Baryon magnetic moments and sigma terms in lattice-regularized chiral perturbation theory

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Abstract

An SU(3) chiral Lagrangian for the lightest decuplet of baryons is constructed on a discrete lattice of spacetime points, and is added to an existing lattice Lagrangian for the lightest octets of mesons and baryons. A nonzero lattice spacing renders all loop integrations finite, and the continuum limit of any physical observable is identical to the result obtained from dimensional regularization. Chiral symmetry and gauge invariance are preserved even at nonzero lattice spacing. Specific calculations discussed here include the non-renormalization of a conserved vector current, the magnetic moments of octet baryons, and the $\pi N$ and $KN$ sigma terms that relate to the nucleon’s strangeness content. The quantitative difference between physics at a nonzero lattice spacing and physics in the continuum limit is easily computed, and it represents an expectation for the size of discretization errors in corresponding lattice QCD simulations.
I. INTRODUCTION AND DISCUSSION

Chiral perturbation theory (ChPT)\cite{1, 2, 3} is a low momentum effective field theory for QCD written as an expansion in small momenta and quark masses, and it has become an invaluable tool for subatomic physics. With only the lightest octets of pseudoscalar mesons and spin-1/2 baryons, ChPT is order-by-order renormalizable and physical results are independent of whichever regularization prescription is chosen.

The addition of the lightest decuplet of spin-3/2 baryons introduces a new physical scale, the mass difference between decuplet and octet baryons, which does not vanish in the chiral limit (i.e. when quark masses vanish). However, this mass difference is similar in size to the pseudoscalar meson masses, so the strict chiral expansion can be generalized to a “small scale expansion” where power counting is now meaningful even if the decuplet baryons are present.\cite{2, 4}

Although physical results are independent of regularization scheme, sometimes one scheme is preferable over another such as when issues other than experimentally-observable quantities are of interest. For example, the convergence of the ChPT expression for an observable is often evaluated by comparing the relative sizes of the contributions that occur at successive orders in the expansion, even though these individual contributions are not physically observable. The work of Refs. \cite{5, 6} discusses this in some detail, and goes one step further by associating the renormalization scale with the physical scale of baryon substructure.

For another example of important non-observables, consider lattice QCD. Numerical simulations must be performed at nonzero lattice spacing, and computed results therefore differ from the desired continuum values. One might expect that the effective theory for a discretization of QCD is a discretization of ChPT, i.e. the most general effective theory, written in terms of hadronic degrees of freedom, that respects chiral symmetry and the other symmetries of discretized QCD and that exists in the same discretized spacetime. Our previous work, Ref. \cite{7}, provides one particular discretization of ChPT along with explicit calculations of differences between results in this theory and results in the continuum.

The present work is mainly intended to exemplify the technique of lattice regularization (with explicit decuplet fields), and to note some of its features. Within a chosen lattice ChPT, it is a simple matter to rigorously determine extrapolations in both quark mass and
lattice spacing. Although less familiar than dimensional regularization, lattice regularization has the advantage that loops integrals can easily be performed numerically, since it is a 4-dimensional theory. The size of lattice spacing artifacts depends on the particular observable that is studied; at lattice spacings typical of lattice QCD simulations, we find that our lattice ChPT Lagrangian leads to noticeable discretization effects for the magnetic moments of octet baryons but negligible discretization effects for the sigma terms.

It is important to interpret our numerical results appropriately. If one is interested in using a lattice chiral Lagrangian to rigorously determine the lattice spacing effects that are present in a specific lattice QCD simulation, then it will be important to use the chiral Lagrangian that properly corresponds to whatever lattice QCD action was employed. Rupak and Shoresh\cite{8} provide a list of five $O(a)$ terms that can appear in the meson chiral Lagrangian appropriate for a Wilson-type lattice QCD action. ($a$ denotes the lattice spacing.) Each of these terms is multiplied by its own parameter which depends on the particular lattice QCD action of interest. For example, all five of the coefficients can be made to vanish by using an “improved” lattice QCD action. A similar study could be performed for the baryon Lagrangian. Our present work does not refer to any specific lattice QCD data, and this leaves the parameters at $O(a)$ and beyond undetermined. These parameters get specified implicitly by our choice of a minimal Lagrangian where the only lattice spacing effects are contained within simple covariant derivatives and field representations. The numerical results for magnetic moments and sigma terms presented in this work are therefore simply examples of lattice spacing effects that arise from a Lagrangian which has not been specially improved in any way. The main point of our present work is to demonstrate the use of lattice regularization for chiral Lagrangian calculations, but we wish to emphasize that this regularization technique can certainly be applied to extensions of our minimal Lagrangian if one wishes to fix the lattice spacing parameters to a particular lattice QCD action.

In Sec. II of the present work, the Lagrangian of Ref. \cite{7} is extended to include the decuplet of spin-3/2 baryons. Section \textbf{III} contains a discussion of the electromagnetic vertex: the non-renormalization of this vertex at vanishing momentum transfer is shown analytically, and the octet baryon magnetic moments are calculated as a function of lattice spacing. In Sec. \textbf{IV} the $KN$ sigma terms of the nucleon’s scalar vertex are determined as a function of lattice spacing, with the necessary chiral counterterms determined from known baryon masses and the known $\pi N$ sigma term. The running of the $\pi N$ sigma term from $q^2 = 0$ to
the Cheng-Dashen point is also determined as a function of lattice spacing.

