Mass Hierarchy and Mixing as Results of Simultaneous Dynamical Breaking of Chiral and Flavor Symmetries

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Abstract

If the hierarchy of quark masses comes out of simultaneous dynamical chiral and flavor symmetry breakings, it is inevitably accompanied by the flavor mixing. Therefore, these phenomena, independent in the standard model and in many of its generalizations, become related since they appear out of the same dynamics. The features of that high energy dynamics may be investigated via accordance with observed mass hierarchy and mixing.

The \( t \)-condensate approach to chiral (and weak) symmetry breaking seems enable reasonably to reproduce the standard model (SM) properties concerning masses of the heavy quark, gauge bosons and the Higgs scalar \( [1] \). This fact could evidence that the physical mechanism based on the Nambu’s idea \( [2] \) is close to a true one, that is really responsible for the origin of SM quark masses.

Limitations of the method consist in leaving aside other quark masses (besides the most heavy \( t \)-one) and neglecting mixing between quark generations. To incorporate the both phenomena new hypotheses and new independent dynamical mechanisms are required. This defect results from the principal idea about a special strong coupling acting exclusively within the \( t \)-generation. This idea is in contrast with total flavor symmetry of all observed interactions.

Therefore, it seems that a more attractive and promising picture would evolve if the most heavy mass can appear via simultaneous chiral and flavor symmetry breakings in a system of \( n \) initially massless and symmetric quark flavors \( (n = 3, 4, \ldots) \). Then a would be ground state with one massive \( (m_1 \neq 0) \) and \( n - 1 \) massless quarks \( (m_2 = m_3 = \cdots = 0) \) turns out the very state to develop mass hierarchy and mixing. A small perturbative violation of the initial symmetries easily produces the necessary development. The phenomena automatically appear together due to a common dynamical mechanism. Properties of the involved perturbative dynamics may be then extracted out of the coincidence with the observed picture for quark mass spectra and, especially, for the weak mixing.

Thus, we are interested to find out appropriate conditions for a phase transition of the type (\( L, R \) are chiralities):

\[
U_L(n) \times U_R(n) \rightarrow U_L(n-1)U_R(n-1)U(1) ,
\]

(1)
in a system of $n$ identical massless quark flavors. As the simplest possible dynamical model let us consider the four-fermion interaction

$$V_{int} = \lambda \sum_{i,k,c,c'}(\bar{\psi}_i^c \psi_k^c)(\bar{\psi}_k^{c'} \psi_i^{c'}) ,$$

\[i, k = 1, 2, \ldots, n \text{ are } L, R \text{ flavors; } cc' = 1, 2, \ldots, N_c \text{ are colors.}\] 

Our system will be one of the Nambu-Yona-Lasinio (NYL) models \[\text{[3].} \] The last ones have been often used before within investigations of the $t$-condensate mechanics \[\text{[1]–[4].} \] The solution of the model is usually looked for as an expansion in $N_c^{-1}$, $N_c \gg 1$, with a dimensionless coupling

$$\beta = \frac{\lambda N_c M^2}{8\pi^2} ,$$

\[\text{taken as a constant at } N_c \to \infty.\]

The factor $M$ is a momentum cut-off of the unrenormalizable model. Let us remind that principal features of chiral violation: initiation of masses and emergence of goldstone bosons, — insignificantly depend on a detailed behaviour near the cut-off region $M$. However, those $N_c^{-1}$ orders ($M_0^0$, $N_c^{-1}$), where consistent treatment is possible in pure NYL models \[\text{[3],}\] are inadequate to achieve our new aims.

Several physical reasons dictate the choice of Eq.(2) as a reasonable trial device.

1. Eq.(2) is an $U_L(n) \times U_R(n)$ symmetric form. Here the flavor groups coincide with chiral ones. Only in such a system simultaneous breaking can occur.

2. The Fierz transformation carries (2) into a combination of (color) vector-vector $L \times R$ couplings conserving flavors. Therefore (2) may be considered as a low-energy local effective limit corresponding to a high-energy theory where all interactions are generated exclusively by vector (pseudovector) fields. Indeed, any number of vector exchanges between massless chiral fermions generates only a vector-vector coupling in the local limit. As it is usually supposed, the gauge theories are favorable candidates in dynamical explaining of SM properties. Therefore the potential (2) may present a rather suitable example for a low-energy limit of such a theory.

