A New Quantum Vortex Operator and Its Correlation Functions

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Abstract

A new local and gauge invariant quantum vortex operator is constructed in three-dimensional gauge field theories. The correlation functions of this operator are evaluated exactly in pure Maxwell theory and by means of a loop expansion in the Abelian Higgs model. In the broken symmetry phase of the latter an explicit expression for the mass of the quantum vortices is obtained from the long distance exponential decay of the two-point function.

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1) Introduction

In several domains of physics a thorough quantum description of topological excitations is frequently needed. This is the case, for instance, of the solitons of nonlinear optics, vortices in superfluids and superconductors, magnetic monopoles and cosmic strings, to cite some examples. In these cases it is highly desirable, not only from the point of view of basic principles and aesthetics, but also from a practical and calculational perspective, to have a theory dealing directly with the quantum creation operators of the relevant topological excitations. A common feature of such kind of excitations is that they cannot in general be expressed polynomially in terms of the basic lagrangian variables and therefore a fundamental question is how to construct the corresponding creation operators.

In the present work we are going to concentrate on vortices of 2+1 dimensional gauge field theories. A quantum vortex operator was constructed in continuum space-time [1] some years ago and its correlation functions were evaluated within some approximation scheme [4]. Other similar constructions appeared previously on the lattice [2] and subsequently on the continuum [3]. A general analysis of these operators in several gauge theories in 2+1D was performed more recently [5], paying special attention to the case of Chern-Simons theories. In this paper, we will obtain a new quantum vortex operator which presents many advantages with respect to the former one [1] and even corrects some defects associated with the previous formulation [1, 7]. The new operator is, for instance, explicitly surface invariant and gauge invariant, even in the presence of a coupling to a Higgs field. There are no cutoffs appearing in the definition of the operator which should be removed at the end of any calculation. In the nontrivial case of the Abelian Higgs model, locality of the correlation function is attained already in the lowest order of a loop expansion, through an exact cancellation of the nonlocal terms. Throughout the paper, we will comment more on each of the several advantages of the present formulation.

In Sect. 2 we start by obtaining an explicitly surface invariant quantum vortex operator in the pure Maxwell theory. This operator is then generalized for the case
of the Abelian Higgs Model where an explicitly gauge invariant form is obtained. The relevant commutation rules are evaluated at the end of the section, both in the Maxwell and Abelian Higgs theories.

In Sect. 3 we evaluate the correlation functions of the new vortex operator, firstly in Maxwell theory, where an exact result can be obtained but there are no genuine vortex excitations and then in the Abelian Higgs model. In this case a loop expansion is made in order to obtain the long distance behavior of the correlation function. Contrary to the previous formulation [4], a mass expansion is no longer needed for the obtainment of the correlation function. In the broken symmetry phase there are true quantum vortex excitations whose mass is explicitly obtained from the long distance behavior of the two-point correlation function.

2) The Vortex Operator and Its Commutation Rules

2.1) The Maxwell Theory

In previous works, a quantum vortex operator was obtained in 2+1D [1, 5] for theories involving an abelian gauge field. In terms of the electric field $E^i = F^{i0}$, it can be expressed as

$$\mu(x) = \exp \left\{ -ib \int_{T_x(C)} d^2 \xi F^{i0}(\xi, x^0) \partial_i \text{arg}(\xi - x) \right\}$$  \hspace{1cm} (2.1)

where $b$ is an arbitrary real parameter with dimension $\text{mass}^{-1/2}$ and $T_x(C)$ is the surface represented in Fig.1.

