ABSTRACT

We discuss the relationship between a standard Shakura & Sunyaev (1973) accretion disk model and the Big Blue Bump (BBB) observed in Type 1 AGN, and propose a new method to estimate black hole masses. We apply this method to a sample of 23 radio–loud Narrow–line Seyfert 1 (RL–NLS1) galaxies, using data from WISE (Wide–field Infrared Survey Explorer), SDSS (Sloan Digital Sky Survey) and GALEX. Our black hole mass estimates are at least a factor ~6 above previous results based on single epoch virial methods, while the Eddington ratios are correspondingly lower. Hence, the black hole masses of RL–NLS1 galaxies are typically above $10^8 M_⊙$, in agreement with the typical black hole mass of blazars.

Key words: accretion discs, galaxies: jets, quasars: emission lines

1 INTRODUCTION

The Spectral Energy Distribution (SED) of Active Galactic Nuclei (AGN) spans several orders of magnitude in frequency and results from the superposition of radiation emitted by different components.

In radio–quiet Type 1 sources, characterized by the presence of broad emission lines in their optical spectrum, the most luminous components are the “Big Blue Bump” (BBB, between ~1 μm and ~3 mm, or log(ν/Hz) ~14.5 – 17) and the “infrared bump” (IR bump, between ~1 mm and ~1 μm or log(ν/Hz) ~11.5–14.5). The former is the most prominent feature in the SED (Sanders et al. 1989; Elvis et al. 1994; Richards et al. 2006), while the latter accounts for 20–40% of the bolometric AGN luminosity. The BBB is thought to be thermal radiation from the accretion disk, while the IR bump is thermal radiation emitted from a dusty torus located a ~1 pc from the black hole (Sanders et al. 1989). Superimposed to the BBB there is often a minor component named “Small Blue Bump” (SBB, extending from 2200˚ A to 4000˚ A) which is likely the blending of several iron lines and hydrogen Balmer continuum (Wills et al. 1982; Vanden Berk et al. 2001). This scheme roughly describes the SED of AGN over at least 5 orders of magnitude in bolometric luminosity (Sanders et al. 1989; Elvis et al. 1994; Richards et al. 2006). It also applies to powerful blazars, although in these cases two more components are needed to describe the entire SED: the “synchrotron hump” (extending from radio to IR/optical wavelengths) and the “Compton hump” (extending from X–rays to TeV energies) which may overwhelm the torus and the BBB radiation. These further components characterize radio–loud sources whose jet is closely aligned to the line of sight, and are due to the synchrotron and inverse Compton processes, respectively.

The common energy production process in AGN is believed to be accretion onto a super–massive black hole ($M$ ~ $10^6–10^9 M_⊙$), through a disk whose observational properties depend (among other parameters) on the black hole mass and accretion rate. This interpretation led several authors to use models of geometrically thin, optically thick accretion disks (Shakura & Sunyaev 1973, hereafter AD model) to fit the optical/UV SED of AGN in order to determine the black hole mass and the accretion rate (e.g. Shields 1978; Malkan & Sargent 1982; Malkan 1983; Zheng et al.)

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NSL1 sources are characterized by the relatively small values of the full width at half maximum (FWHM) of the "broad" component of the Hβ emission line (FWHM(Hβ) < 2000 km s⁻¹), by the presence of strong blended iron lines, and of a prominent soft X-ray excess (Osterbrock & Pogge 1983; Pogge 2000). By estimating the virial black hole mass using the Hβ emission line, and [O iii] width as a surrogate for the bulge stellar velocity dispersion, (Grupe & Mathur 2004) and (Mathur & Grupe 2005) claimed that NSL1 lie systematically below the $M - \sigma_*$ of non-active and active Broad Line galaxies (BLS1). This indicates that the black hole masses of NSL1 are systematically smaller than the black hole masses of BLS1 for a given value of $\sigma_*$. The same considerations apply when considering objects of the same luminosity: NSL1 appear to accrete at a higher Eddington ratio with respect to BLS1, with some objects exceeding the Eddington luminosity (Zhou et al. 2006).

Recently, a few NSL1 sources have been confirmed to be part of a new class of γ-ray emitting sources, as detected by Fermi/LAT (Abdo et al. 2009a,c; Foschini 2011), besides blazars and radio-galaxies. Variability of the γ-ray emission (Calderone et al. 2011) allows to exclude a starburst origin of the γ-rays and confirms the presence of powerful relativistic jets, such as those found in typical blazars. The emerging picture is that γ-NSL1 sources are very similar to powerful blazars, except for their small widths of broad lines, and consequently their small SE virial black hole masses ($10^{6-8} M_\odot$, Yuan et al. 2008). Hence these sources are the best candidates to settle the question on whether very massive black holes ($\gtrsim 10^8 M_\odot$) power relativistic jets. If the SE virial mass estimates will be confirmed this would imply that a large mass is not required to produce a radio-loud AGN. On the other hand, if masses turn out to be systematically underestimated in NSL1, we would then conclude that extragalactic jetted sources preferentially live in large black hole mass systems.

Furthermore we may find that NSL1 too lie on the “canonical” $M - \sigma_*$ relation of broad line sources and accrete close to (albeit below) the Eddington rate. Possible explanations for the observed small widths of the broad lines in NSL1 include: the virial mass scaling relations may need to be modified in order to account for radiation pressure effects (Marconi et al. 2008; Chiaberge & Marconi 2011); the BLR may have a disk-like geometry and be oriented almost face-on, so that the Doppler shifted line velocity, projected along the line of sight, turns out to be small (Decarli et al. 2008); a combination of these effects (Peterson 2011).

In this work we show that the broad-band composite SEDs of AGN are roughly compatible with a simple, non-relativistic AD model (2), and discuss a method to estimate the total disk luminosity using either the continuum (2) or the line luminosities (2) as proxy. Then, we perform a spectroscopic analysis of the SDSS spectra of 23 radio-loud NSL1 in order to disentangle the host galaxy and/or jet...
contribution from the AGN continuum, and estimate line luminosities (§A.1). In § we discuss a new method to estimate the black hole mass and accretion rate, using AD spectrum modeling, and apply this method on the afore-mentioned sample. We discuss our results in §§ and draw our conclusion in §. The observational properties of the Shakura & Sunyaev (1973) AD model are summarized in the appendix.

Throughout the paper, we assume a ΛCDM cosmology with $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$, $\Omega_m = 0.27$, $\Omega_\Lambda = 0.73$.

1.1 Notation

In what follows we will consider a non-relativistic, steady state, geometrically thin, optically thick accretion disk (Shakura & Sunyaev 1973), extending from $R_{in} = 6R_g$ to $R_{out} = 2 \times 10^5 R_g$, where $R_g = GM/c^2$ is the gravitational radius of the black hole. The integrated disk luminosity is $L_d = \int L_\nu \, d\nu = \eta M c^2$, with $\eta \sim 0.1$ (radiative efficiency). The corresponding Eddington ratio is $\ell = L_d/L_{Edd}$ with $L_{Edd} = 1.3 \times 10^{37} (M/10^9 M_\odot)$ erg s$^{-1}$.

The relation between the disk luminosity and its “isotropic equivalent” counterpart is $L_{d,iso} = (2 \cos \theta) L_d$ (Eq. A13), where $\theta$ is the angle between the normal to the disk and the line of sight. For Type 1 AGN we take $\langle 2 \cos \theta \rangle = 1.7$ (Eq. A16).

We will refer to the peak frequency in the $\nu L_{\nu}$ representation as $\nu_p$, and to the luminosity of the peak as $L_{\nu,p}$. These quantities scale with the physical parameters as follows (Eq. A8 and A9):

$$\nu_p \propto M^{-1/2} \dot{M}^{1/4}$$

$$L_{\nu,p} \propto \dot{M}.$$ 

In particular we notice that $L_{\nu,p} \sim 0.5 L_d$ (Eq. A10). The location of the peak (i.e. its luminosity and frequency) determines uniquely the black hole mass and accretion rate. Details about the observational properties of the AD spectrum are given in appendix A.

All spectral slopes $\alpha_\nu$ are defined as $F_\nu \propto \nu^{\alpha_\nu}$.

2 ACCRETION DISK SPECTRUM IN AGN SPECTRA

Richards et al. (2006) built an average Type 1 QSO SED using data from 259 (mainly radio-quiet) sources, observed with instruments ranging from radio wavelengths to X-rays. Individual SEDs have been interpolated between available bands. An average SED is then computed as a geometric mean of individual ones, and is shown in Fig. 1 (red line). Also shown in Fig. 1 are: a spiral galaxy template as given in Mannucci et al. (2001, orange), normalized to have a bolometric luminosity of $10^{45.5}$ erg s$^{-1}$; the location of the Small Blue Bump (SBB, Wills et al. 1985; Vanden Berk et al. 2001); three reference frequencies corresponding to 5100Å, 3000Å and 1350Å (red filled circles), commonly used in calculation of bolometric luminosity (see below); the average spectral slopes found in literature, as measured on composite spectra at near IR, optical/UV (Vanden Berk et al. 2001, green) and far UV wavelengths (Telfer et al. 2002, purple); the rest frame frequency range covered by SDSS, for values of $z = 0$, 0.3, 1, 2 and 3 (thin blue lines). Finally, we show the AD spectrum that best fits the composite Type 1 SED at optical/UV wavelengths (black line). The parameter for the AD model are: $\log(M/M_\odot) = 9$, $\ell = 0.05$ and $\theta = 30^\circ$.

The agreement between the AD model and the composite SED is rather good, therefore the association between the BBB and thermal emission from simple AD model is justified, at least in the interval 1000–5000Å, or $\log(\nu/\text{Hz}) = 14.8$–15.5 (black dotted vertical lines). A few discrepancies between the AD model and the composite Type 1 QSO SED arise:

- at $\log(\nu/\text{Hz}) < 14.7$ a further component emerges in the spectrum, which may be either the host galaxy, the emission from a dusty torus or some other component;
- at $\log(\nu/\text{Hz}) \sim 15$ a Small Blue Bump (SBB) is present, likely due to a blending of iron lines and Hydrogen Balmer continuum;
- at $\log(\nu/\text{Hz}) \gtrsim 15.6$ other physical components contribute to the flux (e.g. a corona).

