Out of plane screening and dipolar interactions in heterostructures

Cheung Chan and T. K. Ng
Department of Physics, Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong, China
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Out-of-plane screening (OPS) is expected to occur generally in metal-semiconductor interfaces but this aspect has been overlooked in previous studies. In this paper we study the effect of OPS in electron-hole bilayer (EHBL) systems. The validity of the dipolar interaction induced by OPS is justified with a RPA calculation. Effect of OPS in electron-hole liquid with close-by screening layers is studied. We find that OPS affects the electronic properties in low density and long wavelength regime. The corresponding zero-temperature phase diagram is obtained within a mean field treatment. We argue that our result is in general relevant to other heterostructures. The case of strongly correlated EHBL is also discussed.

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I. INTRODUCTION

Modern micro-electronics relies to a large degree on surface science, which concerns the material properties near a surface or interface. To enhance the performance of such devices, knowledge of the electronic states near the interfaces is required. Near a surface or interface, electronic reconstruction may alter three key factors - interaction strengths, bandwidths and electron densities [1] which determine electronic states and their properties.

In this paper, we consider another factor - the modification in form of interaction between electrons. For instance, in an insulator-semiconductor-insulator superstructure, if the dielectric constant of the semiconductor is sizably larger than that of insulator (barrier layer), the image charges induced at semiconductor-insulator interface can substantially enhance the binding energy of the excitons confined in the semiconductor layer [2, 3]. In this case, the electrons and holes do not interact via usual Coulomb potential after the effect of the image charges at the semiconductor-insulator interface is taken into account.

Recently, Huang et al. observed non-activated electronic conductivity of a two-dimensional (2D) low density hole system in a heterojunction insulated-gate field-effect transistor [4]. Such non-activated conductivity is unexpected at low charge density strong Coulomb interaction is expected to crystallize the system (Wigner crystal), which is then pinned by disorder resulting in insulating behavior and activated conductivity. Huang et al. attribute the behavior to the screening of Coulomb interactions by the metallic gate, which leads to destruction of the Wigner crystal phase. Physically, the metallic gate which is located at a distance away from the 2D hole gas, provides an out of plane screening (OPS) to the hole-hole interaction, resulting in effective dipolar interaction between holes. Microscopically, when a charge is placed near a metal surface, an image charge of opposite sign will be induced at the surface to screen out the (static) electric field from the charge. From elementary electrostatics, the system can be described equivalently as a dipole formed by the charge and its image charge and the interaction between two charges located near the interface changes from a Coulomb potential $\sim 1/r$ to a dipolar potential $\sim 1/r^3$. This modified interaction, which is generally expected to exist in metal-semiconductor heterostructures, can change the electronic properties near the interface. Unexpectedly, there has been no detailed theoretical study of this effect on electronic properties until recently [5]. The neglect of OPS might be due to dynamical screening of in-plane charges [6]. For high charge density, the screening can effectively reduce both Coulomb and dipolar interactions to short range interactions. However for low charge density electronic liquids in-plane screening is less effective and OPS can lead to a difference, as is observed by Huang et al. [4].

In this paper, we study how OPS affects the electronic properties in systems with two-layer of charges of opposite sign, i.e. the 2D electron-hole bilayer (EHBL) system. We shall study how OPS affects Wigner crystallization and exciton condensate in the system [4, 6] and will also comment on the effect of OPS in interfaces between metals and strongly correlated electron systems [7–9].

II. OPS AND EFFECTIVE INTERACTION BETWEEN CHARGES

In this section we provide the details for the EHBL systems we study and the corresponding OPS effective interaction. We shall assume that the only effect of the metallic screening layers is to provide an image charge for point charges sitting close to it and the effective interaction between charges will be derived from the image charge picture. The validity of this approximation is bounded by the plasma frequency $\omega_p(s)$ of the screening layer, above which the screening layer cannot respond rapidly to the charge fluctuations. Thus our approximation is valid when the plasma frequency of the EHBL layer $\omega_p$ is much less than $\omega_p(s)$, or that the screening layer has density of electric charge much larger than the charge density of the EHBL layers we consider. The image charge picture can be justified by a Random Phase Approximation (RPA) calculation which is shown in the
the distance between the two layers of charges charge together form a dipole. We have assumed that point charge. Thus the point charge and the screening ciently larger than a

For an electron and a hole sitting in different layers, the interlayer interaction is

\[ V^{\text{i}ntra}(\vec{k}) = \frac{2\pi\epsilon^2}{\epsilon_{e,h} k} \left( 1 - e^{-ka} \right). \]  

and its Fourier counterpart is

\[ V^{\text{e},2}(\vec{r}) = -\frac{2\pi\epsilon^2}{\epsilon_{e,h} k} e^{-ka} (1 - e^{-ka})^2. \]

\( \epsilon_{e,h} \) and \( \epsilon_x \) are the intra-layer and inter-layer dielectric constants, respectively.

