Runaway collisions in young star clusters. I. Methods and tests

Marc Freitag1,2*, Frederic A. Rasio2 and Holger Baumgardt3

1 Astronomisches Rechen-Institut, Mönchhofstrasse 12-14, D-69120 Heidelberg, Germany
2 Department of Physics and Astronomy, Northwestern University, Evanston, IL 60208, USA
3 Sternwarte, Universität Bonn, Auf dem Hügel 71, 53121 Bonn, Germany

ABSTRACT
We present the methods and preparatory work for our study of the collisional runaway scenario to form a very massive star (VMS, $M_\star > 400 M_\odot$) at the centre of a young, compact stellar cluster. In the first phase of the process, a very dense central core of massive stars ($M_\star \approx 30 - 120 M_\odot$) forms through mass segregation and gravothermal collapse. This leads to a collisional stage, likely to result in the formation of a VMS (itself a possible progenitor for an intermediate-mass black hole) through a runaway sequence of mergers between the massive stars. In this paper we present the runaway scenario in a general astrophysical context. We then explain the numerical method used to investigate it. Our approach is based on a Monte Carlo code to simulate the stellar dynamics of spherical star clusters using a very large number of particles (a few $10^5$ to several $10^6$). Finally, we report on test computations carried out to ensure that our implementation of the important physics is sound. In a second paper, we present results from more than 100 cluster simulations realized to determine the conditions leading to the collisional formation of a VMS and the characteristics of the runaway sequences.

Key words: Galaxies: Nuclei — Galaxies: Starburst — Galaxies: Star Clusters — Methods: N-Body Simulations, Stellar Dynamics — Stars: Formation

1 INTRODUCTION

Runaway collisions and mergers of massive stars following gravothermal contraction and core collapse in a young, dense star cluster provides a natural path to the formation of a massive object at the centre of the system. This idea goes back to the earliest studies of the quasar/AGN phenomenon (e.g., Spitzer & Saslaw 1966, Colgate 1967, Sanders 1970).

Runaways could easily occur in a variety of observed young star clusters such as the “young populous clusters” (like the Arches and Quintuplet clusters near our Galactic centre) and the “super star clusters” found in all starburst environments, including most galactic mergers. Runaways could easily occur in a variety of observed young star clusters such as the “young populous clusters” (like the Arches and Quintuplet clusters near our Galactic centre) and the “super star clusters” found in all starburst environments, including most galactic mergers. 

The Pistol Star observed in the Quintuplet cluster (Figer et al. 1998) may well be the product of such a runaway, as demonstrated by direct N-body simulations (Portegies Zwart & McMillan 2002). If the massive runaway collision product collapses to a black hole (BH), this scenario also provides a route to the formation of intermediate-mass black holes (IMBH) in star clusters. Dynamical evidence for IMBHs at the centres of some globular clusters has been reported for many years (Gebhardt et al. 2002, Gerssen et al. 2002). Tentative evidence has also been reported for an IMBH in the Galactic centre source IRS 13, which was recently resolved into a small cluster of bright stars (Maillard et al. 2004). This could be the remnant of a much larger cluster that got tidally disrupted as its orbit around the Galactic centre decayed through dynamical friction (Hansen & Milosavljević 2002, Gürkan & Rasio 2003). The very bright ultra-luminous X-ray source (ULX) associated with the young star cluster MGG 11 in the starburst galaxy M82 could also have been produced through runaway collisions (Portegies Zwart et al. 2003). A similar process may be responsible for the formation of IMBHs in larger clusters, such as proto-globular clusters, as well as seed BHs in proto-galactic nuclei. These can later grow through a variety of processes including gas accretion, stellar captures, and mergers. 

* Present address: Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 0HA, UK. E-mail: freitag@ast.cam.ac.uk
The questions we address here are also central to our understanding of low-frequency gravitational-wave (GW) sources and the development of data analysis and detection strategies for these sources. In addition, our work may be viewed as mainly relevant to galactic astronomy, the results could also have applications in extragalactic astronomy and cosmology. The direct injection into the centre of a galaxy of many IMBHs produced by collisional runaways in nearby young star clusters provides an important new channel for building up the mass of a central supermassive BH through mergers (Portegies Zwart & McMillan 2003). It is possible that this process is still ongoing in our own Galactic centre (Hansen & Milosavljevic 2003; Kim et al. 2003; Girkan & Rasio 2003). In contrast, minor mergers of galaxies are unlikely to produce BH mergers, as the smaller BH will rarely experience enough dynamical friction to spiral in all the way to the centre of the more massive galaxy (Volonteri et al. 2003). These ideas are also of critical importance for the design and planning of LISA, since the inspiral of an IMBH into a SMBH provides one of the best sources of low-frequency GWs for direct study of strong field gravity with a space-based interferometer (Cutler & Thorne 2002; Phinney 2003; Collins & Hughes 2004; Miller 2005). Although the SMBHs found in bright quasars and many nearby galactic nuclei are thought to have grown mainly by gas accretion (e.g., ENSO 1992; Fabian & Iwasawa 1999; Richter 2004), current models suggest that LISA will probe most efficiently a cosmological massive BH population of lower mass, which is largely undetected (Menou 2003). LISA will measure their masses with exquisite accuracy, and their mass spectrum will constrain formation scenarios for high-redshift, low-mass galaxies and, more generally, hierarchical models of galaxy formation (e.g., Haehnelt & Kauffmann 2002; Hughes & Holz 2003; Volonteri et al. 2003; Sesana et al. 2004).

GFR04 concentrated on the early dynamical evolution of young, dense star clusters. They performed dynamical Monte Carlo simulations for systems containing up to $10^7$ stars, and followed the rapid mass segregation of massive main-sequence (MS) stars and the development of the Spitzer instability. They showed that, with a realistic initial mass function (IMF), these systems can evolve to core collapse in a small fraction of the initial half-mass relaxation time. If the core-collapse time is less than the lifetime of the most massive MS stars, all stars in the collapsing core may then undergo runaway collisions. The study in GFR04 was limited to the first step in this process, up to the occurrence of core collapse. About 100 simulations were performed for clusters with a wide variety of initial conditions, varying systematically the cluster density profile, stellar IMF, and the number of stars. GFR04’s results confirmed that, for clusters with a moderate initial central concentration and any realistic IMF, the ratio of core-collapse time to initial half-mass relaxation time is typically $\sim 0.1$, in agreement with previous calculations. It was also found that, for all realistic initial conditions, the mass of the collapsing core (at the onset of collapse) is always close to $\sim 10^{-3}$ of the total cluster mass, very similar to the observed correlation between central BH mass and total cluster mass in a variety of environments (Magorrian et al. 1998; Merritt & Ferrarese 2000; Häring & Rix 2004).

In this and a following paper (hereafter Paper II), we go a step further in our study of runaways, by modelling the actual stellar collisions and following the early growth of the massive runaway product.

This paper is organised as follows. In Section 2, we explain in more detail the scenario for forming a massive object through runaway collisions and review the previous works on the subject. In Section 3, we present the numerical method and physical ingredients used in our simulations. Test simulations are presented in Section 4. Finally, in Section 5, we summarise these results and introduce Paper II. In the latter, we will present the results of more than 100 simulations carried out to study runaway collisions in a variety of clusters.

## 2 THE COLLISIONAL RUNAWAY ROUTE

### 2.1 Important quantities

Before reviewing the collisional runaway scenario, it is useful to define some quantities which are often referred to.

If a star with mass $M_1$ and radius $R_1$ travels with relative velocity $V_{rel}$ across a field of stars of mass $M_2$ and radius $R_2$ and number density $n_2$, the collision probability per unit time for this star, i.e., the reciprocal of its collision time, is

$$\frac{1}{t_{coll}} = S_{coll} V_{rel}^2 n_2$$

with

$$S_{coll} = \pi (R_1 + R_2)^2 \left( 1 + \frac{2G(M_1 + M_2)}{(R_1 + R_2)V_{rel}^2} \right).$$

An important velocity scale for collisions between such stars is given by

$$V_c^2 = \frac{2G(M_1 + M_2)}{R_1 + R_2} = (617.5 \text{ km s}^{-1})^2 \frac{M_1 + M_2}{M_\odot} \frac{R_\odot}{R_1 + R_2}. \quad (2)$$

In most situations, $V_{rel}^2 \ll V_c$, and the collision cross section is dominated by gravitational focusing, $S_{coll} \sim 2\pi G(M_1 + M_2)(R_1 + R_2)$. In a system where all stars have mass $M_*$, radius $R_*$, density $n$ and a Maxwellian velocity distribution with 1-D velocity dispersion $\sigma_v$ ($\ll V_c$), the collision time (after which, each star, on average, would have experienced one collision) is then (Binney & Tremaine 1987, Eq. 8-125)

$$t_{coll} \approx 2.1 \times 10^{12} \frac{\text{yr}}{n} \frac{10^6 \text{ pc}^{-3}}{n} \frac{\sigma_v}{30 \text{ km s}^{-1}} \frac{R_\odot}{M_\odot} \frac{M_\odot}{M_*} \kappa \times.$$

Two-body relaxation plays a central role in the evolution of most clusters considered here. The local relaxation time is (Spitzer 1987)

$$t_{rel} = 0.033 \frac{\sigma_v}{G^2 \ln(\gamma_c N_c) n(M_\odot)} \approx 4.8 \times 10^7 \text{ yr} \times$$

$$\frac{10}{\ln(\gamma_c N_c)} \left( \frac{30 \text{ km s}^{-1}}{\sigma_v} \right)^3 \frac{10^6 \text{ pc}^{-3}}{n} \left( \frac{\langle M_\odot \rangle}{M_\odot} \right)^{-2}, \quad (4)$$

where $\langle M_\odot \rangle$ is the average stellar mass and $N_c$ the total number of stars. For systems with a broad stellar mass spectrum, we use $\gamma_c = 0.01$ in the Coulomb logarithm (see Sec. 2.1.4).

The relaxation time defined this way only has a direct meaning for a single-mass population. In case of a mass spectrum,
it serves as a reference time but relaxational evolution may (and does) happen on a small fraction of $t_{\text{cc}}$

The core of a cluster is the central region where density and velocity dispersion are approximately constant. We use the definition of Spitzer (1987, Eq. 1-34) for the core radius,

$$ R_{\text{core}} = \left[9 \sigma_t^2 / (4 \pi G \rho_0) \right]^{1/2} $$

where $\rho = \langle M_* \rangle n$ and underscore 'c' indicates central values.

### 2.2 Basic scenario: summary and expectations

For the reader’s convenience we summarise here the collisional runaway scenario presented in GFR04 for the formation of an intermediate-mass black hole (IMBH, $100 M_\odot < M_{\text{BH}} < 10^5 M_\odot$) at the centre of a dense stellar cluster. In the next subsection we briefly review previous works that have led to the formulation of this scenario or have pioneered its investigation.

The basic idea is to create, through a sequence of collisions, a stellar object much more massive than what normal star formation can produce, with mass of a few hundreds to a few thousands $M_\odot$. We refer to these objects as “very massive star” (VMS). If such a star does not lose too much of its mass to winds by the time it reaches the end of its life, it is likely to undergo complete collapse into a black hole (BH), without mass ejection (Fryer & Kalogera 2001; Heger et al. 2003), hence producing an IMBH.

Stellar collisions are extremely rare in most astrophysical environments. They can only play a role in systems with a high stellar density such as self-gravitating dense clusters. When the most massive stars initially present in a cluster explode as supernovae, the cluster expands significantly as a result of the mass loss and collisions stop (if they were ever occurring). Therefore the formation of a VMS has to occur before massive stars evolve off the MS (including the giant phase which would only make a small difference), i.e., for a cluster containing stars up to $\sim 100 M_\odot$, within a few million years. It is possible that a cluster could be born in a collisional state, i.e., with an average collision time shorter than this, so that it would have been even more collisional while its stars were still on the pre-MS. This has been discussed as a way of creating all stars more massive than $\sim 10 M_\odot$ (Bonnell et al. 1998; Bally & Zinnecker 2005 and references therein). However, we focus here on the simpler case of a gas-free cluster with all stars on the MS initially.

