Signatures of non-universal large scales in conditional structure functions from various turbulent flows

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\textbf{Abstract}. We present a systematic comparison of conditional structure functions in nine turbulent flows. The flows studied include forced isotropic turbulence simulated on a periodic domain, passive grid wind tunnel turbulence in air and in pressurized SF$_6$, active grid wind tunnel turbulence (in both synchronous and random driving modes), the flow between counter-rotating

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discs, oscillating grid turbulence and the flow in the Lagrangian exploration module (in both constant and random driving modes). We compare longitudinal Eulerian second-order structure functions conditioned on the instantaneous large-scale velocity in each flow to assess the ways in which the large scales affect the small scales in a variety of turbulent flows. Structure functions are shown to have larger values when the large-scale velocity significantly deviates from the mean in most flows, suggesting that dependence on the large scales is typical in many turbulent flows. The effects of the large-scale velocity on the structure functions can be quite strong, with the structure function varying by up to a factor of 2 when the large-scale velocity deviates from the mean by \pm 2 standard deviations. In several flows, the effects of the large-scale velocity are similar at all the length scales we measured, indicating that the large-scale effects are scale independent. In a few flows, the effects of the large-scale velocity are larger on the smallest length scales.

1. Introduction

Turbulent flows are composed of a wide range of scales that can be informally classified into ‘large’ and ‘small’ scales. The large scales contain most of the energy in a flow and are unique to each system. The small scales include the inertial and dissipation ranges and are thought to have some degree of universality. An open question in turbulence research pertains to the effects of large scales, including the mean flow and the large fluctuating eddies, on the small scales [1]. Some small-scale quantities have been shown to have a degree of universality because they are nearly identical when properly normalized even in flows with quite different large scales. Examples include Eulerian and Lagrangian scaling exponents [2–4] as well as the form and Reynolds number dependence of the acceleration probability distribution [5–7]. However, other small-scale quantities, such as velocity derivative skewness [8], and coefficients of scaling laws [9, 10], have been found to be different in flows with different large scales.

A valuable tool for understanding the effect of the large scales in a flow on the small scales is to directly condition a small-scale measurement on the state of the large scales. Here we study second-order longitudinal Eulerian structure functions

\[
\langle (\Delta u_r)^2 \rangle = \langle (\langle \mathbf{u}(\mathbf{x} + \mathbf{r}) - \mathbf{u}(\mathbf{x}) \rangle \cdot \mathbf{r} / |\mathbf{r}|)^2 \rangle,
\]

(1)
where \( \mathbf{u} \) is the fluctuating fluid velocity and \( \mathbf{r} \) is the displacement vector between two points. We quantify the effects of the large scales on these structure functions by conditioning on the velocity average

\[
\Sigma \mathbf{u} = (\mathbf{u}(\mathbf{x}) + \mathbf{u}(\mathbf{x} + \mathbf{r}))/2.
\]  

The velocity average contains contributions from all scales larger than \( \mathbf{r} \), but it is dominated by the large scales which contain most of the energy in the flow [11]. We normalize the conditioned structure functions by the unconditioned structure function and plot them against the velocity average. This allows all measured scales to be directly compared and clearly reveals the effect of the large scales on the various scales that exist in the flow, including the small scales. Conditional structure functions can be obtained using data from a wide variety of techniques that have been developed for measuring structure functions, allowing for a comparison of different flows.

