Magnus-induced ratchet effects for skyrmions interacting with asymmetric substrates

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Abstract
We show using numerical simulations that pronounced ratchet effects can occur for ac driven skyrmions moving over asymmetric quasi-one-dimensional substrates. We find a new type of ratchet effect called a Magnus-induced transverse ratchet that arises when the ac driving force is applied perpendicular rather than parallel to the asymmetry direction of the substrate. This transverse ratchet effect only occurs when the Magnus term is finite, and the threshold ac amplitude needed to induce it decreases as the Magnus term becomes more prominent. Ratcheting skyrmions follow ordered orbits in which the net displacement parallel to the substrate asymmetry direction is quantized. Skyrmion ratchets represent a new ac current-based method for controlling skyrmion positions and motion for spintronic applications.

1. Introduction
Ratchet effects can arise when particles placed in an asymmetric environment are subjected to suitable nonequilibrium conditions such as an externally applied ac drive, which produces a net dc motion of the particles [1]. Ratchet effects have been realized for colloidal systems [2], cold atoms in optical traps [3], granular media on sawtooth substrates [4], and the motion of cells crawling on patterned asymmetric substrates [5], and they can be exploited to create devices such as shift registers for domain walls moving in an asymmetric substrate [6]. There has been intense study of ratchet effects for magnetic flux vortices in type-II superconductors interacting with quasi-one-dimensional (q1D) or two-dimensional (2D) asymmetric pinning substrates. The vortices can be effectively described as overdamped particles moving over an asymmetric substrate, and under ratcheting conditions the application of an ac driving current produces a net dc flux flow [7–12]. The simplest vortex ratchet geometry was first proposed by Lee et al [7], where an effectively 2D assembly of vortices was driven over a q1D asymmetrically modulated substrate using an ac driving force oriented along the direction of the substrate asymmetry.

Skyrmions in chiral magnets, initially discovered in MnSi [13] and subsequently identified in numerous other materials at low [14–17] and room temperatures [18–22], have many similarities to superconducting vortices. Skyrmions are spin textures forming topologically stable particle-like objects [17] that can be set into motion by the application of a spin-polarized current [23–27]. As a function of the external driving, skyrmions can exhibit a depinning transition similar to that found for vortices, and from transport measurements it is possible to construct skyrmion velocity versus applied force curves [24, 26–30]. It has been shown that the dynamics of skyrmions interacting with pinning can be captured by an effective particle equation of motion, which produces pinning–depinning and transport properties that agree with those obtained using a continuum-based model [28], just as a particle-based description for vortices can effectively capture the vortex dynamics in the presence of pinning sites [31]. Due to their size scale and the low currents required to move them, skyrmions show tremendous promise for applications in spintronics such as race-track type memory devices [32, 33] originally proposed for domain walls [34], and it may be possible to create skyrmion logic devices similar to those that harness magnetic domain walls [35]. In order to realize such skyrmion applications, new methods must be developed to precisely control skyrmion positions and motion.
The most straightforward approach to a skyrmion ratchet is to utilize the same types of asymmetric substrates known to produce ratchet effects in vortex systems. There are, however, important differences between skyrmion and vortex dynamics due to the strong non-dissipative Magnus component of skyrmion motion [13, 17, 24, 26, 28, 29] that is absent in the vortex system. The Magnus term produces a skyrmion velocity contribution that is perpendicular to the direction of an applied external force. In the presence of pinning, the Magnus term reduces the effectiveness of the pinning by causing the skyrmions to deflect around the edges of attractive pinning sites [26, 28, 29] rather than being captured by the pinning sites as overdamped vortices are. The Magnus term also produces complex winding orbits for skyrmions moving in confined regions [36] or through pinning sites [26, 28, 29]. The ratio of the strength of the Magnus term $\alpha_m$ to the dissipative term $\alpha_d$ can be 10 or higher for skyrmion systems [17]. In contrast, although it is possible for a Magnus effect to appear for vortices in superconductors, it is generally very weak so that the vortex dynamics is dominated by dissipation [31]. A key question is how the Magnus force could impact possible ratchet effects for skyrmions, and whether new types of ratchet phenomena can be realized for skyrmion systems that are not accessible in overdamped systems [40].

