Spontaneous generation of temperature fluctuations in turbulent flows

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In incompressible flows, intermittent temperature fluctuations are expected to be spontaneously generated by viscous dissipation. We experimentally study these fluctuations in a closed von Kármán swirling flow of air at Mach number of order $10^{-3}$ and whose boundaries are maintained at a constant temperature. We observe intermittent peaks of low temperature correlated with pressure drops within the flow and show that they are caused by vorticity filaments. The measured ratio of temperature to pressure fluctuation agrees with the prediction based on adiabatic cooling within vortex cores. Relying on this property, a model for vorticity filaments is presented that captures the spatial structure of its temperature field. This experimental study shows that although the Mach number of the flow is small, there exist regions within the flow where compressible effects cannot be discarded and locally dominate the effect of viscous dissipation.

**Introduction**— Temperature fluctuations spontaneously generated in turbulent flows have not been studied experimentally so far probably because it has been believed that they have a weak effect in most experiments. Two processes are responsible for such temperature fluctuations. First, viscous dissipation of the flow kinetic energy generates heat, as recently studied in detail using numerical simulations [1]. Fluctuations of temperature result from bursts of dissipation that occur at small scales and in a strongly intermittent way in space and time. These fluctuations could therefore provide additional information about small scale intermittency of turbulent flows. There exists a second mechanism other than dissipative processes, which subsists in the limit of a perfect fluid with conserved entropy. It is related to adiabatic cooling or heating induced by pressure fluctuations generated by the flow [2]. Although this mechanism is discarded when the incompressible flow limit is assumed as in the numerical simulations quoted above, it should certainly be taken into account in most astrophysical or geophysical flows. At the laboratory scale, adiabatic cooling is observed within strong coherent vortices such as in the Ranque tube [3] or in von Kármán swirling flows between co-rotating disks [4, 5] where a significant temperature drop can be observed in the vortex core [6]. Generating a von Kármán turbulent swirling flow in air between counter-rotating disks, we show that even in the absence of an externally driven vortex and at moderate values of the Reynolds number, these compressible events contribute significantly to the temperature fluctuations that are generated spontaneously by the flow.

**Experimental set-up and results**— We generate a von Kármán swirling flow in air between two counter-rotating disks in a closed cylinder of diameter 130 mm made out of copper of 2 mm thickness (see fig. 1a). Copper has been chosen for its high thermal conductivity to minimize temperature gradients along the boundaries of the experiment. The top and the bottom faces of the cylinder are closed with circular copper plates of the same thickness as the cylinder. In order to maintain fixed the temperature at the boundaries, a copper tubing of outer diameter 10 mm is welded to the cylindrical part of the container. The inlet and the outlet of the copper tubing are connected to a circulating water bath with thermostat control which maintains the temperature of the water at a given fixed value within ±0.01 K.

We use two loop-controlled brushless DC motors with the angular velocity $\Omega$ ranging from 0 to 2000 rotations per minute (rpm). Each of the motor drives a disk with four curved blades which generate a strong flow inside the cylindrical cavity. The thickness of the disks and the height of the blades are 7.5 mm. Holes drilled on the surface of the cylinder provide access to the probes; one cold-wire temperature probe, one 1D hot-wire velocity probe and one acceleration compensated piezoelectric pressure probe. All probes are placed in the midplane, with the pressure probe flushed to the wall as shown in the top-view sketch of fig. 1a). The power consumption per unit mass by the turbulent flow $⟨\epsilon⟩$ is measured from the power required by the motors to maintain the flow, ranging from 0 to 500 m$^2$/s$^3$. This gives the Kolmogorov length scale $\eta = (\nu^3/⟨\epsilon⟩)^{1/4} \geq 50$ µm with $\nu = 1.5 \times 10^{-5}$ m$^2$/s the kinematic viscosity of air at room temperature. We evaluate the Taylor microscale $\lambda = (15\nu u_{rms}/⟨\epsilon⟩)^{1/2} \sim O(1)$ mm and the Taylor microscale based Reynolds number $Re_\lambda \sim O(10^2)$ using standard estimates for homogeneous and isotropic turbulent flows [7]. The Mach number defined as $Ma = u_{rms}/c$ reaches $4 \times 10^{-3}$ for the current setup where $u_{rms}$ is the RMS of velocity fluctuations and $c$ is the speed of sound in air.

