Clustering, Angular Size and Dark Energy

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The influence of dark matter inhomogeneities on the angular size-redshift test is investigated for a large class of flat cosmological models driven by dark energy plus a cold dark matter component (XCDM). The results are presented in two steps. First, the mass inhomogeneities are modeled by a generalized Zeldovich-Kantowski-Dyer-Roeder (ZKDR) distance which is characterized by a smoothness parameter $\alpha(z)$ and a power index $\gamma$, and, second, we provide a statistical analysis to angular size data for a large sample of milliarcsecond compact radio sources. As a general result, we have found that the $\alpha$ parameter is totally unconstrained by this sample of angular diameter data.

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I. INTRODUCTION

An impressive convergence of recent astronomical observations are suggesting that our world behaves like a spatially flat scenario dominated by cold dark matter (CDM) plus an exotic component endowed with large negative pressure, usually named dark energy [4, 2, 3]. In the framework of general relativity, besides the cosmological constant, there are several candidates for dark energy, among them: a vacuum decaying energy density, or a time varying $\Lambda$ (energy density), a relic scalar field [6], and a Chaplygin Gas [7]. Some recent review articles discussing the history, interpretations, as well as, the major difficulties of such candidates have also been published in the last few years [5].

In the case of X-matter, for instance, the dark energy component is simply described by an equation of state $p_x = \omega_x \rho_x$. The case $\omega = -1$ reduces to the cosmological constant, and together the CDM defines the scenario usually referred to as “cosmic concordance model” (LCDM). The imposition $\omega \geq -1$ is physically motivated by the classical fluid description [8]. However, as discussed by several authors, such an imposition introduces a strong bias in the parameter determination from observational data. In order to take into account this difficulty, superquintessence or phantom dark energy cosmologies have been recently considered where such a condition is relaxed [10]. In contrast to the usual quintessence model, a decoupled phantom component presents an anomalous evolutionary behavior. For instance, the existence of future curvature singularities, a growth of the energy density with the expansion, or even the possibility of a rip-off of the structure of matter at all scales are theoretically expected [11] for a thermodynamic discussion. Although possessing such strange features, the phantom behavior is theoretically allowed by some kinetically scalar field driven cosmology [12], as well as, by brane world models [13], and, perhaps, more important to the present work, a PhantomCDM cosmology provides a better fit to type Ia Supernovae observations than does the $\Lambda$CDM model [14]. Many others observational and theoretical properties phantom driven cosmologies (more generally, of XCDM scenarios) have been successfully confronted to standard results (see, for instance 15, 16, 17, 18, 19).

In this context, one of the most important tasks for cosmologists nowadays is to confront different cosmological scenarios driven by cold dark matter (CDM) plus a given dark energy candidate with the available observational data. As widely known, a key quantity for some cosmological tests is the angular distance-redshift relation, $D_A(z)$, which for a homogeneous and isotropic background, can readily be derived by using the Einstein field equations for the Friedmann-Robertson-Walker (FRW) geometry. From $D_A(z)$ one obtains the expression for the angular diameter $\theta(z)$ which can be compared with the available data for different samples of astronomical objects [20].

