Role of the deep ocean in forming of the global warming slowdown

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Abstract. A slowdown of the increase of the surface air temperature (SAT) at the beginning of the 21 century has provoked the discussion for the phenomenon nature and responsible physical mechanisms. One of the most accepted hypothesis connects the slowdown of SAT rise to anomalously intensive uptake of heat energy by the deep ocean layers. We suppose that verification of the theory can be achieved via the study of the interrelationship of the surface and the deep ocean layers temperatures. Establishing the time shifts and especially the sign of the cross-correlation function of the two temperatures can shed light on the solidity of the theory background. For the aims of the study, we have used a stochastically forced two-box climate model. Analytical estimation of correlation functions and spectral characteristics of the model demonstrates that deep ocean heat uptake cannot serve as a driver of the SAT slowdown (at least in the framework of the model). The cross-correlation function of upper layer and deep ocean temperatures stays nonnegative for any time lag. It means that heat redistribution between layers cannot lead to hiatus forming. Moreover, a comparison of the two-box model and Hasselmann model spectral characteristics evidences that the deep ocean serves as a pacemaker of climate variability in the broad range of frequencies.

1. Introduction

The recent slowdown of the SAT rise named often as a hiatus has occurred for about fifteen years from 1998 to 2012. The phenomenon has initiated numerous studies and speculation of its physical mechanisms. Some publications have even questioned the anthropogenic nature of modern climate warming. Acceleration of the warming process since 2012 and record-breaking SAT in the last decade demonstrate, however, the stability of the global climate warming tendencies.

Despite the recent intensification of global warming, the mechanisms of the hiatus forming remain in the focus of scientific community interest, Medhaug, 2017, [1]. One of the most popular hypotheses connects the SAT slowdown to redistribution of the energy between upper oceanic level and deep ocean (Meehl et al., 2011, [2], Drijfhout, 2014, [3], Volodin and Gritsun, 2018, [4]). The studies of the ocean heat content variation support the heat redistribution suggestion (Yan et al., 2016, [5], Cheng et al., 2019a, [6]). Oceanic heat content has demonstrated a stable linear rise before, during and after the SAT slowdown. The estimated total amount of energy gained every year by the Earth climate system remains relatively constant and equals $\sim 10^{22}$ J/year, Cheng et al., 2019b, [7]. Redistribution of the energy between upper and deeper oceanic layers could lead to the forming of the SAT rise temporal slowdown, Drijfhout, 2018, [8]. On the other hand, analyses of the observation data and model outputs have not demonstrated out of phase variations of the water temperature in the upper layers and the deep ocean, Tokarska et al., 2019, [9].

The proposed research is devoted to the role of the deep ocean in the forming of hiatus. On the base of the simple model, we have attempted to check the hypothesis about oceanic vertical heat redistribution as the main mechanism of the SAT slowdown. In the second section, we briefly discuss the box model of Held et al., 2010, [10] and we present its statistically stable solutions in the condition of external stochastic forcing. The third section contains the study of the correlation, cross-correlation and spectral properties of the model parameters. In the last part of the paper, we discuss the results and present the conclusions.

2. Stochastically forced box model
For the aims of the study, we have used box model Held et al., [10] in the condition of random forcing. The model has become widely accepted for the research of the climate system response to the anthropogenic forcing (Seshadri, 2017, [11], Rohrshneider et al., 2019, [12]). Geoffroy et al., 2013 [13] received valuable results by estimating the coefficients of the model on the base of CMIP5 experiments. In the recent research Soldatenko & Yusupov, 2018, [14], Soldatenko & Colman, 2019, [15], and Williamson et al., 2018, [16], Nijsse et al., 2019, [17] studied some important effects of stochastic external forcing on the base of the two-box model.

