METASTABLE DEFECTS IN GENERIC EXTENSIONS
OF THE STANDARD MODEL

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ABSTRACT
The main features of the recently found classically stable quasi-topological mem-
branes and strings in generic, topologically trivial and weakly coupled two-Higgs
extensions of the standard model are briefly reviewed. A variety of localized so-
lutions in the same model are also presented.

1. Introduction
It is quite conceivable that the Higgs sector of the effective theory of electroweak
interactions will prove to contain more multiplets than just the one doublet of the
Minimal Standard Model (SM). In fact there are well known theoretical arguments
why this might even be desirable. Supersymmetry is one of them. The Minimal
Supersymmetric Standard Model (MSSM) contains two Higgs doublets. Models de-
rived from superstrings as low energy effective theories also have generically a richer
Higgs sector. Finally, if nature employs the mechanism of electroweak baryogenesis
to produce enough $n_B/n_\gamma$ in the Universe, then again an extended Higgs sector is
necessary as an extra source of soft CP-violation.

Let us consider such a model. Assume in addition that it has topologi-
cally trivial target space and vacuum manifold, so that it does not support the existence
of any kind of absolutely stable topological solitons! It was pointed out recently
that it is possible, if the Higgs masses satisfy certain lower bounds, to obtain a
dynamical exclusion of part of the target space, which becomes effectively non-trivial
and leads to the existence of metastable quasi-topological solitons, whose life-times
depend on the specific values of the masses and are typically cosmological. More
importantly it was shown that the necessary conditions may be satisfied for per-
turbatively small values of the coupling constants, consistent with the semiclassical
approximation.

In this lecture I will first describe the method used to search for such solitons
in the context of a toy model, and then I will apply it to find classically stable
wall and string defects in the weakly-coupled generic 2HSM. This is one potentially
interesting realization of a general result which should be taken into account in the

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†In the opposite case topological solitons will exist in addition to the defects discussed below.
analysis of field theoretic models.

2. A toy model

The simplest way to present the method we use to search for classically stable quasi-topological solitons in a field theory, is in the context of the toy model

$$\mathcal{L} = \frac{1}{2} \partial_\mu \Phi^* \partial^\mu \Phi - \frac{\lambda}{4} (\Phi^* \Phi - v^2)^2 + \mu^2 v \text{Re}(\Phi)$$

(1)

in 1+1 dimensions. First of all, the model has a unique vacuum and a trivial target space. Thus, it does not have any topological solitons. Notice on the other hand, that in the limit $\lambda \rightarrow \infty$ all finite energy configurations have the form $\Phi = v \exp i \Theta$. The dynamics of the angle $\Theta$ is obtained by substituting this $\Phi$ into the action. One obtains the sine-Gordon model,

$$\mathcal{L} \rightarrow v^2 \left[ \frac{1}{2} (\partial_\mu \Theta)^2 + \mu^2 \cos \Theta \right]$$

(2)

known to possess absolutely stable topological solitons, in which the angle $\Theta(x)$ changes by $2\pi$ as one moves from $-\infty$ to $+\infty$. It is then intuitively natural to expect that even for finite values of $\lambda$, once it is kept large enough, remnants of these solitons will survive not as absolutely stable solitons, since the topology of the model will be lost, but as local minima of the energy functional. Roughly, the only way an initial configuration with non-zero winding can lose its topological charge and evolve into radiation around the trivial vacuum, is when the Higgs magnitude passes through zero, a process strongly suppressed by a large radial-Higgs mass, i.e. by a large ratio $R \equiv m/\mu \simeq \sqrt{2} \lambda v/\mu$. Indeed, we have checked numerically that for $R \geq 6.1$ the model possesses non-trivial local minima of the energy. These are static solutions of the field equations, characterized by non-zero winding of the phase $\Theta$ (like the topological solitons of the limiting theory), and a space dependent Higgs magnitude, smaller that its asymptotic vacuum value inside the region of width $\sim 1/\mu$ of $\Theta$ variation.

A couple of comments are in order at this point: (a) It is straightforward to check that the solitons exist for arbitrary values of the parameter $\lambda$. Something very desirable since our semiclassical discussion cannot be trusted outside the perturbative regime of small $\lambda$. Indeed, one may by appropriate rescalings pull outside of the action the factor $1/\lambda$ and the only physically relevant parameter left inside is the ratio $R$ defined above. Thus, all conditions about existence and stability of classical solutions are conditions on $R$, while the quantum decay rate is exponentially supressed and of $\mathcal{O}(\exp(-1/\lambda))$ for small $\lambda$. (b) All results are consistent with perturbative unitarity. (c) One may analytically verify these results in a space-compactified version of the model.

