Monopole Solutions in AdS Space

A.R. Lugo, E.F. Moreno and F.A. Schaposnik

Departamento de Física, Universidad Nacional de La Plata
C.C. 67, 1900 La Plata, Argentina

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Abstract

We find monopole solutions for a spontaneously broken SU(2)-Higgs system coupled to gravity in asymptotically anti-de Sitter space. We present new analytic and numerical results discussing, in particular, how the gravitational instability of self-gravitating monopoles depends on the value of the cosmological constant.

1 Introduction

Gravitating monopole solutions to gauge theories have attracted many investigations in the last 25 years [1]-[15]. In particular, the existence of self-gravitating monopoles in spontaneously broken non-Abelian gauge theories, their properties, relation with black hole solutions and their relevance in Cosmology have been thoroughly discussed. Most of these investigations correspond to asymptotically flat space-time but there have been recently several studies for the case in which the cosmological constant $\Lambda$ is non-vanishing [16]-[18]. In particular, we have discussed in [18] the existence of gravitating monopole solutions in the case in which space-time is asymptotically anti-de Sitter (AdS), which in our conventions corresponds to $\Lambda < 0$. Regular (monopole and dyon) and singular (black hole) solutions have been found in this case and the properties of the magnetically charged solutions for vanishing Newton constant $G$ were analysed.

It is the purpose of the present work to complete the investigation initiated in [18] studying in detail the monopole solution in asymptotically AdS space both for vanishing and finite $G$, analytically and numerically. The plan of the paper is the following: we present in Section II the model, the spherically symmetric ansatz and the appropriate boundary conditions leading to gravitating

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monopoles and dyons. Then, in Section III, we discuss, analytically, some relevant properties of the magnetically charged solution and then describe in detail the numerical results both for $G = 0$ and $G \neq 0$. We summarize and discuss our results in Section IV.

2 The model

We consider the action for SU(2) Yang-Mills-Higgs theory coupled to gravity in asymptotically anti-de-Sitter space. The action is defined as

$$S = S_G + S_{ YM} + S_H = \int d^D x \sqrt{|G|}(L_G + L_{ YM} + L_H)$$ (1)

with

$$L_G = \frac{1}{\alpha_0} \left( \frac{1}{2} R - \Lambda \right)$$ (2)

$$L_{ YM} = - \frac{1}{4e^2} F_{\mu \nu}^a F^{a \mu \nu}$$ (3)

$$L_H = - \frac{1}{2} D_\mu H^a D^\mu H^a - V(H)$$ (4)

$$V(H) = \frac{\lambda}{4} (H^a H^a - h_0^2)^2$$ (5)

Here $F_{\mu \nu}^a$, $(a = 1, 2, 3)$ is the field strength,

$$F_{\mu \nu}^a = \partial_\mu A_\nu - \partial_\nu A_\mu + \varepsilon^{abc} A_\mu^b A_\nu^c$$ (6)

and the covariant derivative $D_\mu$ acting on the Higgs triplet $H^a$ is given by

$$D_\mu H^a = \partial_\mu H^a + \varepsilon^{abc} A_\mu^b H^c$$ (7)

We have defined

$$\alpha_0 \equiv 8\pi G$$ (8)

where $G$ is the Newton constant, $e$ the gauge coupling and $\Lambda$ is the cosmological constant (with our conventions $\Lambda < 0$ corresponds, in the absence of matter, to anti-de Sitter space).

The equations of motion that follow from (1) are

$$E_{\mu \nu} + \Lambda G_{\mu \nu} = \alpha_0 (T_{\mu \nu}^YM + T_{\mu \nu}^H)$$

$$D_\mu D^\rho H^a = \frac{\delta V(H)}{\delta H^a}$$

$$\frac{1}{e^2} D_\mu D_{\rho} F_{\mu \rho}^a = \varepsilon^{abc} (D_\mu H^b) H^c$$ (9)
where $E_{\mu\nu}$ is the Einstein tensor and the matter energy-momentum tensor is given by
\[
T^Y_{\mu\nu} = \frac{1}{e^2} ( - F^a_{\mu\rho} F^a_{\nu\sigma} \rho + \frac{1}{2} G_{\mu\nu} F^a_{\rho\sigma} F^a \sigma )
\]
\[
T^H_{\mu\nu} = D_\mu H^a D_\nu H^a + G_{\mu\nu} L_H
\tag{10}
\]