Next we consider EHBL with only one metallic screening layer (see Fig. 1(c)). In this case the two layers of charges have distance \( a/2 + b \) (layer 1) and \( a/2 \) (layer 2) from the screening layer, respectively. The intralayer interactions are thus

\[ V_{1}^{\text{intra}}(\vec{r}) = \frac{\epsilon^2}{\epsilon_1} \left( \frac{1}{r} - \frac{1}{\sqrt{r^2 + (a + 2b)^2}} \right), \]

\[ V_{2}^{\text{intra}}(\vec{r}) = \frac{\epsilon^2}{\epsilon_2} \left( \frac{1}{r} - \frac{1}{\sqrt{r^2 + a^2}} \right). \]

The corresponding Fourier transforms

\[ V_{1}^{\text{intra}}(\vec{k}) = \frac{2\pi\epsilon^2}{\epsilon_{e,h} k} \left( 1 - e^{-k(2b+a)} \right), \]

\[ V_{2}^{\text{intra}}(\vec{k}) = \frac{2\pi\epsilon^2}{\epsilon_{e,h} k} \left( 1 - e^{-ka} \right). \]

The corresponding intralayer interaction is given by

\[ V^{\text{e},1}(\vec{r}) = -\frac{\epsilon^2}{\epsilon_x} \left( \frac{1}{\sqrt{r^2 + b^2}} - \frac{1}{\sqrt{r^2 + (a + b)^2}} \right) \]

and

\[ V^{\text{e},1}(\vec{k}) = -\frac{2\pi\epsilon^2}{\epsilon_{e,h} k} e^{-ka} (1 - e^{-ka})^2. \]

\[ \text{III. COLLECTIVE DENSITY RESPONSES} \]

In this section we study the collective density responses of the EHBL systems we considered. For a two component electronic system, the density-density response of the system is described by a \( 2 \times 2 \) matrix \( \chi_{ij}(q, \omega) \) with \( i, j = 1, 2 \). The density-density response matrix is given in RPA by [10].
\[
\begin{pmatrix}
\chi_{11} & \chi_{12} \\
\chi_{21} & \chi_{22}
\end{pmatrix} = \frac{1}{\kappa} \begin{pmatrix}
(1 - \chi_{02} V_{22}) \chi_{01} & \chi_{01} V_{12} \chi_{02} \\
\chi_{02} V_{21} \chi_{01} & (1 - \chi_{01} V_{11}) \chi_{02}
\end{pmatrix}
\]
(6)

where

\[
\kappa(q, \omega) = (1 - \chi_{01}(q, \omega)V_{11}(q))(1 - \chi_{02}(q, \omega)V_{22}(q)) - \chi_{01}(q, \omega)V_{12}(q)\chi_{02}(q, \omega)V_{21}(q)
\]
(7)

\[V_{ij}(q)\] is the "bare" interaction between \(i^{th}\) and \(j^{th}\) components of the electronic liquid and

\[
\chi_{0i}(q, \omega) = g_s \int \frac{d^2k}{(2\pi)^2} \frac{n_F (\epsilon_k^{(i)}) - n_F (\epsilon_k^{(i)})}{\hbar \omega + \epsilon_k^{(i)} - \epsilon_{k+q}^{(i)}},
\]
(8)

where \(\epsilon_k^{(i)} \sim k^2/2m^{(i)}\) is kinetic energy of species \(i\) particles (of mass \(m^{(i)}\), \(n_F\) is the Fermi-Dirac distribution function and \(g_s = 2\) is spin degeneracy. In the case of two screening layers the interactions \(V_{11(22)}\) and \(V_{12} = V_{21}\) are given by \(V^{\text{intra}}(q)\) (eq. (3)) and \(V^{x,2}(q)\) (eq. (3)), respectively whereas they are given by \(V^{x}(q)\) (eq. (1)) and \(V^{x,1}(q)\) (eq. (5)), respectively if there is only one screening layer.