Note that, in this interpretation of the BBB, the portion of the AD spectrum characterized by the $\alpha_\nu = 1/3$ slope (thick blue line) is hidden by the host galaxy and the torus components, and cannot be revealed directly with observations (although in some case it may be detected in polarized light, Kishimoto et al. 2008). The average slopes at optical/UV and far UV (green and purple lines) are roughly consistent with the slopes near the peak of the AD spectrum. We notice however that fixed spectral features (such as the SBB) may affect the estimation of spectral slopes. Furthermore, the value of the slope likely depends on the width of the wavelength range inside which it is defined. Therefore it is not always possible to infer the presence of an AD spectrum by just checking the spectral slopes at optical/UV wavelengths. By contrast, at near IR the average slope (green line) is inconsistent with an AD spectrum, but this is likely due to the host galaxy component.

We conclude that the AD model provides a reasonable description of the gross properties of Type 1 AGN SED at optical/NUV wavelengths, and the similarity between the predicted spectrum and the average BBB is rather strong. Under this assumption it is possible to infer the black hole mass and the accretion rate by comparing the observed SED with the AD spectrum, as discussed in §. Our black hole mass estimation method requires an estimate of the disk luminosity ($L_d$), as discussed in the following two sections.

2.1 Continuum luminosity as a proxy to disk luminosity

The broad-band similarity among AGN spectra allows to use the continuum luminosity at a given wavelength as a proxy for the bolometric luminosity, that is $L_{bol} = C_{bol} \times \lambda L_\lambda$. In order to explore this relationship Richards et al. (2006) measured the bolometric luminosity for each spectrum (defined to be the integral isotropic luminosity between 100μm and 10 keV) and derived a bolometric correction ($C_{bol}$) based on the continuum luminosity at 3000Å, 5100Å.

1 General relativistic correction are negligible for the purpose of our work, see A4.
Figure 1. Comparison of the composite Type 1 AGN SED (red line) from Richards et al. (2006) and an AD model (black line) for \( \log(M/M_\odot) = 9 \), \( \ell = 0.05 \) and \( \theta = 30^\circ \). Also shown are: a spiral galaxy template as given in Mannucci et al. (2001, orange), normalized to have a bolometric luminosity of \( 10^{45} \) erg s\(^{-1} \); the location of the Small Blue Bump (SBB, Wills et al. 1985; Vanden Berk et al. 2001); three reference frequencies corresponding to 5100 Å, 3000 Å and 1350 Å (red filled circles), commonly used in calculation of bolometric luminosity; the average spectral slopes found in literature, as measured on composite spectra at near IR, optical/UV (Vanden Berk et al. 2001, green) and far UV wavelengths (Telfer et al. 2002, purple); the rest frame frequency range covered by SDSS, for values of \( z = 0, 0.3, 1, 2 \) and \( 3 \) (thin blue lines). Thick blue line highlights the portion of the AD spectrum characterized by the slope \( \alpha = 1/3 \). The rest frame frequency range inside which the AD model reproduces the shape of the AGN composite SED (\( \log(\nu/\text{Hz}) = 14.8-15.5 \)) is shown with dotted black lines.

The resulting values are: \( C_{\text{bol}}(3000\text{ Å}) = 5.62 \pm 1.14 \), \( C_{\text{bol}}(5100\text{ Å}) = 10.33 \pm 2.08 \), \( C_{\text{bol}}(3\mu\text{m}) = 9.12 \pm 2.62 \). The distribution of \( C_{\text{bol}} \) values is relatively narrow, with a relative dispersion of the order of \( \sim 20\% \). Note however, that in particular cases the \( C_{\text{bol}} \) value can differ by as much as 50\% from the mean value. Shen et al. (2011, hereafter S11 catalog) have slightly re-calibrated the \( C_{\text{bol}} \) values, and extended the analysis to 1350 Å, in order to compute bolometric luminosities for all the sources in their sample. Their values are: \( C_{\text{bol}}(5100\text{ Å}) = 9.26 \), \( C_{\text{bol}}(3000\text{ Å}) = 5.15 \) and \( C_{\text{bol}}(1350\text{ Å}) = 3.81 \).

In order to calibrate analogous relations to estimate the disk luminosity \( L_{\text{iso}}^d \) we numerically estimate the bolometric luminosity of the composite SED in Richards et al. (2006), and compare it with the disk luminosity for the AD model shown in Fig. 1. The resulting relation is:

\[
L_{\text{iso}}^d \sim \frac{1}{2} L_{\text{bol}}
\]

Then, we compare \( L_{\text{iso}}^d \) with the luminosities at 5100 Å, 3000 Å and 1350 Å wavelengths, as measured on the composite SED:

\[
\begin{align*}
L_{\text{iso}}^d &\sim 4.4 \nu L_{\nu}(5100\text{ Å}) \\
L_{\text{iso}}^d &\sim 2.4 \nu L_{\nu}(3000\text{ Å}) \\
L_{\text{iso}}^d &\sim 1.8 \nu L_{\nu}(1350\text{ Å})
\end{align*}
\]

The locations of these wavelengths are shown with red filled circles in Fig. 1. Considering the uncertainties (\( \sim 20\% \)) of \( C_{\text{bol}} \) we conclude that our relations (Eq. 1 and 2) are compatible with those of S11. Eq. 2 provides a reliable estimate of \( L_{\text{iso}}^d \) as long as the source continuum is not dominated by other emitting components such as host galaxy starlight or synchrotron radiation from a relativistic jet (for radio-loud sources). In these cases we need alternative luminosity estimators, as discussed in the following section.
2.2 Line luminosities as a proxy to disk luminosity

Relations similar to Eq. 2 can be obtained by using line luminosities. Line ratios are known to be approximately constant among AGN (Francis et al. 1991; Vanden Berk et al. 2001), by setting the Ly$\alpha$ luminosity to 100, relative luminosities of H$\beta$, Mg$\text{ii}$ and C$\text{iv}$ (both narrow and broad components) lines are 22, 34 and 63, respectively, while the total line luminosity is 555.8 (Francis et al. 1991; Celotti et al. 1997). Therefore it is possible to have a rough estimate of the luminosity of all emission lines by measuring the luminosity of a single line. Also, according to the photo-ionization model, the line-emitting gas is ionized by the accretion disk continuum radiation. Therefore we expect (to a first approximation) the disk to line luminosity ratio to be a constant: $L_{\text{iso}}^{d} = \kappa L_{\text{line}}$. This provides a way to estimate the disk luminosity using a single (or a few) line luminosity estimates.

In order to calibrate the $\kappa$ parameter we consider all sources in the S11 catalog having both a continuum and line luminosity estimate for at least one of the combinations: 5100$\text{Å}$–H$\beta$, 3000$\text{Å}$–Mg$\text{ii}$ and 1350$\text{Å}$–C$\text{iv}$. The number of sources in each subsample are 22644, 85514 and 52157 respectively (note that a single source typically belongs to two such subsamples). For each source we estimate $L_{\text{line}}$ using the broad and narrow line luminosities given in S11 and the coefficients given in Francis et al. (1991) and Celotti et al. (1997). Then we compute the $\kappa$ parameter as follows:

$$\kappa = \frac{L_{\text{iso}}^{d}(\text{Eq. 2})}{L_{\text{line}}} \quad (3)$$

where the disk luminosity $L_{\text{iso}}^{d}$ is computed using the continuum luminosity given in S11, and Eq. 2. The distributions of $\kappa$ for the three combinations are approximately log-normal (Fig. 2, upper panel) with median values:

$$\log \kappa(5100\text{Å} – \text{H}\beta) = 1.08 \pm 0.28$$
$$\log \kappa(3000\text{Å} – \text{Mg}\text{ii}) = 1.10 \pm 0.21$$
$$\log \kappa(1350\text{Å} – \text{C}\text{iv}) = 0.92 \pm 0.28$$

The widths of the $\kappa$ distributions in Fig. 2 show that the disk luminosity $L_{\text{iso}}^{d}$ computed using the continuum and the line intensities differs by $\lesssim 0.3$ dex, i.e. a factor $\lesssim 2$. Hence, the relationships between continuum and line luminosities seem quite robust. A possible explanation for the larger dispersion in the 5100$\text{Å}$–H$\beta$ case, with respect to the 3000$\text{Å}$–Mg$\text{ii}$ one, may be that the continuum luminosity at 5100$\text{Å}$ is contaminated by the host galaxy.

Both the continuum and the line luminosities in S11 are affected by uncertainties, therefore the distributions shown in Fig. 2 are likely broadened by measurement errors. Thus, the intrinsic dispersion is expected to be smaller than 0.3 dex (a factor of $\sim 2$) for H$\beta$ and C$\text{iv}$, and 0.2 dex (a factor of $\sim 1.6$) for Mg$\text{ii}$. This is yet another evidence that SEDs in most AGN show some degree of universality: the constancy of the continuum to line luminosity ratio at optical wavelengths implies a constant optical continuum to ionizing UV luminosity ratio.

Since the samples in the S11 catalog are dominated by radio-quiet sources (the great majority are undetected in the FIRST survey), we repeat our analysis on radio-loud sources, i.e. those sources for which the radio-loudness parameter is greater than 100. Interestingly, the $\kappa$ parameters for the radio-loud sub-sample of S11 differ by at most $\sim 5\%$ from the values quoted above.

