A Bayesian Inference of a Relativistic Mean-field Model of Neutron Star Matter from Observations of NICER and GW170817/AT2017gfo

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Abstract

Observations of optical and near-infrared counterparts of binary neutron star mergers not only enrich our knowledge about the abundance of heavy elements in the universe and help reveal the remnant object just after the merger, which is generally known, but can also effectively constrain the dense properties of the nuclear matter and the equation of state (EOS) in the interior of the merging stars. Following the relativistic mean-field description of nuclear matter, we perform a Bayesian inference of the EOS and the properties of the nuclear matter using the first multi-messenger event GW170817/AT2017gfo, together with the NICER mass–radius measurements of pulsars. The kilonova is described by a radiation-transfer model with the dynamical ejecta, and light curves connect with the EOS through the quasi-universal relations between the properties of the ejecta (the ejected mass, velocity, opacity, or electron fraction) and binary parameters (the mass ratio and reduced tidal deformability). It is found that the posterior distributions of the reduced tidal deformability from the AT2017gfo analysis display a bimodal structure, with the first peak enhanced by the GW170817 data, leading to slightly softened posterior EOSs, while the second peak cannot be achieved by a nuclear EOS with saturation properties in their empirical ranges. The inclusion of NICER data results in a stiffened EOS posterior because of the massive pulsar PSR J0740+6620. We provide the results at nuclear saturation density for the nuclear incompressibility, the symmetry energy, and its slope, as well as the nucleon effective mass, from our analysis of the observational data.

Unified Astronomy Thesaurus concepts: Neutron stars (1108); Gravitational waves (678); Pulsars (1306)

1. Introduction

The detection of gravitational waves (GWs) and light from the binary neutron star merger GW170817 marked the first milestone of multi-messenger astronomy (Abbott et al. 2017). The GW signals from coalescing binary neutron stars have been widely used to provide critical insights into the nature of dense nuclear matter and the equation of state (EOS; i.e., the pressure–density relation) of neutron stars (Abbott et al. 2018).

The electromagnetic counterparts of GW sources provide another way of studying the EOS. In particular, the transient optical/infrared/UV event (AT2017gfo) was detected several hours after the merger time of GW170817 (Andreoni et al. 2017; Arcavi et al. 2017; Coulter et al. 2017; Cowperthwaite et al. 2017; Diaz et al. 2017; Drout et al. 2017; Evans et al. 2017; Hu et al. 2017; Kasliwal et al. 2017; Lipunov et al. 2017; Pian et al. 2017; Shappee et al. 2017; Smartt et al. 2017; Tanvir et al. 2017; Troja et al. 2017; Utsumi et al. 2017; Valenti et al. 2017; Pozanenko et al. 2018), the luminosity, spectrum, and light curve of which are consistent with the prediction of the kilonova model, which attributes its emission to the \( r \)-process nucleosynthesis of the ejected neutron-rich matter from the merger. The mass, velocity, and electron fraction of the ejecta are key parameters for understanding the observations of AT2017gfo (e.g., Metzger 2017; Perego et al. 2017; Yu et al. 2018; Ren et al. 2019; Qi et al. 2022), and closely related to the binary parameters (like the mass ratio and radius) and the EOS (e.g., Shibata & Hotokezaka 2019).

Stiffness or softness of the EOS implies larger or smaller stellar radius and orbital separation at merger. A softer EOS and smaller radius result in a more violent collision and more efficient shock heating, which can eject more material with higher velocity and high temperature. The ejected matter with high temperature may trigger the weak interaction and neutrino emission, and further vary the electron fraction of ejecta. Therefore, the EOS affects the input quantities of the kilonova light-curve model, and it is important and interesting to infer the EOS from both the GW and kilonova data.

Merger simulations have revealed some quasi-universal relations between the properties of the ejecta and the binary parameters (mass ratio and reduced tidal deformability) (Nedora et al. 2021). The EOS constraints from kilonova observations have also been investigated (e.g., Margalit & Metzger 2017; Radice et al. 2018b; Coughlin et al. 2019; Brescia et al. 2021; Holmbeck et al. 2022). A group of EOSs from different nuclear many-body frameworks or the parameterizations of EOS such as piecewise polytropes (De et al. 2018; Most et al. 2018; Ecker & Rezzolla 2022), or spectral parameterization (Lindblom 2010; Koliogiannis & Moustakidis 2019) were usually adopted, allowing the study only on the pressure-versus-density function, but not on the physical properties of nuclear matter.

In this work, we perform one of the first studies to connect nuclear matter microscopic parameters to the AT2017gfo data (Villar et al. 2017) of the GW170817 binary neutron star merger. The kilonova is described by a radiation-transfer model depending on which we reproduce important properties of AT2017gfo light curves and explore the underlying phase state of nuclear matter and the EOS. Nuclear matter and the EOS are described by the relativistic mean-field (RMF) model, which encodes a great amount of nuclear physics in a handful of
model parameters. By construction, the RMF effective interactions can facilitate easy incorporation of various nuclear EOS constraints at the nuclear saturation density $n_0$ and moderate values of the isospin asymmetry. In combination with the GW observations of tidal deformability (Abbott et al. 2019) by LIGO/Virgo and the mass and radius measurements of PSR J0030+0451 and PSR J0740+6620 (Miller et al. 2019; Riley et al. 2019a; Miller et al. 2021; Riley et al. 2021a) by the NASA Neutron Star Interior Composition ExploreR (NICER) mission, the inference will be performed directly on key properties like the nuclear incompressibility and the symmetry energy as well as the single particle nucleon effective mass in medium that can be confronted with laboratory studies on nuclear structure and reactions. We do not consider the non-nucleon degree of freedom possibly present in heavy neutron stars since the data we utilize here are mostly from typical stars around or below $1.4M_\odot$ and the main interest of the present study is the EOS parameters around the saturation density $n_0$. Because the stellar radius is controlled mainly by the density dependence of the nuclear symmetry energy around $n_0$ (Lattimer & Prakash 2000), below we also report the most preferred radius and tidal deformability (scaling as the fifth power of the radius) for typical $1.4M_\odot$ stars based on our analysis. See, e.g., Miao et al. (2020), Li et al. (2021a, 2021b), Sun et al. (2023), and Miao et al. (2022a, 2022b) for analyses incorporating strangeness phase transitions in neutron star matter.

