1 Physics issues at LEP2

Recently LEP2, the second stage of the LEP program, has started operation. After a first short run around 130 GeV, the energy has been increased to the nominal threshold for the production of W-boson pairs. This opens the possibility of accurately measuring the W-boson mass \( M_W \). Two methods are advocated for this measurement. The first procedure involves a measurement of the total W-pair cross-section \( \sigma(e^+e^- \rightarrow W^+W^- \rightarrow 4f + n\gamma) \) at an energy close to 161 GeV, i.e., just above the nominal threshold. In this energy region the cross-section \( \sigma(e^+e^- \rightarrow 4f + n\gamma) \) is very sensitive to \( M_W \) through phase-space. Since for this measurement a precise knowledge of the total cross-section is required, this method is model-dependent. During last year’s LEP2 working-group studies the conclusion was reached that an experimental error on the reconstructed W-boson mass \( \Delta M_W \) at LEP2 constitutes a significant improvement on the present hadron-collider measurements \( \Delta M_W \approx 125 \text{MeV} \). Since the mass of the W boson is one of the key parameters of the electroweak theory, such an improved accuracy makes the tests on the Standard Model (SM) of electroweak interactions more stringent.

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THEORETICAL ASPECTS OF GAUGE-BOSON PRODUCTION AT LEP2 AND THE NLC

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A report is given on the theoretical aspects of gauge-boson production processes at LEP2 and the next linear collider (NLC). A short discussion is given on the gauge-boson-related physics issues that play a role at both colliders and on the outcome of the various LEP2/NLC studies that have taken place during the last few years. The main emphasis of this report is on the question of preserving gauge-invariance when dealing with unstable gauge bosons. In this context a strategy is proposed for treating radiative corrections.

The envisaged precision of 40–50 MeV in the determination of \( M_W \) at LEP2 constitutes a significant improvement on the present hadron-collider measurements \( \Delta M_W \approx 125 \text{MeV} \). Since the mass of the W boson is one of the key parameters of the electroweak theory, such an improved accuracy makes the tests on the Standard Model (SM) of electroweak interactions more stringent.

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fermion masses, also $M_W$ is treated as input (fit parameter). From muon decay the mass of the top quark can be calculated as a function of the Higgs-boson mass $M_H$ and the strong coupling $\alpha_s$:

$$G_\mu = \frac{\alpha_s/\sqrt{2}}{M_W^2 (1 - M_Z^2/M_W^2)} \frac{1}{1 - \Delta r(m_t, M_H, \alpha_s)}.$$ 

The so-obtained top-quark mass can then be confronted with the direct bounds from the Tevatron and the indirect ones from the precision measurements at LEP1/SLC. In this way improved limits on $M_H$ can be obtained.

The second piece of information that LEP2 can provide is the structure of the triple gauge-boson couplings (TGC). These couplings appear at tree level in LEP2 processes like $e^+e^- \to 4f$ or $e^+e^- \to 2f + \gamma$, in contrast to LEP1 where they only entered through loop corrections. The largest experimental sensitivity to the TGC is achieved by going to the highest available energy and by investigating angular distributions of the gauge bosons and their subsequent decay products. The theoretical uncertainties associated with these distributions are estimated to be 1–2%, originating from the implementation of ISR and from missing $O(\alpha)$ corrections. Assuming an energy of 190 GeV and an integrated luminosity of 500 pb$^{-1}$/exp, a determination at the level of $\Delta(TGC) = 0.05$–0.1 seems feasible, provided one applies symmetry or operator-hierarchy arguments to reduce the number of independent couplings (to one or two at most). Based on these studies it is safe to say that the Yang–Mills character of the TGC can be established at LEP2. Since the TGC are at the heart of non-abelian theories, this information is essential for a direct confirmation of the SM or for providing a window to physics beyond the SM.

2 Physics issues at the NLC

The afore-mentioned LEP2 TGC studies can much more efficiently be performed at the NLC. This is based on the notion that couplings different from the SM Yang–Mills ones, called anomalous or non-standard couplings, will in general upset the intricate gauge cancelations for longitudinal gauge bosons and lead to a high-energy behavior of the cross-sections increasing with energy and thereby violating unitarity. At LEP2 the energy is too low to see such effects, but at the NLC, with its energy in the range 500–2000 GeV, this feature can be fully exploited. Combined with the increased luminosity at the NLC this allows a determination of the TGC below the percent level, i.e., in the range predicted by specific models. Moreover, the $e\gamma$ and $\gamma\gamma$ modes of the NLC lead to an increased sensitivity to individual couplings (in processes like $e\gamma \to 3f$).

At the NLC one can go one step further. The high energies open the possibility of studying quartic gauge-boson couplings (QGC) in reactions like $e^+e^- \to 6f$ or $\gamma\gamma \to 4f$, thereby entering the realm of the symmetry-breaking mechanism. On top of that one can look for signs of a strongly-interacting symmetry-breaking sector (i.e., resonances or phase shifts), by studying longitudinal gauge-boson interactions (rescattering) in $e^+e^- \to 4f$, $6f$ and $\gamma\gamma \to 4f$. In other words, one of the most important tasks to be performed at the NLC is a detailed investigation of the Higgs sector or any alternative thereof.