In this work we numerically examine a 2D assembly of skyrmions interacting with a q1D asymmetric periodic substrate shown schematically in figure 1. An ac drive is applied either parallel ($F_{ac}^\parallel$) or perpendicular ($F_{ac}^\perp$) to the substrate asymmetry direction, which runs along the $x$ axis. In the overdamped limit, no ratchet effect appears for $F_{ac}^\perp$; however, we find that the Magnus effect produces a novel ratchet effect for $F_{ac}^\parallel$ when it curves the skyrmion trajectories into the asymmetry or $x$ direction. We model the skyrmion dynamics in a sample with periodic boundary conditions using a particle based description [28, 39] and vary the ratio of the Magnus term to the dissipative dynamic terms, the amplitude and frequency of the ac drive, and the strength of the substrate.

Simulation – The equation of motion for a single skyrmion with velocity $\mathbf{v}$ moving in the $x$–$y$ plane is

$$a_\parallel \dot{\mathbf{v}} + a_m \dot{z} \times \mathbf{v} = F_{ac}^\parallel + F_{ac}^\perp.$$  \hspace{1cm} (1)

The damping term $a_d$ keeps the skyrmion velocity aligned with the direction of the net external force, while the Magnus term $a_m$ rotates the velocity toward the direction perpendicular to the net external forces. To examine the role of the Magnus term we vary the relative strength $\alpha_m/a_d$ of the Magnus term to the dissipative term under the constraint $\alpha_\parallel + \alpha_m = 1$, which maintains a constant magnitude of the skyrmion velocity. The substrate force $F_{ac}^\parallel = -VU(\dot{x})\mathbf{X}$ arises from a ratchet potential

$$U(\dot{x}) = U_0 [\sin(2\pi\dot{x}/a) + 0.25 \sin(4\pi\dot{x}/a)],$$  \hspace{1cm} (2)

where $a$ is the periodicity of the substrate and we define the strength of the substrate to be $A_p = 2\pi U_0/a$. The ac driving term is either $F_{ac}^\parallel = F_{ac}^\parallel \cos(\omega t)\dot{x}$ or $F_{ac}^\perp = F_{ac}^\perp \cos(\omega t)\dot{y}$. We measure the time-averaged skyrmion velocities parallel $\langle V \rangle_\parallel \equiv \frac{2\pi}{A_p} (\dot{y} - \dot{x})/o_a$ or perpendicular $\langle V \rangle_\perp \equiv \frac{2\pi}{A_p} (\dot{y} - \dot{x})/o_a$ to the substrate asymmetry direction as we vary the ac amplitude, substrate strength, or $\omega$. Here we focus primarily on samples with $A_p = 1.5$ and $\omega = 2.5 \times 10^{-6}$ inverse simulation time steps, and we consider the sparse limit of a single skyrmion which can also apply to regimes in which skyrmion–skyrmion interactions are negligible.

