Annihilations of superheavy dark matter in superdense clumps

V. Berezinsky,1,2,3 V. Dokuchaev,2,3 Yu. Eroshenko,2,3 M. Kachelrieß,4 and M. Aa. Solberg4

1INFN, Laboratori Nazionali del Gran Sasso, I–67010 Assergi (AQ), Italy
2Center for Astroparticle Physics at LNGS (CFA), I–67010 Assergi (AQ), Italy
3Institute for Nuclear Research of the Russian Academy of Sciences, Moscow, Russia
4Institutt for fysikk, NTNU Trondheim, N–7491 Trondheim, Norway

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Superheavy dark matter (SHDM) exchanges energy with its environment much slower than particles with masses close to the electroweak (EW) scale and has therefore different small-scale clustering properties. Using the neutralino as candidate for the SHDM, we find that free-streaming allows the formation of DM clumps of all masses down to \( \sim 260 m_\chi \) in the case of bino. If small-scale clumps evolve from a non-standard, spiky spectrum of perturbations, DM clumps may form during the radiation dominated era. These clumps are not destroyed by tidal interactions and can be extremely dense. In the case of a bino, a “gravothermal catastrophe” can develop in the central part of the most dense clumps, increasing further the central density and thus the annihilation signal. In the case of a higgsino, the annihilation signal is enhanced by the Sommerfeld effect. As a result annihiliations of superheavy neutralinos in dense clumps may lead to observable fluxes of annihilation products in the form of ultrahigh energy particles, for both cases, higgsinos and binos, as lightest supersymmetric particles.

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I. INTRODUCTION

The case for the existence of non-relativistic, non-baryonic dark matter (DM) in the universe is stronger than ever [1]. But although a wealth of observational data provides compelling evidence for a \( \sim 23\% \) contribution of DM to the total energy density of the universe, its nature is still not known. The most popular DM type are thermal relics, i.e. particles that were at least once during the history of the Universe in chemical equilibrium with the thermal plasma.

The present relic abundance \( \Omega_\chi \) of a thermal relic scales approximately with its annihilation cross section \( \sigma_{ann} \) as \( \Omega_\chi \propto 1/\sigma_{ann} \). Moreover, unitarity bounds annihilations as \( \sigma_{ann} \propto m_\chi^{-2} \) and thus the observed value [2] \( \Omega_{CDM} h^2 = 0.1 \) of the DM abundance constrains the annihilation cross section as \( \langle \sigma_{ann} v \rangle \sim 3 \times 10^{-26}{cm^3/s} \) and limits the mass of any thermal relic as \( m_\chi \lesssim 50 \text{ TeV} \) [3]. Hence thermal relics offer detection prospects both for direct and indirect searches as well as at accelerator experiments.

The assumption that the DM particle was once in chemical equilibrium is however not necessary and does not hold in particular for sufficiently heavy particles. Superheavy particles are generated at the end of inflation and they can play the role of DM particles [4, 5]. Gravitational production at the end of inflation provides the most natural mechanism for the generation of superheavy dark matter (SHDM) [6]. Their decays can result in the observable signal in the form of UHE gamma-rays [4] and UHE neutrinos [7], if they are metastable and long-lived.

While the idea of SHDM is theoretically appealing, the detection of SHDM is challenging. Clearly, accelerator searches and direct detection are not feasible in the case of SHDM. The feasibility of indirect detection of stable SHDM depends on its annihilation rate \( N_{ann} \propto (\rho/m_\chi)^2\langle \sigma_{ann} v \rangle \) that in turn scales naively as \( N_{ann} \propto m_\chi^{-4} \). Since backgrounds like cosmic rays from astrophysical sources or the diffuse photon flux decrease only as \( 1/E^\alpha \) with \( \alpha \lesssim 3 \), indirect detection of DM seems to become more and more difficult for increasing DM masses. The only possibility which overcomes this difficulty is the superdense central region of DM clumps [8], but one needs the realistic scenario for the very high density of DM in the clump center or formation of superdense clumps [3, 10].

We shall use as candidate for SHDM the neutralino in the model of superheavy supersymmetry, as suggested in Ref. [11]. Superheavy supersymmetry is a unique scheme that respects perturbative unitarity despite of coupling particles with mass much larger than the weak scale to the electroweak sector. Within this model, the lightest supersymmetric particle (LSP) that we choose as the neutralino is a natural candidate for SHDM.

Aim of the present work is to study the detection prospects for stable SHDM. Since the annihilation signal from the mean distribution of DM in the halo is far below observational limits, we examine if new effects specific for SHDM exist that can improve the detection chances.

One such effect can result from the early kinetic decoupling of SHDM: While the mass spectrum of DM clumps formed by standard neutralinos with mass in the 100 GeV range has a cutoff at \( M_{min} \sim (10^{-12} - 10^{-4})M_\odot \) [12, 13], the cutoff can be diminished significantly e.g. in the case of ultra-cold WIMPs [10, 14]. We will show that the cutoff is practically absent for a superheavy bino as DM particle, and clumps of all masses are possible beginning formally from \( \sim 260 m_\chi \). This low-mass cutoff increases the diffuse flux of UHE particles produced by annihilations.
Another effect is the formation of superdense clumps, in which the annihilation rate can be strongly enhanced. The formation of superdense clumps is discussed in general in the accompanying paper I [13]. Here we study this problem in more detail for superheavy particles.

