Cosmic evolution in the background of non-minimal coupling in $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity

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Abstract In this manuscript, we are interested to address the issue of cosmic expansion in the background of matter-geometry coupling. For this purpose we consider $f(R, T, Q)$ modified theory (where $R$ is the Ricci Scalar, $T$ is the trace of energy-momentum tensor (EMT) $T_{\mu\nu}$ and $Q = R_{\mu\nu}T^{\mu\nu}$ is interaction of EMT $T_{\mu\nu}$ and Ricci Tensor $R_{\mu\nu}$). We formulate modified field equations in the background of flat Friedmann-Lemaître-Robertson-Walker (FLRW) model which is defined as $d\sigma^2 = dt^2 - a(t)^2(dx^2 + dy^2 + dz^2)$, where $a(t)$ represents the scale factor. In this formalism energy density is found using covariant divergence of modified field equations. $\rho$ involves a contribution from non-minimal matter geometry coupling which helps to study different cosmic eras based on equation of state (EOS). Furthermore, we apply the energy bounds to constrain the model parameters establishing a pathway to discuss the cosmic evolution for best suitable parameters in accordance with recent observations.

Keywords $f(R, T, R_{\mu\nu}T^{\mu\nu})$ gravity · Raychaudhuri equation · Energy conditions, Dark Energy

1 Introduction

Currently our universe is experiencing an accelerated expansion phase and multiple astrophysical researches have been conducted to observe this cosmic scenario. It is highly assumed and considered that this cosmic acceleration is the consequence of an anonymous energy named as DE (Perlmutter et al. 1999). Antagonistic to the gravitational pull, the DE is expanding the universe by having a negative pressure which is completely opposite to the ordinary matter. Many attempts have been made to unveil the reason for accelerated cosmic expansion. The major finding (Ade et al. 2014; Spergel et al. 2007) enlists DE as the major candidate with overall contribution of 68.3%, the other significant 26.8% contribution is from Dark matter despite its elusive and un-explored nature. Baryon, is the major part of visible cosmos which accounts for 4.9% among cosmic ingredients. Despite tremendous researches and observations, late time cosmic acceleration is still a significant as well as challenging area for cosmologists. However, attention is attached to the confirmation through measurements from temperature anisotropies of the existence of DE as puzzling cosmic ingredient with reference to cosmic acceleration by cosmic microwave background radiations (CMBR) (Ade et al. 2014; Spergel et al. 2007), baryon acoustic oscillations (BAO) (Cole et al. 2005), large scale structure (LSS) (Hawkins et al. 2003; Tegmark et al. 2004), weak lensing (Jain and Taylor 2003) and most recent plank’s data (Ade et al. 2014b). Furthermore, to describe the nature of DE several theoretical models are proposed like phantom (Caldwell 2002), quintessence (Sahni and Starobinsky 2000) and fluids with anisotropic equation of state (EoS) (Akarsu and Kilinc 2010). In $\Lambda$ cold dark matter ($\Lambda$CDM) model, the role of DE in GR is played by $\Lambda$. Yet the origin of cosmological constant $\Lambda$ is still under question and $\Lambda$ has two well-known problems known as coincidence and fine-tuning. To express the characteristics of DE, the EoS is proposed which is defined as $\omega_{DE} = \frac{p_{DE}}{\rho_{DE}}$ (the ratio of the pressure to the energy density of DE). The EoS is evaluated by considering that universe is isotropic and homogeneous, and taking the FLRW space-time at the background. $\omega_{DE}$ is a constant
the Lagrangian of Einstein’s equations is to take the function which depends on trace $T$ of the EMT (Poplawski 2006), such that $\Lambda C D M$ model can be taken of the form $R + 2\Lambda(T)$. Finally, by using this idea Harko et al. (2011) proposed the extension of $f(R)$ by replacing the function $f$ with the new dependent parameters $R$ (Ricci scalar) and $T$ (trace of EMT) and non-minimal coupling between matter and geometry allowed in this astonishing theory. The coinciding with constructive geometry matter coupling shows the deviation of test particles from geodesic motion which ruled to additional force as proposed in different theories (Bertolami et al. 2007a; Harko 2008; Sotiriou and Faraoni 2008; Harko et al. 2011).

Due to remarkable growing interest in this theory the efficacy of laws of thermodynamics in $f(R, T)$ gravity have been studied by Sharif and Zubair (2012b, 2013d) and it is concluded that the equilibrium picture of thermodynamics cannot be achieved due to matter geometry interaction. Attempts to reconstruct $f(R, T)$ Lagrangian has also been made under various considerations like the family of holographic DE models by supposing the FLRW universe, (Houndjo and Piattella 2012; Sharif and Zubair 2013c) considering an auxiliary scalar field (Houndjo 2012) and anisotropic solutions (Sharif and Zubair 2012c). Jamil et al. (2012) worked on the reconstruction of cosmological models and they showed that the dust fluid reproduce $\Lambda C D M$, Einstein static universe and de sitter Universe. Al-varenga et al. (2013) discussed the development of matter density perturbations in this theory and they presented the required constraints to get the standard continuity equation in $f(R, T)$ gravity. On the other hand, Sharif and Zubair (2014) reconstructed cosmological models by applying additional constraints for the conserved EMT and studied the stability of the constructed models. Furthermore, the dynamical systems in $f(R, T)$ gravity were explored by Shabani and Farhoudi (2013) that resulted in the development of a vast scale of passable cosmological solutions. Other cosmic issues including compact stars, wormholes and gravitational instability of collapsing stars have been discussed in literature (Moraes et al. 2017; Moraes and Sahoo 2017; Noureen et al. 2015; Noureen and Zubair 2015; Shamir 2015; Zubair et al. 2015, 2016a,b, 2018; Hina Azmat et al. 2018).

