Nuclear Modification of Double Spin Asymmetries

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Abstract

We compute nuclear spin dependent structure functions using a dynamical model for bound nucleon densities and hence calculate nuclear modifications to asymmetries observed in recent doubly polarised deep inelastic scattering experiments. We conclude that while the individual densities are changed substantially by nuclear effects, the asymmetries themselves are largely insensitive to these changes.

Recently a model was proposed [1] to explain the observed differences between free nucleon and bound nucleon structure functions in deep inelastic lepton nucleon scattering (DIS). This model used a dynamical approach, involving modifying a free nucleon input density distribution at a low input scale, $Q^2 = \mu^2 = 0.23$ GeV$^2$, due to nuclear effects, and then evolving the resultant modified bound nucleon densities to the required $Q^2$ scale of the experiment. The model gave satisfactory agreement with available data in a fairly broad $Q^2$ range, from 0.5–30 GeV$^2$.

It is interesting to ask how this model can be extended to a study of spin dependent bound-nucleon densities. The question is not merely academic as, in fact, data on the spin dependent deuteron and neutron structure functions have been obtained [2] from deuteron and $^3$He targets. Nuclear effects in deuteron are known to be small (though measurable), since the deuteron is a loosely bound nucleus. There have been a number of papers [3] dealing with nuclear modifications of spin asymmetries and structure functions in the case of the deuteron. We therefore confine our attention to possible nuclear effects on the double spin asymmetry measurements made with helium nuclei. In this case, it was pointed out by Woloshyn [4] that the protonic contribution to the asymmetry is negligible so that the $^3$He double spin asymmetry is sensitive to the spin dependent neutron structure function, $g_1^n(x, Q^2)$. However, there may be additional modifications due to the presence of the nuclear medium, which we propose to study here. These are especially of importance for checking the validity of the Bjorken Sum rule. Our main conclusion is that the individual (spin independent as well as spin dependent) structure functions undergo substantial modifications due to nuclear effects; however, their ratio—the asymmetry—which is the measured quantity, is largely free from these and so gives hope that the neutron structure function may be unambiguously determined from such a measurement.

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2Depending on the model, corrections due to nuclear effects in deuterium can be as large as 10%.
The quantity of interest is the double spin asymmetry,

\[ A_A^A(x, Q^2) = \frac{g_A^A(x, Q^2)}{F_1^A(x, Q^2)}, \]

where \( g_1 \) and \( F_1 \) are the spin dependent and spin independent structure functions corresponding to the nucleus \( A \). We are therefore interested in studying possible deviations of the measured asymmetry, \( A_{He}^1 \), from the required neutron asymmetry, \( A_n^1 \), due to nuclear effects.

As a starting point we note that the corresponding study of unpolarised bound nucleon densities used the GRV \[5\] density parametrisation as an input. We shall therefore use the GRVs \[6\] spin dependent densities as an input in the corresponding polarised problem. This is essential if we are to retain the definition,

\[
\tilde{q}^f(x) = q^+_f(x) + q^-_f(x);
\]
\[
\tilde{q}^f(x) = q^+_f(x) - q^-_f(x),
\]

where \( q_f^+ \) and \( q_f^- \) are the positive and negative helicity densities of \( f \)-flavour quarks (and similarly for gluons) in either free or bound nucleons.

\section{The Spin Independent Nuclear Densities}

We now quickly review the model before we apply it to the polarised case. This is also useful as the polarised case essentially follows along the lines of the unpolarised problem. The free nucleon input densities are modified by both nuclear swelling and binding effects. Nucleon swelling causes not only a depletion of all parton densities at large- and small-\( x \), but also an enhancement at intermediate-\( x \) \[7\]. The relative increase in the nucleon’s radius is \( \delta_A \), where \( \frac{R_N + \Delta R(A)}{R_N} = 1 + \delta_A \), and is given by,

\[ \delta_A = [1 - P_s(A)]\delta_{vol} + P_s(A)\delta_{vol}/2. \]

The second term corrects for surface effects in the usual manner. Here \( P_s(A) \) is the probability of finding a nucleon on the nuclear surface while \( \delta_{vol} \) parametrises the swelling of the nucleon in the interior of a heavy nucleus and is the only free parameter in the calculation. It was fixed to be \( \delta_{vol} = 0.15 \) in the unpolarised calculation \[1\].