By using the values given in Eq. 4 and the coefficients to compute the line luminosity $L_{\text{line}}$ discussed above, we are able to estimate the total disk luminosity as follows:

$$L_{\text{iso}}^{d} = 12 L(\text{H}\beta) = 303 L(\text{H}\beta)$$
$$L_{\text{iso}}^{d} = 12.5 L(\text{Mg}\text{ii}) = 204 L(\text{Mg}\text{ii}) \quad (5)$$
$$L_{\text{iso}}^{d} = 8.4 L(\text{C}\text{iv}) = 74.1 L(\text{C}\text{iv})$$

We repeat the above analysis on the subsample of NLS1 sources, that are the focus of our study. In particular, we consider the sources common to both the Zhou et al. (2006) and the S11 catalog (1210 sources). The distribution of the $\kappa$ parameter are still log-normal, and are shown in the lower panel of Fig. 2. Median values are now $\sim 0.15$ dex (i.e. a factor $\sim 1.4$) greater:

$$\log \kappa(5100\text{Å} – \text{H}\beta) = 1.23 \pm 0.23$$
$$\log \kappa(3000\text{Å} – \text{Mg}\text{ii}) = 1.24 \pm 0.21$$

The uncertainties are of the same order of magnitude. The class of NLS1 sources is therefore characterized by both a

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Figure 2. Upper panel: distribution of the $\kappa$ parameter (Eq. 3) for the three combinations 5100$\text{Å}$–H$\beta$, 3000$\text{Å}$–Mg$\text{ii}$ and 1350$\text{Å}$–C$\text{iv}$. Both the continuum and line luminosity estimates are those reported in the S11 catalog. The number of sources in each subsample are 22644, 85514 and 52157 respectively (note that a single source typically belongs to two such subsamples). Lower panel: the same as upper panel, for the subsample of Narrow-Line Seyfert 1 sources common to both the S11 and Zhou et al. (2006) catalog.

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2 The radio-loudness parameter provides an indication of whether the AGN SED is dominated by radiation at radio frequencies or optical band. Historically, it is defined as the ratio of 5 GHz to optical B-band luminosity (Kellermann et al. 1989). The values we used here are those given in S11, defined as the ratio of flux densities at 6 cm and 2500$\text{Å}$ (rest frame).

3 The 1350$\text{Å}$–C$\text{iv}$ case is missing since the SDSS wavelength coverage does not allow to observe both the C$\text{iv}$ line and the H$\beta$ line (required to classify the source as a NLS1).
smaller width and a smaller luminosity of lines. The resulting disk luminosities are:

\[ L_{\text{iso}}^{\text{H}} = 424 L(H\beta) \]
\[ L_{\text{iso}}^{\text{Mg}} = 286 L(Mg\text{II}) \]  

(7)

In order to estimate the accretion disk luminosity using a single spectrum we can use either Eq. 2 (whose uncertainties are \(\sim 20\%\)) or Eq. 7 (whose uncertainties are a factor \(\sim 2\)). In cases where the observed continuum radiation is dominated by components other than AD, e.g. synchrotron emission from the jet or host galaxy starlight, Eq. 2 would overestimate the disk luminosity. Therefore Eq. 7 is our preferred choice to estimate \(L_{\text{iso}}^{\text{H}}\).

3 THE SAMPLE

The aim of this work is to estimate the black hole masses of the sample of 23 Radio–Loud, Narrow–Line Seyfert 1 sources (RL–NLS1) given in Yuan et al. (2008, hereafter Y08). We identify each source with a sequential index (#1, #2, etc...), following the same order as in Table 1 of Y08.

All sources have been spectroscopically observed in the SDSS, and 21 over 23 sources are also in the S11 catalog. The IR photometry at 3.4\(\mu\)m, 4.6\(\mu\)m, 11.6\(\mu\)m, 22 \(\mu\)m from WISE (Wide–field Infrared Survey Explorer, Wright et al. 2010) is available for all sources. Finally, 21 over 23 sources have photometric measurements by GALEX (Martin et al. 2007), either in the Medium Imaging Survey (MIS), or the All sky Imaging Survey (AIS). We noticed that when multiple GALEX observations were available, we find significant variability in a few cases (#3, #8, #18) possibly due to the peculiar GALEX observations were available, we find significant variability in a few cases (#3, #8, #18) possibly due to the jet component. In these cases we chose preferably the MIS photometry with lower luminosity.

The redshifts are in the range \(z = 0.1–0.8\), therefore the continuum in the SDSS spectra will likely trace the AD component (\(\beta\)). The FWHM\( (H\beta)\) are less than \(2200 \text{ km s}^{-1}\) as required by the definition of NLS1 given in Zhou et al. (2009). The SE virial black hole masses are in the range \(log(M/M_\odot) = 6–8\), while the Eddington ratio are \(\ell = 0.5–3\) (Yuan et al. 2008). The radio morphology is compact, unresolved on 5\(\prime\) scale, and the radio loudness (Kellermann et al. 1989) is \(\sim 100\) for all sources.

The overall observational properties are very similar to that of blazars (Yuan et al. 2008), and the \(\gamma\)-ray emission from these sources has been predicted, and later detected in 7 RL–NLS1 sources (Abdo et al. 2009b). Calderone et al. (2011) (Foschini 2011), 4 of which are in the Y08 sample. However, these sources show unusually small widths of broad emission lines, and consequently small SE virial black hole mass estimates, when compared to typical blazars.

In order to apply our black hole mass estimation method (\(\Omega\)) to the sources in the sample we need to perform a spectroscopic analysis of the SDSS data. In particular we need to disentangle the host galaxy and/or jet contribution from the AGN continuum, and estimate the emission line luminosities. This procedure is described in the following section.

3.1 Spectral analysis

We used the spectra from the Sloan Digital Sky Survey (SDSS, York et al. 2000), data release 7 (DR7, Abazajian et al. 2009). We dropped spectral bins marked by at least one of the following mask flags: SF\_MASK\_FULLREJECT, SF\_MASK\_LOWFLAT, SF\_MASK\_SCATLIGHT, SF\_MASK\_BRIGHTSKY, SF\_MASK\_ODATA, SF\_MASK\_COMBINE4, SF\_MASK\_BADSKYCHI. Also, we dropped 100 bins at the beginning and end of each spectrum, in order to eventually avoid artifacts from instrument or pipeline.

Each spectrum has been de-reddened using the Galactic extinction values estimated from dust IR emission maps in Schlegel et al. (1998), and the extinction law reported in Cardelli et al. (1989) and O'Donnell (1994). We are currently neglecting any intrinsic reddening in the rest–frame of the source. Then we transformed the spectra to the rest frame by assuming isotropic emission (i.e. multiplying the flux by \(4\pi d_L^2\)). The redshift estimates are provided by the SDSS pipeline. Finally, we rebinned each spectrum by a factor of 3 in order to improve the signal to noise ratio, resulting in a spectral resolution of \(\lambda/\delta\lambda \sim 1450\) (corresponding to \(\sim 200 \text{ km s}^{-1}\)).

The model used to fit the spectra consists of five components:

- a smoothly broken power law to account for the AGN continuum ("AGN continuum" component);
- a spiral host galaxy template from Mannucci et al. (2001) and a power law to (eventually) account for the synchrotron emission from the jet ("galaxy" and "jet" components respectively). The galaxy component has a single free parameter (the overall normalization). The parameters for the jet component are estimated using data from WISE (Wide–field Infrared Survey Explorer, Wright et al. 2010). In particular we use the photometry in the two bands at the longest wavelengths (11\(\mu\)m and 22\(\mu\)m) to estimate the luminosity and the slope of the power law.
- the iron templates from Vestergaard & Wilkes (2001) at UV wavelengths and from Véron-Cetty et al. (2004) at optical wavelengths;
- a Gaussian profile for each emission line listed in Tab. 4. The FWHM of narrow lines are forced to be in the range 200–1000 km s\(^{-1}\), while that of broad lines are forced in the range 1000–3000 km s\(^{-1}\). Furthermore, the FWHM and velocity offset of the H\(\beta\) narrow component is tied to the width and offset of [O\(\text{iii}\)] \(\lambda 4959\) and [O\(\text{iii}\)] \(\lambda 5007\).
- a maximum of 10 additional Gaussian line profiles which are not "a priori" associated to any specific transition. These components are necessary to account for (e.g.) the iron blended emission lines in the range 3100–3500\(\alpha\) (not covered by the above–cited iron templates), or line asymme-

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1 See http://www.sdss.org/dr7/users/fluxcal/spSpec.html

2 The results of the spectral fitting procedure do not change significantly by considering the elliptical galaxy template from Mannucci et al. (2001).

3 In analyzing the source SDSS J094857.32+002225.5 (#5) we also applied an exponential cutoff at log\([\nu/\text{Hz}]=14\) (Abdo et al. 2009b).
4 BLACK HOLE MASS ESTIMATION METHOD

The AGN continuum in the rest frame wavelength range 1000–5000 Å (or log(ν/Hz) = 14.8–15.5), if interpreted as radiation emitted from a Shakura & Sunyaev (1973) accretion disk, allows to constrain an AD model, and to infer the black hole mass. Once we assume proper values for the inner radius of the disk $R_{in}$ and the viewing angle $\theta$ (4.1), the luminosity and frequency of the peak of the AD spectrum uniquely identify a value of the black hole mass. In the following sections we will discuss two methods to locate the peak of the AD spectrum, and infer the black hole mass and accretion rate. An example of the application of both methods to a specific case will be discussed in 4.3.

4.1 Hypotheses

The methods rely on the following hypotheses, which need to be independently verified:

(i) accretion in AGN occurs through steady–state, geometrically thin, optically thick, non–relativistic accretion disks. The emitted spectrum is well described by an AD model (Shakura & Sunyaev 1973);

(ii) once the galaxy and/or jet contribution has been subtracted, the continuum radiation in the range log(ν/Hz) = 14.8–15.5 (4.2) is emitted directly from the accretion disk, i.e. it has not been reprocessed by intervening material, nor it is emitted by some other component;

(iii) the spatial extent of the disk is $R_{in} = 6R_g$, corresponding to a radiative efficiency $\eta \sim 0.1$. The outer radius of the disk $R_{out} = 2 \times 10^3 R_g$ is not critical, since at frequencies much smaller than $v_p$, the AD spectrum will always be hidden by other emitting components. The assumption for $R_{in}$, on the other hand, is more critical, since our black hole mass estimates show a linear dependence on this value (case (v) of 4.2);

(iv) the relation between disk luminosity and its “isotropic equivalent” counterpart is $L_{iso}^{iso} = (2 \cos \theta) L_{p}$. Since we are interested in Type 1 AGN the viewing angle is in the range 0–45 deg (i.e. the aperture of the obscuring torus, Calderone et al. 2012). The averaged de–projection factor is thus (2 cos $\theta$) $\sim 1.7$ (Eq. A16), corresponding to a viewing angle of $\sim$30 deg.

