Mathematical Model for Solar Drying of Grains

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Authors’ contributions

This work was carried out in collaboration between all authors. All authors read and approved the final manuscript.

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ABSTRACT

Mathematical modeling of drying process is a complex procedure that should be carefully done. Moreso, model for solar drying, which is a unique method of drying due to constant fluctuation in the climatic conditions, requires complete integration of the complex phenomena that are involved for accurate prediction of moisture content and drying rate. A mathematical model was developed from heat and mass balance equation considering the physical and thermal properties of the grain, meteorological factors and convective heat transfer during solar drying of grains. The data obtained from the model was compared with experimental results obtained using a solar dryer to dry five selected grains – cowpea, soyabean, groundnut, maize and sorghum at airflow rates of 0.22 m/s, 0.76 m/s and 0.94 m/s. The results from statistical analysis and regression analyses used to compare the results showed that the model is adequate in predicting the moisture content and drying rate of the selected grains as well as other agricultural products with closer physical and thermal properties.

Keywords: Modeling; solar drying; moisture content; drying rate and grains.

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1. INTRODUCTION

Deterioration in quality of agricultural product is due to the activities of micro-organisms which may occur rapidly whenever the moisture content is above a certain critical level. Agricultural products must therefore, be dried until the moisture content is in equilibrium with the normal atmospheric air to such an extent that decrease in quality due to the effect of mould, micro-organisms and insects is negligible. Drying is a complex phenomenon whose mechanisms are not yet understood. In case of solar drying, the complexity becomes more noteworthy due to change of meteorological factors during the entire process. Moreso, the mathematical approach of this complex phenomenon has not been fully developed. Most heat and mass transfer model are temperature dependent almost neglecting the important of the water content which is essential for the prediction of the drying rate during the process. A number of methods of analysis have been developed for the prediction of drying rates [1].

Drying process normally starts with a constant rate period which can be likened to the drying of an open faced body of water. The water and its surroundings, not the grains determine the rate of drying. The process can be represented as an adiabatic evaporation process and the rate of drying can be represented as stated in equation (1).

\[
\frac{dw}{dt} = h_d A(P_s - P_e) = h_d A(\rho_e - \rho_e) = \frac{h_d A(T_s - T_e)}{\rho_w}. \tag{1}
\]

Where,

\[
\frac{dw}{dt} = \text{drying rate, kg of } H_2O/\text{sec}
\]
\[
h_d = \text{convective heat transfer coefficient for water-air interface, W/m}^2\text{K}
\]
\[
A = \text{water surface area, m}^2
\]
\[
h_v = \text{latent heat of vapourization of water at Temperature, } T_s, \text{ J/kg}
\]
\[
h_d = \text{water-vapour or mass transfer coefficient in air}
\]
\[
h_d = \text{water-vapour or mass transfer coefficient at the water-air interface}
\]
\[
T_a = \text{air temperature, K}
\]
\[
T_s = \text{water surface temperature, K}
\]
\[
P_a = \text{partial water pressure in the air, N/m}^2
\]
\[
P_v = \text{water vapour pressure at interface temperature, } T_s, \text{ N/m}^2
\]
\[
\rho_a = \text{partial mass density of at interface in the air, kg/m}^3
\]
\[
\rho_v = \text{mass density of water-vapour at interface temperature, } T_s, \text{ kg/m}^3
\]

However, drying of agricultural products takes place practically in the falling rate period which is mainly by diffusion. The movement of moisture in grains during this period is unsteady. The mass balance of a diffusing substance in an infinitesimal isotropic volume under transient conditions have been described by the fick’s second law stated in equation (2) as given by [2].

\[
\frac{\partial X}{\partial t} = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial X}{\partial r} \right) \tag{2}
\]

Where, X is the water content, t is the time variable, D is the diffusion coefficient or diffusivity and r is the direction through which diffusion occurs. Assuming:

(i) homogeneity of water content inside the grain at the beginning of the drying process
(ii) water content at the surface of the product remains constant
(iii) symmetric condition at the centre of the sphere

and considering the necessary initial and final boundary conditions, an analytical solution of the Fick’s second law can be obtained to calculate the average water content inside the food (M) after a time, t as

\[
\frac{\bar{M} - M_e}{M_a - M_e} = \frac{6}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left( -4 \pi^2 \frac{D_{eff} t}{r^2} \right) \tag{3}
\]

