An Optimal Moving Horizon Estimation for Aerial Vehicular Navigation Application

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Abstract. In this article, an optimal state is estimated using the moving horizon estimation technique (MHE), based on the minimizing the deterministic cost function defined for moving window with a finite number of samples at specific time interval. The optimal moving horizon observer was designed and implemented for the non-linear dynamic problem of aerial vehicle integrated navigation. The low grade commercial inertial measuring instrument (IMU) equipped with accelerometers and gyros sensors instrumented on-board in the strapdown configuration, is employed for collection of the real time experimental data. The data fusion algorithm of moving horizon estimation is realized and the results are collected from the offline algorithm testing on the Matlab software platform. Essential data processing and cleaning of data processing was conducted before algorithm application i.e. solving the multi rate sensors data synching and removing high frequency unwanted contents. Finally, the aerial vehicle dead reckoning integrated navigation was performed with recursive observer using IMU/GPS avionics. Contrary to the widely practiced extended Kalman filter results, recursive observer of MHE exhibited performance enhancement in the response and precision aspect, regardless of environmental noise and failure scenarios.

1. Introduction

Global positioning and satellite system has versatile applications in civil and military fields, with certain deficiencies of the environmental signal attenuations, signals unlock in presence of hurdles i.e. roof, trees for land vehicle navigations, and the low refresh rates, limits its use for certain high dynamic systems. Despite the above mentioned drawbacks, is its precision makes it attractive as the aiding source for high rate IMU systems. In literature the optimal state for data mixing/fusion is computed using the unscented filtering [1], particle filtering and widely practiced extended type filtering [2]. Kalman Filter serves as the optimal state estimator and various approaches are based on the Kalman filtering technique, a data fusion algorithm central difference Kalman filter was designed by Zhao [3]. Errors are introduced due to the linearization of the system model in Kalman filter based techniques and distribution function selection is difficult in the particle type of filtering. Currently the moving horizon estimation serves an optimal tool for handling the state estimation of dynamic systems despite of non-linearities and disturbances. Recursive moving horizon technique minimizes the cost function in the defined window of fixed number of measurements samples, composed of prediction error calculated on recent batch of samples and \textsuperscript{2}nd is the fresh measurements response at current time [4]. In [5], MHE observer was designed and implemented for a simple case of land vehicle 2D navigation. MHE is also termed as the sliding window estimation, as at each time interval a fixed number of sample is processed for optimal state calculation by minimizing the cost function equation.
and on the subsequent interval new measurements are sampled for recursive continuous operation. Here the actual aerial vehicle flight data is used for the optimal state estimation extending MHE based linear observer [6] for nonlinear complex navigation problem, further this optimal state is processed for the navigation parameter computations for onwards use in the flight computer. In literature MHE is being studied for different applications, like the model predictive controllers [7], spacecraft attitude estimation and sensors calibration [8] and integrated navigation filtering [9].

In this article the optimal state is estimated by designing the nonlinear estimator, the mentioned estimator is validated offline in the Matlab environment. For algorithm validation, the real time raw data was collected from an aerial vehicle flight, containing the onboard instrumented inertial measurement unit (IMU) accelerometers and gyro sensors along 3-axis, and GPS receiver installed with smart antenna set. Various supporting avionics for data collection and processing were also installed i.e. analog/digital modules, power source, conditioners and data logger module. Data processing and cleaning was conducted with the raw data from sensors, including the data matching and synching processing with the low rated GPS data, unwanted high frequency noise rejection with the high order Butter worth filtering [10] and highly precise integration scheme of RK-4 [11] replacing the simple integration for avoiding the unwanted computation errors. The algorithm stage comprises of three different stages, one is the dead reckoning stage using the cleaned/processed sensors data and computing the free inertial solution with the singularity avoiding quaternions technique, MHE optimal state estimation based on the nonlinear system model with the measurement matrix, incorporating the optimal state for computing the final navigation parameter and updating the latest states. Finally, for data evaluations two references were available; one is the real time GPS recorded data and the other is the extended Kalman filter implemented side by side with the MHE observer with same starting and data inputs ensured. The complete mutirate package of data cleaning, navigation algorithm implementation, optimal estimation, parameters correction with optimal states, reference extended Kalman filtering and data evaluation was performed on the Matlab platform with homogenous environmental conditions, the data comparison stage witnesses the performance enhancement with MHE.

Consequently, this manuscript is presented comprising the following sections; sections 2 discusses the strap down inertial navigation system model formation, section 3 discusses the moving horizon observer, section 4 is reserved for the algorithm response with the help of the experimental data in comparison to the reference EKF and conventionally article ends with paper conclusion.