The AD model has four parameters: $M$, $\dot{M}$, $R_{in}$, and $\cos \theta$ (4.3). With the assumptions discussed above, the remaining unknown parameters are the black hole mass $M$ and the accretion rate $\dot{M}$.

4.2 Procedure

Usually the localization of the peak of the AD spectrum is not accessible by using a single instrument, requiring optical/UV multiwavelength observations. When these observations are available it is possible to constrain the AD model, and estimate the frequency $v_p$ and luminosity $v_p L_{iso}^{iso}$ of the peak. The latter can then be used to infer the total disk luminosity $L_{iso}^{iso}$ (Eq. A10). Finally, the black hole mass and the accretion rate can be estimated as follows:

$$\frac{M}{10^9 M_\odot} = 1.44 \left(\frac{\nu_p}{10^{15} \text{ Hz}}\right)^{-2} \left(\frac{L_{iso}^{iso}}{(2 \cos \theta) \times 10^{45} \text{ erg s}^{-1}}\right)^{1/2}$$

The uncertainties on these results can be estimated by propagating the uncertainties in the $\nu_p$ and $\nu_p L_{iso}^{iso}$ parameters in the above equations. Hence, whenever the data allow to constrain the location of the peak of the AD spectrum, the accuracy of the black hole mass estimate is determined only by the accuracy of the data points (e.g. Sharrato et al. 2012). When UV observation are not available (or not reliable) the location of the peak cannot be constrained, and we must resort to an alternative method.

4.2.1 The LINE procedure

Here we propose a new method for the AD modeling which relies on broad line luminosities to estimate the total disk luminosity. Fig. 3 illustrates the method. We use Eq. 7 to estimate $L_{iso}^{iso}$. When both the Hβ and Mg II line luminosities were provided by our spectral fitting (4.1), we considered the average of the resulting disk luminosities. This enables us to estimate a value for the luminosity of the peak $\nu_p L_{iso}^{iso}$ (Eq. A10), i.e. to fix a “ceiling” in the $\nu L_{p}$ representation (black dashed line in Fig. 3): the peak of the AD spectrum must lie on this line. Then we use observations from a single instrument (SDSS) to constrain the peak frequency $v_p$, which is related to the black hole mass. In particular, we shift the AD spectrum horizontally (green arrow), until the AD spectrum will always be hidden by other emitting components. The assumption for $R_{in}$, on the other hand, is more critical, since our black hole mass estimates show a linear dependence on this value (case (v) of 4.2).
Table 2. Results of the spectral fitting for the 23 RL–NLS1 sources in Yuan et al. (2008) catalog. Columns are: (1) source numeric identifier; (2) SDSS name of the source; (3) redshift; (4) luminosity and error of the Hβ emission line (both the broad and narrow components); (5) luminosity and error of the Mg ii emission line (both the broad and narrow components); (6) wavelength λ0 and (7) luminosity λ0Lλ0 used to constrain the LINE model (see §4.2.1); (8) jet component (extrapolated from WISE data to wavelength λ0).

| #  | SDSS Name    | z   | log $L(\text{H}\beta)$ erg s$^{-1}$ | log $L(\text{Mg} \text{ ii})$ erg s$^{-1}$ | log $\lambda_0$ | log $\lambda_0L_{\lambda_0}$ | $L_{\lambda_0}$ |
|----|--------------|-----|-----------------------------------|---------------------------------|----------------|-----------------------------|----------------|
| 1  | J081432.11+560956.6 | 0.509 | 42.96 ± 0.01                        | 42.99 ± 0.02                     | 3170 | 45.11                      | —              |
| 2  | J084957.98+510829.0 | 0.583 | 42.29 ± 0.12                        | 42.52 ± 0.04                     | 3039 | 43.58                      | 5.67           |
| 3  | J085001.17+462600.5 | 0.523 | 42.48 ± 0.04                        | 42.51 ± 0.03                     | 3157 | 44.61                      | 0.13           |
| 4  | J090227.16+044309.6 | 0.532 | 42.61 ± 0.03                        | 42.93 ± 0.02                     | 3127 | 44.67                      | 0.07           |
| 5  | J094857.32+002255.5 | 0.584 | 42.81 ± 0.03                        | 42.85 ± 0.03                     | 3039 | 45.15                      | 0.00           |
| 6  | J095317.09+283601.5 | 0.657 | 42.54 ± 0.03                        | 42.84 ± 0.02                     | 2893 | 44.88                      | 0.10           |
| 7  | J103123.73+423439.3 | 0.376 | 42.31 ± 0.02                        | —                               | 3486 | 44.12                      | —              |
| 8  | J103727.45+003635.6 | 0.595 | 42.52 ± 0.05                        | 42.31 ± 0.04                     | 3020 | 44.73                      | —              |
| 9  | J104732.68+472532.1 | 0.708 | 43.10 ± 0.03                        | 43.04 ± 0.04                     | 2674 | 45.15                      | 0.12           |
| 10 | J111005.03+356336.3 | 0.630 | 42.36 ± 0.04                        | 42.59 ± 0.03                     | 2944 | 44.26                      | 0.01           |
| 11 | J113824.54+354337.1 | 0.356 | 42.26 ± 0.01                        | —                               | 3547 | 43.87                      | 0.64           |
| 12 | J114654.28+323652.3 | 0.465 | 42.68 ± 0.01                        | 42.65 ± 0.03                     | 3285 | 44.75                      | —              |
| 13 | J123852.12+394227.8 | 0.622 | 42.41 ± 0.05                        | 42.48 ± 0.03                     | 2961 | 44.59                      | 0.02           |
| 14 | J124634.65+023809.0 | 0.362 | 42.43 ± 0.02                        | 42.65 ± 0.03                     | 3203 | 44.66                      | 0.05           |
| 15 | J130522.75+511640.3 | 0.785 | 43.79 ± 0.01                        | 43.47 ± 0.01                     | 2692 | 45.74                      | 0.47           |
| 16 | J134509.49+313147.8 | 0.501 | 42.43 ± 0.02                        | 42.65 ± 0.03                     | 3203 | 44.46                      | 0.72           |
| 17 | J144318.56+472556.7 | 0.703 | 42.82 ± 0.04                        | 43.24 ± 0.02                     | 2826 | 45.42                      | 0.15           |
| 18 | J150506.48+032630.8 | 0.408 | 41.90 ± 0.03                        | 42.55 ± 0.04                     | 3407 | 44.39                      | 0.14           |
| 19 | J154817.92+351280.0 | 0.478 | 42.84 ± 0.01                        | 42.93 ± 0.02                     | 3249 | 45.05                      | 0.06           |
| 20 | J163323.58+471859.0 | 0.116 | 41.66 ± 0.02                        | —                               | 4303 | 43.74                      | 0.43           |
| 21 | J163401.94+480940.2 | 0.494 | 42.45 ± 0.02                        | 42.02 ± 0.02                     | 3213 | 44.56                      | 0.14           |
| 22 | J164443.53+261913.2 | 0.144 | 41.86 ± 0.01                        | —                               | 4200 | 43.95                      | 0.25           |
| 23 | J172206.03+565451.6 | 0.425 | 42.55 ± 0.01                        | 42.64 ± 0.03                     | 3370 | 44.68                      | 0.04           |

The LINE procedure can be implemented without any fitting procedure, provided we have an estimate for the broad line luminosities and the AGN continuum (§3.4). The search for the peak frequency can be implemented by identifying a wavelength $\lambda_0$ and the corresponding luminosity of the AGN continuum $L_{\lambda_0}$, and requiring the AD spectrum to match this luminosity at the same wavelength. A comparison between the resulting AD model and the AGN continuum can be performed “a posteriori” in order to assess the reliability of the black hole mass estimate (§4.2.4).
4.3 Example of application of the methods

As an example we discuss the case of SDSS J09531.7.09+283601.5 (§6), in Fig. 3. The WISE photometry is shown with black filled circles. The spectral fit (Eq. 6) is shown as a black line, while the AGN continuum component is shown as cyan line. The jet power law extrapolation from IR data is shown as a purple line. The GALEX photometry and their jet subtracted counterparts are shown as open circles and “+” symbols respectively. The disk luminosity \( L_{\text{iso}}^{\text{d}} \) with its uncertainty of a factor 2 is shown as a grey stripe: the peak of the AD model must lie within this region. The grey dashed grid shows the location of peaks for AD models with values of black hole mass and Eddington ratio shown respectively below and above the grey stripe.

By applying our black hole mass estimation methods we identify the LINE and BEST AD models, shown with a red and orange solid line respectively. Both AD models provide a rather good representation of the AGN continuum. The BEST AD model, however, needs a slightly higher luminosity than the LINE model in order to lie above the (jet--subtracted) GALEX photometry. Note that the observed spectrum (black line) has a significantly lower spectral slope (i.e. it is “redder”) than the AGN continuum, because of the host galaxy and jet contributions. Having considered these components in the spectral analysis allows us to reveal the real AGN continuum (cyan line) whose slope agrees with our AD spectrum.

In order to evaluate the uncertainty on the LINE black hole mass we repeat the procedure by requiring the AD model to peak at the top and the bottom of the grey stripe. The resulting AD models (dot--dashed red lines) are found to bracket the real case: the lower one cannot account for the AGN continuum, while the higher one is significantly above the GALEX photometry. This situation often occurred during the analysis of the sources (§6), therefore our black hole mass uncertainties are rather conservative.

Our AD models can be compared with those corresponding to the SE virial masses and bolometric luminosities reported in the Y08 and S11 catalogs (green and blue lines). We consider the disk luminosity as computed using Eq. 4. Note that our peak luminosities are very similar to those of Y08 and S11, since this is the condition we required (on average) to calibrate Eq. 1. However, these models do not provide a good description of the AGN continuum because their line emissions lie \( \sim 0.25 \) dex above our estimates of \( \nu_L \), therefore our black hole mass estimates are 0.5 dex (a factor \( \sim 3 \), Eq. 15) greater than the virial ones. The possible reasons to explain such differences will be discussed in §6.

