Abstract

This paper, for the first time, focuses on the sector-wise analysis of a stock market through multifractal analysis. We have considered Bombay Stock Exchange, India, and identified two time scales, short (< 200 days) and long time-scale (> 200 days) for investment. We infer that long-term investment will be more profitable. For long time scale, sectors can be separated into two categories based on the Hurst exponent values; one corresponds to stable sectors with small fluctuations, and the other with dominance of large fluctuations leading to possible downturns in those sectors.

Keywords: Multifractal detrended fluctuation analysis, Time series analysis, Hurst coefficient, Multifractal spectrum, Singularity strength, Financial market, Short and long-term scale

1. Introduction

Multifractal behavior can be perceived in various complex systems which also includes financial market. The study of multifractality of the time-series of stock prices of financial market has gained a lot of interest recently in order to understand the market behaviour. The multifractal behavior is identified from the presence of multiple Hurst exponents which are extracted from time series data by detrending the fluctuations. The presence of multifractal behavior has been studied and established as robust behavior of the financial time-series for wide variety of data with different frequencies viz. intraday closures, high frequency prices [1][2][3] etc. and for markets in different countries [2][4]. There are several existing methods of studying multifractality with different strengths and weaknesses, most common of which are Multifractal Detrended Fluctuation
Analysis (MFDDA), Multi-ractal Detrended Moving Average (MFDMA), Multi-fractal detrended cross-correlation analysis (MFDCA) etc. The multifractality of financial time-series has been tested for developed and developing countries by comparing the degree of multifractality. As the internal structure of the market is responsible for defining the state of the market, the multifractal nature of financial time series also provides an alternative way of studying the market stability and risks, correlations in stock prices, and signature of crashes, etc.

In this paper, we study sectoral indices of Bombay Stock Exchange (S&P BSE), India. The multifractal behavior of overall Indian stock market has been studied by S Dutta, Kumar et. al., Nargunam et. al and others. Multifractality being an inherent nature of any financial time series data, Indian financial market also does not deviate from it. To understand the behavior of stock market in detail, we look into the underlying structure within the Indian stock market. Instead of looking at the overall market behaviour, in this paper we have done a sector-wise study, which has not been done for any financial market till now. Though the overall Indian market has been categorized as a persistent developing market, we clearly can see two separate categories of sectors in terms of stability and risk assessment.

In Sec. 2, the detailed description of the financial time-series data has been given with the sectors considered for analysis. The algorithm of modified MFDDA has been used along with the overlapping moving window (OMW) and ensemble empirical mode decomposition (EEMD). OMW-EEMD-MFDDA method is used for analysis has been described in Sec. 3. The results are discussed in Sec. 4 and we conclude at Sec. 5 with the anticipation that this study will help the investors to make the strategy to choose right sectors and investment duration for higher chance of profits.

2. Data Description

For the time series analysis, we have collected daily closure prices of all the sectors’ indices of Bombay Stock Exchange (S&P BSE) from 2nd January 2017 to 30th June 2022 (1362 trading days) from the BSE India website. We consider only the sectors, 22 in total, for which data are available for the full period in the given date range. We do not have any missing data points in the considered data-set. Detailed information about the sectors is given in table 1.

3. Methodology

3.1. Logarithmic Return

We compute the logarithmic return of sector indices’ daily closure values with $\Delta t = 1$ day using the formula,

$$ r_i(t) = \ln P_i(t + \Delta t) - \ln P_i(t), \quad i = 1, 2, ..., k. $$

(1)
Table 1: Details of Bombay stock exchange (S&P BSE) sectors considered for the analysis are given here. The first column consists of the symbol of the sectors followed by the name of the corresponding sectors in the second column. The third column shows the total number of available stocks in each sector during the time period considered for analysis. For the rest of the paper, we are going to refer the sectors by their corresponding symbol name provided here.

| Symbol | Sector                                      | # of stocks |
|--------|---------------------------------------------|-------------|
| AU     | Auto                                        | 15          |
| BM     | Basic Materials                             | 189         |
| BX     | Bankex                                      | 10          |
| CD     | Consumer Durables                           | 12          |
| CDGS   | Consumer Discretionary Goods & Services     | 297         |
| CG     | Capital Goods                               | 25          |
| CPSE   | CPSE                                        | 52          |
| EG     | Energy                                      | 27          |
| FMCG   | Fast Moving Consumer Goods                  | 81          |
| FN     | Financials                                  | 139         |
| HC     | Healthcare                                  | 96          |
| ID     | Industrials                                 | 203         |
| II     | India Infrastructure                        | 30          |
| IT     | Information Technology                      | 62          |
| MT     | Metal                                       | 10          |
| ONG    | Oil & Gas                                   | 10          |
| PSU    | PSU                                         | 56          |
| PWR    | Power                                       | 11          |
| RE     | Realty                                      | 10          |
| TC     | Telecom                                     | 17          |
| Teck   | Teck                                        | 28          |
| UT     | Utilities                                   | 24          |

Here $r_i(t)$ represents the logarithmic return price of $i$-th sector at day $t$ with $t = 1, 2, ..., T - \Delta t$. $T$ and $k$ are the total number of trading days and sectors available in the considered time period, respectively.

