Weak microlensing effect and stability of pulsar time scale

M. S. Pshirkov(1); M. V. Sazhin(2)

(1) Pushchino Radio Astronomy Observatory, Astro Space Center, Lebedev Physical Institute, Pushchino, Russia
(2) Sternberg Astronomical Institute, Moscow State University, Moscow, Russia

Abstract

An influence of the weak microlensing effect on the pulsar timing is investigated for pulsar B1937+21. Average residuals of Time of Arrival (TOA) due to the effect would be as large as 10 ns in 20 years observation span. These residuals can be much greater (up to 1 ms in 20 years span) if pulsar is located in globular cluster (or behind it).

1 Introduction

First, the problem of the influence of the weak microlensing effect on the pulsar timing observations was discussed in (Sazhin, 1986). It was considered as interstellar Shapiro effect. The massive body that flies not far from the line pulsar-observer produces changes in the observing frequency of the pulsar similar to glitches. Estimations were made for Crab and Vela pulsars, glitches in these pulsars can be partially explained by the influence of the effect. Substantial contribution to the problem was made by (Larchenkova&Doroshenko, 1995); they mainly investigated the case of microlensing (i.e. the gravitational deflector flies very close to the line observer-pulsar). It was shown that the microlensing effect would cause short-term growth of the residuals and follow-up relaxation. Whole interaction would take less then several years and the maximum amplitude of the residuals would be 20-30 ms. Such remarkable events are very rare, but all the pulsars are affected by the weak microlensing effect to a greater or lesser extent. This effect was considered in (Ohnishi et al., 1996), where timing of millisecond pulsars was proposed as detection method for MACHOs. Growth of number of observed pulsars and time span of observation would make such detection easier. Numerical estimates were made in (Hosokawa, Ohnishi, Fukushima, 1999). They stated that even when the measurement accuracy reaches to 10 ns, probability of the remarkable influence would be in the order of $10^{-1}$ for the pulsar of a few kpc distance from us observed over

*E-mail:pshirkov@prao.ru
ten years. On the other hand there’s well developed formalism for the effect that came from the optics. The weak microlensing effect causes distant sources like quasars from ICRF to “tremble” on the level of tens of mas. It was shown in [Sazhin, 1996, Sazhin, Zharov, Kalinina, 1998] that these angular fluctuations range from a few up to hundreds of microarcseconds and this leads to a small rotation of the celestial reference frame. In [Sazhin, Zharov, Kalinina, 2001] influence of the effect on parallax measurements was considered-apparent parallax can be even negative due to the influence of the effect. Also, the weak microlensing effect can affect VLBI observations [Sazhin, Pshirkov, 2005] and it should be taken into account with new generation of space-based VLBI. In [Kalinina, Pshirkov et al., 2006] some statistical studies with toy-models were made, that was applied later to real model of the Galaxy. In fact, both weak microlensing effect and fly-by effect on timing are very similar and can be considered as manifestation of 4D (four-dimensional) astrometry [Ilyasov et al., 1990].

In this work we tried to apply eikonal formalism that was developed earlier for investigation of weak microlensing effect for use in pulsar timing studies.

The paper is organized as follows. In Section 2 we give a short review of influence of a passing body on pulsar timing in eikonal approximation. In Section 3 we apply a model of distribution of stars in our Galaxy to numerical estimations of their influence on pulsar timing and conclude our consideration in Section 4.

2 Eikonal formalism applied to pulsar timing

Change of phase during the propagation of electromagnetic wave can be obtained as a solution of Hamilton-Jacobi equation for a massless particle:

\[ g^{\mu\nu} \frac{\partial S}{\partial x^\mu} \frac{\partial S}{\partial x^\nu} = 0 \]  

Though \( S \) formally is a function of action, we hereafter identify it as eikonal or wave phase along the trajectory of the ray of light.

In weak field approximation the metric tensor of gravitational field can be written down in a following form

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]  

Here \( \eta_{\mu\nu} \) – is flat Minkowskian metric, \( h_{\mu\nu} \) – small additions to the flat metric that describes gravitational field of spherically symmetric body (star).

