Noise Estimates for Measurements of Weak Lensing from the Lyman-alpha Forest

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ABSTRACT
We have proposed a method for measuring weak lensing using the Lyman-α forest. Here we estimate the noise expected in weak lensing maps and power spectra for different sets of observational parameters. We find that surveys of the size and quality of the ones being done today and ones planned for the future will be able to measure the lensing power spectrum at a source redshift of \( z \approx 2.5 \) with high precision and even be able to image the distribution of foreground matter with high fidelity on degree scales. For example, we predict that Lyman-α forest lensing measurement from the Dark Energy Spectroscopic Instrument survey should yield the mass fluctuation amplitude with statistical errors of 1.5%. By dividing the redshift range into multiple bins some tomographic lensing information should be accessible as well. This would allow for cosmological lensing measurements at higher redshift than are accessible with galaxy shear surveys and correspondingly better constraints on the evolution of dark energy at relatively early times.

Key words: cosmology, gravitational lensing, Lyman-α forest

1 INTRODUCTION
The study of the Ly\( \alpha \) forest in the spectra of high redshift quasars and galaxies has been a great success for cosmology and galaxy formation. It has been used to constrain the mass of neutrinos (Palanque-Delabrouille et al. 2015; Croft et al. 1999) and study the ionization of the intergalactic medium over cosmic history (see McQuinn (2016) for a review). It has recently become possible to map the three dimensional distribution of HI at redshifts of \( z \approx 2.4 \) using the Ly\( \alpha \) forest (Cisewski et al. 2014; Pichon et al. 2001) and measure Baryon Acoustic Oscillations (BAO) (Busca et al. 2013; Slosar et al. 2013).

Weak Gravitational lensing has also provided important cosmological and astrophysical information in recent years and is expected to be an essential probe of dark energy and dark matter in the future (for a review Hoekstra & Jain 2008). The power of weak lensing for measuring dark energy derives primarily from being able to measure the evolution of structure formation which is directly related to the evolution of the energy density in the Universe. This is done by measuring the lensing power spectrum as a function of source redshift. Secondarily, dark energy affects the angular size distance - redshift relation. Extending the source redshift range beyond those of galaxies (\( z \approx 0.4 - 1.4 \)) would allow for further exploration of dark energy and alternative gravity model space. Weak lensing of the CMB is one way of doing this for a source redshift of \( z \approx 1,000 \) (Planck Collaboration et al. 2016) although it is relatively low signal-to-noise and has no range in source redshift. It would be helpful to fill in this gap in source redshift with high signal-to-noise measurements.

The authors recently proposed that weak gravitational lensing could be measured with the Ly\( \alpha \) forest (Croft et al. 2017). A noisy mapping of the HI distribution at high redshift makes it possible to measure lensing by detecting the distortion of the angular correlation function, or power spectrum, from its expected isotropic form. This is very similar to the methods used to measure lensing with the Cosmic Microwave Background (CMB) (Lewis & Challinor 2006) and to proposed methods for measuring weak lensing with 21 cm radiation from high redshift (Metcalf & White 2007; Zahn & Zaldarriaga 2006; Poursioud & Metcalf 2015). In this paper, we will make this proposal more concrete by estimating the noise levels with which we can expect to measure lensing in realistic Ly\( \alpha \) surveys. In a more technical paper to
follow we will show explicitly how a lensing estimator can be constructed in this case.

Future Lyα lensing maps could be cross-correlated with galaxy shear and CMB lensing maps to get as large a range in source redshift as possible for probing the evolution of dark energy and modifications to General Relativity. They could also be cross-correlated with foreground galaxy surveys to measure bias and directly measure the angular size distance redshift relation.

In the next section we review some of the proposed and ongoing Lyα surveys and approximate some of the parameters of these surveys that will be relevant for lensing. In Section 3 we derive the noise estimation in a lensing measurement from Lyα data. We show how to calculate the expected signal in Section 4 and then present estimates for particular sets of survey parameters in Section 5. Section 6 contains a conclusion and discussion of future prospects.

