Electromagnetic ground-wave propagation over the earth’s surface in the transition range

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Abstract. In this paper, the weighted average method is proposed for treating the ground-wave field of a vertical electric dipole over the earth’s surface in the transition range. Analysis and computations in HF/LF ranges are carried out specifically. In order to improve the computation accuracy, the weighted average method is proposed for evaluating the field component in the transition range. It is seen that the computation accuracy for the field component of the ground wave will be improved by using the weighted average method. The proposed method can be extended to the cases of the earth-ionosphere waveguide structure and the multi-layered cases.

1. Introduction
The electromagnetic ground-wave propagation over the surface of the earth and sea has been a subject of interest for over a century. The early analytical solution was formulated for the electromagnetic waves radiated a vertical electric dipole near or on the boundary between the air and the surface of the earth [1-3]. Because of its useful applications in subsurface and close-to-the-surface communications [4, 5], this subject was subsequently retreated widely by many investigators, especially including Van der Pol [6], North [7], Wait [8-10], and King [11-13]. Notably, the details are well summarized in the classic book by Banos [14] and the book by King, Owens, and Wu [15], and further discussions are carried out by Wait [16], and Collin [17].

At large distances from a dipole source to the observation point on the earth’s surface, a spherical-earth model should be adopted. In pioneering works by Watson in 1928, the electromagnetic ground-wave field of a vertical electric dipole over the spherical surface of the earth was expressed in the terms of a very slowly converging series [18]. With the extension of Watson’s theory, remarkable achievements were made by many investigators, such as Bremmer [19-21], Wait [22-24], and Fock [25]. The problem was reexamined by Houdzoumis [26] and Margentis [27].

In [26], the critical distance \( \rho_c \) is defined the boundary between the plane-earth region and the spherical-earth region. For mathematical convenience, the normalized radial distance is expressed as \( \eta \equiv \rho / \rho_c \). It is stated in [28] that the analytical solution for plane-earth model is accurate when \( \eta << 1 \), while the analytical solution for the spherical-earth model is accurate when \( \eta >> 1 \). It was suggested that the average between the plane-earth and spherical-earth formulas was used at \( \eta = 1 \) in Ref. [26].

In the present study, besides the ranges of \( \eta << 1 \) and \( \eta >> 1 \), the rest range can be defined as transition range. It is seen that the transition range is very wide in HF/LF ranges. Unfortunately, the available analytical solutions for both the plane-earth model and spherical-earth model complete analysis and computations for the ground-wave field in the transition range are still in dark. In what
follows, we will attempt to treat analytically the ground-wave field of a vertical electric dipole over the earth’s surface in the transition range by using the weighted average method.

Figure 1. Physical models: (a) Plane-earth model for $\rho << \rho_c$; (b) Spherical-earth model for $\rho >> \rho_c$.

2. Analytical formulation

2.1. The three ranges

The geometry under consideration is shown in Figure 1, where the vertical electric dipoles are located at a height $d$ in the air. The lower half-space (Region 1) is the earth or sea characterized by the permeability $\mu_0$, relatively permittivity $\varepsilon_r$ and conductivity $\sigma$, the upper half-space (Region 2) is occupied by the air characterized by the permeability $\mu_0$, uniform permittivity $\varepsilon_0$ and conductivity $\sigma_1 = 0$. It is conveniently represented by the cylindrical coordinates $(\rho, \varphi, z)$ in the range of $\rho << \rho_C$ where the earth or sea is well approximated by a plane-earth, while by the spherical coordinates $(r, \Theta, \Phi)$ with the origin at the earth’s center when $\rho >> \rho_C$. The critical distance $\rho_c$ is expressed in the form [26]

$$\rho_c = a(k_0d/2)^{-1/3}$$  \hspace{1cm} (1)

It is known that both the radial distance $\rho$ along the plane surface of the earth and sea and the distance $\rho_S$ along the spherical surface are continuous over the critical distance. Then, the transverse magnetic field and associated electric field are written as

$$B_{2\phi}(\rho, z) \rightarrow B_{2\phi}(r, \Theta)$$  \hspace{1cm} (2)

$$B_{2\rho}(\rho, z) \rightarrow B_{2\rho}(r, \Theta)$$  \hspace{1cm} (3)

$$B_{2z}(\rho, z) \rightarrow B_{2z}(r, \Theta)$$  \hspace{1cm} (4)

where $z \rightarrow r-a, \rho \rightarrow \rho_S = a\Theta$.