The initial mass spectrum of DM clumps is determined by the spectrum of cosmological density perturbations. The simplest models of inflation predict a nearly scale-invariant power-law form that is then normalized to the COBE observations. Clumps formed in this case are not very dense, and the small-scale structure of DM enhances the annihilation signal not strongly. A quite different scenario arises, if the potential $V(\phi, T)$ of the inflaton field contains features like a zero derivative $V'(\phi, T)$ for some field value $\phi$. The spectrum of perturbations in this case has a spike and clumps of SHDM can form already in the radiation-dominated (RD) epoch, and they can have very large densities.

For sufficiently dense clumps, the relaxation processes due to gravitational two-body scatterings can initiate a "gravithermal catastrophe" and the density profile of the clump core steepens to an isothermal profile $\rho \propto r^{-2}$ with a much smaller new core radius. While this process was discussed in general in Ref. [11] (hereafter Paper I), we show in this work that for a bino this process can take place and determine the required initial conditions. Annihilations of DM in such dense clumps are strongly amplified because of the density enhancement.

In contrast to a bino, winos and higgsinos are stronger coupled to the thermal plasma. As a result, a "gravithermal catastrophe" does not develop. However, the velocities of DM particles in superdense clumps are very small, leading to an enhancement by the Sommerfeld effect in case of higgsinos or winos. Taking into account both effects, we find that annihilations of SHDM bound in such superdense clumps may lead to observable fluxes of ultrahigh energy particles for all types of neutralinos.

Note that (in contrast to the case with standard power-law spectrum of cosmological perturbations) superdense clumps produced in the RD epoch from a spiky spectrum are not destroyed by tidal forces and their mass function peaks near a definite value. Therefore the fraction of DM in the form of such clumps is $\xi \sim 1/2$.

This article is organized as follows. We start with a discussion of the energy relaxation time of SHLSPs in Sec. II, where we also derive the minimal mass of the SHDM clumps considering free-streaming and the horizon scale at kinetic decoupling. In Sec. III we summarize the evolution of the density profile of superdense clumps. Next we discuss the prospects to detect SHDM clumps in Sec. IV and conclude in Sec. V.

We shall use below the following abbreviations: SHDM for superheavy dark matter and SHLSP for superheavy lightest supersymmetric which in our case is the superheavy neutralino [11]. We fix the mass $m_\chi$ of the SHLSP as $m_\chi = 10^{11}$ GeV, i.e. within the range suggested by gravitational production with $\Omega_\chi h^2 \approx 0.1$ at the end of inflation.

\section{Energy Relaxation and the Minimal Mass of DM Clumps}

\subsection{Relaxation time}

We shall consider here the energy relaxation of SHDM particles interacting with a thermal background at temperature $T$. The consideration is relevant for both ordinary and superdense clumps.

The energy exchange between superheavy neutralinos and light fermions was calculated in [11] for temperatures $T$ below the weak scale, i.e. in the limit $T \ll m_Z \ll M_{\text{SUSY}}$ with $M_{\text{SUSY}} = \min\{M_1, M_2, \mu\}$. Here, $\mu$ denotes the Higgs mixing parameter, and $M_1$ and $M_2$ the U(1) and SU(2) soft SUSY breaking masses. At lowest order in $m_Z^2/M_{\text{SUSY}}^2$, the neutralino masses are simply $\{M_1, M_2, -\mu, \mu\}$, for more details see e.g. [11].

The energy relaxation time is calculated as

$$\tau_{\text{rel}} = \frac{1}{2E_k m_\chi} \sum_i \int_0^\infty d\omega \int d\Omega n_i(\omega)(\delta p)^2 \left( \frac{d\sigma_{\text{el},i}}{d\Omega} \right),$$

where $E_k = (3/2)T$ is the mean kinetic energy of the neutralinos, $\delta p$ the neutralino momentum obtained in one scattering,

$$(\delta p)^2 = 2\omega^2[1 - \cos(\theta)]$$

and the number density of relativistic fermions or bosons with $g$ polarization degrees and energy $\omega$ is

$$n_i = \frac{g_i}{2\pi^2} \frac{\omega^2}{e^{\omega/T} \pm 1} \approx \frac{g_i}{2\pi^2} \omega^2 e^{-\omega/T}.$$ 

Kinetic decoupling occurs when the energy relaxation rate $\tau_{\text{rel}}^{-1}$ becomes smaller than the expansion rate $H$ of the universe. During the radiation dominated (RD) epoch, $H = 1/(2t)$, and

$$H = 1.66\sqrt{g_*} \frac{T^2}{M_{\text{Pl}}},$$

where $g_*$ denotes the number of relativistic degrees of freedom and $M_{\text{Pl}}$ the Planck mass, $M_{\text{Pl}} = 1/\sqrt{G_N} \approx 1.2 \times 10^{19}$ GeV.

