An example of a uniformly accelerated particle detector with non-Unruh response

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Abstract

We propose a scalar background in Minkowski spacetime imparting constant proper acceleration to a classical particle. In contrast to the case of a constant electric field the proposed scalar potential does not create particle-antiparticle pairs. Therefore an elementary particle accelerated by such field is a more appropriate candidate for an "Unruh-detector" than a particle moving in a constant electric field. We show that the proposed detector does not reveal the universal thermal response of the Unruh type.

Key words: Unruh effect, uniformly accelerated detector, external scalar field, vacuum thermalization

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1 Introduction

It was claimed in [1,2] that a detector uniformly accelerated in the empty Minkowski spacetime would respond as if it had been placed in a thermal bath of Fulling-Unruh quanta [2,3] with the temperature \( T_{DU} = a/(2\pi) \), where \( a \) is the proper acceleration of the detector. The temperature \( T \) is now called

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1 In this paper we consider a particular case of 1 + 1 dimensional spacetime.

2 We use natural units \( c = \hbar = 1 \) throughout the paper.
the Davies-Unruh temperature, while the effect of "thermalization" of the Minkowski vacuum in accelerated reference frames is usually referred to as the Unruh effect (more detailed list of publications concerning this issue can be found in [4]). Let us stress that the most striking feature of this prediction is universality of the response of uniformly accelerating detectors. The latter would mean that this response were thermal independently of the structure of the given detector and of the nature of the accelerating force. Such universality, if existed, would manifest that interaction of accelerated detectors with the vacuum of a quantum field depended exclusively on general quantum properties of the vacuum viewed from a uniformly accelerated reference frame. Moreover, if the effect existed the uniformly accelerated reference frames would be physically preferable among all other reference frames in general relativity.

There exist two different aspects of the Unruh problem [4]. The first one is of purely field theoretical nature and has been formulated in the original Unruh work [2]. In the framework of this approach one develops a quantization scheme for the field restricted to a part of Minkowski spacetime, say Fulling quantization in the Rindler wedge [3], and then attempts to interpret the Minkowski vacuum state in terms of particle states arising in the process of this quantization, say Fulling-Unruh particles [2,3]. Another variant of the field theoretical approach, though physically very close to the Unruh one, is based on the Bisognano-Wichmann theorem [5,6]. It is important that the notion of detector in fact is not exploited by both variants of the quantum field theoretical approach to the Unruh problem, and thus this approach (if it was consistent) could give grounds for the aforementioned universality of the response of a uniformly accelerated detector. Indeed, the term "Rindler observer" is used in the context of the Unruh problem exclusively for the sake of convenience and only because the totality of the world lines of all such "observers", which however cannot be realized as trajectories of any physical objects in Minkowski spacetime, completely covers the interior of the Rindler wedge [4]. However, we have shown in Refs. [4,7–9] that both variants of quantum field theoretical approach [2,5] are physically incorrect, because they imply that the quantum field satisfies some special requirement at the boundary of the Rindler manifold which is incompatible with existence of the Minkowski vacuum state in the theory. Nevertheless, this conclusion is based on analysis of two specific sets of arguments [2,5] and, strictly speaking, it does not exclude the possibility of existence of some more profound evidence in favor of the Unruh effect.

Another aspect of the problem is behavior of physical detectors of the given structure uniformly accelerating under action of the given external force. Certainly, this approach is not so general as the first one and there is no hope to prove universality of the Unruh effect in its framework. Therefore this issue is usually discussed in literature with the purpose of illustration of the results obtained by means of quantum field theory, see, e.g., the reviews [10,11].
However, since there is still no compelling evidence for the universal behavior attributed to all uniformly accelerated detectors, the investigation of the response of particular detectors is of great physical interest. Besides discussion of possible experimental observations of the effects of interaction of different quantum systems with external fields, non-universality of thermal response could be proven within such approach just by demonstration of at least a single example of a uniformly accelerated detector which does not reveal the Unruh behavior.

