Dynamic modelling and experimental study of cantilever beam with clearance

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Abstract. Clearances occur in almost all mechanical systems, typically such as the clearance between slide plate of gun barrel and guide. Therefore, to study the clearances of mechanisms can be very important to increase the working performance and lifetime of mechanisms. In this paper, rigid dynamic modelling of cantilever with clearance was done according to the subject investigated. In the rigid dynamic modelling, clearance is equivalent to the spring-dashpot model, the impact of beam and boundary face was also taken into consideration. In ADAMS software, the dynamic simulation was carried out according to the model above. The software simulated the movement of cantilever with clearance under external excitation. Research found: When the clearance is larger, the force of impact will become larger. In order to study how the stiffness of the cantilever’s supporting part influences natural frequency of the system, A Euler beam which is restricted by a draught spring and a torsion spring at its end was raised. Through numerical calculation, the relationship between natural frequency and stiffness was found. When the value of the stiffness is close to the limit value, the corresponding boundary condition is illustrated. An ADAMS experiment was carried out to check the theory and the simulation.

1. Introduction

Any structural dynamic system can be regarded as that consisting of mass, damping and rigidity matrix. Dynamic parameters of structure change following the variation of clearance, resulting in the variation of modal parameters (natural frequency, damping, vibration mode) and the variation of structural dynamic response. So the variation of modal parameters and the variation of structural dynamic response can be looked as the symbol of the variation of structural clearance [1]. It is necessary to study the modelling and experiments of the mechanism with clearances [2-4]. In this paper, rigid dynamic modelling of cantilever with clearance was done according to the subject investigated. In the rigid dynamic modelling, clearance is equivalent to the spring-dashpot model, the impact of beam and boundary face was also taken into consideration. In ADAMS software, the dynamic simulation was carried out according to the model above. The software simulated the movement of cantilever with clearance under external excitation. Research found: When the clearance is larger, the force of impact will become larger. In order to study how the stiffness of the cantilever’s supporting part influences natural frequency of the system, A Euler beam which is restricted by a draught spring and a torsion spring at its end was raised. Through numerical calculation, the relationship between natural frequency and stiffness was found.
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2. Rigid dynamic modelling of cantilever with clearance

In the static condition, the support of cantilever contacts with gland and base. Cantilever will break away from the surface of gland or base when it is under external force. And then it collides with the surface of gland or base repeatedly. Under the effect of external force bending vibration of cantilever occurs after cantilever contacts with the surface of gland or base considering flexibility of cantilever. While cantilever contacts with gland with zero velocity and external force makes the cantilever break away from the surface of gland (limit case) next time, the bending vibration doesn’t occur. On the other hand, while collision force between cantilever and the surface of gland is much higher than external force, bending vibration is not possible to take place [6, 7]. Considering the motion process of cantilever in support and symmetry of force, we can obtain the simplified model as shown in figure 1.

Figure 1. Simplified model of cantilever clearance.

Clearance is equivalent to the spring-dashpot model. There is collision between the end of cantilever and the surface of gland. The motion expression is shown below:

\[ J\ddot{\theta} + 0.5l_c c\dot{\theta} + 0.5l_k k\dot{\theta} = F(t)(1 - l_c) \]  

(1)

According to nonlinear spring-dashpot model put forward by Lankarani et al. [5], the normal contact force can be expressed as the function about puncture depth \( \delta \) considering local deformation of contact point and the duration time of contact process. To obtain the above function viscous damping should be taken into consideration to reflect the energy loss which material damping leads to. The viscous damping is related to the coefficient of resilience and embedded depth. In the contact process, the value of collision force at one point is expressed as:

\[
\begin{align*}
F_n &= K\delta^n + D\dot{\delta} \\
D &= \mu\delta^n \\
\mu &= \frac{3K(1-e^2)}{\delta^2}
\end{align*}
\]  

(2)

Where, \( K \) —— Hertz contact stiffness; \( \mu \) —— the coefficient of viscous damping; \( E \) —— the coefficient of resilience; \( n \) —— index: \( n=3/2 \); \( \delta \) —— relative embedded depth of contact point; \( \dot{\delta} \) —— normal relative velocity of contact point currently; \( \dot{\delta} \) —— the value of normal relative velocity in contact point currently;

The expansion expression of the normal contact force is expressed as:

\[
F_n = K\delta^n + \left[ \frac{3K(1-e^2)}{4\delta^2}\delta^n \right]\dot{\delta}
\]  

(3)
3. The simulation study of cantilever with clearance in ADAMS

According to the fundamental conditions put forward when building the rigid dynamic model of cantilever with clearance in the previous chapter, the motion process of cantilever with clearance is simulated by ADAMS under the effect of external force. ADAMS is analysis software of virtual prototype developed by MDI. ADAMS builds completely parametric geometry model of mechanical system through using interactive graphic environment, part database, constraint database and force database. Its equation solver builds dynamic equation of system by the means of Lagrange equation in multi-rigid-body system theory. It analyzes the characteristics of virtual mechanical system in statics, kinematics and dynamics to output the curves of displacement, velocity, acceleration and reaction. The performance of mechanical system, range of motion, collision detection and peak load can be predicted and the input load of finite element can be calculated.

Nonlinear spring-dashpot model which is used in collision process has been built. We need set contact stiffness, nonlinear coefficient of force, damping ratio, embedded depth in contact point and so on. The values of the above parameters should be only taken properly because the value of collision force needn’t be studied quantitatively. The other parameters use default values apart from embedded depth which value is 0.01.

The length of cantilever is 570mm and the length of support part is 50mm. The obtained model is shown in figure 2 and the enlarged drawing of support part is shown in figure 3. The top and bottom surfaces offset some distance to form the clearance. The variation of offset distance means the variation of clearance size. Unilateral clearance is considered in later content to make the study convenient.

Figure 2. ADAMS model of cantilever with clearance.

Figure 3. Clearance of ADAMS model.
The acceleration curves of MARKER point are shown in figure 4 when the clearance size is 0.2 mm and 0.4 mm. The value of acceleration reflects indirectly the size of collision force. Apparently, collision force increases with the growth of clearance size. The growth of clearance size results in the prolongation of acceleration process before collision and the growth of collision velocity. At the same time, the collision force increases correspondingly accord to equation (3).

4. Conclusions
Clearances occur in almost all mechanical systems, typically such as the clearance between slide plate of gun barrel and guide. Therefore, to study the clearances of mechanisms can be very important to increase the working performance and lifetime of mechanisms. Based on rigid dynamic model of cantilever with clearance, clearance was equivalent to spring-dashpot model in this paper. The rigid motion of support part was simulated by ADAMS through adding collision to the built model. The effect of clearance size on collision force was studied in the simulation, that is: when the clearance is larger, the force of impact will become larger.

Acknowledgements
This work was supported by the National Natural Science Foundation of China (Grant No. 11176024, 51035007).

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