Impact Dynamics on Granular Plate

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Abstract: In this paper, a theoretical and numerical study on the impact of a rubber solid on the free surface of a granular plate is presented, showing a simulation of an aircraft wheel on impact with a flexible landing surface. This physical action, when we use a rheological approach, becomes a fundamental parameter to investigate wear and tear, and consequently strength to micro and macro pavements failure. The study has developed initially from a microscopic point of view and subsequently on macroscale. The effects are strictly linked with material degradation associated with damage evolution. The problem is developed by energetic approach on an elastic-plastic element using the functional energy containing two contributions, bulk and surface. The model simulates the behaviour of flexible runway pavements during the landing phase.

Key words: Impact, FEM (finite element model), airport pavement, aircraft landing.

1. Introduction

The dynamic interaction between a rubber tyre and an infrastructure is the function of many factors: the behaviour of the materials, the configuration and flexibility of the structure, the contact conditions between wheel and infrastructure, the speed of the wheel and the spatial distributions of the geometric features of the wheels.

The paper presents, after a theoretical characterization, the application of a discrete element technique for the study of the two dimensional strain problem of the interaction between a moving rubber wheel and a flexible infrastructure.

The failure of the granular solids corresponds to situations where combinations of brittle fragmentation and visco-plasticity are shown.

The modelling also accounts for size dependency in the strength of the granular solid after fragmentation, and the internal fragment interactions are modelled by non-linear constraints that include Coulomb frictional behaviour. The computational scheme is used to evaluate the time history of the average contact stresses and the distribution of local contact stresses at the interface in the fragmentation zone.

The capabilities for modelling the first fragmentation starts from an initially continuum state. We transit a large-scale fragmentation movement to arrive at a continued evolution of fragments; and the analysis of this process is essential for the successful modelling of all processes observed in the vicinity of the wheels-surface interaction zone. The tensile fragmentation criterion assumes that the material will fragment when the minimum principal stresses reach the tensile strength of the material.

According to Krajcinovic and Lemaitre [1], this particular study will be restricted to micro-cracks, which is the only one of the many classes of micro-defects influencing the mechanical macro response. We follow the volume element theory RVE (representative volume element), and so it is possible to represent a non-homogeneous solid with periodic microstructure. Particularly, in the transition toward the micro-scale, our RVE can be represented by more granular elements joint by means of an asphalt mixture. Considerations are applied to the contact area between two granular elements.

Then we apply the FE (finite element) method,
which includes the aspects described previously to examine the interaction between a stationary flexible structure and aircraft wheels during the landing phase.

Past flexible pavement models used multi-layer elastic analysis, which assumed static loading; whereas, in reality, pavements are subject to loads both static and in motion [2].

Differently from the layered theory, the FE method cannot be a complex and costly analysis tool; thus it is employed only when accurate results are needed. Although involving a more complicated formulation than the multi-layer elastic theory, the application of FE techniques is generally thought to allow an accurate simulation of pavement problems. Furthermore, this method allows to consider almost all controlling parameters (e.g., dynamic loading, discontinuities such as cracks and shoulder joints, visco-elastic and nonlinear elastic behaviour, infinite and stiff foundations, system damping, quasi-static analysis and crack propagation) [3].

For example, the model used in the study by Zaghloul and White [4] employed 3D dynamic finite element to investigate the response of moving loads on pavement structures. Zaghloul [5] employed a visco-elastic model for the asphalt concrete, an extended Drucker-Prager model for granular base course and a Calm Clay model for the clay subgrade soils. Taciroglu [6] simulated the pavement responses using three-dimensional finite element analysis and adopted the $K_\theta$ model, the Uzan model as the nonlinear unbound granular material model and linear subgrade soils model.

Kim [7] found that nonlinearity of unbound layers using the Drucker-Prager plasticity model was not suitable to pavement analysis. Therefore, the Uzan model was adopted for granular materials and cohesive soils for the nonlinear analysis. Mohr-Coulomb failure criterion was employed in the nonlinear finite element analysis.

