A prediction model of bubble size from experimental factors and fluid properties using multiple linear regression algorithm

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Abstract. In a clean water test, the bubble diameter coming out through the circular diffusers was measured by photographic method and binary image analysis. Strong correlation was found between bubble diameter and superficial velocity. The depth of water has been added to existing experimental factors that include pore size and superficial velocity. By using dimensional analysis and multiple linear regression algorithm, a prediction model of bubble diameter was developed considering fluid properties and experimental factors including the depth of water. The developed model (EF-Model) was compared with the existing bubble diameter prediction models for the sample data. As a result of estimating the bubble diameter for the superficial velocity, the EF-Model has a lower RMSE and higher accuracy result than the existing models.

1. Introduction
Bubble experiments are being conducted in various areas, including the chemical sector, biological sector and wastewater and water supply sector. In particular, bubble column experiments are conducted a lot in areas such as mass transfer, wastewater and water supply, and membrane. In the bubble column experiment, the bubble diameter is one of the most fundamental parameter [1]. Bubble experimental designs require determination of many factors such as diffusers, pore sizes, reactors, depth of water, and fluid property. Therefore, consideration of the most fundamental parameters, bubble-scale, must be given first, and then experimental design on the reactor-scale [2].

In previous studies, bubble size has been measured in many different ways. The most commonly used measurement method was digital photography [3-5], and other measures were taken with a capillary suction probe (CSP) [6]. Measured bubbles are being used in the development of bubble diameter prediction models [7,8], and mass transfer analysis [9].

In this study, we measured bubble diameter by using the digital photography method and binary image analysis in clean water. We developed a bubble diameter prediction model that takes both experimental factors and fluid properties into account using the measured bubble diameter and experimental conditions. It is also intended to evaluate developed predictive model by comparing them with existing predictive models.

2. Methods
2.1. Experimental equipments and procedure
A experiment was carried out to measure bubble diameter. The size of glass reactor is 0.6m*0.3m*1.0m (LWH) (Figure 1). The air diffuser used in the experiment is a circular diffuser of Angel Aqua. The flowmeter is RMA-21-SSV of Dwyer. Bubble images for measuring the bubble size were taken by a digital camera. The model of the digital camera is a SONY NEX-5R model. The diameter of the circular diffuser is 10.7cm and the pore size is 30~50μm. The position of the air pump was set higher than the reactor to minimize the loss of the injection flow rate. The experiment was conducted for a total 15 cases with flow rates from 1L/min to 5L/min and diffuser counts from 1 to 3 (Case1-5 : from 1 to 5L/min on 1 diffuser). More than 20 pictures of bubbles were taken per case for measurement. The temperature remained at 25℃.

2.2. Bubble diameter

The bubble size was measured by the image analysis method with Image J. The initial bubble size distribution created at the bottom of the column [10]. So, the bubble images were taken at a distance of 30 cm from the bottom of the reactor where the bubble size distribution was created. The 100 bubbles which were clear shape were measured per case.

The Feret diameter is the distance between two parallel tangents. One of the other optical diameters, the martin diameter, is measured at its shortest distance and can be underestimated. The reason for measuring the Feret diameter is an appropriate optical diameter for the volume equivalent diameter [5]. The minimum of the Feret diameter is denoted by F1, and the maximum is denoted by F2. Substitute the values into equation (1) to obtain the volume equivalent diameter of the spherical volume of the same irregular particle size. The average value was set as the representative bubble size for each case.

Calculating the equivalent diameter by binarizing the bubble image and then measuring the Feret diameter by selecting the non-overlapping bubbles (Figure 2).

\[ d = \left( \frac{F_2^2 - F_1^2}{4} \right)^{1/3} \]  \hspace{1cm} (1)

Figure 1. Experimental setup and diagram.