Nevertheless, the real Universe is not perfectly homogeneous, with light beams experiencing mass inhomogeneities along their way. Actually, from small to intermediate scales ($\lesssim 100$Mpc), there is a lot of structure in form of voids, clumps and clusters which is probed by the propagating light [21]. Since the perturbed metric is unknown, an interesting possibility to account for such an effect is to introduce the smoothness parameter $\alpha$ which is a phenomenological representation of the magnification effects experienced by the light beam. From general grounds, one expects a redshift dependence of $\alpha$ since the degree of smoothness for the pressureless matter is supposed to be a time varying quantity [17, 18]. When $\alpha = 1$ (filled beam), the homogeneous FRW case is fully recovered; $\alpha < 1$ stands for a defocusing effect while $\alpha = 0$ represents a totally clumped universe (empty beam). The distance relation that takes these mass inhomogeneities into account was discussed by Zeldovich [22] followed by Kantowski [23], although a clear-cut application for cosmology was given only in 1972 by Dyer and Roeder [24]. Later on, by considering a perturbed Friedmannian model Tomita [25] performed N-body simulations with the CDM spectrum in order to determine the distribution for $\alpha$ (see also Ref. [26] for a more gen-
eral references may also be found in the textbook by Schneider, Ehlers and Falco, as well as, in Kantowski. Many studies involving the ZKDR distances in dark energy models have been published in the literature. Analytical expressions for a general background in the empty beam approximation ($\alpha = 0$) were derived by Sereno et al. [31]. By assuming that both dominant components may be clustered they also discussed how the critical redshift, i.e., the value of $z$ for which $D_A(z)$ is a maximum (or $\Theta(z)$ minimum), and compared to the homogeneous background results as given by Lima and Alcaniz [32], and, further discussed by Lewis and Ibata [33], and Araújo and Stoeger [34]. More recently, Demianski et al. [35], derived an useful analytical approximate solution for a clumped concordance model (ACDM) valid on the interval $0 \leq z \leq 10$. Additional studies on this subject involving time delay (Lewis and Ibata [33]; Giovi and Amendola [36]; gravitational lensing (Kochanek; Kochanek and Schechter [37]) or even accelerated models driven by particle creation have also been considered [38, 39].

Although carefully investigated in many of their theoretical and observational aspects, an overview in the literature shows that a quantitative analysis on the influence of dark energy in connection with inhomogeneities present in the observed universe still remains to be studied. Analytical expression for a general applied for the $\theta(z)$ statistics with basis on a $\Lambda$CDM cosmology with constant $\alpha$ [40]. It was concluded that the best fit model occurs at $\Omega_M = 0.2$ and $\alpha = 0.8$ whether the characteristic angular size $l$ of the compact radio sources is marginalized. More recently, the smoothness $\alpha$ parameter was constrained through a statistical analysis involving Supernovae Ia data [41]. A $\chi^2$ analysis based on the 182 SNe Ia data of Riess et al. [2] constrained the pair of parameters ($\Omega_M, \alpha$) to be $\Omega_M = 0.33^{+0.09}_{-0.07}$ and $\alpha \geq 0.42$ ($2\sigma$). Such an analysis has also been carried out in the framework of a $\Lambda$CDM cosmology.

In this paper, we focus our attention on X-matter cosmologies with special emphasis to phantom models ($\omega < -1$) by taking into account the presence of a clustered cold dark matter. The mass inhomogeneities will be described by the ZKDR distance characterized by a smoothness parameter $\alpha(z)$ which depends on a positive power index $\gamma$. The main objective is to provide a statistical analysis to angular size data from a large sample of milliarcsecond compact radio sources [42] distributed over a wide range of redshifts ($0.011 \leq z \leq 4.72$) whose distance is defined by the ZKDR equation. As an extra bonus, it will be shown that a pure CDM model ($\Omega_M = 1$) is not compatible with these data even for the empty beam approximation ($\alpha = 0$).

The manuscript is organized as follows. In section 2 we outline the derivation of the ZKDR equation for a X-CDM cosmology. We also provide some arguments (see Appendix) for a locally nonhomogeneous Universe where the homogeneous contribution of the dark matter obeys the relation $\rho_h = \alpha \rho_o (\rho_M / \rho_o)^{\gamma}$ where $\gamma$ is a positive number, $\rho_M$ is the average matter density and $\rho_o$ its present value. In section 3 we analyze the constraints on the free parameters $\alpha$ and $\Omega_M$ from angular size data. We end the paper by summarizing the main results in section 4.

II. THE EXTENDED ZKDR EQUATION

Let us now consider a flat FRW geometry ($c = 1$)

$$ds^2 = dt^2 - R^2(t) \left( dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right), \quad (1)$$

where $R(t)$ is the scale factor. Such a spacetime is supported by the pressureless CDM fluid plus a X-matter component of densities $\rho_M$ and $\rho_x$, respectively. Hence, the total energy momentum tensor, $T^{\mu\nu} = T^{\mu\nu}(M) + T^{\mu\nu}(x)$, can be written as

$$T^{\mu\nu} = [\rho_M + (1 + \omega)\rho_x] U^\mu U^\nu - \omega \rho_x g^{\mu\nu}, \quad (2)$$

