Clock transition by continuous dynamical decoupling of a three-level system

Alexander Stark1,2, Nati Aharon3, Alexander Huck2, Haitham A. R. El-Ella1, Alex Retzker3,
Fedor Jelezko2,4 & Ulrik L. Andersen1

We present a novel continuous dynamical decoupling scheme for the construction of a robust qubit in a three-level system. By means of a clock transition adjustment, we first show how robustness to environmental noise is achieved, while eliminating drive-noise, to first-order. We demonstrate this scheme with the spin sub-levels of the NV-centre’s electronic ground state. By applying drive fields with moderate Rabi frequencies, the drive noise is eliminated and an improvement of 2 orders of magnitude in the coherence time is obtained compared to the pure dephasing time. We then show how the clock transition adjustment can be tuned to eliminate also the second-order effect of the environmental noise with moderate drive fields. A further detailed theoretical investigation suggests an additional improvement of more than 1 order of magnitude in the coherence time which is supported by simulations. Hence, our scheme predicts that the coherence time may be prolonged towards the lifetime-limit using a relatively simple experimental setup.

The reliable and efficient construction and manipulation of qubits is necessary for the implementation of quantum technological applications and quantum information processing. In solid-state and atomic systems, ambient magnetic field fluctuations constitute a serious impediment, which usually limits the coherence time to a small fraction of the inherent lifetime. Pulsed dynamical decoupling1–3 has proven to be very useful in prolonging the coherence time4–12. However, in order to mitigate both environmental and controller noise, composite high-frequency pulse sequences must usually be applied13–17 which require large field strengths18. Continuous dynamical decoupling18–27 offers another possibility of suppressing environmental noise, where diminishing the effect of the controller noise can be achieved by different approaches. In this context, a rotary echo scheme28,29 can be viewed as analogous to pulsed dynamical decoupling. The concatenation of several on-resonance driving fields30–34 is another concept, but inherently connected to a reduction of the dressed energy gap, eventually limiting the performance of the scheme, and in particular reducing the qubit gate operation time.

Multi-state systems allow for yet a different approach. By applying continuous driving fields on a multi-level structure, a fully robust qubit - a qubit that is robust to both external and controller noise - can be obtained35,36. However, with these multi-state schemes, which utilise on-resonance driving fields, it is not possible to achieve robustness against drive noise in a three-level system36. A protected qubit subspace within a three-level configuration can be realised by the application of off-resonant strong driving fields37 making the experimental realisation challenging.

In this report we show how a fully robust qubit can be simply constructed by means of a clock transition adjustment27 using a three-level system. We start with a basic version of our scheme where both continuous on-resonant and off-resonant driving fields are utilised. The on-resonant driving fields result in robustness to environmental noise, whereas the off-resonant driving fields facilitate robustness against drive noise, which typically limits continuous dynamical decoupling schemes. Similar to clock states, which possess a transition that is insensitive to first-order magnetic shifts for a given magnetic field value, the off-resonant driving fields generate a transition that is insensitive to first-order shifts in the drive-field amplitudes. We demonstrate this scheme by

11. Center for Macroscopic Quantum States (bigQ), Department of Physics, Technical University of Denmark, Kgs. Lyngby, 2800, Denmark. 2Institute for Quantum Optics, Ulm University, Albert-Einstein-Allee 11, Ulm, 89081 Germany. 3Racah Institute of Physics, The Hebrew University of Jerusalem, Jerusalem, 91904 Israel. 4Center for Integrated Quantum Science and Technology (IQst), Ulm University, Ulm, 89081 Germany. Alexander Stark and Nati Aharon contributed equally. Correspondence and requests for materials should be addressed to A.S. (email: astark@fysik.dtu.dk)
utilising the ground state spin level of the nitrogen-vacancy centre (NV) in diamond. The states are addressed by a combination of four microwave fields, adjusted to the same Rabi frequency, \( \Omega\), at different transition frequencies, \( \omega_1\) and \( \omega_2\), and with a detuning, \( \Delta\), respectively. All experiments presented here were realized using Qudt Software Suite\(^{38}\). The spin states of the NV centre are initialised and read out by a 532 nm laser identifying spin dependent fluorescence\(^{39–42}\). We are operating in the vicinity of the excited state level anti-crossing\(^{43,44}\) at a magnetic field of 35.8 mT. At this bias field the intrinsic nitrogen nuclear spin becomes polarised through optical pumping and does not contribute to the level structure (more information concerning the setup configuration and the system parameters can be found in Supplementary Information S1 and S2). Our implementation demonstrates that, to first order, drive-noise is eliminated, and compared to the pure dephasing time an improvement of 2 orders of magnitude in the coherence time is obtained even for a moderate drive field strength. Finally, we present an improved version of the scheme, where the clock transition adjustment is extended to also eliminate the second-order effect of the environmental noise. Supported by simulations, our analysis shows that with a moderate driving field strength a further improvement of more than 1 order of magnitude in the coherence time can be obtained. Hence, our scheme allows prolonging the coherence time towards the lifetime limit in a simple experimental setup and without requiring exceptionally strong drive fields. The protocol is applicable to both the optical and microwave domain, and hence to a variety of atomic and solid state systems, such as trapped ions, rare-earth ions, and defect centres.

