Intermittency as a possible underlying mechanism for solar and stellar variability

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Abstract. We briefly discuss the status of the intermittency hypothesis, according to which the grand minima type variability in solar-type stars may be understood in terms of dynamical intermittency. We review concrete examples which establish this hypothesis in the mean-field setting. We discuss some difficulties and open problems regarding the establishment of this hypothesis in more realistic settings as well as its operationally decidability.

1. Introduction

It is now well established that middle-aged solar-type stars show variability on a wide range of time scales, including the intermediate time scales of $\sim 10^3 - 10^4$ years (Weiss 1990). The evidence for the latter comes from a variety of sources, including observational, historical and proxy records. Many solar-type stars seem to show cyclic types of behaviour in their mean magnetic fields (e.g. Weiss 1994, Wilson 1994), which in the case of the Sun have a period of nearly 22 years. Furthermore, the studies of the historical records of the annual mean sunspot data since 1607 AD show the occurrence of epochs of suppressed sunspot activity, such as the Maunder minimum (Eddy 1976, Foukal 1990, Wilson 1994, Ribes & Nesme-Ribes 1993, Hoyt & Schatten 1996). Further research, employing $^{14}C$ (Eddy 1980, Stuiver & Quay 1980, Stuiver & Brazunas 1988, 1989) and $^{10}B$ (Beer et al. 1990, 1994a,b, Weiss & Tobias 1997) as proxy indicators, has provided strong evidence that the occurrence of such epochs of reduced activity (referred to as grand minima) has persisted in the past with similar time scales, albeit irregularly.

These latter, seemingly irregular, variations are important for two reasons. Firstly, the absence of naturally occurring mechanisms in solar and stellar settings, with appropriate time scales (Gough, 1990), makes the explanation of such variations theoretically challenging. Secondly, the time scales of such variations makes them of potential significance in understanding the climatic variability on similar time scales (e.g. Friis-Christensen & Lassen 1991, Beer et al. 1994b, Lean 1994, Stuiver, Grootes & Braziunas 1995, O’Brien et al. 1995, Baliunas & Soon 1995, Butler & Johnston 1996, White et al. 1997). In view of this, a great
deal of effort has gone into trying to understand the mechanism(s) underlying such variations by employing a variety of approaches.

Our aim here is to give a brief account of some recent results that may throw some new light on our understanding of such variations.

2. Theoretical frameworks

Theoretically there are essentially two frameworks within which such variabilities could be studied: stochastic and deterministic.

Here we mainly concentrate on the deterministic approach and recall that given the usual length and nature of the solar and stellar observational data, it is in practice difficult to distinguish between these two frameworks (Weiss 1990). Nevertheless, even if the stochastic features play a significant role in producing such variations, the deterministic components will still be present and are likely to play an important role.

The original attempts at understanding such variabilities were made within the linear theoretical framework. An important example is that of linear mean-field dynamo models (Krause & Rädler 1980) which succeeded in reproducing the nearly 22 year cyclic behaviour. Unfortunately such linear models cannot easily and naturally account for the complicated, irregular looking solar and stellar variability.

The developments in nonlinear dynamical systems theory, over the last few decades, have provided an alternative framework for understanding such variability. Within this nonlinear deterministic framework, irregularities of the grand minima type are probably best understood in terms of various types of dynamical intermittency, characterised by different statistics over different intervals of time. The idea that some type of dynamical intermittency may be responsible for understanding the Maunder minima type variability in the sunspot record goes back at least to the late 1970's (e.g. Tavakol 1978, Ruzmaikin 1981, Zeldovich et al. 1983, Weiss et al. 1984, Spiegel 1985, Feudel et al. 1994). We shall refer to the assumption that grand minima type variability in solar-type starts can be understood in terms of some type of dynamical intermittency as the intermittency hypothesis.

To test this hypothesis one can proceed by adopting either a quantitative or a quantitative approach.