This paper is organized as follows. In Section 2, we introduce the models of the EOS and kilonova that we employed in our analyses. In Section 3, we recall the Bayesian formulation and describe the parameters, priors, and likelihood functions in our analyses. In Section 4, we present our results and a discussion. Finally, we conclude in Section 5.

2. Models of Neutron Star EOS and Kilonova

In this section, we review the adopted models of neutron star EOS and kilonova, including a detailed description of the relations between the kilonova observations and the EOS as well as the stellar properties.

2.1. Neutron Star EOS

The only physics that spherically symmetric neutron stars in hydrostatic equilibrium are sensitive to is the EOS of (neutron-rich) nuclear matter, in the simple case of no strangeness phase transition (Li et al. 2020). In principle, it can be determined by the strong interaction, from solving the first principle QCD. Nevertheless, the complexity of nonperturbative strong interaction makes it difficult to do theoretically, and hence parameterization is widely used to describe the EOS in the analysis of observational data. Presently, the RMF nuclear many-body model is employed in our analyses.

The RMF model starts from a many-body Lagrangian to describe the nucleon–nucleon interactions, which are mediated by scalar ($\sigma$), isoscalar-vector ($\omega$), and isovector-vector ($\rho$) mesons (see, e.g., Li et al. 2008; Zhu et al. 2018, 2019; Traversi et al. 2020),

$$
\mathcal{L} = \overline{\psi} \left( i \gamma_\mu \partial^\mu - MN + g_\sigma \sigma - g_\omega \omega^0 - g_\rho \rho^0 \right) \psi \\
- \frac{1}{2} (\nabla \sigma)^2 - \frac{1}{2} m_\sigma^2 \sigma^2 - \frac{1}{3} g_\sigma \sigma^3 - \frac{1}{4} g_\sigma \sigma^4 \\
+ \frac{1}{2} (\nabla \omega)^2 + \frac{1}{2} m_\omega^2 \omega^2 + \frac{1}{2} g_\omega \omega^2 \Lambda_\omega g_\omega^2 \rho^2 \\
+ \frac{1}{2} (\nabla \rho)^2 + \frac{1}{2} m_\rho^2 \rho^2,
$$

(1)

where $g_\sigma$, $g_\omega$, and $g_\rho$ are the nucleon coupling constants for $\sigma$, $\omega$, and $\rho$ mesons. We also include the nonlinear $\sigma$ self-interactions with two parameters $g_2$ and $g_3$, and the $\omega$–$\rho$ coupling with parameter $\Lambda_\omega$. These six meson coupling parameters can be obtained by fitting the empirical data at the nuclear saturation density $n_0$ (see below in Table 1).
The equation of motion for each meson can be generated by the Euler–Lagrangian equation from the Lagrangian and applying the mean-field approximation:

\[ m_i^2 \sigma_i + g_2 \sigma^2 + g_3 \sigma^3 = g_0 n_s, \]  

where

\[ n_s = \sum_{i=n,p} \frac{1}{\pi^2} \int_{p_F}^\infty \frac{M_i^*}{\sqrt{M_i^{*2} + p_F^2}} p_F^2 dp_F \]  

is the scalar density, \( p_F \) denotes the Fermi momentum, and \( M_i^* = M_i - g_i \sigma \) is the effective mass. The number density of protons and neutrons is represented by \( n_p \) and \( n_n \), respectively. After solving these equations of motion, the energy density and pressure of nuclear matter can be computed by

\[
 e = \sum_{i=n,p} \varepsilon_{kin}^i + \frac{1}{2} m_0^2 \sigma^2 + \frac{3}{4} g_2 \sigma^3 + \frac{1}{4} g_3 \sigma^4 - \frac{1}{2} m_0^2 \omega^2 - \frac{1}{2} m_0^2 \rho^2 - \frac{1}{2} \Lambda_\omega (g_\omega g_\omega \omega^2) - g_\omega \omega (n_n + n_p) + g_\rho \rho (n_p - n_n),
\]

and

\[
 p = \sum_{i=n,p} P_{kin}^i - \frac{1}{2} m_0^2 \omega^2 + \frac{1}{2} m_0^2 \rho^2 + \frac{1}{2} \Lambda_\omega (g_\omega g_\omega \omega^2) + \frac{1}{2} m_0^2 \sigma^2 - \frac{3}{4} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 + \frac{1}{2} m_0^2 \omega^2 + \frac{1}{2} m_0^2 \rho^2 + \frac{1}{2} \Lambda_\omega (g_\omega g_\omega \omega^2). \]

To study the structure of neutron stars, we have to calculate the composition and EOS of cold, neutrino-free, catalyzed matter. We require that the neutron star contains charge-neutral matter consisting of neutrons, protons, and leptons (\( e^+ \), \( \mu^+ \)) in beta equilibrium. Additionally, since we are looking at neutron stars after neutrinos have escaped, we set the neutrino chemical potentials equal to zero. Also, we use ultrarelativistic and nonrelativistic approximations for the electrons and muons, respectively, and their contributions to the energy and pressure are merely added to Equations (6) and (7). Consequently, the energy density and pressure of neutron star matter are simply the functions of nucleon number density.

For completeness, we also write down the expressions of the symmetry energy \( J_0 \), incompressibility \( K_0 \), and symmetry energy slope \( L_0 \) at the saturation density in symmetric nuclear matter

\[
 J_0 = \frac{p_F^2}{6E_F} + \frac{g_0^2}{2[m_0^2 + \Lambda_\omega (g_\omega g_\omega \omega^2)]} (n_p + n_n),
\]

\[
 K_0 = \frac{3p_F^2}{E_F} + \frac{3M_{s}^6 p_F}{E_F} \frac{dM_{s}^6}{dp_F} + \frac{9g_0^2}{m_0^2 + \Lambda_\omega (g_\omega g_\omega \rho^2)} n_0,
\]

\[
 L_0 = 3J_0 + \frac{1}{2} \left( \frac{3\pi^2}{2} n_0 \right)^{2/3} \frac{1}{E_F} \left( \frac{g_0^2}{m_0^2 + \Lambda_\omega (g_\omega g_\omega \rho^2)} \right)^2 \frac{n_0}{E_F} - \left( \frac{K_0}{9E_F} - \frac{1}{3} \right) - \left( \frac{3g_0^2}{m_0^2 + \Lambda_\omega (g_\omega g_\omega \rho^2)} \right)^2 \frac{g_0^2 \Lambda_\omega n_0^2}{m_0^2 + \Lambda_\omega (g_\omega g_\omega \rho^2)}. \]  

To recap, we have six properties of the nuclear matter: the saturation density \( n_0 \), energy per baryon \( E/A \), \( J_0 \), \( K_0 \), \( L_0 \), and effective mass \( M_{s}^6 \), to be reproduced by the six model parameters, \( g_\sigma \), \( g_\omega \), \( g_\rho \), \( g_2 \), \( g_3 \), and \( \Lambda_\omega \). Once the saturation properties of nuclear matter are chosen in their empirical ranges, the six model parameters can be uniquely determined (see the Appendix for more details) for the calculations of neutron stars. In the following analysis, we will directly specify these six saturation properties, rather than the model parameters, to denote the EOS.

