Radial thresholding to mitigate laser guide star aberrations on centre-of-gravity-based Shack–Hartmann wavefront sensors

Olivier Lardière,* Rodolphe Conan, Colin Bradley, Kate Jackson and Peter Hampton

AO Laboratory, Mechanical Engineering Department, University of Victoria, PO Box 3055 STN CSC, Victoria, BC, V8W 3P6, Canada

Accepted 2009 June 8. Received 2009 June 5; in original form 2009 April 14

ABSTRACT
Sodium laser guide stars (LGSs) are elongated sources due to the thickness and the finite distance of the sodium layer. The fluctuations of the sodium layer altitude and atom density profile induce errors on centroid measurements of elongated spots, and generate spurious optical aberrations in closed-loop adaptive optics (AO) systems. According to an analytical model and experimental results obtained with the University of Victoria LGS bench demonstrator, one of the main origins of these aberrations, referred to as LGS aberrations, is not the centre-of-gravity (CoG) algorithm itself, but the thresholding applied on the pixels of the image prior to computing the spot centroids. A new thresholding method, termed ‘radial thresholding’, is presented here, cancelling out most of the LGS aberrations without altering the centroid measurement accuracy.

Key words: instrumentation: adaptive optics – methods: analytical– methods: laboratory.

1 INTRODUCTION
Sodium laser guide star (LGS) adaptive optics (AO) systems allow in theory a full sky coverage; however, there are several limitations. The artificial star is elongated due to the sodium layer thickness (about 10 km) and finite distance (90 km). Consequently, if the laser is launched from the secondary mirror holder, the spots of a Shack–Hartmann wavefront sensor (SH–WFS) are radially elongated from the centre to the edge of the pupil (Fig. 1). The spot elongation is proportional to the telescope diameter and can reach several arcseconds for extremely large telescopes (ELTs).

Moreover, the sodium layer is not static, but fluctuates with a time-scale of about 1 min or less (Davis et al. 2006). Fluctuations of the sodium layer altitude and atom density profile induce errors on centroid measurements of elongated spots and generate spurious aberrations on the wavefront in closed-loop AO systems. These aberrations, referred to as LGS aberrations, can reach several hundred nanometres peak-to-valley (PtV) for ELTs with the classical centre-of-gravity (CoG) centroiding algorithm (Clare, van Dam & Bouchez 2007; Lardière et al. 2008). Some authors proposed new sophisticated centroiding algorithms to mitigate LGS aberrations, such as the matched filtering (Gilles & Ellerbroek 2008; Conan et al. 2009) or the correlation (Poyneer 2003; Thomas et al. 2006, 2008).

However, thanks to the LGS-bench demonstrator built at the University of Victoria (UVic) for ELT projects (Lardière et al. 2008; Conan et al. 2009), we found out that one of the main sources of the LGS aberrations was not the CoG algorithm itself, but simply the threshold applied on the pixels of the SH–WFS images before the centroid computation.

Section 2 reviews the known possible origins of LGS aberrations. A model of the aberrations generated by thresholding is presented in Section 3, as well as a simple new thresholding method, termed ‘radial thresholding’, which mitigates most of LGS aberrations. The experimental results obtained with the UVic bench with the radial thresholding show that the CoG algorithm is still well suited for LGS wavefront sensing on ELTs (Section 4).

2 ORIGINS OF LGS ABERRATIONS
If the LGS spots are radially elongated from the pupil centre, as shown on Fig. 1, the sodium layer fluctuations induce centrosymmetric aberrations, such as focus ($Z_4$) and spherical aberrations ($Z_{11}$, $Z_{22}$, etc.), and also square symmetric aberrations, such as tetrafoils ($Z_{14}$, $Z_{26}$, etc.).