The Basic Scheme

We consider a three-level system with states \(|0\rangle\) and \(|\pm 1\rangle\), where the \(|\pm 1\rangle\) states are dipole coupled to the \(|0\rangle\) state, as illustrated in Fig. 1a. The energy gaps (\(\hbar = 1\)) between the \(|0\rangle\) state and the \(|\pm 1\rangle\) states are \(\omega_1\) and \(\omega_2\), respectively. Our scheme utilises four driving fields which can be expressed by the driving Hamiltonian

\[
H = 2\Omega_2 \left( \cos(\omega_1 t)|0\rangle\langle 1| - \cos(\omega_2 t)|1\rangle\langle 0| + \text{h.c.} \right) \\
+ 2\Omega_1 \left( \cos(\omega_1 + \Delta) t)|0\rangle\langle 1| + \cos(\omega_2 + \Delta) t)|1\rangle\langle 0| + \text{h.c.} \right).
\]

(1)

Moving to the interaction picture (IP) with respect to \(H_0 = \omega_1 |1\rangle\langle 1| - \omega_2 |0\rangle\langle 0|\), changing the basis to \(|0\rangle, |B\rangle, |D\rangle\), with \(|B\rangle = |1\rangle + | -1\rangle\rangle\sqrt{2} \) and \(|D\rangle = |1\rangle - | -1\rangle\rangle\sqrt{2} \), and applying the rotating-wave approximation, we obtain the Hamiltonian

\[
H_I = \sqrt{2}\Omega_2 (|0\rangle\langle D| + |D\rangle\langle 0|) \\
+ \sqrt{2}\Omega_1 (|B\rangle|e^{i\Delta t} + |B\rangle|e^{-i\Delta t}).
\]

(2)

which is illustrated by a level scheme depicted in Fig. 2a. The states \(|0\rangle\) and \(|D\rangle\) are on-resonantly coupled by a single lambda drive with a strength \(\Omega_d = \sqrt{2}\Omega_1\). Under the assumption of \(\Delta > \Omega_2\), an off-resonant coupling between \(|0\rangle\) and \(|B\rangle\) by \(\Omega_B = \sqrt{2}\Omega_1\) is obtained.

The drive, \(\Omega_0\), in Fig. 2a transforms into the dressed states \(|u\rangle, |B\rangle, |d\rangle\), as schematically illustrated in Fig. 2b, with the corresponding eigenvalues \(\{ \pm \Omega_0, 0, -\Omega_0 \}\), respectively. Here, we introduced the states \(|u\rangle = (|0\rangle + |D\rangle)\rangle\sqrt{2} \) and \(|d\rangle = (|0\rangle - |D\rangle)\rangle\sqrt{2} \).

For a strong enough drive, \(\Omega_0\), robustness to magnetic noise is obtained. As the magnetic noise, \(\delta B\), couples between the \(|B\rangle\) state and the \(|u\rangle\) and \(|d\rangle\) states, we can set \(\Omega_0\) such that the power spectrum of the noise, \(S_{B}(\Omega_0)\), is much smaller than \(1/T_1\), where \(T_1\) is the system lifetime. This condition ensures that the first order effect of the magnetic noise is negligible.