Let us introduce now the new external field

$$\tilde{A}_{\mu\nu}(z; x) = b \int_{T_x(C)} d^2 \xi \partial_\mu \text{arg}(\xi - x) \delta^3(z - \xi) - (\mu \leftrightarrow \nu)$$  \hspace{1cm} (2.2)

where $d^2 \xi$ is the surface element of the surface $T_x(C)$. We can express the operator $\mu$ in terms of this and the field intensity tensor as

$$\mu(x) = \exp \left\{ -i \int d^3 z \tilde{A}_{\mu\nu} F^{\mu\nu} \right\}$$  \hspace{1cm} (2.3)
In order to get local correlation functions for $\mu$, one has to introduce a c-number renormalization factor which corresponds to the self-coupling of the external field $\tilde{A}_{\mu\nu}$. Let us start by considering the pure Maxwell theory. In this case,

$$< \mu(x)\mu^\dagger(y) > = Z^{-1} \int DA_\mu \exp \left\{ - \int d^3z \left[ \frac{1}{4}(F_{\mu\nu} + \tilde{A}_{\mu\nu})(F^{\mu\nu} + \tilde{A}^{\mu\nu}) \right] \right\}$$  \hspace{1cm} (2.4)

Observe that since $\tilde{A}_{\mu\nu}$ is not in the form $\partial_{[\mu}\tilde{A}_{\nu]}$ we cannot eliminate the external field from (2.4) by a shift in the functional integration variable as could be done in [1, 4]. The present formulation, therefore corrects this defect of the previous one.

In order to show that the above expression indeed does not depend on the specific choice of the surface $T_x(C)$, let us choose another arbitrary surface $\tilde{T}_x(C)$, also bonded to the curve $C_x$ according to Fig.2. Let us then perform the following change in the functional integration variable in (2.4)

$$A_\mu \rightarrow A_\mu + \Lambda_\mu$$  \hspace{1cm} (2.5)

with

$$\Lambda_\mu = -b\Theta(V(\tilde{T}_x))\partial_\mu arg(\vec{z} - \vec{x})$$  \hspace{1cm} (2.6)

In this expression, $\Theta(V(\tilde{T}_x))$ is the three-dimensional Heaviside function with support inside the volume $V(\tilde{T}_x)$ bounded by $T_x \cup \tilde{T}_x$. It is easy to see that because the singularity of the $arg$ function is outside this volume, we have, under the above change of variable

$$F_{\mu\nu} \rightarrow F_{\mu\nu} - b\partial_\mu \Theta(V(\tilde{T}_x))\partial_\nu arg(\vec{z} - \vec{x}) - (\mu \leftrightarrow \nu)$$  \hspace{1cm} (2.7)

or

$$F_{\mu\nu} \rightarrow F_{\mu\nu} - b \int_{\tilde{T}_x - T_x} d^2\xi \partial_\mu \Theta(V(\tilde{T}_x))\partial_\nu arg(\vec{z} - \vec{x})\delta^3(z - \vec{\xi}) - (\mu \leftrightarrow \nu)$$

$$F_{\mu\nu} \rightarrow F_{\mu\nu} + \tilde{A}_{\mu\nu}(\tilde{T}_x) - \tilde{A}_{\mu\nu}(T_x)$$  \hspace{1cm} (2.8)

From (2.4) and (2.8) it is clear that the correlation function $< \mu\mu^\dagger >$ is surface invariant. As was shown in [4], removal of the cutoffs $\rho$ and $\delta$ immediately leads to local correlation functions.
As we will demonstrate now, it is possible to obtain a simpler form for the vortex operator \( \mu \), in which the presence of no cutoff is needed. In order to achieve this, let us start by performing in (2.4) the following change of functional integration variable

\[ A_\mu \to A_\mu + \tilde{A}_\mu(z; x, y) \]  

(2.9)

where

\[ \tilde{A}_\mu(z; T_x) = b \int_{T_x} d^2 \xi \arg(\xi - x) \delta^3(\xi - x) \]  

(2.10)

The correlation function becomes

\[ \langle \mu(x)\mu^\dagger(y) \rangle = Z^{-1} \int DA_\mu \exp \left\{ -\int d^3 z \left\{ \frac{1}{4}(F_{\mu\nu} + \tilde{A}_{\mu\nu})(F^{\mu\nu} + \tilde{A}^{\mu\nu}) + \mathcal{L}_{GF} + V(\phi) \right\} \right\} \]  

