Effect of thermal radiation on chemically reacting magnetohydrodynamic dusty viscous fluid flow in a porous channel

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Abstract. In this paper, we have studied the combined effects of free convective heat and mass transfer on an unsteady MHD dusty viscous fluid flow in a vertical channel saturated with porous medium subject to the convective boundary condition. The coupled partial differential equations are solved analytically using perturbation technique. The velocity, temperature and concentration fields have been studied for various combinations of physical parameters such as magnetic field, thermal radiation, Biot number and chemical reaction parameters. The expressions for skin friction, Nusselt number and Sherwood number are presented and the results are tabulated. Further, it is observed that the velocity profiles of dusty fluid are higher than the dust particles.

1. Introduction
Makinde and Chinyok[1] have analyzed the transport properties of a dusty fluid in a channel by accounting the electrical conductivity variations. Sivaraj and Rushi Kumar[2] have carried out a theoretical study to predict the influence of electric conductivity variations on Walters-B viscoelastic fluid flow. Sivaraj and Jasmine Benazir[3] have mathematically modeled the electrically conducting flow of a Casson fluid and analyzed the transport properties. Jasmine Benazir and Sivaraj[4] have obtained a numerical solution for chemically reacting Casson fluid over a cone by employing the Crank–Nicolson technique. Mythili and Sivaraj[5] have used the implicit finite difference method to solve the governing equations which represent the chemically reacting Casson fluid flow. Animasaun et al[6] have analyzed the properties of the two chemically reacting species on viscoelastic fluid flow. The significance of thermal radiation on the Couette flow of a chemically reacting Casson fluid over an asymmetric channel has been investigated by Sivaraj and Rushi Kumar[7]. An analytical solution has been obtained by Prakash et al[8] to examine the characteristics of thermal radiation on a heat absorbing fluid flow. A theoretical investigation to explore the important role of nonlinear thermal radiation on the thermal boundary layer of a nanofluid has been carried out by Animasaun and Sandeep[9]. An attempt has been made by Kameswaran et al[10] to examine the influence of non-Darcy flow of a nanofluid over a vertical wavy surface. Stagnation point flow of nanofluid over porous medium has been analyzed by Kameswaran et al[11]. Mythili et al[12] have mathematically modeled the non-Darcy flow of Casson fluid over two different geometries saturated with a porous medium. Makinde and Aziz[13] have investigated the nanofluid boundary layer flow over a sheet subject to convective heating boundary condition. Akbar et al[14] have numerically studied the flow of nanofluid over a cylinder by accounting the convective boundary condition. The
significance of convective boundary condition on the Couette flow of a chemically reacting dusty Walters-B viscoelastic fluid in an asymmetric channel has been investigated by Sivaraj and RushiKumar[15]. Motivated by the above-mentioned investigations, in this paper, we investigate the transport properties of chemically reacting and thermally radiating magnetohydrodynamic flow of a dusty viscous fluid in a porous channel subject to convective boundary condition.

2. Formulation of the problem

The transport properties of time-dependent flow of an incompressible viscous fluid subject to natural convection in a vertical porous channel are modeled as depicted in Fig. 1.

![Flow Geometry](image)

The fluid flow and upright directions are respectively represented by X-axis and Y-axis. The influence of an exterior even magnetic field of strength $B_0$ applied at right angles, thermal radiation and first order chemically reactive species subject to thermal convective boundary conditions are analyzed. The governing equations which define the above-mentioned assumptions can be written as follows:

\[
\frac{\partial U}{\partial t^*} = g \left( \beta_r (T - T_i) + \beta_e (C - C_i) \right) + \nu \frac{\partial^2 U}{\partial Y^2} + \frac{K_i N_0}{\rho} (V - U) - \frac{\nu}{K} U - \frac{\sigma B_0^2}{\rho} U \quad (1)
\]

\[
\frac{\partial V}{\partial t^*} = -\frac{K_i}{m} (U - V) 
\]

\[
\frac{\partial T}{\partial t^*} = \frac{1}{\rho C_p} \left( k \frac{\partial^2 T}{\partial Y^2} - \frac{\partial q_r}{\partial Y} \right) 
\]

\[
\frac{\partial C}{\partial t^*} = D \frac{\partial^2 C}{\partial Y^2} - K_r (C - C_i) 
\]