2. Results and discussion

2.1. ac driving parallel to substrate asymmetry

We first apply the ac driving force parallel to the asymmetry direction, a configuration previously studied for the same type of substrate in vortex systems [7, 12]. In figure 2 we plot $\langle V \rangle_\parallel$ and $\langle V \rangle_\perp$ versus $F_{ac}^\parallel$ for samples with $A_p = 1.5$, so that the maximum magnitude of the substrate force is $F_{ac}^{\parallel\max} = 2.0$ in the negative $x$ direction and $F_{ac}^{\parallel\max} = 1.0$ in the positive $x$ direction. The overdamped limit $\alpha_m/a_d = 0.0$ appears in figure 2(a), where
\( \langle V \rangle_{\parallel} = 0.0 \) and there is a ratchet effect only along the parallel or \( x \)-direction for \( F_{\parallel}^{\text{ac}} > 1.25 \). The skyrmion velocity is quantized and forms a series of steps with \( \langle V \rangle_{\parallel} = n \), where \( n \) is an integer. On the highest step in \( \text{figure 2(a)} \) near \( F_{\parallel}^{\text{ac}} = 2.5, n = 7 \). The quantization indicates that during one ac driving period, the skyrmion moves a net distance of \( na \) in the parallel direction. For very large values of \( F_{\parallel}^{\text{ac}} \), the velocity quantization is lost and the parallel ratchet effect gradually diminishes. In \( \text{figure 2(b)} \) we plot \( \langle V \rangle_{\parallel} \) and \( \langle V \rangle_{\perp} \) for a sample with \( \alpha_{\text{ac}}/\alpha_3 = 1.6 \), showing a nonzero skyrmion velocity component for both the parallel and perpendicular directions. There is again quantization of the parallel velocity \( \langle V \rangle_{\parallel} \) and the Magnus term transfers this quantization into the perpendicular direction even though there is no periodicity of the substrate along the \( y \) direction. Thus, we find \( \langle V \rangle_{\perp} = n\alpha_{\text{ac}}/\alpha_4 \). For \( F_{\parallel}^{\text{ac}} > 5.0 \), we find windows in which the ratcheting effect is lost and \( \langle V \rangle_{\parallel} = \langle V \rangle_{\perp} = 0 \). As \( \alpha_{\text{ac}}/\alpha_4 \) increases, the maximum value of \( n \) decreases, so that at \( \alpha_{\text{ac}}/\alpha_4 = 4.58 \) in \( \text{figure 2(c)} \), only the \( n = 1 \) step appears. At the same time, the width of the non-ratcheting windows increases, as shown in \( \text{figure 2(d)} \) for \( \alpha_{\text{ac}}/\alpha_4 = 7.858 \). These results show that while a ratchet effect does occur for skyrmion systems, an increase in the Magnus term decreases the range of ac drives over which the ratchet effect can be observed.

By lowering the ac frequency \( \omega \) or increasing the substrate strength \( A_\mu \), it is possible to increase the range and magnitude of the skyrmion ratchet effect. In \( \text{figure 3(b), (c)} \) we show representative skyrmion orbits for the system in \( \text{figures 2(b), (c)} \). \( \text{Figure 3(b)} \) shows the orbits at \( \alpha_{\text{ac}}/\alpha_4 = 1.6 \) for \( F_{\parallel}^{\text{ac}} = 2.2 \), corresponding to the \( n = 3 \) step in \( \text{figure 2(b)} \). Here the skyrmion moves along straight lines oriented at an angle to the direction of the applied ac drive. During each ac cycle, the skyrmion first translates a distance \( 3a \) in the positive \( x \) direction and then travels in the reverse direction for a distance of \( a/2 \) before striking the top of the potential barrier, which it cannot overcome in the reverse \( x \) direction for this magnitude of ac drive. It repeats this motion in each cycle. In \( \text{figure 3(c)} \) we show the skyrmion orbit at \( \alpha_{\text{ac}}/\alpha_4 = 4.58 \) for \( F_{\parallel}^{\text{ac}} = 2.0 \), corresponding to the \( n = 1 \) orbit in \( \text{figure 2(c)} \). The angle the skyrmion motion makes with the ac driving direction is much steeper, and the skyrmion is displaced a net distance of \( a \) in the \( x \) direction during each ac driving cycle.

In order to clarify the evolution of the ratchet phases as a function of \( \alpha_{\text{ac}}/\alpha_4 \) and \( F_{\parallel}^{\text{ac}} \), in \( \text{figure 3(a)} \) we plot the ratchet envelopes for the \( n = 1 \) to \( n = 6 \) steps. For the \( n = 1 \) case we show, in blue, all of the regions in the \( F_{\parallel}^{\text{ac}} - \alpha_{\text{ac}}/\alpha_4 \) plane where \( n = 1 \) steps appear. For \( n = 2 \) to \( n = 6 \), to prevent overcrowding of the graph, instead of plotting all ratchet regions we show only the inner step envelope for each \( n = n_0 \) obtained from simulations in which \( F_{\parallel}^{\text{ac}} \) is swept up from zero, and defined to extend from the drive at which \( n \) first reaches \( n_0 \) to the drive at which \( n \) first drops below \( n_0 \). We find that the minimum \( F_{\parallel}^{\text{ac}} \) required to induce a ratchet effect increases linearly with increasing \( \alpha_{\text{ac}}/\alpha_4 \), and that the ratcheting regions form a series of tongue features (shown for \( n = 1 \) only). The ratchet effects persist up to and beyond \( \alpha_{\text{ac}}/\alpha_4 = 10 \). For \( \alpha_{\text{ac}}/\alpha_4 = 0 \), the strongest ratchet effects with the largest values of \( n \) occur near \( F_{\parallel}^{\text{ac}} = 2.3 \), corresponding to a driving force that is slightly higher than the maximum force exerted on the skyrmion by the substrate when it is moving with a negative \( x \) velocity.
component. As $F^\perp_\alpha$ increases above this value, the skyrmion can slip backward over more than one substrate plaquette during each ac cycle, limiting its net forward progress.