### 2.2. Kilonovae

In the present work, we employed a radiation-transfer model (see e.g., Metzger 2017; Yu et al. 2018; Ren et al. 2019; Qi et al. 2022, for more details) to calculate the light curves of the kilonova. The emission luminosity is computed by solving the energy conservation equations of ejecta, where the heating of r-process nucleosynthesis and the cooling of adiabatic expansion are taken into account. Additionally, the source of kilonova is treated as a blackbody and the spectra are given by the blackbody emission.

The ejecta during and after the binary neutron star merger mainly consists of two components, i.e., dynamical ejecta and wind-driven ejecta. The dynamical ejection is driven by the tidal forces during the inspiral and shock heating during the coalescence (Bovard et al. 2017; Radice et al. 2018a; Shibata & Hotokezaka 2019). The tidal forces eject the material primarily along the direction of the equator with relatively low temperature and low electron fraction (smaller than 0.1–0.2). Meanwhile, the shock isotropically ejects and heats the material to a high temperature where the weak interaction can be triggered so that the electron fraction increases. Therefore, the ejecta driven by shock heating has a high electron fraction (\( Y_e > 0.25 \)) and distributes evenly along the inclination \( \theta \). In addition to the dynamic ejecta, the neutrino emissions from the remnant before collapsing into a black hole as well as the viscosity could further drive more ejecta (the so-called wind-driven ejecta) from the disk surrounding the remnant, which is naturally more subject to, e.g., the lifetime of the remnant neutron star. For the present study, we do not consider the wind-driven ejecta when connecting the AT2017gfo observational data with the underlying EOS.

The ejected matter with a low electron fraction that mainly concentrates on the orbital plane will undergo a full r-process nucleosynthesis and produce a large amount of lanthanide elements. The high opacity resulting from the lanthanide elements leads the ejecta on the orbital plane to be the red component. On the other hand, the ejected material along the polar direction is primarily contributed by the shock heating, and will only experience a partial r-process nucleosynthesis, whose lanthanide synthesis is suppressed. Consequently, the polar ejecta has a relatively lower opacity and is called the blue component.
The multiwavelength light curves of AT2017gfo indicate that it cannot be explained by the models with only one single set of parameters, if only the power of r-process nuclei is taken into account (Villar et al. 2017). Therefore, we involved both the red and blue components in our model by implementing a θ-dependent opacity. The ejected material is approximated as being homologously expanding, and the shell structure is formed accordingly. Each shell can further be decomposed into two patches with different opacity, and the interface of these two patches is set to be θ = π/4. Therefore, the opacity can be described by a step function of inclination angle θ:

\[ \kappa = \begin{cases} 
\kappa_{\text{low}}, & \theta \leq \pi/4; \\
\kappa_{\text{high}}, & \theta > \pi/4,
\end{cases} \tag{11} \]

where opacity is denoted by κ, κlow and κhigh are constants and correspond to the blue and red components of the kilonova.

Because of the isotropic distribution of mass, the density is merely the function of radial coordinate r. This distribution is typically described by a power law (see Nagakura et al. 2014), and the density distribution function of the radius can be written as

\[ \rho_{\text{ej}}(R) = \frac{M_{\text{ej}}}{4\pi} (3 - \delta) \frac{R^{-\delta}}{R_{\text{max}}^{3-\delta} - R_{\text{min}}^{3-\delta}}, \tag{12} \]

where the total mass, and the maximal and minimal radius of the ejecta are denoted by \( M_{\text{ej}} \), \( R_{\text{max}} \), and \( R_{\text{min}} \), respectively. The shell with the maximum and minimal radius also represents the maximum and minimum velocity shell through \( R_{\text{max}} = v_{\text{max}} t \) and \( R_{\text{min}} = v_{\text{min}} t \). The index \( \delta \) is a constant between 1 and 3. With this distribution function, the mass of each shell can be calculated by integrating over the radius.

The emission luminosity can be obtained by solving the equation of energy conservation

\[ \frac{dE_{ij}}{dt} = m_{ij} \dot{q}_r \eta_{\text{th}} - \frac{E_{ij}}{R_{ij}} \frac{dR_{ij}}{dt} - L_{ij}, \tag{13} \]

where \( i, j \) denotes the index of patches (indicating that the patch locates at the \( i \)th shell and \( j \)th inclination angular spacing), and \( m_{ij} \) represents the mass of the patch. Because the opacity is a step function and only the two value is available in our computation, the number of patches for each shell is 2 (\( j = 1, 2 \)). The first term on the right-hand side of this equation represents the heating of r-process nucleosynthesis. \( \dot{q}_r \) denotes the radioactive power per unit mass and \( \eta_{\text{th}} \) denotes the thermalization efficiency. They can be written as (Korobkin et al. 2012; Barnes et al. 2016)

\[ \dot{q}_r = 4 \times 10^{18} \left[ \frac{1}{2} - \frac{1}{\pi} \arctan \left( \frac{1 - t_0}{\sigma} \right) \right]^{1.3} \text{ erg s}^{-1} \text{ g}^{-1}, \]

\[ \eta_{\text{th}} = 0.36 \left[ \exp(-0.56 t_{\text{day}}) + \frac{\ln(1 + 0.34 \delta)}{0.34 \delta} \right], \tag{14} \]

where \( t_0 = 1.3 \text{ s}, \sigma = 0.11 \text{ s}, \text{ and } t_{\text{day}} = t \text{ day}^{-1}. \) The second term represents the adiabatic cooling of the ejecta, and it can be simplified by using the relation \( R_i = v_i t / \sqrt{3} \) as \( -\dot{E}_{ij} / t_i \). The last term represents the energy that is carried out by emission, or the luminosity. It can be estimated by

\[ L_{ij} = \frac{E_{ij}}{\max(t_i^j, t_c^j)}, \tag{15} \]

where the light-crossing time is \( t_c = R_i / c \), and the photon diffusion timescale is

\[ t_d^j \approx \frac{3\kappa_j m_{ij}}{\Delta\Omega R_i c}, \tag{16} \]

The diffusion timescale depends on the opacity of the ejecta \( \kappa_j \), which is \( \theta \)-dependent in our model. \( m_{ij} \) denotes the exterior mass of the patch, which sums the mass of the exterior of the \( i \)th shell for the \( j \)th patch.

