A CLASS OF UNIVALENT FUNCTIONS WITH REAL COEFFICIENTS

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Abstract. In this paper we study class $S^+$ of univalent functions $f$ such that $\frac{z}{f(z)}$ has real and positive coefficients. For such functions we give estimates of the Fekete-Szegő functional and sharp estimates of their initial coefficients and logarithmic coefficients. Also, we present necessary and sufficient conditions for $f \in S^+$ to be starlike of order $1/2$.

1. Introduction

Let $A$ be the class of functions $f$ that are analytic in the open unit disc $D = \{z : |z| < 1\}$ of the form $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$. Then the class of starlike functions of order $\alpha$, $0 \leq \alpha < 1$, is defined by

$$S^*(\alpha) = \left\{ f \in A : \Re \frac{zf'(z)}{f(z)} > \alpha, \ z \in D \right\},$$

while $S^* \equiv S^*(0)$ is the well known class of starlike functions mapping the unit disc onto a starlike region $D$, i.e.,

$$w \in f(D) \iff tw \in f(D) \text{ for all } t \in [0,1].$$

More on this classes can be found in [7] and [1].

Further, let $S^+$ denote the class of univalent functions in the unit disc with the next representation

(1) \[ \frac{z}{f(z)} = 1 + b_1 z + b_2 z^2 + \ldots, \quad b_n \geq 0, \ n = 1, 2, 3, \ldots \]

For example, the Silverman class (the class with negative coefficients) is included in the class $S^+$. Namely, that class consists of univalent functions of the form

$$f(z) = z - a_2 z^2 - a_3 z^3 - \ldots, \quad a_n \geq 0, \ n = 2, 3, \ldots$$

which implies that

$$\frac{z}{f(z)} = \frac{1}{1 - a_2 z - a_3 z^2 - \ldots},$$

i.e., $\frac{z}{f(z)}$ has the form (1). Also, the Koebe function $k(z) = \frac{z}{(1+z)^2} \in S^+$. The next characterization is valid for the class $S^+$ (see [2]):

(2) \[ f \in S^+ \quad \iff \quad \sum_{n=2}^{\infty} (n-1)b_n \leq 1. \]

From the relations (1) and (2) we have that

(3) \[ b_2 + 2b_3 \leq 1 \quad (\Rightarrow \ 0 \leq b_2 \leq 1, \ 0 \leq b_3 \leq 1/2). \]

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If we put \( f(z) = z + a_2 z^2 + \ldots \), then by using (1) we easily obtain that
\[
(4) \quad b_1 = -a_2, \quad b_2 = a_2^2 - a_3.
\]
This implies that \( 0 \leq b_1 \leq 2 \). From (1) we obtain
\[
\log \frac{f(z)}{z} = -\log(1 + b_1 z + b_2 z^2 + \cdots),
\]
or
\[
\sum_{n=1}^{\infty} 2\gamma_n z^n = -b_1 z + \left(\frac{1}{2} b_1^2 - b_2\right) z^2 + \left(-\frac{1}{3} b_1^3 + b_1 b_2 - b_3\right) z^3 + \cdots.
\]
(We call \( \gamma_n, n = 1, 2, \ldots \) the logarithmic coefficients of the function \( f \).) From the last relation we have
\[
(5) \quad \left\{ \begin{array}{l}
2\gamma_1 = -b_1, \\
2\gamma_2 = \frac{1}{2} b_1^2 - b_2, \\
2\gamma_3 = -\frac{1}{3} b_1^3 + b_1 b_2 - b_3.
\end{array} \right.
\]

For functions \( f \) in \( S^+ \) we give sharp estimates of their logarithmic coefficients \( \gamma_1, \gamma_2 \) and \( \gamma_3 \) of \( f \) and lower and upper bound of the Fekete-Szegő functional \( (a_3 - \gamma a_2^2) \). Additionally, sharp estimates of coefficients \( a_2, a_3, a_4 \) and \( a_5 \) for functions in a class containing \( S^+ \) is given. At the end the relation between the class \( S^+ \) and the class of starlike functions is studied.