Two-body relaxation provides a mechanism to increase the central density of a cluster and possibly drive it to a collisional state. Although this process of gravothermal core contraction, eventually leading to core collapse, also operates in single-mass clusters, it is much more important for realistic clusters with a broad IMF. The most massive stars will then be subject to dynamical friction and drift to the centre. In all realistic cases, the cluster encounters the Spitzer instability, meaning that this mass-segregation process leads to the formation of central core of massive stars that decouples as a self-gravitating system before the most massive stars can achieve energy equipartition with lighter objects. The system of massive stars then experiences core collapse on its own, very short, timescale. The high concentration of massive stars at the centre of the cluster eventually leads to a high collision rate.

The process of rapid mass segregation and core collapse in clusters with a broad IMF was the subject of GFR04. There, the relaxation-driven evolution of clusters with a variety of structures and IMF was followed. A key finding of this work is that for a given cluster structure but for all power-law mass spectra $\frac{\text{d}N}{\text{d}M} \propto M^{-\alpha}$ with $\alpha \in [1.4, 3]$ as well as for Kroupa (Kroupa et al. 1993) and Miller & Scalo (1979) IMFs, the core-collapse time $t_{\text{cc}}$, expressed in units of the initial half-mass or central relaxation time $t_{\text{rel}}(0)$, see GFR04) only depends on the ratio of the maximum to the average stellar mass, $\mu = M_{\ast \text{max}} / M_*$. Furthermore, for $\mu > 50$, a regime reached by all realistic IMF, the dependence flattens to a constant $t_{\text{cc}} / t_{\text{rel}}(0) \simeq 0.07 - 0.08$ for Plummer models. Even more interestingly, this value, when expressed in units of $t_{\text{cc}}(0)$ appears to be independent of the cluster’s structure$^1$, $t_{\text{cc}} / t_{\text{cc}}(0) \simeq 0.15$. GFR04 also found that, in contrast to core collapse in single-mass clusters, the mass of the contracting core, instead of decreasing to zero, reaches a finite value, representing in all cases a fraction $1.5 - 3 \times 10^{-3}$ of the total cluster mass.

These findings led us to the following two simple expectations:

(i) If the dynamical role of binaries can be neglected (see discussion in Appendix B), any cluster where the core-collapse time, $t_{\text{cc}} \simeq 0.15 t_{\text{cc}}(0)$ (for realistically broad IMF) is shorter than the stellar evolution time of the most massive stars present, $t_* \simeq 3 \text{Myr}$ (if the mass function extends to $\sim 100 M_\odot$) will enter a phase of rapid collisions between the massive stars that drive the core collapse. If the central velocity dispersion at that time is not in excess of $\sim 1000 \text{km s}^{-1}$, collisions are not too disruptive and can lead to growth by mergers (Lai et al. 1993; Freitag & Benz 2002). Since the most massive object will have the largest cross section for further collisions, it is expected to grow in a runaway fashion, i.e., much faster than any other star (see the simple mathematical model in Section 2.5).

(ii) When a runaway occurs, the final mass attained by the central VMS cannot exceed about $3 M_\odot$ (if the mass function extends to $\sim 100 M_\odot$), i.e., with a power-law mass spectrum $\propto M^{-\alpha}$ with $\alpha \geq 3$. We do not find support in our simulations for prediction (ii) but likely because our simulation method may not be able to predict correctly the final mass attained at the end of the runaway.

In paper II, we put these expectations to the test through cluster simulations that include stellar collisions. We shall see that, by and large, our results confirm point (i). For large cluster masses ($\gtrsim 10^5 M_\odot$), cluster evolution may be driven, from the beginning, by collisions, thus accelerating collapse so that runaway may happen at lower densities than predicted by (ii). We do not find support in our simulations for prediction (ii) but likely because our simulation method may not be able to predict correctly the final mass attained at the end of the runaway.

An obvious and legitimate question is whether the condition $t_{\text{cc}} = t_* = 3 \text{Myr}$ is ever fulfilled in real clusters. This is illustrated in Fig. 1 where we show this condition for a variety of King models in a plane representing the (initial) mass $M_\odot(0)$ and half-mass radius $R_\odot(0)$ of the cluster. We

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1 Provided one can define a non-zero $t_{\text{cc}}(0)$, which is not the case for $\gamma$-models with $\gamma < 2$ (Dehnen 1993; Tremaine et al. 1994).
Figure 1. Conditions for rapid core collapse. This diagram shows which cluster masses, radii and initial concentrations will lead to core collapse in less than 3 Myr, i.e., before the most massive stars evolve off the MS (a necessary condition for collisional runaway to occur). We consider clusters that have initially the structure of King models, with various concentrations, parametrised by the dimensionless central potential, $W_0$. The IMF is assumed to be Salpeter between 0.2 and 120 $M_\odot$ ($\langle M_* \rangle \approx 0.69 M_\odot$). In GFR04 we have showed that, for this or other similar IMF, core collapse occurs in 0.15 $t_{cc}(0)$, independent of $W_0$. Each solid line corresponds to the condition $t_{cc} = 3$ Myr for a given value of $W_0$ (see labels on the right side of the frame). Below this line, the core collapse time is shorter, above, it is longer. The dots on the lines correspond to models with 10,000 stars in their (initial) core. For this IMF, only a fraction $\sim 4 \times 10^{-4}$ of the stars are more massive than 50 $M_\odot$, corresponding of $\sim 4$ such stars in the core. The arrows show how much of a decrease in the total mass $M_{cl}$ or half-mass radius $R_h$ leads to a shortening of $t_{cc}$ by a factor of 2.

We also show the position in the $(M_{cl}, R_h)$ plane of a variety of observed clusters. The circles (in red in the on-line color version) are the Milky Way globular clusters from the compilation by Harris (1996, updated on-line at http://physun.physics.mcmaster.ca/Globular.html); the (magenta) pentagons are LMC clusters [Mackey & Gilmour 2003]. Stars (in orange) represent populous young clusters, “super star clusters” and the cluster G1 of M 31. Data for the Arches and Quintuplet clusters are taken from Figer (2004), for NGC 1705-1 and NGC 1569-A from Ho & Filippenko (1996) and for MGG-9 and MGG-11 (in M 82) from McCrady et al. (2003). The shaded ellipse (in green) is the region in which the nuclei of dwarf ellipticals and bulgeless spiral galaxies are observed [Geha et al. 2002, Walcher et al. 2003]. For MW globular clusters, an age of 10 Gyr was assumed and the total mass was increased to correct for the decrease of the stellar average mass due to evolution of massive stars (but no account has been made of tidal stripping of stars).
have assumed a $0.2 - 120 \, M_\odot$ Salpeter IMF ($\langle m \rangle \simeq 0.69 \, M_\odot$, $\mu \simeq 174$). We chose $M_c(0)$ and $R_c(0)$ as parameters because their present-day values are more easily accessible to observation than, say, the central density and because they probably vary less than other quantities during cluster evolution. On the other hand, these quantities do not by themselves determine $t_{cc}(0)$ and, hence, $t_{cc}$; one needs to know how concentrated the cluster was initially (as measured, for instance, by the ratio of core radius to half-mass radius), a piece of information very poorly constrained by observations of evolved clusters. For the sake of simplicity, we restrict our discussion to the King family of cluster models, for which increasing $W_0$ corresponds to increasing concentration. We note that, although most numerical simulations so far have been done with moderate initial concentration, $W_0 \lesssim 6$, observations of young clusters suggest they may initially have very small cores, corresponding to $W_0 \gtrsim 8$ (Campbell et al. 1992; Moffat et al. 1994, but see McCray et al. 2003 concerning the core radius of R136). For initial $W_0$ values ranging from 1 to 12, we have plotted the line indicating clusters with $t_{cc} = 0.15 \, t_{cc}(0) = t_\pi = 3 \, $Myr. Below the line, a cluster with that $W_0$ would have shorter $t_{cc}$. Points of various shapes denote observed clusters (young or old; see caption). We see that, even for a relatively moderate $W_0 = 7$, a few clusters inhabit the region of parameter space for which one expects fast core collapse, possibly leading to collisional runaway.

There is an important caveat to be made, concerning the interpretation of the results of GFR04 and Fig. 1. For a $0.2 - 120 \, M_\odot$ Salpeter IMF, only a fraction $\sim 4 \times 10^{-4}$ of the stars are more massive than $50 \, M_\odot$. To have $10^4$ stars with $\sim 4$ such massive stars within the core of a $W_0 = 3$ or 8 cluster, the total number of stars must be $4.2 \times 10^4$ or $1.9 \times 10^5$, respectively. These values are indicated, for the various $W_0$, as black dots on the corresponding curves of Fig. 1. Actually, our result that, for sufficiently large number of particles, the core collapse occurs on a given fraction (0.15) of the initial central relaxation time $t_{cc}(0)$ does not necessarily imply that only massive stars initially in the core (where the relaxation time is $\approx t_{cc}(0)$) are responsible for the process. If it were the case, $t_{cc}$ would be of order of the dynamical friction timescale in the core, $t_{df,c} \approx \mu^{-1} t_{cc}(0) \lesssim 0.02 \, t_{cc}(0)$ which is much shorter than the value we find. This indicates that massive stars initially outside the core have time to reach the centre.

2.3 Previous works on collisional runaway

Colgate (1967) was the first to discuss the possibility of forming stars much more massive than initially present in a dense cluster through a sequence of stellar collisions. He suggested this mechanism as a way to create a large population of massive stars in a galactic nucleus to explain the quasar luminosities through enhanced super-nova rates. He pointed out that, provided the collisions always result in mergers with little mass loss, a runaway situation should ensue, with one star growing to a very large mass in a short time but also estimated that its growth would actually terminates at $\sim 50 \, M_\odot$ because, assuming a $R_\ast \propto M_\ast$ relation, the runaway object would then become too diffuse to stop $1 \, M_\odot$ impactors ("transparency" problem). A more realistic mass-radius relation for MS stars more massive than $\sim 30 \, M_\odot$ (see Eq. (2) below) actually corresponds to nearly constant projected mass density $M_\ast/R_\ast^2$ so that this argument does not apply if the growing star has time to contract back to normal MS structure between collisions.

Following Colgate (1967), one may gain some insight to the runaway mechanism by considering an idealised situation in which one star of mass $M(t)$ and radius $R(t)$ grows by merging (without mass loss) with stars of mass and radius $m$, $r$, any. One assumes $M \gg m$ (and $R \gg r$) and a constant density $n$ of light stars, with a (3-D) velocity dispersion $\sigma_3$. Then, using equation (4), the growth rate of the massive star reads

$$\frac{dM}{dt} = \frac{m}{t_{coll}} \simeq mn \sigma_3 \pi (r + R)^2 \left(1 + \frac{2G(m + M)}{(r + R)\sigma_3^2}\right) \approx 2\pi G \sigma_3^{-1} m n M R = \frac{M_0}{t_0} \left(\frac{M}{M_0}\right)^{1+\beta},$$

with $t_0^{-1} = 2\pi G \sigma_3^{-1} m n R_0$.

This holds for strong gravitational focusing and a power-law mass-radius relation, $R \approx R_0 (M/M_0)^{1+\beta}$. The solution of this differential equation, for $M(t = 0) = M_0$ is

$$M(t) = \frac{M_0}{t_0} \left(\frac{m}{M_0} \left(1 - t/t_{div}\right)\right)^{1/\beta} \text{ with } t_{div} = \frac{t_0}{\beta}.$$

Hence, in this toy model, $M$ becomes formally infinite after a finite time $t_{div}$, if the exponent $\beta$ is positive. More detailed analysis of the evolution of the whole distribution of stellar masses through use of the so-called "coagulation equation" also leads to the condition $\beta > 0$ for runaway to be possible (Lec 1994; 2003; Malyskhin & Goodman 2001). This kind of approach, ignoring stellar dynamical effects as it does, can only serve as a preliminary guide. A rough estimate of $t_{div}$ in a static cluster core would be

$$t_{div} = \frac{\sigma_3}{2\pi \beta G n m R_0} \approx \beta^{-1} t_{dyn} \frac{R_\ast}{R_0}$$

where $t_{dy}$ is the core dynamical time. One sees that this is a very long timescale because $R_\ast/R_0$ is typically (much) larger than $10^6$.