As the calculations of Ref. [11] show the energy-relaxation time due to elastic scattering of neutralino on the background fermions at $T \ll m_Z$ is larger than the corresponding age of the universe. Thus, it is necessary to include also the elastic scattering of neutralino on weak gauge bosons and the light higgs for $m_Z \ll T \ll M_{\text{SUSY}}$. Since the mass splitting between the lightest neutralino and chargino is of order $O(m_Z^2/M_{\text{SUSY}})$ for a higgsino or wino, inelastic processes like $\chi^0 + \nu_e \rightarrow \chi^+ + e^-$ contribute also to the energy exchange between neutralinos and the plasma. As we will see, elastic interactions with fermions are for $T \gg m_Z$ subdominant, while inelastic ones give the dominant contribution to the energy relaxation of the neutralino. Since the latter are absent for
a bino, the relaxation time is strongly dependent on the nature of the neutralino. We consider in the following only the cases of a bino and a higgsino, noting that a wino behaves in most respects as a higgsino.

1. The Bino as the LSP

The processes dominating the energy exchange between superheavy binos and the thermal plasma at $T \gg m_Z$ are $\chi Z \to \chi Z$, $\chi W^\pm \to \chi W^\pm$ and $\chi h \to \chi h$. In the first process only light higgs exchange contributes at leading order and the squared matrix element is

$$|M_{\chi Z \to \chi Z}|^2 = \frac{e^4 M_I^2 (\mu \sin(2\beta) + M_1)^2}{3 \cos^4(\theta_W)/(\mu^2 - M_1^2)^2},$$

where $\tan\beta = v_1/v_2$ is the ratio of the two Higgs vev and $\theta_W$ the Weinberg angle. The matrix elements of the other relevant processes have the same form, and the total matrix element squared weighted by the relevant polarization degrees is given by $|M|^2 = \sum_i g_i |M_i|^2 = 12|M_{\chi Z \to \chi Z}|^2$. The resulting energy relaxation time follows as

$$\tau_{rel} = \frac{\pi c_W}{32 \alpha^2 T^4 (M_1 + m_{2/3})^2} \sim \frac{A M_{\text{SUSY}}^3}{T^4}.$$ (5)

In the last step, we simplified the energy relaxation time $\tau_{rel}$ introducing a common mass scale $M_{\text{SUSY}}$ and $A = \pi c_W^4/(32 \alpha^2)$.

2. The Higgsino as the LSP

The inelastic process $\chi^0 \nu_e \to \chi^0 \nu_e$ is dominated by $W$ exchange. Its leading contribution to the squared matrix element is given by

$$|M_{\chi^0 \nu_e \to \chi^0 \nu_e}|^2 = \frac{2 e^4 \mu^2 \omega^2 \cos^2(\theta/2)}{s_W^2 (2\omega^2 (1 - \cos(\theta)) + m_W^2)^2}. $$

The energy relaxation time becomes for $\mu \gg T \gg m_W$

$$\tau_{rel} = \frac{6 \pi \sin^4(\theta_W) \mu}{\alpha^2 N_{\text{eff}} T^2 [\ln(4T^2/m_W^2) - 2\gamma + 1]}.$$ (6)

and

$$\tau_{rel} \sim \frac{B \mu m_W}{N_{\text{eff}} T^6}, \quad \text{with} \quad B = \frac{\pi \sin^4(\theta_W)}{160 \alpha^2}$$ (7)

for $m_{\chi^0} \ll T \ll m_W$. The factor $N_{\text{eff}}$ counts the number of fermions contributing to the inelastic processes at temperature $T$.

B. Minimum mass of DM clumps

The growth of small-scale fluctuations can be smeared out by destructive processes such as free-streaming, acoustic oscillations and others [13, 16] (see appendix A in Ref. [20] for a general discussion). For neutralinos with mass close to the electroweak (EW) scale, these processes determine the minimal wave-length in the perturbation spectrum that can grow and thereby also the minimum clump mass. Here we consider SHLSPs and study the effectiveness of these damping effects in the case $m_\chi \gg m_Z$.

We let calculate the cosmological age $t_d$ and the temperature $T_{\text{d}}$ of kinetic decoupling of SHLSPs from the cosmic plasma, i.e. the moment when the relaxation rate $\tau^{-1}_{rel}$ equals the expansion rate $\dot{H}(t_d)$ of the Universe.

In evaluating the condition for decoupling, $\tau^{-1}_{rel} \simeq H(t_d)$, we set $N_{\text{eff}} = 431/4 = 107.75$ as number of relativistic degrees in the standard model (SM) for $T > m_\chi$ and use for the running of coupling and mixing parameters with temperature $T$ the SM relations, $\sin^2 \theta_W(T) = 1/6 + 5\alpha(T)/[9\alpha_s(T)]$. For $M_{\text{SUSY}} = 10^{12}$ GeV we obtain then

$$T_d \simeq \begin{cases} 2\times10^{11} \text{GeV}, \quad \text{bino} \\ 2 \text{ GeV}, \quad \text{higgsino} \end{cases}.$$ (5)

decoupling temperature for a bino and higgsino, respectively. In the former case, $g_e = 298/4$ and $N_{\text{eff}} = 66$. For $t > t_d$, the SHLSP is not longer in thermal equilibrium with the cosmic plasma and its momentum scales as $p \propto 1/a^2$.

