Dynamic Loading of Cogwheels at the Time of Start-Up

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Abstract. Loading of cogwheels of the high-loaded transfers to the start-up moment is considered. It is shown that at torque transfer on a surface of a nave of a cogwheel are generated and extend in material in the radial direction of a wave of tension of torsion. The equations for definition of distribution of tension of torsion on the radius of a cogwheel and for definition of a torsion torque on an outer surface of a cogwheel at the time of the beginning of its rotational motion are received. It is shown that thus overloads have more considerable sizes, than used at modern calculations. Results of calculations for specific input data allow us to draw a conclusion that the dynamic torsion torque at start-up is more, than a rated torsion torque more than twice.

1. Introduction
Due to the continuous growth of power capacity of tooth gears, increase in speed ranges of their work there is a problem of more exact accounting of dynamism of loading. At calculations of tooth gears, it is supposed that loadings from a shaft are transferred to a wheel without changes. However, at the time of start-up, loading has dynamic (shock) character [1–3]. As a result, in material of cogwheels, waves of tension are generated and extended, and characteristics tensely - the deformed state in them are heterogeneous and non-stationary. At torque transfer on a surface of a nave of a cogwheel radial waves of torsion in process of which distribution of the forefront there is a reduction of amplitude of tension owing to increase in a surface of the front are generated. At achievement, rotational motion of the last will begin a forefront wave of tension of an outer surface of a cogwheel.

2. Formulation of problem
The intense deformed condition of material of a cogwheel is defined by the components of tensors of tension \( \sigma_{R} \) and deformations \( \varepsilon_{R} \), which are not depending on coordinates \( \theta \) and \( z \), the being functions only of the coordinate \( R \) and time \( t \). In addition, other components of tensors of tension and deformations are equal to zero. These assumptions reduce process of deformation to distribution in material of radial waves of torsion. The system of equations describing a task includes the movement equation, a condition of continuity and the defining ratio - Hooke's law [4]:

\[
\begin{align*}
\rho \frac{\partial V(R,t)}{\partial t} &= \frac{\partial \sigma_{R0}(R,t)}{\partial R} + 2 \frac{\sigma_{R0}(R,t)}{R} \\
\frac{\partial \varepsilon_{R0}(R,t)}{\partial t} &= \frac{\partial V(R,t)}{\partial R} - \frac{V(R,t)}{R} \\
\frac{\partial \varepsilon_{R0}(R,t)}{\partial t} &= \frac{1}{G} \frac{\partial \sigma_{R0}(R,t)}{\partial t}
\end{align*}
\]

(1)
where \( \rho \) – cogwheel material density; \( V(R,t) \) – circumferential speed of material particles; \( G \) – module of elasticity of the second sort (rigidity modulus).

3. Theory

Entry conditions correspond to unstressed and undeformed condition of material:

\[
\sigma_{R0}(R,0) = \varepsilon_{R0}(R,0) = V(R,0) = 0.
\]

As boundary condition, the dependence considering reduction of tension on a contact area in process of distribution of a front wave deep into of cogwheel material was used:

\[
\sigma_{R0}(R_0,t) = \left( \frac{R_0}{R(t)} \right)^3 \tau_{tor},
\]

where \( \tau_{tor} = \frac{T_{tor}}{W_\rho} \) – torsion tension on a shaft surface; \( T_{tor} \) – the torsion torque transferred from a shaft to a wheel; \( W_\rho \) - polar drag torque of section of a shaft.

The system of equations (1) describing a task represents system of quasi-linear differential equations in partial derivatives of first order of hyperbolic type and therefore, it is reasonable to solve it by method of characteristics [5]. Let's add it with entry conditions (2)-(3) and identical ratios for total differentials of required functions (only 3). As a result, we will receive system of 6 linear algebraic equations with unknown functions:

\[
\frac{\partial \sigma_{R0}}{\partial z}, \frac{\partial \varepsilon_{R0}}{\partial t}, \frac{\partial \varepsilon_{R0}}{\partial z}, \frac{\partial \sigma_{R0}}{\partial t}, \frac{\partial V}{\partial z}, \frac{\partial V}{\partial t}.
\]

Let's demand that it had infinite set of decisions. For this purpose it is necessary that its main determinant equaled to zero. From this condition received three families of the equations of characteristics:

\[
dR = 0, \quad dR = \sqrt{\frac{G}{\rho}} dt = a \ dt, \quad dR = -\sqrt{\frac{G}{\rho}} dt = -a \ dt,
\]

where \( a \) – a propagation velocity of an elastic wave of tension of torsion deep into cogwheel material.

That the decision along characteristics final zero other determinants of system needed to consider equality. For this purpose, we will replace the last column of the main determinant with the corresponding right parts of the equations and we will consider its equality to zero. Consistently substituting in the received equality of the equation of characteristics, we will construct differential ratios between required functions.