Lately, the non-minimal coupling of the EMT and Ricci tensor is introduced, resulting in the modified yet more complicated theory known as $f(R, T, Q)$ gravity (Odintsov and Saez-Gomez 2013; Haghani et al. 2013). Due to complicated non-minimal matter geometry coupling EMT is generally non-conserved and additional force is there. Therefore, it proposes a vast range to explore different cosmic features as thermodynamics properties have already been studied by Sharif and Zubair (2013e). Sharif and Zubair (2013f) also discussed the energy conditions for particular models
of \( f(R, T, Q) \) gravity. They found that the non-minimal coupling becomes the reason of the deviation of test particles from geodesic motion and that gives strength to the non-equilibrium representation of thermodynamics. This induced the idea that the validity of generalized second law of thermodynamics (GSLT) in an expanding universe might lead to the thermal equilibrium in future. Baffou et al. (2016) discussed the stability of de-sitter and power law solution by using perturbation scheme for particular models. In this paper we are interested to discuss the cosmological evolution in \( f(R, T, Q) \) theory, which is based on more general matter-geometry coupling. We pick a particular model of the theory is very effective for non-minimal coupling between geometry and matter. The action of this complicated theory becomes the reason of the deviation of test particles from geodesic motion and gives strength to the non-equilibrium representation of thermodynamics. This induced the idea that the validity of generalized second law of thermodynamics (GSLT) in an expanding universe might lead to the thermal equilibrium in future. Baffou et al. (2016) discussed the stability of de-sitter and power law solution by using perturbation scheme for particular models. In this paper we are interested to discuss the cosmological evolution in \( f(R, T, Q) \) theory, which is based on more general matter-geometry coupling. We pick a particular model of the

\[
A = \frac{1}{2\kappa^2} \int d^4x \sqrt{-g} \left[ f(R, T, R_{\mu\nu}T^{\mu\nu}) + \mathcal{L}_m \right],
\]

where \( \kappa^2 = 8\pi G \), \( f(R, T, Q) \) is a general function which depends on three components, the Ricci scalar \( R \), trace of the EMT \( T \), product of the EMT \( T^{\mu\nu} \) to Ricci tensor \( R_{\mu\nu} \), and \( \mathcal{L}_m \) shows the matter Lagrangian. The EMT for matter is defined as

\[
T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu\nu}}.
\]

If the matter action depends only on the metric tensor other than on its derivatives then the EMT yields

\[
T_{\mu\nu} = \mathcal{E}_{\mu\nu} - \frac{2\delta \mathcal{L}_m}{\delta g^{\mu\nu}}.
\]

The field equations in \( f(R, T, Q) \) gravity can be found by varying the action (1) with respect to \( g_{\mu\nu} \) as

\[
R_{\mu\nu}f_R = \left\{ \frac{1}{2}f - \mathcal{L}_m f_T - \frac{1}{2} \nabla_\alpha \nabla_\beta (f_Q T^{\alpha\beta}) \right\} g_{\mu\nu} + (g_{\mu\nu} \Box - \nabla_\mu \nabla_\nu) f_R + \frac{1}{2} \nabla_\mu T_{\nu}^{\alpha\beta} f_T + \frac{1}{2} \nabla_\alpha T_{\beta}^{\mu\nu} f_T + 2 f_Q R_{\alpha\beta}^{\mu\nu} - \nabla_\alpha \nabla_\beta \left( f_Q T^{\alpha\beta} \right) - G_{\mu\nu} \mathcal{L}_m f_Q - 2(f_T R^{\alpha\beta} + f_Q R^{\alpha\beta}) \frac{\partial^2 \mathcal{L}_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}}
\]

\[
= \left( 1 + f_T + \frac{1}{2} R f_Q \right) T_{\mu\nu}. \tag{4}
\]

The subscripts shows the derivatives with respect to \( R, T, Q \), and box function defined as \( \Box = \nabla_\beta \nabla_\beta \). If we will choose the particular form of Lagrangian then (4) can be shifted towards the well known field equations in \( f(R) \) and \( f(R, T) \) theories. The field equation (4) can be rewritten into the form of effective Einstein field equation (EFE) as

\[
G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T^{\text{eff}}_{\mu\nu}. \tag{5}
\]

This effective form of EFE is similar to GR’s standard field equations. Here \( T^{\text{eff}}_{\mu\nu} \), the effective EMT in \( f(R, T, Q) \) gravity is found to be as

\[
T^{\text{eff}}_{\mu\nu} = \frac{1}{f_R - f_Q} \mathcal{L}_m \left[ \left( 1 + f_T + \frac{1}{2} R f_Q \right) T_{\mu\nu} + \frac{1}{2} \mathcal{L}_m \right]^{\text{eff}} f_Q T^{\alpha\beta} f_T - \nabla_\alpha \nabla_\beta \left( f_Q T^{\alpha\beta} \right) \frac{\partial^2 \mathcal{L}_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}} \right]. \tag{6}
\]

Applying the covariant divergence to the field equation (4), we get

\[
\nabla_\mu T_{\mu\nu} = \frac{2}{2(1 + f_T) + f_Q} \left[ \nabla_\mu \left( f_Q R^{\alpha\mu} T_{\alpha\nu} + \nabla_\nu (\mathcal{L}_m f_T) \right) - \nabla_\nu \left( f_Q R_{\alpha\mu} T^{\alpha\nu} \right) \right]. \tag{7}
\]