Consideration of behavior of a uniformly accelerating physical detector is a rather difficult problem and its treatment in literature is contradictory. The major difficulty is that a uniformly accelerating detector (i.e., an elementary particle or a microscopic bound system) must be considered as a quantum object moving along a definite classical trajectory. Strictly speaking, such an assumption is in contradiction with the uncertainty principle, its range of applicability is very limited and therefore it must be used with proper care. Since a systematic relativistic theory of bound states is still absent, different authors are compelled to use some simplifying assumptions, which are hard to control. Therefore we will consider below the case of an elementary particle detector, which, in our opinion, admits the most consistent analysis at the present moment.

An example of an elementary particle detector accelerated by a homogeneous electric field in 1+1-dimensional spacetime was considered in Refs. [12,13]. The considered detector is described by two equally charged bosons with masses $m$ and $M > m$ interacting with a neutral boson of mass $\mu$. In the absence of the external electric field there obviously exists transition $M \to m + \mu$, if $M^2 > m^2 + \mu^2$. In the presence of the electric field spontaneous excitation of a $m$-boson into a $M$-particle accompanied by the emission of $\mu$-particle is also possible. It was shown in Refs. [12,13], that for the special case of equal accelerations of the charged bosons $(M - m)/m \ll 1$, the ratio of the transition rates of the two processes $m \to M + \mu$ and $M \to m + \mu$ has the Boltzmann form, with temperature parameter coinciding with the Davies-Unruh temperature. Though the latter result holds only for $D = 1 + 1$ spacetime, see Ref. [14], the authors of Refs. [12,13] interpret their treatment as purely quantum mechanical derivation of the Unruh effect since recoil effects have been taken into account. However the presence of the electric field violates stability of detector vacuum. Hence the detector is moving not in empty Minkowski space but in a bath of pairs of $(m, m)$ and $(M, M)$ charged bosons. Moreover, the authors emphasize the fact that the processes of detector transitions and the Schwinger process of creation of boson pairs are in exact equilibrium. Thus relevance of the suggested detector model to the Unruh effect which implies thermal equilibrium between detector and vacuum of $\mu$-boson field remains unclear.
In this paper we consider another mechanism of acceleration of an elementary particle detector, namely acceleration by a stationary scalar background. The most important point of our model is absence of the process of pair creation by the external scalar field. The detector is described by a fermion $F$ of mass $m$ and a boson $B$ of mass $M > m$ interacting with a free massless fermion $\nu$ (which we will call "neutrino"). We consider the $F$-particle as a ground state, and the $B$-particle as an excited state of the detector. Just as in Refs. [12,13] we restrict our consideration to the $1 + 1$-dimensional case.

2 Uniform acceleration by an external scalar field

Let us consider a classical particle in $D = 1 + 1$ spacetime coupled to an external stationary scalar field $\Lambda(z)$. The Lagrange function for this particle can be written in the form $L(z, v) = -m e^{\Lambda(z)} \sqrt{1 - v^2}$, where $v = dz/dt$ denotes velocity of the particle. The conserved canonical Hamilton function reads

$$H = v \frac{\partial L}{\partial v} - L = \frac{m e^{\Lambda(z)}}{\sqrt{1 - v^2}} = \epsilon = \text{const},$$

and thus for any given value $\epsilon$ of energy the velocity $v(t)$ can be represented as a function of its coordinate $z$, $v = v(z)$. The proper acceleration of the particle can be represented as

$$a = \left(\frac{dv}{dt}\right) \left(1 - v^2\right)^{3/2} = \frac{\epsilon}{m} \frac{d(e^{-\Lambda(z)})}{dz}.$$  

Hence the only form of the scalar background $\Lambda(z)$ which can provide a uniform acceleration of the particle, is $\Lambda(z) = -\ln[z - z_s]/R$, where $R$ is a characteristic length parameter of the background. The integration constant $z_s$ may be arbitrary due to translation invariance. It fixes the position of singularity of the external field, which is unavoidable in the case of a uniformly accelerating motion.