Erlingsson [8] conducted three-dimensional finite element analysis of a heavy vehicle simulator to test low volume road structures. A linear elastic material model was used and the single and dual wheel configurations were given.

More recently, the theory of visco-elastoplastic has also been extensively used to analyse HMA (hot mix asphalt) materials. Chehab [9] developed an advanced material characterization procedure including the theoretical models and its supporting experimental testing protocols necessary for predicting the response of asphaltic mixtures subjected to tension loading. The model encompasses the elastic, visco-elastic, plastic and visco-plastic components of asphalt concrete behaviour.

Pirabaroon et al. [10] successfully developed a visco-elastoplastic creep model representing the time-dependency of asphalt mixtures to evaluate their rutting potential and identify factors, which had a significant effect. The creep model parameters were derived from the test results of Asphalt Pavement Analyzer.

In recent years, researchers have successfully applied linear visco-elastic theory to describe the behaviour of HMA materials. Elseifi et al. [11] conducted a comparative study between the elastic FE model and the linear visco-elastic FE model and concluded that it is imperative to incorporate a visco-elastic constitutive model into pavement design methods for improving the accuracy. Yin et al. [12] showed that 3D finite element modelling utilizing viscoelastic material properties provides reasonable prediction of strain response in the field.

Onyango [13] evaluated the existing mechanistic models that predict permanent deformation (rutting) in asphalt mixes by comparing computed permanent deformation to that measured in a full-scale accelerated pavement test.

The aim of this study is to assess the effects of a heavy impact caused by aircraft landing gear wheels on a flexible airport pavement. The paper also presents a 3D FE model developed and calibrated on the data of the present study, which can predict the
pavement performance under the aircraft landing. In conclusion, the paper presents the comparison between the results obtained with a creep FE model and a visco-elastoplastic model. The development of these models required suitable choices of material properties, the tyre contact areas and the associated stresses.

2. Theoretical Background

The impact between two solid elements represents the simplified condition for the generation of shock waves. In the specific case of parallel impact, the two surfaces enter in contact simultaneously and all the points of the two surfaces enter in contact at the same time. The true profile of a shock wave is complex. In Fig. 1, it is possible to observe the difference between the ideal and the true profile where, for the latter, it is clear the dependence form the characteristics of the material and the pressure applied at contact.

An impulsive stress on contact has an initial, middle and final pressure value. Initially, it is a shock wave (discontinuity in compression); the mean reaction is characterized from a slow variation of pressure and the final from a dissolution, which tends to the undisturbed state.

In accordance with Duvall and Graham [15], the shock waves can induce phase transitions in the solid (Fig. 2), then transit from elastic to plastic response (in our case, plasticization of the mixture binding component).

On a theoretical point of view, we classify the problem as the propagation of a shock wave, where a uniform contact pressure is applied to a plane solid surface in an elastic semi space. Given the geometrical origin \( x = 0 \) and the beginning of the phenomenon at time \( t = 0 \), after a laps of time \( t \), the shock front divides the space into two regions: one undisturbed, the other compressed and accelerated. Therefore, the flow equation is reduced to the jump condition:

\[
\Delta P = (v_f - v_o)(v_p - v_o)/V_s \tag{1}
\]

where, \( v_f \) is the wave propagation speed, \( v_o \) and \( v_p \) are the speeds of the particles respectively behind and in front of the shock front, and \( V_s \) is the specific volume of the medium.

The clear result is the introduction of a pressure step, which travels across the medium with changes of shape, which depend on the mechanical proprieties of the element.

In the case of impact, the contact time tends to zero. Therefore, \( t' = \lambda' \) where \( t' \) is the impact time and \( \lambda' \) is the plasticization time. In theory, the problem can be represented as two successive phases.

