On the viewing angle of binary neutron star mergers

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(Dated: July 30, 2018)

The joint detection of the gravitational wave (GW) GW170817 and its electromagnetic (EM) counterparts GRB170817A and kilonova AT 2017gfo has triggered extensive study of the EM emission of binary neutron star mergers. A parameter which is common to and plays a key role in both the GW and the EM analyses is the viewing angle of the binary’s orbit. If a binary is viewed from different angles, the amount of GW energy changes (implying that orientation and distance are correlated) and the EM signatures can vary, depending on the structure of the emission. Information about the viewing angle of the binary orbital plane is therefore crucial to the interpretation of both the GW and the EM data, and can potentially be extracted from either side.

In the first part of this study, we present a systematic analysis of how well the viewing angle of binary neutron stars can be measured from the GW data. We show that if the sky position and the redshift of the binary can be identified via the EM counterpart and an associated host galaxy, then for 50% of the systems the viewing angle can be constrained to $\leq 7^\circ$ uncertainty from the GW data, independent of electromagnetic emission models. On the other hand, if no redshift measurement is available, the measurement of the viewing angle with GW alone is not informative, unless the true viewing angle is close to $90^\circ$. This holds true even if the sky position is measured independently.

Then, we consider the case where some constraints on the viewing angle can be placed from the EM data itself. We show that the EM measurements can then be used in the analysis of GW data to improve the precision of the luminosity distance, and hence of the Hubble constant, by a factor of 2 to 3. That would allow for a 1% measurement of the Hubble constant, independent of the cosmic distance ladder, in the next 5 years.

INTRODUCTION

The discovery of binary neutron star (BNS) merger GW170817 and its electromagnetic (EM) counterparts GRB170817A and kilonova AT 2017gfo, opened the era of multi-messenger astronomy [2, 3].

While much was learned from this first joint detection, the precise EM emission model is still unknown. A key parameter to understand the EM emission is the orbital inclination angle, which strongly impacts the details of the EM signals received at Earth. The gamma-ray burst (GRB) GRB170817A was under-luminous [3, 4], and the X-ray and radio emission that followed were significantly different from those of other GRBs [5-9]. A possible explanation could be that a GRB jet was formed, which we observed from a large inclination angle. However, alternative explanations exist. The observations could also be described by a choked jet, whose cocoon expands leading to a wide-angle, mildly relativistic outflow [10, 11]. If the choked jet is the correct scenario, the viewing angle affects less the light curve and the spectra, and thus cannot be constrained by the available EM data. The inclination angle is also a key ingredient for the interpretation of the kilonova emission and its spectral evolution [12-14].

It would thus be important if the inclination angle of the binary, $\theta_{IN}$, could be measured or at least constrained from the GW data, and then used to study or exclude EM emission mechanisms.

Unfortunately, measurement of the inclination angle with GW data is usually quite poor, due to the well-known degeneracy between the inclination and the luminosity distance [15]. This degeneracy can be resolved if the system has precessing spins, if higher-order harmonics are detectable, or if the merger and ringdown are in band. Neither of these conditions are met for BNS, since neutron stars have small spins, and mass ratios close to one, which suppresses the amplitude of higher-order harmonics [10, 22]. However, in the case of a joint GW-EM detection more information usually exists. If an independent measurement of the luminosity distance is available, e.g. using the redshift inferred from the EM counterpart, the degeneracy is broken and one can expect the uncertainty on the inclination angle to improve. To a smaller extent, knowledge of sky position can also help improve the measurement of the inclination angle, as we discuss below. One could thus envisage a strategy where some information obtained from the EM sector (redshift, sky position) can be folded into the GW analysis to get an improved measurement.

