I. Introduction

Quantum information has allowed us to elucidate quantum physics on the basis of information theory. The understanding of correlation in quantum state has been a central theme in the study. Insights regarding entangled quantum state naturally lead us to apply the basic concepts in quantum information to quantum communication. A well known example is quantum cryptography. A basic resource for quantum communication is an entangled quantum state, which cannot be obtained by local operations and classical communications (LOCC) between parties who share the quantum states.

From another perspective, quantum information in non-inertial frame has also been studied [1,2,3,4,5,6,7,8]. When the measurement based quantum state in an accelerated frame is considered, one of most important aspects about the quantum state is that the entangled quantum states of bosonic system in an accelerated frame have vanishing entanglement at the infinite acceleration limit; however the entangled quantum states of fermionic system in accelerated frame have non-zero entanglement in the same limit.

The quantum discord is regarded as another way to understand the correlation of quantum states. Discord is obtained through the quantum correlation after removing classical correlation so that the resulting correlation displays only the pure correlation with quantumness. Studying quantum discord of a quantum state in an accelerated frame can be important for a better understanding quantum correlation of a state [9,10]. Specifically, the behavior of quantum states at the infinite acceleration limit should be understood since such analysis could provide insight into how quantum states behave near black holes for instance. Discord of the quantum state of fermionic system in an accelerated frame has been studied within single-mode approximation [11].

There are actually different ways to compute quantum discord. In our study, we restrict our measure to geometric discord. It is found that the geometric discord for entangled quantum states of fermionic system in accelerated frame need not vanish even at the infinite acceleration limit when the particle(Alice)-particle(Bob in region I) case or the particle(Alice)-antiparticle(Bob in region II) is considered and it disappears when the particle(Alice)-antiparticle(Bob in region I) case or the particle(Alice)-particle(Bob in region II) one is considered.

II Accelerated Frame

An accelerated frame is best described by invoking Rindler coordinates (τ, ς, y, z), instead of employing the usual Minkowski coordinates (t, x, y, z). Rindler coordinates can be written as

\[ \begin{align*}
ct &= \varsigma \sinh \left( \frac{a\tau}{c} \right), x = \varsigma \cosh \left( \frac{a\tau}{c} \right) \\
ct &= -\varsigma \sinh \left( \frac{a\tau}{c} \right), x = -\varsigma \cosh \left( \frac{a\tau}{c} \right)
\end{align*} \]

where \( a \) denotes the fixed acceleration of the frame and \( c \) is the velocity of light. In fact, Eq(1) covers only the right wedge (called region I) and Eq(2) describes the left wedge (called region II).

The field in Minkowski and Rindler spacetime is written as

\[ \phi = N_M \sum_i (a_{i,M} v_{i,M}^+ + b_{i,M} v_{i,M}^-) \]

\[ = N_R \sum_j (a_{j,I} v_{j,I}^+ + b_{j,I} v_{j,I}^- + a_{j,II} v_{j,II}^+ + b_{j,II} v_{j,II}^-) \]

Here \( a_{i,\Delta}(a_{i,\Delta}) \) and \( b_{i,\Delta}(b_{i,\Delta}) \), which satisfy the anticommutation relations, are the creation (annihilation) operators for the positive and negative energy solutions (particle and antiparticle) and \( \Delta \) denotes \( M, I, II \).
A combination of Minkowski mode, called Unruh mode, can be transformed into single Rindler mode and can annihilate the same Minkowski vacuum, satisfying the relation
\[ A_{\Omega R/L} = \cos r_{\Omega} q_{\Omega L/II} - \sin r_{\Omega} q_{\Omega I/II} \]
where \( r_{\Omega} = (e^{-2\alpha_{\Omega}} + 1)^{-1/2} \). However we may obtain more general relation such as
\[ a_{\Omega,U} = q_L(A_{\Omega,L} \otimes I_R) + q_R(I_L \otimes A_{\Omega,R}), \]
(5)
beyond the single mode approximation. Using this relation, in the case of Grassmann scalar, the Unruh vacuum and the one-particle state is given by
\[ |0_{\Omega,U} = \cos^2 r_{\Omega} |0000 \rangle_{\Omega} - \sin r_{\Omega} \cos r_{\Omega} |0110 \rangle_{\Omega} + \sin r_{\Omega} \cos r_{\Omega} |1001 \rangle_{\Omega} - \sin^2 r_{\Omega} |1111 \rangle_{\Omega} \]
\[ |1_{\Omega,U} = q_R(\cos r_{\Omega} |1000 \rangle_{\Omega} - \sin r_{\Omega} |1110 \rangle_{\Omega}) + q_L(\sin r_{\Omega} |1011 \rangle_{\Omega} + \cos r_{\Omega} |0010 \rangle_{\Omega} \]
(6)
where we have used the notation \(| |p_{\Omega} \rangle_I \rangle q_{\Omega} \rangle j |m_{\Omega} \rangle_I \rangle n_{\Omega} \rangle j \). Here we consider \( q_{\Omega} \) and \( q_L \) as real number. Throughout this paper, we study within the fermionic structure which also underlies physical framework proposed by [14-16].

