New application of Dirac’s representation: N-mode squeezing enhanced operator and squeezed state

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Abstract

It is known that \(\exp[i\lambda(Q_1P_1 - i/2)]\) is a unitary single-mode squeezing operator, where \(Q_1, P_1\) are the coordinate and momentum operators, respectively. In this paper we employ Dirac’s coordinate representation to prove that the exponential operator \(S_n \equiv \exp[i\lambda \sum_{i=1}^{n} (Q_iP_{i+1} + Q_{i+1}P_i)]\), \((Q_{n+1} = Q_1, P_{n+1} = P_1)\), is a n-mode squeezing operator which enhances the standard squeezing. By virtue of the technique of integration within an ordered product of operators we derive \(S_n\)’s normally ordered expansion and obtain new n-mode squeezed vacuum states, its Wigner function is calculated by using the Weyl ordering invariance under similar transformations.

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1 Introduction

Squeezed state has been a hot topic in quantum optics since Stoler [1] put forward the concept of the optical squeezing in 1970’s. \(S_1 = \exp[i\lambda(Q_1P_1 - i/2)]\) is a unitary single-mode squeezing operator, where \(Q_1, P_1\) are the coordinate and momentum operators, respectively, \(\lambda\) is a squeezing parameter. Their variances in the squeezed state \(S_1 |0\rangle = \text{sech}^{1/2} \lambda \exp \left[-\frac{1}{2}a_1^{\dagger 2} \tanh \lambda \right]|0\rangle\) are

\[
\Delta Q_1 = \frac{1}{4} e^{2\lambda}, \quad \Delta P_1 = \frac{1}{4} e^{-2\lambda}, \quad (\Delta Q_1)(\Delta P_1) = \frac{1}{4}.
\]

Some generalized squeezed state have been proposed since then. Among them the two-mode squeezed state not only exhibits squeezing, but also quantum entanglement between the idle-mode and the signal-mode in frequency domain, therefore is a typical entangled states of continuous variable. In recent years, various entangled states have attracted considerable attention and interests of physicists because of their potential uses in quantum communication [2]. Theoretically, the two-mode squeezed state is constructed by acting the two-mode squeezing operator \(S_2 = \exp[\lambda(a_1a_2 - a_1^{\dagger}a_2^{\dagger})]\) on the two-mode vacuum state \(|00\rangle\) [3] [4] [5],

\[
S_2 |00\rangle = \text{sech} \lambda \exp \left[-a_1^{\dagger}a_2^{\dagger} \tanh \lambda \right]|00\rangle.
\]

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We also have \( S_2 = \exp \left[ i\lambda (Q_1 P_2 + Q_2 P_1) \right] \), where \( Q_i \) and \( P_i \) are the coordinate and momentum operators related to Bose operators \((a_i, a_i^\dagger)\) by

\[
Q_i = (a_i + a_i^\dagger)/\sqrt{2}, \quad P_i = (a_i - a_i^\dagger)/(\sqrt{2}i)
\]

In the state \( S_2 |00\rangle \), the variances of the two-mode quadrature operators of light field,

\[
\mathcal{X} = (Q_1 + Q_2)/2, \quad \mathcal{P} = (P_1 + P_2)/2, \quad [\mathcal{X}, \mathcal{P}] = i/2,
\]

take the standard form, i.e.,

\[
\langle 00 | S_2^\dagger \mathcal{X}^2 S_2 |00\rangle = \frac{1}{4}e^{-2\lambda}, \quad \langle 00 | S_2^\dagger \mathcal{P}^2 S_2 |00\rangle = \frac{1}{4}e^{2\lambda}, \quad \text{and} \quad (\Delta \mathcal{X})(\Delta \mathcal{P}) = \frac{1}{4}.
\]

On the other hand, the two-mode squeezing operator has a neat and natural representation in the entangled state \(|\eta\rangle\) representation [6],

\[
S_2 = \int \frac{d^2\eta}{\pi \mu} \frac{|\eta\rangle \langle \eta|}{|\mu\rangle \langle \mu|},
\]

where

\[
|\eta\rangle = \exp\left(-\frac{1}{2} |\eta|^2 + \eta a_1^\dagger - \eta^* a_2^\dagger + a_1^\dagger a_2^\dagger\right)|00\rangle,
\]

makes up a complete set

\[
\int \frac{d^2\eta}{\pi} |\eta\rangle \langle \eta| = 1.
\]

\(|\eta\rangle\) was constructed according to the idea of quantum entanglement initiated by Einstein, Podolsky and Rosen in their argument that quantum mechanics is incomplete [7].

