NANOGrav results and dark first order phase transitions

Andrea Addazi\textsuperscript{1,2,*}, Yi-Fu Cai\textsuperscript{3,4,5,*}, Qingyu Gan\textsuperscript{1}, Antonino Marciano\textsuperscript{6,7,*}, and Kaiqiang Zeng\textsuperscript{1}

\textsuperscript{1}Center for Theoretical Physics, College of Physics, Sichuan University, Chengdu 610065, China; \textsuperscript{2}Istituto Nazionale di Fisica Nucleare, sezione Roma Tor Vergata, Rome I-00133, Italy; \textsuperscript{3}Department of Astronomy, School of Physical Sciences, University of Science and Technology of China, Hefei 230026, China; \textsuperscript{4}Chinese Academy of Sciences, Key Laboratory for Researches in Galaxies and Cosmology, University of Science and Technology of China, Hefei 230026, China; \textsuperscript{5}School of Astronomy and Space Science, University of Science and Technology of China, Hefei 230026, China; \textsuperscript{6}Department of Physics & Center for Field Theory and Particle Physics, Fudan University, Shanghai 200433, China; \textsuperscript{7}Laboratori Nazionali di Frascati, Istituto Nazionale di Fisica Nucleare Via Enrico Fermi 54, Frascati (Roma) I-00044, Italy

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The recent NANOGrav evidence of a common-source stochastic background provides a hint to gravitational waves (GW) radiation from the Early Universe. We show that this result can be interpreted as a GW spectrum produced from first order phase transitions (FOPTs) around a temperature in the keV-MeV window. Such a class of FOPTs at temperatures much below the electroweak scale can be naturally envisaged in several warm dark matter models such as Majoron dark matter.

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1 Introduction

The NANOGrav Collaboration has recently released an analysis of pulsar timing data gathered over the last 12.5 year [1]. An evidence of a stochastic spectrum was found, compatible with gravitational wave (GW) signals with frequency within $f \sim 10^{-6}$-$10^{-5}$ mHz, average GW energy density $\langle \Omega_{GW} h^2 \rangle_{\text{NANOGrav}} \sim 10^{-10}$ and almost flat GW spectrum $\Omega_{GW}(f)h^2 \sim f^{-1.5 \pm 0.5}$ at 1$\sigma$-level. Such a signal may be interpreted as a hint of new physics beyond the standard model (SM), generating GWs from early universe mechanisms. Recently, several attempts were proposed, so as to interpret the NANOGrav observations from cosmic strings [2, 3] and primordial black holes [4, 5]. Indeed, it is well known that in both cases the GW spectrum is almost flat for several orders of magnitude, allowing for a possible cross-check observation in future satellite interferometers such as LISA, TAIJI and TianQin projects (around the mHz GW frequency), and the future terrestrial interferometers beyond LIGO/VIRGO such as the ET project (around 10 Hz or so) [2-5].

GW stochastic backgrounds may notoriously arise also from first order phase transitions (FOPTs) happened in the early universe, which has recently been a subject under massive investigations (see e.g., refs. [6-41]). Most of the FOPTs hitherto analyzed in the literature were around the electroweak (EW) scale, resulting in a test of possible SM exten-
sions of the Higgs sector, leading to GW signals around the mHz frequencies with implications for future space interferometers. Within these scenarios, typically extra scalars beyond the SM are introduced as strongly coupled to the Higgs bosons, compatible with LHC constraints, see e.g., ref. [28].

Nevertheless, FOPT dynamics can be hidden in a dark matter sector, weakly coupled to the Higgs and the other SM fields. In this case, FOPTs can be easily recovered at energies much lower than the EW scale, without any collider constraint. Indeed, there are several dark matter models, beyond the weakly interacting massive particle (WIMP) paradigm, that introduce new hidden scalar fields which may have false or true minima far below the EW domain.

