Gauged Inflation

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Abstract. We propose a model for cosmic inflation which is based on an effective description of strongly interacting, nonsupersymmetric matter within the framework of dynamical abelian projection and centerization. The underlying gauge symmetry is assumed to be SU(N) with $N \gg 1$. Appealing to a thermodynamical treatment, the ground-state structure of the model is determined by a potential for the inflaton field (monopole condensate) which allows for nontrivially BPS saturated and thereby stable solutions. For $T < M_P$ this leads to an apparent decoupling of gravity from the inflaton dynamics. The ground state dynamics implies a heat capacity for the vacuum leading to inflation for temperatures comparable to the mass scale $M$ of the potential. The dynamics has an attractor property. In contrast to the usual slow-roll paradigm we have $m \gg H$ during inflation. As a consequence, density perturbations generated from the inflaton are irrelevant for the formation of large-scale structure, and the model has to be supplemented with an inflaton independent mechanism for the generation of spatial curvature perturbations. Within a small fraction of the Hubble time inflation is terminated by a transition of the theory to its center symmetric phase. Due to the prevailing $Z_N$ symmetry relic vector bosons are stabilized and therefore potential originators of UHECR’s beyond the GZK bound.

1 Introduction

Cosmic Inflation \[1,2,3\] is the concept to explain the almost perfect isotropy, homogeneity, and flatness of the observable universe on large scales and the fact that no topological defects of GUT phase transitions have been observed. To describe this phenomenon in the framework of Friedmann-Robertson-Walker (FRW) geometry one usually assumes cosmology to be driven by one or more real and minimally coupled scalar fields which roll down their respective potentials. If a regime of slow-roll is reached, where the energy density is sufficiently dominated by the potential, the universe undergoes a rapid expansion stretching a causally connected patch to scales much larger than the horizon. As a result, after inflation the observable universe remains highly homogeneous and isotropic as long as regions, which were causally disconnected prior to inflaton, do not enter the horizon. The measured homogeneity and isotropy of the observable universe then implies that inflation must have produced at least 60 e-foldings of the scale factor.

Within the above approach the slow-roll paradigm is exploited to explain the small deviations from perfect isotropy in the cosmic microwave background
(CMB) and also the formation of large-scale structure. The idea is that quantum fluctuations of the inflaton field are so strongly red-shifted during inflation that their wavelengths may leave the horizon. On super-horizon scales these fluctuations transform into classical gaussian noise with a scale invariant spectrum. Upon horizon entry sufficiently long after reheating these classical perturbations are the originators of spatial curvature perturbations which cause the formation of large-scale structure [4]. Crucial for the validity of this picture is the validity of the condition $m \ll H$ following from the slow-roll paradigm. Here $m$ is the mass of fluctuations, and $H$ is the Hubble parameter during inflation. Explaining perturbations by the same field that causes inflation is a rather economic feature. However, $m \ll H$ seems rather artificial since it expresses a hierarchy in the mass scales governing the matter dynamics which drives inflation.

Since inflationary models are not very constrained progress is achieved by relying on a strong principle which determines the dynamics and at the same time explains the usual, phenomenologically driven ad hoc assumptions and desired features. For example, if inflation can be made responsible for the generation of some of the massive and (quasi)stable relics which constitute cold dark matter and upon decay may cause ultra-high energy cosmic rays (UHECR) beyond the Greisen-Zatsepin-Kuzmin (GZK) cutoff [5] then it should also explain why. Or, why does the inflaton potential not allow for moduli whose occurrence after inflation can cause large inhomogeneities and anisotropies by isothermal perturbations [6] being in contradiction with observation? Another question is whether the slow roll of the inflaton field during inflation rather than be imposed can be explained.

In this talk I propose inflation to be caused by the dynamics of strongly interacting matter which is governed by a nonabelian gauge symmetry. For definiteness we assume this symmetry to be SU(N). Within an effective, thermal description, which is based on dynamical abelian projection and centerization at low energy, we are able to positively address the points raised above.

