Direct $qqq$ Force In High Momentum Limit of QCD For Proton Physics

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Abstract

An explicit construction of the proton wave function is outlined in the high momentum limit of QCD dominated by a direct $qqq$ force, one generated by hooking the ends of a $ggg$ vertex to 3 distinct $qgq$ vertices, thus making up a $Y$-shaped diagram (see fig.1). The high degree of $S_3$ symmetry thus involved ensures that the $qqq$ wave function is a mixture of $56,0^+$ and $20,1^+$ components, rather than the traditional $56,0^+$ and $70,0^+$ type. Some results of this paradigm shift are offered.

1. Introduction

This paper (in honor of Pauchy Hwang’s 60th birthday) seeks to exploit the Symmetry theme of this Conference for an explicit construction of the proton wave function in the high momentum limit of QCD. The theoretical framework is provided by a recent paper [1] in the background of the proton’s ‘physics’ [2], wherein the concept of a direct 3-body force is introduced at the quark-gluon level, as a folding of a $ggg$ vertex with 3 distinct $qgq$ vertices, making up a $Y$-shaped diagram (see fig 1). As shown in [1], in the high momentum limit of QCD, where the confining $qq$ force may be neglected, this direct $qqq$ force dominates over the pairwise $qq$ forces, thus offering a new basis for exploring the proton’s wave function, one in which a full-fledged $S_3$ symmetry in several simultaneous d.o.f.’s plays a central role [1].

In this short presentation, we have just enough space (6 pp.) for recapitulating in barest outline the basic dynamics of ref [1], and indicate its extension for an practical construction
of the proton’s wave function in the high momentum limit of QCD (this part is new). This
construction facilitates the effects of $SU(6)$ mixing within the dynamical framework ref [1], so
as to allow for some concrete results on the proton’s ‘physics’. The dynamics is provided by a
covariant Salpeter-like equation [3] governed by what is termed in the literature as the Markov-
Yukawa Transversality Principle (MYTP for short) [4, 5] which specifies that all $qq$ forces are
transverse to the direction of the total hadron 4-momentum $P_{\mu}$—a gauge principle in disguise [6]!
The Salpeter equation [3] has a remarkable property of a 3D-4D interlinkage [7] which can be
adapted to Dirac’s LF Dynamics [8, 9] to take advantage of its bigger stability group. Using the
notation, phase and normalization of ref. [1], the fully antisymmetric ‘spin’-part of the $qqq$ force
in 3D notation reads

$$V_{qqq} = -\frac{g_4^4}{3} \frac{[SSS]}{k_1^2 k_2^2 k_3^2}$$

$$SSS = 2i\Sigma(\eta - \eta') \times (\xi - \xi')(k_1^2 + k_2^2 + k_3^2)/\sqrt{3} - 4[(\eta - \eta') \times (\xi - \xi')]^2$$  \hspace{1cm} (1)$$

Here $\Sigma$ is twice the total hadron spin operator. The procedure is now to insert this term
within an interlinked 3D-4D BSE framework wherein the complete 4D wave function $\Psi$ satisfies a
covariant Salpeter Eq a la MYTP on the light front (LF). The 4D $\Psi$ function – the repository of
all types of transition amplitudes – whose spin dependence is fully described in terms of standard
Dirac matrices, can be related through a sequence of transformations to a 3D scalar function
$\phi$ which (on reduction of the full 4D BSE for $\Psi$) satisfies a 3D Schroedinger-like (albeit LF
covariant) equation. Next, a reconstruction of $\Psi$ in terms of $\phi$ is achieved by Green’s function
 techniques, so that the 4D spin structure of $\Psi$ is recovered. The $qqq$ paper [1] stopped at this
stage with a discussion of the general analytic structure of the 3D wave function $\phi$. In this paper
we seek to bridge the gap for practical applications of this formalism by interpolating between
Ψ and (φ) of [1], a spin-dependent 3D matrix function ψ (see below for its precise definition), which incorporates SU(6) mixing effects to the extent allowed by the dynamics of V_{qqq}, Eq (1.1). The logic is similar to one employed by the Orsay group more than 3 decades ago [10], using a spin (χ)-cum-isospin (φ) representation for SU(6) states [11], but now the dynamics of V_{qqq} automatically determines this mixing. In Sect 2, the structure of the 4D Ψ function is outlined through a sequence of Steps A,B,C of ref [1], using the 3D spin-dependent matrix function ψ as an interpolation between Ψ and φ. Sect 3 has just enough space to list a few results to illustrate the applications.