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5 RESULTS

We analyzed the data from the 23 RL–NLS1 sources of the Y08 catalog. The spectral analysis (§3.1) of each individual source is shown in Fig. B1. The results are summarized in Tab. 2. The fitting models are in good agreement with data with reduced $\chi^2$ in the range 1.16–1.86. Also, the jet contribution at optical/NUV wavelengths is typically negligible, except for the #2, #11, #15, #16, #20 and #22 sources.

The results of our black hole mass estimation methods (§4) are shown graphically in Fig. C1 (adopting the same notation as in Fig. 4). The results are summarized in Tab. 3. The AD models identified by the LINE procedure provide a rather good description of the AGN continuum in 17 over 23 cases (indicated with a blank in the second column of Tab. 3). The remaining 6 sources cannot be modeled with an AD spectrum, and are considered “bad cases” (indicated with a “*” symbol in the second column of Tab. 3; see §6 for a discussion of these sources). These sources will not be considered in the following analysis.

The comparison between the black hole mass estimates for the LINE and BEST AD models are shown in Fig. 5. The mean value for the ratio of the two mass estimates is:

$$\langle \log \frac{M_{[\text{LINE}]} \langle M_{[\text{BEST}]}} \rangle = 0.07 \pm 0.37 \tag{9}$$

The two black hole mass estimates are therefore compatible, within the uncertainties associated to the LINE procedure (§4.2.1).

In Fig. 6 we show the comparison between the black hole masses from the AD models (LINE in upper panels, BEST in lower panels) and the black hole masses from SE virial method, as given in the Y08 (left panels) and S11 (right panels) catalogs. The uncertainty associated to SE
Table 3. Results of our black hole mass estimation method. Columns are: (1) source numeric identifier; (2) flag to indicate if the AD “signature” (i.e. the slope $\alpha_\nu > -1$ at optical wavelengths, see §6) is missing, (3) peak frequency of the AD model, (4) black hole mass estimate (with its uncertainties) and (5) Eddington ratio for the AD model identified by our automatic procedure (LINE model); (6), (7), (8) corresponding quantities for the BEST model; single epoch (SE) virial black hole mass estimate given in the (9) Y08 and (10) S11 catalogs.

| #  | Bad | Method | Best | Y08 | S11 |
|----|-----|--------|------|-----|-----|
|    |     | $\log \nu_p$ Hz | $\log M/M_\odot$ | $\ell$ | $\log \nu_p$ Hz | $\log M/M_\odot$ | $\ell$ | $\log M/M_\odot$ | $\log M/M_\odot$ |
| 1  |     | 15.2  | 8.8 (+ 0.4 ,−0.5 ) 0.022 | 15.5 | 8.4 | 0.084 | 8.0 | 8.1 |
| 2  | *   | —     | —     | —     | —     | —     | —     | 7.4 | 8.0 |
| 3  |     | 15.3  | 8.5 (+ 0.4 ,−0.5 ) 0.014 | 15.1 | 8.8 | 0.006 | 7.2 | 7.5 |
| 4  |     | 15.5  | 8.2 (+ 0.6 ,−0.4 ) 0.059 | 15.3 | 8.5 | 0.010 | 7.7 | 8.0 |
| 5  |     | 15.0  | 9.2 (+ 0.0 ,−0.6 ) 0.006 | 15.1 | 9.1 | 0.010 | 7.5 | 7.8 |
| 6  |     | 15.2  | 8.8 (+ 0.2 ,−0.6 ) 0.012 | 15.5 | 8.3 | 0.002 | 7.8 | 7.9 |
| 7  | *   | —     | —     | —     | —     | —     | —     | 7.3 | 7.6 |
| 8  |     | 15.0  | 9.0 (+ 0.0 ,−0.7 ) 0.004 | 15.0 | 9.1 | 0.004 | 7.3 | 8.5 |
| 9  | *   | —     | —     | —     | —     | —     | —     | 8.1 | 8.2 |
| 10 | *   | —     | —     | —     | —     | —     | —     | 7.1 | 9.0 |
| 11 | *   | —     | —     | —     | —     | —     | —     | 7.1 | 7.6 |
| 12 |     | 15.3  | 8.6 (+ 0.5 ,−0.5 ) 0.020 | 15.1 | 8.8 | 0.008 | 7.8 | 7.9 |
| 13 |     | 15.2  | 8.5 (+ 0.3 ,−0.5 ) 0.012 | 15.4 | 8.2 | 0.037 | 6.8 | 7.5 |
| 14 |     | 15.2  | 8.7 (+ 0.3 ,−0.5 ) 0.009 | 15.2 | 8.7 | 0.011 | 7.3 | 7.7 |
| 15 |     | 15.4  | 8.8 (+ 0.6 ,−0.4 ) 0.110 | 15.2 | 9.2 | 0.049 | 8.5 | 8.5 |
| 16 |     | 15.5  | 8.1 (+ 0.7 ,−0.4 ) 0.037 | 15.1 | 8.8 | 0.008 | 7.5 | 7.6 |
| 17 |     | 15.0  | 9.3 (+ 0.0 ,−0.5 ) 0.009 | 15.6 | 8.4 | 0.193 | 7.8 | 8.3 |
| 18 |     | 15.2  | 8.5 (+ 0.4 ,−0.5 ) 0.009 | 15.4 | 8.3 | 0.021 | 6.6 | 7.6 |
| 19 |     | 15.2  | 8.9 (+ 0.2 ,−0.6 ) 0.014 | 15.3 | 8.7 | 0.027 | 7.9 | 8.0 |
| 20 | *   | —     | —     | —     | —     | —     | —     | 6.3 | — |
| 21 |     | 15.0  | 9.0 (+ 0.0 ,−0.8 ) 0.003 | 15.0 | 8.9 | 0.004 | 7.4 | 7.9 |
| 22 |     | 15.3  | 8.3 (+ 0.6 ,−0.4 ) 0.007 | 15.3 | 8.2 | 0.008 | 6.9 | — |
| 23 |     | 15.3  | 8.6 (+ 0.5 ,−0.5 ) 0.016 | 15.5 | 8.1 | 0.079 | 7.4 | 7.6 |

Figure 5. Comparison between black hole mass estimates obtained using the LINE and BEST procedures. Since the BEST estimate is a manually tuned AD model we give no error associated to the corresponding black hole mass.

The mean values of the ratio of the mass estimates are:

$$\langle \log \frac{M[\text{LINE}]}{M[\text{Y08}]} \rangle = 1.2 \pm 0.5$$

$$\langle \log \frac{M[\text{LINE}]}{M[\text{S11}]} \rangle = 0.8 \pm 0.3$$

$$\langle \log \frac{M[\text{BEST}]}{M[\text{Y08}]} \rangle = 1.1 \pm 0.4$$

$$\langle \log \frac{M[\text{BEST}]}{M[\text{S11}]} \rangle = 0.8 \pm 0.3$$

(10)

The mean values are of the same order (or even greater) than the maximum uncertainty associated to the LINE black hole mass estimate (0.7 dex,§4.2.1), therefore our black hole mass estimates are not compatible with the SE virial ones.

6 DISCUSSION

As discussed in the characteristic disk spectral slope $\alpha_\nu = 1/3$ cannot be directly observed in AGN SED. However for values of the black hole mass $\log(M/M_\odot) \gtrsim 8$ and Eddington ratio $\ell \lesssim 1$, the peak of the AD spectrum is (in principle) observable. Indeed, for 17 over 23 sources considered here, the SDSS continuum show an increasing trend in the $\nu L_\nu$ representation (slope $\alpha_\nu > -1$) at $\log(\nu/\text{Hz}) \gtrsim 14.8$, where the accretion disk is expected to dominate over other emitting components. Assuming that the observed BBB is actually radiation emitted directly from the accretion disk,
the slope $\alpha_v > -1$ at $\log(\nu/\text{Hz}) \gtrsim 14.8$ becomes the “signature” of the presence of the AD component. In the considered sample (§ 3) 17 sources over 23 show such signature (the remaining six “bad” cases will be discussed below). Our spectral fitting procedure (§ 3.1) reveals an emission component that is well described by an AD model, although this has not been included “a priori” in our fitting model. Therefore, observational data are in agreement with hypothesis (i), as discussed in (4.1). Furthermore, the absorption column densities $N_H$, as estimated from X-ray spectral fitting, are compatible with the Galactic values (Yuan et al. 2008; Grupe et al. 2010). Hence, we expect the radiation we observe not to be reprocessed by any intervening medium, as assumed in hypothesis (ii) (4.1). Furthermore, we expect the contribution from other emitting components, such as host galaxy or jet, to be negligible at frequencies where our AD models are constrained ($\log(\nu/\text{Hz}) \gtrsim 14.8$). In particular, the slopes in the AGN continuum component ($\alpha_v > -1$) is incompatible with the ones inferred from the galaxy template of Mannucci et al. (2001). Also, the jet component is expected to decay at frequencies above a cutoff frequency of $\log(\nu/\text{Hz}) \lesssim 15$, as in typical Flat Spectrum radio- quasiars or powerful blazars (Chisellini et al. 2010).

The AD models identified by the LINE procedure (4.2.1) for the 17 “good” sources provide a rather good description of the AGN continuum (Fig. 4). In particular, the AGN continuum slopes in the frequency range covered by SDSS are in good agreement with the ones from LINE AD models (red solid lines). Also, the two limiting solutions (dot–dashed red lines) likely bracket the real case, providing a robust estimate of our uncertainties. The average uncertainty on the black hole mass estimates are of the order of ±0.5 dex (Tab. 3). By taking into account the uncertainties due to hypotheses (iii) and (iv) (radiative efficiency $\eta \sim 0.1$ and viewing angle $\theta \sim 30$ deg) we obtain a maximum uncertainty of ~0.7 dex (4.2.2).