3.2. **EEMD and OMW based Multifractal Detrended Fluctuation Algorithm (EEMD-OMW-MFDFA)**

Multifractal Detrended Fluctuation Algorithm (MF DFA) is a well-accepted and a hugely used method to find the presence of self-similarity and fractionality in any stochastic time series with different power scaling laws [5, 29, 30, 31, 32]. It can be used as an indicator to analyze the dynamics of data-set quantitatively as well as qualitatively. The model was first developed as a Fluctuation Analysis. But this method was incompatible in dealing with the time-series data with a long-term trend expected in real-world data analysis. Peng et al. [33] developed Detrended Fluctuation Analysis (DFA), where the data-set is detrended through polynomial fitting to consider the non-stationarity of the data, which in later years modified to a continuum of power variations of DFA, i.e., MF DFA by Kandelhardt et al. [31, 34]. DFA becomes a particular case of MF DFA with a specific choice of power parameter (power $q = 2$). Instead of using polynomial fitting to detrend the data, we have used ensemble empirical mode decomposition (EEMD) for the same [24, 25, 26]. As our data-set is not very long, we have considered overlapped moving windows (OMW), for a better
analysis. The overall modified algorithm of EEMD-OMW-MFDFA consists of five steps as follows:

- Normalize the data-set by subtracting the average value from each data point and computing the cumulative sum

\[ y(N) = \sum_{k=1}^{N} [x(k) - \bar{x}] \]  

(2)

where \( x(k) \) is the time series of length \( N \) at any point \( k \) and \( \bar{x} \) denotes the average over the entire data-set.

- As the second step, we consider a moving window of segment size \( s \) shifted by 1 data point as overlapped moving window (OMW) and replace the conventional equally divided non-overlapping segmentation to overcome the issue of poor statistics due to lesser number of segments for larger lags \( s \). Hence, total number of epoch will be \( N_s = N - s + 1 \) for time-series of length \( N \).

- In the next step, we replace the local polynomial fitting for detrending the time series by EMD method. Here, the data trends are computed by removing the Intrinsic Mode Functions (IMFs) obtained by EMD method to get stationary data. Conditions and steps included in the calculation of intrinsic mode functions are as follows.

  Conditions:
  1. Difference between the number of extremes and zero crossings should be one, at maximum.
  2. Local mean value of the envelope at any point, defined by the local extrema, must be zero. To get the best-fitted curve between the local extremes, the cubic splines method is considered with the above condition.

  Steps:
  1. Find an IMF which satisfies the conditions as mentioned above.
  2. Compute the remaining time series by subtracting the IMF from the original time series.
  3. Repeat the (i) and (ii) steps until one has the final IMF as a purely residual trend.

A sifting process is considered for the decomposition with the detailed steps as follows:

  1. Figure out all the extrema of the original time series \( x(t) \) and use the maxima \( U(x) \) and minima \( L(x) \) to interpolate an upper and lower envelope, respectively.
2. Calculate the mean \( \mu(t) \) of the envelope: 
\[
\mu(t) = \frac{U(x) + L(x)}{2}
\]
and subtract it from the original time series: 
\[
g(t) = x(t) - \mu(t).
\]

3. If the subtracted time series \( g(t) \) satisfies the conditions mentioned above for IMF, then the sifting process stops. If not, then the procedures from (i) to (iii) are repeated with the subtracted time series \( g(t) \) in place of \( x(t) \).

