The Pricing of Systematic Liquidity Risk in Stock Markets

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resumo | résumé / abstract

A questão de a rendabilidade estar ou não afectada pela liquidez não está ainda resolvida. A ausência de resultados concludentes na investigação empírica sugere que a relação entre a avaliação de activos e a liquidez não tem sido estudada adequadamente na literatura habitual. Considerarmos que os shocks sistemáticos da liquidez poderiam afectar o óptimo comportamento dos agentes nos mercados financeiros. De facto, as flutuações nas diferentes medições da liquidez são significativamente correlacionadas nos activos mais comuns. Em consequência, propomos a construção de um factor de liquidez baseado no rácio de Amihud (2002) e no procedimento de aproximação ortogonal de Fama e French (1993) para o incluir como mais uma variável adicional no seu modelo de três factores.

En le qui concerne la liquidité affectée ou non à la rentabilité des actifs n’est pas encore résolue. L’absence de résultats concluants dans la recherche empirique préalables suggère que la relation évaluation actifs et liquidité adéquatement n’a pas été étudiée dans la littérature standard. Nous considérons que les chocs systématiques de liquidité peuvent affecter le comportement optimal des agents sur les marchés financiers. De fait, des fluctuations dans diverses mesures de liquidité sont significativement reliées entre des actifs. Par conséquent, nous proposons la construction d’un facteur de liquidité basé le rapport d’Amihud (2002) et sur la procédure de rapprochement orthogonal Renommée et de French (1993), pour qu’il puisse être inclus comme une variable additionnelle dans son modèle de trois facteurs.

The question whether liquidity affects asset returns or not remains unresolved thus far. The absence of conclusive results in previous research suggests that asset pricing and liquidity have not been properly addressed in the standard literature. We consider that systematic liquidity shocks affect the optimal behavior of agents in financial markets. Indeed, fluctuations in various measures of liquidity are significantly correlated across common stocks. Accordingly, we propose the construction of a liquidity risk factor based on the ratio of absolute stock returns on euro volume suggested by Amihud (2002) and the approximately orthogonalizing procedure of Fama and French (1993), using it as an augmenting variable in their three-factor model.
1. Introduction

It is generally accepted that asset liquidity influences investors' portfolio decisions because of its close relation to transaction costs. It is reasonable to think that investors who buy illiquid assets require higher expected returns on their investments because a lack of liquidity can be interpreted as an additional risk.

However, the question of whether or not liquidity affects asset returns remains unresolved. While various theoretical models have indicated that liquidity risk is an important factor in explaining returns, empirical studies have failed to find significance for this liquidity risk factor. The reason could be that liquidity-risk measures are weak or proxy total instead of systematic liquidity risk.

This paper contributes to the determination of the role of a liquidity factor in the structure of returns. Our main objective is to re-assess the role of liquidity risk, in the context of a Fama and French (1993) framework, by using the measure of systematic liquidity risk proposed by Amihud (2002).

We use the Fama and French (1993) portfolio sorting procedure to estimate the liquidity factor and then test the importance of this factor in asset returns, by contrasting the performance of four models: the standard CAPM, the Fama and French three-factor model and those two models augmented by the liquidity factor. The data includes all stocks traded on the Spanish stock market from January 1994 to December 2002. Overall, results suggest that liquidity risk is important in explaining the structure of returns in the Spanish stock market.

We consider these results to be of particular importance to professional asset managers and risk managers. Systematic risk prediction is undoubtedly of great importance when making investment decisions, when evaluating financial assets or when structuring portfolios. In short, new evidence on the importance of liquidity risk in asset pricing can be valuable for researchers and have important implications and practical value for investors.

The rest of the paper is organized as follows. Section 2 contains the literature review and provides the theoretical motivation for this analysis. Section 3 describes Amihud (2002)'s measure of liquidity risk, the portfolio formation procedure and the proposed tests to investigate the role of the liquidity factor. Section 4 indicates the data employed and highlights the empirical results obtained from the Spanish stock market. Section 5 has concluding remarks.

2. The role of liquidity in asset pricing

The question of whether liquidity determines expected returns has been widely documented in the financial literature. Using a variety of liquidity measures, studies analyze whether less liquid stocks have higher average returns than expected.

Amihud and Mendelson (1986) were among the first to examine the role of liquidity in asset pricing. They analyzed the relationship between stock returns and bid-ask spreads and found empirical evidence related to the existence of a liquidity premium. However, Eleswarapu and Reingenanum (1993), who extended the sample period by ten years, examined the effect of seasonality on bid-ask spreads and returns. They found that the relationship between bid-ask spreads and asset returns is mainly limited to the month of January.

Brennan and Subrahmanyam (1996) examined whether illiquidity costs caused by adverse selection result in higher expected returns. Instead of using bid-ask spreads as a proxy for liquidity they used estimated variable and fixed transaction costs. They adjusted for risk using the Fama and French (1993) three-factor model and found a concave relationship between

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1 They argue that bid-ask spreads are not an appropriate proxy for liquidity as bid-ask spreads are noisy. Additionally, liquidity costs caused by asymmetric information are captured in the variable component of trading costs.
premiums and variable costs and a convex relationship between premiums and fixed costs, in contrast to the concave relationship found by Amihud and Mendelson (1986). Previous research for the Spanish stock market tends to show quite similar distress results related to the temporal and cross-sectional behavior of bid-ask spreads and other measures of liquidity. For the period 1990-1994, Tapia (1997) analyzes the seasonality of liquidity premium considering the influence of fiscal reasons on trading. The main results of this work indicate the existence of a differential behavior for the liquidity premium, but not for asset with more probability of trading by fiscal reasons. Moreover, when he includes the size variable, his results are weaker. Rubio and Tapia (1998), employing Brennan and Subrahmanyam's (1996) methodology, provide evidence on the relationship between bid-ask spreads and stock returns, analyzing the effect of seasonality. The results show a positive liquidity premium in January, although not significantly different from zero. Nevertheless, the most complete study for the Spanish stock market about bid-ask spreads was published by Blanco (1999). This work is based on the influence of minimum variations in prices on bid-ask spreads. He argues that bid-ask spreads underestimate the temporal and cross-sectional movements in liquidity.

