Teleportation for atomic entangled state by entanglement swapping with separate measurements in cavity QED

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Experimentally feasible scheme for teleportation of atomic entangled state via entanglement swapping is proposed in cavity quantum electrodynamics (QED) without joint Bell-state measurement (BSM). In the teleportation processes the interaction between atoms and a single-mode nonresonant cavity with the assistance of a strong classical driving field substitute the joint measurements. The discussion of the scheme indicates that it can be realized by current technologies.

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I. INTRODUCTION

Quantum entanglement lies at the heart of quantum information theory (QIT), is considered to be a fundamental resource of quantum teleportation. For a long time, it was seen merely as one of the counterintuitive feature of quantum mechanics, import only in the realm of Einstein-Poldolsky-Rosen (EPR) paradox. Only recently has the field of quantum information begin to exploit the applications of its novel feathers. While bipartite entangled state is well known, multipartite entanglement is still under extensive exploration. People soon realized that it isn’t just an extension of bipartite entanglement. For tripartite entangled quantum system, it falls into two classes, namely, GHZ state and W state, respectively. Now great efforts are engaged in investigation of multipartite entanglement with its promising features, such as, decoherence free quantum information processing, multiparty quantum communications and so on.

Quantum teleportation, proposed by Bennett et al. and experimentally realized by Bouwmeester et al. and Boschi et al., is a process to transmit unknown state to a remote location via a quantum channel aided by some classical communication. It attracts extensive public attention due to its promising applications in QIT. Recently, teleportation of entangled states have been suggested different theoretical schemes and experimentally realized by some classical communication. It attracts extensive public attention due to its promising applications in QIT.

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and particle), there is no energy exchange between the atomic system and the cavity, thus the scheme is insensitive to both the cavity decay and the thermal field. Then the interaction between the single-mode cavity and the atoms can be described, in the rotating-wave approximation, as

\[ H_e = \frac{\lambda}{2} \sum_{j=1}^{2} (|e\rangle_j \langle e| + |g\rangle_j \langle g|) \]

\[ + \sum_{i \neq j: i=1}^{2} (S_k^{+} S_k^{+} + S_k^{+} S_k^{+} + H.c.) \]

(1)

where \(\lambda = \epsilon^2/2\delta\) and \(\omega_a\) is the cavity frequency;

\[ S_Z = \frac{1}{2} \sum_{j=1}^{2} (|e\rangle_j \langle e| - |g\rangle_j \langle g|), \]

\[ S_j^+ = |g\rangle_j \langle e|, S_j^- = |e\rangle_j \langle g| \]

with \(|e\rangle_j, |g\rangle_j\) are the excited and ground states of \(j\)th atom, respectively.

If two atoms are simultaneously sent into the cavity then they interact with the cavity. Using the evolution operator of the system \(U(t) = \exp(-\text{i}H_0t)\exp(-\text{i}H_e t)\) with \(H_0 = \sum_{j=1}^{2} \Omega(S_k^{+} + S_k^{-}) \), \(H_e\) is the effective Hamiltonian. it is easy to verify the evolutions by adjusting the interaction time \(\lambda t = \pi/4\) and modulating the driving field \(\Omega t = \pi\).

\begin{align*} 
|g\rangle_1 |g\rangle_2 &\rightarrow \frac{1}{\sqrt{2}}(|g\rangle_1 |g\rangle_2 - \text{i}|e\rangle_1 |e\rangle_2), & (2a) \\
|g\rangle_1 |e\rangle_2 &\rightarrow \frac{1}{\sqrt{2}}(|g\rangle_1 |e\rangle_2 - \text{i}|e\rangle_1 |g\rangle_2), & (2b) \\
|e\rangle_1 |g\rangle_2 &\rightarrow \frac{1}{\sqrt{2}}(|e\rangle_1 |g\rangle_2 - \text{i}|g\rangle_1 |e\rangle_2), & (2c) \\
|e\rangle_1 |e\rangle_2 &\rightarrow \frac{1}{\sqrt{2}}(|e\rangle_1 |e\rangle_2 - \text{i}|g\rangle_1 |g\rangle_2). & (2d) 
\end{align*}

That is to say we have prepared the four maximally bipartite entangled states (EPR) from the reverent product state of the two atoms. From Eq. 2 we can see that the evolutions doesn’t depend on the cavity state, thus this scheme is insensitive to both the cavity decay and the thermal field. Latter we will use the states generated here to serve as quantum channels.

