A Method for Identifying the Friction-induced Vibration Based on The Maximum Information Coefficient

Pengfei Xing 1, Shihao Qiu 1, Guobin Li 1,*, Hongtao Gao 2, Honglin Gao 1, Xiaoliang He 1, Mingji Shang 1 and Hongpeng Zhang 1

1Marine Engineering College, Dalian Maritime University, Dalian, China
2Naval Architecture and Ocean Engineering College, Dalian Maritime University, Dalian, China

*Corresponding author e-mail: guobinli88@163.com

Abstract. A reciprocating running-in experiment is carried out on a friction-abrasion testing machine with disk-pin friction pair. The friction-induced vibration (FIV) signals measured in the experiments are identified by the maximum information coefficient (MIC) method. Experimental investigation shows that the association strength between the identified tangential and normal FIV signal is in a positive correlation with the coefficient of friction. The two-directional FIV signals distribute in the same frequency range, and their root mean square (RMS) variations are in similar accord to the changing of the coefficient of friction and can indicate the wear state evolution of the disk-pin friction pair from the running-in wear to stable wear. Therefore, the FIV signals can be identified by the MIC method.

1. Introduction
Friction-induced vibration (FIV) is excited by the interaction between the friction interface and can provide sufficient information to characterize the tribosystem, such as the running-in state of the friction pairs [1,2]. Compared to other characteristics such as the friction coefficient, the wear surface, and the wear debris, the FIV can be sampled quickly on-site and not need to change the operating condition of the equipment [3-5]. However, the FIV is usually submerged in the noise because of its weakness, and the FIV containing noise cannot characterize the tribosystem accurately. Therefore, to characterize the tribological behavior of the tribosystem by FIV, it is a crucial issue to identify and extract the FIV from the measured vibration signal with strong noise. For this reason, many scholars have conducted various researches on the identification and detection of the FIV, and the advanced mathematical theories and the nonlinear methods have been conducted to extract the FIV in the normal direction, such as the wavelet analysis [6], the multifractal [7], the attractor theory [8] and the recurrence analysis [9], etc. Unfortunately, the FIV in tangential direction has still not received the same concern that as the normal FIV.

According to the driving mechanism of the FIV [10], the fluctuation of the normal contact force can lead to the change of the tangential friction force. Such that both tangential and normal FIV can be excited due to the interaction between the friction interface. In this process, the friction force acts like a coupling force, linking the tangential and normal FIV [11]. Somehow surprisingly, little investigation of the tangential FIV is mentioned so far. Chen and co-authors analyzed that the normal and tangential
FIV under certain conditions through the time-frequency analysis, and found that both of them were with the same or approximate frequency [12]. Liu and co-authors found that the chaotic attractors reconstructed from the tangential and normal FIV show the same convergence trend in the running-in experiment, which can be applied to characterize the variation of the running-in process [13]. Xing and co-authors found that the cross-correlation function calculated from the tangential and normal FIV in the running-in process under the lubricated conditions reached the maximum when t=0s, indicating that the FIV in the tangential and normal direction is without time-delay [14]. Unfortunately, the relevance between the FIV in the normal and tangential directions has still not been thoroughly studied, and the relevance indicators used have certain limitations, such as the Pearson correlation coefficient, which can only measure linear relevance between two variables. However, the FIV has distinct nonlinear characteristics, and the linear correlation coefficient can not accurately describe the relevance between the two nonlinear variables. Therefore, it is proposed to apply an advanced method to investigate the relevance between the tangential and normal FIV in this work.

The maximum information coefficient (MIC) is a statistic proposed to measure the strength of association between two variables by Reshef and co-authors. in 2011 [15]. The MIC cannot only be used to measure the linear relationships and nonlinear relationships between two variables but also to uncover the non-functional dependencies broadly between two variables [16]. Hence, in this work, we propose to study the association between the tangential and normal FIV by utilizing the MIC. On this basis, a new detection method is proposed to extract the FIV in the tangential and the normal directions simultaneously, to take the most advantage of the tribological information contained in the FIV, and to describe the tribological behavior of friction pairs more accurately. This work is organized in the following structure: first, the experimental setup, condition, and the tribological pairs are introduced. Then, the mechanism and the method applied to analyze the experimental data are elaborated in detail. Finally, the new method to detect the FIV are investigated and verified through the vibration measurements.