The most general static spherically symmetric form for the metric in 3 spatial dimensions together with the t’Hooft-Polyakov-Julia-Zee ansatz for the gauge and Higgs fields in the usual vector notation reads
\[
G = -\mu(x) A(x)^2 \, dt^2 + \mu(x)^{-1} \, dr^2 + r^2 \, d\Omega_2
\]
\[
\vec{A} = dt \, e \, h_0 \, J(x) \, \hat{e}_r - d\theta \, (1 - K(x)) \, \hat{e}_\varphi + d\varphi \, (1 - K(x)) \, \sin \theta \, \hat{e}_\theta
\]
\[
\vec{H} = h_0 \, H(x) \, \hat{e}_r
\tag{11}
\]

where we have introduced the dimensionless coordinate $x \equiv e \, h_0 \, r$ and $h_0$ sets the mass scale ($[h_0] = m^1$).

Using this ansatz, the equations of motion take the form
\[
(x \, \mu(x))' = 1 + 3 \, \gamma_0 \, x^2 - \alpha_0 h_0^2 \left( \mu(x) \, V_1 + V_2 + \frac{x^2 \, J'(x)^2}{2 \, A(x)^2} + \frac{J(x)^2 K(x)^2}{\mu(x) A(x)^2} \right)
\tag{12}
\]
\[
x \, A'(x) = \alpha_0 h_0^2 \left( V_1 + \frac{J(x)^2 K(x)^2}{\mu(x)^2 A(x)^2} \right) A(x)
\tag{13}
\]
\[
(\mu(x) A(x) K'(x))' = A(x) \, K(x) \left( \frac{K(x)^2 - 1}{x^2} \right) + H(x)^2 - \frac{J(x)^2}{\mu(x) A(x)^2}
\tag{14}
\]
\[
(x^2 \mu(x) A(x) H'(x))' = A(x) \, H(x) \left( 2 \, K(x)^2 + \frac{\lambda}{e^2} \, x^2 \, (H(x)^2 - 1) \right)
\tag{15}
\]
\[
\mu(x) \left( \frac{x^2 \, J'(x)}{A(x)} \right)' = 2 \, J(x) K(x)^2 \frac{A(x)}{A(x)}
\tag{16}
\]

where, for convenience, we have defined the dimensionless parameter
\[
\gamma_0 = -\frac{\Lambda}{3 e^2 h_0^2}
\tag{17}
\]

and
\[
V_1 = K'(x)^2 + \frac{x^2}{2} H'(x)^2
\]
\[
V_2 = \frac{(K(x)^2 - 1)^2}{2 \, x^2} + \frac{\lambda}{4 e^2} \, x^2 \, (H(x)^2 - 1)^2
\tag{18}
\]
The boundary conditions

Ansatz (11) will lead to well behaved solutions for the matter fields if, at $x = 0$, one imposes

- $H(x)/x$ and $J(x)/x$ are regular;
- $1 - K(x)$ and $K'(x)$ go to zero.
- $\mu(x) \rightarrow 1$

On the other hand we want the system to go asymptotically to anti-de Sitter space which corresponds to the solution of the Einstein equations with $\Lambda < 0$ in absence of matter (see next Section); for this to happen we must impose that the matter energy-momentum tensor vanishes at spatial infinity. From eq.(10) one can see that the appropriate conditions for $x \rightarrow \infty$ are

$$
A(x) \rightarrow 1 \\
K(x) \rightarrow O(x^{-\alpha_1}) \\
H(x) \rightarrow H_\infty + O(x^{-1-\alpha_2}) \\
J(x) \rightarrow J_\infty + O(x^{-\alpha_3})
$$

with $\alpha_i > 0, \ i = 1, 2, 3$.

Note that, being the equations for $A$ and $\mu$ first order, we impose just one condition for each one.

3 The system in AdS space

We shall first consider the case in which the Newton constant $G$ vanishes, so that the gravitational equations decouple from the matter and then analyse the full $G \neq 0$ problem. In the former case we have already studied in [18] the classical equations of motion analytically, showing that monopoles could exist in asymptotically anti-de Sitter spaces and discussed its main properties. We present in the next subsection the numerical evidence that this solutions do exist, thus completing the analysis in [18]. Then, we extend our study to the $G \neq 0$ case and again present both analytical and numerical analysis showing the existence of monopole solutions provided $G$ is smaller than a critical value $G_c$.

