We consider an extended flipped $SU(5)$ model, supplemented by a flavor $U(1)$ symmetry, which yields bi-large neutrino mixings, charged fermion mass hierarchies and CKM mixings. The third leptonic mixing angle $\theta_{13}$ turns out to lie close to 0.07, and neutrino CP violation can be estimated from the observed baryon asymmetry. For lepton flavor violating processes we find the branching ratios, $\text{BR}(\mu \to e\gamma) \sim 10^{-4}$, $\text{BR}(\tau \to e\gamma) \sim 10^{-4}$, $\text{BR}(\tau \to \mu\gamma) \lesssim 5 \cdot 10^{-14}$. The proton lifetime $\tau_{p-x} \approx 10^{34} - 10^{36}$ yrs.

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Neutrino oscillation experiments [1] have provided strong evidence for physics beyond the Standard Model and its minimal supersymmetric extension (MSSM). The two leptonic mixing angles $\theta_{12}, \theta_{23}$ are large, while the third mixing angle $\theta_{13} \lesssim 0.2$. An unsuppressed value of $\theta_{13} (\gtrsim 10^{-2})$ would lead to observable CP violation in the lepton sector. However, if $\theta_{13}$ will turn out to be $\ll 1$, this may hint to some underlying symmetry. Since there are not many models [2] which predict $\theta_{13}$ by symmetry arguments, it is desirable to construct models which predict $\theta_{13}$. The task is not easy because knowledge of mixings arising from the charged lepton sector is also required. Thus, GUTs may play a crucial role and, as we will show in this letter, flipped $SU(5)$ turns out to be an attractive candidate. Augmented with suitable flavor symmetry it enables us to predict $\theta_{13}$. Moreover, it turns out that there is direct relation between the CP asymmetry in leptogenesis and CP violation in neutrino oscillations.

Flipped $SU(5)$ GUT [3, 4] has several desirable features which are conspicuously absent in standard $SU(5)$. For instance, the doublet-triplet (DT) splitting problem is nicely realized via the missing partner mechanism [4], and eventually the dimension five proton decay is naturally suppressed. Finally, the model contains right handed neutrinos (RHN) which play an important role in neutrino oscillations. However, in its minimal form flipped $SU(5)$ does not shed much light on important questions related to fermion masses and mixings. In this letter we address this shortcoming by extending the field content of the model, and by introducing a flavor $U(1)$ symmetry. One of our goals is to realize bi-large neutrino mixings responsible for the atmospheric and solar neutrino oscillations. We also wish to understand how the quark mass hierarchies and CKM mixings arise. The model employs a double seesaw mechanism and we find a prediction for the third leptonic mixing angle, namely

$$\theta_{13} \approx \frac{\Delta m_{31}^{\text{sol}}}{\Delta m_{32}^{\text{atm}}} \tan \theta_{13} \tan \theta_{12} \approx 0.07.$$  

Moreover, the amount of CP violation in the neutrino oscillations can be determined from the observed Baryon asymmetry of the Universe. An $R$-symmetry, in combination with $U(1)$ also plays an essential role. Among other things, the MSSM ‘matter’ parity arises naturally, unwanted dimension five baryon number violating operators, as well lepton flavor violating processes are adequately suppressed, and the MSSM $\mu$ problem can be nicely resolved.

The ‘matter’ sector of minimal flipped $SU(5)$ contains

$$10^{(i)} \equiv 1_i = (q, d^c, \nu^c)_i, \quad \bar{5}_{(i)} \equiv \bar{5}_i = (u^c, l)_i,$$

$$1_{(5)} \equiv 1_i = e^c_i , \quad (i = 1, 2, 3 \text{ is family index}),$$

and the transformation properties under $SU(5) \times U(1)$ and field content of the multiplets is indicated. In the ‘scalar’ (higgs) sector we introduce

$$\phi(5_2) = (T_\phi, h_d), \quad \bar{\phi}(5_{-2}) = (\bar{T}_\phi, h_u),$$

$$H(10_{-1}) = (q_H, D_H^c, \nu_H^c), \quad \bar{H}(10_1) = (\bar{q}_H, \bar{D}_H^c, \bar{\nu}_H^c),$$

where the $\phi, \bar{\phi}$ pair contain the MSSM Higgs doublets, while $H, \bar{H}$ are used for breaking $SU(5) \times U(1)$ to $SU(3)_c \times SU(2)_L \times U(1)_Y$.