2 OBSERVATIONAL DATA SAMPLES

Lensing of the Lyα forest can be carried out with samples of backlights which are quasars, galaxies, or a combination of the two. In Table 1 we have listed some currently available Lyα forest datasets, as well as some for which data collection is ongoing. We also list some planned surveys and a possible survey with a proposed instrument (Mauna Kea Spectroscopic Explorer, MSE). Surveys targeting the Lyα forest from the ground are limited by atmospheric transmission to redshifts \( z > 1.9 \) and by the steep decline in the number of backlights to redshifts below \( z = 3 - 4 \). In this paper we will assume that the mean redshift of Lyα forest pixels is \( z = 2.5 \). We will also make the assumption that all sightlines in our sample fully cover the redshift interval \( \Delta z \) we are considering. Although the redshift range from the Lyα line to Lyβ line is \( \Delta z = 0.55 \) at \( z = 2.5 \), the backlights will not all be at the same redshift, meaning that some spectra will only offer partial coverage. These additional complications will be addressed in future work.

We estimate the mean signal-to-noise ratio for the various observational samples (current and future) listed in Table 1. In order to standardize our results, we assume that all observations are rebinned into pixels with redshift extent \( \Delta z = 0.001 \) (we ignore correlations between pixels, so that the S/N = \( \sqrt{0.001/\langle \Delta z \rangle_1} \), where \( \langle \Delta z \rangle_1 \) is the original pixel size. We also assume a mean redshift of Lyα forest observations of \( z = 2.5 \).

In the table we give the S/N for the observed Lyα forest transmitted flux in the spectra. If \( F \) is this transmitted flux, the quantity \( \delta F = (F/F) - 1 \), can be related to the underlying density field using a linear theory biasing relation. The 3D power spectrum of Lyα forest \( \delta F \) fluctuations is given by (McDonald 2003):

\[
P_3(k) = b^2 (1 + \beta b^2) P_\delta(k),
\]

where \( P_\delta(k) \) is the underlying dark matter power spectrum and \( \mu_{\parallel} \) is the cosine of the angle between the wave vector \( \mathbf{k} \) and the line of sight \( \mathbf{z} \). \( b \) and \( \beta \) are parameters that describe the relative bias between flux and matter fluctuations and the strength of redshift distortions respectively. In the case of galaxies, which are conserved under redshift distortions, \( b = \frac{d\ln D}{d\ln z} \approx \Omega_m(z)^{0.55} \) and \( \beta = 1 \) (Kaiser 1987). For the Lyα forest, which undergoes a non-linear transformation \( F = e^{-\tau} \) between flux \( F \) and optical depth \( \tau \) (which is conserved), \( \beta \) is a separate parameter. At the redshifts \( z = 2 - 3 \) \( \beta \) is approximately 0.2 and \( \beta \) unity (Slosar et al. 2011). As a result, the S/N in the density field will be approximately equal to \( b(1 + \beta) \sim 0.3 \) times the S/N in the observed Lyα forest flux (Slosar et al. 2011), although it is the S/N in the observed flux, given in Table 1 that is relevant for our estimates of lensing reconstruction. Most of the flux S/N values in Table 1 are \( \sim 3 \).

Simulations of the intergalactic medium predict that the transverse HI power spectrum will be damped on small scales by pressure (Peeples et al. 2010b,a). At the redshifts of relevance the Jeans scales is \( R_J \simeq 0.5 \) Mpc. In modeling the flux power spectrum in what follows we include a damping factor of \( e^{-\langle k/v_J \rangle^2} \) although this does not have a strong effect on the lensing results.

Lee et al. (2013) give details for the BOSS DR9 Lyα forest sample. We make the assumption that the signal-to-noise ratio of this sample is similar to that in the final BOSS Data Release. The spectra have been rebinned into vacuum wavelength pixels of size \( \Delta \log_{10}(\lambda) = 10^{-4} \), i.e. \( \Delta v = 69.02 \text{ km s}^{-1} \). The median S/N per pixel values for the quasars in the sample are shown in Figure 3 of Lee et al. Integrating we find that mean S/N for the flux averaged over all quasars in \( \Delta z = 0.001 \) pixels is 2.40. In \( \Delta z = 0.001 \) pixels this is equal to S/N=2.72.

