Radiation-condensation instability in a highly ionized dusty plasma

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Abstract

The dynamics of linear perturbations in a radiatively cooling dusty plasma is considered, with the charge of both dust \(Z_d\) and plasma \(Z_p\) components being allowed to vary with their densities. It is shown that in the long-wavelength limit corresponding to the characteristic cooling length, when the plasma can be treated as quasineutral, the presence of dust particles changes the criteria for radiation instability, regardless the charging process of the dust particles. In particular, the condensation (isobaric) mode is shown to be stabilized (destabilized) if in the equilibrium, the relation between densities of the dust \(n_d\) and plasma \(n\) under the quasineutrality condition, \((d \ln n_d/d \ln n)_q < 1\) \((> 1)\) is satisfied, while the isentropic mode is stabilized (destablized) when the opposite inequalities take place; the isochoric mode is unaltered. Numerical estimates show that these effects can be important in hot phases (\(T \sim 10^6\) K) of the interstellar plasma, and in tokamak plasma near the walls.

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I. INTRODUCTION

It is well known (see, for example, Refs. 1, 2, 3) that the presence of charged dust particles in plasma changes its dispersion properties qualitatively, and that the dust itself can give rise to new plasma instabilities. From intuitive point of view it seems possible that dust must be important in dynamics of radiatively cooling plasmas, not only because it absorbs radiation, but also due to its ability to transform thermal energy of the plasma into radiation in inelastic collisions with electrons and ions. Apparently, it can be of great importance in high temperature ($T \gtrsim 10^6$ K) cosmic and laboratory (in particular, close to tokamak walls) plasmas, where frequent collisions with the ions maintain dust particles sufficiently hot, so that they re-emit thermal energy efficiently\(^4\). The present work is aimed to analyze the effects of charged dust particles on the conditions for the radiation instabilities. It is important to note that this issue has been addressed recently in\(^5,6\), however in these papers dust particles were considered only from the point of view of electrostatic interactions, while the cooling processes were treated as independent on the presence of dust. In our study we explicitly assume that dust particles contribute substantially to the net cooling rate, which brings to the system qualitatively new dynamical features. For the sake of simplicity we will consider here only highly ionized plasmas, when the effects from neutral particles can be neglected. In Sec. II.A we briefly discuss how important can be dust as a cooler of the plasma. In Sec. II.B we formulate a simplified two-fluid model of a collisional and radiatively cooling dusty plasma. In Sec. III.A we describe the steady state equilibrium, with the relevant parameters estimated in Sec. III.B, while in Sec. III.C linearized equations are written. In Sec. III.D the dispersion equation and its solutions are given and criteria of thermal instability in different wavelength ranges are discussed. Sec. IV summarizes the results.
II. BASIC EQUATIONS

A. Radiatively cooling dusty plasma

In an optically thin plasma dust particles, if present, besides their dynamical effects through electrostatic interaction with plasma particles, can also work as a cooling agent via the collisional transfer of thermal energy of plasma component into infrared radiation. If one assumes, as an idealization, that dust particles emit as black body, the energy loss rate due to dust particles varies as \( \propto T_d^4 \), where \( T_d \) is temperature of the dust. For realistic particles this dependence can be steeper. Detailed calculations of the relevant cooling rate are made in Ref. 4. They show that for physical conditions and abundance of dust typical for the interstellar medium (ISM), radiation losses from dust particles dominates cooling from the line emission of heavy elements by two orders of magnitude in the interval \( T = 10^6 - 10^{10} \) K. One should stress, however, that at temperatures \( T \sim 3 \times (10^7 - 10^8) \) K dust particles are efficiently destroyed in collisions with the ions and electrons, and therefore only in temperature range between \( 10^6 \) K and \( 3 \times 10^7 \) K dust contributes sufficiently to the net cooling. Apparently, dust cooling can also be important in tokamak plasma near the walls, where temperature can lie in this interval, or in regions of MARFEs (multifaceted asymmetric radiation from the edge) formation if radiating impurities are introduced into the plasma by sputtering.