We choose below $z_s = 0$ and consider the particle motion in the right half of the space with respect to the singularity, $z > 0$, where $\Lambda(z) = -\ln(z/R)$. With such $\Lambda(z)$ one easily obtains from Eq.(1) for particle trajectories the equation: $z(t) = \sqrt{t^2 + a^{-2}}$, where proper acceleration is equal to $a = \epsilon/(mR)$ and the integration constant is chosen so that the time coordinate of the turning point of the trajectory is $t_0 = 0$. It is worth noting that the totality of obtained trajectories with $0 < \epsilon < \infty$ completely fill the interior of the right (Rindler) wedge of Minkowski spacetime. Therefore acceleration by the scalar background is more appropriate for studying the Unruh problem than
acceleration by a homogeneous electric field. In the latter case acceleration of a particle is determined solely by the field strength and is independent of initial conditions. The totality of such trajectories can cover the whole Minkowski spacetime but of course cannot be restricted to any separate wedge of it.

3 Elementary particle detector in scalar background

We will discuss now behavior of a quantum detector accelerated by the scalar background introduced in the preceding section. As a rule, a kind of "two-level" system is used as a detector while discussing the Unruh effect [10–13]. Such a detector is accelerated by an external force which does not affect however the field responsible for transitions of the "two-level" system and hence the detector can be considered moving in undisturbed vacuum of the latter field. Since the scalar background $\Lambda(z)$ accelerates any massive particle, the field responsible for transitions of our detector should be massless. However it is well known, see, e.g., Ref. [15], that in $D = 1 + 1$ theories a massless scalar particle does not exist even as a mathematical object. Therefore we have to consider a "two-level" system interacting with a massless fermion ("neutrino-antineutrino") field as a model of the detector and hence one of the particles constituting our detector must also be a fermion. We start with discussion of solutions of Klein-Fock-Gordon (KFG) and Dirac equations in the presence of the scalar background $\Lambda(z)$.

The squared Hamilton function (1) for a boson of mass $M$ can be written in the form $H^2 = \Pi^2 + M^2 e^{2\Lambda(z)}$, where $\Pi = \partial L/\partial v$ is momentum of the particle. The KFG equation is obtained from this relation by the standard substitutions $H \rightarrow i\partial/\partial t$, $\Pi \rightarrow -i\partial/\partial z$:

$$\left\{ \frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial z^2} + M^2 e^{2\Lambda(z)} \right\} \Psi_B(t, z) = 0.$$ (3)

Positive frequency ($\epsilon > 0$) solutions of Eq.(3) has the form $\Psi_B(t, z) = \psi_B\epsilon(z)e^{-i\epsilon t}$ where the functions $\psi_B\epsilon(z)$ satisfy the stationary Schrödinger equation

$$\psi''_B\epsilon(z) + \left( \epsilon^2 - \frac{M^2 R^2}{z^2} \right) \psi_B\epsilon(z) = 0.$$ (4)

The penetrability coefficient for the barrier $M^2 R^2 / z^2$, which controls the possibility of quantum tunnelling through the singularity of the background, can
be estimated quasiclassically (see, e.g., Ref. [16]),

\[ D \sim \exp \left\{ -2 \int_{0}^{1/a} dz \sqrt{\frac{M^2R^2}{z^2} - \epsilon^2} \right\} = 0, \tag{5} \]

and is equal to zero since the integral in the argument of the exponential in (5) is logarithmically divergent on its lower limit. It means that the potential barrier which surrounds the singularity \( z = 0 \) is impenetrable even on the quantum level. As a consequence, the boundary condition \( \psi_B(0) = 0 \) should be imposed at the singularity. The field modes satisfying this boundary condition read

\[ \Psi_B(t, z) = \psi_B(z)e^{-i\epsilon t} = \sqrt{\frac{z}{2}} J_{\frac{1}{2}}(\sqrt{M^2R^2 + 1/4}(\epsilon z)) e^{-i\epsilon t}, \tag{6} \]

where \( J_{\kappa}(\epsilon z) \) is the Bessel function, and the normalization constant is defined by the condition

\[ i \int_{0}^{\infty} dz \, \psi_B^*(t, z) \frac{\partial}{\partial t} \psi_B(t, z) = \delta(\epsilon - \epsilon'). \tag{7} \]

