Magnetic field generation in finite beam plasma system

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For finite systems boundaries can introduce remarkable novel features. A well known example is the Casimir effect \cite{1,2} that is observed in quantum electrodynamic systems. In classical systems too novel effects associated with finite boundaries have been observed, for example the surface plasmon mode \cite{3} that appears when the plasma has a finite extension. In this work a novel instability associated with the finite transverse size of a beam flowing through a plasma system has been shown to exist. This instability leads to distinct characteristic features of the associated magnetic field that gets generated. For example, in contrast to the well known unstable Weibel mode of a beam plasma system which generates magnetic field at the skin depth scale, this instability generates magnetic field at the scale length of the transverse beam dimension \cite{4}. The existence of this new instability is demonstrated by analytical arguments and by simulations conducted with the help of a variety of Particle - In - Cell (PIC) codes (e.g. OSIRIS, EPOCH, PICPSI). Two fluid simulations have also been conducted which confirm the observations. Furthermore, laboratory experiments on laser plasma system also provides evidence of such an instability mechanism at work.

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I. INTRODUCTION

The dynamical evolution of magnetic field plays an important role in a variety of contexts ranging from astrophysical phenomena \cite{5} to laboratory plasmas. It is well known that when a high power laser impinges on an overdense plasma target (and/or solid which can get ionized to form a plasma) it generates energetic electrons \cite{6–8}. The current due to these energetic electrons are balanced by the return plasma current from the background electrons. The combination of forward and return shielding currents is known to be susceptible to the Weibel destabilization process as observed from the PIC simulations for periodic infinite beam - plasma system \cite{9}. Such a destabilization leads to spatially separated current filaments at the electron skin depth leading to magnetic field generation at commensurate scales. The choice of infinite periodic system is based on an inherent understanding that the boundary effects due to finite system would merely have a small incremental impact. This, however, turns out to be incorrect. It has been shown here with PIC as well as two fluid simulations that when a beam with finite transverse extent is considered its boundary introduces novel effects. In fact an entirely new instability associated with the finite size of the beam appears which generates magnetic fields at the scale of the beam size right at the very outset. Secondly, the sheared electron flow configuration at the two edges of the finite beam is seen to be susceptible to Kelvin Helmholtz instability \cite{10,11}. Weibel \cite{12} appears only in the bulk region of the beam. Thus, there are three sources of magnetic fluctuations in a finite beam system - the new Finite Boundary System (FBS) and KH instability operating at the edge and the usual Weibel destabilization process occurring in the bulk region. The theoretical description of the FBS instability has been provided which is followed up by evidences from simulations (both PIC and fluid ) and experimental data\cite{13}.

We present a theoretical analysis of the new FBS instability which essentially confirms that the magnetic field appearance observed at the edge of the finite beam right in the beginning is due to a new instability, which has so far not been identified. It is shown that the FBS instability, in fact, arises through the contribution from the boundary. An analytical understanding of the characteristic features of the mode observed in simulations has been provided. This particular mode has direct impact on the long scale magnetic field formation at an early stage of beam plasma system. The experimental confirmation is provided in terms of the spectral properties of the magnetic field produced by laser plasma interactions in the laboratory. The appearance of magnetic fields at scales longer than the skin depth during very early stage can only be accounted for by this instability. The conventional Weibel destabilization route

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of the beam plasma system can account for such long scales much later in time when nonlinear inverse cascade effects have had an opportunity to produce such effects.

The manuscript has been organized as follows. Section II contains the theoretical description of the new effect. In section III the simulation and experimental evidences supporting the FBS effects have been provided. Section IV contains the conclusion.

