Utility-driven Data Analytics on Uncertain Data

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Abstract—Modern Internet of Things (IoT) applications generate massive amounts of data, much of it in the form of objects/items of readings, events, and log entries. Specifically, most of the objects in these IoT data contain rich embedded information (e.g., frequency and uncertainty) and different levels of importance (e.g., unit utility of items, interestingness, cost, risk, or weight). Many existing approaches in data mining and analytics have limitations such as only the binary attribute is considered within a transaction, as well as all the objects/items having equal weights or importance. To solve these drawbacks, a novel utility-driven data analytics algorithm named HUPNU is presented, to extract High-Utility patterns by considering both Positive and Negative unit utilities from Uncertain data. The qualified high-utility patterns can be effectively discovered for risk prediction, manufacturing management, decision-making, among others. By using the developed vertical Probability-Utility list with the Positive-and-Negative utilities (PU±-list) structure, as well as several effective pruning strategies. Experiments showed that the developed HUPNU approach performed great in mining the qualified patterns efficiently and effectively.

Index Terms—Internet of Things, manufacturing data, uncertainty, utility, data analytics.

I. INTRODUCTION

With the increasing prevalence of sensors, mobile phones, actuators, and RFID tags, Internet of Things (IoT) applications generate massive amounts of rich data per day [1], [2], [3]. Examples include the control signals issued to Internet-connected devices like lights or thermostats, measurements from industrial and medical equipment, manufacturing data, or log files from smart-phones that record the complex behavior of sensor-based applications. These applications need to find frequent patterns that represent typical system behavior (e.g., [2], [4], [5]) as well as high risk/utility patterns that represent very important knowledge from normal behavior [6], [7], [8]. For industrial areas [3], [9], various algorithms have been designed by applying data mining methods [3], [10]. These methods can be used to evaluate the complicated and complex data from the industry and solve existing problems.

In some industrial areas, manufacturing schedules can be planned by the discovered information and knowledge, which can then be used to increase and gain utilities, maximally [3], [11]. Pattern mining and analysis, from an industrial research perspective, provides a unique and up-to-date insight into the manufacturing industry [3]. There are many industrial utility-driven manufacturing systems with different IoT applications. Consider an environment with all types of sensors to monitor abnormal conditions. Let each transaction denote the set of sensors showing above normal value at a particular instance, and the value associated with each sensor can be an abnormality or risk measure. Here the number of units can mean the number of abnormal sensors of the same type, then how to discover the high risk patterns is quite useful for manufacturing analytics in Internet of Things. In the retail industry, it can identify the purchase behaviors of customers, which can be used to make specific decisions and improve the service quality of customers. However, existing Frequent-based Pattern Mining (FPM) algorithms [10] ignore that information, and the discovered information or pattern may contain useless knowledge, such as items with a lower utility that may be discovered. Thus, traditional FPM cannot handle the problem of the quantitative databases, and also fails to extract the utility-driven patterns which are insightful for data analytics. A new utility-driven data analytics framework named High-Utility Pattern Mining (HUPM) [6], [7], [8] was presented to provide a solution to the above limitations. It is important to notice that the utility concept can be referenced to reflect multiple participants’ satisfaction [12], utility [11], [13], revenue [14], risk [13], or weight [15].

A. Motivation

To reveal high utility/risk patterns for decision-making, it considers the quantitative database, as well as the unit utility of the items. The analytics of using HUPM instead of FPM is quite helpful for system monitoring, planning, manufacturing management, and decision-making [2], [3], [11]. Most algorithms in traditional HUPM consider all items to have positive unit utilities/risks. When the items have the constraint of negative values, for example sales discount (e.g., buy two get one free) or if a supermarket/retail store sells the products at a loss to stimulate the sales of other relative items (e.g., sell printers to promote the notebook/PC), traditional HUPM cannot mine meaningful information. This situation happens in real-life scenarios, especially when promoting some profitable items for gaining more money in cross-selling conditions. Some prior works have mentioned that traditional HUPM algorithms [6], [8] may generate incomplete and missing information [16] when the dataset consists of negative unit utility objects/items. For analyzing the complex industrial data and manufacturing data [2], [11], [17], it may encounter various challenges,
i.e., the embedded uncertainty, positive risk and negative risk, and other factors. The uncertainty factor exists in many realistic situations, such as the collection of noisy data sources (e.g., GPS, wireless sensor network, RFID, or WiFi systems) [18], [19]. Traditional pattern mining algorithms cannot be utilized straightforwardly to inaccurate or uncertain environments for mining the required knowledge or information. The reason for this is that the utility can be considered as a semantic measure to value how the “utility” of a pattern based on users’ priori experience, goal, and knowledge, and the uncertainty can be considered as an objective measure to value the probability of a pattern as an objective existence; they are two totally different factors. Most existing utility-based algorithms have been studied to handle precise data, while they are unable to deal with data uncertainty [20]. If the uncertain factor was not considered in the mining process, it may find useless or misleading information with low existential probability.

B. Contribution

To solve the above limitations and problems, we attempt to design a novel algorithm named HUPNU to extract high-utility patterns with both positive and negative unit utilities from uncertain data in intelligent environment. Moreover, to deal with realistic situations in real-life applications, the positive and negative unit utilities are considered. The significant contributions of this work are listed below:

- To the best of our knowledge on pattern mining, this is the first study to discuss the problem by considering both the positive and negative unit utilities, and the uncertainty factor to discover the qualified high-utility patterns. This approach is more suitable for realistic situations such as risk prediction, manufacturing management, e-commerce, and decision making.
- A vertical and compact probability-utility list, with both positive and negative utilities (PU\(^\pm\)-list) structure, is developed. This list can keep all the essential knowledge from the database for later mining progress.
- A one-phase method called HUPNU is designed to discover the qualified high-utility patterns using the PU\(^\pm\)-list structure. The multiple database scans and the generate-and-test mechanism can be largely avoided and ignored.
- We also developed several pruning strategies to easily remove the unpromising candidates and reduce the search space size of the qualified high-utility patterns. These pruning strategies can also efficiently reduce the size of the PU\(^\pm\)-list by the designed downward closure property.
- Several experiments were conducted on both synthetic and realistic datasets. Results showed that the developed HUPNU performed great in mining the qualified patterns efficiently and effectively.

This paper is organized as follows: A literature review is given in Section II. Preliminaries and the problem statement are shown in Section III. The designed HUPNU with the several pruning strategies are studied in Section IV. Extensive experiments are conducted in Section V. The conclusion is finally provided in Section VI.

