Influence of cap beam restraint condition on the lateral deformation behavior of large-scale bored piles

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Abstract. The restrained force at the pile cap plays an important role in assessing the stability of bored piles. The constraining force of the cap beam is difficult to calculate due to spatial variability in soil properties, nonlinear soil behavior, and measurement errors. Based on theoretical analysis and field measurements this study proposes an analytical model combining Bayesian theorem and Markov Chain Monte Carlo Simulations (MCMCS) to quantify the restrained force of a pile cap beam. A simplified mechanical model of the constrained effect of the cap beam is established and verified by the results of field measurements. Based on a field excavated slope project, a Bayesian updating modeling approach is established by combing prior information, empirical knowledge, field instrumentation results, and site investigations to calculate the constrained effects of the cap beam. To verify its effectiveness, the model is combined with a finite element model, taking into account the uncertainties of soil moduli and measurement errors. The modeling takes into account the spatial variability and strain dependence of Young’s modulus. By comparisons, it can be found that the predicted displacements of the support pile are in good agreement with the field measurements. The results show that a Bayesian back updating approach can reduce the impact of uncertainty that can guide geotechnical engineering design and construction of similar projects.

1. Introduction
Bored piles have been widely used to stabilize slopes in recent decades. In practice, many approaches have been used to stabilize slopes, such as anchored anti-slide piles, combined piles, H-type piles, and sheet piles [1]. The cap beam of the supporting structure is widely used due to its simplicity and economic considerations. However, the constrained effect of the cap beam is largely due to pile deformation, which can further affect the overall stability of the slope. Thus, accurately assessing the restrained effect of the cap beam is an important task for geotechnical engineers. Many researchers have put a lot of effort into investigating the restrained effect of the cap beam, for example, Broms (1964) [2] proposed a widely used method for evaluating the lateral load capacity of piles [3,4], Mendonca et al. (2000) [5] and Luamba et al. (2019) [6] used the boundary element method and numerical analysis method to analyze capped pile groups, taking into account soil-pile-cap interactions. To our best knowledge, there are a limited number of studies that take into account soil uncertainties and measurement errors associated with the restrained effect of the cap beam.
The uncertainties of geotechnical engineering are reflected in the spatial variability of the soil, errors in soil properties, errors in field measurement, and violations associated with the construction process [7-12]. A Bayesian method is an ideal approach for characterizing uncertainties in geotechnical engineering [13,14]. To reduce these uncertainties, many approaches have been developed, which can be roughly divided into two categories: (1) the first category is characterized by the relationship between soil parameters and the results of on-site or laboratory tests that can account for uncertainties in soil parameters and the transformation model [15-19]; (2) approaches in the second category use field data to reduce uncertainty using Bayesian back analysis [20-24]. Besides the aforementioned methods, the machine learning method was an emerging approach to increase computational efficiency [25-26]. However, these methods, which combine Bayesian and machine learning, did not account for the effect of uncertain factors (i.e. rainfall, water table fluctuation, and the impact of construction disturbances on the result).

This paper aims to study the effect of the constrained force of the cap beam and the spatial variability of the soil on the horizontal displacement of the support pile caused by excavation. A new relationship was proposed between the constrained force of the cap beam and the depth of the excavation, which was used to predict the constrained force at a slope excavation. To combine the measured data with the previous information, a Bayesian framework was included to reduce the effect of the uncertainty of Young’s modulus and measurement error on the slope stability prediction. The uncertainty of Young’s modulus and constrained force was analyzed using the Bayesian back analysis and applied in the field excavation project. Bayesian estimates and field data were presented and discussed in detail.

