Light vector correlator in medium: Wilson coefficients up to dimension 6 operators

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As an improvement of the QCD sum rule method to study modifications of light vector mesons in nuclear matter and/or at finite temperature, we calculate the Wilson coefficients of all independent gluonic non-scalar operators up to dimension 6 in the operator product expansion (OPE) of the vector channel for light quarks. To obtain the gluon part of the light quark OPE from the heavy quark one, we also compute the heavy quark expansion of the relevant quark condensates. Together with the results for the quark operators that are already available in the literature, this completes the OPE of the vector channel in a hot or dense medium for operators up to dimension 6.

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I. INTRODUCTION

Vector mesons in hot and/or dense matter have been in the focus of theoretical and experimental interest already for many years, primarily because some of them (quarkonia) are expected to be indicators of the quark-gluon plasma in heavy-ion collisions [1] while others (light vector mesons, such as \( \rho, \omega \) and \( \phi \)) were predicted to be probes of the partial restoration of chiral symmetry in nuclear matter [2]. Furthermore, due to their decay into di-leptons, which do not interact strongly, they provide a relatively clear experimental signal that does not get distorted by the strongly interacting surrounding matter and therefore in principle allow direct access to the medium to be studied.

The results to be presented in this paper will be relevant for the light vector mesons. Their behavior in nuclear matter has been studied intensively in various experiments during the last two decades and a considerable amount of new information has been obtained [3–5]. About the \( \rho \) meson, the reached consensus seems to point to the conclusion that its peak, which is already rather broad in vacuum, is broadened further due to nuclear matter effects and receives only a small mass shift [6, 7], which is in any case difficult to constrain because of the breadth of the peak (see, however, Ref. [8]). About the \( \omega \) meson, new results have emerged during the last few years, experimentally determining both the mass shift and width of the \( \omega \) peak at normal nuclear matter density with relatively high precision [9–13]. Recently, even the momentum dependence of the \( \omega \) width in nuclear matter has been measured [14]. Finally, even though somewhat scarce, some results are also available for the \( \phi \) meson, for which both width and mass shift at normal nuclear matter density have been obtained [15, 16]. Moreover, the \( \phi \) nuclear transparency ratio including its momentum dependence has also been measured [17, 18]. In the E16 experiment to be performed at the J-PARC facility in Tokai, the width and mass modification of the \( \phi \) are planned to be measured again with improved precision. Moreover, the momentum dependences of these quantities will also be studied [19]. For accurately interpreting these past and future experimental results, it is highly desirable to have a thorough theoretical understanding of how vector mesons are modified in nuclear matter.

As one possible approach, the QCD sum rule method [20] allows one to study the modifications of the vector mesons at both finite temperature and density directly from the first principles of QCD [2, 21–23]. For recent works along this line, see Refs. [24–34]. In all QCD sum rule studies of vector mesons, the results of the operator product expansion (OPE) of the vector channel represent the starting point of any analysis. To compute the OPE in hot and/or dense matter, one needs to take into account non-scalar operators because the existence of a medium breaks Lorentz symmetry. Somewhat surprisingly, the contributions of gluonic non-scalar operators to the OPE of the vector channel for light quarks up to dimension 6 have never been obtained, even though some attempts were made in Ref. [35] and the heavy-quark case was studied in Ref. [36]. As it is well known, one cannot simply take the \( m \rightarrow 0 \) limit of the heavy quark OPE when switching to the light quark case, because some of the gluonic contributions in the heavy quark OPE become part of the quark condensate in the light quark limit [20, 37–39] and therefore need to be subtracted. Such gluonic contributions can be obtained by performing a sort of heavy-quark expansion on the quark condensates, as it was discussed long ago in Ref. [37] for the scalar part of the OPE and more recently in Ref. [38] in relation to the heavy-light-quark pseudoscalar channel (see also [40, 41]). We will follow the same approach and therefore first start from the OPE of non-scalar quark operators, which can (mostly) be found in the literature [35, 42], compute the gluonic contributions of these operators via the heavy-quark expansion and finally subtract these from the heavy quark results of Ref. [36]. By doing this, we compute all gluonic non-scalar contributions to

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the vector channel OPE up to dimension 6. This paper is organized as follows. In section II, we provide the basic definitions of the entities to be discussed in our work. In section III, the OPE for quark operators will be recapitulated, after which the heavy quark expansion for these operators will be discussed. The section concludes with the final OPE result for the gluonic condensates up to dimension 6. Section IV is devoted to the summary and conclusions. For the interested reader, we briefly explain in Appendix A the spin decomposition of gluonic operators in $D$ dimensions that we have used in this study.