First phase: transversal speed at the center of the body remains constant. In this phase, it is necessary to absorb the remaining kinetic energy in the body. Second phase: a concentrated plasticization begins to expand from the core to the external part of the body. The time it takes is given by the expression:

\[
t = \frac{\mu v_o \lambda'^2}{6M'} \tag{2}
\]
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At the maximum movement, the contact zone can be approximated as:
\[ \omega \approx 3p_0^2 r^2 / 4 \mu p_c \] (3)
and in the case of finite movements:
\[ p_{1,2} = \mu \omega^0 \] (4)
in the plane case, the maximum transversal movement assumes the form.

In regards to the mechanical proprieties of the medium subject to the impact actions in the case, the response of solids made of asphalt mixtures can be divided, according to Refs. [16-18], into three groups: elastic, visco-elastic and visco-plastic. In the one-dimensional case, we have:
\[ E_{ii}^v = \varepsilon^{-1} T_{ii} \] (5)
\[ E_{ii}^{we} = A(T_{ii}) \ell^\alpha \] (6)
\[ E_{ii}^{vp} = B(T_{ii}) f(N) \rho^\beta \] (7)
where, \( f(N) \), \( A(T) \) and \( B(T) \) are functions of the stress in the viscous phase. \( A, B, \alpha, \beta \) and \( \varepsilon \) are constants determined at constant temperature.

In the multidimensional case, the equations above become:
\[ [E]^v = [K] [T] \] (8)
\[ [E]^{ve} = A \alpha^\alpha [H] [T^\beta] \] (9)
where, \( K \) is the deformability matrix, \( H = \varepsilon K \) and \( T^\beta = [T_{ii}]^\beta \). Concerning the visco-elastic part, it has to be specified that the critical points which arise in this phase with the load time can be resolved using the Perzyna theory [19]. For an associated visco-plastic flow, we have:
\[ [E]^{vp} = \gamma \phi(t) \partial F / \partial t \] (10)
where, \( \gamma \) is a fluidity parameter associated to the loading times and the number of loading cycles, \( \phi(F) \) is the viscous flow function and \( F \) is the plasticity function \( F(T, k) \) with \( k \) the hardening parameter.

Passing to the numerical implementation, in the case of reduced load intervals, an iterative procedure, as a Newton-Raphson, can be applied.
\[ A[T]^\alpha = [K][A[E]]^\alpha - A \alpha \alpha^\alpha [K][T^\beta]^\alpha - A[E^{vp}]^\alpha \] (11)
and after the rightful developing, stress and strain are approximated as:
\[ [T]^{\alpha+1} = [T]^\alpha + A[T]^\alpha \] (12)
\[ [E^{vp}]^{\alpha+1} = [E^{vp}]^\alpha + A[E^{vp}]^\alpha \] (13)
Therefore, it follows the link between micro-scale effects and material behaviours at macro-scale. So we focus on the micromechanics of the damage processes because the nonlinear response of typical engineering materials is almost entirely dependent on the primary change in the concentration, distribution, orientation and defects in its structural composition.

The relation between the continuum damage mechanics and the fracture mechanics is very complicated. In essence, it is a question of scale. The important role of scale can be clarified by an energetic point of view.

In view of an approximated continuum theory with the physical foundation of micromechanical models, a promising strategy would consist of combining the best features of both models. In this approach, we have only considered the first layer of the pavement package because at micro-scale, damage distribution at the edge of the body where surface degradation is of importance is expected to be significantly different from the damage distribution far from the edge in the body.

We follow the volume element theory RVE, it is possible to represent a non-homogeneous solid with periodic microstructure. Particularly, in the transition toward the micro-scale, our RVE can be represented by more granular elements joint by means of an asphalt mixture, so considerations are applied to the contact area between two granular elements. In this manner, the homogenization problems can be satisfied.

Following Sneddon’s solution [20] type, we model the physics of impact (landing-gear/ground) by means of a rigid frictionless asymmetric concentrated impact, with generic concave profile described by the function \( f(r) \). We find the \( \sigma \) pressure distribution under the concentrated impact and the displacement on the surface, respectively.
3. Materials and Methods

Flexible pavements can often be idealized as closed systems consisting of several layers; so it was decided to model the surface, base, sub-base and sub-grade material using three-dimensional finite elements. The pavement section comprises of asphalt concrete and crushed aggregate, as shown in Fig. 3.