Given the lack of robustness of empirical results, several investigators have re-examined the relationship between liquidity and asset returns using alternative measures of liquidity that allow us to approach the concept of liquidity employed by investors in their financial decisions. In this sense, a large number of papers have focused on the use of liquidity measures based on trading activity, such as trading volume (Brennan et al., 1998), turnover (Datar et al., 1998) or illiquidity ratio (Amihud, 2002), that allow us to obtain a larger series of observations over a longer period of time and to check the robustness of the empirical results.

Brennan, Chordia, and Subrahmanyam (1998) found that the stock volume has a significant negative effect on the cross-section of stock returns and it subsumes the negative effect of size. Spanish market evidence is reported by Miralles and Miralles (2003). However, this liquidity measure has two potential problems. First, the number of shares traded is not by itself a sufficient statistic for the liquidity of a stock since it does not take into account the differences in the number of shares outstanding or the shareholder base. Second, the use of the euro volume has a size bias.

Another related measure is turnover, i.e. the ratio of trading volume to the number of shares outstanding, which we can employ as a measure of the asset trading frequency. Datar et al. (1998), Rouwenhorst (1999), and Chordia et al. (2001) found that cross-sectionally stock returns decrease in stock turnover, which is consistent with a negative relationship between liquidity and expected return.

More recently, Amihud (2002) has proposed a new measure based on trading activity, the aggregate ratio of absolute stock returns on euro volume. In particular, Amihud's illiquidity ratio has a strong theoretical appeal. In this work, Amihud (2002) focuses on the time-series aspects of aggregate liquidity and documents for the US market: a time series relationship between liquidity and expected return on the market level.

In this regard, we have to point out those recent studies that analyze the asset pricing implications of a systematic liquidity risk factor. Chordia et al. (2000) find that the daily relative changes in individual asset liquidity are strongly related to changes in market and industry aggregates. However, market liquidity as a state variable in an asset pricing framework has been investigated by Pastor and Stambaugh (2003). According to earlier studies that document commonality in liquidity, they argue that changes in aggregate liquidity could be non-diversifiable and therefore a priced risk factor. They construct an aggregate monthly liquidity measure and show that monthly portfolio returns command a positive risk premium for the changes in this measure, even after controlling for other systematic risk factors. Martínez et al. (2004), however, do not find evidence relating to this risk factor for the Spanish stock market.

2 A relevant example of these issues can be found in the paper by Rubio and Tapia (1996).
3 The first-order autocorrelation measure in returns conditional on signed volume.
Following these two current streams of research, the main object of this study is to construct a
risk factor based on liquidity and to analyze pricing implications for the Spanish stock market
over the 1994-2002 period. We generate a liquidity factor employing the orthogonal approach
procedure of Fama and French (1993) and analyze whether it should be included as an
augmented variable in the stochastic discount factor. Another significant contribution of this study
is its use of an alternative measure of liquidity based on trading activity of common stocks, the
illiquidity ratio suggested by Amihud (2002), that can be interpreted as the daily price response
associated with one euro of trading volume.

3. Methodology

3.1. The illiquidity ratio

Liquidity is a broad and elusive concept that generally denotes the ability to trade large quantities
quickly, at low cost, and without moving the price (Pastor and Stambaugh, 2003), but liquidity is
not an observable variable. There are many proxies for liquidity, such as the relative bid-ask
spread, adverse selection, depth, or probability of information-based trading. But these are based
on market microstructure data, and are not available for a time series as long as is usually
desirable for studying the effect on expected returns. In contrast, the illiquidity measure used in
this study is calculated from daily data on returns and volume that are readily available over long
periods of time for most markets.

Following previous studies for the US market reported by Amihud (2002) and Acharya and
Pedersen (2003), we use the "illiquidity ratio" for our empirical analysis, as being the best proxy
measure of illiquidity that computes the price response associated with one euro of trading
volume.

The illiquidity ratio of stock $i$ in month $t$ is calculated as (1),

$$\text{ILLIQ}_{it} = \frac{1}{D_{it}} \sum_{d=1}^{D_{it}} \frac{|R_{id}|}{V_{id}}$$

where $R_{id}$ and $V_{id}$ are, respectively, the return and dollar volume on day $d$ in month $t$, and $D_{it}$ is
the number of valid observation days in month $t$ for stock $i$. The reasoning behind this illiquidity
measure is as follows. A stock is illiquid, that is, has a high value of $\text{ILLIQ}_{it}$, if the stock's price
moves a lot in response to little volume.\(^4\)

The advantage of using the illiquidity ratio is that it has a strong theoretical appeal. Hasbrouck
(2002) finds that this measure appears to be the best of the usual proxies employed to capture
Kyle's lambda.

This measure is interpreted as the daily stock price reaction to one euro of trading volume.
Following Amihud (2002), $\text{ILLIQ}_{it}$ can also be interpreted as a measure of consensus belief
among investors about new information. Thus, when investors agree about the implication of
news, the stock price changes without trading, while disagreement induces an increase in trading
volume.

Finally, it should be pointed out that this measure can be easily obtained from databases that
contain daily data on stock return and volume. This makes it available to most stock markets and
enables us to construct a time series of illiquidity over a long period of time, which is necessary
for the study of the effects of illiquidity over time. Moreover it allows checking the robustness of
the available results.