We consider now the physical processes relevant for a spherical region containing DM with total mass $M$ close to the time of horizon crossing. The mass of DM inside the horizon as function of the temperature is given by

$$M = 3.4 \times 10^{16}(T/100 \text{GeV})^{-3}(N_{\text{eff}}/100)^{-3/4} \text{ g}.$$ (9)

In particular, at the temperature of kinetic decoupling given by Eq. (10) the corresponding mass is equal to

$$M_d \approx \begin{cases} 6 \times 10^{-12} \text{ g}, \quad \text{bino} \\ 6 \times 10^{21} \text{ g}, \quad \text{higgsino} \end{cases}.$$ (11)

For a bino, the mass $M_d$ is only 34 times greater than the particle mass $m_\chi \sim 10^{11}$ GeV $\sim 1.78 \times 10^{-13}$ g.

The evolution of fluctuations with mass $M \ll M_d$ and $M \gg M_d$ is rather different after horizon crossing [16]. Fluctuations in DM with mass $M \ll M_d$ run out as sound waves in the radiation plasma. These fluctuations do not have a kick in the peculiar velocities of their DM particles and therefore do not grow logarithmically. After kinetic decoupling their amplitude freezes in until the matter dominated (MD) epoch, and their evolution is analogous to the evolution of entropy perturbations and described by the Meszaros solution (see e.g. [9]). Therefore, there is a steepening in the mass spectrum below $M \sim M_d$.

In the opposite case, $M \gg M_d$, the peculiar velocities just after the horizon crossing are equal to $v_{pH} \simeq \theta_H c/3$.
In contrast to thermal velocities these peculiar velocities are regular and directed toward the center of the fluctuation. The fluctuations grow according to the adiabatic law $\delta \propto \ln(t)$ due to the evolution of the peculiar velocities as $v_s(t) \sim v_{s,0}(t_H)/a(t)$. The free streaming scale $\lambda_{fs}$ of SHDM is very small. Expressed in comoving units, this scale is growing during the RD epoch as

$$\lambda_{fs}(t) = a(t_0) \int_{t_d}^{t} \frac{v(t') dt'}{a(t')} = 2t_d v_d a(t_0) a_d \ln(a(t_0)/a_d),$$  

(13)

where $v(t) = v_{s,0}(t_d)/a(t)$, $v_d = (3T_d/m)$, and the ratio of the scale factors $a(t)/a(t_d)$ is calculated from the Friedmann equation. The corresponding free streaming mass

$$M_{fs}(t) = \frac{4\pi}{3} \rho_c(t_0) \Omega_{m,0} \lambda_{fs}^3(t),$$

(14)

stops growing near the epoch of matter-radiation equality, $t \sim t_{eq}$. For the case of a bino in Eq. (10), the decoupling time is $t_d = 7 \times 10^{-30} s$, and we find

$$M_{fs} = \frac{\pi^{1/4}}{2^{19/3} \sqrt[3]{3}/4} \frac{\rho_c^{1/4} t_d^{3/2}}{G^{3/4}} \left( \frac{T_d}{m_\chi} \right)^{3/2} \ln^3 \left\{ \frac{24}{\pi G \rho_{eq} t_d^2} \right\} \approx 4.6 \times 10^{-11} \text{ g.}$$

(15)

This value is only 260 times larger the particle mass. Thus the free-streaming mass of SHDM defines the cutoff in the mass spectrum. Formally, all mass clumps are possible from $M \gtrsim 260 m_\chi$.

In the case of a higgsino, $M_{fs} \ll m_\chi$, and free-streaming plays no role for the evolutions of density perturbations.

Therefore, the two mass scales $M_d$ and $M_{fs}$ may play the role of $M_{min}$. In the case of a bino, $M_{fs} > M_d$ and the steepening of the mass spectrum starts at $M_{min} \sim M_{fs}$. In the case of a higgsino, $M_{fs}$ is very small and $M_{min} \sim M_d$.

III. CLUMP STRUCTURE FOR STANDARD AND SPIKY PERTURBATIONS

Let us consider first the formation and evolution of clumps of SHDM for a standard power-law spectrum of fluctuations. Clumps of SHDM are produced and evolve according to the usual hierarchical model, as described in Ref. [21] and Paper I, with the essential difference that the minimum clump mass is now the one derived in Sec. IIIB. With an accuracy sufficient for this schematic consideration, we can use $M_{min}$ of order of $260 m_\chi$ for the case of a bino.

The basic features of this scenario for SHDM clumps are the same as for ordinary DM clumps: Most clumps are destroyed by tidal interactions in hierarchical structures and the surviving clumps could be further destructed in the Milky Way (see Paper I). SHDM clumps in this scenario have a rather small density and SHDM particles with their small annihilation cross section are unobservable through their annihilation products. They can be detected only gravitationally as discussed in Paper I.

Let us come now to the case of a spiky perturbation spectrum, or to any other case with fluctuations growing in the RD epoch. In these cases superdense clumps can be produced. Since in the RD dominated epoch large-scale structures are absent, there is no tidal destruction of small clumps. These clumps evolve as isolated objects without any merging. The first stage of evolution, the ordinary gravitational contraction, proceeds in the standard way leading to a $\rho(r) \propto r^{-1.8}$ density profile and a large core with radius $R_c \sim (0.01 - 0.1) R$, where $R$ is the clump radius. At this stage the core size is restricted by tidal forces [21], by a decreasing mode of perturbations [23] or by phase density constraints according to Liouville’s theorem [21]. The “gravothermal instability” sets in, when the density of the core reaches a critical value that is determined mainly by the mass of the DM particle, and leads to an isothermal density profile $\rho(r) \propto r^{-2}$, with a new small core, determined by the properties of SHLSP. This so-called “gravothermal catastrophe” occurs under the influence of two-body gravitational scattering in full analogy of this process to the one in globular clusters. We will discuss this process now in some detail.