In order to further assess the reliability of the LINE black hole mass estimates we considered the BEST AD models, identified by visually tuning the $L^\text{iso}_p$ and $v_0$ parameters in order to achieve the best possible match between the AGN continuum identified in (4.1) and the GALEX photometry. In a few cases we had to relax these requirements, as discussed below:

- the assumption that we can reliably estimate the jet contribution at optical/UV wavelengths by extrapolating a power law from the WISE photometry (4.3.1) may not be correct. For the #15 and #16 sources this assumption does not apply since the power law extrapolation (purple line) lies above the WISE photometry at shorter wavelengths (note that the error bars are smaller than the symbol in the plot). Source #5 would also falls in this class if the cutoff of synchrotron radiation at $\sim 10^{14}$ Hz (4.3.1) is neglected, since jet extrapolation would intercept optical SDSS data. In order to identify the BEST AD model for the #15 and #16 sources we used the continuum observed in SDSS data (black solid lines in Fig. 6) rather than the jet–subtracted one (cyan lines) as requirement at optical wavelengths. For #16 we obtained a good agreement between the BEST model and GALEX photometry. Lack of such agreement for #15 will be discussed below. A similar situation (jet component overestimated at optical/NUV wavelengths) possibly occurs also for source #22. In order to be conservative, for this source we retained the original constraints to identify the BEST model.

For the other sources the jet extrapolation is marginal at optical/NUV wavelengths (Tab. 3), hence the assumption discussed here has a negligible effect.

- for the #5 and #15 sources the GALEX photometry does not follow the extrapolation from the SDSS slope. Therefore a single AD model is not compatible with both the SDSS and GALEX observations. This may be a consequence of source variability, since the SDSS and GALEX data are not simultaneous. Indeed, we found significant variability in GALEX photometry in a few sources (§ 3.1). In particular, source #5 is known to a variable source (Abdo et al. 2009a,b; Foschini et al. 2011). In these cases the BEST model is computed relaxing the requirement of taking GALEX photometry into account, and using only the SDSS data as guidelines.

- for the #8 and #21 sources the SDSS and GALEX data appear to trace the peak of the AD spectrum. For these sources we neglected the jet component. Note that the BEST AD model for these sources provide a robust estimate of the black hole mass, since the peak of the AD spectrum has been directly observed.

The BEST AD models are in good agreement with the LINE ones. In particular, note that the peak in BEST AD spectra lie inside the grey stripes for all sources except #17. Hence, the Eq. 6 are well calibrated. The resulting BEST black hole mass estimates are compatible the LINE ones (Eq. 9 Fig. 5). Also, the scatter in Fig. 5 (0.4 dex) is compatible with the uncertainty on the LINE estimates due to our ignorance on $L^\text{iso}_p$ (0.5 dex, 4.2.1). This provides further support for the reliability of the LINE black hole mass estimates. We conclude that, under the assumptions discussed in 4.1, our LINE procedure provides a reliable estimate of the black hole mass, within the quoted uncertainties.

In six cases the LINE method do not provide an acceptable description of data (sources marked with a ‘$^\ast$’ symbol in the second column of Tab. 3) For these sources the observed SDSS continuum does not show an increasing trend (in the $vL_v$ representation ) at $\log(\nu/\text{Hz}) \gtrsim 14.8$. In two cases the SDSS continuum are dominated by the jet and/or host galaxy emission (#2 and #11), and the AD spectrum is not directly visible. In four cases (#7, #9, #10 and #20) the observed SDSS continuum appears “flat” in the $vL_v$ representation, with no hints for a change of slope. Although the jet–subtracted continuum (cyan line) suggests the presence of an AD spectrum, this decomposition strongly depends on the assumption that the extrapolation of the jet component from IR data is also valid at optical wavelengths. In order to be conservative, we mark these sources as “bad”, and neglect them in our analysis.

6.1 Comparison with SE virial mass estimates

The comparison with the SE virial mass reveals a systematic discrepancy between our mass estimate and those from the Y08 and S11 catalogs (Fig. 6 and 7). Although the discrepancy (≥0.7 dex, Eq. 10) is of the same order of the maximum uncertainty associated to the LINE procedure (4.2.1), it appears systematic. Therefore our black hole mass estimates are not compatible with the virial ones given in the Y08...
Figure 6. Comparison between black hole mass estimates of AD models identified by our procedures (LINE models, upper panels, BEST models lower panels) and single epoch (SE) virial masses as given in the Y08 (left panels) and S11 (right panels) catalogs. The uncertainty associated to SE virial mass is 0.5 dex.

and S11 catalogs. On average, our black hole masses turn out to be a factor \( \sim 6 \) (0.8 dex) greater than virial ones. A possible explanation for this black hole mass discrepancy may involve a radiative efficiency which is a factor \( \sim 6 \) lower than assumed. However, a value of \( \eta \sim 0.02 \) is lower than the minimum efficiency expected for accretion onto a maximally counter–rotating black hole (\( \sim 0.03 \)). Notice that the AD spectrum suggests that the accretion disk is still in the “radiatively efficient” regime: half the gravitational energy gained by matter at each radius is locally emitted as radiation, i.e. it is not advected into the hole.

The mass discrepancy is not due to an inaccurate estimation of the jet contribution at optical/NUV wavelengths. If the actual jet contribution is lower than estimated, the corresponding AGN continuum luminosity (\( \lambda_0 L_{\lambda_0} \)) would correspondingly be higher. The “ceiling” luminosity level, on the other hand, are not affected by the presence of the jet, since it relies on line luminosities. In order to reproduce an higher \( \lambda_0 L_{\lambda_0} \), retaining the same peak luminosity, the LINE AD model must shift to lower frequencies. Therefore, if the actual jet contribution is lower than estimated, we would have obtained greater LINE black hole mass estimates, and greater discrepancy with SE virial masses. Furthermore, the mass discrepancy is not due to having neglected the general relativistic corrections in the AD model. As discussed in [3], the AD model used throughout this work mimics the more sophisticated general relativistic one with \( \eta_{gr} \sim 0.1 \), as long as frequencies below the peak are concerned.

We speculate that a possible explanation for the mass discrepancy is a selection effect in calibrating the SE virial method. The virial method relies on the calibration of both a BLR size–continuum luminosity relation and of a virial factor (Park et al. 2012). However, the sample used to perform the calibration consists of a few dozens of sources: the ones that has been reverberation mapped. As a consequence, the calibration of the method may be biased by selection effects. In particular, the method may provide significantly underestimated black hole masses if the BLR has a flat disk–like geometry, and it is seen almost face–on (Decarli et al. 2011). If these conditions apply, then the discrepancy between our mass estimates and the virial ones would be greater (on average) for AGN showing the smallest widths of broad emission lines, i.e. the class of NLS1 sources (Decarli et al 2008). The black hole mass estimates provided by our method, on the other hand, are scarcely affected by the viewing angle and the geometry of the BLR (§4.2.1).

The Eddington ratios are in the range \( \ell = 0.04–0.2 \) (Tab. 3), significantly below the values reported in Y08. This dis-
crepancy is due both to our greater black hole mass estimates (a factor ~6) and to the fact that we used the disk luminosity \( L_d \) (instead of \( L_{bol} \)) to compute the Eddington ratio (a factor \( \sim 3.4 \), Eq. 1 and A16). Hence, our values of Eddington ratio a factor \( \sim 20 \) smaller than SE virial ones on average. With such small values of \( \ell \) the role of radiation pressure in determining the SE virial masses [Marconi et al. 2008; Chiaberge & Marconi 2011] is expected to be small, not sufficient to explain the black hole mass discrepancy.

6.1.1 The “temperature” argument

If the assumptions discussed in §4.1 apply, then all the independently estimated black hole masses (even the SE virial ones) should produce an AD spectrum compatible with the observed data. This provides a simple way to compare our results with those reported in Y08 and S11. The black hole mass discrepancy arises because the peak frequencies of the LINE and BEST AD spectra are significantly lower then the peak frequencies of the Y08 and S11 ones, while the peak luminosities are compatible. If the Y08 and S11 AD models (blue and green lines in Fig. 7) were the correct ones, there must be a physical process able to shift photon to lower frequencies (i.e. to lower “temperatures”) in order to account for the observed data. However, such a process cannot exist on a thermodynamic basis, since black body spectra (which build up the AD model) already have the lowest temperature corresponding to a given luminosity and emitting surface. Since the luminosities are the same, the only way to reduce the temperature is to increase the emitting surface, that is by increasing the black hole mass.

7 CONCLUSIONS

In this work we analyzed the relationship between the Big Blue Bump (BBB) observed in the SED of Type 1 AGN and a Shakura & Sunyaev (1973) accretion disk model (AD model). The characteristic disk spectral slope \( \alpha = 1/3 \) cannot be directly observed in the AGN SED because of the contributions from other emitting components such as the host galaxy, the torus or (for radio-loud sources) the jet (2). Once these contributions are taken into account, the observations are compatible with the presence of an emitting component which is well described by an AD model. In particular, the peak of such component can be observed directly in the frequency range \( \log(\nu/\text{Hz}) = 14.8-15.5 \). By comparing the average Type 1 AGN SED from Richards et al. (2006) with the AD model we calibrate the relations to estimate the total disk luminosity using the continuum line luminosities (at 1350 Å, 3000 Å and 5100 Å) as proxy (2.4). Furthermore, by using the emission line templates from Francis et al. (1991), we calibrate analogous relations based on the line luminosities of H\( \beta \), Mg ii and C iv (2.2). The latter provide more reliable disk luminosity estimates when the continuum is not dominated by the AD spectrum.

The interpretation of the BBB as being due to the thermal emission from an AD allows to link the AGN observed properties to the properties of the super massive black hole. In particular, the luminosity and frequency at the peak of the AD spectrum uniquely identifies a value for the black hole mass and the accretion rate (4). However, the direct observation of the peak of the AD spectrum requires broadband multiwavelength observations. In order to estimate the black hole mass also for sources with a poor observational coverage, we propose a new method which relies only on spectral observations at optical wavelengths. In particular, we constrain the luminosity of the peak by using the line...
luminosities. Then we constrain the frequency of the peak by requiring the AD model continuum and slope to reproduce the observed AGN continuum beneath the emission lines (A2). The maximum uncertainty on our black hole mass estimates is \( \sim 0.7 \) dex (on average). This uncertainty is greatly reduced if the disk luminosity can be accurately determined, namely, when the peak of the AD spectrum is visible within the frequency range of the data.