In the present study, the transition range is specifically defined by the range of $\eta \in [\eta_1, \eta_2]$, where $\eta_1 = 0.3$ and $\eta_2 = 3$. Obviously, the plane-earth formulas are good approximations in the range of $\eta_1 \leq \eta_2$, while the spherical-earth formulas are good approximations when $\eta_1 \geq \eta_2$ [28]. It is well known that both the plane-earth and spherical-earth formulas for the ground-wave field of a vertical electric dipole over the earth’s surface are available. In the next step, based on the plane-earth and spherical-earth formulas, the ground-wave field can be treated analytically by using the weighted average method.

2.2. The field in the plane-earth and spherical-earth ranges

For the plane-earth model, when both the source dipole and observation point are on or near the planar surface of the earth or sea, the general formulas for the electromagnetic field radiated by a vertical electric dipole are given in (31) - (33) in Ref. [13]. When the inequalities
are satisfied, the general formulas can be simplified greatly. The vertical electric field \( E_{2z} \) is written in the form

\[
E_{2z}(\rho,0) = \frac{\omega \mu_0 B_0 e^{-ik_2\rho}}{2\pi k_2} \left\{ \left[ \frac{ik_2}{r_2} - \frac{1}{r_2^2} \frac{i}{k_2 r_2} \right] - \frac{d}{r_2^2} \left[ \frac{ik_2}{r_2} - \frac{3i}{r_2^2} \frac{1}{k_2 r_2^2} \right] \right\} \left\{ \frac{\pi}{k_1} \frac{r}{k_2 r_2} e^{-i\theta} F(P_0) \right\}
\]

(6)

Here, \( r_2 = (\rho^2 + d^2)^{1/2} \), \( I_h e \) is the electric moment of the antenna,

\[
P_0 = \left( \frac{k_2 \rho}{k_1} \right)^2
\]

(7)

is the Sommerfeld numerical distance, and the function \( F \) is expressed in the form

\[
F(P_0) = \int_0^\infty \frac{e^{it}}{\sqrt{2\pi t}} dt = \frac{1}{2} (1 + i) - C_2(P_0) - iS_2(P_0)
\]

(8)

where \( C_2(P_0) + iS_2(P_0) \) is the Fresnel integral.

For the spherical-earth model (\( \eta \gg \eta_z \)), the vertical electric field \( E_{2z} \) can be expressed in the following form [27]:

\[
E_{2z}(\rho, \Theta) = E_0 V(x, y_0, y_1, q)
\]

(9)

where

\[
E_0 = \frac{i I_h e}{2a} \left\{ \frac{\mu_0}{\epsilon_0} e^{i\theta} \sqrt{\Theta \sin \Theta} \right\}
\]

(10)

\[
V(x, y_0, y_1, q) = e^{\frac{x^2}{4\pi}} \sum_{s=1}^{\infty} e^{-i\epsilon_s} W_2(t_s - y_0) W_2(t_s - y_1)
\]

(11)

where

\[
x = \left( \frac{k_2 a}{2} \right)^{1/3} \Theta
\]

(12)

\[
y_0 = \left( \frac{2}{k_2 a} \right)^{1/3} k_2 d
\]

(13)

\[
y_1 = \left( \frac{2}{k_2 a} \right)^{1/3} k_2 z
\]

(14)

Here \( W_2(x) \) is the Airy function of the second kind, \( \Theta = \rho/a \) is the angular distance from the dipole to the observation point, and both \( d \) and \( z \) are the height of the dipole and that of the observation point, respectively. It should be indicated that the parameters \( t_s (S = 1, 2, \ldots) \) are the roots of the following Stokes equation:

\[
W_2(t_s) - q W_2(t_s) = 0
\]

(15)

where

\[
q = \left( \frac{k_0 a}{2} \right)^{1/3} \Delta g
\]

(16)

where \( \Delta g \) is the normalized surface impedance of the earth or sea. Equation (9) is the ground-wave diffraction formula employed widely in engineering. The ground-wave attenuation factor \( V(x, y_1, y_2, q) \) is a function of the operating frequency, the angular distance from the dipole to the observation point, the earth’s conductivity, the earth’s radius, and the heights of the dipole and the observation point.
2.3. The field in the transition range

In the transition range \( \eta \in [\eta_1, \eta_2] \), the vertical electric field \( E_{2\eta} \) can be represented as the function of \( E_2 \) of the plane-earth model and \( E_2 \) of the spherical-earth model. Considering the case of the observation point being on the earth’s surface \((z \sim 0)\), we write

\[
E_{2\eta} = f[E_2(\rho, 0), E_2(\rho, \Theta)] , \quad \eta \in [\eta_1, \eta_2] \tag{17}
\]