Next we introduce a $U(1)$ flavor symmetry which distinguishes the fermion families. The $U(1)$ breaking is achieved by the GUT singlet superfield $X$ with $U(1)$ charge $Q[X] = -1$. We assume that the scalar component of $X$ develops a VEV such that $\frac{\langle \phi \rangle}{M_{Pl}} \approx \epsilon \approx 0.2$, where $M_{Pl} \approx 2.4 \cdot 10^{18}$ GeV is the ultraviolet cutoff scale. The $\epsilon$ will play an important role as an expansion parameter. Knowledge of the CKM matrix elements fixes the $U(1)$ charges of 10-plets (containing quark doublets $q_i$) as follows: $Q[10_i] = (a + 3, a + 2, a)$, Moreover, the up-type quark and charged lepton Yukawa hierarchies suggest the following selection of $U(1)$ charges: $Q[5_3] = (b + 5, b + 2, b)$, $Q[1_1] = 2a - b$, and also $Q[\bar{\phi}] = 2Q[\phi] = -(a + b)$. The a and b are undetermined numbers for time being.

With the charge assignments given above, we have for the Yukawa couplings:

$$e^{\nu_j} 10, 10_j \phi, \quad e^{D_j} 10, 5_j \phi, \quad e^{5_1} 1_j \phi,$$

$$\theta_{13}, \text{ Rare Processes and Proton Decay in Flipped } SU(5)$$

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with \( t_{ij} = 6 + (i + j - \lambda^2 - \lambda^2) / 2 \), \( r_i = 9 + (i - \lambda)^2 / 2 \),
\[
p_{ij} = 12 + (i - 9j - \lambda^2 + j^2) / 2 .
\]

The last two terms in (3) generate up-type quark and charged lepton masses respectively, yielding the desirable hierarchies: \( \lambda_3 \sim 1 \), \( \lambda_2 : \lambda_1 : \lambda_3 : \epsilon^3 : 1 \) and \( \lambda_c : \lambda_3 : \lambda_2 : \epsilon^3 : \epsilon^2 : 1 \). The CKM mixing angles have the desirable magnitudes (\( V_{us} \sim \epsilon, V_{cb} \sim \epsilon^2, V_{ub} \sim \epsilon^3 \)). However, the first coupling matrix in (3) gives \( m_a \sim \epsilon \), \( m_b \sim \epsilon^2 \) which are both unacceptable. In order to obtain satisfactory values for \( m_d \) and \( m_s \) the field content of the model will be extended. Before doing this, let us note that, in addition to \( U(1) \) invariance, the above couplings also display an underlying \( R \)-symmetry, with the \( R \)-charges of the superfields not completely fixed. The inclusion of the scalar sector will determine some of the charges. Taking for \( H, \bar{H} \) states \( Q[\bar{H}] = a + 1/2 \), \( Q[H] = (a + b + 1)/2 \) the superpotential couplings which resolve the DT splitting problem are \( \epsilon \phi HH + \epsilon \phi \bar{H}H \).

With \(|H| = |\bar{H}| \equiv V \simeq M_{\text{GUT}} \) (in \( \nu^2 \) direction), the color triplets \( T_\alpha \) and \( \bar{T}_\alpha \) acquire masses \( \sim M_{\text{GUT}} \) by pairing with \( D_H \) and \( D_{\bar{H}} \) respectively [4]. The MSSM doublet pair \( h_u, h_d \) remain massless at this stage, as desired. Appearance of the triplet-anti triplet pair below \( M_{\text{GUT}} \) helps to obtain a somewhat reduced value of the strong coupling constant \( \alpha_s(M_Z) \simeq 0.117 \) in good agreement with experiments.

With the above couplings and the \( R \)-symmetry transformations \( \Phi_i \rightarrow e^{i R(\Phi)} \Phi_i \), \( W \rightarrow e^{i R(W)} W \) (\( W \) denotes the superpotential), we assign the \( R \)-charges as follows: \( R(10_i) = R(H) = \alpha, R(5_i) = \beta, R(1_i) = 2\alpha - \beta, R(\bar{\phi}) = \alpha_\phi, R(\bar{\phi}) = \alpha - \beta + \alpha_\phi, R(H) = (\alpha + \beta)/2, R(X) = 0, R(W) = 2\alpha + \alpha_\phi \).