Next we study the collective excitations (i.e. plasmons) in the system. The dispersion of the collective excitations are given by the equation

\[
\kappa(q, \omega(q)) = 0.
\]
(9)

We shall first consider the long wavelength limit \((q \to 0)\) where the equation can be studied analytically. In this limit it is easy to show that

\[
\chi_0(q, \omega) = \frac{n}{m} \left( \frac{\omega}{\omega} \right)^2 + O \left( \left( \frac{\omega}{\omega} \right)^4 \right),
\]
(10)

where \(n = \frac{g_s \pi e^2}{\epsilon_s q^2}\) is carrier density. We have neglected the component index \(i\) for brevity.

We begin with the Coulomb case (no screening layer).

The interactions are respectively \(V_{11,22}(q) = \frac{2\pi e^2}{\epsilon_{1,2} q}\) and \(V_x(q) = -\frac{2\pi e^2}{\epsilon_s q} e^{-q \phi} \sim -\frac{2\pi e^2}{\epsilon_s q} q b^{-1}\). The plasmon equation in \(q \to 0\) limit reads

\[
1 - 2\pi e^2 \left( \frac{n_1}{m_1 \epsilon_1} + \frac{n_2}{m_2 \epsilon_2} \right) \frac{q}{\omega^2} + (2\pi e^2)^2 \times \frac{n_1 n_2}{m_1 m_2} \left[ \frac{1}{\epsilon_1 \epsilon_2} - \frac{1}{\epsilon_s} \right] \left( \frac{q}{\omega^2} \right)^2 = 0.
\]
(11)

We first consider the case \(\epsilon_s^2 = \epsilon_1 \epsilon_2\) such that the term in the square bracket is zero. In this case we need to expand the interlayer interaction to one order higher in \(q\). As a result the last term in eq. (11) is replaced by a term of order \(q^2/\omega^2\) and the plasmon equation at long wavelength limit yields two solutions, which are the out-of-phase mode \((\omega \sim q)\) and in-phase mode \((\omega \sim \sqrt{q})\). Indeed this occurs usually in a 2D electronic systems with both conduction and valence bands where the same dielectric constant \(\epsilon_s^2 = \epsilon_1 \epsilon_2\) is found for all interactions. In the more general case \(\epsilon_s^2 \neq \epsilon_1 \epsilon_2\), which arises quite naturally in the complex environment of EHBL heterostructures, we can easily see from eq. (11) that the plasmon frequency scales as \(\omega \sim \sqrt{q}\). There are two modes of plasmons.

For the OPS case with two screening layers, the interactions are respectively \(V_{11,22} = \frac{2\pi e^2}{\epsilon_{1,2} q}\) and \(V_x(q) = -\frac{2\pi e^2}{\epsilon_s q} e^{-q \phi} \sim -2\pi a^2 e^2 q/\epsilon_s\) for \(q \ll a^{-1}\). Notice the removal of the \(1/q\) singularity in the interactions by OPS. We then obtain after solving the equation the collective modes (up to order \(q^2\))

\[
\omega_{1,2} = \frac{2\pi a e^2}{m_{1,2} \epsilon_{1,2}} \left( \frac{a q a}{4} \right).
\]
(12)

Notice that OPS effectively reduced the long-ranged Coulomb interaction into short-ranged interactions resulting in two collective modes scaling linearly with \(q\). The collective modes represent separate collective motion of the two layers because \(V^{x,2}(q)\) is of higher order in \(q\) than \(V_{11} V_{22}\), and the inter-layer interaction appears only to order \(q^3\). For completeness, we have computed numerically the collective modes spectrums at finite \(q\) as shown in Fig. 2.