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1 In the gravitational-wave literature, the inclination angle is usually the angle between the line of sight and the orbital angular momentum. However, following LIGO and Virgo, we report the angle between the line of sight and the total angular momentum. The difference is only important for large and misaligned spins, which are not expected for binary neutron star mergers.
of the inclination angle. Indeed, using the sky position of the kilonova AT 2017gfo and the redshift of the host galaxy NGC4993, the bound on the viewing angle \( \zeta \equiv \min(\theta_{\text{IN}}, 180^\circ - \theta_{\text{IN}}) \) of GW170817 was improved from \( \zeta \lesssim 56^\circ \) \cite{1} to \( \zeta \lesssim 28^\circ \) \cite{1, 23}.

In the future one might also consider the opposite approach: if the details of the EM emission are well understood, the detection of photons could provide a bound on the inclination angle. For example, detection of a short GRB with a jet break in the afterglow can yield an upper-bound on the binary inclination, since the jet break is expected to be observed only if the viewing angle is smaller than the relativistic jet opening cone \cite{24, 28}. Even if no jet break is observed, the observation of the afterglow spectra, light curves, or even the superluminal motion of the jet put constraints on the viewing angle \cite{7, 29}. These constraints can be used in the GW analysis to get a better measurement of the luminosity distance \cite{30}, which in turn can improve the Hubble constant measurement with GWs \cite{31, 34}.

In this paper we present a systematic study of the measurability of the viewing angle of BNS systems. We first show that, in the absence of a positive detection of an EM counterpart, GWs alone only rarely provide a meaningful constraint on the viewing angle. We then consider the case when the luminosity distance and/or the sky position are independently measured. While little is changed by knowing the sky position, knowledge of the luminosity distance dramatically reduces the 1\( \sigma \) uncertainty for the viewing angle. For 50\% of the BNS detections for which sky position and luminosity distance are measured from the EM sector, the viewing angle uncertainty is below 7\( ^\circ \) (using the projected sensitivities for LIGO and Virgo in the third science run).

We also consider the opposite scenario, and show that if the binary viewing angle is constrained by the EM data, the binary luminosity distance uncertainty can potentially be reduced by a factor of 2 to 3. In turn, that improves the measurement of the Hubble constant with gravitational waves. We show that a 1\% uncertainty on the Hubble constant can then be reached with only \( O(10) \) GW-EM joint detections of BNS, that is, in less than 5 years.

**METHOD AND RESULTS**

We consider two different methods to estimate the distance and inclination of BNS systems. First, we rely on the computationally-expensive stochastic sampler LAL-Inference \cite{33} (specifically on its nested sampling flavor \cite{36}). This is the same algorithm used by the LIGO and Virgo collaborations and delivers posterior distributions for all the unknown parameters on which compact binaries depend. Given the cost of each simulation, LAL-Inference cannot be run on an arbitrarily large number of simulations. We thus use it only on a few specific sources, to show which parameters affect the measurability of luminosity distance and inclination, and how. We then introduce a semi-analytical, faster, approach. After showing that the two give consistent results we use the latter to characterize the population of detectable BNSs.

**Single-event analysis**

All the BNS systems we simulate have component masses 1.4 \( - \) 1.4\( M_\odot \) in the source frame. As mass measured in the detector frame is redshifted by a factor (1 + \( z \)), where \( z \) is the redshift, the sources will appear slightly heavier in the detector frame.

To keep the computational cost reasonable, we make two main simplifying assumptions: we neglect tidal effects and neutron star spins. The former is a very reasonable choice, since tidal effects do not enter the waveform amplitude, and hence are not correlated with the inclination angle. The latter is justified since the spins of known neutron stars in binaries that will merge within a Hubble time is small (the fastest-spinning systems are PSR J0737-3039A \cite{37} and PSR J1946+2052 \cite{38}, which will at most have dimensionless spins of \( \chi \sim 0.04 \) or \( \chi \sim 0.05 \) when they merge). Even GW170817 is consistent with having small spins.

In the work reported here, all the synthetic BNS signals are generated using the IMRPhenomPv2 waveform family \cite{39, 40}, with the reduced order quadrature likelihood approximation \cite{41}. We consider a network consist of the two LIGOs \cite{42} and Virgo \cite{43}, all at design sensitivity. The signals are added into 'zero-noise' (which yields the same results that would be obtained averaging over many noise realizations). We start all analyses at 20 Hz, and use a sampling rate of 8 KHz. We marginalize over calibration errors using the same method described in Ref \cite{44}, using gaussian priors with widths of 3\% for the amplitude and 1.5 degrees phase for all instruments.

