Super-heavy Quarkonia and the Gluon Condensate

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The early idea that a non-perturbative gluon condensate affects the spectrum of heavy quarks is revisited in the light of modern simulation techniques. We evaluate the low lying spectrum of bound states of two heavy quarks for large hypothetical quark mass, $m_Q > m_b$, using non-relativistic QCD and compare with other models to test the consistency.

The concept of a non-perturbative gluon condensate was initially introduced by Shifman, Vainstein, and Zakharov [1]. A non-vanishing gluon condensate can give rise to a shift of energy levels of heavy quarkonia by second order Stark splitting. Leutwyler and Voloshin [2] calculated the level shifts in leading order perturbation theory. Recent improvements like the stochastic vacuum model SVM [3], non-perturbative field strength correlators [3] or a combination of the SVM with lattice calculations [6,7] try to solve some of the problems of the original LV-model. Using NRQCD has the advantage that no apriori knowledge about the heavy quark potential or the vacuum background field is needed. The physical problem is reduced to a simulation of or the vacuum background field is needed. The or knowledge about the heavy quark potential

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For the simulations we use a lattice of size $16^4$ and a gauge coupling of $\beta = 9.17$ to obtain a very fine spacing necessary to resolve small bound states. After generating 360 decorrelated configurations on a Quadrics Q4 we fix to Coulomb gauge with a stopping criterion, $\theta < 10^{-6}$. The simulations require 60 days of Quadrics Q4 time, 70 days on a workstation for gauge-fixing, and more than 150 days of CM2/CM5 time. We apply standard correlated multi-state multi-exponential fits with vector fits for a set of S states and ratio fits for hyperfine and S-P splittings. The error is determined by a jackknife procedure including the uncertainty from the fit range, $[t_{min}, N_T]$. A consistent scale is obtained from $V_{q\bar{q}}$ and the force indicating that scaling violations are indeed small at this high value of the gauge coupling (see ref. [10] for details).

As a first step our results can be compared to leading order perturbation theory which allows to separate the Coulomb from the condensate contribution. Following refs. [10], the level splittings are given to lowest order by the expressions

$$\Delta E_{hfs} \simeq \frac{m(C_F a_s)^4}{3} + \frac{5425\pi C_F^4}{m^4 a_s^4} f_{10} \langle \alpha_s G^2 \rangle$$

$$\Delta E_{SP} \simeq \frac{m(C_F a_s)^2}{16/3} + \frac{64\pi C_F^2}{m^2 a_s^4} f_{21} \langle \alpha_s G^2 \rangle$$

where $f_{nl}^{-1} = 1 + (\lambda_G^{-1}/[m(4/3a_s)^2]) \rho_{nl}$

$$\rho_{10} = 2.48, \quad \rho_{21} = 11.20, \quad C_F = 4/3$$

There are two length scales in the system: the gluon correlation length $\lambda_G$ and the quark correlation length $\lambda_Q$ which is related to the rotational period of the two heavy quarks inside the quarko-
Figure 1. The hyperfine splitting between the ground state of the $^3S_1$ and the $^1S_0$ meson. The line gives the perturbative Coulomb contribution with $\alpha_s(m_Q)$ as shown in fig. 3. Also shown are the $cc$ and $bb$ values.

Figure 2. The S-P splitting from NRQCD (squares), perturbation theory and for the Buchmüller and Tye model.

condensate by a factor $f_{nl}(\lambda_G)$. If we nevertheless take the perturbative results seriously we see from fig. 4 that it is possible to get a value of the condensate compatible with sum rule results. However, the value of $\langle \alpha_s G^2 \rangle$ strongly varies with quark mass. The static approximation ($\lambda_G \to \infty$) should be valid for large quark mass while the approximation of a rapidly varying condensate ($\lambda_G << \lambda_Q$) is expected to hold for intermediate masses, $m_Q < 40$ GeV, because $\lambda_Q$ is decreasing with $m_Q$.

Summary

At large $m_Q$ the hyperfine splitting of the ground state becomes compatible with Coulombic behaviour. However, the S-P splitting from NRQCD is neither in accord with the B+T model nor with leading order perturbative results for LV-type models. On the level of lowest order perturbation theory no consistent picture of the effect of the gluon condensate can be found although there is evidence that the general trend predicted by the analytic results is qualitatively correct. The NRQCD hyperfine splitting of the ground state is decreasing and the S-P splitting is increasing with $m_Q$ for large quark mass consistent with the perturbative dependence on dif-
different powers of $\alpha_s$. It will be interesting to see \cite{Fingberg:1996} if a comparison with more rigorous models like the combined SVM and lattice approach will consolidate the situation. An alternative might be a closed lattice approach. Then one is faced with the difficult problem to switch the condensate on and off during the simulations.

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