A. Gravitational relaxation and evolution of the clumps

The gravothermal instability in globular clusters sets in due to two-body gravitational relaxation. This process can be the dominant one for the superdense clumps from SHDM particles. The other relaxation channel, EW scattering of superheavy neutralinos loses the competition, because its cross section is proportional to $m_\chi^{-2}$ and the interaction has a short range. On the other hand, the very high clump density provides the relaxation time $t_{rel,gr}$ to be shorter than the age of the universe $t_0$.

The two-body gravitational relaxation time determined as $t_{rel,gr} = (1/E)(dE/dt)$ in energy space can be taken from calculations for globular clusters (see [25] and references therein) as

$$t_{rel,gr} \sim \frac{1}{4\pi G^2 m_\chi^2 n \ln(0.4N)},$$

(16)

where $v \sim (GM/R)^{1/2}$ is the virial velocity, and $N$ is the total number of particles in the clump, $N = M/m_\chi$. From Eq. (16) one observes indeed that the relaxation time is inversely proportional the mass squared $m_\chi^2$ of SHDM particle and the density $n$ of the core. The logarithmic term $\ln(0.4N)$ takes into account the long-range character of the gravitational interaction.

The relaxation time shorter than the age $t_0$ of the universe leads to the “gravothermal catastrophe”, which results in an isothermal density profile $\rho(r) \propto r^{-2}$. In this regime the main process responsible for the evolution of the clumps becomes the evaporation of particles.
from the core, and the following calculations are in full analogy with the globular clusters case [25]. The escaping particles have approximately zero total energy, and therefore the energy of the core is approximately constant. The process of the core evolution can be described by the rate of evaporation and the virial theorem, which can be written as

\[ \dot{N}_c/N_c = -a/t_{\text{rel,gr}}, \quad \dot{R}_c/R_c = 2\dot{N}_c/N_c - \dot{E}_c/E_c, \]  

(17)

where \( a \approx 7.4 \cdot 10^{-3} \) for a Maxwellian initial velocity distribution. Joint integration with logarithmic accuracy \([\ln(0.4N) = \text{const in Eq. (17)}]\) and the condition \( E_c = \text{const} \) gives the time evolution for the core mass \( M_c \) and radius \( R_c \):

\[ M_c(t) = m_c N_c(t) = M_{c,i}(1 - (t - t_i)/t_c)^{2/7}, \]  

(18)

\[ R_c(t) = R_{c,i}(1 - (t - t_i)/t_c)^{1/7}, \]  

(19)

where \( t_c = 2/(7\alpha)t_{\text{rel,gr},i} \approx 40t_{\text{rel,gr},i} \) and the subscript \( i \) marks the values at the initial moment of the clump formation. The time \( t_c \) is less than the age of the Universe for clumps above the dotted line in Fig. 1 for \( m_c = 10^{11} \text{ GeV} \) as mass of the DM particle. Thus for clumps above the dotted lines, relaxation results in the “gravothermal catastrophe” producing an isothermal profile \( \rho \propto r^{-2} \) with a tiny new core. It diminishes until the central density becomes sufficiently large and new processes, such as annihilations or the pressure of a degenerate Fermi gas, enter the game (see below).

B. Formation of the new core

When the large initial core loses its stability and contracts under the two-body gravitational forces, the gravitational relaxation time is increasing. It is caused by the increase of the core density as

\[ \rho(t) \propto [1 - (t - t_i)/t_c]^{-10/7}, \]  

(20)

which follows from Eqs. (19). In principle, this phenomenon can stop the contraction and stabilize the core.

After the core collapsed, the singular profile \( \rho \propto r^{-2} \) extends formally down to some small radius \( R_c \). For the density profile \( \rho(r) = \rho_c(r/R_c)^{-2} \), the relative core radius \( x_c = R_c/R \) is given by \( x_c = (\bar{\rho}/3\rho_c)^{1/2} \), where \( \bar{\rho} \) and \( \rho_c \) are the mean and the maximal density of the clump. Do any physical processes exist that prevent the extremely large densities, i.e. a very small radius, of the new core?

The first candidate for such process is given by the EW elastic scattering of SHDM particles i.e. by self-interaction. The relaxation time for this process can be estimated as

\[ t_{\text{rel,XX}}^{-1} \propto \frac{4(2\pi)^{1/2}v\sigma_{\chi \chi}n_c}{3\bar{v}/2}, \]  

(21)

where \( \sigma_{\chi \chi} \) is the cross section of elastic neutralino scattering

\[ \frac{d\sigma_{\chi \chi}}{d\Omega} \approx A v^2/m^2, \]  

(22)

where \( A \) is a constant of order one that depends on the SUSY parameters, \( \alpha = 1/137 \), and \( v \approx (GM/R)^{1/2} \) is the virial velocity. This relaxation time has the same dependence on the core-density (\( \propto n_c \)) as the gravitational relaxation time \([10]\), but is for SHLSPs many magnitudes smaller. Therefore, self-interactions cannot stop the gravitational collapse. One may see this effect in a different way: The core remains transparent for superheavy neutralinos down to extremely small radii. Moreover, [27] proved that elastic scatterings do not change the central distribution of DM. The effect of elastic relaxations in the models of self-interacting DM was studied also, e.g., in [26].