Sanders (1970) investigated the possibility of runaway collisions in dense galactic nuclei (without a central (I)MBH) using a more refined model than Colgate’s. The evolution of a population of stars subject to collisions was followed using a “particles-in-a-box” Monte Carlo method. The outcome of the collisions (occurrence of merger, amount of mass and energy loss) was determined using a generalisation of the semi-analytical method of Spitzer & Saslaw (1964). The structure and dynamics of the cluster were not resolved. Instead, the system was treated as homogeneous within a spherical domain, its size and density being evolved by considering the amount of mass and energy lost through evaporation of stars and collisions, respectively and assuming permanent virial equilibrium. All the gas lost in collisions was recycled into stars. The velocities were picked from a Maxwellian distribution with equipartition between stars of various masses. Thanks to a shallower $M-R$ relation with $\beta = 0.7$, there was no transparency saturation to the growth of a VMS, which was followed from $0.5 \, M_\odot$ to more than $300 \, M_\odot$ in a $10^7 \, M_\odot$ nucleus. A cluster model 10 times more massive, with the
same initial velocity dispersion of $\sim 500 \text{ km s}^{-1}$, did not exhibit any sign of runaway because the velocity dispersion raised above $1000 \text{ km s}^{-1}$ and thus collisions became disruptive.

Lightman & Shapiro (1978), citing unpublished work by Fall and Lightman, exposed the conditions for the onset of a collisional runaway using a simple evaporative analytical model for the contraction of the core of a single-mass globular cluster. They found that the core must evolve to a state containing only a few hundreds to thousands of stars with a velocity dispersion of order $200 \text{ km s}^{-1}$, but such approach lacks physical ingredients such as mass segregation which is key in the evolution of more realistic systems. At any rate, an important point made in this work was that, in the collisional stage, the stars should experience mergers at such high a rate that they should have no time to recover thermal equilibrium between two collisions and could well stay bloated, hence bringing back the transparency problem. The famous paper of Begelman & Rees (1978) introduced the process of runaway collisions to a larger audience as “one of the quickest routes to the formation of a massive object in a dense stellar system”. However, although they mentioned mass-segregation as a way to increase the density of massive stars on a shorter timescale, a self-consistent picture of the evolution a cluster subject to relaxation and collisions was still missing.

Lee (1987) and Quinlan & Shapiro (1990) were first to study the role of collisional mergers in numerical models self-consistently resolving the structure and evolution of stellar clusters (without a central massive BH). They applied very similar simulation methods and assumptions to quite different systems. Using codes that solve the Fokker Planck (FP) equation, they had to keep the stellar mass function discretized into “components” and represented the cluster as a finite set of distribution functions (in energy-space), one for each stellar mass. This approach has the advantage of producing results virtually devoid of noise but imposes a very artificial treatment of stellar evolution and collisions. Stars in the same mass component have to share the exact same properties, including some average age updated as time passes and merger products are added to the components, a possible cause of rejuvenation because the authors assumed complete mixing of the stellar gas during collisions. Collisions were treated as mergers without any mass loss. The mass and orbital energy of mergers of any mass have to be cast into the predefined mass components. Another questionable aspect of FP simulations, when applied to collisional runaways, is the applicability of this formalism to mass components containing a very small number of stars (sometimes less than one). These models also included the dynamical formation of binaries through 3-body interactions and their subsequent hardening (and ejection) as a central source of energy capable of reversing core collapse and turning off collisions in clusters with a relatively low number of stars. It has since been realised that there is a high probability for collisions to occur when these 3-body binaries interact with other stars, leading to a very significant reduction of the heating they provide. Chernoff & Huang (1993), Fregene et al. (2004).

Both Lee (1987) and Quinlan & Shapiro (1990) started with clusters where all stars initially have the same mass (0.7 and 1 $M_\odot$, respectively). In the work of Lee (1987), who was studying globular cluster models, collisions are actually tidal captures assumed to lead to quick merger. He concentrated on cases for which there was no real runaway growth or significant speed-up of core collapse due to mass-segregation. In all models but one, the core collapse was reversed by heating due to 3-body binaries or by mass-loss due to stellar evolution of mergers. As stars more massive than 22.4 $M_\odot$ were not allowed, the simulation had to be stopped for the only case in which conditions for the runaway were met. Lee (1987) suggested that a “typical” proto-galactic nucleus, with $10^6$ stars and a half-mass radius of $\sim 0.4 \text{ pc}$ would be subject to the “merger instability”. The goal of Quinlan & Shapiro (1990) was explicitly to look for the onset of runaway collisions as a way to create a very massive star and, eventually, a seed IMBH that could grow into a massive black hole (MBH, $M_{\text{BH}} \gtrsim 10^5 M_\odot$) at the centre of a galactic nucleus, as suggested by Begelman & Rees (1978). The results of these simulations suggested that runaway collisions would occur provided that the half-mass relaxation time is shorter than $\sim 10^8 \text{ yr}$ (to beat stellar evolution) and $N_c \gtrsim 3 \times 10^6$ (to avoid binary heating). The authors stressed that, as a result of mass segregation, the rise in the central velocity during collapse is only moderate and collisions do not become disruptive. Although not very realistic, these early studies made plausible the idea that successive collisions and mergers of main-sequence stars could lead to the formation of a $\sim 10^8 - 10^9 M_\odot$ object.

Through direct $N$-body simulations of clusters containing 2000 to 65,000 stars, Portegies Zwart et al. (1999a) and Portegies Zwart & McMillan (2002) showed that, in such low-$N_c$ systems, dynamically formed binaries, far from preventing collisions (by heating the cluster and reversing collapse), actually enhance them by increasing the effective cross section. In these small systems, once the few massive stars have segregated to the centre, one of them will repeatedly form a binary with another star and later collide with its companion when an interaction with a third star increases the binary’s eccentricity or induces a chaotic “resonant” interaction. The growth of this star is ultimately stopped by stellar evolution, or by the dissolution of the cluster in the tidal field of the parent galaxy. Given the small number of
stars in these simulations, the maximum mass of the collision product is only \( \sim 200 M_\odot \) when mass loss from stellar winds is negligible.

More recently, Portegies Zwart et al. (2004) have improved on these early N-body simulations, considering models for two young clusters in the galaxy M82: MGG-9 and MGG-11 (McCrady et al. 2003). Most calculations for MGG-11 were performed with 131,072 particles (“128k”), assuming a Salpeter IMF ranging from 1 to 100 \( M_\odot \); those for MGG-9 used the same IMF and about 4 times more particles, in agreement with a higher estimated mass. For two simulations of MGG-11, the record-breaking number of 585,000 particles was used in order to extend the IMF down to a more realistic 0.2 \( M_\odot \) (Salpeter) and 0.1 \( M_\odot \) (Kroupa). The initial conditions used were King models with dimensionless central potential ranging from \( W_0 = 3 \) to \( W_0 = 15 \). The authors found runaway growth of a VMS in all highly concentrated clusters (\( W_0 > 9 \)) with a half-mass dynamical friction timescale for 100 \( M_\odot \) stars shorter than 4 Myr. The mass reached was at least 800 \( M_\odot \) and to 2700 \( M_\odot \), depending in the \( M-R \) relation. Incidentally they noted that no reasonable model for MGG-9 complies with these conditions but that MGG-11 may be in the right domain and suggested this may explain why an ultra-luminous X-ray source is (possibly) associated with the latter but not with MGG-9.

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3 SIMULATION METHODS AND PHYSICAL INGREDIENTS

In the present work, we use a set of stellar dynamical simulations for collisional clusters to establish the conditions and manner under which a runaway occurs. Our basic numerical tool is a Monte Carlo code similar to the one used for GFR04 but developed independently and presenting many differences in its structure. We describe this code briefly in Section 3.1. The reason for using this different code here is that it was originally written to study high-density galactic nuclei including the effects of stellar collisions. In Section 3.2 we explain our treatment of collisions and discuss various relevant aspects of their physics.

3.1 The Monte Carlo code for cluster dynamics

In the past few years, a new Monte Carlo (MC) code, ME(SSY)**2 (for “Monte Carlo Experiments with Spherically SYmmetric Stellar SYstems”) has been developed to follow the long term evolution of dense clusters, with an emphasis on galactic nuclei (Freitag 2001, Freitag & Benz 2001, 2002). This code is based on the scheme first proposed by Henon (1973) to simulate globular clusters but, in addition to relaxation, it also includes collisions, stellar evolution, and, optionally, tidal disruptions and captures of stars by a central MBH through emission of gravitational waves.

The MC technique assumes that the cluster is spherically symmetric and represents it as a set of particles, each of which may be considered as a homogeneous spherical shell of stars sharing the same orbital and stellar properties. Unlike with the MC code used in GFR04, in the present implementation, the number of particles may be lower than the number of stars in the simulated cluster (but the number of stars per particle has to be the same for each particle). Another important assumption is that the system is always in dynamical equilibrium so that orbital timescales need not be resolved and the natural time step is a fraction of the relaxation (or collision) time. Instead of being determined by integration of its orbit, the position of a particle (i.e., the radius \( R \) of the shell) is picked up at random, with a \( R \) probability density that reflects the time spent at \( R \): \( \rho(R) \propto \frac{dP}{dR} \propto \frac{1}{V_j(R)} \) where \( V_j \) is the radial velocity. Unlike the code of GFR04, our scheme adopts time steps that are some small fraction \( f \) of the local relaxation (or collision) time: \( \delta t(R) \approx f \min(T_{rel}, T_{coll}) \) with \( f \) of order or smaller than 0.05. Consequently the central parts of the cluster, where evolution is faster, are updated much more frequently than the outer parts. At each step, a pair of neighbouring particles is selected randomly with probability \( P_{sel} \propto 1/\delta t(R) \). This ensures that a particle stays for an average time \( \delta t(R) \) at \( R \) before being updated. Because particles are evolved one pair at a time and to ensure perfect energy conservation, the (spherical) potential produced by the collection of particles is represented by a binary tree structure allowing both determination of its value at any given \( R \) and its update after modification of the position or mass of a particle in \( C(\ln N_\rho) \) operations, where \( N_\rho \) is the particle number.

The relaxation is treated as a diffusion process, with the classical Chandrasekhar theory (Chandrasekhar 1961, Binney & Tremaine 1987), similarly to what is done in the code used in GFR04. This treatment shares many assumptions with the methods that are based on integrating the FP equation directly. Unlike those methods, ours is based on particles and hence allows us to incorporate further physics more naturally, e.g., a continuous mass spectrum and the inclusion of an anisotropic velocity distribution. The most important addition for the purposes of this paper is stellar collisions. Unlike diffusive relaxation, collisions cannot be treated as a continuous process but are individual events, each of which may importantly affect the orbit, the mass or the mere existence of a particle. When a pair is selected, one computes the collision probability between a star of the first particle and a star of the second,

\[ P_{coll} = S_{coll} V_{rel} n \delta t. \]

\( S_{coll} \) is given by equation 11 and is connected to the maximum impact parameter for collision, \( S_{coll} = \pi b_{max}^2 \). Note

\( \delta t \) is the time step at which events are considered to occur. It is chosen to be small enough to avoid significant changes in the orbit or stellar properties of the particles. The time step is adjusted according to the relaxation time of the system, which is estimated using the classical Chandrasekhar theory (Chandrasekhar 1961, Binney & Tremaine 1987). The relaxation time is defined as the time it takes for a particle to relax to a certain fraction of its initial position. This fraction is typically chosen to be 1/\( e \), where \( e \) is the energy of the particle. The relaxation time is important because it determines the time step at which a particle is considered to be relaxed. A larger relaxation time implies a smaller time step, which in turn leads to a slower relaxation process, but also to a more accurate representation of the relaxation.

