Negative streamwise velocities and other rare events near the wall in turbulent flows

Peter Lenaers, Qiang Li, Geert Brethouwer, Philipp Schlatter, and Ramis Örlü
Linné FLOW Centre, KTH Mechanics, SE-100 44 Stockholm, Sweden
E-mail: Lenaers@mech.kth.se

Abstract. Negative streamwise velocities, extreme wall-normal velocities and high flatness values for the wall-normal fluctuations near the wall are investigated for turbulent channel flow simulations at a series of Reynolds numbers up to \(Re_\tau=1000\) in this paper. Probability density functions of the wall-shear stress and velocity components are presented, as well as joint probability density functions of the velocity components and the pressure. Backflow occurs more often (0.06% at \(Re_\tau=1000\) and further away from the wall into the buffer layer for rising Reynolds number. An oblique vortex outside the viscous sublayer is found to cause this backflow. Extreme \(v\) events occur also more often for rising Reynolds number. Positive and negative velocity spikes appear in pairs, located on the two edges of a strong streamwise vortex: the negative spike occurring in a high speed streak indicating a sweeping motion, while the positive spike is located between a high and low speed streak. These extreme \(v\) events cause high flatness values near the wall \((F(v)=43\) at \(Re_\tau=1000\)).

1. Introduction

The occurrence of negative streamwise velocities in wall-bounded flows has been a debated topic ever since Eckelmann stated that “with certainty, there are no negative velocities near the wall” (Eckelmann, 1974). However, Spalart & Coleman (1997) reported negative wall-shear stress for a zero-pressure gradient (ZPG) turbulent boundary-layer (TBL) simulation. Similar results were determined by Laser Doppler Velocimetry (LDV) measurements by Johansson (1988) in a ZPG TBL. In the experiments clear instantaneous flow reversal has been observed and it is noted that this is not due to measurement noise. Although it is a very counterintuitive phenomenon, the existence of flow reversal will bring along that the practice guides of current measurement techniques need to be revisited in order to allow the detection of instantaneous reverse flow. Because of the presence of pressure in the Navier-Stokes equations, the possibility of negative streamwise velocities cannot be excluded, while this is not the case for a scalar transport problem, e.g. in studies concerning heat transfer, the temperature can not exceed the bounding values set by the boundary conditions.

Another rare event in the near wall region of wall-bounded flows are large fluctuations of the wall-normal velocity component, usually an order of magnitude larger than their local standard deviation. This causes extreme flatness values of the wall-normal velocity which have been reported by several previous studies, see e.g. Xu et al. (1996), Manna & Vacca (2001), and Durst & Beronov (2004). Since the high values were initially only observed in direct
numerical simulations (DNS) but not in experiments, it was thought that these values were non-physical, but in fact numerical artifacts (Durst & Beronov, 2004). However, by specifically designed LDV measurements, Xu et al. (1996) reported that similar high flatness levels are also found in experiments. They further attribute the high flatness values to strong sweep events which only happen very close to the wall and are rare in both time and space. Manna & Vacca (2001) also suggested that these values are caused by rare, but energetic events near the wall.

To investigate these rare events and to try to find whether there is any connection between them, we performed and analysed DNS of turbulent channel flow up to Reynolds number of $Re_{\tau} = 1000$ based on the friction velocity $u_{\tau}$ and channel halfwidth. Because these events occur seldom, long integration times are required to obtain statistically relevant data. Probability density functions (PDF) of wall-shear stress and velocities, and joint PDF (JPDF) of these quantities are calculated and compared. In addition Reynolds number dependencies and trends are studied. Visualisations are used to interpret the physical phenomena causing these events.

2. DNS of incompressible turbulent channel flow

**Code** Plane periodic channel flow is used as the flow case. The channel halfwidth $h$ is used as the reference length and $u_{\tau}$ as the reference velocity which leads to $Re_{\tau} = u_{\tau} h/\nu$ with $\nu$ the kinematic viscosity. The DNS were performed with SIMSON, a fully spectral in-house code described by Chevalier et al. (2007). It uses Fourier expansions in the horizontal directions and Chebyshev polynomials in wall-normal direction to discretise the variables and boundary conditions.