3. The two-Higgs standard model

We will now use the above method to search for analogous solitons in realistic two-Higgs-doublet extensions of the standard model.

The generic 2HSM is described by the Lagrangian

$$\mathcal{L} = -\frac{1}{4} W^a_{\mu \nu} W^{a \mu \nu} - \frac{1}{4} Y_{\mu \nu} Y^{\mu \nu} + |D_\mu H_1|^2 + |D_\mu H_2|^2 - V(H_1, H_2)$$

(3)
where \( W_{\mu}^a = \partial_\mu W^a - \partial_\nu W^a - g e^{abc} W^b_\mu W^c_\nu \) and \( Y_{\mu\nu} = \partial_\mu Y_\nu - \partial_\nu Y_\mu \), the physical \( Z^0 \) and photon fields are \( Z_\mu = W_\mu^3 \cos \theta_W - Y_\mu \sin \theta_W \) and \( A_\mu = W_\mu^3 \sin \theta_W + Y_\mu \cos \theta_W \) and \( \tan \theta_W = g'/g \). Both Higgs doublets have hypercharge equal to one, the covariant derivative is

\[
D_\mu H_I = (\partial_\mu + i/2 g \tau^a W^a_\mu + i/2 g' Y_\mu) H_I
\]

for \( I = 1, 2 \) and the potential reads

\[
V(H_1, H_2) = \lambda_1 \left( |H_1|^2 - \frac{v_1^2}{2} \right)^2 + \lambda_2 \left( |H_2|^2 - \frac{v_2^2}{2} \right)^2 + \lambda_3 \left( |H_1|^2 + |H_2|^2 - \frac{v_1^2 + v_2^2}{2} \right)^2
\]

\[
+ \lambda_4 \left( |H_1|^2 |H_2|^2 - (H_1^0 H_2)(H_2^0 H_1) \right) + \lambda_5 \left[ \text{Re}(H_1^* H_2) - \frac{v_1 v_2}{2} \cos \xi \right]^2
\]

\[
+ \lambda_6 \left[ \text{Im}(H_1^* H_2) - \frac{v_1 v_2}{2} \sin \xi \right]^2
\]

(4)

where \( |H_I|^2 \equiv H_I^0 H_I \). This is the most general potential\(^a\) subject to the condition that both CP invariance and a discrete \( Z_2 \) symmetry \((H_1 \rightarrow -H_1)\) are only broken softly. The softly broken \( Z_2 \) symmetry is there to suppress unacceptably large flavor-changing neutral currents. Assuming all the \( \lambda_i \) are positive, the minimum of the potential is at

\[
<H_1> = e^{-i\xi} \left( \begin{array}{c} 0 \\ v_1/\sqrt{2} \end{array} \right) \quad \text{and} \quad <H_2> = \left( \begin{array}{c} 0 \\ v_2/\sqrt{2} \end{array} \right)
\]

(5)

and apart from the electroweak gauge bosons with masses \( m_\gamma = 0, m_W^2 = g^2 (v_1^2 + v_2^2)/4 \) and \( m_Z = m_W / \cos \theta_W \), the perturbative spectrum contains a charged Higgs boson \( H^+ \), a CP-odd neutral scalar \( A^0 \), and two CP-even neutral scalars \( h^0 \) and \( H^0 \). Again, like in the toy model, the Higgs vacuum is unique and the target space topologically trivial \( (\mathcal{R}^8) \), so that no topological solitons arise.

3.1. The membranes\(^b\)

Let us reduce the large number of parameters by restricting ourselves to \( \lambda_1 = \lambda_2, \lambda_5 = \lambda_6, \xi = 0 \) and \( v_1 = v_2 = v \) (or \( \tan \beta = 1 \)) and consider the limit \( \lambda_1 = \lambda_2 \rightarrow \infty \) and \( \lambda_4 \rightarrow \infty \). The first fixes the magnitudes of the two doublets, while the second restricts them to the form \(^b\)

\[
H_1 = e^{i\Theta/2} \left( \begin{array}{c} 0 \\ v/\sqrt{2} \end{array} \right) \quad \text{and} \quad H_2 = e^{-i\Theta/2} \left( \begin{array}{c} 0 \\ v/\sqrt{2} \end{array} \right).
\]

(6)

In the limit we are considering only \( A^0 \) has finite mass, so only the corresponding field \( \Theta \) can have non-trivial space dependence in finite energy configurations.

