Dynamics of domain wall in a black hole bulk

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As we know there is an idea that domain wall can be regarded as a world volume in a high dimension bulk. The moving of domain wall is admitted as the expansion or collapse of world volume. In the bulk of this paper, it has dilaton field, Maxwell field and gravity. Usually, matter of domain wall is viewed as a perfect fluid. We show the dynamics of domain wall from a magnanimous bulk action with a common factor function \( f(\phi) \). Further-more, we obtain general bulk field equations and the generally modified conjunction conditions. When choosing \( e^{c\phi} \) that inspired from bosonic sector for the effective action of IIA supergravity as \( f(\phi) \), although we try a special solution, it is still very complex to get an accurate solution by solving bulk field equations. In order to conveniently get more concrete physics we naturally attempt another simple situation, namely \( c = 0 \), it can return to bulk done by K. C. K. Chan et al. These bulks are specially interesting and possess non asymptotically flat and non asymptotically (A)dS characteristics. By researching, we find that, in general case, there are three situations of domain wall motion in the bulk: accelerated expansion in all the time, expanding with a constant speed and collapsing into horizon. Whereas the parameter, such as initial density \( \rho_0 \), takes some big value, it will produce a bounce. In addition, there is a fine control situation that domain wall first slows down the expanding speed to a minimum value and then continue to expand quickly.

I. INTRODUCTION

Domain wall is a kind of topological defect considered as a produce of phase transition of vacuum. It can be evolved into stars such as sheetlike or filamentary structures or voids in the university [1]. The domain wall is a hot focus in physics in a recent decades. The works about interaction between black hole and domain wall have been done [2,3]. Beyond that, it also has a popular point that domain wall is a world volume in a high dimension spacetime [4]-[6]. The moving of domain wall is admitted as the expansion or collapse of world volume. So the research of what dynamic of domain wall has in a bulk is very meaningful. Specially, H. S. Reall had done many significant works, such as [7]-[11]. Many people also are interested in it [12]-[21].

In a general way, studying of dynamics of domain wall in a bulk has two methods and many authors obtained the remarked results. One is that people get the dynamics of domain wall via the Israel matching conditions in a straightway given background spacetime. Domain wall and bulk is nonaffecting. This way can deal easily know all situations of movement of domain wall in a certain bulk and many bulks adopt a metric ansatz. These are significant works such as [12],[13]. The second way include two steps due to the reciprocal bond between action of domain wall and bulk metric (bulk spacetime is not a given). So at first one needs to find appropriate bulk metric solutions matching the given action of domain wall. Then using the junction conditions researches the moving of domain wall. For instance, dilaton field is full of bulk and domain wall [7,20,21]. It needs to modify the dilaton couple constants to get a static bulk meeting the field equations in the domain wall action. And then utilizing the evolution equation gets the law of motion of domain wall.

Now on the basis of H. S. Reall and B. H. Lee’s work [7],[12], we plan to begin studying a more general action. Similarly we use a metric ansatz to find some static bulk including all almost charged dilaton gravity background spacetime [22]-[26]. So that it can learn overall movement of domain wall. When we take the factor \( f(\phi) \) a form of \( e^{c\phi} \) inspired from bosonic sector for the effective action of IIA supergravity. We analyze a bulk that was gotten by K. C. K. Chan et al [27] in a special case when \( c = 0 \). This kind of bulk is so special, because of its unusual asymptotics, that we naturally urgently expect to find some unexpected results. As for the influence of other form of \( f(\phi) \) for dynamics of domain wall, we plan to do it in the next work.

Our paper is arranged as follows: in the second Section we begin with a general action for getting a static bulk, finally we choose a special case and review it. In the third section we will deserve the dynamics equation of domain wall and analyze the moving for domain wall. Eventually all results will be researched specially in a five dimension. In the fourth section we will carry on the conclude and discussion.

