Working with Definition of Absolute Value: Study on Prospective Mathematics Teacher of Cokroaminoto Palopo University in Real Analysis Subject

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Abstract. The Definition of absolute value was used to proof the representative theorem only. Furthermore, this definition will be no longer be used. Some students only use the definition to prove a particular theorem, then students use the theorem to solve the problem. Consequently, definitions will be forgotten and students' understanding of concepts will be lacking. This study establish the students of prospective mathematics teacher of Cokroaminoto University to use the definition of Absolute value to solve some real analysis problem. The research unit of this study is the 6th semester prospective mathematics teacher in Cokroaminoto University. This research conducted on semester 6 on academic year of 2016/2017. The study shows that (1) there are some prospective teachers have difficulties to form the problem as definition of absolute value, (2) they cannot conclude the final solution set for two or more solution set. This Research suggest to use Problem Based Learning to improve the prospective mathematics teacher in solving real analysis problem.

1. Introduction

Problem-solving ability are one of the goals that the educational world is hoping today is no exception to mathematics. Problem solving is usually defined as formulating new answers, going beyond the simple application of previously learned rules to create a solution [1]. Therefore the importance of solving mathematical problems hence, education in the sciences must address the crucially important task of teaching students to become more proficient problem-solvers [2]. Problem solving ability can be seen from student ability to solve non-routine problem. Non-routine questions require students to think high-level. Student's High level thinking skills, will give successful problem solving more effective [3]-[5]. One of the courses that presents non-routine issues is real analysis. Real analysis is a compulsory subject in the Mathematics education study program of Cokroaminoto Palopo University. Real analysis is studied by semester VI students and has prerequisite courses.

This course discusses some basic concepts: the properties of the field and the sequence of real numbers, the complete completeness of the real numbers, the Archimedean properties, the set point theory in \( \mathbb{R} \), the sequence of real numbers: the sequence and the limit, the limit theorem, the monoton
sequence, the Cauchy section and sequence, infinity series: series of convergence, special convergence test.

In this case the researcher chooses one subject in the real analysis course that is the absolute value. Absolute value is no longer foreign to some students of mathematics education because the absolute value has been learned during high school.

There are several sections studied in the absolute value of the definition of absolute value, the absolute value theorem, and their use in solving the problem. The current case is that students tend to use the theorem to solve problems because they are simpler. By using the theorem, the students use a single way to solve the problem. That way like substitution method such that good problem-solving skill as the goal of real analysis subject not achieved. The result is student understanding of a concept will be reduced. According to the background mention before, the purpose of this study is to establish the student of prospective mathematics teacher of Cokroaminoto University using the definition of Absolute value to solve some real analysis problem.

In this study the student gave a matter related to the Absolute Value, the problem must be solved by using definition not by theorem. By using definition, the student presenting the heuristic when doing the task. A heuristic is a thinking strategy, something that can be used to identify more information about a problem and how to help you figure out what to do. Heuristic methods, heuristic strategies, or simply heuristics, are ways to make progress on difficult problems. Heuristics are components for problem solving [6]-[8].

2. Literature Review

2.1 Working using definition
A definition in mathematics is the laying down of the mathematical meaning of a particular term, in terms of mathematical objects or ideas that have been previously defined or shown to exist.

2.2 Absolute Value
This research choose absolute value as the topic to see how the student working using definition. Absolute value choosed because this topic are relvant to studnets ablity in solving problem. The problem in this topic neither too difficult nor too easy to solve.

Absolute value topic is derived from the Trichotomy property. we are assured that if \( a^2 \in \mathbb{R} \) and \( a \neq 0 \), then exactly one of the number \( a \) and \( -a \) is positive. The absolute value of \( a \neq 0 \) is defined to be the positive one of these two numbers. The absolute value of 0 is defined to be 0.

