On the minimum mass ratio of W UMa binaries

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ABSTRACT
Using Eggleton’s stellar evolution code, we study the minimum mass ratio (q_{\text{min}}) of W Ursae Majoris (W UMa) binaries that have different primary masses. It is found that the minimum mass ratio of W UMa binaries decreases with increasing mass of the primary if the primary’s mass is less than about 1.3 M_{\odot}, while above this mass the ratio is roughly constant. By comparing the theoretical minimum mass ratio with observational data, it is found that the existence of low-q systems can be explained by the different structure of primaries with different masses. This suggests that the dimensionless gyration radius (k^2_1) and thus the structure of the primary is very important in determining the minimum mass ratio. In addition, we investigate the mass loss during the merging process of W UMa systems and calculate the rotation velocities of the single stars formed by mergers of W UMa binaries due to tidal instability. It is found that in the case of conservation of mass and angular momentum, the merged single stars rotate with an equatorial velocity of about ~588–819 km s^{-1}, which is much larger than their break-up velocities (v_b). This suggests that the merged stars should extend to a very large radius (~3.7–5.3 times the radii of the primaries) or else W UMa systems should lose a large amount of mass (~21–33 per cent of the total mass) during the merging process. If the effect of magnetic braking is considered, the mass loss decreases to ~12–18 per cent of the total mass. This implies that significant angular momentum and mass might be lost from W UMa systems in the course of the merging process, and this kind of mass and angular momentum loss might be driven by the release of orbital energy of the secondaries, similarly to common-envelope evolution.

Key words: instabilities – binaries: close – blue stragglers – stars: evolution – stars: rotation.

1 INTRODUCTION
W Ursae Majoris (W UMa) binaries are eclipsing variables in which two components are in contact or overflowing their Roche limiting surfaces. The components of W UMa binaries share a common convective envelope. In general, W UMa systems have total system masses 1 M_{\odot} \leq (M_1 + M_2) \leq 3 M_{\odot} and orbital periods between 0.22 and 1 d (Gazeas & Niarchos 2006). They are very common systems and can be discovered in fields, open clusters and globular clusters (Kaluzny & Rucinski 1993; Rucinski 1994, 1998, 2000). There is at least one W UMa binary for every 500 main-sequence stars in the solar neighbourhood (Rucinski 2002, 2006). Rucinski (1994) gave the relative frequency of occurrence to be one W UMa system per 275 ± 75 ordinary dwarfs in open clusters. In globular clusters, the relative frequency of occurrence of W UMa systems was also found to be very high (Rucinski 2000).

Eggleton (2006) pointed out that many binaries of short period can be expected to evolve into contact, and there is only a small region in the initial orbital period and mass ratio plane where contact does not occur if Roche-lobe overflow starts while the primary is still in the main-sequence band. W UMa systems have the lowest angular momentum that binaries composed of main-sequence components can have, so they are important sources for testing the angular momentum evolution of binaries (Rucinski 2000; Selam 2004). More importantly, W UMa binaries can be used to study Galactic structure because they have a high spatial frequency of occurrence, ease of detection and provide an absolute magnitude calibration (Rucinski 1997). In addition, W UMa systems are the possible progenitors of blue stragglers/FK Comae type (FK Com) stars (Qian et al. 2005). Some blue stragglers in star clusters are likely formed from mergers of W UMa binaries (Lombardi et al.

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The investigation of mergers of W UMa binaries can help us to understand the formation theory of blue stragglers and FK Com type stars.

Theoretical studies indicate that a W UMa system will merge into a fast-rotating single star due to tidal instability (i.e. Darwin’s instability) when the spin angular momentum of the system is more than a third of its orbital angular momentum (Hut 1980; Eggleton & Kiseleva-Eggleton 2001). The occurrence of tidal instability in W UMa binaries determines the minimum mass ratio \( q_{\text{min}} \) of these systems. W UMa binaries with mass ratio \( q = M_2/M_1 \leq q_{\text{min}} \) should not be observed, since they have merged into fast-rotating stars within a tidal time-scale (about \( 10^{10} \) yr). Therefore, the minimum mass ratio is a very important parameter in investigating the evolution and mergers of W UMa systems.