\(^4\) The estimated variable is multiplied by $10^6$.\(\)
3.2. Mimicking portfolio formation

We propose the construction of an illiquidity-based risk factor, proxied by Amihud's illiquidity ratio, in the context of the Fama and French (1993) framework through the formation of mimicking portfolios. This illiquidity mimicking factor is created by obtaining the difference between the mean return on a set of illiquid stock portfolios (I) and the mean return on a set of very liquid (V) stock portfolios, named IMV (illiquid minus very liquid).\(^5\) The advantage of this construction is that each factor is formed while controlling for the effect of the other Fama and French factors.

For the size and book-to-market portfolio formation procedure, we followed Fama and French (1993). At the end of December in year \(t-1\), the companies were ranked by size and partitioned into small (S) and big (B) companies. Then, the sample companies were ranked by book-to-market and partitioned into three groups, high (H), medium (M) and low (L). Finally, the illiquidity ratio was used to rank companies into very liquid (V), moderately liquid (A) and illiquid (I) companies. We took the average of twelve monthly illiquidity ratios as our measure of the company's illiquidity throughout the year \(t-1\) to avoid the possible effect of seasonality.

Based on the independent sorts and ranking procedure, in year \(t-1\) we constructed eighteen portfolios from the intersection of the two size, three book-to-market and three illiquidity groups \(\{S/L/V, S/L/N, S/L/I, S/M/V, S/M/N, S/M/I, S/H/V, S/H/N, S/H/I, B/L/V, B/L/N, B/L/I, B/M/V, B/M/N, B/M/I, B/H/V, B/H/N, B/H/I\}\).

Following the procedure developed by Fama and French (1993), the size factor SMB (small minus big) was calculated each month as the difference between the simple average of the returns on the nine small company portfolios \(\{S/L/V, S/L/N, S/L/I, S/M/V, S/M/N, S/M/I, S/H/V, S/H/N, S/H/I\}\) and the simple average of the returns on the nine big company portfolios \(\{B/L/V, B/L/N, B/L/I, B/M/V, B/M/N, B/M/I, B/H/V, B/H/N, B/H/I\}\).

The book-to-market factor HML (high minus low) was generated each month as the difference between the simple average of the returns on the six high book-to-market company portfolios \(\{S/H/V, S/H/N, S/H/I, B/H/V, B/H/N, B/H/I\}\) and the simple average of the returns on the six low book-to-market company portfolios \(\{S/L/V, S/L/N, S/L/I, B/L/V, B/L/N, B/L/I\}\).

Also, the illiquidity factor IMV (illiquid minus very liquid) was created each month as the difference between the simple average of the returns on the six illiquid company portfolios \(\{S/L/I, S/M/I, S/H/I, B/L/I, B/M/I, B/H/I\}\) and the simple average of the returns on the six very liquid company portfolios \(\{S/L/V, S/M/V, S/H/V, B/L/V, B/H/V\}\).

3.3. Dependent variable portfolio formation

The next step consists of constructing our dependent variable: 10 illiquidity-based sorted portfolios according to the average illiquidity value of each security in the previous year. \(P_1\) includes the stocks with the smallest illiquidity ratio within the sample and \(P_{10}\) contains the stocks with the largest illiquidity ratio. Portfolio returns were also calculated giving equal weight to each asset within the portfolio. These are the portfolio returns which are employed in testing the illiquidity-based asset pricing models in the next sections.

3.4. Research method

Our approach to determining the role of an illiquidity factor in asset pricing was as follows. First of all, we analyzed the standard CAPM model within a time-series context and for each of the 10 illiquid-based portfolios using (2). However, we also analyzed the available results provided by the Fama and French three-factor model using (3). Finally, we tested the standard CAPM and Fama and French three-factor asset pricing models augmented by the illiquidity factor in (4) and (5),

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5 As suggested by Chan and Faff (2004).
where $r_{jt}$ is the excess return on portfolio $j$, $r_{mt}$ is the excess return on the market portfolio, $SMB_t$ is the mimicking portfolio for the size factor, $HML_t$ is the mimicking portfolio for the book-to-market factor, and $IMV_t$ is a mimicking portfolio for the illiquidity factor, $a_j$ is the intercept of portfolio $j$, and $P_{jm}$, $P_{jsmb}$, $P_{jhml}$ and $P_{jimv}$ are the sensitivities to the risk factors.

We compared the overall significance of the alternative risk specifications and the statistical significance of the estimated factor exposures. And, following Ferson and Harvey (1999), we run a misspecification test for the hypothesis that the liquidity risk factor may be excluded from the regressions.\(^6\)

We also compared the joint significance of the intercept terms. In the CAPM framework the intercept should be zero. Otherwise there are other sources of risk that are not captured by the market factor. In the multivariate framework a significant intercept implies that not all the relevant variables are included as factors or that firm specific risk is still present in the dataset.

Moreover, we observed the standard zero intercept restriction that constitutes the null hypothesis: $H_0: \alpha_j = 0; j = 1, 2, ..., 10$ using the Wald test, asymptotically distributed as a chi-square statistic with degrees of freedom equal to the number of restrictions under the null hypothesis.

While we can estimate the models using a variety of different systems-based methods, we have chosen the generalized method of moments (GMM) approach of MacKinlay and Richardson (1991). These tests may be interpreted, within the context of Grinblatt and Titman (1987), as testing that there is one risk factor that is globally mean-variance efficient. Our estimation technique employs the optimal weighting matrix, which is the inverse of the covariance matrix of the sample moments. Specifically, we impose a heteroskedasticity and autocorrelation consistent covariance matrix in all estimation, which involves a Bartlett kernel with a Newey-West fixed bandwidth and no prewhitening. And following Ferson and Foerster (1994), we use an iterated procedure. The initial weighting matrix is obtained using consistent two-stage least squares initial estimates of the parameter set.

4. Data and Empirical Results from the Spanish stock market

4.1. Data issues

The daily prices and trading volume of all stocks traded on the Spanish stock market from January 1994 through December 2002 were used in this study. This daily data is employed for the monthly calculation of firms’ illiquidity ratios.