The distortions of the density distributions due to swelling, being purely geometrical, conserve the total parton number and momentum of each parton species (i.e., the first and second moments of the distributions are unchanged). Furthermore, the third moments are modified in a well-determined way. Specifically, the first three moments of the parton distributions in a free \( q_N \) and bound \( q_A \) nucleon at \( \mu^2 \) are related by

\[
\frac{\langle q_A(\mu^2) \rangle_1}{\langle q_N(\mu^2) \rangle_1} = \frac{\langle q_A(\mu^2) \rangle_2}{\langle q_N(\mu^2) \rangle_2},
\]

\[
\frac{\langle q_N(\mu^2) \rangle_3 - \langle q_N(\mu^2) \rangle_2^{1/2}}{\langle q_A(\mu^2) \rangle_3 - \langle q_A(\mu^2) \rangle_2^{1/2}} = 1 + \delta_A.
\]
The first two equations imply number and momentum conservation of partons and the last incorporates the swelling effect \[7\].

These are then used as constraint equations to determine the bound-nucleon densities in terms of the known free-nucleon ones. We use the Glück, Reya and Vogt (GRV) parametrisation \[5\] of the free-nucleon distributions at \(Q^2 = \mu^2 = 0.23 \text{GeV}^2\). We find these most appropriate for our purpose as each of their input densities is integrable (there are finite number of partons at \(\mu^2\)).

It is now possible to determine the bound nucleon densities \(q_A\) (for \(q = \text{valence quarks, } u_V, d_V, \text{sea quarks, } S, \text{and gluons, } g\)) in terms of the free densities, \(q_N\). We parametrise the free as well as bound nucleon distributions in the form,

\[ q(x) = N x^\alpha (1 - x)^\beta P(x), \]  

where \(P(x)\) is a polynomial. We take \(P_{A,q}(x) = P_{N,q}(x)\), for simplicity. Then the changes in the three main parameters, \(N, \alpha, \) and \(\beta\), due to swelling, and hence the bound-nucleon densities, are immediately determined by the constraints in eq (3). This fixes the input bound-nucleon densities at \(Q^2 = \mu^2\).

We now discuss the binding effect. The attractive potential describing the nuclear force arises from the exchange of mesons. Hence the energy required for binding is taken away solely from the mesonic component of the nucleon, and not from its other components. At the starting scale, \(Q^2 = \mu^2\), we identify these mesons to be just the sea quarks in the bound nucleon. Hence, the momentum fraction carried by the sea quarks in a nucleon bound in a nucleus at \(Q^2 = \mu^2\) will be reduced. The extent of reduction is determined by the binding energy per nucleon \[1\], which is given by the well-known Weizäcker mass formula.

These nuclear effects completely determine the spin independent input bound-nucleon densities. These are then evolved using the usual Altarelli Parisi evolution equations to obtain the densities at a required value of \(Q^2\).

At the time of interaction, there is a further depletion of the sea densities, which occurs whenever there is nucleon-nucleon interaction, caused by parton–nucleon overlap. When a parton having a momentum fraction, \(x\), of the parent nucleon momentum, \(P_N\), is struck, it is off-shell and localised to a distance, \(\Delta Z \sim 1/(2xP_N)\) (in the Breit frame). For sufficiently small \(x\), \(\Delta Z\) becomes large and can exceed the average 2–nucleon separation\[3\].