We consider two different sky positions, to verify if and to what extent our conclusions depend on the particular value of the detectors’ antenna patterns. In total we create 4 such systems. Their parameters are summarized in Table I. For sources A, B and D, the sky position is near the maximum of LIGO’s antenna pattern, where one would expect most detections to be made \cite{45}.

| Source | SNR | polarization | phase | RA   | DEC   |
|--------|-----|--------------|-------|------|-------|
| A      | 35  | 0.005        |       | 0    | -1.08 | 0.66  |
| B      | 20  | 0.005        |       | 0    | -1.08 | 0.66  |
| C      | 20  | 0.005        | 0     | -1.08| 0.66  |
| D      | 12  | 0.017        | 0     | 0.017| -1.08 | 0.66  |

TABLE I. For all events the GPS time is 1068936994.0 and the two component masses in the source frame are 1.4\( M_\odot \).
The main goal of the present study is to verify how well the inclination angle can be measured. Obviously, this would a priori depend on three main factors: the SNR of the event, the true value of the inclination angle, and the sky location of the event. Each of the sources listed in Table I is re-analyzed for different values of the inclination angle, from nearly face-on ($\theta_{\text{IN}} = 0^\circ$) to edge-on ($\theta_{\text{IN}} = 90^\circ$). Every time the orientation angle is changed, the distance is also varied to keep the SNR fixed at the value given in Table I.

We note that one expects most detections made by advanced detectors to have inclinations close to face-on ($\theta_{\text{IN}} = 0^\circ$) or face-off ($\theta_{\text{IN}} = 180^\circ$), whereas events with inclination close to edge-on ($\theta_{\text{IN}} = 90^\circ$) would be rarer [46]. Why this happens is related to the degeneracy between luminosity distance and inclination. Since that will play a role in the interpretation of the results we present, it’s worth expanding on the subject. Geometrical arguments would suggest that the inclination angles of the population of binaries should be uniform on the sphere, i.e. $p(\theta_{\text{IN}}) \sim \sin(\theta_{\text{IN}})$, while their luminosity distances should be uniform in volume, $p(D_L) \sim D_L^2$ (as long as cosmological effects can be neglected). However, the detected binaries are a subset of the entire population: they are the fraction that produce a GW signal loud enough to be detected. GW emission from a compact binary is not isotropic, instead more energy is emitted along the direction of the orbital angular momentum, while the least amount is emitted parallel to the orbital plane [47]. This implies that edge-on systems, which are the most numerous in the underlying population, will need to be extremely close to be detectable. Conversely, face-on/off systems can be farther away, and still produce a detectable signal. But since far away there is more volume, the population of detectable signals will be dominated by sources with inclination angles close to 0° and 180°. Note that all events LIGO and Virgo have detected thus far (including the binary black holes) are consistent with being face-on/off [15, 23, 48–51].

The analytical form of the inclination angle distribution for sources detectable by advanced detectors was first obtained by Ref. [46], and we will refer to it as the Schutz distribution in this work. Using the Schutz distribution one can calculate the fraction of detectable events that will have viewing angle within a given range. In particular, less than $\sim 7\%$ ($\sim 3\%$) of the detectable events will have viewing angle $> 70^\circ$ ($> 80^\circ$).

All analyses were performed three times: a first time assuming that all parameters are unknown and measured from GW data alone; a second time assuming a counterpart has been found, which provides the sky position (right ascension and declination) of the source; a third time, assuming both sky position and distance are known (the latter by measuring the redshift and using a cosmology to convert redshift to luminosity distance).

For all parameters, we use the same priors used by the LIGO and Virgo collaborations. In particular, the prior on the luminosity distance is uniform in volume while the prior on the inclination angle is isotropic.