2. Back analysis with Bayesian updating

2.1. Bayesian framework

The constrained force caused by the cap beam varies depending on the depth of excavation, soil parameters, and pile properties. Taking into account the uncertainty of the measurement results and soil properties in practical engineering, the force provided by the cap beam is difficult to quantify [27, 28]. The Bayesian method is an ideal method for describing the uncertainty of measurement errors. The method was derived from the general probability theorem. For a given data and prior information, the probability density function of the measurement error is expressed as:

$$f(\delta|\text{Data}, \text{Prior}) = K \int_{\mu,\sigma} f(\delta|\mu,\sigma)f(\text{Data}|\mu,\sigma)f(\mu,\sigma)d\mu d\sigma$$  \hspace{1cm} (1)

where Data is a set of field measurement errors; f(δ|μ,σ) is the probability distribution of the field measurement error taking into account the mean value and standard deviation; f(Data|μ, σ) is the likelihood function that reflects the model fit to the Data; f(μ, σ) is prior distribution of mean and standard deviation; K is the normalized constant. It is assumed that μ and σ are uniformly distributed and independent of each other. Therefore, the preliminary distribution is expressed as follows:

$$f(\mu,\sigma) = \frac{1}{(\mu_{\text{max}} - \mu_{\text{min}}) \times (\sigma_{\text{max}} - \sigma_{\text{min}})}$$  \hspace{1cm} (2)

where μ ∈ (μ_{\text{max}}, μ_{\text{min}}), σ ∈ (σ_{\text{max}}, σ_{\text{min}}). The likelihood function, taking into account the field measurement error, is expressed as:

$$f(\text{Data}|\mu,\sigma) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \times \exp\{-\frac{1}{2}\left[D_i - \mu\right]^2\}$$  \hspace{1cm} (3)

where $D_i = \text{Data}_i$, i = 1, 2, ..., n.

2.2. Bayesian model updating analysis

Soil properties can be updated through the Bayesian model by comparison with field measurements. In this study, Young’s modulus of soil and measurement errors were taken into account in the following
It was assumed that the error probability density function calculated by the model obeys a normal distribution, which can be expressed as:

$$P(\varepsilon|\mu,\sigma,\theta_1,\theta_2,\ldots,\theta_n) = \frac{1}{\sqrt{2\pi}\sigma_e} \exp \left[ -\frac{(\varepsilon - \mu)^2}{2\sigma_e^2} \right]$$ (4)

where $\mu_e$ and $\sigma_e$ are the mean and standard deviation of the calculated errors of the numerical model. It was assumed soil modulus follows a normal distribution as $\mu_e$ and $\sigma_e$. Therefore, the prior probability density function is expressed as:

$$P(\mu_e,\sigma_e,\theta_1,\theta_2,\ldots,\theta_n) = P(\mu_e)P(\sigma_e)P(\theta_1,\theta_2,\ldots,\theta_n)$$ (5)

The likelihood function is the probability density function of the numerical model errors under the condition of the given random variables. In this study, the relationship between the uncertain parameters and the horizontal displacement of the support pile was established using a numerical model. The likelihood function is set for the numerical model error, which is expressed as:

$$P(y|\mu_e,\sigma_e,\theta_1,\theta_2,\ldots,\theta_n) = \frac{1}{\sqrt{2\pi}\sigma_e} \exp \left[ -\frac{(y - g(\theta_1,\theta_2,\ldots,\theta_n) - \mu_e)^2}{2\sigma_e^2} \right]$$ (6)

3. Field instrumented bored piles for an excavated soil slope

3.1. Geological profile

The excavated soil slope, reinforced with 33 bored piles connected by a concrete cap beam, was described in detail by Xu et al. [29]. Three bored piles, namely BP5, BP19, and BP28, were well equipped with distributed optical fiber sensors and conventional inclinometers during excavation. Detailed information on the distributed Brillouin optical time-domain analysis (i.e. BOTDA), sensor calibrations, field installations, and the monitoring process can be found in Xu and Yin (2016) [30] and Xu et al. (2018) [31]. In this study, the bored pile BP5 was analyzed and a representative cross-section of the excavated slope is shown in Figure 1. The excavation width and depth of the slope profile were approximately 20.1 m and 15 m, respectively. The topsoil of the slope was 3 m thick fill soil (i.e. Fill). The second soil layer was about 23 m thick completely decomposed granite (CDG) soil. The bottom soil layer was moderately decomposed granite (MDG). As shown in Figure 1, the slope was excavated in four main stages. Deformation along the distributed optical fiber sensors and lateral displacement along the inclinometer body were recorded at each stage of the excavation.