2. Results over the coefficients

We start the section with a study of the Fekete-Szegő functional for the functions in the class \( S^+ \).

**Theorem 1.** For each \( f \in S^+ \) we have
\[
-1 \leq a_3 - \gamma a_2^2 \leq \begin{cases} 
1 + 2e^{-2\gamma/(1-\gamma)}, & 0 \leq \gamma \leq \frac{\nu_0}{1+\nu_0} = 0.456278 \ldots \\
2(1-\gamma)^{\nu_0+1} + 1, & \frac{\nu_0}{1+\nu_0} \leq \gamma < 1.
\end{cases}
\]
where \( \nu_0 = 0.83927 \ldots \) is the positive real root of the equation
\[
(6) \quad 2(2\nu + 1)e^{-2\nu} = 1.
\]
The lower bound is sharp due to the function \( f_1(z) = \frac{1}{1+z^2} \).

**Proof.** We will use the same method as in the proof of Fekete-Szegő theorem for the class \( S \) (see [1] Theorem 3.8, p. 104)). First, from the relation (1) we have that
\[
-1 \leq a_3 - a_2^2 = -b_2 \leq 0.
\]
Since \( a_2 \) and \( a_3 \) are real we can put (as in that proof) \( a_2 = -2 \int_0^\infty \varphi(t)dt \), where \( \varphi \) is real function and \( |\varphi(t)| \leq e^{-t} \). If we put
\[
\int_0^\infty |\varphi(t)|^2 dt = \left( \nu + \frac{1}{2} \right) e^{-2\nu}, \quad 0 \leq \lambda < \infty,
\]
then by Valiron-Landau lemma we have that \( |a_2| \leq 2(\nu + 1)e^{-\nu} \). Also, we have (as [1]):
\[
a_3 - a_2^2 = 1 - 4 \int_0^\infty |\varphi(t)|^2 dt = 1 - 4 \left( \nu + \frac{1}{2} \right) e^{-2\nu} \leq 0
\]
if and only if \( 0 \leq \nu \leq \nu_0 \), where \( \nu_0 = 0.83927 \ldots \) is the root of the equation (6).
Now, for $0 \leq \gamma < 1$ and for $0 \leq \nu \leq \nu_0$ we have that
\[
\begin{align*}
a_3 - \gamma a_2^2 & \leq 4(1 - \gamma) \left( \int_0^\infty \phi(t)dt \right)^2 - 4 \int_0^\infty [\varphi(t)]^2 dt + 1 \\
& = 4e^{-2\nu} \left[ (1 - \gamma)(\nu + 1)^2 - \left( \nu + \frac{1}{2} \right) \right] + 1 \\
& =: \psi(\nu).
\end{align*}
\]
It is an elementary fact that the function $\psi$ has its maximum $\psi(\gamma/(1 - \gamma))$ if $\frac{\gamma}{1 - \gamma} \in [0, \nu_0]$ and $\psi(\nu_0)$ if $\frac{\gamma}{1 - \gamma} \notin [0, \nu_0]$, which gives the right estimation in the theorem (in the second case we used that $\nu_0$ satisfies the equation (1)).

On the other hand side $a_3 - \gamma a_2^2 \geq a_3 - a_2^2 \geq -1$. \hfill $\square$

Next we give sharp estimates of the first three logarithmic coefficients for functions in $S^+$.

**Theorem 2.** Let $f \in S^+$ and let $\gamma_1, \gamma_2, \gamma_3$ be its logarithmic coefficients. Then

(a) $-1 \leq \gamma_1 \leq 0$;
(b) $-\frac{1}{2} \leq \gamma_2 \leq \frac{(\nu_0 + 1)^2}{(2\nu_0 + 1)} = 0.631464\ldots$;

where $\nu_0 = 0.83927\ldots$ is the solution of the equation (7);
(c) $-\frac{1}{4} \leq \gamma_3 \leq \frac{1}{7}$.

Some of these results are the best possible.