II. RELATED WORK

A. Support-based Pattern Mining

Data mining and analytic technologies are used in many different domains [2], [11], [17], [21] and they provide powerful ways of discovering useful, meaningful, and implicit information from very large datasets. Frequent Pattern Mining (FPM) is the most fundamental concept in retrieving the qualified information using a support-based constraint, and many works have been developed based on the support criteria to mine frequent itemsets or association rules [4], [5], [10]. Other factors, such as interestingness [22] or weights, have also been considered with mining criteria to find the interesting or important patterns in the task of pattern mining. Many algorithms have been designed to find the meaningful patterns from a binary database [10], [22], [23]. However, they only assess whether an item appears in a transaction, and this approach does not consider the useful factors, for example, an event may be occurred in multiple quantities in a record. Quantitative Association-Rule Mining (QARM) [24], [25] was presented instead of the binary value (0 or 1) for discovering more meaningful and useful information.

Different from processing precise data, some pattern mining approaches have been developed to deal with uncertain data, for discovering frequent expected patterns [26] or probabilistic frequent patterns [19] by taking the uncertainty from data into account. The reason is that the uncertainty factor exists in many realistic data sources (e.g., GPS, wireless sensor network, RFID, or WiFi systems). Some details of uncertain data algorithms and applications can be referred to [18].

B. Utility-based Pattern Mining with Efficiency Issues

Although QARM solves the past limitation of traditional association-rule mining, it still does not consider more important and interesting factors such as the unit utilities of the objects/items, which can bring the profitable objects to user in service-oriented manufacturing system [12], [13]. In addition, the support-based constraint of pattern mining is inappropriate for measuring the importance of the items in realistic situations. To tackle these problems, High-Utility Pattern Mining (HUPM) [6], [7], [8] was presented to reveal high-utility patterns. HUPM considers both the unit utility and quantity of the objects to show the high-utility patterns from the quantitative database, which can provide more meaningful results than that of the support-based algorithms. Yao et al. [27] first defined the utility-mining problem by considering the occurred quantity (treated as the internal utility) and unit utility of the objects/items (treated as the external utility) to reveal the itemsets with high utilities. In the past decade, HUPM has been considered as the emerging topic in many tasks of data analytics, and some well-known algorithms are developed, such as the Transaction-Weighted Utility (TWU) model [28], IHUP [5], UP-growth and UP-growth+ [8], HUI-Miner [7], and so on. Many variants of HUPM have also been discussed focusing on mining different forms of utility-oriented patterns. The importance of HUPM is increasing, especially in the current era of big data [29], and more opportunities and challenges are required for discussion and
C. Utility-based Pattern Mining with Effectiveness Issues

In addition to the efficiency issue of utility mining, a number of models have been proposed to address the effectiveness issue of discovering different kinds of high-utility patterns (HUPs). Current HUPM algorithms can successfully handle the temporal data \(^\text{[30, 31]}\), and dynamic data \(^\text{[32, 33, 34]}\). Other interesting effectiveness issues, such as HUPM with discount strategies \(^\text{[35]}\), the concise representation \(^\text{[36]}\), discriminative issue \(^\text{[37]}\), and top-k problem \(^\text{[38]}\) for HUPM, have been extensively studied. Lin et al. first proposed an utility mining model to extract the high-utility patterns from uncertain databases \(^\text{[20]}\). Different from the above utility measures of HUPs, another utility measure for utility-driven pattern mining namely utility occupancy is introduced recently \(^\text{[39]}\). And a comprehensive survey of utility mining has been provided by Gan et al. \(^\text{[13]}\).

All the above HUPM algorithms only consider the positive utilities/risks and quantities of items. However, in some real-world scenarios, the utility/risk/weight values of the objects/items in databases usually can be either positive values or negative values. Therefore, the traditional algorithms of HUPM cannot successfully be applied to handle the databases containing negative values. In the past, the two-phase HUINIV-Mine \(^\text{[16]}\), TS-HOUN \(^\text{[40]}\), and one-phase FHN \(^\text{[41]}\) algorithms are proposed to deal with precise data which containing both positive and negative utility values. However, all the existing negative-based approaches cannot be used to process uncertain data and extract the utility-driven insightful patterns.

III. PRELIMINARIES AND PROBLEM STATEMENT

We first introduce the uncertain database of the defined problem for utility-mining driven in this section. Let \(I = \{i_1, i_2, \ldots, i_m\}\) be a set of objects/items (symbols), and let the uncertain database be a set of transactions such as \(D = \{T_1, T_2, \ldots, T_n\}\), and each object/item in a transaction has an uncertain probability of existence such as \(p(i_k, T_c)\) \(^\text{[18, 19]}\). For each \(T_c\), it has the relationship such that \(i_k \in T_c\).

A positive quantity value is defined as the internal utility, and denoted as \(q(i_k, T_c)\). This quantity value shows the quantity of \(i_k\) in \(T_c\). Let \(i_k \in I\) be related to a positive or negative value, which is defined as the external utility, and denoted as \(pr(i_k)\). A set of external utility of all items in the database is denoted as ptabe = \{\(pr(i_1), pr(i_2), \ldots, pr(i_m)\)\}.

Example 1: Table I is considered as the running example and can be described as follows: It has five transactions \((T_1, T_2, \ldots, T_5)\). Transaction \(T_2\) shows that items \{c\}, \{d\}, and \{e\} are purchased together in \(T_2\), and their quantities are 1, 1, and 2, respectively. We also assume that that unit utilities of the items in the Table I are defined in ptabe as: ptabe = \{\(pr(a):$8, pr(b):$5, pr(c):-2, pr(d):$12, pr(e):$7\)\}. Thus, it is obvious to see that an item \(e\) is sold at loss.

**Definition 1 (utility measure):** The \(u(i, T_c)\) indicates the utility of an item \(i\) in the transaction \(T_c\), which can be calculated as: \(u(i, T_c) = pr(i) \times q(i, T_c)\). The utility of an itemset \(X\) in a transaction \(T_c\), which can be calculated as: \(u(X, T_c) = \sum_{i \in X} u(i, T_c)\). Note that \(X \subseteq I\). Thus, the total utility of \(X\) in a database \(D\) can be denoted as \(u(X)\), which can be calculated as: \(u(X) = \sum_{X \subseteq T_c \land T_c \in D} u(X, T_c)\).