![Figure 1. Schematic illustration of the cross-section of the slope.](image_url)

3.2. Soil parameters

The parameters of soils CDG, MDG, and Fill in the slope are listed in Table 1 [29]. Due to many uncertainties such as disturbance of field construction, rainfall infiltration, and variations of...
groundwater level, Young’s modulus of CDG and MDG soils was treated as a random variable that obeys a normal distribution in the following Bayesian updating approach. The random variables $E_1$ and $E_2$ denote Young’s modulus of CDG and MDG soils, respectively. As indicated in Table 1, the mean values of $E_1$ and $E_2$ are 19 MPa and 400 MPa, with a corresponding standard deviation of 2 MPa and 30 MPa, respectively.

| Parameters          | Notation | CDG | Fill | MDG  |
|---------------------|----------|-----|------|------|
| Model               | –        | HS  | MC   | HS   |
| Unsaturated unit weight (kN/m³) | $\gamma_{unsat}$ | 17.4 | 17.4 | 19.5 |
| Saturated unit weight (kN/m³)    | $\gamma_{sat}$ | 18.4 | 19.5 | 21.5 |
| Secant stiffness (MPa) | $E^{ref}(E^{ref}_{30})$ | 19 | 15 | 400 |
| Tangent oedometer stiffness (kPa) | $E_{oed}^{ref}$ | 25110 | -- | 513000 |
| Unloading/reloading stiffness (kPa) | $E_{ur}^{ref}$ | 177800 | -- | 911100 |
| Cohesion (kPa)       | c        | 26  | 10   | 25   |
| Friction angle (°)   | $\varphi$ | 34  | 28   | 35   |
| Dilatancy angle (°)  | $\psi$  | 0   | 0    | 0    |
| Poisson’s ratio      | $V_{ur}$ | 0.2 | 0.3  | 0.35 |
| Lateral stress coefficient | $K_{0nc}$ | 0.44 | 0.53 | 0.48 |

3.3. Numerical model
As shown in Figure 1, a two-dimensional FEM model was constructed for this excavation case. The constitutive model of CDG and MDG soils was the Hardening Soil model, while the Fill soil was modeled using the Mohr-Coulomb model. The bored pile and cap beam were modeled with elastic plate elements. The soil-pile interaction was considered by setting the interface elements as presented in Figure 1. The excavation was divided into four main stages: 8, 10.5, 13, and 15 m, separately. The constrained force provided by the cap beam was obtained by back analysis, and Young’s modulus of CDG and MDG soils was treated as a random variable updated in the following Bayesian updating approach. The efficient procedure for the numerical model is shown in Figure 2. In this procedure, the numerical model implemented in a Python program was processed as a model function for the displacement of the bored pile in back analysis.

![Figure 2. Simulation model of the case study.](image)
4. Results and Discussion

4.1. Constrained effect of the cap beam

The constrained force is related to soil behavior and excavation progress. The constrained force provided by the cap beam is difficult to measure in field conditions. Figure 3 shows the back analysis procedure. The constrained force provided by the cap beam $F_T$ was first calculated with the given parameters listed in Table 1. The values of the parameters $a_1$, $a_2$, $a_3$, and $a_4$ were 0.403, $-37.1785$, $1140.36$, and $-6441.8$, respectively. Thus, the calculated constrained force $F_T$ after the first stage of excavation was 508.1 kN. In this study, the cap beam force ranges for excavation stages 1 through 4 were [200 kN, 600 kN], [300 kN, 700 kN], [500 kN, 900 kN], and [700 kN, 1100 kN], respectively.