Another effect which can stop the gravitational contraction is SHDM particle annihilation. This effect was studied in Refs. [27] and [28]. In the former the core radius was found from the condition that the characteristic annihilation time in the core should be longer than the time of core formation estimated as hydrodynamical free-fall time \( t_h \approx (G\rho)^{-1/2} \). For an isothermal density profile, the corresponding dimensionless core radius \( x_c \) is given by

\[ x_c^2 \approx \frac{(\sigma_{\text{ann}}v)\rho^{1/2}}{G^{1/2}m}. \]  

(23)

The mass of superheavy particle \( m \) in denominator makes the core radius rather small,

\[ x_c \approx 7.4 \cdot 10^{-13} m_{11}^{-3/2} \left( \frac{\rho}{10^5 \text{ g cm}^{-3}} \right)^{1/4}, \]  

(24)

where \( m_{11} = m/(10^{11} \text{ GeV}) \).

In a more reliable approach [28], the radius of the core was estimated equating the core accretion rate with the annihilation rate inside it. The accretion rate was calculated from the Euler and Poisson equations. The calculated core radius is much smaller than the one from Eq. (23).

However, if SHDM particles are fermions like in the case of neutralinos, there is quite different effect which stops the core contraction at a much larger radius. This effect is the pressure of a degenerate Fermi gas. The maximum density of the core and hence the radius of the core can be derived from the equality of the Fermi momentum of a degenerate gas and the virial momentum of the constituent particles at the radius of the core \( r = r_c \).

\[ p_F = (3\pi^2)^{1/3}(\rho_c/m_\chi)^{1/3} = m_\chi V_c, \]  

(25)

where \( V_c = \sqrt{GM_c/r_c} \) is the virial velocity at the boundary of the core, and \( M_c = (4\pi/3)\rho_c r_c^3 \) is the mass of the
core. For the density profile \( \rho(r) \propto r^{-2} \), the virial velocity is the same at all \( r \) and we can take it for the whole clump with mass \( M \) and radius \( R \). Using core radius from \( x_c = (\bar{\rho}/3\rho_c)^{1/2} \) we obtain

\[
x^2_c = \pi^2 \frac{\bar{\rho}}{m_\chi} \left( \frac{GM}{R} \right)^{-3/2}.
\]

(26)

For a clump with mass \( M \approx 1 \times 10^5 \) g, mean density \( \bar{\rho} \approx 3 \times 10^3 \) g/cm\(^3\) and \( R \approx 3 \) cm, the radius of the core is \( x_c \approx 1 \times 10^{-11} \).

In our calculations below for superheavy neutralinos, we will use for radius of the core Eq. (26). 

C. Properties of superdense clumps from SHDM particles and numerical examples

The detectability of the annihilation signal from DM clumps is determined by the following parameters: The mass \( m_\chi \) of the superheavy neutralino and its mixing parameters, the density profile, e.g. an isothermal profile \( \rho(r) = \rho_c (r/R_c)^{-2} \) in case of a ‘gravithermal catastrophe’, the dimensionless radius of the core \( x_c = R_c/R = (\bar{\rho}/3\rho_c)^{1/2} \) and the maximal density \( \rho_c \). The important parameter, the mean density of a clump \( \bar{\rho} \), is found from the evolution of a primordial perturbation with initial amplitude \( \delta_H \) for a clump of given mass \( M \). The mean densities are shown in Fig. 1 for different \( M \) and \( \delta_H \). The maximum density, and respectively the radius of the core, are found as described above.

As particular examples we consider three sets of clump parameters which we shall use in the next section for the calculation of the annihilation signal from superdense clumps. We discuss first an optimistic example for a bino with mass \( m_\chi = 10^{13} \) GeV. We consider clumps with \( M \approx 10^5 \) g formed from fluctuations with \( \delta_H \approx 0.09 \), marked by a star in Fig. 1. The density profile of such clumps is \( \rho(r) \propto r^{-2} \) (we choose the isothermal value, which is close to the analytical and numerical results \( \beta \approx 1.7 - 2 \)) and the initial core radius \( x_{c,1} \approx 0.01 \). Such clumps have mean density \( \bar{\rho} \approx 1.3 \times 10^3 \) g cm\(^{-3}\), radius \( R \approx 2.6 \) cm, and virial velocity \( v \approx 0.05 \) cm/s. For these parameters, the evolution time of the initial core is \( t_c \approx 0.4t_0 \).

Fermi degeneracy mainly restricts the central density in this case, and the new core radius is \( R_{c,2} \approx 2.9 \times 10^{-11} \) cm \((x_{c,2} \approx 1.1 \times 10^{-11})\).

