Nuclear Effects on the Extraction of $\sin^2 \theta_W$

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Abstract

We study the impact of nuclear effects on the extraction of the weak-mixing angle $\sin^2 \theta_W$ from deep inelastic (anti-)neutrino-nucleus scattering, with special emphasis on the recently announced NuTeV Collaboration $3\sigma$ deviation of $\sin^2 \theta_W$ from its standard model value. We have found that nuclear effects, which are very important in electromagnetic deep inelastic scattering (DIS), are quite small in weak charged current DIS. In neutral current DIS processes, which contain the weak mixing angle, we predict that these effects play also an important role and may dramatically affect the value of $\sin^2 \theta_W$ extracted from the experimental data.

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I. INTRODUCTION

It is a common belief that the standard model (SM) is a low energy remnant of some more fundamental theory. In fact, the recent observation of neutrino oscillations was the first unambiguous indication of the presence of physics beyond the SM, which allows for non-zero neutrino masses and lepton flavor violation processes, forbidden in the SM. This evidence stimulates further searches for deviations from the SM predictions in physical observables related to other sectors of the SM.

The precise determination of the weak-mixing angle $\sin^2 \theta_W$ plays a crucial role in testing the SM of electroweak interactions. Its present value is consistent with all the known electroweak observables [1].

A recent announcement by the NuTeV Collaboration [2] on a $3\sigma$ deviation of the value of $\sin^2 \theta_W$ measured in deep inelastic neutrino-nucleus (Fe) scattering with respect to the fit of the SM predictions to other electroweak measurements [1], may be the sign of new physics beyond the SM. This result takes into account various sources of systematic errors. However, there still remains the question of whether the reported deviation can be accounted for by SM effects not properly implemented in the analysis of the experimental data. In the present paper we examine the role of nuclear effects on the extraction of $\sin^2 \theta_W$, since an iron nuclear target was actually used in the NuTeV experiment.

The observables measured in this experiment are ratios of neutral (NC) to charged (CC) current events, related by a sophisticated Monte Carlo simulation to $\sin^2 \theta_W$. In order to examine the possible impact of nuclear corrections on the extraction of $\sin^2 \theta_W$, we study the corresponding ratios

$$R^\nu_A = \frac{\sigma(\nu_\mu + A \rightarrow \nu_\mu + X)}{\sigma(\nu_\mu + A \rightarrow \mu^- + X)}, \quad (1)$$

$$R^{\bar{\nu}}_A = \frac{\sigma(\bar{\nu}_\mu + A \rightarrow \bar{\nu}_\mu + X)}{\sigma(\bar{\nu}_\mu + A \rightarrow \mu^+ + X)} \quad (2)$$

of neutral current (NC) to charged current (CC) neutrino (anti-neutrino) cross sections for a nuclear target $A$. As is known, neglecting nuclear effects for an isoscalar target, one can extract the weak-mixing angle by using the Llewellyn-Smith relation [3]:

$$R^\nu_N = \frac{\sigma(\nu_\mu(\bar{\nu}_\mu) + N \rightarrow \nu_\mu(\bar{\nu}_\mu) + X)}{\sigma(\nu_\mu(\bar{\nu}_\mu) + N \rightarrow \mu^-(\mu^+) + X)} = \rho^2 \left( \frac{1}{2} - \sin^2 \theta_W + \frac{5}{9} \sin^4 \theta_W (1 + r(-1)) \right), \quad (3)$$

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written in terms of NC and CC (anti-)neutrino-nucleon cross sections. Here,
\[ \rho = \frac{M_W^2}{\cos^2 \theta_W M_Z^2}, \quad r = \frac{\sigma(\nu_\mu + N \rightarrow \mu^+ + X)}{\sigma(\nu_\mu + N \rightarrow \mu^- + X)} \sim \frac{1}{2}. \] (4)

