Testing Gauge-Yukawa-Unified Models By $M_t$

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Abstract

Gauge-Yukawa Unification (GYU) relates the gauge and Yukawa couplings, thereby going beyond the usual GUTs, and it is assumed that the GYU in the third fermion generation implies that its Yukawa couplings are of the same order as the unified gauge coupling at the GUT scale. We re-examine carefully the recent observation that the top-bottom mass hierarchy can be explained to a certain extent in supersymmetric GYU models. It is found that there are equiv-top-mass-lines in the boundary conditions of the Yukawa couplings so that two different GYU models on the same line can not be distinguished by the top mass $M_t$ alone. If they are on different lines, they could be distinguished by $M_t$ in principle, provided that the predicted $M_t$'s are well below the infrared value $M_t$(IR). We find that the ratio $M_t$(IR)/sin$\beta$ depends on tan$\beta$ for large tan$\beta$ and the lowest value of $M_t$(IR) is $\sim$ 188 GeV. We focus our attention on the existing $SU(5)$ GYU models which are obtained by requiring finiteness and reduction of couplings. They, respectively, predict $M_t = (183 + \delta^{MSSM}M_t \pm 5)$ GeV and $(181 + \delta^{MSSM}M_t \pm 3)$ GeV, where $\delta^{MSSM}M_t$ stands for the MSSM threshold correction and is $\sim -2$ GeV for the case that all the MSSM superpartners have the same mass $M_{SUSY}$ with $\mu_{H}/M_{SUSY} << 1$.

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1 Introduction

The great success of the standard model (SM) of the electroweak and strong interactions is spoiled by the presence of the plethora of its free parameters. The traditional way to reduce the number of independent parameters in a theory is to introduce a symmetry. Grand Unified Theories (GUTs) \cite{1,2,3} are representative examples of such attempts. The SU(5) GUTs, for instance, reduce by one the gauge couplings of the SM and provide us with the prediction for one of them \cite{4}. GUTs can also relate the Yukawa couplings among themselves, and in turn might lead to testable predictions for the SM parameters. The prediction of the mass ratio $M_{\tau}/M_b$ in the minimal SU(5) was a successful example of reduction of the independent parameters of the Yukawa sector \cite{5}.

In general, the gauge and Yukawa sectors in GUTs are not related. In searching for a symmetry which could relate the two sectors, one is naturally led to introduce supersymmetry, given that the fields involved have different spins. It, however, turns out that one has to introduce at least N=2 supersymmetry to understand the Yukawa sector as a part of the gauge sector \cite{6}. This is a very strong constraint for the construction of realistic theories \cite{7}, because the models based on extended supersymmetries do not possess a chiral structure. In superstring and composite models, such relations exist also. But in spite of some recent developments, there exist open problems which are partly related to the lack of realistic models.

By a Gauge-Yukawa Unification (GYU) we mean a functional relationship among the gauge and Yukawa couplings, which can be derived from some principle. In contrast to the above mentioned schemes, in the GYU scheme based on the principle of reduction of couplings \cite{8}–\cite{12} and finiteness \cite{13}–\cite{18}, one can write down relations among the gauge and Yukawa couplings in a more concrete fashion. These principles, which are formulated within the framework of perturbatively renormalizable field theory, are not explicit symmetry principles, although they might imply symmetries. The former principle is based on the fact that there may exist renormalization group (RG) invariant relations among couplings which preserve perturbative renormalizability \cite{8}. And the latter one is based on the possibility that these RG invariant relations among couplings lead to finiteness in
perturbation theory [13, 14], even to all orders [10, 11]. Theoretical possibilities of relating couplings discussed here exhibit a generalization of the traditional renormalizability: One can reduce the number of independent couplings without introducing necessarily a symmetry, thereby improving the calculability and predictive power of a given theory and hence generalizing the notion of naturality [19] in a certain sense.

The consequence of a GYU is that in the lowest order in perturbation theory the gauge and Yukawa couplings are related in the form

$$g_i = \kappa_i g_{\text{GUT}}, \quad i = 1, 2, 3, e, \ldots, \tau, b, t$$

(1)

above the unification scale $M_{\text{GUT}}$, where $g_i$ ($i = 1, \ldots, t$) stand for the gauge and Yukawa couplings, and $g_{\text{GUT}}$ is the unified coupling. (We have neglected the Cabibbo-Kobayashi-Maskawa mixing of the quarks.) The constants $\kappa$'s can be explicitly calculated in the GYU scheme based on the principle of reduction of couplings, and it has been found [10]–[12], [18] that various supersymmetric GYU models can predict $M_t$ and $M_b$ that are consistent with the present experimental data [20, 21, 22]. This means that the top-bottom hierarchy could be explained to a certain extent in these models in which one assumes the existence of a GYU at the unification scale $M_{\text{GUT}}$, which should be compared with how the hierarchy of the gauge couplings can be explained if one assumes the existence of a unifying gauge symmetry at $M_{\text{GUT}}$ [4].

It has been observed [10]–[12], [18] that there exists a relatively wide range of $\kappa$'s of $O(1)$ which gives the top-bottom hierarchy of the right order. Of course, the existence of this range is partially related to the infrared behavior of the Yukawa couplings [23, 24]. However, because of the restricted number of analyses which have been performed so far, it was not possible to conclude whether the calculated $\kappa$'s in each case give predictions for $M_t$ and $M_b$, which are consistent with the experimental data just because the large experimental as well as theoretical uncertainties, or whether the top-bottom hierarchy results mainly from the infrared behavior of the Yukawa couplings, and therefore the precise nature of a GYU is not important. It is therefore crucial, in order to test GYU models by more precise measurements of $M_t$ as well as $M_b$, to see which of the cases is indeed realized. More precise and systematic investigations on the range of $\kappa$'s are
moreover indispensable in constructing realistic GYU models and in distinguishing them from each other by experiments.

For this purpose we first calculate the infrared quasi–fixed–point value of $M_t$ \cite{23, 24} (which we denote by $M_t$(IR)) for large $\tan \beta$. We find that the $\tan \beta$ dependence of the ratio $M_t$(IR)/$\sin \beta$ for large $\tan \beta$ ($\gtrsim 40$) is not negligible and the lowest value of $M_t$(IR) is $\simeq 188$ GeV. We also find that there exist equiv-top-mass lines in the space of the boundary conditions of the Yukawa couplings so that two different GYU models on the same line can not be distinguished by $M_t$. (The predictions on other parameters such as $M_b$ varies of course along this line.)

One of our main results is that the present experimental data on $M_t$ and $M_b$ might be interpreted as indicating GYU and different supersymmetric GYU models could be distinguished and tested by a precise measurement of $M_t$ with an uncertainty of few GeV, provided that the models are not on equiv-$M_t$ lines that are very close to each other and the predicted $M_t$'s are well separated from the infrared value. Using the updated experimental data on the SM parameters, we re-examine the $M_t$ prediction of two existing $SU(5)$ GYU models, Finite Unified Theory based \cite{18} and the asymptotically-free minimal supersymmetric GUT with the GYU in the third generation \cite{10}. They predict $M_t = (183 + \delta_{\text{MSSM}} M_t \pm 5)$ GeV and $(181 + \delta_{\text{MSSM}} M_t \pm 3)$ GeV, respectively, where $\delta_{\text{MSSM}} M_t$ stands for the MSSM threshold correction. We find it is $\sim -1\%$ for the case that all the superpartners have the same mass $M_{\text{SUSUY}}$ and $\mu_H/M_{\text{SUSUY}} \ll 1$.

2 The gross behavior of the Yukawa couplings

Before we come to more complete analysis that among other things includes two-loop effects, let us investigate within the one-loop approximation how the low energy values of the Yukawa couplings $g_t$, $g_b$ and $g_\tau$ depend on the GYU boundary condition (1). Since the qualitative behavior of the Yukawa couplings for the energy range relevant to our problem can be understood without $g_1$ and $g_2$, we neglect them.