3.2 Collisional dynamics

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that the total local stellar density \( n \) enters the relation, not \( n_1 \) or \( n_2 \) \cite{Freitag&2002}. A random number is picked from the interval \([0, 1]\) with uniform probability. If it is smaller than \( P_{\text{coll}} \), a collision between the two particles has to be simulated\(^4\). The impact parameter is determined by \( b = b_{\text{max}} \sqrt{X} \), where \( X \) is another random number from the interval \([0, 1]\). The other parameters specifying the collision, i.e., \( M_1, M_2 \) and \( V_{\text{rel}}^\infty \) are already known from the particles' properties. The method for determining the outcome of the collisions is explained in the next section.

### 3.2 Stellar collisions and stellar evolution

In this section, we explain some important aspects of the "stellar micro-physics" of great importance for the runaway scenario in more detail: the outcome of collisions, stellar evolution and the interplay between them. We explain how we deal with these questions in our code and what are the associated uncertainties.

#### 3.2.1 General considerations about collisions

Consider a collision between stars of masses \( M_1, M_2 \) and radii \( R_1, R_2 \). At large separation, their relative velocity is \( V_{\text{rel}}^\infty \) and impact parameter \( b \). If they were point masses, their trajectories would be hyperbolas with separation and velocity at periastron,

\[
d_{\text{min}} = \frac{1}{x + \sqrt{1 + x^2}}, \tag{10}
\]

\[
V_{\text{max}} = \frac{V_{\text{rel}}^\infty}{\sqrt{1 + x^2}}, \tag{11}
\]

with \( x = \left( \frac{V_{\text{rel}}^\infty}{V_{\text{rel}}} \right)^2 \left( \frac{R_1 + R_2}{2b} \right) \).

If one neglects tidal effects until the stars touch, the relative velocity at contact is \( V_{\text{coll}} = \sqrt{V_{\text{rel}}^\infty^2 + V_{\text{rel}}^2} \). Because of gravitational focusing, \( d_{\text{min}} \) is a more useful parameter than \( b \) to describe how central a collision is.

For \( V_{\text{rel}}^\infty \ll V_{\text{rel}} \), gravitational focusing is important, \( S_{\text{coll}} \simeq 2\pi(R_1 + R_2)G(M_1 M_2)/(V_{\text{rel}}^\infty)^2 \) and \( dP/d(d_{\text{min}}) = \text{const.} \) and most collisions result in merger with little mass loss, \( \delta M/M < 0.1 \) \cite{Benz&1987, 1992Lai, 2004Lombardi, 2002Silks, 2002Freitag, 2003Freitag}. When \( V_{\text{rel}}^\infty \ll V_{\text{rel}} \), a regime probably only reached in galactic nuclei in the vicinity of a MBH, the cross section is geometrical, \( S_{\text{coll}} \simeq \pi (R_1 + R_2)^2 \), most encounters have relatively large \( d_{\text{min}} \) \( (dP/d(d_{\text{min}}) \propto d_{\text{min}}) \) and are "fly-bys", i.e., both stars survive and remain unbound. Only nearly head-on collisions are highly disruptive and may lead to destruction of the smaller star or both \cite{Benz&1987, 1992Lai, 2002Freitag, 2003Freitag}.

At low relative velocities, two stars can become bound to each other, i.e., form a binary, through dissipation of orbital energy in tides near periastron, even if \( d_{\text{min}} > R_1 + R_2 \) \cite{Fabian&1973}. For velocities typical of globular clusters \((10 - 50 \text{ km s}^{-1})\), such tidal binaries form up to \( d_{\text{min}}/(R_1 + R_2) \simeq 2 - 3 \) \cite{PortegiesZwart&1992, Kim&Lee}. Their evolution is still a subject of debate and may be very complex, but given the fact that they form with very small pericentre separation and that the stars should swell due to conversion of tidally excited oscillations into heat \cite{McMillanetal, 1998Kumar}, it seems likely that they will promptly merge. Consequently, we could try to account for tidal captures by increasing the effective stellar radii for collisions at low \( V_{\text{rel}}^\infty \) \cite{Lee}, but for simplicity and to stay on the conservative side when testing the runaway scenario, we decided to neglect this effect and only account for genuine collisions with \( d_{\text{min}} < R_1 + R_2 \).

#### 3.2.2 Mass–radius relation

One may expect the \( M–R \) relation to play an important role in determining collision rates through its influence on the cross section. Fig. 2 shows various \( M–R \) relations from the literature. For stars between \( \sim 0.8 \) and \( \sim 150 M_\odot \), we plot the radius at the beginning, middle and end of the MS \cite{BSS}. For \( M > 100 M_\odot \), it seems likely that they will promptly merge. Consequently, we could try to account for tidal captures by increasing the effective stellar radii for collisions at low \( V_{\text{rel}}^\infty \) \cite{Lee}, but for simplicity and to stay on the conservative side when testing the runaway scenario, we decided to neglect this effect and only account for genuine collisions with \( d_{\text{min}} < R_1 + R_2 \).

\[
R_s = 1.6 R_\odot \left( \frac{M_s}{M_\odot} \right)^{0.47}. \tag{12}
\]

Even though this expression is for stars more massive than \( 10^3 M_\odot \), it appears to match nicely with the \( M–R \) relation at \( M_s/M_\odot < 120 \), as shown in Fig. 2.

\(^4\) If each particle represents \( N_{\text{rel}} \) stars, \( N_{\text{rel}} \) identical collisions are assumed to take place. In this way, one may apply the result of the collision (new velocities, stellar masses,\ldots) to the particles themselves. If the collision results in a merger, only one particle is retained in the simulation. If both stars are completely disrupted, both particles are removed.

\(^5\) Data available at http://obswww.unige.ch/~moussavi/evol/stev_database.html
Focusing on low metallicity is reasonable: only if the metallicity is sufficiently low, is it clear that a VMS will be stable while on the MS (Baraffe et al. 2001), that it will experience little evolutionary mass loss, and that it will collapse into a BH as a whole (Fryer & Kalogera 2001).

We do not account for the increase of the stellar radius during the MS evolution and use the size of the star at half its MS lifetime, or at an MS age of 5 Gyr for stars with MS lifetime exceeding 10 Gyr. Although it would be more consistent to let the stellar radii evolve, the extra complication involved in such an improvement is not justified in face of the larger uncertainties introduced by other necessary simplifications, in particular concerning the size and evolution of collision products (see below). By the nature of the scenario studied here, we follow only the first few million years of the cluster evolution. Hence, low-mass stars should be given a size smaller than their “half-MS” radius and closer to the ZAMS value. One can see in Fig. 4 that this only leads to a slight overestimate of the collision cross sections which, being dominated by gravitational focusing, are proportional to the stellar radii. More important is the case of the most massive stars, $M_\ast > 50 M_\odot$, say, which are expected to dominate the collisional process and may significantly evolve over the period of time considered here. Their size increases significantly during the late MS. Neglecting this may lead to an underestimate of their collision rates. A higher, possibly more realistic, size contrast between light and massive stars is likely to facilitate the runaway mechanism. However, by experimenting with large changes in the high-mass $M-R$ relation, ranging from $R_\ast \propto const$ to $R_\ast \propto M_\ast$, we have checked that this is of relatively little importance.

A more questionable simplification is to consider that all stars are initially on the MS. The runaway process has to start before the most massive stars in the IMF ($M_\ast \approx 100 M_\odot$) turn into compact remnants. This gives us at most 3 Myr (see Fig. 4). This is to be compared with the duration of the pre-MS phase. After it has stopped accreting (and hence reached the “birth-line”), a pre-MS star less massive than $\sim 6 - 8 M_\odot$ is several times larger than on the ZAMS. The time required for contraction onto the ZAMS strongly increases with decreasing mass; it is shorter than 3 Myr only for stars more massive than $2 - 3 M_\odot$ (Palla 2002). Consequently, in a young cluster, assuming for simplicity that accretion ceased for all stars at the same instant, low-mass stars are more extended and less dense than high-mass objects. During this phase their collision rate is higher than in the MS phase; however their encounters with more massive but more compact stars may result in the tidal disruption of these low-mass objects rather than a “clean” merger (Zinnecker & Bate 2002). We leave these complications out of our present study. We consider the models presented here as the simplest possible ones that incorporate all the key physical ingredients needed to investigate the runaway scenario.

Figure 2. $M-R$ relations for MS stars from various authors. The $M-R$ for low masses ($0.01-1 M_\odot$, in cyan in the colour version) is from Fig. 3 of Chabrier & Baraffe (2001); $Z = Z_\odot = 0.02$, age of 5 x $10^9$ years) For intermediate masses, we plot radii from the Geneva stellar evolution group for three metallicities: $Z = 0.0004, 0.001, 0.02$ (in blue, magenta and red in the online color version of this figure). Schaller et al. 1992 (Charbonnel et al. 1996, Lejeune & Schaerer 2001). Short-dashed lines correspond to the ZAMS, solid lines to the radius when the star has lived half of its MS lifetime (or $5 \times 10^9$ years for low-mass stars) and long-dashed lines to the end of the MS phase or at an age of $10^{10}$ years at low masses. The maximum mass considered in these series of models is 100 to 150 $M_\odot$. All other $M-R$ relations plotted here are for the ZAMS (Bond et al. 1984; Chabrier & Baraffe 1997; Ishii et al. 1999; Baraffe et al. 2001; Schaerer 2001). Star with higher metallicity have larger radii. At $Z_\odot$, stars more massive than 100 $M_\odot$ have a huge diffuse envelope, due to metal opacity. The relation from Bond et al. 1984 ($R_\ast \propto 1.6 R_\odot (M_\ast/M_\odot)^{0.47}$) neglects this effect. It is established for $M_\ast \gtrsim 10^4 M_\odot$ but matches nearly perfectly the models for $M_\ast \gtrsim 100 M_\odot$.
metallicities are plotted. Data kindly provided by K. Belczynski.

For masses larger than 120 M⊙, extrapolation (linear in the quantities plotted) is used; the fractional amount of He produced is not allowed to exceed 0.56 (Bond et al. 1984). More sophistication could be included in further works, if deemed necessary.

3.2.3 Collisional rejuvenation

The stellar evolution of collision products has only been explored for the case of low-velocity mergers, relevant to globular clusters (Sills et al. 1997, 2001). Such encounters are only mildly supersonic and entropy is nearly conserved. Hence, the structure of the merger can be established by sorting the mass elements from the parent stars according to their entropy (Lombardi et al. 1997, 2002). The main uncertainty about the evolution of these objects is the mechanism, if any, responsible for decreasing the amount of angular momentum as the star relaxes to thermal equilibrium after the collision. For high velocity collisions, significant dissipation occurs and entropy sorting is questionable, not to mention that, in most cases, both stars survive the collision unbound to each other (“fly-bys”).

In view of these difficulties, we used a very simple procedure to set the stellar evolution of mergers, called minimal rejuvenation. We assume that, during a coalescence, the helium cores of both parent stars merge together, while the hydrogen envelopes combine to form the new envelope; no hydrogen is brought to the core. Furthermore, to assign an effective age to the merger, we assume that the mass of the helium core grows linearly with time during the MS and resort to stellar evolution models to provide the relation between the stellar mass and the helium core mass at the terminal age MS. This formalism is also applied to fly-bys during which part of the stellar envelopes is removed. In Fig. 3 we plot the MS lifetime and the total mass of helium produced during the MS phase as functions of the stellar mass (data provided by K. Belczynski; see Hurley et al. 2000). For simplicity and because the dependence on metallicity is weak, we use the Z = 10−4 data for all simulations. In any case, the thermal timescale is always assumed to be shorter than the average time between collisions so that the MS mass–radius relation is applied to collisions products. The validity of this hypothesis during the runaway growth a massive star through repeated mergers is questionable and is discussed in Paper II (Section 2.2 in particular).