The absolute value of a real number \( a \), denoted by \(|a|\), is defined by [6]

\[
|a| = \begin{cases} 
  a, & \text{if } a > 0 \\
  0, & \text{if } a = 0 \\
  -a, & \text{if } a < 0
\end{cases}
\] 

(1)

For example \(|5| = 5\) and \(|-7| = 7\). According to the definition mention above we see that \(|a| = 0\) if and only if \( a = 0 \). Hence we can simplify the theorem become

\[
|a| = \begin{cases} 
  a, & \text{if } a \geq 0 \\
  -a, & \text{if } a < 0
\end{cases}
\] 

(2)

The second definition are used to proof some theorem and solving problem in this research. One example that lecturer do in the class that is “determine the set \( A \) of \( x \in \mathbb{R} \) such that \(|2x + 3| < 7\).”

To solve this problem some students directly use the theorem of

If \( c \geq 0 \), then \(|a| \leq c \) if and only if \(-c \leq a \leq c \) \((3)\)

That theorem make student thinking “substitution”, make student like solving the routine problem. In fact this topic need student thinking critically. This study will show the results of student work in
solving the problem by using the definition. This study also show what is their problem in solving real analysis problem.

2.3 Prospective Teacher
One of the strategies to produce a professional mathematics teacher is to prepare students of mathematics teacher candidate with professional mathematics teacher standard. Students of mathematics teacher candidates have a very important role in meeting the needs of professional math teachers [9]. According to Dedi Supriadi, the ability to be possessed by prospective teachers including prospective teachers of mathematics are: 1) mastery of teaching materials, 2) deep understanding of learners who want to serve later, 3) mastery theory and pedagogy, 4) possession of ability to demonstrate performance, 5) Selection of attitudes, values and personality traits that support the implementation of tasks as teacher teachers, and 6) possession of the ability to perform other professional tasks [9].

Teacher is an important factor that affect the quality of education. Teacher responsibility requires four competencies namely professional competence, pedagogical competence, character and social competence. Prospective teacher education prepare the students to acquire all those competencies including pedagogical competence [10].

3. Method of Research
This research was conducted in the mathematics education study program of Cokroaminoto Palopo University. Subjects in this study were 3 students of the sixth semester. They are selected according to the result of mathematical initial skill test. Subject matter in this research is real analysis. It is chosen based on the curriculum, because real analysis can be programmed in that semester.

The research method used is qualitative research method. Qualitative research methods are often called naturalistic research methods because the research is done on natural conditions (natural settings). In this study, researchers examined the ability of students to solve real analysis problems and the difficulties experienced by students when working on the problem by using the definition of absolute value.

There are 3 (three) stages in the process of research qualitative as follows.

- Orientation stage or description with grand tour question. At this stage the researcher describes what is seen, heard, felt and asked. The data obtained has not been arranged clearly, so the information obtained by a glance and not too deep.
- Reduction or focus stage. At this stage the researcher reduces everything information that has been obtained in the first phase to focus on certain issues, which are subsequently grouped into the various categories defined as the focus of the study.
- Selection Stage. At this stage the researcher outlines the focus that has been set to be more detailed. After the researchers conducted a thorough analysis of the data and information obtained, the researchers can find the theme by way of constructing the data obtained into a building of knowledge, hypothesis or new science.

The end result of qualitative research is not merely to produce data or information that is difficult to obtain by quantitative research methods, but also must be able to produce meaningful information that can be used to overcome the problem and improve human life with hypothesis or new science obtained from research conducted[11].

4. Result And Discussion
4.1 Lecturer Performance in The Class
In this case the lecturer teaches the real analysis course with absolute value definition. The lecturer start the lesson by giving a problem to student like “how many numbers that can be replaced “x” if I give you the equation $|2x + 3| < 7$?. Such problem give challenge for the prospective teacher to high mathematical thinking. This problem drive them to analyze and evaluate the problem. Those processes defined as Higher order thinking [12].
The problem that mentioned before are not measure high mathematical thinking if the prospective teacher using the theorem to solve it. It easy to solve using (3). We just modify the inequality such $2x + 3 < 7$ if and only if $−7 < 2x + 3 < 7$, and by using the simple algebraic operation we get the solution that is $−5 < x < 2$. These process just practice remembering, understanding, and applying as the lower order thinking of the prospective teacher [13].