II. DEFINITIONS

We start from the following correlation function for the vector current $j_\mu(x) = \bar{q}(x)\gamma_\mu q(x)$, expand the OPE results as a function of small quark mass, and keep operator terms up to mass dimension 6.

$$\Pi_{\mu\nu}(q) = \int d^4xe^{iqx} \langle T\{j_\mu(x)j_\nu(0)\} \rangle$$

$$= \Pi^{\text{scalar}}_{\mu\nu} + \Pi^{4.2}_{\mu\nu} + \Pi^{5.2}_{\mu\nu} + \Pi^{6.4}_{\mu\nu}$$

In general, we can classify the correlation function according to spin and dimension of operators which occur in the OPE. $\Pi^{\text{scalar}}_{\mu\nu}$ denotes the contributions from the scalar operators. Among the superscripts of $\Pi_{\mu\nu}$, the first index denotes the dimension and the second one the spin of the corresponding operators. From Lorentz covariance and due to the fact that the vector current is conserved, it can easily be shown that each term in Eq. (2) satisfies the following Lorentz structure [36] within the large $Q^2 = -q^2$ region.

$$\Pi^{\text{scalar}}_{\mu\nu} = (q_\mu q_\nu - q^2 g_{\mu\nu})\Pi^{\text{scalar}}_{\mu\nu},$$

$$\Pi^{4.2}_{\mu\nu} = \frac{1}{Q^2} [I^{4.2}_{\mu\nu} + \frac{1}{Q^2} (q_\mu q_\nu I^{4.2}_{\mu\nu} + q_\mu q_\nu I^{4.2}_{\mu\nu})$$

$$+ g_{\mu\nu} \frac{q_\rho q_\sigma}{Q^2} (I^{4.2}_{\rho\sigma} + J^{4.2}_{\rho\sigma})],$$

$$\Pi^{5.2}_{\mu\nu} = \frac{1}{Q^4} [I^{5.2}_{\mu\nu} + \frac{1}{Q^2} (2q_\mu q_\nu I^{5.2}_{\mu\nu} + q_\mu q_\nu I^{6.2}_{\mu\nu})$$

$$+ g_{\mu\nu} \frac{q_\rho q_\sigma}{Q^2} (I^{5.2}_{\rho\sigma} + J^{5.2}_{\rho\sigma})],$$

$$\Pi^{6.4}_{\mu\nu} = \frac{q_\mu q_\nu}{Q^2} [I^{6.4}_{\mu\nu} + \frac{1}{Q^2} (q_\mu q_\nu I^{6.4}_{\mu\nu} + q_\mu q_\nu I^{6.4}_{\mu\nu})$$

$$+ g_{\mu\nu} \frac{q_\rho q_\sigma}{Q^2} (I^{6.4}_{\rho\sigma} + J^{6.4}_{\rho\sigma})].$$

Here and throughout the whole paper, we use the following convention for the summation of Lorentz indices: $A_{\mu\nu} = \sum g_{\mu\nu} A^{\mu\nu}$. While the heavy quark OPE of the correlator, $\Pi^{h.q.}_{\mu\nu}$, has only gluon condensate contributions, the light quark OPE generally has both gluon and quark condensates. $\Pi^{q}_{\mu\nu} = \Pi^{q}_{\mu\nu} + \Pi^{G}_{\mu\nu}$. The bold superscripts $Q$ and $G$ here represent quark and gluon condensate parts of the light quark OPE, respectively.

For the condensates appearing in this work, we use the following notations.

**Quark condensates**

$$\langle \bar{q}q \rangle \equiv \langle \bar{q}q \gamma_\mu (D_\mu G_{\mu\nu})q \rangle,$$

$$\langle \bar{q}^2 \rangle \equiv \langle \bar{q}^2 \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma q \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma q \rangle,$$

$$A_{\alpha\beta} \equiv \langle \bar{q} \{\bar{D}_\alpha D_\beta \}q \rangle,$$

$$B_{\alpha\beta} \equiv \langle \bar{q} (iD_\alpha \bar{G}_{\beta\mu} \gamma_\mu q) \rangle,$$

$$C_{\alpha\beta} \equiv \langle mQ D_\alpha D_\beta q \rangle,$$

$$F_{\alpha\beta} \equiv \langle \bar{q} (iD_\alpha D_\beta q) \rangle,$$

$$H_{\alpha\beta} \equiv \langle \bar{q}^2 \gamma_\mu \gamma_\nu \bar{q}^2 \gamma_\rho \gamma_\sigma q \rangle,$$

$$K_{\alpha\beta\gamma\delta} \equiv \langle \bar{q}^2 D_\alpha D_\beta D_\gamma D_\delta q \rangle.$$