II. A LATTICE CHIRAL LAGRANGIAN

The standard SU(3) chiral Lagrangian containing pseudoscalar mesons (M), spin-1/2 baryons (B) and spin-3/2 baryons (T) is

\[ \mathcal{L} = \mathcal{L}_M + \mathcal{L}_{MB} + \mathcal{L}_{MT} + \mathcal{L}_{MBT}. \]  

To expand in powers of external momenta, of meson masses and of the T-B mass difference (relative to the larger scales: baryon masses and the chiral symmetry breaking scale \( \Lambda_{\chi} \sim 4\pi F_\pi \)), it is standard practice to write the Lagrangian in terms of heavy baryon fields, \( B_v(x) \) and \( T_{\nu\mu}(x) \), instead of the relativistic fields, \( B(x) \) and \( T_\mu(x) \). The transformation for the spin-1/2 field is

\[ B_v(x) = \exp(i m_{HB} v \cdot x) \frac{1}{2} (1 + \frac{v}{\epsilon}) B(x), \]  

with \( m_{HB} \) chosen near the average octet baryon mass. A similar transformation is used to define the decuplet field. The expanded Lagrangian becomes

\[ \mathcal{L} = \mathcal{L}^{(2)}_M + \mathcal{L}^{(0)}_{MB} + \mathcal{L}^{(1)}_{MB} + \mathcal{L}^{(2)}_{MB} + \mathcal{L}^{(3)}_{MT} + \mathcal{L}^{(1)}_{MBT} + \ldots \]  

where the omitted terms only contribute to octet baryon properties beyond third order. In Euclidean spacetime,

\[ \mathcal{L}^{(2)}_M = \frac{F^2}{4} \text{Tr} \left( \sum_{\mu} \nabla_\mu U \nabla_\mu U - \chi U - \chi U \right), \]  

\[ \mathcal{L}^{(0)}_{MB} = (m_0 - m_{HB}) \text{Tr} \left( \bar{B}_v B_v \right), \]  

\[ \mathcal{L}^{(1)}_{MB} = \sum_{\mu} \left[ \text{Tr} \left( \bar{B}_v v_\mu D_\mu B_v \right) + \mathcal{D} \text{Tr} \left( \bar{B}_v S_\mu [u_\mu, B_v] \right) + \mathcal{F} \text{Tr} \left( \bar{B}_v S_\mu [u_\mu, B_v] \right) \right], \]  

\[ \mathcal{L}^{(2)}_{MB} = \frac{1}{2m_0} \text{Tr} \left( \bar{B}_v (v \cdot Dv \cdot D - D^2) B_v \right) - b_D \text{Tr} \left( \bar{B}_v \chi_+ B_v \right) - b_F \text{Tr} \left( \bar{B}_v \chi_+ B_v \right) \]  

\[ - b_0 \text{Tr} \left( \bar{B}_v B_v \right) \text{Tr} (\chi_+) + \frac{i\mu_D}{4m_0} \sum_{\mu,\nu} \text{Tr} \left( \bar{B}_v [S_\mu, S_\nu] \{\xi F^L_{\mu\nu} \xi + \xi^\dagger F^R_{\mu\nu} \xi, B_v \} \right) \]  

\[ + \frac{i\mu_F}{4m_0} \sum_{\mu,\nu} \text{Tr} \left( \bar{B}_v [S_\mu, S_\nu] \{\xi F^L_{\mu\nu} \xi + \xi^\dagger F^R_{\mu\nu} \xi, B_v \} \right) + \ldots, \]  

\[ \mathcal{L}^{(3)}_{MB} = \ldots, \]  

\[ \mathcal{L}^{(1)}_{MT} = - \sum_{\mu} \sum_{i} \left( \bar{T}^{ijkl}_{\nu\mu} v_\mu D_\nu T^{ijkl}_{\nu\mu} - H \sum_{l} \bar{T}^{ijkl}_{\nu\mu} S_\nu [u_\nu T^{ijkl}_{\nu\mu}] - \Delta \sum_{\mu} \sum_{i} \bar{T}^{ijkl}_{\nu\mu} T^{ijkl}_{\nu\mu} \right), \]  

\[ \mathcal{L}^{(1)}_{MBT} = \frac{g}{2} \sum_{\mu} \sum_{ijklm} \epsilon_{ijkl} \left( \bar{T}^{ijkl}_{\nu\mu} u_\mu B_v^{ij} + \bar{B}_v^{il} u_\mu T^{klm}_{\nu\mu} \right), \]
where $S_\mu = \frac{i}{2} \gamma_5 \sum_{\nu} \sigma_{\mu\nu} v_\nu$ is the Pauli-Lubanski spin vector, $\Delta$ is the decuplet-octet mass difference, and we choose $v_\mu = (0, 0, 0, 1)$ so that the covariant derivatives of Ref. [4] can be retained. The field strength tensors $F^{L}_{\mu\nu}(x)$ and $F^{R}_{\mu\nu}(x)$ correspond to external spin-1 fields. The superscripted indices in Eqs. (9) and (10) are flavour indices:

$$
\begin{pmatrix}
\pi^{11} & \pi^{12} & \pi^{13} \\
\pi^{21} & \pi^{22} & \pi^{23} \\
\pi^{31} & \pi^{32} & \pi^{33}
\end{pmatrix}
= 
\begin{pmatrix}
\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\
\pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\
K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta
\end{pmatrix},
$$

(11)

$$
\begin{pmatrix}
B_{v1} & B_{v2} & B_{v3} \\
B_{v1} & B_{v2} & B_{v3} \\
B_{v1} & B_{v2} & B_{v3}
\end{pmatrix}
= 
\begin{pmatrix}
\frac{1}{\sqrt{2}} \Sigma^0_v + \frac{1}{\sqrt{6}} \Lambda_v & \Sigma^+_v & \Sigma^0_v \\
\Sigma^-_v & -\frac{1}{\sqrt{2}} \Sigma^0_v + \frac{1}{\sqrt{6}} \Lambda_v & n_v \\
\Xi^-_v & \Xi^0_v & -\frac{2}{\sqrt{6}} \Lambda_v
\end{pmatrix},
$$

(12)

$$
\begin{pmatrix}
T_{v\mu}^{111} & T_{v\mu}^{112} & T_{v\mu}^{122} & T_{v\mu}^{222} \\
T_{v\mu}^{113} & T_{v\mu}^{123} & T_{v\mu}^{123} & T_{v\mu}^{233} \\
T_{v\mu}^{133} & T_{v\mu}^{233} & T_{v\mu}^{333}
\end{pmatrix}
= 
\begin{pmatrix}
\Delta^{++}_{v\mu} & \frac{1}{\sqrt{3}} \Delta^+_{v\mu} & \frac{1}{\sqrt{3}} \Delta^0_{v\mu} & \Delta^-_{v\mu} \\
\frac{1}{\sqrt{3}} \Sigma^+_{v\mu} & \frac{1}{\sqrt{3}} \Sigma^0_{v\mu} & \frac{1}{\sqrt{3}} \Sigma^{--}_{v\mu} & \frac{1}{\sqrt{3}} \Xi^0_{v\mu} \\
\frac{1}{\sqrt{3}} \Xi^0_{v\mu} & \frac{1}{\sqrt{3}} \Xi^0_{v\mu} & \frac{1}{\sqrt{3}} \Xi^{--}_{v\mu},
\end{pmatrix}
$$

