Does the Fine-Structure Constant Really Vary in Time?

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ABSTRACT

We discuss how laboratory experiments can be used to place constraints on possible variations of the fine-structure constant $\alpha$ in the observationally relevant redshift interval $z \approx 0-5$, within a rather general theoretical framework. We find a worst case upper limit for $\Delta \alpha / \alpha$ of $8 \times 10^{-6}$ for $z \leq 5$ and $\Delta \alpha / \alpha$ of $0.9 \times 10^{-6}$ for $z \leq 1.6$. The derived limits are at variance with the recent findings by Webb et al., who claim an observed variation of $\Delta \alpha / \alpha = -2.6 \pm 0.4 \times 10^{-5}$ at $1 < z < 1.6$.

Subject headings: cosmology: theory — nuclear reactions, nucleosynthesis, abundances — relativity

1. INTRODUCTION

In an interesting paper, Webb et al. (1998) describe an observational method for investigating possible time and space variations in the fine-structure constant $\alpha$ using quasar absorption systems (see also Drinkwater et al. 1998). Webb et al. present intriguing evidence suggesting that $\alpha$ was smaller in the past (for redshifts $z > 1$). The claimed fractional change is (for $z > 1$) $\Delta \alpha / \alpha = -2.6 \pm 0.4 \times 10^{-5}$

Since a varying value of $\alpha$ can, in principle at least, have significant consequences for cosmology (e.g., concerning recombination), we examine in the present Letter possible variations in $\alpha$ in a broader context, although still under a particular set of assumptions. We also show that in the context of the theory presented in this Letter, the result obtained by Webb et al. appears to be in conflict with results of tests of the equivalence principle.

2. BY HOW MUCH CAN $\alpha$ CHANGE?

The experimental constraints on variations of $\alpha$ were explored extensively by Bekenstein (1982) by constructing a dynamical theory relating variations of $\alpha$ to the electromagnetic fraction of the mass density in the universe. Bekenstein’s (1982) framework for $\alpha$ variability was based on very general assumptions: covariance, gauge invariance, causality, time reversal of electromagnetism, and that gravitation is described by a metric of spacetime which satisfies Einstein’s equations. He obtained the following equation for the temporal variation of $\alpha$ (adopting a Robertson-Walker metric; we correct here a misprint in the original paper): $$(a^3 \dot{\alpha}) = -a (l^2 / \hbar c) \rho_m c^4.$$ (1)

Here $\dot{\alpha} = (\alpha / \alpha_{today})^{1/2}$, $l$ is a length scale of the theory, $\rho_m$ is the total rest mass density of matter, $a$ is the expansion scale factor, and $\zeta = (m_{n,em}) / m_p$ is a dimensionless parameter that measures the fraction of mass in the form of Coulomb energy of an average nucleon $(m_{n,em})$ compared to the free proton mass $m_p$. In order to be able to integrate equation (1), Bekenstein (1982) assumed that $\zeta$ (which is affected by transformations of hydrogen into heavier nuclei in stars) is constant. He was able to show that this assumption is reasonable up to redshifts $z \leq 1$ by the following consideration. The proton conversion rate (via the reaction $4H \rightarrow He^4$) can be estimated from the mean luminosity density (e.g., Davis, Geller, & Huchra 1978) to be $\sim 8 \times 10^{-28}$ cm$^3$ s$^{-1}$. Since a minimal nucleon number density in the universe can be determined from nucleosynthesis to be (e.g., Olive et al. 1981) $\sim 10^{-7}$ cm$^{-3}$, the timescale for proton conversion (and thereby for a change in $\zeta$) is $\tau_{conv} \approx 10^{10}$ s.

The timescale for changes in $\rho_m$ or $a$ is $\sim H_0^{-1} \sim 10^{17}$ s. Thus, for as far back as the luminosity of galaxies does not change significantly (up to $z \leq 1$), the assumption of a constant $\zeta$ is a reasonable one.

However, it is interesting to push this analysis to higher redshifts (e.g., the main change in $\alpha$ claimed by Webb et al. [1998] occurs at $z \approx 1$) by extending Bekenstein’s (1982) analysis with a determination of the behavior of $\zeta$ at higher redshifts. We achieve this as follows. As explained above, the rate of change of the Coulomb contribution to the baryon mass depends on the rate of conversion of hydrogen into helium. Following Bekenstein, we find

$$\zeta = 1.2 \times 10^{-2}(X + 4/3 Y),$$ (2)