The transport properties during primary assumption ($t^* = 0$) are

\[ U = 0, \quad V = 0, \quad T = T_{u1}, \quad C = C_{n1} \text{ for } Y \in (0, d) \quad (5) \]

The transport properties at the limits of the geometry at any time ($t^* > 0$) are

\[ U = 0, \quad V = 0, \quad -k \frac{\partial T}{\partial Y} = h_f (T_{u1} + (T_{w2} - T_{w1}) e^{-\gamma i} - T), \quad C = C_{n1} + (C_{n2} - C_{n1}) e^{-\gamma i} \text{ at } Y = 0 \quad (6) \]

\[ U = 0, \quad V = 0, \quad -k \frac{\partial T}{\partial Y} = h_f (T_{w2} + (T_{w2} - T_{w1}) e^{-\gamma i} - T), \quad C = C_{n2} + (C_{n2} - C_{n1}) e^{-\gamma i} \text{ at } Y = d \quad (7) \]

The change in radiative heat is considered as

\[
\frac{\partial q_r}{\partial Y} = 4(T - T_i) I', \quad I' = \int_0^\infty K_{\lambda h} \frac{\partial q_h}{\partial T} d\lambda_i \quad (8)
\]

where $d$ is dimensional width of the channel, $N_0$ is number of density of the dust particles, $K_i$ is Stokes resistance coefficient, $m$ is mass per unit volume of the dust particles, $K^*$ is dimensional
porous permeability parameter, $K_{x_{1w}}$ is coefficient of radiation absorption near to walls, $e_{b_{11}}$ is Plank’s function, $K_R$ is the chemically reactive factor, $n^*$ is real and positive constant and $h_t$ is heat transfer coefficient.

The following quantities are introduced to obtain the dynamical similarity
\[
x = \frac{X}{d}, \quad y = \frac{Y}{d}, \quad u = \frac{U}{U_0}, \quad v = \frac{V}{U_0}, \quad \theta = \frac{T - T_1}{T_2 - T_1}, \quad \phi = \frac{C - C_1}{C_2 - C_1}, \quad t = \frac{vt^*}{d^2}, \quad n = \frac{n^*d^2}{v}
\]  

(9)

Based on Eq. (9), the basic governing Eqs. (1)-(4) are summarized as following
\[
\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + \frac{\delta}{w}(v - u) - Nu + G_r\theta + G_c\phi
\]  

(10)

\[
\frac{w}{\partial t} = v - u
\]  

(11)

\[
P_r \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} + F\theta
\]  

(12)

\[
S_c \frac{\partial \phi}{\partial t} = \frac{\partial^2 \phi}{\partial y^2} - K_c S_c\phi
\]  

(13)

The corresponding initial and boundary conditions (5)-(7) in dimensionless form are
\[
t = 0: \quad u = 0 = v, \quad \theta = 0, \quad \phi = 0 \quad \text{for} \quad t = 0
\]  

(14)

\[
t > 0: \quad u = 0 = v, \quad \theta' = B_r(\theta - \varepsilon e^{-\alpha t}), \quad \phi = \varepsilon e^{-\alpha t} \quad \text{at} \quad y = 0
\]  

(15)

\[
u = 0 = v, \quad \theta' = B_r(\theta - 1 - \varepsilon e^{-\alpha t}), \quad \phi = 1 + \varepsilon e^{-\alpha t} \quad \text{at} \quad y = 1
\]  

(16)

The mass concentration of dust particle($\delta$), relaxation time parameter for particles($w$), a parameter for thermagnetic field ($M^2$), thermal Grashof number($G_r$), porous permeability coefficient($K_c$), solutal Grashof number($G_c$), Prandtl number($P_r$), Biot number($B_r$), Schmidt number and chemically reactive factor($K_r$) are respectively as follows:

\[
\delta = \frac{mN_0}{\rho}, \quad w = \frac{m}{K_1d^2}, \quad M^2 = \frac{\alpha B_0d^2}{\mu}, \quad G_r = \frac{gB_r(T_2 - T_1)d^2}{vU_0}, \quad 1 = \frac{d^2}{K}, \quad G_c = \frac{gB_c(C_2 - C_1)d^2}{vU_0}, \quad \frac{P_r}{k} = \frac{\mu C_p}{k}, \quad B_r = \frac{h_t d^2}{k}, \quad F = \frac{-4Id^2}{k}, \quad S_c = \frac{v}{D}, \quad K_r = \frac{K_g d^2}{v}
\]