The coherence time of the dressed states is then mainly limited by driving amplitude fluctuations, \(\Omega \rightarrow \Omega (1 + \delta(t))\), where \(\delta(t)\) represents a random noise contribution. To additionally obtain robustness to
Figure 2. Bare, dressed and doubly-dressed states in the double lambda drive. Drive fields are indicated by solid lines, whereas energy separations are marked with dashed lines. (a) Interaction picture representation of Eq. (1) of the bare states in the basis of the drive, $\Omega_1 = \Omega_2 = \Omega$. A simultaneous bright ($\ket{B}$) and dark ($\ket{D}$) state drive can be realized by a phase shift of $-\pi$ in the on-resonant lambda scheme in Fig. 1a. (b) Dressed state picture, where the on-resonant drive is incorporated into the level description. The detuning transforms in this picture to $\Delta_1 = \Delta + \Omega_1$ and $\Delta_2 = \Delta - \Omega_1$. (c) The final doubly-dressed states with incorporated off-resonant drives, where the robust qubit energy gap, $\Omega_{RQ} = \Omega_{B}, \Omega_{D}$, becomes apparent.

Increasing the strength of the driving fields reduces the second-order effect of the magnetic noise but introduces an increased second-order effect of the drive noise, where the robust qubit energy gap, $\Omega_{RQ} = \Omega_{B}, \Omega_{D}$, becomes apparent.

| $\ket{B}$ | $\ket{D}$ |
|----------|-----------|
| $\Omega_B$ | $2\Omega_D$ |
| $\Delta_1$ | $\Delta_2$ |
| $\ket{0}$ | $\ket{0}$ |

The driven states are

$$\ket{B} = \frac{|+1⟩ + |−1⟩}{\sqrt{2}}$$

$$\ket{D} = \frac{|+1⟩ - |−1⟩}{\sqrt{2}}$$

drive-field fluctuations, we consider the effect of the second detuned drive, $\Omega_D$, on the dressed states. We therefore move to the basis of the dressed states and to the IP with respect to $H_{IP} = \Delta \ket{B}\bra{B}$, and obtain

$$H_{IP} = \Omega_B (|u⟩\langle u| - |d⟩\langle d|) - \Delta \ket{B}\bra{B}$$

$$+ \frac{\Omega_D}{\sqrt{2}} (|B⟩\langle u| + |D⟩\langle d| + \text{h.c.}).$$

The eigenstates of $H_{IP}$, denoted by $\{|u⟩, |B⟩, |D⟩\}$, are termed as the doubly-dressed states and their relative level scheme is illustrated in Fig. 2c.

The effect of drive fluctuations can be introduced in $H_{IP}$ by replacing $\Omega_B$ with $\Omega_B(1 + \delta)$. As both driving fields originate from the same source, we can assume that the noise is mostly correlated. Thus, for each set of eigenstates, $|k⟩$ and $|j⟩$, we define the driving coherence time as $T_2^{\text{IP}} = \frac{1}{2} |\epsilon_k^2 - \epsilon_j^2|$, where $\epsilon_k^1$ is the first order term in $\delta_k$ and $\delta_j$ of the eigenvalue expansion of $|k⟩$. For a given driving noise configuration, i.e., for a given relation between $\delta_k$ and $\delta_j$, the driving parameters $\Omega_1$, $\Omega_2$, and $\Delta$ can be chosen such that the driving coherence time of the two negative eigenvalues of $H_{IP}$ is $T_2^{\text{IP}} \gg T_r$. In this case $\epsilon_k^1 \approx \epsilon_j^1$ (see Supplementary Mathematica notebook), which means that the transition frequency of the robust qubit is insensitive to first-order driving fluctuations. Hence, with moderate driving fields, the coherence time of this doubly-dressed qubit is mainly limited by the second-order effect of the magnetic noise $\sim \delta B^2 / 2\mu_0$. Increasing the strength of the driving fields reduces the second-order effect of the magnetic noise but introduces an increased second-order effect of the drive noise, $\sim \delta T_2^{\text{IP}}$. For more details and the explicit forms of the leading terms of the magnetic and Rabi frequency noise see the Supplementary Mathematica notebook.