The minimum mass ratio of W UMa binaries has been investigated by many authors (Rasio 1995; Li, Han & Zhang 2004, 2005; Arbutina 2007, 2009). If the rotation of the secondaries in W UMa systems is neglected, the minimum mass ratio of W UMa-type systems is derived to be about 0.09 (Rasio 1995). If the rotation of the secondaries is taken into account and \( k_1^2 = k_2^2 = 0.06 \) (where \( k_1^2 = I_1/(M_1R_1^2) \), \( k_2^2 = I_2/(M_2R_2^2) \) are the dimensionless gyration radii for the primary and the secondary), the cut-off mass ratio of W UMa systems is derived to be 0.076 (Li & Zhang 2006). If it is assumed that the primary is a radiative main-sequence star (\( k_1^2 \approx 0.075 \)) and the secondary is a fully convective star (\( k_2^2 \approx 0.205 \)), the theoretical minimum mass ratio is derived to be about 0.094–0.109 (Arbutina 2007). These results predict that W UMa binaries with a mass ratio less than the minimum mass ratio should not be observed. However, the mass ratios of some observed W UMa binaries such as V857 Her (\( q = 0.065 \); Qian et al. 2005), AW UMa (\( q = 0.075 \); Rucinski 1992) and SX Crv (\( q = 0.079 \); Zola et al. 2004) are smaller than the theoretical minimum mass ratio. This can be explained as resulting from W UMa systems with slightly evolved primaries or differential rotation of their components (Rasio 1995; Li & Zhang 2006; Arbutina 2007). In order to remove the difference between observations and theoretical predictions, Arbutina (2009) obtained a theoretical minimum mass ratio of about 0.070–0.074 if the effects of rotation, which increase the central concentration, are included. However, little has been done to investigate the minimum ratio of W UMa systems that have different primary masses. Hurley, Pols & Tout (2000) showed that there is a difference in the structure of main-sequence stars with different masses. Because the dimensionless gyration radius of the primary depends on the structure (Li et al. 2005), this may mean that the structure of primaries with different masses needs to be taken into account in determining the minimum mass ratio of W UMa systems.

W UMa systems with \( q \leq q_{\text{min}} \) are unstable and undergo rapid merging, which might result in the formation of rapidly rotating single stars (blue stragglers or FK Com; Webbink 1976; Stepien 1995; Li et al. 2004, 2005). During the merger of a W UMa system, the secondary enters the primary and it, together with the primary, spins up. Therefore, the equatorial velocity of the single star formed by the merger of a W UMa binary would be larger than that of a W UMa binary system, which is 100–200 km s\(^{-1}\) (Selam 2004). However, the rotational velocities of blue stragglers in the field are found to be normal (Carney & Peterson 1981), while the blue stragglers in M67 are rotating slowly compared with main-sequence stars (Mathys 1991). De Marco et al. (2005) measured the rotation velocities of five rapidly rotating blue stragglers, \( v \sin i \), to be about 120, 100, 225, 50 and 50 km s\(^{-1}\), but this information cannot constrain their origin to be stellar collision or binary merger because of the lack of clear theoretical predictions. For FK Com (\( v \sin i \approx 160 \) km s\(^{-1}\)), its angular momentum is about three times smaller than that of the orbital motion in a typical W UMa binary (Rucinski 1990). Therefore, a large amount of mass and angular momentum must be lost from W UMa systems if some blue stragglers or FK Com stars are formed from mergers of W UMa systems. However, it is uncertain how much of the mass is lost during the merging process.