3. Method of solution

The analytical solutions are obtained for Eqs. (10)-(13) by adopting perturbation method. The fluid velocity($u$), fluid temperature($\theta$) and fluid concentration($\phi$) can be represented in power of $\varepsilon$ ($\varepsilon << 1$) as follows:

\[
u(y, t) = u_0(y) + \varepsilon e^{-\alpha t}u_1(y, t) + o(\varepsilon^2); \quad \theta(y, t) = \theta_0(y) + \varepsilon e^{-\alpha t}\theta_1(y, t) + o(\varepsilon^2); \quad \phi(y, t) = \phi_0(y) + \varepsilon e^{-\alpha t}\phi_1(y, t) + o(\varepsilon^2)
\]  

(17)

By means of Eq. (17) followed by neglecting higher powers of $o(\varepsilon^2)$, the Eqs. (10)-(16) becomes

\[
u_0'' - Nu_0 = -(G_r\theta_0 + G_c\phi_0); \quad u_1'' - (N - n)u_1 = -(G_r\theta_1 + G_c\phi_1)
\]  

(18)
\[ \theta'' - F \theta' = 0; \quad \theta'' - (F - nP) \theta' = 0 \tag{19} \]

\[ \phi'' - K_S \phi = 0; \quad \phi'' - (K_r - n) S \phi = 0 \tag{20} \]

The appropriate boundary conditions become

\[ u_0 = 0 = v_0, \quad \theta_0 = \phi_0 = 0, \quad u_1 = 0 = v_1, \quad \theta'_1 = \phi_1 = 1 \text{ at } y = 0 \tag{21} \]

\[ u_0 = 0 = v_0, \quad \theta_0 = \phi_0 = 1, \quad u_1 = 0 = v_1, \quad \theta'_1 = \phi_1 = 1 \text{ at } y = 1 \tag{22} \]

The Eqs. (18)-(20) are solved subject to the boundary conditions stated in Eqs. (21) and (22), and the expressions for the transport properties of the fluid are summarized as follows

\[ u(y,t) = (A_0 e^{\beta_1 y} + A_1 e^{\beta_2 y} + A_2 e^{\beta_3 y} + A_3 e^{\beta_4 y} + A_4 e^{\beta_5 y} + A_{10} e^{\beta_{10} y}) \]

\[ + \varepsilon e^{\gamma t} (A_{15} e^{\beta_{15} y} + A_{16} e^{\beta_{16} y} + A_{17} e^{\beta_{17} y} + A_{18} e^{\beta_{18} y} + A_{19} e^{\beta_{19} y} + A_{20} e^{\beta_{20} y}) \tag{23} \]

\[ v(y,t) = (A_0 e^{\beta_1 y} + A_1 e^{\beta_2 y} + A_2 e^{\beta_3 y} + A_3 e^{\beta_4 y} + A_4 e^{\beta_5 y} + A_{10} e^{\beta_{10} y}) \]

\[ + \varepsilon e^{\gamma t} (A_{15} e^{\beta_{15} y} + A_{16} e^{\beta_{16} y} + A_{17} e^{\beta_{17} y} + A_{18} e^{\beta_{18} y} + A_{19} e^{\beta_{19} y} + A_{20} e^{\beta_{20} y}) \tag{24} \]

\[ \theta(y,t) = (A_0 e^{\beta_1 y} + A_1 e^{\beta_2 y} + A_2 e^{\beta_3 y} + A_3 e^{\beta_4 y} + A_4 e^{\beta_5 y}) \]

\[ + \varepsilon e^{\gamma t} (A_{15} e^{\beta_{15} y} + A_{16} e^{\beta_{16} y} + A_{17} e^{\beta_{17} y} + A_{18} e^{\beta_{18} y} + A_{19} e^{\beta_{19} y} + A_{20} e^{\beta_{20} y}) \tag{25} \]

\[ \phi(y,t) = (A_0 e^{\beta_1 y} + A_1 e^{\beta_2 y} + A_2 e^{\beta_3 y} + A_3 e^{\beta_4 y} + A_4 e^{\beta_5 y}) \]

