Influence of geometrical parameters on convective flows in non-uniformly heated cylindrical fluid layers

A. Evgrafova, A. Sukhanovskii, M. Kuchinskii, E. Popova

1Institute of Continuous Media Mechanics, Academ. Korolyov, 1, Perm, 614013, Russia
2Perm State University, Bukireva St. 15, Perm, 614990, Russia

E-mail: eav@icmm.ru

Abstract. Convection in a fluid layer with a free surface in a cylindrical container non-homogeneously heated from below was studied experimentally. The heater has a circular form and was placed in the center of the vessel. Such a system in the presents of rotation is very promising for studying the nature of tropical cyclones. In current paper we considered the influence of different geometric parameters on the dynamics of convection flows. For it two experimental setups of different sizes were used. Measurements were done for fluids with different values of Prandtl number and heating powers. It was shown that the structure and intensity of mean flow producing by a horizontal temperature gradient are defined by Rayleigh number $Ra$. The basic flow leads to unstable temperature stratification over the heating area and the formation of a system of secondary flows. The formation of secondary flows depends on characteristics of thermal boundary layer and is described by local Rayleigh number $Ra_\delta$.

1. Introduction

Convective flows produced by the non-uniformly heating exist in a large variety of systems of different nature and scale. In particular the very interesting problem is natural convection produced by a localized heat source. In technological processes such flows are commonly observed in heat exchangers, cooling systems of modern electronic equipment and nuclear reactors [1,2]. Concerning atmospheric boundary layer some studies were motivated by processes of impurity propagation from a strong thermal source (for example oil ignition)[3]. Development of urban climatology leads to the problem of convection from so-called urban heat islands and studies of large-scale convective circulation from extended local heat sources with ambient temperature stratification [4,5,6].

The specific of the convection driven by local heat source is producing both vertical and horizontal temperature gradients. As a result of the applied horizontal temperature gradient, a large-scale meridional circulation appears. In the presence of rotation meridional circulation transports angular momentum from the periphery to the center and produces cyclonic vortex. Such system was proposed as a laboratory model of tropical cyclone [7]. PIV measurements showed that the general structure of mean radial and azimuthal flows in the proposed model is similar to the typical structure of a hurricane [8]. In the boundary layer of cyclonic vortex secondary convective structures in the shape of horizontal rolls were observed [8]. There is remarkable similarity of these secondary flows and rainbands in real tropical cyclones [9,10]. Secondary flows may significantly influence on heat and mass transfer in the boundary layer [11,12]. The nature and characteristics of secondary flows over heating area in a non-rotating
cylindrical layer were studied in [13]. Various types of visualization revealed co-existence of radial and transverse convective rolls. Radial rolls are very robust and similar to the longitudinal rolls studied in rectangular tank [14,15]. Transverse rolls in a cylindrical layer are formed periodically and move to the center along with a basic flow. The characteristic frequency of transverse rolls depends on Rayleigh number for a wide range of governing parameters.

It is not clear how strong is the influence of geometrical parameters on the structure and characteristics of basic and secondary flows in cylindrical fluid layer with localized heat source. As a first step to solve this problem we consider two experimental setups. The paper is organized as follows. In section 2 we describe the experimental setup and measurement technique. Experimental results are presented in section 3 and conclusions are given in section 4.

2. Experimental setup
Two cylindrical vessels were used. The first vessel (I) has diameter \( D = 300 \) mm (figure 1 a). The heater is a brass cylindrical plate mounted flush with the bottom. The diameter of the plate \( 2R \) is 104 mm, and its thickness is 10 mm. The second vessel (II) has diameter \( D = 690 \) mm (figure 1 b). The bottom of the larger vessel is the textolite plate with thickness of 20 mm. In the center of plate the brass heater of diameter \( 2R = 195 \) mm was placed flush with the bottom. The temperature of the heaters was controlled by computer. Silicon oils with different values of kinematic viscosity, PMS-20, PMS-10 and PMS-5 (20, 10 and 5 cSt at \( T = 25^0C \)) were used as working fluids. The surface of the fluid was always open. The depth of the layer for the vessel I was fixed \( h = 30 \) mm. For the vessel II two series of experiments were carried out with \( h = 30 \) mm and \( h = 60 \) mm. The room temperature was kept constant by air-conditioning system, and cooling of the fluid was provided mainly by the heat exchange with surrounding air on the free surface and some heat losses through sidewalls. Temperature inside the fluid layer was measured by copper-constantan thermocouple. The velocity field measurements were made with a 2D particle image velocimetry (PIV) system Polis. The measurement, and the processing of the results were performed using the software package Actual Flow.