Alternatively to WIMPs, dark matter particles can have a mass much below the 10 GeV-1 TeV. For example, warm dark matter (WDM) particles typically have masses in a broad range around the 0.1 keV-100 MeV. In WDM models, several new extra scalars with 0.1 keV-100 MeV vacuum expectation values (vevs) related to new symmetries beyond the SM, can be envisaged. An explicit example is the Majoron dark matter model [42-46], analyzed in our previous companion paper1) [14], for which we proposed tests in radio-astronomy experiments, including NANOGrav2).

Indeed, if the FOPT nucleation temperature was around the WDM-scale, then the GW spectrum should be detected around the 1-nHz frequency region, corresponding to Pulsar timing tests [14]. In principle, one would naively expect that the FOPT spectrum cannot work as an explanation for the NANOGrav excess: typically FOPT GW signals do not have a large flat plateau, as in the cosmic strings and primordial black holes (PBH) generation cases. On the other hand, the NANOGrav sensitivity includes only an order of magnitude in the frequency window. This allows us to suspect that also GW stochastic background radiation with a “locally” flat spectrum within the NANOGrav frequency window may provide a possible explanation for the puzzling excess.

In this article, we show that WDM-inspired FOPTs can explain the NANOGrav results, potentially opening a new phenomenological channel for multi-messenger dark matter particle searches. We find examples of FOPTs that are compatible with the NANOGrav stochastic signal within a 95% C.L. We show that, contrary to cosmic strings and PBH genesis mechanisms, the FOPT spectra explaining NANOGrav will elude other GW observations at higher frequencies. Thus, in principle, a comparative analysis of FOPTs, cosmic strings and PBHs can allow us to discriminate them from one another, thanks to future cross-correlated radio- and GW-astronomy analyses.

2 GWs from FOPTs

The GWs originated from FOPTs are characterized by a limited set of parameters. The strength of the FOPT $\alpha \equiv \alpha(T)$ at the bubble nucleation temperature $T_n$ is related to the trace anomaly $[6-9]$ and casts

$$
\alpha = \frac{1}{\rho_g} \left[ V_I - V_F - \frac{T_n}{4} \left( \frac{\partial V_I}{\partial T} - \frac{\partial V_F}{\partial T} \right) \right],
$$

where $\rho_g = (\pi^2/30)g, T_n^4$ is the radiation energy density at the bubble nucleation temperature, $g$, is the number of relativistic degrees of freedom, and $V_{I,F}$ are respectively the effective potentials just before and after the FOPT. Then $V_{I,F}$ do correspond, respectively, to the symmetric and broken phases. The characteristic rate $\beta$ of the phase transition, compared with the Hubble rate, is another important parameter impacting on the GW spectrum, expressed by

$$
\beta = \frac{H}{T_n} = \frac{\partial}{\partial T} \left( \frac{S_3}{T} \right)_{T_n},
$$

where $S_3$ is the thermal-corrected Euclidean action of the scalar field.

The typical contributions to GWs from FOPTs are provided by: (1) bubble-bubble collisions; (2) magnetohydrodynamic (MHD) turbulence; (3) sound shock waves (SW) in the plasma. The latter two effects are generated by the bubble violent expansion inside the early universe plasma. These three contributions are produced in a rapid transient of time, close to the bubble nucleation epoch. Subsequently, they are redshifted by the universe expansion, appearing to today observers as a cosmic gravitational wave stochastic background.

A strong and detectable GW signal can be produced if the bubble wall velocity is high enough. For a supersonic detonation, the velocity reads

$$
v_B = \frac{1}{1 + \alpha} \left( c_s + \sqrt{\frac{\alpha^2 + 2}{3}} \right),
$$

with $c_s = 1/\sqrt{3}$ denoting the characteristic speed of sound in the plasma. Eq. (3) provides a relation between the wall velocity and the FOPT strength magnitude.