To begin with, we eliminate the $\mu$-dependence of the couplings through $(\mu d/d\mu)a_3 =$
\(-\frac{3}{2\pi}\alpha_3^2\) to obtain

\[
-3\alpha_3 \frac{d\rho_t}{d\alpha_3} = \rho_t \left( 6 \rho_t + \rho_b - \frac{7}{3} \right), \\
-3\alpha_3 \frac{d\rho_b}{d\alpha_3} = \rho_b \left( \rho_t + 6 \rho_b + \rho_b \rho_r - \frac{7}{3} \right), \\
-3\alpha_3 \frac{d\rho_r}{d\alpha_3} = \rho_r \left( -\rho_t - 3\rho_b + 3 \rho_r \rho_b + \frac{16}{3} \right),
\]

(2)

where

\[
\alpha_3 = |g_3|^2/4\pi, \quad \alpha_i = |g_i|^2/4\pi, \quad \rho_i = \frac{\alpha_i}{\alpha_3}, \quad i = t, b, \tau, \quad \rho_r = \frac{\alpha_r}{\alpha_b}.
\]

Then we assume a GYU so that the \(\rho_i\)'s may be assumed to be of \(O(1)\) at \(M_{\text{GUT}}\). The terms containing \(\rho_i\)'s in the parenthesis in the evolution equation for \(\rho_r\) are small compared with \(16/3\) if the \(\rho_i\)'s do not increase very much as \(\alpha_3\) varies from \(\alpha_{\text{GUT}}\) to \(\alpha_3(M_{\text{SUSY}})\). Neglecting these terms further, we obtain \(\rho_r \simeq (\kappa_r/\kappa_b)^2(\alpha_3/\alpha_{\text{GUT}})^{-16/9}\) which, with \(\kappa_b/\kappa_r \sim O(1)\) and \(\alpha_3/\alpha_{\text{GUT}} \simeq 2.7\) for \(M_{\text{SUSY}} \sim M_Z\), is about 0.17. We therefore neglect \(\rho_r\) in the evolution of \(\rho_b\) further so that the evolutions of \(\rho_b\) and \(\rho_t\) become symmetric. We then find that, if \(\rho_b/\rho_t \sim O(1)\) at \(M_{\text{GUT}}\), the ratio at low energies roughly remains the same. So we assume that the solution of

\[
-3\alpha_3 \frac{d\rho_t}{d\alpha_3} = \rho_t \left( 7 \rho_t - \frac{7}{3} \right), \\
-3\alpha_3 \frac{d\rho_b}{d\alpha_3} = \rho_b \left( -\gamma \rho_t + 16/3 \right)
\]

(4)

can describe the gross behavior of \(\rho_i\)'s, where we have introduced \(\gamma(<1)\) to take into account approximately the \(\rho_r \rho_b\)-term in the evolution of \(\rho_r\). We find that the solution is given by

\[
\rho_t(M_{\text{SUSY}}) \simeq \frac{1}{3 + (\kappa_t^{-2} - 3)[\alpha_{\text{GUT}}/\alpha_3(M_{\text{SUSY}})]^{7/9}}, \\
\rho_r(M_{\text{SUSY}}) \simeq \kappa_r^2 \left[\frac{\kappa_t^2}{\rho_t(M_{\text{SUSY}})}\right]^{\gamma/7} \left[\frac{\alpha_{\text{GUT}}}{\alpha_3(M_{\text{SUSY}})}\right]^{(16-\gamma)/9},
\]

(5)

where \(\kappa_t (r) = \rho_t (r)(M_{\text{GUT}})\). That the factor \(\alpha_{\text{GUT}}/\alpha_3^{7/9}\) goes to zero as \(\alpha_3\) approaches \(\infty\) comes from the Pendleton-Ross infrared-fixed-point behavior of \(\rho_t\). For the present case, it is about \((1/2.7)^{7/9} \simeq 0.46\) so that the low energy value of \(\rho_t\) can not be explained solely from this fixed point behavior, except for the case that the \(\kappa_t^2\) is very close to
1/3. But this factor is small so that the $\rho_t(M_{\text{SUSY}})$ depends weakly on $\kappa_t^2$, and especially for large $\kappa_t^2$ this dependence disappears practically, which is Hill’s observation of the intermediate-fixed-point [23, 24].

It is, therefore, crucial for the testability of GYU models by the $M_t$ prediction that $M_t$ is sufficiently different from the infrared value. Of course, how much $M_t$ should be away from the infrared value depends on the experimental accuracy. In the next section, we will discuss this problem more in detail. Within the present approximation we may conclude that

$$\left| \frac{\Delta \rho_t}{\rho_t} \right| \simeq (0.92 - 0.22) \left| \frac{\Delta \kappa_t}{\kappa_t} \right| \text{ for } \kappa_t \simeq 0.5 - 1.5 .$$

Since $M_t \propto \sqrt{\rho_t}$, an uncertainty of 2 % in $M_t$ for instance will allow the range of $\kappa_t$ that corresponds to $|\Delta \kappa_t/\kappa_t| \simeq 0.04 - 0.18$. If the uncertainty is of $O(10\%)$, one finds that there is a wide range of the allowed values of $\kappa_t^2$, which qualitatively explain the observation of refs. [10]–[12] [18].

It should be stressed that, to calculate the fermion masses, we need to know the value of $\tan \beta$ [26] in addition to the Yukawa couplings, which should be contrasted to the case of the SM. At the tree level, it can be expressed as

$$\tan \beta = \left[ 2 \left( \frac{M_W^2}{M_t^2} \right) \frac{\alpha_3}{\alpha_2} \rho_r \rho_b - 1 \right]^{1/2} \simeq 111 \sqrt{\rho_r \rho_b} \simeq 60.8 \sqrt{\rho_t} \text{ for } \rho_r \simeq 0.3 \text{ and } \rho_b \simeq \rho_t .$$

From eq. (7), we see that $\tan \beta$ can be predicted from a GYU if we use $\sin^2 \theta_W, \alpha_{\text{EM}}, M_Z$ and $M_t$ as inputs and it will be large for GYU models[1]. Note that the value of $\tan \beta$ does not follow from the Pendleton-Ross nor from the infrared quasi–fixed–point behavior of the Yukawa couplings, because $\rho_r = \rho_r/\rho_b$ does not have the infrared behavior like $\rho_t$, as one can see from eq. (5).

### 3 Testability of a GYU by $M_t$

The gross behavior of the Yukawa couplings discussed in the previous section gives an insight into the GYU physics, and we have seen that the testability of GYU models and the

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1 This result was previously obtained in a different context in ref. [27].
possibility to discriminate among them by $M_t$ crucially depend on the infrared structure of the Yukawa couplings. In this section, we include into the evolution of the couplings: (i) $g_1$ and $g_2$, (ii) two-loop effects and (iii) corrections for the physical masses, where we neglect the non-logarithmic threshold corrections such as the finite corrections coming from the transition from the dimensional reduction scheme to the $\overline{\text{MS}}$ scheme. Then we examine numerically the evolution of the gauge and Yukawa couplings, according to their RG equations [28]. Below $M_{\text{GUT}}$ the evolution of couplings is assumed to be governed by the MSSM. We further assume a unique threshold $M_{\text{SUSY}}$ for all superpartners of the MSSM so that below $M_{\text{SUSY}}$ the SM is the correct effective theory. The uncertainty in the $M_t$ prediction caused by these approximations will be discussed and estimated when considering concrete GYU models in the next section.