However, actual targets such as the iron target of the NuTeV experiment, are not always isoscalar, having a significant neutron excess. In addition, nuclear effects including the EMC effect, nuclear shadowing and Fermi motion corrections are known to be very important for electromagnetic structure functions. These nuclear effects may also modify the CC and NC structure functions, and therefore a detailed study of these effects on the extraction of the weak-mixing angle is essential. In principle any nuclear model which can successfully explain the EMC effect in deep inelastic muon-nucleon scattering can serve for our purposes. For definiteness, in the present work we use a particular nuclear re-scaling model \cite{[4]}. We make complete estimates of nuclear effects on the ratios \( R^{\nu(\bar{\nu})} \) and on the resulting values of \( \sin^2 \theta_W \). In order to reduce the uncertainties related to sea quarks, Paschos-Wolfenstein \cite{[5]} suggested to extract \( \sin^2 \theta_W \) from the relationship

\[ R_N^{-} = \frac{\sigma(\nu_\mu + N \rightarrow \nu_\mu + X) - \sigma(\bar{\nu}_\mu + N \rightarrow \bar{\nu}_\mu + X)}{\sigma(\nu_\mu + N \rightarrow \mu^- + X) - \sigma(\bar{\nu}_\mu + N \rightarrow \mu^+ + X)} = \rho^2 \left( 1 - \sin^2 \theta_W \right). \] (5)

Inspired by the Paschos-Wolfenstein relation, we will also examine nuclear effects on \( \sin^2 \theta_W \) by the following observable for the scattering off a nuclear target \( A \),

\[ R_A^{-} = \frac{\sigma(\nu_\mu + A \rightarrow \nu_\mu + X) - \sigma(\bar{\nu}_\mu + A \rightarrow \bar{\nu}_\mu + X)}{\sigma(\nu_\mu + A \rightarrow \mu^- + X) - \sigma(\bar{\nu}_\mu + A \rightarrow \mu^+ + X)}. \] (6)

Below we present a detailed analysis of nuclear effects starting with a brief summary of the formalism we will use.

II. NUCLEON STRUCTURE FUNCTIONS

In the quark-parton model the nucleon structure functions are determined in terms of the quark \( u(x, Q^2), d(x, Q^2), s(x, Q^2), c(x, Q^2), b(x, Q^2) \) and gluon \( g(x, Q^2) \) distribution functions, which satisfy the QCD \( Q^2 \)-evolution equations. Below we only collect expressions for the relevant proton structure functions. The corresponding neutron structure functions can be obtained from the proton ones by the replacements \( u(x, Q^2) \leftrightarrow d(x, Q^2), \ \bar{u}(x, Q^2) \leftrightarrow \bar{d}(x, Q^2) \).
The structure functions (SF) of CC reactions $\nu(\overline{\nu})N \rightarrow l^-(l^+)X$ are given by

$$F_{1W^+p} = \overline{u}(x)|V_{ud}|^2 + \overline{u}(\xi_b)|V_{ub}|^2\theta(x_b - x) + d(x)|V_{ud}|^2 + d(\xi_c)|V_{cd}|^2\theta(x_c - x) + s(x)|V_{us}|^2 + s(\xi_c)|V_{cs}|^2\theta(x_c - x)$$

$$+ \overline{c}(x)|V_{cd}|^2 + \overline{c}(\xi_b)|V_{cb}|^2\theta(x_b - x) + b(x)|V_{ub}|^2 + b(\xi_c)|V_{cb}|^2\theta(x_c - x),$$

(7)

here $V_{ij}$ are the Cabibbo-Kobayashi-Maskawa mixing matrix elements. The variable

$$\xi_k = \begin{cases} x \left(1 + \frac{m_k^2}{Q^2}\right), & (k = c, b), \\ x, & (k = u, d, s), \end{cases}$$

and the step functions $\theta(x_c - x), \theta(x_b - x)$ take into account rescaling due to heavy quark production thresholds.