At \( \eta = \eta_1 \) and \( \eta = \eta_2 \) the vertical electric field is continuous. Then, we let

\[
E_{2\eta} \big|_{\eta=\eta_1} = E_2(\rho_1, 0) \tag{18}
\]

\[
E_{2\eta} \big|_{\eta=\eta_2} = E_2(a, \Theta_2) \tag{19}
\]

where \( \rho = \eta \rho_\rho, \Theta_2 = \rho_2 / a \) and \( \rho_2 = \eta \rho_c \). At the critical distance \( \rho_c \), the vertical electric field is expressed as the average of the plane-earth formula and spherical-earth formula. Then, we have

\[
E_C = E_{2\eta} \big|_{\eta=\eta_2} = \frac{E_2(c, 0) + E_2(a, \Theta_2)}{2} \tag{20}
\]

where \( \Theta_c = \rho_c / a \).

With Eqs. (18) – (20), the vertical electric field \( E_{2\eta} \) at any place in the transition range can be represented in the following form

\[
E_{2\eta} = f[E_2(\rho, 0), E_2(\rho, \Theta)] = \begin{cases} 
E_2(1 - \frac{\rho - \rho_1}{\rho_c - \rho_1}) + E_C(\frac{\rho - \rho_1}{\rho_c - \rho_1}); & \rho_1 \leq \rho \leq \rho_C \\
E_C(1 - \frac{\rho - \rho_c}{\rho_2 - \rho_c}) + E_2(\frac{\rho - \rho_c}{\rho_2 - \rho_c}); & \rho_C \leq \rho \leq \rho_2 
\end{cases} \tag{21}
\]

It is seen that the vertical electric component is expressed as \( E_z \) in (6) for plane-earth model, \( E_r \) in (9) for spherical-earth model, and \( E_{r2} \) in (21) for transition model, respectively. It is seen that the weighted factor for the plane-earth formula is larger than that for the spherical-earth formula at the propagation distance being in the range of \( \rho \in [\rho_1, \rho_C] \). Correspondingly, the weighted factor for the spherical-earth formula is larger than that for the plane-earth formula at the propagation distance being in the range of \( \rho \in [\rho_1, \rho_2] \). The proposed method in the paper is defined as the weighted average method.

3. Computations and conclusions

In the following computations, we assume that the earth’s radius is taken as \( a = 6, 370 \) km, the current moment of the dipole is \( I = 14 \cdot m \), and Region 1 (seawater) is characterized by \( \varepsilon_r = 80 \) and \( \sigma_1 = 5 \) S/m. Equations (6) and (9) are employed for the plane- and spherical-earth ranges, respectively. In the transition range, the weighted average method is used to treat the ground-wave field components. By using (6), (9), and (21), the magnitudes of the vertical electric component are computed for the three models at \( f = 10 \) MHz, 20 MHz, 100 kHz, and 200 kHz, and shown in figure 2-5, respectively. From above computations, the discussions are carried out and conclusions are drawn as follows:

(i). It is seen that the critical distance \( \rho_c \) is determined by the operating frequency. At the operating frequencies \( f = 10 \) MHz, 20 MHz, 100 kHz, and 200 kHz, the critical distances are \( \rho_c = 72 \) km, 57 km, 338 km, and 268 km, respectively. Correspondingly, the transition ranges can be also determined. The lower the operating frequency, the larger the critical distance is, correspondingly, the wider the transition range is.

(ii). It is seen that neither numerical results by using plane-earth model nor those by using spherical-earth model are accurate in the transition range. In order to improve the computation accuracy, the weighted average method is proposed for evaluating the field component in the transition range. Obviously, with this proposed method for the transition range, the accuracy for evaluating the field component can be improved, the distributions of the field component are also smooth, and there is no existed the discontinuity at the critical distance \( \rho = \rho_c \).
(iii). The proposed method can be extended to the case of earth-ionosphere waveguide structure and the multi-layered cases.

Figure 2. The vertical electric field strength of the ground wave excited by a vertical electric dipole at the plane-earth range, the spherical-earth range, and the transition range at $f = 10$ MHz.

Figure 3. The vertical electric field strength of the ground wave excited by a vertical electric dipole at the plane-earth range, the spherical-earth range, and the transition range at $f = 20$ MHz.

Figure 4. The vertical electric field strength of the ground wave excited by a vertical electric dipole at the plane-earth range, the spherical-earth range, and the transition range at $f = 100$ kHz.

Figure 5. The vertical electric field strength of the ground wave excited by a vertical electric dipole at the plane-earth range, the spherical-earth range, and the transition range at $f = 200$ kHz.