3. In addition (although it is not so significant), among all flavor conserving local couplings only the colorless, scalar form (2) contributes to the equation for fermion masses (the "gap equation"\[3\]) in the leading $N_c^0$ approximation.

Now let us construct NYL gap equations. In the symmetric problem (1) the matrix of quark propagators can be taken in a diagonal form

$$G_{ii}^{-1}(p) = \Sigma_{ii}(p^2) - \hat{p}Z_{ii}(p^2) .$$

$\Sigma$ and $Z$ are quantities to be determined by the equations. In the leading (Fig.1) and next-after-leading (Fig.2) $N_c^{-1}$ approximations\[4\] gap-equations show possible existence of various phase transitions

$$U_L(n) \times U_R(n) \rightarrow U_L(n - n') \times U_R(n - n') U_V(n') ,$$

\[\text{[5]}\]

1In all approximations, besides the leading one, equations for a self-energy $\Sigma$ and for a mass matrix are different.
$n' \leq n$. Any such a solution contains $n'$ massive flavors:

$$m_1 = m_2 = \cdots = m_{n'} \neq 0,$$

(6)

and $n - n'$ massless ones

$$m_{n'+1} = \cdots = m_n = 0.$$

(7)

In the lowest approximation the gap-equations take the well-known \[1\]–\[4\] form

$$m_i = m_i \frac{\beta}{\pi^2 M^2} \int \frac{d^4 p}{m_i^2 + p^2} f(M^2 - |p|^2),$$

(8)

$M^2$ is the momentum cut-off, $f$ is the cut-off function. We transformed the integral (8) into the Euclidean form. The $\beta$ parameter is the usual for NYL the dimensionless coupling constant (3).

We shall confirm now the presence of the phase transitions (5), constructing compound bosons of the model. A $q\bar{q}$ scattering amplitude (Fig.3)

$$F(p_i) = \frac{\beta}{N_c M^2} B(q)$$

(9)

depends on a single variable $q$ in the lowest $N_c^{-1}$ approximation. Summing loops, it is easy to show that chirality conserving ($B_+$) and changing ($B_-$) parts are

$$B_\pm = \frac{1}{2} \left[ \frac{1}{1 - A_1 + A_2} \pm \frac{1}{1 - A_1 - A_2} \right] ,$$

(10)

if we have written the one-loop contribution as

$$A_{\alpha_1\alpha_2} = A_1 \delta_{\alpha_1\alpha_2} + A_2 \delta_{\alpha_1 - \alpha_2} , \quad \alpha_i = (L, R) = \pm 1 .$$

(11)

In the denominators (10) let us remove the part reproducing the gap equation(8). We have

$$1 - A_1 \pm A_2 = 1 - \frac{\beta}{M^2} \int \frac{d^4 p}{\pi^2 i} f(M^2 - |p|^2) + \frac{\beta}{2M^2} \int \frac{d^4 p}{\pi^2 i} \left[ \frac{1}{m_1^2 - p^2} - \frac{1}{m_2^2 - (p - q)^2} \right] f + \frac{\beta}{2M^2} [(m_1 \pm m_2)^2 - q^2] I(m, q) .$$

(12)

We consider the scattering amplitude for quarks with $m_1$ and $m_2$ masses. The function $I(m, q)$ is

$$I(m, q) = \int \frac{d^4 p}{\pi^2 i} \frac{1}{(m_1^2 - p^2)(m_2^2 - (p - q)^2)} f .$$

(13)

The first two terms (12) vanish, when $m_1$ is a root of the equation(8) and this result does not depend on the cut-off choice. All other terms become

$$\frac{\beta}{2M^2} \int \frac{d^4 p}{\pi^2 i} \frac{2pq - q^2 + m_2^2 - m_1^2 + (m_1 \pm m_2)^2 - q^2}{(m_1^2 - p^2)(m_2^2 - (p - q)^2)} .$$

(14)
It is evident from (14) that:

for \( m_1 = m_2 \):

\[
1 - A_1 + A_2 = \frac{\beta}{2M^2} \left[ \frac{1}{2}(-q^2) + (4m_1^2 - q^2)I(m_1, q) \right], \quad (15)
\]

\[
1 - A_1 - A_2 = \frac{\beta}{2M^2} \left[ \frac{1}{2}(-q^2) + (-q^2)I(m_1, q) \right]; \quad (16)
\]

and for \( m_1 \neq 0, m_2 = 0 \):

\[
1 - A_1 \pm A_2 = \frac{\beta}{2M^2} \left[ \frac{1}{2}(-q^2) + (-q^2)I(m_1, q) \right]. \quad (17)
\]