II. CHARGED DILATON BLACK HOLE BULK

In this section, we mainly consider a static bulk spacetime which is started from the charged dilaton gravity rescaled by a \( f(\phi) \) factor. This effective action could be derived from the loop corrections of superstring [11]. And then for analyzing dynamics of domain wall specially, we choose a interesting situation, namely a charged dilaton.
black hole. We start with the total action in a n dimensional bulk

\[ S = \int d^nx \sqrt{-g} f(\phi) [R - \frac{4}{n-2}(\nabla \phi)^2 - V(\phi) - e^{\frac{4\phi}{n-2}} F^2]. \]

Varying equation (1) with respect to the metric \( g^{AB} \), we get the Einstein equation in the n dimensional bulk

\[ R_{AB} = \frac{4}{n-2} (\partial_A \phi \partial_B \phi + \frac{1}{4} g^{AB} V) + 2e^{\frac{4\phi}{n-2}} (F_AF_B^C - \frac{1}{2(n-2)} g^{AB} F^2) - \frac{1}{2(n-2)} g_{AB} F^2 + \frac{1}{2} \nabla_A \nabla_B \phi. \]

Varying equation (1) with respect to Maxwell and dilaton field will also have two equations.

\[ \partial_e (f(\phi) \sqrt{-g} e^{-\frac{4\phi}{n-2}} F^{cd}) = 0, \]

\[ \frac{\partial f(\phi)}{\partial \phi} [R - \frac{4}{n-2} (\nabla \phi)^2 - V(\phi) - e^{\frac{4\phi}{n-2}} F^2] + f(\phi) \left[ \frac{8}{n-2} \nabla^2 \phi - \frac{\partial V(\phi)}{\partial \phi} + \frac{4a}{n-2} e^{\frac{4\phi}{n-2}} F^2 \right] + \frac{8}{n-2} \partial_A f(\phi) g^{AB} \partial_B \phi = 0. \]

The metric take form

\[ ds^2 = -U(r) dt^2 + \frac{1}{U(r)} dr^2 + R^2(r) d\Omega_4^2. \]

We correspond the solution of Maxwell field as an isolated electric charge here.

\[ F_{tr} = f(\phi) e^{\frac{4\phi}{n-2}} \frac{Q}{R^{n-2}}. \]

Taking equation (5) and (6) into equation (2),(4), it will produce a complicated form. Inspired by bosonic sector for the effective action of IIA supergravity we consider the \( f(\phi) \) as a form of \( e^{-\frac{4}{n-2}\phi} \) and n=5, meanwhile equation (2)-(4) are reduced

\[ \frac{27}{R} R'' = 6be^{\frac{4}{n-2}\phi} \phi'' - (12 + 8c^2 e^{\frac{4}{n-2}\phi}) (\phi')^2, \]

\[ \frac{1}{R^2} (6UR'^2 + R(3U' + 4ce^{\frac{4}{n-2}\phi} U') - 3UR'')) = -\frac{6}{R^2} - V - \frac{2e^{\frac{4}{n-2}(-2c+a)\phi} Q^2}{R^6}, \]

\[ 24e^{\frac{4}{n-2}(-c+a)\phi}(a + c)Q^2 = e^{\frac{4}{n-2}\phi} R^4 (72b(-1 + U R^2)) + 9R(R'(8cU' + 3U') - 8cUR'') + R^2(-9 \frac{dV}{d\phi} + 9U' \phi' + 4(3cV - 4bU \phi') + 3cU'') \]

where a prime denotes a derivative with respect to r. Similar to Ref. [27] we consider the metric ansatz

\[ R(r) = \gamma r^N. \]

When we try to solve these field equation, we firstly think of a special solution. So when N take the value of 1 and c \( \neq 0 \), \( \phi \) will have a form

\[ \phi = \text{inversefunction}[C(-\frac{3e^{-\frac{4\phi}{n-2}}}{4c} - \frac{4b}{8e^3})], \]

where C denotes a constant term. It has a complicated form, and for other equations one cannot get an analytical solution. Thus we naturally consider the simpler solutions for different potentials such as c=0, then Ref. [27] got a precise bulk solutions for a different domain wall action in c=0 case. For researching convenience, we review this interesting bulk.