The successor to BOSS, eBOSS (Dawson et al. 2016) is ongoing, and the quasar survey will cover a smaller sky area, planned to eventually be three-quarters of the BOSS area. The majority of eBOSS quasars are at lower redshifts than those for which the Lyα forest is accessible, but there is also a large subset of quasars with \( z > 2.1 \). These Lyα forest eBOSS targets are in two categories: (i) quasars selected using an improved algorithm from those in BOSS, which uses more information on quasar variability, and (ii) re-observation of the faintest BOSS quasars to increase their signal-to-noise ratio. The quasars in group (i) have to a good approximation the same S/N as the BOSS quasars. For quasars in group (ii), we assume that the S/N ratio of the lowest 50% of BOSS quasars will be increased by a factor of approximately \( \sqrt{2} \). Coaddition of eBOSS and BOSS spectra is needed to achieve this. Taking both of these factors into account the mean S/N in \( \Delta z = 0.001 \) is 2.99.

Other surveys covering large fractions of the sky and which will be starting soon are DESI (DESI Collaboration et al. 2016) and WEAVE (Dalton et al. 2012). Both of these involve fiber spectrographs which are more highly multiplexed than BOSS, and this will allow the number of Lyα forest quasars observed to reach more than a million. With the current observing parameters, the S/N per unit wavelength in WEAVE and DESI is expected to be 50% higher than BOSS for objects with the same magnitude (M. Pieri, private communication) WEAVE will include a deep QSO survey (to \( g < 23.2 \)) covering 6000 sq. deg. with a minimum S/N per Å of 0.4, but likely covering a limited redshift range \( \Delta z \sim 0.5 - 1.0 \). A bright QSO survey (\( g < 20 \)) covering all \( z > 2.15 \) quasars over 10000 sq. deg. will also be part of

\footnote{http://mse.cfht.hawaii.edu}
WEAVE, and these brightest will have mean S/N per Å of ~7. Other components of WEAVE include deep exposures of QSOs in the HETDEX fields (450 deg. sq., leading to S/N per Å of > 4). As a result with S/N > 5 these brightest will have mean S/N per Å of 2. The surveys mentioned so far are all exclusively using QSOs to backlight the Ly\text{\alpha} forest. Because of their scarcity this limits the sightline density to be relatively low, with mean separation of the order of 10 arcmins. Lee et al. (2014) showed however that it is possible to map out the Ly\text{\alpha} forest well using star forming galaxies as backlights. Assuming that large aperture telescopes are available to observe these fainter objects this greatly increases the potential sightline density. The CLAMATO survey \cite{Lee2014} is currently ongoing, aiming to cover a ~1 deg. sq. portion of the COSMOS \cite{Scoville2007} field, with ~1000 Ly\text{\alpha} forest sightlines, mostly galaxies. From observations completed so far, the S/N level in the data obeys $dn/(S/N)^{-2.9}$ (K.G. Lee, private communication). A minimum S/N cut of S/N > 1.5 per Å is applied, which leads to a mean S/N of 3.49 in $\Delta z = 0.001$ intervals.

The Prime Focus Spectrograph on the Subaru Telescope will be a powerful resource for high redshift spectroscopy \cite{Takada2012}. High redshift galaxies will be suitable targets for a PFS Ly\text{\alpha} forest survey, and in this case the S/N level in the data is likely to obey $dn/(S/N)^{-3.6}$ (K.G. Lee, private communication). As a result with a S/N > 1.5 cut, the mean S/N is $\Delta z = 0.001$ intervals is 2.7. The wide field of view of PFS will enable sky coverage of ~15 sq. deg. for a possible survey.

Surveys such as CLAMATO have shown that, the space density of galaxies at redshifts $z = 2 - 4$ is high enough to map the Ly\text{\alpha} forest with close to arcminute angular resolution. Given enough resources, future surveys could exploit this over large sky areas. Our final entry in Table 1 is a more speculative one along these lines. The Mauna Kea Spectroscopic Explorer (MSE) is a proposed 10m class multiobject spectroscopic facility. With dedicated use, such a telescope could be expected to make a million sightline survey possible at such arcminute resolution. We assume a mean S/N ratio for the Ly\text{\alpha} forest flux of 3 to be comparable to CLAMATO and PFS.