With only one screening layer, the interactions are \(V_{11}(q) \sim \frac{2\pi e^2}{\epsilon_1} (a + 2b)(a + 2b) q / 2\), \(V_{22}(q) \sim \frac{2\pi e^2}{\epsilon_2} (a - a^2 q / 2)\) and \(V_x(q) = -\frac{2\pi e^2}{\epsilon_s q} e^{-q \phi} (1 - e^{-q a}) \sim\)
$-2\pi ae^2/\epsilon_x$, respectively at small $q$. The collective modes are given by (up to order $q^2$)

$$\omega_1 = \sqrt{\frac{2\pi(a+2b)e^2n_1}{m_1\epsilon_1}} \left( q - \frac{(a+2b)q^2}{4} \right)$$

$$\omega_2 = \sqrt{\frac{2\pi ae^2n_2}{m_2\epsilon_2}} \left( q - \frac{aq^2}{4} \right).$$

Again there are two linear plasmon modes and effect of $V^\pi$ does not enter until $q^3$. The main difference is that the electron-hole layer separation $b$ enters the slope of $\omega_1$ mode ($\propto \sqrt{2b+a}$). The numerically calculated plasmon spectrums are depicted in Fig. 3.

IV. EXCITON CONDENSATION AND WIGNER CRYSTALIZATION

In this section we study exciton condensation and Wigner crystallization in an electron-hole liquid with OPS. The system without OPS has been extensively studied for the search of exciton condensation. We shall consider exciton condensation in a BCS type mean-field theory where the exciton condensation is described by the order parameter $\langle c_{1k}\dagger c_{2k}\rangle$ (1, 2 are layer indices). For simplicity we assume the layers are doped with equal amount of charges (with opposite signs) and the electrons and holes are spin-polarized. Singlet pairing of excitons is implicitly assumed.

The EHBL Hamiltonian in momentum representation is

$$H = \sum_{\alpha k} \xi^\alpha_k c_{\alpha k}^{\dagger} c_{\alpha k} + \sum_{pqk} V^x(k) c_{1p+q}^{\dagger} c_{2q-k} c_{2q} c_{1p}$$

$$+ \frac{1}{2} \sum_{\alpha pqk} V^\alpha(k) c_{\alpha p+k}^{\dagger} c_{\alpha q-k}^{\dagger} c_{\alpha q} c_{\alpha p},$$

where $\alpha = 1, 2$ is the layer index; $c_k (c_k^{\dagger})$ is the momentum $k$ fermion annihilation (creation) operator, $\xi^\alpha_k = \frac{k^2}{2m_\alpha} - \mu_\alpha$ is the electron or hole dispersion and $V^\alpha(k)$ is the intralayer (interlayer) OPS effective interaction. Next we employ the standard Hartree-Fock-Bogoliubov method to derive the mean field equations for exciton condensate. The Hartree-Fock terms $\Sigma^\alpha_k = \sum_q V^\alpha(p-q) \langle c_{\alpha q}^{\dagger} c_{\alpha q} \rangle$ modify the particle dispersions $\xi^\alpha_k \rightarrow \xi^\alpha_k - \Sigma^\alpha_k$ and need to be solved self-consistently. Here we concentrate on the effect of exciton binding on the Fermi surface and shall assume that the self-energy can be captured by introducing effective masses $m^*_\alpha(\epsilon_\alpha)$ and renormalized chemical potentials $\mu^*_\alpha(\epsilon_\alpha)$, i.e. $\xi^\alpha_k - \Sigma^\alpha_k \sim \frac{k^2}{m^*_\alpha} - \mu^*_\alpha$. With this approximation, we obtain the mean field Bogoliubov Hamiltonian

$$H_{MF} = \sum_{k\sigma} (c_k^{\dagger} c_{2k}^{\dagger}) (\xi_k^{1k} - \Delta_k^{1k}) c_{2k} (c_k^{\dagger} c_{2k}^{\dagger}),$$

where

$$\Delta_k = -\sum_q V^x(k-q) \langle c_{1k} c_{2k}\rangle$$

is the exciton order parameter. $H_{MF}$ can be diagonalized easily by the Bogoliubov transformation

$$\begin{pmatrix} c_{1k} \\ c_{2k}^{\dagger} \end{pmatrix} = \begin{pmatrix} u_k & v_k \\ -v_k & u_k \end{pmatrix} \begin{pmatrix} \gamma_{1k} \\ \gamma_{2k} \end{pmatrix},$$

$$\begin{cases} u_k^2 = \frac{1}{2} \left( 1 + \frac{\xi_k}{\epsilon_k} \right), \\
v_k^2 = \frac{1}{2} \left( 1 - \frac{\xi_k}{\epsilon_k} \right), \end{cases}$$

