Gravitational lensing time delays as a tool for testing Lorentz-invariance violation

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ABSTRACT

It is generally expected that quantum gravity theory should yield the model of a space–time foam at short distances leading to Lorentz-invariance violation (LIV) manifested e.g. by energy-dependent modification of the standard relativistic dispersion relation. One direction of research, pursued intensively, is to measure the energy-dependent time-of-arrival delays in photons emitted by astrophysical sources located at cosmological distances. This is tempered, however, by our ignorance of intrinsic emission delays in different energy channels.

In this paper we discuss a test based on gravitational lensing. Monitoring time delays between images obtained in different energy channels, for example optical (low-energy) and TeV photons, may reveal extra delays due to the distorted dispersion relation typical in LIV theories, a test that is free from the systematics inherent in other settings.

Key words: gravitational lensing – cosmology: miscellaneous.

1 INTRODUCTION

Despite the fact that quantum gravity theory still remains elusive, it is generally expected that it should yield the model of a space–time foam at short distances leading to Lorentz-invariance violation (LIV), manifested e.g. by energy-dependent modification of the standard relativistic dispersion relation (Amelino-Camelia et al. 1998; Amelino-Camelia & Piran 2001).

Several years ago it was proposed to use astrophysical objects to look for energy-dependent time-of-arrival delays (Amelino-Camelia et al. 1998). Specifically, gamma-ray bursts (GRBs), highly energetic events visible from cosmological distances, are the most promising sources for constraining LIV theories (Ellis et al. 2003, 2006; Boggs et al. 2004; Rodriguez Martinez & Piran 2006; Jacob & Piran 2007). Indeed, some limits on LIV energy scale have been derived in the above-mentioned papers, and recently corrected (Ellis et al. 2007; Jacob & Piran 2008). Among other sources, BL Lac objects like Mk 501 are considered. It is this particular object from which 20-TeV photons were reported (Amelino-Camelia & Piran 2001). Such objects (also called blazars) have a similar nature to quasars.

The idea of searching for time-of-flight delays is tempered, however, by our ignorance of the intrinsic delay (in the source frame) in different energy channels; see e.g. Ellis et al. (2006). It was also shown (Biesiada & Piórkowska 2007) that lack of detailed knowledge about cosmological models (in the context of accelerating expansion of the Universe) could be another source of systematic effects at high redshifts.

In this paper we discuss a test based on gravitational lensing. Such an idea has already been mentioned in Amelino-Camelia et al. (1998) in the context of GRBs playing the role of high-energy photon sources. More specifically, a source located at a cosmological distance may undergo gravitational lensing by a galaxy lying closer to the observer along the line of sight, with an encounter parameter small enough to produce multiple images (so-called strong lensing; see e.g. Schneider, Ehlers & Falco 1992). Indeed, all known strong-lensing systems (CASTLES Survey: http://www.cfa.harvard.edu/castles/) have a quasar as a source and a galaxy (in most cases elliptical) as a lens. The light signals emitted by the source will be seen delayed by the observer (achromatically in classical general relativity) at the location of the images. This opens up the possibility of studying time delays induced by LIV. Namely, monitoring the time delays between lensed images monitored in different energy channels (e.g. optical or gamma-ray (low-energy) and TeV (high-energy) photons) may reveal extra delays due to the distorted dispersion relation typical in LIV theories. This test is free from the systematics inherent in other settings. The next sections will substantiate our argument further.

2 LIV-INDUCED TIME DELAYS IN DIFFERENT COSMOLOGICAL MODELS

Following Amelino-Camelia & Piran (2001), let us consider a phenomenological approach for LIV by assuming a modified dispersion relation for photons in the form

\[ E^2 - p^2c^2 = \epsilon E^2 \left( \frac{E}{E_{QG}} \right)^n, \]  

(1)
where $\epsilon = \pm 1$ is the ‘sign parameter’ (Amelino-Camelia & Piran 2001) and $\xi_n$ is a dimensionless parameter. As a first guess one may assume $E_{QG}$ equal to the Planck energy, $\xi_1 = 1$ and $\xi_2 = 10^{-7}$ (Jacob & Piran 2007). The dispersion relation (1) essentially corresponds to a power-law expansion (see Ellis et al. 2003; Boggs et al. 2004) so for practical purposes (due to smallness of the expansion parameter $E/E_{QG}$) only the lowest terms of the expansion are relevant. Because in some LIV theories the odd power terms might be forbidden (Burgess et al. 2002), the cases of $n = 1$ and $n = 2$ are usually retained. It should be noted that in some string theories third-order corrections appear as leading ones.