Previous studies of Eulerian structure functions conditioned on the instantaneous large-scale velocity have established the method we present, and have connected the observed conditional dependence to different properties of the flow. Praskovsky et al [12] measured two high-Reynolds-number shear flows in a wind tunnel and towards the end of their paper they presented conditional structure functions showing strong correlations between the large-scale velocity and all length scales of the structure functions. They interpreted these correlations as a result of the variability of the flux of energy through the cascade due to the inhomogeneity of the large-scale flow. Sreenivasan and Dhruva [13] measured Eulerian structure functions conditioned on the instantaneous large-scale velocity at very high Reynolds numbers in the atmospheric boundary layer. They found that, despite the very high Reynolds number, the conditioned structure functions showed a dependence on the large-scale velocity. They compared this dependence to a direct numerical simulation (DNS) of homogeneous isotropic turbulence and experimental measurements from passive grid turbulence which showed very weak dependence on the large-scale velocity. They identified the difference in dependence with the shear found in the atmospheric boundary layer that is absent from the DNS or passive grid turbulence. Kholmyansky and Tsinober [14] also measured conditional structure functions in a very-high Reynolds-number atmospheric boundary layer. They observed a strong dependence and interpreted it as a direct coupling between the large and small scales in turbulence [15]. Blum et al [16] studied a moderate-Reynolds-number flow between oscillating grids and found that both Eulerian and Lagrangian structure functions at all scales show a strong dependence on the large-scale velocity. They demonstrated that system-scale inhomogeneity contributes to the dependence on the large scales, but also identified some of the dependence that is due to variability of the energy injection at large scales, which they called large-scale intermittency.

In this paper, we present a comparison of conditional structure functions from nine different flows that have been widely used for turbulence studies. We include DNS of homogeneous, isotropic turbulence [17], passive grid wind tunnel turbulence in air [18] and in pressurized \( \text{SF}_6 \), active grid wind tunnel turbulence [19] (in both synchronous and random modes), turbulence between counter-rotating discs [20], turbulence between two oscillating grids [16] and turbulence in the Lagrangian exploration module (operating in both constant [21] and random [22] driving modes). The last four flows have been the focus of many Lagrangian measurements because they have nearly zero mean velocity in the region near the geometrical center of the apparatus. Comparison of these flows with more traditional turbulent flows in wind tunnels and DNSs provides important data needed to integrate the different insights that
Table 1. Flow parameters in the various apparatuses. $Re_\lambda$ is the Taylor Reynolds number; $\varepsilon$ is the turbulent kinetic energy dissipation rate per unit mass; $\eta$ is the Kolmogorov length scale; $L$ is the characteristic length; $W$ represents the flow width, which is the smallest system dimension containing the flow.

| Flow                          | $Re_\lambda$ | $\varepsilon$ | $\eta$ | $L$   | $W/L$ | Ref |
|-------------------------------|--------------|---------------|--------|-------|-------|-----|
| DNS$^a$                       | 650          | 1.33          | 0.00141| 3.08  | 2.04  | [17]|
| Passive grid wind tunnel (air)| 66           | 0.448 m$^2$s$^{-3}$ | 295 $\mu$m | 0.0207 m | 44.2 | [18]|
| Passive grid wind tunnel (SF$_6$) | 620         | 0.0098 m$^2$s$^{-3}$ | 70 $\mu$m | 0.14 m | 9.6  |    |
| Active grid wind tunnel (synchronous driving) | 140         | 0.042 m$^2$s$^{-3}$ | 550 $\mu$m | 0.12 m | 7.7  | [19]|
| Active grid wind tunnel (random driving) | 582         | 0.94 m$^2$s$^{-3}$ | 260 $\mu$m | 0.47 m | 1.9  | [19]|
| Oscillating grids             | 260          | $2.31 \times 10^{-3}$ m$^2$s$^{-3}$ | 144 $\mu$m | 0.079 m | 7.1  | [16]|
| Counter-rotating discs        | 690          | 1.15 m$^2$s$^{-3}$ | 30 $\mu$m | 0.071 m | 6.2  | [20]|
| LEM (constant driving)        | 195          | $2.10 \times 10^{-4}$ m$^2$s$^{-3}$ | 260 $\mu$m | 0.094 m | 6.4  | [21]|
| LEM (random driving)          | 210          | $2.70 \times 10^{-4}$ m$^2$s$^{-3}$ | 250 $\mu$m | 0.10 m | 6.0  | [22]|

$^a$ In DNS the choice of units is arbitrary.

have been obtained in these different systems. We hope that as new flows are developed, they can be evaluated and compared using conditional structure functions as a systematic method of identifying the effects of the large scales.