III Geometric discord of quantum state

The quantum discord which was first introduced by Zurek et al for isolating quantum correlation from classical correlation[9][10]. The quantum correlation is obtained by considering the difference between two different ways of writing quantum mutual information. Quantum mutual information of quantum state shared by Alice and Bob can be given by
\[ I(A : B) = S(\rho_A) + S(\rho_B) - S(\rho_{AB}) \]
(7)
where \( S(\rho) = -\text{tr}(\rho \log_2 \rho) \) denotes the von Neumann entropy. Quantum mutual information can also be described by
\[ J(A : B) = \max_{\{\Pi_i\}}(S(\rho_B) - S(\{\Pi_i\})(B|A)) \]
(8)
where \( \{\Pi_i\} \) means a complete set of projection operator. Quantum discord is defined as
\[ D(A : B) = I(A : B) - J(A : B), \]
(9)
which can be interpreted as a pure quantum correlation of quantum state. Quantum discord can be also defined in a geometric way in which one can regard discord as the minimum distance between the state to the set of zero discord states \( \chi[\Pi] \):
\[ D_G = \min_{\chi \in \Omega_0} \| \rho_{AB} - \chi \|^2 \]
where \( \Omega_0 \) denotes the set of zero discord states and \( \| \| \) is the squared Hilbert-Schmidt norm. For computational convenience, the geometric discord is shown to be equivalent to
\[ D_G = \frac{1}{4} (\| x_I^T \|^2 + |T|^2 - \max{\{ \text{eigenvalues}(x_I^T x^T + T T^T) \}}) \]
(10)
where \( x_i = \text{tr}(\rho_{\sigma_i} \otimes I) \) and \( T_{ij} = \text{tr}(\rho_{\sigma_i} \otimes \sigma_j) \) and \( t \) denotes the transpose of vectors or matrices.