III. TELEPORTATION OF BIPARTITE ENTANGLEMENT

In this section we will propose two schemes to teleport the bipartite entangled state via entanglement swapping. The first scheme adopts EPR state as quantum channels while the latter one adopts two nonmaximally bipartite entangled states.

A bipartite nonmaximally entangled atomic quantum state to be teleport can be expressed as

\[ |\psi\rangle_1 = (a|e\rangle_1 |e\rangle_2 + b|g\rangle_1 |g\rangle_2). \]  

(3)

where \(a\) and \(b\) are unknown coefficients and \(|a|^2 + |b|^2 = 1\).

A. Teleportation with EPR state via entanglement swapping

Here we will teleport the bipartite entangled state by two EPR states. Assume the two quantum channels are

\[ |\psi\rangle_{3,4} = \frac{1}{\sqrt{2}}(|ge\rangle_{3,4} - \text{i}|eg\rangle_{3,4}), \]  

(4a)

\[ |\psi\rangle_{5,6} = \frac{1}{\sqrt{2}}(|ge\rangle_{5,6} - \text{i}|eg\rangle_{5,6}), \]  

(4b)

where atoms 3 and 5 belong to Alice and the rest two were sent to Bob. Initially the quantum state of the whole system, consisting 6 atoms, is

\[ |\psi\rangle = \frac{1}{2}(a|e\rangle_1 |e\rangle_2 + b|g\rangle_1 |g\rangle_2) \]

\[ \otimes \ (|ge\rangle_{3,4} - \text{i}|eg\rangle_{3,4}) \otimes (|ge\rangle_{5,6} - \text{i}|eg\rangle_{5,6}), \]  

(5)

which can be rewrite as

\[ |\psi\rangle = \frac{1}{2}\left[|\Phi^{\pm}\rangle_{1,3}|\Phi^{\pm}\rangle_{2,5}(-b|ee\rangle \pm_1 \pm_2 a|gg\rangle)_{4,6} \right. \]

\[ - \text{i}|\Phi^{\pm}\rangle_{1,3}|\Psi^{\pm}\rangle_{2,5}(b|ge\rangle \pm_1 \pm_2 a|eg\rangle)_{4,6} \]

\[ - \text{i}|\Psi^{\pm}\rangle_{1,3}|\Phi^{\pm}\rangle_{2,5}(b|ge\rangle \mp_1 \pm_2 a|ge\rangle)_{4,6} \]

\[ + \left.|\Psi^{\pm}\rangle_{1,3}|\Psi^{\pm}\rangle_{2,5}(b|gg\rangle \mp_1 \mp_2 a|ge\rangle)_{4,6}\right]. \]

(6)

where \(|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|ee\rangle \pm \text{i}|gg\rangle)\) and \(|\Psi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|ge\rangle \pm \text{i}|eg\rangle)\) are four Bell states of atom pairs (1,3) and (2,5).

From the above equation we can see that, if Alice can easy realization joint BSM on atom pairs (1,3) and (2,5), then after been informed the measurement results, Bob can reconstruct the original entangled state by corresponding unitary transformation, thus achieve the goal of teleportation. But the BSM is complex for experimental realization, so here we investigate separate atomic measurements to achieve the goal of joint BSM by using optical cavity.