Thus, integration of quasi-linear system of differential equations of hyperbolic type at the set entry and boundary conditions is reduced to integration of ratios along the respective characteristic directions. The equations of the characteristic directions and ratio along them were received:

- along characteristics \( dR = 0 \):

\[
d\sigma_{R0}(R,t) - G d\varepsilon_{R0}(R,t) = 0;
\]

- along characteristics \( dR = \sqrt{\frac{G}{\rho}} dt = a \ dt \):

\[
d\sigma_{R0}(R,t) - a \rho dV(R,t) + G \frac{V(R,t) dt}{R} + \frac{2 a \sigma_{R0}(R,t) dt}{R} = 0;
\]
along characteristics \[ dR = -\sqrt{\frac{G}{\rho}} dt = -a dt : \]
\[ d\sigma_{R_0}(R,t) + a \rho \ dV(R,t) + \frac{G \ V(R,t) \ dt}{R} - \frac{2 \ a \ \sigma_{R_0}(R,t) \ dt}{R} = 0 ; \] (6)

The forefront wave of torsion tension extends along the characteristic \[ dR = \sqrt{\frac{G}{\rho}} dt = a dt, \] on which the ratio is carried out (5). Besides, on a forefront wave, conditions of a dynamic and kinematic continuity must be satisfied [6]:
\[ \sigma_{R_0}(R,t) = -a \rho V(R,t) \text{ and } V(R,t) = -a \varepsilon_{R_0}(R,t) . \] (7)

Substituting the equations (7) in (5), received the differential equation describing tension variation of torsion on a forefront wave of tension:
\[ 2d\sigma_{R_0}(R,t) = -\frac{a \ \sigma_{R_0}(R,t)}{R(t)} \ dt . \] (8)

The decision (8) taking into account boundary condition (3) has an appearance:
\[ \sigma_{R_0}(R,t) = \tau_{sp} \sqrt{\frac{R_0}{R_0 + at}} = \tau_{sp} \sqrt{\frac{R_0}{R(t)}} . \] (9)

Thus, at distribution of a forefront wave of torsion tension of amplitude of tension decreases under the law (9).

4. Results and discussions

It is known that tension of torsion is distributed on radius under the linear law. Therefore, in case of homogeneous materials of a cogwheel and shaft, in each point of material of system “a shaft – a wheel” torsion tension will also change under the linear law. In this case, on the neutral line, it will be equal to zero, and in the point corresponding to arrival to it of a forefront wave of torsion tension – it will be defined from the decision (9). In process of distribution of the forefront deep into of cogwheel material, his material particles will receive rotational motion with line speed \( V(R,t) \).

At achievement by a forefront wave of tension of torsion of a ring gear (a dividing circle of a cogwheel) the wheel will begin rotational motion. In Figure 1 distribution of tension of torsion is presented on a forefront wave of tension in an instant of its arrival on a dividing circle of a wheel at the following input data: diameter of a shaft is 60 mm, dividing wheel size of 460 mm, a torsion torque equal, \( T = 2000 \text{ Nm} \). It is visible that in the considered case tension falls on 2/3 from initial value at the front. In addition, the more diameter of a cogwheel, the more will be tension drop.
Figure 1. Distribution of tension of torsion on a forefront of wave

At the same time, behind the front of the extending wave there will be a tension drop due to its reduction on a boundary surface (a surface of contact of a shaft and a wheel). Numerical solutions of the corresponding wave tasks [4-6] allow to draw a conclusion that tension drop behind the front happens under the law to the similar law of tension drop at the front, that is corresponds to the equation (9) where as a multiplier before a root it is necessary to use boundary condition:

\[ \sigma_{R0}(R,t) = \left( \frac{R_0}{R(t)} \right)^3 \tau_{kp} \sqrt{\frac{R_0}{R_0 + at}} = \left( \frac{R_0}{R(t)} \right)^2 \tau_{kp}. \]  

(10)

In Figure 2 distribution of tension on wheel radius at the same input data is provided to different instants of time: the curve 1 corresponds to run time a wave from a shaft to an external wheel rim; a curve 2 – time at run by the reflected front wave front from an external wheel rim to a shaft (time is twice more). It is visible that distribution of tension on the radius of a wheel is leveled. It comes as because of partial reflection of the falling wave of tension of torsion from a surface of contact with the second cogwheel entering gearing and because of the subsequent interference of the corresponding waves. The reflected front extends in previously heterogeneous strained and deformed material. Tension at the front of a back wave unloads material in process of distribution of its front to the loaded surface of contact of a shaft and a wheel. Thus, the total state on the forefront of a back wave was presented in the form [7]:

\[ X^\Sigma_H(R,t) = X^\Sigma_H(R,t) + X_I(R,t). \]

(11)

where \( X^\Sigma_H \) – total value of required sizes at the front a back wave as a result of its interaction with the falling wave; \( X^\Sigma_H \) – the amplitude value of required size introduced in considered point by the forefront of a back wave; \( X_I \) – the corresponding parameter of a state in the considered rod point before arrival of the front of a back wave to it.

Then, the dynamic torsion torque in an instant of the beginning of rotational motion of a wheel will be defined so:

\[ T_d = 2\pi \int_{R_0}^{R} r^2 \sigma_{R0}(R,t) dr. \]

Taking into account (10) expression for definition of a dynamic torsion torque has an appearance:

\[ T(t) = 2\pi \int_{R_0}^{R} r^2 \left( \frac{R_0}{R_0 + at} \right)^2 \tau_{kp} dr. \]  

(11)

Results of calculations at the considered earlier input data allow us drawing conclusion: the dynamic torsion torque (at start-up) is significantly more than nominal torque. At run time a wave from a shaft to an external wheel rim (at the time of a start of motion), a torsion torque equal \( T = 5330 \text{ Nm} \), that is 2.66 times more rated, and then at double increase in time a torsion torque equal \( T = 2667 \text{ Nm} \), that is rather close to nominal value of a torsion torque.

5. Conclusions

Thus, the scheme of wave loading of material of cogwheels at the time of start-up is considered. The equations for definition of distribution of tension of torsion on the radius of a cogwheel and for definition of a torsion torque on an outer surface of a cogwheel at the time of the beginning of its
rotational motion are received. It is shown that thus overloads have more considerable sizes than used at modern calculations, but they have short-term character and quickly fade.

References
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