The focus is due to a variation of the sodium layer altitude. This error is not an artefact and must be compensated by refocussing the LGS on the WFS with zoom optics and by updating the offsets of the LGS WFS with a natural guide star (NGS) focus sensor (Herriot et al. 2006). Non-common path errors of the LGS optical train, including the zoom optics, can vary with the sodium layer distance, i.e. the zenithal angle, and induce variable aberrations on the science path too. We assume that these systematic aberrations can be calibrated and virtually negated.

Consequently, aberrations beyond focus are mainly artefacts of the wavefront sensing. According to a model from Clare et al. (2007) and to the first experimental results obtained with the UVic LGS

*E-mail: lardiere@uvic.ca
The spot truncation caused by the field-stop is negligible compared to the truncation induced by the pixel thresholding if the field of view (FOV) of the outermost lenslets is wide enough to make an image of a 20-km-thick sodium profile.

Both kinds of LGS aberrations have been reproduced and characterized in laboratory on the UVic bench with a time series of 88 real sodium profiles (Fig. 6). Beyond the focus, the most significant LGS aberrations detected are the spherical aberration $Z_{11}$ up to 100 nm PtV (30 nm rms), and the tetrafoil $Z_{14}$ with 40 nm PtV (10 nm rms). Moreover, a correlation between the spherical aberration and the profile asymmetry was empirically established (Lardiè et al. 2008). Square-symmetric aberrations, such as $Z_{14}$, should be mitigated by using a polar-coordinate CCD array (Beletic et al. 2008). Square-symmetric aberrations, such as $Z_{14}$, should be mitigated by using a polar-coordinate CCD array (Beletic et al. 2008).

However, we discovered later that $Z_{11}$ mode disappears if no threshold was applied on the pixels of the LGS WFS images before the computation of centroids. The thresholding discards the two extremities of each elongated spot, and consequently truncates radially each spot, as an optical circular field-stop would do. The spot truncation caused by the field-stop is negligible compared to the truncation induced by the pixel thresholding if the field of view (FOV) of the outermost lenslets is wide enough to make an image of a 20-km-thick sodium profile.

With such a large FOV and a polar-coordinate CCD array, as expected for the 30 m telescope (TMT) LGS AO facility (Ellerbroek et al. 2008), the thresholding is likely the main source of the LGS aberrations and deserves a specific study.

### 3.1 Geometrical model of the spot elongation

If the sodium laser is launched on the system optical axis, the spots of a SH-WFS are radially elongated from the pupil centre, as shown on Fig. 2. Let $\rho$ be the distance of a lenslet to the pupil centre, normalized to 1 on the pupil edge. From equation (1) of Lardiè et al. (2008), the geometrical radial size, on the detector plane, of the corresponding spot is proportional to $\rho$, such as $E(\rho) \approx E, \rho$, with

$$E = \frac{f R \sigma_{\text{Na}}}{h_0} \cos(z).$$

In this equation, $h_0$ and $\sigma_{\text{Na}}$ are the mean altitude and the thickness of the sodium layer respectively, $z$ is the zenithal angle, $R$ is the radius of the telescope pupil and $f$ is the resultant focal length of the whole optical system. $E$ is the geometrical radial extent, on the detector plane, of a spot located on the edge of the pupil (i.e. $\rho = 1$). $E(\rho)$ is simply termed spot elongation hereafter, and can be expressed in mm or in pixels.

If $x$ and $y$ are the Cartesian coordinates of a spot in the detector plane, then the geometrical sizes of that spot, projected on the $x$ and $y$ axis are

$$E_x(x,y) = x E(x,y), \quad E_y(x,y) = y E(x,y).$$

![Figure 2. Geometrical model of the spot elongation if the laser is launched from the centre of the pupil.](https://academic.oup.com/mnras/article-abstract/398/3/1461/1266428/320x76-532x299)
Let \( I_0 \) and \( \sigma \) be the maximum intensity and the size of the central spot (i.e. \( \rho = 0 \)), respectively. The extent \( \sigma \), expressed in the same unit than \( E \), is defined by the blurring due to the lenslet diffraction lobe, or to the seeing. Considering this blurring effect, the actual length and width of a spot located at the radius \( \rho \) are, respectively, \( E(\rho) + \sigma \) and \( \sigma \). Let \( I_{\text{max}}(\rho) \) be the maximum intensity of that spot. As the flux is conserved for all spots of the pupil, we have the following equation:

\[
\sigma^2 I_0 \approx \frac{\sigma}{E(\rho) + \sigma} I_{\text{max}}(\rho).
\]