The back analysis was implemented in a Python program interfaced with the Plaxis software. The FEM model was created. The constrained forces provided by the cap beam were specified as a boundary condition that falls within the above range. The constrained force was increased with a step of 2 kN at each calculation step. Then, using the numerical model, the deflections of the bored pile were obtained. To increase the efficiency of the calculation, 10 points of lateral displacement along the bored pile were compared with the results of field measurements. Average errors were obtained for each calculation step. In this study, 200 numerical models were created and executed at each stage of the excavation. A total of 800 numerical models were created and executed for four stages of excavation. The lateral deflections calculated by the numerical model were compared with the results of field measurements. As shown in Figure 4, the errors between the calculated and the measured results were plotted versus the constrained force $F_T$. The results show that the calculated results have minimal errors compared to the results of field measurements, when the constrained forces of the cap beam at four excavation stages were 364, 516, 752, and 984 kN, respectively.
### Figure 4.

Results of the back analysis at different stages of excavation: (a) 1st stage; (b) 2nd stage; (c) 3rd stage; (d) 4th stage.

Figure 5 shows the numerical results and the results of field measurements. It can be seen that, in addition to well-consistent results numerical results and measurements at the points of maximum deflection, there are significant differences that can be observed in the lower part of the bored pile. The difference increases with the depth of excavation. It may be due to the uncertainty of soil parameters and measurement errors. Thus, it is necessary to analyze the influence of uncertainty on the assessment of the characteristics of bored piles.

### Figure 5.

Comparison of simulated displacement and measured displacement along the pile.
4.2. Cap beam force considering uncertainty

The constrained forces obtained by back analysis are uncertain due to field measurement errors and back analysis. In addition, the relationship is based on four stages of excavation, which were too limited by the probabilistic characterization of the constrained force. Therefore, this study adopted the equivalent sampling approach. The probabilistic characterization of the constrained force \( F_T \) was modeled based on prior knowledge and likelihood function within the Bayesian framework. Prior knowledge includes project-specific measurement data from this case study and published studies in the literature.

As previously mentioned, the constrained forces of the cap beam in the four excavation stages were 364, 516, 752, and 984 kN, respectively. It is possible to obtain a cubic function of the relationship between the constrained force and the depth of the excavation, which can be expressed as:

\[
\frac{F_T}{f_0^{\text{L}}yzk_0dz} = 1.797\left(\frac{D}{L}\right)^3 - 0.6764\left(\frac{D}{L}\right)^2 + 0.4359\frac{D}{L} + \delta \tag{7}
\]

where \( D \) and \( L \) are excavation depth and pile length, respectively. \( \delta \) is a random variable obeying a Gaussian distribution. The random variable \( \delta \) was analyzed by integrating a priori information and transforming the integrated information into a large number of equivalent samples for statistical analysis.

A set of prior knowledge with field measurement of the constrained force of the bored pile is summarized in Table 2. The constrained force was normalized and then compared with the Eq. (8) to obtain the information on the random variable \( \delta \). Based on the prior information, the mean value and standard deviation of the random variable \( \delta \) are 0 and 0.05, respectively. Thus, the likelihood function can be expressed as:

\[
f(\text{Data}|\mu,\sigma) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma} \times \exp\left\{-\frac{1}{2}\left[\frac{D_i - 0}{0.05}\right]^2\right\} \tag{8}
\]

where \( D_i \) is the calculated variable \( \delta \) by Eq. (8) as indicated in Table 4.