3.2.4 Implementation and role of stellar evolution and mass loss

ME(SSY)**2 implements a very simple prescription for stellar evolution (Freitag & Benz 2002b). While a star is on the MS, its mass and radius are kept constant. The duration of the MS, TMS, is given by detailed stellar models (Hurley et al. 2000). It is plotted for two metallicities in panel (a) of Fig. 3. For all simulations presented here, we use the data for metallicity Z = 10−4 but it is obvious that the metallicity has little impact on TMS. On the other hand, the amount of mass loss on the MS increases strongly with Z (Vink et al. [2001]). While the possibility of runaway collisions in clusters with high metallicities, such as young populous or “super” clusters observed in our and other galaxies is certainly of great interest, it is unlikely that a high-Z VMS will form an IMBH if it is left to evolve on the MS precisely because it should experience such high mass loss. Indeed, standard prescriptions for mass-loss rates indicate that stars more massive than ~120 M⊙ shed most of their mass on the MS if they are of solar metallicity. At Z ≃ 4 × 10−4, ~120 M⊙ stars lose only ~10% of their mass but objects above ~500 M⊙ should evaporate themselves nearly completely (Lejeune & Schaerer 2001, Kudritzki 2003). Therefore, in this work, we only consider (very) low metallicity. Also recall that, in GFR04, we studied whether mass loss on the MS could decrease the binding energy of the cluster enough to reverse core collapse and found that, even for solar metallicity, this only happens in a very small domain of the parameter space, namely for clusters with a core collapse time already very close to the critical value of 3 Myr.

We assume that all mass lost by the stars is expelled from the cluster. This is probably a reasonable simplification for clusters with a relatively low escape velocity, like globular clusters but a poor one for (proto-)galactic nuclei in which a significant fraction of the gas could be retained. This approximation is discussed in Paper II.

In summary, for this work, the role of stellar evolution is only to set a clock against which core-collapse and collisions have to race. When core collapse requires more time than the MS lifetime of massive stars, it will be terminated.
Runaway collisions

3.2.5 SPH collision simulations

In the MC method, collisions are handled on an event-by-event basis, rather than through average rates and outcomes as in Fokker-Planck codes. To treat collisions with as much realism as possible, we can use the results of 3-D hydro simulations performed with the Smoothed Particle Hydrodynamics (SPH) method (Benz 1990; Monaghan 1992; Rasio & Lombard 1999). A set of some 14000 SPH simulations of collisions between MS stars was performed by Freitag & Benz (2002). The original goal was to include the effects of collisions in stellar dynamical models of galactic nuclei (Freitag & Benz 2002a). This focus determined the parameter range covered in this study: stellar masses from $M_{\text{min}} = 0.1M_\odot$ to $M_{\text{max}} = 74.3M_\odot$, relative velocities in the range $V_{\infty}^{\text{rel}}/V_c \simeq 0.03-0.3$ and impact parameter corresponding to $d_{\text{min}}/(R_1 + R_2) = 0.0-0.9$. Because it proved intractable to summarise the outcome of these simulations (stellar masses, orbital energy and deflection angle) through a set of fitting formulae, we implemented a scheme to interpolate from SPH results in the four-dimensional parameter space ($M_1$, $M_2$, $V_{\text{rel}}$, $d_{\text{min}}$). While this method proves adequate for galactic nuclei simulation in which velocities are high and it is therefore highly unlikely that mergers will lead to formation of a star more massive than $M_{\text{max}}$ (Freitag 2000; Freitag et al. 2004), it is not suited to the problem of runaway growth.

In most cluster simulations reported here and in Paper II, we simply assume that all collisions result in merger with no mass-loss. We will see that this is a satisfying approximation because, in all but the most extreme cases, the central velocity dispersion is and remains relatively small. To go one step beyond this zeroth order "sticky sphere" approximation and test its validity, for a subset of runs, we use the SPH results to allow for non-merging collisions and collisional mass loss in the way described below. For grazing collisions not resulting in a merger, we do not account for the reduction of the relative velocity or non-Keplerian deflection angle.

Complete disruption of both stars is extremely unlikely. Not only does it require a relative velocity of a few $V_c$ but also a nearly head-on geometry. Hence, in this work, we consider only two possible outcomes: merger and "fly-by". Most low-velocity ($V_{\text{rel}} < 0.1V_c$, say) encounters result in mergers, but, in our SPH work, we have not studied those in much detail. Such collisions require a large amount of computation time because, unless the collision is nearly head-on, the pair does not merge immediately after first contact but forms a bound binary which goes through many successive periastron passages until final coalescence. In most cases, the hydrodynamical simulation was stopped before a merged star had formed. But it is very likely that any binary formed through a contact collision will eventually merge because the stars swell as their envelopes are shocked so that each pericentre passage is more dissipative. Hence, we can use our SPH results to establish the condition for merger. We have found the following parameterisation to correctly predict the maximum merger impact parameter for all but a few SPH simulations:

$$
\lambda_{\text{merg}} = c_0 + c_1 q + c_2 M_2 + c_3 l_v + c_4 l_v q + c_5 l_v M_2 + c_6 l_v^2 + c_7 l_v^2 q + c_8 l_v^2 M_2,
$$

with

$$
\lambda = \frac{d_{\text{min}}}{R_1^{(b)} + R_2^{(b)}} \quad q = M_1/M_2 \lesssim 1
$$

and

$$
l_v = \log_{10} \frac{V_{\infty}^{(\infty)}}{V_v^{(b)}}, \quad V_v^{(b)} = \sqrt{\frac{2G(M_1 + M_2)}{R_1^{(b)} + R_2^{(b)}}}.
$$

Index 1 indicates the less massive star; $R_1^{(b)}$ are the radii enclosing half the mass of each star. The numerical coefficient are $(c_0, c_1, \ldots, c_8) = (0.525, -0.107, 6.84 \times 10^{-3}, -2.03, 0.525, 6.16 \times 10^{-4}, 0.132, 0.526, -4.89 \times 10^{-3})$.

To determine the final stellar masses, we make use of the SPH mass loss results in the following way. First when $M_2 > M_{\text{max}} = 74.3M_\odot$, we try to rescale $M_1$ and $M_2$ by some factor $\eta$ bringing $\eta M_2$ to 0.95 $M_{\text{max}}$ while keeping $\eta M_1 > M_{\text{min}}$. This would conserve the mass ratio $q$ but is not possible when $q < q_{\text{min}} = M_2/M_{\text{max}} \simeq 10^{-3}$. Such cases are relatively unimportant because, in typical cases, the VMS grows mostly by merging with $50-120M_\odot$ stars so that $q > 0.025$ even for $M_{\text{VMS}} = 2000M_\odot$. We deal with them by rescaling $M_1$ to 1.05 $M_{\text{min}}$, independently of $M_2$. From the SPH results, we interpolate the fractional mass loss $\delta_M = (\delta M_1 + \delta M_2)/(M_1 + M_2)$ for the $(M_1, M_2, V_{\text{rel}}, d_{\text{min}})$
parameters of the collision (with $M_1$, $M_2$ possibly rescaled), as explained in Freitag & Benz (2005). Inspection of SPH results for fly-bys shows that only in very exceptional cases does the mass of any of the two stars increase. In fact, to a good approximation, we find the mass ratio to be conserved. We thus use this assumption to determine the individual masses from $\delta M$.

4 TEST SIMULATIONS

Before embarking on the large-scale numerical exploration of the runaway scenario to which Paper II is devoted, we first demonstrate that our numerical tools are up to this task. Here we check that we can reliably model the following two key aspects of the scenario: (1) Fast segregation-driven core collapse in clusters with a broad mass function and (2) Influence of collisions in cluster evolution and onset of collisional runaway.

4.1 Important quantities and units

Keeping with the tradition, when not stated otherwise, we are using the $N$-body unit system (Hénon 1971) defined by $G = 1$, $M_{\mathrm{cl}}(0) = 1$ (initial total cluster mass) and $U_{\mathrm{cl}}(0) = -1/2$ (initial cluster potential energy). As time unit, we prefer the “Fokker-Planck” time $T_{\mathrm{FP}}$ to the $N$-body unit $T_{\mathrm{NB}}$ because the former is a relaxation time while the latter is a dynamical time; they are related to each other by $T_{\mathrm{FP}} = N_*/\ln(\gamma_c N_*) T_{\mathrm{NB}}$.

All test computations performed here are based on the Plummer model (Plummer 1911;Binney & Tremaine 1987) for which we recall some important quantities in Table 1.

We refer to GFR04 (Section 3 and Table 1) for more detailed explanations about units and the important physical parameters of a variety of clusters models.

4.2 Core collapse without collisions

4.2.1 Comparison with direct $N$-body simulations

In Freitag & Benz (2001), the ability of ME(SSY)**2 to follow the relaxational evolution of clusters has been successfully tested against other simulation methods for clusters with single-mass, 2-component or continuous mass functions. However, in that paper, no comparison was done with direct $N$-body results and the only continuous mass spectrum considered was a relatively narrow $0.1 - 1.5 M_\odot$ Salpeter IMF ($\mu \equiv M_{\mathrm{max}}/(M_*) \simeq 2.2$). Given the importance of core collapse in clusters with $\mu > 100$ for the runaway scenario and the dearth of published data about this situation, we decided to carry out new comparisons between ME(SSY)**2 and direct $N$-body integrations of the core-collapse evolution of clusters with various IMFs.

The $N$-body simulations reported here were done by H. B. with the collisional Aarseth $N$-body code NBODY4 (Aarseth 1999) on the GRAPE-6 boards of Tokyo University (Makino et al. 2003). Use of GRAPE-6 hardware is essential to perform $N$-body simulations with more than $10^5$ stars within a reasonable amount of computer time. Details on the NBODY4 code can be found in Aarseth (1999) and references therein and Baumgardt & Makino (2003).

We first checked the simplest situation, that of a single-mass cluster. In Fig. 4 we compare the evolution of the Lagrange radii of a Plummer model as obtained with NBODY4 (Baumgardt et al. 2003) and ME(SSY)**2. The agreement between the two methods is excellent if we set the coefficient in the Coulomb logarithm to $\gamma_c = 0.09$ when converting the $N$-body time units, natural to NBODY4, to FP units. Given the run-to-run variations of MC results (for different random sequences), this value is compatible with the one found by Giersz & Heggie (1994) in comparisons between $N$-body runs with various particle numbers ($N_p = 250 - 2000$) and later confirmed by Baumgardt (2001) ($N_p = 128 - 16384$).

| Quantity | Symbol | Value |
|----------|--------|-------|
| Plummer scale | $R_p$ | $3 \pi/16 \simeq 0.589$ |
| Core radius | $R_{\text{core}}$ | 0.417 |
| Half-mass radius | $R_{\text{h}}$ | 0.769 |
| Central density | $\rho_c$ | 1.167 |
| Central 1D velocity dispersion | $\sigma_{v,c}$ | 0.532 |
| Mass within $R_{\text{core}}$ | $M_{\text{core}}$ | 0.193 |
| Central relaxation time | $t_{\text{rc}}$ | 0.0437 |
| Half-mass relaxation time | $t_{\text{rh}}$ | 0.0930 |
Figure 5. Evolution of Lagrange radii during the core collapse of a Plummer model with a mass spectrum. The cluster has a Kroupa mass function (see text) extending from 0.1 to 10$M_\odot$. We compare the results of ME(SSY)$^2$ with $N_p = 10^6$ particles (solid lines) to those of a direct $N$-body run with $N_p = 131,072$ (dotted lines, in magenta in the on-line colour version). A value of $\gamma_c = 0.015$ was used in the Coulomb logarithm when converting $N$-body time units into FP units to get the best overall agreement.

Figure 6. Mass segregation during the core collapse of a Plummer model with a mass spectrum. The data is for the same MC and $N$-body simulations as in Fig. 5. We plot the stellar mass averaged over all particles inside spheres containing the indicated fraction of the total mass of the cluster.