Equations of the model take the form as follows,

$$ C \frac{dT_U}{dt} = -\lambda T_U - \gamma (T_U - T_0) + F(t), \quad (1) $$

$$ C_0 \frac{dT_0}{dt} = \gamma (T_U - T_0), \quad (2) $$

where $T_U$ - upper level temperature, $T_0$ - deep ocean temperature, $C$ and $C_0$ - heat capacity of the upper level and deep ocean, correspondingly, $\gamma$ - coefficient controlling the heat exchange, $\lambda$ - climate feedback parameter. The time unit is equal to one year. External forcing $F(t)$ is assumed proportional to the stationary Gaussian random process. $F(t) = \sigma_F \frac{dW}{dt}, \ W$ - standard Wiener process, $\sigma_F^2$ - variance of the external stochastic forcing.

The equation describing the variability of the deep ocean temperature can be written in the form of the linear damping oscillator

$$ \frac{d^2T_0}{dt^2} + 2a \frac{dT_0}{dt} + bT_0 = mF(t), \quad (3) $$

where,

$$ 2a = \frac{\lambda + \gamma}{C} + \frac{\gamma}{C_0}, \quad b = \frac{\lambda \gamma}{CC_0}, \quad m = \frac{\gamma}{CC_0}. $$

The parameter $b - a^2$ is always negative, in so far as

$$ b - a^2 = -\frac{1}{4} \left[ \left( \frac{\lambda}{C} - \frac{\gamma}{C_0} \right)^2 + \frac{2\lambda \gamma}{C^2} + \frac{\lambda^2}{C^2} + \frac{2\gamma^2}{CC_0} \right] < 0. $$

That peculiarity has determined a unique type of the statistically stable solution that takes the form of Duhamel (convolution) integral,

$$ T_0(t) = \frac{\gamma}{\rho} \int_0^t F(t-u)h_\rho(u)du, \quad (4) $$

where $h_\rho(u)$ is weight function,

$$ h_\rho(u) = \frac{m}{\rho} \exp(-au)sh(\rho u), \quad (5) $$

$$ \rho = \sqrt{a^2 - b}, \quad sh(x) = \frac{\exp(x) - \exp(-x)}{2}. $$

Statistically stable solution for the upper level temperature after some simplification accepted the convolution form,
\[ T_U(t) = \int_0^\infty F(t-u)\varphi(u)du , \]  

where \( \varphi(u) \) is corresponding weight function, 

\[ \varphi(u) = \frac{1}{C} \frac{m^2}{2\rho} \left( \frac{1}{\lambda + \gamma} \exp(-(a + \rho)u) + \frac{1}{\lambda + \gamma} \exp((\rho - a)u) \right) . \]  

### 3. Correlation and spectral characteristics of the model

#### 3.1. Correlation functions and variances

Correlation functions have the general convolution form

\[ B_{\varphi}(\tau) = \sigma_\varphi^2 \int_0^\infty \varphi(u)\varphi(u + \tau)du , \quad B_{\varphi}(\tau) = \sigma_\varphi^2 \int_0^\infty \varphi(u)\varphi(u + \tau)du . \]  

The previous equation allows representing the correlation function of the upper level temperature as

\[ B_{\varphi}(\tau) = \frac{\sigma_\varphi^2 m^2}{8\rho^2} \left[ A_1 \exp(-(a - \rho)\tau) + A_2 \exp(-(a + \rho)\tau) \right] . \]  

where,

\[ A_1 = \frac{1}{(a - \rho)(\gamma + \lambda - C(a - \rho))^2} + \frac{C_0}{aC\gamma^2} , \]

\[ A_2 = \frac{1}{(a + \rho)(\gamma + \lambda - C(a + \rho))^2} + \frac{C_0}{aC\gamma^2} . \]  

The process \( T_U(t) \) is non-differentiable because,

\[ \frac{dB_{\varphi}(\tau)}{d\tau} \bigg|_{t=0} \neq 0 . \]

The correlation function of the deep ocean temperature has a form

\[ B_{\varphi}(\tau) = \sigma_\varphi^2 \int_0^\infty \varphi(u)\varphi(u + \tau)du = \sigma_\varphi^2 \exp(-a\tau)m^2(\rho \cosh(\rho\tau) + a \sinh(\rho\tau)) \frac{4\gamma ab}{4\gamma ab} . \]  

In contrast to \( T_U(t) \) the temperature of the deep ocean is a differentiable process.