Example 2: For example in Table I the utility of \{a\} in \(T_1\) is calculated as: \(u(a, T_1) = 5 \times $8 = $40\). The utility of \{a, e\} in \(T_1\) is calculated as: \(u(a, e, T_1) = u(a, T_1) + u(e, T_1) = 5 \times $8 + 4 \times $7 = $68\). Therefore, the utility of \{a, e\} in the Table I can be summed up as: \(u(\{a, e\}) = u(a, T_1) + u(e, T_1) + (u(a, T_3) + u(e, T_3) = ($40 + $28) + ($32 + $7) = $107\). For the \{a, b, e\}, the total utility of \{a, b, e\} can be calculated as: \(u(\{a, b, e\}) = u(a, T_1) + u(b, T_1) + u(e, T_1) + u(a, T_3) + u(b, T_3) + u(e, T_3) = ($40 + $15 + $28) + ($32 + $15 + $7) = $137\).

Since the discovered patterns are usually rare in realistic scenarios, the probabilistic frequent model \(^\text{[19]}\) cannot be directly applied for any utility-oriented applications \(^\text{[20]}\). The common method of mining uncertain data uses the expected support-based model to mine the interesting patterns. For example, the expected support of \(X\) is to sum up the support value of a pattern \(X\) in a possible world \(W_j\) as: \(expSup(X) = \sum_{i=1}^{D} (\prod_{x_i \in X} p(x_i, T_c))\) \(^\text{[18, 19]}\). The definition of the expected probability measure of the mentioned problem is defined as below.

**Definition 2 (probability measure):** Let \(X\) be a pattern (itemset) and \(T_c\) be a transaction in the database \(D\). The probability of \(X\) in \(T_c\) is denoted as: \(p(X, T_c)\), which can be calculated as: \(p(X, T_c) = \prod_{x_i \in X} p(x_i, T_c)\). Note that \(X \subseteq I\). The probability of \(X\) in \(D\) can thus be denoted as \(Pro(X)\), and defined as \(Pro(X) = \sum_{T_c \in D} (\prod_{x_i \in X} p(x_i, T_c))\).

Example 3: In Table I the probability of \{a\} in \(T_1\) can be calculated as: \(p(a, T_1) = 0.60\). The probability of \(a, e\) in \(T_1\) can then be calculated as: \(p(\{a, e\}, T_1) = p(a, T_1) \times p(e, T_1) = 0.6 \times 0.8 = 0.48\). The probability of \(a, b, e\) in \(D\) can be calculated as: \(Pro(a) = 2.5\), and the probability of \(a, b, e\) in \(D\) can be calculated as: \(Pro(a, b, e) = (p(a, b, e), T_1) + p(\{a, b, e\}, T_3) = 0.24 + 0.75 = 0.99\).

**Definition 3 (Potential High-Utility Itemset, PHUI):** Let \(X\) be a pattern (itemset) in an uncertain database \(D\). We can say that \(X\) is a PHUI if it satisfies two conditions: \(1) u(X) \geq minUtil, and 2) Pro(X) \geq minPro \times |D|\), in which \(minUtil\) is a minimum utility threshold and \(minPro\) is a minimum probability threshold. We can then conclude that interesting desired PHUI has a high expected probability and a high utility value.

**Problem Statement.** With an uncertain database, a utility table (with a positive or negative utility value of each item),
a minimum utility threshold \((\text{minUtil})\), and a minimum probability threshold \((\text{minPro})\), the problem of utility-driven data analytics from uncertain data is to discover the complete set of PHUIs.

For example, if the \(\text{minPro}\) and \(\text{minUtil}\) are respectively set as \(\text{minPro} = 0.25\) and \(\text{minUtil} = 20\), the derived PHUIs from Table I are \(\{\{a\}:$96, 2.50\}; \{\{b\}:$40, 2.50\}; \{\{d\}:$96, 2.40\}; \{\{e\}:$77, 3.55\}; \{\{a, b\}:$102, 1.30\}; \{\{a, e\}:$50, 1.51\}; \{\{b, c\}:$103, 2.15\}; \{\{c, e\}:$35, 2.225\}; \{\{d, e\}:$166, 2.22\}; \{\{b, e\}:$48, 1.475\}\}. Here, \(\{\{b\}:$40, 2.50\}\) indicates that the utility value and expected probability of pattern \(\{b\}\) are \$40 and 2.50, respectively.

IV. PROPOSED APPROACH FOR MINING PHUIs

In this section, an utility-driven data analytics framework named HUPNU is presented to discover the Potential High-Utility Itemsets (PHUIs) from an uncertain database. We further design the Probability-Utility list with positive and negative utilities (\(\text{PU}^\pm\)-list). In addition, several pruning strategies are presented here to reduce the search space of the potential HUIs. More details are described below.

A. Positive and Negative Unit Utilities

The monotonic/anti-monotonic proprieties cannot be held in utility mining [28]. In this situation, the utility of a pattern may be higher, lower, or equal to any of its subset patterns. The search space to discover the meaningful and useful patterns may become large if many items exist in the database. The TWU model [28] was presented to avoid the problem of “combinational exploration”, which aims at reducing the search space for mining the high-utility itemsets. Several extensions are then extensively studied [7, 28, 8] to improve the mining performance. However, those approaches, including the TWU model, do not consider both the positive and negative unit utilities of items, which are addressed in this paper. Moreover, the existing works do not consider the above situations in the uncertain database for discovering the PHUIs. We define the following properties for the addressed problem as follows:

\textbf{Property 1:} We first assume that \(pu(X)\) and \(nu(X)\) are respectively the sum of positive and negative utility of an item \(X\) in a database. Thus, we can obtain that the utility of \(X\) is calculated as: \(u(X) = pu(X) + nu(X)\), where \(nu(X) \leq u(X) \leq pu(X)\) holds.

From the above-stated property, we can conclude that \(u(X)\) and \(nu(X)\) cannot be straightforwardly used as the over-estimated utility of a pattern \((X)\). Moreover, even \(pu(u)\) is the upper-bound value of \(X\), the downward closure property for the superset of \(X\) cannot be held since both the positive and negative unit utilities of the items are considered in the addressed problem. We then re-utilize the traditional TWU property to establish a new over-estimated value of the discovered pattern.

\textbf{Definition 4:} In HUPM, the transaction utility \((TU)\) is defined as: \(TU(T_c) = \sum_{i \in T_c} u(i, T_c)\). In this paper, by considering both positive and negative unit utility of items, we then re-define the transaction utility as: \(\text{RTU}(T_c) = \sum_{i \in T_c \land \text{pro}(i) > 0} u(i, T_c)\) \(\). The transaction-weight utilization of an itemset \(X\) is also then redefined as \(\text{RTWU}: \text{RTWU}(X) = \sum_{X \subseteq T \land T \in E} \text{RTU}(T)\). The above-stated definitions can be used to hold the downward closure property for mining the required PHUIs. Note that \(\text{RTWU}(X) \geq u(X)\).