The purpose of this paper is to study the minimum mass ratio of W UMa systems and the mass loss during the merging process. Employing Eggleton’s stellar evolution code, we study the structure of primaries with different masses, and then determine the minimum mass ratio of W UMa binaries. We compare the theoretical minimum mass ratio with observational data and find that the existence of low-\( q \) systems can be explained by the different structure of primaries with different masses. We find that the structure of the primary is important in determining the minimum mass ratio of a W UMa binary. In addition, we investigate the mass loss during the merging process of W UMa systems. We find that W UMa systems should lose a large amount of mass, to avoid the merged stars rotating faster than their break-up velocities. If the effect of magnetic braking is considered, the angular momentum loss may be more efficient and the mass loss may decrease.

### 2 The Minimum Mass Ratio

Rasio (1995) predicted that the dynamical stability limit should depend on the structure of W UMa systems with given masses \( M_1 \) and \( M_2 \). We use a stellar evolution code to determine the dimensionless gyration radii \( k^2 \) of stars with different masses and ages, and study the effect of the interior structure (the dimensionless gyration radius) of components with different masses on the minimum mass ratio. This code was originally developed by Eggleton (1971, 1972) and Eggleton, Faulkner & Flannery (1973) and has been updated with the latest input physics during the last three decades (e.g. Han, Pols & Eggleton 1994; Pols et al. 1995, 1998; Nelson & Eggleton 2001; Eggleton & Kiseleva-Eggleton 2002). Observations of W UMa binaries show a well-defined short-period limit of about 0.22 d (Rucinski 2008), which is equivalent to a lower mass limit for the primary of approximately 0.6 \( M_\odot \) (Stepien 2006). We therefore calculate the dimensionless gyration radii \( k^2 \) of stars with solar metallicity (\( Z = 0.02 \)) and with mass \( M \) between 0.5 and 2.5 \( M_\odot \).

Fig. 1 shows the relationship of \( k^2 \) versus \( M \) for stars of age 10 Myr, 1 Gyr, 2 Gyr and 5 Gyr, respectively. It is seen from Fig. 1...
that \( k^2 \) decreases with increasing age, and this can help explain the existence of low-\( q \) W UMa systems as suggested by Li & Zhang (2006) and Arbutina (2007). In addition, we also found that \( k^2 \) decreases with increasing mass of the star if the star’s mass is less than about 1.3 \( M_\odot \), and above this mass \( k^2 \) is roughly constant. This might be caused by differences in the structure of main-sequence stars with different masses. We display the relationship of the fractional radius of the convection zone \((R_{cz}/R)\) and \( k^2 \) for stars of age 10 Myr in Fig. 2 (where \( R_{cz} \) is the thickness of the convective zone and \( R \) the stellar radius). The dotted line represents a relation between \( R_{cz}/R \) and \( k^2 \) for stars with a convective envelope and the solid line represents the relation between \( R_{cz}/R \) and \( k^2 \) for stars with a convective core. The change between convective envelope and core occurs at 1.36 \( M_\odot \).

Figure 2. The relation between the fractional radius of the convection zone \((R_{cz}/R)\) and \( k^2 \) for stars of age 10 Myr. The dotted line represents a relation between \( R_{cz}/R \) and \( k^2 \) for stars with a convective envelope and the solid line represents the relation between \( R_{cz}/R \) and \( k^2 \) for stars with a convective core. The change between convective envelope and core occurs at 1.36 \( M_\odot \).

Figure 3. The relation between \( q_{min} \) and the mass of the primary of a W UMa binary. Filled stars represent the observed W UMa binaries with extreme mass ratios and the filled circle represents TZ Boo.

between the theoretical minimum mass ratio and the mass of the primary for W UMa systems, which is shown in Fig. 3. It is seen in Fig. 3 that the minimum mass ratio decreases with the evolutionary degree of a W UMa system, as suggested by Rasio (1995) and Li & Zhang (2006). This suggests that the dynamical stability limit of W UMa systems indeed depends on the evolutionary status of such systems. It is also found that the minimum mass ratio of young W UMa binaries with an age of 10 Myr decreases with increasing mass of the primary if the primary’s mass is less than about 1.3 \( M_\odot \); above this mass the ratio is roughly constant.