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References
[1] Sommerfeld A N 1909 Propagation of waves in wireless telegraphy *Annals of Physics* 28 665-736
[2] Sommerfeld A N 1926 On the propagation of waves in wireless telegraphy *Annals of Physics* 81
[3] Sommerfeld A N 1935 In dia differential und integralgleichungen der meckanik und physik Frank P and Mises RV (Eds.), Braunschweig: F. Vieweg and Son II: 932 – 933

[4] Bhattacharya D and Chatterjee B 2015 Subsurface Communications with Special Reference to Antenna Structure Ite Journal of Research 19(10) 582-589

[5] Chang C H, Kuo C Y, Shum C K, et al. 2016 Global surface and subsurface geostrophic currents from multi-mission satellite altimetry and hydrographic data, 1996-2011 Journal of Marine Science & Technology 24(6) 1181-1193

[6] Pol B V D 1935 Theory of the reflection of light from a point source by a finitely conducting flat mirror: with application to Radiotelegraphy Physica 2(1-2) 843 – 853

[7] Norton K A 1937 The propagation of radio waves over the surface of the earth and in the upper atmosphere Proceedings of the Institute of Radio Engineers 25(1203-1236)

[8] Wait J R 1953 Propagation of radio waves over a stratified ground Geophysics 18(416-422)

[9] Wait J R 1961 The electromagnetic fields of a horizontal dipole in the presence of a conducting half-space Canadian Journal Physics 39(1017-1027)

[10] Wait J R 1970 Electromagnetic Waves in Stratified Media (2nd, Ed.) New York: Pergamon Press

[11] King R W P 1984 New formulas for the electromagnetic field of a vertical electric dipole in a dielectric or conducting half-space near its horizontal interface Journal of Applied Physics 53(8476-8482) 1982 erratum 56 3366

[12] King R W P 1985 Electromagnetic surface waves: New formulas and applications IEEE Transactions on Antennas & Propagation AP-33(11) 1204-1212

[13] King R W P 1990 Electromagnetic field of a vertical dipole over a imperfect conducting half-space Radio Science 25 149 – 160

[14] Banos A J 1966 Dipole Radiation in the Presence of a Conducting Half-Space Oxford: Pergamon Press

[15] King R W P, Margaret O and Wu T T 1992 Lateral Electromagnetic Waves, Theory and Applications to Communications, Geophysical Exploration, and Remoting Sensing, New York: Springer-Verlag

[16] Wait J R 1998 The ancient and modern history of EM ground-wave propagation Antennas & Propagation Magazine IEEE 40 7-24

[17] Collin R E 2004 Hertzian dipole radiation over a lossy earth or sea: Some early and late 20th century controversies Antennas & Propagation Magazine IEEE 46 64-79

[18] Watson G N 1928 The diffraction of radio waves by the earth in Proceedings of the Royal Society of London A-95(83-99)

[19] Bremmer H 1949 Terrestrial Radio Waves, New York: Elsevier

[20] Bremmer H 1954 The extension of Sommerfeld’s formula for the propagation of radio waves over a flat earth to different conductivities of the soil Physica 20 441-460

[21] Bremmer H 1958 Applications of operational calculus to groundwave propagation, particularly for long waves Ire Transactions on Antennas & Propagation AP-6 267-274

[22] Wait J R 1956 Radiation from a vertical electric dipole over a curved stratified ground Wait J R. Radiation from a vertical antenna over a curved stratified ground Journal of Research of the National Bureau of Standards 56 232-239

[23] Wait J R 1956 Radiation and propagation from a vertical antenna over a spherical earth Journal of Research of the National Bureau of Standards 56 237-244

[24] Wait J R 1957 The transient behavior of the electromagnetic ground wave on a spherical earth Ire Transactions on Antennas & Propagation AP-5 198-202

[25] Fock V A 1965 Electromagnetic Diffraction and Propagation Problems Oxford: Pergamon

[26] Houdzoumis V A 1994 Scattering of electromagnetic missiles, Part I; Vertical electric dipole radiation over spherical earth, Part II Ph.D. dissertation, Harvard University, Cambridge, MA
[27] Margetis D 1999 Studies in classical electromagnetic radiation and Bose- Einstein condensation Ph.D. dissertation, *Harvard University, Cambridge, MA*

[28] King R W P and Harrison C W 1998 Electromagnetic ground-wave field of vertical antennas for communication at 1 to 30 MHz *Electromagnetic Compatibility IEEE Transactions on* **40** 337-342