Repeated crossing of two concentric spherical thin-shells with charge

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Interaction / collision of two concentric spherical thin-shells of linear fluid resulting in collapse has been considered recently. We show that addition of finely tuned electric charges on the shells apart from the cosmological constant serves to delay the collapse indefinitely, yielding an ever colliding system of two concentric fluid shells. Given the finely tuned charges this provides an example of a perpetual two-body motion in general relativity.

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I. INTRODUCTION

Due to frictional energy loss, perpetual motion in Newtonian mechanics remains a myth, unless periodic pumping of the lost energy is supplemented for. It remains still challenging to provide a prototype example of perpetual motion in physics. A stable system of two (or more) objects may emerge as a result of presumably attained perpetual motion. We direct our attention next to general relativity in search for evidence which governs the dynamics of heavenly objects; their stability, evolution, chaotic behavior etc. Our two-body problem will be an exceptionally simple, yet non-trivial one; two concentric spherical thin-shells each made of a gas fluid. Even in such a simple system of concentric binary shells one confronts naturally with repeated passing through / crossing of the shells. This is a highly non-linear process that admittedly realistic solution can hardly be found unless simplifying assumptions are imposed. The exact two-body problem can be solved in Newtonian mechanics. The reason is the consistent definition of a center of mass of the system and absence of gravitational radiation. In general relativity on the other hand the existence of gravitational radiation prevents us to define reliably center of mass of objects in motion. We note that special system at rest are available in general relativity; the charged masses along a line found by Majumdar and Papapetrou is one such example [1–3]. Interaction of two (or more) objects otherwise turns into a highly inelastic process of collision associated with radiation. In such a process the total energy of interacting masses alongside with gravitational and other fields, such as electromagnetic, scalar, spinor, axion etc. transmute gravity into the formation of a newly curved spacetime. We have exact solutions of colliding waves in general relativity that describes evolution / head-on collision of two null fields [4]. We recall that when two null planar shells (both thin and thick) collide the result is a naked singularity. For such an exact process see [5] and references cited therein. When we focus our attention to non-null fields moving slower than light speed we encounter serious technical problems. Collision of shells, branes and bubbles in general relativity have been considered before. Some relatively older works are given in Ref. [6–16] while some more recent studies are given in Ref. [17–31]. In addition to the collision between two shells, Wang and Gao have studied recently the collision between a static spherically symmetric thin-shell wormhole and a thin shell [32]. In their study they have applied the formalism given by Langlois, et al on the conservation laws for collisions of branes and shells in general relativity [33].

In this paper we restrict ourselves to a relatively simple problem of interacting / colliding / crossing of two spherical shells both made of a linear gas. Such a gas / fluid is described by the equation of state (EoS) $P = w \sigma$ in which $P$ and $\sigma$ are the pressure and the energy density of the shells, respectively, and $w$ is a constant. The energy conditions (weak, strong and dominant) are satisfied for $-\frac{1}{3} < w < 1$. The recent interest in such a problem of spherical shell-crossings provides the main motivation for the present study [30, 31]. The process, as advocated by those authors will be a rather special one, namely, the four velocity of each shell remains unchanged during the collision process. Yet the mass / energy (and electric charge added in this paper) of each shell will effect the other shell to create a new spacetime in analogy with the colliding null fields. For the energy conservation and junction conditions we appeal to EoS and the Israel conditions [34–38]. The thin-shell of gas taking part in a collision process is said to satisfy transparency condition provided that only gravitational interaction is active during the crossing.

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The conclusion drawn with the addition of charge to the shells is that oppositely charged shells tend to form a perpetually colliding pair at almost equal distances so that the system points toward a non-collapsing one. Iteration of the exact solution beyond certain number of crossings becomes technically out of our reach. Yet the numerical analysis to certain orders of collisions provides convincing evidence that the pattern tends with finely tuned charges to a non-collapsing system of shells crossing each other indefinitely. It is observed that the positive effect played by the negative cosmological constant in [30, 31] is extended further by the presence of opposite charges on the shells.

Organization of the paper is as follows. In Section II we introduce our formalism. The transparency of collision is described in Section III. The effect of charge is introduced in Section IV. Our paper ends with Conclusion in Section V.