We collected the physical parameters of some W UMa systems (listed in Table 1) with extreme mass ratios from the literature. The observed systems are also plotted in Fig. 3 with filled stars. TZ Boo is indicated with a different symbol (filled circle) because it has been found to be a quadruple system and its spectra have been contaminated by the third and fourth bodies (Pribulla et al. 2009). In addition, we do not include V857 Her, which has the lowest mass ratio of \( q = 0.065 \) (Qian et al. 2006), in our analysis as the primary mass is unknown. We predict that its mass must be greater than 1.25 \( M_\odot \). It is seen in Fig. 3 that W UMa systems are above the theoretical curves except for TZ Boo, i.e. they are located in the stable region although some of these systems have mass ratios lower than the minimum mass ratio predicted by previous theory. The fact that TZ Boo is located in the unstable region might be due to the uncertainty of our stellar models, in which metallicity effects are not considered. Another probable reason for the anomaly is the presence of additional companion(s).

3 Mergers of W UMA binaries

Li et al. (2008) argued that the mass ratio of W UMa systems becomes smaller and smaller owing to their dynamical evolution. Therefore, they should merge into fast-rotating stars due to Darwin’s instability if their mass ratios become smaller than the cut-off mass ratio of W UMa systems. When a W UMa binary begins to coalesce into a fast-rotating single star due to Darwin’s instability, its orbital angular momentum can be expressed as \( J_{orb} = 3J_{spin} \) (Hut 1980; Eggleton & Kiseleva-Eggleton 2001). We assumed that a W UMa binary is in synchronous rotation (i.e. \( \omega_{spin,1} = \omega_{spin,2} = \omega_{orb} = \omega_0 = 2\pi/P_{orb} \), where \( \omega_{spin,1} \) and \( \omega_{spin,2} \) are the spin angular velocities of the two components and \( \omega_{orb} \) is the orbital angular velocity of the
1. The physical parameters of W UMa binaries.

| Stars         | $q_{mb}$ | $M_1$ (M$_\odot$) | $R_1$ (R$_\odot$) | $P$ (d) | $v_{e}$ (km s$^{-1}$) | $v_{b}$ (km s$^{-1}$) | $R_{ex}$ (R$_\odot$) | $\delta M$ (M$_\odot$) | $\delta M_{mb}$ (M$_\odot$) | References |
|---------------|----------|-------------------|-------------------|--------|------------------------|------------------------|----------------------|-------------------|--------------------------|------------|
| AW UMa       | 0.078    | 1.79              | 1.9               | 0.4387 | 819.31                 | 432.85                 | 9.73                 | 0.645             | 0.349                    | (1)        |
| SX Crv       | 0.0787   | 1.246             | 1.347             | 0.3166 | 804.60                 | 419.98                 | 6.67                 | 0.439             | 0.238                    | (2)        |
| V870 Ara     | 0.082    | 1.503             | 1.67              | 0.3997 | 778.28                 | 414.27                 | 8.22                 | 0.518             | 0.280                    | (3)        |
| FP Boo       | 0.096    | 1.614             | 2.31              | 0.6405 | 673.88                 | 365.01                 | 9.73                 | 0.460             | 0.249                    | (4)        |
| DN Bootis    | 0.103    | 1.428             | 1.71              | 0.4476 | 709.84                 | 399.05                 | 7.59                 | 0.439             | 0.238                    | (5)        |
| CK Boo       | 0.107    | 1.442             | 1.521             | 0.3352 | 840.94                 | 425.18                 | 7.99                 | 0.551             | 0.299                    | (6)        |
| FG Hya       | 0.111    | 1.444             | 1.405             | 0.3278 | 792.26                 | 442.69                 | 6.96                 | 0.514             | 0.279                    | (7)        |
| GR Vir       | 0.122    | 1.376             | 1.49              | 0.347  | 788.40                 | 419.64                 | 7.34                 | 0.492             | 0.266                    | (2)        |
| ε CrA        | 0.128    | 1.72              | 2.12              | 0.5914 | 655.84                 | 393.33                 | 8.69                 | 0.487             | 0.264                    | (6)        |
| DZ Psc       | 0.135    | 1.352             | 1.469             | 0.3661 | 730.89                 | 418.92                 | 6.71                 | 0.444             | 0.241                    | (2)        |
| V776 Cas     | 0.138    | 1.75              | 1.821             | 0.4404 | 751.89                 | 428.08                 | 8.56                 | 0.597             | 0.324                    | (7)        |
| V410 Aur     | 0.144    | 1.304             | 1.397             | 0.3663 | 691.20                 | 421.89                 | 6.04                 | 0.402             | 0.218                    | (9)        |
| V677 Cen     | 0.142    | 1.06              | 1.19              | 0.325  | 646.60                 | 412.13                 | 4.94                 | 0.309             | 0.168                    | (8)        |
| AH Cnc       | 0.149    | 1.21              | 1.36              | 0.3605 | 681.99                 | 411.89                 | 5.80                 | 0.368             | 0.199                    | (10)       |
| TZ Boo       | 0.153    | 0.72              | 0.97              | 0.2976 | 587.95                 | 376.22                 | 3.56                 | 0.180             | 0.098                    | (6)        |