| Cases | Constrained force(kN) | Excavation depth(m) | Pile length (m) | Normalized Constrained force | Calculated variable \( \delta \) by Eq.20 | Reference |
|-------|----------------------|---------------------|----------------|-----------------------------|-------------------------------------|-----------|
| 1     | 160                  | 3.59                | 30             | 0.025                       | -0.020                              | Zhao (2014) |
| 2     | 264.95               | 6                   | 22             | 0.072                       | -0.033                              | Wu (2013)  |
| 3     | 295                  | 5.5                 | 20             | 0.065                       | -0.041                              | Liang (2014) |
| 4     | 338.3                | 8.5                 | 21.9           | 0.099                       | -0.073                              | Xu et al. (1998) |
| 5     | 351                  | 5.95                | 32.5           | 0.071                       | 0.003                               | Wu (2013)  |
| 6     | 354                  | 7                   | 24             | 0.089                       | -0.025                              | Wu (2008)  |
| 7     | 723.8                | 7.9                 | 21             | 0.243                       | 0.079                               | Ji (2015)  |
| 8     | 915                  | 9                   | 20             | 0.348                       | 0.126                               | Lu et al (2002) |
| 9     | 109.3                | 2.5                 | 26.6           | 0.030                       | -0.006                              | Li (2009)  |
| 10    | 219.2                | 3                   | 22             | 0.069                       | 0.018                               | Gao (2009) |
| 11    | 300                  | 6                   | 19.5           | 0.092                       | -0.030                              | Yang (2001) |
|       | 294.8                | 3.5                 | 22             | 0.088                       | 0.029                               |           |
| 12    | 319.1                | 5.5                 | 22             | 0.096                       | 0.001                               | Tang (2017) |
| 13    | 376.7                | 7.5                 | 22             | 0.113                       | -0.028                              |           |

| Table 2. Prior information. |
The equivalent samples were calculated using the Markov Chain Monte Carlo Simulation method. Figure 6 shows the error distribution of the 3000 equivalent samples based on the Bayesian method, among which the errors of 2216 samples (73.9%) are in the range from −0.08 to 0.06, the errors of 446 samples (14.9%) are greater than 0.06, and the errors of 337 samples (11.2%) are less than −0.08. To highlight the comparison results, the error value was calculated based on the field measurement value and the calculated value was used instead of the absolute value.

![Figure 6](image)

**Figure 6.** The errors distribution between equivalent samples and field measurement error.

Figure 7(a) shows the comparison between the error samples and the probability density function curve of the equivalent samples. Measurement errors range from −0.08 to 0.06, which is in good agreement with the equivalent samples in Figure 6. Figure 7(b) shows the cumulative probability density functions of the equivalent samples and the measurement error. The results show that the equivalent samples are in good agreement with the measurement results, thus the equivalent samples represent the error samples.

![Figure 7](image)

**Figure 7.** Validation of the probability distribution for the field measurement error of retaining pile crown beam obtained from equivalent samples (a) Probability density function of field measurement error; (b) CDF of field measurement errors.

The normalized coefficient of the constrained force was calculated taking into account the equivalent samples of the measurement error. Figure 8 shows 3000 equivalent samples calculated using the
Bayesian method for the normalization coefficient of the excavation depth and the constrained force by the cap beam. The normalized coefficients of the constrained force and the excavation depth, after taking into account the measurement error, can be obtained as:

$$\frac{F_T}{\int_0^L \gamma z k_d dz_T} = 1.780\left(\frac{D}{L}\right)^3 - 0.486\left(\frac{D}{L}\right)^2 + 0.209\frac{D}{L} + 0.043$$  \hspace{1cm} (9)

![Figure 8. Equation of equivalent sample regression.](image)

To test the reliability of Eq. (10), the random sampling method was used to estimate the constrained force. The excavation depth and the constrained force were taken as input parameters to the numerical model. Figure 9 shows the comparison of the results of random sampling calculation and field measurements. It can be seen that the predicted results are in good agreement with the measured data. As the excavation depth increases, the maximum lateral displacement of the pile in the predicted results gradually increases, which is in good agreement with the results of the theoretical analysis.