(13)

where $T_{v\mu}^{ijk}$ is understood to be completely symmetric in $i, j, k$. In the Lagrangian, the pseudoscalar mesons are represented nonlinearly,

$$
U(x) = \xi^2(x) = \exp(-i\lambda^a \pi^a(x)/F),
$$

(14)

where $\lambda^a$ is a Gell-Mann matrix, and the current quark mass matrix, $\mathcal{M}$, enters via

$$
\chi = 2B\mathcal{M},
$$

(15)

$$
\chi^+ = \xi^\dagger \xi^\dagger + \xi \chi^\dagger \xi.
$$

(16)

So far the definitions of this section have been general, and have not assumed a spacetime lattice at all. We now consider the derivative structures in the Lagrangian, and it is here that the expressions become lattice dependent. As in Ref. [7], we choose an isotropic hypercubic lattice. Nearest neighbour sites are separated by a distance $a$, and $a_\mu$ will be used to denote a vector of length $a$ in the positive $\mu$ direction.

First, we recall the non-decuplet definitions from Ref. [4]. Denoting the external spin-1 fields corresponding to $F^{L}_{\mu\nu}(x)$ and $F^{R}_{\mu\nu}(x)$ by $L_{\mu}(x)$ and $R_{\mu}(x)$ respectively, the derivatives that appear in the lowest-order meson Lagrangian, Eq. (14), are taken to be

$$
\nabla_{\mu}^{(+)} U(x) = \frac{1}{a} \left[ R_{\mu}(x) U(x + a_\mu) L_{\mu}(x) - U(x) \right],
$$

(17)
and the meson derivative needed for the baryon Lagrangian is

\[ u_\mu(x) = \frac{i}{2} \xi^\dagger(x) \nabla^{(\pm)}_{\mu} U(x) \xi(x) - \frac{i}{2} \xi(x) \nabla_{\mu}^{(\pm)} U^\dagger(x) \xi(x), \]  

(18)

where

\[ \nabla_{\mu}^{(\pm)} U(x) = \frac{1}{2a} \left[ R_\mu U(x + a_\mu) L_{\mu}^\dagger(x) - R_{\mu}^\dagger U(x - a_\mu) L_{\mu}(x - a_\mu) \right]. \]  

(19)

The covariant derivative for the octet baryon field is

\[ aD_4 B_\nu(x) = B_\nu(x) - \sum_{X,Y = L,R} \Gamma^X_4(x - a_4) B_\nu(x - a_4) \Gamma^Y_4(x - a_4), \]  

(20)

\[ aD_j B_\nu(x) = \frac{1}{2} \sum_{X,Y = L,R} \left[ \Gamma^X_4(x) B_\nu(x + a_j) \Gamma^Y_4(x - a_j) - \Gamma^X_4(x - a_j) B_\nu(x - a_j) \Gamma^Y_4(x - a_j) \right], \]  

(21)

in the temporal and spatial directions respectively, where

\[ 2\Gamma^X_{\mu}(x) = \begin{cases} 
\xi(x) L_{\mu}(x) \xi^\dagger(x + a_\mu), & \text{for } X = L, \\
\xi^\dagger(x) R_{\mu}(x) \xi(x + a_\mu), & \text{for } X = R. 
\end{cases} \]  

(22)

Not defined explicitly in Ref. [7] but required for the present work are the field strength tensors, which we discretize as follows so that the chiral transformation properties and the lattice symmetries are respected:

\[ 4ia^2 F^X_{\mu\nu}(x) = 4 - X_{\mu}(x) X_{\nu}(x + a_\mu) X^\dagger_{\nu}(x + a_\mu) X^\dagger_{\nu}(x) - X_{\nu}(x) X^\dagger_{\mu}(x - a_\mu + a_\nu) X^\dagger_{\mu}(x - a_\mu) X_{\mu}(x - a_\mu) - X_{\mu}(x - a_\mu) X^\dagger_{\mu}(x - a_\mu - a_\nu) X^\dagger_{\nu}(x - a_\mu - a_\nu) X_{\nu}(x - a_\mu - a_\nu) - X^\dagger_{\mu}(x - a_\mu) X_{\mu}(x - a_\mu - a_\nu) X^\dagger_{\nu}(x + a_\mu - a_\nu) X^\dagger_{\nu}(x), \]  

(23)

for \( X = L \) and \( R \). Finally, consider the covariant derivative of the decuplet field. Since we have chosen \( v_\mu = (0, 0, 0, 1) \), only the temporal derivative is needed and we define it to be

\[ aD_4 T^{ijk}_{\nu\mu}(x) = T^{ijk}_{\nu\mu}(x) - \sum_l \left[ \Gamma^L_{4i\nu}(x - a_4) + \Gamma^R_{4i\nu}(x - a_4) \right] T^{ijk}_{\nu\mu}(x - a_4) \] 

\[ - \sum_l \left[ \Gamma^L_{4j\nu}(x - a_4) + \Gamma^R_{4j\nu}(x - a_4) \right] T^{ijk}_{\mu\nu}(x - a_4) \] 

\[ - \sum_l \left[ \Gamma^L_{4k\nu}(x - a_4) + \Gamma^R_{4k\nu}(x - a_4) \right] T^{ijk}_{\nu\mu}(x - a_4). \]  

(24)

The chiral invariance of this Lagrangian can readily be verified by adding the chiral transformation of the decuplet field, \( T^{ijk}_{\nu\mu} \to o_{iL}o_{jM}o_{kN} T^{rLm}_{\nu\mu} \), to the set of chiral transformations given in Ref. [4].
When performing calculations, it is important to remember that the decuplet propagator must always be accompanied by the projector which eliminates spuriously spin-1/2 components from the propagating field. For the present Lagangian, the product of propagator and projector is

$$-\frac{P_{\mu\nu}^{3/2}}{\Gamma_{TT}} = \frac{ia(\delta_{\mu\nu} - v_{\mu}v_{\nu} + \frac{4}{3}S_{\mu}S_{\nu})}{\sin(ap_{4}) - i[a\Delta + 2\sin^2(ap_{4}/2)]}.$$