Hence,

\[
I_{\text{max}}(\rho) \approx \frac{I_0}{1 + \frac{\sigma}{\sigma}}.
\]

### 3.2 Centroid errors versus threshold

Let \( P_0(r) \) be the projection of the sodium layer vertical profile on the SH–WFS detector plane, for a lenslet located on the edge of the pupil (i.e. \( \rho = 1 \)). This projection is not linear and takes in account the perspective effect which stretches the lower part of the sodium layer and compress the upper part. Thus, the one-dimensional function \( P_0(r) \) is the radial intensity profile of the geometrical image of the most elongated spot. The variable \( r \) is the radius position defined inside the subimage plane of the considered spot only. If a threshold of value \( \text{Thres} \) is applied on the spot image, the thresholded profile, obtained according to equation (1), is noted \( P_0(t) \), with \( t = \text{Thres}/I_{\text{max}}(\rho) \) the threshold value normalized to the local spot maximum intensity. For any spot profile and any threshold \( t \in [0; 1] \), we can define the dimensionless function \( g(t) \) such as

\[
g(t) = \frac{1}{E} \left( \int_0^r P_0(r) \, dr - \int_0^1 P_0(r) \, dr \right).
\]

This function provides, for any spot of the pupil, the radial centroid error induced by the threshold \( t \), expressed as a fraction of the elongation of the considered spot. In absolute units (pixel or \( \mu \)m), the radial centroid error scales as the spot elongation, and is \( g(t) E(\rho) \). We note that \( g(t) = 0 \) for symmetric profiles. Fig. 3 plots a geometrical spot profile \( P_0(r) \) with its function \( g(t) \). This profile is among the most asymmetric sodium profiles found in the 88-profile sequence of Fig. 6. The projection of the radial error \( g(t) E(\rho) \) on the x- and y-axis provides \( \delta_x(x, y) \) and \( \delta_y(x, y) \), the errors made on the wavefront slopes in x- and y-directions, respectively, for a spot located in coordinates \( (x, y) \):

\[
\delta_x(x, y) = E_x(x, y) g(t) = x \, E(t),
\]

\[
\delta_y(x, y) = E_y(x, y) g(t) = y \, E(t).
\]

Hence, the wavefront error (WFE) \( Z(x, y) \) caused by thresholding is deduced from the slopes, such as

\[
\frac{\partial Z}{\partial x} = x \, E(t),
\]

\[
\frac{\partial Z}{\partial y} = y \, E(t).
\]

Using a polar coordinate system \( (\rho, \theta) \), such as \( x = \rho \cos \theta \) and \( y = \rho \sin \theta \), we can use the following conversion formulae,

\[
\frac{\partial Z}{\partial \rho} = \frac{x}{\rho} \frac{\partial Z}{\partial x} + \frac{y}{\rho} \frac{\partial Z}{\partial y},
\]

\[
\frac{\partial Z}{\partial \theta} = -y \frac{\partial Z}{\partial x} + x \frac{\partial Z}{\partial y},
\]

and to obtain

\[
\frac{\partial Z}{\partial \rho} = \rho \, E(t),
\]

\[
\frac{\partial Z}{\partial \theta} = 0.
\]

Thus, the wavefront distortion generated by thresholding is always a surface of revolution whatever \( g(t) \), i.e. whatever the sodium profile. In terms of Zernike polynomials (Noll 1976), this result proves that the thresholding can only induce a combination of focus \( (Z_0) \) and spherical aberrations \( (Z_{11}, Z_{22}, Z_{37}, \text{etc.}) \).