where $U^\mu = \delta^\mu_t$ is the hydrodynamics 4-velocity of the comoving volume elements. In this framework, the independent components of the Einstein Field Equations (EFE)

$$G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 8\pi G T^{\mu\nu}, \quad (3)$$

take the following forms:

$$\left( \frac{\dot{R}}{R} \right)^2 = H_0^2 \left[ \Omega_M \left( \frac{R_0}{R} \right)^3 + \Omega_x \left( \frac{R_0}{R} \right)^{3(1+\omega)} \right]. \quad (4)$$

$$\frac{\dot{R}}{R} = -\frac{1}{2} H_0^2 \left[ \Omega_M \left( \frac{R_0}{R} \right)^3 + (3\omega + 1)\Omega_x \left( \frac{R_0}{R} \right)^{3(1+\omega)} \right], \quad (5)$$

where an overdot denotes derivative with respect to time and $H_0 = 100h$Kms$^{-1}$Mpc$^{-1}$ is the Hubble parameter. By the flat condition, $\Omega_x = 1 - \Omega_M$, is the present day dark energy density parameter. As one may check from (2)-(5), the case $\omega = -1$ describes effectively the favored “cosmic concordance model” (ACDM).

On the other hand, in the framework of a conformally flat FRW metric, the optical scalar equation in the geometric optics approximation reads (Optical shear neglected) [43]

$$\sqrt{A'' + \frac{1}{2} R_{\mu\nu} k^\mu k^\nu \sqrt{A}} = 0, \quad (6)$$

where $A$ is the beam cross sectional area, plicas means derivative with respect to the affine parameter describing the null geodesics, and $k^\mu$ is a 4-vector tangent to the photon trajectory whose divergence determines the optical scalar expansion [17, 31, 32]. The circular frequency of the light ray as seen by the observer with 4-velocity
$U^a$ is $\omega = U^a k_\alpha$, while the angular diameter distance, $D_A$, is proportional to $\sqrt{A}$ [22].

As widely known, there is no an acceptable averaging procedure for smoothing out local inhomogeneities. After Dyer and Roeder [24], it is usual to introduce a phenomenological parameter, $\alpha(z) = 1 - \frac{\rho_{id}}{\rho_{mz}}$, called the “smoothness” parameter. For each value of $z$, such a parameter quantifies the portion of matter in clumps ($\rho_{id}$) relative to the amount of background matter which is uniformly distributed ($\rho_{mz}$). As a matter of fact, such authors examined only the case for constant $\alpha$, however, the basic consequence of the structure formation process is that it must be a function of the redshift. Combining equations (2), (3) and (6), after a straightforward but lengthy algebra one finds that the angular diameter distance, $D_A(z)$, obeys the following differential equation

$$(1 + z)^2 \frac{d^2 D_A}{dz^2} + (1 + z) \frac{dD_A}{dz} + \mathcal{H}D_A = 0,$$ (7)

which satisfies the boundary conditions:

$$\begin{cases}
D_A(0) = 0, \\
\left. \frac{dD_A}{dz} \right|_{z=0} = 1.
\end{cases}$$ (8)

The functions $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$ in equation (7) read

$$\mathcal{F} = \Omega_M (1 + z)^3 + (1 - \Omega_M)(1 + z)^{3(\omega+1)},$$

$$\mathcal{G} = \frac{7}{2} \Omega_M (1 + z)^3 + \frac{3\omega + 7}{2} (1 - \Omega_M)(1 + z)^{3(\omega+1)},$$

$$\mathcal{H} = \frac{3\alpha(z)}{2} \Omega_M (1 + z)^3 + \frac{3(\omega + 1)}{2} (1 - \Omega_M)(1 + z)^{3(\omega+1)}.$$

The smoothness parameter $\alpha(z)$, appearing in the expression of $\mathcal{H}$, assumes the form below (see Appendix A for a detailed discussion)

$$\alpha(z) = \frac{\beta_\alpha (1 + z)^{3\gamma}}{1 + \beta_\gamma (1 + z)^{3\gamma}},$$ (10)

where $\beta_\alpha$ and $\gamma$ are constants. Note that the fraction $\alpha_0 = \beta_\alpha/(1 + \beta_\gamma)$ is the present day value of $\alpha(z)$. In Fig. 1 we show the general behavior of $\alpha(z)$ for some selected values of $\beta_\alpha$ and $\gamma$.

