Economic prospects of Steel Reinforced Functionally Graded Concrete (SRFGC) beam structures

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Abstract. Engineering materials, for instance, ceramics and concrete, are manufactured with uniform properties purposefully. Because the uniformity is a convenient measurement to ensure the safety design calculation of a structure to be built. The non-uniform material property of concrete can be resulted from mixing, placing, consolidation, and curing processes during the manufacturing of structural members. Therefore, using concrete as a material could lead to an uneconomical use of natural resources because of non-uniformity problems that occur in the structural members. Ideally, in the Functionally Graded Concrete (FGC) material, the properties (elasticity, density, rigidity) of concrete material have to be smoothly graded in the projected directions (longitudinally, transversally, diagonally) by following a governing equation. Hence, FGC has significant advantages over the existing concrete materials in the prominence of cement optimization. In recent progress, many researchers have been studied the mechanical behaviors and production methodologies of FGC enthusiastically. In the state-of-the-art of the FGC developments, two objectives need to be achieved: 1. The manufacturing procedure of FGC material, and 2. The method of analysis to design FGC material. In current developments, the technologies for manufacturing FGC material are still not available for the practitioners. The objective of this paper is to present a method of analysis to design a Steel Reinforced Functionally Concrete Graded (SRFGC) beam element based on the SNI (Indonesian National Standard) of Structural Concrete Requirements for Buildings code 2847:2013. The effects of the non-uniformity of concrete strengths are incorporated into the analysis. Because of the crack-opening assumption in the tensile region of the concrete, the neutral axis position is no longer at the mid-height of the beam cross-section. Hence, an iterative technique has to be employed in the design. An iterative technique is necessary because the internal forces of the beam have to be calculated at the cross-section being designed. The application of the method of analysis is highlighted by designing various type of beam cross-sections, which show the economic benefits of SRFGC beam compared to both the normal and high strength concrete’s material.
1. Functionally graded material

Concrete and ceramic are the industrial materials which are designed and manufactured intentionally to have homogeneous mechanical properties. The material homogeneity is a worthwhile criterion to ensure the safety of members of a structure. The non-homogeneous material property is a result of method of mixing, placing method, consolidation process, and curing process, including the effects of segregation and accumulation of the aggregates during the mixing. Thus, the non-homogeneity could show uneconomical use of natural resources due to high-stress concentrations that occur in the boundary between structural elements. On the opposing to the homogeneity assumed in the design and analyses, the steel Reinforced Concrete (RC) structure element in the built structures are mainly found as a graded concrete material [1,2].

Functionally Graded Material (FGM) is a new phrasing of a composition between two or more different materials where the essential properties are graded over a particular orientation to gain some desired functional properties [3,4]. In the compositions of FGM, the two or more material properties are merged functionally to enhance the individual material property. In nature, bamboo is one among typical materials that show a radial gradient as a result of the evolutionary process to adapt to their growing environment.

Compared to the experimental works, there have been a vast number of researches doing the analyses of FGM numerically [5-14], from where the analyses of FGM can be used to analyze the Functionally Graded Concrete (FGC).

By today, FGC has not been implemented widely in construction projects. Two major problems in implementing FGC are: there is no producing method established yet, and there is no building code available for designing FGC elements in structures. Variation of elastic moduli and strengths of concretes in FGC element needs an accurate method of analysis to predict its strength and behavior [15].

In this paper, the method and analysis of the ACI 318M-11 [16] to design a Steel Reinforced Functionally Concrete Graded (SRFGC) beam with various cross-sections subjected to a constant bending moment are highlighted. By using the iteration method, the SRFGC can be designed similarly with the conventional steel RC beam. In the end, a comparison study on the cost of material is showed to highlight the economic prospects of the SRFGC.

2. Functionally graded concrete (FGC)

Researches on FGC, however, are still inadequate. The efforts [4,17-18] to manufacture an FGC material had confronted one challenging difficulty: to produce a smooth graded between two different properties of concrete strengths. The continuity of graded concrete has not been obtained. Besides, laminated or composite concrete strengths were found. In the laminated or composite concrete strengths, the stress concentration between the concrete layers will degrade the quality of the FGC material.