### 3.3 Wavefront aberrations induced by uniform thresholding

If the threshold value \( \text{Thres} \) is constant for the whole pupil, then \( t = \text{Thres}/I_{\text{max}}(\rho) \) varies with \( \rho \). From equations (5) and (10), we obtain in that case

\[
\frac{\partial Z}{\partial \rho} \approx \rho \, E \left( \frac{\text{Thres}}{I_0} \left( 1 + \frac{E}{\sigma} \right) \right).
\]

Moreover, we assume that \( g(t) \) can be expressed, otherwise approximated, as a polynomial of degree \( n \) such as

\[
g(t) \approx \sum_{i=1}^{n} g_i t^i
\]

with the sum starting at \( i = 1 \) because \( g(0) = 0 \) by definition (equation 6). Using the binomial theorem, we can find, after integration of equation (11)

\[
Z(\rho) \approx \sum_{i=1}^{n} g_i \left( \frac{\text{Thres}}{I_0} \right)^i \frac{1}{\rho^{i+2}} \int_{j=0}^{i} i! \frac{1}{j!} \frac{1}{{\rho^j}^{i+2}}.
\]

We can check that \( Z(\rho) = 0 \) if the spots are not elongated \( (E = 0) \), or if the threshold is null, or again if the spot profile is symmetric \( (g_i = 0, \forall i) \). If \( n \) is the order of the polynomial \( g(t) \), the radial order of the wavefront aberration is \( n + 2 \). A uniform threshold applied on a triangular or trapezoidal (i.e. \( n = 1 \)) asymmetric spot profile induces already aberrations of the third radial order:

\[
Z(\rho) \approx \frac{E}{2} g_1 \left( \frac{\text{Thres}}{I_0} \right) \rho^2 \left( \frac{E^2}{\sigma^2} \right) \rho^{i+2}.
\]

The same threshold applied on a more complex asymmetric sodium profile will induce higher order aberrations on the wavefront. The projection of \( \rho^2 \) on the Zernike polynomials base is a pure focus:

\[
\rho^2 = \frac{1}{2\sqrt{Z_4}}.
\]
while $\rho^3$ is a combination of focus and all spherical aberrations ($Z_{11}$, $Z_{22}$, $Z_{37}$, etc.):

$$\rho^3 \approx 0.3 \left( Z_4 + 0.14 Z_{11} - 0.015 Z_{22} + 0.003 Z_{37} + \cdots \right). \quad (16)$$

The focus term is corrected by the NGS focus sensor already required to offset the altitude variation of the sodium layer. Beyond the focus, the main aberration induced by a uniform thresholding is $Z_{11}$ mode, higher order spherical aberrations being negligible. The $Z_{11}$ error is fluctuating with the time, due to the variations of the sodium profile asymmetry, and is an issue for LGS AO systems.

### 3.4 Wavefront aberrations induced by radial thresholding

If the threshold value is now not uniform over the whole pupil, but defined for each subimage proportionally to the maximum intensity of the local spot, then the normalized threshold $t = \text{Thres}/I_{\text{max}}(\rho)$ becomes constant, as well as $g(t)$. From equation 10, we deduce that

$$Z(\rho) = \frac{E}{2} g(t) \rho^2. \quad (17)$$

Consequently, such a thresholding method induces only a focus error and no spherical aberrations at all, whatever the sodium profile structure. As the threshold value scales as the inverse of the radius, this thresholding is referred to as ‘radial thresholding’.