4. Once the sifting process stops, say after the \( n \)-th steps, we subtract the final IMF to obtain the pure trend of the data-set \( r_n(t) \): 
\[
r_n(t) = x(t) - \sum_{i=1}^{n} g_i(t) \tag{3}
\]

A disadvantage of considering EMD is mode mixing \([26, 39]\). IMF may consist of different frequency components. On the other hand, the same frequency or frequencies of similar magnitudes can be found in different IMFs. To overcome this, we consider the ensemble EMD (EEMD) \([26, 40]\). For the modification, we now add different white noises \( w_i(t) \) to original data set \( x(t) \): 
\[
x_i(t) = x(t) + w_i(t) \quad \text{where} \quad i = 1, ..., M \quad \text{for ensemble size} \quad M \quad \text{and compute the IMFs using EMD method which satisfies all the conditions. Finally, we consider ensemble mean of all the IMFs to get the final one, and we calculate the trend by subtracting the ensemble-averaged IMF from the original time series.}

To choose the white noise we have considered: 
\[
\epsilon_n = \frac{\epsilon}{\sqrt{N}} \quad \text{with} \quad \epsilon_n, \quad \text{the final standard deviation of the error,} \quad \text{is calculated from the difference between the original time series data and the corresponding IMFs, and} \quad \epsilon \quad \text{is the amplitude of the added noise. We have considered the ensemble number as 100 and the amplitude of added white noise as 0.2 times the data set’s standard deviation} \quad \text{[35, 38, 39, 41].}
\]

Another disadvantage of the EMD method is it leads to numerical errors, which results in incorrect decomposition analysis. To discard the spurious IMF, we include an extra threshold condition based on correlation with the original signal, which will be significant for relevant IMFs \([41, 42, 43]\). We only consider the IMFs which are having correlations larger than the threshold correlation value. The threshold correlation value \( \mu_{TH} \) is:
\[
\mu_{TH} = \frac{\max_i(\mu_i)}{10 \times \max_i(\mu_i) - 3} \quad i = 1, 2, ..., k \tag{3}
\]

Here \( \mu_i \) is the correlation coefficients between \( i \)-th IMF and the original data set and \( k \) is the total number of IMFs.

• Finally, the \( q \)-th order fluctuation function \( F_q(s) \) is computed as follows considering all segments \( s \):
\[
F_q(s) = \left[ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(s, v)]^{q/2} \right]^{1/q} \quad \text{for} \ q \neq 0 \tag{4}
\]
\[
F_0(s) = \exp \left[ \frac{1}{2N_s} \sum_{v=1}^{2N_s} \ln[F^2(s, v)]^{1/2} \right] \quad \text{for} \ q = 0 \tag{5}
\]
according to L’Hôpital’s rule.

- The fluctuation function $F_q(s)$ varies as power law of the segment size $s$ with a function of power $q$ for a multifractal time-series [44].

$$F_q(s) \sim s^{H(q)} \quad (6)$$

Here function $H(q)$ is defined as the generalized Hurst index or self-similarity exponent of the multifractal data-set. For a mono-fractal time-series scaling behavior is independent of power $q$ and shows the same behavior for small ($q < 0$) and large ($q > 0$) fluctuations.

3.3. Multifractal Strength Analysis

To analyze the strength of multifractality in the considered time-series, we compute multifractal scaling exponent $\tau(q)$ using generalized Hurst exponent calculated from eqn. (6) by

$$\tau(q) = qH(q) - 1 \quad (7)$$

This can also provide the information about the shape of the singularity spectrum through Legendre Transformation [45] [46] [47] [48]. Singularity index or Hölder exponent $\alpha$ can be computed by differentiating a smooth $\tau(q)$ function:

$$\alpha = \tau'(q) = H(q) + qH'(q)$$

and singularity spectrum is related to the singularity strength as $f(\alpha) = q\alpha - \tau(q)$ [49]. The width of the inverted parabolic singularity spectrum curve ($\Delta \alpha$) refers to the strength of the multifractality and complexity of the time-series: $\Delta \alpha = \alpha_{max} - \alpha_{min}$ with $\alpha_{max}$ and $\alpha_{min}$ as the maximum and minimum value of the $\alpha$. In other words, it also denotes the uneven distribution and presence of more severe fluctuation in the data-set.

