Modelling Plane Geometry: the connection between Geometrical Visualization and Algebraic Demonstration

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Abstract. The teaching and learning of Mathematics contents have been challenging along the history of the education, both for the teacher, in his dedicated task of teaching, as for the student, in his arduous and constant task of learning. One of the topics that are most discussed in these contents is the difference between the concepts of proof and demonstration. This work presents an interesting discussion about such concepts considering the use of the mathematical modeling approach for teaching, applied to some examples developed in the classroom with a group of students enrolled in the discipline of Geometry of the Mathematics curse of UFVJM.

1. Introduction
Mathematics science presents several factors that make it hardly understandable by the students when is taught traditionally. This traditional teaching focused on the teacher as a transmitter of knowledge leads the students to reach their goals by they own effort. In this way, the mathematical subjects, the teaching methodologies and the interpersonal relationship between teacher and students do not have any relation with the student’s academic life, without any contextualization.

Is very known in the literature that the use of mathematical modeling approach to teach Plane Geometry helps the students in their geometrical visualization process. However, we must emphasize the contributions of this visualization process to the students without distancing them to the algebraic thinking. Although some dynamic geometry software can not perform demonstrations, the didactical technique of modeling the behavior of the geometrical construction lead the students to search for the proof of a given theorem.

In this context, based on the concepts of Didactic Engineering (DE) as a research methodology, this work presents the teaching of area concept in Plane Geometry using mathematical modeling approach and GeoGebra, seeking to emphasize the importance of understanding the distinction between geometric visualization and algebraic demonstration. We divided the activities of this research into three stages. A diagnostic questionnaire was initially applied in the first stage, followed by the intervention and the student submission to some dynamical activities with the software in the second stage, and finally, was applied a diagnostic questionnaire for a better analysis of the data in the third and last stage.
2. Modelling Approach and Research Methodology

2.1. Investigating the Problem

According to [1], the definition of area is a mathematical perception that makes possible to verify and measure the occupation of a particular surface. The teaching of this concept, in plane surfaces, has shown to be of a great social relevance and thus, contributed to the formation of the individual. However, [2] and [3] affirm that this topic is worked on schools demanding from the student’s some competencies that involve calculations.

The teaching of mathematics related to the memorization of mathematical concepts does not value the construction process of knowledge, such as observation, investigation, experimentation, and formalization of ideas. According to [4], the conceptual difficulties expressed by the students present several factors related to their learning process. Among them, the errors made in the geometric and numerical conceptions are evidenced, as was also observed in this research, through diagnostic questionnaires applied in the classroom. Further details about these questionnaires can be found in [8].

This research, which was part of the first stage of DE, showed that the main mistakes made by the students participating in this research were related to the association of graphic representation and their properties. The Plane Geometry class, offered in the first year of the Mathematics course in UFVJM, is one of the first disciplines that introduces the abstract thinking, focused on theorems demonstrations, definitions and propositions. In this way, students were expected to have difficulties in writing a text using appropriate formal language.

Due to these difficulties, it is necessary to discuss new pedagogical practices aimed to improving students’ learning, allowing them to establish cognitive correspondences between the formulas and the graphical and geometrical representations of plane figures. In this sense, the computational resource through modeling approach can represent the bridge between the formal treatment of the concepts associated with the figures and their graphic representations.

Researchers in Mathematics Education, such as Bassanezi and D’Ambrsio, discuss the improvement and renewal of teaching mathematics in the schools as well as the way in which it can be developed, seeking justifications for the insertion of modeling approach in the students’ curriculum [5]. Among the mathematical subjects that can be inserted in this proposal, Geometry stands out as an exponent, since it presents several options of contextualization. In this way, the use of technological resources for teaching Mathematics stands out as an excellent contribution to the teaching-learning process, particularly in the topic area of plane figures, which is the subject discussed below.

2.2. Didactical Activities with Modeling Approach

The didactical activities were elaborated and planned with detail, based on the principles of modeling approach, with the help of GeoGebra software. In these models, the visual resources obtained from the constructions with GeoGebra are used with profusion since there are several ways to calculate the area of a plane figure. In this case, the geometric verification is necessary so the teacher can reach the main objective, improving the learning of concepts and the formalization of students’ ideas.

The proposed model is composed of theoretical and practical contents discussed by the students. The theoretical contents portray the definitions and propositions related to each chosen figure, besides proposing ideas for the demonstration of propositions. Only one practical exercise was included since most of the time was devoted to the demonstrations of the polygons areas. Also, the models were designed and elaborated to lead the student from the experimentation stage to the concepts formalization.

The visual resources from the GeoGebra constructions have been extensively explored, since there are several ways of calculating the area of a plane figure, that is, the geometric proof is necessary to achieve the primary objective of improving the learning of concepts and
formalization of the students’ ideas. The following two activities present the use of modeling approach in problems that deals with areas of plane figures subject.

**Activity 1**

We made a distinction between congruent surfaces and equivalent surfaces, concepts that the students had difficulties to visualize geometrically. Two congruent triangles were constructed, and the students verified, through the command "Area [< Polygon >]”, that the triangles had the same area. As an example of equivalent surfaces, we constructed two quadrilaterals composed of two triangles $T_1$ and $T_2$, interconnected by distinct segments, as can be observed in Figure 1.