We can assume, without loss of generality, the quantum state of atom pair (1,3) is

\[ |\Phi^{+}\rangle = \frac{1}{\sqrt{2}}(|ee\rangle + \text{i}|gg\rangle)_{1,3} \]  

(7)

Sending the atom pair to the cavity, they will simultaneously interact with the cavity mode, by adjusting the
interaction time $\lambda t = \pi/4$ and modulating the driving field $\Omega = \pi$, the state of the atom pair will evolve, according to Eq. (2), as

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|ee\rangle + i|gg\rangle)_{1,3} \rightarrow \frac{1}{2}(|ee\rangle - i|gg\rangle)_{1,3}$$

$$+ i(|gg\rangle - i|ee\rangle)_{1,3} = |e\rangle_1|e\rangle_3.$$

(8)

Similarly the other three Bell-states of the atom pairs would evolve into

$$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|ee\rangle + i|gg\rangle)_{1,3} \rightarrow -i|g\rangle_1|g\rangle_3,$$

(9a)

$$|\Psi^\pm\rangle = \frac{1}{\sqrt{2}}(|ge\rangle + i|eg\rangle)_{1,3} \rightarrow |g\rangle_1|e\rangle_3,$$

(9b)

$$|\Psi^\mp\rangle = \frac{1}{\sqrt{2}}(|ge\rangle - i|eg\rangle)_{1,3} \rightarrow -i|e\rangle_1|g\rangle_3.$$  

(9c)

In this way, we can achieve the goal of BSM with unit probability of success by separate atomic measurements, and the four Bell-states of atom pair (2, 5) can also be distinguished successfully in the same way.

After four separate measurements on atoms 1, 2, 3 and 5, the entanglement of the entangled atom pairs (1, 2), (3, 4) and (5, 6) disappears and the new entanglement between atoms 4 and 6 is set up which means the entanglement swapping happens. Then after Alice informs her measurement results to Bob; he can reconstruct the original atomic entangled state by corresponding unitary transformation.

### B. Teleportation with nonmaximally bipartite entangled state via entanglement swapping

Here we will teleport the bipartite entangled state by two bipartite nonmaximally entangled atomic states. Assume the quantum channels are

$$|\psi\rangle_{3,4} = (\alpha_1|ge\rangle_{3,4} - i\beta_1|eg\rangle_{3,4})$$

(10a)

$$|\psi\rangle_{5,6} = (\alpha_2|ge\rangle_{5,6} - i\beta_2|eg\rangle_{5,6})$$

(10b)

where $\alpha_i$ and $\beta_i$ are unknown coefficients and $|\alpha_i|^2 + |\beta_i|^2 = 1$ with $i = 1, 2$. We can assume $|\alpha_i| > |\beta_i|$ without loss of generality. So initially the quantum state of the system is

$$|\psi\rangle_{3,4} = (a|e\rangle_1|e\rangle_2 + b|g\rangle_1|g\rangle_2)(\alpha_1|ge\rangle_{3,4}

- i\beta_1|eg\rangle_{3,4})(\alpha_2|ge\rangle_{5,6} - i\beta_2|eg\rangle_{5,6})$$

(11)

which can be rewrite as

$$|\psi\rangle = \frac{1}{2}\left[-|\Phi^\pm\rangle_{1,3}|\Phi^\pm\rangle_{2,5}(\alpha_1\alpha_2|ee\rangle \pm_1 \mp_2 \beta_1\beta_2|gg\rangle)_{4,6}

- i|\Psi^\pm\rangle_{1,3}|\Psi^\pm\rangle_{2,5}(\alpha_1\beta_2|be\rangle \mp_1 \pm_2 \beta_1\alpha_2|ge\rangle)_{4,6}

- i|\Psi^\pm\rangle_{1,3}|\Psi^\pm\rangle_{2,5}(\beta_1\alpha_2|bg\rangle \mp_1 \pm_2 \alpha_1\beta_2|eg\rangle)_{4,6}

+ |\Psi^\pm\rangle_{1,3}|\Psi^\pm\rangle_{2,5}(\beta_1\beta_2|gg\rangle \mp_1 \pm_2 \alpha_1\alpha_2|ee\rangle)_{4,6}\right].$$

(12)

where the notes $\pm_1$ and $\mp_1$ correspond to the $i$th BSM.