### Table 1. Some relevant parameters for future Ly\text{\alpha} forest observational datasets (see Section 2 for details).

| Dataset       | When          | Area          | $N_{\text{spectra}}$ | mean separation | mean S/N in flux |
|---------------|---------------|---------------|----------------------|-----------------|------------------|
| BOSS DR12    | 2016          | 10,000 sq. deg.| 160,000              | 15 arcmin       | 2.72             |
| eBOSS        | 2014–2018     | 7,500 sq. deg.| 270,000              | 10 arcmin       | 2.99             |
| CLAMATO      | 2014–2018     | 0.8 sq. deg.  | 1,000                | 1.7 arcmin      | 3.49             |
| WEAVE        | 2018–2025     | 6,000 sq. deg.| 400,000              | 7.5 arcmin      | 3.26             |
| DESI         | 2018–2023     | 14,000 sq. deg.| 770,000              | 8.1 arcmin      | 3.26             |
| Subaru PFS   | 2019–2022     | 15 sq. deg.   | 7,400                | 2.7 arcmin      | 2.70             |
| MSE          | 2025–         | 1,000 sq. deg.| 1,000,000            | 1.9 arcmin      | 3                |

3 ESTIMATING THE NOISE IN THE LENSING

Quasars and galaxies will be randomly, not regularly, distributed across the sky and distributed in redshift. We will present a lensing estimator that deals with these complications in a more technical paper \cite{McTaff2017}, but here we will estimate the noise by imagining the sources are distributed in a regular rectangular grid on the sky and all at a redshift beyond the range considered. In this case we can borrow some of the techniques used for lensing of the CMB and 21 cm emission without significant modification (see \cite{Hu2001} Zahn & Zaldarriaga2006).

The noise in the gravitational potential is given by $N_P(L) = \left( \sum_j \int_{\ell_{\min}}^{\ell_{\max}} \frac{d^2 \ell}{(2\pi)^2} C_{\ell,j} \cdot L \cdot C_{\ell,j} \cdot (\ell - L_{\ell,j})^2 \right) \frac{2C_{\ell,j}^2}{C_{\ell,j}^{\text{tot}}}$, \hspace{1cm} (2)

where $C_{\ell,j} = \frac{P_{0F}(\sqrt{\ell/D})^2 + (j2\pi/L)^2}{D^2 L}$ \hspace{1cm} (3)

is an approximation of the the angular/radial power spectrum of the flux. The total power spectrum in the denominator of (2) is given by $C_{\ell,j}^{\text{tot}} = C_{\ell,j} + C_{\ell,j}^{N}$ \hspace{1cm} (4)

where $C_{\ell,j}^{N}$ is the power spectrum of noise from errors in the density estimate in each pixel. The discrete form of this estimator and noise (on a rectangular grid) is also derived in \cite{Pourtsidis2015}.

The smallest angular scale that can be probed is set by the density of back lights on the sky. $\ell_{\max} \approx \pi/\delta\theta$ where $\delta\theta$ is the average angular distance between back lights. The largest scale is set by the dimensions of the survey $\ell_{\min} \approx A_{\text{survey}}^{1/2}$ where $A_{\text{survey}}$ is the area of the survey in steradians. The maximum number of radial modes, $j_{\text{max}}$, is set by the number of pixels used in each of the spectra.

If $\sigma_\tau$ is the fractional error in the estimate of the absorption within one pixel then the noise in the angular density

2 http://clamato.lbl.gov
power spectrum normalization, denoted lensing measurement. The estimated signal-to-noise for the sample variance and the second comes from noise in the

\[ \kappa(\theta, z_s) = \frac{3}{2} \frac{H_0^2 \Omega_m}{c^2} \int_0^{x_s} d\chi \frac{f(\chi) f(x_s - \chi)}{a(\chi)f(\chi)} \delta(\theta, \chi) \]  

(8)

\[ \bar{f}(\chi) = \begin{cases} R \sinh(\chi/R), & \Omega_C < 0 \\ \chi, & \Omega_C = 0 \\ R \sin(\chi/R), & \Omega_C > 0 \end{cases} \]  

(9)

where \( \chi \) is the coordinate distance, \( x_s \) is the coordinate distance to the source at redshift \( z_s \), \( a(\chi) \) is the expansion parameter and \( \delta(\theta, \chi) \) is the overdensity \((\rho - \bar{\rho})/\bar{\rho}\). In the non-flat case, \( \Omega_C = 1 - \Omega_m - \Omega_L \) and \( R = \frac{1}{H_0 \sqrt{\Omega_C}} \).