Equation \((1)\) can be solved in the following form: we take an exact solution [Weinberg, 1972] and then take its asymptotic when the impact parameter of the propagating ray is much larger then the Schwarzschild radius \( r_g = \frac{2GM}{c^2} \) \( (M\)-mass of the deflector)

\[ \psi = \psi_I + \frac{r_s \omega}{c} \text{arch} \left( \frac{r}{\rho} \right) \]  

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Here $\psi$ is full change of the phase along the trajectory, $\psi_l$ - change of the phase along the trajectory that corresponds to the propagation in the flat space and time, $r_g$ - Schwarzschild radius of the deflector, $\omega$ - frequency of the electromagnetic wave, $r$ - some point on the trajectory, $\rho$ - impact parameter (i.e. minimal distance between deflector $D$ and curve of photon propagation).

Figure 1: Schematic picture of origin of the weak microlensing effect. $D$-deflecting object.

Only the second term in (3) is a matter of interest to us, though it’s only a small addition to the usual change of phase during the propagation. The complete phase shift can be obtained as a sum of two solutions. The first is a phase shift during propagation from the source of the electromagnetic waves (which is located in $r_{in}$) to the closest approach to the deflector (we set the point of origin to the center of the deflector):

$$\delta \psi_+ = \psi(r = r_{in}) - \psi(r = \rho) = \frac{r_g \omega}{c} \text{arch} \left( \frac{r_{in}}{\rho} \right)$$  (4)

The second is a phase shift during propagation from the closest approach to the deflector to the observers (at $r_{out}$):

$$\delta \psi_- = \psi(r = r_{out}) - \psi(r = \rho) = \frac{r_g \omega}{c} \text{arch} \left( \frac{r_{out}}{\rho} \right)$$  (5)

And the total phase shift is:
\[ \Delta \psi = \frac{r_g \omega}{c} \ln \left( \frac{4 r_{in} r_{out}}{\rho^2} \right) \]  

(6)

We treated the deflector as a motionless body in this solution. In fact, all stars, including MACHOs of our Galaxy are moving. Approximate solution of space-time metric in the case of moving deflector and trajectory of photon in such a variable gravitational field was calculated in Kopeikin, Schaffer, 1999.\]

The metric which originates from a moving body and small perturbations of photon trajectory in gravitational field of this body, differ in \( \left( \frac{v}{c} \right)^3 \) terms from our solutions and we will omit this difference. To describe the motion we take \( \rho \) (impact parameter) as function of \( t \) only:

\[ \Delta \psi_2 - \Delta \psi_1 = \frac{r_g \omega}{c} \ln \left( \frac{r_{in1} r_{out1}}{r_{in2} r_{out2}} \frac{\rho_2^2}{\rho_2^2} \right) = \frac{r_g \omega}{c} \left[ \ln \left( \frac{r_{out2}}{r_{out1}} \right) + \ln \left( \frac{r_{in2}}{r_{in1}} \right) + \ln \left( \frac{\rho_2}{\rho_2} \right) \right] \]

(7)

Indices denotes values at different epochs \( t_1 \) and \( t_2 \). The first two terms are negligibly small, so we can rewrite expression (7):

\[ \Delta \psi_2 - \Delta \psi_1 = \frac{r_g \omega}{c} \ln \left( \frac{\rho_2^2}{\rho_2^2} \right) \]

(8)

Also we can write out time dependence of \( \rho(t) \):

\[ \rho(t) = \sqrt{\rho_0^2 + v^2(t - t_0)^2} \]

Here, \( t \) is time span of observations (we set the epoch of initial observations equal to 0), \( t_0 \) - is the epoch of the closest approach of the deflector to the line of propagation.

\[ \delta T = \frac{\Delta \psi_2 - \Delta \psi_1}{\omega} = \frac{r_g}{c} \ln \left( \frac{\rho_1^2}{\rho_2^2} \right) = -\frac{r_g}{c} \ln \left( \frac{\rho_0^2 + v^2(t - t_0)^2}{\rho_0^2 + v^2(t_1 - t_0)^2} \right) \]

(9)

We can set the first epoch \( t_1 \) equal to zero and discard the second index, \( t_2 \equiv t \):

\[ \delta T = -\frac{r_g}{c} \ln \left( \frac{\rho_0^2 + v^2(t - t_0)^2}{\rho_0^2 + v^2t_0^2} \right) \]

Hereafter phrases like "deflector s close to pulsar" mean we observe close angular coincidence of the bodies, not in 3D space.

\[ \delta T = -\frac{r_g}{c} \ln \left( \frac{\theta_0^2 + \mu^2(t - t_0)^2}{\theta_0^2 + \mu^2t_0^2} \right) \]

(10)
Figure 2: The plane of the deflector: red circle represents apparent position of the pulsar. Deflector with proper motion $\mu$ and impact parameter $\theta_0$ is passing by near this position.