By solving Equation (13) and summing the luminosity of each shell for a specific patch and at a specific time step, we obtain the bolometric luminosity as a function of time:

\[ L_{\text{bol}} = \sum_i L_{ij}, \tag{17} \]

Additionally, the blackbody spectrum is assumed for the emission and the effective temperature can be calculated through this bolometric luminosity:

\[ T_{\text{eff}} = \left( \frac{L_{\text{bol}}}{\sigma_B \Delta\Omega R_{\text{ph}}^2} \right)^{1/4}, \tag{18} \]

where \( \sigma_B \) is the Stephan–Boltzmann constant and \( R_{\text{ph}} \) is the radius of the photosphere, which is defined as the radius where the exterior optical depth \( \tau_{\text{ext}} = R_{\text{ph}} \) is unitary. This radius can be calculated analytically with the density distribution Equation (12) as

\[ R_{\text{ph}}^\tau = \left[ R_{\text{max}}^{3-\delta} - \frac{4\pi}{M_{\text{ej}}\kappa_3 - \delta} (R_{\text{max}}^{3-\delta} - R_{\text{max}}^{3-\delta}) \right]^{1/\delta}. \tag{19} \]

However, \( R_{\text{ph}}^\tau \) may be smaller than \( R_{\text{min}} \) at the later time of evolution. We, therefore, define this photosphere radius as \( R_{\text{ph}} = \max[R_{\text{ph}}^\tau, R_{\text{min}}] \). Eventually, the flux with frequency \( \nu \) that is measured by the observer is obtained by summing up the contributions from all the rays,

\[ F_{\nu} = \frac{2h\nu^3}{c^2} \int \frac{1}{\exp(h\nu/kT_{\text{eff}}) - 1} \frac{R_{\text{ph}}^2}{D^2} n \cdot d\Omega, \tag{20} \]

where \( n \) and \( n_{\Omega} \) are the unit vector along the line of sight, and the unit vector of the solid angle, respectively. Subsequently, we determine the monochromatic AB magnitude by \( M_r = -2.5 \log_{10}(F_r / 3631.7) \).

Having determined the models for describing the EOS and kilonovae, we will perform a Bayesian analysis of the EOS by exploiting the data of AT2017gfo (Villar et al. 2017), GW170817 (Abbott et al. 2019), and NICER pulsars (Miller et al. 2019; Riley et al. 2019a; Miller et al.; 2021; Riley et al. 2021a). Before that, an introduction to the details of the Bayesian analysis will be presented in the next section.

3. Observational Constraints and Bayesian Analysis

Given a model hypothesis with a set of parameters \( \theta \), and some data \( d \), the posterior probability can be obtained by
applying the Bayes theorem,
\[ p(\theta|d) = \frac{\mathcal{L}(d|\theta)p(\theta)}{\int \mathcal{L}(d|\theta)p(\theta)d\theta}, \tag{21} \]
where \( \mathcal{L}(d|\theta) \) denotes the likelihood of the data \( d \) given a set of parameters \( \theta \) and their corresponding prior probability \( p(\theta) \). The denominator is the evidence of data \( d \) and acts as a normalization factor. The evidence can be obtained by integrating the numerator over all of the parameter space. In reality, however, the parameter space has a nontrivial number of dimensions, and may lead to a severe problem that is often referred to as the curse of dimensionality. One can only resort to the statistical computational techniques, e.g., Markov Chain Monte Carlo or nested sampling methods, to approximate the evidence or the marginalized distributions. In our analysis, the python package BILBY (Ashton et al. 2019; Romero-Shaw et al. 2020) and the nested sampler pymultinest (Buchner et al. 2014) will be implemented to generate the posterior samples and estimate the marginalized distributions.

To incorporate the data of the kilonova light curve, the GW, and the NICER mass–radius measurements, we take the total likelihood function as the form of
\[ \mathcal{L}(d|\theta) = \mathcal{L}_{\text{AT2017gfo}} \times \mathcal{L}_{\text{GW170817}} \times \mathcal{L}_{\text{NICER}}. \tag{22} \]
More details of the likelihood are described below in Section 3.2.

### 3.1. Parameters and Priors

As explained above in Section 2.1, the parameters of RMF models can be directly related to the saturation properties of the nuclear matter. Therefore, the six saturation properties will be treated as free parameters in our Bayesian analysis. In practice, the first two properties, \( n_0 \) and \( E/A \), are well determined and have much smaller uncertainties compared with the remaining four properties. We fix their value to be \( n_0 = 0.16 \text{fm}^{-3} \) and \( E/A = 16 \text{MeV} \). The prior distribution of the remaining four properties are denoted as \( \theta_{\text{eos}} \) and are set with a uniform distribution with the ranges listed in Table 1.

Furthermore, there are eight parameters in our kilonova model: the ejected mass \( M_{\text{ej}} \), the index of the mass distribution \( \delta \), the minimal and maximal velocity \( v_{\text{min}} \) and \( v_{\text{max}} \), the low and high opacity value \( n_{\text{low}} \) and \( n_{\text{high}} \), the luminosity distance of the source \( D \), and the viewing angle \( \theta_{\text{view}} \). In our analysis, the distance and viewing angle are fixed as \( D = 40 \text{ Mpc} \) and \( \theta_{\text{view}} = \pi/6 \). We denote the remaining six parameters as \( \theta_{\text{kn}} \) and compute their posterior distribution in the following analysis.