![Figure 9. Comparison of prediction results using the regression equation with measurement data.](image)

4.3. Bayesian updating results

In the four stages of excavation, the constraint forces were obtained using Eq. (9). In this Bayesian updating, the uncertainties in Young’s modulus of the soil and measurement errors were taken into account in the following analysis. Young’s modulus of CDG and MDG was strain-dependent, so it has a significant effect on the pile deformations. Because of excavations, plastic deformations occurred at
different stages. The disturbance area in the slope can be divided into different regions based on the deformation cloud map in the deterministic model, as shown in Figure 10. Young’s modulus of all soil disturbance areas was treated as a random variable with the same normal distribution, which was updated by the Bayesian method. The input parameters for each region were $E_{A1}$, $E_{A2}$, $E_{A3}$, $E_{A4}$, $E_{A5}$, $E_{A6}$, and $E_{A7}$. In addition, this paper uses data on the stress-strain behavior of CDG that can be found in Xu [32]. Data regression equations show that $m$ and $\gamma_r$ are 0.772 and 0.095. $E_1$ and $E_2$ are the initial Young’s modulus without regard to stress-strain behavior, which is used to obtain the input parameters by multiplying the normalized Young’s modulus.

![Figure 10](image)

**Figure 10.** Results of parametric back analysis: (a) 1st stage of excavation; (b) 2nd stage of excavation; (c) 3rd stage of excavation; (d) 4th stage of excavation.

It was assumed that $E_1$ is a Gaussian random variable, the mean value of which is 22.6 MPa. The standard deviation of $E_1$ was taken to be 2.38 MPa. It was assumed that $E_2$ is also a Gaussian random variable, the mean value of which is 319.6 MPa. The standard deviation of $E_2$ was taken to be 23.9 MPa. It was assumed that the error of the numerical model has a Gaussian distribution. The mean value was taken to be 0 mm to show that the numerical results are unbiased. The standard deviation of the numerical error was taken to be 1 mm. The Bayesian updating for Young’s modulus began with the first excavation stage and ended with the third excavation stage.

**Table 3.** The updating process of the statistical characteristics of CDG elastic modulus.

| Random variable | The excavation stage | Mean (MPa) | Standard deviation | 90% Confidence interval |
|-----------------|----------------------|------------|--------------------|-------------------------|
| $E_1$ (CDG)     | 1                    | 18.840     | 1.817              | [16.956, 20.724]         |
|                 | 2                    | 18.627     | 1.648              | [16.764, 20.490]         |
|                 | 3                    | 18.339     | 1.435              | [16.505, 20.173]         |
Table 4. The updating process of the statistical characteristics of MDG elastic modulus.

| Random variable | The excavation stage | Mean (GPa) | Standard deviation | 90% Confidence interval |
|-----------------|----------------------|------------|--------------------|-------------------------|
|                 | 1                    | 0.399779   | 0.028215           | [0.359801, 0.439757]    |
| $E_2$ (MDG)     | 2                    | 0.399321   | 0.026978           | [0.359389, 0.439253]    |
|                 | 3                    | 0.400029   | 0.022980           | [0.360026, 0.440032]    |

Values of posterior statistical characteristics (such as mean and standard deviation) of the input parameters were obtained from equivalent samples generated by the MCMCS. Figure 11 shows the distribution of the input parameters for the first Bayesian updating. As shown in Figure 11, the mean of all variables is different due to the disturbance of field construction. In addition, Table 3 and Table 4 show the results of updating $E_1$ and $E_2$, respectively. As indicated in Table 3 and Table 4, the standard deviation of CDG gradually decreases, and the 90% confidence interval of CDG decreases gradually by combining more field measurements. The mean value of MDG soil remains stable while the standard deviation is gradually decreasing. This indicates that the uncertainties of geotechnical parameters are mitigated by the Bayesian updating approach.