2 The model

2.1 Effective description of thermalized pure SU(N) gauge theory

We consider FRW geometry. The matter sector is assumed to be effectively described, minimally coupled, and gauged pure SU(N) thermodynamics. We appeal to the concept of dynamical abelian projection and centerization of the fundamental gauge symmetry. In this approach, $N-1$ species of condensed magnetic monopoles give mass to $N-1$ species of abelian gauge bosons at temperatures $T$ larger than a dynamically generated mass scale $M$. For $T \sim M$ the maximally abelian gauge symmetry $U(1)^N$ reduces to the discrete center symmetry $Z_N$ so that gauge bosons do not participate in the dynamics anymore. Since it seems to be impossible to apply the framework of ref. [7] to the case of arbitrarily large $N$ we simplify the picture by assuming a single complex $U(1)$ gauge symmetry, but we keep the feature of reduction to the center symmetry...
for $T \sim M$. The action then assumes the following form

$$S = \int d^4x \sqrt{-g} \left[ -\frac{1}{16\pi G} R - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{D}_\mu \phi \mathcal{D}^\mu \phi - V(\bar{\phi} \phi) \right], \quad (1)$$

where $\mathcal{D}_\mu \equiv \partial_\mu + ieA_\mu$ denotes the gauge covariant derivative, and $V$ is an effective potential to be constructed below. A thermal treatment needs a continuation of (1) to Euclidean signature with a compact time coordinate $0 \leq \tau \leq \beta \equiv 1/T$ along which the inflaton field $\phi$ solely varies. Speaking of analytical continuation, let me stress at this point the fact that observables such as energy-density or the Hubble parameter being time-independent in the Euclidean have a trivial continuation to Minkowskian signature.

We now construct the potential $V$. Disregarding gravity and the gauge sector for the moment, we want $\phi, \bar{\phi}$ to be stable solutions to the scalar field equations. This criterion is automatically satisfied if we assume $\phi, \bar{\phi}$ to be BPS saturated \[1\] since it is this class of field configurations possessing lowest Euclidean action in a given topological sector. As we will show later, a consequence of BPS saturation is an efficient decoupling of gravity away from the Planck scale. On the other hand, for a global, thermal description of the ground state of the system we want only a single, dynamically generated mass scale $M$ to appear in the potential. According to the above picture of successive symmetry reduction the potential is gauge invariant for $T > M$ and $Z_N$ symmetric for $T \sim M$. It will be shown below that this implies the validity of a Born-Oppenheimer approximation in the regime $T > M$ such that the back-reaction gauge field onto the scalar dynamics can be neglected.

The above constraints determine the potential uniquely \[12\], and we have

$$V(\bar{\phi} \phi) \equiv \bar{V}^{1/2} V^{1/2} = \frac{M^6}{\phi \bar{\phi}} + \lambda^2 M^{-2(N-3)} (\bar{\phi} \phi)^N - 2 \lambda M^{6-N} \frac{1}{\phi \bar{\phi}} \text{Re} \phi^N, \quad \text{or}$$

$$V^{1/2} = \frac{M^3}{\phi} - \lambda \frac{\phi^{N-1}}{M^{N-3}}, \quad \lambda \sim 1. \quad (2)$$

The corresponding BPS equations read

$$\partial_\tau \phi = \bar{V}^{1/2}, \quad \partial_\tau \bar{\phi} = V^{1/2}. \quad (3)$$

Note that adding a constant to the potential would destroy the feature that BPS saturated solutions exist.

### 2.2 Ground state solution: BPS saturated thermodynamics

I now come to the solution of the ground state dynamics for $T > M$, that is, the regime of continuous gauge symmetry where $V = M^6 / \phi \bar{\phi}$. Obviously, $\bar{V}^{1/2}$ and $V^{1/2}$ are only determined up to phases $e^{i\theta}$ and $e^{-i\bar{\theta}}$, respectively, which at first

\[1\] $T > M$ and $T \sim M$ are merely labels and not to be taken literally. For example, letting $N \to \infty$ we have $T > M/2\pi$ in the regime of continuous gauge symmetry.
sight expresses the freedom to choose the gauge in which one would like to solve (3). At finite temperature and for a bosonic field the physical gauge is the one that yields periodic solutions. Relying on the definition (2), it turns out that only for the choice \( \delta = \pm \frac{\pi}{2} \) there are periodic solutions to (3) which read

\[
\phi^{(n)}(\tau) = \sqrt{\frac{M^3 \beta}{2|n| \pi}} e^{2n\pi i \frac{\tau}{\beta}}, \quad (n \in \mathbb{Z}).
\]