The Dirac equation in the accelerating scalar background reads

\[ \left[ i \left( \gamma_0 \frac{\partial}{\partial t} - \gamma_1 \frac{\partial}{\partial z} \right) - me^{A(z)} \right] \Psi_F(t, z) = 0, \tag{8} \]

where \( \Psi_F \) is a two-component function. We adopt the following representation for the Dirac \( \gamma \)-matrices, \( \gamma_0 = \sigma_3, \gamma_1 = i\sigma_1 \), where \( \sigma_i \) are the standard Pauli matrices. Due to the impenetrability of the potential barrier at the point \( z = 0 \) (see the Eq.(5)), one should impose at this point the boundary condition of vanishing of the local scalar current \( s = \bar{\Psi}_F \Psi_F, s(t, 0) = 0 \). The positive- (negative-) frequency modes, which obey such a boundary condition and are normalized by the relations

\[ \int_{0}^{\infty} dz \, \Psi_{F\epsilon}^{(\pm)\dagger} \Psi_{F\epsilon'}^{(\pm)} = \delta(\epsilon - \epsilon'), \quad \int_{0}^{\infty} dz \, \Psi_{F\epsilon}^{(\pm)\dagger} \Psi_{F\epsilon'}^{(\mp)} = 0, \tag{9} \]

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\[ \Psi_{F\epsilon}^{(\pm)}(t, z) = \psi_{F\epsilon}(z)e^{\mp i\epsilon t} = \frac{\sqrt{\epsilon z}}{2} \begin{pmatrix} J_{mR-1/2}(\epsilon z) \pm J_{mR+1/2}(\epsilon z) \\ J_{mR-1/2}(\epsilon z) \mp J_{mR+1/2}(\epsilon z) \end{pmatrix} e^{\mp i\epsilon t}. \tag{10} \]
Finally, we write down the positive- (negative-) frequency solutions for the free massless Dirac equation

\[ \Psi^{(\pm)}_{\nu p}(t, z) = \psi_{\nu p}(z) e^{\mp i|p|t} = \frac{1}{2\sqrt{\pi}} \left( \frac{1}{\pm i \text{sign}(p)} \right) e^{i(pz \mp |p|t)}, \quad (11) \]

\(-\infty < p < +\infty\). Since massless particles do not interact with the scalar background the functions (11) do not obey any boundary condition at the point \( z = 0 \). We use the sets of modes (6),(10),(11) for quantization of respectively the massive boson \( \Phi_B \), fermion \( \Phi_F \) and neutrino \( \Phi_\nu \) fields.

Let the Lagrangian of interaction between the three fields be of the form \( \mathcal{L}_{\text{int}} = \lambda (\bar{\Phi}_F \Phi_\nu + \bar{\Phi}_\nu \Phi_F)\Phi_B \). We will consider, first, the decay of the massive fermion \( \Phi_F^{\epsilon} \to \Phi_B^{\epsilon'} + \nu_p \) which does not occur in the absence of the scalar background. Let the initial state of the fermion be a wave packet \( \left| \Phi_F^{\epsilon} \right> = \int_{\epsilon}^{\infty} dE C_i(E) \bar{f}_E \left| \text{vac} \right> \) with the spectral weight function \( C_i(E) \) normalized by the condition \( \int_{\epsilon}^{\infty} |C_i(E)|^2 dE = 1 \) and \( \bar{f}_E \) is fermion creation operator. Then the matrix element of the decay in the first order of perturbation theory is given by

\[ \langle \Phi_B^{\epsilon'}, \nu_p | \Phi_F^{\epsilon} \rangle = -i\lambda \int_{-\infty}^{\infty} dt \int_{0}^{\infty} dz \int_{0}^{\infty} dE \, C_i(E) \bar{\Psi}_B^{\epsilon'}(t, z) \Psi^{(+)}_{\nu p}(t, z) \bar{\Psi}^{(+)}_{F \epsilon}(t, z). \quad (12) \]

Substituting Eqs.(6),(10),(11) into (12) we obtain

\[ \langle \Phi_B^{\epsilon'}, \nu_p | \Phi_F^{\epsilon} \rangle = -i\lambda C_i(\epsilon' + |p|) I(\epsilon', p), \quad (13) \]

where

\[ I(\epsilon', p) = \int_{0}^{\infty} dz \psi_{\Phi_B^{\epsilon'}}^*(z) \bar{\psi}^{(+)}_{\nu p}(z) \psi^{(+)}_{F \epsilon' + |p|}(z). \quad (14) \]

