Non-holomorphic Corrections from Threebranes in F Theory

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We construct solutions of type IIB supergravity corresponding to 7 branes, an $O_7$ plane and 3 branes. By considering a probe moving in this background, with constant coupling and an AdS$_5$ component in its geometry, we are able to reproduce the exact low energy effective action for $\mathcal{N} = 2$ super Yang-Mills theory with gauge group $SU(2)$ and $N_f = 4$ massless flavors. After turning on a mass for the flavors we find corrections to the AdS$_5$ geometry. In addition, the coupling shows a power law dependence on the energy scale of the theory. The origin of the power law behaviour of the coupling is traced back to instanton corrections. Instanton corrections to the four derivative terms in the low energy effective action are correctly obtained from a probe analysis. We study how these instanton corrections are reflected in the background geometry by calculating the quark-antiquark potential. Finally we consider a solution corresponding to an asymptotically free field theory. Again, the leading form of the four derivative terms in the low energy effective action are in complete agreement with field theory expectations.
1. Introduction

Recent progress in string theory has lead to a deep and powerful connection between Yang-Mills theory and string theory, something which was expected more than twenty years ago\[1\]. In particular, according to Maldacena\[2\] the large $N$ and large 't Hooft coupling dynamics of Yang-Mills theory is captured by supergravity. This relationship can be motivated by studying a brane in full string theory. One then considers a low energy limit which decouples the field theory from gravity, and at the same time one considers the near horizon limit of the corresponding supergravity solution.

In this paper, we study the low energy limit of $\mathcal{N} = 2$ super Yang-Mills theory, by realizing it as the world volume theory on a Dirichlet three brane moving near seven branes, that is, threebranes in F theory. In this way we provide an alternative field theory interpretation which is obtained by probing with this brane the near horizon supergravity solutions. We begin by identifying the exact effective complex coupling\[3\] of the low energy field theory with the complex coupling of type IIB supergravity. The remaining supergravity equations then determine a unique metric. This supergravity solution corresponds to a sevenbrane background. We then consider introducing a large number $N$ of coincident threebranes into the problem. The supergravity solution for the threebranes plus sevenbranes has the same complex coupling as the pure sevenbrane background\[4\]. The presence of the threebranes does however deform the geometry and switch on a flux for the self-dual five form. The flux and deformed metric are determined by solving the Laplace equation on the background generated by the sevenbranes\[4\]. In the large $N$ limit ('t Hooft limit) both curvature and string loop corrections to the background are suppressed. The field theory of interest is then realized as a Born Infeld action describing the worldvolume dynamics of a threebrane probe moving in this geometry. In the supergravity description, we are studying the two-body interaction between the source and probe threebranes. We will thus compare the worldvolume theory of the probe to the low energy effective action for the field theory with gauge group $SU(2)$ as suggested by the work of \[5\].

The first example we consider is the theory with $N_f = 4$ massless flavors. This theory is a finite conformal field theory and as expected the geometry contains an $AdS_5$ factor. By requiring that the effective action in field theory has an exact $SL(2,\mathbb{Z})$ symmetry, we are able to fix the form of the six derivative terms\[6\]. After performing a field redefinition on the field theory side \[6, 7, 8\], the form of the field theory effective action and probe action agree up to and including six derivative contributions. This computation is a simple
extension of the result reported in [6] for the $\mathcal{N} = 4$ case. The new feature is the identification of the explicit $N$ dependence of the background coupling which is needed to exhibit the expected overall $N$ dependence of the probe action.

The next example we study is the theory with $N_f = 4$ massive flavors. The presence of the flavor masses explicitly breaks the conformal invariance. Indeed, the coupling of this theory picks up a dependence on the energy scale of the theory due to instanton corrections. Solutions of this type are particularly interesting from the point of view of the gravity/field theory connection. The radial direction of the AdS$_5$ space plays the role of an energy scale in the conformal field theory case[9]. However, all beta functions in the conformal field theory vanish and the evolution under the renormalization group is trivial. Examples of quantum field theories that are not conformal have a much richer renormalization group evolution and may provide valuable insights into the role of the "fifth dimension" [10]. In this case, we are able to determine the first corrections to the AdS$_5$ geometry. This approximate background leads to corrections to the four derivative terms in the probe action that are consistent with the form of instanton corrections to the four derivative terms in the field theory. Our results strongly suggest that the probe worldvolume action reproduces the exact Wilsonian effective action of the field theory. We also compute the quark-antiquark potential in this geometry.

Our third and final example considers the pure gauge theory. We are able to find an approximate solution, valid when the separation between the probe and source is large. This geometry reproduces the correct semiclassical structure of the four derivative terms. It would thus seem that the background we have found is capable of describing perturbative field theory on the probe world volume. By explicitly computing the magnitude of the square of the Ricci tensor, we are able to explain why this is indeed the case.

When this work was near completion we received [11], [12]. In these papers solutions of IIB supergravity corresponding to non-conformal field theories were constructed. These solutions preserve the $SO(6)$ invariance of the AdS$_5 \times S_5$ solution, and thus are similar but not identical to the solution we obtain in the $N_f = 4$ massless flavors case.