4 THE LENSING SIGNAL

The lensing potential is related to the density contrast along the line of sight in the weak lensing Born approximation by

\[ C_{\ell, j} \simeq \frac{1}{D^2L N} \frac{V}{\Omega} \frac{A_{\text{survey}}}{\sigma_i^2} \frac{\sigma_i^2}{\sigma_{\max}(l_{\max} - l_{\min})^2} \]

(5)

\[ \simeq \frac{\sigma_i^2}{\sigma_{\max}(l_{\max} - l_{\min})^2 \sigma_{\min}^2} \]

(6)

where \( N \) is the total number of data points and \( V \) is the total volume within which the HI absorption is being measured. If there are correlations between pixels then \( C_{\ell, j} \) would have a \( j \) dependence. These could easily arise from the fitting of the continuum. We will ignore this complication here.

An estimate for the error in the band power spectrum is derived in the appendix and can be expressed as

\[ \Delta C_D(L) = \sqrt{\frac{4\pi}{A_{\text{survey}}L D_L}} \left[ C_D(L) + N_D(L) \right] \]

(7)

where \( \Delta L \) is the width of the band. The first term of this is the sample variance and the second comes from noise in the lensing measurement. The estimated signal-to-noise for the power spectrum normalization, denoted \( \Sigma \), is also derived in the appendix. This will be a convenient way to summarize the total signal-to-noise for a survey and for determining how well the linear growth factor can be measured.

5 NOISE ESTIMATES

Using the estimated parameters given in table 1, we have calculated the lensing error, equation (2), and the power spectrum error, equation (7), for each of the surveys. In each case we use \( \delta z = 0.001 \) for the pixel size and a constant signal-to-noise per pixel. We assume a standard redshift of 2.5 and redshift ranges of \( \Delta z = 0.5 \) and \( \Delta z = 0.1 \). The results are displayed in terms of displacement power spectrum for aesthetic reasons. Plots of the convergence or lensing potential power spectra would be equivalent. We also show the total signal-to-noise, equation (8), for each case in table 2.

Figure 2 shows the estimated errors for the estimated parameters of the BOSS survey, from Table 1. Because of the relatively low density of backlights on the sky and the relatively low signal-to-noise spectra this is the lowest lensing signal-to-noise case we investigate, but even here gravitational lensing could be detectable and some power spectrum information accessible in these data. If the spectra are broken up into bins of \( \Delta z = 0.1 \) the power spectrum would not be measurable in individual bins.
The situation is more promising for eBOSS, figure 3, where the density of sources is higher. In this case, if the full range redshift range of $\Delta z = 0.5$ is used a good power spectrum should be recoverable and even a moderately good spectrum within individual $\Delta z = 0.1$ bins although probably not good enough to measure the expected evolution in the power spectrum. For BOSS and eBOSS, the expected lensing power spectrum never exceeds the noise per mode, $N_D$, and for this reason it would not be possible to create a high fidelity map of the convergence. For comparison, we have included the expected lensing power spectrum at $z = 2$ and $z = 3$ to give a sense of whether the evolution of the power spectrum could be detected.

The CLAMATO survey will cover a smaller area of sky with a higher density of backlights. From figure 4 it can be seen that the range in multipole is shifted to smaller scales relative to the BOSSs. In this case the noise per mode is below the expected power spectrum for a range of $L$. This indicates that it would be possible to map the convergence with a resolution given by the scale at which $N_D(L)$ and $C_D(L)$ intersect. Even when the spectra are divided up into bins of $\Delta z = 0.1$ good signal-to-noise is retained indicating that some tomographic information should be recoverable. The uncertainty in the power spectrum measurement where $N_D(L) < C_D(L)$ is dominated by sample variance owing to the small survey area.

WEAVE and DESI surveys will have larger areas than CLAMATO and higher backlight densities than the BOSSs. One can see in figures 5 and 6 that this results in an excellent power spectrum measurement at least within a wide redshift range. $N_D(L)$ is not low enough in these cases to map the convergence except perhaps for WEAVE at the...
largest scales. This is largely a consequence of the lower backlight density relative to CLAMATO. Subaru PFS will have an area smaller than WEAVE or DESI, but larger that CLAMATO with a backlight density that is similar to CLAMATO. Here lens mapping will be possible within small redshift ranges and tomographic information should be accessible (figure 7). Finally, figure 8 shows that a very ambitious MSE survey would produce an excellent measurement of the power spectrum in bins as small as $\Delta z = 0.1$ and large high fidelity maps with a resolution of $\sim 1^\circ$.