We recall that, with a GYU boundary condition at $M_{\text{GUT}}$ alone, the value of $\tan \beta$ can not be determined. Usually, $\tan \beta$ is determined in the Higgs sector, which however depends strongly on the supersymmetry breaking terms. Here we avoid this by using the tau mass $M_\tau$ as input. (This means that we partly fix the Higgs sector indirectly.) That is, assuming that

$$M_Z \ll M_t \ll M_{\text{SUSY}}, \quad (8)$$

we require the matching condition at $M_{\text{SUSY}}$ [28],

$$\alpha_i^{\text{SM}} = \alpha_i \sin^2 \beta, \quad \alpha_b^{\text{SM}} = \alpha_b \cos^2 \beta, \quad \alpha_\tau^{\text{SM}} = \alpha_\tau \cos^2 \beta,$$

$$\alpha_\lambda = \frac{1}{4} \left( 3 \alpha_1 + \alpha_2 \right) \cos^2 2\beta, \quad (9)$$

to be satisfied [3], where $\alpha_i^{\text{SM}} (i = t, b, \tau)$ are the SM Yukawa couplings and $\alpha_\lambda$ is the Higgs coupling. This is our definition of $\tan \beta$, and eq. (9) fixes $\tan \beta$, because with a given set of the input parameters [20],

$$M_\tau = 1.777 \text{ GeV}, \quad M_Z = 91.188 \text{ GeV}, \quad (10)$$

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2The corrections coming from this transition of renormalization scheme will be partly taken account in the next section. Unless it is explicitly stated, these corrections are not considered below.

3There are MSSM threshold corrections to this matching condition [30, 31], which will be discussed later.
with \[29\]

\[
\alpha_{EM}^{-1}(M_Z) = 127.9 + \frac{8}{9\pi} \log \frac{M_t}{M_Z},
\]

\[
\sin^2 \theta_W(M_Z) = 0.2319 - 3.03 \times 10^{-5}T - 8.4 \times 10^{-8}T^2,
\]

\( T = M_t/[\text{GeV}] - 165, \)

the matching condition (9) and the GYU boundary condition at \( M_{\text{GUT}} \) can be satisfied only for a specific value of \( \tan \beta \). Here \( M_\tau, M_t, M_Z \) are pole masses, and the couplings are defined in the \( \overline{\text{MS}} \) scheme with six flavors.

The translation from a Yukawa coupling into the corresponding mass follows according to

\[
m_i = \frac{1}{\sqrt{2}} g_i(\mu) \, v(\mu), \quad i = t, b, \tau \quad \text{with} \quad v(M_Z) = 246.22 \, \text{GeV},
\]

where \( m_i(\mu) \)'s are the running masses satisfying the respective evolution equation of two-loop order. The pole masses can be calculated from the running ones, of course. For the top mass, we use \[28, 30\]

\[
M_t = m_t(M_t) \left[ 1 + 4 \frac{\alpha_3(M_t)}{\pi} + 10.95 \left( \frac{\alpha_3(M_t)}{\pi} \right)^2 + k_t \frac{\alpha_t(M_t)}{\pi} \right],
\]

where \( k_t \simeq -0.3 \) for the range of parameters we are concerned with in this paper \[30\]. Note that both sides of eq. (13) contains \( M_t \) so that \( M_t \) is defined only implicitly. Therefore, its determination requires an iteration method. As for the tau and bottom masses, we assume that \( m_\tau(\mu) \) and \( m_b(\mu) \) for \( \mu \leq M_Z \) satisfy the evolution equation governed by the \( SU(3)_C \times U(1)_{\text{EM}} \) theory with five flavors and use

\[
M_b = m_b(M_b) \left[ 1 + 4 \frac{\alpha_3(5f)}{3 \pi} + 12.4 \left( \frac{\alpha_3(5f)}{\pi} \right)^2 \right],
\]

\[
M_\tau = m_\tau(M_\tau) \left[ 1 + \frac{\alpha_{EM}(5f)}{\pi} \right],
\]

where the experimental value of \( m_b(M_b) \) is \((4.1 - 4.5) \, \text{GeV} \) \[20\]. The couplings with five flavors entered in eq. (14) \( \alpha_3(5f) \) and \( \alpha_{EM}(5f) \) are related to \( \alpha_3 \) and \( \alpha_{EM} \) by

\[
\alpha_{3(5f)}^{-1}(M_Z) = \alpha_3^{-1}(M_Z) - \frac{1}{3\pi} \ln \frac{M_t}{M_Z},
\]

\[
\alpha_{EM(5f)}^{-1}(M_Z) = \alpha_{EM}^{-1}(M_Z) - \frac{8}{9\pi} \ln \frac{M_t}{M_Z}.
\]

(15)
Using the input values given in eqs. (10) and (11), we find

\begin{align*}
m_\tau(M_\tau) &= 1.771 \text{ GeV}, \\
m_\tau(M_Z) &= 1.746 \text{ GeV}, \\
\alpha_{\text{EM}(5f)}^{-1}(M_\tau) &= 133.7, \quad (16)
\end{align*}

and from eq. (12) we obtain

\begin{align*}
\alpha_{\tau}^{\text{SM}}(M_Z) &= \frac{g_\tau^2}{4\pi} = 8.005 \times 10^{-6}, \quad (17)
\end{align*}

which we use as an input parameter instead of \( M_\tau \).

With these assumptions specified above, we compute the infrared quasi–fixed–point values of the top quark mass \( M_t(\text{IR}) \) \cite{23, 24} as function of \( \kappa_b^2(= \kappa_t^2) \) for three different \( M_{\text{SUSY}} \)'s, where we fix \( \kappa_t^2 = 6 \). (Since \( \alpha_{\text{GUT}} \simeq 0.04 \), \( \kappa_t^2 = 6 \) means that \( g_t(M_{\text{GUT}}) \simeq 1.7 \).

The values of \( M_t(\text{IR}) \) will be increased by \( \sim +1 \text{ GeV} \) if we use \( \kappa_t^2 = 8 \) \( (g_t(M_{\text{GUT}}) \simeq 2.) \)

This is shown in fig. 1, where the solid, dashed and dot-dashed lines correspond to \( M_t(\text{IR}) \) with \( M_{\text{SUSY}} = 1000, 500 \) and \( 300 \text{ GeV} \). If the predicted values of \( M_t \) are sufficiently below
Figure 2: $M_t(\text{IR})/\sin \beta$ as a function of $\tan \beta$ for $M_{\text{SUSY}} = 500$ GeV.

$M_t(\text{IR})$, there will be a chance to discriminate different boundary conditions of the Yukawa couplings at $M_{\text{GUT}}$ and hence to distinguish different GYU models. Fig. 1 also shows that a $M_t$ below $\sim 188$ GeV cannot be solely understood as a consequence of the infrared quasi–fixed–point behavior of the Yukawa couplings in the MSSM.

Note that the values for $\tan \beta$ in the parameter range in fig. 1 are large ($\gtrsim 40$). In this regime, the ratio $M_t(\text{IR})/\sin \beta$ depends on $\tan \beta$ as one can see from fig. 2. We find that for $7 \lesssim \tan \beta \lesssim 40$,

$$M_t(\text{IR})/\sin \beta \simeq \begin{cases} 
(203 - 199) \text{ GeV} \\
(201 - 197) \text{ GeV } \text{ for } M_{\text{SUSY}} = (199 - 195) \text{ GeV} \\
1 \text{ TeV} \\
500 \text{ GeV} \\
300 \text{ GeV} 
\end{cases}$$

while for $\tan \beta \gtrsim 40$, $M_t(\text{IR})$ for $M_{\text{SUSY}} = 500$ GeV may be approximated as

$$M_t(\text{IR}) \simeq 195 - 0.3 \Delta t - 0.01 (\Delta t)^2 ,$$

$$\Delta t = \tan \beta - 50 .$$

The result above is consistent with the previous one, $M_t/\sin \beta \simeq (190 - 210)$ GeV [24].