(2.11)

where

\[ \tilde{A}_{\mu\nu} = \tilde{A}_{\mu\nu} - \partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu \]

\[ \tilde{A}_{\mu\nu} = b \int_{T_x(\mathcal{C})} d^2 \xi_\nu \partial_\mu \arg(\xi - x) \delta^3(z - \xi) - \int_{T_x(\mathcal{C})} d^2 \xi_\nu \arg(\xi - z) \partial_\mu(\xi) \delta^3(z - \xi) - (\mu \leftrightarrow \nu) \]  

(2.12)

The vortex operator becomes

\[ \mu(x) = \exp \left\{ -i \int d^3 z \tilde{A}_{\mu\nu} F^{\mu\nu} \right\} \]  

(2.13)

and we may, therefore, write now

\[ \mu(x) = \exp \left\{ -ib \int_{C_x(\mathcal{C})} d^2 \tilde{\xi} \partial_i \left[ F^{i0}(\tilde{\xi}, x^0) \arg(\tilde{\xi} - \tilde{x}) \right] \right\} \]  

(2.14)

Observe that this reduces to (2.1) because in the absence of matter \( \partial_i F^{i0} = 0 \).

Let us take the general expression for the vortex operator, given by (2.14). Observing that we have the surface integral of a total derivative, we may use Stokes’ theorem to write

\[ \mu(x) = \exp \left\{ -ib \int_{C_x} d\xi^i \epsilon^{ij} F^{ij}(\xi, x^0) \arg(\xi - \tilde{x}) \right\} \]  

(2.15)

where \( C_x \) is the curve defined in Fig. 1. Taking the limit in which the cutoffs \( \rho \) and \( \delta \) go to zero, we can express the vortex operator in a simple form, in terms of a line integral

\[ \mu(x) = \exp \left\{ -i2\pi b \int\limits_{\tilde{x}, L} d\xi^i \epsilon^{ij} F^{ij}(\tilde{\xi}, x^0) \right\} \]  

(2.16)
This can also be put in a covariant form, namely,

\[ \mu(x) = \exp \left\{ -i\pi b \int_{\vec{x},L}^\infty d\xi \mu_{\epsilon\alpha\beta} F^{\alpha\beta}(\xi, x^0) \right\} \]  

(2.17)

This form of the vortex operator is a natural extension for 2+1D of the corresponding soliton operator first introduced in 1+1D by Mandelstam [3, 4], namely

\[ \mu_{1+1} = \exp \left\{ -ib \int_{x}^{\infty} d\xi \mu_{\epsilon_{\mu\nu}} \partial_{\nu} \phi \right\} \]  

(2.18)

where \( \phi \) is a scalar field, the Sine-Gordon field, for instance.

Finally, it is extremely convenient, again, to express the vortex operator in terms of an external field

\[ \mu(x) = \exp \left\{ \frac{1}{2} \int d^3z \tilde{B}_{\alpha\beta} F^{\alpha\beta}(z) \right\} \]  

(2.19)

where the external field \( \tilde{B}_{\mu\nu} \) is given by

\[ \tilde{B}_{\alpha\beta} = i2\pi b \int_{\vec{x},L}^\infty d\xi \mu_{\epsilon_{\mu\alpha\beta}} \delta^3(z - \xi) \]  

(2.20)

It is obvious that we can perform the same transformations we did in order to arrive at the above expression, also to the quadratic self-coupling \( \bar{A}_{\mu\nu} \) term in the correlation function. We can therefore substitute \( \bar{A}_{\mu\nu} \) for \( \tilde{B}_{\mu\nu} \) in (2.11). Observe that also here we cannot eliminate \( \tilde{B}_{\mu\nu} \) from (2.11) by a shift in the functional integration variable.

2.2) The Abelian Higgs Model

One has to consider now what happens when the gauge field is coupled to a charged field. Let us take the Abelian Higgs model as a paradigm. This is defined by

\[ \mathcal{L}_{AH} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + |D_{\mu} \phi|^2 - V(\phi) \]  

(2.21)

As usual, \( D_{\mu} = \partial_{\mu} + ieA_{\mu} \), where \( e \) is the gauge coupling constant.