where $E_k = \sqrt{(\xi_k)^2 + \Delta_k^2}$, $\xi_k = \frac{1}{2} (\xi_k^1 + \xi_k^2) \equiv \frac{k^2}{2m_\alpha} - \mu$, where $m^{-1}_\alpha = (m^{-1}_1 + m^{-1}_2)/2$ and $\mu = (\mu^*_1 + \mu^*_2)/2$. The ground state wavefunction is

$$|\psi_G\rangle = \prod_k (u_k + v_k c_{1k}^{\dagger} c_{2k}^{\dagger}) |0\rangle,$$

where $\Delta_k$ is determined by the self-consistent equation

$$\Delta_k = -\frac{1}{2} \sum_q V^x(k-q) \frac{\Delta_k}{E_k}.$$
limit, the exciton pairing is diminished due to the repulsive nature of the interlayer OPS potential at short distance (see Fig. 1(f)). This leads to a linear dependence of $V^\ast(k)$ versus $k$ at small $k$ (see eq. (23)). In this case, the gap equation, eq. (20), is of the form
$$
\int_0^{k_F} \frac{k}{\sqrt{k^2 + \Delta^2}} d^2 k
$$
constant for small gap $\Delta$, where $k_F \sim 1/r_s$ and larger $r_s$ (smaller $k_F$) implies a smaller $\Delta$ to satisfy the equation. The electrons and holes need to be placed closer to each other to produce a large enough $\Delta$ and leads to the drop of $b_s(r_s)$ at large $r_s$. As the screening separation $a$ increases, the transition line shifts upward as the interlayer OPS potential is strengthened which enhances pairing. For $a = 25$ (comparable with $b$), the image charge effect becomes negligible and potential becomes essentially Coulomb-like which permits exciton formation for all $r_s$ we considered (cf. Fig. 1 in Ref. [12]). The main effect of OPS potential is to suppress exciton pairing at low density.

Previous numerical study of the same EHBL with no screening layer [12] reveals also an excitonic Wigner crystal phase at large $r_s$. Wigner crystal is commonly formed in low density (i.e. large $r_s$) electron liquid because of domination of Coulomb repulsive potential energy ($\sim 1/r_s$) over kinetic energy ($\sim 1/r_s^2$). To minimize the potential energy the electron wavefunction “crystalizes” to ensure maximum separation between electrons which yields the Wigner crystal phase. Here we argue that OPS suppresses the Wigner crystal phase in two ways. Firstly, as shown above, exciton formation is suppressed at large $r_s$ and thus the excitonic Wigner crystal is unlikely to form. On the other hand, electronic Wigner crystals in separated layers are also prohibited since introduction of OPS reduces the (intralayer) potential energy and changes its scaling form to $\sim 1/r_s^3$ (dipolar interaction, see eq. (A.11)) at large particle separation $r \gg a$. In this case kinetic energy again dominates at large $r_s$ and an usual electron/hole liquid phase should occur. The situation is similar to the case as found in Ref. [4] where the electronic Wigner crystal phase is destroyed by screening. We note, however that our simple study cannot rule out the possibility of having a Wigner crystal phase at some intermediate values of $r_s$ where the kinetic and potential energies are of comparable magnitudes.

We now consider the situation of EHBL with only one screening layer which may be easier to realize experimentally (Fig. 1(c)). In this case we adopt eq. (25) for interlayer interaction, where $V^\ast(k)$ scales as constant at small $k$. We can again consider the gap equation and argue similarly that the exciton phase boundary would also drop at large $r_s$, as in the two-layer screening case. Indeed we have solved the gap equations and find that the phase diagram is qualitatively the same as the two OPS layer case except that the area under the phase boundary $b_s(r_s)$ is larger (see Fig. 4 (open symbols)).

For the Wigner crystal phase, the “asymmetric” OPS introduces some complications. First we note that an
excitonic Wigner crystal phase is also unlikely to occur at large $r_s$. However the system may form a hybrid phase where a Wigner crystal is formed at layer 1 and electron/hole liquid phase remains for layer 2 because screening mainly affects layer 2. To examine this possibility we check the effective intralayer interaction after taking into account the screening effect of the other charged layer (see eq. $\Delta$ in Appendix and discussions thereafter). We see that the effective intralayer interaction is mainly dominated by $V_{1,2}^{\text{intra}}(q)$, and screening from the other layer is not important. Therefore, we expect that at large $r_s$ kinetic energy again dominates and the both layers are in the electron/hole liquid phase. Notice, however that $V_{1,2}^{\text{intra}}(q)$ has a dipolar form only when $r_s \sim r/a_B > b/a_B$ for layer 1. Thus for some large enough $b/a_B$, a hybrid phase (Wigner crystal at layer 1, electron/hole liquid at layer 2) may still occur at some intermediate densities $b/a_B \gg r_s \gg 1$.