(25)

III. GAUGE INVARIANCE AND OCTET BARYON MAGNETIC MOMENTS

To calculate the electromagnetic form factors of an octet baryon, one simply identifies the photon field, \(A_\mu(x)\), with the external spin-1 fields of Sec. II, <small>where \(Q = \text{diag}(2/3, -1/3, -1/3)\).</small> From Eqs. (3-10), it is easy to see that the leading order contribution to an octet baryon’s electromagnetic vertex is contained within \(\mathcal{L}_{MB}^{(1)}\) and the next-to-leading order contribution within \(\mathcal{L}_{MB}^{(2)}\). The resulting matrix element is

\[
\langle B(p') | J_{\mu}^{em} | B(p) \rangle = ieB_{\nu}(k') \left[ v_{\mu}Q_{B} \left( 1 - i \sin(ak_{4}') - 2 \sin^2 \left( \frac{ak_{4}'}{2} \right) \right) \right] + \frac{i\mu_{B}^{LO}}{m_{0}} \sum_{\nu} [S_{\mu}, S_{\nu}] \sin(aq_{\nu}) \left( 1 - \frac{i}{2} \sin(aq_{\mu}) - \sin^2 \left( \frac{aq_{\mu}}{2} \right) \right) B_{\nu}(k)\right]
\]

(27)

where \(q = p' - p\) is the momentum transfer, \(Q_{B}\) denotes the electric charge of the baryon, and \(\mu_{B}^{LO}\) are the leading order expressions for the magnetic moments:

\[
\mu_{p}^{LO} = \frac{\mu_{D}}{3} + \mu_{F}, \quad \mu_{\Sigma^{-}}^{LO} = \frac{\mu_{D}}{3} - \mu_{F}, \quad \mu_{\Lambda^{0}}^{LO} = -\mu_{\Lambda}^{LO} = \frac{\mu_{D}}{3}, \quad \mu_{n}^{LO} = \mu_{\Xi^{-}}^{LO} = -\frac{2\mu_{D}}{3}, \quad \mu_{\Lambda^{0}}^{LO} = \mu_{\Xi^{0}}^{LO} = \frac{\mu_{D}}{\sqrt{3}}.
\]

(28-32)

Notice from Eq. (27) that the term containing \(Q_{B}\) is purely temporal and the \(\mu_{B}^{LO}\) term is purely spatial with our chosen frame, since \(v_{\mu} = (0, 0, 0, 1) \Rightarrow S \cdot v = 0\).

Returning to the full Lagrangian of Eq. (3), one finds the corrections to the matrix element of Eq. (27) that are shown diagrammatically in Fig. 1. As will now be shown, evaluation of
FIG. 1: One-loop contributions to an octet baryon’s electromagnetic vertex. Dashed, solid, double and wavy lines represent mesons, octet baryons, decuplet baryons and photons respectively. $\delta Z$ denotes the contribution to wave function renormalization that arises from the diagrams in Fig. 3. these diagrams leads to the renormalization of $\mu_B^{LO}$, and to the non-renormalization of $Q_B$ as required by gauge invariance.

A. The non-renormalization of electric charge

To verify the non-renormalization of $Q_B$, it is sufficient to work at vanishing momentum transfer. Also, the momentum of each external baryon is simply $(0, 0, 0, m_B)$ plus corrections which are of negligibly high order in the chiral expansion. For definiteness we will discuss the proton’s charge in this subsection; the extension to other octet baryons is straightforward.

In this limit, the contribution of the diagram in Fig. 1(a) to the matrix element in Eq. (27)
FIG. 2: One-loop contributions to the wave function renormalization of an octet baryon. Dashed, solid and double lines represent mesons, octet baryons and decuplet baryons respectively.

with a $\pi^\pm$ in the loop is

$$
\mathcal{M}_{\pi^\pm}^{(a)} = \int_{-\pi/a}^{\pi/a} \frac{d^4q}{(2\pi)^4} \left( \frac{-ie\nu_{\mu}}{2F^2} \right) \left( \frac{a^2}{2 \sum_{\lambda} (1 - \cos(aq_{\lambda})) + a^2x_{\pi}^2} \right)
$$

$$
= \frac{-i a^2 e\nu_{\mu}}{4F^2} \int_0^\infty dz \int_{-\pi/a}^{\pi/a} \frac{d^4q}{(2\pi)^4} \exp \left[ -z \left( 4 + \frac{a^2x_{\pi}^2}{2} - \sum_{\lambda} \cos(aq_{\lambda}) \right) \right]
$$

$$
= \frac{-ie\nu_{\mu}}{4a^2F^2} W_4(a^2x_{\pi}^2),
$$

(33)

where $x_M$ is related to the meson mass $^6$

$$
m_M = \frac{2}{a} \arcsinh \left( \frac{ax_M}{2} \right),
$$

(34)

and $W_4(\epsilon^2)$ is the integral transform of the fourth power of a Bessel function $^7$,

$$
W_n(\epsilon^2) \equiv \int_0^\infty dx I_0^n(x) \exp \left[ -x \left( n + \frac{\epsilon^2}{2} \right) \right].
$$

(35)

Summing over all internal mesons gives

$$
\mathcal{M}^{(a)} \equiv \sum_M \mathcal{M}_M^{(a)} = \frac{-ie\nu_{\mu}}{4a^2F^2} \left[ \frac{7}{12} + \left( 1 - \frac{a^2x_{\pi}^2}{16} \right) W_4(a^2x_{\pi}^2) + 2 \left( 1 - \frac{a^2x_K^2}{16} \right) W_4(a^2x_K^2) \right.
$$

$$
- \frac{5a^2x_{\eta}^2}{48} W_4(a^2x_{\eta}^2) \right].
$$

(36)

Notice that this contribution is quadratically divergent as $a \to 0$. Dimensional regularization does not show power divergences, and Fig. 2(a) vanishes exactly in that scheme.