2. Non-Linear Strapdown INS Error Model
In this article the navigation reference coordinate system followed is the north east down or in other words NED geographic reference coordinate system. The sensors installed on the vehicle are considered in the body reference coordinate system and the transformation from body to navigation frame is realized with the quaternion rotation matrix formation [12].

$$
\begin{bmatrix}
q_0 \\
q_1 \\
q_2 \\
q_3
\end{bmatrix} = \begin{bmatrix}
0 & -\omega_y & -\omega_z & \omega_x \\
\omega_y & 0 & \omega_z & -\omega_x \\
\omega_z & -\omega_y & 0 & \omega_x \\
-\omega_x & \omega_y & -\omega_z & 0
\end{bmatrix}
\begin{bmatrix}
q_0 \\
q_1 \\
q_2 \\
q_3
\end{bmatrix}.
$$

Using the quaternion rotation matrix formation, the kinematical transformation and the velocity transformation are converted to quaternion matrix formulation as follow:

$$
\begin{bmatrix}
q_0 \\
q_1 \\
q_2 \\
q_3
\end{bmatrix} = \begin{bmatrix}
q_0^2 + q_1^2 - q_2^2 - q_3^2 \quad 2(q_1 q_2 + q_3 q_0) \quad 2(q_1 q_3 - q_2 q_0) \\
2(q_1 q_2 - q_3 q_0) \quad q_0^2 + q_2^2 - q_1^2 - q_3^2 \quad 2(q_2 q_3 + q_1 q_0) \\
2(q_1 q_3 + q_2 q_0) \quad 2(q_2 q_3 - q_1 q_0) \quad q_0^2 + q_3^2 - q_1^2 - q_2^2
\end{bmatrix}^T
$$

The strap down differential equations [13] are given as;

$$
\delta \dot{V}^n = f^n \times \Phi^n + \nabla^p - (2 \delta \omega_{en}^n \times V^n) - (2 \omega_{en}^n + \omega_{en}^n) \times \delta V^n
$$

$$
\delta L = \frac{\delta V_{n}}{R_m + h}, \delta \lambda = \frac{\delta V_{e}}{L} \sec \lambda + \frac{V_{e}}{(R_n + h)^2} \tan L \sec L \delta L, \delta \hat{h} = \delta V_{n}
$$

$$
\dot{\Phi}^n = \delta \omega_{en}^n + \delta \omega_{en}^n - (\omega_{en}^n + \omega_{en}^n) \times \Phi^n + \epsilon^n
$$

2
Where the state vector is $x_{\text{state}} = (\phi_n, \phi_e, \phi_d, \delta v_n, \delta v_e, \delta v_d, \delta \text{Lat}, \delta \text{Lon}, \delta \text{Alt})$ with $\phi_n, \phi_e, \phi_d$ are the attitude, $\delta v_n, \delta v_e, \delta v_d$ are the velocity error states and $\delta \text{Lat}, \delta \text{Lon}, \delta \text{Alt}$ are the position states.

$Z_p(t) = [(L_1 - L_2) R_{e_0}^n; (\lambda_1 - \lambda_2) R_{e_0}^e \cos L; h_1 - h_2; V_{in} - V_{Gin}; V_{le} - V_{Gle}; V_{in} - V_{Gin}]^T$ (3)

The observation/measurement matrix of system depends on the number of available measurement sources. Here the three position and three velocities parameters are received from the GPS receiver forming the system observation matrix. The state space compact representation takes the form;

$$X(k+1) = f(x(k), u(k)) + w(k), Z(k) = h(x(k), u(k)) + v(k)$$ (4)

3. Optimal Moving Horizon State Estimator

This section explains the structure of MHE state estimator, which is based on minimizing the estimation cost function over a moving fixed sized window of samples. MHE is the convex optimization framework, incorporating the current measurement information along with the preceding measurements set to estimate the upcoming optimal state. MHE can incorporate the system constraints directly and can be formulated for nonlinear problems, the system cost function comprised of prediction error based on current data and arrival cost summarizing the past samples, given as [8];

$$J_t = \sum_{k=0}^{\tau} \hat{v}_k R^{-1} \hat{v}_k + w_k^T Q^{-1} w_k + (x_0 - x_0) P_{\text{C}_0}^{-1} (x_0 - x_0)^T$$ (5)

The equation Eq. (5) can further be expressed in two times interval to formulate the recursive state estimator valid for continuous recursive state estimation.

$$J_t = \sum_{k=t-N+1}^{t} (\hat{v}_k R^{-1} \hat{v}_k + w_k^T Q^{-1} w_k) + \sum_{k=0}^{N-1} (\hat{v}_k R^{-1} \hat{v}_k + w_k^T Q^{-1} w_k) + (x_0 - x_0) P_{\text{C}_0}^{-1} (x_0 - x_0)^T$$ (6)