2. $\mathcal{N} = 2$ Super Yang-Mills Theory with gauge group $SU(2)$ and $N_f = 4$ Massless Multiplets

In this section we begin by collecting the known field theory results on the low energy effective action of $\mathcal{N} = 2$ super Yang-Mills theory with gauge group $SU(2)$ and $N_f = 4$ massless matter multiplets. These are then compared to the result obtained from studying a probe threebrane in supergravity.
2.1. Field Theory Results

The perturbative beta function for $\mathcal{N} = 2$ super Yang-Mills theory with gauge group $SU(N_c)$ and $N_f$ flavor hypermultiplets is proportional to $(2N_c - N_f)$. Thus, for $N_c = 2$ and $N_f = 4$, the perturbative beta function vanishes. If in addition all of the flavors of matter are massless, we obtain a finite conformally invariant theory\textsuperscript{[13]}. The exact effective coupling of the theory has the form

$$\tau = \tau_1 + i\tau_2 = \tau_{cl} + \frac{i}{\pi} \sum_{n=0,2,4,...} c_n e^{in\pi\tau_{cl}} = \tau_{cl} + \frac{i}{\pi} \sum_{n=0,2,4,...} c_n q^n,$$

where $\tau_{cl}$ is the classical coupling of the theory. The coefficient $c_0$ is a one loop perturbative correction, which in the Pauli-Villars scheme, has the value $c_0 = 4 \log(2)$\textsuperscript{[14]}. The coefficients $c_n$ with $n > 0$ and even come from nonperturbative (instanton) effects. The two instanton coefficient has been computed and has the value $c_2 = -7/(2^63^5)$. The leading contribution to the low energy effective action comprises all terms with the equivalent of two derivatives or four fermions\textsuperscript{[15]} and is determined in terms of the effective coupling. The next-to-leading contribution to the low energy effective action contains all terms with the equivalent of four derivatives or eight fermions\textsuperscript{[15]}. Using the scale invariance and $U(1)_R$ symmetry of the model, Dine and Seiberg argued that the four derivative term is one loop exact\textsuperscript{[10]}. In\textsuperscript{[17]} the vanishing of instanton corrections to the four derivative terms was explicitly verified and a rigorous proof of this non-renormalization theorem has recently been given in\textsuperscript{[18]}. The one loop contribution to the four derivative terms has been considered in\textsuperscript{[19]}. The result for the low energy effective action, up to and including four derivative terms, in $\mathcal{N} = 2$ superspace, is given by

$$8\pi (S^{(2)}_{eff} + S^{(4)}_{eff}) = \Im \int d^4xd^4\theta \left(\frac{1}{2} A^2\right) + \frac{3}{128\pi^2} \int d^4xd^4\theta d^4\bar{\theta} \log A \log \bar{A},$$

(2.1)

where $A$ is an $\mathcal{N} = 2$ Abelian chiral superfield. The number of terms that contribute to the low energy effective action at each order, for six derivative terms or higher, increases rapidly and a direct approach to these terms is not feasible. An elegant approach to study these terms has been developed in\textsuperscript{[6]} for $\mathcal{N} = 4$ super Yang-Mills\textsuperscript{[20]}, based on the conjectured $SL(2, Z)$ duality of the theory. This duality was used to fix the form of the effective action up to six derivatives. The theory that we are studying is also believed to have an exact $SL(2, Z)$ duality\textsuperscript{[13]}, and under this assumption the analysis of\textsuperscript{[6]} applies.
The unique $SL(2, Z)$ invariant form for the six derivative terms is

$$8\pi S^6_{\text{eff}} = \left(\frac{3}{128\pi^2}\right)^2 \lambda^{(6)} \int d^4 x d^4 \theta d^4 \bar{\theta} \left(\frac{1}{\sqrt{\tau}} \frac{D_{\alpha} D_{\alpha} \log(A) D_{\alpha} D_{\alpha} \log(A)}{A}\right) \frac{D_{\hat{\alpha}1} D_{\hat{\alpha}1} D_{\hat{\beta}2} D_{\hat{\beta}2} \log \tilde{A}}{\tau A^2}$$

$$+ \frac{i}{2} \left(\frac{3}{128\pi^2}\right)^2 \int d^4 x d^4 \theta d^4 \bar{\theta} \left(\log(A) \frac{D_{\alpha} D_{\alpha} \log(A)}{\tau A^2} \right) $$

$$- \log(A) \frac{D_{\alpha} D_{\alpha} \log(A)}{\tau A^2},$$

(2.2)

Under duality, the second term above mixes with the two and four derivative terms and consequently its coefficient is fixed. The requirement of self duality does not fix $\lambda^{(6)}$, since duality maps this term into itself at lowest order. These are the field theory results that we wish to compare to gravity.

On the gravity side, we will consider a probe moving in a background to be specified below. The probe worldvolume dynamics is captured by a Born-Infeld action. The Born-Infeld action itself is self-dual, but the duality does not act on the separation of the branes. This separation is parametrized by the Higgs fields which belong to the same supermultiplet as the gauge fields. This implies, as pointed out in [6], that the Higgs fields that realize $\mathcal{N} = 2$ supersymmetry linearly must be related by a nonlinear gauge field dependent redefinition to the separation. It is interesting to note that a similar field redefinition is needed to map the linear realization of conformal symmetry in super Yang-Mills theory into the isometry of the Anti de-Sitter spacetime of the supergravity description [4]. We refer the reader to [3] for the detailed form of the field redefinitions. The result after performing the field redefinitions, in terms of component fields, reads

$$S_{\text{eff}} = \int d^4 x \left( -\frac{1}{4g^2} \partial_m \bar{\varphi} \partial^m \varphi - \frac{1}{8g^2} (F_{\alpha\beta} F^{\alpha\beta} + \bar{F}_{\alpha\beta} \bar{F}^{\alpha\beta}) \right) + \left(\frac{3}{128\pi^2}\right)^2 \times$$

$$\times \frac{1}{32\pi} \frac{F_{\alpha\beta} F^{\alpha\beta} \bar{F}_{\hat{\beta}\hat{\alpha}} \bar{F}^{\hat{\beta}\hat{\alpha}} + (\partial_m \bar{\varphi} \partial^m \varphi)(\partial_n \bar{\varphi} \partial^n \varphi) - F^{\beta\alpha} \partial_m \varphi \sigma^m \alpha \beta \bar{F}^{\hat{\beta}\hat{\alpha}} \partial_n \bar{\varphi} \sigma^n \beta \hat{\alpha}}{\varphi^2 \bar{\varphi}^2}$$

$$- \frac{g^2}{256\pi^2} \left(\frac{3}{128\pi^2}\right)^2 \frac{2 F_{\alpha\beta} F^{\alpha\beta} \bar{F}_{\hat{\beta}\hat{\alpha}} \bar{F}^{\hat{\beta}\hat{\alpha}} (F^{\rho\sigma} F_{\rho\sigma} + \bar{F}^{\rho\sigma} \bar{F}_{\rho\sigma})}{\varphi^4 \bar{\varphi}^4},$$