We consider a NV centre electron spin with a pure dephasing time of $T_2^* \approx 21\mu s$ resulting from magnetic noise described by an Ornstein-Uhlenbeck random process with a correlation time of $\tau_c \approx 15\mu s$. In this case, with $\Omega_1 = \Omega_2 = 2\pi \cdot 6$ MHz, a coherence time of $\approx 1$ ms is obtained (see Fig. 3), which is limited by the second-order effect of both magnetic and drive noise. Further increase of the drive strength, $\Omega$, would allow for a further improvement in the coherence time up to the point where the second-order drive noise is too strong and begins to dominate, which in our case is at $\Omega_1 = \Omega_2 \approx 2\pi \cdot 10$ MHz (see inset of Fig. 3).

**Experimental Results**

The experimental implementation of the proposed scheme follows the protocol illustrated in Fig. 1b. Four drive fields are applied on the bare basis states $\{|−1⟩, |0⟩, |1⟩\}$ of the NV centre (see Fig. 1a). The field amplitudes of the on-resonant drives are adjusted to yield an identical Rabi frequency, $\Omega$, for both transitions, $\omega_1$ and $\omega_2$ (see Suppl. B). The off-resonant drives are obtained by adding a detuning, $\Delta$, to the resonant drives, resulting in the total field

$$\Omega_{tot}(t) = \Omega \cdot \cos(\omega_0 t) - \cos(\omega_0 t)$$

$$+ \cos(\omega_1 + \Delta) t + \cos(\omega_2 + \Delta) t$$

(4)

synthesised by a signal generator ($\Omega_1 = \Omega_2 = \Omega = \Omega_2 \cdot 2.27$ MHz) and thereby implementing Eq. (1).

The interaction with the field couples the drive, $\Omega$, to the bare spin states (see Fig. 1a). On-resonant drives induce Rabi oscillations at a rate $\Omega_{D}$, and a positive (negative) detuning results in ‘red’ (‘blue’) detuned AC-Stark shifted energy levels of the $|d⟩$ and $|B⟩$, ($|u⟩$ and $|D⟩$) states, as shown in Fig. 2b. In combination, these drives create the doubly-dressed states, which are depicted in Fig. 2c. The appearing energy levels of the doubly-dressed states, $|B⟩$, $|D⟩$, and $|\tilde{B}⟩$, are eventually all coupled to the drive fields (see Fig. 2).
By adjusting the detuning, $\Delta$, a configuration can be obtained, in which two states (either $|\uparrow\rangle \leftrightarrow |\uparrow\rangle$ or $|\downarrow\rangle \leftrightarrow |\downarrow\rangle$) experience the same drive noise, $\delta \Omega$. This eliminates the energy gap fluctuations (due to $\delta \Omega$) between the two considered states and reflects the robustness of the qubit against drive strength fluctuations, $\delta \Omega$. The large energy gap in the dressed states, $\Omega_{\text{D}}$, which originates from the on-resonant drives, ensures a sufficient decoupling from external magnetic noise contributions, $\delta B$, as it also increases the energy gap of the robust qubit, $\Omega_{\text{RQ}}$ (see Fig. 2c).

To determine the performance of the scheme the detuning-dependent coherence times of the protected states have to be recorded, yielding the optimal AC-Stark shifted energy levels that are least sensitive to drive fluctuations. The measurement is performed in the dressed-state basis and is analogous to a free induction decay (FID) or Ramsey measurement (see Fig. 1b). Here, by the application of an on-resonant $\pi/2$ pulse with $\Omega_{\text{res}}$, a superposition is created between the $|\uparrow\rangle$ and $|\downarrow\rangle$ states (which is also a superposition of the $|\uparrow\rangle$ and $|\downarrow\rangle$ states, but with a different initial phase factor in the $|\downarrow\rangle$ state). In the next step, the double lambda drive (depicted in Fig. 1a) is applied on the superposition states as a function of interaction time $\tau$, revealing the present energy gaps (in Fig. 2c) as a coherent evolution. By mapping the coherences to populations with a consecutive repeat signal, $S(\tau)$, contains frequency components proportional to the energy gaps of the doubly-dressed states (between the states in Fig. 2c). The robust state is identified by the longest measured coherence time in Fig. 4 as energy fluctuations in the robust state are significantly suppressed while the oscillations induced by the other energy gaps decay quickly.

