Initiation mechanisms of radial drilling-fracturing considering shale hydration and reservoir dip

Yuxin Chen | Yunhong Ding | Chong Liang | Dawei Zhu | Yu Bai | Chunmei Zou

PetroChina Research Institute of Petroleum Exploration & Development, Beijing, China

Correspondence
Yu Bai, PetroChina Research Institute of Petroleum Exploration & Development, Beijing 100083, China.
Email: s16020266@s.upc.edu.cn

Funding information
This study was financially supported by the Key and Core Technology Projects of China National Petroleum Corporation (Grant No. 2020B-4118)

Abstract
Radial drilling-fracturing is an innovative fracturing technology that achieves superior stimulation effects. This paper develops a model for radial drilling-fracturing in shale reservoirs, which can predict fracture initiation pressure and failure mode of shale. Compared with published models, this model additionally considers shale hydration and inclinations of boreholes and reservoir. Then, the influences of 7 factors are studied and main conclusions are as follows. First, with the angle between radial borehole and formation strike increasing, matrix failure pressure declines only around 90° and 270°. Bedding tensile failure pressure ascends and then falls back. Bedding shear failure pressure rapidly descends, then reaches a plateau, and finally rebounds. A small or large angle is favorable for bedding tensile failure. Shale tends to crack with bedding shear failure under a moderate angle and with matrix failure only at 90° and 270°. Second, with the enlargement of azimuth difference between the radial borehole and main wellbore, matrix failure pressure periodically fluctuates, forming an inverted W-shape curve. Bedding tensile failure pressure and bedding shear failure pressure both ascend, then slide downward, and finally rise again. Besides, a small azimuth difference inclines shale to bedding shear failure, and the other failure modes tend to occur when azimuth difference is around 90°. Third, pressures required for all failure modes descend as the dip of shale formation increases. However, the difference in descending rate leads to that the increase of dip angle can incline shale to bedding shear failure. Besides, pressures required for bedding tensile failure and matrix failure decline linearly with the increasing reservoir temperature. Temperature has no impact on bedding shear failure. Hence, matrix failure and bedding tensile failure are more prone to occur in the high-temperature reservoir. The research provides a reference for field applications of radial drilling-fracturing in the shale reservoir.
1 | INTRODUCTION

Hydraulic fracturing is an important stimulation technology, which can be applied to develop the hydrocarbon reservoir and geothermal energy.\textsuperscript{1-4} Due to the domination of in situ stress, fractures produced by hydraulic fracturing mainly propagate along the maximum horizontal in situ stress,\textsuperscript{5} which often fails to establish effective communication with the target zone. In order to control the directional propagation of fractures artificially, radial drilling-fracturing, an innovative technology of hydraulic fracturing, is proposed by researchers.

As shown in Figure 1, radial drilling-fracturing requires drilling several radial boreholes with a microdrill jet or high-pressure water jet\textsuperscript{6-8} and then carrying out hydraulic fracturing operation. Compared with radial drilling or hydraulic fracturing applied alone, radial drilling-fracturing succeeds the merits of both technologies and additionally generates a favorable synergism. On the one hand, these radial boreholes, with diameter of 25-50 mm and length of 10-100 m,\textsuperscript{9-11} can penetrate the damaged zone near the wellbore, guide hydraulic fractures toward the target area. Besides, the radial borehole can reduce fracture initiation pressure (FIP). Liu et al\textsuperscript{12} made a comparison between radial drilling-fracturing and conventional perforation fracturing through a series of triaxial hydraulic fracturing experiments. According to their experimental results, FIP of conventional perforation fracturing is 8.75 MPa. FIP of radial drilling-fracturing is only 7.94 MPa, which is 9.3\% lower than that of conventional perforation fracturing. On the other hand, the subsequent hydraulic fracturing can generate numerous artificial fractures, which further enlarge wellbore-reservoir contact area and significantly boost the well production. Many field applications of radial drilling-fracturing\textsuperscript{13-16} demonstrate that this technology can efficiently develop the low-permeability reservoir, coalbed methane reservoir, lithologic trap reservoir and dead-oil area, etc. Hence, radial drilling-fracturing has an extensive application prospect and should be studied in advance.

In the hydraulic fracturing design, FIP is a cardinal parameter to determine the pumping schedule and the relevant stimulation parameters. Thus, it is essential to predict FIP accurately for a successful hydraulic fracturing operation. Many researches have been conducted on fracture initiation through analytical, experimental, and numerical methods. FIP was first introduced by Haimson et al\textsuperscript{17} to determine in situ stress in the stratum. In addition, they established a primary model for fracture initiation in the vertical wellbore with open-hole completion. Through a series of scaled model tests, Ketterij et al\textsuperscript{18} investigated the relationship between the perforation parameters and fracture initiation. They concluded that the fracture is most likely to initiate at the perforation in general. Hossain et al\textsuperscript{19} built a classic model for predicting fracture initiation from an arbitrarily orientated wellbore, based on three possible types of generated fractures, longitudinal, transverse, and complex multiple fractures. Besides, they studied the effects of perforation and wellbore orientation on fracture initiation by the nondimensional parameter method. The roles of microstructure and fluid infiltration in fracture initiation were studied by Lhomme et al\textsuperscript{20} through various laboratory-scale experiments. It was discovered that fracture initiation was closely related to the fluid seepage mechanism and rock microstructure. Jin et al\textsuperscript{21} classified the fracture initiation in naturally fractured formation into three modes, including tensional fracture along natural fractures, shear fracture along natural fractures, and tensional fracture initiating from rock body. With the emergence of radial drilling-fracturing, scholars gradually focus on the fracture initiation in radial drilling-fracturing. By utilizing ABAQUS software, Gong

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{FIGURE_1.png}
\caption{Schematic diagram of radial drilling-fracturing}
\end{figure}
et al\textsuperscript{22} analyzed the fracture initiation in radial drilling-fracturing. They found that, in terms of FIP, azimuth is the most important factor, followed by the radial borehole length and diameter. Guo et al\textsuperscript{23} built a theoretical model of two radial boreholes and experimentally proved that the radial borehole row generates an excellent guidance on hydraulic fractures.\textsuperscript{24} In addition, they also proposed the technology of refracturing guided by radial boreholes,\textsuperscript{25} and the feasibility of this new technology was demonstrated theoretically. Liu et al\textsuperscript{26} developed a generic model for determining FIP and fracture initiation location in radial drilling-fracturing. For the tree-type hydraulic fracturing in coalbed, Zuo et al\textsuperscript{27} established a fracture initiation model where the coal seam joints and gas pressure are considered. Liu et al\textsuperscript{28} verified the guiding effect of radial holes on directional propagation of hydraulic fracture by the extended finite element method and true triaxial experiments. On the basis of Liu’s model, Chen et al\textsuperscript{29} analytically studied the interference between radial boreholes by analyzing the varieties of FIP with different borehole parameters.

These previously published fracture initiation models for radial drilling-fracturing neglect the influence of microfractures in the rock and only consider the failure of rock matrix. However, as illustrated in Figure 2, shale is characterized by abundant beddings, which can be regarded as microfractures. Based on triaxial compression tests, Niandou et al\textsuperscript{30} pointed out that shale beddings directly control the fracture initiation of shale. Meanwhile, these previously published models neglect the physico-chemical actions between rock and fracturing fluid, but the shale hydration can affect the shale’s mechanical strength and cause additional hydration stress. Besides, the former models for radial drilling-fracturing assume that the main wellbore is vertical and the radial borehole is horizontal. In practice, the main wellbore is not always vertical, and the radial borehole is not necessarily horizontal, especially in dipping reservoirs. The radial borehole is much longer than the perforation, with a maximum length of 100 m. As illustrated in Figure 3, in the dipping reservoir, the horizontal radial borehole probably protrudes into the barrier, and this will result in the failure of hydraulic fracturing. Therefore, it is important to ensure that the radial borehole is parallel to the reservoir, which entails the tilt of the radial borehole. Hence, the deviation of main wellbore, the tilt of radial borehole, and the dip of shale reservoir should also be taken into account in the fracture initiation model.