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\(^a\)Relaxing these conditions is straightforward but beyond the scope of the present note.

\(^b\)We made use of the gauge freedom of the model to assign half of the relative phase \( \Theta \) between the two doublets on each one of them.
The energy functional for the field $\Theta$ takes the sine-Gordon form

$$E = \frac{v^2}{2} \int d^3x \left[ \frac{1}{2} (\nabla \Theta)^2 - m_A^2 \cos \Theta \right] + \text{gauge} \quad (7)$$

where the gauge contribution is minimum when $W^a_\mu = Y_\mu = 0$. The limiting sine-Gordon model possesses absolutely stable topological walls, of thickness of $O(m_A^{-1})$ and energy per unit area $\mathcal{E}/A = 4v^2m_A = 2m_H^2m_A\sin^2\theta_W/\pi\alpha$ ($\alpha$ is the fine structure constant). We thus expect that for finite but "large enough values of the parameters $\lambda_1 = \lambda_2$ and $\lambda_4$" there will arise classically stable solutions of the full equations of the model which should look very similar to the sine-Gordon solution.

The search for solutions and the study of their stability for finite values of the parameters $\lambda_1 = \lambda_2$ and $\lambda_4$ was carried out numerically in the gauge $W^a_x = Y_x = 0$. The profile of the solution for $m_h = 2.5m_A$, $m_{H^0} = 5.0m_A$ and $m_{H^+} = 4.0m_A$ is shown in Figure 1.

![Figure 1](image_url)

All gauge fields as well as the upper (charged) components of both Higgs doublets vanish. The neutral components of the doublets differ very little from their form in the limiting theory. This fact is also shown in Figure 2, where we plot the neutral components of the two Higgs doublets in the complex plane. Their phases wind around in opposite directions as one crosses the wall, starting at zero and joining at $\pm \pi$.

Figure 3 depicts the boundaries of classical stability region in the $(m_{H^0}, m_{H^+})$ plane for three different values of $m_h$ all in units of $m_A$. Classically-stable membranes
exist above the indicated lines. Also given is the energy density in units of $v^2 m_A$, for some selected points close to the boundaries. For values of the Higgs masses consistent with present experimental bounds and with perturbative unitarity (for instance for $m_A \sim 50$ GeV, $m_h \sim 125$ GeV and $m_{H^0}, m_{H^+} \gtrsim 200$ GeV) the theory supports the existence of classically stable wall "defects", with typical thickness and energy density of $O(m_A^{-1})$ and $O(4m_W^3) \sim 10^{10} gr/cm^2$ respectively. Their decay rate per unit area has an exponential dependence on the parameters of the theory and is thus very sensitive to their precise values.

Figure 2

They are so heavy that if their life-times are cosmological they will be as disastrous as the ordinary topological domain walls arising in models with spontaneously broken discrete symmetries. The presence of even an isolated wall of cosmic dimensions in our Universe today is clearly excluded. Assuming, as it seems quite natural, that their production in the early universe is not extremely suppressed, the curves of Figure 3 should in this case be interpreted as giving the upper bounds of the Higgs masses consistent with the cosmological data. If, on the other hand, the membranes are short lived (as it will happen if the values of the masses happen to be close to a boundary line), they are potentially useful either as initial seeds for galaxy formation, or by contributing to the electroweak baryogenesis. For these statements to become more quantitative a thorough study is necessary of their production mechanism, of their
decay rate and of their dynamics.

3.2. The string

It is possible to apply the same method to find stable string solutions in the 2HSM. The details are given in Reference [6]. Here I briefly present the main steps.

Let us go back to (3), (4) and consider the limit $\lambda_1, \lambda_2 \to \infty$ and $\lambda_5 \to \infty$. Among the Higgs fields only $H^+$ has finite mass in this limit.

![Graph showing Higgs field configurations](image)

**Figure 3**

The generic finite energy Higgs field configuration is then

$$H_1 = \begin{pmatrix} 0 \\ v_1/\sqrt{2} \end{pmatrix} \quad \text{and} \quad H_2 = U \begin{pmatrix} 0 \\ v_2/\sqrt{2} \end{pmatrix},$$

(8)
with
\[ U(x) = e^{-i \xi n(x) \cdot \tau} \] (9)

The only dynamical field left in the above limit is the unit vector \( n(x) \), which for finite energy must satisfy \( n_3(x) = 1 \) at spatial infinity.