Variances of the upper level and deep ocean temperatures are as follows,

\[ \text{Var}[T_U] = \sigma_\varphi^2 \frac{C\gamma + C_0\lambda}{2C\gamma(C\gamma + C_0(\gamma + \lambda))} . \]  

\[ \text{Var}[T_0] = \frac{\sigma_\varphi^2 m^2}{4ab} = \sigma_\varphi^2 \frac{\gamma}{2\lambda(C\gamma + C_0(\gamma + \lambda))} . \]  

Soldatenko & Colman, 2019, [15] received expression for the variance of upper level temperature identical to (16) on the base of the Fokker-Planck equation solving. The ratio of the two variances is proportional to the ratio of heat capacities.
\[
\frac{\text{Var}[T_U]}{\text{Var}[T_H]} = 1 + \frac{C_0 \lambda}{C \gamma}.
\] (14)

In case \( \gamma = 0 \) system (1-2) has become equivalent to the classical Hasselmann model, [18] for the upper ocean level, where the variance of the temperature, \( \text{Var}[T_H] = \frac{\sigma_U^2}{2C \lambda} \). Ratio of the upper level temperature variances of the two models,

\[
\frac{\text{Var}[T_U]}{\text{Var}[T_H]} = \frac{C \gamma + C_0 \lambda}{C \gamma + C_0 (\gamma + \lambda)} < 1.
\] (15)

Deep ocean layers support the stabilization of the system and decrease variability of the upper level.

The correlation function of the upper level temperature (Fig.1) demonstrates two intervals characterized by the very distinct rates of exponential decay. Worth to note that the deterministic forcing of the system (1-2) is characterized by the existence of the two relaxation time scales, Geoffroy et al., 2013, [13]. The correlation function of the deep ocean temperature seems as well exponential but into the vicinity of the zero delay behavior changes and derivative of the function is equal to zero (Fig.1.b).

![Fig.1.](image)

a) The correlation function of the model parameters. Red-line – upper level temperature, blue-line – deep ocean temperature. Note the logarithmic scale for the y-axis.

b) Correlation function of the deep ocean temperature at limited time lags (linear scale). CMIP5 ensemble-averaged parameters (Geoffroy et al, 2013, [13]) were used; \( \lambda = 1.13, \ \gamma = 0.67, \ C = 8.0, \ C_0 = 107.0 \).

3.2. Spectral properties of the temperature

Spectral densities of the upper level and deep ocean temperature follow the equations

\[
S_{T_U}(\omega) = \frac{\sigma_U^2 \left( \gamma^2 + C_0 \omega^2 \right)}{2C_0 \gamma \lambda \omega^2 + C_0^2 \omega^4 \left( \lambda^2 + C^2 \omega^2 \right) + \gamma^4 \lambda^2 + \left( C + C_0 \right) \omega^2},
\] (16)
$$S_{T_0}(\omega) = \frac{\sigma^2 \lambda^2 (a + \rho)^2 + \omega^2 (a + \rho)^2 + \omega^2)}{((a - \rho)^2 + \omega^2)((a + \rho)^2 + \omega^2)}.$$  \hfill (17)

Worth to mention that spectral density corresponding to Hasselmann model, \(C \frac{dT_H}{dt} = -\lambda T_H + F(t)\) is described by the relationship, \(S_{T_H} = \frac{\sigma^2}{\lambda^2 + C^2 \omega^2}\). Into ultra-low frequency limit (\(\omega \to 0\)) spectral densities of \(T_u\) and \(T_0\) as well as \(T_H\) are the same and equal, \(S_{T_u|\omega=0} = S_{T_0|\omega=0} = S_{T_H|\omega=0} = \frac{\sigma^2}{\lambda^2}\).

Equality of the spectral densities on the zero frequency for these parameters was demonstrated in the recent paper of Williamson et al., 2018, [16].