Value $\theta_0$ depends on location of pulsar in Galaxy and its proper motion. The higher is density of deflectors in the neighborhood of pulsar on the celestial sphere, the smaller that value would be. We take into consideration only deflectors between the pulsar and the observer, because they make the largest contribution on the effect.

3 Estimates for B1937-21

We chose two pulsars J1643-1224 and B1937+21 for further estimates, because they’re quite distant and located in populated regions of our Galaxy (B1937+21: $G_l = 57.51, G_b = -0.29, r_p = 3.6kpc$; J1643-1224: $G_l = 5.67, G_b = 21.22, r_p = 4.86kpc$) [ATNF-psrcat, 1999, Manchester et al., 2005], so probability that effect would have place is much higher than for other millisecond pulsars. It’s essential to define values $\theta_0$ and $t_e$ - average duration of influence. They can be approximately found in such way [Kalinina, Pshirkov et al., 2006]: stars are nearly uniformly distributed in the neighborhood of the pulsar on the celestial sphere; the angular distance to the nearest star, which would affect the pulsar timing depends on the location of pulsar.

We calculated the density of the stars in the neighborhood, using accepted model of the disk of our Galaxy [Bahcall, 1986].

$$N(\theta, \phi) = \int_0^{r_p} n(\xi, \theta, \phi)\xi^2 d\xi$$

$$N(\theta, \phi)$$ – sought density in the direction of the pulsar, which is assigned by the angles $\theta, \phi$. $\theta$-
angle between the line of sight and the Galactic plane, $\phi$- angle between the projection of the line observer-pulsar to the galactic plane and the line Solar system-Galactic center; $\xi$- distance from the observer.

$$n(r, z) = n_0 \exp\left(\frac{-r - R_0}{3500}\right) \exp\left(\frac{-z}{325}\right) \text{pc}^{-3}$$

$n_0 = 0.1$ - density of the stars in Sun’s neighborhood, $r$ - distance from the axis of the Galaxy, $z$-distance from the Galactic plane, $R_0 = 8000 \text{pc}$ - distance between the Solar system and the Galactic center, $3500 \text{pc}$ and $325 \text{pc}$ - radial and vertical scales of the model, accordingly.

$$r(\xi, \theta, \phi) = \sqrt{R_0^2 + \xi^2 \cos^2(\theta) - 2R_0 \xi \cos(\phi) \cos(\theta)}$$

$$z(\xi, \theta, \phi) = \xi \sin(\theta)$$

$$N(\theta, \phi) = \int_0^{\rho} n_0 \xi^2 \exp\left(\frac{16}{7}\right) \exp\left(\frac{-\sqrt{R_0^2 + \xi^2 \cos^2(\theta) - 2R_0 \xi \cos(\phi) \cos(\theta)}}{3500}\right) \exp\left(\frac{\xi \sin(\theta)}{325}\right) d\xi$$

(12)

Average angular distance $\theta_1$ between the pulsar and the closest deflector (star) can be found with taking into account $N(\theta, \phi)$. Values $\theta_0$ and $t_e$ were calculated using Monte-Carlo simulation:

- a circle of $\theta_1$ were circumscribed around the pulsar on the celestial sphere, then a large amount (1000) of test deflectors with proper motion $\mu$ were started from this circle under random angles $\alpha$. As a result we found distributions for values $\theta_0$ and $t_e$, and their averages, that were used in following estimates.
- Only known distribution of stars in our Galaxy was used in our estimates and if we take into account possible influence of Dark Matter, then sought values can be lower in 2-3 times, because mass of DM doesn’t exceed mass of ordinary matter more than 4-5 times. Also, we set mass of deflectors equal to $M_{\odot}$. Values that are essential for further estimations (J1643-1224, B1937+21) are given in the table below.

| PSR       | $\theta_1$ | $\mu$    | $\theta_0$ | $t_e$ |
|-----------|------------|----------|------------|------|
| J1643-1224| 7.3"       | ~10 mas/yr| 4.7"       | 470 yr |
| B1937+21  | 2.5"       | ~10 mas/yr| 1.5"       | 150 yr |

We can see the influence of the effect on the residuals, but only trends of cubic order and higher will survive during usual fitting procedure (Baker & Hellings, 1986). Linear and quadratic terms will redefine apparent period of pulsar $P$ and its first derivative $\dot{P}$ and can’t be found.