The $G = 0$ case

Taking the $\alpha_0 h_0^2 \rightarrow 0$ limit, one easily finds for the metric the solution

$$
A(x) = 1 \\
\mu(x) = 1 + \gamma_0 x^2 - \frac{a}{x}
$$

with $\alpha_i > 0, \ i = 1, 2, 3$. 

Note that, being the equations for $A$ and $\mu$ first order, we impose just one condition for each one.
which is nothing but the vacuum solution of the Einstein equations with a cosmological constant (assumed negative), and corresponds to a neutral Schwarzschild black hole in AdS space. Concerning the integration constant $a$, it is related to the mass of the black hole and will be put to zero in what follows, in agreement with the condition imposed on $\mu$ at $x = 0$. This metric, in turn, acts as a (AdS) background with radius $r_0$, 

$$r_0 = \sqrt{-3/\Lambda} \quad (21)$$

for the Yang-Mills-Higgs system.

For simplicity we study eqs. (16) in the BPS limit which corresponds to $\lambda/e^2 = 0$ with $h_0$ fixed.

$$\left( \mu(x) K'(x) \right)' = K(x) \left( \frac{K(x)^2 - 1}{x^2} + H(x)^2 - \frac{J(x)^2}{\mu(x)} \right)$$

$$\left( x^2 \mu(x) H'(x) \right)' = 2 H(x) K(x)^2$$

$$\mu(x) \left( x^2 J'(x) \right)' = 2 J(x) K(x)^2 \quad (22)$$

The total amount of matter $M$ associated to the solution of (22) is defined as (see for example [13])

$$M = \int_{\Sigma_t} d^3x \sqrt{g^{(3)}} T_{00} \quad (23)$$

where $g^{(3)}$ is the determinant of the induced metric on surfaces $\Sigma_t$ of constant time $t$ with normal vector $e_0 = \mu(x)^{-1/2} \partial_t$ and $T_{00} \equiv e_0^\mu e_0^\nu T_{\mu\nu} = T_{tt}/\mu(x)$ is the local energy density as seen by an observer moving on the flux lines of $\partial_t$. For the spherically symmetric configuration we are considering, it takes the form

$$M = \frac{4\pi h_0}{e} \int_0^\infty dx \frac{x^2}{(1 + \gamma_0 x^2)^{3/2}} \frac{T_{tt}}{e^2 h_0^4} \quad (24)$$

We quote for completeness the explicit expressions for $T_{tt} = T_{tt}^{(YM)} + T_{tt}^{(H)}$

$$\frac{T_{tt}^{(YM)}}{e^2 h_0^4} = \frac{\mu(x)}{2} J'(x)^2 + \frac{J(x)^2 K(x)^2}{x^2} + \mu(x) K'(x)^2 + \frac{\mu(x)}{x^4} \frac{K(x)^2 - 1}{x^2}$$

$$\frac{T_{tt}^{(H)}}{e^2 h_0^4} = \frac{\mu(x)^2}{2} H'(x)^2 + \frac{\mu(x)}{x^2} H(x)^2 K(x)^2 \quad (25)$$

It is not difficult to see from these expression that the boundary conditions imposed through eqs. (19) are precisely those required for finiteness of $M$.

As stated above, analytical arguments showing the possibility of monopole solutions were presented in [18]. To begin with, let us note that is possible to perform a power series expansion for large $x$ and calculate the coefficients in the expansion recursively. One can consistently propose an expansion of the form

$$K(x) = -\frac{K_{\nu+1}}{x^{\nu+1}} \left( 1 - \frac{k_\nu}{x^2} + \ldots \right)$$
\[ H(x) = 3 \frac{H_3}{x^3} \sum_{k=0}^{\nu} \frac{(-)^k}{2k+3} \frac{1}{x^{2k}} + H_{\infty} \left( 1 + \frac{h_{\nu}}{x^{2\nu+4}} + \ldots \right) \]
\[ J(x) = J_1 \frac{1}{x} + J_{\infty} \left( 1 + \frac{j_{\nu}}{x^{2\nu+4}} + \ldots \right) \quad (26) \]

and, after insertion in eqs. (22), one can determine the coefficients \( k_{\nu}, h_{\nu} \) and \( j_{\nu} \) recursively. For positive integer \( \nu \) one has
\[
\begin{align*}
  k_{\nu} &= \frac{\nu^2 + 3 \nu + 3 + J_{\infty}^2}{2 (2\nu + 3)} \\
  h_{\nu} &= \frac{K_{\nu+1}^2}{(\nu + 2) (2\nu + 1)} \\
  j_{\nu} &= \frac{K_{\nu+1}^2}{(\nu + 2) (2\nu + 3)} \quad (27)
\end{align*}
\]

Similar expressions can be obtained for \( \nu \) a positive semi-integer.

