Kinematical Observables in Semi-Invisible Decays

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Invisible particles frequently appear in final state in studying physics at colliders. Experimental precision is also low in measuring missing energy. In this paper, we propose a general approach for studying process involving invisible particles. We provided two kinematical observables which are sensitive to production kinematics in different regions, and hence are complementary. Usage of our approach is illustrated by three examples. It is shown that our observables are still useful in case that the significance $S/B$ is relatively low.

Observation of the Higgs scalar $h(125)$ [1, 2] implies the final building block of the Standard Model (SM) was found. However, it does not mean one have completely understand our universe [3]. On the one hand, physical properties of the mass eigenstate $h(125)$ are still not measured at a requisite precision [4, 5], for instances $CP$ nature, potential form etc. On the other hand, cosmological and astronomical observations convincingly indicate that matter particles predicted by the SM are only a few percent of our universe, new particles with a broad range of mass have to exist in [6, 7]. In general, particles beyond the SM are neutral with respect to charges in the SM [8], and hence weakly interacting with the particles that have been observed [9, 10]. Therefore, invisible they are usually in experimental detectors, and appear as missing energy of which the experimental precision is low.

This is not only a general property in beyond standard model (BSM), the neutrinos as the lightest sector in the SM are also invisible at current colliders. Experimental investigations on processes involving neutrinos turn out to be rather hard, particularly at a hadron collider [11]. The situation becomes worse when the collider energy is increased, where electroweak radiations and hadronization effects can further reduce measurement precision on the missing energy [12].

However, experimental searches involving those invisible particles are particularly important [13]. For instance, the lightest neutrino unavoidably appear in hunting for supersymmetry particles [14]; observation of neutrino oscillations not only proves that neutrinos are massive, but also implies lepton flavor number violated processes [15–18], which could give constraints to the lepton mixing angles and the neutrino mass ratios, and also provide an alternative window for probing properties of new physics.

Therefore, essentially important the frequently happening semi-invisible decays are [19–21]. In this letter we report a general method for studying processes involving invisible particles. Without loss of generality, we assume that the invisible particle $D$ emerges from decay of a mother particle $M$ which is generated in some certain mechanism. Furthermore, the associated particle $S$ is taken to be visible. Fig. 1 is a sketch plot of such a configuration. Theoretically, mass of the invisible particle $D$ can be uniquely determined if the rest frame of the mother particle $M$ can be precisely reconstructed. However, in practice $M$ are produced in pair, and hence it is nearly impossible to identify the missing momentum. On the other hand, since the invisibility of $D$, spin correlation effects which are powerful probes of production and decay dynamics are covered up. Here we introduce a new method to study the semi-invisible decay modes.

Under narrow width approximation, the total cross section with subsequent decay $M(p_M, \lambda_M) \to S(p_S) + D(p_D)$ can be written in general as,

$$\frac{d\sigma}{d\Phi_M d\Phi_S} = \frac{1}{2m_M \Gamma_M} \mathcal{P}^{\lambda_M}_{\lambda_M}(p_M) \mathcal{D}^{\lambda_M}_{\lambda_M}(p_M^*, p_S^*),$$

