DYNAMICS OF TOPOLOGICAL DEFECTS IN ELEKTRONWEEK THEORY

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Embedded defects proposed long ago (Z-vortices and Nambu monopoles) have
been successfully searched for in 3D equilibrium lattice studies within the standard
model near the electroweak phase transition and the crossover (which follows it for
realistic Higgs mass). Gauge independent lattice–vortex operators are proposed.
Vortex condensation (percolation) is found to characterize the high–temperature
phase. Small vortex clusters are thermally activated with non–negligible density on
the low–temperature side only at higher Higgs mass, where preliminary evidence
supports their semiclassical nature.

1 Introduction
In this contribution some recent work\cite{1,2} has been reviewed dealing with the
topological signatures in the dimensionally reduced lattice \( SU(2) \) Higgs model.
This model provides an effective representation of the electroweak theory at
temperatures near the electroweak scale in some range of Higgs masses.\cite{3,4}

Lattice studies have shown\cite{5,6,7} that the electroweak theory does not have
the true thermal phase transition (for realistic Higgs mass) necessary to explain
the generation of baryon asymmetry within the standard model. Qualitative
properties of the model (without extensions) are still of interest, in order to
discuss the high temperature properties of gauge–matter systems in general and
in view of alternative scenarios of baryogenesis. Topological defects (vortices)
and their condensation (percolation) are expected to play an important role in
these contexts. Therefore, independently of quantitative estimates about the
effectiveness of string–mediated baryogenesis, a first step towards the study of
embedded topological defects seemed to be worthwhile. It has turned out that,
for Higgs masses not much below the endpoint of the phase transition\cite{6}
and in the so–called crossover region above, these defects not only condense in the
symmetric phase as they do at lower Higgs mass. These defects (Z–vortices\cite{8})
can be thermally generated, also in the broken phase, as semiclassical objects with non–vanishing density while they have been shown to be unstable at zero and finite temperature.

2 The lattice model and defect operators

The 3D lattice model is defined by the action

\[ S = \beta_G \sum_p \left( 1 - \frac{1}{2} \text{Tr} U_p \right) - \beta_H \sum_{x, \mu} \text{Re} \left( \phi_x^\dagger U_{x,\mu} \phi_{x+\hat{\mu}} \right) + \sum_x \left( \phi_x^\dagger \phi_x + \beta_R \left( \phi_x^\dagger \phi_x - 1 \right)^2 \right) \]

(1)

with the 2–component complex isospinor \( \phi_x \) for the Higgs field. The lattice couplings are related to the continuum parameters of the 3D SU(2) Higgs model \( g_3, \lambda_3 \) and \( m_3 \). The lattice gauge coupling \( \beta_G = \frac{4}{a g_3^2} \) (with \( g_3^2 \approx g_4^2 T \)) gives the lattice spacing in units of temperature, and the hopping parameter \( \beta_H \) substitutes \( m_2^2 \) driving the transition. The parameter of the phase transition or the crossover can be translated into a temperature and a Higgs mass \( M_H \approx M_H^* \) where the parameter \( M_H^* \) is used to parametrize the Higgs self–coupling

\[ \beta_R = \frac{\lambda_3}{g_3^2} \beta_G^2 = \frac{1}{8} \left( \frac{M_H^*}{80 \text{ GeV}} \right)^2 \frac{\beta_R}{\beta_G}. \]

(2)

The Z–vortex corresponds to the Abrikosov-Nielsen-Olesen vortex solution related to the Abelian subgroup of SU(2) and embedded into the SU(2) gauge field of the electroweak theory. We use gauge invariant lattice definitions for Z–vortices and Nambu monopoles as extended topological objects of size \( ka \). The construction for elementary \( (k=1) \) defects proceeds as follows. A composite adjoint unit vector field \( n_x = n^a_x \sigma^a, n^a_x = -(\phi_x^+ \sigma^a \phi_x)/(\phi_x^+ \phi_x) \) is introduced which allows to define the gauge invariant flux \( \bar{\theta}_p \) through the plaquette \( p = \{x, \mu\} \)

\[ \bar{\theta}_p = \text{arg} \left( \text{Tr} \left[ \left( 1 + n_x \right) V_{x,\mu} V_{x+\hat{\mu},\nu} V_{x+\hat{\nu},\mu} V_{x,\nu}^+ \right] \right) \]