The relation (1) leads to a Hamiltonian,

$$\mathcal{H} = \int p^2 c^2 \left[ 1 + \epsilon \left( \frac{E}{\xi_n E_{QG}} \right)^n \right],$$  \hspace{1cm} (2)

from which the time-dependent group velocity $v(t) = \partial \mathcal{H}/\partial p$ can be inferred.

The comoving distance travelled by a photon to the Earth is

$$r(t) = \int_{t_{\text{fountain}}}^t v(t') \, dt' = \int_0^z v(z') \frac{dz'}{H(z')(1+z')},$$  \hspace{1cm} (3)

where in the last equation a standard time-redshift parametrization was taken into account. $H(z)$ here denotes the Universe’s expansion rate (the so-called Hubble function). Starting from this point, our considerations will have a cosmological connotation. The reason for this is simple: the modifications due to LIV theories are extremely small, so one has to look for sources located at cosmological distances (such as quasars or GRBs), which are far enough away to compensate for the smallness of LIV corrections. This means that cosmological background geometry should be taken into account.

From now on we will assume a flat Friedmann–Robertson–Walker model with $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$ – the so-called concordance model, as supported by observations (Spergel et al. 2003). Some alternatives to the concordance model could in principle also be discussed (Biesiada & Piorkowska 2007).

Expressing the group velocity in terms of redshift, we obtain

$$v(z) \simeq c(1+z) \left[ 1 + \epsilon \frac{n+1}{2} \left( \frac{E}{\xi_n E_{QG}} \right)^n (1+z)^n \right].$$  \hspace{1cm} (4)

The time of flight for a photon of energy $E$ is equal to

$$t_{\text{LIV}} = \int_0^z \left[ 1 + \epsilon \frac{n+1}{2} \left( \frac{E}{\xi_n E_{QG}} \right)^n (1+z)^n \right] \frac{dz'}{H(z')}.$$  \hspace{1cm} (5)

In the first term one easily recognizes the time of flight for photons in the standard relativistic cosmology (i.e. without LIV). Owing to the very small magnitude of LIV corrections, it also represents the time of flight for low-energy photons fairly well. Below, therefore, we neglect LIV corrections at low energy.

In consequence, the time delay between a low-energy and a high-energy photon is equal to

$$\Delta t_{\text{LIV}} = \frac{n+1}{2} \left( \frac{E}{\xi_n E_{QG}} \right)^n \int_0^z (1+z)^n \frac{dz'}{H(z')}.$$  \hspace{1cm} (6)

where we restricted our attention to ‘infraluminal’ motion of high-energy photons (i.e. low-energy photons arrive earlier at the observer). Generalization to ‘superluminal’ motion is straightforward: the same value with an opposite sign (time delays become early arrivals).

The idea of observational strategy emerging from (6) is again simple: monitor an appropriate (i.e. emitting both low- and high-energy photons) cosmological source at different energy channels and try to detect this time delay. Some attempts along these lines have already been undertaken (Ellis et al. 2003, 2006; Boggs et al. 2004; Albert et al. 2008). However there remains an unavoidable uncertainty regarding intrinsic time delays: there is no reason that low- and high-energy signals should be emitted simultaneously, and when detecting distinct signals (peaks in the light curve) at different energies we have no idea which one was sent first.

Our method outlined below, invoking gravitational lensing, allows us to remove this ambiguity. Before describing it, let us recall now that in cosmology one distinguishes three types of distances.