The $\tan \beta$ dependence of $M_t/\sin \beta$ for large $\tan \beta$ appears, because $\alpha_b$ and $\alpha_\tau$ nontrivially
contribute to the evolution of $\alpha_t$.

Next we would like to come to the equiv-$M_t$ lines in the $\kappa_t^2 - \kappa_b^2$ plane. If one concentrates only on the $M_t$ prediction from GYUs and discard others, there have to exist such lines, because $M_t$ as a function of $\kappa_t^2$ and $\kappa_b^2$ defines a smooth surface. Strictly speaking, we should talk about equiv-$M_t$ surfaces, because these equiv-$M_t$ lines exist for a given $M_{SUSY}$ which can be seen as a parameter, too. In fig. 3 we show such lines for $M_t = 182, 179, 176$ GeV, where we have fixed $M_{SUSY}$ at 500 GeV. Fig. 3 shows that GYU models with $\kappa_b \lesssim 1$ can be better distinguished. It is clear that to discriminate two models on the same line, we have to look at other predictions, e.g., $M_b$ and $\alpha_3(M_Z)$.

The matching condition (9) suffers from the threshold corrections coming from the MSSM superpartners:

$$\alpha_i^{SM} \to \alpha_i^{SM}(1 + \Delta_i^{SUSY}) \, , \, i = 1, 2, \ldots, \tau \, ,$$

It was shown that these threshold effects to the gauge couplings can be effectively parametrized

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4 We assume that $\kappa_b^2 = \kappa_\tau^2$. 

Figure 3: The equiv-$M_t$ lines for $M_t = 182$ (dot-dashed), 179 (solid) and 176 GeV (dashed) with $M_{SUSY} = 500$ GeV.
by just one energy scale [32]. Accordingly, we can identify our $M_{\text{SUSY}}$ with that defined in ref. [32]. This ensures that there are no further one-loop threshold corrections to $\alpha_3(M_Z)$ when we calculate it as a function of $\alpha_{\text{EM}}(M_Z)$ and $\sin^2\theta_W(M_Z)$.

The same scale $M_{\text{SUSY}}$ does not describe threshold corrections to the Yukawa couplings, and they could cause large corrections to the fermion mass prediction [30, 31]. For $m_b$, for instance, the correction can be as large as 50% for very large values of $\tan\beta$, especially in models with radiative gauge symmetry breaking and with supersymmetry softly broken by the universal breaking terms. As we will see when discussing concrete $SU(5)$ GYU models in the next section, these models predict (with these corrections suppressed) values for the bottom quark mass that are rather close to the experimentally allowed region so that there is room only for small corrections. Consequently, GYU models in which $SU(2) \times U(1)$ gauge symmetry is broken radiatively favor non-universal soft breaking terms [34]. It is interesting to note that the consistency of the GYU hypothesis is closely related to the fine structure of supersymmetry breaking and also to the Higgs sector, because these superpartner corrections to $m_b$ can be kept small for appropriate supersymmetric spectrum characterized by very heavy squarks and/or small $\mu_H$ describing the mixing of the two Higgs doublets in the superpotential.

To get an idea about the magnitude of the correction, let us consider the case that all the superpartners have the same mass $M_{\text{SUSY}}$ with $M_{\text{SUSY}} \gg \mu_H$ and $\tan\beta \approx 50$, and vary $\mu_H/M_{\text{SUSY}}$ from $-0.2$ to $0.15$. We quote $\Delta$’s at $\mu = M_{\text{SUSY}}$ from ref. [31]:

$$2\pi \Delta_t \approx -\frac{4}{3} \alpha_3 - \frac{1}{8} \alpha_b,$$

$$2\pi \Delta_b \approx -\frac{4}{3} \alpha_3 + \frac{1}{4} \alpha_b F_2(M^2_{\text{SUSY}}, M^2_t) + (\frac{4}{3} \alpha_3 + \frac{1}{2} \alpha_2 - \frac{1}{2} \alpha_t) \frac{\mu_H}{M_{\text{SUSY}}} \tan\beta$$,

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5It is, of course possible, to compute the MSSM correction to $M_t$ directly, i.e., without constructing an effective theory below $M_{\text{SUSY}}$. In this approach, too, large corrections have been reported [33]. In the present paper, evidently, we are following the effective theory approach as e.g. refs. [30, 31].

6The solution with small $\mu_H$ is favored by the experimental data and cosmological constraints [34]. The sign of this correction is determined by the relative sign of $\mu_H$ and the gluino mass parameter, $M_3$, and is correlated with the chargino exchange contribution to the $b \to s \gamma$ decay [30]. The later has the same sign as the Standard Model and the charged Higgs contributions when the supersymmetric corrections to $m_b$ are negative.
\[ 2\pi \Delta r \simeq -\frac{1}{4} \alpha_2 - \frac{1}{8} \alpha_r + \frac{1}{4} \alpha_2 \frac{\mu_H}{M_{\text{SUSY}}} \tan \beta, \]

where

\[ F_2(M^2_{\text{SUSY}}, M^2_t) \simeq -\frac{1}{2} + \frac{M^2_t}{M^2_{\text{SUSY}}} + \frac{M^4_t}{M^4_{\text{SUSY}}} \ln \left( \frac{M^2_t}{M^2_{\text{SUSY}}} \right) \text{ for } M_t \ll M_{\text{SUSY}}, \]

and terms proportional to \( \cos \beta, \cot \beta \) etc. are suppressed. The result is presented in table 1, where it is assumed: \( M_{\text{SUSY}} = 500 \text{ GeV} \) and a GYU boundary condition \( \alpha_3 = \alpha_2 = \alpha_1 = \alpha_t = \alpha_b = \alpha_r = \alpha_{\text{GUT}}. \)

| \( \mu_H/M_{\text{SUSY}} \) | \( M_t [\text{GeV}] \) | \( m_b(M_b) [\text{GeV}] \) |
|----------------|----------------|----------------|
| -0.2           | 175.4          | 5.02           |
| -0.1           | 175.4          | 4.80           |
| 0.1            | 175.4          | 4.31           |
| 0.15           | 175.4          | 4.18           |

Table 1. The \( \mu_H/M_{\text{SUSY}} \) dependence of the top and bottom quark mass predictions.

Without the threshold corrections \( \Delta \)'s, we obtain \( M_t = 177.2 \text{ GeV} \) and \( m_b(M_b) = 4.62 \) GeV, and so the MSSM correction to the \( M_t \) prediction is \( \sim -1\% \) in this case. Comparing with the results of \([31, 33]\), this may appear to be underestimated. Note however that there is a nontrivial interplay among the corrections between the \( M_t \) and \( M_b \) predictions for a given GYU boundary condition at \( M_{\text{GUT}} \) and the fixed pole tau mass, which has not been taken into account in refs. \([31, 33]\). Here we will not go into details of this MSSM correction, and we leave this problem to future work. In the following discussions, the MSSM threshold correction to the \( M_t \) prediction, which will become calculable once the superpartner spectra are known, will be denoted by

\[ \delta^{\text{MSSM}} M_t. \]  

\[ (20) \]

4 Comparison of the Gauge-Yukawa Unified models based on \( SU(5) \)

There are two gauge-Yukawa unified models based on \( SU(5) \) we would like to consider here; the first one \([10]\) is an asymptotically-free unified theory (AFUT), and the second
one (FUT) is finite to all orders. Here we would like to address the question whether it is possible to distinguish these models experimentally.

