Systematic study of the $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ gauge symmetry

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Abstract

We carry a systematic study of possible extensions of the standard model based on the gauge group $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$. We consider both models with particles with exotic electric charges and models which do not contain exotic electric charges neither in the gauge boson sector nor in the fermion sector. For the first case an infinite number of models can, in principle, be constructed, while the restriction to non-exotic electric charges only allows for eight different anomaly-free models. Four of them are three-family models in the sense that anomalies cancel by an interplay between the three families, and another two are one-family models where anomalies cancel family by family as in the standard model. The remaining two are two-family models.

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I. INTRODUCTION

The standard model (SM), based on the gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, with $SU(2)_L \otimes U(1)_Y$ hidden and $SU(3)_c$ confined, can be extended in several different ways: first, by adding new fermion fields (adding a right-handed neutrino field constitutes its simplest extension and has profound consequences, as the implementation of the see-saw mechanism, and the enlarging of the possible number of local abelian symmetries that can be gauged simultaneously); second, by augmenting the scalar sector to more than one Higgs representation, and third by enlarging the local gauge group. In this last direction $SU(4)_L \otimes U(1)_X$ as a flavor group has been considered in the literature \[1, 2, 3, 4, 5, 6, 7, 8, 9\] which, among its best features, provides with an alternative to the problem of the number $N_f$ of fermion families in Nature, in the sense that anomaly cancellation is achieved when $N_f = N_c = 3$, $N_c$ being the number of colors of $SU(3)_c$ \[2\]. Moreover, this gauge structure has been used recently in order to implement the little Higgs mechanism \[6, 7\].

In this work a systematic study of the $SU(3)_c \otimes SU(4)_L \otimes U(1)_X$ local gauge symmetry (hereafter the 3-4-1 theory) shows that, by restricting the fermion field representations to particles without exotic electric charges and by paying due attention to anomaly cancellation, a few different models are obtained, while by relaxing the condition of nonexistence of exotic electric charges, an infinite number of models can be generated, all of them with particles with exotic electric charges (quarks with electric charges $5/3$ and $-4/3$, for example), as the ones considered in Refs. \[1, 2, 3, 4, 5\].

An analysis along the same lines has previously been carried out for the extension of the SM based on the gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_X$ \[10\] (the so-called 3-3-1 models), with the result that there exit only ten different anomaly free models with particles with only ordinary electric charges. Two of them are one family models and have been partially studied in Ref. \[11\]. Eight are models for three families, and the supersymmetric version of one of them has been considered in Ref. \[12\], while some interesting phenomenology of these models has been studied in Ref. \[13\]. Well motivated 3-3-1 models which include particles with exotic electric charges have also been considered in the literature \[14\].

This paper is organized as follows. In Sec. II we introduce the characteristics of the gauge group; in Sec. III we present the eight different anomaly-free models without exotic electric charges that can be constructed and study their scalar and gauge boson sectors; in
we discuss models with exotic electric charges, and in the last section we present our conclusions.

II. THE MODEL

We assume that the electroweak group is $SU(4)_L \otimes U(1)_X \supset SU(3)_L \otimes U(1)_Z \supset SU(2)_L \otimes U(1)_Y$, where the gauge structure $SU(3)_L \otimes U(1)_Z$ refers to the one presented in Ref. [10]. We also assume that the left handed quarks (color triplets), left-handed leptons (color singlets) and scalars, transform either under the 4 or the $\bar{4}$ fundamental representations of $SU(4)_L$.

As in the SM, $SU(3)_c$ is vectorlike.

In $SU(4)_L \otimes U(1)_X$, the most general expression for the electric charge generator is a linear combination of the four diagonal generators of the gauge group

$$Q = a T_{3L} + \frac{1}{\sqrt{3}} b T_{8L} + \frac{1}{\sqrt{6}} c T_{15L} + X I_4,$$

where $T_{iL} = \lambda_i L / 2$, being $\lambda_i$ the Gell-Mann matrices for $SU(4)_L$ normalized as $\text{Tr}(\lambda_i \lambda_j) = 2 \delta_{ij}$, $I_4 = Dg(1, 1, 1, 1)$ is the diagonal $4 \times 4$ unit matrix, and $a$, $b$ and $c$ are free parameters to be fixed next. Notice that we can absorb an eventual coefficient for $X$ in its definition.

If we assume that the usual isospin $SU(2)_L$ of the SM is such that $SU(2)_L \subset SU(4)_L$, and we demand for accommodating each family of SM fermions into different fundamental representations 4 or $\bar{4}$ of $SU(4)_L$, then $a = 1$ and we have just a two-parameter set of models, all of them characterized by the values of $b$ and $c$. So, Eq. (1) allows for an infinite number of models in the context of the 3-4-1 theory, each one associated to particular values of the parameters $b$ and $c$, with characteristic signatures that make them different from each other.