Before considering the construction of the vortex operator in the Abelian Higgs Model, let us introduce the following representation of the Higgs field which is going to be very convenient

\[ \phi = \frac{1}{\sqrt{2}} \rho \ e^{i\theta} \]  

(2.22)
In terms of $\rho$ and $\theta$ we can rewrite the lagrangian as

$$\mathcal{L}_{AH} = -\frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \frac{1}{2} e^2 \rho^2 (A_\mu + \frac{1}{e} \partial_\mu \theta) (A^\mu + \frac{1}{e} \partial^\mu \theta) + \frac{1}{2} \partial_\mu \rho \partial^\mu \rho - V(\rho) \quad (2.23)$$

This is clearly invariant under gauge transformations

$$A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$$
$$\theta \rightarrow \theta - e \Lambda \quad (2.24)$$

Introducing a new vector field

$$W_\mu = A_\mu + \frac{1}{e} \partial_\mu \theta \quad (2.25)$$

and using the field equation for $\theta$

$$\Box \theta = -e \partial_\mu A^\mu \quad (2.26)$$

which allows us to write

$$\partial_\mu W^\mu = 0 \quad (2.27)$$

we cast the lagrangian in the form

$$\mathcal{L}_{AH} = -\frac{1}{4} W_{\mu\nu}W_{\mu\nu} + \frac{1}{2} e^2 \rho^2 W_\mu \left[ \frac{-\Box \delta_{\mu\nu} + \partial^\mu \partial^\nu}{\Box} \right] W_\nu + \frac{1}{2} \partial_\mu \rho \partial^\mu \rho - V(\rho)$$

or

$$\mathcal{L}_{AH} = -\frac{1}{4} W_{\mu\nu} \left[ 1 + \frac{e^2 \rho^2}{\Box} \right] W_{\mu\nu} + \frac{1}{2} \partial_\mu \rho \partial^\mu \rho - V(\rho) \quad (2.28)$$

In this form, the gauge invariance of the lagrangian becomes explicit.

We can now construct the vortex creation operator appropriate for the Abelian Higgs model. It is clear from the analysis done in the previous subsection that a local operator correlation function can be obtained by adding to $W_{\mu\nu}$ the external field intensity tensor $\bar{A}_{\mu\nu}$, similarly to (2.11):

$$\langle \mu(x) \mu^\dagger(y) \rangle = Z^{-1} \int DW_\mu D\rho \exp \left\{ - \int d^3 z \left[ \frac{1}{4} (W_{\mu\nu} + \bar{A}_{\mu\nu}) \left[ 1 + \frac{e^2 \rho^2}{\Box} \right] (W^{\mu\nu} + \bar{A}^{\mu\nu}) 
+ \mathcal{L}_{GF} + \frac{1}{2} \partial_\mu \rho \partial^\mu \rho - V(\rho) \right] \right\} \quad (2.29)$$
Surface independence of the above expression can be shown by applying the same transformation as in (2.7). Also in (2.29), we can exchange \( A_{\mu\nu} \) for \( \tilde{B}_{\mu\nu} \) using the same procedure utilized in the last subsection thereby making surface independence explicit.

From (2.29), we can infer the form of the quantum vortex operator in the Abelian Higgs model, namely

\[
\mu(x) = \exp \left\{ \frac{1}{2} \int d^3 z \tilde{B}_{\alpha\beta} \left[ 1 + \frac{e^2 \rho^2}{-\Box} \right] F^{\alpha\beta}(z) \right\} \tag{2.30}
\]

In what follows, we are going to study the properties of the vortex operator both in the Maxwell and Abelian Higgs theories.