An interesting question naturally arises: is the \( n \)-mode exponential operator

\[
S_n = \exp \left[ i\lambda \sum_{i=1}^{n} (Q_i P_{i+1} + Q_{i+1} P_i) \right], \quad (Q_{n+1} = Q_1, \ P_{n+1} = P_1), \ n \geq 2,
\]

a squeezing operator? If yes, what kind of squeezing for \( n \)-mode quadratures of field it can engenders? To answer these questions we must know what is the normally ordered expansion of \( S_n \) and what is the state \( S_n |0\rangle \) (\(|0\rangle\) is the \( n \)-mode vacuum state)? In this work we shall analyse \( S_n \) in detail. But how to disentangle the exponential of \( S_n \)? Since the terms in the set \( Q_i P_{i+1} \) and \( Q_{i+1} P_i \) \((i = 1, 2, \cdots, n)\) do not make up a closed Lie algebra, the problem of what is \( S_n \)'s normally ordered form seems difficult. Thus we appeal to Dirac’s coordinate representation and the technique of integration within an ordered product (IWOP) of operators [8, 9] to solve this problem. Our work is arranged as follows: firstly we use the IWOP technique to derive the normally ordered expansion of \( S_n \) and obtain the explicit form of \( S_n |0\rangle \); then we examine the variances of the \( n \)-mode quadrature operators in the state \( S_n |0\rangle \), we find that \( S_n \) causes squeezing which is stronger than the standard squeezing. Thus \( S_n \) is an \( n \)-mode squeezing-enhanced operator. The Wigner function of \( S_n |0\rangle \) is calculated by using the Weyl ordering invariance under similar transformations. Some examples are discussed in the last section.
2 Normal Product Form of $S_n$ derived by Dirac’s coordinate representation

In order to disentangle operator $S_n$, let $A$ be

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 1 \\ 1 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 1 & 0 & \cdots & 1 & 0 \end{pmatrix},$$

(8)

then $S_n$ in (7) is compactly expressed as

$$S_n = \exp[i\lambda Q_i A_{ij} P_j],$$

(9)

here and henceforth the repeated indices represent Einstein’s summation notation. Using the Baker-Hausdorff formula,

$$e^{A}B e^{-A} = B + [A, B] + \frac{1}{2!} [A, [A, B]] + \frac{1}{3!} [A, [A, [A, B]]] + \cdots ,$$

we have

$$S_n^{-1} Q_k S_n = Q_k - \lambda Q_i A_{ik} + \frac{1}{2!} \lambda^2 [Q_i A_{ij} P_j, Q_l A_{lk}] + \cdots$$

$$= Q_i (e^{-\lambda A_{ij}})_{ik} = (e^{-\lambda \tilde{A}})_{ik} Q_i,$$

(10)

$$S_n^{-1} P_k S_n = P_k + \lambda A_{ki} P_i + \frac{1}{2!} \lambda^2 [A_{ki} P_j, Q_l A_{lm} P_m] + \cdots$$

$$= (e^{\lambda A})(\tilde{A})_{ki} P_i.$$  

(11)

From Eq. (10) we see that when $S_n$ acts on the n-mode coordinate eigenstate $|\bar{q}\rangle$, where $\bar{q} = (q_1, q_2, \cdots, q_n)$, it squeezes $|\bar{q}\rangle$ in this way:

$$S_n |\bar{q}\rangle = |\Lambda|^{1/2} |\Lambda \bar{q}\rangle, \quad \Lambda = e^{-\lambda \tilde{A}}, \quad |\Lambda| \equiv \det \Lambda.$$  

(12)

Thus $S_n$ has the representation on the Dirac’s coordinate basis $|\bar{q}\rangle [10]$,

$$S_n = \int d^n q S_n |\bar{q}\rangle \langle \bar{q}| = |\Lambda|^{1/2} \int d^n q |\Lambda \bar{q}\rangle \langle \bar{q}|, \quad S_n^\dagger = S_n^{-1},$$