2. Experiments

2.1. Experimental equipment
The disk-pin running-in experiments have been conducted on a CFT-1 wear test rig, and the structure of the test rig is shown in Fig. 1. The load device 1 pressurizes the upper sample 5 through special springs to make it compact the lower sample 6. The load applied on the upper sample 5 can be regulated from 0 to 200N, and automatically adjusted to keep it stable by the load device 1. The upper sample 5 is fixed under the load device 1, which remains stationary during the experiment. The lower sample 6 is fixed on a movable bench firmly. The bench is driven by the motor 4 through an eccentric mechanism to reciprocate in the stroke of 5mm. The rotational speed of the motor 4 can be controlled from 0-2000rpm. The friction torque sensor 2 measures the friction torque and records it in the data acquisition system of the test rig in the form of friction coefficient. The tri-axial acceleration sensor 3, model PCB-356A24 with a range of ±500g and a sensitivity of 10mV/g, is fixed on the lower sample to measure the normal and tangential vibration signal. The data acquisition system (model INV-3062T2) is used to store the vibration measurements into the computer. The surface roughness of the disk-pin friction pair is measured by the confocal laser scanning microscope (model OLS 4000).
2.2. Friction pairs
In this running-in experiment, the disk-pin specimens are used as the friction pairs. More specifically, the lower sample is a disk specimen made up of alloy cast iron and prepared by a wire cut electric discharge machine from a cylinder liner of an engine. The disk specimen is with Φ30mm diameter, a 300-400HV hardness, and the surface roughness Sa=1.7μm. Its metal micro-structure is graphite and pearlite matrix with P, Mn, Si, C, Fe and more. The upper sample is a pin specimen composed of alloy cast iron comprising a marginal number of phosphates and obtained in the same way from the piston ring matched the prior referred cylinder liner. The pin specimen is with Φ3mm diameter, a 600-700HV hardness, and the surface roughness Sa=0.6μm. Its metal micro-structure is a graphite, sorbite, and pearlite matrix involving P, Mo, Si, C, Fe and more.

2.3. Experimental condition
The reciprocating running-in experiment of the disk-pin friction pair was conducted with continuous drop lubrication under 293K and 45% relative humidity. A common marine lubricant CD40 is used as the lubricating oil, whose viscosity is 139.6cSt at 40°C and 12.5cSt at 100°C, and the density is 0.8957 g/cm3. In this experiment, the rotational speed of the motor is set to 600rpm, and the computed relative velocity between the friction pair is 0.1m/s. The pressure load applied on the pin specimen is 70N, and the calculated contact pressure between the friction pair is 9.9MPa. The data acquisition system is set to sample 4096 points in 0.4s. A time-series of vibration signals are collected in every 2min. The duration of the running-in experiment is 120min.

3. Method for detecting FIV signals
3.1. Maximal information coefficient
The maximum information coefficient (MIC) is depending on the opinion that if there is an association between a pair of variable data, then a grid can be drawn on the scatter plot of the pair variable data that partitions the data to encapsulate that relationship [15]. The value of the MIC of pair variables falls between 0-1. The MIC tends to 0 means that the two variables are independent statistically, and tends to 1 indicating a strong association between the two. The MIC can measure the linear relationships, the nonlinear relationships, and the non-function dependency relationships between two variables, i.e., it is with broad generality. The MIC can give similar scores (0 to 1) to different types of noisy relationships equally, i.e., it is with wide equitability. The MIC is computed depending on mutual information (MI) [17] and meshing methods and is more accurate than the MI. Given two variables A and B, the calculation steps of MIC can be summarized succinctly, as shown in Fig.2.
Divide the value range of variable $A$ into $x$ segments
Divide the value range of variable $B$ into $y$ segments
Define a division $G$, the $G$ is a grid with a size of $x \times y$
Calculate the $MI$ of variables $A$ and $B$
Where $p(a,b)$ is the joint probability density of $A$ and $B$, $p(a)$ and $p(b)$ are the boundary probability densities of $A$ and $B$.

$$MI_{A,B} = \sum_{a \in A} \sum_{b \in B} p(a,b) \log \frac{p(a,b)}{p(a)p(b)}$$

Convert the value of $MI$ into the interval $[0,1]$ by normalization factor $\{x,y\}_{\min}$.

$$D_{x,y} = \frac{\log \min[x,y]}{\log\min[x,y]}$$

$$D|G$$

Given two variables $A = \{a_i, i=1,2,\ldots,n\}$ and $B = \{b_i, i=1,2,\ldots,n\}$ where $n$ is the number of samples
Choose the maximum value of $MI(A, B)$ in different meshing ways of dividing $G$ as the $MI$ value of a division $G$.

$$M(D_{x,y}) = \max_{G} MI(D|G)$$

where $D|G$ denotes data $D$ are divided by $G$.