\textbf{Example 4:} For example, the \(\text{RTU}(T_2)\) is \$26. Consider two patterns \{a\} and \{a, b, c\}, the \(\text{RTWU}\{\{a\}\} = \$185\) and \(\text{RTWU}\{\{a, b, c\}\} = \$216\); both of them are the over-estimated values of \(u(\{a\}) = \$96\) and \(u(\{a, b, c\}) = \$137\).

B. Probability Utility \((\text{PU}^\pm\)-List Structure

In the developed HUPNU algorithm, the processing order of the items in the database is defined as \(\succ\), which holds the proprieties as: (1) the items are then sorted as the \(\text{RTWU}\)-ascending order; and (2) negative items always succeed positive ones. The designed Probability Utility \((\text{PU}^\pm\)-List structure used in the HUPNU algorithm is stated as follows:

\textbf{Definition 5 (\(\text{PU}^\pm\)-list):} Let \(X\) be an itemset in the database. The \(\text{PU}^\pm\)-list of \(X\) is denoted as: \(\text{XPU}\), and it consists of five elements: (1) \(\text{tid}\) represents the transaction ID in the database; (2) \(\text{pro}\) is the expected probability of \(X\) in \(T_{\text{tid}}\), and \(\text{pro}(X, T_{\text{tid}}) \geq 0\); (3) \(\text{pu}\) shows the positive utility of \(X\) in \(T_{\text{tid}}\), and \(u(X, T_{\text{tid}}) \geq 0\); (4) \(\text{nu}\) represents the negative utility of \(X\) in \(T_{\text{tid}}\), and \(u(X, T_{\text{tid}}) < 0\); (5) \(\text{rpu}\) represents \(\sum_{i \in T_{\text{tid}} \land \text{pro}(i) > 0} u(i, T_{\text{tid}}) \geq 0\), which keeps only a positive utility value for the remaining items.

\begin{align*}
\text{tid} & \quad \text{pro} & \quad \text{pu} & \quad \text{nu} & \quad \text{rpu} \\
T_1 & \quad 1.0 & \quad 6 & \quad 0 & \quad 13 \\
T_2 & \quad 0.55 & \quad 6 & \quad 0 & \quad 33 \\
T_3 & \quad 1.0 & \quad 24 & \quad 0 & \quad 9 \\
T_4 & \quad 0.45 & \quad 0 & \quad 0 & \quad 0 \\
T_5 & \quad 0.70 & \quad 0 & \quad 0 & \quad 0 \\
\end{align*}

Fig. 1. Constructed \(\text{PU}^\pm\)-list of pattern \(\{a\}\)

\begin{align*}
\text{tid} & \quad \text{pro} & \quad \text{pu} & \quad \text{nu} & \quad \text{rpu} \\
T_1 & \quad 1.0 & \quad 6 & \quad 0 & \quad 13 \\
T_2 & \quad 0.40 & \quad 3 & \quad 0 & \quad 10 \\
T_3 & \quad 0.55 & \quad 6 & \quad 0 & \quad 33 \\
T_4 & \quad 0.45 & \quad 6 & \quad 0 & \quad 9 \\
T_5 & \quad 0.60 & \quad 14 & \quad 0 & \quad 4 \\
T_6 & \quad 0.90 & \quad 21 & \quad 0 & \quad 0 \\
\end{align*}

Fig. 2. Constructed \(\text{PU}^\pm\)-list of the running example

\textbf{Example 5:} The search space of the developed HUPNU approach can be shown as the utility-based Set-enumeration tree [37], called a \(\text{PU}^\pm\)-tree, based on the developed \(\text{PU}^\pm\)-list. Since \(\{\text{RTWU}(a)\} = \$185\); \(\text{RTWU}(b) = \$259\); \(\text{RTWU}(c) = \$202\); \(\text{RTWU}(d) = \$231\); and \(\text{RTWU}(e) = \$285\), the designed processing order \(\succ\) of the running example can be represented as: \(\{a \succ d \succ b \succ c\}\). Thus, the constructed \(\text{PU}^\pm\)-list for all items in Table I is shown in Fig. 2. We have \{a\}. \(\text{PUL} = \)
can be concluded that \( \text{Pro} \) tid

is empty, the PU

is defined as

\[Ey.pu \]

and the computational cost of the run time can be reduced. If

\[Py \]

as the inputs, and returns

\[Pyz.PUL \]

Algorithm 1 Construction procedure

| Input: \( P, Py, Pz \) |
|-----------------------|
| Output: \( Pyz \) with its \( Pyz.PUL \) |
| 1. \( Pyz.PUL \leftarrow \emptyset \) |
| 2. set \( Probability = \text{SUM}(Y.pro) \), \( Utility = \text{SUM}(Y.rpu) + \text{SUM}(Y.rpu) \) |
| 3. for each tuple \( ex \in Pyz.PUL \) do |
| 4. if \( \exists ez \in P.z.PUL \) and \( ey.tid = ez.tid \) then |
| 5. if \( P.PUL \neq \emptyset \) then |
| 6. search each element \( e \in P.PUL \) such that \( e.tid = ey.tid \) |
| 7. \( ey \leftarrow <ey.tid, ey.pro \times ez.pro/ey.pro, ey.pro + ez.pu - e.pu, ey.nu + ez.nu - e.nu, ez.rpu> \) |
| 8. else |
| 9. \( ey \leftarrow <ey.tid, ey.pro \times ez.pro, ey.pro + ez.pu, ey.nu + ez.nu, ez.rpu> \) |
| 10. \( Pyz.PUL \leftarrow Pyz.PUL \cup \{ ez \} \) |
| 11. else |
| 12. \( Probability = Probability - ey.pro, Utility = Utility - ey.pu - ey.rpu \) |
| 13. if \( Probability < minPro \times |D| \) or \( Utility < minUtil \) then |
| 14. \( return null \) |
| 15. end if |
| 16. end if |
| 17. end if |
| 18. end for |
| 19. return \( Pyz \) |

Lemma 2 (Upper-bound probability of PHUI): The summed up probability of any node in the PU\(^+\)-tree is greater than the summed up probabilities of its supersets.

\[ X^{k-1} \subset X^k \] \[ RTWU(X^k) = \sum_{X^k \subset T, \forall T_c \in D} tu(T_c) \leq \sum_{X^{k-1} \subset T, \forall T_c \in D} tu(T_c) = RTWU(X^{k-1}) \] holds. \( \square \)
SUM($X^{k-1}.pu$) and SUM($X^{k-1}.rpu$) are less than $minUtil$, any supersets of $X$ cannot be a PHUI. Those nodes in the tree can be directly ignored and pruned to reduce the search space for mining the PHUIs.