When such an expansion are assumed, non-trivial solutions exist if and only if
\[ H_{\infty}^2 = \nu(\nu + 1)\gamma_0, \quad \nu = \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots \quad (28) \]

If one relaxes a power behavior like in (26), then one again gets a relation like (28) but with \( \nu \) a real number, the leading exponent in the asymptotic expansion of \( K(x) \). It is important to note that, having AdS space a natural scale \( r_0 \), the system trades the principle arbitrary \( h_0 \) dimensionfull parameter for the AdS radius \( r_0 = \sqrt{-3/\Lambda} \) which now sets the scale,
\[ |\vec{H}(\infty)|^2 = \nu(\nu + 1)\gamma_0 h_0^2 = \nu(\nu + 1)(er_0)^{-1} \quad (29) \]

Then, using (28) for \( \nu \) real and a given \( \Lambda \) is equivalent to consider an integer \( \nu \) (for example with \( \nu = 1 \)) provided \( r_0 \) (i.e. \( \Lambda \)) is changed accordingly.

To obtain a detailed profile of the monopole solution, we solved numerically the differential equations. For simplicity we considered the \( J = 0 \) case corresponding to a purely magnetic solution. The equations of motion read
\[
\begin{align*}
  (\mu(x)K'(x))' &= K(x) \left( \frac{K(x)^2 - 1}{x^2} + H(x)^2 \right) \quad (30) \\
  (x^2\mu(x)H'(x))' &= 2H(x)K(x)^2 \quad (31) \\
  \mu(x) &= 1 + \gamma_0 x^2 \quad (32)
\end{align*}
\]

We employed a relaxation method for boundary value problems [19]. Such method determines the solution by starting with an initial guess and improving it iteratively. The natural initial guess was the exact Prassad-Sommerfield solution [20] (which corresponds to \( \gamma_0 = 0 \)). We have found regular monopole solutions for any value of the cosmological constant \( \Lambda \). We present in Figure 1
the solution profile for different values of $\Lambda$. A distinctive feature of solutions for $\Lambda \neq 0$ compared with the flat-space Prasad-Sommerfield solution concerns the asymptotic behavior of the fields. Indeed, when $\Lambda \neq 0$, the Higgs field approaches its v.e.v. faster than in the Prasad-Sommerfield (PS) case,

$$H(x) \sim H_\infty + \frac{C_{\Lambda \neq 0}}{x^3}, \quad x \gg 1$$

(33)

$$H_{PS}(x) \sim H_\infty - \frac{1}{x}, \quad x \gg 1$$

(34)

As a result of this change of the asymptotic behavior, one can see that the radius $R_c$ of the monopole core decreases. Indeed, as can be seen in Fig.1, when $\gamma_0$ (the cosmological constant) grows, $R_c$ becomes smaller as the magnetic field concentrates near the origin.

We have also computed numerically the monopole mass which can be written as

$$M = \frac{4\pi}{e^2} \frac{1}{r_0} f_{\gamma_0}(\lambda/e^2)$$

(35)

where, extending the usual flat space notation, we have introduced the dimensionless function $f_{\gamma_0}(\lambda/e^2)$. In the present case, the cosmological constant provides a natural scale and this has been exploited in (35). This formula can be written in units of the mass scale $h_0$ as

$$\frac{M}{h_0} = \frac{4\pi}{e} E$$

(36)

where $E$ is a dimensionless function of $\gamma_0$,

$$E = \sqrt{\gamma_0} f_{\gamma_0}(\lambda/e^2)$$

(37)

We present in Figure 2 a plot for $E$ as a function of $\gamma_0$ where one can see that $\lim_{\gamma_0 \to 0} \sqrt{\gamma_0} f_{\gamma_0}(0) = 1$ which is the correct result for Prasad-Sommerfield monopoles in flat space.

