A Bayesian search for gravitational waves from the Vela Pulsar in Virgo VSR2 data.

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Abstract.
The Virgo detector undertook its second science run (VSR2) from July 2009 to January 2010, providing unprecedented sensitivity to gravitational waves at frequencies below 40 Hz. The VSR2 dataset presented an ideal opportunity [1] to search for gravitational waves from the Vela pulsar (B0833-45, J0835-4510), for which gravitational wave emission is expected at ∼22 Hz assuming it is a non axi-symmetric rotator. We give a summary of the results obtained in [1], describing the Bayesian method more fully and presenting further details of the data used.

1. Introduction
A recent paper by the Virgo collaboration and the LIGO Scientific Collaboration [1] presented three searches for gravitational waves from the Vela pulsar (PSR B0833-45, PSR J0835-4510) in data from the Virgo interferometer during the Virgo Science Run 2 (VSR2). These searches assumed the rigid triaxial neutron star (NS) model [2], which predicts gravitational wave emission at twice the rotation frequency of the pulsar. The searches also assumed that the gravitational wave signal is phase locked to the observed radio pulses. Here we will give a summary of the results obtained, describing the Bayesian method more fully, the results from this method, and present further details of the data used.

PSR B0833-45 was discovered in 1968 [3], in the Vela supernova remnant, providing some of the first direct observational evidence that established pulsars as rotating NSs. Since the initial detection of radio pulses, weak visible and X-ray pulses have been detected, and strong γ-rays pulses seen.

Certain properties of Vela mark it out as an appealing target to search for gravitational waves. It is young (with a characteristic age of ∼11000 y) and has a high spin-down (f ≃ −1.56 × 10⁻¹¹ Hz s⁻¹), meaning that there is a large amount of energy lost from the system which could be powering gravitational wave emission. Vela is a young pulsar and glitches regularly, with a glitch rate of 1/3 y⁻¹. This would present a problem if Vela glitched during VSR2 as the gravitational wave phase evolution during a glitch is not known. Vela’s relative proximity of ∼300 pc also marks it out as a prime candidate for gravitational wave detection.

A useful upper limit can be placed on the gravitational wave strain from a spinning star called the spin-down limit [4]. This limit is computed by assuming that all the rotational energy lost from a source is due to gravitational wave emission.
which for Vela is $h_{sd}^0 = 3.59 \times 10^{-24}$. This scenario is unrealistic as we know that pulsar spin-down is due to a number of mechanisms, including magnetic dipole radiation. However, the spin-down limit does provide us with a useful benchmark. If we can place an upper limit on the gravitational wave emission below the spin-down limit, then we can constrain the fraction of spin-down energy due to gravitational waves. The spin-down limit for Vela is the highest for all known pulsars. However, Vela is at a disadvantage compared to many other pulsars as the rotation frequency of Vela ($\sim 11$ Hz) [5], and hence the frequency of gravitational waves ($\sim 22$ Hz), is near the low frequency limit of current detectors. This low rotation frequency, means that a search for gravitational waves from Vela gains no advantage from including data from the LIGO detectors, as their noise floors at low frequencies are significantly higher than that of the Virgo detector. Figure 1 shows the spin-down limit of Vela ($3.29 \times 10^{-24}$) on the same axes as the noise curves for the current gravitational wave detector network. It is clear that our best chance of detecting a signal will be using data from the Virgo detector alone, where the advanced seismic isolation system results in a lower noise floor at $\sim 22$ Hz [6].

![Figure 1](image-url)

**Figure 1.** The RMS strain sensitivity noise curves for the LIGO and Virgo interferometers (scaled for the length of the VSR2 run, but not for antenna pattern attenuation), with the spin-down limit plotted for the Vela pulsar at twice its spin frequency. The points at which the dashed line intersects the noise curves shows the noise floor in Virgo data at twice the spin frequency of the Vela pulsar is more than 100 times lower than in LIGO data. [7, 8].