**Strategy 3:** For an itemset $X$ in the PU$^\pm$-list, if the $X.pul$ is null or the $pro(X)$ value is less than $minPro \times |D|$, $X$ cannot be considered a PHUI; none of its superset (node) is a PHUI. Therefore, the construction procedure of PU$^\pm$-lists of $X$’s supersets can be ignored.

The efficient Estimated Utility Co-occurrence Pruning (EUCP) strategy [42] is also utilized here for the designed HUPNU algorithm. Thus, the Estimated Utility Co-occurrence Structure (EUCS) is built to keep the RTWU values of the 2-itemsets. More information can be found in [42].

**Strategy 6:** Let $X$ be a 2-itemset, which is also one of the nodes in the Set-enumeration PU$^\pm$-tree. While the depth-first search is performed, if the RTWU of $X$ is no greater than the $minUtil$ based on the built EUCS, $X$ and any supersets of $X$ are not considered to be the PHUI; the construction progress of the PU$^\pm$-tree for $X$ and the supersets of it can be ignored.

Based on the above proposed pruning strategies, the designed HUPNU is shown in detail below.

### D. The Procedure of HUPNU

In this section, the main procedure of the developed HUPNU is shown in Algorithm 2. First, it examines the uncertain database to find the values of RTWU (with the redefined RTU) and $pro(i)$ of each item (Line 1). The expected support and RTWU of each item in the set $I^*$ are then checked against the $minPro \times |D|$ and $minUtil$, respectively, and the satisfied items are then discovered and obtained. In this step, the other items can be ignored directly since they could not be the potential HUI (Line 2). The database is then scanned again (Line 4) to re-order the items in the transactions according to the designed order as $\succ$ (Line 3). In addition, the items in the transactions are then re-ordered based on the total order $\succ$ while performing the database scan. After that, the PU$^\pm$-list of each 1-item $i \in I^*$ is constructed, respectively, and the depth-first search is recursively performed by the Search procedure with the empty itemset $\emptyset$, the set of single items $I^*$, $minPro$, $minUtil$, and the EUCS (Line 5).

**Algorithm 2 HUPNU main procedure**

**Input:** $D$, $minPro$, $minUtil$, $ptable$.

**Output:** The set of PHUIs.

1. scan uncertain database $D$ to calculate the RTWU(i) and $pro(i)$ of each item $i \in I$;
2. $I^*$ ← each item $i$ such that $pro(i) \geq minPro \times |D| \land RTWU(i) \geq minUtil$;
3. sort the items in the set of $I^*$ as $\succ$ order;
4. scan database $D$ to build the PU$^\pm$-list of each item $i \in I^*$ and construct EUCS;
5. call Search($\emptyset, I^*, minPro, minUtil, EUCS$).
6. return PHUIs

The search procedure of the HUPNU is described in Algorithm 3. For each extension $Py$ of $P$, if the probability of $Py$ is no less than $minPro \times |D|$, and the summed up actual utility of $Py$ in the PU$^\pm$-list (denoted as $SUM(Y.pu) + SUM(Y.nu)$) is no less than $minUtil$, then $Py$ can be considered to be a PHUI (Lines 2 to 4). The developed pruning Strategies 3 and 4 are then performed to check whether the extension ($Py$) can be explored (Line 5). This progress can be executed by integrating $Py$ with all extensions $P_{z}$ of $P$, such that $z \succ y$ and $RTWU\{y, z\} \geq minUtil$ (Line 8, pruning Strategy 6), to generate the extensions ($P_{yz}$) containing $|Py| + 1$ items. After that, the PU$^\pm$-list of $P_{yz}$ can be built by the Search procedure to join the PU$^\pm$-lists of $P$, $Py$, and $P_{z}$ (Lines 9 to 13). Note that the promising PU$^\pm$-lists can only be later explored (Line 12, pruning Strategy 5). A recursive Search procedure of $P_{yz}$ is then performed to obtain its utility and explore the extension(s) (Line 16).

### V. Experimental Study

In this section, we evaluate the performance of the developed HUPNU algorithm. All the algorithms were implemented with Java language, and executed on an Intel Core-i5 processor running on Microsoft Windows 7 64 bits operation system with 4GB of main memory. Memory usage was measured by Java API. To the best of our knowledge, this is the first paper that considers both the positive and negative unit utilities of items in an uncertain database; none of the previous works handle this topic. Thus, the developed HUPNU along with several designed pruning strategies were compared in the experiments. HUPNU$P_{12}$ denotes that pruning Strategies 1 and 2 were involved in the HUPNU; HUPNU$P_{123}$ considers pruning Strategies 1, 2, and 3; HUPNU$P_{1234}$ takes the pruning...
Strategies 1, 2, 3, and 4; and HUPNU_{All} is concerned with all the pruning Strategies (1 to 5) for evaluation. Experiments were carried out on five realistic datasets (i.e., kosarak, accidents, psumb, retail, and mushroom) and one synthetic dataset [43]. The individual characteristics of each of the six datasets are given below.

- **kosarak:** it has 990,002 transactions, and the number of distinct items is 41,270. The average length and the maximum length of transactions are 8.09 and 2,498, respectively.
- **accidents:** it has 340,183 transactions and 468 distinct items. For the all transactions, the average length is 33.8 and the maximum length is 51.
- **psumb:** it has total 49,046 transactions and 2,113 distinct items. It is a very dense dataset since the average length of each transaction is 74.
- **retail:** this dataset contains 88,162 transactions and total 16,470 distinct items. The average length of transactions is 10.3, and the maximum length is 76.
- **mushroom:** it has 8,124 transactions with 119 distinct items. It is a dense dataset since both the average length and the maximum length are 23.
- **T1014D100K [43]:** this dataset has 100,000 transactions with 870 items. These transactions have average 10.1 items, and the maximum length is 29.

The external utilities of different items for the six datasets were generated in the range of [1,000, 1,000] using a log-normal distribution. In addition, the quantities of the items were randomly generated in the range of [1, 5]. These settings are similar to the previous well-known algorithms [7], [22], [8] for HUPM. Moreover, the unique probabilities of the items were randomly assigned in the range of (0.0, 1.0). In the experiments, we evaluated the implemented algorithms in terms of runtime, number of visited nodes (or patterns), and scalability. The results are given below.

### A. Evaluation of Runtime

The runtime of the four implemented algorithms were then compared under the variants of minUtil and minPro thresholds. Results in terms of the two thresholds are shown in Fig. 3 and Fig. 4 respectively.