**The $G \neq 0$ case**

In order to study the asymptotic behavior of the solutions to eqs.(13) we have taken as independent metric functions $A(x)$ and $\tilde{\mu}(x) = A(x)\mu(x)$. Moreover, identifying $h_0 = (e r_0)^{-1} (\gamma_0 = 1)$, the equations to study become, in the BPS limit,

$$A(x) \left( x \tilde{\mu}(x) \right)' = (1 + 3 x^2) A(x)^2 - \alpha_0 h_0^2 \left( A(x)^2 V_2 + \frac{x^2}{2} J'(x)^2 \right)$$

$$x \tilde{\mu}(x)^2 A'(x) = \alpha_0 h_0^2 \left( \tilde{\mu}(x)^2 V_1 + J(x)^2 K(x)^2 \right) A(x)$$

$$(\tilde{\mu}(x) K'(x))' = K(x) \left( A(x) \frac{K(x)^2 - 1}{x^2} + A(x) H(x)^2 - \frac{J(x)^2}{\tilde{\mu}} \right)$$

$$(x^2 \tilde{\mu}(x) H'(x))' = 2 A(x) H(x) K(x)^2$$

$$(x \tilde{\mu}(x) A(x) \left( x^2 J'(x) \right))' = x^2 \tilde{\mu}(x) A'(x) J'(x) + 2 A(x)^2 K(x)^2 J(x)$$

(38)
where, in the first one, we have combined eqs. (12) and (13).

We consider for simplicity the purely magnetic case, \( J = 0 \), and propose a power series expansion of the form

\[
\tilde{\mu}(x) = x^2 + \sum_{m=0}^{\infty} \frac{\tilde{\mu}_m}{x^m}, \quad \tilde{\mu}_0 \equiv 1
\]

\[
A(x) = \sum_{m=0}^{\infty} \frac{A_m}{x^m}, \quad A_0 \equiv 1
\]

\[
K(x) = \left\{ \sum_{m=0}^{\infty} \frac{K_m}{x^m}, \quad K_0 \equiv 0 \right\}
\]

\[
H(x) = \sum_{m=0}^{\infty} \frac{H_m}{x^m}, \quad H_0 \equiv H_\infty, \ H_1 \equiv 0
\]

Again, coefficients can be determined recursively. The leading coefficients in the expansions for \( K \) and \( H \) coincide with those already presented for \( \alpha_0 = 0 \) (eq.(26)). We then just quote the corresponding ones for the metric functions (for \( \nu \) a positive integer)

\[
\tilde{\mu}(x) = 1 + x^2 + \frac{\tilde{\mu}_1}{x} + \frac{\alpha_0 h_0^2}{x^2} - \frac{A_6}{x^4} + \ldots
\]

\[
A(x) = 1 + \frac{A_6}{x^6} + \ldots
\]

where

\[
A_6 = -\alpha_0 h_0^2 \left( \frac{3}{4} H_3^2 + \frac{2}{3} K_2^2 \delta_{\nu,1} \right)
\]

We see that function \( A \) is completely determined, to all orders, in terms of \( H \) and \( K \) coefficients. As an example, and from the numerical results described below, one finds for \( \nu = 1 \) and \( \alpha_0 = 0.1 \) that \( \tilde{\mu}_1 = -0.24, \ K_2 = 0.73, \ H_3 = -0.32 \) and then \( A_6 = -0.87 \).

Concerning the asymptotic expansion for \( \tilde{\mu} \) corresponds to a Reissner-Nördstrom metric (with cosmological constant), with the free parameter \( -\tilde{\mu}_1 \) related to the gravitationary mass and the coefficient of the \( 1/x^2 \) term, which arises for charged solutions, precisely corresponding to the \( Q_m = 1 \) magnetic solution we are considering. As seen from afar, and for an appropriate set of parameters, the metric can be identified with that of a magnetically charged black hole as that described in [18, 21]. Concerning the expansion for \( K \), it has one free coefficient \( (K_{\nu+1}) \), while for the expansion for \( H \) two coefficients remain free \( (H_\infty \text{ and } H_3) \).