Movement of moisture in grains can also be likened to that of heat conduction in a solid, and the following equation according to [1] was used to predict the drying rate:

Within the Solid:

\[
\frac{dw}{dt} = \frac{d_w \gamma A \delta M/\delta X}{100} \tag{4}
\]

At the surface:

\[
-d_w \gamma \left( \frac{\delta M}{\delta X} \right) = S(M_s - M_e) \gamma \tag{5}
\]

\[
\frac{\delta M}{\delta t} = d_w \left( \frac{\delta^2 M}{\delta X^2} \right) \tag{6}
\]

where, \( d_w \) = diffusivity of water, m\(^2\)/s, \( \gamma = \) density of grain, kg/m\(^3\), M = moisture content, at time t, db, %, \( M_s = \) moisture content at surface, db, %, \( M_e = \) equilibrium moisture content at the relative humidity of air, db, %, S = surface
2. MODEL DEVELOPMENT

Modeling solar drying process is based on equations that rule the heat and mass transfer phenomena, analogous to those involved in convective heating. According to [5], the global energy balance to a solar dryer is given as:

Rate of energy gained by the dried product is equal to the absorbed radiant energy less the convective heat loss, evaporative heat loss and the radiative heat loss as shown in equation (8)

\[
\frac{d(MC_pT)}{dt} = \alpha A_p q_s - h A_s (T - T_a) - \frac{d(\lambda M_w)}{dt} - A e \sigma (T^4 - T_0^4). \tag{8}
\]

Thus, equation (8) is modified to accommodate the effect of conductive heat loss to give

\[
\frac{d(MC_pT)}{dt} = \alpha A_p q_s - h A_s (T - T_a) - \frac{KA_s(T_i-T_o)}{x} - \frac{d(\lambda M_w)}{dt} - A e \sigma (T^4 - T_0^4). \tag{10}
\]

Where, \(K = \) thermal conductivity of drying plate, W/m K, \(A_s = \) surface area of the product available for heat transfer, m\(^2\), \(T_i = \) temperature of the inner wall of the dryer, K, \(T_o = \) temperature of the outer wall of the dryer, K and \(x = \) thickness of drying plate, m.

Equation (10) was solved by a finite difference method to obtain the temperature of the product at any time \(i\), as shown in equation (11).

\[
T_{i+1} = \left[\alpha A_p q_s - h A_s (T - T_a) - \frac{KA_s(T_i-T_o)}{x} - \frac{d(\lambda M_w)}{dt} - A e \sigma (T_i^4 - T_0^4)\right] \frac{\Delta t}{m d (1+M_s) C_p} + T_i. \tag{11}
\]
2.1 Assumption for the Model

Where i is the time step, \( m_i \) is the mass of dry matter (kg) and \( \Delta t \) is the time increment.

However, the major focus of the model is the moisture content of the grain and hence the moisture content is made the independent variable. In addition to this, the model also incorporates thermal storage to solve the problem of moisture re-absorption at night. Thus, using the Euclid finite difference method, two drying equations; one to predict drying rate during the day and the other to predict drying rate at night were incorporated into the model and are stated in Equation (12) and Equation (13) as

\[
M_{i+1}\text{day} = \left[ aA_p q_s - hA_s (T - T_a) - \frac{KA_s (T_i - T_a)}{x} \Delta \sigma (T^4 - T_{a}^4) - \left( \frac{T_{i+1} - T_i}{\Delta t} M C_p \right) \right] \frac{\Delta t}{\lambda} + M_i \quad (12)
\]

\[
M_{i+1}\text{night} = \left[ m_p C_p (T_s - T_a) - hA_s (T - T_a) - \frac{KA_s (T_i - T_0)}{x} \Delta \sigma (T^4 - T_{a}^4) - \left( \frac{T_{i+1} - T_i}{\Delta t} M C_p \right) \right] \frac{\Delta t}{\lambda} + M_i \quad (13)
\]

Eventually, a single drying equation (Equation 14) was developed to handle both the day and night by combining equation (12) and (13) and assuming \((1 - \beta)\) and \((1 - \delta)\) as coefficient of the two equations as follows:

\[
M_{i+1} = (1 - \beta) M_{i+1}\text{day} + (1 - \delta) M_{i+1}\text{night}.
\]