Experimentally [19-21], these studies have revealed that the ultimate strengths of the FGC were restricted by the lowest concrete strength of the constituents in the FGC, but the stiffness of the FGC is close to the stiffness of the highest concrete strength in the mixture.

2.1. SRFCG beam

The application of FGM concept to the RC beam can be achieved by producing SRFCG. By varying two different types of the compressive strengths in the vertical direction of the cross-section of a beam, a potential approach to reduce the unneccessity of concrete strength at the bottom fiber under the tensile stress and increasing the necessity of concrete strength at the top fiber under compressive stress. Preferably, there will be no reduction of the beam’s bending strength, and at the same time can potentially reduce the cost of RC material. Similar ideas could be applied for the improvement of other building members.
The Young’s modulus of the FGC beam is expressed as follow:

\[ E_c(y) = E_{cb} \left[ 1 + \frac{E_{ct} - E_{cb}}{E_{cb}} \left( \frac{y}{h} \right)^p \right] \]  

(1)

where \( E_c(y) \) is the concrete elastic modulus at \( y \)-ordinate; \( E_{cb} \) is the concrete elastic modulus at the bottom of the beam cross-section; \( E_{ct} \) is the concrete elastic modulus at the top of the beam cross-section; \( h \) is the height of the cross-section and \( p \) is the gradient of the modulus variation. \( y \) is measured from the bottom fiber of the cross-section.

2.2. Design of SRFGC beam cross section

The kinematic assumptions of SRGFC beam cross-section under a bending moment is illustrated in Fig. 1. The compression forces consist of the un-cracked concrete portion and compression steel bar, while the tensile force is only given by the steel bar in the cracked concrete portion. The design of the beam cross-section is calculated iteratively by balancing the moment strengths resulted by the concrete and steel.

In the case of a beam subjected to a bending moment, the compressive force is resisted by the concrete part above the neutral axis and the compressive rebar, and the tensile force is resisted by the tensile rebar below the neutral axis. Beam cross-section design is iteratively calculated by taking into consideration the balance of moment applied to the concrete and rebar steel. The elastic modulus ratio used in this calculation can be defined as a function of the distance from the bottom fiber as follows.

\[ n(y) = \frac{E_s}{E_c(y)} \]  

(2)

The equilibrium state of stress between concrete and rebar steel is reached by assuming the yielding of compressive concrete and tensile steel bar which can be expressed as follows, except for the tensile concrete part below the neutral axis is not considered.

\[ T_s = C_c + C_s \]  

(3)

where,

\[ T_s = f_y \sum_{i=1}^{n_c} A_{si} \]

\[ C_s = E_s \left( 1 - \frac{\beta_c d'}{a} \right) \times 0.003 \times \sum_{i=1}^{n_s} A_{si} \]
\[ C_c = 0.85 f'_{Ct} \int_{\beta c} b \, d\bar{y} \]

where, \( T_s \) is the total tensile force from the steel bars; \( C_s \) is the total compressive force from the steel bars; \( C_c \) is the total compressive force from the uncracked concrete portion; \( f'_{Ct} \) is the stress of the concrete fiber at the top of the beam’s cross-section; \( c \) is the height of the uncracked steel reinforced FGC section from the neutral axis; \( \bar{y}, \bar{y}_c \), and \( \bar{y}_t \) are measured from the neutral axis of the cracked beam cross section; \( nS, nS' \) are the number of tensile and compressive steel bar and \( b, A_s \) and \( A_t \) are the width of the uncracked concrete, \( i \)-th tensile steel bar and compressive steel bar, respectively.