B. Two fluids

We will consider a simplified two-fluid approach, with the dust charge instantaneously (on times much shorter than other characteristic time scales of the system) settled on the equilibrium \( Z_d = Z_d(n, n_d, T) \), where \( n \) and \( n_d \) are the plasma and dust densities, \( T \) is the plasma temperature. To outline the main physical effects, we will study the simple case of potential perturbations in the planar geometry. The full set of equations in this case is

\[
\partial_t n + \partial_x (n v) = 0,
\]
\[
\rho \partial_t v + \rho v \partial_x v = -\partial_x p - e Z_p n \partial_x \phi - f_0 (v - u),
\]

(2)

\[
\frac{3}{2} d_t p - \frac{5}{2} p d_t n = -\mathcal{L}(T, n, n_d),
\]

(3)

\[
\partial_t n_d + \partial_x (n_d u) = 0,
\]

(4)

\[
\rho_d \partial_t u + \rho_d u \partial_x u = -e Z_d n_d \partial_x \phi + f_0 (v - u),
\]

(5)

\[
p = k_B n T,
\]

(6)

\[
\partial_x^2 \phi = -4 \pi e (Z_d n_d + Z_p n),
\]

(7)

where \( v, u \) are velocities of the plasma and dust particles, respectively, \( f_0 \) [g cm\(^{-3}\)s\(^{-1}\)] is the coefficient describing the friction between dust and plasma, \( \rho = m n \) and \( \rho_d = m_d n_d \) are the mass densities of the plasma and dust components, \( p \), the gas pressure, \( \phi \), the electrostatic potential, \( Z_d \), the dust charge (negative for negatively charged dust), \( Z_p \), the mean plasma charge defined as \( Z_p n = (Z_i n - n_e) = (Z_i - x)n \) with \( Z_i \) being the ion charge, and \( x = n_e / n \); note that for positively charged dust particles \( x \geq Z_i \); here an adiabatic index \( \gamma = 5/3 \) has been assumed and the net cooling function \( \mathcal{L} \) explicitly depending on number density of dust particles; the dust component is assumed kinetically cold, so that the dust pressure term in Eq. (6) is omitted; \( \partial_a \) and \( d_a \) denote partial and Lagrangian derivatives with respect to \( a \). In general, dust particles contribute both to heating (e.g., via photoemission of electrons from grain surface, see for recent discussion Ref. 8), and cooling (via collisional transfer of kinetic energy of plasma particles into heat of dust grains and subsequent re-emission in, as a rule, infrared range\(^4\)). It is obvious that the charge separation are important only on the spatial scales comparable (or shorter) to Debye length, on longer lengths plasma can be treated as quasineutral (see, below).
III. DISPERSION RELATION

A. Unperturbed state

We assume that in an unperturbed state radiative cooling is balanced by energy input from external sources (e.g. hard emission, Ohmic heating etc.), so that

\[ \mathcal{L} = 0, \]  

(note that in most cases the net cooling function can be presented in the form \( \mathcal{L} = L(T, n, n_d) - \Gamma(T, n, n_d)n \), where \( L(T, n, n_d)n \) is the radiative cooling rate, \( \Gamma(T, n, n_d)n \), the heating rate due to external energy sources]. We assume also that unperturbed plasma is quasineutral

\[ Z_{d0}n_{d0} + Z_{p0}n_0 = 0. \]  

We will further assume that the dust and plasma charges adjust the corresponding equilibria instantaneously \( Z_d = Z_d(n, n_d, T) \) and \( Z_p = Z_p(n, n_d, T) \), i.e. that the charging characteristic times are shorter than other relevant times.

B. Evaluation of the relevant numbers

The problem is characterized by three time scales: thermal, frictional, and electrostatic. Thermal time scale is written as

\[ \tau_0 = \frac{k_B T_0}{L_0}, \]  

here \( L_0 \) is the radiative cooling rate taken at the equilibrium, \( k_B \), Boltzmann constant. Frictional times for the plasma and dust particles

\[ \tau_f = \frac{\rho_0}{f_0}, \quad \tau_{f,d} = \frac{\rho_{d0}}{f_0}, \]  

respectively, and electrostatic time (the inverse plasma frequency)
\[ \tau_e = \sqrt{\frac{m}{4\pi e^2 n}}. \]  

(12)