### A. charged dilaton black hole bulk

When \( c = 0 \), the equations (7,9) rewrite

\[ 1 \frac{d^2 R}{R \frac{dr^2}{} \frac{d\phi}{dr}} = -\frac{4}{(n-2)^2} \frac{d\phi}{dr}^2, \]

\[ \frac{1}{R^{n-2}} \frac{d}{dr} \left[ \frac{dV}{d\phi} \frac{d\phi}{dr} \right] = \frac{n-2}{8} \frac{dV}{d\phi} + a e^{\frac{4\phi}{n-2}} \times \frac{Q^2}{R^{2(n-2)}}, \]

\[ \frac{1}{R^{n-2}} \frac{d}{dr} \left[ U \frac{d}{dr} (R^{n-2}) \right] = (n-2)(n-3) \frac{R^{-2}}{} - V - 2e^{\frac{4\phi}{n-2}} \frac{Q^2}{R^{2(n-2)}}, \]

where \( \gamma \) and N are constants. From (12), we can get

\[ \phi(r) = \phi_0 + \phi_1 \log r. \]

Then we just use two kinds of potential. It gets four kinds of spacetime solutions (non asymptotically flat and non asymptotically (A)dS). For details, see ref [27].

Type I, \( V(\phi) = 0 \)

For the solutions

\[ U(r) = \frac{(4 + a^2)^2}{(2 + a^2)^2} r^{2\frac{4}{4+a^2}} - \frac{4(4 + a^2)^2 M r^{2\frac{4+a^2}{4+a^2}}}{3a^2 \gamma^3}, \]

\[ N = \frac{a^2}{4 + a^2}, \]

\[ r_h = \frac{4M (2 + a^2)^2}{3a^2 (4 + a^2) \gamma^{\frac{4+a^2}{4+a^2}}}, \]

\[ \phi(r) = \frac{3}{4a} \log \frac{2Q (2 + a^2)}{12 \gamma^4} + \frac{6a}{2(4 + a^2)} \log r, \]
where $M$ is mass of black hole and $Q$ is the electric charge. It denotes that when potential disappear, bulk will be a non asymptotically flat spacetime.

Type II, $V(\phi) = 2\Lambda e^{2\phi}$

(i) For the first solutions

$$U_1(r) = r^{\frac{2a^2}{1+a^2}}\left[\frac{2(1 + a^2)^2}{(1-a^2)^2(2+a^2)} - \frac{4(1 + a^2)M}{3\gamma^3} \times r^{-\frac{2(2+a^2)}{1+a^2}} + 2Q^2(1+a^2)^2}{3(2+a^2)^2}\right].$$

(ii) For the second solutions

$$N = \frac{1}{1+a^2},$$

$$\phi_1 = -\frac{3a}{2(1+a^2)},$$

$$\Lambda = -\frac{6a}{2(1+a^2)}e^{\frac{2a\phi_0}{3a}},$$

$$b = \frac{2}{3a}.$$  

When $a^2 < 1$, it has two horizons, $a^2 > 1$, it just has one horizon.

(iii) For the third solutions

$$U_3(r) = \frac{(4 + a^2)^2}{\gamma^2(2 + a^2)^2} e^{\frac{2a\phi_0}{3a}} - \frac{4(4 + a^2)M}{3\gamma^3} \times r^{-\frac{2(2+a^2)}{1+a^2}} + 2\Lambda Q^2(1+a^2)^2}{a^2\gamma^2(4 - 4a^2)3\gamma^3},$$

$$N = \frac{a^2}{4 + a^2},$$

$$\phi_1 = \frac{6a}{2(4 + a^2)},$$

$$b = -\frac{2}{a}.$$  

Its horizon defined is pretty difficult because of depending of the constant $a$ when we take $U(r) = 0$.

### III. Dynamics of Domain Wall

Now we start from the general formalisms about the domain wall. Before that, we introduce some useful notations. Using $X^A = (t, r, X, Y, Z)$ denotes bulk coordinate and $x^\mu(\tau, x, y, z)$ denotes internal coordinate of domain wall. When moving domain wall is embedded into bulk spacetime parallelly along one space dimension $r$, we have $X^A(x^\mu) = X^A(t, \tau, x, y, z)$. Assuming the position of domain wall in the bulk is $r(\tau)$. This metric will exist in the range $r < r(\tau)$ and the unit normal point to the range. Because of the reflection symmetry, we always take the value of $\{\}$ to calculate result. Next defining a four vector $30$

