Archery Bow Design Equation

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Abstract—Bow and arrow is one of the ancient combat tool used for defense, hunting and in recent times, used in sports. There have been several improvements in archery since then. Bows, as we know today are made from several components, materials and are available in different shapes and designs. There have been several variations in the bow design, broadly categorized under traditional bow or long bow, recurve bow, cross bow and compound bow. They may look very different and may have different efficiency but their underlying principle of operations have not changed much in past decades. There have been several studies and patents filed for new designs of the bows to improve the efficiency and several accessories have also been introduced to improve targeting, stability, and power. In this study using laws of physics, a design equation has been established which can be used as basis for deriving various aspects of the bow design and can be used for creating new bow designs with added efficiency. The bow design equation in this study is named as ‘RV bow design equation’ which is split into three parts to cover the aspect of arrow mass, energy stored in the bow, and energy transferred to arrow to give designers holistic view of the bow design and ability to monitor several parameters independently and in conjunction to each other. This equation will facilitate designers to make informed decisions about several aspects of the bow and they can use this equation to maximize the energy or velocity output of the arrow with a given design and material used for construction.

Index Terms—Archery, Arrow Speed, Bow Design, Energy.

I. INTRODUCTION

Archery is said to have been into existence for approximately 10,000 years [1]. Many civilizations have used archery for hunting as well as for warfare. The two main components of archery are the bow and the arrow. The bow is a curved instrument which can store potential energy, when the bow is drawn. The arrow is a long stick like projectile that is shot by the bow.

The form and basics of construction of a bow and arrow had not changed significantly until 1960s when American engineers started working on the modern form of the compound bow [1].

Traditionally, the bows were made from materials like wood, horns or bones; sometimes from a combination of wood and horn or wood and metal. With the advancement, the modern day bows are made from laminated wood, plastic, fiberglass or carbon fibers [2]. To improve performance and accuracy, several accessories have been developed so far. However, there has been no extensive study in the field of archery involving theoretical physics.

Based on theoretical physics, this study leads to the derivation of an equation that can be used as a base to explain different aspects of bow function.

II. DERIVATION OF DESIGN EQUATION

The design equation for the bow is derived by the calculating the output velocity of the arrow. In Fig.1 below, three positions of an arrow have been described.

![Diagram of the bow]

P1 is the initial position of the bow when the bow string is at 90 degrees to the horizontal, and is at rest i.e. no arrow is loaded and string is at initial tension.

P3 is the position at maximum draw length. At this position, the bow string is at maximum tension before the string is released for shooting the arrow.

P2 is intermediate position between P1 and P3. This represents the intermediate state to depict the state of arrow after string release and before the arrow loses contact with bow string.

A. Definition of Variables

V= Output velocity of arrow
u=Initial velocity of arrow
d= Brace height (minimum distance between bow string and bow raiser)
t= Time taken by arrow to come at given position from full draw position
L= Draw length of the bow (above brace height) at given point of time (t)
α= Angle made by the bow string from horizontal at a given position at a given time (t)
a= acceleration of arrow at a given position at given time (t)
F=Tension in the string at a given position at given point of time (t)
v = velocity of the arrow at a given position at a given point of time (t)
M = Mass of the arrow
T1 = Tension in bow string at the time of rest
T2 = Tension in bow string at the time of full draw
D= Full draw length of the bow (above brace height)
\( \theta \) = Angle made by bow string from horizontal at full draw length
H = End to end height of the bow

**B. Assumptions**

During the derivation of the equation we have made the following assumptions:
1. Bow string is non-stretchable and hence the length of the string is not changing.
2. Bow string has no elastic property hence no energy is stored in the string.
3. All of the bow energy is stored in the bow limbs as potential energy by way of limb deformation at the time of drawing of bow.
4. Bow design is symmetrical and arrow is shot from the center of the bow.
5. Mass of the arrow is constant and is located at center of mass of the arrow.

**C. Calculation**

By Newton’s First Law of Motion

\[ F = Ma \]  

(1)

By Newton’s First Equation of Motion

\[ v = u + at \]  

(2)

Since the arrow at full draw length is at rest, hence the initial velocity of the arrow is Zero so, \( u = 0 \)

Equation (2) is now transformed as

\[ v = at \]  

(3)

Using (1), substituting the value of ‘a’ in (3)

\[ v = \frac{F}{M} t \]  

(4)

By Newton’s Second Equation of Motion

\[ L = ut + \frac{a t^2}{2} \]  

(5)

Since, \( u = 0 \) as initial velocity of arrow is zero

\[ L = \frac{a t^2}{2} \]  

(6)

Using (3) and (6)

\[ L = v \frac{t}{2} \]  

(7)

Using (4) and (7)

\[ L = \frac{m v^2}{2F} \]  

(8)

Equation (8) can be rearranged as follows

\[ v^2 = \frac{2L}{M} v \]  

(9)

To calculate the force applied in direction of the arrow, adding the ‘cos’ component of the force applied by the bow string from above and below the arrow respectively.