### 4.1 Asymptotically Free Unified Theory

The field content is minimal: three generations of quarks and leptons are accommodated by three chiral supermultiplets in $\Psi^I(10)$ and $\Phi^I(\overline{5})$, where $I$ runs over the three generations. A $\Sigma(24)$ is used to break $SU(5)$ down to $SU(3)_C \times SU(2)_L \times U(1)_Y$, and $H(5)$ and $\overline{H}(\overline{5})$ to describe the two Higgs supermultiplets appropriate for electroweak symmetry breaking. The superpotential of the model is given by

$$W = \frac{g_t}{4} \epsilon^{\alpha \beta \gamma \delta \tau} \Psi_3^\alpha \Psi_3^\beta H_\tau + \sqrt{2} g_b \overline{H}^\alpha \Psi_3^\beta \Psi_3^\gamma + \frac{g_\lambda}{3} \Sigma_3^\alpha \Sigma_3^\beta \Sigma_3^\gamma + g_f \overline{H}^\alpha \Sigma_3^\beta H_\beta$$

$$+ m_1 \Sigma_3^\alpha \Sigma_3^\gamma + m_2 \overline{H}^\alpha H_\alpha , \quad (21)$$

where $\alpha, \beta, \ldots$ are the $SU(5)$ indices, and we have suppressed the Yukawa couplings of the first two generations. In this approximation, there are two Yukawa and two Higgs couplings. If one applies the principle of reduction of couplings and require the theory to be asymptotically free, one finds that $\kappa_t$ and $\kappa_b = \kappa_f$ are strongly constrained. In the one-loop approximation, they can be written as

$$\kappa_{t,b}^2 = \sum_{m,n=0}^\infty r_{t,b}^{(m,n)} [\tilde{\alpha}_{\lambda}]^m [\tilde{\alpha}_f]^n , \quad (22)$$

where

$$\tilde{\alpha}_i \equiv \alpha_i/\alpha , \ i = \lambda, f , \quad (23)$$

and the first expansion coefficients $r_{t,b}^{(m,n)}$ are given in table 2.
The coupling $\tilde{\alpha}_\lambda$ is allowed to vary from 0 to $15/7$, while $\tilde{\alpha}_f$ may vary from 0 to a maximum $\tilde{\alpha}_{f_{\text{max}}}$ which depends on $\tilde{\alpha}_\lambda$. For small $\tilde{\alpha}_\lambda$, it is given by

$$\tilde{\alpha}_{f_{\text{max}}} = 560/521 - 0.1313 \ldots \tilde{\alpha}_\lambda - 0.0212 \ldots [\tilde{\alpha}_\lambda]^2 - 0.0058 \ldots [\tilde{\alpha}_\lambda]^3$$

$$-0.0019 \ldots [\tilde{\alpha}_\lambda]^4 + O([\tilde{\alpha}_\lambda]^5), \quad (24)$$

and, it vanishes at $\tilde{\alpha}_\lambda = 15/7$. Therefore, each equation of (22) describes a two-dimensional surface with boundary. As we can see from eq. (22), along with the $r^{(m,n)}_{t,b}$'s given in table 2, the $\tilde{\alpha}_\lambda$-dependence of $\kappa$'s are rather weak, so we show in table 3 the predictions for different values of $\tilde{\alpha}_f$, where $\tilde{\alpha}_\lambda$ is fixed at zero. ($M_{\text{SUSY}} = 500$ GeV)

| $\tilde{\alpha}_f$ | $\kappa_t^2$ | $\kappa_b^2$ | $\alpha_3(M_Z)$ | $\tan \beta$ | $M_{\text{GUT}}$ [GeV] | $m_b(M_b)$ [GeV] | $M_t$ [GeV] |
|-------------------|-------------|-------------|----------------|-------------|----------------|----------------|-------------|
| 0.2               | 1.315       | 0.925       | 0.122          | 52.2        | $1.74 \times 10^{16}$ | 4.60           | 180.9       |
| 0.6               | 1.187       | 0.816       | 0.121          | 51.1        | $1.73 \times 10^{16}$ | 4.63           | 179.8       |
| 1.0               | 1.001       | 0.642       | 0.121          | 49.0        | $1.71 \times 10^{16}$ | 4.69           | 179.1       |
| 1.075             | 0.972       | 0.572       | 0.121          | 47.9        | $1.71 \times 10^{16}$ | 4.72           | 177.8       |

Table 3. The predictions of AFUT for $M_{\text{SUSY}} = 500$ GeV (before the threshold corrections from the superheavy particles).
¿From table 3 we see that the predicted values for $M_t$ are well below the infrared values and the width of the $M_t$ prediction on the asymptotically free surface is at most 4 GeV. But this uncertainty can be shrunk as we will do now.

We turn our discussion to proton decay to further constrain the parameter space of AFUT. First we recall that if one includes the threshold effects of superheavy particles, the GUT scale $M_{\text{GUT}}$ at which $\alpha_1$ and $\alpha_2$ are supposed to meet is related to $M_H$, the mass of the superheavy $SU(3)_C$-triplet Higgs supermultiplets contained in $H$ and $\bar{H}$. To see this, we write the one-loop relation

$$\alpha_2^{-1}(\mu) = \alpha^{-1}(\Lambda) + \frac{1}{2\pi} \left( \ln \frac{\Lambda}{\mu} - 6 \ln \frac{\Lambda}{M_V} + 2 \ln \frac{\Lambda}{M_\Sigma} \right),$$

$$\alpha_1^{-1}(\mu) = \alpha^{-1}(\Lambda) + \frac{1}{2\pi} \left( \frac{33}{5} \ln \frac{\Lambda}{\mu} - 10 \ln \frac{\Lambda}{M_V} + 2 \ln \frac{\Lambda}{M_H} \right),$$

where $M_\Sigma$ and $M_V$ stand for the masses of the superheavy Higgs supermultiplets contained in $24$ and the superheavy gauge supermultiplets, and $\mu > M_{\text{SUSY}}$ and $\Lambda > M_V, M_H, M_\Sigma$. Then from $\alpha_1^{-1}(M_{\text{GUT}}) = \alpha_2^{-1}(M_{\text{GUT}})$, we find that

$$M_{\text{GUT}} = [M_V]^{5/7} [M_H]^{-1/14} [M_\Sigma]^{5/14}.$$

Using the tree-level mass relations,

$$\frac{M_V}{M_\Sigma} = 2\sqrt{2} \frac{g}{g_\lambda}, \quad \frac{M_H}{M_\Sigma} = 2 \frac{g_f}{g_\lambda},$$

which follows from the assumption that the mass parameter $m_2$ in the superpotential is fine tuned so that the $SU(2)_L$-doublet Higgs supermultiplets remain light, we can rewrite eq. (26) as

$$M_H = [\tilde{\alpha}_f]^{15/28} [\tilde{\alpha}_\lambda]^{-5/28} M_{\text{GUT}}.$$

As known, $M_H$ controls the nucleon decay which is mediated by dimension five operators, and non-observation of the nucleon decay requires $M_H \gtrsim 10^{17}$ GeV for $\tan \beta \simeq 50$. Since $M_{\text{GUT}} \simeq 1.7 \times 10^{16}$ GeV and $\tilde{\alpha}_f \lesssim 1.1$ as one can see from eq. (22) and table 2, the value of $\tilde{\alpha}_\lambda$ has to be less than $\sim 4.4 \times 10^{-5}$. Therefore, the reduction solutions that are consistent with the nucleon decay constraint are very close to the boundary of

\[7\text{For } \kappa_6^2 = 0.64 \text{ and } M_{\text{SUSY}} = 500 \text{ GeV, for instance, the } M_t(\text{IR}) \text{ is } \simeq 195 \text{ GeV.} \]
Figure 4: $M_t$ against $\kappa_t^2$ ($\kappa_b^2 = 0.64$) with $M_{\text{SUSY}} = 1000$ (solid), 500 (dashed) and 300 (dod-dashed) GeV.

the asymptotically surface, i.e., $\tilde{\alpha}_f$ has to be very close to 1.075 (see table 3). Comparing $\kappa$’s for large $\tilde{\alpha}_f$ in table 3, we assume in the following discussions that for AFUT

$$\kappa_t^2 = 1.0 \text{ and } \kappa_b^2 = 0.64 \ ,$$

with an uncertainty of 5 (10) % for $\kappa_t^2$ ($\kappa_b^2$).