It is important to see that any modified theory which involve non-minimal coupling between geometry and matter does not obey the ideal continuity equation. This complicated theory \( f(R, T, Q) \) also involves this type of non-minimal coupling so it also deviate from standard behavior of continuity equation. Here, non-minimal coupling between matter and geometry induces extra force acting on massive particles,
whose equation of motion is given by Haghani et al. (2013).

\[ \frac{d^2 \chi^k}{ds^2} + \Gamma^{k}_{\mu\nu} u^\mu u^\nu = f^k, \]

where

\[ f^k = \frac{h^\lambda_{\nu}}{(\rho + p)(1 + 2f_T + Rf_{RT})} \nabla_{\nu} \rho \]

\[ \times \left[ (f_T + Rf_{RT}) - (1 + 3f_T) \nabla_{\nu} p \right. \]

\[ - (\rho + p) f_{RT} R^{\sigma\rho}(\nabla_{\nu} h_{\sigma\rho} - 2 \nabla_{\rho} h_{\sigma\nu}) \]

\[ - f_{RT} R_{\sigma\rho} h^{\sigma\rho} \nabla_{\nu} (\rho + p) \right]. \]

It has been found that the impact of non-minimal coupling is always present independent of the choice matter Lagrangian, the extra force does not vanish even with the Lagrangian \( \mathcal{L}_m = p \) as compared to the results presented in Koivisto (2006), Bertolami et al. (2007a, 2008b). Haghani et al. (2013) also presented the Lagrange multiplier approach and found the conservation of matter EMT. Moreover, if one eliminates the dependence of \( Q \), it results in divergence equation of \( f(R, T) \) theory as given below

\[ \nabla^\alpha T_{\alpha\beta} = \frac{f_T}{1 - f_T} \left[ (\Theta_{\alpha\beta} + T_{\alpha\beta}) \nabla^\alpha \ln f_T \right. \]

\[ - \frac{1}{2} g_{\alpha\beta} \nabla^\alpha T + \left. \nabla^\alpha \Theta_{\alpha\beta} \right]. \]

Alvarenga et al. (2013) shown that choice of a specific model within these theories can guarantee the conservation of EMT and continuity equation is valid for the model \( f(R, T) = f_1(R) + f_2(T) \), where \( f_2(T) = \alpha T^\frac{1}{1+2\omega} + \beta \). In this manuscript, we are interested to explore the role of non-minimal coupling in cosmic evolution so we opt the non-conserved dynamical equation and evaluate necessary parameters.

We consider the isotropic and homogeneous flat FLRW metric defined as

\[ ds^2 = dt^2 - a^2(t)(dx^2 + dy^2 + dz^2), \]

where \( a(t) \) represents the scale factor. The effective energy density and pressure for this metric is found to be the components of \( T^\text{eff}_{\mu\nu} \), which assumes the form of perfect fluid as

\[ T_{\mu\nu} = (\rho + p) u_\mu u_\nu - p g_{\mu\nu} \]  

(8)

where \( p \) represent pressure, \( \rho \) for proper density and \( u_\mu \) is for 4-velocity. In FLRW background, \( \rho_{\text{eff}} \) and \( p_{\text{eff}} \) can be found as

\[ \rho_{\text{eff}} = \frac{1}{f_T - f_Q L_m} \left[ \rho + (\rho - L_m) f_T + \frac{1}{2}(f - Rf_R) \right. \]

\[ - 3H \partial_t f_R - \frac{3}{2}(3H^2 - \dot{H}) \rho f_Q - \frac{3}{2}(3H^2 + \dot{H}) p f_Q \]

\[ + \frac{3}{2} H \partial_t [H - \rho f_Q] \right] . \]

(9)

\[ p_{\text{eff}} = \frac{1}{f_T - f_Q L_m} \left[ p + (p + L_m) f_T + \frac{1}{2}(Rf_R - f) \right. \]

\[ + \frac{1}{2}(\dot{H} + 3H^2)(\rho - p) f_Q + \partial_t f_R + 2H \partial_t f_R \]

\[ + \frac{1}{2} \partial_t [H - \rho f_Q] + 2H \partial_t [H - \rho f_Q] \right] . \]

(10)

where \( R = -6(\dot{H} + 3H^2), H = \frac{\dot{a}}{a} \) is for Hubble parameter and upper dot for the time derivative. Here, we ignored those terms which involved the second derivative of matter Lagrangian with respect to \( g_{\mu\nu} \). In the case of perfect fluid the matter Lagrangian can either be \( L_m = \rho \) or \( L_m = -p \).

3 \( f(R, T, Q) = R + \alpha Q + \beta T \) gravity

We are interested to explore the cosmic evolution using matter conservation equation of more generic modified theory. Here, we will set \( L_m = \rho \) and we will take the simplest model \( f(R, T, Q) = R + \alpha Q + \beta T \) where \( \alpha, \beta \) are coupling parameters. In this model, choice of \( \alpha = 0 \) results in minimal coupling of the form \( f(R, T) = R + \beta T \) (Harko et al. 2011), such model has been widely studied in the formalism of \( f(R, T) \) gravity (for review see Moraes et al. 2017; Moraes and Sahoo 2017; Nour et al. 2015; Nour and Zubair 2015; Shamir 2015; Zubair et al. 2015, 2018; Hina Azmat et al. 2018). Moreover, the choice of \( \alpha = \beta = 0 \), results in Eistein’s formalism of GR.