Figure 7. Evolution of Lagrange radii during the core collapse of a Plummer model with a broad mass spectrum. The cluster has a Kroupa mass function (see text) extending from 0.1 to 100$M_\odot$. We compare the results of ME(SSY)$^2$ with $N_p = 1.25 \times 10^6$ particles (solid lines) to the averaged results of two direct $N$-body runs with $N_p = 131,072$ (dotted lines, in magenta in the on-line colour version). Our standard value of $\gamma_c = 0.01$ was used in the Coulomb logarithm when converting $N$-body time units into FP units.

Figure 8. Mass segregation during the core collapse of a Plummer model with a broad mass spectrum. The data is for the same MC and $N$-body simulations as in Fig. 7. See caption of Fig. 6 for explanations about the plotted quantities.
\(\gamma_c = 0.11\); it is also compatible with the one chosen by Drukier et al. (1999) to adjust their FP results to N-body simulations, \(\gamma_c = 0.10\).

After this test-run, we considered a cluster consisting of stars with masses from 0.1 to 10 \(M_\odot\). The masses are distributed according to the IMF advocated by Kroupa (2001). For our simulations, the “Kroupa IMF” corresponds to a piecewise power law: \(dN_i/dM_i \propto M_i^{-\alpha}\) with \(\alpha = 1.3\) below 0.5 \(M_\odot\) and \(\alpha = 2.3\) for higher masses. This produces a mass ratio \(\mu \approx 19.3\). Fig. 5 depicts the Lagrange radii evolution for ME(SSY)**2 and NBODY4 simulations of this model. Converting time units with \(\gamma_c = 0.015\), we observe an excellent agreement between the results of both codes until, at \(t \approx 0.043\) \(T_{FP}\), shortly before the moment of deepest collapse in the N-body run, it starts showing slower contraction of the innermost regions. This departure from the MC results is probably due to small-number effects, such as large-angle scatterings or binary formation, that naturally kick in in the shrinking core of the N-body system (computed with \(N_p = 131\,072\) but are, by nature of the MC approach, absent from the ME(SSY)**2 run. Eventually, at \(t \approx 0.047\) \(T_{FP}\), 3-body binaries reverse core collapse in the N-body simulation. Hénon (1975) explained why a smaller \(\gamma_c\) value is appropriate for systems with a mass spectrum and, from multi-mass N-body simulations with \(N_p = 250 – 1000\), Giersz & Heggie (1996) indeed found \(\gamma_c \approx 0.015 – 0.025\).

In such multi-mass clusters, core collapse is driven by mass segregation. Fig. 5 allows us to witness this process by plotting the evolution of the average mass within spheres containing various fractions of the total cluster mass. Again the MC results match those of the N-body run closely.

We now consider a mass spectrum of realistic breadth, i.e., a Kroupa IMF extending from 0.1 to 100 \(M_\odot\), yielding \(\mu \approx 157\). To reduce numerical noise without increasing \(N_p\) to an impractically high value, we realised two N-body simulations of this system with \(N_p = 131\,072\) (starting from different realisations of the initial cluster) and averaged their results. Comparison of the Lagrange radii and average stellar mass evolutions are shown in Figs 6 and 7. This time, we stuck to \(\gamma_c = 0.01\), the value we traditionally use to convert from FP time to physical time in our MC simulations of multi-mass clusters. This value turns out to yield a perfect match between the MC core-collapse time and the time of maximum contraction of the inner regions in the N-body runs. However, \(\gamma_c \approx 0.025\) would have allowed a better agreement in the early shapes of the Lagrange radii curves. At any rate, the agreement between ME(SSY)**2 and NBODY4 results is not as good as for previous cases. In particular, it appears that mass segregation is faster and stronger but more progressive in the N-body run. In the MC simulation, segregation accelerates at late times and the average stellar mass in the innermost regions (inside the 1% Lagrange radius) reaches values similar to those found in the N-body run near the moment of collapse. A better understanding of the cause of the differences between the two simulation methods in the regime of broad IMF may be reached in future investigations thanks, in particular, to N-body simulations with higher \(N_p\) and, hence, less noise and less affected by small-number effects in the central regions. For the moment, we note that the ME(SSY)**2 evolution of this broad-IMF cluster is qualitatively similar to that shown by the direct N-body integration and that good quantitative agreement is obtained for the aspects most important to the present investigation of the collisional runaway, namely the core collapse time and the degree of mass segregation reached in the central regions during late collapse. The core collapse time we obtain with ME(SSY)**2 is \(t_{cc} \approx 7.3 \times 10^{-3}\) \(T_{FP}\) = 7.9 \(\times 10^{-3}\) \(t_{FP}(0) = 0.17\) \(t_{FP}(0)\), in agreement with the value of \(t_{cc} \approx 0.15\) \(t_{FP}(0)\) found in GFR04 for systems with \(\mu \geq 50\).

As for the value of \(\gamma_c\), we decided to stay on the conservative side by keeping it at 0.01. This is probably slightly too small, hence predicting a core-collapse evolution too slow with respect to other time scales, most importantly that for stellar evolution, but the difference in relaxation time compared to, say, \(\gamma_c \approx 0.025\) is smaller than 10% for \(N_c \geq 10^6\).

### 4.2.2 Comparison with the gaseous model

The direct N-body method represents the most accurate but most computationally expensive way of simulating the secular evolution of a stellar cluster, subject to relaxation. Unfortunately, precisely because it treats gravitation in such a direct fashion, it offers no or little clean and easy way of establishing the global behaviour that a system may exhibit in the limit of very large number of stars (\(N_c \gg 10^5\)). For the moment, we note that the ME(SSY)**2 evolution of this broad-IMF cluster is qualitatively similar to that shown by the direct N-body integration and that good quantitative agreement is obtained for the aspects most important to the present investigation of the collisional runaway, namely the core collapse time and the degree of mass segregation reached in the central regions during late collapse. The core collapse time we obtain with ME(SSY)**2 is \(t_{cc} \approx 7.3 \times 10^{-3}\) \(T_{FP}\) = 7.9 \(\times 10^{-3}\) \(t_{FP}(0) = 0.17\) \(t_{FP}(0)\), in agreement with the value of \(t_{cc} \approx 0.15\) \(t_{FP}(0)\) found in GFR04 for systems with \(\mu \geq 50\).

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4.3 Runaway collisions: Comparison with Quinlan & Shapiro (1990)

In [Freitag & Benz 2002], we have already checked that ME(SSY)**2 produces the correct rate of collisions, both as a function of the distance to the centre of the cluster and of the masses of stars, for multi-mass clusters. We have also reproduced the overall results of collisional models of MBH-hosting galactic nuclei considered by Duncan & Shapiro (1983) and Murphy et al. (1991). In such environments, however, collisions occur at very high relative velocities and do not lead to runaway growth [Freitag et al. 2004].

To our knowledge, Quinlan & Shapiro (1990) were the first to study in a systematic way the process of collisional runaway accounting self-consistently for cluster dynamics and secular evolution and how collisions themselves affect it.

We have run simulations for all 6 “E” models of QS90, assuming, in most cases, sticky-sphere collisions and complete rejuvenation for each merger as these authors did. Initial conditions are high-density Plummer clusters with all stars of solar type, with various total masses and sizes. Table 1 lists the characteristics of these models. Stellar evolution is included. QS90 implemented it in a simplified way, similar to ours. However, while we can assign an individual age to each MC particle (which represents 4.2 to 36 stars, depending on the model), in the FP scheme of QS90 only a small number of discrete mass component are present (for stars of mass 2⊙ with i = 1, 2, . . . , 8), each of which represents some homogeneous population. Stellar evolution can only be treated in statistical way, with each component having an average age. Similarly, collisions can only be accounted for statistically by integrating merger rates within and between mass components and distributing the number of merger products over components in a way that conserve total mass but not necessarily individual stellar masses because not all possible merger masses are represented. Unlike QS90, we have allowed for collisions between MS stars and compact remnants by assuming complete disruption of the MS star but no change to the mass or velocity of the compact object. Certainly an oversimplification, this prescription obviously corresponds to the most unfavourable possible one.
and 2 $\odot$ than 3 $\odot$ mass bin; white dwarfs are not plotted. We used 5 negligible contribution of neutron stars is included in the first dwarfs and a similar number of neutron stars have formed. The evolution with stellar collisions. Same simulation as in Fig. 10 number of stars in various mass bins during cluster simulations for comparison with Quinlan & Shapiro (1990).

| Name         | Name in QS90 | $N_*$       | $N_p$       | $R_{NB}$ (pc) | $t_{cc}$ (FP) | $t_{cc}$ (Myr) |
|--------------|--------------|-------------|-------------|---------------|---------------|---------------|
| QS90-E4A     | E4A          | $1.8 \times 10^7$ | $5 \times 10^5$ | 0.41          | 0.134         | 141           |
| QS90-E4Ahrr  | E4A          | $1.8 \times 10^7$ | $2.1 \times 10^6$ | 0.41          | 0.129         | 136           |
| QS90-E4Asph$^{(a)}$ | E4A | $1.8 \times 10^7$ | $5 \times 10^5$ | 0.41          | 0.275         | 289           |
| QS90-E4B     | E4B          | $3.1 \times 10^7$ | $5 \times 10^5$ | 0.71          | 0.182         | 553           |
| QS90-E4B$^{(b)}$ | E4B | $3.1 \times 10^7$ | $5 \times 10^5$ | 0.71          | 0.172         | 522           |
| QS90-E4B$^{(c)}$ | E4B | $3.1 \times 10^7$ | $5 \times 10^5$ | 0.71          | 0.189         | 574           |
| QS90-E2A     | E2A          | $6.2 \times 10^6$ | $5 \times 10^5$ | 0.565         | 0.371         | 397           |
| QS90-E2B     | E2B          | $1.1 \times 10^7$ | $5 \times 10^5$ | 1.00          | $^{(d)}$      | ...           |
| QS90-E1A     | E1A          | $2.1 \times 10^6$ | $5 \times 10^5$ | 0.765         | 0.758         | 803           |
| QS90-E1B     | E1B          | $3.6 \times 10^6$ | $5 \times 10^5$ | 1.31          | 1.23          | 3680          |
| QS90-E4B$^{(a)}$ | E1B | $3.6 \times 10^6$ | $5 \times 10^5$ | 1.31          | 1.24          | 3710          |

$^{(a)}$ Collision prescriptions based on SPH data. Minimal rejuvenation.
$^{(b)}$ Other random sequence than for QS90-E4B.
$^{(c)}$ Time step four times larger than for QS90-E4B.
$^{(d)}$ Core collapse stopped by stellar evolution.
$^{(e)}$ Other random sequence than for QS90-E1B.

Figure 11. Number of stars in various mass bins during cluster evolution with stellar collisions. Same simulation as in Fig. 10 (QS90-E4A). Dotted lines are the results of QS90 (their Fig. 2a) for mass-component of mass 1, 2, 4, 8 and $\geq 16 \odot$. The curves for 1 and 2 $\odot$ are nearly indistinguishable from ours (for 1--1.4 and 2--3 $\odot$ bins). We obtain a lower number of stars more massive than 3 $\odot$ than QS90. At the end of our simulation, ~ 200 white dwarfs and a similar number of neutron stars have formed. The (negligible) contribution of neutron stars is included in the first mass bin; white dwarfs are not plotted. We used 5 $\times 10^5$ particles for our simulation so each of them represents 36 stars.

as far as collisional runaway is concerned. To have the same ratios between the time scale for relaxation and that for other processes (collisions, stellar evolution) as QS90, we adopt their $\gamma_c = 0.4$ value for this set of simulations.

In contrast to QS90 we find core collapse and collisional runaways in all cases but E2B whose core contraction is stopped by the mass loss due to stellar evolution of collision products.