The pavement structure in the application is based on the structure as found for the runway of the Reggio Calabria airport. The considered pavement consists of a 100 mm thick asphalt concrete layer as the surfacing course, a 150 mm thick of bitumen-treated mixture as the base course, a 210 mm thick granular layer as the sub-base course and a compacted soil subgrade.

3.1 Contact Area and Associated Stress

The tyre contact areas considered in the model were Airbus 321 tyre [21]. The most common way of applying wheel loads in a finite element analysis is to apply pressure loads to a circular or rectangular equivalent contact area with uniform tyre pressure [22].

For the FEM (finite element model), the contact area \( A_c \) was represented as an ellipse having a length \( L \) and a width \( L' = 0.6L \), as shown in Fig. 4.

The contact area can be calculated as:

\[
A_c = \frac{P}{p} \quad (14)
\]

where, \( P \) is the wheel load and \( p \) is the tyre pressure.

Considering the wheel load of the principal gear of an Airbus 321 [21], the contact area is:

\[
A_c = \frac{199,510}{1.36} = 146,700 \text{ mm}^2 \quad (15)
\]

![Pavement section](image)

Fig. 3  Pavement section.

![Contact area between tyre and pavement](image)

Fig. 4  Contact area between tyre and pavement.

| Table 1  | Airbus 321 characteristics. |
|-----------------|-----------------------------|
| Maximum ramp weight | 93,400 kg |
| Percentage of weight on main gear group | 95.4% |
| Nose gear tire size | 30 × 8.8 R15 |
| Nose gear tire pressure | 10.8 bar |
| Main gear tire size | 1,270 × 455 R22 |
| Main gear tire pressure | 13.6 bar |

![Normal acceleration during landing phase](image)

Fig. 5  Normal acceleration during landing phase [23].

![Aircraft landing gear model](image)

Fig. 6  Aircraft landing gear model [24].

To evaluate the pavement load in exceptional condition, the dynamic parameters of an “hard” landing (Fig. 5), that caused the break of some gear components, were considered [23].

Starting from this and considering the damping effect of the gear system (Fig. 6), it is possible to calculate the acceleration graph during the hard landing (Fig. 7).

As shown in Fig. 7, the peak acceleration value, during the hard landing, is 1.99 m/s\(^2\). This value of acceleration was used in the FEM to calculate the
maximum wheel load. Under the new load, the contact area becomes:

$$A_c = \frac{397.025}{1.36} = 291,930 \text{ mm}^2$$  (16)

From Eq. (16), the footprint dimensions are: $L = 747$ mm and $L' = 498$ mm.

### 3.2 Pavement Materials

The pavement configuration is shown in Fig. 3 and the material properties of pavement layers are given in Table 2.

Elastic properties (modulus of elasticity and Poisson’s ratio) were obtained by conducting laboratory testing on HMA materials and field non-destructive evaluation of granular and subgrade materials.

While an elastic constitutive model was assumed for the granular layers and the base course, a time hardening creep model was incorporated to simulate the viscoelastic behaviour of the HMA surface layer [3]:

$$\dot{\varepsilon} = F(\sigma, T, t)$$  (17)

Creep strain ($\dot{\varepsilon}$) is a function of stress ($\sigma$), time ($t$) and temperature ($T$).

