COSMOLOGICAL INFLATION,
MICROWAVE BACKGROUND ANISOTROPIES AND
LARGE SCALE STRUCTURE OF THE UNIVERSE

D. S. Salopek

Department of Physics, University of Alberta, Edmonton, Canada T6G 2J1

Abstract

Cosmological inflation provides the simplest and most promising mechanism for generating fluctuations for structure formation. Using powerful Hamilton-Jacobi methods, I will describe (1) how to compute density fluctuations and cosmic microwave anisotropies arising from inflation, and (2) improvements of the Zel’dovich approximation describing gravitational collapse. I compare these results with the latest cosmological observations.

Keywords: cosmology / inflation / large scale structure of the Universe / microwave background anisotropies / semiclassical gravity

in Proceedings of the
International School of Astrophysics “D. Chalonge”
Third Course: Current Topics in Astrofundamental Physics
Erice, Italy, September 4-16, 1994
Edited by N. Sanchez (Kluwer Academic Publishers)
I. INTRODUCTION AND REVIEW

The field of cosmology remains one of the most exciting areas in physics. A new branch of astronomy was recently opened with the detection of the cosmic microwave background anisotropy in 1992 by the Cosmic Background Explorer (COBE) satellite [1]. Of equal importance to astronomy is the tremendous growth in (1) galaxy redshift surveys and (2) measurements of intermediate-angle microwave anisotropies. Although one cannot predict the precise outcome, the impact in theoretical cosmology should be quite pronounced.

The inflationary scenario is perhaps the most promising mechanism for explaining microwave background fluctuations as well as fluctuations for galaxy formation. Before discussing theoretical strategies, I will first review the observational clues that suggest that we are on the right track.

A. Observational Clues Supporting Inflation

1. Critical Density of the Universe

A galaxy survey compiled by the Infrared Astronomical Satellite (IRAS) can be employed in conjunction with various redshift surveys to compute the mean density of the Universe. One finds [2]

$$\Omega^{0.6}/b_{IRAS} = 1.0 \pm 0.2$$

(1.1)

where $\Omega \equiv \rho/\rho_{\text{crit}}$ and $b_{IRAS}$ is the biasing parameter for IRAS galaxies. Provided that $b_{IRAS} \sim 1$ (which is quite reasonable to assume), it is consistent to assume that the Universe is at critical density which is the strongest prediction of inflation.

2. Large Angle Microwave Anisotropies

Using statistical arguments in 1992, the COBE team announced that they had detected a microwave anisotropy using one year of DMR (Differential Microwave Radiometer) data
Soon after, an MIT group confirmed their results [3]. The Tenerife experiment was actually the first to measure the anisotropy in one patch of the sky [1].

With two years of data, the COBE team determined that the spectral index for scalar perturbations, which describes the slope of the power spectrum, was $n = 1.17 \pm 0.31$ [3]. The simplest inflation models give $n < 1$. In this limited sense inflation is consistent with COBE.

3. Hubble Parameter?

Is the present value of the Hubble parameter $H_0$ consistent with inflation? Unfortunately, this is still an open question. In order for the age, $t_0 = 2/(3H_0)$, of an Einstein-deSitter universe to be larger than that of the oldest stars, one requires that $H_0 \leq 50$ km s$^{-1}$Mpc$^{-1}$. At this conference, the Hubble parameter controversy was reviewed by Tammann [1]. Personally, I prefer a new technique utilizing the Sunyaev-Zel’dovich (SZ) effect which describes how hot gas within a cluster of galaxies produces a temperature deficit $\Delta T_{SZ}$ in the cosmic microwave background. The SZ effect has been observed by Birkinshaw et al [7] and by a group at Cavendish in Cambridge [4]. Throughout this article, I will assume that $H_0 = 50$ km s$^{-1}$Mpc$^{-1}$, which is consistent with measurements of the SZ effect.

4. Structure Formation?

Is the observed pattern of galaxies consistent with inflation? Within an order of magnitude, the answer is yes. However, much attention has been given to the fact that the observations deviate from the standard cold-dark-matter (CDM) model [8] with an $n = 1$ spectrum by a factor of about three over about a decade in wavenumber. My own view is that the discrepancy is not very serious and that higher order nonlinear corrections, slight variations in the models, increases in the amount of data, etc., will resolve this issue. I believe that most of the basic ingredients have been correctly identified and that wholesale
modifications such as abandoning inflation, invoking defect models, etc., are probably not necessary.

For example, there are many inflationary models, and one now wishes to resolve which is the correct one. Here I will focus on one class of models which is particularly easy to deal with: power-law inflation. These models yield substantial amounts of primordial gravitational waves and they also give rise to more power at large scales. They alleviate the problems of large scale structure.

As the data for galaxy clustering and microwave background anisotropies (both intermediate and large angle) continue to improve over the next few years, we will be better able to identify the correct inflationary model. In this way, it is possible to probe the earliest epochs of the Universe.

B. Theoretical Strategy

Faced a with an increasing number of observations, it proves extremely useful to have a theoretical formalism that encompasses virtually all aspects of cosmology. In this way, theory attempts to put some order in what otherwise looks like chaos. I strongly advocate using the Hamilton-Jacobi formalism for general relativity. The reasons are quite numerous:

1. **Inflation requires a Quantum Theory of Gravity.** However, the full theory does not exist. Since it is generally believed that tensor fluctuations arising from inflation began initially in the ground state, it is absolutely essential to quantize the gravitational field. At the present, one should perhaps be content with a semi-classical approximation where one approximates the wavefunctional by

\[ \Psi \sim e^{iS/\hbar}, \]  

(1.2)

where \( \hbar \) is Planck’s constant. The phase factor \( S \) is often referred to as the generating functional. It satisfies the Hamilton-Jacobi equation for general relativity. If \( S \) is real,
then this approximation would be entirely classical. However, by allowing $S$ to be complex, one can indeed describe quantum phenomena, including fluctuations beginning in the ground state.

(2) Solving the Constraint Equations of General Relativity. For example, the momentum constraint associated with the $G^0_i$ Einstein equation is rather trivial to satisfy, since it legislates that the generating functional be invariant under reparametrizations of the spatial coordinates.

(3) Covariant Description of the Gravitational Field. Using HJ theory, one can perform hypersurface-invariant computations. In this sense, one can resolve the problem of time: time is arbitrary. Moreover, the HJ formalism includes almost everything that you wanted to know about gauge-invariant variables but were afraid to ask. For an alternative approach describing covariant perturbations of general relativity, see ref. \[15\].

(4) Finding Exact Solutions. HJ theory is useful in deriving exact solutions for cosmological situations of physical interest: during inflation, during the radiation/matter dominated epochs, etc. Here I will review exact solutions of the long-wavelength equations and the perturbation equations for power-law inflation.

(5) Applying the Zel’dovich Approximation to General Relativity.

HJ methods allow one to discuss the Zel’dovich approximation \[16\] in a relativistic framework \[17\], \[12\], \[18\]. The Zel’dovich approximation describes the formation of sheet-like structures which appear to be quite common in our Universe.