(2.3)

where we have set $\tau = i \frac{\sqrt{2}}{g^2}$. The six derivative terms for the scalars are not displayed since they depend on the arbitrary constant $\lambda^{(6)}$. The value of $\lambda^{(6)}$ as well as the structure of the effective action given above can be checked by explicitly computing instanton corrections to the six derivative terms. We hope to return to this in the near future [21]. Notice that all acceleration terms were eliminated by the field redefinition, something first noted in [5].
2.2. Supergravity Results

The supergravity background relevant for the study of $\mathcal{N} = 2$ supersymmetric field theory is generated by sevenbranes and a large number of threebranes\cite{1}, i.e. threebranes in F theory\cite{1}. To construct this background it is convenient to start with a solution for the sevenbranes by themselves\cite{1}. The sevenbrane solution is described in terms of non-zero metric, dilaton and axion fields. As usual, the metric and axion are combined into a single complex coupling $\tau = \chi + ie^{-\phi} = \tau_1 + i\tau_2$. The coupling $\tau$ is identified with the modular parameter of the elliptic fiber of the F theory compactification. The $(8,9)$ plane is taken to be orthogonal to the sevenbranes. In terms of the complex coordinate $z = x^8 + ix^9$ we make the following ansatz for the metric

$$ds^2 = e^{\varphi(z,\bar{z})}dzd\bar{z} + (dx^7)^2 + ... + (dx^1)^2 - (dx^0)^2. \quad (2.4)$$

The parameter $z$ is to be identified with the Higgs field appearing in the low energy effective action of the $\mathcal{N} = 2$ field theory. With this ansatz, the type IIB supergravity equations of motion reduce to\cite{23}

$$\partial\bar{\partial}\tau = \frac{2\partial\tau\bar{\partial}\bar{\tau}}{\tau - \bar{\tau}}, \quad (2.5)$$

The complex coupling $\tau$ is identified with the low energy effective coupling of the $\mathcal{N} = 2$ field theory. Supersymmetry constrains the effective coupling of the field theory to be a function of $z$, so that the first equation in (2.5) is automatically satisfied. The general solution to the second equation in (2.5) is

$$\varphi(z,\bar{z}) = \log(\tau_2) + F(z) + \bar{F}(\bar{z}). \quad (2.6)$$

The functions $F(z)$ and $\bar{F}(\bar{z})$ should be chosen in order that (2.4) yields a sensible metric. For the case that we are considering, the explicit form for the metric transverse to the sevenbranes is

$$ds^2 = e^{\varphi(z,\bar{z})}dzd\bar{z} = \tau_2|da|^2. \quad (2.7)$$

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\footnote{Supergravity backgrounds corresponding to $\mathcal{N} = 1$ field theories have been considered in\cite{22}.}
where $a$ is the quantity that appears in the Seiberg-Witten solution\[4\]. This specifies the solution for the sevenbranes by themselves.

Next following \[4\], we introduce threebranes into the problem\[3\]. The presence of the threebranes modifies the metric and switches on a non-zero flux for the self dual RR five-form field strength. The world volume coordinates of the threebranes are $x^0, x^1, x^2, x^3$. One obtains a valid solution\[4\] by making the following ansatz for the metric

$$ds^2 = f^{-1/2}dx^2 + f^{1/2}g_{ij}dx^idx^j$$  \hspace{1cm} (2.8)

and the following ansatz for the self-dual 5-form field strength

$$F_{0123i} = -\frac{1}{4}\partial_i f^{-1}.$$  \hspace{1cm} (2.9)

The complex field $\tau$ is unchanged by the introduction of the threebranes. Inserting the above ansatz into the IIB supergravity equations of motion, one finds that $f$ satisfies the following equation of motion

$$\frac{1}{\sqrt{g}}\partial_i(\sqrt{g}g^{ij}\partial_j f) = -(2\pi)^4 N \delta^6(x-x^0) \sqrt{g}.$$  \hspace{1cm} (2.10)

This last equation corresponds to the case in which all of the three branes are located at the same point. In the limit that $N \to \infty$ the curvature becomes small almost everywhere and the supergravity solution can be used to reliably compute quantities in the field theory limit as explained in\[4\].

In the case of $N_f = 4$ massless hypermultiplets, (2.10) is explicitly given by

$$\left[\tau_2 \partial_y^2 + 4\partial_a \partial_{\bar{a}}\right] f = -(2\pi)^4 N \delta^{(4)}(y)\delta^{(2)}(a).$$  \hspace{1cm} (2.11)

The solution is given by\[8\]

$$f = \frac{4N\pi}{[y^2 + \tau_2|a|^2]^2}.$$

To reproduce the low energy effective action of the field theory, we now consider the dynamics of a threebrane probe moving in this geometry. It is well known that the probe

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2 See also \[24\] where this solution was independently discovered.