### 2.3) Commutation Rules

Let us evaluate in this subsection the commutation rules of \( \mu \) which characterize it as a vortex operator. First of all, consider the commutator with the topological charge, which in 2+1D happens to be the magnetic flux along the xy-plane,

\[
Q = \int d^2 z J^0 = \int d^2 z \epsilon^{ij} \partial_i A_j
\]

\[
J^\mu = \epsilon^{\mu\nu\alpha} \partial_\nu A_\alpha \tag{2.31}
\]

Observe that we can write the vortex operator as \( \mu \equiv e^A \), where both in the pure Maxwell theory and the Abelian Higgs model, we have

\[
A = -i\pi b \int_{x,L}^\infty d\xi \epsilon_{ij} \Pi^j(\xi, x^0) \tag{2.32}
\]

where \( \Pi^i \) is the momentum canonically conjugated to \( A_i \).

Using the fact that \([A, J^0] \) is a c-number, we can write

\[
[J^0(y), \mu(x)] = -\mu(x)[A(x), J^0(y)] \tag{2.33}
\]

Indeed,

\[
[A(x), J^0(y)] = i2\pi b \int_{x,L}^\infty d\xi \epsilon_{ij} \epsilon^{kl} \partial_k^{(y)} [\Pi^j(\xi, t), A_i(\vec{y}, t)]
\]

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\[ [A(x), J^0(y)] = -2\pi b \int_{x,L}^\infty d\xi^i \partial_i \delta^2(\vec{\xi} - \vec{y}) = -2\pi b \delta^2(\vec{x} - \vec{y}) \]  \hspace{1cm} (2.34)

where we have used the fact that \([\Pi^j(\vec{\xi}, t), A_i(\vec{y}, t)] = i\delta^j_i \delta^2(\vec{\xi} - \vec{y})\). Inserting (2.34) in (2.33) and integrating over \(y\), we get

\[ [Q, \mu(x)] = 2\pi b \mu(x) \]  \hspace{1cm} (2.35)

This shows that \(\mu\) creates states carrying \(2\pi b\) units of topological charge (magnetic flux).

There is another commutation relation which characterizes \(\mu\) as a vortex creation operator. This is the one with the gauge field potential \(A_i\). Once again, following exactly the same procedure above, we obtain

\[ [\mu(\vec{x}, t), A_i(\vec{y}, t)] = 2\pi b \mu(\vec{x}, t) \int_{x,L}^\infty d\xi^j \epsilon^{ij} \delta^2(\vec{\xi} - \vec{y}) \]

This can be written as

\[ [\mu(\vec{x}, t), A_i(\vec{y}, t)] = \lim_{\rho, \delta \to 0} \oint_{C_x} \epsilon_{ij} d\xi^j \text{arg}(\vec{\xi} - \vec{x}) \delta^2(\vec{\xi} - \vec{y}) \]

By the use of Stokes’ theorem, this can be written as

\[ [\mu(x), A_i(y)] = \mu(x)b \int_{T_x(C)} d^2\vec{\xi} \partial_i \delta^2(\vec{\xi} - \vec{y}) \text{arg}(\vec{\xi} - \vec{x}) \]

\[ = \mu(x) b \left[ \partial_i \delta^2(\vec{y} - \vec{x}) \Theta(\vec{y}; T_x(C)) + \text{arg}(\vec{y} - \vec{x}) \right] \int_{T_x(C)} d^2\xi \partial_i \delta^2(\vec{\xi} - \vec{y}) \]

By using Stokes’ theorem once more, the last surface integral above can be transformed in a line integral which vanishes in the limit when the \(\rho\) and \(\delta\) cutoffs are removed

\[ \oint_{C_x} d\xi^j \epsilon^{ij} \delta^2(\vec{\xi} - \vec{y}) \stackrel{\rho, \delta \to 0}{\longrightarrow} 0 \]

We therefore conclude that the commutator of the \(\mu\) operator with the gauge potential is,

\[ [\mu(x), A_i(y)] = \mu(x)b \partial_i \delta^{(y)} \text{arg}(\vec{y} - \vec{x}) \]  \hspace{1cm} (2.36)

where we have already taken the limit in which the cutoffs \(\rho\) and \(\delta\) vanish and the Heaviside function in the first term becomes identically one. This relation again shows that the \(\mu\) operator does indeed create a vortex field configuration.
3) The Quantum Vortex Correlation Functions