Columns:
- Stars – GCVS name of star; $q$ – mass ratio; $M_1$ – mass of the primary; $R_1$ – radius of the primary; $P$ – orbital period; $v_e$, $v_b$ – equatorial velocity and break-up velocity.
- Columns (5) and (6) show the ranges of the expanded fast-rotating single star formed by the merger of a W UMa binary in the case of conservation of mass and angular momentum; $R_{ex}$ – radius of the expanded fast-rotating single star formed by the merger of a W UMa binary without mass loss; $\delta M$ and $\delta M_{mb}$ – lost mass during the merging process without and with magnetic braking.

References:
- (1) Pribulla et al. (1999);
- (2) Gazeas et al. (2005);
- (3) Szalai et al. (2007);
- (4) Gazeas et al. (2006);
- (5) Şenavcı et al. (2008);
- (6) Yakut & Eggleton (2005);
- (7) Zola et al. (2005);
- (8) Maceroni & vant Veer (1996);
- (9) Yang, Qian & Zhu (2005);
- (10) Zhang, Zhang & Deng (2005).

The total angular momentum of a merging W UMa binary can therefore be approximately expressed as:

$$ J_{tot} = J_{orb} + J_{spin} \approx 4J_{spin} = 4\left(\frac{k_2^2}{k_1^2}M_1R_1^2 + k_2^2M_2R_2^2\right)\omega_0, $$

where $M_1$, $M_2$, and $R_1$, $R_2$ are the masses and radii of the primary and the secondary in solar units, and $k_1$, $k_2$ the dimensionless gyration radii for both components. We assume that the dimensionless gyration radii for both components of a W UMa binary are equal ($k_2^2 = k_2^2$) as shown by Li et al. (2005). When a W UMa binary with a mass ratio of $q$ merges into a single star, we have:

$$ J_{tot} = \frac{4(1 + q^{1.92})}{1 + q} k_1^2(M_1 + M_2)R_1^2\omega_0, $$

where $q = M_2/M_1$ is the mass ratio.

The angular momentum of the fast-rotating single star formed by the merger of a W UMa binary can be written as:

$$ J_s = k_2^2MR^2\omega = k_2^2MRv_e, $$

where $M$ and $R$ are the mass and radius of the fast-rotating single star formed by the merger of the W UMa system in solar units, $k$ is the dimensionless gyration radius and $\omega$, $v_e$ the spin angular velocity and equatorial velocity of the fast-rotating star formed from the merger.