As shown in Fig. 3 and Fig. 4, the runtimes of all implemented algorithms decreased along with the increasing of the minUtil or minPro threshold. Specifically, the implemented algorithms with variants of pruning strategies greatly improved the performance, up to nearly one or two orders of magnitude faster than the baseline approach. For example, HUPNU_{P1234}, which adopts all the efficient pruning strategies, outperformed the other variants of the designed approach. The reason is that the HUPNU_{P1234} algorithm is concerned with all the pruning strategies, and the unpromising candidates can be greatly reduced. Therefore, the traversal procedure to explore the unpromising patterns in the enumeration tree can be avoided, as well as the costly join operation to generate the huge unpromising candidates of the PU^{±}-lists. When the minUtil and minPro values were set lower, more and longer patterns were mined, and there was a greater computational cost in terms of the runtime that was required. This situation can be easily observed in a very dense dataset, for example in accidents and psmb.

The developed PU^{±}-list can also easily help the variants of the algorithms to directly mine the required patterns without candidate generation. The list structure can effectively reduce the multiple database scans. We can also observe that the designed pruning strategies can greatly help with reducing the number of unpromising patterns. The required memory usage of the variants of algorithms is much less, but due to the page limit, we omit some details of the results. However, in the general pattern-mining approach, it can be easily concluded that more memory usage was required when the threshold was set lower. The designed PU^{±}-list could actually solve this limitation by using a more compressed search space, and achieve effectiveness and efficiency for mining the PHUIs.

### B. Evaluation of Number of Patterns (Visited Nodes)

In this section, the numbers of visited nodes (also known as the candidate patterns) in the designed PU^{±}-tree are compared to evaluate the effects of pruning strategies. Note that the number of visited nodes of the four variants of the HUPNU algorithm is denoted as N_1, N_2, N_3, and N_4, respectively. According to the same parameter settings in Fig. 3 and Fig. 4, the final results of the patterns are respectively shown in Table II and Table III in terms of minUtil and minPro thresholds.

From Table II and Table III, it can be seen that N_1 > N_2 > N_3 > N_4 > PHUIs in terms of varied minUtil and minPro. We can then draw a conclusion such that: (1) The number of designed PHUIs discovered was fewer in the uncertain dataset compared to the number of candidate patterns. (2) The set of the discovered PHUIs was more meaningful; the designed HUPNU discover more concrete and useful patterns with both positive and negative unit utilities of item constraints than the traditional algorithms in HUPM. (3) The search space of the HUPNU was very large without any pruning strategies, but the developed strategies could reduce the its size. (4) The results were reasonable, and even the size of the search space could be reduced by different pruning strategies, the completeness, and the correctness of the final PHUIs could still be held.

For the designed pruning strategies 2, 3, and 4 of the implemented HUPNU_{P1} algorithm, they hold the upper-bound values for the utility and probability of the patterns, and thus combinational exploration could be avoided. However, some unpromising patterns could not be effectively filtered, which can be obviously seen from the gap between the number of final PHUIs and N_1. It can also be seen that PU-Prune had a great performance in removing unpromising patterns, which could be observed in N_1 and N_2. This strategy avoids the construction progress for the numerous unpromising patterns. We can also see that the EUCP strategy made a great effort to reduce the search space. The relationships of N_1 > N_2 > N_3 > N_4 were correctly held. When the threshold value was set lower, for example, minUtil or minPro, the gap between different implemented algorithms of the visited patterns could
Fig. 3. Runtime under varied minUtil with a fixed minPro.

Fig. 4. Runtime under varied minPro with a fixed minUtil.
become huge, and the effectiveness of the designed pruning strategies is held.

C. Evaluation of Scalability

From Fig. 5, the scalability was carried out on a realistic BMS-POS dataset under varied dataset sizes. The threshold values were set as $\minPro = 0.0001$ and $\minUtil = 10k$, and the dataset size was varied from 100k to 500k. In Fig. 5(a), we can then find that the runtimes of the four implemented algorithms linearly increased along with the increasing size of the dataset. We can see that the runtime of HUPNU$_{P_{123}}$ was close to that of HUPNU$_{P_{123}}$, and both of them were significantly faster than HUPNU$_{P_{1}}$. We also can see that HUPNU$_{P_{1234}}$ outperformed the other implemented algorithms. When the size of the dataset increases, the gap between the implemented algorithms becomes larger but they still remain stable with linear growth. The memory usage of four implemented algorithms is shown in Fig. 5(b). HUPNU$_{P_{1}}$ requires the most memory usage; HUPNU$_{P_{123}}$ and HUPNU$_{P_{1234}}$ has similar results, and requires the least memory usage. Fig. 5(c) shows the number of visited patterns of the four implemented algorithms and the final PHUs. The results show the effectiveness and efficiency of the designed pruning strategies, and when the size of the dataset becomes larger, the gap of those algorithms becomes huge.

VI. CONCLUSION

In this paper, we present a HUPNU algorithm by jointly considering the uncertainty and utility (both positive and negative) factors, to reveal the qualified high-utility patterns. This is the first work concerning these realistic factors in some real-life situations, such as Internet of Things data and manufacturing data. A vertical structure named PU$^+$-list was designed to keep necessary information, such as probability, and the positive and negative utilities of the items for later mining progress. Based on the above properties, the HUPNU algorithm could directly produce the qualified high-utility patterns in one phase. Moreover, several efficient pruning strategies were also developed to greatly reduce the search space for mining the promising patterns, and thus the computation could be sped up efficiently. Extensive experiments were carried out on several synthetic/realistic datasets to show the efficiency and effectiveness of the designed algorithm in terms of runtime, number of discovered qualified patterns, and scalability.