To resolve the problem of the \( d \) and \( s \) quark masses, we introduce the following vector-like ‘matter’ states
\[
F_\alpha(5_2) = (\bar{D}_F, L_F)_\alpha \quad \text{and} \quad \bar{F}_\alpha(5_{-2}) = (\bar{D}^c_F, \bar{L}_F)_\alpha ,
\]
where \( \alpha = 1, 2 \) labels the two pairs \( F, \bar{F} \). The couplings of these states with the \( 10_i \) plets will induce mixing of \( d^c \) and \( \bar{D}_F^c \) states. This allows us to improve the light down-type quark masses. Also, the relation \( M_U = m_{D_F}^2 \) (arising from the second coupling of (3)) is violated, which will be important for realizing large \( \nu_\mu - \nu_\tau \) mixing. An additional singlet scalar superfield \( S \) also plays an important role.

The relevant couplings are given by
\[
\bar{H}10 \bar{F}_\phi + (\bar{H}H)^2(10FH + 5FH) + SFF + \bar{H}10F \phi \quad ,
\]
where generation indices are suppressed, and the \( R \)-charges are as follows: \( R(\bar{F}_\phi) = -R(F_\phi) = (\alpha - \beta)/2 \), \( R(S) = \alpha_\phi + 2\alpha = 3(\alpha + \beta)/2 \). With \( U(1) \) charge assignments \( Q[S] = 7 + 3(3a + b)/2 \), \( Q[F_\phi] = -(5a + b + 7/2, 5a + b + 9/2) \), \( Q[F_\phi] = (a - b - 7, a - b - 5)/2 \) and \( b = -3a - 4/3 \), the couplings in (5) can be written in family space:
\[
\begin{array}{cccc}
1 & 1 & \epsilon & \bar{H}_\phi / M_{\text{GUT}} \\
0 & 0 & \epsilon & \bar{H}_\phi / M_{\text{GUT}} \\
0 & 0 & 0 & \bar{H}_\phi / M_{\text{GUT}} \\
\end{array} 
\]

(6)

Assuming that \( \langle S \rangle \sim \epsilon^2 M_{\text{GUT}} \) \( (\epsilon^2 \equiv M_{\text{GUT}} / M_{\text{P}}) \) and substituting all appropriate VEVs, from the first matrix couplings in (3) and (6) with field embeddings given in (1), (2), (4), the ‘big’ \( 5 \times 5 \) down quark mass matrix has the form
\[
\begin{array}{cccc}
q_1 & q_2 & q_3 \\
\epsilon^2 & \epsilon^2 & \epsilon^2 \\
\epsilon^2 & \epsilon^2 & \epsilon^2 \\
\epsilon^2 & \epsilon^2 & \epsilon^2 \\
\end{array} 
\]

(8)

Integrating out the heavy \( D_F^c, \bar{D}_F \) states, (8) reduces to the \( 3 \times 3 \) matrix
\[
M_d \simeq q_2 \left( \begin{array}{ccc}
\epsilon^2 & \epsilon^2 & \epsilon^2 \\
\epsilon^2 & \epsilon^2 & \epsilon^2 \\
\epsilon^2 & \epsilon^2 & 1 \\
\end{array} \right) h_d \quad ,
\]

(9)

which yields the desired hierarchies \( m_{u_d} \sim \epsilon \), \( m_{u_s} \sim \epsilon^2 \).