Figure 2(b) is nonzero only if the loop meson is $\pi^\pm$ or $K^\pm$. For the pion loop, the lattice regularized expression is

$$
\mathcal{M}_{\pi^\pm}^{(b)} = \frac{iea^2\nu_{\mu}}{4F^2} \int_{-\pi/a}^{\pi/a} \frac{d^4q}{(2\pi)^4} \sin^2(aq_{\lambda}) \left[ 4 + a^2x_{\pi}^2/2 - \sum_{\lambda} \cos(aq_{\lambda}) \right]^2
$$

$$
= \frac{ie\nu_{\mu}}{4a^2F^2} \int_0^\infty dzI_0^3(z)I_0'(z) \exp \left[ -z \left( 4 + \frac{a^2x_{\pi}^2}{2} \right) \right]
$$

$$
= \frac{-ie\nu_{\mu}}{4a^2F^2} \left[ \frac{1}{4} - \left( \frac{a^2x_{\pi}^2}{8} \right) W_4(a^2x_{\pi}^2) \right].
$$

(37)
Repeating this calculation for the kaon loop leads to
\[ \mathcal{M}^{(b)} = \sum_{M} \mathcal{M}_{M}^{(b)} = \frac{-ie v_{\mu}}{4a^{2}F^{2}} \left[ \frac{3}{4} - \left( 1 + \frac{a^{2}x_{k}^{2}}{8} \right) W_{4}(a^{2}x_{k}^{2}) - 2 \left( 1 + \frac{a^{2}x_{K}^{2}}{8} \right) W_{4}(a^{2}x_{K}^{2}) \right]. \] (38)

The one-loop correction shown in Fig. 2(a) evaluates as follows:
\[ \delta Z^{(a)} = 1 + \frac{1}{a} \frac{1}{\delta Z^{(a)}}. \] (40)

where the external proton momentum is \( m_0 v + k \) and
\[ \delta Z^{(a)} = \frac{1}{3a^{2}F^{2}} \left[ 1 - \frac{9}{64}a^{2}x_{k}^{2}W_{4}(a^{2}x_{k}^{2}) - \frac{9}{32}a^{2}x_{K}^{2}W_{4}(a^{2}x_{K}^{2}) - \frac{5}{64}a^{2}x_{\eta}^{2}W_{4}(a^{2}x_{\eta}^{2}) \right]. \] (41)

This is precisely what was required to facilitate the expected non-renormalization, since Eqs. (33), (38) and (41) give
\[ \mathcal{M}^{(a)} + \mathcal{M}^{(b)} + ie v_{\mu} \delta Z^{(a)} = 0. \] (42)

Turning to the contributions that depend on \( \mathcal{D} \) and \( \mathcal{F} \), one finds
\[ \mathcal{M}_{\pi^{0}}^{(c)} = \frac{1}{2} (\mathcal{D} + \mathcal{F})^{2} G_{0}(a^{2}x_{\pi}^{2}), \] (43)
\[ \mathcal{M}_{K^{0}}^{(c)} = (\mathcal{D} - \mathcal{F})^{2} G_{0}(a^{2}x_{K}^{2}), \] (44)
\[ \mathcal{M}_{\eta}^{(c)} = \frac{1}{6} (\mathcal{D} - 3\mathcal{F})^{2} G_{0}(a^{2}x_{\eta}^{2}), \] (45)

where
\[ G_{M}(\epsilon^{2}) = \frac{ie a^{2}v_{\mu}}{4F^{2}} \int_{\pi/a}^{\pi/a} \frac{d^{4}q}{(2\pi)^{4}} \frac{[\cos(aq_{4}) - i \sin(aq_{4})] \sum_{k=1}^{3} \sin^{2}(aq_{k})}{[4 + \epsilon^{2}/2 - \sum_{\lambda} \cos(aq_{\lambda})][\sin(aq_{4}) - iaM - 2i \sin^{2}(aq_{4}/2)]^{2}}. \] (46)

The contribution of Fig. 3(d) with a charged pion in the loop is
\[ \mathcal{M}_{\pi^{+}}^{(d)} = \frac{ie a^{2}v_{\mu}}{2F^{2}} \mathcal{D}^{2} \left[ \int_{\pi/a}^{\pi/a} \frac{d^{4}q}{(2\pi)^{4}} v_{\mu} \sin(aq_{4}) (\rho_{\lambda} \cos(aq_{\lambda}) \sum_{\lambda} \cos(aq_{\lambda}) \delta Z^{(a)}(a^{2}x_{\pi}^{2})) \right] \]
\[ = \frac{ie a^{2}v_{\mu}}{2F^{2}} \mathcal{D}^{2} \left[ \int_{-\pi/a}^{\pi/a} \frac{d^{4}q}{(2\pi)^{4}} \frac{\sin(aq_{4}) \sum_{k=1}^{3} \sin^{2}(aq_{k})}{[4 + a^{2}x_{\pi}^{2}/2 - \sum_{\lambda} \cos(aq_{\lambda})]} \right] \]
\[ = (\mathcal{D} + \mathcal{F})^{2} G_{0}(a^{2}x_{\pi}^{2}), \] (47)
where we have used $S_4 = 0$ and $S_1^2 = S_2^2 = S_3^2 = -1/4$ as well as integration by parts.