In summary, the previous fracture initiation models for radial drilling-fracturing fail to consider shale beddings, shale hydration as well as inclinations of the main wellbore, radial borehole, and shale reservoir. Thus, these previous models are inapplicable for the radial drilling-fracturing in the shale reservoir. In this paper, the shale failure is classified into three modes: Shale matrix failure, tensile failure along bedding, and shear failure along bedding. Based on this, we establish an improved fracture initiation model for radial drilling-fracturing applied in the shale reservoir, which can predict FIP and the specific failure mode of shale. Compared with the former models, this novel model additionally considers the shale hydration and the inclinations of the main wellbore, radial borehole, and shale reservoir. Combining this model, we perform an array of sensitivity analyses on different factors, including the azimuth of the radial borehole, the angle between the radial borehole and formation strike, the dip angle of shale formation, azimuth difference between the main wellbore and the radial borehole, deviation angle of the main wellbore, reservoir temperature, and pore pressure.

2  |  MODEL ESTABLISHMENT

In this part, the stress distribution around the radial borehole is obtained by superposition of stress induced by main wellbore and radial borehole. The shale fracture criteria for each mode are given, respectively, in order to determine FIP and the specific failure mode of shale.

2.1  |  Model assumptions and coordinate systems

It is stipulated in the model that tensile stress is negative and compressive stress is positive. Besides, the assumptions made in the model derivation are as follows.

1. Due to the small curvature radius of radial drilling, the radial borehole is regarded as a small open borehole intersecting with the cased main wellbore.
2. The formation rock is assumed to be homogeneous, isotropic, and linearly elastic porous medium, which is suitable for simplified calculations and widely adopted by the previous studies.
3. The Young’s moduli of the formation and the cement sheath are in the same order of magnitude. Thus, it is assumed that the formation and the cement sheath have the same Young’s modulus for the calculation simplification.
4. The fluid flow in the medium is radial flow and follows Darcy’s law.

As illustrated in Figure 4, radial drilling-fracturing is applied in shale reservoir, and the drilled radial borehole is parallel to the shale reservoir to prevent it from protruding.
into the barrier. The in situ stress coordinate system \( (x_s, y_s, z_s) \) is defined by three mutually perpendicular in situ stresses. The rectangular coordinate system \( (x_1, y_1, z_1) \) is derived from rotating coordinate system \( (x_s, y_s, z_s) \) through \( \beta_M \) degree around \( z_s \)-axis. Then, the main wellbore rectangular coordinate system \( (x_0, y_0, z_0) \) is constructed by rotating coordinate system \( (x_1, y_1, z_1) \) through \( \gamma \) degree around \( y_1 \)-axis. By a similar approach, the radial borehole rectangular coordinate system \( (x, y, z) \) can be established by rotating coordinate system \( (x_0, y_0, z_0) \) through \( \alpha \) and \( \omega \) degrees successively. Moreover, the cylindrical coordinate systems of the main wellbore and radial borehole are \( (\theta_0, r_0, z_0) \) and \( (\theta, r, z) \) correspondingly. The origins of the coordinate systems \( (x, y, z), (x_0, y_0, z_0), (r, \theta, z), \) and \( (r_0, \theta_0, z_0) \) are set at the intersection point of two borehole axes. Point M is the research point where the stress is computed.

Angles \( \beta_M \) and \( \beta_R \) are the azimuths of the main wellbore and radial borehole, respectively. Angle \( \gamma \), the deviation angle of the main wellbore, is the angle between vertical in situ stress and the axis of main wellbore. Angle \( \alpha \) is the angle between the \( x_0 \)-axis and the project line of the radial borehole axis on the main wellbore cross section. Angle \( \omega \), the tilt angle of the radial borehole, is the angle between the radial borehole axis and the main wellbore.
cross section. Angle $\eta$ is the strike angle of the shale formation, and angle $\zeta$ is the dip angle of the shale formation. Meanwhile, in the cylindrical coordinate system ($\theta$, $r$, $z$), the intersection angle between a particular bedding plane and the wall of radial borehole is constant and noted as $\delta$. Besides, the radial borehole is parallel to the reservoir, and angle $\alpha$ and angle $\omega$ can be represented by other angles through simple derivation.

$$\alpha = \arccot(cot(\beta_R - \beta_M)/\cos\gamma)$$

$$\omega = \arctan((\sin(\beta_R - \eta)\tan\zeta) + \arctan(\cos(\beta_R - \beta_M)\tan\gamma)$$

where $\alpha$ is the angle between the $x_0$-axis and the project line of the radial borehole axis on the main wellbore cross section, degrees; $\beta_R$ is the azimuth of the radial borehole, degrees; $\beta_M$ is the azimuth of the radial borehole, degrees; $\gamma$ is the deviation angle of the main wellbore, degrees; $\omega$ is the tilt angle of the radial borehole, degrees; $\eta$ is the strike angle of the shale formation, degrees; $\zeta$ is the dip angle of the shale formation, degrees.

$$\sigma_{In-Situ}^{r_0} = \sigma_{In-Situ}^{x_0 y_0} \left(1 + \frac{3}{r_0^2} - \frac{4}{r_0^2} \right) \sin2\theta_0 + \frac{\sigma_{In-Situ}^{x_0 y_0} - \sigma_{In-Situ}^{y_0 x_0}}{2} \left(1 + \frac{3}{r_0^2} - \frac{4}{r_0^2} \right) \cos2\theta_0 + \frac{\sigma_{In-Situ}^{x_0 y_0} + \sigma_{In-Situ}^{y_0 x_0}}{2} \left(1 - \frac{R_s^2}{r_0^2} \right) \sin2\theta_0$$

$$\sigma_{\theta_0}^{in-Situ} = \frac{\sigma_{In-Situ}^{x_0 y_0} + \sigma_{In-Situ}^{y_0 x_0}}{2} - \frac{\sigma_{In-Situ}^{x_0 y_0} - \sigma_{In-Situ}^{y_0 x_0}}{2} \left(1 + \frac{3}{r_0^2} - \frac{4}{r_0^2} \right) \cos2\theta_0 + \frac{\sigma_{In-Situ}^{x_0 y_0} + \sigma_{In-Situ}^{y_0 x_0}}{2} \left(1 + \frac{3}{r_0^2} - \frac{4}{r_0^2} \right) \sin2\theta_0$$

$$\sigma_{z_0}^{in-Situ} = \sigma_{In-Situ}^{x_0 y_0} \left(1 + \frac{2}{r_0^2} \right) \cos\theta_0 + \frac{\sigma_{In-Situ}^{x_0 y_0} - \sigma_{In-Situ}^{y_0 x_0}}{2} \left(1 + \frac{2}{r_0^2} \right) \sin\theta_0$$

where $\nu$, is Poisson’s ratio of shale; $R_s$ is the external radius of the casing, m; $r_0$ and $\theta_0$ are coordinates of point M in the coordinate system ($r_0$, $\theta_0$, $z_0$), m and degrees; $\sigma_{In-Situ}^{x_0 y_0}$, $\sigma_{In-Situ}^{y_0 x_0}$, $\sigma_{In-Situ}^{x_0 z_0}$, $\sigma_{In-Situ}^{y_0 z_0}$, $\sigma_{In-Situ}^{z_0 x_0}$, and $\sigma_{In-Situ}^{z_0 y_0}$ are the stress components caused by in situ stress in the coordinate system ($x_0$, $y_0$, $z_0$), MPa.