In section 2, we briefly describe each of the nine flows that we study. Section 3 presents the data, and in section 4 we discuss some possible interpretations of the conditional dependence observed in these flows.

2. Description of the flows studied

Parameters for each of the nine flows are summarized in table 1. We define a characteristic velocity by $u = (\langle u_i u_i \rangle / 3)^{1/2}$ where $u_i$ is the fluctuating velocity, and a characteristic length scale by $L = u^3 / \varepsilon$, where $\varepsilon$ is the dissipation rate of turbulent kinetic energy per unit mass. The Taylor Reynolds number reported is $Re_\lambda \equiv u_\lambda / \nu = (15 u L / \nu)^{1/2}$, where $\lambda$ is the Taylor microscale and $\nu$ is the kinematic viscosity. The Kolmogorov length scale is $\eta = (\nu^3 / \varepsilon)^{1/4}$. We briefly describe each system with further details available in the referenced works.

The DNS data set analyzed has an $Re_\lambda$ of approximately 650, with $2048^3$ grid points and periodic boundary conditions [17]. The turbulence is maintained stationary by stochastic forcing at large scales. The domain size is $2.1L$. Statistics are calculated from 15 snapshots of the simulation. The pseudospectral algorithm of Rogallo [23] was used. The time stepping is second-order Runge–Kutta. Aliasing errors were controlled by a combination of truncation and phase shifting techniques.

One wind tunnel was used for passive and active grid turbulence measurements in air. The wind tunnel is $0.914 \times 0.914$ m$^2$ and 9.1 m long, with a horizontal, open circuit design. In the passive grid configuration it produced a flow with $Re_\lambda = 66$. The grid mesh was $M = 2.54$ cm and the mean flow was 10.3 m s$^{-1}$. Measurements were taken using hot-wire anemometry at $x/M = 60$, which is $x/L = 74$ [18].

In the active grid configuration, the grid mesh was $M = 11.4$ cm. A mean flow of 3.3 m s$^{-1}$ was used for the synchronous grid experiments where the grid bars and wings were all rotated at
a constant rate. A mean flow of 7.0 m \text{s}^{-1} was used for the random grid experiments where the rotation direction of individual grid bars and wings was switched at random intervals. The active grid configuration produced a flow with \( Re_\lambda = 140 \) for the synchronous mode and \( Re_\lambda = 582 \) for the random mode of operation. Measurements were taken using hot-wire anemometry at \( x/M = 62 \), which is \( x/L = 59 \) in synchronous mode and \( x/L = 15 \) in random mode [19]. For the active and passive grid turbulence measurements in air, the characteristic velocity was defined using only one component, \( u = \langle u_1^2 \rangle^{1/2} \), but these flows are sufficiently isotropic that this makes no significant difference from the definition used in the other flows. The energy dissipation rate was determined from the derivative of the streamwise velocity component, 
\[
\varepsilon = 15\nu \langle (\frac{\partial u_1}{\partial x})^2 \rangle.
\]

Data from a passive grid at high Reynolds number was obtained using a new turbulence facility that can use pressurized sulfur hexafluoride (SF\(_6\)). A biplanar grid of crossed square bars of classical construction generated the turbulence. The solidity of the grid was 35\% and the spacing of the bars was 107 mm. The measurement section in the tunnel had a cross-sectional area of 1.9 m\(^2\) and was 8.8 m long. The flow was driven in this recirculating tunnel with a 240 kW fan up to speeds of 5 m \text{s}^{-1} and was temperature regulated by a flat plate heat exchanger to \( \pm 0.5 \) K. Fluid velocities were measured with hot-wire anemometers stationed 67 times the grid spacing, or 7.1 m, downstream of the grid. The wires were 2.5 \( \mu \text{m} \) in diameter, 450 \( \mu \text{m} \) long and responded at about 80 kHz. The signals from the wires were low-pass-filtered with a cut-off frequency of 30 kHz, sampled at 60 kHz, and between 8 and 16 million data points were collected from each hot wire. The turbulence intensity was 2.5\% and the integral length scale was 9 cm. The energy dissipation rate was estimated from the derivative of the streamwise velocity component, 
\[
\varepsilon = 15\nu \langle (\frac{\partial u_1}{\partial x})^2 \rangle.
\]

