | [6]       | 1.52E−02   | 5.47E−02    | 2            | 2        | −                    | −                |
|          | IPMA      | 2E−18      | 1E−12       | 1E−13        | 1E−25    | 0.749                | 100%             |
| 2        | [1]       | 1.000000   | 0.990284    | −            | −        | −                    | 75%              |
|          | [4]       | 1.000000   | 1.000000    | −            | −        | 4.768                | −                |
|          | [5]       | 1E−10      | 1E−10       | −            | −        | 7.626                | −                |
|          | IPMA      | 1          | 1           | 1            | 0        | 2.104                | 100%             |
| 3        | [1]       | −30        | −25         | −            | −        | −                    | 75%              |
|          | [4]       | −30        | −29         | −            | −        | 2.881                | 95%              |
|          | IPMA      | −30        | −29         | 29.86        | 0.2040   | 0.542                | 96%              |
| 4        | [4]       | −6.000000  | −6.000000   | −            | −        | 7.015                | 100%             |
|          | IPMA      | −6         | −6          | −6           | 0        | 0.704                | 100%             |
| 5        | [4]       | 0          | 0           | 0            | 0        | 4.047                | 100%             |
|          | [7]       | −          | 4.68E−17    | 2.64E−17     | −        | −                    | −                |
|          | IPMA      | 0          | 0           | 0            | 0        | 0.614                | 100%             |
|          | [8]       | −          | −           | −            | −        | 0.27                 | −                |
|          | IPMA      | 0          | 0           | 0            | 0        | 1.529                | 100%             |
| 6        | [4]       | 1E−17      | 1E−17       | −            | −        | 23.375               | 100%             |
|          | IPMA      | 1E−21      | 1E−18       | 1E−19        | 1E−35    | 1.043                | 100%             |
| 7        | [12]      | 3.693E−07  | 2.738       | 6.375E−01    | 7.788E−01| 29                   | −                |
|          | [4]       | 1E−10      | 1E−10       | −            | −        | 29.893               | 100%             |
|          | [6]       | 3.59E−02   | 9.94E−02    | 10−2         | −        | −                    | −                |
|          | [8]       | −          | 10−2        | −            | 0.42     | −                    | −                |
|          | IPMA      | 1E−13      | 1E−11       | 1E−11        | 1E−21    | 1.675                | 100%             |
| 8        | [1]       | 1E−6       | 1E−6        | −            | −        | −                    | 100%             |
|          | [12]      | 4.667      | 2.92E02     | 6.690E01     | 5.512E01 | 50.0                 | −                |
|          | [7]       | −          | 0.002234    | 0.002645     | −        | −                    | −                |
|          | IPMA      | 1E−19      | 1E−17       | 1E−18        | 1E−30    | 10.013               | 100%             |
| 9        | [8]       | −          | 1E−2        | −            | 0.21     | −                    | −                |
|          | IPMA      | 0          | 0           | 0            | 3.52     | 100%                 | −                |
| 10       | [8]       | −          | −2094.57    | −            | 0.06     | −                    | −                |
|          | IPMA      | −2094.90   | −2094.90    | −2094.90     | 0        | 11.12                | 100%             |
|          | [8]       | −          | −2094.58    | −            | 1.97     | −                    | −                |
|          | IPMA      | −2094.90   | −2094.90    | −2094.90     | 0        | 5.99                 | 100%             |

This table shows performance of IPMA compared with other algorithms through the above indicators in 50 cycles.
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2. Schaffer function: \( \text{max} f_2 = 0.5 - \frac{\sin^2(\sqrt{X_1^2 + X_2^2} - 0.5)}{1 + 0.001(X_1^2 + X_2^2)^2} \), \(-100 \leq X_i \leq 100, i = 1,2 \). The optimal solution is \((X_1, X_2) = (0, 0)\) and the optimal maximum value is 1.

3. \( \text{min} f_3 = \sum_{i \in \{1,2,..,5\}} \text{integer}(X_i) \), \(-5.12 \leq X_i \leq 5.12, i = 1,...,5\). The function in the section \((-5.12, -5)\) has a global minimum value \(-30\).