2. Structure of Full 4D Ψ

We collect some essential material from [1]. The LF momenta in 3D form are:

\[ p_{iz}; p_{0} = \frac{M p_{i+}}{P_{+}} - \frac{M p_{i-}}{2P_{-}} \]
\[ \hat{p}_{i} \equiv \{ p_{i\perp}, p_{iz} \} \]  \hspace{1cm} (2.1)

\[ \sqrt{2}\xi = p_{3} - p_{2}; \quad \sqrt{6}\eta = -2p_{3} + p_{1} + p_{2}; \]  \hspace{1cm} (2.2)

There are three main steps for the Ψ - φ interconnection

Step A: Define an auxiliary 4D scalar function Φ [1] :

\[ \Psi = \Pi_{123}S_{F_{i}}^{-1}(-p_{i})\Phi(p_{1}p_{2}p_{3})W(P) \]  \hspace{1cm} (2.3)

where [12], [1]

\[ W(P) = [\chi' \phi' + \chi'' \phi'']/\sqrt{2} \]  \hspace{1cm} (2.4)

\[ |\chi' >; |\chi'' > = \left[ \frac{M - i\gamma_{\mu}P}{2M}[i\gamma_{5}; i\tilde{\gamma}_{\mu}/\sqrt{3}]C/\sqrt{2} \right] \otimes [1; \gamma_{5}\tilde{\gamma}_{\mu}]u(P) \]  \hspace{1cm} (2.5)

Step B: Set up the Master Eq for Φ with Gordon reduction.

Step C: Make a reduction of the Master Eq for 4D Φ to one for 3D φ; then reconstruct Φ in terms of φ, via Green’s fn method [13] adapted to the LF formalism [1]

The final result for reconstructed 4D spinor Ψ in terms 3D scalar φ is

\[ \Psi(\xi, \eta) = \Pi_{123}S_{F}(p_{i})D_{123}W(P)\sum_{123}^{\phi}(\hat{\xi}, \hat{\eta})/(2\pi i)^{2} \]  \hspace{1cm} (2.6)
where
\[
\frac{1}{D_{123}} = \int \frac{P_2 dq_{12} - dP_3}{4M^2(2i\pi)^2\Delta_1\Delta_2\Delta_3}
\]  
(2.7)
and the 3D wave fn $\phi$ satisfies a 6D Differential equation in coordinate space with $S_3$ symmetric variables
\[
\sqrt{2}s_3 = r_1 - r_2; \quad \sqrt{6}t_3 = -2r_3 + r_1 + r_2
\]  
(2.8)

Now to the specific contribution of the present paper, viz., an interpolating function $\psi$ between $\Psi$ and $\phi$, so as to incorporate the $SU(6)$ effects. This is achieved by introducing a new 3D spin-dependent scalar matrix function $\psi$, via the representation:
\[
\psi = \psi_s + i\sqrt{2}\Sigma \eta \times \xi \psi_a
\]  
(2.9)
where $\xi, \eta$ are given by Eq (2.2), and the norm of the second term is for later convenience. In the notation of ref [10], the simplest interpretation of the two scalar functions $\psi_s$ and $\psi_a$ is that they stand for the symmetric $(56; 0^+)$ and antisymmetric $20; 1^+$ states respectively, each with $J = 1/2$, instead of a 56-70 mixture [10]. A second difference from ref [10] is that these two functions are now dynamically linked by two coupled equations (c.f., the single equation for $\phi$, Eq (5.18) in ref [1]). With a little approximation of angular averaging over certain terms, the $\psi_s$ and $\psi_a$ equations get almost decoupled and satisfy two similar equations represented symbolically as
\[
D_{123}[\psi_s; \psi_a] = \int V_s[4(\eta \times \xi)^2 - 4X \pm 2X](\psi_s; \psi_a);
\]  
(2.10)
where $D_{123}$ is given by (2.7), $V_s$ the strength of the fully symmetric part of the $qqq$ force, and
\[
X = \rho(\eta \times \xi \hat{P}/\sqrt{3}; \quad \rho = \xi^2 + \eta^2
\]  
(2.11)

If now as a first approximation, the (smaller) term $\pm 2X$ in Eq.(2.10)is dropped, the ratio of the $\psi_s$ and $\psi_0$ components becomes almost independent of the dynamics, except for an overall constant ratio, while the dynamics is almost entirely contained in a common function $\phi$ satisfying an equation of the form (2.10). Thus
\[
\psi_s = \cos \beta \phi; \quad \psi_a = \sin \beta \phi
\]  
(2.12)
where the phase factor $\beta$, plays the role of the mixing angle $\phi$ of ref [10], and the scalar function $\phi$ may now be identified with the quantity $\phi$ of ref.[1].
3. Results and Discussion

Fig. 2 is a generic diagram for different types of matrix elements with appropriate insertions for $A$. Thus the spin matrix elements $g_i$ of ref.[2] are obtained with $A = i \gamma^\mu \gamma^5 \lambda_i$ where the $\lambda$'s are the Gellmann matrixes as given in [14]. The 56-20 mixing affects $g_3$ but not $g_8$ and $g_0$. The results in lowest order are

$$g_3 = (5/3) \cos^2 \beta - \sin^2 \beta; \quad g_8 = g_0 = 1/\sqrt{3} \quad (2.13)$$

Note that only $g_3$ depends on $\beta$, and agrees with the observed value 1.248 for $\beta \approx 24^\circ$. The other two quantities (at $\approx 0.58$) agree with [2]. Of the latter, only $g_8$ is affected by the 2-gluon anomaly, but not $g_8$.

The result for the fractional correction to $g_A^{(0)}$ due to the 2-gluon anomaly will be given in a more detailed communication on the lines of a recent preliminary analysis [15].

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