There are a total of 24 gauge bosons in the gauge group under consideration, 15 of them associated with $SU(4)_L$ which can be written as

$$\frac{1}{2} \lambda_\alpha A^\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} D^0_{1\mu} & W^+_{\mu} & K^{(b+1)/2}_{1\mu} & X^{(3+b+2c)/6}_{\mu} \\ W^-_{\mu} & D^0_{2\mu} & K^{(b-1)/2}_{1\mu} & V^{(-3+b+2c)/6}_{\mu} \\ K^{-(b+1)/2}_{1\mu} & K^{-(b-1)/2}_{1\mu} & D^0_{3\mu} & Y^{-(b-c)/3}_{\mu} \\ X^{-(3+b+2c)/6}_{\mu} & V^{(3-b-2c)/6}_{\mu} & Y^{(b-c)/3}_{\mu} & D^0_{4\mu} \end{pmatrix},$$

where $D^0_{1\mu} = A^\mu_3 / \sqrt{2} + A^\mu_8 / \sqrt{6} + A^\mu_{15} / \sqrt{12}$; $D^0_{2\mu} = -A^\mu_3 / \sqrt{2} + A^\mu_8 / \sqrt{6} + A^\mu_{15} / \sqrt{12}$; $D^0_{3\mu} = -2A^\mu_8 / 6 + A^\mu_{15} / \sqrt{12}$, and $D^0_{4\mu} = -3A^\mu_{15} / \sqrt{12}$. The upper indices in the gauge bosons in
the former expression stand for the electric charge of the corresponding particle, some of them functions of the $b$ and $c$ parameters as they should be.

Different from the SM where only the abelian $U(1)_Y$ factor is anomalous, in the 3-4-1 theory both, $SU(4)_L$ and $U(1)_X$ are anomalous ($SU(3)_c$ is vectorlike). So, special combinations of multiplets must be used in each particular model in order to cancel the possible anomalies, and obtain renormalizable models. The triangle anomalies we must take care of are: $[SU(4)_L]^3$, $[SU(3)_c]^2U(1)_X$, $[SU(4)_L]^2U(1)_X$, $[grav]^2U(1)_X$ and $[U(1)_X]^3$.

Let us now see how the charge operator in Eq. (1) acts on the representations $4$ and $\bar{4}$ of $SU(4)_L$:

$$Q[4] = Dg\left(\frac{1}{2} + \frac{b}{6} + \frac{c}{12} + X, -\frac{1}{2} + \frac{b}{6} + \frac{c}{12} + X, -\frac{2b}{6} + \frac{c}{12} + X, -\frac{3c}{12} + X\right),$$

$$Q[\bar{4}] = Dg\left(-\frac{1}{2} - \frac{b}{6} - \frac{c}{12} + X, \frac{1}{2} - \frac{b}{6} - \frac{c}{12} + X, \frac{2b}{6} - \frac{c}{12} + X, \frac{3c}{12} + X\right).$$

(3)

III. MODELS WITHOUT EXOTIC ELECTRIC CHARGES

Notice, from Eq. (3), that if we demand for gauge bosons with electric charges $0, \pm 1$ only, there are not more than four different possibilities for the simultaneous values of $b$ and $c$; they are: $b = c = 1; b = c = -1; b = 1, c = -2$, and $b = -1, c = 2$.

It is clear that, if we accommodate the known left-handed quark and lepton isodoublets in the two upper components of $4$ and $\bar{4}$ (or $\bar{4}$ and $4$), do not allow for electrically charged antiparticles in the two lower components of the multiplets (antiquarks violate $SU(3)_c$, and $e^+, \mu^+$ and $\tau^+$ violate lepton number at tree level) and forbid the presence of exotic electric charges in the possible models, then the electric charge of the third and fourth components in $4$ and $\bar{4}$ must be equal either to the charge of the first and/or second component, which in turn implies that $b$ and $c$ can take only the four sets of values stated above. So, these four sets of values for $b$ and $c$ are necessary and sufficient conditions in order to exclude exotic electric charges in the fermion sector too.

A further analysis also shows that models with $b = c = -1$ are equivalent, via charge conjugation, to models with $b = c = 1$. Similarly, models with $b = -1, c = 2$ are equivalent to models with $b = 1, c = -2$. So, with the constraints imposed, we have only two different sets of models; those for $b = c = 1$ and those for $b = 1, c = -2$.

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3-4-1 models without exotic electric charges of both classes have been proposed in Refs. as viable models to implement the little Higgs mechanism.

A. Models for $b = c = 1$

Let us start defining the following complete sets of spin 1/2 Weyl spinors (complete in the sense that each set contains its own charged antiparticles):

- $S_1^q = \{(u, d, D, D')_L \sim [3, 4, -\frac{1}{12}],
  u^c_L \sim [3, 1, \frac{1}{3}],
  d^c_L \sim [3, 1, \frac{1}{3}],
  D^c_L \sim [3, 1, \frac{1}{3}])\}.$

- $S_2^q = \{(d, u, U, U')_L \sim [3, 4, \frac{5}{12}],
  u^c_L \sim [3, 1, -\frac{2}{3}],
  d^c_L \sim [3, 1, \frac{1}{3}],
  U^c_L \sim [3, 1, -\frac{2}{3}],
  U^c_L \sim [3, 1, -\frac{2}{3}])\}.$

- $S_3^q = \{(\nu^0_e, e^-, E^-, E^{\prime -})_L \sim [1, 4, -\frac{3}{4}],
  e^+_L \sim [1, 1, 1],
  E^+_L \sim [1, 1, 1],
  E^{\prime +}_L \sim [1, 1, 1]).\}$

- $S_4^q = \{(E^+, N^0_1, N^0_2, N^0_3)_L \sim [1, 4, \frac{3}{4}],
  E^-_L \sim [1, 1, -1]).\}$

- $S_5^q = \{(e^-, \nu^0_e, N^0_1, N^0_2, N^0_3)_L \sim [1, 4, -\frac{1}{4}],
  e^+_L \sim [1, 1, 1]).\}$

- $S_6^q = \{(N^0_1, E^+_1, E^+_2, E^+_3)_L \sim [1, 4, \frac{3}{4}],
  E^-_{1L} \sim [1, 1, 1],
  E^-_{2L} \sim [1, 1, 1],
  E^-_{3L} \sim [1, 1, 1].\}$

Taking into account that each set includes charged particles together with their corresponding antiparticles, and since $SU(3)_c$ is vectorlike, the anomalies $[\text{grav}]^2 U(1)_X$, $[SU(3)_c]^3$ and $[SU(3)_c]^3 U(1)_X$ automatically vanish. So, we only have to take care of the remaining three anomalies whose values are shown in Table I. From this table several anomaly free models can be constructed. Let us see.