II. THE FORMALISM

Let’s start with two concentric timelike shells identified as \( \Sigma_i \) (inner shell) and \( \Sigma_o \) (outer shell) within a spherically symmetric bulk spacetime. These shells divide the bulk into three different regions given by \( \mathcal{M}_1 \) (inside the inner shell), \( \mathcal{M}_2 \) (between the two shells) and \( \mathcal{M}_3 \) (outside the outer shell) each expressed with the following line element

\[
\left. ds^2_n \right|_{\Sigma_i} = -f_a (r_a) \left( dt^2_a + \frac{dr^2_a}{f_a (r_a)} + r^2_a d\Omega^2_a \right)
\]

in which \( a = 1, 2, 3 \) and \( d\Omega^2_a = d\theta^2_a + \sin^2 \theta_a d\phi^2_a \). Also the induced line element on the shells can be written as

\[
ds^2_i = -dt^2_i + r^2_i d\Omega^2_i
\]

and

\[
ds^2_o = -dt^2_o + r^2_o d\Omega^2_o
\]

in which \( \tau_{i/o} \) is the proper time on each shell and \( r_{i/o} \) is the location of each shell. Note that the two shells are specified with different coordinates. In the vicinity of inner shell one finds \( \Sigma_i = \Sigma_i^{(1)} = \Sigma_i^{(2)}, \ r_1 = r_2 = r_i, \ \theta_1 = \theta_2 = \theta_i, \ \phi_1 = \phi_2 = \phi, \ while \ t_1 \neq t_2 \) and we identify

\[
-f_1 (r_1) \left. dt^2_i \right|_{\Sigma_i^{(1)}} + \left. \frac{dr_i^2}{f_1 (r_1)} \right|_{\Sigma_i^{(1)}} - f_2 (r_2) \left. dt^2_i \right|_{\Sigma_i^{(2)}} + \left. \frac{dr_i^2}{f_2 (r_2)} \right|_{\Sigma_i^{(2)}} = -dr_i^2
\]

which in turn implies that

\[
\left( \frac{dt_{1,2}}{d\tau_i} \right)^2 = \left( \frac{f_{1,2} (r_i)}{f_{1,2}^2 (r_i)} \right)^2.
\]

The normal unit four vectors to the inner shell on either side (from \( \mathcal{M}_1 \) toward \( \mathcal{M}_2 \) ) are given by

\[
\left( n^{(1,2)}_i \right) = \frac{1}{\sqrt{\Delta^{(1,2)}}} \left( -\frac{dr_i}{d\tau_i}, 1, 0, 0 \right),
\]

where

\[
\Delta^{(1,2)} = \frac{f_{1,2}^2 (r_i)}{f_{1,2} (r_i) + \left( \frac{dr_i}{d\tau_i} \right)^2}.
\]

The second fundamental form or the extrinsic curvature tensor of the inner shell is obtained with the nonzero components given by

\[
\left( K^{(1,2)}_i \right) = \frac{2 \frac{dr_i}{d\tau_i} + f_{1,2}'}{2 \sqrt{f_{1,2} (r_i) + \left( \frac{dr_i}{d\tau_i} \right)^2}},
\]
in which a 'prime' denotes $\frac{d}{dr}$. From the Israel junction conditions

$$[K^B_A] - [K] \delta_A^B = -8\pi G S_A^B,$$

with $[Z] = Z^{(2)} - Z^{(1)}$, $[K] = \text{trace } [K^B_A]$ and $S_A^{B(i)} = \text{diag} (-\sigma_i, P_i, P_i)$ which is the energy momentum tensor on the inner shell, one finds

$$\sigma_i = \frac{1}{4\pi G} \left( \frac{\sqrt{f_1(r_i)} + \left( \frac{dr_i}{d\tau_i} \right)^2}{r_i} - \frac{\sqrt{f_2(r_i)} + \left( \frac{dr_i}{d\tau_i} \right)^2}{r_i} \right),$$

$$P_i = \frac{1}{8\pi G} \left( \frac{\frac{d^2 r_i}{d\tau_i^2} + f_3}{2\sqrt{f_2(r_i)} + \left( \frac{dr_i}{d\tau_i} \right)^2} - \frac{\frac{d^2 r_i}{d\tau_i^2} + f_3}{2\sqrt{f_2(r_i)} + \left( \frac{dr_i}{d\tau_i} \right)^2} \right).$$

