Modified gravity inspired DGP brane cosmology

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Abstract

We consider a DGP brane scenario where a scalar field is present on the brane through the introduction of a scalar potential, itself motivated by the notion of modified gravity. This theory predicts that the mass appearing in the gravitational potential is modified by the addition of the mass of the scalar field. The cosmological implications that such a scenario entails are examined and shown to be consistent with a universe expanding with power-law acceleration.

1 Introduction

The idea that extra dimensions can be probed by gravitons and eventually non-standard matter has been the dominant trend in the recent past. These models usually yield the correct Newtonian \(1/r\) potential at large distances because the gravitational field is quenched on sub-millimeter transverse scales. This quenching appears either due to finite extension of the transverse dimensions [1, 2] or due to sub-millimeter transverse curvature scales induced by negative cosmological constants [3, 4, 5, 6, 7, 8]. A feature common to these type of models is that they predict deviations from the usual 4D gravity at short distances. The model proposed by Dvali, Gabadadze and Porrati (DGP) [9, 10] is very different in that it predicts deviations from the standard 4D gravity over large distances. The transition between four and higher-dimensional gravitational potentials in the DGP model arises because of the presence of both the brane and bulk Einstein terms in the action. An interesting observation was made in [11, 12] where it was shown that the DGP model allows for an embedding of the standard Friedmann cosmology in the sense that the cosmological evolution of the background metric on the brane can entirely be described by the standard Friedmann equation plus energy conservation on the brane. This was later generalized to arbitrary number of transverse dimensions in [13]. For a comprehensive review of the phenomenology of DGP cosmology, the reader is referred to [14].

An interesting observation made a few years ago is that the expansion of our universe is currently undergoing a period of acceleration which is directly measured from the light-curves of several hundred type Ia supernovae [15, 16] and independently from observations of the cosmic microwave background (CMB) by the WMAP satellite [17] and other CMB experiments [18]. However, the mechanism responsible for this acceleration is not well understood and many authors introduce a mysterious cosmic fluid, the so called dark energy, to explain this effect [19]. Recently, it has been shown that such an accelerated expansion could be the result of a modification to the Einstein-Hilbert action [20]. One such modification has been proposed in [21] where a term of the form \(R^{-1}\) was added to...
the usual Einstein-Hilbert action. It was then shown that this term could give rise to accelerating solutions of the field equations without dark energy.

In this paper, we focus attention on the DGP brane model and introduce a scalar field on the brane. The potential describing such a scalar field is taken to be that appearing in modified theories of gravity when the term $R^{-1}$ is added to the usual Einstein-Hilbert action. This model predicts that for such a potential, the mass density should be modified by the addition of the mass density of the corresponding scalar field on the brane. We obtain the evolution of the metric on the spacetime by solving the field equations in the limit of small curvature, predicting a power-law acceleration on the brane. The components of the metric in Gaussian normal coordinates are also calculated and presented.

2 DGP model with a brane scalar field

We start by writing the action for the DGP model with a scalar field on the brane part of the action

$$S = \frac{m_3^3}{2} \int d^5x \sqrt{-g} R + \int d^4x \sqrt{-q} \left[ \frac{m_3^3}{2} R - \frac{1}{2} (\nabla \Phi)^2 - V(\Phi) \right] + S_m[q_{\mu\nu}, \psi_m].$$

where the first term in (1) corresponds to the Einstein-Hilbert action in 5D for the 5-dimensional bulk metric $g_{AB}$, with the Ricci scalar denoted by $R$. Similarly, the second term is the Einstein-Hilbert action for a scalar field $\Phi$ corresponding to the induced metric $q_{\mu\nu}$ on the brane, where $R$ is the relevant scalar curvature and $m_3$ and $m_4$ are reduced Planck masses in four and five dimensions respectively and $S_m$ is the matter action on the brane with matter field $\psi_m$. The induced metric $q_{\mu\nu}$ is defined as usual from the bulk metric $g_{AB}$ by

$$q_{\mu\nu} = \delta^A_{\mu} \delta^B_{\nu} g_{AB}.$$  

It would now be possible to write the field equations resulting from this action, yielding, in $d−1$ spatial dimensions