At this point, it is interesting to compare Eq. (7) together the subsidiary definitions (8)-(10) with other treatments appearing in the literature. For $\gamma = 0$ (constant $\alpha$) and $\omega = -1$ (ΛCDM) it reduces to Eq. (2) as given by Alcaniz et al. [40]. In fact, for $\omega = -1$ the function $\mathcal{H}$ is given by $\mathcal{H} = \frac{3\alpha(z)}{4} \Omega_M (1 + z)^3$. Further, recalling the existence of a simple relation between the luminosity distance, and the angular-diameter distance (from Etherington principle [44], $D_L = (1 + z)^2 D_A$), it is easy to see that Eq. (3) of Santos et al. [41] is recovered.

A more general expression for ΛCDM model (by including the curvature term) has been derived by Demianski et al. [5]. As one may check, for $\alpha$ constant, by identifying $\omega \equiv m/3 - 1$, our Eq. (7) is exactly Eq.(10) as presented by Giovi and Amendola [36] in their time delay studies (see also Eq. (2) of Sereno et al. [45]). Different from other approaches appearing in the literature (see for instance, Refs. [29, 30]), we stress that in this paper the $\alpha$ parameter is always smaller than unity. In addition, the $\alpha$ parameter may also depend on the direction along the line of sight (for a discussion of such effects see Linder [18], Sereno et al. [15], Wang [46]).

Let us now discuss the integration of the ZKDR equation with emphasis in the so-called phantom dark energy model ($\omega < -1$). In what follows, assuming that $\omega$ is a constant, we have applied for all graphics a simple Runge-Kutta scheme (see, for instance, the rksuite package from www.netlib.org).

In Figure 2 one can see how the equation of state parameter, $\omega$, affects the angular diameter distance. For fixed values of $\Omega_M = 0.3$, $\beta_\alpha = 0.5$ and $\gamma = 0$, all the distances increase with the redshift when $\omega$ diminishes and enters in the phantom regime ($\omega < -1$). For comparison we have also plotted the case for ΛCDM cosmology ($\omega = -1$).

In Fig. 3 we show the effect of the $\gamma$ parameter on the angular diameter distance for a specific phantom cosmolo-
ogy with $\omega = -1.3$, as requested by some recent analyzes of Supernovae data. For this plot we have considered $\beta_o = 0.5$. As shown in Appendix A, $\beta_o = (\rho_h/\rho_{cl})_{z=0}$, is the present ratio between the homogeneous ($\rho_h$) and the clumped ($\rho_{cl}$) fractions. It was fixed in such a way that $\alpha_o$ assumes the value 0.33. Until redshifts of the order of 2, the distance grows for smaller values of $\gamma$, and after that, it decreases following nearly the same behavior.

In Fig. 4 we display the influence of the $\beta_o$ parameter on the angular diameter distance for two distinct sets of $\gamma$ values. The cosmological framework is defined $\Omega_M = 0.3$ and the same equation of state parameter $\omega = -1.3$ (phantom cosmology). For each branch (a subset of 3 curves with fixed $\gamma$) the distance increases for smaller values of $\beta_o$, as should be expected.

III. ZKDR DISTANCE AND ANGULAR SIZE STATISTICS

As we have seen, in order to apply the angular diameter distance to a more realistic description of the universe it is necessary to take into account local inhomogeneities in the distribution of matter. Similarly, such a statement remains true for any cosmological test involving angular diameter distances, as for instance, measurements of angular size, $\theta(z)$, of distant objects. Thus, instead of the standard FRW homogeneous diameter distance one must consider the solutions of the ZKDR equation.

Here we are concerned with angular diameters of light sources described as rigid rods and not isophotal diameters. In the FRW metric, the angular size of a light source of proper length $l$ (assumed free of evolutionary effects) and located at redshift $z$ can be written as

$$\theta(z) = \frac{l}{D_A(z)},$$

where $l = 100h$ is the angular size scale expressed in milliarcsecond (mas) while $l$ is measured in parsecs for compact radio sources (see below).

