Improvement students’ achievement in elementary linear algebra through APOS theory approach

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Abstract. This study aimed to improve students’ achievement in elementary linear algebra through APOS theory approach. This study is pre-experimental one-shot case study design. The sample were 32 students who participating in the lecture of chemistry mathematics II academic year 2018/2019 at Andalas University. Instrument used in this study was problem solving test in elementary linear algebra which consisted of four item test that measures the ability to solve daily life problem with system of linear equation, determine determinant of the matrix, determine invers of matrix, and determine eigen value and eigen vector of matrix. Descriptive statistics and t-test with α = 0.05 level of significance were used for data analysis. The results show that: (1) APOS theory approach can improve students’ achievement in elementary linear algebra significantly and (2) there was no a significant gender gap in students’ achievement in elementary linear algebra.

1. Introduction

Elementary linear algebra (ELA) is one of the subjects that are taught in department of chemistry at Andalas University with courses name Mathematics Chemistry II. Based on the experience of teaching the ELA since the year 2012 was obtained information that the average of student learning outcomes around 50 to 60 with the percentage of students have passed above “B” grade always under 40%. Low learning outcomes in the ELA does not just happen in Indonesia, but also in other countries (for example see [1, 2]), One of the reasons is most of students have difficulty in making mathematical model for a problem in everyday life [3].

There are some things that are suspected as the cause of the low achievement in ELA, based on the interviews of some students at Andalas University was obtained information: (1) text book that used as a lecture’s reference book difficult to understand by the student; (2) the students felt that the practice of usage of definitions and theorems in the classroom has not been adequately; (3) students were less motivated in learning ELA.

In order for text book can help students in studying a course, text books must meet the quality of a good text book. Gracin [4] argues that there are five indicators of a good text book, namely: (1) contents (text book contains definitions, theorems, examples and exercises are in accordance with the syllabus); (2) mathematical activities (text book load of mathematical activities in the form of representations, modelling, calculation, interpretation, argumentation, and reasoning); (3) complexity levels (examples of solved problems, exercises, and mathematics’ activities in the text book begins from the most simple to the most complex); (4) Answer forms (involving various forms of answers in
examples and exercises, e.g. open-ended, closed-ended, multiple choice, and true or false); (5) Contextual features (real-world experiences are incorporated into the tasks in textbook).

Many studies show that there was a correlation between learning motivation with learning outcomes, for example Herges et al. [5] argues that improving learning outcomes can be done through increased motivation to learn. There are several strategies to increase the motivation of learning, for example: (1) using active learning approach or cooperative learning [6-8], because, according to Stanberry [9], active learning can promote greater comprehension for students in science, technology, engineering, and mathematics (STEM) and (2) using ICT in learning mathematics [10], because ICT really help students in learning ELA [11,12].

Mathematics at the college level should be studied with the following stages: (1) understanding the definition; (2) understanding the theorems or lemmas and its proof process; (3) do exercise in using of definitions, lemmas, or theorems in mathematical problem solving. In practice, during the lecture, the lecturer often forget provides time for students to do the exercises. In order to become a reliable problem solver in mathematics is required creativity, intuition, and experience. It all can be obtained through a lot of practice in the classroom.

APOS theory approach is a teaching and learning approach that is specifically for the learning of mathematics at the college level that integrates the use of computer/ICT, cooperative learning, and pay attention to the mental construction of college students in understanding a mathematical concept. APOS theory postulated that the mathematical knowledge and skills is the result of student-owned construction (through construction of mental actions, process, object, and schemas are abbreviated with APOS) and the results of the interaction of the students with others in understanding mathematical ideas [13]. Interactions between students (students with less clever and clever) and interaction between students with lecturer is intended to develop what is meant by Vygotsky referred to as zone of proximal development.