In fig. 4 we show $M_t$ as a function of $\kappa_t^2$ with $M_{\text{SUSY}} = 1000$, 500 and 300 GeV, where $\kappa_b^2$ is fixed at the predicted value 0.64. From this we see that around $\kappa_t^2 = 1$ the $M_t$ prediction is sensitive against the change of $\kappa_t^2$ and is stable against the change of $M_{\text{SUSY}}$. Fig. 5 shows the sensitivity of the $M_t$ prediction against $\kappa_b^2$ with $\kappa_t^2$ fixed at 1.0 ($M_{\text{SUSY}} = 500$ GeV). From these figures we may conclude that accurate measurements on $M_t$ with an uncertainty of less than $\sim$ few GeV would exclude or confirm the predicted region in the $\kappa_t^2 - \kappa_b^2$ plane in AFUT if there would be no theoretical uncertainty.
Figure 5: $M_t$ against $\kappa_b^2$ ($\kappa_t^2 = 1.0$) with $M_{\text{SUSY}} = 500$ GeV.
The threshold effects of the superheavy particles has also an influence on the coupling unification\(^8\), and we would like to calculate them. One finds in one-loop order that
\[
\alpha_3(M_{\text{GUT}}) = \alpha_{\text{GUT}}^{-1} + \frac{1}{2\pi} \left[ \frac{6}{7} \ln \frac{M_V}{M_\Sigma} - \frac{9}{7} \ln \frac{M_H}{M_\Sigma} \right]
\]
\[
= \alpha_{\text{GUT}}^{-1} + \frac{1}{2\pi} \left[ \frac{3}{14} \ln \tilde{\alpha}_\lambda - \frac{9}{14} \ln \tilde{\alpha}_f \right],
\]
where we have used the mass relations (27), and \(\alpha_{\text{GUT}}^{-1} = \alpha_2^{-1}(M_{\text{GUT}}) = \alpha_1^{-1}(M_{\text{GUT}})\). Since \(\tilde{\alpha}_f \approx 1\) and \(\tilde{\alpha}_\lambda \lesssim 4.4 \times 10^{-5}\) because of the proton decay constraint (as discussed above) and \(\alpha_{\text{GUT}} \approx 0.04\), we obtain
\[
\alpha_3(M_{\text{GUT}}) \gtrsim 1.014 \alpha_{\text{GUT}}.
\]
In table 4 we present the predictions with the threshold effects (i.e., \(\alpha_3(M_{\text{GUT}}) = 1.014 \alpha_{\text{GUT}}\)), where we have used \(\kappa_t^2 = 1.0\) and \(\kappa_b^2 = 0.64\). The numbers in the parenthesis are those obtained without the threshold effects, i.e., \(\alpha_3(M_{\text{GUT}}) = \alpha_{\text{GUT}}\).

| \(M_{\text{ SUSY}}\) [TeV] | \(\alpha_3(M_Z)\) | \(m_b(M_b)\) [GeV] | \(M_t\) [GeV] |
|--------------------------|-----------------|-----------------|-------------|
| 0.5                      | 0.127(0.121)    | 4.89(4.69)      | 180.6(177.8) |
| 1                        | 0.125(0.119)    | 4.87(4.68)      | 180.5(177.7) |

**Table 4.** The threshold effects of the superheavy particles.

As we can see from table 4, the threshold corrections have significant effects, especially on \(\alpha_3(M_Z)\) and \(m_b(M_b)\)\(^8\). The predicted values for \(\alpha_3(M_Z)\) are slightly larger than the central experimental value 0.118 ± 0.006, but prefers those obtained from the electroweak data and \(e^+e^-\) jets experiments\(^2\). Since \(\alpha_3(M_Z)\) decreases as \(M_{\text{ SUSY}}\) increases, AFUT needs a relatively large \(M_{\text{ SUSY}}\).

Now we come to the final value of \(M_t\) for AFUT, and we collect the uncertainties: The MSSM threshold correction is denoted by \(\delta^{\text{MSSM}}M_t\) as we have discussed in the previous

\(^8\)It has been argued\(^32, 40\) that there might be gravitationally induced effects that cause corrections to the coupling unification. Here we do not consider them, because it has not been well established how to introduce gravitational effects into the framework of renormalizable quantum field theory.

\(^9\)As for \(m_b(M_b)\), we see that only negative corrections to \(m_b(M_b)\) of at most 20 % (which we have mentioned in the previous section) are allowed.
section (see eq. (20)). The uncertainties involved in $\kappa_t^2$ and $\kappa_b^2$ may lead to another $\sim \pm 1.4$ GeV (see eq. (29) and figs. 4 and 5). The threshold effects of the superheavy particles are included only in the gauge sector, but not in the Yukawa sector. If we assume that its magnitude is similar to that in the gauge sector (eq. (31)), it will be $\lesssim 2\%$ in $\kappa_b^2$ and $\kappa_t^2$, leading to an uncertainty of $\sim \pm 0.4$ GeV in $M_t$. The finite corrections coming from the conversion from the dimensional reduction (DR) scheme to the ordinary $\overline{MS}$ at $M_Z$ may be included in the gauge sector $\mathbb{32}$ (about $1\%$ for $\alpha_3$ and $0.2\%$ for $\alpha_2$). As for the Yukawa sector, we assume that it is similar to that for $\alpha_3$, which gives rise to an uncertainty of $\sim \pm 1$ GeV in $M_t$. From these considerations, we finally obtain

$$M_t = (181 + \delta^{\text{MSSM}} M_t \pm 3) \text{ GeV}$$

for AFUT.

## 4.2 Finite Unified Theory

From the classification of finite theories given in ref. [14], one finds that using $SU(5)$ as gauge group there exist only two candidate models which can accommodate three generations. But only one of them contains a $\mathbb{24}$ which can be used for the spontaneous symmetry breaking SSB of $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$. For the other one, one has to incorporate another way, such as the Wilson flux breaking to achieve the desired SSB of $SU(5)$. Here we focus our attention on the first model.

The field content is: three $(\mathbf{5} + \mathbf{10})$'s for three generations, four pairs of $(\mathbf{5} + \bar{\mathbf{5}})$-Higgses and one $\mathbb{24}$. To ensure finiteness to all orders [17], we have to find isolated, non-degenerate solutions to the reduction equations [8] of the model, which are consistent with the one-loop finiteness conditions. In the previous studies of refs. [15], however, no attempt was made to find isolated, non-degenerate solutions, but rather the opposite; the freedom offered by the degeneracy has been used in order to make specific ansätze that could lead to phenomenologically acceptable predictions (see for another attempt ref. [16] in which the dimensional regularization plays the fundamental rôle). Here we would like to follow the treatment of ref. [18] in which an isolated, non-degenerate reduction solution exists thanks to certain discrete symmetries in the superpotential. The solution corresponds
to the Yukawa matrices without intergenerational mixing (which is reasonable as a first approximation), and yields in the one-loop approximation

\[ \kappa_1 = \kappa_2 = \kappa_3, \]
\[ \kappa_t = \kappa_c = \kappa_u = \sqrt{8/5}, \]
\[ \kappa_b = \kappa_s = \kappa_d = \kappa_\tau = \kappa_\mu = \kappa_e = \sqrt{6/5}. \]