**Proof.**

(a) It is evident since $\gamma_1 = -\frac{1}{2}b_1$ (from (6)) and $0 \leq b_1 \leq 2$. The functions $f_1(z) = \frac{z}{1+ez}$ and $f_2(z) = \frac{z}{1+ez}$ show that the result is the best possible.

(b) From (6) and (8) we have that
\[
\gamma_2 = \frac{1}{2} \left( \frac{1}{2}b_1^2 - b_2 \right) = \frac{1}{2} \left( a_3 - \frac{1}{2}a_2^2 \right)
\]
and the result directly follows from Theorem 1 for $\gamma = \frac{1}{2}$. For the function $f_1(z) = \frac{z}{1+ez}$ we have that $\log f_1(z) = -\log(1 + z^2) = -z^2 + \ldots$, which means that left hand side estimate is the best possible.

We were not able to prove sharpness of the right hand side of the inequality (the upper bound of $\gamma_2$), but it is worth pointing that the estimate goes in a line with the sharp estimate coresponding to the univalent functions, known to be (see [1, Theorem 3.8] or [7, p.136])
\[
|\gamma_2| \leq \frac{1}{2}(1 + 2e^{-2}) = 0.635\ldots.
\]

(c) From (5) we have
\[
2\gamma_3 = -\frac{1}{3}b_1^3 + b_1b_2 - b_3 =: u(b_1),
\]
where
\[
u(t) = -\frac{1}{3}t^3 + b_2t - b_3, \quad 0 \leq t \leq 2.
\]
Since $u'(t) = -t^2 + b_2$ and $u'(t) = 0$ for $t_0 = \sqrt{b_2}$, then the function $u$ attains its maximum $u(t_0) = u(\sqrt{b_2}) = \frac{2}{3}b_2^{3/2} - b_3 \leq \frac{2}{3}(1 - 2b_3)^{3/2} - b_3 \leq \frac{2}{3}$,
because $b_2 \leq 1 - 2b_3$ (see (3)) and the last function is a decreasing function of $b_3$, $0 \leq b_3 \leq \frac{1}{2}$.

This provides that $\gamma_3 \leq \frac{1}{3}$. For the function $f_3(z) = \frac{z}{1 + z + z^2}$ we have

$$\log \frac{f_3(z)}{z} = -\log(1 + z + z^2) = -z - \frac{1}{2}z^2 + \frac{2}{3}z^3 + \cdots,$$

i.e. $\gamma_3 = \frac{1}{3}$.

As for lower bound for $\gamma_3$, by using (5) and (4), we have

$$-2\gamma_3 = \frac{1}{3}b_1^3 - b_1b_2 + b_3 = \frac{1}{3}b_1^3 - b_1(b_1^2 - a_3) + b_3 = -\frac{2}{3}b_1^3 + a_3b_1 + b_3 = v(b_1),$$

where

$$v(t) = -\frac{2}{3}t^3 + a_3t + b_3, \quad (0 \leq t \leq 2).$$

From here we have

$$v'(t) = -2t^2 + a_3.$$

If $a_3 \leq 0$, then $v'(t) \leq 0$, and if $a_3 > 0$ then we can write

$$v'(t) = -2(b_1^2 - a_3) - a_3 = -2b_2 - a_3$$

and also we have $v'(t) < 0$, since $0 \leq b_2 \leq 1$. It means that the function $v$ is a decreasing function, which gives that

$$-2\gamma_3 \leq v(0) = b_3 \leq \frac{1}{2},$$

i.e $\gamma_3 \geq -\frac{1}{4}$. The function $f_4(z) = \frac{z}{1 + z^2}$ shows that the result is the best possible.

Let $U(\lambda)$, $0 < \lambda \leq 1$, denote the class of functions $f \in \mathcal{A}$ which satisfy the condition

$$(7) \quad \left| \left( \frac{z}{f(z)} \right)^2 f'(z) - 1 \right| < \lambda \quad (z \in \mathbb{D}).$$

We put $U(1) \equiv \mathcal{U}$. More about classes $\mathcal{U}$ and $U(\lambda)$ we can find in [5], [7], [3] and [4].