Stock return in month $t$ is calculated as the ratio between its price in month $t$ and in month $t-1$, adjusted by dividends, splits and new issues. Market return is calculated as an equally-weighted portfolio comprised of all stocks available either in a given month of the sample while the monthly Treasury Bill rate observed in the secondary market is used as the risk-free rate.

In order to construct the Fama-French risk factors, we have used the number of shares traded at the end of each sample year and the accounting information from the balance sheets of each

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\(^6\) We are grateful to an anonymous referee for bringing this to our attention.
firm at the end of each sample year. The market value is calculated by multiplying the number of shares of each firm in December of the previous year by their price at the end of each month. The book value for any firm in month \( t \) is given by its value at the end of the previous year, remaining constant from January to December. Then, the book-to-market ratio in all months of year \( t \) is calculated by dividing the book value at the end of December in previous year by the market value at that date.

4.2. Empirical Results

4.2.1. Background and descriptive statistics

Table 1 reports the average characteristics of the distribution of the market return factor, the Fama-French factors, and the illiquidity-based systematic factor. The correlation coefficients between them are presented in Panel B. It is interesting to point out that the average market risk premium is positive, and hence consistent with the assumption of risk aversion. The mean return for the derived size (SMB) factor is negative. In this order, there is evidence in the early anomalies literature that the small firm effect may not be stable over time and may depend on factors such as business cycles. Moreover, recent evidence suggests that the size effect may have gone in reverse. The average return on the HML factor is positive, and the average return on the IMV factor is also positive. Finally, the correlations between the three last factors are low and correlation with the market factor is quite similar to previous results shown for the Spanish market (Menéndez, 2000; Nieto, 2004; Martínez et al., 2004).

| Panel A: Descriptive statistics | Mean | Volatility | Skewness | Kurtosis |
|---------------------------------|------|------------|----------|----------|
| MKT                             | 0.8762 | 6.2296   | -0.0023  | 3.9763   |
| SMB                             | -0.1928 | 3.5307 | 0.7985 | 3.7127 |
| HML                             | 0.2866 | 3.2991 | 0.6307 | 3.4945 |
| IMV                             | 0.2118 | 11.043 | 0.9130 | 6.0562 |

| Panel B: Correlation coefficients | MKT | SMB | HML | IMV |
|-----------------------------------|-----|-----|-----|-----|
| MKT                               | 1.000 | 0.304 | 0.287 | 0.122 |
| SMB                               | 1.000 | 0.077 | 0.155 | 0.219 |
| HML                               | 1.000 | 1.000 | 0.155 | 0.219 |
| IMV                               | 1.000 | 1.000 | 1.000 | 1.000 |

Note: In this Table, Panel A reports the mean, volatility, skewness, and kurtosis for the excess market return (MKT) and for the mimicking portfolio factor returns of size (SMB), book-to-market (HML) and illiquidity (IMV). Panel B reports correlations between the excess market returns and the SMB, HML, and IMV factor returns. Data are monthly covering the period from January 1994 to December 2002.

4.2.2. Main asset pricing test results

For the purpose of comparison, we first run the CAPM and Fama-French model. Results are reported in Table 2, Panels A and B respectively. Several aspects of these results deserve to be mentioned. First, for the CAPM and not the Fama and French three-factor model, the risk-adjusted
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average return (alpha) of the P10 portfolio is significantly higher than the alpha for the P1 portfolio. Average risk-adjusted returns of stocks with high liquidity exceed those ones with low liquidity. Pastor and Stambaugh (2003) interpret the result as the average liquidity premium existing in the US market. A joint test will be performed later in the paper.

Second, the market factor is significantly related to the excess returns of the liquidity portfolios, in both the CAPM and Fama-French model. However, SMB and HML are less significant in explaining the excess returns of the liquidity portfolios.7

Table 2 – CAPM and Fama-French model performances of sorted by illiquidity

Panel A: Standard CAPM

|       | P1     | P2     | P3     | P4     | P5     | P6     | P7     | P8     | P9     | P10    |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \(\alpha\) | 0.38   | 0.84*  | -0.71* | -0.29  | -0.53  | -0.19  | 0.01   | -0.07  | -0.16  | 0.77   |
|       | (1.08) | (2.79) | (-2.08)| (-0.86)| (-1.56)| (-0.63)| (0.02) | (-0.25)| (-0.40)| (1.72) |
| \(\beta_m\) | 1.00*  | 0.89   | 0.96   | 0.96   | 0.90   | 1.26   | 0.82   | 0.84   | 1.30   | 1.01   |
|       | (7.74) | (10.1) | (13.0) | (6.50) | (10.0) | (10.6) | (8.31) | (10.7) | (5.59) | (4.84) |
| Adj. R\(^2\) | 72.67  | 67.70  | 67.64  | 66.91  | 65.29  | 74.32  | 53.42  | 66.35  | 68.19  | 50.82  |

Panel B: Fama-French model

|       | P1     | P2     | P3     | P4     | P5     | P6     | P7     | P8     | P9     | P10    |
|-------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| \(\alpha\) | 0.60*  | 1.06*  | 0.45   | 0.35   | -0.59  | -0.47  | 0.22   | -1.04* | -0.20  | -0.32  |
|       | (1.94) | (4.49) | (1.22) | (0.84) | (-1.75)| (-1.43)| (0.61) | (-3.48)| (-0.53)| (-0.66)|
| \(\beta_m\) | 1.03*  | 0.97*  | 1.04*  | 1.00*  | 0.98*  | 1.22*  | 0.87*  | 0.81*  | 1.14*  | 0.90*  |
|       | (8.23) | (9.45) | (15.2) | (8.51) | (12.9) | (11.4) | (6.97) | (12.1) | (9.84) | (5.25) |
| \(\beta_{smb}\) | -0.12  | -0.45* | -0.45* | -0.21  | -0.46* | 0.34*  | -0.27  | 0.19   | 0.87*  | 0.59*  |
|       | (-0.72)| (-3.19)| (-2.69)| (-1.08)| (-4.20)| (2.53) | (-0.98)| (1.08) | (3.93) | (2.14) |
| \(\beta_{hml}\) | -0.20  | -0.17  | -0.12  | -0.27  | -0.22  | -0.29  | -0.33* | -0.01  | 0.87*  | 0.77*  |
|       | (-1.01)| (-0.64)| (-0.59)| (-0.85)| (-1.16)| (-1.20)| (-1.88)| (-0.09)| (2.66) | (1.99) |
| Adj. R\(^2\) | 72.59  | 72.44  | 71.48  | 67.68  | 70.13  | 76.66  | 55.16  | 66.34  | 81.49  | 59.51  |