In the case of narrow spectral weight function of the initial packet, i.e. when all probable values of the energy of the \( F \)-particle are very close to \( \epsilon \), we can substitute \( |C_i(E)|^2 \approx \delta(E - \epsilon) \). Then we get for the differential probability of
the decay
\[ dW_{F \rightarrow B}(\epsilon', p|\epsilon) = |\langle B_{\epsilon'}, \nu_p|F\rangle|^2 d\epsilon' dp = \]
\[ = \lambda^2 \delta(\epsilon - \epsilon' - |p|) |I(\epsilon', p)|^2 d\epsilon' dp. \] (15)

It is easy to see that the integral (14) is divergent at \( p \to 0 \). Indeed, the asymptotic behavior of the modes (6), (10) is given by the following expressions

\[ \Psi_{B\epsilon}(t, z) \sim \frac{1}{\sqrt{\pi \epsilon}} \cos(\epsilon z - \alpha) e^{-i\epsilon t}, \quad \alpha = \frac{\pi}{2} \left( \sqrt{M^2 R^2 + \frac{1}{4} + \frac{1}{2}} \right), \] (16)

\[ \Psi_{F\epsilon}^{(+)}(t, z) \sim \frac{1}{\sqrt{\pi}} \begin{pmatrix} \cos(\epsilon z - \beta) \\ -\sin(\epsilon z - \beta) \end{pmatrix} e^{-i\epsilon t}, \quad \beta = \frac{\pi}{2} \left( mR + \frac{1}{2} \right). \]

Note that the asymptotic expressions (16) in fact coincide with \( \Psi \)-functions of free massless particles. This is because the mass defect at long distance from the singularity equals to the mass of the particle and the latter moves almost with the speed of light. Substituting expressions (16) into the integral (14) at \( p = 0 \), we arrive to a linear divergency of the integral (14) at \( z \to \infty \). It means that \( I(\epsilon', p) \sim 1/p, p \to 0 \), and we get for the probability (15)
\[ dW_{F \rightarrow B}(\epsilon', p|\epsilon) \sim \frac{\lambda^2}{\epsilon} \delta(\epsilon - \epsilon' - |p|) \frac{dp}{p^2}, \quad p \to 0. \] (17)

From this expression it is evident that the total probability of the decay is divergent. This fact has a simple physical explanation. Indeed, suppose the decay occurs after reflection from the singularity, when the wave packet \( |F\rangle \) is moving to the right. Due to special properties of the fermion current \( \bar{\Psi}_F \Psi_F \) in \( D = 1 + 1 \) dimensions (see, e.g., Ref. [17]), the neutrino with energy \( |p| \) is emitted strictly to the left. Thus, due to the energy conservation law, the boson \( B \) with the energy \( \epsilon' = \epsilon - |p| \) can be emitted either to the right (in such a case the momentum transferred to the external field is \( \Delta P = 2|p| \)) or to the left (\( \Delta P = 2\epsilon \)). It is easy to see that the second process can occur only at distances \( z \lesssim 1/\epsilon \) from the singularity. Hence only the first process contribute to the total probability at long times since \( p \) can be arbitrary small. If the time of observation \( \tau \) is finite, then the neutrinos with the energies \( |p| \lesssim \tau \) can not be emitted. Thus for the total probability at long time we obtain
\[ W_{F \rightarrow B}(\epsilon) \sim \frac{\lambda^2}{\epsilon} \int \frac{dp}{p^2} = \frac{\lambda^2}{\epsilon} \tau, \quad \tau \to \infty. \] (18)
It means that there exists the decay rate $R_{F \rightarrow B} \sim \lambda^2 / \epsilon$ (more rigorous derivation of this result will be given in a forthcoming publication).

The lowest-order amplitude of the process $B_{\epsilon'} \rightarrow F_{\epsilon} + \bar{\nu}_p$ reads

$$\langle F_{\epsilon}, \bar{\nu}_p | B \rangle =$$

$$= -i\lambda \int_{-\infty}^{\infty} dt \int_{0}^{\infty} dz \int_{0}^{\infty} dE C_i(E) \Psi_{B E}(t, z) \bar{\Psi}_{F \epsilon}(t, z) \Psi_{\nu p}(t, z).$$

(19)

It is easy to see that the decay rate for this process can be calculated by exactly the same method which have been used for calculation of $R_{F \rightarrow B}$ and give the result $R_{B \rightarrow F} \sim \lambda^2 / \epsilon'$.