The structure functions $F_{2W^+p}$ and $F_{3W^+p}$ are obtained from (7) by the replacements of the quark distribution functions $q(x, Q^2)$ indicated in the curly brackets:

$$F_{2W^+p}(x, Q^2) = F_{1W^+p}(x, Q^2)\{q(x, Q^2) \rightarrow 2\xi_kq(\xi_k, Q^2)\},$$

(8)

$$F_{3W^+p}(x, Q^2) = 2 F_{1W^+p}(x, Q^2)\{\overline{q}(x, Q^2) \rightarrow -\overline{q}(x, Q^2)\}.\tag{9}$$

The structure functions of the NC reactions $\nu(\overline{\nu})N \rightarrow \nu(\overline{\nu})X$ are

$$F_{1Zp} = \frac{1}{2} \left\{ \left[ (g_V^u)^2 + (g_A^u)^2 \right] (u(x) + \overline{u}(x) + c(x) + \overline{c}(x)) + \left[ (g_V^d)^2 + (g_A^d)^2 \right] (d(x) + \overline{d}(x) + s(x) + \overline{s}(x)) \right\},$$

(10)

$$F_{2Zp} = 2xF_{1Zp},$$

(11)

$$F_{3Zp} = 2\left[ g_V^u g_A^u (u(x) - \overline{u}(x) + c(x) - \overline{c}(x)) + g_V^d g_A^d (d(x) - \overline{d}(x) + s(x) - \overline{s}(x)) \right].$$

(12)
In the SM the vector and axial-vector quark couplings are given by

\[ g^V_u = \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W, \quad g^V_d = -\frac{1}{2} + \frac{2}{3} \sin^2 \theta_W, \quad g^A_u = \frac{1}{2}, \quad g^A_d = -\frac{1}{2}. \]

In our analysis we adopt the CTEQ6 Set-2 parton distribution functions of Ref. [6].

III. NUCLEAR EFFECT ON NUCLEON STRUCTURE FUNCTIONS

Here, we summarize our approach for calculating the structure functions of a given nuclear target starting from the free nucleon quark distribution functions discussed in the previous section.

A. Nuclear Parton Distributions in the extended \( x \)–rescaling Model

Since the discovery of the EMC effect [7] in deep inelastic muon-nucleus scattering, various models [8, 9] have been proposed for its explanation. In our present work, we choose one of the successful EMC models usually referred as the extended \( x \)–rescaling model [4]. We extend this model to describe the CC and NC structure functions in (anti-)neutrino-nucleus deep inelastic scattering.

Let \( K_{pA}(x,Q^2) = xP_{A(N)}(x,Q^2), p = V, S, G \) be the momentum distributions of valence quarks(V), sea quarks(S) and gluon (G) in the nucleus A (or nucleon N), respectively.

The \( x \)–rescaling model [4] is based on the fact that in a nucleus the Bjorken variable \( x \) of the nucleon structure functions is rescaled due to the binding energy of nucleons in the nuclear environment. In Ref. [4] it was found that the universal \( x \)–rescaling violates conservation of nuclear momentum. In the extended \( x \)–rescaling model this problem is fixed by introducing different \( x \)–rescaling parameters for the momentum distributions of valence quarks and sea quarks (gluons) in the nucleon structure function, i.e.,

\[ K^V_A(x, Q^2) = K^V_N(\delta^V(x), x, Q^2). \] \hspace{2cm} (13)

For the \( x \)–rescaling parameters of valence quarks, sea quarks and gluons we take the values \( \delta^V = 1.026 \) and \( \delta^S = \delta^G = 0.945 \) [4]. Actually, only one of these two parameters is free while the other can be determined from the momentum conservation sum rule.
B. Nuclear shadowing of sea quark and gluon distributions

The x-rescaling with the above given parameters explains the EMC effect in the region of medium values of $x$. As is well known, in the low $x$ region the effect of nuclear shadowing must also be taken into account. This amounts to a depletion of the nuclear structure functions at low $x$, due to the destructive interference of diffractive channels induced by final state interactions \[10\]. In this picture, shadowing corrections to structure functions are the same for neutrino or for charged lepton scattering. At low $Q^2$ we expect other contributions, such as those present in a vector-meson-dominance model, and in principle in this case there could be differences between the shadowing corrections to structure functions in neutrino and in charged lepton scattering. Nevertheless, a detailed analysis with all the important contributions taken into account has been performed \[11\], with the conclusion that the total shadowing in neutrino induced reactions is comparable in magnitude to shadowing in charged lepton induced reactions. Therefore for our purposes it is enough to take the shadowing corrections used for nuclear structure functions in lepton charged scattering, and apply them to the nuclear structure functions of neutrino induced reactions. For definiteness we use the approach presented in Ref. \[12\], in which this effect is incorporated into the structure functions by introducing a nuclear shadowing factor in the sea quark $q_s$ and gluon $g$ distribution functions

$$q_s(x, Q^2) \rightarrow R_{sh}^A(x) \cdot q_s(x, Q^2), \quad g(x, Q^2) \rightarrow R_{sh}^A(x) \cdot g(x, Q^2).$$