For a flat FLRW universe, the non-zero components of FLRW equation for \( p_{\text{eff}} = p + p_{\text{DE}} \) and \( \rho_{\text{eff}} = \rho + \rho_{\text{DE}} \) are

\[ 3H^2 = \rho_{\text{eff}}, \quad -2\dot{H} - 3H^2 = p_{\text{eff}}, \]

(11)

where dots being time derivative and components of \( \rho_{\text{DE}} \) and \( p_{\text{DE}} \) are given as follows

\[ \rho_{\text{DE}} = \frac{1}{2\alpha \rho - 2} \left[ \rho (\beta + 6\alpha H^2 + 4\alpha \dot{H}) \right. \]

\[ + p(-5\beta + 12\alpha H^2 - 2\alpha \rho + 4\rho \dot{H}) \]

\[ - \alpha(4H\dot{p} + \alpha - 2(\dot{p} + \dot{H})) \right]. \]

(12)

and effective EoS \( \omega_{\text{eff}} \) is
cosmic evolution in the background of non-minimal coupling in \( f(R, T, R_{\mu\nu}T^{\mu\nu}) \) gravity

\[
\omega_{\text{eff}} = \frac{-\rho(\beta + 6aH^2 + 4\alpha \dot{H}) + \rho(-2 - 5\beta + 4\alpha(3H^2 + \dot{H})) - \alpha(4H(\dot{\rho} + \dot{p}) - \rho + \dot{p})}{3\beta \rho - (2 + \beta - 12aH^2)\rho + 3aH(\dot{\rho} - \dot{p})}.
\]  

(13)

The EoS of DE is, \( \omega_{\text{DE}} = \frac{\rho_{\text{DE}}}{p_{\text{DE}}} \)

\[
\omega_{\text{DE}} = \frac{-\rho(\beta + 6aH^2 + 4\alpha \dot{H}) + \rho(-2 - 5\beta + 12aH^2) - 2\alpha \rho + 4\alpha \dot{H}) - \alpha(4H(\dot{\rho} + \dot{p}) - \rho + \dot{p})}{3\beta \rho - (2 - 12aH^2)\rho - 2\alpha \rho^2 + 3aH(\dot{\rho} - \dot{p})},
\]  

(14)

and conservation equation (7) takes the form

\[
\dot{\rho} + 3H(p + \rho) = \frac{-18\alpha H^3(\rho + \rho)H + 3(\beta - 3\alpha H)\dot{\rho} + (\beta - 3\alpha H)\dot{\rho} - 9\alpha H^2(\dot{\rho} + \dot{p})}{2(1 + \beta - 6aH^2 - 3aH)}.
\]  

(15)

Now the above equations are expressed in terms of redshift by using relation \( a(t) = \frac{1}{1+z} \), where \( \frac{d^2}{dt^2} = -(1+z)H \frac{d^2}{d^2} \)

where \( p = p(z) \) and \( \rho = \rho(z) \). Where prime is for derivative with respect to redshift parameter \( z \).

\[
3H(p + \rho) - (1 + z)H \rho' = \frac{1}{2(1 + \beta + 3aH(-2H + (1 + z)H))} \times \left[-(H(1 + z)\beta(3p' + \rho')
+ 9\alpha H^2(2p + 2\rho - (1 + z)(p' + \rho'))
+ 3(1 + z)\alpha a H H'(-2p - 2\rho + (1 + z)(p' + \rho')))\right].
\]  

(16)

The revolutionary field equation \( G_{\mu\nu} = 8\pi GT_{\mu\nu} \) shows the connectedness of matter content of universe with geometry of the fabric of space-time, represented in Einstein’s general theory of relativity. The LHS of the previously stated field equation show the Einstein tensor, which satisfy the Bianchi identities \( \nabla_{\mu}G_{\rho\mu} \equiv 0 \) and RHS shows the EMT. If the covariante derivative of EMT is zero \( (\nabla_{\mu}T_{\rho\mu} = 0) \) then it shows the conservation of matter in every part of the universe. EFE can be explored on different choices of metric \( g_{\mu\nu} \) and EMT \( T_{\mu\nu} \). Although matter and geometry are on same footing but GR does not allow us to check the possible effects of non-minimal coupling between them. These limitations of GR vanished in recently developed theories like \( f(R, T) \) and \( f(R, T, Q) \) theories. In these theories EMT is not conserved \( (\nabla_{\mu}T_{\rho\mu} \neq 0) \), we use this result to find the value of energy density. Such formation of energy density from the non-conserved EMT helps to study the role of non-minimal coupling in cosmic expansion. Before finding the value of \( \rho(z) \) we should know the relation of \( H(z) \). A lot of relations exists in literature with the requirement of their theoretical consistency and observational viability. But here we will take the power law expansion in terms of red shift given as \( H(z) = H_0(1+z)\frac{\dot{z}}{z} \), where \( m \) is the power law exponent.

Power law cosmology appears as a good phenomenological explanation of the cosmic evolution, it can describe the cosmic history including radiation epoch, the dark matter epoch and the accelerating DE dominated epoch. Further, these solutions provide the scale factor evolution for the standard fluids such as dust matter case \( (m = 2/3) \) or radiation dominated eras \( (m = 1/2) \). Also, \( m \geq 1 \) predicts a late-time accelerating Universe. It provides an interesting alternative to deal with the problems like (age, flatness and horizon problems) associated with the standard model. Evolution of power law model has been discussed in various articles (Lohiya and Sethi 1999; Sethi et al. 1999; Batra et al. 2000; Gehlaut et al. 2003; Dev et al. 2002, 2008), for instance it addresses the horizon, flatness and age problems for the parametric value \( m \geq 1 \) (Sethi et al. 2005). These type of solutions are found to be consistent with various data sets including nucleosynthesis (Kaplinghat et al. 1999; Lohiya and Sethi 1999), with the age of high-redshift objects such as globular clusters (Kaplinghat et al. 1999; Lohiya and Sethi 1999), with the SNeIa data (Sethi et al. 2005; Dev et al. 2008), and with X-ray gas mass fraction measurements of galaxy clusters (Allen et al. 2004; Zhu et al. 2008). In the framework of power law cosmology, Alcaniz et al. (2005) have discussed the angular size-redshift data of compact radio sources, the gravitational lensing statistics and SNeIa magnitude-redshift relation (Lohiya and Sethi 1999; Dev et al. 2002).