In Sec. 2, I will describe the most useful data sets available to cosmologists. Hamilton-Jacobi theory will be reviewed in Sec. 3. Several hypersurface-invariant approximation schemes will be discussed including the spatial gradient expansion and the quadratic curvature approximation. In Sec. 4, I discuss the phenomenological consequences of various inflation models. Applications of HJ theory to the Zel’dovich approximation are given in Sec. 5. Attempts to go beyond the semi-classical approximation using one-loop calculations will be summarized in Sec. 6. Finally, I conclude in Sec. 7.

(Unless otherwise stated, units will be chosen so that $c = k = 8\pi G = 8\pi / m_p^2 = \hbar = 1$.)

5
II. DESCRIPTION OF COSMOLOGICAL DATA

For a quantitative analysis, the two best cosmological data sets are: (1) the large angle (> 30\degree) microwave anisotropies determined by COBE and (2) galaxy correlation functions. A third data set, (3) microwave background measurements at intermediate angles, shows wide variations, and it presently does not set useful quantitative limits.

A. Large Angle Microwave Anisotropy

In Fig.(1), I show the correlation function in the temperature

\[ C(\alpha) = \langle \Delta T(x) \Delta T(x') \rangle \]  \hspace{1cm} (2.1)

with the monopole and dipole removed. It is computed using the two-year COBE data set. For angles less than 60\degree, the error bars are much smaller than than the values of measurements, and one is impressed by the precision of the experiment. However when one factors out all the enthusiasm and publicity, there are basically two numbers that follow from this graph: the amplitude and the slope.

The slope is often given in terms of the spectral index, which they determine to be

\[ n = 1.17 \pm 0.31 \] \cite{5}. In their initial analysis, they included the quadrupole component after subtracting the contribution of the Milky Way. Since this subtraction is quite tricky and since cosmic variance for the quadrupole is large, one may wish to repeat the calculations without including the quadrupole, in which case one finds \[ n = 0.96 \pm 0.36 \]. In either case, their results provide a test of inflation, and they are consistent with the simplest models of inflation which yield \( n < 1 \).

The amplitude is given in terms of the temperature anisotropy

\[ \sigma_{sky}(10^0) = 30.5 \pm 2.7 \mu K \] \hspace{1cm} (2.2)

for a Gaussian beam of 10\degree FWHM (full-width-at-half-maximum). This value does not differ much from the first-year results, but the 10% error bars represent an improvement by
a factor of two. When one fits various inflations, I will disregard COBE’s determination of the spectral index, and use only $\sigma_{sky}(10^0)$ since it has the smallest error limits.

**B. Large Scale Structure of the Universe**

Over the past five years, there has been tremendous improvement in the quantity and quality of galaxy clustering data. Frenk [20] has given an excellent review of the situation at this conference. Here I will touch on the main points.

Recently, Peacock and Dodds [2] have compiled the most useful galaxy data in a user-friendly form. They consider eight data sets, including: one angular survey, (1) the APM (Automatic Plate Machine) and seven redshift surveys (2) Abell clusters, (3) Radio galaxies, (4) Abell vs. IRAS (Infrared galaxies), (5) CfA (Center for Astrophysics), (6) APM/Stromlo, (7) Radio vs. IRAS and (8) IRAS. Corrections are made for nonlinear evolution and for redshift space distortions. In addition, before they merge the data, they rescale each set using a biasing parameter $b_i, i = 1, ..., 8$. A redshift space correction implies that the IRAS biasing parameter is constrained by eq.(1.1). If we accept the inflationary model with $\Omega = 1$, we see that it is consistent to assume that the biasing parameter $b_{IRAS}$ for IRAS galaxies is unity. Hence, from now on we will assume that that IRAS galaxies trace the mass: $b_{IRAS} = 1$. I wish to emphasize that the biasing parameter is no longer a free parameter.

Using all data sets, they determine the linear power spectrum

$$P_\rho(k) \equiv \frac{k^3}{2\pi^2} \int d^3 x e^{-ik\cdot x} \left< \frac{\delta \rho(x) \delta \rho(0)}{\rho} \right>$$

for the density perturbation $\delta \rho$ at the present epoch. It is $k^3/(2\pi^2)$ multiplied by the Fourier transform of the two-point correlation function in the density. It is plotted in Fig.(2). The precision is quite remarkable, considering how much scatter there was in each of the individual data sets. For comparison, the 1.8-power-law usually associated with optical galaxies is also shown. At smaller scales where the 1.8-power-law is valid, optical galaxies are definitely biased tracers.
1. Intermediate Angle Anisotropies

Wright et al [21] have compiled a list of microwave background anisotropy measurements. Their results are plotted in Fig.(3). The solid line depicts the standard prediction of $n = 1$ CDM with no gravitational wave contribution. Unfortunately, there is much scatter in the measurements for $\ell > 20$. In addition, one suspects that there is contamination from foreground objects.

Although this is a promising line of research (for a review, see ref. [22]), the present quality of intermediate-angle data does not lead to any firm quantitative conclusions. For this reason, when considering fits of inflationary models, I will utilize only the (1) large angle microwave normalization and (2) galaxy clustering data.

III. HAMILTON-JACOBI THEORY

Theoretical computations of various inflation models can be described in very elegant terms using Hamilton-Jacobi methods. Some very deep theoretical issues can be addressed. Previously, there had been some confusion in the choice of gauge. For example, what should one choose for the time hypersurface? Other problems include choosing the initial wavefunctional.

A. Resolving the Question of Time for Semi-Classical Gravity

Before getting into intricacies of HJ theory, I would like to demonstrate how it is possible to illuminate the role of time in semi-classical gravity. A simple analogy from potential theory illustrates the basic point which is easy to understand.

1. Potential Theory

The fundamental problem in potential theory is: given a force field $g^i(u_k)$ which is a function of $n$ variables $u_k$, what is the potential $\Phi \equiv \Phi(u_k)$ (if it exists) whose gradient
returns the force field,

$$\frac{\partial \Phi}{\partial u_i} = g_i^j (u_k) \quad (3.1)$$

Not all force fields are derivable from a potential. Provided that the force field satisfies the integrability relation,

$$0 = \frac{\partial g^i}{\partial u_j} - \frac{\partial g^j}{\partial u_i} = \left[ \frac{\partial}{\partial u_j}, \frac{\partial}{\partial u_i} \right] \Phi, \quad (3.2)$$

(i.e., it is curl-free), one may find a solution which is conveniently expressed using a line-integral

$$\Phi(u_k) = \int_C \sum_j dv_j \ g^j(v_l). \quad (3.3)$$

If the two endpoints are fixed, all contours return the same answer. In practice, I will employ the simplest contour that one can imagine: a line connecting the origin to the observation point $u_k$. Using $s$, $0 \leq s \leq 1$, to parameterize the contour, the line-integral may be rewritten as

$$\Phi(u_k) = \sum_{j=1}^n \int_0^1 ds \ u_j \ g^j(su_k). \quad (3.4)$$

Infinite dimensional line integrals appear in solutions of the Hamilton-Jacobi equation for general relativity. Basically, each contour corresponds to a specific choice of time-hypersurface. They all yield the same answer provided certain consistency relations analogous to eq. (3.2) are met.

