Dynamical evolution of bulge shapes

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Figure rotation substantially increases the fraction of stochastic orbits in triaxial systems. This increase is most dramatic in systems with shallow cusps showing that it is not a direct consequence of scattering by a central density cusp or black hole. In a recent study of stationary triaxial potentials (Valluri & Merritt 1998) it was found that the most important elements that define the structure of phase space are the two dimensional resonant tori. The increase in the fraction of stochastic orbits in models with figure rotation is a direct consequence of the destabilization of these resonant tori.

The presence of a large fraction of stochastic orbits in a triaxial bulge will result in the evolution of its shape from triaxial to axisymmetric. The timescales for evolution can be as short as a few crossing times in the bulges of galaxies and evolution is accelerated by figure rotation. This suggests that low luminosity ellipticals and the bulges of early type spirals are likely to be predominantly axisymmetric.

1. Introduction

It is now widely believed that the effects of central black holes and cusps on the dynamics of triaxial galaxies are well understood: the box orbits which form the backbone of triaxial elliptical galaxies become chaotic due to scattering by the divergent central force (e.g. Gerhard & Binney 1985). The scattering of these orbits then results in the evolution of the triaxial galaxy to an axisymmetric one whose dynamics is dominated by well behaved families of regular orbits. Thus most studies of elliptical galaxies still focus on the nature of the regular orbits. Recent investigations of the structure of phase space in triaxial ellipticals have shown that phase space is rich in regular and chaotic regions even in the absence of black holes and steep cusps.

Studying the effects of central black holes on galaxies has taken on renewed importance because of the discovery that many if not most bulge dominated galaxies have central black holes. The existence of central black holes as the end products of the QSO and AGN phenomena is justified by energetic arguments. But less is known about the interplay between the growth of a black hole and the shape of its host galaxy. Most models for the fueling of QSO and AGN require a high degree of triaxiality to transport fuel to the center and to simultaneously transport angular momentum outwards (Rees 1990). Understanding the interplay between black hole growth and galaxy shape is one motivation for studying the behavior of orbits in triaxial potentials.

There have been several studies of the effect of figure rotation on the orbits of stars in triaxial galaxies. Most studies have focused on the behavior of the periodic orbits in the plane perpendicular to the rotation axis. Some authors (Martinet & Udry 1990) found that increasing figure rotation resulted in a decrease in the phase space occupied by the unstable \(x_3\) family and consequently a reduction in the overall chaos. Others (Udry & Pfenniger 1988 and Udry 1991) found that increasing figure rotation had negligible effect on the stochasticity of orbits in 3-dimensional models. More recently it has been shown (Tsuchiya et al. 1993) that orbits of all 4 major families in a perfect ellipsoidal model (completely integrable when stationary) became stochastic when figure rotation is added. Rapidly rotating triaxial bars can be almost completely regular
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(Pfenniger & Friedli 1991) although more slowly rotating bars and bars with high central concentrations generally contain a large fraction of stochastic orbits that eventually destroy the bars (Norman et al. 1996; Sellwood & Moore 1999).

We use the frequency analysis technique (Laskar 1990) to study the behavior of orbits in a family of triaxial density models with figure rotation. The models have a density law that fits the observed luminosity profiles of ellipticals and the bulges of spirals and is given by Dehnen’s law

\[
\rho(m) = \frac{(3 - \gamma)M}{4\pi abc}m^{-\gamma}(1 + m)^{-(4-\gamma)}, \quad 0 \leq \gamma < 3 \tag{1.1}
\]

where

\[
m^2 = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2}, \quad a \geq b \geq c \geq 0 \tag{1.2}
\]

and \(M = 1\) is the total mass. The parameter \(\gamma\) determines the slope of the central density cusp and \(a, b, c\) are the semi-axes of the model. In some cases we also introduced a central point mass \(M_h\) representing a nuclear black hole. The figure rotates about its short axis and the degree of figure rotation can be small (as in the case of giant ellipticals) or reasonably large as in the case of bulges. The co-rotation radius \(R_\Omega\) is parameterized in units of the half-mass radius of the model and ranges from \(R_\Omega = 25\) (slowly rotating) to \(R_\Omega = 3\) (rapidly rotating). Frequency analysis was restricted to \(\sim 10^4\) orbits in each model. Orbits were launched from the equi-effective-potential surface corresponding to the half-mass radius. (Thus all orbits have the same Jacobi Integral, \(E_J = E - \frac{1}{2}\omega_\times r^2\)). The initial conditions for the orbits were selected in two different ways to study orbits from all four major families.