The in situ stress components are as below in the in situ stress coordinate system ($x_0$, $y_0$, $z_0$).

$$\begin{bmatrix}
\sigma_{x_0} & \tau_{x_0 y_0} & \tau_{x_0 z_0} \\
\tau_{x_0 y_0} & \sigma_{y_0} & \tau_{y_0 z_0} \\
\tau_{x_0 z_0} & \tau_{y_0 z_0} & \sigma_{z_0}
\end{bmatrix} = \begin{bmatrix}
\sigma_H & 0 & 0 \\
0 & \sigma_h & 0 \\
0 & 0 & \sigma_v
\end{bmatrix}$$

where $\sigma_H$ is the maximum horizontal in situ stress, MPa; $\sigma_h$ is the minimum horizontal in situ stress, MPa; $\sigma_v$ is the vertical in situ stress, MPa; $\sigma_x$, $\sigma_y$, and $\sigma_z$ are the stress components in the coordinate system ($x_0$, $y_0$, $z_0$), MPa.

Therefore, the stress distribution caused by in situ stress can be obtained in the coordinate system ($x_0$, $y_0$, $z_0$) based on the previous studies.17

In the process of hydraulic fracturing, the internal pressure of the borehole, $P_w$, will rise to a high level and yield stress around the borehole. However, in terms of the
cased main wellbore, only partial internal pressure can act on the shale surface through the stiff casing. Thus, the stresses caused by the internal pressure of the main wellbore can be written as follows in the coordinate system \((r_0, \theta_0, z_0)\).\(^{31}\)

\[
\sigma_{r0}^{\text{IP-MB}} = \frac{R_2^2}{r_0^2} P_1 \\
\sigma_{\theta0}^{\text{IP-MB}} = -\frac{R_2^2}{r_0^2} P_1
\]

(6)

Of which:

\[
P_1 = \frac{1 + \nu_r}{E_r} \frac{2(1 - \nu_r)}{R_i^2} R^2 P_w / \left[ \frac{1 + \nu_r}{E_r} \frac{1 + \nu_r R_i^2 + (1 - 2\nu_r)R_2^2}{R_2^2 - R_i^2} \right].
\]

where \(E_r\) is Young’s modulus of the casing, MPa; \(\nu_r\) is Poisson’s ratio of the casing; \(R_i\) is the internal radius of the casing; \(m; E_r\) is Young’s modulus of shale, GPa; \(P_w\) is the internal pressure of borehole, MPa.

### 2.3 Stress caused by radial borehole

It is different from the cased main wellbore that radial borehole is completed with open hole. Therefore, the internal pressure of radial borehole can directly act on the shale surface, and the stresses caused by the internal pressure of radial borehole are as below in the coordinate system \((r, \theta, z)\).\(^{31}\)

\[
\sigma_r^{\text{IP-RB}} = \frac{R_2^2}{r^2} P_w \\
\sigma_\theta^{\text{IP-RB}} = -\frac{R_2^2}{r^2} P_w
\]

(7)

where \(R\) is the radius of the radial borehole, m; \(r\) is the coordinate of point M in the cylindrical coordinate system \((r, \theta, z)\), m; \(\sigma_r^{\text{FP-RB}}\) and \(\sigma_\theta^{\text{FP-RB}}\) are stress components caused by the internal pressure of radial borehole in the coordinate system \((r, \theta, z)\), MPa.

Moreover, under the high internal pressure, the fracturing fluid will infiltrate into the surrounding shale from the radial borehole and causes additional stress components near the radial borehole. As it is assumed that the fluid flow in the medium is radial flow and follows Darcy’s law, the stress components generated by the infiltration of fracturing fluid can be given in the cylindrical coordinate system \((r, \theta, z)\).\(^{32}\)

\[
\sigma_r^{\text{FP-RB}} = \kappa \left( P_w - P_p \right) \left( \frac{\varepsilon (1 - 2\nu_r)}{2 (1 - \nu_r)} \left( 1 - \frac{R_2^2}{r^2} \right) - \phi \right) \\
\sigma_\theta^{\text{FP-RB}} = \kappa \left( P_w - P_p \right) \left( \frac{\varepsilon (1 - 2\nu_r)}{2 (1 - \nu_r)} \left( 1 + \frac{R_2^2}{r^2} \right) - \phi \right) \\
\sigma_z^{\text{FP-RB}} = \kappa \left( \frac{\varepsilon (1 - 2\nu_r)}{1 - \nu_r} - \phi \right) \left( P_w - P_p \right)
\]

(8)

where \(\phi\) is shale porosity; \(\varepsilon\) is Biot’s coefficient; \(P_p\) is the initial pore pressure, MPa; \(\kappa\) is the permeability coefficient; \(\sigma_r^{\text{FP-RB}}, \sigma_\theta^{\text{FP-RB}}, \text{ and } \sigma_z^{\text{FP-RB}}\) are stress components caused by the infiltration of fracturing fluid in the coordinate system \((r, \theta, z)\), MPa.

### 2.4 Total stress around radial borehole

To obtain the total stress around the radial borehole, the stress components caused by the cased main wellbore need to be transformed to cylindrical coordinate system \((r, \theta, z)\) from cylindrical coordinate system \((r_0, \theta_0, z_0)\). In accordance with Equations (5) and (6), the total stress components caused by the main wellbore are as follows in the coordinate system \((r_0, \theta_0, z_0)\).

\[
\sigma_{r_0}^{\text{MB}} = \tau_{r_\theta_0}^{\text{MB}} \left( 1 + \frac{3 \varepsilon}{r_0^2} - \frac{3 \varepsilon}{r_0^4} \right) \frac{R_2^2}{r_0^2} \sin \theta_0 + \frac{\tau_{\theta_\theta_0}^{\text{MB}}}{2} \frac{\sin \theta_0}{\tau_{r_\theta_0}^{\text{MB}}} - \frac{2 \varepsilon}{r_0^2} \cos \theta_0 + \frac{\tau_{r_\theta_0}^{\text{MB}}}{2} \frac{\sin \theta_0}{\tau_{r_\theta_0}^{\text{MB}}} - \frac{2 \varepsilon}{r_0^2} \sin \theta_0
\]

(9)
where \( \sigma_{r0}^{MB}, \sigma_{\theta0}^{MB}, \sigma_{z0}^{MB}, \tau_{r\theta0}^{MB}, \tau_{r\theta0}^{MB}, \) and \( \tau_{rz0}^{MB} \) are the total stress components caused by the main wellbore in the coordinate system \((r_0, \theta_0, z_0)\), MPa.

Then, the total stress components caused by the main wellbore are converted to coordinate systems \((x_0, y_0, z_0)\), \((x, y, z)\), and \((r, \theta, z)\) successively. Finally, the total stress components caused by the cased main wellbore can be acquired in coordinate system \((r, \theta, z)\) by the transformation matrix 10.

Thus, by the stress superposition principle, the total stress around radial borehole can be directly obtained in coordinate systems \((r, \theta, z)\).

\[
\begin{align*}
\sigma_{r}^{Total} &= \sigma_{r}^{MB} + \frac{R^2}{r^2} P_w + \kappa \left( P_w - P_\rho \right) \left( \frac{\epsilon (1 - 2v_r)}{2 (1 - v_r)} \left( 1 - \frac{R^2}{r^2} \right) - \phi \right) \\
\sigma_{\theta}^{Total} &= \sigma_{\theta}^{MB} + \frac{R^2}{r^2} P_w + \kappa \left( P_w - P_\rho \right) \left( \frac{\epsilon (1 - 2v_\theta)}{2 (1 - v_\theta)} \left( 1 + \frac{R^2}{r^2} \right) - \phi \right) \\
\sigma_{z}^{Total} &= \sigma_{z}^{MB} + \kappa \left( \frac{\epsilon (1 - 2v_\rho)}{1 - v_\rho} \right) \left( P_w - P_\rho \right)
\end{align*}
\]

where \( \sigma_{r}^{Total}, \sigma_{\theta}^{Total}, \sigma_{z}^{Total}, \tau_{r\theta}^{Total}, \tau_{r\theta}^{Total}, \) and \( \tau_{rz}^{Total} \) are the total stress components around the radial borehole in the coordinate system \((r, \theta, z)\), MPa.