We applied our method to the sample of 23 radio–loud Narrow line Seyfert 1 galaxies (RL–NLS1) of Yuan et al. (2008). The method provides reliable black hole mass estimates for 17 sources over 23, if our interpretation of the BBB is correct (A5, 8). The remaining six sources are either dominated by synchrotron radiation from the jet, or do not show “hints” for the presence of an AD–like emitting component. The resulting black hole mass estimates are a factor \( \sim 6 \) (on average) greater than the corresponding single epoch virial mass estimates, and the Eddington ratios are a factor \( \sim 20 \) below. The discrepancy between our black hole mass estimates and the single epoch virial ones may be due to selection effects occurred in the calibration of the BLR–continuum relation and of the virial factor.

The black hole masses estimated in this paper for the sample of RL–NLS1 are in the interval \( \log (M/M_\odot) = 8–9 \), and the Eddington ratios are \( \ell = 0.04–0.2 \). Therefore, very radio–loud NLS1 appear not to be extreme in terms of black hole masses and Eddington ratios. Their black hole masses are similar to those of blazars. We find no evidence for jetted sources with mass below \( 10^8 M_\odot \).

ACKNOWLEDGMENTS

We acknowledge M. Vestergaard and B.J. Wilkes for having provided the UV iron template. We thank R. Decarli for fruitful discussion.

APPENDIX A: ACCRETION DISK MODEL

A1 Shakura & Sunyaev Accretion Disc (AD)

Here we review the properties of the AD model for a geometrically thin, optically thick accretion disk (Shakura & Sunyaev 1973, hereafter SS73 model) adopted in our analysis of the SED.

The amount of gravitational energy released from each annulus of the disk is given by

\[
F(R) = \frac{3}{8\pi} \left( \frac{R}{R_g} \right)^{-3} \left[ 1 - \left( \frac{R}{R_{\text{in}}} \right)^{-1/2} \right] \frac{M c^2}{R_g^2} \tag{A1}
\]

where \( R \) is the distance from the black hole, \( R_g = GM/c^2 \) is the gravitational radius of the black hole, \( R_{\text{in}} \) is the inner radius of the disk. By introducing the adimensional parameters:

\[
x = \frac{R}{R_{\text{in}}} \quad \eta = \frac{R_g}{2R_{\text{in}}} \tag{A2}
\]

we rewrite the emitted flux as \( F(R) = \tilde{F}(x) \mathcal{P} \), with

\[
\tilde{F}(x) = \frac{3}{\pi} x^{-3} (1 - x^{-1/2}) \quad \mathcal{P} = \eta^2 \frac{M c^2}{R_g^2} \tag{A3}
\]

where all physical quantities are cast into the \( \mathcal{P} \) parameter, while \( \tilde{F}(x) \) accounts for dimensionless flux distribution. The total disk luminosity is given by

\[
L_d = 2 \int_{R_{\text{in}}}^{R_{\text{out}}} 2\pi R \ F(R) \ dR = \int_1^{x_{\text{out}}} x^{-2} (1 - x^{-1/2}) \ dx \eta \dot{M} c^2 \tag{A4}
\]

where \( x_{\text{out}} = R_{\text{out}}/R_{\text{in}} \) is the normalized outer radius of the disk. The quantity in squared parentheses is equal to 1 (provided \( R_{\text{out}} \gg R_{\text{in}} \)), therefore the parameter \( \eta \) as defined above is the radiative efficiency of the disk. By assuming \( R_{\text{in}} = 6R_g \) (appropriate for a non–rotating black hole) we obtain \( \eta \sim 0.1 \).

The maximum amount of energy flux is (by differentiating Eq. A3):

\[
\text{MAX}[F(R)] = F(R_{\text{max}}) = \frac{64}{27} \left( \frac{3}{8\pi} \right)^7 \mathcal{P} \tag{A5}
\]

and it is emitted at a radius \( R_{\text{max}} = 49/36 \ R_g \). The assumption of optical thickness implies that each annulus emits radiation as a black body with temperature: \( T(R) = [F(R)/\sigma]^{1/4} \). The maximum temperature is therefore (Eq. A5):

\[
T_{\text{max}} \ [\text{K}] = 3.46 \times 10^4 \left( \frac{\eta}{0.1} \right)^{3/4} \left( \frac{M}{10^9 M_\odot} \right)^{-1/2} \left( \frac{\dot{M}}{\dot{M}_0 \text{yr}^{-1}} \right)^{1/4} \tag{A6}
\]

The emitted spectrum is a superposition of black body spectra:

\[
L_\nu = 2 \int_{R_{\text{in}}}^{R_{\text{out}}} 2\pi R \ dR \ \pi B[\nu, T(R)] \]

\[
= 4\pi^2 R_{\text{in}}^2 \mathcal{P}^{3/4} \int_1^{x_{\text{out}}} x \ dx \ B \left[ \frac{\nu}{P^{1/4}} \right]^{1/4} \tag{A7}
\]

where \( B[\nu, T(R)] \) is the Planck function. The spectrum profile is completely determined by the dimensionless integral, the only dependences on physical parameters (\( \mathcal{P} \)) being the characteristic frequency (\( \propto \mathcal{P}^{1/4} \)) and the overall normalization (\( \propto R_{\text{in}}^{3/4} \)). The disk spectra are therefore self–similar, and the peak frequency and luminosity scale as:

\[
\nu_{\text{p}} \ [\text{Hz}] = A \left( \frac{\eta}{0.1} \right)^{3/4} \left( \frac{M}{10^9 M_\odot} \right)^{-1/2} \left( \frac{\dot{M}}{\dot{M}_0 \text{yr}^{-1}} \right)^{1/4} \tag{A8}
\]

\[
\frac{\nu_{\text{p}} \ L_{\nu_{\text{p}}}}{[\text{erg s}^{-1}]} = B \left( \frac{\eta}{0.1} \right) \left( \frac{M}{M_\odot \text{yr}^{-1}} \right)^{1/4} \tag{A9}
\]

where \( \nu_{\text{p}} \) is the frequency of the peak in the \( \nu L_\nu \) representation, \( \log A = 15.25 \) and \( \log B = 45.36 \). By introducing the Eddington ratio \( \ell = L_d/L_{\text{Edd}} \) (with \( L_{\text{Edd}} = 1.3 \times 10^{47} (M/10^9 M_\odot) \text{ erg s}^{-1} \)), the previous equations can be rewritten as:

\[
\frac{\nu_{\text{p}} \ L_{\nu_{\text{p}}}}{[\text{Hz}]} = A \left( \frac{\eta}{0.1} \right)^{1/2} \left( \frac{M}{10^9 M_\odot} \right)^{-1/4} \left( \frac{\ell}{0.04} \right)^{1/4} \tag{A9}
\]

\[
\frac{\nu_{\text{p}} \ L_{\nu_{\text{p}}}}{[\text{erg s}^{-1}]} = B \left( \frac{M}{10^9 M_\odot} \right) \left( \frac{\ell}{0.04} \right)^{1/4} \tag{A9}
\]

Notice that, for a given value of \( \eta \), an estimate of the luminosity and peak frequency allows to determine the physical parameters \( M \) and \( \dot{M} \).

The spectrum of an AD is shown in Fig. 11 (black solid
line): the superposition of black body spectra, weighted by the surface of emitting annuli produces a “flat” spectrum with slope \( \alpha_v \sim 1/3 \); at highest frequencies, the Wien spectrum from the inner ring dominates, and the overall spectrum decays exponentially.

The self–similarity of AD spectra implies the existence of relations among quantities at the peak frequencies in the \( L_{\nu} \) and the \( \nu L_{\nu} \) representations (\( \nu_{\nu'} \) and \( \nu_p \), respectively), and the disk luminosity \( L_d \):

\[
\frac{\nu_p}{\nu_{\nu'}} = 3.1 \quad \frac{L_{\nu_p}}{L_{\nu_{\nu'}}} = 0.66 \quad \frac{\nu_p}{\nu_{\nu'}} L_{\nu_p} \frac{L_{\nu_p}}{L_d} = 0.5 \quad (A10)
\]

Also, note that the peak luminosity \( \nu_p L_{\nu_p} \) is independent from the actual value of \( R_{\text{out}} \), as long as \( R_{\text{out}} \gtrsim 10 R_{\text{in}} \). The relation between the maximum temperature in the disk and the color temperature of the AD spectrum (i.e. the black body temperature associated to the peak frequency \( \nu_{\nu'} \)) is

\[ T_{\text{max}} = 3.5 T_{\text{col}}. \]

### A2 Peak shift

The physical parameters \( \eta, M, \dot{M} \) uniquely identify the frequency and luminosity of the spectral peak (Eq. [A3] or [A4]). Variations of one or more of these parameters will shift the peak along specific directions, whose slope in a \( \log \nu L_{\nu} \) vs. \( \log \nu \) plot is given by:

\[
\alpha = \frac{d \log \nu_p L_{\nu_p}}{d \log \nu_p} \quad (A11)
\]

Here is a list of peak shift relations used in this work:

1. (i) vertical shift (\( \alpha = \infty \)): variations in \( M, \dot{M} \) with constant \( M/M^2 \) ratio (fixed \( \eta \));
2. (ii) horizontal shift (\( \alpha = 0 \)): variations in \( M \) with constant \( \dot{M} \) (fixed \( \eta \));
3. (iii) \( \alpha = 4 \): variations in \( M, \dot{M} \) (fixed \( \eta \));
4. (iv) \( \alpha = -4 \): variations in \( M \) and \( \dot{M} \), with constant \( \ell \propto M/M^2 \) (fixed \( \eta \));
5. (v) no shift: variations in all parameters, with constant \( \eta \dot{M} \) and \( M/M^2 \).