The six input kilonova parameters \( \theta_{\text{kn}} \) describing the properties of the ejecta do relate to the binary parameters. Indeed, the quasi-universal relations are extracted by fitting the data of simulations (see Nedora et al. 2021, 2022, for more details) and the ejected mass \( M_{\text{ej}} \), the mean velocity \( v_{\text{mean}} \), and the electron fraction \( Y_e \) are expressed as functions of binary parameters (mass ratio \( q \) and reduced tidal parameter \( \Lambda \)). However, these relations are not exact and deviations from the fitted formulations are expected in realistic situations. We introduce three deviation parameters to account for the uncertainty of the relations accordingly (Breschi et al. 2021). Consequently, the \( M_{\text{ej}} \), \( v_{\text{mean}} \), and \( Y_e \) are expressed with three additional deviation parameters \( \alpha_m \), \( \alpha_v \), and \( \alpha_e \) as
\[
\log_{10} M_{\text{ej}} = (1 + \alpha_m) \log_{10} M_{\text{ej}}^{\text{fit}}(q, \Lambda), \tag{23}
\]
\[
v_{\text{mean}} = (1 + \alpha_v) v_{\text{mean}}^{\text{fit}}(q, \Lambda), \tag{24}
\]
\[
Y_e = (1 + \alpha_e) Y_e^{\text{fit}}(q, \Lambda). \tag{25}
\]
The three deviation parameters \( \theta_{\text{dev}} = (\alpha_m, \alpha_v, \alpha_e) \) will be treated as input parameters in our Bayesian analysis, and their priors follow the Gaussian distribution with vanished means and standard deviations of 0.2.

In our kilonova models that take \( v_{\text{min}} \) and \( v_{\text{max}} \) as the input parameters, the mean velocity can be expressed in terms of the minimal and maximal velocity as
\[
v_{\text{mean}} = \frac{(3 - \delta)(v_{\text{max}}^{\delta} - v_{\text{min}}^{\delta})}{(4 - \delta)(v_{\text{max}}^{1-\delta} - v_{\text{min}}^{1-\delta})}. \tag{26}
\]
The electron fraction \( Y_e \) can be mapped into the mean opacity \( \bar{\kappa} \) of the ejecta by the relation in Tanaka et al. (2020). The \( \bar{\kappa} \) can be written as
\[
\bar{\kappa} = \frac{\sqrt{2}}{2} n_{\text{high}} + \left( 1 - \frac{\sqrt{2}}{2} \right) n_{\text{low}}. \tag{27}
\]
Meanwhile, once an EOS is determined from the EOS parameters \( \theta \), the binary properties (mass ratio \( q \) and \( \Lambda \)) can be determined with given masses. Therefore, all of the kilonova parameters \( \theta_{\text{kn}} \) can be mapped into the EOS parameters \( \theta_{\text{eos}} \) with three deviation parameters and two binary property parameters (we use the mass ratio \( q \) and the chirp mass \( M \) in our analysis) by utilizing these above relations.

Finally, two additional parameters \( \theta_{\text{NICER}} \) are required for the NICER data, which represent the central pressure of PSR J0030+0451 and PSR J0740+6620. All of the parameters and their prior distributions are summarily listed in Table 1.

### 3.2. Observational Data and Likelihood

#### 3.2.1. AT2017gfo

The observed light curves of AT2017gfo (Villar et al. 2017) will be fitted by our kilonova model. In reproducing the light curve of AT2017gfo, we only consider the dynamical ejecta as the source of \( r \)-process nucleosynthesis, since the effects of EOS on other parts of ejecta are mild (see, e.g., Perego et al. 2017; Yu et al. 2018; Ren et al. 2019; Breschi et al. 2021). In Figure 1, we plot the light curves of the kilonova model with the best-fitting parameters of \( \theta_{\text{kn}} \). The observational data (circles) or limits (triangles) are taken from Villar et al. (2017). Note that the solid lines that represent the model predictions deviate from the observational data significantly after 4 days of the merger event for most of the bands (only the \( K \), \( H \), and \( J \) bands are compatible). This might be a consequence of only two components (red and blue) being taken into account in our model. As generally believed, a third component should be incorporated to account for it (e.g., Perego et al. 2017; Villar et al. 2017; Yu et al. 2018; Ren et al. 2019; Breschi et al. 2021; Qi et al. 2022), including the energy or material injection from the central black hole hyperaccretion systems or magnetars, which is independent of the EOS. Therefore, we will not address this further in the following.

After obtaining the posterior samples of kilonova parameters \( \theta_{\text{kn}} \), we approximate their posterior distribution with the
Gaussian kernel density estimation (KDE). In the following, the posterior distribution will be treated as the likelihood of the EOS parameters $\theta_{\text{eos}}$ and the deviation parameters $\theta_{\text{dev}}$.

3.2.2. GW170817

The GW170817 likelihood is calculated through a high-precision interpolation of the likelihood developed in Hinterer et al. (2020) from fitting the strain data released by LIGO/Virgo, which is encapsulated in the python package toast,

$$\mathcal{L}_{\text{GW170817}} = F(\Lambda_1, \Lambda_2, \mathcal{M}, q),$$

where the chirp mass is $M = (M_1 M_2)^{3/5} / (M_1 + M_2)^{1/5}$, the mass ratio is $q = M_1 / M_2$, and $\Lambda_1(M_1)$ and $\Lambda_2(M_2)$ denote the tidal deformability (mass) of the individual star, respectively. $\Lambda_1$ and $\Lambda_2$ are connected with the reduced tidal deformability $\tilde{\Lambda}$ by

$$\tilde{\Lambda} = \frac{16}{13} \frac{(q + 12) q^2 \Lambda_1 + (1 + 12q) \Lambda_2}{(1 + q)^5}.$$

The tidal deformability, mass, and radius of a star can be computed by solving the perturbed tidal field equation (Flanagan & Hinderer 2008; Hinderer 2008; Hinderer et al. 2010) and the TOV equation simultaneously. Once the EOS and the central pressure of the star are determined, one can integrate both equations from the stellar center to the surface, where the pressure vanishes.