All characteristic times (frictional and electrostatic) relevant to the dust component are connected with the corresponding plasma times by the intrinsic constants \( \delta = n_{d0}/n_0 \), the concentration of dust particles, and \( \mu = m_d n_{d0}/mn_0 \equiv \rho_{d0}/\rho_0 \), the mass density ratio of the dust and plasma components:

\[ \tau_{f,d} = \mu \tau_f, \quad \tau_{e,d} = \delta^{-1} \mu^{-1/2} \tau_e. \]  

(13)

It is readily seen that two dimensionless parameters appear in the problem

\[ \kappa = \frac{\tau_0}{\tau_e} = \frac{k_B T_0}{L_0} \sqrt{\frac{4\pi e^2 n}{m}}, \quad \text{and} \quad \nu = \frac{\tau_0}{\tau_f} = \frac{k_B T_0 f_0}{L_0 \rho_0}, \]  

(14)

which characterize the role of electrostatic and frictional forces in dynamics of radiation-condensation instability.

C. Linearized equations

The full set of linearized dynamical equations for a nonmagnetized dusty plasma is written as

\[ \partial_t n + n_0 \partial_x v = 0, \]  

(15)

\[ mn_0 \partial_t v + \partial_x p + eZ_{p0} n_0 \partial_x \phi + f_0 (v - u) = 0, \]  

(16)

\[ \frac{3}{2} d_t p - \frac{5}{2n_0} d_t n + (\partial_{\alpha} \mathcal{L})n + (\partial_{\alpha} \mathcal{L}) n_d + (\partial_\tau \mathcal{L}) T = 0, \]  

(17)

\[ \partial_t n_d + n_{d0} \partial_x u = 0, \]  

(18)

\[ m_d n_{d0} \partial_t u + eZ_{d0} n_{d0} \partial_x \phi - f_0 (v - u) = 0, \]  

(19)

\[ \frac{p}{p_0} = \frac{n}{n_0} + \frac{T}{T_0}, \]  

(20)
\[ \partial^2_x \phi = -4 \pi e [Z_{d0} n_d + (\partial_n Z_d) n_{d0} n + (\partial_T Z_d) n_{d0} T + (\partial_{n_d} Z_d) n_{d0} n_d + Z_{p0} n + (\partial_n Z_p) n_{p0} n + (\partial_T Z_p) n_{p0} T + (\partial_{n_d} Z_p) n_{d0} n_d, \] (21)

where the unperturbed variables are given with subscript zero, \( a_0 \), while the perturbations are without subscript, \( a \).

In order to evaluate the interrelation between electrostatic and radiative effects it is useful to write the linearized Poisson equation (21) in a non-dimensional form

\[ \partial^2_x \bar{\phi} = -\kappa^2 \{ \delta [Z_{d0} \bar{n}_d + (\partial_n Z_d) \bar{n} + (\partial_{n_d} Z_d) \bar{n}_d + (\partial_T Z_d) \bar{T}] + (\partial_n Z_p) \bar{n} + (\partial_{n_d} Z_p) \bar{n}_d + (\partial_T Z_p) \bar{T} + Z_{p0} \bar{n}, \] (22)

here all variables with bar are normalized to unperturbed values: \( \bar{a} = a/a_0; \ \bar{\tau} = t/\tau_0, \ \bar{\xi} = x/\tau_0 c_0 \), where \( c_0 \) is the isothermal sound speed, the potential is normalized to \( m c_0^2/e, \kappa \) is defined above.

For typical interstellar or tokamak hot plasma at \( T \sim 10^6 \) K, \( \kappa \) can be estimated as

\[ \kappa^2 \sim \frac{3 \cdot 10^{30}}{n_0}. \] (23)

It is readily seen from these estimates that motions on electrostatic and radiation scales can be separated due to a huge difference in corresponding times and lengths. This simply means that electrostatic forces acting between the charged dust and plasma components keep dusty plasma quasineutral on much shorter time scales than the relevant hydrodynamical scales. Formally it can be expressed in the form of perturbation procedure applied to the above dimensionless linearized equations with dynamical variables as a series

\[ a = a^0 + \kappa^{-2} a^1 + ..., \] (24)

and potential as

\[ \phi = \kappa^{-2} \phi^1 + \kappa^{-4} \phi^2 + ... \] (25)