where $X$ and $Y$ are the mass fractions in hydrogen and helium, respectively. In order to determine the conversion rate, we adopted three possible star formation rate (SFR) histories for the universe: (1) one that follows the Madau et al. (1996) curve, (2) one that rises from the present up to a peak at $z \sim 1$ and then is flat at the peak value up to $z \sim 5$ (as is perhaps suggested by the COBE Diffuse Infrared Background experiment; Hauser et al. 1998; Calzetti & Heckman 1998), and (3) a SFR that is constant (at the peak value) for all redshifts (as a limiting case). These star formation histories were used to synthesize a mean stellar population of the universe as a function of time, using the Bruzual & Charlot (1993) evolutionary models. The bolometric luminosity from the mean stellar population was then used to calculate the hydrogen-to-helium conversion rate. In Figure 1, we show the proton conversion rate as a function of redshift up to $z = 5$ for the three assumed SFR histories. As can be seen from the figure, the timescale for proton conversion satisfies $\tau_{conv} \approx 10^{10}$ s up to $z = 5$. Since $\tau_{conv} \gg H_0^{-1}$, we can still take $\zeta$ to be constant in integrating equation (1). Indeed, direct calculation of equation (2) shows that it varies by less than 1% over the redshift interval that we consider. Thus we
obtain
\[ \dot{e} = -\zeta(\ell^2 c^3/\hbar)\rho_m(t - t_c), \]  
(3)

where \( t_c \) is an (unknown) integration constant. From equation
(2) with \( X = 0.74 \) and \( Y = 0.26 \), we find for \( \zeta \) the value
1.3 \times 10^{-2} \text{ (see also Bekenstein 1982).} \ We will consider in
the following the case of a low \( \Omega (\Omega = 0.2, \text{see below}) \) universe.
The results, however, would not change significantly in an
\( \Omega = 1 \) universe. Equation (3) can be integrated in time by
taking into account that for \( z < 4(\Omega^{-1} - 1) \) one has
\( \rho_m = \Omega_{m,0}\rho_c(t_0/it)^3 \), where \( \Omega_{m,0} \) is the baryon fraction at the
present time \( t_0 \) and \( \rho_c \) is the critical density \( \rho_c \equiv 3H_0^2/(8\pi G) \),
and that \( \vert \epsilon - 1 \vert \ll 1 \). We find
\[ e(z) - 1 = -\frac{3\zeta^2}{8\pi} \left( \frac{L_p}{\ell} \right)^2 \Omega_{m,0} H_0^2 t_0^2 \times \left[ 1 \right] \left[ \frac{L_0}{t_0} - \frac{L_0}{t(z)} \right], \]  
(4)

where \( L_p = (G\hbar/c^3)^{1/2} \) is the Planck length. We have verified that
the above analytical approximation is applicable to the redshift interval of interest. As an example, in Figure 2 we plot for
the case \( t_c = t_0 \) the difference between the expression of
equation (4) and the result of a direct numerical integration of
equation (3). Equation (4) has two quite different regimes depend-
ning on the value of \( t_c/t_0 \). Let us consider first the case
\( \vert t_c \vert \leq t_0 \), which is perhaps the most physically founded since
there is no a priori reason for having \( \vert t_c \vert \gg t_0 \). We find
\[ \vert e(z \leq 5) - 1 \vert \leq 6.2 \times 10^{-3} \Omega_{m,0} \left( \frac{L_p}{\ell} \right)^2. \]  
(5)

In order to obtain an absolute upper limit on the variability,
we will assume that all of the matter density is nucleonic by
taking \( \Omega = \Omega_{m,0} = 0.2 \) (e.g., Garnavich et al. 1998; Perlmutter
et al. 1998). We can also use the results of the Eötvös-Dicke-
Braginsky experiments (Eötvös, Parker, & Fekete 1922; Roll,
Krotkov, &Dicke 1964; Braginsky & Pamov 1972), designed
to test the equivalence principle, to infer \( llL_p \leq 10^{-1} \). Thus,
our final upper limit is
\[ \vert \epsilon(z \leq 5) - 1 \vert \leq 1.2 \times 10^{-9}. \]  
(6)

If we assume instead that \( \vert t_c \vert \gg t_0 \), we can set an upper limit to
\( \vert t_c \vert \) by considering constraints on \( \vert \epsilon/\epsilon \vert \). From Shlyakhter’s
(1976) analysis of the Oklo natural reactor, we have \( \vert \epsilon/\epsilon \vert < 0.5 \times 10^{-7} H_0 \). More recently, Damour & Dyson (1996) have
derived the less strong but more robust limit of \( \vert \epsilon/\epsilon \vert < 3.4 \times 10^{-7} H_0 \),
which we will adopt in the following. When used in combination with equation (3) and with \( \Omega_{m,0} = 0.2 \), this gives
\[ \vert (llL_p)^2 t_c H_0 \vert < 6.6 \times 10^{-3}. \]  
(7)