A. Two-party entangled state

We consider a generalized \( \Phi^+ \) state such as
\[ |\Phi^+ \rangle = \cos \alpha |00 \rangle + \sin \alpha |11 \rangle \]
(11)
Suppose two parties, Alice and Bob, initially prepare a generalized \( \Phi^+ \) state in an inertial frames and thereafter Bob moves in an accelerated frame. Since Bob has an accessible part due to his acceleration, the physical state that Alice and Bob(assuming particle in Bob’s region I) may share is given by \( \rho_{AB}^{\Phi^+} \), if we go beyond the single mode approximation. Likewise, we denote the physical state of Alice’s particle and Bob’s antiparticle in Bob’s region I, Alice’s particle and Bob’s particle in Bob’s region II, and Alice’s particle and Bob’s antiparticle in Bob’s region II as \( \rho_{AB}^{\Phi^+, \Phi^+, \Phi^+} \) and \( \rho_{AB}^{\Phi^+, \Phi^+, \Phi^+} \) respectively. The geometric discord of \( \rho_{AB}^{\Phi^+, \Phi^+, \Phi^+} \) can be shown to be
\[ D_G(\rho_{AB}^{\Phi^+, \Phi^+, \Phi^+}) = \frac{1}{4} [\cos^2 2\alpha + \cos 2\gamma \cos 2\alpha + (q_{LL}^2 - q_{RR}^2 \cos 2\gamma) \sin^2 2\alpha + q_{RR}^2 \cos^2 2\gamma \sin^2 2\alpha - \max(\cos^2 2\alpha + \cos 2\gamma \cos 2\alpha + (q_{LL}^2 - q_{RR}^2 \cos 2\gamma) \sin^2 2\alpha, q_{RR}^2 \cos^2 2\gamma \sin^2 2\alpha)] \]
\[ D_G(\rho_{AB}^{\Phi^+, \Phi^+, \Phi^+}) = \frac{1}{4} [\cos^2 2\alpha + \cos 2\gamma \cos 2\alpha + (q_{LL}^2 - q_{RR}^2 \cos 2\gamma) \sin^2 2\alpha + q_{RR}^2 \cos^2 2\gamma \sin^2 2\alpha - \max(\cos^2 2\alpha + \cos 2\gamma \cos 2\alpha + (q_{LL}^2 - q_{RR}^2 \cos 2\gamma) \sin^2 2\alpha, q_{RR}^2 \cos^2 2\gamma \sin^2 2\alpha)] \]
\[ D_G(\rho_{AB}^{\Phi^+, \Phi^+, \Phi^+}) = \frac{1}{4} [\cos^2 2\alpha + \cos 2\gamma \cos 2\alpha + (q_{LL}^2 - q_{RR}^2 \cos 2\gamma) \sin^2 2\alpha + q_{RR}^2 \cos^2 2\gamma \sin^2 2\alpha - \max(\cos^2 2\alpha + \cos 2\gamma \cos 2\alpha + (q_{LL}^2 - q_{RR}^2 \cos 2\gamma) \sin^2 2\alpha, q_{RR}^2 \cos^2 2\gamma \sin^2 2\alpha)] \]
\[ D_G(\rho_{AB}^{\Phi^+, \Phi^+, \Phi^+}) = \frac{1}{4} [\cos^2 2\alpha + \cos 2\gamma \cos 2\alpha + (q_{LL}^2 - q_{RR}^2 \cos 2\gamma) \sin^2 2\alpha + q_{RR}^2 \cos^2 2\gamma \sin^2 2\alpha - \max(\cos^2 2\alpha + \cos 2\gamma \cos 2\alpha + (q_{LL}^2 - q_{RR}^2 \cos 2\gamma) \sin^2 2\alpha, q_{RR}^2 \cos^2 2\gamma \sin^2 2\alpha)] \]
(12)
The geometric discord for the quantum states in the various regimes can be numerically evaluated and the behavior with acceleration is shown in Fig 1-2. Fig. 1 illustrates the quantum discord for the quantum states $\rho_{ABI}^+$ and $\rho_{ABII}^+$ and Fig. 2 shows the quantum discord for the quantum states $\rho_{ABI}^-\Phi_+$ and $\rho_{ABII}^-\Phi_+$. From the plots, we see that the quantum discord for $\rho_{ABI}^+$ and $\rho_{ABII}^+$ never vanish even at the infinite acceleration, which implies that quantum states $\rho_{ABI}^+$ and $\rho_{ABII}^+$ have non-zero quantum correlation even at the infinite acceleration. In case of $\rho_{ABI}^-\Phi_+$ and $\rho_{ABII}^-\Phi_+$, their quantum discord disappear at the infinite acceleration, meaning that there is no quantum correlation of $\rho_{ABI}^-\Phi_+$ and $\rho_{ABII}^-\Phi_+$ in the limit of infinite acceleration. It should be noted that as we have seen in the entanglement behavior of $\rho_{ABI}^\Phi_+$, $\rho_{ABII}^\Phi_+\Phi_+$, and $\rho_{ABII}^-\Phi_+$, the quantum discord of quantum state $\rho_{ABI}^\Phi_+$ ($\rho_{ABII}^\Phi_+$) coincides with that of $\rho_{ABII}^\Phi_+$ ($\rho_{ABII}^\Phi_+$) at the infinite acceleration.

So far we have considered the one-particle state described by $|1\Omega\rangle_U$. The one-particle state may also take the form

$$
|1\Omega\rangle_U = q_L(\cos\gamma_\Omega|0100\rangle_\Omega - \sin\gamma_\Omega|0111\rangle_\Omega) + q_R(\sin\gamma_\Omega|1110\rangle_\Omega + \cos\gamma_\Omega|0010\rangle_\Omega)
$$

(13)

FIG. 1: (Color online) The geometric discord of quantum states $\rho_{ABI}^\Phi_+$ and $\rho_{ABII}^\Phi_+$ when $\alpha = \frac{\pi}{4}$ and $\alpha = \frac{\pi}{12}$. The solid(dotted) lines (from top to bottom) denote the geometric discord of $\rho_{ABI}^\Phi_+$ and $\rho_{ABII}^\Phi_+$ at $q_R = 1$, $q_R = 0.75$ $q_R = 0.5$ and $q_R = 0.25$ respectively. The quantum states such as $\rho_{ABI}^\Phi_+$ and $\rho_{ABII}^\Phi_+$ show the non-vanishing geometric discord even at the infinite acceleration. Here $\gamma = \frac{\pi}{4}$ denotes the infinite acceleration.

In this case, one has a generalized $\Phi^-$ state given by

$$
|\Phi^-angle = \cos\alpha|00\rangle + \sin\alpha|11\rangle
$$

(14)

As in the previous analysis, suppose that two parties Alice and Bob prepare the new generalized $\Phi^-$ state in inertial frames and Bob moves in an accelerated frame afterwards. Since Bob has unaccessible part due to his acceleration, if we go beyond the single mode approximation, the physical state $\rho_{ABI}^\Phi_-$ of Alice and Bob(using particle in Bob’s region I) share is obtained by tracing out all other parts. In this way, one obtains the physical state $\rho_{ABII}^\Phi_-$ of Alice and Bob(using anti-particle in Bob’s region I), the physical state $\rho_{ABI}^\Phi_-$ of Alice and antiBob(using particle in Bob’s region II), and the physical state $\rho_{ABII}^\Phi_-$ of Alice and antiBob(using anti-particle in Bob’s region II). Using $\rho_{ABI}^\Phi_-$, $\rho_{ABII}^\Phi_-$, and $\rho_{ABII}^\Phi_-$, one can compute the mutual information, classical communication and geometric discord. It turns out that the behavior of quantum discord $\rho_{ABI}^\Phi_-$, $\rho_{ABII}^\Phi_-$, and $\rho_{ABII}^\Phi_-$ is the same as that of $\rho_{ABI}^\Phi_+\Phi_+$, $\rho_{ABII}^\Phi_+\Phi_+$, and $\rho_{ABII}^\Phi_+$ respectively.