Agreement for model E4A is very satisfying: in our model QS90-E4A, we find core collapse and runaway to happen at $t_{cc} \approx 140$ Myr $\approx 1.44 t_{bh}(0)$, to compare with
Runaway collisions

QS90’s value of \( \sim 130 \) Myr. Without collisions, a single-mass Plummer model would experience core collapse at \( t_{cc} \simeq 17 - 18 t_{rh}(0) \) so mergers clearly play an important role in the cluster evolution from the beginning, by dissipating energy and creating a mass spectrum, hence allowing mass segregation. For this reason, one cannot predict the occurrence of runaway in those clusters just from the relaxational value of the core-collapse time, \( t_{cc|\text{rel}} \). Also, even though all stars are initially of one solar mass, it is not sufficient to have \( t_{cc|\text{rel}} < 10 \) Gyr for collisional runaway to kick in. Paradoxically, if merger products are formed before deep collapse they may be able to stop collapse as they evolve off the MS much earlier, as happens in our case E2B (QS90-E2B). Fig. 11 shows the evolution of the Lagrange radii our model E4A. We have also redone the simulation with the more realistic prescriptions for collision outcomes based on SPH simulations (QS90-E4Asph). As can be seen in Fig. 10 when collisional mass-loss and, more importantly, fly-by, non-merging collisions are accounted for, the cluster evolution is significantly slower. This is because, in clusters with such a high velocity dispersion \( (\sigma_{v,c}(0) \simeq 231 \text{ km s}^{-1}) \), only a minority of collisions result in mergers, amounting to a reduction in the effective collision cross sections. In Fig. 11, we have plotted the evolution of the number of stars of various masses during the evolution of QS90-E4A. The evolution of the number of stars of mass 2 to 3 \( M_\odot \) in our run is nearly identical to that of the 2 \( M_\odot \) component of QS90 but we obtain significantly fewer stars more massive than 3 \( M_\odot \). This demonstrates that the rate of collision between 1 \( M_\odot \) (and hence, of formation of first-generation mergers) is the same in both simulations. Because QS90’s FP method imposes a completely different (and much less physical) way of dealing with mergers of mergers, it is not surprising no close agreement is reached on that matter. The \( M-R \) relation cannot be blamed, however, as they assume \( R_e/R_\odot = (M_e/M_\odot)^{0.55} \), which is close to our relation. The drop in the number of stars more massive than 16 \( M_\odot \) at the end of our simulation corresponds to the run-away growth of a VMS by merging between massive stars. When we stopped our run, the VMS had reached 651 \( M_\odot \). We refer to Paper II for a discussion of physical processes that may terminate the VMS growth, most of which are lacking from ME(SSY)**2. How mergers create a population of massive stars which come to dominate the central region of the cluster is further indicated by Fig. 12. Here, we follow the contribution to the central density of stars in various mass ranges. Unlike QS90, we cannot determine the density at \( R = 0 \) but have to sum the mass in some small spherical volume to estimate it. Our values are therefore lower limits. Still, the agreement for the lowest two mass bins \( (1 - 1.4 M_\odot \) and \( 2 - 3 M_\odot \)) is very satisfying, but again, we get considerably fewer higher-mass stars.

Although 3 times less massive, model E4B has the same velocity dispersion as E4A and, hence a very similar initial value of \( t_{rh}/t_{coll} \) (up to small differences in Coulomb logarithms). Therefore, one could have expected collisions to play the same role and to get the same value of \( t_{cc}/t_{rh}(0) \). However, stellar evolution introduce still another time scale in the problem and because \( t_{rh}(0) \) is longer (in years) for model E4A, more collision products have time to evolve off the MS, which delays core collapse. At \( t = 0.13 T_F \), the fractional number of white dwarfs in our E4A run is \( \sim 10^{-5} \) while it reaches \( \sim 10^{-3} \) in run E4B. In QS90’s run for this case, stellar evolution was able to stop core collapse around \( t \simeq 700 \) Myr, before any star more massive than 32 \( M_\odot \) formed. In contrast, our models produced a runaway merger after 520 to 570 Myr already. The evolution of number of stars of different masses is plotted in Fig. 13 and compared with QS90’s results (for the contraction phase).

Again, the agreement with QS90 is excellent for the \( 2 - 3 M_\odot \) mass bin but poorer for others. This time, our run produces more massive stars (after \( t \approx 300 \) Myr), which eventually lead to runaway. Clearly, because the evolution is driven by a positive feedback loop between mass segregation and collisions and because only stellar evolution of merger products themselves can revert it, differences in the treatment of collisions may have very important effects.

For their other “E” models, QS90 only indicate the end state in a very succinct way, giving no details about the evolution. For model E2A, we find runaway at \( t \approx 400 \) Myr, to be compared with QS90’s value of \( t \approx 490 \) Myr, a satisfying agreement, given how different the approaches are. Model E2B is prevented from entering the run-away phase by stellar evolution in both QS90’s and our simulation. In our simulation (QS90-E2B), a few particles grow to 8 \( M_\odot \) and one to 11 \( M_\odot \) during the phase of central concentration. QS90 report that the most massive stars formed have 8 \( M_\odot \) in this case. In QS90, the core collapse of model E1A is reversed by the formation of, and energy input from 3-body binaries, a process we do no take into consideration (see discussion in Appendix A), but the fact that they form stars as massive as 64 \( M_\odot \) indicates that they nearly reach the conditions for runaway. For this cluster we obtain runaway at \( t \approx 800 \) Myr. Finally, E1B is another case for which QS90 find the core collapse is interrupted by stellar evolution but our simulation produces runaway at \( t \approx 0.37 \) Gyr. In one of our simulations for this model, the runaway star is destroyed.
in a collision with a compact object, just after it has reached a mass of $157 \, M_\odot$. Another runaway sequence starts shortly afterwards.

Since QS90, runaway collisions in clusters have been studied by use of the direct $N$-body method (Portegies Zwart et al. 1999a; Portegies Zwart & McMillan 2002; Portegies Zwart et al. 2003). Unfortunately, these simulations were either limited to $N_p \lesssim 10^3$, in which case binaries play a crucial role or, for the few cases reaching $N_p \approx 6 \times 10^5$ and experiencing runaway, done for clusters with high initial concentration (King parameter $W_0 \gtrsim 9$) and, hence probably dominated by small-number effects in the innermost regions. In Paper II, we report briefly on the simulations we have done with initial conditions similar to those of some models of Portegies Zwart et al. 2004).

5 SUMMARY

The results presented here are mostly tests to ensure that the Monte Carlo stellar dynamics code we use, ME(SSY)**2 (Freitag & Benz 2001, 2002a), correctly treats the key processes at play in the runaway route, which are:

(i) Mass segregation-induced core collapse, driven by 2-body relaxation in cluster with a broad, realistic mass function (Salpeter of Kroupa, with $M_{\text{min}} = 0.1 - 0.2 \, M_\odot$ and $M_{*, \text{max}} \approx 100 \, M_\odot$)

(ii) Effects of collisions in the evolution of the cluster and occurrence of collisional runaway.

This paper is a companion to Paper II (Freitag et al. 2003), in which we present the results of our large set of simulations to determine the conditions for, and characteristics of collisional runaway in young stellar clusters. In the scenario we investigate, the collisional phase is brought up by the concentration, through relaxation, of massive stars in the centre of a cluster. A star much more massive than $100 \, M_\odot$, formed in a quick sequence of collisions may not only be a progenitor for an IMBH but is in itself an exotic object of considerable interest.

To address point (i), we performed simulations of clusters with ME(SSY)**2, NBODY4 and the gaseous-model code SPEDI. The only physical process included in the MC and gaseous-model runs was 2-body relaxation, treated in the standard Fokker-Planck approximation, i.e., as sum of uncorrelated small-angle two-body scatterings whose rate is determined by local conditions at each point in the cluster. The $N$-body code treats Newtonian gravitation in an essential exact way, without assumptions about the nature of relaxation. ME(SSY)**2 produces results in close agreement with their $N$-body counterparts for $\mu = M_{*, \text{max}} / \langle M_\ast \rangle \lesssim 20$ at least. Realistic IMF correspond to $\mu \gtrsim 100$, however. There are clear discrepancies between the MC and $N$-body simulations in this regime. Most noticeably $N$-body results show an initially stronger but more progressive concentration of massive stars in the central regions. Nevertheless, the most important characteristics of the core collapse as a path to the collisional runaway stage, namely the time for it to happen, $t_{cc}$, and the magnitude of central mass segregation during deep collapse are similar in both type of simulations. Furthermore, SPEDI yields results very close to those of ME(SSY)**2, except for a value of $t_{cc} \approx 13\%$ shorter.

Turning now to point (ii), we note that only very few numerical simulations of the evolution of clusters subject to relaxation and collisions, up to and including the runaway stage have been published. Putting aside the recent $N$-body runaway simulations (Portegies Zwart et al. 1999a, Portegies Zwart & McMillan 2002, Portegies Zwart et al. 2004), not suitable for clear-cut comparison with ME(SSY)**2 results as small-$N$ effects may play a strong role in them, we are left with the older FP simulations of Quinlan & Shapiro (1990, QS90). These models lack realism because they start with a single-mass population of $1 \, M_\odot$ stars. Collisions are assumed to result in perfect mergers, with no mass loss. In most cases considered, the cluster is dense enough to promote stellar mergers at early times. The more massive stars thus formed accelerate collapse by mass segregation, hence further increasing the merger rate. These same merger products can also terminate the process of central density buildup through their mass when they evolve off the MS, well before the original $1 \, M_\odot$ would have done so.

We have simulated the 6 models of QS90 for which these authors have assumed that the gas lost through stellar evolution escapes the cluster (rather than staying in it and forming new stars). Although collisional runaway happens more often in our simulations than in QS90 runs, we obtain good general agreement with the relatively scarce data published by QS90. For the two situations in which they report core collapse uninterrupted by stellar evolution or 3-body binaries, we obtain core collapse times $8\%$ longer and $18\%$ shorter than theirs, which we find satisfying considering the widely different numerical methods and, especially, treatments of collisions, of critical importance here. The production rate of $2 - 3 \, M_\odot$ objects in our runs is nearly exactly the same as for QS90’s $2 \, M_\odot$ mass bin but noticeable differences exist for higher-mass objects, which, resulting from longer merger sequences, are more affected by differences in treatments of collisions. For a large part, the discrepancies between ours and QS90’s certainly originates in this. Our treatment of collisions being much more direct and accurate than the one allowed by the FP code of QS90, the differences found in these comparisons do not cast doubt on the ability of ME(SSY)**2 to deal with collisional cluster evolution.

We have also re-simulated one of the QS90 models with the highest velocity dispersion using our more realistic treatment of collisions, based on SPH simulations (Freitag & Benz 2002a) and found a significantly longer core-collapse time due to the collisions being less effective at producing higher-mass stars. This stresses the importance of using collision prescriptions which account for fly-bys and mass loss, in the high-velocity regime. This question is investigated in more depth in Paper II.

APPENDIX A: NEGLECT OF BINARIES

In principle, “hard” binary stars, either primordial or formed during core collapse through 3-body processes, may play an important role in the cluster evolution. During gravitational encounters with single stars, a hard binary is likely to shrink, thus allowing the single star (possibly a former member of the binary if an exchange has occurred) to emerge with increased kinetic energy. Through this dynamical heating, hard binaries may suspend or even reverse core collapse.
Runaway collisions

Figure A1. Core collapse of a $W_0 = 3$ King cluster with Salpeter mass function, computed with SPEDI. The mass spectrum is discretized into 15 components with the indicated individual stellar masses ($m_i$) and mass fraction ($f_{m_i}$). On the top panel, we represent the contribution of each mass component to the central mass density. The dotted line is the total density. The bottom panel shows the velocity dispersion of each component. The dotted line is the mass-averaged dispersion.