The measured coherence times, $T_2^{(\{u,d\})}$, are extracted by fitting a sinusoidal exponential decay to $S(\tau)$ and plotted as a function of detuning, $\Delta$, in Fig. 4 (for a detailed explanation of the procedure see Supplementary Information S3). The asymmetric shape of the curve provides insights about the appearing dynamics. Starting from external magnetic noise contributions, $\delta B$, where first-order drive noise of the system is eliminated. By including magnetic noise (solid orange line) with a pure dephasing time of $T_2^{\uparrow\downarrow}=2\mu$s, the coherence time reaches a limit, which is set by the second-order effects of the magnetic and drive noise, and constitutes the maximum improvement of the scheme ($T_2^{\uparrow\downarrow}=6\mu$s). Since the coherence time as function of $\Delta$ has a Lorentzian shape, the FWHM of the peak is $\approx 1/2T_2^{\uparrow\downarrow}$. Inset: Plot of the coherence time obtained for different drive fields, $\Omega = \Omega_1 = \Omega_2$. For increased drive strengths the second-order magnetic noise is reduced but the second-order drive noise is increased. Hence, the coherence time is improved up to $T_2^{\uparrow\downarrow}=1.2$ ms for $\Omega_1 = \Omega_2=2\pi\cdot10$ MHz.

Figure 3. Expected coherence times for a drive $\Omega_1 = \Omega_2=2\pi\cdot6$ MHz and $\delta_1 = \delta_2 = 0.005$. Under the assumption of no magnetic noise, the blue dotted curve predicts the position of optimal detuning, $\Delta/2\pi\approx 19.35$ MHz, where first-order drive noise of the system is eliminated. By including magnetic noise (solid orange line) with a pure dephasing time of $T_2^{\uparrow\downarrow}=2\mu$s, the coherence time reaches a limit, which is set by the second-order effects of the magnetic and drive noise, and constitutes the maximum improvement of the scheme ($T_2^{\uparrow\downarrow}=6\mu$s). Since the coherence time as function of $\Delta$ has a Lorentzian shape, the FWHM of the peak is $\approx 1/2T_2^{\uparrow\downarrow}$. Inset: Plot of the coherence time obtained for different drive fields, $\Omega = \Omega_1 = \Omega_2$. For increased drive strengths the second-order magnetic noise is reduced but the second-order drive noise is increased. Hence, the coherence time is improved up to $T_2^{\uparrow\downarrow}=1.2$ ms for $\Omega_1 = \Omega_2=2\pi\cdot10$ MHz.
The measured free induction decay time, $T_2^* = (1.78 \pm 24) \mu s$, of the bare states sets the theoretical limited for the coherence time of $\sim 190 \mu s$ when the drive noise is eliminated. By selecting a four times higher noise, $\delta \Omega_B \approx 4 \delta \Omega_0$, the theoretical dependence of the coherence time on the detuning plotted in Fig. 4 is obtained. The analytical function of the coherence time is given by $T_2^*[(\delta)] = \sqrt{2}/(\sqrt{2}/(\sqrt{2}/(90)) \mu s$. In addition, simulations with magnetic and driving noise models were performed for several detuning values, which reproduce the experimental results very well. These simulation results are presented in Fig. 4 (for more details on the simulations see Supplementary Information S6). In the following, we clarify how quantitatively the noise parameters are grasped in the experiment.

It appears that the DC and AC components of the magnetic noise, $\delta B$, have equal contributions to both transitions, $\omega_2$ and $\omega_2$, as we observe (within the error bar) the same values for $T_2$ and $T_2 = (215 \pm 31) \mu s$ (see Supplementary Information S2C and D). However, by comparing the coherence time of the Rabi drives, we obtain a drastic difference, $T_2^*[(\delta)] = (62 \pm 12) \mu s$ and $T_2^*[(\delta)] = (159 \pm 24) \mu s$, which hints at a drive frequency dependent noise spectrum. As all the fields are produced by the same signal generator, it is valid to consider correlations in the noise. The combination of both effects can truly cause the noise imbalance between $\delta \Omega_B$ and $\delta \Omega_0$, which directly affects the position of the peak with respect to the detuning, $\Delta$. As this is a setup specific setting, the AC-Stark shifts have to be adjusted to compensate for this value. However, the coherence time improving effect, as theoretically predicted, is expected to be within the range of $50 \text{ MHz}$ at the utilised drive field, $\Omega$, as larger detunings have a negligible energy shift on the states. A further and more detailed investigation of the drive noise is presented in Supplementary Informations S4 and S5.