Based on Equation (11), the stress distribution on the wall of the radial borehole can be obtained by setting \( r \) to \( R \), which is the radius of the radial borehole, and consecutively changing the values of \( \theta \) and \( z \). In addition, the three principal stresses on the radial borehole wall, \( \sigma_1, \sigma_2, \) and \( \sigma_3 \), can be calculated by Equation (12)

\[
\begin{align*}
\sigma_1 &= \sigma_r^{Total} \\
\sigma_2 &= \frac{1}{2} \left[ \left( \sigma_{\theta}^{Total} + \sigma_{z}^{Total} \right) + \sqrt{\left( \sigma_{\theta}^{Total} - \sigma_{z}^{Total} \right)^2 + 4 \left( \tau_{r\theta}^{Total} \right)^2} \right] \\
\sigma_3 &= \frac{1}{2} \left[ \left( \sigma_{\theta}^{Total} + \sigma_{z}^{Total} \right) - \sqrt{\left( \sigma_{\theta}^{Total} - \sigma_{z}^{Total} \right)^2 + 4 \left( \tau_{r\theta}^{Total} \right)^2} \right]
\end{align*}
\]

The directions of principal stresses are illustrated in Figure 5. The three principal stresses are perpendicular to each other, and \( \sigma_1 \) is always perpendicular to the axis of the radial borehole. Besides, angle \( \psi \) is the angle between \( \sigma_2 \) and the radial borehole axis, and angle \( \rho \) is the included angle between radial borehole axis and the horizontal plane. Angle \( \psi \) and \( \rho \) are calculated by Equations (13) and (14) separately.

\[
\psi = \frac{1}{2} \tan^{-1} \left( \frac{2 \sigma_{z}^{Total}}{\sigma_{\theta}^{Total} - \sigma_{z}^{Total}} \right)
\]

\[
\rho = \arctan (\sin(\beta_R - \eta) \tan \zeta)
\]

Therefore, the direction vectors of principal stresses in the coordinate system \((x, y, z)\) can be easily computed by several spatial angles. The direction vectors of \( \sigma_1, \sigma_2, \) and \( \sigma_3 \) are notated as \( \lambda_1, \lambda_2, \) and \( \lambda_3 \) correspondingly, and the expressions are as below.

\[
\begin{align*}
\lambda_1 &= a_1 i + a_2 j + a_3 k \\
\lambda_2 &= -\sin \beta_R \cos \psi - \cos \beta_R \sin \psi \sin \theta \\
\lambda_3 &= \cos \beta_R \cos \psi - \sin \beta_R \sin \psi \sin \theta
\end{align*}
\]
\[
\begin{align*}
\lambda_2 &= b_1 i + b_2 j + b_3 k \\
b_1 &= \cos \beta \cos \rho \cos \psi + \sin \beta \sin \psi \\
b_2 &= \sin \beta \cos \rho \cos \psi - \cos \beta \sin \psi \\
b_3 &= \sin \beta \sin \rho \cos \psi + \cos \beta \cos \psi \\
\end{align*}
\]

\[
\begin{align*}
\lambda_3 &= c_1 i + c_2 j + c_3 k \\
c_1 &= -\cos \beta \cos \rho \sin \psi + \sin \beta \sin \psi \\
c_2 &= -\sin \beta \cos \rho \sin \psi - \cos \beta \sin \psi \\
c_3 &= -\sin \rho \sin \psi + \cos \rho \cos \psi \\
\end{align*}
\]

Moreover, the normal vector to shale bedding plane can be obtained in the in situ coordinate system \((x, y, z)\) by Equation (18).

\[
\lambda_b = d_1 i + d_2 j + d_3 k \\
d_1 = \sin \psi \sin \zeta \\
d_2 = -\cos \psi \sin \zeta \\
d_3 = \cos \zeta \\
\]

The angles between the normal vector to shale bedding plane and direction vectors of three principal stresses are recorded as \(\varphi_1, \varphi_2, \varphi_3\). These included angles can be acquired individually according to the included angle formula.  

\[
\varphi_n = \cos^{-1}\left(\frac{\lambda_b \lambda_n}{|\lambda_b| |\lambda_n|}\right) \\
\]

2.5 | Fracture initiation of radial drilling-fracturing

2.5.1 | The effect of shale hydration

Due to the strong water sensitivity, the hydration swelling of shale occurs after it contacts with fracturing fluid. This phenomenon is called shale hydration and should be considered in the fracture initiation criteria of shale. So far, the research on shale hydration has mainly focused on the wellbore stability of drilling. In the light of relevant researches, the effect of shale hydration can be signified by the hydration stress, which is calculated by Equation (20).

\[
P_\pi = I_m \frac{R' T}{V} \ln \frac{A_f}{A_{sh}} \times 10^{-6} \\
\]

where \(P_\pi\) is the hydration stress, MPa; \(I_m\) is membrane efficiency; \(R'\) is the gas constant, 8.314 J mol\(^{-1}\) K\(^{-1}\); \(T\) is the temperature, K; \(V\) is the partial molar volume of the water, \(1.8 \times 10^{-5}\) m\(^3\) mol\(^{-1}\); \(A_f\) is fracturing fluid water activity; \(A_{sh}\) is shale water activity.

2.5.2 | Fracture initiation criteria of shale

The failure of shale can be divided into three modes, including shale matrix failure, tensile failure along bedding, and shear failure along bedding. Their corresponding fracture initiation criteria are as follows.

1. Shale matrix failure

Shale is a porous material, and the total stress should be substituted by the effective stress. Thus, the maximum tensile stress of point \(M (r, \theta, z)\) can be calculated by Equation (21).

\[
s_t(r, \theta, z) = -[s_3(r, \theta, z) - \tau P_p - P_z] \\
\]

The condition of shale matrix failure reads.

\[
s_t(r, \theta, z) > \sigma_f \\
\]

where \(\sigma_f\) is the tensile strength of the shale matrix, MPa.

When Equation (22) is met, shale matrix failure takes place, and the internal pressure at this moment is named as matrix failure pressure, \(P_{MF}\).

2. Tensile failure along bedding

In the process of hydraulic fracturing, fracturing fluid flows into shale beddings and generates normal stress acting on the bedding plane. The effective normal stress on the research point \(M (r, \theta, z)\) can be computed by Equation (23).

\[
s_n(r, \theta, z) = -[s_3(r, \theta, z) cos^2 \varphi_3 + s_2(r, \theta, z) cos^2 \varphi_2 + s_1(r, \theta, z) cos^2 \varphi_1 - \tau P_p - P_z] \\
\]

Tensile failure initiates along the bedding plane when the effective normal stress exceeds the tensile strength of bedding plane.

\[
s_n(r, \theta, z) > \sigma_{BT} \\
\]

where \(\sigma_{BT}\) is the tensile strength of the shale bedding, MPa.

When Equation (24) is satisfied, tensile failure occurs in shale bedding, and the internal pressure required to satisfy Equation (24) is notated as bedding tensile failure pressure, \(P_{BT}\).
3. Shear failure along bedding

In addition to tensile failure, shear failure may also take place in shale bedding. Based on the Mohr-Coulomb criterion, the condition of shear failure along bedding can be deduced as follows.\(^\text{(21)}\)

\[
\sigma_1(r, \theta, z) - \sigma_3(r, \theta, z) > \frac{2(c_B + \sigma_3(r, \theta, z)\tan a_B)}{(1 - \tan \alpha \cot \phi_1)\sin 2\phi_1}
\]

\[(25)\]

where \(a_B\) is the internal friction angle of shale bedding, degrees; \(c_B\) is the cohesion force of shale bedding, MPa.

