Semiclassical Gravity and Mesoscopic Physics *

B. L. Hu

Department of Physics, University of Maryland, College Park, MD 20742, USA †
Institute for Advanced Study, Princeton, New Jersey 08540, USA
Department of Physics, Hong Kong University of Science and Technology,
Clear Water Bay, Kowloon, Hong Kong

(UMDPP 96-50, IASSNS-HEP-95/xxx, October, 1995)

Abstract

Developments in theoretical cosmology in the recent decades show a close connection with particle physics, quantum gravity and unified theories. Answers or hints to many fundamental questions in cosmology like the homogeneity and isotropy of the Universe, the sources of structure formation and entropy generation, and the initial state of the Universe can be traced back to the activities of quantum fields and the dynamics of spacetime from the Grand Unification time to the Planck time at $10^{-43}$ sec. A closer depiction of this primordial state of the Universe requires at least a semiclassical theory of gravity and the consideration of non-equilibrium statistical processes involving quantum fields. This critical state is intermediate between the well-known classical epoch successfully described by Einstein’s Theory of General Relativity and the completely unknown realm of quantum gravity. Many issues special to this stage such as the transition from quantum to classical spacetime via decoherence, cross-over behavior at the Planck scale, tunneling and particle creation, or growth of density contrast from vacuum fluctuations share some basic concerns of mesoscopic physics for condensed matter, atoms or nuclei, in the quantum/classical and the micro/macro interfaces, or the discrete/continuum and the stochastic/deterministic transitions. We point out that underlying these issues are three main factors: quantum coherence, fluctuations and correlation. We discuss how a deeper understanding of these aspects of fields and spacetimes can help one to address some basic problems, such as Planck scale metric fluctuations, cosmological phase transition and structure formation, and the black hole entropy, end-state and information paradox.

*Invited talk at the International Symposium on Quantum Classical Correspondence, Drexel University, Philadelphia, Sept. 8-11, 1994. To appear in the Proceedings edited by D. H. Feng and B. L. Hu (International Press, Boston, 1996)
†permanent address. e-mail:hu@umdhep.umd.edu
1 Introduction

This conference covers four rapidly developing areas of physics: the foundational aspects of quantum mechanics, quantum gravity and cosmology, mesoscopic physics and chaos. Though apparently disjoint, they share the common concern of how to correctly understand the quantum and classical descriptions with regard to their individual particularities and mutual consistency. In this talk I'd like to address two aspects: Quantum cosmology [1, 2] and mesoscopic physics [3]. (Another work on using the concept of decoherence to explore the relation between quantum and classical chaos is reported by Shiokawa [3] in this conference.) Specifically, I want to show how semiclassical gravity can be viewed as mesoscopic physics. By semiclassical gravity I mean the theory where gravity is treated classically by Einstein’s Theory of General Relativity, and matter field quantum mechanically by quantum field theory. This is represented by the discipline of quantum field theory in curved spacetime [3], which provides a good description of gravity and matter below the Planck energy $10^{19}$ GeV (or Planck time $10^{-43}$ sec, Planck length $10^{-33}$ cm) and constitutes the theoretical framework for understanding quantum processes in the early universe and black hole physics from the Planck energy to the Grand Unified Theory (GUT) energy $10^{15}$ GeV). Mesoscopic physics deals with problems where the characteristic interaction scales or sample sizes are intermediate between the microscopic and the macroscopic. For the experts they refer to a specific set of problems in condensed matter and atomic/optical physics (see, e.g., [3]). For the present discussion, I will adopt a more general definition, with 'meso' referring to the interface between macro and micro on the one hand and the interface between classical and quantum on the other. These two aspects will often bring in the continuum / discrete and the deterministic / stochastic factors. I will show how issues concerning the micro / macro interface and the quantum to classical transition arise in quantum cosmology and semiclassical gravity in a way categorically similar to the new problems arising from condensed matter and atomic/optical physics (and, at a higher energy level, particle/nuclear physics, at the quark-gluon and nucleon interface). I will show that many issues are related to the coherence and correlation properties of quantum systems, and involve stochastic notions, such as noise, fluctuations, dissipation and diffusion in the treatment of transport, scattering and propaga-

---

1 Another meaning of mesosopia can be defined with respect to structures and interactions. Instead of dwelling on these individual processes in their specific context, one can refer to the general category of problems which exist in between two distinct levels of matter structure or interaction scales, such as between the molecular and atomic scales, the QED lepton-hadron and nuclear scale, the nuclear and particle (quark-gluon) scales, the QCD and GUT (grand unification theory) scale (assuming deserts in-between), and of course, from GUT to QG (quantum gravity) scale, which is depicted by semiclassical gravity. The distinct levels of interaction are not arbitrarily picked, they obey theories of a fundamental (QED, QCD) or quasi-fundamental (atomic, nuclear interaction) nature. The meso scales between them have common traits. They usually fall in the range where the approximations taken from either level (e.g., low energy QCD versus perturbative hadron physics) fail, and new structure depicted by new collective variables and new language are called for to study its behavior. The new problems encountered in condensed matter and nuclear/particle physics fall under such a conceptual category. So do the problems of extending semiclassical gravity towards quantum gravity or projecting quantum gravity (e.g., superstring theory) onto low energy particle physics (the standard model).
tion processes. The advantage of making such a comparison between these two apparently disjoint disciplines is twofold: The theory of mesoscopic processes which can be tested in laboratories with the newly developed nanotechnology can enrich our understanding of the basic issues common to these disciplines while being extended to the realm of general relativity and quantum gravity. The formal techniques developed and applied to problems in quantum field theory and spacetime geometry can be adopted to treat condensed matter and atomic/optical systems with more rigor, accuracy and completeness. Many conceptual and technical challenges are posed by mesoscopic processes in both areas.

2 Problems in Semiclassical Gravity and Mesoscopic Physics where Quantum and Statistical Mechanics Play a Fundamental Role

For many years I have maintained the somewhat unconventional view that cosmology and astrophysics should be considered as ‘condensed matter’ physics \( ^3 \). This refers to both classical and quantum processes. \( ^4 \) (For a recent account of a similar viewpoint, see \( ^5 \).) At face value, this statement could be taken to mean that these subjects are applications of general relativity (GR) and quantum field theory (QFT) (for early universe and late stage black hole processes), as condensed matter physics is with respect to quantum mechanics and statistical physics. Less trivially, it is how the viewpoint and approach to important problems in cosmology and astrophysics which I want to make a distinction from the way traditionally pursued in GR or QFT. By nature, cosmology and astrophysics are interdisciplinary subjects. Unlike GR or QFT and their mathematical derivatives such as superstring theory, conformal field theory or canonical gravity via old and new variables, which rely on formal constructions and abstractions, sometimes even to the extent of defying reality, these disciplines involve many physical processes and many different effects to paint even an approximate picture. In essence, their growth has always been strongly influenced or enriched by other major currents of theoretical and observational physics.