To give

\[
M_{i+1} = \frac{1}{\lambda} \left[ aA_p q_s - hA_s (T_p - T_a) - kA (T_2 - T_1) \Delta t - A_p \epsilon \sigma \left( -T_{a}^4 + T_{sky}^4 \right) \right.
\]

\[
- \left[ \frac{T_i}{m_0 C_p} + A_p f_r \left( T_i - \frac{\Delta u_i}{A_p} \right) (T_f - T_i) - \Delta t \left( m_f - m_i \right) m_0 C_p \right] \frac{\Delta t}{t(1 - m_i)} \right]
\]

\[
+ \frac{1}{\lambda} \left[ \beta \left[ m_p C_p (T_s - T_a) - hA_s (T_p - T_a) - kA (T_2 - T_1) \Delta t - A_p \epsilon \sigma \left( -T_{a}^4 + T_{sky}^4 \right) \right. \right.
\]

\[
- \left. \left[ \frac{T_i}{m_0 C_p} + A_p f_r \left( T_i - \frac{\Delta u_i}{A_p} \right) (T_f - T_i) - \Delta t \left( m_f - m_i \right) m_0 C_p \right] \frac{\Delta t}{t(1 - m_i)} \right]
\]

\[
(14)
\]

Where, \( \beta \) = energy source factor from solar incidence on the dryer and \( \delta \) = energy source factor from heat storage chamber. If the initial temperature, \( T_i \) of the grain, at time, i is known, then the temperature, \( T_{i+1} \) of the grain at known time, \( i + 1 \), is estimated using equation (15).

\[
T_{i+1} = T_i + \frac{Q_u}{m_0 C_p} \quad (15)
\]

Where \( Q_u \) = heat gained by the grain, \( m_0 \) = initial mass of the wet grain and \( C_p \) = specific heat capacity of the grain.

2.1 Assumption for the Model

In developing the model, the following assumptions were made:

1. No chemical reaction takes place during drying
2. The material undergoes shrinkage as drying progresses
3. There is uniform distribution of air throughout the dryer
4. Thermal conduction between two grain particles is negligible
5. Moisture diffusion within grain particles is negligible

2.2 Input for the Model

The physical and thermal properties of the selected grains such as geometrical mean diameter, surface area, bulk density, true density, sphericity, porosity, specific heat capacity, thermal conductivity and thermal diffusivity were obtained from literature. These data are the essential inputs for the model. They also served as guide in designing the solar dryer Fig. 1 used for the experimental testing.
2.3 Algorithm for the Model

The steps (algorithms) for solving this model are presented in the flow chart shown in Fig. 1. Once the initial estimates of moisture content and temperature of the product are known, the next moisture content is calculated from the model using Maple mathematical programming following the algorithms.
3. RESULTS AND DISCUSSION

3.1 Effect of Airflow Rates on the Moisture Content of Grains using the Dryer and Model

Fig. 3 showed the variation in moisture contents of cowpea, soyabean, groundnut, maize and sorghum respectively at airflow rates of 0.22 m/s, 0.76 m/s and 0.94 m/s using the developed solar dryer. The plots showed that the moisture content in the dryer decreased with increase in airflow rate. The rate of decrease becomes more pronounced at 0.94 m/s airflow rate. This implies that the rate of moisture removal increases with increase in airflow rate. The reason for the rapid decrease in moisture content at 0.94 m/s airflow rate was as a result of the faster rate of moisture removal from the grain surface as moist air moves from the interstitial part to the grain surface. However, toward the end of drying the rate of moisture reduction decreases showing that more time is required to remove the little amount of moisture still left in the core of the grain. This is in agreement with previous report during drying of cocoa [7,8] and coffee [9]. Similarly, Fig. 4 showed the variation in moisture contents of the selected grains at airflow rates of 0.22 m/s, 0.76 m/s and 0.94 m/s using the model. Just like for the dryer, the plot showed that the moisture content decreased with increase in airflow rate. However, unlike during drying using the dryer, reduction of moisture seems to increase faster toward the end of the drying period using the model. The increase in moisture reduction toward the end of the drying period was more pronounced when the airflow rate was 0.94 m/s for all the selected grains. This does not follow the expected constant moisture reduction during drying of grains. The deviation from the expected pattern might be due to constant fluctuation in the climatic condition during this period which is may not be accurately represented in the model.
Fig. 3. Variation in moisture content of (a) cowpea (b) soyabean (c) groundnut (d) maize (e) sorghum using the developed solar dryer at different airflow rates with time.
Fig. 4. Variation in moisture content with time of (a) cowpea (b) soyabean (c) groundnut (d) maize and (e) sorghum using the model at different airflow rate