The ratio of the two box (1-2) and Hasselmann model spectral densities of the upper level temperature takes the form

$$\text{RatS}(\omega) = \frac{S_{T_u}(\omega)}{S_{T_H}(\omega)} = \frac{2C_0 \gamma \omega^3 + C_0 \omega^3 (\lambda^2 + C^2 \omega^2) + \gamma^2 (\lambda^2 + (C + C_0)^2 \omega^2)}{2C_0 \gamma \lambda \omega^3 + C_0 \omega^3 (\lambda^2 + C^2 \omega^2) + \gamma^2 (\lambda^2 + (C + C_0)^2 \omega^2)}.$$ \hfill (18)

It is obvious that \(\text{RatS}(\omega) < 1\) for any value of frequency \(\omega\). It means that the variability of \(T_u\) in model (1-2) is less than variability of \(T_H\) in any frequency range (Fig.2). Note that ratio \(\text{RatS}(\omega)\) reaches an extreme value (maximum) on frequency \(\omega_{Cr,1}\) coinciding with the frequency of deep ocean undamped oscillator (see equation 3),

$$\omega_{Cr,1} = \sqrt{b} = \frac{\gamma \lambda}{\sqrt{C C_0}}.$$ \hfill (19)

$$\text{RatS}(\omega_{Cr,1}) = \left(\frac{C \gamma + C_0 \lambda}{C \gamma + C_0 (\gamma + \lambda)}\right)^2 = \frac{\text{Var}[T_u]}{\text{Var}[T_H]}, \text{ (see eq. 15).}$$ \hfill (20)

Fig.2. Spectral density of temperature. Red-line – upper level temperature in the model (1-2), blue-line – deep ocean, purple line - Hasselmann model. Parameter values are the same as at Fig.1.

3.3. Cross-correlation function
Cross-correlation function of $T_U(t)$ and $T_0(t)$ can be easily constructed on the base of the equation

$$B_{T_U,T_0}(\tau) = E[T_U(t)T_0(t+\tau)] = \sigma_T^2 \int_0^\infty h_{T_U}(u)h_{T_0}(u+\tau)du. \quad (21)$$

If the temperature of the upper level leads, i.e. $\tau > 0$,

$$B_{T_U,T_0}(\tau) = \frac{\sigma_T^2}{8a^2\rho\lambda C} \left[ e^{(\rho-a)\tau} \left( \frac{C_0}{C_0} + a - \rho \right) - e^{-(\rho+a)\tau} \left( \frac{C_0}{C_0} + a + \rho \right) \right]. \quad (22)$$

If the upper level temperature lags, i.e. $\tau < 0$, it is convenient using equality $B_{T_U,T_0}(-\tau) = B_{T_U,T_0}(\tau)$, to write

$$B_{T_U,T_0}(\tau) = \frac{\sigma_T^2}{8a^2\rho\lambda C} \left[ e^{(\rho-a)\tau} \left( \frac{C_0}{C_0} - a + \rho \right) - e^{-(\rho+a)\tau} \left( \frac{C_0}{C_0} - a - \rho \right) \right]. \quad (23)$$

At the zero-lag cross-correlation function can be represented as $B_{T_U,T_0}(0) = \frac{\sigma_T^2\gamma}{2\lambda(C_0\gamma + C_0(\gamma + \lambda))}$. The coefficient of correlation of $T_U(t)$ and $T_0(t)$ follows the equation,

$$r_{T_U,T_0}^2 = \frac{1}{1 + \frac{C_0\lambda}{C_0\gamma}} \frac{Var[T_0]}{Var[T_U]} \quad (24)$$

Worth to note that covariance of $T_U(t)$ and $T_0(t)$ is nonnegative for any values of time lag (Fig.3), which contradicts the suggestion of cooling of upper oceanic level as the result of intensive deep ocean heat uptake.

Fig.3. a) Cross-correlation function of the upper layer and deep ocean temperature.

b) The phase shift of the external forcing and deep ocean temperature as a function of frequency.