* Indicates statistical significance at the 0.05 level

Note: At the beginning of each month from January 1994 to December 2002, stocks are sorted in ascending order based on their illiquidity measures, \(ILLIQ\). Based on each sorting, stocks are grouped into equally-weighted decile portfolios and held for 12 months. P1 denotes the lowest \(ILLIQ\) decile portfolio (the most liquid decile) and P10 is the highest \(ILLIQ\) decile portfolio (the least liquid decile).

Panel A presents parameter estimates of the capital asset pricing model (CAPM):

\[ r_t = \alpha + \beta_{m,t} \beta_m + \epsilon_t \]

And Panel B reports parameter estimates of the Fama and French three factor model:

\[ r_t = \alpha + \beta_{m,t} \beta_m + \beta_{smb,t} \beta_{smb} + \beta_{hml,t} \beta_{hml} + \epsilon_t \]

where \(r_t\) is the excess return on portfolio \(j\), \(\epsilon_t\) is the excess return on the market portfolio, \(SMB\) is the mimicking portfolio for the size factor and \(HML\) is the mimicking portfolio for the book-to-market factor. \(\alpha\) is the intercept of portfolio \(j\), and \(\beta_m, \beta_{smb}\) and \(\beta_{hml}\) are the sensitivities to the risk factors. Numbers in parentheses are t-statistics. The adjusted R-squares are reported in percentages.

7 This is consistent with previous empirical evidence from the Spanish stock market. See Nieto (2004), among others.
Table 3 reports estimates of the standard CAPM and Fama-French model augmented by the illiquidity factor. Panel A in Table 3 reveals that eight of the ten betas are statistically significant for the illiquidity (IMV) factor. Notably, there is a strong pattern that the illiquid portfolios have positive or at least less negative IMV betas, and three of these cases are significantly positive at the 5% level. In contrast, three of the very liquid portfolios have significantly negative IMV betas. Panel B of Table 3 shows that the illiquidity factor exhibits significant explanatory power on the time-series variation of average returns after controlling for Fama-French factors.

Table 3 also reports the adjusted $R^2$-squares for the time series regressions. It may safely be argued that there is a relevant improvement in the variability of portfolio returns explained by the illiquidity adjusted models. Finally, the $F$-test for the hypothesis that the illiquidity factor may be excluded from the regression is reported. In the CAPM and Fama and French models argumented by the illiquidity factor, the $F$-tests for 8 and 7 of the 10 portfolios produced $p$-values below 0.05.
Table 3 - CAPM and Fama-French model augmented by an illiquidity factor

Panel A: Standard CAPM augmented by an illiquidity factor

|   | P1  | P2  | P3  | P4  | P5  | P6  | P7  | P8  | P9  | P10 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\alpha$ | 1.52 | -0.01 | -0.70 | 0.62 | 0.74 | -0.17 | 0.31 | 0.08 | 0.44 | 0.23 |
| (1.24) | (-0.04) | (-1.37) | (0.47) | (1.50) | (-0.47) | (0.88) | (0.20) | (0.91) | (0.59) |
| $\beta_m$ | 1.04* | 0.92* | 1.01* | 0.99* | 0.92* | 1.25* | 0.84* | 0.80* | 1.23* | 0.93* |
| (11.6) | (9.27) | (14.0) | (10.3) | (11.2) | (10.1) | (8.33) | (10.6) | (9.72) | (8.22) |
| $\beta_{imv}$ | -0.13* | -0.10* | -0.15* | -0.09 | -0.07* | 0.02 | -0.07* | 0.10* | 0.23* | 0.26* |
| (-7.18) | (-4.49) | (-4.12) | (-1.62) | (-2.55) | (0.78) | (-2.22) | (4.57) | (3.71) | (8.40) |
| Adj. $R^2$ | 81.65 | 74.09 | 80.04 | 70.87 | 68.00 | 74.16 | 55.63 | 73.81 | 83.48 | 75.76 |
| $F$-test | 29.3 | 15.3 | 37.0 | 8.88 | 5.90 | 0.65 | 3.89 | 17.5 | 54.7 | 60.6 |

