Research of the Magnetic Field in Optical Pump Magnetic Resonance Experiment

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Abstract. In the experiment of optical pumping magnetic resonance, the calculation of horizontal and vertical magnetic fields is related to the accuracy of the final results, therefore, the construction of correct calculation method of horizontal and vertical magnetic field is very important for the experiment. First, the article analyses the relation between magnetic field of the Helmholtz coil and current. Then, based on the widely used optical pumping magnetic resonance experiment instrument, the article analyzes the differences of the winding method of coil for horizontal and vertical magnetic field (parallel and series), and gets correct calculation methods and the corresponding formulas of the horizontal and vertical magnetic field for different cases. Finally, the results are proved by the experiment of optical pumping magnetic resonance. The research can not only improve the accuracy of the optical pumping magnetic resonance experiment, but also enhance the scientific research and innovation ability of the students.

Introduction

Optical pump magnetic resonance experiment is a very important experiment of modern physics experiment. The relation formula between the magnetic field and the current of the Helmholtz coil used in the experiments which is given in the existing modern physics experimental textbook is not consistent, and for how to use the formula did not make a detailed description, which will easily lead to erroneous calculations. For example, the calculation of the horizontal magnetic field and the vertical magnetic field in the "modern physics experiment" gives the same formula:\[B = \frac{16\pi N}{5^2} \cdot I \cdot 10^{-7}(T)\] (1)

While "modern physics experiment" gives the formula of the horizontal magnetic field is\[B = \frac{32\pi N}{5^2} \cdot I \cdot 10^{-7}(T)\] (2)

Now, optical pump magnetic resonance instruments used in the domestic colleges and universities are almost produced by Beijing Dahua Radio Instrument Factory, according to the instruction provided by the factory we can know: two horizontal magnetic field coils of the experimental instrument are connected in parallel, the digital ammeter displays the sum of the currents flowing through the two coils; and the two vertical magnetic field coils are connected in series, the digital ammeter displays the current flowing through a single coil current[3]. Therefore, when calculating the magnitude of the vertical magnetic field we can directly use Helmholtz coil magnetic field calculation formula. And to calculate the magnitude of the horizontal magnetic field, we firstly should divide the magnitude of the current meter by two before substituting it into the Helmholtz coil magnetic field calculation formula. In the actual optical pump magnetic resonance experiment, it is necessary to explain the calculation formula of horizontal magnetic field and
vertical magnetic field in detail, so that it will not lead to wrong calculation results.

**Theoretical Analysis**

**Principle Analysis of Optical Pump Magnetic Resonance**

In the optical pump magnetic resonance experiment, we should use circularly polarized light to excite gaseous rubidium atoms, so that the atomic energy level deviates from the Boltzmann distribution under heat balance. When the total magnetic field is zero and reverse, the degeneracy and re-splitting of the Zeeman sub-level occurs, with the optical pumping signal appearing[4]. After that, the superfine structural energy levels’ Zeeman split of the atoms which the nuclear magnetic moment is not zero occurs[5]. If the energy provided by the external environment (which is usually emitted by electromagnetic waves) is exactly the same as the energy level difference of Zeeman split aroused by the magnetic field, then the resonance transition between the atoms in the Zeeman level occurs, the external energy is absorbed to the maximum extent, then the magnetic resonance phenomenon occurs[6-7]. At this time there is:

\[ h \cdot \nu = g_F \cdot \mu_B \cdot B . \]  

Where \( h \) is the Planck constant; \( \nu \) is the frequency of the incident electromagnetic wave, the unit is Hz; \( g_F \) is the Lande factor; \( \mu_B \) is the wave magnet; \( B \) is the total magnetic induction intensity of the applied magnetic field, and the unit is T.

**Helmholtz Coil Magnetic Field Calculation Formula Derivation**

The Helmholtz coil consists of a pair of identical current-carrying coils which parallel to each other and are coaxial, and the distance between the coils is equal to the effective radius of the coil. It is widely used, because it can produce a small range of weak and uniform magnetic field.