Different \( n \) label different topologies. We restrict ourselves to \( n = 1 \) in the following. There are two important observations. First, calculating the ratio of the masses of scalar and vector excitations in the background of (4), we obtain

\[
m_\phi/m_A \geq \sqrt{\frac{6}{e}}.
\]

Hence, the scalar field is slow as compared to the gauge field if \( e \) is not larger than unity, and therefore the Born-Oppenheimer approximation leading to (4) is justified. Second, the ratio of scalar mass to temperature is

\[
m_\phi/T = \sqrt{\frac{6}{e}} \times 2\pi \sim 15.4.
\]

This means that \( V \) indeed is an effective potential since fluctuations in \( \phi \) are irrelevant and therefore are contained in the shape of \( V \) and in the BPS saturated solution \( \phi^{(1)} \).

Let us now look at the gauge field dynamics of the ground state. We have to solve

\[
\partial_\mu \left[ \sqrt{\tilde{g}} F_{\mu \nu} \right] = j_\nu,
\]

where \( j_\mu \equiv ie \delta_{\mu 0} [\tilde{D}_\tau \phi^{(1)} - (\phi^{(1)} \tilde{D}_\tau) \phi^{(1)}] \). If \( A_\mu \) is pure gauge \( \tilde{D}_\tau \phi^{(1)} \) vanishes. Accordingly, the solution to the gauge field dynamics boils down to finding an \( A_\mu = \delta_{\mu 0} A(\tau) \) such that \( \tilde{D}_\tau \phi^{(1)} = 0 \). This is most easily done in the unitary gauge \( \phi^{(1)} = |\phi^{(1)}| \) where due to (4) we have \( \partial_\tau \phi^{(1)} = 0 \). Hence, \( \tilde{D}_\tau \phi^{(1)} = 0 \) for \( A(\tau) \equiv 0 \) in this gauge.

Inserting the solution to the ground state dynamics into the matter sector of the euclidean action, the only nonvanishing term is the potential with

\[
V(|\phi^{(1)}|^2) = 2\pi M^3 T.
\]

This does not depend on euclidean time and therefore is interpreted as a \( T \) dependent cosmological constant \( \Lambda = \Lambda(T) \) in Minkowskian signature.

### 2.3 Excitations

We already realized that there are no scalar excitations for \( T > M \). The excitations of the gauge field are massive \( (m_A = e|\phi^{(1)}|) \) due to the Higgs mechanism, and therefore we have three polarization states. We assume that the gauge coupling \( e \) is small enough to permit a calculation of the energy density and the pressure based on a perturbation of the black-body expression. We obtain

\[
\varepsilon_A = \frac{\pi^2}{10} T^4 - \frac{e^2}{16\pi} M^3 T, \quad p_A = \frac{\pi^2}{30} T^4 - \frac{e^2}{24\pi} M^3 T.
\]

For example, taking \( e \sim 0.1 \), we would trust (7) since \( M \leq 2\pi T \).

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2 By physical gauge we mean the one that maximizes the physics contained in the scalar sector alone. To determine this gauge, the periodicity criterion is applied.

3 Ground state is assumed to be locally Lorentz invariant.
2.4 Energy-momentum conservation in a Friedmann universe

Taking into account the equation of state $p_A = -\varepsilon_A$ for the vacuum, energy and pressure of the universe are given as

$$\varepsilon = \varepsilon_A + A, \quad p = p_A - A.$$  

(8)

In terms of $A$ this reads

$$\varepsilon = \frac{A^4}{160\pi^2 M^{12}} + \kappa_1 A, \quad p = \frac{A^4}{480\pi^2 M^{12}} - \kappa_2 A,$$  