\[ + \varepsilon e^{\gamma t} (A_{15} e^{\beta_{15} y} + A_{16} e^{\beta_{16} y} + A_{17} e^{\beta_{17} y} + A_{18} e^{\beta_{18} y} + A_{19} e^{\beta_{19} y} + A_{20} e^{\beta_{20} y}) \tag{26} \]

where \( N = K^{-1} + M^2 \); \( \beta_1 = \beta_2 = \sqrt{K, S} ; \beta_3 = \beta_4 = \sqrt{F} ; \beta_5 = \beta_6 = \sqrt{N} \);

\[ \beta_7 = \beta_8 = \sqrt{K, S} ; \beta_9 = \beta_{10} = \sqrt{F - nP} ; \beta_{11} = \beta_{12} = \sqrt{N - n} ; A_i = (e^{\beta_i - \beta_{10}})^{-1} \]

\[ A_0 = e^{\beta_1} \left( e^{\beta_1 - \beta_2} \right)^{-1} ; A_1 = -B_i (\beta_3 - \beta_2)^{-1} \left( e^{\beta_2 - \beta_3} \right)^{-1} ; A_2 = -B_i (\beta_3 - \beta_2)^{-1} \left( e^{\beta_2 - \beta_3} \right)^{-1} \]

\[ A_3 = A_4 \left( \beta_2 - 2 \beta_2 \right)^{-1} ; A_6 = -A_5 \left( \beta_2 - N \right)^{-1} ; A_7 = -A_5 \left( \beta_2 - N \right)^{-1} ; A_8 = -A_5 \beta_4 - N \]

\[ A_7^* = A_9 \left( e^{\beta_4 - \beta_5} \right) + A_9 \left( e^{\beta_4 - \beta_5} \right) + A_9 \left( e^{\beta_4 - \beta_5} \right) + A_9 \left( e^{\beta_4 - \beta_5} \right) ; A_9 = A_9^* \left( e^{\beta_4 - \beta_5} \right)^{-1} \]

\[ A_{10} = A_{10} \left( e^{\beta_4 - \beta_5} \right) + A_{10} \left( e^{\beta_4 - \beta_5} \right) + A_{10} \left( e^{\beta_4 - \beta_5} \right) + A_{10} \left( e^{\beta_4 - \beta_5} \right) ; A_{10} = -A_{10}^* \left( e^{\beta_4 - \beta_5} \right)^{-1} \]

\[ A_{11} = -A_{11}^* \left( e^{\beta_4 - \beta_5} \right) ; A_{12} = -A_{11}^* \left( e^{\beta_4 - \beta_5} \right) ; A_{13} = \frac{B_i \left( e^{\beta_2 - \beta_3} - 1 \right)}{\left( \beta_2 - \beta_3 \right) \left( e^{\beta_2 - \beta_3} \right)} ; A_{14} = \frac{B_i \left( e^{\beta_2 - \beta_3} - 1 \right)}{\left( \beta_2 - \beta_3 \right) \left( e^{\beta_2 - \beta_3} \right)} \]

\[ A_{15} = -A_{16} \left( \beta_2 - N - n \right) ; A_{16} = -A_{16} \left( \beta_2 - N - n \right) ; A_{17} = -A_{16} \left( \beta_2 - N - n \right) ; A_{18} = -A_{16} \left( \beta_2 - N - n \right) \]

\[ A_{19} = -A_{16} \left( e^{\beta_2 - \beta_3} \right) + A_{16} \left( e^{\beta_2 - \beta_3} \right) + A_{16} \left( e^{\beta_2 - \beta_3} \right) + A_{16} \left( e^{\beta_2 - \beta_3} \right) ; A_{19} = A_{19} \left( e^{\beta_2 - \beta_3} \right)^{-1} \]

\[ A_{20} = A_{20} \left( e^{\beta_4 - \beta_5} \right) + A_{20} \left( e^{\beta_4 - \beta_5} \right) + A_{20} \left( e^{\beta_4 - \beta_5} \right) + A_{20} \left( e^{\beta_4 - \beta_5} \right) ; A_{20} = -A_{20} \left( e^{\beta_4 - \beta_5} \right)^{-1} \]