There are two three family structures which are:

- Model A = $2S_1^q \oplus S_2^q \oplus 3S_5^q$. (This model has been analyzed in Ref. [8]).

- Model B = $S_1^q \oplus 2S_2^q \oplus 3S_5^q$.

We find only one two family structure given by: Model C = $S_1^q \oplus S_2^q \oplus S_5^q \oplus S_5^q$.

A one family model can not be directly extracted from $S_i, \ i = 1, 2, ..., 6$, but we can check that the following particular arrangement is an anomaly free one family structure: Model D = $S_1^q \oplus (e^-, \nu^0_e, N^0, N^0)_L \oplus (E^+_1, N^0_1, N^0_2, N^0_3)_L \oplus (N^0_4, E^+_1, E^+_2)_L \oplus E^+_2$. As it can
be checked, this model reduces to the model in Ref. [11] for the breaking chain \( SU(4)_L \otimes U(1)_X \rightarrow SU(3)_L \otimes U(1)_\alpha \otimes U(1)_X \rightarrow SU(3)_L \otimes U(1)_Z \), for the value \( \alpha = 1/12 \). In an analogous way, other one family models with more exotic charged leptons can also be constructed.

### B. Models for \( b = 1, c = -2 \)

As in the previous case, let us define the following complete sets of spin 1/2 Weyl spinors:

- \( S^q_1 = \{ (u, d, D, U)_L \sim [3, 4, \frac{1}{6}], \ u^c_L \sim [3, 1, -\frac{2}{3}], \ d^c_L \sim [3, 1, \frac{1}{3}], \ D^c_L \sim [3, 1, \frac{4}{3}], \ U^c_L \sim [3, 1, -\frac{4}{3}] \} \).
- \( S^q_2 = \{ (d, u, U, D)_L \sim [3, 4, \frac{1}{6}], \ u^c_L \sim [3, 1, -\frac{2}{3}], \ d^c_L \sim [3, 1, \frac{1}{3}], \ U^c_L \sim [3, 1, -\frac{3}{3}], \ D^c_L \sim [3, 1, \frac{3}{3}] \} \).
- \( S^q_3 = \{ (\nu^0_e, e^-, E^-, N^0)_L \sim [1, 4, -\frac{1}{2}], \ e^+_L \sim [1, 1, 1], \ E^+_L \sim [1, 1, 1] \} \).
- \( S^q_4 = \{ (e^-, \nu^0_e, N^0, E^-)_L \sim [1, 4, -\frac{1}{2}], \ e^+_L \sim [1, 1, 1], \ E^+_L \sim [1, 1, 1] \} \).
- \( S^q_5 = \{ (E^+, N^0_1, N^0_2, e^+)_L \sim [1, 4, \frac{1}{2}], \ E^-_L \sim [1, 1, -1], \ e^-_L \sim [1, 1, -1] \} \).
- \( S^q_6 = \{ (N^0_3, E^+, e^+, N^0_4)_L \sim [1, 4, \frac{1}{2}], \ E^-_L \sim [1, 1, -1], \ e^-_L \sim [1, 1, -1] \} \).

For these sets the anomalies \([grav]^2U(1)_X\), \([SU(3)_c]^3\) and \([SU(3)_c]^2U(1)_X\) vanish. The other anomalies are shown in Table III. Again, several anomaly free models can be constructed from this table. Let us see.

We find two three family structures which are:

- Model \( E = 2S^q_1 \oplus S^q_2 \oplus 3S^q_3 \). (This model has been studied in Ref. [9]).
- Model \( F = S^q_1 \oplus 2S^q_2 \oplus 3S^q_3 \).
We again find only one two family structure given by: Model \( G = S_1^q \oplus S_2^q \oplus S_3^q \oplus S_4^q \).

Two one family models can be constructed using \( S_i^q \), \( i = 1, \ldots, 6 \). They are:

- Model \( H = S_2^q \oplus 2S_3^q \oplus S_5^q \).
- Model \( I = S_1^q \oplus 2S_4^q \oplus S_6^q \).

### C. The Scalar and Gauge Boson Sectors

#### 1. Models for \( b = c = 1 \)

If our aim is to break the symmetry following the pattern \( SU(3)_c \otimes SU(4)_L \otimes U(1)_X \to SU(3)_c \otimes SU(3)_L \otimes U(1)_X \to SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_Q \), we must introduce, at least, the following three Higgs scalars \( \phi_1[1, 4, -3/4] \) with a vacuum expectation value (VEV) aligned in the direction \( \langle \phi_1 \rangle = (v, 0, 0, 0)^T \); \( \phi_2[1, 4, -1/4] \) with a VEV aligned as \( \langle \phi_2 \rangle = (0, 0, V, 0)^T \) and \( \phi_3[1, 4, -1/4] \) with a VEV aligned as \( \langle \phi_3 \rangle = (0, 0, V', V')^T \), with the hierarchy \( V \sim V' \gg v \sim 174 \) GeV (the electroweak breaking scale).