The couplings in (6) and (7) yield the neutrino Dirac \( 5 \times 5 \) matrix
\[
\begin{array}{cccc}
\nu_1 & \nu_2 & \nu_3 \\
\epsilon_{\alpha} h_u & \epsilon_{\alpha} h_u & \epsilon_{\alpha} h_u \\
\epsilon_{\alpha} h_u & \epsilon_{\alpha} h_u & \epsilon_{\alpha} h_u \\
\epsilon_{\alpha} h_u & \epsilon_{\alpha} h_u & \epsilon_{\alpha} h_u \\
\epsilon_{\alpha} h_u & \epsilon_{\alpha} h_u & \epsilon_{\alpha} h_u \\
\end{array} 
\]

(10)

Integrating out the heavy \( L, \bar{L} \) states, we obtain
\[
M_D = \left( \begin{array}{ccc}
\nu_1 & \nu_2 & \nu_3 \\
\epsilon_{\alpha} & \epsilon_{\alpha} & \epsilon_{\alpha} \\
\epsilon_{\alpha} & \epsilon_{\alpha} & \epsilon_{\alpha} \\
\end{array} \right) h_u .
\]

(11)

This modified form for the Dirac mass matrix will be important for bi-large neutrino mixings. With only the
\(\nu^c\) RHN states in (11), one expects \(\theta_{23} \sim e^3\) (the ratio of (2,3) and (3,3) elements). However, imagine that \(\nu^c\) state decouples with some new singlet state \(N_3\) at high scale. Then \(\theta_{23}\) will be determined by the ratio of (2,2) and (3,2) elements which is naturally large \(\sim e_G/\epsilon^2 \sim 1\). By the same token the \(\nu^c\) state should decouple. This decoupling mechanism was discussed in [5] and successfully applied within various GUTs [6–8]. For large solar mixing an important role is played by the strong mixing between the \(L_2\) and \(l_2\) states. Therefore, for bi-large neutrino mixings we introduce three additional GUT-singlet RHN \(N_i\). They will generate a suitable neutrino mass texture through the double seesaw mechanism, after introducing the scalar superfield \(S\). With \(R\) and \(H(1)\) charged gives by \(R(N_{1,3}) = 3\alpha + \beta, R(N_2) = (3\alpha + \beta)/4, Q(S) = 3(3\alpha + \beta)/4, Q[N_1] = -17/6, Q[N_2] = -1/3, Q[N_3] = 1/6, Q[\bar{S}] = -3/2\), the relevant couplings involving \(N_i\) states read

\[
10_1(N_1 + S\epsilon N_2 + e^3 N_3)H + 10_2(SN_2 + e^2 N_3)\bar{H} + 10_3N_3\bar{H} + (\bar{H}H)^2 N_2 N_2, \tag{12}
\]

where for higher order operators the cut off scale \(M_{pl}\) has been omitted. The \(9 \times 9\) mass matrix for neutral fermions is given by

\[
\nu, \nu^c, \begin{pmatrix}
\nu \\
\nu^c \\
N
\end{pmatrix}
\begin{pmatrix}
0 & M_D & 0 \\
M_D^T & 0 & M \\
0 & M_T^T & M_N
\end{pmatrix}, \tag{13}
\]

where \(M\) and \(M_N\) are given by \(10 \cdot N\) and \(N \cdot N\) couplings of eq. (12). Integrating out the heavy \(\nu^c\), \(N\) states leads to the light neutrino mass matrix given by the double seesaw formula: \(m_\nu = \frac{m_D}{M_T} M_N^{-1} \frac{1}{M_T} m_D\). Substituting \(\epsilon^2 \sim e_G\) and assuming \(\langle S \rangle \sim e_G^0 M_{pl}\), we find

\[
m_\nu = \left( \begin{array}{ccc}
e^6 & e^3 & e^3 \\
e^3 & \beta^2 & \alpha \beta \\
e^3 & \alpha \beta & \alpha^2
\end{array} \right) \frac{1}{\epsilon G^2} \left(\begin{array}{ccc}
\bar{\alpha} & \bar{\beta} & 0 \\
0 & \beta^2 & 0 \\
0 & 0 & 0
\end{array} \right) m', \tag{14}
\]

with \(m \sim \frac{(\epsilon^6)^2}{\epsilon G^2} = (0.01 - 0.1)\) eV. This is indeed the desired form for the leading part of the neutrino mass matrix responsible for large atmospheric neutrino mixing angle (provided by \(\alpha \sim \beta\)). Note that the scale \(m\) in (14) has the correct magnitude.