REFERENCES

[1] L. Atzori, A. Iera, and G. Morabito, “The internet of things: A survey,” Computer Networks, vol. 54, no. 15, pp. 2787–2805, 2010.
[2] C. W. Tsai, C. F. Lai, M. C. Chiang, L. T. Yang et al., “Data mining for internet of things: A survey,” IEEE Communications Surveys and Tutorials, vol. 16, no. 1, pp. 77–97, 2014.
[3] F. Chen, P. Deng, J. Wan, D. Zhang, A. V. Vasilakos, and X. Rong, “Data mining for the internet of things: literature review and challenges,” International Journal of Distributed Sensor Networks, vol. 11, no. 8, p. 431047, 2015.
[4] R. Agrawal, R. Srikant et al., “Fast algorithms for mining association rules,” in Proceedings of the 20th International Conference on Very Large Databases, vol. 1215, 1994, pp. 487–499.
[5] J. Han, J. Pei, Y. Yin, and R. Mao, “Mining frequent patterns without candidate generation: A frequent-pattern tree approach,” Data Mining and Knowledge Discovery, vol. 8, no. 1, pp. 53–87, 2004.
[6] C. F. Ahmed, S. K. Tanbeer, B. S. Jeong, and Y. K. Lee, “Efficient tree structures for high utility pattern mining in incremental databases,” IEEE Transactions on Knowledge and Data Engineering, vol. 21, no. 12, pp. 1708–1721, 2009.
[7] M. Liu and J. Qu, “Mining high utility itemsets without candidate generation,” in Proceedings of the 21st ACM International Conference on Information and Knowledge Management. ACM, 2012, pp. 55–64.
[8] V. S. Tseng, B. E. Shie, C. W. Wu, and P. S. Yu, “Efficient algorithms for mining high utility itemsets from transactional databases,” IEEE Transactions on Knowledge and Data Engineering, vol. 25, no. 8, pp. 1772–1786, 2013.
[9] Z. Sheng, S. Yang, Y. Yu, A. Vasilakos, J. Mccann, and K. Leung, “A survey on the ief protocol suite for the internet of things: Standards, challenges, and opportunities,” IEEE Wireless Communications, vol. 20, no. 6, pp. 91–98, 2013.
[10] P. Fournier-Viger, J. C. W. Lin, B. Vo, T. T. Chi, J. Zhang, and H. B. Le, “A survey of itemset mining,” Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery, vol. 7, no. 4, 2017.
[11] U. Yun, G. Lee, and E. Yoon, “Efficient high utility pattern mining for establishing manufacturing plans with sliding window control,” IEEE Transactions on Industrial Electronics, vol. 64, no. 9, pp. 7239–7249, 2017.
[12] F. Tao, Y. Cheng, L. Zhang, and D. Zhao, “Utility modelling, equilibrium, and coordination of resource service transaction in service-oriented manufacturing system,” Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture, vol. 226, no. 6, pp. 1099–1117, 2012.
[13] W. Gan, J. C. W. Lin, P. Fournier-Viger, H. C. Chao, V. S. Tseng, and P. S. Yu, “A survey of utility-oriented pattern mining,” arXiv preprint arXiv:1805.10511, 2018.
[14] Y. W. Teng, C. H. Tai, P. S. Yu, and M. S. Chen, “Revenue maximization on the multi-grade product,” in Proceedings of the SIAM International Conference on Data Mining. SIAM, 2018, pp. 576–584.
[15] V. H. Cai, A. W. C. Fu, C. Cheng, and W. Kwong, “Mining association rules with weighted items,” in International Database Engineering and Applications Symposium. IEEE, 1998, pp. 68–77.
[16] C. J. Chu, V. S. Tseng, and T. Liang, “An efficient algorithm for mining high utility itemsets with negative item values in large databases,” Applied Mathematics and Computation, vol. 215, no. 2, pp. 767–778, 2009.
[17] J. Wan, S. Tang, D. Li, S. Wang, C. Liu, H. Abbas, and A. V. Vasilakos, “A manufacturing big data solution for active preventive maintenance,” IEEE Transactions on Industrial Informatics, vol. 13, no. 4, pp. 2039–2047, 2017.
[18] C. C. Aggarwal and P. S. Yu, “A survey of uncertain data algorithms and applications,” IEEE Transactions on Knowledge and Data Engineering, vol. 21, no. 5, pp. 609–623, 2009.
[19] T. Bernecker, H. P. Kriegel, M. Renz, F. Verhein, and A. Zuefel, “Probabilistic frequent itemset mining in uncertain databases,” in Proceedings of the 15th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. ACM, 2009, pp. 119–128.
[20] J. C. W. Lin, W. Gan, P. Fournier-Viger, T. P. Hong, and V. S. Tseng, “Efficient algorithms for mining high-utility itemsets in uncertain databases,” Knowledge-Based Systems, vol. 96, pp. 171–187, 2016.
[21] Y. Liu, X. Weng, J. Wan, X. Yue, H. Song, and A. V. Vasilakos, “Exploring data validity in transportation systems for smart cities,” IEEE Communications Magazine, vol. 55, no. 5, pp. 26–33, 2017.
[22] L. Geng and H. J. Hamilton, “Interestest measures for data mining: A survey,” ACM Computing Surveys, vol. 38, no. 3, p. 9, 2006.
[23] J. M. Luna, A. Cano, M. Pechenizkiy, and S. Ventura, “Speeding-up association rule mining with inverted index compression,” IEEE Transactions on Cybernetics, vol. 46, no. 12, pp. 3059–3072, 2016.
[24] R. Srikant and R. Agrawal, “Mining sequential patterns: Generalizations and performance improvements,” in International Conference on Extending Database Technology. Springer, 1996, pp. 1–17.
[25] T. P. Hong, C. S. Ku, and S. C. Chi, “Mining association rules from quantitative data,” Intelligent Data Analysis, vol. 3, no. 5, pp. 363–376, 1999.
[26] C. K. Chui, B. Kao, and E. Hung, “Mining frequent itemsets from uncertain data,” in Pacific-Asia Conference on Knowledge Discovery and Data Mining. Springer, 2007, pp. 47–58.
[27] H. Yao, H. J. Hamilton, and C. J. Butz, “A foundational approach to mining itemset utilities from databases,” in Proceedings of the SIAM International Conference on Data Mining. SIAM, 2004, pp. 482–486.
### TABLE II
 Derived patterns under varied minUtil and fixed minPro

| Dataset | Pattern | Threshold of minUtil |
|---------|---------|----------------------|
| N1 | 109,475,579 | 79,746,008 |
| N2 | 13,399,980 | 12,180,884 |
| N3 | 12,322,701 | 10,611,814 |
| N4 | 8,067,669 | 6,811,140 |
| PHUs | 45,613 | 28,977 |
| N1 | 194,512 | 163,169 |
| N2 | 147,052 | 119,802 |
| N3 | 146,426 | 119,140 |
| N4 | 144,957 | 117,671 |
| PHUs | 6,331 | 5,462 |
| N1 | 2,322,059 | 1,814,071 |
| N2 | 1,265,985 | 801,074 |
| N3 | 1,238,118 | 795,362 |
| N4 | 1,236,179 | 793,423 |
| PHUs | 8,648 | 4,123 |
| N1 | 96,139,509 | 48,350,235 |
| N2 | 21,876,139 | 17,735,518 |
| N3 | 21,078,117 | 17,429,475 |
| N4 | 20,960,880 | 17,332,324 |
| PHUs | 19,278 | 13,584 |
| N1 | 1,558,452 | 947,145 |
| N2 | 828,898 | 527,865 |
| N3 | 700,121 | 440,243 |
| N4 | 698,969 | 439,522 |
| PHUs | 147,088 | 85,133 |
| N1 | 14,095,513 | 7,194,329 |
| N2 | 6,199,289 | 2,365,166 |
| N3 | 4,619,226 | 1,655,006 |
| N4 | 4,454,919 | 1,570,317 |
| PHUs | 37,242 | 20,710 |
| N1 | 144,957 | 107,740 |
| N2 | 106,668 | 71,740 |
| N3 | 85,133 | 56,590 |
| N4 | 82,898 | 52,786 |
| PHUs | 14,496 | 8,248 |
| N1 | 1,236,179 | 883,884 |
| N2 | 883,884 | 640,104 |
| N3 | 698,969 | 439,522 |
| N4 | 698,969 | 439,522 |
| PHUs | 147,088 | 85,133 |