$$e^A_\mu = \frac{\partial X^A}{\partial x^\mu},$$

the induced metric is

$$h_{\mu\nu} = g_{AB}e^A_\mu e^B_\nu.$$  

When we take the general bulk metric ansatz

$$dS^2 = -A(r)dt^2 + B(r)dr^2 + R(r)^2d\Omega^2.$$  

the velocity of the domain wall is

$$V^A = \left(\sqrt{\frac{B^2t^2 + 1}{A}}, r, 0, 0, 0\right),$$

where a dot donates the derivative with respect to $\tau$ and we use $g_{AB}V^AV_B = -1$. The normal vector is

$$n^A = (-\sqrt{\frac{B}{A}}, \frac{1 + B\tau^2}{B}, 0, 0, 0),$$

where we use the conditions:

$$g_{AB}n^An^B = 1,$$

$$n^AV_A = 0.$$  

And we get

$$dS^2 = -d\tau^2 + R^2(r(\tau))d\Omega^2.$$  

The extrinsic curvature tensor is

$$K_{\mu\nu} = e^A_\mu e^B_\nu K_{AB} = e^A_\mu e^B_\nu \nabla_A n_B.$$  

Explicitly, the extrinsic curvature components are $31$

$$K_{\mu\nu} = -\frac{\hat{h}_{\mu\nu}R'}{R}\sqrt{1 + B\tau^2},$$

$$K_{\tau\tau} = \frac{1}{\sqrt{AB}}\frac{d}{dr}(\sqrt{1 + B\tau^2}).$$  

We use the following action to describe the dynamics of a domain wall in the background of the charged dilaton black hole (1).

$$S_{DW} = \int d^{n-1}x\sqrt{-h}f(\phi)\{K\} + \int d^{n-1}x\sqrt{-h}L_m(\phi, \tilde{h}_{\mu\nu}).$$  

\[46\]
Where we introduce an assumption about the induced metric $h_{\mu \nu}$ \[30\]

\[\tilde{h}_{\mu \nu} = e^{2\xi(\phi)}h_{\mu \nu}, \] (47)

and we consider the matter in the domain wall as perfect fluid. So the energy-momentum tensor can be $T^{\mu \nu} = (\rho + p)V^\mu V^\nu + p h^{\mu \nu}$.

When varying the metric $g^{AB}$ for total action, the n-1 dimension field equation is

\[f(\phi)\{K_{AB} - K h_{AB}\} + \frac{1}{2} n \partial_B f(\phi) - n \partial f(\phi)g_{AB} = \tau_{AB},\] (48)

where $\tau_{AB} = -2 \frac{\delta L}{\delta h_{AB} + h_{AB} L_m}$. Thus we gain the generalized Israel matching conditions \[32\].

And varying dilaton field $\phi$, the scalar field equation is

\[\frac{4}{n - 3} \{f(\phi) n \cdot \partial \phi\} - \frac{\partial f(\phi)}{\partial \phi} \{K\} = \frac{dL_M}{d\phi}. \] (49)

When $c = 0$, the boundary equations have

\[\{K_{AB} - h_{AB} K\} = \tau_{AB}, \] (50)

\[\frac{4}{n - 3} n \cdot \partial \phi = \frac{dL_M}{d\phi}. \] (51)

So, we get three boundary equations

\[K_{ij} = -\frac{h_{ij}}{R} R' \sqrt{1 + \frac{B'}{B}} = \frac{1}{2} (\tau_{ij} - \frac{1}{3} \tau h_{ij}), \] (52)

\[K_{\tau \tau} = \frac{1}{\sqrt{AB}} \frac{d}{dr} \left(\sqrt{A(1 + Br^2)}\right) = -\frac{1}{2} (\tau_{\tau \tau} - \frac{1}{3} \tau h_{\tau \tau}), \] (53)

\[\frac{8}{n - 3} n \cdot \partial \phi = \xi' \tau. \] (54)

When $n = 5$, we can expand equation \[2\], it will have

\[\frac{d}{dr} \left(\frac{R}{R'}\right) = 1 + \frac{4R^2 \phi'^2}{9R'^2}. \] (55)

Using metric \[5\], $n = 5$ and energy-momentum tensor $T^{\mu \nu}$, We will deform these three equations as a new face

\[\sqrt{U(r)} + r^2 = \frac{R}{R'} \phi, \] (56)

\[\phi' = \frac{9}{4} \frac{R'}{R} \xi'(3w - 1), \] (57)

\[\dot{\rho} + 3H(P + \rho) = \frac{4\rho \phi'^2}{9H}, \] (58)

where the $w = \frac{\rho}{p}$, $H = \frac{\dot{R}}{R}$.