Now, the \( SU(4)_L \) gauge boson sector for \( b = c = 1 \), as can be seen from Eq. (2), is given by

\[
\frac{1}{2} \lambda_\alpha A_\mu^\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix}
D^0_{1\mu} & W^+_{\mu} & K^+_{\mu} & X^+_{\mu} \\
W^-_{\mu} & D^0_{2\mu} & K^0_{1\mu} & V^0_{\mu} \\
K^-_{\mu} & K^0_{1\mu} & D^0_{3\mu} & Y^0_{\mu} \\
X^-_{\mu} & V^0_{\mu} & Y^0_{\mu} & D^0_{4\mu}
\end{pmatrix}
\]

After breaking the symmetry with \( \langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi_3 \rangle \) and using for the covariant derivative for 4-plets \( iD^\mu = i\partial^\mu - g\lambda_\alpha A^\alpha_\mu/2 - g'XB^\mu \), where \( g \) and \( g' \) are the \( SU(4)_L \) and \( U(1)_X \) gauge coupling constants, respectively, we get the following mass terms for the charged gauge bosons: \( M_{W^\pm}^2 = g^2v^2/2 \), \( M_{K^\pm}^2 = g^2(v^2 + V^2)/2 \), \( M_{X^\pm}^2 = g^2(v^2 + V^2)/2 \), \( M_{K^0_{1\mu}}^2 = g^2V^2/2 \), \( M_{Y^0_{\mu}}^2 = g^2V^2/2 \).
\[ M^2_{V^0(Y^0)} = g^2 V^2/2 \quad \text{and} \quad M^2_{Y^0(Y^0)} = g^2 (V^2 + V'^2)/2. \] Since \( W^\pm \) does not mix with \( K^\pm \) or with \( X^\pm \) we have that \( v \approx 174 \text{ GeV} \) as in the SM.

For the four neutral gauge bosons we get mass terms of the form

\[
M = \frac{g^2}{2} \left\{ V^2 \left( \frac{g' B^\mu}{2g} - \frac{2 A_8^\mu}{\sqrt{3}} + \frac{A_{15}^\mu}{\sqrt{6}} \right)^2 + V'^2 \left( \frac{g' B^\mu}{2g} - \frac{3 A_{15}^\mu}{\sqrt{6}} \right)^2 \right. \\
+ v^2 \left( A_3^\mu + \frac{A_8^\mu}{\sqrt{3}} + \frac{A_{15}^\mu}{\sqrt{6}} - \frac{3g' B^\mu}{2g} \right)^2 \right\}. \quad (4)
\]

\( M \) is a \( 4 \times 4 \) matrix with one zero eigenvalue corresponding to the photon. Once the photon field has been identified, we remain with a \( 3 \times 3 \) mass matrix for three neutral gauge bosons \( Z^\mu, Z'^\mu \) and \( Z''^\mu \). For the particular case \( V' = V \), the field \( Z''^\mu = A_8^\mu/\sqrt{3} - \sqrt{2/3} A_{15}^\mu \) decouples from the other two and acquires a squared mass \((g^2/2)V^2\). By diagonalizing the remaining \( 2 \times 2 \) mass matrix we get other two physical neutral gauge bosons which are defined through the mixing angle \( \theta \) between \( Z_\mu \) and \( Z'_\mu \):

\[
Z_1^\mu = Z_\mu \cos \theta + Z'_\mu \sin \theta, \quad Z_2^\mu = -Z_\mu \sin \theta + Z'_\mu \cos \theta, \quad (5)
\]

where

\[
\tan(2\theta) = -\frac{2\sqrt{2} C_W}{\sqrt{1 + 2\delta^2} \left[ 1 + \frac{2 V^2}{v^2} C_W^4 - \frac{2}{1 + 2 \delta^2} C_W^2 \right]}, \quad (6)
\]

with \( \delta = g'/(2g) \).

The photon field \( A^\mu \) and the fields \( Z_\mu \) and \( Z'_\mu \) are given by

\[
A^\mu = S_W A_3^\mu + C_W \left[ \frac{T_W}{\sqrt{3}} \left( A_8^\mu + \frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2/2)^{1/2} B^\mu \right],
\]

\[
Z^\mu = C_W A_3^\mu - S_W \left[ \frac{T_W}{\sqrt{3}} \left( A_8^\mu + \frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2/2)^{1/2} B^\mu \right],
\]

\[
Z''^\mu = \sqrt{\frac{2}{3}} (1 - T_W^2/2)^{1/2} \left( A_8^\mu + \frac{A_{15}^\mu}{\sqrt{2}} \right) - \frac{T_W}{\sqrt{2}} B^\mu. \quad (7)
\]

\( S_W = 2\delta/\sqrt{6\delta^2 + 1} \) and \( C_W \) are the sine and cosine of the electroweak mixing angle respectively, and \( T_W = S_W/C_W \). We can also identify the \( Y \) hypercharge associated with the SM abelian gauge boson as

\[
Y^\mu = \frac{T_W}{\sqrt{3}} \left( A_8^\mu + \frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2/2)^{1/2} B^\mu. \quad (8)
\]
2. Models for $b = 1, c = -2$