(3)

via the projected links \( V_{x,\mu}(U, n) = U_{x,\mu} + n_x U_{x,\mu} n_{x+\hat{\mu}} \). The plaquette angle \( \chi_p \) is constructed with the help of the Abelian link angles \( \chi_{x,\mu} = \text{arg} \left( \phi_x^+ V_{x,\mu} \phi_{x+\hat{\mu}} \right) \) as usual: \( \chi_p = \chi_{x,\mu} + \chi_{x+\hat{\mu},\nu} - \chi_{x+\hat{\nu},\mu} - \chi_{x,\nu} \). The Z–vorticity number \( \sigma_p \) of plaquette \( p \) and the monopole charge \( j_c \) carried by the cube \( c \) (defined in terms of the fluxes \( \bar{\theta}_p \) penetrating the surface \( \partial c \)) are given by

\[ \sigma_p = \frac{1}{2\pi} \left( \chi_p - \bar{\theta}_p \right), \quad j_c = -\frac{1}{2\pi} \sum_{p \in \partial c} \bar{\theta}_p. \]

(4)
A $Z$–vortex is formed by links $l = \{x, \rho\}$ of the dual lattice ($l$ dual to $p$) which carry a non–zero vortex number: $^{*}\sigma_{x,\rho} = \varepsilon_{\rho\mu\nu}\sigma_{x,\mu\nu}/2$. $Z$–vortex trajectories are either closed or begin/end on Nambu (anti–) monopoles: $\sum_{\mu=1}^{3} (^{*}\sigma_{x-\hat{\mu},\mu} -^{*}\sigma_{x,\mu}) = ^{*}j_{x}$. Extended monopoles (vortices) on $k^{3}$ cubes ($k^{2}$ plaquettes) are constructed analogously replacing the elementary plaquettes in terms of $V_{x,\mu}$ by Wilson loops of corresponding size.

We call a vortex cluster a set of connected dual links carrying non–zero vorticity (vortex trajectories). A bond percolation algorithm (known from cluster algorithms for spin models) has been used to separate the various disconnected $Z$–vortex clusters that coexist in a lattice configuration.

We have measured the total densities $\rho_{m} = \sum_{c} |j_{c}|/L^{3}$ of Nambu monopoles and $\rho_{v} = \sum_{p} |^{*}\sigma_{p}|/(3 L^{3})$ of vortex links as well as the percolation probability of $Z$–vortex trajectories $C = \lim_{r \rightarrow \infty} C(r)$ derived from the cluster correlation function $C(r) = \sum_{x,y,i} \delta_{x^{*}\sigma^{(i)}(x)} \delta_{y^{*}\sigma^{(i)}(y)} \cdot \delta(|x-y|-r)/\sum_{x,y} \delta(|x-y|-r)$. A cluster $^{*}\sigma^{(i)}$ contributes to the correlator if the vortex lines pass through both points $x$ and $y$. The average number of $Z$–vortex clusters and the average number of dual $Z$–vortex links per cluster have been measured to characterize the structural change near the percolation transition across the electroweak crossover.

3 Monte Carlo results

3.1 At thermal first order phase transition

We have scanned the phase transition with elementary defects at $M_{*H}^{*} = 30$ GeV (strong first order) and 70 GeV (weak first order) for $\beta_{G} = 12$ and lattice volume $16^{3}$. At the lower Higgs mass we have observed a discontinuity of the densities $\rho_{m,v}$ jumping to zero at the critical temperature $T_{c}$. The percolation probability $C$ has a finite jump to zero at $T_{c}$. The same study near the endpoint of the first order transition is summarized in Fig. 1 for increasing

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Densities of elementary Nambu monopoles $\rho_{m}$ and $Z$–vortices $\rho_{v}$ (left) and percolation probability $C$ (right) vs. \(\beta_{H}\) at $M_{*H}^{*} = 70$ GeV and $\beta_{G} = 12$.}
\end{figure}

$\beta_{H}$ (decreasing temperature). Now the percolation probability continuously vanishes towards $\beta_{Hc} = 0.34355$. There are inflection points of the densities
\( \rho_m \) and \( \rho_v \) at this value of \( \beta_H \) where the corresponding objects are approximately half as abundant compared to the symmetric phase. For \( \beta_H > \beta_{Hc} \) the densities decrease exponentially. Within our accuracy and using the mentioned percolation definition the critical temperature \( T_c \) and the percolation temperature \( T_{perc} \) coincide in this model.