The struck parton must return to the parent nucleon within the interaction time, as required by the uncertainty principle. However, while it extends outside the parent nucleon, it can interact with other nucleons in the nucleus. Such an interaction between two nucleons caused by parton–nucleon overlap results in loss of energy of the parent nucleon, mimicking exactly the effect of binding. Hence we call this the second binding effect and assume its strength to be the same as that due to the usual binding. This immediately fixes the loss in sea quarks (valence quarks are not depleted due to the requirement of quantum number conservation) due to this effect to be

\[ S'_A(x, Q^2) = K'(A)S_A(x, Q^2), \]  

\[ \text{(5a)} \]

\(^3\)Although the spatial extent of a single coloured parton cannot exceed the range of QCD confinement, the struck parton can combine with a wee parton and form a colourless scalar with vacuum quantum numbers which can then escape from the nucleon.
where the depletion factor is,

\[
K'(A) = \begin{cases} 
1, & \text{when } x > x_0; \\
1 - 2\beta(x_0x^{-1} - 1), & \text{when } x_A < x < x_0; \\
1 - 2\beta(x_0x_A^{-1} - 1), & \text{when } x < x_A,
\end{cases}
\]  
(5b)

where \(\beta\) is the same as in usual binding, viz.,

\[
\beta = \frac{U(\mu^2)}{M_N\langle S_N(\mu^2)\rangle_2} = 0.037/2 ,
\]  
(6)

\(U(\mu^2)\) being the binding energy between each pair of nucleons, which is known. The limiting values, \(x_0 = 1/(2M_N d_N)\) (where \(d_N\) is the average correlation distance between two neighbouring nucleons in the lab frame), and \(x_A = 1/(4\overline{R}_A M_N)\) (where \(2\overline{R}_A \simeq 1.4R_A\) is the average thickness of the nucleus), determine the starting and saturation values respectively of this shadowing effect; the latter occurs when the struck quark wave function completely overlaps the nucleus in the \(z\)-direction. In general, a parton with a momentum fraction, \(x\), \(x_A \leq x \leq x_0\), can overlap \((n - 1)\) other nucleons, where \(n = 1/(2M_N d_N x) = x_0 / x\). Due to the applicability of the superposition principle to the scalar field interaction with various nucleons, the loss of energy due to interaction with each of the nucleons over which the struck quark wave function extends, is equal and additive, and thus explains the depletion factor in eq (5). Since this effect acts on the intermediate state of the probe–target interaction, it does not participate in the QCD evolution of the initial state.

Nuclear modification due to binding and swelling at the input scale \(Q^2 = \mu^2\), and parton-nucleon overlap due to the second binding effect at the \(Q^2\) scale of the scattering together determine the structure function, \(F_1^A(x, Q^2)\), of a nucleon bound in a nucleus \(A\). The model gives good agreement with available data [1].

We now proceed to an analysis of the corresponding spin dependent densities.

\section{The Spin Dependent Nuclear Densities}

The same nuclear effects of binding and swelling affect the spin dependent densities also. This is because they influence the positive and negative helicity densities, out of which the spin independent and spin dependent densities are composed (see eq (2)). The entire swelling effect can now be rephrased as the effect of swelling on individual helicity densities, so that equations analogous to (3) are valid for the spin dependent densities, \(\tilde{q}(x)\), as well. This can be seen as follows: Swelling simply rearranges the parton distributions in the bound nucleon; there is no change in the number of each parton species. In particular, each helicity type is also conserved, i.e.,

\[
\int q_A^+(x, \mu^2)dx = \int q_N^+(x, \mu^2)dx , \quad \int q_A^-(x, \mu^2)dx = \int q_N^-(x, \mu^2)dx .
\]

Hence, their sum and difference is also conserved. The former is contained in the first equation of the equation set (3); the latter implies, for the polarised combination,

\[
\langle \tilde{q}_A(\mu^2) \rangle_1 = \langle \tilde{q}_N(\mu^2) \rangle_1 .
\]  
(7a)
Note that $\langle q(\mu^2) \rangle_n = \langle q^+(\mu^2) \rangle_n + \langle q^-(\mu^2) \rangle_n$ for every moment, $n$, for both the free and bound nucleon, and similarly for the spin dependent density as well. Similarly, since the momentum carried by each helicity density is unchanged, momentum conservation between the free and bound nucleon also holds for the sum and difference of the helicity densities. The corresponding equation for the sum is the second equation in (3); the equation for the helicity difference is

$$\langle \tilde{q}_A(\mu^2) \rangle_2 = \langle \tilde{q}_N(\mu^2) \rangle_2 .$$

(7b)