3.2.3. PSR J0030+0451 and PSR J0740+6620

The mass–radius measurements of two pulsars PSR J0030+0451 and PSR J0740+6620 by NICER collaborations have set strong constraints on the EOS. At a 68% confidence level, the mass and radius of PSR J0030+0451 are $M = 1.34_{-0.15}^{+0.15} M_\odot, R = 12.71_{-1.39}^{+1.14} \text{km}$ by Riley et al. (2019a), or $M = 1.44_{-0.14}^{+0.15} M_\odot, R = 13.02_{-1.06}^{+1.24} \text{km}$ by Miller et al. (2019); and the results of PSR J0740+6620 are $M = 2.072_{-0.006}^{+0.007} M_\odot, R = 12.39_{-1.30}^{+1.25} \text{km}$ by Miller et al. (2021), or $M = 2.062_{-0.091}^{+0.082} M_\odot, R = 13.71_{-1.50}^{+2.61} \text{km}$ by Miller et al. (2021). We implement ST+PST model samples of PSR J0030+0451 (Riley et al. 2019b) and the NICER and XMM samples of PSR J0740+6620 (Riley et al. 2021b) with the KDE methods to generate the posterior distributions, which will be treated as the likelihood in our analysis. Note that the central pressure for these two pulsars is included and treated as input parameters when calculating the NICER likelihood. The masses and radii will be computed by solving the TOV equation with the EOS given central pressures.

4. Results and Discussion

The EOS is connected with the properties of the ejecta through the quasi-universal relations, which are the functions of the mass ratio $q$ and the reduced tidal deformability $\tilde{\Lambda}$. In our analysis, the parameters of the kilonova model $\theta_{\text{km}}$ will first be sampled. Their posterior results are listed in Table 2. The posterior distributions of the ejecta parameters will be approximated by implementing the Gaussian KDE method, and these posterior will further be used as the likelihood of kilonova observations when sampling the binary parameters or EOS parameters.

4.1. GW170817: Mass Ratio and Tidal Deformability

The quasi-universal relations with their deviation, Equations (23)–(25), describe the properties of the ejecta in terms of the binary parameters (the mass ratio $q$ and the reduced tidal deformability $\tilde{\Lambda}$). We first compute the posterior samples of $q$ and $\tilde{\Lambda}$ by implementing the nested sampler, and compare the results to those of AT2017gfo, GW170817, and GW170817/AT2017gfo in Figure 2. We report in detail the median values and the 90% confidence intervals in Table 3.

Figure 2 illustrates the posterior distributions of the mass ratio $q$ (upper panel) and reduced tidal deformability $\tilde{\Lambda}$ (lower panel) by fitting the AT2017gfo light-curve data, the GW170817 likelihood, and the combined data of kilonova and GW, and they are represented by the red, orange, and blue lines, respectively. The solid lines represent the histogram of the samples and the dashed lines represent the distributions fitted by Gaussian KDE. Note that the KDE results of the distributions deviate from the histogram when $q$ is close to 1, for a stiff boundary are set at $q = 1$, and cases with $q < 1$ do not exist. However, the Gaussian KDE function may extend to the region of $q < 1$ and result in a decline close to $q = 1$. In the upper panel of the distributions of mass ratio, it can be seen that

![Figure 1](image-url)
in comparison to the results from the GW170817 data, AT2017gfo favors a smaller mass ratio. In the lower panel of the $\Lambda$ distributions, an interesting aspect of the kilonova data is reported. The result of the kilonova fitting displays a bimodal structure (Breschi et al. 2021): The first and the dominant peak locate around $\Lambda = 114$, while the secondary one is around $\Lambda = 1610$. Because of the second peak, the 90% confidence interval upper limit of $\Lambda$ is considerably larger than the GW170817 and GW170817/AT2017gfo results (shown in Table 3). Moreover, the first peak is close to that of GW170817 posterior distribution and results in a significant enhancement around the $\Lambda = 213$ region in the result of the combined data. Nevertheless, the secondary peak is suppressed by the GW170817 data and disappeared. In spite of the consistency of the location of the dominant peaks, GW170817 results show a longer tail with a larger value of $\Lambda$. Consequently, the AT2017gfo data strongly favor a smaller tidal deformability and softer EOS, which will be shown in the following sections.

4.2. Parameters of the Nuclear EOS and the Properties of the Neutron Star

The EOS is specified by four parameters in our analysis, which are the symmetry energy $J_0$, incompressibility $K_0$, symmetry energy slope $L_0$ and effective mass ratio $M_N^e/M_N$. In our process of sampling, we first calculate the coupling constants from these saturation properties, and calculate the neutron star core EOS after adding the lepton contribution by solving the equations of motion, Equations (2)-(4). We then join the core EOS with the usual BPS crust one (Baym et al. 1971). The mass, radius, and tidal deformability of neutron stars will be obtained with the whole stellar EOS and the likelihood of various cases are yielded.

We report the posterior distributions of EOS parameters $\theta_{eos}$ in Figure 3, and list the median values and the 90% confidence intervals in Table 4. The results of four different analyses with the data of AT2017gfo (red), GW170817/AT2017gfo (blue), GW170817 + NICER (orange), and GW170817/AT2017gfo + NICER (green) are reported in Figure 3. The same as in Figure 2, the histograms are denoted by the solid lines and the approximated distributions of KDE are denoted by dashed lines.

The symmetry energy shown in Figure 3 (leftmost panel) from different analyses shows similar distributions. This similarity can also be found for the confidence intervals listed in Table 4, and represent the insensitivity of symmetry energy $J_0$ on these observational data. All of our analyses favor smaller incompressibility except for the GW170817 + NICER case (orange) (see the second panel of Figure 3). For example, the median value of $K_0$ is around 250 MeV for the GW170817 + NICER case, while it is around 230 MeV for the other three analyses. Such a deviation is a consequence of the massive NICER observational data in our likelihood requires a preference for a soft EOS and a smaller radius for neutron stars.

This difference between GW170817/AT2017gfo and NICER analysis tends to be a larger value and a softer EOS, which will be shown in the following sections. Similarly, a larger nucleon effective mass, which is preferred by the AT2017gfo and GW170817/AT2017gfo analyses results in a softer EOS (Hornick et al. 2018). The introduction of the NICER observational data in our likelihood requires a stiffer EOS and smaller effective mass. Therefore, the distribution of GW170817 + NICER (orange) favors a smaller effective mass, and the ratio decreases from 0.79 of the GW170817 and AT2017gfo analyses to 0.72 of the GW170817 + NICER analyses. Finally, a trade-off of effective mass is

![Figure 2. The posterior distributions of $q$ (upper) and $\Lambda$ (lower) for AT2017gfo (red), GW170817 (orange), and the GW170817/AT2017gfo (blue) result. The solid lines represent the histogram of the posterior samples, and the dashed lines are smoothed by the Gaussian KDE methods from the histogram.](image-url)

![Table 3](image-url)

| Likelihood            | $q$                       | $\Lambda$                       |
|-----------------------|----------------------------|---------------------------------|
| AT2017gfo             | 1.0530^{+0.0072}_{-0.0076} | 127.1568^{+144.4992}_{-112.6891}|
| GW170817              | 1.1871^{+0.2656}_{-0.1706}  | 350.6256^{+230.0389}_{-243.0834}|
| GW170817/AT2017gfo    | 1.0513^{+0.0916}_{-0.0452}  | 213.5724^{+80.5000}_{-80.1237}  |

Note: The large value of the confidence interval upper limit of $\Lambda$ for AT2017gfo is the result of the second peak in the posterior distributions.
achieved by the GW170817/AT2017gfo + NICER analyses, which balanced the soft EOS preference of GW170817/AT2017gfo and the stiff one of PSR J0740+6620 in the NICER data.