4. Results and Analysis

For the fluctuation analysis, we have log normalized the return data of the daily closing prices of each stock. We show the same for Au sector, in Fig. 1 (bottom), compared to the return data set (up). All the other sectors also show similar structure. Earlier the presence of two scaling behavior (periods shorter and longer than 24 hours) has been found for price time series [50] [51] [52] [53] [54]. In our analysis of sectorwise analysis, we similarly found two separate time-scales with a shorter timescale between 10 days to 200 working days, and a longer time scale between 200 and 1000 working days. We show the shorter time scale plots in Fig 2. Panel (a) shows the fluctuation function $F_q(s)$ for $q = 2$. As the graph shows, in the log-log scale, it possibly fits a straight line and thus indicates a presence of scaling with $q$. One can notice the non-existence of negative $q$ power in Fig 2 (b) $\tau(q)$ vs $q$ and (c) $H(q)$ vs $q$, which was also reported in an earlier paper [55]. On the other hand, for long-time scale, the whole range of $q$, both negative and positive values, are present (see Fig. 3). As for the shorter segments $s$, the average variance is not well defined, which leads to the non-existence of a negative power value of those variances. But, on advantage, it
establishes a threshold value to mark the change in the multifractal behavior in the market. Scaling exponent from the Fluctuation function at $q = 2$ also helps to calculate the Hurst exponent ($H_2$) for stationary time series from eqn. [7]. The generalized Hurst exponent is dependent on $q$ and decreases as $q$ increases, which is a characteristic of multifractal features of the data set [55]. Renyi exponent $\tau(q)$ is also non-linear, indicating multifractality. Finally, we obtain the single-humped multifractal spectrum $f(\alpha)$ ($\alpha=$singularity strength) by the Legendre transform, indicating multifractality. The width of the multifractal spectrum denotes the power or degree of multifractality. We also calculate the asymmetry of the spectrum by the ratio $B = (\delta\alpha_L - \delta\alpha_R)/\delta\alpha_L + \delta\alpha_R$ where $\delta\alpha_L = \alpha_0 - \alpha_{min}$ and $\delta\alpha_R = \alpha_{max} - \alpha_0$ are the width of left and right branches of the singularity spectrum. These two parameters decides the pattern of high and low fluctuations [11, 50]. $\alpha_0$, $\alpha_{max}$, $\alpha_{min}$ are the values of $\alpha$ at maximum value of $f(\alpha)$, maximum and minimum $\alpha$. The value of $B$ ranges from $-1$ to $1$ [57, 58]. It denotes the skewness of the multifractal spectrum. $B = 0$, $> 0$, $< 0$ signifies the symmetric (low and high fluctuation are almost equally responsible for the dynamics), left-skewed (large fluctuation is more dominant), and right skewed (dominance of low fluctuation is more clearly visible) multifractal spectrum, respectively. For the shorter time scale, the multifractal spectrum is vast, which signifies very strong multifractality or high volatility and the occasional burst of the stock market. One can conclude that this shorter period is not suitable for the investment point of view. In contrast, we have the full singularity spectrum for large time scales, and less width denotes a more stable market suitable for investment for more extended time scales. Table 2 shows the most important parameters of the multifractal spectrum: $\Delta\alpha$, $H_2$, $dH$, and $B$ [20, 32, 41, 59]. $\alpha_{max}$ and $\alpha_{min}$ denote the most extreme and smoothest event in the considered data-set and $\alpha_0$ (=value of $\alpha$ at $f(\alpha)_{max}$) provides the information about the structure of the process, e.g. lower value will signify more correlated process.

Figure 1: Daily closing prices (top) and normalized log-returns (bottom) of the full time period for Auto sector is shown here.
which loose its fine structure and appears to be more regular [41]. In addition, left skewed multifractal spectrum indicates the dominance of large fluctuations which can lead to extreme events [32] [59]. A right skewed spectrum, in contrast, signifies presence of more small fluctuations and thus is a sign of stable sector. AU, BX, CG, FMCG, HC, IT, RE, Teck, PWR, CPSE, EG, TC, and UT sectors with Hurst exponent value greater than 0.5, indicates the presence of persistent fluctuations or positive auto-correlation [38]. On the other hand, CD, MT, ONG, PSU, BM, FN, and ID have anti-persistence behavior or negative correlation. In other words, a positive (negative) return on a specific day will mostly be followed by a negative (positive) return on the next working day. The width of the generalized Hurst exponents (dH) is computed using all H(q) values over the range of qϵ[−10,10]. This width signifies the strength of the multifractality in a sector; the higher the values, the higher the multifractality [60]. AU sector (dH = 1.38) is the most multifractal among all the sectors present in Indian market, followed by PSU (dH = 1.25) and CG (dH = 1.15). On the other hand, RE (dH = 0.22), IT (dH = 0.26), CDGS (dH = 0.3) have the lower degree of multifractality in the considered period. This information also gives an idea of long-range dependency [13] [20] [21] [61] [62]. We can conclude that RE, IT, and CDGS show the lowest level of dependency among all. Value of B parameter included in Table 2 thus helps one to study the dynamics of the market quantitatively, as well as qualitatively [32] [59].