Panel B: Fama-French model augmented by an illiquidity factor

|   | P1  | P2  | P3  | P4  | P5  | P6  | P7  | P8  | P9  | P10 |
|---|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\alpha$ | 1.46 | -0.13 | -1.05* | 0.42 | 0.71 | -0.21 | 0.35 | -0.17 | 0.31 | 0.18 |
| (1.10) | (-0.36) | (-2.66) | (0.49) | (1.47) | (-0.57) | (1.03) | (-0.60) | (0.69) | (0.50) |
| $\beta_m$ | 0.99* | 0.95* | 1.00* | 0.99* | 0.98* | 1.22* | 0.86* | 0.83* | 1.17* | 0.95* |
| (9.92) | (8.59) | (14.8) | (9.71) | (12.8) | (11.1) | (6.99) | (14.3) | (12.0) | (7.49) |
| $\beta_{smb}$ | 0.36* | -0.24 | -0.01 | 0.03 | -0.40* | 0.31 | -0.16 | -0.19 | 0.44* | -0.13 |
| (2.31) | (-1.36) | (-0.07) | (0.18) | (-3.07) | (1.75) | (-0.40) | (-1.10) | (2.66) | (-0.61) |
| $\beta_{hml}$ | 0.18 | -0.01 | 0.22 | -0.07 | -0.17 | -0.31 | -0.24 | -0.32* | 0.54* | 0.19 |
| (0.85) | (-0.01) | (1.43) | (-0.37) | (-1.04) | (-1.07) | (-0.96) | (-2.29) | (2.31) | (0.59) |
| $\beta_{imv}$ | -0.18* | -0.07* | -0.16* | -0.09 | -0.02 | 0.01 | -0.04 | -0.14* | 0.15* | 0.27* |
| (-6.95) | (-2.53) | (-4.45) | (-1.58) | (-0.69) | (0.22) | (-0.73) | (3.98) | (2.32) | (5.57) |
| Adj. $R^2$ | 83.06 | 74.40 | 80.13 | 69.95 | 69.77 | 76.25 | 55.02 | 74.87 | 85.75 | 75.75 |
| $F$-test | 35.6 | 5.30 | 25.3 | 5.21 | 5.23 | 0.34 | 0.04 | 0.82 | 20.0 | 17.7 |
| (0.00) | (0.02) | (0.00) | (0.02) | (0.56) | (0.82) | (0.36) | (0.00) | (0.00) | (0.00) |

* Indicates statistical significance at the 0.05 level.

Note: At the beginning of each month from January 1994 to December 2002, stocks are sorted in ascending order based on their illiquidity measures, $I_{IIIQ}$. Based on each sorting, stocks are grouped into equally-weighted decile portfolios and held for 12 months. $P_1$ denotes the lowest $I_{IIIQ}$ decile portfolio (the most liquid decile) and $P_{10}$ is the highest $I_{IIIQ}$ decile portfolio (the least liquid decile).

Panel A presents parameter estimates of the CAPM augmented by an illiquidity factor:

$$r_{jt} = \alpha_j + \beta_{jr} r_{mt} + \beta_{jimv} JMV_t + \mu_j$$

Panel B reports parameter estimates of the Fama and French three factor model augmented by an illiquidity factor:

$$r_{jt} = \alpha_j + \beta_{jrf} r_{mt} + \beta_{jrb} SMB_t + \beta_{jrh} HML_t + \beta_{jrmv} IMV_t + \mu_j$$

where $r_{jt}$ is the excess return on portfolio $j$, $r_{mt}$ is the excess return on the market portfolio, $SMB_j$ is the mimicking portfolio for the size factor, $HML_j$ is the mimicking portfolio for the book-to-market factor, and $IMV_j$ is a mimicking portfolio for the illiquidity factor, $\alpha_j$ is the intercept of portfolio $j$, and $\beta_{jrf}$, $\beta_{jrb}$, $\beta_{jrh}$ and $\beta_{jrmv}$ are the sensitivities to the risk factors. Numbers in parentheses are t-statistics. The adjusted $R$-squares are reported in percentages. The $F$-test and its p-value for the hypothesis that the illiquidity factor may be excluded from the regression are reported.
Of course, the fact that we have found an apparent improvement in equity pricing using the previous illiquidity factor does not imply that the liquidity adjusted models are the “correct” models. We should also test whether the intercepts in the regressions above are jointly equal to zero.

### Table 4 – Test for the joint significance of the intercept terms. Portfolios sorted by liquidity

| Model                                      | Wald test | p-value |
|--------------------------------------------|-----------|---------|
| Standard CAPM                              | 22.673    | 0.012   |
| Fama and French model                      | 36.765    | 0.000   |
| CAPM augmented by IMV                       | 15.276    | 0.122   |
| Fama and French model augmented by IMV     | 18.581    | 0.045   |

1 Under the null hypothesis asymptotically distributed $\chi^2_{10}$.

Note: Comparison of competing models: the standard CAPM, the Fama and French three-factor model, and both of them augmented by an illiquidity factor, named IMV. The joint significance of the intercept terms is analyzed employing the Wald test with ten portfolios sorted by liquidity for the period January 1994 – December 2002. Portfolios are equally weighted.

Table 4 reports the results of the Wald test that analyzes whether portfolio intercepts are jointly equal to zero. This test also indicates the risk specification suitable for the Spanish market. In other words, whether the models completely capture average returns when used as asset pricing models.

The Wald test is rejected with a significance level of 5% for all asset models considered except the third, the illiquidity-based CAPM. It is relevant to point out that we obtained the best risk specification using these portfolios, and adding the illiquidity factor to the CAPM. In addition, with a significance level of 1%, we cannot reject the null hypothesis for asset models augmented by the illiquidity risk factor.

### 4.2.3. Robustness check

It may not be surprising that an illiquidity factor like IMV that is formed on ILLIQ can explain the returns on (decile) portfolios formed on the same measure. In other words, the fact that the liquidity-adjusted model can account for the illiquidity-based portfolio returns that the CAPM and the Fama-French three-factor model may not be convincing proof that the liquidity-adjusted model performs better than the CAPM and the three-factor model.8

Therefore, before drawing some overall conclusions regarding the asset pricing role of liquidity, it is instructive to conduct a robustness check. In particular, we replicate the tests of the previous section using portfolios sorted by a random selection of stocks. Results are reported in Table 5. For comparison, the CAPM and the Fama-French model are also estimated in relation to these portfolios.

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8 The authors gratefully acknowledge the suggestion of an anonymous referee.