The cumulative signal to noise ratio is useful statistic to gauge the overall significance of detection and the precision possible in power spectrum normalization from observations. We define this quantity in Appendix A, and in Table 2, we show calculated values for the different surveys we have considered. We can see that forest lensing is likely to be highly competitive with galaxy shear surveys.

6 CONCLUSION

Weak gravitational lensing should be measurable with high precision using the Ly$\alpha$ forest data from surveys of the scale and quality being done today and in the near future. Our estimates show that the lensing power spectrum could be measured for source redshifts of $z \simeq 2 - 3$ in this way and that the evolution of clustering over this range should be measurable. This would provide weak lensing information for a range of source redshifts that is not accessible with galaxy shear surveys or lensing of the CMB and allow for probing early dark energy and certain alternative gravity theories.

Cross-correlating Ly$\alpha$ lensing maps with foreground surveys or galaxy lensing surveys boosts the signal-to-noise and provides access to unique information. With the ability to map the convergence with high fidelity comes the possibility of another kind of cosmological probe. The convergence map for one redshift and the convergence map for another will have the same foreground matter, but different $\chi_s$ in equation (8) which is a function of cosmological parameters. With additional redshift information on the foreground, one might be able to constrain the foreground density and the cosmology simultaneously. Such a measurement would not suffer from cosmic variance and be a more direct measure of the angular size / redshift relation. This possibility will be further investigated in the future.

ACKNOWLEDGMENTS

We thank Andreu Font, Mat Pieri and K.G. Lee for their help with understanding the various samples of observational data and for their willingness to give approximate estimates of future data properties. AR and RBM have been supported partly through project GLENCO, funded under the European Seventh Framework Programme, Ideas, Grant Agreement n. 259349. RACC is supported by NASA ATP award NNX17AK56G.
Table 2. The estimated cumulative signal-to-noise for the lensing power spectrum, $S$, (defined in the appendix) for different Lyα surveys and redshift bin width. For comparison a galaxy shear survey that covers the whole sky, has intrinsic galaxy ellipticities of 0.3, a source redshift of 0.9, 30 galaxies per arcmin$^2$ and no other noise would have an $S$ of $\sim 12$.

REFERENCES

Busca N. G., Delubac T., Rich J., Bailey S., Font-Ribera A., Kirkby D., Le Golf J.-M., Pieri M. M., Slosar A., Aubourg É., others. 2013, A&A, 552, A96
Cisewski J., Croft R. A. C., Freeman P. E., Genovese C. R., Khandai N., Ozbek M., Wasserman L., 2014, MNRAS, 440, 2599
Croft R., Romeo A., Metcalf R. B., 2017
Croft R. A. C., Hu W., Davé R., 1999, Physical Review Letters, 83, 1092
Dalton G., et al., 2012, in Ground-based and Airborne Instrumentation for Astronomy IV Vol. 8446 of Proc.SPIE, WEAVE: the next generation wide-field spectroscopy facility for the William Herschel Telescope. p. 84460P
Dawson K. S., et al., 2016, AJ, 151, 44
DESI Collaboration Aghamousa A., Aguilar J., Ahlen S., Alam S., Allen L. E., Allende Prieto C., Annis J., Bailey S., Ballard C., et al. 2016, ArXiv e-prints
Eisenstein D. J., Hu W., 1999, ApJ, 511, 5
Hoekstra H., Jain B., 2008, Annual Review of Nuclear and Particle Science, 58, 99
Hu W., 2001, ApJS, 145, 69
Lee K.-G., et al., 2013, ApJL, 795, L12
McQuinn M., 2016, Ann.Rev.Astronomy& Astrophysics, 54, 313
Metcalfe R. B., Croft R., Romeo A., 2017
Palanque-Delabrouille N., Vérard C., Baur J., Magneville C., Rossi G., Lesgourgues J., Borde A., Burtin E., LeGoff J.-M., Rich J., Viel M., Weinberg D., 2015, JCAP, 11, 011
Peacock J. A., Dodds S. J., 1996, MNRAS, 280, L19
Peeples M. S., Weinberg D. H., Davé R., Fardal M. A., Katz N., 2010a, MNRAS, 404, 1281
Peeples M. S., Weinberg D. H., Davé R., Fardal M. A., Katz N., 2010b, MNRAS, 404, 1295
Pichon C., Vergely J. L., Rollinde E., Colombi S., Petitjean P., 2001, MNRAS, 326, 597
Planck Collaboration Ade P. A. R., Aghanim N., Arnaud M., Ashdown M., Aumont J., Baccigalupi C., Banday A. J., Barreiro R. B., Bartlett J. G., et al. 2016, A&A, 594, A15
Pourtsidou A., Metcalf R. B., 2015, MNRAS, 448, 2368
Scoville N., et al., 2007, ApJS, 172, 1
Slosar A., et al., 2011, JCAP, 9, 001
Slosar A., Iršič V., Kirkby D., Bailey S., Busca N. G., Delubac T., et al., 2013, JCAP, 4, 026
Takada M., 2012, Journal of Geophysical Research (Biogeosciences), p. 111502