Let $U^+(\lambda)$, $0 < \lambda \leq 1$, denote the class of functions $f$ satisfy the conditions (1) and (7). By using (2) we can conclude that $U^+(1) \equiv \mathcal{S}^+$. For example, the function

$$f_\lambda(z) = \frac{z}{1 + (1 + \lambda)z + \lambda z^2} = z - (1 + \lambda)z^2 + (1 + \lambda + \lambda^2)z^3 - (1 + \lambda + \lambda^2 + \lambda^3)z^4 + \cdots$$

belongs to the class $U^+(\lambda)$ and it is extremal in many cases.

Also, if $f \in U^+(\lambda)$ and has the form (1) then by (2):

$$\sum_{n=2}^{\infty} (n-1)b_n \leq \lambda,$$
which implies the appropriate inequalities:
\begin{equation}
0 \leq b_2 \leq \lambda, \quad b_2 + 2b_3 \leq \lambda, \quad b_2 + 2b_3 + 3b_4 \leq \lambda, \ldots.
\end{equation}

For the coefficients of functions from the class $\mathcal{U}^+(\lambda)$, the next theorem is valid.

**Theorem 3.** If $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$ belongs to the class $\mathcal{U}^+(\lambda)$, $0 < \lambda \leq 1$, then we have
\begin{equation}
-(1 + \lambda) \leq a_2 \leq 0,
\end{equation}
\begin{equation}
-\lambda \leq a_3 \leq 1 + \lambda + \lambda^2,
\end{equation}
\begin{equation}
-\lambda \leq a_4 \leq \frac{4\lambda}{3} \sqrt{\frac{2\lambda}{3}},
\end{equation}
\begin{equation}
a_5 \geq \begin{cases} 
-\lambda/3, & 0 < \lambda \leq 4/27 \\
-9\lambda^2/4, & 4/27 \leq \lambda \leq 1.
\end{cases}
\end{equation}

All these inequalities are sharp.

**Proof.** For $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$ and $f \in \mathcal{U}(\lambda)$, $0 < \lambda \leq 1$ it is shown in [4] the next sharp inequalities:
\begin{equation}
|a_2| \leq 1 + \lambda, \quad |a_3| \leq 1 + \lambda + \lambda^2, \quad |a_4| \leq 1 + \lambda + \lambda^2 + \lambda^3.
\end{equation}

In the same paper the authors conjectured that $|a_n| \leq \sum_{k=0}^{n-1} \lambda^k$. Since the function $f_\lambda$ defined by (8) belongs to the class $\mathcal{U}^+(\lambda)$, then the lower bounds for $a_2$ and $a_4$ and the upper bounds for $a_3$ are valid and sharp. We only need to prove the lower bounds for $a_3$ and $a_5$ and the upper bounds for $a_2$ and $a_4$.

If $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$ and $f$ has the form (9), then by comparing the coefficients we easily conclude that
\begin{equation}
a_2 = -b_1,
\end{equation}
\begin{equation}
a_3 = -b_2 + b_1^2,
\end{equation}
\begin{equation}
a_4 = -b_3 + 2b_1b_2 - b_1^3,
\end{equation}
\begin{equation}
a_5 = -b_4 + b_2^2 + 2b_1^2b_3 - 3b_2^2b_2 + b_1^4.
\end{equation}

From $a_2 = -b_1$ and $b_1 \geq 0$, we have $a_2 \leq 0$. Also, by using (9) and (10), we obtain
\begin{equation}
-3 = b_2 - b_1^2 \leq b_2 \leq \lambda,
\end{equation}
which implies $a_3 \geq -\lambda$. The function $f_b(z) = \frac{z}{1 + \lambda z^2} (= z - \lambda z^3 + \cdots)$ shows that two previous results are the best possible.