Fig. 2a and 2b show the time series of temperature and pressure fluctuations about their mean values, obtained for $\Omega = 2000$ rpm. Sharp negative peaks are observed in both signals, examples of which are shown in the inset of figure 1. The intermittent pressure drops in a turbulent flow have been fairly well-studied and so is the phys-
FIG. 1. Sketch of the von Kármán flow configuration driven by two counter-rotating disks. Temperature of the boundaries is maintained constant using a circulating water bath. The pressure, velocity and temperature probes are all placed in the midplane.

One immediate question that arises is whether the negative peaks observed in pressure and temperature signals are correlated and result from the same type of structures in the turbulent flow. To check that, we evaluate the joint PDFs between pressure and temperature signals. To do so, we place the temperature probe close to the pressure probe ($d \approx 3$ mm apart) near the boundary. The joint PDF between pressure and temperature is defined as,

$$\Pi(p_0, T_0, \Delta t) dp_0 dT_0 = \text{prob}\left(p(t) \in [p_0, p_0 + dp_0], T(t + \Delta t) \in [T_0, T_0 + dT_0]\right)$$

for a time lag $\Delta t$ between the pressure and temperature signals. The correlation between pressure and temperature shows a maximum for a non zero time lag $\Delta t_{\text{max}} \sim 1$ ms which can be attributed to the advection of fluctuations between the two probes by the mean flow. This gives a characteristic advection velocity of $u_c = d/\Delta t_{\text{max}} = 3$ m/s. Fig. 3b shows the joint PDF $\Pi(p_0, T_0, \Delta t = \Delta t_{\text{max}})$ obtained for a rotation rate of $\Omega = 2000$ rpm. We observe that the joint PDFs are skewed towards negative values of both pressure and temperature. The joint PDFs also become increasingly skewed with increasing rotation rate $\Omega$. We conclude
that the vorticity filaments are responsible not only for the distribution of pressure drops but also for the temperature drops.

![Graph](image)

**FIG. 3.** (a) PDFs of temperature fluctuations normalized by their respective RMS values for $\Omega = 1200$ rpm (red), 1600 rpm (blue) and 2000 rpm (green). (b) The joint PDF of pressure and temperature fluctuations when the two probes are 3 mm apart for $\Omega = 2000$ rpm. The isocontours of probability are plotted on a logarithmic scale.

**Theoretical model**—In order to describe the profile of the temperature drops, we use a model of steady, weakly compressible, viscous and axisymmetric strong vortex in an ideal gas. The governing equations are,

\[
\frac{1}{r} \frac{d}{dr} \left( r \frac{d}{dr} \right) + w = 0 \quad (2a)
\]

\[
\frac{dp}{dr} = \frac{\rho_\infty u_r^2}{r} \quad (2b)
\]

\[
u_r \frac{d(\rho u_\theta)}{dr} = \nu \frac{d}{dr} \left( r \frac{d}{dr} \right) \left( \frac{1}{r} \frac{d}{dr} (r u_\theta) \right) \quad (2c)
\]

\[
u_r \frac{dT}{dr} = \frac{\alpha}{r} \frac{d}{dr} \left( \frac{dT}{dr} \right) + \frac{\nu^2}{c_p} \left( \frac{d}{dr} \left( \frac{d}{dr} \right) \right)^2 + \frac{u_r}{\rho_\infty c_p} \frac{dp}{dr} \quad (2d)
\]

\[ho T_\infty + T \rho_\infty = 0, \quad (2e)
\]

where the first three equations constitute the hydrodynamic part and the last two equations constitute the thermodynamic part of the governing equations with $\rho_\infty$ and $T_\infty$ the density and the temperature far from the vortex, $\alpha$ the thermal diffusivity and $c_p$ the specific heat capacity at constant pressure. The velocity field in cylindrical coordinates is written as,

\[
u = u_r(r) \hat{r} + u_\theta(r) \hat{\theta} + zw(r) \hat{z},
\]

and the assumption of a strong vortex is,

\[
|u_\theta| \gg |u_r|, \quad |u_\theta| \gg |u_z|.
\]