3 It is well known\[25\] the case of constant $\tau$ corresponds to a IIB orientifold background. Threebranes in a IIB orientifold background were first considered in \[24\].
has a low energy effective action which matches that of the corresponding low energy field theories. Here we are interested in checking the form predicted by the probe for the higher order corrections. The leading low energy effective action plus corrections for the bosons in the background described above, is obtained by expanding the self-dual action:

$$S = \frac{T_3}{2} \int d^4x \left[ \sqrt{\text{det}(G_{mn} + e^{-\frac{1}{2}\phi}F_{mn})} + \chi F \wedge F \right].$$  \hspace{1cm} (2.12)

$T_3$ has no dependence on the string coupling constant. We obtain for the scalar terms obtained from the expansion of (2.12) after setting $y = 0$:

$$S \sim \frac{1}{2} \int d^4x \left( \tau_2 \partial_m a \partial^n \bar{a} \right) - \frac{f}{2} \tau_2^2 (\partial_m a \partial^m a) (\partial_n \bar{a} \partial^n \bar{a}) + \ldots$$

$$= \frac{1}{2} \int d^4x \left( \tau_2 \partial_m a \partial^n \bar{a} \right) - \frac{2N\pi}{(a\bar{a})^2} (\partial^n a \partial_m a) (\partial^n \bar{a} \partial_m \bar{a}) + \ldots$$

$$+ \frac{8\pi^2N^2}{\tau_2(a\bar{a})^4} (\partial_p a \partial^n \bar{a}) (\partial^n a \partial_p a) (\partial^n \bar{a} \partial_n \bar{a}) + \ldots \right).$$  \hspace{1cm} (2.13)

Notice that each term in this action comes multiplied by a different power of $N$. As things stand, the $2n$ derivative term will come with a coefficient of $\tau_2^n f^{n-1} \sim N^{n-1}$. The full effective action for the probe interacting with $N$ coincident source threebranes, should come with an overall factor of $N$. This is achieved by noting that the coupling of the background, $\tau_2$ should be identified with

$$\tau_2 \equiv \frac{N\tau_{SW}}{\lambda} = \frac{1}{g_s},$$  \hspace{1cm} (2.14)

where $\tau_{SW}$ is the Seiberg-Witten effective coupling for the field theory of interest. With this identification, the string coupling is $O(\frac{1}{N})$ and explicitly goes to zero as $N \to \infty$. Notice also that $Ng_s = N g^2_{YM} \sim \lambda$ so that the large 't Hooft coupling limit corresponds to large $\lambda$. After making the $N$ dependence of $\tau_2$ explicit, we find that the probe action is indeed proportional to $N$. We will not always show this dependence explicitly in what follows. Following [6], we find the Taylor expansion of (2.12) exactly matches the super Yang-Mills effective action (2.3) after identifying

$$a\bar{a} = \frac{1}{T_3\lambda}\phi\bar{\phi}, \quad (F_{s,\alpha\beta}F_{s}^{\alpha\beta}) = \frac{1}{4T_3\lambda}(F_{f,\alpha\beta}F_{f}^{\alpha\beta}).$$
where $F_{s,\alpha\beta}$ is the field strength appearing on the probe worldvolume and $F_{f,\alpha\beta}$ is the field strength of the field theory. Note that $\tau^2$ appearing in (2.13) is the classical coupling plus all instanton corrections. The fact that the four derivative terms are independent of $\tau^2$ shows that the supergravity result explicitly reproduces the nonrenormalization theorem for these terms\cite{30}.

3. $\mathcal{N} = 2$ Super Yang-Mills Theory with gauge group $SU(2)$ and $N_f = 4$ Massive Multiplets

In this section we consider the supergravity background corresponding to the case where all flavor multiplets of the field theory on the probe worldvolume have a mass. In this case, both the effective coupling and the four derivative terms get contributions from instantons. We are able to show that the supergravity solution is capable of producing what is expected for the one instanton correction. We are not however able to fix the coefficient of this correction. The dilaton of the supergravity solution is no longer a constant and there are corrections to the AdS$_5$ geometry reflecting the fact that the field theory is no longer conformally invariant. We compute the quark-antiquark potential and show that its form is remarkably similar to that for a quark-antiquark pair in the $\mathcal{N} = 4$ theory at finite temperature.

3.1. Field Theory Results

The masses of the quark flavors breaks the conformal invariance that is present in massless theory. In this case, the effective coupling does pick up a dependence on the energy scale as a result of instanton corrections. At high enough energies we expect these corrections can be neglected and the theory flows to the conformal field theory corresponding to the case of massless flavors. Indeed, the perturbative beta function still vanishes and the coupling goes to a constant at high energies. We will focus attention on the two and four derivative terms appearing in the low energy effective action. These terms are completely specified by a holomorphic prepotential $\mathcal{F}$ and a real function $\mathcal{H}$

$$S_{\text{eff}} = \frac{1}{2i} \int d^4x \left( \int d^4\theta \mathcal{F}(A) - \int d^4\bar{\theta} \bar{\mathcal{F}}(\bar{A}) \right) + \int d^4x \int d^4\theta d^4\bar{\theta} \mathcal{H}(A, \bar{A}).$$

In what follows, we will only account for the one instanton corrections to both the prepotential and the four derivative terms. The prepotential does not receive any loop corrections
for $N_f = 4$. The one instanton correction to the prepotential was computed in [31]. The one instanton corrected prepotential is

$$F = \frac{1}{2} \tau_{cl} A^2 - \frac{i \tau_{cl} q}{2\pi A^2} m_1 m_2 m_3 m_4.$$  

This corresponds to a low energy effective coupling

$$\tau = \tau_{cl} - \frac{3iq\tau_{cl}}{\pi \varphi^4} m_1 m_2 m_3 m_4$$  

The one loop correction to the real function $H$ is [19]

$$H = \frac{3}{256\pi^2} \log^2 \left( \frac{A\bar{A} \langle A \rangle \langle \bar{A} \rangle}{\langle A \bar{A} \rangle} \right),$$  

and the one instanton correction is given by

$$H(\varphi, \bar{\varphi}) = \frac{-qm_1 m_2 m_3 m_4}{8\pi^2 \varphi^4} \log \varphi.$$  