In this scenario, energy density is found by solving (16) as

\[
\rho(z) = e^{-\frac{(1 + m)(6(1 + \beta)\log(1 + z) + m(-1 - 3m + \beta)(1 + 3m)\log[m(2 + \rho - 3m)] + 3H^2(1 + 3m)\alpha(1 - m(1 + m(1 + 3m)))}}{2 + 6(1 + \beta) + 3H^2(1 + m)\alpha(1 - m(1 + 3m))}} c,
\]  

(17)
where c is constant of integration. As energy density is found to be in an exponential form so it will remain positive for all values of unknowns parameters like $\alpha$, $\beta$, $\omega$, $m$, $z$. It will only depend on constant of integration c when we take negative value of c then energy density will be negative or less than zero otherwise for all positive values of c energy density will remain positive. One can also get the relation between time and redshift as

$$t = \left( \frac{1}{1 + z} \right)^{\frac{1}{2}}. \quad (18)$$

Using the value of $\rho(z)$, one can get $\rho_{\text{eff}}$ and $p_{\text{eff}}$ in terms of redshift as and we take $c = 10$ and $\omega = 1$

\[ \rho_{\text{eff}} = \frac{10(6H_0^2m(1 + z)\frac{2}{3} \alpha + m(2 - 2\beta))}{1 - (1 + z)} \frac{d(c + 3m + \beta)}{d\tau} \alpha + \beta), \]

\[ p_{\text{eff}} = \frac{1}{(1 + z)^{\frac{6(1 + \rho)}{4(1 + m + \beta)}} - 10a(6H_0^2m(1 + z)\frac{2}{3} \alpha + m(2 - 2\beta))^{\frac{4c + 3m + \beta}{1 + m + \beta}}} \times (3H_0^4(16 + 21m)(1 + z)\frac{2}{3} \alpha^2 - 12H_0^2m(1 + z)\frac{2}{3} \alpha(2 + \beta) - m(1 + \beta)(1 + 3\beta)). \]

and effective EoS in term of redshift can be written as

$$\omega_{\text{eff}} = \frac{48H_0^4(1 + z)\frac{2}{3} \alpha^2 + m(-1 - 63H_0^4(1 + z)\frac{2}{3} \alpha^2 - 2\beta + 3\beta^2 + 12H_0^2(1 + z)\frac{2}{3} \alpha(2 + \beta))}{m(1 + 3H_0^2(1 + z)\frac{2}{3} \alpha - \beta)(-1 + 6H_0^2(1 + z)\frac{2}{3} \alpha + \beta)}.$$

Cosmic acceleration can be measured through a dimensionless cosmological function known as the deceleration parameter $q$. Here, $q$ is given by

$$q = \frac{a\ddot{a}}{\dot{a}^2} = \frac{1}{m} - 1 \quad (22)$$

$q$ characterizes the accelerating or decelerating behavior of cosmos, here, $q < 0$ explains an accelerating epoch, whereas $q > 0$ describes decelerating epoch. In power law cosmology we require $m > 0$ to restrict $q$ as $q > -1$. Graphical representation of effective components $\rho_{\text{eff}}$, EoS $\omega_{\text{eff}}$ are shown in Fig. 1. In this discussion, we choose the following values of unknown parameters $\alpha = 10$, $\beta = -5$ (as per the ranges set in Table 1), and $m = 1.066658$. For this value of $m$, deceleration parameter is $-0.0624924$ which favors the expanding behavior of cosmos. We set the parameters in a way to keep the positivity of $\rho_{\text{eff}}$. It can be seen that $\rho_{\text{eff}}$ is positive and increasing function as shown on right plot and $\omega_{\text{eff}}$ approaches to $-1$ at $z = 0$ representing the $\Lambda$CDM epoch in accordance with recent observations from Plank's data (Ade et al. 2014a; Spergel et al. 2007).

4 Energy conditions

The EFE $G_{\mu\nu} = 8\pi GT_{\mu\nu}$ describe the relation between space-time geometry and matter content. The LHS of this equation represent geometry and RHS corresponds the matter distribution. The suppressed idea is that the energy matter distribution tells us that how space time is curved and how gravity plays his role. Therefore, if we apply any condition on $T_{\mu\nu}$ then it will be immediately referred to the conditions on Einstein Tensor $G_{\mu\nu}$ (Hawking and Ellis 1973). Matter energy distribution is responsible for casual and geodesic structure of space-time. For this purpose energy conditions ensure that the causality principle is appreciated and acceptable physical sources have to be studied (Hawking and Ellis 1973; Wald 1984). The energy conditions are based on Raychaudhuri equations and can be taken from the expansion given by

$$\frac{d\theta}{d\tau} = -\frac{\theta^2}{2} - \sigma_{\mu\nu}\sigma^{\mu\nu} + \omega_{\mu\nu}\omega^{\mu\nu} - R_{\mu\nu}k^{\mu}k^{\nu} \quad (23)$$