### 4 LABORATORY RESULTS OF THE RADIAL THRESHOLDING

#### 4.1 Experimental setup

The radial thresholding is easy to implement on real SH–WFSs, and has been validated in laboratory with the UVic LGS demonstrator bench and compared to the results obtained without threshold or with an uniform threshold. Two signal-to-noise ratios (S/N) have been considered, 140 and 18, the latter being expected for real LGSs. At S/N = 140, the read-out noise is negligible, and thresholding could not be necessary. However, this favourable case is useful to properly distinguish the thresholding effect from the noise, and to validate our model.

The read-out noise of the SH–WFS camera is 0.43 ADU rms. As the offset frame of the detector is subtracted to the raw images, some pixels may have negative values due to the read-out noise. Consequently, a 0-ADU threshold discards the negative values, and does not mean that no threshold is applied on the image. We will have to distinguish both cases in our following experimental study at low S/N.

The FOV of the circular field stop of the bench corresponds to a sodium layer thickness of 20 km for the most elongated spot. As the mean thickness of the sodium layer is about 10 km, the spot truncation effect due to the field-stop should be negligible compared to the spot truncation caused by the thresholding.

Fig. 1 displays a 29 x 29 radially elongated spot image obtained with the UVic bench for S/N = 18. On this image, the maximum spot elongation $E$ is 8 pixels, and the spot width $\sigma = 2$ pixels. Fig. 4 plots a horizontal cut of this image ($y = 0$). A uniform threshold at 1.3 ADU, i.e. $3 \sigma_{\text{RON}}$ for the bench, is shown as an example. This graph already points up that a uniform thresholding is not suited for LGS spots, since edge spots are fainter than the central spots. At low S/N, a uniform threshold might even reach the maximum intensity of some spots located on the edge. Fig. 5 illustrates this issue for S/N = 18.

---

**Figure 4.** Horizontal radial cut of the 29 x 29 elongated spot SH–WFS frame of Fig. 1 obtained with the UVic LGS-bench at S/N = 18. The red solid line illustrates a uniform threshold set at three times the read-out noise ($\sigma_{\text{RON}} = 0.43$ ADU on the bench). The radial cut is a binning of the three central pixel lines of the frame. According to the spot elongation, the theoretical intensity ratio between the central spots and the edge spots is 4. The actual ratio is around 7 here due to the Gaussian profile of the collimated beam used on the bench, and also to the field curvature of the imaging lens. These effects exaggerate slightly the WFEs induced by thresholding.

**Figure 5.** Truncation effect induced by thresholding on the four outer spots at S/N = 18, for three different threshold levels. The 0-ADU level is the detector offset. Pixel values might be negative due to read-out noise.

#### 4.2 Performance at high flux

Fig. 6 plots the LGS aberrations for $Z_4$, $Z_{11}$, $Z_{22}$ and $Z_{14}$ modes versus the time, measured during the whole sodium profile sequence, with no threshold, with a 13-ADU uniform threshold and a radial threshold (4 per cent of local maximum) at S/N = 140, in open loop without turbulence. Such a high uniform threshold, not suited for real AO systems, is chosen here to exaggerate the thresholding effects on the wavefront and validate the model. Fig. 7 plots the corresponding temporal rms fluctuation of all Zernike modes beyond focus up to $Z_{38}$, for S/N = 140 and for different threshold values (both uniform and radial).

As the main focus error source is the altitude variation of the sodium layer, the focus obtained without thresholding has been subtracted from those obtained with thresholding. This subtraction highlights the extra focus error induced by thresholding. This error reaches 350 nm PtV with the uniform threshold, and 200 nm PtV with the radial threshold. Moreover, a correlation is visible between the focus and $Z_{11}$ modes induced by the uniform thresholding. This correlation confirms that $Z_4$ and $Z_{11}$ modes arise together. This result is in good agreement with the model (equations 13 to 16). Although the ratio $Z_4/Z_{11}$ depends on the coefficients $g_i$, i.e. the sodium profile structure and asymmetry, we can attempt to quantify it for