$$m_4^2 \left( R_{AB} - \frac{1}{2} g_{AB} R \right) + m_2^2 \delta^\mu_A \delta^\nu_B \left( R^{(d−1)}_{\mu\nu} - \frac{1}{2} q_{\mu\nu} R^{(d−1)} \right) \delta(y) = \delta^\mu_A \delta^\nu_B \left( T_{\mu\nu} + \mathcal{T}_{\mu\nu} \right) \delta(y),$$

where $T_{\mu\nu}$ is the energy-momentum tensor in the matter frame, and

$$\mathcal{T}_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - \frac{1}{2} q_{\mu\nu} q^{\alpha\beta} \partial_\alpha \Phi \partial_\beta \Phi - V(\Phi) q_{\mu\nu},$$

with the equation of motion for $\Phi$ becoming

$$\nabla_\mu \nabla^\mu \Phi = \frac{dV(\Phi)}{d\Phi}.$$  

Note that $\Phi$ lives on the brane. The corresponding junction conditions, relating the extrinsic curvature to the energy-momentum tensor, become $[11, 12]$

$$\lim_{\epsilon \to +0} [K_{\mu\nu}]_{y=+\epsilon} = \lim_{\epsilon \to -0} [K_{\mu\nu}]_{y=-\epsilon} = \frac{1}{m_4^2} \left( T_{\mu\nu} + \mathcal{T}_{\mu\nu} - \frac{1}{d-1} q_{\mu\nu} q^{\alpha\beta} (T_{\alpha\beta} + \mathcal{T}_{\alpha\beta}) \right) \bigg|_{y=0} - \frac{m_2^2}{m_4^2} \left( R^{(d-1)}_{\mu\nu} - \frac{1}{2(d-1)} q_{\mu\nu} q^{\alpha\beta} R^{(d-1)}_{\alpha\beta} \right) \bigg|_{y=0}.$$  

In order to get a qualitative picture of how gravity works for the DGP braneworld, let us take small metric fluctuations around flat, empty space and look at gravitational perturbations, $h_{AB}$, that is

$$g_{AB} = \eta_{AB} + h_{AB},$$

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where $\eta_{AB}$ is the five-dimensional Minkowski metric. Choosing the harmonic gauge in the bulk

$$\partial^A h_{AB} = \frac{1}{2} \partial_B h_A^A,$$  

(8)

the $\mu 5$-components of this gauge condition lead to $h_{\mu 5} = 0$, so that the surviving components are $h_{\mu \nu}$ and $h_{55}$. The latter component is solved by the following equation

$$\Box^{(5)} h_5^5 = \Box^{(5)} h^\mu_\mu,$$  

(9)

where $\Box^{(5)}$ is the five-dimensional d’Alembertian. The $\mu \nu$-component of the field equations (3) become, after a little manipulation [10]

$$m_4^3 \Box^{(5)} h_{\mu \nu} + m_3^2 \left( \Box^{(4)} h_{\mu \nu} - \partial_\mu \partial_\nu h^5_5 \right) \delta(y) = -2\delta(y) \left[ T_{\mu \nu} + T_{\mu \nu} - \frac{1}{d - 1} \eta_{\mu \nu} \eta^{\alpha \beta} (T_{\alpha \beta} + T_{\alpha \beta}) \right],$$  

(10)

where $\Box^{(4)}$ is the four-dimensional (brane) d’Alembertian, and we take the brane to be located at $y = 0$. This yields the equation for the gravitational potential of mass densities $\rho(\vec{r}) = M \delta(\vec{r})$ and $\rho_\Phi(\vec{r}) = M_\Phi \delta(\vec{r})$ on the brane

$$m_4^3 \left( \Box^{(4)} + \partial^2_y \right) U(\vec{r}, y) + m_3^2 \delta(y) \Box^{(4)} U(\vec{r}, y) = \frac{2}{3} (M + M_\Phi) \delta(\vec{r}) \delta(y).$$  

(11)