### B. Hamilton-Jacobi Equation for General Relativity

The Hamilton-Jacobi equation for general relativity is derived using a Hamiltonian formulation of gravity. One first writes the line element using the ADM 3+1 split,

$$ds^2 = \left( -N^2 + \gamma^{ij} N_i N_j \right) dt^2 + 2N_i dt \ dx^i + \gamma_{ij} dx^i \ dx^j, \quad (3.5)$$
where $N$ and $N_i$ are the lapse and shift functions, respectively, and $\gamma_{ij}$ is the 3-metric. Hilbert’s action for gravity interacting with a scalar field becomes

$$I = \int d^4x \left( \pi^\phi \dot{\phi} + \pi^j \dot{\gamma}_{ij} - N\mathcal{H} - N^i \mathcal{H}_i \right). \quad (3.6)$$

The lapse and shift functions are Lagrange multipliers that ensure that the energy constraint $\mathcal{H}(x)$ and the momentum constraint $\mathcal{H}_i(x)$ vanish.

The object of chief importance is the generating functional

$$S \equiv S[\gamma_{ij}(x), \phi(x)]. \quad (3.7)$$

For each universe with field configuration $[\gamma_{ij}(x), \phi(x)]$ it assigns a number which can be complex. The generating functional is the ‘phase’ of the wavefunctional in the semi-classical approximation, eq.(1.2). For the applications that we are considering, the prefactor before the exponential is not important, although it has interesting consequences for quantum cosmology [23]. The probability $\mathcal{P}$ of finding a field configuration is given by the square of the wavefunctional:

$$\mathcal{P} \equiv |\Psi|^2. \quad (3.8)$$

Replacing the conjugate momenta by functional derivatives of $S$ with respect to the fields,

$$\pi^{ij}(x) = \frac{\delta S}{\delta \gamma_{ij}(x)}, \quad \pi^\phi(x) = \frac{\delta S}{\delta \phi(x)}, \quad (3.9)$$

and substituting into the energy constraint equation, one obtains the Hamilton-Jacobi equation,

$$\mathcal{H}(x) = \gamma^{-1/2} \frac{\delta S}{\delta \gamma_{ij}(x)} \frac{\delta S}{\delta \gamma_{kl}(x)} [2\gamma_{il}(x)\gamma_{jk}(x) - \gamma_{ij}(x)\gamma_{kl}(x)]$$

$$+ \frac{1}{2} \gamma^{-1} \left( \frac{\delta S}{\delta \phi(x)} \right)^2 + \gamma^{1/2} V(\phi(x))$$

$$- \frac{1}{2} \gamma^{1/2} R + \frac{1}{2} \gamma^{1/2} \gamma^{ij} \phi_{,i} \phi_{,j} = 0, \quad (3.10)$$
which describes how $S$ evolves in superspace. $R$ is the Ricci scalar associated with the 3-metric, and $V(\phi)$ is the scalar field potential. In addition, one must also satisfy the momentum constraint

$$\mathcal{H}_i(x) = -2 \left( \gamma_{ik} \frac{\delta S}{\delta \gamma_{kj}(x)} \right)_{,j} + \frac{\delta S}{\delta \gamma_{ik}(x)} \gamma_{ik,i} + \frac{\delta S}{\delta \phi(x)} \phi_{,i} = 0 ,$$  \hspace{1cm} (3.11)

which legislates that $S$ be invariant under reparametrizations of the spatial coordinates. Since neither the lapse function nor the shift function appears in eqs. (3.10,3.11) the temporal and spatial coordinates are arbitrary. HJ theory is covariant.

C. Spatial Gradient Expansion

It appears initially that finding solutions to the HJ equation will be a hopeless task since one is essentially describing an ensemble of evolving universes. However, it is not very difficult to obtain approximate solutions. In the first attempt to solve eq.(3.10), I will expand the generating functional

$$S = S^{(0)} + S^{(2)} + S^{(4)} + \ldots ,$$  \hspace{1cm} (3.12)

in a series of terms according to the number of spatial gradients that they contain. As a result, the Hamilton-Jacobi equation can likewise be grouped into terms with an even number of spatial derivatives:

$$\mathcal{H} = \mathcal{H}^{(0)} + \mathcal{H}^{(2)} + \mathcal{H}^{(4)} + \ldots .$$  \hspace{1cm} (3.13)

The invariance of the generating functional under spatial coordinate transformations (see eq.(3.11)) suggests a solution of the form,

$$S^{(0)}[\gamma_{ij}(x), \phi(x)] = -2 \int d^3 x \gamma^{1/2} H(\phi(x)) ,$$  \hspace{1cm} (3.14)

for the zeroth order term $S^{(0)}$. The function $H \equiv H(\phi)$ satisfies the HJ equation of order zero,
\[ H^2 = \frac{2}{3} \left( \frac{\partial H}{\partial \phi} \right)^2 + \frac{V(\phi)}{3}, \] (3.15)

which is a nonlinear ordinary differential equation. Note that \( S^{(0)} \) contains no spatial gradients. It describes the evolution of long-wavelength fields—these are inhomogeneous field configurations where the spatial gradients of the inhomogeneities are so small that they do not alter the evolution significantly from that of a homogeneous model. In fact, long-wavelength fields are an essential feature of all structure formation scenarios arising in cosmology.

We will examine in detail the special case of inflation with an exponential potential [24]

\[ V(\phi) = V_0 \exp \left( -\sqrt{\frac{2}{p}} \phi \right), \] (3.16)

which has the exact solution

\[ H(\phi) = \left[ \frac{V_0}{3 (1 - 1/(3p))} \right]^{1/2} \exp \left( -\frac{\phi}{\sqrt{2p}} \right). \] (single scalar field) (3.17)

It yields power-law inflation where the scale factor evolves according to \( a(t) \propto t^p \).

In order to compute higher order terms in the spatial gradient expansion, one introduces a change of variables, \( (\gamma_{ij}, \phi) \rightarrow (f_{ij}, u) : \)

\[ u = \int \frac{d\phi}{-2 \frac{\partial H}{\partial \phi}}, \quad f_{ij} = \Omega^{-2}(u) \gamma_{ij}, \] (3.18)

where the conformal factor \( \Omega \equiv \Omega(u) \) is defined through

\[ \frac{d \ln \Omega}{du} \equiv -2 \frac{\partial H}{\partial \phi} \frac{\partial \ln \Omega}{\partial \phi} = H. \] (3.19)

in which case the equation for \( S^{(2m)} \) becomes

\[ \left. \frac{\delta S^{(2m)}}{\delta u(x)} \right|_{f_{ij}} + \mathcal{R}^{(2m)}[u(x), f_{ij}(x)] = 0. \] (3.20)

The remainder term \( \mathcal{R}^{(2m)} \) depends on some quadratic combination of the previous order terms (it may be written explicitly). For example, for \( m = 1 \), it is

\[ \mathcal{R}^{(2)} = \frac{1}{2} \gamma^{ij} \gamma_{ij} \phi, i \phi, j - \frac{1}{2} \gamma^{1/2} R. \] (3.21)
Eq. (3.20) has the form of an infinite dimensional gradient. It may be integrated using a line integral analogous to eq. (3.4):

\[ S^{(2m)} = - \int d^3x \int_0^1 ds \ u(x) \ \mathcal{R}^{(2m)}[su(x), f_{ij}(x)] ; \]  

