Abstract

In this work we consider a simple, Bohr-Sommerfeld (Old quantum atomic) theory of the magnetic monopole. We consider the system, simply called magnetic monopole “atom”, consisting of the practically standing, massive magnetic monopole as the “nucleus” and electron rotating around magnetic monopole. At this system we apply quasi-classical, Bohr-Sommerfeld quantum atomic theory. Precisely, we apply firstly, by the electron rotation, Bohr-Sommerfeld momentum quantization postulate. Secondly we use equivalence between total centrifugal force acting at rotating electron and classical magnetostatic interaction between rotating electron and magnetic monopole. It yields result practically equivalent to the Dirac quantization relation between electrical and magnetic charge.

In this work we shall consider a simple, Bohr-Sommerfeld (Old quantum atomic) theory of the magnetic monopole. We shall consider the system, simply called magnetic monopole “atom”, consisting of the practically standing, massive magnetic monopole as the “nucleus” and electron rotating around magnetic monopole. At this system we shall apply quasi-classical, Bohr-Sommerfeld quantum atomic theory. Precisely, we shall apply firstly, by the electron rotation, Bohr-Sommerfeld momentum quantization postulate. Secondly we shall use equivalence between total centrifugal force acting at rotating electron and classical magnetostatic interaction between rotating electron and magnetic monopole. It yields result practically equivalent to the Dirac quantization relation between electrical and magnetic charge.

As it is well known Dirac \cite{1} introduced concept of the magnetic monopole starting, roughly speaking, from the rotation of the electron around magnetic monopole described by Dirac relativistic equation of the electron. It yields the following Dirac electric/magnetic charge quantization condition

\[
\frac{1}{4\pi\varepsilon_0 c^2}eq = \frac{n}{2}\hbar
\]  

(1)
for \( n = 1, 2, \ldots \). Here \( e \) represents the electron electrical charge, \( q \) - magnetic monopole magnetic charge, \( c \) - speed of light, \( \epsilon_0 \) - vacuum electric permittivity correlated with vacuum magnetic permittivity \( \mu_0 = \frac{1}{\epsilon_0 c^2} \), and \( \hbar \) - reduced Planck constant.

According to Dirac theory distance between electron and magnetic monopole can be arbitrary. Dirac theory does not give any prediction on the magnetic monopole mass, but according to contemporary quantum field theories it can be expected that this mass is much larger than electron mass.

Now shall consider the system, simply called magnetic monopole ”atom”, consisting of the practically standing, massive magnetic monopole as the ”nucleus” and electron rotating around magnetic monopole. At this system we shall apply quasi-classical, Bohr-Sommerfeld quantum atomic theory.

Firstly we shall apply Bohr-Sommerfeld momentum quantization postulate

\[
mvR = n\hbar
\]

for \( n = 1, 2, \ldots \), where \( m, v \) and \( R \) represent the electron mass, speed and cyclical orbit radius.

Secondly we shall apply equivalence between centrifugal force acting at rotating electron and classical magnetostatic interaction between rotating electron and magnetic monopole

\[
\frac{mv^2}{R} = \frac{\mu_0 (ev)q}{4\pi R^2} = \frac{1}{4\pi \epsilon_0 c^2} \frac{(ev)q}{R^2}.
\]

As it is not hard to see right hand of (3) is, in fact, analogous to classical magnetostatic interaction between two electrical charges \( e_1 \) and \( e_2 \) moving perpendicularly at the direction of the mutual distance \( R \), with speeds \( v_1 \) and \( v_2 \)

\[
F = \frac{\mu_0 (e_1 v_1)(e_2 v_2)}{4\pi R^2}
\]

where one moving electrical charge in (4) is changed by standing magnetic monopole charge in (3).

As it is not hard to see (3) can be simply transformed in the equation

\[
mvR = \frac{\mu_0}{4\pi}eq = \frac{1}{4\pi \epsilon_0 c^2}eq.
\]

It, according to (2), yields

\[
\frac{1}{4\pi \epsilon_0 c^2}eq = n\hbar
\]

for \( n = 1, 2, \ldots \), that is practically equivalent, to Dirac electric/magnetic charge quantization condition (1). Precisely speaking, left hand of (1) is identical to left hand of (6), while right hand of (1) represents one half of the right hand of (6). This difference can be usually explained by fact that equations system (2), (3) is practically non-relativistic.