Looking at the development of theoretical cosmology and astrophysics in the last forty years, if one wants to name one important branch of physics in each decade which enhanced its growth, one could say that in the fifties, it was nuclear physics which laid down the

\( ^2 \) Although I have worked on quantum processes at the somewhat remote Planck scale \( (10^{-43} \text{ sec}, 10^{-33} \text{ cm}) \), and I share the enthusiasm with other advocates on the importance of explaining most common observations from their quantum origin (the most ambitious of all in this attempt is the ‘getting everything from nothing’ philosophy – structure and dynamics from quantum noise or vacuum fluctuations) I do think that many of the observed cosmological phenomena arose from nonlinear processes (hydrodynamic, kinetic or mechanical, including chaos) at very late times. I don’t think a single stroke, no matter how appealing the theory appears (e.g., superstring theories purported by those as the theory of everything TOE, or a certain proposal of the boundary condition of the wave function of the universe) can explain everything we see in the universe today. The appreciation and explanation of the hierarchy of structures and interactions in the natural world require more in-depth analysis on the cross-over between each ‘elementary’ level and the composite levels derived therefrom. See, e.g., \( ^6 \)
background for processes later observed in neutron stars, and explained nucleosynthesis in the early universe (between 1 sec and 3 minutes). In the sixties, it was relativity (of course, to begin with, general relativity was responsible for the inception of modern cosmology and astrophysics in the 20’s and 30’s – the three solar system tests, Hubble expansion, Friedmann and Schwarzschild solutions were their cornerstones, followed by detailed studies of gravitational collapse in the 30’s and discoveries of exact solutions in the 50’s ). The observation of microwave background radiation, quasars and pulsars, which ushered in the golden age of contemporary relativistic cosmology and astrophysics. In the seventies, it was quantum field theory and semiclassical gravity theory which drew the attention of the importance of quantum processes in addressing some of the most important issues in theoretical cosmology, such as the singularity and the horizon problems (possible singularity avoidance and horizon removal due to quantum processes such as trace anomaly and particle creation [9, 10]) and which provided the theoretical basis for the discovery of the celebrated Hawking effect [11] in black holes. In the eighties, it was from particle physics considerations that inflationary universe [12] was introduced, which quickly became the focus of current cosmological research activities. With it, the recognition that the early universe can provide a cheap and convenient laboratory to test out possible particle physics theories from the energy range of the Standard Model to that of the Grand Unified Theories provided further impetus to the establishment of a new field of particle astrophysics, not unlike the efforts in the fifties by the nuclear physicists in the establishment of nuclear astrophysics.

What about the nineties? In my opinion, statistical mechanics, especially nonequilibrium statistical mechanics, kinetic theory, stochastic mechanics, chaos, fractals, and complexity studies, will provide new stimulus and show new avenues for the development of both physical cosmology and gravitation physics.

In recent reviews I have enumerated some important quantum processes in the early universe which require a nonequilibrium statistical mechanical treatment. I discussed processes such as phase transition, particle creation, and entropy generation in the Waseda University Conference on Quantum Physics and the Universe [13]. In the Los Alamos Conference on Fluctuations and Order [14], I have expanded this list and discussed specifically the relevance of noise and fluctuations to the problems of decoherence in the transition from quantum to semiclassical gravity, particle creation, entropy generation, galaxy formation, and suggested that the ‘Birth of the Universe’ be viewed in the light of large fluctuation phenomenon. Let me display that list and add a few more entries [15, 16]:

**TABLE 1**

1. Cosmological particle creation as parametric amplification of vacuum fluctuations
2. Thermal radiance from accelerated observers, moving mirrors, and black holes as fluctuation-dissipation phenomena
3. Entropy generation from quantum stochastic and kinetic processes
4. Phase transitions in the early universe as noise-induced processes
5. Galaxy formation from primordial quantum fluctuations
6. Anisotropy dissipation from particle creation as backreaction processes
7. Dissipation in quantum cosmology and the issue of the initial state
8. Decoherence, backreaction and the semiclassical limit of quantum gravity
9. Inflationary multiverse network, cellula automata, and complexity
10. Percolation, nucleation, and spinodal decomposition
11. Topology change in spacetime and loss of quantum coherence problems
12. Stochastic spacetime, coarsening and continuum limit
13. Spacetime foam, bubble and froth: the kinetics of topology and geometry
14. Regge-Ponzano quantum gravity, spin-network
15. Gravitational entropy, singularity and time asymmetry
16. ‘Birth’ of the universe as a spacetime fluctuation and tunneling phenomenon

3 Mesoscopic Physics – Fundamental Issues at the Quantum/ Classical and Micro/ Macro Interfaces

To practitioners in condensed matter and atomic/optical physics, mesoscopics refers to rather specific problems where, for example, the sample size is comparable to the probing scale (nanometers), or the interaction time is comparable to the time of measurement (femtosecond), or that the electron wavefunction correlated over the sample affects its transport properties, or that the fluctuation pattern is reproducible and sample specific. Let me select a partial list of these processes defined experimentally or phenomenologically:

TABLE 2

1. Large n Rydberg atom: quantum chaos. Extent of validity of quasiclassical approximations, traces or fingerprints of classical chaos in corresponding quantum systems.
2. Aharonov-Bohm effect and Berry phase. Phases of wavefunctions and topology of field configurations. A-B oscillations in loops and conductance fluctuations in wires.
3. Quantum Hall effect: quantum correlations, topology effects of electrons in lower-dimensional systems.
4. Localization: electron transport in stochastic media. Interference between diffusion paths, coherent backscattering.
5. Quantum transport: Landauer scattering, coherent versus dissipative transport. Universal fluctuations in conductance. Sample specific and reproducible fluctuation patterns.
6. Quantum optics: dephasing, quantum trajectories, squeezing and uncertainty.
7. Sonoluminescence: dynamical Casimir effect and micro to macro energy transfer.
8. Discrete to continuum transitions: lattice gauge theory and infrared limit.
9. Finite size effect: correction to universality behavior at critical point in bulk systems, correlation length affected by finite size.
10. Fluctuations and noise: effect of quantum and thermal fluctuations. Nonequilibrium
noise, fluctuation-dissipation, coherent and incoherent tunnelling, resonant tunneling in quantum wells.