We see that OPS becomes important for low density electronic systems due to change in scaling of the potential energy. Generally speaking, for heterostructures, insulating behavior resulting from low carrier density can be avoided by addition of metallic screening layers [4]. This method may be preferred over other methods like increasing carrier density by dopants since dopants act like impurities and introduce unnecessary scattering at low temperature.

V. STRONGLY CORRELATED EHBL

In strongly correlated materials, the basic electronic properties are determined by the bandwidth, the on-site Coulomb interactions $U$ and the charge transfer energy $E_c$. If such a ultra-thin film, originally a Mott insulator, is placed close to a metal surface, $U$ and $E_c$ can be strongly reduced by OPS [2]. When the bandwidth exceeds the suppressed $U$ and $E_c$, the insulating film can undergo an insulator-metal phase transition. Furthermore, if a heterostructure is formed, structural relaxation and local electronic states may exist at the interfaces. For instance, in an interface formed by YBa$_2$Cu$_3$O$_7$ (YBCO) cuprate and metal [8] [9], the CuO$_2$ plane near the interface (depletion layer) is intrinsically doped by electronic reconstruction resulting in a strongly correlated electron system with OPS interaction induced by the metal. We shall consider here how OPS would affect the properties of this system.

The mean field analysis on effect of OPS can also be performed for strongly correlated EHBL systems [13, 14] with a two-layer $t$-$J$ type model. We assume here that the suppression of $U$ and $E_c$ induced by OPS are not strong enough to destroy strong correlation, otherwise we can simply apply the usual electron-hole liquid picture described in previous section. Therefore the setting is similar to that shown in Fig.1(b) except that the electron-hole liquid is replaced by a strongly correlated EHBL with holons and doublons and the excitons are formed by holon-doublon pairs instead of electron-hole pairs. A mean field calculation similar to that of Ref. [13] can be carried out by applying the slave-boson mean field theory to the two-layer $t$-$J$ model. The main difference is that the on-site interlayer interaction $V_0 \sum_i b_{i1}^\dagger b_{i1}^\dagger b_{i2} b_{i2}$ is replaced by the OPS effective interaction $\sum_{ij} V_{ij}^{\text{intra}} b_{i1}^\dagger b_{i2}^\dagger b_{j2} b_{j1}$, where $b_{i\alpha}$ ($b_{i\alpha}^\dagger$) is the bosonic holon ($\alpha = 1$) or doublon ($\alpha = 2$) annihilation (creation) operator of layer $\alpha$ at site $i$. The OPS interaction is then decoupled as

$$\sum_{ij} V_{ij}^{\text{intra}} b_{i1}^\dagger b_{i2}^\dagger b_{j2} b_{j1} \approx \sum_{p q k} V_{k}^{\text{intra}} b_{1p}^\dagger b_{2q}^\dagger b_{2q+k} b_{1p-k}$$

$$\approx \sum_p \Delta_p^b (b_{1p} b_{2p} + b_{2p}^\dagger b_{1p}^\dagger) - \sum_p \Delta_p^b (b_{1p} b_{2p}^\dagger),$$

where $\Delta_p^b = \sum q V_{x}^{p \rightarrow q} (b_{1q} b_{2q})$ is the exciton pairing. Assuming that $\Delta_p^b = \Delta(b(p))$ is homogeneous in space, we obtain a mean field Hamiltonian which is of the same form as in previous study [13] for on-site interaction $V_0$ with $(b_{1i} b_{2i}) \sim \sum_k (b_{1k} b_{2k})$. Since in terms of exciton pairing the attract-repel behavior renders the interlayer OPS interaction resembling an on-site interaction (see Fig. 1(f)), we expect that the mean field phase diagrams in both case are qualitatively the same. The introduction of OPS interaction solely shifts the exciton phase boundary due to a reduction of interaction strength, as in the case of usual electron-hole liquid.