(i) Comoving distance:

$$r(z) = c \int_0^z \frac{dz'}{H(z')} = \frac{c}{H_0} \tilde{r}(z),$$  \hspace{1cm} (7)

where by $\tilde{r}(z)$ we denoted a reduced (dimensionless) comoving distance, i.e. a comoving distance expressed as a fraction of the Hubble horizon $d_H = c/H_0$.

(ii) Angular diameter distance:

$$D_\Lambda(z) = \frac{1}{1+z} r(z).$$  \hspace{1cm} (8)

(iii) Luminosity distance:

$$D_L(z) = (1+z) r(z).$$  \hspace{1cm} (9)

Angular diameter distance is the one used in gravitational lensing theory (because gravitational lensing deals with light deflection, i.e. essentially with angles). The luminosity distance is a measure invoked while using standard candles (e.g. Type Ia supernovae). The point is that both distance measures are related to the comoving distance by a factor $1+z$ (see above). The comoving distance, then, is closely related to the time of flight $r(z) = c t$. It is in fact the distance to the source measured in light-years. Therefore we can rewrite the time of flight in LIV theory (5) in terms of comoving distance: $r_{\text{LIV}}(z) = c t_{\text{LIV}}$. Of course the comoving distance to the source is fixed – there are photons of different energies that travel with different speeds. It is, however, useful (for later calculations) to think of them as travelling with the same speed $c$ but along different comoving distances $r(z)$ and $r_{\text{LIV}}(z)$.

### 3 GRAVITATIONAL LENSING TIME DELAYS

Gravitational lensing of quasars and extragalactic radio sources at high redshifts by foreground galaxies is now well-established and has developed into a mature branch of both theoretical and observational astrophysics (Schneider et al. 1992). Misalignment of the source, the lens and observer typically results in multiple images, the angular positions and magnification ratios of which allow reconstruction of the lensing mass distribution. In particular they provide an independent confirmation of dark matter in galaxies and became an important tool for investigating the dark matter distribution. Another important ingredient of gravitational lensing is the time delay between lensed images of the source. This effect originates as a competition between the Shapiro time delay from the gravitational field and the geometric delay due to the bending of light rays and is best understood in terms of the Fermat principle. In other words, the intervening mass between the source and the observer introduces an effective index of refraction, thereby increasing the light travel time.

In general, the light travel time can be calculated as

$$t(x) = \frac{1+z_l}{c} \frac{D_L D_s}{D_{ls}} \left[ \frac{1}{2} (x - \beta)^2 - \psi(x) \right].$$  \hspace{1cm} (10)
where $x$ and $\beta$ are positions (as projected on the celestial sphere) of the image and the source, $\psi(x)$ is the projected gravitational potential (i.e., the actual potential integrated along the line of sight), and $D_l$ and $D_s$ are angular diameters to the lens and the source located at redshifts $z_l$ and $z_s$, respectively ($D_h$ is the angular-distance distance between lens and source). From now on we adopt the notation (standard in gravitational lensing theory) in which $D$ denotes angular-distance distance and subscripts refer to the components of the lensing system (i.e. the source or the lens).

The lensing is called strong if the source position happens to lie within the so-called Einstein ring, a circle of radius $\theta_E$ (defining the proper deflection scale of a given lens). In this case multiple images appear and, since lensing galaxies are often ellipticals, the number of images is usually equal to four (Ratnatunga, Ostrander & Griffiths 1995; CASTLES Survey) or five (Inada et al. 2005); the issue of image multiplicity is discussed in Schneider et al. (1992).

However, a surprisingly realistic (Schneider et al. 1992) model of the lens potential is that of a singular isothermal sphere (SIS). Indeed lensing by ellipticals can be modelled by a variant called the singular isothermal ellipsoid (SIE). Therefore for the purpose of illustrating our ideas we shall restrict our attention to the SIS model, since generalization to the SIE is rather straightforward and would not change our conclusions.