We recall the recent laboratory PREX-II experiment that measured the neutron skin of 208Pb and implied the symmetry energy slope as $L_0 = 106 \pm 37$ MeV (Reed et al. 2021). The large central value from the PREX-II measurement deviates significantly from our analysis of the observational data (about 34.4 MeV) as can be seen in Table 4. Nevertheless, considering the large deviation in the $L_0$ distribution from PREX-II, different analyses from laboratory experiments and astrophysical data could be compatible with each other. For example, the joint analysis of PREX-II and the more recent CREX (Adhikari et al. 2022) suggests low symmetry energy slopes, i.e., $L_0 = 15.3^{+46.8}_{-41.5}$ (Zhang & Chen 2022), which is similar to our present results.

Figure 4 compares and contrasts the 90% confidence intervals of the EOSs for all the analyses. The contour of the GW170817/AT2017gfo + NICER analysis is denoted by the green-shaded region, while the other contours are denoted by the dashed lines. The median results for each posterior distribution are denoted by the dashed lines. The median results for each posterior sample and the smoothed distribution functions by the Gaussian KDE are represented by solid and dashed lines, respectively. The results from different analyses, AT2017gfo (red), GW170817/AT2017gfo (blue), GW170817 + NICER (orange), and GW170817/AT2017gfo + NICER (green), are denoted by different colors.

Moreover, the interval contours and the median lines of AT2017gfo and GW170817/AT2017gfo almost overlap with each other. Given the proximity of the dominant peak of the $\Lambda$ distributions from the AT2017gfo and GW170817 analyses shown in Figure 2, this similarity in EOS is a result of that and implies the consistency of the AT2017gfo and GW170817 data. On the other hand, the analysis of the NICER data favors a stiffer EOS because of the massive pulsar. The medium region that fulfills the small $\Lambda$ and large maximum mass $M_{\text{TOV}}$ is significantly enhanced in the distribution of the analysis that takes all data into account. Meanwhile, the very soft and very stiff EOSs is disfavored. Note that the upper bound of $M_{\text{TOV}}$ in our analyses is around 2.1 $M_\odot$, to the 90% posterior credible level, which is incompatible with GW190814 (Abbott et al. 2020) if its $\sim 2.6 M_\odot$ low-mass component is assumed to be a neutron star without phase transitions (see also the discussion in Li et al. 2021a; Nathanail et al. 2021). And the tension could in principle be resolved in the two-family scenario, which interprets the $\sim 2.6 M_\odot$ component of GW190814 as a quark star and the GW170817 event as a binary neutron star merger (Bombaci et al. 2021).

Finally, we report the mass–radius relations and the tidal deformability intervals of each analysis in Figure 5, and list the radius and tidal deformability of 1.4 $M_\odot$ stars $R_{1.4}$ and $\Lambda_{1.4}$ in the last two rows of Table 4, respectively. The analysis with GW170817 and AT2017gfo gives a smaller radius for stars around 1.4 $M_\odot$ compared with the analysis with NICER data. The median value of $R_{1.4}$ increases from $\sim 11.4$ km for AT2017gfo and GW170817/AT2017gfo to 12.4 km for GW170817 + NICER, and further decreases to 11.6 km for GW170817/AT2017gfo + NICER because of the trade-off. We mention here that the radius results are similar to the ones obtained with a chiral effective-field-theory description of nuclear matter (Capano et al. 2020). The tidal deformability $\Lambda_{1.4}$ has a similar behavior (increases from $\sim 250$ to 440 and

| Parameters         | AT2017gfo          | GW170817/AT2017gfo | GW170817 + NICER | GW170817/AT2017gfo + NICER |
|-------------------|--------------------|--------------------|-----------------|---------------------------|
| $J_0$ (MeV)       | $32.921^{+3.807}_{-2.617}$ | $33.086^{+2.697}_{-2.697}$ | $32.728^{+2.517}_{-2.162}$ | $33.041^{+1.722}_{-1.722}$ |
| $K_0$ (MeV)       | $231.597^{+10.176}_{-9.538}$ | $250.580^{+9.804}_{-9.804}$ | $250.818^{+9.762}_{-9.762}$ | $230.289^{+2.548}_{-2.096}$ |
| $L_0$ (MeV)       | $33.754^{+11.658}_{-10.340}$ | $35.353^{+11.604}_{-10.340}$ | $53.164^{+24.703}_{-23.970}$ | $34.459^{+15.243}_{-12.531}$ |
| $M_N^*/M_N$       | $0.788^{+0.022}_{-0.021}$   | $0.790^{+0.008}_{-0.007}$   | $0.716^{+0.046}_{-0.046}$   | $0.760^{+0.050}_{-0.039}$   |
| $R_{1.4}$ (km)    | $11.407^{+0.228}_{-0.228}$  | $11.530^{+0.264}_{-0.264}$  | $12.382^{+0.511}_{-0.511}$  | $11.636^{+0.211}_{-0.211}$  |
| $\Lambda_{1.4}$   | $255.049^{+11.816}_{-8.011}$ | $251.290^{+12.407}_{-25.402}$ | $440.869^{+123.422}_{-107.743}$ | $300.294^{+26.978}_{-16.784}$ |
the median results of the posterior distributions. Each analysis are the same as those in Figure 3. The dashed lines represent the GW170817 analysis, while the other contours are represented by dashed lines. The colors of each analysis are the same as those in Figure 3. The dashed-dotted lines denote the median results of the posterior distributions.