---

© 2009 The Authors. Journal compilation © 2009 RAS, MNRAS 398, 1461–1467
a linear fit of the functions $g(t)$, i.e. for a sequence of trapezoidal or triangular sodium profiles. In this simple case, the ratio $Z_4/Z_{11}$ expected by the model is about 7.3 with $E/\sigma = 4$ (equations 14 to 16), while the observed ratio is about 4.9 for the PtV amplitudes (Fig. 6). The model and the experiments provide the same order of magnitude for the ratio $Z_4/Z_{11}$. The difference is likely due to the assumption made about the profile shape. A higher order fit of the functions $g(t)$ would be required for each profile, as well as further measurements using a symmetric sodium profile as a reference.

Fig. 6 shows clearly that the radial thresholding mitigates the spherical aberrations ($Z_4$ and $Z_{22}$) induced by the sodium layer variations, as if no thresholding was applied. The variations for the $Z_{11}$ mode reaches 90 nm PtV with uniform thresholding, against 10 nm PtV with radial thresholding or without thresholding at all. The remaining error is likely the component induced by the noise and by the circular field-stop. Moreover, Fig. 7(a) shows that the rms fluctuation of $Z_{11}$ mode scales as the level of the uniform threshold, which is in agreement with the model too (equation 13).

As expected, the radial thresholding does not mitigate the square symmetric LGS aberration $Z_{14}$. However, the $Z_{14}$ error is modified by the thresholding, and is almost null without thresholding (Figs 6 and 7a). Hence, the thresholding would likely induce other effects not considered in our model. The pixel boundaries or the spot overlap may have more or less impact depending on the threshold value. Assuming that polar coordinate CCD arrays should mitigate square symmetric modes, we will not investigate with more details the other possible sources of this mode in this paper.

At S/N = 140, the best performances are obtained with a radial threshold set at 4 per cent of the local spot maximum, $\sigma_{\text{RON}} = 0.43$ ADU.

4.3 Performance at low flux

Fig. 8 plots the temporal rms fluctuation of Zernike modes from $Z_4$ to $Z_{20}$, for S/N = 18 and for different threshold values, both uniform and radial.

At low S/N, edge spots are very faint, a few ADU. Consequently, a uniform threshold, even low, can easily exceed the maximum of

![Figure 7. LGS-induced aberrations measured on the UVic LGS-bench at S/N = 140 when a uniform threshold (a) or a radial threshold (b) is applied on LGS spot images ($\sigma_{\text{RON}} = 0.43$ ADU).](image)

![Table 1. rms WFEs, in nm rms, and achievable Strehl ratios (S), in per cent for $\lambda = 1$ μm, obtained at S/N = 140 and 18, during the sodium profile sequence with the CoG algorithm without pixel thresholding, with uniform thresholding, and with radial thresholding. The threshold levels are optimized for S/N = 18. The uniform threshold values are 13 and 0 ADU for S/N = 140 and 18, respectively, while the radial threshold values are 4 and 12 per cent for S/N = 140 and 18, respectively. All Zernike modes beyond the focus (from $Z_5$ to $Z_{20}$) are added up and spatially averaged over the pupil. The bench accuracy is the ultimate centroiding performance achieved by the bench on non-elongated spots. Read-out noise is 0.43 ADU rms.](table)
LGS-induced aberrations measured on the UVic LGS-bench at S/N = 18 when a uniform threshold (a) or a radial threshold (b) is applied prior to computing centroids (σRON = 0.43 ADU).

Figure 8. LGS-induced aberrations measured on the UVic LGS-bench at S/N = 18 when a uniform threshold (a) or a radial threshold (b) is applied prior to computing centroids (σRON = 0.43 ADU).