Thus in what follows we turn to the dimensional variables and equations, with the quasineutrality condition instead of solving Poisson equation.
D. Dispersion equation

In this framework the quasineutrality equation can be written in terms of $n_d$

$$n_d = -[Z_{d0} + (\partial_{n_d}Z_d)n_{d0} + (\partial_{n_d}Z_p)n_0]^{-1}[(\partial_nZ_d)n_{d0} + Z_{p0} + (\partial_nZ_p)n_0]n +$$

$$- [Z_{d0} + (\partial_{n_d}Z_d)n_{d0} + (\partial_{n_d}Z_p)n_0]^{-1}[(\partial_TZ_d)n_{d0} + (\partial_TZ_p)n_0]T.$$  \hspace{1cm} (26)

From Eq. (26) it follows that $(\partial_Tn_d)_q = -[Z_{d0} + (\partial_{n_d}Z_d)n_{d0} + (\partial_{n_d}Z_p)n_0]^{-1}[(\partial_TZ_d)n_{d0} + (\partial_TZ_p)n_0]$, which will be taken as zero in order that the quasineutrality to hold for purely isochoric perturbations; here the subscript $q$ denotes that the derivative is taken over the quasineutrality state.

Combining Equations of motion 16 and 19 in order to eliminate undefined $\phi$ we arrive finally for perturbations in the form $a \propto \exp(\Omega t + ikx)$ to the eigenmatrix $M$ for the eigenvector $(n', T', v', u')$

$$M = \begin{pmatrix}
\Omega & 0 & ikn_0 & 0 \\
0 & ik\frac{\rho_0}{n_0} & \frac{\rho_0}{n_0} \Omega & \rho_0 \Omega \\
-\frac{\rho_0}{n_0} \Omega + \mathcal{L}_n + A\mathcal{L}_{n_d} & \frac{3}{2} \frac{\rho_0}{T_0} \Omega + \mathcal{L}_T & 0 & 0 \\
A\Omega & 0 & 0 & ikn_{d0}
\end{pmatrix}, \hspace{1cm} (27)
$$

where $A = -[Z_{d0} + (\partial_{n_d}Z_d)n_{d0} + (\partial_{n_d}Z_p)n_0]^{-1}[(\partial_nZ_d)n_{d0} + Z_{p0} + (\partial_nZ_p)n_0] \equiv (dn_d/dn)_q$.

Finally the compatibility condition $\text{det}(M) = 0$ leaves the dispersion equation

$$\Omega^3 + \frac{2}{3} \frac{T_0}{\rho_0} \mathcal{L}_T \Omega^2 + \frac{5}{2} k^2 c_d^2 \Omega + \frac{2}{3} k^2 c_d^2 \left[ \frac{T_0}{\rho_0} \mathcal{L}_T - \frac{n_0}{\rho_0} (\mathcal{L}_n + A\mathcal{L}_{n_d}) \right] = 0,$$  \hspace{1cm} (28)

where $c_d = c_0/\sqrt{1 + \mu}$, $c_0$ being the isothermal sound speed in a plasma gas. It is seen that this equation is identical to the standard dispersion equation\(^9\), when all terms correspondent to dust component are omitted.

D1. Isobaric mode

In general, there are three solutions of Eq. 28. One corresponds to the so-called isobaric (condensation) mode, which formally can be obtained from Eq. 28 putting $k \to \infty$ and $\text{Im}(\Omega) = 0$ (see discussion in Refs. 9, 10), i.e.
The condition for instability of this mode is also straightforwardly follows from the Hurwitz criterion that the last coefficient of the third order polynomial in Eq. 28 is negative, i.e. 
\[ n_0(L_n + A L_n d) - T_0 L_T > 0. \]