In this case, in order both to break the symmetry following the scheme $SU(3)_c \times SU(4)_L \times U(1)_X \rightarrow SU(3)_c \times SU(3)_L \times U(1)_X \rightarrow SU(3)_c \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_c \times U(1)_Q$ and, at the same time, to give mass to the fermion fields (a problem that, in any case, is model dependent), the following four Higgs scalars must be introduced \cite{9} : $\phi_1[1, \bar{4}, -1/2]$ with a Vacuum Expectation Value (VEV) aligned in the direction $\langle \phi_1 \rangle = (0, v, 0, 0)^T$; $\phi_2[1, \bar{4}, -1/2]$ with a VEV aligned as $\langle \phi_2 \rangle = (0, 0, V, 0)^T$; $\phi_3[1, 4, -1/2]$ with a VEV aligned in the direction $\langle \phi_3 \rangle = (v', 0, 0, 0)^T$, and $\phi_4[1, 4, -1/2]$ with a VEV aligned as $\langle \phi_4 \rangle = (0, 0, 0, V')^T$, with the hierarchy $V \sim V' \gg \sqrt{v^2 + v'^2} \simeq 174$ GeV.

For $b = 1$ and $c = -2$, the 15 gauge fields associated with $SU(4)_L$ can be written as (see Eq. (2))

$$\frac{1}{2} \lambda_\alpha A^\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} D^0_{1\mu} & W^+_{\mu} & K^+_\mu & X^0_{\mu} \\ W^-_{\mu} & D^0_{2\mu} & K^0_{1\mu} & V^-_{\mu} \\ K^-_{\mu} & K^0_{1\mu} & D^0_{3\mu} & Y^-_{\mu} \\ X^0_{\mu} & V^+_{\mu} & Y^+_{\mu} & D^0_{4\mu} \end{pmatrix}. $$

After breaking the symmetry with $\langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi_3 \rangle + \langle \phi_4 \rangle$, we get the following mass terms for the charged gauge bosons: $M^2_{W^\pm} = g^2(v^2 + v'^2)/2$, $M^2_{K^\pm} = g^2(v^2 + V^2)/2$, $M^2_{V^\pm} = g^2(v^2 + V'^2)/2$, $M^2_{Y^\pm} = g^2(V^2 + V'^2)/2$, $M^2_{K^0_{1\mu}(K^0_{1\mu}) = g^2(v^2 + V^2)/2$, and $M^2_{X^0_{\mu}(X^0_{\mu}) = g^2(v'^2 + V'^2)/2}$.

Since $W^\pm$ does not mix with the other charged bosons we have that $\sqrt{v^2 + v'^2} \approx 174$ GeV.

For the four neutral gauge bosons we now get mass terms of the form

$$M = \frac{g^2}{2} \left\{ V^2 \left( \frac{g' B^\mu}{g} - 2 A^\mu_8 \sqrt{3} + A^\mu_{15} \sqrt{6} \right)^2 + V'^2 \left( \frac{g' B^\mu}{g} + 3 A^\mu_{15} \sqrt{6} \right)^2 \right\} + v^2 \left( A_3^\mu + A_8^\mu \sqrt{3} + A_{15}^\mu \sqrt{6} - g' B^\mu \right)^2 + v'^2 \left( A_3^\mu + A_8^\mu \sqrt{3} + A_{15}^\mu \sqrt{6} \right)^2 \} = \frac{g^2}{2} \left( V^2 + V'^2 \right), \quad (9)$$

which is a $4 \times 4$ matrix with one zero eigenvalue corresponding to the photon. After extracting the photon, a $3 \times 3$ mass matrix remains for three neutral gauge bosons $Z^\mu$, $Z'^\mu$ and $Z''^\mu$. Let us choose $V' = V$ and $v' = v$ in order to simplify matters. For this particular case the field $Z''^\mu = 2 A_8^\mu/\sqrt{3} + A_{15}^\mu/\sqrt{3}$ decouples from the other two and acquires a squared mass $(g^2/2)(V^2 + v^2)$. By diagonalizing the remaining $2 \times 2$ mass matrix we get other two physical neutral gauge bosons which are defined through the mixing angle $\theta$ between $Z_\mu$ and
\[ Z_{\mu}^1, \text{ which is now given by} \]
\[ \tan(2\theta) = \frac{S_W^{2\mu} \sqrt{C_{2W}^{2\mu}}}{(1 + S_W^{2\mu})^2 + \frac{g^2}{v^2} C_W^{4\mu} - 2}, \]  \hspace{1cm} (10)

where \( S_W = g'/\sqrt{2g'^2 + g^2} \) is the sine of the electroweak mixing angle.

The photon field \( A^\mu \) and the fields \( Z^\mu \) and \( Z'^\mu \) are then given by
\[
A^\mu = S_W A_8^\mu + C_W \left[ T_W^{\frac{1}{\sqrt{3}}} \left( A_8^\mu - 2 \frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^\mu \right],
\]
\[
Z^\mu = C_W A_8^\mu - S_W \left[ T_W^{\frac{1}{\sqrt{3}}} \left( A_8^\mu - 2 \frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^\mu \right],
\]
\[
Z'^\mu = \frac{1}{\sqrt{3}} (1 - T_W^2)^{1/2} \left( A_8^\mu - 2 \frac{A_{15}^\mu}{\sqrt{2}} \right) - T_W B^\mu,
\]  \hspace{1cm} (11)

and the \( Y \) hypercharge associated with the SM abelian gauge boson is
\[ Y^\mu = T_W^{\frac{1}{\sqrt{3}}} \left( A_8^\mu - 2 \frac{A_{15}^\mu}{\sqrt{2}} \right) + (1 - T_W^2)^{1/2} B^\mu. \]  \hspace{1cm} (12)

### IV. MODELS WITH EXOTIC ELECTRIC CHARGES

We now relax the condition of non-existence of exotic electric charges in the 3-4-1 extension of the SM. As mentioned in Sec. II, the parameters \( b \) and \( c \) in Eq. (1) are now arbitrary and we then have, in principle, an infinite number of possible embeddings of the SM gauge group into \( SU(3)_c \otimes SU(4)_L \otimes U(1)_X \). A particular embedding depends on the physical motivations of the 3-4-1 model to be constructed.