Here we assumed that the gluon and the sea quark distribution functions receive the same nuclear shadowing. For the nuclear shadowing factor we use the parameterization proposed in Ref. \[12\]

$$R_{sh}^A(x) = \left\{ \begin{array}{ll} 1 + a \ln A \ln(x/0.15), & (x < 0.15), \\ 1 + b \ln A \ln(x/0.15) \ln(x/0.3), & (0.15 \leq x \leq 0.3) \\ 1, & (x > 0.3). \end{array} \right\}$$  (14)

This parameterization gives the summary of those features of nuclear shadowing which were important for explaining this nuclear effect. The parameters $a$, $b$ in Eq. (14) can be determined from the experimental data on nuclear shadowing in $^{56}$Fe and $^{40}$Ca \[6, 13\]. We find the values $a = 0.013$ and $b = -0.02$. 

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C. Nuclear structure functions and Fermi motion of nucleons

We introduce the average nucleon structure functions in a nucleus \( A \) in a conventional way as

\[
F_i(A)(x, Q^2) = \frac{1}{A} [F_iA(x, Q^2) - \frac{1}{2} (N - Z)(F^n_i(x, Q^2) - F^p_i(x, Q^2))],
\]

where \( i = 1, 2, 3 \), and \( A = Z + N \) with \( N \) and \( Z \) being the number of neutron and proton in the nucleus \( A \). The functions \( F_iA(x, Q^2) \) are the nuclear structure functions. The second term compensates for the neutron excess in a nucleus \( A \). The functions \( F^p_i(x, Q^2) \) and \( F^n_i(x, Q^2) \) are free proton and neutron structure functions, calculated according to Eqs. (7)-(12). The nuclear structure functions \( F_iA(x, Q^2) \), with Fermi motion corrections in a nucleus, can be written as a convolution

\[
F_iA(x, Q^2) = \sum_\lambda \int \frac{d^3p}{(2\pi)^3} |\psi_\lambda(\vec{p})|^2 z F_i^{N(A)}(\frac{x}{z}, Q^2).
\]

Here the bound nucleon structure functions \( F_i^{N(A)} \) for the nucleon \( N \) in the single-particle nucleon state with wave function \( \psi_\lambda(\vec{p}) \) are given by Eqs. (7)-(12) with the substitutions introduced in the previous section, in order to incorporate \( x \)-rescaling and nuclear shadowing.

In Eq. (16) we use the variable \( z = (p_0 + p_3)/m_N, p_0 = m_N + \epsilon_\lambda \), where \( \epsilon_\lambda \) is the binding energy of a nucleon in the single-particle state \( \lambda \). The single-particle wave function \( \psi_\lambda(\vec{p}) \) of the nucleon in momentum space satisfies the light-cone normalization condition:

\[
\int \frac{d^3p}{(2\pi)^3} |\psi_\lambda(\vec{p})|^2 z = 1.
\]

In the following calculation, \( \epsilon_\lambda \) and \( \psi_\lambda(\vec{p}) \) are taken from Ref. [4].

Previously, in Ref. [4], the above described approach was applied to the analysis of the electromagnetic structure functions of deep inelastic muon-nucleus scattering in order to explain the EMC effect [7]. In Fig. 1, we show the results of this analysis in the form of a ratio of the nuclear structure functions \( F_2^\gamma \) of \( ^{56}\text{Fe} \) and deuteron. The experimental points correspond to the data of the EMC Collaboration [7]. As seen from Fig. 1 the theoretical curve derived in the adopted approach fit the experimental data with good precision. Fig. 1 also shows that nuclear effects on the structure functions are significant.