We have $R(r) = \gamma r^N$, $\phi(r) = \phi_0 + \phi_1 \log r$, so solving \[57\] we get the expression of $\xi(\phi)$

\[\xi(\phi) = \frac{4}{9N(3w - 1)} \phi + \phi_2. \] (59)

Thus we obtained the Friedmann equations from the other boundary equations.

\[H^2 = \frac{1}{36} \rho^2 - \frac{N^2}{\gamma^2} U(r), \] (60)

\[\dot{\rho} + 3H(P + \rho) = -\frac{4}{9} \rho \phi'^2 H \left(\frac{R}{R'}\right)^2. \] (61)

Solving equation \[60\] and \[61\], we get the expressions

\[\rho = \rho_1 \frac{R}{R'} \left(\frac{R}{R'} - \frac{4}{9} \phi'^2 H \left(\frac{R}{R'}\right)^2\right) \] (62)

\[\dot{R}^2 - Y(R) = 0, \] (63)

where $Y(R) = \frac{1}{36} (\rho_1 R^2 - \frac{4}{9\gamma^2} (\phi'^2 H \left(\frac{R}{R'}\right)^2))^2 - R^2 \frac{N^2}{\gamma^2} U(r)$.

From the equation \[63\], we know that it decides the evolution of scale factor of universe. And it shows a transpositional process of energy from the potential $Y(R)$ to kinetic energy for a particle.

\section{IV. Domain Wall Evolution}

When we have the evolution equation for scale factor, we can analyze the movement of domain wall and know the fate of the domain wall in a 5 dimensional black hole bulk.

Before this, we need to do a skillful analysis. Assuming we consider the moving of domain wall in the range $R > 1$. When we can get its solution $Y(R) > 0$, domain wall may be expanding in all the time. It can keep away from horizon, otherwise the fate of domain wall will be fallen into black hole eventually. From a physical point of view, the domain wall universe has an initial coordinate horizon, otherwise the fate of domain wall will be fallen.

Now we take general situation $\gamma = 1, \rho_1 = 1, \phi_0 = 0$. This wouldn't change property of $Y(R)$.

Type I's solution

\[U(r) = \frac{(4 + a^2)^2 r^{\frac{3a + 2}{3a}}}{(2 + a^2)^2} - \frac{4(4 + a^2) M}{3a^2} \frac{r^{3a + 2}}{r^{a + 2} - 2}. \] (64)

From the monotony of $U(r)$, the horizon just has one. At the same time due to power of the first term is more than 0, while $r \to \infty$ this bulk spacetime is non-asymptotically flat.

Then it has

\[Y(R) = \frac{4a^2 M}{3(4 + a^2)} R^{-\frac{3a}{2} - 2} + \frac{\rho_1^2}{36} R^{-4 - 2w - \frac{a^2}{2}} - \frac{a^4}{(2 + a^2)^2}. \] (65)

For simpleness, we can take the form of $Y(R)$

\[Y(R) = R^{C_1} + R^{C_2} - C_3. \] (66)
By analyzing, $C_1 < -2$, $C_2 < 0$, and then $w > \frac{2}{a}$, it shows that $Y(R)$ decreases monotonously. Because of the constant term is less than 0. When $R \to \infty$, $Y(R) < 0$. When $C_2 > 0$, then $w < \frac{-2}{a} - \frac{4}{3a^2}$. When $a^2 < 1$, $w$ doesn’t have a solution. When $a^2 > 1$, $w < -3$, $Y(R)$ will be dominated by $R^{C_2}$. So $Y(R)$ increase monotonously.

Case (i). For $C_2 < 0$, $w > \frac{-2}{a}$. As shown in FIG. 1, $Y(R)$ will quickly decrease to 0. it has nothing to do with value of $a$. In this situation domain wall has a bad fate. It cannot run out horizon, and it has a short life. In a ward, it will be fallen into black hole finally. But it has a special situation. When $R_1$ is very big, $Y(R)$ will bigger than 0 in some range of $R$. For this, domain wall will have two situations. One is when $R < 0$, domain wall will collapse into horizon in all the time. When $R > 0$, domain wall will firstly expand to a maximum and then collapse into the horizon.