Let us now discuss the constraints from angular size measurements of high $z$ objects on the cosmological parameters. The present analysis is based on the angular size data for milliarcsecond compact radio sources compiled by Gurvits et al. [42] (see also [20] for applications to the unclustered FRW case). This sample is composed by 145 sources at low and high redshifts (0.011 $\leq z \leq$ 4.72) distributed into 12 bins with 12-13 sources per bin (for more details see Gurvits et al. [42]). In Figure 5 we show the binned data of the median angular size plotted as a function of redshift $z$ to the case with $\gamma = 0$ and some selected values of $\Omega_M$ and $\alpha_0 = \beta_0/(1-\beta_o) = $ constant. As can be seen there, for a given value of $\Omega_M$ the corresponding curve is slightly modified for different values of the smoothness parameter $\alpha$.

Now, in order to constrain the cosmic parameters, we first fix the central value of the Hubble parameter ob-
tained by the Hubble Space Telescope (HST) key project $H_o = 72 \pm 8$ km s$^{-1}$ Mpc$^{-1}$ (Freedman et al. [47]). Nowadays, this HST result has been confirmed by many different classes of estimators like the Sunyaev-Zeldovich effect and the ages of old high redshifts galaxies [48]. This value is also in accordance with the 3 years release of the WMAP team [2], however, it is greater than the recent determination by Sandage and collaborators [49]. Following standard lines, the confidence regions are constructed through a $\chi^2$ minimization

$$\chi^2(l, \omega, \alpha) = \sum_{i=1}^{12} \frac{|\theta(z_i, l, \omega, \alpha) - \theta_{oi}|^2}{\sigma_i^2},$$

(12)

where $\theta(z_i, l, \omega, \alpha)$ is defined from Eq. (7) and $\theta_{oi}$ are the observed values of the angular size with errors $\sigma_i$ of the $i$th bin in the sample. The confidence regions are defined by the conventional two-parameters $\chi^2$ levels. In this analysis, the intrinsic length $l_i$ is considered a kind of “nuisance” parameter, and, as such, we have also marginalized over it.

In Fig. 6 we show confidence regions in the $\omega - \alpha$ plane fixing $\Omega_M = 0.263$, and assuming a Gaussian prior on the $\omega$ parameter, i.e., $\omega = -1 \pm 0.3$ (in order to accelerate the universe). The “×” indicates the best fit model that occurs at $\omega = -1.03$ and $\alpha \simeq 0.9$.

In Fig. 7 the confidence regions are shown in the $\Omega_M - \alpha$ plane. We have now assumed a Gaussian prior on $\Omega_M$, i.e., $\Omega_M = 0.3 \pm 0.1$ from the large scale structure. From Figs. 6 and 7, it is also perceptible that while the parameters $\omega$ and $\Omega_M$ are strongly restricted, the entire interval of $\alpha$ is still allowed. This shows the impossibility of tightly constraining the smoothness parameter $\alpha$ with the current angular size data. This result is in good agreement with the one found by Lima and Alcaniz [4] where the same data set were used to investigate constraints on quintessence scenarios in homogeneous background, and is also in line with the one obtained by Barber et al. [50] who argued in favor of $\alpha_0 = \alpha(z = 0)$ near unity (see also Alcaniz, Lima and Silva [40] for constraints on a clustered $\Lambda$CDM model).

IV. SUMMARY AND CONCLUDING REMARKS

All cosmological distances must be notably modified whether the space-time is filled by a smooth dark energy component with negative pressure plus a clustered dark matter. Here we have addressed the question of how the angular diameter distance of extragalactic objects are modified by assuming a slightly inhomogeneous universe. The present study complements our previous results [20] by considering that the inhomogeneities can be described by the Zeldovich-Kantowski-Dyer-Roeder distance (in this connection see also, Giov and Amendola [30]; Lewis and Ibata [33]; Sereno et al. [43]: Demianski et al. [55]). The dark energy component was described by the equation of state $p_x = \omega \rho_x$. A special emphasis was
given to the case of phantom cosmology ($\omega < -1$) when the dominant energy condition is violated. The effects of the local clustered distribution of dark matter have been described by the “smoothness” phenomenological parameter $\alpha(z)$, and a simple argument for its functional redshift dependence was given in the Appendix A (see also Figure 1).