**Cases** Simulations at $Re_{\tau} = 180, 590$, and 1000 were performed. The computational domain and resolutions for the various cases are summarised in Table 1. After reaching a statistically steady state, the statistics are averaged over a time period $T$. During this time period, a certain number of instantaneous velocity fields $N_f$ is collected. Both the simulations at $Re_{\tau} = 180$ and 590 were performed with two different resolutions to investigate resolution effects. To distinguish between the cases and their different resolutions, an $a$, $b$, or $c$ is added to their name, each letter corresponding with similar grid spacing in viscous units.

| $Re_{\tau}$ nom. | $Re_{\tau}$ calc. | $(L_x \times L_y \times L_z)/h$ | $N_x \times N_y \times N_z$ | $\Delta x_i^+$ | $N_f$ | $u_{\tau} T/h$ |
|-----------------|-----------------|---------------------|---------------------|----------------|------|---------------|
| 180a            | 178.8           | $4\pi \times 2 \times \frac{3}{2} \pi$ | $128 \times 129 \times 128$ | 18, 4, 6       | 301  | 128           |
| 180b            | 178.9           | $4\pi \times 2 \times \frac{3}{2} \pi$ | $192 \times 129 \times 160$ | 12, 4, 5       | 261  | 111           |
| 590b            | 586.4           | $2\pi \times 2 \times \pi$ | $384 \times 257 \times 384$ | 10, 7, 5       | 43   | 30            |
| 590c            | 587.1           | $2\pi \times 2 \times \pi$ | $768 \times 385 \times 768$ | 5, 5, 2, 5     | 49   | 34            |
| 1000b           | 1000.2          | $8\pi \times 2 \times 3\pi$ | $2560 \times 385 \times 1920$ | 10, 5, 5       | 2    | 13            |

**Validation** Figure 1 shows that the present channel results agree well with previous DNS data by Moser et al. (1999) and del Álamo et al. (2004), while Figure 2 shows the evolution of $\langle u^+ \rangle$, $u_{\text{rms}}^+$, $F(u)$, and $F(v)$ for the $180b$ case at $y^+ = 1$ as a function of the number of independent points $N_p$ used. To calculate $N_p$ it is assumed that the integral length scale in the spanwise
3. Results

3.1. Negative streamwise velocities

To investigate the existence of negative streamwise velocities, the PDF of the wall-shear stress $\tau_w = \nu \partial u / \partial y$, which is shown in Figure 3a), was studied. For each case, $N_f$ instantaneous velocity fields (see Table 1) were taken to calculate the PDF of $\tau_w$ (all PDFs are normalised such that their integral is equal to 1). Our results are compared with ZPG TBL data from Schlatter & Örlü (2010) at similar Reynolds numbers. The PDFs show rare negative $\tau_w$ caused by negative streamwise velocities near the wall. We observe a Reynolds number dependence with the tails of the PDF widening for higher Reynolds number. Furthermore, the difference in the PDF of the 180a and 180b case is negligible. The left tail of the 590b case is a bit wider than that of the 590c indicating that negative velocities occur more frequent in the lower resolution case. The PDFs from the turbulent channel and the ZPG TBL collapse onto each other when the wall-shear stress is scaled in viscous units, indicating that the existence of negative streamwise velocities near the wall and the physical phenomena causing it are general for wall bounded flows.
Figure 2: a) Evolution of \( (u)^+ \) (blue) and \( 3u_{rms}^+ \) (green) as a function of the number of independent points \( N_p \) used to calculate these quantities for the 180b case at \( y^+ = 1 \). The horizontal lines denote the ±0.5% confidence interval for both variables with the mean calculated. b) Evolution of \( 4F(u) \) (blue) and \( F(v) \) (green). The vertical lines denote the ±1.5% confidence interval for \( F(u) \) and ±2.5% interval for \( F(v) \).