Therefore, in equation (11), the mass is modified by the addition of the mass of the scalar field. The resulting gravitational potential for $r \ll \ell_{DGP}$ is given by [9, 11]

$$U(\vec{r}) = -\frac{(M + M_\Phi)}{6\pi m_3^2 r} \left[ 1 + \left( \gamma - \frac{2}{\pi} \right) \frac{r}{\ell_{DGP}} + \frac{r}{\ell_{DGP}} \ln \left( \frac{r}{\ell_{DGP}} \right) + \mathcal{O} \left( \frac{r}{\ell_{DGP}} \right)^2 \right],$$  

(12)

and

$$U(\vec{r}) = -\frac{(M + M_\Phi)}{6\pi^2 m_3^4 r^2} \left[ 1 - 2 \left( \frac{r}{\ell_{DGP}} \right)^{-2} + \mathcal{O} \left( \frac{r}{\ell_{DGP}} \right)^{-4} \right],$$  

(13)

for $r \gg \ell_{DGP}$, where $\gamma = 0.577$ is the Euler constant and $\ell_{DGP} = m_3^2 / 2m_4^3$ is the transition scale between the four and five-dimensional behavior of gravitational potential that the DGP scenario predicts.

### 3 DGP cosmology with a brane scalar field

Although the DGP model predicts deviations to gravity at large distances, it could account for the standard cosmological equations of motion at any distance scale on the brane. It is therefore appropriate to start by writing the from of the line element in brane gravity, that is

$$ds^2 = g_{\mu \nu} dx^\mu dx^\nu + b^2(y, t) dy^2 = -n^2(y, t) dt^2 + a^2(y, t) \gamma_{ij} dx^i dx^j + b^2(y, t) dy^2,$$  

(14)

where $\gamma_{ij}$ is a maximally symmetric 3-dimensional metric where $k = -1, 0, 1$ parameterizes the spatial curvature. Building on the results of [22], the cosmological evolution equations of a 3-brane in a 5-dimensional bulk resulting from equations (3) and (6) were presented in the first two references in [20]. Here, we will follow [11, 12] and only give the results relevant to the present work for a brane of dimension $\nu + 1$ and the scalar field $\Phi$. A detailed discussion on the derivation of these results can be found in the said references. Adopting the Gaussian normal system gauge

$$b^2(y, t) = 1,$$  

(15)

the field equations on the brane for metric (14) and $d = \nu + 1$ spatial dimensions are

$$G^{(\nu)}_{00} = \frac{1}{2} \nu (\nu - 1) n^2 \left( \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) = \frac{1}{m_{\nu - 1}^2} T_{00},$$  

(16)
\( G_{ij}^{(\nu)} = (\nu - 1) \left( \frac{\dot{n} \dot{a}}{n^3 a} - \frac{\dot{a}}{n^2 a} \right) q_{ij} - \frac{1}{2} (\nu - 1)(\nu - 2)n^2 \left( \frac{\ddot{a}^2}{n^2 a^2} + \frac{k}{a^2} \right) q_{ij} = \frac{1}{m_{\nu - 1}^\nu} T_{ij}, \) 

\( \nabla_\mu \nabla^\mu \Phi = \frac{dV(\Phi)}{d\Phi}. \)

The junction conditions (6) for an ideal fluid on the brane, given by

\( T_{\mu\nu} = (\rho + p)u_\mu u_\nu + p q_{\mu\nu}, \)

together with energy conservation resulting from the vanishing of

\( G_{05} = \nu \left( \frac{n' \dot{a}}{na} - \frac{\dot{a}}{a} \right) = 0, \)

in the bulk, leads to

\( (\dot{\rho} + \dot{\rho}_\Phi) a\big|_{y=0} = -\nu (\rho + \rho_\Phi + p + p_\Phi) \dot{a}\big|_{y=0}, \)

where

\( \rho_\Phi = \left[ \frac{1}{2} \dot{\Phi}^2 + n^2 V(\Phi) \right]_{y=0}, \)

\( p_\Phi = a^2 \left[ \frac{1}{2n^2} \dot{\Phi}^2 - V(\Phi) \right]_{y=0}. \)

One may now proceed to obtain the cosmological equations by taking the gauge

\( n(0, t) = 1, \)

and performing the transformation

\( t = \int^{t} n(0, \tau) d\tau, \)

of the time coordinate. This gauge is convenient because it gives the usual cosmological time on the brane. Consequently, we find that our basic dynamical variable is \( a(y, t) \) with \( n(y, t) \) given by