Improved Scheme

So far, our scheme shows how to eliminate the first-order effect of the drive fluctuations, $\delta \Omega$, where for moderate drive fields the coherence time is mainly limited by the second-order effect of the magnetic noise $\sim \delta B^2/\Omega$. However, the second-order effect of the magnetic noise can be suppressed in a similar way as demonstrated for the elimination of the first-order drive fluctuations, $\delta \Omega$.

To see this, we consider the on-resonant drive ($\Omega_1$ in Fig. 2). For the dressed states, the second-order effect of the magnetic noise is given by $\sim \delta B^2/\Omega_1 (|u\rangle - |d\rangle) (|u\rangle - |d\rangle)$, which describes the fluctuation of the robust energy gap (between $|B\rangle$ and $|d\rangle$) with $-\delta B^2/\Omega_1$. By introducing a one-photon detuning, $\Delta_0$, which denotes a detuning of the coupling between $|D\rangle$ and $|0\rangle$, the symmetry is broken. In this case the second-order effect of the magnetic noise is given by $\sim \delta B^2/\Omega_1 (|u\rangle - |d\rangle) (|u\rangle - |d\rangle) - \delta B (|B\rangle)$. By adjusting the one-photon detuning, $\Delta_0$, we can set $b = c$, and achieve a clock transition that is insensitive to magnetic field fluctuations, $\delta B$ (up to second-order).

We now combine this idea with the presented elimination of the first-order drive fluctuations, $\delta \Omega$, to obtain a true clock transition. Including the one-photon detuning, $\Delta_0$ in the driving fields of both $\Lambda$ systems and magnetic noise, which is given by $\delta B_x = \delta B (|1\rangle + |+\rangle - |1\rangle - |1\rangle)$, Eq. (3) results in

$$H'_{\mu} = \Omega_0 (|u\rangle\langle u| - |d\rangle\langle d|) - \Delta |B\rangle\langle B| - \Delta_0 |0\rangle\langle 0| + \frac{\Omega_0}{\sqrt{2}} (|B\rangle\langle u| + |B\rangle\langle d|) + \text{h.c.}) + \frac{\delta B}{\sqrt{2}} (e^{i\Delta_0} |B\rangle\langle u| + |B\rangle\langle d|) + \text{h.c.},$$

Equation (5)
and the IP is now obtained with respect to $H'_{02} = \Delta|B\rangle\langle B| + \Delta_0|0\rangle\langle 0|$. We continue by moving to the basis of the eigenstates of the drives (the double dressed states)

$$H''_{II} \approx E_a|\tilde{u}\rangle\langle \tilde{u}| + E_B|\tilde{B}\rangle\langle \tilde{B}| + E_d|\tilde{d}\rangle\langle \tilde{d}|$$
$$+ \frac{\Delta B}{\sqrt{2}} (c^{\Delta}(\alpha|\tilde{B}\rangle\langle \tilde{u}| + \beta|\tilde{B}\rangle\langle \tilde{d}|) + h.c.),$$

(6)

where $E_i$ are the eigenvalues, and $\alpha$ and $\beta$ are real coefficients. The drive noise is treated as before, where we require $\epsilon^{\tilde{B}}_i \approx \epsilon^{\tilde{d}}_i$. This gives us one constraint on $\Delta$ and $\Delta_0$. The second constraint comes from the elimination of the second-order effect of the magnetic noise. Moving to the IP with respect to $H'_{03} = -\Delta|B\rangle\langle B|$ we obtain the time independent Hamiltonian

$$H''_{III} \approx E_a|\tilde{u}\rangle\langle \tilde{u}| + (E_B + \Delta)|\tilde{B}\rangle\langle \tilde{B}| + E_d|\tilde{d}\rangle\langle \tilde{d}|$$
$$+ \frac{\Delta B}{\sqrt{2}} (c^{\Delta}(\alpha|\tilde{B}\rangle\langle \tilde{u}| + \beta|\tilde{B}\rangle\langle \tilde{d}|) + h.c.),$$

(7)

This enables the calculation of the second-order contribution of the magnetic noise to the eigenvalues, which are the $b$ and $c$ coefficients, as a function of $\Delta$ and $\Delta_0$. The two constraints, $\epsilon^{\tilde{B}}_i \approx \epsilon^{\tilde{d}}_i$ and $b \approx c$, allow us to determine the optimal values of $\Delta$ and $\Delta_0$. For more details and the explicit forms of the leading terms of the magnetic and Rabi frequency noise, see Supplementary Mathematica notebook.