The force equilibrium of the cross-section is then can be computed by,

\[ E_s \left( 1 - \frac{\beta d'}{a} \right) \times 0.003 \times \sum_{i=1}^{10} A_{S,i} + 0.85 f'_{Ct} \int_{\beta c} b \, d\bar{y} + f_s \sum_{i=1}^{10} A_{S,i} = 0 \]  

From where the value of \( a \) can be obtained by iteratively solving the above equilibrium equation.

By using the equivalent block ultimate stress of steel bar and concrete (compressive side), the bending-resisting moments of the SRFGC beam cross-section can be determined from the lowest value between:

\[ M_{Sr} = C_s \left( d - d' \right) \left( 1 - \frac{\beta d'}{a} \right) \times 0.003 \times \sum_{i=1}^{10} A_{S,i} \left( d - d' \right) \]

\[ M_{Cr} = C_C \left( d - \frac{a}{2} \right) = 0.85 f'_{Ct} \int_{\beta c} b \, d\bar{y} \left( d - \frac{a}{2} \right) \]

where, \( M_{Sr} \) and \( M_{Cr} \) are the equivalent resisting moments of steel and concrete, respectively.

3. Material cost comparison study

In this study, a homogenous RC beam strength is at 69MPa and 24 MPa, the SRFGC beams of varying concrete compressive strengths from the bottom to the top of the beam cross section which is designed based on the equivalent block ultimate stress design method (see Fig. 1), to calculate the cost of material for comparison. Two types of cross sections (see Fig. 3) are considered: rectangular and trapezoidal cross sections.

3.1. Parameter of material properties

The graded modulus function in Eq. (1) of the concrete compressive strengths is selected to follow the degree of polynomial order of \( p = 1 \) (linear), 2 (quadratic), and 3 (cubic) cases variation. Fig. 2 shows two homogeneous (\( p = 0 \)) RC beams of 69 MPa and 24 MPa concrete compressive strengths, namely Case A and Case C, respectively. Case B-1, B-2 and B-3 in Fig. 2 show SRFGC cross-sections of functionally graded concrete compressive strengths which vary from 24 MPa at the bottom fiber and 69 MPa at the top fiber of the beam cross-section following the polynomial order of \( p \).

The following equation determines the Young’s modulus of concretes.

\[ For \ F_c \leq 36 \text{ N/mm}^2 : E_c = 2.10 \times 10^4 \left( \frac{\gamma_c}{23} \right)^{\frac{v}{2}} \left( \frac{F_c}{20} \right)^{\frac{v}{2}} \]

\[ For \ F_c > 36 \text{ N/mm}^2 : E_c = 3.35 \times 10^4 \left( \frac{\gamma_c}{24} \right)^{\frac{v}{2}} \left( \frac{F_c}{60} \right)^{\frac{v}{3}} \]

where \( E_c \) is the elastic modulus of concrete in N/mm\(^2\); \( F_c \) is the compressive design strength of concrete in N/mm\(^2\) and \( \gamma_c \) is the weight density of concrete in kN/m\(^3\). The weight densities of \( \gamma_c = 23 \text{ kN/m}^3 \) and \( \gamma_c = 25 \text{ kN/m}^3 \) for concrete with compressive strengths of \( F_c = 24 \text{ N/mm}^2 \) and \( F_c = 69 \text{ N/mm}^2 \), respectively. The weight densities of \( \gamma_s = 78 \text{ kN/m}^3 \) is assigned to the steel bar.
3.2. Volume fraction of FGC beam

To calculate the cost of material for each case, the volume fraction of the respective material constituent in the SRFGC should be computed from the weight density distribution in Eq. (1). The SRFGC weight density distribution of concrete part can then be given as,

$$\gamma_c(y) = \gamma_{cb} \left[ 1 + \frac{\gamma_{ct} - \gamma_{cb}}{\gamma_{cb}} \left( \frac{y}{h} \right)^b \right]$$

(8)

where $\gamma_c(y)$ is the weight density of concrete at $y$ ordinate; $\gamma_{cb}$ is the weight density of concrete at the bottom fiber of the beam; $\gamma_{ct}$ is the weight density of concrete at the top fiber of the beam.