Table 5 - Robustness check

| Panel A: Standard CAPM |       |       |       |       |       |       |       |       |       |
|------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|                        | R1    | R2    | P3    | P4    | P5    | P6    | P7    | P8    | P9    |
| \( \alpha \)           | 0.88  | -0.10 | -0.75*| 0.19  | 0.51  | 0.02  | 0.14  | -0.05 | -0.05 |
|                        | (1.10)| (-0.26)| (-2.12)| (0.33)| (1.30)| (0.07)| (0.46)| (-0.21)| (-0.14)|
| Adj. \( R^2 \)         | 43.27 | 73.38 | 70.12 | 48.47 | 71.99 | 80.52 | 64.72 | 71.73 | 44.69 |
|                        |       |       |       |       |       |       |       |       |       |

| Panel B: Fama and French model |       |       |       |       |       |       |       |       |       |
|\( \alpha \)                   | 1.26  | -0.21 | -1.00*| 0.50  | 0.75  | -0.16 | 0.38  | -0.02 | 0.38  |
|                        | (1.00)| (-0.60)| (-2.57)| (0.58)| (1.50)| (-0.45)| (1.13)| (-0.08)| (0.81)|
| Adj. \( R^2 \)              | 41.30 | 86.43 | 86.82 | 47.62 | 71.83 | 80.34 | 78.50 | 69.83 | 42.04 |

| Panel C: Standard CAPM augmented by an illiquidity factor |       |       |       |       |       |       |       |       |       |
|\( \alpha \)                   | 1.52  | -0.01 | -0.70 | 0.62  | 0.74  | -0.17 | 0.31  | 0.08  | 0.44  |
|                        | (1.24)| (-0.04)| (-1.37)| (0.47)| (1.50)| (-0.47)| (0.88)| (0.20)| (0.91)|
| \( \beta_m \)               | 1.13* | 1.11* | 1.24* | 0.93* | 0.87* | 1.01* | 0.79* | 0.91* | 0.54  |
|                        | (5.25)| (21.6)| (8.56)| (6.22)| (12.2)| (11.7)| (15.2)| (5.75)| (4.92)|
| \( \beta_{inv} \)           | 0.23* | 0.14* | 0.10* | 0.06  | -0.02 | -0.03 | -0.05*| -0.06*| -0.03 |
|                        | (2.63)| (4.51)| (3.19)| (0.32)| (-1.43)| (-1.09)| (-1.98)| (-1.73)| (-0.80)|
| Adj. \( R^2 \)              | 49.62 | 78.27 | 73.54 | 47.93 | 72.18 | 80.46 | 67.91 | 73.36 | 46.01 |
| F-test                    | 12.1  | 35.2  | 6.61  | 0.01  | 1.29  | 2.12  | 6.11  | 5.43  | 1.05  |
|                        | (0.00)| (0.00)| (0.01)| (0.89)| (0.25)| (0.15)| (0.01)| (0.02)| (0.00)|

| Panel D: Fama-French model augmented by an illiquidity factor |       |       |       |       |       |       |       |       |       |
|\( \alpha \)                   | 1.46  | -0.13 | -1.05*| 0.42  | 0.71  | -0.21 | 0.35  | -0.17 | 0.31  |
|                        | (1.10)| (-0.36)| (-2.66)| (0.49)| (1.47)| (-0.57)| (1.03)| (-0.60)| (0.69)|
| \( \beta_m \)               | 1.12* | 1.05* | 1.08* | 0.84* | 0.86* | 0.98* | 0.81* | 0.82* | 0.50  |
|                        | (5.31)| (15.8)| (16.8)| (6.81)| (11.6)| (11.7)| (13.4)| (9.65)| (7.35)|
| \( \beta_{smb} \)           | 0.09  | 0.47* | 1.31* | 0.70* | 0.11  | 0.25* | -0.17*| 0.66* | 0.30  |
|                        | (0.28)| (3.65)| (6.52)| (3.44)| (0.61)| (1.99)| (-1.78)| (2.29)| (1.63)|
| \( \beta_{hml} \)           | 0.16  | 0.07  | 0.30  | 0.23  | 0.04  | -0.05 | -0.01 | -0.50*| -0.29*|
|                        | (0.45)| (1.23)| (0.96)| (0.23)| (-0.41)| (-0.10)| (1.79)| (2.02)| (0.59)|
| \( \beta_{inv} \)           | 0.21* | 0.08* | -0.05 | -0.08*| -0.04 | -0.06*| -0.03 | -0.16*| -0.07 |
|                        | (1.95)| (2.07)| (-1.28)| (-2.28)| (-1.44)| (-1.14)| (2.49)| (-1.69)| (-1.80)|
| Adj. \( R^2 \)              | 46.18 | 88.29 | 87.16 | 48.28 | 72.17 | 81.30 | 78.66 | 79.70 | 45.20 |
| F-test                    | 6.08  | 9.91  | 2.48  | 1.71  | 1.68  | 3.89  | 1.42  | 28.2  | 4.22  |
|                        | (0.01)| (0.00)| (0.12)| (0.19)| (0.19)| (0.05)| (0.23)| (0.00)| (0.04)|

* Indicates statistical significance at the 0.05 level.

Note: At the beginning of each month from January 1994 to December 2002, stocks are sorted randomly and grouped into equally-weighted decile portfolios and held for 12 months.

Panel A presents alpha estimates of the standard CAPM. Panel B reports alpha estimates of the Fama and French model. Panel C presents parameter estimates of the standard CAPM augmented by an illiquidity factor. And Panel D reports parameter estimates of the Fama and French model augmented by an illiquidity factor.

Numbers in parentheses are t-statistics. The adjusted \( R^2 \) squares are reported in percentages. The F-test and its p-value for the hypothesis that the illiquidity factor may be excluded from the regression are reported.
Panels C and D of Table 5 indicate that the illiquidity factor loadings are significant for 6 of the 10 portfolios. Adjusted $R^2$s and $F$-test results also indicate that the liquidity-adjusted models perform better than the CAPM and the three-factor model. But it is interesting to point out that only the liquidity-adjusted CAPM presents insignificant risk-adjusted average returns across all portfolios.

Finally, Table 6 reports the results of the Wald test that analyzes whether portfolio intercepts are jointly equal to zero. Again, the liquidity-adjusted CAPM obtains the best risk specification.