For the considered drive noise, $\delta_1 = \delta_2 = 0.005$, in Fig. 3, and for a moderate drive of $\Omega_1 = \Omega_2 = 2\pi \cdot 2$ MHz, the optimal detunings would be obtained by $\Delta = 2\pi \cdot 8.9956$ MHz and $\Delta_0 = 2\pi \cdot 1.7386$ MHz. We simulated the present driving configuration under the effect of the same magnetic noise model considered for the simulations of the experiment in Fig. 4 (with $T_2^* \approx 2$ µs). The results of the simulation are shown in the inset of Fig. 5 indicating an improvement of more than 1 order of magnitude in the coherence time compared to the original scheme. There are two limiting factors on the coherence time. The first factor are the higher-order terms of the noise; the second-order term of the driving noise and the fourth order term of the magnetic noise. With the parameters considered in the simulation these terms result in a limit of ~10 ms on the coherence time. The second factor is the amplitude mixing between the eigenstates due to fast rotating terms, which introduces first order driving noise. In our case the mixing is ~0.2%, which means that the coherence time is limited to 500 times the driving noise limited coherence time (~20 µs here), and hence the limit on the coherence time is ~10 ms. Therefore, taking both factors into account we conclude that the coherence time is limited to ~5.3 ms, which is in agreement with the simulation results. For low drive field strengths, the drive noise contributions (second-order terms and amplitude mixing) are small and the coherence time is mainly limited by the fourth-order magnetic noise. For higher drive fields the fourth-order magnetic noise becomes negligible and the coherence time is mainly limited by the drive noise contributions. The estimated maximal coherence time for different drive field strengths is shown in Fig. 5. The amplitude mixing can also be decreased by increasing the Zeeman splitting. Given the noise parameters, one can optimise the driving parameters with respect to these factors and obtain the optimal coherence time.
Conclusion
In this work we presented and experimentally demonstrated a new scheme for the creation of a robust qubit in a three-level system by means of a clock transition adjustment. The basic scheme is based on the application of continuous resonant and off-resonant drive fields. The resonant drive fields provide robustness to environmental noise, whereas the off-resonant drive fields eliminate the first-order effect of the drive noise by tuning a clock like transition that is insensitive to first-order shifts of the drive-field amplitudes. For the case of the NV centre in diamond, we achieved an improvement of ~2 orders of magnitude in the coherence time compared to the pure dephasing time while utilising moderate drive fields. In the optimal version of the scheme, the clock transition adjustment is extended to also eliminate the second-order effect of the environmental noise without necessitating strong drive fields. Hence, our scheme proposes the possibility to prolong the coherence time towards the lifetime limit using a relatively simple experimental setup and without requiring extremely strong drive fields. However, further measurements are encouraged to verify the performance of the improved scheme.

This scheme facilitates the sensing of AC magnetic fields, and in particular, high frequency fields in the GHz regime, where the sensitivity would be solely limited by the coherence time of the robust qubit. While this work has focused solely on the NV centre, we believe that this scheme is applicable to a variety of atomic and solid-state systems with optical or microwave transitions, such as trapped ions, rare-earth ions, and other defect centres. We therefore believe that the scheme has potential applications in a wide range of tasks in the fields of quantum information science and technology, and in particular quantum sensing.

Data Availability
The authors declare that all relevant data supporting the findings of this study are available within the paper (and its Supplementary information file). Any raw data can be obtained from the corresponding authors on reasonable request.

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**Author Contributions**

N.A. and A.R. conceived the idea and developed the theory. A.A., A.H., F.I. and U.L.A. designed and supervised the project. A.S. performed, planned and developed the concept of the experiment with support from H.A.R.E.-E. and A.H. A.S. and N.A. analysed the data. N.A. planned and carried out the simulations. A.S. and N.A. took the lead in writing the manuscript with support from A.H.. All authors contributed to the interpretation of the results, provided critical feedback and helped to shape the research, analysis and manuscript.

**Additional Information**

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