There has been only one previous targeted search for gravitational waves from Vela. That search used 16 days of data from the Cryogenic Laser Interferometer Observatory (CLIO)
detector in Japan taken in 2007 and produced a strain upper limit of $5.3 \times 10^{-20}$, well above the spin-down limit of $\sim 3.3 \times 10^{-24}$ [9].

The method used for the current search is outlined in Section 2. Section 3 looks at the VSR2 data, highlighting a nuisance noise artefact present in the VSR2 data. The timing model used in the analysis is presented in Section 4. Section 5 briefly describes the recovery of hardware injections in the VSR2 data. The results from the analysis make up Section 6, and the discussion of the results finishes off the paper in Section 7.

2. The Bayesian search method

Three almost independent analysis methods were used to perform a search for gravitational waves from Vela during VSR2. The importance of using different and independent search methods lies not just in the obvious need for corroborating results and cross-checking the analyses’ outputs, but also in the situation where different methods can have specific strengths making them more suitable than others in certain situations. The three methods are: a complex heterodyne method using Bayesian formalism and a Markov chain Monte Carlo [10]; a time-domain matched filter method using the $F$-statistic and $G$-statistic [11]; and a frequency domain matched filter method using the signal’s sidereal modulation Fourier components [12]. A fuller description of the Bayesian method follows.

The Bayesian analysis method used was developed by [13], and has previously been used to search for gravitational waves from many other pulsars e.g. [10, 14]. It consists of two distinct stages. The first stage performs a precise complex heterodyne to remove the known spin phase evolution from the data and drastically down-samples the data in the process (from 16,384 Hz to 1/60 Hz). In order to remove the phase evolution from the signal we assume that the gravitational wave signal is phase-locked to its radio signal. The down-sampling stage is necessary in order to make the process computationally efficient. This Bayesian parameter estimation is the second stage of the search, and uses a Markov Chain Monte Carlo (MCMC) algorithm to explore the posterior probability volume and produce marginalised posteriors for the unknown signal parameters $h_0, \phi_0, \psi, \cos \iota$, defined below.

The heterodyned data undergoes a process aimed at removing any particularly large outliers, by simply removing data points whose absolute value is greater than five times the standard deviation of the data. This test is run on the heterodyned data twice, to account for any extreme outliers that might skew the standard deviation of the data and so degrade the effectiveness of the cleaning.

Once heterodyned, the expected gravitational wave signal has the form [1]

$$h'(t) = h_0 \left( \frac{1}{4} F_+ (1 + \cos^2 \iota) \cos \phi_0 + \frac{1}{2} F_\times \cos \iota \sin \phi_0 \right) +$$

$$ih_0 \left( \frac{1}{4} F_+ (1 + \cos^2 \iota) \sin \phi_0 - \frac{1}{2} F_\times \cos \iota \cos \phi_0 \right),$$

where $F_+$ and $F_\times$ are the antenna beam patterns of the interferometer to plus and cross polarisations, $h_0$ is the gravitational wave amplitude, $\psi$ is the gravitational wave polarisation angle, $\iota$ is the inclination of the pulsar’s rotation axis with respect to the line of sight, and $\phi_0$ is the initial phase of the signal.

The parameter estimation assumes the data is stationary over 30-minute segments of contiguous data, and divides the data into as many 30-minute sections as possible. If there is a section of data of 5 minutes or longer on the end of a longer contiguous section of data already included, then these smaller segments are also included.

For this search two separate Bayesian parameter estimation runs are performed. The first run has uniform priors on all parameters; this assumes complete ignorance of the parameters.
prior to the analysis. The second run assumes uniform priors on the parameters $h_0$ and $\phi_0$, but with Gaussian priors on $\cos \epsilon$ and $\psi$. The motivation for using Gaussian priors on $\cos \epsilon$ and $\psi$ comes from observations of the pulsar wind tori as described in [15], and the premise that the orientation of the wind tori matches the orientation of the pulsar spin axis.