Eq. (17) is only valid for constant stress and temperature. Kraus [25] explains the Bailey-Norton law which is capable of modelling primary and secondary creep. The formulation is based on a basic assumption that material depends on the present stress state explicitly. In this approach, strain is represented in Eq. (18).

$$\dot{\varepsilon} = \frac{A}{m+1} \sigma^n t^{m+1}$$  (18)

$A$, $m$ and $n$ are constants that are a function of temperature. Strain rate can be obtained by differentiating Eq. (18) to obtain:

$$\dot{\varepsilon} = \frac{\partial \varepsilon}{\partial t} = A \sigma^n t^m$$  (19)

where, $\dot{\varepsilon}$ = uniaxial equivalent creep strain rate, $\sigma$ = uniaxial equivalent deviatoric stress (Mises equivalent stress), $t$ = total time, $A$, $n$, $m$ = user defined functions of temperature.

$A$ and $n$ must be positive with $-1 < m \leq 0$.

In this research, the $m$ value was set at -0.5 while $n$ value is 0.67 and $A$ is $1e-9$.

### 3.3 Finite Element Model

The finite element mesh developed has the following dimensions: 10 m in $x$ and $y$ directions and 2.5 m in the $z$ direction.

The degree of mesh refinement is the most important factor in estimating an accurate stress field in the pavement: the finest mesh is required near the loads to capture the stress and strain gradients. The mesh presented has 126,245 nodes and 29,900 quadratic hexahedral elements of type C3D20R (continuum 3-dimensional 20 node elements with reduced integration). Quadratic elements yield better solutions than linear interpolation elements [26]. The three-dimensional view of FEM considered for the analysis is shown in Fig. 8.

The loads (vertical and horizontal) were uniformly applied to the element, which was created to be the same size as the wheel imprint of an A321 (Eq. (16)). In this analysis, the surface was considered to be free from any discontinuities (with no cracks) or unevenness, and the interface between layers was considered to be fully bonded (with no gaps).
Since the boundary conditions have a significant influence in predicting the response of the model, the model was constrained at the bottom: X-Symm on the sides parallel to y-axis and Y-Symm on the sides parallel to x-axis.

All layers were considered perfectly bonded to one another so that the nodes at the interface of two layers had the same displacements in all three (x, y, z) directions. Assuming perfect bond at the layer interfaces, it implies that there will be no slippage at the interface. This assumption is more applicable to HMA layers, since the possibility of slippage is greater at the sub-base/subgrade interface [12].

4. Results and Discussion

The results of the non-linear FE analysis are illustrated in the following figures. In particular, the results obtained with the creep FE model were compared with the results obtained with a visco-elastoplastic model. In the visco-elastoplastic model, the material behaviour is broken down into three parts: elastic, plastic and viscous.

Fig. 9 shows a one-dimensional idealization of this material model with the elastic-plastic and the elastic-viscous networks in parallel.

Fig. 10 shows the Von Mises stress distribution for the two considered FE models at the landing aircraft impact instant.

Fig. 10  Mises stress at the instant of impact: (a) creep model and (b) visco-elastoplastic model.
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Fig. 11  Displacement contours at the instant of impact: (a) creep model and (b) elasto-visco-plastic model.

Fig. 12  Predicted deflection profiles (y-direction).

Fig. 11 presents the results of pavement surface deflection along transversal direction. The two graphs of Figs. 10a and 10b are shown with the same scale to facilitate visual comparison of deflection magnitude.

Finally, in Fig. 12, the predicted transversal surface deflection profiles along the transverse centre line are plotted.

The visco-elastic characteristic of the second FEM is reflected in the wider deflection of the profile.

5. Conclusions

Finite element analysis of pavement structures, if validated, can be extremely useful, because it can be used directly to estimate pavement response parameters without resorting to potentially costly field experiments.

If accurate correlations between the theoretically-calculated and the field-measured response parameters can be obtained, then the FEM can be used to simulate pavement response utilizing measurements from strain gages. In particular, the proposed model has clearly confirmed the need and importance of 3-Dimensional finite element analysis on flexible pavements to consider the behaviour of the structure under high stress.

The two considered models (creep and visco-elastoplastic) showed different predicted results in terms of displacement, and the second had a wider deflection profile.

Future research advancements can be done in the direction of a better profiling of the pavement behaviour under stress, in function of different combinations of variables as loads in number and time, tyre type and pressure.

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