Figure 2. Bubble diameter measurement.
It is difficult to define the speed of a fluid when air bubbles rise in a liquid-filled reactor or tank. So to express the true velocity of a fluid, a superficial velocity is used. It is also known as the apparent velocity. Superficial velocity is calculated by dividing the flow rate \( Q \) by the cross section area \( A \) of the diffuser as in equation (2).

\[
U_s = \frac{Q}{A}
\]  

(2)

Measurement bubble diameter and distribution and superficial velocity are summarized in Table 1. In general, the diameter of the bubble increases as superficial velocity increases (Figure 3a). Standard deviation also increases as superficial velocity increases (Figure 3b). Because an increase in superficial velocity results in more coalescence and break-up [11]. That is, superficial velocity is strongly correlated with bubble diameter and distribution.

**Table 1.** Bubble diameter measurement and superficial velocity.

| Case | Diameter (mm) | Median (mm) | Max (mm) | Min (mm) | Superficial velocity(cm/s) | Standard deviation |
|------|---------------|-------------|----------|----------|---------------------------|--------------------|
| 1    | 1.130         | 1.015       | 2.432    | 0.838    | 0.185                     | 0.328              |
| 2    | 2.156         | 2.093       | 3.792    | 0.803    | 0.371                     | 0.769              |
| 3    | 2.827         | 2.747       | 5.404    | 1.077    | 0.556                     | 0.831              |
| 4    | 2.879         | 2.825       | 4.431    | 1.474    | 0.741                     | 0.686              |
| 5    | 3.092         | 3.008       | 4.932    | 1.253    | 0.927                     | 0.884              |
| 6    | 0.567         | 0.499       | 1.337    | 0.365    | 0.093                     | 0.222              |
| 7    | 1.049         | 0.912       | 2.933    | 0.666    | 0.185                     | 0.445              |
| 8    | 1.805         | 1.475       | 5.502    | 0.763    | 0.278                     | 0.981              |
| 9    | 2.084         | 2.096       | 3.877    | 0.969    | 0.371                     | 0.664              |
| 10   | 2.361         | 2.316       | 4.836    | 0.529    | 0.463                     | 0.964              |
| 11   | 0.591         | 0.610       | 0.708    | 0.311    | 0.062                     | 0.089              |
| 12   | 0.845         | 0.789       | 1.767    | 0.422    | 0.124                     | 0.276              |
| 13   | 1.402         | 1.291       | 3.115    | 0.485    | 0.185                     | 0.597              |
| 14   | 1.884         | 1.870       | 3.389    | 0.474    | 0.247                     | 0.603              |
| 15   | 2.215         | 2.222       | 4.522    | 0.921    | 0.309                     | 0.724              |

**Figure 3.** Superficial velocity is correlated with diameter (a) and standard deviation (b).

In order to compare and verify measured bubble diameter, data were extracted from literatures measuring bubble diameters for superficial velocity. The data with the experimental conditions clearly stated to the depth of water developed a bubble diameter prediction model by aggregating the
experimental data [9,10,12]. Data that did not show the depth of water conditions were used as sample data for verification of the predictive model through the depth of water assumptions [13-15].

### 2.3 Correlation for bubble diameter

Bubble diameter prediction models developed in previous studies were developed to match fluid property rather than experimental factors such as depth of water (H) and pore size (p). Models not considering the pore size [3,8], models not considering fluid properties [7], models not taking into superficial velocity($U_s$) [13]. Also, there was a calculated model by equating buoyancy and surface tension [12] (Table 2).

| Reference | Prediction models | Unconsidered factors |
|-----------|-------------------|----------------------|
| 1 Akita K and Yoshida F 1974[3] | $d = 26D \left( \frac{gD^2 \rho_l}{\sigma} \right)^{(-0.5)} \left( \frac{\rho_l^2 gD^3}{\mu^2} \right)^{(-0.12)} \left( \frac{U_s}{gd} \right)^{(-0.1)}$ | Pore size(p) |
| 2 Sarrafi A et al. 1999[12] | $d = \left( \frac{6p\sigma}{g(\rho_l - \rho_g)} \right)^{1/3}$ | Viscosity($\mu$) |
| 3 Azad M and Syeda SR 2006[7] | $d = \left( \frac{U_s}{\sqrt{gp}} \right)^{1/3}$ | Surface tension($\sigma$) |
| 4 Pohorecki R et al. 2005[8] | $d = 0.289 \rho^{-0.552} \mu^{-0.048} \sigma^{0.442} U_s^{-0.124}$ | Pore size(p) |
| 5 Vogelpohl A 1986[13] | $d = \left( \frac{6p\sigma}{\rho_l g} \right)^{1/3} + \left( \frac{81\mu Q}{\pi \rho g} \right) + \left( \frac{135Q^2}{4\pi^2 g} \right)^{1/5} \right)^{1/4}$ | Superficial velocity($U_s$) |