(9)

where

$$\kappa_1 \equiv 1 - \frac{e^2}{32\pi^2}, \quad \kappa_2 \equiv 1 + \frac{e^2}{48\pi^2}.$$  

(10)

Energy-momentum is conserved if the scale factor $a$ satisfies

$$\frac{d}{da} (\varepsilon a^3) = -3a^2 p.$$  

(11)

Upon use of (9) eq. (11) yields

$$\frac{d}{da} A = \frac{A}{a} \frac{A^3}{40\pi^2 M^{12}} + 3(\kappa_1 - \kappa_2) \frac{\kappa_1}{40\pi^2 M^{12}}.$$  

(12)

2.5 Attractor property of inflaton dynamics

The solution to eq. (12) can be given analytically only for the inverse function $a/a_0 = (a/a_0)(A)$. We have

$$\frac{a}{a_0} = \left( \frac{A}{A_0} \right)^{\frac{\kappa_1}{\kappa_2 - \kappa_1}} \left( \frac{A^3 + 120\pi^2(\kappa_1 - \kappa_2) M^{12}}{A^3 + 120\pi^2(\kappa_1 - \kappa_2) M^{12}} \right)^{\frac{2\kappa_1 - 3\kappa_2}{2\kappa_2 - \kappa_1}}.$$  

(13)

Using (8) and (13) it can be shown that for $A \gg M^4$ there is radiation scaling, $\varepsilon \sim a^{-4}$, whereas for $A \sim M^4$ we have $\varepsilon \sim \text{const}$. The points $A/M^4$, where $a/a_0$ has undergone 60 e-foldings, turn out to be between 1.34 and 1.44 for the three very different initial conditions $A_0/M^4 = 10^2, 10^4, 10^7$! So the (inflationary) regime, where the cosmological constant dominates the energy density, does practically not depend on the initial conditions prescribed at the borderline of applicability of our model. Hence, we demand that inflation be terminated at $A/M^4 = 1.4$ by a transition to the center symmetric phase. This phase transition sets in at the first point of inflexion $|\phi_c|$ of $V$. Setting $V(|\phi_c|^2) = A = 1.4 M^4$, yields $N=34$. We will work with this value in the following.
2.6 Decoupling of gravity from inflaton dynamics

Let me now come back to the question how well the effect of gravitational back-reaction onto the inflaton dynamics is already contained in the shape of the potential considering the BPS saturated solution \( \phi^{(1)} \). To decide on this we look at the consequences of this BPS saturation. Taking the covariant divergence of eqs. (3) and appealing again to (3), we obtain the following right-hand sides

\[
\partial_\tau \bar{V}^{1/2} + 3H \bar{V}^{1/2} = \frac{\partial \bar{V}^{1/2}}{\partial \phi} V^{1/2} + 3H \bar{V}^{1/2} = \frac{\partial V^{1/2}}{\partial \phi} \frac{\partial \overline{V}^{1/2}}{\partial \phi} + 3H \bar{V}^{1/2} = \bar{V}^{1/2} \frac{\partial V}{\partial \phi} \frac{\partial \overline{V}^{1/2}}{\partial \phi} + 3H \bar{V}^{1/2}.
\]

The terms \( \frac{\partial V}{\partial \phi} \) and \( \frac{\partial V}{\partial \phi} \) in the respective right-hand sides for \( \bar{\phi} \) and \( \phi \) represent the usual sources in the scalar equations of motion. The terms \( 3H \bar{V}^{1/2} \) and \( 3H \bar{V}^{1/2} \) are in excess. Therefore, we must investigate whether the ratio

\[
R \equiv \frac{3H \bar{V}^{1/2}}{\frac{\partial V}{\partial \phi}}
\]

is smaller than unity. For the radiation dominated epoch and during inflation we respectively obtain

\[
R_r \sim \frac{3 \sqrt{\frac{2}{8}} \pi^3 M^3 T^2 / M_P}{(2\pi M T)^{3/2}} = \sqrt{\frac{9}{5}} \frac{T}{M_P}, \quad R_i \sim 2\sqrt{18} \frac{M}{M_P} < 10^{-3}
\]

for \( M \) being smaller than the GUT scale \( \sim 10^{16} \text{ GeV} \). If \( T \) is of order \( M_P \) \( R_r \) is of order unity. So decoupling of gravity becomes effective if the initial temperature is smaller than the Planck mass. Since there is an attractor property of the dynamics in the sub-Planckian regime we have to assume that Planckian physics drives the universe towards temperatures lower than \( M_P \) to be able to apply our model. Summarizing, we have shown that back-reacting gravity is contained in the BPS saturated scalar dynamics if initial conditions are set below the Planckian regime.