The Einstein-ring radius for the SIS model is

$$\theta_E = 4\pi D_h \frac{\sigma^2}{c^2},$$  

(11)

where $\sigma$ denotes the one-dimensional velocity dispersion of stars in the lensing galaxy. If the lensing is strong, i.e. $\beta := |\beta| < \theta_E$, then two collinear images A and B form on the opposite side of the lens, at radial distances $R_A = \beta + \theta_E$ and $R_B = \theta_E - \beta$, having time delays between the images

$$\Delta t_{\text{SIS}} = \frac{1 + z_l}{2c} \left( R_A^2 - R_B^2 \right),$$  

(12)

which according to the above-mentioned relations for the SIS model can also be written as

$$\Delta t_{\text{SIS}} = \frac{2(1 + z_l)}{c} \frac{D_l D_s}{D_h} \theta_E \beta + \frac{8\pi}{H_0} \hat{r}_l \beta \frac{\sigma^2}{c^2}. $$  

(13)

In the last equation $\hat{r}_l$ denotes the reduced comoving distance to the lens. The equation (12) is commonly used by the gravitational lensing community because it reduces the time delay problem to one of relative astrometry of images, whereas $\beta$ is much harder to assess (it must be small in order for strong lensing to occur) and the Einstein-ring radius is not a directly observable quantity (although image separation represents the Einstein radius fairly well). However, equation (13) is more useful from a theoretical point of view. In particular, it shows explicitly that the time delay between images is created at the lens location ($\hat{r}_l$ factor). Let us stress again that this time delay is achromatic in general relativity.

## 4 LIV-INDUCED TIME DELAYS AND GRAVITATIONAL LENSING TIME DELAYS

Let us now imagine a source at cosmological distance emitting low-energy and high-energy (in TeV range) photons, which undergoes gravitational lensing by a foreground galaxy. Let us also assume that the LIV-type distorted dispersion relation (1) holds. The observer would again notice time delays between images, but this time it would be a combined effect of gravitational lensing and LIV, and therefore no longer achromatic. This idea was formulated originally (mentioned) in Amelino-Camelia et al. (1998), but to our best knowledge it has not been developed further.

It is rather straightforward to calculate this using the above-mentioned fictitious ‘LIV comoving distance’ $\hat{r}_{\text{LIV}}(z_l)$, namely

$$\Delta t_{\text{LIV,SIS}} = \frac{8\pi}{H_0} \hat{r}_{\text{LIV}}(z_l) \beta \frac{\sigma^2}{c^2},$$  

(14)

where

$$\hat{r}_{\text{LIV}}(z_l) = \hat{r}_l + H_0 \frac{n + 1}{2} \left( \frac{E}{H_0} \frac{E_{\text{QG}}}{E_{\text{QG}}} \right)^n \int_0^{z_l} \left( 1 + z' \right)^n \frac{dz'}{H(z')}.$$  

(15)

Because the LIV effect is extremely small, let us restrict ourselves further to the $n = 1$ case:

$$\hat{r}_{\text{LIV}}(z_l) = \hat{r}_l + H_0 \frac{E}{E_{\text{QG}}} \int_0^{z_l} \left( 1 + z' \right) \frac{dz'}{H(z')}.$$  

(16)

One should note that besides the phenomenology in which the quantum-gravity scale $E_{\text{QG}}$ is assumed to be close to the Planck scale, there exist brane-inspired approaches where this is not necessarily so. For example Farakos, Mavromatos & Pasipoularides (2009) have constructed a brane world model with asymmetry in warp factors in front of temporal and spatial parts of the metric. Such a model predicts that photons propagating on a three-dimensional brane would feel a modified dispersion relation with quadratic energy dependence, just like the $n = 2$ case in our notation. However, our $E_{\text{QG}}$ term acquires a different meaning there: a combination of model parameters rather than simply the quantum-gravity energy scale, so the quadratic correction in such a theory is not automatically smaller than the linear correction in another theory. Extension of our results to $n = 2$ (or to any other value of $n$) is straightforward, so we will proceed with $n = 1$.