### 3.1 Mesoscopic Physics

1. Quantum / classical correspondence, Deterministic / stochastic dynamics

   This issue rests at the foundation of quantum mechanics. Recent work has shown how nonequilibrium statistical mechanics can provide deeper insight into the issue of quantum to classical transition [17, 18]. A particularly interesting application of the phenomena of decoherence is to study the relation between quantum and classical chaos [19]. The study of quantum chaos has been focused on searching for the clues or traces (scars and fingerprints) of the quantum counterpart of a chaotic classical system. How a quantum system evolves or changes over into a classical chaotic system is not easily addressed. It is of interest to study how decoherence of a quantum system by interaction with an environment or by the action of noise can quantitatively depict the emergence of classical chaotic behavior [4, 20] and thus provide the missing link between these rather disparate domains.

2. Micro / macro structures and dynamics

   This is of course the general theme of statistical mechanics, but the issues take on a special significance when coupled with the quantum / classical interface. A well known example of macroscopic quantum processes is quantum tunneling with dissipation [21]. Another example is sonoluminescence. Viewed as dynamical Casimir effect [22], the (macroscopic) classical collapse of a dielectric media excites the (microscopic) quantum fluctuations resulting in the emission of light. We shall see many semiclassical gravity processes are of this nature.

3. Medium energy scale, intermediate wave zone

   This is the scale where existing approximation schemes from either end (high and low energy, near and far zone, short and long wavelength regime) via WKB, Born-Oppenheimer, or similar methods fail. It is also the range where neither few-body dynamics or statistical description can cope with. In the sense defined in footnote 1, this issue is at the heart of mesoscopia. It is not just a time-honored problem in mathematical physics in terms of coming up with techniques to deal with this difficult range, but it bears on deeper theoretical physics issues, such as the choice or emergence of a suitable set of collective variables for the best description of the intermediate scale physics, the conditions for the relative closure (decoupling and renormalizability) of effective theories and the cross-over behavior between the low and high energy theories.

4. Discrete / continuum limit and extended / localized state transitions
Examples are numerous in this category, such as localization of wave scattering in random media (e.g., Sheng in [3]), melting or coarsening transition. Formally there is a correspondence between the Einstein lagrangian describing the evolution of spacetime (geometrodynamics) and that describing the dynamics of an interface [23, 24] (zero cosmological constant corresponds to adding surfactants) Correspondingly, galaxy formation from gravitational instability can be viewed as an interface growth problem [25].

5. Finite size versus bulk behavior

This started with phase transition studies in the early 70’s concerning the correction in the infrared behavior of the order parameter fields in systems of finite size compared to bulk systems [26]. The central issue is how the transition from micro scales to macro scales can be understood with the running of coupling constants and interaction parameters from the ultraviolet into the critical regime, and how the finite size of the sample changes the scaling behavior.

6. Quantum Transport

Traditional transport theory applied to mesoscopic structures is based on near-equilibrium or linear response approximations (e.g., Landauer formula). New nanodevice operations involve nonlinear, fast-response and far-from-equilibrium processes. This recognition calls for new developments in the theory of transport. One serious approach via the Keldysh method using Wigner functions (e.g., Buot in [3]) is closely related to similar formalisms developed for nonequilibrium quantum fields aimed at problems in the early universe [27].

7. Correlations and Fluctuations

Again examples are many: Sample specific fluctuation patterns, universal conductance fluctuations, strongly correlated systems [3].

3.2 Problems in Semiclassical Gravity

Let us now take a look at problems in semiclassical gravity of a similar nature.

1. A necessary task for any proponent theory of quantum gravity (in addition to showing its intrinsic viability in, say, addressing the issue of time, etc) is to demonstrate that it has the correct limit of semiclassical gravity, or explain how the classical spacetime picture as depicted by the theory of general relativity emerges. At the heart of this problem is decoherence, a necessary but not sufficient condition. Quantum decoherence was studied for minisuperspace models of quantum cosmology since the mid-80’s [28]. Amongst the interesting outcomes were the derivation of the semiclassical Einstein equations, the proof that the no-boundary or tunnelling boundary conditions of the wavefunction of the universe
can lead to an inflationary epoch, with the de Sitter invariant vacuum prevailing. The assumptions which go into this transition are discussed in detail in [31].

2. Semiclassical gravity is at the **micro / macro interface**. Here gravity is treated macroscopically by general relativity, while matter described by quantum field is by nature microscopic. Many of the semiclassical gravity processes has this property. Examples are Casimir energy of quantum fields in curved or topologically nontrivial spacetimes, cosmological particle creation from vacuum fluctuations of quantum fields. The latter is, in our assessment, closely related to sonoluminescence viewed as dynamical Casimir effect, in that the dynamics of the universe (governed by Einstein’s equation) parametrically excites the vacuum fluctuations of the quantum fields into particle pairs. Particle creation provides an energy transfer mechanism from the macroscopic (geometrodynamics) to the microscopic (particles and fields). When the backreaction of quantum field processes is included, the dynamics of spacetime is driven by the expectation value of the energy momentum tensor operator of the quantum matter field. (This is the reverse, i.e., micro to macro, process.)

What is the proper tool or language for depicting the microscopic structure of spacetime based on our knowledge of the low energy physics? The hope is that it is describable by suitable generalizations of the well-proven quantum field theory (e.g., by string field theory) in a non-perturbative way (not based on the linearized theory of spin 2 particle). Conceptually we have taken a different route here: moving up in energy scale rather than down, i.e., the construction or deduction of the attributes of a microscopic theory of gravity from analyzing the fluctuations in the critical regime from the macroscopic theory (Einstein’s general relativity and its generalized semiclassical form). To highlight the conceptual contrast, we can even say that in certain ways Einstein’s general relativity theory is the hydrodynamic limit of a more fundamental microscopic theory [32]. This ‘bottom-up’ rather than ‘top-down’ approach we have taken has the simple advantage that one starts from a more familiar terrain, that of semiclassical gravity and its fluctuations. Although we don’t entertain the hope of deducing completely the high energy or microscopic structure of spacetime (we recommend a serious look at superstring theories [33], despite its seeming retreat, and the knot-theory representation of quantum relativity of Ashtekar, Rovelli and Smolin [34]), we think the study of critical phenomena at the Planck scale via the approach we have taken will reveal some special properties of the sub-structure. Difference in viewpoints notwithstanding, it is the next logical step and a worthwhile challenge.