![Figure 1. Models of two equivalent surfaces](image)

During this activity, it was suggested that the students change the arrangement of the points indicated in the Figure. With this suggestion, they found that, although the quadrilaterals are distinct, they are also equivalent surfaces. In addition, they reported that with the aid of the modeling approach there was a better understanding when the main differences between the concepts of congruence and equivalence were approached practically and dynamically.

**Activity 2**

In this activity was presented the formula to calculate the rectangle area and an idea of how to obtain the respective equation. Students were asked if they would be able to obtain the formula for the area using a formal demonstration method, initially without the aid of the software. Later, the modeling, shown in Figure 3, was used to integrate the use of the software and its tools with the demonstration of geometric propositions.

On the proposed questioning, one of the students considered that it was enough to subdivide the rectangle into unit squares and soon after conjecture the formula using the postulates. The demonstration, presented in Figure 2, was done on the board by the student himself so that the other students could share the idea, exchange information and to interact. After completing the explanation, the students were asked if the demonstration was correct and some realized that there was an error.

The students pointed out that the mistake was to consider that $m$ and $n$ are both natural and for that they used as an example a rectangle whose sides measure was 3.65cm. Therefore, in the form that was performed, where only the case where the segments are commensurable...
Figure 2. Demonstration produced by a student

was considered, the demonstration would be incomplete. To finish the demonstration suggested by the student, we must consider the case where the segments are incommensurate.

Another way of proving the formula of the rectangle area was presented, using the previous postulates and with the aid of mathematical modeling approach through some auxiliary constructions, as shown in Figure 3. Using the GeoGebra tool, which gives dynamism to the model. In this case, the students considered that the demonstration was conducted more simply.

The students’ observation was expected since the activity was based on a research that reinforces the need to correlate the manipulation of dynamic geometry software with the demonstration of geometric propositions in the teaching and learning process [6].

Figure 3. Support Model - Rectangle Area

Activity 3

In this activity was presented an alternative to the demonstration of the circle area, through modeling approach and the concept of infinity. In the model elaborated for this didactic activity, the student should first click on the “Animate” button and verify that as the number of sides
of the polygon inscribed in the circle increases, the polygon area tends to circle area, as can be seen in Figure 4.

![Figure 4. Support Model - Rectangle Area](image)

It should be emphasized that while demonstrating the formulas, students were instructed to manipulate the constructions carried out in the software to verify the veracity of the statements and to propose new ideas for future demonstrations.

Subsequently, were taken some construction steps with the students:

- Create a slider. In this case: name “n”, initial value 3, final value 30, and increment 1.
- In the input field, enter the command: sequence 
  
  \[(\cos(360^{\circ}/n \cdot i), \sin(360^{\circ}/n \cdot i)), i, 1, n)\]

- Animate the slider “n”.

We emphasized that in demonstrating the formulas of the remaining figures, students were instructed to manipulate the constructions carried out in the software to verify the veracity of the affirmations and to propose new ideas for the future demonstrations.

For example, during the demonstration of the area formula of a circular segment, a student questioned whether the fact that the central angle was acute, obtuse or straight could interfere in the final equation. These questions arise through the manipulation of the constructed figures, which would hardly be done in the classroom without the use of computational resources.

3. Results Discussion

Along Activity 1, by changing the arrangement of the points indicated in the models built in GeoGebra, the students were enthusiastic to verify that, although the quadrilaterals are different, they are also similar surfaces. In addition, they reported that with the aid of mathematical modeling approach there was a better understanding when the main differences between the concepts of congruence and equivalence were approached in a practical and dynamic way.

In general, it was found that the objective of the proposed activities was successfully achieved because, when requested, the students responded promptly and safely. The improvement of the formal mathematical writing is a factor of extreme importance since in the classroom the students presented great difficulty in developing a demonstration.

One of the main benefits gained by the use of mathematical modeling approach and the application of didactic sequences in the teaching of Geometry are precision and visualization, exploration and discovery and proof of theorems. Although dynamic geometry software can not
perform demonstrations, the technique of experimenting with assumptions leads the student to search for the proof of a theorem.

It was verified that the dynamics that GeoGebra brought to the classes of Plane Geometry, promoted the communication of the students among themselves, and between the teacher and the students. Collective work proved to be beneficial, allowing students to share information about construction processes and demonstrations.

4. Final Considerations

From the results generated in observations in the classroom, it was found that the main obstacles faced by students regarding the teaching of areas are due to the precipitous introduction of formulas lead to apparent reasoning.

In addition, it was verified that the use of mathematical modeling approach allied to computational resources in the teaching-learning process of Plane Geometry, in the investigated group, contributed significantly in several aspects, such as:

- Greater interaction among students;
- The development of skills and competencies related to Geometric Design;
- The development of demonstration techniques;
- Significant improvement as to the essay questions, appropriate to the formal mathematical writing.
- The benefit of the process of graphical and geometric visualization, without there being a distancing of the algebraic thought;

In addition to what has been presented, it is worth mentioning that the experiences acquired in this research, confers the training of researchers and readers albeit in a restricted way, contributing to future discussions about the use of Mathematical Modeling in the teaching of Plane Geometry.

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