The term involving \( t_c \) in equation (4) becomes dominant, and we obtain
\[ \vert \epsilon(z \leq 5) - 1 \vert \leq 8 \times 10^{-6}. \]  
(8)

The above limits become stronger as the redshift decreases.
For example, at the redshift of interest for the Webb et al.
(1998) result \( (z = 1.6) \), one would find instead \( \vert \epsilon(z = 1.6) - 1 \vert \leq 1.6 \times 10^{-10} \) if \( \vert t_c/t_0 \leq 1 \) or \( \vert \epsilon(z = 1.6) - 1 \vert \leq 9 \times 10^{-7} \) otherwise. Both limits, regardless of the value of \( t_c \), are
at variance with the Webb et al. (1998) result.

3. DISCUSSION

The analysis of experimental constraints for the variation of
the fine-structure constant within the framework of rather gen-
eral dynamical theories provides upper limits to any change in
\( \alpha \) that are at variance with the Webb et al. (1998) detection.
We have verified that varying the cosmological parameters \( \alpha \),
\( \Omega \), and \( \Omega_{m,0} \) within the framework of rather gen-
eral dynamical theories provides upper limits to any change in
\( \alpha \) that are at variance with the Webb et al. (1998) detection.
the values of $\Omega, H_0$ (primordial helium fraction) does not affect our constraints significantly.

We should note that variations in $\alpha$ in the context of the theory discussed in the present Letter result in no modifications to the Planckian spectrum of black body radiation. In fact, one could think of varying $\alpha$ as a varying permittivity of the vacuum. This would simply be equivalent to changing the expansion factor $a(t)$ to a different function, which cannot modify the Planck spectrum.

Variations of the fine-structure constant $\alpha$ could be driven also by a different dynamics. In theories with extra dimensions (e.g., Kaluza-Klein or superstring theories), variations in the observed value of $\alpha$ can be induced by variations in the length scale of the compact dimensions (see, e.g., Marciano 1984; Barrow 1987). The effects of extra dimensions were discussed in detail by Barrow (1987), who found upper limits to $|\dot{\alpha}/\alpha|$ of $\lesssim 10^{-6}H_0$ for both Kaluza-Klein and superstring theories. This upper limit is also much smaller than the claimed detection by Webb et al. (1998). However, note that in these theories the variations of all physical constants are interrelated.

Finally, we would like to mention that the calculations leading to equation (4) would have to be modified if a significant component of the dark matter in the universe were to decay electromagnetically at redshift $z \approx 1$ (e.g., Sciama 1982, 1997). We hope that the present Letter will inspire more attempts to determine possible variations of $\alpha$ observationally.

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REFERENCES

Barrow, J. D. 1987, Phys. Rev. D, 35, 1805
Bekenstein, J. D. 1982, Phys. Rev. D, 25, 1527
Braginsky, V. B., & Panov, V. I. 1972, Zh. Eksp. Teor. Fiz. 61, 873 [Soviet Phys.—JETP 34, 463 (1972)]
Bruzual, G., & Charlot, S. 1993, ApJ, 405, 538
Calzetti, D., & Heckman, T. M. 1998, ApJ, submitted
Damour, T., & Dyson, F. 1996, Nucl. Phys., B480, 37
Davis, M., Geller, M. J., & Huchra, J. 1978, ApJ, 221, 1
Drinkwater, M. J., Webb, J. K., Barrow, J. D., & Flambaum, V. V. 1998, MNRAS, 295, 457
Eötvös, R. V., Pekar, V., & Fekete, E. 1922, Ann. Physik, 68, 11
Garnavich, P. M., et al. 1998, ApJ, 493, L53
Hauser, M. G., et al. 1998, ApJ, in press
Madau, P., Ferguson, H. C., Dickinson, M. E., Giavalisco, M., Steidel, C. S., & Fucnter, A. 1996, MNRAS, 283, 1388
Marciano, W. 1984, Phys. Rev. Lett., 52, 1984
Olive, K. A., Schramm, D. N., Steigman, G., Turner, M. S., & Yang, J. 1981, ApJ, 264, 557
Perlmutter, S., et al. 1998, Nature, 391, 51
Roll, P. G., Krotkov, R., & Dicke, R. H. 1964, Ann. Phys., 26, 442
Sciama, D. W. 1982, MNRAS, 198, 1
———. 1997, MNRAS, 289, 945
Shlyakhter, A. I. 1976, Nature, 264, 340
Webb, J. K., Flambaum, V. V., Churchill, C. W., Drinkwater, M. J., & Barrow, J. D. 1998, preprint (astro-ph/9803165)