According to Biot-Savart's law, we can know the relation between the magnetic field and the current along the central axis of the toroidal coil is:

\[ dB = \frac{\mu_0 \cdot I_0 \cdot dl}{2 \pi \cdot (R^2 + x^2)^{3/2}}. \]  

We can get the magnetic field on the central axis of a single circular coil by integrating formula (4) along the loop:

\[ B = \frac{\mu_0 \cdot I_0 \cdot R^2}{2 \cdot (R^2 + x^2)^{3/2}}. \]  

Where \( \mu_0 \) is the vacuum permeability; \( I_0 \) is the effective current flowing through a single coil, the unit is A; \( R \) is the effective radius of the coil, the unit is m; \( x \) is the coordinate of the distance from the center of the coil on the central axis, the unit is m.

Therefore, the magnetic field at the Helmholtz coil midpoint is[8]:

\[ B = \frac{\mu_0 \cdot I_0 \cdot R^2 \cdot N}{2 \cdot \left( R^2 + \left( \frac{R}{2} \right)^2 \right)^{3/2}} + \frac{\mu_0 \cdot I_0 \cdot R^2 \cdot N}{2 \cdot \left( R^2 + \left( -\frac{R}{2} \right)^2 \right)^{3/2}} \]

\[ = \frac{32 \pi}{5^3} \cdot \frac{N}{R} \cdot I_0 \cdot 10^{-7}. \]

Where \( N \) is the number of turns of the coil.
Calculation and Analysis of Horizontal Magnetic Field

Using the Helmholtz Coil Magnetic Field Calculation Formula to Analysis. Since the two horizontal magnetic field coils of the optical pump magnetic resonance tester are connected in parallel, the horizontal current $I_{\parallel 0}$ displayed on the digital ammeter is regarded as the sum of the currents flowing through the two coils:

$$I_{\parallel 0} = 2I_{\parallel 1}$$  \hspace{1cm} (7)

Substitute formula (7) into formula (6), and we can know the relation between the magnetic induction intensity of the horizontal magnetic field and the current displayed on the digital ammeter is:

$$B_{\parallel} = \frac{16\pi}{5^2} \cdot \frac{N}{R} \cdot I_{\parallel 0} \cdot 10^{-7}$$  \hspace{1cm} (8)

Where $B_{\parallel}$ is the magnetic induction intensity of the horizontal magnetic field, the unit is T; $N$ is the number of turns of the horizontal field coil; $R$ is the radius of the horizontal field coil, the unit is m.

Theoretical Analysis of Using the Resonance Cancellation Method to Calculate the Magnitude of the Horizontal Magnetic Field [9]. Adjust the vertical field of the current magnitude to offset the vertical component of the magnetic field; then adjust the horizontal and triangular wave sweeps (DC component $B_{\text{DC}}$, AC component $B_{\text{AC}}$). The magnitude should satisfy [10]:

$$|B_{\parallel}| \geq |B_{\text{AC}}| + |B_{\text{DC}}| + |B_{\parallel 0}|$$  \hspace{1cm} (9)

And let the horizontal magnetic field, the scanning field and the horizontal component $B_{\text{parallel}}$ of the geomagnetic field be in the same direction [11]. Finally, adjust the RF field frequency so that the resonant position occurs at the peak of the corresponding triangular wave, record the frequency as $\nu_1$, as shown in Figure 1 [9][12], where $B_{\parallel 0}$ is always the total magnetic field in the horizontal direction [13-14].

Then we can know from formula (3):

$$h\nu_1 = g_f \mu_0 (B_{\parallel 0} + B_{\parallel} + B_{\text{AC}} + B_{\text{DC}})$$  \hspace{1cm} (10)

Figure 1. The Diagram of $B_{\parallel 0}, B_{\parallel}, B_{\text{DC}}$ Resonating with the Same Direction.