4. \( \text{min} f_4 = -5 \sin X_1 \sin X_2 \sin X_3 \sin X_4 \sin X_5 - \sin(5X_1) \sin(5X_2) \sin(5X_3) \sin(5X_4) \sin(5X_5) \), \(0 \leq X_i \leq \pi, i = 1,2,...,5\). The optimal solution is \((X_1,X_2,...,X_5) = (\pi/2, \pi/2, ..., \pi/2)\) and the optimal minimum value is \(-6\).

5. Generalized Rastrigin function: \( f \). The function has many local minimum solutions, but it has only one global minimum solution \((X_1, X_2) = (0, 0)\) and the global minimum value is 0, when \(n = 2, m = 5\) and \(n = 10\).

6. Quartic function: \( \text{min} f_6 = \sum_{i=1}^{n} iX_i^2 - 100 \leq X_i \leq 100, i = 1,2,...,n \). The optimal solution is \((X_1, X_2,...,X_n) = (0,0,...,0)\) and the optimal minimum value is 0, when \(n = 20\).

7. Ackley’s function: \( \text{min} f_7 = -20 \exp\left[-0.2 \sqrt{\frac{1}{n} \sum_{i=1}^{n} X_i^2}\right] - \exp\left[\frac{1}{n} \sum_{i=1}^{n} \cos(2\pi X_i/n)\right] + 20 - e, |X_i| \leq 32 \). The optimal solution is \((X_1,X_2,...,X_n) = (0,0,...,0)\) and the optimal minimum value is 0, when \(n = 20\).

8. Generalized Rosen Brock’s function:


\[
\text{min } f_2 = \frac{\sum_{i=1}^{n-1} [100(X_{i+1} - X_i)^2 + (1 - X_i)^2]}{2.048}, \text{subject to } |X_i| \leq 2.048.
\]

The optimal solution is \((X_1, X_2, \ldots, X_n) = (1, 1, \ldots, 1)\) and the optimal minimum value is 0, when \(n = 3\).

9. Griewank function: \(\text{min } f_9 = 1/4000 \sum_{i=1}^{n} X_i^2 - \prod_{i=1}^{n} \cos (X_i/\sqrt{i}) + 1, X_i \in (-600, 600)\). The optimal minimum value is 0, when \(n = 30\).

10. Schwefel function: \(\text{min } f_{10} = - \sum_{i=1}^{n} X_i \sin \sqrt{|X_i|}, X_i \in (-500, 500)\). The optimal minimum value is \(-209.69\), when \(n = 5\). The optimal minimum value is \(-209.69\), when \(n = 50\). To verify the performance of the IPMA, we choose a computer whose CPU is made by AMD and has an operating system of Windows 7. The mathematical software Matlab7.0 was used for the testing. Each function is applied 50 times. We present the parameters in the process of solving the functions for this paper in the table 1.

In Table 1, the data of function \(f_1\) show that the improved population algorithm is better than frog-leaping algorithm in terms of precision. Other algorithms find it very difficult to achieve the optimal solution of function \(f_2\), but the IPMA can easily do so, which indicates that the algorithm’s ability of escaping from local optimal solution is very ideal. Function \(f_3\) can test whether the algorithm’s ability of global optimization is good. It is used to measure the efficiency of the algorithm, so we know the efficiency of the IPMA is very high compared with other algorithms from the data of function \(f_3\). The data functions \(f_4, f_5, f_6, f_7, f_8\) and \(f_{10}\) reflect that the IPMA is a better algorithm from a certain angle. Through cooperation of the best value, the worst value, the average value, the variance, and the convergence rate, the IPMA is found to be better than the other algorithms in the calculation of stability and precision.

**Conclusions**

In this paper, based on the local search mechanism and crossover operator, we put forward an IPMA. The parameters of the IPMA are simple and easy to realize. By testing the 10 functions, we find that the IPMA is better than the basic PMA and is superior to many intelligent algorithms. The PMA is good at global searches, but it does not perform well in local search and its precision is low. We introduce local search in population and crossover operator to improve these deficits. Therefore, the IPMA is increased to avoid falling into local optimum capacity; its calculation speed and precision are also improved.

**Author Contributions**

Conceived and designed the experiments: XYL. Performed the experiments: YQZ. Analyzed the data: YQZ. Contributed reagents/materials/analysis tools: YQZ. Wrote the paper: YQZ.

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