In order to further check our nuclear model, we also apply it to weak charged current neutrino-iron DIS, which do not depend on the weak mixing angle. In this case there are no free parameters. The comparison with data is shown in Fig. 2. We see that the
results with nuclear corrections (solid lines) for high and medium values of $x$ are in excellent agreement with experimental data [15]. At small $x$ the agreement is a bit worse but still quite reasonable. We also show the results without nuclear corrections (dashed lines). The surprising conclusion is that these nuclear corrections are negligible in charged current DIS, even at large $x \sim 0.65$ values, where in the electromagnetic case there is a large effect. At small $x$ the data is not very precise, but the trend indicates that the shadowing corrections are negligible, and there is even the possibility of antishadowing. This region certainly deserves further experimental and theoretical analysis.

Having our approach verified in the case of the electromagnetic and weak charged processes (structure functions) we apply it to the analysis of nuclear effects in neutral current (anti-)neutrino-nucleus deep inelastic scattering, and study their impact on the extraction of the weak-mixing angle $\sin^2 \theta_W$.

**IV. NUCLEAR EFFECTS ON THE EXTRACTION OF $\sin^2 \theta_W$**

In our numerical analysis we study the influence of nuclear effects on the extraction of $\sin^2 \theta_W$ from the observables $R_A^{\nu(p)}$ and $R_A^{-}$, taking into account some kinematical cut-offs.
FIG. 2: The comparison of our results for the charged current DIS differential cross sections at $E_\nu = 150$ GeV with the experimental data [15]. Our results are given for $y > 0.31$ since the CTEQ6 parton distributions which we used are available only for $Q^2 > 1.3$ GeV$^2$.

specific for the NuTeV experiment.

The differential cross sections for CC and NC (anti-)neutrino-nucleus deep inelastic scattering, in terms of the structure functions defined in Eq. (15), are given by [16]

$$\frac{d^2\sigma^{\nu,\bar{\nu}}_{CC}(A)}{dx dy} = \frac{G_F^2}{\pi} m_N^2 E_{\nu,\bar{\nu}} \left\{ xy^2 F_1^{W^\pm(A)}(x, Q^2) + \left(1 - y - \frac{xy m_N}{2 E_{\nu,\bar{\nu}}} \right) F_2^{W^\pm(A)}(x, Q^2) \pm \left(y - \frac{y^2}{2} \right) x F_3^{W^\pm(A)}(x, Q^2) \right\},$$

for the CC reaction, and

$$\frac{d^2\sigma^{\nu,\bar{\nu}}_{NC}(A)}{dx dy} = \frac{G_F^2}{\pi} m_N^2 E_{\nu,\bar{\nu}} \left\{ xy^2 F_1^{Z(A)}(x, Q^2) + \left(1 - y - \frac{xy m_N}{2 E_{\nu,\bar{\nu}}} \right) F_2^{Z(A)}(x, Q^2) \pm \left(y - \frac{y^2}{2} \right) x F_3^{Z(A)}(x, Q^2) \right\},$$

for the NC reaction.
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In the event selection, the NuTeV Collaboration applied the cut off

$$20\text{GeV} \leq E_{\text{cal}} \leq 180\text{GeV},$$

(20)

for a visible energy deposit to the calorimeter $E_{\text{cal}}$. The lower limit ensures full efficiency of the trigger, allows for an accurate vertex determination and reduces cosmic ray background.

Therefore we calculate the observables $R^{\nu(\overline{\nu})}_A$ and $R^{-}_A$ imposing the same cut off on the energy $E_h$ of the final hadronic state $X$, assuming $E_h = E_{\text{cal}}$. Since $E_h \approx \nu$ we can write the kinematical variables averaged over the (anti-)neutrino flux as

$$x = \frac{Q^2}{2M_N E_{\text{cal}}} \leq 1, \quad y = \frac{E_{\text{cal}}}{\langle E_{\nu(\overline{\nu})} \rangle} \leq 1.$$  

(21)

For the average energies of the neutrino and antineutrino beams we take the values $<E_{\nu}> = 120$ GeV and $<E_{\overline{\nu}}> = 112$ GeV, as in the NuTeV experiment [17].