Note also that, the energy conservation for the inner shell demands that

$$\frac{d\sigma_i}{dr_i} = -\frac{2}{r_i} (P_i + \sigma_i).$$

A similar analysis for the outer shell yields

$$\left( \frac{dt_{2,3}}{d\tau_o} \right)^2 = \frac{f_{2,3}(r_o) + \left( \frac{dr_o}{d\tau_o} \right)^2}{f_{2,3}^2(r_o)},$$

$$\sigma_o = \frac{1}{4\pi G} \left( \frac{\sqrt{f_2(r_o)} + \left( \frac{dr_o}{d\tau_o} \right)^2}{r_o} - \frac{\sqrt{f_3(r_o)} + \left( \frac{dr_o}{d\tau_o} \right)^2}{r_o} \right),$$

$$P_o = \frac{1}{8\pi G} \left( \frac{\frac{d^2 r_o}{d\tau_o^2} + f_3}{2\sqrt{f_2(r_o)} + \left( \frac{dr_o}{d\tau_o} \right)^2} - \frac{\frac{d^2 r_o}{d\tau_o^2} + f_3}{2\sqrt{f_2(r_o)} + \left( \frac{dr_o}{d\tau_o} \right)^2} \right),$$

and

$$\frac{d\sigma_o}{dr_o} = -\frac{2}{r_o} (P_o + \sigma_o)$$

in which the label $o$ stands for the outer shell and $r_o$ is the location of the outer shell.

Next, we consider the EoS on each shell to be that of a linear gas i.e.,

$$P_{i/o} = w_{i/o} \sigma_{i/o},$$

in which $w_{i/o}$ is a constant. Considering the EoS (18) with the energy conservation (13) and (17) one finds

$$\sigma_{i/o} = \frac{C}{r_{i/o}^{2(w_{i/o}+1)}}.$$
in which $C$ is an integration constant. To compare our numerical results with [30, 31] we set $C = \frac{m_{o,i}l^{2w_i}}{4\pi G}$ in which $m_{o,i}$ is a new constant and $\frac{1}{r^2} = -3\Lambda$ in which $\Lambda$ is the cosmological constant to appear in our metric functions. For inner shell one finds the equation of motion, using (11), as

$$\left(\frac{dr_i}{d\tau_i}\right)^2 + V_i = 0 \quad (20)$$

with

$$V_i = \frac{f_2 + f_1}{2} - \frac{(f_2 - f_1)^2}{4\nu_i^2} - \frac{\nu_i^2}{4} \quad (21)$$

in which

$$\nu_i = 4\pi G r_i \sigma_i = \frac{m_i l^{2w_i}}{r_i^{2w_i+1}} \quad (22)$$

where $\sigma_i$ is given in (19). A similar equation can be found for the outer shell such that

$$\left(\frac{dr_o}{d\tau_o}\right)^2 + V_o = 0 \quad (23)$$

in which

$$V_o = \frac{f_3 + f_2}{2} - \frac{(f_3 - f_2)^2}{4\nu_o^2} - \frac{\nu_o^2}{4} \quad (24)$$

with

$$\nu_o = 4\pi G r_o \sigma_o = \frac{m_o l^{2w_o}}{r_o^{2w_o+1}} \quad (25)$$

The time evolution of the two thin shells are with respect to two different proper times i.e., $\tau_o$ and $\tau_i$ and therefore we can not study their collision in terms of a common time. Therefore we use the relations (5) and (20)/(23) to write

$$\frac{dr_i}{d\tau_i} = \frac{dr_i}{dt_2} \sqrt{\frac{f_2(r_i) - V_i}{f_2^2(r_i)}} \quad (26)$$

and

$$\frac{dr_o}{d\tau_o} = \frac{dr_o}{dt_2} \sqrt{\frac{f_2(r_o) - V_o}{f_2^2(r_o)}} \quad (27)$$

Finally we plug these into (20) and (23) to determine the evolution of the thin-shells in terms of the common time $t_2$ given by

$$\left(\frac{dr_{i/o}}{dt_2}\right)^2 + \frac{f_2^2(r_{i/o})}{f_2(r_{i/o})} \frac{V_{i/o}}{V_{i/o}} = 0 \quad (28)$$