Similarly,

$$\mathcal{M}_{K^+}^{(d)} = \frac{2}{3}(D^2 + 3F^2)G_0(a^2x_K^2). \quad (48)$$

Next, consider the wave function renormalization of Fig. 2(b). For a $\pi^0$ loop, the contribution to the two-point function is

$$\delta \Gamma^{(b)}_{pp} = -\frac{ia}{2F^2(D + F)^2} \int_{\min(\pi/a, \pi/a - k)}^{\min(\pi/a, \pi/a + k)} \frac{d^4q}{(2\pi)^4} \times \frac{\sum_\rho S_\rho \sin(\alpha q_\rho) \sum_\sigma S_\sigma \sin(\alpha q_\sigma)}{\sin(\alpha q_4 + \alpha k_4) - 2i \sin^2(\alpha (q_4 + k_4)/2)[4 + a^2x_\pi^2/2 - \sum_\lambda \cos(\alpha q_\lambda)]} \equiv \frac{1}{\delta Z^{(b)}_{\pi^0}} \left[\frac{\sin(\alpha k_4) - 2i \sin^2(\alpha k_4) - a\delta X}{ia}\right] \quad (49)$$

where the external baryon momentum is $m_0v + k$ and $\delta X$ produces a mass renormalization. The limits of integration have been chosen to ensure that the momentum of each internal propagator remains within the lattice’s Brillouin zone over the entire domain of $q$ integration. As it happens, the $k$-dependences in the limits of integration do not affect wave function renormalization, and the result is

$$ie\nu_\mu \delta Z^{(b)}_{\pi^0} = -\frac{1}{2}(D + F)^2G_0(a^2x_\pi^2), \quad (50)$$

which exactly cancels the $\pi^0$ loop from Eq. (43). In the same way, replacing the $\pi^0$ in $\delta Z$ by each of the other mesons serves to exactly cancel Eqs. (44), (45), (47) and (48).

Finally, the decuplet contributions of Figs. 1(e), 1(f) and 2(c) are found to cancel in essentially the same manner as did the octet contributions:

$$\mathcal{M}_{\pi^+}^{(c)} = \frac{4}{9}C^2G_\Delta(a^2x_\pi^2), \quad (51)$$

$$ie\delta Z_{\pi^+}^{(c)} = -\frac{8}{9}C^2G_\Delta(a^2x_\pi^2), \quad (52)$$

$$\mathcal{M}_{\pi^0}^{(c)} = -ie\delta Z_{\pi^0}^{(c)} = \frac{4}{9}C^2G_\Delta(a^2x_\pi^2), \quad (53)$$

$$\mathcal{M}_{K^0}^{(c)} = -ie\delta Z_{K^0}^{(c)} = \frac{2}{9}C^2G_\Delta(a^2x_K^2), \quad (54)$$

$$\mathcal{M}_{\pi^+}^{(f)} = -\frac{4}{9}C^2G_\Delta(a^2x_\pi^2), \quad (55)$$

$$\mathcal{M}_{K^+}^{(f)} = -ie\delta Z_{K^+}^{(f)} = \frac{1}{9}C^2G_\Delta(a^2x_K^2). \quad (56)$$

Thus, all corrections to the proton’s electric charge at first loop order in the chiral expansion sum to zero, and the non-renormalization of electric charge is confirmed.
B. The octet baryon magnetic moments

The one-loop corrections to octet baryon magnetic moments come from Figs. 1(d) and 1(f). These diagrams were discussed in the previous subsection, but only for vanishing momentum transfer which is not sufficient to obtain magnetic moments.

It is convenient to choose the Breit frame where the incoming baryon has momentum $m_0 v - q/2$ and the outgoing baryon has momentum $m_0 v + q/2$, and to choose the integration momentum to be the internal baryon’s momentum. This choice displays symmetries in the integrand that help to simplify the calculations. Notice that higher order corrections to the external baryon momenta have already been omitted, and that $m_0$ is equal to the physical mass in these loop diagrams since the difference is of higher chiral order.

The evaluation of Figs. 1(d) and 1(f) for arbitrary momentum transfer $q$ gives

$$M^{(d)} + M^{(f)} = -\frac{2e}{am_N} \sum_\nu [S_\mu, S_\nu] \sin \left( \frac{aq_\nu}{2} \right) \delta F_2(q),$$

where $\delta F_2$ is the loop correction to the Pauli form factor. The magnetic moments are

$$\mu_B = \mu_B^{LO} + \left[ \delta F_2^{\text{oct}}(0) \right]_B + \left[ \delta F_2^{\text{dec}}(0) \right]_B,$$

where the last two terms are from Figs. 1(d) and 1(f) respectively. Both of these terms rely on a single integral with $\mu \neq 4, \nu \neq 4$ and $\mu \neq \nu$, namely

$$H_M(\epsilon^2) = \frac{-ia^3 m_N}{2F^2} \int_{-\pi/a}^{\pi/a} \frac{d^4q}{(2\pi)^4} \frac{\sin^2(aq_\mu) \cos(aq_\nu)}{[4 + \epsilon^2/2 - \sum_\lambda \cos(aq_\lambda)]^2 \left[ \sin(aq_4) - iaM - 2i \sin^2(aq_4/2) \right]},$$

but with one key difference. For the decuplet diagram it is $H_\Delta$ that enters, and the mass splitting $\Delta$ ensures that the integrand has no singularities on the domain of integration. In that case, a convenient form for numerical evaluation is

$$H_\Delta(\epsilon^2) = \frac{a^3 m_N}{4F^2} \int_{-\pi/a}^{\pi/a} \frac{d^4q}{(2\pi)^4} \frac{\cos(aq_\mu) \cos(aq_\nu)(1 + a\Delta - \cos(aq_4))}{[4 + \epsilon^2/2 - \sum_\lambda \cos(aq_\lambda)] \left[ (1 + a\Delta)(1 - \cos(aq_4)) + a^2 \Delta^2/2 \right]}.$$

For the octet diagram there is a singularity, and we must integrate around it according to the usual “+i\epsilon” prescription for field theory. Some details of this procedure in the context of lattice regularization can be found in the appendices of Ref. [7], so here we simply provide our final expression:

$$H_0(\epsilon^2) = \frac{a^3 m_N}{4F^2} \left[ \int_{-\pi/a}^{\pi/a} \frac{d^4q}{(2\pi)^4} \frac{\cos(aq_\mu) \cos(aq_\nu)}{[4 + \epsilon^2/2 - \sum_\lambda \cos(aq_\lambda)]} \right].$$
This too is easily evaluated numerically.

Our final expression for the octet baryon magnetic moments is

$$\mu_B = c_1^B \mu_D + c_2^B \mu_F + c_3^B H_0(a^2 x_\pi^2) + c_4^B H_0(a^2 x_K^2) + c_5^B H_\Delta(a^2 x_\pi^2) + c_6^B H_\Delta(a^2 x_K^2),$$

where the coefficients are listed in Table I. $H_M$ diverges as $a \to 0$, but the offending terms can be absorbed into renormalized definitions of $\mu_D$ and $\mu_F$. The result is identical to that obtained from dimensional regularization. However lattice regularization also allows us to compute at nonzero $a$, and it is interesting to consider lattice spacings that are typical of lattice QCD simulations.