In particular, case (v) is used in [6.1.1] to show that if the actual radiative efficiency \( \eta \) is greater than hypothesized in [4.1], then our black hole mass estimate is a lower limit. The physical interpretation of case (v) is depicted in Fig. [A1].

By following the black hole mass estimation method outlined in [4.2.1], we identify an AD model (black line) in agreement with observed data (brown line): the peak lies at the “ceiling” luminosity level determined by broad line luminosities (Eq. [5]) and the AD spectrum is in agreement with the observed continuum. This model relies on the assumption of radiative efficiency \( \eta \sim 0.1 \). However, the actual value of efficiency may be different. By increasing the \( \eta \) parameter (i.e. decreasing the inner radius of the disk \( R_{\text{in}} \)) the peak shifts to higher frequencies and luminosities as radiation comes from the inner, hotter radii (blue line). This new model would no longer be in agreement with the “ceiling” luminosity argument, therefore we must decrease the accretion rate (green line), leaving \( M \) and \( \eta \) unchanged. The obtained spectrum is not in agreement with the observed continuum, therefore we must decrease the “temperature” of the spectrum (6.1.1), by increasing \( M \) (red line). The final AD model is again in agreement with observed data, but has higher values of \( \eta \) and \( M \) (and a lower value of \( \dot{M} \)).

### A3 Observational properties

The emission from the whole (geometrically thin) disk is anisotropic since the observed flux is proportional to the projected area seen by the observer, i.e. \( F_{\nu} \propto \cos \theta \), where \( \theta \) is the viewing angle. By requiring \( \int_{\text{Sph}} F_{\nu} D_L^2 d\Omega = L_{\nu} \) we obtain:

\[
L_{\nu} = \frac{2\pi D_L^2 F_{\nu_o}}{(1+z) \cos \theta} \quad (A12)
\]

where \( D_L \) is the luminosity distance, \( \nu_o = \nu/(1+z) \) is the observed frequency and \( F_{\nu_o} \) is the observed flux density. Note that the luminosity–flux relation for a thin disk is different from the AGN case, in particular the relation between the “isotropic equivalent” luminosity and the real luminos-
The observed spectrum is therefore (from Eq. [A11], Eq. [A12]):
\[ F_\nu = \frac{4\pi h\nu^3}{c^2 D_L^2} (1 + z) \cos \theta \int_{R_{in}}^{R_{out}} \frac{R dR}{\exp(\frac{h\nu}{kT}) - 1} \]  
(A14)

The model for the observed spectrum has four parameters: \( M, \dot{M}, R_{in} \) and \( \cos \theta \) (the value of \( R_{out} \) is not important here) which are related to quantities in Eq. (A14) through the temperature distribution given in Eq. (A11) Not all parameter can be constrained observationally, since the viewing angle is degenerate with both \( \dot{M} \) and \( M \). Hence we are forced to make a simplifying assumption about the inclination angle:

- (i) the innermost stable circular orbit (isco) depends on the spin parameter \( a \) of the black hole, giving \( R_{isco} = 6R_g \). The maximum spin of an accreting black hole is \( a = 0.998 \) (Thorne 1974), with \( R_{isco} = 1.24R_g \).

The binding energy of a particle at \( R_{isco} \) in units of the particle rest–mass is (e.g. Cunningham 1975):
\[ \eta_{gr} = 1 - \sqrt{1 - \frac{2}{3} \frac{R_g}{R_{isco}}} \]  
(A17)

i.e. \( \eta_{gr}(a = 0) \approx 0.06 \) and \( \eta_{gr}(a = 0.998) \approx 0.32 \). This is expected to be the maximum possible value for the radiative efficiency (compare Eq. [A3]).

- (ii) the different radial distribution of energy flux (Page & Thorne 1974; Zhang et al. 1997) with respect to Eq. (A1). The resulting spectrum is still the superposition of black body spectra;

- (iii) the spectrum received by distant observers is influenced by gravitational redshift, Doppler boost and gravitational bending of light (Cunningham 1973).

Li et al. (2005) have developed a package to synthesize the observed spectrum for an optically thick, geometrically thin accretion disk around a Kerr black hole, taking into account all these effects. The code is available as the model KERRBB within the X-ray data reduction package XSPEC (Arnaud 1996). In the following we will compare the spectral profiles of both the “classical” and “relativistic” models, and show that the differences are negligible for the purpose of our work.

We compute the accretion disk flux, as received by an observer at a given distance, using both the the SS73 and KERRBB models. The black hole mass, accretion rate and distance of the observer will be kept fixed for all the considered models.

We consider five SS73 AD models, by varying the inner radius of the disk \( R_{in} \). The values of \( R_{in} \) has been chosen in order to reduce the discrepancies between SS73 and KERRBB models (see below). We take the model with \( R_{in} = 6R_g \) as a reference spectrum, and normalize all other SS73 spectra by the luminosity of its peak (\( \nu_0 F_{\nu, ref} \)). These spectra are shown with solid lines in Fig. A2. Note that the only dependence on the viewing angle \( \theta \) for the SS73 model is due to the projected area seen by the observer, i.e. to a factor \( \cos \theta \). By plotting spectra normalized by \( \cos \theta \) we completely remove this dependence.

Then we consider five groups of KERRBB models, by varying the spin of the black hole: \( a = -1, 0, 0.4, 0.7 \) and 0.998. These values span the entire range of allowed values for the spin of an accreting black hole (Thorne 1974). For each value of the spin, we consider three different viewing angles: \( \theta = 0^\circ, 30^\circ \) and \( 45^\circ \). All spectra are normalized by the luminosity \( \nu_0 F_{\nu, ref} \) of the reference SS73 model discussed above, and by \( \cos \theta \). These spectra are shown with dotted, dashed and dot–dashed lines in Fig. A2. Note that the KERRBB models show a residual dependence on the viewing angle, due to light bending and Doppler boosting.

The values of \( R_{in} \) has been chosen in order to allow the SS73 spectra to resemble as close as possible the KERRBB spectra, at given values of spin. The profile of the normalized spectra are indeed very similar (spectra of the same color), the differences being at most \( \pm 0.1 \) dex for the highest value of spin (\( a = 0.998 \)). The (empirical) relation between the innermost stable circular orbit \( R_{isco} \) and \( R_{in} \) in the SS73 model and the radiative efficiency of the corresponding KERRBB model is:
\[ \frac{R_{in}}{R_g} = \frac{1}{2\eta_{gr}} + 1.25 \]  
(A18)

where \( \eta_{gr} \) is given by Eq. [A17].

From the observational point of view, the SS73 AD model with \( R_{in} = 6R_g \) (used throughout this work) mimics the KERRBB model with spin \( a \sim 0.7 \) (\( \eta_{gr} \sim 0.1 \)), as long as frequencies below the peak are concerned. Therefore, the results of our work are not influenced by having neglected the general relativistic corrections in modeling the accretion disk spectrum.

REFERENCES

Abazajian K. N., et al., 2009, ApJS, 182, 543
Abdo A. A., et al., 2009a, ApJ, 699, 976
Abdo A. A., et al., 2009b, ApJ, 707, 727
Abdo A. A., et al., 2009c, ApJ, 707, L142
Arnaud K. A., 1996, in Jacoby G. H., Barnes J., eds, Astronomical Data Analysis Software and Systems V Vol. 101 of Astronomical Society of the Pacific Conference Series, XSPEC: The First Ten Years pp. 17
Assel R. J., Frank S., Grier C. J., Kochanek C. S., Denney K. D., Peterson B. M., 2012, ApJ, 753, L2
**Figure A2.** Comparison between the SS73 (Shakura & Sunyaev 1973, solid lines) and the KERRBB (Li et al. 2005, dotted, dashed and dot–dashed lines) accretion disk spectra. The SS73 spectrum with $R_{\text{in}} = 6 R_g$ is used as a reference spectrum: all other spectra are normalized by its peak luminosity. We consider three viewing angles $\theta = 0^\circ$, $30^\circ$ and $45^\circ$, and normalize all spectra by $\cos \theta$. This completely removes the dependence of the SS73 on the viewing angle. The KERRBB show a residual dependence on the viewing angle (dotted, dashed and dot–dashed lines). The colors identify a value for the inner radius of the SS73 model (respectively $R_{\text{in}}/R_g = 14.5$, $10$, $7.9$, $6$ and $2.8$) and for the black hole spin of the KERRBB model (respectively $a = -1$, $0$, $0.4$, $0.7$, $0.998$). The inner radii for the SS73 models have been chosen in order to allow the SS73 spectra to resemble as close as possible the KERRBB spectra, at given values of the spin. The resulting empirical relation between $R_{\text{in}}$ and the radiative efficiency of the KERRBB model (Eq. A17) is given in Eq. A18.
APPENDIX B: FIGURES: SPECTRAL FITTING

This appendix is a collection of the figures related to the spectral fitting procedure discussed in §3.1. On the left panels we show the whole rest frame wavelength range, while on the right panels we show a detailed view on the Hβ, [O iii] and Mg ii regions. The SDSS data and associated uncertainties are shown with black squares and grey lines respectively. Also shown are the fitting models (red lines), as well as the individual components: the AGN continuum (black), the galaxy template (cyan), the jet component (as extrapolated from WISE photometry, purple), the iron templates (orange), the broad (blue) and narrow (green) emission lines, and the additional emission lines (grey). In lower part of left panels we show the residuals in units of data uncertainties. The red lines show the cumulative $\chi^2_{\text{red}}$ (values on right axis).

APPENDIX C: FIGURES: BLACK HOLE MASS ESTIMATION

This appendix is a collection of the figures related to the black hole mass estimation procedures described in §4, adopting the same notation as in Fig. 4.
Figure B1. Results of the spectral fitting procedure (§3.1, App. B).
Figure B1. (continued)
Figure B1. (continued)
Figure B1. (continued)
Figure B1. (continued)
Figure B1. (continued)
Figure B1. (continued)
Figure B1. (continued)
Figure C1. Results of the black hole mass estimation procedures (App. C). Notation is the same as in Fig. 4.
Figure C1. (continued)
Figure C1. (continued)
Figure C1. (continued)