The extension of the third of the equations in (3) to the spin dependent case is not as straightforward. Every helicity density, $q^h(x)$, $(h = +, -)$, spreads out over a larger size, or, equivalently, gets pinched in momentum space, according to Heisenberg’s uncertainty relation, $\Delta p \Delta x = 1$. Applying this to each helicity type, for each flavour, we have,

$$\frac{\langle q^+(\mu^2) \rangle_3 - \langle q^+_A(\mu^2) \rangle_2^{1/2}}{\langle \tilde{q}^+_A(\mu^2) \rangle_3 - \langle \tilde{q}^+_A(\mu^2) \rangle_2^{1/2}} = 1 + \delta_A ; \quad \frac{\langle q^-_N(\mu^2) \rangle_3 - \langle q^-_A(\mu^2) \rangle_2^{1/2}}{\langle \tilde{q}^-_A(\mu^2) \rangle_3 - \langle \tilde{q}^-_A(\mu^2) \rangle_2^{1/2}} = 1 + \delta_A .$$

(8)

However, for later convenience, we prefer to use analogous expressions for the sum and difference, $q_f$ and $\tilde{q}_f$, rather than for the individual helicity densities. Hence, the third of the constraints arising from swelling, i.e., the third of eq (3) and its spin dependent counterpart read,

$$\frac{\langle q_N(\mu^2) \rangle_3 - \langle q_N(\mu^2) \rangle_2^{1/2}}{\langle q_A(\mu^2) \rangle_3 - \langle q_A(\mu^2) \rangle_2^{1/2}} = 1 + \delta_A ; \quad \frac{\langle \tilde{q}_N(\mu^2) \rangle_3 - \langle \tilde{q}_N(\mu^2) \rangle_2^{1/2}}{\langle \tilde{q}_A(\mu^2) \rangle_3 - \langle \tilde{q}_A(\mu^2) \rangle_2^{1/2}} = 1 + \delta_A .$$

(7c)

The error involved between the exact expressions, eq (8), and their approximations, eq (7c), is a term proportional to $(1 - (1 + \delta_A)^2)$ and is of order $\delta_A$. This term mixes spin dependent and spin independent moments; however, since $\delta_A$ is small (about 10%), these errors are small, and can be ignored. We are therefore justified in using eq (7c) rather than eq (8) to constrain the second moments of the parton densities. The three sets of equations, (7a–c), thus provide the three sets of constraint equations, analogous to the set (3), with which we can fix the input bound nucleon spin dependent densities.

The modified input densities are thus determined, given a set of valid input free nucleon distributions, which we take to be the Glück, Reya, and Vogelsang ‘standard’ set (GRVs) [8]. These densities can also be parametrised in a form similar to eq (4); in fact, every spin dependent density is a factor of the form of the RHS of eq (4) times the corresponding unpolarised density. Hence there are again three constraint equations which serve to fix the three main parameters, $\alpha$, $\beta$, and $N$ for the corresponding bound nucleon spin dependent densities.

Binding causes loss of energy in the sea: this is due to loss of mesons from the nucleon. Since these mesons are spin-0 bosons, it is clear that no spin is lost from the sea due to binding (equal numbers of positive and negative helicity partners are lost). Hence we see that binding changes the sum, but not the difference of the helicity densities[4].

We thus obtain the input polarised densities analogous to the unpolarised ones. These are then evolved to the scale of interest.

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[4] It is possible that $\rho$, etc., mesons also participate in this interaction, leading to a change in the polarised sea densities, but this component is small and we neglect it.
At the time of interaction, the second binding effect applies to struck partons with momentum fraction \( x \leq x_0 \), as in the unpolarised case. The mechanism for this depletion is independent of the helicity of the quark, and so this effect is identical in both the spin independent as well as spin dependent cases. Hence, the spin dependent structure function, \( g_A^1(x, Q^2) \) can now be computed by evolving the modified input spin dependent densities to the required value of \( Q^2 \), and including the second binding effect.