Residuals of TOA due to the weak microlensing effect can be written as follows:

$$\delta T_{\text{post fit}} = Ct^3 + Dt^4 + Et^5$$

(13)

$C, D, E$ are coefficients in Taylor’s series of function (5) where $t = 0$.

Plotted coefficient $C$ depending on $t_0$ is represented in fig. 4 (plotted for B1937+21).
One can see from the plot that the fastest increase of residuals takes place when the epoch of the initial observation are 50-150 years away from the epoch $t_0$, because the third derivative have maximum in that interval maximal. If the initial observation coincides with the closest approach, then only fourth and higher orders term will affect timing and the residuals will increase much slowly. Magnitude of the residuals after subtraction of linear and quadratic terms can be expressed as follows: $\delta T_{\text{postfit}} = -\frac{r_g}{c} \ln \left( \frac{\frac{\theta_0^2}{\theta_0^2 + \mu^2 t_0^2}}{\frac{\theta_0^2}{\theta_0^2 + \mu^2 t_0^2}} \right) - A(0)t - B(0)t^2$, where $A(0)$, $B(0)$ -linear and quadratic coefficients at $t = 0$.

$$A(0) = \frac{2r_g}{c} \frac{\mu^2 t_0}{\theta^2 + \mu^2 t_0^2}$$

$$B(0) = -\frac{r_g}{c} \frac{\mu^2}{\theta^2 + \mu^2 t_0^2} + \frac{2r_g}{c} \frac{\mu^4 t_0^4}{(\theta^2 + \mu^2 t_0^2)^2}$$

The plot in fig. 5 shows magnitude of the residuals at different $t_0$ (0, 50, 100 years; blue, green and red graphs accordingly). Module of that magnitude depends only on module $t_0$.

Residuals of 10 ns magnitude due to the effect of weak microlensing will appear with probability of $\sim 50\%$ if time span of observations exceeds 20 years.

We can also calculate Allan variance (AVAR) for pulsar time scale with time residuals caused by the effect.

TOA residuals due to the effect can be significant, if $\theta_1$ (angular distance between the pulsar and the nearest affecting body) is much smaller than average. The plot Fig.7 represents situation when $\theta_0 = 0.1\text{mas}$. This situation has $\sim 0.5\%$ chance of probability in case of B1937+21; Probability reduces like $\theta_0$ inverse squared. The magnitude of the residuals can be as a great as 800-1000 ns in the same 20 years span.

However, if we used in fitting procedure terms of cubic and higher orders, then the magnitude of the effect can be effectively set to 0.

The magnitude can be much greater for pulsars in GC (or pulsars behind GC) ($n = 10^3 - 4 \text{pc}^{-3}$, $L$ (length of path of ray in GC) = 10pc, $d$ (distance to GC) = 1 – 10kpc). $t_e$ and $\theta_0$ can be much smaller because the density of stars in GC is large, The magnitude of the effect will be much greater (the same 1 ms in 20 years span). Time of one significant interaction will be quite small (20-30 years). Complete investigation of the question can be found in (Sazhin, Saphonova, 1993, Larchenkova & Kopeikin, 2006).

### 4 Conclusions

So, we can make several conclusions: average TOA residuals due to a weak microlensing effect is about 10 ns (B1937+21) in 20 years span. TOA residuals can be effectively set to zero by using higher order terms in fitting procedure (not for pulsars in globular clusters. Residuals can be
much greater if pulsar is located in a globular cluster, so the pulsars in globular clusters can’t be recommended for using in PT scale.

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Figure 3: TOA residuals, caused by the effect; no fitting conducted yet.

Figure 4: Graph for the coefficient $C(t)$ shows that the influence of higher-power order items should be taken into consideration.
Figure 5: TOA residuals due to weak microlensing effect. The blue curve corresponds to $t_0 = 0$, green one to $t_0 = 50$ years and red one to $t_0 = 100$ years.

Figure 6: Allan Variance (AVAR) which appears from a weak microlensing effect.
Figure 7: Strong influence of the effect. TOA residuals can reach 1 ms in 20 years observations span.