The parameters of a more typical example (marked by a cross in Fig. 1) are \( M \approx 10^{15} \) g and \( \delta_H \approx 0.07 \). These clumps have the same density profile and initial core radius, \( \rho(r) \propto r^{-2} \) and \( x_{c,1} \approx 0.01 \). But now \( \bar{\rho} \approx 6.3 \times 10^{-11} \) g cm\(^{-3}\), \( R \approx 1.6 \times 10^6 \) cm, \( v \approx 0.65 \) cm/s, and no singularity forms in the clumps.

In the case of a higgsino with the same mass, \( m_\chi = 10^{11} \) GeV, an optimistic choice of parameters corresponds to clumps with \( M = M_d \approx 6 \times 10^{21} \) g. The density profile of such clumps is \( \rho(r) \propto r^{-2} \) and the core radius \( x_{c,1} \approx 0.01 \). Also in this case no gravithermal catastrophe develops. These clumps have \( \bar{\rho} \approx 10^{-6} \) g cm\(^{-3}\), \( R \approx 10^6 \) cm, and \( v \approx 600 \) cm/s.

IV. ANNIHILATION

A. Annihilation rate

The annihilation rate \( \dot{N}_{\text{ann}} \) of neutralinos in a single clump is

\[
\dot{N}_{\text{ann}} = \frac{1}{2} \int_0^R 4\pi r^2 dr \frac{n^2(r)}{\langle \sigma v \rangle} = \frac{3}{8\pi} \frac{\langle \sigma v \rangle}{m_\chi} \frac{M^2}{R^3} S,
\]

(27)

where \( n(r) = \rho(r)/m_\chi \) is the number density of neutralinos as function of the distance to the core of the cloud. The function \( S \) was determined in Ref. [21] and depends on the distribution of DM in the clump. In particular, the function is \( S = 1 \) for the simplest case of an uniform clump and \( S \approx 4/(9x_c) \) for an isothermal profile \( \rho \propto r^{-2} \) with a small core size, \( x_c \ll 1 \).

The resulting flux \( I_i \) of particles of type \( i = N, \gamma, \nu \) from DM annihilations summed over all DM clumps in the Galactic halo is given by

\[
I_i(E) = \frac{1}{2} \dot{N}_{\text{ann}} \frac{1}{m_\chi} \frac{dN_i}{dx},
\]

(28)

d\( dN_i/dx \) is the differential number of particles of type \( i \) produced per annihilation with energy \( E = xm_\chi \). We calculate these spectra as described in Ref. [22] for the case of a non-supersymmetric evolution of the fragmentation functions \( dN_i/dE \).
The function $\mathcal{H}$ contains the information about the smooth DM distribution in the halo,

$$\mathcal{H} = \int_0^\pi d\zeta \sin \zeta \int_0^{r_{\text{max}}(\zeta)} ds \frac{\xi \rho_h(r, \zeta)}{M},$$

where $\xi$ is the fraction of DM in form of neutralino clumps, $n_H(r) = \xi \rho_h/M$ is the number density of clumps at distance $s$ from the Sun along the line-of-sight (l.o.s.), and $\zeta$ is the angle between the direction in the sky and the galactic center (GC). Finally, $r_{\text{max}} = (R_h^2 - r_\odot^2 \sin^2 \zeta)^{1/2} + r_\odot \cos \zeta$ is the distance to the border of the DM halo of radius $r_h$ and $r_\odot = 8.5$ kpc is the distance of the Sun to the Galactic center.

As distributions of the DM in the galactic halo we use the Navarro-Frenk-White profile [32],

$$\rho_h(R) = \frac{\rho_0}{(R/R_s)^\alpha (1 + R/R_s)^2},$$

with $\alpha = 1$, scale radius $R_h = 20$ kpc, $R_s = 200$ kpc as the size of the DM halo and $\rho_h(r_\odot) = 0.3$ GeV/cm$^3$ as the DM density at the position of the Sun.

Annihilations may proceed in the clumps with different parameters because the a priori unknown position and height of the putative spike in the spectrum of perturbation. The formation and the properties of superdense clumps were considered in Paper I [15]. Following the formalism of Ref. [15], the clumps density follows as shown in Fig. 1. For illustration we use in our calculations the three sets of clumps parameters presented in the previous section.

### B. Cross section

Analytical approximations for the annihilation cross section of a neutralino valid in the limit $M_{\text{SUSY}} > m_Z$ were presented in Ref. [11] using lowest order perturbation theory. We use as annihilation cross section for both binos and higgsino the subprocesses $\chi \chi \rightarrow ZZ$ and $\chi \chi \rightarrow W^+W^-$ in the case of a higgsino,

$$\langle \sigma_{\text{ann}}v \rangle \simeq 2 \times 10^{-42}m_{11}^{-2} \text{ cm}^3 \text{ s}^{-1}. \quad (31)$$

Annihilations into $Zh^0$ and $ZA$ can increase the cross-section, depending on the values of the SUSY breaking parameters.