(Heggie & Hut 2003, and references therein). Obviously, this might prevent the core from ever entering the high-density collisional phase needed in the runaway scenario. However, when the finite size of stars is taken into account in the numerical study of single-binary and binary-binary interactions, it appears that the collision of at least two of the stars is a likely event (see Fregeau et al. 2004 for the most recent cross-section computations and references about this question). Although using point-mass dynamics, Giersz & Spurzem (2003) studied the statistics of binary-single and binary-binary encounters occurring in their simulations of clusters containing primordial binaries and found that of order 50% of these interactions should indeed lead to stellar mergers, for typical globular cluster parameters. It is therefore possible that binaries actually foster collisions rather than preventing them. This is exactly what was found in N-body simulations of relatively small clusters (most of them with $N_c ≤ 131 072$, one run with $N_c ≃ 585 000$) by Portegies Zwart and collaborators (Portegies Zwart et al. 1999, Portegies Zwart & McMillan 2002, Portegies Zwart et al. 2004). Furthermore, dissipative processes, such as collisions or tidal interactions, occurring during binary interactions may significantly reduce the heating effect of hard binaries, compared with the point-mass approximation (McMillan 1986, Goodman & Hernquist 1991, Chernoff & Huang 1996). To our knowledge, however, the impact of this on core collapse has not yet been studied explicitly.

Even if binaries probably do not prevent core collapse and collisions, it does not follow that they will not impede or modify the runaway process. Indeed, the cross section for collision of a single star with a binary is of order $\pi G a M (V_{\text{rel}})^{-2} \propto M$ where $a$ is the binary semi-major axis and $M$ the mass of the three stars, while it is approximately $\pi G R M (V_{\text{rel}})^{-2} \propto M^\alpha$ with $\alpha ≃ 1.5$ for the collision with a single more massive star of mass $M (> 50 M_\odot)$ and radius $R$. According to mathematical modelling through the coagulation equation, runaway is expected only if the cross section scales like $M^\zeta$ with $\zeta > 1$ (Lee 1993, 2000, Malyshkin & Goodman 2001). This condition is not obeyed in the case of binaries competing with each other for interaction and merger with single stars. However, the coagulation equation is clearly at best a crude idealisation for the complex stellar dynamical situation of interest here.

To simplify the problem, we have assumed in this work that no primordial binaries were present. We now examine if we are then justified to neglect binary process altogether.

Figure A2. Same gaseous model simulation as on Fig. A1. Here we plot, for each mass component, the central collision rate per star (top panel) and integrated number of collision or collision probability (bottom panel) for a star near the centre. To avoid double counting, the rate for component $i$ includes collisions with all components $j ≤ i$. The steep (black) dash-dotted line is an estimate of the formation rate of 3-body binaries (per star) obtained by using average stellar density and mass in equation (A1). Note that this simulation do not include the effects of collisions or binary formation but only 2-body relaxation. Collision and binary formation rates have been estimated a posteriori assuming the cluster is made of $10^6$ stars and its size is $R_{NB} = 0.2 \text{ pc}$. 

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consistently, in spite of the possibility of forming binaries dynamically. We do not consider binary formed by tidal interactions because their formation requires close passage at a distance of a few stellar radii at most. Their formation rate cannot then be significantly higher than collision rate and their post-capture evolution, though still a controversial issue, is likely to lead to merger anyway. We instead consider the question of binaries formed through (point-mass) 3-body interactions.

The creation rate of such binaries is (Binney & Tremaine 1987, Eq. [8-7]; see also appendix of Ivanova et al. 2002)

$$\dot{n}_{3b} \approx C_{3b} n^3 \frac{G^2 M^5}{\sigma_v^9}, \quad (A1)$$

where \(n\) is the stellar number density and \(C_{3b} \approx 0.75\) (Goodman & Hut 1993; Heggie & Hut 2003).

We compare this with the collision rate,

$$\dot{n}_{\text{coll}} = n t^{-1}_{\text{coll}} \approx 4 \pi n^2 R_{\text{rel}}^2 \left(1 + \frac{2GM_s}{R_s (V_{\infty})^2}\right) V_{\infty} \approx 8 \pi n^2 GM_s R_s \sigma_v \quad (A2)$$

One gets

$$\frac{\dot{n}_{\text{coll}}}{\dot{n}_{3b}} \approx \frac{500}{n R_{\text{rel}}^2 (\frac{\sigma_v}{V_{\infty}})^8} \quad (A3)$$

with \(V_{\infty} = (2GM_s/R_s)^{1/2} = 618 \text{ km s}^{-1}\). For typical 100 M\(_\odot\) stars (dominating the central regions), \(R_s \approx 14 R_{\odot}\), \(V_c \approx 1670 \text{ km s}^{-1}\) and

$$\frac{\dot{n}_{\text{coll}}}{\dot{n}_{3b}} \bigg|_{100 M_{\odot}} \approx 7 \left(\frac{n}{10^5 \text{ pc}^{-3}}\right)^{-1} \left(\frac{\sigma_v}{20 \text{ km s}^{-1}}\right)^8. \quad (A4)$$

Thus, one cannot clearly exclude that the formation of 3-body binaries will not compete with direct collision, at least for systems with a relatively low velocity dispersion.

Estimating \textit{a posteriori} the central values of \(\dot{n}_{3b}\) from MC runs is very difficult because of the steep dependences on \(n\) and \(\sigma_v\), which make the estimate extremely noisy. Thus, to see how \(\dot{n}_{\text{coll}}\) and \(\dot{n}_{3b}\) evolve during core collapse, we resort to the SPEDI gas-model code presented in Section 4.2.2. In Figure 1, we follow the evolution of central densities and velocity dispersions in a 15-component SPEDI core-collapse simulation. Assuming \(N_s = 10^6\) and \(R_{NB} = 0.2 \text{ pc}\), we can now compute what the central collision and 3-body formation rates would have been during the core collapse. Because the evolution speeds up near the moment of core collapse, we use the central potential instead of time as independent variable in Fig. 1, where we plot the instantaneous and time-integrated rates for all mass components. For this size and star number, the cluster should become collisional before the first binary forms. How this depends on the cluster parameters is expressed by either of the scalings

$$\frac{\dot{n}_{\text{coll}}}{\dot{n}_{3b}} \propto N_s^3 R_{c1}^{-1} \left(\ln \Lambda T_{sh}\right)^{-2/3} N_s^{10/3} \quad (A5)$$

where \(R_{c1}\) is some characteristic cluster size (e.g., the half-mass radius). The strong dependence on \(N_s\) suggests that 3-body binaries, which appear to dominate the collision process in N-body simulations, may be of little importance in larger systems with \(N_s \gtrsim 10^6\), such as very massive young clusters or proto-galactic nuclei. This analysis is in agreement with the results of Portegies Zwart et al. (2003) who find the dynamically-formed binaries to play a lesser role in comparison with \textit{earlier, lower-N simulations} (Portegies Zwart & McMillan 2004).

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REFERENCES

Aarseth S. J., 1999, PASP, 111, 1333
Amaro-Seoane P., Freitag M., Spurzem R., 2004, MNRAS, 352, 655
Bally J., Zinnecker H., 2005, AJ, 129, 2281
Baraffe I., Heger A., Woosley S. E., 2001, ApJ, 550, 890
Baumgardt H., 2001, MNRAS, 325, 1323
Baumgardt H., Heggie D. C., Hut P., Makino J., 2003, MNRAS, 341, 247
Baumgardt H., Makino J., 2003, MNRAS, 340, 227
Baumgardt H., Makino J., Ebisuzaki T., 2004a, ApJ, 613, 1133
—, 2004b, ApJ, 613, 1143
Begelman M. C., Rees M. J., 1978, MNRAS, 185, 847
Belczynski K., Kalogera V., Bulik T., 2002, ApJ, 572, 407
Benz W., 1990, in Numerical Modelling of Nonlinear Stellar Pulsations Problems and Prospects, Buchler J. R., ed., pp. 269–288
Benz W., Hills J. G., 1987, ApJ, 323, 614
—, 1992, ApJ, 389, 546
Binney J., Tremaine S., 1987, Galactic Dynamics. Princeton University Press
Blandford R. D., 2004, in Coevolution of Black Holes and Galaxies, from the Carnegie Observatories Centennial Symposium., Ho L., ed., Cambridge University Press, p. 154
Bond J. R., Arnett W. D., Carr B. J., 1984, ApJ, 280, 825
Bonnell I. A., Bate M. R., Zinnecker H., 1998, MNRAS, 298, 93
Campbell B., Hunter D. A., Holtzman J. A., Lauer T. R., Shayer E. J., Code A., Faber S. M., Groth E. J., Light R. M., Lynds R., O’Neil E. J., Westphal J. A., 1992, AJ, 104, 1721
Chabrier G., Baraffe I., 2000, ARA&A, 38, 337
Chandrasekhar S., 1960, Principles of stellar dynamics. New York: Dover, Enlarged ed.
Miller G. E., Scalo J. M., 1979, ApJS, 41, 513
Miller M. C., 2005, ApJ, 618, 426
Moffat A. F. J., Drissen L., Shara M. M., 1994, ApJ, 436, 183
Monaghan J. J., 1992, ARA&A, 30, 543
Murphy B. W., Cohn H. N., Durisen R. H., 1991, ApJ, 370, 60
Palla F., 2002, in Physics of Star Formation in Galaxies, Lectures of the 29th Advanced Course of the Swiss Society for Astronomy and Astrophysics (SSAA), Maeder A., Meynet G., eds., pp. 9–133
Phinney E. S., 2003, AAS/High Energy Astrophysics Division, 7, #27.03
Plummer H. C., 1911, MNRAS, 71, 460
Podsiadlowski P., 1996, MNRAS, 279, 1104
Portegies Zwart S. F., Baumgardt H., Hut P., Makino J., McMillan S. L. W., 2004, Nat, 428, 724
Portegies Zwart S. F., Makino J., McMillan S. L. W., Hut P., 1999a, A&A, 348, 117
—, 1999b, A&A, 348, 117
Portegies Zwart S. F., McMillan S. L. W., 2002, ApJ, 576, 899
Portegies Zwart S. F., Meinen A. T., 1993, A&A, 280, 174
Quinlan G. D., Shapiro S. L., 1990, ApJ, 356, 483
Rasio F. A., Lombardi J. C., 1999, Journal of Computational and Applied Mathematics, 109, 213
Richstone D., 2004, in Coevolution of Black Holes and Galaxies, from the Carnegie Observatories Centennial Symposia, Ho L., ed., Cambridge University Press, p. 281
Sanders R. H., 1970, ApJ, 162, 791
Schaerer D., 2002, A&A, 382, 28
Schaller G., Schaerer D., Meynet G., Maeder A., 1992, A&AS, 96, 269
Sesana A., Haardt F., Madau P., Volonteri M., 2004, ApJ, 611, 623
Sills A., Adams T., Davies M. B., Bate M. R., 2002, MNRAS, 332, 49
Sills A., Faber J. A., Lombardi J. C., Rasio F. A., Warren A. R., 2001, ApJ, 548, 323
Sills A., Lombardi J. C., Bailer J. D., Demarque P., Rasio F. A., Shapiro S. L., 1997, ApJ, 487, 290
Soltan A., 1982, MNRAS, 200, 115
Spitzer L. J., Saslaw W. C., 1966, ApJ, 143, 400
Spitzer L., 1987, Dynamical evolution of globular clusters. Princeton University Press
Spurzem R., Takahashi K., 1995, MNRAS, 272, 772
Stothers R. B., Chin C., 1997, ApJ, 489, 319
Tremaine S., Richstone D. O., Byun Y., Dressler A., Faber S. M., Grillmair C., Kormendy J., Lauer T. R., 1994, AJ, 107, 634
van der Marel R. P., Gerssen J., Guhathakurta P., Peterson R. C., Gebhardt K., 2002, AJ, 124, 3255
Vink J. S., de Koter A., Lamers H. J. G. L. M., 2001, A&A, 369, 574
Volonteri M., Haardt F., Madau P., 2003, ApJ, 582, 559
Walcher C. J., van der Marel R. P., McLaughlin D., Rix H.-W., Böker T., Häring N., Ho L. C., Sarzi M., Shields J. C., 2005, ApJ, 618, 237
Wyithe J. S. B., Loeb A., 2003, ApJ, 595, 614
Yu Q., Tremaine S., 2002, MNRAS, 335, 965
Zinnecker H., Bate M. R., 2002, in ASP Conf. Ser. 267: Hot Star Workshop III: The Earliest Phases of Massive Star Birth, Crowther P. A., ed., p. 209