3. The third issue relating to the treatment of intermediate energy range is a common but nontrivial one. In the quantum / classical gravity context, this refers to the regime where one can no longer assume the wave function of the universe to be in the WKB form [35]. New phenomena are expected to arise here. This problem exists in all decoherence considerations of quantum to classical transition, i.e., whether the WKB wavefunction has

---

3 After all, this is the approach physicists have undertaken for centuries, in extracting new physics of a smaller scale from the contradictions of old physics at a larger scale – molecular kinetic theory from thermodynamics, quantum physics from atomic spectroscopy, particle physics from nuclear processes, etc.
a classical attribute (no in general – there is no peaking in the phase space [36]) or whether
the coherent state has the most classical character (yes in general – it is the state which
has the lowest entropy [37]). In terms of viewing classical persistent structures as arising
from coarse-graining the quantum histories, a more difficult related issue is what collective
variables [3] will be most suited for the description of the intermediate scale physics, and
how one constructs them from the accessible low energy phenomenology.

When considered in the context of decoherence studies, be it the ‘choice’ of an environ-
ment (degree of subjective or objective prerogative) in the environment -induced approach
[17] or realizing the projection operators in the decoherent history approach [18], if the Planck
scale is not given but left as an emergent parameter, this will take on the difficult question
of what gives rise to the Planck scale as the regime where classical behavior emerges, and
to what extent the scales of consistent and stable structures depend on coarse-graining the
progenatory quantum system. If general relativity is viewed as hydrodynamics [32], this
issue takes on the question of whether above the Planck scale, there can exist intermediate
scales which admit stable and persistent structures, similar to the many scales which exist in
the kinetic theory regime before the system settles into the long wavelength hydrodynamics
regime [38], or the existence of resonances as excited states of composite particles, as wit-
nessed by the profusion of ‘elementary’ particles in the 60’s and 70’s.

4. Discrete and continuum spacetime Lattice gravity [39] and simplicial geometry [40]
such as depicted by Regge calculus is an important line of inquiry bridging discrete models
of quantum gravity to classical general relativity.

5. Finite size effect in cosmology refers to the effect of spacetime geometry and topology
on the infrared behavior of quantum fields near a phase transition. O’Connor and I took
up this study ten years ago [41]. The effect is, strictly speaking, neither due specifically to
topology nor curvature, but the presence of a finite size in the background spacetime or in
the fluctuation field operator. It has an effect on the nature of phase transition (first or
second order) and how it is approached and consummated (dynamics or energetics). Here,
the role of quantum fluctuations in inducing phase transitions is of basic importance.

6. Nonequilibrium quantum fields and nonlinear quantum transport. Calzetta and I [27]
had developed a kinetic theory for nonequilibrium quantum fields, using the Schwinger-
Keldysh (closed-time-path) formalism, the nPI effective action method with Winger func-
tions. (See also [12] and compare with Buot in [8]). This method has been applied to an
analysis of reheating by particle creation [43] and first order phase transitions [44] in the
inflationary universe, and to heavy-ion collisions in nuclear reactions [45]. Recent summary
of work in these directions can be found in meetings of thermal field theory and its applica-
tions [46].

7. Quantum fluctuations in the inflaton field can be viewed as noise which seeds galaxy
formation [47] and entropy generation. Quantum origin of noise and fluctuations has been
discussed by [48, 49, 50]. In attempting to deal with more exotic ideas such as the ‘birth’ of the Universe, one encounters difficulty even in explaining the meaning of words such as tunnelling from ‘nothing’. But in terms of mesoscopic physics, it is possible to visualize it as a large fluctuation problem in stochastic processes, or a phase transition problem in critical dynamics. We think it is important to first understand the physics of these processes in a more familiar setting before trying to formulate it in the uncharted waters of quantum cosmology. Mesoscopic physics is such an area where new physics can be learned and ideas tested out.

4 Common Points – Quantum Coherence, Fluctuations, Correlations

Viewed in a more theoretical light, we can decipher three aspects which underlie most of the processes named above. They are quantum coherence, fluctuations and correlations. All mesoscopic processes involve one or more of these aspects.

1. Fluctuations and decoherence: quantum / classical correspondence:

Decoherence: Fluctuations and noise in the environment is what is responsible for decoherence in the system, which is a necessary condition for quantum to classical transition. Classicality: Classical description in terms of definite trajectories in phase space requires correlations between conjugate variables. Noise and fluctuations destroy this correlation. The observed classical reality as an emergent phenomenon from quantum description has intrinsic stochastic behavior [51, 52].

2. Coherence and dissipation

This is the counterpart to 1. above, as fluctuation and dissipation are balanced by the fluctuation-dissipation relation. (We found that this relation exists for general, including non-equilibrium, conditions as it originates from the unitarity of the closed system from which the open system is defined [50, 53]). Coherence of electron wavefunction in a mesoscale sample makes it sensitive to circuit geometry, as manifest in the detection of Aharonov-Bohm effects. Anderson localization can be understood as due to coherent back-scattering from random sources. Coherence in quantum systems is altered by dissipative effects, as occurs in macroscopic quantum phenomena [21], e.g., competition between coherence and dissipation manifests in tunneling at finite temperature.

3. Correlation

Strongly correlated system has very different transport properties related to scattering or diffusion processes. Long range order established near the critical point invokes higher order
correlations which determine the nature of phase transitions. Correlation between particles and a quantum field is a determining factor for many atomic / optical processes.

Let me mention some current research directions in semiclassical cosmology and black hole physics which also involve these aspects in a fundamental way.

A. The above-mentioned issue of decoherence and transition from quantum cosmology to semiclassical gravity has been treated with the theory of quantum open systems and the method of influence functional. The recent work of Calzetta, Matacz, Sinha and me \[50, 53, 54\] (see also \[55\]) show that an Einstein- Langevin equation is a natural generalization of the semiclassical Einstein equation at the quantum gravity transition. The latter is a mean field theory obtained by taking a noise average of the former. This provides a new platform for one to investigate fluctuation processes related to particle creation at the Planck scale. These work also show that the backreaction of particle creation resulting in the dissipation of the background spacetime can be viewed as a manifestation of a fluctuation- dissipation relation. One can use these results to explore deeper issues such as metric fluctuations in spacetime, both curved and flat, near the Planck scale. Kuo and Ford \[56\] calculated fluctuations in the Casimir energy and considered possible experimental observations. Nicholas Philips \[57\] has been calculating the fluctuations and variance in the energy momentum tensor of quantum fields in curved spacetimes. I have speculated that the variance may come out to be largely independent of the local geometry and topology. (somewhat similar to the universal fluctuations of conductance observed in mesoscopic processes, which is independent of sample size and shape). If this were true then it will reveal some simple yet basic properties of Planck scale physics.