The influence of the dark energy component was quantified by considering the angular diameters for sample of milliarcsecond radio sources (Fig. 5) as described by Gurvits et al.\textsuperscript{[42]} By marginalizing over the characteristic angular size $l$, fixing $\Omega_M = 0.263$, and assuming a Gaussian prior on the EOS parameter, i.e., $\omega = -1 \pm 0.3$, the best fit model occurs at $\omega = -1.03$ and $\alpha = 0.9$. This phantom model coincides with the central value recently determined by the Supernova Legacy Survey (Astier et al.\textsuperscript{[3]}). On the other hand, fixing $\omega = -1.023$ and assuming a Gaussian prior for $\Omega_M$, that is, $\Omega_M = 0.3 \pm 0.1$, we obtained the best fit values ($\Omega_M = 0.29$, $\alpha = 0.9$).

Finally, in order to improve the present results, a statistical study is necessary for determining the intrinsic length of the compact radio sources. Further, unlike to what happens with SNe data\textsuperscript{[41]}, the angular diameter sample of compact radio sources of Gurvits et al.\textsuperscript{[42]} does not provide useful constraints on the $\alpha$ parameter (see Figs. 6 e 7). Naturally, these results reinforce the interest for measurements of angular size from compact radio sources at intermediary and high redshifts in order to constrain the $\alpha$ parameter with basis on the ZKDR distance.

**APPENDIX A: ON THE REDSHIFT DEPENDENCE OF $\alpha(z)$**

In this Appendix we discuss the functional redshift dependence of the smoothness parameter, $\alpha(z)$, adopted in this work. By definition

$$\alpha(z) = 1 - \frac{\rho_{cl}(z)}{\rho_M(z)},$$

(A1)

where $\rho_{cl}$ denotes the clumped fraction of the total matter density, $\rho_M$, present in the considered FRW type Universe. This means that the ratio between the homogeneous ($\rho_h$) and the clumped fraction can be written as $\rho_h/\rho_{cl} = \alpha(z)/(1 - \alpha(z))$. How this ratio depends on the redshift? In this concern, we first remember that $\alpha(z)$ lies on the interval [0,1]. Secondly, in virtue of the structure formation process, one expects that the degree of homogeneity must increase for higher redshifts, or equivalently, the clumped fraction should be asymptotically vanishing at early times, say, for $z \geq 100$. This means that $\alpha(z) \rightarrow 1$ at high $z$. On the other hand, $\alpha$ must be zero for a completely clustered matter which is disproved at low redshifts by the data of galaxy clusters\textsuperscript{[3]}. It thus follows that at present ($z = 0$), the related fraction assume an intermediate value, say, $\beta_0$. In addition, it is also natural to suppose that the redshift dependence...
of the total matter density, $\rho_M$, must play an important role in the evolution of their fractions. In this way, for the sake of generality, we will assume a power law

$$\frac{\rho_h}{\rho_{cl}} \equiv \frac{\alpha(z)}{1 - \alpha(z)} = \beta_o (\rho_M/\rho_0)^{\gamma} \tag{A2}$$

where $\beta_o = (\rho_h/\rho_{cl})_{z=0}$ and $\gamma$ are dimensionless numbers. Finally, inserting $\rho_M(z)$, and solving for $\alpha(z)$ we obtain:

$$\alpha(z) = \frac{\beta_o (1 + z)^{3\gamma}}{1 + \beta_o (1 + z)^{3\gamma}}, \tag{A3}$$

which is the expression adopted in this work (see Eq. 10).

As one may check, for positive values of $\gamma$, the smoothness function (A3) has all the physically desirable properties above discussed. In particular, the limit for high values of $z$ does not depend on the values of $\beta_o$ and $\gamma$ (both of the order of unity). Note also that if the clumped and homogeneous portions are contributing equally at present ($\beta_o = 1$), we see that $\alpha(z = 0) = 1/2$ regardless of the value of $\gamma$. Figure 1 display the general behavior of $\alpha(z)$ with the redshift for different choices of $\beta_o$ and $\gamma$. The above functional dependence should be compared with the other ones discussed in the literature (see, for instance, [17, 18, 39] and Refs. therein). One of the most interesting features of (A3) is that its validity is not restricted to a given redshift interval.

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