where $\theta$, $\sigma_{\mu\nu}$ and $\omega_{\mu\nu}$ shows expansion, shear and rotation respectively. These parameters are related to the congruence explained by the null vector field $k^{\mu}$. The shear is a spatial tensor with $\sigma^2 \equiv \sigma_{\mu\nu}\sigma^{\mu\nu} \geq 0$, thus it is obvious from Raychaudhuri equations that for any hypersurface orthogonal congruences, which forces $\omega \equiv 0$, the condition for attractive gravity reduce to $R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$. However, in GR, through the EFE we can write $T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$. In the context of modified theory, we used the effective EMT which is shown in (5) and positivity condition, $R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$ in the Raychaudhuri equations gives the following form of NEC $T_{\mu\nu}^{\text{eff}}k^{\mu}k^{\nu} \geq 0$ and for ordinary matter we can also write $T_{\mu\nu}^{\text{mat}}k^{\mu}k^{\nu} \geq 0$. It is simple to prove that the previous conditions impose energy density positive in all local
frame of references by using local Lorentz transformation. Energy conditions describes the behavior of the similarity of light-like, time-like and space-like curves. It is generally used in GR to find and study the singularities of space-time. The energy conditions, NEC (Null energy condition), WEC (Weak energy condition), SEC (Strong energy condition) and DEC (Dominant energy condition) in terms of EMT are given by (Visser and Barcelo 2000; Barcelo and Visser 2002).

\[ \begin{align*}
\text{NEC} & \iff \rho_{\text{eff}} + p_{\text{eff}} \geq 0, \\
\text{WEC} & \iff \rho_{\text{eff}} \geq 0, \quad \rho_{\text{eff}} + p_{\text{eff}} \geq 0, \\
\text{SEC} & \iff \rho_{\text{eff}} + 3p_{\text{eff}} \geq 0, \quad \rho_{\text{eff}} + p_{\text{eff}} \geq 0, \\
\text{DEC} & \iff \rho_{\text{eff}} \geq 0, \quad \rho_{\text{eff}} \pm p_{\text{eff}} \geq 0.
\end{align*} \]

Now, we will discuss the energy conditions for our particular model of \( f(R, T, Q) \) gravity which is \( R + \alpha Q + \beta T \) by considering FLRW metric. WEC is found to be of the following form

\[ \text{WEC: } \rho_{\text{eff}} = \rho + \frac{1}{2\alpha \rho - 2} \left[ -\frac{\beta \rho (1 + \beta - 12 \alpha H^2)}{2 \alpha \rho - 2} \right] \geq 0, \quad (24) \]

NEC yields as

\[ \text{NEC: } \rho_{\text{eff}} + p_{\text{eff}} = \frac{1}{2\alpha \rho - 2} \left[ -2 \rho (1 + \beta - 6 \alpha H^2) \right. \\
+ 2(1 + z)\alpha H H' + \rho \left( -2(1 + \beta) \right. \\
+ 6 \alpha H^2 + 4(1 + z)\alpha H H' \left. \right) \\
+ (1 + z)\alpha H \left( (1 + z)H' (p - \rho') \right. \\
+ \left. H \left( 8p' + (1 + z)(p'' - \rho'') \right) \right] \geq 0. \quad (25) \]

SEC yields as

\[ \text{SEC: } \rho_{\text{eff}} + 3p_{\text{eff}} = \frac{1}{2\alpha \rho - 2} \left[ -2 \rho (1 + 2 \beta + 3 \alpha H^2 \\
- 6(1 + z)\alpha H H' - 6 \rho (1 + 2 \beta \right. \\
- 6 \alpha H^2 + 2(1 + z)\alpha H H' \left. \right) \\
+ 3(1 + z)\alpha H \left( (1 + z)H' (p - \rho') \right. \\
+ \left. H \left( 6p' + 2\rho' + (1 + z)(p'' - \rho'') \right) \right] \right] \geq 0. \quad (26) \]

DEC yields as

\[ \text{DEC: } \rho_{\text{eff}} - p_{\text{eff}} = \frac{1}{2\alpha \rho - 2} \left[ 2 \rho (1 + 9 \alpha H^2 \\
- 2(1 + z)\alpha H H' + 2 \rho (1 + 4 \beta \right. \\
- 6 \alpha H^2 + 2(1 + z)\alpha H H' \left. \right) \\
- (1 + z)\alpha H \left( (1 + z)H' (p - \rho') \right. \\
+ \left. H \left( 2p' + 6\rho' + (1 + z)(p'' - \rho'') \right) \right] \right] \geq 0. \quad (27) \]

Inequalities (24)–(27) depends on five parameters \( \alpha, \beta, m, z, c \). In this approach, we fix two parameters and find the valid regions by varying the possible ranges of other parameters. We prefer to fix the constant of integration as \( c = 10 \) and range of \( z \) will be from \(-0.9\) to \( 10 \) and show the results for WEC and NEC. The validity region for different cases are shown in Table 1 in which we took the different values of \( m \) to show the relation between \( \alpha, \beta \) and \( m \). Initially, we fix the value of \( m = 1.1 \) for WEC then range for alpha is \( \alpha \geq 1.01 \) and for beta is \( 1.1 \leq \beta \leq 270 \) and \( -550 \leq \beta \leq -2.8 \). If we take value of \( m = 10 \) then the ranges of \( \beta \) will also increase like for \( \alpha \geq 1.01 \), it requires \( 1.1 \leq \beta \leq 270 \) and \(-550 \leq \beta \leq -2.8 \).
In this setup, we show different ranges of $\beta$ depending on the particular ranges of $\alpha$ and results are shown in Table 1. If we choose $m = 1.1$ with $\alpha \geq 1.01$ then range of $\beta$ is $(-1.3 \leq \beta \leq 0.9)$. If we fix $m = 2$ and $\alpha \geq 1.01$ then range of $\beta$ is $(-2.9 \leq \beta \leq 0.9)$. From this discussion we can conclude that range of $\beta$ for $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$ lies between $-2.9$ and $0.9$ for any value of $\alpha \geq 1.01$ and $m > 1$. Finally, in the last two columns of Table 1 we show the combine validity region for WEC and NEC. Same in this case if we increase the value of $m$ then the ranges of $\alpha$ also increases. Keep in notice that in common region, range of $\alpha$ is also restricted and very short. For $m = 1.1$ range of $\alpha$ is $0 \leq \alpha \leq 0.00000016$ and range of $\beta$ is $(-0.64 \leq \beta \leq 0.11)$. If we fix $m = 10$ then range of $\alpha$ is $0 \leq \alpha \leq 0.00001$ and range of $\beta$ is $(-0.55 \leq \beta \leq 0.22)$ for $\rho_{\text{eff}}$.