II. THEORETICAL DESCRIPTION

An equilibrium configuration of the beam plasma system in 2-D $x−y$ plane is considered as shown in Fig.1. The central region II from $−a ≤ y ≤ a$ carries the beam current and an oppositely flowing background plasma current which balance each other. In region I and region III the plasma is static and at rest. The charge neutralization in equilibrium is achieved by balancing the total electron density by the background ion density, viz., $\sum_\alpha n_{0\alpha} = n_{0i}$ in all the three regions. The electron flow velocity in region I and III is zero, whereas in region II it satisfies the condition of zero current, i.e. $\sum_\alpha n_{0\alpha} v_{0\alpha x} = 0$. Here the suffix $\alpha$ stands for $b$ for beam electrons and $p$ for the plasma electrons.

The linearized perturbation of this equilibrium is considered with variations in $y$ and $t$ only. The flow is confined in $x−y$ plane, so we have $B_{1z}$, $E_{1x}$ and $E_{1y}$ (where the suffix $x,y,z$ denotes the components) only as the perturbed dominant fields. Eliminating all the perturbed fields in terms of $E_{1x}$ leads to the following differential equation:

$$\left[f_2 E'_{1x}\right]' - g_2 E_{1x} = 0 \quad (1)$$

Here

$$f_2 = 1 + \frac{S_4}{\omega^2} - \frac{S_3^2}{\omega^2(S_1 - \omega^2)} \quad (2)$$

$$g_2 = S_2 - \omega^2 \quad (3)$$

where

$$S_1 = \sum_\alpha \frac{n_{0\alpha}}{n_{0\gamma 0\alpha}}; \quad (4)$$

$$S_2 = \sum_\alpha \frac{n_{0\alpha}}{n_{0\gamma 0\alpha}^3}; \quad (5)$$

$$S_3 = \sum_\alpha \frac{n_{0\alpha} v_{0\alpha x}}{n_{0\gamma 0\alpha}}; \quad (6)$$

$$S_4 = \sum_\alpha \frac{n_{0\alpha} v_{0\alpha x}^2}{n_{0\gamma 0\alpha}}; \quad (7)$$

It should be noted that $S_3$ and $S_4$ are finite only when there is an equilibrium flow in the two fluid electron depiction. Furthermore, if the flow velocities of the two electron species are equal and opposite then $S_3 = 0$.

The homogeneous limit of Califano et.al. [14, 15] can be easily recovered if we take Fourier transform of Eq.(1). The homogeneous equation yields the dispersion relation for the Weibel growth rate. We now seek the possibilities for obtaining purely growing modes in a finite system. For this purpose we multiply Eq.(1) by $E_{1x}$, replace $\omega^2 = -\gamma^2$ (for purely growing modes) and integrate over $y$ over region II, i.e. from $−a$ to $a$. This yields:

$$\int_{-a}^{a} [f_2 E'_{1x}]' - g_2 E_{1x}^2 \, dy = 0 \quad (8)$$

Upon integrating by parts we obtain

$$f_2 \left[E_{1x} E'_{1x}\right]_{-a}^{a} - \int_{-a}^{a} \left\{f_2 [E'_{1x}]^2 + g_2 E_{1x}^2\right\} \, dy = 0 \quad (9)$$

In region II, $f_2$ and $g_2$ being constant, we can take them outside the integral. Thus Eq.(9) can be written as

$$f_2 \left[E_{1x} E'_{1x}\right]_{-a}^{a} - f_2 \int_{-a}^{a} [E'_{1x}]^2 \, dy - g_2 \int_{-a}^{a} E_{1x}^2 \, dy = 0 \quad (10)$$
If the boundary term is absent, as in the case of infinite homogeneous system, the Eq.\ref{10} can be satisfied for a finite value of $E_{1x}$ provided second and third terms have opposite signs. The integrand being positive definite this is possible provided $g_2$ and $f_2$ have opposite signs. The definitions of $g_2$ and $f_2$ in terms of $\gamma^2$ are

$$f_2 = 1 + \frac{S_2^2}{\gamma^2(S_1 + \gamma^2)} - \frac{S_4}{\gamma^2}$$

$$g_2 = S_2 + \gamma^2$$

Since $g_2$ is positive, the only possible way for $f_2$ to be negative is to have $S_4/\gamma^2$ dominate over the first two terms of $f_2$. Thus, the conventional Weibel gets driven by $S_4$. It is also obvious that $S_3$ provides a stabilizing contribution making it more difficult for $f_2$ to become negative. A finite value of $S_3$ implies a non-symmetric flow configuration, i.e., one for which the two electron species have different flow speeds.