By integrating Eq. (8) along to the depth of the SRFGC beam’s cross-section, one can obtain the total volume per unit depth $V$ of the concrete part of the SRFGC from,

$$V = \int_0^h \gamma_{cb} \left[ 1 + \frac{\gamma_{ct} - \gamma_{cb}}{\gamma_{cb}} \left( \frac{y}{h} \right)^b \right] b(y) \, dy$$

(9)

where, $b(y)$ is the $y$-variable width of the SRFGC beam cross-section.

3.3. Comparison of material cost

The average unit price of homogeneous concrete material and steel bar available in the standard material construction in Japan at present (2019) are shown in Table 1.

| Table 1. Concrete and steel bar unit price (2019 average price market in Japan) |
|--------------------------|--------------------------|--------------------------|
| Strength $F_c'$ (MPa)    | Price (Yen/m$^3$)        | Steel bar SD-345 (Yen/Ton) |
|--------------------------|--------------------------|--------------------------|
| 24                       | 17,000                   | 67,000                   |
| 69                       | 34,700                   |                           |

As the basis of the comparison study, the high strength concrete of RC beam (Case-A) is designed to determine the reference bending moment for loading, its dimension of the cross-section and required reinforcing steel bars are designed based on the ACI 318M-11 [1]. The results are then used for designing the other cases to find the optimal beam cross-section dimensions and necessary reinforcing steel bars. The equivalent block stress of concrete is determined from $f_{c'} = F_c / 3$, and the equivalent block stress of steel bar is limited by $f_s = 215 \, \text{N/mm}^2$ (Steel Grades Carbon Steel SD345). The design is optimized by finding a suitable loading of bending moments that are close to Eqs. (5-6),
the dimensions of the cross-section and requirement of reinforcing steel bars within the equivalent block stress limit of concrete and steel bar materials.

Two beam cross-sections, as shown in Fig. 3 are selected for comparison study. A rectangular and a trapezoidal cross-section with the dimension of $b \times h$ and $(b + b') \times h$, respectively, being reinforced by double steel bars.

![Figure 3. Various SRFGC beams cross-sections](image)

The results of design and analysis of various the RC and SRFGC beam subjected to bending moment are tabulated in Tables 2 and 3. The beam cross-section dimensions, required steel bars, and the total cost of material are also shown in the tables.

**Table 2.** Material cost comparison between RC and SRFGC rectangular cross-section beam subjected to bending moment ($M_{\text{design}} = 870$ kN.m)

| Case | $b \times h$ (in mm) | Concrete volume ($m^3$) per unit length | $f'c = 28$ MPa | $f'c = 69$ MPa | Steel bars | Total price (yen/ $m^3$) |
|------|---------------------|----------------------------------------|----------------|----------------|-------------|-------------------------|
| A    | 400 x 400           | 0.0000                                 | 0.1600         | 0              | 1 x 2 x Ø16 | 7,143 (+17.5%)           |
|      |                     |                                        |                |                | 1 x 4 x Ø29 |                         |
| B-1  | 400 x 420           | 0.0840                                 | 0.0840         | 1              | 1 x 2 x Ø16 | 5,934 (+2.4%)            |
|      |                     |                                        |                |                | 1 x 4 x Ø29 |                         |
| B-2  | 400 x 430           | 0.1173                                 | 0.0587         | 2              | 1 x 2 x Ø16 | 5,621 (-7.5%)            |
|      |                     |                                        |                |                | 1 x 4 x Ø29 |                         |
| B-3  | 400 x 440           | 0.1350                                 | 0.0450         | 3              | 1 x 2 x Ø16 | 5,447 (-10.4%)           |
|      |                     |                                        |                |                | 1 x 4 x Ø29 |                         |
| C    | 400 x 570           | 0.2640                                 | 0.0000         | 0              | 1 x 2 x Ø16 | 6,079                   |
|      |                     |                                        |                |                | 1 x 4 x Ø29 |                         |