We show the graphical description for validity regions of $\rho_{\text{eff}} \geq 0$ and $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$ in Figs. 2–3. In Fig. 2, we show the validity region for $\rho_{\text{eff}} \geq 0$ $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$ for the particular choice $m = 10$. Figure 3, shows the region which validate the NEC for $m = 10$. Right side of Fig. 3 presents the common region for both $\rho_{\text{eff}} \geq 0$ and $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$ at $z = 0$ and $m = 10$. Yellow color shows the region of $\rho_{\text{eff}} \geq 0$ and blue color shows the region for $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$. The validity regions of energy conditions are shown in Table 1.

One can represent energy conditions in the combined form as

$$\beta A_1 + \alpha H A_2 \geq A_3$$

where $A_{i,s}$ purely depend on energy conditions which are

| $m$ | $\rho_{\text{eff}} \geq 0$ | $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$ | $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$, $\rho_{\text{eff}} \geq 0$ |
|-----|----------------|----------------|----------------------------------|
|     | Validity of $\alpha$ | Validity of $\beta$ | Validity of $\alpha$ | Validity of $\beta$ | Validity of $\alpha$ | Validity of $\beta$ |
| 1.1 | $\alpha \geq 1.01$ | $1.1 \leq \beta \leq 270$ | $\alpha \geq 1.01$ | $-1.3 \leq \beta \leq 0.9$ | $0 \leq \alpha \leq 0.00000016$ | $-0.64 \leq \beta \leq 0.11$ |
| 2   | $\alpha \geq 1.01$ | $1.1 \leq \beta \leq 1500$ | $\alpha \geq 1.01$ | $-2.9 \leq \beta \leq 0.9$ | $0 \leq \alpha \leq 0.000013$ | $-0.58 \leq \beta \leq 0.29$ |
| 10  | $\alpha \geq 1.01$ | $1.1 \leq \beta \leq 9000$ | $\alpha \geq 1.01$ | $-17.8 \leq \beta \leq 0.9$ | $0 \leq \alpha \leq 0.00001$ | $-0.55 \leq \beta \leq 0.22$ |

$\beta \leq 9000$) and $(-17900 \leq \beta \leq -50)$. We can see that WEC is valid only for positive values of $\alpha$ whereas $\beta$ needs some particular range for different values of $\alpha$ and $m$. If we take small value of $m$ then validity range is also small for $\beta$, likewise if we increase the starting value of $\alpha$ then range of $\beta$ also increases. Choice of $m$ and particular range of $\beta$ are directly proportional to each other while $\alpha \geq 1.01$ and $\alpha$ has also direct relation with $\beta$. If we will take larger value of $\alpha$ then we have to choose the larger value for $\beta$ and vice versa, like if we choose $m = 2$ then $\alpha = 10$ and $\beta = -5$ but if decrease the value of alpha as $\alpha = 1.001$ then $\beta$ will be $-8$.

$\rho_{\text{eff}} + p_{\text{eff}} \geq 0$ is also valid for positive values of $\alpha$. In this setup, we show different ranges of $\beta$ depending on the particular ranges of $\alpha$ and results are shown in Table 1. If we choose $m = 1.1$ with $\alpha \geq 1.01$ then range of $\beta$ is $(-1.3 \leq \beta \leq 0.9)$. If we fix $m = 2$ and $\alpha \geq 1.01$ then range of $\beta$ is $(-2.9 \leq \beta \leq 0.9)$. From this discussion we can conclude that range of $\beta$ for $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$ lies between $-2.9$ and $0.9$ for any value of $\alpha \geq 1.01$ and $m > 1$. Finally, in the last two columns of Table 1 we show the combine validity region for WEC and NEC. Same in this case if we increase the value of $m$ then the ranges of $\alpha$ also increases. Keep in notice that in common region, range of $\alpha$ is also restricted and very short. For $m = 1.1$ range of $\alpha$ is $0 \leq \alpha \leq 0.00000016$ and range of $\beta$ is $(-0.64 \leq \beta \leq 0.11)$. If we fix $m = 10$ then range of $\alpha$ is $0 \leq \alpha \leq 0.00001$ and range of $\beta$ is $(-0.55 \leq \beta \leq 0.22)$ for $\rho_{\text{eff}}$.