Combining all massive quarks between themselves and massive with massless ones we obtain the number of compound massless bosons in the model: \( n'(4n - 3n') \). It is just the number one can obtain calculating Goldstone modes associated with the transition (5):

\[
2n^2 - 2(n - n')^2 - n'^2 = n'(4n - 3n'). \quad (18)
\]

Thus, gap-equations of the NYL model evidence possibility of the transitions (5). They can be traced in both consistent approximations of the NYL models \((N_c^0, N_c^{-1})\). Then, the remaining question is: what a state will be vacuum one (i.e. the lowest energy state)?

The leading \( N_c^0 \) approximation determines as the vacuum state the flavor symmetric solution \( n' = n \). This can be proved by constructing a corresponding effective potential. In the next \( N_c^{-1} \) approximation it was stated that the \( n' = 1 \) solution may be the vacuum state at some conditions. It was confirmed in [3] by lattice calculations. But [3] did not take into account significant part (17) of Goldstone modes (when massive and massless quarks are combined). Nevertheless, next \( N_c^{-1} \) approximations show that the dependence of \( n' \) inevitably appears in the NYL gap equations. Prompting by these considerations and possible essential contributions from higher \( N_c \) approximations, a general form of the gap-equation could be imagined as

\[
1 - \beta^{-1} = \frac{m_1^2}{M^2} \ln \frac{M^2}{m_1^2} + \frac{n'}{N_c} f_1 \left( N_c, \frac{m_1^2}{M^2} \right) + \frac{1}{N_c} f_2 \left( N_c, \frac{m_1^2}{M^2} \right). \quad (19)
\]

Here \( f_1 \) and \( f_2 \) are some functions which cannot be calculated in NYL models for high \( N_c^{-1} \) approximations. They essentially depend on the region near the cut-off \( M \). The term \( f_1 \) is crucial. If \( f_1 > 0 \) and weakly (less than \( m_1^2/M^2 \)) depends on \( m_1^2/M^2 \), a region \( (\beta) \) appears where only the \( n' = 1 \) solution becomes possible.

Therefore, let us assume that in a flavor symmetric, massless system the vacuum state with one massive and \( n - 1 \) massless quarks becomes possible. That state will realize simultaneous breaking of flavor and chiral symmetries. Then, a single mechanism relates mass spectra with mixing properties. Such a relation permits unambiguously to propose and investigate those interactions at high energies which could be responsible for the observed mass and CKM matrix properties. In the considered approximation of the local low-energy interaction (2) we shall propose and investigate necessary properties of the coupling constant \( \lambda(\beta) \).
In order to insert other masses into a symmetric system, it is necessary to resolve degeneracy of \((n-1)\) massless quarks. That means that flavor symmetries must be violated, coupling constants should depend on flavor indices

\[
\lambda \to \lambda_{ii',kk'}, \quad \beta \to \beta_{ii',kk'},
\]

\(ii'\) are \(L\) flavors, \(k,k'\) are \(R\) flavors. But certainly, the complete absence of flavor symmetries is not a situation, which will be interesting for our aims. The flavor violation must be small in comparison with the symmetric (flavor independent) and strong coupling part \(\beta\). Moreover, the only interesting situation seems to be that one, when flavor numbers remain conserved in (20). There will be the case when mixing can also appear spontaneously as a result of the same phase transition and will not be inserted via interaction constants. Therefore, we choose

\[
\beta_{ii',kk'} = \beta_{ik} \delta_{ii'} \delta_{kk'}, \quad \beta_{ik} = \beta + \delta \beta_{ik}, \quad |\delta \beta_{ik}| \ll \beta.
\]

The flavor dependent part \(\delta \beta_{ik}\) could be perturbative one.