2.2. ac driving perpendicular to substrate asymmetry

In figure 4 we plot $\langle V \rangle_\parallel$ and $\langle V \rangle_\perp$ versus $F^\perp_\alpha$ for $A_p = 1.5$. The inset in figure 4(a) shows that for $\alpha_m/\alpha_d = 0$, $\langle V \rangle_\parallel = \langle V \rangle_\perp = 0$, indicating that there is no ratchet effect in either direction. When the Magnus term is finite, as in figure 4(a) where $\alpha_m/\alpha_d = 0.855$, pronounced ratcheting occurs in both the parallel and perpendicular directions. We describe this as a Magnus-induced transverse ratchet effect. Here $\langle V \rangle_\parallel$ is quantized at integer values, with the largest step in figure 4(a) falling at $n = 5$, and $\langle V \rangle_\perp = \langle V \rangle_\parallel = \alpha_m/\alpha_d$. As $\alpha_m/\alpha_d$ increases, figures 4(b), (c) shows that the maximum value of $F^\perp_\alpha$ at which ratcheting occurs decreases, as does the maximum value of $n$ and the widths of the ratcheting windows. In addition to the integer steps in $\langle V \rangle_\parallel$, we also observe fractional steps, as shown in the inset of figure 4(c) for $\alpha_m/\alpha_d = 4.58$ for $F^\perp_\alpha = 2.0$ from figure 2(c), where the skyrmion translates a distance $a$ in the $x$ direction during every ac drive cycle.

Figure 3. (a) A plot of $F^\perp_\alpha$ versus $\alpha_m/\alpha_d$ indicating all $n = 1$ (blue) ratcheting regions, and the inner envelopes of the $n = 2$ (orange), 3 (purple), 4 (green), 5 (red) and 6 (yellow) ratcheting regions. The boundaries of the inner envelopes are defined as the point at which the ratchet velocity first reaches the value $n$ followed by the point at which it first drops below $n$. The $y$-axis marked $A$ and the dashed lines marked $B$–$D$ are the values of $\alpha_m/\alpha_d$ for which the velocity curves in figure 2 were obtained. The minimum ac amplitude required to produce a ratchet effect increases with increasing $\alpha_m/\alpha_d$.

(b), (c) Skyrmion trajectory images. White (green) regions are high (low) areas of the substrate potential. Thin black lines indicate forward motion of the skyrmion and thick purple lines indicate backward motion. The red circle is the skyrmion. (b) The $n = 3$ orbit at $\alpha_m/\alpha_d = 1.6$ for $F^\perp_\alpha = 2.2$ from figure 2(b). The skyrmion moves in straight lines and translates $3a$ in the $x$ direction during one ac drive cycle. (c) The $n = 1$ orbit at $\alpha_m/\alpha_d = 4.58$ for $F^\perp_\alpha = 2.0$ from figure 2(c), where the skyrmion translates a distance $a$ in the $x$ direction during every ac drive cycle.
The skyrmion orbits for perpendicular ac driving differ markedly from those that appear under parallel ac driving, since the Magnus term induces a velocity component perpendicular to the direction of the external driving force. As a result, perpendicular ac drives induce parallel skyrmion motion that interacts with the substrate asymmetry. Figure 5(a) shows a skyrmion orbit on the \( V_{\parallel} \) (red) and perpendicular \( V_{\perp} \) (blue) to the substrate asymmetry versus \( F_{ac} \) for samples with \( A_\chi = 1.5 \). At \( \alpha_m/\alpha_d = 0.855 \), ratcheting occurs in both the parallel and perpendicular directions for \( F_{ac} > 1.5 \), and \( \langle V_{\parallel} \rangle = \perp \) with integer \( n \). Inset: at \( \alpha_m/\alpha_d = 0 \), there is no ratchet effect. (b) \( \alpha_m/\alpha_d = 4.0 \). (c) \( \alpha_m/\alpha_d = 7.018 \). Inset: a blowup of \( \langle V_{\parallel} \rangle \) from the main panel at the third \( n = 1 \) step showing that additional fractional steps appear at velocity values such as \( n/m = 1/2 \).