For generating the sub-leading part of the neutrino mass matrix, responsible for large solar neutrino mixing, we employ the mechanism of single RHN dominance [9]. We introduce an additional right-handed state \(N\) and scalar superfield \(S\) with \(R\) and \(U(1)\) charges \(R[N] = (3\alpha + \beta) k/2, R[S] = (3\alpha + \beta)(k + 1)/2, Q[N] = -3(k + 2)/6, Q[S] = (22 + k)/3\), where \(k\) is an integer. The relevant couplings are

\[
\kappa(hH)^k N_1 \bar{\phi} H + \kappa' N F_2 \bar{\phi} S + c S S' (hH)^{k-1} N^2, \tag{15}
\]

where \(\kappa, \kappa'\) are dimensionless couplings. We will assume that \((S') \sim e_G^{2k+1} M_{pl}\). Recalling that the \(L_2\) state (from \(F_2\) strongly mixes with \(l_2\), integrating out \(N\) gives the sub-leading contribution to the neutrino mass matrix:

\[
m_\nu^{(1)}(\alpha^2 + \bar{\alpha} \bar{\beta} \beta^2 \alpha) m', \tag{16}
\]

with \(m' \sim \frac{\kappa^2(h^0)^2}{\epsilon G^2} = 5 \cdot (10^{-3} - 10^{-2})\) eV (for \(\kappa \sim \kappa' \sim 1/5\)) to explain the solar neutrino anomaly.

With the neutrino mass matrix \(m_\nu = m_\nu^{(0)} + m_\nu^{(1)}\), with entries \(m_\nu^{(0)}\) and \(m_\nu^{(1)}\) given by (14) and (16) respectively [10, 13], the two mixing angles \(\theta_{12}\) and \(\theta_{23}\) are naturally large, while the third leptonic mixing angle is

\[
\theta_{13} \equiv |U_{e3}| \sim \sqrt{\frac{\Delta m^2_{atm}}{\Delta m^2_{solar}}} \tan \theta_{12} \tan \theta_{23} + \tan^2 \theta_{12}. \tag{17}
\]

Since the contribution to \(U_{e3}\) from the charged lepton sector is of order \(e^2\) and can be safely ignored, the model predicts the third leptonic mixing angle to be \(\theta_{13} \approx 0.07\).

The mass of the RHN state \(M_N \sim M_N^{(0)} + M_N^{(1)}\) for \(k = 1(2)\) is of order 50 GeV(0.5 keV). A keV mass sterile neutrino may contribute to the dark energy budget of the universe [14].

If the last coupling in (15) is generated by exchange of some additional states, then \(k = 0\) is also possible. This gives \(M_N \sim 10^9\) GeV, a scale preferred by leptogenesis, and which allows one more prediction. The lepton asymmetry is created by the out of equilibrium decay of \(N\), in which the states \(l_i, \nu^c\) and \(N_2\) are also involved. There is only one CP violating phase in this system which also appears in the light neutrino mass matrix, whose dominant part is generated via \(\nu^c\), \(N_2\) states. This allows one [15] to relate the CP asymmetry \(e_N\) and the leptonic Jarlskog invariant \(\mathcal{J}^l\). For \(M_N \sim 10^9\) GeV we have

\[
\frac{n_B}{s} \sim -4.8 \cdot 10^{-9} \mathcal{J}^l. \tag{18}
\]

To obtain \(n_B/s \approx 9 \cdot 10^{-11}\) we need \(\mathcal{J}^l \sim -0.02\). CP violation of this size can be tested experimentally.

The RHNs provide a source for lepton flavor violating rare processes such as \(l_\alpha \rightarrow l_\beta \gamma\) [16]. Below \(M_G\), there are two right-handed states \(\nu^c\) and \(N\). The latter couples with the light neutrinos so weakly (\(\sim \kappa_2^2\) and \(\kappa_2^2\) for \(k = 1\) and \(k = 2\) resp.) that it plays no role in rare processes. As far as the state \(\nu^c\) is concerned, its mass generated via mixing with \(N_2\) is \(M_G\). Assuming \(N = 1\) SUGRA and universality of soft scalar masses at \(M_G\), the non-universal contributions are generated at the weak scale. With the Yukawa couplings \(e_G h_{\nu^c} \nu_5^c (e^3 l_1 + l_2 + l_3)\) one expects

\[
\text{BR}(\mu \rightarrow e\gamma) \sim \text{BR}(\tau \rightarrow e\gamma) \sim e^6 \text{BR}(\tau \rightarrow \mu\gamma). \tag{19}
\]
For tan β ~ 60 (suggested by the charged fermion sector), we find BR(µ → eγ) ≤ 5 × 10^{-14} (with sparticle masses m_S ≳ 100 GeV), which is well below the current experimental bound [17], but within striking range of ongoing (and planned) experiments [18]. From (19) the processes τ → µγ, τ → eγ are adequately suppressed, consistent with the recent experimental bounds [19].