Also, Figs. 5, 6 and 7 showed the comparison in the moisture content of grains using the experimental dryer and the developed model at airflow rates of 0.22 m/s, 0.76 m/s and 0.94 m/s respectively. From the figures, the moisture contents of the grains decrease with time for both experimental drying and developed model. Fig. 5 shows that the moisture contents of the grains during the experimental drying and using the developed model at 0.22 m/s were close except during drying of soyabeans. This was as a result of prevailing high solar insolation during this drying period. The plots also showed little deviation between the moisture content obtained from experimental drying and the model at the beginning of the drying. However, the deviation tends to increase toward the end of the drying period. Also, increase in deviation of moisture content between experimental and model was observed as the airflow rate increase from 0.22 m/s to 0.94 m/s. However, this can be normalized by incorporating the pattern of moisture removal as equilibrium drying is approached into the developed model equation.

3.2 Effect of Airflow Rates on the Drying Rate of Grains using the Dryer and Model

The drying rates of the five selected grains with time using the solar dryer at airflow rates of 0.22 m/s, 0.76 m/s and 0.94 m/s are shown in Fig. 8.
The plots showed that there was an initial short increase in the drying rate at the beginning of the experimental drying test followed by a long falling rate period that continued until equilibrium is attained. The Figure also showed that the drying rates of the different grains increased as the airflow rate increases. Statistical Analysis of Variance of means and differences table (Table 1) showed that there was no significant effect of airflow rate on drying rate of cowpea, groundnut and maize but there were significance difference ($P<0.001$) when drying soyabean and sorghum at the three different airflow rates. This means that the rate of drying depends on the type of grains being dried. The differences in drying rates which was observed during drying of

![Figure 5](image_url)

**Fig. 5.** Experimental and predicted moisture contents of grains at 0.22 m/s
soyabean and sorghum could be due to their smaller surface areas of these grains compared to the other selected grains as well as the structural and molecular compositions of these grains in relation to others. Thus, the effect of increase the airflow rate is readily seen in smaller grains like sorghum than larger and starchier grains like maize.
Fig. 7. Experimental and predicted moisture contents of grains at 0.75 m/s
Table 1. Effect of method and airflow rate used on the drying rate of selected grains

| Source           | DoF | Type III SS | Mean Square | F-Value | Pr>F  |
|------------------|-----|-------------|-------------|---------|-------|
| Cowpea Airflow   | 2   | 3.20751905  | 1.60375952  | 24.12   | <0.0001 |
| Method           | 1   | 0.02542937  | 0.02542937  | 0.38    | 0.5375 |
| Method*Airflow   | 2   | 0.35934444  | 0.17967222  | 2.70    | 0.0712 |
| Soyabean Airflow | 2   | 0.90705397  | 0.45352698  | 4.35    | 0.0150 |
| Method           | 1   | 0.00086429  | 0.00086429  | 0.01    | 0.9276 |
| Method*Airflow   | 2   | 3.44053333  | 1.72026667  | 16.49   | <0.0001 |
| Groundnut Airflow| 2   | 2.92821111  | 1.46410556  | 9.06    | 0.0002 |
| Method           | 1   | 1.79286429  | 1.79286429  | 11.10   | 0.0011 |
| Method*Airflow   | 2   | 0.15901429  | 0.07950714  | 0.49    | 0.6125 |
| Maize Airflow    | 2   | 3.12868730  | 1.56434365  | 14.18   | <0.0001 |
| Method           | 1   | 0.63431429  | 0.63431429  | 5.75    | 0.0180 |
| Method*Airflow   | 2   | 0.21706190  | 0.10853095  | 0.98    | 0.3769 |
| Sorghum Airflow  | 2   | 8.00949048  | 4.00474524  | 53.24   | <0.0001 |
| Method           | 1   | 0.16866746  | 0.16866746  | 2.24    | 0.1369 |
| Method*Airflow   | 2   | 0.13574444  | 0.06787222  | 0.90    | 0.4084 |