3 Numerical solutions

3.1 Flat universe

Let us for definiteness assume \( M = 10^{13} \text{ GeV} \) and Planckian energy density of the initially dominating radiation and solve the Friedmann equation

\[
H^2(t) = \left( \frac{\dot{a}(t)}{a(t)} \right)^2 = \frac{8\pi G}{3} \left( \frac{A^4}{160\pi^2 M^{12}} + \kappa_1 A - \frac{3k}{8\pi G a^2(t)} \right)
\]
numerically for the cases of closed, flat, and open universes corresponding to \( k = +1, 0, -1 \), respectively. The results are shown in Fig. 1.

![Graph showing the evolution of open, flat, and closed universes.](image)

**Fig. 1.** Evolution of open, flat and closed universes. We have set \( a_0, r = 1/8.65 \times 10^6 M_P^{-1} \) which is very near to the critical radius for a closed universe.

### 3.2 Closed universes need not collapse.

Let me now raise the question under which conditions a closed universe of Planckian initial condition can escape gravitational collapse. To answer this we need to investigate whether the Hubble parameter

\[
H = \sqrt{\frac{8\pi G}{3} \left( \frac{A^4}{160\pi^2 M_{12}^4} + \kappa_1 A - \frac{3}{8\pi G a^2(t)} \right)}
\]

\( (18) \)

can be prevented from becoming zero for suitably chosen initial radii \( a_0 \). As a consequence of the finite heat capacity of the vacuum and with \( M = 10^{13} \) GeV it turns out that for

\[
a_0 > \frac{1}{1.3} \times 10^6 M_P^{-1} \sim 10^7 M_P^{-1}
\]

\( (19) \)

\( H \) always remains positive, and therefore the corresponding closed universe does not collapse. However, eq. \( (19) \) expresses a hierarchy originated by a hierarchy in the scales of gravity and matter. This stands as a fact as long as we have no unified quantum description of gravity and matter at our disposal to decide whether such a hierarchy can be dynamically generated or not.

### 4 Universe after inflation

#### 4.1 Termination of inflation and reheating

In this last part of my talk I will briefly mention what our model has to say concerning the epochs following quasiexponential expansion. A termination of
inflation is induced by the transition of the SU(N) gauge theory from its phase of maximally abelian dynamics to that governed by the discrete center symmetry $Z_N$. In our model this happens around the point of inflexion $|\phi_c|$ where inflaton fluctuations become massless and therefore large. Even solutions to the euclidean mean field equations yield a time dependent inflaton amplitude indicating the breakdown of thermal equilibrium in this regime. Therefore, we have to solve the dynamics in Minkowskian signature. Fluctuations around the mean field grow like $\delta \phi \sim e^{mt}$ for tachyonic masses. Numerically, $m$ is of order $M$. On the other hand, the Hubble parameter $H$, which can be taken inflation-valued, is $H \sim \frac{M}{H M} M \sim 10^{-6} M$. This implies a duration of the tachyonic regime of about $10^{-6}$ inflationary Hubble times. Moreover, during the subsequent regime of reheating (positive mass squared for the fluctuations) $\phi$ performs damped oscillations about its vacuum value $\phi = M$. If there were only a few oscillations then this regime would last again only about $10^{-6}$ Hubble times since the frequency of oscillation should be comparable to the mass of excitations $= NM = 34 M \text{[17]}$. There seems to be numerical evidence that the assumption of only a few oscillations is justified \text{[14,15,16]}. Therefore, inflation would be terminated very rapidly in our scenario.

