Radar methods for determining the thickness of dielectric layers

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Abstract. This paper describes two methods, the amplitude method and the method based on geometric optics. The methods were applied to practical data obtained by georadar tests. As a result, the refractive index and thickness of the investigated planar layered medium were calculated. When using the amplitude method, the refractive index of the investigated medium is 1.3, the thickness is 0.11 m. When using the method based on geometric optics, the refractive index of the investigated medium is 1.43, the thickness is 0.102 m. After comparing the results of these methods, it was found that the error of the amplitude method is higher than that of the geometrical optics method.

1. Introduction

One of the most important tasks of ultra-wideband (UWB) location is the development of effective mathematical software for determining the characteristics of layered media, such as the thickness and nature of the artificial cover of each layer, based on the results of measurements of the local wave projections [1–3]. A typical example is the determination of the thickness of the layers and the nature of the artificial cover of the roadway. In this article, a comparison of the amplitude method and the method developed by the authors of the article is considered. These methods will be applied to calculate the thickness and refractive index of a medium with known thickness and refractive index.

In practical calculations, the propagation of electromagnetic waves created by the sounding pulse of GPR is considered in the framework of the laws of geometric optics for plane waves at large distances from the source. Accordingly, the principles of Fermat, Huygens, Fresnel and Snellius law are used in determining the propagation velocities and amplitudes of the waves. In addition, the assumption that the medium is not frequency dependent is used [4, 5].

The propagating wave is reflected only from the interfaces of different media, in which a change in the relative permittivity or conductivity is observed. The reflection coefficient R at normal incidence of the wave at the interface is calculated by the formula:

\[ R = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \]  

where \( \varepsilon_1 \) – dielectric constant of the first layer; \( \varepsilon_2 \) – dielectric permittivity of the second layer.

A refracted wave can be detected by GPR if the dielectric permittivity of the first layer is greater than that of the second. The refracted pulse preserves in any case the polarity of the incident signal [6, 7]. The refractive index \( T \) is calculated by the formula:
The propagation of waves of different types, using a two-layer medium as an example, is shown in Figure 1.

\[ T = 1 - R = \frac{2\sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}} \] (2)

Figure 1. Electromagnetic wave propagation in a two-layer medium: a – direct wave from the transmitting antenna to the receiving antenna, b – direct wave propagating in the medium, c – reflected wave from the media interface, d – refracted wave.

The most important parameters for GPR survey are the actual part of dielectric permittivity, specific conductivity and the speed of wave propagation in the medium, which depends on the dielectric permittivity:

\[ \varepsilon = \varepsilon' - j\varepsilon'' \] (3)

\[ \sigma = \omega\varepsilon'\varepsilon_0 \] (4)

\[ V = \frac{c}{\sqrt{\varepsilon'}} \] (5)

where \( \varepsilon, \varepsilon', \) and \( \varepsilon'' \) – the complex permittivity, the real and imaginary parts of it, respectively; \( \sigma \) – specific conductivity, cm/m; \( \omega \) – frequency of the applied electromagnetic field, Hz; \( V \) – velocity of propagation of electromagnetic waves, m/s.

The value of magnetic permeability in GPR for most ground environments is close to unity and does not depend on the field frequency.

2. Amplitude method

According to (1), the ratio of the amplitude of the incident wave at the interface to the amplitude of the reflected wave depends on the dielectric permittivity of these media. Knowing the dielectric permittivity of the upper medium, we can determine thus the dielectric permittivity and thickness of the lower medium.

Before surveying the site, you must make a metal sheet rationing. This is necessary to determine the level of air wave amplitudes.

Then a GPR test will be performed at some height from the surface of the medium under study. It will be possible to highlight the reflection of the air wave from the surface of the medium on the amplitude map. Then the real part of the dielectric permittivity is searched by the ratio of amplitudes:
\[ \sqrt{\varepsilon_1} = k_1 \cdot \frac{1 + \frac{A_0}{A_m}}{1 - \frac{A_0}{A_m}} \]  

(6)

where \( \varepsilon_1 \) – is the real part of the relative permittivity of the first layer; \( A_m \) – amplitude of the wave reflected from the metal plate; \( A_0 \) – amplitude of the wave reflected from the ground surface; \( k_1 \) – an empirical coefficient that takes into account errors in the determination of amplitudes and other.

The formulas for calculating the dielectric permittivities of the underlying layers are given in [8, 9], as well as a procedure for determining the correction factor.

3. Method based on geometric optics

To accurately determine the parameters of the obstacle it is sufficient to know, the information obtained at only two points - at the point when the receiver is combined with the transmitter and when it is separated (Figure 2).