It is well known that the enlargement of the symmetry group of the SM usually leads to fermions in large multiplets having fractionary electric charges different from \( \pm 2/3 \) and \( \pm 1/3 \) for exotic quarks and integer electric charges different from 0 and \( \pm 1 \) for exotic leptons, and to new gauge bosons with electric charges larger than 1 and/or fractionary (leptoquarks). Several 3-4-1 models have been constructed in the literature [1, 2, 3, 4, 5] which contain exotic electric charges only in the quark sector, while leptons have ordinary electric charges and gauge bosons have integer electric charges. We will restrict ourselves to this type of models and, for the sake of comparison with the models in the previous section, we will sketch the model in Ref. [2].

We start by noticing from Eq. (2) that if we demand for gauge bosons with integer electric charges, then a first condition is that the \( b \) parameter must be different from zero and that
the allowed values for it are the (positive or negative) odd integers. Now, the 3-4-1 model
in Ref. [2] is constructed with the goal of including right-handed neutrinos in the theory in
such a way that each family of leptons belongs to the same multiplet in the representation 4
of SU(4)_L and includes the charge conjugate fields of a SM SU(2)_L doublet; so, the correct
embedding of SU(2)_L doublets implies to start with \((\nu_a, e_a, \nu_c^a, e^c_a)_L \sim [1, 4, X_{aL}]\) (where c
stands for charge conjugation, and \(a = 1, 2, 3\) is a family index). This condition and Eq. (1)
provide three simultaneous equations from which the values of \(b, c\) and of the hypercharge
\(X_{aL}\) of these three multiplets can be obtained. The results are: \(b = -1, c = -4,\) and
\(X_{aL} = 0.\)

With \(b\) and \(c\) fixed we can again resort to Eq. (1) in order to see which the quark
content of a 4-plet and a \(\bar{4}\)-plet in the model must be. For a correct embedding of SU(2)_L
doublets, the first and second members in a 4-plet must have electric charges \(2/3\) and \(-1/3,\)
respectively, while in a \(\bar{4}\)-plet these charges must be \(-1/3\) and \(2/3.\) Again, these condition
and Eq. (1) provide simultaneous equations from which the electric charge of the third and
fourth components and the value of the hypercharge \(X\) of a quark multiplet can be obtained.
In this way, for the model under consideration, the electric charge content of a quark 4-plet is
\((2/3, -1/3, 2/3, 5/3)\) and \(X_q = 2/3,\) and for a quark \(\bar{4}\)-plet we have \((-1/3, 2/3, -1/3, -4/3)\)
and \(X_{\bar{q}} = -1/3.\)

Cancellation of the \([SU(3)_c]^3\) anomaly (or equivalently, the vectorlike character of SU(3)_c)
requires the inclusion of the charge conjugate of each quark field as an SU(4)_L \(\otimes U(1)_X\)
singlet for which the hypercharge \(X\) coincides with the electric charge. Cancellation of
the \([SU(4)_L]^3\) anomaly requires to have equal number of 4-plets and \(\bar{4}\)-plets, which in turn
implies that we must have one family of quarks transforming under the representation 4 and
two families of quarks transforming under the representation \(\bar{4}\) of SU(4)_L so, since \(X_{aL} = 0,\)
the \(X\) values obtained for the quark 4-plets and \(\bar{4}\)-plets automatically ensure cancellation of
the \([SU(4)_L]^2U(1)_X\) anomaly (provided \(N_c = N_f,\) where \(N_c = 3\) is the number of colors of
SU(3)_c and \(N_f\) is the number of fermion families). Then, it is now just a matter of counting
to check that the model is also free of the anomalies \([SU(3)_c]^2U(1)_X, [grav]^2U(1)_X,\) and
\([U(1)_X]^3.\) The complete anomaly free fermion content of the model is then

\[
L_{aL} = (\nu_a, e_a, \nu^c_a, e^c_a)_L \sim [1, 4, 0], \quad a = 1, 2, 3; \\
Q_{1L} = (u_1, d_1, U_1, J_1)_L \sim [3, 4, 2/3];
\]
masses can be neglected. In this approximation, after breaking the symmetry with $v \neq 0$, the other charged bosons we have that
\[ SU_\phi \]
by introducing the following four Higgs scalars \[2\]:

\[
\langle \phi \rangle = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \quad \langle V \rangle = \begin{pmatrix} v \\ v' \\ v'' \end{pmatrix}
\]

where $U, J, D, J'$ are exotic quarks of electric charges $2/3, 5/3, -1/3, -4/3$, respectively.