Using the experimental values of $\mu_p$ and $\mu_n$ to fix the parameters $\mu_D$ and $\mu_F$, the other seven magnetic moments become predictions of the theory, and are plotted as a function of the lattice cutoff $\pi/a$ in Fig. 3. The plot assumes standard experimental values for the coefficients appearing within loops: $D = 0.75$, $F = 0.50$, and $C = 1.5$. These values could be varied within experimental uncertainties, but such details do not significantly affect our present interest: the size of discretization effects.

As seen in Fig. 3, each magnetic moment smoothly approaches the corresponding dimensional regularized result at $\pi/a \to \infty$. These limiting values are given in Table II. The
FIG. 3: Octet baryon magnetic moments (in units of $\mu_N$) as functions of lattice spacing. Lagrangian parameters are fixed by requiring the proton and neutron magnetic moments to equal their experimental values at all lattice spacings.

TABLE II: A comparison of the magnetic moments at $\pi/a = 6.0$ GeV, corresponding to $a = 0.10$ fm, and their values in the continuum limit.

| $B$               | $\Sigma^+$ | $\Sigma^0$ | $\Sigma^-$ | $\Xi^0$ | $\Xi^-$ | $\Lambda$ | $\Lambda \Sigma^0$ |
|-------------------|------------|------------|------------|--------|--------|-----------|-------------------|
| $\mu_B(a = 0)$    | 1.64       | 0.12       | -1.40      | -0.14  | -0.98  | -0.12     | 1.11              |
| $\mu_B(a = 0.1 \text{fm}) / \mu_B(a = 0)$ | 1.11       | 1.94       | 0.97       | 2.58   | 1.01   | 1.95      | 1.05              |

agreement with experiment is not particularly impressive at this chiral order, as has been known for some time[3]. The situation is dramatically improved at next chiral order[10, 11] or by the methods of Refs. [5, 6]. At present, we focus on the discretization effects rather than a precise comparison to experiment.

For $\pi/a = 6$ GeV, Table II shows that the relative sizes of discretization effects vary from a few percent to a factor of 2 or more, depending on which magnetic moment is
chosen. The large variation is somewhat misleading: the absolute discretizations are quite comparable for all magnetic moments, as is evident from Fig. [3] However, even $O(10\% \to 30\%)$ discretization uncertainties are significant in this context, since chiral corrections are typically of this order too. If ChPT is being employed as a way to determine chiral effects in lattice QCD, then these discretization effects must be considered. Notice that $\pi/a = 6$ GeV corresponds to a lattice spacing of 0.1 fm, which is typical of modern lattice QCD simulations.

IV. THE $\pi N$ AND $KN$ SIGMA TERMS

Sigma terms are the scalar form factors of a baryon multiplied by the quark mass,

$$\sigma_{\pi N}(t) = \hat{m} \langle N(p') | \bar{u}u + \bar{d}d | N(p) \rangle,$$

$$\sigma_{KN}^{(1)}(t) = \frac{1}{2}(\hat{m} + m_s) \langle N(p') | \bar{u}u + \bar{s}s | N(p) \rangle,$$

$$\sigma_{KN}^{(2)}(t) = \frac{1}{2}(\hat{m} + m_s) \langle N(p') | -\bar{u}u + 2\bar{d}d + \bar{s}s | N(p) \rangle,$$

where $\hat{m} = (m_u + m_d)/2$. The sigma terms vanish in the chiral limit and are therefore useful in discussions of chiral symmetry and its breaking. Furthermore, they offer a probe of the nucleon’s strangeness content,

$$y \equiv \frac{2 \langle N(p) | \bar{s}s | N(p) \rangle}{\langle N(p) | \bar{u}u + \bar{d}d | N(p) \rangle} = \left( \frac{\hat{m}}{\hat{m} + m_s} \right) \left( \frac{3\sigma_{KN}^{(1)}(0) + \sigma_{KN}^{(2)}(0)}{\sigma_{\pi N}(0)} \right) - 1,$$

and this quantity continues to be of great interest to many researchers. (See Ref. [3] and references therein.)

The lattice-regularized Lagrangian leads to

$$\sigma_{\pi N} = -x_\pi^2 (2b_0 + b_D + b_F) + x_\pi^2 \sum_{j=1}^{3} \sum_\phi a_j^\phi K_j(\phi, \phi),$$

$$\sigma_{KN}^{(1)} = -2x_K^2 (b_0 + b_D) + x_K^2 \sum_{j=1}^{3} \sum_\phi b_j^\phi K_j(\phi, \phi) - \frac{x_K^2}{12}(D^2 - 2DF - 3F^2)K_1(\pi, \eta) + \frac{x_K^2}{6}K_2(\pi, \eta),$$

$$\sigma_{KN}^{(2)} = -2x_K^2 (b_0 - b_F) + x_K^2 \sum_{j=1}^{3} \sum_\phi c_j^\phi K_j(\phi, \phi) + \frac{x_K^2}{4}(D^2 - 2DF - 3F^2)K_1(\pi, \eta) - \frac{x_K^2}{2}K_2(\pi, \eta),$$
TABLE III: Coefficients that appear in the nucleon sigma terms.