Since the EM emission only depends on the absolute value of $\cos \theta_{\text{IN}}$, we quote results for the viewing angle $\zeta (\equiv \min(\theta_{\text{IN}}, 180^\circ - \theta_{\text{IN}}))$ [3], rather than the inclination angle $\theta_{\text{IN}}$ itself. In Fig. 1 we report the 1-σ uncertainty (in degrees) for the measurement of the viewing angle $\zeta$ for all sources, as a function of the true inclination angle. We first discuss the case where no information is available from the EM side (dot-dashed lines). We find that the uncertainties are roughly constant until the true inclination angle gets above $\sim 80^\circ$. For the sources A, B, and C, the uncertainty happens to be nearly the same: $\sim 15^\circ$, while it is a little wider for source D $\sim 19.5^\circ$. We have verified that for all these configurations the viewing angle posterior is not very informative. It is similar to the Schutz distribution, with some extra support at 30° and a depletion of support close to edge-on. This is shown in Fig. 2 for Source B, and in the Appendix for the other sources.

The fact that the viewing angle uncertainty is the same for small to moderate inclinations can be explained by a combination of priors and the well-known degeneracy between luminosity distance and inclination. Since the emission of GW is larger toward the direction of the system angular momentum, one can obtain a similar signal at the detector by increasing the viewing angle while moving the source closer.

As the true inclination increases from zero, the true distance has to decrease to maintain the same SNR. However, smaller distances are disfavored by the prior. Thus,

\[\text{FIG. 1. Viewing angle uncertainty as a function of binary inclination angle.}\]
the Bayesian code will prefer to keep the viewing angle posterior close to face-on and overestimate the distance to compensate. This can be done because of the correlation between the two parameters: the eventual small decrease in the likelihood introduced by biasing both luminosity distance and inclination is more than compensated for by the better prior value at larger distances. This behavior can be sustained till the true inclination angle is close to edge-on. At that point, the degeneracy is reduced, and the likelihood penalty for keeping the posterior at face-on cannot be compensated for by the prior: both luminosity distance and viewing angle posteriors are centered at the true value, and typically better measured.

For the low SNR event, source D, one gets exactly the Schutz distribution for small to moderate inclinations. Unlike for the other sources, in this case the uncertainty increases as the inclination angle gets close to 90°. This happens because when the true inclination is close to edge-on a significant posterior peak still survives at 30°: since the SNR is low, the extra likelihood to be gained with more support at edge-on is comparable with the prior penalty, and a bimodal distribution arises (see Appendix).

We find that similar conclusions can be drawn if the sky position of the source can be considered as known (dashed lines in Fig. 1). It is still the case that the posteriors for small to moderate inclinations are similar to the Schutz distribution, and the main difference is that the distance-inclination degeneracy is resolved at smaller inclinations, $\theta_{\text{JN}} \lesssim 70°$.

It is only when both sky position and distance are known (solid lines in Fig. 1) that the uncertainties are much smaller for all sources and all inclinations. In this case the posteriors for the viewing angle are centered around the true value for all systems. The uncertainties reach a minimum when the true source is edge-on, since in that case the cross polarization of the GW signals is zero, which reduces the residual degeneracy between inclination and polarization angle $\psi$ (the only two unknown angles left in the amplitude of the GW signal).

It is interesting to verify how precisely the BNS sources can be localized as a function of their orientation. This is shown in Fig. 3 for the runs in which all parameters are considered unknown. We find that the 90% credible interval (in deg$^2$) is roughly constant for small to moderate inclination angles and for all sources. Source A is more localized in contrast with source D because of its higher SNR. The difference between source B and C (which have the same network SNR) is that source C has a SNR roughly split in equal amounts in the three interferometers, while source B has most of the SNR in the two LIGOs, while being sub-threshold SNR in Virgo (4.2 for the face-on orientation). Since most of the sky resolution for GW sources comes from triangulation and by requiring phase and amplitude consistency across the network [54,55], it helps if the source is above threshold in more detectors (Incidentally, we stress that the same is not true for the viewing angle uncertainty: as Fig. 1 shows one gets the same uncertainties for sources B and C. Likewise, the uncertainty for the luminosity distance is the same for the two sources up to viewing angles of $\sim 70°$). We note that the edge-on sources are relatively poorly-localized. This can be explained as follow: the sky position (right ascension and declination) and orientation (inclination, polarization angle) all enter the frequency-domain GW amplitude and phase (e.g. Eqs. 4.1-4.5 in Ref. [57]). When the inclination angle is close to 90°, some of these terms are suppressed, reducing the number of constraints that can be used to enforce phase and amplitude consistency, leading to larger uncertainties.