3.1 Mergers of W UMa binaries without angular momentum loss

We assumed that the merged star and the primary have the same radius and the same dimensionless gyration radius ($k_2^2 = k_2^2$ and $R_1 = R_2$), because the mass ratio of a W UMa system is very low at the beginning of a merger. If the total angular momentum and total mass are conserved in the course of the merger ($J_s = J_{tot} = M_1R_1\omega_0$, $M = M_1 + M_2$), using equations (2) and (3) we can obtain:

$$ \omega \approx \frac{4(1 + q^{1.92})}{1 + q} \frac{\omega_0}{v_e}. $$

and

$$ v_e = R\omega \approx \frac{4(1 + q^{1.92})}{1 + q} R_1\omega_0. $$

In addition, we can calculate the break-up velocities ($v_b$) of the single stars formed by mergers of W UMa systems if W UMa systems with extreme mass ratios merge into fast-rotating stars.

The break-up velocity ($v_b$) of a single star formed by the merger can be written as:

$$ v_b \approx \left(\frac{GM_1}{R_1}\right)^{1/2}. $$

Based on equations (5) and (6), the equatorial velocities and break-up velocities for single stars formed from mergers of W UMa systems ranges from 588–819 km s$^{-1}$. If the angular momentum and mass of W UMa systems are conserved in the course of a merger, the single stars formed by mergers of a W UMa binaries will rotate at a velocity faster than their break-up velocities. This suggests that the applicability of the assumptions ($J_s = J_{tot} = R_1 \simeq R$) is unreasonable. In the course of the merger, W UMa systems should lose a large amount of mass and angular momentum, or else the merged stars should expand to a very large radius compared with that of a main-sequence star with the same mass.

It is noted that FK Com stars are rapidly rotating G-type giants that might result from the merger of close (W UMa) binaries (Webbink 1976; Bopp & Rucinski 1981; Bopp & Stencel 1981). We assumed that such a fast-rotating star expands to a very large radius ($R = R_{ex} \gg R_1$) and $J_s = J_{tot}$. Using equations (2) and (3), we can calculate the radius of the expanded merged star formed by the merger of a W UMa binary:

$$ R_{ex} \approx \frac{4(1 + q^{1.92})}{1 + q} \frac{R_1^2\omega_0}{v_e}. $$

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It is found that the radius of the expanded single star depends on the equatorial velocity. Rucinski (1990) showed that the rotational velocity \((v \sin i)\) of a FK Com star is about 160 km s\(^{-1}\). We therefore assumed that the equatorial velocity of the expanded merged star is 160 km s\(^{-1}\). We then found that the expanded merged stars formed by mergers of W UMa binaries have radii of about \(\sim 3.6–9.7 R_\odot\) (listed in Table 1) and about \(\sim 3.7–5.3\) times the radii of the primaries.

### 3.2 Mass loss without magnetic braking

If the merged stars do not expand, then W UMa systems should lose a large amount of mass and angular momentum in the course of the merger. The mass loss \((\delta M)\) during the merging process can be calculated by using the following formula:

\[
\delta M = k_s^2 (M_1 + M_2 - \delta M) R_s v_s + \delta M \rho \omega \rho v_{\text{esc}},
\]

where \(k_s\), \(R_s\) and \(v_s\) are the dimensionless gyration radius, radius and equatorial velocity of the merged star; \(R_{\text{esc}}\) and \(v_{\text{esc}}\) are the radius and velocity of the stellar wind that escapes from the system. We assumed that the merged star and the primary have the same radius and the same dimensionless gyration radius \((k_s^2 \approx k_s^2\) and \(R_s \approx R_s\)). De Marco et al. (2005) gave a mean value of the rotational velocity for five fast-rotating blue stragglers of about 160 km s\(^{-1}\). We therefore assumed that the equatorial velocity of the merged star is \(v_s \approx 160\) km s\(^{-1}\). If the effect of magnetic braking is not considered, we can assume \(R_{\text{esc}} \approx R_s \approx R_t\) and \(v_{\text{esc}} \approx v_s \approx 160\) km s\(^{-1}\). Then, using equations (2) and (8), we obtain

\[
\delta M = 4^{(1+q^{1.92})} \frac{R_t \omega_h}{1+q} \frac{k_s^2(M_1 + M_2)}{160} - 1
\]