**Gluon condensates**

$$\langle G^2 \rangle \equiv \langle \bar{q}^2 C_{\mu\nu} G^{\mu\nu} \rangle,$$

$$\langle G^3 \rangle \equiv \langle \bar{q} f^{abc} G^{\mu\nu} G^{\rho\sigma} G^{\lambda\mu} \rangle,$$

$$\langle j^2 \rangle \equiv \langle \bar{q}^2 (D_\mu G_{\alpha\nu}) (D_\nu G_{\alpha\mu}) \rangle,$$

$$G_{2\alpha\beta} \equiv \langle \bar{q}^2 C_{\mu\nu} G^{\mu\nu}_{\alpha\beta \mu\nu} \rangle,$$

$$X_{\alpha\beta} \equiv \langle \bar{q}^2 D_\mu D_\nu G_{\alpha\mu\nu} \rangle,$$

$$Y_{\alpha\beta} \equiv \langle \bar{q}^2 D_\mu G_{\alpha\mu\nu} G_{\beta\nu\mu} \rangle,$$

$$Z_{\alpha\beta} \equiv \langle \bar{q}^2 D_\mu D_\nu G_{\alpha\mu\nu} \rangle,$$

$$G_{4\alpha\beta\gamma\delta} \equiv \langle \bar{q}^2 G_{\alpha\mu\nu} G_{\beta\mu\nu} G_{\gamma\mu\nu} G_{\delta\mu\nu} \rangle.$$

Here, $m$ is quark mass corresponding to the $q(x)$ field. $G_{\alpha\beta\gamma\delta} = \epsilon_{\alpha\beta\gamma\delta} G_{\mu\nu} G^{\rho\sigma} G^{\lambda\mu} G^{\kappa\nu}$. Note that throughout this work, the heavy quark OPE of the correlator, $\Pi^{h.q.}_{\mu\nu}$, has only gluon condensate contributions, the light quark OPE generally has both gluon and quark condensates, $\Pi^{q}_{\mu\nu} \equiv \Pi^{q}_{\mu\nu} + \Pi^{G}_{\mu\nu}$. The bold superscripts $Q$ and $G$ here represent quark and gluon condensate parts of the light quark OPE, respectively.

III. OPE CALCULATION AND RESULTS

A. Light quark OPE for quark condensates

In this work, we calculate the OPE for quark operators up to dimension 6. After taking the small quark mass limit, which means that we expand the quark propagators for small $m^2/q^2$ and keep all terms up to total
mass dimension 6, we get,

\[ \left\{ \Pi^{\text{scalar}} \right\} : \]

\[ \Pi^{\text{scalar}} = \frac{2m(qq)}{Q^4} - \frac{8m^3(qq)}{3Q^6} - \frac{4(qj)_4}{9Q^6} - \frac{2(jj_y^2)}{Q^6}, \]  

(23)

\[ J_{\mu}^{5,2} = (4 - 15 \frac{m^2}{Q^2}) F_{\mu\nu}, \]  

(24)

\[ J_{\mu}^{5,4} = (4 - 15 \frac{m^2}{Q^2}) F_{\mu\nu}, \]  

(25)

\[ J_{\mu}^{5,2} = 7 \frac{A_{\mu\nu}}{2} B_{\mu\nu} - 13 C_{\mu\nu} + 4 H_{\mu\nu}, \]  

(26)

\[ J_{\mu}^{5,4} = -3 \frac{A_{\mu\nu}}{2} B_{\mu\nu} - 5 C_{\mu\nu} - 4 H_{\mu\nu}, \]  

(27)

\[ I_{\mu\nu,\lambda}^{5,4} = -16i K_{\mu\nu,\lambda}, \]  

(28)

\[ J_{\mu\nu,\lambda}^{5,4} = 16i K_{\mu\nu,\lambda}, \]  

(29)

Note that the four-quark condensate terms \( \langle j_j^2 \rangle \) and \( H_{\mu\nu} \) do not play any role for the computations of this paper as their redefinition in terms of the heavy quark expansion leads only to gluonic terms of order \( \alpha_s^2 \). We therefore will not consider them in what follows. We have confirmed that the scalar result is consistent with that of Refs. [20, 37] and the spin-2 and spin-4 parts agree with those of Refs. [23, 42, 45].