Hu et al. (2006) used DNS to calculate the wall-shear stress in a turbulent channel and found that the probability of negative wall-shear stress increases with Reynolds number which is consistent with our results. A comparison of the present results for the wall-shear stress fluctuations scaled with their rms value, and DNS data by Hu et al. (2006) is shown in Figure 3b) where also data from micro-pillar experiments in a ZPG TBL flow by Grosse & Schröder (2009) is added. The comparison is very good, confirming the general behaviour of wall turbulence. Note that their range of measurements is smaller, so they do not mention any negative wall-shear stress.

Figure 3: Comparison of the PDF of \( \tau_w \) of present DNS with literature data. (a) Present results (lines) and (◦) DNS of ZPG TBL (Schlatter & Örlü, 2010) at \( Re_\tau = u_\tau \cdot \delta_{99} / \nu = 191, 590, \) and 1000s. b) PDF of the wall-shear stress \( \tau_w \) at \( Re_\tau = 180, 590, 1000 \) (lines) compared with (◦) DNS data by Hu et al. (2006) (at \( Re_\tau = 180, 720, \) and 1440 respectively), and experimental data by Grosse & Schröder (2009) (×: at \( Re_\tau = u_\tau \cdot \delta_{99} / \nu = 3105, \) ♦: at \( Re_\tau = 4378 \)).

To study how far the negative velocity extends into the flow, and how significant the fraction of negative velocities is as a function of wall distance \( y^+ \), the PDFs of the streamwise velocity \( u \) at selected \( y^+ \)-values were also investigated. The PDFs of the streamwise velocity at \( y^+ = 2 \) and \( y^+ = 5 \) are depicted in Figure 4. The figures show a clear Reynolds number dependence on the tails as similar as those in the wall-shear stress, see Figure 3a), the negative tail of the PDF is wider for higher Reynolds number at all \( y \)-positions indicating that negative velocities occur more often for higher Reynolds number. Moreover, backflow also occurs farther away from the
wall for higher Reynolds number: the PDF at $y^+ = 5$ shows negative velocities for the 590 and 1000b cases, but no longer for the 180 cases.

![Figure 4: PDF of the streamwise velocity $u$ at a) $y^+ = 2$ and b) $y^+ = 5$.](image)

Summarising, more negative velocities occur for higher Reynolds number as seen in Figure 5a) which depicts the percentage of negative velocity as a function of the distance to the wall. Here, the percentage is calculated by taking the ratio of points in the DNS with negative velocity and the total number of points at each wall-normal position. For the 180 cases approximately 0.01% of velocities are negative near the wall. This rises to 0.05% for the 590 and to 0.06% for the 1000b case. They also occur further away from the wall for higher Reynolds number; up to $y^+ \approx 4.5$ for the 180 cases, $y^+ \approx 6.5$ for the 590, and $y^+ \approx 8.5$ for the 1000b case. Small but consistent differences are observed between the cases at the same Reynolds number but different resolutions with the higher resolution cases showing slightly lower percentages of backflow.

Figure 5b) shows the percentage of backflow at the wall as a function of Reynolds number. This percentage is calculated based on the wall-shear stress. This percentage increases with Reynolds number and the difference in results between resolutions is negligible. The data from Hu et al. (2006) shows a similar trend although at slightly higher values. The TBL results of Schlatter & Örlü (2010) also show a similar trend but for higher values. Exemplarily, the effect of insufficient spatial resolution, a common problem in near-wall measurements, is shown using the DNS data by Schlatter & Örlü (2010) at $Re_\tau = u_\tau \cdot \delta_{99}/\nu = 1240$. In order to mimic the finite length of e.g. a hot-wire sensor by means of the DNS data, the raw time series are filtered along the spanwise direction with a physical-space top-hat filter. There is a clear reduction of the percentage of backflow with increasing length of the sensing element, which goes along with observations on the fluctuation wall-shear stress (Örlü & Schlatter (2011)). Hence insufficient spatial (but also temporal) resolution might be one of the reasons why experimental investigations detect less or no instantaneous backflow.