(13)

since $\int d^n q |\bar{q}\rangle \langle \bar{q}| = 1$. Using the expression of $|\bar{q}\rangle$ in Fock space

$$|\bar{q}\rangle = e^{-n/4} : \exp \left[-\frac{1}{2} \bar{q} \tilde{q} + \sqrt{2} \bar{q} \hat{a} + \frac{1}{2} \hat{a} \hat{a}^\dagger \right] |\bar{0}\rangle,$$

$$\hat{a}^\dagger = (a_1^\dagger, a_2^\dagger, \cdots, a_n^\dagger),$$

(14)

and the normally ordered form of n-mode vacuum projector $|\bar{0}\rangle \langle \bar{0}| = : \exp[-\hat{a}^\dagger \hat{a}] :$, we can put $S_n$ into the normal ordering form,

$$S_n = \pi^{-n/2} |\Lambda|^{1/2} \int d^n q : \exp \left[-\frac{1}{2} \bar{q}(1 + \tilde{\Lambda})\bar{q} + \sqrt{2} \bar{q} (\tilde{\Lambda} \hat{a}^\dagger + a) \right]$$

$$-\frac{1}{2} (\bar{a} \hat{a} + \hat{a}^\dagger \hat{a}^\dagger) - \hat{a}^\dagger a :.$$  

(15)

To perform the integration in Eq. (15) by virtue of the IWOP technique, using the mathematical formula

$$\int d^n x \exp[-\bar{x}F x + \bar{x}v] = \pi^{n/2} (\det F)^{-1/2} \exp \left[\frac{1}{4} \bar{x}F^{-1} v \right],$$

(16)
then we derive

\[ S_n = \left( \frac{\det \Lambda}{\det N} \right)^{1/2} \exp \left[ \frac{1}{2} \hat{a}^\dagger \left( \Lambda N^{-1} \Lambda - I \right) \hat{a} \right] \times \exp \left[ \frac{1}{2} \hat{a}^\dagger \left( \Lambda N^{-1} - I \right) \hat{a} \right], \]

(17)

where \( N = (1 + \bar{\Lambda} \Lambda)/2 \). Eq. (17) is just the normal product form of \( S_n \).

### 3 Squeezing property of \( S_n |0\rangle \)

Operating \( S_n \) on the n-mode vacuum state \( |0\rangle \), we obtain the squeezed vacuum state

\[ S_n |0\rangle = \left( \frac{\det \Lambda}{\det N} \right)^{1/2} \exp \left[ \frac{1}{2} \hat{a}^\dagger \left( \Lambda N^{-1} \Lambda - I \right) \hat{a} \right] |0\rangle. \]

(18)

Now we evaluate the variances of the n-mode quadratures. The quadratures in the n-mode case are defined as

\[ X_1 = \frac{1}{\sqrt{2n}} \sum_{i=1}^{n} Q_i, \quad X_2 = \frac{1}{\sqrt{2n}} \sum_{i=1}^{n} P_i, \]

(19)

obeying \([X_1, X_2] = \frac{i}{2}\). Their variances are \((\Delta X_i)^2 = \langle X_i^2 \rangle - \langle X_i \rangle^2, i = 1, 2\). Noting the expectation values of \( X_1 \) and \( X_2 \) in the state \( S_n |0\rangle \), \( \langle X_1 \rangle = \langle X_2 \rangle = 0 \), then using Eqs. (10) and (11) we see that the variances are

\[ (\Delta X_1)^2 = \langle 0 | S_n^{-1} X_1^2 S_n |0\rangle = \frac{1}{2n} \langle 0 | S_n^{-1} \sum_{i=1}^{n} Q_i \sum_{j=1}^{n} Q_j S_n |0\rangle \]

\[ = \frac{1}{2n} \langle 0 | \sum_{i=1}^{n} Q_i (e^{-\Lambda \Lambda})_{ki} \sum_{j=1}^{n} (e^{-\Lambda \Lambda})_{jl} Q_j |0\rangle \]

\[ = \frac{1}{2n} \sum_{i,j} (e^{-\Lambda \Lambda})_{ki}(e^{-\Lambda \Lambda})_{jl} \langle 0 | Q_i Q_j |0\rangle \]

\[ = \frac{1}{4n} \sum_{i,j} (e^{-\Lambda \Lambda})_{ki}(e^{-\Lambda \Lambda})_{jl} \langle 0 | a_k a_l^\dagger |0\rangle \]

\[ = \frac{1}{4n} \sum_{i,j} (e^{-\Lambda \Lambda})_{ki}(e^{-\Lambda \Lambda})_{jl} \delta_{kl} = \frac{1}{4n} \sum_{i,j} (\Lambda \Lambda)_{ij}, \]