B. Heavy Quark Expansion

Next, we need to evaluate the gluonic components of the heavy quark condensates to obtain the light quark OPE for gluon condensates from the heavy quark one [37]. For this purpose, we follow the techniques of the HQE (heavy quark expansion) introduced in Ref. [46] (in Ref. [38], the same procedure was referred to as “operator mixing”, and was discussed as the relation between normal-ordered and non-normal-ordered operators). The essence of the method can be summarized as follows. In momentum space, quark bilinear condensates can be represented by a closed one loop and are calculated in Fock-Schweringer gauge as,

\[ \langle \bar{q}O[D_{\mu}]q \rangle = -i \int \frac{d^D p}{(2\pi)^D} \langle \text{Tr}_{C,D}[O[-i p_{\mu} - i \hat{A}_{\mu}]S(p)] \rangle. \]  

(30)

For more details such as the detailed meaning of \( \hat{A}_{\mu} \), see for instance Ref. [38]. Here, the loop integral is computed in \( D = 4 - \epsilon \) dimensions with dimensional regularization. In doing this, some care is however needed, because inconsistencies can occur depending on which dimension (\( D \) or 4) is used for the spin decomposition of operators. For example, HQE’s scalar part of \( O_{\alpha\beta} \) and that of its scalar decomposed one in 4 dimensions, \( \frac{1}{4} g_{\alpha\beta} O_{\mu\nu} \), will generally have a different result for non-logarithmic terms. The reason why this inconsistency occurs is that direct HQE using Eq. (30) involves the spin decomposition process in \( D \) dimension and not in 4. We have explicitly checked that all direct HQEs of general quark operators are consistent with those of their scalar parts as long as all calculations (including the spin decomposition) are performed in \( D \) dimensions, which is the strategy that we will adopt in this work. For the above consistency to hold, it should be noted that the spin decomposition of gluonic operators should also be carried out in \( D \) dimensions. Details of this procedure are given in the Appendix A. This treatment differs from that of Ref. [38], where furthermore \( \text{Tr}[f] = D \) was used in some instances to avoid the above-mentioned problem. We however find that this approach is not valid in every case.

Employing the strategy explained in the preceding paragraph, the HQE results for the condensates appearing in this work are obtained as follows.

\[ \langle qq \rangle = - \frac{(G^2)}{48\pi^2 m} - \frac{(G^2)}{1440\pi^2 m^3} - \frac{(j_j^2)}{120\pi^2 m^3}, \]  

(31)

\[ \langle qj_q \rangle = - \frac{(j_j^2)}{24\pi^2} \log \frac{\mu^2}{m^2}, \]  

(32)

\[ A_{\alpha\beta} = \frac{Y_{\alpha\beta}}{24\pi^2} \log \frac{\mu^2}{m^2} - \frac{X_{\alpha\beta}}{48\pi^2} \log \frac{\mu^2}{m^2} - \frac{Z_{\alpha\beta}}{16\pi^2} \log \frac{\mu^2}{m^2}, \]  

(33)

\[ B_{\alpha\beta} = \frac{m^2 G_{2\alpha\beta}}{8\pi^2} \left( \log \frac{\mu^2}{m^2} - 1 \right) - \frac{X_{\alpha\beta}}{48\pi^2} \left( \log \frac{\mu^2}{m^2} - 2 \right) + \frac{Z_{\alpha\beta}}{16\pi^2} \log \frac{\mu^2}{m^2} - \frac{2}{16\pi^2} \log \frac{\mu^2}{m^2}, \]  

(34)

\[ C_{\alpha\beta} = \frac{m^2 G_{2\alpha\beta}}{48\pi^2} \log \frac{\mu^2}{m^2} - \frac{X_{\alpha\beta}}{240\pi^2} \log \frac{\mu^2}{m^2} + \frac{Y_{\alpha\beta}}{480\pi^2} \log \frac{\mu^2}{m^2} - \frac{Z_{\alpha\beta}}{480\pi^2} \log \frac{\mu^2}{m^2}, \]  

(35)

\[ F_{\alpha\beta} = \frac{G_{2\alpha\beta}}{480\pi^2} \log \frac{\mu^2}{m^2} - \frac{X_{\alpha\beta}}{960\pi^2 m^2} + \frac{Y_{\alpha\beta}}{120\pi^2 m^2} - \frac{Z_{\alpha\beta}}{160\pi^2 m^2}, \]  

(36)

\[ K_{\alpha\beta,\gamma\delta} = \frac{11}{480\pi^2} \log \frac{\mu^2}{m^2} G_{\alpha\beta,\gamma\delta}. \]  

(37)

Here, we have used dimensional regularization in combination with the \( \overline{\text{MS}} \) scheme. \( \mu \) is the renormalization scale. In the above expansion, we ignored all gluonic operators with dimension larger than 6. Note that there are a number of terms on the right hand side of these equations that diverge in the small \( m \) limit. These terms will get canceled by respective terms of the heavy quark OPE to be discussed in the next subsection. Let us further-more mention that the results for the scalar condensates [Eqs. (31) and (32)] completely agree with those given in Ref. [37].