The one anti-instanton contribution is given by the complex conjugate of the one instanton correction. The pure scalar two and four derivative terms appearing in the low energy effective action, after performing the field redefinition needed to compare to the brane result, are easily obtained by using the formulas quoted in [8],[30]. The results are

$$S = \int d^4x \left( K_{\varphi\bar{\varphi}} \partial_\mu \varphi \partial^\mu \varphi + \tilde{H}_{\varphi\bar{\varphi}\varphi\bar{\varphi}} (\partial^m \varphi) (\partial_m \varphi) (\partial^n \varphi) (\partial_n \varphi) \right),$$  

where

$$K_{\varphi\bar{\varphi}} \equiv \text{Im} \left( \frac{\partial^2 F}{\partial \varphi^2} \right) = \tau_2 = \frac{4\pi^2}{g_{cl}^2} - \frac{6\pi}{g_{cl}^2} m_1 m_2 m_3 m_4 \left[ \frac{q}{a^4} + \frac{q}{\bar{a}^4} \right]$$  

and

$$\tilde{H}_{\varphi\bar{\varphi}\varphi\bar{\varphi}} = 16 \left( \frac{\partial^4 H}{\partial \varphi \partial \varphi \partial \bar{\varphi} \partial \bar{\varphi}} - \frac{\partial^3 H}{\partial \varphi \partial \varphi \partial \bar{\varphi}} (K_{\varphi\bar{\varphi}})^{-1} \frac{\partial K_{\varphi\bar{\varphi}}}{\partial \varphi} - \frac{\partial K_{\varphi\bar{\varphi}}}{\partial \bar{\varphi}} (K_{\varphi\bar{\varphi}})^{-1} \frac{\partial^3 H}{\partial \varphi \partial \varphi \partial \bar{\varphi}} \right)$$

$$+ 2 \frac{\partial K_{\varphi\bar{\varphi}}}{\partial \varphi} (K_{\varphi\bar{\varphi}})^{-1} \frac{\partial^2 H}{\partial \varphi \partial \bar{\varphi}} (K_{\varphi\bar{\varphi}})^{-1} \frac{\partial K_{\varphi\bar{\varphi}}}{\partial \bar{\varphi}}$$

$$= \frac{3}{8\pi^2} \frac{1}{\varphi^2 \bar{\varphi}^2} + \frac{40m_1 m_2 m_3 m_4}{\pi^2} \left[ \frac{q}{\varphi^2 \bar{\varphi}^2} + \frac{q}{\bar{\varphi}^2 \varphi^2} \right].$$  

Notice that for large $\varphi$ the fall off of the four derivative terms is like $|\varphi|^{-4}$. This has an interesting supergravity interpretation.
3.2. Supergravity Results

The first step in the supergravity analysis entails solving (2.10) for the background geometry, with the complex coupling $\tau$ given in (3.1). The coupling $\tau$ is only valid for large $|\varphi|$. For small $|\varphi|$ higher instanton corrections cannot be neglected. For this reason, we will construct a solution to (2.10) which is valid for large $|\varphi|$. Towards this end, split $\tau_2$ into two pieces as follows

$$\tau_2 = V_1 - V_2, \quad V_1 = \frac{4\pi^2}{g_{cl}^2} \equiv \tau_{2cl}, \quad V_2 = \frac{3\tau_{2cl}}{2\pi} m_1 m_2 m_3 m_4 \left[ \frac{q}{a^4} + \frac{\bar{q}}{\bar{a}^4} \right].$$

We can now solve (2.10) perturbatively by writing $f = f_0 + f_1 + ...$ where

$$\left[ V_1 \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial a \partial \bar{a}} \right] f_0 = -N(2\pi)^4 \delta^{(4)}(y) \delta^{(2)}(a), \quad (3.3)$$

$$\left[ V_1 \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial a \partial \bar{a}} \right] f_n = V_2 \frac{\partial^2}{\partial y^2} f_{n-1}. \quad (3.4)$$

To find the leading corrections to the four derivative terms, it is sufficient to focus attention on $f_0$ and $f_1$. The solution for $f_0$ is

$$f_0 = \frac{4N\pi}{[y^2 + \tau_{2cl}|a|^2]^2}.$$

The function $f_1$ satisfies

$$\left[ \tau_{2cl} \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial a \partial \bar{a}} \right] f_1 = \frac{3m_1 m_2 m_3 m_4 \tau_{2cl}}{2\pi} \left( \frac{q}{a^4} + \frac{\bar{q}}{\bar{a}^4} \right) \left( -\frac{16N\pi}{[y^2 + \tau_{2cl} a \bar{a}]^3} + \frac{96N\pi y^2}{[y^2 + \tau_{2cl} a \bar{a}]^4} \right).$$

We will look for solutions to this equation that preserve rotational symmetry in the $y_i$ variables. To do this it is useful to move into radial coordinates. Denoting the angular variable in the $a, \bar{a}$ plane by $\theta$ and the radial coordinate in the $a, \bar{a}$ plane by $r$ and in the $y_i$ plane by $\rho$, we find

$$\left[ \tau_{2cl} \frac{\partial^2}{\partial \rho^2} + \tau_{2cl} \frac{3}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} \right] f_1 = \frac{3m_1 m_2 m_3 m_4 \tau_{2cl}}{\pi} \cos(4\theta) \left( -\frac{16N\pi}{[\rho^2 + \tau_{2cl} r^2]^{3/2}} + \frac{96N\pi \rho^2}{[\rho^2 + \tau_{2cl} r^2]^2} \right).$$

\footnote{The factors $q$ and $\bar{q}$ appearing in $V_2$ are pure phases and can be absorbed into a convenient choice for $\theta = 0$.}
By inspection, it is clear that the angular dependence of \( f_1 \) is given by 
\[
\cos(4\theta)g(r, \rho).
\]
The function \( g \) satisfies
\[
\left[ \tau_2^{(0)} \frac{\partial^2}{\partial \rho^2} + \tau_2^{(0)} \frac{3}{\rho} \frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} - \frac{16}{r^2} \right] g
= \frac{3m_1m_2m_3m_4\tau_2^{(2)}}{\pi} \frac{1}{r^4} \left( -\frac{16N\pi}{[\rho^2 + \tau_2^{(0)} r^2]^3} + \frac{96N\pi\rho^2}{[\rho^2 + \tau_2^{(0)} r^2]^4} \right).
\]
(3.5)