We can distinguish the four Bell states by sending the two atoms into the cavity and choosing an appropriate interaction time. After operate separate measurements on atoms 1, 2, 3 and 5, the state of the rest two atoms will project into one of the following state

$$\frac{1}{2}(\alpha_1\alpha_2b|e\rangle_4|e\rangle_6 \pm_1 \pm_2 \beta_1\beta_2a|g\rangle_4|g\rangle_6),$$

(13a)

$$\frac{1}{2}(\alpha_1\beta_2b|e\rangle_4|g\rangle_6 \pm_1 \mp_2 \beta_1\alpha_2a|g\rangle_4|e\rangle_6),$$

(13b)

$$\frac{1}{2}(\beta_1\alpha_2b|g\rangle_4|e\rangle_6 \mp_1 \pm_2 \alpha_1\beta_2a|e\rangle_4|g\rangle_6),$$

(13c)

$$\frac{1}{2}(\beta_1\beta_2b|g\rangle_4|g\rangle_6 \mp_1 \mp_2 \alpha_1\alpha_2a|e\rangle_4|e\rangle_6).$$

(13d)

To realize the teleportation based on cavity QED, Bob must prepare another single-mode high quality factor resonant optical cavity, which is initially in the vacuum states and a photon detector is also necessary.

Bob sends one of his two atoms to the cavity and it will interact with the cavity mode. According to the Jaynes-Cummings model, the Hamiltonian of the resonant interaction system is

$$H = \omega_a(a^+a + S_Z) + \varepsilon(aS_+ + a^+S_-),$$

(14)

where $a^+$ and $a$ are the creation and annihilation operators for the cavity mode, respectively. So, the initial state of the two particles and the cavity is

$$|\Psi(0)\rangle_{4,6,C} = \frac{1}{2}(\alpha_1\alpha_2b|e\rangle_4|e\rangle_6 - \beta_1\beta_2a|g\rangle_4|g\rangle_6)|0\rangle_C.$$  

(15)

Without loss of generality, let Bob sends particle 6 into the cavity, the state of the two-particle and the cavity will evolve as

$$|\Psi(t)\rangle_{4,6,C} = \frac{1}{2}[(\alpha_1\alpha_2b|e\rangle_4|e\rangle_6 - \beta_1\beta_2a|g\rangle_4|g\rangle_6)|0\rangle_C - i\sin\varepsilon t|g\rangle_6

\otimes |1\rangle_C - \beta_1\beta_2a|g\rangle_4|g\rangle_6)|0\rangle_C].$$

(16)

Taking the interacting time $\cos ct = |\beta_1\beta_2|/|\alpha_1\alpha_2|$, we can get the state of the quantum system after interaction as

$$|\Psi(t)\rangle_{4,6,C} = \frac{1}{2}[|\beta_1\beta_2(b|e\rangle_4|e\rangle_6 + a|g\rangle_4|g\rangle_6)

\otimes |0\rangle_C - i\sin\varepsilon t|g\rangle_6\otimes |1\rangle_C].$$

(17)

Here we have discard the phase factor, which can be removed by a simple rotation operator. From Eq. (17) we can see if Bob can detect a photon in the cavity then we know our teleportation process fail. If he can’t detect any photon in the cavity, that is, the cavity is still
in the vacuum state $|0\rangle_C$ and the atoms are in the state $(b|e\rangle_4|e\rangle_6 + a|g\rangle_4|g\rangle_6)$, which related to the original entangled state of atoms 1 and 2 up to a corresponding unitary transformation and by the classical information Bob can discriminate the collapsed state and reconstruct the original state. In other words, separate local measurements can also achieve the goal of joint measurement and thus teleportation with separate local measurements could succeed. We also note that after teleportation the entanglement of atoms $(1,2)$, $(3,4)$ and $(5,6)$ disappear and the new entanglement between the atoms 4 and 6 is set up which means the entanglement swapping do happens.