2.1. Quantitative approach

Given the complexity of the underlying equations, the most direct approach to the study of dynamo equations is numerical. Ideally one would like to start with the full 3-D dynamo models with the least number of simplifying assumptions and approximations. There have been a great deal of effort in this direction over the last two decades (e.g. Gilman 1983, Nordlund et al. 1992, Brandenburg et

\footnote{It is worth bearing in mind that one can always produce complicated looking behaviour within the linear framework, by combining many simpler behaviours. The crucial point is that in this case complexity in behaviour requires a complicated underlying mechanism. Furthermore, there are qualitative differences, in terms of spectra and other dynamical indicators, between complicated dynamical behaviours produced by linearly complex and nonlinearly chaotic systems.}
The difficulty of dealing with small scale turbulence has meant that a detailed fully self-consistent model is beyond the range of the computational resources currently available, although important attempts have been made to understand turbulent dynamos in stars (e.g. Cattaneo, Hughes & Weiss 1991, Nordlund et al. 1992, Moss et al. 1995, Brandenburg et al. 1996, Cattaneo & Hughes 1996) and accretion discs (e.g. Brandenburg et al. 1995, Hawley et al. 1996). Such studies have had to be restricted to the geometry of a Cartesian box, which in essence makes them local dynamos, whereas magnetic fields in astrophysical objects are observed to exhibit large scale structure, related to the shape of the object, and thus can only be captured fully by global dynamo models (Tobias 1998). Furthermore, despite great advancements in numerical capabilities, these models still involve approximations and parametrisations and are extremely expensive numerically, especially if the aim is to make a comprehensive search for possible ranges of dynamical modes of behaviours as a function of control parameters.

An alternative approach, which is much cheaper numerically, has been to employ mean-field dynamo models. Despite their idealised nature, these models reproduce some features of more complicated models and allow us to analyse certain global properties of magnetic fields in the Sun. For example, the dependence of various outcomes of these models (such as parity, time dependence, cycle period, etc.) on global properties, including boundary conditions, have been shown to be remarkably similar to those produced by full three-dimensional simulations of turbulent models (Brandenburg 1999a,b). This gives some motivation for using these models for our studies below.

A number of attempts have recently been made to numerically study such models, or their truncations, to see whether they are capable of producing the grand minima type behaviours. There are a number of problems with these attempts. Firstly, the developments in dynamical systems theory over the last two decades have uncovered a number of theoretical mechanisms for intermittency, each with their dynamical and statistical signatures. Secondly, the simplifications and approximations involved in these models, make it difficult to decide whether a particular type of behaviour obtained in a specific model is in fact generic. And finally, the characterisation of such numerically obtained behaviours as “intermittent” is often phenomenological and based on simple observations of the resulting time series (e.g. Zeldovich et al. 1983, Jones et al. 1985, Schmalz & Stix 1991, Feudel et al. 1993, Covas et al. 1997a,b,c, Tworkowski et al. 1998, and references therein), rather than a concrete dynamical understanding coupled with measurements of the predicted dynamical signatures and scalings. There are, however, examples where the presence of various forms of intermittency has been established concretely in such dynamo models, by using various signatures and scalings (Brooke 1997, (Covas & Tavakol 1997, Covas et al. 1997c, Brooke et al. 1998, Covas & Tavakol 1998, Covas et al. 1999b).

\(^2\)Which at times would require extremely long runs to transcend transients.
2.2. Qualitative approach

Given the inevitable approximations and simplifications involved in dynamo modelling (specially given the turbulent nature of the regimes underlying such dynamo behaviours and hence the parametrisations necessary for their modelling in practice), a great deal of effort has recently gone into the development of approaches that are in some sense generic. The main idea is to start with various qualitative features that are thought to be commonly present in such settings and then to study the generic dynamical consequences of such assumptions.

Such attempts essentially fall into the following categories. Firstly, there are the low dimensional ODE models that are obtained using the Normal Form approach (Spiegel 1994, Tobias et al. 1995, Knobloch et al. 1996). These models are robust and have been successful in accounting for certain aspects of the dynamos, such as several types of amplitude modulation of the magnetic field energy, with potential relevance for solar variability of the Maunder minima type.