**Population analysis**

Having gained an understanding of which parameters can impact the measurability of the viewing angle, we would like to determine what fraction of the BNSs for which an EM counterpart is found will yield significant viewing angle constraints. Unfortunately, it is computationally prohibitive to run LALINFEREN CE on large sets of hundreds or thousands of events.

We thus build an approximate Bayesian estimator for the two binary parameters of interest: the inclination angle and the luminosity distance.

This algorithm assumes that sky position, chirp mass, and mass ratio of the binaries are known. These assumptions can be justified as follows. First, if an EM coun-
counterpart is found, it will typically provide a precise sky position, unless only the GRB signal is detected. Second, the mass parameters, which are inferred from the phasing of the GW waveform do not significantly couple to distance and inclination, which are primarily measured from the amplitude of the signal. Since the arrival phase (or rather, the phase difference between detector pairs) and the signal-to-noise ratio are measured, the only unknown parameter left is the orientation of the binary in the plane of sky, \( \psi \) ("polarisation") \([68]\). We modify the Bayesian estimator of Ref. \([59]\) to use the events’ signal-to-noise ratios and the relative phase differences to reconstruct posteriors for luminosity distance and viewing angle, while numerically marginalizing over the polarization (see Appendix for more details).

We have verified that the standard deviations we obtain for the viewing angle using the approximate code are very similar to the estimates obtained with LALInference for the sources described in the previous section. For example, in Fig. 4 we show the uncertainties for source D when the sky position is known, or when both luminosity distance and sky position are known.

Having shown that the approximate Bayesian estimator gives results which are consistent with LALInference, we proceed and use the former for large populations of BNSs. We consider three observing scenarios \([69]\): (i) A network with the two LIGOs and Virgo (HLV) at the expected LIGO-Virgo third observing run (O3) sensitivity (\(\sim 2019+\)); (ii) HLV at the design sensitivity (\(\sim 2021+\)). (iii) To check how things will change when the network of ground-based detectors grows, we also consider a five-detector network made of the two LIGOs, Virgo, LIGO India \([61]\) and Kagra \([62, 63]\) (HLVJI), all at their design sensitivity (\(\sim 2024+\)). For each scenario we simulate 1.4-1.4M\(_{\odot}\) BNSs with random orientation, distributed uniformly in comoving volume. Throughout this work, a standard Λ-CDM Planck cosmology is assumed: \(\Omega_M = 0.308, \Omega_{\Lambda} = 0.692, h_0 = 0.678 [61]\). A BNS is considered detected if the measured network signal-to-noise ratio \(\sim [65]\) is greater than 12. Following the approach of Ref. \([59]\), we add Gaussian noise to the measured SNR ratio and relative phases.

We estimate the distance and inclination for 1000 detections, for each network.

For the simulations in which we assume redshift information exists, we convert the redshift to luminosity distance using the Planck cosmological parameters, and marginalize the posterior over the luminosity distance with a Dirac \(\delta\) centered at the true distance. We note that in practice this conversion might suffer from two sources of uncertainty: the redshift may not be a true measure of the luminosity distance due to the peculiar motion of the source, and the value of the Hubble constant is not precisely known. To account for the former, we introduce a 250 km/s Gaussian uncertainty around the true source redshift to account for the uncertainty due to the source peculiar motion. We also use a top-hat prior on the Hubble constant, from 65 to 75 km/s/Mpc to cover the range of currently estimated values \([64, 66]\). Other cosmological parameters don’t play a significant role for the redshift conversion, given that advanced detectors will only detect BNS up to redshift of \(z < 0.1\).