In this study, fluid properties were fixed through the clean water test, and the more experimental factors, depth of water and pore size, were considered with other factors. The bubble diameter prediction model was developed by dimensional analysis and multiple linear regression algorithm (Equation 3-5).

\[
\begin{align*}
\pi_1 &= \frac{d}{p} \quad \pi_2 = \frac{A}{p^2} \quad \pi_3 = \frac{0p}{\mu p} \quad \pi_4 = \frac{H}{p} \quad \pi_5 = \frac{\sigma p}{\mu^2} \quad \pi_6 = \frac{\rho p^2}{\mu^2} \\
\pi_2 &= \frac{0p}{\mu p} \times \frac{p^2}{A} \quad \pi_5 = \frac{\rho p^2}{\mu^2} \times \frac{\mu^2}{\rho p^2} \quad \pi_4 = \frac{H}{p} \quad \pi_5 = \frac{\rho p^2}{\sigma} \quad \pi_4 = \frac{H}{p}
\end{align*}
\]

\[
d = 10^{(0.071)} \left( \frac{\rho U_p}{\mu} \right)^{(0.317)} \left( \frac{H}{p} \right)^{(-0.420)} \left( \frac{\rho p^2}{\sigma} \right)^{(-0.721)}
\]

### 3. Results and discussion

#### 3.1 Comparison with prediction models

In order to compare and verify the five bubble diameter prediction models and the developed model (EF-Model) in this study, experimental conditions and factors of the sample data were substituted in each model. Models were compared by graph for superficial velocity that strong correlation with bubble diameters.
Figure 4. Comparison with prediction models (EF-Model : a / Model 1-5 : b-f).

Table 3. RMSE of the prediction models.

|        | EF-Model | Model 1 | Model 2 | Model 3 | Model 4 | Model 5 |
|--------|----------|---------|---------|---------|---------|---------|
| RMSE   | 1.029    | 1.697   | 2.532   | 4.578   | 4.131   | 2.508   |

As shown in Table 3, EF-Model developed with the depth of water in this study was most similarly predicted with an RMSE of 1.029. As expected, models without taking into account superficial velocity were not suitable for predictive models (Models 2, 5). Models that did not take into account the pore size gradually decreased in bubble diameter as superficial velocity increased (Models 1, 4). This is the opposite of the correlation between the superficial velocity and bubble diameter that was verified in the previous experiment. Model 3 has the largest RMSE, considering superficial velocity and pore size but not including fluid properties (Figure 4).
3.2. Discussion

It was found that developing bubble diameter prediction models with only fluid properties or experimental factors was less predictable. For increasing the accuracy of the bubble diameter prediction model, it is recommended that the fluid properties and the experimental factors applied in the experiment together. Depending on the experimental conditions and circumstances, there may be plenty of experimental factors. This study found that the depth of water which is one of the experimental factors was applied and it can reduce predictive errors.

4. Conclusion

The bubble diameter prediction model (EF-Model), considering experimental factors and fluid properties, had the lowest RMSE of 1.029, and the graph also showed that it was well-predictable compared to other existing models. The model with the highest RMSE is Model 3, which excludes fluid properties. Models 1 and 4 showed an opposite tendency for bubble diameters to decrease as superficial velocity increased. Models 2 and 5 did not include a superficial velocity and were therefore not suitable for prediction. Later on, additional experimental factors will be reflected to develop a bubble diameter prediction model that is more accurate and can reduce errors in the design.

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