At first sight, this GYU boundary condition seems to lead to unacceptable predictions of the fermion masses. But this is not the case, because each generation has an own pair of \((5 + 5)\)-Higgses: We may use the fact that mass terms do not influence the \(\beta\)-functions in a certain class of renormalization schemes, and introduce appropriate mass terms that permit us to perform a rotation in the Higgs sector such that only one pair of Higgs doublets, coupled to the third family, remains light and acquires a non-vanishing VEV (in a similar way to what was done by León et al. [41]). Note that the effective coupling of the Higgs doublets to the first family after the rotation is very small avoiding in this way a potential problem with the proton lifetime [42]. Thus, effectively, we have at low energies the MSSM with only one pair of Higgs doublets. In other words, the effective GYU boundary condition under this assumption becomes

\[ \kappa_1 = \kappa_2 = \kappa_3, \kappa_t = \sqrt{8/5}, \kappa_b = \kappa_\tau = \sqrt{6/5}, \quad (33) \]

where the Yukawa couplings of the first two generations should be regarded as free parameters. The predictions of \(M_t\) and \(m_b(M_b)\) for various \(M_{\text{SUSY}}\) are given in table 5.

| \(M_{\text{SUSY}}\) [GeV] | \(\alpha_3(M_Z)\) | \(\tan \beta\) | \(M_{\text{GUT}}\) [GeV] | \(m_b(M_b)\) [GeV] | \(M_t\) [GeV] |
|------------------------|-----------------|-------------|-----------------|----------------|----------------|
| 300                    | 0.123           | 54.2        | \(2.06 \times 10^{16}\) | 4.53           | 182.2          |
| 500                    | 0.122           | 54.3        | \(1.75 \times 10^{16}\) | 4.53           | 182.6          |
| \(10^3\)               | 0.119           | 54.4        | \(1.41 \times 10^{16}\) | 4.53           | 183.0          |

**Table 5.** The predictions for different \(M_{\text{SUSY}}\) for FUT.

Similar to the case of AFUT, only negative MSSM corrections of at most \(~10\ %\) to \(m_b(M_b)\) is allowed, implying that FUT also favors non-universal soft symmetry breaking
terms. The predicted $M_t$ values are well below the infrared value, for instance 194 GeV for $M_{\text{SUSY}} = 500$ GeV (see fig. 1), so that the $M_t$ prediction must be sensitive against the change of $\kappa_i^2$ as well as $\kappa_b^2$. This is shown in figs. 6 and 7. For fig. 6 we have fixed $\kappa_b^2$ at 1.2 and used $M_{\text{SUSY}} = 1000$ (solid), 500 (dashed) and 300 (dot-dashed) GeV, while we have fixed $\kappa_i^2$ at 1.6 and used $M_{\text{SUSY}} = 500$ in fig. 7.

The nice feature of FUT is that there is no finite range in the $\kappa_i^2 - \kappa_b^2$ plane; it predicts a point. But the structure of the threshold effects from the superheavy particles is involved in this case, compared to AFUT. They are not arbitrary and probably determinable to a certain extent, because the mixing of the superheavy Higgses is strongly dictated by the fermion mass matrix of the MSSM. To bring these threshold effects under control is beyond the scope of the present paper, and we would like to leave it to future work. Here we assume that the magnitude of these effects is the same as that for AFUT and so $\sim \pm 3$ GeV in $M_t$ (see table 4). The superheavy particle threshold effects in the Yukawa sector

Figure 6: $M_t$ versus $\kappa_i^2$ for $M_{\text{SUSY}} = 1000$ (solid), 500 (dashed) and 300 GeV (dot-dashed) with $\kappa_b^2 = 1.2$ fixed.
Figure 7: $M_t$ versus $\kappa_b^2$ for $M_{\text{SUSY}} = 500$ with $\kappa_t^2 = 1.6$ fixed.
can be estimated from figs. 6 and 7 if we assume that its magnitude is also the same as that of AFUT, that is, 2% in $\kappa_1^2$ and $\kappa_2^2$. This gives an uncertainty of $\pm 0.4$ GeV in $M_t$. The MSSM threshold correction is denoted $\delta^{\text{MSSM}} M_t$ as for the case of AFUT (see also eq. (20)). Thus, including all the uncertainties we have discussed above, we may conclude that

$$M_t = (183 + \delta^{\text{MSSM}} M_t + \pm 5) \text{ GeV},$$

(34)

for FUT, where the finite corrections coming from the conversion from the DR scheme to the ordinary $\overline{\text{MS}}$ in the gauge sector \cite{39} are included, and those in the Yukawa sector are included as an uncertainty of $\sim \pm 1$ GeV.

Comparing this with the $M_t$ prediction of AFUT given in eq. (32), we see that at present the two existing GYU models based on $SU(5)$ cannot be theoretically distinguished by $M_t$. To distinguish them, it is therefore important to reduce the uncertainties in $M_t$. Also important is the structure of the supersymmetry breaking, because, as we see from tables 4 and 5, the two models predict different $m_b$. Since the accurate prediction on $m_b$ depends strongly on the soft supersymmetry breaking terms, we have to clarify this subject more in detail in order to distinguish between AFUT and FUT, which will be our future work.

As a last remark we would like to mention that even the soft supersymmetry breaking terms can be controlled by the reduction of couplings and finiteness \cite{43}.

### 5 Conclusion

The electroweak and strong interactions can be unified in GUTs \cite{1,2,3}, thereby relating the apparently independent gauge couplings of the SM. The observed hierarchy of these couplings, $\alpha_1 < \alpha_2 < \alpha_3$, can be understood if one assumes that the unifying gauge symmetry is broken at a $M_{\text{GUT}}$ which is much larger than the electroweak scale \cite{4}. The top-bottom mass hierarchy at low energies could be explained to a certain extent if one assumes the existence of a GYU at $M_{\text{GUT}}$ \cite{18,10,12}. Of course, the observed top-bottom hierarchy, $M_t/m_b(M_b) \simeq (37 - 47)$, is not a proof for a GYU, but it may indicate a unification that goes beyond the usual grand unification.
We have seen that different GYU models could be distinguished and tested by a precise measurement of $M_t$ if the models are not on the equiv-$M_t$ lines that are very close to each other and the predicted $M_t$’s are well separated from the infrared value. We, therefore, have analyzed the infrared quasi–fixed–point behavior of the $M_t$ prediction in some detail. We have also seen that the infrared value, $M_t$(IR), depends on $\tan \beta$ and its lowest value is $\sim 188$ GeV. Comparing this with the experimental value [21], $M_t = (180 \pm 12)$ GeV [21], we may conclude that the present data on $M_t$ cannot be satisfyingly explained solely from the infrared quasi–fixed–point behavior of the top Yukawa coupling. Two GYU models on the same equiv-$M_t$ line predict in general different values for $M_b$ and $\alpha_3(M_Z)$, and so their precise knowledge will allow us to further shrink the allowed range of the GYU boundary conditions.

The main conclusion of our calculations in AFUT and FUT is that they predict $M_t = (181 + \delta_{\text{MSSM}} M_t \pm 3)$ GeV and $(183 + \delta_{\text{MSSM}} M_t \pm 5)$ GeV, respectively, where $\delta_{\text{MSSM}} M_t$ stands for the MSSM threshold correction. We found it is $\sim -2$ GeV for the case that all the superpartners have the same mass $M_{\text{SUSUY}}$ and $\mu_H/M_{\text{SUSUY}} \ll 1$. These predictions are consistent with the present experimental data. Clearly, to exclude or verify these GYU models, the experimental as well as theoretical uncertainties have to be further reduced. One of the largest theoretical uncertainties for FUT results from the not-yet-calculated threshold effects of the superheavy particles. Since the structure of the superheavy particles in FUT is basically fixed, it will be possible to bring these threshold effects under control, which will reduce the uncertainty of the $M_t$ prediction (5 GeV) down to $\sim 2$ GeV.

Here we have been regarding $\delta_{\text{MSSM}} M_t$ as unknown because we have no sufficient information on the superpartner spectra. Recently, however, we have found that the idea of reduction of couplings can be applied to dimensionfull parameters too. As a result, it becomes possible to predict the superpartner spectra to a certain extent and then to calculate $\delta_{\text{MSSM}} M_t$. 
References

[1] J.C. Pati and A. Salam, Phys. Rev. Lett. 31 (1973) 661.