(20)

similarly we have

\[ (\Delta X_2)^2 = \langle 0 | S_n^{-1} X_2^2 S_n |0\rangle = \frac{1}{4n} \sum_{i,j} \left[ (\Lambda \Lambda)^{-1} \right]_{ij}, \]

(21)

Eqs. (20) - (21) are the quadrature variance formula in the transformed vacuum state acted by the operator \( \exp[\hat{A} \lambda Q_i A_i^\dagger P_j] \). By observing that \( A \) in (11) is a symmetric matrix, we see

\[ \sum_{i,j} [(A + \hat{A})^\dagger]_{ij} = 2^{2l} n, \]

(22)

then using \( A \hat{A} = \Lambda \hat{A} \), so \( \bar{\Lambda} \Lambda = e^{-\Lambda (A + \hat{A})} \), a symmetric matrix, we have

\[ \sum_{i,j=1}^{n} (\Lambda \Lambda)_{ij} = \sum_{l=0}^{\infty} \frac{(-\lambda)^l}{l!} \sum_{i,j} [(A + \hat{A})^\dagger]_{ij} = n \sum_{l=0}^{\infty} \frac{(-\lambda)^l}{l!} 2^{2l} = ne^{-4\lambda}, \]

(23)
and
\[ \sum_{i,j=1}^{n} (\bar{\Lambda}\Lambda)^{-1}_{ij} = ne^{4\lambda}. \]  

(24)

It then follows
\[ (\triangle X_1)^2 = \frac{1}{4n} \sum_{i,j=1}^{n} (\bar{\Lambda}\Lambda)_{ij} = \frac{e^{-4\lambda}}{4}, \]

(25)

\[ (\triangle X_2)^2 = \frac{1}{4n} \sum_{i,j=1}^{n} \left[ (\bar{\Lambda}\Lambda)^{-1} \right]_{ij} = \frac{e^{4\lambda}}{4}. \]

(26)

This leads to \((\triangle X_1)(\triangle X_2) = \frac{1}{4}\), which shows that \(S_n\) is a correct n-mode squeezing operator for the n-mode quadratures in Eq. (19). Furthermore, Eqs. (25) and (26) clearly indicate that the squeezed vacuum state \(S_n|0\rangle\) may exhibit stronger squeezing \((e^{-4\lambda})\) in one quadrature than that \((e^{-2\lambda})\) of the usual two-mode squeezed vacuum state. This is a way of enhancing squeezing.

4 The Wigner function of \(S_n|0\rangle\)

Wigner distribution functions [12] of quantum states are widely studied in quantum statistics and quantum optics. Now we derive the expression of the Wigner function of \(S_n|0\rangle\). Here we take a new method to do it. Recalling that in Ref. [13] we have introduced the Weyl ordering form of single-mode Wigner operator \(\Delta_1(q_1, p_1),\)
\[ \Delta_1(q_1, p_1) = \delta(q_1 - Q_1) \delta(p_1 - P_1) : \]

(27)

its normal ordering form is
\[ \Delta_1(q_1, p_1) = \frac{1}{\pi} : \exp \left[ -(q_1 - Q_1)^2 - (p_1 - P_1)^2 \right] : \]

(28)

where the symbols : : and : : : denote the normal ordering and the Weyl ordering, respectively. Note that the order of Bose operators \(a_1\) and \(a_1^\dagger\) within a normally ordered product and a Weyl ordered product can be permuted. That is to say, even though \([a_1, a_1^\dagger] = 1\), we can have : \(a_1 a_1^\dagger : = : a_1^\dagger a_1 :\)
and : \(a_1 a_1^\dagger : = : a_1^\dagger a_1 :\). The Weyl ordering has a remarkable property, i.e., the order-invariance of Weyl ordered operators under similar transformations, which means
\[ U : (\circ \circ \circ) : U^{-1} = : U (\circ \circ \circ) U^{-1} : \]

(29)

as if the “fence” : : : did not exist.