Masses for quarks and the symmetry breaking pattern $SU(3)_c \otimes SU(4)_L \otimes U(1)_X \to SU(3)_c \otimes SU(3)_L \otimes U(1)_Y$ are achieved by introducing the following four Higgs scalars \[2\]: $\phi_1[1, 4, -1]$ with a Vacuum Expectation Value (VEV) aligned in the direction $\langle \phi_1 \rangle = (0, 0, 0, V')^T$; $\phi_2[1, 4, 0]$ with a VEV aligned as $\langle \phi_2 \rangle = (0, 0, V, 0)^T$; $\phi_3[1, 4, 1]$ with a VEV aligned in the direction $\langle \phi_3 \rangle = (0, v', 0, 0)^T$, and $\phi_4[1, 4, 0]$ with a VEV aligned as $\langle \phi_4 \rangle = (v, 0, 0, 0)^T$, with the hierarchy $V \sim V' \gg v \sim v'$.

To give masses to leptons, the Higgs decouplet $\phi_5[1, 10, 0]$ with the VEV
\[
\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & w \\ 0 & w & 0 \end{pmatrix}
\]

must be introduced.

With $b = -1$ and $c = -4$, the 15 gauge fields associated with $SU(4)_L$ can be written as
\[
\frac{1}{2} \lambda_\alpha A_\alpha = \frac{1}{\sqrt{2}} \begin{pmatrix} D^0_{1\mu} & W^+_{\mu} & K^0_{\mu} & X^-_{\mu} \\ W^-_{\mu} & D^0_{2\mu} & K^0_{1\mu} & V_-^- \\ K^0_{\mu} & K^+_{1\mu} & D^0_{3\mu} & Y^-_\mu \\ X^+_{\mu} & V^+_{\mu} & Y^+_{\mu} & D^0_{4\mu} \end{pmatrix}
\]

In view of the fact that the scalar $\phi_5$ is required only to give masses to leptons, $w$ can be assumed small as compared with $v$ and $v'$ and its contribution to the gauge boson masses can be neglected. In this approximation, after breaking the symmetry with $\langle \phi_1 \rangle + \langle \phi_2 \rangle + \langle \phi_3 \rangle + \langle \phi_4 \rangle$, the following masses for the charged gauge bosons are obtained: $M_{W\pm}^2 = g^2(v^2 + v'^2)/2$, $M_{X\pm}^2 = g^2(v^2 + V'^2)/2$, $M_{K^0_{\mp}}^2 = g^2(v^2 + V^2)/2$, $M_{K^\mp_{\pm}}^2 = g^2(V^2 + V'^2)/2$, $M_{K^0_{(K^\mp)}}^2 = g^2(v^2 + V^2)/2$, and $M_{V^{++/(V^--)}}^2 = g^2(v^2 + V'^2)/2$. Since $W^{\pm}$ does not mix with the other charged bosons we have that $\sqrt{v^2 + v'^2} \approx 174$ GeV.

The mass matrix for the neutral gauge bosons, in the basis $(A_3^u, A_8^u, A_{15}^u, B^u)$, is given by

\[2\]
By restricting the gauge and fermion field representations to particles without neutral gauge bosons, we can diagonalize it perturbatively. The standard procedure [15] gives, at the first order in the perturbation parameter \( q = (v/V)^2 \), one eigenvalue of order \( v^2 \) associated to the neutral gauge boson of the SM and two eigenvalues of order \( V^2 \) associated to the two new neutral gauge bosons predicted by the model.

For this model, the photon field \( A^\mu \) and the fields \( Z_\mu, Z'_\mu \) and \( Z''_\mu \) are given by

\[
A^\mu = S_W A^\mu_3 + C_W \left[ \frac{T_W}{\sqrt{3}} \left( -A^\mu_8 - 2\sqrt{2}A^\mu_{15} \right) + (1 - 3T^2_W)^{1/2} B^\mu \right],
\]

\[
Z^\mu = C_W A^\mu_3 - S_W \left[ \frac{T_W}{\sqrt{3}} \left( -A^\mu_8 - 2\sqrt{2}A^\mu_{15} \right) + (1 - 3T^2_W)^{1/2} B^\mu \right],
\]

\[
Z'_\mu = \frac{1}{3} (1 - 3T^2_W)^{1/2} \left( -A^\mu_8 - 2\sqrt{2}A^\mu_{15} \right) - \sqrt{3}T_W B^\mu,
\]

\[
Z''_\mu = \frac{2\sqrt{2}}{3} A^\mu_8 - \frac{1}{3} A^\mu_{15},
\]

where the sine of the electroweak mixing angle is \( S_W = \delta/\sqrt{1 + 4\delta^2} \). We can also identify the \( Y \) hypercharge associated with the SM abelian gauge boson as

\[
Y^\mu = \frac{T_W}{\sqrt{3}} \left( -A^\mu_8 - 2\sqrt{2}A^\mu_{15} \right) + (1 - 3T^2_W)^{1/2} B^\mu.
\]

V. CONCLUSIONS

In this paper we have done a detailed study of the \( SU(3)_c \otimes SU(4)_L \otimes U(1)_X \) gauge symmetry. By restricting the gauge and fermion field representations to particles without
exotic electric charges, eight different anomaly-free models have been identified. Four of them are three-family models in the sense that anomalies cancel by an interplay between the three families. Another two are one-family models where anomalies cancel family by family as in the SM. The remaining two are two-family models. Three-family models of this class have been recently proposed as viable models to implement the little Higgs mechanism.[6, 7]

If we allow for particles with exotic electric charges, we end up with an infinite number of models. In this case, a particular model depends on the physical motivation of the 3-4-1 extension of the SM to be constructed. Examples of such a class of models are the ones studied in Refs. [1, 2, 3, 4, 5]. In the main text we have presented the main features of the model developed in Ref. [2]. An interesting variation of this model, allowing the inclusion of the see-saw mechanism for neutrino masses and flavor mixing in the neutrino sector, is studied in Ref. [3].