Now, we can assume that observations at low energy would essentially provide a time delay between images equal to $\Delta t_{\text{SIS}}$, whereas monitoring of the same images in a high-energy (TeV) channel would provide $\Delta t_{\text{LIV,SIS}}$. These two measurements would differ by

$$\Delta t_{\text{LIV,SIS}} - \Delta t_{\text{SIS}} = \frac{8\pi}{H_0} \frac{\sigma^2}{c^2} \frac{E}{E_{\text{QG}}} \int_0^{z_l} \left( 1 + z' \right) \frac{dz'}{H(z')}.$$  

(17)

Rigorously one should instead calculate $\Delta t_{\text{LIV,SIS}}(E_1) - \Delta t_{\text{LIV,SIS}}(E_2)$ and write $\Delta t = E_1 - E_2$ in the numerator of (17), but due to our assumption that $E_2$ is many orders of magnitude smaller than $E_1$ and thus having negligible LIV correction, the above expression is a good approximation. Let us make an estimate for the above LIV effect using a real strong-lensing system. HST 14176+5226 can serve as an example. This system was discovered with the Hubble Space Telescope (HST) (Ratnatunga et al. 1995) and further confirmed to be a gravitational lens (Crampton et al. 1996). The lensed source is a quasar at redshift $z_l = 3.4$, whereas the lens is an elliptical galaxy having redshift $z_s = 0.809$. The lens model that best fits the observed images was based on a singular isothermal ellipsoid (Ratnatunga, Griffiths & Ostrander 1999), giving an Einstein radius $\theta_E = 1.489$ arcsec and $\beta = 0.13$ arcsec $= 8.4 \times 10^{-7}$ rad.

Optical spectroscopy of the lensing galaxy in the HST 14176+5226 system (Ohyama et al. 2002) provided measurements of the velocity dispersion in the lensing galaxy. These measurements have been confirmed by Treu & Koopmans (2004), who performed spectroscopic observations on the Keck telescope as part of the Lenses Structure and Dynamics (LSD) survey. The result is $\sigma = 290 \pm 8 \text{ km s}^{-1}$.

Substituting these data in (17) gives $\Delta t_{\text{LIV,SIS}} - \Delta t_{\text{SIS}}$ equal to $3.7 \times 10^{-9}$ s for 5-TeV photons and $1.5 \times 10^{-8}$ s for 20-TeV ones.
The model presented above was the simplest one, because its aim was to illustrate ideas. Our purpose in choosing the HST 14176+5226 system as an example was similar. In reality one would encounter systems with different numbers of images or different separations, e.g. SDSS J1004+4112 (Inada et al. 2003) with an image separation of 14.6 arcsec or the recently discovered system SDSS J1029+2623, where a quasar at $z_s = 2.197$ is doubly imaged by a massive galaxy cluster at $z_l = 0.55$ with an image separation of 22.5 arcsec (Inada et al. 2006).

Closing this section, it is interesting to ask how the LIV effects might modify image configurations. It might be suspected that they do so, since, from Fermat’s principle, perspective images are located at stationary points of the wavefront travel-time function (given by equation 10). Therefore, since LIV modifies the time of flight in an energy-dependent way (due to the modified dispersion relation) then one expects the images seen at different energies to be located at different positions. It is easy to see that for the SIS lens (generalizations to other mass profiles are also rather straightforward) the difference between Einstein radii for high- and low-energy photons $\Delta \theta_{E,LIV} := \theta_{E,LIV} - \theta_E$ would be given by the formula

$$\Delta \theta_{E,LIV} = \theta_E \frac{E}{E_{QG}} \left( \frac{I_1^{(1)}(z_1, z_s)}{r(z_1)} - \frac{I_1^{(1)}(z_s)}{r(z_s)} \right),$$

where $I_1^{(1)}(z_1, z_s) := \int_{z_s}^{z_1} [(1 + z')/H(z')] dz'$. For realistic lens configurations like HST 14176+5226, this would give negligibly small corrections of order $10^{-16}$ arcsec. Hence even if LIV were operating this would not be able to change the position of macro-images in a detectable way. However it cannot be excluded that such minute differences may become relevant while studying caustic crossing, possibly leading to different magnification patterns due to microlensing at different energies.