![Figure 1. Experimental models, dimensions and location of the coordinate system.](image)

We use the set of the non-dimensional parameters which are commonly used for similar problems. These are Prandtl number \( Pr \) and Rayleigh number \( Ra \):

\[
Pr = \frac{\nu}{\kappa} \quad (1)
\]

\[
Ra = \frac{g\beta h^3 \Delta T}{\nu^2} \quad (2)
\]

where \( g \) is the gravitational acceleration, \( h \) is the layer depth, \( \beta \) is the coefficient of thermal expansion, \( \Delta T \) - temperature difference between temperatures of the heater and the room, \( \nu \) is the coefficient of kinematic viscosity.
3. Results

Detailed description of the basic flow structure can be found in [13] but brief description of the general structure of the large-scale flow is necessary for a better understanding of the results. The heat flux in the central part of the bottom is a source of the intensive upward motion above the heater. Warm fluid cools at the free surface and moves toward the periphery where the cooled fluid moves downward along the side wall. Large-scale advective flow occupies the whole vessel (figure 2). Experimental measurements of velocity fields in a vertical cross-section over the heating area showed that instantaneous fields are irregular and asymmetric. Along with the main updraft in the center there are less intensive but pronounced upgoing convective flows close to the periphery of the heater.

![Large-scale circulation](image)

Figure 2. Large-scale circulation.

At first we wanted to compare the large-scale flow structure for vessels I and II. If the structure of the flow does not depend on the size of the vessel (for the same ratio of the layer depth to the heater diameter and value of Prandtl number) than non-dimensional velocity field should be the same for the same values of Rayleigh number. Non-dimensional radial velocity fields are shown in figure 3. Velocities are averaged in the azimuthal direction and in time. Positive values describe divergent flow, and negative values convergent flow. As a characteristic velocity we chose ratio of kinematic viscosity and characteristic size (depth of the layer) $v^* = v/h$. One can see that structures of mean flow have a good qualitative agreement. For the quantitative comparison we present mean radial velocity profiles (figure 4). Profiles in both case are plotted for $r/R=0.5$. The agreement is satisfactory, the discrepancy (about 20%) can be explained by different ratio of the heater size and vessel diameter which also important geometric parameter. We can conclude that for the described parameters the variation of the geometric sizes of the vessel for fixed Rayleigh number does not substantially change the structure of the main large-scale flow.

![Mean radial velocity fields](image)

Figure 3. Mean radial velocity fields for $Ra = 1.1 \cdot 10^7$, $Pr=66$: a - $D = 300$ mm, $h = 30$ mm; b - $D = 690$ mm, $h = 60$ mm.

The large-scale advective flow in the lower part of the layer leads to the formation of boundary layer with potentially unstable temperature stratification above the heater and makes possible the generation of the secondary convective flows. The structure and specifics of secondary
flows over the heater in the case of non-rotating cylindrical layer are described in detail in [13]. Secondary flows visualized by shadowgraph technique are presented in figure 5. Regimes for the weak heating are characterized by appearance of ring-like rolls. An increase of the heat flux leads to more complex convective patterns - the superposition of the spiral and a system of radial rolls on the periphery. The authors in [13] showed that radial rolls are robust and stationary. Ring-like rolls and spirals are orientated in transverse direction to the main flow and appear periodically on the heater periphery.