Shear failure along bedding occurs when the shear stress surpasses the shear strength, and the required pressure is recorded as bedding shear failure pressure, \(P_{BS}\).

2.5.3 | The prediction of theoretical fracture initiation pressure

The calculation process of theoretical fracture initiation pressure is shown in Figure 6. First, set an initial internal pressure, \(P_w\), and input it into the established model. By continuously changing \(\theta\) and \(z\), the stress distribution on the wall of radial borehole can be obtained. Then, according to the three fracture initiation criteria, determine whether the shale matrix or shale bedding will crack. The current internal pressure will be recorded as the initiation pressure of the present failure mode if a specific failure mode takes place. Otherwise, further increase the internal pressure and conduct trial calculation again until the initiation pressures of all failure modes are obtained. The actual fracture initiation pressure is determined by the minimum value of the initiation pressure of the three fracture modes. Finally, theoretical FIP is obtained.

\[
P_f = \min(P_{MF}, P_{BT}, P_{BS})
\]

\[(26)\]

3 | CASE CALCULATION AND SENSITIVITY ANALYSIS

In this section, the influences of several factors on the fracture initiation of radial drilling-fracturing in the shale reservoir are analyzed through a series of sensitivity analysis. Since the main improvement of this model is additionally considering the shale hydration as well as the inclinations of the wellbore and reservoir, the factors related to the improvement are mainly selected for analysis. The azimuth of radial borehole is also analyzed as the factor is the key parameter of borehole layout. These analyzed factors include azimuth of radial borehole, angle between radial borehole and formation strike, the dip angle of shale formation, azimuth difference between radial borehole and main wellbore, the deviation of main wellbore, reservoir temperature, and pore pressure. The input values of basic parameters are shown in Table 1, in which the shale mechanical parameters are the measured data of Longmaxi shale formation.
### 3.1 Influence of radial borehole azimuth

In this section, the azimuth of the radial borehole is changed incessantly to investigate its influence on fracture initiation. In addition, to avoid being affected by the inclinations of the main wellbore, radial borehole, and shale reservoir, it is assumed that the main wellbore is vertical and the shale reservoir is horizontal, namely $\gamma = 0^\circ$ and $\zeta = 0^\circ$.

As illustrated in Figure 7, the theoretical FIP rises from 36.9 MPa to 42.8 MPa when the azimuth of radial borehole rises from $0^\circ$ to $90^\circ$. Besides, the pressures required for three failure modes all ascend with the increasing radial borehole azimuth, but their ascending characteristics are different. When the azimuth of the radial borehole grows from $0^\circ$ to $30^\circ$, the bedding tensile failure pressure swiftly ascends from 36.9 MPa to 42.4 MPa. As the azimuth of the radial borehole exceeds $30^\circ$, the ascending rate slows down significantly, and the bedding tensile failure pressure nearly remains stable, merely enlarging 0.8 MPa with the radial borehole azimuth rising from $30^\circ$ to $90^\circ$. Both matrix failure pressure and bedding shear failure pressure keep a positive correlation to the radial borehole azimuth in the whole process. The growth rate of matrix failure pressure is greater than that of bedding shear failure pressure. With the radial borehole azimuth increasing from $0^\circ$ to $90^\circ$, matrix failure pressure ascends by 27% to 48.5 MPa, and bedding shear failure pressure ascends by 14.5% to 48.1 MPa. Furthermore, the difference in variation characteristics leads to the change of shale failure mode. It can be concluded from Figure 6 that a small or large azimuth of radial borehole, $\beta < 20^\circ$ or $\beta > 40^\circ$, can predispose shale to bedding tensile failure, and shale matrix failure is apt to occur when the radial borehole azimuth is moderate, $20^\circ \leq \beta \leq 40^\circ$.

#### TABLE 1 Basic parameters

| Parameter                                      | Value | Unit |
|-----------------------------------------------|-------|------|
| Parameters of in situ stress                  |       |      |
| Maximum horizontal in situ stress, $\sigma_H$ | 48    | MPa  |
| Minimum horizontal in situ stress, $\sigma_h$ | 40    | MPa  |
| Vertical in situ stress, $\sigma_v$           | 50    | MPa  |
| Parameters of reservoir                       |       |      |
| Young’s modulus of shale, $E_r$               | 14.09 | GPa  |
| Poisson’s ratio of the shale, $\nu_r$         | 0.367 | -    |
| Biot’s coefficient, $\xi$                    | 0.85  | -    |
| Shale porosity, $\phi$                       | 0.11  | -    |
| Permeability coefficient, $\chi$             | 0.9   | -    |
| Initial pore pressure, $P_p$                 | 18    | MPa  |
| Internal friction angle of shale bedding, $a_B$ | 33.86 | degrees |
| Cohesion force of shale bedding, $c_B$        | 8.98  | MPa  |
| Tensile strength of shale matrix, $\sigma_T$  | 11.67 | MPa  |
| Tensile strength of shale bedding, $\sigma_{BT}$ | 2    | MPa  |
| Membrane efficiency, $I_m$                   | 0.1   | -    |
| Reservoir temperature, $T$                   | 363   | K    |
| Shale water activity, $A_{sh}$               | 0.915 | -    |
| Parameters of boreholes                      |       |      |
| Internal radius of the main wellbore casing, $R_i$ | 0.165 | m   |
| External radius of the main wellbore casing, $R_e$ | 0.18  | m   |
| Radius of the radial borehole, $R$            | 0.015 | m   |
| Length of the radial borehole, $L$            | 20    | m    |
| Young’s modulus of the casing, $E_c$          | 135   | GPa  |
| Poisson’s ratio of the casing, $\nu_c$       | 0.22  | -    |
| Angle between the radial borehole wall and bedding plane, $\omega$ | 12 | degrees |
| Parameters of fracturing fluid                |       |      |
| Fracturing fluid water activity, $A_f$       | 0.78  | -    |

### 3.2 Influence of angle between radial borehole and formation strike

By changing formation strike angle $\eta$ and investigating the variation of FIP, the influence of the angle between the radial borehole and formation strike $\beta_R-\eta$ is studied in this section. To eliminate the effects of the other factors, other parameters are stipulated as follows: $\gamma = 0^\circ$, $\beta_R = 30^\circ$, and $\zeta = 10^\circ$.

As shown in Figure 8, matrix failure pressure is little affected by the angle between radial borehole and formation strike. Matrix failure pressure basically maintains at 40.5 MPa and only has a slight decline of 0.1 MPa when the included angle is around $90^\circ$ and $180^\circ$. However, the
change of bedding tensile failure pressure and bedding shear failure pressure is more prominent. With the angle between the radial borehole and formation strike increasing from 0° to 90°, bedding tensile failure pressure initially remains unchanged, then increases rapidly, and reaches its peak, 54.4 MPa. With the further increase of included angle, bedding tensile failure pressure gradually falls back and finally remains constant. On the whole, the changing curve of bedding tensile failure pressure is symmetrical to the axis where the included angle equals 180°. With the increasing angle between radial borehole and formation strike, bedding shear failure pressure swiftly declines at first and touches the minimum of 33.7 MPa when the included angle is 140°. Then, bedding shear failure pressure reaches a plateau where it fluctuates around 34.5 MPa. When the angle between radial borehole and formation strike transcends 230°, bedding shear failure pressure begins to quickly ascend and finally returns to its summit, 66.3 MPa. With the increase of the included angle from 0 to 360°, the following failure modes occur successively in shale: tensile failure along bedding, matrix failure, shear failure along bedding, matrix failure, and tensile failure along bedding. Therefore, bedding tensile failure tends to occur when the angle between radial borehole and formation strike is considerably small or large, and bedding shear failure is apt to occur under a moderate included angle. Only when the included angle is around 90° or 180°, shale is prone to shale matrix failure.