### 4.2 Adiabatic density perturbations due to the inflaton field?

Usually, one estimates the magnitude of adiabatic density perturbations stemming from the inflaton field by exploiting $m \ll H$ during inflation which is a consequence of the slow-roll paradigm for the dynamics in Minkowskian signature. In our model this condition is not met for the bulk of inflation. On the contrary, we have $m \sim M_p/H M$. Thanks to the heat capacity of the vacuum thermal equilibrium is intact during the bulk of inflation and only gets destroyed with the onset of the phase transition. Therefore, we may estimate the size of fluctuations by looking at the occupation numbers of scalar excitations being of the order $10^{-7} \text{[18]}$. Hence, there are no fluctuations during the bulk of inflation. A rough estimate for the density contrast arising from the few excited scalar modes that may leave the horizon at a very late stage of inflation and are assumed to enter it as classical perturbations well after reheating yields

$$\frac{\delta T}{T} \sim \frac{\delta \rho}{\rho} = \left. \frac{\partial V}{\partial \phi} \delta \phi \right|_{\phi = \phi_c} < 10^{-9}.$$  \hspace{1cm} (20)

This is much too low to explain the measured anisotropy $\delta T \sim 10^{-5}$ of the CMB. Therefore, the required density perturbations do not originate from the fluctuations of the inflaton field in our model. Recently, it was proposed by Lyth and Wands \text{[19]} that the spatial curvature perturbations required for the formation of the large scale structure can originate from an extra scalar field not driving inflation – the curvaton. Our model has to be supplemented with such a mechanism for the generation of density perturbations.

\footnote{4 The energy density of vector excitations is suppressed by a factor $(2\pi)^{-4}$, and hence we assume their fluctuations too be negligible.}
There are two final points which I would like to stress. First, gauged inflation excludes the post-inflationary occurrence of moduli. Moduli fluctuations are associated with a mass scale much smaller than the one of the field driving and terminating inflation. Therefore, in contradiction to observation, they have the potential to introduce large isothermal density perturbations. Typical candidates are the Goldstone modes of the spontaneous breakdown of a continuous, global symmetry \cite{7}. In our model we have a spontaneous breakdown of a $Z_N$ symmetry during tachyonic preheating. However, (a) this symmetry originated from a gauge symmetry \cite{19}, and (b), it is not continuous. Second, the vector bosons surviving inflation are coupled in a $Z_N$ symmetric way to the matter generated during reheating. Thus, their decay is mediated by high dimensional operators which makes them quasi-stable and therefore candidates for the originators of UHECR’s beyond the GZK bound and cold dark matter \cite{20}.

5 Summary

In this talk I presented a model for cosmic inflation which is based on the effective, thermal description of an SU(N) pure gauge theory at large N and FRW gravity. As a consequence of the BPS saturation of the ground state, which is a dynamical monopole condensate, gravitational and gauge field dynamics do not back-react. Cosmic evolution is driven by a competition between the finite heat capacity of the vacuum and the radiation of massive gauge field quanta. This leads to inflation for temperatures comparable to the mass scale of the gauge theory within large variations of the initial conditions. A closed universe of Planckian initial conditions need not collapse if its initial radius is larger than a critical value determined by the hierarchy between $M_P$ and the scale of the matter sector. In contrast to the usual slow-roll paradigm inflation is realized at $m \gg H$ although the time variation of the inflaton field is nil. Density perturbations stemming from adiabatic inflaton fluctuations during inflation are irrelevant for the formation of large-scale structure and CMB anisotropy. Hence, the model needs to be supplemented with a mechanism to generate them. This mechanism is available \cite{18}.

Inflation is very rapidly terminated in the course of a transition from the phase of maximal abelian gauge symmetry to the phase of discrete center symmetry. Due to the latter there are no moduli excitations which possibly could introduce large isothermal density perturbations after inflation. Inflationary relics are massive vector bosons which are protected by high dimensional operators from decaying into matter generated during reheating. Therefore, these vector particles may originate UHECR’s beyond the GZK bound. On the other hand, they do contribute to the cold dark matter of the universe.

Acknowledgements

I would like to thank the organizers for providing the framework for a very stimulating conference.
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