(3.22)

the conformal 3-metric \( f_{ij}(x) \) is held constant during the integration which may be performed explicitly in many cases of interest. For example, at second order in spatial gradients, one finds that

\[ S^{(2)}[f_{ij}(x), u(x)] = \int d^3xf^{1/2} \left[ j(u)\tilde{R} + k(u) f^{ij}u_{,i}u_{,j} \right] . \]  

(3.23)

Here \( \tilde{R} \) is the Ricci curvature of the conformal 3-metric \( f_{ij} \), eq. (3.18); the \( u \)-dependent coefficients \( j \) and \( k \) are

\[ j(u) = \int_0^u \frac{\Omega(u')}{2} \ du', \quad k(u) = H(u) \ \Omega(u) , \]  

(3.24)

where \( F \) is an arbitrary constant. Provided spatial gauge invariance is maintained, all contours for the line integral give the same result, eq. (3.23). \( S^{(2)} \) is useful in deriving the Zel’dovich approximation from general relativity (see Sec. 5).

**D. Quadratic Curvature Approximation**

In the context of inflation, a finite number of terms in the spatial gradient expansion is insufficient. As a result, John Stewart and I have developed a method where we effectively sum an infinite subset of terms. We make an Ansatz for the generating functional of the form

\[ S = S^{(0)} + S^{(2)} + \mathcal{Q} \]  

(3.25)

where the functional \( \mathcal{Q} \) is quadratic in the Ricci tensor of the 3 metric:

\[ \mathcal{Q} = \int d^3xf^{1/2} \left[ \tilde{R} \ \tilde{S}(u, \tilde{D}^2) \ \tilde{R} + \tilde{R}^{ij} \ \tilde{T}(u, \tilde{D}^2) \ \tilde{R}_{ij} - \frac{3}{8} \tilde{R} \ \tilde{T}(u, \tilde{D}^2) \ \tilde{R} \right] . \]  

(3.26)
Here $\tilde{S}(u, \tilde{D}^2)$ and $\tilde{T}(u, \tilde{D}^2)$ are the scalar and tensor operators which are functions of $u$ and $\tilde{D}^2$. By using a convenient line-integral in superspace, one may rewrite the HJ equation in its integral form, which is useful because it enables one to safely integrate certain terms by parts. In substituting the Ansatz, one collects all quadratic terms in the Ricci curvature, one neglects higher order terms, and finally one obtains the nonlinear Riccati equations:

$$0 = \frac{\partial \tilde{S}}{\partial u} + \frac{1}{8\Omega^3} \left( \frac{\partial H}{\partial \phi} \right)^2 \left( \frac{\Omega}{2} - Hj + 8H \tilde{S} \tilde{D}^2 \right)^2,$$

(3.27)

$$0 = \frac{\partial \tilde{T}}{\partial u} + \frac{2}{\Omega^3} \left( j + \tilde{T} \tilde{D}^2 \right)^2.$$

(3.28)

By making a Riccati transformation, $(S, T) \rightarrow (w, y)$, these may be reduced to linear equations for $w$ and $y$:

$$0 = \frac{\partial^2 w}{\partial u^2} + \left( 3H(u) + 2 \frac{\partial}{\partial u} \left[ \ln \left( \frac{1}{H} \frac{\partial H}{\partial \phi} \right) \right] \right) \frac{\partial w}{\partial u} - \Omega^{-2}(u) \tilde{D}^2 w,$$

(3.29)

(scalar perturbations)

(3.30)

$$0 = \frac{\partial^2 y}{\partial u^2} + 3H(u) \frac{\partial y}{\partial u} - \Omega^{-2}(u) \tilde{D}^2 y.$$  

(tensor perturbations)

(3.31)

(3.32)

For power-law inflation, the solution to both of these equations may be expressed in terms of Hankel functions. At early times, far within the Hubble radius, the solution is chosen to be a positive frequency mode which describes the ground state (Bunch-Davies vacuum [25]). More details are given in ref. [10].

**IV. PHENOMENOLOGICAL CONSEQUENCES**

Before one can compare theoretical models with galaxy data, one must make an assumption for the form of the dark matter. Here I will consider the simplest cold-dark-matter model [8]. I will assume a baryon fraction, $\Omega_B = 0.03$, consistent with nucleosynthesis. In addition, it is convenient to display the results in Fourier space where $k$ is the comoving wavenumber.
In Fig.(4), I display several power spectra for scalar fluctuations,  
\[ P_\zeta(k) = P_\zeta(k_0) \left( \frac{k}{k_0} \right)^{n-1}, \]  
which may give rise to structure formation. This form arises from power-law inflation model where the spectral index for scalar perturbations is given by  
\[ n = 1 - 2/(p - 1). \]  
I have normalized the spectra using COBE’s 2-year data set, eq.(2.2). If there were no gravitational wave contribution to COBE’s signal (which is indeed true for \( n=1 \)), all the lines would join at \( k_0 = 10^{-4}\text{Mpc}^{-1} \), the comoving scale effectively probed by COBE. Actually, as \( n \) decreases, the gravitational wave contribution increases, leaving less fluctuations available for the scalar perturbations: \( P_\zeta(k_0) \) decreases as \( n \) decreases.

The variable \( \zeta \) has proven to be very useful since it is independent of time provided the wavelength of a mode exceeds the Hubble radius. This is true during the inflationary epoch, during heating of the Universe and even during the radiation and matter dominated \( [26] \), provided the wavelength of the fluctuations exceeds the Hubble radius, \( k/(Ha) \ll 1 \). In the matter-dominated era, Bardeen’s gauge-invariant variable, \( \Phi_H \), and the density perturbation, \( \delta \rho/\rho \) are related to \( \zeta \) through  
\[ \Phi_H(k) = T(k) \zeta(k)/5, \]  
\[ \delta \rho(k)/\rho = \frac{k^2 \tau^2}{30} T(k) \zeta(k), \]  
where \( T(k) \) is the cold-dark-matter transfer function and \( \tau \) is conformal time \( \tau = \int dt/a(t) \).

Power spectra for the linear density perturbation are shown in Fig.(5). One sees that cold-dark-matter with \( n = 1 \) is not consistent with the observed data for \( k > 10^{-1.6}\text{Mpc}^{-1} \) (short distances), and that the discrepancy is of the order of a factor of three.

It is useful to note that the model with \( n = 0.8 \) yields a 50% gravitational wave contribution to \( \sigma_{sky}^2(10^0) \). Looking at Fig.(5), we see that it too is ruled out. For this reason,
we can state: for power-law inflation, at most 50% of COBE’s signal can be the result of gravitational waves.

For $n = 0.9$, gravitational waves contribute 35% to COBE’s signal. This model yields an improved fit, although it is not quite perfect either since it underestimates the data by a small factor (less than two) for $k < 10^{-1.6} \text{ Mpc}^{-1}$ (large distances). This discrepancy is the central problem in large scale structure. However, I do not feel that it is very severe for $n = 0.9$, and I will wait for more data before suggesting radical alterations to the cosmology. In fact, one can further alleviate the problem by adopting a smaller value of the Hubble parameter. A more thorough discussion is given in ref. [27].