3.3 Influence of shale formation dip angle

The dip angle of formation is one of the essential geological parameters of shale reservoir. It affects the layout of boreholes and the selection of hydraulic fracturing parameters. In this part, the influence of shale formation dip angle is studied and other parameters are assumed to be $\gamma = 0^\circ$, $\beta_R = 30^\circ$, and $\eta = -60^\circ$.

FIP curves of shale formation with different dip angles are shown in Figure 9. Pressures required for all failure modes decline with the increasing dip angle of shale formation. And the descending rate of bedding shear failure pressure is the fastest among the three failure modes. With the increase of formation dip angle from 0° to 25°, bedding shear failure pressure decreases from 44 MPa to 25.1 MPa, and bedding tensile failure pressure smoothly declines from 42 MPa to 35.1 MPa. However, the curve of matrix failure pressure shows different characteristic. There is no noticeable change in matrix failure pressure at first, and matrix failure pressure declines only 0.2 MPa as the dip angle of shale formation reaches 15°. The declining rate of matrix failure pressure is significantly accelerated when the dip angle transcends 15°. The matrix failure pressure decreases rapidly from 40.3 MPa to 30.7 MPa with the dip angle rising from 15° to 25°. Bedding shear failure pressure declines most dramatically, followed by shale matrix failure and bedding tensile failure. Therefore, the increase of dip angle will make shale prone to bedding shear failure and weaken the tendency to bedding tensile failure.

3.4 Influence of azimuth difference between radial borehole and main wellbore

The azimuth of main wellbore is a relevant parameter of drilling design. In this part, the influence of the azimuth difference between the radial borehole and main wellbore, $\beta_R - \beta_M$, is investigated. Meanwhile, to avoid the influence
of the other factors, it is assumed that $\zeta = 0^\circ$, $\beta_R = 30^\circ$, and $\gamma = 20^\circ$.

As illustrated in Figure 10, pressures required for bedding tensile failure and bedding shear failure have a similar varying trend with the change in the azimuth difference between the radial borehole and main wellbore. With the increase of the azimuth difference from $0^\circ$ to $180^\circ$, the two curves initially go up and climb to their summits where the azimuth difference equals $90^\circ$. Then, they begin to slide downward and revert to rise again when azimuth difference between the radial borehole and main wellbore is around $150^\circ$. In addition, the variation range of bedding shear failure pressure is much wider than that of bedding tensile failure pressure. Matrix failure pressure periodically fluctuates with the increasing azimuth difference, forming an inverted W-shape curve. With the increase of azimuth difference, the failure mode of shale is shear failure along bedding at first. Then, it converts to tensile failure along bedding when the azimuth difference is beyond $60^\circ$ and reverts to shear failure along bedding when the azimuth difference exceeds $120^\circ$. Meanwhile, compared with the other two failure modes, pressure required for bedding shear failure is quite low at the outset, but it varies more dramatically with the change of azimuth difference. Thus, a small azimuth difference inclines shale to bedding shear failure, and the other two failure modes tend to occur when azimuth difference is around $90^\circ$.

### 3.5 Influence of main wellbore deviation angle

In practice, the main wellbore generally has a certain deviation angle, whose effect is neglected by previous models for radial drilling-fracturing. To investigate the effect of the main wellbore deviation angle, the deviation angle of main wellbore is changed continuously, and other parameters are assumed to be $\zeta = 0^\circ$, $\beta_R = 30^\circ$, and $\beta_M = 90^\circ$.

Figure 11 reveals the changing trend of pressures required for three failure modes with the change in deviation angle of main wellbore. With the increase of the main wellbore deviation angle from $0^\circ$ to $40^\circ$, pressures required for bedding shear failure and bedding tensile failure increase mildly at the beginning. The pressure decreases, respectively, when the deviation angle of main wellbore exceeds $10^\circ$ and $15^\circ$. In addition, the descending rate of bedding shear failure pressure is much higher than that of bedding tensile failure pressure. On the other hand, shale matrix failure pressure initially stays stable when the deviation angle of main wellbore is between $0^\circ$ to $20^\circ$. However, as long as the deviation angle goes beyond $20^\circ$, shale matrix failure pressure declines swiftly with the increasing deviation angle. In general, with the increase of deviation angle from $0^\circ$ to $40^\circ$, bedding shear failure exhibits the maximum pressure drop among the three failure modes. Meanwhile, the shale failure mode is shale matrix failure when the deviation angle of main wellbore is between $0^\circ$ to $35^\circ$, but it is converted to shear failure along bedding when the deviation angle of main wellbore exceeds $35^\circ$. Therefore, the increase of the main wellbore deviation angle predisposes shale to crack with bedding shear failure.

### 3.6 Influence of reservoir temperature

Reservoir temperature is an essential factor that can affect the shale hydration degree and the magnitude of FIP.
However, the published fracture initiation models for radial drilling-fracturing have not considered the impact of reservoir temperature on fracture initiation. In this part, to study the influence of reservoir temperature, other parameters are assumed as follows: $\beta_R = 30^\circ$, $\beta_M = 90^\circ$, $\zeta = 10^\circ$, $\eta = -60^\circ$, and $\gamma = 10^\circ$.

The curve of FIP vs reservoir temperature is shown in Figure 12. Pressures required for bedding tensile failure and matrix failure both present a negative linear correlation with the increasing reservoir temperature. Pressures descend 0.2 MPa for every 50 K increase of reservoir temperature. With the rise of reservoir temperature from 273 K to 473 K, matrix failure pressure declines from 41 MPa to 39 MPa, and bedding tensile failure pressure descends from 41.5 MPa to 39.5 MPa. No matter how reservoir temperature changes, the pressure required for bedding shear failure remains unchanged at 39.5 MPa. It is apparent that reservoir temperature has no impact on bedding shear failure. According to the different changing characteristics of the pressures required for three failure modes, it can be concluded that a high reservoir temperature can reduce theoretical FIP and induce matrix failure and bedding tensile failure of shale.

### 3.7 Influence of pore pressure

As is concluded in many previous studies, formation pore pressure is an indispensable parameter in the design of hydraulic fracturing and can affect the fracture initiation of radial drilling-fracturing. However, these previous studies fail to consider shale beddings and only analyze the influence of pore pressure on matrix failure. Hence, the influence of pore pressure on three failure modes is investigated in this part. To analyze the shale fracture initiation with different pore pressures and avert the effects of other factors, other calculation parameters are as follows: $\beta_R = 30^\circ$, $\beta_M = 90^\circ$, $\zeta = 10^\circ$, $\eta = -60^\circ$, and $\gamma = 10^\circ$.

As shown in Figure 13, the pressures required for three failure modes all decrease with the increase of pore pressure. There is an excellent linear relationship between the pressures required for bedding tensile failure as well as bedding shear failure and pore pressure. For every 1 MPa rise in pore pressure, the bedding shear failure pressure drops 0.4 MPa, and the bedding tensile failure pressure declines 1.5 MPa. Meanwhile, with the increasing pore pressure, the matrix failure pressure evenly decreases in the beginning, dropping 2.8 MPa for every 1 MPa rise in pore pressure. When pore pressure outstrips 18 MPa, the decrease rate of matrix failure pressure gets fast. The pressures required for matrix failure and bedding tensile failure are less than that of bedding shear failure, leading to the conversion of the shale failure mode. Overall, the declining rate of bedding shear failure is much less than that of the other modes. Thus, the increase of pore pressure can reduce theoretical FIP and change the failure mode. Shale tends to crack with the bedding shear failure under a low pore pressure, and the increase of pore pressure can incline shale prone to matrix failure and bedding tensile failure.