For the cooling function in the form \( L = \Lambda(T)n^2 + \Lambda_d(T)nn_d - \Gamma n \), one can write the growth rate as
\[
\Omega \sim \frac{2}{5p_0} \left[ n_0(L_n + A L_n d) - T_0 L_T \right].
\] (29)

\[ \eta_d = \Lambda_d(T)nn_d/L^c \] is the relative contribution of dust cooling. It is readily seen that for strongly dynamically coupled ions and dust when \( n_d = \delta n \), electrostatic forces do not affect radiation instability. When \( (d \ln n_d/d \ln n)_q < 1 \), i.e. quasineutrality and dust charging requires that the dust particles escape compressed regions, the condensation mode is stabilized because the corresponding condition
\[
\frac{\partial \ln L^c}{\partial \ln T} < 1 - \frac{d \ln n_d}{d \ln n} - 1 \eta_d,
\] (31)
fulfills in a narrower temperature interval than when electrostatic effects disappear. Contrary, if quasineutrality and dust charging requires an enhancement of dust particles in compressed regions, i.e. \( (d \ln n_d/d \ln n)_q > 1 \), the condensation instability is enhanced. Qualitatively it can be understood as due to an enhancement of the net cooling rate associated with dust cooling.

\[ D2. \text{ Isentropic (acoustic) mode} \]

The other two modes in the short-wavelength limit correspond to overstable acoustic motions with the wave numbers \( \pm k \) and the growth rate much smaller than the wave frequency: \( |Re(\Omega)| \ll |Im(\Omega)|^{9,10} \). In order to find the solution in this case we will expand it as
\[
\Omega = \pm i \left( \frac{5}{3} \right)^{1/2} k c_d + \Omega_r,
\] (32)
where $\Omega_r$ is the growth rate of the amplitude of acoustic waves, for which we obtain to the first order to $k$

$$\Omega_r \sim -\frac{2}{15p_0} \left[ T_0\mathcal{L}_T + \frac{3}{2}n_0(\mathcal{L}_n + A\mathcal{L}_{n_d}) \right], \quad (33)$$

or

$$\Omega_r \sim -\frac{2\mathcal{L}_c}{15p_0} \left\{ \frac{\partial \ln \mathcal{L}_c}{\partial \ln T} + \frac{3}{2} \left[ 1 + \left( \frac{d\ln n_d}{d\ln n} \right)_q - 1 \right] \eta_d \right\}. \quad (34)$$

It is seen from (34) that contrary to the isobaric mode, isentropic perturbations are destabilized if $(d\ln n_d/d\ln n)_q < 1$, since the restrictions on the cooling rate $\mathcal{L}_c$ is here weaker than in the absence of electrostatic forces. Instead, isentropic perturbations are stabilized when $(d\ln n_d/d\ln n)_q > 1$. Such a behavior is qualitatively clear because the instability of isentropic mode is physically connected with overheating of adiabatically compressed regions, and as soon as dust cooling decreases in compressed regions when $(d\ln n_d/d\ln n)_q < 1$, it results in an additional overheating.

**D3. Isochoric mode**

The instability criterion for the isochoric mode corresponding to the long-wavelength limit $k \to 0$: $\mathcal{L}_T < 0$, remains obviously unaltered by the electrostatic forces and additional dust cooling, because this limit describes perturbations with $T \neq 0$ and $n_d \simeq n \simeq 0$.

**V. SUMMARY**

In the present paper we considered radiation instability in an optically thin dusty plasma with dust particles contributing substantially to the radiative energy losses. We have shown that combination of the two effects: electrostatic interaction of the dust and plasma components and their tight dynamical coupling (expressed in quasineutrality), and additional energy losses connected with dust particles, strongly modifies the criteria for the instability. This influence differs for different instability regimes: the isobaric (condensation) mode is
shown to be destabilized in dusty plasmas if the dust particles are excessively attracted in the regions of gas compression, \((d \ln n_d/d \ln n)_q > 1\), and stabilized in the opposite case; in the short-wavelength limit corresponding to the isentropic (adiabatic) mode effects from the presence of dust work differently: they quench the instability when \((d \ln n_d/d \ln n)_q > 1\), and destabilize it in the opposite case; the condition for the instability remains unaltered in a long-wavelength range where isochoric mode dominates.