We can also calculate the probability of successful teleportation. The other three states in Eq. (13a) and the states of (13d) have the same successful teleportation probability, that is $P_a = P_d = |\beta_1\beta_2|^2$. As to the states (13b) and (13c) they also have the same probability of successful teleportation, but we have to break them into two different classes in calculating the probability of successful teleportation. While $|\alpha_1\beta_2| \geq |\beta_1\alpha_2|$, the successful teleportation probability of them is $P_b = P_e = |\beta_1\alpha_2|^2$, otherwise $P_b = P_e = |\alpha_1\beta_2|^2$. So, the total probability of successful teleportation in this scheme is $P = P_a + P_b + P_e + P_d = 2|\beta_1|^2$ in the case of $|\alpha_1\beta_2| \geq |\beta_1\alpha_2|$, otherwise $P = 2|\beta_2|^2$. We also note that if $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$, then $P_a = P_d = |\beta|^4$ and $P_b = P_e = |\alpha|^2$, thus $P = 2|\beta|^2$ which proves to be the maximally probability for entanglement teleportation 20.

IV. TELEPORTATION OF MULTIPARTITE ENTANGLEMENT

We also note that the scheme in section 3 can directly be generalized to teleport multipartite entanglement. In this section, we will consider the generalization of our schemes.

We first consider the former scheme, in order to teleport a $N$-atom entangled state we need $N$ EPR pairs, thus the initial state of the system is

$$|\Psi\rangle = \left(a|e\rangle_1 \cdots |e\rangle_n + b|g\rangle_1 \cdots |g\rangle_n\right)$$

$$\otimes \prod_{j=1}^n \sqrt{\frac{1}{2}}(|e\rangle_{n+j}|e\rangle_{2n+j} + |g\rangle_{n+j}|g\rangle_{2n+j}) \quad (18)$$

where atoms 1, 2, $\cdots$, $2n$ belong to Alice and atoms $(2n+1), (2n+2) \cdots (3n)$ are sent to Bob. We can distinguish the four Bell states of atom pairs $(j, n+j)$ one by one by using the cavity, thus can achieve unity probability and fidelity teleport the $N$-partite entangled state of atoms $1, 2, \cdots, n$ to another $N$ atoms of $(2n+1), (2n+2) \cdots (3n)$. After teleportation, the new entanglement of atoms $(2n+1), (2n+2) \cdots (3n)$ is set up.

The generalization of the latter scheme is familiar with the former one. In order to teleport a $N$-atom entangled state one need to set up $N$ bipartite nonmaximally entangled states. After the distinguish of the four Bell-states of atom pairs $(j, n+j)$ one by one by using the cavity, Bob only need send one of his atoms to the resonant cavity and take a proper interaction time, if he can’t detect photon in the cavity then he can reconstruct the original state by corresponding unitary transformation which he can know by the aid of classical information he received from Alice.

V. REMARKS AND CONCLUSIONS

Here, we will give a brief discussion on the experimental realization of our scheme. For the large-detuned cavity, it is free of the effects of the cavity decay and thermal field. Meanwhile it is noted that the atomic state evolution is independent of the cavity field state, thus based on cavity QED techniques presently 13, 14, 15 it might be realizable. In our scheme, the two atoms must be simultaneously interaction with the cavity. But in real case, we can’t achieve simultaneousness in perfect precise. Calculation on the error suggests that it only slightly affects the fidelity of the reconstate 21. For the resonant cavity, In order to realize the teleportation successfully, the relationship between the teleportation time and the excited atom lifetime should take into consideration. The time required to complete the teleportation should be much shorter than that of atom radiation. Hence, atom with a sufficiently long excited lifetime should be chosen. For the Rydberg atoms with principal quantum numbers 50 and 51, the interaction time is on the order is much shorter than the atomic radiative time 13. So our scheme is realizable by using available cavity QED techniques.

In conclusion, simple and physical schemes for teleportation of bipartite entangled atomic state based on cavity QED are proposed. The first scheme adopts two EPR state as quantum channels and the latter one adopts two bipartite nonmaximally entangled states as quantum channel. We also consider the generation of EPR state in section 2, in the forth section we generalize the two schemes to the case of multipartite entanglement teleportation. The presented schemes are achieve with separate atomic measurements instead of any types of joint measurement, which are difficult for experimental realization. The probability of successful teleportation is obtained and proves to be unit or maximal within current known knowledge. In addition, our scheme is insensitive to both the thermal field and the cavity decay, thus feasible within current cavity QED techniques.

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