Fig. 2 Figure on the left represents the validity region for $\rho_{\text{eff}} \geq 0$ whereas the figure on the right side shows the validity region for $\rho_{\text{eff}} + p_{\text{eff}} \geq 0$. Herein, we set $H_0 = 67.3$. 
under discussion for WEC, we found the values of $A_{i,s}$

$$A^\text{WEC}_1 = \frac{3p - \rho}{2}, \quad A^\text{WEC}_2 = 6\rho H + \frac{3}{2}(1 + z)H(p' - \rho)$$

$$A^\text{WEC}_3 = \rho$$

for NEC, we can found as

$$A^\text{NEC}_1 = -\rho - \rho,$$

$$A^\text{NEC}_2 = p(6H - 2(1 + z)H') + \rho(3H + 2(1 + z)H') + \frac{1}{2}[(1 + z)H'(p' - \rho')$$

$$+ H(8p' + (1 + z)(p'' - \rho''))],$$

$$A^\text{NEC}_3 = p + \rho$$

for SEC, we can found as

$$A^\text{SEC}_1 = -2\rho - p,$$

$$A^\text{SEC}_2 = 3p(6H - 2(1 + z)H') + \rho(-3H + 6(1 + z)H') + \frac{3}{2}[(1 + z)H'(p' - \rho')$$

$$+ H(6p' + 2\rho' + (1 + z)(p'' - \rho''))],$$

$$A^\text{SEC}_3 = \rho + 3p$$

for DEC, we can found as

$$A^\text{DEC}_1 = 4p,$$

$$A^\text{DEC}_2 = p(-6H + 2(1 + z)H') + \rho(9H - 2(1 + z)H') - \frac{1}{2}[(1 + z)H'(p' - \rho')$$

$$+ H(2p' + 6\rho' + (1 + z)(p'' - \rho''))],$$

$$A^\text{DEC}_3 = \rho - p.$$
the early stages of cosmic evolution specifically the inflationary paradigm. The non-minimal theories as that of this theory imply the violation of equivalence principle. Similar behavior is also suggested in cosmological study, e.g., Bertolami et al. (2007b) showed that data from Abell cluster A586 supports the interaction between dark matter and energy which does imply the violation of equivalence principle. Thus it would be interesting to test these models with non-minimal coupling and explore their implications in cosmology, gravitational collapse as well as in gravitational waves.

In this article, we have constructed a cosmological scenario from the complicated non-minimal matter geometry coupling in the $f(R, T, Q)$ gravity. We consider a simplest case of non-minimal coupling in this modified theory in the form of model $f(R, T, Q) = R + \alpha T + \beta Q$. Dynamical equations are presented in Sect. 3, where we consider the power law cosmology to find an expression for energy density $\rho$. Using Eq. (17), it is obvious to find the expressions of effective EMT and its components. In power law cosmology, one can represent the cosmic history depending on the choice of parameter $m$. Here, we set parameter $m$ according to the evolution of $q$ as per recent observational data. In Fig. 1, we set $m = 1.0666580$ with $q = -0.0624924$ to see the evolution of $\rho_{\text{eff}}$ and $\omega_{\text{eff}}$, it is found that WEC is satisfied and $\omega_{\text{eff}} \rightarrow -1$ validating the current cosmic epoch (Ade et al. 2014a; Spergel et al. 2007). It is to be noted that we set the choice of parameters $\alpha$ and $\beta$ as per validity ranges expressed in Table 1, where we develop the constraints on these parameters for different values of $m$ satisfying WEC and NEC. The energy conditions bounds depend on five parameters $\alpha$, $\beta$, $m$, $z$, $c$. In this approach, we fixed two parameters and find the valid regions by varying the possible ranges of other parameters. We prefer to fix the constant of integration as $c = 10$ and range of $z$ will be from $-0.9$ to $10$ and show the results for WEC and NEC. Since we want to represent the accelerated cosmic expansion so we have to fix $m > 1$, in this regard, three different values of $m$ are selected as $m = 1.1$, $m = 2$ and $m = 10$. Using this set of values we find the validity conditions on all ranges of $\alpha$ and $\beta$. It appeared as more comprehensive way to fix the coupling parameters in accordance with energy bounds. Evolution of WEC and NEC versus redshift $z$ is presented in Figs. 2 and 3.

In literature, observational constraints have been developed on the choice of power law exponent $m$, cosmological parameters $q$ and $H_0$. Chakkrit et al. (2011) explored the phantom power law cosmology using cosmological observations from Cosmic Microwave Background (CMB), Baryon Acoustic Oscillations (BAO) and observational Hubble data, they found the best fit value of power law exponent as $m \approx -6.51^{+0.24}_{-0.25}$. Kumar (2012) found the constraints on Hubble and deceleration parameters from the latest $H(z)$ and SNeIa data as $q = -0.18^{+0.12}_{-0.12}$, $H_0 = 68.43^{+2.84}_{-2.80}$ km s$^{-1}$ Mpc$^{-1}$ and $q = -0.38^{+0.05}_{-0.05}$. $H_0 = 69.18^{+0.55}_{-0.54}$ km s$^{-1}$ Mpc$^{-1}$ respectively. The combination of $H(z)$ and SNeIa data yields the constraints $q = -0.34^{+0.05}_{-0.05}$, $H_0 = 69.18^{+0.55}_{-0.54}$ km s$^{-1}$ Mpc$^{-1}$. The consistent observational constraints on both of the parameters $q$ and $H_0$ according to latest 28 points of $H(z)$ are found as $q = -0.0451^{+0.0614}_{-0.0625}$, $H_0 = 65.2299^{+2.4862}_{-2.4607}$ in case of Union 2.1 SN data, these parameters take the values $q = -0.3077^{+0.1045}_{-0.1036}$, $H_0 = 68.7702^{+1.4052}_{-1.3754}$ Rani et al. (2015). Using the data set of Kumar (2012) and Rani et al. (2015), we choose the parameter $m$ and develop the ranges of $\omega_{\text{eff}}$ as shown in Table 2. For $m = 1.221$, $\omega_{\text{eff}}$ is found to be $-1.31601$ which agrees with the observational results of Planck+WMAP+H0 (Ade et al. 2014a; Spergel et al. 2007). Also, for the choice of $m = 1.0473$ and $m = 1.4445$, results of $\omega_{\text{eff}}$ are consistent with the observational data of 95% (WMAP5+BAO+SN) (Komatsu et al. 2009) and WMAP9 (Hinshaw et al. 2013) as shown in Table 2.

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