In this way we reproduced simply and satisfactorily, by Bohr-Sommerfeld (Old, non-relativistic) quantum atomic theory, Dirac electric/magnetic charge quantization condition obtained originally by of the Dirac relativistic equation of electron.
Further, as it is not hard to see, equations system (2), (3) with \( v \) and \( R \) as the variables is incomplete or insufficient for unambiguous determination of \( v \) and \( R \). It, precisely (2), admits only a functional relation between \( v \) and \( R \\
v = n \frac{\hbar}{mR} \tag{7}
\) for \( n = 1, 2, \ldots \), or
\[ R = n \frac{\hbar}{mv}. \tag{8} \]
for \( n = 1, 2, \ldots \). Last expression, according to de Broglie relation, implies
\[ 2\pi R = n \frac{h}{mv} = n\lambda \tag{9} \]
for \( n = 1, 2, \ldots \), which means that \( n \)-th electron orbit has \( n \) electron de Broglie wavelength.

According to (8), for \( n = 1 \) and \( v = c \) (we shall use this condition relatively roughly since, as it has been pointed out, (2), (3) is a non-relativistic equations system), we can obtain lower limit of \( R \), \( R_{\text{min}} \), that equals
\[ R_{\text{min}} = \frac{\hbar}{mc} = \lambda_{\text{red}} \tag{10} \]
representing the electron reduced Compton wavelength \( \lambda_{\text{red}} \approx 10^{-13}\text{[m]} \). Since (reduced) Compton wave length can be independently, usually defined as the radius of a system rotating with speed of light and with angular momentum equivalent to (reduced) Planck constant, our result is very satisfactory.

Finally, as it has been discussed previously, correspondence between (3) and (4) admits that magnetic charge \( q \) can be formally considered as the product of the electrical charge and speed. So, suppose
\[ q = kce \tag{11} \]
where \( k \) represents a parameter that can be determined later. It, introduced in (6), for \( n = 1 \), yields
\[ \frac{ke^2}{4\pi\varepsilon_0c} = \hbar \tag{12} \]
or, after simple transformations,
\[ k = \left( \frac{e^2}{4\pi\varepsilon_0\hbar c} \right)^{-1} = \alpha^{-1} = 137 \tag{13} \]
where \( \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137} \) represents the fine structure constant. Since \( k \) is much larger than 1 we cannot interpret \( kc \) in (11) as a speed, but we can interpret
\[ e_k = ke = \frac{c}{\alpha} \tag{14} \]
in (11) as the renormalized electrical charge so that magnetic charge (11) can be presented in the form
\[ q = ce_k = \frac{ce}{\alpha}. \tag{15} \]
In conclusion we shall shortly repeat and point out the following. In this work we consider a simple, Bohr-Sommerfeld (Old quantum atomic) theory of the magnetic monopole. We consider the system, simply called magnetic monopole "atom", consisting of the practically standing, massive magnetic monopole as the "nucleus" and electron rotating around magnetic monopole. At this system we apply quasi-classical, Bohr-Sommerfeld quantum atomic theory. Precisely, we apply firstly, by the electron rotation, Bohr-Sommerfeld momentum quantization postulate. Secondly we use equivalence between total centrifugal force acting at rotating electron and classical magnetostatic interaction between rotating electron and magnetic monopole. It yields result practically equivalent to the Dirac quantization relation between electrical and magnetic charge.

References

[1] P. A. M. Dirac, Proc. Roy. Soc. (London) A133 (1931) 60
In this work we consider the attractive classical magnetic force between two moving electrical charges where, in the first case, one or, in the second case, both moving electric charges are formally changed by one or two magnetically charged massive magnetic monopoles. In the first case we obtain a system, simply called magnetic monopole "atom", consisting of the practically standing, massive magnetic monopole as the "nucleus" and electrically charged particle with small mass rotating around magnetic monopole. Equivalence of mentioned magnetic force with centrifugal force, after application of the Bohr-Sommerfeld angular momentum quantization postulate, implies a relation between electric and magnetic charge practically identical to the Dirac theory of the magnetic monopole. In the second case we obtain a system, simply called magnetic monopole "rotator", consisting of two monopoles rotating around their mass center. Equivalence of mentioned magnetic force with centrifugal force, after application of the Bohr-Sommerfeld total angular momentum quantization postulate, yields "rotator" total energy spectrum. Energy of the total separation of monopoles equals relativistic rest energy of the single monopole and this fact is called magnetic monopole half-confinement or quasi-confinement (in a conceptual analogy with quark theory). Finally we roughly estimate magnetic monopole mass that is 18769 (quadrate of the inverse value of the fine structure constant) times larger than electron mass.