We next comment on the possibility of forming spatially inhomogeneous phases. One example of such inhomogeneity is charge corrugation in the form of stripes. By applying mean field theory to $t$-$J$ model with long range Coulomb interaction $V_c \sum_{i \neq j} \frac{1}{r_{ij}} m_i m_j$, it is shown that stripes are preferred to minimize the exchange $J$ term [15]. In particular, it is the decoupling of the exchange term into the anti-ferromagnetic channel $m_i$ that drives the stripe formation, while the Coulomb interaction controls the spacing evolution of stripes with doping. Moreover, the stripes spacing increases as the doping $\delta$ decreases. The effect of OOPS on stripes is twofold. Firstly, OOPS weakens the on-site repulsion $U$ [16] and thus the superexchange $J \sim t^2/U$ term is enhanced (assuming that strong correlation is still intact). Consequently, the stripes phase is strengthened. On the other hand, Coulomb interaction tends to smooth out the charge density, while a dipolar interaction ($V \sim 1/r^3$ for large $r$) would be less effective and a more inhomogeneous phase would be preferred. Notice that extreme charge inhomogeneity like phase separation [17] is not likely since the OOPS interaction scales like $1/r$ for small $r$ and still suppresses phase separation.
VI. SUMMARY

We have constructed a dipolar interaction for OPS effect of metallic layer in heterostructures and have justified the construction by a RPA calculation. The OPS interaction is expected to be present rather generally at interfaces with metallic layers. We apply the OPS interaction to EHBL system and find that OPS mainly affects the electronic properties in the low density regime. Our conclusion is not restricted to EHBL since the behavior is mainly due to the modification of the interaction scaling from $1/r$ (Coulomb) to $1/r^3$ (dipole) at distance of large $r$. OPS might be employed to eliminate Wigner-crystal like behavior at low temperatures. For strongly correlated electron systems, OPS mainly affects the magnetic channel by reducing the Hubbard $U$ and charge transfer energy $E_c$. The reduction of $U$ may drive the system into usual electron liquid. Furthermore, the reduction in channel by reducing the Hubbard $U$ is the effective intralayer interaction 

\[ V^{\text{eff}}_{\text{intra}}(q) = V_2(q) + \frac{V_{2s}(q) \chi_{0s} V_{s2}(q)}{1 - \chi_{0s} V_{s2}(q)} \]

\[ \approx 2\pi e^2 \frac{1}{q} \left( 1 - e^{-aq} \right) \]

where $V_2(q) = V_{ss}(q) = 2\pi e^2/q$ are the bare Coulomb interactions of layer 2 and screening layer $s$, $V_{2s}(q) = 2\pi e^2 e^{-aq}/q$ is the interlayer Coulomb interaction between the layers, and $\chi_{0s} = \chi_{0s}(q \to 0, \omega = 0) = -N_F$ is the $q \to 0$ static density-density response function of layer $s$, $N_F$ is density of states at the Fermi surface. Notice we have assumed that $q$ is small (long wavelength limit) in writing down the interactions and the calculation by RPA.

\[ F_0 = \frac{2\pi e^2 N_F}{q} + 2\pi e^2 N_F \approx 1 . \]

This approximation is valid if the charge density $n_2$ of layer 2 is much less than the density of the screening layer $n_s$ and $q \ll \sqrt{n_2} \ll \sqrt{n_s}$. We shall take the same limit in the following derivations. This gives eq. (2).

Similarly we can construct the interlayer interaction $V^{\text{eff}}_{\text{inter}}(q)$ with two-layer OPS (Fig. 1(b)) taking into account the screening of system 2' (i.e. integrated out all the screening by 2, s1 and s2):
In the small $q$ limit, $V_{x,2}^{\perp}$ and $V_{\text{intra}}^{1,2}$ scale as $q$ and constant respectively. The second term due to screening is of higher order in $q$ and thus it cannot alter the scaling of the $V_{\text{intra}}^{1}(q)$ term ($\sim$ constant). One can repeat the analysis for the one-layer OPS case (see Fig. 1(c)) and the scaling of the effective intralayer interaction $V_{\text{intra}}^{1,2}(q)$ in the lowest order of $q$ is not affected by screening of the opposite charged layer.

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