Dependence of transverse roll formation frequency $f$ for on Rayleigh number for different fluids and aspect ratios is presented in figure 6a. One can see that frequency $f$ grows with an increase of the Rayleigh number for both vessels but there is significant quantitative discrepancy between measurement for different values of the layer depth. For a correct description of

![Figure 4](image1.png)

**Figure 4.** Mean radial velocity profiles at $r/R = 0.5$. $Ra = 11 \cdot 10^6$. $Pr = 66$.

![Figure 5](image2.png)

**Figure 5.** Visualization of secondary flows: a - $Ra = 0.2 \cdot 10^5$, b - $Ra = 1.2 \cdot 10^6$. 
Figure 6. Characteristic frequency $f$ versus Rayleigh number (a) and local Rayleigh number (b).

secondary flow regimes, the local governing parameter should be used, for example the local Rayleigh number, defined through the characteristics of the thermal boundary layer, because processes in a boundary layer with unstable temperature stratification are connected but do not directly depend on global parameters. From the temperature measurements for $Ra = 1.1 \times 10^7$ (vessel II) we estimated that height of thermally unstable layer has value $\delta \approx 6$ mm. Replacing $h$ by fixed value of $\delta$ (6 mm) for calculation of local Rayleigh number $Ra_\delta$ we see that all experimental points lie on the same curve. This result needs to be discussed. There are two possible explanations. The first one is that characteristic frequency $f$ does not depend on $\delta$ and is defined only by physical properties of the fluid and applied temperature difference. The second one required that $\delta$ is constant or changes weakly for all experimental realisations presented in figure 6b. In order to check that we plan to study the properties of thermal boundary layer for a wide range of experimental parameters.

It is also found that variation in fluid viscosity for fixed model does not change the structure and dynamics of secondary flows. In figure 7 instantaneous radial velocity fields over the heater at the height $z = 3$ mm are presented for different Prandtl numbers but constant $D$, $h$ and $Ra_\delta$. The values of velocity and the shape of fields are very similar.

4. Conclusions

The influence of geometrical parameters on the structure and characteristics of basic and secondary flows from a localized heat source in a cylindrical layer was studied experimentally. Two experimental setups of different sizes were used. Measurements were done for fluids with different values of Prandtl number and heating powers. It was shown that the structure and intensity of mean flow producing by a horizontal temperature gradient are defined by Rayleigh number $Ra$. The basic flow leads to unstable temperature stratification over the heating area and the formation of a system of secondary flows. The formation of secondary flows depends on characteristics of thermal boundary layer and are described by local Rayleigh number $Ra_\delta$. For calculation of $Ra_\delta$ we used the assumption that for the described experiments $\delta$ is constant or has only slight deviation from the constant value. In order to check that we plan to study the properties of thermal boundary layer for a wide range of experimental parameters.

The financial support of the grant RFBR 16-31-00150 is gratefully acknowledged.
Figure 7. Instantaneous radial velocity fields at $z = 3$ mm ($Ra_{\delta} = 3.1 \times 10^4$, D = 300 mm, h=30 mm): a - $Pr = 110$, b - $Pr = 209$.

References
[1] Chiu WKS et al. 2000 Phys. Fluids 12 2128
[2] Aydin O and Yang WJ 2000 Int. J. Numerical Methods for Heat and Fluid Flow 10 (5) 518
[3] Shvarts FG and Shklyaev VA 2009 Computational continuum mechanics 1 96
[4] Lu J et al 1997 J. Ap. Met. 36 1377
[5] Kurbatskii AF 2001 J. App. Met. 40 1748
[6] Arnfield AJ 2003 Int. J. Climatol. 23 1-26
[7] Bogatyrev GP 1990 Letters of JETP 51 557-559
[8] Sukhanovskii et al. 2016 Quart J.R.Met.Soc. 316 23-33
[9] Moon Y, Nolan DS. 2009 J. Atmos. Sci. 72 164-190
[10] Moon Y, Nolan DS. 2009 J. Atmos. Sci. 72 191-215
[11] Morrison I et al. 2005 J. Atmos. Sci. 62 2662
[12] Zhang JA et al. 2008 Boundary-Layer Meteor. 128 173
[13] Sukhanovskii et al. 2016 Physica D 316 23-33
[14] Batalov et al. 2007 Fluid Dyn. 42 540-549
[15] Sukhanovsky et al. 2012 Eur. Phys. J. B 85 12