### TABLE III
 Derived patterns under varied minPro and fixed minUtil

| Dataset | Pattern | Threshold of minPro |
|---------|---------|----------------------|
| N1 | 79,746,008 | 73,064,334 |
| N2 | 12,180,884 | 11,778,729 |
| N3 | 10,611,814 | 10,375,295 |
| N4 | 6,811,140 | 6,028,964 |
| PHUs | 35,662 | 24,589 |
| N1 | 194,512 | 150,179 |
| N2 | 147,052 | 109,522 |
| N3 | 146,426 | 109,127 |
| N4 | 144,957 | 107,740 |
| PHUs | 6,331 | 4,552 |
| N1 | 2,322,059 | 1,892,203 |
| N2 | 1,265,985 | 904,469 |
| N3 | 1,238,118 | 885,941 |
| N4 | 1,236,179 | 883,884 |
| PHUs | 8,648 | 5,582 |
| N1 | 37,793,976 | 37,017,817 |
| N2 | 16,931,811 | 16,454,728 |
| N3 | 16,526,029 | 16,131,265 |
| N4 | 16,507,082 | 16,090,920 |
| PHUs | 14,406 | 13,048 |
| N1 | 974,145 | 923,770 |
| N2 | 527,865 | 485,700 |
| N3 | 440,243 | 406,109 |
| N4 | 439,522 | 405,048 |
| PHUs | 85,133 | 75,769 |
| N1 | 2,322,059 | 6,992,960 |
| N2 | 2,365,166 | 2,195,551 |
| N3 | 1,655,006 | 1,519,652 |
| N4 | 1,570,317 | 1,384,445 |
| PHUs | 20,710 | 19,278 |

EQUIVLED PATTERNS UNDER VARIED minUtil and FIXED minPro
Fig. 5. Scalability evaluation.

[28] Y. Liu, W. k. Liao, and A. Choudhary, “A two-phase algorithm for fast discovery of high utility itemsets,” in Pacific-Asia Conference on Knowledge Discovery and Data Mining. Springer, 2005, pp. 689–695.

[29] C. W. Tsai, C. F. Lai, H. C. Chao, and A. V. Vasilakos, “Big data analytics: a survey,” Journal of Big Data, vol. 2, no. 1, p. 21, 2015.

[30] J. C. W. Lin, W. Gan, T. P. Hong, and V. S. Tseng, “Efficient algorithms for mining up-to-date high-utility patterns,” Advanced Engineering Informatics, vol. 29, no. 3, pp. 648–661, 2015.

[31] J. C. W. Lin, J. Zhang, P. Fournier-Viger, T. P. Hong, and J. Zhang, “A two-phase approach to mine short-period high-utility itemsets in transactional databases,” Advanced Engineering Informatics, vol. 33, pp. 29–43, 2017.

[32] J. C. W. Lin, W. Gan, and T. P. Hong, “A fast updated algorithm to maintain the discovered high-utility itemsets for transaction modification,” Advanced Engineering Informatics, vol. 29, no. 3, pp. 562–574, 2015.

[33] W. Gan, J. C. W. Lin, P. Fournier-Viger, H. C. Chao, T. P. Hong, and H. Fujita, “A survey of incremental high-utility itemset mining,” Wiley Interdisciplinary Reviews: Data Mining and Knowledge Discovery, vol. 8, no. 2, 2018.

[34] J. C. W. Lin, W. Gan, and T. P. Hong, “A fast maintenance algorithm of the discovered high-utility itemsets with transaction deletion,” Intelligent Data Analysis, vol. 20, no. 4, pp. 891–913, 2016.

[35] J. C. W. Lin, W. Gan, P. Fournier-Viger, T. P. Hong, and V. S. Tseng, “Fast algorithms for mining high-utility itemsets with various discount strategies,” Advanced Engineering Informatics, vol. 30, no. 2, pp. 109–126, 2016.

[36] V. S. Tseng, C. W. Wu, P. Fournier-Viger, and P. S. Yu, “Efficient algorithms for mining the concise and lossless representation of high utility itemsets,” IEEE Transactions on Knowledge and Data Engineering, vol. 27, no. 3, pp. 726–739, 2015.

[37] J. C. W. Lin, W. Gan, P. Fournier-Viger, T. P. Hong, and H. C. Chao, “FDHUP: Fast algorithm for mining discriminative high utility patterns,” Knowledge and Information Systems, vol. 51, no. 3, pp. 873–909, 2017.

[38] V. S. Tseng, C. W. Wu, P. Fournier-Viger, and P. S. Yu, “Efficient algorithms for mining top-k high utility itemsets,” IEEE Transactions on Knowledge and Data Engineering, vol. 28, no. 1, pp. 54–67, 2016.

[39] W. Gan, J. C. W. Lin, P. Fournier-Viger, H. C. Chao, and P. S. Yu, “HUOPM: High utility occupancy pattern mining,” arXiv preprint arXiv:1812.10926, 2018.

[40] G. C. Lan, T. P. Hong, J. P. Huang, and V. S. Tseng, “On-shelf utility mining with negative item values,” Expert Systems with Applications, vol. 41, no. 7, pp. 3450–3459, 2014.

[41] J. C. W. Lin, P. Fournier-Viger, and W. Gan, “FHN: An efficient algorithm for mining high-utility itemsets with negative unit profits,” Knowledge-Based Systems, vol. 111, pp. 283–298, 2016.

[42] P. Fournier-Viger, C. W. Wu, S. Zida, and V. S. Tseng, “FHM: Faster high-utility itemset mining using estimated utility co-occurrence pruning,” in International Symposium on Methodologies for Intelligent Systems. Springer, 2014, pp. 83–92.

[43] R. Agrawal and R. Srikant, “Quest synthetic data generator. http://www.Almaden.ibm.com/cs/quest/syndata.html,” 1994.