The MIC is defined as:

$$MIC(D_{x,y}) = \max_{G} \left[ D_{x,y} - \frac{MI(D|G)}{\log \min[x,y]} \right]$$

where $B(n)$ is the upper limit of the $x \times y$ grid
Generally, $\epsilon > 0, \epsilon < 1$

Figure 2. The calculation steps of MIC.

3.2. Numerical verification
Hoffmann and co-authors [10] investigated the physical driving mechanism of FIV by establishing a mathematical model of a minimal two freedom degree and pointed that the friction force links the normal and tangential FIV at the contact interface. We employ this mathematical model to investigate the MIC features of the normal and tangential FIV to provide a theoretical basis for the proposed detection method. By ignoring the static term in the interaction, the resulting equations of the mathematical model read:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{y} \end{bmatrix} + \begin{bmatrix} k_{11} & k_{12} - \mu k_3 \\ k_{21} & k_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

where $m$ is the mass coefficient, $k_{ij}$ $(i,j=1,2)$ is the stiffness coefficient, $\mu$ is the friction coefficient.

Assume that $m=1$, $k_{11}=3$, $k_{12}=k_{21}=-0.866$, $k_{22}=9$, and the system is with a constant friction coefficient $\mu$ between 0 to 1.5. To solve the equilibrium Eqs. (1). The Runge-Kutta method was adopted. Taking $\mu=0.2$ as an example, the calculated FIV in the tangential and normal direction is shown in Fig. 3. It can be seen from Fig.3 that the relationship between the two variables is not apparent.

Figure 3. The calculated FIV when $\mu=0.2$: (a) tangential FIV (b) normal FIV.

The MIC of tangential and normal FIV signals under different friction coefficients is calculated, as shown in Fig. 4. As can be seen from Fig. 4 that the MIC of the two calculated FIV variables gradually decreases with the decrease of friction coefficient, i.e., there is an apparent positive correlation between the friction coefficient and the MIC. Therefore, based on this relevance, we propose a method for detecting FIV by using MIC in this work.
3.3. Detection method
The original vibration signals are measured in tangential and normal directions through a triaxial acceleration sensor in the running-in experiments. The harmonic wavelet packet transform (HWPT) is used to draw out the vibration signals in different frequency bands. The MIC of tangential vibration signals and normal ones in the different frequency bands is calculated at the different times. Then the variation trend of the MIC of vibration signals reconstructed from different frequency bands with the time can be obtained. When the trend of MIC with time is consistent with that of friction coefficient with time, the corresponding frequency band is extracted as the FIV. The workflow of the detection method is depicted in Fig. 5.

Figure 5. The calculating workflow of the detection method.

4. Results and discussions
Based on the MIC, the FIV signals in tangential and normal were identified to validate the proposed method for detecting FIV. Their variation trend in the running-in experiment was discussed thoroughly in the following manner.
4.1. Analysis of the wear state of the disk-pin friction pair

Fig. 6 depicts a changing of the friction coefficient during the running-in experiment of the disk-pin friction pair. The variation trend of the friction coefficient has two distinct stages with the running-in experiment. At the first stage, i.e., within the first 42mins, the friction coefficient is high initially and fall off gradually, implying that the wear state of disk-pin friction pair transforms from severe wear to slim wear; at the second stage, i.e., nearly 42mins later, the friction coefficient fluctuates steadily at around 0.1077, indicating that the disk-pin friction pair has entered stable wear. Therefore, the changing of the friction coefficient suggests that the wear state evolved from a running-in wear to a stable wear.

![Figure 6](image-url)

**Figure 6.** Changing of friction coefficient during the experiment.

4.2. The detection of FIV signals

4.2.1. MIC analysis of the original vibration signals. Fig. 7 displays the time-domain waveform of the original vibration signals in 1min, 40min, 80min, and 120min in the process of the running-in. Fig. 7 displayed that there are apparent periodic components contained in the original vibration signals, and the tangential vibration signals show more robust periodic variation characteristics than the normal ones. Fig. 8 shows the variation of the MIC with time, and the MIC is low and remains almost stable as the running-in process progresses. It can be seen from Fig. 8 that the association strength of the two original vibration signals has no correlation to the friction coefficient and weak at each time during the running-in experiments. Therefore, original vibration signals cannot reflect the characteristics of FIV.