**A. Discussion with Grishchuk**

In this conference, Grishchuk [28] has stated that he believes that the tensor fluctuations arising from virtually all inflation models will dominate the contribution to the microwave background anisotropy observed by COBE. Unfortunately, I disagree with this claim. I feel that he has not adequately treated heating of the Universe after inflation. In my Ph.D. thesis (see ref. [26]), I computed the evolution of the metric fluctuations during the transition from the inflationary epoch to the radiation-dominated era for a phenomenological model. Although the maximum temperature of the radiation varies sensitively on the damping coefficient $\Gamma$, there was basically no effect on the scalar perturbations: $\zeta$ was constant in time. More precisely, on a comoving slice where the matter is at rest, the fluctuations in the metric, (both scalar and tensor) were constant. These metric fluctuations were essentially dormant whenever the physical wavelength of a given mode exceeded the Hubble radius. The anisotropies we observe now in large angle microwave experiments have basically survived intact since the modes left the Hubble radius during inflation.
V. APPLYING THE ZEL’DOVICH APPROXIMATION TO GENERAL RELATIVITY

In a Newtonian framework, Zel’dovich [16] showed how to extrapolate the results of cosmological perturbation theory for a matter-dominated Universe to the quasilinear regime. Recently, we (Croudace, Parry, Salopek and Stewart — hereafter CPSS [17]) have been able to derive the Zel’dovich approximation and its higher order corrections using general relativity. The simplicity of our results should prove to be a useful tool in cosmology. Here, I will review how to use Hamilton-Jacobi methods in solving Einstein’s equations. We hope to apply our techniques to the interpretation of redshift surveys. An alternative approach to incorporating the Zel’dovich approximation within general relativity has been suggested by Matarrese et al [29]; see also ref. [15].

A. Gradient Expansion of 3-metric

We assume from the beginning that cold-dark-matter is described by collisionless, dust-like particles whose 4-velocity, $U^\mu$, is given by the 4-gradient of the potential $\chi$:

$$ U^\mu = -\chi^{,\mu}. $$

(5.1)

Throughout, we will observe the line-element on a time-hypersurface where $\chi$ is uniform:

$$ ds^2 = -d\chi^2 + \gamma_{ij}(\chi, q)dq^i dq^j. $$

(5.2)

This choice is referred to as comoving, synchronous gauge, in which case the 3-metric $\gamma_{ij}$ has physical significance: it gives the physical distance, $ds = (\gamma_{ij}dq^i dq^j)^{1/2}$, between two comoving observers separated by an infinitesimal comoving distance $dq^i$. The evolution equation for the 3-metric then becomes

$$ \frac{\partial \gamma_{ij}}{\partial \chi} = 2\gamma^{-1/2} \frac{\delta S}{\delta \gamma_{kl}(x)} \left( 2\gamma_{ik}\gamma_{jl} - \gamma_{ij}\gamma_{kl} \right), $$

(5.3)

where the generating functional $S$ is given by an expression analogous to eq.(3.12).
During the matter-dominated era, the peculiar velocity of the matter is typically small compared to the speed of light. Hence we are justified in assuming that all spatial gradients are small compared to unity,

\[ R \ll 1 \quad \text{(5.4)} \]

where \( R \) is the Ricci curvature of the 3-metric. In fact, if we retain only the long-wavelength piece, \( S^{(0)} \) (analogous to eq.(3.14)), in the evolution equation (5.3), we determine that the 3-metric evolves according to

\[ \gamma^{(1)}_{ij}(\chi, q) = a^2(\chi) k_{ij}(q), \quad \text{where} \quad a(\chi) \equiv \chi^{2/3} \quad \text{(5.5)} \]

which is accurate to first order in spatial gradients. The ‘seed metric’ \( k_{ij}(q) \) is an arbitrary function that is independent of time; it describes the initial fluctuations whose wavelength is larger than the Hubble radius. This result is very old — it was known to Lifshitz and Khalatnikov [30] (see also Tomita [31]).

One may solve for higher order terms by adopting an iterative method. For example, accurate to third order in spatial gradients, we use eq.(3.23) to find

\[ \gamma^{(3)}_{ij}(\chi, q) = a^2(\chi) \left[ k_{ij}(q) + \frac{9}{20} a(\chi) \left( \hat{R}(q) k_{ij}(q) - 4 \hat{R}_{ij}(q) \right) \right] \quad \text{(3rd order)} \quad \text{(5.6)} \]

where \( \hat{R}_{ij} \) is the Ricci tensor of the seed metric \( k_{ij} \).

Unfortunately, after a sufficient amount of time, the third order expression, eq.(5.6), leads to nonsensical results: the determinant of the 3-metric can actually become negative. A similar problem occurs when one naively approximates a non-negative function \( \cos^2(x) \sim 1 - x^2 \) with the first two terms of a Taylor expansion — the approximate function falls below zero when \( x > 1 \). One can easily remedy this problem by expanding \( \cos(x) \sim 1 - x^2/2 \), and then approximating \( \cos^2(x) \) by the square of this result — this technique guarantees a positive result. In an analogous way, we can improve the expansions eqs.(5,6) by expressing the results as a ‘square.’ The improved 3-third order result

\[ \tilde{\gamma}^{(3)}_{ij}(\chi, q) = a^2(\chi) \left\{ k_{il} + \frac{9}{40} a(\chi) \left[ \hat{R} k_{il} - 4 \hat{R}_{il} \right] \right\} k^{lm} \quad \text{(5.7)} \]

\[ \left\{ k_{jm} + \frac{9}{40} a(\chi) \left[ \hat{R} k_{jm} - 4 \hat{R}_{jm} \right] \right\} \quad \text{(5.8)} \]
yields the relativistic generalization of the Zel’dovich approximation.

In Fig.(6), we compare our approximations with an exact Szekeres solution with azimuthal symmetry [32]. It is remarkable that the improved 3rd order expansion yields the exact result. Higher order corrections to the Zel’dovich approximation are discussed in refs. [17], [18], [12].

VI. ONE-LOOP APPROXIMATION APPLIED TO INFLATION

This field is still in its infancy, but several notable steps have already been taken. I will focus my attention on curvature-coupled models, but I will also mention other avenues that are being pursued.

One of the main problems with inflation is that there are too many models. Some of these are very heavily constrained by observations, but can theoretical arguments be used to restrict the models further?

In the standard model of particle physics with energies below 1 TeV, renormalizability arguments proved to very effective in limiting the class of permissible Lagrangians. For example, in the Higgs sector, one considers only those scalar field potential which were polynomials of degree 4 or less, otherwise the theory is not renormalizable. In an analogous way, it would be very useful to have a renormalizable theory of inflation. However, this is quite a daunting task because this would require a quantum theory of the gravitational field! Many great minds have been wrecked on these treacherous shores, so we will have to proceed cautiously.