The rate of fluid velocity \( (\tau) \), the rate of heat transfer \( (Nu) \) and the rate of mass transfer (Sh) at wall \( y = 0 \) and wall \( y = 1 \) are given by

\[ \tau_{0y} = -u'|_{y=0}, \tau_{1y} = -u'|_{y=1}; \quad \tau_{p0} = -v'|_{y=0}, \tau_{p1} = -v'|_{y=1} ; \]

\[ Nu_0 = -\theta'|_{y=0}, Nu_1 = -\theta'|_{y=1}; \quad Sh_0 = -\phi'|_{y=0}, Sh_1 = -\phi'|_{y=1} \tag{27} \]

4. Result and discussions
Figures 2 and 3 depict the flow, heat and mass transfer characteristics of the problem. Throughout the computations we employ \( w = 10 \), \( G_r = 2 \), \( M = 2 \), \( G_c = 1 \), \( K = 2 \), \( P_r = 0.71 \), \( S_c = 0.96 \), \( Bi = 0.1 \), \( \varepsilon = 0.02 \), \( t = 1 \), \( F = 2 \), and \( K_r = 2 \) unless otherwise stated.

Table 1. Variations of \( M \) and \( w \) on \( \tau_{f0} \), \( \tau_{p0} \), \( \tau_{f1} \), and \( \tau_{p1} \). (PP – Physical Parameter)

| PP | Values | \( \tau_{f0} \) | \( \tau_{p0} \) | \( \tau_{f1} \) | \( \tau_{p1} \) |
|----|--------|----------------|----------------|----------------|----------------|
| \( M \) | 0        | -0.1944        | -0.1796       | 0.3349         | 0.3215         |
|       | 2        | -0.1392        | -0.1276       | 0.2714         | 0.2611         |
|       | 4        | -0.0764        | -0.0687       | 0.1904         | 0.1839         |
| \( w \) | 1        | -0.1392        | -0.1407       | 0.2714         | 0.2727         |
|       | 2        | -0.1392        | -0.1431       | 0.2714         | 0.2749         |
|       | 3        | -0.1392        | -0.1479       | 0.2714         | 0.2792         |

Table 2. Effect of \( F \) and \( Bi \) on \( \tau_{f0} \), \( \tau_{p0} \), \( \tau_{f1} \), \( \tau_{p1} \), \( Nu_0 \), and \( Nu_1 \).
5. Conclusions
Analytical solutions are obtained for the magnetohydrodynamic flow of thermally radiating and chemically reacting dusty viscous fluid in an irregular channel subject to the convective boundary condition. The important observations of this investigation are as follows: The dusty fluid velocity is higher than the dust particles velocity. The flow and heat transfer profiles strictly decrease for increasing $F$ and Bi. An increase in $K_r$ diminishes the flow and mass transfer profiles. Velocity profiles enhance for increasing $K$, Gr and Gc but they decrease for increasing $M$.

References
[1] Makinde OD and Chinyok T 2010 Comput Math Appl 60 660.
[2] Sivaraj R and Rushi Kumar B 2013 Int J Heat Mass Transfer 61 119.
[3] Sivaraj R and Jasmine Benazir A 2015 Special Topics Reviews Porous Media 6 267–281.
[4] Jasmine Benazir A and Sivaraj R 2016 Adv Intelligent Systems Comput 436 537.
[5] Mythili D and Sivaraj R 2016 J Mol Liq 216 466.
[6] Animasaun IL, Raju CSK and Sandeep N 2016 Alexandria Engng J 55 1595.
[7] Sivaraj R and Rushi Kumar B 2012 Int J Heat Mass Transfer 55 3076.
[8] Prakash J, Rushi Kumar B, Sivaraj R 2014 Walailak J Science Tech 11 939.
[9] Animasaun IL and Sandeep N 2016 Powder Tech 301 858.
[10] Kameswaran PK, Vasu B, Murthy PVSN and Gorla RSR 2016 Int Commun Heat Mass Transfer 77 78.
[11] Kameswaran PK, Sutradya A, Murthy PVSN and Sibanda P 2016 J Nanofluids 5 310.
[12] Mythili D, Sivaraj R and Rashidi MM 2017 Int J Numerical Methods Heat Fluid Flow 27 156.
[13] Makinde OD and Aziz A Int J Thermal Sci 50 1326.
[14] Akbar NS, Nadeem S, Haq RU and Khan ZH 2013 Chinese J Aeronautics 26 1389.
[15] Sivaraj R and Rushi Kumar B 2012 Int J Heat Mass Transfer 55 3076.