![Figure 2](image)

Figure 2. Schemes of obstacle probing with combined (a) and separated (b) receiver and transmitter.

Electrical length of the investigated area for monostatic and bistatic location schemes:

\[ \tau_{511} = t_1 - t_0 = 2 \frac{nd}{c} \]  

(7)

\[ \tau_{512} = t_3 - t_2 = f(z_0, a, d, n) = f(z_0, a, d, \frac{\tau_{211}c}{2d}) \]  

(8)

where \( z_0 \) – the range to the leading edge of the underlying surface, \( a \) – the distance between the antennas, \( d \) – the thickness of the layer under study, \( n \) – the refractive index of the layer under study.

4. Comparison of methods

The full-scale experiment was carried out to determine the thickness and refractive index of a single-layer medium. The gas-concrete block with thickness of 10 cm and refractive index 1.5 was taken as the investigated medium. The geometry of the experiment was carried out according to the scheme shown in Figure 3.
Let us use the method of amplitudes to determine the parameters of the layer. First, we normalized the signal to the metal sheet and determined the amplitude of the reflected signal from the metal sheet. The Figure 4 shows a graph of the dependence of the radiation intensity on the signal duration.

The maximum value of the amplitude from the metal sheet is $A_m = 0.3554$ relative unit.

Next, the medium under study was probed, and the graph of the dependence of intensity on duration is shown in Figure 5.
The maximum value of the amplitude from the front boundary of the medium under study is $A_0 = 0.04942$ relative unit.

Using formula 6, we obtain that the refractive index of the investigated medium is 1.3.

Next, write down the formula for the dependence of the refractive index and the thickness of the layer [10, 11]:

$$ h = \frac{ct}{2n} $$

where $h$ – thickness of the medium under study, $c$ – speed of light, $t$ – the time of the wave that passed inside the layer, $n$ – refractive index of the medium under study.

After substituting all values, we obtain that the thickness of the layer under study is 0.11 m.

Next, we will use the method developed by the authors of the article. When using it it is necessary to register a signal with two probing schemes, monostatic and bistatic. Figure 6 shows the graphs of the dependence of the radiation intensity on the duration.

**Figure 5.** The graph of the radiation intensity versus duration.
Figure 6. Diagram of radiation intensity dependence on duration (a) monostatic sensing scheme, (b) bistatic sensing scheme.

A model of signal propagation in planar layered media is implemented, which allows us to calculate the delay of signals when an electromagnetic wave passes through a layer in the case of bistatic and monostatic location. It is shown as a function that depends on the refractive index of the medium, the distance between the antennas, the height of the antenna above the layer and the thickness of the layer. As a result of application of the developed method, plots of dependence of the signal delay on the layer thickness (Figure 7a) and of the refractive index on the layer thickness (Figure 7b) were shown.
Figure 7. a) Plot of the signal delay versus layer thickness, b) plot of the refractive index versus layer thickness.

The solution of the search for the thickness and refractive index of the medium is reduced to a graphical solution of the problem. The red line in Figure 7a corresponds to the function depending on signal delay in the bistatic mode of location, and the blue line corresponds to the value of signal delay in the monostatic mode of location. Values of intersection points d1=0.102 m. and d2=0.295 m. of these lines correspond to the solution of the layer thickness of search problem. To determine the correct solution it is necessary to analyze Figure 7b. Figure 7b shows the refractive index depending on the thickness of the medium layer. The values of the refractive index n1=1.43 and n2=0.49 correspond to the obtained values of the layer thickness d1 and d2. The refractive index cannot be lower than 1, therefore, the solutions n2, d2 are incorrect.

5. Conclusion

Comparison of the characteristics of use of two considered methods allows a number of conclusions to be made. The method based on geometric optics is preferable for obtaining information about the structure of planar layered media. As a result of the application of the methods considered in the article, the characteristics of the medium under study, namely the refractive index and thickness, were determined. Using the amplitude method, the refractive index of the gas concrete block is 1.3 and the thickness is 0.11 m. When using the method based on geometric optics, the refractive index of the medium under study is 1.43 and the thickness is 0.102 m. The measurement error for thickness and refractive index for the amplitude method is 10% and 13.4% respectively, the measurement error of the method based on geometric optics is 2% and 5.7%. It is also important that the method is stable to measurement errors. There may not be a sharp jump in amplitudes at the boundaries of media with insignificant changes in dielectric permittivity, and as a result of inhomogeneity of the medium interference and re-reflections appear. As a result, false jumps appear that are not related to the media interfaces. As a consequence, the amplitude method cannot be applied, because it is necessary to know the exact location of the reflected signal and its values. Whereas method based on geometrical optics is stable to the error of measurement of electrical length of layers.
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