![Figure 4](image_url)

**Figure 4.** The 90% confidence interval of EOS for all four analyses. The green-shaded region represents the result of the GW170817/AT2017gfo analysis, while the other contours are represented by dashed lines. The colors of each analysis are the same as those in Figure 3. The dashed-dotted lines denote the median results of the posterior distributions.

The 90% confidence interval of the EOS for all four analyses. The green-shaded region represents the result of the GW170817/AT2017gfo analysis, while the other contours are represented by dashed lines. The colors of each analysis are the same as those in Figure 3. The dashed-dotted lines denote the median results of the posterior distributions.

![Figure 5](image_url)

**Figure 5.** Same as Figure 4, but for the radius (upper panel) and tidal deformability (lower panel) as functions of the stellar mass. The vertical black line represents the 1.4 $M_\odot$ line.

Even since the first detection of the multi-messenger signal of the GW170817 binary neutron star merger, a large number of works have investigated its implications on the neutron star EOS. The effects of the matter of the binary system imprinted into the GW signal as the tidal deformability contributions, and one may extract from it and constrain the EOS by analyzing the GW signals. On the other hand, the transient kilonova event of AT2017gfo can also shed light on the neutron star EOS through the properties of dynamical ejecta.

In this work, we implemented the quasi-universal relations between the binary properties (the mass ratio $q$ and reduced tidal deformability $\tilde{\Lambda}$) and the properties of the ejecta (the ejected mass, velocity, and electron fraction), and combined the observational data of AT2017gfo to constrain the neutron star EOS. The reduced tidal deformability of binary can be directly related to the saturation properties of nuclear matter in the framework of the RMF model. Thereafter, we performed a Bayesian analysis of the EOS and the saturation properties (the symmetry energy $J_0$, incompressibility $K_0$, symmetry energy slope $L_0$, and effective mass ratio $M_N^*/M_N$) with the AT2017gfo light-curve data. Our analysis shows a bimodal structure of the $\tilde{\Lambda}$ distribution, where the dominant peak corresponds to softer EOS and a smaller radius of stars. This dominant peak is enhanced by the GW170817 results, while the second peak is suppressed and disappeared in the distribution of GW170817/AT2017gfo.

We proceeded to perform a joint analysis with various observational data combinations (GW170817/AT2017gfo, GW170817 + NICER, and GW170817/AT2017gfo + NICER). The 90% confidence interval of the EOS of AT2017gfo and GW170817/AT2017gfo almost overlapped with each other, implying the consistency of GW170817 and AT2017gfo. However, the introduction of NICER data makes the posterior distributions strongly favor a stiff EOS with a larger stellar radius, since the massive pulsar PSR J0740+6620 in the NICER data demands stiff EOSs to be consistent with it. As a result, both the very stiff and very soft EOSs are excluded due to their incapability of reproducing the AT2017gfo data or PSR J0740+6620 data. When combining all the observational data, the properties of the nuclear matter at saturation are found to be $J_0 = 33.0410^{+1.7220}_{-1.5490}$ MeV, $K_0 = 230.2890^{+2.2203}_{-2.0966}$ MeV, $L_0 = 34.4599^{+18.2543}_{-12.5515}$ MeV, and $M_N^*/M_N = 0.7604^{+0.0020}_{-0.0019}$, at the 90% confidence level. Correspondingly, the radius and the tidal deformability for the 1.4 $M_\odot$ neutron stars are $11.6367^{+0.0121}_{-0.0107}$ km and $300.2940^{+26.9738}_{-36.7643}$, respectively. Additional future joint multi-messenger observations on neutron stars, binary evolution, and their mergers are expected to further constrain their EOSs.

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Appendix A
Detailed Derivation of the Properties of the Nuclear Matter from the RMF Model Parameters

We sum up the energy density and pressure Expressions (6) and (7) at saturation density \((p = 0)\) to yield a simplified expression:

\[
e + p = (E/A + M_N)n_0 = \sum_{i=A,p} e_i^{\text{kin}} + \sum_{i=A,p} p_i^{\text{kin}} + g_\omega \omega n_0.
\]

(A1)

The left-hand side is known from \(E/A\) and \(n_0\), and the kinetic terms only depend on Fermi momentum \(p_F\) and effective mass. Combining this equation with the \(\omega\) equation of motion, we can express \(\omega\) and \(g_\omega\) in terms of the known quantities

\[
\omega = \sqrt{\frac{(E/A + M_N)n_0 - \sum(e_i^{\text{kin}} + p_i^{\text{kin}})}{m_\omega^2}},
\]

(A2)

\[
g_\omega = \frac{m_\omega^2 \omega}{n_0}.
\]

(A3)

The expressions of \(\Lambda_\nu\) and \(g_\rho\) (\(\rho\) vanishes for symmetric nuclear matter) can also be obtained in the same way by combining Equations (4), (8), and (10)

\[
\begin{align*}
\Lambda_\nu &= -\frac{m^2 \sigma}{3 \beta \gamma \rho_0^2} \omega^2, \\
\frac{g_\nu^q}{g_\rho} &= \sqrt{\frac{m^2}{\beta - 1 - \Lambda_\nu(g_\omega \omega)^2}},
\end{align*}
\]

(A4)

(A5)

where \(\alpha\) and \(\beta\) are written as

\[
\alpha = L_0 - 3J_0 - \frac{1}{2} \left( \frac{3\pi^2 n_0}{2} \right)^{2/3} \frac{1}{E_F}
\times \left( \frac{g_\sigma^2}{m_\sigma^2 + \Lambda_\nu(g_\omega \omega)^2} \frac{n_0}{E_F} - \frac{K_0}{9E_F} - \frac{1}{3} \right),
\]

(A6)

\[
\beta = \frac{2J_0}{n_0} - \frac{P_\rho^2}{3E_F n_0}.
\]

(A7)

The determination of the last three parameters relies on Equations (7) and (2) and the derivative of Equation (2). Their expressions can be written as

\[
\sigma = \sqrt{\frac{C - 6B + 12A}{m^2}},
\]

(A8)

\[
g_2 = \frac{-3C + 15B - 24A}{\sigma^3},
\]

(A9)

\[
g_3 = \frac{2C - 8B + 12A}{\sigma^4},
\]

(A10)

\[
g_\sigma = \frac{(g_\rho \sigma)}{\sigma},
\]

(A11)

where \(A, B,\) and \(C\) are

\[
A = \sum_{i=A,p} p_i^{\text{kin}} + \frac{1}{2} m_\omega^2 \omega^2,
\]

(A12)
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