![FIG. 1: Y(R) versus R for the Type I solutions's, R > 1.](image)

Case (ii). For $a^2 > 1$, $w < -3$. $Y(R)$ fastly increases. From FIG. 2, when $R > 0$, domain wall moving will generate accelerated expansion. And it can’t stop. When $R < 0$, domain wall collapses immediately into horizon.

![FIG. 2: Y(R) versus R for the Type I solutions, R > 1.](image)

Type II’s solutions

(i) the metric $U(r)$ is

$$U(r) = \frac{2(1 + a^2)e^{\frac{4Mw}{Q^2}}}{3(2 + a^2)}r^{\frac{-4}{1 + a^2}} - \frac{4}{3}(1 + a^2)Mr^{\frac{-2}{1 + a^2}} + r^{\frac{-2}{1 + a^2}} \frac{2(1 + a^2)^2}{(1 - a^2)(2 + a^2)}.$$  

$$\Lambda = \frac{-3a^2 - \frac{4Mw}{Q^2}}{1 - a^2}.$$  

From this bulk solution, we know $a \neq 1$, $a > 0$. And this spacetime is nonflat and non-(A)dS. When $a^2 < 1$, $\Lambda < 0$, and $a^2 > 1$, $\Lambda > 0$, we have

$$Y(R) = -\frac{2Q^2R^{-2(2+a^2)}}{3(2 + a^2)} + \frac{4M}{3(1 + a^2)} + \frac{\rho_1^2}{36}R^{-4-2a^2-6w^2}$$

$$- \frac{1}{(1-a^2)(2+a^2)}.$$  

(69)

Similarly, for simpleness, we take the form of $Y(R)$

$$Y(R) = -R^{C_1} + R^{C_2} + R^{C_3} + C_4.$$  

(70)

By comparing $C_1 < C_2 < 0$, it shows $R^{C_1} < R^{C_2} < 1$. So the first two terms of $Y(R)$ are dominated by $R^{C_2}$.

For $\Lambda < 0$, $a^2 < 1$. If $C_3 > 0$, $w < -1$, $Y(R)$ will be dominated by $R^{C_3}$, and it will increase monotonously. If $C_3 < 0$, $w < -\frac{3}{2}$, $Y(R)$ will decrease monotonously. And $C_4$ is very small, $Y(R)$ must be less than 0 quickly.

For $\Lambda > 0$, $a^2 > 1$. If $C_3 > 0$, $w > -\frac{3}{2}$, we cannot define range. If $C_3 < 0$, $w > -1$, $Y(R)$ will decreases monotonously. Although $Y(R)$ decrease all the time, because of $C_4 > 0$, $Y(R)$ eventually close into a constant.

Case (i). For $\Lambda < 0$, $a^2 < 1$, $w < -1$, in FIG. 3 $Y(R)$ fastly increases. When $R > 0$, domain wall moving will produce expansion increasing. And it can’t stop. When $R < 0$, domain wall collapses immediately into horizon.

![FIG. 3: Y(R) versus R for the first Type II’s solutions, R > 1.](image)

Case (ii). For $\Lambda > 0$, $a^2 > 1$. If $w > -1$, this situation is interesting. When $R > 0$, domain wall will expand in all the time. And its velocity closes to a constant eventually. The other is domain wall collapses into horizon slowly.

![FIG. 4: Y(R) versus R for the first Type II’s solutions, R > 1.](image)
(ii) the metric $U(r)$ is

$$
U(r) = \frac{1}{r^{2+a}} \left( 4 + a^2 \right)^2 \left( -1 + \frac{4a^2 \phi_0}{a^2} Q^2 \right) - \frac{4(4 + a^2) M r^{-2(2 + a^2)}}{3a^2},
$$

$$
\Lambda = \frac{-2(2 + a^2)e^{2a \phi_0} Q^2}{(4 - a^2)} + \frac{12 e^{2a \phi_0}}{4 - a^2}.
$$

In this situation, $a \neq 2$. And when $a^2 < 4$, the bulk spacetime is a black hole. Or it is just a naked singularity. This bulk solution is nonflat and non-(A)dS. Because of the existence of $Q$, $\Lambda$ is not ensured.