For a more detailed description of the gravitational wave signal and the complex heterodyne method see [1, 13].

3. The Data used in the Search
The Virgo science run 2 (VSR2) started on 7th July 2009 21:00:00 UTC and finished on 8th January 2010 22:00:01 UTC. A representative noise curve for this run is shown in Figure 1. Segments of data from the run have various flags assigned to them which indicate the state of the interferometer during those times. The flag that indicates the best possible quality data is known as “science mode”; all the data from “science mode” segments that pass the contiguity and outlier tests, described in Section 2, was used in the search. For VSR2 data, the outlier and contiguity tests removed a few percent of the data. The heterodyned data, with the contiguity and outlier tests applied is shown in Figure 2. A similar plot is shown in Figure 6 from [1]; however, the data from that figure are further processed, including an averaging of the data, to arrive at the data in Figure 2.

![Heterodyned data for psr J0835-4510, for detector V1, from 931035721 to 947023167](image)

Figure 2. A plot of the heterodyned VSR2 data, with the GPS seconds along the $x$-axis, and the strain on the $y$-axis. The blue and red dots represent the real and the imaginary parts of the complex data respectively. The time period covered by this plot is the entire of VSR2, i.e. from 7th July 2009 21:00:00 UTC to 8th January 2010 22:00:01 UTC.
Post-run analysis of the VSR2 data found a previously unknown noise artefact in the frequency band at which we expect gravitational wave signals from Vela [1]. The noise presented itself in the form of a pair of wondering “lines” which can clearly be seen in Figure 3. The cause of the lines was found to be a chiller pump, and measures have been taken to remove the effects on the detector data of this pump in future science runs. It is estimated that the effect of the chiller pump noise lines degraded the sensitivity of the searches by 20% [1], for a more detailed investigation into this noise artefact see [16].

![Spectrogram for V1:h_16384Hz; 2009/7/7 21:11:47 to 2009/10/5 6:27:52 UTC.](image)

**Figure 3.** A spectrogram of the whole of VSR2. The expected frequency of gravitational waves from Vela is plotted in black at $\sim 22.4$ Hz. The chiller lines are clear to see at $\sim 22.4$ Hz and $\sim 22.35$ Hz. It is also clear to see the variation in the detector noise over time.

4. The timing model
The complex heterodyne stage of the search which removes the expected phase evolution of the gravitational wave signal from the data needs a precise model of the phase evolution of the expected signal from the pulsar in question. To determine this phase model we assume that the gravitational waves are phase locked with electromagnetic pulses emitted from the pulsar. Detailed timing of these pulses is carried out by several radio telescopes, and it is from this data the phase model is formed [1]. The time of arrival data (TOAs) of the radio pulses were supplied by two different observatories: the Mt. Pleasant Observatory near Hobart in Tasmania (which has two antennas, the 26-m Mt Pleasant antenna and the 14-m Vela antenna built specifically for observing the Vela pulsar), and the Hartebeesthoek Radio Astronomy Observatory (HartRAO) near Johannesburg in South Africa.
Tempo2 software [17] was used to fit a modelled phase evolution to the TOAs. Best fit values for the right ascension, declination, proper motion, rotation frequency, and first and second derivatives of the rotation frequency were produced over the epoch of VSR2. TOAs were used from 1st June 2009 to 31st March 2010. By ensuring that the TOAs span VSR2, we aim to produce an accurate fit to the data and therefore an accurate model of the gravitational wave phase evolution over the same period. The parameters of the timing model produced for VSR2 are shown in Table 1.

| Parameter              | Value                                           |
|------------------------|-------------------------------------------------|
| Right Acension         | $08^h 35^m 20.7543822 \pm 2.5 \times 10^{-05}$ |
| Declination            | $-45^\circ 10' 32.95068'' \pm 6.8 \times 10^{-4}$ |
| $f_{\text{rot}} [\text{Hz}]$ | $11.19105730233076 \pm 8.6 \times 10^{-11}$ |
| $\dot{f}_{\text{rot}} [\text{Hz/s}]$ | $-1.55838759952784 \times 10^{-11} \pm 3.9 \times 10^{-18}$ |
| $\ddot{f}_{\text{rot}} [\text{Hz/s}^2]$ | $4.90687976149349 \times 10^{-22} \pm 8.7 \times 10^{-26}$ |

Table 1. Table showing the parameters for the fitted timing model for Vela TOAs from 1st June 2009 to 31st March 2010.

