Distance-regular Cayley graphs with small valency

Edwin R. van Dam
Department of Econometrics and O.R., Tilburg University, The Netherlands

Mojtaba Jazaeri †
Department of Mathematics, Shahid Chamran University of Ahvaz, Ahvaz, Iran
School of Mathematics, Institute for Research in Fundamental Sciences (IPM), P.O. Box: 19395-5746, Tehran, Iran

Received 28 March 2019, accepted 9 May 2019, published online 20 September 2019

Abstract

We consider the problem of which distance-regular graphs with small valency are Cayley graphs. We determine the distance-regular Cayley graphs with valency at most 4, the Cayley graphs among the distance-regular graphs with known putative intersection arrays for valency 5, and the Cayley graphs among all distance-regular graphs with girth 3 and valency 6 or 7. We obtain that the incidence graphs of Desarguesian affine planes minus a parallel class of lines are Cayley graphs. We show that the incidence graphs of the known generalized hexagons are not Cayley graphs, and neither are some other distance-regular graphs that come from small generalized quadrangles or hexagons. Among some “exceptional” distance-regular graphs with small valency, we find that the Armanios-Wells graph and the Klein graph are Cayley graphs.

Keywords: Cayley graph, distance-regular graph.
Math. Subj. Class.: 05E30

*We thank Sasha Gavrilyuk, who, in the final stages of writing this paper, at a conference in Plzeň, pointed us to [27] for what he called Higman’s method applied to generalized polygons.

†The research of Mojtaba Jazaeri was in part supported by a grant from School of Mathematics, Institute for Research in Fundamental Sciences (IPM) (No. 95050039).

E-mail addresses: edwin.vandam@uvt.nl (Edwin R. van Dam), m.jazaeri@scu.ac.ir, m.jazaeri@ipm.ir (Mojtaba Jazaeri)

© This work is licensed under https://creativecommons.org/licenses/by/4.0/
Razdaljno-regularni Cayleyjevi grafi z majhno valenco

Edwin R. van Dam
Department of Econometrics and O.R., Tilburg University, The Netherlands

Mojtaba Jazaeri †
Department of Mathematics, Shahid Chamran University of Ahvaz, Ahvaz, Iran
School of Mathematics, Institute for Research in Fundamental Sciences (IPM),
P.O. Box: 19395-5746, Tehran, Iran

Prejeto 28. marca 2019, sprejeto 9. maja 2019, objavljeno na spletu 20. septembra 2019

Povzetek

Problem, ki ga obravnavamo, sprašuje, kateri razdaljno-regularni grafi z majhno valenco so Cayleyjevi grafi. Poiščemo Cayleyjeve grafe med razdaljno-regularnimi grafi z valenco kvečjemu 4, med razdaljno-regularnimi grafi za znane domnevne presečne tabele z valenco 5 in med vsemi razdaljno-regularnimi grafi z ožino 3 in valenco 6 ali 7. Ugotovimo, da so incidenčni grafi Desarguesovih afinityh ravnin brez enega razreda vzporednih premic Cayleyjevi grafi. Pokažemo, da incidenčni grafi znanih posplošenih šestkotnikov niso Cayleyjevi grafi; prav tako niso Cayleyjevi nekateri drugi razdaljno-regularni grafi, ki izhajajo iz majhnih posplošenih štirkotnikov ali šestkotnikov. Med “izjemnimi” razdaljno-regularnimi grafi z majhno valenco smo našli dva, ki sta Cayleyjeva, in sicer sta to Armanios-Wellsov graf in Kleinov graf.

Ključne besede: Cayleyjev graf, razdaljno-regularen graf.

Math. Subj. Class.: 05E30

* Avtorja se zahvaljujeta Sashi Gavrilyuku, ki ju je v zaključni fazi pisanja tega članka na konferenci v Plznju seznanil s člankom [27], ki govori o – kakor temu on pravi – uporabi Higmanove metode na posplošenih mno- gokotnikih.

† Raziskave Mojtabe Jazaerija so bile delno podprte z nepovratnimi sredstvi s strani School of Mathematics, Institute for Research in Fundamental Sciences (IPM) (št. 95050039).

E-poštna naslova: edwin.vandam@uvt.nl (Edwin R. van Dam), m.jazaeri@scu.ac.ir, m.jazaeri@ipm.ir (Mojtaba Jazaeri)

© (C) To delo je objavljeno pod licenco https://creativecommons.org/licenses/by/4.0/