We investigate the stability of the Positive Mass Theorem for three-dimensional axisymmetric manifolds. It is widely known that asymptotically flat manifolds with nonnegative scalar curvature have nonnegative ADM mass, and that the only asymptotically flat manifold with nonnegative scalar curvature and zero ADM mass is Euclidean space. We will show that asymptotically flat axisymmetric manifolds that have nonnegative scalar curvature, small ADM mass, and satisfy an additional technical assumption, are close to Euclidean space in a Sobolev sense: the components of the metric tensor in cylindrical coordinates are Sobolev close to the corresponding background Euclidean metric arising from those same coordinates. Some geometric consequences of this form of stability will also be explored. (Received February 05, 2019)