I would say that our present understanding of inflationary theory is comparable to that of weak interactions when Fermi introduced his 4-fermion theory in the 1930’s. Fermi’s theory was an outstanding example of phenomenological work, but only thirty years later in the late 1960’s was a renormalizable theory of the weak interactions formulated. For the same reason, I suspect that it may be several decades before we obtain a satisfactory quantum formulation of the gravitational field, whether it be a renormalizable theory or even a finite
theory as suggested by superstrings (see, e.g., ref. [33], [34]).

HJ theory, which is accurate to lowest order in Planck’s constant \( (\hbar^0) \) has proven to be a very effective, but can we expand to higher order in \( \hbar \)? For instance, expanding the wavefunctional to next order in \( \hbar \) corresponds to the one-loop approximation. Some partial progress has been made in this area provided one considers only matter fields running in the loops.

### A. Economical Formulation of Cosmological Inflation

It is possible to construct an economical model of cosmological inflation where the GUT Higgs is identified with the inflaton which is also the Brans-Dicke scalar. One begins with the action for the 4-metric \( g_{\mu \nu} \) interacting with a scalar field \( \phi \):

\[
I = \int \sqrt{-g} \left\{ \left[ \frac{m^2}{16\pi} - \frac{1}{2} \xi \phi^2 \right] (4)R - \frac{1}{2} g^{\mu \nu} \phi,_{\mu} \phi,_{\nu} - \frac{\lambda}{4} (\phi^2 - \sigma^2)^2 \right\}. \tag{6.1}
\]

The \(-\xi (4)R \phi^2 / 2\) term must appear in order to cancel infinities when one considers a \( \lambda \phi^4 / 4\) self-interaction in curved space-time. This theory is relatively easy to understand. The coefficient of the Ricci 4-curvature \( (4)R \) can be interpreted in terms of the effective value of the Planck mass

\[
G_{\text{eff}}^{-1}(\phi) \equiv m_{\text{P, eff}}^2(\phi) \equiv m^2 - 8\pi \xi \phi^2. \tag{6.2}
\]

Here \( m^2 \) is the bare contribution to \( m_{\text{P, eff}}^2 \) whereas \(-8\pi \xi \phi^2\) is the Higgs contribution. In order that the Universe inflate sufficiently, one requires that [26]

\[
\xi < 0.002 \quad \text{if} \quad m \sim m_{\text{P}}. \tag{6.3}
\]

Although this is a rather crude constraint, it is nonetheless very vexing for chaotic inflation. If this constraint is violated, Newton’s constant \( G_{\text{eff}} \) can become negative. Classically, it would have to remain negative because it must cross a singularity in order to reach positive values.
1. Variable Planck Mass Model and Induced Gravity

The simplest way to satisfy eq.(6.3) is to take $\xi$ to be negative. Then there are essentially two cases to consider. In Induced Gravity, one legislates that the bare value $m$ vanish, and that the present value of the Planck scale is determined when the scalar field rolls to its minimum $\sigma$ in the potential. This is a particularly attractive model requiring the fewest number of parameters, although it may not produce a radiation-dominated Universe after inflation (which would be a disaster). In the second case which is adopted here one assumes that $m \sim m_P$, and that the present value of the Planck scale obtains only a small contribution from $\phi$ at the minimum of its potential, although it was much much larger during the inflationary era when $\phi >> \sigma$. This situation is called the Variable Planck Mass Model.

Since the gravitational sector of eq.(6.1) is a bit messy, it proves very convenient to employ a conformal transformation of the metric

$$g_{\mu\nu} = \Omega^2(\phi) \tilde{g}_{\mu\nu}, \quad \Omega(\phi) = m_P / m_{\text{eff}}(\phi).$$

(6.4)

This transformation suggested by Brans and Dicke [35] is easy to interpret— it is just a change of units from a variable ruler $m_{\text{eff}}(\phi)$ to a uniform ruler $m_P$ (which is the current value of the Planck scale). Much of the physics that is discussed below is similar to changing units in the same way that one converts from feet to metres. The new Lagrangian density

$$\tilde{\mathcal{L}} = \frac{m_P^2}{16\pi} (\tilde{R} - \frac{1}{2} \tilde{g}^{\mu\nu} \chi_{,\mu} \chi_{,\nu} - V_{\text{eff}}(\phi(\chi)))$$

(6.5)

describes standard Einstein gravity for the new metric $\tilde{g}_{\mu\nu}$. The effective potential given by

$$V_{\text{eff}}(\phi) = \Omega^4(\phi) V(\phi)$$

(6.6)

is very flat as $|\phi| \to \infty$ which is very desirable for inflation. $\chi$ is a function of $\phi$, and it is given in ref. [26]. For induced gravity, its expression is quite simple

$$\phi = \sigma e^{\alpha x}, \quad V_{\text{eff}}(\chi) = \frac{\lambda}{4} \left( \frac{m_P^2}{8\pi|\xi|} \right)^2 (1 - e^{-2\alpha x})^2, \quad \alpha = \left( \frac{8\pi|\xi|}{1 + 6|\xi|} \right)^{1/2} m_P^{-1}.$$  

(6.7)
The effective potential for both models is plotted in Fig.(7).

The fluctuation spectrum for the Variable Planck Mass model is essentially the same as for induced gravity:

\[ P_\zeta(k) = \frac{1}{8\pi^2} \frac{\lambda}{\xi^2} N_I^2(k), \quad \text{where} \quad N_I(k) = 60 - \ln \left[ k/(10^{-4}\text{Mpc}^{-1}) \right]. \quad (6.8) \]

Normalizing according to the COBE observations, one obtains

\[ \frac{\lambda}{\xi^2} = 3.9 \times 10^{-10} \pm 18\% \quad b_\rho = 0.91 \pm 9\%. \quad (6.9) \]

Gravity waves contribute about 0.3% to \( \sigma_{sky}^2(10^0) \). If \( \phi \) is the GUT Higgs, then \( \lambda \sim 0.05 \) is consistent with radiative corrections and one can fit COBE if \( \xi \sim -10^4 \). The spectral index for scalar fluctuations is very close to unity, \( n = 0.967 \). In order to reconcile this result with galaxy data, one would have to alter the standard cold-dark-matter transfer function. For example, a mixed-dark-matter model \[36\] could be invoked to explain Fig.(2).

The main advantage of having large negative \( \xi \) is that one-loop radiative corrections do not destroy the required flatness in the effective potential \[28\]. For example, one can couple a gauge boson \( A_\mu \) through the minimal prescription replacing

\[ \partial_\mu \rightarrow \partial_\mu - i e A_\mu, \quad (6.10) \]

and by adding kinetic terms \( F_{\mu\nu} F^{\mu\nu} \) to the original action \[5,1\]. After the conformal transformation is performed, the gauge field \( A_\mu \) does not transform— it is conformally invariant. Its mass is given by the Higgs mechanism

\[ m_A(\phi) = e\phi \Omega(\phi) \quad (6.11) \]

which is scaled by the additional factor \( \Omega(\phi) \). If \( m \neq 0 \), then for small values of \( \phi \), one recovers the usual coupling expression, but for large values of \( \phi \), the gauge boson mass is independent of \( \phi \): the gauge boson decouples from the Higgs field. As a result the 1-loop radiative potential \[28\]

\[ V_{\text{rad}}(\phi) = \frac{3}{64\pi^2} m_A^4(\phi) \ln \left( \frac{m_A^2(\phi)}{\mu^2} \right) \quad (6.12) \]
remains very flat as $\phi \to \infty$ ($\mu$ is the renormalization scale.) The above argument works for fermions as well. By altering the gravitational sector, one has a robust method of producing a flat potential that proves favorable for the inflationary scenario.