All the models presented here have in common two new neutral currents. One of them mixes with the SM neutral current which is also included as a part of each model. This mixing, however, is model dependent and has been calculated for models A and E of Sec. III in Refs. [8] and [9], respectively, using experimental results from the CERN LEP, SLAC Linear Collider and atomic parity violation data. This calculation also shows that, for these two models, the mass of the new neutral gauge boson $Z'_2$ which mixes with the one of the SM satisfies $0.7 \text{ TeV} \leq M_{Z'_2}$, which is compatible with the bound obtained from $p\bar{p}$ collisions at the Fermilab Tevatron.

It is worth noticing that both for the particular 3-4-1 model with exotic electric charges presented in Sec. IV and for all the four three-family models without exotic electric charges (models A, B, E and F in Sec. III), universality for the known leptons in the three families is present at the tree level in the weak basis, up to mixing with the exotic fields. Since the mass scale of the new neutral gauge boson $Z'$ and of the exotic particles is of the order of $V$ (the scale of the $SU(4)_L \otimes U(1)_X$ symmetry), this mixing will suppress tree-level flavor changing neutral currents (FCNC) effects in the lepton sector. For the quarks, instead, one family transform differently from the other two and, as a result, there can be potentially large FCNC effects in the hadronic sector.

Finally, let us mention that all the 3-4-1 models without exotic electric charges identified in this work are relatively new in the literature. In particular, the phenomenology of models
B, D, F, H, I, has not yet been studied, as far as we know.

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[1] M.B. Voloshin, Sov. J. Nucl. Phys. 48, 512 (1988).

[2] F. Pisano and T.A. Tran, ICTP preprint IC/93/200 (1993); V. Pleitez, arXiv: [hep-ph/9302287](http://arxiv.org/abs/hep-ph/9302287).

R. Foot, H.N. Long and T.A. Tran, Phys. Rev. D50, R34 (1994); F. Pisano and V. Pleitez, Phys. Rev. D51, 3865 (1995).

[3] I. Cotaescu, Int. J. Mod. Phys. A12, 1483 (1997).

[4] Fayyazuddin and Riazuddin, Phys. Rev. D30, 1041 (1984).

[5] Fayyazuddin and Riazuddin, JHEP 0412, 013 (2004).

[6] D.E. Kaplan and M. Schmaltz, JHEP 0310, 039 (2003)

[7] O.C.W. Kong, arXiv: [hep-ph/0308148](http://arxiv.org/abs/hep-ph/0308148). O.C.W. Kong, J. Korean Phys. Soc. 45, S404 (2004); O.C.W. Kong, Phys. Rev. D70, 075021 (2004).

[8] W.A. Ponce, D.A. Gutiérrez and L.A. Sánchez, Phys. Rev. D69, 055007 (2004).

[9] L.A. Sánchez, F.A. Pérez and W.A. Ponce, Eur. Phys. J. C35, 259 (2004).

[10] W.A. Ponce, J.B. Flórez, and L.A. Sánchez, Int. J. Mod. Phys. A17, 643 (2002); W.A. Ponce, Y. Giraldo and L.A. Sánchez, in *Proceedings of the VIII Mexican Workshop of Particles and Fields*, Zacatecas, Mexico, 2001. Edited by J.L. Díaz-Cruz et al. (AIP Conf. Proceed. Vol. 623, N.Y., 2002), pp. 341-346.

[11] L.A. Sánchez, W.A. Ponce and R. Martínez, Phys. Rev. D64, 075013 (2001); J.M. Mira, W.A. Ponce, D.A. Restrepo and L.A. Sánchez, Phys. Rev. D67, 075002 (2003); R. Martínez, W.A. Ponce and L.A. Sánchez, Phys. Rev. D65, 055013 (2002); R. Martínez, N. Poveda and J.-A. Rodríguez, Phys. Rev. D69, 075013 (2004).

[12] L.A. Sánchez, W.A. Ponce and J.M. Mira, Eur. Phys. J. C42, 205 (2005).

[13] D.L. Anderson and M. Sher, Phys. Rev. D72, 095014 (2005); P.V. Dong, H.N. Long, D.T. Nhung and D.V. Soa, Phys. Rev. D73, 035004 (2006); P.V. Dong, H.N. Long, and D.V. Soa,
Phys. Rev. D\textbf{73}, 075005 (2006); A. Palcu, Mod. Phys. Lett. A \textbf{21}, 1203 (2006).

[14] M. Singer, J.W.F. Valle and J. Schechter, Phys. Rev. D\textbf{22}, 738 (1980); F. Pisano and V. Pleitez, Phys. Rev. D\textbf{46}, 410 (1992); P.H. Frampton, Phys. Rev. Lett. \textbf{69}, 2887 (1992); J.C. Montero, F. Pisano and V. Pleitez, Phys. Rev. D\textbf{47}, 2918 (1993); R. Foot, O.F. Hernandez, F. Pisano and V. Pleitez, Phys. Rev. D\textbf{47}, 4158 (1993); V. Pleitez and M.D. Tonasse, Phys. Rev. D\textbf{48}, 2353 (1993); D. Ng, Phys. Rev. D\textbf{49}, 4805 (1994); L. Epele, H. Fanchiotti, C. García Canal and D. Gómez Dumm, Phys. Lett. B \textbf{343} 291 (1995).

[15] L.I. Schiff. \textit{Quantum Mechanics}, 3rd ed. (McGraw-Hill, New York, 1968), pp. 248-250.