where $\mathcal{P}^{\lambda_M}_{\lambda_M}(p_M)$ and $\mathcal{D}^{\lambda_M}_{\lambda_M}(p_M^*, p_S^*)$ are production and decay helicity density matrix of the mother particle $M$, and the helicity values $\lambda_M$ are summed over; futhermore $p_M^*$ and $p_S^*$ are momentum of the particle $M$ and $S$ in the rest frame of $M$, respectively; and the decay helicity density matrix is calculated also in this frame where spin polarization effects are maximum. However, practically...
useful differential cross section is represented in terms of kinematical variables which can be directly measured in experiments. Then moving into the Lab. frame, the differential cross section is given as
\[
\frac{d\sigma}{d\Phi_M d\Phi_S} = \frac{\mathcal{P}^{\lambda M}_M(p_M)}{2m_M \Gamma_M} D^{\lambda M}_{\lambda M}(p^*_M, p^*_S) \mathcal{J}(p_M; p^*_S, p_S),
\]
where \(\mathcal{J}(p_M; p^*_S, p_S)\) is the Jacobi factor which stands for the variable transformation from \(p^*_S\) to \(p_S\). In consideration of that \(D^{\lambda M}_{\lambda M}(p^*_M, p^*_S)\) is un-measurable due to the missing momentum \(p_D\), the polarization effects have to be averaged, and then we have,
\[
\frac{d\sigma}{d\Phi_M d\Phi_S} = \frac{\mathcal{P}(p_M)}{2m_M \Gamma_M} \overline{D}(p^*_M, p^*_S) \mathcal{J}(p_M; p^*_S, p_S).
\]
In this form, the production helicity density matrix \(\mathcal{P}^{\lambda M}_M(p_M)\) is completely factorized out. For given production mechanism of the mother particle \(M\) with momentum \(p_M\), the production helicity density matrix \(\mathcal{P}^{\lambda M}_M(p_M)\) is unique, and hence can provide a template for studying the decay dynamics. Furthermore, the decay helicity density matrix \(D^{\lambda M}_{\lambda M}\) in which the decay dynamics is completely encoded is also factorized out in the sense that it is independent of \(p_M\). On the other hand, the Jacobi factor \(\mathcal{J}(p_M; p^*_S, p_S)\) can be directly calculated once \(p_M\) and \(p_S\) are given (\(p^*_S\) is expressed in terms of \(p_M\) and \(p_S\)).

Therefore, factorization in Eq. (3) implies that the kinematical distributions of the particle \(S\) can be investigated by integrating over the momenta \(p_S\) for given \(p_M\). Based on this consideration we introduce following two observables,
\[
m_D(p_M) = \sqrt{\max \{m_D^2\}} \overline{D}(p^*_S) \mathcal{J}(p^*_S, p_S),
\]
\[
m_D(p_M) = \sqrt{\min \{m_D^2\}} \overline{D}(p^*_S) \mathcal{J}(p^*_S, p_S),
\]
where \(m_D^2 = (p_M - p_S)^2\). Then the boundary values are simply given as
\[
m_D^\pm = \sqrt{m_S^2 + m_D^2 - 2E_M E_S(\pm \beta_M \beta_S)}.
\]
Here \(E_S\) and \(\beta_S\) are functions of \(m_D^2\), \(p_S^2\), as well as \(p_M\). Neglecting the \(p_M\) and \(p^*_S\) dependences of \(m_D^\pm\), in the massless limit \(m_S = 0\) we have
\[
m_D^\pm = \sqrt{m_D^2 + \frac{1 \pm \beta_M}{1 \pm \beta_M} (m_D^2 - m_M^2)}.
\]

We can see that in this case \(m_D^+\) is maximally sensitive to \(m_D\) when \(\beta_M = 0\), while \(\beta_M = 1\) the most sensitive region is for \(m_D^-\). In this sense \(m_D^+\) and \(m_D^-\) are two complementary observables for studying the whole kinematical region.

Differential distribution with respect to the variables \(m_D^\pm\) can be written as,
\[
\frac{d\sigma}{d\Phi_M d\Phi_S} = \int d\Phi_M \frac{\overline{D}(p_M)}{2m_M \Gamma_M} \mathcal{B}(p_M, m_D^\pm),
\]
where the weight function \(\mathcal{B}(p_M, m_D^\pm)\) is given as
\[
\mathcal{B} = \int d\Phi_S \overline{D}(p^*_S) \mathcal{J}(p^*_S, p_S) \delta^2 \left( m_D^\pm - \mathcal{K}^\pm(p^*_M, p^*_S) \right).
\]
Here \(\mathcal{K}^\pm(p^*_M, p^*_S)\) are kernel functions that give the boundary values \(m_D^\pm\) by integrating over the kinematical variables \(p_S\) for given \(p_M\). Therefore, the distribution shapes of \(m_D^\pm\) encode the decay dynamics (production dynamics given by \(\overline{D}(p_M)\) is universal). As long as the kernel functions don’t spread too much, \(m_D^\pm\) have maximum sensitivities to \(m_D\). We illustrate usage of our observables with three examples below.