$$
Y(R) = \frac{4a^2 M}{3(4 + a^2) R^{-2(2 + a^2)}} + \frac{\rho^2}{36} R^{-2(2 + a^2)} - \frac{(1 - 1) a^4}{(4 - a^2)(2 + a^2)}. \quad (73)
$$

For analyzing

$$
Y(R) = R^{C_1} + R^{C_2} + C_3. \quad (74)
$$

Obviously, $C_1 < 0$. When $a^2 > 4$, $C_3 > 0$. If $C_2 > 0, w > -1$, $Y(R)$ is dominated by $R^{C_2}$, and it will increase monotonously. If $C_2 < 0, w > -2$, $Y(R)$ will decrease monotonously. $Y(R) > 0$ and it closes to a constant finally.

When $a^2 < 4, C_3 < 0$. If $C_2 > 0$, we cannot have a solution. If $C_2 < 0, w > -1, Y(R)$ will decrease monotonously. And $Y(R) < 0$ quickly. This situation is not hold.

Case (i). For $a^2 > 4$. Because of property of coupling constant $a$, the bulk is a naked singularity. If $w < -1$, domain wall moving has two situations. One is expansion, the other collapses into a origin.

If $w > -\frac{2}{3}$, domain wall may expand always until velocity is a constant or collapse immediately into an origin.

(iii) the metric $U(r)$ is

$$
U(r) = \frac{(4 + a^2)^2}{(2 + a^2)^2} e^{\frac{a}{a^2}} - \frac{2(4 + a^2) M r^{-2(2 + a^2)}}{a^2}.
$$

$$
+ \frac{1 - 7}{3} \frac{\frac{4}{2 + a^2} (2 + a^2) \frac{a^2}{2} (4 + a^2)^2 Q^2}{a^2(4 - 4a^2)} + \frac{\frac{4}{2 + a^2} \Lambda}{2 + a^2}, \quad (75)
$$

It shows that $a \neq 1$ and this spacetime is nonflat and non-(A)dS because of the existing of $a$. At the same time it cannot sure the situation of horizon.

$$
Y(R) = -\frac{a^4}{2(a^2)^2} + \frac{2(4 + a^2) M}{a^2} \frac{R^{-2 + \frac{2}{a^2}}}{\Lambda^2} - \frac{1}{3} \frac{2 - 2 + a^2}{2 \Lambda} \frac{(2 + a^2) \frac{a^2}{2} (4 + a^2)^2 Q^2}{a^2(1 - a^2)}
$$

$$
+ \frac{\rho^2}{36} R^{-4 - \frac{a^2}{3} - 6 \omega}. \quad (76)
$$

For analyzing and simplification, we don’t consider the situation of limit of $\Lambda$.

$$
Y(R) = R^{C_1} - m R^2 + R^{C_2} - C_3. \quad (77)
$$

(a) $\Lambda > 0$

When $a^2 > 1, -2 < C_1 < 2, m < 0, R^{C_1} < R^2$. So the first two terms of $Y(R)$ will be dominated by the second term. If $C_2 < 0, w > -\frac{2}{3}$, $Y(R)$ will be dominated by $R^{C_2}$. $Y(R)$ will increase. If $C_2 > 0, w < -2, Y(R)$ will increase monotonously.

When $0 < a^2 < 1, C_1 > 2, m > 0$. So the first two terms will be dominated by the first term. If $C_2 < 0, w > -2, Y(R)$ will be dominated by $R^{C_1}$, and increase monotonously. For $C_2 > 0$, if $C_2 > C_1$, it doesn’t have a solution. $C_2 < C_1$, and then $w > -\frac{2}{3}$, $Y(R)$ will be dominated by $R^{C_1}$ and increases monotonously.

(b) $\Lambda < 0$

If $a^2 > 1, -2 < C_1 < 2, m > 0, R^{C_1} < R^2$. So the first two terms of $Y(R)$ will be dominated by the second term. So it just when $C_2 > 2, w < -\frac{2}{3}$, $Y(R)$ is dominated by $R^{C_1}$. $Y(R)$ will increase monotonously.