3 Gauge-invariant treatment of unstable gauge bosons

3.1 Lowest order

The above-described physics issues all involve an investigation of processes with photons and/or fermions in the initial and final state. If complete sets of graphs contributing to a given process are taken into account, the associated matrix elements are in principle gauge-invariant. However, the gauge bosons that appear as intermediate particles can give rise to poles $1/(k^2 - M^2)$ if they are treated as stable particles. This can be cured by introducing the finite decay width in one way or another, while at the same time preserving gauge independence and, through a proper high-energy behavior, unitarity. In field theory, such widths arise naturally from the imaginary parts of higher-order diagrams describing the gauge-boson self-energies, resummed to all orders. This procedure has been used with great success in the past: indeed, the $Z$ resonance can be described to very high numerical accuracy. However, in doing a Dyson summation of self-energy graphs, we are singling out only a very limited subset of all the possible higher-order diagrams. It is therefore not surprising that one often ends up with a result that retains some gauge dependence.

Till recently two approaches were popular
in the construction of lowest-order LEP2/NLC Monte Carlo generators. The first one involves the systematic replacement $1/(k^2 - M^2) \rightarrow 1/(k^2 - M^2 + \imath \Gamma)$, also for $k^2 < 0$. Here $\Gamma$ denotes the physical width of the gauge boson with mass $M$ and momentum $k$. This scheme is called the ‘fixed-width scheme’. As in general the resonant diagrams are not gauge-invariant by themselves, this substitution will destroy gauge invariance. Moreover, it has no physical motivation, since in perturbation theory the propagator for space-like momenta does not develop an imaginary part. Consequently, unitarity is violated in this scheme. To improve on the latter another approach can be adopted, involving the use of a running width $\imath \Gamma(k^2)$ instead of the constant one $\imath \Gamma$ (‘running-width scheme’). This, however, still cannot cure the problem with gauge invariance.

At this point one might ask oneself the legitimate question whether the gauge-breaking terms are numerically relevant or not. After all, the gauge breaking is caused by the finite decay width and is, as such, in principle suppressed by powers of $\Gamma/M$. From LEP1 we know that gauge breaking can be negligible for all practical purposes. However, the presence of small scales can amplify the gauge-breaking terms. This is for instance the case for almost collinear space-like photons or longitudinal gauge bosons at high energies, involving scales of $\mathcal{O}(p_{\nu}^2/E_{\nu}^2)$ (with $p_{\nu}$ the momentum of the involved gauge boson). In these situations the external current coupled to the photon or to the longitudinal gauge boson becomes approximately proportional to $p_{\nu}$. In other words, in these regimes sensible theoretical predictions are only possible if the amplitudes with external currents replaced by the corresponding gauge-boson momenta fulfill appropriate Ward identities.

In order to substantiate these statements, a truly gauge-invariant scheme is needed. It should be stressed, however, that any such scheme is arbitrary to a greater or lesser extent: since the Dyson summation must necessarily be taken to all orders of perturbation theory, and we are not able to compute the complete set of all Feynman diagrams to all orders, the various schemes differ even if they lead to formally gauge-invariant results. Bearing this in mind, we need some physical motivation for choosing a particular scheme. In this context two options can be mentioned, which fulfill the criteria of gauge invariance and physical motivation. The first option is the so-called ‘pole scheme’. In this scheme one decomposes the complete amplitude according to the pole structure by expanding around the poles (e.g. $f(k^2)/(k^2 - M^2) = f(M^2)/(k^2 - M^2) + \text{finite terms}$). As the physically observable residues of the poles are gauge-invariant, gauge invariance is not broken if the finite width is taken into account in the pole terms $\propto 1/(k^2 - M^2)$. It should be noted, however, that there exists some controversy in the literature about the ‘correct’ procedure for doing this and about the range of validity of the pole scheme, especially in the vicinity of thresholds. The second option is based on the philosophy of trying to determine and include the minimal set of Feynman diagrams that is necessary for compensating the gauge violation caused by the self-energy graphs. This is obviously the theoretically most satisfying solution, but it may cause an increase in the complexity of the matrix elements and a consequent slowing down of the numerical calculations. For the gauge bosons we are guided by the observation that the lowest-order decay widths are exclusively given by the imaginary parts of the fermion loops in the one-loop self-energies. It is therefore natural to perform a Dyson summation of these fermionic one-loop self-energies and to include the other possible one-particle-irreducible fermionic one-loop corrections (‘fermion-loop scheme’).