In our analysis, we will study FOPTs related to non-runaway bubbles. It is known that in these cases, most

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1) A 0.1-100 keV FOPT takes place between the big bang nucleosynthesis (BBN, $T_{BBN} \sim 0.1-1$ MeV) and the recombination ($T_{CMB} \sim 1$ eV) epochs. It does not alter the predictions for neither the BBN nor the cosmic microwave background (CMB) radiation: the BBN happens much before the keV-scale, producing the right amount of light nuclei ratios; the CMB radiation is emitted when the keV-FOPT already ended, with no bubbles having survived and with a GW spectrum produced that is much below the CMB GW limits [14].

2) An alternative possibility of low GW frequency signals is related to solitonsynthesis of Q-balls predicted in asymmetric dark matter scenarios [47].
of FOPTs predict a dominance of the sound waves and turbulence contributions over the collision GW spectrum \[6, 7, 9, 48, 49\].

3 Majoron dark matter

As an example of WDM model which can generate FOPTs, we may consider a minimal extension of the SM symmetry with an extra global \(U_L(1)\) symmetry. We introduce a new complex scalar singlet field that is a singlet of the SM gauge group and is coupled to neutrinos and the Higgs boson as:

\[ f H \bar{v}_R + h \sigma v_R v_R^c + V(\sigma, H) + h.c., \tag{4} \]

with \(h, f\) coupling matrices, here we assign \(L = 1\) for RH neutrinos and \(L = -2\) for the singlet scalar. The potential \(V(\sigma, H)\) induces a vev for the scalar field that spontaneously breaks the extra \(U_L(1)\) once \(\langle \sigma \rangle = v'\). After the symmetry breaking, RH neutrinos acquire a Majorana mass \(m_R = h v'\) \[42-44\].

It is worth to note that, while the real part of the complex scalar singlet gets a mass proportional to \(v'\), the imaginary part remains perturbatively massless, as a Nambu-Goldstone boson of \(U_L(1)\). The imaginary part field \(J\) of \(\sigma = \phi + i J\) is dubbed Majoron and, if the symmetry is softly broken by non-perturbative quantum gravity effects, it can acquire a tiny mass \[45\]. Thus the Majoron can provide a candidate of either WDM \[45, 46\] or Cold Bose-Einstein DM \[59\].

The scalar sector of the model has a potential \(V(\sigma, H) = V_0(\sigma, H) + V_1(\sigma) + V_2(\sigma, H)\),

\[ V_0(\sigma, H) = \lambda_1 \left( |\sigma|^2 - \frac{\nu^2}{2} \right)^2 + \lambda_H \left( |H|^2 - \frac{\nu^2}{2} \right)^2 \]

\[ + \lambda_{H} \left( |\sigma|^2 - \frac{\nu^2}{2} \right) \left( |H|^2 - \frac{\nu^2}{2} \right). \tag{6} \]

In eq. (5), the \(V_{1,2}\) potentials are higher order effective operators which can efficiently catalyze FOPTs.

Resorting to a wide literature devoted to the argument, two possibilities can be considered: (1) the case of five-dimensional (5-D) operators, which softly break the \(U_L(1)\) symmetry (see refs. \[45, 60-65\]); (2) the case of six-dimensional (6-D) operators, with 5-D terms suppressed (see refs. \[66, 67\]).

The first possibility corresponds to

\[ V^{(5)}_2(\sigma, H) = \frac{\beta_1}{\Lambda} (H^\dagger H)^2 \sigma + \frac{\beta_2}{\Lambda} (H^\dagger H) \sigma^2 \sigma^* + \frac{\beta_3}{\Lambda} (H^\dagger H) \sigma^3 + h.c., \tag{8} \]