For induced gravity with $m = 0$, the mass of the gauge boson $m_A = e m_P / \sqrt{8\pi |\xi|}$ is independent of the scalar field, and hence the scalar field does not couple directly to radiation. Hence, there may be a problem with producing a radiation-dominated era at the end of inflation.

However, for the Variable Planck Mass model, heating is efficient, and the maximum temperature

$$T_{\text{max}} \sim \epsilon \left( \frac{15}{128\pi^4 g_{\text{eff}}^{-1} \xi^2} \right)^{\frac{1}{4}} m_P = 1.2 \times 10^{-4} m_P, \quad (6.13)$$

reached after inflation is rather high and in order not produce magnetic monopoles one requires that the GUT scale satisfy

$$\sigma > 1.5 \times 10^{-4} m_P. \quad (6.14)$$

This result is computed for the SU(5) model, and it is slightly model dependent (the effective number of degrees of freedom was taken to be $g_{\text{eff}} = 160.75$ and the efficiency factor for heating was $\epsilon \sim 0.5$).

**B. Quantum Scale of Inflation**

In the context of quantum cosmology, Barvinsky and Kamenshchik considered the Variable Planck Mass model using a 1-loop approximation. They point out that the initial value of the scalar field is a free parameter which can be constrained using arguments of quantum cosmology.

At tree-level, they find that the wavefunction for the scalar field in a minisuperspace model is not normalizable. However, in a 1-loop approximation, they find that it is strongly peaked with values that are consistent the measurement of the microwave background
anisotropy. This is an interesting development for quantum cosmology, and it should be pursued further. Moreover, the wavefunction of the Universe can actually be measured [9]. For the latest experimental results concerning the statistics of the large angle microwave background measurements, consult Kogut [37] who was able to rule out some toy non-Gaussian models.

C. Other One-Loop Effects

Unfortunately, space limitations do not permit a detailed discussion of other one-loop effects in inflation. For completeness, I will mention the main areas:

1. stochastic inflation [38] - [41] — in this proceedings, Linde [42] has given an enthusiastic discussion;

2. heating of the Universe after inflation is an interesting topic [43];

3. $R^2$-inflation [44], [9], [26] — here, one-loop effects drive inflation;

4. computation of the curvature coupling constant $\xi$ — consult Hill and Salopek [45] as well as Buchbinder et al [46] and Reuter [47];

5. generation of primordial magnetic fields from inflation through the breaking of conformal invariance (Dolgov [48]);

6. string field theory and inflation — whether string theory is compatible with inflation is still an open question. In this volume, related issues are discussed by Norma Sanchez [49]. See also ref. [34].

I suspect that the next major stage in the development of inflation theory will be a careful treatment of one-loop effects. Although, this subject is technically quite challenging, many problems are indeed tractable. I feel that the fascinating theory of one-loop effects on inflation will be of interest for many years to come.
VII. CONCLUSIONS

The inflationary scenario provides a relatively good understanding of many cosmological observations, including galaxy clustering and microwave anisotropies. As the data continues to improve, one hopes to determine the correct inflationary model.

In order to facilitate the comparison of models with observations, some very powerful theoretical machinery has been developed. In fact, Hamilton-Jacobi theory provides an elegant means to solve many problems appearing in cosmology.

Here I have reviewed how to compute galaxy correlations and microwave anisotropies arising from power-law inflation where the scale factor evolves according to $a(t) \propto t^p$. This model helps to alleviate the problems of large scale structure. For example, if $p = 21$, the spectral index for scalar perturbations is found to be $n = 0.9$. This models yields a 35% gravity wave contribution to $a^2_{sky}(10^0)$ (determined by COBE). Although the fit to large scale clustering is not perfect, I feel that with improvements in the quantity and quality of data, one should obtain a better fit.

Another nice application of Hamilton-Jacobi theory is incorporating the Zel’dovich approximation within general relativity. The Zel’dovich approximation describes the formation of sheet-like structures which appear to be quite common in our Universe.

At the moment, the future for the inflationary scenario appears quite bright. Already researchers are considering going beyond the semi-classical approximation provided by Hamilton-Jacobi theory: they are now investigating one-loop corrections to inflation.

VIII. ACKNOWLEDGMENTS

I would like to thank Norma Sanchez for organizing an enjoyable conference. Some of the work that was reviewed here was done in collaboration with John Stewart at Cambridge University. I acknowledge partial support from the Natural Sciences and Engineering Research Council of Canada, and the Canadian Institute for Theoretical Astrophysics in Edmonton.
REFERENCES

[1] E.L. Wright et al, Astrophys. J. Lett. 396, L13 (1992); G.F. Smoot et al, Astrophys. J. Lett. 396, L1 (1992).

[2] J.A. Peacock and S.J. Dodds, Mon. Not. Roy. Astron. Soc. (1994, in press).

[3] K. Ganga, E. Cheng, S. Meyer and L. Page, Astrophys. J., 410, L57 (1993).

[4] A.N. Lasenby, these proceedings; M. Jones, R. Saunders, P. Alexander, M. Birkinshaw, N. Dillon, K. Grainge, S. Hancock, A. Lasenby, D. Lefebvre, G. Pooley, P. Scott, D. Titterington and D. Wilson, Nature 365, 320 (1993).

[5] C.L. Bennett et al, COBE preprint (1994); K.M. Gorski et al, Astrophys. J. Lett. 430, L89 (1994).

[6] G.A. Tammann, these proceedings.

[7] M. Birkinshaw, J.P. Hughes and K.A. Arnaud, Astrophys. J. 379, 466 (1991).

[8] P.J.E. Peebles, Astrophys. J. 263, L1 (1982); J.R. Bond and G. Efstathiou, Astrophys. J. Lett. 285, L45 (1984); Mon. Not. R. Astr. Soc. 226, 655 (1987).

[9] D.S. Salopek, Phys. Rev. Lett., 69, 3602 (1992); Inter. J. Mod. Physics D 3, 257 (1994); Proc. of the International School of Astrophysics, “D. Chalonge”, Second Course: Current Topics in Astrofundamental Physics, Erice, Italy, September 6-13, 1992, p. 23, ed. N. Sanchez and A. Zichichi (World Scientific Publishers, 1993).

[10] D.S. Salopek and J.M. Stewart, 'Hypersurface-Invariant Approach to Cosmological Perturbations,' Phys. Rev. D (1994, in press).

[11] J. Parry, D.S. Salopek and J.M. Stewart, Phys. Rev. D 49, 2872 (1994).

[12] D.S. Salopek, J.M. Stewart and K.M. Croudace, Mon. Not. Roy. Astr. Soc. (in press, 1994).
[13] D.S. Salopek and J.M. Stewart, Class. Quantum Grav. 9, 1943 (1992).