Figure 2: Multifractal analysis for Short term (upto 200 days) period. (a) Fluctuation functions for various sectors (see color code), (b) Generalized Hurst exponent depending on q, (c) Mass exponent τ(q) vs q. (d) Multifractal spectrum.
5. Conclusion

This paper analyzes multifractality of sectoral daily indices of Bombay stock exchange, India. We replace MFDFA method to OMW-EEMD-MFDFA which makes our method more efficient and analysis robust. Firstly, multifractality in finance is not new, but so far, it has not been analyzed to provide information on sector-wise data in a market. Assessing the economic and financial condition is essential for policymakers, mutual funds, portfolio managers, and investors. It is important to note here that a market’s overall efficiency may not always be the same as the efficiency found at the sectoral level. This motivates the authors of these sector-specific studies, also the first comprehensive sector-wise analysis using multifractality. Thus the present work focuses on the sectoral efficiency of 22 sectors present in the S&P BSE market in the considered period using multifractality analysis.

Secondly, this study also proves the existence of long-range correlation and randomness, which depend on the chosen time scales. In this paper, we have figured out the two market dynamics, 10 to 200 days as short and more than 200 days as the long term. It is beneficial for the investors to get the behavior of the individual sectors in the market and how they behave on these two separate scales. Moreover, at shorter time scale, negative $q$ power of variances vanish. At both the scales, decreasing Hurst exponent function with $q$ and non-linear Renyi
Table 2: Sector-wise values of width of singularity spectrum (\(\Delta \alpha\), column 2), Hurst exponents (\(H_2\), column 6), spread in Hurst exponent (\(dH\), column 7), and asymmetry ratio (\(B\), column 8)

| Sectors | \(\Delta \alpha\) | \(\sigma_{\text{max}}\) | \(\alpha_0\) | \(\alpha_{\text{min}}\) | \(H_2\) | \(dH\) | \(B\) |
|---------|----------------|-----------------|----------|----------------|------|------|-----|
| AU      | 1.71           | 1.92            | 0.99     | 0.21           | 0.72 | 1.38 | -0.09 |
| BM      | 0.64           | 0.69            | 0.4      | 0.05           | 0.33 | 0.46 | 0.09 |
| BX      | 1.07           | 1.54            | 0.97     | 0.47           | 0.86 | 0.75 | -0.08 |
| CD      | 1.21           | 1.37            | 0.59     | 0.16           | 0.47 | 0.95 | -0.29 |
| CDGS    | 0.46           | 0.83            | 0.73     | 0.37           | 0.66 | 0.3  | 0.57 |
| CG      | 1.55           | 1.51            | 0.75     | -0.04          | 0.5  | 1.15 | -0.02 |
| CPSE    | 1.4            | 1.61            | 0.92     | 0.21           | 0.71 | 1.09 | 0.02 |
| EG      | 0.58           | 1.07            | 0.82     | 0.49           | 0.82 | 0.35 | 0.13 |
| FMCG    | 1.27           | 1.49            | 0.85     | 0.22           | 0.67 | 0.98 | 0.0  |
| FN      | 0.71           | 0.86            | 0.54     | 0.15           | 0.44 | 0.55 | 0.11 |
| HC      | 0.85           | 1.16            | 0.83     | 0.31           | 0.75 | 0.55 | 0.23 |
| ID      | 0.71           | 1.01            | 0.35     | 0.31           | 0.47 | 0.48 | -0.42 |
| II      | 0.87           | 1.33            | 1        | 0.46           | 0.87 | 0.65 | 0.26 |
| IT      | 0.4            | 0.76            | 0.67     | 0.36           | 0.62 | 0.26 | 0.51 |
| MT      | 0.89           | 0.59            | 0.32     | -0.29          | 0.18 | 0.65 | 0.37 |
| ONG     | 1.04           | 1.23            | 0.52     | 0.19           | 0.45 | 0.68 | -0.36 |
| PSU     | 1.65           | 1.6             | 0.71     | -0.04          | 0.47 | 1.25 | -0.08 |
| PWR     | 1.01           | 1.43            | 0.87     | 0.42           | 0.8  | 0.7  | -0.11 |
| RE      | 0.41           | 0.86            | 0.71     | 0.45           | 0.84 | 0.22 | 0.23 |
| TC      | 1.51           | 1.75            | 0.84     | 0.23           | 0.67 | 1.12 | -0.19 |
| Teck    | 0.82           | 1.2             | 1.01     | 0.38           | 0.84 | 0.63 | 0.54 |
| UT      | 0.66           | 1.2             | 1.03     | 0.54           | 0.91 | 0.44 | 0.50 |

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