**Measure the viewing angle:** In Figure 5 we show the cumulative distribution for the 1σ viewing angle uncertainty, for the HLV network at design sensitivity. We see that if the sky positions and redshifts of the BNSs

\[ \text{FIG. 4. Standard deviation for the viewing angle of source D against the true value of the inclination angle as measured with the full LALInference code (lines) and with the approximate Bayesian estimator (symbols). Triangles correspond to the case when both distance and sky position are known, while circles assume that only the sky position is known.} \]
were precisely known, the viewing angle for half of them would be constrained to $< 6^\circ$. This uncertainty increases by $\sim 1^\circ$ if we include uncertainties in the peculiar velocity and in the cosmology.

If instead only sky positions are identified, without any information about the luminosity distance, only $< 5\%$ of BNSs will have inclination uncertainty below $10^\circ$. These numbers are $\sim 10\%$ worse for O3, and $\sim 10\%$ better when the detector network extends to HLVJ.

**Measure the luminosity distance:** Next, we wish to explore the situation where the EM sector provides a measurement or bound on the viewing angle, and show how that can be used to measure more precisely the luminosity distance of the binary using GW data. We consider various possibilities for the quality of the EM-based $\zeta$ measurement. The ideal scenario, in which the viewing angle is *precisely* measured; an uncertain measurement; and an upper bound. In the first case, we use a Dirac $\delta$ centered at the true viewing angle as prior for the inclination angle. If an uncertain measurement is available we treat the $\zeta$ prior in the GW analysis as a normal distribution centered at the value, and consider different widths of the distribution. Last, if the GRB jet opening angle $\theta_{\text{jet}}$ is measured, we use that to place an upper bound on the viewing angle $\zeta \leq \theta_{\text{jet}}$.

In Figure 5 we show the cumulative distribution of the 1$\sigma$ fractional luminosity distance uncertainty for simulated events detected by HLV at the design sensitivity. The blue curve shows the results for the worst case scenario, when no viewing angle information is available. The green curves are obtained by assuming an uncertain Gaussian measurement from the EM sector, with standard deviation given in the legend. Finally, the orange line is the optimal situation in which the viewing angle is perfectly known. In this case, the bound on the luminosity distance would be improved by a factor of $\sim 3$ compared to the worst case scenario.

Finally, we report results for the case when a short GRB is detected and the jet opening angle is measured, which provides an upper bound for the viewing angle, $\theta_{\text{jet}}$. We consider various combinations of $(\theta_{\text{JN}}, \theta_{\text{jet}})$, and for each pair we simulate 1000 detections. In Figure 7 we show median fractional luminosity distance uncertainty for the 1000 BNSs having the value of $\theta_{\text{JN}}$ and short GRB jet opening angle $\theta_{\text{jet}}$.

**FIG. 5.** Cumulative distribution function of 1$\sigma$ viewing angle uncertainty for BNS detected by Advanced LIGO-Virgo at design sensitivity.

**FIG. 6.** Cumulative distribution function of fractional luminosity distance uncertainty, which is calculated as the 1$\sigma$ uncertainty divided by the true value, for BNS detected by Advanced LIGO-Virgo at design sensitivity.

**FIG. 7.** Median value of the fractional luminosity distance uncertainty of short GRB associated BNSs measured by Advanced LIGO-Virgo at design sensitivity. Each point represents the median uncertainty of 1000 BNSs with inclination angle $\theta_{\text{JN}}$ and short GRB jet opening angle $\theta_{\text{jet}}$. 
DISCUSSION

In this paper we have shown that 50\% of the BNSs detected by Advanced LIGO+Virgo for which sky position and redshift are known (from EM observations) will yield a 1\sigma uncertainty on the viewing angle of 7° or less. We emphasize that this result is independent of any EM emission model. It therefore implies that the viewing angle measurements obtained this way can be used to constrain the EM mechanism. The sky location and redshift of BNS can be solely determined by the EM counterparts or, in some spectacular events, solely by the host. This is because in some well localized events the host groups can be uniquely identified without the help of EM counterparts.