Furthermore, the mass hierarchy property even requires the mass matrix \(M_{ik}\) to be represented as some perturbative series in interactions distinguishing flavor indices. The last statement is known for years as "the radiative mechanism" proposed for hierarchy explanations [7]. If the 4-fermion form (2) is a local low-energy limit for some high energy vector (pseudovector) theory, the perturbative and flavor dependent part of \(\beta_{ik}\) must have a form:

\[
\beta_{ik} = \beta + \delta \beta_{ik} = \beta + \sum_{n,m=1} a_{nm} g_{iL}^n g_{kR}^m.
\]

And \(a_{nm}\) coefficients are assumed as decreasing with \(n, m\) numbers, similarly to coefficients of any perturbation theory in \(g_{L}, g_{R} (\sim c_n (4\pi)^{n-m}, c_n \sim 1)\). The formula (22) becomes evident if one considers a local limit of any number of vector exchanges between massless \(L\) and \(R\) fermions \((g_{L} \neq g_{R} \sim 1, \text{Fig.4})\). Considered vector interactions conserve \(R\) and \(L\) flavors.

The next step is the sequential perturbative solution [8] of the gap equation with \(\beta_{ik}, (22)\) at conditions:

\[
|\delta \beta_{ik}| \ll \beta, \quad m_1 \gg m_2 \gg m_3 \ldots.
\]

It is not a simple task for a non-linear equation. But it can be seen that, when \(\beta_{ik}\) is determined by Eq.(22), for the mass matrix \(M_{ik}\) one obtains the solution

\[
M_{ik} = \frac{m_1}{n} R_{ik} + \sum_{n,m=0} b_{nm} g_{iL}^n g_{kR}^m
\]

in any order of \(N_{c}^{-1}\) approximations. The \(R\) matrix elements are: \(R_{ik} = 1, \text{for all } i,k\) indices ("democratic matrix", Y.Koide, H.Fritzsch). Coefficients \(b_{nm}\) have the perturbative properties similar to that ones of \(a_{nm}\). They can depend on \(g_{iL}, g_{kR}\) via transposition symmetric combinations (as \(\Sigma_{i=1} g_{iL}, \Sigma_{i=1} g_{iL}^2 \ldots\)).

The flavor symmetric zero order approximation \(\delta \beta_{ik} = 0\) for the \(M_{ik}\) matrix (when only one eigenvalue \(m_1\) differs from zero) must be taken here in the most general form \((m_1/n)R_{ik}\), since the flavor basis has been already fixed by Eq.(21): all interactions are diagonal in
flavor indices ($L$ and $R$). Therefore, mixing becomes inevitable in the situation considered in spite of the flavor conserving interactions. This seems to be an attractive property of the simultaneous flavor-chirality breaking.

Diagonalization of mass matrices for up and down quark families by means of unitary matrices $U_L$ and $U_R$:

$$\Sigma_{ik}^{(u,d)} = U_L^{+(u,d)} \Sigma_{\text{diag}}^{(u,d)} U_R^{(u,d)} ,$$

and calculation of the weak mixing matrix $V_{CKM}$

$$V_{CKM} = U_L^{(d)} U_L^{(u)}$$

may be fulfilled for the general form (24) [9]. One obtains the hierarchy spectrum of masses. It is the direct consequence of the perturbative structure (22) of $M_{ik}(u)$. An impressive result arises for the mixing matrix: it automatically acquires a form qualitatively similar to the observed one (for any coefficients $a_{nm}$ with a perturbative pattern) when and only when $L$ constants $g_{iL}^{(u,d)}$ are independent of $u,d$ indices:

$$g_{iL}^{(u)} = g_{iL}^{(d)} \equiv g_{iL} , \quad g_{kR}^{(u)} \neq g_{kR}^{(d)} .$$

The important point is the following. If at high energies one has some flavor dependent (but neutral, conserving flavor) interaction of the chiral type

$$\sim g_{iL}^{(u,d)} \bar{\psi}_{iL}^{(u,d)} \gamma_\mu \psi_{iL}^{(u,d)} Z_\mu'(x) , \quad \sim g_{kR}^{(u,d)} \bar{\psi}_{kR}^{(u,d)} \gamma_\mu \psi_{kR}^{(u,d)} Z_\mu'(x) ,$$

the conditions (27) are not new, additional ones, but those of the $SU_L(2)$ weak symmetry. When $g_{iL}^{(u)} \neq g_{iL}^{(d)}$ a direct, explicit violation of the weak isospin would be inserted.

The statement "qualitatively similar" means the following.

1. The diagonal coefficients $V_{us}, V_{cs}, V_{tb}$ differ from unity by the second orders of $g_{iL}, g_{kR}$ perturbations.