For finite system something interesting happens when boundary contributions are retained.

The value of $[E_{1x}E_{1x}']_{a}^{|a}$ should be positive as $E_{1x}^2$ should increase as one enters region II from region I (at $y = a$) and it should decrease at $y = a$. Thus the sign of first term will be determined by the sign of $f_2$. Another way to understand the positivity of the sign of $[E_{1x}E_{1x}']_{a}^{|a}$ is by realizing that this term is essentially the radiative flux moving outside region II which can only be positive. This is seen by casting it in the form of the Poynting flux by expressing the derivative of $E_{1x}$ in terms of $B_{1z}$.

There exists the possibility then that the first term of Eq.\ref{10} has a finite contribution to balance the second and the third terms. Thus even if $f_2$ and $g_2$ have same signs (positive) Eq.\ref{10} can be satisfied for a finite $E_{1x}$. It should be noted that the instability driven in this case is different from the Weibel mode as the boundary terms are responsible for it. It should be noted that Equation\ref{10} can be satisfied most easily by the boundary contribution provided the variations in $E_{1x}$ in the bulk is minimal so as to have minimal (close to zero) contribution from the second term. Thus the instability driven by the boundary term would have a preference for long scale excitation.

It should be noted that contrary to the Weibel mode, $S_3$ aids the FBS instability. A finite and large $S_3$ may easily render $f_2$ to be positive which is required for this instability.

### III. EVIDENCES

PIC simulations using OSIRIS\cite{16} \cite{17} and EPOCH\cite{18} were carried out for the case of a forward beam current and a compensating return plasma current of a finite extent, at $t = 0$ shown as the equilibrium configuration in Fig.1. The simulations were carried out and analyzed extensively both 2-D and 3-D for various parameters. Here, for the sake of brevity we will present plots from our 2-D studies only. The simulation parameters for which the results are presented are as follows.

- $\omega_{pe}$ moving along $\hat{x}$ with a velocity of $0.9c$ in a central region of $10c/\omega_{pe}$ to $15c/\omega_{pe}$ electrons and ions both with density $n_0$ moving with a velocity of $0.1c$ was considered. Here, $n_0$ is the density of background ions which are at rest everywhere. In the remaining region from $y = 0$ to $20c/\omega_{pe}$ electrons and ions both with density $n_0$ are at rest.

Thus, the plasma everywhere is neutral with electron density balancing the density of background plasma ions. In the central beam region the beam current is exactly compensated by the return shielding current. Snapshots at various times for this particular case have been depicted in Fig.2 and Fig.3 in the form of 2-D color plots for the $z$ component of the magnetic field and the charge density respectively.

From these plots it is clearly evident that there are three distinct phases of evolution. During the first phase from $t = 0.12\omega_{pe}^{-1}$ to $t = 30\omega_{pe}^{-1}$ perturbed $z$ component of magnetic field along $\hat{z}$ (transverse to both flow and inhomogeneity) appear at the edges with opposite polarity. This magnetic field has no $x$ dependence and is a function of $y$ alone. This is consistent with the analytical choice. The magnetic field perturbations are seen to grow with time and also expand in $y$ from the edges in both the directions at the speed of light. The electron density perturbations, which also appear at the edge, on the other hand, remain confined at the edge during this phase. This first phase of the evolution, thus can be characterized by the appearance of magnetic field perturbations with variations only along $\hat{y}$, the transverse direction. This suitably fits with the analytical description. Keeping in view that the structures do not seem to vary with respect to the $x$, the 1-D profiles along $y$ have been shown in Fig.1 for $E_{1x}$. As predicted for the FBS mode theoretically, $E_{1x}$ shows minimal variation inside the beam region. The PIC simulations were also repeated for the case where $S_3 = 0$ was taken by a choice of symmetric flow. In this case we observed that the FBS mode did not appear. This shows that $S_3$ plays a destabilizing role for this instability unlike is role in the Weibel mode.
During the second phase from \( t = 30\omega_{pe}^{-1} \) the Kelvin Helmholtz (KH) like perturbations appear at the edge of the current, and at a subsequently much later time viz., at around \( t = 30\omega_{pe}^{-1} \) one can observe the appearance of Weibel like perturbations in the bulk of the central region of the current flow. Both the KH and the Weibel mode have variations along both \( \hat{y} \) and \( \hat{x} \) directions.