Change the direction of the horizontal magnetic field individually, keep the direction of the sweep unchanged, adjust the frequency of the RF field again that let the resonance occurs at the peak, record the frequency as $\nu_2$ [14].
Then we can know from formula (3):
\[ -\hbar \nu_\perp = g_F \mu_B \cdot (B_{\parallel \|} - B_\perp + B_{AC} + B_{DC}). \] (11)

Subtract the formula (10) from the formula (11), so there is:
\[ B_\parallel = \frac{h \cdot (\nu_\perp + \nu_\parallel)}{2 \cdot g_F \cdot \mu_B}. \] (12)

**Calculation of Vertical Magnetic Field**

**Use the Helmholtz Coil Magnetic Field Calculation Formula to Analysis.** The two vertical magnetic field coils of the optical pump magnetic resonance tester are connected in series, and the vertical current \( I_{\perp \|} \) displayed on the digital ammeter is regard as the current flowing through a single coil:
\[ I_{\perp \|} = I_{\perp 0}. \] (13)

Substitute the equation (13) into the equation (6), and we can know the relation between the magnetic induction intensity of the vertical magnetic field and the magnitude of the current displayed on the digital ammeter is:
\[ B_\perp = \frac{32\pi \cdot N_\perp \cdot I_{\perp \|}}{5^3 \cdot R_\perp} \cdot 10^{-7}. \] (14)

Where \( B_\perp \) is the magnetic induction intensity of the vertical magnetic field, the unit is T; \( R_\perp \) is the radius of the coil of the vertical magnetic field, the unit is m; \( N_\perp \) is the number of turns of the vertical magnetic field.

**Theoretical Analysis of Using the Resonance Cancellation Method to Calculate the Magnitude of the Vertical Magnetic Field [15].** Firstly set the direction of the horizontal magnetic field and the scanning field to make it be in the same direction with the geomagnetic field horizontal component[11]. Then adjust the magnitude of the vertical magnetic field coil current to equate to the number which is \( n \) times the current corresponding to the vertical component of the geomagnetic field:
\[ B_\perp = n B_{\perp 0}. \] (15)

Where \( B_{\perp 0} \) is the vertical component of the geomagnetic field.

And set the direction of the vertical field to be the same as the vertical component of the geomagnetic field. Finally, adjust the frequency of the RF field to make the resonance occur at the peak, record the frequency as \( \nu_3 \).

We can know from formula (3) that:
\[ \sqrt{B_{\parallel \|}^2 + (B_\perp + B_{\perp \perp})^2} = \frac{\hbar \nu_3}{g_F \mu_B}. \] (16)

Change the direction of the vertical field to make it be opposite to the vertical component of the geomagnetic field, adjust the RF field frequency again to make the resonance occur at the crest, record the frequency as \( \nu_4 \).

We can know from formula (3) that:
\[ \sqrt{B_{\parallel \|}^2 + (B_\perp - B_{\perp \perp})^2} = \frac{\hbar \nu_4}{g_F \mu_B}. \] (17)

Subtract formula (16) from formula (17), and according to formula (15) we can know:
Experimental Verification

The instrument used in the experiment is a DH807 optical pump magnetic resonance tester produced by Beijing Dahua Radio Instrument Factory, the instrument can produce the horizontal magnetic field, the scanning field and the vertical magnetic field, and it can convert the optical signal at the time of resonance into an electrical signal then put into oscilloscope. The experiment also used Tektronix TDS1002B digital oscilloscope and Shengpu F06A digital synthesis function signal generator (frequency in the range of 0 ~ 2MHz continuously adjustable)[12].

Horizontal Magnetic Field

In the process of optical pump magnetic resonance experiment, we obtain the resonance frequency data \( \nu_1 \) and \( \nu_2 \) at different peak field current conditions by measuring the rubidium \(^{87}\)Rb isotope magnetic resonance signal, and then according to the theoretical value of \( g_F \) (the theoretical value of \(^{87}\)Rb is 0.5)[16], substitute it into formula (12) then we can obtain the magnetic induction intensity of the corresponding horizontal magnetic field, and finally compare with the result of the formula (8) to verify accuracy of the theoretical formula.

In the experiment, the peak value of the sweep field is 980mV, and the maximum of the current which offsets the vertical component of the geomagnetic field by adjusting the maximum amplitude of the pumping signal is 0.044A[9][17]. The measured data and the calculated results of the horizontal magnetic field are shown in Table 1, where the number of turns of the horizontal field coil is 250 and the effective radius is 0.2478 meters.