**B. Two-party mixed entangled state**

Until now, we have considered quantum discord of pure states in fermionic system when one of parties travels with a uniform acceleration. We next briefly consider
a more general scenario when the two parties share a mixed state. Naturally, we find that how the behavior of quantum discord depends on the degree of mixedness. More specifically, we consider a Werner state, where a white noise component is added to a maximally entangled states. Suppose two parties Alice and Bob initially prepare a Werner state

\[ \rho_W = F|\Phi^+(\alpha = \pi/4)\rangle\langle\Phi^+(\alpha = \pi/4)| + \frac{1 - F}{4}I, \]  

\[ (15) \]

in inertial frame where the maximally entangled state \( |\Phi^+(\alpha = \pi/4)\rangle \) corresponds to \( \alpha = \pi/4 \) in Eq. \[ \text{[11]}, \] and then Bob moves in an uniformly accelerated frame.

FIG. 3: (Color online) The geometric discord of quantum states \( \rho_{AB_I^+}^W \) and \( \rho_{AB_{II}^-}^W \) when \( F = 0.9 \) and \( F = 0.6 \). The solid(dotted) lines (from top to bottom) denote the geometric discord of \( \rho_{AB_I^+}^W \) at \( q_R = 1, q_R = 0.75 \), \( q_R = 0.5 \) and \( q_R = 0.25 \) respectively. The quantum states such as \( \rho_{AB_I^+}^W \) and \( \rho_{AB_{II}^-}^W \) show the non-vanishing geometric discord even at the infinite acceleration. Here \( \gamma = \frac{\pi}{4} \) denotes the infinite acceleration.

Since Bob has unaccessible part due to his acceleration, the physical state shared between Alice and Bob(using particle in Bob’s region I)(Alice and Bob(using anti-particle in Bob’s region I)) are given by \( \rho_{AB_I^+}^W \) and \( \rho_{AB_{II}^-}^W \) beyond single mode approximation. Moreover, the physical states shared between Alice and Bob(using particle in Bob’s region II)(Alice and Bob(using anti-particle in Bob’s region II)) are \( \rho_{AB_{II}^+}^W \) and \( \rho_{AB_{II}^-}^W \). The geometric discord of \( \rho_{AB_I^+}^W, \rho_{AB_{II}^+}^W, \rho_{AB_I^-}^W \) and \( \rho_{AB_{II}^-}^W \) are found in Fig. 3 and Fig. 4. We observe that all quantum states show non-vanishing quantum mutual information and classical information even at the infinite acceleration but the geometric discsords of the quantum states such as \( \rho_{AB_I^+}^W \) and \( \rho_{AB_{II}^-}^W \) disappear at the infinite acceleration.

FIG. 4: (Color online) The geometric discord of quantum states \( \rho_{AB_I^+}^W \) and \( \rho_{AB_{II}^-}^W \) when \( F = 0.9 \) and \( F = 0.6 \). The solid(dotted) lines (from top to bottom) denote the geometric discord of \( \rho_{AB_I^+}^W \) at \( q_R = 1, q_R = 0.75 \), \( q_R = 0.5 \) and \( q_R = 0.25 \) respectively. The geometric discord of the quantum states such as \( \rho_{AB_I^+}^W \) and \( \rho_{AB_{II}^-}^W \) disappear at the infinite acceleration. Here \( \gamma = \frac{\pi}{4} \) denotes the infinite acceleration.

III. Discussion and Conclusion

We have investigated the geometric discord of entangled states such as \( \Phi^+, \Phi^- \) and the Werner state for fermionic systems. We have found that beyond the single-mode approximation, the geometric discord for entangled quantum states of fermionic system in accelerated frame does not vanish even at the infinite acceleration limit if the quantum state of Alice’s particle and Bob’s particle in Bob’s region I or Alice’s particle and Bob’s anti-particle in Bob’s region II but the quantum state of Alice’s particle and Bob’s anti-particle in Bob’s region I or Alice’s particle and Bob’s particle in Bob’s region II have vanishing geometric discord in the same infinite acceleration limit. We also observed that the quantum discord of \( \Phi^+ \) and \( \Phi^- \) states behaves in the analogous way.

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