This equation admits a power series solution. To set up the solution, notice that both \( r \) and \( \rho \) have the dimensions of length \((L)\). It is not difficult to see that \( g \) has dimension \( L^{-8} \). Thus, by dimensional analysis, it must have an expansion of the form
\[
g = \sum_m c_m r^m \rho^{8-m}.
\]
(3.6)

For consistency, we require that \( g \to 0 \) at least as \( r^{-4} \) as \( r \to \infty \). If this is not the case, \( f_1 \) is not a small correction to \( f_0 \). Thus, we restrict \( m \geq 4 \) in (3.6). With this restriction, after inserting (3.6) into (3.5), one finds for the first few \( c_m \):
\[
c_m = 0, \quad m < 8, \quad c_8 = \alpha m_1m_2m_3m_4N, \\
c_9 = 0, \quad c_{10} = -6m_1m_2m_3m_4N \left[ \frac{1 + \alpha}{(\tau_2^{cl})^3} \right].
\]

The full solution is not needed, since only \( f_1 \) at \( y = 0 \) enters the probe action. Notice that this solution for \( f_1 \) is labeled by an arbitrary parameter \( \alpha \) which cannot be fixed by the above iterative calculation. It is an interesting open question to see if \( \alpha \) can be fixed by a more sophisticated analysis [21]. The correction to the leading term in \( f \)
\[
f(y = 0, a, \bar{a}) = \frac{4N\pi}{(\tau_2^{cl}a\bar{a})^2} + \alpha m_1m_2m_3m_4N \frac{1}{2(a\bar{a})^2} \left( \frac{q}{a^4} + \frac{\bar{q}}{\bar{a}^4} \right) + O\left( \frac{1}{|a|^{12}} \right)
\]
represents a correction to the AdS\(_5\) geometry. This correction is expected because we are no longer dealing with a conformal field theory. Notice that the AdS\(_5\) geometry is recovered in the limit of large energies \((|a| \to \infty)\) and in the limit of massless matter \( m_i \to 0 \). Expanding the probe action in this background, we find that the pure scalar terms read
\[ S = \frac{T_3}{2} \int \left[ \left( \frac{\tau_{2cl}}{2\pi} - \frac{3\tau_{2cl}}{2\pi} m_1 m_2 m_3 m_4 \left( \frac{q}{a^4} + \frac{\bar{q}}{a^4} \right) \right) \partial_m a \partial^m \bar{a} 
+ \left( 2N \pi \right) \left( \frac{\alpha N m_1 m_2 m_3 m_4 (\tau_{2cl})^2}{4(a\bar{a})^2} \left( \frac{q}{a^4} + \frac{\bar{q}}{a^4} \right) \right) \partial_n a \partial^n a \partial_m \bar{a} \partial^m \bar{a} \right]. \]

Notice that the correction to the four derivative terms has the structure of the one instanton corrections computed using field theory. We have not been able to fix \( \alpha \) with our asymptotic analysis, so that the coefficient of this correction could not be checked. As reviewed above, instanton effects explicitly break the conformal symmetry of the field theory. The breaking of the \( SO(2,4) \) conformal symmetry in the field theory is reflected in the corrections to the \( \text{AdS}_5 \) geometry, which break the \( SO(2,4) \) isometry of the \( \text{AdS}_5 \) space. Note that the coupling runs with a power law. Solutions of type-0 string theories with a power law running for the coupling have been studied in [32]. Power law running of the coupling has also played a prominent role in gauge-coupling unification in theories with large internal dimensions [33]. The presence of the large internal dimensions is reflected in the fact that massive Kaluza-Klein modes run in loops of the four dimensional theory. This gives rise to a power law running of the couplings. In [11] it was suggested that this effect may be responsible for the power law running of couplings in the IIB background discussed in that study. In our case, there is no need for effects due to large internal dimensions and the power law running of the coupling is simply explained by instanton effects in the four dimensional field theory.

Before leaving this section we would like to make some comments on the supergravity interpretation of the leading \( |a|^{-4} \) behaviour of the four derivative terms. The operator on the worldvolume which couples to the dilaton is given by [34]
\[ \mathcal{O}_\phi = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu}. \]

By expanding the Born-Infeld action one finds a four derivative term in the effective potential that has the form
\[ \mathcal{O}_\phi \mathcal{O}_\phi \left[ \frac{2N \pi}{(a\bar{a})^2} + \frac{\alpha N m_1 m_2 m_3 m_4 (\tau_{2cl})^2}{4(a\bar{a})^2} \left( \frac{q}{a^4} + \frac{\bar{q}}{a^4} \right) \right]. \]

The leading term of \( |a|^{-4} \) comes from the static massless propagator in the six dimensional transverse space. This term is due to exchange of a dilaton and appears because the supergravity modes which couple to constant gauge fields on the brane have zero momentum along the brane [34].
3.3. Instanton Effects and the static Quark-Antiquark Potential from Supergravity

In this section we study the static quark-antiquark potential, in the large $|\varphi|$ region where the background geometry described above is valid. This corresponds to studying the effects of instantons on the static quark-antiquark potential in the large $N$ field theory. There is a large body of evidence from lattice calculations that indicate that instanton effects play a major role in the physics of light hadrons\cite{35}. Below we will argue that the supergravity description provides a powerful new approach to these questions.