[2] H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

[3] H. Fritzsch and P. Minkowski, Ann. Phys. 93 (1975) 193; H. Georgi, in Particles and Fields–1974, ed. C.E. Carlson, American Institute of Physics, New York

[4] H. Georgi, H. Quinn, S. Weinberg, Phys. Rev. Lett. 33 (1974) 451.

[5] A. Buras, J. Ellis, M.K. Gaillard and D. Nanopoulos, Nucl. Phys. B135 (1978) 66.

[6] P. Fayet, Nucl. Phys. B149 (1979) 134.

[7] F. del Aguila, M. Dugan, B. Grinstein, L. Hall, G.G. Ross and P. West, Nucl. Phys. B250 (1985) 225.

[8] W. Zimmermann, Commun. Math. Phys. 97 (1985) 211; R. Oehme and W. Zimmermann, Commun. Math. Phys. 97 (1985) 569.

[9] J. Kubo, K. Sibold and W. Zimmermann, Nucl. Phys. B259 (1985) 331.

[10] J. Kubo, M. Mondragón and G. Zoupanos, Nucl. Phys. B424 (1994) 291.

[11] J. Kubo, M. Mondragón, N.D. Tracas and G. Zoupanos, Phys. Lett. B342 (1995) 155.

[12] J. Kubo, M. Mondragón, S. Shoda and G. Zoupanos, Gauge-Yukawa Unification in SO(10) SUSY GUTs, Max-Planck-Institute preprint, MPI-Ph/95-125, to appear in Nucl. Phys. B.

[13] A.J. Parkes and P.C. West, Phys. Lett. B138 (1984) 99; Nucl. Phys. B256 (1985) 340; D.R.T. Jones and A.J. Parkes, Phys. Lett. B160 (1985) 267; D.R.T. Jones and L. Mezincescu, Phys. Lett. B136 (1984) 242; B138 (1984) 293; A.J. Parkes, Phys. Lett. B156 (1985) 73; I. Jack and D.R.T. Jones, Phys. Lett. B333 (1994) 372.
[14] S. Hamidi, J. Patera and J.H. Schwarz, Phys. Lett. B141 (1984) 349; X.D. Jiang and X.J. Zhou, Phys. Lett. B197 (1987) 156; B216 (1989) 160.

[15] S. Hamidi and J.H. Schwarz, Phys. Lett. B147 (1984) 301; D.R.T. Jones and S. Raby, Phys. Lett. B143 (1984) 137; J.E. Björkman, D.R.T. Jones and S. Raby, Nucl. Phys. B259 (1985) 503; J. León et al, Phys. Lett. B156 (1985) 66; D.I. Kazakov, On a possible explanation of the origin of the quark mass spectrum, Dubna preprint, JINR-E2-94-162.

[16] A.V. Ermushev, D.I. Kazakov and O.V. Tarasov, Nucl. Phys. B281 (1987) 72; D.I. Kazakov, Mod. Phys. Lett. A2 (1987) 663; Phys. Lett. B179 (1986) 352.

[17] C. Lucchesi, O. Piguet and K. Sibold, Helv. Phys. Acta. 61 (1988) 321.

[18] D. Kapetanakis, M. Mondragón and G. Zoupanos, Zeit. f. Phys. C60 (1993) 181; M. Mondragón and G. Zoupanos, Nucl. Phys. B (Proc. Suppl) 37C (1995) 98.

[19] H. Georgi and A. Pais, Phys. Rev. D10 (1974) 539.

[20] Particle Data Group, L. Montanet et al., Phys. Rev. D50 (1994) 1173.

[21] CDF Collaboration, F. Abe et al., Phys. Rev. Lett. 74 (1995) 2626; D0 Collaboration, S. Abachi et al., Phys. Rev. Lett. 74 (1995) 2632.

[22] K. Hagiwara, Talk given at the Yukawa International Seminar 95: From the Standard Model to Grand Unified Theories, Kyoto, 21-25 Aug. 1995.

[23] C.T. Hill, Phys. Rev. D24 (1981) 691; C.T. Hill, C.N. Leung and S. Rao, Nucl. Phys. B262 (1985) 517

[24] W.A. Bardeen, M. Carena, S. Pokorski and C.E.M. Wagner, Phys. Lett. B320 (1994) 110; M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B419 (1994) 213.

[25] B. Pendleton and G.G Ross, Phys. Lett. B98 (1981) 291.
[26] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. 67 (1982) 1889; 68 (1982) 927.

[27] M. Bando, T. Kugo, N. Maekawa and H. Nakano, Mod. Phys. Lett. A7 (1992) 3379.

[28] H. Arason et al., Phys. Rev. D46 (1992) 3945; V. Barger, M.S. Berger and P. Ohmann, Phys. Rev. D47 (1993) 1093, and references therein.

[29] P.H. Chankowski, Z. Pluciennik and S. Pokorski, Nucl. Phys. B439 (1995) 23.

[30] L. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D50 (1994) 7048; M. Carena, M. Olechowski, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B426 (1994) 269.

[31] B.D. Wright, *Yukawa Coupling Thresholds: Application to the MSSM and the Minimal Supersymmetric SU(5) GUT*, University of Wisconsin-Madison report, MAD/PH/812 (hep-ph/9404217).

[32] P. Langacker and N. Polonsky, Phys. Rev. D47 (1993) 4028.

[33] D. Pierce, in *Proc. of SUSY 94*, Ann Arbor, Michigan, 1994, eds. C. Kolda and J. Wells, p.418; A. Donini, *ONE-LOOP CORRECTIONS TO THE TOP, STOP AND GLUINO MASSES IN THE MSSM*, CERN report, CERN-TH-95-287 (hep-ph/9511289); J. Feng, N. Polonsky and S. Thomas, *The Light Higgsino-Gaugino Window*, University of Munich report, LMU-TPW-95-18; N. Polonsky, *On Supersymmetric b – τ Unification, Gauge Unification, and Fixed Points*, University of Munich report, LMU-TPW-96-04.

[34] F.M. Borzumati, M. Olechowski and S. Pokorski, Phys. Lett. B349 (1995) 311; H. Murayama, M. Olechowski and S. Pokorski, *Viable t – b – τ Yukawa Unification in SUSY SO(10)*, MPI preprint No. MPI-PhT/95-100, hep-ph/9510327.

[35] S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150; N. Sakai, Zeit. f. Phys. C11 (1981) 153.
[36] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. Lett. 69 (1993) 1992; J. Ellis, S. Kelley and D. V. Nanopoulos, Nucl. Phys. B373 (1992) 55; Y. Yamada, Z. Phys. C60 (1993) 83

[37] N. Sakai and T. Yanagida, Nucl. Phys. B197 (1982) 533; S. Weinberg, Phys. Rev. D26 (1982) 287.

[38] J. Hisano, H. Murayama and T. Yanagida, Nucl. Phys. B402 (1993) 46.

[39] I. Antoniadis, C. Kounnas and K. Tamvakis, Phys. Lett. B119 (1982) 377; A. Schüler, S. Sakakibatra and J.G Körner, Phys. Lett. B194 (1987) 125.

[40] L.J. Hall and U. Sarid, Phys. Rev. Lett. 70 (1993) 2673; A. Vayonakis, Phys. Lett. B307 (1993) 318; T. Dasgupta, P. Mamales and P. Nath, Phys. Rev. D52 (1995) 5366; D. Ring, S. Urano and R. Arnowitt, Phys. Rev. D52 (1995) 6623.

[41] J. León et al, Phys. Lett. B156 (1985) 66.

[42] N. Deshpande, Xiao-Gang, He and E. Keith, Phys. Lett. B332 (1994) 88.

[43] I. Jack and D.R.T. Jones, Phys. Lett. B349 (1995) 294.