If in addition one takes the remaining couplings of the Higgs potential and the gauge couplings to zero, one is left with an effective O(3) non-linear \( \sigma \)–model for the unit vector field \( n(x) \). It is well known that this model possesses topological solitons characterized by an \( S^2 \rightarrow S^2 \) winding number, an arbitrary size \( \rho \) and an energy which due to the scale invariance of the model does not depend on \( \rho \).

What will happen to such a soliton solution of the limiting theory if one tries to relax the couplings \( \lambda_1, \lambda_2 \) and \( \lambda_5 \) to finite values? To leading order one only has to consider the effect of the potential and of the gauge interactions on the size of the soliton. Using a scaling argument one may show that the potential tends to shrink the soliton and the question then is if the gauge interaction is able to halt this shrinking. In Reference [6] it was shown that for large enough radial Higgs masses the two opposing forces come to equilibrium and a classically stable string solution arises. This is actually true even for perturbatively small values of the parameters, again consistent with perturbative unitarity and with present day phenomenological bounds on the Higgs masses.

Such strings may be produced either cosmologically at the electroweak phase transition or with smaller probability as finite length (of order a few over \( m_W \)) string loops in large accelerators such as LHC. We do not have at this point control over their production probability and their decay rate, necessary to analyse their phenomenological implications. Naively, one expects that so energetic Higgs lumps will decay characteristically into a very large number of jets.

3.3. No stable localized spherically symmetric solitons

In a similar fashion, let us go back to the generic two-Higgs model, and set for simplicity \( v_1 = v_2 = v \) and \( g', \lambda_3, \lambda_4, \lambda_5, \lambda_6 \) as well as \( \xi \) all equal to zero. In the simplified model consider the limit \( \lambda_1 = \lambda_2 \rightarrow \infty \). All finite energy Higgs configurations must then have the form:

\[ H_1 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \quad \text{and} \quad H_2 = U \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \] (10)

with \( U \) a general SU(2) matrix. The dynamics of \( U \) is described by

\[ \mathcal{L} \sim \text{Tr} \partial_\mu U^{-1} \partial^\mu U + \text{gauge part} \] (11)

One may follow the steps of the string case and consider the limit \( g = 0 \). The model reduces to the global SU(2) non-linear \( \sigma \)-model. Finite energy configurations of this model have \( U = \text{constant at infinity} \) and are thus classified according to \( \pi_3(S^3) \). Contrary to the previous two-dimensional case the limiting model does not support the existence of such solutions. Any such non-trivial configuration shrinks indefinitely to a singular configuration.

Our starting point is not so promising. In contrast to the previous cases, the limiting theory does not possess stable solitons. Actually, once we allow for finite values of \( \lambda_1 \) and \( \lambda_2 \) the shrinking tendency of any non-trivial Higgs configuration will
be enhanced. One might still hope though, that this will be halted by the gauge interaction, in much the same way this happened in the case of the strings.

The results of our search for spherically symmetric solutions and for generic values of all parameters of the model are described in detail in Reference [9]. There is a wide variety of solutions none of which was found to be stable. Figure 4 is a graphical representation of the solution branches in the special case of equal masses $m_1 = m_2 \equiv m$ for the two radial Higgses and equal values $v_1 = v_2$ for their corresponding vevs.

![Figure 4](image-url)

$S_0$ corresponds to the sphaleron solution of the SM embedded in the 2HSM, $S_1$ to the first deformed sphaleron and so on. The $W_n$ are new branches with no analog in the SM. The integer number shown on each branch represents the number of unstable modes of the corresponding solution. The interested reader may find a discussion of the characteristics of the various branches of solutions and of their potential phenomenological role in Reference [9].

I would like to finish with a comment about the possibility of another kind of localized solitons in the 2HSM. It was argued above that there exists a limit of the parameters of the model in which the only remaining degrees of freedom are the components of a unit vector $n$. Furthermore, any finite energy configuration must have $n_3 = 1$ at spatial infinity. In three dimensions such a configuration defines a map from $S^3$ into $S^2$, known to be classified according to the Hopf homotopy group $\pi_3(S^2)$. These maps are at best axially symmetric and can be thought of as a finite length straight string twisted by an integral multiple of $2\pi$ around its axis and glued at the two ends. A situation in between the infinite string (shown to exist) and the
spherically symmetric case, where we failed. It is thus possible that in this case the
gauge fields will manage to halt the shrinking and that non-trivial axially symmetric
stable Hopf configurations will exist.

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