Figure 4. The skyrmion velocity parallel \( \langle V \rangle_\parallel \) (red) and perpendicular \( \langle V \rangle_\perp \) (blue) to the substrate asymmetry versus \( F_{ac} \) for samples with \( A_\chi = 1.5 \). (a) At \( \alpha_m/\alpha_d = 0.855 \), ratcheting occurs in both the parallel and perpendicular directions for \( F_{ac} > 1.5 \), and \( \langle V_{\parallel} \rangle = \perp \) with integer \( n \). Inset: at \( \alpha_m/\alpha_d = 0 \), there is no ratchet effect. (b) \( \alpha_m/\alpha_d = 4.0 \). (c) \( \alpha_m/\alpha_d = 7.018 \). Inset: a blowup of \( \langle V_{\parallel} \rangle \) from the main panel at the third \( n = 1 \) step showing that additional fractional steps appear at velocity values such as \( n/m = 1/2 \).

Figure 5. Skyrmion trajectory images from the system in figure 4. White (green) regions are high (low) areas of the substrate potential. Black lines indicate the motion of the skyrmion; the red circle is the skyrmion. (a) The \( n = 2 \) step at \( \alpha_m/\alpha_d = 0.855 \) for \( F_{ac} = 2.0 \). (b) The \( n = 1 \) step at \( \alpha_m/\alpha_d = 4.0 \) for \( F_{ac} = 0.7 \). (c) A non-ratcheting orbit at \( \alpha_m/\alpha_d = 4.0 \) for \( F_{ac} = 0.76 \). (d) For higher ac drives the orbits become more complicated, as shown here for \( \alpha_m/\alpha_d = 4.0 \) at \( F_{ac} = 1.97 \).

The skyrmion orbits for perpendicular ac driving differ markedly from those that appear under parallel ac driving, since the Magnus term induces a velocity component perpendicular to the direction of the external driving force. As a result, perpendicular ac drives induce parallel skyrmion motion that interacts with the substrate asymmetry. Figure 5(a) shows a skyrmion orbit on the \( \langle V \rangle_\parallel = 2 \) step for \( F_{ac} = 2.0 \) and
The minimum ac force needed to induce a ratchet effect increases linearly with increasing substrate potential barriers. In contrast, for a parallel ac drive the Magnus term curves the skyrmion trajectories far enough to run along the direction parallel to the asymmetry. The minimum ac amplitude required to generate a ratchet effect decreases as the strength of the Magnus term increases, opposite to what we found for parallel ac driving in figure 3(a). Here, the skyrmion trajectories are more strongly curved into the parallel direction as \( \alpha_m/\alpha_d \) increases, so that a lower ac amplitude is needed to push the skyrmions over the substrate potential barriers. In contrast, for a parallel ac drive the Magnus term curves the skyrmion trajectories out of the parallel direction, so that a larger ac amplitude must be applied for the skyrmions to hop over the substrate maxima. We also find that as \( \alpha_m/\alpha_d \) increases, the minimum ac force needed to produce a ratchet effect drops below the pinning force exerted by the substrate. This cannot occur in overdamped systems, and is an indication that the Magnus term induces some inertia-like behavior. The threshold ac force value \( F^\text{th} \) for ratcheting to occur can be fit to \( F^\text{th}_n \propto (\alpha_m/\alpha_d)^{-1} \) for perpendicular driving, while for parallel driving, \( F^\text{th}_m \propto \alpha_m/\alpha_d \). As \( \alpha_m/\alpha_d \) increases, the extent of the regions where ratcheting occurs decreases; however, by increasing the substrate strength, the regions of ratcheting broaden in extent and the magnitude of the ratchet effect increases. In figure 6(b) we show the evolution of the \( n = 1, 2, 3 \) and 4 ratchet regions as a function of \( F^\text{th}_m \) and \( A_p \), for fixed \( \alpha_m/\alpha_d = 4.92 \). Here, as \( A_p \) increases, the magnitude of the ratchet effect increases, forming a series of tongues. The minimum ac force needed to induce a ratchet effect increases linearly with increasing \( A_p \). We find the same behavior of the ratchet regions when we vary \( A_p \) for different parallel drives \( F^\text{th}_m \) (not shown). These results indicate that skyrmions can exhibit a new type of Magnus-induced ratchet effect that appears when the driving force is applied perpendicular to the asymmetry direction of the substrate.