For n-mode case, the Weyl ordering form of the Wigner operator is
\[ \Delta_n(q, p) = \delta(q - \bar{Q}) \delta(p - \bar{P}) : \]

(30)

where \(\bar{Q} = (Q_1, Q_2, \cdots, Q_n)\) and \(\bar{P} = (P_1, P_2, \cdots, P_n)\). Then according to the Weyl ordering invariance under similar transformations and Eqs. (10) and (11) we have
\[ S_n^{-1} \Delta_n(q, p) S_n = \Delta_n(e^{\lambda A} q, e^{-\lambda A} p) : \]

(31)
Thus using Eqs. (27) and (31) the Wigner function of \( S_n \) \(|0\rangle\) is

\[
\langle 0 | S_n^{-1} \Delta_n (\vec{q}, \vec{p}) S_n | 0 \rangle = \frac{1}{\pi^n} \langle 0 : \exp[-(e^{\lambda A} \vec{q} - \vec{Q})^2 - (e^{-\lambda A} \vec{p} - \vec{P})^2] : | 0 \rangle
\]

\[
= \frac{1}{\pi^n} \exp[-(e^{\lambda A} \vec{q})^2 - (e^{-\lambda A} \vec{p})^2]
\]

\[
= \frac{1}{\pi^n} \exp \left[ -\vec{q} e^{\lambda A} \vec{q} - \vec{p} e^{-\lambda A} \vec{p} \right]
\]

\[
= \frac{1}{\pi^n} \exp \left[ -\vec{q} (\Lambda \Lambda)^{-1} \vec{q} - \vec{p} \Lambda \Lambda \vec{p} \right], \quad (32)
\]

From Eq. (32) we see that once the explicit expression of \( \Lambda \Lambda = \exp[-\lambda(A + \hat{A})] \) is deduced, the Wigner function of \( S_n |0\rangle \) can be calculated.

5 Some examples of calculating the Wigner function

For \( n = 2 \), form Eq. (7) we have \( S_2' = \exp[\pm 2\lambda (Q_1 P_2 + Q_2 P_1)] \) which exhibits clearly the stronger squeezing than the usual two-mode squeezing operator \( S_2' \). For \( n = 3 \), the three-mode operator \( S_3 \), from Eq. (9) we see that the matrix \( A \) is

\[
A \Lambda = \begin{pmatrix}
  u & v & v \\
  v & u & v \\
  v & v & u
\end{pmatrix}, \quad u = \frac{2}{3} e^{2\lambda} + \frac{1}{3}, \quad v = \frac{1}{3} e^{-2\lambda} - \frac{1}{3}
\]

(33)

and \( (\Lambda \Lambda)^{-1} \) is obtained by replacing \( \lambda \) with \( -\lambda \) in \( \Lambda \Lambda \). Thus the squeezing state \( S_3 |000\rangle \) is

\[
S_3 |000\rangle = A_3 \exp \left[ \frac{1}{6} A_1 \sum_{i=1}^{3} a_i^{\dagger 2} - \frac{2}{3} A_2 \sum_{i<j}^{3} a_i^{\dagger} a_j^{\dagger} \right] |000\rangle, \quad (34)
\]

where

\[
A_1 = (1 - \text{sech}2\lambda) \tanh \lambda, \quad A_2 = \frac{\sinh 3\lambda}{2 \cosh \lambda \cosh 2\lambda}, \quad A_3 = \text{sech} \lambda \cosh^{-1/2} 2\lambda. \quad (35)
\]

In particular, for the case of the infinite squeezing \( \lambda \rightarrow \infty \), Eq. (36) reduces to

\[
S_3 |000\rangle \sim \exp \left\{ \frac{1}{6} \left[ \sum_{i=1}^{3} a_i^{\dagger 2} - \frac{4}{3} \sum_{i<j}^{3} a_i^{\dagger} a_j^{\dagger} \right] \right\} |000\rangle \equiv | \rangle_{s_3}, \quad (36)
\]

which is just the common eigenvector of the three compatible Jacobian operators in three-body case with zero eigenvalues \([14]\), i.e.,

\[
(P_1 + P_2 + P_3) | \rangle_{s_3} = 0, \quad (Q_3 - Q_2) | \rangle_{s_3} = 0,
\]

\[
\left( \frac{\mu_3 Q_3 + \mu_2 Q_2}{\mu_3 + \mu_2} - Q_1 \right) | \rangle_{s_3} = 0, \quad \left( \mu_i = \frac{m_i}{m_1 + m_2 + m_3} \right), \quad (37)
\]

as common eigenvector

\[
[P_1 + P_2 + P_3, Q_3 - Q_2] = 0, \quad \left[ \frac{\mu_3 Q_3 + \mu_2 Q_2}{\mu_3 + \mu_2} - Q_1, P_1 + P_2 + P_3 \right] = 0. \quad (38)
\]
Since the common eigenvector of three compatible Jacobian operators is an entangled state, the state $|s_3\rangle$ is also an entangled state.