Further, from (10) we have
\begin{equation}
a_4 = -b_3 + 2b_1b_2 - b_1^3 \leq 2b_2b_1 - b_1^3 =: w(b_1),
\end{equation}
where $0 \leq b_1 \leq 1 + \lambda$ (since $b_1 = -a_2 \leq 1 + \lambda$). It is an elementary fact to get that the function $w$ has its maximum $\frac{4b_2}{3} \sqrt{\frac{2b_2}{3}}$ for $b_1 = \sqrt{\frac{2b_2}{3}}$. It means that
\begin{equation}
a_4 \leq \frac{4b_2}{3} \sqrt{\frac{2b_2}{3}} \leq \frac{4\lambda}{3} \sqrt{\frac{2\lambda}{3}},
\end{equation}
since $0 \leq b_2 \leq \lambda$. The function
\begin{equation}
f_7(z) = \frac{z}{1 + \sqrt{\frac{2\lambda}{3}} z + \lambda z^2}
\end{equation}
shows that the result is the best possible.
Finally, from (10) we also have

\[-a_5 = b_4 - b_2^2 - 2b_1b_3 + 3b_2b_2 - b_1^2\]
\[\leq b_4 + 3b_2b_2 - b_1^2\]
\[\leq \frac{1}{3}(\lambda - b_2) + 3b_2b_2 - b_1^2\]
\[\leq \left\{\begin{array}{ll}
\lambda/3, & 0 < \lambda \leq 4/27 \\
9\lambda^2/4, & 4/27 < \lambda \leq 1
\end{array}\right.,\]

where we used the relation (9) and the same method as in the previous case. The functions

\[f_2(z) = \frac{z}{1 + \sqrt{\frac{3\lambda}{2}z + \lambda z^2}}\]

and

\[f_8(z) = \frac{z}{1 + \frac{\lambda}{3}z^4}\]

show that the result is the best possible.

For \(\lambda = 1\) from the previous theorem we have

**Corollary 1.** Let \(f(z) = z + a_2z^2 + a_3z^3 + \cdots\) belong to the class \(S^+\). Then we have the next sharp inequalities

\[-2 \leq a_2 \leq 0,\]
\[-1 \leq a_3 \leq 3,\]
\[-4 \leq a_4 \leq \frac{4}{3}\sqrt{2},\]
\[-\frac{9}{4} \leq a_5 \leq 5.

We note that upper bound for \(a_5\) follows from de Brange’s theorem.

### 3. Relation with starlike functions

In this section we study the relation between the class \(S^+\) and the class of starlike functions.

**Theorem 4.** Let \(f \in A\) and satisfy the condition (11). Then the condition

\[\sum_{n=1}^{\infty} (2n-1)b_n \leq 1\]

is necessary and sufficient for \(f\) to be in the class \(S^*(1/2)\).

**Proof.** The sufficient condition follows from the result given in the paper of Reade, Silverman and Todorov [6].

Let’s prove the necessary case. If \(f \in S^*(1/2)\), then

\[\left|\frac{zf'(z)}{f(z)} - 1\right| < 1 \quad (z \in \mathbb{D})\]

or equivalently

\[\left|\frac{z}{f'(z)} - z \left(\frac{z}{f(z)}\right)'\right| < 1 \quad (z \in \mathbb{D})\]
and from here
\[
\frac{\sum_{n=1}^{\infty} nb_n z^n}{1 - \sum_{n=2}^{\infty} (n-1)b_n z^n} < 1 \quad (z \in \mathbb{D}).
\]
If \( z = r \) \((0 < r < 1)\) we have from the last inequality that
\[
\frac{\sum_{n=1}^{\infty} nb_n r^n}{1 - \sum_{n=2}^{\infty} (n-1)b_n r^n} < 1,
\]
which implies the condition
\[
\sum_{n=1}^{\infty} (2n-1)b_n r^n < 1.
\]
Finally, when \( r \to 1 \) we have
\[
\sum_{n=1}^{\infty} (2n-1)b_n \leq 1,
\]
i.e., the relation \((*)\).