The learning cycle in the APOS theory approach include: (1) computer laboratory activities, (2) classroom discussion, and (3) exercise. Through the laboratory activity, students can reduce abstract mathematical concept becomes more concrete, class discussion activities provide an opportunity to students to ask questions, share strategies or ways in solving mathematics. Arguing in classroom discussion is an exercise that is very valuable in attempt to improve ability of students in deductive reasoning, whereas the activities of exercise are intended to give student opportunity to apply definitions and theorems in mathematics problem solving.

Based on the characteristics of the APOS theory approach and the problems in teaching and learning of ELA, this study aimed to answer the questions research as follows: (1) is APOS theory approach can improve students achievement in ELA? If so, whether that improvement is significant?, (2) is there a gender gap in ELA achievement?

2. Methods
This study using quantitative research methods with pre-experimental one-shot case study design. To answer the first question research, several stages were done are as follows: (1) develop valid, practice, and effective learning materials based on APOS theory approach (2) developing a problem solving tests in ELA and its rubric scoring, (3) carry out lecture ELA based on APOS theory approach, (4) carry out a problem solving test in ELA, (5) analyze students score in problem solving test. To answer the second question research, several stages were done are as follows: (1) calculating the average and standard deviation of the overall score, (2) do a Shapiro-Wilk normality test for overall score, (3) do a one-sample t test at level of significance $\alpha = 0.05$ for overall score, and to answer the third question research, several stage were done as follows: (1) do a Shapiro-Wilk normality test for male and female score, (2) do a homogeneity test for male versus female score, (3) do a two-independent sample t-test. All data were analyzed using SPSS 17.0 at level of significance $\alpha = 0.05$. 


3. Result dan Discussion
Based on the results of the test in the ELA against 32 students was gained a mean score, standard deviation, and the percentage of students that have passed above “B” grade is stated in table 1.

Table 1. Mean score, standard deviation and number or percentage of students that have passed above “B” grade.

| Number of Student | $\bar{X}$ | SD  | Number and percentage of students that have passed above “B” grade. |
|-------------------|----------|-----|---------------------------------------------------------------|
| Overall           | 32       | 68.7500 | 15.85010                                                     |
|                   |          |       | 17 (53.125)                                                  |

Table 1 showed that the average of students score that learning ELA based on APOS theory approach more than 60.

Next, to find out whether the mean score 68.7500 different from 60 significantly, Shapiro-Wilk normality test and one -sample t-test were done for overall score, the result were stated in table 2 and table 3.

Table 2. Result of normality test for overall score with Shapiro-Wilk

| Data              | Shapiro-Wilk Statistic | df | Sig.          | decision                  |
|-------------------|------------------------|----|---------------|---------------------------|
| Overall Score     | .941                   | 32 | .078          | so overall score was normal (.078 >0.05) |

Table 3. Result of one -sample t-test on overall score

| Data            | t-statistic | df | Sig. (2-tailed) | decision                  |
|-----------------|-------------|----|-----------------|---------------------------|
| Overall Score   | .941        | 31 | .004            | 68.7500 different from 60 significantly (.004 <0.05) |

Based on the results of the test in the ELA against 14 male and 18 female students, a mean score, standard deviation, and the percentage of students that have passed above “B” grade is stated in table 4.

Table 4. Mean score, standard deviation and number or percentage of male and female students that have passed above “B” grade.

| Gender | Number of Student | $\bar{X}$ | SD | Number and percentage of students that have passed above “B” grade. |
|--------|-------------------|----------|----|---------------------------------------------------------------|
| Male   | 14                | 68.5714  | 18.23232 | 6 (18.75) |
| Female | 18                | 68.8889  | 14.27828 | 11 (34.375) |

Futhermore, to find out whether the average of male and female score respectively 68.5714 and 68.8889 different significantly, a Shapiro-Wilk normality test, Levene's homogeneity test of variant, and two independent sample t-test were done for male and female score, The result were stated in table 5, table 6 and table 7.