\( n(y, t) = \frac{\dot{a}(y, t)}{\dot{a}(0, t)}. \)

The basic set of cosmological equations in the present setting for a brane scalar field without a cosmological constant in the bulk now become

\[
\begin{align*}
\lim_{\epsilon \to +0} [\partial_y a]_{y=+\epsilon}^{y=-\epsilon}(t) &= \frac{m_{\nu - 1}}{2m_{\nu + 1}} (\nu - 1) \left[ \frac{\ddot{a}^2(0, t)}{a(0, t)} + \frac{k}{a(0, t)} \right]_{y=0} \\
&\quad - \frac{(\rho + \rho_\Phi)a(0, t)}{\nu m_{\nu + 1}}_{y=0}, \\
I^+ &= \left[ \ddot{a}^2(0, t) - a^2(0, t) + k \right] a^{\nu - 1}(0, t)_{y>0}, \\
I^- &= \left[ \ddot{a}^2(0, t) - a^2(0, t) + k \right] a^{\nu - 1}(0, t)_{y<0}, \\
\nabla_\mu \nabla^\mu \Phi &= \frac{dV(\Phi)}{d\Phi}, \\
n(y, t) &= \frac{\dot{a}(y, t)}{\dot{a}(0, t)}. 
\end{align*}
\]
It is appropriate at this point to discuss the cosmology in the DGP model by taking $I^+ = I^-$. The cosmological equations in this framework for a $(\nu - 1)$-dimensional space are given by

\[ \frac{\dot{a}^2(0, t) + k}{a^2(0, t)} = \frac{2(\rho + \rho_\Phi)}{\nu(\nu - 1)m_\nu^{-1}}, \tag{32} \]

\[ \ddot{\Phi} + 3 \frac{\dot{a}(0, t)}{a(0, t)} \dot{\Phi} + \frac{dV(\Phi)}{d\Phi} = 0, \tag{33} \]

\[ I = \left[ \dot{a}^2(0, t) - a^2(y, t) + k \right] a^{\nu-1}(y, t), \tag{34} \]

\[ n(y, t) = \frac{\dot{a}(y, t)}{a(0, t)}. \tag{35} \]

Equations (34) and (35) may now be used, taking $\nu = 3$, to obtain the components of the metric

\[ a^2(y, t) = a^2(0, t) + (\dot{a}^2(0, t) + k) y^2 + 2 \sqrt{(\dot{a}^2(0, t) + k)a^2(0, t) - I y}, \tag{36} \]

\[ n(y, t) = \left[ a(0, t) + \dot{a}(0, t)y^2 + a(0, t) y \frac{a(0, t)\dot{a}(0, t) + \ddot{a}(0, t) + k}{\sqrt{(\dot{a}^2(0, t) + k)a^2(0, t) - I}} \right] \frac{1}{a(y, t)}. \tag{37} \]

The form of these equations becomes particularly simple for $I = 0$, that is

\[ a(y, t) = a(0, t) + \sqrt{\dot{a}^2(0, t) + k} y, \tag{38} \]

\[ n(y, t) = 1 + \frac{\dot{a}(0, t)}{\sqrt{\dot{a}^2(0, t) + k}} y. \tag{39} \]

4 $R^{-1}$ inspired DGP scenario

To progress further, the form of the potential $V(\Phi)$ should be specified. Motivated by theories of modified gravity where a $R^{-1}$ term is present in the action and with an eye on the effects that such a modification may have on the DGP brane scenarios we take the potential as [21]

\[ V(\Phi) \simeq \mu^2 m_3^2 \exp\left( -\sqrt{3/2}\Phi/m_3 \right). \tag{40} \]