\begin{table}[h]
\centering
\begin{tabular}{cccccccc}
\hline
Survey & $\Delta z = 0.1$ & $\Delta z = 0.5$ \\
\hline
BOSS DR12 & 1.3 & 6.0 \\
eBOSS & 3.7 & 15.9 \\
CLAMATO & 2.7 & 7.7 \\
WEAVE & 7.0 & 28.0 \\
DESI & 4.2 & 28.8 \\
Subaru PFS & 7.3 & 12.5 \\
MSE & 53 & 162 \\
\hline
\end{tabular}
\caption{The estimated cumulative signal-to-noise for the lensing power spectrum, $S$, (defined in the appendix) for different Lyα surveys and redshift bin width. For comparison a galaxy shear survey that covers the whole sky, has intrinsic galaxy ellipticities of 0.3, a source redshift of 0.9, 30 galaxies per arcmin$^2$ and no other noise would have an $S$ of $\sim 12$.}
\end{table}
To forecast how well the power spectrum can be recovered we start with the likelihood for Fourier modes with a Gaussian prior
\[
L = \frac{1}{(2\pi)^N \prod_{\ell} \sqrt{N_{\ell} C_{\ell}}} \exp \left[ -\frac{1}{2} \sum_{\ell} \frac{(\hat{a}_{\ell} - a_{\ell})^2}{N_{\ell}} \right] \exp \left[ -\frac{1}{2} \sum_{\ell} \frac{a_{\ell}^2}{C_{\ell}} \right].
\] (A1)

Here \(C_{\ell}\) is the signal power spectrum and \(N_{\ell}\) is the noise per mode. The actual amplitudes are \(a_{\ell}\) and \(\hat{a}_{\ell}\). In the real case with an irregular survey shape, non-isotropic noise and randomly distributed backlights there will be cross-correlations between different modes and this must be done more carefully, but for our purposes we will consider them uncorrelated.

We can calculate, marginalizing this over the mode values by integrating over the \(a_{\ell}'s\)
\[
\hat{L} = \frac{1}{(2\pi)^{N/2} \prod_{\ell} \sqrt{N_{\ell} C_{\ell}}} \exp \left[ -\frac{1}{2} \sum_{\ell} \frac{\hat{a}_{\ell}^2}{C_{\ell} + N_{\ell}} \right].
\] (A2)

The error for a parameter is often estimated with the Fisher information for that parameter which is the average of the second derivative of the log of the likelihood with respect to that parameter. In this case we will take the normalization of the power spectrum \(A\) to be our parameter. The information for this parameter is
\[
S^2 \equiv -\left\langle \frac{\partial^2 \ln L}{\partial^2 A} \right\rangle = \frac{1}{2} \sum_{\ell} \frac{C_{\ell}^2}{(N_{\ell} + C_{\ell})^2} \simeq \frac{A_{\text{survey}}}{4\pi} \int d\ell \frac{\ell C_{\ell}^2}{(N_{\ell} + C_{\ell})^2} \int d\ell \frac{C_{\ell}^2}{(N_{\ell} + C_{\ell})^2}.
\] (A3)

In converting from the sum to the integral the density of modes in a finite size survey is approximated as \(\Delta \ell \simeq 2\pi/\sqrt{A_{\text{survey}}}\). This can be applied to the whole spectrum, in which case this is the square of signal-to-noise for the power spectrum normalization. It can also be applied to bands in \(\ell\) which, if the noise and spectra are approximated as linear within the bands, is equation 7 for the lensing spectrum.