3.1) The Maxwell Theory

Let us evaluate here the euclidean correlation functions of the vortex operator introduced in the last section. Using (2.19), we can write

\[< \mu(x)\mu^\dagger(y) > = Z^{-1} \int DA_\mu \exp \left\{ - \int d^3z \left[ \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \mathcal{L}_{GF} + \frac{1}{2} \tilde{B}_{\mu\nu}(z;x,y) F_{\mu\nu} + \frac{1}{4} \tilde{B}_{\mu\nu} \tilde{B}^{\mu\nu} \right] \right\} \]

(3.1)

where \(\tilde{B}_{\mu\nu}(z;x,y) = i\tilde{B}_{\mu\nu}(z;x) - i\tilde{B}_{\mu\nu}(z;y)\) and \(\tilde{B}_{\mu\nu}(z;x)\) is given by (2.20). The i factor in the previous expression appears after the analytic continuation to the euclidean space. The last term in the exponent in (3.1) is the renormalization factor, equivalent to the one appearing in (2.4), which will eliminate the unphysical self-interaction of the the line \(L\) which occurs in the definition of the \(\mu\) operator.

The quadratic functional integral in (3.1) can be easily performed with the help of the euclidean propagator of the electromagnetic field and the result is

\[< \mu(x)\mu^\dagger(y) > = \exp \left\{ \frac{1}{8} \int d^3zd^3z' \tilde{B}_{\mu\nu}(z) \tilde{B}_{\alpha\beta}(z') P^\mu_\alpha P^\nu_\beta D^\lambda_\rho(z - z') - \frac{1}{4} \int d^3z \tilde{B}^\mu_\nu \tilde{B}^{\mu\nu} \right\} \]

(3.2)

where \(P^\mu_\alpha = \partial^\mu \delta^\alpha_\nu - \partial^\nu \delta^\mu_\alpha\) and

\[D^\lambda_\rho = [-\Box \delta^\lambda_\rho + (1 - \xi^{-1})\partial^\lambda \partial^\rho] \left[ \frac{1}{(-\Box)^2} \right] \]

is the euclidean \(A_\mu\) propagator. To comply with the gauge condition (2.27) we must actually choose \(\xi \to \infty\).

We immediately see that the second term of \(D\) vanishes anyway when contracted with the \(P's\) and only the gauge invariant first part gives a contribution to the correlation function. Inserting (2.20) in (3.2), contracting the two \(P's\) and integrating over \(z\) and \(z'\) we get

\[< \mu(x)\mu^\dagger(y) > = \exp \left\{ \frac{b^2}{2} \sum_{i,j=1}^2 \lambda_i \lambda_j \left[ \int_{x_1}^{x_2} d\xi_k \epsilon^{k\mu\nu} \int_{x_1}^{x_2} d\eta_l \epsilon^{l\alpha\beta} \frac{1}{4} \left[ \partial_\nu \delta^\beta_\alpha - \partial_\mu \delta^\alpha_\beta \right] + \right] \right\} \]