This is the form of the potential one encounters when studying theories of modified gravity with a Lagrangian of the form $\mathcal{L}(R) = R - \frac{\mu^4}{R}$ and we shall concentrate in the limit of small $R$ in the Einstein frame. Our aim is to obtain explicitly the components of the metric on the brane, i.e. equations (36) and (37) in the limit of small curvature, and show that the results are consistent with an accelerating universe. Note that one may obtain an effective mass for the scalar field, $M_{\Phi}^{\text{eff}}$, from the second derivative of the potential $V(\Phi)$ in the Einstein frame in the usual 4D gravity with $\mu \sim H_0 \simeq 10^{-32}$ Gev and evaluating $V''(\Phi)$ around $R \sim H_0^2$, noting that $\Phi/m_3 \sim 1$. This leads to an effective mass squared of order $\mu^2$, where $\mu$ is the cosmological constant in $R^{-1}$ gravity [21]. Hence, for models of modified gravity with any function of the Ricci scalar, $\mathcal{L}(R)$, within the framework of the discussion at hand, we must take into account the contribution of the vacuum energy (cosmological constant) to the gravitational potential.

To proceed, we consider the evolution of the scale factor with time on the brane. From equations (32) and (33) for the spatially flat FRW metric and setting $\nu = 3$, we can write

\[ 3H_0^2 = \frac{1}{m_3^2} (\rho + \rho_\Phi), \tag{41} \]
\[ \ddot{\Phi} + 3H_0\dot{\Phi} + \frac{dV(\Phi)}{d\Phi} = 0, \]  
(42)

where \( H_0 \equiv \frac{\dot{a}(0,t)}{a(0,t)} \) and for \( n(0,t) = 1 \) (on the brane) we have

\[ \rho_\Phi = \frac{1}{2}\dot{\Phi}^2 + V(\Phi). \]  
(43)

We must now solve the system of equations (41) and (42) with \( \rho = 0 \). Substituting potential (40) into equations (41) and (42), we obtain the evolution of the scale factor on the brane

\[ a(0,t) \propto t^{4/3}, \]  
(44)

together with that of the scalar field

\[ \Phi \propto -\frac{4}{3} \ln t. \]  
(45)

As can be seen, equation (44) predicts a power-law acceleration on the brane. This result is consistent with the observational results similar to quintessence with the equation of state parameter \(-1 < w_{DE} < -\frac{1}{3}\), [23].

To continue, we find the evolution of \( a(y,t) \) and \( n(y,t) \) everywhere in spacetime. Thus, using equations (44) and substituting into equations (36) and (37), one finds

\[ a^2(y,t) = C^2 \left[ t^{8/3} + \frac{16}{9} t^{2/3} y^2 \right] + 2 \sqrt{\frac{16}{9}} C^4 t^{10/3} - I y. \]  
(46)

and

\[ n(y,t) = C \left[ t^{4/3} + \frac{4}{9} y^2 t^{-2/3} + t^{4/3} y \sqrt{\frac{16}{9}} C^2 t^{4/3} \frac{20}{9} C^2 t^{4/3} \frac{4}{9} \right] \frac{1}{a(y,t)}. \]  
(47)

In the particular case \( I = 0 \), we obtain

\[ a(y,t) = C \left[ t^{4/3} + \frac{4}{3} t^{1/3} y \right], \]  
(48)

and

\[ n(y,t) = C \left[ 1 + \frac{y}{3t} \right], \]  
(49)

where \( C \) is the proportionality constant. Note that for \( y = 0 \), equations (48) and (49) reduce to (44) and \( n(0,t) = 1 \) respectively. There appears coordinate singularities on the space-like hypercone \( y = \pm 3t \). This is presumably a consequence of the fact that the orthogonal geodesics emerging from the brane (which we used to set up our Gaussian normal system, \( b^2 = 1 \)) do not cover the full five-dimensional spacetime.

5 Conclusions

Brane models provide a somewhat exotic, yet interesting extension of our parameter space for gravitational theories. In this work we have considered the DGP model with a brane scalar field. The scalar field was motivated and inspired by a desire to study the effects of modified gravity, represented by a term like \( R^{-1} \). We have shown that this model predicts that the mass in the gravitational potential is modified by the addition of the mass of such a scalar field. The cosmological evolution of this model was also studied by solving the relevant dynamical equations and the components of the metric was obtained in the limit of small curvature. The evolution of the universe in such a scenario seems to be consistent with the present observations, predicting an accelerated expansion.
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