In the study, the volume of the concrete is calculated from the fraction volume which is given by Eq. (9) multiplied by the corresponding weight density and unit price shown in Table 1. The price of steel bars per unit length, which is calculated from the unit price are also shown in Table 1.
Table 3. Material cost comparison between RC and SRFGC trapezoidal cross-section beams subjected to bending moment ($M_{\text{design}} = 870$ kN.m)

| Case | $b \times b' \times h$ (in mm) | Concrete volume ($m^3$) per unit length | $f'c = 28$ MPa | $f'c = 69$ MPa | $p$ | Steel bars | Total price (yen/ $m^3$) |
|------|-------------------------------|------------------------------------------|----------------|----------------|-----|------------|---------------------|
| A    | 350 x 400 x 460               | 0.0000                                   | 0.1725         | 0              | 2 x Ø16 | 4 x Ø29     | 7,577 (+25.2%)      |
| B-1  | 350 x 400 x 480               | 0.0843                                   | 0.0807         | 0.0136         | 2 x Ø16 | 4 x Ø29     | 5,824 (-3.8%)       |
| B-2  | 350 x 400 x 490               | 0.1169                                   | 0.0556         | 0.0221         | 2 x Ø16 | 4 x Ø29     | 5,507 (-9.0%)       |
| B-3  | 350 x 400 x 510               | 0.1368                                   | 0.0432         | 0.0487         | 2 x Ø16 | 4 x Ø29     | 5,416 (-10.5%)      |
| C    | 350 x 400 x 660               | 0.2625                                   | 0.0000         | 0              | 2 x Ø16 | 4 x Ø29     | 6,053               |

4. Summary and discussion

From Tables 2 and 3, in general, Cases B-3, which are the cubical SRFGC cross-sections, shows the lowest cost of material compared to both high-strength RC (Case A) and the normal RC (Case C). The high strength RC (Case A) has highest material price compared to the normal RC (Case C). In Figs. 4 and 5, the lowest cost of material was found in Case B-3 for all type of cross-sections where the gradation function of compressive strength is following the 3rd degree ($p = 3$) of the polynomial.
The following conclusions can be drawn from these findings.

1) When comparing the cost of material, the SRFGC beams (Case B-1, B-2 and B-3) exhibit more economical than both the normal RC beams (Case C) and high-strength RC beams (Case A). Moreover, among the SRFGC beams, the cost of material of gradation \( p = 3 \) (Case B-3) shows the lowest and the most economical case.

2) The most effective graded function of the SRFGC beam was found to be \( p = 3 \). Therefore, the difficulty to create a linear (\( p = 1 \)) SRFGC beams in practice is not necessary.

3) One additional benefit of using FGC in the SRFGC was pointed in previous study [10,17,20], where the compressive strength of the FGC was limited by the lowest compressive strength part and the rigidity of the FGC was following the highest compressive strength part.

Finally, the results of comparisons between the weight per unit length of the most economical (Case B-3) SRFGC beams to the normal RC beam (Case C) and high-strength RC beam (Case A) are summarized in Table 4. From the Table, it was shown that the weight of the SRFGC beam (Case B-3) is lighter than that of the normal RC beam and almost the same weight with the high-strength RC beam in any type of cross-sectional shapes under the same bending moment. Especially for the normal concrete, the weight is mostly reduced by 29 % and very close when compared to the high-strength beam (Case A).

| Comparison between Case B-3 with | Cross-section | Rectangle (%) | Trapezoïd (%) |
|---------------------------------|---------------|---------------|---------------|
| Case A (High strength concrete)  | +4.0          | -3.2          |
| Case C (Normal concrete)        | -28.8         | -28.5         |

Therefore, the SFRGC of Case B-3 exhibits the lower weight due to smaller beam cross-section required compared to the normal concrete (see Table 4 for Case C), while cheaper material costs compared to both the normal and high strength RC beams (see Tables 2 and 3 for Cases A and C).

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