### 4 Conclusions and Recommendations

In this paper, we develop a novel fracture initiation model for radial drilling-fracturing applied in the shale reservoir. Compared with the published analytical models, this new model additionally considers shale hydration as well as the inclinations of the main wellbore, radial borehole, and shale reservoir. This model classifies the shale failure into three failure modes to investigate the influence of
shale beddings. Then, the influences of different factors on the fracture initiation are studied through a series of sensitivity analyses. This model can be utilized to predict the theoretical FIP and specific failure mode of shale for the radial drilling-fracturing optimization and provide guidance for field operation. The main conclusions are as follows.

1. Matrix failure pressure and bedding shear failure pressure gradually increase with the increasing azimuth of the radial borehole. Bedding tensile failure pressure first rises swiftly and then remains constant with the increasing azimuth. Shale is prone to matrix failure and bedding shear failure with a moderate azimuth of radial borehole and tends to crack with bedding tensile failure when the azimuth is small or large enough.

2. With the increment of the angle between radial borehole and formation strike, bedding tensile failure pressure initially ascends, reaches its peak at 180°, and finally falls back. Bedding shear failure pressure rapidly descends, reaches a plateau between 120° to 230°, and rebounds in the end. Matrix failure is less affected by the included angle, whose FIP only declines slightly around 90° and 270°. Thus, bedding tensile failure is apt to occur when the included angle is considerably small or large, and bedding shear failure tends to take place with a moderate included angle. Only when the angle is around 90° or 270°, shale tends to crack with matrix failure.

3. As the dip angle of shale formation increases, pressures required for bedding tensile failure and bedding shear failure gradually decline. The matrix failure pressure remains stable in the beginning and decreases rapidly when the dip angle exceeds 15°. The increasing dip angle disposes shale to crack with bedding shear failure and weakens the tendency for bedding tensile failure.

4. With the increase of azimuth difference between the radial borehole and main borehole, pressures required for bedding tensile failure and bedding shear failure ascend at first, then decline gradually and finally revert to rise again. Matrix failure pressure fluctuates regularly, forming an inverted W-shape curve. Shale tends to break in bedding shear failure mode when azimuth difference is small, and in the other two failure modes when azimuth difference is around 90°.

5. With the augment of main wellbore deviation, pressures required for bedding shear failure and bedding tensile failure slightly rise at first and then turn down. Matrix failure pressure initially remains stable and significantly declines when the deviation angle exceeds 20°. Because bedding shear failure drops fastest in FIP among all failure modes, the increase of main wellbore deviation can incline shale to bedding shear failure.

6. Pressures required for bedding tensile failure and matrix failure linearly descend with the increasing reservoir temperature, but reservoir temperature has no influence on bedding shear failure. Thus, a high reservoir temperature can reduce theoretical FIP and induce matrix failure and bedding tensile failure.

7. As the pore pressure enlarges, pressures required for bedding tensile failure and shear failure evenly decline, but matrix failure pressure evenly descends in the beginning and accelerates its decline when pore pressure exceeds 18 MPa. The increase of pore pressure can reduce theoretical FIP and convert the failure mode from bedding shear failure to matrix failure or bedding tensile failure.

### NOMENCLATURE

- $\sigma_v$: Vertical in situ stress, MPa
- $\sigma_h$: Minimum horizontal in situ stress, MPa
- $\sigma_H$: Maximum horizontal in situ stress, MPa
- $\beta_M$: Azimuth of the main borehole, degrees
- $\beta_R$: Azimuth of the radial borehole, degrees
- $\omega$: Tilt angle of the radial borehole, degrees
- $\zeta$: Dip angle of the shale formation, degrees
- $\eta$: Strike angle of the shale formation, degrees
- $\gamma$: Deviation angle of the main wellbore, degrees
- $\delta$: Intersection angle between the borehole wall and bedding plane, degrees
- $\alpha$: Angle between the $x_0$-axis and the project line of the radial borehole axis on the main wellbore cross section, degrees
- $\nu$: Poisson’s ratio of the shale
- $\nu_c$: Poisson’s ratio of the casing
- $R_i$: Internal radius of the casing, m
- $R_e$: External radius of the casing, m
- $E_r$: Young’s modulus of shale, GPa
- $E_c$: Young’s modulus of the casing, GPa
- $P_w$: Internal pressure of the wellbore, MPa
- $R$: Radius of the radial borehole, m
- $\phi$: Shale porosity
- $\epsilon$: Biot’s coefficient
- $\kappa$: Permeability coefficient
- $P_p$: Initial pore pressure, MPa
- $\sigma_1$, $\sigma_2$, and $\sigma_3$: Three principal stresses on the research point, MPa
- $\psi$: Angle between $\sigma_2$ and the radial borehole axis, degrees
- $\rho$: Included angle between radial borehole axis and the horizontal plane, degrees
\( \lambda_b \) Normal direction vector to shale bedding plane
\( \lambda_1, \lambda_2 \) and \( \lambda_3 \) Direction vectors of three principal stresses
\( P_H \) Hydration stress, MPa
\( I_m \) Membrane efficiency
\( A_{wh} \) Shale water activity
\( A_f \) Fracturing fluid water activity
\( T \) Reservoir temperature, K
\( R' \) Gas constant, 8.314 J mol\(^{-1}\) K\(^{-1}\)
\( V \) Partial molar volume of the water, 1.8 \times 10^{-5} \text{ m}^3/\text{mol}
\( \sigma_T \) Tensile strength of the shale matrix, MPa
\( \sigma_{BT} \) Tensile strength of the radial borehole, MPa
\( \sigma_p \) Effective normal stress on the radial borehole wall, MPa
\( a_B \) Internal friction angle of shale bedding, degrees
\( c_B \) Cohesion force of shale bedding, MPa
\( \varphi_1, \varphi_2 \), and \( \varphi_3 \) Angles between the direction vectors of three principal stresses and the normal direction vector to the bedding plane, degrees
\( P_{MF} \) Matrix failure pressure, MPa
\( P_{BT} \) Bedding tensile failure pressure, MPa
\( P_{RS} \) Bedding shear failure pressure, MPa
\( P_J \) Theoretical fracture initiation pressure, MPa
\( r \) and \( \theta \) Coordinates of research point in the coordinate system \((r, \theta, z)\), m and degrees
\( r_0 \) and \( \theta_0 \) Coordinates of research point in the coordinate system \((r_0, \theta_0, z_0)\), m and degrees
\( \sigma_{x_z}, \sigma_{y_z}, \sigma_{z_z}, \tau_{x_y}, \tau_{x_z}, \text{ and } \tau_{y_z} \) Stress components in the coordinate system \((x, y, z)\), MPa
\( \sigma_{r_0}, \sigma_{\theta_0}, \sigma_{z_0}, \tau_{r_\theta}, \tau_{r_z}, \text{ and } \tau_{\theta_z} \) Stress components caused by in situ stress in the coordinate system \((x_0, y_0, z_0)\), MPa
\( \sigma_{IP-RB}, \sigma_{IP-RB}, \sigma_{IP-RB}, \sigma_{IP-RB}, \sigma_{IP-RB}, \text{ and } \sigma_{IP-RB} \) Stress components caused by the internal pressure of radial borehole in the coordinate system \((r, \theta, z)\), MPa
\( \sigma_{PP}, \sigma_{PP}, \sigma_{PP}, \sigma_{PP}, \sigma_{PP}, \text{ and } \sigma_{PP} \) Stress components caused by the infiltration of fracturing fluid in the coordinate system \((r, \theta, z)\), MPa
\( \sigma_{MB}, \sigma_{MB}, \sigma_{MB}, \sigma_{MB}, \sigma_{MB}, \text{ and } \sigma_{MB} \) Total stress components caused by the main wellbore in the coordinate system \((r_0, \theta_0, z_0)\), MPa
\( \sigma_{r}, \sigma_{r}, \sigma_{r}, \sigma_{r}, \sigma_{r}, \text{ and } \sigma_{r} \) Total stress components around the radial borehole in the coordinate system \((r, \theta, z)\), MPa