Vela is known as a particularly glitchy pulsar, with a glitch rate of $\sim 1/3 \text{y}^{-1}$, and as the gravitational wave phase evolution is uncertain during a glitch, it was important to ensure that Vela did not glitch during VSR2. The observations of Vela by Hartesbessthoeck and Mt. Pleasant radio antennae show no evidence for a glitch during VSR2.

5. Validation with hardware injections

As a test for each of the three analyses’ pipelines presented in [1], 13 gravitational wave signals were injected into the VSR2 data by moving one of the interferometer mirrors. Each of the three methods were then used to search for these signals, labelled pulsar00 to pulsar12. All three search methods were able to detect the signal and recover its parameters accurately when the SNR of the injected signal was sufficiently large. The results from the searches for three of the pulsars (pulsars 03, 05, 08), for each of the three search methods are presented in [1]. We present the probability distribution functions (PDFs) for the four unknown signal parameters for pulsar03 in Figure 4. These plots show the PDFs in blue, and also show the injected value for each parameter plotted as black vertical dashed lines on the plots. The injected parameter values have been accurately recovered, with the PDF for the $h_0$ parameter showing largest discrepancy of $\sim 3\%$ between injected value and the peak in the PDF, which is well within the calibration uncertainty for the Virgo interferometer.

6. Results

The headline result of the three different search methods are upper limits on the gravitational wave signal amplitude $h_0$. Although the Bayesian and frequentist methods are not asking directly equivalent questions, we find that the upper limits from the three methods, presented in table 2, are very similar.

The output of the Bayesian method’s parameter estimation runs are posterior probability distribution functions (PDFs) for each of the signal parameters. Each PDF is calculated by marginalising over the other signal parameters, meaning that at each value of the parameter for which the PDF has been calculated, the posterior probability is calculated considering all plausible combinations of values for the other signal parameters. The PDF of most interest is for the amplitude parameter $h_0$, as this is the strength of the gravitational wave signal. Where the $h_0$ PDF indicates there is no signal present, e.g. the PDF peaks at a $h_0$ value of zero, we
MCMC 10000000 iterations, JPULSAR03, V1

Figure 4. The PDFs for the hardware injection pulsar03. The black vertical dashed lines on each plot shows the injected value for that parameter, it can be seen that the injected parameter values have been accurately recovered. The PDF for the $h_0$ parameter appears as the worst match to the injected value, however the discrepancy is only $\sim 3\%$.

| Analysis method                                      | 95% upper limit for $h_0$          |
|-----------------------------------------------------|-----------------------------------|
| Bayesian heterodyne, restricted priors              | $(2.1 \pm 0.1) \times 10^{-24}$   |
| Bayesian heterodyne, unrestricted priors            | $(2.4 \pm 0.1) \times 10^{-24}$   |
| $G$-statistic                                       | $(2.2 \pm 0.1) \times 10^{-24}$   |
| $F$-statistic                                       | $(2.4 \pm 0.1) \times 10^{-24}$   |
| MF on signal Fourier components, 2 d.o.f.           | $(1.9 \pm 0.1) \times 10^{-24}$   |
| MF on signal Fourier components, 4 d.o.f.           | $(2.2 \pm 0.1) \times 10^{-24}$   |

Table 2. Table showing the upper limits obtained by the three different search methods [1].

find the $h_0$ value that bounds 95% of the probability density. This value is a 95% credible upper limit on $h_0$. The upper limit on $h_0$ can also be used to place an inferred upper limit on the equatorial ellipticity of the NS.