4 Long-time behavior of the accelerated detector

It was shown in the preceding section that at long times $\tau$ ($1/\epsilon \ll \tau \ll \epsilon / \lambda^2$) the processes of neutrino, antineutrino emission are characterized by decay rates. It means that master equations can be used to study the long-time behavior of the detector. One can easily obtain from Eq.(17) that the average neutrino energy emitted for time $\tau$ is of the order $\langle |p| \rangle \sim \lambda^2 / \epsilon \ln \epsilon \tau$. Hence, with regard for the upper limit for $\tau$ admitted in the framework of perturbation theory, we have $\langle |p| \rangle \ll \epsilon$. This result means that neutrinos are effectively emitted in zero mode at long times and thus the master equations for the problem constitute a set of differential equations rather than of integrodifferential ones.

Denote by $n_F, n_B$ the average numbers of fermions and bosons with the energy $\epsilon$ and by $n_{\nu}, n_{\bar{\nu}}$ the average numbers of neutrinos and antineutrinos in the zero mode. Let $R_{F \rightarrow B} = R_{B \rightarrow F} = R$. Then the corresponding master equations, which take into account effects of quantum statistics, read

$$\frac{1}{R} \frac{dn_F}{dt} = n_B(1 - n_F)(1 - n_{\bar{\nu}}) - n_F(1 + n_B)(1 - n_{\nu}),$$

$$\frac{1}{R} \frac{dn_{\nu}}{dt} = n_F(1 + n_B)(1 - n_{\nu}), \quad n_F + n_{\nu} - n_{\bar{\nu}} = 1, \quad n_F + n_B = 1.$$ 

(20)

The results of numerical solution for the system (20) are presented in Fig. 1. The initial conditions adopted for the presented solution are $n_F(0) = 1$ and $n_B(0) = n_{\nu}(0) = n_{\bar{\nu}}(0) = 0$. 


It is seen from Fig. 1 that, while \( t \ll \mathcal{R}^{-1} \), the initial fermion decays into boson and neutrino. For \( t \geq \mathcal{R}^{-1} \) the number of neutrinos in the zero mode tends to unity exponentially fast, \( 1 - n_\nu(t) \propto \sqrt{t} e^{-\mathcal{R}t} \). However, when \( n_\nu \) comes to unity, this channel owing to the Pauli principle becomes closed. The number of antineutrinos is initially growing much more slowly, since the antineutrinos are created by boson decay, and no bosons are initially present. After \( n_\nu \) comes to unity, the created boson decays into fermion and antineutrino. Finally, the number of antineutrinos also comes to unity, and this channel also becomes closed. Asymptotically the resulting state contains a fermion and a \( \nu \bar{\nu} \)-pair in the zero mode. However, this state is achieved very slowly, \( 1 - n_F(t) \approx 1 - n_\nu(t) \approx n_B(t) \approx (2\mathcal{R}t)^{-1/2} \) as \( t \to +\infty \).

Thus the long time behavior of the detector accelerated by scalar background crucially differs from the behavior one could predict on the basis of the Unruh conjecture. Indeed, instead of arriving to some "thermal equilibrium" state, the detector creates a \( \nu \bar{\nu} \)-pair and returns to its initial state. Note that, if we change the initial conditions for the detector \( n_F = 1, n_B = 0 \to n_F = 0, n_B = 1 \), we will have boson in the final state instead of the fermion. It means that the detector remembers its initial state and hence one by no means can talk about equilibrium in the final state of this particular case of detector. From this we conclude that the thermal Unruh response is not a universal property of all uniformly accelerated detectors but can be revealed by detectors of some special nature, see Refs. [11,12].

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Fig. 1. Long-time behavior of the occupation numbers $n_F$ (a solid line), $n_B$ (a dash line), $n_\nu$ (a dash dot line), and $n_{\nu}$ (a dot line).