The energy of a quark-antiquark pair can be read off of the expectation value of a Wilson loop. This Wilson loop is identified with a fundamental string ending on the boundary of the asymptotically AdS$_5$ space\cite{36}. The Wilson loop configuration is thus obtained by minimizing the Nambu-Goto action\cite{5}

\[ S = \frac{1}{2\pi} \int d\tau d\sigma \sqrt{\det(g_{MN}\partial_\alpha x^M \partial_\beta x^N)} \]

The metric $g_{MN}$ felt by the strings is not the Einstein metric (2.8), but rather the string frame metric. We are interested in a static string configuration and take $\sigma = x^1$ and $\tau = x^0$. The string is at a fixed $x^2, x^3, x^4, x^5, x^6, x^7$ and $\theta$ where $\theta$ is the angular variable in the $(8,9)$ plane. In terms of the variable $r^2 \equiv a\bar{a}$, the Nambu-Goto action takes the form

\[ S = \frac{T}{2\pi} \int d\sigma \sqrt{(\partial_\sigma r)^2 + \frac{1}{f\tau_2}} = \frac{T}{2\pi} \int d\sigma \sqrt{(\partial_\sigma r)^2 + \frac{r^4}{a} - \frac{b}{a^2}} \]

\[ a = \frac{4N\pi}{\tau_{2cl}}, \quad b = \cos(4\theta)m_1m_2m_3m_4N\left(\frac{\alpha\tau_{2cl}}{2} - \frac{6}{\tau_{2cl}}\right) \]

where $T = \int d\tau$. We have dropped terms of $O(r^{-4})$ in the square root above. The solution to the Euler-Lagrange equations of motion following from this action is obtained in the usual way: The action does not depend explicitly on $\sigma$ so that the Hamiltonian in the $\sigma$ direction is a constant of the motion

\[ \frac{r^4/a - b/a^2}{\sqrt{(\partial_\sigma r)^2 + r^4/a - b/a^2}} = \text{const} = \sqrt{r^4_0/a - b/a^2} \]

\[ ^5 \text{In this expression } \alpha, \beta = \tau, \sigma, \text{ and } M, N = 0, 1, ..., 9 \]
where $r_0$ is the minimal value of $r$. By symmetry we have $r(\sigma = 0) = r_0$. It is now straightforward to obtain

$$\sigma = \sqrt{\frac{r_0^4}{a} - \frac{b}{a^2}} \int_{r_0}^{r} dy \frac{dy}{\sqrt{(\frac{y^4}{a} - \frac{b}{a^2})(\frac{y^4}{a} + \frac{r_0^4}{a})}}.$$

The string endpoints are at the boundary of the asymptotically AdS$_5$ space ($r \to \infty$) so that we can trade the integration constant $r_0$ for the distance $L$ between the quark and anti-quark

$$L = 2\sqrt{\frac{r_0^4}{a} - \frac{b}{a^2}} \int_{r_0}^{\infty} dy \frac{dy}{\sqrt{(\frac{y^4}{a} - \frac{b}{a^2})(\frac{y^4}{a} + \frac{r_0^4}{a})}}.$$

The energy is now computed by evaluating our action at this classical solution. After subtracting twice the self-energy of a quark, we obtain the following result for the quark-antiquark potential

$$E = \frac{1}{\pi \sqrt{(\frac{r_0^4}{a} - \frac{b}{a^2})}} \int_{r_0}^{\infty} dy \left( \frac{\sqrt{(\frac{y^4}{a} - \frac{b}{a})}}{\sqrt{y^4 + r_0^4}} - 1 \right).$$

To extract the dependence of this energy on the quark-antiquark separation $L$ we need to determine $r_0$ as a function of $L$. The expression for the energy given above is identical to the static potential for the quark-antiquark pair in the $\mathcal{N} = 4$ theory at finite temperature [37]. From the results of [37] we know that $E(L)$ has the form

$$E = -\frac{c_1}{L} - c_2 L^3.$$

The constant $c_1$ is positive. In the limit that $m_i \to 0$, $c_2 \to 0$ and we regain the $\frac{1}{L}$ dependence, a fact which is determined by conformal invariance. The sign of the constant $c_2$ is dependent on $\theta$. For $c_2$ positive (negative) we have a screening (antiscreening) of the quark-antiquark pair due to the instantons. This expression can’t be trusted for very large $L$: for larger and larger $L$ the Wilson loop is able to move further and further into the bulk. Our solution is however only valid for large $r$, so that the Wilson loop begins to explore regions in the bulk for which our solution is not valid. The long distance behaviour of the quark-antiquark potential could be extracted from the exact supergravity background.
4. Pure Gauge $\mathcal{N} = 2$ Super Yang-Mills Theory with Gauge Group $SU(2)$

In this section we obtain the leading correction to the four derivative terms in both the field theory and the supergravity descriptions.

4.1. Field Theory Results

In the case where there are no flavor multiplets, the perturbative beta function does not vanish and the field theory is asymptotically free. We will focus attention on the perturbative contributions to the two and four derivative terms appearing in the low energy effective action. The one loop results for the Kähler metric and real function $H$ are

$$K_{\varphi \bar{\varphi}} \sim \frac{\log(\varphi \bar{\varphi}/\Lambda^2)}{\varphi \bar{\varphi}},$$

$$H(A, \bar{A}) \sim \log \left(\frac{A}{\Lambda}\right) \log \left(\frac{\bar{A}}{\Lambda}\right).$$

This leads to the following four derivative term for the scalars, after performing the field redefinition $^8$

$$S = \int d^4 x (\partial^m \varphi \partial_m \varphi)(\partial^n \bar{\varphi} \partial_n \bar{\varphi}) \frac{8 + 4 \log \left(\frac{\varphi \bar{\varphi}}{\Lambda^2}\right) + \left[\log \left(\frac{\varphi \bar{\varphi}}{\Lambda^2}\right)\right]^2}{\varphi^2 \bar{\varphi}^2 \left[\log \left(\frac{\varphi \bar{\varphi}}{\Lambda^2}\right)\right]^2} \left[\frac{1}{(\varphi \bar{\varphi})^2} + O\left(\frac{1}{(\varphi \bar{\varphi})^2 \log |\varphi|}\right)\right].$$

The $|\varphi|^{-4}$ fall off at large separations (large $|\varphi|$) suggests that the dominant interaction between the branes is again due to the exchange of massless supergravity modes propagating in the six dimensional space transverse to the three branes.