B. Structure formation from quantum fluctuations in the early universe

This important problem involves the fluctuation and decoherence aspects. The current folklore is that the classical fluctuations (noise) arise from the high frequency quantum fluctuations of a quantized free field, with no due considerations of the quantum origin of noise and the decoherence process. We have raised serious doubts on such a practice \[48, 49, 50\]. Recently Calzetta and I have shown by way of an interacting field theory model (a scalar - gravitational field coupling) \[58\] how taking the correct procedure can actually produce an improved result which alleviates the necessity of assuming an unnaturally small coupling constant. In the traditional and we think erroneous treatment, the fluctuations part of the quantum field is implicitly viewed as completely decohered and acts as a stochastic classical source in the equation governing the density perturbations. We pointed out that it is the noise term resulting from the decoherence of the mean field which enters into the Langevin equation. The quantum fluctuations are instrumental to the decoherence of the mean field but are themselves not the direct source of classical noise. There is an interplay of decoherence and fluctuations here, which contributes to the density contrast proportional to an extra factor of the coupling constant (thus in a $\lambda\phi^4$ model for the inflaton, the $\lambda$ can be as
high as $10^{-6}$ rather than assuming an unnaturally small $10^{-12}$). This subtle but important point is missed out in the conventional treatment.

C. Correlation dynamics, nonlocality of quantum fields and black hole information paradox

While working on a formalism of nonequilibrium quantum field theory first from the correlation dynamics (Boltzmann-BBGKY) \[27\] and then from the open system viewpoint (Langevin) \[59\], it became increasing clear to us that even though sophisticated techniques (e.g., Feynman perturbation) and concepts (e.g., renormalization theory) in quantum field theory have been developed, some simple problems in particle field interaction have not been understood well enough yet. This is because historically we are more interested in the energetics aspects of the systems, such as transition amplitudes, scattering cross sections, energy level shifts due to interactions of particle and fields. The statistical mechanics aspects of particle and fields has not been explore to the same extent. Problems such as the correlation, noise, fluctuations, and dissipative dynamics of a system are traditionally dealt with in statistical mechanics, but rarely for quantum fields and particle-field interactions. Studies of quantum processes in strong field and curved spacetime conditions such as in the early universe and black holes have led us to such inquiries. (Other areas such as quark-gluon plasma, heavy ion collisions and high field atomic / optical processes have also seen such a recent demand.) Questions such as the noise, dissipation and entropy a particle-field system – how they are related to particle creation and quantum fluctuations on the one hand and the correlation dynamics and kinetics on the other – requires a deeper probe into the stochastic properties of quantum fields. Even from the limited results our investigations have produced, one can see the rich physics and potential this nascent subject of nonequilibrium field theory holds which describes the statistical mechanics of particles and quantum field systems.

As sample problems, let me mention the derivation of Hawking and Unruh radiation from statistical field theory, viewed as kinematical effects without invoking a priori geometric notions or constructs \[60, 61\]. The work of Raval, Anglin and I on the statistical properties of particle (detectors and probes) - field systems \[62, 63\] and that of Raval, Johnson and I on the backreaction of particle creation (Unruh effect) on the accelerated detector and moving mirror problems are precursors to the problems of black hole entropy, backreaction and information paradox. On this issue, I have been of the opinion that the correlations in the quantum field has a strong role to play in where the information from the black hole can be stored and transferred. To see this in a clearer light, Calzetta and I have constructed a platform to describe the statistical mechanical properties of interacting quantum fields, i.e., in terms of the dynamics of the correlation functions, after the Boltzmann-BBGKY scheme \[14, 27\]. The main theme contained in two recent papers \[63, 66\] can be summarized as follows: Starting from the thesis that the full dynamics of an interacting quantum

---

\[4\] This can perhaps be compared to the difference between diamagnetism and paramagnetism, the former requires the field first to polarize the media before interacting with it, thus the extra factor of interaction involved.
field may be described by means of the Dyson-Schwinger equations governing the infinite hierarchy of Wightman functions which measure the correlations of the field, we showed how this hierarchy of equations can be obtained from the variation of the infinite particle irreducible, or ‘master’ effective action (MEA). Truncation of this hierarchy gives rise to a quantum subdynamics governing a finite number of correlation functions (which constitute the ‘system’), and expression of the higher order correlation functions (which constitute the ‘environment’) in terms of the lower-order ones by functional relations (‘slaving’ or ‘factorization’) induces dissipation in the dynamics of the subsystem driven by the stochastic fluctuations of the environment, which we call the ‘correlation noises’. These two aspects are related by the fluctuation-dissipation relation. This is the quantum field equivalent of the BBGKY hierarchy in Boltzmann’s theory. Any subsystem involving a finite number of correlation functions defines an effective theory, which is, by this reasoning, intrinsically dissipative. The relation of loop expansion and correlation order is expounded. We see that ordinary quantum field theory which involves only the mean field and a two-point function, or any finite-loop effective action in a perturbative theory are, by nature, effective theories which possess these properties. Histories defined by lower-order correlation functions can be decohered by the noises from the higher order functions and acquire classical stochastic attributes. The present scheme invoking the correlation order is a natural way to describe the quantum to classical transition for a closed system as it avoids ad hoc stipulation of the system-environment split. It is through decoherence that the subsystem variables become classical and the subdynamics becomes stochastic.

The application of such a conceptual scheme (correlation dynamics of interacting field) for addressing the black hole information paradox problem is currently under study. A sketch of the main ideas (including scaling properties of fields at infrared limit as induced by the black hole) are described in [67], where the reader will also find discussions of metric fluctuations and Einstein-Langevin equation. At a simpler level, recently Anglin et al [68] has used a particle - (free) field model to illustrate recoherence, and drew implications of their results on the black hole information paradox.

Here we have only presented a sampling of current problems in semiclassical gravity which share with mesoscopic physics some common concerns of the basic issues in both fields. We hope this sketch can induce more thoughts and discussions and bring out more interesting insights beneficial to the development of both disciplines.

Acknowledgement  Many ideas I discussed or proposed here are based on results obtained from work done over the past ten years in collaboration with Esteban Calzetta, Salman Habib, Andrew Matacz, Denjoe O’Connor, Juan Pablo Paz, Apan Raval, Kazutomu Shiokawa, Sukanya Sinha, Chris Stephens and Yuhong Zhang. I thank them for sharing the pleasure and excitement of searches and insights. (Any misrepresentation or misconception is sheerly due to my own ignorance or misjudgement.) I have also benefited from discussions with Prof. Zhao Bin Zu of Academia Sinica, China, and Profs. Ping Sheng, Zhao Qing
References