![Figure 7](image-url)

**Figure 7.** The waveforms of original vibration signal: (a) 1min (b) 40min (c) 80min (d) 120min.
4.2.2. **MIC analysis of the identified FIV signals.** A 7-layer harmonic wavelet packet transform is applied to decompose the original vibration signals into 128 frequency bands, i.e., the bandwidth of each band is 40Hz. According to the calculation flow displayed in Fig. 5, the MIC of the decomposed signals in each band is calculated. By repeated calculation, the final bandwidth of 120Hz is selected to retain the information of FIV to the greatest extent, i.e., the reconstructed frequency bands are 1-120Hz, 121-240Hz, ..., 4920-5040Hz. Fig. 9 displays the MIC of signals in different frequency bands. As can be seen from Fig. 9, the MIC in the same frequency range is the largest, which implies that the FIV signals in tangential and normal have the same frequency range.

![Figure 9. The MIC of signals in different frequency bands: (a) 1min (b) 40min (c) 80min (d) 120min.](image)

The identified FIV signals are determined in the frequency range of 2880-3000Hz. Fig. 10 depicts the variation of the MIC of the tangential and normal FIV signals with time. As displayed in Fig. 10, the variation trend of the MIC has two distinct stages during the running-in experiment. In the first stage (within the first 42mins), the MIC is high initially and decreases gradually, implying that the association strength of the tangential and normal FIV is evolving from a strong correlation to a weak correlation. In the second stage (nearly 42mins later), the MIC fluctuates steadily at around 0.2483, indicating that the association strength of the tangential and normal FIV becomes stable. The variation trend of MIC of
FIV signals is in accordance with the changing of friction coefficient in the experiment. Therefore, the
tangential and normal vibration signals, whose association strength is in positive correlation with the
friction coefficient, can be identified from the original ones by the MIC method.

![Figure 10](image.png)

**Figure 10.** The variation of MIC of FIV signals in tangential and normal.

The time-domain waveform of the identified FIV signals is displayed in Figs. 11(a) - (d). Comparing
with the waveforms of the original signals displayed in Fig. 7(a)-(d), three visible distinct can be found:
(1) the periodic variation trend disappears in the time-domain waveforms of the identified FIV signals,
(2) the amplitudes of the FIV signals weaken significantly in both tangential and normal direction, and
(3) the amplitude of the identified FIV signals changes in the same direction at different times.

![Figure 11](image.png)

**Figure 11.** The waveforms of identified FIV signal: (a) 1min (b) 40min (c) 80min (d) 120min.

4.3. **RMS analysis of the identified FIV signals**
The FIV signals can supply information reflecting the changing of the wear state between the friction
pair [19]. Accordingly, the FIV signals can be identified by investigating the change of the wear states.
In this work, the change of the wear state between the disk-pin friction pair is evaluated by the coefficient
of friction. Then, the change of the identified FIV signals under different wear states of the disk-pin
friction pair is discussed based on this foundation. Fig. 12 depicts the change of the RMS of FIV signals
in both directions in the running-in experiment of the disk-pin friction pair. It can be seen from Fig. 12
that the variation trend of RMS of the identified FIV signals in both directions displays two variation
stages in the whole running-in experiment. In the first stages (within the first 42mins), the RMS of
identified tangential and normal FIV signals evolves from high to low, indicating that FIV excited by
the interaction of the friction pair develops from strong vibration to weak vibration. Severe wear implies a strong vibration, while mild wear indicates a weak vibration. Therefore, the evolution process of the identified FIV signals in this stage implies that the wear state of the disk-pin friction pair transforms from severe wear to mild wear, which is coherent to the conclusion of the friction coefficient investigation. In the second stage (nearly 42mins later), the RMS of the identified FIV signals fluctuates smoothly at 0.00091 (the tangential FIV) and 0.00076 (the normal FIV) approximately, suggesting that the FIV excited by the interaction of the disk-pin friction pair is weak and stable, i.e., the friction pair wear evolve to a stable wear stage with mild wear.

![RMS variation of FIV signals: (a) tangential signal (b) normal signal.](image)

5. Conclusion
The reciprocating running-in wear experiment of the disk-pin friction pair was performed on a friction-abrasion testing machine. The FIV signals in tangential and normal directions were identified by the MIC method. Depending on the present study, the following conclusions can be drawn as:

1. The tangential and normal vibration signals, whose association strength is in positive correlation with the friction coefficient, can be identified from the original ones by the MIC method.

2. The MIC variation trend of the identified FIV signals in tangential and normal is in accordance to the changing of the friction coefficient, and the identified FIV signals in tangential and normal distribute in the same frequency range.