In order to get the detailed profile of the solutions, we have again to solve numerically the equations of motion. For simplicity, we have considered the BPS limit, \( \lambda/e^2 = 0 \). Employing the same relaxation method as for the \( G = 0 \) case we have found a self-gravitating monopole solution satisfying the boundary conditions previously discussed. Solutions are similar to those corresponding to
In particular, we have found a maximum value
for the gravitational interaction strength $\alpha_0$ such that above $\alpha^*_0$ the solution
cesses to exist. This effect, already encountered in asymptotically flat space, can
be understood noting that as $\alpha_0$ increases from 0 to its critical value, the ratio
$M = \text{mass}/\text{radius}$ for the monopole solution also increases until it becomes
gravitationally unstable. Now, as the cosmological constant $|\Lambda|$ increases, the
radius of the monopole decreases (the behavior for $\alpha_0 \neq 0$ is analogous to that
depicted in Fig.1 for $\alpha_0 = 0$) while the mass of the monopole increases (the
behavior for $\alpha_0 \neq 0$ is analogous to that in Fig.2) so that $M$ is a monotonically
growing function of $-\Lambda$ or, what is the same, of $\gamma_0$. This explains why the
critical value $\alpha^*_0(\gamma_0)$ for $\gamma_0 > 0$, is smaller than the asymptotically flat one,
$\alpha^*_0(\gamma_0) < \alpha^*_0(0)$: the critical value $M_c$ at which the solution collapses is reached
before, in the $\alpha$ domain, for $\gamma_0 > 0$ than for $\gamma_0 = 0$. As an example, for $\lambda = 0$
and $\gamma_0 = 1$ the critical $\alpha_0$-value is $\alpha^*_0(1) = 1.374$ to be compared with the
asymptotically flat space value $\alpha^*_0(0) = 5.549$.

Concerning the solution for the metric, as can be seen in Fig. 3, $\mu/(1 + \gamma_0 x^2)$
has a minimum which decreases as the strength of the gravitation interaction
grows and tends to zero as $\alpha_0 \to \alpha^*_0$. A similar behavior can be seen to occur for
$A(x)$ which has also a minimum at the origin which tends to zero as $\alpha_0 \to \alpha^*_0$. As
in the case of asymptotically flat space, $A$ develops a step-like behavior which
becomes more and more sharp as $\alpha^*_0$ is approached. The position of the center
of the step function can be used to determine the corresponding value of the
horizon, which for AdS spaces results from the solution of a quartic algebraic
equation. [21],[18]

4 Discussion

In this work we have studied in detail the monopole and dyon solutions to
Yang-Mills-Higgs theory coupled to gravity for asymptotically anti-de Sitter
space presented in ref.[18]. We have first considered the case in which the
Newton constant $\alpha_0$ vanishes so that the Einstein equations decouple leading
to a Schwarzschild black hole in AdS space. This metric acts as a background
for dyon solutions which were studied in detail making both an analytical and a
numerical analysis. A distinctive feature of AdS solutions in this case is that the
monopole radius is smaller than that corresponding to the $\Lambda = 0$ case. Apart
from this property, qualitatively, the Higgs field and magnetic field behavior
is very similar to that corresponding to the 't Hooft-Polyakov solution. More
interesting is the behavior of solutions where gravity is effectively coupled to
the matter fields. In first place, as it happens in asymptotically flat space, a
critical value for the Newton constant exists above which no regular monopole
or dyon solution can be found. This effect was explained in asymptotically flat
spaces [11]-[13] by noting that as $\alpha_0$ grows the mass of the monopole grows and
its radius decreases so that it finally becomes gravitationally unstable. Now,
the presence of a cosmological constant enhances this effect and for this reason,
the critical value we find, $\alpha^*_0(\Lambda)$ is smaller than the asymptotically flat one.
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Figure 1: Plot of the functions $K(r)$ and $H(r)$ (in dimensionless variables) for the monopole solution with $\lambda = 0$ and $\alpha_0 = 0$. The solid line corresponds to the solution with $\gamma_0 = 1.0$ and the dashed line corresponds to the BPS flat space solution.
Figure 2: Energy of the monopole configuration as a function of $\gamma_0$ for $h_0 = 1$ and different values of $\lambda$: $\lambda = 0$ (solid line), $\lambda = 10$ (dashed line) and $\lambda = 20$ (dotted line) in the $\alpha_0 = 0$ case.
Figure 3: The solution for the metric function $\mu(r)/(1 + r^2)$ for fixed $\gamma_0 = 1$ and $\lambda = 0$. The solid line corresponds to $\alpha_0 = 1$ and the dashed one to $\alpha_0 = \alpha^c_0 \approx 1.371$. 
Figure 4: The solution for the metric function $A(r)$ for fixed $\gamma_0 = 1$ and $\lambda = 0$. The solid line corresponds to $\alpha_0 = 1$ and the dashed one to $\alpha_0 = \alpha_0^c$. 