We have shown that BNSs without independent redshift information yield uninteresting inclination constraints unless the system is close to edge-on. This is because the distance-inclination degeneracy dominates the uncertainty. Without an independent estimate of the redshift (and hence luminosity distance), the degeneracy can only be broken when the binary is close to edge-on. Unfortunately, edge-on binaries are harder to detect and localize than face-on binaries; only ~ 3\% of events will have viewing angle > 80°. Even after advanced detectors start observing one BNS a week, only one or two events per year will be close to edge-on. And yet, there are several reasons why binaries with large orbital inclinations should be sought out, beside yielding better inclination and luminosity distance measurements (Figure 1 and Ref. [53]). For example, simulations suggest there may be interesting EM features along the equatorial plane [12]. EM follow-up observations will have to be properly planned in order to find the counterparts for these rare but valuable sources.

The jet break in short GRB afterglows has long been used to study the jet opening angle [28]. The uncertainties in the inferred opening angle are usually a few degrees. It is interesting to verify how well GWs can constrain the BNS’s viewing angle for those sources for which a GRB could be detected. To answer this question we selected a subset of the BNSs in Figure 5 only keeping the sources with \( \theta_{\text{IN}} < 25° \). We find that half of this subsample of sources has 1\sigma viewing angle uncertainty of 8° or less, if their sky locations and redshifts are constrained. That level of precision would be comparable with the jet opening angle uncertainty inferred from the afterglow jet break, allowing for a compelling comparison between the two (under the assumption that the total orbital angular momentum aligns with the jet).

We have also shown how one can expect a factor of 2 to 3 improvement in the fractional luminosity distance uncertainty, if the binary inclination angle is independently measured. As the luminosity distance is the main source of uncertainty when measuring the Hubble constant with GWs [68], a better luminosity distance measurement translates into a better \( H_0 \) measurement. Since the \( H_0 \) uncertainty scales as \( 1/\sqrt{N} \) [32, 69, 70], where \( N \) is the number of detections, a factor of 3 improvement in distance uncertainty implies a factor of 9 fewer events are required to achieve any given \( H_0 \) precision. Instead of the 200 BNS detections required as estimated in Ref. [70], only O(10) BNSs would be required to reach a 1\% \( H_0 \) uncertainty, if the binary inclination angles were independently constrained. This is a number of detections likely to be made within 5 years [60].

In Figure 7 we have shown that the median value of the fractional luminosity distance uncertainty for BNSs can be as small as 5 to 7\% if the jet opening angle of an associated GRB is measured and it is smaller than 30°. This result is consistent with what was found by Ref. [30]. We notice that the luminosity distance uncertainty is not significantly improved if \( \theta_{\text{jet}} \) is larger than 30°. This is because, as shown in the Single-event analysis Section, the viewing angle posteriors are equal to the Schutz distribution for true viewing angle smaller than ~ 70°. Since the Schutz distribution peaks around 30° (black dashed line in Fig. 2), an EM-based upper limit only helps if it bounds the viewing angle to be smaller than ~ 30°. Luckily, the jet opening angles of short GRBs almost never exceed 30° [28, 71, 72]. We thus expect that observations of associated GRB jet opening angles will significantly improve the luminosity distance measurement for BNSs.

We acknowledge valuable discussions with K G Arun, Juan Calderon Bustillo, Maria Haney, Daniel Holz, Vivien Raymond, Om Sharan Salafia, B.S. Sathyaprakash, and John Veitch. H.-Y.C. was supported by the Black Hole Initiative at Harvard University, through a grant from the John Templeton Foundation. S.V. acknowledges support of the MIT physics department through the Solomon Buchsbaum Research Fund, the National Science Foundation, and the LIGO Laboratory. LIGO was constructed by the California Institute of Technology and Massachusetts Institute of Technology with funding from the National Science Foundation and operates under cooperative agreement PHY-0757058. R.N. was supported in part by NSF grant AST1312651. The authors acknowledge the LIGO Data Grid clusters. We are grateful for computational resources provided by Cardiff University, and funded by an STFC grant supporting UK Involvement in the Operation of Advanced LIGO. LIGO Document Number P1800196.