C. Light Quark OPE for Gluon condensates

After these preparations, we can now obtain the light quark OPE for gluon condensate from the following formula.

\[ \Pi^{\text{G}}_{\mu\nu} = \lim_{m^2/q^2 \rightarrow 0} \left\{ \Pi^{h,q}_{\mu\nu} \right\} - \Pi^{\text{G}}_{\mu\nu}, \]  

(38)

The subscript G in \( \Pi^{\text{G}}_{\mu\nu} \) stands for the replacement of quark condensates into gluon ones via the heavy quark expansion. Each correlation function on the r.h.s has mass singularities as seen in the last subsection, but they should be canceled on the l.h.s. To use the above formula, we need the light quark limit of the heavy quark OPE, \( \lim_{m^2/q^2 \rightarrow 0} \{ \Pi^{h,q}_{\mu\nu} \} \), which can easily be extracted from
the formulas given in Ref. [36]. The results read

\[
\lim_{m^2/q^2 \to 0} \Pi^{h,\psi} = \frac{1}{\pi^2 q^2} \left( \frac{1}{48} + \frac{1}{36 q^2} \right) (G^2)
+ \frac{1}{\pi^2 q^2} \left( \frac{1}{20 m^2} + \frac{1}{540} \right) (G^2)
+ \frac{3}{\pi^2 q^2} \left( \frac{1}{60 m^2} + \frac{1}{1620} + \frac{1}{54} \log \frac{Q^2}{m^2} \right) (j^2)
\]

\[
J_{\mu \nu}^{\alpha, 2} = \frac{1}{\pi^2 q^2} \left( \frac{1}{60 m^2} + \frac{1}{1620} + \frac{1}{54} \log \frac{Q^2}{m^2} \right) G_{\mu \nu}.
\]

\[
J_{\mu \nu}^{\alpha, 4} = \frac{7}{24 \pi^2} + \frac{1}{12 \pi^2} \log \frac{Q^2}{m^2} G_{\mu \nu}.
\]

Note that, by independently checking the computations of Ref. [36], we found that both formulas given in Eq. (20) of that reference should be multiplied by a factor of 1/2, which was already mentioned in Ref. [33]. This fact is taken into account for the above results.

Finally, substituting the results of Eqs. (31-37) into Eqs. (23-29), and thereafter using Eq. (38), we see that indeed all mass singularities cancel and obtain the following final expression:

\[
\left\{ \Pi_{\mu \nu} \right\} = \frac{1}{\pi^2 q^2} \left( \frac{1}{48} + \frac{1}{36 q^2} \right) (G^2)
+ \frac{1}{\pi^2 q^2} \left( \frac{1}{324} + \frac{1}{54} \log \frac{Q^2}{m^2} \right) (j^2)
\]

\[
J_{\mu \nu}^{\alpha, 2} = \frac{1}{\pi^2 q^2} \left( \frac{1}{60 m^2} + \frac{1}{1620} + \frac{1}{54} \log \frac{Q^2}{m^2} \right) G_{\mu \nu}.
\]

From these results, one can now easily extract the transverse and longitudinal part of the correlator and can derive the corresponding sum rules. While the values of scalar and some twist-2 non-scalar gluonic condensates in nuclear matter were discussed already a long time ago [2, 47], it is also possible to give rough estimates for the twist-4 gluonic condensates which were the main target of this work [36]. With these estimates, it will be possible to study the consequences of our results on the behavior of vector mesons at finite density. We expect that the twist-4 gluonic condensates could have some non-negligible effect in particular on the modification of the vector meson masses at non-zero momenta, namely their dispersion relations [48]. While the effect of gluonic condensates will likely be rather small for the \( \rho \) and \( \omega \) channels, where the finite density modifications of the OPE are dominated by quark condensate terms [35], their relative importance will increase for the \( \phi \) meson case, where finite density effects due to quark condensates are suppressed [27]. This could be relevant for the future interpretation of experimentally measured spectra, which always involve vector mesons that move with some finite velocity relative to the surrounding nuclear matter.