The other approach is to single out the main generic ingredients of such models and to study their dynamical consequences. For axisymmetric dynamo models, these ingredients consist of the presence of invariant subspaces, non-normal parameters and non-skew property. The dynamics underlying such systems has recently been studied in (Covas et al., 1997c, 1999b; Ashwin et al. 1999). This has led to a number of novel phenomena, including a new type of intermittency, referred to as in–out intermittency, which we shall briefly discuss in section 4.

3. Models

The standard mean-field dynamo equation is given by

\[ \frac{\partial B}{\partial t} = \nabla \times (u \times B + \alpha B - \eta_t \nabla \times B), \]

where \( B \) and \( u \) are the mean magnetic field and mean velocity respectively and the turbulent magnetic diffusivity \( \eta_t \) and the coefficient \( \alpha \) arise from the correlation of small scale turbulent velocities and magnetic fields (Krause & Rädler, 1980). In axisymmetric geometry, eq. (1) is solved by splitting the magnetic field into meridional and azimuthal components, \( B = B_p + B_\phi \), and expressing these components in terms of scalar field functions \( B_p = \nabla \times A_\hat{\phi} \), \( B_\phi = B_\hat{\phi} \).

In the following we shall also employ a family of truncations of the one dimensional version of equation (1), along with a time dependent form of \( \alpha \), obtained by using a spectral expansions of the form:

\[ \frac{dA_n}{dt} = -n^2 A_n + \frac{D}{2} (B_{n-1} + B_{n+1}) + \sum_{m=1}^{N} \sum_{l=1}^{N} F(n, m, l) B_m C_l, \]

\[ \frac{dB_n}{dt} = -n^2 B_n + \sum_{m=1}^{N} G(n, m) A_m, \]  

(2)
\[
\frac{dC_n}{dt} = -\nu n^2 C_n - \sum_{m=1}^{N} \sum_{l=1}^{N} \mathcal{H}(n, m, l) A_m B_l.
\]

where \(A_n, B_n\) and \(C_n\) are derived from the spectral expansion of the magnetic field \(B\) and \(\alpha\) respectively, \(\mathcal{F}, \mathcal{H}\) and \(\mathcal{G}\) are coefficients expressible in terms of \(m, n\) and \(l\), \(N\) is the truncation order, \(D\) is the dynamo number and \(\nu\) is the Prandtl number (see Covas et al. 1997a,b,c for details).

4. Different forms of intermittency in ODE and PDE dynamo models

Recent detailed studies of axisymmetric mean field dynamo models have produced concrete evidence for the presence of various forms of dynamical intermittency in such models. We shall give a brief overview of these results in this section.

![Figure 1](image-url)

Figure 1. Example of crisis induced intermittency in a shell dynamo with a cut, with \(r_0 = 0.2, C_\alpha = 25.5, C_\Omega = -10^4, \theta_0 = 45^\circ\). See Covas et al., 1999a for details of the model.

4.1. Crisis (or attractor merging) intermittency

A particular form of this type of intermittency, discovered by Grebogi, Ott & Yorke (Grebogi et al. 1982, 1987), is the so called “attractor merging crisis”, where as a system parameter is varied, two or more chaotic attractors merge to
form a single attractor. There is both experimental and numerical evidence for this type of intermittency (see for example Ott (1993) and references therein). We have found concrete evidence for the presence of such a behaviour in a 6-dimensional truncation of mean-field dynamo model of the type (Covas & Tavakol 1997) and more recently, in a PDE model of type (Covas & Tavakol (1999) for details). Fig. shows an example of the latter which clearly demonstrates the merging of two attractors, with different time averages for energy and parity. For a concrete characterisation and scaling, see Covas & Tavakol (1999).