[1] For a summary of work up to 1980, see, e.g., the reviews of B. S. De Witt and S. W. Hawking, in *General Relativity, an Einstein Centenary Survey* ed. S. W. Hawking and W. Israel (Cambridge University Press, Cambridge, 1979) and essays in *Quantum Gravity 2, A Second Oxford Symposium*, eds. C. J. Isham, R. Penrose and D. W. Sciama (Claredon Press, Oxford, 1981); For more recent work, see, e.g., A. Ashtekar and J. Stachel, eds. *Conceptual Problems in Quantum Gravity* (Birkhäuser, Boston, 1991) C. J. Isham, *Conceptual and Geometrical Problems in Canonical Quantum Gravity*, in *Recent Aspects of Quantum Fields* eds. H. Mitter and H. Gausterer (Springer, Berlin 1992); K. Kuchar, *Canonical Quantum Gravity*, in *General Relativity and Gravitation 1992*, eds R. J. Gleiser et al (IOP, London, 1993) For a recent review of a general nature, see, C. J. Isham, “Structural Issues in Quantum Gravity”, Plenary talk at GR14, Florence, August, 1995. [gr-qc/9510063](http://arxiv.org/abs/gr-qc/9510063)

[2] See, e.g., S. W. Hawking in *Relativity, Groups and Topology II*, *Les Houches, Session XL, 1983*, (Elsevier Science Publishers B. V., 1984); J. B. Hartle, in *Gravitation and Quantizations*, *Les Houches 1992* eds. B. Julia and J. Zinn-Justin (North Holland, Amsterdam, 1994). S. Coleman, J. B. Hartle, T. Piran and S. Weinberg, eds. *Quantum Cosmology and Baby Universes* (World Scientific, Singapore, 1991)

[3] See, e.g., B. L. Altshuler, P. A. Lee and R. A. Webb, eds, *Mesoscopic Phenomena in Solids* (North Holland, Amsterdam, 1991). B. K. Kramer, ed., *Quantum Coherence in Mesoscopic Systems* (Plenum Press, New York, 1991). W. P. Kirk and M. A. Reed, eds, *Nanostructures and Mesoscopic Systems* (Academic Press, San Diego, 1992). F. A. Buot, Phys. Rep. 234, 73-174 (1993). S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, 1995). P. Sheng, *Introduction to Wave Scattering, Localization and Mesoscopic Phenomena* (Academic Press, New York, 1995).

[4] K. Shiookawa and B. L. Hu, article in this volume. Details are in [chao-dyn/9503009](http://arxiv.org/abs/chao-dyn/9503009), Phys. Rev. E 52, 2497 (1995)

[5] N. Birrell and P. W. C. Davies, *Quantum Fields in Curved Spaces* (Cambridge University Press, Cambridge, 1982)
[6] B. L. Hu, “Cosmology as ‘Condensed Matter’ Physics” in Proc. Third Asia Pacific Physics Conference, ed. Y. W. Chan et al (World Scientific, Singapore, 1988) Vol. 1, p. 301. gr-qc/9511076

[7] B. L. Hu, ”Fluctuation, Dissipation and Irreversibility in Cosmology” in The Physical Origin of Time-Asymmetry, Huelva, Spain, 1991 eds. J. J. Halliwell, J. Perez-Mercader and W. H. Zurek (Cambridge University Press, Cambridge, 1994).

[8] L. Smolin, “Cosmology as a Problem in Critical Phenomena” in Complex Systems and Binary Networks eds. L. Lopez-Pena et al (Springer, Berlin, 1995) gr-qc/9505022

[9] L. Parker, Phys. Rev. 183, 1057 (1969). R. U. Sexl and H. K. Urbantke, Phys. Rev. 179, 1247 (1969). Ya. B. Zel’dovich, Pis’ma Zh. Eksp. Teor. Fiz, 12, 443 (1970) [Sov. Phys. - JETP Lett. 12, 307(1970)]

[10] Ya. Zel’dovich and A. Starobinsky, Zh. Eksp. Teor. Fiz 61, 2161 (1971) [Sov. Phys.-JETP 34, 1159 (1971)] L. Grishchuk, Ann. N. Y. Acad. Sci. 302, 439 (1976). B. L. Hu and L. Parker, Phys. Lett. 63A, 217 (1977). B. L. Hu and L. Parker, Phys. Rev. D17, 933 (1978). F. V. Fischetti, J. B. Hartle and B. L. Hu, Phys. Rev. D20, 1757 (1979). J. B. Hartle and B. L. Hu, Phys. Rev. D20, 1772 (1979); 21, 2756 (1980) J. B. Hartle, Phys. Rev. D23, 2121 (1981)

[11] S.W. Hawking, Commun. Math. Phys. 43, 199 (1975). W.G. Unruh, Phys. Rev. D 14, 870 (1976).

[12] A. H. Guth, Phys. Rev. D 23, 347 (1981). A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. 48, 1220 (1982). A. D. Linde, Phys. Lett. 114B, 431 (1982). Phys. Lett. 162B, 281 (1985).

[13] B. L. Hu, “Quantum Statistical Processes in the Early Universe” in Quantum Physics and the Universe, Proc. Waseda Conference, Aug. 1992 eds. M. Namiki et al (Pergamon Press, Tokyo, 1993). Vistas in Astronomy 37, 391 (1993). gr-qc/9302031

[14] B. L. Hu and A. Matacz, “Quantum Noise in Gravitation and Cosmology” in Proc. International Workshop on Fluctuations and Order: A New Synthesis, Los Alamos, Sept. 1993, Proceedings edited by Marko Millonas (Springer-Verlag, Berlin, 1995). University of Maryland Preprint umdpp94-44. astro-ph/9312012

[15] B. L. Hu, “Quantum Statistical Fields in Gravitation and Cosmology” in Proc. Third International Workshop on Thermal Field Theory and Applications, eds. R. Kobes and G. Kunstatter (World Scientific, Singapore, 1994) gr-qc/9403061

[16] B. L. Hu, “Nonequilibrium Quantum Fields in Cosmology: Comments on Selected Current Topics” in Second Paris Cosmology Colloquium Observatorie de Paris, June 2-4, 1994 eds H. J. de Vega and N. Sanchez (World Scientific, Singapore, 1995) gr-qc/9409053
[17] W. H. Zurek, Phys. Rev. D24, 1516 (1981); D26, 1862 (1982); in Frontiers of Nonequilibrium Statistical Physics, ed. G. T. Moore and M. O. Scully (Plenum, N. Y., 1986); Physics Today 44, 36 (1991); E. Joos and H. D. Zeh, Z. Phys. B59, 223 (1985); A. O. Caldeira and A. J. Leggett, Phys. Rev. A 31, 1059 (1985); W. G. Unruh and W. H. Zurek, Phys. Rev. D40, 1071 (1989). B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. D45, 2843 (1992); D47, 1576 (1993); J. P. Paz, S. Habib and W. H. Zurek, Phys. Rev. D47, 488 (1993). W. H. Zurek, J. P. Paz and S. Habib, Phys. Rev. Lett. 70, 1187 (1993); W. H. Zurek, Prog. Theor. Phys. 89, 281 (1993).