Subject to, $\hat{x}_k \in \Omega_1, \hat{v}_k \in \Omega_2, \hat{w}_k \in \Omega_3$, N is the sample size considered at a time, depending on the computation capacity of the resources available onboard, P is the positive definite covariance weighting matrix, Q and R are the system matrices. The pictorial framework of MHE estimator can be explained in Fig.1. By minimizing the Eq. (6), the least squares optimal state solution is given as [6],

$$x(k) = F^N x(k-1) + F^N (P_0 + P_N)^{-1} M_N ^T * \text{Wt}*[Z(k-N) - M_N x(k-N))]^T$$ (7)

Putting $M_N = [(HF^N)^T, (HF^{N-1})^T, ..., (H)^T]^T$, and solving, optimal state equation forms as;

$$\hat{x}(k+1) = (F - L_N HF) \hat{x}(k) + L_N y(k+1) - L_N (y(k-N) - HF^N \hat{x}(k))$$ (8)

Where, $L_N = F^N P_{C_{N-1}} (HF^N)^T * (I + (HF^N) P_{C_{N-1}} (HF^N)^T)^{-1} + R^t$, and $L_N = a * F^N P_{C_{N}} (HF^N)^T$. F is the system transition matrix and H is the output observation matrix, a is the tuning parameter with value variation from 0–1. L_N is the system gain matrix and the $(y(k-N) - HF^N \hat{x}(k))$ can be viewed as the appropriate weighting matrix. The covariance propagation matrix is derived as;

$$P_{C_N} = (P_{C_{N-1}} - P_{C_{N-1}} (HF^N)^T * (I + (HF^N) P_{C_{N-1}} (HF^N)^T)^{-1} * (HF^N) P_{C_{N-1}}) + Q_n$$ (9)

For comparison purpose the extended Kalman filter (EKF) was implemented, in brief the implemented Kalman filter [3] can be expressed as;

**Time Update**

$$\hat{x}_{k+1} = \Phi_{k+1} \hat{x}_k,$$

$$P_{k+1} = \Phi_k P_k \Phi_k^T + Q_k$$

**Measurements**

$$K_k = P_k H_k^T \left[ H_k P_k H_k^T + R_k \right]^{-1}$$

$$\hat{x}_k = \hat{x}_k + K_k [z_k - H_k \hat{x}_k]$$

$$P_k = [I - K_k H_k] P_k$$
4. Experimental Setup and Analysis

The algorithm was validated with the actual flight test data of a small unmanned vehicle. This experiment was conducted with the low-grade commercial IMU system, installed with the commercial-grade GPS receiver with the data rate of 1hz, for continuous visibility multiple smart antennas was fabricated on the vehicle body. The continuous clean power supply was ensured with help of onboard source and dc-dc power conditioner.

4.1 Performance Analysis

In order to assess the performance of the MHE state estimator, a prototype of the GPS/IMU integrated navigation was implemented for aerial vehicle dynamic navigation. Figure.2 shows the system configuration, in parallel an extended Kalman filter was implemented with equal inputs, initializations and environmental disturbances for meaningful comparison. Contrary to the spherical model of the earth, ellipsoidal earth model was implemented, also the latitude dependent gravity model was exploited to avoid the errors. SI unit’s data processing and representation are used throughout the article. The navigation parameters velocities and positions were initialized from the available GPS values and for the attitude initialization, the coarse alignment routine was followed. MHE and Kalman filter were both aided with the available GPS 6 measurements, for more insight following results are presented. In figures.3~5, the velocity error state and velocity parameter response was presented in the normal flight, under the similar data input, initializations and disturbance condition, it is shown that the MHE velocity states error with respect to GPS was optimal as compared to the extended Kalman filter response because MHE has more data information by considering a batch of samples for next state estimate.
In the failure scenario simulation as shown in Fig.6, GPS data to the algorithm was stopped and the measurement vector was updated from the previously calculated states. The GPS off interval is...
highlighted for easier comparative analysis, also in the GPS off failure scenario simulation MHE proved to be more robust as compared EKF. EKF response was comparable to the MHE during a normal profile run, but in the failure mode, the course divergence & slower convergence was observed.

5. Conclusion
In this manuscript, moving horizon estimation technique is implemented for the small aerial vehicle navigation. The quaternions based nonlinear state and observation models, and focal moving horizon observer were developed for GPS/IMU integrated navigation system. In the navigation, the three set of integrations i.e. quaternions coefficient integration, velocity integration and position integration were replaced with the high order Runge-Kutta integration. Experiments and simulation analysis demonstrate that the proposed moving horizon based estimator enhance the navigation accuracy for the GPS/IMU integrated navigation as indicated by results, for normal and failure scenarios. Future research will focus on the GPS/IMU coupling schemes i.e. tightly coupled systems, resources optimization and implementation for a multisensory-based fusion schemes, and real-time on-board algorithm validation.

6. References
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