**Remark 1.** Since the class of convex functions is the subset of the class \( S^*(1/2) \), then if a function \( f \) is convex and
\[
\frac{z}{f(z)} = 1 + b_1 z + b_2 z^2 + \ldots
\]
with \( b_n \geq 0 \) for \( n = 1, 2, \ldots \), we have
\[
\sum_{n=1}^{\infty} (2n-1)b_n \leq 1.
\]
The converse is not true. Namely, for the function
\[
f(z) = \frac{z}{1 + \frac{1}{4} z^2},
\]
we have that
\[
\frac{z}{f(z)} = 1 + \frac{1}{3} z^2
\]
and
\[
\sum_{n=1}^{\infty} (2n-1)b_n = 1,
\]
but
\[
1 + \frac{z f''(z)}{f'(z)} = \frac{1 - \frac{2}{9} z^2 + \frac{1}{9} z^4}{1 - \frac{1}{9} z^4} < 0
\]
for \( z = r(0 < r < 1) \) and \( r \) close to 1.

**Theorem 5.** Let \( f \in S^+ \) and let \( b_1 = 0 \), then \( f \in S^* \).

*Proof.* Since \( f \in S^+ \), then \( \sum_{n=2}^{\infty} (n-1)b_n \leq 1 \), and since \( b_1 = 0 \), then also \( \sum_{n=2}^{\infty} (n-1)b_n \leq 1 = 1 - b_1 \), which implies, by result of Reade et al. ([6]) (see the previous sited result in Theorem 4), that \( f \in S^* \).

We note that if \( b_1 = 0 \), then \( \text{Re} \left( \frac{f(z)}{z} \right) > \frac{1}{2} \) \((z \in \mathbb{D})\) since
\[
|z/f(z) - 1| \leq |z|^2 \sum_{n=2}^{\infty} b_n \leq \sum_{n=2}^{\infty} (n-1)b_n \leq |z|^2 < 1 \quad (z \in \mathbb{D}).
\]
But under the condition of this theorem we do not have that \( f \in S^*(1/2) \). For example, for the function \( f_1(z) = \frac{z}{1+z} \) we have \( b_1 = 0 \), but \( \sum_{n=1}^{\infty} (2n-1)b_n = 3 \), which means that \( f_1 \notin S^*(1/2) \) (by the previous theorem).

\[ \square \]

**Theorem 6.** Let \( f \in S^+ \). Then the function

\begin{equation}
(12) \quad g(z) = z + \frac{1}{2} \left( \frac{z}{f(z)} - 1 - b_1 z \right)
\end{equation}

is univalent in \( \mathbb{D} \). More precisely, \( \text{Re} g'(z) > 0 \) \( (z \in \mathbb{D}) \), \( g \in S^* \) and \( g \in \mathcal{U} \).

**Proof.** It is well-known that if \( f(z) = z + a_2 z^2 + a_3 z^3 + \cdots \) and \( \sum_{n=2}^{\infty} n |a_n| \leq 1 \), then \( \text{Re} f'(z) > 0 \) \( (z \in \mathbb{D}) \) and \( f \in S^* \) with \( \left| \frac{zf'(z)}{f(z)} - 1 \right| < 1 \) \( (z \in \mathbb{D}) \). It is easily to prove those statement (in the second case better to consider the form \( |zf'(z) - f(z)| < |f(z)| \)).

By \((12)\) we have

\[ g(z) = z + \sum_{n=2}^{\infty} \frac{1}{2} b_n z^n. \]

Since \( f \in S^+ \) implies \( \sum_{n=2}^{\infty} (n-1)b_n \leq 1 \) and since \( \frac{n}{2(n-1)} \leq 1 \) for \( n \geq 2 \), then

\[ \sum_{n=2}^{\infty} n \left( \frac{1}{2} b_n \right) = \sum_{n=2}^{\infty} (n-1)b_n \frac{n}{2(n-1)} \leq \sum_{n=2}^{\infty} (n-1)b_n \leq 1. \]

By previous remarks we have \( \text{Re} g'(z) > 0 \) \( (z \in \mathbb{D}) \) and \( g \in S^* \). Also, \( g \in \mathcal{U} \) by the result given in [5]. \( \square \)

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