These observations with characteristics three phase developments have been repeatedly observed in both 2-D and 3-D configurations from a variety of simulations carried out with different PIC as well as fluid codes.

A comparison of the evolution of the magnetic field spectra for the periodic infinite system as well as the finite beam case has been shown in its snapshots at various times in Fig. 5. It is clear from the figure that for the periodic infinite case the peak of spectral power appears at the electron skin depth scale initially. The spectral power only subsequently cascades towards longer scales via inverse cascade mechanism. On the other hand for the finite case it can be clearly observed that the spectral peak appears at the beam size right from the very beginning. In laboratory laser plasma experiments the electron beam width would be finite, typically commensurate with the dimension of the laser focal spot. The measured magnetic field in a series of experiments where the electron beam gets generated at the critical density surface and propagates in the overdense plasma medium, clearly show that right from the very beginning viz. \( t=0.12\omega_{pe}^{-1} \) time scales the spectra maximizes at the longest measurable scale and not the electron skin depth scale as one would expect from a Weibel like destabilization process. These experiments [13] provide yet another evidence for the existence of a FBS instability.

### IV. SUMMARY

While the KH and the Weibel modes are well known and have been discussed extensively in the literature, the FBS mode has neither been observed and nor been described anywhere. We thus report the first observation of this mode which relies entirely on the systems with finite boundary effects. The implications of this particular instability on magnetic field generation needs to be evaluated in different contexts. For instance, it is likely that the finite size jets emanating from astrophysical objects are susceptible to this particular instability. This work also suggests that the finite size considerations in many other systems need to be looked afresh to unravel new effects which might have been overlooked so far.

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FIG. 1: Schematics of 2D- equilibrium geometry of the beam plasma system where beam is finite in transverse direction
FIG. 2: Snapshots of evolution of magnetic field $B$ in the units of $(m_e c \omega_p/e)$ with time $t$ in unit of $\omega_p^{-1}$: (a) At time $t = 18.0$, Fig. shows the emergence of $B_z$ field with the opposite polarity at the edge of the beam (b) At time $t = 30.0$, Fig. shows the initial development of K-H vortices at the edges of the beam (c) Non-linear stage of K-H instability and the bulk region shows the Weibel (current filamentation) instability. (d) Finally, the Magnetic field structures at the later time $t = 42.0$ due to the presence of all three instabilities in the considered system.
FIG. 3: Snapshots of the evolution of electron density [in units of $n_0$] with time $t$ [in the unit of $\omega_{pe}^{-1}$]: (a) The electron density configuration at initial time $t = 18.0$. (b) At time $t = 30.0$, Fig. shows the initial development of K-H vortices at the edges of the beam. (c) The formation of K-H vortices at time $t = 36.0$ at the edge of the beam which is highlighted by a red box and the bulk region shows the Weibel (current filamentation) instability. (d) During the non-linear stage of K-H instabilities at time $t = 42.0$, the fig. shows the merging of K-H vortices which is highlighted by a red box.
FIG. 4: Initial time evolution of $E_{1x}$ showing the minimal variation inside the beam region.
FIG. 5: Magnetic field spectra evolution with $k$: Case(a)-Finite beam-plasma system where field spectra peaks at the focal width of the beam; Case(b)-Infinite beam-plasma system where peak of the field spectra remains at the electron skin depth i.e. Weibel scale.