If $0 < a^2 < 1, C_1 > 2, M < 0$, so the first two terms of $Y(R)$ will be dominated by the $R^{C_1}$. $C_2 < 0$, then $w > -2, Y(R)$ will be dominated by $R^{C_1}$ and increases monotonously. For $C_2 > 0$. If $C_2 > C_1$, $w$ not has a solution. $C_2 < C_1$, then $w > -\frac{2}{3}$, $Y(R)$ will be dominated by $R^{C_1}$ and increases monotonously.

Case (i). It shows that for $\Lambda > 0, a^2 > 1, w > -\frac{2}{3}$, when $w$ increases, $Y(R)$ fastly increases. Domain wall has two situations. One is in all the time expanding, the other is collapsing into horizon. And when $a^2 < 1, w > -\frac{2}{3}$, domain wall also has two situations. One is always expanding, the other is collapsing to singularity. And the first situation expand quickly very much.

![FIG. 5: Y(R) versus R for the second Type II's solutions, R > 1.](image)

![FIG. 6: Y(R) versus for the third Type II’s solutions, L = 1. R > 1. Left picture is for a^2 > 1, right picture is for a^2 < 1.](image)
Case (ii). For $\Lambda < 0$. If $a^2 > 1$, $w < -\frac{2}{3}$, when $w$ continuously decreases, $Y(R)$ increases more and more faster. Domain wall has two situations. One is in the expanding, the other is collapsing to horizon. If $a^2 < 1$, $w > -\frac{2}{3}$, domain wall has two situations. One is always expanding quickly, the other is collapsing to horizon.

![Fig. 7: Y(R) versus R for the third Type II’s solutions, $\Lambda = -1$. $R > 1$.](image)

Case (iii). There always exists a situation for the form of equation (84) that $Y(R)$ is not increase in all the time when we adjust other parameters such as $M$, $Q$ or $\Lambda$. Domain wall will firstly expands decreasingly and reaches a minimum value and then start accelerated expansion.

![Fig. 8: Y(R) versus R for the special Type II’s solutions.](image)

Now we summarize that due to coupling constant $a$ and state parameter $w$, domain wall always can cross horizon of whatever kinds of black hole. Out of horizon there are three kinds of different evolution experience for domain wall. First is domain wall will undergo accelerated expansion all the time. Second is domain wall will go through a decelerated expansion until a constant velocity. Third is domain wall will process a rebound. The rate of expansion for domain wall slows down and the wall expand into a maximum value of $r$, after that, the wall starts to contract until collapse into a singularity point.

V. CONCLUSION AND DISCUSSION

In this paper, the bulk has dilaton field, Maxwell field and gravity. Usually, matter of domain wall is viewed as a perfect fluid. We show the dynamics of domain wall from a magnificent bulk action with a common factor function $f(\phi)$. Further-more, we obtain general bulk field equations and the generally modified conjunction conditions. When choosing $e^{\phi}$ that inspired from bosonic sector for the effective action of IIA supergravity as $f(\phi)$ [11], although we try a special solution, it is still very complex to get an accurate solution by solving bulk field equations. In order to conveniently get more concrete physics we naturally attempt another simple situation, namely $c = 0$, it can return to bulk done by K. C. K. Chan et al [21]. These bulks are specially interesting and possess non asymptotically flat and non asymptotically (A)dS characteristics.

Then by researching, it shows that for different dilaton potentials when dilaton coupling constant $a$ and the ratio of pressure to density $w$ are adjusted, domain wall always can cross horizon. Aimed at expanding, it includes two situations. One is expansion quickly and another is motion of constant velocity [21]. It can be seen from those pictures that in fact on account of dilaton field, expansion will be produced very early and sharply. Unless we regulate the other parameters i.e., $\Lambda, \gamma, M, Q$, it will arise the result of Fig. 8. At the same time, for the expansion we can analyze the stability of domain wall by calculating $Y''(R)$ [33-39]. Obviously, when domain wall exist matter, it is not stability because of $Y''(R) > 0$ in all the situations of expansion. But if we take a big value for these parameter such as $p_1$, domain wall can come true bounce.

For extensional works, Firstly there must exist bulk solutions of other form of $f(\phi)$. So that we can analyze a matched movement of domain wall. Second, we are interested in the perturbation of domain wall. Third if it can depend on regulating parameters for the purpose of ending inflation. Fourth, we can research on the inflation such as hybrid inflation. And for it, we consider that the inflaton is Higgs field and use the dilaton field for ending inflation.

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