| \( j \) | \( \phi \) | \( a_j^\phi \) | \( b_j^\phi \) | \( c_j^\phi \) |
|---|---|---|---|---|
| 1 | \( \pi \) | \((3/2)(\mathcal{D} + \mathcal{F})^2\) | \((3/4)(\mathcal{D} + \mathcal{F})^2\) | \((3/4)(\mathcal{D} + \mathcal{F})^2\) |
| 1 | \( K \) | \((1/6)(5\mathcal{D}^2 - 6\mathcal{D}\mathcal{F} + 9\mathcal{F}^2)\) | \((7/6)\mathcal{D}^2 - \mathcal{D}\mathcal{F} + (5/2)\mathcal{F}^2\) | \((3/2)(\mathcal{D} - \mathcal{F})^2\) |
| 1 | \( \eta \) | \((1/18)(\mathcal{D} - 3\mathcal{F})^2\) | \((5/36)(\mathcal{D} - 3\mathcal{F})^2\) | \((5/36)(\mathcal{D} - 3\mathcal{F})^2\) |
| 2 | \( \pi \) | \(3/2\) | \(3/4\) | \(3/4\) |
| 2 | \( K \) | \(3/2\) | \(5/2\) | \(3/2\) |
| 2 | \( \eta \) | \(3/4\) | \(3/2\) | \(25/36\) |
| 3 | \( \pi \) | \(2\) | \(1\) | \(1\) |
| 3 | \( K \) | \(1/4\) | \(1/3\) | \(1/2\) |
| 3 | \( \eta \) | \(0\) | \(0\) | \(0\) |

where the integrals are

\[
K_1(\phi_1, \phi_2) = \frac{1}{16aF^2} \int_{-\pi}^{\pi} \frac{d^4\theta}{(2\pi)^4} \left[ 4 + a^2 x_{\phi_1}^2 / 2 - \sum_{\lambda} \cos \theta_{\lambda} \right] \frac{\sum_{k=1}^{3} \sin^2 \theta_k}{\sum_{k=1}^{3} \sin^2 \theta_k} \tag{70}
\]

\[
K_2(\phi_1, \phi_2) = \frac{1}{8aF^2} \int_{-\pi}^{\pi} \frac{d^4\theta}{(2\pi)^4} \left[ 3 + a^2 x_{\phi_1}^2 / 2 - \sum_{k=1}^{3} \cos \theta_k \right] \frac{\sum_{k=1}^{3} \sin^2 \theta_k}{\sum_{k=1}^{3} \sin^2 \theta_k} \left( 1 - \cos \theta_4 \right) \tag{71}
\]

\[
K_3(\phi, \phi) = -\frac{C^2}{12aF^2} \int_{-\pi}^{\pi} \frac{d^4\theta}{(2\pi)^4} \left[ 4 + a^2 x_{\phi}^2 / 2 - \sum_{\lambda} \cos \theta_{\lambda} \right] \frac{\sum_{k=1}^{3} \sin^2 \theta_k}{\sum_{k=1}^{3} \sin^2 \theta_k} \left( 1 + a\Delta - \cos \theta_4 \right) \sum_{k=1}^{3} \sin^2 \theta_k \tag{72}
\]

and their coefficients are defined in Table III.

Following Ref. [7], the octet baryon masses and \( \sigma_{\pi N}(0) \) are constrained to their physical values at all lattice spacings. These five observables are functions of five parameters: \( m_0, b_0, b_D, b_F \) and \( \mathcal{D} \). The other axial coupling is \( \mathcal{F} \equiv 1.267 - \mathcal{D} \). With these parameters fixed, predictions are obtained for the \( KN \) sigma terms as shown in Fig. 4. Notice that the \( KN \) sigma terms approach their continuum values very quickly: at \( \pi/a = 1 \text{ GeV} \) the difference is \( O(20\%) \) and at \( \pi/a = 6 \text{ GeV} \) the difference is \( O(1\%) \).

The momentum dependences of the scalar form factors are parameter-free, so all lattice spacing effects are exclusively from loop diagrams. For definiteness, consider the running of the \( \pi N \) sigma term to the Cheng-Dashen point, \( q^2 = -2m_\pi^2 \) in Euclidean notation. This is obtained from the scalar vertex with incoming momentum \( q = iQ/a \) where \( Q \equiv \)
FIG. 4: Kaon-nucleon sigma terms (in units of GeV) as functions of lattice spacing. Lagrangian parameters are fixed by requiring the octet baryon masses and $\sigma_{\pi N}(0)$ to equal their experimental values at all lattice spacings.

$(0, 0, \sqrt{2}a x_\pi, 0)$. Numerical results can be obtained directly from the integral expression,

$$\sigma_{\pi N}(-2 x_\pi^2) - \sigma_{\pi N}(0) = x_\pi^2 \sum_{j=1}^{3} \sum_{\phi} a_j^\phi \left[ \tilde{K}_j(\phi) - K_j(\phi, \phi) \right],$$

(73)

with

$$\tilde{K}_1(\phi) = \frac{1}{16aF^2} \int_{-\pi}^{\pi} \frac{d^4\theta}{(2\pi)^4} \frac{\sum_{k=1}^{3} \left[ \sin^2 \theta_k \cosh^2 Q_k + \cos^2 \theta_k \sinh^2 Q_k \right]}{\left[ (4 + a^2 x_\pi^2 / 2 - \sum_{\lambda} \cos \theta_\lambda \cosh Q_{\lambda k})^2 + (\sum_{k=1}^{3} \sin \theta_k \sinh Q_{k})^2 \right]},$$

(74)

$$\tilde{K}_2(\phi) = \frac{1}{8aF^2} \int_{-\pi}^{\pi} \frac{d^4\theta}{(2\pi)^4} \frac{(1 - \cos \theta_4)}{\left[ (4 + a^2 x_\pi^2 / 2 - \sum_{\lambda} \cos \theta_\lambda \cosh Q_{\lambda k})^2 + (\sum_{k=1}^{3} \sin \theta_k \sinh Q_{k})^2 \right]},$$

(75)

$$\tilde{K}_3(\phi) = \frac{-C^2}{12aF^2} \int_{-\pi}^{\pi} \frac{d^4\theta}{(2\pi)^4} \frac{\sum_{k=1}^{3} \left[ \sin^2 \theta_k \cosh^2 Q_k + \cos^2 \theta_k \sinh^2 Q_k \right]}{\left[ (4 + a^2 x_\pi^2 / 2 - \sum_{\lambda} \cos \theta_\lambda \cosh Q_{\lambda k})^2 + (\sum_{k=1}^{3} \sin \theta_k \sinh Q_{k})^2 \right]}.$$
FIG. 5: The difference (in units of MeV) between the pion-nucleon sigma term at the Cheng-Dashen point and at $q^2 = 0$, as a function of lattice spacing.

$$\sigma_{\pi N}(-2m^2_\pi) - \sigma_{\pi N}(0)$$

and the results are plotted in Fig. 5. In this case, lattice spacing effects are at the few percent level for $\pi/a = 6$ GeV.

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