The flow between counter-rotating discs is inside a 48.3 cm diameter cylindrical water tank. The two counter-rotating discs are separated by 43.9 cm. At a propeller frequency of 3.5 Hz, the flow reaches a high Reynolds number of \( Re_\lambda = 690 \). Nearly neutrally buoyant polystyrene particles 25 \( \mu \text{m} \) in diameter and 1.06 g \text{cm}^{-3} in density were seeded as tracer particles. Their trajectories were recorded using three cameras at a frame rate of 27 000 images \text{s}^{-1}. Two pulsed frequency-doubled Nd:YAG lasers (wavelength 532 nm) with a combined power of roughly 150 W illuminated a (5 cm)\(^3\) observation volume at the center of the tank. Particle velocities were measured through three-dimensional (3D) particle tracking techniques [9]. The energy dissipation rate was determined from the inertial range scaling of the second-order velocity structure functions, and was checked with the third-order velocity structure function (the four-fifths law) and the mixed velocity–acceleration structure function [20].

The oscillating grids are inside a \( 1 \times 1 \times 1.5 \) m\(^3\) octangular prism tank. The two grids were spaced 56.2 cm apart with an 8 cm mesh spacing. A typical grid frequency was 3 Hz with a 12 cm peak-to-peak stroke. They produced a flow with \( Re_\lambda = 260 \). Polystyrene tracer particles of diameter 136 \( \mu \text{m} \) = 0.94\( \eta \) were recorded using four high-speed digital cameras recording 450 frames \text{s}^{-1}. The video data were filtered through real-time image compression circuits, allowing for continuous data recording. An approximately (5 cm)\(^3\) cubic detection volume was illuminated in the center of the tank using a pulsed 50 W Nd:YAG laser (wavelength 532 nm). Particle velocities were determined using 3D particle tracking techniques [16]. The energy dissipation rate was estimated from the inertial range of the third-order structure function.

The Lagrangian exploration module (LEM) is a regular icosahedron with an edge length of 40 cm and a volume of 140 liters. A propeller is installed on each of the 12 vertices. The propellers are driven by 12 independently controlled DC motors. Here we report data
corresponding to a motor frequency of 1.67 Hz. When the motor speeds were maintained constant, they generated a water flow with \( Re_\lambda = 195 \) at the center of the apparatus. When the motor speeds were randomly adjusted within \( \pm 40\% \) of the nominal speed of 1.67 Hz, the Reynolds number of the flow was \( Re_\lambda = 210 \). Hollow glass spheres with diameter between 60 and 70 µm and average density between 1 and 1.2 g cm\(^{-3}\) were used as tracer particles. A 15 × 10 × 10 cm\(^3\) detection volume was illuminated with a 30 W Nd:YAG laser (wavelength 532 nm). Tracer particle motions in the detection volume were recorded with three high-speed cameras at a frame rate of 1000 frames per second. Particle velocities were measured from 3D particle tracking techniques [22]. The energy dissipation rate was determined from the inertial range scaling of the second-order velocity structure functions.

3. Data

We focus on the second-order longitudinal Eulerian structure functions, labeled \( \langle (\Delta u_r)^2 \rangle \) in equation (1). From the fluctuating velocities measured at two points separated by a distance \( r \), the component of the velocity difference parallel to the separation vector is extracted and the second moment gives the longitudinal structure function [5]. Note that different experimental techniques access this quantity in different ways. Hot-wire measurements in the wind tunnels use fixed probes, so the separation vector has a fixed direction. Particle-tracking measurements sample randomly positioned tracer particles and average over separations with all directions. Figure 1 shows the second-order longitudinal structure function for each of the flows we consider. The structure functions are compensated by inertial range Kolmogorov (1941) scaling, \( \langle \varepsilon r \rangle^{2/3} \), to better compare the various flows. Many of the flows are at modest Reynolds numbers and show, at most, a very small region of inertial range scaling. A larger separation of scales is clearly visible in the higher-Reynolds-number data in figures 1(a), (c), (e) and (g).