Finally, we display the equivalent neutron bound-nucleon structure functions at an arbitrary scale, \( Q^2 > \mu^2 \) (with \( R = \sigma_L/\sigma_T = 0 \)):

\[
\begin{align*}
F_{1n/A}(x, Q^2) &= \frac{1}{18} \left[ u_v^A(x, Q^2) + 4 d_v^A(x, Q^2) + K'(A) S_A(x, Q^2) \right], \\
g_{1n/A}(x, Q^2) &= \frac{1}{18} \left[ \bar{u}_v^A(x, Q^2) + 4 \bar{d}_v^A(x, Q^2) + K'(A) \bar{S}_A(x, Q^2) \right].
\end{align*}
\]

\( q^A(x, \mu^2) \) incorporates the effect of swelling on every input parton density, \( q^N(x, \mu^2) \), as well as that of binding for the unpolarised densities, and the corresponding \( Q^2 \)-dependent quantities that appear here are these input densities, evolved suitably to the required scale. \( K'(A) \) incorporates the second binding effect, at \( Q^2 \), as discussed above. The experimentally measured asymmetry, and quantity of interest, are the ratios, at the scale \( Q^2 \), for the neutron bound in the helium nucleus and for a free neutron:

\[
\begin{align*}
A_{\text{meas}} &= \frac{g_{1n/\text{He}}}{F_{1n/\text{He}}}, \\
A_{\text{reqd}} &= \frac{g_{1n}}{F_{1n}},
\end{align*}
\]

and can thus be computed. (We use the free and bound nucleon unpolarised structure functions from \[1\]). Note that the input spin dependent densities (which are taken from \[8\]) were actually fitted to both the free proton as well as deuteron and \(^3\)He spin dependent data; however, we use them here as the free nucleon parametrisations (which is permissible especially in view of the large error bars on presently available data). Furthermore, the smearing effect of Fermi motion (at large \( x \)) is neglected in this work for simplicity. Hence our results are not valid at large \( x \).

In fig. 1 we give the results of our computations for the measured (bound nucleon) and required (free nucleon) spin dependent structure function, \( g_{1n}^A \) for typical values of \( Q^2 \), \( Q^2 = 1, 4 \text{ GeV}^2 \). We see that the deviations of the bound neutron structure function can be as large as 10–15% at small \( x \) and about 6% at intermediate \( x \) values. The data points plotted on this graph correspond to the values extracted at \( Q^2 = 4 \text{ GeV}^2 \) from a measurement of the asymmetry by the E142 Collaboration \[8\] (with \( R = 0 \)) and indicate the size of the error bars in currently available data. In fig. 2, we plot the asymmetries at \( Q^2 = 4 \text{ GeV}^2 \). The data points here correspond exactly to the E142 data and therefore go over a range of \( Q^2 \) with a mean of about 2 GeV\(^2\); however, the asymmetry is not very sensitive to \( Q^2 \) in the \( x \) range of the available data. Notice that in this case, nuclear effects cause not more than 5% deviation in the asymmetry at both small and intermediate values of \( x \). The deviation is slightly larger at larger \( x \), \( x > 0.4 \), but this is due to the fact that the neutron spin dependent structure function changes sign near this value, and hence this deviation cannot be considered to be significant.

In short, we see that nuclear effects, though significant, equally affect both the spin dependent as well as the spin independent structure functions in such a way that the measured
asymmetries are to a great extent independent of them. Since it is the asymmetry rather than the structure function which is measured in a polarised experiment, much smaller errors on data are required before these small deviations due to nuclear effects become observable in such experiments. On the other hand, as already stated, this seems to make possible clean and unambiguous extraction of the relevant free nucleon structure functions from a measurement of double spin asymmetries with such light nuclear targets.

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Figure Captions

Fig. 1 The free and bound nucleon spin dependent structure function for $Q^2 = 1.4 \text{ GeV}^2$ as a function of $x$ are shown as solid and dashed lines respectively. The structure function data are extracted at $Q^2 = 4 \text{ GeV}^2$ from the asymmetries measured by the E142 collaboration.

Fig. 2 The bound and free nucleon asymmetries for $Q^2 = 4 \text{ GeV}^2$ as a function of $x$ are shown as solid and dashed lines respectively. The data are from the E142 collaboration.
Fig. 2