![Graph of Drying Time vs. Airflow Rate](a)

![Graph of Drying Rate vs. Airflow Rate](b)

![Graph of Drying Rate vs. Time](c)
Fig. 8. Variation of drying rate with drying time using the dryer at different airflow rates for (a) cowpea (b) soyabean (c) groundnut (d) maize (e) sorghum
Similarly, Fig. 9 showed the drying rate using the developed mathematical model for the selected grains at airflow rates of 0.22 m/s, 0.76 m/s and 0.94 m/s. The plot showed that there was an initial constant drying rate period followed by a falling rate period which tends to increased slightly toward the end of the drying period. The deviations of these plots from those obtained during experimental solar drying was due to the sudden change in atmospheric condition as time change from day to night which may have not be adequate accommodated in the model. But generally, there is no significant difference effect of airflow rate on the drying rate using the model.

3.3 Validation of the Model

The developed model was validated by comparing data obtained from the model with data recorded during the experimental drying of the selected grains by linear regression analysis using SAS Statistical software. But then, visual comparison between these plots and the plots of moisture content and drying rate with time during the experimental drying test shows that the two methods are close in drying pattern. These drying patterns follow similar established drying curves in literatures [10,11,12]. Although, at the beginning of the drying experiment the rate of drying using the dryer was faster as predicted by the model but toward the end of the drying period, moisture reduction was faster as predicted by the model. The statistical Analysis of Variance of means and differences table (Table 1) shows that there was no significant difference in the drying rates using the dryer and the prediction of model for cowpea, groundnut and maize. However, there was significant difference during drying of soyabean and sorghum using dryer and the prediction of the model. The reason for this might be due to the small size of these two types of grain. Both grains have smaller surface area compared to other selected grains. Thus, the combined effect of their relative sizes and the constant fluctuation in the climatic condition may likely be responsible for the difference in the drying rates using the dryer and the model. The result of the statistical Analysis of Variance of means and difference conducted on the moisture content of grains at 0.22 m/s, 0.76 m/s and 0.94 m/s airflow rates revealed that there was no significant effect \( (P \leq 0.005) \) between the prediction of the model and drying with the developed dryer for all the selected grains. Similarly, there was no interaction of the main factors of airflow rate and method on the moisture content. Although, significant effect of the air velocity on the moisture content of cowpea and sorghum was observed but the effect only comes up when the two methods were considered at once. It follows thus, that there is no significance difference between using the dryer and model. And hence, the model is appropriate to be applied for solar drying of grains and pulses.
3.4 Plots of Best Fit

The model was further validated by the plots of regression analysis carried out on the measured (dryer) and predicted (model). Fig. 7, Fig. 8, and Fig. 9 are the goodness of fit plots between measured and predicted values when drying cowpea, soyabean, groundnut, maize, and sorghum at 0.22 m/s airflow rate.
soyabean, groundnut, maize and sorghum at airflow rates 0.22 m/s, 0.76 m/s and 0.94 m/s respectively. The fitted curves for prediction of the moisture content has good values compared with the experimental ones and hence further show that the model can adequately predict the drying rate in the solar dryer.

Fig. 11. Goodness of fit curve for drying (a) cowpea (b) soyabean (c) groundnut (d) maize (e) sorghum at 0.76 m/s airflow rate
Fig. 12. Goodness of fit curve for drying (a) cowpea (b) soyabean (c) groundnut (d) maize (e) sorghum at 0.94 m/s airflow rate

4. CONCLUSION

Though, little variance was noted between the drying plots of the measured (dryer) and predicted (model) due to constant fluctuation in the solar radiation and the smaller sized of soyabean and sorghum grains, nevertheless, the developed model has proven to be suitable for predicting the moisture content and drying rate during solar drying of grains, especially larger sized. However, the model can be improved upon to accommodate the constant fluctuation in weather conditions, drying when approaching equilibrium and to handle smaller sized grains.
COMPETING INTERESTS

Authors have declared that no competing interests exist.

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