2. Frequency mapping and resonant tori

Laskar’s (1990) frequency analysis technique is based on the idea that regular orbits have 3 isolating integrals of motion which are related to 3 fundamental frequencies. A filtered Fourier transform technique can be used to accurately determine these 3 frequencies \((\omega_x, \omega_y, \omega_z)\). While stochastic orbits do not really have fixed frequencies, quantities resembling frequencies which measure their local behavior can be used to determine how they diffuse in frequency space. Regular orbits come in three types: (1) Orbits in regions that maintain their regular character in spite of departures of the potential from integrable form; (2) orbits associated with stable resonant tori; (3) orbits associated with stable periodic orbits, or “boxlets”.

The use of frequency mapping has shown that even in weakly chaotic systems, it is the resonant tori that provide the skeletal structure to regular phase space (Valluri & Merritt 1998). Frequency mapping provides the simplest method for finding resonant tori. They are families of orbits which satisfy a condition: \(l\omega_x + m\omega_y + n\omega_z = 0\) with \((l, m, n)\) integers. Such orbits are restricted to 2-dimensional surfaces in phase space and we refer to them as thin orbits. Thin boxes are the most generic box orbits in non-integrable triaxial potentials. They avoid the center because they are two-dimensional surfaces. They generate families of 3-D boxes whose maximum thickness is determined by the strength of the central cusp or black hole (Merritt 1999). The closed periodic boxlet orbits lie at the intersection of two or more resonance zones. High order resonances also exist for tube orbit families. Unlike the well known thin tube families around the long and short axes, thin resonant tubes are often surrounded by unstable regions, making it difficult to find them without a technique like frequency mapping.
3. Results: Destruction of the Resonant Tori

A box or boxlet orbit reverses its sense of progression around the rotation axis every time it reaches a turning point. In a rotating frame this means that the path described during the prograde segment of the orbit is not retraced during the retrograde segment. This “envelope doubling” is a consequence of the Coriolis forces on the two segments being different (de Zeeuw & Merritt 1983). Envelope doubling effectively thickens the thin box orbits driving them closer to the center. This results in a narrowing of the stable portion of the resonance layer and renders a large fraction of the orbits stochastic. The degree of “thickening” increases with increasing figure rotation and results in a corresponding rise in the fraction of stochastic box-like orbits.

Figure 1 (a) shows a plot of a quantity measuring the diffusion rates of $10^4$ orbits started at rest at the half-mass equi-potential surface in a non-rotating triaxial model with central cusp slope $\gamma = 0.5$. Only one octant of the surface is plotted. The grey scale is proportional to the logarithm of the diffusion rate: the dark regions indicate initial conditions corresponding to stochastic orbits, the white regions correspond to regular orbits. Figure 1 (b) shows the same set of orbits started from the equi-effective-potential surface of a model with $R_0 = 8$. Rotation results in the broadening of the unstable regions with a resultant narrowing of the stable (white) regions. It also gives rise to new unstable and stable resonances which are seen in Figure 1 (b) as dark striations within the white regions. The increase in the number of resonances and their broadening results in greater overlap of nearby stochastic layers eventually leading to the onset of global stochasticity (e.g. Chirikov 1979).

Contrary to the finding of Tsuchiya et al. (1993) we find that figure rotation has a strong destabilizing effect on inner-long axis tubes. The low angular momentum $z$-tubes and the outer $x$-tubes also become more stochastic. The high angular momentum $z$-tubes are much less affected. The increased stochasticity of tube orbits can be attributed largely to the increase in the width of the stochastic layers associated with the resonant tube.
orbit families. We emphasize that for the tube orbits it is the destabilization of resonant tubes and not scattering by divergent central forces that determines their stability.

4. Conclusions

It is a popular misconception that in the presence of figure rotation box orbits in a triaxial elliptical will loop around the center due to Coriolis forces thereby reducing stochasticity. We find that on the contrary stochasticity increases with increasing figure rotation primarily because the thin box orbits and resonant tubes, which play a crucial role in structuring phase space, are broadened and destabilized by the “envelope doubling” effect.

Models for the fueling of AGN and QSOs require triaxial central potentials which aid accretion onto a black hole, but the same black holes would tend to destroy triaxiality. Low luminosity (\(M_B > -19\)) ellipticals and the bulges of spirals are expected to evolve into axisymmetric shapes on time scales much shorter than the age of the Universe (Valluri & Merritt 1998). If the peanut-shaped bulges in nearby galaxies are in fact triaxial they are probably dynamically young or are composed of only tube like orbits.

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