Parameter values are the same as at Fig.1.
Cross-correlation function $\mathcal{R}_{T_u,T_u}(\tau)$ has unique positive extreme (Fig.1). Time lag $\tau_{es}$ corresponding to the extreme follows the equation

$$\tau_{es} = \frac{\ln \left| \frac{(\rho + 3a)C + \gamma + \lambda}{(\rho - 3a)C + \gamma + \lambda} \right|}{\rho}.$$  \hspace{1cm} (25)

Using typical values of model parameters (Geoffrey et al., 2013, [13]) accepted in our paper earlier for driving the figures we can get an estimation of $\tau_{es} \approx 15$ years.

3.4. The phase shift of the external forcing and deep ocean temperature response

The phase shift of the external stochastic forcing and deep ocean response follows the expression

$$\Delta \varphi_{T_u,T_u}(\omega) = \arctan \left[ \frac{\int_0^\infty h_1(u) \sin(\omega u) du}{\int_0^\infty h_1(u) \cos(\omega u) du} \right] = \arctan \left( \frac{-2a\omega}{b - \omega^2} \right).$$ \hspace{1cm} (26)

In the high-frequency range phase shift is close to 180°. On the contrary, in low-frequency limit ($\omega \to 0$) external forcing $F(t)$ and deep ocean temperature $T_u(t)$ are in phase (Fig.3.b). The critical value of frequency that divides positively and negatively correlated spectral components of $F(t)$ and $T_u(t)$ is determined by equation (19) and so on is equal to the internal frequency of the deep ocean.

Typical values of parameters (Geoffrey et al., 2013), [13] $\lambda = 1.13$, $\gamma = 0.67$, $C = 8$, $C_0 = 107$ correspond to critical frequency, $\omega_c \approx \frac{1}{28}$, meaning time scale $\approx 150$ years. It follows that the heat uptake on the upper boundary of atmosphere and the deep ocean temperature positively correlate only on the time scales exceeding one or two centuries. Observed for several recent decades energy disbalance on the top boundary of the atmosphere cannot be responsible for the synchronous anomalous heat uptake of the deep ocean. It seems doubtful that the relatively short-term SAT rise hiatus of the early 21-st century can be determined by the response of the deep ocean to external anthropogenically induced forcing.

4. Discussion and conclusions

Mechanisms of hiatus forming stay in close connection to the more general climate theory problem – detection and (if possible) partition of the effects of the external deterministic forcing and the background internal climate variability. The variability, in turn, can reflect nonlinear interactions of the climate system components or results from stochastic forcing of any nature. In spite of a number of well-known large-scale climatically important patterns like North Atlantic Oscillations, Atlantic Multidecadal Oscillations, El Nino-Southern Oscillation, and some others their scales of temporal variability are still debatable, Mann et al., 2020, [19]. In this relation, model (1-2) in the conditions of stochastic forcing can be considered as a simple paradigmatic model that expends the classical Hasselmann approach. It is important to note that the spectral density of the box-model upper layer temperature has a unique maximum at zero frequency (see eq. 16). It means that the presence of the model deep ocean layer leads to the transform of the upper layer spectra to low-frequency white noise, which can seriously affect our ability to detect any other climatic signal. The second important difference of the upper layer temperature from the Hasselmann scheme consists in the strong suppression of the spectral density in the broad frequency range corresponding to time scale from ~50 to ~1500 years.

Finally, we can formulate the following conclusions.

1. Correlation of the upper level and deep ocean temperatures in the two-box Held et al., 2010, [10] model in the conditions of stochastic forcing is always positive. Moreover, the covariance of
temperatures on any time shifts are nonnegative. It means that the hypothesis about the early 21st-century slowdown of SAT rise determined by the anomalous deep ocean heat uptake seems to have some caveats.

2. The existence of the deep ocean in the model (1-2) decreases the variability of the upper level temperature in the entire frequency range. Comparison to the classical Hasselmann model demonstrates that this damping is highly frequency-selective taking the extreme values at the internal frequency of the deep ocean, which corresponds roughly, to one and a half of the century.

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