In this work we shall consider the attractive classical magnetic force between two moving electrical charges where, in the first case, one or, in the second case, both moving electric charges are formally changed by one or two magnetically charged massive magnetic monopoles. In the first case we shall obtain a system, simply called magnetic monopole "atom", consisting of the practically standing, massive magnetic monopole as the "nucleus" and electrically charged particle with small mass rotating around magnetic monopole. Equivalence of mentioned magnetic
force with centrifugal force, after application of the Bohr-Sommerfeld angular momentum quantization postulate, implies a relation between electric and magnetic charge practically identical (neglecting 0.5 coefficient) to the Dirac theory of the magnetic monopole. In the second case we obtain a system, simply called magnetic monopole "rotator", consisting of two monopoles rotating around their mass center. Equivalence of mentioned magnetic force with centrifugal force, after application of the Bohr-Sommerfeld total angular momentum quantization postulate, yields "rotator" total energy spectrum. Energy of the total separation of monopoles equals relativistic rest energy of the single monopole and this fact is called magnetic monopole half-confinement or quasi-confinement (in a conceptual analogy with quark theory). Finally we shall roughly estimate magnetic monopole mass that is 18769 (quadrate of the inverse value of the fine structure constant) times larger than electron mass.

Consider, in the first case, formally the attractive classical magnetic force between two moving electrical charges where one moving electrical charge is formally changed by a magnetic charge of a massive magnetic monopole. Suppose that this force is equivalent to centrifugal force, i.e.

\[
\frac{\mu_0 (ev)q}{4\pi R^2} = \frac{mv^2}{R}.
\]

Here \(\mu_0 = \frac{1}{\varepsilon_0 c^2}\) represents the vacuum magnetic permittivity, \(\varepsilon_0\) - vacuum electric permittivity, \(c\) - speed of light, \(m\) - mass of the particle with electric charge \(e\) much smaller than large mass of the magnetic monopole with magnetic charge \(q\), \(v\) - speed of the electrically charged particle and \(R\) radius of the circular orbit of the electrically charged particle around massive magnetic monopole in the center of this orbit. In this way we obtain a system, simply called magnetic monopole "atom", consisting of the practically standing, massive magnetic monopole as the "nucleus" and electrically charged particle with small mass rotating around magnetic monopole.

Expression (1) can be simply transformed in the following expression

\[
\frac{1}{4\pi\varepsilon_0 c^2}eq = mvR
\]

whose right hand obviously represents the angular momentum of the rotating electrically charged particle. It, after application of the Bohr-Sommerfeld angular momentum quantization postulate

\[
mvR = n\hbar
\]

where \(\hbar\) represents the reduced Planck constant and \(n = 1, 2, \ldots\), quantum number, turns out in

\[
\frac{1}{4\pi\varepsilon_0 c^2}eq = n\hbar
\]

for \(n = 1, 2, \ldots\), that is, supposing that \(e\) represents the electrical charge of the electron, practically (neglecting additional factor \(\frac{1}{2}\) at right hand of (4) ) equivalent to Dirac electric/magnetic charge quantization relation \([1]\).

According to (4), for \(n = 1\), it follows

\[
q = 4\pi\varepsilon_0 \hbar c^2 \left(\frac{1}{e}\right) = e_p c \left(\frac{1}{e}\right)
\]

2
where \( e_P = (4\pi\varepsilon_0\hbar c)^{\frac{1}{2}} \) represents the Planck electrical charge. Suppose, in full agreement with previous discussions, that Planck magnetic charge \( q_P \) is simply product of the Planck electric charge \( e_P \) and speed of light \( c \), i.e.

\[
q_P = ce_P
\]

so that (5) can be rewritten in the following form

\[
q = q_P \frac{e_P}{e}.
\]

that implies

\[
\frac{q}{q_P} = \frac{e_P}{e}.
\]

Last expression obviously states that quotient of the magnetic charge and Planck magnetic charge is inversely proportional to the quotient of the electric charge and Planck electric charge.

For \( e = e_P \) (8) yields \( q = q_P = ce_P \).

But when \( e \) represents the electron electric charge approximately 11 times smaller than \( e_P \), then (8) yields \( q \) approximately 11 times larger than \( q_P \). In the same case (5) be transformed in the following form

\[
q = \frac{ce}{\alpha} = 137ce.
\]

where \( \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = \frac{1}{137} \) represents the fine structure constant.

Consider, in the second case, now, in analogy with (1), the attractive classical magnetic force between two magnetic monopole with the same mass \( M \). Suppose that these monopole rotates with the same speed and mutual distance \( 2R \) around their common mass center so that attractive classical magnetic force acting at one monopole is equivalent to centrifugal force, i.e.

\[
\frac{\mu_0}{4\pi} \frac{q^2}{(2R)^2} \equiv \frac{1}{4\pi\varepsilon_0c^2} \frac{q^2}{(2R)^2} = \frac{Mv^2}{R}.
\]

In this way we obtain a system, simply called magnetic monopole ”rotator”, consisting of two magnetic monopoles rotating around their mass center.