To measure the effects of the large scales on each flow, we condition the structure functions on a quantity that is representative of the large scales. We use a velocity average, \( \Sigma u \) in equation (2), because it is a convenient and measurable quantity that reflects primarily the instantaneous state of the large scales. It is appropriate to consider longitudinal and transverse versions of the velocity average, denoted by \( \Sigma u_\parallel \) and \( \Sigma u_\perp \), for velocity components in directions parallel and perpendicular to the separation vector \( r \), respectively.

Statistics of the longitudinal velocity average have received some attention in recent years due to the demonstration by Hosokawa [24] that

\[
\langle \Delta u_r (\Sigma u_\parallel) \rangle = \frac{\varepsilon r}{15}
\]

(3)

for \( r \) in the inertial range. This relation can be derived from the four-fifths law for the third-order longitudinal structure function in classical Kolmogorov theory. From equation (3) we can conclude that, in general, the mixed moment \( \langle \Delta u_r (\Sigma u_\parallel) \rangle \) does not vanish, which in turn implies that \( \Delta u_r \) and \( \Sigma u_\parallel \) are not statistically independent. This consideration, however, does not apply to the transverse velocity average \( \Sigma u_\perp \).

Mouri and Hori [11] argue that the velocity average represents motions on scales \( r \) and larger. This allows equation (3) to be satisfied without requiring a correlation between small-scale and large-scale motions since correlations near scale \( r \) can be responsible. However, they also note that while the velocity average has contributions from scales near \( r \), its variance is dominated by the large scales. This follows from the fact that the single point fluctuating velocity has contributions from all scales, but its variance is dominated by the large scales.
Figure 1. Eulerian second-order longitudinal velocity structure functions compensated for by $(\varepsilon r)^{2/3}$. (a) DNS, (b) passive grid wind tunnel (air), (c) passive grid wind tunnel (pressurized SF$_6$), (d) active grid (synchronous driving), (e) active grid (random driving), (f) oscillating grids, (g) counter rotating discs, (h) LEM (constant driving) and (i) LEM (random driving).

The effect of taking the two-point average is to filter scales smaller than $r$. Thus for equation (3) to be controlled by scales near $r$, there must be a strong cancellation that removes the dominant contribution from the large scales to the velocity average.

In this paper, we use conditioning on the velocity average as a way of conditioning on the instantaneous state of the large scales. In this context, the connection between the velocity average and the large scales is justified because the large scales dominate in determining the conditional bin for each sample since they dominate the variance of the velocity average. Blum et al [16] provide some discussion of why correlations between sums and differences of the same measurements like equation (3) do not explain the conditional dependence they observe. Here we add two additional pieces of evidence. Firstly, we minimize any possible effects of correlations between sums and differences of the same measurements by conditioning the longitudinal structure functions on the transverse component of the velocity average. Secondly,
Figure 2. The Eulerian second-order longitudinal structure functions are conditioned on the transverse velocity average ($\Sigma u_\perp$) and are plotted against $\Sigma u_\perp$. Symbols represent the following separation distances $r/\eta$: $+$ = 4, $\circ$ = 8, $*$ = 16, $\times$ = 32, $\Box$ = 64, $\diamond$ = 128, $\triangle$ = 256, $\triangledown$ = 512, $\rhd$ = 1024. (a) DNS, (b) passive grid wind tunnel (air), (c) passive grid wind tunnel (pressurized SF$_6$), (d) active grid (synchronous driving), (e) active grid (random driving), (f) oscillating grids, (g) counter rotating discs, (h) LEM (constant driving) and (i) LEM (random driving).

the large differences between the flows in our results indicates that the conditional dependence cannot be explained by a universal correlation.