Appendix

LALInference inclination angle posteriors

In Figs. 2, 8, 9, 10 we show the posteriors on the viewing angle obtained with LALInference when no EM
information is provided. Thin lines (all overlapping) correspond to small and moderate inclination angles. We use thicker lines for \( \theta_{\text{in}} = 74^\circ, 78^\circ, 83^\circ, 88^\circ, 90^\circ \) sources. For those, the viewing angle posterior is usually significantly different from the prior. We stress that source A is quite loud, with a network SNR of 35, similar to GW170817\(^4\). For the weakest event, source D, the SNR in so low that the two polarizations cannot be disentangled even partially, and the posteriors are exactly equal to the Schutz distribution, inferred from the analytic expression in Ref. [46], is 19.4\(^\circ\).

**Approximate Bayesian estimator**

We use each individual detector’s signal-to-noise ratio, \( \bar{\rho} \), and the relative phase difference between detector pairs, \( \Delta \eta \), to reconstruct the distance \( D \) and inclination \( \theta_{\text{in}} \) posterior:

\[
\begin{align*}
\bar{f}(D, \theta_{\text{in}} | \bar{\rho}, \Delta \eta) & \propto f(D, \theta_{\text{in}}) f(\bar{\rho}, \Delta \eta | D, \theta_{\text{in}}) \\
& = f(D, \theta_{\text{in}}) \frac{1}{f(D, \theta_{\text{in}})} \int f(\bar{\rho}, \Delta \eta | D, \theta_{\text{in}}, \psi) f(D, \theta_{\text{in}}, \psi) d\psi \\
& = \int f(\bar{\rho}, \Delta \eta | D, \theta_{\text{in}}, \psi) f(D, \theta_{\text{in}}, \psi) d\psi. \quad (1)
\end{align*}
\]

The prior can be written as

\[
f(D, \theta_{\text{in}}, \psi) = D^2 \sin \theta_{\text{in}} H[D_h(\theta_{\text{in}}, \psi) - D] \quad (2)
\]

where \( H[D_h(\theta_{\text{in}}, \psi) - D] \) is a Heaviside function that cuts off at the maximum distance \( D_h(\theta_{\text{in}}, \psi) \) a binary with inclination and orientation \((\theta_{\text{in}}, \psi)\) can be detected. The likelihood \( f(\bar{\rho}, \Delta \eta | D, \theta_{\text{in}}, \psi) \) is calculated from a chi-square minimization process: \( f(\bar{\rho}, \Delta \eta | D, \theta_{\text{in}}, \psi) \sim \exp(-\Delta \chi^2_{\rho} / 2 \sigma^2_{\rho}) \exp(-\Delta \chi^2_{\Delta \eta} / 2 \sigma^2_{\Delta \eta}) \), where

\[
\Delta \chi^2_{\rho} = \sum_i (\rho_{\text{measured}, i} - \rho_i(D, \theta_{\text{in}}, \psi))^2. \quad (3)
\]

The index \( i \) goes through all detectors and

\[
\Delta \chi^2_{\Delta \eta} = \sum_j (\Delta \eta_{\text{measured}, j} - \Delta \eta_j(\theta_{\text{in}}, \psi))^2, \quad (4)
\]

where \( j \) goes through all non-degenerate detector pairs\(^4\). The variances \( \sigma^2_{\rho} \) and \( \sigma^2_{\Delta \eta} \) are taken from Equation 12 and 14 of Ref. [59]. We grid the \((D, \theta_{\text{in}}, \psi)\) parameter space to evaluate Equation 2 and 3 in each grid cell, and integrate over \( \psi \) to construct the posterior given in Equation 1.

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\(^4\) The relative phase is independent of the source distance.
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