By using Eq. (32), the Wigner function is

$$\langle 0 | S_3^{-1} \Delta_3 (q, p) S_3 | 0 \rangle = \frac{1}{\pi^3} \exp \left\{ -\frac{2}{3} (\cosh 4\lambda + 2 \cosh 2\lambda) \sum_{i=1}^{3} |\alpha_i|^2 \right\} \times \exp \left\{ -\frac{1}{3} (\sinh 4\lambda - 2 \sinh 2\lambda) \sum_{i=1}^{3} \alpha_i^2 \right\} - \frac{2}{3} \sum_{j>i=1}^{3} \left[ (\cosh 4\lambda - \cosh 2\lambda) \alpha_i \alpha_i^* + (\sinh 2\lambda + \sinh 4\lambda) \alpha_i \alpha_j \right] + c.c. \right\}. \quad (39)$$

For $n = 4$ case, the four-mode operator $S_4$ is

$$S_4 = \exp\{i \lambda [(Q_1 + Q_3) (P_1 + P_3) + (Q_2 + Q_4) (P_1 + P_3)]\} \quad (40)$$

the matrix $A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$, thus we have

$$\Lambda \tilde{\Lambda} = \begin{pmatrix} r & t & s & t \\ t & r & t & s \\ s & t & r & t \\ t & s & t & r \end{pmatrix}, \quad (41)$$

where $r = \cosh^2 2\lambda, s = \sinh^2 2\lambda, t = -\sinh 2\lambda \cosh 2\lambda$. Then substituting Eq. (41) into Eq. (32) we obtain

$$\langle 0 | S_4^{-1} \Delta_4 (q, p) S_4 | 0 \rangle = \frac{1}{\pi^4} \exp \left\{ -2 \cosh^2 2\lambda \sum_{i=1}^{4} |\alpha_i|^2 + (M + M^*) \tanh^2 2\lambda + (R^* + R) \tanh 2\lambda \right\}, \quad (42)$$

where $M = \alpha_1 \alpha_3^* + \alpha_2 \alpha_4^*$, $R = \alpha_1 \alpha_2^* + \alpha_1 \alpha_4^* + \alpha_2 \alpha_3^* + \alpha_3 \alpha_4$. This form differs evidently from the Wigner function of the direct-product of usual two two-mode squeezed states' Wigner functions. In addition, using Eq. (11) we can check Eqs. (25) and (26). Further, using Eq. (11) we have

$$N^{-1} = \frac{1}{2} \begin{pmatrix} 2 & \tanh 2\lambda & 0 & \tanh 2\lambda \\ \tanh 2\lambda & 2 & \tanh 2\lambda & 0 \\ 0 & \tanh 2\lambda & 2 & \tanh 2\lambda \\ \tanh 2\lambda & 0 & \tanh 2\lambda & 2 \end{pmatrix}, \quad \det N = \cosh^2 2\lambda. \quad (43)$$

Then substituting Eqs. (43) into Eq. (17) yields the four-mode squeezed state [11] [13],

$$S_4 \langle 0000 \rangle = \sech 2\lambda \exp \left\{ -\frac{1}{2} \left( a_1^i + a_3^i \right) \left( a_2^i + a_4^i \right) \tanh 2\lambda \right\} \langle 0000 \rangle, \quad (44)$$

from which one can see that the four-mode squeezed state is not the same as the direct product of two two-mode squeezed states in Eq. (11).

In summary, by virtue of Dirac’s coordinate representation and the IWOP technique: we have shown that an n-mode squeezing operator $S_n = \exp\{i \lambda \sum_{i=1}^{n} (Q_i P_{i+1} + Q_{i+1} P_i)\}$, $(Q_{n+1} = Q_1, P_{n+1} = P_1)$, is an n-mode squeezing operator which enhances the stronger squeezing for the n-mode quadratures [10]. $S_n$'s normally ordered expansion and new n-mode squeezed vacuum states are obtained.

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