(3.3)
\[
\partial_\nu \partial'_\beta \delta^{\alpha\mu} - \partial_\nu \partial'_\alpha \delta^{\beta\mu} \right] F(\xi - \eta) - \int_{x_i}^\infty d\xi_1 \int_{x_j}^\infty d\eta_1 \delta^3(\xi - \eta) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right) \right)
\]
where \( x_1 \equiv x, \ x_2 \equiv y, \ \lambda_1 \equiv 1 \) and \( \lambda_2 \equiv -1 \). The last term is the renormalization factor and
\[
F(x) = \frac{1}{-\Box_E} = \frac{1}{4\pi|x|} \tag{3.4}
\]
The first four terms above are equal and give, each one, a contribution
\[
\int_{x_i}^\infty d\xi_1 \int_{x_j}^\infty d\eta_1 \left[ \delta^{ij}(-\Box_E) - \partial^i(\xi_E) \partial^j(\eta) \right] F(\xi - \eta) \tag{3.5}
\]
We see that the first term above, which contains all the line dependence and therefore the nonlocality, is canceled by the last one in (3.3). The second one, produces four terms, \( F(A - B) \), where \( A \) and \( B \) correspond to the limits of the two integrals. Since \( \sum_i \lambda_i = 0 \), however, only the \( F(x_i - x_j) \) contribution to the sum in (3.3) survives. We finally conclude that
\[
< \mu(x)\mu^\dagger(y) > = \exp \left\{ -\frac{b^2}{2} \sum_{ij} \lambda_i \lambda_j F(x_i - x_j) \right\} = \exp \left\{ \frac{b^2}{2} F(x - y) - b^2 F(\epsilon) \right\} \tag{3.6}
\]
Introducing the renormalized field
\[
\mu_R = \mu \exp \left\{ \frac{b^2}{2} F(\epsilon) \right\}
\]
we obtain the final result for the vortex correlation function in Maxwell’s theory:
\[
< \mu(x)\mu^\dagger(y) >_R = \exp \left\{ \frac{b^2}{4\pi|x - y|} \right\} \tag{3.7}
\]
The behavior of this correlation function at infinity shows that there are no genuine vortex excitations in pure Maxwell theory. This happens because
\[
< \mu^\dagger | x - \eta \rightarrow \infty > | \mu > |^2 = 1 \tag{3.8}
\]
This relation clearly shows that \( < 0|\mu > = 1 \), indicating that the vortex states in this case are not orthogonal to the vacuum.
3.2) The Abelian Higgs Model

Let us consider now the nontrivial case of the Abelian Higgs Model, which is described by the lagrangian given either by (2.21) or (2.28). Let us concentrate first on the broken phase, where $g^2 < 0$, $\rho \equiv \rho_0 = (4\pi |g|^2 / \lambda)$ and the gauge field acquires a mass $M = e\rho_0$ after the shift $\rho \rightarrow \rho + \rho_0$ is performed. The vortex correlation function is now given by

$$
< \mu(x)\mu^\dagger(y) > = Z^{-1} \int D\rho D\rho' \exp \left\{ - \int d^3z \left[ \frac{1}{4} (W_{\mu\nu} + \tilde{B}_{\mu\nu}) \left[ 1 + \frac{e^2 (\rho + \rho_0)^2}{-\Box} \right] \right] 
+ \frac{1}{2} \partial_\mu \rho \partial_\mu \rho' + V(\rho) \right\} = \exp \{ \Lambda(x, y) - S_R[\tilde{B}_{\mu\nu}] \} 
$$

(3.9)

In this expression, $\Lambda(x, y)$ is the sum of all Feynman graphs with the field $\tilde{B}_{\mu\nu}$ in the external legs and $S_R[\tilde{B}_{\mu\nu}]$ is the self-coupling of this external field. We are going to evaluate (3.9) by means of a loop expansion. Furthermore, we are going to consider the long distance limit of (3.9). In this limit, it can be shown [4, 5] that only two legs graphs contribute to $\Lambda(x, y)$.

The relevant vertex for the leading term (0-loop) in this expansion comes from the linear coupling of $\tilde{B}_{\mu\nu}$ with the gauge field $W_\mu$, which can be inferred from expression (3.9), namely

$$
\frac{1}{2} \tilde{B}_{\mu\nu} P_\mu^\alpha \left[ 1 + \frac{M^2}{-\Box} \right] W^\alpha \quad (3.10)
$$

Under the above mentioned conditions, the leading contribution to $\Lambda(x, y)$ is given by the graph of Fig. 3 and the correlation function is given by

$$
< \mu(x)\mu^\dagger(y) > = \exp \left\{ \frac{1}{8} \int d^3zd^3z' \tilde{B}^{\mu\nu}(z) \tilde{B}_{\mu\nu}(z') \tilde{P}_\alpha^\mu \tilde{P}_\rho^\alpha \left[ 1 + \frac{M^2}{-\Box} \right] \left[ 1 + \frac{M^2}{-\Box} \right] \right\}
$$

(3.11)