**Example 1: Lepton flavor violating decay of the \(\tau\)-lepton.** The \(\tau\)-lepton, as the heaviest particle in the lepton sector, is expected to be a natural probe for lepton number violation. The present best limit of \(\mathcal{B}(\tau \rightarrow 3\mu) < 2.1 \times 10^{-8}\) at 90\% CL was obtained by the Belle experiment [22], and a slightly mild upper limit of \(6.9 \times 10^{-8}\) at 90\% CL was obtained by the CMS experiment [23]. On the other hand, invisible decay modes of the \(\tau\)-lepton, \(\tau \rightarrow \ell \alpha\), can also be essential [24–26] when new \(Z^\prime\) gauge bosons [27–29] or axion-like particles [30–32] exist in nature. Because of large irreducible backgrounds from the leptonic decay modes \(\tau \rightarrow \nu_\tau \ell_\tau\), experimental constrains on these decay modes are relatively weak. The present upper limits of the branching fractions are obtained by the ARGUS collaboration [33], and are several percent of the corresponding leptonic decay modes. The upper limits in Ref. [33] are obtained by tagging one side of the \(\tau\)-lepton pair decays in 3-prong mode. A new method was proposed in Ref. [34].

Here we study how our observables behaves for the decay process \(\tau \rightarrow \mu X\) at the energies of the Belle II experiment [35–38]. The interaction Lagrangian is simply given a usual Yukawa interaction, \(\mathcal{L}_Y = -g_X X \tau_\mu + h.c.\) and was implemented in FeynRules [39]. Events are generated at \(\sqrt{s} = 10.58\) GeV by using MadGraph [40].

and \(m_D^-\). As expected \(m_D^-\) is more sensitive to the mass.
of the invisible scalar, since the τ-lepton has a relatively large boost factor $\sim 0.942$. Furthermore, while $m_D$ is closing to the background when $m_X$ decrease, $m_{T0}$ is leaving. This implies a complementary property of our observables for measuring $m_X$. Fig. 2(c) gives the density distributions in the $(m_D^+, m_D^-)$ plane. One can see a clean linear correlation in the signal events, however a broad distributions in the background events. Fig. 3 shows the distributions for different significance $S/B$ with 100 total number of events and $10^4$ events for the templates of $\tau^\pm$. We can see that even for $S/B = 1/9$ there is still a possibility to observe the signal events.

**Example II: Lepton flavor violating decay of the Higgs**. Next we study a lepton flavor violating decay of the Higgs boson, $h \rightarrow \tau^+\tau^-$, which is predicted in many BSMs, such as supersymmetry [41, 42], models with flavor symmetries [43] or warped extra dimensions models [44–46] and others [47, 48]. The current upper limits on the branching ratios are 0.47% and 0.28% [49, 50] for $h \rightarrow e\mu$ and $h \rightarrow \mu\tau$, respectively. Lepton flavor violating Z boson decays are also predicted by BSMs [51–53], and have been investigated by ATLAS [54]. The major backgrounds are $h/Z \rightarrow \tau^+\tau^-$ and $h/Z \rightarrow \mu^+\mu^-$. In most of the investigated channels, the backgrounds is significantly larger than the signal [49, 50]. Furthermore, the situation become very worse when we consider the case that both $Z$ and $h$ decay in a lepton flavor violating way. Therefore, it is very important to have an alternative way for probing lepton flavor violating decays in both $Z$ and $h$ channels. In our approach, the observables can be defined by following equation,