### III. TRANSPARENCY OF THE COLLISION PROCESS

In this section we consider a transparent collision between the two shells which has been studied by Ida and Nakao in [39]. According to the condition of transparency, the four velocities of the shells are conserved during the collision i.e. the four velocities are continuous. Without going through the detail of the calculation we follow [39] and consider the two shells passing through each other and divide the bulk spacetime into three parts which are labeled as $\mathcal{M}_1$, $\mathcal{M}_4$ and $\mathcal{M}_3$ such that the inner shell after the collision was the outer shell before the collision and vice versa. Therefore the
FIG. 1: Time evolution of each shell independently are depicted at the top figure. Time evolution of the inner and outer shells when both shells are chargeless but interact gravitationally are shown at the bottom figure. We choose $Q_a = 0$, $M_1 = 0.00$, $M_2 = 0.025$, $M_3 = 0.05$, $m_{o/i} = 0.0136$, $w_{o/i} = 0.2$, $l = 1$ and the initial position of both shells set to be $r_{o/i} (t_2 = 0) = 1.50$. The vertical axis is logarithmic.

FIG. 2: Time evolution of each shell independently are depicted are shown at the top figure. Time evolution of the inner and outer shells when one shell (initially inner) is charged while the other is chargeless are shown at the bottom figure. We have $Q_1 = 0$, $Q_2 = Q_3 = 0.15$, $M_1 = 0.00$, $M_2 = 0.025$, $M_3 = 0.05$, $m_{o/i} = 0.0136$, $w_{o/i} = 0.2$, $l = 1$ and the initial position of both shells set to be $r_{o/i} (t_2 = 0) = 1.50$. The vertical axis is logarithmic.
FIG. 3: Time evolution of each shell independently are depicted at the top. Time evolution of the inner and outer shells when both shells are charged are given at the bottom figure. Precisely, \( Q_1 = 0, Q_2 = -Q_3 = -0.15, M_1 = 0.00, M_2 = 0.025, M_3 = 0.05, m_{o/i} = 0.0136, w_{o/i} = 0.2, l = 1 \) and the initial position of both shells set to be \( r_{o/i}(t_2 = 0) = 1.50 \). The vertical axis is logarithmic.

The spacetime on the sides of \( \Sigma_{o/i} \) are given by (1) but \( a = 1, 4 \) for \( \Sigma_o \) and \( a = 4, 3 \) for \( \Sigma_i \). Applying the transparent collision conditions one finds the line element of \( \mathcal{M}_4 \) given by (1) with

\[
f_4(r) = \frac{\nu_i^2 + \nu_o^2}{2} + \frac{f_1 + f_3 - f_2}{2} + \frac{(\nu_i^2 - f_1)(\nu_o^2 - f_3)}{2f_2} + \frac{\epsilon_1\epsilon_o}{2f_2} \sqrt{\nu_i^4 - 2(f_1 + f_2)\nu_i^2 + (f_1 - f_2)^2} \sqrt{\nu_o^4 - 2(f_2 + f_3)\nu_o^2 + (f_2 - f_3)^2}
\]

in which \( \epsilon_{i/o} = \pm 1 \) stands for the direction of the motion of the shells, i.e., if the shell is moving radially outward/inward \( \epsilon_{i/o} = +1/-1 \). Let’s add that after the collision the equation found in previous sections are all valid provided the substitutions \( o \leftrightarrow i \), and \( 2 \rightarrow 4 \) are made in all equations. For instance, the equation of motion of the shells after the collision is given by

\[
\left( \frac{dr_{o/i}}{dt_2} \right)^2 + \frac{f_4^2(r_{o/i})V_{o/i}}{f_4(r_{o/i}) - V_{o/i}} = 0
\]

in which

\[
V_{o/i} = \frac{f_4 + f_1/3}{2} - \frac{(f_4 - f_1/3)^2}{4\nu_{o/i}^2} - \frac{\nu_{o/i}^2}{4}
\]

with

\[
\nu_{o/i} = 4\pi Gr_{o/i}\sigma_{o/i} = \frac{m_{o/i}l^{2\omega_{o/i}}}{r_{o/i}^{2\omega_{o/i}+1}}.
\]