In this expression, $D_M^{\mu\nu}(z - z')$ is the massive gauge field propagator, which is given by

$$
D_M^{\mu\nu}(z - z') = \frac{\delta^{\mu\nu}}{-\Box + M^2} + \text{gauge terms} \quad (3.12)
$$

and the last term is the self-interaction of the external field.
We may evaluate (3.11) by following exactly the same steps as we did in the last subsection, Eqs.(3.2) to (3.7). The only difference is that now $F(x)$ in (3.3), (3.4) and (3.5) is exchanged by
\[
F_M(x) = \left[ 1 + \frac{M^2}{-\Box_E} \right] \left[ \frac{1}{-\Box_E + M^2} \right] \left[ 1 + \frac{M^2}{-\Box_E} \right]
\]
and the self-interaction term, the last one in (3.11), contains the $M^2$ dependent piece. As a consequence, the last term in (3.3), which contains a delta function becomes multiplied by $1 + M^2/(-\Box_E)$.

In (3.13), $m$ is a mass regulator introduced in order to give meaning to the inverse Fourier transform of $\frac{1}{k^4}$, which appears corresponding to $\frac{1}{(-\Box_E)^2}$.

Observe that in the same way as in the case of pure Maxwell theory, the first term in (3.3) which contains all the nonlocal dependence is exactly canceled by the renormalization term. Following the same steps as we did from (3.6) to (3.7) it is easy to see that all the $m$ dependence vanishes because $\sum_i \lambda_i = 0$ and we get
\[
< \mu(x)\mu^\dagger(y) > \xrightarrow{\|x-y\| \to \infty} \exp \left\{ -\mathcal{M}|x-y| + \frac{b^2}{4\pi|x-y|} \right\}
\]
In this expression, $\mathcal{M} = \frac{b^2 M^2}{8\pi} = \frac{b^2 e^2 \rho_0^2}{8\pi}$ is the quantum vortex mass in the order of approximation we are working at. We now see that the vortex operator indeed creates genuine excitations which are orthogonal to the vacuum. This follows from the fact that $< \mu\mu^\dagger > \xrightarrow{\|x-y\| \to \infty} 0$ and hence $< \mu > = 0$.

In the symmetric phase, where $\rho_0 \to 0$ the vortex mass $\mathcal{M}$ vanishes and only the second term contributes to (3.14). The correlation function has the same long distance behavior as the exact one in pure Maxwell theory. This indicates that, as expected, there are no genuine vortex excitations in the symmetric phase.
4) Concluding Remarks

There are many advantages in the new formulation for the quantization of three-dimensional vortices presented here. The vortex operator is explicitly surface invariant. There are no cutoffs appearing in the definition of the operator. Even in the case of the Abelian Higgs model there are only gauge invariant degrees of freedom appearing in the definition of the operator, in such a way that gauge invariance is always explicit. Perhaps the nicest feature of this formulation, however, is the fact that a mass expansion is not required for the obtainment of the vortex correlation function in the nontrivial case of the Abelian Higgs model. The mass expansion was a common characteristic of the previous formulation. Also there is an exact cancelation of all the nonlocal terms already in lowest order in the loop expansion.

There are many possible applications for the formulation developed here. We can envisage the case of vortices in superconductors and maybe also superfluids. It would be interesting anyway to study the temperature dependence of the correlation functions studied here in connection with these applications.

We have also established a generalization of the present formulation for the case of quantum cosmic strings in 3+1D which is going to be published elsewhere. In any of the above mentioned cases, however, in order to obtain quantities like the scattering amplitudes of topological excitations as well as cross sections for processes involving these excitations, the knowledge of an asymptotic theory for the corresponding creation operators will be needed. This is so far not known and is a very interesting field of research.
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Figure Captions

Fig. 1 - Surface used in the definition of the external field $A_{\mu\nu}(z;x)$ and the operator $\mu$

Fig. 2 - The surface $\tilde{T}_x(C)$

Fig. 3 - Leading graph contributing to the vortex correlation function. The external legs are $\tilde{B}_{\mu\nu}$’s. The wavy line is a $W_\mu$ propagator. The vertex is defined by 3.10.
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