5 DISCUSSIONS

5.1 Weighted centre-of-gravity algorithm

Considering the issues caused by the thresholding on LGS spots, we could suggest to use the weighted CoG algorithm, which does not require a prior thresholding, instead of the conventional CoG algorithm. Basically, this algorithm consists in weighting the pixels of the spot image by a reference image of the spot (Nicolle et al. 2004). If the spot image matches perfectly the reference image, this technique is equivalent to compute the CoG of the square of the image intensity. However, the CoG of the square of an elongated spot image differs from the actual centre of gravity of that spot, if the spot profile is asymmetric. Consequently, the weighted CoG algorithm cannot be used on LGS AO systems as it is.

5.2 Case study of non-radial spot elongation

The theoretical model presented in Section 3 assumes that the laser is launched on the optical axis. This configuration is optimal for ELTs because it minimizes the spot elongation and the LGS aberrations. However, on a 10-m class telescope, the laser can preferably be launched from the side of the telescope pupil, like for the Keck telescope (Wizinowich et al. 2006). The LGS spots of a SH–WFS are no longer radially elongated from the pupil centre, but from an edge of the pupil.

The model is still valid for this case, providing that the origin of the coordinate system (ρ, θ) or (x, y) coincides with the position of laser launch telescope. Equation (13), expressing the wavefront 

\[ Z(\rho, \theta) \]

remains unchanged and the aberrations generated by a uniform thresholding are still centro-symmetric for a virtual giant pupil centred on the laser axis, with a diameter twice the distance between the laser axis and the optical axis. The only difference is the domain of definition of the function \( Z(\rho, \theta) \), which must be shifted to match the telescope pupil.

Fig. 9 displays the portion of the wavefront \( Z(\rho, \theta) \) seen by a circular pupil located at the right of the laser axis, and plots the PtV amplitudes of this wavefront projected on Zernike modes. Assuming a linear fit of the function \( g(t) \), we can consider \( Z(\rho) \propto \rho^2 + 8/3\rho^2 \) for reproducing the LGS aberrations induced by an uniform thresholding (equation 14 with \( E/\sigma = 4 \) ), and \( Z(\rho) \propto \rho^2 \) for reproducing the LGS aberrations induced by a radial thresholding (equation 17).

As the aberrations are not centro-symmetric, the Zernike decomposition of \( \rho^2 \) features, in addition to the focus and spherical aberrations, many other modes, such as tip and tilt (\( Z_3, Z_9 \)), astigmatism (\( Z_5, Z_7, Z_8 \)), coma (\( Z_7, Z_9 \)) and some trefoil (\( Z_6, Z_{10} \)). Moreover, compared to the centrally projected laser configuration, the PtV amplitude of these aberrations are about seven times greater.

On the other hand, the decomposition of \( \rho^3 \) features only tip and tilt modes in addition to the focus, meaning that the radial thresholding removes all the LGS aberrations beyond the focus. Tip-tilt and focus modes are not issues, since they will be corrected by the NGS tip-tilt and focus sensor already necessary to determine

Figure 9. Typical LGS aberrations arising when the laser is launched from the side of the pupil (\( x = 0 \) and \( y = 0 \)) with a uniform thresholding (top) and for a radial thresholding (bottom). These aberrations are compared, in terms of Zernike modes, to those arising when the laser is centrally projected. Aberration amplitudes are PtV in an arbitrary unit.
the atmospheric tip-tilt, unseen by a LGS WFS, and to offset the sodium layer distance variations, respectively.  

Finally, the radial thresholding is even more advised if the laser is launched from the side of the pupil, since the LGS aberrations induced by the uniform thresholding are much higher and are spread in more modes. The side-projected laser configuration has not been tested on the UVic bench, but could be implemented if the need arises for any LGS AO project.

6 CONCLUSION

The fluctuations of the sodium layer vertical profile induce centro-symmetric and square symmetric aberrations on LGS SH–WFSs. Centro-symmetric aberrations, such as focus and spherical aberrations, are due to a spot truncation by either a circular field-stop or by a threshold applied on the image pixels. Square symmetric aberrations, such as $Z_{14}$, are due to a spot truncation by the pixel boundaries and a sampling effect. Some extra aberrations may arise due to uncalibrated non-common path errors of the LGS train varying with the sodium layer distance.