Hence (9) can be modified to calculate the velocity of the arrow at given time ‘t’ is:

\[ v^2 = \frac{2L}{M} F \cos \alpha \]  

(10)

By movement of arrow by distance of ‘dL’, the velocity of the arrow is changed by ‘dv’, tension in the bow string is changed by ‘dF’ and angle of bow string made from horizontal is changed by ‘d\( \alpha \)’.

Equation (10) can be integrated as below

\[ \int_0^V v^2 dv = \int_0^D L dL \int_{T1}^{T2} F dF \int_0^{90} \cos \alpha d\alpha \]  

(11)

Limits are explained as below:

Velocity of the arrow will range from initial position of rest 0 (Zero) to a final velocity ‘V’.

Draw length will range from initial full draw length of ‘D’ to Zero, as the bow string will return to initial position.

Angle from horizontal will range from, angle made by bow string at full draw length ‘0’ to 90 degrees as the bow string returns back to its original position.

Tension in the bow string will change from Tension at full draw length ‘T2’ to tension at initial position of rest of bow to ‘T1’.

By integrating (11) with applied limits

\[ \frac{v^3}{3} = \frac{4}{M} \left[ \frac{D^2}{2} \left( \frac{T_1^2}{2} - \frac{T_2^2}{2} \right) \right] \left[ 1 - \sin \theta \right] \]  

(12)

Equation (12) can be rearranged as

\[ v^3 = \frac{3}{M} \left[ T_2^2 - T_1^2 \right] D^2 \left[ 1 - \sin \theta \right] \]  

(13)

Since tension in bow string at full draw ‘T2’ is always greater than tension in bow string at initial state of bow ‘T1’, and value of ‘Sin’ is between -1 and +1 , right side of (13) is always positive , hence we can conclude that left side of (13) is also positive. So final velocity of arrow ‘V’ is always positive.

**III. EXPLANATION OF DESIGN EQUATION**

Design Equation derived (13) is named as RV Bow Design Equation. This design equation can be divided into 3 parts.

**A. Arrow Constant**

Following equation is called Arrow constant:

\[ \text{Arrow constant} = \frac{3}{M} \]  

(14)

This constant confirms if the mass of the arrow increases the output velocity of the arrow decreases and vice-versa. Since at any point of time the mass of the arrow to be shot is constant hence it is called the arrow constant. In addition to it, this constant can be used to device or design an arrow with right mass for a given design of the bow.

As given the limitation of choice of material to construct the bow and design of the bow, every bow is constructed to operate in an optimal range of velocity of the arrow. Hence, choosing a right mass of the arrow can help operate the bow in the optimal range. Additionally, Arrow constant is always...
a positive value.

**B. Bow energy coefficient**

Following is the formula for bow energy coefficient

\[
\text{Bow energy coefficient} = [T_2^2 - T_1^2] \]  \hspace{1cm} (15)

This coefficient represents the potential energy stored in the bow.

Although by design of bow or limitations of the human body, the maximum draw length ‘D’ for the bow is limited; however, we can increase the maximum energy stored in the bow by increasing tension at the maximum draw length which can be achieved by various creative design.

Additionally, (15) can be used to calculate energy stored in any number of equipment used additionally on bow to store the energy, which can then be delivered to arrow via bow string.

Bow energy coefficient is always a positive value.

**C. Energy transfer coefficient**

Following is the formula for energy transfer coefficient:

\[
\text{Energy transfer coefficient} = D^2 \left[1 - \sin \theta \right] \]  \hspace{1cm} (16)

This is the coefficient that accounts for the fraction of the potential energy transferred to the arrow. As the angle value varies between 0 to 90 degrees, smaller the value of \( \sin \theta \) greater will the energy transferred to the arrow.

In the traditional bow, the angle between bow string and horizontal increases as the height of the bow ‘H’ increases. Hence, higher the height of the bow lower would be the energy transferred to the arrow as the angle between the bow string and the horizontal is larger.

In a non-traditional bow, the longer bow height with smaller bow string angle to horizontal can be achieved as well, as a part of the bow design.

It is observed that longer the draw length ‘D’, energy transferred to the arrow will be more, as the arrow will be in contact with the bow string for a longer duration; hence accelerating the arrow for longer time.

As per the material used to construct bow and the limitation of the human body, maximum draw length is limited. For a normal average human, maximum average draw length is for man 70.625 cm and for woman 63.75 cm (for man 28.25 inch and for woman 25.50 inch) [3]. Using Fig.1, total draw length of the bow is ‘b+D’. Since total draw length of the bow is humanly fixed, we can maximize duration of acceleration by keeping ‘b’ to a minimum.

Additionally, Energy transfer coefficient is always a positive value.

**IV. BOW ENERGY STORAGE**

Energy storage can be calculated using the RV Bow Design Equation (13). Fig.2 describes the positions of bow limbs for potential energy storage.

B1 is the natural position of the bow limb, this is the position of the limb when the bow is un-stringed.