IV. SUMMARY AND CONCLUSIONS

In this work, we have for the first time computed the Wilson coefficients, at leading order in \( \alpha_s \), of dimension 6, spin-2 and spin-4 gluonic operators in the OPE of the vector correlator for light quarks. We have also obtained...
the leading order \( \alpha_s \) Wilson coefficient of the dimension 4, spin-2 gluon operator (including its \( m^2 \) correction) which has so far never been correctly given in the literature. For self-adjoint mesons, this completes the vector channel OPE for all possible scalar and non-scalar operators up to dimension 6 that can have non-zero expectation values in a hot and/or dense medium that is invariant under parity and time reversal.

To reach our final results, given in Eqs. (46-52), we followed the (standard) procedure of starting from the OPE expression of gluonic operators for arbitrary quark masses (which is usually used for the OPE of the heavy quark correlator), taking its small quark mass limit and subtracting from it the contributions that become part of the quark condensates in this limit. This subtraction cancels all mass singularities for \( m \to 0 \), which appear at the intermediate steps of the computation and leads to a well-behaved final expression.

In the future, we plan to apply our results to the QCD sum rule analyses of light vector mesons in nuclear matter and/or at finite temperature. As the non-scalar operators that we have studied in this work affect the momentum dependence of the mesons in a non-trivial way (i.e. they modify the dispersion relation observed in vacuum), it will be especially interesting to investigate the behavior of the vector mesons with non-zero momentum and to provide post- and predictions for past and future experiments that measure vector mesons in nuclei.

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Appendix A: Spin decomposition of gluonic operators in \( D \) dimensions

In this appendix, we provide formulas of the spin decompositions of gluonic operators in \( D \) dimensions used in our work.

1. Decomposition of \( \langle g^2 C_{\kappa \alpha}^a G_{\lambda \beta}^{ab} \rangle \)

From the symmetry properties of \( \langle g^2 C_{\kappa \alpha}^a G_{\lambda \beta}^{ab} \rangle \), its Lorentz structure can be decomposed into

\[
\langle g^2 C_{\kappa \alpha}^a G_{\lambda \beta}^{ab} \rangle = \alpha_{\kappa \lambda \alpha \beta} + g_{\kappa \lambda} b_{\alpha \beta} - g_{\kappa \beta} b_{\alpha \lambda} + g_{\kappa \alpha} b_{\lambda \beta} - g_{\kappa \beta} b_{\lambda \alpha}, \quad (A1)
\]

with

\[
c_{\kappa \lambda \alpha \beta} = g_{\kappa \lambda} g_{\alpha \beta} - g_{\kappa \beta} g_{\alpha \lambda}. \quad (A2)
\]

Contracting the Lorentz indices with one or two metric tensors, the variable \( a \) and the tensor \( b_{\alpha \beta} \) are determined as

\[
a = \frac{1}{(D - 1)(D - 2)} (G^2), \quad (A3)
\]

\[
b_{\alpha \beta} = \frac{1}{(D - 2)} G_{2 \alpha \beta}. \quad (A3)
\]

2. Decomposition of \( \langle g^3 f^{abc} C_{\mu \alpha}^a G_{\alpha \beta}^{ab} G_{\nu \rho}^{bc} \rangle \)

Let us first study the scalar part of this operator. Its Lorentz structure can be reduced as

\[
\langle g^3 f^{abc} C_{\mu \alpha}^a G_{\alpha \beta}^{ab} G_{\nu \rho}^{bc} \rangle |_{\text{scalar}} = a (g_{\mu \alpha} e_{\nu \rho \beta} - g_{\mu \beta} e_{\nu \rho \alpha} - g_{\mu \rho} e_{\nu \alpha \beta} + g_{\mu \sigma} e_{\nu \alpha \beta}). \quad (A4)
\]

Contracting the Lorentz indices, \( a \) is determined as

\[
a = \frac{1}{D(D - 1)(D - 2)} (G^3). \quad (A5)
\]

Next, we consider the spin-2 part, that we represent as one general spin-2 tensor \( a_{\alpha \beta} \), which is symmetric and traceless. Using again the symmetries of \( \langle f^{abc} C_{\mu \alpha}^a G_{\alpha \beta}^{ab} G_{\nu \rho}^{bc} \rangle \), we obtain

\[
\langle g^3 f^{abc} C_{\mu \alpha}^a G_{\alpha \beta}^{ab} G_{\nu \rho}^{bc} \rangle |_{\text{spin-2}} = a_{\alpha \beta} e_{\nu \rho \beta} - a_{\beta \rho} e_{\nu \alpha \beta} - a_{\beta \sigma} e_{\nu \alpha \rho} + a_{\alpha \sigma} e_{\nu \rho \beta}.
\]