The cut off, Eq. (20), characteristic for the NuTeV experimental events, does not exclude the region of small values of $Q^2$ where the QCD parton picture is not really applicable. Therefore it is not possible to calculate the total cross sections of the CC and NC reactions for the ratios $R^{\nu(\overline{\nu})}_A$ and $R^{-}_A$ in a theoretically controllable way. Given that we study these ratios at fixed values of $Q^2$,

$$Q^2 = 2m_N \langle E_{\nu(\overline{\nu})} \rangle xy,$$

(22)

which allows us to examine the $Q^2$ dependence of the nuclear effects. This method may be helpful if the experimental events concentrate around some known average value of $\langle Q^2 \rangle$. In the NuTeV experiment $\langle Q^2 \rangle \sim 20\text{GeV}^2$, but this value was obtained from the Monte Carlo event simulation. The actual kinematics of the selected CC and NC events is poorly known, except for the above mentioned energy deposit cut off (20). Despite the fact that the average $Q^2$ is 20 GeV$^2$, a substantial fraction of events may correspond to relatively low values of $Q^2$. Therefore our results are not directly applicable to the NuTeV experimental data, but only indicate some general features of nuclear effects on the extraction of $\sin^2 \theta_W$ from $R^{\nu(\overline{\nu})}_A$ and $R^{-}_A$ relevant to this experiment, namely that in general they are sizable. The actual role of nuclear effects can only be revealed by their inclusion in the corresponding Monte Carlo event simulation.

The results of our analysis are summarized in Tables I and II, and graphically in Fig. 3, where we define

$$\delta R^i = R^i_A - R^i_N, \quad \text{with} \quad i = \nu, \overline{\nu}, -. $$

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as differences between the NC/CC ratios calculated with \((R^i_A)\) and without \((R^i_N)\) nuclear corrections. The quantity \(\delta \sin^2 \theta_W\) in Tables I and II, and in Fig. 3, are the net effect of nuclear corrections on \(\sin^2 \theta_W\) extracted from \(R^\nu\) (solid curve) or \(R^-\) (dashed curve). The ratio \(R^\nu\) makes no appreciable influence on \(\sin^2 \theta_W\), being weakly sensitive to its variations in the region of its possible values, and therefore the corresponding values of \(\delta \sin^2 \theta_W\) are not presented in Table 1.

We estimate \(\delta \sin^2 \theta_W\) in the following way. First, we calculate the NC/CC ratio \(R^\nu_N\) without nuclear effects, taking for \(\sin^2 \theta_W\) the central NuTeV value \(\sin^2 \theta_W = \sin^2 \theta_W^{(N)} = 0.2277\). Then we calculate the ratio \(R^\nu_A\) with nuclear effects in the way described in section 3, fitting \(\sin^2 \theta_W\) in order to get the value of \(R^\nu_A\) equal to \(R^\nu_N\) calculated in the first
step without nuclear corrections. Thus we obtain the values of $\delta \sin^2 \theta_W$ from the equations

$$R_A'^\nu (\sin^2 \theta_W^{(A)}) = R_N'^\nu (\sin^2 \theta_W^{(N)}),$$

$$\delta \sin^2 \theta_W = \sin^2 \theta_W^{(A)} - \sin^2 \theta_W^{(N)}.\quad (24)$$

The same procedure is applied for the estimation of the nuclear correction $\delta \sin^2 \theta_W$ from the Paschos-Wolfenstein ratio $R_A'$. Therefore in order to get the actual value of $\sin^2 \theta_W$, which is the value $\sin^2 \theta_W^{(A)}$ extracted with nuclear effects, the value $\sin^2 \theta_W^{(N)}$, obtained without these effects, must be shifted by $\delta \sin^2 \theta_W$. Note that the value $\sin^2 \theta_W^{(N)}$ corresponds to the case reported by the NuTeV Collaboration. In fact, the NuTeV analysis is based on the parton model equations (7)-(12) for neutrino-nucleon CC and NC DIS, applied to an iron target nucleus. In this approach one assumes that the parton distribution functions (PDF) are the effective PDF in iron which absorb all the nuclear target effects, and are the same for any DIS process with the same target. The effective PDF in iron were extracted from CC data [15] and then used to calculate the NC structure functions. However as it follows from our analysis the PDF per nucleon extracted from the CC data should be very close to those of a free nucleon, since nuclear effects in CC DIS are negligible. Thus the analysis based on the so extracted PDF deals with practically free nucleon PDF risking to lose the nuclear effects which do not manifest themselves in the CC DIS but can be important in the NC neutrino-iron DIS process. Recall that these effects are very important in charged lepton DIS processes.