Figure 2 shows longitudinal structure functions for each flow conditioned on the transverse component of the velocity average. The data from the passive grid in air in figure 2(b) are nearly flat for all length scales, indicating a weak dependence of the structure functions at any $r$ on the large scales. This flow is at a very low Reynolds number of $Re_\lambda = 66$; however, the much-higher-Reynolds-number data ($Re_\lambda = 620$) from the passive grid in SF$_6$ in figure 2(c) also shows very little dependence. In the higher-$Re_\lambda$ passive grid data, there is a weak dependence on length scale with the smallest scale depending on the large-scale
velocity slightly more than other scales. Another flow with little dependence of the structure functions on the large-scale velocity is counter-rotating discs in figure 2(g). The other six flows all show somewhat stronger dependence on the large scales. The upward curvature indicates that large velocity differences preferentially occur when the magnitude of the velocity average is large.

The DNS data in figure 2(a) show a weak dependence of structure functions on the transverse velocity average when the velocity average is near zero. For $\Sigma u_\perp$ less than twice the rms velocity, the curves at all length scales are almost flat except for a weak slope which is sensitive to sampling effects. For larger values of the velocity average, the curves are seen to rise, with the largest rise occurring for small values of $r$. These DNS data differ somewhat from other published results [13] in which both DNS and passive grid wind tunnel data show no dependence on the large-scale velocity. The reason for this apparent difference is still not understood, but it is possible that the particular features of how the large scales were forced may have played a role [25].

Figures 2(d) and (e) illustrate the difference in the effects of the large scales when the active grid wind tunnel is driven in the synchronous and random modes. In the random mode, the curvature is significantly larger even though the Reynolds number is much larger. A possible explanation may be that $L$ is much larger in the random mode (see table 1), rendering the flow less homogeneous and therefore more sensitive to large-scale effects. This explanation is consistent with the differences between the data from the active grid wind tunnel and the passive grid wind tunnel in figure 2(b). Again, the flow with greater homogeneity has less curvature even though the Reynolds number is lower.

A surprise in the data presented here is that data from counter-rotating discs shown in figure 2(g) show very little dependence on the large scales. This can be compared to the much stronger dependence in the oscillating grid flow shown in figure 2(f), even though the oscillating grid flow is more homogeneous and isotropic. To investigate this further, we conditioned not on the transverse velocity average but on different components of the velocity average that are fixed in the laboratory frame. Figure 3 reveals that the disparity between these two flows disappears when the second-order structure functions for both flows are conditioned on the component of the velocity average in the axial direction, $\Sigma u_z$, which is vertical in the laboratory frame.

**Figure 3.** Structure functions conditioned on the vertical component of the velocity average, $\Sigma u_z$. Symbols represents the following separation distances $r/\eta$: $+ = 4$, $\circ = 8$, $* = 16$, $\times = 32$, $\square = 64$, $\diamond = 128$, $\triangle = 256$, $\nabla = 512$, $\triangledown = 1024$. (a) Counter-rotating discs; (b) oscillating grids.
Here, the dependence on the large-scale velocity for the counter-rotating disc flow in figure 3(a) is approximately the same as that in the oscillating grid data in figure 3(b). Further evidence about the effects of the direction of the large-scale velocity comes from conditioning structure functions from each of these flows on directions perpendicular to the axial direction (data not shown). Here the counter-rotating disc flow shows almost no dependence and the oscillating grid flow shows dependence similar to the axial dependence. It appears that the direction of the large-scale conditioning can play a significant role for systems where large-scale anisotropy is important.

Figures 2(h) and (i) show that the Lagrangian exploration module in constant and random driving modes has structure functions with strong dependence on the large-scale velocity. Unlike the different driving modes of the active grid, the different driving modes of the Lagrangian exploration module show similar curvatures as well as similar Reynolds numbers.