### 3.2 In the crossover region

From a phenomenological point of view the crossover region, investigated in our studies at \( M_H^* = 100 \, \text{GeV} \), is more interesting because it is not excluded by experimental evidence. The Monte Carlo results presented in Figs. 2-3 show the existence of a network of Z–vortices on the high–temperature side of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figures/figure2.png}
\caption{Same as Fig. 1 for defects of size \( k=2 \) at \( M_H^* = 100 \, \text{GeV}, \beta_G = 16 \) and lattice \( 32^3 \).}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figures/figure3.png}
\caption{Average length \( L \) per cluster (vortex size \( k = 2 \)) for the same ensemble as in Fig. 2.}
\end{figure}

the crossover, with finite probability of percolating (at \( T > T_{perc} \)), while only smaller clusters occur below \( T_{perc} \), however much more abundantly than at lower Higgs mass. Some of these clusters are closed, others open (with Nambu monopoles at the ends). We have checked that there is an universal percolation temperature for vortices of extension \( k a \) at respective \( \beta_G^{(k)} = k \beta_G^{(1)} \) keeping the physical volume fixed. The estimate is \( T_{perc} = 170 \) or 130 GeV (without or with \( t \)-quarks) with a Higgs mass \( M_H = 94 \) or 103 GeV. We note that the crossover/percolation transition is also accompanied by interesting physics in so far as the spectral evolution in various channels is characterized by strong mixing of gauge and scalar degrees of freedom.

The physical implications of the decay of the percolating network of Z–strings into many small clusters seem to be interesting and worth to be studied more in detail. For a cosmological context the kinetics of this phase transition is interesting. It will differ from the bubble dominated transition of the strong first order transition and needs real time simulations.

We propose to identify some fraction of the density \( \rho_v \) on the “broken phase” side of the crossover as the density of sphalerons. This conjecture is supported by the signature of a classical lattice sphaleron with respect to the
new (Z–vortex and Nambu monopole) degrees of freedom. Fig. 4 shows one of the solutions with a Nambu monopole–antimonopole pair in the center. Note also that the average cluster size is $\mathcal{L} = 2$ (Fig. 3).

Figure 4: Classical sphaleron as a Nambu monopolium bound by a Z-vortex string. The clouds show the suppression in Higgs field modulus.

Why do we believe that the lattice defect operators (4) detect not just lattice artifacts? Encouraging although preliminary quantitative information is provided by investigations of the continuum limit (this requires the calculation of the density of extended defects having various sizes on finer lattices) and by measuring local averages of gauge field action and Higgs field near the Z–vortex soul. Fig. 6 shows this for elementary vortices (non–zero vs. zero vorticity plaquettes) for a rather coarse lattice. When vortices are condensed in the symmetric phase, their local averages differ only slightly from no–vortex averages. Local action and Higgs field are distinctly different from the bulk in the broken phase. The correlations between the position of the vortices and decreasing Higgs field values (larger cloud densities) are exemplified in Fig. 5, where we show a snapshot using $k = 3$ vortices (blocked from a $48^3$ lattice) for $\beta_G = 24$ at a temperature slightly below $T_{\text{perc}} (\beta_H = 0.3630)$. This Figure

Figure 5: Clusters of extended ($k = 3$) Z-vortices just below $T_{\text{perc}}$. Nambu monopoles are not shown. The Higgs modulus is visualized as in Fig. 4.

Figure 6: Average squared Higgs field modulus (left) and gauge field energy (right) inside and outside an elementary ($k=1$) Z–vortex on both sides of the percolation transition, ($M_H = 100$ GeV, $\beta_G = 8$).
supports the semiclassical nature of the defects.

4 Outlook
For the next future, the systematic exploration of the continuum limit with lattices of comparable physical size has to be completed. For the broken phase we expect to obtain well–defined size distribution and internal profile of the embedded $\mathbb{Z}$–vortices while, for the condensed phase, this will probably not be possible.

We plan to extend our considerations to more realistic models with $\theta_W \neq 0$. In order to clarify the connection between the dynamics of vortices with the evolution of Chern–Simons number, the 3–dimensional studies presented here have to be complemented by Euclidean and real–time simulations. This seems to be an interesting piece of physics whether or not it finally leads to a viable mechanism of baryogenesis.

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