We assumed \(k_s^2\) to be about 0.075. Based on equation (9), the mass lost during the merging process of W UMa systems can be calculated and listed in Table 1. It is found that the distribution of mass loss has a range \(\sim 0.18–0.65 M_\odot\), which is about \(\sim 21–33\) per cent of the total mass.

### 3.3 Mass loss with magnetic braking

Stars that have convective envelopes can be magnetically braked and slowed down faster than just angular momentum loss in the stellar wind would account for (Tout & Pringle 1992). The stellar wind is forced by the magnetic field to corotate out to the Alfvén radius \((R_A)\) then escapes freely \((R_{\text{esc}} = R_A)\). Then the same mass lost from the systems will remove more angular momentum than that escaped from the surface of the systems. We studied the mass loss \((\delta M_{\text{mag}})\) in the merging process of a W UMa binary when the effect of magnetic braking is included. We assumed that the stellar wind is in synchronous rotation with the merged stars \((v_{\text{esc}}/R_A = v_s/R_t)\) and \(v_s \approx 160\) km s\(^{-1}\). The mass loss can then be written as

\[
\delta M_{\text{mag}} = 4^{(1+q^{1.92})} \frac{R_t \omega_h}{1+q} \frac{k_s^2(M_1 + M_2)}{160} - 1
\]

The Alfvén radius can be expressed as (Tout & Pringle 1992)

\[
\frac{R_A}{R_t} = 1.1 \gamma^{-1/4} (\gamma/10^{-2})^{1/2},
\]

in which

\[
f = \omega/\omega_h,
\]

where \(\gamma\) is the efficiency of dynamo regeneration \((\sim 10^{-2})\) and \(\omega\) and \(\omega_h\) are the angular velocity and break-up angular velocity of the star. The break-up angular velocity reads

\[
\omega_h = \left(\frac{GM_1}{R_t^3}\right)^{1/2},
\]

and the angular velocity reads

\[
\omega = \frac{G^{1/2}(M_1 + M_2)^{1/2}}{A^{3/2}}.
\]

Based on equations (11), (13) and (14), we obtain

\[
f = (1 + q)^{1/2} \left(\frac{0.49q^{-2/3} \ln(1 + q^{-1/3})}{0.6q^{-2/3} + \ln(1 + q^{-1/3})}\right)^{3/2}.
\]

If \(q \sim 0.1\), we obtain

\[
f \approx 0.4611.
\]

From equations (11) and (16), we find

\[
\frac{R_A}{R_t} \approx 1.335.
\]

Hence, the mass loss \((\delta M_{\text{mag}})\) is determined by using equations (10) and (17) and considering the effect of magnetic braking, and is listed in Table 2. The distribution of the mass loss has a range from \(\sim 0.10–0.35 M_\odot\) in the course of a merger, which is about \(\sim 12–18\) per cent of the total mass.

### 4 DISCUSSION AND CONCLUSIONS

In this paper, we investigated the minimum mass ratio of W UMa systems that have different primary masses. In addition, we studied the mass loss during mergers of W UMa systems.