To make that problem meaningful for the students, the lecturer solve the problem using definition of absolute value. By using the definition the prospective teacher think more high than use theorem. Problems must be challenging to attract students, but not too difficult that frustrates students. When students get stuck or stalled, lecturers need to intervene so that students can solve problems, and not avoid them. However, in some cases, the lecturer must find a solution to the problem. Lecturers should, whenever possible, listen to what students have found and try to build on their ideas [14].

The lecturer leads the students to develop thinking skills, problem-solving skills, and students’ intellectual skills, because the problems that given to students aim to acquire problem-solving skills [15].

4.2 Students Work by Definition of Absolute Value

The first lesson on the absolute value topic is proving the theorem. There is no Real Analysis book which show complete proof process even though Bartle Book. One example from bartle ini proving the theorem $|ab| = |a||b|$. If either $a$ or $b$ is $0$, then both sides are equal to $0$. There are four other cases to consider. If $a > 0; b > 0$, then $ab > 0$, so that $|ab| = ab = |a||b|$. If $a > 0, b < 0$, then $ab < 0$, so that $|ab| = −ab = a(−b) = |a||b|$. The remaining cases are treated similarly. The solution mention before likely easy, but hard to understand.

A problem given to students as follows

Find the set solution of $B = \{x \in \mathbb{R}; |x − 1| < |x|\}$.

The figure below show that the prospective teacher use the definition of absolute value as initial stage of problem solving.

![Figure 1](image1.png)

**Figure 1.** Prospective teachers define the problem

The prospective teacher define $|x − 1|$ as two algebraic form that is $x − 1$ then $x > 1$ and $−x − 1$ then $x < 1$. It means that, the prospective teacher have a good understanding in using definition of absolute value. The same case are doing by prospective teacher when define $|x|$.

![Figure 2](image2.png)

**Figure 2.** Prospective teachers result

The figure above show that, the prospective teacher thinking about how to find the solution set of the algebraic form that found before. The prospective teacher use probability to solve the problem. More prospective teachers only can reach this steps. They can not find the solution set although they need one step more. The next step is shown bellow.

![Figure 3](image3.png)

**Figure 3.** Prospective final result
According to the figure this prospective teacher can find the solution set of the given problem that is 
\( x > \frac{1}{2} \). This problem are not too difficult than others because it only exactly 1 solution set in that 
probability. There are some problem that need the prospective teacher to think more critically. Such 
how to find the intersection of solution set.

4.3 *Prospective teacher difficulties in solving problem using absolute value definition.*

There are some problem are given to the prospective teacher. The following list are type of problem 
found in solving using definition of absolute value.

- A little of prospective teacher can not using the definition absolute value. This is the first step 
in solving the problem. It means that this prospective teacher cannot solve the problem at all.
- More student can solve the problem until step 2. They only gain the new algebraic form using 
the definition of absolute value. More student are understand the use of definition.
- More of the prospective teacher can solve the problem complete.

5. Discussion

Students quickly understand about the definition of absolute value depends on the learning method 
that chosen by the lecturer in teaching. The challenging problem make the student motivated to 
thinking and solve it. In line with that, The following characteristics for a thinking classroom: frequent 
questions, engagement in active learning, challenging students beyond their comfort zones and making 
them aware of the uncertainty in conclusions. In the framework of this study, three strategies are going 
to be considered as fostering critical thinking in higher education settings: explicitly targeting critical 
thinking components, active learning, and reflection [16].
The deficiency were found that is a little of prospective teacher cannot solve the problem at all. The 
researcher assume that they need more time and practice to solving such problem.
The use of sections to divide the text of the paper is optional and left as a decision for the author. Where the author wishes to divide the paper into sections the formatting shown in table 2 should be 
used.

6. Conclusion

The study shows that (1) there are some prospective teachers have difficulties to form the problem as 
definition of absolute value, (2) they cannot conclude the final solution set for two or more solution 
set. This research suggest to choose the suitable learning method to improve the prospective 
mathematics teacher in solving real analysis problem.

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