We may then conclude that, within a time-series context, this paper presents evidence showing that an illiquidity risk factor plays a relevant role in explaining the average returns in the Spanish market.

### 5. Conclusions

In this paper, we have analyzed the role of liquidity as an additional factor in asset pricing. The motivation for our study was provided by the growing interest in liquidity that has emerged in the asset pricing literature over recent years.

Our empirical results support the recent evidence found in US market data and allow us to affirm that aggregate illiquidity should be a key ingredient of asset pricing models. Our results indicate that time-varying expected excess asset returns in the Spanish stock market, from January 1994 through December 2002, can be explained by an illiquidity-based CAPM model.

However, it must be recognized that our sample period is short in comparison to the available evidence on asset pricing. The results should be taken as valid just for the period being studied, and more general conclusions should be left for future research when longer series of data will be readily available. We have to point out that one feature of the methodology which reduces its appeal is the complexity surrounding the construction of the size, book-to-market and illiquidity factors. This is particularly so in smaller markets where extensive and reliable data over sufficiently long time-series are difficult to compile.

Overall, it can be stated that the main goal of the paper has been achieved. However, the observed results suggest that further empirical work would be beneficial. In particular, it would be of interest to explain the cross-sectional variation in illiquidity.
The Pricing of Systematic Liquidity Risk in Stock Markets

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References

Acharya, V.; Pedersen, L. (2003), Asset Pricing with Liquidity Risk, Working Paper, London Business School.

Amihud, Y. (2002), «Illiquidity and stock returns: cross-section and time-series effects>, Journal of Financial Markets, 5, 31-56.

Amihud, Y.; Mendelson, H. (1986), «Asset pricing and the bid-ask spread>, Journal of Financial Economics, 17, 223-249.

Blanco, R. (1999), «Análisis de la Liquidez en el Mercado Español de Renta Variable e Impacto de las Regulaciones sobre Variaciones Mínimas de Precios>, El Mercado Español de Renta Variable. Análisis de la Liquidez e Influencia del Mercado de Derivados, Banco de España, Servicio de Publicaciones, 66, Madrid.

Brennan, M. J.; Subrahmanyam, A. (1996), «Market microstructure and asset pricing: on the compensation for illiquidity in stock returns>, Journal of Financial Economics, 41, 341-364.

Brennan, M. J.; Chordia, T.; Subrahmanyam, A. (1998), «Alternative factor specifications, security characteristics, and the cross-section of expected stock returns>, Journal of Financial Economics, 49, 345-373.

Chan, H.; Faff, R. (2004), «Asset Pricing and the Illiquidity Premium>, The Financial Review, forthcoming.

Chordia, T.; Roll, R.; Subrahmanyam, A. (2000), «Commonality in liquidity>, Journal of Financial Economics, 56, 3-28.

Chordia, T.; Subrahmanyam, A.; Anshuman, V. (2001), «Trading Activity and Expected Stock Returns>, Journal of Financial Economics, 59, 3-32.

Datar, V.; Naik, N.; Radcliffe, R. (1998), «Liquidity and stock returns: An alternative test>, Journal of Financial Markets, 1, 203-219.

Eleswarapu, V.; Reinganum, M. (1993), «The Seasonal Behavior of the Liquidity Premium in Asset Pricing>, Journal of Financial Economics, 34, 373-386.

Fama, E. F.; French, K. R. (1993), «Common risk factors in the returns on stocks and bonds>, Journal of Financial Economics, 33, 3-56.

Ferson, W.; Foerster, S. (1994), «Finite sample properties of the generalized method of moments in tests of conditional asset pricing models>, Journal of Financial Economics, 36, 29-55.

Ferson, W.; Harvey, C. (1999), «Conditioning Variables and the Cross Section of Stock Returns>, Journal of Finance, 54, 1325-1360.

Grinblatt, M.; Titman, S. (1987), «The relation between mean-variance efficiency and arbitrage pricing>, Journal of Business, 60, 97-112.

Hasbrouck, J. (2002), Inferring Trading Costs from Daily Data: US Equities from 1962 to 2001, Working Paper, Stern School of Business, New York University.

Mackinlay, A.; Richardson, M. (1991) «Using generalized method of moments to test mean-variance efficiency>, Journal of Finance, 46, 511-527.

Martínez, M. A.; Nieto, B.; Rubio, G.; Tapia, M. (2004), «Asset Pricing and Systematic Liquidity Risk: An empirical investigation of the Spanish Stock Market>, International Review of Economics and Finance, forthcoming.

Menéndez, S. (2000), «Determinantes fundamentales de la rentabilidad de las acciones>, Revista Española de Financiación y Contabilidad, 106, 1015-1031.
Miralles, J. L.; Miralles, M. M. (2003), «Actividad negociadora y esperanza de rentabilidad en la bolsa de valores española», *Revista de Economía Financiera*, 1, 11-32.

Nieto, B. (2004), «Evaluating multi-beta pricing models: an empirical analysis with Spanish market data», *Revista de Economía Financiera*, 2, 80-108.

Pastor, L.; Stambaugh, R. (2003), «Liquidity risk and expected stock returns», *Journal of Political Economy*, 111, 642-685.

Rouwenhorst, K. (1999), «Local return factors and turnover in emerging markets», *Journal of Finance*, 54, 1439-1464.

Rubio, G.; Tapia, M. (1996), «Adverse Selection, Volume, and Transactions around Dividend Announcements in a Continuous Auction System», *European Financial Management Journal*, 2, 127-153.

Rubio, G.; Tapia, M. (1998), «The Liquidity Premium in Equity Pricing under a Continuous Auction System», *European Journal of Finance*, 4, 1-28.

Tapia, M. (1997), «Resultados Preliminares sobre la Estacionalidad de la Prima por Liquidez en España: Efectos Fiscales», *Información Comercial Española*, 764, 65-75.