Table 5. Result of normality test for male score and female score with Shapiro-Wilk

| Data    | Shapiro-Wilk Statistic | df | Sig. | decision                  |
|---------|------------------------|----|------|---------------------------|
| Male    | .939                   | 13 | .449 | male score was normal (.449 >0.05) |
| Female  | .925                   | 18 | .156 | female score was normal (.156 >0.05) |
Table 6. Result of the Levene's Test for Equality of Variances

| Data                    | Levene-statistic | df1 | df2 | Sig.     | decision                                    |
|-------------------------|------------------|-----|-----|----------|---------------------------------------------|
| Male score vs Female Score | 1.019            | 1   | 30  | .321     | variance of male score equal to female score (.321 > 0.05) |

Table 7. Result of two independent sample t-test on male and female score

| Data                    | t-statistic | df | Sig. (2-tailed) | decision                                      |
|-------------------------|-------------|----|-----------------|-----------------------------------------------|
| Male Score vs Female Score | -.055       | 30 | .956            | mean score of male and female not different significantly (.956 > 0.05) |

Based on table 1 and table 3, mean score of achievement in ELA was increased significantly from 60 to 68.7500, there were at least five reasons why APOS theory approach can improve students achievement in ELA.

First, learning materials that used in the lecture of ELA in this research have been designed specifically based on APOS theory approach, specifically for the learning materials on the topic of system of linear equation (SLE), the materials already meet the criteria valid, practical and effective [14], so the learning materials already meet the five criteria of a good learning materials as has been said by Gracin [4]. Thus, the learning materials that was developed based on APOS theory approach is very helpful for student in understanding ELA.

Second, APOS theory approach using the learning cycle ACE, i.e. there are special sessions for exercise, in this exercise session students are given the opportunity to practice the definitions, theorems, or lemmas in mathematical problem solving. This is a good opportunity for students to hone their skills in ELA. If there are students who have difficulty in this exercise sessions, the students can ask the lecturer or his group, so ultimately student is expected to complete the math problems. A structured and adequate exercise will help students in honing their skills in ELA, and [15,16].

Third, teaching and learning based on APOS theory approach using the computer, in this study computer is used by students to check the truth of the answers of the exercise. The existence of a computer in teaching and learning will allow students can try and try again to solve the exercise up to gained the correct answer, This is in accordance with the opinion of Zakaria & Khalid [10] that the use of ICT/computer in teaching and learning can motivate students to learn ELA. In addition, according to Ulus [11] and Caglayan [12], the use of ICT/computer can assist students in understanding the ELA, so finally can improve students achievement in ELA.

Fourth, in APOS theory approach, students actively involved during the learning process, that is active in understanding definitions and theorems nor in resolving the problems in ELA. For example, in finding the characteristics of SLE that has a single solution, many solution, and have no solution, students are actively involved in doing some activity in student's worksheet, then together with his group, students made conclusions about the characteristics of SLE with one, many, or have no solution. According to Stanberry [9], active learning approach can improve students understanding in STEM.

Fifth, APOS theory approach using cooperative learning, so that every student had a great opportunity to interact with one another in order to develop own skills in ELA, so every students is expected have no difficulties in learning ELA. This will result in the rise of the learning motivation of students in ELA. According to Entonado & Garcia [8] cooperative learning can enhance the learning motivation of students.

Based on table 4 and table 7, was obtained that the mean score of male and female student greater than 60 and mean score of female students greater than male students, but this differences not
This shows that APOS theory approach suitable for male and female students. The findings of this study is consistent with the research of Ajai & Imoko [17] and Manalaysy [18], that is female students outperformed their male counterparts but the difference is not statistically significant. But, especially for learning mathematics in school level, the opposite results occur, for example Tao & Michalopoulos [19] and Contini, Di Tommaso & Mendolia [20] found that female students tend to underperform with respect to male students.

4. Conclusion
Based on the result and discussion of this study can be concluded that: (1) APOS theory teaching and learning approach can improve student achievement in elementary linear algebra significantly; (2) There is no gender gap in elementary linear algebra achievement.

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