Arbutina (2009) has investigated the theoretical stability limit of W UMa binaries and considered the effects of rotation, which would increase the central concentration. Arbutina gave the minimum mass ratio of W UMa binaries to be 0.070–0.074. Considering the different structure of different primaries, we found that the minimum mass ratio of young W UMa binaries with an age of 10 Myr decreases with increasing mass of the primary if the primary’s mass is less than about 1.3 M\(_\odot\); above this mass the ratio is roughly constant. The minimum mass ratio of binaries with \(M_1 \geq 1.3 M_\odot\) is lower than the value given by Arbutina (2009). This is mainly because Arbutina considered the effect of rotation based on a \(n = 3\) polytrope (which has \(k_s^2 \approx 0.075\)). By comparing the theoretical minimum mass ratio with observational data, it is found that observational systems are in the stable region, with the exception of TZ Boo. This means that these observed W UMa systems are dynamically stable and the existence of low-\(q\) systems can be explained by the different structure of primaries with different masses. This suggests that the dimensionless gyration radius and thus the structure of the primary are very important in determining the minimum mass ratio. A W UMa system with a less massive primary will merge at a larger mass ratio. Therefore, it is necessary to consider different structures of primaries for the study of the dynamical stability of W UMa systems.

Previous theoretical studies have argued that a W UMa binary would eventually merge into a single star (Webbink 1976, 1985; Tutukov & Yungelson 1987; Mateo et al. 1990). Assuming that angular momentum and mass are not lost from W UMa systems during the merging process, it is found that the merged stars rotate faster than their break-up velocities, which is
unreasonable. One possible explanation is that merged stars expand to a very large radius which is about $\sim 3.7-5.3$ times the radii of the primaries. These expanded merged stars can be observed, like for example FK Com type stars. We needs to obtain the parameters of FK Com type stars and make comparisons with our results.

Another possible explanation is that during the merger, W UMa systems lose a large amount of mass and angular momentum. Chen & Han (2008) predicted that a large amount of mass ($\sim 0.5 M_\odot$) must be lost from W UMa systems in the course of a merger for the theoretical model to match observations. However, they did not give a physical mechanism for the mass loss. We calculated that the mass loss during the merging process of a W UMa system would be $\sim 21-33$ percent of the total mass. If the effect of magnetic braking is considered, the angular momentum loss due to mass loss may be more efficient and the mass loss will decrease to $\sim 12-18$ percent of the total mass. Our results give lower values than predicted by Chen & Han (2008). This might be due to the lack of W UMa systems with $M_1 \geq 2.0 M_\odot$ in our sample, as these systems would lose more mass during the merging process.

There is a significant difference between the theoretical predictions and observations of the rotation velocities of blue stragglers and FK Com type giants. In the colour–magnitude diagrams of globular clusters, some W UMa systems are, in fact, in the region of the blue stragglers, and there are 20 W UMa binaries observed among about 900 blue stragglers (Rucinski 2000; Von Braun & Mateo 2002; Von Braun 2003, and references therein). This implies that the formation of some blue stragglers is related to mergers of W UMa systems. In addition, Taam & Sandquist (2000) suggested that binaries in the common-envelope phase at short orbital periods must eventually merge into a single star, since the massive component cannot be on the giant branch. Our study indicates that significant angular momentum and mass might be lost from a W UMa system in the course of the merging process, and this kind of mass and angular momentum loss might be driven by the release of orbital energy of the secondaries. This is similar to common-envelope evolution. When the secondary of a W UMa system with high spin velocity merges into the primary, the orbital energy is deposited into the envelope, disrupting it. This energy can cause only some of the mass of the envelope to be ejected, because for stars near the main sequence the binding energy of the envelope is too large for energy from the orbital motion to eject it completely (Taam & Sandquist 2000). There are other possible mechanisms for angular momentum and mass loss: first, a circumstellar disc. The mass lost from the system might form a circumstellar disc, since the rotational velocity is fastest at the equator. This circumstellar disc would extract angular momentum from the single stars formed by mergers of W UMa binaries. Secondly, for FK Com type giants, radii are much larger than those of main-sequence stars with the same masses and their dimensionless gyration radii have risen to a higher value because their envelopes have become convective. This could lead to a decrease in the rotation velocities of FK Com type giants.

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