$$m_D^2(\mu^\pm) = (p_h - p_{\mu^\pm})^2. \quad (10)$$

Fig. 4(a) is the scatter plot in the $(m_D^+, m_D^-)$ plane for various channels, and $10^4$ events for both $h/Z$ and $\mu^\pm$ are used. The very narrow distributions of $m_D^2(\mu^-)$ and $m_D^2(\mu^+)$ for the $h \rightarrow \tau^+\mu^-$ and $h \rightarrow \mu^+(\tau^+)\mu^-(\tau^-)$ channels are amplified in Fig. 4(b) and Fig. 4(c), respectively. We can clearly see that signal and backgrounds are separated well, except for $m_D^2(\mu^+)$ for the channels $h \rightarrow \tau^+\mu^-$ and $h \rightarrow \tau^+\tau^-$. However, this degenerate can be completed lifted by investigating correlations between $m_D^2(\mu^+)$ and $m_D^2(\mu^-)$. For the background decay channel $h \rightarrow \tau^+\tau^-$, $m_D^2(\mu^+)$ and $m_D^2(\mu^-)$ distributions are always represented by the purple region. On the other
hand, \( m^±_D(\mu^+) \) and \( m^±_D(\mu^-) \) distributions for the signal decay channel \( h \to \tau^+\mu^- \) are represented by green and black regions. Therefore, the signal events can be distinguished by the distribution of \( m^±_D(\mu^-) \). Similar property is expected for possible signal decay process \( Z \to \tau^+\mu^- \).

\[ \text{Fig. 4. Scatter plots for decay channels } h/Z \to (\mu^+/\tau^+)(\mu^-/\tau^-). \quad 10^4 \text{ events are used for both the template of } h/Z \text{ and the visible particle } \mu^\pm. \quad \text{Fig. 4(b) and Fig. 4(c) are amplified representation of the very narrow distributions regions in Fig. 4(a)}. \]

**Heavy neutrino.** Next we give an example in which the invisible particle appears in a second subsequent decay. We illustrate this example by introducing a heavy neutrino which is necessary in the well-known seesaw mechanism. Here we consider a relatively light neutrino \([55–57]\) emerging from decay of a top-quark \([58–60]\),

\[ \bar{t} \to \bar{b} + W^-, \quad W^+ \to \mu^- + \bar{\nu}_N. \quad (11) \]

Furthermore, its interaction with the \( W^- \) boson is assumed to be right-handed \([61]\). Off-shell effect of \( W^- \) when \( m_N > m_W \) is also taken into account. Observables are defined as the boundary values of \( m^2_D \) defined as

\[ m^2_D = (p_t - p_b - p_\ell)^2. \quad (12) \]

Fig. 5 shows the distributions of \( m^2_D \) for different mass of \( N \). Except for the case \( m_N = 50 \text{GeV} \), distributions of \( m^2_D \) are separated from the background from \( \bar{t} \to \bar{b} + \mu^- + \bar{\nu}_\mu \). Again, degenerate between backgrounds and signals with \( m_N \) around \( 50 \text{GeV} \) is expected to be lifted by studying alternative definitions of \( m^2_D \). For instances \( m^2_D = (p_t - p_l)^2 \) or \( m^2_D = (p_l - p_\ell)^2 \) in case of that \( m_N > m_W \). On the other hand, since the top-quark pair can be produced in both gluon and quark channels, it is also possible to separate signals and backgrounds by studying \( m^2_D \) channel by channel. We leave those studies in a future work.

\[ \text{Fig. 5. Scatter plots for decay channels } \bar{t} \to \bar{b} + \mu^- + \bar{\nu}_N \text{ and } t \to b + \mu^+ + \bar{\nu}_\mu. \quad 10^4 \text{ events are used for both the template of } t/\bar{t} \text{ and the visible particles } b/\bar{b} \text{ and } \mu^\pm. \]

In summary we provided a general approach for studying semi-invisible decay process. The observables \( m^2_D \) are complementary in the sense they are sensitive to production kinematics in different regions, and most importantly are useful in case of that the significance \( S/B \) is relatively low.
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