Suppose also that Bohr-Somerfeld total (of both monopoles) angular momentum quantization postulate analogous to (3) is satisfied

\[
2MvR = n\hbar
\]

for \( n = 1, 2, \ldots \). After simple calculations, we obtain the following solution of the equations system (10), (11)

\[
v_n = \frac{1}{4\alpha^2} v_{nH} = \frac{1}{4\alpha} \frac{c}{n} \quad (12)
\]

\[
R_n = (4\alpha^2)(\frac{m}{M})R_{nH} = (4\alpha)(\frac{m}{M})\lambda_e n = (4\alpha)\lambda_q n.
\]

Here

\[
v_{nH} = \frac{1}{4\pi\varepsilon_0\hbar c} \frac{1}{n}
\]

and

\[
R_{nH} = \frac{4\pi\varepsilon_0\hbar c}{me^2} n^2
\]
represent the electron speed and circular orbit radius for quantum number \( n \) in the Bohr-Sommerfeld theory of hydrogen atom, for \( n = 1, 2, \ldots \), \( \lambda_e = \frac{\hbar}{mc} \) - electron Compton wavelength, and \( \lambda_q = \frac{\hbar}{Mc} \) - magnetic monopole Compton wavelength.

Theory of relativity and quantum field theory imply that speed of the magnetic monopole cannot be larger than \( c \) and that rotation radius of the magnetic monopole Compton wavelength cannot be smaller than magnetic monopole Compton wavelength. It according to (12), (13), implies the following condition

\[
(4\alpha)n \geq 1
\]

or

\[
n \geq \frac{1}{4\alpha} = \frac{137}{4} \simeq 34.25 \simeq 34 \equiv n_0.
\]

It represents an interesting result. Namely, it points out that simple relativistic and quantum field theoretical correction of the Bohr-Sommerfeld Old, quasi-classical quantum theory of the magnetic monopole needs the effective shift of the ground state from 1 toward \( n_0 = 34 \).

Total energy of the rotating monopoles, as it is not hard to see, equals

\[
E_n = \frac{M v_n^2}{2} + \frac{M v_n^2}{2} - \frac{1}{4\pi\varepsilon_0 c^2} \frac{q^2}{2R_n} = -Mv_n^2 = -M\left(\frac{1}{(4\alpha)^2} \frac{c^2}{n^2}\right)
\]

for \( n = 1, 2, \ldots \), or, precisely, according to the relativistic and quantum field theoretical limits, for \( n = n_0, n_0 + 1, n_0 + 2, \ldots \).

For shifted ground state, i.e. for \( n = n_0 \), total energy (16) has minimal, negative value

\[
E_{n_0} \simeq -Mc^2
\]

while in the limit when \( n \) tends toward infinity this total energy tends toward zero

\[
E_\infty \simeq 0.
\]

It implies that energy of the separation of the pair of monopoles interacting in the described way equals

\[
\Delta E = E_\infty - E_{n_0} \simeq Mc^2.
\]

But it, as it is not hard to see, represents total relativistic rest energy of the single monopole

\[
E_0 = Mc^2.
\]

In this way we obtain a situation that can be metaphorically entitled magnetic monopole half-confinement or quasi-confinement (in a conceptual analogy with quark theory).

Finally, we shall roughly estimate magnetic monopole mass by expression

\[
Mc^2 = \frac{\mu_0}{4\pi} \frac{q^2}{R} \equiv \frac{1}{4\pi\varepsilon_0 c^2} \frac{q^2}{R^2}
\]

according to which magnetic monopole rest energy is identical to magnetic monopole self-interaction by magnetic force at distance \( R \) representing magnetic monopole or electron classical radius. It, according to (9), implies

\[
M = \left(\frac{1}{\alpha^2}\right)m = 18769m
\]
where \( m \) represents the electron mass.

In conclusion we can shortly repeat and point out the following. In this work we consider the attractive classical magnetic force between two moving electrical charges where, in the first case, one or, in the second case, both moving electric charges are formally changed by one or two magnetically charged massive magnetic monopoles. In the first case we obtain a system, simply called magnetic monopole "atom", consisting of the practically standing, massive magnetic monopole as the "nucleus" and electrically charged particle with small mass rotating around magnetic monopole. Equivalence of mentioned magnetic force with centrifugal force, after application of the Bohr-Sommerfeld angular momentum quantization postulate, implies a relation between electric and magnetic charge practically identical to the Dirac theory of the magnetic monopole. In the second case we obtain a system, simply called magnetic monopole "rotator", consisting of two monopoles rotating around their mass center. Equivalence of mentioned magnetic force with centrifugal force, after application of the Bohr-Sommerfeld total angular momentum quantization postulate, yields "rotator" total energy spectrum. Energy of the total separation of monopoles equals relativistic rest energy of the single monopole and this fact is called magnetic monopole half-confinement or quasi-confinement (in a conceptual analogy with quark theory). Finally we roughly estimate magnetic monopole mass that is 18769 (quadrate of the inverse value of the fine structure constant) times larger than electron mass.

References

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