2. $V_{us}(V_{cd})$ contains terms proportional to the first order of the perturbations (for the numbers of generations $n = 3, 4$. Why it is reasonable to consider $n = 4$ — see below).

$$V_{us} \sim \frac{\langle 2nd \rangle}{\langle 1st \rangle} + \langle 1st \rangle , \quad n = 3 ; \quad V_{us} \sim \frac{\langle 3rd \rangle + \langle 1st \rangle \langle 2nd \rangle + \langle 1st \rangle^3}{\langle 1st \rangle^2 + \langle 2nd \rangle} , \quad n = 4 .$$

3. $V_{st}(V_{cb})$ also is proportional to the first order perturbations but in the different form:

$$V_{st} \sim \langle 1 \rangle , \quad n = 3 ; \quad V_{st} \sim \frac{\langle 2nd \rangle}{\langle 1st \rangle} + \langle 1st \rangle , \quad n = 4 .$$

Because of such a difference $V_{us}$ turns out numerically larger than $V_{st}$. The excessive factor contains the number of vector components $-4$, besides unknown factors assumed to be of the unity order.

Thus, $|V_{us}| > |V_{st}|$ again might be an evidence for the vector character of high energy interactions. But the difference is only numerical here in contrast to usually assumed differences in perturbation orders [10].

6
4. 

\[ |V_{ub}| \leq \langle 3rd \rangle , \quad \text{the same with } |V_{td}|. \]  

(30)

Thus, the discussed mechanism reproduces observed differences in hierarchy steps:

\[ \frac{|V_{us}|}{|V_{ud}|} \gg \frac{|V_{ub}|}{|V_{us}|}. \]  

(31)

The interesting point here is that the necessary condition for such a property seems to be the simultaneous presence of high energy interactions dependent on flavors (perturbative, like responsible for \( \delta \beta_{ik} \) in Eq.(28)) and independent ones (may be strong, responsible for the symmetric part \( \lambda \) in Eq.(2)) [9(2)].

There are also some other properties and consequences of the mechanism.

- There are no direct relations between mass ratios and mixing elements but there is correspondence of values with equal orders of perturbations.

- The \( CP \) violation may naturally be included in the mixing matrix [9(2)]. The ratios \( \text{Im}V_{cd}/\text{Re}V_{cd}, \text{Im}V_{ts}/\text{Re}V_{ts} \) can be evaluated exclusively through mass ratios. Their numerical values closely approach experimentally acceptable figures.

  In establishing of this property a vector character of the interaction distinguishing flavors is crucial again. The result is correct for any \( a_{nm} \) coefficients.

- The interesting point is that the 4th generation of quarks (if it exists) insignificantly influences the properties of the three generations.

  In addition the solution of the gap-equation with the hierarchy spectrum permits also to obtain some parametric suppression of flavor changing neutral currents \( \sim N^{-1}_c f_1(m_2/m_1)^{1/2}, \) where \( f_1 \) is the function from Eq.(18)). Besides, the Goldstone bosons (18) necessary contain the highest flavor.

All these facts permit us to state our final conclusion: the observed form of the \( V_{CKM} \) mixing matrix may evidence that quark generations are distinguished due to new chiral vector interactions (neutral in flavors), very likely without the scalar Yukawa couplings. The new interaction could be gauge one, its scale \( M \) is higher or much higher than 1 TeV.
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FIGURE CAPTIONS

**Fig.1.** The gap equation in the leading $N_c$ approximation.

**Fig.2.** The self-energy and gap equations in the next after leading $N_c$ approximation.

**Fig.3.** The $q\bar{q}$ scattering amplitude in the leading $N_c$ approximation.

**Fig.4.** The diagram for $R - L$ interactions through vector exchanges (perturbative $g_i(L)$, $\tilde{g}_k(R)$ and unperturbative $g$). The hatched area symbolizes the necessary strong coupling independent of flavors.
\[ \Sigma_{ii} = \sum_{i} \mathbf{R}_{ii} \]

Fig. 1

\[ \Sigma_{ij}(\rho) = \sum_{i} \mathbf{R}_{ij}(\rho) \]

Fig. 2

\[ \mathbf{R}_{ij} = \sum_{i} \mathbf{R}_{ij}(\rho) \]

Fig. 3

Fig. 4