The set of governing eqns. 2 forms an underdetermined system of equations and thus has an infinite number of steady vortex solutions. This system can be closed with some assumption on the velocity field which has resulted in a variety of vortex solutions proposed till date [14–20]. On assuming $w$ to be constant we obtain the vortex solution for a Burgers vortex. Assuming also small, finite values of $\nu$ and $\alpha$, we obtain what we call an adiabatic Burgers vortex. The corresponding pressure and temperature profiles are given by,

\[
p(r) = \left[ \frac{\rho_\infty \Gamma_\infty^2}{8\pi^2 r_c^5} \left( \frac{1 - \exp(-r^2/r_c^2)}{r^2/r_c^2} \right)^2 \right]
\]

\[
+ 2\text{Ei}\left( \frac{r^2}{r_c^2} \right) - 2\text{Ei}\left( \frac{r^2}{r_c^2} \right) \right], \quad (3a)
\]

\[
T(r) = \left[ \frac{\Gamma_\infty^2}{8\pi^2 r_c^2 c_p} \left( \frac{1 - \exp(-r^2/r_c^2)}{r^2/r_c^2} \right)^2 \right]
\]

\[
+ 2\text{Ei}\left( \frac{r^2}{r_c^2} \right) - 2\text{Ei}\left( \frac{r^2}{r_c^2} \right) \right], \quad (3b)
\]

where $\Gamma_\infty$ is the circulation far from the vortex, $\rho_\infty$ is the pressure far from the vortex, $\text{Ei}$ is the exponential integral which can be computed numerically [21] and $r_c = 2 \sqrt{\nu/\alpha}$ is the characteristic width of the vortex core which originates from a balance between viscous diffusion and angular momentum transport (eq. 2c). We observe from eqn. 2c that for small values of $\nu$ and $\alpha$, temperature fluctuations are driven by adiabatic pressure fluctuations. Eqns. 3 also predict that the amplitudes of pressure ($\Delta p$) and temperature ($\Delta T$) satisfy,

\[
\frac{\Delta p}{\Delta T} = \left( \frac{\gamma}{\gamma - 1} \right) \frac{p_\infty}{T_\infty} \approx 1.2 \text{ Pa/mK}, \quad (4)
\]

where $\gamma = c_p/c_v$ is the ratio of specific heats ($\gamma \approx 1.4$ for air) and $p_\infty$ is the ambient pressure. Fig. 4 shows the value of pressure, denoted by $p_{\text{max}}(T)$, that maximizes the conditional PDF $\Pi(p(T, \Delta T_{\text{max}}))$ for each value of $T$ (data extracted from fig. 3b). We observe that for negative values of temperature fluctuations, $p_{\text{max}}$ depends linearly on $T$ with a slope of 1.2 Pa/mK as predicted by...
From this observation we conclude that concurrent negative fluctuations in pressure and temperature are associated with adiabatic cooling. We can further check the validity of our model by comparing the vortex temperature profile predicted by eqn. 3b with the profile observed experimentally. We use temperature measurements performed in the bulk of the mid-plane and apply the method of coherent averaging to extract the mean temperature profile of the negative drops. Fig. 4b shows the obtained mean temperature profile along with the theoretical profile predicted for adiabatic Burgers vortex. The time coordinate is transformed into space coordinate $r = u_c t$ using the characteristic advection velocity $u_c$ obtained previously. The fitting procedure involves three free parameters; the characteristic width of the vortex $r_c$, the constant prefactor (which governs the amplitude) and the value of the temperature field far from the filament ($T_\infty$). Note that a good agreement is also found by fitting the pressure measurement. However, we found a larger value of $r_c$ for the pressure profile, which can be attributed to the spatial averaging caused by probe dimensions as reported by Abry et al. In our experiments, the dimensions of the pressure and temperature probes are 1 mm and 1 $\mu$m respectively. Since the physical dimension of the measuring element of the temperature probe is smaller than any length scale associated with the turbulent flow, the temperature profile of the structure would be more accurate at estimating $r_c$. From the fit of the temperature profile, we obtain $r_c = 2.2$ mm $\sim O(\lambda)$ and we observe that this model is able to capture the structure of vorticity filaments.

**Conclusion** — We have experimentally shown that vorticity filaments in turbulent flows contribute to temperature fluctuations even for small Mach numbers of order $10^{-3}$ through adiabatic effects. This is evidenced by the correlation between the negative peaks observed in both pressure and temperature fluctuations. An analytical model of an adiabatic Burgers vortex is shown to be in agreement with the experimentally observed temperature profile of vorticity filaments. Vorticity filaments being characteristic of all turbulent flows, this process cannot be disregarded without caution in a variety of contexts. Not only when studying temperature fluctuations, but also when studying a field that is coupled to the temperature. Examples of which are quite numerous: propagation of acoustic or electromagnetic waves in turbulent flows, nucleation of liquid droplets in a turbulent flow of a vapor phase, as in cloud physics for instance, or in any other flow in which temperature dependent chemical reaction can take place such as in the interstellar medium. Whether this effect can also back react on the small scales of the turbulent velocity field deserves further studies.

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