while the second scenario amounts to

\[ V^{(6)}_2(\sigma, H) = \frac{\gamma_1}{\Lambda^2} \sigma^b + \frac{\gamma_2}{\Lambda^2} \sigma^9 \sigma^5 + \frac{\gamma_3}{\Lambda^2} (\sigma^*)^2 \sigma^4 + \frac{\gamma_4}{\Lambda^2} (\sigma^*)^3 \sigma^3 + h.c., \tag{9} \]

\[ V^{(6)}_2(\sigma, H) = \frac{\delta_1}{\Lambda^2} (H^\dagger H)^2 \sigma^2 + \frac{\delta_2}{\Lambda^2} (H^\dagger H)^2 \sigma^* \sigma \]

\[ + \frac{\delta_3}{\Lambda^2} (H^\dagger H) \sigma^3 \sigma^* + \frac{\delta_4}{\Lambda^2} (H^\dagger H)(\sigma \sigma^*)^2 + \frac{\delta_5}{\Lambda^2} (H^\dagger H) \sigma^4 + h.c.. \tag{10} \]

The leading order thermal corrections to the effective potential cast

\[ V_{\text{eff}}(\sigma, T) \approx CT^2 (\sigma^2 \sigma) + V(\sigma, H), \tag{11} \]

with

\[ C = \frac{1}{4} \left( m_{\sigma}^2 + \frac{m_{\nu}^2}{\nu^2} + h^2_H + h^2_R - 24k \right), \tag{12} \]

in which including 5-D operators one finds

\[ K = K^{(5)} = (\lambda_2 + \lambda_3) v' + \beta_2 \frac{v'}{\Lambda}, \tag{13} \]

while including 6-D operators, one obtains

\[ K = K^{(6)} = \frac{1}{\Lambda^2} \left( (\delta_2 + \delta_3 + \gamma_2 + \gamma_3 + \gamma_4) v^2 + (\delta_2 + \delta_3) v^2 \right). \tag{14} \]

When exactly \(L\)-preserving operators are taken into account, the only terms allowed are the ones associated to \(\gamma_4, \delta_2, \delta_4\). In the following, we will consider the \(L\)-preserving case, reducing the large parameter space we have just introduced.

4 GW signals and NANOGrav

In Figure 1, we show several GW spectra from FOPTs which lie in the NANOGrav 12.5 year sensitivity. Comparisons with NANOGrav 11 year, PPTA and EPTA are also displayed. The GW spectra are performed following the methodology explained in refs. \[8, 9\]. In particular, we explored the case of non-runaway bubbles, dominated by the sound waves and turbulence contributions to the GW stochastic background \[6-9\]. Several FOPT spectra with nucleation temperature
around the keV-range are considered. We show here that many possible GW signals “enter” within the NANOGrav 12.5 year region with an almost flat spectra, compatible with the..125 region with an almost flat spectra, compatible with the..125 region with an almost flat spectra, compatible with the..125 region with an almost flat spectra, compatible with the.125 region with an almost flat spectra, compatible with the.

In this article, we reported that the strong evidence of a single-spectrum stochastic background found by NANOGrav 12.5 year can be explained as a GW signal from FOPTs around the warm dark matter physics scale. Indeed, reasons to suspect that FOPTs may be much below the electroweak scale are inspired by several WDM scenarios, where the typical new physics scale appears around the keV-MeV energy window rather than in the electroweak domain. Our result inspires a different paradigm towards indirect dark matter searches beyond WIMPs. On the other hand, interestingly, radio-signals from FOPTs can be discriminated by ones from PBHs or cosmic strings from future multimessenger cross-checks with GW direct detection experiments, including LISA, TAIJI, TianQin and ET. Thus, the NANOGrav result may inaugurate a new era towards a multimessenger approach to fundamental physics and dark matter.

Note added: During the preparation of our article, another explanation of NANOGrav from FOPTs appeared on arXiv [73]. While the authors of ref. [73] considered 0.1-100 MeV physics, we performed our analysis within the 0.1 keV-100 MeV-window which is compatible with warm dark matter models.

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5 Conclusions

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