[18] R. B. Griffiths, J. Stat. Phys. 36, 219 (1984); R. Omnés, J. Stat Phys. 53, 893, 933, 957 (1988); Ann. Phys. (N. Y.) 201, 354 (1990); Rev. Mod. Phys. 64, 339 (1992); The Interpretation of Quantum Mechanics, (Princeton University Press, Princeton (1994)). J. B. Hartle, “Quantum Mechanics of Closed Systems” in Directions in General Relativity Vol. 1, eds B. L. Hu, M. P. Ryan and C. V. Vishveswara (Cambridge Univ., Cambridge, 1993); M. Gell-Mann and J. B. Hartle, in Complexity, Entropy and the Physics of Information, ed. by W. H. Zurek (Addison-Wesley, Reading, 1990); J. B. Hartle and M. Gell- Mann, Phys. Rev. D47, 3345 (1993). J. P. Paz and S. Sinha, Phys. Rev. D44, 1038 (1991). H. F. Dowker and J. J. Halliwell, Phys. Rev. D46, 1580 (1992). T. Brun, Phys. Rev. D47, 3383 (1993). J. Tammely, Phys. Rev. D48, 5730 (1993). J. P. Paz and W. H. Zurek, Phys. Rev. D48, 2728 (1993).

[19] M. C. Gutzwiller in Chaos in Classical and Quantum Mechanics (Springer-Verlag 1990). W. Dieter Heiss, Chaos and Quantum Chaos, Lecture Notes in Physics Vol. 411 (Springer-Verlag 1992). K. Nakamura, Quantum Chaos (Cambridge University Press, Cambridge, 1993).

[20] W. H. Zurek and J. P. Paz, Phys. Rev. Lett. 72, 2508 (1994).

[21] A. O. Caldeira and A. J. Leggett, Ann. Phys. (N. Y.) 149, 374 (1993).

[22] J. Schwinger, Proc. Nat. Acad. Sci. USA 90, 7285 (1993)

[23] See, e.g., Fluctuating Geometries in Statistical Mechanics and Field Theory, eds F. David, P. Ginsparg and J. Jinn-Justin, Les Houches Session LXII 1994 (North-Holland, Amsterdam, 1996)

[24] R. K. P. Zia, “Driven Diffusive Systems” in Phase Transitions and Critical Phenomena Vol. 20, eds. C. Domb and J. Lebowitz (Academic Press, New York, 1995)

[25] See, e.g., F. Family and T. Vicsek, Dynamics of Fractal Surfaces (World Scientific, Singapore, 1991); R. Julien et al (eds), Surface Disordering: Growth, Roughening, and Phase Transitions, Les Houches Workshop, 1992 (Nova Science, Commack, N. Y. 1992)

[26] See, e.g., M. N. Barber, in Phase Transitions and Critical Phenomena Vol. 8, eds. C. Domb and J. Lebowitz (Academic Press, New York, 1983); J. L. Cardy, ed Finite Size Scaling (North Holland, Amsterdam 1988)
[27] E. Calzetta and B. L. Hu, Phys. Rev. D37, 2878 (1988)

[28] J. J. Halliwell, “A Bibliography of Quantum Cosmology” Int. J. Mod. Phys. A5, 2473 (1990)

[29] J. B. Hartle and S. W. Hawking, Phys. Rev. D28, 1960 (1983)

[30] A. Vilenkin, Phys. Rev. D27, 2848 (1983), D30, 509 (1984); Phys. Lett. 117B, 25 (1985)

[31] J. P. Paz and S. Sinha, Phys. Rev. D44, 1038 (1991); ibid D45, 2823 (1992).

[32] B. L. Hu, “General Relativity as Hydrodynamics” unpublished talks (1994)

[33] M. B. Green, J. H. Schwarz and E. Witten, Superstring Theory (Cambridge University, Cambridge, 1990). E. Witten, Physics Today (1996)

[34] A. Ashtekar, Lectures on Non-Perturbative Canonical Gravity (World Scientific, Singapore, 1991); A. Ashtekar and J. Lewandowski, in Knot Theory and Quantum Gravity ed. J. Baez (Oxford University, London, 1995); C. Rovelli and L. Smolin, “Spin Networks in Quantum Gravity”, Phys. Rev. D (1996)

[35] E. Calzetta, Phys. Rev. D42, 4066 (1990).

[36] S. Habib and R. Laflamme, Phys. Rev. 42, 4056 (1990).

[37] J. P. Paz, S. Habib and W. H. Zurek, Phys. Rev. 47, 488 (1993).

[38] H. Spohn, Large Scale Dynamics of Interacting Particles (Springer-Verlag, Berlin 1991)

[39] J. Ambjørn, B. Durhuus and J. Frölich, Nucl. Phys. B257 [FS14], 433 (1985); B275 [FS17], 161 (1986); J. Ambjørn, B. Durhuus, J. Frölich and P. Orland, Nucl. Phys. B270 [FS16], 457 (1986); F. David, Nucl. Phys. B257 [FS14], 45, 543 (1985); Phys. Lett. 159B, 303 (1985) V. A. Kazakov, Phys. Lett. 150B, 282 (1985); V. A. Kazakov, I. K. Kostov and A. A. Migdal, Phys. Lett. 157B, 295 (1985); E. Brezin and V. A. Kazakov, Phys. Lett. 236B, 144 (1990) D. J. Gross and A. A. Migdal, Phys. Rev. Lett. 64, 717 (1990); Nucl. Phys. B340, 333 (1990) M. R. Douglas and S. H. Shenker, Nucl. Phys. B335, 635 (1990). J. Ambjørn, B. Durhuus and T. Jonsson, Mod. Phys. Lett. A6, 1133 (1991); M. E. Agishtein adn A. A. Migdal, Mod. Phys. Lett. A6, 1863 (1991)

For recent reviews, see, e.g., Statistical Mechanics of Membranes and Surfaces eds D. Nelson et al (World Scientific, Singapore, 1989); Two-Dimensional Quantum Gravity and Random Surfaces eds D. J. Gross, T. Piran and S. Weinberg (World Scientific, Singapore, 1992), and [23].