For the LEP2 process $e^+e^- \rightarrow 4f$ this amounts to adding the fermionic triple gauge-boson vertex corrections. The complete set of fermionic contributions forms a gauge-independent subset and obeys all Ward identities exactly, even with resummed propagators. As mentioned above, the validity of the Ward identities guarantees a proper behavior of the cross-sections in the presence of collinear photons and at high energies in the presence of longitudinal gauge-boson modes. On top of that, within the fermion-loop scheme the appropriately renormalized matrix elements for the generic LEP2 process $e^+e^- \rightarrow 4f$ can be formulated in terms of effective Born matrix elements using the familiar language of running couplings.

A numerical comparison of the various schemes confirms the importance of not violating the Ward identities. For the LEP2 process $e^+e^- \rightarrow e^-\bar{\nu}_e\bar{u}d$, a process that is particularly important for TGC studies, the impact of violating the $U(1)$ electromagnetic gauge invari-
ance was demonstrated. Of the above-mentioned schemes only the running-width scheme violates $U(1)$ gauge invariance. The associated gauge-breaking terms are enhanced in a disastrous way by a factor of $\mathcal{O}(s/m^2)$, in view of the fact that the electron may emit a virtual (space-like) photon with $p^2$ as small as $m^2$. A similar observation can be made at high energies (NLC) when some of the intermediate gauge bosons become effectively longitudinal. There too the running-width scheme becomes unreliable.

In processes involving more intermediate gauge bosons, e.g., $e^+e^- \to 6f$, also the fixed-width scheme breaks down at high energies as a result of breaking $SU(2)$ gauge invariance.

### 3.2 Radiative corrections

By employing the fermion-loop scheme all one-particle-irreducible fermionic one-loop corrections can be embedded in the tree-level matrix elements. This results in running couplings, propagator functions, vertex functions, etc. However, there is still the question about the bosonic corrections. A large part of these bosonic corrections, as e.g. the leading QED corrections, factorize and can be treated by means of a convolution, using the fermion-loop-improved cross-sections in the integration kernels. This allows the inclusion of higher-order QED corrections and soft-photon exponentiation. In this way various important effects can be covered, as e.g. the large negative soft-photon corrections near the nominal W-pair threshold, the distortion of angular distributions as a result of hard-photon boost effects, and the average energy loss due to radiated photons. Nevertheless, the remaining bosonic corrections can be large, especially at high energies.

In order to include these corrections one might attempt to extend the fermion-loop scheme. In the context of the background-field method a Dyson summation of bosonic self-energies can be performed without violating the Ward identities. However, the resulting matrix elements depend on the quantum gauge parameter at the loop level that is not completely taken into account. As mentioned before, the perturbation series has to be truncated; in that sense the dependence on the quantum gauge parameter could be viewed as a parametrization of the associated ambiguity.

As a more appealing strategy one might adopt a hybrid scheme, adding the remaining bosonic loop corrections by means of the pole scheme. This is gauge-invariant and contains the well-known bosonic corrections for the production of on-shell gauge bosons (in particular W-boson pairs). Moreover, if the quality of the pole scheme were to degrade in certain regions of phase-space, the associated error is reduced by factors of $\alpha/\pi$. It should be noted that the application of the pole scheme to photonic corrections requires some special care, because in that case terms proportional to $\log(k^2 - M^2)/(k^2 - M^2)$ complicate the pole expansion.

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### References

1. Z. Kunszt et al., in *Physics at LEP2*, eds. G. Altarelli, T. Sjöstrand and F. Zwirner, (CERN 96-01, Genèве, 1996) Vol. 1, p. 141, [hep-ph/9602352](http://arxiv.org/abs/hep-ph/9602352).
2. W. Beenakker et al., in *Physics at LEP2*, eds. G. Altarelli, T. Sjöstrand and F. Zwirner, (CERN 96-01, Genèве, 1996) Vol. 1, p. 79, [hep-ph/9602351](http://arxiv.org/abs/hep-ph/9602351).
3. W. Beenakker and G.J. van Oldenborgh, *Phys. Lett.* **B381** (1996) 248.
4. M. Rijssenbeek, these proceedings.
5. G. Gounaris et al., in *Physics at LEP2*, eds. G. Altarelli, T. Sjöstrand and F. Zwirner, (CERN 96-01, Genèве, 1996) Vol. 1, p. 525, [hep-ph/9601233](http://arxiv.org/abs/hep-ph/9601233).
6. P.M. Zerwas (ed.), $e^+e^-$ collisions at 500 GeV: The physics potential (DESY 93-123C, Hamburg, 1993).
7. M. Veltman, *Physica* **29** (1963) 186.
8. R.G. Stuart, *Phys. Lett.* **B262** (1991) 113.
9. A. Aeppli, G.J. van Oldenborgh and D. Wyler, *Nucl. Phys.* **B428** (1994) 126.
10. E.N. Argyres et al., *Phys. Lett.* **B358** (1995) 339.
11. W. Beenakker et al., [hep-ph/9612260](http://arxiv.org/abs/hep-ph/9612260).
12. W. Beenakker and A. Denner, *Int. J. Mod. Phys.* **A9** (1994) 4837.
13. A. Denner and S. Dittmaier, *Phys. Rev.* **D54** (1996) 4499.