The PDFs using uniform priors on the parameters can be seen in Figure 5. The important PDF from Figure 5 is the $h_0$ PDF, as this tells us how likely different signal strengths are; the
most likely value for $h_0$ is at zero for this PDF, i.e. there is no evidence to suggest there is a signal present. The 95% credible upper limit on $h_0$ can be seen as the vertical dashed line, it is $\sim 2.4 \times 10^{-24}$.

![MCMC 1000000 iterations, J0835–4510, V1](image)

**Figure 5.** The PDF outputs for Vela from VSR2, produced by MCMC with 1 000 000 iterations [1]. The MCMC was run with uniform priors on all parameters. The 95% upper limit on $h_0$ derived from this MCMC is $\sim 2.4 \times 10^{-24}$.

The PDFs obtained using Gaussian priors on $\psi$ and $\cos(\iota)$ can be seen in Figure 6. Again, the $h_0$ PDF indicates that there is no signal present. It should also be clear that the PDFs for $\psi$ and $\cos(\iota)$ are strongly peaked compared to those in Figure 5, because of the priors placed on these parameters. The 95% credible upper limit is again plotted as a vertical dashed line at $\sim 2.1 \times 10^{-24}$.

7. Discussion

With the analysis of VSR2 data we have been able to surpass the spin-down limit for Vela with three almost independent search methods [1]. Each search method was used to perform two searches making different assumptions about our knowledge of the signal parameters $\psi$ and $\iota$. Vela is only the second pulsar for which this benchmark has been achieved. We now discuss the results obtained with the Bayesian method in more detail.

For the analysis where we assume no prior knowledge of the unknown signal parameters we have placed a 95% credible upper limit of $2.4 \times 10^{-24}$ on $h_0$, corresponding to $\sim 73\%$ of the spin-down limit of $3.3 \times 10^{-24}$. In the analysis where we assume some prior knowledge of the
Figure 6. The PDF outputs for Vela from VSR2, produced by MCMC with 1 000 000 iterations [1]. The MCMC was run with Gaussian priors on the orientation parameters $\psi$ and $\cos(\iota)$. The 95% upper limit on $h_0$ derived from this MCMC is $\sim 2.1 \times 10^{-24}$. Parameters $\psi$ and $\iota$ we are able to place a slightly lower 95% credible upper limit on $h_0$ of $2.1 \times 10^{-24}$. This slightly lower upper limit corresponds to $\sim 64\%$ of the spin-down limit.

The upper limits placed on $h_0$ can be used to infer an upper limit on the equatorial ellipticity of the NS using the canonical value for a NS’s moment of inertia of $I = 10^{38}$ kg m$^2$, this yields corresponding upper limits on the ellipticity for the two different analyses of $\sim 1.2 \times 10^{-3}$ and $\sim 1.3 \times 10^{-3}$, both of which are greater by several orders of magnitude than the maximum allowable equatorial ellipticity predicted by standard NS equations of state. However, these upper limits represent conservative estimates, as the theoretical predictions for $I$ range from $\sim 1 - 3 \times 10^{38}$ kg m$^2$ [10]. It is also useful to represent the upper limits as a fraction of the total spin-down energy, by inferring an upper limit on the power emitted as gravitational waves by $\dot{E}_{GW} = \frac{32\pi G I^2 \epsilon^2 f^6}{5c}$; this gives values of $52\%$ and $45\%$. Although we have surpassed the spin-down limit, these results remain far from those for the Crab pulsar [10], where upper limits on the fraction of spin-down energy lost as gravitational waves are $\sim 2\%$; however, tighter upper limits for Vela are expected to be obtained with the analysis of Virgo VSR4 data, which was collected from 3rd June 2011 to 3rd September 2011.
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