Let’s add that, in Eq. (30) a sub \( o \) implies that the shell was initially the outer shell. After the first collision, although we still use the same label physically the outer shell becomes the new inner shell. Therefore following to each collision
the order of the shells changes and no matter what we set them initially (which is indicated by their sub-indices o or i) they can be inner or outer shells with time. Finally we would like to add that the above formalism is applicable for a generic spherically symmetric bulk spacetime with the line element given in (1). Therefore in addition to the vacuum spherically symmetric bulk one may also consider the case with energy momentum tensor which due to spherical symmetry reads as

$$T_{\mu}^\nu = \text{diag} \left( -T, -T, \tilde{T}, \tilde{T} \right)$$ \hspace{1cm} (33)

in which $T$ and $\tilde{T}$ are found by applying the Einstein equations in separate regions.

IV. INTERACTING CHARGED SHELLS

As stated in Introduction, our aim is to investigate the effect of electric / magnetic charge on the collapsing shells introduced by [30, 31]. To accomplish this we consider the spacetimes to be Reissner-Nordström AdS with a line element given by (1) while

$$f_a = 1 - \frac{2M_a}{r} + \frac{Q_a}{r^2} + \frac{r^2}{l^2}$$

in which $a = 1, 2, 3$ refer to the inside, in between and outside of the shells, respectively, before each collision. After each collision the metric function of inside / $a = 1$ and outside / $a = 3$ remain unaltered but the metric between the shells changes as given by (29) and this sequence either goes on forever or ends with a collapse. In Fig.1 we reproduced the case reported in [30, 31] with $Q_a = 0, M_1 = 0.00, M_2 = 0.025, M_3 = 0.05, m_{a/i} = 0.0136, \omega_{a/i} = 0.2, l = 1$ and initial position of both shells are set to be at $r_{a/i}(t_2 = 0) = 1.50$. As it was reported in [30, 31] the two shells after making three collisions eventually collapse and a black hole is formed. In Fig. 2 we solve numerically the same two-shells system with charges on one of the shells only. We choose $Q_1 = 0, Q_2 = Q_3 = 0.15$ and the other parameters including the initial conditions are the same as in Fig. 1. The charges on the shells can be calculated by using Gauss’s law which yields the charge on the inner and outer shells to be $q_i = 0.15$ and $q_o = 0$ initially. We observe that adding positive charge on one of the shells and keeping the rest of configuration the same causes the system collide two more times but eventually collapsing to make a charged black hole as the former case. In our Fig. 3 we have added charges on both shells. Initially the charge on inner and outer shells are chosen to be $q_i = -0.015$ and $q_o = 0.030$ which in turn implies $Q_1 = 0, Q_2 = -Q_3 = -0.015$. The other parameters and initial positions are as in Figs. 1 and 2. We observe that the shells initially collide quite the same as the other two cases but owing to the opposite charges on the shells the collapse does not occur and the shells after making finite number of crossings return almost to their initial conditions. This is what we may interpret as a stable configuration which repeats itself periodically but eternally. Let’s end this section by adding that at the top of each figure, for comparison, we also present the motion of each shell independent of the other.

We would like to add that, in this specific study our aim was to show that a collapsing system of repeated crossing of two concentric chargeless spherical thin-shells may be stabilized without altering the masses of the shells but adding enough opposite charges on the shells. As the nature of the collisions and the equations are highly non-linear we could not find an analytic correlations between the masses and charges necessary but as we have shown numerically in the specific examples, such charges exist. Correlation between masses and added charges can only be obtained by more number of collisions and tabular account for both quantities.

V. CONCLUSION

The dynamics of two concentric spherical binary thin-shells made of gas satisfying a linear equation of state is revisited. As a new element in this paper we added electric charge to the shells and investigated their evolution / collision in a suitable time variable. The problem was considered previously without charge but with a negative cosmological constants in Ref. [30, 31]. The confining boundary role of AdS in [30, 31] is shown to be valid also in the presence of finely-tuned opposite charges on the shells. Adding charge on one shell did not change the ultimate collapse of the binary-shell system but given the finely tuned charges on both shells with opposite signs we obtain a non-collapsing, permanently colliding system. Such a system may have impact in cosmological formations as an
example of self-sustaining two-body problem with perpetual motion.

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