Since the relative velocities of SHLSPs in DM clumps are small, $\beta = v \ll 1$, factors $g^2/\beta$ or $\ln(g^2/\beta)$ can lead to a break-down of perturbation theory. This effect, first studied by Sommerfeld for Coulomb interactions, was generalized in Ref. [30] to the exchange of massive non-abelian gauge bosons relevant for neutralino annihilations. In the case of a wino or higgsino, the small mass splitting $\delta m = m_s - m_{\lambda_\pm}$ between the lightest neutralino and the lightest chargino means that the charginos produced in $\chi \chi \rightarrow \lambda_{\pm}^{\pm} \lambda_{\mp}^{\mp}$ have the same small velocity as the neutralinos. Therefore multiple photon, $Z$ and $W^\pm$ exchange between the charginos becomes important.

The resulting enhancement of the annihilation cross section can be calculated non-relativistically and is, neglecting bound-state effects, characterized by two parameters [31]: The ratio $\varepsilon = m_W/m_\chi$ determines, if the annihilation proceeds in the Coulomb ($\varepsilon \ll 1$) or in the Yukawa ($\varepsilon \gg 1$) regime, while the ratio $x = g^2_{\text{eff}}/\beta$ of the squared effective coupling constant and the velocity determines, if factors $g_{\text{eff}}^2/\beta$ or $\ln(g_{\text{eff}}^2/\beta)$ lead to a break-down of perturbation theory. Here, the effective coupling constant $g_{\text{eff}}$ includes all pre-factors in front of the Yukawa potential, as e.g. mixing factors.

The Sommerfeld factor $\mathcal{R}$ as ratio of the perturbative and non-perturbative annihilation cross section is given in the Coulomb case by

$$\mathcal{R} = \frac{\sigma_{\text{ann,v}}} {\sigma_{\text{pert}}} \sim \frac{\eta}{1 - \exp(-\eta)} \quad (32)$$

with $\eta = \pm g_{\text{eff}}^2/(2\beta)$. Using in the case of a higgsino the parameters for the optimistic example given in Sec. IV.A we find a strong enhancement of the annihilation rate, $\mathcal{R} \sim 10^{10}$. On the other hand, higgsino are relatively tightly coupled to the thermal plasma, leading to a relatively large value of $M_d$. Thus there is no gravothermal catastrophe for higgsino clump and we use $S = 1$.

For a bino, the Sommerfeld effect is not effective, $\mathcal{R} = 1$. Using the above parameters for the optimistic example, we find $S \approx 4/(9\epsilon_{2,2}) \sim 4 \times 10^{10}$. Hence, annihilations of binos in clumps formed during the RD epoch can be enhanced by the factor $S \sim 4 \times 10^{10}$ compared to the annihilation signal computed for a smooth DM distribution inside clumps.

The maximal fluxes of photons, nucleons and neutrinos from neutralino annihilations in Galactic halo together with experimental data for a neutralino with $10^{11}$ GeV.

![FIG. 2: The maximal fluxes $I_i(E)$ of photons, nucleons and neutrinos from neutralino annihilations in Galactic halo together with experimental data for a neutralino with $10^{11}$ GeV.](image-url)
given in the optimistic example for a bino, the flux shown in Fig. [2] has been rescaled by the factor $10^{-14}$, while for a higgsino the flux was rescaled by $10^{-5}$. Thus the fluxes of secondaries overshoot the experimental data by many orders of magnitude for the most optimistic scenarios. This offers the potential to test different DM masses as well as different scenarios for the formation of superdense clumps. At present, annihilations of SHLSPS are mainly restricted by experimental limits on the photon fraction [55] and a galactic anisotropy of the UHECR flux [36]. However, in the future neutrino searches at lower energies by a km$^2$ neutrino telescope as ICECUBE may become competitive.

V. CONCLUSIONS

We have studied the properties of SHDM clumps in two different cosmological scenarios, one with power-law and one with spiky density perturbations. As superheavy DM particles we have considered the superheavy neutralino, which properties were studied within superheavy supersymmetry in Ref. [11].

For standard power-law fluctuations, SHDM clumps are formed in the DM dominated epoch in hierarchical structures, when a small clump is hosted by a bigger clump, which in turn is submerged into an even bigger one, etc. Small clumps are tidally disrupted in such structures and only a small fraction of the clumps survives. The surviving clumps can be further destroyed by tidal interaction in the Galaxy.

In contrast, clumps in spiky density perturbations are born in the RD dominated epoch, when hierarchical structures are not yet formed. They evolve as the single isolated objects without tidal destruction and merging.

For the standard cosmological scenario the density of SHDM particles in a clump is low and the annihilation signal is weak, caused by the too small annihilation cross section. These clumps can be detected only when clumps are passing by gravitational wave detectors.

In superdense clumps, an isothermal profile may continue up to very small radius of the new core, if the clump went through the “gravithermal catastrophe”. The density of particles in the core and nearby is very large and this enhances the annihilation signal. Another reason for the increase of the annihilation signal in comparison with clumps of EW mass particles is the small $M_{\text{min}}$ in the mass distribution of the clumps. The cutoff $M_{\text{min}}$ is smaller for a bino, but a higgsino gains from the Sommerfeld factor, which increases the annihilation cross section.

As a result, we found that the annihilation rate of stable superheavy neutralinos may be large enough to be detectable, if primordial density perturbations are spiky. Hence the search for photons or a galactic anisotropy in UHECRs [36] as well as the search for UHE neutrinos offers not only the potential to identify the DM candidate but also to learn about the inflationary potential.

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