Contracting with two metric tensors and using formula (A.3) of Ref. [36], we get

\[
a_{\mu \alpha} = \frac{1}{(D - 2)(D - 3)} \langle g^3 f^{abc} C_{\mu \alpha}^a G_{\alpha \beta}^{ab} G_{\nu \rho}^{bc} \rangle = \frac{1}{2(D - 2)(D - 3)} (X_{\mu \alpha} + 2Z_{\mu \alpha}). \quad (A7)
\]

3. Decomposition of \( \langle g^2 C_{\mu_1 \nu_1} D_\alpha D_\beta G_{\mu_2 \nu_2}^{a} \rangle \)

We start again with the scalar part, whose Lorentz structure can be reduced as

\[
\langle g^2 C_{\mu_1 \nu_1} D_\alpha D_\beta G_{\mu_2 \nu_2}^{a} \rangle |_{\text{scalar}} = a g_{\alpha \beta} e_{\mu_2 \nu_1 \nu_2} + b (g_{\alpha \nu_2} e_{\mu_2 \mu_1 \nu_1 \nu_2} - g_{\alpha \mu_2} e_{\mu_1 \nu_1 \nu_2}) + d (g_{\nu_2 \nu_1} e_{\mu_1 \mu_2 \nu_1 \nu_2} - g_{\nu_2 \mu_1} e_{\mu_1 \nu_1 \nu_2}). \quad (A8)
\]

Contracting the Lorentz indices and making use of the fact that anti-symmetrizing Eq. (A8) within the indices attached to the covariant derivatives (\( \alpha \) and \( \beta \)) leads to an operator with three gluon fields that we discussed in
the previous subsection, one can derive the following result for \(a, \ b\) and \(d\):

\[
a = \frac{2}{(D + 2)(D + 1)(D - 1)} \left( \frac{1}{D - 2} C_2^2 - \frac{1}{D - 1} j^2 \right),
\]

\[
b = \frac{1}{(D + 2)(D + 1)(D - 1)} \left( \frac{1}{D - 2} C_2^2 - \frac{1}{D - 1} j^2 \right),
\]

\[
d = -\frac{1}{(D + 2)(D - 1)} \left( \frac{3}{D - 2} C_2^2 + \frac{1}{D - 1} j^2 \right).
\]

Next, we study the spin-2 part, which we parametrize by symmetric and traceless tensors \(a_{1\alpha\beta}^1, a_{2\alpha\beta}^2, \ldots\). Using the symmetries of \(\langle g^2 G_{\mu_1\nu_1} D_{\alpha} D_{\beta} G_{\mu_2\nu_2}^a \rangle\), we get

\[
\langle g^2 G_{\mu_1\nu_1} D_{\alpha} D_{\beta} G_{\mu_2\nu_2}^a \rangle_{\text{spin-2}} =
\]

\[
a_{1\mu_1\nu_2}^1 g_{\nu_1\nu_2} g_{\alpha\beta} + a_{1\mu_1\nu_2}^1 g_{\mu_1\nu_2} g_{\alpha\beta} + a_{2\mu_1\nu_2}^2 g_{\nu_1\nu_2} g_{\alpha\beta} + a_{2\mu_1\nu_2}^2 g_{\mu_1\nu_2} g_{\alpha\beta} + b_{1\mu_1\nu_2}^1 g_{\nu_1\nu_2} g_{\alpha\beta} + b_{1\mu_1\nu_2}^1 g_{\mu_1\nu_2} g_{\alpha\beta} + b_{2\mu_1\nu_2}^2 g_{\nu_1\nu_2} g_{\alpha\beta} + b_{2\mu_1\nu_2}^2 g_{\mu_1\nu_2} g_{\alpha\beta} + b_{1\mu_1\nu_2}^1 c_{\mu_1\nu_1\nu_2} + b_{2\mu_1\nu_2}^2 c_{\mu_1\nu_1\nu_2} - b_{1\mu_1\nu_2}^1 c_{\mu_1\nu_2\nu_1} - b_{2\mu_1\nu_2}^2 c_{\mu_1\nu_2\nu_1} + b_{1\mu_1\nu_2}^1 g_{\nu_1\nu_2} g_{\alpha\beta} + b_{2\mu_1\nu_2}^2 g_{\nu_1\nu_2} g_{\alpha\beta} + d_{1\mu_1\nu_2}^1 g_{\nu_1\nu_2} g_{\alpha\beta} + d_{1\mu_1\nu_2}^1 g_{\mu_1\nu_2} g_{\alpha\beta} + d_{2\mu_1\nu_2}^2 g_{\nu_1\nu_2} g_{\alpha\beta} - d_{1\mu_1\nu_2}^1 g_{\mu_1\nu_2} g_{\alpha\beta} - d_{2\mu_1\nu_2}^2 g_{\mu_1\nu_2} g_{\alpha\beta} - d_{2\mu_1\nu_2}^2 c_{\mu_1\nu_1\nu_2} + d_{1\mu_1\nu_2}^1 c_{\mu_1\nu_2\nu_1} + d_{1\mu_1\nu_2}^1 g_{\nu_1\nu_2} g_{\alpha\beta} + d_{2\mu_1\nu_2}^2 g_{\nu_1\nu_2} g_{\alpha\beta} - d_{2\mu_1\nu_2}^2 g_{\mu_1\nu_2} g_{\alpha\beta}.
\]