A major point which is seen throughout figure 2 is that separation of scales alone is not sufficient to produce small scales that are independent of the large scales. For most of the flows, there is relatively little dependence of the conditional structure functions on the length scale, $r$, so the dependence on the large scales does not diminish with decreasing scale. The DNS shows more dependence on the length scale than the other flows, with the structure function at the smallest scales curving upward more strongly than larger scales. In the experiments where there is scale dependence, the smallest scales usually curve up most strongly. This is exactly the opposite of what would be expected if limited separation of scales were the cause. From this observation, we infer that the trend of the conditional structure functions depending strongly on the large-scale velocity is likely to persist to yet higher Reynolds numbers despite the wider range of scales that would be present.

Figure 4 shows longitudinal structure functions again, but this time conditioned on the longitudinal component of the velocity average. These data are largely similar to figure 2 conditioned on the transverse velocity average, but conditioning on the longitudinal velocity average reveals a few new insights. The similarity between figures 2 and 4 is evidence that kinematic correlations between sums and differences of the same measurements are not a large factor in the observed conditional dependence since only figure 4 uses the same components for differences and sums. Data from the wind tunnels (panels (b)–(e) of figures 2 and 4) show the largest differences between conditioning on the transverse and longitudinal velocity averages. Note that the structure function minimum has been shifted towards negative large-scale velocity by over one standard deviation. A major factor here is the orientation of the hot-wire probes used for wind tunnel measurements. They are fixed in space so that the positive longitudinal direction is always downstream in the wind tunnel. The other data are taken from particle tracking or simulations and the data are averaged over many different random longitudinal directions. The wind tunnel data show a correlation between larger downstream velocity and larger structure functions. For both passive grids this dependence is weak, but for the active grid the structure functions conditioned on the downstream velocity show a stronger dependence that is somewhat different at different length scales. The longitudinally conditioned data from the Lagrangian exploration module in figures 4(h) and (i) show a stronger dependence for larger length scales, in contrast with the DNS data in figure 4(a) and all flows in figure 2.

To distinguish between the effects of true Navier–Stokes dynamics and the effects of a kinematic nature (such as incompressibility) it is useful to make comparisons with Gaussian random fields [26]. Figure 5 shows the second-order structure function conditioned
Figure 4. The Eulerian second-order longitudinal structure functions conditioned on the longitudinal velocity average ($\langle \Sigma u_\parallel \rangle$), and plotted versus $\Sigma u_\parallel$. Symbols represent the following separation distances $r/\eta$: + = 4, ◦ = 8, * = 16, × = 32, □ = 64, ◇ = 128, △ = 256, ◊ = 512, ▽ = 1024. (a) DNS, (b) passive grid wind tunnel (air), (c) passive grid wind tunnel (pressurized SF$_6$), (d) active grid (synchronous driving), (e) active grid (random driving), (f) oscillating grids, (g) counter rotating discs, (h) LEM (constant driving), (i) LEM (random driving).

on the longitudinal velocity average for Gaussian random fields. The Gaussian random fields are constructed as vector fields where each component of the velocity vector is Gaussian, independent and identically distributed, with all velocity gradients also being Gaussian. The velocity components are also subject to the condition of being divergence free and having the same energy spectrum as the DNS velocity field at a given Reynolds number. The result is almost perfectly flat, showing no influence of the large-scale velocity on any length scale. This provides a baseline for evaluating conditional structure functions and confirms the argument presented in [16] that the observed conditional dependence is not a simple kinematic correlation between averages and differences of the same measurements.
We also studied the effect of conditioning on the velocity at the midpoint between the two points rather than the velocity average. The midpoint velocity is inaccessible to particle tracking experiments, but can be studied in numerical simulations. Conditioning the DNS data on the midpoint makes very little difference for small $r$ as expected, since here the two velocities are nearly the same. At larger $r$ there are some small quantitative differences between conditioning on the velocity average and the midpoint velocity, but the main features of the conditional structure functions in figure 2(a) are unchanged. We also considered how the conditional dependence changes if the conditioning point is selected away from the midpoint. For Gaussian random fields, this introduces a kinematic correlation that is avoided by conditioning on the midpoint.