[40] T. Regge, Nuovo Cimento 19, 558 (1961). G. Ponzano and T. Regge, “Semiclassical limit of Racah coefficients” in Spectroscopic and Group Theoretical Methods in Physics ed. F. Bloch (North Holland, Amsterdam, 1968) For recent work, see, e.g., J. B. Hartle,
J. Math. Phys. 26, 804 (1985); 27, 287 (1986); 30, 452 (1989). H. W. Hamber, in *Critical Phenomena, Random Systems, Gauge Theories* 1984 Les Houches Summer School, eds K. Osterwalder and R. Stora (North Holland, Amsterdam, 1986). H. W. Hamber, Nucl. Phys. B (Proc. Suppl) 20, 728 (1991); 25A, 150 (1992); Phys. Rev. D45, 507 (1992); Nucl. Phys. B400, 347 (1993); R. M. Williams and P. A. Tucker, Class. Quant. Grav. 9, 1409 (1992). H. W. Hamber and R. M. Williams, Phys. Rev. D47, 510 (1993); Nucl. Phys. 415, 463 (1994) J. W. Barrett and T. J. Foxon, Class. Quant. Grav. 11, 543 (1994). J. Iwasaki, “A reformulation of the Ponzano- Regge quantum gravity models in terms of surfaces” (1994) J. W. Barrett, “Quantum Gravity as topological quantum field theory” (1995)

[41] B. L. Hu and D. J. O'Connor, Phys. Rev. D36, 1701 (1987).

[42] F. Copper, S. Habib, Y. Kluger, E. Motolla, J. P. Paz and P. R. Anderson, Phys. Rev. D50, 2848 (1994)

[43] D. Boyanovsky, H. J. de Vega and R. Holman, Phys. Rev. D49, 2769 (1994); D. Boyanovsky, H. J. de Vega, R. Holman, D.-S. Lee and A. Singh, Phys. Rev. D51, 4419 (1995) For a recent review, see, D. Boyanovsky, H. J. de Vega and R. Holman, in *Second Paris Cosmology Colloquium* Observatorie de Paris, June 2-4, 1994 eds H. J. de Vega and N. Sanchez (World Scientific, Singapore, 1995) and references therein.

[44] E. Calzetta, “Spinodal Decomposition in Quantum Field Theory” Ann. Phys. (N.Y.) 190, 32 (1989). M. Gleiser, G. C. Marques and R. O. Ramos, Phys. Rev. D48, 1571 (1993); M. Gleiser and R. O. Ramos, Phys. Rev. D50, 2441 (1994). For a recent review, see, M. Gleiser, in *Second Paris Cosmology Colloquium* Observatorie de Paris, June 2-4, 1994 eds H. J. de Vega and N. Sanchez (World Scientific, Singapore, 1995) and references therein.

[45] See, e.g., H. H. Gutbrod and J. Rafelski, eds, *Particles Production in Highly Excited Matter* (Plenum, New York, 1993). L. P. Csernai, *Introduction to Relativistic Heavy Ion Collisions* (Wiley, New York, 1994)

[46] *Proc. Second International Workshop on Thermal Fields and Their Applications*, eds. H. Ezawa et al (North-Holland, Amsterdam, 1991). *Proc. Third International Workshop on Thermal Field Theory and Applications*, eds. R. Kobes and G. Kunstatter (World Scientific, Singapore, 1994)

[47] A. A. Starobinsky, in *Field Theory, Quantum Gravity and Strings*, ed. H. J. de Vega and N. Sanchez (Springer, Berlin 1986); J. M. Bardeen and G. J. Bublik, Class. Quan. Grav. 4, 473 (1987); S. J. Rey, Nucl. Phys. B284, 706 (1987).

[48] B. L. Hu and Y. Zhang, “Coarse-Graining, Scaling, and Inflation” Univ. Maryland Preprint 90-186 (1990); B. L. Hu, in *Relativity and Gravitation: Classical and Quantum
Proc. SILARG VII, Cocoyoc, Mexico 1990. eds. J. C. D’ Olivo et a (World Scientific, Singapore 1991).

[49] B. L. Hu, J. P. Paz and Y. Zhang, “Quantum Origin of Noise and Fluctuation in Cosmology” in The Origin of Structure in the Universe Conference at Chateau du Pont d’Oye, Belgium, April, 1992, ed. E. Gunzig and P. Nardone (NATO ASI Series) (Plenum Press, New York, 1993) p. 227

[50] E. Calzetta and B. L. Hu, Phys. Rev. D49, 6636 (1994)

[51] M. Gell- Mann and J. B. Hartle, Phys. Rev. D47, 3345 (1993).

[52] B. L. Hu, “Statistical Mechanics and Quantum Cosmology”, in Proc. Second International Workshop on Thermal Fields and Their Applications, eds. H. Ezawa et al (North-Holland, Amsterdam, 1991).

[53] B. L. Hu, Physica A158, 399 (1989); B. L. Hu and S. Sinha, Phys. Rev. D51, 1587 (1995).

[54] B. L. Hu and A. Matacz, Phys. Rev. D 51, 1577 (1995)

[55] A. Campos and E. Verdaguer, Phys. Rev. D53, (1996)

[56] C.-I. Kuo and L. H. Ford, Phys. Rev. D47, 4510 (1993)

[57] N. G. Phillips and B. L. Hu, “Energy Momentum Fluctuations of Quantum Fields in Curved Spacetimes” Maryland preprint (1996)

[58] E. Calzetta and B. L. Hu, “Quantum Fluctuations, Decoherence of the Mean Field and Structure Formation in the Early Universe” gr-qc/9505046, Phys. Rev. D52, (1995)

[59] B. L. Hu, J. P. Paz and Y. Zhang, Phys. Rev. D45, 2843 (1992); D47, 1576 (1993)

[60] J. R. Anglin, Phys. Rev. D47, 4525 (1993)

[61] B. L. Hu and A. Matacz, Phys. Rev. D49, 6612 (1994)

[62] Alpan Raval, B. L. Hu and J. R. Anglin, “Stochastic Theory of Accelerated Detectors in a Quantum Field” (1995) gr-qc/9510002

[63] Alpan Raval and B. L. Hu, “Quantum Field Correlations Around an Accelerated Detector” (1995)

[64] R. Balescu, Equilibrium and Nonequilibrium Statistical Mechanics (John Wiley, New York, 1975)
[65] E. Calzetta and B. L. Hu, “Decoherence of Correlation Histories” in Directions in General Relativity, Vol II: Brill Festschrift, eds B. L. Hu and T. A. Jacobson (Cambridge University Press, Cambridge, 1993) gr-qc/9302013

[66] E. Calzetta and B. L. Hu, “Correlations, Decoherence, Dissipation and Noise in Quantum Field Theory”, in Heat Kernel Techniques and Quantum Gravity, ed. S. Fulling (Texas A& M Press, College Station 1995). hep-th/9501040

[67] B. L. Hu, “Correlation Dynamics of Quantum Fields and Black Hole Information Paradox” Erice Lectures, Sept. 1995. gr-qc/9511073

[68] J. R. Anglin, R. Laflamme, W. H. Zurek and J. P. Paz, Phys. Rev. D52, 2221 (1995)