### 3.2. High flatness and extreme wall-normal velocities

Near the wall, high flatness values for the wall-normal fluctuations are observed. The wall-limiting values of the flatness of the wall-normal fluctuations are shown in Figure 6a). The flatness of $\tau$ reaches approximately a value of 27 for the 180 cases, 36 for the 590 cases, and 43 for the 1000b case. The error bars at $y^+ = 1$ indicate 5% confidence intervals as discussed in Section 2. A clear Reynolds number dependence is seen with increasing flatness values for rising Reynolds number, while the difference between resolutions is negligible. This contradicts results by Durst & Beronov (2004) who saw a clear increase in flatness values for increasing resolutions.

These high flatness factors are caused by very high fluctuations in the wall-normal velocities near the wall as discussed by Xu et al. (1996). Wall-normal velocities larger than 10 times the local rms-value were defined extreme as indicated by the vertical dashed lines in Figure 6b) where
Figure 5: a) Percentage of negative velocities as a function of the distance to the wall. The horizontal lines indicate the percentage of negative wall-shear stress of the respective simulations. b) Percentage of backflow at the wall. Red ◦-symbols: 180a and 590b cases, with the ×-symbols the 180b, 590c, and 1000b cases. ZPG TBL results by Schlatter & Örlü (2010) (solid line), and turbulent channel data by Hu et al. (2006) (△). ◦: backflow ratio for spatially averaged DNS data (Schlatter & Örlü, 2010) to mimic a finite length of the sensing element, with $L^+ = [5.4, 21.7, 38.0, 54.3, 70.6, 87.0]$ in the direction of the arrow.

the PDF of $v$ at $y^+ = 2$ is shown for all simulations. It is found that these extreme velocities occur near the wall. They occur more often for higher Reynolds number. The widening of the tails in Figure 6b) is not as clear as in Figure 4a), this is because the PDF is normalised with the RMS of the wall-normal velocity which increases as the Reynolds number increases at a fixed wall-normal position. However, if the PDF of the wall-normal velocity is scaled in viscous units (not shown), the tail of the PDF widens much more. The very long tails in the PDF of $v$ reveal the intermittent behaviour of the flow field at this wall-normal position.

These extreme wall-normal velocities appear to be very rare and only occur in a region close to the wall as shown in Figure 6c) where the percentage of the extreme $v$ events is shown as a function of wall-normal position. As with the negative streamwise velocities, extreme wall-normal velocities occur more often for higher Reynolds number, although the difference between Reynolds numbers is smaller than for the negative velocity events (from about 0.04% for the 180 cases, up to 0.05% for the 1000b case). Analysis of the data shows that they do occur further in field than the negative streamwise velocities, up to about $y^+ \approx 25$ for all cases. Also similar to the negative streamwise velocities, the cases with higher resolutions show slightly lower percentages of extreme wall-normal velocities.

In Figure 6d), the wall limiting value of the flatness of $v$ are compared with ZPG TBL results by Schlatter & Örlü (2010). The values at the wall are calculated by a zeroth order extrapolation in $y^+$ direction.