Figure 11. Distribution of a random variable at the first Bayesian updating: (a) posterior distribution of $E_1$; (b) posterior distribution of $E^{41}_{MDG}$; (c) posterior distribution of $E^{42}_{MDG}$; (d) posterior distribution of $E^{43}_{MDG}$; (e) posterior distribution of $E^{44}_{MDG}$; (f) posterior distribution of $E^{45}_{MDG}$; (g) posterior distribution of $E^{46}_{MDG}$; (h) posterior distribution of $E^{47}_{MDG}$.

Figure 12 shows the process of updating the posterior probability density function for $E_1$ and $E_2$ from the first to the third excavation stage. As can be seen from Figure 12(a), the probability density curve
for Young’s modulus gradually narrows. In Figure 12(b), the posterior distribution probability density curve gradually narrows, but the mean value remains almost stable. This shows that the Bayesian back analysis can effectively identify geotechnical parameters. To analyze the effect of the Bayesian updating approach, the coefficient of variance (COV) of each updated posterior distribution sample was used to characterize the discreteness of the updated samples, as indicated in Table 5. The COV for $E_1$ and $E_2$ gradually decrease with excavation stages. Discretization of posterior samples decreases with additional excavation information. The study shows that the proposed method can be used to accurately analyze key parameters and guide geotechnical engineering design and construction.

Table 5. COV for modulus of elasticity of CDG and MDG with Bayesian updating.

| Random variable | The excavation stage | COV(%) |
|-----------------|----------------------|--------|
| $E_1$(CDG)      | 2                    | 9.644  |
|                 | 3                    | 8.847  |
|                 | 4                    | 7.825  |
| $E_2$(MDG)      | 2                    | 7.058  |
|                 | 3                    | 6.756  |
|                 | 4                    | 5.745  |

Figure 12. Posterior distribution of the modulus of elasticity in step-by-step update: (a) posterior distribution of CDG modulus; (b) posterior distribution of MDG elastic modulus.

The Bayesian updating procedure was repeated until the third stage of the excavation. The displacements of the support pile were calculated using a numerical model based on recognized parameters. Figure 13 shows the maximum horizontal displacement of the support pile at each stage of excavation. The maximum horizontal displacement of the pile measured in the field and predicted by the numerical model are compared. The maximum and the minimum values of Young’s modulus at the 90% confidence interval after each Bayesian updating are treated as input parameters. The calculation shows that the results of the prediction of the maximum lateral displacement fluctuate more and more sharply with the increase in the stages of excavation. The filled symbols in Figure 13 show the prediction for the future excavation stage using the Bayesian updating approach. As the number of uses of the Bayesian updating increases, the prediction results become more accurate. This study reveals the uncertainty in the prediction of the numerical model and shows that the Bayesian updating approach can reduce the effect of uncertainty.
The first update
The second update
The third update
The first update
The second update
The third update
The first update
The second update
The third update

Figure 13. Comparison of prediction results using the Bayesian updating and measured data.

5. Conclusions
Uncertainty in geotechnical engineering has a significant impact on the deformation analysis of support piles. In this study, the maximum constrained forces of the cap beam were investigated as a function of the depths of the excavation. The Bayesian updating procedure was proposed and applied in the field slope excavation project. The main conclusions are summarized as follows:
(1) A simplified mechanical model of the slope was created. A cubic function was established between the depth of the excavation and the constrained force.
(2) Taking into account the errors of the field measurement data, the Bayesian method was adopted to update the formulas for constrained force and depth of excavation through the equivalent samples. An improved approach has been proposed for estimating the constrained force of the cap beam for different levels of excavation. The approach was verified with literature examples and field measurement results in this study. This proves that the proposed approach has good accuracy for estimating the constrained force of the cap beam.
(3) Uncertainties in geotechnical parameters can be reduced by Bayesian updating based on field measurements. With an increase in the number of excavations stages, the range of parameter fluctuations decreases. The predicted displacements of the support pile are progressively aligned with the field measurements through the Bayesian updating process. This shows that the updating approach can be used as an efficient tool for reducing the impact of uncertainty and guiding geotechnical engineering design and construction.

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