4.2. Supergravity Results

The problem of finding the relevant background geometry corresponding to the asymptotically free gauge theory is considerably more complicated. The Laplace equation (2.10) becomes (we will show all $N$ dependence in this section)

$$\left[\frac{N}{\lambda} \left(\frac{8\pi}{g_{cl}^2} + \frac{6}{\pi} + \frac{2}{\pi} \log \left(\frac{a\bar{a}}{\Lambda^2}\right)\right) \frac{\partial^2}{\partial y^2} + \frac{\partial}{\partial a} \frac{\partial}{\partial \bar{a}}\right] f = -(2\pi)^4 N \delta^{(4)}(y) \delta^{(2)}(a).$$

Performing a Fourier transform on the $y$ variables and working in the large $|a|$ region (which corresponds to the semi-classical regime of the field theory) we find
\[
\left[ \frac{\partial}{\partial a} \frac{\partial}{\partial \bar{a}} - k^2 N \frac{8\pi}{\lambda} \frac{6}{\pi} + \frac{2}{\pi} \log \left( \frac{a\bar{a}}{\Lambda^2} \right) \right] f = 0.
\]

We have not been able to solve this equation exactly. However an approximate solution in the large \(|a|\) region is given by

\[ f \approx Ne^{-2k\sqrt{N}a}/(\sqrt{2\pi} + \frac{1}{2} \log(|a|)). \]

Using this approximate solution we obtain

\[ f(y = 0, a, \bar{a}) = \int d^4 k \frac{12\pi^2 \lambda^2}{16N(a\bar{a})^2 \left( \log \left[ \frac{\sqrt{N}}{\lambda} \exp \left( \frac{2\pi^2 k^2}{g_{cl}^2} + \frac{1}{2} \right) \right] \right)^2}. \]

This result determines the coefficient of the four derivative terms

\[ \tau_2^2 f(y = 0, a, \bar{a}) \sim \frac{N}{(a\bar{a})^2} + O \left( \frac{1}{(a\bar{a})^2 \log |a|} \right). \]

This is exactly the same behaviour as obtained from the field theory analysis.

The complex coupling \(\tau\) in this supergravity background has a logarithmic dependence on \(a\bar{a}\) corresponding to the logarithmic dependence of the field theory coupling on the energy scale. Gravity solutions that have couplings with this logarithmic dependence have been constructed in type-0 theories[38].

We should now address the validity of this computation. There are two potential sources of corrections to the supergravity background - string loop effects and curvature corrections. At large \(N\) and large \('t\) Hooft coupling both of these types of corrections are small and supergravity is a reliable description of the background. As the \('t\) Hooft coupling decreases, curvature corrections become important and the uncorrected supergravity can no longer be trusted[39]. The uncorrected supergravity does not correctly describe the large \(N\) perturbative field theory. We would like to determine wheather the uncorrected supergravity is a valid description for the perturbative field theory living on the probe, which is a different question. The simplest way to assess the validity of the supergravity description is simply to compute the square of the Ricci tensor. This calculation should be carried out in string frame because we are interested in the region in which the dilaton is going to zero. We will show both the Einstein and string frame results in what follows.
In the large $|a|$ region the Einstein frame metric that we have computed above takes the form

$$g^{(e)}_{MN}dx^M dx^N = \frac{\sqrt{N}}{\lambda} \left( a\bar{a} \log |a| \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{a\bar{a} \log |a|} \left( \delta_{ij} dx^{i'} dx^{j'} + \lambda \log |a| |d\bar{a}d\bar{a}| \right) \right),$$

where we have rescaled $x^i \to x'^i$ where $x' = x\lambda/\sqrt{N}$. The corresponding string frame metric is

$$g^{(s)}_{MN}dx^M dx^N = \frac{1}{\sqrt{\lambda}} \left( a\bar{a}(\log |a|)^{1/2} \eta_{\mu\nu} dx^\mu dx^\nu + \frac{1}{a\bar{a}(\log |a|)^{3/2}} \left( \delta_{ij} dx^{i'} dx^{j'} + \log |a| |d\bar{a}d\bar{a}| \right) \right).$$

The leading contribution to the square of the Ricci tensor in the Einstein frame is a constant

$$R_{MN}R^{MN} = \frac{32}{N}.$$ 

In the string frame, the leading contribution to the square of the Ricci tensor diverges logarithmically for large $|a|

$$R_{MN}R^{MN} = \frac{32}{\lambda \log |a|}.$$ 

To interpret these results, note that the Yang-Mills coupling squared is $g^2 = \lambda(N \log |a|)^{-1}$, so that the 't Hooft coupling is $\lambda_T \equiv g^2 N = \lambda/\log |a|$. We see that the square of the Ricci tensor in the string frame is inversely proportional to the 't Hooft coupling, so that we recover the well known result that curvature effects in the background are small at large $\lambda_T$. The perturbative probe field theory is valid for $|a| >> 1$. It is clear that in the $N \to \infty$ limit, large $\lambda_T$ and large $|a|$ are compatible, i.e. in the large $N$ limit, the 't Hooft coupling is large even when the probe worldvolume field theory is perturbative. Moving to smaller $|a|$ one would need to correct the asymptotic solution for the background that we have found. The supergravity solution in this region captures the strong coupling dynamics of the asymptotically free gauge theory on the probe.

Finally, we note that the effects that we have computed in this section are linear in both the number of source branes and the number of probe branes. The supergravity will not capture effects which do not have this linear dependence.

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In what follows $\mu, \nu = 0, 1, 2, 3$; $i, j = 4, 5, 6, 7$ and $M, N = 0, 1, ..., 9$. 
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[39] For nice discussions of this see: E. Witten, *New Perspectives in the Quest for Unification*, hep-ph/9812208;
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