3.3. Correlations

Both negative streamwise as extreme wall-normal velocities occur more often with higher Reynolds number. However, they do not appear to be correlated looking at their JPDF (not shown). However, the JPDF of $u$ and $p$, and $v$ and $p$ shown in Figure 7 show interesting features. Note that the contour line are evaluated on a logarithmic scale to better show the rare events. Figure 7a) shows a clear correlation between negative $u$ and negative $p$. Note that negative $p$ is a necessary but not sufficient prerequisite for the presence of vortical motion. There appears to be a sharp cutoff line (indicated by the dashed line) running from negative $p$, negative $u$ events, towards positive $p$, moderate $u$ events. In Figure 7b) the extreme negative $v$ events are distributed symmetric around $p = 0$. Note that the positive extreme $v$ are correlated with
Figure 6: a) Flatness factor of the wallnormal fluctuations for the various simulations. The error bars at $y^+ = 1$ indicate 5% confidence intervals. b) PDF of the wall-normal velocity $v$ at $y^+ = 2$ scaled with $v_{rms}$. The vertical lines designate extreme velocities $\pm 10v_{rms}$. c) Percentage of extreme velocities as a function of the distance to the wall. d) Flatness factor at the wall as a function of Reynolds number. Red ◦: 180a and 590b cases, ×: 180b, 590c, and 1000b cases. Solid line: ZPG TBL results (Schlatter & Örlü, 2010).

4. Visualisation

Figure 8a) shows an typical instantaneous streamwise velocity field of the 590b case. The background shows the wall-shear stress with high (red) and low speed streaks (blue) visible.
Regions of negative velocity (indicated by dark blue patches and white contour lines) are clearly visible and located in low-speed streaks. They occupy an area of order $30 \times 30$ viscous units and extend into the flow up to $y^+ \approx 6.5$, see also Figure 5a). Furthermore, the patches of negative velocity seem to be always located in a region below a comparably strong viscous-orientated vortical structure, indicated by the green contours of $\lambda_2^+ = -0.046$ as hinted on by the JPDF of $u$ and $p$. These contours are plotted up till $y^+ \approx 25$. Due to the obliqueness of the orientation of the vortex, there will be induced flow in both (negative) spanwise and (negative) streamwise direction, i.e. the transverse flow and backflow, respectively. This is further confirmed in Figure 9a) where a vortex is clearly visible in the $yz$-plane above the patch of negative velocity at $y^+ \approx 15$. Note that in this representation only the in-plane vortical motion can be identified, however, the vortex axis is oblique with respect to the $yz$ plane. In addition, the rotation of these oblique vortices is consistent with those streamwise vortices which are generated by instability of the streaks (Schoppa & Hussain, 2002) and there might be a connection between the two.

The extreme $v$ events were previously studied by Xu et al. (1996) where they reported that the extreme events that contribute to the high flatness of wall-normal velocity very close to the wall can be characterised by a positive and negative velocity spike (defined by $|v'/v_{rms}| \geq 5$) which always appear in pairs. In addition, the dimension of each positive/negative spike is about 50 wall units in streamwise and 30 wall units in spanwise directions respectively. These findings are confirmed in the present simulation and similar events were observed in the ZPG TBL simulations by Schlatter & Örlü (2010). Here, a typical example of both positive and negative wall-normal velocity spike taken from the 590b case is shown in Figure 8b) at $y^+ \approx 1$. Regions of extreme high/low wall-normal velocities ($|v| \geq 5v_{rms}$) are designated by the white and black contour lines respectively. In addition, vortices using the $\lambda_2$ criterion up to $y^+ = 25$ with a contour value of $\lambda_2 = -0.12$ are also shown in the same figure. It is clearly seen that the present spikes also appear in pairs and for each spike, the spanwise length is about 20 wall units which is slightly narrower than those found by Xu et al. (1996), while the streamwise length is similar for both studies being around 50 wall units. Moreover, the negative wall-normal velocity spike is associated with a high speed region indicating a strong sweep motion (downwash motion towards the wall) and consequently a large contribution to the Reynolds shear stress, whereas the positive velocity spike is located somewhere between a high and low speed streak. This behaviour is consistent with the results found by Xu et al. (1996) who further claimed that the positive spike has no significant contribution to the Reynolds shear stress. However, this is not in agreement with the present results. Although, the positive velocity spike is located between both positive and negative streamwise velocity fluctuation regions, the contribution from the region where $u' < 0, v' > 0$ can be significant, e.g. by examining the JPDF of $(u', v')$ close to the wall at e.g. $y^+ \approx 1$ (not shown here), it is found that the Reynolds stress mainly comes from both the sweep ($u' > 0, v' < 0$) and ejection motion ($u' < 0, v' > 0$), in which the contribution from ejection is slightly less than that from sweep.