3. The RMS evolution process of the identified normal and tangential FIV signals is in consonance to the changing of the friction coefficient and can characterize the changing of the wear stage from a running-in wear to a stable wear.

Acknowledgments
This work was financially supported by the National Natural Science Foundation of China (Grant No. 51879020 and 51679022), and the Innovative Talents Training Project for the Doctoral Students of Dalian Maritime University (Grant No. BSCXXM006).

References
[1] D. Sun, G.B. Li, H.J. Wei, H.F. Liao, Experimental study on the chaotic attractor evolvement of the friction vibration in a running-in process, Tribology International, 88 (2015) 290–297.
[2] Y.K. Zhou, Z. Hua, X. Zuo, The behavior of intrinsic randomness and dynamic abrupt changes of friction force signal during the friction process, Journal of Tribology, (2016).
[3] Machado Tiago Henrique, Alves Diogo Stuani, Cavalec Katia Lucchesi, Investigation about journal bearing wear effect on rotating system dynamic response in time domain, Tribology International, (2018)
[4] O.A. Hassina, N. Weib, H. Towosyfyana, F. Gu, A.D. Balla, 'Journal bearing lubrication monitoring based on spectrum cluster analysis of vibration signals, in: Comadem, 2015.
[5] C. Ding, H. Zhu, G.D. Sun, Y.K. Zhou, X. Zuo, Chaotic characteristics and attractor evolution of friction noise during friction process, friction, 6 (2018) 47–61.
[6] G.B. Li, Y.H. Lin, H.Z. Wang, H.J. Wei, G.Y. Wang, Harmonic wavelet packet analysis of
friction-induced vibration, Tribology Transactions, 54 (2011) 895–901.

[7] J.M. Li, H.J. Wei, L. Fan, L.D. Wei, Multifractal Detrended Fluctuation Analysis of Frictional Vibration Signals in the Running-in Wear Process, Tribol Lett, 65 (2017) 50.

[8] Y.K. Zhou, H. Zhu, X. Zuo, Dynamic evolutionary consistency between friction force and friction temperature from the perspective of morphology and structure of phase trajectory, Tribology International, 94 (2016) 606–615.

[9] H. Zhu, X. Zuo, Y.K. Zhou, Recurrence evolvement of brass surface profile in lubricated wear process, Wear, 352–353 (2016) 9–17.

[10] N. Hoffmann, M. Fischer, R. Allgaier, L. Gaul, A minimal model for studying properties of the mode-coupling type instability in friction induced oscillations, Mechanics Research Communications, 29 (2002) 197–205.

[11] N. Hoffmann, L. Gaul, Effects of damping on mode-coupling instability in friction induced oscillations, ZAMM - Journal of Applied Mathematics and Mechanics / Zeitschrift Für Angewandte Mathematik Und Mechanik, 83 (2003) 524–534.

[12] G.X. Chen, Z.R. Zhou, Experimental observation of the initiation process of friction-induced vibration under reciprocating sliding conditions, Wear, 259 (2005) 277–281.

[13] T. Liu, G.B. Li, H.J. Wei, D. Sun, Experimental observation of cross correlation between tangential friction vibration and normal friction vibration in a running-in process, Tribology International, 97 (2016) 77–88.

[14] P.F. Xing, G.B. Li, T. Liu, H.T. Gao, G.Y. Wang, A Detection Method for Friction Vibration Based on Harmonic Wavelet Packet Transform and Cross-Correlation Analysis, (2018).

[15] D.N. Reshef, Y.A. Reshef, H.K. Finucane, S.R. Grossman, G. McVean, P.J. Turnbaugh, E.S. Lander, M. Mitzenmacher, P.C. Sabeti, Detecting novel associations in large data sets, Science, 334 (2011) 1518–1524.

[16] X.H. Tang, J.C. Wang, J.G. Lu, G. Liu, J.D. Chen, Improving Bearing Fault Diagnosis Using Maximum Information Coefficient Based Feature Selection, Applied Sciences, 8 (2018) 2143.

[17] T.M. Cover, J.A. Thomas, Elements of Information Theory 2nd Edition, 2 edition, Wiley-Interscience, Hoboken, N.J, 2006.

[18] G.S. Chen, Handbook of friction-vibration interactions, Elsevier, 2014.

[19] D. Sun, G.B. Li, H.J. Wei, H.F. Liao, T. Liu, Investigation on Frictional Vibration Behavior of Tribological Pairs Under Different Wear States, J. Tribol, 137 (2015).