[14] D.S. Salopek, 43, 3214 (1991); ibid 45, 1139 (1992); D.S. Salopek and J.R. Bond, Phys. Rev. D 42, 3936 (1990).

[15] G.F.R. Ellis, these proceedings; G.F.R. Ellis and M. Bruni, Phys. Rev. D 40, 1804 (1989).

[16] Y.B. Zel’довich, A & A, 5, 84 (1970).

[17] K.M. Croudace, J. Parry, D.S. Salopek and J.M. Stewart, Astrophys. J. 423, 22 (1994).

[18] D.S. Salopek, J.M. Stewart, K.M. Croudace and J. Parry, in Proc. 37th Yamada Conference, June 8-12, 1993, Tokyo, Japan, ed. K. Sato (Universal Academic Press, Inc. 1994).

[19] D.S. Salopek, ’Resolving the Question of Time for Semiclassical Gravity,’ in Proc. of the Lake Louise Winter Institute, Particle Physics and Cosmology, Feb. 20-26, 1994, Eds. B. Campbell and F. Khanna (1995), (astro-ph/9408053).

[20] C.S. Frenk, these proceedings.

[21] E.L. Wright, G.F. Smoot, C.L. Bennett and P.M. Lubin, Astrophys. J. (in press, 1994).

[22] G.F. Smoot, these proceedings.

[23] A.O. Barvinsky and A.Y. Kamenshchik, Phys. Lett. B332, 270 (1994); A.O. Barvinsky, Physics Reports 230, 237 (1993); Phys. Lett. B241, 201 (1990).

[24] F. Lucchin and S. Matarrese, Phys. Rev. D 32, 1316 (1985).

[25] T.S. Bunch, and P.C.W. Davies, Proc. R. Soc. Lond. A. 360, 117 (1978).

[26] D.S. Salopek, J.R. Bond and J.M. Bardeen, Phys. Rev. D 40, 1753 (1989).

[27] D.S. Salopek, submitted to Mon. Not. Roy. Astr. Soc. (1994).
[28] L.P. Grishchuk, these proceedings.

[29] Matarrese, S., Pantano, O. & Saez, D., Phys. Rev. D, 47, 1311 (1993).

[30] E.M. Lifshitz & I.M. Khalatnikov, Usp. Fiz. Nauk, 80, 391 (1964), [Sov. Phys. Usp., 6, 495 (1964)].

[31] Tomita, K., Prog. Theor. Phys., 54, 730 (1975).

[32] P. Szekeres, Commun. Math. Phys., 41, 55 (1975).

[33] M. Gasperini, N. Sanchez and G. Veneziano, Nucl. Phys. B 364, 365 (1991).

[34] B.A. Campbell, A. Linde and K.A. Olive, Nucl. Phys. 355, 146 (1991); F.C. Adams et al, Phys. Rev. D 47, 426 (1993).

[35] C. Brans and R. H. Dicke, Phys. Rev. 124, 925 (1961).

[36] J.A. Holtzman, Astrophys J. Supp. 71, 1 (1989); A. van Dalen and R.K. Schaefer, Astrophys. J., 398, 33.

[37] A. Kogut, these proceedings.

[38] A.A. Starobinsky, in Current Topics in Field Theory, Quantum Gravity, and Strings, ed. H.T. de Vega and N. Sanchez (Springer: New York, 1986), p.107.

[39] J.M. Stewart, Class. Quantum Grav. 8, 909 (1991).

[40] D.S. Salopek and J.R. Bond, Phys. Rev. D 43, 1005 (1991).

[41] A. Linde, D. Linde and A. Mezhlinian, Phys. Rev. D 49, 1783 (1994).

[42] A. Linde, these proceedings.

[43] D. Boyanovsky, H.J. DeVega and R. Holman, Phys. Rev. D 49, 2769 (1994); L. Kofman, A. Linde and A. Starobinsky, (U. of Hawaii preprint, 1994)

[44] A.A. Starobinsky, in Quantum Gravity, p.103, ed. M.A. Markov and P. West (New York:
[45] C.T. Hill and D.S. Salopek, Annals of Phys. 213, 21-30 (1992).

[46] I.L. Buchbinder, S.D. Odinstsov and I.L. Shapiro, Rev. Nuovo Cimento 12, 1 (1989).

[47] M. Reuter, Phys. Rev. D 49, 6379 (1994).

[48] A. Dolgov, Phys. Rev. D 48, 2499 (1993).

[49] N. Sanchez, these proceedings.
IX. FIGURE CAPTIONS

Fig.(1): COBE has measured the angular correlation function $C(\alpha)$ for the temperature anisotropy with the monopole and dipole removed. Below about 60 degrees, the measurements are clearly much larger than the error bars. In fact, COBE claims a 10 standard deviation detection. (Taken from Bennett et al [5].)  

Fig.(2): The observed galaxy power spectrum is shown. For comparison, the 1.8 power-law that is usually associated with optical galaxies is also shown.

Fig.(3): A plot of recent microwave background measurements is shown. Since one is observing the celestial sphere, it is useful to decompose into spherical harmonics, $Y_{\ell m}$, where $\ell$ is harmonic the index, and $T_{\ell}^2$ is the average amplitude for a particular $\ell$. The solid curve depicts the standard cold-dark-matter model prediction. For $\ell > 20$, there is much scatter in the observational data, and more data are required. (Taken from Wright et al [21].)

Fig.(4): Primordial scalar perturbations of the metric are described by the function $\zeta$. The power spectra for zeta are shown for various choices of the the spectral index $n = 1 - 2/(p - 1)$ arising from power-law inflation. They have been normalized using COBE’s 2-yr data set.

Fig.(5): For the present epoch, power spectra for the linear density perturbation $\delta\rho/\rho$ are shown for various cold-dark-matter (CDM) models. The points with error-bars are the observed data with $b_{IRAS} = 1$. Standard CDM utilizes a Zel’dovich spectrum with a spectral index $n = 1.0$. It is unsatisfactory at short scales. The dark line depicts the best fit of the power-law inflation models with $n = 0.9$. It yields a 35% gravitational wave contribution to the large-angle microwave background anisotropy $\sigma_{sky}^2(10^0)$.

Fig.(6): The various orders of the gradient expansion are compared with the exact Szekeres solution (bold curve) for the evolution of the 3-metric component $\gamma_{33}$ in terms of the scale factor $a(\chi) \equiv \chi^{2/3}$. Pancake formation occurs when $\gamma_{33} = 0$. The thin curve is the first order term (long-wavelength), whereas the dotted graph is the third order result. The improved third order expansion yields the exact result.

Fig.(7): Inflation can be reconciled with particle physics using the (b) Variable Planck Mass
model which is closely related to (a) Induced Gravity. The effective potentials are plotted as a function of $\chi \equiv \chi(\phi)$. For large positive values of $\chi$, the effective potential is extremely flat as required by inflation. One-loop radiative corrections do not destroy the flatness of the potential.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9412012v2
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9412012v2
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9412012v2
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/astro-ph/9412012v2