**CONFLICT OF INTEREST**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

**ORCID**

Yuxin Chen [https://orcid.org/0000-0001-9030-4937]
Dawei Zhu [https://orcid.org/0000-0003-0288-3110]
Yu Bai [https://orcid.org/0000-0001-9515-8692]

**REFERENCES**

1. Guo TK, Tang SJ, Sun J, et al. A coupled thermal-hydraulic-mechanical modeling and evaluation of geothermal extraction in the enhanced geothermal system based on analytic hierarchy process and fuzzy comprehensive evaluation. Appl Energy. 2020;258(113981):1-12.
2. Davies R, Foulger G, Bindley A, Styles P. Induced seismicity and hydraulic fracturing for the recovery of hydrocarbons. Mar Pet Geol. 2013;45(4):171-185.
3. Guo TK, Zhang SC, Wang L, et al. Optimization of proppant size for frac-pack completion based on a new equipment. J Petrol Sci Eng. 2012;96–97:1-9.
4. Guo TK, Wang X, Li Z, et al. Numerical simulation study on fracture propagation of zipper and synchronous fracturing in hydrogen energy development. Int J Hydrogen Energy. 2019;44(11):5270-5285.
5. Zhang Y, Zhang J, Yuan B, Yin S. In-situ stresses controlling hydraulic fracture propagation and fracture breakdown pressure. J Petrol Sci Eng. 2018;164:164-173.
6. Wang B, Li G, Huang Z, Li J, Zheng D, Li H. Hydraulics Calculations and Field Application of Radial Jet Drilling. SPE Drilling & Completion; 2016.
7. Dickinson W, Dickinson R. Horizontal radial drilling system. In: SPE California Regional Meeting. Society of Petroleum Engineers; 1985.
8. Dickinson W, Dykstra H, Nordlund R, et al. Coiled-tubing radials placed by water-Jet drilling: field results, theory, and practice. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers; 1993.
9. Balch RS, Ruan T, Savage M, et al. Field testing and validation of a mechanical alternative to radial jet drilling for improving recovery in mature oil wells. In: SPE Western Regional Meeting. Society of Petroleum Engineers; 2016.
10. Li Y, Wang C, Shi L, et al. Application and development of drilling and completion of the ultrashort-radius radial well by high pressure jet flow techniques. In: Proceedings of the International Oil and Gas Conference and Exhibition. Society of Petroleum Engineers; 2000.
11. Ursegov S, Bazylev A, Taraskin E. First results of cyclic stimulations of vertical wells with radial horizontal bores in heavy oil carbonates (Russian). In: Proceedings of the SPE Russian Oil and Gas Technical Conference and Exhibition. Society of Petroleum Engineers; 2008.
12. Liu QL, Tian SZ, Li GS, et al. Fracture initiation and propagation characteristics for radial drilling-fracturing: an experimental study. In: Unconventional Resources Technology Conference, Houston, Texas, USA; 2018.
13. Tan HY, Liu JY, Li XP, Zhang LH, Cai J. A simulation method for permeability of porous media based on multiple fractal model. Int J Eng Sci. 2015;95:76-84.
14. Megorden MP, Jiang H, Bentley PJD. Improving hydraulic fracture geometry by directional drilling in a coal seam gas formation. In: Proceedings of the SPE Unconventional Resources Conference and Exhibition—Asia Pacific. Society of Petroleum Engineers; 2013.

15. Ragab AMS. Improving well productivity in an Egyptian oil field using radial drilling technique. J Petrol Gas Eng. 2013;4(5):103-117.

16. Cinelli SD, Kamel AH. Novel technique to drill horizontal laterals revitalizes aging field. In: SPE/IADC Drilling Conference. Society of Petroleum Engineers; 2013.

17. Haimson B, Fairhurst C. Initiation and extension of hydraulic fractures in rocks. Soc Petrol Eng J. 1967;7(03):310-318.

18. Keiterij RB, Pater CJ. Impact of perforations on hydraulic fracture tortuosity. SPE Prod Facil. 1999;14(02):117-130.

19. Hossain MM, Rahman MK, Rahman SS. Hydraulic fracture initiation and propagation: roles of wellbore trajectory, perforation and stress regimes. J Petrol Sci Eng. 2000;27:129-149.

20. Lhomme TP, Pater CJ, Helfferich PH. Experimental study of hydraulic fracture initiation in Colton sandstone. In: SPE/ISRM Rock Mechanics Conference; 2002.

21. Jin Y, Zhang XD, Chen M. Initiation pressure models for hydraulic fracturing of vertical wells in naturally fractured formation. Acta Petrolei Sinica. 2005;26(6):113-118.

22. Gong DG, Qu ZQ, Guo TK, Tian Y, Tian KH. Variation rules of fracture initiation pressure and fracture starting point of hydraulic fracture in radial well. J Petrol Sci Eng. 2016;140:41-56.

23. Guo TK, Liu BY, Qu ZQ, et al. Study on initiation mechanisms of hydraulic fracture guided by vertical multi-radial boreholes. Rock Mech Rock Eng. 2017;50(7):1767-1785.

24. Guo TK, Rui ZH, Qu ZQ, Qi N. Experimental study of directional propagation of hydraulic fracture guided by multi-radial slim holes. J Petrol Sci Eng. 2018;166:592-601.

25. Guo TK, Gong FC, Qu ZQ, Tian X, Liu B. Study on fracture initiation mechanisms of hydraulic refracturing guided by directional boreholes. J Energy Resour Technol. 2018;140(8): Article ID 082901.

26. Liu QL, Tian SC, Li GS, et al. An analytical model for fracture initiation from radial lateral borehole. J Petrol Sci Eng. 2018;164:206-218.

27. Zuo SJ, Ge ZL, Deng K, Zheng J, Wang H. Fracture initiation pressure and failure modes of tree-type hydraulic fracturing in gas-bearing coal seams. J Nat Gas Sci Eng. 2020;77:103260.

28. Liu XQ, Qu ZQ, Guo TK, et al. An innovative technology of directional propagation of hydraulic fracture guided by radial holes in fossil hydrogen energy development. Int J Hydrogen Energy. 2019;44(11):5286-5302.

29. Chen YX, Ding YH, Liang C, et al. An analytical model for fracture initiation from a particular radial borehole in hydraulic fracturing guided by multiradial boreholes. Geofluids. 2021;2021:Article ID 6657788, 18 pages.

30. Niandou H, Shao JF, Henry JP, Fourmaintraux D. Laboratory investigation of the mechanical behaviour of Tournemire shale. Int J Rock Mech Min Sci. 1997;34(1):3-16.

31. Timoshenko S, Goodier J. Theory of Elasticity. McGraw-Hill book Company; 1951.

32. Lubinski A. The theory of elasticity for porous bodies displaying a strong pore structure. In Proceedings of the 2nd US National Congress of Applied Mechanics; 1954:247-256.

33. Jaeger JG, Cook NGW, Zimmerman RW. Fundamental of Rock Mechanics. Blackwell Publishers; 2007.

34. Velzer V, Schutts A. Plane and Solid Geometry. Springer; 2009.

35. Chenevert ME, Pernot V. Control of shale swelling pressures using inhibitive water-base muds. In: SPE Annual Technical Conference and Exhibition. Society of Petroleum Engineers; 1998.

36. Chen GZ, Chenevert ME, Sharma MM, et al. A study of wellbore stability in shales including poroelastic, chemical, and thermal effects. J Petrol Sci Eng. 2003;38:167-176.

How to cite this article: Chen Y, Ding Y, Liang C, Zhu D, Bai Y, Zou C. Initiation mechanisms of radial drilling-fracturing considering shale hydration and reservoir dip. Energy Sci Eng. 2021;9:2099–2114. https://doi.org/10.1002/ese3.969