4.2. Type I-Intermittency

This form of intermittency, first discovered by Pomeau and Manneville in the early 1980’s (Pomeau & Manneville 1980), has been extensively studied analytically, numerically and experimentally (see Bussac & Meunier 1982, Richter et al. 1994 and references therein). It is identified by long almost regular phases interspersed by (usually) shorter chaotic bursts. In particular, this type of intermittency has been found in a 12–D truncated dynamo model of type (Covas et al. 1997c), and more recently in a PDE dynamo model of type (Covas & Tavakol 1999). Fig. gives an example of such time series, where the irregular interruptions of the laminar phases by chaotic bursts can easily be seen. For a concrete characterisation, including the scaling for the average length of laminar phases see Covas & Tavakol (1999).

4.3. On-Off and In-Out Intermittency

An important feature of systems with symmetry (as in the case of solar and stellar dynamos) is the presence of invariant submanifolds. It may happen that attractors in such invariant submanifolds may become unstable in transverse directions. When this happens, one possible outcome could be that the trajectories can come arbitrarily close to this submanifold but also have intermittent large deviations from it. This form of intermittency is referred as on-off intermittency (Platt et al. 1993a,b). Examples of this type of intermittency have been found in dynamo models, both phenomenologically (Schmitt et al., 1996) and concretely in truncated dynamo models of the type (Covas et al. 1997c).

A generalisation of on-off intermittency, the in-out intermittency, discovered recently (Ashwin et al. 1999) is expected to be generic for axisymmetric dynamo settings. The crucial distinguishing feature of this type of intermittency is that, as opposed to on-off intermittency, there can be different invariant sets associated with the transverse attraction and repulsion to the invariant submanifold, which are not necessarily chaotic. This gives rise to identifiable signatures and scalings (Ashwin et al. 1999).

Concrete evidence for the occurrence of this type of intermittency has been found recently in both PDE and truncated dynamo models of the types (Covas et al. 1999a,b) respectively (see Covas et al. (1999a,b) for details).

5. Intermittency hypothesis: theory and observation

In the previous section, we have summarised concrete evidence for the presence of four different types of dynamical intermittency in both truncated and PDE
mean-field dynamo models. From a theoretical point of view, the intermittency hypothesis may therefore be said to have been established, at least within this family of mean-field models. What remains to be seen is whether these types of intermittency still persist in more realistic models. An encouraging development in this connection is the discovery of a type of intermittency which is expected to occur generically in axisymmetric dynamo settings, independently of the details of specific models. Despite these developments, testing the intermittency hypothesis poses a number of difficulties in practice:

1. Observationally, all precise dynamical characterisation of solar and stellar variability are constrained by the length and the quality of the available observational data. This is particularly true of the intermediate (and of course longer) time scale variations. Such a characterisation is further constrained by the fact that some of the indicators of such mechanisms, such as scalings, require very long and high quality data.

2. Theoretically, there is now a large number of such mechanisms, some of which share similar signatures and scalings, which could potentially complicate the process of differentiation between the different mechanisms.

Figure 2. Example of Type-I intermittency in a shell dynamo with a cut, with $r_0 = 0.7$, $C_\alpha = 28.0$, $C_\Omega = -10^4$, $\theta_0 = 45^\circ$. See Covas et al., 1999a for details of the model.
3. An important feature of real dynamo settings is the inevitable presence of noise. This calls for a theoretical and numerical study of effects of noise on the dynamics, on the one hand (e.g. Meinel & Brandenburg 1990, Moss et al. 1992, Ossendrijver & Hoyng 1996, Ossendrijver, Hoyng & Schmitt 1996) and on the signatures and scalings of various mechanisms of intermittency on the other.

These issues raise a number of interesting questions. Is, for example, the intermittency hypothesis operationally decidable at present? Will it be operationally decidable in foreseeable future?

In this connection it is worth bearing in mind that some types of intermittency do possess signatures that are rather easily identifiable. Nevertheless, we believe the answer to these difficult questions can only be realistically contemplated once a more clear picture has emerged of all the possible types of intermittency that can occur in more realistic solar-type dynamo models (and ultimately real dynamos) and once their precise signatures and scalings, in presence of noise, have been identified.

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