Table 1. Resonant Frequency and Magnetic Induction Intensity at Different Horizontal Field Current.

| Horizontal current / A | 0.3  | 0.4  | 0.5  | 0.6  | 0.7  | 0.8  |
|------------------------|------|------|------|------|------|------|
| Resonance frequency /kHz | \( \nu_1 \) | 1587 | 1915 | 2240 | 2565 | 2889 | 3217 |
|                         | \( \nu_2 \) | 384  | 699  | 1023 | 1351 | 1678 | 2002 |
| Horizontal magnetic field /\( x10^{-4} \) (T) | Experimental value [formula(12)] | 1.361 | 1.814 | 2.268 | 2.721 | 3.175 | 3.629 |
|                         | Theoretical value [formula(8)]   | 1.408 | 1.868 | 2.331 | 2.798 | 3.263 | 3.729 |
| Relative error          | 3.491% | 2.940% | 2.798% | 2.808% | 2.771% | 2.762% |

From above we can know: the formula of using Helmholtz coil magnetic field calculation formula and the magnitude of the horizontal current displayed on the instrument to calculate the horizontal magnetic field is formula (8).

Measurement of the Vertical Magnetic Field

In the experiment, the peak value of the sweep peak is 504mV, the peak frequency of RF peak is 2V, the horizontal field current is 0.1A, the vertical field current \( I_{\perp_{\text{地}}} \) which offsets the vertical component of the geomagnetic field is 0.044A, and adjust the vertical field current \( I_{\perp_{\text{地}}} \) to n times (n = 1, 2, 3, 4, 5, 6)(N = 1, 2, 3, 4, 5, 6); adjust the RF frequency to obtain the peak resonance signal of \(^{87}\)Rb isotope[17]. The experimental data measured and the calculated values of the vertical magnetic field are shown in Table 2. Wherein the number of turns of the vertical field coil is 100 and the effective radius is 0.153 m.
Table 2. Resonant Frequency and Magnetic Induction Intensity at Different Vertical Field Current.

| n   | Vertical current / A | Resonance frequency /kHz | Vertical magnetic field /×10⁻⁴ (T) | Experimental value [formula(12)] | Theoretical value [formula(8)] | Relative error |
|-----|----------------------|---------------------------|-------------------------------------|----------------------------------|--------------------------------|----------------|
|     | 0.044                | v₃                        | 842                                 | 0.247                            | 0.259                          | 4.630%         |
|     | 0.088                | v₄                        | 768                                 | 0.505                            | 0.517                          | 2.436%         |
|     | 0.132                |                           | 927                                 | 0.737                            | 0.776                          | 4.938%         |
|     | 0.176                |                           | 1043                                | 0.998                            | 0.103                          | 3.492%         |
|     | 0.22                 |                           | 1180                                | 1.25                             | 1.29                           | 3.016%         |
|     | 0.264                |                           | 1327                                | 1.504                            | 1.551                          | 3.051%         |

From above we can know: the formula of using Helmholtz coil magnetic field calculation formula and the magnitude of the vertical current displayed on the instrument to calculate the vertical magnetic field is formula (14).

Conclusion

From the experimental measurement and theoretical calculation we can know: Helmholtz coil magnetic field calculation formulas of horizontal magnetic field and vertical magnetic field Helmholtz coil magnetic field were (8) and (14). Whether the horizontal magnetic field or the vertical magnetic field, the relative errors of the theoretical calculation and experimental measurement are within the allowable error range.

In this article, the calculation formulas of horizontal and vertical magnetic field in optical pump magnetic resonance experiment are theoretically deduced and experimentally verified, and we explain the two cases in detail. It is helpful for us to use the correct formula to calculate the magnitude of the magnetic induction intensity, and improve the accuracy of the optical pump magnetic resonance experiment. At the same time, the experimental verification method in the article enriches the research of optical pump magnetic resonance experiment and enhance the scientific research and innovation ability of the students.

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