The symmetry R × U(1) plays another important role. It forbids Z_2 'matter' parity violating operators such as $i\delta H, \phi F, \bar{\phi} F, \bar{\phi} F, 1 \cdot 5 \cdot 5 H$, etc., which are otherwise allowed by flipped SU(5). Thus, 'matter' parity emerges automatically in our scheme and we have a stable cold dark matter candidate (LSP).

We now turn to the discussion of proton decay. Since the color triplets from scalar superfields have mass terms $\epsilon M_{GUT} (T_{\theta} D_H^\theta + T_{\bar{\theta}} D_H^{\bar{\theta}})$, the emergence of appropriate $d = 5$ B-violating operators require matter couplings with $D_H^\theta, D_H^{\bar{\theta}}$ fragments. However, such couplings are strongly suppressed ($< \epsilon^2 G < 5 \cdot 10^{-8}$) and are not relevant for nucleon decay. The Planck scale suppressed B-violating operators $\Gamma_{\text{kin}} M_{\text{Pl}}^{-1} 10, 10, 10 K_n \Delta$, $i\epsilon_{\text{kin}} M_{\text{Pl}}^{-1} 10, 10, 10 F_n \Delta$, $\Gamma_{\text{kin}} M_{\text{Pl}}^{-1} 10, 10, 10 K_n \Delta$ are allowed by flipped SU(5), but $R \cdot U(1)$ helps to suppress them. One can check out that the operators $q_i q_j q_{2,3}$ and $d \bar{c} \bar{c} c$ (arising from $\Gamma$, $\bar{\Gamma}$ and $R$ resp.) are suppressed: $\Gamma_{112} (H_{M_{\text{Pl}}}^{(1)}) 112 \sim \epsilon^2 \Gamma_{1123} = \epsilon^2 \Gamma_{1123} < 5 \cdot 10^{-10}$, $R_{321} = \epsilon^2 \Gamma_{1123} < 2 \cdot 10^{-5}$. Thus, $d = 5$ nucleon decay rates are adequately suppressed.

Observable proton decay arises from $d = 6$ operators mediated by $X, Y$ gauge bosons. Because of extra triplets (coming from $\phi, \bar{\phi}, H, \bar{H}$) with masses $\sim \epsilon M_{GUT}$, the meeting point (scale identified with $M_{GUT}$) of SU(3)_c and SU(2)_L gauge couplings is reduced by a factor 1.2 compared to the value determined in minimal flipped SU(5). Moreover, due to the additional $F, \bar{F}$ states (constituting complete SU(5) multiplets) the unified coupling $\alpha_G$ is increased by factor 1.3. Thus, the proton lifetime is reduced by a factor 5 (or so) compared to the minimal scheme. Following [20] this leads us to predict $\tau_{p \rightarrow \pi^0 e^+ e^-} \approx 10^{34.6-36}$ yrs.

To summarize, we have proposed an extension of flipped SU(5) which preserves the successful features of the minimal scheme. The extension consists of vector-like 'matter' states, and a new symmetry R × U(1) which insures that matter parity is automatic, rare decay processes are adequately suppressed, unwanted $d = 5$ baryon number violation is absent, and the MSSM $\mu$ term can be generated through one of two distinct mechanisms [21].

The extension also enables us to reproduce observed charged fermion mass hierarchies and the CKM mixing elements. Neutrino mass scales compatible with present observations are also reproduced as well as bi-large mixings in the neutrino sector. The latter allows to relate the cosmological CP phase with neutrino oscillation's CP violation (estimated to be few percent). This prediction together with $\theta_{13} \simeq 0.07$ and $\tau_{p \rightarrow \pi^0 e^+ e^-} \approx 10^{35.6 \pm 1}$ yrs hopefully can be tested in the near future.

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