In a simplest case of a dilute hot plasma with dust particles charged predominantly collisionally, i.e. \(j_p \to 0\), \((d \ln n_d/d \ln n)_q\) approaches unity from below as (see Appendix)
\[
\left(\frac{d \ln n_d}{d \ln n}\right)_q = 1 - \left(\frac{m_i}{m_e}\right)^{1/2} j_p,
\]
and therefore the isobaric mode is quenched, while the isentropic is weakly enhanced.

As \(j_p\) approaches the critical value \(j_{pc} = (m_i/m_e)^{1/2} - 1\), when dust particles change the charge from negative to positive, \((d \ln n_d/d \ln n)_q \to -\infty\) at \(j_p = j_{pc} - 0\), and \((d \ln n_d/d \ln n)_q \to +\infty\) at \(j_p = j_{pc} + 0\), (see Appendix). Therefore, from the left side of the critical point, \(j_p = j_{pc} - 0\) where the charging is dominated by collisions, the isobaric mode is strongly quenched, while the isentropic is strongly enhanced. Contrary, from the right side of the critical point, \(j_p = j_{pc} + 0\) where the grains are charged predominantly by photoionization, the isobaric mode is enhanced, while the isentropic one is suppressed.

One can therefore conclude that in hot and dilute astrophysical plasmas embedded in a strong radiation field, where radiation dominates dust charging, the thermally unstable isobaric mode is enhanced. Under the conditions of an exposing radiation field with an intensity well below the critical value (such as in astrophysical plasmas far from strong radiation sources and dusty plasma sheaths near the tokamak walls) when the collisional processes are the principal charging mechanisms, isentropic mode is destabilized, while the isobaric one is quenched.
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APPENDIX

Generally, the equation for dust charge is written as (e.g. Refs. 1, 7, 11, 12)

\[ j_p + (1 - y)(1 - Qy) = \left( \frac{m_i}{m_e} \right)^{1/2} (1 + Qy) \exp(y), \tag{A1} \]

where \( y = e(\phi_s - \langle \phi \rangle)/kT \), \( \phi_s \) is the grain surface potential, \( \langle \phi \rangle \) is the average potential in plasma, \( j_p = I_p/(ev_i a^2 n) \), the photoionization rate for the grains in units of the ion charge flux on the surface, \( Q = 4\pi \lambda_D^2 n_d C \), \( C = a(1 + a/\lambda_D) \) is the capacity of a dust grain, \( \lambda_D = \sqrt{kT/[4\pi(n_e + n)e^2]} \), the Debye length; for a dilute dusty plasma, \( 4\pi a^3 n_d/3 \ll 1 \) and \( a \ll \lambda_D \), \( Z_d = kTya/e^2 \) and \( C = a \). It is readily seen from Eq. A1 (see discussion in Refs. 1, 7, 12) that at \( j_p \rightarrow 0 \) dust charge is negative, \( Z_d < 0 \), however, when radiation flux increases (or \( n \) decreases) dust grains become charged positively\(^{12}\). The dependence \( n_d(n) \) along the quasineutrality condition can be found as the solution of Eq. A1 along with the quasineutrality condition \(^{9} \) written here as

\[ 1 - x = -\frac{y}{y_0} \zeta, \tag{A2} \]

where for a dilute plasma \( y_0 = e^2/(k_B Ta) \), and \( \zeta = n_d/n \). With these notations \( Q = \zeta/[(1 + x)y_0] \). Equations A1 and A2 must be solved with a complementary equation for the balance of electrons

\[ a_x x = j_p \zeta - \left( \frac{m_i}{m_e} \right)^{1/2} (1 + Qy) \exp(y) + b_e x_a x + \bar{j}_p x_a, \tag{A3} \]

which we assume to be dominated by photoemission of electrons from dust grains, photoionization of neutrals by collisions with thermal electrons, photoionization of neutrals by
external radiation, radiative recombination, and sticking of electrons on dust grains. Here
\( a_r = \alpha_r/(4\pi v_ia^2) \), \( \alpha_r \) is radiative recombination coefficient, \( b_e = \beta_e/(4\pi v_ia^2) \), \( \beta_e \) is the rate
of collisional ionizations of neutrals, \( \tilde{j}_p = \tilde{I}_p/(v_i a^2 n) \) is the photoionization rate of neutrals,
\( x_a = n_a/n \), their fraction.