Both kinds of aberrations have been reproduced in the laboratory with UVic LGS bench. Square symmetric aberrations should be negated with a polar coordinate CCD array. Considering such a detector and a FOV per subaperture wide enough to be able to image a 20-km-thick sodium layer, the residual LGS aberrations are mainly caused by the threshold applied on the pixels of elongated spots prior to computing the centroids.

This statement is confirmed by the theoretical model and experimental results presented in this paper. The aberrations induced by the thresholding contain focus and spherical aberration modes if the laser is launched from the pupil centre. Their amplitudes scale as the spot elongation (i.e. the telescope diameter), the sodium profile asymmetry and the threshold value.

These aberrations disappear if the threshold value is not static and uniform over the pupil, but defined dynamically and independently for each lenslet, proportionally to the maximum intensity of the local spot. The residual LGS aberrations are only a focus error if the laser is centrally projected, or a combination of tip, tilt and focus if the laser is launched beside the pupil. These residual modes are not issues since they will be corrected by the NGS tip-tilt and focus sensor already required in any LGS AO system.

This new thresholding method, termed radial thresholding, is very simple and add no extra computational time compared to the uniform thresholding. It can be implemented on all current LGS SH–WFS in operation. Moreover, the radial thresholding makes the CoG algorithm still well-suited for ELTs, where the LGS spot elongation is greater. Unlike the matched filter or the correlation centroiding algorithms, the CoG is a simple, well-proven and fast algorithm which requires no special calibration processes or reference images, and has no repercussions on the design of the LGS AO system.

ACKNOWLEDGMENTS

The authors are grateful to TMT consortium. The TMT Project gratefully acknowledges the support of the TMT partner institutions. They are the Association of Canadian Universities for Research in Astronomy (ACURA), the California Institute of Technology and the University of California. This work was supported as well by the Gordon and Betty Moore Foundation, the Canada Foundation for Innovation, the Ontario Ministry of Research and Innovation, the National Research Council of Canada, the Natural Sciences and Engineering Research Council of Canada, the British Columbia Knowledge Development Fund, the Association of Universities for Research in Astronomy (AURA) and the US National Science Foundation. We would also like to thank Laurent Jolissaint, the reviewer of this paper, for his useful comments.

REFERENCES

Beletic J. W. et al., 2005, Exp. Astron., 19, 103
Clare R. M., van Dam M. A., Bouchez A. H., 2007, Opt. Express, 15, 4711
Conan R., Lardière O., Herriot G., Bradley C., Jackson K., 2009, Appl. Opt., 48, 1198
Davis D. S., Hickson P., Herriot G., She C.-Y., 2006, Opt. Lett., 31, 3369
Ellerbroek B. et al., 2008, Proc. SPIE, 7015, 70150R
Gilles L., Ellerbroek B., 2008, Opt. Lett., 33, 1159
Herriot G., Hickson P., Ellerbroek B., Vérán J.-P., Ye C. S., Clare R., Looze D., 2006, Proc. SPIE, 6272, 62721I
Lardière O., Conan R., Bradley C., Jackson K., Herriot G., 2008, Opt. Express, 16, 5527
Nicolle M., Fusco T., Rousset G., Michau V., 2004, Opt. Lett., 29, 2743
Noll R. J., 1976, J. Opt. Soc. Am., 66, 207
Poyneer L. A., 2003, Appl. Opt., 42, 5807
Thomas S., Fusco T., Tokovinin A., Nicolle M., Michau V., Rousset G., 2006, MNRAS, 371, 323
Thomas S., Adkins S., Gavel D., Fusco T., Michau V., 2008, MNRAS, 387, 173
Wizinowich P. et al., 2006, PASP, 118, 297

This paper has been typeset from a \TeX/\LaTeX file prepared by the author.