2.3. Frequency dependence

In figure 7(a) we plot a heat map of the parallel ratchet velocity \( \langle V \rangle_p \) for perpendicular ac driving as a function of \( F^\text{th}_m \) and \( \omega \) in a sample with \( A_p = 1.5 \) and \( \alpha_m/\alpha_d = 3.042 \). The ratchet effect extends over a wider range of \( F^\text{th}_m \) at low ac driving frequencies, while for higher \( \omega \) the ratcheting regions form a series of tongues. The ratchet magnitude is non-monotonic as a function of \( \omega \), and in several regions it increases in magnitude with increasing \( \omega \), such as on the first tongue where the maximum ratcheting effect occurs near \( \omega = 10^{-3} \) inverse simulation.
time steps before decreasing again as $\omega$ increases further. We find very similar ratchet behaviors for parallel ac driving, as shown in figure 7(b) where we plot $\langle V\rangle_\parallel$ as a function of $F_{ac\parallel}$ and $\omega$ and observe a series of tongues.

2.4. Discussion
All of the ratchet effects we describe should be robust against distortions and imperfections in the substrate pattern. The q1D nature of the substrate renders the ratchet behavior relatively insensitive to point-like imperfections of the type that are the most likely to be present in an actual physical realization of the potential. 1D imperfections such as variations in the distance between neighboring substrate minima that are correlated along the entire length of the q1D potential could reduce the magnitude of the ratchet effect, but would not destroy it unless the distortions were severe enough to destroy the periodicity of the substrate. Distortions of this size should be easily preventable and would not be expected to occur in a real sample.

The minimum substrate spacing $a$ for which our ratchet predictions are expected to remain valid is the same as the minimum skyrmion–skyrmion spacing at which the particle-based model breaks down; below this spacing, distortions of the skyrmion shape could affect the ratcheting. In principle there is no maximum spacing $a$ at which our predictions cease to apply since the ratchet mechanism does not require collective skyrmion–skyrmion interactions to be present. The limit of very large $a$ would require large driving forces and/or low ac driving frequencies to produce a ratchet effect.

3. Summary
Our results show that skyrmions are an ideal system in which to realize ratchet effects in the presence of asymmetric substrates. As expected, they undergo ratcheting when an ac drive is applied parallel to the substrate asymmetry direction; however, the skyrmions also exhibit a unique ratchet feature not found in overdamped systems, which is the transverse ratchet effect. Here, when the ac drive is applied perpendicular to the substrate asymmetry direction, the Magnus term curves the skyrmion orbits and drives them partially parallel to the asymmetry, generating ratchet motion. The asymmetric substrates we consider could be created using methods similar to those employed to create q1D asymmetric potentials in superconductors and related systems, such as nanofabricated asymmetric thickness modulation, asymmetric regions of radiation damage, asymmetric doping, or blind holes arranged in patterns containing density gradients. Since the skyrmion ratchet effects persist down to low frequencies, it should be possible to directly image the ratcheting motion, while for higher frequencies the existence of ratchet transport can be deduced from transport studies. Such ratchet effects offer a

![Figure 7. The magnitude of $\langle V\rangle_\parallel$ in samples with $A_p = 1.5$ and $\alpha_{ac}/\alpha_0 = 3.042$. (a) As a function of $F_{ac\perp}$ and $\omega$ in units of $10^{-3}$ inverse simulation time steps, $\langle V\rangle_\parallel$ has clear tongue structures, and its magnitude is non-monotonic for varied frequency. (b) $\langle V\rangle_\parallel$ as a function of $F_{ac\parallel}$ and $\omega$ has similar features.](image-url)
new method for controlling skyrmion motion that could be harnessed in skyrmion applications. We also expect that skyrmions could exhibit a rich variety of other ratchet behaviors under different conditions, such as more complicated substrate geometries, use of asymmetric ac driving, or collective effects due to skyrmion–skyrmion interactions.

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