The cause of the near wall wall-normal velocity spikes is related to the turbulence phenomena outside the viscous sublayer (Xu et al., 1996). According to these authors, the large negative spikes are results of the downwash motions coming from the near-wall vortex (being centred around $y^+ \approx 15$) which is a possible product of interaction of a vortex living in the buffer region (about $y^+ \approx 30$) with the wall. However, no explanation is provided for the positive spike. In the present case, the phenomena appear to be different. There is always a quasi-streamwise vortex lying above the extreme events, see the $yz$ plane cut through a pair of extreme $v$ events as shown in Figure 9b). Such a streamwise vortex gives rise to the upwash and downwash motions on its two sides which leads to both the positive and negative extreme $v$ fluctuations. However, further analysis is ongoing in order to further explain the origin and mechanism of these events.
Figure 8: Top view of part of the flow field of the 590 case. a) 480 viscous units in $x$-direction, 360 in $z$-direction. The background colour shows the wall-shear stress, with the dark blue patches encircled by white lines regions of negative $\tau_w$, the green structures isocontours of negative $\lambda_2^+=-0.046$. b) 190 viscous units in $x$-direction, 480 in $z$-direction. The background colour shows $u$, the white and black contour lines regions with extreme positive and negative $v$ respectively. The green structures are isocontours of negative $\lambda_2^+=-0.12$.

Figure 9: a) Part of a $yz$-plane showing a patch of negative $u$ (red contour line) with background colour representing $u$. b) Part of a $yz$-plane showing extreme $v$ events (red ◦) with background colour representing $v$.

5. Conclusions

DNS data of turbulent channel flow at three different friction Reynolds number $Re_\tau=180, 590$ and 1000 has been used to investigate rare events near the wall, i.e. negative streamwise velocities and extreme wall-normal velocities. A refinement in resolution is performed to investigate resolution effects.

Negative streamwise velocity is indeed found in the flow field in the vicinity of the wall and this flow reversal is not due to numerical artifacts. With an increase of Reynolds number, the percentage of flow reversal increases from 0.01% for $Re_\tau=180$ to 0.06% for $Re_\tau=1000$. They also extend further away from the wall, up to $y^+ \approx 4.5$ for $Re_\tau=180$, and $y^+ \approx 8.5$ for $Re_\tau=1000$. These regions of negative $u$ are of the order of $30 \times 30$ viscous units, and on top of them strong oblique vortices are found, which induce transverse flow and backflow. The simulations with higher resolution suggest a slightly lower percentage, but no qualitative different results are found suggesting that the resolution is sufficient.

Near the wall, extreme wall-normal velocity fluctuations are also observed. These extreme fluctuations cause high values of the flatness near the wall of up to $F(v) = 43$ for $Re_\tau=1000$. The resolution effects on the wall-limiting values of flatness are negligible but these values
are increasing with increasing Reynolds number. Similar to the negative $u$ events, the higher the Reynolds number, the more frequently the extreme velocities are. In addition, changing the resolution has similar effects, i.e. with higher resolution, the percentage of extreme high wall-normal velocity fluctuations slightly decreases. The present results are consistent with the previous studies, e.g. by Hu et al. (2006) and Xu et al. (1996). Positive and negative velocity spikes appear in pairs, and are located on the two sides of a strong streamwise vortex. The negative spike occurs in a high speed streak indicating a sweeping motion, while the positive spike is located between a high and low speed streak. Similar results found in ZPG TBL DNS and experiments suggest these events are universal for wall-bounded flows.

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