Taking all possible contractions, we can derive specific expressions for the tensors \(a_{1\alpha\beta}^1, a_{2\alpha\beta}^2, \ldots\). These read

\[
a_{1\alpha\beta}^1 = \frac{1}{(D + 4)(D + 1)(D - 1)} \left[ \frac{2D^2 + D - 7}{D - 3} X_{\alpha\beta} + \frac{D^2 + 3D - 2}{D - 1} Y_{\alpha\beta} + \frac{D^2 + 3D - 2}{D - 1} Z_{\alpha\beta} \right],
\]

\[
a_{1\alpha\beta}^2 = \frac{1}{(D + 4)(D + 1)(D - 1)} \left[ \frac{D^2 - D + 1}{D - 3} X_{\alpha\beta} + \frac{4}{D} Y_{\alpha\beta} + \frac{2(2D^2 + 3D - 6)}{D(D - 3)} Z_{\alpha\beta} \right],
\]

\[
b_{1\alpha\beta}^1 = \frac{1}{D + 1} \left[ \frac{2}{D - 3} X_{\alpha\beta} + \frac{D - 1}{D(D - 2)} Y_{\alpha\beta} + \frac{D^2 - 3D + 3}{D(D - 1)(D - 3)} Z_{\alpha\beta} \right],
\]

\[
b_{1\alpha\beta}^2 = \frac{1}{(D + 4)(D + 1)(D - 1)} \left[ \frac{D^2 - D + 1}{2D - 3} X_{\alpha\beta} + \frac{2D^2 + 3D - 6}{D(D - 3)} Z_{\alpha\beta} \right],
\]

\[
d_{1\alpha\beta}^1 = \frac{1}{(D + 1)(D - 2)} \left[ \frac{3}{2D - 3} X_{\alpha\beta} + \frac{D - 1}{D} Y_{\alpha\beta} - \frac{4D - 3}{D(D - 3)} Z_{\alpha\beta} \right],
\]

\[
d_{1\alpha\beta}^2 = \frac{1}{(D + 4)(D + 1)(D - 2)} \left[ \frac{-3(2D + 1)}{2D - 3} X_{\alpha\beta} + \frac{2}{D} Y_{\alpha\beta} - \frac{D^3 + 3D^2 + 6}{D(D - 3)} Z_{\alpha\beta} \right].
\]

Note that the \(D = 4\) limit of this result differs from that of Eq. (D.4) of Ref. [36], which contains several typos.

Finally, we consider the spin-4 part of this operator, for which we need a symmetric and traceless tensor \(a_{\alpha\beta\gamma\delta}\). Making again use of the symmetries of \(\langle g^2 G_{\mu_1\nu_1} D_{\alpha} D_{\beta} G_{\mu_2\nu_2}^a \rangle\), we obtain

\[
\langle g^2 G_{\mu_1\nu_1} D_{\alpha} D_{\beta} G_{\mu_2\nu_2}^a \rangle_{\text{spin-4}} =
\]

\[
g_{\mu_1\nu_2} a_{\mu_1\nu_2\alpha\beta} - g_{\mu_1\nu_2} a_{\mu_1\nu_2\alpha\beta} + g_{\nu_1\nu_2} a_{\mu_1\nu_2\alpha\beta} - g_{\nu_1\nu_2} a_{\mu_1\nu_2\alpha\beta}.
\]

Contracting two suitable Lorentz indices, we derive

\[
a_{\alpha\beta\gamma\delta} = \frac{1}{D - 2} G_{4\alpha\beta\gamma\delta}.
\]

[1] T. Matsui and H. Satz, Phys. Lett. B 178, 416 (1986).
[2] T. Hatsuda and S. H. Lee, Phys. Rev. C 46, no. 1, R34