5 DISCUSSION AND CONCLUSIONS

In this paper we discussed a method (first mentioned in Amelino-Camelia et al. 1998) to test LIV effects by monitoring time delays between images of gravitationally lensed quasars in low- and high-energy channels. In standard theory (general relativity) the result should be the same: gravitational lensing is essentially achromatic. On the other hand, in the presence of LIV effects time delays lose this property: high-energy photons should arrive at different times compared with low-energy ones. Therefore time delays between images should be different at different energies (e.g. optical or gamma-rays and TeV photons). We tacitly assuming (the following approach is taken by the rest of the LIV-studying community) that LIV effects are manifested only in the high-energy domain (where the small-scale ‘foamy’ structure of space–time reveals itself), whereas the overall background geometry of space–time shaped by the low-energy content of the Universe is that of general relativity (more precisely the flat Friedmann–Robertson–Walker model as suggested by cosmological data) with light deflection (i.e. geodesic motions) defined in a standard way.

Because this method is directional in nature, it removes the assumptions about intrinsic time delays of signals at different energies. In fact, time delays between images at different energies could be established in different experiments (at unrelated observing sessions) performed on a given lensing system. The only demand is that they are accurate enough (taken with a sufficient temporal resolution). Since the time delay between images is produced at the lens location, the result does not depend very strongly on the cosmological model. Lenses are located at modest redshifts where all realistic cosmological models essentially agree.

One may ask whether appropriate lensing systems (i.e. having sources emitting both low- and high-energy photons) exist. It is an observational fact that very high-energy emission ($E > 100$ GeV) has been detected from over a dozen blazars (Wagner 2008), which have a similar nature to quasars. Quasars, on the other hand, are the sources in all known strong-lensing systems – the CASTLES data base contains 100 such systems. It is a matter of coordinating strong lensing surveys with experiments in high-energy astrophysics, such as the Astro-rivelatore Gamma a Immagini LEggero (AGILE), Gamma-Ray Large Area Space Telescope (GLAST) or Major Atmospheric Gamma-ray Imaging Cherenkov Telescope (MAGIC) experiments (Persic et al. 2008), and the future will certainly bring the discovery of lensed high-energy sources. Angular resolution of high-energy experiments is gradually being improved. For example a recently launched AGILE instrument (Tavani et al. 2007) has been designed to obtain accurate localization ($\sim 2$–3 arcmin) of transient events by the Gamma Ray Imaging Detector (GRID)–Super AGILE (SA) combination. Therefore it becomes close to the image separation of the SDSS J1004+4112 or SDSS J1029+2623 systems. Depending on the exposure and the diffuse background, its flux sensitivity threshold can reach values of $(10–20) \times 10^{-8}$ photon cm$^{-2}$ s$^{-1}$ at energies higher than 100 MeV, with an effective area above 200 cm$^2$ at 30 MeV. Moreover it has excellent timing capability, with overall photon absolute time tagging of uncertainty less than 2 ps and very small dead times ($< 200$ µs for the GRID, ~5 µs for the sum of the SA readout units, and ~20 µs for each of the individual CsI bars). In fact, the AGILE instrument is optimized in the range below 1 GeV, hence it is not representative of the TeV-range experiments needed to probe LIV, but it clearly shows these methods of improvement of sensitivity, timing and angular resolution in high-energy astrophysics.

An order-of-magnitude estimate for the effect discussed in this paper is not encouraging at present. For a typical lensing system like HST 14176+5226 it is of the order of nanoseconds. However, bearing in mind that high-energy astrophysical sources display rapid variability (indeed the intrinsic variability in the relativistic shocks powering these sources is enhanced by a Lorentz factor typically of order of $10^2$) and that e.g. light curves of GRBs are already sampled with millisecond resolution (and AGILE went down to microseconds), one should not reject the idea presented above on the grounds that it is not within the scope of present-day observational technology.

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