Assuming that \(|1 - x| \ll 1\), and taking into account that for hot plasma \( T \geq 10^6 \text{ K} \)
\( a_r, b_e \sim 10^{-12} - 10^{-11} \ll \sqrt{m_i/m_e} \), one can reduce Eqs. [A1]-[A3] to the following system

\[
\frac{(1 - x) y_0}{\zeta} = \ln \left( \frac{m_i/m_e}{1 + j_p} \right), \tag{A4}
\]

\[
x = \frac{j_p \zeta + \tilde{j}_p x_a}{(m_i/m_e)^{1/2} \zeta}, \tag{A5}
\]

which has for \( x_a \ll 1 \) one physically meaningful root

\[
\zeta \simeq \left( (m_i/m_e)^{1/2} - j_p \right) y_0 (m_i/m_e)^{1/2} \left[ \ln \left( \frac{m_i/m_e}{1 + j_p} \right) \right]^{-1}. \tag{A6}
\]

The other root vanishes at \( j_p \) and thus is unphysical. It is seen that at \( j_p = j_{pc} - 1 = \left[ (m_i/m_e)^{1/2} - 1 \right] - 0 \) the solution \( \zeta \to +\infty \) logarithmically, while from the right side \( j_p = j_{pc} + 1 \)
as \( \zeta \to -\infty \). The derivative

\[
\left( \frac{dn_d}{dn} \right)_{q} = n \left( \frac{d \zeta}{dn} \right) + \zeta \sim -O \left[ \ln \left( \frac{m_i/m_e}{1 + j_p} \right) \right]^{-2} + O \left[ \ln \left( \frac{m_i/m_e}{1 + j_p} \right) \right]^{-1}, \tag{A7}
\]

and hence always goes to \(-\infty\) at \( j_p \sim j_{pc} \). Note, that at this point \( x \sim 1 - (m_e/m_i)^{1/2} \), so
that \(|1 - x| \ll 1\) holds. The logarithmic derivative

\[
\left( \frac{n d n_d}{n_d d n} \right)_{q} \sim -j_p \left[ \ln \left( \frac{m_i/m_e}{1 + j_p} \right) \right]^{-1} + 1, \tag{A8}
\]

and goes to \(-\infty\) from the left side of the point \( j_p = (m_i/m_e)^{1/2} - 1 \), and to \(+\infty\) from the
right side.

It is readily seen that in the limit \( j_p \to 0 \), when charging is predominantly collisional,
and \( a \ll \lambda_D \)

\[
\zeta \simeq (m_e/m_i)^{1/2} \left[ (m_i/m_e)^{1/2} - j_p \right] \left[ \ln (m_i/m_e)^{1/2} - j_p \right], \tag{A9}
\]

and

\[
\left( \frac{d \ln n_d}{d \ln n} \right)_{q} \simeq 1 - (m_e/m_i)^{1/2} j_p. \tag{A10}
\]
REFERENCES

[1] C. K. Goertz, Rev. Geophys. 27, 271 (1989)

[2] N. N. Rao, P. K. Shukla, M. Y. Yu, Planet. Space Sci., 38, 543 (1990)

[3] A. Barkan, R. L. Merlino, N. D’Angelo, Phys. Plasmas, 2, 3563 (1995)

[4] J. R. Burke, J. Silk, Astrophys. J., 190, 1 (1974)

[5] A. Kopp, A. Schröer, G. T. Birk and P. K. Shukla, Phys. Plasmas, 4, 4414 (1997)

[6] G. T. Birk, H. Wiechen, Phys. Plasmas, 8, 5057 (2001)

[7] B. T. Draine, E. E. Salpeter, Astrophys. J., 231, 77 (1979)

[8] E. L. O. Bakes, A. G. G. M. Tielens, Astrophys. J., 427, 822 (1994)

[9] G. B. Field, Astrophys. J., 142, 531 (1965)

[10] B. Meerson, Rev. Mod. Phys., 68, 215 (1996)

[11] D. A. Mendis, M. Rosenberg, Annu. Rev. Astron. Astrophys., 32, 419 (1994)

[12] U. de Angelis, Phys. Scr., 45, 465 (1992)