From Tables I and II and Fig. 3, it is seen that nuclear effects on $R'^\nu$, $R'^F$, $R'^-$ and $\sin^2 \theta_W$ are large and strongly depend on the value of $Q^2$. Moreover, at certain values of $Q^2$ the nuclear correction $\delta \sin^2 \theta_W$ changes its sign. These transition values are $Q^2 \simeq 10$ GeV$^2$ and $Q^2 \simeq 8$ GeV$^2$ for the extraction methods using $R_A'^\nu$ and $R_A'$ respectively. In the NuTeV experiment the average value of $Q^2$ is about 20 GeV$^2$, which lies in the region of positive values of $\delta \sin^2 \theta_W$. For this reason it might be thought that nuclear effects enhance the NuTeV deviation of $\sin^2 \theta_W$ from its SM value. However as we already noted, a substantial fraction of the NuTeV events may have quite small values of $Q^2$ providing negative nuclear correction $\delta \sin^2 \theta_W$ to the weak-mixing angle $\sin^2 \theta_W$.

The following note is also in order. Given the kinematical cut off in Eq. (20), the $Q^2$-region of negative $\delta \sin^2 \theta_W$ corresponds to the values of the Bjorken variable $x < 0.25$, where we expect strong nuclear shadowing effects. This is the case for the parameterization of the shadowing factor given in Eq. (14). In our analysis shadowing is a dominant nuclear effect in the region $Q^2 < 5$ GeV$^2$ corresponding to $x < 0.13$. In more sophisticated models
FIG. 3: The $Q^2$ dependence of the nuclear modification to the weak mixing angle $\delta \sin^2 \theta_W$ extracted from $R^e$ (solid curve) and from $R^-$ (dashed curve).

shadowing may be accompanied with antishadowing, acting in such a way that in the CC DIS their common effect is small due to self-compensation while in the NC DIS this effect could be larger than in our model, enhancing the nuclear corrections to the weak mixing angle $\delta \sin^2 \theta_W$ at the small x-values. Thus a more careful study of these effects in neutrino-nucleus scattering is required.

Recently, in Ref. [18], it was observed that corrections from higher-twist effects of nuclear shadowing to $\sin^2 \theta_W$ extracted via the Paschos-Wolfenstein relation, may well be of the same size as the deviation from its global fit value reported by the NuTeV Collaboration [2].

V. SUMMARY AND CONCLUSIONS

In this work, we have shown that taking into account nuclear effects such as nuclear $x$-rescaling, nuclear shadowing and Fermi motion in the nucleon structure functions of CC and NC (anti-)neutrino-nucleus scattering, may significantly affect the extracted value of the weak-mixing angle $\sin^2 \theta_W$.

The procedure used by the NuTeV Collaboration [2] in order to take into account nuclear effects was to extract effective nuclear parton distribution functions from charged current DIS data, which then were used in order to obtain the value of $\sin^2 \theta_W$ from their data.
Although in principle this is a reasonable idea, in practice our results indicate that it might be difficult to implement, since nuclear effects play a minor role, if at all, in charged current DIS, while in NC DIS they are important. Unfortunately there is no direct way of comparing our theoretical predictions with the results of the NuTeV Collaboration presented in the form of ratios of NC to CC experimental event candidates with poorly identified kinematics. Uncertainties in kinematics are pertinent to the experiments measuring NC (anti-)neutrino scattering since the final state neutrinos are not detectable. As a consequence, among the NuTeV events there might be a substantial fraction of low $Q^2$ events, despite the fact that in this range their probability decreases with decreasing $Q^2$. Thus in theoretical estimations of ratios like $R^\nu$, $R^\bar{\nu}$, $R^-$, the kinematical integration in the total NC and CC cross sections extends to the low $Q^2$ region including $Q^2 = 0$. This region poses the problem of calculating the structure functions in a theoretically controllable way, since the QCD parton model, presently the only firmly motivated one, is not applicable below $Q^2 \sim \text{few GeV}^2$, where QCD has a strong non-perturbative behaviour.

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