B2 is at rest position of the bow limb when the bow is stringed. When the bow is stringed the bow limbs are deformed or bent to an angle ‘\( \sigma \)’ from natural position B1.

B3 is the position when the bow is drawn to the maximum draw length. At this position, the bow limb is bent to an angle ‘\( \beta \)’ form the natural position B1.

**A. Variable definition**

- \( x \) = displacement of far end of the bow limb at position B2 from position B1
- \( y \) = displacement of far end of the bow limb at position B3 from position B2
- \( \beta \) = Bend angle of the bow limb at position B3 from position B1
- \( \sigma \) = Bend angle of the bow limb at position B2 from position B1
- \( r \) = length of the bow limb from the bending point to the far end
- \( K \) = spring constant for the limb

**B. Assumptions**

1. Bow limbs are deformed uniformly on both sides
2. All the potential energy of the bow is stored in the skewed limbs of bow.

**C. Calculations**

By Hooke’s law, bow at position B2

\[
T_1 = K x \]  \hspace{1cm} (17)

When the bow is deformed to position B3

\[
T_2 = K (x + y) \]  \hspace{1cm} (18)

Equation (15), (17) and (18) are combined to calculate Bow Energy coefficient

Bow Energy Coefficient = \(|K(x + y)|^2 - (Kx)^2\)  \hspace{1cm} (19)

Above equation is rearranged as

Bow energy coefficient = \(K^2(y^2 + 2xy)\)  \hspace{1cm} (20)

When the bow limb deflection is small

\[
x = r \sigma \]  \hspace{1cm} (21)
\[
(x + y) = r \beta \]  \hspace{1cm} (22)

Equation (20), (21) and (22) are combined
Bow Energy coefficient = \( K^2r^2(\beta^2 - \sigma^2) \)  \( (23) \)

**D. Explanation**

Equation (20) is the general equation for potential energy stored in the bow. As we can observe higher the bend of the bow due to draw ‘\( y \)’ more will the potential energy stored in the bow.

This equation also will help to optimize the bow bend required for the stringing the bow.

Differentiating (20) to maximize the potential energy stored.

\[ K^2 \frac{d(y^2 + 2xy)}{dy} = 0 \]  \( (24) \)

\[ 2y + 2x = 0 \]

\[ x = -y \text{ or } x = y \]  \( (25) \)

Ignoring the negative sign as Bow energy coefficient is always positive;

Using (20) and (25)

Max Bow energy coefficient = \( K^2(x^2 + 2x^2) \)

Rearranging the above equation

Max Bow energy coefficient = \( K^2(3x^2) \)

Max Bow Energy coefficient = \( 3K^2x^2 \)  \( (26) \)

This implies \( \beta = 2\sigma \)  \( (27) \)

This implies that for the bow to store maximum potential energy it has to be strung to half the maximum displacement value of the bow limb.

Using (23) and (27), Max Bow Energy coefficient = \( K^2r^2((2\sigma)^2 - \sigma^2) \). Rearranging above equation:

Max Bow Energy coefficient = \( 3K^2r^2\sigma^2 \)  \( (28) \)

Equation (28) states that maximum potential energy is stored when bow limb is strung at half the maximum angle of the limb at full draw.

In addition to above, it is also observed that higher the length of the bow limb ‘\( r \)’, higher will be the energy stored in the bow limb. However, it was established earlier in energy transfer coefficient (16) larger the bow limb, lower will be the energy transfer coefficient; as the angle ‘\( \theta \)’ between the bow string and horizontal at the full draw will increase. This will lead to lesser energy transfer to the arrow i.e. although we have achieved higher potential energy stored in the bow limb by increasing the length of the limb, lesser component of that energy is delivered to the arrow.

This is to note that longer bow limb and smaller angle between bow string and horizontal at full draw can be achieved concurrently in a non-traditional bow design.

**V. OBSERVATIONS**

**A. Optimal bow design**

Using Fig.1 and Fig.2 optimal bow design points can be summarized as below:

1. Keep ‘\( d \)’ as small as possible.
2. Total bow draw length \( (b+D) \) is humanly fixed value for a given bow design or archer. So it can be assumed that ‘\( D \)’ for a bow is constant.
3. Bow must be strung at half of the maximum deflection value of the bow limb.

Given the above consideration RV Bow design equation can be represented as below for traditional bow design:

Equation (13) and (28) are combined:

\[ V^3 = \frac{9}{\pi} [K^2r^2\sigma^2]D^2[1 - \sin \theta] \]  \( (29) \)

Using (29), it can be observed that for the bow limb, material used should have high spring constant ‘\( K \)’. Another interesting observation is once the material for bow limb is selected most part of the above equation becomes constant for a given arrow to be shot as ‘\( M \)’, ‘\( K \)’ and ‘\( D \)’ become constants.

We can further maximize (29), by creative design solutions for the bow.

**VI. CONCLUSION**

RV Bow Design Equation (13) can be used as design base for any type of bow design. This equation can also be used to create new designs for the bows or, optimization of a given bow design. Its application depends on the creativity of the user and limited by available materials and technology of the time.

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