4. Discussion

We provide here a discussion of some possible interpretations of these data. The data clearly reveal that the small scales have a different dependence on the large-scale velocity in the different flows. However, the causes of the differences are difficult to conclusively identify.

A central question is what allows the passive grid wind tunnel flows to have less dependence of the conditional structure functions on the large scales than the other flows have. The DNS is an intermediate case with very little dependence on the large-scale velocity when it is small (less than 2 standard deviations) but significant dependence beyond this. One possible hypothesis is that the flows with weak dependence on the large scales are homogeneous, not just across the detection volume but also well beyond it. Table 1 reports the number of integral length scales that span the flow in its narrowest dimension, $W/L$. The passive grid flow in air that has the largest $W/L$ ratio also shows the least dependence on the large scales. For DNS with periodic boundary conditions, $W/L$ is defined using the size of the periodic box, and it
is possible that the limited size of this domain contributes. The other flows have smaller $W/L$ and stronger dependence on the large scales. In highly homogeneous flows, any fluid entering the observation volume would likely be statistically similar and therefore less likely to show a dependence on the large scales. In less homogeneous flows, fluid could be swept into the detection volume from regions with different energy, leading to dependence of the structure functions on the large-scale velocity that is responsible for the sweeping.

A related hypothesis is that structure functions depend on the large scales whenever significant energy exists at scales much larger than $L$. In the passive grid wind tunnel experiments, energy is injected at a fairly well-defined scale and then measurements are made in decaying turbulence before there is time for much energy to be transferred up to larger scales. But in DNS and in the zero-mean-flow systems designed for Lagrangian measurement (panels (a) and (f)–(i) in figures 1, 2 and 4), the flow is usually driven continuously and the flow in the region studied is statistically stationary. In these circumstances the largest length scales are dictated by the system dimensions, which are typically much larger than the integral scale, and these scales can accumulate energy. The existence of energy in these large scales can lead to variation in the energy flux down the cascade either through fluctuations in the energy flux into and out of these scales or through transport of more energetic turbulence from near the energy injection mechanism into the detection volume. Such energy fluctuations may result in the dependence of all scales on the largest length scales shown in figures 2 and 4.

Another question is: why do the small scales in some flows show a stronger dependence on the large-scale velocity than larger scales (figures 2(a), (c) and (f)), while in other flows all scales collapse (figures 2(d), (e) and (i)). One possible consideration here is that variability of the energy injection at large scales of the type described in the previous paragraph will produce temporal variations in the energy dissipation rate, which would cause the dissipation scale to fluctuate. If the dissipation scale is smaller when the large-scale velocity is larger, it would increase the structure functions at small scales when the large-scale velocity is large. This is exactly what is seen in figures 2(a) and (f). However, more work is needed to understand this effect since it is not seen in all data sets.

We also note an intriguing connection between conditional Eulerian and Lagrangian statistics. Lagrangian accelerations have been conditioned on the instantaneous large-scale velocity and found to have a significant dependence in both a counter-rotating disc experiment and isotropic DNS [4], [27–29]. The results are quite similar to the smallest scales of the Eulerian structure functions presented here. However, here we also have data from the passive grid wind tunnel where the conditional dependence is nearly absent. It would be very interesting to probe the dependence of accelerations on the large-scale velocity in a wider variety of flows to see if similar flow dependence is observed there.

5. Conclusions

We have presented Eulerian structure functions conditioned on the large-scale velocity for nine different turbulent flows. Some flows show relatively little dependence on the large-scale velocity while most flows show a significant dependence that contains information about how the unique large scales of each flow affect the small scales. This systematic comparison of conditional structure functions in different turbulent flows provides a reference that can be used to compare the effects of the large scales in new flows as they are developed.
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